Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.3-Tangent/103-4.3.2.1-a+b-tan-^m-c+d-tan-^n

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [1328]. This is test number [103].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (1328)	0.00 (0)
Mathematica	90.89 (1207)	9.11 (121)
Maple	84.04 (1116)	15.96 (212)
Fricas	71.54 (950)	28.46 (378)
Mupad	62.88 (835)	37.12 (493)
Maxima	43.45 (577)	56.55 (751)
Giac	38.93 (517)	61.07 (811)
Sympy	22.21 (295)	77.79 (1033)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

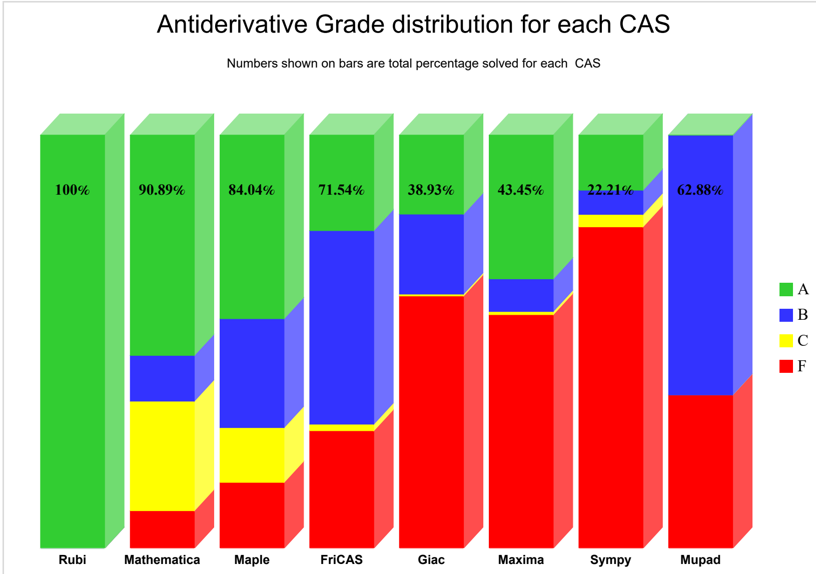
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

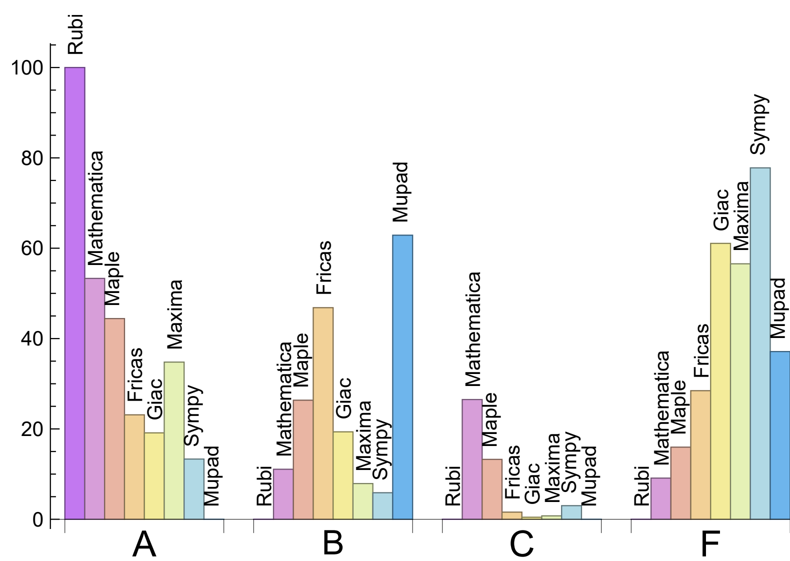
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	53.31	11.07	26.51	9.11
Maple	44.43	26.36	13.25	15.96
Maxima	34.79	7.91	0.75	56.55
Fricas	23.12	46.84	1.58	28.46
Giac	19.13	19.35	0.45	61.07
Sympy	13.33	5.87	3.01	77.79
Mupad	N/A	62.88	0.00	37.12

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	121	85.95 %	14.05 %	0.00 %
Maple	212	98.58 %	1.42 %	0.00 %
Fricas	378	36.77 %	63.23 %	0.00 %
Giac	811	55.36 %	26.26 %	18.37 %
Maxima	751	49.93 %	4.13 %	45.94 %
Sympy	1033	83.54 %	7.36 %	9.10 %
Mupad	493	96.75 %	3.25 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

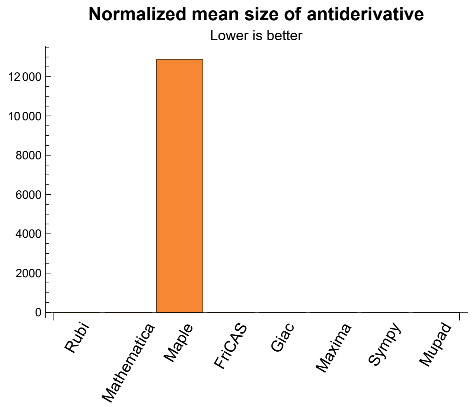
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

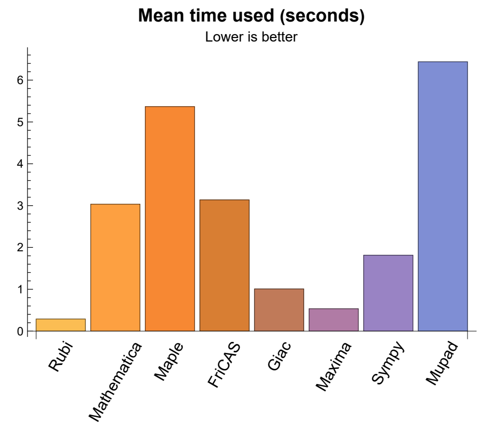
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.29	176.87	1.00	161.00	1.00
Mathematica	3.03	323.55	1.87	166.00	1.08
Maple	5.37	2637683.52	12867.27	276.50	1.36
Maxima	0.54	281.16	2.16	161.00	1.07
Fricas	3.14	2019.61	8.84	358.00	2.32
Sympy	1.81	489.88	4.09	170.00	1.82
Giac	1.01	341.92	2.53	222.00	1.83
Mupad	6.44	569.35	3.08	144.00	1.16

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
12000
10000
8000
6000
4000
2000
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
6
5
4
3
2
1
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {179, 185, 310, 311, 393, 394, 398, 399, 405, 410, 675, 676, 677, 678, 679, 680, 788, 1050, 1059, 1062, 1089, 1099, 1100, 1110, 1129, 1130, 1134, 1135, 1136, 1137, 1138, 1143, 1144, 1145, 1146, 1149, 1150, 1151, 1152, 1153, 1155, 1156, 1161, 1166, 1170, 1171, 1172, 1216, 1318}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

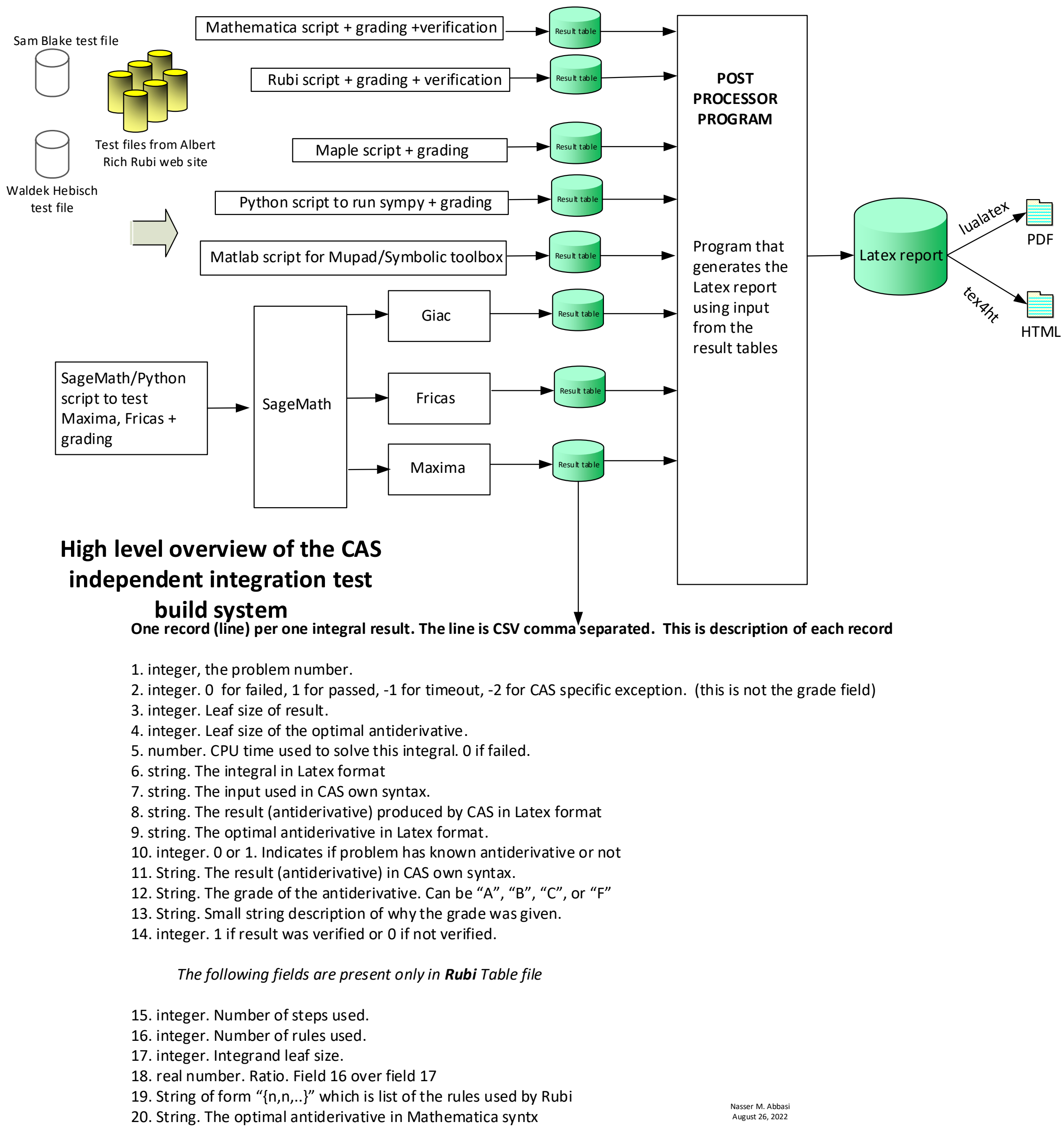
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27,
28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55,
56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83,
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886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906,
907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927,

928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 13, 14, 15, 16, 18, 27, 28, 30, 37, 38, 40, 42, 49, 50, 51, 52, 59, 60, 61, 62, 63, 66, 67, 68, 69, 70, 71, 72, 73, 76, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 149, 150, 151, 154, 156, 157, 158, 159, 160, 161, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 193, 194, 195, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 275, 283, 284, 285, 297, 307, 309, 318, 319, 343, 345, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 412, 413, 414, 415, 416, 417, 418, 424, 425, 426, 429, 504, 506, 508, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 530, 532, 534, 536, 537, 538, 543, 544, 545, 546, 547, 552, 553, 582, 583, 589, 590, 608, 609, 610, 611, 612, 613, 614, 615, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 641, 642, 643, 644, 645, 646, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 682, 684, 686, 688, 689, 696, 697, 698, 699, 700, 701, 708, 710, 712, 714, 719, 722, 724, 725, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 891, 892, 893, 894, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 910, 911, 912, 913, 914, 915, 916, 917, 921, 922, 923, 927, 928, 929, 930, 932, 933, 935, 936, 937, 940, 941, 942, 944, 945, 946, 948, 949, 950, 951, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 972, 973, 974, 975, 977, 978, 979, 980, 981, 983, 984, 985, 986, 987, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1052, 1067, 1069, 1070, 1075, 1076, 1080, 1082, 1083, 1084, 1086, 1088, 1092, 1093, 1094, 1098, 1101, 1102, 1103, 1105, 1106, 1107, 1108, 1109, 1111, 1112, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1140, 1141, 1142, 1144, 1147, 1148, 1150, 1154, 1157, 1158, 1159, 1160, 1162, 1164, 1165, 1167, 1168, 1193, 1194, 1212, 1214, 1215, 1220, 1221, 1222, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1235, 1236, 1237, 1238, 1239, 1241, 1242, 1243, 1244, 1245, 1247, 1248, 1249, 1250, 1251, 1252, 1257, 1258, 1263, 1265, 1267, 1268, 1269, 1270, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1291, 1292, 1293, 1294, 1295, 1298, 1299, 1300, 1301, 1302, 1303, 1305, 1306, 1307, 1309, 1310, 1311, 1324, 1325, 1326, 1327, 1328 }

B grade: { 17, 19, 24, 25, 26, 29, 31, 32, 33, 34, 35, 36, 39, 41, 43, 44, 45, 46, 47, 48, 53, 54, 55, 56, 57, 58, 64, 65, 74, 75, 83, 96, 155, 162, 163, 188, 196, 308, 310, 311, 313, 328, 329, 616, 675, 676, 677, 678, 679, 680, 723, 788, 790, 890, 895, 908, 909, 918, 919, 920, 924, 925, 926, 931, 934, 938, 939, 943, 947, 952, 970, 976, 982, 988, 1012, 1027, 1036, 1050, 1053, 1054, 1055, 1059, 1060, 1061, 1062, 1063, 1065, 1066, 1068, 1071, 1072, 1073, 1074, 1077, 1078, 1079, 1081, 1085, 1087, 1089, 1090, 1091, 1095, 1096, 1097, 1099, 1100, 1104, 1110, 1113, 1126, 1127, 1135, 1136, 1137, 1138, 1139, 1143, 1145, 1146, 1149, 1151, 1152, 1153, 1155, 1156, 1161, 1163, 1166, 1169, 1170, 1171, 1172, 1181, 1182, 1234, 1240, 1246, 1264, 1266, 1271, 1272, 1280, 1290, 1297, 1318, 1320 }

C grade: { 8, 9, 10, 11, 12, 20, 21, 22, 23, 138, 139, 146, 147, 148, 152, 153, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 279, 287, 290, 291, 292, 293, 294, 295, 296, 300, 301, 302, 303, 304, 305, 306, 336, 337, 338, 339, 340, 341, 342, 344, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 419, 420, 421, 422, 423, 427, 428, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 505, 507, 509, 529, 531, 533, 535, 539, 540, 541, 542, 548, 549, 550, 551, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 584, 585, 586, 587, 588, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 640, 647, 671, 672, 673, 674, 681, 683, 685, 687, 690, 691, 692, 693, 694, 695, 707, 709, 711, 713, 720, 721, 726, 727, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 873, 1191, 1192, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1213, 1216, 1217, 1218, 1219, 1223, 1224, 1225, 1226, 1253, 1254, 1255, 1256, 1259, 1260, 1261, 1262, 1289, 1296, 1308 }

F grade: { 89, 189, 190, 191, 192, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 276, 277, 278, 280, 281, 282, 286, 288, 289, 298, 299, 312, 314, 315, 316, 317, 320, 321, 322, 323, 324, 325, 326, 327, 330, 331, 332, 333, 334, 335, 411, 702, 703, 704, 705, 706, 715, 716, 717, 718, 755, 789, 791, 792, 793, 794, 795, 796, 797, 885, 886, 887, 888, 889, 971, 1048, 1049, 1051, 1056, 1057, 1058, 1064, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1304, 1312, 1313, 1314, 1315, 1316, 1317, 1319, 1321, 1322, 1323 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 92, 93, 94, 95, 99, 100, 101, 102, 107, 108, 109, 110, 111, 112, 113, 117, 118, 119, 120, 121, 122, 126, 127, 128, 129, 130, 131, 134, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 272, 273, 274, 275, 279, 280, 281, 282, 283, 287, 290, 291, 292, 293, 294, 297, 300, 301, 302, 303, 304, 307, 343, 344, 345, 346, 347, 348, 364, 365, 366, 367, 368, 369, 378, 379, 380, 384, 385, 386, 389, 390, 391, 395, 396, 397, 400, 401, 402, 406, 407, 408, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 505, 507, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 671, 672, 673, 674, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 929, 930, 931, 932, 933, 934, 935, 937, 938, 939, 940, 941, 942, 943, 944, 945, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1013, 1014, 1015, 1016, 1019, 1021, 1022, 1023, 1024, 1027, 1028, 1029, 1030, 1031, 1032, 1037, 1038, 1039, 1040, 1041, 1042, 1044, 1045, 1046, 1052, 1053, 1054, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1098, 1099, 1100, 1104, 1105, 1106, 1110, 1111, 1112, 1116, 1117, 1118, 1122, 1123, 1124, 1128, 1129, 1130, 1134, 1135, 1136, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228 }

B grade: { 89, 90, 91, 96, 97, 98, 103, 104, 105, 106, 114, 115, 116, 123, 124, 125, 132, 133, 135, 136, 137, 140, 141, 142, 143, 144, 147, 148, 149, 150, 151, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 336, 337, 338, 339, 340, 341, 342, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 370, 371, 372, 373, 374, 375, 376, 377, 504, 506, 511, 512, 513, 514, 515, 519, 520, 521, 522, 527, 528, 529, 530, 531, 532, 533, 537, 538, 539, 540, 541, 542, 546, 547, 548, 549, 550, 551, 554, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 1002, 1010, 1011, 1012, 1017, 1018, 1020, 1025, 1026, 1033, 1034, 1035, 1036, 1097, 1101, 1102, 1103, 1107, 1108, 1109, 1113, 1114, 1115, 1119, 1120, 1121, 1125, 1126, 1127, 1131, 1132, 1133, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1268, 1272 }

C grade: { 138, 139, 145, 146, 152, 153, 381, 382, 383, 387, 388, 392, 393, 394, 398, 399, 403, 404, 405, 409, 410, 508, 509, 510, 516, 517, 518, 523, 524, 525, 526, 534, 535, 536, 543, 544, 545, 552, 553, 627, 628, 629, 630, 631, 681, 682, 683, 684, 685, 688, 689, 690, 691, 692, 693, 694, 695, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 928, 936, 946, 1043, 1051 }

F grade: { 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 276, 277, 278, 284, 285, 286, 288, 289, 295, 296, 298, 299, 305, 306, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 411, 675, 676, 677, 678, 679, 680, 686, 687, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 1047, 1048, 1049, 1050, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1265, 1266, 1267, 1269, 1270, 1271, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 156, 157, 162, 163, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 290, 291, 292, 293, 294, 295, 296, 297, 300, 301, 302, 303, 304, 305, 306, 307, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 412, 413, 414, 415, 416, 417, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 482, 484, 485, 486, 487, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 730, 734, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 891, 892, 896, 897, 898, 899, 904, 905, 906, 907, 914, 915, 916, 917, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 995, 1002, 1003, 1004, 1005, 1006, 1010, 1012, 1013, 1014, 1015, 1019, 1020, 1027, 1030, 1036, 1037, 1041, 1046, 1054, 1065, 1066, 1067, 1071, 1072, 1073, 1077, 1078, 1079, 1083, 1084, 1089, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1203, 1204, 1205, 1206, 1207, 1209, 1210, 1211, 1212, 1213, 1216, 1217, 1218, 1219, 1223, 1225, 1226 }

B grade: { 7, 135, 136, 137, 140, 141, 142, 143, 144, 147, 148, 149, 150, 151, 154, 155, 158, 159, 160, 161, 189, 190, 191, 192, 197, 198, 199, 200, 206, 207, 208, 209, 418, 481, 483, 488, 489, 490, 491, 492, 719, 722, 723, 724, 725, 728, 729, 731, 732, 733, 751, 752, 753, 754, 757, 758, 759, 762, 763, 764, 890, 908, 918, 992, 993, 994, 999, 1000, 1001, 1007, 1008, 1009, 1011, 1016, 1017, 1018, 1024, 1025, 1026, 1033, 1034, 1035, 1043, 1044, 1045, 1085, 1090, 1091, 1095, 1096, 1097, 1103, 1121, 1157, 1163, 1202, 1208, 1214, 1215, 1220, 1221, 1222, 1224, 1227, 1228 }

C grade: { 138, 139, 145, 146, 152, 153, 720, 721, 726, 727 }

F grade: { 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 193, 194, 195, 196, 201, 202, 203, 204, 205, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 288, 289, 298, 299, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 755, 756, 760, 761, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 893, 894, 895, 900, 901, 902, 903, 909, 910, 911, 912, 913, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 996, 997, 998, 1021, 1022, 1023, 1028, 1029, 1031, 1032, 1038, 1039, 1040, 1042, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1068, 1069, 1070, 1074, 1075, 1076, 1080, 1081, 1082, 1086, 1087, 1088, 1092, 1093, 1094, 1098, 1099, 1100, 1101, 1102, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1158, 1159, 1160, 1161, 1162, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328 }

2.1.5 FriCAS

A grade: { 4, 5, 6, 7, 8, 9, 16, 17, 18, 19, 20, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 234, 241, 242, 243, 244, 245, 274, 275, 336, 337, 338, 339, 340, 341, 342, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 370, 371, 372, 373, 374, 375, 376, 377, 412, 413, 414, 415, 416, 417, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 495, 496, 497, 499, 500, 501, 893, 894, 897, 899, 900, 901, 902, 903, 905, 907, 909, 910, 911, 912, 913, 914, 916, 920, 921, 922, 923, 924, 925, 926, 927, 929, 930, 931, 932, 933, 934, 935, 937, 938, 939, 940, 941, 942, 944, 945, 947, 948, 949, 950, 951, 953, 954, 955, 956, 957, 958, 962, 963, 968, 969, 974, 975, 979, 980, 981, 985, 986, 987, 990, 991, 996, 997, 998, 1004, 1005, 1006, 1013, 1014, 1015, 1020, 1021, 1022, 1023, 1028, 1029, 1030, 1031, 1032, 1033, 1037, 1038, 1039, 1040, 1041, 1042, 1044, 1045, 1046, 1052, 1053, 1054, 1066, 1067, 1068, 1069, 1070, 1074, 1075, 1076, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1090, 1091, 1092, 1094, 1191, 1192, 1193, 1194, 1197, 1198, 1199, 1200, 1203, 1204, 1205, 1206, 1209, 1210, 1211, 1212, 1213, 1219 }

B grade: { 1, 2, 3, 10, 11, 12, 13, 14, 15, 21, 22, 23, 25, 35, 45, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 140, 141, 142, 143, 144, 147, 148, 149, 150, 151, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 235, 236, 237, 238, 239, 240, 246, 247, 272, 273, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 290, 291, 292, 293, 294, 295, 296, 297, 300, 301, 302, 303, 304, 305, 306, 307, 343, 344, 345, 346, 347, 348, 364, 365, 366, 367, 368, 369, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 418, 419, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 498, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 681, 682, 683, 684, 685, 689, 691, 693, 719, 722, 723, 724, 725, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 890, 891, 895, 896, 904, 908, 917, 918, 919, 943, 952, 959, 960, 961, 964, 965, 966, 967, 970, 971, 972, 973, 976, 977, 978, 982, 983, 984, 988, 989, 992, 993, 994, 995, 999, 1000, 1001, 1002, 1003, 1007, 1008, 1009, 1010, 1011, 1012, 1016, 1017, 1018, 1019, 1024, 1025, 1026, 1027, 1034, 1035, 1036, 1043, 1051, 1065, 1071, 1072, 1073, 1077, 1078, 1079, 1089, 1093, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1195, 1196, 1201, 1202, 1207, 1208, 1214, 1215, 1216, 1217, 1218, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1230, 1231, 1232, 1233, 1249, 1250, 1251, 1252, 1255, 1256, 1257, 1262 }

C grade: { 138, 139, 145, 146, 152, 153, 671, 672, 673, 674, 720, 721, 726, 727, 892, 898, 906, 915, 928, 936, 946 }

F grade: { 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 288, 289, 298, 299, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 411, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 675, 676, 677, 678, 679, 680, 686, 687, 688, 690, 692, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 1047, 1048, 1049, 1050, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1229, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1253, 1254, 1258, 1259, 1260, 1261, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328 }

2.1.6 Sympy

A grade: { 5, 6, 7, 8, 9, 14, 16, 17, 18, 19, 20, 22, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 412, 413, 414, 415, 416, 417, 421, 422, 423, 424, 425, 426, 427, 428, 432, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 495, 499, 900, 909, 910, 911, 912, 919, 920, 921, 922, 923, 924, 925, 926, 928, 929, 930, 931, 932, 933, 935, 936, 937, 938, 939, 940, 941, 944, 945, 946, 947, 948, 949, 950, 953, 954, 955, 964, 970, 976, 982, 988, 1065, 1066, 1067, 1068, 1069, 1070, 1073, 1074, 1075, 1076, 1080, 1081, 1082, 1084, 1085, 1086, 1087, 1088, 1092, 1093, 1094, 1098, 1099, 1191, 1192, 1199, 1205 }

B grade: { 1, 2, 3, 4, 10, 11, 12, 13, 15, 21, 23, 25, 35, 418, 419, 420, 429, 430, 431, 433, 496, 497, 498, 500, 501, 502, 890, 891, 893, 894, 895, 896, 897, 899, 901, 902, 903, 904, 905, 907, 908, 913, 914, 916, 917, 918, 927, 934, 942, 943, 951, 952, 958, 1043, 1044, 1045, 1046, 1051, 1052, 1053, 1054, 1071, 1072, 1077, 1078, 1079, 1083, 1089, 1090, 1091, 1095, 1096, 1097, 1193, 1197, 1198, 1203, 1204 }

C grade: { 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 892, 898, 906, 915, 1194, 1195, 1200, 1201, 1206, 1207, 1209, 1210, 1211, 1212, 1213, 1216, 1217, 1218, 1219 }

F grade: { 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 956, 957, 959, 960, 961, 962, 963, 965, 966, 967, 968, 969, 971, 972, 973, 974, 975, 977, 978, 979, 980, 981, 983, 984, 985, 986, 987, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 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1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328 }

2.1.7 Giac

A grade: { 5, 6, 7, 17, 18, 29, 30, 31, 32, 33, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 135, 136, 137, 140, 141, 142, 143, 144, 147, 148, 149, 150, 151, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 343, 344, 345, 346, 347, 348, 364, 365, 366, 367, 368, 369, 377, 378, 379, 380, 384, 385, 386, 387, 388, 389, 390, 391, 395, 396, 397, 399, 400, 401, 406, 407, 408, 409, 410, 417, 429, 439, 440, 449, 450, 451, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 473, 474, 475, 476, 477, 478, 479, 480, 484, 485, 486, 487, 494, 495, 496, 497, 498, 499, 500, 501, 502, 671, 672, 673, 674, 684, 892, 893, 897, 899, 901, 905, 907, 911, 912, 914, 916, 921, 922, 927, 928, 929, 930, 933, 935, 936, 937, 940, 941, 944, 945, 946, 947, 948, 949, 950, 953, 954, 955, 958, 964, 1046, 1054, 1069, 1070, 1080, 1081, 1084, 1085, 1086, 1087, 1092, 1100, 1194, 1200, 1206, 1207, 1209, 1210, 1211, 1212, 1213, 1217 }

B grade: { 1, 2, 3, 4, 8, 9, 10, 11, 12, 13, 14, 15, 16, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 34, 35, 36, 37, 39, 50, 51, 214, 215, 216, 217, 222, 223, 224, 230, 231, 232, 336, 337, 338, 339, 340, 341, 342, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 370, 371, 372, 373, 374, 375, 376, 398, 402, 412, 413, 414, 415, 416, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 430, 431, 432, 433, 434, 435, 436, 437, 438, 441, 442, 443, 444, 445, 446, 447, 448, 452, 453, 454, 455, 472, 481, 482, 483, 488, 489, 490, 491, 492, 493, 655, 656, 657, 658, 659, 660, 661, 662, 665, 667, 668, 669, 670, 890, 891, 894, 895, 896, 898, 900, 902, 903, 904, 906, 908, 909, 910, 913, 915, 917, 918, 919, 920, 923, 924, 925, 926, 931, 932, 934, 938, 939, 942, 943, 951, 952, 992, 1000, 1009, 1065, 1066, 1067, 1068, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1082, 1083, 1088, 1089, 1090, 1091, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1191, 1192, 1193, 1195, 1196, 1197, 1198, 1199, 1201, 1202, 1203, 1204, 1205, 1208, 1214, 1215, 1216, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228 }

C grade: { 138, 139, 145, 146, 152, 153 }

F grade: { 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 218, 219, 220, 221, 225, 226, 227, 228, 229, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 381, 382, 383, 392, 393, 394, 403, 404, 405, 411, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632,

633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 663, 664, 666, 675, 676, 677, 678, 679, 680, 681, 682, 683, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 956, 957, 959, 960, 961, 962, 963, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 993, 994, 995, 996, 997, 998, 999, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328
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2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 188, 189, 213, 214, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 290, 291, 292, 293, 294, 295, 296, 297, 300, 301, 302, 303, 304, 305, 306, 307, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 610, 611, 635, 636, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 721, 722, 801, 802, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 994, 995, 996, 997, 998, 1003, 1004, 1005, 1006, 1012, 1013, 1014, 1015, 1019, 1020, 1021, 1022, 1023, 1027, 1028, 1029, 1030, 1031, 1032, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1051, 1052, 1053, 1054, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1139, 1140, 1157, 1158, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1259, 1260, 1261, 1262 }

C grade: { }

F grade: { 186, 187, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 288, 289, 298, 299, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 411, 608, 609, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 675, 676, 677, 678, 679, 680, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 992, 993, 999, 1000, 1001, 1002, 1007, 1008, 1009, 1010, 1011, 1016, 1017, 1018, 1024, 1025, 1026, 1033, 1034, 1035, 1047, 1048, 1049, 1050, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1135, 1136, 1137, 1138, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1241, 1258, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	104	81	81	192	197	252	73
N.S.	1	1.00	1.02	0.79	0.79	1.88	1.93	2.47	0.72
time (sec)	N/A	0.074	0.328	0.065	0.524	0.894	0.624	1.681	3.813

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	81	68	70	159	168	204	63
N.S.	1	1.00	0.98	0.82	0.84	1.92	2.02	2.46	0.76
time (sec)	N/A	0.061	0.218	0.049	0.523	0.604	0.237	1.104	3.707

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	74	60	59	120	121	156	51
N.S.	1	1.00	1.10	0.90	0.88	1.79	1.81	2.33	0.76
time (sec)	N/A	0.046	0.169	0.047	0.510	0.499	0.195	0.901	3.689

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	53	47	48	85	85	107	39
N.S.	1	1.00	1.08	0.96	0.98	1.73	1.73	2.18	0.80
time (sec)	N/A	0.029	0.116	0.035	0.550	0.444	0.161	0.662	3.748

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	43	39	37	47	44	58	25
N.S.	1	1.00	1.26	1.15	1.09	1.38	1.29	1.71	0.74
time (sec)	N/A	0.014	0.024	0.031	0.491	0.614	0.125	0.495	3.777

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	23	17	18	24	18	17
N.S.	1	1.00	1.00	1.21	0.89	0.95	1.26	0.95	0.89
time (sec)	N/A	0.006	0.008	0.000	0.289	0.664	0.087	0.461	3.761

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	27	24	37	17	20	34	19
N.S.	1	1.00	1.42	1.26	1.95	0.89	1.05	1.79	1.00
time (sec)	N/A	0.015	0.023	0.152	0.490	0.538	0.086	0.474	3.779

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	54	35	49	51	46	75	27
N.S.	1	1.00	1.69	1.09	1.53	1.59	1.44	2.34	0.84
time (sec)	N/A	0.032	0.127	0.148	0.490	0.438	0.147	0.605	3.734

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	68	48	58	83	80	102	47
N.S.	1	1.00	1.36	0.96	1.16	1.66	1.60	2.04	0.94
time (sec)	N/A	0.048	0.187	0.210	0.513	0.396	0.157	0.551	3.790

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	72	53	71	125	128	128	57
N.S.	1	1.00	1.12	0.83	1.11	1.95	2.00	2.00	0.89
time (sec)	N/A	0.067	0.208	0.203	0.488	0.547	0.191	0.606	3.872

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	84	61	83	156	158	158	70
N.S.	1	1.00	1.01	0.73	1.00	1.88	1.90	1.90	0.84
time (sec)	N/A	0.081	0.389	0.197	0.514	0.554	0.269	0.675	3.978

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	84	76	93	197	206	186	79
N.S.	1	1.00	0.84	0.76	0.93	1.97	2.06	1.86	0.79
time (sec)	N/A	0.102	0.380	0.188	0.493	0.510	0.296	0.673	4.065

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	108	82	95	217	219	274	86
N.S.	1	1.00	0.96	0.73	0.85	1.94	1.96	2.45	0.77
time (sec)	N/A	0.098	0.331	0.051	0.502	0.428	0.320	1.141	3.702

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	73	70	82	174	168	222	73
N.S.	1	1.00	0.78	0.75	0.88	1.87	1.81	2.39	0.78
time (sec)	N/A	0.080	0.223	0.044	0.503	0.423	0.397	0.888	3.802

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	76	61	68	137	136	170	60
N.S.	1	1.00	1.19	0.95	1.06	2.14	2.12	2.66	0.94
time (sec)	N/A	0.042	0.262	0.043	0.500	0.423	0.219	0.658	3.787

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	51	49	55	93	88	116	40
N.S.	1	1.00	0.82	0.79	0.89	1.50	1.42	1.87	0.65
time (sec)	N/A	0.034	0.174	0.044	0.492	0.450	0.165	0.608	3.742

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	100	40	41	56	53	66	29
N.S.	1	1.00	2.63	1.05	1.08	1.47	1.39	1.74	0.76
time (sec)	N/A	0.014	0.633	0.017	0.635	0.431	0.143	0.466	3.679

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	30	39	42	19	22	68	30
N.S.	1	1.00	0.81	1.05	1.14	0.51	0.59	1.84	0.81
time (sec)	N/A	0.030	0.050	0.193	0.501	0.509	0.129	0.600	3.733

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	100	49	56	58	51	85	29
N.S.	1	1.00	2.63	1.29	1.47	1.53	1.34	2.24	0.76
time (sec)	N/A	0.046	0.563	0.137	0.520	0.426	0.162	0.629	3.826

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	64	64	68	94	87	116	53
N.S.	1	1.00	1.10	1.10	1.17	1.62	1.50	2.00	0.91
time (sec)	N/A	0.065	0.224	0.208	0.512	0.397	0.260	0.818	3.840

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	105	78	83	139	136	146	68
N.S.	1	1.00	1.42	1.05	1.12	1.88	1.84	1.97	0.92
time (sec)	N/A	0.083	0.495	0.209	0.493	0.471	0.226	0.860	3.812

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	79	90	97	174	168	180	80
N.S.	1	1.00	0.85	0.97	1.04	1.87	1.81	1.94	0.86
time (sec)	N/A	0.103	0.406	0.217	0.497	0.417	0.983	0.954	3.915

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	124	106	109	219	218	212	92
N.S.	1	1.00	1.11	0.95	0.97	1.96	1.95	1.89	0.82
time (sec)	N/A	0.122	0.830	0.199	0.666	0.432	0.328	1.134	4.156

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	296	83	95	214	207	274	87
N.S.	1	1.00	2.35	0.66	0.75	1.70	1.64	2.17	0.69
time (sec)	N/A	0.121	1.333	0.052	0.869	0.450	0.369	0.915	3.792

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	228	72	82	177	173	222	73
N.S.	1	1.00	2.53	0.80	0.91	1.97	1.92	2.47	0.81
time (sec)	N/A	0.053	1.283	0.047	0.493	0.449	0.289	0.700	3.804

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	178	62	69	134	131	170	59
N.S.	1	1.00	2.09	0.73	0.81	1.58	1.54	2.00	0.69
time (sec)	N/A	0.044	0.891	0.051	0.495	0.478	0.224	0.590	3.728

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	119	51	76	97	94	118	41
N.S.	1	1.00	1.89	0.81	1.21	1.54	1.49	1.87	0.65
time (sec)	N/A	0.025	0.774	0.023	0.504	0.424	0.170	0.497	3.683

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	95	60	53	83	66	123	39
N.S.	1	1.00	1.58	1.00	0.88	1.38	1.10	2.05	0.65
time (sec)	N/A	0.074	0.895	0.198	0.540	0.474	0.209	0.743	3.771

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	144	62	56	91	71	119	38
N.S.	1	1.00	2.09	0.90	0.81	1.32	1.03	1.72	0.55
time (sec)	N/A	0.080	1.110	0.195	0.513	0.463	0.256	0.846	3.809

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	126	75	68	94	87	116	53
N.S.	1	1.00	1.77	1.06	0.96	1.32	1.23	1.63	0.75
time (sec)	N/A	0.076	0.785	0.220	0.503	0.493	0.220	1.019	3.782

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	251	91	83	139	136	146	68
N.S.	1	1.00	2.49	0.90	0.82	1.38	1.35	1.45	0.67
time (sec)	N/A	0.107	0.926	0.217	0.577	0.465	0.278	1.075	3.794

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	254	112	94	174	165	180	80
N.S.	1	1.00	2.35	1.04	0.87	1.61	1.53	1.67	0.74
time (sec)	N/A	0.131	0.965	0.202	0.548	0.460	0.416	1.289	3.905

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	359	132	109	219	218	212	92
N.S.	1	1.00	2.85	1.05	0.87	1.74	1.73	1.68	0.73
time (sec)	N/A	0.150	1.649	0.204	0.590	0.442	0.365	1.612	4.105

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	349	93	108	254	246	326	100
N.S.	1	1.00	2.18	0.58	0.68	1.59	1.54	2.04	0.62
time (sec)	N/A	0.185	1.738	0.055	0.501	0.404	1.090	1.015	3.764

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	294	82	95	217	218	274	87
N.S.	1	1.00	2.53	0.71	0.82	1.87	1.88	2.36	0.75
time (sec)	N/A	0.064	2.182	0.050	0.615	0.424	0.330	0.847	3.732

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	231	72	82	174	170	222	72
N.S.	1	1.00	2.14	0.67	0.76	1.61	1.57	2.06	0.67
time (sec)	N/A	0.060	0.957	0.056	0.535	0.439	0.320	0.594	3.722

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	176	61	108	137	138	170	59
N.S.	1	1.00	1.98	0.69	1.21	1.54	1.55	1.91	0.66
time (sec)	N/A	0.039	0.908	0.042	0.572	0.433	0.209	0.505	3.691

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	159	82	67	137	105	157	64
N.S.	1	1.00	1.85	0.95	0.78	1.59	1.22	1.83	0.74
time (sec)	N/A	0.127	1.272	0.201	0.516	0.412	0.328	0.914	3.800

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	151	80	67	58	51	163	63
N.S.	1	1.00	2.13	1.13	0.94	0.82	0.72	2.30	0.89
time (sec)	N/A	0.063	1.545	0.176	0.499	0.404	0.193	1.020	4.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	133	87	68	138	107	150	65
N.S.	1	1.00	1.29	0.84	0.66	1.34	1.04	1.46	0.63
time (sec)	N/A	0.152	1.325	0.214	0.505	0.390	0.598	1.206	3.966

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	240	100	83	139	136	146	68
N.S.	1	1.00	2.33	0.97	0.81	1.35	1.32	1.42	0.66
time (sec)	N/A	0.107	0.826	0.213	0.501	0.358	0.268	1.409	3.977

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	245	123	96	174	168	180	80
N.S.	1	1.00	1.83	0.92	0.72	1.30	1.25	1.34	0.60
time (sec)	N/A	0.140	0.746	0.201	0.515	0.364	1.728	1.466	4.009

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	359	152	109	219	218	212	92
N.S.	1	1.00	2.53	1.07	0.77	1.54	1.54	1.49	0.65
time (sec)	N/A	0.211	2.831	0.213	0.489	0.372	0.409	0.850	4.271

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	363	176	123	254	246	245	107
N.S.	1	1.00	2.24	1.09	0.76	1.57	1.52	1.51	0.66
time (sec)	N/A	0.223	1.389	0.233	0.505	0.364	6.052	0.905	4.680

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	840	88	0	221	219	116	125
N.S.	1	1.00	6.46	0.68	0.00	1.70	1.68	0.89	0.96
time (sec)	N/A	0.104	6.580	0.133	0.000	0.381	0.353	2.430	4.070

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	235	77	0	175	197	104	106
N.S.	1	1.00	2.16	0.71	0.00	1.61	1.81	0.95	0.97
time (sec)	N/A	0.086	3.812	0.098	0.000	0.383	0.303	1.903	3.979

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	196	65	0	138	134	87	91
N.S.	1	1.00	2.18	0.72	0.00	1.53	1.49	0.97	1.01
time (sec)	N/A	0.066	1.984	0.095	0.000	0.379	0.230	0.966	4.006

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	174	56	0	93	114	70	73
N.S.	1	1.00	2.35	0.76	0.00	1.26	1.54	0.95	0.99
time (sec)	N/A	0.049	1.237	0.101	0.000	0.378	0.207	0.725	4.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	86	48	0	55	88	60	61
N.S.	1	1.00	1.72	0.96	0.00	1.10	1.76	1.20	1.22
time (sec)	N/A	0.039	0.362	0.093	0.000	0.369	0.173	0.657	3.947

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	45	47	0	32	65	58	29
N.S.	1	1.00	1.36	1.42	0.00	0.97	1.97	1.76	0.88
time (sec)	N/A	0.018	0.113	0.074	0.000	0.367	0.097	0.502	3.942

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	45	48	0	32	60	60	29
N.S.	1	1.00	1.36	1.45	0.00	0.97	1.82	1.82	0.88
time (sec)	N/A	0.011	0.123	0.001	0.000	0.383	0.092	0.457	3.951

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	87	54	0	55	92	72	72
N.S.	1	1.00	1.85	1.15	0.00	1.17	1.96	1.53	1.53
time (sec)	N/A	0.042	0.362	0.300	0.000	0.372	0.161	0.522	4.042

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	286	68	0	99	116	91	96
N.S.	1	1.00	4.09	0.97	0.00	1.41	1.66	1.30	1.37
time (sec)	N/A	0.066	0.708	0.244	0.000	0.381	0.190	0.585	3.986

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	414	77	0	134	138	105	110
N.S.	1	1.00	4.60	0.86	0.00	1.49	1.53	1.17	1.22
time (sec)	N/A	0.086	0.946	0.282	0.000	0.363	0.231	0.797	3.973

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	365	89	0	181	196	116	126
N.S.	1	1.00	3.38	0.82	0.00	1.68	1.81	1.07	1.17
time (sec)	N/A	0.104	3.397	0.283	0.000	0.379	0.269	0.791	4.091

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	882	91	0	198	262	111	132
N.S.	1	1.00	6.21	0.64	0.00	1.39	1.85	0.78	0.93
time (sec)	N/A	0.158	6.526	0.147	0.000	0.374	0.350	2.089	4.061

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	300	79	0	151	214	98	114
N.S.	1	1.00	2.42	0.64	0.00	1.22	1.73	0.79	0.92
time (sec)	N/A	0.137	1.822	0.114	0.000	0.377	1.004	1.652	4.032

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	273	70	0	113	177	79	100
N.S.	1	1.00	2.62	0.67	0.00	1.09	1.70	0.76	0.96
time (sec)	N/A	0.114	1.672	0.129	0.000	0.382	0.288	1.049	4.026

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	135	60	0	66	148	69	84
N.S.	1	1.00	1.71	0.76	0.00	0.84	1.87	0.87	1.06
time (sec)	N/A	0.098	0.430	0.112	0.000	0.369	0.304	0.902	4.015

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	68	62	0	43	117	72	39
N.S.	1	1.00	1.15	1.05	0.00	0.73	1.98	1.22	0.66
time (sec)	N/A	0.057	0.220	0.126	0.000	0.402	0.161	0.680	3.969

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	66	60	0	32	73	70	46
N.S.	1	1.00	1.12	1.02	0.00	0.54	1.24	1.19	0.78
time (sec)	N/A	0.030	0.110	0.089	0.000	0.357	0.126	0.532	3.969

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	68	62	0	43	117	72	39
N.S.	1	1.00	1.11	1.02	0.00	0.70	1.92	1.18	0.64
time (sec)	N/A	0.023	0.194	0.000	0.000	0.354	0.135	0.460	3.944

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	135	67	0	66	150	81	97
N.S.	1	1.00	1.90	0.94	0.00	0.93	2.11	1.14	1.37
time (sec)	N/A	0.102	0.383	0.276	0.000	0.373	0.250	0.750	3.941

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	276	82	0	114	180	109	125
N.S.	1	1.00	2.85	0.85	0.00	1.18	1.86	1.12	1.29
time (sec)	N/A	0.141	1.982	0.301	0.000	0.377	0.278	0.881	3.958

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	319	90	0	151	216	109	135
N.S.	1	1.00	2.61	0.74	0.00	1.24	1.77	0.89	1.11
time (sec)	N/A	0.162	1.624	0.296	0.000	0.381	0.457	1.612	4.007

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	264	94	0	167	257	111	140
N.S.	1	1.00	1.64	0.58	0.00	1.04	1.60	0.69	0.87
time (sec)	N/A	0.210	4.264	0.187	0.000	0.377	0.423	2.665	3.949

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	239	83	0	120	214	91	122
N.S.	1	1.00	1.67	0.58	0.00	0.84	1.50	0.64	0.85
time (sec)	N/A	0.190	2.697	0.158	0.000	0.368	0.495	2.217	3.809

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	118	75	0	77	184	80	110
N.S.	1	1.00	0.99	0.63	0.00	0.65	1.55	0.67	0.92
time (sec)	N/A	0.163	0.483	0.158	0.000	0.363	0.326	1.391	3.851

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	91	74	0	54	156	81	49
N.S.	1	1.00	0.99	0.80	0.00	0.59	1.70	0.88	0.53
time (sec)	N/A	0.088	0.277	0.130	0.000	0.359	0.241	1.073	3.769

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	91	75	0	54	151	80	49
N.S.	1	1.00	1.03	0.85	0.00	0.61	1.72	0.91	0.56
time (sec)	N/A	0.069	0.454	0.124	0.000	0.377	0.213	0.959	3.757

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	91	74	0	54	153	81	49
N.S.	1	1.00	1.08	0.88	0.00	0.64	1.82	0.96	0.58
time (sec)	N/A	0.044	0.381	0.120	0.000	0.364	0.221	0.836	3.775

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	93	75	0	54	155	80	50
N.S.	1	1.00	1.06	0.85	0.00	0.61	1.76	0.91	0.57
time (sec)	N/A	0.035	0.252	0.122	0.000	0.357	0.177	0.597	3.752

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	118	81	0	77	187	93	120
N.S.	1	1.00	1.20	0.83	0.00	0.79	1.91	0.95	1.22
time (sec)	N/A	0.154	0.470	0.342	0.000	0.383	0.301	1.012	3.868

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	379	95	0	125	218	119	145
N.S.	1	1.00	2.85	0.71	0.00	0.94	1.64	0.89	1.09
time (sec)	N/A	0.215	4.992	0.329	0.000	0.372	0.356	0.954	4.054

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	429	95	0	135	248	100	132
N.S.	1	1.00	2.51	0.56	0.00	0.79	1.45	0.58	0.77
time (sec)	N/A	0.259	0.906	0.214	0.000	0.387	0.460	4.517	3.911

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	126	87	0	88	219	89	128
N.S.	1	1.00	0.86	0.59	0.00	0.60	1.49	0.61	0.87
time (sec)	N/A	0.236	0.502	0.182	0.000	0.386	1.829	2.079	3.980

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	98	89	0	65	189	92	59
N.S.	1	1.00	0.77	0.70	0.00	0.51	1.48	0.72	0.46
time (sec)	N/A	0.125	0.266	0.158	0.000	0.365	0.270	1.392	3.937

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	95	87	0	54	156	88	60
N.S.	1	1.00	0.75	0.69	0.00	0.43	1.24	0.70	0.48
time (sec)	N/A	0.127	0.487	0.146	0.000	0.365	0.607	1.173	3.898

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	69	89	0	43	117	87	92
N.S.	1	1.00	0.59	0.77	0.00	0.37	1.01	0.75	0.79
time (sec)	N/A	0.083	0.373	0.152	0.000	0.370	0.202	1.099	3.899

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	94	87	0	54	156	88	60
N.S.	1	1.00	0.85	0.79	0.00	0.49	1.42	0.80	0.55
time (sec)	N/A	0.058	0.356	0.164	0.000	0.359	0.295	1.016	3.897

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	98	89	0	65	189	92	60
N.S.	1	1.00	0.84	0.77	0.00	0.56	1.63	0.79	0.52
time (sec)	N/A	0.051	0.255	0.000	0.000	0.363	0.218	0.780	3.894

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	123	94	0	88	221	101	142
N.S.	1	1.00	1.02	0.78	0.00	0.73	1.84	0.84	1.18
time (sec)	N/A	0.218	0.446	0.332	0.000	0.392	0.438	1.274	3.839

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	444	109	0	136	252	129	165
N.S.	1	1.00	2.79	0.69	0.00	0.86	1.58	0.81	1.04
time (sec)	N/A	0.303	2.957	0.390	0.000	0.381	0.375	1.244	4.163

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	105	94	120	330	0	0	109
N.S.	1	1.00	0.62	0.56	0.71	1.96	0.00	0.00	0.65
time (sec)	N/A	0.230	2.380	0.263	0.493	0.490	0.000	0.000	0.338

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	95	92	120	285	0	0	100
N.S.	1	1.00	0.75	0.72	0.94	2.24	0.00	0.00	0.79
time (sec)	N/A	0.142	1.798	0.203	0.479	0.457	0.000	0.000	4.147

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	97	58	84	233	0	0	63
N.S.	1	1.00	1.28	0.76	1.11	3.07	0.00	0.00	0.83
time (sec)	N/A	0.055	0.768	0.168	0.492	0.413	0.000	0.000	0.221

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	77	53	83	187	0	0	54
N.S.	1	1.00	1.15	0.79	1.24	2.79	0.00	0.00	0.81
time (sec)	N/A	0.040	0.471	0.163	0.497	0.423	0.000	0.000	4.037

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	64	36	60	159	0	0	39
N.S.	1	1.00	1.39	0.78	1.30	3.46	0.00	0.00	0.85
time (sec)	N/A	0.017	0.201	0.151	0.496	0.464	0.000	0.000	3.948

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	A	B	F	F	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	0	230	107	336	0	0	61
N.S.	1	1.00	0.00	2.95	1.37	4.31	0.00	0.00	0.78
time (sec)	N/A	0.132	0.709	4.585	0.486	0.559	0.000	0.000	0.171

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	197	586	134	476	0	0	97
N.S.	1	1.00	1.77	5.28	1.21	4.29	0.00	0.00	0.87
time (sec)	N/A	0.172	4.477	0.899	0.503	0.454	0.000	0.000	4.016

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	144	904	179	519	0	0	124
N.S.	1	1.00	0.99	6.23	1.23	3.58	0.00	0.00	0.86
time (sec)	N/A	0.292	2.161	0.899	0.493	0.474	0.000	0.000	4.070

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	166	111	138	354	0	0	120
N.S.	1	1.00	0.83	0.56	0.69	1.78	0.00	0.00	0.60
time (sec)	N/A	0.334	2.826	0.204	0.498	0.395	0.000	0.000	4.231

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	162	76	102	315	0	0	84
N.S.	1	1.00	1.60	0.75	1.01	3.12	0.00	0.00	0.83
time (sec)	N/A	0.069	1.516	0.189	0.488	0.384	0.000	0.000	0.301

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	148	70	102	258	0	0	74
N.S.	1	1.00	1.61	0.76	1.11	2.80	0.00	0.00	0.80
time (sec)	N/A	0.058	1.003	0.169	0.486	0.386	0.000	0.000	4.059

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	79	54	83	215	0	0	61
N.S.	1	1.00	1.10	0.75	1.15	2.99	0.00	0.00	0.85
time (sec)	N/A	0.028	0.528	0.153	0.485	0.474	0.000	0.000	0.184

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	201	231	107	357	0	0	76
N.S.	1	1.00	2.54	2.92	1.35	4.52	0.00	0.00	0.96
time (sec)	N/A	0.121	1.248	0.770	0.528	0.466	0.000	0.000	4.087

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	178	631	132	501	0	0	112
N.S.	1	1.00	1.26	4.48	0.94	3.55	0.00	0.00	0.79
time (sec)	N/A	0.275	1.447	0.837	0.506	0.463	0.000	0.000	4.197

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	190	1142	178	546	0	0	136
N.S.	1	1.00	1.03	6.21	0.97	2.97	0.00	0.00	0.74
time (sec)	N/A	0.394	2.083	0.867	0.491	0.461	0.000	0.000	4.072

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	176	131	156	412	0	0	142
N.S.	1	1.00	0.86	0.64	0.76	2.02	0.00	0.00	0.70
time (sec)	N/A	0.334	2.355	0.194	0.486	0.496	0.000	0.000	4.295

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	170	96	120	371	0	0	107
N.S.	1	1.00	1.31	0.74	0.92	2.85	0.00	0.00	0.82
time (sec)	N/A	0.082	2.823	0.187	0.491	0.486	0.000	0.000	4.209

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	154	89	120	312	0	0	98
N.S.	1	1.00	1.29	0.75	1.01	2.62	0.00	0.00	0.82
time (sec)	N/A	0.072	1.422	0.180	0.508	0.448	0.000	0.000	0.321

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	140	73	101	271	0	0	84
N.S.	1	1.00	1.39	0.72	1.00	2.68	0.00	0.00	0.83
time (sec)	N/A	0.043	1.054	0.157	0.482	0.440	0.000	0.000	4.076

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	148	329	126	406	0	0	98
N.S.	1	1.00	1.42	3.16	1.21	3.90	0.00	0.00	0.94
time (sec)	N/A	0.193	1.314	0.971	0.523	0.526	0.000	0.000	0.232

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	170	608	133	511	0	0	114
N.S.	1	1.00	1.49	5.33	1.17	4.48	0.00	0.00	1.00
time (sec)	N/A	0.194	1.538	0.822	0.498	0.501	0.000	0.000	4.110

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	182	677	181	558	0	0	139
N.S.	1	1.00	1.21	4.48	1.20	3.70	0.00	0.00	0.92
time (sec)	N/A	0.287	2.135	0.829	0.493	0.446	0.000	0.000	4.004

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	200	926	216	659	0	0	171
N.S.	1	1.00	1.05	4.87	1.14	3.47	0.00	0.00	0.90
time (sec)	N/A	0.386	2.650	0.855	0.500	0.441	0.000	0.000	0.220

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	166	92	120	321	0	0	107
N.S.	1	1.00	1.28	0.71	0.92	2.47	0.00	0.00	0.82
time (sec)	N/A	0.058	1.854	0.166	0.499	0.430	0.000	0.000	4.142

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	123	131	156	389	0	0	144
N.S.	1	1.00	0.61	0.65	0.78	1.94	0.00	0.00	0.72
time (sec)	N/A	0.316	1.798	0.192	0.493	0.441	0.000	0.000	0.450

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	122	113	138	343	0	0	129
N.S.	1	1.00	0.71	0.66	0.80	1.99	0.00	0.00	0.75
time (sec)	N/A	0.233	1.899	0.201	0.485	0.451	0.000	0.000	4.182

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	129	93	120	289	0	0	99
N.S.	1	1.00	1.02	0.74	0.95	2.29	0.00	0.00	0.79
time (sec)	N/A	0.134	1.130	0.189	0.504	0.472	0.000	0.000	4.167

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	113	73	101	238	0	0	83
N.S.	1	1.00	1.15	0.74	1.03	2.43	0.00	0.00	0.85
time (sec)	N/A	0.067	0.851	0.173	0.841	0.449	0.000	0.000	0.261

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	83	53	83	235	0	0	54
N.S.	1	1.00	1.24	0.79	1.24	3.51	0.00	0.00	0.81
time (sec)	N/A	0.042	0.274	0.198	0.526	0.455	0.000	0.000	4.087

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	84	59	81	238	0	0	60
N.S.	1	1.00	1.18	0.83	1.14	3.35	0.00	0.00	0.85
time (sec)	N/A	0.028	0.245	0.158	0.504	0.441	0.000	0.000	0.227

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	133	668	122	459	0	0	80
N.S.	1	1.00	1.34	6.75	1.23	4.64	0.00	0.00	0.81
time (sec)	N/A	0.186	1.141	5.846	0.502	0.451	0.000	0.000	0.193

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	153	1368	160	546	0	0	134
N.S.	1	1.00	1.09	9.70	1.13	3.87	0.00	0.00	0.95
time (sec)	N/A	0.285	2.327	1.075	0.526	0.468	0.000	0.000	4.087

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	170	1370	203	617	0	0	155
N.S.	1	1.00	0.94	7.61	1.13	3.43	0.00	0.00	0.86
time (sec)	N/A	0.375	2.768	1.124	0.494	0.456	0.000	0.000	4.116

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	149	131	157	388	0	0	138
N.S.	1	1.00	0.73	0.64	0.77	1.89	0.00	0.00	0.67
time (sec)	N/A	0.325	1.866	0.178	0.509	0.480	0.000	0.000	4.204

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	165	113	139	336	0	0	129
N.S.	1	1.00	0.95	0.65	0.80	1.93	0.00	0.00	0.74
time (sec)	N/A	0.237	1.349	0.174	0.497	0.434	0.000	0.000	0.276

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	123	91	121	273	0	0	93
N.S.	1	1.00	0.92	0.68	0.91	2.05	0.00	0.00	0.70
time (sec)	N/A	0.141	1.268	0.187	0.506	0.407	0.000	0.000	0.202

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	124	75	103	272	0	0	84
N.S.	1	1.00	1.19	0.72	0.99	2.62	0.00	0.00	0.81
time (sec)	N/A	0.091	1.094	0.182	0.507	0.392	0.000	0.000	4.046

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	124	72	101	271	0	0	72
N.S.	1	1.00	1.27	0.73	1.03	2.77	0.00	0.00	0.73
time (sec)	N/A	0.058	0.644	0.174	0.501	0.375	0.000	0.000	3.985

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	124	78	94	272	0	0	83
N.S.	1	1.00	1.19	0.75	0.90	2.62	0.00	0.00	0.80
time (sec)	N/A	0.042	0.611	0.149	0.505	0.372	0.000	0.000	3.994

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	196	697	139	500	0	0	109
N.S.	1	1.00	1.48	5.28	1.05	3.79	0.00	0.00	0.83
time (sec)	N/A	0.260	2.052	1.238	0.526	0.377	0.000	0.000	4.052

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	214	1399	184	601	0	0	178
N.S.	1	1.00	1.18	7.73	1.02	3.32	0.00	0.00	0.98
time (sec)	N/A	0.378	2.007	0.913	0.499	0.383	0.000	0.000	4.020

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	214	1397	221	678	0	0	186
N.S.	1	1.00	0.97	6.35	1.00	3.08	0.00	0.00	0.85
time (sec)	N/A	0.481	4.330	1.014	0.552	0.388	0.000	0.000	3.940

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	149	131	157	342	0	0	140
N.S.	1	1.00	0.73	0.64	0.77	1.67	0.00	0.00	0.68
time (sec)	N/A	0.336	2.012	0.218	0.586	0.383	0.000	0.000	3.915

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	149	111	139	283	0	0	129
N.S.	1	1.00	0.85	0.63	0.79	1.61	0.00	0.00	0.73
time (sec)	N/A	0.245	1.615	0.172	0.532	0.378	0.000	0.000	0.251

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	135	93	121	283	0	0	93
N.S.	1	1.00	1.02	0.70	0.91	2.13	0.00	0.00	0.70
time (sec)	N/A	0.157	1.204	0.191	0.510	0.368	0.000	0.000	3.864

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	135	94	119	283	0	0	88
N.S.	1	1.00	1.02	0.71	0.89	2.13	0.00	0.00	0.66
time (sec)	N/A	0.110	1.106	0.184	0.522	0.370	0.000	0.000	0.197

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	135	91	116	284	0	0	91
N.S.	1	1.00	1.08	0.73	0.93	2.27	0.00	0.00	0.73
time (sec)	N/A	0.073	0.778	0.180	0.506	0.376	0.000	0.000	0.203

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	137	97	119	283	0	0	106
N.S.	1	1.00	1.03	0.73	0.89	2.13	0.00	0.00	0.80
time (sec)	N/A	0.057	0.745	0.144	0.549	0.365	0.000	0.000	3.806

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	188	724	161	511	0	0	132
N.S.	1	1.00	1.18	4.55	1.01	3.21	0.00	0.00	0.83
time (sec)	N/A	0.349	2.068	0.934	0.503	0.374	0.000	0.000	0.248

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	263	1426	202	612	0	0	201
N.S.	1	1.00	1.23	6.66	0.94	2.86	0.00	0.00	0.94
time (sec)	N/A	0.482	3.603	0.906	0.527	0.383	0.000	0.000	4.057

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	150	116	137	294	0	0	129
N.S.	1	1.00	0.93	0.72	0.85	1.81	0.00	0.00	0.80
time (sec)	N/A	0.077	2.263	0.176	0.510	0.369	0.000	0.000	3.975

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	125	321	218	391	0	154	114
N.S.	1	1.00	1.17	3.00	2.04	3.65	0.00	1.44	1.07
time (sec)	N/A	0.108	2.400	0.242	0.514	0.394	0.000	0.649	4.933

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	148	305	201	326	0	124	65
N.S.	1	1.00	1.80	3.72	2.45	3.98	0.00	1.51	0.79
time (sec)	N/A	0.082	0.906	0.102	0.551	0.400	0.000	0.523	4.602

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	85	290	183	256	0	89	128
N.S.	1	1.00	1.39	4.75	3.00	4.20	0.00	1.46	2.10
time (sec)	N/A	0.051	0.714	0.101	0.510	0.389	0.000	0.514	4.258

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	C	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	87	273	160	222	0	67	30
N.S.	1	1.00	2.18	6.82	4.00	5.55	0.00	1.68	0.75
time (sec)	N/A	0.029	0.631	0.153	0.529	0.358	0.000	0.531	4.290

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	C	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	138	294	179	355	0	89	50
N.S.	1	1.00	2.23	4.74	2.89	5.73	0.00	1.44	0.81
time (sec)	N/A	0.058	1.078	0.115	0.519	0.386	0.000	0.581	4.363

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	140	310	200	410	0	109	70
N.S.	1	1.00	1.61	3.56	2.30	4.71	0.00	1.25	0.80
time (sec)	N/A	0.088	1.200	0.096	0.512	0.374	0.000	0.712	4.937

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	105	325	220	478	0	130	87
N.S.	1	1.00	0.95	2.95	2.00	4.35	0.00	1.18	0.79
time (sec)	N/A	0.121	4.144	0.097	0.507	0.403	0.000	0.928	5.285

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	90	322	218	360	0	154	143
N.S.	1	1.00	0.84	3.01	2.04	3.36	0.00	1.44	1.34
time (sec)	N/A	0.105	0.532	0.110	0.524	0.383	0.000	0.579	4.577

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	78	306	201	300	0	124	65
N.S.	1	1.00	0.95	3.73	2.45	3.66	0.00	1.51	0.79
time (sec)	N/A	0.076	0.240	0.112	0.521	0.374	0.000	0.531	4.258

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	64	291	183	233	0	89	126
N.S.	1	1.00	1.05	4.77	3.00	3.82	0.00	1.46	2.07
time (sec)	N/A	0.050	0.066	0.110	0.517	0.378	0.000	0.500	3.944

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	50	274	160	175	0	67	30
N.S.	1	1.00	1.25	6.85	4.00	4.38	0.00	1.68	0.75
time (sec)	N/A	0.027	0.045	0.139	0.512	0.366	0.000	0.545	4.217

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	C	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	39	295	179	328	0	89	50
N.S.	1	1.00	0.63	4.76	2.89	5.29	0.00	1.44	0.81
time (sec)	N/A	0.053	0.116	0.117	0.524	0.376	0.000	0.627	4.301

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	41	311	201	394	0	109	70
N.S.	1	1.00	0.47	3.57	2.31	4.53	0.00	1.25	0.80
time (sec)	N/A	0.085	0.118	0.100	0.491	0.372	0.000	0.620	4.618

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	41	326	220	459	0	130	87
N.S.	1	1.00	0.37	2.96	2.00	4.17	0.00	1.18	0.79
time (sec)	N/A	0.112	0.173	0.098	0.509	0.396	0.000	0.662	4.906

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	145	342	242	460	0	192	115
N.S.	1	1.00	1.04	2.44	1.73	3.29	0.00	1.37	0.82
time (sec)	N/A	0.146	2.577	0.115	0.513	0.402	0.000	0.915	4.640

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	127	326	223	392	0	163	97
N.S.	1	1.00	1.12	2.88	1.97	3.47	0.00	1.44	0.86
time (sec)	N/A	0.122	3.042	0.111	0.511	0.379	0.000	0.768	4.542

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	102	311	204	322	0	129	74
N.S.	1	1.00	1.13	3.46	2.27	3.58	0.00	1.43	0.82
time (sec)	N/A	0.088	1.632	0.117	0.510	0.396	0.000	0.591	4.281

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	C	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	157	295	184	278	0	92	59
N.S.	1	1.00	2.38	4.47	2.79	4.21	0.00	1.39	0.89
time (sec)	N/A	0.066	1.509	0.132	0.517	0.379	0.000	0.600	4.153

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	C	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	147	291	183	365	0	93	55
N.S.	1	1.00	2.23	4.41	2.77	5.53	0.00	1.41	0.83
time (sec)	N/A	0.074	1.057	0.099	0.517	0.379	0.000	0.678	4.293

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	87	312	206	423	0	117	80
N.S.	1	1.00	0.94	3.35	2.22	4.55	0.00	1.26	0.86
time (sec)	N/A	0.108	1.419	0.099	0.515	0.381	0.000	0.661	4.375

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	381	327	228	492	0	139	95
N.S.	1	1.00	3.23	2.77	1.93	4.17	0.00	1.18	0.81
time (sec)	N/A	0.140	7.023	0.099	0.517	0.389	0.000	0.787	4.662

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	150	358	267	517	0	223	137
N.S.	1	1.00	0.84	2.00	1.49	2.89	0.00	1.25	0.77
time (sec)	N/A	0.215	3.024	0.141	0.531	0.417	0.000	0.871	4.975

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	138	342	248	447	0	194	119
N.S.	1	1.00	0.91	2.25	1.63	2.94	0.00	1.28	0.78
time (sec)	N/A	0.176	2.473	0.161	0.565	0.383	0.000	0.774	4.772

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	122	327	229	376	0	161	96
N.S.	1	1.00	0.95	2.53	1.78	2.91	0.00	1.25	0.74
time (sec)	N/A	0.143	3.317	0.155	0.576	0.380	0.000	0.664	4.441

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	154	311	208	340	0	130	81
N.S.	1	1.00	1.44	2.91	1.94	3.18	0.00	1.21	0.76
time (sec)	N/A	0.118	2.585	0.123	0.556	0.374	0.000	0.623	4.378

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	156	309	202	357	0	115	77
N.S.	1	1.00	1.95	3.86	2.52	4.46	0.00	1.44	0.96
time (sec)	N/A	0.073	1.688	0.109	0.548	0.362	0.000	0.716	4.329

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	87	305	206	422	0	117	80
N.S.	1	1.00	0.80	2.80	1.89	3.87	0.00	1.07	0.73
time (sec)	N/A	0.138	2.583	0.113	0.528	0.378	0.000	0.689	4.442

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	377	325	228	492	0	139	95
N.S.	1	1.00	2.86	2.46	1.73	3.73	0.00	1.05	0.72
time (sec)	N/A	0.175	7.164	0.115	0.565	0.377	0.000	0.853	4.658

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	416	341	245	549	0	156	119
N.S.	1	1.00	2.62	2.14	1.54	3.45	0.00	0.98	0.75
time (sec)	N/A	0.211	7.697	0.115	0.543	0.411	0.000	0.911	5.007

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	275	126	0	646	0	242	180
N.S.	1	1.00	0.88	0.40	0.00	2.07	0.00	0.78	0.58
time (sec)	N/A	0.229	1.908	0.210	0.000	0.383	0.000	0.648	6.213

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	164	109	0	562	0	197	160
N.S.	1	1.00	0.57	0.38	0.00	1.96	0.00	0.69	0.56
time (sec)	N/A	0.187	1.977	0.160	0.000	0.395	0.000	0.600	6.164

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	150	93	0	550	0	177	137
N.S.	1	1.00	0.58	0.36	0.00	2.12	0.00	0.68	0.53
time (sec)	N/A	0.159	1.031	0.162	0.000	0.379	0.000	0.515	5.885

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	129	72	0	304	0	110	71
N.S.	1	1.00	1.59	0.89	0.00	3.75	0.00	1.36	0.88
time (sec)	N/A	0.073	1.145	0.176	0.000	0.390	0.000	0.568	4.230

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	147	102	0	541	0	173	128
N.S.	1	1.00	0.56	0.39	0.00	2.06	0.00	0.66	0.49
time (sec)	N/A	0.154	0.874	0.170	0.000	0.403	0.000	0.597	5.923

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	155	116	0	677	0	202	147
N.S.	1	1.00	0.54	0.40	0.00	2.36	0.00	0.70	0.51
time (sec)	N/A	0.209	1.370	0.140	0.000	0.378	0.000	0.590	6.058

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	273	133	0	776	0	220	171
N.S.	1	1.00	0.87	0.42	0.00	2.47	0.00	0.70	0.54
time (sec)	N/A	0.260	2.517	0.153	0.000	0.396	0.000	0.644	6.661

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	346	142	0	705	0	269	225
N.S.	1	1.00	0.98	0.40	0.00	2.00	0.00	0.76	0.64
time (sec)	N/A	0.344	2.463	0.180	0.000	0.409	0.000	0.655	5.637

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	233	126	0	600	0	226	201
N.S.	1	1.00	0.71	0.39	0.00	1.84	0.00	0.69	0.62
time (sec)	N/A	0.313	1.456	0.168	0.000	0.391	0.000	0.669	5.565

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	231	108	0	604	0	206	177
N.S.	1	1.00	0.77	0.36	0.00	2.01	0.00	0.68	0.59
time (sec)	N/A	0.277	1.073	0.171	0.000	0.378	0.000	0.748	5.668

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	231	118	0	593	0	203	179
N.S.	1	1.00	0.78	0.40	0.00	2.00	0.00	0.68	0.60
time (sec)	N/A	0.276	1.085	0.171	0.000	0.379	0.000	0.667	5.550

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	227	116	0	563	0	205	147
N.S.	1	1.00	0.76	0.39	0.00	1.88	0.00	0.69	0.49
time (sec)	N/A	0.250	0.948	0.191	0.000	0.388	0.000	0.612	5.204

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	228	118	0	584	0	198	168
N.S.	1	1.00	0.76	0.39	0.00	1.94	0.00	0.66	0.56
time (sec)	N/A	0.266	1.050	0.182	0.000	0.387	0.000	0.615	5.478

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	231	131	0	730	0	221	185
N.S.	1	1.00	0.71	0.40	0.00	2.24	0.00	0.68	0.57
time (sec)	N/A	0.342	1.159	0.164	0.000	0.398	0.000	0.666	5.963

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	346	149	0	837	0	246	209
N.S.	1	1.00	0.98	0.42	0.00	2.37	0.00	0.70	0.59
time (sec)	N/A	0.378	2.365	0.152	0.000	0.404	0.000	0.694	6.280

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	236	141	0	620	0	251	240
N.S.	1	1.00	0.64	0.38	0.00	1.68	0.00	0.68	0.65
time (sec)	N/A	0.431	3.693	0.192	0.000	0.398	0.000	0.931	5.485

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	234	125	0	614	0	231	217
N.S.	1	1.00	0.68	0.36	0.00	1.79	0.00	0.67	0.63
time (sec)	N/A	0.387	1.559	0.180	0.000	0.402	0.000	0.765	5.778

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	232	116	0	615	0	216	158
N.S.	1	1.00	0.71	0.35	0.00	1.87	0.00	0.66	0.48
time (sec)	N/A	0.395	1.146	0.187	0.000	0.380	0.000	0.806	4.162

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	158	103	0	359	0	160	155
N.S.	1	1.00	1.01	0.66	0.00	2.29	0.00	1.02	0.99
time (sec)	N/A	0.240	2.385	0.199	0.000	0.383	0.000	0.727	4.217

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	225	120	0	575	0	207	157
N.S.	1	1.00	0.77	0.41	0.00	1.97	0.00	0.71	0.54
time (sec)	N/A	0.249	2.626	0.209	0.000	0.378	0.000	0.713	4.180

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	234	133	0	596	0	227	206
N.S.	1	1.00	0.68	0.39	0.00	1.74	0.00	0.66	0.60
time (sec)	N/A	0.382	1.312	0.246	0.000	0.389	0.000	0.733	5.683

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	234	147	0	742	0	252	227
N.S.	1	1.00	0.64	0.40	0.00	2.02	0.00	0.68	0.62
time (sec)	N/A	0.451	2.230	0.201	0.000	0.388	0.000	0.822	6.119

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	267	472	0	559	0	0	-1
N.S.	1	1.00	1.52	2.68	0.00	3.18	0.00	0.00	-0.01
time (sec)	N/A	0.355	4.113	0.349	0.000	0.365	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	249	229	0	479	0	0	-1
N.S.	1	1.00	1.84	1.70	0.00	3.55	0.00	0.00	-0.01
time (sec)	N/A	0.228	3.959	0.190	0.000	0.378	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	229	349	0	413	0	0	278
N.S.	1	1.00	2.20	3.36	0.00	3.97	0.00	0.00	2.67
time (sec)	N/A	0.175	1.227	0.192	0.000	0.386	0.000	0.000	7.314

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	B	B	F	F(-2)	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	0	121	379	206	0	0	89
N.S.	1	1.00	0.00	2.47	7.73	4.20	0.00	0.00	1.82
time (sec)	N/A	0.042	1.123	0.205	0.628	0.364	0.000	0.000	5.121

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	B	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	0	157	549	329	0	0	-1
N.S.	1	1.00	0.00	1.91	6.70	4.01	0.00	0.00	-0.01
time (sec)	N/A	0.096	2.439	0.196	0.628	0.369	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	B	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	0	197	974	355	0	0	-1
N.S.	1	1.00	0.00	1.64	8.12	2.96	0.00	0.00	-0.01
time (sec)	N/A	0.168	2.701	0.188	0.672	0.372	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	B	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	0	359	1151	424	0	0	-1
N.S.	1	1.00	0.00	2.33	7.47	2.75	0.00	0.00	-0.01
time (sec)	N/A	0.260	3.667	0.187	0.735	0.387	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	213	449	0	673	0	0	-1
N.S.	1	1.00	0.84	1.77	0.00	2.65	0.00	0.00	-0.00
time (sec)	N/A	0.526	3.097	0.217	0.000	0.375	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	234	405	0	600	0	0	-1
N.S.	1	1.00	1.08	1.87	0.00	2.76	0.00	0.00	-0.00
time (sec)	N/A	0.440	3.344	0.171	0.000	0.382	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	203	363	0	525	0	0	-1
N.S.	1	1.00	1.15	2.06	0.00	2.98	0.00	0.00	-0.01
time (sec)	N/A	0.345	1.827	0.174	0.000	0.383	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	255	326	0	451	0	0	-1
N.S.	1	1.00	2.45	3.13	0.00	4.34	0.00	0.00	-0.01
time (sec)	N/A	0.172	2.037	0.184	0.000	0.382	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	160	320	555	353	0	0	-1
N.S.	1	1.00	1.93	3.86	6.69	4.25	0.00	0.00	-0.01
time (sec)	N/A	0.088	1.746	0.188	0.629	0.364	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	160	370	1008	395	0	0	-1
N.S.	1	1.00	1.34	3.11	8.47	3.32	0.00	0.00	-0.01
time (sec)	N/A	0.135	1.869	0.172	0.730	0.365	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	166	412	1198	450	0	0	-1
N.S.	1	1.00	0.84	2.08	6.05	2.27	0.00	0.00	-0.01
time (sec)	N/A	0.361	2.788	0.174	0.777	0.388	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	224	457	2883	491	0	0	-1
N.S.	1	1.00	0.95	1.94	12.27	2.09	0.00	0.00	-0.00
time (sec)	N/A	0.450	3.261	0.189	0.989	0.377	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	232	494	0	769	0	0	-1
N.S.	1	1.00	0.90	1.91	0.00	2.98	0.00	0.00	-0.00
time (sec)	N/A	0.522	4.389	0.196	0.000	0.411	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	258	450	0	694	0	0	-1
N.S.	1	1.00	1.18	2.05	0.00	3.17	0.00	0.00	-0.00
time (sec)	N/A	0.445	3.291	0.171	0.000	0.385	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	192	407	0	621	0	0	-1
N.S.	1	1.00	1.05	2.24	0.00	3.41	0.00	0.00	-0.01
time (sec)	N/A	0.345	3.411	0.180	0.000	0.384	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	267	365	0	542	0	0	-1
N.S.	1	1.00	1.92	2.63	0.00	3.90	0.00	0.00	-0.01
time (sec)	N/A	0.260	2.612	0.196	0.000	0.400	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	215	388	0	626	0	0	-1
N.S.	1	1.00	1.55	2.79	0.00	4.50	0.00	0.00	-0.01
time (sec)	N/A	0.261	2.375	0.190	0.000	0.383	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	162	372	1072	408	0	0	-1
N.S.	1	1.00	1.33	3.05	8.79	3.34	0.00	0.00	-0.01
time (sec)	N/A	0.134	2.007	0.168	0.684	0.369	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	187	414	1284	466	0	0	-1
N.S.	1	1.00	1.20	2.65	8.23	2.99	0.00	0.00	-0.01
time (sec)	N/A	0.179	3.717	0.172	0.682	0.385	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	188	459	3186	509	0	0	-1
N.S.	1	1.00	0.93	2.27	15.77	2.52	0.00	0.00	-0.00
time (sec)	N/A	0.365	3.088	0.168	0.961	0.369	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	207	501	3508	566	0	0	-1
N.S.	1	1.00	0.87	2.10	14.68	2.37	0.00	0.00	-0.00
time (sec)	N/A	0.474	4.986	0.188	1.703	0.383	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	224	667	0	658	0	0	-1
N.S.	1	1.00	1.03	3.06	0.00	3.02	0.00	0.00	-0.00
time (sec)	N/A	0.441	3.476	0.202	0.000	0.441	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	222	623	0	580	0	0	-1
N.S.	1	1.00	1.25	3.52	0.00	3.28	0.00	0.00	-0.01
time (sec)	N/A	0.341	2.278	0.182	0.000	0.436	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	210	580	0	577	0	0	-1
N.S.	1	1.00	1.50	4.14	0.00	4.12	0.00	0.00	-0.01
time (sec)	N/A	0.253	2.014	0.203	0.000	0.439	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	132	350	0	301	0	0	242
N.S.	1	1.00	1.50	3.98	0.00	3.42	0.00	0.00	2.75
time (sec)	N/A	0.083	1.413	0.199	0.000	0.388	0.000	0.000	6.709

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	140	352	0	299	0	407	171
N.S.	1	1.00	1.65	4.14	0.00	3.52	0.00	4.79	2.01
time (sec)	N/A	0.129	1.638	0.194	0.000	0.390	0.000	0.780	6.089

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	160	395	0	355	0	504	-1
N.S.	1	1.00	1.33	3.29	0.00	2.96	0.00	4.20	-0.01
time (sec)	N/A	0.167	1.644	0.200	0.000	0.388	0.000	1.424	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	159	401	0	403	0	529	-1
N.S.	1	1.00	0.99	2.49	0.00	2.50	0.00	3.29	-0.01
time (sec)	N/A	0.266	1.851	0.210	0.000	0.397	0.000	1.571	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	191	473	0	456	0	552	-1
N.S.	1	1.00	0.96	2.39	0.00	2.30	0.00	2.79	-0.01
time (sec)	N/A	0.355	1.411	0.191	0.000	0.492	0.000	2.045	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	240	815	0	612	0	0	-1
N.S.	1	1.00	1.10	3.74	0.00	2.81	0.00	0.00	-0.00
time (sec)	N/A	0.449	4.278	0.178	0.000	0.619	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	217	771	0	613	0	0	-1
N.S.	1	1.00	1.20	4.26	0.00	3.39	0.00	0.00	-0.01
time (sec)	N/A	0.361	2.133	0.184	0.000	0.524	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	158	464	0	321	0	0	-1
N.S.	1	1.00	1.24	3.65	0.00	2.53	0.00	0.00	-0.01
time (sec)	N/A	0.132	1.985	0.168	0.000	0.487	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	164	463	0	320	0	0	-1
N.S.	1	1.00	1.29	3.65	0.00	2.52	0.00	0.00	-0.01
time (sec)	N/A	0.129	1.625	0.177	0.000	0.448	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	158	464	0	320	0	878	-1
N.S.	1	1.00	1.26	3.71	0.00	2.56	0.00	7.02	-0.01
time (sec)	N/A	0.179	1.911	0.185	0.000	0.446	0.000	1.698	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	190	509	0	382	0	976	-1
N.S.	1	1.00	1.17	3.14	0.00	2.36	0.00	6.02	-0.01
time (sec)	N/A	0.272	2.030	0.191	0.000	0.471	0.000	6.102	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	186	549	0	435	0	1000	-1
N.S.	1	1.00	0.93	2.73	0.00	2.16	0.00	4.98	-0.00
time (sec)	N/A	0.375	1.257	0.180	0.000	0.437	0.000	7.802	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	270	1007	0	623	0	0	-1
N.S.	1	1.00	1.05	3.92	0.00	2.42	0.00	0.00	-0.00
time (sec)	N/A	0.557	3.968	0.188	0.000	0.516	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	241	963	0	621	0	0	-1
N.S.	1	1.00	1.11	4.42	0.00	2.85	0.00	0.00	-0.00
time (sec)	N/A	0.464	2.810	0.195	0.000	0.668	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	177	575	0	332	0	0	-1
N.S.	1	1.00	1.07	3.46	0.00	2.00	0.00	0.00	-0.01
time (sec)	N/A	0.188	2.219	0.169	0.000	0.578	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	171	576	0	331	0	0	-1
N.S.	1	1.00	1.04	3.51	0.00	2.02	0.00	0.00	-0.01
time (sec)	N/A	0.183	2.146	0.189	0.000	0.516	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	175	575	0	331	0	0	-1
N.S.	1	1.00	1.04	3.42	0.00	1.97	0.00	0.00	-0.01
time (sec)	N/A	0.285	1.909	0.194	0.000	0.677	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	171	576	0	330	0	1348	-1
N.S.	1	1.00	1.06	3.56	0.00	2.04	0.00	8.32	-0.01
time (sec)	N/A	0.285	2.266	0.187	0.000	0.610	0.000	4.829	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	203	620	0	393	0	1446	-1
N.S.	1	1.00	1.02	3.12	0.00	1.97	0.00	7.27	-0.01
time (sec)	N/A	0.369	1.349	0.173	0.000	0.740	0.000	18.884	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	199	661	0	445	0	1472	-1
N.S.	1	1.00	0.84	2.78	0.00	1.87	0.00	6.18	-0.00
time (sec)	N/A	0.482	1.566	0.198	0.000	0.772	0.000	17.822	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	181	211	0	645	0	256	655
N.S.	1	1.00	0.53	0.62	0.00	1.88	0.00	0.75	1.91
time (sec)	N/A	0.322	1.828	0.194	0.000	0.797	0.000	0.598	5.716

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	163	201	0	494	0	231	616
N.S.	1	1.00	0.51	0.63	0.00	1.55	0.00	0.72	1.93
time (sec)	N/A	0.328	1.266	0.160	0.000	0.690	0.000	0.708	5.438

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	162	190	0	492	0	215	622
N.S.	1	1.00	0.54	0.64	0.00	1.65	0.00	0.72	2.08
time (sec)	N/A	0.262	1.167	0.181	0.000	0.774	0.000	0.694	5.163

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	163	190	0	493	0	215	599
N.S.	1	1.00	0.54	0.63	0.00	1.63	0.00	0.71	1.98
time (sec)	N/A	0.322	1.080	0.201	0.000	0.718	0.000	0.607	5.291

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	161	189	0	486	0	215	587
N.S.	1	1.00	0.53	0.62	0.00	1.60	0.00	0.71	1.94
time (sec)	N/A	0.316	1.152	0.168	0.000	0.703	0.000	0.638	5.313

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	201	199	0	642	0	231	630
N.S.	1	1.00	0.63	0.62	0.00	2.00	0.00	0.72	1.96
time (sec)	N/A	0.283	1.718	0.150	0.000	0.765	0.000	1.229	5.364

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	236	210	0	787	0	241	611
N.S.	1	1.00	0.68	0.61	0.00	2.27	0.00	0.69	1.76
time (sec)	N/A	0.358	2.035	0.144	0.000	0.682	0.000	0.792	5.754

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	379	379	210	246	0	684	0	268	674
N.S.	1	1.00	0.55	0.65	0.00	1.80	0.00	0.71	1.78
time (sec)	N/A	0.456	3.593	0.167	0.000	0.634	0.000	0.796	5.459

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	192	235	0	521	0	244	668
N.S.	1	1.00	0.54	0.66	0.00	1.46	0.00	0.68	1.87
time (sec)	N/A	0.366	1.841	0.181	0.000	0.704	0.000	0.699	5.240

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	194	225	0	521	0	228	642
N.S.	1	1.00	0.58	0.67	0.00	1.55	0.00	0.68	1.91
time (sec)	N/A	0.402	1.758	0.203	0.000	0.719	0.000	0.938	5.255

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	190	225	0	521	0	228	653
N.S.	1	1.00	0.57	0.67	0.00	1.56	0.00	0.68	1.95
time (sec)	N/A	0.354	1.701	0.180	0.000	0.616	0.000	0.758	5.176

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	194	225	0	521	0	226	640
N.S.	1	1.00	0.58	0.67	0.00	1.55	0.00	0.67	1.90
time (sec)	N/A	0.394	1.592	0.171	0.000	0.701	0.000	0.727	5.060

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	189	223	0	515	0	228	630
N.S.	1	1.00	0.56	0.66	0.00	1.52	0.00	0.67	1.86
time (sec)	N/A	0.392	1.705	0.197	0.000	0.683	0.000	0.848	5.190

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	205	233	0	681	0	238	660
N.S.	1	1.00	0.57	0.65	0.00	1.90	0.00	0.66	1.84
time (sec)	N/A	0.368	3.985	0.214	0.000	0.626	0.000	2.054	5.257

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	281	244	0	836	0	254	652
N.S.	1	1.00	0.74	0.64	0.00	2.19	0.00	0.67	1.71
time (sec)	N/A	0.450	2.106	0.179	0.000	0.808	0.000	1.179	5.462

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	5.550	1.087	0.000	0.000	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	16.660	1.067	0.000	0.000	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	1.134	1.063	0.000	0.000	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	1.110	1.005	0.000	0.000	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	1.081	1.064	0.000	0.000	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	12.140	1.049	0.000	0.000	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	8.065	1.014	0.000	0.000	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	13.988	1.028	0.000	0.000	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	7.414	0.995	0.000	0.000	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	16.900	1.035	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	3.797	1.094	0.000	0.000	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	14.549	1.014	0.000	0.000	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	6.039	1.074	0.000	0.000	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	2.865	1.033	0.000	0.000	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	5.535	1.081	0.000	0.000	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	3.796	1.009	0.000	0.000	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	6.349	1.181	0.000	0.000	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	11.636	1.084	0.000	0.000	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	6.761	1.047	0.000	0.000	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	12.186	1.026	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	6.408	1.040	0.000	0.000	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	11.458	0.990	0.000	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-2)	F(-1)	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	7.300	1.083	0.000	0.000	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	12.335	1.069	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	A	A	B	F	F	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	0	190	190	393	0	0	235
N.S.	1	1.00	0.00	0.81	0.81	1.68	0.00	0.00	1.00
time (sec)	N/A	0.208	180.002	0.174	0.534	0.633	0.000	0.000	0.626

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	A	A	B	F	F	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	0	156	153	294	0	0	195
N.S.	1	1.00	0.00	0.84	0.83	1.59	0.00	0.00	1.05
time (sec)	N/A	0.095	180.003	0.115	0.508	0.674	0.000	0.000	4.070

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	A	A	A	F	F	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	0	150	153	239	0	0	176
N.S.	1	1.00	0.00	0.86	0.88	1.37	0.00	0.00	1.01
time (sec)	N/A	0.090	180.005	0.096	0.521	0.551	0.000	0.000	4.276

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	223	133	135	188	0	0	184
N.S.	1	1.00	1.43	0.85	0.87	1.21	0.00	0.00	1.18
time (sec)	N/A	0.057	1.019	0.078	0.523	0.513	0.000	0.000	3.968

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	A	B	F	F	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	0	0	233	387	0	0	328
N.S.	1	1.00	0.00	0.00	0.90	1.49	0.00	0.00	1.26
time (sec)	N/A	0.210	0.706	0.421	0.530	0.616	0.000	0.000	4.305

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	A	B	F	F	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	0	0	261	555	0	0	806
N.S.	1	1.00	0.00	0.00	0.87	1.86	0.00	0.00	2.70
time (sec)	N/A	0.293	180.005	0.354	0.532	0.571	0.000	0.000	4.200

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	A	B	F	F(-2)	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	0	0	306	678	0	0	417
N.S.	1	1.00	0.00	0.00	0.94	2.07	0.00	0.00	1.28
time (sec)	N/A	0.395	180.004	0.398	0.533	0.603	0.000	0.000	4.408

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	81	133	136	226	0	0	171
N.S.	1	1.00	0.52	0.85	0.87	1.45	0.00	0.00	1.10
time (sec)	N/A	0.059	0.410	0.084	0.536	0.528	0.000	0.000	4.558

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	A	A	B	F	F	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	0	210	208	501	0	0	243
N.S.	1	1.00	0.00	0.84	0.83	2.00	0.00	0.00	0.97
time (sec)	N/A	0.232	180.004	0.102	0.535	0.504	0.000	0.000	4.385

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	A	A	B	F	F	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	0	175	172	409	0	0	218
N.S.	1	1.00	0.00	0.86	0.85	2.01	0.00	0.00	1.07
time (sec)	N/A	0.123	180.004	0.096	0.529	0.570	0.000	0.000	4.065

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	A	A	B	F	F	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	0	169	172	350	0	0	198
N.S.	1	1.00	0.00	0.88	0.90	1.82	0.00	0.00	1.03
time (sec)	N/A	0.116	180.003	0.103	0.527	0.604	0.000	0.000	4.318

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	294	151	153	262	0	0	195
N.S.	1	1.00	1.68	0.86	0.87	1.50	0.00	0.00	1.11
time (sec)	N/A	0.077	1.256	0.097	0.517	0.795	0.000	0.000	3.987

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	411	0	233	446	0	0	369
N.S.	1	1.00	1.62	0.00	0.92	1.76	0.00	0.00	1.45
time (sec)	N/A	0.309	4.346	0.417	0.548	1.161	0.000	0.000	4.303

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	587	0	258	622	0	0	855
N.S.	1	1.00	1.86	0.00	0.82	1.97	0.00	0.00	2.71
time (sec)	N/A	0.392	7.756	0.361	0.554	1.459	0.000	0.000	0.528

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	A	B	F(-1)	F	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	0	0	304	744	0	0	460
N.S.	1	1.00	0.00	0.00	0.95	2.32	0.00	0.00	1.43
time (sec)	N/A	0.386	180.003	0.392	0.548	1.203	0.000	0.000	4.406

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	82	153	154	280	0	0	208
N.S.	1	1.00	0.46	0.86	0.87	1.58	0.00	0.00	1.18
time (sec)	N/A	0.087	0.727	0.097	0.525	0.967	0.000	0.000	4.544

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	180.002	0.405	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	5.284	0.761	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	125	211	208	479	0	0	266
N.S.	1	1.00	0.44	0.75	0.74	1.70	0.00	0.00	0.94
time (sec)	N/A	0.322	1.421	0.117	0.535	1.158	0.000	0.000	4.625

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	115	192	190	418	0	0	228
N.S.	1	1.00	0.49	0.81	0.80	1.76	0.00	0.00	0.96
time (sec)	N/A	0.202	0.898	0.099	0.541	1.493	0.000	0.000	0.362

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	82	172	172	315	0	0	210
N.S.	1	1.00	0.38	0.81	0.81	1.48	0.00	0.00	0.99
time (sec)	N/A	0.118	0.710	0.094	0.517	1.321	0.000	0.000	4.477

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	140	146	154	327	0	0	172
N.S.	1	1.00	0.79	0.82	0.87	1.84	0.00	0.00	0.97
time (sec)	N/A	0.089	0.640	0.117	0.531	1.167	0.000	0.000	0.241

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	141	158	152	315	0	0	197
N.S.	1	1.00	0.77	0.86	0.83	1.71	0.00	0.00	1.07
time (sec)	N/A	0.077	0.451	0.082	0.538	0.943	0.000	0.000	4.391

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	195	0	249	580	0	0	559
N.S.	1	1.00	0.68	0.00	0.87	2.03	0.00	0.00	1.95
time (sec)	N/A	0.303	1.289	0.405	0.543	0.959	0.000	0.000	0.248

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	179	0	286	717	0	0	887
N.S.	1	1.00	0.55	0.00	0.87	2.19	0.00	0.00	2.71
time (sec)	N/A	0.393	1.507	0.382	0.528	0.961	0.000	0.000	4.569

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	290	158	151	305	0	0	190
N.S.	1	1.00	1.58	0.86	0.82	1.66	0.00	0.00	1.03
time (sec)	N/A	0.080	1.102	0.087	0.561	1.330	0.000	0.000	0.215

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	34.660	0.360	0.000	0.000	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	14.648	0.535	0.000	0.000	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	145	212	209	443	0	0	263
N.S.	1	1.00	0.51	0.75	0.74	1.57	0.00	0.00	0.93
time (sec)	N/A	0.316	1.637	0.126	0.530	1.341	0.000	0.000	4.536

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	130	190	191	366	0	0	214
N.S.	1	1.00	0.55	0.80	0.81	1.54	0.00	0.00	0.90
time (sec)	N/A	0.231	1.024	0.117	0.522	1.219	0.000	0.000	0.186

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	128	168	173	346	0	0	217
N.S.	1	1.00	0.60	0.79	0.81	1.62	0.00	0.00	1.02
time (sec)	N/A	0.150	0.779	0.109	0.599	1.011	0.000	0.000	0.682

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	130	171	171	360	0	0	193
N.S.	1	1.00	0.63	0.83	0.83	1.76	0.00	0.00	0.94
time (sec)	N/A	0.114	0.708	0.105	0.508	1.399	0.000	0.000	3.971

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	128	177	164	346	0	0	217
N.S.	1	1.00	0.60	0.83	0.77	1.62	0.00	0.00	1.02
time (sec)	N/A	0.099	0.464	0.095	0.524	1.575	0.000	0.000	4.423

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	189	0	265	633	0	0	822
N.S.	1	1.00	0.60	0.00	0.85	2.02	0.00	0.00	2.63
time (sec)	N/A	0.420	1.690	0.457	0.596	1.466	0.000	0.000	3.977

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	354	233	0	310	792	0	0	893
N.S.	1	1.00	0.66	0.00	0.88	2.24	0.00	0.00	2.52
time (sec)	N/A	0.508	2.067	0.457	0.531	1.587	0.000	0.000	5.993

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	331	177	164	336	0	0	233
N.S.	1	1.00	1.55	0.83	0.77	1.58	0.00	0.00	1.09
time (sec)	N/A	0.105	1.544	0.105	0.511	0.972	0.000	0.000	3.989

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	159	0	0	0	0	0	-1
N.S.	1	1.00	3.70	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	0.866	0.405	0.000	0.000	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	44	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.074	0.404	0.000	0.000	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	1065	0	0	0	0	0	-1
N.S.	1	1.00	5.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.367	8.947	1.125	0.000	0.000	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	900	0	0	0	0	0	-1
N.S.	1	1.00	7.09	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.185	8.286	1.026	0.000	0.000	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	2.597	0.504	0.000	0.000	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	159	0	0	0	0	0	-1
N.S.	1	1.00	3.70	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.812	0.380	0.000	0.000	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.134	15.866	1.137	0.000	0.000	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.259	5.878	1.233	0.000	0.000	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.459	16.623	1.217	0.000	0.000	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.662	22.753	1.428	0.000	0.000	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	44	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.081	0.372	0.000	0.000	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	123	0	0	0	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.767	0.625	0.000	0.000	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	2.245	0.951	0.000	0.000	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	1.157	1.310	0.000	0.000	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	180.002	1.322	0.000	0.000	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	35.579	1.240	0.000	0.000	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	5.479	0.780	0.000	0.000	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.251	43.489	0.896	0.000	0.000	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.139	33.168	1.089	0.000	0.000	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	16.301	0.992	0.000	0.000	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	153	0	0	0	0	0	-1
N.S.	1	1.00	2.19	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	4.965	0.779	0.000	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	128	0	0	0	0	0	-1
N.S.	1	1.00	2.61	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.498	0.802	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	10.666	1.577	0.000	0.000	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.174	18.727	1.738	0.000	0.000	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	11.357	1.653	0.000	0.000	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	5.209	1.726	0.000	0.000	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	5.257	1.701	0.000	0.000	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	3.213	1.640	0.000	0.000	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	117	318	141	241	0	314	144
N.S.	1	1.00	1.02	2.77	1.23	2.10	0.00	2.73	1.25
time (sec)	N/A	0.109	0.848	0.492	0.499	1.485	0.000	0.732	5.466

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	105	303	122	196	0	290	98
N.S.	1	1.00	1.13	3.26	1.31	2.11	0.00	3.12	1.05
time (sec)	N/A	0.076	0.465	0.276	0.520	1.473	0.000	0.638	4.836

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	92	288	106	167	0	249	125
N.S.	1	1.00	1.28	4.00	1.47	2.32	0.00	3.46	1.74
time (sec)	N/A	0.050	0.110	0.234	0.496	1.539	0.000	0.603	4.464

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	74	272	80	133	0	232	65
N.S.	1	1.00	1.48	5.44	1.60	2.66	0.00	4.64	1.30
time (sec)	N/A	0.030	0.092	0.250	0.513	1.132	0.000	0.616	4.302

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	64	293	102	205	0	253	84
N.S.	1	1.00	0.86	3.96	1.38	2.77	0.00	3.42	1.14
time (sec)	N/A	0.060	0.144	0.202	0.502	1.362	0.000	0.662	4.577

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	68	308	121	236	0	275	103
N.S.	1	1.00	0.69	3.14	1.23	2.41	0.00	2.81	1.05
time (sec)	N/A	0.093	0.161	0.200	0.516	0.970	0.000	0.735	5.190

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	68	323	143	261	0	295	120
N.S.	1	1.00	0.56	2.67	1.18	2.16	0.00	2.44	0.99
time (sec)	N/A	0.125	0.196	0.201	0.526	0.876	0.000	0.775	5.805

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	187	187	219	775	0	293	125
N.S.	1	1.00	0.70	0.70	0.81	2.88	0.00	1.09	0.46
time (sec)	N/A	0.199	1.290	0.253	0.509	1.489	0.000	0.788	5.305

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	52	172	203	774	0	274	104
N.S.	1	1.00	0.21	0.70	0.83	3.15	0.00	1.11	0.42
time (sec)	N/A	0.161	0.397	0.259	0.524	1.178	0.000	0.684	4.641

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	175	170	200	698	0	249	104
N.S.	1	1.00	0.72	0.70	0.82	2.86	0.00	1.02	0.43
time (sec)	N/A	0.158	0.462	0.249	0.523	1.039	0.000	0.758	4.380

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	53	155	183	671	0	222	86
N.S.	1	1.00	0.24	0.70	0.82	3.02	0.00	1.00	0.39
time (sec)	N/A	0.140	0.264	0.220	0.515	1.150	0.000	0.662	4.065

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	232	159	182	790	0	225	86
N.S.	1	1.00	1.05	0.72	0.82	3.56	0.00	1.01	0.39
time (sec)	N/A	0.143	1.874	0.190	0.520	1.139	0.000	0.725	4.252

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	229	174	205	823	0	249	100
N.S.	1	1.00	0.93	0.70	0.83	3.33	0.00	1.01	0.40
time (sec)	N/A	0.168	1.395	0.192	0.509	0.944	0.000	0.717	4.347

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	375	369	209	358	0	446	185
N.S.	1	1.00	1.79	1.76	1.00	1.70	0.00	2.12	0.88
time (sec)	N/A	0.226	4.541	0.439	0.516	1.020	0.000	1.019	7.039

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	729	354	187	321	0	405	176
N.S.	1	1.00	3.92	1.90	1.01	1.73	0.00	2.18	0.95
time (sec)	N/A	0.195	6.105	0.321	0.518	1.117	0.000	0.992	5.959

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	332	339	171	273	0	379	143
N.S.	1	1.00	2.08	2.12	1.07	1.71	0.00	2.37	0.89
time (sec)	N/A	0.168	2.778	0.365	0.521	1.348	0.000	0.861	5.149

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	315	323	149	231	0	344	137
N.S.	1	1.00	2.28	2.34	1.08	1.67	0.00	2.49	0.99
time (sec)	N/A	0.133	1.842	0.331	0.506	1.091	0.000	0.728	4.618

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	553	309	130	210	0	313	100
N.S.	1	1.00	4.73	2.64	1.11	1.79	0.00	2.68	0.85
time (sec)	N/A	0.110	6.088	0.266	0.499	0.869	0.000	0.713	4.259

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	314	305	122	239	0	299	118
N.S.	1	1.00	2.75	2.68	1.07	2.10	0.00	2.62	1.04
time (sec)	N/A	0.111	1.757	0.207	0.499	0.891	0.000	0.730	4.196

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	272	303	129	244	0	299	102
N.S.	1	1.00	2.32	2.59	1.10	2.09	0.00	2.56	0.87
time (sec)	N/A	0.122	4.167	0.207	0.511	1.033	0.000	0.868	4.318

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	271	323	147	275	0	322	128
N.S.	1	1.00	1.92	2.29	1.04	1.95	0.00	2.28	0.91
time (sec)	N/A	0.154	2.261	0.211	0.532	0.915	0.000	0.850	5.075

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	175	338	168	304	0	340	130
N.S.	1	1.00	1.06	2.05	1.02	1.84	0.00	2.06	0.79
time (sec)	N/A	0.188	0.898	0.205	0.528	1.012	0.000	0.966	5.752

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	110	312	134	261	0	284	124
N.S.	1	1.00	0.99	2.81	1.21	2.35	0.00	2.56	1.12
time (sec)	N/A	0.272	0.955	0.312	0.516	0.830	0.000	0.615	4.446

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	107	298	117	223	0	263	78
N.S.	1	1.00	1.23	3.43	1.34	2.56	0.00	3.02	0.90
time (sec)	N/A	0.147	0.349	0.286	0.528	1.051	0.000	0.571	4.225

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	98	304	115	223	0	253	103
N.S.	1	1.00	1.10	3.42	1.29	2.51	0.00	2.84	1.16
time (sec)	N/A	0.138	0.189	0.168	0.515	1.078	0.000	0.578	4.246

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	107	304	115	221	0	260	78
N.S.	1	1.00	1.32	3.75	1.42	2.73	0.00	3.21	0.96
time (sec)	N/A	0.135	0.490	0.187	0.508	1.098	0.000	0.578	4.342

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	122	319	128	291	0	284	124
N.S.	1	1.00	1.10	2.87	1.15	2.62	0.00	2.56	1.12
time (sec)	N/A	0.265	1.782	0.168	0.527	0.913	0.000	0.651	4.436

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	130	333	151	326	0	305	130
N.S.	1	1.00	0.96	2.47	1.12	2.41	0.00	2.26	0.96
time (sec)	N/A	0.329	1.470	0.165	0.515	0.978	0.000	0.754	4.821

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	226	191	226	1840	0	254	376
N.S.	1	1.00	0.80	0.68	0.80	6.55	0.00	0.90	1.34
time (sec)	N/A	0.340	3.551	0.162	0.533	1.503	0.000	0.782	4.689

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	229	197	229	1855	0	264	375
N.S.	1	1.00	0.82	0.71	0.82	6.65	0.00	0.95	1.34
time (sec)	N/A	0.353	2.169	0.144	0.508	1.639	0.000	0.718	4.607

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	192	200	227	1763	0	260	367
N.S.	1	1.00	0.69	0.72	0.82	6.34	0.00	0.94	1.32
time (sec)	N/A	0.324	1.421	0.178	0.505	1.356	0.000	0.651	4.578

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	337	197	226	1808	0	261	365
N.S.	1	1.00	1.20	0.70	0.80	6.43	0.00	0.93	1.30
time (sec)	N/A	0.349	0.739	0.195	0.532	2.629	0.000	0.683	4.646

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	203	212	240	2384	0	291	415
N.S.	1	1.00	0.66	0.69	0.78	7.79	0.00	0.95	1.36
time (sec)	N/A	0.482	1.383	0.168	0.517	2.182	0.000	0.752	4.715

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	203	227	265	2398	0	309	424
N.S.	1	1.00	0.61	0.69	0.80	7.24	0.00	0.93	1.28
time (sec)	N/A	0.657	4.169	0.173	0.517	2.011	0.000	0.798	4.983

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	346	355	211	502	0	345	176
N.S.	1	1.00	1.83	1.88	1.12	2.66	0.00	1.83	0.93
time (sec)	N/A	0.492	6.256	0.191	0.533	1.223	0.000	0.881	4.948

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	183	338	191	475	0	326	178
N.S.	1	1.00	1.11	2.05	1.16	2.88	0.00	1.98	1.08
time (sec)	N/A	0.388	3.057	0.182	0.551	1.141	0.000	0.850	4.856

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	192	349	193	470	0	326	153
N.S.	1	1.00	1.17	2.13	1.18	2.87	0.00	1.99	0.93
time (sec)	N/A	0.363	3.248	0.185	0.543	1.086	0.000	0.789	4.796

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	127	349	191	440	0	323	177
N.S.	1	1.00	0.77	2.13	1.16	2.68	0.00	1.97	1.08
time (sec)	N/A	0.387	2.497	0.180	0.537	0.881	0.000	0.769	4.768

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	253	349	193	420	0	314	152
N.S.	1	1.00	1.57	2.17	1.20	2.61	0.00	1.95	0.94
time (sec)	N/A	0.349	0.869	0.208	0.511	0.851	0.000	0.729	4.677

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	217	349	187	407	0	318	172
N.S.	1	1.00	1.32	2.12	1.13	2.47	0.00	1.93	1.04
time (sec)	N/A	0.373	1.084	0.237	0.531	0.932	0.000	0.745	4.776

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	173	364	200	493	0	341	176
N.S.	1	1.00	0.92	1.93	1.06	2.61	0.00	1.80	0.93
time (sec)	N/A	0.496	1.482	0.190	0.529	0.958	0.000	0.855	4.822

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	368	379	219	524	0	366	192
N.S.	1	1.00	1.71	1.76	1.02	2.44	0.00	1.70	0.89
time (sec)	N/A	0.651	6.297	0.199	0.514	0.852	0.000	0.995	5.016

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	150	242	0	1099	0	278	135
N.S.	1	1.00	0.57	0.92	0.00	4.16	0.00	1.05	0.51
time (sec)	N/A	0.384	1.858	0.423	0.000	0.995	0.000	0.684	6.026

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	100	230	0	1074	0	262	120
N.S.	1	1.00	0.48	1.11	0.00	5.16	0.00	1.26	0.58
time (sec)	N/A	0.182	0.359	0.155	0.000	1.236	0.000	0.672	4.574

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	78	206	0	981	0	224	90
N.S.	1	1.00	0.47	1.24	0.00	5.91	0.00	1.35	0.54
time (sec)	N/A	0.114	0.073	0.101	0.000	1.011	0.000	0.645	4.100

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	78	2750	0	1016	0	0	90
N.S.	1	1.00	0.47	16.67	0.00	6.16	0.00	0.00	0.55
time (sec)	N/A	0.181	0.093	18.539	0.000	0.974	0.000	0.000	0.164

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	124	11217	0	1206	0	0	149
N.S.	1	1.00	0.56	50.76	0.00	5.46	0.00	0.00	0.67
time (sec)	N/A	0.344	0.493	0.903	0.000	0.781	0.000	0.000	0.189

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	169	16923	0	1336	0	0	198
N.S.	1	1.00	0.62	61.99	0.00	4.89	0.00	0.00	0.73
time (sec)	N/A	0.470	0.893	0.890	0.000	1.305	0.000	0.000	0.198

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	118	241	0	950	0	246	107
N.S.	1	1.00	0.37	0.76	0.00	2.99	0.00	0.77	0.34
time (sec)	N/A	0.307	0.748	0.156	0.000	1.166	0.000	0.717	5.109

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	86	229	0	911	0	224	88
N.S.	1	1.00	0.32	0.86	0.00	3.42	0.00	0.84	0.33
time (sec)	N/A	0.156	0.176	0.120	0.000	1.093	0.000	0.715	4.137

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	67	217	0	805	0	266	73
N.S.	1	1.00	0.27	0.88	0.00	3.26	0.00	1.08	0.30
time (sec)	N/A	0.135	0.031	0.231	0.000	1.521	0.000	0.792	0.184

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	102	8262	0	1058	0	330	121
N.S.	1	1.00	0.35	28.69	0.00	3.67	0.00	1.15	0.42
time (sec)	N/A	0.249	0.264	0.798	0.000	1.153	0.000	1.096	3.893

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	151	13941	0	1202	0	359	175
N.S.	1	1.00	0.44	40.29	0.00	3.47	0.00	1.04	0.51
time (sec)	N/A	0.429	0.441	0.867	0.000	1.177	0.000	1.199	0.191

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	188	362	0	971	0	286	144
N.S.	1	1.00	0.51	0.98	0.00	2.63	0.00	0.78	0.39
time (sec)	N/A	0.385	0.895	0.130	0.000	1.220	0.000	0.836	6.895

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	112	350	0	949	0	268	129
N.S.	1	1.00	0.36	1.11	0.00	3.01	0.00	0.85	0.41
time (sec)	N/A	0.242	0.640	0.150	0.000	1.451	0.000	0.802	5.244

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	90	326	0	912	0	237	99
N.S.	1	1.00	0.33	1.20	0.00	3.37	0.00	0.87	0.37
time (sec)	N/A	0.159	0.232	0.083	0.000	1.349	0.000	0.786	4.375

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	78	2377	0	878	0	0	85
N.S.	1	1.00	0.31	9.40	0.00	3.47	0.00	0.00	0.34
time (sec)	N/A	0.191	0.192	0.720	0.000	1.152	0.000	0.000	0.153

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	4055	9665	0	1071	0	0	142
N.S.	1	1.00	13.21	31.48	0.00	3.49	0.00	0.00	0.46
time (sec)	N/A	0.313	27.710	0.723	0.000	1.128	0.000	0.000	3.975

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	4084	14663	0	1200	0	0	193
N.S.	1	1.00	11.31	40.62	0.00	3.32	0.00	0.00	0.53
time (sec)	N/A	0.463	21.984	0.770	0.000	1.258	0.000	0.000	3.981

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	155	245	0	1105	0	252	118
N.S.	1	1.00	0.68	1.08	0.00	4.87	0.00	1.11	0.52
time (sec)	N/A	0.294	1.514	0.161	0.000	1.037	0.000	0.836	5.995

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	100	233	0	1080	0	237	103
N.S.	1	1.00	0.58	1.35	0.00	6.24	0.00	1.37	0.60
time (sec)	N/A	0.132	0.402	0.114	0.000	1.276	0.000	0.772	0.884

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	79	221	0	987	0	214	82
N.S.	1	1.00	0.51	1.42	0.00	6.33	0.00	1.37	0.53
time (sec)	N/A	0.105	0.073	0.091	0.000	1.360	0.000	0.750	4.147

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	245	6656	0	1199	0	346	119
N.S.	1	1.00	1.38	37.39	0.00	6.74	0.00	1.94	0.67
time (sec)	N/A	0.213	17.772	0.676	0.000	1.035	0.000	1.417	0.178

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	321	11278	0	1343	0	375	173
N.S.	1	1.00	1.35	47.39	0.00	5.64	0.00	1.58	0.73
time (sec)	N/A	0.356	19.142	0.737	0.000	1.334	0.000	1.562	0.191

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	110	245	0	1064	0	254	118
N.S.	1	1.00	0.46	1.02	0.00	4.41	0.00	1.05	0.49
time (sec)	N/A	0.271	2.614	0.128	0.000	1.327	0.000	0.873	4.416

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	90	233	0	1028	0	237	103
N.S.	1	1.00	0.48	1.25	0.00	5.50	0.00	1.27	0.55
time (sec)	N/A	0.163	0.220	0.150	0.000	1.256	0.000	0.684	4.134

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	67	209	0	920	0	282	69
N.S.	1	1.00	0.47	1.46	0.00	6.43	0.00	1.97	0.48
time (sec)	N/A	0.090	0.035	0.132	0.000	1.148	0.000	0.776	0.259

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	83	2043	0	990	0	0	85
N.S.	1	1.00	0.52	12.69	0.00	6.15	0.00	0.00	0.53
time (sec)	N/A	0.152	0.097	1.020	0.000	1.423	0.000	0.000	0.157

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	125	8963	0	1184	0	0	147
N.S.	1	1.00	0.58	41.69	0.00	5.51	0.00	0.00	0.68
time (sec)	N/A	0.284	0.344	0.776	0.000	1.174	0.000	0.000	4.171

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	440	13527	0	1309	0	0	197
N.S.	1	1.00	1.64	50.29	0.00	4.87	0.00	0.00	0.73
time (sec)	N/A	0.428	18.043	0.856	0.000	1.188	0.000	0.000	4.219

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	100	327	0	898	0	238	101
N.S.	1	1.00	0.32	1.05	0.00	2.89	0.00	0.77	0.32
time (sec)	N/A	0.239	1.729	0.132	0.000	1.261	0.000	0.724	0.553

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	80	315	0	805	0	216	86
N.S.	1	1.00	0.31	1.23	0.00	3.13	0.00	0.84	0.33
time (sec)	N/A	0.141	0.112	0.121	0.000	1.044	0.000	0.678	0.281

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	66	303	0	766	0	282	71
N.S.	1	1.00	0.28	1.26	0.00	3.19	0.00	1.18	0.30
time (sec)	N/A	0.119	0.026	0.136	0.000	0.916	0.000	0.746	4.207

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	101	7175	0	1016	0	346	117
N.S.	1	1.00	0.36	25.62	0.00	3.63	0.00	1.24	0.42
time (sec)	N/A	0.269	0.259	0.650	0.000	1.003	0.000	0.864	0.173

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	4071	12089	0	1156	0	375	170
N.S.	1	1.00	12.01	35.66	0.00	3.41	0.00	1.11	0.50
time (sec)	N/A	0.391	21.950	1.221	0.000	1.064	0.000	0.913	4.085

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.820	0.398	0.000	0.000	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	95	82	81	80	97	947	76
N.S.	1	1.00	1.02	0.88	0.87	0.86	1.04	10.18	0.82
time (sec)	N/A	0.077	0.372	0.069	0.537	1.641	0.152	2.636	4.104

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	79	71	70	69	83	716	65
N.S.	1	1.00	1.03	0.92	0.91	0.90	1.08	9.30	0.84
time (sec)	N/A	0.064	0.217	0.028	0.505	1.243	0.115	1.477	4.008

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	67	60	59	58	70	515	54
N.S.	1	1.00	1.12	1.00	0.98	0.97	1.17	8.58	0.90
time (sec)	N/A	0.048	0.147	0.033	0.510	1.901	0.103	1.088	4.043

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	51	49	47	47	56	327	43
N.S.	1	1.00	1.16	1.11	1.07	1.07	1.27	7.43	0.98
time (sec)	N/A	0.029	0.113	0.022	0.512	1.335	0.076	0.716	4.051

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	38	38	37	35	41	174	32
N.S.	1	1.00	1.31	1.31	1.28	1.21	1.41	6.00	1.10
time (sec)	N/A	0.013	0.026	0.025	0.523	1.513	0.068	0.721	4.058

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	22	16	27	24	18	21
N.S.	1	1.00	1.00	1.29	0.94	1.59	1.41	1.06	1.24
time (sec)	N/A	0.006	0.007	0.017	0.282	1.126	0.043	0.460	3.978

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	24	22	38	35	42	42	79
N.S.	1	1.00	1.50	1.38	2.38	2.19	2.62	2.62	4.94
time (sec)	N/A	0.016	0.029	0.161	0.547	1.009	0.143	0.525	4.152

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	51	33	48	59	70	83	70
N.S.	1	1.00	1.76	1.14	1.66	2.03	2.41	2.86	2.41
time (sec)	N/A	0.032	0.132	0.117	0.523	1.208	0.346	0.583	4.102

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	66	46	58	72	83	113	83
N.S.	1	1.00	1.43	1.00	1.26	1.57	1.80	2.46	1.80
time (sec)	N/A	0.050	0.291	0.158	0.521	0.981	0.496	0.631	4.074

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	70	51	71	89	97	140	96
N.S.	1	1.00	1.17	0.85	1.18	1.48	1.62	2.33	1.60
time (sec)	N/A	0.065	0.301	0.157	0.511	0.773	0.802	0.618	3.982

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	82	59	82	100	110	169	107
N.S.	1	1.00	1.09	0.79	1.09	1.33	1.47	2.25	1.43
time (sec)	N/A	0.082	0.406	0.149	0.524	0.648	1.033	0.767	4.109

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	82	74	93	111	124	198	116
N.S.	1	1.00	0.88	0.80	1.00	1.19	1.33	2.13	1.25
time (sec)	N/A	0.099	0.376	0.154	0.533	0.921	1.730	0.793	4.142

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	110	121	110	109	139	1315	146
N.S.	1	1.00	0.92	1.01	0.92	0.91	1.16	10.96	1.22
time (sec)	N/A	0.120	0.644	0.038	0.510	0.999	0.184	2.303	4.092

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	113	100	91	90	134	1262	90
N.S.	1	1.00	1.15	1.02	0.93	0.92	1.37	12.88	0.92
time (sec)	N/A	0.096	0.455	0.042	0.509	0.792	0.151	1.723	4.074

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	99	83	78	77	94	675	108
N.S.	1	1.00	1.57	1.32	1.24	1.22	1.49	10.71	1.71
time (sec)	N/A	0.039	0.319	0.029	0.510	0.881	0.111	0.971	4.066

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	74	62	58	58	85	554	57
N.S.	1	1.00	1.28	1.07	1.00	1.00	1.47	9.55	0.98
time (sec)	N/A	0.032	0.294	0.031	0.523	0.755	0.098	0.874	4.018

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	69	47	41	44	48	201	136
N.S.	1	1.00	1.77	1.21	1.05	1.13	1.23	5.15	3.49
time (sec)	N/A	0.015	0.104	0.020	0.549	1.047	0.071	0.555	4.094

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	43	38	49	56	70	50	61
N.S.	1	1.00	1.23	1.09	1.40	1.60	2.00	1.43	1.74
time (sec)	N/A	0.027	0.060	0.176	0.512	0.928	0.213	0.675	4.187

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	82	46	58	67	85	98	79
N.S.	1	1.00	2.00	1.12	1.41	1.63	2.07	2.39	1.93
time (sec)	N/A	0.048	0.162	0.136	0.517	1.171	0.462	0.747	4.072

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	92	61	78	86	131	154	97
N.S.	1	1.00	1.59	1.05	1.34	1.48	2.26	2.66	1.67
time (sec)	N/A	0.072	0.304	0.197	0.513	1.195	0.679	0.842	4.012

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	103	75	91	107	126	191	112
N.S.	1	1.00	1.32	0.96	1.17	1.37	1.62	2.45	1.44
time (sec)	N/A	0.101	0.736	0.160	0.519	1.012	1.005	0.898	4.016

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	122	87	111	128	178	248	129
N.S.	1	1.00	1.24	0.89	1.13	1.31	1.82	2.53	1.32
time (sec)	N/A	0.126	0.566	0.172	0.527	1.141	1.430	1.060	3.993

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	121	103	124	141	172	287	145
N.S.	1	1.00	1.01	0.86	1.03	1.18	1.43	2.39	1.21
time (sec)	N/A	0.142	1.010	0.191	0.540	1.094	2.307	1.153	3.966

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	161	153	137	136	194	1997	182
N.S.	1	1.00	1.10	1.04	0.93	0.93	1.32	13.59	1.24
time (sec)	N/A	0.136	0.814	0.052	0.523	1.083	0.199	2.998	3.878

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	97	124	114	113	160	1485	154
N.S.	1	1.00	1.03	1.32	1.21	1.20	1.70	15.80	1.64
time (sec)	N/A	0.065	1.053	0.036	0.516	1.239	0.150	1.703	3.845

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	100	97	95	94	128	993	135
N.S.	1	1.00	1.03	1.00	0.98	0.97	1.32	10.24	1.39
time (sec)	N/A	0.062	0.576	0.038	0.517	1.414	0.113	1.323	3.826

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	79	74	78	71	94	603	106
N.S.	1	1.00	1.10	1.03	1.08	0.99	1.31	8.38	1.47
time (sec)	N/A	0.039	0.248	0.026	0.524	1.659	0.090	0.804	3.824

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	79	59	71	78	92	72	75
N.S.	1	1.00	1.27	0.95	1.15	1.26	1.48	1.16	1.21
time (sec)	N/A	0.058	0.212	0.177	0.533	2.293	0.325	1.013	3.863

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	80	62	74	97	114	88	78
N.S.	1	1.00	1.16	0.90	1.07	1.41	1.65	1.28	1.13
time (sec)	N/A	0.066	0.154	0.175	0.510	1.043	0.657	1.140	3.913

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	96	74	92	99	146	171	102
N.S.	1	1.00	1.16	0.89	1.11	1.19	1.76	2.06	1.23
time (sec)	N/A	0.101	0.330	0.211	0.532	1.353	0.953	1.254	4.065

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	120	90	117	126	177	236	124
N.S.	1	1.00	1.15	0.87	1.12	1.21	1.70	2.27	1.19
time (sec)	N/A	0.135	1.153	0.167	0.518	1.062	1.379	1.503	3.961

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	118	111	135	152	207	301	145
N.S.	1	1.00	0.91	0.85	1.04	1.17	1.59	2.32	1.12
time (sec)	N/A	0.164	1.649	0.190	0.522	0.960	2.326	1.850	3.929

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	152	131	158	173	241	370	166
N.S.	1	1.00	0.97	0.83	1.01	1.10	1.54	2.36	1.06
time (sec)	N/A	0.198	0.996	0.200	0.524	1.094	3.051	2.812	4.103

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	190	209	171	170	277	3385	225
N.S.	1	1.00	1.05	1.15	0.94	0.94	1.53	18.70	1.24
time (sec)	N/A	0.176	1.215	0.060	0.517	0.867	0.217	5.195	3.974

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	122	176	149	148	214	2281	221
N.S.	1	1.00	0.95	1.38	1.16	1.16	1.67	17.82	1.73
time (sec)	N/A	0.106	0.766	0.045	0.518	0.992	0.171	2.830	3.979

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	123	135	124	123	187	1886	168
N.S.	1	1.00	0.95	1.04	0.95	0.95	1.44	14.51	1.29
time (sec)	N/A	0.101	1.699	0.044	0.524	1.307	0.159	2.122	3.921

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	105	107	113	100	131	1071	167
N.S.	1	1.00	1.02	1.04	1.10	0.97	1.27	10.40	1.62
time (sec)	N/A	0.077	0.380	0.029	0.523	0.727	0.119	0.937	3.890

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	94	85	89	101	133	90	92
N.S.	1	1.00	1.02	0.92	0.97	1.10	1.45	0.98	1.00
time (sec)	N/A	0.130	0.415	0.202	0.520	1.009	0.525	1.426	3.974

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	94	83	88	114	133	102	94
N.S.	1	1.00	0.97	0.86	0.91	1.18	1.37	1.05	0.97
time (sec)	N/A	0.130	0.280	0.170	0.532	1.409	0.972	1.766	3.956

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	90	90	99	126	170	132	102
N.S.	1	1.00	0.91	0.91	1.00	1.27	1.72	1.33	1.03
time (sec)	N/A	0.143	0.318	0.210	0.513	1.596	1.363	1.855	4.008

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	125	103	122	131	187	246	127
N.S.	1	1.00	1.07	0.88	1.04	1.12	1.60	2.10	1.09
time (sec)	N/A	0.184	1.280	0.191	0.529	1.251	2.194	1.896	3.983

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	147	126	149	162	252	335	150
N.S.	1	1.00	1.04	0.89	1.06	1.15	1.79	2.38	1.06
time (sec)	N/A	0.234	2.164	0.187	0.523	1.570	3.013	1.854	3.974

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	154	155	170	186	265	416	174
N.S.	1	1.00	0.91	0.91	1.00	1.09	1.56	2.45	1.02
time (sec)	N/A	0.262	0.451	0.202	0.518	1.017	5.073	1.318	4.064

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	178	179	195	214	340	506	202
N.S.	1	1.00	0.90	0.90	0.98	1.08	1.72	2.56	1.02
time (sec)	N/A	0.301	0.514	0.237	0.526	1.414	6.671	1.373	4.248

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	167	138	146	181	947	158	165
N.S.	1	1.00	1.08	0.90	0.95	1.18	6.15	1.03	1.07
time (sec)	N/A	0.386	1.763	0.191	0.523	1.160	1.225	2.446	4.258

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	155	119	123	159	821	129	138
N.S.	1	1.00	1.24	0.95	0.98	1.27	6.57	1.03	1.10
time (sec)	N/A	0.264	0.624	0.179	0.528	1.723	0.794	1.960	4.078

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	107	92	99	134	677	100	109
N.S.	1	1.00	1.10	0.95	1.02	1.38	6.98	1.03	1.12
time (sec)	N/A	0.149	0.449	0.115	0.522	1.628	0.702	1.249	4.075

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	91	79	85	111	554	86	94
N.S.	1	1.00	1.15	1.00	1.08	1.41	7.01	1.09	1.19
time (sec)	N/A	0.094	0.422	0.139	0.524	1.301	0.570	0.902	4.110

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	77	78	68	72	89	405	73	78
N.S.	1	1.17	1.18	1.03	1.09	1.35	6.14	1.11	1.18
time (sec)	N/A	0.070	0.096	0.111	0.530	1.527	0.509	0.644	4.046

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	66	63	68	63	255	73	76
N.S.	1	1.00	1.43	1.37	1.48	1.37	5.54	1.59	1.65
time (sec)	N/A	0.042	0.132	0.092	0.549	1.411	0.462	0.516	4.025

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	76	62	69	62	241	74	73
N.S.	1	1.00	1.69	1.38	1.53	1.38	5.36	1.64	1.62
time (sec)	N/A	0.034	0.064	0.087	0.516	1.269	0.412	0.593	4.182

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	91	80	84	98	626	88	95
N.S.	1	1.00	1.38	1.21	1.27	1.48	9.48	1.33	1.44
time (sec)	N/A	0.054	0.212	0.269	0.530	1.021	0.892	0.577	4.130

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	96	94	100	140	1080	116	108
N.S.	1	1.00	1.19	1.16	1.23	1.73	13.33	1.43	1.33
time (sec)	N/A	0.123	0.496	0.260	0.589	0.918	1.509	0.719	4.157

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	106	114	122	180	1336	155	137
N.S.	1	1.00	0.99	1.07	1.14	1.68	12.49	1.45	1.28
time (sec)	N/A	0.213	0.677	0.272	0.668	1.103	2.382	0.750	4.197

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	131	137	145	207	1533	187	153
N.S.	1	1.00	0.98	1.03	1.09	1.56	11.53	1.41	1.15
time (sec)	N/A	0.336	1.793	0.289	1.021	1.834	3.241	0.788	4.261

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	242	168	206	387	3279	251	213
N.S.	1	1.00	1.01	0.70	0.86	1.62	13.72	1.05	0.89
time (sec)	N/A	0.508	6.239	0.158	0.664	1.370	1.508	2.436	4.384

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	182	142	180	340	2837	221	185
N.S.	1	1.00	0.92	0.72	0.91	1.73	14.40	1.12	0.94
time (sec)	N/A	0.353	3.023	0.171	1.079	1.075	1.228	1.903	4.327

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	329	125	164	288	2312	201	158
N.S.	1	1.00	2.12	0.81	1.06	1.86	14.92	1.30	1.02
time (sec)	N/A	0.208	2.777	0.156	0.633	0.885	1.024	1.173	4.271

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	121	251	114	155	226	1992	181	137
N.S.	1	1.06	2.20	1.00	1.36	1.98	17.47	1.59	1.20
time (sec)	N/A	0.115	1.109	0.158	1.031	1.007	0.891	0.906	4.252

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	161	101	137	153	1314	165	126
N.S.	1	1.00	1.83	1.15	1.56	1.74	14.93	1.88	1.43
time (sec)	N/A	0.081	0.732	0.120	1.013	0.870	0.790	0.731	4.072

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	181	104	139	157	1476	173	133
N.S.	1	1.00	2.21	1.27	1.70	1.91	18.00	2.11	1.62
time (sec)	N/A	0.069	0.493	0.130	0.959	0.847	0.759	0.588	4.167

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	106	97	131	154	1260	159	121
N.S.	1	1.00	1.29	1.18	1.60	1.88	15.37	1.94	1.48
time (sec)	N/A	0.060	1.365	0.102	0.510	0.864	0.721	0.474	4.085

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	154	126	164	235	2927	206	152
N.S.	1	1.00	1.44	1.18	1.53	2.20	27.36	1.93	1.42
time (sec)	N/A	0.152	0.721	0.296	0.506	0.875	1.670	0.921	4.289

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	136	140	200	323	4070	235	183
N.S.	1	1.00	0.91	0.93	1.33	2.15	27.13	1.57	1.22
time (sec)	N/A	0.256	0.782	0.295	0.526	1.246	1.971	0.883	4.351

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	146	166	240	386	5222	272	222
N.S.	1	1.00	0.77	0.88	1.27	2.04	27.63	1.44	1.17
time (sec)	N/A	0.388	1.281	0.329	0.505	1.177	2.897	0.936	4.462

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	243	203	308	628	0	345	284
N.S.	1	1.00	0.86	0.72	1.09	2.22	0.00	1.22	1.00
time (sec)	N/A	0.563	3.814	0.206	0.512	1.036	0.000	2.509	4.353

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	694	186	293	549	0	325	263
N.S.	1	1.00	2.90	0.78	1.23	2.30	0.00	1.36	1.10
time (sec)	N/A	0.386	6.333	0.240	0.517	1.114	0.000	2.257	4.252

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	351	169	279	481	0	304	240
N.S.	1	1.00	1.92	0.92	1.52	2.63	0.00	1.66	1.31
time (sec)	N/A	0.219	2.203	0.187	0.504	1.347	0.000	1.431	4.206

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	269	158	262	317	0	282	236
N.S.	1	1.00	1.81	1.06	1.76	2.13	0.00	1.89	1.58
time (sec)	N/A	0.165	5.812	0.170	0.508	0.844	0.000	0.920	4.121

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	200	150	256	326	0	263	224
N.S.	1	1.00	1.55	1.16	1.98	2.53	0.00	2.04	1.74
time (sec)	N/A	0.132	4.038	0.153	0.526	1.077	0.000	0.810	4.092

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	234	147	253	328	0	275	224
N.S.	1	1.00	1.81	1.14	1.96	2.54	0.00	2.13	1.74
time (sec)	N/A	0.114	3.556	0.167	0.510	1.517	0.000	0.711	4.009

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	127	140	248	321	0	265	215
N.S.	1	1.00	1.04	1.15	2.03	2.63	0.00	2.17	1.76
time (sec)	N/A	0.111	1.911	0.135	0.521	1.093	0.000	0.490	4.309

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	209	182	290	494	0	328	256
N.S.	1	1.00	1.24	1.08	1.73	2.94	0.00	1.95	1.52
time (sec)	N/A	0.293	4.377	0.369	0.512	1.039	0.000	0.932	4.679

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	178	201	349	585	0	357	293
N.S.	1	1.00	0.84	0.95	1.65	2.77	0.00	1.69	1.39
time (sec)	N/A	0.432	4.929	0.378	0.501	1.029	0.000	1.124	4.797

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	1281	249	444	886	0	472	387
N.S.	1	1.00	4.07	0.79	1.41	2.81	0.00	1.50	1.23
time (sec)	N/A	0.610	6.529	0.279	0.510	1.152	0.000	2.688	4.891

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	788	234	433	784	0	451	373
N.S.	1	1.00	3.08	0.91	1.69	3.06	0.00	1.76	1.46
time (sec)	N/A	0.388	6.407	0.286	0.559	1.106	0.000	2.087	4.679

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	236	211	410	510	0	409	359
N.S.	1	1.00	1.13	1.01	1.97	2.45	0.00	1.97	1.73
time (sec)	N/A	0.276	2.617	0.178	0.506	1.094	0.000	1.316	4.639

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	387	206	402	526	0	400	359
N.S.	1	1.00	2.05	1.09	2.13	2.78	0.00	2.12	1.90
time (sec)	N/A	0.233	6.238	0.219	0.499	1.064	0.000	1.098	4.641

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	324	190	389	531	0	376	326
N.S.	1	1.00	1.92	1.12	2.30	3.14	0.00	2.22	1.93
time (sec)	N/A	0.199	6.213	0.184	0.496	1.154	0.000	1.000	4.531

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	319	191	394	528	0	401	355
N.S.	1	1.00	1.85	1.11	2.29	3.07	0.00	2.33	2.06
time (sec)	N/A	0.166	6.241	0.244	0.505	1.270	0.000	0.829	4.448

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	176	183	385	511	0	370	333
N.S.	1	1.00	1.07	1.11	2.33	3.10	0.00	2.24	2.02
time (sec)	N/A	0.169	2.863	0.191	0.495	1.220	0.000	0.613	4.277

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	243	246	444	793	0	476	385
N.S.	1	1.00	1.08	1.09	1.96	3.51	0.00	2.11	1.70
time (sec)	N/A	0.454	2.069	0.468	0.510	1.505	0.000	1.037	4.869

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	241	264	516	925	0	502	430
N.S.	1	1.00	0.87	0.95	1.86	3.33	0.00	1.81	1.55
time (sec)	N/A	0.583	3.579	0.464	0.509	1.271	0.000	1.118	5.548

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	65	41	39	46	46	40	49
N.S.	1	1.00	2.10	1.32	1.26	1.48	1.48	1.29	1.58
time (sec)	N/A	0.030	0.039	0.078	0.488	1.380	0.187	0.478	4.108

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	67	55	53	83	190	64	64
N.S.	1	1.00	1.34	1.10	1.06	1.66	3.80	1.28	1.28
time (sec)	N/A	0.045	0.279	0.079	0.500	1.017	0.289	0.464	3.951

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	84	69	73	120	442	74	84
N.S.	1	1.00	1.22	1.00	1.06	1.74	6.41	1.07	1.22
time (sec)	N/A	0.066	0.609	0.108	0.489	0.998	0.380	0.671	4.017

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	87	83	93	157	790	84	104
N.S.	1	1.00	0.99	0.94	1.06	1.78	8.98	0.95	1.18
time (sec)	N/A	0.087	1.184	0.096	0.491	1.345	0.483	0.592	4.120

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	65	41	39	46	46	40	49
N.S.	1	1.00	2.10	1.32	1.26	1.48	1.48	1.29	1.58
time (sec)	N/A	0.027	0.040	0.069	0.494	0.845	0.178	0.484	4.118

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	67	55	53	83	190	64	64
N.S.	1	1.00	1.34	1.10	1.06	1.66	3.80	1.28	1.28
time (sec)	N/A	0.045	0.279	0.085	0.498	0.981	0.281	0.550	4.084

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	86	69	73	120	442	74	84
N.S.	1	1.00	1.25	1.00	1.06	1.74	6.41	1.07	1.22
time (sec)	N/A	0.067	0.714	0.087	0.496	0.854	0.378	0.618	4.089

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	95	83	93	157	790	83	104
N.S.	1	1.00	1.08	0.94	1.06	1.78	8.98	0.94	1.18
time (sec)	N/A	0.084	0.691	0.098	0.489	0.980	0.502	0.529	4.069

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	456	456	167	430	0	1682	0	0	371
N.S.	1	1.00	0.37	0.94	0.00	3.69	0.00	0.00	0.81
time (sec)	N/A	0.541	19.725	0.432	0.000	1.405	0.000	0.000	17.636

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	140	370	0	1879	0	0	354
N.S.	1	1.00	0.88	2.33	0.00	11.82	0.00	0.00	2.23
time (sec)	N/A	0.227	1.121	0.143	0.000	1.584	0.000	0.000	8.485

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	115	385	0	1586	0	0	231
N.S.	1	1.00	0.30	1.01	0.00	4.15	0.00	0.00	0.60
time (sec)	N/A	0.260	0.269	0.146	0.000	0.968	0.000	0.000	5.466

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	100	327	0	1740	0	0	290
N.S.	1	1.00	0.94	3.08	0.00	16.42	0.00	0.00	2.74
time (sec)	N/A	0.123	0.110	0.118	0.000	1.212	0.000	0.000	4.792

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	87	363	0	1465	0	0	213
N.S.	1	1.00	0.24	1.01	0.00	4.09	0.00	0.00	0.59
time (sec)	N/A	0.225	0.035	0.508	0.000	1.397	0.000	0.000	4.178

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	111	15730	0	3585	0	0	682
N.S.	1	1.00	0.96	135.60	0.00	30.91	0.00	0.00	5.88
time (sec)	N/A	0.202	0.130	7.793	0.000	1.622	0.000	0.000	4.062

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	415	415	139	26754	0	3539	0	0	832
N.S.	1	1.00	0.33	64.47	0.00	8.53	0.00	0.00	2.00
time (sec)	N/A	0.413	0.767	0.865	0.000	2.759	0.000	0.000	4.044

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	166	45074	0	4130	0	0	1910
N.S.	1	1.00	0.88	238.49	0.00	21.85	0.00	0.00	10.11
time (sec)	N/A	0.423	1.177	1.138	0.000	2.055	0.000	0.000	4.834

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	338	712	0	4429	0	0	1355
N.S.	1	1.00	1.62	3.41	0.00	21.19	0.00	0.00	6.48
time (sec)	N/A	0.310	4.731	0.174	0.000	1.608	0.000	0.000	41.577

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	170	588	0	4373	0	0	1229
N.S.	1	1.00	0.94	3.25	0.00	24.16	0.00	0.00	6.79
time (sec)	N/A	0.261	1.675	0.139	0.000	2.362	0.000	0.000	20.206

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	158	663	0	4304	0	0	1141
N.S.	1	1.00	1.17	4.91	0.00	31.88	0.00	0.00	8.45
time (sec)	N/A	0.162	1.121	0.130	0.000	1.625	0.000	0.000	9.325

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	116	538	0	4234	0	0	1112
N.S.	1	1.00	0.91	4.20	0.00	33.08	0.00	0.00	8.69
time (sec)	N/A	0.153	0.342	0.125	0.000	1.855	0.000	0.000	6.821

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	106	641	0	4150	0	0	1099
N.S.	1	1.00	0.95	5.77	0.00	37.39	0.00	0.00	9.90
time (sec)	N/A	0.122	0.112	0.120	0.000	1.658	0.000	0.000	5.425

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	111	22251	0	8401	0	0	2260
N.S.	1	1.00	0.96	191.82	0.00	72.42	0.00	0.00	19.48
time (sec)	N/A	0.215	0.224	0.849	0.000	3.717	0.000	0.000	4.670

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	186	35890	0	8973	0	0	2500
N.S.	1	1.00	1.25	240.87	0.00	60.22	0.00	0.00	16.78
time (sec)	N/A	0.301	0.396	1.094	0.000	2.274	0.000	0.000	4.740

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	168	54149	0	9142	0	0	2500
N.S.	1	1.00	0.89	286.50	0.00	48.37	0.00	0.00	13.23
time (sec)	N/A	0.468	1.587	1.193	0.000	2.608	0.000	0.000	4.876

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	198	756	0	6847	0	0	2384
N.S.	1	1.00	0.94	3.58	0.00	32.45	0.00	0.00	11.30
time (sec)	N/A	0.317	3.794	0.145	0.000	6.310	0.000	0.000	47.857

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	205	823	0	6733	0	0	2165
N.S.	1	1.00	1.30	5.21	0.00	42.61	0.00	0.00	13.70
time (sec)	N/A	0.214	2.236	0.147	0.000	5.547	0.000	0.000	24.999

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	138	704	0	6683	0	0	2191
N.S.	1	1.00	0.87	4.46	0.00	42.30	0.00	0.00	13.87
time (sec)	N/A	0.196	0.756	0.137	0.000	5.709	0.000	0.000	11.834

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	121	792	0	6582	0	0	2100
N.S.	1	1.00	0.90	5.91	0.00	49.12	0.00	0.00	15.67
time (sec)	N/A	0.173	0.423	0.115	0.000	5.956	0.000	0.000	6.828

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	220	28373	0	13263	0	0	2500
N.S.	1	1.00	1.59	205.60	0.00	96.11	0.00	0.00	18.12
time (sec)	N/A	0.341	0.209	1.457	0.000	9.112	0.000	0.000	10.738

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	233	45710	0	13829	0	0	2500
N.S.	1	1.00	1.54	302.72	0.00	91.58	0.00	0.00	16.56
time (sec)	N/A	0.345	0.540	1.097	0.000	10.302	0.000	0.000	10.360

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	268	66439	0	14030	0	0	2500
N.S.	1	1.00	1.40	346.04	0.00	73.07	0.00	0.00	13.02
time (sec)	N/A	0.529	1.466	1.486	0.000	10.554	0.000	0.000	10.777

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	185	88284	0	14895	0	0	2500
N.S.	1	1.00	0.78	372.51	0.00	62.85	0.00	0.00	10.55
time (sec)	N/A	0.712	3.726	1.642	0.000	9.738	0.000	0.000	10.914

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	143	958	0	9021	0	0	2862
N.S.	1	1.00	0.86	5.74	0.00	54.02	0.00	0.00	17.14
time (sec)	N/A	0.245	0.995	0.147	0.000	11.827	0.000	0.000	11.473

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	162	445	0	2309	0	0	938
N.S.	1	1.00	0.71	1.94	0.00	10.08	0.00	0.00	4.10
time (sec)	N/A	0.378	4.180	0.177	0.000	0.980	0.000	0.000	11.485

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	500	500	144	846	0	1886	0	0	791
N.S.	1	1.00	0.29	1.69	0.00	3.77	0.00	0.00	1.58
time (sec)	N/A	0.570	17.479	0.143	0.000	0.962	0.000	0.000	7.942

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	159	377	0	2245	0	0	827
N.S.	1	1.00	1.14	2.69	0.00	16.04	0.00	0.00	5.91
time (sec)	N/A	0.179	0.773	0.135	0.000	0.956	0.000	0.000	4.993

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	424	424	108	797	0	1774	0	0	730
N.S.	1	1.00	0.25	1.88	0.00	4.18	0.00	0.00	1.72
time (sec)	N/A	0.283	0.213	0.140	0.000	0.883	0.000	0.000	4.863

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	107	339	0	2122	0	0	781
N.S.	1	1.00	1.23	3.90	0.00	24.39	0.00	0.00	8.98
time (sec)	N/A	0.098	0.068	0.622	0.000	1.250	0.000	0.000	4.754

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	87	777	0	1725	0	0	708
N.S.	1	1.00	0.22	1.93	0.00	4.29	0.00	0.00	1.76
time (sec)	N/A	0.255	0.033	0.149	0.000	1.467	0.000	0.000	5.139

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	111	20194	0	4447	0	0	2028
N.S.	1	1.00	0.96	174.09	0.00	38.34	0.00	0.00	17.48
time (sec)	N/A	0.183	0.163	1.028	0.000	1.431	0.000	0.000	4.638

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	461	461	142	39033	0	4050	0	0	2145
N.S.	1	1.00	0.31	84.67	0.00	8.79	0.00	0.00	4.65
time (sec)	N/A	0.499	0.839	1.057	0.000	2.351	0.000	0.000	7.256

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	203	58397	0	4908	0	0	2500
N.S.	1	1.00	1.05	301.02	0.00	25.30	0.00	0.00	12.89
time (sec)	N/A	0.391	3.278	1.270	0.000	1.666	0.000	0.000	0.587

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	213	878	0	5852	0	0	2930
N.S.	1	1.00	0.76	3.11	0.00	20.75	0.00	0.00	10.39
time (sec)	N/A	0.489	5.999	0.172	0.000	1.159	0.000	0.000	14.853

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	171	846	0	5793	0	0	2282
N.S.	1	1.00	0.76	3.74	0.00	25.63	0.00	0.00	10.10
time (sec)	N/A	0.316	5.374	0.161	0.000	1.408	0.000	0.000	8.304

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	243	826	0	5709	0	0	2869
N.S.	1	1.00	1.47	5.01	0.00	34.60	0.00	0.00	17.39
time (sec)	N/A	0.218	1.535	0.138	0.000	0.973	0.000	0.000	6.024

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	119	818	0	5654	0	0	2239
N.S.	1	1.00	0.95	6.54	0.00	45.23	0.00	0.00	17.91
time (sec)	N/A	0.156	0.161	0.124	0.000	1.368	0.000	0.000	5.633

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	100	799	0	5638	0	0	2844
N.S.	1	1.00	0.86	6.89	0.00	48.60	0.00	0.00	24.52
time (sec)	N/A	0.140	0.174	0.129	0.000	1.516	0.000	0.000	6.305

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	105	809	0	5624	0	0	2236
N.S.	1	1.00	0.88	6.74	0.00	46.87	0.00	0.00	18.63
time (sec)	N/A	0.124	0.105	0.105	0.000	1.256	0.000	0.000	5.920

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	165	39358	0	11665	0	0	2500
N.S.	1	1.00	1.10	262.39	0.00	77.77	0.00	0.00	16.67
time (sec)	N/A	0.336	1.210	1.737	0.000	2.077	0.000	0.000	4.437

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	184	60231	0	12983	0	0	2500
N.S.	1	1.00	0.96	313.70	0.00	67.62	0.00	0.00	13.02
time (sec)	N/A	0.452	3.510	1.553	0.000	2.551	0.000	0.000	4.561

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	259	93020	0	13156	0	0	2500
N.S.	1	1.00	1.07	385.98	0.00	54.59	0.00	0.00	10.37
time (sec)	N/A	0.607	5.718	2.256	0.000	2.181	0.000	0.000	4.641

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	556	1000	0	10005	0	0	2500
N.S.	1	1.00	1.91	3.44	0.00	34.38	0.00	0.00	8.59
time (sec)	N/A	0.546	6.863	0.223	0.000	1.910	0.000	0.000	14.333

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	216	986	0	9896	0	0	2500
N.S.	1	1.00	0.96	4.36	0.00	43.79	0.00	0.00	11.06
time (sec)	N/A	0.357	5.331	0.148	0.000	1.567	0.000	0.000	11.238

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	220	968	0	9834	0	0	2500
N.S.	1	1.00	1.28	5.63	0.00	57.17	0.00	0.00	14.53
time (sec)	N/A	0.259	1.519	0.133	0.000	1.613	0.000	0.000	9.452

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	122	967	0	9804	0	0	2500
N.S.	1	1.00	0.78	6.16	0.00	62.45	0.00	0.00	15.92
time (sec)	N/A	0.216	0.177	0.151	0.000	1.613	0.000	0.000	8.464

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	103	953	0	9790	0	0	2500
N.S.	1	1.00	0.66	6.15	0.00	63.16	0.00	0.00	16.13
time (sec)	N/A	0.192	0.164	0.127	0.000	1.838	0.000	0.000	9.629

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	108	955	0	9767	0	0	2500
N.S.	1	1.00	0.71	6.28	0.00	64.26	0.00	0.00	16.45
time (sec)	N/A	0.176	0.099	0.113	0.000	2.027	0.000	0.000	8.860

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	237	115830	0	20287	0	0	2500
N.S.	1	1.00	1.22	594.00	0.00	104.04	0.00	0.00	12.82
time (sec)	N/A	0.505	2.252	2.849	0.000	3.588	0.000	0.000	4.892

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	232	175534	0	22339	0	0	2500
N.S.	1	1.00	0.95	716.47	0.00	91.18	0.00	0.00	10.20
time (sec)	N/A	0.647	4.881	4.659	0.000	4.461	0.000	0.000	4.953

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	108	1211	0	14477	0	0	2500
N.S.	1	1.00	0.56	6.24	0.00	74.62	0.00	0.00	12.89
time (sec)	N/A	0.287	0.265	0.135	0.000	3.218	0.000	0.000	20.892

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	106	211	158	2847	0	0	130
N.S.	1	1.00	0.52	1.04	0.78	14.09	0.00	0.00	0.64
time (sec)	N/A	0.121	0.618	0.073	0.489	1.398	0.000	0.000	5.936

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	94	200	147	2757	0	0	114
N.S.	1	1.00	0.51	1.09	0.80	14.98	0.00	0.00	0.62
time (sec)	N/A	0.099	0.258	0.057	0.488	1.632	0.000	0.000	5.057

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	79	189	135	2729	0	0	153
N.S.	1	1.00	0.48	1.14	0.81	16.44	0.00	0.00	0.92
time (sec)	N/A	0.080	0.085	0.051	0.507	1.470	0.000	0.000	4.573

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	61	178	124	2640	0	0	141
N.S.	1	1.00	0.41	1.19	0.83	17.60	0.00	0.00	0.94
time (sec)	N/A	0.065	0.041	0.086	0.491	0.739	0.000	0.000	4.374

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	69	189	135	2883	0	0	102
N.S.	1	1.00	0.42	1.14	0.81	17.37	0.00	0.00	0.61
time (sec)	N/A	0.082	0.113	0.052	0.498	0.957	0.000	0.000	4.544

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	72	200	146	2861	0	0	114
N.S.	1	1.00	0.39	1.09	0.79	15.55	0.00	0.00	0.62
time (sec)	N/A	0.098	0.110	0.054	0.489	1.224	0.000	0.000	5.131

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	72	211	159	3096	0	0	128
N.S.	1	1.00	0.36	1.04	0.79	15.33	0.00	0.00	0.63
time (sec)	N/A	0.112	0.115	0.054	0.502	1.675	0.000	0.000	5.890

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	133	250	217	5080	0	0	995
N.S.	1	1.00	0.50	0.93	0.81	18.96	0.00	0.00	3.71
time (sec)	N/A	0.177	1.131	0.069	0.490	1.746	0.000	0.000	6.988

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	120	238	205	5043	0	0	986
N.S.	1	1.00	0.48	0.96	0.82	20.25	0.00	0.00	3.96
time (sec)	N/A	0.159	0.731	0.054	0.524	1.080	0.000	0.000	5.758

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	99	212	186	4967	0	0	954
N.S.	1	1.00	0.44	0.95	0.83	22.27	0.00	0.00	4.28
time (sec)	N/A	0.128	0.293	0.062	0.500	2.704	0.000	0.000	4.866

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	85	200	173	4881	0	0	937
N.S.	1	1.00	0.42	0.98	0.85	23.93	0.00	0.00	4.59
time (sec)	N/A	0.107	0.124	0.065	0.504	1.067	0.000	0.000	4.536

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	166	200	173	5093	0	0	949
N.S.	1	1.00	0.81	0.98	0.85	24.97	0.00	0.00	4.65
time (sec)	N/A	0.101	1.016	0.058	0.526	2.140	0.000	0.000	4.489

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	77	212	185	5149	0	0	968
N.S.	1	1.00	0.35	0.95	0.83	23.09	0.00	0.00	4.34
time (sec)	N/A	0.130	0.221	0.054	0.495	1.409	0.000	0.000	5.065

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	81	231	206	5441	0	0	983
N.S.	1	1.00	0.33	0.93	0.83	21.85	0.00	0.00	3.95
time (sec)	N/A	0.149	0.216	0.076	0.519	1.853	0.000	0.000	5.690

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	164	308	280	7332	0	0	1796
N.S.	1	1.00	0.50	0.94	0.85	22.35	0.00	0.00	5.48
time (sec)	N/A	0.307	3.049	0.055	0.514	3.165	0.000	0.000	10.117

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	144	279	258	7272	0	0	1729
N.S.	1	1.00	0.48	0.93	0.86	24.32	0.00	0.00	5.78
time (sec)	N/A	0.274	1.414	0.054	0.496	3.301	0.000	0.000	8.239

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	120	250	238	7179	0	0	1742
N.S.	1	1.00	0.44	0.92	0.88	26.39	0.00	0.00	6.40
time (sec)	N/A	0.245	0.832	0.059	0.507	3.742	0.000	0.000	6.091

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	101	225	216	7117	0	0	1674
N.S.	1	1.00	0.41	0.92	0.88	29.05	0.00	0.00	6.83
time (sec)	N/A	0.207	0.360	0.054	0.497	3.322	0.000	0.000	4.975

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	192	226	214	7395	0	0	1767
N.S.	1	1.00	0.78	0.92	0.87	30.18	0.00	0.00	7.21
time (sec)	N/A	0.215	1.976	0.057	0.494	3.563	0.000	0.000	4.656

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	91	225	215	7355	0	0	1752
N.S.	1	1.00	0.37	0.92	0.88	30.02	0.00	0.00	7.15
time (sec)	N/A	0.216	0.306	0.062	0.503	2.943	0.000	0.000	5.020

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	103	243	235	7742	0	0	1777
N.S.	1	1.00	0.38	0.90	0.87	28.67	0.00	0.00	6.58
time (sec)	N/A	0.251	0.521	0.056	0.513	3.325	0.000	0.000	6.275

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	103	267	259	7771	0	0	1795
N.S.	1	1.00	0.34	0.89	0.87	25.99	0.00	0.00	6.00
time (sec)	N/A	0.273	0.603	0.056	0.504	3.199	0.000	0.000	8.398

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	104	289	279	8178	0	0	1821
N.S.	1	1.00	0.32	0.89	0.86	25.09	0.00	0.00	5.59
time (sec)	N/A	0.299	0.713	0.058	0.503	3.478	0.000	0.000	10.097

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	61	178	124	2640	0	0	141
N.S.	1	1.00	0.41	1.19	0.83	17.60	0.00	0.00	0.94
time (sec)	N/A	0.065	0.054	0.000	0.487	1.317	0.000	0.000	0.002

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	82	202	136	2694	0	0	94
N.S.	1	1.00	0.51	1.25	0.84	16.63	0.00	0.00	0.58
time (sec)	N/A	0.068	0.160	0.115	0.494	0.994	0.000	0.000	4.585

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	83	273	126	2696	0	0	120
N.S.	1	1.00	0.40	1.31	0.61	12.96	0.00	0.00	0.58
time (sec)	N/A	0.109	0.087	0.229	0.496	1.059	0.000	0.000	4.784

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	84	285	172	2778	0	0	122
N.S.	1	1.00	0.39	1.33	0.80	12.98	0.00	0.00	0.57
time (sec)	N/A	0.102	0.165	0.181	0.508	1.381	0.000	0.000	4.715

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	243	284	225	7470	0	0	2500
N.S.	1	1.00	0.81	0.95	0.75	24.90	0.00	0.00	8.33
time (sec)	N/A	0.561	1.865	0.151	0.488	9.636	0.000	0.000	10.438

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	222	255	202	7356	0	0	2500
N.S.	1	1.00	0.82	0.94	0.75	27.14	0.00	0.00	9.23
time (sec)	N/A	0.384	0.876	0.154	0.494	9.301	0.000	0.000	6.179

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	155	241	185	7198	0	0	2500
N.S.	1	1.00	0.62	0.96	0.74	28.79	0.00	0.00	10.00
time (sec)	N/A	0.282	0.184	0.133	0.510	8.283	0.000	0.000	7.616

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	227	225	171	7094	0	0	2500
N.S.	1	1.00	0.98	0.97	0.74	30.58	0.00	0.00	10.78
time (sec)	N/A	0.170	0.294	0.125	0.504	8.680	0.000	0.000	5.259

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	204	224	170	7205	0	0	2500
N.S.	1	1.00	0.88	0.97	0.73	31.06	0.00	0.00	10.78
time (sec)	N/A	0.161	0.179	0.159	0.486	11.530	0.000	0.000	6.722

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	225	225	170	7202	0	0	2500
N.S.	1	1.00	0.97	0.97	0.73	31.04	0.00	0.00	10.78
time (sec)	N/A	0.164	0.192	0.121	0.501	10.978	0.000	0.000	5.310

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	131	241	185	7748	0	0	2500
N.S.	1	1.00	0.52	0.96	0.74	30.99	0.00	0.00	10.00
time (sec)	N/A	0.282	0.547	0.109	0.498	10.956	0.000	0.000	5.330

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	222	255	201	7820	0	0	2500
N.S.	1	1.00	0.82	0.94	0.74	28.86	0.00	0.00	9.23
time (sec)	N/A	0.398	1.886	0.122	0.506	11.563	0.000	0.000	5.900

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	248	277	223	8348	0	0	2500
N.S.	1	1.00	0.83	0.92	0.74	27.83	0.00	0.00	8.33
time (sec)	N/A	0.536	3.772	0.121	0.508	10.652	0.000	0.000	6.914

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	442	311	311	14604	0	0	2500
N.S.	1	1.00	1.11	0.78	0.78	36.60	0.00	0.00	6.27
time (sec)	N/A	0.683	1.913	0.138	0.509	14.899	0.000	0.000	9.318

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	375	296	296	14311	0	0	2500
N.S.	1	1.00	1.05	0.83	0.83	39.97	0.00	0.00	6.98
time (sec)	N/A	0.500	1.215	0.141	0.508	13.871	0.000	0.000	12.623

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	157	281	277	14258	0	0	2500
N.S.	1	1.00	0.49	0.88	0.87	44.84	0.00	0.00	7.86
time (sec)	N/A	0.334	1.725	0.132	0.500	11.668	0.000	0.000	6.762

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	151	276	268	14125	0	0	2500
N.S.	1	1.00	0.48	0.88	0.86	45.27	0.00	0.00	8.01
time (sec)	N/A	0.296	1.304	0.118	0.513	12.670	0.000	0.000	11.566

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	182	278	270	14310	0	0	2500
N.S.	1	1.00	0.58	0.88	0.85	45.28	0.00	0.00	7.91
time (sec)	N/A	0.287	0.730	0.129	0.506	10.155	0.000	0.000	6.795

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	166	281	277	14311	0	0	2500
N.S.	1	1.00	0.52	0.89	0.87	45.15	0.00	0.00	7.89
time (sec)	N/A	0.340	0.637	0.162	0.515	15.376	0.000	0.000	11.260

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	195	296	315	16081	0	0	2500
N.S.	1	1.00	0.54	0.83	0.88	44.92	0.00	0.00	6.98
time (sec)	N/A	0.504	1.834	0.116	0.496	10.991	0.000	0.000	7.760

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	397	397	230	310	342	16034	0	0	2500
N.S.	1	1.00	0.58	0.78	0.86	40.39	0.00	0.00	6.30
time (sec)	N/A	0.683	4.501	0.118	0.499	12.862	0.000	0.000	9.636

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	493	493	723	373	449	23042	0	0	2500
N.S.	1	1.00	1.47	0.76	0.91	46.74	0.00	0.00	5.07
time (sec)	N/A	0.977	6.380	0.187	0.494	15.240	0.000	0.000	20.447

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	444	444	403	356	434	22791	0	0	2500
N.S.	1	1.00	0.91	0.80	0.98	51.33	0.00	0.00	5.63
time (sec)	N/A	0.739	5.352	0.152	0.500	18.467	0.000	0.000	20.800

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	373	347	420	22644	0	0	2500
N.S.	1	1.00	0.94	0.88	1.06	57.18	0.00	0.00	6.31
time (sec)	N/A	0.574	4.924	0.168	0.504	14.420	0.000	0.000	9.840

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	390	390	325	339	408	22603	0	0	2500
N.S.	1	1.00	0.83	0.87	1.05	57.96	0.00	0.00	6.41
time (sec)	N/A	0.594	4.257	0.166	0.512	13.352	0.000	0.000	20.836

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	326	339	402	22816	0	0	2500
N.S.	1	1.00	0.85	0.88	1.04	59.26	0.00	0.00	6.49
time (sec)	N/A	0.536	3.783	0.155	0.497	12.895	0.000	0.000	9.161

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	259	343	411	22777	0	0	2500
N.S.	1	1.00	0.67	0.88	1.06	58.55	0.00	0.00	6.43
time (sec)	N/A	0.529	5.518	0.168	0.502	14.057	0.000	0.000	16.563

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	235	347	419	22843	0	0	2500
N.S.	1	1.00	0.59	0.88	1.06	57.68	0.00	0.00	6.31
time (sec)	N/A	0.574	2.864	0.165	0.517	12.886	0.000	0.000	9.638

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	444	444	358	356	464	25393	0	0	2500
N.S.	1	1.00	0.81	0.80	1.05	57.19	0.00	0.00	5.63
time (sec)	N/A	0.743	4.168	0.146	0.514	19.738	0.000	0.000	13.708

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	493	493	495	372	502	25401	0	0	2500
N.S.	1	1.00	1.00	0.75	1.02	51.52	0.00	0.00	5.07
time (sec)	N/A	0.961	6.198	0.165	0.508	14.825	0.000	0.000	18.880

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	282	1092020	0	0	0	0	-1
N.S.	1	1.00	1.22	4727.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.119	1.903	1.263	0.000	0.000	0.000	0.000	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	217	1091177	0	0	0	0	-1
N.S.	1	1.00	1.18	5930.31	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.486	1.563	0.564	0.000	0.000	0.000	0.000	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	189	1089481	0	0	0	0	286
N.S.	1	1.00	1.25	7215.11	0.00	0.00	0.00	0.00	1.89
time (sec)	N/A	0.359	0.561	0.440	0.000	0.000	0.000	0.000	6.908

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	123	1085178	0	0	0	0	224
N.S.	1	1.00	1.07	9436.33	0.00	0.00	0.00	0.00	1.95
time (sec)	N/A	0.095	0.098	0.456	0.000	0.000	0.000	0.000	6.083

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	154	1089777	0	0	0	0	-1
N.S.	1	1.00	1.11	7840.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.291	0.391	0.461	0.000	0.000	0.000	0.000	0.000

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	161	1090997	0	0	0	0	-1
N.S.	1	1.00	0.89	6027.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.243	0.597	1.075	0.000	0.000	0.000	0.000	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	197	1092009	0	0	0	0	-1
N.S.	1	1.00	0.89	4941.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.542	1.497	0.405	0.000	0.000	0.000	0.000	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	313	1347974	0	0	0	0	-1
N.S.	1	1.00	1.12	4814.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.416	3.896	0.760	0.000	0.000	0.000	0.000	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	850	1346578	0	0	0	0	-1
N.S.	1	1.00	3.76	5958.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.087	6.115	1.183	0.000	0.000	0.000	0.000	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	219	1345543	0	0	0	0	-1
N.S.	1	1.00	1.18	7234.10	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.891	2.547	1.089	0.000	0.000	0.000	0.000	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	203	1343730	0	0	0	0	-1
N.S.	1	1.00	1.34	8840.33	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.433	0.801	1.101	0.000	0.000	0.000	0.000	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	175	1344189	0	0	0	0	-1
N.S.	1	1.00	1.21	9270.27	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.319	0.289	1.648	0.000	0.000	0.000	0.000	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	176	1345207	0	0	0	0	-1
N.S.	1	1.00	1.02	7775.76	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.438	0.865	1.375	0.000	0.000	0.000	0.000	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	197	1346038	0	0	0	0	-1
N.S.	1	1.00	0.88	6009.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.607	1.687	1.254	0.000	0.000	0.000	0.000	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	229	1346975	0	0	0	0	-1
N.S.	1	1.00	0.86	5063.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.773	3.018	0.444	0.000	0.000	0.000	0.000	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	349	1347722	0	0	0	0	-1
N.S.	1	1.00	1.05	4059.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.927	3.415	0.497	0.000	0.000	0.000	0.000	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	300	1310426	0	0	0	0	-1
N.S.	1	1.00	1.08	4730.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.574	3.201	0.475	0.000	0.000	0.000	0.000	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	264	1345303	0	0	0	0	-1
N.S.	1	1.00	1.14	5823.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.507	2.018	0.309	0.000	0.000	0.000	0.000	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	221	1308215	0	0	0	0	-1
N.S.	1	1.00	1.18	6958.59	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	1.026	0.834	0.322	0.000	0.000	0.000	0.000	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	244	27882	0	0	0	0	-1
N.S.	1	1.00	1.33	152.36	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	1.026	2.288	18.491	0.000	0.000	0.000	0.000	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	170	16922	0	0	0	0	-1
N.S.	1	1.00	0.93	92.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.512	0.912	0.790	0.000	0.000	0.000	0.000	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	194	33401	0	0	0	0	-1
N.S.	1	1.00	0.89	152.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.678	1.723	0.798	0.000	0.000	0.000	0.000	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	249	34250	0	0	0	0	-1
N.S.	1	1.00	0.92	126.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.883	3.691	0.887	0.000	0.000	0.000	0.000	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	300	50542	0	0	0	0	-1
N.S.	1	1.00	0.94	158.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.069	4.216	0.897	0.000	0.000	0.000	0.000	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	270	945945	0	0	0	0	-1
N.S.	1	1.00	1.16	4077.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.716	4.180	1.028	0.000	0.000	0.000	0.000	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	221	945569	0	0	0	0	-1
N.S.	1	1.00	1.18	5029.62	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.494	1.976	1.172	0.000	0.000	0.000	0.000	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	189	943433	0	0	0	0	-1
N.S.	1	1.00	1.24	6206.80	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.134	0.813	0.546	0.000	0.000	0.000	0.000	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	123	940502	0	0	0	0	2500
N.S.	1	1.00	1.07	8178.28	0.00	0.00	0.00	0.00	21.74
time (sec)	N/A	0.099	0.104	0.648	0.000	0.000	0.000	0.000	43.186

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	124	940263	0	0	0	0	1716
N.S.	1	1.00	1.14	8626.27	0.00	0.00	0.00	0.00	15.74
time (sec)	N/A	0.099	0.101	0.699	0.000	0.000	0.000	0.000	13.298

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	157	944864	0	0	0	0	-1
N.S.	1	1.00	1.07	6427.65	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.153	0.421	0.637	0.000	0.000	0.000	0.000	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	172	945613	0	0	0	0	-1
N.S.	1	1.00	0.96	5253.41	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.260	1.618	0.570	0.000	0.000	0.000	0.000	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	197	947049	0	0	0	0	-1
N.S.	1	1.00	0.86	4135.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.376	2.803	0.615	0.000	0.000	0.000	0.000	0.000

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	270	764550	0	0	0	0	-1
N.S.	1	1.00	1.08	3058.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.161	3.178	2.179	0.000	0.000	0.000	0.000	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	321	798950	0	0	0	0	-1
N.S.	1	1.00	1.65	4097.18	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.892	1.262	0.974	0.000	0.000	0.000	0.000	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	182	762478	0	0	0	0	-1
N.S.	1	1.00	1.18	4951.16	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.315	1.290	2.292	0.000	0.000	0.000	0.000	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	163	798605	0	0	0	0	-1
N.S.	1	1.00	1.09	5359.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.314	1.240	1.001	0.000	0.000	0.000	0.000	0.000

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	183	762494	0	0	0	0	-1
N.S.	1	1.00	1.15	4795.56	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.333	0.945	1.017	0.000	0.000	0.000	0.000	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	202	799513	0	0	0	0	-1
N.S.	1	1.00	1.05	4142.55	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.449	4.441	0.985	0.000	0.000	0.000	0.000	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	223	764302	0	0	0	0	-1
N.S.	1	1.00	0.93	3171.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.595	5.718	1.007	0.000	0.000	0.000	0.000	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	407	1493684	0	0	0	0	-1
N.S.	1	1.00	1.28	4711.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.594	6.170	1.318	0.000	0.000	0.000	0.000	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	468	1491834	0	0	0	0	-1
N.S.	1	1.00	1.86	5943.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.315	6.240	1.260	0.000	0.000	0.000	0.000	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	189	1489186	0	0	0	0	-1
N.S.	1	1.00	0.88	6958.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.517	3.416	1.321	0.000	0.000	0.000	0.000	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	198	1488901	0	0	0	0	-1
N.S.	1	1.00	0.99	7481.91	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.452	3.039	1.303	0.000	0.000	0.000	0.000	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	194	1489210	0	0	0	0	-1
N.S.	1	1.00	0.92	7057.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.471	3.866	1.304	0.000	0.000	0.000	0.000	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	235	1488925	0	0	0	0	-1
N.S.	1	1.00	1.11	7023.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.499	1.448	1.243	0.000	0.000	0.000	0.000	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	294	1490722	0	0	0	0	-1
N.S.	1	1.00	1.11	5625.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.666	2.783	1.285	0.000	0.000	0.000	0.000	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	483	1491406	0	0	0	0	-1
N.S.	1	1.00	1.62	5004.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.804	6.288	3.446	0.000	0.000	0.000	0.000	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	480	0	0	0	489	207
N.S.	1	1.00	1.00	5.39	0.00	0.00	0.00	5.49	2.33
time (sec)	N/A	0.083	0.186	2.950	0.000	0.000	0.000	0.574	6.118

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	480	0	0	0	489	89
N.S.	1	1.00	1.00	5.39	0.00	0.00	0.00	5.49	1.00
time (sec)	N/A	0.071	0.148	1.869	0.000	0.000	0.000	0.522	6.212

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	101	435	0	0	0	1061	205
N.S.	1	1.00	1.13	4.89	0.00	0.00	0.00	11.92	2.30
time (sec)	N/A	0.072	0.182	0.679	0.000	0.000	0.000	0.695	6.325

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	101	435	0	0	0	269	129
N.S.	1	1.00	1.13	4.89	0.00	0.00	0.00	3.02	1.45
time (sec)	N/A	0.073	0.153	0.715	0.000	0.000	0.000	0.616	6.083

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	480	0	0	0	479	204
N.S.	1	1.00	1.00	5.39	0.00	0.00	0.00	5.38	2.29
time (sec)	N/A	0.086	0.186	1.168	0.000	0.000	0.000	0.527	6.441

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	101	435	0	0	0	1061	201
N.S.	1	1.00	1.13	4.89	0.00	0.00	0.00	11.92	2.26
time (sec)	N/A	0.077	0.178	0.816	0.000	0.000	0.000	0.686	6.364

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	480	0	0	0	479	89
N.S.	1	1.00	1.00	5.39	0.00	0.00	0.00	5.38	1.00
time (sec)	N/A	0.075	0.152	1.211	0.000	0.000	0.000	0.505	6.092

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	101	435	0	0	0	413	219
N.S.	1	1.00	1.13	4.89	0.00	0.00	0.00	4.64	2.46
time (sec)	N/A	0.076	0.157	0.668	0.000	0.000	0.000	0.520	6.527

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	479	0	0	0	0	207
N.S.	1	1.00	1.00	5.04	0.00	0.00	0.00	0.00	2.18
time (sec)	N/A	0.072	0.098	0.876	0.000	0.000	0.000	0.000	5.570

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	479	0	0	0	0	191
N.S.	1	1.00	1.00	5.04	0.00	0.00	0.00	0.00	2.01
time (sec)	N/A	0.068	0.067	0.852	0.000	0.000	0.000	0.000	5.644

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	103	434	0	0	0	641	205
N.S.	1	1.00	1.08	4.57	0.00	0.00	0.00	6.75	2.16
time (sec)	N/A	0.070	0.069	0.669	0.000	0.000	0.000	0.943	5.430

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	103	434	0	0	0	0	189
N.S.	1	1.00	1.08	4.57	0.00	0.00	0.00	0.00	1.99
time (sec)	N/A	0.074	0.073	0.683	0.000	0.000	0.000	0.000	5.595

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	479	0	0	0	498	207
N.S.	1	1.00	1.00	5.04	0.00	0.00	0.00	5.24	2.18
time (sec)	N/A	0.069	0.109	0.874	0.000	0.000	0.000	1.358	5.613

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	103	434	0	0	0	641	205
N.S.	1	1.00	1.08	4.57	0.00	0.00	0.00	6.75	2.16
time (sec)	N/A	0.071	0.076	1.030	0.000	0.000	0.000	0.892	5.676

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	479	0	0	0	498	191
N.S.	1	1.00	1.00	5.04	0.00	0.00	0.00	5.24	2.01
time (sec)	N/A	0.070	0.064	1.099	0.000	0.000	0.000	1.237	5.712

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	103	434	0	0	0	209	189
N.S.	1	1.00	1.08	4.57	0.00	0.00	0.00	2.20	1.99
time (sec)	N/A	0.071	0.068	0.698	0.000	0.000	0.000	0.849	5.697

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	C	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	466	466	153	341	0	73878	0	435	2137
N.S.	1	1.00	0.33	0.73	0.00	158.54	0.00	0.93	4.59
time (sec)	N/A	0.509	0.429	0.251	0.000	3.887	0.000	0.714	12.191

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	C	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	465	465	204	328	0	74435	0	424	2111
N.S.	1	1.00	0.44	0.71	0.00	160.08	0.00	0.91	4.54
time (sec)	N/A	0.386	0.429	0.199	0.000	3.244	0.000	0.715	11.414

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	C	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	467	467	162	340	0	73744	0	430	2050
N.S.	1	1.00	0.35	0.73	0.00	157.91	0.00	0.92	4.39
time (sec)	N/A	0.480	0.284	0.203	0.000	3.914	0.000	0.695	11.668

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	C	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	525	525	104	350	0	74494	0	457	2500
N.S.	1	1.00	0.20	0.67	0.00	141.89	0.00	0.87	4.76
time (sec)	N/A	0.391	0.247	0.205	0.000	3.661	0.000	1.397	5.624

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	21046	0	0	0	0	0	-1
N.S.	1	1.00	129.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.188	36.817	0.487	0.000	0.000	0.000	0.000	0.000

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	7362	0	0	0	0	0	-1
N.S.	1	1.00	45.17	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.178	126.529	0.380	0.000	0.000	0.000	0.000	0.000

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	6316	0	0	0	0	0	-1
N.S.	1	1.00	38.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.174	34.421	0.384	0.000	0.000	0.000	0.000	0.000

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	32685	0	0	0	0	0	-1
N.S.	1	1.00	200.52	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.163	118.636	0.387	0.000	0.000	0.000	0.000	0.000

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	6198	0	0	0	0	0	-1
N.S.	1	1.00	38.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.151	92.906	0.410	0.000	0.000	0.000	0.000	0.000

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	23355	0	0	0	0	0	-1
N.S.	1	1.00	143.28	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.188	179.197	0.406	0.000	0.000	0.000	0.000	0.000

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	525	525	371	131	0	3638	0	0	1015
N.S.	1	1.00	0.71	0.25	0.00	6.93	0.00	0.00	1.93
time (sec)	N/A	0.623	16.148	0.366	0.000	1.203	0.000	0.000	18.885

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	373	373	442	124	0	3010	0	0	890
N.S.	1	1.00	1.18	0.33	0.00	8.07	0.00	0.00	2.39
time (sec)	N/A	0.397	1.089	0.236	0.000	1.168	0.000	0.000	11.794

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	439	439	313	82	0	3577	0	0	881
N.S.	1	1.00	0.71	0.19	0.00	8.15	0.00	0.00	2.01
time (sec)	N/A	0.252	0.824	0.226	0.000	1.293	0.000	0.000	8.729

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	346	88	0	2859	0	18	830
N.S.	1	1.00	1.09	0.28	0.00	8.99	0.00	0.06	2.61
time (sec)	N/A	0.220	0.240	0.242	0.000	1.198	0.000	1.427	8.078

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	415	415	294	60	0	3383	0	0	863
N.S.	1	1.00	0.71	0.14	0.00	8.15	0.00	0.00	2.08
time (sec)	N/A	0.217	0.215	0.559	0.000	1.477	0.000	0.000	6.880

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	744	0	0	0	0	0	2133
N.S.	1	1.00	1.85	0.00	0.00	0.00	0.00	0.00	5.31
time (sec)	N/A	0.356	0.862	0.273	0.000	0.000	0.000	0.000	15.563

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	546	546	464	0	0	0	0	0	2500
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	4.58
time (sec)	N/A	0.446	3.396	0.243	0.000	0.000	0.000	0.000	21.080

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	300	90	0	0	0	0	1540
N.S.	1	1.00	0.91	0.27	0.00	0.00	0.00	0.00	4.68
time (sec)	N/A	0.261	1.038	0.216	0.000	0.000	0.000	0.000	10.090

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	365	89	0	16583	0	0	924
N.S.	1	1.00	1.12	0.27	0.00	50.71	0.00	0.00	2.83
time (sec)	N/A	0.251	0.817	0.210	0.000	103.067	0.000	0.000	7.496

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	415	415	224	60	0	0	0	0	1229
N.S.	1	1.00	0.54	0.14	0.00	0.00	0.00	0.00	2.96
time (sec)	N/A	0.276	0.310	0.323	0.000	0.000	0.000	0.000	8.218

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	415	415	294	60	0	3320	0	0	863
N.S.	1	1.00	0.71	0.14	0.00	8.00	0.00	0.00	2.08
time (sec)	N/A	0.226	0.278	0.401	0.000	1.551	0.000	0.000	7.009

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	415	415	251	58	0	0	0	0	817
N.S.	1	1.00	0.60	0.14	0.00	0.00	0.00	0.00	1.97
time (sec)	N/A	0.220	0.260	0.306	0.000	0.000	0.000	0.000	7.239

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	415	415	313	57	0	16201	0	0	1048
N.S.	1	1.00	0.75	0.14	0.00	39.04	0.00	0.00	2.53
time (sec)	N/A	0.236	0.403	0.278	0.000	81.709	0.000	0.000	8.484

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	106	102	0	0	0	0	2500
N.S.	1	1.00	0.32	0.30	0.00	0.00	0.00	0.00	7.44
time (sec)	N/A	0.271	0.246	0.216	0.000	0.000	0.000	0.000	6.594

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	106	101	0	0	0	0	2500
N.S.	1	1.00	0.31	0.30	0.00	0.00	0.00	0.00	7.40
time (sec)	N/A	0.256	0.164	0.214	0.000	0.000	0.000	0.000	6.437

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	191	0	0	0	0	0	-1
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.451	1.684	0.510	0.000	0.000	0.000	0.000	0.000

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	141	0	0	0	0	0	-1
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.208	0.804	0.430	0.000	0.000	0.000	0.000	0.000

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	116	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.420	0.369	0.000	0.000	0.000	0.000	0.000

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	99	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.272	0.266	0.000	0.000	0.000	0.000	0.000

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	142	0	0	0	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.188	0.528	0.519	0.000	0.000	0.000	0.000	0.000

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	198	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.378	2.682	0.658	0.000	0.000	0.000	0.000	0.000

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	18.929	0.394	0.000	0.000	0.000	0.000	0.000

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.740	0.365	0.000	0.000	0.000	0.000	0.000

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	8.261	0.400	0.000	0.000	0.000	0.000	0.000

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.144	34.727	0.346	0.000	0.000	0.000	0.000	0.000

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	1.588	0.392	0.000	0.000	0.000	0.000	0.000

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	249	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.417	2.308	0.342	0.000	0.000	0.000	0.000	0.000

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	135	0	0	0	0	0	-1
N.S.	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.197	1.123	0.341	0.000	0.000	0.000	0.000	0.000

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	138	0	0	0	0	0	-1
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.228	0.308	0.000	0.000	0.000	0.000	0.000

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	117	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.190	0.233	0.000	0.000	0.000	0.000	0.000

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	118	0	0	0	0	0	-1
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.162	0.248	0.000	0.000	0.000	0.000	0.000

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	154	0	0	0	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.159	0.272	0.287	0.000	0.000	0.000	0.000	0.000

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	190	0	0	0	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.320	0.851	0.288	0.000	0.000	0.000	0.000	0.000

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	212	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.394	2.040	0.352	0.000	0.000	0.000	0.000	0.000

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.156	1.482	0.379	0.000	0.000	0.000	0.000	0.000

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.158	1.630	0.429	0.000	0.000	0.000	0.000	0.000

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.135	1.492	0.384	0.000	0.000	0.000	0.000	0.000

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.155	1.516	0.380	0.000	0.000	0.000	0.000	0.000

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	116	788	139	287	0	0	-1
N.S.	1	1.00	1.78	12.12	2.14	4.42	0.00	0.00	-0.02
time (sec)	N/A	0.077	1.083	13.366	0.480	0.977	0.000	0.000	0.000

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	C	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	67	734	127	228	0	0	-1
N.S.	1	1.00	1.49	16.31	2.82	5.07	0.00	0.00	-0.02
time (sec)	N/A	0.056	0.609	13.114	0.517	0.667	0.000	0.000	0.000

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	C	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	111	246	114	191	0	0	22
N.S.	1	1.00	3.96	8.79	4.07	6.82	0.00	0.00	0.79
time (sec)	N/A	0.037	0.534	12.310	0.519	0.664	0.000	0.000	5.414

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	71	420	127	278	0	0	102
N.S.	1	1.00	1.51	8.94	2.70	5.91	0.00	0.00	2.17
time (sec)	N/A	0.057	0.773	26.532	0.487	0.667	0.000	0.000	5.645

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	136	475	142	329	0	0	-1
N.S.	1	1.00	2.09	7.31	2.18	5.06	0.00	0.00	-0.02
time (sec)	N/A	0.076	1.111	25.324	0.543	0.827	0.000	0.000	0.000

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	104	1482	158	340	0	0	-1
N.S.	1	1.00	1.14	16.29	1.74	3.74	0.00	0.00	-0.01
time (sec)	N/A	0.120	3.102	13.481	0.533	0.785	0.000	0.000	0.000

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	125	790	145	297	0	0	-1
N.S.	1	1.00	1.76	11.13	2.04	4.18	0.00	0.00	-0.01
time (sec)	N/A	0.096	1.211	14.171	0.531	1.211	0.000	0.000	0.000

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	C	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	70	736	131	236	0	0	-1
N.S.	1	1.00	1.43	15.02	2.67	4.82	0.00	0.00	-0.02
time (sec)	N/A	0.076	1.162	14.093	0.494	0.980	0.000	0.000	0.000

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	C	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	83	426	131	290	0	0	-1
N.S.	1	1.00	1.69	8.69	2.67	5.92	0.00	0.00	-0.02
time (sec)	N/A	0.080	1.301	25.821	0.492	0.922	0.000	0.000	0.000

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	90	485	146	340	0	0	-1
N.S.	1	1.00	1.27	6.83	2.06	4.79	0.00	0.00	-0.01
time (sec)	N/A	0.099	1.428	26.895	0.494	0.682	0.000	0.000	0.000

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	152	517	161	391	0	0	-1
N.S.	1	1.00	1.67	5.68	1.77	4.30	0.00	0.00	-0.01
time (sec)	N/A	0.122	3.348	27.299	0.501	0.775	0.000	0.000	0.000

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	147	1482	158	340	0	0	-1
N.S.	1	1.00	1.39	13.98	1.49	3.21	0.00	0.00	-0.01
time (sec)	N/A	0.147	2.207	14.354	0.492	0.653	0.000	0.000	0.000

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	125	790	145	297	0	0	-1
N.S.	1	1.00	1.42	8.98	1.65	3.38	0.00	0.00	-0.01
time (sec)	N/A	0.124	1.797	14.082	0.482	0.613	0.000	0.000	0.000

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	125	748	144	281	0	0	-1
N.S.	1	1.00	1.95	11.69	2.25	4.39	0.00	0.00	-0.02
time (sec)	N/A	0.084	1.886	30.175	0.494	0.521	0.000	0.000	0.000

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	147	485	148	341	0	0	-1
N.S.	1	1.00	1.71	5.64	1.72	3.97	0.00	0.00	-0.01
time (sec)	N/A	0.129	2.031	28.401	0.504	0.492	0.000	0.000	0.000

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	164	520	161	390	0	0	-1
N.S.	1	1.00	1.55	4.91	1.52	3.68	0.00	0.00	-0.01
time (sec)	N/A	0.163	2.528	29.317	0.501	0.467	0.000	0.000	0.000

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	213	1225	0	469	0	0	-1
N.S.	1	1.00	0.97	5.57	0.00	2.13	0.00	0.00	-0.00
time (sec)	N/A	0.180	1.145	14.221	0.000	0.549	0.000	0.000	0.000

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	174	710	0	471	0	0	-1
N.S.	1	1.00	0.87	3.55	0.00	2.36	0.00	0.00	-0.00
time (sec)	N/A	0.141	1.008	14.073	0.000	0.490	0.000	0.000	0.000

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	126	2687	0	269	0	0	-1
N.S.	1	1.00	1.85	39.51	0.00	3.96	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.942	13.597	0.000	0.493	0.000	0.000	0.000

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	174	4362	0	470	0	0	-1
N.S.	1	1.00	0.87	21.81	0.00	2.35	0.00	0.00	-0.00
time (sec)	N/A	0.148	1.140	13.288	0.000	0.510	0.000	0.000	0.000

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	213	4381	0	548	0	0	-1
N.S.	1	1.00	0.96	19.73	0.00	2.47	0.00	0.00	-0.00
time (sec)	N/A	0.176	1.162	27.552	0.000	0.473	0.000	0.000	0.000

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	232	1261	0	509	0	0	-1
N.S.	1	1.00	0.92	5.00	0.00	2.02	0.00	0.00	-0.00
time (sec)	N/A	0.254	1.246	13.891	0.000	0.529	0.000	0.000	0.000

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	232	778	0	511	0	0	-1
N.S.	1	1.00	1.00	3.35	0.00	2.20	0.00	0.00	-0.00
time (sec)	N/A	0.225	0.943	12.458	0.000	0.525	0.000	0.000	0.000

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	224	6915	0	509	0	0	-1
N.S.	1	1.00	0.96	29.55	0.00	2.18	0.00	0.00	-0.00
time (sec)	N/A	0.224	0.915	13.384	0.000	0.454	0.000	0.000	0.000

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	222	6919	0	510	0	0	-1
N.S.	1	1.00	0.95	29.57	0.00	2.18	0.00	0.00	-0.00
time (sec)	N/A	0.213	0.789	14.193	0.000	0.605	0.000	0.000	0.000

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	232	6924	0	508	0	0	-1
N.S.	1	1.00	0.99	29.59	0.00	2.17	0.00	0.00	-0.00
time (sec)	N/A	0.228	1.209	13.559	0.000	1.003	0.000	0.000	0.000

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	232	6941	0	602	0	0	-1
N.S.	1	1.00	0.91	27.33	0.00	2.37	0.00	0.00	-0.00
time (sec)	N/A	0.251	1.664	28.615	0.000	1.522	0.000	0.000	0.000

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	235	844	0	522	0	0	-1
N.S.	1	1.00	0.86	3.09	0.00	1.91	0.00	0.00	-0.00
time (sec)	N/A	0.320	1.387	13.242	0.000	1.113	0.000	0.000	0.000

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	231	9445	0	518	0	0	-1
N.S.	1	1.00	0.87	35.37	0.00	1.94	0.00	0.00	-0.00
time (sec)	N/A	0.309	1.026	14.033	0.000	1.265	0.000	0.000	0.000

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	154	5851	0	307	0	0	-1
N.S.	1	1.00	1.09	41.50	0.00	2.18	0.00	0.00	-0.01
time (sec)	N/A	0.203	1.670	15.081	0.000	1.684	0.000	0.000	0.000

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	224	7710	0	520	0	0	-1
N.S.	1	1.00	1.01	34.73	0.00	2.34	0.00	0.00	-0.00
time (sec)	N/A	0.211	2.061	14.421	0.000	1.126	0.000	0.000	0.000

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	235	9482	0	522	0	0	-1
N.S.	1	1.00	0.85	34.48	0.00	1.90	0.00	0.00	-0.00
time (sec)	N/A	0.323	2.079	13.631	0.000	1.038	0.000	0.000	0.000

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	149	1146	1134	380	0	0	-1
N.S.	1	1.00	0.86	6.59	6.52	2.18	0.00	0.00	-0.01
time (sec)	N/A	0.313	1.385	41.190	0.632	0.919	0.000	0.000	0.000

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	125	1041	961	326	0	0	-1
N.S.	1	1.00	0.89	7.44	6.86	2.33	0.00	0.00	-0.01
time (sec)	N/A	0.220	1.061	43.424	0.590	0.982	0.000	0.000	0.000

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	104	575	540	282	0	0	-1
N.S.	1	1.00	1.02	5.64	5.29	2.76	0.00	0.00	-0.01
time (sec)	N/A	0.130	0.878	43.475	0.583	0.781	0.000	0.000	0.000

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	118	400	374	217	0	0	-1
N.S.	1	1.00	1.71	5.80	5.42	3.14	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.770	46.088	0.582	1.171	0.000	0.000	0.000

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	B	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	0	576	0	470	0	0	-1
N.S.	1	1.00	0.00	4.00	0.00	3.26	0.00	0.00	-0.01
time (sec)	N/A	0.225	43.240	41.445	0.000	0.758	0.000	0.000	0.000

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	223	2093	0	610	0	0	-1
N.S.	1	1.00	1.27	11.96	0.00	3.49	0.00	0.00	-0.01
time (sec)	N/A	0.269	2.913	41.987	0.000	0.990	0.000	0.000	0.000

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	161	1147	1181	402	0	0	-1
N.S.	1	1.00	0.74	5.26	5.42	1.84	0.00	0.00	-0.00
time (sec)	N/A	0.429	1.919	47.147	0.645	0.742	0.000	0.000	0.000

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	135	1042	995	362	0	0	-1
N.S.	1	1.00	0.97	7.50	7.16	2.60	0.00	0.00	-0.01
time (sec)	N/A	0.191	1.262	41.651	0.600	0.990	0.000	0.000	0.000

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	105	575	545	302	0	0	-1
N.S.	1	1.00	1.02	5.58	5.29	2.93	0.00	0.00	-0.01
time (sec)	N/A	0.139	1.398	45.718	0.603	0.786	0.000	0.000	0.000

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	255	571	0	491	0	0	-1
N.S.	1	1.00	1.77	3.97	0.00	3.41	0.00	0.00	-0.01
time (sec)	N/A	0.225	1.628	44.899	0.000	0.938	0.000	0.000	0.000

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	228	2091	0	637	0	0	-1
N.S.	1	1.00	1.06	9.68	0.00	2.95	0.00	0.00	-0.00
time (sec)	N/A	0.409	2.553	48.268	0.000	0.977	0.000	0.000	0.000

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	167	1557	3165	466	0	0	-1
N.S.	1	1.00	0.75	7.01	14.26	2.10	0.00	0.00	-0.00
time (sec)	N/A	0.431	2.676	46.271	0.794	1.076	0.000	0.000	0.000

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	161	1149	1267	409	0	0	-1
N.S.	1	1.00	0.91	6.53	7.20	2.32	0.00	0.00	-0.01
time (sec)	N/A	0.237	1.983	47.778	0.632	0.798	0.000	0.000	0.000

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	142	1044	1059	366	0	0	-1
N.S.	1	1.00	1.00	7.35	7.46	2.58	0.00	0.00	-0.01
time (sec)	N/A	0.186	1.552	46.198	0.598	0.951	0.000	0.000	0.000

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	176	888	0	570	0	0	-1
N.S.	1	1.00	0.98	4.96	0.00	3.18	0.00	0.00	-0.01
time (sec)	N/A	0.320	2.580	49.075	0.000	0.836	0.000	0.000	0.000

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	241	740	0	648	0	0	-1
N.S.	1	1.00	1.35	4.13	0.00	3.62	0.00	0.00	-0.01
time (sec)	N/A	0.310	2.406	44.735	0.000	0.769	0.000	0.000	0.000

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	318	2352	0	722	0	0	-1
N.S.	1	1.00	1.43	10.59	0.00	3.25	0.00	0.00	-0.00
time (sec)	N/A	0.394	7.717	51.206	0.000	0.939	0.000	0.000	0.000

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	159	374	0	388	0	0	-1
N.S.	1	1.00	0.88	2.07	0.00	2.14	0.00	0.00	-0.01
time (sec)	N/A	0.326	1.646	43.976	0.000	1.185	0.000	0.000	0.000

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	139	287	0	331	0	0	-1
N.S.	1	1.00	0.99	2.05	0.00	2.36	0.00	0.00	-0.01
time (sec)	N/A	0.222	1.342	43.382	0.000	1.061	0.000	0.000	0.000

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	140	257	0	333	0	0	-1
N.S.	1	1.00	1.33	2.45	0.00	3.17	0.00	0.00	-0.01
time (sec)	N/A	0.174	0.998	43.815	0.000	0.793	0.000	0.000	0.000

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	138	270	0	329	0	0	-1
N.S.	1	1.00	1.28	2.50	0.00	3.05	0.00	0.00	-0.01
time (sec)	N/A	0.134	1.095	50.529	0.000	0.744	0.000	0.000	0.000

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	210	621	0	620	0	0	-1
N.S.	1	1.00	1.17	3.45	0.00	3.44	0.00	0.00	-0.01
time (sec)	N/A	0.311	1.915	48.805	0.000	1.067	0.000	0.000	0.000

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	248	801	0	703	0	0	-1
N.S.	1	1.00	1.14	3.69	0.00	3.24	0.00	0.00	-0.00
time (sec)	N/A	0.415	2.350	48.243	0.000	1.386	0.000	0.000	0.000

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	186	445	0	418	0	0	-1
N.S.	1	1.00	0.84	2.01	0.00	1.89	0.00	0.00	-0.00
time (sec)	N/A	0.431	1.800	48.074	0.000	1.330	0.000	0.000	0.000

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	156	344	0	355	0	0	-1
N.S.	1	1.00	0.86	1.89	0.00	1.95	0.00	0.00	-0.01
time (sec)	N/A	0.336	1.530	46.774	0.000	0.996	0.000	0.000	0.000

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	158	468	0	357	0	0	-1
N.S.	1	1.00	1.09	3.23	0.00	2.46	0.00	0.00	-0.01
time (sec)	N/A	0.234	1.317	45.207	0.000	0.985	0.000	0.000	0.000

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	156	482	0	355	0	0	-1
N.S.	1	1.00	1.06	3.28	0.00	2.41	0.00	0.00	-0.01
time (sec)	N/A	0.183	1.636	45.433	0.000	0.710	0.000	0.000	0.000

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	158	484	0	356	0	0	-1
N.S.	1	1.00	1.07	3.29	0.00	2.42	0.00	0.00	-0.01
time (sec)	N/A	0.186	0.907	40.987	0.000	0.909	0.000	0.000	0.000

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	226	1111	0	650	0	0	-1
N.S.	1	1.00	1.02	5.03	0.00	2.94	0.00	0.00	-0.00
time (sec)	N/A	0.413	2.649	48.086	0.000	0.918	0.000	0.000	0.000

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	268	1265	0	743	0	0	-1
N.S.	1	1.00	1.04	4.90	0.00	2.88	0.00	0.00	-0.00
time (sec)	N/A	0.504	2.940	45.044	0.000	1.002	0.000	0.000	0.000

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	199	499	0	431	0	0	-1
N.S.	1	1.00	0.77	1.93	0.00	1.67	0.00	0.00	-0.00
time (sec)	N/A	0.543	2.101	46.740	0.000	0.640	0.000	0.000	0.000

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	165	398	0	368	0	0	-1
N.S.	1	1.00	0.75	1.82	0.00	1.68	0.00	0.00	-0.00
time (sec)	N/A	0.451	3.034	47.438	0.000	0.579	0.000	0.000	0.000

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	171	644	0	370	0	0	-1
N.S.	1	1.00	0.94	3.54	0.00	2.03	0.00	0.00	-0.01
time (sec)	N/A	0.333	1.857	43.175	0.000	0.656	0.000	0.000	0.000

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	167	543	0	366	0	0	-1
N.S.	1	1.00	0.89	2.89	0.00	1.95	0.00	0.00	-0.01
time (sec)	N/A	0.332	1.770	47.201	0.000	0.688	0.000	0.000	0.000

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	171	660	0	369	0	0	-1
N.S.	1	1.00	0.93	3.59	0.00	2.01	0.00	0.00	-0.01
time (sec)	N/A	0.239	1.287	48.037	0.000	1.237	0.000	0.000	0.000

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	158	661	0	368	0	0	-1
N.S.	1	1.00	0.85	3.55	0.00	1.98	0.00	0.00	-0.01
time (sec)	N/A	0.242	2.749	48.945	0.000	1.293	0.000	0.000	0.000

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	241	1578	0	662	0	0	-1
N.S.	1	1.00	0.93	6.12	0.00	2.57	0.00	0.00	-0.00
time (sec)	N/A	0.525	3.824	46.564	0.000	0.969	0.000	0.000	0.000

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	925	0	0	0	0	0	-1
N.S.	1	1.00	6.65	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.270	8.360	0.781	0.000	0.000	0.000	0.000	0.000

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.135	2.639	0.658	0.000	0.000	0.000	0.000	0.000

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	166	0	0	0	0	0	-1
N.S.	1	1.00	4.49	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.055	0.753	0.546	0.000	0.000	0.000	0.000	0.000

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.196	2.803	0.834	0.000	0.000	0.000	0.000	0.000

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.351	16.141	0.516	0.000	0.000	0.000	0.000	0.000

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	5.624	0.681	0.000	0.000	0.000	0.000	0.000

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.120	11.320	0.753	0.000	0.000	0.000	0.000	0.000

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	5.635	0.673	0.000	0.000	0.000	0.000	0.000

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.121	6.536	0.660	0.000	0.000	0.000	0.000	0.000

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	5.216	0.616	0.000	0.000	0.000	0.000	0.000

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	66	4321	162	0	0	0	-1
N.S.	1	1.00	0.33	21.39	0.80	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.154	0.236	19.129	0.524	0.000	0.000	0.000	0.000

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	65	2253	151	0	0	0	-1
N.S.	1	1.00	0.35	12.24	0.82	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.166	18.575	0.506	0.000	0.000	0.000	0.000

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	153	2125	139	0	0	0	-1
N.S.	1	1.00	0.92	12.80	0.84	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.317	16.994	0.511	0.000	0.000	0.000	0.000

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	164	492	128	0	0	0	86
N.S.	1	1.00	1.09	3.28	0.85	0.00	0.00	0.00	0.57
time (sec)	N/A	0.086	0.237	14.324	0.527	0.000	0.000	0.000	5.454

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	194	1112	139	0	0	0	99
N.S.	1	1.00	1.17	6.70	0.84	0.00	0.00	0.00	0.60
time (sec)	N/A	0.098	0.348	48.648	0.510	0.000	0.000	0.000	5.729

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	194	1208	152	0	0	0	-1
N.S.	1	1.00	1.05	6.57	0.83	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.585	49.217	0.542	0.000	0.000	0.000	0.000

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	207	1272	165	0	0	0	-1
N.S.	1	1.00	1.02	6.30	0.82	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.136	0.885	53.507	0.561	0.000	0.000	0.000	0.000

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	215	7157	221	0	0	0	-1
N.S.	1	1.00	0.80	26.71	0.82	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.213	1.569	18.403	0.505	0.000	0.000	0.000	0.000

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	202	6328	209	0	0	0	-1
N.S.	1	1.00	0.81	25.41	0.84	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.188	1.453	18.643	0.503	0.000	0.000	0.000	0.000

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	199	3512	190	0	0	0	-1
N.S.	1	1.00	0.89	15.75	0.85	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.160	0.427	18.971	0.507	0.000	0.000	0.000	0.000

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	170	3089	177	0	0	0	-1
N.S.	1	1.00	0.83	15.14	0.87	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.137	0.500	16.588	0.537	0.000	0.000	0.000	0.000

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	173	1757	177	0	0	0	-1
N.S.	1	1.00	0.85	8.61	0.87	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.136	0.765	52.487	0.497	0.000	0.000	0.000	0.000

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	77	1733	191	0	0	0	-1
N.S.	1	1.00	0.35	7.77	0.86	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.159	0.268	56.151	0.505	0.000	0.000	0.000	0.000

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	81	1975	212	0	0	0	-1
N.S.	1	1.00	0.33	7.93	0.85	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.184	0.287	59.005	0.544	0.000	0.000	0.000	0.000

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	80	1903	224	0	0	0	-1
N.S.	1	1.00	0.30	7.10	0.84	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.209	0.399	55.305	0.543	0.000	0.000	0.000	0.000

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	229	9273	262	0	0	0	-1
N.S.	1	1.00	0.77	31.01	0.88	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.319	2.207	17.662	0.507	0.000	0.000	0.000	0.000

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	225	8774	240	0	0	0	-1
N.S.	1	1.00	0.83	32.50	0.89	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.272	2.180	18.478	0.501	0.000	0.000	0.000	0.000

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	194	4528	220	0	0	0	-1
N.S.	1	1.00	0.79	18.48	0.90	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.244	1.032	17.936	0.521	0.000	0.000	0.000	0.000

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	188	4227	218	0	0	0	-1
N.S.	1	1.00	0.77	17.25	0.89	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.245	3.204	52.275	0.504	0.000	0.000	0.000	0.000

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	100	2370	221	0	0	0	-1
N.S.	1	1.00	0.41	9.67	0.90	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.249	0.348	50.568	0.579	0.000	0.000	0.000	0.000

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	102	2472	243	0	0	0	-1
N.S.	1	1.00	0.38	9.09	0.89	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.275	0.529	64.469	0.512	0.000	0.000	0.000	0.000

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	101	2598	265	0	0	0	-1
N.S.	1	1.00	0.34	8.69	0.89	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.318	0.620	54.520	0.519	0.000	0.000	0.000	0.000

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	303	11313	207	0	0	0	-1
N.S.	1	1.00	1.12	41.75	0.76	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.458	0.386	19.338	0.517	0.000	0.000	0.000	0.000

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	264	10343	189	0	0	0	-1
N.S.	1	1.00	1.06	41.37	0.76	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.329	0.431	20.344	0.492	0.000	0.000	0.000	0.000

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	227	2278	174	0	0	0	-1
N.S.	1	1.00	0.98	9.82	0.75	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.204	0.227	16.048	0.495	0.000	0.000	0.000	0.000

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	204	1892	174	0	0	0	-1
N.S.	1	1.00	0.88	8.16	0.75	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.204	0.153	17.457	0.496	0.000	0.000	0.000	0.000

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	226	1900	175	0	0	0	-1
N.S.	1	1.00	0.97	8.19	0.75	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.203	0.171	16.658	0.551	0.000	0.000	0.000	0.000

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	193	5450	189	0	0	0	-1
N.S.	1	1.00	0.77	21.80	0.76	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.343	0.410	55.597	0.532	0.000	0.000	0.000	0.000

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	455	37365	318	0	0	0	-1
N.S.	1	1.00	1.14	93.88	0.80	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.759	6.179	22.496	0.537	0.000	0.000	0.000	0.000

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	424	28980	303	0	0	0	-1
N.S.	1	1.00	1.19	81.18	0.85	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.560	6.121	23.149	0.523	0.000	0.000	0.000	0.000

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	368	18333	283	0	0	0	-1
N.S.	1	1.00	1.16	57.65	0.89	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.383	1.721	20.422	0.505	0.000	0.000	0.000	0.000

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	301	20286	276	0	0	0	-1
N.S.	1	1.00	0.96	64.40	0.88	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.348	4.784	17.352	0.524	0.000	0.000	0.000	0.000

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	380	20172	274	0	0	0	-1
N.S.	1	1.00	1.21	64.45	0.88	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.338	6.101	17.250	0.508	0.000	0.000	0.000	0.000

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	279	20282	284	0	0	0	-1
N.S.	1	1.00	0.87	63.58	0.89	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.398	2.801	16.298	0.516	0.000	0.000	0.000	0.000

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	239	21870	322	0	0	0	-1
N.S.	1	1.00	0.67	61.26	0.90	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.581	0.614	59.641	0.517	0.000	0.000	0.000	0.000

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	493	493	564	101140	456	0	0	0	-1
N.S.	1	1.00	1.14	205.15	0.92	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.082	6.263	28.122	0.527	0.000	0.000	0.000	0.000

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	444	444	530	79208	439	0	0	0	-1
N.S.	1	1.00	1.19	178.40	0.99	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.837	6.178	32.519	0.530	0.000	0.000	0.000	0.000

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	499	50668	426	0	0	0	-1
N.S.	1	1.00	1.26	127.95	1.08	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.629	6.171	29.167	0.523	0.000	0.000	0.000	0.000

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	392	392	462	50771	418	0	0	0	-1
N.S.	1	1.00	1.18	129.52	1.07	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.658	6.146	19.072	0.536	0.000	0.000	0.000	0.000

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	492	50714	409	0	0	0	-1
N.S.	1	1.00	1.28	131.72	1.06	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.612	6.149	20.925	0.524	0.000	0.000	0.000	0.000

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	398	50673	415	0	0	0	-1
N.S.	1	1.00	1.03	131.62	1.08	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.589	2.647	22.537	0.511	0.000	0.000	0.000	0.000

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	387	50764	425	0	0	0	-1
N.S.	1	1.00	0.98	128.19	1.07	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.654	6.134	20.981	0.530	0.000	0.000	0.000	0.000

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	220	21305	0	0	0	0	-1
N.S.	1	1.00	0.84	81.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.614	2.303	36.899	0.000	0.000	0.000	0.000	0.000

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	185	10794	0	0	0	0	-1
N.S.	1	1.00	0.84	48.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.301	0.784	29.770	0.000	0.000	0.000	0.000	0.000

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	174	10498	0	0	0	0	-1
N.S.	1	1.00	0.97	58.65	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.329	0.162	37.431	0.000	0.000	0.000	0.000	0.000

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	143	2597	0	0	0	0	-1
N.S.	1	1.00	0.92	16.75	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.162	38.506	0.000	0.000	0.000	0.000	0.000

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	209	5190	0	0	0	0	-1
N.S.	1	1.00	0.99	24.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.440	0.862	35.276	0.000	0.000	0.000	0.000	0.000

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	261	12857	0	0	0	0	-1
N.S.	1	1.00	1.07	52.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.518	0.790	37.311	0.000	0.000	0.000	0.000	0.000

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	255	25460	0	0	0	0	-1
N.S.	1	1.00	0.83	83.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.847	3.299	35.276	0.000	0.000	0.000	0.000	0.000

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	219	23629	0	0	0	0	-1
N.S.	1	1.00	0.83	89.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.665	1.582	35.006	0.000	0.000	0.000	0.000	0.000

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	196	12725	0	0	0	0	-1
N.S.	1	1.00	0.92	59.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.507	1.267	33.802	0.000	0.000	0.000	0.000	0.000

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	175	11638	0	0	0	0	-1
N.S.	1	1.00	0.95	62.91	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.379	0.502	32.694	0.000	0.000	0.000	0.000	0.000

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	237	6349	0	0	0	0	-1
N.S.	1	1.00	1.12	29.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.556	0.600	34.230	0.000	0.000	0.000	0.000	0.000

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	292	13888	0	0	0	0	-1
N.S.	1	1.00	1.19	56.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.923	2.098	38.457	0.000	0.000	0.000	0.000	0.000

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	312	16676	0	0	0	0	-1
N.S.	1	1.00	1.09	58.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.023	5.999	35.702	0.000	0.000	0.000	0.000	0.000

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	329	49804	0	0	0	0	-1
N.S.	1	1.00	0.92	139.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.132	5.487	35.865	0.000	0.000	0.000	0.000	0.000

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	274	33758	0	0	0	0	-1
N.S.	1	1.00	0.88	108.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.935	3.567	35.886	0.000	0.000	0.000	0.000	0.000

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	214	33357	0	0	0	0	-1
N.S.	1	1.00	0.83	128.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.729	1.804	35.138	0.000	0.000	0.000	0.000	0.000

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	190	16676	0	0	0	0	-1
N.S.	1	1.00	0.86	75.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.535	1.131	32.751	0.000	0.000	0.000	0.000	0.000

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	253	27482	0	0	0	0	-1
N.S.	1	1.00	1.04	113.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.092	2.087	36.014	0.000	0.000	0.000	0.000	0.000

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	241	15065	0	0	0	0	-1
N.S.	1	1.00	0.97	60.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.044	1.120	35.208	0.000	0.000	0.000	0.000	0.000

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	284	17291	0	0	0	0	-1
N.S.	1	1.00	0.98	59.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.584	2.520	40.204	0.000	0.000	0.000	0.000	0.000

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	320	18203	0	0	0	0	-1
N.S.	1	1.00	0.95	54.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.586	3.998	43.032	0.000	0.000	0.000	0.000	0.000

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	193	8425	0	0	0	0	-1
N.S.	1	1.00	0.88	38.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.319	3.546	32.688	0.000	0.000	0.000	0.000	0.000

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	177	6056	0	0	0	0	-1
N.S.	1	1.00	0.95	32.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.199	0.658	38.720	0.000	0.000	0.000	0.000	0.000

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	144	2048	0	0	0	0	-1
N.S.	1	1.00	0.97	13.74	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.166	35.233	0.000	0.000	0.000	0.000	0.000

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	143	1631	0	0	0	0	-1
N.S.	1	1.00	0.92	10.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.184	33.718	0.000	0.000	0.000	0.000	0.000

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	209	4705	0	0	0	0	-1
N.S.	1	1.00	0.99	22.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.180	1.178	36.236	0.000	0.000	0.000	0.000	0.000

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	243	11480	0	0	0	0	-1
N.S.	1	1.00	0.98	46.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.548	2.468	35.543	0.000	0.000	0.000	0.000	0.000

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	241	9931	0	0	0	0	-1
N.S.	1	1.00	0.86	35.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.654	5.779	37.943	0.000	0.000	0.000	0.000	0.000

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	229	9450	0	0	0	0	-1
N.S.	1	1.00	0.98	40.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.498	5.925	38.644	0.000	0.000	0.000	0.000	0.000

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	203	4830	0	0	0	0	-1
N.S.	1	1.00	1.02	24.27	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.379	1.035	38.346	0.000	0.000	0.000	0.000	0.000

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	183	4836	0	0	0	0	-1
N.S.	1	1.00	0.97	25.59	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.363	2.050	39.409	0.000	0.000	0.000	0.000	0.000

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	202	4822	0	0	0	0	-1
N.S.	1	1.00	1.04	24.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.345	1.835	33.996	0.000	0.000	0.000	0.000	0.000

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	341	14605	0	0	0	0	-1
N.S.	1	1.00	1.34	57.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.908	2.064	36.477	0.000	0.000	0.000	0.000	0.000

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	319	15849	0	0	0	0	-1
N.S.	1	1.00	1.03	51.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.163	6.219	36.788	0.000	0.000	0.000	0.000	0.000

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	504	41735	0	0	0	0	-1
N.S.	1	1.00	1.49	123.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.876	6.301	37.546	0.000	0.000	0.000	0.000	0.000

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	296	26450	0	0	0	0	-1
N.S.	1	1.00	0.97	86.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.720	4.098	38.190	0.000	0.000	0.000	0.000	0.000

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	255	20642	0	0	0	0	-1
N.S.	1	1.00	1.01	81.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.546	2.082	27.445	0.000	0.000	0.000	0.000	0.000

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	214	19740	0	0	0	0	-1
N.S.	1	1.00	0.85	78.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.513	5.244	38.882	0.000	0.000	0.000	0.000	0.000

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	218	20598	0	0	0	0	-1
N.S.	1	1.00	0.91	86.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.510	4.157	37.081	0.000	0.000	0.000	0.000	0.000

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	209	19720	0	0	0	0	-1
N.S.	1	1.00	0.82	77.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.572	4.296	37.438	0.000	0.000	0.000	0.000	0.000

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	141	0	0	0	0	0	-1
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.303	0.948	0.536	0.000	0.000	0.000	0.000	0.000

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	107	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.499	0.424	0.000	0.000	0.000	0.000	0.000

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	88	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.221	0.308	0.000	0.000	0.000	0.000	0.000

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	145	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.281	0.636	2.320	0.000	0.000	0.000	0.000	0.000

Problem 884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	192	0	0	0	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.546	0.889	2.602	0.000	0.000	0.000	0.000	0.000

Problem 885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.191	3.406	0.378	0.000	0.000	0.000	0.000	0.000

Problem 886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.204	3.925	0.359	0.000	0.000	0.000	0.000	0.000

Problem 887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.176	4.631	0.362	0.000	0.000	0.000	0.000	0.000

Problem 888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.195	5.830	0.395	0.000	0.000	0.000	0.000	0.000

Problem 889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.188	5.827	0.351	0.000	0.000	0.000	0.000	0.000

Problem 890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	55	23	48	83	114	83	36
N.S.	1	1.00	2.20	0.92	1.92	3.32	4.56	3.32	1.44
time (sec)	N/A	0.048	0.214	0.070	0.496	1.104	0.181	0.559	4.782

Problem 891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	36	45	31	34	54	68	54	26
N.S.	1	1.44	1.80	1.24	1.36	2.16	2.72	2.16	1.04
time (sec)	N/A	0.046	0.148	0.071	0.539	0.993	0.167	0.484	4.728

Problem 892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	13	20	24	13	12
N.S.	1	1.00	1.00	1.08	1.08	1.67	2.00	1.08	1.00
time (sec)	N/A	0.018	0.014	0.022	0.522	1.123	0.075	0.434	4.693

Problem 893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	32	20	0	19	42	33	23
N.S.	1	1.00	1.39	0.87	0.00	0.83	1.83	1.43	1.00
time (sec)	N/A	0.054	0.116	0.118	0.000	1.153	0.082	0.450	4.775

Problem 894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	45	22	0	35	100	65	21
N.S.	1	1.00	1.80	0.88	0.00	1.40	4.00	2.60	0.84
time (sec)	N/A	0.055	0.556	0.151	0.000	0.879	0.161	0.532	4.810

Problem 895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	56	21	0	48	141	96	20
N.S.	1	1.00	2.24	0.84	0.00	1.92	5.64	3.84	0.80
time (sec)	N/A	0.051	0.473	0.182	0.000	1.224	0.192	0.634	4.791

Problem 896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	80	50	72	133	182	133	80
N.S.	1	1.00	1.38	0.86	1.24	2.29	3.14	2.29	1.38
time (sec)	N/A	0.071	1.132	0.082	0.486	1.188	0.322	0.709	4.804

Problem 897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	52	54	72	102	138	102	80
N.S.	1	1.00	0.85	0.89	1.18	1.67	2.26	1.67	1.31
time (sec)	N/A	0.067	0.985	0.072	0.498	1.274	0.261	0.623	4.723

Problem 898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	29	28	37	71	94	166	27
N.S.	1	1.00	0.76	0.74	0.97	1.87	2.47	4.37	0.71
time (sec)	N/A	0.044	0.061	0.042	0.499	1.170	0.175	0.565	4.636

Problem 899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	36	45	31	34	35	44	35	26
N.S.	1	1.44	1.80	1.24	1.36	1.40	1.76	1.40	1.04
time (sec)	N/A	0.046	0.158	0.066	0.551	1.482	0.121	0.502	4.542

Problem 900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	74	38	0	69	100	125	48
N.S.	1	1.00	1.35	0.69	0.00	1.25	1.82	2.27	0.87
time (sec)	N/A	0.080	0.762	0.170	0.000	1.094	0.164	0.536	4.618

Problem 901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	34	39	0	21	51	54	28
N.S.	1	1.00	1.21	1.39	0.00	0.75	1.82	1.93	1.00
time (sec)	N/A	0.069	0.262	0.182	0.000	1.035	0.125	0.591	4.597

Problem 902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	53	39	0	39	107	106	56
N.S.	1	1.00	0.91	0.67	0.00	0.67	1.84	1.83	0.97
time (sec)	N/A	0.076	0.969	0.188	0.000	0.886	0.227	0.687	4.616

Problem 903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	58	39	0	54	151	140	67
N.S.	1	1.00	0.94	0.63	0.00	0.87	2.44	2.26	1.08
time (sec)	N/A	0.078	1.069	0.224	0.000	1.268	0.263	0.813	4.716

Problem 904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	93	81	123	177	245	177	96
N.S.	1	1.00	1.06	0.92	1.40	2.01	2.78	2.01	1.09
time (sec)	N/A	0.083	1.341	0.093	0.486	0.922	0.516	0.887	4.895

Problem 905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	63	75	106	146	201	146	117
N.S.	1	1.00	0.77	0.91	1.29	1.78	2.45	1.78	1.43
time (sec)	N/A	0.069	1.214	0.081	0.524	1.089	0.431	0.776	4.707

Problem 906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	41	38	55	115	156	371	39
N.S.	1	1.00	0.69	0.64	0.93	1.95	2.64	6.29	0.66
time (sec)	N/A	0.048	0.137	0.045	0.523	0.942	0.316	0.724	4.734

Problem 907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	52	54	72	84	112	84	80
N.S.	1	1.00	0.85	0.89	1.18	1.38	1.84	1.38	1.31
time (sec)	N/A	0.063	1.034	0.074	0.531	1.197	0.280	0.615	4.749

Problem 908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	55	21	48	48	66	48	34
N.S.	1	1.00	2.20	0.84	1.92	1.92	2.64	1.92	1.36
time (sec)	N/A	0.048	0.204	0.074	0.529	1.548	0.145	0.566	4.730

Problem 909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	234	44	0	125	133	184	64
N.S.	1	1.00	3.30	0.62	0.00	1.76	1.87	2.59	0.90
time (sec)	N/A	0.084	1.005	0.174	0.000	1.013	0.218	0.571	4.783

Problem 910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	115	52	0	84	167	159	76
N.S.	1	1.00	1.39	0.63	0.00	1.01	2.01	1.92	0.92
time (sec)	N/A	0.089	1.124	0.194	0.000	1.101	0.296	0.636	4.875

Problem 911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	34	50	0	21	51	72	59
N.S.	1	1.00	0.68	1.00	0.00	0.42	1.02	1.44	1.18
time (sec)	N/A	0.076	0.278	0.194	0.000	1.303	0.186	0.806	4.820

Problem 912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	64	53	0	39	107	140	77
N.S.	1	1.00	0.74	0.61	0.00	0.45	1.23	1.61	0.89
time (sec)	N/A	0.086	1.160	0.230	0.000	1.159	0.278	0.876	4.846

Problem 913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	58	52	0	54	151	174	88
N.S.	1	1.00	0.64	0.58	0.00	0.60	1.68	1.93	0.98
time (sec)	N/A	0.088	1.338	0.286	0.000	1.531	0.356	1.094	4.862

Problem 914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	74	96	140	190	264	190	95
N.S.	1	1.00	0.74	0.96	1.40	1.90	2.64	1.90	0.95
time (sec)	N/A	0.073	1.224	0.094	0.503	1.186	0.687	0.992	4.739

Problem 915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	49	46	72	159	219	650	82
N.S.	1	1.00	0.64	0.60	0.94	2.06	2.84	8.44	1.06
time (sec)	N/A	0.051	0.248	0.060	0.502	0.832	0.457	1.225	4.583

Problem 916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	63	75	106	128	175	128	117
N.S.	1	1.00	0.77	0.91	1.29	1.56	2.13	1.56	1.43
time (sec)	N/A	0.069	1.041	0.080	0.505	1.023	0.417	0.782	4.545

Problem 917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	80	50	72	97	131	97	80
N.S.	1	1.00	1.38	0.86	1.24	1.67	2.26	1.67	1.38
time (sec)	N/A	0.071	0.945	0.069	0.534	1.124	0.301	0.729	4.604

Problem 918	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	85	22	64	61	82	61	78
N.S.	1	1.00	3.40	0.88	2.56	2.44	3.28	2.44	3.12
time (sec)	N/A	0.048	0.597	0.071	0.501	1.156	0.199	0.628	4.585

Problem 919	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	194	57	0	179	175	200	85
N.S.	1	1.00	2.04	0.60	0.00	1.88	1.84	2.11	0.89
time (sec)	N/A	0.091	2.082	0.164	0.000	1.012	0.282	0.655	4.678

Problem 920	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	279	60	0	143	199	217	93
N.S.	1	1.00	2.76	0.59	0.00	1.42	1.97	2.15	0.92
time (sec)	N/A	0.098	1.740	0.191	0.000	1.015	0.314	0.735	4.844

Problem 921	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	121	65	0	99	212	193	103
N.S.	1	1.00	1.06	0.57	0.00	0.87	1.86	1.69	0.90
time (sec)	N/A	0.094	1.047	0.239	0.000	1.218	0.361	0.853	4.767

Problem 922	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	34	66	0	21	51	88	76
N.S.	1	1.00	0.68	1.32	0.00	0.42	1.02	1.76	1.52
time (sec)	N/A	0.072	0.284	0.228	0.000	0.763	0.264	0.960	4.718

Problem 923	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	53	66	0	39	107	174	98
N.S.	1	1.00	0.61	0.76	0.00	0.45	1.23	2.00	1.13
time (sec)	N/A	0.082	1.011	0.281	0.000	1.135	0.357	1.199	4.873

Problem 924	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	376	57	0	137	143	200	82
N.S.	1	1.00	3.96	0.60	0.00	1.44	1.51	2.11	0.86
time (sec)	N/A	0.096	1.600	0.189	0.000	1.186	0.247	0.645	4.724

Problem 925	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	214	44	0	93	100	184	61
N.S.	1	1.00	3.01	0.62	0.00	1.31	1.41	2.59	0.86
time (sec)	N/A	0.087	1.429	0.170	0.000	1.462	0.206	0.597	4.623

Problem 926	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	130	38	0	41	68	125	45
N.S.	1	1.00	2.36	0.69	0.00	0.75	1.24	2.27	0.82
time (sec)	N/A	0.083	0.852	0.155	0.000	1.229	0.193	0.519	4.681

Problem 927	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	32	20	0	19	37	33	19
N.S.	1	1.00	1.39	0.87	0.00	0.83	1.61	1.43	0.83
time (sec)	N/A	0.055	0.135	0.153	0.000	1.055	0.077	0.452	4.697

Problem 928	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	29	64	0	49	117	46	32
N.S.	1	1.00	0.78	1.73	0.00	1.32	3.16	1.24	0.86
time (sec)	N/A	0.048	0.034	0.134	0.000	1.421	0.138	0.483	4.587

Problem 929	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	81	78	0	61	178	118	66
N.S.	1	1.00	0.93	0.90	0.00	0.70	2.05	1.36	0.76
time (sec)	N/A	0.079	0.772	0.190	0.000	1.457	0.202	0.549	4.751

Problem 930	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	101	91	0	73	214	123	77
N.S.	1	1.00	0.81	0.73	0.00	0.59	1.73	0.99	0.62
time (sec)	N/A	0.113	0.807	0.207	0.000	1.167	0.238	0.607	4.901

Problem 931	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	374	60	0	111	155	217	90
N.S.	1	1.00	3.70	0.59	0.00	1.10	1.53	2.15	0.89
time (sec)	N/A	0.097	1.551	0.206	0.000	1.028	0.328	0.712	4.683

Problem 932	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	113	52	0	57	122	159	73
N.S.	1	1.00	1.36	0.63	0.00	0.69	1.47	1.92	0.88
time (sec)	N/A	0.088	1.037	0.184	0.000	0.989	0.249	0.604	4.713

Problem 933	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	34	39	0	21	46	54	28
N.S.	1	1.00	1.21	1.39	0.00	0.75	1.64	1.93	1.00
time (sec)	N/A	0.071	0.283	0.179	0.000	0.924	0.135	0.594	4.740

Problem 934	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	51	22	0	35	88	65	21
N.S.	1	1.00	2.04	0.88	0.00	1.40	3.52	2.60	0.84
time (sec)	N/A	0.053	0.398	0.174	0.000	1.044	0.142	0.514	4.608

Problem 935	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	102	78	0	61	172	118	66
N.S.	1	1.00	1.01	0.77	0.00	0.60	1.70	1.17	0.65
time (sec)	N/A	0.105	0.664	0.166	0.000	1.048	0.191	0.537	4.884

Problem 936	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	39	92	0	73	221	61	38
N.S.	1	1.00	0.61	1.44	0.00	1.14	3.45	0.95	0.59
time (sec)	N/A	0.063	0.059	0.133	0.000	1.287	0.246	0.567	4.730

Problem 937	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	135	105	0	85	258	137	88
N.S.	1	1.00	1.18	0.92	0.00	0.75	2.26	1.20	0.77
time (sec)	N/A	0.090	0.965	0.206	0.000	1.212	0.351	0.673	5.222

Problem 938	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	569	84	0	173	246	287	139
N.S.	1	1.00	3.69	0.55	0.00	1.12	1.60	1.86	0.90
time (sec)	N/A	0.117	3.623	0.271	0.000	1.017	0.454	1.087	4.745

Problem 939	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	923	71	0	127	202	253	138
N.S.	1	1.00	6.89	0.53	0.00	0.95	1.51	1.89	1.03
time (sec)	N/A	0.105	6.913	0.237	0.000	1.525	0.393	1.026	5.691

Problem 940	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	143	65	0	72	168	193	102
N.S.	1	1.00	1.25	0.57	0.00	0.63	1.47	1.69	0.89
time (sec)	N/A	0.098	1.174	0.224	0.000	1.282	0.343	0.852	4.727

Problem 941	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	34	50	0	21	46	72	55
N.S.	1	1.00	0.68	1.00	0.00	0.42	0.92	1.44	1.10
time (sec)	N/A	0.075	0.276	0.214	0.000	1.606	0.201	0.793	4.715

Problem 942	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	53	39	0	39	95	106	56
N.S.	1	1.00	0.91	0.67	0.00	0.67	1.64	1.83	0.97
time (sec)	N/A	0.077	0.671	0.195	0.000	1.215	0.196	0.738	4.744

Problem 943	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	56	21	0	48	129	96	20
N.S.	1	1.00	2.24	0.84	0.00	1.92	5.16	3.84	0.80
time (sec)	N/A	0.053	0.361	0.178	0.000	1.025	0.176	0.676	4.639

Problem 944	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	115	91	0	73	207	123	77
N.S.	1	1.00	0.88	0.69	0.00	0.56	1.58	0.94	0.59
time (sec)	N/A	0.113	0.668	0.239	0.000	1.184	0.220	0.588	4.830

Problem 945	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	111	105	0	85	258	137	87
N.S.	1	1.00	0.69	0.65	0.00	0.53	1.60	0.85	0.54
time (sec)	N/A	0.124	0.971	0.185	0.000	0.846	0.338	0.666	5.250

Problem 946	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	49	118	0	97	296	72	57
N.S.	1	1.00	0.54	1.30	0.00	1.07	3.25	0.79	0.63
time (sec)	N/A	0.066	0.065	0.183	0.000	1.344	0.369	0.693	4.720

Problem 947	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	455	87	0	142	243	285	170
N.S.	1	1.00	2.84	0.54	0.00	0.89	1.52	1.78	1.06
time (sec)	N/A	0.116	3.863	0.290	0.000	1.150	0.526	1.255	6.951

Problem 948	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	151	79	0	87	209	227	146
N.S.	1	1.00	1.03	0.54	0.00	0.60	1.43	1.55	1.00
time (sec)	N/A	0.104	1.476	0.254	0.000	1.613	0.446	1.122	6.072

Problem 949	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	34	66	0	21	46	88	69
N.S.	1	1.00	0.68	1.32	0.00	0.42	0.92	1.76	1.38
time (sec)	N/A	0.072	0.389	0.201	0.000	1.180	0.257	0.984	4.725

Problem 950	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	53	53	0	39	95	140	75
N.S.	1	1.00	0.61	0.61	0.00	0.45	1.09	1.61	0.86
time (sec)	N/A	0.085	0.967	0.231	0.000	1.396	0.272	0.883	4.774

Problem 951	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	75	39	0	54	139	140	64
N.S.	1	1.00	1.21	0.63	0.00	0.87	2.24	2.26	1.03
time (sec)	N/A	0.080	0.750	0.233	0.000	1.403	0.234	0.785	4.714

Problem 952	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	74	22	0	61	167	125	21
N.S.	1	1.00	2.96	0.88	0.00	2.44	6.68	5.00	0.84
time (sec)	N/A	0.054	0.447	0.236	0.000	1.690	0.243	0.724	4.648

Problem 953	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	134	105	0	85	246	140	88
N.S.	1	1.00	0.83	0.65	0.00	0.52	1.52	0.86	0.54
time (sec)	N/A	0.122	0.776	0.224	0.000	1.632	0.276	0.640	5.171

Problem 954	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	139	119	0	97	296	149	98
N.S.	1	1.00	0.72	0.62	0.00	0.50	1.53	0.77	0.51
time (sec)	N/A	0.141	0.955	0.285	0.000	1.560	0.388	0.718	6.049

Problem 955	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	133	132	0	109	333	160	109
N.S.	1	1.00	0.60	0.59	0.00	0.49	1.49	0.72	0.49
time (sec)	N/A	0.151	1.425	0.239	0.000	1.838	0.476	0.825	7.097

Problem 956	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	61	66	70	88	0	0	155
N.S.	1	1.00	0.66	0.72	0.76	0.96	0.00	0.00	1.68
time (sec)	N/A	0.116	1.322	0.359	0.282	1.215	0.000	0.000	6.212

Problem 957	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	37	47	47	60	0	0	87
N.S.	1	1.00	0.62	0.78	0.78	1.00	0.00	0.00	1.45
time (sec)	N/A	0.099	0.923	0.232	0.284	1.355	0.000	0.000	4.793

Problem 958	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	20	27	42	20	29
N.S.	1	1.00	1.00	0.88	0.80	1.08	1.68	0.80	1.16
time (sec)	N/A	0.069	0.423	0.189	0.292	0.916	1.685	1.150	0.456

Problem 959	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	110	78	110	261	0	0	81
N.S.	1	1.00	1.16	0.82	1.16	2.75	0.00	0.00	0.85
time (sec)	N/A	0.124	0.753	0.356	0.518	1.071	0.000	0.000	0.302

Problem 960	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	136	117	159	287	0	0	132
N.S.	1	1.00	0.99	0.85	1.15	2.08	0.00	0.00	0.96
time (sec)	N/A	0.134	1.099	0.319	0.506	1.266	0.000	0.000	4.854

Problem 961	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	149	156	200	299	0	0	179
N.S.	1	1.00	0.82	0.86	1.10	1.65	0.00	0.00	0.99
time (sec)	N/A	0.144	1.490	0.338	0.519	1.013	0.000	0.000	4.979

Problem 962	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	66	70	104	0	0	95
N.S.	1	1.00	1.00	0.70	0.74	1.11	0.00	0.00	1.01
time (sec)	N/A	0.117	1.365	0.318	0.302	1.151	0.000	0.000	8.622

Problem 963	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	80	47	48	75	0	0	168
N.S.	1	1.00	1.29	0.76	0.77	1.21	0.00	0.00	2.71
time (sec)	N/A	0.106	1.033	0.299	0.292	1.544	0.000	0.000	6.788

Problem 964	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	54	22	20	41	44	20	47
N.S.	1	1.00	2.00	0.81	0.74	1.52	1.63	0.74	1.74
time (sec)	N/A	0.071	0.545	0.157	0.276	0.881	1.752	0.463	0.198

Problem 965	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	114	77	110	267	0	0	83
N.S.	1	1.00	1.20	0.81	1.16	2.81	0.00	0.00	0.87
time (sec)	N/A	0.125	0.908	0.316	0.512	0.710	0.000	0.000	0.287

Problem 966	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	136	97	157	303	0	0	134
N.S.	1	1.00	0.93	0.66	1.08	2.08	0.00	0.00	0.92
time (sec)	N/A	0.131	1.410	0.322	0.538	0.762	0.000	0.000	4.888

Problem 967	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	152	116	200	317	0	0	181
N.S.	1	1.00	0.79	0.60	1.04	1.64	0.00	0.00	0.94
time (sec)	N/A	0.149	1.878	0.329	0.503	0.864	0.000	0.000	4.937

Problem 968	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	100	66	70	123	0	0	97
N.S.	1	1.00	1.06	0.70	0.74	1.31	0.00	0.00	1.03
time (sec)	N/A	0.103	1.915	0.325	0.283	0.792	0.000	0.000	8.226

Problem 969	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	78	47	48	92	0	0	83
N.S.	1	1.00	1.26	0.76	0.77	1.48	0.00	0.00	1.34
time (sec)	N/A	0.101	1.369	0.265	0.289	0.969	0.000	0.000	9.948

Problem 970	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	70	22	20	56	44	0	120
N.S.	1	1.00	2.59	0.81	0.74	2.07	1.63	0.00	4.44
time (sec)	N/A	0.077	0.679	0.219	0.308	0.941	4.430	0.000	0.313

Problem 971	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	A	A	B	F	F	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	0	95	133	272	0	0	109
N.S.	1	1.00	0.00	0.76	1.06	2.18	0.00	0.00	0.87
time (sec)	N/A	0.134	180.004	0.296	0.499	1.499	0.000	0.000	0.355

Problem 972	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	138	95	159	311	0	0	135
N.S.	1	1.00	0.95	0.65	1.09	2.13	0.00	0.00	0.92
time (sec)	N/A	0.135	1.549	0.288	0.520	1.792	0.000	0.000	0.279

Problem 973	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	152	114	200	325	0	0	181
N.S.	1	1.00	0.79	0.59	1.04	1.68	0.00	0.00	0.94
time (sec)	N/A	0.148	2.439	0.341	0.509	1.410	0.000	0.000	4.930

Problem 974	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	94	66	73	78	0	0	113
N.S.	1	1.00	1.04	0.73	0.81	0.87	0.00	0.00	1.26
time (sec)	N/A	0.107	1.291	0.253	0.289	1.438	0.000	0.000	5.282

Problem 975	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	91	45	47	50	0	0	77
N.S.	1	1.00	1.57	0.78	0.81	0.86	0.00	0.00	1.33
time (sec)	N/A	0.101	0.987	0.264	0.301	1.400	0.000	0.000	4.860

Problem 976	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	64	22	20	45	44	0	65
N.S.	1	1.00	2.56	0.88	0.80	1.80	1.76	0.00	2.60
time (sec)	N/A	0.073	0.483	0.203	0.289	1.555	1.458	0.000	4.803

Problem 977	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	117	102	131	284	0	0	113
N.S.	1	1.00	0.94	0.82	1.06	2.29	0.00	0.00	0.91
time (sec)	N/A	0.127	1.263	0.272	0.497	1.657	0.000	0.000	5.030

Problem 978	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	141	120	176	308	0	0	156
N.S.	1	1.00	0.84	0.72	1.05	1.84	0.00	0.00	0.93
time (sec)	N/A	0.142	1.662	0.338	0.523	1.261	0.000	0.000	5.001

Problem 979	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	146	139	217	320	0	0	202
N.S.	1	1.00	0.70	0.66	1.03	1.52	0.00	0.00	0.96
time (sec)	N/A	0.152	2.072	0.376	0.525	1.605	0.000	0.000	5.014

Problem 980	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	94	64	71	65	0	0	98
N.S.	1	1.00	1.04	0.71	0.79	0.72	0.00	0.00	1.09
time (sec)	N/A	0.117	2.007	0.259	0.279	1.166	0.000	0.000	5.146

Problem 981	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	93	47	46	65	0	0	98
N.S.	1	1.00	1.55	0.78	0.77	1.08	0.00	0.00	1.63
time (sec)	N/A	0.111	1.404	0.241	0.278	1.091	0.000	0.000	5.034

Problem 982	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	72	22	20	59	46	0	93
N.S.	1	1.00	2.67	0.81	0.74	2.19	1.70	0.00	3.44
time (sec)	N/A	0.076	0.799	0.203	0.301	1.554	7.322	0.000	5.094

Problem 983	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	138	121	147	310	0	0	139
N.S.	1	1.00	0.88	0.78	0.94	1.99	0.00	0.00	0.89
time (sec)	N/A	0.141	2.058	0.339	0.501	1.342	0.000	0.000	0.490

Problem 984	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	145	139	192	334	0	0	182
N.S.	1	1.00	0.73	0.70	0.96	1.68	0.00	0.00	0.91
time (sec)	N/A	0.161	1.976	0.333	0.517	1.429	0.000	0.000	5.192

Problem 985	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	160	158	233	346	0	0	229
N.S.	1	1.00	0.66	0.65	0.96	1.43	0.00	0.00	0.95
time (sec)	N/A	0.174	2.675	0.379	0.537	1.522	0.000	0.000	5.071

Problem 986	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	98	66	68	80	0	0	121
N.S.	1	1.00	1.07	0.72	0.74	0.87	0.00	0.00	1.32
time (sec)	N/A	0.118	3.000	0.274	0.293	1.907	0.000	0.000	5.456

Problem 987	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	95	47	46	80	0	0	123
N.S.	1	1.00	1.53	0.76	0.74	1.29	0.00	0.00	1.98
time (sec)	N/A	0.107	2.101	0.258	0.289	1.789	0.000	0.000	5.552

Problem 988	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	72	22	20	72	46	0	118
N.S.	1	1.00	2.67	0.81	0.74	2.67	1.70	0.00	4.37
time (sec)	N/A	0.074	1.069	0.183	0.281	1.512	9.296	0.000	5.493

Problem 989	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	149	140	169	330	0	0	165
N.S.	1	1.00	0.79	0.74	0.90	1.76	0.00	0.00	0.88
time (sec)	N/A	0.163	2.525	0.325	0.486	1.246	0.000	0.000	0.636

Problem 990	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	182	158	214	354	0	0	208
N.S.	1	1.00	0.79	0.68	0.93	1.53	0.00	0.00	0.90
time (sec)	N/A	0.172	3.662	0.335	0.504	1.500	0.000	0.000	5.641

Problem 991	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	171	177	255	366	0	0	255
N.S.	1	1.00	0.62	0.65	0.93	1.34	0.00	0.00	0.93
time (sec)	N/A	0.184	3.296	0.301	0.492	1.111	0.000	0.000	5.140

Problem 992	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	87	154	710	380	0	325	-1
N.S.	1	1.00	0.56	1.00	4.61	2.47	0.00	2.11	-0.01
time (sec)	N/A	0.121	2.868	0.437	0.599	1.071	0.000	1.450	0.000

Problem 993	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	129	122	481	304	0	0	-1
N.S.	1	1.00	1.22	1.15	4.54	2.87	0.00	0.00	-0.01
time (sec)	N/A	0.093	1.996	0.348	0.595	1.518	0.000	0.000	0.000

Problem 994	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	74	96	114	229	0	0	62
N.S.	1	1.00	1.17	1.52	1.81	3.63	0.00	0.00	0.98
time (sec)	N/A	0.077	0.945	0.332	0.606	1.289	0.000	0.000	5.770

Problem 995	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	65	37	67	0	0	34
N.S.	1	1.00	1.00	1.59	0.90	1.63	0.00	0.00	0.83
time (sec)	N/A	0.067	0.686	0.331	0.540	1.670	0.000	0.000	0.710

Problem 996	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	68	74	0	80	0	0	135
N.S.	1	1.00	0.76	0.82	0.00	0.89	0.00	0.00	1.50
time (sec)	N/A	0.083	0.873	0.322	0.000	1.339	0.000	0.000	6.314

Problem 997	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	90	85	0	92	0	0	160
N.S.	1	1.00	0.66	0.62	0.00	0.68	0.00	0.00	1.18
time (sec)	N/A	0.091	1.269	0.373	0.000	1.399	0.000	0.000	6.150

Problem 998	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	105	95	0	104	0	0	183
N.S.	1	1.00	0.58	0.52	0.00	0.57	0.00	0.00	1.01
time (sec)	N/A	0.112	1.696	0.377	0.000	2.261	0.000	0.000	6.730

Problem 999	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	101	186	949	455	0	0	-1
N.S.	1	1.00	0.64	1.17	5.97	2.86	0.00	0.00	-0.01
time (sec)	N/A	0.112	3.426	0.331	0.622	1.505	0.000	0.000	0.000

Problem 1000	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	133	128	689	379	0	307	-1
N.S.	1	1.00	1.18	1.13	6.10	3.35	0.00	2.72	-0.01
time (sec)	N/A	0.096	2.853	0.345	0.585	1.818	0.000	1.354	0.000

Problem 1001	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	100	122	277	304	0	0	-1
N.S.	1	1.00	0.94	1.15	2.61	2.87	0.00	0.00	-0.01
time (sec)	N/A	0.097	1.507	0.343	0.570	2.063	0.000	0.000	0.000

Problem 1002	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	119	269	139	357	0	0	-1
N.S.	1	1.00	1.12	2.54	1.31	3.37	0.00	0.00	-0.01
time (sec)	N/A	0.093	1.925	0.375	0.617	1.846	0.000	0.000	0.000

Problem 1003	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	69	64	37	71	0	0	62
N.S.	1	1.00	1.60	1.49	0.86	1.65	0.00	0.00	1.44
time (sec)	N/A	0.075	1.199	0.448	0.545	1.555	0.000	0.000	5.523

Problem 1004	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	79	75	92	84	0	0	159
N.S.	1	1.00	0.88	0.83	1.02	0.93	0.00	0.00	1.77
time (sec)	N/A	0.088	1.649	0.354	0.614	1.125	0.000	0.000	6.105

Problem 1005	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	101	85	146	97	0	0	182
N.S.	1	1.00	0.74	0.62	1.07	0.71	0.00	0.00	1.34
time (sec)	N/A	0.099	2.169	0.368	0.631	1.426	0.000	0.000	6.514

Problem 1006	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	115	97	200	110	0	0	205
N.S.	1	1.00	0.63	0.53	1.10	0.60	0.00	0.00	1.13
time (sec)	N/A	0.113	2.807	0.395	0.613	1.133	0.000	0.000	7.300

Problem 1007	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	110	164	1249	534	0	0	-1
N.S.	1	1.00	0.65	0.98	7.43	3.18	0.00	0.00	-0.01
time (sec)	N/A	0.117	4.346	0.339	0.789	1.243	0.000	0.000	0.000

Problem 1008	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	169	186	949	455	0	0	-1
N.S.	1	1.00	1.06	1.17	5.97	2.86	0.00	0.00	-0.01
time (sec)	N/A	0.109	3.117	0.330	0.646	1.142	0.000	0.000	0.000

Problem 1009	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	155	153	479	380	0	314	-1
N.S.	1	1.00	1.01	0.99	3.11	2.47	0.00	2.04	-0.01
time (sec)	N/A	0.106	1.861	0.326	0.589	0.886	0.000	1.668	0.000

Problem 1010	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	113	300	203	378	0	0	-1
N.S.	1	1.00	0.74	1.96	1.33	2.47	0.00	0.00	-0.01
time (sec)	N/A	0.118	2.439	0.396	0.593	1.327	0.000	0.000	0.000

Problem 1011	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	109	350	440	405	0	0	-1
N.S.	1	1.00	0.70	2.26	2.84	2.61	0.00	0.00	-0.01
time (sec)	N/A	0.117	2.700	0.456	0.647	1.173	0.000	0.000	0.000

Problem 1012	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	90	75	41	75	0	0	65
N.S.	1	1.00	2.09	1.74	0.95	1.74	0.00	0.00	1.51
time (sec)	N/A	0.071	2.135	0.373	0.631	0.806	0.000	0.000	7.765

Problem 1013	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	100	87	100	90	0	0	161
N.S.	1	1.00	1.11	0.97	1.11	1.00	0.00	0.00	1.79
time (sec)	N/A	0.085	2.982	0.391	0.609	1.168	0.000	0.000	6.470

Problem 1014	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	112	99	158	105	0	0	184
N.S.	1	1.00	0.82	0.73	1.16	0.77	0.00	0.00	1.35
time (sec)	N/A	0.096	3.615	0.388	0.638	1.106	0.000	0.000	7.330

Problem 1015	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	128	110	216	120	0	0	207
N.S.	1	1.00	0.70	0.60	1.19	0.66	0.00	0.00	1.14
time (sec)	N/A	0.110	5.317	0.374	0.569	0.987	0.000	0.000	8.148

Problem 1016	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	340	328	843	416	0	0	-1
N.S.	1	1.00	1.67	1.61	4.13	2.04	0.00	0.00	-0.00
time (sec)	N/A	0.129	8.564	0.416	0.587	1.005	0.000	0.000	0.000

Problem 1017	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	155	299	578	358	0	0	-1
N.S.	1	1.00	1.01	1.95	3.78	2.34	0.00	0.00	-0.01
time (sec)	N/A	0.109	3.240	0.430	0.587	1.108	0.000	0.000	0.000

Problem 1018	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	123	267	343	337	0	0	-1
N.S.	1	1.00	1.16	2.52	3.24	3.18	0.00	0.00	-0.01
time (sec)	N/A	0.095	2.129	0.386	0.556	0.961	0.000	0.000	0.000

Problem 1019	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	64	63	27	68	0	0	49
N.S.	1	1.00	1.56	1.54	0.66	1.66	0.00	0.00	1.20
time (sec)	N/A	0.062	0.786	0.366	0.552	1.100	0.000	0.000	5.377

Problem 1020	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	64	82	17	71	0	0	112
N.S.	1	1.00	1.45	1.86	0.39	1.61	0.00	0.00	2.55
time (sec)	N/A	0.069	0.776	0.352	0.566	1.340	0.000	0.000	5.480

Problem 1021	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	73	109	0	119	0	0	135
N.S.	1	1.00	0.78	1.16	0.00	1.27	0.00	0.00	1.44
time (sec)	N/A	0.084	1.071	0.382	0.000	1.117	0.000	0.000	5.627

Problem 1022	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	99	118	0	131	0	0	158
N.S.	1	1.00	0.71	0.84	0.00	0.94	0.00	0.00	1.13
time (sec)	N/A	0.095	1.659	0.405	0.000	0.952	0.000	0.000	5.963

Problem 1023	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	115	130	0	143	0	0	183
N.S.	1	1.00	0.62	0.70	0.00	0.77	0.00	0.00	0.98
time (sec)	N/A	0.112	2.233	0.418	0.000	0.789	0.000	0.000	6.421

Problem 1024	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	386	409	979	451	0	0	-1
N.S.	1	1.00	1.51	1.60	3.84	1.77	0.00	0.00	-0.00
time (sec)	N/A	0.148	9.794	0.386	0.697	0.989	0.000	0.000	0.000

Problem 1025	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	348	379	706	385	0	0	-1
N.S.	1	1.00	1.71	1.86	3.46	1.89	0.00	0.00	-0.00
time (sec)	N/A	0.131	8.959	0.355	0.579	0.813	0.000	0.000	0.000

Problem 1026	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	184	348	414	385	0	0	-1
N.S.	1	1.00	1.19	2.25	2.67	2.48	0.00	0.00	-0.01
time (sec)	N/A	0.116	5.474	0.367	0.611	0.801	0.000	0.000	0.000

Problem 1027	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	87	62	37	71	0	0	108
N.S.	1	1.00	2.02	1.44	0.86	1.65	0.00	0.00	2.51
time (sec)	N/A	0.076	1.524	0.344	0.540	0.742	0.000	0.000	0.768

Problem 1028	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	89	70	0	81	0	0	114
N.S.	1	1.00	0.99	0.78	0.00	0.90	0.00	0.00	1.27
time (sec)	N/A	0.081	1.192	0.365	0.000	0.934	0.000	0.000	5.051

Problem 1029	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	89	109	0	119	0	0	117
N.S.	1	1.00	0.65	0.80	0.00	0.87	0.00	0.00	0.85
time (sec)	N/A	0.097	1.350	0.402	0.000	0.904	0.000	0.000	0.393

Problem 1030	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	103	95	48	95	0	0	138
N.S.	1	1.00	1.02	0.94	0.48	0.94	0.00	0.00	1.37
time (sec)	N/A	0.086	1.711	0.366	0.623	0.825	0.000	0.000	5.542

Problem 1031	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	93	130	0	143	0	0	163
N.S.	1	1.00	0.63	0.88	0.00	0.97	0.00	0.00	1.11
time (sec)	N/A	0.102	2.648	0.383	0.000	1.061	0.000	0.000	5.911

Problem 1032	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	151	141	0	155	0	0	186
N.S.	1	1.00	0.78	0.73	0.00	0.80	0.00	0.00	0.96
time (sec)	N/A	0.115	3.535	0.391	0.000	1.522	0.000	0.000	6.525

Problem 1033	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	459	490	1159	466	0	0	-1
N.S.	1	1.00	1.51	1.61	3.81	1.53	0.00	0.00	-0.00
time (sec)	N/A	0.169	13.029	0.339	0.612	1.258	0.000	0.000	0.000

Problem 1034	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	390	460	826	400	0	0	-1
N.S.	1	1.00	1.54	1.82	3.26	1.58	0.00	0.00	-0.00
time (sec)	N/A	0.150	11.511	0.348	0.610	1.277	0.000	0.000	0.000

Problem 1035	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	205	429	472	400	0	0	-1
N.S.	1	1.00	1.00	2.10	2.31	1.96	0.00	0.00	-0.00
time (sec)	N/A	0.135	9.698	0.352	0.595	1.217	0.000	0.000	0.000

Problem 1036	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	91	75	41	75	0	0	114
N.S.	1	1.00	2.12	1.74	0.95	1.74	0.00	0.00	2.65
time (sec)	N/A	0.075	2.612	0.382	0.559	0.970	0.000	0.000	5.291

Problem 1037	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	106	71	120	84	0	0	136
N.S.	1	1.00	1.18	0.79	1.33	0.93	0.00	0.00	1.51
time (sec)	N/A	0.086	2.177	0.388	0.635	1.191	0.000	0.000	5.467

Problem 1038	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	102	83	0	93	0	0	137
N.S.	1	1.00	0.75	0.61	0.00	0.68	0.00	0.00	1.01
time (sec)	N/A	0.095	1.899	0.394	0.000	1.076	0.000	0.000	5.477

Problem 1039	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	106	118	0	131	0	0	128
N.S.	1	1.00	0.57	0.63	0.00	0.70	0.00	0.00	0.69
time (sec)	N/A	0.114	2.091	0.397	0.000	1.179	0.000	0.000	5.356

Problem 1040	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	130	130	0	143	0	0	140
N.S.	1	1.00	0.56	0.56	0.00	0.61	0.00	0.00	0.60
time (sec)	N/A	0.133	2.905	0.414	0.000	1.364	0.000	0.000	5.529

Problem 1041	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	117	105	76	119	0	0	163
N.S.	1	1.00	0.76	0.68	0.49	0.77	0.00	0.00	1.06
time (sec)	N/A	0.106	3.697	0.353	0.629	1.082	0.000	0.000	6.127

Problem 1042	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	115	151	0	167	0	0	186
N.S.	1	1.00	0.58	0.76	0.00	0.84	0.00	0.00	0.93
time (sec)	N/A	0.120	5.530	0.373	0.000	0.886	0.000	0.000	6.703

Problem 1043	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	180	1784	905	247	2225	0	229
N.S.	1	1.00	1.34	13.31	6.75	1.84	16.60	0.00	1.71
time (sec)	N/A	0.118	4.441	1.075	0.595	1.102	1.742	0.000	9.227

Problem 1044	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	110	129	576	155	979	0	230
N.S.	1	1.00	1.11	1.30	5.82	1.57	9.89	0.00	2.32
time (sec)	N/A	0.108	2.963	1.294	0.567	1.145	0.994	0.000	1.683

Problem 1045	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	72	78	287	81	311	0	112
N.S.	1	1.00	1.12	1.22	4.48	1.27	4.86	0.00	1.75
time (sec)	N/A	0.093	1.624	0.997	0.560	0.932	0.531	0.000	0.388

Problem 1046	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	51	25	25	28	70	23	40
N.S.	1	1.00	1.96	0.96	0.96	1.08	2.69	0.88	1.54
time (sec)	N/A	0.063	0.417	0.215	0.516	1.166	0.211	1.215	4.855

Problem 1047	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	79	0	0	0	0	0	-1
N.S.	1	1.00	1.52	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.092	58.693	2.902	0.000	0.000	0.000	0.000	0.000

Problem 1048	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F(-2)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.088	180.002	2.694	0.000	0.000	0.000	0.000	0.000

Problem 1049	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.088	124.566	2.745	0.000	0.000	0.000	0.000	0.000

Problem 1050	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	87	142	0	0	0	0	0	-1
N.S.	1	1.32	2.15	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.071	13.836	2.669	0.000	0.000	0.000	0.000	0.000

Problem 1051	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	B	B	F	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	0	5385	0	255	2225	0	332
N.S.	1	1.00	0.00	40.19	0.00	1.90	16.60	0.00	2.48
time (sec)	N/A	0.125	61.692	1.279	0.000	0.938	1.642	0.000	9.873

Problem 1052	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	161	129	0	163	979	0	229
N.S.	1	1.00	1.63	1.30	0.00	1.65	9.89	0.00	2.31
time (sec)	N/A	0.104	14.757	1.211	0.000	1.035	0.866	0.000	1.361

Problem 1053	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	131	78	0	89	313	0	112
N.S.	1	1.00	2.05	1.22	0.00	1.39	4.89	0.00	1.75
time (sec)	N/A	0.093	41.633	0.939	0.000	0.932	0.463	0.000	0.372

Problem 1054	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	95	25	25	38	71	23	46
N.S.	1	1.00	3.65	0.96	0.96	1.46	2.73	0.88	1.77
time (sec)	N/A	0.060	1.969	0.203	0.541	1.119	0.217	1.166	0.174

Problem 1055	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	133	0	0	0	0	0	-1
N.S.	1	1.00	2.56	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.087	16.047	0.989	0.000	0.000	0.000	0.000	0.000

Problem 1056	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F(-2)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.088	180.009	2.579	0.000	0.000	0.000	0.000	0.000

Problem 1057	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F(-2)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.085	180.000	0.991	0.000	0.000	0.000	0.000	0.000

Problem 1058	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F(-2)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.085	180.003	2.445	0.000	0.000	0.000	0.000	0.000

Problem 1059	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	88	141	0	0	0	0	0	-1
N.S.	1	1.31	2.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.077	65.040	3.546	0.000	0.000	0.000	0.000	0.000

Problem 1060	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	88	141	0	0	0	0	0	-1
N.S.	1	1.31	2.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	6.241	0.836	0.000	0.000	0.000	0.000	0.000

Problem 1061	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	86	141	0	0	0	0	0	-1
N.S.	1	1.32	2.17	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.067	1.550	1.224	0.000	0.000	0.000	0.000	0.000

Problem 1062	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	86	141	0	0	0	0	0	-1
N.S.	1	1.32	2.17	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.069	6.026	0.879	0.000	0.000	0.000	0.000	0.000

Problem 1063	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	88	141	0	0	0	0	0	-1
N.S.	1	1.31	2.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.077	106.732	0.938	0.000	0.000	0.000	0.000	0.000

Problem 1064	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F(-2)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	88	0	0	0	0	0	0	-1
N.S.	1	1.31	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	180.001	3.088	0.000	0.000	0.000	0.000	0.000

Problem 1065	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	331	100	114	205	184	333	125
N.S.	1	1.00	3.01	0.91	1.04	1.86	1.67	3.03	1.14
time (sec)	N/A	0.064	3.396	0.115	0.506	0.959	0.388	0.597	5.417

Problem 1066	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	263	76	87	141	122	228	76
N.S.	1	1.00	3.29	0.95	1.09	1.76	1.52	2.85	0.95
time (sec)	N/A	0.050	1.856	0.110	0.524	1.051	0.322	0.530	5.069

Problem 1067	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	66	50	55	67	53	110	38
N.S.	1	1.00	1.43	1.09	1.20	1.46	1.15	2.39	0.83
time (sec)	N/A	0.021	0.044	0.085	0.546	0.927	0.254	0.460	4.884

Problem 1068	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	102	70	0	44	87	90	45
N.S.	1	1.00	2.17	1.49	0.00	0.94	1.85	1.91	0.96
time (sec)	N/A	0.030	0.455	0.174	0.000	0.944	0.134	0.448	5.010

Problem 1069	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	94	91	0	57	162	117	70
N.S.	1	1.00	1.18	1.14	0.00	0.71	2.02	1.46	0.88
time (sec)	N/A	0.044	0.577	0.202	0.000	1.068	0.188	0.540	5.020

Problem 1070	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	150	112	0	80	258	140	111
N.S.	1	1.00	1.34	1.00	0.00	0.71	2.30	1.25	0.99
time (sec)	N/A	0.059	0.747	0.212	0.000	1.286	0.254	0.667	5.103

Problem 1071	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	948	176	187	371	313	670	217
N.S.	1	1.00	6.20	1.15	1.22	2.42	2.05	4.38	1.42
time (sec)	N/A	0.139	7.266	0.126	0.490	1.122	0.610	0.759	4.925

Problem 1072	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	261	135	149	286	236	512	139
N.S.	1	1.00	2.25	1.16	1.28	2.47	2.03	4.41	1.20
time (sec)	N/A	0.112	3.193	0.136	0.503	1.131	0.441	0.657	4.917

Problem 1073	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	175	94	98	158	119	301	75
N.S.	1	1.00	2.24	1.21	1.26	2.03	1.53	3.86	0.96
time (sec)	N/A	0.065	1.479	0.109	0.508	1.093	0.357	0.557	5.115

Problem 1074	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	155	93	0	88	170	131	112
N.S.	1	1.00	2.07	1.24	0.00	1.17	2.27	1.75	1.49
time (sec)	N/A	0.061	1.008	0.244	0.000	1.132	0.264	0.565	5.624

Problem 1075	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	134	123	0	83	258	176	93
N.S.	1	1.00	1.47	1.35	0.00	0.91	2.84	1.93	1.02
time (sec)	N/A	0.101	1.072	0.208	0.000	0.727	0.240	0.639	5.241

Problem 1076	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	256	152	0	114	401	215	148
N.S.	1	1.00	1.98	1.18	0.00	0.88	3.11	1.67	1.15
time (sec)	N/A	0.117	1.066	0.227	0.000	0.741	0.388	0.781	5.443

Problem 1077	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	1564	269	269	577	476	1117	356
N.S.	1	1.00	8.23	1.42	1.42	3.04	2.51	5.88	1.87
time (sec)	N/A	0.237	8.152	0.187	0.499	0.933	0.807	1.032	5.189

Problem 1078	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	733	210	224	469	381	904	223
N.S.	1	1.00	5.20	1.49	1.59	3.33	2.70	6.41	1.58
time (sec)	N/A	0.140	6.394	0.160	0.499	0.898	0.730	0.942	5.064

Problem 1079	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	219	155	150	288	223	597	122
N.S.	1	1.00	2.05	1.45	1.40	2.69	2.08	5.58	1.14
time (sec)	N/A	0.091	3.473	0.095	0.513	1.080	0.501	0.746	5.070

Problem 1080	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	236	133	0	204	260	186	175
N.S.	1	1.00	1.83	1.03	0.00	1.58	2.02	1.44	1.36
time (sec)	N/A	0.107	2.042	0.242	0.000	0.770	0.445	0.734	5.664

Problem 1081	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	305	159	0	131	393	226	184
N.S.	1	1.00	2.24	1.17	0.00	0.96	2.89	1.66	1.35
time (sec)	N/A	0.209	2.030	0.245	0.000	1.042	0.488	0.817	5.662

Problem 1082	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	260	197	0	143	552	286	183
N.S.	1	1.00	1.86	1.41	0.00	1.02	3.94	2.04	1.31
time (sec)	N/A	0.155	1.605	0.275	0.000	0.879	0.460	0.952	5.607

Problem 1083	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	229	116	144	216	257	247	99
N.S.	1	1.00	1.99	1.01	1.25	1.88	2.23	2.15	0.86
time (sec)	N/A	0.246	4.560	0.271	0.518	0.910	10.036	0.603	7.425

Problem 1084	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	176	95	117	86	92	127	64
N.S.	1	1.00	1.66	0.90	1.10	0.81	0.87	1.20	0.60
time (sec)	N/A	0.089	1.807	0.209	0.495	1.041	4.221	0.536	5.754

Problem 1085	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	95	83	93	44	44	71	43
N.S.	1	1.00	2.11	1.84	2.07	0.98	0.98	1.58	0.96
time (sec)	N/A	0.052	0.574	0.198	0.491	1.258	0.830	0.463	5.175

Problem 1086	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	206	117	0	131	248	178	2639
N.S.	1	1.00	1.61	0.91	0.00	1.02	1.94	1.39	20.62
time (sec)	N/A	0.104	1.268	0.295	0.000	1.163	2.611	0.507	9.310

Problem 1087	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	372	177	0	188	610	296	1384
N.S.	1	1.00	2.14	1.02	0.00	1.08	3.51	1.70	7.95
time (sec)	N/A	0.282	1.515	0.408	0.000	1.252	5.907	0.553	9.235

Problem 1088	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	435	248	0	275	1192	445	1952
N.S.	1	1.00	1.86	1.06	0.00	1.18	5.09	1.90	8.34
time (sec)	N/A	0.461	2.021	0.558	0.000	1.352	14.339	0.687	10.002

Problem 1089	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	1936	175	249	303	374	389	133
N.S.	1	1.00	13.63	1.23	1.75	2.13	2.63	2.74	0.94
time (sec)	N/A	0.258	7.662	0.281	0.484	1.262	16.932	0.641	6.817

Problem 1090	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	253	153	218	143	156	232	139
N.S.	1	1.00	2.72	1.65	2.34	1.54	1.68	2.49	1.49
time (sec)	N/A	0.133	2.354	0.225	0.488	1.120	1.750	0.593	5.183

Problem 1091	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	302	139	184	128	143	186	135
N.S.	1	1.00	4.03	1.85	2.45	1.71	1.91	2.48	1.80
time (sec)	N/A	0.106	2.481	0.237	0.507	0.862	1.674	0.552	5.126

Problem 1092	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	385	163	0	336	510	342	1334
N.S.	1	1.00	1.91	0.81	0.00	1.66	2.52	1.69	6.60
time (sec)	N/A	0.243	2.992	0.450	0.000	1.352	14.843	0.585	8.769

Problem 1093	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	476	227	0	513	966	496	1984
N.S.	1	1.00	1.76	0.84	0.00	1.89	3.56	1.83	7.32
time (sec)	N/A	0.385	4.071	0.702	0.000	1.076	31.062	0.689	10.329

Problem 1094	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	633	298	0	609	1792	648	2653
N.S.	1	1.00	1.77	0.83	0.00	1.71	5.02	1.82	7.43
time (sec)	N/A	0.642	5.223	1.033	0.000	1.299	73.486	0.828	11.672

Problem 1095	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	595	239	409	311	377	470	314
N.S.	1	1.00	4.44	1.78	3.05	2.32	2.81	3.51	2.34
time (sec)	N/A	0.182	2.801	0.339	0.503	1.551	4.032	0.798	5.617

Problem 1096	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	317	216	386	316	391	472	297
N.S.	1	1.00	2.54	1.73	3.09	2.53	3.13	3.78	2.38
time (sec)	N/A	0.182	4.530	0.312	0.522	0.905	4.205	0.739	5.509

Problem 1097	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	315	200	331	278	354	364	281
N.S.	1	1.00	3.03	1.92	3.18	2.67	3.40	3.50	2.70
time (sec)	N/A	0.155	3.587	0.282	0.535	1.484	4.049	0.653	5.372

Problem 1098	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	474	239	0	719	821	496	1910
N.S.	1	1.00	1.74	0.88	0.00	2.63	3.01	1.82	7.00
time (sec)	N/A	0.331	7.460	0.636	0.000	1.123	54.147	0.706	10.105

Problem 1099	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	354	354	4395	304	0	918	1583	806	2640
N.S.	1	1.00	12.42	0.86	0.00	2.59	4.47	2.28	7.46
time (sec)	N/A	0.533	8.171	1.010	0.000	1.214	102.385	0.834	12.085

Problem 1100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	448	448	5726	374	0	1151	0	782	2500
N.S.	1	1.00	12.78	0.83	0.00	2.57	0.00	1.75	5.58
time (sec)	N/A	0.833	8.531	1.686	0.000	1.235	0.000	1.033	13.521

Problem 1101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	219	740	0	532	0	277	200
N.S.	1	1.00	1.46	4.93	0.00	3.55	0.00	1.85	1.33
time (sec)	N/A	0.292	4.551	0.384	0.000	1.321	0.000	0.666	8.889

Problem 1102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	155	704	0	413	0	227	90
N.S.	1	1.00	1.55	7.04	0.00	4.13	0.00	2.27	0.90
time (sec)	N/A	0.166	2.977	0.247	0.000	1.042	0.000	0.565	7.069

Problem 1103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	88	685	4714	318	0	185	854
N.S.	1	1.00	1.28	9.93	68.32	4.61	0.00	2.68	12.38
time (sec)	N/A	0.098	1.223	0.291	0.702	0.909	0.000	0.475	7.196

Problem 1104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	339	128	0	714	0	360	763
N.S.	1	1.00	2.42	0.91	0.00	5.10	0.00	2.57	5.45
time (sec)	N/A	0.244	5.235	0.801	0.000	1.516	0.000	0.524	7.739

Problem 1105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	281	269	0	1115	0	473	2500
N.S.	1	1.00	1.33	1.27	0.00	5.28	0.00	2.24	11.85
time (sec)	N/A	0.406	1.987	0.454	0.000	1.366	0.000	0.574	8.167

Problem 1106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	329	382	0	1478	0	632	2500
N.S.	1	1.00	1.18	1.36	0.00	5.28	0.00	2.26	8.93
time (sec)	N/A	0.663	2.902	0.439	0.000	1.802	0.000	0.687	10.253

Problem 1107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	271	910	0	810	0	337	309
N.S.	1	1.00	1.50	5.03	0.00	4.48	0.00	1.86	1.71
time (sec)	N/A	0.365	7.610	0.292	0.000	1.860	0.000	0.963	13.956

Problem 1108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	221	872	0	666	0	288	196
N.S.	1	1.00	1.69	6.66	0.00	5.08	0.00	2.20	1.50
time (sec)	N/A	0.215	3.977	0.273	0.000	1.460	0.000	0.806	10.316

Problem 1109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	111	852	0	520	0	241	2869
N.S.	1	1.00	1.13	8.69	0.00	5.31	0.00	2.46	29.28
time (sec)	N/A	0.150	2.051	0.254	0.000	1.120	0.000	0.559	16.322

Problem 1110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	376	140	0	808	0	407	847
N.S.	1	1.00	2.46	0.92	0.00	5.28	0.00	2.66	5.54
time (sec)	N/A	0.227	4.523	0.374	0.000	1.080	0.000	0.574	7.342

Problem 1111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	272	284	0	1012	0	462	1580
N.S.	1	1.00	1.30	1.36	0.00	4.84	0.00	2.21	7.56
time (sec)	N/A	0.440	1.955	0.423	0.000	1.260	0.000	0.663	7.757

Problem 1112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	311	362	0	1268	0	624	2500
N.S.	1	1.00	1.14	1.32	0.00	4.63	0.00	2.28	9.12
time (sec)	N/A	0.682	2.855	0.418	0.000	1.806	0.000	0.755	9.210

Problem 1113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	528	1043	0	1115	0	423	400
N.S.	1	1.00	2.44	4.83	0.00	5.16	0.00	1.96	1.85
time (sec)	N/A	0.433	9.918	0.325	0.000	2.541	0.000	1.350	27.628

Problem 1114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	271	1003	0	941	0	374	257
N.S.	1	1.00	1.63	6.04	0.00	5.67	0.00	2.25	1.55
time (sec)	N/A	0.271	7.387	0.279	0.000	1.024	0.000	1.120	19.220

Problem 1115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	208	970	0	752	0	315	2500
N.S.	1	1.00	1.59	7.40	0.00	5.74	0.00	2.40	19.08
time (sec)	N/A	0.199	3.291	0.263	0.000	1.096	0.000	0.677	28.125

Problem 1116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	260	237	0	1053	0	464	2500
N.S.	1	1.00	1.41	1.28	0.00	5.69	0.00	2.51	13.51
time (sec)	N/A	0.288	2.147	0.324	0.000	0.992	0.000	0.620	9.358

Problem 1117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	291	302	0	1112	0	520	2500
N.S.	1	1.00	1.34	1.39	0.00	5.12	0.00	2.40	11.52
time (sec)	N/A	0.425	2.122	0.364	0.000	0.982	0.000	0.784	9.188

Problem 1118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	324	452	0	1282	0	602	2500
N.S.	1	1.00	1.14	1.59	0.00	4.50	0.00	2.11	8.77
time (sec)	N/A	0.672	2.898	0.419	0.000	1.345	0.000	0.822	9.338

Problem 1119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	168	793	0	443	0	243	119
N.S.	1	1.00	1.33	6.29	0.00	3.52	0.00	1.93	0.94
time (sec)	N/A	0.195	3.778	0.309	0.000	0.849	0.000	0.681	6.548

Problem 1120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	95	759	0	353	0	185	67
N.S.	1	1.00	1.28	10.26	0.00	4.77	0.00	2.50	0.91
time (sec)	N/A	0.104	2.140	0.292	0.000	1.110	0.000	0.587	5.914

Problem 1121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	71	733	6772	285	0	156	2947
N.S.	1	1.00	1.54	15.93	147.22	6.20	0.00	3.39	64.07
time (sec)	N/A	0.052	1.097	0.360	0.700	0.806	0.000	0.513	6.402

Problem 1122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	222	150	0	1018	0	376	2500
N.S.	1	1.00	1.43	0.97	0.00	6.57	0.00	2.43	16.13
time (sec)	N/A	0.211	1.548	0.396	0.000	1.417	0.000	0.572	8.306

Problem 1123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	275	232	0	1429	0	494	2500
N.S.	1	1.00	1.24	1.05	0.00	6.47	0.00	2.24	11.31
time (sec)	N/A	0.419	2.245	0.351	0.000	1.575	0.000	0.640	9.169

Problem 1124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	324	357	0	1777	0	669	2500
N.S.	1	1.00	1.09	1.20	0.00	5.96	0.00	2.24	8.39
time (sec)	N/A	0.661	3.418	0.362	0.000	1.424	0.000	0.834	11.142

Problem 1125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	219	1056	0	620	0	247	182
N.S.	1	1.00	1.58	7.60	0.00	4.46	0.00	1.78	1.31
time (sec)	N/A	0.220	4.371	0.290	0.000	1.245	0.000	0.783	6.587

Problem 1126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	189	1030	0	600	0	209	142
N.S.	1	1.00	2.05	11.20	0.00	6.52	0.00	2.27	1.54
time (sec)	N/A	0.157	3.519	0.294	0.000	0.872	0.000	0.715	6.385

Problem 1127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	158	1015	0	550	0	192	2500
N.S.	1	1.00	2.08	13.36	0.00	7.24	0.00	2.53	32.89
time (sec)	N/A	0.117	2.621	0.313	0.000	1.002	0.000	0.641	14.427

Problem 1128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	297	257	0	1589	0	476	2500
N.S.	1	1.00	1.45	1.25	0.00	7.75	0.00	2.32	12.20
time (sec)	N/A	0.328	3.413	0.347	0.000	1.530	0.000	0.811	15.809

Problem 1129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	388	399	0	2298	0	588	2500
N.S.	1	1.00	1.38	1.42	0.00	8.18	0.00	2.09	8.90
time (sec)	N/A	0.540	4.918	0.320	0.000	2.274	0.000	0.939	15.104

Problem 1130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	468	596	0	2674	0	785	2500
N.S.	1	1.00	1.27	1.62	0.00	7.27	0.00	2.13	6.79
time (sec)	N/A	0.862	7.868	0.379	0.000	4.597	0.000	1.404	13.881

Problem 1131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	232	920	0	983	0	314	255
N.S.	1	1.00	1.47	5.82	0.00	6.22	0.00	1.99	1.61
time (sec)	N/A	0.315	7.367	0.304	0.000	1.497	0.000	1.016	9.145

Problem 1132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	218	899	0	929	0	250	221
N.S.	1	1.00	1.72	7.08	0.00	7.31	0.00	1.97	1.74
time (sec)	N/A	0.238	5.138	0.289	0.000	1.042	0.000	0.903	8.904

Problem 1133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	198	885	0	843	0	228	2500
N.S.	1	1.00	1.82	8.12	0.00	7.73	0.00	2.09	22.94
time (sec)	N/A	0.189	4.025	0.317	0.000	1.016	0.000	0.809	25.068

Problem 1134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	371	364	0	2699	0	547	2500
N.S.	1	1.00	1.39	1.36	0.00	10.11	0.00	2.05	9.36
time (sec)	N/A	0.453	6.822	0.472	0.000	4.025	0.000	1.091	93.425

Problem 1135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F(-1)	B	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	1004	518	0	3287	0	701	-1
N.S.	1	1.00	2.86	1.48	0.00	9.36	0.00	2.00	-0.00
time (sec)	N/A	0.703	9.870	0.485	0.000	10.194	0.000	1.333	0.000

Problem 1136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F(-1)	B	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	446	446	1160	743	0	3845	0	1061	-1
N.S.	1	1.00	2.60	1.67	0.00	8.62	0.00	2.38	-0.00
time (sec)	N/A	1.072	10.928	0.422	0.000	20.585	0.000	1.288	0.000

Problem 1137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	589	1079	0	1073	0	0	-1
N.S.	1	1.00	2.24	4.10	0.00	4.08	0.00	0.00	-0.00
time (sec)	N/A	0.678	8.003	0.717	0.000	0.995	0.000	0.000	0.000

Problem 1138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	559	866	0	773	0	0	-1
N.S.	1	1.00	2.24	3.46	0.00	3.09	0.00	0.00	-0.00
time (sec)	N/A	0.630	6.289	0.518	0.000	0.787	0.000	0.000	0.000

Problem 1139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	442	866	0	509	0	0	2101
N.S.	1	1.00	2.93	5.74	0.00	3.37	0.00	0.00	13.91
time (sec)	N/A	0.272	3.745	0.517	0.000	0.863	0.000	0.000	20.353

Problem 1140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	182	877	0	369	0	0	1724
N.S.	1	1.00	1.50	7.25	0.00	3.05	0.00	0.00	14.25
time (sec)	N/A	0.145	3.440	4.225	0.000	0.700	0.000	0.000	19.833

Problem 1141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	249	1180	0	437	0	0	-1
N.S.	1	1.00	1.41	6.67	0.00	2.47	0.00	0.00	-0.01
time (sec)	N/A	0.214	4.379	0.690	0.000	0.871	0.000	0.000	0.000

Problem 1142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	302	2226	0	521	0	0	-1
N.S.	1	1.00	1.19	8.76	0.00	2.05	0.00	0.00	-0.00
time (sec)	N/A	0.550	5.822	0.753	0.000	0.862	0.000	0.000	0.000

Problem 1143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	686	1518	0	1512	0	0	-1
N.S.	1	1.00	2.09	4.61	0.00	4.60	0.00	0.00	-0.00
time (sec)	N/A	0.887	10.302	0.503	0.000	1.079	0.000	0.000	0.000

Problem 1144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	574	1234	0	1142	0	0	-1
N.S.	1	1.00	1.82	3.92	0.00	3.63	0.00	0.00	-0.00
time (sec)	N/A	0.871	7.205	0.543	0.000	0.912	0.000	0.000	0.000

Problem 1145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	507	1702	0	796	0	0	-1
N.S.	1	1.00	2.59	8.68	0.00	4.06	0.00	0.00	-0.01
time (sec)	N/A	0.473	7.855	0.595	0.000	0.890	0.000	0.000	0.000

Problem 1146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	518	1169	0	928	0	0	-1
N.S.	1	1.00	2.66	5.99	0.00	4.76	0.00	0.00	-0.01
time (sec)	N/A	0.459	7.882	0.516	0.000	0.918	0.000	0.000	0.000

Problem 1147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	240	1275	0	506	0	0	-1
N.S.	1	1.00	1.39	7.37	0.00	2.92	0.00	0.00	-0.01
time (sec)	N/A	0.222	4.484	0.703	0.000	0.985	0.000	0.000	0.000

Problem 1148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	302	1657	0	590	0	0	-1
N.S.	1	1.00	1.34	7.36	0.00	2.62	0.00	0.00	-0.00
time (sec)	N/A	0.302	5.967	0.726	0.000	1.635	0.000	0.000	0.000

Problem 1149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-2)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	415	415	849	1851	0	1965	0	0	-1
N.S.	1	1.00	2.05	4.46	0.00	4.73	0.00	0.00	-0.00
time (sec)	N/A	1.139	13.200	0.529	0.000	1.370	0.000	0.000	0.000

Problem 1150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	645	1503	0	1533	0	0	-1
N.S.	1	1.00	1.71	3.98	0.00	4.06	0.00	0.00	-0.00
time (sec)	N/A	1.139	8.952	0.530	0.000	1.692	0.000	0.000	0.000

Problem 1151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	539	2130	0	1151	0	0	-1
N.S.	1	1.00	2.10	8.29	0.00	4.48	0.00	0.00	-0.00
time (sec)	N/A	0.683	6.295	0.574	0.000	1.311	0.000	0.000	0.000

Problem 1152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	549	2647	0	1196	0	0	-1
N.S.	1	1.00	2.20	10.59	0.00	4.78	0.00	0.00	-0.00
time (sec)	N/A	0.661	8.316	0.556	0.000	1.834	0.000	0.000	0.000

Problem 1153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	560	3161	0	1105	0	0	-1
N.S.	1	1.00	2.18	12.30	0.00	4.30	0.00	0.00	-0.00
time (sec)	N/A	0.655	8.690	0.513	0.000	1.221	0.000	0.000	0.000

Problem 1154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	292	2559	0	674	0	0	-1
N.S.	1	1.00	1.30	11.37	0.00	3.00	0.00	0.00	-0.00
time (sec)	N/A	0.310	6.695	0.531	0.000	1.434	0.000	0.000	0.000

Problem 1155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	602	1295	0	809	0	0	-1
N.S.	1	1.00	3.01	6.48	0.00	4.04	0.00	0.00	-0.00
time (sec)	N/A	0.436	7.588	0.656	0.000	1.548	0.000	0.000	0.000

Problem 1156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	505	983	0	557	0	0	-1
N.S.	1	1.00	3.34	6.51	0.00	3.69	0.00	0.00	-0.01
time (sec)	N/A	0.276	5.728	0.650	0.000	1.038	0.000	0.000	0.000

Problem 1157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	147	212	3834	271	0	0	473
N.S.	1	1.00	1.79	2.59	46.76	3.30	0.00	0.00	5.77
time (sec)	N/A	0.070	2.921	0.851	0.707	1.234	0.000	0.000	10.046

Problem 1158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	195	1738	0	398	0	0	1508
N.S.	1	1.00	1.12	9.99	0.00	2.29	0.00	0.00	8.67
time (sec)	N/A	0.207	3.423	0.908	0.000	1.482	0.000	0.000	19.623

Problem 1159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	249	2946	0	506	0	0	-1
N.S.	1	1.00	1.29	15.26	0.00	2.62	0.00	0.00	-0.01
time (sec)	N/A	0.311	4.010	0.819	0.000	1.349	0.000	0.000	0.000

Problem 1160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	309	5218	0	590	0	0	-1
N.S.	1	1.00	1.18	19.92	0.00	2.25	0.00	0.00	-0.00
time (sec)	N/A	0.550	5.597	0.819	0.000	1.060	0.000	0.000	0.000

Problem 1161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	718	1815	0	974	0	0	-1
N.S.	1	1.00	3.44	8.68	0.00	4.66	0.00	0.00	-0.00
time (sec)	N/A	0.487	11.644	0.589	0.000	1.727	0.000	0.000	0.000

Problem 1162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	209	2564	0	571	0	0	-1
N.S.	1	1.00	1.62	19.88	0.00	4.43	0.00	0.00	-0.01
time (sec)	N/A	0.149	5.179	0.593	0.000	1.319	0.000	0.000	0.000

Problem 1163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	337	1291	8970	619	0	0	-1
N.S.	1	1.00	2.61	10.01	69.53	4.80	0.00	0.00	-0.01
time (sec)	N/A	0.147	4.644	0.674	1.048	0.817	0.000	0.000	0.000

Problem 1164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	267	3196	0	694	0	0	-1
N.S.	1	1.00	1.38	16.47	0.00	3.58	0.00	0.00	-0.01
time (sec)	N/A	0.337	4.644	0.724	0.000	1.383	0.000	0.000	0.000

Problem 1165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	333	4835	0	1004	0	0	-1
N.S.	1	1.00	1.24	17.97	0.00	3.73	0.00	0.00	-0.00
time (sec)	N/A	0.585	7.301	0.659	0.000	1.440	0.000	0.000	0.000

Problem 1166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	788	7870	0	1097	0	0	-1
N.S.	1	1.00	2.26	22.55	0.00	3.14	0.00	0.00	-0.00
time (sec)	N/A	0.872	9.352	0.635	0.000	2.224	0.000	0.000	0.000

Problem 1167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	283	2968	0	873	0	0	-1
N.S.	1	1.00	1.56	16.40	0.00	4.82	0.00	0.00	-0.01
time (sec)	N/A	0.258	7.054	0.621	0.000	1.217	0.000	0.000	0.000

Problem 1168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	278	2907	0	1030	0	0	-1
N.S.	1	1.00	1.55	16.24	0.00	5.75	0.00	0.00	-0.01
time (sec)	N/A	0.235	6.411	0.613	0.000	1.573	0.000	0.000	0.000

Problem 1169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	394	2448	0	992	0	0	-1
N.S.	1	1.00	2.10	13.02	0.00	5.28	0.00	0.00	-0.01
time (sec)	N/A	0.310	4.968	0.729	0.000	1.050	0.000	0.000	0.000

Problem 1170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	687	4889	0	1325	0	0	-1
N.S.	1	1.00	2.48	17.65	0.00	4.78	0.00	0.00	-0.00
time (sec)	N/A	0.590	8.970	0.709	0.000	1.390	0.000	0.000	0.000

Problem 1171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	354	354	803	7061	0	1572	0	0	-1
N.S.	1	1.00	2.27	19.95	0.00	4.44	0.00	0.00	-0.00
time (sec)	N/A	0.855	9.368	0.649	0.000	1.267	0.000	0.000	0.000

Problem 1172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	444	444	928	10145	0	1801	0	0	-1
N.S.	1	1.00	2.09	22.85	0.00	4.06	0.00	0.00	-0.00
time (sec)	N/A	1.194	10.281	0.727	0.000	1.401	0.000	0.000	0.000

Problem 1173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.125	7.820	0.575	0.000	0.000	0.000	0.000	0.000

Problem 1174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.238	16.170	2.931	0.000	0.000	0.000	0.000	0.000

Problem 1175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	4.017	0.559	0.000	0.000	0.000	0.000	0.000

Problem 1176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.053	1.753	0.509	0.000	0.000	0.000	0.000	0.000

Problem 1177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.204	26.780	3.270	0.000	0.000	0.000	0.000	0.000

Problem 1178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.384	8.134	3.476	0.000	0.000	0.000	0.000	0.000

Problem 1179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.750	27.939	3.366	0.000	0.000	0.000	0.000	0.000

Problem 1180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.300	50.217	3.306	0.000	0.000	0.000	0.000	0.000

Problem 1181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	422	0	0	0	0	0	-1
N.S.	1	1.00	3.55	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	18.177	3.007	0.000	0.000	0.000	0.000	0.000

Problem 1182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	171	0	0	0	0	0	-1
N.S.	1	1.00	2.19	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	6.287	0.635	0.000	0.000	0.000	0.000	0.000

Problem 1183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.178	14.661	1.228	0.000	0.000	0.000	0.000	0.000

Problem 1184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.336	28.487	3.531	0.000	0.000	0.000	0.000	0.000

Problem 1185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-1)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.629	47.349	3.994	0.000	0.000	0.000	0.000	0.000

Problem 1186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.136	27.043	3.003	0.000	0.000	0.000	0.000	0.000

Problem 1187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	2.538	2.685	0.000	0.000	0.000	0.000	0.000

Problem 1188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	20.448	2.782	0.000	0.000	0.000	0.000	0.000

Problem 1189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	11.138	2.837	0.000	0.000	0.000	0.000	0.000

Problem 1190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.110	17.142	2.712	0.000	0.000	0.000	0.000	0.000

Problem 1191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	130	159	153	151	240	2046	141
N.S.	1	1.00	0.93	1.14	1.09	1.08	1.71	14.61	1.01
time (sec)	N/A	0.107	1.108	0.090	0.541	1.133	0.148	1.346	5.273

Problem 1192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	96	97	96	95	143	968	91
N.S.	1	1.00	1.10	1.11	1.10	1.09	1.64	11.13	1.05
time (sec)	N/A	0.056	0.458	0.079	0.658	1.168	0.108	0.753	5.217

Problem 1193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	59	51	53	52	73	355	55
N.S.	1	1.00	1.40	1.21	1.26	1.24	1.74	8.45	1.31
time (sec)	N/A	0.018	0.040	0.062	0.585	0.953	0.099	0.480	5.167

Problem 1194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	66	83	92	78	524	98	94
N.S.	1	1.00	1.14	1.43	1.59	1.34	9.03	1.69	1.62
time (sec)	N/A	0.052	0.130	0.171	0.526	1.117	0.469	0.476	5.678

Problem 1195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	190	141	184	231	2878	241	152
N.S.	1	1.00	1.71	1.27	1.66	2.08	25.93	2.17	1.37
time (sec)	N/A	0.102	2.288	0.176	0.526	1.453	0.868	0.540	5.485

Problem 1196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	243	206	339	510	0	426	279
N.S.	1	1.00	1.39	1.18	1.94	2.91	0.00	2.43	1.59
time (sec)	N/A	0.183	4.581	0.269	0.615	1.494	0.000	0.655	5.646

Problem 1197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	221	307	259	257	445	4557	259
N.S.	1	1.00	1.03	1.43	1.20	1.20	2.07	21.20	1.20
time (sec)	N/A	0.181	2.552	0.112	0.629	0.974	0.204	2.682	5.347

Problem 1198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	185	189	164	162	258	2258	230
N.S.	1	1.00	1.41	1.44	1.25	1.24	1.97	17.24	1.76
time (sec)	N/A	0.128	1.131	0.100	0.564	0.635	0.140	1.281	5.276

Problem 1199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	96	97	95	94	143	968	91
N.S.	1	1.00	1.08	1.09	1.07	1.06	1.61	10.88	1.02
time (sec)	N/A	0.057	0.457	0.080	0.526	0.756	0.115	0.755	5.182

Problem 1200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	108	117	126	133	1025	127	115
N.S.	1	1.00	1.05	1.14	1.22	1.29	9.95	1.23	1.12
time (sec)	N/A	0.083	0.173	0.197	0.556	0.697	0.645	0.567	5.733

Problem 1201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	321	200	234	310	4258	332	208
N.S.	1	1.00	2.55	1.59	1.86	2.46	33.79	2.63	1.65
time (sec)	N/A	0.157	2.149	0.224	0.566	0.933	0.961	0.627	7.123

Problem 1202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	291	304	437	715	0	614	367
N.S.	1	1.00	1.36	1.42	2.04	3.34	0.00	2.87	1.71
time (sec)	N/A	0.248	3.735	0.263	0.532	1.029	0.000	0.729	7.302

Problem 1203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	299	495	382	380	711	8276	494
N.S.	1	1.00	0.99	1.64	1.26	1.26	2.35	27.40	1.64
time (sec)	N/A	0.358	6.425	0.135	0.590	1.322	0.272	5.776	5.383

Problem 1204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	221	307	252	250	445	4557	259
N.S.	1	1.00	1.01	1.40	1.15	1.14	2.03	20.81	1.18
time (sec)	N/A	0.183	2.299	0.130	0.535	0.946	0.182	2.670	5.270

Problem 1205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	130	159	148	146	240	2046	142
N.S.	1	1.00	0.90	1.10	1.03	1.01	1.67	14.21	0.99
time (sec)	N/A	0.118	1.033	0.092	0.710	1.190	0.146	1.336	5.226

Problem 1206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	126	164	175	206	1712	176	178
N.S.	1	1.00	0.90	1.17	1.25	1.47	12.23	1.26	1.27
time (sec)	N/A	0.189	0.729	0.198	0.620	1.383	0.846	0.719	5.587

Problem 1207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	535	281	312	509	6730	448	271
N.S.	1	1.00	2.33	1.22	1.36	2.21	29.26	1.95	1.18
time (sec)	N/A	0.243	4.625	0.305	0.601	1.413	1.375	0.795	7.819

Problem 1208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	327	422	539	907	0	830	466
N.S.	1	1.00	1.37	1.77	2.26	3.79	0.00	3.47	1.95
time (sec)	N/A	0.356	5.768	0.321	0.656	1.426	0.000	0.948	8.005

Problem 1209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	160	221	229	305	2516	238	235
N.S.	1	1.00	0.84	1.16	1.21	1.61	13.24	1.25	1.24
time (sec)	N/A	0.333	1.301	0.234	0.641	1.613	1.373	0.987	5.776

Problem 1210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	126	164	178	211	1712	177	177
N.S.	1	1.00	0.88	1.14	1.24	1.47	11.89	1.23	1.23
time (sec)	N/A	0.181	0.775	0.229	0.567	1.411	0.874	0.742	5.485

Problem 1211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	108	117	126	137	1025	126	115
N.S.	1	1.00	1.05	1.14	1.22	1.33	9.95	1.22	1.12
time (sec)	N/A	0.086	0.159	0.219	0.530	0.812	0.633	0.571	5.600

Problem 1212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	65	82	91	79	524	97	93
N.S.	1	1.00	1.10	1.39	1.54	1.34	8.88	1.64	1.58
time (sec)	N/A	0.049	0.122	0.176	0.559	1.243	0.471	0.469	5.494

Problem 1213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	143	133	180	207	8053	201	173
N.S.	1	1.00	1.21	1.13	1.53	1.75	68.25	1.70	1.47
time (sec)	N/A	0.108	0.368	0.259	0.546	1.058	11.754	0.509	5.754

Problem 1214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	302	202	389	766	0	542	309
N.S.	1	1.00	1.65	1.10	2.13	4.19	0.00	2.96	1.69
time (sec)	N/A	0.318	3.349	0.463	0.581	1.509	0.000	0.566	6.969

Problem 1215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	529	309	808	1941	0	1111	609
N.S.	1	1.00	1.90	1.11	2.90	6.96	0.00	3.98	2.18
time (sec)	N/A	0.631	6.895	0.806	0.647	2.760	0.000	0.703	12.025

Problem 1216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	C	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	1789	364	379	713	8928	589	347
N.S.	1	1.00	6.28	1.28	1.33	2.50	31.33	2.07	1.22
time (sec)	N/A	0.539	7.101	0.364	0.560	1.630	2.077	1.059	9.380

Problem 1217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	538	281	314	500	6730	447	272
N.S.	1	1.00	2.41	1.26	1.41	2.24	30.18	2.00	1.22
time (sec)	N/A	0.253	4.792	0.277	0.550	1.659	1.501	0.834	7.752

Problem 1218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	320	200	233	300	4258	331	208
N.S.	1	1.00	2.54	1.59	1.85	2.38	33.79	2.63	1.65
time (sec)	N/A	0.156	2.167	0.208	0.543	1.184	1.008	0.649	6.953

Problem 1219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	189	141	181	228	2878	241	153
N.S.	1	1.00	1.70	1.27	1.63	2.05	25.93	2.17	1.38
time (sec)	N/A	0.109	2.249	0.209	0.532	1.329	0.890	0.542	5.609

Problem 1220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	306	203	389	721	0	541	374
N.S.	1	1.00	1.66	1.10	2.11	3.92	0.00	2.94	2.03
time (sec)	N/A	0.357	2.237	0.392	0.544	1.630	0.000	0.581	7.355

Problem 1221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	556	289	885	2231	0	1395	725
N.S.	1	1.00	1.92	1.00	3.05	7.69	0.00	4.81	2.50
time (sec)	N/A	0.707	6.948	0.679	0.626	2.905	0.000	0.678	10.380

Problem 1222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	457	457	840	419	1842	4767	0	1759	1421
N.S.	1	1.00	1.84	0.92	4.03	10.43	0.00	3.85	3.11
time (sec)	N/A	1.243	7.300	1.387	0.693	4.388	0.000	0.867	34.607

Problem 1223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	2775	553	646	1259	0	1066	578
N.S.	1	1.00	6.83	1.36	1.59	3.10	0.00	2.63	1.42
time (sec)	N/A	0.530	7.050	0.495	0.619	1.242	0.000	1.205	10.251

Problem 1224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	327	421	534	870	0	830	466
N.S.	1	1.00	1.36	1.75	2.22	3.62	0.00	3.46	1.94
time (sec)	N/A	0.361	5.741	0.298	0.563	1.268	0.000	0.948	8.287

Problem 1225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	292	303	422	672	0	614	367
N.S.	1	1.00	1.32	1.37	1.91	3.04	0.00	2.78	1.66
time (sec)	N/A	0.276	4.308	0.274	0.572	1.141	0.000	0.745	7.415

Problem 1226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	244	208	327	491	0	422	279
N.S.	1	1.00	1.38	1.18	1.85	2.77	0.00	2.38	1.58
time (sec)	N/A	0.203	4.525	0.278	0.671	1.056	0.000	0.650	5.661

Problem 1227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	409	311	806	1855	0	1112	719
N.S.	1	1.00	1.43	1.09	2.82	6.49	0.00	3.89	2.51
time (sec)	N/A	0.666	5.599	0.717	0.602	2.904	0.000	0.679	12.644

Problem 1228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	457	457	833	419	1858	4817	0	1758	1417
N.S.	1	1.00	1.82	0.92	4.07	10.54	0.00	3.85	3.10
time (sec)	N/A	1.283	7.302	1.324	0.697	6.130	0.000	0.905	18.886

Problem 1229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	194	1096	0	0	0	0	2500
N.S.	1	1.00	0.93	5.24	0.00	0.00	0.00	0.00	11.96
time (sec)	N/A	0.386	2.317	0.515	0.000	0.000	0.000	0.000	21.263

Problem 1230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	149	890	0	16199	0	0	2500
N.S.	1	1.00	0.95	5.67	0.00	103.18	0.00	0.00	15.92
time (sec)	N/A	0.243	0.588	0.433	0.000	99.739	0.000	0.000	9.663

Problem 1231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	120	632	0	8653	0	0	845
N.S.	1	1.00	0.98	5.18	0.00	70.93	0.00	0.00	6.93
time (sec)	N/A	0.154	0.147	0.449	0.000	11.891	0.000	0.000	7.649

Problem 1232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	158	693	0	16728	0	0	2500
N.S.	1	1.00	0.93	4.08	0.00	98.40	0.00	0.00	14.71
time (sec)	N/A	0.329	0.340	0.554	0.000	102.595	0.000	0.000	9.764

Problem 1233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	276	992	0	32681	0	0	2500
N.S.	1	1.00	1.19	4.29	0.00	141.48	0.00	0.00	10.82
time (sec)	N/A	0.539	2.077	0.516	0.000	240.135	0.000	0.000	11.425

Problem 1234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	747	1277	0	0	0	0	2500
N.S.	1	1.00	2.18	3.73	0.00	0.00	0.00	0.00	7.31
time (sec)	N/A	1.067	6.353	0.538	0.000	0.000	0.000	0.000	35.523

Problem 1235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	247	1748	0	0	0	0	2500
N.S.	1	1.00	0.96	6.83	0.00	0.00	0.00	0.00	9.77
time (sec)	N/A	0.510	3.464	0.532	0.000	0.000	0.000	0.000	79.384

Problem 1236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	202	1366	0	0	0	0	2500
N.S.	1	1.00	1.04	7.01	0.00	0.00	0.00	0.00	12.82
time (sec)	N/A	0.306	1.706	0.450	0.000	0.000	0.000	0.000	21.863

Problem 1237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	140	918	0	0	0	0	2823
N.S.	1	1.00	0.93	6.12	0.00	0.00	0.00	0.00	18.82
time (sec)	N/A	0.215	0.560	0.419	0.000	0.000	0.000	0.000	17.414

Problem 1238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	168	957	0	0	0	0	2500
N.S.	1	1.00	0.99	5.63	0.00	0.00	0.00	0.00	14.71
time (sec)	N/A	0.339	0.407	0.558	0.000	0.000	0.000	0.000	13.014

Problem 1239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	316	1413	0	0	0	0	2500
N.S.	1	1.00	1.32	5.91	0.00	0.00	0.00	0.00	10.46
time (sec)	N/A	0.625	3.291	0.542	0.000	0.000	0.000	0.000	12.952

Problem 1240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	2093	1872	0	0	0	0	2500
N.S.	1	1.00	6.14	5.49	0.00	0.00	0.00	0.00	7.33
time (sec)	N/A	1.301	6.340	0.608	0.000	0.000	0.000	0.000	36.703

Problem 1241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	413	2491	0	0	0	0	-1
N.S.	1	1.00	1.28	7.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.634	6.210	0.503	0.000	0.000	0.000	0.000	0.000

Problem 1242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	262	1892	0	0	0	0	2500
N.S.	1	1.00	1.13	8.19	0.00	0.00	0.00	0.00	10.82
time (sec)	N/A	0.409	2.144	0.483	0.000	0.000	0.000	0.000	68.218

Problem 1243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	233	1251	0	0	0	0	2500
N.S.	1	1.00	1.24	6.65	0.00	0.00	0.00	0.00	13.30
time (sec)	N/A	0.300	1.130	0.454	0.000	0.000	0.000	0.000	34.414

Problem 1244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	199	1278	0	0	0	0	2500
N.S.	1	1.00	1.02	6.55	0.00	0.00	0.00	0.00	12.82
time (sec)	N/A	0.614	0.591	0.555	0.000	0.000	0.000	0.000	13.247

Problem 1245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	333	1863	0	0	0	0	2500
N.S.	1	1.00	1.37	7.67	0.00	0.00	0.00	0.00	10.29
time (sec)	N/A	0.768	5.120	0.539	0.000	0.000	0.000	0.000	16.399

Problem 1246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	F(-1)	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	2946	2485	0	0	0	0	2500
N.S.	1	1.00	8.30	7.00	0.00	0.00	0.00	0.00	7.04
time (sec)	N/A	1.295	6.472	0.546	0.000	0.000	0.000	0.000	44.603

Problem 1247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	235	3444	0	0	0	0	2500
N.S.	1	1.00	0.95	13.89	0.00	0.00	0.00	0.00	10.08
time (sec)	N/A	0.492	3.447	0.432	0.000	0.000	0.000	0.000	16.985

Problem 1248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	178	2772	0	0	0	0	3017
N.S.	1	1.00	1.00	15.57	0.00	0.00	0.00	0.00	16.95
time (sec)	N/A	0.290	1.106	0.473	0.000	0.000	0.000	0.000	9.566

Problem 1249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	129	2010	0	14968	0	0	2287
N.S.	1	1.00	0.96	15.00	0.00	111.70	0.00	0.00	17.07
time (sec)	N/A	0.172	0.238	0.433	0.000	55.508	0.000	0.000	7.234

Problem 1250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	101	1399	0	8324	0	0	2909
N.S.	1	1.00	0.99	13.72	0.00	81.61	0.00	0.00	28.52
time (sec)	N/A	0.114	0.152	0.462	0.000	4.676	0.000	0.000	7.353

Problem 1251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	158	1468	0	23416	0	0	2500
N.S.	1	1.00	0.93	8.64	0.00	137.74	0.00	0.00	14.71
time (sec)	N/A	0.303	0.442	0.523	0.000	95.663	0.000	0.000	10.268

Problem 1252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	258	2144	0	47721	0	0	2500
N.S.	1	1.00	1.06	8.79	0.00	195.58	0.00	0.00	10.25
time (sec)	N/A	0.614	2.432	0.558	0.000	174.501	0.000	0.000	15.261

Problem 1253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	415	5655	0	0	0	0	2500
N.S.	1	1.00	1.31	17.84	0.00	0.00	0.00	0.00	7.89
time (sec)	N/A	0.635	6.360	0.514	0.000	0.000	0.000	0.000	23.382

Problem 1254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	287	4572	0	0	0	0	2500
N.S.	1	1.00	1.33	21.17	0.00	0.00	0.00	0.00	11.57
time (sec)	N/A	0.351	1.842	0.533	0.000	0.000	0.000	0.000	13.180

Problem 1255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	124	3311	0	35751	0	0	2500
N.S.	1	1.00	0.83	22.07	0.00	238.34	0.00	0.00	16.67
time (sec)	N/A	0.221	0.186	0.458	0.000	240.231	0.000	0.000	12.224

Problem 1256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	113	2268	0	20798	0	0	2500
N.S.	1	1.00	0.82	16.43	0.00	150.71	0.00	0.00	18.12
time (sec)	N/A	0.172	0.202	0.437	0.000	36.059	0.000	0.000	11.430

Problem 1257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	247	2353	0	62414	0	0	2500
N.S.	1	1.00	1.17	11.15	0.00	295.80	0.00	0.00	11.85
time (sec)	N/A	0.647	2.131	0.569	0.000	208.288	0.000	0.000	20.908

Problem 1258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	628	3448	0	0	0	0	-1
N.S.	1	1.00	2.00	10.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.020	6.269	0.596	0.000	0.000	0.000	0.000	0.000

Problem 1259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	368	8858	0	0	0	0	2500
N.S.	1	1.00	1.27	30.54	0.00	0.00	0.00	0.00	8.62
time (sec)	N/A	0.717	3.662	0.550	0.000	0.000	0.000	0.000	35.673

Problem 1260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	284	6646	0	0	0	0	2500
N.S.	1	1.00	1.30	30.35	0.00	0.00	0.00	0.00	11.42
time (sec)	N/A	0.461	1.588	0.494	0.000	0.000	0.000	0.000	24.497

Problem 1261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	127	5088	0	0	0	0	2500
N.S.	1	1.00	0.65	26.09	0.00	0.00	0.00	0.00	12.82
time (sec)	N/A	0.343	0.217	0.514	0.000	0.000	0.000	0.000	22.063

Problem 1262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	115	3236	0	35738	0	0	2500
N.S.	1	1.00	0.62	17.40	0.00	192.14	0.00	0.00	13.44
time (sec)	N/A	0.275	0.182	0.508	0.000	148.694	0.000	0.000	22.926

Problem 1263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	323	3329	0	0	0	0	-1
N.S.	1	1.00	1.19	12.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.029	5.215	0.575	0.000	0.000	0.000	0.000	0.000

Problem 1264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	425	2536	5222	0	0	0	0	-1
N.S.	1	1.00	5.97	12.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.786	6.275	0.661	0.000	0.000	0.000	0.000	0.000

Problem 1265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	565	0	0	0	0	0	-1
N.S.	1	1.00	1.68	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.843	6.029	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	1518	0	0	0	0	0	-1
N.S.	1	1.00	5.88	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.640	6.121	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	261	0	0	0	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.691	2.145	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	167	1278217828	0	0	0	0	-1
N.S.	1	1.00	1.02	7841827.17	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.187	291.364	0.000	0.000	0.000	0.000	0.000

Problem 1269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	224	0	0	0	0	0	-1
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.509	1.738	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	266	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.814	4.858	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	2566	0	0	0	0	0	-1
N.S.	1	1.00	7.78	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.187	6.144	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	1526	1613618305	0	0	0	0	-1
N.S.	1	1.00	5.91	6254334.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.568	6.093	305.833	0.000	0.000	0.000	0.000	0.000

Problem 1273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	292	0	0	0	0	0	-1
N.S.	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.823	1.667	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	231	0	0	0	0	0	-1
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.562	1.929	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	264	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.889	3.346	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	498	0	0	0	0	0	-1
N.S.	1	1.00	1.27	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.420	6.268	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	773	0	0	0	0	0	-1
N.S.	1	1.00	1.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.089	8.215	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	550	0	0	0	0	0	-1
N.S.	1	1.00	1.62	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.841	6.704	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	432	0	0	0	0	0	-1
N.S.	1	1.00	1.64	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.895	2.990	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	1187	0	0	0	0	0	-1
N.S.	1	1.00	4.35	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.384	6.207	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	350	0	0	0	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.077	5.577	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	440	0	0	0	0	0	-1
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.594	6.036	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	438	0	0	0	0	0	-1
N.S.	1	1.00	1.66	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.849	3.425	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	294	0	0	0	0	0	-1
N.S.	1	1.00	1.35	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.846	1.985	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	167	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.168	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F(-2)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	166	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.167	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	232	0	0	0	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.511	1.512	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	308	0	0	0	0	0	-1
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.851	2.219	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	1877	0	0	0	0	0	-1
N.S.	1	1.00	5.27	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.910	6.355	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	1503	0	0	0	0	0	-1
N.S.	1	1.00	5.51	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.194	6.195	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	231	0	0	0	0	0	-1
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.538	2.060	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	224	0	0	0	0	0	-1
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.459	1.693	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	243	0	0	0	0	0	-1
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.493	1.115	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	349	0	0	0	0	0	-1
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.848	4.949	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	417	417	601	0	0	0	0	0	-1
N.S.	1	1.00	1.44	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.321	6.482	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-1)	F(-1)	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	470	470	2261	0	0	0	0	0	-1
N.S.	1	1.00	4.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.619	6.753	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	1883	0	0	0	0	0	-1
N.S.	1	1.00	5.43	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.211	6.426	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	350	0	0	0	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.053	5.844	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	264	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.876	3.150	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	266	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.798	4.932	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	316	0	0	0	0	0	-1
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.855	2.101	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	433	433	610	0	0	0	0	0	-1
N.S.	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.329	6.482	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F(-2)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	596	596	1050	0	0	0	0	0	-1
N.S.	1	1.00	1.76	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.079	6.642	180.000	0.000	0.000	0.000	0.000	0.000

Problem 1304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.232	2.128	0.220	0.000	0.000	0.000	0.000	0.000

Problem 1305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	234	189	0	0	0	0	0	-1
N.S.	1	1.09	0.88	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.328	2.066	0.223	0.000	0.000	0.000	0.000	0.000

Problem 1306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	135	0	0	0	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.152	0.268	0.167	0.000	0.000	0.000	0.000	0.000

Problem 1307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	120	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.232	0.107	0.000	0.000	0.000	0.000	0.000

Problem 1308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	118	0	0	0	0	0	-1
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.196	0.173	0.075	0.000	0.000	0.000	0.000	0.000

Problem 1309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	178	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.274	0.598	0.189	0.000	0.000	0.000	0.000	0.000

Problem 1310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	299	266	0	0	0	0	0	-1
N.S.	1	0.99	0.88	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.563	4.327	0.272	0.000	0.000	0.000	0.000	0.000

Problem 1311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	455	455	670	0	0	0	0	0	-1
N.S.	1	1.00	1.47	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.056	6.288	0.383	0.000	0.000	0.000	0.000	0.000

Problem 1312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.255	8.369	0.150	0.000	0.000	0.000	0.000	0.000

Problem 1313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.192	0.809	0.140	0.000	0.000	0.000	0.000	0.000

Problem 1314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.188	5.445	0.138	0.000	0.000	0.000	0.000	0.000

Problem 1315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.246	9.838	0.132	0.000	0.000	0.000	0.000	0.000

Problem 1316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.263	20.863	0.131	0.000	0.000	0.000	0.000	0.000

Problem 1317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	5.745	0.180	0.000	0.000	0.000	0.000	0.000

Problem 1318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	981	0	0	0	0	0	-1
N.S.	1	1.00	7.43	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.166	8.800	0.166	0.000	0.000	0.000	0.000	0.000

Problem 1319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	2.934	0.143	0.000	0.000	0.000	0.000	0.000

Problem 1320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	173	0	0	0	0	0	-1
N.S.	1	1.00	3.20	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.055	0.911	0.116	0.000	0.000	0.000	0.000	0.000

Problem 1321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.151	18.966	0.190	0.000	0.000	0.000	0.000	0.000

Problem 1322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.228	6.412	0.201	0.000	0.000	0.000	0.000	0.000

Problem 1323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.184	1.664	0.316	0.000	0.000	0.000	0.000	0.000

Problem 1324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	163	0	0	0	0	0	-1
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.204	0.977	0.231	0.000	0.000	0.000	0.000	0.000

Problem 1325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	136	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.172	0.589	0.167	0.000	0.000	0.000	0.000	0.000

Problem 1326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	117	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.331	0.110	0.000	0.000	0.000	0.000	0.000

Problem 1327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	166	0	0	0	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.213	0.762	0.326	0.000	0.000	0.000	0.000	0.000

Problem 1328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	231	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.281	1.937	0.421	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [992] had the largest ratio of [35]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	3	1.00	22	0.136
2	A	5	3	1.00	22	0.136
3	A	4	3	1.00	22	0.136
4	A	3	3	1.00	22	0.136
5	A	2	2	1.00	20	0.100
6	A	2	1	1.00	13	0.077
7	A	2	2	1.00	20	0.100
8	A	3	3	1.00	22	0.136
9	A	4	3	1.00	22	0.136
10	A	5	3	1.00	22	0.136
11	A	6	3	1.00	22	0.136
12	A	7	3	1.00	22	0.136
13	A	6	4	1.00	24	0.167
14	A	5	4	1.00	24	0.167
15	A	3	3	1.00	24	0.125
16	A	3	3	1.00	22	0.136
17	A	2	2	1.00	15	0.133
18	A	3	2	1.00	22	0.091
19	A	3	3	1.00	24	0.125
20	A	4	4	1.00	24	0.167
21	A	5	4	1.00	24	0.167
22	A	6	4	1.00	24	0.167
23	A	7	4	1.00	24	0.167
24	A	6	5	1.00	24	0.208
25	A	4	4	1.00	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	4	4	1.00	22	0.182
27	A	3	3	1.00	15	0.200
28	A	5	4	1.00	22	0.182
29	A	5	4	1.00	24	0.167
30	A	4	4	1.00	24	0.167
31	A	5	5	1.00	24	0.208
32	A	6	5	1.00	24	0.208
33	A	7	5	1.00	24	0.208
34	A	7	6	1.00	24	0.250
35	A	5	4	1.00	24	0.167
36	A	5	4	1.00	22	0.182
37	A	4	3	1.00	15	0.200
38	A	6	5	1.00	22	0.227
39	A	5	4	1.00	24	0.167
40	A	6	5	1.00	24	0.208
41	A	5	4	1.00	24	0.167
42	A	6	5	1.00	24	0.208
43	A	7	6	1.00	24	0.250
44	A	8	6	1.00	24	0.250
45	A	6	4	1.00	24	0.167
46	A	5	4	1.00	24	0.167
47	A	4	4	1.00	24	0.167
48	A	3	3	1.00	24	0.125
49	A	3	2	1.00	24	0.083
50	A	2	2	1.00	22	0.091
51	A	2	2	1.00	15	0.133
52	A	4	4	1.00	22	0.182
53	A	4	4	1.00	24	0.167
54	A	5	4	1.00	24	0.167
55	A	6	4	1.00	24	0.167
56	A	6	5	1.00	24	0.208
57	A	5	5	1.00	24	0.208
58	A	4	4	1.00	24	0.167
59	A	6	6	1.00	24	0.250
60	A	3	3	1.00	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	3	3	1.00	22	0.136
62	A	3	2	1.00	15	0.133
63	A	4	4	1.00	22	0.182
64	A	5	5	1.00	24	0.208
65	A	6	5	1.00	24	0.208
66	A	6	5	1.00	24	0.208
67	A	5	4	1.00	24	0.167
68	A	7	7	1.00	24	0.292
69	A	4	4	1.00	24	0.167
70	A	4	4	1.00	24	0.167
71	A	4	3	1.00	22	0.136
72	A	4	2	1.00	15	0.133
73	A	5	4	1.00	22	0.182
74	A	6	5	1.00	24	0.208
75	A	6	4	1.00	24	0.167
76	A	8	7	1.00	24	0.292
77	A	5	4	1.00	24	0.167
78	A	5	5	1.00	24	0.208
79	A	5	4	1.00	24	0.167
80	A	5	3	1.00	22	0.136
81	A	5	2	1.00	15	0.133
82	A	6	4	1.00	22	0.182
83	A	7	5	1.00	24	0.208
84	A	6	6	1.00	26	0.231
85	A	5	5	1.00	26	0.192
86	A	3	3	1.00	26	0.115
87	A	3	3	1.00	24	0.125
88	A	2	2	1.00	17	0.118
89	A	6	6	1.00	24	0.250
90	A	8	8	1.00	26	0.308
91	A	8	8	1.00	26	0.308
92	A	7	7	1.00	26	0.269
93	A	4	4	1.00	26	0.154
94	A	4	4	1.00	24	0.167
95	A	3	3	1.00	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	6	6	1.00	24	0.250
97	A	8	8	1.00	26	0.308
98	A	9	9	1.00	26	0.346
99	A	7	6	1.00	26	0.231
100	A	5	4	1.00	26	0.154
101	A	5	4	1.00	24	0.167
102	A	4	3	1.00	17	0.176
103	A	7	7	1.00	24	0.292
104	A	7	7	1.00	26	0.269
105	A	8	8	1.00	26	0.308
106	A	9	8	1.00	26	0.308
107	A	5	3	1.00	17	0.176
108	A	7	6	1.00	26	0.231
109	A	6	6	1.00	26	0.231
110	A	5	5	1.00	26	0.192
111	A	4	4	1.00	26	0.154
112	A	3	3	1.00	24	0.125
113	A	3	3	1.00	17	0.176
114	A	7	7	1.00	24	0.292
115	A	8	8	1.00	26	0.308
116	A	9	8	1.00	26	0.308
117	A	7	7	1.00	26	0.269
118	A	6	6	1.00	26	0.231
119	A	5	5	1.00	26	0.192
120	A	4	4	1.00	26	0.154
121	A	4	4	1.00	24	0.167
122	A	4	3	1.00	17	0.176
123	A	8	8	1.00	24	0.333
124	A	9	9	1.00	26	0.346
125	A	10	9	1.00	26	0.346
126	A	7	6	1.00	26	0.231
127	A	6	6	1.00	26	0.231
128	A	5	5	1.00	26	0.192
129	A	5	5	1.00	26	0.192
130	A	5	4	1.00	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	5	3	1.00	17	0.176
132	A	9	8	1.00	24	0.333
133	A	10	9	1.00	26	0.346
134	A	6	3	1.00	17	0.176
135	A	5	3	1.00	26	0.115
136	A	4	3	1.00	26	0.115
137	A	3	3	1.00	26	0.115
138	A	2	2	1.00	26	0.077
139	A	3	3	1.00	26	0.115
140	A	4	3	1.00	26	0.115
141	A	5	3	1.00	26	0.115
142	A	5	3	1.00	26	0.115
143	A	4	3	1.00	26	0.115
144	A	3	3	1.00	26	0.115
145	A	2	2	1.00	26	0.077
146	A	3	3	1.00	26	0.115
147	A	4	3	1.00	26	0.115
148	A	5	3	1.00	26	0.115
149	A	6	4	1.00	28	0.143
150	A	5	4	1.00	28	0.143
151	A	4	4	1.00	28	0.143
152	A	3	3	1.00	28	0.107
153	A	3	3	1.00	28	0.107
154	A	4	4	1.00	28	0.143
155	A	5	4	1.00	28	0.143
156	A	7	5	1.00	28	0.179
157	A	6	5	1.00	28	0.179
158	A	5	5	1.00	28	0.179
159	A	4	4	1.00	28	0.143
160	A	4	4	1.00	28	0.143
161	A	4	4	1.00	28	0.143
162	A	5	5	1.00	28	0.179
163	A	6	5	1.00	28	0.179
164	A	13	9	1.00	28	0.321
165	A	12	9	1.00	28	0.321

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	11	8	1.00	28	0.286
167	A	3	3	1.00	28	0.107
168	A	11	8	1.00	28	0.286
169	A	12	9	1.00	28	0.321
170	A	13	9	1.00	28	0.321
171	A	14	10	1.00	28	0.357
172	A	13	10	1.00	28	0.357
173	A	12	9	1.00	28	0.321
174	A	12	9	1.00	28	0.321
175	A	12	9	1.00	28	0.321
176	A	12	9	1.00	28	0.321
177	A	13	10	1.00	28	0.357
178	A	14	10	1.00	28	0.357
179	A	14	10	1.00	28	0.357
180	A	13	9	1.00	28	0.321
181	A	16	13	1.00	28	0.464
182	A	7	7	1.00	28	0.250
183	A	14	11	1.00	28	0.393
184	A	13	9	1.00	28	0.321
185	A	14	10	1.00	28	0.357
186	A	9	9	1.00	28	0.321
187	A	9	9	1.00	28	0.321
188	A	7	7	1.00	28	0.250
189	A	2	2	1.00	28	0.071
190	A	3	3	1.00	28	0.107
191	A	5	5	1.00	28	0.179
192	A	6	5	1.00	28	0.179
193	A	11	10	1.00	28	0.357
194	A	10	10	1.00	28	0.357
195	A	9	9	1.00	28	0.321
196	A	7	7	1.00	28	0.250
197	A	3	3	1.00	28	0.107
198	A	4	4	1.00	28	0.143
199	A	7	6	1.00	28	0.214
200	A	8	6	1.00	28	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	11	9	1.00	28	0.321
202	A	10	9	1.00	28	0.321
203	A	9	9	1.00	28	0.321
204	A	8	8	1.00	28	0.286
205	A	8	8	1.00	28	0.286
206	A	4	3	1.00	28	0.107
207	A	5	4	1.00	28	0.143
208	A	7	5	1.00	28	0.179
209	A	8	5	1.00	28	0.179
210	A	10	9	1.00	28	0.321
211	A	9	9	1.00	28	0.321
212	A	8	8	1.00	28	0.286
213	A	3	3	1.00	28	0.107
214	A	4	4	1.00	28	0.143
215	A	5	5	1.00	28	0.179
216	A	6	5	1.00	28	0.179
217	A	7	5	1.00	28	0.179
218	A	10	10	1.00	28	0.357
219	A	9	9	1.00	28	0.321
220	A	4	3	1.00	28	0.107
221	A	4	4	1.00	28	0.143
222	A	5	5	1.00	28	0.179
223	A	6	6	1.00	28	0.214
224	A	7	6	1.00	28	0.214
225	A	11	10	1.00	28	0.357
226	A	10	9	1.00	28	0.321
227	A	5	3	1.00	28	0.107
228	A	5	4	1.00	28	0.143
229	A	6	5	1.00	28	0.179
230	A	6	5	1.00	28	0.179
231	A	7	6	1.00	28	0.214
232	A	8	6	1.00	28	0.214
233	A	25	14	1.00	26	0.538
234	A	24	14	1.00	26	0.538
235	A	23	13	1.00	26	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	23	13	1.00	26	0.500
237	A	23	13	1.00	26	0.500
238	A	24	14	1.00	26	0.538
239	A	25	14	1.00	26	0.538
240	A	26	15	1.00	26	0.577
241	A	25	15	1.00	26	0.577
242	A	24	14	1.00	26	0.538
243	A	24	14	1.00	26	0.538
244	A	24	14	1.00	26	0.538
245	A	24	14	1.00	26	0.538
246	A	25	15	1.00	26	0.577
247	A	26	15	1.00	26	0.577
248	A	4	4	1.00	28	0.143
249	A	4	4	1.00	28	0.143
250	A	4	4	1.00	28	0.143
251	A	4	4	1.00	28	0.143
252	A	4	4	1.00	28	0.143
253	A	4	4	1.00	28	0.143
254	A	4	4	1.00	28	0.143
255	A	4	4	1.00	28	0.143
256	A	4	4	1.00	28	0.143
257	A	4	4	1.00	28	0.143
258	A	4	4	1.00	28	0.143
259	A	4	4	1.00	28	0.143
260	A	4	4	1.00	28	0.143
261	A	4	4	1.00	28	0.143
262	A	4	4	1.00	28	0.143
263	A	4	4	1.00	28	0.143
264	A	4	4	1.00	28	0.143
265	A	4	4	1.00	28	0.143
266	A	4	4	1.00	28	0.143
267	A	4	4	1.00	28	0.143
268	A	4	4	1.00	28	0.143
269	A	4	4	1.00	28	0.143
270	A	4	4	1.00	28	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	4	4	1.00	28	0.143
272	A	8	8	1.00	26	0.308
273	A	6	6	1.00	26	0.231
274	A	6	6	1.00	24	0.250
275	A	5	5	1.00	17	0.294
276	A	11	7	1.00	24	0.292
277	A	12	8	1.00	26	0.308
278	A	13	9	1.00	26	0.346
279	A	5	5	1.00	17	0.294
280	A	9	9	1.00	26	0.346
281	A	7	7	1.00	26	0.269
282	A	7	7	1.00	24	0.292
283	A	6	6	1.00	17	0.353
284	A	13	9	1.00	24	0.375
285	A	13	9	1.00	26	0.346
286	A	13	9	1.00	26	0.346
287	A	6	6	1.00	17	0.353
288	A	3	3	1.00	26	0.115
289	A	4	4	1.00	28	0.143
290	A	9	9	1.00	26	0.346
291	A	8	8	1.00	26	0.308
292	A	7	7	1.00	26	0.269
293	A	6	6	1.00	24	0.250
294	A	6	6	1.00	17	0.353
295	A	13	9	1.00	24	0.375
296	A	13	9	1.00	26	0.346
297	A	6	6	1.00	17	0.353
298	A	3	3	1.00	26	0.115
299	A	4	4	1.00	28	0.143
300	A	9	9	1.00	26	0.346
301	A	8	8	1.00	26	0.308
302	A	7	7	1.00	26	0.269
303	A	7	7	1.00	24	0.292
304	A	7	6	1.00	17	0.353
305	A	15	9	1.00	24	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	14	9	1.00	26	0.346
307	A	7	6	1.00	17	0.353
308	A	2	2	1.00	24	0.083
309	A	2	2	1.00	24	0.083
310	A	6	6	1.00	26	0.231
311	A	5	5	1.00	26	0.192
312	A	4	4	1.00	26	0.154
313	A	2	2	1.00	24	0.083
314	A	6	4	1.00	26	0.154
315	A	7	5	1.00	26	0.192
316	A	8	5	1.00	26	0.192
317	A	9	5	1.00	26	0.192
318	A	2	2	1.00	24	0.083
319	A	6	4	1.00	26	0.154
320	A	3	3	1.00	28	0.107
321	A	3	3	1.00	28	0.107
322	A	3	3	1.00	28	0.107
323	A	3	3	1.00	28	0.107
324	A	3	3	1.00	26	0.115
325	A	6	6	1.00	24	0.250
326	A	5	5	1.00	24	0.208
327	A	3	3	1.00	24	0.125
328	A	3	3	1.00	22	0.136
329	A	2	2	1.00	15	0.133
330	A	5	5	1.00	22	0.227
331	A	6	6	1.00	24	0.250
332	A	4	4	1.00	26	0.154
333	A	4	4	1.00	26	0.154
334	A	4	4	1.00	26	0.154
335	A	4	4	1.00	26	0.154
336	A	5	3	1.00	23	0.130
337	A	4	3	1.00	23	0.130
338	A	3	3	1.00	23	0.130
339	A	2	2	1.00	23	0.087
340	A	3	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	4	3	1.00	23	0.130
342	A	5	3	1.00	23	0.130
343	A	16	12	1.00	25	0.480
344	A	15	12	1.00	25	0.480
345	A	15	12	1.00	25	0.480
346	A	14	11	1.00	25	0.440
347	A	13	10	1.00	25	0.400
348	A	14	11	1.00	25	0.440
349	A	8	5	1.00	25	0.200
350	A	7	5	1.00	25	0.200
351	A	6	5	1.00	25	0.200
352	A	5	5	1.00	25	0.200
353	A	4	4	1.00	25	0.160
354	A	4	4	1.00	25	0.160
355	A	4	4	1.00	25	0.160
356	A	5	5	1.00	25	0.200
357	A	6	5	1.00	25	0.200
358	A	7	6	1.00	25	0.240
359	A	6	6	1.00	25	0.240
360	A	6	5	1.00	25	0.200
361	A	6	6	1.00	25	0.240
362	A	7	6	1.00	25	0.240
363	A	10	10	1.00	25	0.400
364	A	17	14	1.00	25	0.560
365	A	18	15	1.00	25	0.600
366	A	17	14	1.00	25	0.560
367	A	18	15	1.00	25	0.600
368	A	18	15	1.00	25	0.600
369	A	20	16	1.00	25	0.640
370	A	9	9	1.00	25	0.360
371	A	8	7	1.00	25	0.280
372	A	8	8	1.00	25	0.320
373	A	8	7	1.00	25	0.280
374	A	8	8	1.00	25	0.320
375	A	8	7	1.00	25	0.280

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	9	8	1.00	25	0.320
377	A	10	8	1.00	25	0.320
378	A	11	10	1.00	21	0.476
379	A	9	8	1.00	21	0.381
380	A	6	5	1.00	19	0.263
381	A	9	7	1.00	19	0.368
382	A	11	9	1.00	21	0.429
383	A	13	9	1.00	21	0.429
384	A	14	11	1.00	21	0.524
385	A	12	9	1.00	21	0.429
386	A	11	8	1.00	12	0.667
387	A	16	13	1.00	21	0.619
388	A	19	15	1.00	21	0.714
389	A	19	13	1.00	21	0.619
390	A	17	11	1.00	21	0.524
391	A	14	9	1.00	19	0.474
392	A	16	12	1.00	19	0.632
393	A	18	14	1.00	21	0.667
394	A	20	15	1.00	21	0.714
395	A	10	9	1.00	21	0.429
396	A	8	7	1.00	21	0.333
397	A	7	6	1.00	12	0.500
398	A	11	9	1.00	21	0.429
399	A	13	11	1.00	21	0.524
400	A	10	9	1.00	21	0.429
401	A	8	7	1.00	21	0.333
402	A	5	4	1.00	19	0.210
403	A	9	7	1.00	19	0.368
404	A	12	10	1.00	21	0.476
405	A	14	11	1.00	21	0.524
406	A	14	10	1.00	21	0.476
407	A	12	8	1.00	21	0.381
408	A	11	7	1.00	12	0.583
409	A	19	15	1.00	21	0.714
410	A	19	15	1.00	21	0.714

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	7	4	1.00	23	0.174
412	A	6	3	1.00	19	0.158
413	A	5	3	1.00	19	0.158
414	A	4	3	1.00	19	0.158
415	A	3	3	1.00	19	0.158
416	A	2	2	1.00	17	0.118
417	A	2	1	1.00	10	0.100
418	A	2	2	1.00	17	0.118
419	A	3	3	1.00	19	0.158
420	A	4	3	1.00	19	0.158
421	A	5	3	1.00	19	0.158
422	A	6	3	1.00	19	0.158
423	A	7	3	1.00	19	0.158
424	A	6	4	1.00	21	0.190
425	A	5	4	1.00	21	0.190
426	A	3	3	1.00	21	0.143
427	A	3	3	1.00	19	0.158
428	A	2	2	1.00	12	0.167
429	A	3	2	1.00	19	0.105
430	A	3	3	1.00	21	0.143
431	A	4	4	1.00	21	0.190
432	A	5	4	1.00	21	0.190
433	A	6	4	1.00	21	0.190
434	A	7	4	1.00	21	0.190
435	A	7	6	1.00	21	0.286
436	A	4	4	1.00	21	0.190
437	A	4	3	1.00	19	0.158
438	A	3	3	1.00	12	0.250
439	A	4	3	1.00	19	0.158
440	A	4	3	1.00	21	0.143
441	A	4	4	1.00	21	0.190
442	A	5	5	1.00	21	0.238
443	A	6	5	1.00	21	0.238
444	A	7	5	1.00	21	0.238
445	A	8	6	1.00	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
446	A	5	5	1.00	21	0.238
447	A	5	3	1.00	19	0.158
448	A	4	4	1.00	12	0.333
449	A	5	4	1.00	19	0.210
450	A	5	4	1.00	21	0.190
451	A	5	4	1.00	21	0.190
452	A	5	5	1.00	21	0.238
453	A	6	6	1.00	21	0.286
454	A	7	6	1.00	21	0.286
455	A	8	6	1.00	21	0.286
456	A	8	7	1.00	21	0.333
457	A	7	7	1.00	21	0.333
458	A	6	6	1.00	21	0.286
459	A	5	5	1.00	21	0.238
460	A	4	4	1.17	21	0.190
461	A	2	2	1.00	19	0.105
462	A	2	2	1.00	12	0.167
463	A	3	3	1.00	19	0.158
464	A	4	4	1.00	21	0.190
465	A	5	5	1.00	21	0.238
466	A	6	6	1.00	21	0.286
467	A	8	6	1.00	21	0.286
468	A	7	6	1.00	21	0.286
469	A	6	6	1.00	21	0.286
470	A	5	5	1.06	21	0.238
471	A	3	3	1.00	21	0.143
472	A	3	3	1.00	19	0.158
473	A	3	3	1.00	12	0.250
474	A	4	4	1.00	19	0.210
475	A	5	5	1.00	21	0.238
476	A	6	6	1.00	21	0.286
477	A	8	7	1.00	21	0.333
478	A	7	7	1.00	21	0.333
479	A	6	6	1.00	21	0.286
480	A	4	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
481	A	4	4	1.00	21	0.190
482	A	4	3	1.00	19	0.158
483	A	4	4	1.00	12	0.333
484	A	5	5	1.00	19	0.263
485	A	6	5	1.00	21	0.238
486	A	8	7	1.00	21	0.333
487	A	7	7	1.00	21	0.333
488	A	5	5	1.00	21	0.238
489	A	5	5	1.00	21	0.238
490	A	5	4	1.00	21	0.190
491	A	5	3	1.00	19	0.158
492	A	5	4	1.00	12	0.333
493	A	6	5	1.00	19	0.263
494	A	7	5	1.00	21	0.238
495	A	2	2	1.00	12	0.167
496	A	3	3	1.00	12	0.250
497	A	4	4	1.00	12	0.333
498	A	5	4	1.00	12	0.333
499	A	2	2	1.00	12	0.167
500	A	3	3	1.00	12	0.250
501	A	4	4	1.00	12	0.333
502	A	5	4	1.00	12	0.333
503	A	14	10	1.00	23	0.435
504	A	11	8	1.00	23	0.348
505	A	12	8	1.00	23	0.348
506	A	8	5	1.00	21	0.238
507	A	11	7	1.00	14	0.500
508	A	11	6	1.00	21	0.286
509	A	16	12	1.00	23	0.522
510	A	13	8	1.00	23	0.348
511	A	11	8	1.00	23	0.348
512	A	12	8	1.00	23	0.348
513	A	9	6	1.00	23	0.261
514	A	9	5	1.00	21	0.238
515	A	8	5	1.00	14	0.357

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
516	A	11	6	1.00	21	0.286
517	A	12	7	1.00	23	0.304
518	A	13	8	1.00	23	0.348
519	A	13	8	1.00	23	0.348
520	A	10	7	1.00	23	0.304
521	A	10	5	1.00	21	0.238
522	A	9	6	1.00	14	0.429
523	A	12	7	1.00	21	0.333
524	A	12	7	1.00	23	0.304
525	A	13	8	1.00	23	0.348
526	A	14	8	1.00	23	0.348
527	A	10	6	1.00	14	0.429
528	A	12	9	1.00	23	0.391
529	A	14	10	1.00	23	0.435
530	A	10	7	1.00	23	0.304
531	A	12	8	1.00	23	0.348
532	A	7	4	1.00	21	0.190
533	A	11	7	1.00	14	0.500
534	A	11	6	1.00	21	0.286
535	A	17	13	1.00	23	0.565
536	A	14	9	1.00	23	0.391
537	A	11	7	1.00	23	0.304
538	A	10	7	1.00	23	0.304
539	A	9	6	1.00	23	0.261
540	A	8	5	1.00	23	0.217
541	A	8	5	1.00	21	0.238
542	A	8	5	1.00	14	0.357
543	A	12	7	1.00	21	0.333
544	A	13	8	1.00	23	0.348
545	A	14	9	1.00	23	0.391
546	A	11	8	1.00	23	0.348
547	A	10	7	1.00	23	0.304
548	A	9	6	1.00	23	0.261
549	A	9	6	1.00	23	0.261
550	A	9	5	1.00	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
551	A	9	6	1.00	14	0.429
552	A	13	8	1.00	21	0.381
553	A	14	8	1.00	23	0.348
554	A	10	6	1.00	14	0.429
555	A	13	8	1.00	21	0.381
556	A	12	8	1.00	21	0.381
557	A	11	8	1.00	21	0.381
558	A	10	7	1.00	21	0.333
559	A	11	8	1.00	21	0.381
560	A	12	8	1.00	21	0.381
561	A	13	8	1.00	21	0.381
562	A	14	9	1.00	23	0.391
563	A	13	9	1.00	23	0.391
564	A	12	9	1.00	23	0.391
565	A	11	8	1.00	23	0.348
566	A	11	8	1.00	23	0.348
567	A	12	9	1.00	23	0.391
568	A	13	9	1.00	23	0.391
569	A	15	10	1.00	23	0.435
570	A	14	10	1.00	23	0.435
571	A	13	10	1.00	23	0.435
572	A	12	9	1.00	23	0.391
573	A	12	9	1.00	23	0.391
574	A	12	9	1.00	23	0.391
575	A	13	10	1.00	23	0.435
576	A	14	10	1.00	23	0.435
577	A	15	10	1.00	23	0.435
578	A	10	7	1.00	21	0.333
579	A	10	7	1.00	23	0.304
580	A	10	7	1.00	23	0.304
581	A	10	7	1.00	24	0.292
582	A	17	14	1.00	23	0.609
583	A	16	13	1.00	23	0.565
584	A	15	12	1.00	23	0.522
585	A	14	11	1.00	23	0.478

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
586	A	14	11	1.00	23	0.478
587	A	14	11	1.00	23	0.478
588	A	15	12	1.00	23	0.522
589	A	16	13	1.00	23	0.565
590	A	17	14	1.00	23	0.609
591	A	17	13	1.00	23	0.565
592	A	16	13	1.00	23	0.565
593	A	15	12	1.00	23	0.522
594	A	15	12	1.00	23	0.522
595	A	15	12	1.00	23	0.522
596	A	15	12	1.00	23	0.522
597	A	16	13	1.00	23	0.565
598	A	17	13	1.00	23	0.565
599	A	18	14	1.00	23	0.609
600	A	17	14	1.00	23	0.609
601	A	16	13	1.00	23	0.565
602	A	16	13	1.00	23	0.565
603	A	16	13	1.00	23	0.565
604	A	16	13	1.00	23	0.565
605	A	16	13	1.00	23	0.565
606	A	17	13	1.00	23	0.565
607	A	18	13	1.00	23	0.565
608	A	14	10	1.00	25	0.400
609	A	13	10	1.00	25	0.400
610	A	11	9	1.00	25	0.360
611	A	7	5	1.00	25	0.200
612	A	8	6	1.00	25	0.240
613	A	10	8	1.00	25	0.320
614	A	10	7	1.00	25	0.280
615	A	15	10	1.00	25	0.400
616	A	14	10	1.00	25	0.400
617	A	13	9	1.00	25	0.360
618	A	12	9	1.00	25	0.360
619	A	8	6	1.00	25	0.240
620	A	9	7	1.00	25	0.280

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
621	A	10	7	1.00	25	0.280
622	A	11	7	1.00	25	0.280
623	A	16	10	1.00	25	0.400
624	A	15	10	1.00	25	0.400
625	A	14	10	1.00	25	0.400
626	A	13	9	1.00	25	0.360
627	A	13	9	1.00	25	0.360
628	A	9	7	1.00	25	0.280
629	A	10	7	1.00	25	0.280
630	A	11	7	1.00	25	0.280
631	A	12	7	1.00	25	0.280
632	A	14	11	1.00	25	0.440
633	A	13	10	1.00	25	0.400
634	A	12	9	1.00	25	0.360
635	A	7	5	1.00	25	0.200
636	A	7	5	1.00	25	0.200
637	A	9	7	1.00	25	0.280
638	A	10	8	1.00	25	0.320
639	A	11	9	1.00	25	0.360
640	A	14	10	1.00	25	0.400
641	A	13	9	1.00	25	0.360
642	A	8	6	1.00	25	0.240
643	A	8	6	1.00	25	0.240
644	A	8	6	1.00	25	0.240
645	A	9	7	1.00	25	0.280
646	A	10	8	1.00	25	0.320
647	A	15	11	1.00	25	0.440
648	A	14	10	1.00	25	0.400
649	A	9	7	1.00	25	0.280
650	A	9	7	1.00	25	0.280
651	A	9	7	1.00	25	0.280
652	A	9	7	1.00	25	0.280
653	A	10	7	1.00	25	0.280
654	A	11	8	1.00	25	0.320
655	A	7	4	1.00	25	0.160

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
656	A	7	4	1.00	25	0.160
657	A	7	4	1.00	25	0.160
658	A	7	4	1.00	25	0.160
659	A	7	4	1.00	25	0.160
660	A	7	4	1.00	25	0.160
661	A	7	4	1.00	25	0.160
662	A	7	4	1.00	25	0.160
663	A	7	4	1.00	25	0.160
664	A	7	4	1.00	25	0.160
665	A	7	4	1.00	25	0.160
666	A	7	4	1.00	25	0.160
667	A	7	4	1.00	25	0.160
668	A	7	4	1.00	25	0.160
669	A	7	4	1.00	25	0.160
670	A	7	4	1.00	25	0.160
671	A	32	18	1.00	23	0.783
672	A	30	18	1.00	23	0.783
673	A	28	16	1.00	23	0.696
674	A	30	18	1.00	23	0.783
675	A	9	5	1.00	25	0.200
676	A	9	5	1.00	25	0.200
677	A	9	5	1.00	25	0.200
678	A	9	5	1.00	25	0.200
679	A	9	5	1.00	25	0.200
680	A	9	5	1.00	25	0.200
681	A	16	10	1.00	23	0.435
682	A	15	10	1.00	23	0.435
683	A	14	8	1.00	23	0.348
684	A	12	7	1.00	21	0.333
685	A	13	7	1.00	14	0.500
686	A	19	10	1.00	21	0.476
687	A	20	10	1.00	23	0.435
688	A	12	7	1.00	14	0.500
689	A	12	7	1.00	14	0.500
690	A	13	7	1.00	14	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
691	A	13	7	1.00	14	0.500
692	A	11	6	1.00	14	0.429
693	A	11	6	1.00	14	0.429
694	A	12	7	1.00	14	0.500
695	A	12	7	1.00	14	0.500
696	A	8	6	1.00	23	0.261
697	A	7	5	1.00	23	0.217
698	A	6	4	1.00	23	0.174
699	A	5	3	1.00	21	0.143
700	A	8	6	1.00	23	0.261
701	A	9	7	1.00	23	0.304
702	A	7	4	1.00	23	0.174
703	A	7	4	1.00	23	0.174
704	A	7	4	1.00	23	0.174
705	A	7	4	1.00	23	0.174
706	A	7	4	1.00	23	0.174
707	A	8	6	1.00	21	0.286
708	A	8	6	1.00	21	0.286
709	A	6	4	1.00	21	0.190
710	A	5	3	1.00	19	0.158
711	A	5	3	1.00	12	0.250
712	A	8	6	1.00	19	0.316
713	A	10	8	1.00	21	0.381
714	A	11	9	1.00	21	0.429
715	A	9	5	1.00	23	0.217
716	A	9	5	1.00	23	0.217
717	A	9	5	1.00	23	0.217
718	A	9	5	1.00	23	0.217
719	A	5	4	1.00	24	0.167
720	A	4	4	1.00	24	0.167
721	A	3	3	1.00	24	0.125
722	A	4	4	1.00	24	0.167
723	A	5	4	1.00	24	0.167
724	A	6	5	1.00	26	0.192
725	A	5	5	1.00	26	0.192

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
726	A	4	4	1.00	26	0.154
727	A	4	4	1.00	26	0.154
728	A	5	5	1.00	26	0.192
729	A	6	5	1.00	26	0.192
730	A	6	6	1.00	26	0.231
731	A	5	5	1.00	26	0.192
732	A	5	5	1.00	26	0.192
733	A	5	5	1.00	26	0.192
734	A	6	6	1.00	26	0.231
735	A	13	10	1.00	26	0.385
736	A	12	9	1.00	26	0.346
737	A	4	4	1.00	26	0.154
738	A	12	9	1.00	26	0.346
739	A	13	10	1.00	26	0.385
740	A	14	11	1.00	26	0.423
741	A	13	10	1.00	26	0.385
742	A	13	10	1.00	26	0.385
743	A	13	10	1.00	26	0.385
744	A	13	10	1.00	26	0.385
745	A	14	11	1.00	26	0.423
746	A	14	10	1.00	26	0.385
747	A	16	13	1.00	26	0.500
748	A	7	7	1.00	26	0.269
749	A	15	12	1.00	26	0.462
750	A	14	10	1.00	26	0.385
751	A	7	6	1.00	28	0.214
752	A	6	6	1.00	28	0.214
753	A	4	4	1.00	28	0.143
754	A	3	3	1.00	28	0.107
755	A	8	8	1.00	28	0.286
756	A	10	10	1.00	28	0.357
757	A	8	7	1.00	28	0.250
758	A	5	5	1.00	28	0.179
759	A	4	4	1.00	28	0.143
760	A	8	8	1.00	28	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
761	A	10	10	1.00	28	0.357
762	A	8	6	1.00	28	0.214
763	A	6	5	1.00	28	0.179
764	A	5	4	1.00	28	0.143
765	A	9	9	1.00	28	0.321
766	A	9	9	1.00	28	0.321
767	A	10	10	1.00	28	0.357
768	A	7	6	1.00	28	0.214
769	A	6	6	1.00	28	0.214
770	A	5	5	1.00	28	0.179
771	A	4	4	1.00	28	0.143
772	A	9	9	1.00	28	0.321
773	A	10	10	1.00	28	0.357
774	A	8	7	1.00	28	0.250
775	A	7	7	1.00	28	0.250
776	A	6	6	1.00	28	0.214
777	A	5	5	1.00	28	0.179
778	A	5	4	1.00	28	0.143
779	A	10	10	1.00	28	0.357
780	A	11	11	1.00	28	0.393
781	A	9	7	1.00	28	0.250
782	A	8	7	1.00	28	0.250
783	A	7	6	1.00	28	0.214
784	A	7	6	1.00	28	0.214
785	A	6	5	1.00	28	0.179
786	A	6	4	1.00	28	0.143
787	A	11	10	1.00	28	0.357
788	A	6	6	1.00	26	0.231
789	A	5	5	1.00	26	0.192
790	A	3	3	1.00	24	0.125
791	A	7	5	1.00	26	0.192
792	A	8	6	1.00	26	0.231
793	A	4	4	1.00	26	0.154
794	A	5	5	1.00	26	0.192
795	A	5	5	1.00	26	0.192

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
796	A	5	5	1.00	26	0.192
797	A	5	5	1.00	26	0.192
798	A	14	9	1.00	21	0.429
799	A	13	9	1.00	21	0.429
800	A	12	9	1.00	21	0.429
801	A	11	8	1.00	21	0.381
802	A	12	9	1.00	21	0.429
803	A	13	9	1.00	21	0.429
804	A	14	9	1.00	21	0.429
805	A	15	10	1.00	23	0.435
806	A	14	10	1.00	23	0.435
807	A	13	10	1.00	23	0.435
808	A	12	9	1.00	23	0.391
809	A	12	9	1.00	23	0.391
810	A	13	10	1.00	23	0.435
811	A	14	10	1.00	23	0.435
812	A	15	10	1.00	23	0.435
813	A	15	11	1.00	23	0.478
814	A	14	11	1.00	23	0.478
815	A	13	10	1.00	23	0.435
816	A	13	10	1.00	23	0.435
817	A	13	10	1.00	23	0.435
818	A	14	11	1.00	23	0.478
819	A	15	11	1.00	23	0.478
820	A	17	14	1.00	23	0.609
821	A	16	13	1.00	23	0.565
822	A	15	12	1.00	23	0.522
823	A	15	12	1.00	23	0.522
824	A	15	12	1.00	23	0.522
825	A	16	13	1.00	23	0.565
826	A	18	14	1.00	23	0.609
827	A	17	14	1.00	23	0.609
828	A	16	13	1.00	23	0.565
829	A	16	13	1.00	23	0.565
830	A	16	13	1.00	23	0.565

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
831	A	16	13	1.00	23	0.565
832	A	17	14	1.00	23	0.609
833	A	19	15	1.00	23	0.652
834	A	18	15	1.00	23	0.652
835	A	17	14	1.00	23	0.609
836	A	17	14	1.00	23	0.609
837	A	17	14	1.00	23	0.609
838	A	17	14	1.00	23	0.609
839	A	17	14	1.00	23	0.609
840	A	11	8	1.00	25	0.320
841	A	11	9	1.00	25	0.360
842	A	9	7	1.00	25	0.280
843	A	8	6	1.00	25	0.240
844	A	12	10	1.00	25	0.400
845	A	14	11	1.00	25	0.440
846	A	12	8	1.00	25	0.320
847	A	11	8	1.00	25	0.320
848	A	10	8	1.00	25	0.320
849	A	9	7	1.00	25	0.280
850	A	13	10	1.00	25	0.400
851	A	14	10	1.00	25	0.400
852	A	15	11	1.00	25	0.440
853	A	13	8	1.00	25	0.320
854	A	12	8	1.00	25	0.320
855	A	11	8	1.00	25	0.320
856	A	10	8	1.00	25	0.320
857	A	14	10	1.00	25	0.400
858	A	14	10	1.00	25	0.400
859	A	15	11	1.00	25	0.440
860	A	16	11	1.00	25	0.440
861	A	11	9	1.00	25	0.360
862	A	10	8	1.00	25	0.320
863	A	8	6	1.00	25	0.240
864	A	8	6	1.00	25	0.240
865	A	13	10	1.00	25	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
866	A	14	11	1.00	25	0.440
867	A	11	9	1.00	25	0.360
868	A	10	8	1.00	25	0.320
869	A	9	7	1.00	25	0.280
870	A	9	7	1.00	25	0.280
871	A	9	7	1.00	25	0.280
872	A	14	10	1.00	25	0.400
873	A	15	11	1.00	25	0.440
874	A	12	9	1.00	25	0.360
875	A	11	8	1.00	25	0.320
876	A	10	8	1.00	25	0.320
877	A	10	8	1.00	25	0.320
878	A	10	8	1.00	25	0.320
879	A	10	8	1.00	25	0.320
880	A	8	6	1.00	23	0.261
881	A	7	5	1.00	23	0.217
882	A	6	4	1.00	21	0.190
883	A	9	7	1.00	23	0.304
884	A	10	8	1.00	23	0.348
885	A	8	5	1.00	23	0.217
886	A	10	6	1.00	23	0.261
887	A	10	6	1.00	23	0.261
888	A	10	6	1.00	23	0.261
889	A	10	6	1.00	23	0.261
890	A	3	3	1.00	29	0.103
891	A	4	4	1.44	29	0.138
892	A	3	3	1.00	27	0.111
893	A	3	3	1.00	29	0.103
894	A	3	3	1.00	29	0.103
895	A	3	3	1.00	29	0.103
896	A	4	3	1.00	31	0.097
897	A	4	3	1.00	31	0.097
898	A	3	2	1.00	31	0.065
899	A	4	4	1.44	29	0.138
900	A	4	3	1.00	31	0.097

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
901	A	3	3	1.00	31	0.097
902	A	4	3	1.00	31	0.097
903	A	4	3	1.00	31	0.097
904	A	4	3	1.00	31	0.097
905	A	4	3	1.00	31	0.097
906	A	3	2	1.00	31	0.065
907	A	4	3	1.00	31	0.097
908	A	3	3	1.00	29	0.103
909	A	4	3	1.00	31	0.097
910	A	4	3	1.00	31	0.097
911	A	3	3	1.00	31	0.097
912	A	4	3	1.00	31	0.097
913	A	4	3	1.00	31	0.097
914	A	4	3	1.00	31	0.097
915	A	3	2	1.00	31	0.065
916	A	4	3	1.00	31	0.097
917	A	4	3	1.00	31	0.097
918	A	3	3	1.00	29	0.103
919	A	4	3	1.00	31	0.097
920	A	4	3	1.00	31	0.097
921	A	4	3	1.00	31	0.097
922	A	3	3	1.00	31	0.097
923	A	4	4	1.00	31	0.129
924	A	4	3	1.00	31	0.097
925	A	4	3	1.00	31	0.097
926	A	4	3	1.00	31	0.097
927	A	3	3	1.00	29	0.103
928	A	3	3	1.00	31	0.097
929	A	5	4	1.00	31	0.129
930	A	5	4	1.00	31	0.129
931	A	4	3	1.00	31	0.097
932	A	4	3	1.00	31	0.097
933	A	3	3	1.00	31	0.097
934	A	3	3	1.00	29	0.103
935	A	5	4	1.00	31	0.129

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
936	A	4	3	1.00	31	0.097
937	A	6	4	1.00	31	0.129
938	A	4	3	1.00	31	0.097
939	A	4	3	1.00	31	0.097
940	A	4	3	1.00	31	0.097
941	A	3	3	1.00	31	0.097
942	A	4	3	1.00	31	0.097
943	A	3	3	1.00	29	0.103
944	A	5	4	1.00	31	0.129
945	A	5	4	1.00	31	0.129
946	A	5	3	1.00	31	0.097
947	A	4	3	1.00	31	0.097
948	A	4	3	1.00	31	0.097
949	A	3	3	1.00	31	0.097
950	A	4	3	1.00	31	0.097
951	A	4	3	1.00	31	0.097
952	A	3	3	1.00	29	0.103
953	A	5	4	1.00	31	0.129
954	A	5	4	1.00	31	0.129
955	A	5	4	1.00	31	0.129
956	A	4	3	1.00	33	0.091
957	A	4	3	1.00	33	0.091
958	A	3	3	1.00	31	0.097
959	A	5	5	1.00	33	0.152
960	A	6	5	1.00	33	0.152
961	A	7	5	1.00	33	0.152
962	A	4	3	1.00	33	0.091
963	A	4	3	1.00	33	0.091
964	A	3	3	1.00	31	0.097
965	A	5	5	1.00	33	0.152
966	A	6	6	1.00	33	0.182
967	A	7	6	1.00	33	0.182
968	A	4	3	1.00	33	0.091
969	A	4	3	1.00	33	0.091
970	A	3	3	1.00	31	0.097

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
971	A	6	6	1.00	33	0.182
972	A	6	5	1.00	33	0.152
973	A	7	6	1.00	33	0.182
974	A	4	3	1.00	33	0.091
975	A	4	3	1.00	33	0.091
976	A	3	3	1.00	31	0.097
977	A	6	6	1.00	33	0.182
978	A	7	6	1.00	33	0.182
979	A	8	6	1.00	33	0.182
980	A	4	3	1.00	33	0.091
981	A	4	3	1.00	33	0.091
982	A	3	3	1.00	31	0.097
983	A	7	6	1.00	33	0.182
984	A	8	6	1.00	33	0.182
985	A	9	6	1.00	33	0.182
986	A	4	3	1.00	33	0.091
987	A	4	3	1.00	33	0.091
988	A	3	3	1.00	31	0.097
989	A	8	6	1.00	33	0.182
990	A	9	6	1.00	33	0.182
991	A	10	6	1.00	33	0.182
992	A	6	5	1.00	35	0.143
993	A	5	5	1.00	35	0.143
994	A	4	4	1.00	35	0.114
995	A	2	2	1.00	35	0.057
996	A	3	3	1.00	35	0.086
997	A	4	3	1.00	35	0.086
998	A	5	3	1.00	35	0.086
999	A	6	6	1.00	35	0.171
1000	A	5	5	1.00	35	0.143
1001	A	5	5	1.00	35	0.143
1002	A	5	5	1.00	35	0.143
1003	A	2	2	1.00	35	0.057
1004	A	3	3	1.00	35	0.086
1005	A	4	3	1.00	35	0.086

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1006	A	5	3	1.00	35	0.086
1007	A	6	5	1.00	35	0.143
1008	A	6	6	1.00	35	0.171
1009	A	6	5	1.00	35	0.143
1010	A	6	6	1.00	35	0.171
1011	A	6	5	1.00	35	0.143
1012	A	2	2	1.00	35	0.057
1013	A	3	3	1.00	35	0.086
1014	A	4	3	1.00	35	0.086
1015	A	5	3	1.00	35	0.086
1016	A	7	6	1.00	35	0.171
1017	A	6	6	1.00	35	0.171
1018	A	5	5	1.00	35	0.143
1019	A	2	2	1.00	35	0.057
1020	A	2	2	1.00	35	0.057
1021	A	3	3	1.00	35	0.086
1022	A	4	3	1.00	35	0.086
1023	A	5	3	1.00	35	0.086
1024	A	8	6	1.00	35	0.171
1025	A	7	6	1.00	35	0.171
1026	A	6	5	1.00	35	0.143
1027	A	2	2	1.00	35	0.057
1028	A	3	3	1.00	35	0.086
1029	A	4	3	1.00	35	0.086
1030	A	3	3	1.00	35	0.086
1031	A	4	4	1.00	35	0.114
1032	A	5	4	1.00	35	0.114
1033	A	9	6	1.00	35	0.171
1034	A	8	6	1.00	35	0.171
1035	A	7	5	1.00	35	0.143
1036	A	2	2	1.00	35	0.057
1037	A	3	3	1.00	35	0.086
1038	A	4	3	1.00	35	0.086
1039	A	5	3	1.00	35	0.086
1040	A	6	3	1.00	35	0.086

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1041	A	4	3	1.00	35	0.086
1042	A	5	4	1.00	35	0.114
1043	A	4	3	1.00	31	0.097
1044	A	4	3	1.00	31	0.097
1045	A	4	3	1.00	31	0.097
1046	A	3	3	1.00	29	0.103
1047	A	3	3	1.00	31	0.097
1048	A	3	3	1.00	31	0.097
1049	A	3	3	1.00	31	0.097
1050	A	3	3	1.32	31	0.097
1051	A	4	3	1.00	31	0.097
1052	A	4	3	1.00	31	0.097
1053	A	4	3	1.00	31	0.097
1054	A	3	3	1.00	29	0.103
1055	A	3	3	1.00	31	0.097
1056	A	3	3	1.00	31	0.097
1057	A	3	3	1.00	31	0.097
1058	A	3	3	1.00	31	0.097
1059	A	3	3	1.31	33	0.091
1060	A	3	3	1.31	33	0.091
1061	A	3	3	1.32	33	0.091
1062	A	3	3	1.32	33	0.091
1063	A	3	3	1.31	33	0.091
1064	A	3	3	1.31	33	0.091
1065	A	4	4	1.00	26	0.154
1066	A	3	3	1.00	26	0.115
1067	A	2	2	1.00	24	0.083
1068	A	2	2	1.00	26	0.077
1069	A	3	3	1.00	26	0.115
1070	A	4	3	1.00	26	0.115
1071	A	5	5	1.00	28	0.179
1072	A	4	4	1.00	28	0.143
1073	A	3	3	1.00	26	0.115
1074	A	3	2	1.00	28	0.071
1075	A	3	3	1.00	28	0.107

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1076	A	4	4	1.00	28	0.143
1077	A	6	5	1.00	28	0.179
1078	A	5	4	1.00	28	0.143
1079	A	4	3	1.00	26	0.115
1080	A	3	3	1.00	28	0.107
1081	A	5	5	1.00	28	0.179
1082	A	4	4	1.00	28	0.143
1083	A	5	5	1.00	28	0.179
1084	A	4	4	1.00	28	0.143
1085	A	2	2	1.00	26	0.077
1086	A	5	5	1.00	28	0.179
1087	A	4	4	1.00	28	0.143
1088	A	5	4	1.00	28	0.143
1089	A	5	5	1.00	28	0.179
1090	A	3	3	1.00	28	0.107
1091	A	3	3	1.00	26	0.115
1092	A	4	4	1.00	28	0.143
1093	A	5	5	1.00	28	0.179
1094	A	6	5	1.00	28	0.179
1095	A	4	4	1.00	28	0.143
1096	A	4	4	1.00	28	0.143
1097	A	4	3	1.00	26	0.115
1098	A	5	4	1.00	28	0.143
1099	A	6	5	1.00	28	0.179
1100	A	7	5	1.00	28	0.179
1101	A	6	6	1.00	30	0.200
1102	A	5	5	1.00	30	0.167
1103	A	4	4	1.00	28	0.143
1104	A	8	5	1.00	30	0.167
1105	A	9	6	1.00	30	0.200
1106	A	10	6	1.00	30	0.200
1107	A	7	6	1.00	30	0.200
1108	A	6	5	1.00	30	0.167
1109	A	5	4	1.00	28	0.143
1110	A	8	5	1.00	30	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1111	A	9	6	1.00	30	0.200
1112	A	10	6	1.00	30	0.200
1113	A	8	6	1.00	30	0.200
1114	A	7	5	1.00	30	0.167
1115	A	6	4	1.00	28	0.143
1116	A	9	6	1.00	30	0.200
1117	A	9	6	1.00	30	0.200
1118	A	10	7	1.00	30	0.233
1119	A	5	5	1.00	30	0.167
1120	A	4	4	1.00	30	0.133
1121	A	3	3	1.00	28	0.107
1122	A	8	5	1.00	30	0.167
1123	A	9	6	1.00	30	0.200
1124	A	10	6	1.00	30	0.200
1125	A	5	5	1.00	30	0.167
1126	A	4	4	1.00	30	0.133
1127	A	4	4	1.00	28	0.143
1128	A	9	6	1.00	30	0.200
1129	A	10	7	1.00	30	0.233
1130	A	11	7	1.00	30	0.233
1131	A	5	5	1.00	30	0.167
1132	A	5	5	1.00	30	0.167
1133	A	5	4	1.00	28	0.143
1134	A	10	6	1.00	30	0.200
1135	A	11	7	1.00	30	0.233
1136	A	12	7	1.00	30	0.233
1137	A	9	9	1.00	32	0.281
1138	A	9	9	1.00	32	0.281
1139	A	7	7	1.00	32	0.219
1140	A	3	3	1.00	32	0.094
1141	A	4	4	1.00	32	0.125
1142	A	6	5	1.00	32	0.156
1143	A	10	9	1.00	32	0.281
1144	A	10	10	1.00	32	0.312
1145	A	8	8	1.00	32	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1146	A	8	8	1.00	32	0.250
1147	A	4	3	1.00	32	0.094
1148	A	5	4	1.00	32	0.125
1149	A	11	9	1.00	32	0.281
1150	A	11	10	1.00	32	0.312
1151	A	9	9	1.00	32	0.281
1152	A	9	9	1.00	32	0.281
1153	A	9	9	1.00	32	0.281
1154	A	5	3	1.00	32	0.094
1155	A	8	8	1.00	32	0.250
1156	A	7	7	1.00	32	0.219
1157	A	2	2	1.00	32	0.062
1158	A	4	4	1.00	32	0.125
1159	A	5	5	1.00	32	0.156
1160	A	6	5	1.00	32	0.156
1161	A	8	8	1.00	32	0.250
1162	A	3	3	1.00	32	0.094
1163	A	3	3	1.00	32	0.094
1164	A	5	5	1.00	32	0.156
1165	A	6	6	1.00	32	0.188
1166	A	7	6	1.00	32	0.188
1167	A	4	3	1.00	32	0.094
1168	A	4	4	1.00	32	0.125
1169	A	5	5	1.00	32	0.156
1170	A	6	5	1.00	32	0.156
1171	A	7	6	1.00	32	0.188
1172	A	8	6	1.00	32	0.188
1173	A	3	3	1.00	28	0.107
1174	A	4	4	1.00	28	0.143
1175	A	3	3	1.00	28	0.107
1176	A	2	2	1.00	26	0.077
1177	A	6	4	1.00	28	0.143
1178	A	7	5	1.00	28	0.179
1179	A	8	5	1.00	28	0.179
1180	A	5	5	1.00	28	0.179

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1181	A	4	4	1.00	28	0.143
1182	A	3	3	1.00	26	0.115
1183	A	5	4	1.00	28	0.143
1184	A	6	5	1.00	28	0.179
1185	A	7	6	1.00	28	0.214
1186	A	3	3	1.00	30	0.100
1187	A	3	3	1.00	30	0.100
1188	A	3	3	1.00	30	0.100
1189	A	3	3	1.00	30	0.100
1190	A	3	3	1.00	30	0.100
1191	A	4	3	1.00	23	0.130
1192	A	3	3	1.00	23	0.130
1193	A	2	2	1.00	21	0.095
1194	A	2	2	1.00	23	0.087
1195	A	3	3	1.00	23	0.130
1196	A	4	3	1.00	23	0.130
1197	A	5	4	1.00	25	0.160
1198	A	4	4	1.00	25	0.160
1199	A	3	3	1.00	23	0.130
1200	A	4	4	1.00	25	0.160
1201	A	3	3	1.00	25	0.120
1202	A	4	4	1.00	25	0.160
1203	A	6	5	1.00	25	0.200
1204	A	5	4	1.00	25	0.160
1205	A	4	3	1.00	23	0.130
1206	A	5	5	1.00	25	0.200
1207	A	5	5	1.00	25	0.200
1208	A	4	4	1.00	25	0.160
1209	A	6	6	1.00	25	0.240
1210	A	5	5	1.00	25	0.200
1211	A	4	4	1.00	25	0.160
1212	A	2	2	1.00	23	0.087
1213	A	3	2	1.00	25	0.080
1214	A	4	3	1.00	25	0.120
1215	A	5	4	1.00	25	0.160

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1216	A	6	6	1.00	25	0.240
1217	A	5	5	1.00	25	0.200
1218	A	3	3	1.00	25	0.120
1219	A	3	3	1.00	23	0.130
1220	A	4	3	1.00	25	0.120
1221	A	5	4	1.00	25	0.160
1222	A	6	4	1.00	25	0.160
1223	A	6	6	1.00	25	0.240
1224	A	4	4	1.00	25	0.160
1225	A	4	4	1.00	25	0.160
1226	A	4	3	1.00	23	0.130
1227	A	5	4	1.00	25	0.160
1228	A	6	4	1.00	25	0.160
1229	A	10	7	1.00	27	0.259
1230	A	9	6	1.00	27	0.222
1231	A	8	5	1.00	25	0.200
1232	A	11	6	1.00	27	0.222
1233	A	12	7	1.00	27	0.259
1234	A	13	8	1.00	27	0.296
1235	A	11	7	1.00	27	0.259
1236	A	10	6	1.00	27	0.222
1237	A	9	5	1.00	25	0.200
1238	A	11	6	1.00	27	0.222
1239	A	12	7	1.00	27	0.259
1240	A	13	8	1.00	27	0.296
1241	A	12	7	1.00	27	0.259
1242	A	11	6	1.00	27	0.222
1243	A	10	5	1.00	25	0.200
1244	A	12	7	1.00	27	0.259
1245	A	12	7	1.00	27	0.259
1246	A	13	8	1.00	27	0.296
1247	A	10	7	1.00	27	0.259
1248	A	9	6	1.00	27	0.222
1249	A	8	5	1.00	27	0.185
1250	A	7	4	1.00	25	0.160

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1251	A	11	6	1.00	27	0.222
1252	A	12	7	1.00	27	0.259
1253	A	10	7	1.00	27	0.259
1254	A	9	6	1.00	27	0.222
1255	A	8	5	1.00	27	0.185
1256	A	8	5	1.00	25	0.200
1257	A	12	7	1.00	27	0.259
1258	A	13	8	1.00	27	0.296
1259	A	10	7	1.00	27	0.259
1260	A	9	6	1.00	27	0.222
1261	A	9	6	1.00	27	0.222
1262	A	9	5	1.00	25	0.200
1263	A	13	8	1.00	27	0.296
1264	A	14	8	1.00	27	0.296
1265	A	14	9	1.00	29	0.310
1266	A	13	8	1.00	29	0.276
1267	A	11	8	1.00	29	0.276
1268	A	7	4	1.00	29	0.138
1269	A	8	5	1.00	29	0.172
1270	A	9	6	1.00	29	0.207
1271	A	14	9	1.00	29	0.310
1272	A	13	8	1.00	29	0.276
1273	A	12	8	1.00	29	0.276
1274	A	8	5	1.00	29	0.172
1275	A	9	6	1.00	29	0.207
1276	A	10	6	1.00	29	0.207
1277	A	15	9	1.00	29	0.310
1278	A	14	9	1.00	29	0.310
1279	A	13	8	1.00	29	0.276
1280	A	13	8	1.00	29	0.276
1281	A	9	6	1.00	29	0.207
1282	A	10	6	1.00	29	0.207
1283	A	13	8	1.00	29	0.276
1284	A	12	8	1.00	29	0.276
1285	A	7	4	1.00	29	0.138

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1286	A	7	4	1.00	29	0.138
1287	A	8	5	1.00	29	0.172
1288	A	9	6	1.00	29	0.207
1289	A	14	9	1.00	29	0.310
1290	A	13	8	1.00	29	0.276
1291	A	8	5	1.00	29	0.172
1292	A	8	5	1.00	29	0.172
1293	A	8	5	1.00	29	0.172
1294	A	9	6	1.00	29	0.207
1295	A	10	6	1.00	29	0.207
1296	A	15	10	1.00	29	0.345
1297	A	14	9	1.00	29	0.310
1298	A	9	6	1.00	29	0.207
1299	A	9	6	1.00	29	0.207
1300	A	9	6	1.00	29	0.207
1301	A	9	6	1.00	29	0.207
1302	A	10	6	1.00	29	0.207
1303	A	11	6	1.00	29	0.207
1304	A	7	4	1.00	25	0.160
1305	A	7	5	1.09	25	0.200
1306	A	6	4	1.00	25	0.160
1307	A	5	3	1.00	23	0.130
1308	A	5	3	1.00	12	0.250
1309	A	8	5	1.00	25	0.200
1310	A	9	6	0.99	25	0.240
1311	A	10	7	1.00	25	0.280
1312	A	7	4	1.00	27	0.148
1313	A	7	4	1.00	27	0.148
1314	A	7	4	1.00	27	0.148
1315	A	7	4	1.00	27	0.148
1316	A	7	4	1.00	27	0.148
1317	A	4	4	1.00	30	0.133
1318	A	8	5	1.00	30	0.167
1319	A	5	4	1.00	30	0.133
1320	A	4	3	1.00	28	0.107

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1321	A	8	5	1.00	30	0.167
1322	A	8	6	1.00	30	0.200
1323	A	8	5	1.00	27	0.185
1324	A	7	4	1.00	27	0.148
1325	A	7	4	1.00	27	0.148
1326	A	5	3	1.00	25	0.120
1327	A	8	5	1.00	27	0.185
1328	A	9	5	1.00	27	0.185

Chapter 3

Listing of integrals

Local contents

3.1 $\int \tan^5(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=102

$$-iax-\frac{a\log(\cos(c+dx))}{d}+\frac{ia\tan(c+dx)}{d}-\frac{a\tan^2(c+dx)}{2d}-\frac{ia\tan^3(c+dx)}{3d}+\frac{a\tan^4(c+dx)}{4d}+\frac{ia\tan^5(c+dx)}{5d}$$

```
[Out] -I*a*x-a*ln(cos(d*x+c))/d+I*a*tan(d*x+c)/d-1/2*a*tan(d*x+c)^2/d-1/3*I*a*tan
(d*x+c)^3/d+1/4*a*tan(d*x+c)^4/d+1/5*I*a*tan(d*x+c)^5/d
```

Rubi [A]
time = 0.07, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3609, 3606, 3556}

$$\frac{ia\tan^5(c+dx)}{5d}+\frac{a\tan^4(c+dx)}{4d}-\frac{ia\tan^3(c+dx)}{3d}-\frac{a\tan^2(c+dx)}{2d}+\frac{ia\tan(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}-iax$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (-I)*a*x - (a*Log[Cos[c + d*x]])/d + (I*a*Tan[c + d*x])/d - (a*Tan[c + d*x]
^2)/(2*d) - ((I/3)*a*Tan[c + d*x]^3)/d + (a*Tan[c + d*x]^4)/(4*d) + ((I/5)*
a*Tan[c + d*x]^5)/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \tan^5(c+dx)(a+ia\tan(c+dx))\,dx &= \frac{ia\tan^5(c+dx)}{5d}+\int \tan^4(c+dx)(-ia+a\tan(c+dx))\,dx \\ &= \frac{a\tan^4(c+dx)}{4d}+\frac{ia\tan^5(c+dx)}{5d}+\int \tan^3(c+dx)(-a-ia\tan(\\ &= -\frac{ia\tan^3(c+dx)}{3d}+\frac{a\tan^4(c+dx)}{4d}+\frac{ia\tan^5(c+dx)}{5d}+\int \tan^2(c \\ &= -\frac{a\tan^2(c+dx)}{2d}-\frac{ia\tan^3(c+dx)}{3d}+\frac{a\tan^4(c+dx)}{4d}+\frac{ia\tan^5(c}{5d} \\ &= -iax+\frac{ia\tan(c+dx)}{d}-\frac{a\tan^2(c+dx)}{2d}-\frac{ia\tan^3(c+dx)}{3d}+\frac{a\,\text{ta}}{} \\ &= -iax-\frac{a\log(\cos(c+dx))}{d}+\frac{ia\tan(c+dx)}{d}-\frac{a\tan^2(c+dx)}{2d}-\end{aligned}$$

Mathematica [A]
time = 0.33, size = 104, normalized size = 1.02

$$-\frac{ia\text{ArcTan}(\tan(c+dx))}{d}+\frac{ia\tan(c+dx)}{d}-\frac{ia\tan^3(c+dx)}{3d}+\frac{ia\tan^5(c+dx)}{5d}-\frac{a(4\log(\cos(c+dx))+2\tan^2(c+dx)-\tan^4(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-I)*a*ArcTan[Tan[c + d*x]])/d + (I*a*Tan[c + d*x])/d - ((I/3)*a*Tan[c + d
*x]^3)/d + ((I/5)*a*Tan[c + d*x]^5)/d - (a*(4*Log[Cos[c + d*x]] + 2*Tan[c +
 d*x]^2 - Tan[c + d*x]^4))/(4*d)
```

Maple [A]
time = 0.06, size = 81, normalized size = 0.79

method	result
derivativedivides	$\frac{a\left(i\tan(dx+c)+\frac{i\left(\tan^5(dx+c)\right)}{5}+\frac{\left(\tan^4(dx+c)\right)}{4}-\frac{i\left(\tan^3(dx+c)\right)}{3}-\frac{\left(\tan^2(dx+c)\right)}{2}+\frac{\ln\left(1+\tan^2(dx+c)\right)}{2}-i\arctan(\tan(dx+c))\right.}{d}$
default	$\frac{a\left(i\tan(dx+c)+\frac{i\left(\tan^5(dx+c)\right)}{5}+\frac{\left(\tan^4(dx+c)\right)}{4}-\frac{i\left(\tan^3(dx+c)\right)}{3}-\frac{\left(\tan^2(dx+c)\right)}{2}+\frac{\ln\left(1+\tan^2(dx+c)\right)}{2}-i\arctan(\tan(dx+c))\right.}{d}$
risch	$\frac{2iac}{d}-\frac{2a\left(75\,e^{8i(dx+c)}+150\,e^{6i(dx+c)}+200\,e^{4i(dx+c)}+100\,e^{2i(dx+c)}+23\right)}{15d\left(e^{2i(dx+c)}+1\right)^5}-\frac{a\ln\left(e^{2i(dx+c)}+1\right)}{d}$
norman	$\frac{ia\tan(dx+c)}{d}-\frac{a\left(\tan^2(dx+c)\right)}{2d}+\frac{a\left(\tan^4(dx+c)\right)}{4d}-iax-\frac{ia\left(\tan^3(dx+c)\right)}{3d}+\frac{ia\left(\tan^5(dx+c)\right)}{5d}+\frac{a\ln(1+\tan}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a*(I*tan(d*x+c)+1/5*I*tan(d*x+c)^5+1/4*tan(d*x+c)^4-1/3*I*tan(d*x+c)^3-
1/2*tan(d*x+c)^2+1/2*ln(1+tan(d*x+c)^2)-I*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.52, size = 81, normalized size = 0.79

$$-\frac{-12i\,a\tan\left(dx+c\right)^{5}-15\,a\tan\left(dx+c\right)^{4}+20i\,a\tan\left(dx+c\right)^{3}+30\,a\tan\left(dx+c\right)^{2}+60i\,(dx+c)a-30\,a\log\left(\tan\left(dx+c\right)^{2}+1\right)-60i\,a\tan\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(-12*I*a*tan(d*x + c)^5 - 15*a*tan(d*x + c)^4 + 20*I*a*tan(d*x + c)^3
 + 30*a*tan(d*x + c)^2 + 60*I*(d*x + c)*a - 30*a*log(tan(d*x + c)^2 + 1) -
60*I*a*tan(d*x + c))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(86) = 172.
time = 0.89, size = 192, normalized size = 1.88

$$-\frac{150\,ae^{(8i\,dx+8i\,c)}+300\,ae^{(6i\,dx+6i\,c)}+400\,ae^{(4i\,dx+4i\,c)}+200\,ae^{(2i\,dx+2i\,c)}+15\left(ae^{(10i\,dx+10i\,c)}+5\,ae^{(8i\,dx+8i\,c)}+10\,ae^{(6i\,dx+6i\,c)}+10\,ae^{(4i\,dx+4i\,c)}+5\,ae^{(2i\,dx+2i\,c)}+a\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)+46\,a}{15\left(de^{(10i\,dx+10i\,c)}+5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}+10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/15*(150*a*e^(8*I*d*x + 8*I*c) + 300*a*e^(6*I*d*x + 6*I*c) + 400*a*e^(4*I
*d*x + 4*I*c) + 200*a*e^(2*I*d*x + 2*I*c) + 15*(a*e^(10*I*d*x + 10*I*c) + 5
*a*e^(8*I*d*x + 8*I*c) + 10*a*e^(6*I*d*x + 6*I*c) + 10*a*e^(4*I*d*x + 4*I*c
) + 5*a*e^(2*I*d*x + 2*I*c) + a)*log(e^(2*I*d*x + 2*I*c) + 1) + 46*a)/(d*e^
(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) +
10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(85) = 170.
time = 0.62, size = 197, normalized size = 1.93

$$-\frac{a\log\left(e^{2idx}+e^{-2ic}\right)}{d}+\frac{-150ae^{8ic}e^{8idx}-300ae^{6ic}e^{6idx}-400ae^{4ic}e^{4idx}-200ae^{2ic}e^{2idx}-46a}{15de^{10ic}e^{10idx}+75de^{8ic}e^{8idx}+150de^{6ic}e^{6idx}+150de^{4ic}e^{4idx}+75de^{2ic}e^{2idx}+15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5*(a+I*a*tan(d*x+c)),x)
```

```
[Out] -a*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-150*a*exp(8*I*c)*exp(8*I*d*x) - 30
0*a*exp(6*I*c)*exp(6*I*d*x) - 400*a*exp(4*I*c)*exp(4*I*d*x) - 200*a*exp(2*I
*c)*exp(2*I*d*x) - 46*a)/(15*d*exp(10*I*c)*exp(10*I*d*x) + 75*d*exp(8*I*c)*
exp(8*I*d*x) + 150*d*exp(6*I*c)*exp(6*I*d*x) + 150*d*exp(4*I*c)*exp(4*I*d*x
) + 75*d*exp(2*I*c)*exp(2*I*d*x) + 15*d)
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(86) = 172$.
time = 1.68, size = 252, normalized size = 2.47

$$-\frac{15\,ae^{(10i\,dx+10i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+75\,ae^{(8i\,dx+8i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+150\,ae^{(6i\,dx+6i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+150\,ae^{(4i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+75\,ae^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+150\,ae^{(8i\,dx+8i\,c)}+300\,ae^{(6i\,dx+6i\,c)}+400\,ae^{(4i\,dx+4i\,c)}+200\,ae^{(2i\,dx+2i\,c)}+15\,a\log\left(e^{(2i\,dx+2i\,c)}+1\right)+46\,a}{15\left(de^{(10i\,dx+10i\,c)}+5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}+10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/15*(15*a*e^(10*I*d*x + 10*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 75*a*e^(8*
I*d*x + 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 150*a*e^(6*I*d*x + 6*I*c)*log
(e^(2*I*d*x + 2*I*c) + 1) + 150*a*e^(4*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*
c) + 1) + 75*a*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 150*a*e^(
8*I*d*x + 8*I*c) + 300*a*e^(6*I*d*x + 6*I*c) + 400*a*e^(4*I*d*x + 4*I*c) +
200*a*e^(2*I*d*x + 2*I*c) + 15*a*log(e^(2*I*d*x + 2*I*c) + 1) + 46*a)/(d*e^
(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) +
10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) + d)
```

Mupad [**B**]
time = 3.81, size = 73, normalized size = 0.72

$$\frac{a\tan(c+d\,x)\ 1\mathrm{i}-\frac{a\tan(c+d\,x)^2}{2}-\frac{a\tan(c+d\,x)^3\,1\mathrm{i}}{3}+\frac{a\tan(c+d\,x)^4}{4}+\frac{a\tan(c+d\,x)^5\,1\mathrm{i}}{5}+a\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5*(a + a*tan(c + d*x)*1i),x)
```

```
[Out] (a*tan(c + d*x)*1i - (a*tan(c + d*x)^2)/2 - (a*tan(c + d*x)^3*1i)/3 + (a*ta
n(c + d*x)^4)/4 + (a*tan(c + d*x)^5*1i)/5 + a*log(tan(c + d*x) + 1i))/d
```

3.2 $\int \tan^4(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=83

$$ax - \frac{ia\log(\cos(c+dx))}{d} - \frac{a\tan(c+dx)}{d} - \frac{ia\tan^2(c+dx)}{2d} + \frac{a\tan^3(c+dx)}{3d} + \frac{ia\tan^4(c+dx)}{4d}$$

```
[Out] a*x-I*a*ln(cos(d*x+c))/d-a*tan(d*x+c)/d-1/2*I*a*tan(d*x+c)^2/d+1/3*a*tan(d*
x+c)^3/d+1/4*I*a*tan(d*x+c)^4/d
```

Rubi [A]
time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3609, 3606, 3556}

$$\frac{ia\tan^4(c+dx)}{4d} + \frac{a\tan^3(c+dx)}{3d} - \frac{ia\tan^2(c+dx)}{2d} - \frac{a\tan(c+dx)}{d} - \frac{ia\log(\cos(c+dx))}{d} + ax$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] a*x - (I*a*Log[Cos[c + d*x]])/d - (a*Tan[c + d*x])/d - ((I/2)*a*Tan[c + d*x
]^2)/d + (a*Tan[c + d*x]^3)/(3*d) + ((I/4)*a*Tan[c + d*x]^4)/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\int \tan^4(c+dx)(a+ia\tan(c+dx))\,dx = \frac{ia\tan^4(c+dx)}{4d} + \int \tan^3(c+dx)(-ia+a\tan(c+dx))\,dx$$
$$= \frac{a\tan^3(c+dx)}{3d} + \frac{ia\tan^4(c+dx)}{4d} + \int \tan^2(c+dx)(-a-ia\tan($$
$$= -\frac{ia\tan^2(c+dx)}{2d} + \frac{a\tan^3(c+dx)}{3d} + \frac{ia\tan^4(c+dx)}{4d} + \int \tan(c$$
$$= ax - \frac{a\tan(c+dx)}{d} - \frac{ia\tan^2(c+dx)}{2d} + \frac{a\tan^3(c+dx)}{3d} + \frac{ia\tan^4}{}$$
$$= ax - \frac{ia\log(\cos(c+dx))}{d} - \frac{a\tan(c+dx)}{d} - \frac{ia\tan^2(c+dx)}{2d} + \frac{a\,\mathrm{t}}{}$$

Mathematica [A]
time = 0.22, size = 81, normalized size = 0.98

$$\frac{a\mathrm{ArcTan}(\tan(c+dx))}{d} - \frac{a\tan(c+dx)}{d} + \frac{a\tan^3(c+dx)}{3d} - \frac{ia(4\log(\cos(c+dx)) + 2\tan^2(c+dx) - \tan^4(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (a*ArcTan[Tan[c + d*x]])/d - (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
- ((I/4)*a*(4*Log[Cos[c + d*x]] + 2*Tan[c + d*x]^2 - Tan[c + d*x]^4))/d
```

Maple [A]
time = 0.05, size = 68, normalized size = 0.82

method	result	size
derivativedivides	$\frac{a\left(-\tan(dx+c)+\frac{i\left(\tan^4(dx+c)\right)}{4}+\frac{\left(\tan^3(dx+c)\right)}{3}-\frac{i\left(\tan^2(dx+c)\right)}{2}+\frac{i\ln\left(1+\tan^2(dx+c)\right)}{2}+\arctan(\tan(dx+c))\right)}{d}$	68
default	$\frac{a\left(-\tan(dx+c)+\frac{i\left(\tan^4(dx+c)\right)}{4}+\frac{\left(\tan^3(dx+c)\right)}{3}-\frac{i\left(\tan^2(dx+c)\right)}{2}+\frac{i\ln\left(1+\tan^2(dx+c)\right)}{2}+\arctan(\tan(dx+c))\right)}{d}$	68
norman	$ax-\frac{a\tan(dx+c)}{d}+\frac{a\left(\tan^3(dx+c)\right)}{3d}-\frac{ia\left(\tan^2(dx+c)\right)}{2d}+\frac{ia\left(\tan^4(dx+c)\right)}{4d}+\frac{ia\ln\left(1+\tan^2(dx+c)\right)}{2d}$	79
risch	$-\frac{2ac}{d}-\frac{4ia\left(6\,\mathrm{e}^{6i(dx+c)}+9\,\mathrm{e}^{4i(dx+c)}+8\,\mathrm{e}^{2i(dx+c)}+2\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^4}-\frac{ia\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{d}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a*(-tan(d*x+c)+1/4*I*tan(d*x+c)^4+1/3*tan(d*x+c)^3-1/2*I*tan(d*x+c)^2+1
/2*I*ln(1+tan(d*x+c)^2)+arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.52, size = 70, normalized size = 0.84

$$-\frac{-3i\,a\tan\left(dx+c\right)^{4}-4\,a\tan\left(dx+c\right)^{3}+6i\,a\tan\left(dx+c\right)^{2}-12\left(dx+c\right)a-6i\,a\log\left(\tan\left(dx+c\right)^{2}+1\right)+12\,a\tan\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(-3*I*a*tan(d*x + c)^4 - 4*a*tan(d*x + c)^3 + 6*I*a*tan(d*x + c)^2 -
12*(d*x + c)*a - 6*I*a*log(tan(d*x + c)^2 + 1) + 12*a*tan(d*x + c))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(71) = 142.
time = 0.60, size = 159, normalized size = 1.92

$$\frac{-24i\,ae^{(6i\,dx+6i\,c)}-36i\,ae^{(4i\,dx+4i\,c)}-32i\,ae^{(2i\,dx+2i\,c)}-3\left(i\,ae^{(8i\,dx+8i\,c)}+4i\,ae^{(6i\,dx+6i\,c)}+6i\,ae^{(4i\,dx+4i\,c)}+4i\,ae^{(2i\,dx+2i\,c)}+i\,a\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-8i\,a}{3\left(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(-24*I*a*e^(6*I*d*x + 6*I*c) - 36*I*a*e^(4*I*d*x + 4*I*c) - 32*I*a*e^(2
*I*d*x + 2*I*c) - 3*(I*a*e^(8*I*d*x + 8*I*c) + 4*I*a*e^(6*I*d*x + 6*I*c) +
6*I*a*e^(4*I*d*x + 4*I*c) + 4*I*a*e^(2*I*d*x + 2*I*c) + I*a)*log(e^(2*I*d*x
 + 2*I*c) + 1) - 8*I*a)/(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) +
6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(70) = 140.
time = 0.24, size = 168, normalized size = 2.02

$$-\frac{ia\log\left(e^{2idx}+e^{-2ic}\right)}{d}+\frac{-24iae^{6ic}e^{6idx}-36iae^{4ic}e^{4idx}-32iae^{2ic}e^{2idx}-8ia}{3de^{8ic}e^{8idx}+12de^{6ic}e^{6idx}+18de^{4ic}e^{4idx}+12de^{2ic}e^{2idx}+3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4*(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I*a*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-24*I*a*exp(6*I*c)*exp(6*I*d*x) -
 36*I*a*exp(4*I*c)*exp(4*I*d*x) - 32*I*a*exp(2*I*c)*exp(2*I*d*x) - 8*I*a)/(
3*d*exp(8*I*c)*exp(8*I*d*x) + 12*d*exp(6*I*c)*exp(6*I*d*x) + 18*d*exp(4*I*c
)*exp(4*I*d*x) + 12*d*exp(2*I*c)*exp(2*I*d*x) + 3*d)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(71) = 142.
time = 1.10, size = 204, normalized size = 2.46

$$\frac{-3i\,ae^{(8i\,dx+8i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-12i\,ae^{(6i\,dx+6i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-18i\,ae^{(4i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-12i\,ae^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-24i\,ae^{(6i\,dx+6i\,c)}-36i\,ae^{(4i\,dx+4i\,c)}-32i\,ae^{(2i\,dx+2i\,c)}-3i\,a\log\left(e^{(2i\,dx+2i\,c)}+1\right)-8i\,a}{3\left(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3*(-3*I*a*e^(8*I*d*x + 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 12*I*a*e^(6*
I*d*x + 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 18*I*a*e^(4*I*d*x + 4*I*c)*lo
g(e^(2*I*d*x + 2*I*c) + 1) - 12*I*a*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*
I*c) + 1) - 24*I*a*e^(6*I*d*x + 6*I*c) - 36*I*a*e^(4*I*d*x + 4*I*c) - 32*I*
a*e^(2*I*d*x + 2*I*c) - 3*I*a*log(e^(2*I*d*x + 2*I*c) + 1) - 8*I*a)/(d*e^(8
*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e
^(2*I*d*x + 2*I*c) + d)
```

Mupad [B]
time = 3.71, size = 63, normalized size = 0.76

$$\frac{\frac{a\tan(c+d\,x)^3}{3}-\frac{a\tan(c+d\,x)^2\,1{}\mathrm{i}}{2}-a\tan(c+d\,x)+\frac{a\tan(c+d\,x)^4\,1{}\mathrm{i}}{4}+a\,\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)\,1{}\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + a*tan(c + d*x)*1i),x)
```

```
[Out] ((a*tan(c + d*x)^3)/3 - (a*tan(c + d*x)^2*1i)/2 - a*tan(c + d*x) + (a*tan(c
 + d*x)^4*1i)/4 + a*log(tan(c + d*x) + 1i)*1i)/d
```

3.3 $\int \tan^3(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=67

$$iax+\frac{a\log(\cos(c+dx))}{d}-\frac{ia\tan(c+dx)}{d}+\frac{a\tan^2(c+dx)}{2d}+\frac{ia\tan^3(c+dx)}{3d}$$

```
[Out] I*a*x+a*ln(cos(d*x+c))/d-I*a*tan(d*x+c)/d+1/2*a*tan(d*x+c)^2/d+1/3*I*a*tan(
d*x+c)^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3609, 3606, 3556}

$$\frac{ia\tan^3(c+dx)}{3d}+\frac{a\tan^2(c+dx)}{2d}-\frac{ia\tan(c+dx)}{d}+\frac{a\log(\cos(c+dx))}{d}+iax$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] I*a*x + (a*Log[Cos[c + d*x]])/d - (I*a*Tan[c + d*x])/d + (a*Tan[c + d*x]^2)
/(2*d) + ((I/3)*a*Tan[c + d*x]^3)/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\int \tan^3(c+dx)(a+ia\tan(c+dx))\,dx = \frac{ia\tan^3(c+dx)}{3d} + \int \tan^2(c+dx)(-ia+a\tan(c+dx))\,dx$$
$$= \frac{a\tan^2(c+dx)}{2d} + \frac{ia\tan^3(c+dx)}{3d} + \int \tan(c+dx)(-a-ia\tan(c$$
$$= iax - \frac{ia\tan(c+dx)}{d} + \frac{a\tan^2(c+dx)}{2d} + \frac{ia\tan^3(c+dx)}{3d} - a\int \mathrm{t}$$
$$= iax + \frac{a\log(\cos(c+dx))}{d} - \frac{ia\tan(c+dx)}{d} + \frac{a\tan^2(c+dx)}{2d} + \frac{ia}{}$$

Mathematica [A]
time = 0.17, size = 74, normalized size = 1.10

$$\frac{ia\text{ArcTan}(\tan(c+dx))}{d} - \frac{ia\tan(c+dx)}{d} + \frac{ia\tan^3(c+dx)}{3d} + \frac{a(2\log(\cos(c+dx))+\tan^2(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3*(a + I*a*Tan[c + d*x]),x]
[Out] (I*a*ArcTan[Tan[c + d*x]])/d - (I*a*Tan[c + d*x])/d + ((I/3)*a*Tan[c + d*x]
^3)/d + (a*(2*Log[Cos[c + d*x]] + Tan[c + d*x]^2))/(2*d)
```

Maple [A]
time = 0.05, size = 60, normalized size = 0.90

method	result	size
derivativedivides	$\frac{a\left(-i\tan(dx+c)+\frac{i\left(\tan^3(dx+c)\right)}{3}+\frac{\left(\tan^2(dx+c)\right)}{2}-\frac{\ln\left(1+\tan^2(dx+c)\right)}{2}+i\arctan(\tan(dx+c))\right)}{d}$	60
default	$\frac{a\left(-i\tan(dx+c)+\frac{i\left(\tan^3(dx+c)\right)}{3}+\frac{\left(\tan^2(dx+c)\right)}{2}-\frac{\ln\left(1+\tan^2(dx+c)\right)}{2}+i\arctan(\tan(dx+c))\right)}{d}$	60
norman	$iax+\frac{a\left(\tan^2(dx+c)\right)}{2d}-\frac{ia\tan(dx+c)}{d}+\frac{ia\left(\tan^3(dx+c)\right)}{3d}-\frac{a\ln\left(1+\tan^2(dx+c)\right)}{2d}$	66
risch	$-\frac{2iac}{d}+\frac{2a\left(9\,\mathrm{e}^{4i(dx+c)}+9\,\mathrm{e}^{2i(dx+c)}+4\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}+\frac{a\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{d}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*a*(-I*tan(d*x+c)+1/3*I*tan(d*x+c)^3+1/2*tan(d*x+c)^2-1/2*ln(1+tan(d*x+c
)^2)+I*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.51, size = 59, normalized size = 0.88

$$-\frac{-2i\,a\tan\left(dx+c\right)^{3}-3\,a\tan\left(dx+c\right)^{2}-6i\left(dx+c\right)a+3\,a\log\left(\tan\left(dx+c\right)^{2}+1\right)+6i\,a\tan\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(-2*I*a*tan(d*x + c)^3 - 3*a*tan(d*x + c)^2 - 6*I*(d*x + c)*a + 3*a*lo
g(tan(d*x + c)^2 + 1) + 6*I*a*tan(d*x + c))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(57) = 114.
time = 0.50, size = 120, normalized size = 1.79

$$\frac{18\,ae^{\left(4i\,dx+4i\,c\right)}+18\,ae^{\left(2i\,dx+2i\,c\right)}+3\,\left(ae^{\left(6i\,dx+6i\,c\right)}+3\,ae^{\left(4i\,dx+4i\,c\right)}+3\,ae^{\left(2i\,dx+2i\,c\right)}+a\right)\log\left(e^{\left(2i\,dx+2i\,c\right)}+1\right)+8\,a}{3\,\left(de^{\left(6i\,dx+6i\,c\right)}+3\,de^{\left(4i\,dx+4i\,c\right)}+3\,de^{\left(2i\,dx+2i\,c\right)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(18*a*e^(4*I*d*x + 4*I*c) + 18*a*e^(2*I*d*x + 2*I*c) + 3*(a*e^(6*I*d*x
 + 6*I*c) + 3*a*e^(4*I*d*x + 4*I*c) + 3*a*e^(2*I*d*x + 2*I*c) + a)*log(e^(2*
I*d*x + 2*I*c) + 1) + 8*a)/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c)
 + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(56) = 112.
time = 0.20, size = 121, normalized size = 1.81

$$\frac{a\log\left(e^{2idx}+e^{-2ic}\right)}{d}+\frac{18ae^{4ic}e^{4idx}+18ae^{2ic}e^{2idx}+8a}{3de^{6ic}e^{6idx}+9de^{4ic}e^{4idx}+9de^{2ic}e^{2idx}+3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3*(a+I*a*tan(d*x+c)),x)
```

```
[Out] a*log(exp(2*I*d*x) + exp(-2*I*c))/d + (18*a*exp(4*I*c)*exp(4*I*d*x) + 18*a*
exp(2*I*c)*exp(2*I*d*x) + 8*a)/(3*d*exp(6*I*c)*exp(6*I*d*x) + 9*d*exp(4*I*c
)*exp(4*I*d*x) + 9*d*exp(2*I*c)*exp(2*I*d*x) + 3*d)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(57) = 114.
time = 0.90, size = 156, normalized size = 2.33

$$\frac{3\,ae^{\left(6i\,dx+6i\,c\right)}\log\left(e^{\left(2i\,dx+2i\,c\right)}+1\right)+9\,ae^{\left(4i\,dx+4i\,c\right)}\log\left(e^{\left(2i\,dx+2i\,c\right)}+1\right)+9\,ae^{\left(2i\,dx+2i\,c\right)}\log\left(e^{\left(2i\,dx+2i\,c\right)}+1\right)+18\,ae^{\left(4i\,dx+4i\,c\right)}+18\,ae^{\left(2i\,dx+2i\,c\right)}+3\,a\log\left(e^{\left(2i\,dx+2i\,c\right)}+1\right)+8\,a}{3\,\left(de^{\left(6i\,dx+6i\,c\right)}+3\,de^{\left(4i\,dx+4i\,c\right)}+3\,de^{\left(2i\,dx+2i\,c\right)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3*(3*a*e^(6*I*d*x + 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 9*a*e^(4*I*d*x
+ 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 9*a*e^(2*I*d*x + 2*I*c)*log(e^(2*I*
d*x + 2*I*c) + 1) + 18*a*e^(4*I*d*x + 4*I*c) + 18*a*e^(2*I*d*x + 2*I*c) + 3
*a*log(e^(2*I*d*x + 2*I*c) + 1) + 8*a)/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*
d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Mupad [B]
time = 3.69, size = 51, normalized size = 0.76

$$-\frac{a\tan(c+d\,x)\ 1\mathrm{i}-\frac{a\tan(c+d\,x)^2}{2}-\frac{a\tan(c+d\,x)^3\,1\mathrm{i}}{3}+a\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + a*tan(c + d*x)*1i),x)
```

```
[Out] -(a*tan(c + d*x)*1i - (a*tan(c + d*x)^2)/2 - (a*tan(c + d*x)^3*1i)/3 + a*lo
g(tan(c + d*x) + 1i))/d
```

3.4 $\int \tan^2(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=49

$$-ax+\frac{ia\log(\cos(c+dx))}{d}+\frac{a\tan(c+dx)}{d}+\frac{ia\tan^2(c+dx)}{2d}$$

```
[Out] -a*x+I*a*ln(cos(d*x+c))/d+a*tan(d*x+c)/d+1/2*I*a*tan(d*x+c)^2/d
```

Rubi [A]
time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3609, 3606, 3556}

$$\frac{ia\tan^2(c+dx)}{2d}+\frac{a\tan(c+dx)}{d}+\frac{ia\log(\cos(c+dx))}{d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] -(a*x) + (I*a*Log[Cos[c + d*x]])/d + (a*Tan[c + d*x])/d + ((I/2)*a*Tan[c +
d*x]^2)/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))\,dx = \frac{ia\tan^2(c+dx)}{2d} + \int \tan(c+dx)(-ia+a\tan(c+dx))\,dx$$
$$= -ax + \frac{a\tan(c+dx)}{d} + \frac{ia\tan^2(c+dx)}{2d} - (ia)\int \tan(c+dx)\,dx$$
$$= -ax + \frac{ia\log(\cos(c+dx))}{d} + \frac{a\tan(c+dx)}{d} + \frac{ia\tan^2(c+dx)}{2d}$$

Mathematica [A]
time = 0.12, size = 53, normalized size = 1.08

$$-\frac{a\text{ArcTan}(\tan(c+dx))}{d} + \frac{a\tan(c+dx)}{d} + \frac{ia(2\log(\cos(c+dx)) + \tan^2(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] -((a*ArcTan[Tan[c + d*x]])/d) + (a*Tan[c + d*x])/d + ((I/2)*a*(2*Log[Cos[c
+ d*x]] + Tan[c + d*x]^2))/d
```

Maple [A]
time = 0.04, size = 47, normalized size = 0.96

method	result	size
derivativedivides	$\frac{a\left(\frac{i\left(\tan^2(dx+c)\right)}{2}+\tan(dx+c)-\frac{i\ln\left(1+\tan^2(dx+c)\right)}{2}-\arctan(\tan(dx+c))\right)}{d}$	47
default	$\frac{a\left(\frac{i\left(\tan^2(dx+c)\right)}{2}+\tan(dx+c)-\frac{i\ln\left(1+\tan^2(dx+c)\right)}{2}-\arctan(\tan(dx+c))\right)}{d}$	47
norman	$\frac{a\tan(dx+c)}{d} - ax + \frac{ia\left(\tan^2(dx+c)\right)}{2d} - \frac{ia\ln\left(1+\tan^2(dx+c)\right)}{2d}$	50
risch	$\frac{2ac}{d} + \frac{2ia\left(2\,e^{2i(dx+c)}+1\right)}{d\left(e^{2i(dx+c)}+1\right)^2} + \frac{ia\ln\left(e^{2i(dx+c)}+1\right)}{d}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a*(1/2*I*tan(d*x+c)^2+tan(d*x+c)-1/2*I*ln(1+tan(d*x+c)^2)-arctan(tan(d*
x+c)))
```

Maxima [A]
time = 0.55, size = 48, normalized size = 0.98

$$-\frac{-i\,a\tan\left(dx+c\right)^2 + 2\left(dx+c\right)a + i\,a\log\left(\tan\left(dx+c\right)^2+1\right) - 2\,a\tan\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(-I*a*tan(d*x + c)^2 + 2*(d*x + c)*a + I*a*log(tan(d*x + c)^2 + 1) - 2
*a*tan(d*x + c))/d
```

Fricas [A]
time = 0.44, size = 85, normalized size = 1.73

$$\frac{4i\,ae^{(2i\,dx+2i\,c)} + \left(i\,ae^{(4i\,dx+4i\,c)} + 2i\,ae^{(2i\,dx+2i\,c)} + i\,a\right)\log\left(e^{(2i\,dx+2i\,c)} + 1\right) + 2i\,a}{de^{(4i\,dx+4i\,c)} + 2\,de^{(2i\,dx+2i\,c)} + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] (4*I*a*e^(2*I*d*x + 2*I*c) + (I*a*e^(4*I*d*x + 4*I*c) + 2*I*a*e^(2*I*d*x +
2*I*c) + I*a)*log(e^(2*I*d*x + 2*I*c) + 1) + 2*I*a)/(d*e^(4*I*d*x + 4*I*c)
+ 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(41) = 82$.
time = 0.16, size = 85, normalized size = 1.73

$$\frac{ia\log\left(e^{2idx} + e^{-2ic}\right)}{d} + \frac{4iae^{2ic}e^{2idx} + 2ia}{de^{4ic}e^{4idx} + 2de^{2ic}e^{2idx} + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2*(a+I*a*tan(d*x+c)),x)
```

```
[Out] I*a*log(exp(2*I*d*x) + exp(-2*I*c))/d + (4*I*a*exp(2*I*c)*exp(2*I*d*x) + 2*
I*a)/(d*exp(4*I*c)*exp(4*I*d*x) + 2*d*exp(2*I*c)*exp(2*I*d*x) + d)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(43) = 86$.
time = 0.66, size = 107, normalized size = 2.18

$$\frac{i\,ae^{(4i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)} + 1\right) + 2i\,ae^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)} + 1\right) + 4i\,ae^{(2i\,dx+2i\,c)} + i\,a\log\left(e^{(2i\,dx+2i\,c)} + 1\right) + 2i\,a}{de^{(4i\,dx+4i\,c)} + 2\,de^{(2i\,dx+2i\,c)} + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] (I*a*e^(4*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 2*I*a*e^(2*I*d*x +
2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 4*I*a*e^(2*I*d*x + 2*I*c) + I*a*log(e
^(2*I*d*x + 2*I*c) + 1) + 2*I*a)/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x +
2*I*c) + d)
```

Mupad [B]
time = 3.75, size = 39, normalized size = 0.80

$$\frac{a\left(2\tan(c+d\,x)-\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,2\mathrm{i}+\tan(c+d\,x)^2\,1\mathrm{i}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + a*tan(c + d*x)*1i),x)
[Out] (a*(2*tan(c + d*x) - log(tan(c + d*x) + 1i)*2i + tan(c + d*x)^2*1i))/(2*d)
```

3.5 $\int \tan(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=34

$$-iax-\frac{a\log(\cos(c+dx))}{d}+\frac{ia\tan(c+dx)}{d}$$

```
[Out] -I*a*x-a*ln(cos(d*x+c))/d+I*a*tan(d*x+c)/d
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {3606, 3556}

$$\frac{ia\tan(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}-iax$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (-I)*a*x - (a*Log[Cos[c + d*x]])/d + (I*a*Tan[c + d*x])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \tan(c+dx)(a+ia\tan(c+dx))\,dx&=-iax+\frac{ia\tan(c+dx)}{d}+a\int \tan(c+dx)\,dx\\&=-iax-\frac{a\log(\cos(c+dx))}{d}+\frac{ia\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 43, normalized size = 1.26

$$-\frac{ia\text{ArcTan}(\tan(c+dx))}{d}-\frac{a\log(\cos(c+dx))}{d}+\frac{ia\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]*(a + I*a*Tan[c + d*x]),x]
[Out] ((-I)*a*ArcTan[Tan[c + d*x]])/d - (a*Log[Cos[c + d*x]])/d + (I*a*Tan[c + d*
x])/d
```

Maple [A]
time = 0.03, size = 39, normalized size = 1.15

method	result	size
norman	$\frac{ia\tan(dx+c)}{d} - iax + \frac{a\ln\left(1+\tan^2(dx+c)\right)}{2d}$	37
derivativedivides	$\frac{a\left(i\tan(dx+c)+\frac{\ln\left(1+\tan^2(dx+c)\right)}{2}-i\arctan(\tan(dx+c))\right)}{d}$	39
default	$\frac{a\left(i\tan(dx+c)+\frac{\ln\left(1+\tan^2(dx+c)\right)}{2}-i\arctan(\tan(dx+c))\right)}{d}$	39
risch	$\frac{2iac}{d} - \frac{2a}{d\left(e^{2i(dx+c)}+1\right)} - \frac{a\ln\left(e^{2i(dx+c)}+1\right)}{d}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*a*(I*tan(d*x+c)+1/2*ln(1+tan(d*x+c)^2)-I*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.49, size = 37, normalized size = 1.09

$$-\frac{2i(dx+c)a - a\log\left(\tan(dx+c)^2+1\right) - 2i\,a\tan(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
[Out] -1/2*(2*I*(d*x + c)*a - a*log(tan(d*x + c)^2 + 1) - 2*I*a*tan(d*x + c))/d
```

Fricas [A]
time = 0.61, size = 47, normalized size = 1.38

$$-\frac{\left(ae^{(2i\,dx+2i\,c)}+a\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)+2\,a}{de^{(2i\,dx+2i\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
[Out] -((a*e^(2*I*d*x + 2*I*c) + a)*log(e^(2*I*d*x + 2*I*c) + 1) + 2*a)/(d*e^(2*I
*d*x + 2*I*c) + d)
```

Sympy [A]
time = 0.13, size = 44, normalized size = 1.29

$$-\frac{2a}{de^{2ic}e^{2idx}+d}-\frac{a\log\left(e^{2idx}+e^{-2ic}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c)),x)
[Out] -2*a/(d*exp(2*I*c)*exp(2*I*d*x) + d) - a*log(exp(2*I*d*x) + exp(-2*I*c))/d
```

Giac [A]
time = 0.50, size = 58, normalized size = 1.71

$$-\frac{ae^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+a\log\left(e^{(2i\,dx+2i\,c)}+1\right)+2\,a}{de^{(2i\,dx+2i\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] -(a*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + a*log(e^(2*I*d*x + 2
*I*c) + 1) + 2*a)/(d*e^(2*I*d*x + 2*I*c) + d)
```

Mupad [B]
time = 3.78, size = 25, normalized size = 0.74

$$\frac{a\left(\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)+\tan(c+d\,x)\ 1\mathrm{i}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + a*tan(c + d*x)*1i),x)
[Out] (a*(log(tan(c + d*x) + 1i) + tan(c + d*x)*1i))/d
```

3.6 $\int (a + ia\tan(c + dx))\,dx$

Optimal. Leaf size=19

$$ax - \frac{ia\log(\cos(c + dx))}{d}$$

```
[Out] a*x-I*a*ln(cos(d*x+c))/d
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3556}

$$ax - \frac{ia\log(\cos(c + dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + I*a*Tan[c + d*x],x]
```

```
[Out] a*x - (I*a*Log[Cos[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a + ia\tan(c + dx))\,dx &= ax + (ia)\int \tan(c + dx)\,dx \\ &= ax - \frac{ia\log(\cos(c + dx))}{d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 1.00

$$ax - \frac{ia\log(\cos(c + dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + I*a*Tan[c + d*x],x]
```

```
[Out] a*x - (I*a*Log[Cos[c + d*x]])/d
```

Maple [A]
time = 0.00, size = 23, normalized size = 1.21

method	result	size
default	$ax+\frac{ia\ln\left(1+\tan^{2}\left(dx+c\right)\right)}{2d}$	23
norman	$ax+\frac{ia\ln\left(1+\tan^{2}\left(dx+c\right)\right)}{2d}$	23
derivativedivides	$\frac{a\left(\frac{i\ln\left(1+\tan^{2}\left(dx+c\right)\right)}{2}+\arctan\left(\tan\left(dx+c\right)\right)\right)}{d}$	28
risch	$-\frac{ia\ln\left(\mathrm{e}^{2i\left(dx+c\right)}+1\right)}{d}-\frac{2ac}{d}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+I*a*tan(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+1/2*I*a/d*ln(1+tan(d*x+c)^2)
```

Maxima [A]
time = 0.29, size = 17, normalized size = 0.89

$$ax+\frac{i\,a\log\left(\sec\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+I*a*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + I*a*log(sec(d*x + c))/d
```

Fricas [A]
time = 0.66, size = 18, normalized size = 0.95

$$-\frac{i\,a\log\left(e^{(2i\,dx+2i\,c)}+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+I*a*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] -I*a*log(e^(2*I*d*x + 2*I*c) + 1)/d
```

Sympy [A]
time = 0.09, size = 24, normalized size = 1.26

$$-\frac{ia\log\left(e^{2idx}+e^{-2ic}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+I*a*tan(d*x+c),x)
```

```
[Out] -I*a*log(exp(2*I*d*x) + exp(-2*I*c))/d
```

Giac [A]
time = 0.46, size = 18, normalized size = 0.95

$$ax - \frac{i\,a\log\left(|\cos\left(dx + c\right)|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+I*a*tan(d*x+c),x, algorithm="giac")
```

```
[Out] a*x - I*a*log(abs(cos(d*x + c)))/d
```

Mupad [B]
time = 3.76, size = 17, normalized size = 0.89

$$\frac{a\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + a*tan(c + d*x)*1i,x)
```

```
[Out] (a*log(tan(c + d*x) + 1i)*1i)/d
```

3.7 $\int \cot(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=19

$$iax+\frac{a\log(\sin(c+dx))}{d}$$

```
[Out] I*a*x+a*ln(sin(d*x+c))/d
```

Rubi [A]
time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3612, 3556}

$$\frac{a\log(\sin(c+dx))}{d}+iax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] I*a*x + (a*Log[Sin[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)(a+ia\tan(c+dx))\,dx &= iax+a\int \cot(c+dx)\,dx\\ &= iax+\frac{a\log(\sin(c+dx))}{d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 27, normalized size = 1.42

$$iax+\frac{a(\log(\cos(c+dx))+\log(\tan(c+dx)))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] I*a*x + (a*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]]))/d
```

Maple [A]
time = 0.15, size = 24, normalized size = 1.26

method	result	size
derivativedivides	$\frac{ia(dx+c)+a\ln(\sin(dx+c))}{d}$	24
default	$\frac{ia(dx+c)+a\ln(\sin(dx+c))}{d}$	24
risch	$-\frac{2iac}{d}+\frac{a\ln\left(e^{2i(dx+c)}-1\right)}{d}$	27
norman	$iax+\frac{a\ln(\tan(dx+c))}{d}-\frac{a\ln(1+\tan^2(dx+c))}{2d}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(I*a*(d*x+c)+a*ln(sin(d*x+c)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(17) = 34.
time = 0.49, size = 37, normalized size = 1.95

$$-\frac{-2i\,(dx+c)a+a\log\left(\tan\left(dx+c\right)^{2}+1\right)-2\,a\log\left(\tan\left(dx+c\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(-2*I*(d*x + c)*a + a*log(tan(d*x + c)^2 + 1) - 2*a*log(tan(d*x + c)))
/d
```

Fricas [A]
time = 0.54, size = 17, normalized size = 0.89

$$\frac{a\log\left(e^{(2i\,dx+2i\,c)}-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] a*log(e^(2*I*d*x + 2*I*c) - 1)/d
```

Sympy [A]
time = 0.09, size = 20, normalized size = 1.05

$$\frac{a\log\left(e^{2idx}-e^{-2ic}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] a*log(exp(2*I*d*x) - exp(-2*I*c))/d
```

Giac [A]
time = 0.47, size = 34, normalized size = 1.79

$$-\frac{2\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)-a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -(2*a*log(tan(1/2*d*x + 1/2*c) + I) - a*log(tan(1/2*d*x + 1/2*c)))/d
```

Mupad [B]
time = 3.78, size = 19, normalized size = 1.00

$$\frac{a\,\mathrm{atan}(2\tan(c+d\,x)+1\mathrm{i})\;2\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + a*tan(c + d*x)*1i),x)
```

```
[Out] (a*atan(2*tan(c + d*x) + 1i)*2i)/d
```

3.8 $\int \cot^2(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=32

$$-ax-\frac{a\cot(c+dx)}{d}+\frac{ia\log(\sin(c+dx))}{d}$$

```
[Out] -a*x-a*cot(d*x+c)/d+I*a*ln(sin(d*x+c))/d
```

Rubi [A]
time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.136$, Rules used = {3610, 3612, 3556}

$$-\frac{a\cot(c+dx)}{d}+\frac{ia\log(\sin(c+dx))}{d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] -(a*x) - (a*Cot[c + d*x])/d + (I*a*Log[Sin[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{a\cot(c+dx)}{d} + \int \cot(c+dx)(ia-a\tan(c+dx))\,dx$$
$$= -ax - \frac{a\cot(c+dx)}{d} + (ia)\int \cot(c+dx)\,dx$$
$$= -ax - \frac{a\cot(c+dx)}{d} + \frac{ia\log(\sin(c+dx))}{d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.13, size = 54, normalized size = 1.69

$$-\frac{a\cot(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)}{d} + \frac{ia(\log(\cos(c+dx))+\log(\tan(c+dx)))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] -((a*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2])/d) + (I
*a*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]]))/d
```

Maple [A]
time = 0.15, size = 35, normalized size = 1.09

method	result	size
derivativedivides	$\frac{ia\ln(\sin(dx+c))+a(-\cot(dx+c)-dx-c)}{d}$	35
default	$\frac{ia\ln(\sin(dx+c))+a(-\cot(dx+c)-dx-c)}{d}$	35
risch	$\frac{2ac}{d} - \frac{2ia}{d\left(e^{2i(dx+c)}-1\right)} + \frac{ia\ln\left(e^{2i(dx+c)}-1\right)}{d}$	48
norman	$\frac{-\frac{a}{d}-ax\tan(dx+c)}{\tan(dx+c)} + \frac{ia\ln(\tan(dx+c))}{d} - \frac{ia\ln\left(1+\tan^2(dx+c)\right)}{2d}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(I*a*ln(sin(d*x+c))+a*(-cot(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.49, size = 49, normalized size = 1.53

$$-\frac{2\,(dx+c)a + i\,a\log\left(\tan\left(dx+c\right)^2+1\right) - 2i\,a\log\left(\tan\left(dx+c\right)\right) + \frac{2\,a}{\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*(d*x + c)*a + I*a*log(tan(d*x + c)^2 + 1) - 2*I*a*log(tan(d*x + c))
 + 2*a/tan(d*x + c))/d
```

Fricas [A]
time = 0.44, size = 51, normalized size = 1.59

$$\frac{\left(i\,ae^{(2i\,dx+2i\,c)}-i\,a\right)\log\left(e^{(2i\,dx+2i\,c)}-1\right)-2i\,a}{de^{(2i\,dx+2i\,c)}-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((I*a*e^(2*I*d*x + 2*I*c) - I*a)*log(e^(2*I*d*x + 2*I*c) - 1) - 2*I*a)/(d*e
^(2*I*d*x + 2*I*c) - d)
```

Sympy [A]
time = 0.15, size = 46, normalized size = 1.44

$$-\frac{2ia}{de^{2ic}e^{2idx}-d}+\frac{ia\log\left(e^{2idx}-e^{-2ic}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+I*a*tan(d*x+c)),x)
```

```
[Out] -2*I*a/(d*exp(2*I*c)*exp(2*I*d*x) - d) + I*a*log(exp(2*I*d*x) - exp(-2*I*c)
)/d
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(30) = 60$.
time = 0.61, size = 75, normalized size = 2.34

$$-\frac{4i\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)-2i\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{-2i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(4*I*a*log(tan(1/2*d*x + 1/2*c) + I) - 2*I*a*log(tan(1/2*d*x + 1/2*c))
 - a*tan(1/2*d*x + 1/2*c) - (-2*I*a*tan(1/2*d*x + 1/2*c) - a)/tan(1/2*d*x +
 1/2*c))/d
```

Mupad [B]
time = 3.73, size = 27, normalized size = 0.84

$$-\frac{a\left(\cot(c+d\,x)+2\,\mathrm{atan}(2\tan(c+d\,x)+1\mathrm{i})\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + a*tan(c + d*x)*1i),x)
[Out] -(a*(cot(c + d*x) + 2*atan(2*tan(c + d*x) + 1i)))/d
```

3.9 $\int \cot^3(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=50

$$-iax-\frac{ia\cot(c+dx)}{d}-\frac{a\cot^2(c+dx)}{2d}-\frac{a\log(\sin(c+dx))}{d}$$

```
[Out] -I*a*x-I*a*cot(d*x+c)/d-1/2*a*cot(d*x+c)^2/d-a*ln(sin(d*x+c))/d
```

Rubi [A]
time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3610, 3612, 3556}

$$-\frac{a\cot^2(c+dx)}{2d}-\frac{ia\cot(c+dx)}{d}-\frac{a\log(\sin(c+dx))}{d}-iax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (-I)*a*x - (I*a*Cot[c + d*x])/d - (a*Cot[c + d*x]^2)/(2*d) - (a*Log[Sin[c +
 d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{a\cot^2(c+dx)}{2d} + \int \cot^2(c+dx)(ia-a\tan(c+dx))\,dx$$
$$= -\frac{ia\cot(c+dx)}{d} - \frac{a\cot^2(c+dx)}{2d} + \int \cot(c+dx)(-a-ia\tan(c+$$
$$= -iax - \frac{ia\cot(c+dx)}{d} - \frac{a\cot^2(c+dx)}{2d} - a\int \cot(c+dx)\,dx$$
$$= -iax - \frac{ia\cot(c+dx)}{d} - \frac{a\cot^2(c+dx)}{2d} - \frac{a\log(\sin(c+dx))}{d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.19, size = 68, normalized size = 1.36

$$-\frac{ia\cot(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)}{d} - \frac{a(\cot^2(c+dx)+2\log(\cos(c+dx))+2\log(\tan(c+dx)))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + I*a*Tan[c + d*x]),x]
[Out] ((-I)*a*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2])/d -
(a*(Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[Tan[c + d*x]]))/(2*d)
```

Maple [A]
time = 0.21, size = 48, normalized size = 0.96

method	result	size
derivativedivides	$\frac{ia(-\cot(dx+c)-dx-c)+a\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)}{d}$	48
default	$\frac{ia(-\cot(dx+c)-dx-c)+a\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)}{d}$	48
risch	$\frac{2iac}{d}+\frac{2a\left(2\,e^{2i(dx+c)}-1\right)}{d\left(e^{2i(dx+c)}-1\right)^2}-\frac{a\ln\left(e^{2i(dx+c)}-1\right)}{d}$	60
norman	$\frac{-\frac{a}{2d}-\frac{ia\tan(dx+c)}{d}-iax\left(\tan^2(dx+c)\right)}{\tan(dx+c)^2}-\frac{a\ln(\tan(dx+c))}{d}+\frac{a\ln\left(1+\tan^2(dx+c)\right)}{2d}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(I*a*(-cot(d*x+c)-d*x-c)+a*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c))))
```

Maxima [A]
time = 0.51, size = 58, normalized size = 1.16

$$-\frac{2i\,(dx+c)a - a\log\left(\tan\left(dx+c\right)^2+1\right) + 2\,a\log\left(\tan\left(dx+c\right)\right) + \frac{2i\,a\tan(dx+c)+a}{\tan(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*I*(d*x + c)*a - a*log(tan(d*x + c)^2 + 1) + 2*a*log(tan(d*x + c)) +
 (2*I*a*tan(d*x + c) + a)/tan(d*x + c)^2)/d
```

Fricas [A]
time = 0.40, size = 83, normalized size = 1.66

$$\frac{4\,ae^{(2i\,dx+2i\,c)} - \left(ae^{(4i\,dx+4i\,c)} - 2\,ae^{(2i\,dx+2i\,c)} + a\right)\log\left(e^{(2i\,dx+2i\,c)} - 1\right) - 2\,a}{de^{(4i\,dx+4i\,c)} - 2\,de^{(2i\,dx+2i\,c)} + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] (4*a*e^(2*I*d*x + 2*I*c) - (a*e^(4*I*d*x + 4*I*c) - 2*a*e^(2*I*d*x + 2*I*c)
 + a)*log(e^(2*I*d*x + 2*I*c) - 1) - 2*a)/(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2
*I*d*x + 2*I*c) + d)
```

Sympy [A]
time = 0.16, size = 80, normalized size = 1.60

$$-\frac{a\log\left(e^{2idx} - e^{-2ic}\right)}{d} + \frac{4ae^{2ic}e^{2idx} - 2a}{de^{4ic}e^{4idx} - 2de^{2ic}e^{2idx} + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+I*a*tan(d*x+c)),x)
```

```
[Out] -a*log(exp(2*I*d*x) - exp(-2*I*c))/d + (4*a*exp(2*I*c)*exp(2*I*d*x) - 2*a)/
(d*exp(4*I*c)*exp(4*I*d*x) - 2*d*exp(2*I*c)*exp(2*I*d*x) + d)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(44) = 88.
time = 0.55, size = 102, normalized size = 2.04

$$-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 16\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right) + 8\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right) - 4i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - \frac{12\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 4i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/8*(a*tan(1/2*d*x + 1/2*c)^2 - 16*a*log(tan(1/2*d*x + 1/2*c) + I) + 8*a*l
og(tan(1/2*d*x + 1/2*c)) - 4*I*a*tan(1/2*d*x + 1/2*c) - (12*a*tan(1/2*d*x +
 1/2*c)^2 - 4*I*a*tan(1/2*d*x + 1/2*c) - a)/tan(1/2*d*x + 1/2*c)^2)/d
```

Mupad [B]
time = 3.79, size = 47, normalized size = 0.94

$$-\frac{a\,\mathrm{atan}(2\tan(c+d\,x)+1\mathrm{i})\;2\mathrm{i}}{d}-\frac{\frac{a}{2}+a\tan(c+d\,x)\;1\mathrm{i}}{d\tan\left(c+d\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + a*tan(c + d*x)*1i),x)
```

```
[Out] - (a*atan(2*tan(c + d*x) + 1i)*2i)/d - (a/2 + a*tan(c + d*x)*1i)/(d*tan(c +
 d*x)^2)
```

3.10 $\int \cot^4(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=64

$$ax+\frac{a\cot(c+dx)}{d}-\frac{ia\cot^2(c+dx)}{2d}-\frac{a\cot^3(c+dx)}{3d}-\frac{ia\log(\sin(c+dx))}{d}$$

```
[Out] a*x+a*cot(d*x+c)/d-1/2*I*a*cot(d*x+c)^2/d-1/3*a*cot(d*x+c)^3/d-I*a*ln(sin(d
*x+c))/d
```

Rubi [A]
time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.136$, Rules used = {3610, 3612, 3556}

$$-\frac{a\cot^3(c+dx)}{3d}-\frac{ia\cot^2(c+dx)}{2d}+\frac{a\cot(c+dx)}{d}-\frac{ia\log(\sin(c+dx))}{d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] a*x + (a*Cot[c + d*x])/d - ((I/2)*a*Cot[c + d*x]^2)/d - (a*Cot[c + d*x]^3)/
(3*d) - (I*a*Log[Sin[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{a\cot^3(c+dx)}{3d} + \int \cot^3(c+dx)(ia - a\tan(c+dx))\,dx$$
$$= -\frac{ia\cot^2(c+dx)}{2d} - \frac{a\cot^3(c+dx)}{3d} + \int \cot^2(c+dx)(-a - ia\tan(c$$
$$= \frac{a\cot(c+dx)}{d} - \frac{ia\cot^2(c+dx)}{2d} - \frac{a\cot^3(c+dx)}{3d} + \int \cot(c+dx)($$
$$= ax + \frac{a\cot(c+dx)}{d} - \frac{ia\cot^2(c+dx)}{2d} - \frac{a\cot^3(c+dx)}{3d} - (ia)\int \text{co}$$
$$= ax + \frac{a\cot(c+dx)}{d} - \frac{ia\cot^2(c+dx)}{2d} - \frac{a\cot^3(c+dx)}{3d} - \frac{ia\log(\sin}{d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.21, size = 72, normalized size = 1.12

$$-\frac{a\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(c+dx)\right)}{3d} - \frac{ia(\cot^2(c+dx) + 2\log(\cos(c+dx)) + 2\log(\tan(c+dx)))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] -1/3*(a*Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/d
 - ((I/2)*a*(Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[Tan[c + d*x]]))/d
```

Maple [A]
time = 0.20, size = 53, normalized size = 0.83

method	result	size
derivativedivides	$\frac{ia\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+a\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)}{d}$	53
default	$\frac{ia\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+a\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)}{d}$	53
risch	$-\frac{2ac}{d}+\frac{2ia\left(9\,e^{4i(dx+c)}-9\,e^{2i(dx+c)}+4\right)}{3d\left(e^{2i(dx+c)}-1\right)^3}-\frac{ia\ln\left(e^{2i(dx+c)}-1\right)}{d}$	72
norman	$\frac{ax\left(\tan^3(dx+c)\right)+\frac{a\left(\tan^2(dx+c)\right)}{d}-\frac{a}{3d}-\frac{ia\tan(dx+c)}{2d}}{\tan(dx+c)^3}-\frac{ia\ln(\tan(dx+c))}{d}+\frac{ia\ln\left(1+\tan^2(dx+c)\right)}{2d}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(I*a*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+a*(-1/3*cot(d*x+c)^3+cot(d*x+c)
+d*x+c))
```

Maxima [A]
time = 0.49, size = 71, normalized size = 1.11

$$\frac{6\,(dx+c)a+3i\,a\log\left(\tan\left(dx+c\right)^{2}+1\right)-6i\,a\log\left(\tan\left(dx+c\right)\right)+\frac{6\,a\tan\left(dx+c\right)^{2}-3i\,a\tan\left(dx+c\right)-2\,a}{\tan\left(dx+c\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(6*(d*x + c)*a + 3*I*a*log(tan(d*x + c)^2 + 1) - 6*I*a*log(tan(d*x + c)
) + (6*a*tan(d*x + c)^2 - 3*I*a*tan(d*x + c) - 2*a)/tan(d*x + c)^3)/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(56) = 112.
time = 0.55, size = 125, normalized size = 1.95

$$\frac{18i\,ae^{(4i\,dx+4i\,c)}-18i\,ae^{(2i\,dx+2i\,c)}-3\left(i\,ae^{(6i\,dx+6i\,c)}-3i\,ae^{(4i\,dx+4i\,c)}+3i\,ae^{(2i\,dx+2i\,c)}-i\,a\right)\log\left(e^{(2i\,dx+2i\,c)}-1\right)+8i\,a}{3\left(de^{(6i\,dx+6i\,c)}-3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(18*I*a*e^(4*I*d*x + 4*I*c) - 18*I*a*e^(2*I*d*x + 2*I*c) - 3*(I*a*e^(6*
I*d*x + 6*I*c) - 3*I*a*e^(4*I*d*x + 4*I*c) + 3*I*a*e^(2*I*d*x + 2*I*c) - I*
a)*log(e^(2*I*d*x + 2*I*c) - 1) + 8*I*a)/(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*
I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(54) = 108.
time = 0.19, size = 128, normalized size = 2.00

$$-\frac{ia\log\left(e^{2idx}-e^{-2ic}\right)}{d}+\frac{18iae^{4ic}e^{4idx}-18iae^{2ic}e^{2idx}+8ia}{3de^{6ic}e^{6idx}-9de^{4ic}e^{4idx}+9de^{2ic}e^{2idx}-3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I*a*log(exp(2*I*d*x) - exp(-2*I*c))/d + (18*I*a*exp(4*I*c)*exp(4*I*d*x) -
18*I*a*exp(2*I*c)*exp(2*I*d*x) + 8*I*a)/(3*d*exp(6*I*c)*exp(6*I*d*x) - 9*d*
exp(4*I*c)*exp(4*I*d*x) + 9*d*exp(2*I*c)*exp(2*I*d*x) - 3*d)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(56) = 112.
time = 0.61, size = 128, normalized size = 2.00

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-3i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+48i\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)-24i\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{-44i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+3i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*d*x + 1/2*c)^3 - 3*I*a*tan(1/2*d*x + 1/2*c)^2 + 48*I*a*log(
tan(1/2*d*x + 1/2*c) + I) - 24*I*a*log(tan(1/2*d*x + 1/2*c)) - 15*a*tan(1/2
*d*x + 1/2*c) - (-44*I*a*tan(1/2*d*x + 1/2*c)^3 - 15*a*tan(1/2*d*x + 1/2*c)
^2 + 3*I*a*tan(1/2*d*x + 1/2*c) + a)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B]
time = 3.87, size = 57, normalized size = 0.89

$$\frac{2\,a\,\mathrm{atan}(2\tan(c+d\,x)+1\mathrm{i})}{d}-\frac{-a\tan(c+d\,x)^2+\frac{1\mathrm{i}\,a\tan(c+d\,x)}{2}+\frac{a}{3}}{d\tan\left(c+d\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + a*tan(c + d*x)*1i),x)
```

```
[Out] (2*a*atan(2*tan(c + d*x) + 1i))/d - (a/3 + (a*tan(c + d*x)*1i)/2 - a*tan(c
+ d*x)^2)/(d*tan(c + d*x)^3)
```

3.11 $\int \cot^5(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=83

$$iax+\frac{ia\cot(c+dx)}{d}+\frac{a\cot^2(c+dx)}{2d}-\frac{ia\cot^3(c+dx)}{3d}-\frac{a\cot^4(c+dx)}{4d}+\frac{a\log(\sin(c+dx))}{d}$$

```
[Out] I*a*x+I*a*cot(d*x+c)/d+1/2*a*cot(d*x+c)^2/d-1/3*I*a*cot(d*x+c)^3/d-1/4*a*co
t(d*x+c)^4/d+a*ln(sin(d*x+c))/d
```

Rubi [A]
time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3610, 3612, 3556}

$$-\frac{a\cot^4(c+dx)}{4d}-\frac{ia\cot^3(c+dx)}{3d}+\frac{a\cot^2(c+dx)}{2d}+\frac{ia\cot(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}+iax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] I*a*x + (I*a*Cot[c + d*x])/d + (a*Cot[c + d*x]^2)/(2*d) - ((I/3)*a*Cot[c +
d*x]^3)/d - (a*Cot[c + d*x]^4)/(4*d) + (a*Log[Sin[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{a\cot^4(c+dx)}{4d} + \int \cot^4(c+dx)(ia-a\tan(c+dx))\,dx$$
$$= -\frac{ia\cot^3(c+dx)}{3d} - \frac{a\cot^4(c+dx)}{4d} + \int \cot^3(c+dx)(-a-ia\tan(c$$
$$= \frac{a\cot^2(c+dx)}{2d} - \frac{ia\cot^3(c+dx)}{3d} - \frac{a\cot^4(c+dx)}{4d} + \int \cot^2(c+dx$$
$$= \frac{ia\cot(c+dx)}{d} + \frac{a\cot^2(c+dx)}{2d} - \frac{ia\cot^3(c+dx)}{3d} - \frac{a\cot^4(c+dx)}{4d}$$
$$= iax + \frac{ia\cot(c+dx)}{d} + \frac{a\cot^2(c+dx)}{2d} - \frac{ia\cot^3(c+dx)}{3d} - \frac{a\cot^4(c}{4c}$$
$$= iax + \frac{ia\cot(c+dx)}{d} + \frac{a\cot^2(c+dx)}{2d} - \frac{ia\cot^3(c+dx)}{3d} - \frac{a\cot^4(c}{4c}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.39, size = 84, normalized size = 1.01

$$-\frac{ia\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(c+dx)\right)}{3d} + \frac{a(2\cot^2(c+dx)-\cot^4(c+dx)+4\log(\cos(c+dx))+4\log(\tan(c+dx)))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-1/3*I)*a*Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2
])/d + (a*(2*Cot[c + d*x]^2 - Cot[c + d*x]^4 + 4*Log[Cos[c + d*x]] + 4*Log[
Tan[c + d*x]]))/(4*d)
```

Maple [A]
time = 0.20, size = 61, normalized size = 0.73

method	result	si
derivativedivides	$\frac{ia\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+a\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)}{d}$	6
default	$\frac{ia\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+a\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)}{d}$	6
risch	$-\frac{2iac}{d}-\frac{4a\left(6\,e^{6i(dx+c)}-9\,e^{4i(dx+c)}+8\,e^{2i(dx+c)}-2\right)}{3d\left(e^{2i(dx+c)}-1\right)^4}+\frac{a\ln\left(e^{2i(dx+c)}-1\right)}{d}$	8
norman	$\frac{\frac{ia\left(\tan^3(dx+c)\right)}{d}+iax\left(\tan^4(dx+c)\right)-\frac{a}{4d}+\frac{a\left(\tan^2(dx+c)\right)}{2d}-\frac{ia\tan(dx+c)}{3d}}{\tan(dx+c)^4}+\frac{a\ln(\tan(dx+c))}{d}-\frac{a\ln\left(1+\tan^2(dx+c)\right)}{2d}$	1(

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(I*a*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+a*(-1/4*cot(d*x+c)^4+1/2*cot(
d*x+c)^2+ln(sin(d*x+c))))
```

Maxima [A]
time = 0.51, size = 83, normalized size = 1.00

$$-\frac{-12i\,(dx+c)a+6\,a\log\left(\tan\left(dx+c\right)^{2}+1\right)-12\,a\log\left(\tan\left(dx+c\right)\right)-\frac{12i\,a\tan(dx+c)^{3}+6\,a\tan(dx+c)^{2}-4i\,a\tan(dx+c)-3\,a}{\tan(dx+c)^{4}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(-12*I*(d*x + c)*a + 6*a*log(tan(d*x + c)^2 + 1) - 12*a*log(tan(d*x +
 c)) - (12*I*a*tan(d*x + c)^3 + 6*a*tan(d*x + c)^2 - 4*I*a*tan(d*x + c) - 3
*a)/tan(d*x + c)^4)/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(71) = 142$.
time = 0.55, size = 156, normalized size = 1.88

$$-\frac{24\,ae^{(6i\,dx+6i\,c)}-36\,ae^{(4i\,dx+4i\,c)}+32\,ae^{(2i\,dx+2i\,c)}-3\left(ae^{(8i\,dx+8i\,c)}-4\,ae^{(6i\,dx+6i\,c)}+6\,ae^{(4i\,dx+4i\,c)}-4\,ae^{(2i\,dx+2i\,c)}+a\right)\log\left(e^{(2i\,dx+2i\,c)}-1\right)-8\,a}{3\left(de^{(8i\,dx+8i\,c)}-4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}-4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(24*a*e^(6*I*d*x + 6*I*c) - 36*a*e^(4*I*d*x + 4*I*c) + 32*a*e^(2*I*d*x
 + 2*I*c) - 3*(a*e^(8*I*d*x + 8*I*c) - 4*a*e^(6*I*d*x + 6*I*c) + 6*a*e^(4*I
*d*x + 4*I*c) - 4*a*e^(2*I*d*x + 2*I*c) + a)*log(e^(2*I*d*x + 2*I*c) - 1) -
 8*a)/(d*e^(8*I*d*x + 8*I*c) - 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4
*I*c) - 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(70) = 140$.
time = 0.27, size = 158, normalized size = 1.90

$$\frac{a\log\left(e^{2idx}-e^{-2ic}\right)}{d}+\frac{-24ae^{6ic}e^{6idx}+36ae^{4ic}e^{4idx}-32ae^{2ic}e^{2idx}+8a}{3de^{8ic}e^{8idx}-12de^{6ic}e^{6idx}+18de^{4ic}e^{4idx}-12de^{2ic}e^{2idx}+3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+I*a*tan(d*x+c)),x)
```

```
[Out] a*log(exp(2*I*d*x) - exp(-2*I*c))/d + (-24*a*exp(6*I*c)*exp(6*I*d*x) + 36*a
*exp(4*I*c)*exp(4*I*d*x) - 32*a*exp(2*I*c)*exp(2*I*d*x) + 8*a)/(3*d*exp(8*I
*c)*exp(8*I*d*x) - 12*d*exp(6*I*c)*exp(6*I*d*x) + 18*d*exp(4*I*c)*exp(4*I*d
*x) - 12*d*exp(2*I*c)*exp(2*I*d*x) + 3*d)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(71) = 142$.
time = 0.68, size = 158, normalized size = 1.90

$$-\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-8i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-36\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+384\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)-192\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+120i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{400\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-120i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-36\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+8i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/192*(3*a*tan(1/2*d*x + 1/2*c)^4 - 8*I*a*tan(1/2*d*x + 1/2*c)^3 - 36*a*ta
n(1/2*d*x + 1/2*c)^2 + 384*a*log(tan(1/2*d*x + 1/2*c) + I) - 192*a*log(tan(
1/2*d*x + 1/2*c)) + 120*I*a*tan(1/2*d*x + 1/2*c) + (400*a*tan(1/2*d*x + 1/2
*c)^4 - 120*I*a*tan(1/2*d*x + 1/2*c)^3 - 36*a*tan(1/2*d*x + 1/2*c)^2 + 8*I*
a*tan(1/2*d*x + 1/2*c) + 3*a)/tan(1/2*d*x + 1/2*c)^4)/d
```

Mupad [B]
time = 3.98, size = 70, normalized size = 0.84

$$\frac{a\,\mathrm{atan}(2\tan(c+d\,x)+1\mathrm{i})\;2\mathrm{i}}{d}-\frac{-1\mathrm{i}\,a\tan(c+d\,x)^3-\frac{a\tan(c+d\,x)^2}{2}+\frac{1\mathrm{i}\,a\tan(c+d\,x)}{3}+\frac{a}{4}}{d\tan\left(c+d\,x\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(a + a*tan(c + d*x)*1i),x)
```

```
[Out] (a*atan(2*tan(c + d*x) + 1i)*2i)/d - (a/4 + (a*tan(c + d*x)*1i)/3 - (a*tan(
c + d*x)^2)/2 - a*tan(c + d*x)^3*1i)/(d*tan(c + d*x)^4)
```

3.12 $\int \cot^6(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=100

$$-ax-\frac{a\cot(c+dx)}{d}+\frac{ia\cot^2(c+dx)}{2d}+\frac{a\cot^3(c+dx)}{3d}-\frac{ia\cot^4(c+dx)}{4d}-\frac{a\cot^5(c+dx)}{5d}+\frac{ia\log(\sin(c+dx))}{d}$$

```
[Out] -a*x-a*cot(d*x+c)/d+1/2*I*a*cot(d*x+c)^2/d+1/3*a*cot(d*x+c)^3/d-1/4*I*a*cot
(d*x+c)^4/d-1/5*a*cot(d*x+c)^5/d+I*a*ln(sin(d*x+c))/d
```

Rubi [A]
time = 0.10, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3610, 3612, 3556}

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{ia\cot^4(c+dx)}{4d}+\frac{a\cot^3(c+dx)}{3d}+\frac{ia\cot^2(c+dx)}{2d}-\frac{a\cot(c+dx)}{d}+\frac{ia\log(\sin(c+dx))}{d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] -(a*x) - (a*Cot[c + d*x])/d + ((I/2)*a*Cot[c + d*x]^2)/d + (a*Cot[c + d*x]^
3)/(3*d) - ((I/4)*a*Cot[c + d*x]^4)/d - (a*Cot[c + d*x]^5)/(5*d) + (I*a*Log
[Sin[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \cot^6(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{a\cot^5(c+dx)}{5d} + \int \cot^5(c+dx)(ia-a\tan(c+dx))\,dx$$
$$= -\frac{ia\cot^4(c+dx)}{4d} - \frac{a\cot^5(c+dx)}{5d} + \int \cot^4(c+dx)(-a-ia\tan(c$$
$$= \frac{a\cot^3(c+dx)}{3d} - \frac{ia\cot^4(c+dx)}{4d} - \frac{a\cot^5(c+dx)}{5d} + \int \cot^3(c+dx$$
$$= \frac{ia\cot^2(c+dx)}{2d} + \frac{a\cot^3(c+dx)}{3d} - \frac{ia\cot^4(c+dx)}{4d} - \frac{a\cot^5(c+dx}{5d}$$
$$= -\frac{a\cot(c+dx)}{d} + \frac{ia\cot^2(c+dx)}{2d} + \frac{a\cot^3(c+dx)}{3d} - \frac{ia\cot^4(c+d}{4d}$$
$$= -ax - \frac{a\cot(c+dx)}{d} + \frac{ia\cot^2(c+dx)}{2d} + \frac{a\cot^3(c+dx)}{3d} - \frac{ia\cot^4(}{4}$$
$$= -ax - \frac{a\cot(c+dx)}{d} + \frac{ia\cot^2(c+dx)}{2d} + \frac{a\cot^3(c+dx)}{3d} - \frac{ia\cot^4(}{4}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.38, size = 84, normalized size = 0.84

$$-\frac{a\cot^5(c+dx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\tan^2(c+dx)\right)}{5d} + \frac{ia(2\cot^2(c+dx)-\cot^4(c+dx)+4\log(\cos(c+dx))+4\log(\tan(c+dx)))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] -1/5*(a*Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x]^2])/d
 + ((I/4)*a*(2*Cot[c + d*x]^2 - Cot[c + d*x]^4 + 4*Log[Cos[c + d*x]] + 4*Lo
g[Tan[c + d*x]]))/d
```

Maple [A]
time = 0.19, size = 76, normalized size = 0.76

method	result
derivativedivides	$\frac{ia\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+a\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-dx-c\right)}{d}$
default	$\frac{ia\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+a\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-dx-c\right)}{d}$
risch	$\frac{2ac}{d}-\frac{2ia\left(75\,e^{8i(dx+c)}-150\,e^{6i(dx+c)}+200\,e^{4i(dx+c)}-100\,e^{2i(dx+c)}+23\right)}{15d\left(e^{2i(dx+c)}-1\right)^5}+\frac{ia\ln\left(e^{2i(dx+c)}-1\right)}{d}$

norman	$\frac{-\frac{a}{5d}-ax\left(\tan^5(dx+c)\right)+\frac{a\left(\tan^2(dx+c)\right)}{3d}-\frac{a\left(\tan^4(dx+c)\right)}{d}-\frac{ia\tan(dx+c)}{4d}+\frac{ia\left(\tan^3(dx+c)\right)}{2d}}{\tan(dx+c)^5}+\frac{ia\ln(\tan(dx+c))}{d}-\frac{ia\ln(1$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(I*a*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+a*(-1/5*cot(d*
x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.49, size = 93, normalized size = 0.93

$$-\frac{60(dx+c)a+30i\,a\log\left(\tan(dx+c)^2+1\right)-60i\,a\log\left(\tan(dx+c)\right)+\frac{60\,a\tan(dx+c)^4-30i\,a\tan(dx+c)^3-20\,a\tan(dx+c)^2+15i\,a\tan(dx+c)+12\,a}{\tan(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(60*(d*x + c)*a + 30*I*a*log(tan(d*x + c)^2 + 1) - 60*I*a*log(tan(d*x
 + c)) + (60*a*tan(d*x + c)^4 - 30*I*a*tan(d*x + c)^3 - 20*a*tan(d*x + c)^2
 + 15*I*a*tan(d*x + c) + 12*a)/tan(d*x + c)^5)/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(86) = 172$.
time = 0.51, size = 197, normalized size = 1.97

$$\frac{-150i\,ae^{(8i\,dx+8i\,c)}+300i\,ae^{(6i\,dx+6i\,c)}-400i\,ae^{(4i\,dx+4i\,c)}+200i\,ae^{(2i\,dx+2i\,c)}-15\left(-i\,ae^{(10i\,dx+10i\,c)}+5i\,ae^{(8i\,dx+8i\,c)}-10i\,ae^{(6i\,dx+6i\,c)}+10i\,ae^{(4i\,dx+4i\,c)}-5i\,ae^{(2i\,dx+2i\,c)}+i\,a\right)\log\left(e^{(2i\,dx+2i\,c)}-1\right)-46i\,a}{15\left(de^{(10i\,dx+10i\,c)}-5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}-10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/15*(-150*I*a*e^(8*I*d*x + 8*I*c) + 300*I*a*e^(6*I*d*x + 6*I*c) - 400*I*a*
e^(4*I*d*x + 4*I*c) + 200*I*a*e^(2*I*d*x + 2*I*c) - 15*(-I*a*e^(10*I*d*x +
10*I*c) + 5*I*a*e^(8*I*d*x + 8*I*c) - 10*I*a*e^(6*I*d*x + 6*I*c) + 10*I*a*e
^(4*I*d*x + 4*I*c) - 5*I*a*e^(2*I*d*x + 2*I*c) + I*a)*log(e^(2*I*d*x + 2*I*
c) - 1) - 46*I*a)/(d*e^(10*I*d*x + 10*I*c) - 5*d*e^(8*I*d*x + 8*I*c) + 10*d
*e^(6*I*d*x + 6*I*c) - 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) -
 d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(83) = 166$.
time = 0.30, size = 206, normalized size = 2.06

$$\frac{ia\log\left(e^{2idx}-e^{-2ic}\right)}{d}+\frac{-150iae^{8ic}e^{8idx}+300iae^{6ic}e^{6idx}-400iae^{4ic}e^{4idx}+200iae^{2ic}e^{2idx}-46ia}{15de^{10ic}e^{10idx}-75de^{8ic}e^{8idx}+150de^{6ic}e^{6idx}-150de^{4ic}e^{4idx}+75de^{2ic}e^{2idx}-15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+I*a*tan(d*x+c)),x)
```

```
[Out] I*a*log(exp(2*I*d*x) - exp(-2*I*c))/d + (-150*I*a*exp(8*I*c)*exp(8*I*d*x) +
 300*I*a*exp(6*I*c)*exp(6*I*d*x) - 400*I*a*exp(4*I*c)*exp(4*I*d*x) + 200*I*
a*exp(2*I*c)*exp(2*I*d*x) - 46*I*a)/(15*d*exp(10*I*c)*exp(10*I*d*x) - 75*d*
exp(8*I*c)*exp(8*I*d*x) + 150*d*exp(6*I*c)*exp(6*I*d*x) - 150*d*exp(4*I*c)*
exp(4*I*d*x) + 75*d*exp(2*I*c)*exp(2*I*d*x) - 15*d)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(86) = 172.
time = 0.67, size = 186, normalized size = 1.86

$$\frac{6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-15i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-70\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+180i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1920i\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)+960i\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+660\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{-2192i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-660\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+180i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+70\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-15i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-6\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*(6*a*tan(1/2*d*x + 1/2*c)^5 - 15*I*a*tan(1/2*d*x + 1/2*c)^4 - 70*a*ta
n(1/2*d*x + 1/2*c)^3 + 180*I*a*tan(1/2*d*x + 1/2*c)^2 - 1920*I*a*log(tan(1/
2*d*x + 1/2*c) + I) + 960*I*a*log(tan(1/2*d*x + 1/2*c)) + 660*a*tan(1/2*d*x
 + 1/2*c) + (-2192*I*a*tan(1/2*d*x + 1/2*c)^5 - 660*a*tan(1/2*d*x + 1/2*c)^
4 + 180*I*a*tan(1/2*d*x + 1/2*c)^3 + 70*a*tan(1/2*d*x + 1/2*c)^2 - 15*I*a*t
an(1/2*d*x + 1/2*c) - 6*a)/tan(1/2*d*x + 1/2*c)^5)/d
```

Mupad [B]
time = 4.07, size = 79, normalized size = 0.79

$$-\frac{2\,a\,\mathrm{atan}\left(2\,\tan\left(c+d\,x\right)+1\mathrm{i}\right)}{d}-\frac{a\,{\tan\left(c+d\,x\right)}^4-\frac{1\mathrm{i}\,a\,{\tan\left(c+d\,x\right)}^3}{2}-\frac{a\,{\tan\left(c+d\,x\right)}^2}{3}+\frac{1\mathrm{i}\,a\,\tan\left(c+d\,x\right)}{4}+\frac{a}{5}}{d\,{\tan\left(c+d\,x\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(a + a*tan(c + d*x)*1i),x)
```

```
[Out] - (2*a*atan(2*tan(c + d*x) + 1i))/d - (a/5 + (a*tan(c + d*x)*1i)/4 - (a*tan
(c + d*x)^2)/3 - (a*tan(c + d*x)^3*1i)/2 + a*tan(c + d*x)^4)/(d*tan(c + d*x
)^5)
```

3.13 $\int \tan^4(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=112

$$2a^2x-\frac{2ia^2\log(\cos(c+dx))}{d}-\frac{2a^2\tan(c+dx)}{d}-\frac{ia^2\tan^2(c+dx)}{d}+\frac{2a^2\tan^3(c+dx)}{3d}+\frac{ia^2\tan^4(c+dx)}{2d}-\frac{a^2\tan^5(c+dx)}{5d}$$

```
[Out] 2*a^2*x-2*I*a^2*ln(cos(d*x+c))/d-2*a^2*tan(d*x+c)/d-I*a^2*tan(d*x+c)^2/d+2/
3*a^2*tan(d*x+c)^3/d+1/2*I*a^2*tan(d*x+c)^4/d-1/5*a^2*tan(d*x+c)^5/d
```

Rubi [A]
time = 0.10, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3624, 3609, 3606, 3556}

$$-\frac{a^2\tan^5(c+dx)}{5d}+\frac{ia^2\tan^4(c+dx)}{2d}+\frac{2a^2\tan^3(c+dx)}{3d}-\frac{ia^2\tan^2(c+dx)}{d}-\frac{2a^2\tan(c+dx)}{d}-\frac{2ia^2\log(\cos(c+dx))}{d}+2a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] 2*a^2*x - ((2*I)*a^2*Log[Cos[c + d*x]])/d - (2*a^2*Tan[c + d*x])/d - (I*a^2
*Tan[c + d*x]^2)/d + (2*a^2*Tan[c + d*x]^3)/(3*d) + ((I/2)*a^2*Tan[c + d*x]
^4)/d - (a^2*Tan[c + d*x]^5)/(5*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
```

```
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \tan^4(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{a^2\tan^5(c+dx)}{5d} + \int \tan^4(c+dx)\left(2a^2+2ia^2\tan(c+dx)\right)\,dx$$
$$= \frac{ia^2\tan^4(c+dx)}{2d} - \frac{a^2\tan^5(c+dx)}{5d} + \int \tan^3(c+dx)\left(-2ia^2+2a\right.$$
$$= \frac{2a^2\tan^3(c+dx)}{3d} + \frac{ia^2\tan^4(c+dx)}{2d} - \frac{a^2\tan^5(c+dx)}{5d} + \int \tan^2($$
$$= -\frac{ia^2\tan^2(c+dx)}{d} + \frac{2a^2\tan^3(c+dx)}{3d} + \frac{ia^2\tan^4(c+dx)}{2d} - \frac{a^2\tan}{}$$
$$= 2a^2x - \frac{2a^2\tan(c+dx)}{d} - \frac{ia^2\tan^2(c+dx)}{d} + \frac{2a^2\tan^3(c+dx)}{3d} +$$
$$= 2a^2x - \frac{2ia^2\log(\cos(c+dx))}{d} - \frac{2a^2\tan(c+dx)}{d} - \frac{ia^2\tan^2(c+dx}{d}$$

Mathematica [A]
time = 0.33, size = 108, normalized size = 0.96

$$\frac{2a^2\text{ArcTan}(\tan(c+dx))}{d} - \frac{2a^2\tan(c+dx)}{d} + \frac{2a^2\tan^3(c+dx)}{3d} - \frac{a^2\tan^5(c+dx)}{5d} - \frac{ia^2(4\log(\cos(c+dx))+2\tan^2(c+dx)-\tan^4(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (2*a^2*ArcTan[Tan[c + d*x]])/d - (2*a^2*Tan[c + d*x])/d + (2*a^2*Tan[c + d*
x]^3)/(3*d) - (a^2*Tan[c + d*x]^5)/(5*d) - ((I/2)*a^2*(4*Log[Cos[c + d*x]]
+ 2*Tan[c + d*x]^2 - Tan[c + d*x]^4))/d
```

Maple [A]
time = 0.05, size = 82, normalized size = 0.73

method	result
derivativedivides	$\frac{a^2\left(-2\tan(dx+c)-\frac{\left(\tan^5(dx+c)\right)}{5}+\frac{i\left(\tan^4(dx+c)\right)}{2}+\frac{2\left(\tan^3(dx+c)\right)}{3}-i\left(\tan^2(dx+c)\right)+i\ln\left(1+\tan^2(dx+c)\right)+2\arctan(\tan(d\right.}{d}$
default	$\frac{a^2\left(-2\tan(dx+c)-\frac{\left(\tan^5(dx+c)\right)}{5}+\frac{i\left(\tan^4(dx+c)\right)}{2}+\frac{2\left(\tan^3(dx+c)\right)}{3}-i\left(\tan^2(dx+c)\right)+i\ln\left(1+\tan^2(dx+c)\right)+2\arctan(\tan(d\right.}{d}$

risch	$-\frac{4a^2c}{d}-\frac{2ia^2\left(135\,\mathrm{e}^{8i(dx+c)}+300\,\mathrm{e}^{6i(dx+c)}+370\,\mathrm{e}^{4i(dx+c)}+200\,\mathrm{e}^{2i(dx+c)}+43\right)}{15d\left(\mathrm{e}^{2i(dx+c)}+1\right)^5}-\frac{2ia^2\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{d}$
norman	$2a^2x-\frac{2a^2\tan(dx+c)}{d}+\frac{2a^2\left(\tan^3(dx+c)\right)}{3d}-\frac{a^2\left(\tan^5(dx+c)\right)}{5d}-\frac{ia^2\left(\tan^2(dx+c)\right)}{d}+\frac{ia^2\left(\tan^4(dx+c)\right)}{2d}+\frac{ia}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a^2*(-2*tan(d*x+c)-1/5*tan(d*x+c)^5+1/2*I*tan(d*x+c)^4+2/3*tan(d*x+c)^3
-I*tan(d*x+c)^2+I*ln(1+tan(d*x+c)^2)+2*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.50, size = 95, normalized size = 0.85

$$-\frac{6\,a^2\tan\left(dx+c\right)^5-15i\,a^2\tan\left(dx+c\right)^4-20\,a^2\tan\left(dx+c\right)^3+30i\,a^2\tan\left(dx+c\right)^2-60\,(dx+c)a^2-30i\,a^2\log\left(\tan\left(dx+c\right)^2+1\right)+60\,a^2\tan\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/30*(6*a^2*tan(d*x + c)^5 - 15*I*a^2*tan(d*x + c)^4 - 20*a^2*tan(d*x + c)
^3 + 30*I*a^2*tan(d*x + c)^2 - 60*(d*x + c)*a^2 - 30*I*a^2*log(tan(d*x + c)
^2 + 1) + 60*a^2*tan(d*x + c))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(100) = 200.
time = 0.43, size = 217, normalized size = 1.94

$$-\frac{2\left(135i\,a^2e^{(8i\,dx+8i\,c)}+300i\,a^2e^{(6i\,dx+6i\,c)}+370i\,a^2e^{(4i\,dx+4i\,c)}+200i\,a^2e^{(2i\,dx+2i\,c)}+43i\,a^2+15\left(i\,a^2e^{(10i\,dx+10i\,c)}+5i\,a^2e^{(8i\,dx+8i\,c)}+10i\,a^2e^{(6i\,dx+6i\,c)}+10i\,a^2e^{(4i\,dx+4i\,c)}+5i\,a^2e^{(2i\,dx+2i\,c)}+i\,a^2\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)\right)}{15\left(de^{(10i\,dx+10i\,c)}+5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}+10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/15*(135*I*a^2*e^(8*I*d*x + 8*I*c) + 300*I*a^2*e^(6*I*d*x + 6*I*c) + 370*
I*a^2*e^(4*I*d*x + 4*I*c) + 200*I*a^2*e^(2*I*d*x + 2*I*c) + 43*I*a^2 + 15*(
I*a^2*e^(10*I*d*x + 10*I*c) + 5*I*a^2*e^(8*I*d*x + 8*I*c) + 10*I*a^2*e^(6*I
*d*x + 6*I*c) + 10*I*a^2*e^(4*I*d*x + 4*I*c) + 5*I*a^2*e^(2*I*d*x + 2*I*c)
+ I*a^2)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*
I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*
e^(2*I*d*x + 2*I*c) + d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(100) = 200.
time = 0.32, size = 219, normalized size = 1.96

$$-\frac{2ia^2\log\left(e^{2idx}+e^{-2ic}\right)}{d}+\frac{-270ia^2e^{8ic}e^{8idx}-600ia^2e^{6ic}e^{6idx}-740ia^2e^{4ic}e^{4idx}-400ia^2e^{2ic}e^{2idx}-86ia^2}{15de^{10ic}e^{10idx}+75de^{8ic}e^{8idx}+150de^{6ic}e^{6idx}+150de^{4ic}e^{4idx}+75de^{2ic}e^{2idx}+15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -2*I*a**2*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-270*I*a**2*exp(8*I*c)*exp(8
*I*d*x) - 600*I*a**2*exp(6*I*c)*exp(6*I*d*x) - 740*I*a**2*exp(4*I*c)*exp(4*
I*d*x) - 400*I*a**2*exp(2*I*c)*exp(2*I*d*x) - 86*I*a**2)/(15*d*exp(10*I*c)*
exp(10*I*d*x) + 75*d*exp(8*I*c)*exp(8*I*d*x) + 150*d*exp(6*I*c)*exp(6*I*d*x
) + 150*d*exp(4*I*c)*exp(4*I*d*x) + 75*d*exp(2*I*c)*exp(2*I*d*x) + 15*d)
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(100) = 200$.
time = 1.14, size = 274, normalized size = 2.45

$$-\frac{2\left(15i\,a^2e^{(10i\,dx+10i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+75i\,a^2e^{(8i\,dx+8i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+150i\,a^2e^{(6i\,dx+6i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+150i\,a^2e^{(4i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+75i\,a^2e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+135i\,a^2e^{(8i\,dx+8i\,c)}+300i\,a^2e^{(6i\,dx+6i\,c)}+370i\,a^2e^{(4i\,dx+4i\,c)}+200i\,a^2e^{(2i\,dx+2i\,c)}+15i\,a^2\log\left(e^{(2i\,dx+2i\,c)}+1\right)+43i\,a^2\right)}{15\left(de^{(10i\,dx+10i\,c)}+5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}+10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2/15*(15*I*a^2*e^(10*I*d*x + 10*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 75*I*a
^2*e^(8*I*d*x + 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 150*I*a^2*e^(6*I*d*x
+ 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 150*I*a^2*e^(4*I*d*x + 4*I*c)*log(e
^(2*I*d*x + 2*I*c) + 1) + 75*I*a^2*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I
*c) + 1) + 135*I*a^2*e^(8*I*d*x + 8*I*c) + 300*I*a^2*e^(6*I*d*x + 6*I*c) +
370*I*a^2*e^(4*I*d*x + 4*I*c) + 200*I*a^2*e^(2*I*d*x + 2*I*c) + 15*I*a^2*lo
g(e^(2*I*d*x + 2*I*c) + 1) + 43*I*a^2)/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*
I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*
e^(2*I*d*x + 2*I*c) + d)
```

Mupad [**B**]
time = 3.70, size = 86, normalized size = 0.77

$$\frac{\frac{2\,a^2\,\tan(c+d\,x)^3}{3}-2\,a^2\,\tan(c+d\,x)-\frac{a^2\,\tan(c+d\,x)^5}{5}+a^2\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,2\mathrm{i}-a^2\,\tan(c+d\,x)^2\,1\mathrm{i}+\frac{a^2\,\tan(c+d\,x)^4\,1\mathrm{i}}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] (a^2*log(tan(c + d*x) + 1i)*2i - 2*a^2*tan(c + d*x) - a^2*tan(c + d*x)^2*1i
 + (2*a^2*tan(c + d*x)^3)/3 + (a^2*tan(c + d*x)^4*1i)/2 - (a^2*tan(c + d*x)
^5)/5)/d
```

3.14 $\int \tan^3(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=93

$$2ia^2x+\frac{2a^2\log(\cos(c+dx))}{d}-\frac{2ia^2\tan(c+dx)}{d}+\frac{a^2\tan^2(c+dx)}{d}+\frac{2ia^2\tan^3(c+dx)}{3d}-\frac{a^2\tan^4(c+dx)}{4d}$$

```
[Out] 2*I*a^2*x+2*a^2*ln(cos(d*x+c))/d-2*I*a^2*tan(d*x+c)/d+a^2*tan(d*x+c)^2/d+2/
3*I*a^2*tan(d*x+c)^3/d-1/4*a^2*tan(d*x+c)^4/d
```

Rubi [A]
time = 0.08, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3624, 3609, 3606, 3556}

$$-\frac{a^2\tan^4(c+dx)}{4d}+\frac{2ia^2\tan^3(c+dx)}{3d}+\frac{a^2\tan^2(c+dx)}{d}-\frac{2ia^2\tan(c+dx)}{d}+\frac{2a^2\log(\cos(c+dx))}{d}+2ia^2x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (2*I)*a^2*x + (2*a^2*Log[Cos[c + d*x]])/d - ((2*I)*a^2*Tan[c + d*x])/d + (a
^2*Tan[c + d*x]^2)/d + (((2*I)/3)*a^2*Tan[c + d*x]^3)/d - (a^2*Tan[c + d*x]
^4)/(4*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \tan^3(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{a^2\tan^4(c+dx)}{4d}+\int \tan^3(c+dx)\left(2a^2+2ia^2\tan(c+dx)\right)\,dx$$
$$= \frac{2ia^2\tan^3(c+dx)}{3d}-\frac{a^2\tan^4(c+dx)}{4d}+\int \tan^2(c+dx)\left(-2ia^2+2\right.$$
$$= \frac{a^2\tan^2(c+dx)}{d}+\frac{2ia^2\tan^3(c+dx)}{3d}-\frac{a^2\tan^4(c+dx)}{4d}+\int \tan(c$$
$$= 2ia^2x-\frac{2ia^2\tan(c+dx)}{d}+\frac{a^2\tan^2(c+dx)}{d}+\frac{2ia^2\tan^3(c+dx)}{3d}-$$
$$= 2ia^2x+\frac{2a^2\log(\cos(c+dx))}{d}-\frac{2ia^2\tan(c+dx)}{d}+\frac{a^2\tan^2(c+dx}{d}$$

Mathematica [A]

time = 0.22, size = 73, normalized size = 0.78

$$\frac{a^2(24i\text{ArcTan}(\tan(c+dx))+24\log(\cos(c+dx))-24i\tan(c+dx)+12\tan^2(c+dx)+8i\tan^3(c+dx)-3\tan^4(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3*(a + I*a*Tan[c + d*x])^2,x]
[Out] (a^2*((24*I)*ArcTan[Tan[c + d*x]] + 24*Log[Cos[c + d*x]] - (24*I)*Tan[c + d
*x] + 12*Tan[c + d*x]^2 + (8*I)*Tan[c + d*x]^3 - 3*Tan[c + d*x]^4))/(12*d)
```

Maple [A]

time = 0.04, size = 70, normalized size = 0.75

method	result
derivativedivides	$\frac{a^2\left(-2i\tan(dx+c)-\frac{\left(\tan^4(dx+c)\right)}{4}+\frac{2i\left(\tan^3(dx+c)\right)}{3}+\tan^2(dx+c)-\ln\left(1+\tan^2(dx+c)\right)+2i\arctan(\tan(dx+c))\right)}{d}$
default	$\frac{a^2\left(-2i\tan(dx+c)-\frac{\left(\tan^4(dx+c)\right)}{4}+\frac{2i\left(\tan^3(dx+c)\right)}{3}+\tan^2(dx+c)-\ln\left(1+\tan^2(dx+c)\right)+2i\arctan(\tan(dx+c))\right)}{d}$
risch	$-\frac{4ia^2c}{d}+\frac{2a^2\left(21\,e^{6i(dx+c)}+36\,e^{4i(dx+c)}+29\,e^{2i(dx+c)}+8\right)}{3d\left(e^{2i(dx+c)}+1\right)^4}+\frac{2a^2\ln\left(e^{2i(dx+c)}+1\right)}{d}$

norman	$\frac{a^2(\tan^2(dx+c))}{d} - \frac{a^2(\tan^4(dx+c))}{4d} + 2ia^2x - \frac{2ia^2\tan(dx+c)}{d} + \frac{2ia^2(\tan^3(dx+c))}{3d} - \frac{a^2\ln(1+\tan^2(dx+c))}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a^2*(-2*I*tan(d*x+c)-1/4*tan(d*x+c)^4+2/3*I*tan(d*x+c)^3+tan(d*x+c)^2-l
n(1+tan(d*x+c)^2)+2*I*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.50, size = 82, normalized size = 0.88

$$-\frac{3\,a^2\tan(dx+c)^4 - 8i\,a^2\tan(dx+c)^3 - 12\,a^2\tan(dx+c)^2 - 24i\,(dx+c)a^2 + 12\,a^2\log\left(\tan(dx+c)^2+1\right) + 24i\,a^2\tan(dx+c)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/12*(3*a^2*tan(d*x + c)^4 - 8*I*a^2*tan(d*x + c)^3 - 12*a^2*tan(d*x + c)^
2 - 24*I*(d*x + c)*a^2 + 12*a^2*log(tan(d*x + c)^2 + 1) + 24*I*a^2*tan(d*x
+ c))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(83) = 166.
time = 0.42, size = 174, normalized size = 1.87

$$\frac{2\left(21\,a^2e^{(6i\,dx+6i\,c)} + 36\,a^2e^{(4i\,dx+4i\,c)} + 29\,a^2e^{(2i\,dx+2i\,c)} + 8\,a^2 + 3\left(a^2e^{(8i\,dx+8i\,c)} + 4\,a^2e^{(6i\,dx+6i\,c)} + 6\,a^2e^{(4i\,dx+4i\,c)} + 4\,a^2e^{(2i\,dx+2i\,c)} + a^2\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)\right)}{3\left(de^{(8i\,dx+8i\,c)} + 4\,de^{(6i\,dx+6i\,c)} + 6\,de^{(4i\,dx+4i\,c)} + 4\,de^{(2i\,dx+2i\,c)} + d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 2/3*(21*a^2*e^(6*I*d*x + 6*I*c) + 36*a^2*e^(4*I*d*x + 4*I*c) + 29*a^2*e^(2*
I*d*x + 2*I*c) + 8*a^2 + 3*(a^2*e^(8*I*d*x + 8*I*c) + 4*a^2*e^(6*I*d*x + 6*
I*c) + 6*a^2*e^(4*I*d*x + 4*I*c) + 4*a^2*e^(2*I*d*x + 2*I*c) + a^2)*log(e^(
2*I*d*x + 2*I*c) + 1))/(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6
*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A]
time = 0.40, size = 168, normalized size = 1.81

$$\frac{2a^2\log\left(e^{2idx}+e^{-2ic}\right)}{d} + \frac{42a^2e^{6ic}e^{6idx} + 72a^2e^{4ic}e^{4idx} + 58a^2e^{2ic}e^{2idx} + 16a^2}{3de^{8ic}e^{8idx} + 12de^{6ic}e^{6idx} + 18de^{4ic}e^{4idx} + 12de^{2ic}e^{2idx} + 3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] 2*a**2*log(exp(2*I*d*x) + exp(-2*I*c))/d + (42*a**2*exp(6*I*c)*exp(6*I*d*x)
 + 72*a**2*exp(4*I*c)*exp(4*I*d*x) + 58*a**2*exp(2*I*c)*exp(2*I*d*x) + 16*a
**2)/(3*d*exp(8*I*c)*exp(8*I*d*x) + 12*d*exp(6*I*c)*exp(6*I*d*x) + 18*d*exp
(4*I*c)*exp(4*I*d*x) + 12*d*exp(2*I*c)*exp(2*I*d*x) + 3*d)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(83) = 166$.
time = 0.89, size = 222, normalized size = 2.39

$$\frac{2\left(3a^2e^{(8i\,dx+8i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+12\,a^2e^{(6i\,dx+6i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+18\,a^2e^{(4i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+12\,a^2e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+21\,a^2e^{(6i\,dx+6i\,c)}+36\,a^2e^{(4i\,dx+4i\,c)}+29\,a^2e^{(2i\,dx+2i\,c)}+3\,a^2\log\left(e^{(2i\,dx+2i\,c)}+1\right)+8\,a^2\right)}{3\left(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2/3*(3*a^2*e^(8*I*d*x + 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 12*a^2*e^(6*I
*d*x + 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 18*a^2*e^(4*I*d*x + 4*I*c)*log
(e^(2*I*d*x + 2*I*c) + 1) + 12*a^2*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I
*c) + 1) + 21*a^2*e^(6*I*d*x + 6*I*c) + 36*a^2*e^(4*I*d*x + 4*I*c) + 29*a^2
*e^(2*I*d*x + 2*I*c) + 3*a^2*log(e^(2*I*d*x + 2*I*c) + 1) + 8*a^2)/(d*e^(8*
I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^
(2*I*d*x + 2*I*c) + d)
```

Mupad [B]
time = 3.80, size = 73, normalized size = 0.78

$$-\frac{2\,a^2\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)-a^2\,\tan(c+d\,x)^2+\frac{a^2\,\tan(c+d\,x)^4}{4}+a^2\,\tan(c+d\,x)\,2\mathrm{i}-\frac{a^2\,\tan(c+d\,x)^3\,2\mathrm{i}}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] -(2*a^2*log(tan(c + d*x) + 1i) + a^2*tan(c + d*x)*2i - a^2*tan(c + d*x)^2 -
 (a^2*tan(c + d*x)^3*2i)/3 + (a^2*tan(c + d*x)^4)/4)/d
```

3.15 $\int \tan^2(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=64

$$-2a^2x+\frac{2ia^2\log(\cos(c+dx))}{d}+\frac{a^2\tan(c+dx)}{d}-\frac{i(a+ia\tan(c+dx))^3}{3ad}$$

```
[Out] -2*a^2*x+2*I*a^2*ln(cos(d*x+c))/d+a^2*tan(d*x+c)/d-1/3*I*(a+I*a*tan(d*x+c))
^3/a/d
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3624, 3558, 3556}

$$\frac{a^2\tan(c+dx)}{d}+\frac{2ia^2\log(\cos(c+dx))}{d}-2a^2x-\frac{i(a+ia\tan(c+dx))^3}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -2*a^2*x + ((2*I)*a^2*Log[Cos[c + d*x]])/d + (a^2*Tan[c + d*x])/d - ((I/3)*
(a + I*a*Tan[c + d*x])^3)/(a*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{i(a+ia\tan(c+dx))^3}{3ad} - \int (a+ia\tan(c+dx))^2\,dx$$
$$= -2a^2x + \frac{a^2\tan(c+dx)}{d} - \frac{i(a+ia\tan(c+dx))^3}{3ad} - (2ia^2)\int \tan($$
$$= -2a^2x + \frac{2ia^2\log(\cos(c+dx))}{d} + \frac{a^2\tan(c+dx)}{d} - \frac{i(a+ia\tan(c}{3ad}$$

Mathematica [A]
time = 0.26, size = 76, normalized size = 1.19

$$-\frac{2a^2\operatorname{ArcTan}(\tan(c+dx))}{d} + \frac{2a^2\tan(c+dx)}{d} - \frac{a^2\tan^3(c+dx)}{3d} + \frac{ia^2(2\log(\cos(c+dx)) + \tan^2(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (-2*a^2*ArcTan[Tan[c + d*x]])/d + (2*a^2*Tan[c + d*x])/d - (a^2*Tan[c + d*x
]^3)/(3*d) + (I*a^2*(2*Log[Cos[c + d*x]] + Tan[c + d*x]^2))/d
```

Maple [A]
time = 0.04, size = 61, normalized size = 0.95

method	result	size
derivativedivides	$\frac{a^2\left(2\tan(dx+c)-\frac{(\tan^3(dx+c))}{3}+i(\tan^2(dx+c))-i\ln(1+\tan^2(dx+c))-2\arctan(\tan(dx+c))\right)}{d}$	61
default	$\frac{a^2\left(2\tan(dx+c)-\frac{(\tan^3(dx+c))}{3}+i(\tan^2(dx+c))-i\ln(1+\tan^2(dx+c))-2\arctan(\tan(dx+c))\right)}{d}$	61
norman	$\frac{ia^2(\tan^2(dx+c))}{d} - 2a^2x + \frac{2a^2\tan(dx+c)}{d} - \frac{a^2(\tan^3(dx+c))}{3d} - \frac{ia^2\ln(1+\tan^2(dx+c))}{d}$	75
risch	$\frac{4a^2c}{d} + \frac{2ia^2(15\,e^{4i(dx+c)}+18\,e^{2i(dx+c)}+7)}{3d(e^{2i(dx+c)}+1)^3} + \frac{2ia^2\ln(e^{2i(dx+c)}+1)}{d}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a^2*(2*tan(d*x+c)-1/3*tan(d*x+c)^3+I*tan(d*x+c)^2-I*ln(1+tan(d*x+c)^2)-
2*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.50, size = 68, normalized size = 1.06

$$-\frac{a^2\tan(dx+c)^3 - 3i\,a^2\tan(dx+c)^2 + 6\,(dx+c)a^2 + 3i\,a^2\log\left(\tan(dx+c)^2+1\right) - 6\,a^2\tan(dx+c)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(a^2*tan(d*x + c)^3 - 3*I*a^2*tan(d*x + c)^2 + 6*(d*x + c)*a^2 + 3*I*a
^2*log(tan(d*x + c)^2 + 1) - 6*a^2*tan(d*x + c))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(56) = 112$.
time = 0.42, size = 137, normalized size = 2.14

$$-\frac{2\left(-15i\,a^2e^{(4i\,dx+4i\,c)}-18i\,a^2e^{(2i\,dx+2i\,c)}-7i\,a^2+3\left(-i\,a^2e^{(6i\,dx+6i\,c)}-3i\,a^2e^{(4i\,dx+4i\,c)}-3i\,a^2e^{(2i\,dx+2i\,c)}-i\,a^2\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)\right)}{3\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(-15*I*a^2*e^(4*I*d*x + 4*I*c) - 18*I*a^2*e^(2*I*d*x + 2*I*c) - 7*I*a^
2 + 3*(-I*a^2*e^(6*I*d*x + 6*I*c) - 3*I*a^2*e^(4*I*d*x + 4*I*c) - 3*I*a^2*e
^(2*I*d*x + 2*I*c) - I*a^2)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(6*I*d*x + 6
*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(54) = 108$.
time = 0.22, size = 136, normalized size = 2.12

$$\frac{2ia^2\log\left(e^{2idx}+e^{-2ic}\right)}{d}+\frac{30ia^2e^{4ic}e^{4idx}+36ia^2e^{2ic}e^{2idx}+14ia^2}{3de^{6ic}e^{6idx}+9de^{4ic}e^{4idx}+9de^{2ic}e^{2idx}+3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] 2*I*a**2*log(exp(2*I*d*x) + exp(-2*I*c))/d + (30*I*a**2*exp(4*I*c)*exp(4*I*
d*x) + 36*I*a**2*exp(2*I*c)*exp(2*I*d*x) + 14*I*a**2)/(3*d*exp(6*I*c)*exp(6
*I*d*x) + 9*d*exp(4*I*c)*exp(4*I*d*x) + 9*d*exp(2*I*c)*exp(2*I*d*x) + 3*d)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(56) = 112$.
time = 0.66, size = 170, normalized size = 2.66

$$-\frac{2\left(-3i\,a^2e^{(6i\,dx+6i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-9i\,a^2e^{(4i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-9i\,a^2e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-15i\,a^2e^{(4i\,dx+4i\,c)}-18i\,a^2e^{(2i\,dx+2i\,c)}-3i\,a^2\log\left(e^{(2i\,dx+2i\,c)}+1\right)-7i\,a^2\right)}{3\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2/3*(-3*I*a^2*e^(6*I*d*x + 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 9*I*a^2*e
^(4*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 9*I*a^2*e^(2*I*d*x + 2*I*
c)*log(e^(2*I*d*x + 2*I*c) + 1) - 15*I*a^2*e^(4*I*d*x + 4*I*c) - 18*I*a^2*e
```

```
^(2*I*d*x + 2*I*c) - 3*I*a^2*log(e^(2*I*d*x + 2*I*c) + 1) - 7*I*a^2)/(d*e^(
6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Mupad [B]
time = 3.79, size = 60, normalized size = 0.94

$$-\frac{\frac{a^2\tan(c+d\,x)^3}{3}-2\,a^2\tan(c+d\,x)+a^2\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,2\mathrm{i}-a^2\tan(c+d\,x)^2\,1\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] -(a^2*log(tan(c + d*x) + 1i)*2i - 2*a^2*tan(c + d*x) - a^2*tan(c + d*x)^2*1
i + (a^2*tan(c + d*x)^3)/3)/d
```

3.16 $\int \tan(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=62

$$-2ia^2x - \frac{2a^2\log(\cos(c+dx))}{d} + \frac{ia^2\tan(c+dx)}{d} + \frac{(a+ia\tan(c+dx))^2}{2d}$$

```
[Out] -2*I*a^2*x-2*a^2*ln(cos(d*x+c))/d+I*a^2*tan(d*x+c)/d+1/2*(a+I*a*tan(d*x+c))
^2/d
```

Rubi [A]
time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3608, 3558, 3556}

$$\frac{ia^2\tan(c+dx)}{d} - \frac{2a^2\log(\cos(c+dx))}{d} - 2ia^2x + \frac{(a+ia\tan(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (-2*I)*a^2*x - (2*a^2*Log[Cos[c + d*x]])/d + (I*a^2*Tan[c + d*x])/d + (a +
I*a*Tan[c + d*x])^2/(2*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && !LtQ[m, 0]
```

Rubi steps

$$\int \tan(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{(a+ia\tan(c+dx))^2}{2d} - i\int (a+ia\tan(c+dx))^2\,dx$$
$$= -2ia^2x + \frac{ia^2\tan(c+dx)}{d} + \frac{(a+ia\tan(c+dx))^2}{2d} + (2a^2)\int \tan($$
$$= -2ia^2x - \frac{2a^2\log(\cos(c+dx))}{d} + \frac{ia^2\tan(c+dx)}{d} + \frac{(a+ia\tan(c+}{2d}$$

Mathematica [A]
time = 0.17, size = 51, normalized size = 0.82

$$\frac{a^2(-4i\text{ArcTan}(\tan(c+dx)) - 4\log(\cos(c+dx)) + 4i\tan(c+dx) - \tan^2(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]*(a + I*a*Tan[c + d*x])^2,x]
[Out] (a^2*((-4*I)*ArcTan[Tan[c + d*x]] - 4*Log[Cos[c + d*x]] + (4*I)*Tan[c + d*x
] - Tan[c + d*x]^2))/(2*d)
```

Maple [A]
time = 0.04, size = 49, normalized size = 0.79

method	result	size
derivativedivides	$\frac{a^2\left(2i\tan(dx+c) - \frac{(\tan^2(dx+c))}{2} + \ln(1+\tan^2(dx+c)) - 2i\arctan(\tan(dx+c))\right)}{d}$	49
default	$\frac{a^2\left(2i\tan(dx+c) - \frac{(\tan^2(dx+c))}{2} + \ln(1+\tan^2(dx+c)) - 2i\arctan(\tan(dx+c))\right)}{d}$	49
norman	$-\frac{a^2(\tan^2(dx+c))}{2d} - 2ia^2x + \frac{2ia^2\tan(dx+c)}{d} + \frac{a^2\ln(1+\tan^2(dx+c))}{d}$	58
risch	$\frac{4ia^2c}{d} - \frac{2a^2(3\,e^{2i(dx+c)}+2)}{d(e^{2i(dx+c)}+1)^2} - \frac{2a^2\ln(e^{2i(dx+c)}+1)}{d}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*a^2*(2*I*tan(d*x+c)-1/2*tan(d*x+c)^2+ln(1+tan(d*x+c)^2)-2*I*arctan(tan(
d*x+c)))
```

Maxima [A]
time = 0.49, size = 55, normalized size = 0.89

$$-\frac{a^2\tan(dx+c)^2 + 4i(dx+c)a^2 - 2a^2\log\left(\tan(dx+c)^2+1\right) - 4i\,a^2\tan(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a^2*tan(d*x + c)^2 + 4*I*(d*x + c)*a^2 - 2*a^2*log(tan(d*x + c)^2 + 1
) - 4*I*a^2*tan(d*x + c))/d
```

Fricas [A]
time = 0.45, size = 93, normalized size = 1.50

$$-\frac{2\left(3\,a^2e^{(2i\,dx+2i\,c)}+2\,a^2+\left(a^2e^{(4i\,dx+4i\,c)}+2\,a^2e^{(2i\,dx+2i\,c)}+a^2\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)\right)}{de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2*(3*a^2*e^(2*I*d*x + 2*I*c) + 2*a^2 + (a^2*e^(4*I*d*x + 4*I*c) + 2*a^2*e^
(2*I*d*x + 2*I*c) + a^2)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(4*I*d*x + 4*I*
c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A]
time = 0.17, size = 88, normalized size = 1.42

$$-\frac{2a^2\log\left(e^{2idx}+e^{-2ic}\right)}{d}+\frac{-6a^2e^{2ic}e^{2idx}-4a^2}{de^{4ic}e^{4idx}+2de^{2ic}e^{2idx}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -2*a**2*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-6*a**2*exp(2*I*c)*exp(2*I*d*x
) - 4*a**2)/(d*exp(4*I*c)*exp(4*I*d*x) + 2*d*exp(2*I*c)*exp(2*I*d*x) + d)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(54) = 108$.
time = 0.61, size = 116, normalized size = 1.87

$$-\frac{2\left(a^2e^{(4i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+2\,a^2e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+3\,a^2e^{(2i\,dx+2i\,c)}+a^2\log\left(e^{(2i\,dx+2i\,c)}+1\right)+2\,a^2\right)}{de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*(a^2*e^(4*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 2*a^2*e^(2*I*d*x
 + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 3*a^2*e^(2*I*d*x + 2*I*c) + a^2*lo
g(e^(2*I*d*x + 2*I*c) + 1) + 2*a^2)/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x
 + 2*I*c) + d)
```

Mupad [B]
time = 3.74, size = 40, normalized size = 0.65

$$\frac{a^2 \left(4 \ln\left(\tan(c+d\,x)+1\mathrm{i}\right)-\tan(c+d\,x)^2+\tan(c+d\,x)\ 4\mathrm{i}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + a*tan(c + d*x)*1i)^2,x)
[Out] (a^2*(4*log(tan(c + d*x) + 1i) + tan(c + d*x)*4i - tan(c + d*x)^2))/(2*d)
```

3.17 $\int (a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=38

$$2a^2x-\frac{2ia^2\log(\cos(c+dx))}{d}-\frac{a^2\tan(c+dx)}{d}$$

```
[Out] 2*a^2*x-2*I*a^2*ln(cos(d*x+c))/d-a^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {3558, 3556}

$$-\frac{a^2\tan(c+dx)}{d}-\frac{2ia^2\log(\cos(c+dx))}{d}+2a^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] 2*a^2*x - ((2*I)*a^2*Log[Cos[c + d*x]])/d - (a^2*Tan[c + d*x])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+ia\tan(c+dx))^2\,dx &= 2a^2x-\frac{a^2\tan(c+dx)}{d}+\left(2ia^2\right)\int\tan(c+dx)\,dx\\ &= 2a^2x-\frac{2ia^2\log(\cos(c+dx))}{d}-\frac{a^2\tan(c+dx)}{d}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 100 vs. 2(38) = 76.
time = 0.63, size = 100, normalized size = 2.63

$$-\frac{a^2\sec(c)\sec(c+dx)\left(4\text{ArcTan}(\tan(3c+dx))\cos(c)\cos(c+dx)-4dx\cos(2c+dx)+\cos(dx)\left(-4dx+i\log\left(\cos^2(c+dx)\right)\right)+i\cos(2c+dx)\log\left(\cos^2(c+dx)\right)+2\sin(dx)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -1/2*(a^2*Sec[c]*Sec[c + d*x]*(4*ArcTan[Tan[3*c + d*x]]*Cos[c]*Cos[c + d*x]
 - 4*d*x*Cos[2*c + d*x] + Cos[d*x]*(-4*d*x + I*Log[Cos[c + d*x]^2]) + I*Cos
[2*c + d*x]*Log[Cos[c + d*x]^2] + 2*Sin[d*x]))/d
```

Maple [A]
time = 0.02, size = 40, normalized size = 1.05

method	result	size
derivativedivides	$\frac{a^{2}\left(-\tan \left(d x+c\right)+i \ln \left(1+\tan^{2}\left(d x+c\right)\right)+2 \arctan \left(\tan \left(d x+c\right)\right)\right)}{d}$	40
default	$\frac{a^{2}\left(-\tan \left(d x+c\right)+i \ln \left(1+\tan^{2}\left(d x+c\right)\right)+2 \arctan \left(\tan \left(d x+c\right)\right)\right)}{d}$	40
norman	$2 a^{2} x-\frac{a^{2} \tan \left(d x+c\right)}{d}+\frac{i a^{2} \ln \left(1+\tan^{2}\left(d x+c\right)\right)}{d}$	42
risch	$-\frac{4 a^{2} c}{d}-\frac{2 i a^{2}}{d \left(\mathrm{e}^{2 i \left(d x+c\right)}+1\right)}-\frac{2 i a^{2} \ln \left(\mathrm{e}^{2 i \left(d x+c\right)}+1\right)}{d}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a^2*(-tan(d*x+c)+I*ln(1+tan(d*x+c)^2)+2*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.63, size = 41, normalized size = 1.08

$$a^{2} x+\frac{\left(d x+c-\tan \left(d x+c\right)\right) a^{2}}{d}+\frac{2 i\, a^{2} \log \left(\sec \left(d x+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] a^2*x + (d*x + c - tan(d*x + c))*a^2/d + 2*I*a^2*log(sec(d*x + c))/d
```

Fricas [A]
time = 0.43, size = 56, normalized size = 1.47

$$-\frac{2 \left(i\, a^{2}+\left(i\, a^{2} e^{\left(2 i\, d x+2 i\, c\right)}+i\, a^{2}\right) \log \left(e^{\left(2 i\, d x+2 i\, c\right)}+1\right)\right)}{d e^{\left(2 i\, d x+2 i\, c\right)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2*(I*a^2 + (I*a^2*e^(2*I*d*x + 2*I*c) + I*a^2)*log(e^(2*I*d*x + 2*I*c) + 1
))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A]
time = 0.14, size = 53, normalized size = 1.39

$$-\frac{2ia^2}{de^{2ic}e^{2idx}+d}-\frac{2ia^2\log\left(e^{2idx}+e^{-2ic}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -2*I*a**2/(d*exp(2*I*c)*exp(2*I*d*x) + d) - 2*I*a**2*log(exp(2*I*d*x) + exp
(-2*I*c))/d
```

Giac [A]
time = 0.47, size = 66, normalized size = 1.74

$$-\frac{2\left(i\,a^2e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+i\,a^2\log\left(e^{(2i\,dx+2i\,c)}+1\right)+i\,a^2\right)}{de^{(2i\,dx+2i\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*(I*a^2*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + I*a^2*log(e^(2
*I*d*x + 2*I*c) + 1) + I*a^2)/(d*e^(2*I*d*x + 2*I*c) + d)
```

Mupad [B]
time = 3.68, size = 29, normalized size = 0.76

$$\frac{a^2\left(-\tan(c+d\,x)+\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,2\mathrm{i}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] (a^2*(log(tan(c + d*x) + 1i)*2i - tan(c + d*x)))/d
```

3.18 $\int \cot(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=37

$$2ia^2x+\frac{a^2\log(\cos(c+dx))}{d}+\frac{a^2\log(\sin(c+dx))}{d}$$

```
[Out] 2*I*a^2*x+a^2*ln(cos(d*x+c))/d+a^2*ln(sin(d*x+c))/d
```

Rubi [A]
time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3622, 3556}

$$\frac{a^2\log(\sin(c+dx))}{d}+\frac{a^2\log(\cos(c+dx))}{d}+2ia^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (2*I)*a^2*x + (a^2*Log[Cos[c + d*x]])/d + (a^2*Log[Sin[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3622

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^2/((a_.) + (b_.)*tan[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[d*(2*b*c - a*d)*(x/b^2), x] + (Dist[d^2/b, In
t[Tan[e + f*x], x], x] + Dist[(b*c - a*d)^2/b^2, Int[1/(a + b*Tan[e + f*x])
, x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 +
 b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)(a+ia\tan(c+dx))^2\,dx &= 2ia^2x+a^2\int \cot(c+dx)\,dx-a^2\int \tan(c+dx)\,dx\\ &= 2ia^2x+\frac{a^2\log(\cos(c+dx))}{d}+\frac{a^2\log(\sin(c+dx))}{d}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 30, normalized size = 0.81

$$\frac{a^2(2idx+2\log(\cos(c+dx))+\log(\tan(c+dx)))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^2,x]
[Out] (a^2*((2*I)*d*x + 2*Log[Cos[c + d*x]] + Log[Tan[c + d*x]]))/d
```

Maple [A]
time = 0.19, size = 39, normalized size = 1.05

method	result	size
risch	$-\frac{4ia^2c}{d}+\frac{a^2\ln\left(\mathrm{e}^{4i(dx+c)}-1\right)}{d}$	31
derivativedivides	$\frac{a^2\ln(\cos(dx+c))+2ia^2(dx+c)+a^2\ln(\sin(dx+c))}{d}$	39
default	$\frac{a^2\ln(\cos(dx+c))+2ia^2(dx+c)+a^2\ln(\sin(dx+c))}{d}$	39
norman	$2ia^2x+\frac{a^2\ln(\tan(dx+c))}{d}-\frac{a^2\ln(1+\tan^2(dx+c))}{d}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(a^2*ln(cos(d*x+c))+2*I*a^2*(d*x+c)+a^2*ln(sin(d*x+c)))
```

Maxima [A]
time = 0.50, size = 42, normalized size = 1.14

$$\frac{2i\,(dx+c)a^2-a^2\log\left(\tan\left(dx+c\right)^2+1\right)+a^2\log\left(\tan\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
[Out] (2*I*(d*x + c)*a^2 - a^2*log(tan(d*x + c)^2 + 1) + a^2*log(tan(d*x + c)))/d
```

Fricas [A]
time = 0.51, size = 19, normalized size = 0.51

$$\frac{a^2\log\left(e^{(4i\,dx+4i\,c)}-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
[Out] a^2*log(e^(4*I*d*x + 4*I*c) - 1)/d
```

Sympy [A]
time = 0.13, size = 22, normalized size = 0.59

$$\frac{a^2\log\left(e^{4idx}-e^{-4ic}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] a**2*log(exp(4*I*d*x) - exp(-4*I*c))/d
```

Giac [A]
time = 0.60, size = 68, normalized size = 1.84

$$\frac{a^2 \log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-4\,a^2 \log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)+a^2 \log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+a^2 \log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (a^2*log(tan(1/2*d*x + 1/2*c) + 1) - 4*a^2*log(tan(1/2*d*x + 1/2*c) + I) +
a^2*log(tan(1/2*d*x + 1/2*c) - 1) + a^2*log(tan(1/2*d*x + 1/2*c)))/d
```

Mupad [B]
time = 3.73, size = 30, normalized size = 0.81

$$-\frac{a^2\left(2\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)-\ln\left(\tan(c+d\,x)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] -(a^2*(2*log(tan(c + d*x) + 1i) - log(tan(c + d*x))))/d
```

3.19 $\int \cot^2(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=38

$$-2a^2x-\frac{a^2\cot(c+dx)}{d}+\frac{2ia^2\log(\sin(c+dx))}{d}$$

```
[Out] -2*a^2*x-a^2*cot(d*x+c)/d+2*I*a^2*ln(sin(d*x+c))/d
```

Rubi [A]
time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3623, 3612, 3556}

$$-\frac{a^2\cot(c+dx)}{d}+\frac{2ia^2\log(\sin(c+dx))}{d}-2a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -2*a^2*x - (a^2*Cot[c + d*x])/d + ((2*I)*a^2*Log[Sin[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cot^2(c+dx)(a+ia\tan(c+dx))^2\,dx &= -\frac{a^2\cot(c+dx)}{d}+\int \cot(c+dx)\left(2ia^2-2a^2\tan(c+dx)\right)\,dx\\ &= -2a^2x-\frac{a^2\cot(c+dx)}{d}+\left(2ia^2\right)\int \cot(c+dx)\,dx\\ &= -2a^2x-\frac{a^2\cot(c+dx)}{d}+\frac{2ia^2\log(\sin(c+dx))}{d}\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 100 vs. 2(38) = 76.
time = 0.56, size = 100, normalized size = 2.63

$$\frac{a^2\csc(c)\csc(c+dx)\left(4dx\cos(2c+dx)+\cos(dx)\left(-4dx+i\log\left(\sin^2(c+dx)\right)\right)-i\cos(2c+dx)\log\left(\sin^2(c+dx)\right)+2\sin(dx)+4\text{ArcTan}(\tan(3c+dx))\sin(c)\sin(c+dx)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*Csc[c]*Csc[c + d*x]*(4*d*x*Cos[2*c + d*x] + Cos[d*x]*(-4*d*x + I*Log[S
in[c + d*x]^2]) - I*Cos[2*c + d*x]*Log[Sin[c + d*x]^2] + 2*Sin[d*x] + 4*Arc
Tan[Tan[3*c + d*x]]*Sin[c]*Sin[c + d*x]))/(2*d)
```

Maple [**A**]
time = 0.14, size = 49, normalized size = 1.29

method	result	size
derivativedivides	$\frac{-a^2(dx+c)+2ia^2\ln(\sin(dx+c))+a^2(-\cot(dx+c)-dx-c)}{d}$	49
default	$\frac{-a^2(dx+c)+2ia^2\ln(\sin(dx+c))+a^2(-\cot(dx+c)-dx-c)}{d}$	49
risch	$\frac{4a^2c}{d}-\frac{2ia^2}{d\left(e^{2i(dx+c)}-1\right)}+\frac{2ia^2\ln\left(e^{2i(dx+c)}-1\right)}{d}$	54
norman	$\frac{-\frac{a^2}{d}-2a^2x\tan(dx+c)}{\tan(dx+c)}+\frac{2ia^2\ln(\tan(dx+c))}{d}-\frac{ia^2\ln(1+\tan^2(dx+c))}{d}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(d*x+c)+2*I*a^2*ln(sin(d*x+c))+a^2*(-cot(d*x+c)-d*x-c))
```

Maxima [**A**]
time = 0.52, size = 56, normalized size = 1.47

$$-\frac{2\,(dx+c)a^2+i\,a^2\log\left(\tan\left(dx+c\right)^2+1\right)-2i\,a^2\log\left(\tan\left(dx+c\right)\right)+\frac{a^2}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(2*(d*x + c)*a^2 + I*a^2*log(tan(d*x + c)^2 + 1) - 2*I*a^2*log(tan(d*x + c
)) + a^2/tan(d*x + c))/d
```

Fricas [A]
time = 0.43, size = 58, normalized size = 1.53

$$-\frac{2\left(i\,a^2+\left(-i\,a^2e^{(2i\,dx+2i\,c)}+i\,a^2\right)\log\left(e^{(2i\,dx+2i\,c)}-1\right)\right)}{de^{(2i\,dx+2i\,c)}-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2*(I*a^2 + (-I*a^2*e^(2*I*d*x + 2*I*c) + I*a^2)*log(e^(2*I*d*x + 2*I*c) -
1))/(d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [A]
time = 0.16, size = 51, normalized size = 1.34

$$-\frac{2ia^2}{de^{2ic}e^{2idx}-d}+\frac{2ia^2\log\left(e^{2idx}-e^{-2ic}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -2*I*a**2/(d*exp(2*I*c)*exp(2*I*d*x) - d) + 2*I*a**2*log(exp(2*I*d*x) - exp
(-2*I*c))/d
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(36) = 72.
time = 0.63, size = 85, normalized size = 2.24

$$-\frac{8i\,a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)-4i\,a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{-4i\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(8*I*a^2*log(tan(1/2*d*x + 1/2*c) + I) - 4*I*a^2*log(tan(1/2*d*x + 1/2
*c)) - a^2*tan(1/2*d*x + 1/2*c) - (-4*I*a^2*tan(1/2*d*x + 1/2*c) - a^2)/tan
(1/2*d*x + 1/2*c))/d
```

Mupad [B]
time = 3.83, size = 29, normalized size = 0.76

$$-\frac{a^2\left(\cot(c+d\,x)+4\,\mathrm{atan}(2\tan(c+d\,x)+1\mathrm{i})\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + a*tan(c + d*x)*1i)^2,x)
[Out] -(a^2*(cot(c + d*x) + 4*atan(2*tan(c + d*x) + 1i)))/d
```

3.20 $\int \cot^3(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=58

$$-2ia^2x-\frac{2ia^2\cot(c+dx)}{d}-\frac{a^2\cot^2(c+dx)}{2d}-\frac{2a^2\log(\sin(c+dx))}{d}$$

```
[Out] -2*I*a^2*x-2*I*a^2*cot(d*x+c)/d-1/2*a^2*cot(d*x+c)^2/d-2*a^2*ln(sin(d*x+c))
/d
```

Rubi [A]
time = 0.07, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3623, 3610, 3612, 3556}

$$-\frac{a^2\cot^2(c+dx)}{2d}-\frac{2ia^2\cot(c+dx)}{d}-\frac{2a^2\log(\sin(c+dx))}{d}-2ia^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (-2*I)*a^2*x - ((2*I)*a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]^2)/(2*d) - (2
*a^2*Log[Sin[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cot^3(c+dx)(a+ia\tan(c+dx))^2\,dx &= -\frac{a^2\cot^2(c+dx)}{2d}+\int \cot^2(c+dx)\left(2ia^2-2a^2\tan(c+dx)\right)\,dx\\ &= -\frac{2ia^2\cot(c+dx)}{d}-\frac{a^2\cot^2(c+dx)}{2d}+\int \cot(c+dx)\left(-2a^2-2ia\right.\\ &= -2ia^2x-\frac{2ia^2\cot(c+dx)}{d}-\frac{a^2\cot^2(c+dx)}{2d}-\left(2a^2\right)\int \cot(c+d\\ &= -2ia^2x-\frac{2ia^2\cot(c+dx)}{d}-\frac{a^2\cot^2(c+dx)}{2d}-\frac{2a^2\log(\sin(c+dx}{d}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.22, size = 64, normalized size = 1.10

$$-\frac{a^2\left(\cot^2(c+dx)+4i\cot(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)+4(\log(\cos(c+dx))+\log(\tan(c+dx)))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + I*a*Tan[c + d*x])^2,x]
[Out] -1/2*(a^2*(Cot[c + d*x]^2 + (4*I)*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1
/2, -Tan[c + d*x]^2] + 4*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]])))/d
```

Maple [A]
time = 0.21, size = 64, normalized size = 1.10

method	result	size
derivativedivides	$\frac{-a^2\ln(\sin(dx+c))+2ia^2(-\cot(dx+c)-dx-c)+a^2\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)}{d}$	64
default	$\frac{-a^2\ln(\sin(dx+c))+2ia^2(-\cot(dx+c)-dx-c)+a^2\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)}{d}$	64
risch	$\frac{4ia^2c}{d}+\frac{2a^2\left(3\,e^{2i(dx+c)}-2\right)}{d\left(e^{2i(dx+c)}-1\right)^2}-\frac{2a^2\ln\left(e^{2i(dx+c)}-1\right)}{d}$	66

norman	$\frac{-\frac{a^2}{2d}-\frac{2ia^2\tan(dx+c)}{d}-2ia^2x(\tan^2(dx+c))}{\tan(dx+c)^2}+\frac{a^2\ln(1+\tan^2(dx+c))}{d}-\frac{2a^2\ln(\tan(dx+c))}{d}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(-a^2*ln(sin(d*x+c))+2*I*a^2*(-cot(d*x+c)-d*x-c)+a^2*(-1/2*cot(d*x+c)^2
-ln(sin(d*x+c))))
```

Maxima [A]
time = 0.51, size = 68, normalized size = 1.17

$$-\frac{4i\,(dx+c)a^2-2\,a^2\log\left(\tan\left(dx+c\right)^2+1\right)+4\,a^2\log\left(\tan\left(dx+c\right)\right)+\frac{4i\,a^2\tan(dx+c)+a^2}{\tan(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
[Out] -1/2*(4*I*(d*x + c)*a^2 - 2*a^2*log(tan(d*x + c)^2 + 1) + 4*a^2*log(tan(d*x
 + c)) + (4*I*a^2*tan(d*x + c) + a^2)/tan(d*x + c)^2)/d
```

Fricas [A]
time = 0.40, size = 94, normalized size = 1.62

$$\frac{2\left(3\,a^2e^{(2i\,dx+2i\,c)}-2\,a^2-\left(a^2e^{(4i\,dx+4i\,c)}-2\,a^2e^{(2i\,dx+2i\,c)}+a^2\right)\log\left(e^{(2i\,dx+2i\,c)}-1\right)\right)}{de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
[Out] 2*(3*a^2*e^(2*I*d*x + 2*I*c) - 2*a^2 - (a^2*e^(4*I*d*x + 4*I*c) - 2*a^2*e^(
2*I*d*x + 2*I*c) + a^2)*log(e^(2*I*d*x + 2*I*c) - 1))/(d*e^(4*I*d*x + 4*I*c
) - 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A]
time = 0.26, size = 87, normalized size = 1.50

$$-\frac{2a^2\log\left(e^{2idx}-e^{-2ic}\right)}{d}+\frac{6a^2e^{2ic}e^{2idx}-4a^2}{de^{4ic}e^{4idx}-2de^{2ic}e^{2idx}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+I*a*tan(d*x+c))**2,x)
[Out] -2*a**2*log(exp(2*I*d*x) - exp(-2*I*c))/d + (6*a**2*exp(2*I*c)*exp(2*I*d*x)
 - 4*a**2)/(d*exp(4*I*c)*exp(4*I*d*x) - 2*d*exp(2*I*c)*exp(2*I*d*x) + d)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(52) = 104$.
time = 0.82, size = 116, normalized size = 2.00

$$-\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-32\,a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)+16\,a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-8i\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{24\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-8i\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/8*(a^2*tan(1/2*d*x + 1/2*c)^2 - 32*a^2*log(tan(1/2*d*x + 1/2*c) + I) + 1
6*a^2*log(tan(1/2*d*x + 1/2*c)) - 8*I*a^2*tan(1/2*d*x + 1/2*c) - (24*a^2*ta
n(1/2*d*x + 1/2*c)^2 - 8*I*a^2*tan(1/2*d*x + 1/2*c) - a^2)/tan(1/2*d*x + 1/
2*c)^2)/d
```

Mupad [B]
time = 3.84, size = 53, normalized size = 0.91

$$-\frac{\frac{a^2}{2}+a^2\tan(c+d\,x)\;2\mathrm{i}}{d\tan\left(c+d\,x\right)^2}-\frac{a^2\,\mathrm{atan}(2\tan(c+d\,x)+1\mathrm{i})\;4\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] - (a^2*tan(c + d*x)*2i + a^2/2)/(d*tan(c + d*x)^2) - (a^2*atan(2*tan(c + d*
x) + 1i)*4i)/d
```

3.21 $\int \cot^4(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=74

$$2a^2x+\frac{2a^2\cot(c+dx)}{d}-\frac{ia^2\cot^2(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-\frac{2ia^2\log(\sin(c+dx))}{d}$$

```
[Out] 2*a^2*x+2*a^2*cot(d*x+c)/d-I*a^2*cot(d*x+c)^2/d-1/3*a^2*cot(d*x+c)^3/d-2*I*
a^2*ln(sin(d*x+c))/d
```

Rubi [A]
time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {3623, 3610, 3612, 3556}

$$-\frac{a^2\cot^3(c+dx)}{3d}-\frac{ia^2\cot^2(c+dx)}{d}+\frac{2a^2\cot(c+dx)}{d}-\frac{2ia^2\log(\sin(c+dx))}{d}+2a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] 2*a^2*x + (2*a^2*Cot[c + d*x])/d - (I*a^2*Cot[c + d*x]^2)/d - (a^2*Cot[c +
d*x]^3)/(3*d) - ((2*I)*a^2*Log[Sin[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{a^2\cot^3(c+dx)}{3d} + \int \cot^3(c+dx)\left(2ia^2 - 2a^2\tan(c+dx)\right)\,dx$$
$$= -\frac{ia^2\cot^2(c+dx)}{d} - \frac{a^2\cot^3(c+dx)}{3d} + \int \cot^2(c+dx)\left(-2a^2 - 2i\right.$$
$$= \frac{2a^2\cot(c+dx)}{d} - \frac{ia^2\cot^2(c+dx)}{d} - \frac{a^2\cot^3(c+dx)}{3d} + \int \cot(c+$$
$$= 2a^2x + \frac{2a^2\cot(c+dx)}{d} - \frac{ia^2\cot^2(c+dx)}{d} - \frac{a^2\cot^3(c+dx)}{3d} - (2$$
$$= 2a^2x + \frac{2a^2\cot(c+dx)}{d} - \frac{ia^2\cot^2(c+dx)}{d} - \frac{a^2\cot^3(c+dx)}{3d} - \frac{2i}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.50, size = 105, normalized size = 1.42

$$-\frac{a^2\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(c+dx)\right)}{3d} + \frac{a^2\cot(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)}{d} - \frac{ia^2(\cot^2(c+dx)+2\log(\cos(c+dx))+2\log(\tan(c+dx)))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -1/3*(a^2*Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])
/d + (a^2*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2])/d
- (I*a^2*(Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[Tan[c + d*x]]))/d
```

Maple [A]
time = 0.21, size = 78, normalized size = 1.05

method	result	size
derivativedivides	$\frac{-a^2(-\cot(dx+c)-dx-c)+2ia^2\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+a^2\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)}{d}$	78

default	$\frac{-a^2(-\cot(dx+c)-dx-c)+2ia^2\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+a^2\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)}{d}$	78
risch	$-\frac{4a^2c}{d}+\frac{2ia^2\left(15\,e^{4i(dx+c)}-18\,e^{2i(dx+c)}+7\right)}{3d\left(e^{2i(dx+c)}-1\right)^3}-\frac{2ia^2\ln\left(e^{2i(dx+c)}-1\right)}{d}$	78
norman	$\frac{-\frac{a^2}{3d}+2a^2x\left(\tan^3(dx+c)\right)+\frac{2a^2\left(\tan^2(dx+c)\right)}{d}-\frac{ia^2\tan(dx+c)}{d}}{\tan(dx+c)^3}+\frac{ia^2\ln\left(1+\tan^2(dx+c)\right)}{d}-\frac{2ia^2\ln(\tan(dx+c))}{d}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(-cot(d*x+c)-d*x-c)+2*I*a^2*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+a^
2*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.49, size = 83, normalized size = 1.12

$$\frac{6\,(dx+c)a^2+3i\,a^2\log\left(\tan\left(dx+c\right)^2+1\right)-6i\,a^2\log\left(\tan\left(dx+c\right)\right)+\frac{6\,a^2\tan(dx+c)^2-3i\,a^2\tan(dx+c)-a^2}{\tan(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(6*(d*x + c)*a^2 + 3*I*a^2*log(tan(d*x + c)^2 + 1) - 6*I*a^2*log(tan(d*
x + c)) + (6*a^2*tan(d*x + c)^2 - 3*I*a^2*tan(d*x + c) - a^2)/tan(d*x + c)^
3)/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(68) = 136.
time = 0.47, size = 139, normalized size = 1.88

$$-\frac{2\left(-15i\,a^2e^{(4i\,dx+4i\,c)}+18i\,a^2e^{(2i\,dx+2i\,c)}-7i\,a^2+3\left(i\,a^2e^{(6i\,dx+6i\,c)}-3i\,a^2e^{(4i\,dx+4i\,c)}+3i\,a^2e^{(2i\,dx+2i\,c)}-i\,a^2\right)\log\left(e^{(2i\,dx+2i\,c)}-1\right)\right)}{3\left(de^{(6i\,dx+6i\,c)}-3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(-15*I*a^2*e^(4*I*d*x + 4*I*c) + 18*I*a^2*e^(2*I*d*x + 2*I*c) - 7*I*a^
2 + 3*(I*a^2*e^(6*I*d*x + 6*I*c) - 3*I*a^2*e^(4*I*d*x + 4*I*c) + 3*I*a^2*e^
(2*I*d*x + 2*I*c) - I*a^2)*log(e^(2*I*d*x + 2*I*c) - 1))/(d*e^(6*I*d*x + 6*
I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(66) = 132.
time = 0.23, size = 136, normalized size = 1.84

$$-\frac{2ia^2\log\left(e^{2idx}-e^{-2ic}\right)}{d}+\frac{30ia^2e^{4ic}e^{4idx}-36ia^2e^{2ic}e^{2idx}+14ia^2}{3de^{6ic}e^{6idx}-9de^{4ic}e^{4idx}+9de^{2ic}e^{2idx}-3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -2*I*a**2*log(exp(2*I*d*x) - exp(-2*I*c))/d + (30*I*a**2*exp(4*I*c)*exp(4*I
*d*x) - 36*I*a**2*exp(2*I*c)*exp(2*I*d*x) + 14*I*a**2)/(3*d*exp(6*I*c)*exp(
6*I*d*x) - 9*d*exp(4*I*c)*exp(4*I*d*x) + 9*d*exp(2*I*c)*exp(2*I*d*x) - 3*d)
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(68) = 136$.
time = 0.86, size = 146, normalized size = 1.97

$$\frac{a^2 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 - 6i\,a^2 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + 96i\,a^2 \log\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + i\right) - 48i\,a^2 \log\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right) - 27\,a^2 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - \frac{-88i\,a^2 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 27\,a^2 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 6i\,a^2 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 - 6*I*a^2*tan(1/2*d*x + 1/2*c)^2 + 96*I*a^
2*log(tan(1/2*d*x + 1/2*c) + I) - 48*I*a^2*log(tan(1/2*d*x + 1/2*c)) - 27*a
^2*tan(1/2*d*x + 1/2*c) - (-88*I*a^2*tan(1/2*d*x + 1/2*c)^3 - 27*a^2*tan(1/
2*d*x + 1/2*c)^2 + 6*I*a^2*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*c)
^3)/d
```

Mupad [**B**]
time = 3.81, size = 68, normalized size = 0.92

$$\frac{2\,a^2\,\mathrm{cot}(c+d\,x)}{d}+\frac{4\,a^2\,\mathrm{atan}(2\,\tan(c+d\,x)+1\mathrm{i})}{d}-\frac{a^2\,\mathrm{cot}(c+d\,x)^3}{3\,d}-\frac{a^2\,\mathrm{cot}(c+d\,x)^2\,1\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] (2*a^2*cot(c + d*x))/d + (4*a^2*atan(2*tan(c + d*x) + 1i))/d - (a^2*cot(c +
 d*x)^2*1i)/d - (a^2*cot(c + d*x)^3)/(3*d)
```

3.22 $\int \cot^5(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=93

$$2ia^2x+\frac{2ia^2\cot(c+dx)}{d}+\frac{a^2\cot^2(c+dx)}{d}-\frac{2ia^2\cot^3(c+dx)}{3d}-\frac{a^2\cot^4(c+dx)}{4d}+\frac{2a^2\log(\sin(c+dx))}{d}$$

```
[Out] 2*I*a^2*x+2*I*a^2*cot(d*x+c)/d+a^2*cot(d*x+c)^2/d-2/3*I*a^2*cot(d*x+c)^3/d-
1/4*a^2*cot(d*x+c)^4/d+2*a^2*ln(sin(d*x+c))/d
```

Rubi [A]
time = 0.10, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3623, 3610, 3612, 3556}

$$-\frac{a^2\cot^4(c+dx)}{4d}-\frac{2ia^2\cot^3(c+dx)}{3d}+\frac{a^2\cot^2(c+dx)}{d}+\frac{2ia^2\cot(c+dx)}{d}+\frac{2a^2\log(\sin(c+dx))}{d}+2ia^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (2*I)*a^2*x + ((2*I)*a^2*Cot[c + d*x])/d + (a^2*Cot[c + d*x]^2)/d - (((2*I)
/3)*a^2*Cot[c + d*x]^3)/d - (a^2*Cot[c + d*x]^4)/(4*d) + (2*a^2*Log[Sin[c +
 d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \cot^5(c+dx)(a+ia\tan(c+dx))^2\,dx &= -\frac{a^2\cot^4(c+dx)}{4d} + \int \cot^4(c+dx)\left(2ia^2 - 2a^2\tan(c+dx)\right)\,dx \\
&= -\frac{2ia^2\cot^3(c+dx)}{3d} - \frac{a^2\cot^4(c+dx)}{4d} + \int \cot^3(c+dx)\left(-2a^2 - 2\right. \\
&= \frac{a^2\cot^2(c+dx)}{d} - \frac{2ia^2\cot^3(c+dx)}{3d} - \frac{a^2\cot^4(c+dx)}{4d} + \int \cot^2(c \\
&= \frac{2ia^2\cot(c+dx)}{d} + \frac{a^2\cot^2(c+dx)}{d} - \frac{2ia^2\cot^3(c+dx)}{3d} - \frac{a^2\cot^4(}{4d} \\
&= 2ia^2x + \frac{2ia^2\cot(c+dx)}{d} + \frac{a^2\cot^2(c+dx)}{d} - \frac{2ia^2\cot^3(c+dx)}{3d} - \\
&= 2ia^2x + \frac{2ia^2\cot(c+dx)}{d} + \frac{a^2\cot^2(c+dx)}{d} - \frac{2ia^2\cot^3(c+dx)}{3d} -
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.41, size = 79, normalized size = 0.85

$$
-\frac{a^2\left(8i\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(c+dx)\right)+3\left(-4\cot^2(c+dx)+\cot^4(c+dx)-8(\log(\cos(c+dx))+\log(\tan(c+dx)))\right)\right)}{12d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -1/12*(a^2*((8*I)*Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c +
d*x]^2] + 3*(-4*Cot[c + d*x]^2 + Cot[c + d*x]^4 - 8*(Log[Cos[c + d*x]] + Lo
g[Tan[c + d*x]]))))/d
```

Maple [A]
time = 0.22, size = 90, normalized size = 0.97

method	result
risch	$-\frac{4ia^2c}{d} - \frac{2a^2\left(21\,\mathrm{e}^{6i(dx+c)}-36\,\mathrm{e}^{4i(dx+c)}+29\,\mathrm{e}^{2i(dx+c)}-8\right)}{3d\left(\mathrm{e}^{2i(dx+c)}-1\right)^4} + \frac{2a^2\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{d}$

derivativedivides	$\frac{-a^2\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+2ia^2\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+a^2\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}\right.}{d}$
default	$\frac{-a^2\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+2ia^2\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+a^2\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}\right.}{d}$
norman	$\frac{\frac{a^2\left(\tan^2(dx+c)\right)}{d}-\frac{a^2}{4d}-\frac{2ia^2\tan(dx+c)}{3d}+\frac{2ia^2\left(\tan^3(dx+c)\right)}{d}+2ia^2x\left(\tan^4(dx+c)\right)}{\tan(dx+c)^4}+\frac{2a^2\ln(\tan(dx+c))}{d}-\frac{a^2\ln(1+\tan^2(}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+2*I*a^2*(-1/3*cot(d*x+c)^3+cot
(d*x+c)+d*x+c)+a^2*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c))))
```

Maxima [A]
time = 0.50, size = 97, normalized size = 1.04

$$-\frac{-24i\,(dx+c)a^2+12\,a^2\log\left(\tan\left(dx+c\right)^2+1\right)-24\,a^2\log\left(\tan\left(dx+c\right)\right)-\frac{24i\,a^2\tan(dx+c)^3+12\,a^2\tan(dx+c)^2-8i\,a^2\tan(dx+c)-3\,a^2}{\tan(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/12*(-24*I*(d*x + c)*a^2 + 12*a^2*log(tan(d*x + c)^2 + 1) - 24*a^2*log(ta
n(d*x + c)) - (24*I*a^2*tan(d*x + c)^3 + 12*a^2*tan(d*x + c)^2 - 8*I*a^2*ta
n(d*x + c) - 3*a^2)/tan(d*x + c)^4)/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(83) = 166.
time = 0.42, size = 174, normalized size = 1.87

$$-\frac{2\left(21\,a^2e^{(6i\,dx+6i\,c)}-36\,a^2e^{(4i\,dx+4i\,c)}+29\,a^2e^{(2i\,dx+2i\,c)}-8\,a^2-3\left(a^2e^{(8i\,dx+8i\,c)}-4\,a^2e^{(6i\,dx+6i\,c)}+6\,a^2e^{(4i\,dx+4i\,c)}-4\,a^2e^{(2i\,dx+2i\,c)}+a^2\right)\log\left(e^{(2i\,dx+2i\,c)}-1\right)\right)}{3\left(de^{(8i\,dx+8i\,c)}-4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}-4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(21*a^2*e^(6*I*d*x + 6*I*c) - 36*a^2*e^(4*I*d*x + 4*I*c) + 29*a^2*e^(2
*I*d*x + 2*I*c) - 8*a^2 - 3*(a^2*e^(8*I*d*x + 8*I*c) - 4*a^2*e^(6*I*d*x + 6
*I*c) + 6*a^2*e^(4*I*d*x + 4*I*c) - 4*a^2*e^(2*I*d*x + 2*I*c) + a^2)*log(e^
(2*I*d*x + 2*I*c) - 1))/(d*e^(8*I*d*x + 8*I*c) - 4*d*e^(6*I*d*x + 6*I*c) +
6*d*e^(4*I*d*x + 4*I*c) - 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A]
time = 0.98, size = 168, normalized size = 1.81

$$\frac{2a^2\log\left(e^{2idx}-e^{-2ic}\right)}{d}+\frac{-42a^2e^{6ic}e^{6idx}+72a^2e^{4ic}e^{4idx}-58a^2e^{2ic}e^{2idx}+16a^2}{3de^{8ic}e^{8idx}-12de^{6ic}e^{6idx}+18de^{4ic}e^{4idx}-12de^{2ic}e^{2idx}+3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] 2*a**2*log(exp(2*I*d*x) - exp(-2*I*c))/d + (-42*a**2*exp(6*I*c)*exp(6*I*d*x
) + 72*a**2*exp(4*I*c)*exp(4*I*d*x) - 58*a**2*exp(2*I*c)*exp(2*I*d*x) + 16*
a**2)/(3*d*exp(8*I*c)*exp(8*I*d*x) - 12*d*exp(6*I*c)*exp(6*I*d*x) + 18*d*ex
p(4*I*c)*exp(4*I*d*x) - 12*d*exp(2*I*c)*exp(2*I*d*x) + 3*d)
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(83) = 166$.
time = 0.95, size = 180, normalized size = 1.94

$$-\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-16i\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-60\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+768\,a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)-384\,a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+240i\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{800\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-240i\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-60\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+16i\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/192*(3*a^2*tan(1/2*d*x + 1/2*c)^4 - 16*I*a^2*tan(1/2*d*x + 1/2*c)^3 - 60
*a^2*tan(1/2*d*x + 1/2*c)^2 + 768*a^2*log(tan(1/2*d*x + 1/2*c) + I) - 384*a
^2*log(tan(1/2*d*x + 1/2*c)) + 240*I*a^2*tan(1/2*d*x + 1/2*c) + (800*a^2*ta
n(1/2*d*x + 1/2*c)^4 - 240*I*a^2*tan(1/2*d*x + 1/2*c)^3 - 60*a^2*tan(1/2*d*
x + 1/2*c)^2 + 16*I*a^2*tan(1/2*d*x + 1/2*c) + 3*a^2)/tan(1/2*d*x + 1/2*c)^
4)/d
```

Mupad [**B**]
time = 3.92, size = 80, normalized size = 0.86

$$\frac{a^2\,\mathrm{atan}(2\tan(c+d\,x)+1\mathrm{i})\;4\mathrm{i}}{d}-\frac{-a^2\tan(c+d\,x)^3\,2\mathrm{i}-a^2\tan(c+d\,x)^2+\frac{a^2\tan(c+d\,x)\,2\mathrm{i}}{3}+\frac{a^2}{4}}{d\tan\left(c+d\,x\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] (a^2*atan(2*tan(c + d*x) + 1i)*4i)/d - ((a^2*tan(c + d*x)*2i)/3 + a^2/4 - a
^2*tan(c + d*x)^2 - a^2*tan(c + d*x)^3*2i)/(d*tan(c + d*x)^4)
```

3.23 $\int \cot^6(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=112

$$-2a^2x-\frac{2a^2\cot(c+dx)}{d}+\frac{ia^2\cot^2(c+dx)}{d}+\frac{2a^2\cot^3(c+dx)}{3d}-\frac{ia^2\cot^4(c+dx)}{2d}-\frac{a^2\cot^5(c+dx)}{5d}+\frac{2ia^2\log(s$$

```
[Out] -2*a^2*x-2*a^2*cot(d*x+c)/d+I*a^2*cot(d*x+c)^2/d+2/3*a^2*cot(d*x+c)^3/d-1/2
*I*a^2*cot(d*x+c)^4/d-1/5*a^2*cot(d*x+c)^5/d+2*I*a^2*ln(sin(d*x+c))/d
```

Rubi [A]
time = 0.12, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3623, 3610, 3612, 3556}

$$-\frac{a^2\cot^5(c+dx)}{5d}-\frac{ia^2\cot^4(c+dx)}{2d}+\frac{2a^2\cot^3(c+dx)}{3d}+\frac{ia^2\cot^2(c+dx)}{d}-\frac{2a^2\cot(c+dx)}{d}+\frac{2ia^2\log(\sin(c+dx))}{d}-2a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -2*a^2*x - (2*a^2*Cot[c + d*x])/d + (I*a^2*Cot[c + d*x]^2)/d + (2*a^2*Cot[c
 + d*x]^3)/(3*d) - ((I/2)*a^2*Cot[c + d*x]^4)/d - (a^2*Cot[c + d*x]^5)/(5*d
) + ((2*I)*a^2*Log[Sin[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \cot^6(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{a^2\cot^5(c+dx)}{5d} + \int \cot^5(c+dx)\left(2ia^2-2a^2\tan(c+dx)\right)\,dx$$

$$= -\frac{ia^2\cot^4(c+dx)}{2d} - \frac{a^2\cot^5(c+dx)}{5d} + \int \cot^4(c+dx)\left(-2a^2-2i\right.$$

$$= \frac{2a^2\cot^3(c+dx)}{3d} - \frac{ia^2\cot^4(c+dx)}{2d} - \frac{a^2\cot^5(c+dx)}{5d} + \int \cot^3(c$$

$$= \frac{ia^2\cot^2(c+dx)}{d} + \frac{2a^2\cot^3(c+dx)}{3d} - \frac{ia^2\cot^4(c+dx)}{2d} - \frac{a^2\cot^5(c}{5d}$$

$$= -\frac{2a^2\cot(c+dx)}{d} + \frac{ia^2\cot^2(c+dx)}{d} + \frac{2a^2\cot^3(c+dx)}{3d} - \frac{ia^2\cot}{}$$

$$= -2a^2x - \frac{2a^2\cot(c+dx)}{d} + \frac{ia^2\cot^2(c+dx)}{d} + \frac{2a^2\cot^3(c+dx)}{3d} -$$

$$= -2a^2x - \frac{2a^2\cot(c+dx)}{d} + \frac{ia^2\cot^2(c+dx)}{d} + \frac{2a^2\cot^3(c+dx)}{3d} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.83, size = 124, normalized size = 1.11

$$-\frac{a^2\cot^5(c+dx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\tan^2(c+dx)\right)}{5d} + \frac{a^2\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(c+dx)\right)}{3d} + \frac{ia^2\left(2\cot^2(c+dx)-\cot^4(c+dx)+4\log(\cos(c+dx))+4\log(\tan(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -1/5*(a^2*Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x]^2])
/d + (a^2*Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])
/(3*d) + ((I/2)*a^2*(2*Cot[c + d*x]^2 - Cot[c + d*x]^4 + 4*Log[Cos[c + d*x]
] + 4*Log[Tan[c + d*x]]))/d
```

Maple [A]
time = 0.20, size = 106, normalized size = 0.95

method	result
risch	$\frac{4a^2c}{d} - \frac{2ia^2(135\,e^{8i(dx+c)} - 300\,e^{6i(dx+c)} + 370\,e^{4i(dx+c)} - 200\,e^{2i(dx+c)} + 43)}{15d(e^{2i(dx+c)}-1)^5} + \frac{2ia^2\ln(e^{2i(dx+c)}-1)}{d}$
derivativedivides	$\frac{-a^2\left(-\frac{(\cot^3(dx+c))}{3}+\cot(dx+c)+dx+c\right)+2ia^2\left(-\frac{(\cot^4(dx+c))}{4}+\frac{(\cot^2(dx+c))}{2}+\ln(\sin(dx+c))\right)+a^2\left(-\frac{(\cot^5(dx+c))}{5}\right.}{d}$
default	$\frac{-a^2\left(-\frac{(\cot^3(dx+c))}{3}+\cot(dx+c)+dx+c\right)+2ia^2\left(-\frac{(\cot^4(dx+c))}{4}+\frac{(\cot^2(dx+c))}{2}+\ln(\sin(dx+c))\right)+a^2\left(-\frac{(\cot^5(dx+c))}{5}\right.}{d}$
norman	$\frac{\frac{ia^2(\tan^3(dx+c))}{d}-\frac{a^2}{5d}-2a^2x(\tan^5(dx+c))+\frac{2a^2(\tan^2(dx+c))}{3d}-\frac{2a^2(\tan^4(dx+c))}{d}-\frac{ia^2\tan(dx+c)}{2d}}{\tan(dx+c)^5}+\frac{2ia^2\ln(\tan(dx+c))}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+2*I*a^2*(-1/4*cot(d*x+c)^4+1
/2*cot(d*x+c)^2+ln(sin(d*x+c)))+a^2*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot
(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.67, size = 109, normalized size = 0.97

$$-\frac{60\,(dx+c)a^2 + 30i\,a^2\log\left(\tan(dx+c)^2+1\right) - 60i\,a^2\log\left(\tan(dx+c)\right) + \frac{60\,a^2\tan(dx+c)^4 - 30i\,a^2\tan(dx+c)^3 - 20\,a^2\tan(dx+c)^2 + 15i\,a^2\tan(dx+c) + 6\,a^2}{\tan(dx+c)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/30*(60*(d*x + c)*a^2 + 30*I*a^2*log(tan(d*x + c)^2 + 1) - 60*I*a^2*log(t
an(d*x + c)) + (60*a^2*tan(d*x + c)^4 - 30*I*a^2*tan(d*x + c)^3 - 20*a^2*ta
n(d*x + c)^2 + 15*I*a^2*tan(d*x + c) + 6*a^2)/tan(d*x + c)^5)/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(100) = 200.
time = 0.43, size = 219, normalized size = 1.96

$$-\frac{2\left(135i\,a^2e^{(8i\,dx+8i\,c)} - 300i\,a^2e^{(6i\,dx+6i\,c)} + 370i\,a^2e^{(4i\,dx+4i\,c)} - 200i\,a^2e^{(2i\,dx+2i\,c)} + 43i\,a^2 + 15\left(-i\,a^2e^{(10i\,dx+10i\,c)} + 5i\,a^2e^{(8i\,dx+8i\,c)} - 10i\,a^2e^{(6i\,dx+6i\,c)} + 10i\,a^2e^{(4i\,dx+4i\,c)} - 5i\,a^2e^{(2i\,dx+2i\,c)} + i\,a^2\right)\log\left(e^{(2i\,dx+2i\,c)}-1\right)\right)}{15\left(de^{(10i\,dx+10i\,c)} - 5\,de^{(8i\,dx+8i\,c)} + 10\,de^{(6i\,dx+6i\,c)} - 10\,de^{(4i\,dx+4i\,c)} + 5\,de^{(2i\,dx+2i\,c)} - d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/15*(135*I*a^2*e^(8*I*d*x + 8*I*c) - 300*I*a^2*e^(6*I*d*x + 6*I*c) + 370*
I*a^2*e^(4*I*d*x + 4*I*c) - 200*I*a^2*e^(2*I*d*x + 2*I*c) + 43*I*a^2 + 15*(
-I*a^2*e^(10*I*d*x + 10*I*c) + 5*I*a^2*e^(8*I*d*x + 8*I*c) - 10*I*a^2*e^(6*
I*d*x + 6*I*c) + 10*I*a^2*e^(4*I*d*x + 4*I*c) - 5*I*a^2*e^(2*I*d*x + 2*I*c)
```

```
 + I*a^2)*log(e^(2*I*d*x + 2*I*c) - 1))/(d*e^(10*I*d*x + 10*I*c) - 5*d*e^(8
*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) - 10*d*e^(4*I*d*x + 4*I*c) + 5*d
*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(100) = 200$.
time = 0.33, size = 218, normalized size = 1.95

$$\frac{2ia^2\log\left(e^{2idx} - e^{-2ic}\right)}{d} + \frac{-270ia^2e^{8ic}e^{8idx} + 600ia^2e^{6ic}e^{6idx} - 740ia^2e^{4ic}e^{4idx} + 400ia^2e^{2ic}e^{2idx} - 86ia^2}{15de^{10ic}e^{10idx} - 75de^{8ic}e^{8idx} + 150de^{6ic}e^{6idx} - 150de^{4ic}e^{4idx} + 75de^{2ic}e^{2idx} - 15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] 2*I*a**2*log(exp(2*I*d*x) - exp(-2*I*c))/d + (-270*I*a**2*exp(8*I*c)*exp(8*
I*d*x) + 600*I*a**2*exp(6*I*c)*exp(6*I*d*x) - 740*I*a**2*exp(4*I*c)*exp(4*I
*d*x) + 400*I*a**2*exp(2*I*c)*exp(2*I*d*x) - 86*I*a**2)/(15*d*exp(10*I*c)*e
xp(10*I*d*x) - 75*d*exp(8*I*c)*exp(8*I*d*x) + 150*d*exp(6*I*c)*exp(6*I*d*x)
 - 150*d*exp(4*I*c)*exp(4*I*d*x) + 75*d*exp(2*I*c)*exp(2*I*d*x) - 15*d)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(100) = 200$.
time = 1.13, size = 212, normalized size = 1.89

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^5 - 15i\,a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^4 - 55\,a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 + 180i\,a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - 1920i\,a^2\log\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + i\right) + 960i\,a^2\log\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right) + 630\,a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + \frac{-2192i\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 630\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 + 180i\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 55\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 15i\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 3\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 15*I*a^2*tan(1/2*d*x + 1/2*c)^4 - 55*
a^2*tan(1/2*d*x + 1/2*c)^3 + 180*I*a^2*tan(1/2*d*x + 1/2*c)^2 - 1920*I*a^2*
log(tan(1/2*d*x + 1/2*c) + I) + 960*I*a^2*log(tan(1/2*d*x + 1/2*c)) + 630*a
^2*tan(1/2*d*x + 1/2*c) + (-2192*I*a^2*tan(1/2*d*x + 1/2*c)^5 - 630*a^2*tan
(1/2*d*x + 1/2*c)^4 + 180*I*a^2*tan(1/2*d*x + 1/2*c)^3 + 55*a^2*tan(1/2*d*x
 + 1/2*c)^2 - 15*I*a^2*tan(1/2*d*x + 1/2*c) - 3*a^2)/tan(1/2*d*x + 1/2*c)^5
)/d
```

Mupad [B]
time = 4.16, size = 92, normalized size = 0.82

$$-\frac{4\,a^2\,\mathrm{atan}\left(2\,\tan(c+d\,x)+1{}\mathrm{i}\right)}{d}-\frac{2\,a^2\,\tan(c+d\,x)^4-a^2\,\tan(c+d\,x)^3\,1{}\mathrm{i}-\frac{2\,a^2\,\tan(c+d\,x)^2}{3}+\frac{a^2\,\tan(c+d\,x)\,1{}\mathrm{i}}{2}+\frac{a^2}{5}}{d\,\tan\left(c+d\,x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] - (4*a^2*atan(2*tan(c + d*x) + 1i))/d - ((a^2*tan(c + d*x)*1i)/2 + a^2/5 -
(2*a^2*tan(c + d*x)^2)/3 - a^2*tan(c + d*x)^3*1i + 2*a^2*tan(c + d*x)^4)/(d
*tan(c + d*x)^5)
```

3.24 $\int \tan^3(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=126

$$4ia^3x+\frac{4a^3\log(\cos(c+dx))}{d}-\frac{4ia^3\tan(c+dx)}{d}+\frac{2a^3\tan^2(c+dx)}{d}+\frac{4ia^3\tan^3(c+dx)}{3d}-\frac{11a^3\tan^4(c+dx)}{20d}-$$

```
[Out] 4*I*a^3*x+4*a^3*ln(cos(d*x+c))/d-4*I*a^3*tan(d*x+c)/d+2*a^3*tan(d*x+c)^2/d+
4/3*I*a^3*tan(d*x+c)^3/d-11/20*a^3*tan(d*x+c)^4/d-1/5*tan(d*x+c)^4*(a^3+I*a
^3*tan(d*x+c))/d
```

Rubi [A]
time = 0.12, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3637, 3673, 3609, 3606, 3556}

$$-\frac{\tan^4(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{5d}-\frac{11a^3\tan^4(c+dx)}{20d}+\frac{4ia^3\tan^3(c+dx)}{3d}+\frac{2a^3\tan^2(c+dx)}{d}-\frac{4ia^3\tan(c+dx)}{d}+\frac{4a^3\log(\cos(c+dx))}{d}+4ia^3x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (4*I)*a^3*x + (4*a^3*Log[Cos[c + d*x]])/d - ((4*I)*a^3*Tan[c + d*x])/d + (2
*a^3*Tan[c + d*x]^2)/d + (((4*I)/3)*a^3*Tan[c + d*x]^3)/d - (11*a^3*Tan[c +
 d*x]^4)/(20*d) - (Tan[c + d*x]^4*(a^3 + I*a^3*Tan[c + d*x]))/(5*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \tan^3(c+dx)(a+ia\tan(c+dx))^3\,dx &= -\frac{\tan^4(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{5d}+\frac{1}{5}a\int \tan^3(c+dx)(a+ia \\
&= -\frac{11a^3\tan^4(c+dx)}{20d}-\frac{\tan^4(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{5d}+\frac{1}{5}a\int \\
&= \frac{4ia^3\tan^3(c+dx)}{3d}-\frac{11a^3\tan^4(c+dx)}{20d}-\frac{\tan^4(c+dx)\left(a^3+ia^3\tan\right.}{5d} \\
&= \frac{2a^3\tan^2(c+dx)}{d}+\frac{4ia^3\tan^3(c+dx)}{3d}-\frac{11a^3\tan^4(c+dx)}{20d}-\tan^4 \\
&= 4ia^3x-\frac{4ia^3\tan(c+dx)}{d}+\frac{2a^3\tan^2(c+dx)}{d}+\frac{4ia^3\tan^3(c+dx)}{3d} \\
&= 4ia^3x+\frac{4a^3\log(\cos(c+dx))}{d}-\frac{4ia^3\tan(c+dx)}{d}+\frac{2a^3\tan^2(c+d}{d}
\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 296 vs. $2(126) = 252$.
time = 1.33, size = 296, normalized size = 2.35

$$\frac{a^3\sec(c)\sec^5(c+dx)\left(105\cos(2c+3dx)+150idx\cos(2c+3dx)+105\cos(4c+3dx)+150idx\cos(4c+3dx)+30idx\cos(4c+5dx)+30idx\cos(6c+5dx)+75\cos(2c+3dx)\log\left(\cos^2(c+dx)\right)+75\cos(4c+3dx)\log\left(\cos^2(c+dx)\right)+15\cos(4c+5dx)\log\left(\cos^2(c+dx)\right)+15\cos(6c+5dx)\log\left(\cos^2(c+dx)\right)+75\cos(dx)\left(3+4idx+2\log\left(\cos^2(c+dx)\right)\right)+75\cos(2c+dx)\left(3+4idx+2\log\left(\cos^2(c+dx)\right)\right)-470i\sin(dx)+360i\sin(2c+dx)-280i\sin(2c+3dx)+135i\sin(4c+3dx)-83i\sin(4c+5dx)\right)}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3*(a + I*a*Tan[c + d*x])^3,x]
```

[Out] (a^3*Sec[c]*Sec[c + d*x]^5*(105*Cos[2*c + 3*d*x] + (150*I)*d*x*Cos[2*c + 3*d*x] + 105*Cos[4*c + 3*d*x] + (150*I)*d*x*Cos[4*c + 3*d*x] + (30*I)*d*x*Cos[4*c + 5*d*x] + (30*I)*d*x*Cos[6*c + 5*d*x] + 75*Cos[2*c + 3*d*x]*Log[Cos[c + d*x]^2] + 75*Cos[4*c + 3*d*x]*Log[Cos[c + d*x]^2] + 15*Cos[4*c + 5*d*x]*Log[Cos[c + d*x]^2] + 15*Cos[6*c + 5*d*x]*Log[Cos[c + d*x]^2] + 75*Cos[d*x]*(3 + (4*I)*d*x + 2*Log[Cos[c + d*x]^2]) + 75*Cos[2*c + d*x]*(3 + (4*I)*d*x + 2*Log[Cos[c + d*x]^2]) - (470*I)*Sin[d*x] + (360*I)*Sin[2*c + d*x] - (280*I)*Sin[2*c + 3*d*x] + (135*I)*Sin[4*c + 3*d*x] - (83*I)*Sin[4*c + 5*d*x]))/(240*d)

Maple [A]
time = 0.05, size = 83, normalized size = 0.66

method	result
derivativedivides	$\frac{a^3 \left(-4i\tan(dx+c)-\frac{i\left(\tan^5(dx+c)\right)}{5}-\frac{3\left(\tan^4(dx+c)\right)}{4}+\frac{4i\left(\tan^3(dx+c)\right)}{3}+2\left(\tan^2(dx+c)\right)-2\ln\left(1+\tan^2(dx+c)\right)+4i\arctan(\right.}{d}$
default	$\frac{a^3 \left(-4i\tan(dx+c)-\frac{i\left(\tan^5(dx+c)\right)}{5}-\frac{3\left(\tan^4(dx+c)\right)}{4}+\frac{4i\left(\tan^3(dx+c)\right)}{3}+2\left(\tan^2(dx+c)\right)-2\ln\left(1+\tan^2(dx+c)\right)+4i\arctan(\right.}{d}$
risch	$-\frac{8ia^3c}{d}+\frac{2a^3\left(240\,\mathrm{e}^{8i(dx+c)}+585\,\mathrm{e}^{6i(dx+c)}+695\,\mathrm{e}^{4i(dx+c)}+385\,\mathrm{e}^{2i(dx+c)}+83\right)}{15d\left(\mathrm{e}^{2i(dx+c)}+1\right)^5}+\frac{4a^3\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{d}$
norman	$\frac{2a^3\left(\tan^2(dx+c)\right)}{d}-\frac{3a^3\left(\tan^4(dx+c)\right)}{4d}+4ia^3x-\frac{4ia^3\tan(dx+c)}{d}+\frac{4ia^3\left(\tan^3(dx+c)\right)}{3d}-\frac{ia^3\left(\tan^5(dx+c)\right)}{5d}-$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*a^3*(-4*I*tan(d*x+c)-1/5*I*tan(d*x+c)^5-3/4*tan(d*x+c)^4+4/3*I*tan(d*x+c)^3+2*tan(d*x+c)^2-2*ln(1+tan(d*x+c)^2)+4*I*arctan(tan(d*x+c)))

Maxima [A]
time = 0.87, size = 95, normalized size = 0.75

$$-\frac{12i\,a^3\tan\left(dx+c\right)^5+45\,a^3\tan\left(dx+c\right)^4-80i\,a^3\tan\left(dx+c\right)^3-120\,a^3\tan\left(dx+c\right)^2-240i\,(dx+c)a^3+120\,a^3\log\left(\tan\left(dx+c\right)^2+1\right)+240i\,a^3\tan\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")

[Out] -1/60*(12*I*a^3*tan(d*x + c)^5 + 45*a^3*tan(d*x + c)^4 - 80*I*a^3*tan(d*x + c)^3 - 120*a^3*tan(d*x + c)^2 - 240*I*(d*x + c)*a^3 + 120*a^3*log(tan(d*x + c)^2 + 1) + 240*I*a^3*tan(d*x + c))/d

Fricas [A]
time = 0.45, size = 214, normalized size = 1.70

$$\frac{2\left(240\,a^3e^{(8i\,dx+8i\,c)}+585\,a^3e^{(6i\,dx+6i\,c)}+695\,a^3e^{(4i\,dx+4i\,c)}+385\,a^3e^{(2i\,dx+2i\,c)}+83\,a^3+30\left(a^3e^{(10i\,dx+10i\,c)}+5\,a^3e^{(8i\,dx+8i\,c)}+10\,a^3e^{(6i\,dx+6i\,c)}+10\,a^3e^{(4i\,dx+4i\,c)}+5\,a^3e^{(2i\,dx+2i\,c)}+a^3\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)\right)}{15\left(de^{(10i\,dx+10i\,c)}+5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}+10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 2/15*(240*a^3*e^(8*I*d*x + 8*I*c) + 585*a^3*e^(6*I*d*x + 6*I*c) + 695*a^3*e
^(4*I*d*x + 4*I*c) + 385*a^3*e^(2*I*d*x + 2*I*c) + 83*a^3 + 30*(a^3*e^(10*I
*d*x + 10*I*c) + 5*a^3*e^(8*I*d*x + 8*I*c) + 10*a^3*e^(6*I*d*x + 6*I*c) + 1
0*a^3*e^(4*I*d*x + 4*I*c) + 5*a^3*e^(2*I*d*x + 2*I*c) + a^3)*log(e^(2*I*d*x
 + 2*I*c) + 1))/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e
^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) + d
)
```

Sympy [A]
time = 0.37, size = 207, normalized size = 1.64

$$\frac{4a^3\log\left(e^{2idx}+e^{-2ic}\right)}{d}+\frac{480a^3e^{8ic}e^{8idx}+1170a^3e^{6ic}e^{6idx}+1390a^3e^{4ic}e^{4idx}+770a^3e^{2ic}e^{2idx}+166a^3}{15de^{10ic}e^{10idx}+75de^{8ic}e^{8idx}+150de^{6ic}e^{6idx}+150de^{4ic}e^{4idx}+75de^{2ic}e^{2idx}+15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] 4*a**3*log(exp(2*I*d*x) + exp(-2*I*c))/d + (480*a**3*exp(8*I*c)*exp(8*I*d*x
) + 1170*a**3*exp(6*I*c)*exp(6*I*d*x) + 1390*a**3*exp(4*I*c)*exp(4*I*d*x) +
 770*a**3*exp(2*I*c)*exp(2*I*d*x) + 166*a**3)/(15*d*exp(10*I*c)*exp(10*I*d*
x) + 75*d*exp(8*I*c)*exp(8*I*d*x) + 150*d*exp(6*I*c)*exp(6*I*d*x) + 150*d*e
xp(4*I*c)*exp(4*I*d*x) + 75*d*exp(2*I*c)*exp(2*I*d*x) + 15*d)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(112) = 224.
time = 0.91, size = 274, normalized size = 2.17

$$\frac{2\left(30\,a^3e^{(10i\,dx+10i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+150\,a^3e^{(8i\,dx+8i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+300\,a^3e^{(6i\,dx+6i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+300\,a^3e^{(4i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+150\,a^3e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+240\,a^3e^{(8i\,dx+8i\,c)}+585\,a^3e^{(6i\,dx+6i\,c)}+695\,a^3e^{(4i\,dx+4i\,c)}+385\,a^3e^{(2i\,dx+2i\,c)}+30\,a^3\log\left(e^{(2i\,dx+2i\,c)}+1\right)+83\,a^3\right)}{15\left(de^{(10i\,dx+10i\,c)}+5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}+10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2/15*(30*a^3*e^(10*I*d*x + 10*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 150*a^3*e
^(8*I*d*x + 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 300*a^3*e^(6*I*d*x + 6*I*
c)*log(e^(2*I*d*x + 2*I*c) + 1) + 300*a^3*e^(4*I*d*x + 4*I*c)*log(e^(2*I*d*
x + 2*I*c) + 1) + 150*a^3*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1)
+ 240*a^3*e^(8*I*d*x + 8*I*c) + 585*a^3*e^(6*I*d*x + 6*I*c) + 695*a^3*e^(4*
I*d*x + 4*I*c) + 385*a^3*e^(2*I*d*x + 2*I*c) + 30*a^3*log(e^(2*I*d*x + 2*I*
c) + 1) + 83*a^3)/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x + 8*I*c) + 10*d
*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) +
 d)
```

Mupad [B]
time = 3.79, size = 87, normalized size = 0.69

$$-\frac{4\,a^3\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)+a^3\,\tan(c+d\,x)\,4\mathrm{i}-2\,a^3\,\tan(c+d\,x)^2-\frac{a^3\,\tan(c+d\,x)^3\,4\mathrm{i}}{3}+\frac{3\,a^3\,\tan(c+d\,x)^4}{4}+\frac{a^3\,\tan(c+d\,x)^5\,1\mathrm{i}}{5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] -(4*a^3*log(tan(c + d*x) + 1i) + a^3*tan(c + d*x)*4i - 2*a^3*tan(c + d*x)^2
 - (a^3*tan(c + d*x)^3*4i)/3 + (3*a^3*tan(c + d*x)^4)/4 + (a^3*tan(c + d*x)
^5*1i)/5)/d
```

3.25 $\int \tan^2(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=90

$$-4a^3x+\frac{4ia^3\log(\cos(c+dx))}{d}+\frac{2a^3\tan(c+dx)}{d}-\frac{ia(a+ia\tan(c+dx))^2}{2d}-\frac{i(a+ia\tan(c+dx))^4}{4ad}$$

```
[Out] -4*a^3*x+4*I*a^3*ln(cos(d*x+c))/d+2*a^3*tan(d*x+c)/d-1/2*I*a*(a+I*a*tan(d*x
+c))^2/d-1/4*I*(a+I*a*tan(d*x+c))^4/a/d
```

Rubi [A]
time = 0.05, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3624, 3559, 3558, 3556}

$$\frac{2a^3\tan(c+dx)}{d}+\frac{4ia^3\log(\cos(c+dx))}{d}-4a^3x-\frac{i(a+ia\tan(c+dx))^4}{4ad}-\frac{ia(a+ia\tan(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] -4*a^3*x + ((4*I)*a^3*Log[Cos[c + d*x]])/d + (2*a^3*Tan[c + d*x])/d - ((I/2
)*a*(a + I*a*Tan[c + d*x])^2)/d - ((I/4)*(a + I*a*Tan[c + d*x])^4)/(a*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && !LeQ
[m, -1] && !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \tan^2(c+dx)(a+ia\tan(c+dx))^3\,dx &= -\frac{i(a+ia\tan(c+dx))^4}{4ad}-\int (a+ia\tan(c+dx))^3\,dx\\ &= -\frac{ia(a+ia\tan(c+dx))^2}{2d}-\frac{i(a+ia\tan(c+dx))^4}{4ad}-(2a)\int (a+\\ &= -4a^3x+\frac{2a^3\tan(c+dx)}{d}-\frac{ia(a+ia\tan(c+dx))^2}{2d}-\frac{i(a+ia\,t\\ &= -4a^3x+\frac{4ia^3\log(\cos(c+dx))}{d}+\frac{2a^3\tan(c+dx)}{d}-\frac{ia(a+ia\,ta}{2}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 228 vs. 2(90) = 180.
time = 1.28, size = 228, normalized size = 2.53

$$-\frac{a^3\sec(c)\sec^4(c+dx)\left(-5i\cos(3c+2dx)+8dx\cos(3c+2dx)+2dx\cos(3c+4dx)+2dx\cos(5c+4dx)+2\cos(c)\left(-4i+6dx-3i\log\left(\cos^2(c+dx)\right)\right)+\cos(c+2dx)\left(-5i+8dx-4i\log\left(\cos^2(c+dx)\right)\right)-4i\cos(3c+2dx)\log\left(\cos^2(c+dx)\right)-i\cos(3c+4dx)\log\left(\cos^2(c+dx)\right)-i\cos(5c+4dx)\log\left(\cos^2(c+dx)\right)+15\sin(c)-13\sin(c+2dx)+7\sin(3c+2dx)-5\sin(3c+4dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] -1/8*(a^3*Sec[c]*Sec[c + d*x]^4*((-5*I)*Cos[3*c + 2*d*x] + 8*d*x*Cos[3*c +
2*d*x] + 2*d*x*Cos[3*c + 4*d*x] + 2*d*x*Cos[5*c + 4*d*x] + 2*Cos[c]*(-4*I +
 6*d*x - (3*I)*Log[Cos[c + d*x]^2]) + Cos[c + 2*d*x]*(-5*I + 8*d*x - (4*I)*
Log[Cos[c + d*x]^2]) - (4*I)*Cos[3*c + 2*d*x]*Log[Cos[c + d*x]^2] - I*Cos[3
*c + 4*d*x]*Log[Cos[c + d*x]^2] - I*Cos[5*c + 4*d*x]*Log[Cos[c + d*x]^2] +
15*Sin[c] - 13*Sin[c + 2*d*x] + 7*Sin[3*c + 2*d*x] - 5*Sin[3*c + 4*d*x]))/d
```

Maple [A]
time = 0.05, size = 72, normalized size = 0.80

method	result
derivativedivides	$\frac{a^3\left(4\tan(dx+c)-\frac{i\left(\tan^4(dx+c)\right)}{4}-\left(\tan^3(dx+c)\right)+2i\left(\tan^2(dx+c)\right)-2i\ln\left(1+\tan^2(dx+c)\right)-4\arctan(\tan(dx+c))\right)}{d}$
default	$\frac{a^3\left(4\tan(dx+c)-\frac{i\left(\tan^4(dx+c)\right)}{4}-\left(\tan^3(dx+c)\right)+2i\left(\tan^2(dx+c)\right)-2i\ln\left(1+\tan^2(dx+c)\right)-4\arctan(\tan(dx+c))\right)}{d}$
risch	$\frac{8a^3c}{d}+\frac{2ia^3\left(12\,e^{6i(dx+c)}+23\,e^{4i(dx+c)}+18\,e^{2i(dx+c)}+5\right)}{d\left(e^{2i(dx+c)}+1\right)^4}+\frac{4ia^3\ln\left(e^{2i(dx+c)}+1\right)}{d}$

norman	$-4a^3x+\frac{4a^3\tan(dx+c)}{d}-\frac{a^3(\tan^3(dx+c))}{d}+\frac{2ia^3(\tan^2(dx+c))}{d}-\frac{ia^3(\tan^4(dx+c))}{4d}-\frac{2ia^3\ln(1+\tan^2(dx+c)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a^3*(4*tan(d*x+c)-1/4*I*tan(d*x+c)^4-tan(d*x+c)^3+2*I*tan(d*x+c)^2-2*I*
ln(1+tan(d*x+c)^2)-4*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.49, size = 82, normalized size = 0.91

$$-\frac{i\,a^3\tan\left(dx+c\right)^4+4\,a^3\tan\left(dx+c\right)^3-8i\,a^3\tan\left(dx+c\right)^2+16\,(dx+c)a^3+8i\,a^3\log\left(\tan\left(dx+c\right)^2+1\right)-16\,a^3\tan\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(I*a^3*tan(d*x + c)^4 + 4*a^3*tan(d*x + c)^3 - 8*I*a^3*tan(d*x + c)^2
+ 16*(d*x + c)*a^3 + 8*I*a^3*log(tan(d*x + c)^2 + 1) - 16*a^3*tan(d*x + c))
/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(76) = 152$.
time = 0.45, size = 177, normalized size = 1.97

$$-\frac{2\left(-12i\,a^3e^{(6i\,dx+6i\,c)}-23i\,a^3e^{(4i\,dx+4i\,c)}-18i\,a^3e^{(2i\,dx+2i\,c)}-5i\,a^3+2\left(-i\,a^3e^{(8i\,dx+8i\,c)}-4i\,a^3e^{(6i\,dx+6i\,c)}-6i\,a^3e^{(4i\,dx+4i\,c)}-4i\,a^3e^{(2i\,dx+2i\,c)}-i\,a^3\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)\right)}{de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -2*(-12*I*a^3*e^(6*I*d*x + 6*I*c) - 23*I*a^3*e^(4*I*d*x + 4*I*c) - 18*I*a^3
*e^(2*I*d*x + 2*I*c) - 5*I*a^3 + 2*(-I*a^3*e^(8*I*d*x + 8*I*c) - 4*I*a^3*e^
(6*I*d*x + 6*I*c) - 6*I*a^3*e^(4*I*d*x + 4*I*c) - 4*I*a^3*e^(2*I*d*x + 2*I*
c) - I*a^3)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6
*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(76) = 152$.
time = 0.29, size = 173, normalized size = 1.92

$$\frac{4ia^3\log\left(e^{2idx}+e^{-2ic}\right)}{d}+\frac{24ia^3e^{6ic}e^{6idx}+46ia^3e^{4ic}e^{4idx}+36ia^3e^{2ic}e^{2idx}+10ia^3}{de^{8ic}e^{8idx}+4de^{6ic}e^{6idx}+6de^{4ic}e^{4idx}+4de^{2ic}e^{2idx}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] 4*I*a**3*log(exp(2*I*d*x) + exp(-2*I*c))/d + (24*I*a**3*exp(6*I*c)*exp(6*I*
d*x) + 46*I*a**3*exp(4*I*c)*exp(4*I*d*x) + 36*I*a**3*exp(2*I*c)*exp(2*I*d*x
) + 10*I*a**3)/(d*exp(8*I*c)*exp(8*I*d*x) + 4*d*exp(6*I*c)*exp(6*I*d*x) + 6
*d*exp(4*I*c)*exp(4*I*d*x) + 4*d*exp(2*I*c)*exp(2*I*d*x) + d)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(76) = 152$.
time = 0.70, size = 222, normalized size = 2.47

$$-\frac{2\left(-2i\,a^{3}e^{(8i\,dx+8i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-8i\,a^{3}e^{(6i\,dx+6i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-12i\,a^{3}e^{(4i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-8i\,a^{3}e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-12i\,a^{3}e^{(6i\,dx+6i\,c)}-23i\,a^{3}e^{(4i\,dx+4i\,c)}-18i\,a^{3}e^{(2i\,dx+2i\,c)}-2i\,a^{3}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-5i\,a^{3}\right)}{de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2*(-2*I*a^3*e^(8*I*d*x + 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 8*I*a^3*e^(
6*I*d*x + 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 12*I*a^3*e^(4*I*d*x + 4*I*c
)*log(e^(2*I*d*x + 2*I*c) + 1) - 8*I*a^3*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x
 + 2*I*c) + 1) - 12*I*a^3*e^(6*I*d*x + 6*I*c) - 23*I*a^3*e^(4*I*d*x + 4*I*c
) - 18*I*a^3*e^(2*I*d*x + 2*I*c) - 2*I*a^3*log(e^(2*I*d*x + 2*I*c) + 1) - 5
*I*a^3)/(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x +
 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Mupad [B]
time = 3.80, size = 73, normalized size = 0.81

$$-\frac{a^{3}\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,4\mathrm{i}-4\,a^{3}\,\tan(c+d\,x)-a^{3}\,{\tan(c+d\,x)}^{2}\,2\mathrm{i}+a^{3}\,{\tan(c+d\,x)}^{3}+\frac{a^{3}\,{\tan(c+d\,x)}^{4}\,1\mathrm{i}}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] -(a^3*log(tan(c + d*x) + 1i)*4i - 4*a^3*tan(c + d*x) - a^3*tan(c + d*x)^2*2
i + a^3*tan(c + d*x)^3 + (a^3*tan(c + d*x)^4*1i)/4)/d
```

3.26 $\int \tan(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=85

$$-4ia^3x-\frac{4a^3\log(\cos(c+dx))}{d}+\frac{2ia^3\tan(c+dx)}{d}+\frac{a(a+ia\tan(c+dx))^2}{2d}+\frac{(a+ia\tan(c+dx))^3}{3d}$$

```
[Out] -4*I*a^3*x-4*a^3*ln(cos(d*x+c))/d+2*I*a^3*tan(d*x+c)/d+1/2*a*(a+I*a*tan(d*x
+c))^2/d+1/3*(a+I*a*tan(d*x+c))^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3608, 3559, 3558, 3556}

$$\frac{2ia^3\tan(c+dx)}{d}-\frac{4a^3\log(\cos(c+dx))}{d}-4ia^3x+\frac{a(a+ia\tan(c+dx))^2}{2d}+\frac{(a+ia\tan(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (-4*I)*a^3*x - (4*a^3*Log[Cos[c + d*x]])/d + ((2*I)*a^3*Tan[c + d*x])/d + (
a*(a + I*a*Tan[c + d*x])^2)/(2*d) + (a + I*a*Tan[c + d*x])^3/(3*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && !LtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \tan(c+dx)(a+ia\tan(c+dx))^3\,dx &= \frac{(a+ia\tan(c+dx))^3}{3d} - i\int (a+ia\tan(c+dx))^3\,dx\\ &= \frac{a(a+ia\tan(c+dx))^2}{2d} + \frac{(a+ia\tan(c+dx))^3}{3d} - (2ia)\int (a+ia\\ &= -4ia^3x + \frac{2ia^3\tan(c+dx)}{d} + \frac{a(a+ia\tan(c+dx))^2}{2d} + \frac{(a+ia\,\mathrm{ta}}{3}\\ &= -4ia^3x - \frac{4a^3\log(\cos(c+dx))}{d} + \frac{2ia^3\tan(c+dx)}{d} + \frac{a(a+ia\tan}{2d}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 178 vs. $2(85) = 170$.
time = 0.89, size = 178, normalized size = 2.09

$$-\frac{ia^3\sec(c)\sec^3(c+dx)\left(6dx\cos(2c+3dx)+6dx\cos(4c+3dx)+9\cos(dx)\left(-i+2dx-i\log\left(\cos^2(c+dx)\right)\right)+9\cos(2c+dx)\left(-i+2dx-i\log\left(\cos^2(c+dx)\right)\right)-3i\cos(2c+3dx)\log\left(\cos^2(c+dx)\right)-3i\cos(4c+3dx)\log\left(\cos^2(c+dx)\right)-24\sin(dx)+15\sin(2c+dx)-13\sin(2c+3dx)\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((-1/12*I)*a^3*Sec[c]*Sec[c + d*x]^3*(6*d*x*Cos[2*c + 3*d*x] + 6*d*x*Cos[4*
c + 3*d*x] + 9*Cos[d*x]*(-I + 2*d*x - I*Log[Cos[c + d*x]^2]) + 9*Cos[2*c +
d*x]*(-I + 2*d*x - I*Log[Cos[c + d*x]^2]) - (3*I)*Cos[2*c + 3*d*x]*Log[Cos[
c + d*x]^2] - (3*I)*Cos[4*c + 3*d*x]*Log[Cos[c + d*x]^2] - 24*Sin[d*x] + 15
*Sin[2*c + d*x] - 13*Sin[2*c + 3*d*x]))/d
```

Maple [A]
time = 0.05, size = 62, normalized size = 0.73

method	result	size
derivativedivides	$\frac{a^3\left(4i\tan(dx+c)-\frac{i\left(\tan^3(dx+c)\right)}{3}-\frac{3\left(\tan^2(dx+c)\right)}{2}+2\ln\left(1+\tan^2(dx+c)\right)-4i\arctan(\tan(dx+c))\right)}{d}$	62
default	$\frac{a^3\left(4i\tan(dx+c)-\frac{i\left(\tan^3(dx+c)\right)}{3}-\frac{3\left(\tan^2(dx+c)\right)}{2}+2\ln\left(1+\tan^2(dx+c)\right)-4i\arctan(\tan(dx+c))\right)}{d}$	62
norman	$-\frac{3a^3\left(\tan^2(dx+c)\right)}{2d}-4ia^3x+\frac{4ia^3\tan(dx+c)}{d}-\frac{ia^3\left(\tan^3(dx+c)\right)}{3d}+\frac{2a^3\ln\left(1+\tan^2(dx+c)\right)}{d}$	76
risch	$\frac{8ia^3c}{d}-\frac{2a^3\left(24\,e^{4i(dx+c)}+33\,e^{2i(dx+c)}+13\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}-\frac{4a^3\ln\left(e^{2i(dx+c)}+1\right)}{d}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a^3*(4*I*tan(d*x+c)-1/3*I*tan(d*x+c)^3-3/2*tan(d*x+c)^2+2*ln(1+tan(d*x+
c)^2)-4*I*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.49, size = 69, normalized size = 0.81

$$-\frac{2i\,a^3\tan\left(dx+c\right)^3+9\,a^3\tan\left(dx+c\right)^2+24i\left(dx+c\right)a^3-12\,a^3\log\left(\tan\left(dx+c\right)^2+1\right)-24i\,a^3\tan\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/6*(2*I*a^3*tan(d*x + c)^3 + 9*a^3*tan(d*x + c)^2 + 24*I*(d*x + c)*a^3 -
12*a^3*log(tan(d*x + c)^2 + 1) - 24*I*a^3*tan(d*x + c))/d
```

Fricas [A]
time = 0.48, size = 134, normalized size = 1.58

$$-\frac{2\left(24\,a^3e^{(4i\,dx+4i\,c)}+33\,a^3e^{(2i\,dx+2i\,c)}+13\,a^3+6\left(a^3e^{(6i\,dx+6i\,c)}+3\,a^3e^{(4i\,dx+4i\,c)}+3\,a^3e^{(2i\,dx+2i\,c)}+a^3\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)\right)}{3\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -2/3*(24*a^3*e^(4*I*d*x + 4*I*c) + 33*a^3*e^(2*I*d*x + 2*I*c) + 13*a^3 + 6*
(a^3*e^(6*I*d*x + 6*I*c) + 3*a^3*e^(4*I*d*x + 4*I*c) + 3*a^3*e^(2*I*d*x + 2
*I*c) + a^3)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(
4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A]
time = 0.22, size = 131, normalized size = 1.54

$$-\frac{4a^3\log\left(e^{2idx}+e^{-2ic}\right)}{d}+\frac{-48a^3e^{4ic}e^{4idx}-66a^3e^{2ic}e^{2idx}-26a^3}{3de^{6ic}e^{6idx}+9de^{4ic}e^{4idx}+9de^{2ic}e^{2idx}+3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] -4*a**3*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-48*a**3*exp(4*I*c)*exp(4*I*d*
x) - 66*a**3*exp(2*I*c)*exp(2*I*d*x) - 26*a**3)/(3*d*exp(6*I*c)*exp(6*I*d*x
) + 9*d*exp(4*I*c)*exp(4*I*d*x) + 9*d*exp(2*I*c)*exp(2*I*d*x) + 3*d)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(73) = 146$.
time = 0.59, size = 170, normalized size = 2.00

$$-\frac{2\left(6\,a^{3}e^{(6i\,dx+6i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+18\,a^{3}e^{(4i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+18\,a^{3}e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+24\,a^{3}e^{(4i\,dx+4i\,c)}+33\,a^{3}e^{(2i\,dx+2i\,c)}+6\,a^{3}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+13\,a^{3}\right)}{3\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2/3*(6*a^3*e^(6*I*d*x + 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 18*a^3*e^(4*
I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 18*a^3*e^(2*I*d*x + 2*I*c)*lo
g(e^(2*I*d*x + 2*I*c) + 1) + 24*a^3*e^(4*I*d*x + 4*I*c) + 33*a^3*e^(2*I*d*x
 + 2*I*c) + 6*a^3*log(e^(2*I*d*x + 2*I*c) + 1) + 13*a^3)/(d*e^(6*I*d*x + 6*
I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Mupad [B]
time = 3.73, size = 59, normalized size = 0.69

$$\frac{4\,a^{3}\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)+a^{3}\tan(c+d\,x)\,4\mathrm{i}-\frac{3\,a^{3}\tan(c+d\,x)^{2}}{2}-\frac{a^{3}\tan(c+d\,x)^{3}\,1\mathrm{i}}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] (4*a^3*log(tan(c + d*x) + 1i) + a^3*tan(c + d*x)*4i - (3*a^3*tan(c + d*x)^2
)/2 - (a^3*tan(c + d*x)^3*1i)/3)/d
```

3.27 $\int (a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=63

$$4a^3x-\frac{4ia^3\log(\cos(c+dx))}{d}-\frac{2a^3\tan(c+dx)}{d}+\frac{ia(a+ia\tan(c+dx))^2}{2d}$$

```
[Out] 4*a^3*x-4*I*a^3*ln(cos(d*x+c))/d-2*a^3*tan(d*x+c)/d+1/2*I*a*(a+I*a*tan(d*x+
c))^2/d
```

Rubi [A]
time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {3559, 3558, 3556}

$$-\frac{2a^3\tan(c+dx)}{d}-\frac{4ia^3\log(\cos(c+dx))}{d}+4a^3x+\frac{ia(a+ia\tan(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] 4*a^3*x - ((4*I)*a^3*Log[Cos[c + d*x]])/d - (2*a^3*Tan[c + d*x])/d + ((I/2)
*a*(a + I*a*Tan[c + d*x])^2)/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rubi steps

$$
\begin{aligned}
\int (a+ia\tan(c+dx))^3\,dx &= \frac{ia(a+ia\tan(c+dx))^2}{2d} + (2a)\int (a+ia\tan(c+dx))^2\,dx \\
&= 4a^3x - \frac{2a^3\tan(c+dx)}{d} + \frac{ia(a+ia\tan(c+dx))^2}{2d} + \left(4ia^3\right)\int \tan(c+dx)\,dx \\
&= 4a^3x - \frac{4ia^3\log(\cos(c+dx))}{d} - \frac{2a^3\tan(c+dx)}{d} + \frac{ia(a+ia\tan(c+dx))^2}{2d}
\end{aligned}
$$

Mathematica [A]
time = 0.77, size = 119, normalized size = 1.89

$$
\frac{a^3\sec(c)\sec^2(c+dx)\left(2dx\cos(3c+2dx)+\cos(c+2dx)\left(2dx-i\log\left(\cos^2(c+dx)\right)\right)+\cos(c)\left(-i+4dx-2i\log\left(\cos^2(c+dx)\right)\right)-i\cos(3c+2dx)\log\left(\cos^2(c+dx)\right)+3\sin(c)-3\sin(c+2dx)\right)}{2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Sec[c]*Sec[c + d*x]^2*(2*d*x*Cos[3*c + 2*d*x] + Cos[c + 2*d*x]*(2*d*x
- I*Log[Cos[c + d*x]^2]) + Cos[c]*(-I + 4*d*x - (2*I)*Log[Cos[c + d*x]^2])
- I*Cos[3*c + 2*d*x]*Log[Cos[c + d*x]^2] + 3*Sin[c] - 3*Sin[c + 2*d*x]))/(2
*d)
```

Maple [A]
time = 0.02, size = 51, normalized size = 0.81

method	result	size
derivativedivides	$\frac{a^3\left(-3\tan(dx+c)-\frac{i\left(\tan^2(dx+c)\right)}{2}+2i\ln\left(1+\tan^2(dx+c)\right)+4\arctan(\tan(dx+c))\right)}{d}$	51
default	$\frac{a^3\left(-3\tan(dx+c)-\frac{i\left(\tan^2(dx+c)\right)}{2}+2i\ln\left(1+\tan^2(dx+c)\right)+4\arctan(\tan(dx+c))\right)}{d}$	51
norman	$4a^3x-\frac{3a^3\tan(dx+c)}{d}-\frac{ia^3\left(\tan^2(dx+c)\right)}{2d}+\frac{2ia^3\ln\left(1+\tan^2(dx+c)\right)}{d}$	59
risch	$-\frac{8a^3c}{d}-\frac{2ia^3\left(4\,e^{2i(dx+c)}+3\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{4ia^3\ln\left(e^{2i(dx+c)}+1\right)}{d}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a^3*(-3*tan(d*x+c)-1/2*I*tan(d*x+c)^2+2*I*ln(1+tan(d*x+c)^2)+4*arctan(t
an(d*x+c)))
```

Maxima [A]
time = 0.50, size = 76, normalized size = 1.21

$$
a^3x + \frac{3\left(dx+c-\tan\left(dx+c\right)\right)a^3}{d} + \frac{i\,a^3\left(\frac{1}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)^2-1\right)\right)}{2\,d} + \frac{3i\,a^3\log\left(\sec\left(dx+c\right)\right)}{d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 3*(d*x + c - tan(d*x + c))*a^3/d + 1/2*I*a^3*(1/(sin(d*x + c)^2 - 1
) - log(sin(d*x + c)^2 - 1))/d + 3*I*a^3*log(sec(d*x + c))/d
```

Fricas [A]
time = 0.42, size = 97, normalized size = 1.54

$$-\frac{2\left(4i\,a^3e^{(2i\,dx+2i\,c)}+3i\,a^3+2\left(i\,a^3e^{(4i\,dx+4i\,c)}+2i\,a^3e^{(2i\,dx+2i\,c)}+i\,a^3\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)\right)}{de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -2*(4*I*a^3*e^(2*I*d*x + 2*I*c) + 3*I*a^3 + 2*(I*a^3*e^(4*I*d*x + 4*I*c) +
2*I*a^3*e^(2*I*d*x + 2*I*c) + I*a^3)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(4*
I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A]
time = 0.17, size = 94, normalized size = 1.49

$$-\frac{4ia^3\log\left(e^{2idx}+e^{-2ic}\right)}{d}+\frac{-8ia^3e^{2ic}e^{2idx}-6ia^3}{de^{4ic}e^{4idx}+2de^{2ic}e^{2idx}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**3,x)
```

```
[Out] -4*I*a**3*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-8*I*a**3*exp(2*I*c)*exp(2*I
*d*x) - 6*I*a**3)/(d*exp(4*I*c)*exp(4*I*d*x) + 2*d*exp(2*I*c)*exp(2*I*d*x)
+ d)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(55) = 110$.
time = 0.50, size = 118, normalized size = 1.87

$$-\frac{2\left(2i\,a^3e^{(4i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+4i\,a^3e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+4i\,a^3e^{(2i\,dx+2i\,c)}+2i\,a^3\log\left(e^{(2i\,dx+2i\,c)}+1\right)+3i\,a^3\right)}{de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2*(2*I*a^3*e^(4*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 4*I*a^3*e^(2
*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 4*I*a^3*e^(2*I*d*x + 2*I*c)
+ 2*I*a^3*log(e^(2*I*d*x + 2*I*c) + 1) + 3*I*a^3)/(d*e^(4*I*d*x + 4*I*c) +
2*d*e^(2*I*d*x + 2*I*c) + d)
```

Mupad [B]
time = 3.68, size = 41, normalized size = 0.65

$$-\frac{a^3\,\left(6\tan(c+d\,x)-\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,8\mathrm{i}+\tan(c+d\,x)^2\,1\mathrm{i}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] -(a^3*(6*tan(c + d*x) - log(tan(c + d*x) + 1i)*8i + tan(c + d*x)^2*1i))/(2*
d)
```

3.28 $\int \cot(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=60

$$4ia^3x+\frac{3a^3\log(\cos(c+dx))}{d}+\frac{a^3\log(\sin(c+dx))}{d}-\frac{a^3+ia^3\tan(c+dx)}{d}$$

```
[Out] 4*I*a^3*x+3*a^3*ln(cos(d*x+c))/d+a^3*ln(sin(d*x+c))/d+(-a^3-I*a^3*tan(d*x+c
))/d
```

Rubi [A]
time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3637, 3670, 3556, 3612}

$$-\frac{a^3+ia^3\tan(c+dx)}{d}+\frac{a^3\log(\sin(c+dx))}{d}+\frac{3a^3\log(\cos(c+dx))}{d}+4ia^3x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (4*I)*a^3*x + (3*a^3*Log[Cos[c + d*x]])/d + (a^3*Log[Sin[c + d*x]])/d - (a^
3 + I*a^3*Tan[c + d*x])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3670

```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)]))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[B*(d/
b), Int[Tan[e + f*x], x], x] + Dist[1/b, Int[Simp[A*b*c + (A*b*d + B*(b*c -
 a*d))*Tan[e + f*x], x]/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d,
e, f, A, B}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \cot(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{a^3+ia^3\tan(c+dx)}{d} + a\int \cot(c+dx)(a+ia\tan(c+dx))(a+$$

$$= -\frac{a^3+ia^3\tan(c+dx)}{d} + a\int \cot(c+dx)\left(a^2+4ia^2\tan(c+dx)\right)$$

$$= 4ia^3x + \frac{3a^3\log(\cos(c+dx))}{d} - \frac{a^3+ia^3\tan(c+dx)}{d} + a^3\int \cot(c$$

$$= 4ia^3x + \frac{3a^3\log(\cos(c+dx))}{d} + \frac{a^3\log(\sin(c+dx))}{d} - \frac{a^3+ia^3\tan}{d}$$

Mathematica [A]
time = 0.89, size = 95, normalized size = 1.58

$$\frac{a^3\sec(c)\sec(c+dx)\left(\cos(dx)\left(8idx+3\log\left(\cos^2(c+dx)\right)+\log\left(\sin^2(c+dx)\right)\right)+\cos(2c+dx)\left(8idx+3\log\left(\cos^2(c+dx)\right)+\log\left(\sin^2(c+dx)\right)\right)-4i\sin(dx)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Sec[c]*Sec[c + d*x]*(Cos[d*x]*((8*I)*d*x + 3*Log[Cos[c + d*x]^2] + Log
[Sin[c + d*x]^2]) + Cos[2*c + d*x]*((8*I)*d*x + 3*Log[Cos[c + d*x]^2] + Log
[Sin[c + d*x]^2]) - (4*I)*Sin[d*x]))/(4*d)
```

Maple [A]
time = 0.20, size = 60, normalized size = 1.00

method	result	size
norman	$4ia^3x-\frac{ia^3\tan(dx+c)}{d}+\frac{a^3\ln(\tan(dx+c))}{d}-\frac{2a^3\ln(1+\tan^2(dx+c))}{d}$	57
derivativedivides	$\frac{-ia^3(\tan(dx+c)-dx-c)+3a^3\ln(\cos(dx+c))+3ia^3(dx+c)+a^3\ln(\sin(dx+c))}{d}$	60
default	$\frac{-ia^3(\tan(dx+c)-dx-c)+3a^3\ln(\cos(dx+c))+3ia^3(dx+c)+a^3\ln(\sin(dx+c))}{d}$	60
risch	$-\frac{8ia^3c}{d}+\frac{2a^3}{d\left(e^{2i(dx+c)}+1\right)}+\frac{3a^3\ln\left(e^{2i(dx+c)}+1\right)}{d}+\frac{a^3\ln\left(e^{2i(dx+c)}-1\right)}{d}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-I*a^3*(tan(d*x+c)-d*x-c)+3*a^3*ln(cos(d*x+c))+3*I*a^3*(d*x+c)+a^3*ln(
sin(d*x+c)))
```

Maxima [A]
time = 0.54, size = 53, normalized size = 0.88

$$\frac{4i\,(dx+c)a^3 - 2\,a^3\log\left(\tan\left(dx+c\right)^2+1\right) + a^3\log\left(\tan\left(dx+c\right)\right) - i\,a^3\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (4*I*(d*x + c)*a^3 - 2*a^3*log(tan(d*x + c)^2 + 1) + a^3*log(tan(d*x + c))
- I*a^3*tan(d*x + c))/d
```

Fricas [A]
time = 0.47, size = 83, normalized size = 1.38

$$\frac{2\,a^3 + 3\,\left(a^3e^{(2i\,dx+2i\,c)} + a^3\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right) + \left(a^3e^{(2i\,dx+2i\,c)} + a^3\right)\log\left(e^{(2i\,dx+2i\,c)}-1\right)}{de^{(2i\,dx+2i\,c)} + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] (2*a^3 + 3*(a^3*e^(2*I*d*x + 2*I*c) + a^3)*log(e^(2*I*d*x + 2*I*c) + 1) + (
a^3*e^(2*I*d*x + 2*I*c) + a^3)*log(e^(2*I*d*x + 2*I*c) - 1))/(d*e^(2*I*d*x
+ 2*I*c) + d)
```

Sympy [A]
time = 0.21, size = 66, normalized size = 1.10

$$\frac{2a^3}{de^{2ic}e^{2idx} + d} + \frac{a^3\left(\log\left(e^{2idx} - e^{-2ic}\right) + 3\log\left(e^{2idx} + e^{-2ic}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] 2*a**3/(d*exp(2*I*c)*exp(2*I*d*x) + d) + a**3*(log(exp(2*I*d*x) - exp(-2*I*
c)) + 3*log(exp(2*I*d*x) + exp(-2*I*c)))/d
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(57) = 114.
time = 0.74, size = 123, normalized size = 2.05

$$\frac{3\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-8\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)+3\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-\frac{3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2i\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (3*a^3*log(tan(1/2*d*x + 1/2*c) + 1) - 8*a^3*log(tan(1/2*d*x + 1/2*c) + I)
+ 3*a^3*log(tan(1/2*d*x + 1/2*c) - 1) + a^3*log(tan(1/2*d*x + 1/2*c)) - (3*
a^3*tan(1/2*d*x + 1/2*c)^2 - 2*I*a^3*tan(1/2*d*x + 1/2*c) - 3*a^3)/(tan(1/2
*d*x + 1/2*c)^2 - 1))/d
```

Mupad [B]
time = 3.77, size = 39, normalized size = 0.65

$$-\frac{a^3\,\left(4\,\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)-\ln\left(\tan(c+d\,x)\right)+\tan(c+d\,x)\,1{}\mathrm{i}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] -(a^3*(4*log(tan(c + d*x) + 1i) + tan(c + d*x)*1i - log(tan(c + d*x))))/d
```

3.29 $\int \cot^2(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=69

$$-4a^3x+\frac{ia^3\log(\cos(c+dx))}{d}+\frac{3ia^3\log(\sin(c+dx))}{d}-\frac{\cot(c+dx)\,(a^3+ia^3\tan(c+dx))}{d}$$

```
[Out] -4*a^3*x+I*a^3*ln(cos(d*x+c))/d+3*I*a^3*ln(sin(d*x+c))/d-cot(d*x+c)*(a^3+I*
a^3*tan(d*x+c))/d
```

Rubi [A]

time = 0.08, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3634, 3670, 3556, 3612}

$$\frac{3ia^3\log(\sin(c+dx))}{d}+\frac{ia^3\log(\cos(c+dx))}{d}-\frac{\cot(c+dx)\,(a^3+ia^3\tan(c+dx))}{d}-4a^3x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] -4*a^3*x + (I*a^3*Log[Cos[c + d*x]])/d + ((3*I)*a^3*Log[Sin[c + d*x]])/d -
(Cot[c + d*x]*(a^3 + I*a^3*Tan[c + d*x]))/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3634

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-a^2)*(b*c - a*d)*(a + b*Tan[e + f*x]
)^(m - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] + Di
st[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[b*(b*c*(m - 2) - a*d*(m - 2*n - 4)) + (a*b*c*(m - 2)
+ b^2*d*(n + 1) - a^2*d*(m + n - 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3670

```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)]))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[B*(d/
b), Int[Tan[e + f*x], x], x] + Dist[1/b, Int[Simp[A*b*c + (A*b*d + B*(b*c -
 a*d))*Tan[e + f*x], x]/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d,
e, f, A, B}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \cot^2(c+dx)(a+ia\tan(c+dx))^3\,dx &= -\frac{\cot(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{d}-\int \cot(c+dx)(a+ia\tan(\\ &= -\frac{\cot(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{d}-\left(ia^3\right)\int \tan(c+dx)\,dx-\\ &= -4a^3x+\frac{ia^3\log(\cos(c+dx))}{d}-\frac{\cot(c+dx)\left(a^3+ia^3\tan(c+dx\right.}{d}\\ &= -4a^3x+\frac{ia^3\log(\cos(c+dx))}{d}+\frac{3ia^3\log(\sin(c+dx))}{d}-\frac{\cot(c+}{}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 144 vs. 2(69) = 138.
time = 1.11, size = 144, normalized size = 2.09

$$\frac{a^3\csc\left(\frac{c}{2}\right)\csc(c+dx)\sec\left(\frac{c}{2}\right)\left(14dx\cos(2c+dx)-i\cos(2c+dx)\log\left(\cos^2(c+dx)\right)+\cos(dx)\left(-14dx+i\log\left(\cos^2(c+dx)\right)+3i\log\left(\sin^2(c+dx)\right)\right)-3i\cos(2c+dx)\log\left(\sin^2(c+dx)\right)+4\sin(dx)+12\text{ArcTan}(\tan(4c+dx))\sin(c)\sin(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Csc[c/2]*Csc[c + d*x]*Sec[c/2]*(14*d*x*Cos[2*c + d*x] - I*Cos[2*c + d*
x]*Log[Cos[c + d*x]^2] + Cos[d*x]*(-14*d*x + I*Log[Cos[c + d*x]^2] + (3*I)*
Log[Sin[c + d*x]^2]) - (3*I)*Cos[2*c + d*x]*Log[Sin[c + d*x]^2] + 4*Sin[d*x
] + 12*ArcTan[Tan[4*c + d*x]]*Sin[c]*Sin[c + d*x]))/(8*d)
```

Maple [A]
time = 0.20, size = 62, normalized size = 0.90

method	result	size
derivativedivides	$\frac{ia^3\ln(\cos(dx+c))-3a^3(dx+c)+3ia^3\ln(\sin(dx+c))+a^3(-\cot(dx+c)-dx-c)}{d}$	62
default	$\frac{ia^3\ln(\cos(dx+c))-3a^3(dx+c)+3ia^3\ln(\sin(dx+c))+a^3(-\cot(dx+c)-dx-c)}{d}$	62
norman	$\frac{-\frac{a^3}{d}-4a^3x\tan(dx+c)}{\tan(dx+c)}+\frac{3ia^3\ln(\tan(dx+c))}{d}-\frac{2ia^3\ln(1+\tan^2(dx+c))}{d}$	68

risch	$\frac{8a^3c}{d}-\frac{2ia^3}{d\left(\mathrm{e}^{2i(dx+c)}-1\right)}+\frac{ia^3\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{d}+\frac{3ia^3\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{d}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(I*a^3*ln(cos(d*x+c))-3*a^3*(d*x+c)+3*I*a^3*ln(sin(d*x+c))+a^3*(-cot(d*
x+c)-d*x-c))
```

Maxima [A]
time = 0.51, size = 56, normalized size = 0.81

$$-\frac{4\,(dx+c)a^3+2i\,a^3\log\left(\tan\left(dx+c\right)^2+1\right)-3i\,a^3\log\left(\tan\left(dx+c\right)\right)+\frac{a^3}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -(4*(d*x + c)*a^3 + 2*I*a^3*log(tan(d*x + c)^2 + 1) - 3*I*a^3*log(tan(d*x +
 c)) + a^3/tan(d*x + c))/d
```

Fricas [A]
time = 0.46, size = 91, normalized size = 1.32

$$\frac{-2i\,a^3+\left(i\,a^3e^{(2i\,dx+2i\,c)}-i\,a^3\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-3\left(-i\,a^3e^{(2i\,dx+2i\,c)}+i\,a^3\right)\log\left(e^{(2i\,dx+2i\,c)}-1\right)}{de^{(2i\,dx+2i\,c)}-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] (-2*I*a^3 + (I*a^3*e^(2*I*d*x + 2*I*c) - I*a^3)*log(e^(2*I*d*x + 2*I*c) + 1
) - 3*(-I*a^3*e^(2*I*d*x + 2*I*c) + I*a^3)*log(e^(2*I*d*x + 2*I*c) - 1))/(d
*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [A]
time = 0.26, size = 71, normalized size = 1.03

$$-\frac{2ia^3}{de^{2ic}e^{2idx}-d}+\frac{a^3\cdot\left(3i\log\left(e^{2idx}-e^{-2ic}\right)+i\log\left(e^{2idx}+e^{-2ic}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] -2*I*a**3/(d*exp(2*I*c)*exp(2*I*d*x) - d) + a**3*(3*I*log(exp(2*I*d*x) - ex
p(-2*I*c)) + I*log(exp(2*I*d*x) + exp(-2*I*c)))/d
```

Giac [A]
time = 0.85, size = 119, normalized size = 1.72

$$-\frac{-2i\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)+16i\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)-2i\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)-6i\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{-6i\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(-2*I*a^3*log(tan(1/2*d*x + 1/2*c) + 1) + 16*I*a^3*log(tan(1/2*d*x + 1
/2*c) + I) - 2*I*a^3*log(tan(1/2*d*x + 1/2*c) - 1) - 6*I*a^3*log(tan(1/2*d*
x + 1/2*c)) - a^3*tan(1/2*d*x + 1/2*c) - (-6*I*a^3*tan(1/2*d*x + 1/2*c) - a
^3)/tan(1/2*d*x + 1/2*c))/d
```

Mupad [B]
time = 3.81, size = 38, normalized size = 0.55

$$-\frac{a^3\,\left(\ln\left(\mathrm{tan}\left(c+d\,x\right)+1{}\mathrm{i}\right)\,4{}\mathrm{i}+\mathrm{cot}\left(c+d\,x\right)-\ln\left(\mathrm{tan}\left(c+d\,x\right)\right)\,3{}\mathrm{i}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] -(a^3*(log(tan(c + d*x) + 1i)*4i + cot(c + d*x) - log(tan(c + d*x))*3i))/d
```

3.30 $\int \cot^3(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=71

$$-4ia^3x-\frac{2ia^3\cot(c+dx)}{d}-\frac{4a^3\log(\sin(c+dx))}{d}-\frac{a\cot^2(c+dx)(a+ia\tan(c+dx))^2}{2d}$$

```
[Out] -4*I*a^3*x-2*I*a^3*cot(d*x+c)/d-4*a^3*ln(sin(d*x+c))/d-1/2*a*cot(d*x+c)^2*(
a+I*a*tan(d*x+c))^2/d
```

Rubi [A]
time = 0.08, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {3626, 3623, 3612, 3556}

$$-\frac{2ia^3\cot(c+dx)}{d}-\frac{4a^3\log(\sin(c+dx))}{d}-4ia^3x-\frac{a\cot^2(c+dx)(a+ia\tan(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (-4*I)*a^3*x - ((2*I)*a^3*Cot[c + d*x])/d - (4*a^3*Log[Sin[c + d*x]])/d - (
a*Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^2)/(2*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3626

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*b*(a + b*Tan[e + f*x])^(m - 1)*((c +
 d*Tan[e + f*x])^(n + 1)/(f*(m - 1)*(a*c - b*d))), x] + Dist[2*(a^2/(a*c -
b*d)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2]
```

Rubi steps

$$\begin{aligned}\int \cot^3(c+dx)(a+ia\tan(c+dx))^3\,dx &= -\frac{a\cot^2(c+dx)(a+ia\tan(c+dx))^2}{2d}+(2ia)\int \cot^2(c+dx)(a+\\ &= -\frac{2ia^3\cot(c+dx)}{d}-\frac{a\cot^2(c+dx)(a+ia\tan(c+dx))^2}{2d}+(2ia)\\ &= -4ia^3x-\frac{2ia^3\cot(c+dx)}{d}-\frac{a\cot^2(c+dx)(a+ia\tan(c+dx))^2}{2d}\\ &= -4ia^3x-\frac{2ia^3\cot(c+dx)}{d}-\frac{4a^3\log(\sin(c+dx))}{d}-\frac{a\cot^2(c+d}{}\end{aligned}$$

Mathematica [A]
time = 0.78, size = 126, normalized size = 1.77

$$\frac{a^3\csc\left(\frac{c}{2}\right)\csc^2(c+dx)\sec\left(\frac{c}{2}\right)\left(3i\cos(c)-3i\cos(c+2dx)+\left(-1-4idx-2\log\left(\sin^2(c+dx)\right)+2\cos(2(c+dx))\left(2idx+\log\left(\sin^2(c+dx)\right)\right)\right)\sin(c)\right)(\cos(3dx)+i\sin(3dx))}{4d(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Csc[c/2]*Csc[c + d*x]^2*Sec[c/2]*((3*I)*Cos[c] - (3*I)*Cos[c + 2*d*x]
+ (-1 - (4*I)*d*x - 2*Log[Sin[c + d*x]^2] + 2*Cos[2*(c + d*x)]*((2*I)*d*x +
 Log[Sin[c + d*x]^2]))*Sin[c])*(Cos[3*d*x] + I*Sin[3*d*x]))/(4*d*(Cos[d*x]
+ I*Sin[d*x])^3)
```

Maple [A]
time = 0.22, size = 75, normalized size = 1.06

method	result	size
risch	$\frac{8ia^3c}{d}+\frac{2a^3\left(4\,e^{2i(dx+c)}-3\right)}{d\left(e^{2i(dx+c)}-1\right)^2}-\frac{4a^3\ln\left(e^{2i(dx+c)}-1\right)}{d}$	66
derivativedivides	$\frac{-ia^3(dx+c)-3a^3\ln(\sin(dx+c))+3ia^3(-\cot(dx+c)-dx-c)+a^3\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)}{d}$	75

default	$\frac{-ia^3(dx+c)-3a^3\ln(\sin(dx+c))+3ia^3(-\cot(dx+c)-dx-c)+a^3\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)}{d}$	75
norman	$\frac{-\frac{a^3}{2d}-\frac{3ia^3\tan(dx+c)}{d}-4ia^3x(\tan^2(dx+c))}{\tan(dx+c)^2}-\frac{4a^3\ln(\tan(dx+c))}{d}+\frac{2a^3\ln(1+\tan^2(dx+c))}{d}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-I*a^3*(d*x+c)-3*a^3*ln(sin(d*x+c))+3*I*a^3*(-cot(d*x+c)-d*x-c)+a^3*(-
1/2*cot(d*x+c)^2-ln(sin(d*x+c))))
```

Maxima [A]
time = 0.50, size = 68, normalized size = 0.96

$$-\frac{8i\,(dx+c)a^3-4\,a^3\log\left(\tan\left(dx+c\right)^2+1\right)+8\,a^3\log\left(\tan\left(dx+c\right)\right)+\frac{6i\,a^3\tan(dx+c)+a^3}{\tan(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(8*I*(d*x + c)*a^3 - 4*a^3*log(tan(d*x + c)^2 + 1) + 8*a^3*log(tan(d*x
 + c)) + (6*I*a^3*tan(d*x + c) + a^3)/tan(d*x + c)^2)/d
```

Fricas [A]
time = 0.49, size = 94, normalized size = 1.32

$$\frac{2\left(4\,a^3e^{(2i\,dx+2i\,c)}-3\,a^3-2\left(a^3e^{(4i\,dx+4i\,c)}-2\,a^3e^{(2i\,dx+2i\,c)}+a^3\right)\log\left(e^{(2i\,dx+2i\,c)}-1\right)\right)}{de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 2*(4*a^3*e^(2*I*d*x + 2*I*c) - 3*a^3 - 2*(a^3*e^(4*I*d*x + 4*I*c) - 2*a^3*e
^(2*I*d*x + 2*I*c) + a^3)*log(e^(2*I*d*x + 2*I*c) - 1))/(d*e^(4*I*d*x + 4*I
*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A]
time = 0.22, size = 87, normalized size = 1.23

$$-\frac{4a^3\log\left(e^{2idx}-e^{-2ic}\right)}{d}+\frac{8a^3e^{2ic}e^{2idx}-6a^3}{de^{4ic}e^{4idx}-2de^{2ic}e^{2idx}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] -4*a**3*log(exp(2*I*d*x) - exp(-2*I*c))/d + (8*a**3*exp(2*I*c)*exp(2*I*d*x)
 - 6*a**3)/(d*exp(4*I*c)*exp(4*I*d*x) - 2*d*exp(2*I*c)*exp(2*I*d*x) + d)
```

Giac [A]
time = 1.02, size = 116, normalized size = 1.63

$$-\frac{a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-64\,a^{3}\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)+32\,a^{3}\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-12i\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{48\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-12i\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^{3}}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(a^3*tan(1/2*d*x + 1/2*c)^2 - 64*a^3*log(tan(1/2*d*x + 1/2*c) + I) + 3
2*a^3*log(tan(1/2*d*x + 1/2*c)) - 12*I*a^3*tan(1/2*d*x + 1/2*c) - (48*a^3*t
an(1/2*d*x + 1/2*c)^2 - 12*I*a^3*tan(1/2*d*x + 1/2*c) - a^3)/tan(1/2*d*x +
1/2*c)^2)/d
```

Mupad [B]
time = 3.78, size = 53, normalized size = 0.75

$$-\frac{\frac{a^3}{2}+a^3\,\mathrm{tan}\left(c+d\,x\right)\,3{}\mathrm{i}}{d\,{\mathrm{tan}\left(c+d\,x\right)}^2}-\frac{a^3\,\mathrm{atan}\left(2\,\mathrm{tan}\left(c+d\,x\right)+1{}\mathrm{i}\right)\,8{}\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] - (a^3*tan(c + d*x)*3i + a^3/2)/(d*tan(c + d*x)^2) - (a^3*atan(2*tan(c + d*
x) + 1i)*8i)/d
```

3.31 $\int \cot^4(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=101

$$4a^3x+\frac{2a^3\cot(c+dx)}{d}-\frac{4ia^3\log(\sin(c+dx))}{d}-\frac{ia\cot^2(c+dx)(a+ia\tan(c+dx))^2}{2d}-\frac{\cot^3(c+dx)(a+ia\tan$$

```
[Out] 4*a^3*x+2*a^3*cot(d*x+c)/d-4*I*a^3*ln(sin(d*x+c))/d-1/2*I*a*cot(d*x+c)^2*(a
+I*a*tan(d*x+c))^2/d-1/3*cot(d*x+c)^3*(a+I*a*tan(d*x+c))^3/d
```

Rubi [A]
time = 0.11, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3629, 3626, 3623, 3612, 3556}

$$\frac{2a^3\cot(c+dx)}{d}-\frac{4ia^3\log(\sin(c+dx))}{d}+4a^3x-\frac{\cot^3(c+dx)(a+ia\tan(c+dx))^3}{3d}-\frac{ia\cot^2(c+dx)(a+ia\tan(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] 4*a^3*x + (2*a^3*Cot[c + d*x])/d - ((4*I)*a^3*Log[Sin[c + d*x]])/d - ((I/2)
*a*Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^2)/d - (Cot[c + d*x]^3*(a + I*a*Ta
n[c + d*x])^3)/(3*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3626

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*b*(a + b*Tan[e + f*x])^(m - 1)*((c +
 d*Tan[e + f*x])^(n + 1)/(f*(m - 1)*(a*c - b*d))), x] + Dist[2*(a^2/(a*c -
b*d)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2]
```

Rule 3629

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-d)*(a + b*Tan[e + f*x])^m*((c + d*Ta
n[e + f*x])^(n + 1)/(f*m*(c^2 + d^2))), x] + Dist[a/(a*c - b*d), Int[(a + b
*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cot^4(c+dx)(a+ia\tan(c+dx))^3\,dx &= -\frac{\cot^3(c+dx)(a+ia\tan(c+dx))^3}{3d}+i\int \cot^3(c+dx)(a+ia\,\mathrm{ta}\\ &= -\frac{ia\cot^2(c+dx)(a+ia\tan(c+dx))^2}{2d}-\frac{\cot^3(c+dx)(a+ia\,\mathrm{tan}}{3d}\\ &= \frac{2a^3\cot(c+dx)}{d}-\frac{ia\cot^2(c+dx)(a+ia\tan(c+dx))^2}{2d}-\frac{\cot^3(c}{}\\ &= 4a^3x+\frac{2a^3\cot(c+dx)}{d}-\frac{ia\cot^2(c+dx)(a+ia\tan(c+dx))^2}{2d}-\\ &= 4a^3x+\frac{2a^3\cot(c+dx)}{d}-\frac{4ia^3\log(\sin(c+dx))}{d}-\frac{ia\cot^2(c+dx}{}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 251 vs. $2(101) = 202$.
time = 0.93, size = 251, normalized size = 2.49

$$\frac{a^3\csc\left(\frac{c}{2}\right)\csc^3(c+dx)\sec\left(\frac{c}{2}\right)(\cos(3dx)+i\sin(3dx))\left(9i\cos(2c+dx)-36dx\cos(2c+dx)-12dx\cos(2c+3dx)+12dx\cos(4c+3dx)+9\cos(dx)\left(-i+4dx-i\log\left(\sin^2(c+dx)\right)\right)+9i\cos(2c+dx)\log\left(\sin^2(c+dx)\right)+3i\cos(2c+3dx)\log\left(\sin^2(c+dx)\right)-3i\cos(4c+3dx)\log\left(\sin^2(c+dx)\right)-24\sin(dx)-48\mathrm{ArcTan}(\tan(4c+dx))\sin(c)\sin^3(c+dx)-15\sin(2c+dx)+13\sin(2c+3dx)\right)}{24d(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Csc[c/2]*Csc[c + d*x]^3*Sec[c/2]*(Cos[3*d*x] + I*Sin[3*d*x])*((9*I)*Co
s[2*c + d*x] - 36*d*x*Cos[2*c + d*x] - 12*d*x*Cos[2*c + 3*d*x] + 12*d*x*Cos
```

```
[4*c + 3*d*x] + 9*Cos[d*x]*(-I + 4*d*x - I*Log[Sin[c + d*x]^2]) + (9*I)*Cos
[2*c + d*x]*Log[Sin[c + d*x]^2] + (3*I)*Cos[2*c + 3*d*x]*Log[Sin[c + d*x]^2
] - (3*I)*Cos[4*c + 3*d*x]*Log[Sin[c + d*x]^2] - 24*Sin[d*x] - 48*ArcTan[Ta
n[4*c + d*x]]*Sin[c]*Sin[c + d*x]^3 - 15*Sin[2*c + d*x] + 13*Sin[2*c + 3*d*
x]))/(24*d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [A]
time = 0.22, size = 91, normalized size = 0.90

method	result
risch	$-\frac{8a^3c}{d}+\frac{2ia^3\left(24\,\mathrm{e}^{4i(dx+c)}-33\,\mathrm{e}^{2i(dx+c)}+13\right)}{3d\left(\mathrm{e}^{2i(dx+c)}-1\right)^3}-\frac{4ia^3\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{d}$
derivativedivides	$\frac{-ia^3\ln(\sin(dx+c))-3a^3(-\cot(dx+c)-dx-c)+3ia^3\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+a^3\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)-\right.}{d}$
default	$\frac{-ia^3\ln(\sin(dx+c))-3a^3(-\cot(dx+c)-dx-c)+3ia^3\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+a^3\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)-\right.}{d}$
norman	$\frac{-\frac{a^3}{3d}+4a^3x\left(\tan^3(dx+c)\right)+\frac{4a^3\left(\tan^2(dx+c)\right)}{d}-\frac{3ia^3\tan(dx+c)}{2d}}{\tan(dx+c)^3}-\frac{4ia^3\ln(\tan(dx+c))}{d}+\frac{2ia^3\ln\left(1+\tan^2(dx+c)\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-I*a^3*ln(sin(d*x+c))-3*a^3*(-cot(d*x+c)-d*x-c)+3*I*a^3*(-1/2*cot(d*x+
c)^2-ln(sin(d*x+c)))+a^3*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.58, size = 83, normalized size = 0.82

$$\frac{24\,(dx+c)a^3+12i\,a^3\log\left(\tan\left(dx+c\right)^2+1\right)-24i\,a^3\log\left(\tan\left(dx+c\right)\right)+\frac{24\,a^3\tan(dx+c)^2-9i\,a^3\tan(dx+c)-2\,a^3}{\tan(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6*(24*(d*x + c)*a^3 + 12*I*a^3*log(tan(d*x + c)^2 + 1) - 24*I*a^3*log(tan
(d*x + c)) + (24*a^3*tan(d*x + c)^2 - 9*I*a^3*tan(d*x + c) - 2*a^3)/tan(d*x
 + c)^3)/d
```

Fricas [A]
time = 0.46, size = 139, normalized size = 1.38

$$-\frac{2\left(-24i\,a^3e^{(4i\,dx+4i\,c)}+33i\,a^3e^{(2i\,dx+2i\,c)}-13i\,a^3+6\left(i\,a^3e^{(6i\,dx+6i\,c)}-3i\,a^3e^{(4i\,dx+4i\,c)}+3i\,a^3e^{(2i\,dx+2i\,c)}-i\,a^3\right)\log\left(e^{(2i\,dx+2i\,c)}-1\right)\right)}{3\left(de^{(6i\,dx+6i\,c)}-3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -2/3*(-24*I*a^3*e^(4*I*d*x + 4*I*c) + 33*I*a^3*e^(2*I*d*x + 2*I*c) - 13*I*a
^3 + 6*(I*a^3*e^(6*I*d*x + 6*I*c) - 3*I*a^3*e^(4*I*d*x + 4*I*c) + 3*I*a^3*e
^(2*I*d*x + 2*I*c) - I*a^3)*log(e^(2*I*d*x + 2*I*c) - 1))/(d*e^(6*I*d*x + 6
*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [A]
time = 0.28, size = 136, normalized size = 1.35

$$-\frac{4ia^3\log\left(e^{2idx}-e^{-2ic}\right)}{d}+\frac{48ia^3e^{4ic}e^{4idx}-66ia^3e^{2ic}e^{2idx}+26ia^3}{3de^{6ic}e^{6idx}-9de^{4ic}e^{4idx}+9de^{2ic}e^{2idx}-3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] -4*I*a**3*log(exp(2*I*d*x) - exp(-2*I*c))/d + (48*I*a**3*exp(4*I*c)*exp(4*I
*d*x) - 66*I*a**3*exp(2*I*c)*exp(2*I*d*x) + 26*I*a**3)/(3*d*exp(6*I*c)*exp(
6*I*d*x) - 9*d*exp(4*I*c)*exp(4*I*d*x) + 9*d*exp(2*I*c)*exp(2*I*d*x) - 3*d)
```

Giac [A]
time = 1.08, size = 146, normalized size = 1.45

$$\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-9i\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+192i\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)-96i\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-51\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{-176i\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-51\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+9i\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(a^3*tan(1/2*d*x + 1/2*c)^3 - 9*I*a^3*tan(1/2*d*x + 1/2*c)^2 + 192*I*a
^3*log(tan(1/2*d*x + 1/2*c) + I) - 96*I*a^3*log(tan(1/2*d*x + 1/2*c)) - 51*
a^3*tan(1/2*d*x + 1/2*c) - (-176*I*a^3*tan(1/2*d*x + 1/2*c)^3 - 51*a^3*tan(
1/2*d*x + 1/2*c)^2 + 9*I*a^3*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2*
c)^3)/d
```

Mupad [B]
time = 3.79, size = 68, normalized size = 0.67

$$\frac{4\,a^3\,\mathrm{cot}\left(c+d\,x\right)}{d}+\frac{8\,a^3\,\mathrm{atan}\left(2\,\mathrm{tan}\left(c+d\,x\right)+1{}\mathrm{i}\right)}{d}-\frac{a^3\,{\mathrm{cot}\left(c+d\,x\right)}^3}{3\,d}-\frac{a^3\,{\mathrm{cot}\left(c+d\,x\right)}^2\,3{}\mathrm{i}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] (4*a^3*cot(c + d*x))/d + (8*a^3*atan(2*tan(c + d*x) + 1i))/d - (a^3*cot(c +
 d*x)^2*3i)/(2*d) - (a^3*cot(c + d*x)^3)/(3*d)
```

3.32 $\int \cot^5(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=108

$$4ia^3x+\frac{4ia^3\cot(c+dx)}{d}+\frac{2a^3\cot^2(c+dx)}{d}-\frac{3ia^3\cot^3(c+dx)}{4d}+\frac{4a^3\log(\sin(c+dx))}{d}-\frac{\cot^4(c+dx)\,(a^3+ia^3\tan(c+dx))}{4d}$$

```
[Out] 4*I*a^3*x+4*I*a^3*cot(d*x+c)/d+2*a^3*cot(d*x+c)^2/d-3/4*I*a^3*cot(d*x+c)^3/
d+4*a^3*ln(sin(d*x+c))/d-1/4*cot(d*x+c)^4*(a^3+I*a^3*tan(d*x+c))/d
```

Rubi [A]
time = 0.13, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3634, 3672, 3610, 3612, 3556}

$$-\frac{3ia^3\cot^3(c+dx)}{4d}+\frac{2a^3\cot^2(c+dx)}{d}+\frac{4ia^3\cot(c+dx)}{d}+\frac{4a^3\log(\sin(c+dx))}{d}-\frac{\cot^4(c+dx)\,(a^3+ia^3\tan(c+dx))}{4d}+4ia^3x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (4*I)*a^3*x + ((4*I)*a^3*Cot[c + d*x])/d + (2*a^3*Cot[c + d*x]^2)/d - (((3*
I)/4)*a^3*Cot[c + d*x]^3)/d + (4*a^3*Log[Sin[c + d*x]])/d - (Cot[c + d*x]^4
*(a^3 + I*a^3*Tan[c + d*x]))/(4*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3634

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-a^2)*(b*c - a*d)*(a + b*Tan[e + f*x]
)^(m - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] + Di
st[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[b*(b*c*(m - 2) - a*d*(m - 2*n - 4)) + (a*b*c*(m - 2)
+ b^2*d*(n + 1) - a^2*d*(m + n - 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3672

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
 b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
 -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cot^5(c+dx)(a+ia\tan(c+dx))^3\,dx &= -\frac{\cot^4(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{4d}-\frac{1}{4}\int \cot^4(c+dx)(a+ia\\ &= -\frac{3ia^3\cot^3(c+dx)}{4d}-\frac{\cot^4(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{4d}-\frac{1}{4}\int\\ &= \frac{2a^3\cot^2(c+dx)}{d}-\frac{3ia^3\cot^3(c+dx)}{4d}-\frac{\cot^4(c+dx)\left(a^3+ia^3\tan\right.}{4d}\\ &= \frac{4ia^3\cot(c+dx)}{d}+\frac{2a^3\cot^2(c+dx)}{d}-\frac{3ia^3\cot^3(c+dx)}{4d}-\frac{\cot^4(}{}\\ &= 4ia^3x+\frac{4ia^3\cot(c+dx)}{d}+\frac{2a^3\cot^2(c+dx)}{d}-\frac{3ia^3\cot^3(c+dx)}{4d}\\ &= 4ia^3x+\frac{4ia^3\cot(c+dx)}{d}+\frac{2a^3\cot^2(c+dx)}{d}-\frac{3ia^3\cot^3(c+dx)}{4d}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 254 vs. $2(108) = 216$.
time = 0.96, size = 254, normalized size = 2.35

$$\frac{a^3\csc\left(\frac{c}{2}\right)\csc^4(c+dx)\sec\left(\frac{c}{2}\right)\left(-15i\cos(c)+13i\cos(c+2dx)+7i\cos(3c+2dx)-5i\cos(3c+4dx)+8\sin(c)+12idx\sin(c)+6\log\left(\sin^2(c+dx)\right)\sin(c)+5\sin(c+2dx)+8idx\sin(c+2dx)+4\log\left(\sin^2(c+dx)\right)\sin(c+2dx)-5\sin(3c+2dx)-8idx\sin(3c+2dx)-4\log\left(\sin^2(c+dx)\right)\sin(3c+2dx)-2idx\sin(3c+4dx)-\log\left(\sin^2(c+dx)\right)\sin(3c+4dx)+2idx\sin(5c+4dx)+\log\left(\sin^2(c+dx)\right)\sin(5c+4dx)\right)}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Csc[c/2]*Csc[c + d*x]^4*Sec[c/2]*((-15*I)*Cos[c] + (13*I)*Cos[c + 2*d*
x] + (7*I)*Cos[3*c + 2*d*x] - (5*I)*Cos[3*c + 4*d*x] + 8*Sin[c] + (12*I)*d*
x*Sin[c] + 6*Log[Sin[c + d*x]^2]*Sin[c] + 5*Sin[c + 2*d*x] + (8*I)*d*x*Sin[
c + 2*d*x] + 4*Log[Sin[c + d*x]^2]*Sin[c + 2*d*x] - 5*Sin[3*c + 2*d*x] - (8
*I)*d*x*Sin[3*c + 2*d*x] - 4*Log[Sin[c + d*x]^2]*Sin[3*c + 2*d*x] - (2*I)*d
*x*Sin[3*c + 4*d*x] - Log[Sin[c + d*x]^2]*Sin[3*c + 4*d*x] + (2*I)*d*x*Sin[
5*c + 4*d*x] + Log[Sin[c + d*x]^2]*Sin[5*c + 4*d*x]))/(16*d)
```

Maple [A]
time = 0.20, size = 112, normalized size = 1.04

method	result
risch	$-\frac{8ia^3c}{d}-\frac{2a^3\left(12\,\mathrm{e}^{6i(dx+c)}-23\,\mathrm{e}^{4i(dx+c)}+18\,\mathrm{e}^{2i(dx+c)}-5\right)}{d\left(\mathrm{e}^{2i(dx+c)}-1\right)^4}+\frac{4a^3\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{d}$
derivativedivides	$\frac{-ia^3(-\cot(dx+c)-dx-c)-3a^3\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+3ia^3\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+a^3\left(-\frac{(c}{}\right.}{d}$
default	$\frac{-ia^3(-\cot(dx+c)-dx-c)-3a^3\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+3ia^3\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+a^3\left(-\frac{(c}{}\right.}{d}$
norman	$\frac{-\frac{a^3}{4d}+\frac{2a^3\left(\tan^2(dx+c)\right)}{d}-\frac{ia^3\tan(dx+c)}{d}+\frac{4ia^3\left(\tan^3(dx+c)\right)}{d}+4ia^3x\left(\tan^4(dx+c)\right)}{\tan(dx+c)^4}+\frac{4a^3\ln(\tan(dx+c))}{d}-\frac{2a^3\ln(1+\tan^2}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-I*a^3*(-cot(d*x+c)-d*x-c)-3*a^3*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+3*
I*a^3*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+a^3*(-1/4*cot(d*x+c)^4+1/2*cot(d
*x+c)^2+ln(sin(d*x+c))))
```

Maxima [A]
time = 0.55, size = 94, normalized size = 0.87

$$-\frac{-16i\,(dx+c)a^3+8\,a^3\log\left(\tan\left(dx+c\right)^2+1\right)-16\,a^3\log\left(\tan\left(dx+c\right)\right)+\frac{-16i\,a^3\tan(dx+c)^3-8\,a^3\tan(dx+c)^2+4i\,a^3\tan(dx+c)+a^3}{\tan(dx+c)^4}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(-16*I*(d*x + c)*a^3 + 8*a^3*log(tan(d*x + c)^2 + 1) - 16*a^3*log(tan(
d*x + c)) + (-16*I*a^3*tan(d*x + c)^3 - 8*a^3*tan(d*x + c)^2 + 4*I*a^3*tan(
d*x + c) + a^3)/tan(d*x + c)^4)/d
```

Fricas [A]
time = 0.46, size = 174, normalized size = 1.61

$$-\frac{2\left(12\,a^3e^{(6i\,dx+6i\,c)}-23\,a^3e^{(4i\,dx+4i\,c)}+18\,a^3e^{(2i\,dx+2i\,c)}-5\,a^3-2\left(a^3e^{(8i\,dx+8i\,c)}-4\,a^3e^{(6i\,dx+6i\,c)}+6\,a^3e^{(4i\,dx+4i\,c)}-4\,a^3e^{(2i\,dx+2i\,c)}+a^3\right)\log\left(e^{(2i\,dx+2i\,c)}-1\right)\right)}{de^{(8i\,dx+8i\,c)}-4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}-4\,de^{(2i\,dx+2i\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -2*(12*a^3*e^(6*I*d*x + 6*I*c) - 23*a^3*e^(4*I*d*x + 4*I*c) + 18*a^3*e^(2*I
*d*x + 2*I*c) - 5*a^3 - 2*(a^3*e^(8*I*d*x + 8*I*c) - 4*a^3*e^(6*I*d*x + 6*I
*c) + 6*a^3*e^(4*I*d*x + 4*I*c) - 4*a^3*e^(2*I*d*x + 2*I*c) + a^3)*log(e^(2
*I*d*x + 2*I*c) - 1))/(d*e^(8*I*d*x + 8*I*c) - 4*d*e^(6*I*d*x + 6*I*c) + 6*
d*e^(4*I*d*x + 4*I*c) - 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A]
time = 0.42, size = 165, normalized size = 1.53

$$\frac{4a^3\log\left(e^{2idx}-e^{-2ic}\right)}{d}+\frac{-24a^3e^{6ic}e^{6idx}+46a^3e^{4ic}e^{4idx}-36a^3e^{2ic}e^{2idx}+10a^3}{de^{8ic}e^{8idx}-4de^{6ic}e^{6idx}+6de^{4ic}e^{4idx}-4de^{2ic}e^{2idx}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] 4*a**3*log(exp(2*I*d*x) - exp(-2*I*c))/d + (-24*a**3*exp(6*I*c)*exp(6*I*d*x
) + 46*a**3*exp(4*I*c)*exp(4*I*d*x) - 36*a**3*exp(2*I*c)*exp(2*I*d*x) + 10*
a**3)/(d*exp(8*I*c)*exp(8*I*d*x) - 4*d*exp(6*I*c)*exp(6*I*d*x) + 6*d*exp(4*
I*c)*exp(4*I*d*x) - 4*d*exp(2*I*c)*exp(2*I*d*x) + d)
```

Giac [A]
time = 1.29, size = 180, normalized size = 1.67

$$-\frac{3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-24i\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-108\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1536\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)-768\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+456i\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{1600\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-456i\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-108\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24i\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/192*(3*a^3*tan(1/2*d*x + 1/2*c)^4 - 24*I*a^3*tan(1/2*d*x + 1/2*c)^3 - 10
8*a^3*tan(1/2*d*x + 1/2*c)^2 + 1536*a^3*log(tan(1/2*d*x + 1/2*c) + I) - 768
*a^3*log(tan(1/2*d*x + 1/2*c)) + 456*I*a^3*tan(1/2*d*x + 1/2*c) + (1600*a^3
*tan(1/2*d*x + 1/2*c)^4 - 456*I*a^3*tan(1/2*d*x + 1/2*c)^3 - 108*a^3*tan(1/
2*d*x + 1/2*c)^2 + 24*I*a^3*tan(1/2*d*x + 1/2*c) + 3*a^3)/tan(1/2*d*x + 1/2
*c)^4)/d
```

Mupad [B]
time = 3.91, size = 80, normalized size = 0.74

$$\frac{a^3\,\mathrm{atan}\left(2\,\tan\left(c+d\,x\right)+1{}\mathrm{i}\right)\,8{}\mathrm{i}}{d}-\frac{-a^3\,{\tan\left(c+d\,x\right)}^3\,4{}\mathrm{i}-2\,a^3\,{\tan\left(c+d\,x\right)}^2+a^3\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}+\frac{a^3}{4}}{d\,{\tan\left(c+d\,x\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] (a^3*atan(2*tan(c + d*x) + 1i)*8i)/d - (a^3*tan(c + d*x)*1i + a^3/4 - 2*a^3
*tan(c + d*x)^2 - a^3*tan(c + d*x)^3*4i)/(d*tan(c + d*x)^4)
```

3.33 $\int \cot^6(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=126

$$-4a^3x-\frac{4a^3\cot(c+dx)}{d}+\frac{2ia^3\cot^2(c+dx)}{d}+\frac{4a^3\cot^3(c+dx)}{3d}-\frac{11ia^3\cot^4(c+dx)}{20d}+\frac{4ia^3\log(\sin(c+dx))}{d}-c$$

```
[Out] -4*a^3*x-4*a^3*cot(d*x+c)/d+2*I*a^3*cot(d*x+c)^2/d+4/3*a^3*cot(d*x+c)^3/d-1
1/20*I*a^3*cot(d*x+c)^4/d+4*I*a^3*ln(sin(d*x+c))/d-1/5*cot(d*x+c)^5*(a^3+I*
a^3*tan(d*x+c))/d
```

Rubi [A]
time = 0.15, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3634, 3672, 3610, 3612, 3556}

$$-\frac{11ia^3\cot^4(c+dx)}{20d}+\frac{4a^3\cot^3(c+dx)}{3d}+\frac{2ia^3\cot^2(c+dx)}{d}-\frac{4a^3\cot(c+dx)}{d}+\frac{4ia^3\log(\sin(c+dx))}{d}-\frac{\cot^5(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{5d}-4a^3x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] -4*a^3*x - (4*a^3*Cot[c + d*x])/d + ((2*I)*a^3*Cot[c + d*x]^2)/d + (4*a^3*C
ot[c + d*x]^3)/(3*d) - (((11*I)/20)*a^3*Cot[c + d*x]^4)/d + ((4*I)*a^3*Log[
Sin[c + d*x]])/d - (Cot[c + d*x]^5*(a^3 + I*a^3*Tan[c + d*x]))/(5*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3634

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-a^2)*(b*c - a*d)*(a + b*Tan[e + f*x]
)^(m - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] + Di
st[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[b*(b*c*(m - 2) - a*d*(m - 2*n - 4)) + (a*b*c*(m - 2)
+ b^2*d*(n + 1) - a^2*d*(m + n - 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3672

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
 b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
 -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \cot^6(c+dx)(a+ia\tan(c+dx))^3\,dx &= -\frac{\cot^5(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{5d}-\frac{1}{5}\int \cot^5(c+dx)(a+ia \dots \\
&= -\frac{11ia^3\cot^4(c+dx)}{20d}-\frac{\cot^5(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{5d}-\frac{1}{5}\int \dots \\
&= \frac{4a^3\cot^3(c+dx)}{3d}-\frac{11ia^3\cot^4(c+dx)}{20d}-\frac{\cot^5(c+dx)\left(a^3+ia^3\tan \dots\right.}{5d} \dots \\
&= \frac{2ia^3\cot^2(c+dx)}{d}+\frac{4a^3\cot^3(c+dx)}{3d}-\frac{11ia^3\cot^4(c+dx)}{20d}-\dots \\
&= -\frac{4a^3\cot(c+dx)}{d}+\frac{2ia^3\cot^2(c+dx)}{d}+\frac{4a^3\cot^3(c+dx)}{3d}-\dots \\
&= -4a^3x-\frac{4a^3\cot(c+dx)}{d}+\frac{2ia^3\cot^2(c+dx)}{d}+\frac{4a^3\cot^3(c+dx)}{3d} \dots \\
&= -4a^3x-\frac{4a^3\cot(c+dx)}{d}+\frac{2ia^3\cot^2(c+dx)}{d}+\frac{4a^3\cot^3(c+dx)}{3d} \dots
\end{aligned}
$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 359 vs. $2(126) = 252$.
time = 1.65, size = 359, normalized size = 2.85

$$
\frac{a^3\csc(c)\csc^5(c+dx)(\cos(3dx)+i\sin(3dx))\left(-225i\cos(2c+dx)+600dx\cos(2c+dx)-105i\cos(2c+3dx)+300dx\cos(2c+3dx)+105i\cos(4c+3dx)-300dx\cos(4c+3dx)-60dx\cos(4c+5dx)+60dx\cos(6c+5dx)-75\cos(dx)\left(-3i+8dx-2i\log\left(\sin^2(c+dx)\right)\right)-150i\cos(2c+dx)\log\left(\sin^2(c+dx)\right)-75i\cos(2c+3dx)\log\left(\sin^2(c+dx)\right)+75i\cos(4c+3dx)\log\left(\sin^2(c+dx)\right)+15i\cos(4c+5dx)\log\left(\sin^2(c+dx)\right)-15i\cos(6c+5dx)\log\left(\sin^2(c+dx)\right)+470\sin(dx)+960\text{ArcTan}(\tan(4c+dx))\sin(c)\sin^5(c+dx)+360\sin(2c+dx)-280\sin(2c+3dx)-135\sin(4c+3dx)+83\sin(4c+5dx)\right)}{240d(\cos(dx)+i\sin(dx))^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Csc[c]*Csc[c + d*x]^5*(Cos[3*d*x] + I*Sin[3*d*x])*((-225*I)*Cos[2*c +
d*x] + 600*d*x*Cos[2*c + d*x] - (105*I)*Cos[2*c + 3*d*x] + 300*d*x*Cos[2*c
+ 3*d*x] + (105*I)*Cos[4*c + 3*d*x] - 300*d*x*Cos[4*c + 3*d*x] - 60*d*x*Cos
[4*c + 5*d*x] + 60*d*x*Cos[6*c + 5*d*x] - 75*Cos[d*x]*(-3*I + 8*d*x - (2*I)
*Log[Sin[c + d*x]^2]) - (150*I)*Cos[2*c + d*x]*Log[Sin[c + d*x]^2] - (75*I)
*Cos[2*c + 3*d*x]*Log[Sin[c + d*x]^2] + (75*I)*Cos[4*c + 3*d*x]*Log[Sin[c +
 d*x]^2] + (15*I)*Cos[4*c + 5*d*x]*Log[Sin[c + d*x]^2] - (15*I)*Cos[6*c + 5
*d*x]*Log[Sin[c + d*x]^2] + 470*Sin[d*x] + 960*ArcTan[Tan[4*c + d*x]]*Sin[c
]*Sin[c + d*x]^5 + 360*Sin[2*c + d*x] - 280*Sin[2*c + 3*d*x] - 135*Sin[4*c
+ 3*d*x] + 83*Sin[4*c + 5*d*x]))/(240*d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [A]

time = 0.20, size = 132, normalized size = 1.05

method	result
risch	$\frac{8a^3c}{d} - \frac{2ia^3\left(240\,\mathrm{e}^{8i(dx+c)}-585\,\mathrm{e}^{6i(dx+c)}+695\,\mathrm{e}^{4i(dx+c)}-385\,\mathrm{e}^{2i(dx+c)}+83\right)}{15d\left(\mathrm{e}^{2i(dx+c)}-1\right)^5} + \frac{4ia^3\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{d}$
derivativedivides	$\frac{-ia^3\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)-3a^3\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+3ia^3\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}\right.}{d}$
default	$\frac{-ia^3\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)-3a^3\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+3ia^3\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}\right.}{d}$
norman	$\frac{-\frac{a^3}{5d}-4a^3x\left(\tan^5(dx+c)\right)+\frac{4a^3\left(\tan^2(dx+c)\right)}{3d}-\frac{4a^3\left(\tan^4(dx+c)\right)}{d}-\frac{3ia^3\tan(dx+c)}{4d}+\frac{2ia^3\left(\tan^3(dx+c)\right)}{d}}{\tan(dx+c)^5} + \frac{4ia^3\ln(\tan(dx+c)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-I*a^3*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))-3*a^3*(-1/3*cot(d*x+c)^3+cot
(d*x+c)+d*x+c)+3*I*a^3*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+
a^3*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c))
```

Maxima [A]

time = 0.59, size = 109, normalized size = 0.87

$$-\frac{240\,(dx+c)a^3 + 120i\,a^3\log\left(\tan\left(dx+c\right)^2+1\right) - 240i\,a^3\log\left(\tan\left(dx+c\right)\right) + \frac{240\,a^3\tan(dx+c)^4-120i\,a^3\tan(dx+c)^3-80\,a^3\tan(dx+c)^2+45i\,a^3\tan(dx+c)+12\,a^3}{\tan(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(240*(d*x + c)*a^3 + 120*I*a^3*log(tan(d*x + c)^2 + 1) - 240*I*a^3*lo
g(tan(d*x + c)) + (240*a^3*tan(d*x + c)^4 - 120*I*a^3*tan(d*x + c)^3 - 80*a
^3*tan(d*x + c)^2 + 45*I*a^3*tan(d*x + c) + 12*a^3)/tan(d*x + c)^5)/d
```

Fricas [A]
time = 0.44, size = 219, normalized size = 1.74

$$-\frac{2\left(240i\,a^{3}e^{(8i\,dx+8i\,c)}-585i\,a^{3}e^{(6i\,dx+6i\,c)}+695i\,a^{3}e^{(4i\,dx+4i\,c)}-385i\,a^{3}e^{(2i\,dx+2i\,c)}+83i\,a^{3}+30\left(-i\,a^{3}e^{(10i\,dx+10i\,c)}+5i\,a^{3}e^{(8i\,dx+8i\,c)}-10i\,a^{3}e^{(6i\,dx+6i\,c)}+10i\,a^{3}e^{(4i\,dx+4i\,c)}-5i\,a^{3}e^{(2i\,dx+2i\,c)}+i\,a^{3}\right)\log\left(e^{(2i\,dx+2i\,c)}-1\right)\right)}{15\left(de^{(10i\,dx+10i\,c)}-5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}-10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -2/15*(240*I*a^3*e^(8*I*d*x + 8*I*c) - 585*I*a^3*e^(6*I*d*x + 6*I*c) + 695*
I*a^3*e^(4*I*d*x + 4*I*c) - 385*I*a^3*e^(2*I*d*x + 2*I*c) + 83*I*a^3 + 30*(
-I*a^3*e^(10*I*d*x + 10*I*c) + 5*I*a^3*e^(8*I*d*x + 8*I*c) - 10*I*a^3*e^(6*
I*d*x + 6*I*c) + 10*I*a^3*e^(4*I*d*x + 4*I*c) - 5*I*a^3*e^(2*I*d*x + 2*I*c)
 + I*a^3)*log(e^(2*I*d*x + 2*I*c) - 1))/(d*e^(10*I*d*x + 10*I*c) - 5*d*e^(8
*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) - 10*d*e^(4*I*d*x + 4*I*c) + 5*d
*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [A]
time = 0.36, size = 218, normalized size = 1.73

$$\frac{4ia^{3}\log\left(e^{2idx}-e^{-2ic}\right)}{d}+\frac{-480ia^{3}e^{8ic}e^{8idx}+1170ia^{3}e^{6ic}e^{6idx}-1390ia^{3}e^{4ic}e^{4idx}+770ia^{3}e^{2ic}e^{2idx}-166ia^{3}}{15de^{10ic}e^{10idx}-75de^{8ic}e^{8idx}+150de^{6ic}e^{6idx}-150de^{4ic}e^{4idx}+75de^{2ic}e^{2idx}-15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] 4*I*a**3*log(exp(2*I*d*x) - exp(-2*I*c))/d + (-480*I*a**3*exp(8*I*c)*exp(8*
I*d*x) + 1170*I*a**3*exp(6*I*c)*exp(6*I*d*x) - 1390*I*a**3*exp(4*I*c)*exp(4
*I*d*x) + 770*I*a**3*exp(2*I*c)*exp(2*I*d*x) - 166*I*a**3)/(15*d*exp(10*I*c
)*exp(10*I*d*x) - 75*d*exp(8*I*c)*exp(8*I*d*x) + 150*d*exp(6*I*c)*exp(6*I*d
*x) - 150*d*exp(4*I*c)*exp(4*I*d*x) + 75*d*exp(2*I*c)*exp(2*I*d*x) - 15*d)
```

Giac [A]
time = 1.61, size = 212, normalized size = 1.68

$$\frac{6\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-45i\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-190\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+660i\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-7680i\,a^{3}\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)+3840i\,a^{3}\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+2460\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{-8768i\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-2460\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+660i\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+190\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-45i\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-6\,a^{3}}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/960*(6*a^3*tan(1/2*d*x + 1/2*c)^5 - 45*I*a^3*tan(1/2*d*x + 1/2*c)^4 - 190
*a^3*tan(1/2*d*x + 1/2*c)^3 + 660*I*a^3*tan(1/2*d*x + 1/2*c)^2 - 7680*I*a^3
*log(tan(1/2*d*x + 1/2*c) + I) + 3840*I*a^3*log(tan(1/2*d*x + 1/2*c)) + 246
0*a^3*tan(1/2*d*x + 1/2*c) + (-8768*I*a^3*tan(1/2*d*x + 1/2*c)^5 - 2460*a^3
*tan(1/2*d*x + 1/2*c)^4 + 660*I*a^3*tan(1/2*d*x + 1/2*c)^3 + 190*a^3*tan(1/
2*d*x + 1/2*c)^2 - 45*I*a^3*tan(1/2*d*x + 1/2*c) - 6*a^3)/tan(1/2*d*x + 1/2
*c)^5)/d
```

Mupad [B]
time = 4.11, size = 92, normalized size = 0.73

$$-\frac{8\,a^3\,\mathrm{atan}(2\tan(c+d\,x)+1\mathrm{i})}{d}-\frac{4\,a^3\tan(c+d\,x)^4-a^3\tan(c+d\,x)^3\,2\mathrm{i}-\frac{4\,a^3\tan(c+d\,x)^2}{3}+\frac{a^3\tan(c+d\,x)\,3\mathrm{i}}{4}+\frac{a^3}{5}}{d\tan\left(c+d\,x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] - (8*a^3*atan(2*tan(c + d*x) + 1i))/d - ((a^3*tan(c + d*x)*3i)/4 + a^3/5 -
(4*a^3*tan(c + d*x)^2)/3 - a^3*tan(c + d*x)^3*2i + 4*a^3*tan(c + d*x)^4)/(d
*tan(c + d*x)^5)
```

3.34 $\int \tan^3(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal. Leaf size=160

$$8ia^4x+\frac{8a^4\log(\cos(c+dx))}{d}-\frac{8ia^4\tan(c+dx)}{d}+\frac{4a^4\tan^2(c+dx)}{d}+\frac{8ia^4\tan^3(c+dx)}{3d}-\frac{67a^4\tan^4(c+dx)}{60d}-$$

```
[Out] 8*I*a^4*x+8*a^4*ln(cos(d*x+c))/d-8*I*a^4*tan(d*x+c)/d+4*a^4*tan(d*x+c)^2/d+
8/3*I*a^4*tan(d*x+c)^3/d-67/60*a^4*tan(d*x+c)^4/d-1/6*tan(d*x+c)^4*(a^2+I*a
^2*tan(d*x+c))^2/d-7/15*tan(d*x+c)^4*(a^4+I*a^4*tan(d*x+c))/d
```

Rubi [A]
time = 0.19, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3637, 3675, 3673, 3609, 3606, 3556}

$$-\frac{67a^4\tan^4(c+dx)}{60d}-\frac{7\tan^4(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{15d}+\frac{8ia^4\tan^3(c+dx)}{3d}+\frac{4a^4\tan^2(c+dx)}{d}-\frac{8ia^4\tan(c+dx)}{d}+\frac{8a^4\log(\cos(c+dx))}{d}+8ia^4x-\frac{\tan^4(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (8*I)*a^4*x + (8*a^4*Log[Cos[c + d*x]])/d - ((8*I)*a^4*Tan[c + d*x])/d + (4
*a^4*Tan[c + d*x]^2)/d + (((8*I)/3)*a^4*Tan[c + d*x]^3)/d - (67*a^4*Tan[c +
 d*x]^4)/(60*d) - (Tan[c + d*x]^4*(a^2 + I*a^2*Tan[c + d*x])^2)/(6*d) - (7*
Tan[c + d*x]^4*(a^4 + I*a^4*Tan[c + d*x]))/(15*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rule 3675

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[
e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && G
tQ[m, 1] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \tan^3(c+dx)(a+ia\tan(c+dx))^4\,dx = -\frac{\tan^4(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{6d} + \frac{1}{6}a\int \tan^3(c+dx)(a+$$

$$= -\frac{\tan^4(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{6d} - \frac{7\tan^4(c+dx)\left(a^4+ia\right.}{15d}$$

$$= -\frac{67a^4\tan^4(c+dx)}{60d} - \frac{\tan^4(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{6d} - \frac{7}{}$$

$$= \frac{8ia^4\tan^3(c+dx)}{3d} - \frac{67a^4\tan^4(c+dx)}{60d} - \frac{\tan^4(c+dx)\left(a^2+ia^2\right.}{6d}$$

$$= \frac{4a^4\tan^2(c+dx)}{d} + \frac{8ia^4\tan^3(c+dx)}{3d} - \frac{67a^4\tan^4(c+dx)}{60d} - \tan$$

$$= 8ia^4x - \frac{8ia^4\tan(c+dx)}{d} + \frac{4a^4\tan^2(c+dx)}{d} + \frac{8ia^4\tan^3(c+dx}{3d}$$

$$= 8ia^4x + \frac{8a^4\log(\cos(c+dx))}{d} - \frac{8ia^4\tan(c+dx)}{d} + \frac{4a^4\tan^2(c+}{d}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 349 vs. 2(160) = 320.
time = 1.74, size = 349, normalized size = 2.18

$$\frac{a^4\sec(c)\sec^6(c+dx)\left(345\cos(3c+2dx)+450idx\cos(3c+2dx)+120\cos(3c+4dx)+180idx\cos(3c+4dx)+120\cos(5c+4dx)+180idx\cos(5c+4dx)+30idx\cos(5c+6dx)+30idx\cos(7c+6dx)+225\cos(3c+2dx)\log\left(\cos^2(c+dx)\right)+90\cos(3c+4dx)\log\left(\cos^2(c+dx)\right)+90\cos(5c+4dx)\log\left(\cos^2(c+dx)\right)+15\cos(5c+6dx)\log\left(\cos^2(c+dx)\right)+15\cos(7c+6dx)\log\left(\cos^2(c+dx)\right)+15\cos(c+2dx)\left(23+30idx+15\log\left(\cos^2(c+dx)\right)\right)+10\cos(c)\left(49+60idx+30\log\left(\cos^2(c+dx)\right)\right)+860i\sin(c)-780i\sin(c+2dx)+510i\sin(3c+2dx)-366i\sin(3c+4dx)+150i\sin(5c+4dx)-86i\sin(5c+6dx)\right)}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (a^4*Sec[c]*Sec[c + d*x]^6*(345*Cos[3*c + 2*d*x] + (450*I)*d*x*Cos[3*c + 2*
d*x] + 120*Cos[3*c + 4*d*x] + (180*I)*d*x*Cos[3*c + 4*d*x] + 120*Cos[5*c +
4*d*x] + (180*I)*d*x*Cos[5*c + 4*d*x] + (30*I)*d*x*Cos[5*c + 6*d*x] + (30*I
)*d*x*Cos[7*c + 6*d*x] + 225*Cos[3*c + 2*d*x]*Log[Cos[c + d*x]^2] + 90*Cos[
3*c + 4*d*x]*Log[Cos[c + d*x]^2] + 90*Cos[5*c + 4*d*x]*Log[Cos[c + d*x]^2]
+ 15*Cos[5*c + 6*d*x]*Log[Cos[c + d*x]^2] + 15*Cos[7*c + 6*d*x]*Log[Cos[c +
 d*x]^2] + 15*Cos[c + 2*d*x]*(23 + (30*I)*d*x + 15*Log[Cos[c + d*x]^2]) + 1
0*Cos[c]*(49 + (60*I)*d*x + 30*Log[Cos[c + d*x]^2]) + (860*I)*Sin[c] - (780
*I)*Sin[c + 2*d*x] + (510*I)*Sin[3*c + 2*d*x] - (366*I)*Sin[3*c + 4*d*x] +
(150*I)*Sin[5*c + 4*d*x] - (86*I)*Sin[5*c + 6*d*x]))/(240*d)
```

Maple [A]
time = 0.06, size = 93, normalized size = 0.58

method	result

derivativedivides	$\frac{a^4\left(-8i\tan(dx+c)+\frac{\left(\tan^6(dx+c)\right)}{6}-\frac{4i\left(\tan^5(dx+c)\right)}{5}-\frac{7\left(\tan^4(dx+c)\right)}{4}+\frac{8i\left(\tan^3(dx+c)\right)}{3}+4\left(\tan^2(dx+c)\right)-4\ln\left(1+\tan^2(dx+c\right.\right.}{d}$
default	$\frac{a^4\left(-8i\tan(dx+c)+\frac{\left(\tan^6(dx+c)\right)}{6}-\frac{4i\left(\tan^5(dx+c)\right)}{5}-\frac{7\left(\tan^4(dx+c)\right)}{4}+\frac{8i\left(\tan^3(dx+c)\right)}{3}+4\left(\tan^2(dx+c)\right)-4\ln\left(1+\tan^2(dx+c\right.\right.}{d}$
risch	$-\frac{16ia^4c}{d}+\frac{4a^4\left(270\,\mathrm{e}^{10i(dx+c)}+855\,\mathrm{e}^{8i(dx+c)}+1350\,\mathrm{e}^{6i(dx+c)}+1125\,\mathrm{e}^{4i(dx+c)}+486\,\mathrm{e}^{2i(dx+c)}+86\right)}{15d\left(\mathrm{e}^{2i(dx+c)}+1\right)^6}+\frac{8a^4\ln\left(\mathrm{e}^{2i(dx+c)}\right.}{d}$
norman	$\frac{4a^4\left(\tan^2(dx+c)\right)}{d}-\frac{7a^4\left(\tan^4(dx+c)\right)}{4d}+\frac{a^4\left(\tan^6(dx+c)\right)}{6d}+8ia^4x-\frac{8ia^4\tan(dx+c)}{d}+\frac{8ia^4\left(\tan^3(dx+c)\right)}{3d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a^4*(-8*I*tan(d*x+c)+1/6*tan(d*x+c)^6-4/5*I*tan(d*x+c)^5-7/4*tan(d*x+c)
^4+8/3*I*tan(d*x+c)^3+4*tan(d*x+c)^2-4*ln(1+tan(d*x+c)^2)+8*I*arctan(tan(d*
x+c)))
```

Maxima [A]
time = 0.50, size = 108, normalized size = 0.68

$$\frac{10\,a^4\tan\left(dx+c\right)^6-48i\,a^4\tan\left(dx+c\right)^5-105\,a^4\tan\left(dx+c\right)^4+160i\,a^4\tan\left(dx+c\right)^3+240\,a^4\tan\left(dx+c\right)^2+480i\left(dx+c\right)a^4-240\,a^4\log\left(\tan\left(dx+c\right)^2+1\right)-480i\,a^4\tan\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/60*(10*a^4*tan(d*x + c)^6 - 48*I*a^4*tan(d*x + c)^5 - 105*a^4*tan(d*x + c
)^4 + 160*I*a^4*tan(d*x + c)^3 + 240*a^4*tan(d*x + c)^2 + 480*I*(d*x + c)*a
^4 - 240*a^4*log(tan(d*x + c)^2 + 1) - 480*I*a^4*tan(d*x + c))/d
```

Fricas [A]
time = 0.40, size = 254, normalized size = 1.59

$$\frac{4\left(270\,a^4e^{(10i\,dx+10i\,c)}+855\,a^4e^{(8i\,dx+8i\,c)}+1350\,a^4e^{(6i\,dx+6i\,c)}+1125\,a^4e^{(4i\,dx+4i\,c)}+486\,a^4e^{(2i\,dx+2i\,c)}+86\,a^4+30\left(a^4e^{(12i\,dx+12i\,c)}+6\,a^4e^{(10i\,dx+10i\,c)}+15\,a^4e^{(8i\,dx+8i\,c)}+20\,a^4e^{(6i\,dx+6i\,c)}+15\,a^4e^{(4i\,dx+4i\,c)}+6\,a^4e^{(2i\,dx+2i\,c)}+a^4\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)\right)}{15\left(de^{(12i\,dx+12i\,c)}+6\,de^{(10i\,dx+10i\,c)}+15\,de^{(8i\,dx+8i\,c)}+20\,de^{(6i\,dx+6i\,c)}+15\,de^{(4i\,dx+4i\,c)}+6\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 4/15*(270*a^4*e^(10*I*d*x + 10*I*c) + 855*a^4*e^(8*I*d*x + 8*I*c) + 1350*a^
4*e^(6*I*d*x + 6*I*c) + 1125*a^4*e^(4*I*d*x + 4*I*c) + 486*a^4*e^(2*I*d*x +
 2*I*c) + 86*a^4 + 30*(a^4*e^(12*I*d*x + 12*I*c) + 6*a^4*e^(10*I*d*x + 10*I
*c) + 15*a^4*e^(8*I*d*x + 8*I*c) + 20*a^4*e^(6*I*d*x + 6*I*c) + 15*a^4*e^(4
*I*d*x + 4*I*c) + 6*a^4*e^(2*I*d*x + 2*I*c) + a^4)*log(e^(2*I*d*x + 2*I*c)
+ 1))/(d*e^(12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*I*c) + 15*d*e^(8*I*d*
x + 8*I*c) + 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x + 4*I*c) + 6*d*e^(2
*I*d*x + 2*I*c) + d)
```

Sympy [A]
time = 1.09, size = 246, normalized size = 1.54

$$\frac{8a^4\log\left(e^{2idx}+e^{-2ic}\right)}{d}+\frac{1080a^4e^{10ic}e^{10idx}+3420a^4e^{8ic}e^{8idx}+5400a^4e^{6ic}e^{6idx}+4500a^4e^{4ic}e^{4idx}+1944a^4e^{2ic}e^{2idx}+344a^4}{15de^{12ic}e^{12idx}+90de^{10ic}e^{10idx}+225de^{8ic}e^{8idx}+300de^{6ic}e^{6idx}+225de^{4ic}e^{4idx}+90de^{2ic}e^{2idx}+15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] 8*a**4*log(exp(2*I*d*x) + exp(-2*I*c))/d + (1080*a**4*exp(10*I*c)*exp(10*I*
d*x) + 3420*a**4*exp(8*I*c)*exp(8*I*d*x) + 5400*a**4*exp(6*I*c)*exp(6*I*d*x
) + 4500*a**4*exp(4*I*c)*exp(4*I*d*x) + 1944*a**4*exp(2*I*c)*exp(2*I*d*x) +
 344*a**4)/(15*d*exp(12*I*c)*exp(12*I*d*x) + 90*d*exp(10*I*c)*exp(10*I*d*x)
 + 225*d*exp(8*I*c)*exp(8*I*d*x) + 300*d*exp(6*I*c)*exp(6*I*d*x) + 225*d*ex
p(4*I*c)*exp(4*I*d*x) + 90*d*exp(2*I*c)*exp(2*I*d*x) + 15*d)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(142) = 284.
time = 1.02, size = 326, normalized size = 2.04

$$\frac{4\left(30\,a^4e^{(12i\,dx+12i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+180\,a^4e^{(10i\,dx+10i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+450\,a^4e^{(8i\,dx+8i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+600\,a^4e^{(6i\,dx+6i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+450\,a^4e^{(4i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+180\,a^4e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+270\,a^4e^{(10i\,dx+10i\,c)}+855\,a^4e^{(8i\,dx+8i\,c)}+1350\,a^4e^{(6i\,dx+6i\,c)}+1125\,a^4e^{(4i\,dx+4i\,c)}+486\,a^4e^{(2i\,dx+2i\,c)}+30\,a^4\log\left(e^{(2i\,dx+2i\,c)}+1\right)+86\,a^4\right)}{15\left(de^{(12i\,dx+12i\,c)}+6\,de^{(10i\,dx+10i\,c)}+15\,de^{(8i\,dx+8i\,c)}+20\,de^{(6i\,dx+6i\,c)}+15\,de^{(4i\,dx+4i\,c)}+6\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 4/15*(30*a^4*e^(12*I*d*x + 12*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 180*a^4*e
^(10*I*d*x + 10*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 450*a^4*e^(8*I*d*x + 8*
I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 600*a^4*e^(6*I*d*x + 6*I*c)*log(e^(2*I*
d*x + 2*I*c) + 1) + 450*a^4*e^(4*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1
) + 180*a^4*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 270*a^4*e^(1
0*I*d*x + 10*I*c) + 855*a^4*e^(8*I*d*x + 8*I*c) + 1350*a^4*e^(6*I*d*x + 6*I
*c) + 1125*a^4*e^(4*I*d*x + 4*I*c) + 486*a^4*e^(2*I*d*x + 2*I*c) + 30*a^4*l
og(e^(2*I*d*x + 2*I*c) + 1) + 86*a^4)/(d*e^(12*I*d*x + 12*I*c) + 6*d*e^(10*
I*d*x + 10*I*c) + 15*d*e^(8*I*d*x + 8*I*c) + 20*d*e^(6*I*d*x + 6*I*c) + 15*
d*e^(4*I*d*x + 4*I*c) + 6*d*e^(2*I*d*x + 2*I*c) + d)
```

Mupad [B]
time = 3.76, size = 100, normalized size = 0.62

$$-\frac{8\,a^4\,\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)-4\,a^4\,{\tan(c+d\,x)}^2+\frac{7\,a^4\,{\tan(c+d\,x)}^4}{4}-\frac{a^4\,{\tan(c+d\,x)}^6}{6}+a^4\,\tan(c+d\,x)\,8{}\mathrm{i}-\frac{a^4\,{\tan(c+d\,x)}^3\,8{}\mathrm{i}}{3}+\frac{a^4\,{\tan(c+d\,x)}^5\,4{}\mathrm{i}}{5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] -(8*a^4*log(tan(c + d*x) + 1i) + a^4*tan(c + d*x)*8i - 4*a^4*tan(c + d*x)^2
 - (a^4*tan(c + d*x)^3*8i)/3 + (7*a^4*tan(c + d*x)^4)/4 + (a^4*tan(c + d*x)
^5*4i)/5 - (a^4*tan(c + d*x)^6)/6)/d
```

3.35 $\int \tan^2(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal. Leaf size=116

$$-8a^4x+\frac{8ia^4\log(\cos(c+dx))}{d}+\frac{4a^4\tan(c+dx)}{d}-\frac{ia(a+ia\tan(c+dx))^3}{3d}-\frac{i(a+ia\tan(c+dx))^5}{5ad}-\frac{i(a^2+ia^2}{}$$

```
[Out] -8*a^4*x+8*I*a^4*ln(cos(d*x+c))/d+4*a^4*tan(d*x+c)/d-1/3*I*a*(a+I*a*tan(d*x
+c))^3/d-1/5*I*(a+I*a*tan(d*x+c))^5/a/d-I*(a^2+I*a^2*tan(d*x+c))^2/d
```

Rubi [A]
time = 0.06, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3624, 3559, 3558, 3556}

$$\frac{4a^4\tan(c+dx)}{d}+\frac{8ia^4\log(\cos(c+dx))}{d}-8a^4x-\frac{i(a^2+ia^2\tan(c+dx))^2}{d}-\frac{i(a+ia\tan(c+dx))^5}{5ad}-\frac{ia(a+ia\tan(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] -8*a^4*x + ((8*I)*a^4*Log[Cos[c + d*x]])/d + (4*a^4*Tan[c + d*x])/d - ((I/3
)*a*(a + I*a*Tan[c + d*x])^3)/d - ((I/5)*(a + I*a*Tan[c + d*x])^5)/(a*d) -
(I*(a^2 + I*a^2*Tan[c + d*x])^2)/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \tan^2(c+dx)(a+ia\tan(c+dx))^4\,dx &= -\frac{i(a+ia\tan(c+dx))^5}{5ad} - \int (a+ia\tan(c+dx))^4\,dx \\
&= -\frac{ia(a+ia\tan(c+dx))^3}{3d} - \frac{i(a+ia\tan(c+dx))^5}{5ad} - (2a)\int (a+ \\
&= -\frac{ia(a+ia\tan(c+dx))^3}{3d} - \frac{i(a+ia\tan(c+dx))^5}{5ad} - \frac{i(a^2+ia^2 \mathrm{t}}{} \\
&= -8a^4x + \frac{4a^4\tan(c+dx)}{d} - \frac{ia(a+ia\tan(c+dx))^3}{3d} - \frac{i(a+ia\,\mathrm{t}}{} \\
&= -8a^4x + \frac{8ia^4\log(\cos(c+dx))}{d} + \frac{4a^4\tan(c+dx)}{d} - \frac{ia(a+ia\,\mathrm{ta}}{3}
\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 294 vs. 2(116) = 232.
time = 2.18, size = 294, normalized size = 2.53

$$-\frac{a^4\sec(c)\sec^5(c+dx)\left(-90i\cos(2c+3dx)+150dx\cos(2c+3dx)-90i\cos(4c+3dx)+150dx\cos(4c+3dx)+30dx\cos(4c+5dx)+30dx\cos(6c+5dx)+30\cos(dx)\left(-7i+10dx-5i\log\left(\cos^2(c+dx)\right)\right)+30\cos(2c+dx)\left(-7i+10dx-5i\log\left(\cos^2(c+dx)\right)\right)-75i\cos(2c+3dx)\log\left(\cos^2(c+dx)\right)-75i\cos(4c+3dx)\log\left(\cos^2(c+dx)\right)-15i\cos(4c+5dx)\log\left(\cos^2(c+dx)\right)-15i\cos(6c+5dx)\log\left(\cos^2(c+dx)\right)-445\sin(dx)+345\sin(2c+dx)-275\sin(2c+3dx)+120\sin(4c+3dx)-79\sin(4c+5dx)\right)}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] -1/120*(a^4*Sec[c]*Sec[c + d*x]^5*((-90*I)*Cos[2*c + 3*d*x] + 150*d*x*Cos[2
*c + 3*d*x] - (90*I)*Cos[4*c + 3*d*x] + 150*d*x*Cos[4*c + 3*d*x] + 30*d*x*C
os[4*c + 5*d*x] + 30*d*x*Cos[6*c + 5*d*x] + 30*Cos[d*x]*(-7*I + 10*d*x - (5
*I)*Log[Cos[c + d*x]^2]) + 30*Cos[2*c + d*x]*(-7*I + 10*d*x - (5*I)*Log[Cos
[c + d*x]^2]) - (75*I)*Cos[2*c + 3*d*x]*Log[Cos[c + d*x]^2] - (75*I)*Cos[4*
c + 3*d*x]*Log[Cos[c + d*x]^2] - (15*I)*Cos[4*c + 5*d*x]*Log[Cos[c + d*x]^2
] - (15*I)*Cos[6*c + 5*d*x]*Log[Cos[c + d*x]^2] - 445*Sin[d*x] + 345*Sin[2*
c + d*x] - 275*Sin[2*c + 3*d*x] + 120*Sin[4*c + 3*d*x] - 79*Sin[4*c + 5*d*x
]))/d
```

Maple [A]
time = 0.05, size = 82, normalized size = 0.71

method	result

derivativedivides	$\frac{a^4\left(8\tan(dx+c)+\frac{\left(\tan^5(dx+c)\right)}{5}-i\left(\tan^4(dx+c)\right)-\frac{7\left(\tan^3(dx+c)\right)}{3}+4i\left(\tan^2(dx+c)\right)-4i\ln\left(1+\tan^2(dx+c)\right)-8\arctan(\tan(\right.}{d}$
default	$\frac{a^4\left(8\tan(dx+c)+\frac{\left(\tan^5(dx+c)\right)}{5}-i\left(\tan^4(dx+c)\right)-\frac{7\left(\tan^3(dx+c)\right)}{3}+4i\left(\tan^2(dx+c)\right)-4i\ln\left(1+\tan^2(dx+c)\right)-8\arctan(\tan(\right.}{d}$
risch	$\frac{16a^4c}{d}+\frac{4ia^4\left(210\,e^{8i(dx+c)}+555\,e^{6i(dx+c)}+655\,e^{4i(dx+c)}+365\,e^{2i(dx+c)}+79\right)}{15d\left(e^{2i(dx+c)}+1\right)^5}+\frac{8ia^4\ln\left(e^{2i(dx+c)}+1\right)}{d}$
norman	$-8a^4x+\frac{8a^4\tan(dx+c)}{d}-\frac{7a^4\left(\tan^3(dx+c)\right)}{3d}+\frac{a^4\left(\tan^5(dx+c)\right)}{5d}+\frac{4ia^4\left(\tan^2(dx+c)\right)}{d}-\frac{ia^4\left(\tan^4(dx+c)\right)}{d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a^4*(8*tan(d*x+c)+1/5*tan(d*x+c)^5-I*tan(d*x+c)^4-7/3*tan(d*x+c)^3+4*I*
tan(d*x+c)^2-4*I*ln(1+tan(d*x+c)^2)-8*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.62, size = 95, normalized size = 0.82

$$\frac{3\,a^4\tan\left(dx+c\right)^5-15i\,a^4\tan\left(dx+c\right)^4-35\,a^4\tan\left(dx+c\right)^3+60i\,a^4\tan\left(dx+c\right)^2-120\left(dx+c\right)a^4-60i\,a^4\log\left(\tan\left(dx+c\right)^2+1\right)+120\,a^4\tan\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/15*(3*a^4*tan(d*x + c)^5 - 15*I*a^4*tan(d*x + c)^4 - 35*a^4*tan(d*x + c)^
3 + 60*I*a^4*tan(d*x + c)^2 - 120*(d*x + c)*a^4 - 60*I*a^4*log(tan(d*x + c)
^2 + 1) + 120*a^4*tan(d*x + c))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(98) = 196.
time = 0.42, size = 217, normalized size = 1.87

$$-\frac{4\left(-210i\,a^4e^{(8i\,dx+8i\,c)}-555i\,a^4e^{(6i\,dx+6i\,c)}-655i\,a^4e^{(4i\,dx+4i\,c)}-365i\,a^4e^{(2i\,dx+2i\,c)}-79i\,a^4+30\left(-i\,a^4e^{(10i\,dx+10i\,c)}-5i\,a^4e^{(8i\,dx+8i\,c)}-10i\,a^4e^{(6i\,dx+6i\,c)}-10i\,a^4e^{(4i\,dx+4i\,c)}-5i\,a^4e^{(2i\,dx+2i\,c)}-i\,a^4\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)\right)}{15\left(de^{(10i\,dx+10i\,c)}+5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}+10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -4/15*(-210*I*a^4*e^(8*I*d*x + 8*I*c) - 555*I*a^4*e^(6*I*d*x + 6*I*c) - 655
*I*a^4*e^(4*I*d*x + 4*I*c) - 365*I*a^4*e^(2*I*d*x + 2*I*c) - 79*I*a^4 + 30*
(-I*a^4*e^(10*I*d*x + 10*I*c) - 5*I*a^4*e^(8*I*d*x + 8*I*c) - 10*I*a^4*e^(6
*I*d*x + 6*I*c) - 10*I*a^4*e^(4*I*d*x + 4*I*c) - 5*I*a^4*e^(2*I*d*x + 2*I*c
) - I*a^4)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(
8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*
d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(97) = 194.
time = 0.33, size = 218, normalized size = 1.88

$$\frac{8ia^4\log\left(e^{2idx}+e^{-2ic}\right)}{d}+\frac{840ia^4e^{8ic}e^{8idx}+2220ia^4e^{6ic}e^{6idx}+2620ia^4e^{4ic}e^{4idx}+1460ia^4e^{2ic}e^{2idx}+316ia^4}{15de^{10ic}e^{10idx}+75de^{8ic}e^{8idx}+150de^{6ic}e^{6idx}+150de^{4ic}e^{4idx}+75de^{2ic}e^{2idx}+15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] 8*I*a**4*log(exp(2*I*d*x) + exp(-2*I*c))/d + (840*I*a**4*exp(8*I*c)*exp(8*I
*d*x) + 2220*I*a**4*exp(6*I*c)*exp(6*I*d*x) + 2620*I*a**4*exp(4*I*c)*exp(4*
I*d*x) + 1460*I*a**4*exp(2*I*c)*exp(2*I*d*x) + 316*I*a**4)/(15*d*exp(10*I*c
)*exp(10*I*d*x) + 75*d*exp(8*I*c)*exp(8*I*d*x) + 150*d*exp(6*I*c)*exp(6*I*d
*x) + 150*d*exp(4*I*c)*exp(4*I*d*x) + 75*d*exp(2*I*c)*exp(2*I*d*x) + 15*d)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(98) = 196.
time = 0.85, size = 274, normalized size = 2.36

$$-\frac{4\left(-30i\,a^4e^{(10i\,dx+10i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-150i\,a^4e^{(8i\,dx+8i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-300i\,a^4e^{(6i\,dx+6i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-300i\,a^4e^{(4i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-150i\,a^4e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-210i\,a^4e^{(8i\,dx+8i\,c)}-555i\,a^4e^{(6i\,dx+6i\,c)}-655i\,a^4e^{(4i\,dx+4i\,c)}-365i\,a^4e^{(2i\,dx+2i\,c)}-30i\,a^4\log\left(e^{(2i\,dx+2i\,c)}+1\right)-79i\,a^4\right)}{15\left(de^{(10i\,dx+10i\,c)}+5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}+10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -4/15*(-30*I*a^4*e^(10*I*d*x + 10*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 150*I
*a^4*e^(8*I*d*x + 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 300*I*a^4*e^(6*I*d*
x + 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 300*I*a^4*e^(4*I*d*x + 4*I*c)*log
(e^(2*I*d*x + 2*I*c) + 1) - 150*I*a^4*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x +
2*I*c) + 1) - 210*I*a^4*e^(8*I*d*x + 8*I*c) - 555*I*a^4*e^(6*I*d*x + 6*I*c)
 - 655*I*a^4*e^(4*I*d*x + 4*I*c) - 365*I*a^4*e^(2*I*d*x + 2*I*c) - 30*I*a^4
*log(e^(2*I*d*x + 2*I*c) + 1) - 79*I*a^4)/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^
(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5
*d*e^(2*I*d*x + 2*I*c) + d)
```

Mupad [B]
time = 3.73, size = 87, normalized size = 0.75

$$-\frac{\frac{7\,a^4\,\tan(c+d\,x)^3}{3}-8\,a^4\,\tan(c+d\,x)-\frac{a^4\,\tan(c+d\,x)^5}{5}+a^4\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,8\mathrm{i}-a^4\,\tan(c+d\,x)^2\,4\mathrm{i}+a^4\,\tan(c+d\,x)^4\,1\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] -(a^4*log(tan(c + d*x) + 1i)*8i - 8*a^4*tan(c + d*x) - a^4*tan(c + d*x)^2*4
i + (7*a^4*tan(c + d*x)^3)/3 + a^4*tan(c + d*x)^4*1i - (a^4*tan(c + d*x)^5)
/5)/d
```

3.36 $\int \tan(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal. Leaf size=108

$$-8ia^4x-\frac{8a^4\log(\cos(c+dx))}{d}+\frac{4ia^4\tan(c+dx)}{d}+\frac{a(a+ia\tan(c+dx))^3}{3d}+\frac{(a+ia\tan(c+dx))^4}{4d}+\frac{(a^2+ia^2\tan(c+dx))^2}{d}$$

```
[Out] -8*I*a^4*x-8*a^4*ln(cos(d*x+c))/d+4*I*a^4*tan(d*x+c)/d+1/3*a*(a+I*a*tan(d*x
+c))^3/d+1/4*(a+I*a*tan(d*x+c))^4/d+(a^2+I*a^2*tan(d*x+c))^2/d
```

Rubi [A]
time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3608, 3559, 3558, 3556}

$$\frac{4ia^4\tan(c+dx)}{d}-\frac{8a^4\log(\cos(c+dx))}{d}-8ia^4x+\frac{(a^2+ia^2\tan(c+dx))^2}{d}+\frac{a(a+ia\tan(c+dx))^3}{3d}+\frac{(a+ia\tan(c+dx))^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (-8*I)*a^4*x - (8*a^4*Log[Cos[c + d*x]])/d + ((4*I)*a^4*Tan[c + d*x])/d + (
a*(a + I*a*Tan[c + d*x])^3)/(3*d) + (a + I*a*Tan[c + d*x])^4/(4*d) + (a^2 +
 I*a^2*Tan[c + d*x])^2/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \tan(c+dx)(a+ia\tan(c+dx))^4\,dx &= \frac{(a+ia\tan(c+dx))^4}{4d} - i\int (a+ia\tan(c+dx))^4\,dx \\
&= \frac{a(a+ia\tan(c+dx))^3}{3d} + \frac{(a+ia\tan(c+dx))^4}{4d} - (2ia)\int (a+ia \\
&= \frac{a(a+ia\tan(c+dx))^3}{3d} + \frac{(a+ia\tan(c+dx))^4}{4d} + \frac{(a^2+ia^2\tan(c}{d} \\
&= -8ia^4x + \frac{4ia^4\tan(c+dx)}{d} + \frac{a(a+ia\tan(c+dx))^3}{3d} + \frac{(a+ia\,\mathrm{ta}}{4} \\
&= -8ia^4x - \frac{8a^4\log(\cos(c+dx))}{d} + \frac{4ia^4\tan(c+dx)}{d} + \frac{a(a+ia\,\mathrm{tan}}{3a}
\end{aligned}
$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 231 vs. 2(108) = 216.
time = 0.96, size = 231, normalized size = 2.14

$$
-\frac{ia^4\sec(c)\sec^4(c+dx)\left(-12i\cos(3c+2dx)+24dx\cos(3c+2dx)+6dx\cos(3c+4dx)+6dx\cos(5c+4dx)+12\cos(c+2dx)\left(-i+2dx-i\log\left(\cos^2(c+dx)\right)\right)+3\cos(c)\left(-7i+12dx-6i\log\left(\cos^2(c+dx)\right)\right)-12i\cos(3c+2dx)\log\left(\cos^2(c+dx)\right)-3i\cos(3c+4dx)\log\left(\cos^2(c+dx)\right)-3i\cos(5c+4dx)\log\left(\cos^2(c+dx)\right)+42\sin(c)-38\sin(c+2dx)+18\sin(3c+2dx)-14\sin(3c+4dx)\right)}{12d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((-1/12*I)*a^4*Sec[c]*Sec[c + d*x]^4*((-12*I)*Cos[3*c + 2*d*x] + 24*d*x*Cos
[3*c + 2*d*x] + 6*d*x*Cos[3*c + 4*d*x] + 6*d*x*Cos[5*c + 4*d*x] + 12*Cos[c
+ 2*d*x]*(-I + 2*d*x - I*Log[Cos[c + d*x]^2]) + 3*Cos[c]*(-7*I + 12*d*x - (
6*I)*Log[Cos[c + d*x]^2]) - (12*I)*Cos[3*c + 2*d*x]*Log[Cos[c + d*x]^2] - (
3*I)*Cos[3*c + 4*d*x]*Log[Cos[c + d*x]^2] - (3*I)*Cos[5*c + 4*d*x]*Log[Cos[
c + d*x]^2] + 42*Sin[c] - 38*Sin[c + 2*d*x] + 18*Sin[3*c + 2*d*x] - 14*Sin[
3*c + 4*d*x]))/d
```

Maple [A]
time = 0.06, size = 72, normalized size = 0.67

method	result
derivativedivides	$\frac{a^4\left(8i\tan(dx+c)+\frac{\left(\tan^4(dx+c)\right)}{4}-\frac{4i\left(\tan^3(dx+c)\right)}{3}-\frac{7\left(\tan^2(dx+c)\right)}{2}+4\ln\left(1+\tan^2(dx+c)\right)-8i\arctan(\tan(dx+c))\right)}{d}$
default	$\frac{a^4\left(8i\tan(dx+c)+\frac{\left(\tan^4(dx+c)\right)}{4}-\frac{4i\left(\tan^3(dx+c)\right)}{3}-\frac{7\left(\tan^2(dx+c)\right)}{2}+4\ln\left(1+\tan^2(dx+c)\right)-8i\arctan(\tan(dx+c))\right)}{d}$

risch	$\frac{16ia^4c}{d} - \frac{4a^4(30\,e^{6i(dx+c)}+63\,e^{4i(dx+c)}+50\,e^{2i(dx+c)}+14)}{3d(e^{2i(dx+c)}+1)^4} - \frac{8a^4\ln(e^{2i(dx+c)}+1)}{d}$
norman	$-\frac{7a^4(\tan^2(dx+c))}{2d} + \frac{a^4(\tan^4(dx+c))}{4d} - 8ia^4x + \frac{8ia^4\tan(dx+c)}{d} - \frac{4ia^4(\tan^3(dx+c))}{3d} + \frac{4a^4\ln(1+\tan^2(dx+c}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a^4*(8*I*tan(d*x+c)+1/4*tan(d*x+c)^4-4/3*I*tan(d*x+c)^3-7/2*tan(d*x+c)^
2+4*ln(1+tan(d*x+c)^2)-8*I*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.53, size = 82, normalized size = 0.76

$$\frac{3\,a^4\tan{(dx+c)}^4 - 16i\,a^4\tan{(dx+c)}^3 - 42\,a^4\tan{(dx+c)}^2 - 96i\,(dx+c)a^4 + 48\,a^4\log\left(\tan{(dx+c)}^2+1\right) + 96i\,a^4\tan{(dx+c)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/12*(3*a^4*tan(d*x + c)^4 - 16*I*a^4*tan(d*x + c)^3 - 42*a^4*tan(d*x + c)^
2 - 96*I*(d*x + c)*a^4 + 48*a^4*log(tan(d*x + c)^2 + 1) + 96*I*a^4*tan(d*x
+ c))/d
```

Fricas [A]
time = 0.44, size = 174, normalized size = 1.61

$$-\frac{4\left(30\,a^4e^{(6i\,dx+6i\,c)} + 63\,a^4e^{(4i\,dx+4i\,c)} + 50\,a^4e^{(2i\,dx+2i\,c)} + 14\,a^4 + 6\left(a^4e^{(8i\,dx+8i\,c)} + 4\,a^4e^{(6i\,dx+6i\,c)} + 6\,a^4e^{(4i\,dx+4i\,c)} + 4\,a^4e^{(2i\,dx+2i\,c)} + a^4\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)\right)}{3\left(de^{(8i\,dx+8i\,c)} + 4\,de^{(6i\,dx+6i\,c)} + 6\,de^{(4i\,dx+4i\,c)} + 4\,de^{(2i\,dx+2i\,c)} + d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -4/3*(30*a^4*e^(6*I*d*x + 6*I*c) + 63*a^4*e^(4*I*d*x + 4*I*c) + 50*a^4*e^(2
*I*d*x + 2*I*c) + 14*a^4 + 6*(a^4*e^(8*I*d*x + 8*I*c) + 4*a^4*e^(6*I*d*x +
6*I*c) + 6*a^4*e^(4*I*d*x + 4*I*c) + 4*a^4*e^(2*I*d*x + 2*I*c) + a^4)*log(e
^(2*I*d*x + 2*I*c) + 1))/(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) +
 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A]
time = 0.32, size = 170, normalized size = 1.57

$$-\frac{8a^4\log\left(e^{2idx}+e^{-2ic}\right)}{d} + \frac{-120a^4e^{6ic}e^{6idx} - 252a^4e^{4ic}e^{4idx} - 200a^4e^{2ic}e^{2idx} - 56a^4}{3de^{8ic}e^{8idx} + 12de^{6ic}e^{6idx} + 18de^{4ic}e^{4idx} + 12de^{2ic}e^{2idx} + 3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] -8*a**4*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-120*a**4*exp(6*I*c)*exp(6*I*d
*x) - 252*a**4*exp(4*I*c)*exp(4*I*d*x) - 200*a**4*exp(2*I*c)*exp(2*I*d*x) -
 56*a**4)/(3*d*exp(8*I*c)*exp(8*I*d*x) + 12*d*exp(6*I*c)*exp(6*I*d*x) + 18*
d*exp(4*I*c)*exp(4*I*d*x) + 12*d*exp(2*I*c)*exp(2*I*d*x) + 3*d)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(94) = 188$.
time = 0.59, size = 222, normalized size = 2.06

$$-\frac{4\left(6\,a^{4}e^{(8i\,dx+8i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+24\,a^{4}e^{(6i\,dx+6i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+36\,a^{4}e^{(4i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+24\,a^{4}e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+30\,a^{4}e^{(6i\,dx+6i\,c)}+63\,a^{4}e^{(4i\,dx+4i\,c)}+50\,a^{4}e^{(2i\,dx+2i\,c)}+6\,a^{4}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+14\,a^{4}\right)}{3\left(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -4/3*(6*a^4*e^(8*I*d*x + 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 24*a^4*e^(6*
I*d*x + 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 36*a^4*e^(4*I*d*x + 4*I*c)*lo
g(e^(2*I*d*x + 2*I*c) + 1) + 24*a^4*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*
I*c) + 1) + 30*a^4*e^(6*I*d*x + 6*I*c) + 63*a^4*e^(4*I*d*x + 4*I*c) + 50*a^
4*e^(2*I*d*x + 2*I*c) + 6*a^4*log(e^(2*I*d*x + 2*I*c) + 1) + 14*a^4)/(d*e^(
8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*
e^(2*I*d*x + 2*I*c) + d)
```

Mupad [B]
time = 3.72, size = 72, normalized size = 0.67

$$\frac{8\,a^{4}\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)-\frac{7\,a^{4}\,\tan(c+d\,x)^{2}}{2}+\frac{a^{4}\,\tan(c+d\,x)^{4}}{4}+a^{4}\,\tan(c+d\,x)\,8\mathrm{i}-\frac{a^{4}\,\tan(c+d\,x)^{3}\,4\mathrm{i}}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] (8*a^4*log(tan(c + d*x) + 1i) + a^4*tan(c + d*x)*8i - (7*a^4*tan(c + d*x)^2
)/2 - (a^4*tan(c + d*x)^3*4i)/3 + (a^4*tan(c + d*x)^4)/4)/d
```

3.37 $\int (a+ia\tan(c+dx))^4\,dx$

Optimal. Leaf size=89

$$8a^4x-\frac{8ia^4\log(\cos(c+dx))}{d}-\frac{4a^4\tan(c+dx)}{d}+\frac{ia(a+ia\tan(c+dx))^3}{3d}+\frac{i\left(a^2+ia^2\tan(c+dx)\right)^2}{d}$$

```
[Out] 8*a^4*x-8*I*a^4*ln(cos(d*x+c))/d-4*a^4*tan(d*x+c)/d+1/3*I*a*(a+I*a*tan(d*x+
c))^3/d+I*(a^2+I*a^2*tan(d*x+c))^2/d
```

Rubi [A]
time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3559, 3558, 3556}

$$-\frac{4a^4\tan(c+dx)}{d}-\frac{8ia^4\log(\cos(c+dx))}{d}+8a^4x+\frac{i\left(a^2+ia^2\tan(c+dx)\right)^2}{d}+\frac{ia(a+ia\tan(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] 8*a^4*x - ((8*I)*a^4*Log[Cos[c + d*x]])/d - (4*a^4*Tan[c + d*x])/d + ((I/3)
*a*(a + I*a*Tan[c + d*x])^3)/d + (I*(a^2 + I*a^2*Tan[c + d*x])^2)/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rubi steps

$$\begin{aligned}\int (a+ia\tan(c+dx))^4\,dx &= \frac{ia(a+ia\tan(c+dx))^3}{3d} + (2a)\int (a+ia\tan(c+dx))^3\,dx \\ &= \frac{ia(a+ia\tan(c+dx))^3}{3d} + \frac{i\left(a^2+ia^2\tan(c+dx)\right)^2}{d} + \left(4a^2\right)\int (a+ia\tan(c+ \\ &= 8a^4x - \frac{4a^4\tan(c+dx)}{d} + \frac{ia(a+ia\tan(c+dx))^3}{3d} + \frac{i\left(a^2+ia^2\tan(c+dx)\right)^2}{d} \\ &= 8a^4x - \frac{8ia^4\log(\cos(c+dx))}{d} - \frac{4a^4\tan(c+dx)}{d} + \frac{ia(a+ia\tan(c+dx))^3}{3d} +\end{aligned}$$

Mathematica [A]
time = 0.91, size = 176, normalized size = 1.98

$$\frac{a^4\sec(c)\sec^3(c+dx)\left(6dx\cos(2c+3dx)+6dx\cos(4c+3dx)+3\cos(dx)\left(-2i+6dx-3i\log\left(\cos^2(c+dx)\right)\right)+3\cos(2c+dx)\left(-2i+6dx-3i\log\left(\cos^2(c+dx)\right)\right)-3i\cos(2c+3dx)\log\left(\cos^2(c+dx)\right)-3i\cos(4c+3dx)\log\left(\cos^2(c+dx)\right)-21\sin(dx)+12\sin(2c+dx)-11\sin(2c+3dx)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (a^4*Sec[c]*Sec[c + d*x]^3*(6*d*x*Cos[2*c + 3*d*x] + 6*d*x*Cos[4*c + 3*d*x]
 + 3*Cos[d*x]*(-2*I + 6*d*x - (3*I)*Log[Cos[c + d*x]^2]) + 3*Cos[2*c + d*x]
*(-2*I + 6*d*x - (3*I)*Log[Cos[c + d*x]^2]) - (3*I)*Cos[2*c + 3*d*x]*Log[Co
s[c + d*x]^2] - (3*I)*Cos[4*c + 3*d*x]*Log[Cos[c + d*x]^2] - 21*Sin[d*x] +
12*Sin[2*c + d*x] - 11*Sin[2*c + 3*d*x]))/(6*d)
```

Maple [A]
time = 0.04, size = 61, normalized size = 0.69

method	result	size
derivativedivides	$\frac{a^4\left(-7\tan(dx+c)+\frac{\left(\tan^3(dx+c)\right)}{3}-2i\left(\tan^2(dx+c)\right)+4i\ln\left(1+\tan^2(dx+c)\right)+8\arctan(\tan(dx+c))\right)}{d}$	61
default	$\frac{a^4\left(-7\tan(dx+c)+\frac{\left(\tan^3(dx+c)\right)}{3}-2i\left(\tan^2(dx+c)\right)+4i\ln\left(1+\tan^2(dx+c)\right)+8\arctan(\tan(dx+c))\right)}{d}$	61
norman	$8a^4x-\frac{7a^4\tan(dx+c)}{d}+\frac{a^4\left(\tan^3(dx+c)\right)}{3d}-\frac{2ia^4\left(\tan^2(dx+c)\right)}{d}+\frac{4ia^4\ln\left(1+\tan^2(dx+c)\right)}{d}$	75
risch	$-\frac{16a^4c}{d}-\frac{4ia^4\left(18\,\mathrm{e}^{4i(dx+c)}+27\,\mathrm{e}^{2i(dx+c)}+11\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}-\frac{8ia^4\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{d}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a^4*(-7*tan(d*x+c)+1/3*tan(d*x+c)^3-2*I*tan(d*x+c)^2+4*I*ln(1+tan(d*x+c
)^2)+8*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.57, size = 108, normalized size = 1.21

$$a^4x+\frac{(\tan(dx+c)^3+3\,dx+3\,c-3\,\tan(dx+c))a^4}{3\,d}+\frac{6\,(dx+c-\tan(dx+c))a^4}{d}+\frac{2i\,a^4\left(\frac{1}{\sin(dx+c)^2-1}-\log\left(\sin(dx+c)^2-1\right)\right)}{d}+\frac{4i\,a^4\log(\sec(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 1/3*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^4/d + 6*(d*x
+ c - tan(d*x + c))*a^4/d + 2*I*a^4*(1/(sin(d*x + c)^2 - 1) - log(sin(d*x +
 c)^2 - 1))/d + 4*I*a^4*log(sec(d*x + c))/d
```

Fricas [A]
time = 0.43, size = 137, normalized size = 1.54

$$-\frac{4\left(18i\,a^4e^{(4i\,dx+4i\,c)}+27i\,a^4e^{(2i\,dx+2i\,c)}+11i\,a^4+6\left(i\,a^4e^{(6i\,dx+6i\,c)}+3i\,a^4e^{(4i\,dx+4i\,c)}+3i\,a^4e^{(2i\,dx+2i\,c)}+i\,a^4\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)\right)}{3\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -4/3*(18*I*a^4*e^(4*I*d*x + 4*I*c) + 27*I*a^4*e^(2*I*d*x + 2*I*c) + 11*I*a^
4 + 6*(I*a^4*e^(6*I*d*x + 6*I*c) + 3*I*a^4*e^(4*I*d*x + 4*I*c) + 3*I*a^4*e^
(2*I*d*x + 2*I*c) + I*a^4)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(6*I*d*x + 6*
I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A]
time = 0.21, size = 138, normalized size = 1.55

$$-\frac{8ia^4\log\left(e^{2idx}+e^{-2ic}\right)}{d}+\frac{-72ia^4e^{4ic}e^{4idx}-108ia^4e^{2ic}e^{2idx}-44ia^4}{3de^{6ic}e^{6idx}+9de^{4ic}e^{4idx}+9de^{2ic}e^{2idx}+3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**4,x)
```

```
[Out] -8*I*a**4*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-72*I*a**4*exp(4*I*c)*exp(4*
I*d*x) - 108*I*a**4*exp(2*I*c)*exp(2*I*d*x) - 44*I*a**4)/(3*d*exp(6*I*c)*ex
p(6*I*d*x) + 9*d*exp(4*I*c)*exp(4*I*d*x) + 9*d*exp(2*I*c)*exp(2*I*d*x) + 3*
d)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(77) = 154.
time = 0.51, size = 170, normalized size = 1.91

$$-\frac{4\left(6i\,a^4e^{(6i\,dx+6i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+18i\,a^4e^{(4i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+18i\,a^4e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+18i\,a^4e^{(4i\,dx+4i\,c)}+27i\,a^4e^{(2i\,dx+2i\,c)}+6i\,a^4\log\left(e^{(2i\,dx+2i\,c)}+1\right)+11i\,a^4\right)}{3\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -4/3*(6*I*a^4*e^(6*I*d*x + 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 18*I*a^4*e
^(4*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 18*I*a^4*e^(2*I*d*x + 2*I
*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 18*I*a^4*e^(4*I*d*x + 4*I*c) + 27*I*a^4*
e^(2*I*d*x + 2*I*c) + 6*I*a^4*log(e^(2*I*d*x + 2*I*c) + 1) + 11*I*a^4)/(d*e
^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Mupad [B]
time = 3.69, size = 59, normalized size = 0.66

$$\frac{\frac{a^4\tan(c+d\,x)^3}{3}-7\,a^4\tan(c+d\,x)+a^4\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,8\mathrm{i}-a^4\tan(c+d\,x)^2\,2\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] (a^4*log(tan(c + d*x) + 1i)*8i - 7*a^4*tan(c + d*x) - a^4*tan(c + d*x)^2*2i
 + (a^4*tan(c + d*x)^3)/3)/d
```

3.38 $\int \cot(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal. Leaf size=86

$$8ia^4x+\frac{7a^4\log(\cos(c+dx))}{d}+\frac{a^4\log(\sin(c+dx))}{d}-\frac{(a^2+ia^2\tan(c+dx))^2}{2d}-\frac{3(a^4+ia^4\tan(c+dx))}{d}$$

```
[Out] 8*I*a^4*x+7*a^4*ln(cos(d*x+c))/d+a^4*ln(sin(d*x+c))/d-1/2*(a^2+I*a^2*tan(d*
x+c))^2/d-3*(a^4+I*a^4*tan(d*x+c))/d
```

Rubi [A]
time = 0.13, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3637, 3675, 3670, 3556, 3612}

$$-\frac{3(a^4+ia^4\tan(c+dx))}{d}+\frac{a^4\log(\sin(c+dx))}{d}+\frac{7a^4\log(\cos(c+dx))}{d}+8ia^4x-\frac{(a^2+ia^2\tan(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (8*I)*a^4*x + (7*a^4*Log[Cos[c + d*x]])/d + (a^4*Log[Sin[c + d*x]])/d - (a^
2 + I*a^2*Tan[c + d*x])^2/(2*d) - (3*(a^4 + I*a^4*Tan[c + d*x]))/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3670

```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)]))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[B*(d/
b), Int[Tan[e + f*x], x], x] + Dist[1/b, Int[Simp[A*b*c + (A*b*d + B*(b*c -
 a*d))*Tan[e + f*x], x]/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d,
e, f, A, B}, x] && NeQ[b*c - a*d, 0]
```

Rule 3675

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[
e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && G
tQ[m, 1] &&  !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)(a+ia\tan(c+dx))^4\,dx &= -\frac{(a^2+ia^2\tan(c+dx))^2}{2d}+\frac{1}{2}a\int \cot(c+dx)(a+ia\tan(c+dx)) \\ &= -\frac{(a^2+ia^2\tan(c+dx))^2}{2d}-\frac{3(a^4+ia^4\tan(c+dx))}{d}+\frac{1}{2}a\int \cot(c \\ &= -\frac{(a^2+ia^2\tan(c+dx))^2}{2d}-\frac{3(a^4+ia^4\tan(c+dx))}{d}+\frac{1}{2}a\int \cot(c \\ &= 8ia^4x+\frac{7a^4\log(\cos(c+dx))}{d}-\frac{(a^2+ia^2\tan(c+dx))^2}{2d}-\frac{3(a^4+ \\ &= 8ia^4x+\frac{7a^4\log(\cos(c+dx))}{d}+\frac{a^4\log(\sin(c+dx))}{d}-\frac{(a^2+ia^2\,\mathrm{ta}}{2}\end{aligned}$$

Mathematica [A]

time = 1.27, size = 159, normalized size = 1.85

$$\frac{a^4\sec(c)\sec^2(c+dx)\left(16idx\cos(3c+2dx)+7\cos(3c+2dx)\log\left(\cos^2(c+dx)\right)+\cos(3c+2dx)\log\left(\sin^2(c+dx)\right)+\cos(c+2dx)\left(16idx+7\log\left(\cos^2(c+dx)\right)+\log\left(\sin^2(c+dx)\right)\right)+2\cos(c)\left(2+16idx+7\log\left(\cos^2(c+dx)\right)+\log\left(\sin^2(c+dx)\right)\right)+16i\sin(c)-16i\sin(c+2dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (a^4*Sec[c]*Sec[c + d*x]^2*((16*I)*d*x*Cos[3*c + 2*d*x] + 7*Cos[3*c + 2*d*x
]*Log[Cos[c + d*x]^2] + Cos[3*c + 2*d*x]*Log[Sin[c + d*x]^2] + Cos[c + 2*d*
```

```
x]*((16*I)*d*x + 7*Log[Cos[c + d*x]^2] + Log[Sin[c + d*x]^2]) + 2*Cos[c]*(2
 + (16*I)*d*x + 7*Log[Cos[c + d*x]^2] + Log[Sin[c + d*x]^2]) + (16*I)*Sin[c
] - (16*I)*Sin[c + 2*d*x]))/(8*d)
```

Maple [A]
time = 0.20, size = 82, normalized size = 0.95

method	result	size
norman	$\frac{a^4(\tan^2(dx+c))}{2d}+8ia^4x-\frac{4ia^4\tan(dx+c)}{d}+\frac{a^4\ln(\tan(dx+c))}{d}-\frac{4a^4\ln(1+\tan^2(dx+c))}{d}$	73
derivativedivides	$\frac{a^4\left(\frac{(\tan^2(dx+c))}{2}+\ln(\cos(dx+c))\right)-4ia^4(\tan(dx+c)-dx-c)+6a^4\ln(\cos(dx+c))+4ia^4(dx+c)+a^4\ln(\sin(dx+c))}{d}$	82
default	$\frac{a^4\left(\frac{(\tan^2(dx+c))}{2}+\ln(\cos(dx+c))\right)-4ia^4(\tan(dx+c)-dx-c)+6a^4\ln(\cos(dx+c))+4ia^4(dx+c)+a^4\ln(\sin(dx+c))}{d}$	82
risch	$-\frac{16ia^4c}{d}+\frac{2a^4(5\,e^{2i(dx+c)}+4)}{d(e^{2i(dx+c)}+1)^2}+\frac{a^4\ln(e^{2i(dx+c)}-1)}{d}+\frac{7a^4\ln(e^{2i(dx+c)}+1)}{d}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
[Out] 1/d*(a^4*(1/2*tan(d*x+c)^2+ln(cos(d*x+c)))-4*I*a^4*(tan(d*x+c)-d*x-c)+6*a^4
*ln(cos(d*x+c))+4*I*a^4*(d*x+c)+a^4*ln(sin(d*x+c)))
```

Maxima [A]
time = 0.52, size = 67, normalized size = 0.78

$$\frac{a^4\tan(dx+c)^2+16i(dx+c)a^4-8a^4\log\left(\tan(dx+c)^2+1\right)+2a^4\log\left(\tan(dx+c)\right)-8i\,a^4\tan(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
[Out] 1/2*(a^4*tan(d*x + c)^2 + 16*I*(d*x + c)*a^4 - 8*a^4*log(tan(d*x + c)^2 + 1
) + 2*a^4*log(tan(d*x + c)) - 8*I*a^4*tan(d*x + c))/d
```

Fricas [A]
time = 0.41, size = 137, normalized size = 1.59

$$\frac{10\,a^4e^{(2i\,dx+2i\,c)}+8\,a^4+7\left(a^4e^{(4i\,dx+4i\,c)}+2\,a^4e^{(2i\,dx+2i\,c)}+a^4\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)+\left(a^4e^{(4i\,dx+4i\,c)}+2\,a^4e^{(2i\,dx+2i\,c)}+a^4\right)\log\left(e^{(2i\,dx+2i\,c)}-1\right)}{de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
[Out] (10*a^4*e^(2*I*d*x + 2*I*c) + 8*a^4 + 7*(a^4*e^(4*I*d*x + 4*I*c) + 2*a^4*e^
(2*I*d*x + 2*I*c) + a^4)*log(e^(2*I*d*x + 2*I*c) + 1) + (a^4*e^(4*I*d*x + 4
```

```
*I*c) + 2*a^4*e^(2*I*d*x + 2*I*c) + a^4)*log(e^(2*I*d*x + 2*I*c) - 1))/(d*e
^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A]
time = 0.33, size = 105, normalized size = 1.22

$$\frac{a^4\left(\log\left(e^{2idx}-e^{-2ic}\right)+7\log\left(e^{2idx}+e^{-2ic}\right)\right)}{d}+\frac{10a^4e^{2ic}e^{2idx}+8a^4}{de^{4ic}e^{4idx}+2de^{2ic}e^{2idx}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] a**4*(log(exp(2*I*d*x) - exp(-2*I*c)) + 7*log(exp(2*I*d*x) + exp(-2*I*c)))/
d + (10*a**4*exp(2*I*c)*exp(2*I*d*x) + 8*a**4)/(d*exp(4*I*c)*exp(4*I*d*x) +
 2*d*exp(2*I*c)*exp(2*I*d*x) + d)
```

Giac [A]
time = 0.91, size = 157, normalized size = 1.83

$$\frac{14\,a^4\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-32\,a^4\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)+14\,a^4\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+2\,a^4\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-\frac{21\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-16i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-46\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+16i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+21\,a^4}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/2*(14*a^4*log(tan(1/2*d*x + 1/2*c) + 1) - 32*a^4*log(tan(1/2*d*x + 1/2*c)
 + I) + 14*a^4*log(tan(1/2*d*x + 1/2*c) - 1) + 2*a^4*log(tan(1/2*d*x + 1/2*
c)) - (21*a^4*tan(1/2*d*x + 1/2*c)^4 - 16*I*a^4*tan(1/2*d*x + 1/2*c)^3 - 46
*a^4*tan(1/2*d*x + 1/2*c)^2 + 16*I*a^4*tan(1/2*d*x + 1/2*c) + 21*a^4)/(tan(
1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

Mupad [B]
time = 3.80, size = 64, normalized size = 0.74

$$\frac{a^4\tan(c+d\,x)^2}{2\,d}-\frac{8\,a^4\,\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)}{d}+\frac{a^4\,\ln\left(\tan(c+d\,x)\right)}{d}-\frac{a^4\tan(c+d\,x)\,4{}\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] (a^4*tan(c + d*x)^2)/(2*d) - (a^4*tan(c + d*x)*4i)/d - (8*a^4*log(tan(c + d
*x) + 1i))/d + (a^4*log(tan(c + d*x)))/d
```

3.39 $\int \cot^2(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal. Leaf size=71

$$-8a^4x+\frac{4ia^4\log(\cos(c+dx))}{d}+\frac{4ia^4\log(\sin(c+dx))}{d}-\frac{\cot(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{d}$$

```
[Out] -8*a^4*x+4*I*a^4*ln(cos(d*x+c))/d+4*I*a^4*ln(sin(d*x+c))/d-cot(d*x+c)*(a^2+
I*a^2*tan(d*x+c))^2/d
```

Rubi [A]
time = 0.06, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3634, 12, 3622, 3556}

$$\frac{4ia^4\log(\sin(c+dx))}{d}+\frac{4ia^4\log(\cos(c+dx))}{d}-8a^4x-\frac{\cot(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] -8*a^4*x + ((4*I)*a^4*Log[Cos[c + d*x]])/d + ((4*I)*a^4*Log[Sin[c + d*x]])/
d - (Cot[c + d*x]*(a^2 + I*a^2*Tan[c + d*x])^2)/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3622

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^2/((a_.) + (b_.)*tan[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[d*(2*b*c - a*d)*(x/b^2), x] + (Dist[d^2/b, In
t[Tan[e + f*x], x], x] + Dist[(b*c - a*d)^2/b^2, Int[1/(a + b*Tan[e + f*x])
, x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 +
 b^2, 0]
```

Rule 3634

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-a^2)*(b*c - a*d)*(a + b*Tan[e + f*x]
)^(m - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] + Di
```

```
st[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[b*(b*c*(m - 2) - a*d*(m - 2*n - 4)) + (a*b*c*(m - 2)
+ b^2*d*(n + 1) - a^2*d*(m + n - 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rubi steps

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^4\,dx = -\frac{\cot(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{d} - \int -4ia^2\cot(c+dx)(a+$$

$$= -\frac{\cot(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{d} + \left(4ia^2\right)\int \cot(c+dx)(a+$$

$$= -8a^4x - \frac{\cot(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{d} + \left(4ia^4\right)\int \cot(c+$$

$$= -8a^4x + \frac{4ia^4\log(\cos(c+dx))}{d} + \frac{4ia^4\log(\sin(c+dx))}{d} - \frac{\cot(c+$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 151 vs. 2(71) = 142.
time = 1.55, size = 151, normalized size = 2.13

$$\frac{a^4\csc(c)\csc(c+dx)\sec(c)\sec(c+dx)\left(6dx\cos(4c+2dx)-i\cos(4c+2dx)\log\left(\cos^2(c+dx)\right)+\cos(2dx)\left(-6dx+i\log\left(\cos^2(c+dx)\right)+i\log\left(\sin^2(c+dx)\right)\right)-i\cos(4c+2dx)\log\left(\sin^2(c+dx)\right)+2\sin(2dx)+4\text{ArcTan}(\tan(5c+dx))\sin(2c)\sin(2(c+dx))\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (a^4*Csc[c]*Csc[c + d*x]*Sec[c]*Sec[c + d*x]*(6*d*x*Cos[4*c + 2*d*x] - I*Co
s[4*c + 2*d*x]*Log[Cos[c + d*x]^2] + Cos[2*d*x]*(-6*d*x + I*Log[Cos[c + d*x
]^2] + I*Log[Sin[c + d*x]^2]) - I*Cos[4*c + 2*d*x]*Log[Sin[c + d*x]^2] + 2*
Sin[2*d*x] + 4*ArcTan[Tan[5*c + d*x]]*Sin[2*c]*Sin[2*(c + d*x)]))/(4*d)
```

Maple [A]
time = 0.18, size = 80, normalized size = 1.13

method	result	size
risch	$\frac{16a^4c}{d}-\frac{4ia^4}{d\left(e^{2i(dx+c)}+1\right)\left(e^{2i(dx+c)}-1\right)}+\frac{4ia^4\ln\left(e^{4i(dx+c)}-1\right)}{d}$	67
derivativedivides	$\frac{a^4(\tan(dx+c)-dx-c)+4ia^4\ln(\cos(dx+c))-6a^4(dx+c)+4ia^4\ln(\sin(dx+c))+a^4(-\cot(dx+c)-dx-c)}{d}$	80
default	$\frac{a^4(\tan(dx+c)-dx-c)+4ia^4\ln(\cos(dx+c))-6a^4(dx+c)+4ia^4\ln(\sin(dx+c))+a^4(-\cot(dx+c)-dx-c)}{d}$	80

norman	$\frac{\frac{a^4\left(\tan^2(dx+c)\right)}{d}-\frac{a^4}{d}-8a^4x\tan(dx+c)}{\tan(dx+c)}+\frac{4ia^4\ln(\tan(dx+c))}{d}-\frac{4ia^4\ln\left(1+\tan^2(dx+c)\right)}{d}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(tan(d*x+c)-d*x-c)+4*I*a^4*ln(cos(d*x+c))-6*a^4*(d*x+c)+4*I*a^4*ln
(sin(d*x+c))+a^4*(-cot(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.50, size = 67, normalized size = 0.94

$$-\frac{8\,(dx+c)a^4+4i\,a^4\log\left(\tan\left(dx+c\right)^2+1\right)-4i\,a^4\log\left(\tan\left(dx+c\right)\right)-a^4\tan\left(dx+c\right)+\frac{a^4}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -(8*(d*x + c)*a^4 + 4*I*a^4*log(tan(d*x + c)^2 + 1) - 4*I*a^4*log(tan(d*x +
 c)) - a^4*tan(d*x + c) + a^4/tan(d*x + c))/d
```

Fricas [A]
time = 0.40, size = 58, normalized size = 0.82

$$-\frac{4\left(i\,a^4+\left(-i\,a^4e^{(4i\,dx+4i\,c)}+i\,a^4\right)\log\left(e^{(4i\,dx+4i\,c)}-1\right)\right)}{de^{(4i\,dx+4i\,c)}-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -4*(I*a^4 + (-I*a^4*e^(4*I*d*x + 4*I*c) + I*a^4)*log(e^(4*I*d*x + 4*I*c) -
1))/(d*e^(4*I*d*x + 4*I*c) - d)
```

Sympy [A]
time = 0.19, size = 51, normalized size = 0.72

$$-\frac{4ia^4}{de^{4ic}e^{4idx}-d}+\frac{4ia^4\log\left(e^{4idx}-e^{-4ic}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] -4*I*a**4/(d*exp(4*I*c)*exp(4*I*d*x) - d) + 4*I*a**4*log(exp(4*I*d*x) - exp
(-4*I*c))/d
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(65) = 130$.
time = 1.02, size = 163, normalized size = 2.30

$$-\frac{-8i\,a^4\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)+32i\,a^4\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)-8i\,a^4\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)-8i\,a^4\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{-8i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+8i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^4}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/2*(-8*I*a^4*log(tan(1/2*d*x + 1/2*c) + 1) + 32*I*a^4*log(tan(1/2*d*x + 1
/2*c) + I) - 8*I*a^4*log(tan(1/2*d*x + 1/2*c) - 1) - 8*I*a^4*log(tan(1/2*d*
x + 1/2*c)) - a^4*tan(1/2*d*x + 1/2*c) - (-8*I*a^4*tan(1/2*d*x + 1/2*c)^3 -
 5*a^4*tan(1/2*d*x + 1/2*c)^2 + 8*I*a^4*tan(1/2*d*x + 1/2*c) + a^4)/(tan(1/
2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)))/d
```

Mupad [**B**]
time = 4.00, size = 63, normalized size = 0.89

$$\frac{a^4\tan(c+d\,x)}{d}-\frac{a^4\cot(c+d\,x)}{d}-\frac{a^4\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,8\mathrm{i}}{d}+\frac{a^4\,\ln\left(\tan(c+d\,x)\right)\,4\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] (a^4*tan(c + d*x))/d - (a^4*cot(c + d*x))/d - (a^4*log(tan(c + d*x) + 1i)*8
i)/d + (a^4*log(tan(c + d*x))*4i)/d
```

3.40 $\int \cot^3(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal. Leaf size=103

$$-8ia^4x-\frac{a^4\log(\cos(c+dx))}{d}-\frac{7a^4\log(\sin(c+dx))}{d}-\frac{\cot^2(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}-\frac{3i\cot(c+dx)\left(a^4\right.}{}$$

```
[Out] -8*I*a^4*x-a^4*ln(cos(d*x+c))/d-7*a^4*ln(sin(d*x+c))/d-1/2*cot(d*x+c)^2*(a^
2+I*a^2*tan(d*x+c))^2/d-3*I*cot(d*x+c)*(a^4+I*a^4*tan(d*x+c))/d
```

Rubi [A]
time = 0.15, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3634, 3674, 3670, 3556, 3612}

$$-\frac{7a^4\log(\sin(c+dx))}{d}-\frac{a^4\log(\cos(c+dx))}{d}-\frac{3i\cot(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{d}-8ia^4x-\frac{\cot^2(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (-8*I)*a^4*x - (a^4*Log[Cos[c + d*x]])/d - (7*a^4*Log[Sin[c + d*x]])/d - (C
ot[c + d*x]^2*(a^2 + I*a^2*Tan[c + d*x])^2)/(2*d) - ((3*I)*Cot[c + d*x]*(a^
4 + I*a^4*Tan[c + d*x]))/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3634

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-a^2)*(b*c - a*d)*(a + b*Tan[e + f*x]
)^(m - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] + Di
st[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[b*(b*c*(m - 2) - a*d*(m - 2*n - 4)) + (a*b*c*(m - 2)
+ b^2*d*(n + 1) - a^2*d*(m + n - 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
```

```
^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3670

```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)]))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[B*(d/
b), Int[Tan[e + f*x], x], x] + Dist[1/b, Int[Simp[A*b*c + (A*b*d + B*(b*c -
 a*d))*Tan[e + f*x], x]/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d,
e, f, A, B}, x] && NeQ[b*c - a*d, 0]
```

Rule 3674

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(
a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
- 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*
d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \cot^3(c+dx)(a+ia\tan(c+dx))^4\,dx &= -\frac{\cot^2(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}-\frac{1}{2}\int \cot^2(c+dx)(a+ic \\ &= -\frac{\cot^2(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}-\frac{3i\cot(c+dx)\left(a^4+ia^4\right.}{d} \\ &= -\frac{\cot^2(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}-\frac{3i\cot(c+dx)\left(a^4+ia^4\right.}{d} \\ &= -8ia^4x-\frac{a^4\log(\cos(c+dx))}{d}-\frac{\cot^2(c+dx)\left(a^2+ia^2\tan(c+d\right.}{2d} \\ &= -8ia^4x-\frac{a^4\log(\cos(c+dx))}{d}-\frac{7a^4\log(\sin(c+dx))}{d}-\frac{\cot^2(c+}{}\end{aligned}$$

Mathematica [A]
time = 1.33, size = 133, normalized size = 1.29

$$\frac{a^4\csc^2(c+dx)\left(-2-16idx+8i\cot(c)-8i\cos(c+2dx)\csc(c)-\log\left(\cos^2(c+dx)\right)-7\log\left(\sin^2(c+dx)\right)+\cos(2(c+dx))\left(16idx+\log\left(\cos^2(c+dx)\right)+7\log\left(\sin^2(c+dx)\right)\right)\right)(\cos(4dx)+i\sin(4dx))}{4d(\cos(dx)+i\sin(dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (a^4*Csc[c + d*x]^2*(-2 - (16*I)*d*x + (8*I)*Cot[c] - (8*I)*Cos[c + 2*d*x]*
Csc[c] - Log[Cos[c + d*x]^2] - 7*Log[Sin[c + d*x]^2] + Cos[2*(c + d*x)]*((1
6*I)*d*x + Log[Cos[c + d*x]^2] + 7*Log[Sin[c + d*x]^2]))*(Cos[4*d*x] + I*Si
n[4*d*x]))/(4*d*(Cos[d*x] + I*Sin[d*x])^4)
```

Maple [A]
time = 0.21, size = 87, normalized size = 0.84

method	result
norman	$\frac{-\frac{a^4}{2d}-\frac{4ia^4\tan(dx+c)}{d}-8ia^4x(\tan^2(dx+c))}{\tan(dx+c)^2}-\frac{7a^4\ln(\tan(dx+c))}{d}+\frac{4a^4\ln(1+\tan^2(dx+c))}{d}$
risch	$\frac{16ia^4c}{d}+\frac{2a^4(5\,e^{2i(dx+c)}-4)}{d(e^{2i(dx+c)}-1)^2}-\frac{7a^4\ln(e^{2i(dx+c)}-1)}{d}-\frac{a^4\ln(e^{2i(dx+c)}+1)}{d}$
derivativedivides	$\frac{-a^4\ln(\cos(dx+c))-4ia^4(dx+c)-6a^4\ln(\sin(dx+c))+4ia^4(-\cot(dx+c)-dx-c)+a^4\left(-\frac{(\cot^2(dx+c))}{2}-\ln(\sin(dx+c))\right)}{d}$
default	$\frac{-a^4\ln(\cos(dx+c))-4ia^4(dx+c)-6a^4\ln(\sin(dx+c))+4ia^4(-\cot(dx+c)-dx-c)+a^4\left(-\frac{(\cot^2(dx+c))}{2}-\ln(\sin(dx+c))\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^4*ln(cos(d*x+c))-4*I*a^4*(d*x+c)-6*a^4*ln(sin(d*x+c))+4*I*a^4*(-cot
(d*x+c)-d*x-c)+a^4*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c))))
```

Maxima [A]
time = 0.50, size = 68, normalized size = 0.66

$$-\frac{16i\,(dx+c)a^4-8\,a^4\log\left(\tan\left(dx+c\right)^2+1\right)+14\,a^4\log\left(\tan\left(dx+c\right)\right)+\frac{8i\,a^4\tan(dx+c)+a^4}{\tan(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/2*(16*I*(d*x + c)*a^4 - 8*a^4*log(tan(d*x + c)^2 + 1) + 14*a^4*log(tan(d
*x + c)) + (8*I*a^4*tan(d*x + c) + a^4)/tan(d*x + c)^2)/d
```

Fricas [A]
time = 0.39, size = 138, normalized size = 1.34

$$\frac{10\,a^4e^{(2i\,dx+2i\,c)}-8\,a^4-\left(a^4e^{(4i\,dx+4i\,c)}-2\,a^4e^{(2i\,dx+2i\,c)}+a^4\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-7\left(a^4e^{(4i\,dx+4i\,c)}-2\,a^4e^{(2i\,dx+2i\,c)}+a^4\right)\log\left(e^{(2i\,dx+2i\,c)}-1\right)}{de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] (10*a^4*e^(2*I*d*x + 2*I*c) - 8*a^4 - (a^4*e^(4*I*d*x + 4*I*c) - 2*a^4*e^(2
*I*d*x + 2*I*c) + a^4)*log(e^(2*I*d*x + 2*I*c) + 1) - 7*(a^4*e^(4*I*d*x + 4
*I*c) - 2*a^4*e^(2*I*d*x + 2*I*c) + a^4)*log(e^(2*I*d*x + 2*I*c) - 1))/(d*e
^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A]
time = 0.60, size = 107, normalized size = 1.04

$$\frac{a^4\left(-7\log\left(e^{2idx}-e^{-2ic}\right)-\log\left(e^{2idx}+e^{-2ic}\right)\right)}{d}+\frac{10a^4e^{2ic}e^{2idx}-8a^4}{de^{4ic}e^{4idx}-2de^{2ic}e^{2idx}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] a**4*(-7*log(exp(2*I*d*x) - exp(-2*I*c)) - log(exp(2*I*d*x) + exp(-2*I*c)))
/d + (10*a**4*exp(2*I*c)*exp(2*I*d*x) - 8*a**4)/(d*exp(4*I*c)*exp(4*I*d*x)
- 2*d*exp(2*I*c)*exp(2*I*d*x) + d)
```

Giac [A]
time = 1.21, size = 150, normalized size = 1.46

$$-\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+8\,a^4\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-128\,a^4\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)+8\,a^4\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+56\,a^4\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-16i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{84\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-16i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^4}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/8*(a^4*tan(1/2*d*x + 1/2*c)^2 + 8*a^4*log(tan(1/2*d*x + 1/2*c) + 1) - 12
8*a^4*log(tan(1/2*d*x + 1/2*c) + I) + 8*a^4*log(tan(1/2*d*x + 1/2*c) - 1) +
 56*a^4*log(tan(1/2*d*x + 1/2*c)) - 16*I*a^4*tan(1/2*d*x + 1/2*c) - (84*a^4
*tan(1/2*d*x + 1/2*c)^2 - 16*I*a^4*tan(1/2*d*x + 1/2*c) - a^4)/tan(1/2*d*x
+ 1/2*c)^2)/d
```

Mupad [B]
time = 3.97, size = 65, normalized size = 0.63

$$\frac{8\,a^4\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)}{d}-\frac{a^4\cot(c+d\,x)^2}{2\,d}-\frac{7\,a^4\,\ln\left(\tan(c+d\,x)\right)}{d}-\frac{a^4\cot(c+d\,x)\,4\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] (8*a^4*log(tan(c + d*x) + 1i))/d - (a^4*cot(c + d*x)*4i)/d - (a^4*cot(c + d
*x)^2)/(2*d) - (7*a^4*log(tan(c + d*x)))/d
```

3.41 $\int \cot^4(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal. Leaf size=103

$$8a^4x+\frac{4a^4\cot(c+dx)}{d}-\frac{8ia^4\log(\sin(c+dx))}{d}-\frac{a\cot^3(c+dx)(a+ia\tan(c+dx))^3}{3d}-\frac{i\cot^2(c+dx)\left(a^2+ia^2\,\mathrm{t}\right.}{d}$$

```
[Out] 8*a^4*x+4*a^4*cot(d*x+c)/d-8*I*a^4*ln(sin(d*x+c))/d-1/3*a*cot(d*x+c)^3*(a+I
*a*tan(d*x+c))^3/d-I*cot(d*x+c)^2*(a^2+I*a^2*tan(d*x+c))^2/d
```

Rubi [A]
time = 0.11, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3626, 3623, 3612, 3556}

$$\frac{4a^4\cot(c+dx)}{d}-\frac{8ia^4\log(\sin(c+dx))}{d}+8a^4x-\frac{i\cot^2(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{d}-\frac{a\cot^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] 8*a^4*x + (4*a^4*Cot[c + d*x])/d - ((8*I)*a^4*Log[Sin[c + d*x]])/d - (a*Cot
[c + d*x]^3*(a + I*a*Tan[c + d*x])^3)/(3*d) - (I*Cot[c + d*x]^2*(a^2 + I*a^
2*Tan[c + d*x])^2)/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3626

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*b*(a + b*Tan[e + f*x])^(m - 1)*((c +
 d*Tan[e + f*x])^(n + 1)/(f*(m - 1)*(a*c - b*d))), x] + Dist[2*(a^2/(a*c -
b*d)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2]
```

Rubi steps

$$\begin{aligned}\int \cot^4(c+dx)(a+ia\tan(c+dx))^4\,dx &= -\frac{a\cot^3(c+dx)(a+ia\tan(c+dx))^3}{3d}+(2ia)\int \cot^3(c+dx)(a+\\ &= -\frac{a\cot^3(c+dx)(a+ia\tan(c+dx))^3}{3d}-\frac{i\cot^2(c+dx)\left(a^2+ia^2\,\mathrm{t}\right.}{d}\\ &= \frac{4a^4\cot(c+dx)}{d}-\frac{a\cot^3(c+dx)(a+ia\tan(c+dx))^3}{3d}-\frac{i\cot^2(c}{}\\ &= 8a^4x+\frac{4a^4\cot(c+dx)}{d}-\frac{a\cot^3(c+dx)(a+ia\tan(c+dx))^3}{3d}-\\ &= 8a^4x+\frac{4a^4\cot(c+dx)}{d}-\frac{8ia^4\log(\sin(c+dx))}{d}-\frac{a\cot^3(c+dx)}{}\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 240 vs. 2(103) = 206.
time = 0.83, size = 240, normalized size = 2.33

$$\frac{a^4\csc(c)\csc^3(c+dx)(\cos(4dx)+i\sin(4dx))\left(6i\cos(2c+dx)-36dx\cos(2c+dx)-12dx\cos(2c+3dx)+12dx\cos(4c+3dx)+\cos(dx)\left(-6i+36dx-9i\log\left(\sin^2(c+dx)\right)\right)+9i\cos(2c+dx)\log\left(\sin^2(c+dx)\right)+3i\cos(2c+3dx)\log\left(\sin^2(c+dx)\right)-3i\cos(4c+3dx)\log\left(\sin^2(c+dx)\right)-21\sin(dx)-48\mathrm{ArcTan}(\tan(5c+dx))\sin(c)\sin^3(c+dx)-12\sin(2c+dx)+11\sin(2c+3dx)\right)}{6d(\cos(dx)+i\sin(dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (a^4*Csc[c]*Csc[c + d*x]^3*(Cos[4*d*x] + I*Sin[4*d*x])*((6*I)*Cos[2*c + d*x
] - 36*d*x*Cos[2*c + d*x] - 12*d*x*Cos[2*c + 3*d*x] + 12*d*x*Cos[4*c + 3*d*
x] + Cos[d*x]*(-6*I + 36*d*x - (9*I)*Log[Sin[c + d*x]^2]) + (9*I)*Cos[2*c +
 d*x]*Log[Sin[c + d*x]^2] + (3*I)*Cos[2*c + 3*d*x]*Log[Sin[c + d*x]^2] - (3
*I)*Cos[4*c + 3*d*x]*Log[Sin[c + d*x]^2] - 21*Sin[d*x] - 48*ArcTan[Tan[5*c
+ d*x]]*Sin[c]*Sin[c + d*x]^3 - 12*Sin[2*c + d*x] + 11*Sin[2*c + 3*d*x]))/(
6*d*(Cos[d*x] + I*Sin[d*x])^4)
```

Maple [**A**]
time = 0.21, size = 100, normalized size = 0.97

method	result
risch	$-\frac{16a^4c}{d}+\frac{4ia^4(18\,\mathrm{e}^{4i(dx+c)}-27\,\mathrm{e}^{2i(dx+c)}+11)}{3d(\mathrm{e}^{2i(dx+c)}-1)^3}-\frac{8ia^4\ln(\mathrm{e}^{2i(dx+c)}-1)}{d}$
derivativedivides	$\frac{a^4(dx+c)-4ia^4\ln(\sin(dx+c))-6a^4(-\cot(dx+c)-dx-c)+4ia^4\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+a^4\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\right.}{d}$
default	$\frac{a^4(dx+c)-4ia^4\ln(\sin(dx+c))-6a^4(-\cot(dx+c)-dx-c)+4ia^4\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+a^4\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\right.}{d}$
norman	$\frac{-\frac{a^4}{3d}+8a^4x(\tan^3(dx+c))+\frac{7a^4\left(\tan^2(dx+c)\right)}{d}-\frac{2ia^4\tan(dx+c)}{d}}{\tan(dx+c)^3}-\frac{8ia^4\ln(\tan(dx+c))}{d}+\frac{4ia^4\ln(1+\tan^2(dx+c))}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(d*x+c)-4*I*a^4*ln(sin(d*x+c))-6*a^4*(-cot(d*x+c)-d*x-c)+4*I*a^4*(
-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+a^4*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.50, size = 83, normalized size = 0.81

$$\frac{24\,(dx+c)a^4+12i\,a^4\log\left(\tan\left(dx+c\right)^2+1\right)-24i\,a^4\log\left(\tan\left(dx+c\right)\right)+\frac{21\,a^4\tan(dx+c)^2-6i\,a^4\tan(dx+c)-a^4}{\tan(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*(24*(d*x + c)*a^4 + 12*I*a^4*log(tan(d*x + c)^2 + 1) - 24*I*a^4*log(tan
(d*x + c)) + (21*a^4*tan(d*x + c)^2 - 6*I*a^4*tan(d*x + c) - a^4)/tan(d*x +
 c)^3)/d
```

Fricas [A]
time = 0.36, size = 139, normalized size = 1.35

$$-\frac{4\left(-18i\,a^4e^{(4i\,dx+4i\,c)}+27i\,a^4e^{(2i\,dx+2i\,c)}-11i\,a^4+6\left(i\,a^4e^{(6i\,dx+6i\,c)}-3i\,a^4e^{(4i\,dx+4i\,c)}+3i\,a^4e^{(2i\,dx+2i\,c)}-i\,a^4\right)\log\left(e^{(2i\,dx+2i\,c)}-1\right)\right)}{3\left(de^{(6i\,dx+6i\,c)}-3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -4/3*(-18*I*a^4*e^(4*I*d*x + 4*I*c) + 27*I*a^4*e^(2*I*d*x + 2*I*c) - 11*I*a
^4 + 6*(I*a^4*e^(6*I*d*x + 6*I*c) - 3*I*a^4*e^(4*I*d*x + 4*I*c) + 3*I*a^4*e
^(2*I*d*x + 2*I*c) - I*a^4)*log(e^(2*I*d*x + 2*I*c) - 1))/(d*e^(6*I*d*x + 6
*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [A]
time = 0.27, size = 136, normalized size = 1.32

$$-\frac{8ia^4\log\left(e^{2idx}-e^{-2ic}\right)}{d}+\frac{72ia^4e^{4ic}e^{4idx}-108ia^4e^{2ic}e^{2idx}+44ia^4}{3de^{6ic}e^{6idx}-9de^{4ic}e^{4idx}+9de^{2ic}e^{2idx}-3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] -8*I*a**4*log(exp(2*I*d*x) - exp(-2*I*c))/d + (72*I*a**4*exp(4*I*c)*exp(4*I
*d*x) - 108*I*a**4*exp(2*I*c)*exp(2*I*d*x) + 44*I*a**4)/(3*d*exp(6*I*c)*exp
(6*I*d*x) - 9*d*exp(4*I*c)*exp(4*I*d*x) + 9*d*exp(2*I*c)*exp(2*I*d*x) - 3*d
)
```

Giac [A]
time = 1.41, size = 146, normalized size = 1.42

$$\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-12i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+384i\,a^4\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)-192i\,a^4\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-87\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{-352i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-87\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^4}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(a^4*tan(1/2*d*x + 1/2*c)^3 - 12*I*a^4*tan(1/2*d*x + 1/2*c)^2 + 384*I*
a^4*log(tan(1/2*d*x + 1/2*c) + I) - 192*I*a^4*log(tan(1/2*d*x + 1/2*c)) - 8
7*a^4*tan(1/2*d*x + 1/2*c) - (-352*I*a^4*tan(1/2*d*x + 1/2*c)^3 - 87*a^4*ta
n(1/2*d*x + 1/2*c)^2 + 12*I*a^4*tan(1/2*d*x + 1/2*c) + a^4)/tan(1/2*d*x + 1
/2*c)^3)/d
```

Mupad [B]
time = 3.98, size = 68, normalized size = 0.66

$$\frac{7\,a^4\cot(c+d\,x)}{d}+\frac{16\,a^4\,\mathrm{atan}(2\tan(c+d\,x)+1\mathrm{i})}{d}-\frac{a^4\cot(c+d\,x)^3}{3\,d}-\frac{a^4\cot(c+d\,x)^2\,2\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] (7*a^4*cot(c + d*x))/d + (16*a^4*atan(2*tan(c + d*x) + 1i))/d - (a^4*cot(c
+ d*x)^2*2i)/d - (a^4*cot(c + d*x)^3)/(3*d)
```

3.42 $\int \cot^5(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal. Leaf size=134

$$8ia^4x+\frac{4ia^4\cot(c+dx)}{d}+\frac{8a^4\log(\sin(c+dx))}{d}-\frac{ia\cot^3(c+dx)(a+ia\tan(c+dx))^3}{3d}-\frac{\cot^4(c+dx)(a+ia\tan}{4d}$$

```
[Out] 8*I*a^4*x+4*I*a^4*cot(d*x+c)/d+8*a^4*ln(sin(d*x+c))/d-1/3*I*a*cot(d*x+c)^3*
(a+I*a*tan(d*x+c))^3/d-1/4*cot(d*x+c)^4*(a+I*a*tan(d*x+c))^4/d+cot(d*x+c)^2
*(a^2+I*a^2*tan(d*x+c))^2/d
```

Rubi [A]

time = 0.14, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3629, 3626, 3623, 3612, 3556}

$$\frac{4ia^4\cot(c+dx)}{d}+\frac{8a^4\log(\sin(c+dx))}{d}+8ia^4x+\frac{\cot^2(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{d}-\frac{\cot^4(c+dx)(a+ia\tan(c+dx))^4}{4d}-\frac{ia\cot^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (8*I)*a^4*x + ((4*I)*a^4*Cot[c + d*x])/d + (8*a^4*Log[Sin[c + d*x]])/d - ((
I/3)*a*Cot[c + d*x]^3*(a + I*a*Tan[c + d*x])^3)/d - (Cot[c + d*x]^4*(a + I*
a*Tan[c + d*x])^4)/(4*d) + (Cot[c + d*x]^2*(a^2 + I*a^2*Tan[c + d*x])^2)/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3626

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*b*(a + b*Tan[e + f*x])^(m - 1)*((c +
 d*Tan[e + f*x])^(n + 1)/(f*(m - 1)*(a*c - b*d))), x] + Dist[2*(a^2/(a*c -
b*d)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2]
```

Rule 3629

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-d)*(a + b*Tan[e + f*x])^m*((c + d*Ta
n[e + f*x])^(n + 1)/(f*m*(c^2 + d^2))), x] + Dist[a/(a*c - b*d), Int[(a + b
*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cot^5(c+dx)(a+ia\tan(c+dx))^4\,dx &= -\frac{\cot^4(c+dx)(a+ia\tan(c+dx))^4}{4d}+i\int \cot^4(c+dx)(a+ia\,\mathrm{ta}\\ &= -\frac{ia\cot^3(c+dx)(a+ia\tan(c+dx))^3}{3d}-\frac{\cot^4(c+dx)(a+ia\tan}{4d}\\ &= -\frac{ia\cot^3(c+dx)(a+ia\tan(c+dx))^3}{3d}-\frac{\cot^4(c+dx)(a+ia\tan}{4d}\\ &= \frac{4ia^4\cot(c+dx)}{d}-\frac{ia\cot^3(c+dx)(a+ia\tan(c+dx))^3}{3d}-\frac{\cot^4(c}{}\\ &= 8ia^4x+\frac{4ia^4\cot(c+dx)}{d}-\frac{ia\cot^3(c+dx)(a+ia\tan(c+dx))^3}{3d}\\ &= 8ia^4x+\frac{4ia^4\cot(c+dx)}{d}+\frac{8a^4\log(\sin(c+dx))}{d}-\frac{ia\cot^3(c+dx}{}\end{aligned}$$

Mathematica [A]
time = 0.75, size = 245, normalized size = 1.83

$$\frac{a^4\csc(c)\csc^4(c+dx)\left(-42i\cos(c)+38i\cos(c+2dx)+18i\cos(3c+2dx)-14i\cos(3c+4dx)+21\sin(c)+36idx\sin(c)+18\log\left(\sin^2(c+dx)\right)\sin(c)+12\sin(c+2dx)+24idx\sin(c+2dx)+12\log\left(\sin^2(c+dx)\right)\sin(c+2dx)-12\sin(3c+2dx)-24idx\sin(3c+2dx)-12\log\left(\sin^2(c+dx)\right)\sin(3c+2dx)-6idx\sin(3c+4dx)-3\log\left(\sin^2(c+dx)\right)\sin(3c+4dx)+6idx\sin(5c+4dx)+3\log\left(\sin^2(c+dx)\right)\sin(5c+4dx)\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (a^4*Csc[c]*Csc[c + d*x]^4*((-42*I)*Cos[c] + (38*I)*Cos[c + 2*d*x] + (18*I)
*Cos[3*c + 2*d*x] - (14*I)*Cos[3*c + 4*d*x] + 21*Sin[c] + (36*I)*d*x*Sin[c]
 + 18*Log[Sin[c + d*x]^2]*Sin[c] + 12*Sin[c + 2*d*x] + (24*I)*d*x*Sin[c + 2
*d*x] + 12*Log[Sin[c + d*x]^2]*Sin[c + 2*d*x] - 12*Sin[3*c + 2*d*x] - (24*I
)*d*x*Sin[3*c + 2*d*x] - 12*Log[Sin[c + d*x]^2]*Sin[3*c + 2*d*x] - (6*I)*d*
x*Sin[3*c + 4*d*x] - 3*Log[Sin[c + d*x]^2]*Sin[3*c + 4*d*x] + (6*I)*d*x*Sin
[5*c + 4*d*x] + 3*Log[Sin[c + d*x]^2]*Sin[5*c + 4*d*x]))/(12*d)
```

Maple [A]
time = 0.20, size = 123, normalized size = 0.92

method	result
risch	$-\frac{16 i a^{4} c}{d}-\frac{4 a^{4} \left(30 \,\mathrm{e}^{6 i \left(d x +c \right)}-63 \,\mathrm{e}^{4 i \left(d x +c \right)}+50 \,\mathrm{e}^{2 i \left(d x +c \right)}-14\right)}{3 d \left(\mathrm{e}^{2 i \left(d x +c \right)}-1\right)^{4}}+\frac{8 a^{4} \ln \left(\mathrm{e}^{2 i \left(d x +c \right)}-1\right)}{d}$
norman	$\frac{-\frac{a^{4}}{4 d}+\frac{7 a^{4} \left(\tan^{2}\left(d x +c \right)\right)}{2 d}-\frac{4 i a^{4} \tan \left(d x +c \right)}{3 d}+\frac{8 i a^{4} \left(\tan^{3}\left(d x +c \right)\right)}{d}+8 i a^{4} x \left(\tan^{4}\left(d x +c \right)\right)}{\tan \left(d x +c \right)^{4}}+\frac{8 a^{4} \ln \left(\tan \left(d x +c \right)\right)}{d}-\frac{4 a^{4} \ln \left(1+\tan\right.}{d}$
derivativedivides	$\frac{a^{4} \ln \left(\sin \left(d x +c \right)\right)-4 i a^{4} \left(-\cot \left(d x +c \right)-d x -c \right)-6 a^{4} \left(-\frac{\left(\cot^{2}\left(d x +c \right)\right)}{2}-\ln \left(\sin \left(d x +c \right)\right)\right)+4 i a^{4} \left(-\frac{\left(\cot^{3}\left(d x +c \right)\right)}{3}+\cot \left(d x +c \right)+\right.}{d}$
default	$\frac{a^{4} \ln \left(\sin \left(d x +c \right)\right)-4 i a^{4} \left(-\cot \left(d x +c \right)-d x -c \right)-6 a^{4} \left(-\frac{\left(\cot^{2}\left(d x +c \right)\right)}{2}-\ln \left(\sin \left(d x +c \right)\right)\right)+4 i a^{4} \left(-\frac{\left(\cot^{3}\left(d x +c \right)\right)}{3}+\cot \left(d x +c \right)+\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*ln(sin(d*x+c))-4*I*a^4*(-cot(d*x+c)-d*x-c)-6*a^4*(-1/2*cot(d*x+c)^
2-ln(sin(d*x+c)))+4*I*a^4*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+a^4*(-1/4*co
t(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c))))
```

Maxima [A]
time = 0.52, size = 96, normalized size = 0.72

$$-\frac{-96 i \,{\left(d x + c\right)} a^{4} + 48 \, a^{4} \log\left(\tan\left(d x + c\right)^{2} + 1\right) - 96 \, a^{4} \log\left(\tan\left(d x + c\right)\right) + \frac{-96 i \, a^{4} \tan\left(d x + c\right)^{3} - 42 \, a^{4} \tan\left(d x + c\right)^{2} + 16 i \, a^{4} \tan\left(d x + c\right) + 3 \, a^{4}}{\tan\left(d x + c\right)^{4}}}{12 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/12*(-96*I*(d*x + c)*a^4 + 48*a^4*log(tan(d*x + c)^2 + 1) - 96*a^4*log(ta
n(d*x + c)) + (-96*I*a^4*tan(d*x + c)^3 - 42*a^4*tan(d*x + c)^2 + 16*I*a^4*
tan(d*x + c) + 3*a^4)/tan(d*x + c)^4)/d
```

Fricas [A]
time = 0.36, size = 174, normalized size = 1.30

$$-\frac{4 \, {\left(30 \, a^{4} e^{\left(6 i \, d x + 6 i \, c\right)} - 63 \, a^{4} e^{\left(4 i \, d x + 4 i \, c\right)} + 50 \, a^{4} e^{\left(2 i \, d x + 2 i \, c\right)} - 14 \, a^{4} - 6 \, {\left(a^{4} e^{\left(8 i \, d x + 8 i \, c\right)} - 4 \, a^{4} e^{\left(6 i \, d x + 6 i \, c\right)} + 6 \, a^{4} e^{\left(4 i \, d x + 4 i \, c\right)} - 4 \, a^{4} e^{\left(2 i \, d x + 2 i \, c\right)} + a^{4}\right)} \log\left(e^{\left(2 i \, d x + 2 i \, c\right)} - 1\right)\right)}}{3 \, {\left(d e^{\left(8 i \, d x + 8 i \, c\right)} - 4 \, d e^{\left(6 i \, d x + 6 i \, c\right)} + 6 \, d e^{\left(4 i \, d x + 4 i \, c\right)} - 4 \, d e^{\left(2 i \, d x + 2 i \, c\right)} + d\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -4/3*(30*a^4*e^(6*I*d*x + 6*I*c) - 63*a^4*e^(4*I*d*x + 4*I*c) + 50*a^4*e^(2
*I*d*x + 2*I*c) - 14*a^4 - 6*(a^4*e^(8*I*d*x + 8*I*c) - 4*a^4*e^(6*I*d*x +
6*I*c) + 6*a^4*e^(4*I*d*x + 4*I*c) - 4*a^4*e^(2*I*d*x + 2*I*c) + a^4)*log(e
^(2*I*d*x + 2*I*c) - 1))/(d*e^(8*I*d*x + 8*I*c) - 4*d*e^(6*I*d*x + 6*I*c) +
 6*d*e^(4*I*d*x + 4*I*c) - 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A]
time = 1.73, size = 168, normalized size = 1.25

$$\frac{8a^4\log\left(e^{2idx}-e^{-2ic}\right)}{d}+\frac{-120a^4e^{6ic}e^{6idx}+252a^4e^{4ic}e^{4idx}-200a^4e^{2ic}e^{2idx}+56a^4}{3de^{8ic}e^{8idx}-12de^{6ic}e^{6idx}+18de^{4ic}e^{4idx}-12de^{2ic}e^{2idx}+3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] 8*a**4*log(exp(2*I*d*x) - exp(-2*I*c))/d + (-120*a**4*exp(6*I*c)*exp(6*I*d*
x) + 252*a**4*exp(4*I*c)*exp(4*I*d*x) - 200*a**4*exp(2*I*c)*exp(2*I*d*x) +
56*a**4)/(3*d*exp(8*I*c)*exp(8*I*d*x) - 12*d*exp(6*I*c)*exp(6*I*d*x) + 18*d
*exp(4*I*c)*exp(4*I*d*x) - 12*d*exp(2*I*c)*exp(2*I*d*x) + 3*d)
```

Giac [A]
time = 1.47, size = 180, normalized size = 1.34

$$-\frac{3\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-32i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-180\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3072\,a^4\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)-1536\,a^4\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+864i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{3200\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-864i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-180\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+32i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a^4}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/192*(3*a^4*tan(1/2*d*x + 1/2*c)^4 - 32*I*a^4*tan(1/2*d*x + 1/2*c)^3 - 18
0*a^4*tan(1/2*d*x + 1/2*c)^2 + 3072*a^4*log(tan(1/2*d*x + 1/2*c) + I) - 153
6*a^4*log(tan(1/2*d*x + 1/2*c)) + 864*I*a^4*tan(1/2*d*x + 1/2*c) + (3200*a^
4*tan(1/2*d*x + 1/2*c)^4 - 864*I*a^4*tan(1/2*d*x + 1/2*c)^3 - 180*a^4*tan(1
/2*d*x + 1/2*c)^2 + 32*I*a^4*tan(1/2*d*x + 1/2*c) + 3*a^4)/tan(1/2*d*x + 1/
2*c)^4)/d
```

Mupad [B]
time = 4.01, size = 80, normalized size = 0.60

$$\frac{a^4\,\mathrm{atan}\left(2\,\tan\left(c+d\,x\right)+1{}\mathrm{i}\right)\,16{}\mathrm{i}}{d}-\frac{-a^4\,{\tan\left(c+d\,x\right)}^3\,8{}\mathrm{i}-\frac{7\,a^4\,{\tan\left(c+d\,x\right)}^2}{2}+\frac{a^4\,\tan\left(c+d\,x\right)\,4{}\mathrm{i}}{3}+\frac{a^4}{4}}{d\,{\tan\left(c+d\,x\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] (a^4*atan(2*tan(c + d*x) + 1i)*16i)/d - ((a^4*tan(c + d*x)*4i)/3 + a^4/4 -
(7*a^4*tan(c + d*x)^2)/2 - a^4*tan(c + d*x)^3*8i)/(d*tan(c + d*x)^4)
```

3.43 $\int \cot^6(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal. Leaf size=142

$$-8a^4x-\frac{8a^4\cot(c+dx)}{d}+\frac{4ia^4\cot^2(c+dx)}{d}+\frac{23a^4\cot^3(c+dx)}{15d}+\frac{8ia^4\log(\sin(c+dx))}{d}-\frac{\cot^5(c+dx)\,(a^2+ia}{5d}$$

```
[Out] -8*a^4*x-8*a^4*cot(d*x+c)/d+4*I*a^4*cot(d*x+c)^2/d+23/15*a^4*cot(d*x+c)^3/d
+8*I*a^4*ln(sin(d*x+c))/d-1/5*cot(d*x+c)^5*(a^2+I*a^2*tan(d*x+c))^2/d-3/5*I
*cot(d*x+c)^4*(a^4+I*a^4*tan(d*x+c))/d
```

Rubi [A]

time = 0.21, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3634, 3674, 3672, 3610, 3612, 3556}

$$\frac{23a^4\cot^3(c+dx)}{15d}+\frac{4ia^4\cot^2(c+dx)}{d}-\frac{8a^4\cot(c+dx)}{d}+\frac{8ia^4\log(\sin(c+dx))}{d}-\frac{3i\cot^4(c+dx)\,(a^4+ia^4\tan(c+dx))}{5d}-8a^4x-\frac{\cot^5(c+dx)\,(a^2+ia^2\tan(c+dx))^2}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] -8*a^4*x - (8*a^4*Cot[c + d*x])/d + ((4*I)*a^4*Cot[c + d*x]^2)/d + (23*a^4*
Cot[c + d*x]^3)/(15*d) + ((8*I)*a^4*Log[Sin[c + d*x]])/d - (Cot[c + d*x]^5*
(a^2 + I*a^2*Tan[c + d*x])^2)/(5*d) - (((3*I)/5)*Cot[c + d*x]^4*(a^4 + I*a^
4*Tan[c + d*x]))/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3634

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-a^2)*(b*c - a*d)*(a + b*Tan[e + f*x]
)^(m - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] + Di
st[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[b*(b*c*(m - 2) - a*d*(m - 2*n - 4)) + (a*b*c*(m - 2)
+ b^2*d*(n + 1) - a^2*d*(m + n - 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3672

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
 b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
 -1] && NeQ[a^2 + b^2, 0]
```

Rule 3674

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(
a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
- 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*
d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \cot^6(c+dx)(a+ia\tan(c+dx))^4\,dx &= -\frac{\cot^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{5d}-\frac{1}{5}\int \cot^5(c+dx)(a+ia\, \\
&= -\frac{\cot^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{5d}-\frac{3i\cot^4(c+dx)\left(a^4+ia^4\right.}{5d} \\
&= \frac{23a^4\cot^3(c+dx)}{15d}-\frac{\cot^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{5d}-\frac{3i\cot}{} \\
&= \frac{4ia^4\cot^2(c+dx)}{d}+\frac{23a^4\cot^3(c+dx)}{15d}-\frac{\cot^5(c+dx)\left(a^2+ia^2\tan\right.}{5d} \\
&= -\frac{8a^4\cot(c+dx)}{d}+\frac{4ia^4\cot^2(c+dx)}{d}+\frac{23a^4\cot^3(c+dx)}{15d}-\frac{\cot^5}{} \\
&= -8a^4x-\frac{8a^4\cot(c+dx)}{d}+\frac{4ia^4\cot^2(c+dx)}{d}+\frac{23a^4\cot^3(c+dx)}{15d} \\
&= -8a^4x-\frac{8a^4\cot(c+dx)}{d}+\frac{4ia^4\cot^2(c+dx)}{d}+\frac{23a^4\cot^3(c+dx)}{15d}
\end{aligned}
$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 359 vs. $2(142) = 284$.
time = 2.83, size = 359, normalized size = 2.53

$$
\frac{a^4\csc(c)\csc^5(c+dx)(\cos(4dx)+i\sin(4dx))\left(-210i\cos(2c+dx)+600dx\cos(2c+dx)-90i\cos(2c+3dx)+300dx\cos(2c+3dx)+90i\cos(4c+3dx)-300dx\cos(4c+3dx)-60dx\cos(4c+5dx)+60dx\cos(6c+5dx)-30\cos(dx)\left(-7i+20dx-5i\log\left(\sin^2(c+dx)\right)\right)-150i\cos(2c+dx)\log\left(\sin^2(c+dx)\right)-75i\cos(2c+3dx)\log\left(\sin^2(c+dx)\right)+75i\cos(4c+3dx)\log\left(\sin^2(c+dx)\right)+15i\cos(4c+5dx)\log\left(\sin^2(c+dx)\right)-15i\cos(6c+5dx)\log\left(\sin^2(c+dx)\right)+445\sin(dx)+960\mathrm{ArcTan}(\tan(5c+dx))\sin(c)\sin^5(c+dx)+345\sin(2c+dx)-275\sin(2c+3dx)-120\sin(4c+3dx)+79\sin(4c+5dx)\right)}{120d(\cos(dx)+i\sin(dx))^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (a^4*Csc[c]*Csc[c + d*x]^5*(Cos[4*d*x] + I*Sin[4*d*x])*((-210*I)*Cos[2*c +
d*x] + 600*d*x*Cos[2*c + d*x] - (90*I)*Cos[2*c + 3*d*x] + 300*d*x*Cos[2*c +
 3*d*x] + (90*I)*Cos[4*c + 3*d*x] - 300*d*x*Cos[4*c + 3*d*x] - 60*d*x*Cos[4
*c + 5*d*x] + 60*d*x*Cos[6*c + 5*d*x] - 30*Cos[d*x]*(-7*I + 20*d*x - (5*I)*
Log[Sin[c + d*x]^2]) - (150*I)*Cos[2*c + d*x]*Log[Sin[c + d*x]^2] - (75*I)*
Cos[2*c + 3*d*x]*Log[Sin[c + d*x]^2] + (75*I)*Cos[4*c + 3*d*x]*Log[Sin[c +
d*x]^2] + (15*I)*Cos[4*c + 5*d*x]*Log[Sin[c + d*x]^2] - (15*I)*Cos[6*c + 5*
d*x]*Log[Sin[c + d*x]^2] + 445*Sin[d*x] + 960*ArcTan[Tan[5*c + d*x]]*Sin[c]
*Sin[c + d*x]^5 + 345*Sin[2*c + d*x] - 275*Sin[2*c + 3*d*x] - 120*Sin[4*c +
 3*d*x] + 79*Sin[4*c + 5*d*x]))/(120*d*(Cos[d*x] + I*Sin[d*x])^4)
```

Maple [**A**]
time = 0.21, size = 152, normalized size = 1.07

method	result

risch	$\frac{16a^4c}{d}-\frac{4ia^4\left(210\,\mathrm{e}^{8i(dx+c)}-555\,\mathrm{e}^{6i(dx+c)}+655\,\mathrm{e}^{4i(dx+c)}-365\,\mathrm{e}^{2i(dx+c)}+79\right)}{15d\left(\mathrm{e}^{2i(dx+c)}-1\right)^5}+\frac{8ia^4\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{d}$
norman	$\frac{-\frac{a^4}{5d}-8a^4x\left(\tan^5(dx+c)\right)+\frac{7a^4\left(\tan^2(dx+c)\right)}{3d}-\frac{8a^4\left(\tan^4(dx+c)\right)}{d}-\frac{ia^4\tan(dx+c)}{d}+\frac{4ia^4\left(\tan^3(dx+c)\right)}{d}}{\tan(dx+c)^5}+\frac{8ia^4\ln(\tan(dx+c}{d}$
derivativedivides	$\frac{a^4(-\cot(dx+c)-dx-c)-4ia^4\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)-6a^4\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+4ia^4\left(-\right.}{d}$
default	$\frac{a^4(-\cot(dx+c)-dx-c)-4ia^4\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)-6a^4\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+4ia^4\left(-\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(-cot(d*x+c)-d*x-c)-4*I*a^4*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))-6*a
^4*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+4*I*a^4*(-1/4*cot(d*x+c)^4+1/2*cot(
d*x+c)^2+ln(sin(d*x+c)))+a^4*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)
-d*x-c))
```

Maxima [A]

time = 0.49, size = 109, normalized size = 0.77

$$-\frac{120\,(dx+c)a^4+60i\,a^4\log\left(\tan\left(dx+c\right)^2+1\right)-120i\,a^4\log\left(\tan\left(dx+c\right)\right)+\frac{120\,a^4\tan(dx+c)^4-60i\,a^4\tan(dx+c)^3-35\,a^4\tan(dx+c)^2+15i\,a^4\tan(dx+c)+3\,a^4}{\tan(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/15*(120*(d*x + c)*a^4 + 60*I*a^4*log(tan(d*x + c)^2 + 1) - 120*I*a^4*log
(tan(d*x + c)) + (120*a^4*tan(d*x + c)^4 - 60*I*a^4*tan(d*x + c)^3 - 35*a^4
*tan(d*x + c)^2 + 15*I*a^4*tan(d*x + c) + 3*a^4)/tan(d*x + c)^5)/d
```

Fricas [A]

time = 0.37, size = 219, normalized size = 1.54

$$-\frac{4\left(210i\,a^4e^{(8i\,dx+8i\,c)}-555i\,a^4e^{(6i\,dx+6i\,c)}+655i\,a^4e^{(4i\,dx+4i\,c)}-365i\,a^4e^{(2i\,dx+2i\,c)}+79i\,a^4+30\left(-i\,a^4e^{(10i\,dx+10i\,c)}+5i\,a^4e^{(8i\,dx+8i\,c)}-10i\,a^4e^{(6i\,dx+6i\,c)}+10i\,a^4e^{(4i\,dx+4i\,c)}-5i\,a^4e^{(2i\,dx+2i\,c)}+i\,a^4\right)\log\left(e^{(2i\,dx+2i\,c)}-1\right)\right)}{15\left(de^{(10i\,dx+10i\,c)}-5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}-10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -4/15*(210*I*a^4*e^(8*I*d*x + 8*I*c) - 555*I*a^4*e^(6*I*d*x + 6*I*c) + 655*
I*a^4*e^(4*I*d*x + 4*I*c) - 365*I*a^4*e^(2*I*d*x + 2*I*c) + 79*I*a^4 + 30*(
-I*a^4*e^(10*I*d*x + 10*I*c) + 5*I*a^4*e^(8*I*d*x + 8*I*c) - 10*I*a^4*e^(6*
I*d*x + 6*I*c) + 10*I*a^4*e^(4*I*d*x + 4*I*c) - 5*I*a^4*e^(2*I*d*x + 2*I*c)
 + I*a^4)*log(e^(2*I*d*x + 2*I*c) - 1))/(d*e^(10*I*d*x + 10*I*c) - 5*d*e^(8
```

```
*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) - 10*d*e^(4*I*d*x + 4*I*c) + 5*d
*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [A]
time = 0.41, size = 218, normalized size = 1.54

$$\frac{8ia^4\log\left(e^{2idx}-e^{-2ic}\right)}{d}+\frac{-840ia^4e^{8ic}e^{8idx}+2220ia^4e^{6ic}e^{6idx}-2620ia^4e^{4ic}e^{4idx}+1460ia^4e^{2ic}e^{2idx}-316ia^4}{15de^{10ic}e^{10idx}-75de^{8ic}e^{8idx}+150de^{6ic}e^{6idx}-150de^{4ic}e^{4idx}+75de^{2ic}e^{2idx}-15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] 8*I*a**4*log(exp(2*I*d*x) - exp(-2*I*c))/d + (-840*I*a**4*exp(8*I*c)*exp(8*
I*d*x) + 2220*I*a**4*exp(6*I*c)*exp(6*I*d*x) - 2620*I*a**4*exp(4*I*c)*exp(4
*I*d*x) + 1460*I*a**4*exp(2*I*c)*exp(2*I*d*x) - 316*I*a**4)/(15*d*exp(10*I*
c)*exp(10*I*d*x) - 75*d*exp(8*I*c)*exp(8*I*d*x) + 150*d*exp(6*I*c)*exp(6*I*
d*x) - 150*d*exp(4*I*c)*exp(4*I*d*x) + 75*d*exp(2*I*c)*exp(2*I*d*x) - 15*d)
```

Giac [A]
time = 0.85, size = 212, normalized size = 1.49

$$\frac{3\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-30i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-155\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+600i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-7680i\,a^4\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)+3840i\,a^4\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+2370\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{-8768i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-2370\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+600i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+155\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-30i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a^4}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^4*tan(1/2*d*x + 1/2*c)^5 - 30*I*a^4*tan(1/2*d*x + 1/2*c)^4 - 155
*a^4*tan(1/2*d*x + 1/2*c)^3 + 600*I*a^4*tan(1/2*d*x + 1/2*c)^2 - 7680*I*a^4
*log(tan(1/2*d*x + 1/2*c) + I) + 3840*I*a^4*log(tan(1/2*d*x + 1/2*c)) + 237
0*a^4*tan(1/2*d*x + 1/2*c) + (-8768*I*a^4*tan(1/2*d*x + 1/2*c)^5 - 2370*a^4
*tan(1/2*d*x + 1/2*c)^4 + 600*I*a^4*tan(1/2*d*x + 1/2*c)^3 + 155*a^4*tan(1/
2*d*x + 1/2*c)^2 - 30*I*a^4*tan(1/2*d*x + 1/2*c) - 3*a^4)/tan(1/2*d*x + 1/2
*c)^5)/d
```

Mupad [B]
time = 4.27, size = 92, normalized size = 0.65

$$-\frac{16\,a^4\,\mathrm{atan}\left(2\,\tan\left(c+d\,x\right)+1\mathrm{i}\right)}{d}-\frac{8\,a^4\,{\tan\left(c+d\,x\right)}^4-a^4\,{\tan\left(c+d\,x\right)}^3\,4\mathrm{i}-\frac{7\,a^4\,{\tan\left(c+d\,x\right)}^2}{3}+a^4\,\tan\left(c+d\,x\right)\,1\mathrm{i}+\frac{a^4}{5}}{d\,{\tan\left(c+d\,x\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] - (16*a^4*atan(2*tan(c + d*x) + 1i))/d - (a^4*tan(c + d*x)*1i + a^4/5 - (7*
a^4*tan(c + d*x)^2)/3 - a^4*tan(c + d*x)^3*4i + 8*a^4*tan(c + d*x)^4)/(d*ta
n(c + d*x)^5)
```

3.44 $\int \cot^7(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal. Leaf size=162

$$-8ia^4x-\frac{8ia^4\cot(c+dx)}{d}-\frac{4a^4\cot^2(c+dx)}{d}+\frac{8ia^4\cot^3(c+dx)}{3d}+\frac{67a^4\cot^4(c+dx)}{60d}-\frac{8a^4\log(\sin(c+dx))}{d}-$$

```
[Out] -8*I*a^4*x-8*I*a^4*cot(d*x+c)/d-4*a^4*cot(d*x+c)^2/d+8/3*I*a^4*cot(d*x+c)^3
/d+67/60*a^4*cot(d*x+c)^4/d-8*a^4*ln(sin(d*x+c))/d-1/6*cot(d*x+c)^6*(a^2+I*
a^2*tan(d*x+c))^2/d-7/15*I*cot(d*x+c)^5*(a^4+I*a^4*tan(d*x+c))/d
```

Rubi [A]
time = 0.22, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3634, 3674, 3672, 3610, 3612, 3556}

$$\frac{67a^4\cot^4(c+dx)}{60d}+\frac{8ia^4\cot^3(c+dx)}{3d}-\frac{4a^4\cot^2(c+dx)}{d}-\frac{8ia^4\cot(c+dx)}{d}-\frac{8a^4\log(\sin(c+dx))}{d}-\frac{7i\cot^5(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{15d}-8ia^4x-\frac{\cot^6(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (-8*I)*a^4*x - ((8*I)*a^4*Cot[c + d*x])/d - (4*a^4*Cot[c + d*x]^2)/d + (((8
*I)/3)*a^4*Cot[c + d*x]^3)/d + (67*a^4*Cot[c + d*x]^4)/(60*d) - (8*a^4*Log[
Sin[c + d*x]])/d - (Cot[c + d*x]^6*(a^2 + I*a^2*Tan[c + d*x])^2)/(6*d) - ((
(7*I)/15)*Cot[c + d*x]^5*(a^4 + I*a^4*Tan[c + d*x]))/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3634

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-a^2)*(b*c - a*d)*(a + b*Tan[e + f*x]
)^(m - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] + Di
st[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[b*(b*c*(m - 2) - a*d*(m - 2*n - 4)) + (a*b*c*(m - 2)
+ b^2*d*(n + 1) - a^2*d*(m + n - 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3672

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
 b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
 -1] && NeQ[a^2 + b^2, 0]
```

Rule 3674

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(
a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
- 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*
d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \cot^7(c+dx)(a+ia\tan(c+dx))^4\,dx &= -\frac{\cot^6(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{6d} - \frac{1}{6}\int \cot^6(c+dx)(a+ia \\
&= -\frac{\cot^6(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{6d} - \frac{7i\cot^5(c+dx)\left(a^4+ia\right.}{15d} \\
&= \frac{67a^4\cot^4(c+dx)}{60d} - \frac{\cot^6(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{6d} - \frac{7i\,\mathrm{co}}{} \\
&= \frac{8ia^4\cot^3(c+dx)}{3d} + \frac{67a^4\cot^4(c+dx)}{60d} - \frac{\cot^6(c+dx)\left(a^2+ia^2\,\mathrm{t}\right.}{6d} \\
&= -\frac{4a^4\cot^2(c+dx)}{d} + \frac{8ia^4\cot^3(c+dx)}{3d} + \frac{67a^4\cot^4(c+dx)}{60d} - \frac{\mathrm{co}}{} \\
&= -\frac{8ia^4\cot(c+dx)}{d} - \frac{4a^4\cot^2(c+dx)}{d} + \frac{8ia^4\cot^3(c+dx)}{3d} + \frac{67a}{} \\
&= -8ia^4x - \frac{8ia^4\cot(c+dx)}{d} - \frac{4a^4\cot^2(c+dx)}{d} + \frac{8ia^4\cot^3(c+dx)}{3d} \\
&= -8ia^4x - \frac{8ia^4\cot(c+dx)}{d} - \frac{4a^4\cot^2(c+dx)}{d} + \frac{8ia^4\cot^3(c+dx)}{3d}
\end{aligned}
$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 363 vs. 2(162) = 324.
time = 1.39, size = 363, normalized size = 2.24

$$\frac{a^4\csc(c)\csc^6(c+dx)\left(860i\cos(c)-780i\cos(c+2dx)-510i\cos(3c+2dx)+366i\cos(3c+4dx)+150i\cos(5c+4dx)-86i\cos(5c+6dx)-490\sin(c)-600idx\sin(c)-300\log\left(\sin^2(c+dx)\right)\sin(c)-345\sin(c+2dx)-450idx\sin(c+2dx)-225\log\left(\sin^2(c+dx)\right)\sin(c+2dx)+345\sin(3c+2dx)+450idx\sin(3c+2dx)+225\log\left(\sin^2(c+dx)\right)\sin(3c+2dx)+120\sin(3c+4dx)+180idx\sin(3c+4dx)+90\log\left(\sin^2(c+dx)\right)\sin(3c+4dx)-120\sin(5c+4dx)-180idx\sin(5c+4dx)-90\log\left(\sin^2(c+dx)\right)\sin(5c+4dx)-30idx\sin(5c+6dx)-15\log\left(\sin^2(c+dx)\right)\sin(5c+6dx)+30idx\sin(7c+6dx)+15\log\left(\sin^2(c+dx)\right)\sin(7c+6dx)\right)}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (a^4*Csc[c]*Csc[c + d*x]^6*((860*I)*Cos[c] - (780*I)*Cos[c + 2*d*x] - (510*
I)*Cos[3*c + 2*d*x] + (366*I)*Cos[3*c + 4*d*x] + (150*I)*Cos[5*c + 4*d*x] -
 (86*I)*Cos[5*c + 6*d*x] - 490*Sin[c] - (600*I)*d*x*Sin[c] - 300*Log[Sin[c
+ d*x]^2]*Sin[c] - 345*Sin[c + 2*d*x] - (450*I)*d*x*Sin[c + 2*d*x] - 225*Lo
g[Sin[c + d*x]^2]*Sin[c + 2*d*x] + 345*Sin[3*c + 2*d*x] + (450*I)*d*x*Sin[3
*c + 2*d*x] + 225*Log[Sin[c + d*x]^2]*Sin[3*c + 2*d*x] + 120*Sin[3*c + 4*d*
x] + (180*I)*d*x*Sin[3*c + 4*d*x] + 90*Log[Sin[c + d*x]^2]*Sin[3*c + 4*d*x]
 - 120*Sin[5*c + 4*d*x] - (180*I)*d*x*Sin[5*c + 4*d*x] - 90*Log[Sin[c + d*x
]^2]*Sin[5*c + 4*d*x] - (30*I)*d*x*Sin[5*c + 6*d*x] - 15*Log[Sin[c + d*x]^2
]*Sin[5*c + 6*d*x] + (30*I)*d*x*Sin[7*c + 6*d*x] + 15*Log[Sin[c + d*x]^2]*S
in[7*c + 6*d*x]))/(240*d)
```

Maple [**A**]
time = 0.23, size = 176, normalized size = 1.09

method	result
risch	$\frac{16ia^4c}{d}+\frac{4a^4(270\,\mathrm{e}^{10i(dx+c)}-855\,\mathrm{e}^{8i(dx+c)}+1350\,\mathrm{e}^{6i(dx+c)}-1125\,\mathrm{e}^{4i(dx+c)}+486\,\mathrm{e}^{2i(dx+c)}-86)}{15d(\mathrm{e}^{2i(dx+c)}-1)^6}-\frac{8a^4\ln(\mathrm{e}^{2i(dx+c)}-$
norman	$\frac{-\frac{a^4}{6d}+\frac{7a^4(\tan^2(dx+c))}{4d}-\frac{4a^4(\tan^4(dx+c))}{d}-\frac{4ia^4\tan(dx+c)}{5d}+\frac{8ia^4(\tan^3(dx+c))}{3d}-\frac{8ia^4(\tan^5(dx+c))}{d}-8ia^4x(\tan^6(dx+c))}{\tan(dx+c)^6}-$
derivativedivides	$a^4\left(-\frac{(\cot^2(dx+c))}{2}-\ln(\sin(dx+c))\right)-4ia^4\left(-\frac{(\cot^3(dx+c))}{3}+\cot(dx+c)+dx+c\right)-6a^4\left(-\frac{(\cot^4(dx+c))}{4}+\frac{(\cot^2(dx+c))}{2}+\mathrm{l}\right.$
default	$a^4\left(-\frac{(\cot^2(dx+c))}{2}-\ln(\sin(dx+c))\right)-4ia^4\left(-\frac{(\cot^3(dx+c))}{3}+\cot(dx+c)+dx+c\right)-6a^4\left(-\frac{(\cot^4(dx+c))}{4}+\frac{(\cot^2(dx+c))}{2}+\mathrm{l}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))-4*I*a^4*(-1/3*cot(d*x+c)^3+cot(
d*x+c)+d*x+c)-6*a^4*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+4*I
*a^4*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c)+a^4*(-1/6*cot(d*
x+c)^6+1/4*cot(d*x+c)^4-1/2*cot(d*x+c)^2-ln(sin(d*x+c))))
```

Maxima [A]
time = 0.50, size = 123, normalized size = 0.76

$$-\frac{480i\,(dx+c)a^4-240\,a^4\log\left(\tan\left(dx+c\right)^2+1\right)+480\,a^4\log\left(\tan\left(dx+c\right)\right)-\frac{-480i\,a^4\tan(dx+c)^5-240\,a^4\tan(dx+c)^4+160i\,a^4\tan(dx+c)^3+105\,a^4\tan(dx+c)^2-48i\,a^4\tan(dx+c)-10\,a^4}{\tan(dx+c)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/60*(480*I*(d*x + c)*a^4 - 240*a^4*log(tan(d*x + c)^2 + 1) + 480*a^4*log(
tan(d*x + c)) - (-480*I*a^4*tan(d*x + c)^5 - 240*a^4*tan(d*x + c)^4 + 160*I
*a^4*tan(d*x + c)^3 + 105*a^4*tan(d*x + c)^2 - 48*I*a^4*tan(d*x + c) - 10*a
^4)/tan(d*x + c)^6)/d
```

Fricas [A]
time = 0.36, size = 254, normalized size = 1.57

$$\frac{4\left(270\,a^4e^{(10i\,dx+10i\,c)}-855\,a^4e^{(8i\,dx+8i\,c)}+1350\,a^4e^{(6i\,dx+6i\,c)}-1125\,a^4e^{(4i\,dx+4i\,c)}+486\,a^4e^{(2i\,dx+2i\,c)}-86\,a^4-30\left(a^4e^{(12i\,dx+12i\,c)}-6\,a^4e^{(10i\,dx+10i\,c)}+15\,a^4e^{(8i\,dx+8i\,c)}-20\,a^4e^{(6i\,dx+6i\,c)}+15\,a^4e^{(4i\,dx+4i\,c)}-6\,a^4e^{(2i\,dx+2i\,c)}+a^4\right)\log\left(e^{(2i\,dx+2i\,c)}-1\right)\right)}{15\left(de^{(12i\,dx+12i\,c)}-6\,de^{(10i\,dx+10i\,c)}+15\,de^{(8i\,dx+8i\,c)}-20\,de^{(6i\,dx+6i\,c)}+15\,de^{(4i\,dx+4i\,c)}-6\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 4/15*(270*a^4*e^(10*I*d*x + 10*I*c) - 855*a^4*e^(8*I*d*x + 8*I*c) + 1350*a^
4*e^(6*I*d*x + 6*I*c) - 1125*a^4*e^(4*I*d*x + 4*I*c) + 486*a^4*e^(2*I*d*x +
 2*I*c) - 86*a^4 - 30*(a^4*e^(12*I*d*x + 12*I*c) - 6*a^4*e^(10*I*d*x + 10*I
```

```
*c) + 15*a^4*e^(8*I*d*x + 8*I*c) - 20*a^4*e^(6*I*d*x + 6*I*c) + 15*a^4*e^(4
*I*d*x + 4*I*c) - 6*a^4*e^(2*I*d*x + 2*I*c) + a^4)*log(e^(2*I*d*x + 2*I*c)
- 1))/(d*e^(12*I*d*x + 12*I*c) - 6*d*e^(10*I*d*x + 10*I*c) + 15*d*e^(8*I*d*
x + 8*I*c) - 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x + 4*I*c) - 6*d*e^(2
*I*d*x + 2*I*c) + d)
```

Sympy [A]

time = 6.05, size = 246, normalized size = 1.52

$$-\frac{8a^4\log\left(e^{2idx}-e^{-2ic}\right)}{d}+\frac{1080a^4e^{10ic}e^{10idx}-3420a^4e^{8ic}e^{8idx}+5400a^4e^{6ic}e^{6idx}-4500a^4e^{4ic}e^{4idx}+1944a^4e^{2ic}e^{2idx}-344a^4}{15de^{12ic}e^{12idx}-90de^{10ic}e^{10idx}+225de^{8ic}e^{8idx}-300de^{6ic}e^{6idx}+225de^{4ic}e^{4idx}-90de^{2ic}e^{2idx}+15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] -8*a**4*log(exp(2*I*d*x) - exp(-2*I*c))/d + (1080*a**4*exp(10*I*c)*exp(10*I
*d*x) - 3420*a**4*exp(8*I*c)*exp(8*I*d*x) + 5400*a**4*exp(6*I*c)*exp(6*I*d*
x) - 4500*a**4*exp(4*I*c)*exp(4*I*d*x) + 1944*a**4*exp(2*I*c)*exp(2*I*d*x)
- 344*a**4)/(15*d*exp(12*I*c)*exp(12*I*d*x) - 90*d*exp(10*I*c)*exp(10*I*d*x
) + 225*d*exp(8*I*c)*exp(8*I*d*x) - 300*d*exp(6*I*c)*exp(6*I*d*x) + 225*d*e
xp(4*I*c)*exp(4*I*d*x) - 90*d*exp(2*I*c)*exp(2*I*d*x) + 15*d)
```

Giac [A]

time = 0.91, size = 245, normalized size = 1.51

$$-\frac{5\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-48i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-240\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+880i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2835\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-30720\,a^4\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)+15360\,a^4\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-10080i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{37632\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-10080i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-2835\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+880i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+240\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-48i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5\,a^4}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/1920*(5*a^4*tan(1/2*d*x + 1/2*c)^6 - 48*I*a^4*tan(1/2*d*x + 1/2*c)^5 - 2
40*a^4*tan(1/2*d*x + 1/2*c)^4 + 880*I*a^4*tan(1/2*d*x + 1/2*c)^3 + 2835*a^4
*tan(1/2*d*x + 1/2*c)^2 - 30720*a^4*log(tan(1/2*d*x + 1/2*c) + I) + 15360*a
^4*log(tan(1/2*d*x + 1/2*c)) - 10080*I*a^4*tan(1/2*d*x + 1/2*c) - (37632*a^
4*tan(1/2*d*x + 1/2*c)^6 - 10080*I*a^4*tan(1/2*d*x + 1/2*c)^5 - 2835*a^4*ta
n(1/2*d*x + 1/2*c)^4 + 880*I*a^4*tan(1/2*d*x + 1/2*c)^3 + 240*a^4*tan(1/2*d
*x + 1/2*c)^2 - 48*I*a^4*tan(1/2*d*x + 1/2*c) - 5*a^4)/tan(1/2*d*x + 1/2*c)
^6)/d
```

Mupad [B]

time = 4.68, size = 107, normalized size = 0.66

$$-\frac{a^4\,\mathrm{atan}\left(2\,\tan\left(c+d\,x\right)+1{}\mathrm{i}\right)\,16{}\mathrm{i}}{d}-\frac{a^4\,{\tan\left(c+d\,x\right)}^5\,8{}\mathrm{i}+4\,a^4\,{\tan\left(c+d\,x\right)}^4-\frac{a^4\,{\tan\left(c+d\,x\right)}^3\,8{}\mathrm{i}}{3}-\frac{7\,a^4\,{\tan\left(c+d\,x\right)}^2}{4}+\frac{a^4\,\tan\left(c+d\,x\right)\,4{}\mathrm{i}}{5}+\frac{a^4}{6}}{d\,{\tan\left(c+d\,x\right)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^7*(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] - (a^4*atan(2*tan(c + d*x) + 1i)*16i)/d - ((a^4*tan(c + d*x)*4i)/5 + a^4/6
- (7*a^4*tan(c + d*x)^2)/4 - (a^4*tan(c + d*x)^3*8i)/3 + 4*a^4*tan(c + d*x)
^4 + a^4*tan(c + d*x)^5*8i)/(d*tan(c + d*x)^6)
```

3.45 $\int \frac{\tan^6(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=130

$$\frac{5x}{2a}+\frac{3i\log(\cos(c+dx))}{ad}-\frac{5\tan(c+dx)}{2ad}+\frac{3i\tan^2(c+dx)}{2ad}+\frac{5\tan^3(c+dx)}{6ad}-\frac{3i\tan^4(c+dx)}{4ad}-\frac{\tan^5(c+d}{2d(a+ia\tan(c}$$

```
[Out] 5/2*x/a+3*I*ln(cos(d*x+c))/a/d-5/2*tan(d*x+c)/a/d+3/2*I*tan(d*x+c)^2/a/d+5/
6*tan(d*x+c)^3/a/d-3/4*I*tan(d*x+c)^4/a/d-1/2*tan(d*x+c)^5/d/(a+I*a*tan(d*x
+c))
```

Rubi [A]

time = 0.10, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3631, 3609, 3606, 3556}

$$-\frac{\tan^5(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{3i\tan^4(c+dx)}{4ad}+\frac{5\tan^3(c+dx)}{6ad}+\frac{3i\tan^2(c+dx)}{2ad}-\frac{5\tan(c+dx)}{2ad}+\frac{3i\log(\cos(c+dx))}{ad}+\frac{5x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (5*x)/(2*a) + ((3*I)*Log[Cos[c + d*x]])/(a*d) - (5*Tan[c + d*x])/(2*a*d) +
(((3*I)/2)*Tan[c + d*x]^2)/(a*d) + (5*Tan[c + d*x]^3)/(6*a*d) - (((3*I)/4)*
Tan[c + d*x]^4)/(a*d) - Tan[c + d*x]^5/(2*d*(a + I*a*Tan[c + d*x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3631

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((c + d*Tan[e + f*x])^(n - 1)/(
2*a*f*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a^2), Int[(c + d*Tan[e + f*x])
^(n - 2)*Simp[a*c^2 + a*d^2*(n - 1) - b*c*d*n - d*(a*c*(n - 2) + b*d*n)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\tan^6(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{\tan^5(c+dx)}{2d(a+ia\tan(c+dx))} + \frac{\int \tan^4(c+dx)(5a-6ia\tan(c+dx))\,dx}{2a^2}$$

$$= -\frac{3i\tan^4(c+dx)}{4ad} - \frac{\tan^5(c+dx)}{2d(a+ia\tan(c+dx))} + \frac{\int \tan^3(c+dx)(6ia+5a\tan(c+\ldots}{2a^2}$$

$$= \frac{5\tan^3(c+dx)}{6ad} - \frac{3i\tan^4(c+dx)}{4ad} - \frac{\tan^5(c+dx)}{2d(a+ia\tan(c+dx))} + \frac{\int \tan^2(c+dx)(-\ldots}{\ldots}$$

$$= \frac{3i\tan^2(c+dx)}{2ad} + \frac{5\tan^3(c+dx)}{6ad} - \frac{3i\tan^4(c+dx)}{4ad} - \frac{\tan^5(c+dx)}{2d(a+ia\tan(c+dx))} + \ldots$$

$$= \frac{5x}{2a} - \frac{5\tan(c+dx)}{2ad} + \frac{3i\tan^2(c+dx)}{2ad} + \frac{5\tan^3(c+dx)}{6ad} - \frac{3i\tan^4(c+dx)}{4ad} - \ldots$$

$$= \frac{5x}{2a} + \frac{3i\log(\cos(c+dx))}{ad} - \frac{5\tan(c+dx)}{2ad} + \frac{3i\tan^2(c+dx)}{2ad} + \frac{5\tan^3(c+dx)}{6ad} - \ldots$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 840 vs. $2(130) = 260$.
time = 6.58, size = 840, normalized size = 6.46

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^6/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (5*x*Cos[c]*Sec[c + d*x]*(Cos[d*x] + I*Sin[d*x]))/(2*(a + I*a*Tan[c + d*x])
) + (3*ArcTan[Tan[d*x]]*Cos[c]*Sec[c + d*x]*(Cos[d*x] + I*Sin[d*x]))/(d*(a
+ I*a*Tan[c + d*x])) + (((3*I)/2)*Cos[c]*Log[Cos[c + d*x]^2]*Sec[c + d*x]*(
Cos[d*x] + I*Sin[d*x]))/(d*(a + I*a*Tan[c + d*x])) + (Cos[2*d*x]*Sec[c + d*
x]*((-1/4*I)*Cos[c] - Sin[c]/4)*(Cos[d*x] + I*Sin[d*x]))/(d*(a + I*a*Tan[c
+ d*x])) + (Sec[c + d*x]^3*((I/6)*Cos[c] - Sin[c]/6)*(9*Cos[c] - (2*I)*Sin[
c])*(Cos[d*x] + I*Sin[d*x]))/(d*(Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])
*(a + I*a*Tan[c + d*x])) + (Sec[c + d*x]^5*((-1/4*I)*Cos[c] + Sin[c]/4)*(Co
```

```
s[d*x] + I*Sin[d*x]))/(d*(a + I*a*Tan[c + d*x])) + (((5*I)/2)*x*Sec[c + d*x
]*Sin[c]*(Cos[d*x] + I*Sin[d*x]))/(a + I*a*Tan[c + d*x]) + ((3*I)*ArcTan[Ta
n[d*x]]*Sec[c + d*x]*Sin[c]*(Cos[d*x] + I*Sin[d*x]))/(d*(a + I*a*Tan[c + d*
x])) - (3*Log[Cos[c + d*x]^2]*Sec[c + d*x]*Sin[c]*(Cos[d*x] + I*Sin[d*x]))/
(2*d*(a + I*a*Tan[c + d*x])) + (Sec[c + d*x]*(-1/4*Cos[c] + (I/4)*Sin[c])*(
Cos[d*x] + I*Sin[d*x])*Sin[2*d*x])/(d*(a + I*a*Tan[c + d*x])) + (((7*I)/6)*
Sec[c + d*x]^2*(Cos[d*x] + I*Sin[d*x])*(-Cos[c - d*x] + Cos[c + d*x] - I*Si
n[c - d*x] + I*Sin[c + d*x]))/(d*(Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2]
)*(a + I*a*Tan[c + d*x])) - ((I/6)*Sec[c + d*x]^4*(Cos[d*x] + I*Sin[d*x])*(
-Cos[c - d*x] + Cos[c + d*x] - I*Sin[c - d*x] + I*Sin[c + d*x]))/(d*(Cos[c/
2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(a + I*a*Tan[c + d*x])) + (x*Sec[c + d
*x]*(Cos[d*x] + I*Sin[d*x])*(-3*Sec[c] - I*(3*Cos[c] + (3*I)*Sin[c])*Tan[c]
))/(a + I*a*Tan[c + d*x])
```

Maple [A]
time = 0.13, size = 88, normalized size = 0.68

method	result
derivativedivides	$\frac{-2\tan(dx+c)-\frac{i\left(\tan^4(dx+c)\right)}{4}+\frac{\left(\tan^3(dx+c)\right)}{3}+i\left(\tan^2(dx+c)\right)-\frac{11i\ln(\tan(dx+c)-i)}{4}-\frac{1}{2(\tan(dx+c)-i)}-\frac{i\ln(\tan(dx+c)+i)}{4}}{da}$
default	$\frac{-2\tan(dx+c)-\frac{i\left(\tan^4(dx+c)\right)}{4}+\frac{\left(\tan^3(dx+c)\right)}{3}+i\left(\tan^2(dx+c)\right)-\frac{11i\ln(\tan(dx+c)-i)}{4}-\frac{1}{2(\tan(dx+c)-i)}-\frac{i\ln(\tan(dx+c)+i)}{4}}{da}$
risch	$\frac{11x}{2a}-\frac{i\mathrm{e}^{-2i(dx+c)}}{4da}+\frac{6c}{da}-\frac{2i\left(9\,\mathrm{e}^{4i(dx+c)}+10\,\mathrm{e}^{2i(dx+c)}+7\right)}{3da\left(\mathrm{e}^{2i(dx+c)}+1\right)^4}+\frac{3i\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{da}$
norman	$\frac{\frac{5x}{2a}+\frac{\tan^5(dx+c)}{3da}+\frac{5x\left(\tan^2(dx+c)\right)}{2a}-\frac{3i}{2da}-\frac{5\tan(dx+c)}{2da}-\frac{5\left(\tan^3(dx+c)\right)}{3da}+\frac{3i\left(\tan^4(dx+c)\right)}{4da}-\frac{i\left(\tan^6(dx+c)\right)}{4da}}{1+\tan^2(dx+c)}-\frac{3i\ln\left(1+\tan^2(d\right.}{2da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-2*tan(d*x+c)-1/4*I*tan(d*x+c)^4+1/3*tan(d*x+c)^3+I*tan(d*x+c)^2-11/
4*I*ln(tan(d*x+c)-I)-1/2/(tan(d*x+c)-I)-1/4*I*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(110) = 220.

time = 0.38, size = 221, normalized size = 1.70

$$\frac{66\,dxe^{(10i\,dx+10i\,c)}+3\,(88\,dx-i)e^{(8i\,dx+8i\,c)}+12\,(33\,dx-7i)e^{(6i\,dx+6i\,c)}+2\,(132\,dx-49i)e^{(4i\,dx+4i\,c)}+2\,(33\,dx-34i)e^{(2i\,dx+2i\,c)}-36\,\left(-i\,e^{(10i\,dx+10i\,c)}-4i\,e^{(8i\,dx+8i\,c)}-6i\,e^{(6i\,dx+6i\,c)}-4i\,e^{(4i\,dx+4i\,c)}-i\,e^{(2i\,dx+2i\,c)}\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-3i}{12\,\left(ade^{(10i\,dx+10i\,c)}+4\,ade^{(8i\,dx+8i\,c)}+6\,ade^{(6i\,dx+6i\,c)}+4\,ade^{(4i\,dx+4i\,c)}+ade^{(2i\,dx+2i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(66*d*x*e^(10*I*d*x + 10*I*c) + 3*(88*d*x - I)*e^(8*I*d*x + 8*I*c) + 1
2*(33*d*x - 7*I)*e^(6*I*d*x + 6*I*c) + 2*(132*d*x - 49*I)*e^(4*I*d*x + 4*I*
c) + 2*(33*d*x - 34*I)*e^(2*I*d*x + 2*I*c) - 36*(-I*e^(10*I*d*x + 10*I*c) -
 4*I*e^(8*I*d*x + 8*I*c) - 6*I*e^(6*I*d*x + 6*I*c) - 4*I*e^(4*I*d*x + 4*I*c
) - I*e^(2*I*d*x + 2*I*c))*log(e^(2*I*d*x + 2*I*c) + 1) - 3*I)/(a*d*e^(10*I
*d*x + 10*I*c) + 4*a*d*e^(8*I*d*x + 8*I*c) + 6*a*d*e^(6*I*d*x + 6*I*c) + 4*
a*d*e^(4*I*d*x + 4*I*c) + a*d*e^(2*I*d*x + 2*I*c))
```

Sympy [A]
time = 0.35, size = 219, normalized size = 1.68

$$\frac{-18ie^{4ic}e^{4idx}-20ie^{2ic}e^{2idx}-14i}{3ade^{8ic}e^{8idx}+12ade^{6ic}e^{6idx}+18ade^{4ic}e^{4idx}+12ade^{2ic}e^{2idx}+3ad}+\begin{cases}-\frac{ie^{-2ic}e^{-2idx}}{4ad} & \text{for}\ ade^{2ic}\neq 0\\ x\left(\frac{(11e^{2ic}-1)e^{-2ic}}{2a}-\frac{11}{2a}\right) & \text{otherwise}\end{cases}+\frac{11x}{2a}+\frac{3i\log\left(e^{2idx}+e^{-2ic}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+I*a*tan(d*x+c)),x)
```

```
[Out] (-18*I*exp(4*I*c)*exp(4*I*d*x) - 20*I*exp(2*I*c)*exp(2*I*d*x) - 14*I)/(3*a*
d*exp(8*I*c)*exp(8*I*d*x) + 12*a*d*exp(6*I*c)*exp(6*I*d*x) + 18*a*d*exp(4*I
*c)*exp(4*I*d*x) + 12*a*d*exp(2*I*c)*exp(2*I*d*x) + 3*a*d) + Piecewise((-I*
exp(-2*I*c)*exp(-2*I*d*x)/(4*a*d), Ne(a*d*exp(2*I*c), 0)), (x*((11*exp(2*I*
c) - 1)*exp(-2*I*c)/(2*a) - 11/(2*a)), True)) + 11*x/(2*a) + 3*I*log(exp(2*
I*d*x) + exp(-2*I*c))/(a*d)
```

Giac [A]
time = 2.43, size = 116, normalized size = 0.89

$$-\frac{\frac{33i\,\log(\tan(dx+c)-i)}{a}+\frac{3i\,\log(i\,\tan(dx+c)-1)}{a}+\frac{3\,(-11i\,\tan(dx+c)-9)}{a(\tan(dx+c)-i)}+\frac{3i\,a^3\tan(dx+c)^4-4\,a^3\tan(dx+c)^3-12i\,a^3\tan(dx+c)^2+24\,a^3\tan(dx+c)}{a^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(33*I*log(tan(d*x + c) - I)/a + 3*I*log(I*tan(d*x + c) - 1)/a + 3*(-1
1*I*tan(d*x + c) - 9)/(a*(tan(d*x + c) - I)) + (3*I*a^3*tan(d*x + c)^4 - 4*
a^3*tan(d*x + c)^3 - 12*I*a^3*tan(d*x + c)^2 + 24*a^3*tan(d*x + c))/a^4)/d
```

Mupad [B]
time = 4.07, size = 125, normalized size = 0.96

$$\frac{\tan(c+d\,x)^3}{3\,a\,d}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{4\,a\,d}-\frac{2\tan(c+d\,x)}{a\,d}-\frac{1\mathrm{i}}{2\,a\,d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}+\frac{\tan(c+d\,x)^2\,1\mathrm{i}}{a\,d}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,11\mathrm{i}}{4\,a\,d}-\frac{\tan(c+d\,x)^4\,1\mathrm{i}}{4\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6/(a + a*tan(c + d*x)*1i),x)
```

```
[Out] (tan(c + d*x)^2*1i)/(a*d) - (log(tan(c + d*x) + 1i)*1i)/(4*a*d) - (2*tan(c
+ d*x))/(a*d) - 1i/(2*a*d*(tan(c + d*x)*1i + 1)) - (log(tan(c + d*x) - 1i)*
11i)/(4*a*d) + tan(c + d*x)^3/(3*a*d) - (tan(c + d*x)^4*1i)/(4*a*d)
```

3.46 $\int \frac{\tan^5(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=109

$$-\frac{5ix}{2a}+\frac{2\log(\cos(c+dx))}{ad}+\frac{5i\tan(c+dx)}{2ad}+\frac{\tan^2(c+dx)}{ad}-\frac{5i\tan^3(c+dx)}{6ad}-\frac{\tan^4(c+dx)}{2d(a+ia\tan(c+dx))}$$

```
[Out] -5/2*I*x/a+2*ln(cos(d*x+c))/a/d+5/2*I*tan(d*x+c)/a/d+tan(d*x+c)^2/a/d-5/6*I
*tan(d*x+c)^3/a/d-1/2*tan(d*x+c)^4/d/(a+I*a*tan(d*x+c))
```

Rubi [A]
time = 0.09, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3631, 3609, 3606, 3556}

$$-\frac{\tan^4(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{5i\tan^3(c+dx)}{6ad}+\frac{\tan^2(c+dx)}{ad}+\frac{5i\tan(c+dx)}{2ad}+\frac{2\log(\cos(c+dx))}{ad}-\frac{5ix}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (((-5*I)/2)*x)/a + (2*Log[Cos[c + d*x]])/(a*d) + (((5*I)/2)*Tan[c + d*x])/(
a*d) + Tan[c + d*x]^2/(a*d) - (((5*I)/6)*Tan[c + d*x]^3)/(a*d) - Tan[c + d*
x]^4/(2*d*(a + I*a*Tan[c + d*x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3631

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((c + d*Tan[e + f*x])^(n - 1)/(
2*a*f*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a^2), Int[(c + d*Tan[e + f*x])
^(n - 2)*Simp[a*c^2 + a*d^2*(n - 1) - b*c*d*n - d*(a*c*(n - 2) + b*d*n)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^5(c+dx)}{a+ia\tan(c+dx)}\,dx &= -\frac{\tan^4(c+dx)}{2d(a+ia\tan(c+dx))}+\frac{\int \tan^3(c+dx)(4a-5ia\tan(c+dx))\,dx}{2a^2}\\ &= -\frac{5i\tan^3(c+dx)}{6ad}-\frac{\tan^4(c+dx)}{2d(a+ia\tan(c+dx))}+\frac{\int \tan^2(c+dx)(5ia+4a\tan(c+d\cdots}{2a^2}\\ &= \frac{\tan^2(c+dx)}{ad}-\frac{5i\tan^3(c+dx)}{6ad}-\frac{\tan^4(c+dx)}{2d(a+ia\tan(c+dx))}+\frac{\int \tan(c+dx)(-4a-\cdots}{2\cdots}\\ &= -\frac{5ix}{2a}+\frac{5i\tan(c+dx)}{2ad}+\frac{\tan^2(c+dx)}{ad}-\frac{5i\tan^3(c+dx)}{6ad}-\frac{\tan^4(c+dx)}{2d(a+ia\tan(c+d\cdots}\\ &= -\frac{5ix}{2a}+\frac{2\log(\cos(c+dx))}{ad}+\frac{5i\tan(c+dx)}{2ad}+\frac{\tan^2(c+dx)}{ad}-\frac{5i\tan^3(c+dx)}{6ad}-\cdots\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 235 vs. $2(109) = 218$.
time = 3.81, size = 235, normalized size = 2.16

$$\frac{\sec(c+dx)(\cos(dx)+i\sin(dx))\left(6dx\sin(c)+3i\cos(2dx)\sin(c)+12i\log\left(\cos^2(c+dx)\right)\sin(c)+2i\sec^2(c+dx)\sin(c)+24\text{ArcTan}(\tan(dx))(-i\cos(c)+\sin(c))+28i\sec(c+dx)\sin(dx)-4i\sec^3(c+dx)\sin(dx)+3\cos(c)\left(-2idx-\cos(2dx)+4\log\left(\cos^2(c+dx)\right)+2\sec^2(c+dx)+i\sin(2dx)\right)+3\sin(c)\sin(2dx)+4\sec^2(c+dx)\sin(c)\tan(c)-28\sec(c+dx)\sin(dx)\tan(c)+4\sec^3(c+dx)\sin(dx)\tan(c)\right)}{12d(a+ia\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]*(Cos[d*x] + I*Sin[d*x])*(6*d*x*Sin[c] + (3*I)*Cos[2*d*x]*Sin[
c] + (12*I)*Log[Cos[c + d*x]^2]*Sin[c] + (2*I)*Sec[c + d*x]^2*Sin[c] + 24*A
rcTan[Tan[d*x]]*((-I)*Cos[c] + Sin[c]) + (28*I)*Sec[c + d*x]*Sin[d*x] - (4*
I)*Sec[c + d*x]^3*Sin[d*x] + 3*Cos[c]*((-2*I)*d*x - Cos[2*d*x] + 4*Log[Cos[
c + d*x]^2] + 2*Sec[c + d*x]^2 + I*Sin[2*d*x]) + 3*Sin[c]*Sin[2*d*x] + 4*Se
c[c + d*x]^2*Sin[c]*Tan[c] - 28*Sec[c + d*x]*Sin[d*x]*Tan[c] + 4*Sec[c + d*
x]^3*Sin[d*x]*Tan[c]))/(12*d*(a + I*a*Tan[c + d*x]))
```

Maple [A]
time = 0.10, size = 77, normalized size = 0.71

method	result	size

derivativedivides	$\frac{2i\tan(dx+c)-\frac{i\left(\tan^3(dx+c)\right)}{3}+\frac{\left(\tan^2(dx+c)\right)}{2}+\frac{i}{2\tan(dx+c)-2i}-\frac{9\ln(\tan(dx+c)-i)}{4}+\frac{\ln(\tan(dx+c)+i)}{4}}{da}$	77
default	$\frac{2i\tan(dx+c)-\frac{i\left(\tan^3(dx+c)\right)}{3}+\frac{\left(\tan^2(dx+c)\right)}{2}+\frac{i}{2\tan(dx+c)-2i}-\frac{9\ln(\tan(dx+c)-i)}{4}+\frac{\ln(\tan(dx+c)+i)}{4}}{da}$	77
risch	$-\frac{9ix}{2a}-\frac{e^{-2i(dx+c)}}{4da}-\frac{4ic}{da}-\frac{2\left(6\,e^{4i(dx+c)}+9\,e^{2i(dx+c)}+7\right)}{3da\left(e^{2i(dx+c)}+1\right)^3}+\frac{2\ln\left(e^{2i(dx+c)}+1\right)}{da}$	10
norman	$\frac{-\frac{1}{da}+\frac{\tan^4(dx+c)}{2da}-\frac{5ix}{2a}-\frac{5ix\left(\tan^2(dx+c)\right)}{2a}+\frac{5i\tan(dx+c)}{2da}+\frac{5i\left(\tan^3(dx+c)\right)}{3da}-\frac{i\left(\tan^5(dx+c)\right)}{3da}}{1+\tan^2(dx+c)}-\frac{\ln(1+\tan^2(dx+c))}{da}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(2*I*tan(d*x+c)-1/3*I*tan(d*x+c)^3+1/2*tan(d*x+c)^2+1/2*I/(tan(d*x+c)
-I)-9/4*ln(tan(d*x+c)-I)+1/4*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.38, size = 175, normalized size = 1.61

$$\frac{-54i\,dxe^{(8i\,dx+8i\,c)}-3\,(54i\,dx+17)e^{(6i\,dx+6i\,c)}-81\,(2i\,dx+1)e^{(4i\,dx+4i\,c)}+(-54i\,dx-65)e^{(2i\,dx+2i\,c)}+24\left(e^{(8i\,dx+8i\,c)}+3\,e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+e^{(2i\,dx+2i\,c)}\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-3}{12\left(ade^{(8i\,dx+8i\,c)}+3\,ade^{(6i\,dx+6i\,c)}+3\,ade^{(4i\,dx+4i\,c)}+ade^{(2i\,dx+2i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(-54*I*d*x*e^(8*I*d*x + 8*I*c) - 3*(54*I*d*x + 17)*e^(6*I*d*x + 6*I*c)
 - 81*(2*I*d*x + 1)*e^(4*I*d*x + 4*I*c) + (-54*I*d*x - 65)*e^(2*I*d*x + 2*I
*c) + 24*(e^(8*I*d*x + 8*I*c) + 3*e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*
c) + e^(2*I*d*x + 2*I*c))*log(e^(2*I*d*x + 2*I*c) + 1) - 3)/(a*d*e^(8*I*d*x
 + 8*I*c) + 3*a*d*e^(6*I*d*x + 6*I*c) + 3*a*d*e^(4*I*d*x + 4*I*c) + a*d*e^(
2*I*d*x + 2*I*c))
```

Sympy [A]
time = 0.30, size = 197, normalized size = 1.81

$$\frac{-12e^{4ic}e^{4idx}-18e^{2ic}e^{2idx}-14}{3ade^{6ic}e^{6idx}+9ade^{4ic}e^{4idx}+9ade^{2ic}e^{2idx}+3ad}+\begin{cases}-\frac{e^{-2ic}e^{-2idx}}{4ad} & \text{for}\ ade^{2ic}\neq 0\\ x\left(\frac{\left(-9ie^{2ic}+i\right)e^{-2ic}}{2a}+\frac{9i}{2a}\right) & \text{otherwise}\end{cases}-\frac{9ix}{2a}+\frac{2\log\left(e^{2idx}+e^{-2ic}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+I*a*tan(d*x+c)),x)
```

```
[Out] (-12*exp(4*I*c)*exp(4*I*d*x) - 18*exp(2*I*c)*exp(2*I*d*x) - 14)/(3*a*d*exp(
6*I*c)*exp(6*I*d*x) + 9*a*d*exp(4*I*c)*exp(4*I*d*x) + 9*a*d*exp(2*I*c)*exp(
2*I*d*x) + 3*a*d) + Piecewise((-exp(-2*I*c)*exp(-2*I*d*x)/(4*a*d), Ne(a*d*e
xp(2*I*c), 0)), (x*((-9*I*exp(2*I*c) + I)*exp(-2*I*c)/(2*a) + 9*I/(2*a)), T
rue)) - 9*I*x/(2*a) + 2*log(exp(2*I*d*x) + exp(-2*I*c))/(a*d)
```

Giac [A]
time = 1.90, size = 104, normalized size = 0.95

$$\frac{\frac{3\,\log(\tan(dx+c)+i)}{a}-\frac{27\,\log(i\,\tan(dx+c)+1)}{a}+\frac{3\,(9\,\tan(dx+c)-7i)}{a(\tan(dx+c)-i)}-\frac{2\left(2i\,a^2\tan(dx+c)^3-3\,a^2\tan(dx+c)^2-12i\,a^2\tan(dx+c)\right)}{a^3}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(3*log(tan(d*x + c) + I)/a - 27*log(I*tan(d*x + c) + 1)/a + 3*(9*tan(d
*x + c) - 7*I)/(a*(tan(d*x + c) - I)) - 2*(2*I*a^2*tan(d*x + c)^3 - 3*a^2*t
an(d*x + c)^2 - 12*I*a^2*tan(d*x + c))/a^3)/d
```

Mupad [B]
time = 3.98, size = 106, normalized size = 0.97

$$\frac{\tan(c+d\,x)^2}{2\,a\,d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)}{4\,a\,d}+\frac{\tan(c+d\,x)\;2\mathrm{i}}{a\,d}-\frac{1}{2\,a\,d\,\left(1+\tan\left(c+d\,x\right)\;1\mathrm{i}\right)}-\frac{9\,\ln\left(\tan(c+d\,x)-\mathrm{i}\right)}{4\,a\,d}-\frac{\tan(c+d\,x)^3\;1\mathrm{i}}{3\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + a*tan(c + d*x)*1i),x)
```

```
[Out] log(tan(c + d*x) + 1i)/(4*a*d) - (9*log(tan(c + d*x) - 1i))/(4*a*d) + (tan(
c + d*x)*2i)/(a*d) - 1/(2*a*d*(tan(c + d*x)*1i + 1)) + tan(c + d*x)^2/(2*a*
d) - (tan(c + d*x)^3*1i)/(3*a*d)
```

3.47 $\int \frac{\tan^4(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=90

$$-\frac{3x}{2a}-\frac{2i\log(\cos(c+dx))}{ad}+\frac{3\tan(c+dx)}{2ad}-\frac{i\tan^2(c+dx)}{ad}-\frac{\tan^3(c+dx)}{2d(a+ia\tan(c+dx))}$$

```
[Out] -3/2*x/a-2*I*ln(cos(d*x+c))/a/d+3/2*tan(d*x+c)/a/d-I*tan(d*x+c)^2/a/d-1/2*t
an(d*x+c)^3/d/(a+I*a*tan(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3631, 3609, 3606, 3556}

$$-\frac{\tan^3(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{i\tan^2(c+dx)}{ad}+\frac{3\tan(c+dx)}{2ad}-\frac{2i\log(\cos(c+dx))}{ad}-\frac{3x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (-3*x)/(2*a) - ((2*I)*Log[Cos[c + d*x]])/(a*d) + (3*Tan[c + d*x])/(2*a*d) -
 (I*Tan[c + d*x]^2)/(a*d) - Tan[c + d*x]^3/(2*d*(a + I*a*Tan[c + d*x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3631

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((c + d*Tan[e + f*x])^(n - 1)/(
2*a*f*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a^2), Int[(c + d*Tan[e + f*x])
^(n - 2)*Simp[a*c^2 + a*d^2*(n - 1) - b*c*d*n - d*(a*c*(n - 2) + b*d*n)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^4(c+dx)}{a+ia\tan(c+dx)}\,dx &= -\frac{\tan^3(c+dx)}{2d(a+ia\tan(c+dx))}+\frac{\int \tan^2(c+dx)(3a-4ia\tan(c+dx))\,dx}{2a^2} \\
&= -\frac{i\tan^2(c+dx)}{ad}-\frac{\tan^3(c+dx)}{2d(a+ia\tan(c+dx))}+\frac{\int \tan(c+dx)(4ia+3a\tan(c+dx)}{2a^2} \\
&= -\frac{3x}{2a}+\frac{3\tan(c+dx)}{2ad}-\frac{i\tan^2(c+dx)}{ad}-\frac{\tan^3(c+dx)}{2d(a+ia\tan(c+dx))}+\frac{(2i)\int \tan(c+}{a} \\
&= -\frac{3x}{2a}-\frac{2i\log(\cos(c+dx))}{ad}+\frac{3\tan(c+dx)}{2ad}-\frac{i\tan^2(c+dx)}{ad}-\frac{\tan^3(c+dx}{2d(a+ia\tan(c+}
\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 196 vs. $2(90) = 180$.
time = 1.98, size = 196, normalized size = 2.18

$$\frac{\cos(c)\sec(c+dx)(\cos(dx)+i\sin(dx))\left(-6dx-4i\log\left(\cos^2(c+dx)\right)+8dx\sec^2(c)-2i\sec^2(c+dx)+4\sec(c)\sec(c+dx)\sin(dx)+\sin(2dx)+\mathrm{ArcTan}(\tan(dx))(-8-8i\tan(c))+2idx\tan(c)+4\log\left(\cos^2(c+dx)\right)\tan(c)+2\sec^2(c+dx)\tan(c)+4i\sec(c)\sec(c+dx)\sin(dx)\tan(c)-i\sin(2dx)\tan(c)-8dx\tan^2(c)+\cos(2dx)(i+\tan(c))\right)}{4d(a+ia\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (Cos[c]*Sec[c + d*x]*(Cos[d*x] + I*Sin[d*x])*(-6*d*x - (4*I)*Log[Cos[c + d*
x]^2] + 8*d*x*Sec[c]^2 - (2*I)*Sec[c + d*x]^2 + 4*Sec[c]*Sec[c + d*x]*Sin[d
*x] + Sin[2*d*x] + ArcTan[Tan[d*x]]*(-8 - (8*I)*Tan[c]) + (2*I)*d*x*Tan[c]
+ 4*Log[Cos[c + d*x]^2]*Tan[c] + 2*Sec[c + d*x]^2*Tan[c] + (4*I)*Sec[c]*Sec
[c + d*x]*Sin[d*x]*Tan[c] - I*Sin[2*d*x]*Tan[c] - 8*d*x*Tan[c]^2 + Cos[2*d*
x]*(I + Tan[c])))/(4*d*(a + I*a*Tan[c + d*x]))
```

Maple [A]
time = 0.10, size = 65, normalized size = 0.72

method	result	size
derivativedivides	$\frac{\tan(dx+c)-\frac{i\left(\tan^2(dx+c)\right)}{2}+\frac{7i\ln(\tan(dx+c)-i)}{4}+\frac{1}{2\tan(dx+c)-2i}+\frac{i\ln(\tan(dx+c)+i)}{4}}{da}$	65

default	$\frac{\tan(dx+c)-\frac{i\left(\tan^2(dx+c)\right)}{2}+\frac{7i\ln(\tan(dx+c)-i)}{4}+\frac{1}{2\tan(dx+c)-2i}+\frac{i\ln(\tan(dx+c)+i)}{4}}{da}$	65
risch	$-\frac{7x}{2a}+\frac{i\mathrm{e}^{-2i(dx+c)}}{4da}-\frac{4c}{da}+\frac{2i}{da\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}-\frac{2i\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{da}$	78
norman	$\frac{\frac{i}{da}+\frac{\tan^3(dx+c)}{da}-\frac{3x}{2a}-\frac{3x\left(\tan^2(dx+c)\right)}{2a}+\frac{3\tan(dx+c)}{2da}-\frac{i\left(\tan^4(dx+c)\right)}{2da}}{1+\tan^2(dx+c)}+\frac{i\ln(1+\tan^2(dx+c))}{da}$	111

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(tan(d*x+c)-1/2*I*tan(d*x+c)^2+7/4*I*ln(tan(d*x+c)-I)+1/2/(tan(d*x+c)
-I)+1/4*I*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.38, size = 138, normalized size = 1.53

$$-\frac{14\,dxe^{(6i\,dx+6i\,c)}+(28\,dx-i)e^{(4i\,dx+4i\,c)}+2\,(7\,dx-5i)e^{(2i\,dx+2i\,c)}+8\left(i\,e^{(6i\,dx+6i\,c)}+2i\,e^{(4i\,dx+4i\,c)}+i\,e^{(2i\,dx+2i\,c)}\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-i}{4\left(ade^{(6i\,dx+6i\,c)}+2\,ade^{(4i\,dx+4i\,c)}+ade^{(2i\,dx+2i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(14*d*x*e^(6*I*d*x + 6*I*c) + (28*d*x - I)*e^(4*I*d*x + 4*I*c) + 2*(7*
d*x - 5*I)*e^(2*I*d*x + 2*I*c) + 8*(I*e^(6*I*d*x + 6*I*c) + 2*I*e^(4*I*d*x
+ 4*I*c) + I*e^(2*I*d*x + 2*I*c))*log(e^(2*I*d*x + 2*I*c) + 1) - I)/(a*d*e^
(6*I*d*x + 6*I*c) + 2*a*d*e^(4*I*d*x + 4*I*c) + a*d*e^(2*I*d*x + 2*I*c))
```

Sympy [A]
time = 0.23, size = 134, normalized size = 1.49

$$\begin{cases}\frac{ie^{-2ic}e^{-2idx}}{4ad} & \text{for}\ ade^{2ic}\neq 0\\ x\left(\frac{(1-7e^{2ic})e^{-2ic}}{2a}+\frac{7}{2a}\right) & \text{otherwise}\end{cases}+\frac{2i}{ade^{4ic}e^{4idx}+2ade^{2ic}e^{2idx}+ad}-\frac{7x}{2a}-\frac{2i\log\left(e^{2idx}+e^{-2ic}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise((I*exp(-2*I*c)*exp(-2*I*d*x)/(4*a*d), Ne(a*d*exp(2*I*c), 0)), (x*
((1 - 7*exp(2*I*c))*exp(-2*I*c)/(2*a) + 7/(2*a)), True)) + 2*I/(a*d*exp(4*I
*c)*exp(4*I*d*x) + 2*a*d*exp(2*I*c)*exp(2*I*d*x) + a*d) - 7*x/(2*a) - 2*I*l
og(exp(2*I*d*x) + exp(-2*I*c))/(a*d)
```

Giac [A]
time = 0.97, size = 87, normalized size = 0.97

$$-\frac{-\frac{7i\log(\tan(dx+c)-i)}{a}-\frac{i\log(-i\tan(dx+c)+1)}{a}+\frac{2\left(ia\tan(dx+c)^2-2a\tan(dx+c)\right)}{a^2}-\frac{-7i\tan(dx+c)-5}{a(\tan(dx+c)-i)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(-7*I*log(tan(d*x + c) - I)/a - I*log(-I*tan(d*x + c) + 1)/a + 2*(I*a*
tan(d*x + c)^2 - 2*a*tan(d*x + c))/a^2 - (-7*I*tan(d*x + c) - 5)/(a*(tan(d*
x + c) - I)))/d
```

Mupad [B]
time = 4.01, size = 91, normalized size = 1.01

$$\frac{\ln\left(\tan(c+dx)-\mathrm{i}\right)\,7\mathrm{i}}{4\,a\,d}+\frac{\ln\left(\tan(c+dx)+1\mathrm{i}\right)\,1\mathrm{i}}{4\,a\,d}+\frac{\tan(c+dx)}{a\,d}+\frac{1\mathrm{i}}{2\,a\,d\,\left(1+\tan\left(c+dx\right)\,1\mathrm{i}\right)}-\frac{\tan(c+dx)^2\,1\mathrm{i}}{2\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + a*tan(c + d*x)*1i),x)
```

```
[Out] (log(tan(c + d*x) - 1i)*7i)/(4*a*d) + (log(tan(c + d*x) + 1i)*1i)/(4*a*d) +
 tan(c + d*x)/(a*d) + 1i/(2*a*d*(tan(c + d*x)*1i + 1)) - (tan(c + d*x)^2*1i
)/(2*a*d)
```

3.48 $\int \frac{\tan^3(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=74

$$\frac{3ix}{2a}-\frac{\log(\cos(c+dx))}{ad}-\frac{3i\tan(c+dx)}{2ad}-\frac{\tan^2(c+dx)}{2d(a+ia\tan(c+dx))}$$

```
[Out] 3/2*I*x/a-ln(cos(d*x+c))/a/d-3/2*I*tan(d*x+c)/a/d-1/2*tan(d*x+c)^2/d/(a+I*a
*tan(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3631, 3606, 3556}

$$-\frac{\tan^2(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{3i\tan(c+dx)}{2ad}-\frac{\log(\cos(c+dx))}{ad}+\frac{3ix}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (((3*I)/2)*x)/a - Log[Cos[c + d*x]]/(a*d) - (((3*I)/2)*Tan[c + d*x])/(a*d)
- Tan[c + d*x]^2/(2*d*(a + I*a*Tan[c + d*x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3631

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((c + d*Tan[e + f*x])^(n - 1)/(
2*a*f*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a^2), Int[(c + d*Tan[e + f*x])
^(n - 2)*Simp[a*c^2 + a*d^2*(n - 1) - b*c*d*n - d*(a*c*(n - 2) + b*d*n)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{\tan^2(c+dx)}{2d(a+ia\tan(c+dx))}+\frac{\int \tan(c+dx)(2a-3ia\tan(c+dx))\,dx}{2a^2}$$
$$=\frac{3ix}{2a}-\frac{3i\tan(c+dx)}{2ad}-\frac{\tan^2(c+dx)}{2d(a+ia\tan(c+dx))}+\frac{\int \tan(c+dx)\,dx}{a}$$
$$=\frac{3ix}{2a}-\frac{\log(\cos(c+dx))}{ad}-\frac{3i\tan(c+dx)}{2ad}-\frac{\tan^2(c+dx)}{2d(a+ia\tan(c+dx))}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 174 vs. $2(74) = 148$.
time = 1.24, size = 174, normalized size = 2.35

$$-\frac{i\cos(c)\sec(c+dx)(\cos(dx)+i\sin(dx))\left(-6dx-2i\log\left(\cos^2(c+dx)\right)+4dx\sec^2(c)+4\sec(c)\sec(c+dx)\sin(dx)+\sin(2dx)+\text{ArcTan}(\tan(dx))(-4-4i\tan(c))-2idx\tan(c)+2\log\left(\cos^2(c+dx)\right)\tan(c)+4i\sec(c)\sec(c+dx)\sin(dx)\tan(c)-i\sin(2dx)\tan(c)-4dx\tan^2(c)+\cos(2dx)(i+\tan(c))\right)}{4d(a+ia\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-1/4*I)*Cos[c]*Sec[c + d*x]*(Cos[d*x] + I*Sin[d*x])*(-6*d*x - (2*I)*Log[C
os[c + d*x]^2] + 4*d*x*Sec[c]^2 + 4*Sec[c]*Sec[c + d*x]*Sin[d*x] + Sin[2*d*
x] + ArcTan[Tan[d*x]]*(-4 - (4*I)*Tan[c]) - (2*I)*d*x*Tan[c] + 2*Log[Cos[c
+ d*x]^2]*Tan[c] + (4*I)*Sec[c]*Sec[c + d*x]*Sin[d*x]*Tan[c] - I*Sin[2*d*x]
*Tan[c] - 4*d*x*Tan[c]^2 + Cos[2*d*x]*(I + Tan[c])))/(d*(a + I*a*Tan[c + d*
x]))
```

Maple [**A**]
time = 0.10, size = 56, normalized size = 0.76

method	result	size
derivativedivides	$\frac{-i\tan(dx+c)-\frac{i}{2(\tan(dx+c)-i)}+\frac{5\ln(\tan(dx+c)-i)}{4}-\frac{\ln(\tan(dx+c)+i)}{4}}{da}$	56
default	$\frac{-i\tan(dx+c)-\frac{i}{2(\tan(dx+c)-i)}+\frac{5\ln(\tan(dx+c)-i)}{4}-\frac{\ln(\tan(dx+c)+i)}{4}}{da}$	56
risch	$\frac{5ix}{2a}+\frac{e^{-2i(dx+c)}}{4da}+\frac{2ic}{da}+\frac{2}{da\left(e^{2i(dx+c)}+1\right)}-\frac{\ln\left(e^{2i(dx+c)}+1\right)}{da}$	77
norman	$\frac{\frac{1}{2da}+\frac{3ix}{2a}+\frac{3ix\left(\tan^2(dx+c)\right)}{2a}-\frac{3i\tan(dx+c)}{2da}-\frac{i\left(\tan^3(dx+c)\right)}{da}}{1+\tan^2(dx+c)}+\frac{\ln\left(1+\tan^2(dx+c)\right)}{2da}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-I*tan(d*x+c)-1/2*I/(tan(d*x+c)-I)+5/4*ln(tan(d*x+c)-I)-1/4*ln(tan(d
*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.38, size = 93, normalized size = 1.26

$$\frac{10i\,dxe^{(4i\,dx+4i\,c)}+(10i\,dx+9)e^{(2i\,dx+2i\,c)}-4\left(e^{(4i\,dx+4i\,c)}+e^{(2i\,dx+2i\,c)}\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)+1}{4\left(ade^{(4i\,dx+4i\,c)}+ade^{(2i\,dx+2i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(10*I*d*x*e^(4*I*d*x + 4*I*c) + (10*I*d*x + 9)*e^(2*I*d*x + 2*I*c) - 4*
(e^(4*I*d*x + 4*I*c) + e^(2*I*d*x + 2*I*c))*log(e^(2*I*d*x + 2*I*c) + 1) +
1)/(a*d*e^(4*I*d*x + 4*I*c) + a*d*e^(2*I*d*x + 2*I*c))
```

Sympy [A]
time = 0.21, size = 114, normalized size = 1.54

$$\begin{cases}\frac{e^{-2ic}e^{-2idx}}{4ad} & \text{for}\ ade^{2ic}\neq 0\\ x\left(\frac{(5ie^{2ic}-i)e^{-2ic}}{2a}-\frac{5i}{2a}\right) & \text{otherwise}\end{cases}+\frac{2}{ade^{2ic}e^{2idx}+ad}+\frac{5ix}{2a}-\frac{\log\left(e^{2idx}+e^{-2ic}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise((exp(-2*I*c)*exp(-2*I*d*x)/(4*a*d), Ne(a*d*exp(2*I*c), 0)), (x*((
5*I*exp(2*I*c) - I)*exp(-2*I*c)/(2*a) - 5*I/(2*a)), True)) + 2/(a*d*exp(2*I
*c)*exp(2*I*d*x) + a*d) + 5*I*x/(2*a) - log(exp(2*I*d*x) + exp(-2*I*c))/(a*
d)
```

Giac [A]
time = 0.73, size = 70, normalized size = 0.95

$$-\frac{\frac{\log(\tan(dx+c)+i)}{a}-\frac{5\log(-i\tan(dx+c)-1)}{a}+\frac{4i\tan(dx+c)}{a}+\frac{5\tan(dx+c)-3i}{a(\tan(dx+c)-i)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(log(tan(d*x + c) + I)/a - 5*log(-I*tan(d*x + c) - 1)/a + 4*I*tan(d*x
+ c)/a + (5*tan(d*x + c) - 3*I)/(a*(tan(d*x + c) - I)))/d
```

Mupad [B]
time = 4.00, size = 73, normalized size = 0.99

$$\frac{5\,\ln\left(\tan(c+d\,x)-\mathrm{i}\right)}{4\,a\,d}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)}{4\,a\,d}-\frac{\tan(c+d\,x)\ 1\mathrm{i}}{a\,d}+\frac{1}{2\,a\,d\,\left(1+\tan\left(c+d\,x\right)\ 1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + a*tan(c + d*x)*1i),x)
```

```
[Out] (5*log(tan(c + d*x) - 1i))/(4*a*d) - log(tan(c + d*x) + 1i)/(4*a*d) - (tan(
c + d*x)*1i)/(a*d) + 1/(2*a*d*(tan(c + d*x)*1i + 1))
```

3.49 $\int \frac{\tan^2(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=50

$$\frac{x}{2a}+\frac{i\log(\cos(c+dx))}{ad}-\frac{i}{2d(a+ia\tan(c+dx))}$$

```
[Out] 1/2*x/a+I*ln(cos(d*x+c))/a/d-1/2*I/d/(a+I*a*tan(d*x+c))
```

Rubi [A]

time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3621, 3556}

$$-\frac{i}{2d(a+ia\tan(c+dx))}+\frac{i\log(\cos(c+dx))}{ad}+\frac{x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] x/(2*a) + (I*Log[Cos[c + d*x]])/(a*d) - (I/2)/(d*(a + I*a*Tan[c + d*x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3621

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(a*c + b*d)^2*((a + b*Tan[e + f*x])^
m/(2*a^3*f*m)), x] + Dist[1/(2*a^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[
a*c^2 - 2*b*c*d + a*d^2 - 2*b*d^2*Tan[e + f*x], x], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LeQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^2(c+dx)}{a+ia\tan(c+dx)}\,dx &= -\frac{i}{2d(a+ia\tan(c+dx))}+\frac{\int(a-2ia\tan(c+dx))\,dx}{2a^2}\\ &= \frac{x}{2a}-\frac{i}{2d(a+ia\tan(c+dx))}-\frac{i\int\tan(c+dx)\,dx}{a}\\ &= \frac{x}{2a}+\frac{i\log(\cos(c+dx))}{ad}-\frac{i}{2d(a+ia\tan(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.36, size = 86, normalized size = 1.72

$$\frac{-1+2idx+2\log(\cos^2(c+dx))+(i-2dx+2i\log(\cos^2(c+dx)))\tan(c+dx)+4\text{ArcTan}(\tan(dx))(-i+\tan(c+dx))}{4ad(-i+\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (-1 + (2*I)*d*x + 2*Log[Cos[c + d*x]^2] + (I - 2*d*x + (2*I)*Log[Cos[c + d*
x]^2])*Tan[c + d*x] + 4*ArcTan[Tan[d*x]]*(-I + Tan[c + d*x]))/(4*a*d*(-I +
Tan[c + d*x]))
```

Maple [A]
time = 0.09, size = 48, normalized size = 0.96

method	result	size
derivativedivides	$\frac{-\frac{3i\ln(\tan(dx+c)-i)}{4}-\frac{1}{2(\tan(dx+c)-i)}-\frac{i\ln(\tan(dx+c)+i)}{4}}{da}$	48
default	$\frac{-\frac{3i\ln(\tan(dx+c)-i)}{4}-\frac{1}{2(\tan(dx+c)-i)}-\frac{i\ln(\tan(dx+c)+i)}{4}}{da}$	48
risch	$\frac{3x}{2a}-\frac{ie^{-2i(dx+c)}}{4da}+\frac{2c}{da}+\frac{i\ln\left(e^{2i(dx+c)}+1\right)}{da}$	56
norman	$\frac{\frac{x}{2a}+\frac{x\left(\tan^2(dx+c)\right)}{2a}-\frac{i}{2da}-\frac{\tan(dx+c)}{2da}}{1+\tan^2(dx+c)}-\frac{i\ln\left(1+\tan^2(dx+c)\right)}{2da}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-3/4*I*ln(tan(d*x+c)-I)-1/2/(tan(d*x+c)-I)-1/4*I*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.37, size = 55, normalized size = 1.10

$$\frac{\left(6\,dxe^{(2i\,dx+2i\,c)}+4i\,e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-i\right)e^{(-2i\,dx-2i\,c)}}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(6*d*x*e^(2*I*d*x + 2*I*c) + 4*I*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2
*I*c) + 1) - I)*e^(-2*I*d*x - 2*I*c)/(a*d)
```

Sympy [A]
time = 0.17, size = 88, normalized size = 1.76

$$\begin{cases} -\frac{ie^{-2ic}e^{-2idx}}{4ad} & \text{for } ade^{2ic} \neq 0 \\ x\left(\frac{(3e^{2ic}-1)e^{-2ic}}{2a} - \frac{3}{2a}\right) & \text{otherwise} \end{cases} + \frac{3x}{2a} + \frac{i\log\left(e^{2idx} + e^{-2ic}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise((-I*exp(-2*I*c)*exp(-2*I*d*x)/(4*a*d), Ne(a*d*exp(2*I*c), 0)), (x
*((3*exp(2*I*c) - 1)*exp(-2*I*c)/(2*a) - 3/(2*a)), True)) + 3*x/(2*a) + I*l
og(exp(2*I*d*x) + exp(-2*I*c))/(a*d)
```

Giac [A]
time = 0.66, size = 60, normalized size = 1.20

$$-\frac{\frac{3i\log(\tan(dx+c)-i)}{a} + \frac{i\log(i\tan(dx+c)-1)}{a} + \frac{-3i\tan(dx+c)-1}{a(\tan(dx+c)-i)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(3*I*log(tan(d*x + c) - I)/a + I*log(I*tan(d*x + c) - 1)/a + (-3*I*tan
(d*x + c) - 1)/(a*(tan(d*x + c) - I)))/d
```

Mupad [B]
time = 3.95, size = 61, normalized size = 1.22

$$-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,3\mathrm{i}}{4\,a\,d} - \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{4\,a\,d} - \frac{1\mathrm{i}}{2\,a\,d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + a*tan(c + d*x)*1i),x)
```

```
[Out] - (log(tan(c + d*x) - 1i)*3i)/(4*a*d) - (log(tan(c + d*x) + 1i)*1i)/(4*a*d)
 - 1i/(2*a*d*(tan(c + d*x)*1i + 1))
```

3.50 $\int \frac{\tan(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=33

$$-\frac{ix}{2a}-\frac{1}{2d(a+ia\tan(c+dx))}$$

```
[Out] -1/2*I*x/a-1/2/d/(a+I*a*tan(d*x+c))
```

Rubi [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {3607, 8}

$$-\frac{1}{2d(a+ia\tan(c+dx))}-\frac{ix}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + I*a*Tan[c + d*x]),x]
[Out] ((-1/2*I)*x)/a - 1/(2*d*(a + I*a*Tan[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(c+dx)}{a+ia\tan(c+dx)}\,dx&=-\frac{1}{2d(a+ia\tan(c+dx))}-\frac{i\int 1\,dx}{2a}\\&=-\frac{ix}{2a}-\frac{1}{2d(a+ia\tan(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 45, normalized size = 1.36

$$\frac{i-2dx+(1-2idx)\tan(c+dx)}{4ad(-i+\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (I - 2*d*x + (1 - (2*I)*d*x)*Tan[c + d*x])/(4*a*d*(-I + Tan[c + d*x]))
```

Maple [A]
time = 0.07, size = 47, normalized size = 1.42

method	result	size
risch	$-\frac{ix}{2a}-\frac{e^{-2i(dx+c)}}{4da}$	26
derivativedivides	$\frac{\frac{i}{2\tan(dx+c)-2i}-\frac{\ln(\tan(dx+c)-i)}{4}+\frac{\ln(\tan(dx+c)+i)}{4}}{da}$	47
default	$\frac{\frac{i}{2\tan(dx+c)-2i}-\frac{\ln(\tan(dx+c)-i)}{4}+\frac{\ln(\tan(dx+c)+i)}{4}}{da}$	47
norman	$\frac{-\frac{1}{2da}-\frac{ix}{2a}-\frac{ix\left(\tan^2(dx+c)\right)}{2a}+\frac{i\tan(dx+c)}{2da}}{1+\tan^2(dx+c)}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(1/2*I/(tan(d*x+c)-I)-1/4*ln(tan(d*x+c)-I)+1/4*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.37, size = 32, normalized size = 0.97

$$\frac{\left(-2i\,dxe^{(2i\,dx+2i\,c)}-1\right)e^{(-2i\,dx-2i\,c)}}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(-2*I*d*x*e^(2*I*d*x + 2*I*c) - 1)*e^(-2*I*d*x - 2*I*c)/(a*d)
```

Sympy [A]
time = 0.10, size = 65, normalized size = 1.97

$$\begin{cases} -\frac{e^{-2ic}e^{-2idx}}{4ad} & \text{for } ade^{2ic} \neq 0 \\ x\left(\frac{(-ie^{2ic}+i)e^{-2ic}}{2a} + \frac{i}{2a}\right) & \text{otherwise} \end{cases} - \frac{ix}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise((-exp(-2*I*c)*exp(-2*I*d*x)/(4*a*d), Ne(a*d*exp(2*I*c), 0)), (x*(
(-I*exp(2*I*c) + I)*exp(-2*I*c)/(2*a) + I/(2*a)), True)) - I*x/(2*a)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(25) = 50.
time = 0.50, size = 58, normalized size = 1.76

$$-\frac{\frac{\log(\tan(dx+c)-i)}{a} - \frac{\log(-i\tan(dx+c)+1)}{a} - \frac{\tan(dx+c)+i}{a(\tan(dx+c)-i)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(log(tan(d*x + c) - I)/a - log(-I*tan(d*x + c) + 1)/a - (tan(d*x + c)
+ I)/(a*(tan(d*x + c) - I)))/d
```

Mupad [B]
time = 3.94, size = 29, normalized size = 0.88

$$-\frac{x\,1\mathrm{i}}{2\,a} - \frac{1}{2\,a\,d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + a*tan(c + d*x)*1i),x)
```

```
[Out] - (x*1i)/(2*a) - 1/(2*a*d*(tan(c + d*x)*1i + 1))
```

3.51 $\int \frac{1}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=33

$$\frac{x}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}$$

```
[Out] 1/2*x/a+1/2*I/d/(a+I*a*tan(d*x+c))
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {3560, 8}

$$\frac{x}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(-1),x]
```

```
[Out] x/(2*a) + (I/2)/(d*(a + I*a*Tan[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+ia\tan(c+dx)}\,dx&=\frac{i}{2d(a+ia\tan(c+dx))}+\frac{\int 1\,dx}{2a}\\&=\frac{x}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 45, normalized size = 1.36

$$\frac{1-2idx+(-i+2dx)\tan(c+dx)}{4ad(-i+\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(-1),x]
```

```
[Out] (1 - (2*I)*d*x + (-I + 2*d*x)*Tan[c + d*x])/(4*a*d*(-I + Tan[c + d*x]))
```

Maple [A]
time = 0.00, size = 48, normalized size = 1.45

method	result	size
risch	$\frac{x}{2a}+\frac{i\mathrm{e}^{-2i(dx+c)}}{4da}$	26
derivativedivides	$\frac{-\frac{i\ln(\tan(dx+c)-i)}{4}+\frac{1}{2\tan(dx+c)-2i}+\frac{i\ln(\tan(dx+c)+i)}{4}}{da}$	48
default	$\frac{-\frac{i\ln(\tan(dx+c)-i)}{4}+\frac{1}{2\tan(dx+c)-2i}+\frac{i\ln(\tan(dx+c)+i)}{4}}{da}$	48
norman	$\frac{\frac{x}{2a}+\frac{i}{2da}+\frac{x\left(\tan^2(dx+c)\right)}{2a}+\frac{\tan(dx+c)}{2da}}{1+\tan^2(dx+c)}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/4*I*ln(tan(d*x+c)-I)+1/2/(tan(d*x+c)-I)+1/4*I*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.38, size = 32, normalized size = 0.97

$$\frac{\left(2\,dxe^{(2i\,dx+2i\,c)}+i\right)e^{(-2i\,dx-2i\,c)}}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*d*x*e^(2*I*d*x + 2*I*c) + I)*e^(-2*I*d*x - 2*I*c)/(a*d)
```

Sympy [A]
time = 0.09, size = 60, normalized size = 1.82

$$\begin{cases} \frac{ie^{-2ic}e^{-2idx}}{4ad} & \text{for } ade^{2ic} \neq 0 \\ x\left(\frac{(e^{2ic}+1)e^{-2ic}}{2a} - \frac{1}{2a}\right) & \text{otherwise} \end{cases} + \frac{x}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise((I*exp(-2*I*c)*exp(-2*I*d*x)/(4*a*d), Ne(a*d*exp(2*I*c), 0)), (x*
((exp(2*I*c) + 1)*exp(-2*I*c)/(2*a) - 1/(2*a)), True)) + x/(2*a)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(25) = 50.
time = 0.46, size = 60, normalized size = 1.82

$$-\frac{\frac{i\log(\tan(dx+c)-i)}{a} - \frac{i\log(-i\tan(dx+c)+1)}{a} + \frac{-i\tan(dx+c)-3}{a(\tan(dx+c)-i)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(I*log(tan(d*x + c) - I)/a - I*log(-I*tan(d*x + c) + 1)/a + (-I*tan(d*
x + c) - 3)/(a*(tan(d*x + c) - I)))/d
```

Mupad [B]
time = 3.95, size = 29, normalized size = 0.88

$$\frac{x}{2\,a} + \frac{1\mathrm{i}}{2\,a\,d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*tan(c + d*x)*1i),x)
```

```
[Out] x/(2*a) + 1i/(2*a*d*(tan(c + d*x)*1i + 1))
```

3.52 $\int \frac{\cot(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=47

$$-\frac{ix}{2a}+\frac{\log(\sin(c+dx))}{ad}+\frac{1}{2d(a+ia\tan(c+dx))}$$

```
[Out] -1/2*I*x/a+ln(sin(d*x+c))/a/d+1/2/d/(a+I*a*tan(d*x+c))
```

Rubi [A]
time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3632, 3560, 8, 3556}

$$\frac{1}{2d(a+ia\tan(c+dx))}+\frac{\log(\sin(c+dx))}{ad}-\frac{ix}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-1/2*I)*x)/a + Log[Sin[c + d*x]]/(a*d) + 1/(2*d*(a + I*a*Tan[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3632

```
Int[1/(((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Tan[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Tan[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{a+ia\tan(c+dx)}\,dx = -\left(i\int \frac{1}{a+ia\tan(c+dx)}\,dx\right) + \frac{\int \cot(c+dx)\,dx}{a}$$
$$= \frac{\log(\sin(c+dx))}{ad} + \frac{1}{2d(a+ia\tan(c+dx))} - \frac{i\int 1\,dx}{2a}$$
$$= -\frac{ix}{2a} + \frac{\log(\sin(c+dx))}{ad} + \frac{1}{2d(a+ia\tan(c+dx))}$$

Mathematica [A]
time = 0.36, size = 87, normalized size = 1.85

$$\frac{-i+2dx-2i\log\left(\sin^2(c+dx)\right)+\operatorname{ArcTan}(\tan(dx))(-4-4i\tan(c+dx))+\left(-1+2idx+2\log\left(\sin^2(c+dx)\right)\right)\tan(c+dx)}{4ad(-i+\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (-I + 2*d*x - (2*I)*Log[Sin[c + d*x]^2] + ArcTan[Tan[d*x]]*(-4 - (4*I)*Tan[
c + d*x]) + (-1 + (2*I)*d*x + 2*Log[Sin[c + d*x]^2])*Tan[c + d*x])/(4*a*d*(
-I + Tan[c + d*x]))
```

Maple [A]
time = 0.30, size = 54, normalized size = 1.15

method	result	size
derivativedivides	$\frac{-\frac{i}{2(\tan(dx+c)-i)}-\frac{3\ln(\tan(dx+c)-i)}{4}+\ln(\tan(dx+c))-\frac{\ln(\tan(dx+c)+i)}{4}}{da}$	54
default	$\frac{-\frac{i}{2(\tan(dx+c)-i)}-\frac{3\ln(\tan(dx+c)-i)}{4}+\ln(\tan(dx+c))-\frac{\ln(\tan(dx+c)+i)}{4}}{da}$	54
risch	$-\frac{3ix}{2a}+\frac{e^{-2i(dx+c)}}{4da}-\frac{2ic}{da}+\frac{\ln\left(e^{2i(dx+c)}-1\right)}{ad}$	55
norman	$\frac{\frac{1}{2da}-\frac{ix}{2a}-\frac{ix\left(\tan^2(dx+c)\right)}{2a}-\frac{i\tan(dx+c)}{2da}}{1+\tan^2(dx+c)}+\frac{\ln(\tan(dx+c))}{da}-\frac{\ln\left(1+\tan^2(dx+c)\right)}{2da}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/2*I/(tan(d*x+c)-I)-3/4*ln(tan(d*x+c)-I)+ln(tan(d*x+c))-1/4*ln(tan
(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.37, size = 55, normalized size = 1.17

$$\frac{\left(-6i\,dxe^{(2i\,dx+2i\,c)}+4\,e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}-1\right)+1\right)e^{(-2i\,dx-2i\,c)}}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(-6*I*d*x*e^(2*I*d*x + 2*I*c) + 4*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x +
2*I*c) - 1) + 1)*e^(-2*I*d*x - 2*I*c)/(a*d)
```

Sympy [A]
time = 0.16, size = 92, normalized size = 1.96

$$\begin{cases}\frac{e^{-2ic}e^{-2idx}}{4ad} & \text{for } ade^{2ic}\neq 0\\ x\left(\frac{(-3ie^{2ic}-i)e^{-2ic}}{2a}+\frac{3i}{2a}\right) & \text{otherwise}\end{cases}-\frac{3ix}{2a}+\frac{\log\left(e^{2idx}-e^{-2ic}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise((exp(-2*I*c)*exp(-2*I*d*x)/(4*a*d), Ne(a*d*exp(2*I*c), 0)), (x*((
-3*I*exp(2*I*c) - I)*exp(-2*I*c)/(2*a) + 3*I/(2*a)), True)) - 3*I*x/(2*a) +
 log(exp(2*I*d*x) - exp(-2*I*c))/(a*d)
```

Giac [A]
time = 0.52, size = 72, normalized size = 1.53

$$-\frac{\frac{3\log(\tan(dx+c)-i)}{a}+\frac{\log(i\tan(dx+c)-1)}{a}-\frac{4\log(\tan(dx+c))}{a}-\frac{3\tan(dx+c)-5i}{a(\tan(dx+c)-i)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(3*log(tan(d*x + c) - I)/a + log(I*tan(d*x + c) - 1)/a - 4*log(tan(d*x
 + c))/a - (3*tan(d*x + c) - 5*I)/(a*(tan(d*x + c) - I)))/d
```

Mupad [B]
time = 4.04, size = 72, normalized size = 1.53

$$\frac{\ln\left(\tan(c+d\,x)\right)}{a\,d}-\frac{\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)}{4\,a\,d}+\frac{1}{2\,a\,d\,\left(1+\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}-\frac{3\,\ln\left(\tan(c+d\,x)-\mathrm{i}\right)}{4\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + a*tan(c + d*x)*1i),x)
```

```
[Out] 1/(2*a*d*(tan(c + d*x)*1i + 1)) - log(tan(c + d*x) + 1i)/(4*a*d) - (3*log(t
an(c + d*x) - 1i))/(4*a*d) + log(tan(c + d*x))/(a*d)
```

3.53 $\int \frac{\cot^2(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=70

$$-\frac{3x}{2a}-\frac{3\cot(c+dx)}{2ad}-\frac{i\log(\sin(c+dx))}{ad}+\frac{\cot(c+dx)}{2d(a+ia\tan(c+dx))}$$

```
[Out] -3/2*x/a-3/2*cot(d*x+c)/a/d-I*ln(sin(d*x+c))/a/d+1/2*cot(d*x+c)/d/(a+I*a*ta
n(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3633, 3610, 3612, 3556}

$$-\frac{3\cot(c+dx)}{2ad}-\frac{i\log(\sin(c+dx))}{ad}+\frac{\cot(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{3x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (-3*x)/(2*a) - (3*Cot[c + d*x])/(2*a*d) - (I*Log[Sin[c + d*x]])/(a*d) + Cot
[c + d*x]/(2*d*(a + I*a*Tan[c + d*x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3633

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-a)*((c + d*Tan[e + f*x])^(n + 1)/(2*f*(b*
c - a*d)*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*
Tan[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &
& NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^2(c+dx)}{a+ia\tan(c+dx)}\,dx &= \frac{\cot(c+dx)}{2d(a+ia\tan(c+dx))} - \frac{\int \cot^2(c+dx)(-3a+2ia\tan(c+dx))\,dx}{2a^2}\\ &= -\frac{3\cot(c+dx)}{2ad} + \frac{\cot(c+dx)}{2d(a+ia\tan(c+dx))} - \frac{\int \cot(c+dx)(2ia+3a\tan(c+dx)}{2a^2}\\ &= -\frac{3x}{2a} - \frac{3\cot(c+dx)}{2ad} + \frac{\cot(c+dx)}{2d(a+ia\tan(c+dx))} - \frac{i\int \cot(c+dx)\,dx}{a}\\ &= -\frac{3x}{2a} - \frac{3\cot(c+dx)}{2ad} - \frac{i\log(\sin(c+dx))}{ad} + \frac{\cot(c+dx)}{2d(a+ia\tan(c+dx))}\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 286 vs. $2(70) = 140$.
time = 0.71, size = 286, normalized size = 4.09

$$\frac{\csc\left(\frac{c}{2}\right)\csc(c+dx)\sec\left(\frac{c}{2}\right)\sec(c+dx)\left(8\cos(c)-9\cos(c+2dx)+2idx\cos(c+2dx)+\cos(3c+2dx)-2idx\cos(3c+2dx)-2\cos(c+2dx)\log\left(\sin^2(c+dx)\right)+2\cos(3c+2dx)\log\left(\sin^2(c+dx)\right)+10i\sin(c)-4dx\sin(c)-4i\log\left(\sin^2(c+dx)\right)\sin(c)+16i\text{ArcTan}(\tan(dx))\sin(c)(\cos(c+dx)+i\sin(c+dx))\sin(c+dx)-7i\sin(c+2dx)-2dx\sin(c+2dx)-2i\log\left(\sin^2(c+dx)\right)\sin(c+2dx)-i\sin(3c+2dx)+2dx\sin(3c+2dx)+2i\log\left(\sin^2(c+dx)\right)\sin(3c+2dx)\right)}{32ad(-i+\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (Csc[c/2]*Csc[c + d*x]*Sec[c/2]*Sec[c + d*x]*(8*Cos[c] - 9*Cos[c + 2*d*x] +
 (2*I)*d*x*Cos[c + 2*d*x] + Cos[3*c + 2*d*x] - (2*I)*d*x*Cos[3*c + 2*d*x] -
 2*Cos[c + 2*d*x]*Log[Sin[c + d*x]^2] + 2*Cos[3*c + 2*d*x]*Log[Sin[c + d*x]
^2] + (10*I)*Sin[c] - 4*d*x*Sin[c] - (4*I)*Log[Sin[c + d*x]^2]*Sin[c] + (16
*I)*ArcTan[Tan[d*x]]*Sin[c]*(Cos[c + d*x] + I*Sin[c + d*x])*Sin[c + d*x] -
(7*I)*Sin[c + 2*d*x] - 2*d*x*Sin[c + 2*d*x] - (2*I)*Log[Sin[c + d*x]^2]*Sin
[c + 2*d*x] - I*Sin[3*c + 2*d*x] + 2*d*x*Sin[3*c + 2*d*x] + (2*I)*Log[Sin[c
 + d*x]^2]*Sin[3*c + 2*d*x]))/(32*a*d*(-I + Tan[c + d*x]))
```

Maple [**A**]
time = 0.24, size = 68, normalized size = 0.97

method	result	size

derivativedivides	$\frac{\frac{5i\ln(\tan(dx+c)-i)}{4}-\frac{1}{2(\tan(dx+c)-i)}-\frac{1}{\tan(dx+c)}-i\ln(\tan(dx+c))-\frac{i\ln(\tan(dx+c)+i)}{4}}{da}$	68
default	$\frac{\frac{5i\ln(\tan(dx+c)-i)}{4}-\frac{1}{2(\tan(dx+c)-i)}-\frac{1}{\tan(dx+c)}-i\ln(\tan(dx+c))-\frac{i\ln(\tan(dx+c)+i)}{4}}{da}$	68
risch	$-\frac{5x}{2a}-\frac{i\mathrm{e}^{-2i(dx+c)}}{4da}-\frac{2c}{da}-\frac{2i}{da\left(\mathrm{e}^{2i(dx+c)}-1\right)}-\frac{i\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{da}$	78
norman	$\frac{-\frac{1}{da}-\frac{3x\tan(dx+c)}{2a}-\frac{3x\left(\tan^3(dx+c)\right)}{2a}-\frac{3\left(\tan^2(dx+c)\right)}{2da}-\frac{i\tan(dx+c)}{2da}}{\tan(dx+c)(1+\tan^2(dx+c))}-\frac{i\ln(\tan(dx+c))}{da}+\frac{i\ln(1+\tan^2(dx+c))}{2da}$	125

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(5/4*I*ln(tan(d*x+c)-I)-1/2/(tan(d*x+c)-I)-1/tan(d*x+c)-I*ln(tan(d*x+
c))-1/4*I*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.38, size = 99, normalized size = 1.41

$$-\frac{10\,dxe^{(4i\,dx+4i\,c)}-(10\,dx-9i)e^{(2i\,dx+2i\,c)}+4\left(i\,e^{(4i\,dx+4i\,c)}-i\,e^{(2i\,dx+2i\,c)}\right)\log\left(e^{(2i\,dx+2i\,c)}-1\right)-i}{4\left(ade^{(4i\,dx+4i\,c)}-ade^{(2i\,dx+2i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(10*d*x*e^(4*I*d*x + 4*I*c) - (10*d*x - 9*I)*e^(2*I*d*x + 2*I*c) + 4*(
I*e^(4*I*d*x + 4*I*c) - I*e^(2*I*d*x + 2*I*c))*log(e^(2*I*d*x + 2*I*c) - 1)
 - I)/(a*d*e^(4*I*d*x + 4*I*c) - a*d*e^(2*I*d*x + 2*I*c))
```

Sympy [A]
time = 0.19, size = 116, normalized size = 1.66

$$\begin{cases} -\frac{ie^{-2ic}e^{-2idx}}{4ad} & \text{for}\; ade^{2ic}\neq 0 \\ x\left(\frac{\left(-5e^{2ic}-1\right)e^{-2ic}}{2a}+\frac{5}{2a}\right) & \text{otherwise} \end{cases}-\frac{2i}{ade^{2ic}e^{2idx}-ad}-\frac{5x}{2a}-\frac{i\log\left(e^{2idx}-e^{-2ic}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise((-I*exp(-2*I*c)*exp(-2*I*d*x)/(4*a*d), Ne(a*d*exp(2*I*c), 0)), (x
*((-5*exp(2*I*c) - 1)*exp(-2*I*c)/(2*a) + 5/(2*a)), True)) - 2*I/(a*d*exp(2
*I*c)*exp(2*I*d*x) - a*d) - 5*x/(2*a) - I*log(exp(2*I*d*x) - exp(-2*I*c))/(
a*d)
```

Giac [A]
time = 0.58, size = 91, normalized size = 1.30

$$-\frac{-\frac{10i\log(\tan(dx+c)-i)}{a}+\frac{2i\log(-i\tan(dx+c)+1)}{a}+\frac{8i\log(\tan(dx+c))}{a}+\frac{\tan(dx+c)^2-13i\tan(dx+c)-8}{\left(-i\tan(dx+c)^2-\tan(dx+c)\right)a}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/8*(-10*I*log(tan(d*x + c) - I)/a + 2*I*log(-I*tan(d*x + c) + 1)/a + 8*I*
log(tan(d*x + c))/a + (tan(d*x + c)^2 - 13*I*tan(d*x + c) - 8)/((-I*tan(d*x
 + c)^2 - tan(d*x + c))*a))/d
```

Mupad [B]
time = 3.99, size = 96, normalized size = 1.37

$$\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,5\mathrm{i}}{4\,a\,d}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{4\,a\,d}-\frac{\frac{1}{a}+\frac{\tan(c+d\,x)\,3\mathrm{i}}{2\,a}}{d\,\left(\tan\left(c+d\,x\right)^2\,1\mathrm{i}+\tan\left(c+d\,x\right)\right)}-\frac{\ln\left(\tan(c+d\,x)\right)\,1\mathrm{i}}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + a*tan(c + d*x)*1i),x)
```

```
[Out] (log(tan(c + d*x) - 1i)*5i)/(4*a*d) - (log(tan(c + d*x) + 1i)*1i)/(4*a*d) -
 ((tan(c + d*x)*3i)/(2*a) + 1/a)/(d*(tan(c + d*x) + tan(c + d*x)^2*1i)) - (
log(tan(c + d*x))*1i)/(a*d)
```

3.54 $\int \frac{\cot^3(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=90

$$\frac{3ix}{2a}+\frac{3i\cot(c+dx)}{2ad}-\frac{\cot^2(c+dx)}{ad}-\frac{2\log(\sin(c+dx))}{ad}+\frac{\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}$$

```
[Out] 3/2*I*x/a+3/2*I*cot(d*x+c)/a/d-cot(d*x+c)^2/a/d-2*ln(sin(d*x+c))/a/d+1/2*co
t(d*x+c)^2/d/(a+I*a*tan(d*x+c))
```

Rubi [A]
time = 0.09, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3633, 3610, 3612, 3556}

$$-\frac{\cot^2(c+dx)}{ad}+\frac{3i\cot(c+dx)}{2ad}-\frac{2\log(\sin(c+dx))}{ad}+\frac{\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}+\frac{3ix}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (((3*I)/2)*x)/a + (((3*I)/2)*Cot[c + d*x])/(a*d) - Cot[c + d*x]^2/(a*d) - (
2*Log[Sin[c + d*x]])/(a*d) + Cot[c + d*x]^2/(2*d*(a + I*a*Tan[c + d*x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3633

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-a)*((c + d*Tan[e + f*x])^(n + 1)/(2*f*(b*
c - a*d)*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*
Tan[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &
& NeQ[c^2 + d^2, 0] && !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^3(c+dx)}{a+ia\tan(c+dx)}\,dx &= \frac{\cot^2(c+dx)}{2d(a+ia\tan(c+dx))} - \frac{\int \cot^3(c+dx)(-4a+3ia\tan(c+dx))\,dx}{2a^2}\\
&= -\frac{\cot^2(c+dx)}{ad} + \frac{\cot^2(c+dx)}{2d(a+ia\tan(c+dx))} - \frac{\int \cot^2(c+dx)(3ia+4a\tan(c+dx}{2a^2}\\
&= \frac{3i\cot(c+dx)}{2ad} - \frac{\cot^2(c+dx)}{ad} + \frac{\cot^2(c+dx)}{2d(a+ia\tan(c+dx))} - \frac{\int \cot(c+dx)(4a-3}{2a}\\
&= \frac{3ix}{2a} + \frac{3i\cot(c+dx)}{2ad} - \frac{\cot^2(c+dx)}{ad} + \frac{\cot^2(c+dx)}{2d(a+ia\tan(c+dx))} - \frac{2\int \cot(c+dx}{a}\\
&= \frac{3ix}{2a} + \frac{3i\cot(c+dx)}{2ad} - \frac{\cot^2(c+dx)}{ad} - \frac{2\log(\sin(c+dx))}{ad} + \frac{\cot^2(c+dx)}{2d(a+ia\tan(c+}\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 414 vs. 2(90) = 180.
time = 0.95, size = 414, normalized size = 4.60

$$\frac{\csc\left(\frac{c}{2}\right)\csc^2(c+dx)\sec\left(\frac{c}{2}\right)\sec(c+dx)\left(-3\cos(2c+dx)+6idx\cos(2c+dx)+7\cos(2c+3dx)+2idx\cos(2c+3dx)+\cos(4c+3dx)-2idx\cos(4c+3dx)+\cos(dx)\left(-5-6idx-12\log\left(\sin^2(c+dx)\right)\right)+12\cos(2c+dx)\log\left(\sin^2(c+dx)\right)+4\cos(2c+3dx)\log\left(\sin^2(c+dx)\right)-4\cos(4c+3dx)\log\left(\sin^2(c+dx)\right)-25i\sin(dx)+2dx\sin(dx)-4i\log\left(\sin^2(c+dx)\right)\sin(dx)+64\text{ArcTan}(\tan(dx))\sin(c)(\cos(c+dx)+i\sin(c+dx))\sin^2(c+dx)+i\sin(2c+dx)-2dx\sin(2c+dx)+4i\log\left(\sin^2(c+dx)\right)\sin(2c+dx)+9i\sin(2c+3dx)-2dx\sin(2c+3dx)+4i\log\left(\sin^2(c+dx)\right)\sin(2c+3dx)-i\sin(4c+3dx)+2dx\sin(4c+3dx)-4i\log\left(\sin^2(c+dx)\right)\sin(4c+3dx)\right)}{64ad(-i+\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (Csc[c/2]*Csc[c + d*x]^2*Sec[c/2]*Sec[c + d*x]*(-3*Cos[2*c + d*x] + (6*I)*d
*x*Cos[2*c + d*x] + 7*Cos[2*c + 3*d*x] + (2*I)*d*x*Cos[2*c + 3*d*x] + Cos[4
*c + 3*d*x] - (2*I)*d*x*Cos[4*c + 3*d*x] + Cos[d*x]*(-5 - (6*I)*d*x - 12*Lo
g[Sin[c + d*x]^2]) + 12*Cos[2*c + d*x]*Log[Sin[c + d*x]^2] + 4*Cos[2*c + 3*
d*x]*Log[Sin[c + d*x]^2] - 4*Cos[4*c + 3*d*x]*Log[Sin[c + d*x]^2] - (25*I)*
Sin[d*x] + 2*d*x*Sin[d*x] - (4*I)*Log[Sin[c + d*x]^2]*Sin[d*x] + 64*ArcTan[
Tan[d*x]]*Sin[c]*(Cos[c + d*x] + I*Sin[c + d*x])*Sin[c + d*x]^2 + I*Sin[2*c
 + d*x] - 2*d*x*Sin[2*c + d*x] + (4*I)*Log[Sin[c + d*x]^2]*Sin[2*c + d*x] +
 (9*I)*Sin[2*c + 3*d*x] - 2*d*x*Sin[2*c + 3*d*x] + (4*I)*Log[Sin[c + d*x]^2
]*Sin[2*c + 3*d*x] - I*Sin[4*c + 3*d*x] + 2*d*x*Sin[4*c + 3*d*x] - (4*I)*Lo
g[Sin[c + d*x]^2]*Sin[4*c + 3*d*x]))/(64*a*d*(-I + Tan[c + d*x]))
```

Maple [A]
time = 0.28, size = 77, normalized size = 0.86

method	result
derivativedivides	$\frac{\frac{i}{2\tan(dx+c)-2i}+\frac{7\ln(\tan(dx+c)-i)}{4}-\frac{1}{2\tan(dx+c)^2}+\frac{i}{\tan(dx+c)}-2\ln(\tan(dx+c))+\frac{\ln(\tan(dx+c)+i)}{4}}{da}$
default	$\frac{\frac{i}{2\tan(dx+c)-2i}+\frac{7\ln(\tan(dx+c)-i)}{4}-\frac{1}{2\tan(dx+c)^2}+\frac{i}{\tan(dx+c)}-2\ln(\tan(dx+c))+\frac{\ln(\tan(dx+c)+i)}{4}}{da}$
risch	$\frac{7ix}{2a}-\frac{e^{-2i(dx+c)}}{4da}+\frac{4ic}{da}+\frac{2}{da(e^{2i(dx+c)}-1)^2}-\frac{2\ln(e^{2i(dx+c)}-1)}{ad}$
norman	$\frac{-\frac{\tan^2(dx+c)}{da}+\frac{i\tan(dx+c)}{da}-\frac{1}{2da}+\frac{3ix(\tan^2(dx+c))}{2a}+\frac{3ix(\tan^4(dx+c))}{2a}+\frac{3i(\tan^3(dx+c))}{2da}}{\tan(dx+c)^2(1+\tan^2(dx+c))}+\frac{\ln(1+\tan^2(dx+c))}{da}-\frac{2\ln(\tan}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(1/2*I/(tan(d*x+c)-I)+7/4*ln(tan(d*x+c)-I)-1/2/tan(d*x+c)^2+I/tan(d*x
+c)-2*ln(tan(d*x+c))+1/4*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.36, size = 134, normalized size = 1.49

$$\frac{14i\,dxe^{(6i\,dx+6i\,c)}+(-28i\,dx-1)e^{(4i\,dx+4i\,c)}-2\,(-7i\,dx-5)e^{(2i\,dx+2i\,c)}-8\left(e^{(6i\,dx+6i\,c)}-2\,e^{(4i\,dx+4i\,c)}+e^{(2i\,dx+2i\,c)}\right)\log\left(e^{(2i\,dx+2i\,c)}-1\right)-1}{4\left(ade^{(6i\,dx+6i\,c)}-2\,ade^{(4i\,dx+4i\,c)}+ade^{(2i\,dx+2i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(14*I*d*x*e^(6*I*d*x + 6*I*c) + (-28*I*d*x - 1)*e^(4*I*d*x + 4*I*c) - 2
*(-7*I*d*x - 5)*e^(2*I*d*x + 2*I*c) - 8*(e^(6*I*d*x + 6*I*c) - 2*e^(4*I*d*x
 + 4*I*c) + e^(2*I*d*x + 2*I*c))*log(e^(2*I*d*x + 2*I*c) - 1) - 1)/(a*d*e^(
6*I*d*x + 6*I*c) - 2*a*d*e^(4*I*d*x + 4*I*c) + a*d*e^(2*I*d*x + 2*I*c))
```

Sympy [A]
time = 0.23, size = 138, normalized size = 1.53

$$\begin{cases} -\frac{e^{-2ic}e^{-2idx}}{4ad} & \text{for } ade^{2ic} \neq 0 \\ x\left(\frac{(7ie^{2ic}+i)e^{-2ic}}{2a} - \frac{7i}{2a}\right) & \text{otherwise} \end{cases} + \frac{2}{ade^{4ic}e^{4idx} - 2ade^{2ic}e^{2idx} + ad} + \frac{7ix}{2a} - \frac{2\log\left(e^{2idx} - e^{-2ic}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise((-exp(-2*I*c)*exp(-2*I*d*x)/(4*a*d), Ne(a*d*exp(2*I*c), 0)), (x*(
(7*I*exp(2*I*c) + I)*exp(-2*I*c)/(2*a) - 7*I/(2*a)), True)) + 2/(a*d*exp(4*
I*c)*exp(4*I*d*x) - 2*a*d*exp(2*I*c)*exp(2*I*d*x) + a*d) + 7*I*x/(2*a) - 2*
log(exp(2*I*d*x) - exp(-2*I*c))/(a*d)
```

Giac [A]
time = 0.80, size = 105, normalized size = 1.17

$$\frac{\frac{\log(\tan(dx+c)+i)}{a} + \frac{7\log(i\tan(dx+c)+1)}{a} - \frac{8\log(\tan(dx+c))}{a} - \frac{7\tan(dx+c)-9i}{a(\tan(dx+c)-i)} + \frac{2\left(6\tan(dx+c)^2+2i\tan(dx+c)-1\right)}{a\tan(dx+c)^2}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(log(tan(d*x + c) + I)/a + 7*log(I*tan(d*x + c) + 1)/a - 8*log(tan(d*x
+ c))/a - (7*tan(d*x + c) - 9*I)/(a*(tan(d*x + c) - I)) + 2*(6*tan(d*x + c)
^2 + 2*I*tan(d*x + c) - 1)/(a*tan(d*x + c)^2))/d
```

Mupad [B]
time = 3.97, size = 110, normalized size = 1.22

$$\frac{7\ln(\tan(c+dx)-\mathrm{i})}{4ad} + \frac{\ln(\tan(c+dx)+1\mathrm{i})}{4ad} - \frac{2\ln(\tan(c+dx))}{ad} - \frac{\frac{1}{2a} + \frac{3\tan(c+dx)^2}{2a} - \frac{\tan(c+dx)\,1\mathrm{i}}{2a}}{d\left(\tan(c+dx)^3\,1\mathrm{i} + \tan(c+dx)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + a*tan(c + d*x)*1i),x)
```

```
[Out] (7*log(tan(c + d*x) - 1i))/(4*a*d) + log(tan(c + d*x) + 1i)/(4*a*d) - (2*lo
g(tan(c + d*x)))/(a*d) - (1/(2*a) - (tan(c + d*x)*1i)/(2*a) + (3*tan(c + d*
x)^2)/(2*a))/(d*(tan(c + d*x)^2 + tan(c + d*x)^3*1i))
```

3.55 $\int \frac{\cot^4(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=108

$$\frac{5x}{2a}+\frac{5\cot(c+dx)}{2ad}+\frac{i\cot^2(c+dx)}{ad}-\frac{5\cot^3(c+dx)}{6ad}+\frac{2i\log(\sin(c+dx))}{ad}+\frac{\cot^3(c+dx)}{2d(a+ia\tan(c+dx))}$$

```
[Out] 5/2*x/a+5/2*cot(d*x+c)/a/d+I*cot(d*x+c)^2/a/d-5/6*cot(d*x+c)^3/a/d+2*I*ln(s
in(d*x+c))/a/d+1/2*cot(d*x+c)^3/d/(a+I*a*tan(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3633, 3610, 3612, 3556}

$$-\frac{5\cot^3(c+dx)}{6ad}+\frac{i\cot^2(c+dx)}{ad}+\frac{5\cot(c+dx)}{2ad}+\frac{2i\log(\sin(c+dx))}{ad}+\frac{\cot^3(c+dx)}{2d(a+ia\tan(c+dx))}+\frac{5x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (5*x)/(2*a) + (5*Cot[c + d*x])/(2*a*d) + (I*Cot[c + d*x]^2)/(a*d) - (5*Cot[
c + d*x]^3)/(6*a*d) + ((2*I)*Log[Sin[c + d*x]])/(a*d) + Cot[c + d*x]^3/(2*d
*(a + I*a*Tan[c + d*x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3633

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-a)*((c + d*Tan[e + f*x])^(n + 1)/(2*f*(b*
c - a*d)*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*
Tan[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &
& NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^4(c+dx)}{a+ia\tan(c+dx)}\,dx &= \frac{\cot^3(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{\int \cot^4(c+dx)(-5a+4ia\tan(c+dx))\,dx}{2a^2}\\
&=-\frac{5\cot^3(c+dx)}{6ad}+\frac{\cot^3(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{\int \cot^3(c+dx)(4ia+5a\tan(c+d\ldots}{2a^2}\\
&=\frac{i\cot^2(c+dx)}{ad}-\frac{5\cot^3(c+dx)}{6ad}+\frac{\cot^3(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{\int \cot^2(c+dx)(5a\ldots}{2\ldots}\\
&=\frac{5\cot(c+dx)}{2ad}+\frac{i\cot^2(c+dx)}{ad}-\frac{5\cot^3(c+dx)}{6ad}+\frac{\cot^3(c+dx)}{2d(a+ia\tan(c+dx))}-\int \ldots\\
&=\frac{5x}{2a}+\frac{5\cot(c+dx)}{2ad}+\frac{i\cot^2(c+dx)}{ad}-\frac{5\cot^3(c+dx)}{6ad}+\frac{\cot^3(c+dx)}{2d(a+ia\tan(c+dx))}\ldots\\
&=\frac{5x}{2a}+\frac{5\cot(c+dx)}{2ad}+\frac{i\cot^2(c+dx)}{ad}-\frac{5\cot^3(c+dx)}{6ad}+\frac{2i\log(\sin(c+dx))}{ad}+\ldots\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 365 vs. 2(108) = 216.
time = 3.40, size = 365, normalized size = 3.38

$$\frac{\csc(c)(\cos(dx)+i\sin(dx))\left(14\csc(c+dx)-28\cos(c-dx)\csc(2(c+dx))+14i\sec(c+dx)-24dx\sec(c+dx)+24dx\cos^2(c)\sec(c+dx)+3\cos(c)\cos(2dx)\sec(c+dx)\sin(c)+30dx\sec(c+dx)\sin^2(c)-3i\cos(2dx)\sec(c+dx)\sin^2(c)+12i\log\left(\sin^2(c+dx)\right)\sec(c+dx)\sin^2(c)+24\text{ArcTan}(\tan(dx))\sec(c+dx)\sin(c)(-i\cos(c)+\sin(c))+2\csc^2(c+dx)\sec(c+dx)(\cos(c)+i\sin(c))(i\cos(c)+2\sin(c))-3idx\sec(c+dx)\sin(2c)+6\log\left(\sin^2(c+dx)\right)\sec(c+dx)\sin(2c)-3i\cos(c)\sec(c+dx)\sin(c)\sin(2dx)-3\sec(c+dx)\sin^2(c)\sin(2dx)-28i\csc(2(c+dx))\sin(c-dx)+2\csc^3(c+dx)(-1+\cos(c-dx)\sec(c+dx)+i\sec(c+dx)\sin(c-dx))\right)}{12ad(-i+\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (Csc[c]*(Cos[d*x] + I*Sin[d*x])*(14*Csc[c + d*x] - 28*Cos[c - d*x]*Csc[2*(c
 + d*x)] + (14*I)*Sec[c + d*x] - 24*d*x*Sec[c + d*x] + 24*d*x*Cos[c]^2*Sec[
c + d*x] + 3*Cos[c]*Cos[2*d*x]*Sec[c + d*x]*Sin[c] + 30*d*x*Sec[c + d*x]*Si
n[c]^2 - (3*I)*Cos[2*d*x]*Sec[c + d*x]*Sin[c]^2 + (12*I)*Log[Sin[c + d*x]^2
]*Sec[c + d*x]*Sin[c]^2 + 24*ArcTan[Tan[d*x]]*Sec[c + d*x]*Sin[c]*((-I)*Cos
[c] + Sin[c]) + 2*Csc[c + d*x]^2*Sec[c + d*x]*(Cos[c] + I*Sin[c])*(I*Cos[c]
 + 2*Sin[c]) - (3*I)*d*x*Sec[c + d*x]*Sin[2*c] + 6*Log[Sin[c + d*x]^2]*Sec[
c + d*x]*Sin[2*c] - (3*I)*Cos[c]*Sec[c + d*x]*Sin[c]*Sin[2*d*x] - 3*Sec[c +
```

```
 d*x]*Sin[c]^2*Sin[2*d*x] - (28*I)*Csc[2*(c + d*x)]*Sin[c - d*x] + 2*Csc[c
+ d*x]^3*(-1 + Cos[c - d*x]*Sec[c + d*x] + I*Sec[c + d*x]*Sin[c - d*x])))/(
12*a*d*(-I + Tan[c + d*x]))
```

Maple [A]
time = 0.28, size = 89, normalized size = 0.82

method	result
derivativedivides	$\frac{-\frac{9i\ln(\tan(dx+c)-i)}{4}+\frac{1}{2\tan(dx+c)-2i}-\frac{1}{3\tan(dx+c)^3}+\frac{i}{2\tan(dx+c)^2}+2i\ln(\tan(dx+c))+\frac{2}{\tan(dx+c)}+\frac{i\ln(\tan(dx+c)+i)}{4}}{da}$
default	$\frac{-\frac{9i\ln(\tan(dx+c)-i)}{4}+\frac{1}{2\tan(dx+c)-2i}-\frac{1}{3\tan(dx+c)^3}+\frac{i}{2\tan(dx+c)^2}+2i\ln(\tan(dx+c))+\frac{2}{\tan(dx+c)}+\frac{i\ln(\tan(dx+c)+i)}{4}}{da}$
risch	$\frac{9x}{2a}+\frac{i\mathrm{e}^{-2i(dx+c)}}{4da}+\frac{4c}{da}+\frac{2i\left(6\,\mathrm{e}^{4i(dx+c)}-9\,\mathrm{e}^{2i(dx+c)}+7\right)}{3da\left(\mathrm{e}^{2i(dx+c)}-1\right)^3}+\frac{2i\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{da}$
norman	$\frac{\frac{i\left(\tan^3(dx+c)\right)}{da}-\frac{1}{3da}+\frac{5\left(\tan^4(dx+c)\right)}{2da}+\frac{5x\left(\tan^3(dx+c)\right)}{2a}+\frac{5x\left(\tan^5(dx+c)\right)}{2a}+\frac{5\left(\tan^2(dx+c)\right)}{3da}+\frac{i\tan(dx+c)}{2da}}{\tan(dx+c)^3(1+\tan^2(dx+c))}+\frac{2i\ln(\tan(dx+c))}{da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-9/4*I*ln(tan(d*x+c)-I)+1/2/(tan(d*x+c)-I)-1/3/tan(d*x+c)^3+1/2*I/ta
n(d*x+c)^2+2*I*ln(tan(d*x+c))+2/tan(d*x+c)+1/4*I*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.38, size = 181, normalized size = 1.68

$$\frac{54\,dxe^{(8i\,dx+8i\,c)}-3\,(54\,dx-17i)e^{(6i\,dx+6i\,c)}+81\,(2\,dx-i)e^{(4i\,dx+4i\,c)}-(54\,dx-65i)e^{(2i\,dx+2i\,c)}-24\,\left(-i\,e^{(8i\,dx+8i\,c)}+3i\,e^{(6i\,dx+6i\,c)}-3i\,e^{(4i\,dx+4i\,c)}+i\,e^{(2i\,dx+2i\,c)}\right)\log\left(e^{(2i\,dx+2i\,c)}-1\right)-3i}{12\,\left(ade^{(8i\,dx+8i\,c)}-3\,ade^{(6i\,dx+6i\,c)}+3\,ade^{(4i\,dx+4i\,c)}-ade^{(2i\,dx+2i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(54*d*x*e^(8*I*d*x + 8*I*c) - 3*(54*d*x - 17*I)*e^(6*I*d*x + 6*I*c) +
81*(2*d*x - I)*e^(4*I*d*x + 4*I*c) - (54*d*x - 65*I)*e^(2*I*d*x + 2*I*c) -
24*(-I*e^(8*I*d*x + 8*I*c) + 3*I*e^(6*I*d*x + 6*I*c) - 3*I*e^(4*I*d*x + 4*I
*c) + I*e^(2*I*d*x + 2*I*c))*log(e^(2*I*d*x + 2*I*c) - 1) - 3*I)/(a*d*e^(8*
```

```
I*d*x + 8*I*c) - 3*a*d*e^(6*I*d*x + 6*I*c) + 3*a*d*e^(4*I*d*x + 4*I*c) - a*
d*e^(2*I*d*x + 2*I*c))
```

Sympy [A]
time = 0.27, size = 196, normalized size = 1.81

$$\frac{12ie^{4ic}e^{4idx}-18ie^{2ic}e^{2idx}+14i}{3ade^{6ic}e^{6idx}-9ade^{4ic}e^{4idx}+9ade^{2ic}e^{2idx}-3ad}+\begin{cases}\frac{ie^{-2ic}e^{-2idx}}{4ad} & \text{for } ade^{2ic}\neq 0\\ x\left(\frac{(9e^{2ic}+1)e^{-2ic}}{2a}-\frac{9}{2a}\right) & \text{otherwise}\end{cases}+\frac{9x}{2a}+\frac{2i\log\left(e^{2idx}-e^{-2ic}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+I*a*tan(d*x+c)),x)
```

```
[Out] (12*I*exp(4*I*c)*exp(4*I*d*x) - 18*I*exp(2*I*c)*exp(2*I*d*x) + 14*I)/(3*a*d
*exp(6*I*c)*exp(6*I*d*x) - 9*a*d*exp(4*I*c)*exp(4*I*d*x) + 9*a*d*exp(2*I*c)
*exp(2*I*d*x) - 3*a*d) + Piecewise((I*exp(-2*I*c)*exp(-2*I*d*x)/(4*a*d), Ne
(a*d*exp(2*I*c), 0)), (x*((9*exp(2*I*c) + 1)*exp(-2*I*c)/(2*a) - 9/(2*a)),
True)) + 9*x/(2*a) + 2*I*log(exp(2*I*d*x) - exp(-2*I*c))/(a*d)
```

Giac [A]
time = 0.79, size = 116, normalized size = 1.07

$$-\frac{\frac{27i\log(\tan(dx+c)-i)}{a}-\frac{3i\log(-i\tan(dx+c)+1)}{a}-\frac{24i\log(\tan(dx+c))}{a}+\frac{3(-9i\tan(dx+c)-11)}{a(\tan(dx+c)-i)}+\frac{2i\left(22\tan(dx+c)^3+12i\tan(dx+c)^2-3\tan(dx+c)-2i\right)}{a\tan(dx+c)^3}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(27*I*log(tan(d*x + c) - I)/a - 3*I*log(-I*tan(d*x + c) + 1)/a - 24*I
*log(tan(d*x + c))/a + 3*(-9*I*tan(d*x + c) - 11)/(a*(tan(d*x + c) - I)) +
2*I*(22*tan(d*x + c)^3 + 12*I*tan(d*x + c)^2 - 3*tan(d*x + c) - 2*I)/(a*tan
(d*x + c)^3))/d
```

Mupad [B]
time = 4.09, size = 126, normalized size = 1.17

$$-\frac{\ln(\tan(c+dx)-\mathrm{i})\,9\mathrm{i}}{4ad}+\frac{\ln(\tan(c+dx)+1\mathrm{i})\,1\mathrm{i}}{4ad}+\frac{\ln(\tan(c+dx))\,2\mathrm{i}}{ad}+\frac{\frac{\tan(c+dx)\,1\mathrm{i}}{6a}-\frac{1}{3a}+\frac{3\tan(c+dx)^2}{2a}+\frac{\tan(c+dx)^3\,5\mathrm{i}}{2a}}{d\left(\tan(c+dx)^4\,1\mathrm{i}+\tan(c+dx)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + a*tan(c + d*x)*1i),x)
```

```
[Out] (log(tan(c + d*x) + 1i)*1i)/(4*a*d) - (log(tan(c + d*x) - 1i)*9i)/(4*a*d) +
 (log(tan(c + d*x))*2i)/(a*d) + ((tan(c + d*x)*1i)/(6*a) - 1/(3*a) + (3*tan
(c + d*x)^2)/(2*a) + (tan(c + d*x)^3*5i)/(2*a))/(d*(tan(c + d*x)^3 + tan(c
+ d*x)^4*1i))
```

3.56 $\int \frac{\tan^6(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=142

$$-\frac{25x}{4a^2}-\frac{6i\log(\cos(c+dx))}{a^2d}+\frac{25\tan(c+dx)}{4a^2d}-\frac{3i\tan^2(c+dx)}{a^2d}-\frac{25\tan^3(c+dx)}{12a^2d}+\frac{3i\tan^4(c+dx)}{2a^2d(1+i\tan(c+dx))}-\frac{}{4a}$$

```
[Out] -25/4*x/a^2-6*I*ln(cos(d*x+c))/a^2/d+25/4*tan(d*x+c)/a^2/d-3*I*tan(d*x+c)^2
/a^2/d-25/12*tan(d*x+c)^3/a^2/d+3/2*I*tan(d*x+c)^4/a^2/d/(1+I*tan(d*x+c))-1
/4*tan(d*x+c)^5/d/(a+I*a*tan(d*x+c))^2
```

Rubi [A]
time = 0.16, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3639, 3676, 3609, 3606, 3556}

$$\frac{3i\tan^4(c+dx)}{2a^2d(1+i\tan(c+dx))}-\frac{25\tan^3(c+dx)}{12a^2d}-\frac{3i\tan^2(c+dx)}{a^2d}+\frac{25\tan(c+dx)}{4a^2d}-\frac{6i\log(\cos(c+dx))}{a^2d}-\frac{25x}{4a^2}-\frac{\tan^5(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (-25*x)/(4*a^2) - ((6*I)*Log[Cos[c + d*x]])/(a^2*d) + (25*Tan[c + d*x])/(4*
a^2*d) - ((3*I)*Tan[c + d*x]^2)/(a^2*d) - (25*Tan[c + d*x]^3)/(12*a^2*d) +
(((3*I)/2)*Tan[c + d*x]^4)/(a^2*d*(1 + I*Tan[c + d*x])) - Tan[c + d*x]^5/(4
*d*(a + I*a*Tan[c + d*x])^2)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^6(c+dx)}{(a+ia\tan(c+dx))^2}\,dx &= -\frac{\tan^5(c+dx)}{4d(a+ia\tan(c+dx))^2} - \frac{\int \frac{\tan^4(c+dx)(-5a+7ia\tan(c+dx))}{a+ia\tan(c+dx)}\,dx}{4a^2} \\
&= \frac{3i\tan^4(c+dx)}{2a^2d(1+i\tan(c+dx))} - \frac{\tan^5(c+dx)}{4d(a+ia\tan(c+dx))^2} + \frac{\int \tan^3(c+dx)\left(-48ia^2 \cdots\right.}{8 \cdots} \\
&= -\frac{25\tan^3(c+dx)}{12a^2d} + \frac{3i\tan^4(c+dx)}{2a^2d(1+i\tan(c+dx))} - \frac{\tan^5(c+dx)}{4d(a+ia\tan(c+dx))^2} + \int \cdots \\
&= -\frac{3i\tan^2(c+dx)}{a^2d} - \frac{25\tan^3(c+dx)}{12a^2d} + \frac{3i\tan^4(c+dx)}{2a^2d(1+i\tan(c+dx))} - \frac{\tan^5(\cdots}{4d(a+ia\,\mathrm{t}\cdots} \\
&= -\frac{25x}{4a^2} + \frac{25\tan(c+dx)}{4a^2d} - \frac{3i\tan^2(c+dx)}{a^2d} - \frac{25\tan^3(c+dx)}{12a^2d} + \frac{3i\tan^4(\cdots}{2a^2d(1+i\,\mathrm{ta}\cdots} \\
&= -\frac{25x}{4a^2} - \frac{6i\log(\cos(c+dx))}{a^2d} + \frac{25\tan(c+dx)}{4a^2d} - \frac{3i\tan^2(c+dx)}{a^2d} - \frac{25\tan^3(c\cdots}{12a^2\cdots}
\end{aligned}
$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 882 vs. 2(142) = 284.
time = 6.53, size = 882, normalized size = 6.21

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^6/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (-25*x*Cos[2*c]*Sec[c + d*x]^2*(Cos[d*x] + I*Sin[d*x])^2)/(4*(a + I*a*Tan[c
 + d*x])^2) - (6*ArcTan[Tan[d*x]]*Cos[2*c]*Sec[c + d*x]^2*(Cos[d*x] + I*Sin
[d*x])^2)/(d*(a + I*a*Tan[c + d*x])^2) + (((5*I)/4)*Cos[2*d*x]*Sec[c + d*x]
^2*(Cos[d*x] + I*Sin[d*x])^2)/(d*(a + I*a*Tan[c + d*x])^2) - ((3*I)*Cos[2*c
]*Log[Cos[c + d*x]^2]*Sec[c + d*x]^2*(Cos[d*x] + I*Sin[d*x])^2)/(d*(a + I*a
*Tan[c + d*x])^2) + (Cos[4*d*x]*Sec[c + d*x]^2*((-1/16*I)*Cos[2*c] - Sin[2*
c]/16)*(Cos[d*x] + I*Sin[d*x])^2)/(d*(a + I*a*Tan[c + d*x])^2) + (Sec[c]*Se
c[c + d*x]^4*(3*Cos[c] - I*Sin[c])*((-1/3*I)*Cos[2*c] + Sin[2*c]/3)*(Cos[d*
x] + I*Sin[d*x])^2)/(d*(a + I*a*Tan[c + d*x])^2) - (((25*I)/4)*x*Sec[c + d*
x]^2*Sin[2*c]*(Cos[d*x] + I*Sin[d*x])^2)/(a + I*a*Tan[c + d*x])^2 - ((6*I)*
ArcTan[Tan[d*x]]*Sec[c + d*x]^2*Sin[2*c]*(Cos[d*x] + I*Sin[d*x])^2)/(d*(a +
 I*a*Tan[c + d*x])^2) + (3*Log[Cos[c + d*x]^2]*Sec[c + d*x]^2*Sin[2*c]*(Cos
[d*x] + I*Sin[d*x])^2)/(d*(a + I*a*Tan[c + d*x])^2) + (5*Sec[c + d*x]^2*(Co
s[d*x] + I*Sin[d*x])^2*Sin[2*d*x])/(4*d*(a + I*a*Tan[c + d*x])^2) + (Sec[c
+ d*x]^2*(-1/16*Cos[2*c] + (I/16)*Sin[2*c])*(Cos[d*x] + I*Sin[d*x])^2*Sin[4
*d*x])/(d*(a + I*a*Tan[c + d*x])^2) - (((13*I)/6)*Sec[c]*Sec[c + d*x]^3*(Co
s[d*x] + I*Sin[d*x])^2*(-Cos[2*c - d*x] + Cos[2*c + d*x] - I*Sin[2*c - d*x]
 + I*Sin[2*c + d*x]))/(d*(a + I*a*Tan[c + d*x])^2) + ((I/6)*Sec[c]*Sec[c +
d*x]^5*(Cos[d*x] + I*Sin[d*x])^2*(-Cos[2*c - d*x] + Cos[2*c + d*x] - I*Sin[
2*c - d*x] + I*Sin[2*c + d*x]))/(d*(a + I*a*Tan[c + d*x])^2) + (x*Sec[c + d
*x]^2*(Cos[d*x] + I*Sin[d*x])^2*(6 + (6*I)*Tan[c] + I*(6*Cos[2*c] + (6*I)*S
in[2*c])*Tan[c]))/(a + I*a*Tan[c + d*x])^2
```

Maple [A]

time = 0.15, size = 91, normalized size = 0.64

method	result
derivativedivides	$\frac{4\tan(dx+c)-\frac{\left(\tan^3(dx+c)\right)}{3}-i\left(\tan^2(dx+c)\right)+\frac{i}{4(\tan(dx+c)-i)^2}+\frac{49i\ln(\tan(dx+c)-i)}{8}+\frac{11}{4(\tan(dx+c)-i)}-\frac{i\ln(\tan(dx+c)+i)}{8}}{d\,a^2}$
default	$\frac{4\tan(dx+c)-\frac{\left(\tan^3(dx+c)\right)}{3}-i\left(\tan^2(dx+c)\right)+\frac{i}{4(\tan(dx+c)-i)^2}+\frac{49i\ln(\tan(dx+c)-i)}{8}+\frac{11}{4(\tan(dx+c)-i)}-\frac{i\ln(\tan(dx+c)+i)}{8}}{d\,a^2}$
risch	$-\frac{49x}{4a^2}+\frac{5i\mathrm{e}^{-2i(dx+c)}}{4a^2d}-\frac{i\mathrm{e}^{-4i(dx+c)}}{16a^2d}-\frac{12c}{a^2d}+\frac{2i\left(9\,\mathrm{e}^{4i(dx+c)}+18\,\mathrm{e}^{2i(dx+c)}+13\right)}{3d\,a^2\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}-\frac{6i\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{a^2d}$
norman	$\frac{-\frac{25x}{4a}+\frac{10\left(\tan^5(dx+c)\right)}{3da}-\frac{\tan^7(dx+c)}{3da}-\frac{25x\left(\tan^2(dx+c)\right)}{2a}-\frac{25x\left(\tan^4(dx+c)\right)}{4a}+\frac{9i}{2da}+\frac{25\tan(dx+c)}{4da}+\frac{125\left(\tan^3(dx+c)\right)}{12da}+\frac{6i\left(\tan^2(\right.}{da}}{a(1+\tan^2(dx+c))^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(4*tan(d*x+c)-1/3*tan(d*x+c)^3-I*tan(d*x+c)^2+1/4*I/(tan(d*x+c)-I)^
2+49/8*I*ln(tan(d*x+c)-I)+11/4/(tan(d*x+c)-I)-1/8*I*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.37, size = 198, normalized size = 1.39

$$-\frac{588\,dxe^{(10i\,dx+10i\,c)}+12\,(147\,dx-29i)e^{(8i\,dx+8i\,c)}+3\,(588\,dx-251i)e^{(6i\,dx+6i\,c)}+(588\,dx-587i)e^{(4i\,dx+4i\,c)}+288\,\left(i\,e^{(10i\,dx+10i\,c)}+3i\,e^{(8i\,dx+8i\,c)}+3i\,e^{(6i\,dx+6i\,c)}+i\,e^{(4i\,dx+4i\,c)}\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-51i\,e^{(2i\,dx+2i\,c)}+3i}{48\,\left(a^2de^{(10i\,dx+10i\,c)}+3\,a^2de^{(8i\,dx+8i\,c)}+3\,a^2de^{(6i\,dx+6i\,c)}+a^2de^{(4i\,dx+4i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/48*(588*d*x*e^(10*I*d*x + 10*I*c) + 12*(147*d*x - 29*I)*e^(8*I*d*x + 8*I
*c) + 3*(588*d*x - 251*I)*e^(6*I*d*x + 6*I*c) + (588*d*x - 587*I)*e^(4*I*d*
x + 4*I*c) + 288*(I*e^(10*I*d*x + 10*I*c) + 3*I*e^(8*I*d*x + 8*I*c) + 3*I*e
^(6*I*d*x + 6*I*c) + I*e^(4*I*d*x + 4*I*c))*log(e^(2*I*d*x + 2*I*c) + 1) -
51*I*e^(2*I*d*x + 2*I*c) + 3*I)/(a^2*d*e^(10*I*d*x + 10*I*c) + 3*a^2*d*e^(8
*I*d*x + 8*I*c) + 3*a^2*d*e^(6*I*d*x + 6*I*c) + a^2*d*e^(4*I*d*x + 4*I*c))
```

Sympy [A]
time = 0.35, size = 262, normalized size = 1.85

$$\frac{18ie^{4ic}e^{4idx}+36ie^{2ic}e^{2idx}+26i}{3a^2de^{6ic}e^{6idx}+9a^2de^{4ic}e^{4idx}+9a^2de^{2ic}e^{2idx}+3a^2d}+\begin{cases}\frac{\left(80ia^2de^{4ic}e^{-2idx}-4ia^2de^{2ic}e^{-4idx}\right)e^{-6ic}}{64a^4d^2} & \text{for } a^4d^2e^{6ic}\neq 0\\ x\left(\frac{\left(-49e^{4ic}+10e^{2ic}-1\right)e^{-4ic}}{4a^2}+\frac{49}{4a^2}\right) & \text{otherwise}\end{cases}-\frac{49x}{4a^2}-\frac{6i\log\left(e^{2idx}+e^{-2ic}\right)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] (18*I*exp(4*I*c)*exp(4*I*d*x) + 36*I*exp(2*I*c)*exp(2*I*d*x) + 26*I)/(3*a**
2*d*exp(6*I*c)*exp(6*I*d*x) + 9*a**2*d*exp(4*I*c)*exp(4*I*d*x) + 9*a**2*d*e
xp(2*I*c)*exp(2*I*d*x) + 3*a**2*d) + Piecewise(((80*I*a**2*d*exp(4*I*c)*exp
(-2*I*d*x) - 4*I*a**2*d*exp(2*I*c)*exp(-4*I*d*x))*exp(-6*I*c)/(64*a**4*d**2
), Ne(a**4*d**2*exp(6*I*c), 0)), (x*((-49*exp(4*I*c) + 10*exp(2*I*c) - 1)*e
xp(-4*I*c)/(4*a**2) + 49/(4*a**2)), True)) - 49*x/(4*a**2) - 6*I*log(exp(2*
I*d*x) + exp(-2*I*c))/(a**2*d)
```

Giac [A]
time = 2.09, size = 111, normalized size = 0.78

$$-\frac{\frac{6i\,\log(\tan(dx+c)+i)}{a^2}-\frac{294i\,\log(\tan(dx+c)-i)}{a^2}+\frac{3\left(147i\,\tan(dx+c)^2+250\,\tan(dx+c)-107i\right)}{a^2(\tan(dx+c)-i)^2}+\frac{16\left(a^4\tan(dx+c)^3+3i\,a^4\tan(dx+c)^2-12\,a^4\tan(dx+c)\right)}{a^6}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/48*(6*I*log(tan(d*x + c) + I)/a^2 - 294*I*log(tan(d*x + c) - I)/a^2 + 3*
(147*I*tan(d*x + c)^2 + 250*tan(d*x + c) - 107*I)/(a^2*(tan(d*x + c) - I)^2
) + 16*(a^4*tan(d*x + c)^3 + 3*I*a^4*tan(d*x + c)^2 - 12*a^4*tan(d*x + c))/
a^6)/d
```

Mupad [B]
time = 4.06, size = 132, normalized size = 0.93

$$\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,49\mathrm{i}}{8\,a^2\,d}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{8\,a^2\,d}+\frac{4\tan(c+d\,x)}{a^2\,d}-\frac{\tan(c+d\,x)^2\,1\mathrm{i}}{a^2\,d}-\frac{\tan(c+d\,x)^3}{3\,a^2\,d}+\frac{\frac{5}{2\,a^2}+\frac{\tan(c+d\,x)\,11\mathrm{i}}{4\,a^2}}{d\,\left(\tan\left(c+d\,x\right)^2\,1\mathrm{i}+2\tan\left(c+d\,x\right)-\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] (log(tan(c + d*x) - 1i)*49i)/(8*a^2*d) - (log(tan(c + d*x) + 1i)*1i)/(8*a^2
*d) + (4*tan(c + d*x))/(a^2*d) - (tan(c + d*x)^2*1i)/(a^2*d) - tan(c + d*x)
^3/(3*a^2*d) + ((tan(c + d*x)*11i)/(4*a^2) + 5/(2*a^2))/(d*(2*tan(c + d*x)
+ tan(c + d*x)^2*1i - 1i))
```

3.57 $\int \frac{\tan^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=124

$$\frac{15ix}{4a^2}-\frac{4\log(\cos(c+dx))}{a^2d}-\frac{15i\tan(c+dx)}{4a^2d}-\frac{2\tan^2(c+dx)}{a^2d}+\frac{5i\tan^3(c+dx)}{4a^2d(1+i\tan(c+dx))}-\frac{\tan^4(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

```
[Out] 15/4*I*x/a^2-4*ln(cos(d*x+c))/a^2/d-15/4*I*tan(d*x+c)/a^2/d-2*tan(d*x+c)^2/
a^2/d+5/4*I*tan(d*x+c)^3/a^2/d/(1+I*tan(d*x+c))-1/4*tan(d*x+c)^4/d/(a+I*a*t
an(d*x+c))^2
```

Rubi [A]
time = 0.14, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3639, 3676, 3609, 3606, 3556}

$$\frac{5i\tan^3(c+dx)}{4a^2d(1+i\tan(c+dx))}-\frac{2\tan^2(c+dx)}{a^2d}-\frac{15i\tan(c+dx)}{4a^2d}-\frac{4\log(\cos(c+dx))}{a^2d}+\frac{15ix}{4a^2}-\frac{\tan^4(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (((15*I)/4)*x)/a^2 - (4*Log[Cos[c + d*x]])/(a^2*d) - (((15*I)/4)*Tan[c + d*
x])/(a^2*d) - (2*Tan[c + d*x]^2)/(a^2*d) + (((5*I)/4)*Tan[c + d*x]^3)/(a^2*
d*(1 + I*Tan[c + d*x])) - Tan[c + d*x]^4/(4*d*(a + I*a*Tan[c + d*x])^2)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx &= -\frac{\tan^4(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\int \frac{\tan^3(c+dx)(-4a+6ia\tan(c+dx))}{a+ia\tan(c+dx)}\,dx}{4a^2}\\
&= \frac{5i\tan^3(c+dx)}{4a^2d(1+i\tan(c+dx))}-\frac{\tan^4(c+dx)}{4d(a+ia\tan(c+dx))^2}+\frac{\int \tan^2(c+dx)\left(-30ia^2\right.}{8a}\\
&= -\frac{2\tan^2(c+dx)}{a^2d}+\frac{5i\tan^3(c+dx)}{4a^2d(1+i\tan(c+dx))}-\frac{\tan^4(c+dx)}{4d(a+ia\tan(c+dx))^2}+\frac{\int \tan}{}\\
&= \frac{15ix}{4a^2}-\frac{15i\tan(c+dx)}{4a^2d}-\frac{2\tan^2(c+dx)}{a^2d}+\frac{5i\tan^3(c+dx)}{4a^2d(1+i\tan(c+dx))}-\frac{\mathrm{ta}}{4d(a+}\\
&= \frac{15ix}{4a^2}-\frac{4\log(\cos(c+dx))}{a^2d}-\frac{15i\tan(c+dx)}{4a^2d}-\frac{2\tan^2(c+dx)}{a^2d}+\frac{5i\tan^3(c}{4a^2d(1+i\,\mathrm{ta}}
\end{aligned}
$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 300 vs. 2(124) = 248.
time = 1.82, size = 300, normalized size = 2.42

$$
\frac{\sec^2(c+dx)(\cos(dx)+i\sin(dx))^2\left(64idx-16\cos(2dx)-16\cos(2c-dx)\sec(c)\sec(c+dx)+16\cos(2c+dx)\sec(c)\sec(c+dx)-128idx\sin^2(c)+60dx\sin(2c)-i\cos(4dx)\sin(2c)+32i\log\left(\cos^2(c+dx)\right)\sin(2c)+8i\sec^2(c+dx)\sin(2c)+64\mathrm{ArcTan}(\tan(dx))(-i\cos(2c)+\sin(2c))+16i\sin(2dx)-\sin(2c)\sin(4dx)-16i\sec(c)\sec(c+dx)\sin(2c-dx)+16i\sec(c)\sec(c+dx)\sin(2c+dx)-64dx\tan(c)+\cos(2c)\left(-60idx+\cos(4dx)+32\log\left(\cos^2(c+dx)\right)+8\sec^2(c+dx)-i\sin(4dx)-64dx\tan(c)\right)\right)}{16a^2d(-i+\tan(c+dx))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^2*(Cos[d*x] + I*Sin[d*x])^2*((64*I)*d*x - 16*Cos[2*d*x] - 16*
Cos[2*c - d*x]*Sec[c]*Sec[c + d*x] + 16*Cos[2*c + d*x]*Sec[c]*Sec[c + d*x]
- (128*I)*d*x*Sin[c]^2 + 60*d*x*Sin[2*c] - I*Cos[4*d*x]*Sin[2*c] + (32*I)*L
og[Cos[c + d*x]^2]*Sin[2*c] + (8*I)*Sec[c + d*x]^2*Sin[2*c] + 64*ArcTan[Tan
[d*x]]*((-I)*Cos[2*c] + Sin[2*c]) + (16*I)*Sin[2*d*x] - Sin[2*c]*Sin[4*d*x]
 - (16*I)*Sec[c]*Sec[c + d*x]*Sin[2*c - d*x] + (16*I)*Sec[c]*Sec[c + d*x]*S
in[2*c + d*x] - 64*d*x*Tan[c] + Cos[2*c]*((-60*I)*d*x + Cos[4*d*x] + 32*Log
[Cos[c + d*x]^2] + 8*Sec[c + d*x]^2 - I*Sin[4*d*x] - 64*d*x*Tan[c])))/(16*a
^2*d*(-I + Tan[c + d*x])^2)
```

Maple [A]
time = 0.11, size = 79, normalized size = 0.64

method	result
derivativedivides	$\frac{-2 i \tan(dx+c)-\frac{\left(\tan^2(dx+c)\right)}{2}-\frac{9i}{4(\tan(dx+c)-i)}+\frac{1}{4(\tan(dx+c)-i)^2}+\frac{31\ln(\tan(dx+c)-i)}{8}+\frac{\ln(\tan(dx+c)+i)}{8}}{d\,a^2}$
default	$\frac{-2 i \tan(dx+c)-\frac{\left(\tan^2(dx+c)\right)}{2}-\frac{9i}{4(\tan(dx+c)-i)}+\frac{1}{4(\tan(dx+c)-i)^2}+\frac{31\ln(\tan(dx+c)-i)}{8}+\frac{\ln(\tan(dx+c)+i)}{8}}{d\,a^2}$
risch	$\frac{31ix}{4a^2}+\frac{e^{-2i(dx+c)}}{a^2d}-\frac{e^{-4i(dx+c)}}{16a^2d}+\frac{8ic}{a^2d}+\frac{2\,e^{2i(dx+c)}+4}{d\,a^2\left(e^{2i(dx+c)}+1\right)^2}-\frac{4\ln\left(e^{2i(dx+c)}+1\right)}{a^2d}$
norman	$\frac{\frac{3}{da}-\frac{\tan^6(dx+c)}{2da}+\frac{15ix}{4a}+\frac{4\left(\tan^2(dx+c)\right)}{da}+\frac{15ix\left(\tan^2(dx+c)\right)}{2a}+\frac{15ix\left(\tan^4(dx+c)\right)}{4a}-\frac{15i\tan(dx+c)}{4da}-\frac{25i\left(\tan^3(dx+c)\right)}{4da}-\frac{2i\left(\tan^5 \right.}{d}}{a(1+\tan^2(dx+c))^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(-2*I*tan(d*x+c)-1/2*tan(d*x+c)^2-9/4*I/(tan(d*x+c)-I)+1/4/(tan(d*x
+c)-I)^2+31/8*ln(tan(d*x+c)-I)+1/8*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.38, size = 151, normalized size = 1.22

$$\frac{124i\,dxe^{(8i\,dx+8i\,c)}-8\,(-31i\,dx-6)e^{(6i\,dx+6i\,c)}+(124i\,dx+95)e^{(4i\,dx+4i\,c)}-64\left(e^{(8i\,dx+8i\,c)}+2\,e^{(6i\,dx+6i\,c)}+e^{(4i\,dx+4i\,c)}\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)+14\,e^{(2i\,dx+2i\,c)}-1}{16\left(a^2de^{(8i\,dx+8i\,c)}+2\,a^2de^{(6i\,dx+6i\,c)}+a^2de^{(4i\,dx+4i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(124*I*d*x*e^(8*I*d*x + 8*I*c) - 8*(-31*I*d*x - 6)*e^(6*I*d*x + 6*I*c)
 + (124*I*d*x + 95)*e^(4*I*d*x + 4*I*c) - 64*(e^(8*I*d*x + 8*I*c) + 2*e^(6*
I*d*x + 6*I*c) + e^(4*I*d*x + 4*I*c))*log(e^(2*I*d*x + 2*I*c) + 1) + 14*e^(
2*I*d*x + 2*I*c) - 1)/(a^2*d*e^(8*I*d*x + 8*I*c) + 2*a^2*d*e^(6*I*d*x + 6*I
*c) + a^2*d*e^(4*I*d*x + 4*I*c))
```

Sympy [A]
time = 1.00, size = 214, normalized size = 1.73

$$\frac{2e^{2ic}e^{2idx}+4}{a^2de^{4ic}e^{4idx}+2a^2de^{2ic}e^{2idx}+a^2d}+\begin{cases}\frac{\left(16a^2de^{4ic}e^{-2idx}-a^2de^{2ic}e^{-4idx}\right)e^{-6ic}}{16a^4d^2} & \text{for } a^4d^2e^{6ic}\neq 0\\ x\left(\frac{\left(31ie^{4ic}-8ie^{2ic}+i\right)e^{-4ic}}{4a^2}-\frac{31i}{4a^2}\right) & \text{otherwise}\end{cases}+\frac{31ix}{4a^2}-\frac{4\log\left(e^{2idx}+e^{-2ic}\right)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] (2*exp(2*I*c)*exp(2*I*d*x) + 4)/(a**2*d*exp(4*I*c)*exp(4*I*d*x) + 2*a**2*d*
exp(2*I*c)*exp(2*I*d*x) + a**2*d) + Piecewise((((16*a**2*d*exp(4*I*c)*exp(-2
*I*d*x) - a**2*d*exp(2*I*c)*exp(-4*I*d*x))*exp(-6*I*c)/(16*a**4*d**2), Ne(a
**4*d**2*exp(6*I*c), 0)), (x*((31*I*exp(4*I*c) - 8*I*exp(2*I*c) + I)*exp(-4
*I*c)/(4*a**2) - 31*I/(4*a**2)), True)) + 31*I*x/(4*a**2) - 4*log(exp(2*I*d
*x) + exp(-2*I*c))/(a**2*d)
```

Giac [A]
time = 1.65, size = 98, normalized size = 0.79

$$\frac{\frac{2\log(\tan(dx+c)+i)}{a^2}+\frac{62\log(\tan(dx+c)-i)}{a^2}-\frac{8\left(a^2\tan(dx+c)^2+4i\,a^2\tan(dx+c)\right)}{a^4}-\frac{93\tan(dx+c)^2-150i\tan(dx+c)-61}{a^2(\tan(dx+c)-i)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(2*log(tan(d*x + c) + I)/a^2 + 62*log(tan(d*x + c) - I)/a^2 - 8*(a^2*t
an(d*x + c)^2 + 4*I*a^2*tan(d*x + c))/a^4 - (93*tan(d*x + c)^2 - 150*I*tan(
d*x + c) - 61)/(a^2*(tan(d*x + c) - I)^2))/d
```

Mupad [B]
time = 4.03, size = 114, normalized size = 0.92

$$\frac{31\ln(\tan(c+d\,x)-\mathrm{i})}{8\,a^2\,d}+\frac{\ln(\tan(c+d\,x)+1\mathrm{i})}{8\,a^2\,d}-\frac{\tan(c+d\,x)\,2\mathrm{i}}{a^2\,d}-\frac{\tan(c+d\,x)^2}{2\,a^2\,d}+\frac{\frac{9\tan(c+d\,x)}{4\,a^2}-\frac{2\mathrm{i}}{a^2}}{d\,\left(\tan(c+d\,x)^2\,1\mathrm{i}+2\tan(c+d\,x)-\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] (31*log(tan(c + d*x) - 1i))/(8*a^2*d) + log(tan(c + d*x) + 1i)/(8*a^2*d) -
(tan(c + d*x)*2i)/(a^2*d) - tan(c + d*x)^2/(2*a^2*d) + ((9*tan(c + d*x))/(4
*a^2) - 2i/a^2)/(d*(2*tan(c + d*x) + tan(c + d*x)^2*1i - 1i))
```

3.58 $\int \frac{\tan^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=104

$$\frac{9x}{4a^2}+\frac{2i\log(\cos(c+dx))}{a^2d}-\frac{9\tan(c+dx)}{4a^2d}+\frac{i\tan^2(c+dx)}{a^2d(1+i\tan(c+dx))}-\frac{\tan^3(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

```
[Out] 9/4*x/a^2+2*I*ln(cos(d*x+c))/a^2/d-9/4*tan(d*x+c)/a^2/d+I*tan(d*x+c)^2/a^2/
d/(1+I*tan(d*x+c))-1/4*tan(d*x+c)^3/d/(a+I*a*tan(d*x+c))^2
```

Rubi [A]

time = 0.11, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3639, 3676, 3606, 3556}

$$\frac{i\tan^2(c+dx)}{a^2d(1+i\tan(c+dx))}-\frac{9\tan(c+dx)}{4a^2d}+\frac{2i\log(\cos(c+dx))}{a^2d}+\frac{9x}{4a^2}-\frac{\tan^3(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (9*x)/(4*a^2) + ((2*I)*Log[Cos[c + d*x]])/(a^2*d) - (9*Tan[c + d*x])/(4*a^2
*d) + (I*Tan[c + d*x]^2)/(a^2*d*(1 + I*Tan[c + d*x])) - Tan[c + d*x]^3/(4*d
*(a + I*a*Tan[c + d*x])^2)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
```

```
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx &= -\frac{\tan^3(c+dx)}{4d(a+ia\tan(c+dx))^2} - \frac{\int \frac{\tan^2(c+dx)(-3a+5ia\tan(c+dx))}{a+ia\tan(c+dx)}\,dx}{4a^2} \\
&= \frac{i\tan^2(c+dx)}{a^2d(1+i\tan(c+dx))} - \frac{\tan^3(c+dx)}{4d(a+ia\tan(c+dx))^2} + \frac{\int \tan(c+dx)\left(-16ia^2 -\right.}{8a} \\
&= \frac{9x}{4a^2} - \frac{9\tan(c+dx)}{4a^2d} + \frac{i\tan^2(c+dx)}{a^2d(1+i\tan(c+dx))} - \frac{\tan^3(c+dx)}{4d(a+ia\tan(c+dx))^2} - \\
&= \frac{9x}{4a^2} + \frac{2i\log(\cos(c+dx))}{a^2d} - \frac{9\tan(c+dx)}{4a^2d} + \frac{i\tan^2(c+dx)}{a^2d(1+i\tan(c+dx))} - \frac{}{4d(a}
\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 273 vs. 2(104) = 208.
time = 1.67, size = 273, normalized size = 2.62

$$-\frac{\sec^2(c+dx)(\cos(dx)+i\sin(dx))^2\left(-32dx-12i\cos(2dx)-8i\cos(2c-dx)\sec(c)\sec(c+dx)+8i\cos(2c+dx)\sec(c)\sec(c+dx)+64dx\sin^2(c)+32\mathrm{ArcTan}(\tan(dx))(\cos(2c)+i\sin(2c))+36idx\sin(2c)+\cos(4dx)\sin(2c)-16\log\left(\cos^2(c+dx)\right)\sin(2c)-12\sin(2dx)-i\sin(2c)\sin(4dx)+8\sec(c)\sec(c+dx)\sin(2c-dx)-8\sec(c)\sec(c+dx)\sin(2c+dx)-32idx\tan(c)+\cos(2c)\left(36dx+i\cos(4dx)+16i\log\left(\cos^2(c+dx)\right)+\sin(4dx)-32idx\tan(c)\right)\right)}{16a^2d(-i+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -1/16*(Sec[c + d*x]^2*(Cos[d*x] + I*Sin[d*x])^2*(-32*d*x - (12*I)*Cos[2*d*x
] - (8*I)*Cos[2*c - d*x]*Sec[c]*Sec[c + d*x] + (8*I)*Cos[2*c + d*x]*Sec[c]*
Sec[c + d*x] + 64*d*x*Sin[c]^2 + 32*ArcTan[Tan[d*x]]*(Cos[2*c] + I*Sin[2*c]
) + (36*I)*d*x*Sin[2*c] + Cos[4*d*x]*Sin[2*c] - 16*Log[Cos[c + d*x]^2]*Sin[
2*c] - 12*Sin[2*d*x] - I*Sin[2*c]*Sin[4*d*x] + 8*Sec[c]*Sec[c + d*x]*Sin[2*
c - d*x] - 8*Sec[c]*Sec[c + d*x]*Sin[2*c + d*x] - (32*I)*d*x*Tan[c] + Cos[2
*c]*(36*d*x + I*Cos[4*d*x] + (16*I)*Log[Cos[c + d*x]^2] + Sin[4*d*x] - (32*
I)*d*x*Tan[c]))))/(a^2*d*(-I + Tan[c + d*x])^2)
```

Maple [A]
time = 0.13, size = 70, normalized size = 0.67

method	result
derivativedivides	$\frac{-\tan(dx+c)-\frac{17i\ln(\tan(dx+c)-i)}{8}-\frac{i}{4(\tan(dx+c)-i)^2}-\frac{7}{4(\tan(dx+c)-i)}+\frac{i\ln(\tan(dx+c)+i)}{8}}{d\,a^2}$
default	$\frac{-\tan(dx+c)-\frac{17i\ln(\tan(dx+c)-i)}{8}-\frac{i}{4(\tan(dx+c)-i)^2}-\frac{7}{4(\tan(dx+c)-i)}+\frac{i\ln(\tan(dx+c)+i)}{8}}{d\,a^2}$
risch	$\frac{17x}{4a^2}-\frac{3i\mathrm{e}^{-2i(dx+c)}}{4a^2d}+\frac{i\mathrm{e}^{-4i(dx+c)}}{16a^2d}+\frac{4c}{a^2d}-\frac{2i}{d\,a^2\left(\mathrm{e}^{2i(dx+c)}+1\right)}+\frac{2i\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{a^2d}$
norman	$\frac{\frac{9x}{4a}-\frac{\tan^5(dx+c)}{da}+\frac{9x\left(\tan^2(dx+c)\right)}{2a}+\frac{9x\left(\tan^4(dx+c)\right)}{4a}-\frac{3i}{2da}-\frac{9\tan(dx+c)}{4da}-\frac{15\left(\tan^3(dx+c)\right)}{4da}-\frac{2i\left(\tan^2(dx+c)\right)}{da}}{a(1+\tan^2(dx+c))^2}-\frac{i\ln(1+\tan^2(}{a^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(-tan(d*x+c)-17/8*I*ln(tan(d*x+c)-I)-1/4*I/(tan(d*x+c)-I)^2-7/4/(ta
n(d*x+c)-I)+1/8*I*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.38, size = 113, normalized size = 1.09

$$\frac{68\,dxe^{(6i\,dx+6i\,c)}+4\,(17\,dx-11i)e^{(4i\,dx+4i\,c)}-32\left(-i\,e^{(6i\,dx+6i\,c)}-i\,e^{(4i\,dx+4i\,c)}\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-11i\,e^{(2i\,dx+2i\,c)}+i}{16\left(a^2de^{(6i\,dx+6i\,c)}+a^2de^{(4i\,dx+4i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(68*d*x*e^(6*I*d*x + 6*I*c) + 4*(17*d*x - 11*I)*e^(4*I*d*x + 4*I*c) -
32*(-I*e^(6*I*d*x + 6*I*c) - I*e^(4*I*d*x + 4*I*c))*log(e^(2*I*d*x + 2*I*c)
 + 1) - 11*I*e^(2*I*d*x + 2*I*c) + I)/(a^2*d*e^(6*I*d*x + 6*I*c) + a^2*d*e^
(4*I*d*x + 4*I*c))
```

Sympy [A]
time = 0.29, size = 177, normalized size = 1.70

$$\begin{cases}\frac{(-48ia^2de^{4ic}e^{-2idx}+4ia^2de^{2ic}e^{-4idx})e^{-6ic}}{64a^4d^2} & \text{for } a^4d^2e^{6ic}\neq 0\\ x\left(\frac{(17e^{4ic}-6e^{2ic}+1)e^{-4ic}}{4a^2}-\frac{17}{4a^2}\right) & \text{otherwise}\end{cases} - \frac{2i}{a^2de^{2ic}e^{2idx}+a^2d}+\frac{17x}{4a^2}+\frac{2i\log\left(e^{2idx}+e^{-2ic}\right)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise((((-48*I*a**2*d*exp(4*I*c)*exp(-2*I*d*x) + 4*I*a**2*d*exp(2*I*c)*e
xp(-4*I*d*x))*exp(-6*I*c)/(64*a**4*d**2), Ne(a**4*d**2*exp(6*I*c), 0)), (x*
((17*exp(4*I*c) - 6*exp(2*I*c) + 1)*exp(-4*I*c)/(4*a**2) - 17/(4*a**2)), Tr
ue)) - 2*I/(a**2*d*exp(2*I*c)*exp(2*I*d*x) + a**2*d) + 17*x/(4*a**2) + 2*I*
log(exp(2*I*d*x) + exp(-2*I*c))/(a**2*d)
```

Giac [A]
time = 1.05, size = 79, normalized size = 0.76

$$-\frac{-\frac{2i\log(\tan(dx+c)+i)}{a^2}+\frac{34i\log(\tan(dx+c)-i)}{a^2}+\frac{16\tan(dx+c)}{a^2}+\frac{-51i\tan(dx+c)^2-74\tan(dx+c)+27i}{a^2(\tan(dx+c)-i)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/16*(-2*I*log(tan(d*x + c) + I)/a^2 + 34*I*log(tan(d*x + c) - I)/a^2 + 16
*tan(d*x + c)/a^2 + (-51*I*tan(d*x + c)^2 - 74*tan(d*x + c) + 27*I)/(a^2*(t
an(d*x + c) - I)^2))/d
```

Mupad [B]
time = 4.03, size = 100, normalized size = 0.96

$$-\frac{\ln(\tan(c+d\,x)-\mathrm{i})\,17\mathrm{i}}{8\,a^2\,d}+\frac{\ln(\tan(c+d\,x)+1\mathrm{i})\,1\mathrm{i}}{8\,a^2\,d}-\frac{\tan(c+d\,x)}{a^2\,d}-\frac{\frac{3}{2\,a^2}+\frac{\tan(c+d\,x)\,7\mathrm{i}}{4\,a^2}}{d\,\left(\tan(c+d\,x)^2\,1\mathrm{i}+2\tan(c+d\,x)-\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] (log(tan(c + d*x) + 1i)*1i)/(8*a^2*d) - (log(tan(c + d*x) - 1i)*17i)/(8*a^2
*d) - tan(c + d*x)/(a^2*d) - ((tan(c + d*x)*7i)/(4*a^2) + 3/(2*a^2))/(d*(2*
tan(c + d*x) + tan(c + d*x)^2*1i - 1i))
```

3.59 $\int \frac{\tan^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=79

$$-\frac{3ix}{4a^2}+\frac{\log(\cos(c+dx))}{a^2d}-\frac{3}{4a^2d(1+i\tan(c+dx))}-\frac{\tan^2(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

```
[Out] -3/4*I*x/a^2+ln(cos(d*x+c))/a^2/d-3/4/a^2/d/(1+I*tan(d*x+c))-1/4*tan(d*x+c)
^2/d/(a+I*a*tan(d*x+c))^2
```

Rubi [A]
time = 0.10, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3639, 3670, 3556, 12, 3607, 8}

$$-\frac{3}{4a^2d(1+i\tan(c+dx))}+\frac{\log(\cos(c+dx))}{a^2d}-\frac{3ix}{4a^2}-\frac{\tan^2(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (((-3*I)/4)*x)/a^2 + Log[Cos[c + d*x]]/(a^2*d) - 3/(4*a^2*d*(1 + I*Tan[c +
d*x])) - Tan[c + d*x]^2/(4*d*(a + I*a*Tan[c + d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3670

```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)]))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[B*(d/
b), Int[Tan[e + f*x], x], x] + Dist[1/b, Int[Simp[A*b*c + (A*b*d + B*(b*c -
 a*d))*Tan[e + f*x], x]/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d,
e, f, A, B}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx &= -\frac{\tan^2(c+dx)}{4d(a+ia\tan(c+dx))^2} - \frac{\int \frac{\tan(c+dx)(-2a+4ia\tan(c+dx))}{a+ia\tan(c+dx)}\,dx}{4a^2} \\
&= -\frac{\tan^2(c+dx)}{4d(a+ia\tan(c+dx))^2} + \frac{i\int -\frac{6ia^2\tan(c+dx)}{a+ia\tan(c+dx)}\,dx}{4a^3} - \frac{\int \tan(c+dx)\,dx}{a^2} \\
&= \frac{\log(\cos(c+dx))}{a^2d} - \frac{\tan^2(c+dx)}{4d(a+ia\tan(c+dx))^2} + \frac{3\int \frac{\tan(c+dx)}{a+ia\tan(c+dx)}\,dx}{2a} \\
&= \frac{\log(\cos(c+dx))}{a^2d} - \frac{\tan^2(c+dx)}{4d(a+ia\tan(c+dx))^2} - \frac{3}{4d\left(a^2+ia^2\tan(c+dx)\right)} - \\
&= -\frac{3ix}{4a^2} + \frac{\log(\cos(c+dx))}{a^2d} - \frac{\tan^2(c+dx)}{4d(a+ia\tan(c+dx))^2} - \frac{3}{4d\left(a^2+ia^2\tan(c+\right.}
\end{aligned}$$

Mathematica [A]
time = 0.43, size = 135, normalized size = 1.71

$$\frac{\sec^2(c+dx)\left(8+\cos(2(c+dx))\left(-1-4idx-8\log\left(\cos^2(c+dx)\right)\right)+16i\text{ArcTan}(\tan(dx))(\cos(2(c+dx))+i\sin(2(c+dx)))+i\sin(2(c+dx))+4dx\sin(2(c+dx))-8i\log\left(\cos^2(c+dx)\right)\sin(2(c+dx))\right)}{16a^2d(-i+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^2*(8 + Cos[2*(c + d*x)]*(-1 - (4*I)*d*x - 8*Log[Cos[c + d*x]^
2]) + (16*I)*ArcTan[Tan[d*x]]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) + I*S
in[2*(c + d*x)] + 4*d*x*Sin[2*(c + d*x)] - (8*I)*Log[Cos[c + d*x]^2]*Sin[2*
(c + d*x)]))/(16*a^2*d*(-I + Tan[c + d*x])^2)
```

Maple [A]
time = 0.11, size = 60, normalized size = 0.76

method	result
derivativedivides	$\frac{\frac{5i}{4(\tan(dx+c)-i)}-\frac{1}{4(\tan(dx+c)-i)^2}-\frac{7\ln(\tan(dx+c)-i)}{8}-\frac{\ln(\tan(dx+c)+i)}{8}}{d\,a^2}$
default	$\frac{\frac{5i}{4(\tan(dx+c)-i)}-\frac{1}{4(\tan(dx+c)-i)^2}-\frac{7\ln(\tan(dx+c)-i)}{8}-\frac{\ln(\tan(dx+c)+i)}{8}}{d\,a^2}$
risch	$-\frac{7ix}{4a^2}-\frac{e^{-2i(dx+c)}}{2a^2d}+\frac{e^{-4i(dx+c)}}{16a^2d}-\frac{2ic}{a^2d}+\frac{\ln(e^{2i(dx+c)}+1)}{a^2d}$
norman	$\frac{-\frac{1}{da}-\frac{3ix}{4a}-\frac{3(\tan^2(dx+c))}{2da}-\frac{3ix(\tan^2(dx+c))}{2a}-\frac{3ix(\tan^4(dx+c))}{4a}+\frac{3i\tan(dx+c)}{4da}+\frac{5i(\tan^3(dx+c))}{4da}}{a(1+\tan^2(dx+c))^2}-\frac{\ln(1+\tan^2(dx+c))}{2a^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(5/4*I/(tan(d*x+c)-I)-1/4/(tan(d*x+c)-I)^2-7/8*ln(tan(d*x+c)-I)-1/8
*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.37, size = 66, normalized size = 0.84

$$\frac{\left(-28i\,dxe^{(4i\,dx+4i\,c)}+16\,e^{(4i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-8\,e^{(2i\,dx+2i\,c)}+1\right)e^{(-4i\,dx-4i\,c)}}{16\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(-28*I*d*x*e^(4*I*d*x + 4*I*c) + 16*e^(4*I*d*x + 4*I*c)*log(e^(2*I*d*x
 + 2*I*c) + 1) - 8*e^(2*I*d*x + 2*I*c) + 1)*e^(-4*I*d*x - 4*I*c)/(a^2*d)
```

Sympy [A]
time = 0.30, size = 148, normalized size = 1.87

$$\begin{cases} \frac{(-16a^2de^{4ic}e^{-2idx}+2a^2de^{2ic}e^{-4idx})e^{-6ic}}{32a^4d^2} & \text{for } a^4d^2e^{6ic} \neq 0 \\ x\left(\frac{(-7ie^{4ic}+4ie^{2ic}-i)e^{-4ic}}{4a^2}+\frac{7i}{4a^2}\right) & \text{otherwise} \end{cases} - \frac{7ix}{4a^2} + \frac{\log\left(e^{2idx}+e^{-2ic}\right)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise(((-16*a**2*d*exp(4*I*c)*exp(-2*I*d*x) + 2*a**2*d*exp(2*I*c)*exp(-
4*I*d*x))*exp(-6*I*c)/(32*a**4*d**2), Ne(a**4*d**2*exp(6*I*c), 0)), (x*((-7
*I*exp(4*I*c) + 4*I*exp(2*I*c) - I)*exp(-4*I*c)/(4*a**2) + 7*I/(4*a**2)), T
rue)) - 7*I*x/(4*a**2) + log(exp(2*I*d*x) + exp(-2*I*c))/(a**2*d)
```

Giac [A]
time = 0.90, size = 69, normalized size = 0.87

$$-\frac{\frac{2\log(\tan(dx+c)+i)}{a^2}+\frac{14\log(\tan(dx+c)-i)}{a^2}-\frac{21\tan(dx+c)^2-22i\tan(dx+c)-5}{a^2(\tan(dx+c)-i)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/16*(2*log(tan(d*x + c) + I)/a^2 + 14*log(tan(d*x + c) - I)/a^2 - (21*tan
(d*x + c)^2 - 22*I*tan(d*x + c) - 5)/(a^2*(tan(d*x + c) - I)^2))/d
```

Mupad [B]
time = 4.02, size = 84, normalized size = 1.06

$$-\frac{7\ln\left(\tan(c+d\,x)-\mathrm{i}\right)}{8\,a^2\,d}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)}{8\,a^2\,d}-\frac{\frac{5\tan(c+d\,x)}{4\,a^2}-\frac{1\mathrm{i}}{a^2}}{d\,\left(\tan\left(c+d\,x\right)^2\,1\mathrm{i}+2\tan\left(c+d\,x\right)-\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] - (7*log(tan(c + d*x) - 1i))/(8*a^2*d) - log(tan(c + d*x) + 1i)/(8*a^2*d) -
 ((5*tan(c + d*x))/(4*a^2) - 1i/a^2)/(d*(2*tan(c + d*x) + tan(c + d*x)^2*1i
 - 1i))
```

3.60 $\int \frac{\tan^2(c+dx)}{(a+ia\tan(c+dx))^2} \, dx$

Optimal. Leaf size=59

$$-\frac{x}{4a^2}+\frac{3i}{4a^2d(1+i\tan(c+dx))}-\frac{i}{4d(a+ia\tan(c+dx))^2}$$

```
[Out] -1/4*x/a^2+3/4*I/a^2/d/(1+I*tan(d*x+c))-1/4*I/d/(a+I*a*tan(d*x+c))^2
```

Rubi [A]
time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3621, 3607, 8}

$$\frac{3i}{4a^2d(1+i\tan(c+dx))}-\frac{x}{4a^2}-\frac{i}{4d(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -1/4*x/a^2 + ((3*I)/4)/(a^2*d*(1 + I*Tan[c + d*x])) - (I/4)/(d*(a + I*a*Tan
[c + d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rule 3621

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(a*c + b*d)^2*((a + b*Tan[e + f*x])^
m/(2*a^3*f*m)), x] + Dist[1/(2*a^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[
a*c^2 - 2*b*c*d + a*d^2 - 2*b*d^2*Tan[e + f*x], x], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LeQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{i}{4d(a+ia\tan(c+dx))^2} + \frac{\int \frac{a-2ia\tan(c+dx)}{a+ia\tan(c+dx)}\,dx}{2a^2}$$
$$= \frac{3i}{4a^2d(1+i\tan(c+dx))} - \frac{i}{4d(a+ia\tan(c+dx))^2} - \frac{\int 1\,dx}{4a^2}$$
$$= -\frac{x}{4a^2} + \frac{3i}{4a^2d(1+i\tan(c+dx))} - \frac{i}{4d(a+ia\tan(c+dx))^2}$$

Mathematica [A]
time = 0.22, size = 68, normalized size = 1.15

$$\frac{\sec^2(c+dx)(-4i+(i+4dx)\cos(2(c+dx))+(1+4idx)\sin(2(c+dx)))}{16a^2d(-i+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + I*a*Tan[c + d*x])^2,x]
[Out] (Sec[c + d*x]^2*(-4*I + (I + 4*d*x)*Cos[2*(c + d*x)] + (1 + (4*I)*d*x)*Sin[
2*(c + d*x)]))/(16*a^2*d*(-I + Tan[c + d*x])^2)
```

Maple [A]
time = 0.13, size = 62, normalized size = 1.05

method	result	size
risch	$-\frac{x}{4a^2}+\frac{i e^{-2i(dx+c)}}{4a^2d}-\frac{i e^{-4i(dx+c)}}{16a^2d}$	44
derivativedivides	$\frac{\frac{i}{4(\tan(dx+c)-i)^2}+\frac{i\ln(\tan(dx+c)-i)}{8}+\frac{3}{4(\tan(dx+c)-i)}-\frac{i\ln(\tan(dx+c)+i)}{8}}{d\,a^2}$	62
default	$\frac{\frac{i}{4(\tan(dx+c)-i)^2}+\frac{i\ln(\tan(dx+c)-i)}{8}+\frac{3}{4(\tan(dx+c)-i)}-\frac{i\ln(\tan(dx+c)+i)}{8}}{d\,a^2}$	62
norman	$\frac{\frac{i\left(\tan^2(dx+c)\right)}{da}-\frac{x}{4a}-\frac{x\left(\tan^2(dx+c)\right)}{2a}-\frac{x\left(\tan^4(dx+c)\right)}{4a}+\frac{i}{2da}+\frac{\tan(dx+c)}{4da}+\frac{3\left(\tan^3(dx+c)\right)}{4da}}{a(1+\tan^2(dx+c))^2}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d/a^2*(1/4*I/(tan(d*x+c)-I)^2+1/8*I*ln(tan(d*x+c)-I)+3/4/(tan(d*x+c)-I)-1
/8*I*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.40, size = 43, normalized size = 0.73

$$-\frac{\left(4\,dxe^{(4i\,dx+4i\,c)}-4i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-4i\,dx-4i\,c)}}{16\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*d*x*e^(4*I*d*x + 4*I*c) - 4*I*e^(2*I*d*x + 2*I*c) + I)*e^(-4*I*d*x
 - 4*I*c)/(a^2*d)
```

Sympy [A]
time = 0.16, size = 117, normalized size = 1.98

$$\begin{cases} \frac{(16ia^2de^{4ic}e^{-2idx}-4ia^2de^{2ic}e^{-4idx})e^{-6ic}}{64a^4d^2} & \text{for } a^4d^2e^{6ic}\neq 0 \\ x\left(\frac{(-e^{4ic}+2e^{2ic}-1)e^{-4ic}}{4a^2}+\frac{1}{4a^2}\right) & \text{otherwise} \end{cases} - \frac{x}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise(((16*I*a**2*d*exp(4*I*c)*exp(-2*I*d*x) - 4*I*a**2*d*exp(2*I*c)*ex
p(-4*I*d*x))*exp(-6*I*c)/(64*a**4*d**2), Ne(a**4*d**2*exp(6*I*c), 0)), (x*(
(-exp(4*I*c) + 2*exp(2*I*c) - 1)*exp(-4*I*c)/(4*a**2) + 1/(4*a**2)), True))
 - x/(4*a**2)
```

Giac [A]
time = 0.68, size = 72, normalized size = 1.22

$$-\frac{\frac{2i\,\log(-i\,\tan(dx+c)+1)}{a^2}-\frac{2i\,\log(-i\,\tan(dx+c)-1)}{a^2}+\frac{3i\,\tan(dx+c)^2-6\,\tan(dx+c)+5i}{a^2(\tan(dx+c)-i)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/16*(2*I*log(-I*tan(d*x + c) + 1)/a^2 - 2*I*log(-I*tan(d*x + c) - 1)/a^2
+ (3*I*tan(d*x + c)^2 - 6*tan(d*x + c) + 5*I)/(a^2*(tan(d*x + c) - I)^2))/d
```

Mupad [B]
time = 3.97, size = 39, normalized size = 0.66

$$-\frac{x}{4\,a^2}-\frac{\frac{3\tan(c+d\,x)}{4}-\frac{1}{2}\mathrm{i}}{a^2\,d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + a*tan(c + d*x)*1i)^2,x)
[Out] - x/(4*a^2) - ((3*tan(c + d*x))/4 - 1i/2)/(a^2*d*(tan(c + d*x)*1i + 1)^2)
```

3.61 $\int \frac{\tan(c+dx)}{(a+ia\tan(c+dx))^2} dx$

Optimal. Leaf size=59

$$-\frac{ix}{4a^2}-\frac{1}{4d(a+ia\tan(c+dx))^2}+\frac{1}{4d\left(a^2+ia^2\tan(c+dx)\right)}$$

```
[Out] -1/4*I*x/a^2-1/4/d/(a+I*a*tan(d*x+c))^2+1/4/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3607, 3560, 8}

$$\frac{1}{4d\left(a^2+ia^2\tan(c+dx)\right)}-\frac{ix}{4a^2}-\frac{1}{4d(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((-1/4*I)*x)/a^2 - 1/(4*d*(a + I*a*Tan[c + d*x])^2) + 1/(4*d*(a^2 + I*a^2*T
an[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{1}{4d(a+ia\tan(c+dx))^2} - \frac{i\int \frac{1}{a+ia\tan(c+dx)}\,dx}{2a}$$
$$= -\frac{1}{4d(a+ia\tan(c+dx))^2} + \frac{1}{4d\left(a^2+ia^2\tan(c+dx)\right)} - \frac{i\int 1\,dx}{4a^2}$$
$$= -\frac{ix}{4a^2} - \frac{1}{4d(a+ia\tan(c+dx))^2} + \frac{1}{4d\left(a^2+ia^2\tan(c+dx)\right)}$$

Mathematica [A]
time = 0.11, size = 66, normalized size = 1.12

$$\frac{\sec^2(c+dx)((1+4idx)\cos(2(c+dx)) - (i+4dx)\sin(2(c+dx)))}{16a^2d(-i+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^2*((1 + (4*I)*d*x)*Cos[2*(c + d*x)] - (I + 4*d*x)*Sin[2*(c + 
d*x)]))/(16*a^2*d*(-I + Tan[c + d*x])^2)
```

Maple [A]
time = 0.09, size = 60, normalized size = 1.02

method	result	size
risch	$-\frac{ix}{4a^2} - \frac{e^{-4i(dx+c)}}{16a^2d}$	26
derivativedivides	$\frac{-\frac{i}{4(\tan(dx+c)-i)} + \frac{1}{4(\tan(dx+c)-i)^2} - \frac{\ln(\tan(dx+c)-i)}{8} + \frac{\ln(\tan(dx+c)+i)}{8}}{d\,a^2}$	60
default	$\frac{-\frac{i}{4(\tan(dx+c)-i)} + \frac{1}{4(\tan(dx+c)-i)^2} - \frac{\ln(\tan(dx+c)-i)}{8} + \frac{\ln(\tan(dx+c)+i)}{8}}{d\,a^2}$	60
norman	$\frac{-\frac{ix}{4a} + \frac{\tan^2(dx+c)}{2da} - \frac{ix\left(\tan^2(dx+c)\right)}{2a} - \frac{ix\left(\tan^4(dx+c)\right)}{4a} + \frac{i\tan(dx+c)}{4da} - \frac{i\left(\tan^3(dx+c)\right)}{4da}}{a(1+\tan^2(dx+c))^2}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(-1/4*I/(tan(d*x+c)-I)+1/4/(tan(d*x+c)-I)^2-1/8*ln(tan(d*x+c)-I)+1/
8*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.36, size = 32, normalized size = 0.54

$$\frac{\left(-4i\,dxe^{(4i\,dx+4i\,c)}-1\right)e^{(-4i\,dx-4i\,c)}}{16\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(-4*I*d*x*e^(4*I*d*x + 4*I*c) - 1)*e^(-4*I*d*x - 4*I*c)/(a^2*d)
```

Sympy [A]
time = 0.13, size = 73, normalized size = 1.24

$$\begin{cases} -\frac{e^{-4ic}e^{-4idx}}{16a^2d} & \text{for } a^2de^{4ic}\neq 0\\ x\left(\frac{(-ie^{4ic}+i)e^{-4ic}}{4a^2}+\frac{i}{4a^2}\right) & \text{otherwise}\end{cases} - \frac{ix}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise((-exp(-4*I*c)*exp(-4*I*d*x)/(16*a**2*d), Ne(a**2*d*exp(4*I*c), 0)
), (x*((-I*exp(4*I*c) + I)*exp(-4*I*c)/(4*a**2) + I/(4*a**2)), True)) - I*x
/(4*a**2)
```

Giac [A]
time = 0.53, size = 70, normalized size = 1.19

$$-\frac{\frac{\log(\tan(2\,dx+2\,c)-i)}{a^2}-\frac{\log(-i\,\tan(2\,dx+2\,c)+1)}{a^2}-\frac{\tan(2\,dx+2\,c)+i}{a^2(\tan(2\,dx+2\,c)-i)}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/16*(log(tan(2*d*x + 2*c) - I)/a^2 - log(-I*tan(2*d*x + 2*c) + 1)/a^2 - (
tan(2*d*x + 2*c) + I)/(a^2*(tan(2*d*x + 2*c) - I)))/d
```

Mupad [B]
time = 3.97, size = 46, normalized size = 0.78

$$-\frac{x\,1\mathrm{i}}{4\,a^2}+\frac{\tan(c+d\,x)}{4\,a^2\,d\,\left(\tan\left(c+d\,x\right)^2\,1\mathrm{i}+2\tan\left(c+d\,x\right)-\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] tan(c + d*x)/(4*a^2*d*(2*tan(c + d*x) + tan(c + d*x)^2*1i - 1i)) - (x*1i)/(
4*a^2)
```

3.62 $\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=61

$$\frac{x}{4a^2}+\frac{i}{4d(a+ia\tan(c+dx))^2}+\frac{i}{4d\left(a^2+ia^2\tan(c+dx)\right)}$$

```
[Out] 1/4*x/a^2+1/4*I/d/(a+I*a*tan(d*x+c))^2+1/4*I/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3560, 8}

$$\frac{i}{4d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{x}{4a^2}+\frac{i}{4d(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(-2),x]
```

```
[Out] x/(4*a^2) + (I/4)/(d*(a + I*a*Tan[c + d*x])^2) + (I/4)/(d*(a^2 + I*a^2*Tan[
c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx &= \frac{i}{4d(a+ia\tan(c+dx))^2}+\frac{\int \frac{1}{a+ia\tan(c+dx)}\,dx}{2a}\\ &= \frac{i}{4d(a+ia\tan(c+dx))^2}+\frac{i}{4d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{\int 1\,dx}{4a^2}\\ &= \frac{x}{4a^2}+\frac{i}{4d(a+ia\tan(c+dx))^2}+\frac{i}{4d\left(a^2+ia^2\tan(c+dx)\right)}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 68, normalized size = 1.11

$$-\frac{\sec^2(c+dx)(4i+(i+4dx)\cos(2(c+dx))+(1+4idx)\sin(2(c+dx)))}{16a^2d(-i+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(-2),x]
```

```
[Out] -1/16*(Sec[c + d*x]^2*(4*I + (I + 4*d*x)*Cos[2*(c + d*x)] + (1 + (4*I)*d*x)
*Sin[2*(c + d*x)]))/(a^2*d*(-I + Tan[c + d*x])^2)
```

Maple [A]
time = 0.00, size = 62, normalized size = 1.02

method	result	size
risch	$\frac{x}{4a^2}+\frac{i\mathrm{e}^{-2i(dx+c)}}{4a^2d}+\frac{i\mathrm{e}^{-4i(dx+c)}}{16a^2d}$	44
derivativedivides	$\frac{-\frac{i\ln(\tan(dx+c)-i)}{8}-\frac{i}{4(\tan(dx+c)-i)^2}+\frac{1}{4\tan(dx+c)-4i}+\frac{i\ln(\tan(dx+c)+i)}{8}}{d\,a^2}$	62
default	$\frac{-\frac{i\ln(\tan(dx+c)-i)}{8}-\frac{i}{4(\tan(dx+c)-i)^2}+\frac{1}{4\tan(dx+c)-4i}+\frac{i\ln(\tan(dx+c)+i)}{8}}{d\,a^2}$	62
norman	$\frac{\frac{x}{4a}+\frac{x\left(\tan^2(dx+c)\right)}{2a}+\frac{x\left(\tan^4(dx+c)\right)}{4a}+\frac{i}{2da}+\frac{3\tan(dx+c)}{4da}+\frac{\tan^3(dx+c)}{4da}}{a(1+\tan^2(dx+c))^2}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(-1/8*I*ln(tan(d*x+c)-I)-1/4*I/(tan(d*x+c)-I)^2+1/4/(tan(d*x+c)-I)+
1/8*I*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.35, size = 43, normalized size = 0.70

$$\frac{\left(4\,dxe^{(4i\,dx+4i\,c)}+4i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-4i\,dx-4i\,c)}}{16\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*d*x*e^(4*I*d*x + 4*I*c) + 4*I*e^(2*I*d*x + 2*I*c) + I)*e^(-4*I*d*x
- 4*I*c)/(a^2*d)
```

Sympy [A]
time = 0.14, size = 117, normalized size = 1.92

$$\begin{cases} \frac{\left(16ia^2de^{4ic}e^{-2idx}+4ia^2de^{2ic}e^{-4idx}\right)e^{-6ic}}{64a^4d^2} & \text{for } a^4d^2e^{6ic} \neq 0 \\ x\left(\frac{\left(e^{4ic}+2e^{2ic}+1\right)e^{-4ic}}{4a^2} - \frac{1}{4a^2}\right) & \text{otherwise} \end{cases} + \frac{x}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise((((16*I*a**2*d*exp(4*I*c)*exp(-2*I*d*x) + 4*I*a**2*d*exp(2*I*c)*ex
p(-4*I*d*x))*exp(-6*I*c)/(64*a**4*d**2), Ne(a**4*d**2*exp(6*I*c), 0)), (x*(
(exp(4*I*c) + 2*exp(2*I*c) + 1)*exp(-4*I*c)/(4*a**2) - 1/(4*a**2)), True))
+ x/(4*a**2)
```

Giac [A]
time = 0.46, size = 72, normalized size = 1.18

$$-\frac{\frac{2i\log(i\tan(dx+c)+1)}{a^2} - \frac{2i\log(i\tan(dx+c)-1)}{a^2} + \frac{-3i\tan(dx+c)^2-10\tan(dx+c)+11i}{a^2(\tan(dx+c)-i)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/16*(2*I*log(I*tan(d*x + c) + 1)/a^2 - 2*I*log(I*tan(d*x + c) - 1)/a^2 +
(-3*I*tan(d*x + c)^2 - 10*tan(d*x + c) + 11*I)/(a^2*(tan(d*x + c) - I)^2))/
d
```

Mupad [B]
time = 3.94, size = 39, normalized size = 0.64

$$\frac{x}{4\,a^2} - \frac{\frac{\tan(c+d\,x)}{4} - \frac{1}{2}\mathrm{i}}{a^2\,d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] x/(4*a^2) - (tan(c + d*x)/4 - 1i/2)/(a^2*d*(tan(c + d*x)*1i + 1)^2)
```

3.63 $\int \frac{\cot(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=71

$$-\frac{3ix}{4a^2}+\frac{\log(\sin(c+dx))}{a^2d}+\frac{3}{4a^2d(1+i\tan(c+dx))}+\frac{1}{4d(a+ia\tan(c+dx))^2}$$

```
[Out] -3/4*I*x/a^2+ln(sin(d*x+c))/a^2/d+3/4/a^2/d/(1+I*tan(d*x+c))+1/4/d/(a+I*a*t
an(d*x+c))^2
```

Rubi [A]
time = 0.10, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3640, 3677, 3612, 3556}

$$\frac{3}{4a^2d(1+i\tan(c+dx))}+\frac{\log(\sin(c+dx))}{a^2d}-\frac{3ix}{4a^2}+\frac{1}{4d(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (((-3*I)/4)*x)/a^2 + Log[Sin[c + d*x]]/(a^2*d) + 3/(4*a^2*d*(1 + I*Tan[c +
d*x])) + 1/(4*d*(a + I*a*Tan[c + d*x])^2)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(c+dx)}{(a+ia\tan(c+dx))^2}\,dx &= \frac{1}{4d(a+ia\tan(c+dx))^2}+\frac{\int \frac{\cot(c+dx)(4a-2ia\tan(c+dx))}{a+ia\tan(c+dx)}\,dx}{4a^2}\\
&= \frac{3}{4a^2d(1+i\tan(c+dx))}+\frac{1}{4d(a+ia\tan(c+dx))^2}+\frac{\int \cot(c+dx)\left(8a^2-6ia\right.}{8a^4}\\
&= -\frac{3ix}{4a^2}+\frac{3}{4a^2d(1+i\tan(c+dx))}+\frac{1}{4d(a+ia\tan(c+dx))^2}+\frac{\int \cot(c+dx)\,dx}{a^2}\\
&= -\frac{3ix}{4a^2}+\frac{\log(\sin(c+dx))}{a^2d}+\frac{3}{4a^2d(1+i\tan(c+dx))}+\frac{1}{4d(a+ia\tan(c+dx))^2}
\end{aligned}$$

Mathematica [A]
time = 0.38, size = 135, normalized size = 1.90

$$\frac{\sec^2(c+dx)\left(-8+\cos(2(c+dx))\left(-1-4idx-8\log\left(\sin^2(c+dx)\right)\right)+16i\mathrm{ArcTan}(\tan(dx))(\cos(2(c+dx))+i\sin(2(c+dx)))+i\sin(2(c+dx))+4dx\sin(2(c+dx))-8i\log\left(\sin^2(c+dx)\right)\sin(2(c+dx))\right)}{16a^2d(-i+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^2*(-8 + Cos[2*(c + d*x)]*(-1 - (4*I)*d*x - 8*Log[Sin[c + d*x]
^2]) + (16*I)*ArcTan[Tan[d*x]]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) + I*
Sin[2*(c + d*x)] + 4*d*x*Sin[2*(c + d*x)] - (8*I)*Log[Sin[c + d*x]^2]*Sin[2
*(c + d*x)]))/(16*a^2*d*(-I + Tan[c + d*x])^2)
```

Maple [A]
time = 0.28, size = 67, normalized size = 0.94

method	result
derivativedivides	$\frac{-\frac{3i}{4(\tan(dx+c)-i)}-\frac{1}{4(\tan(dx+c)-i)^2}-\frac{7\ln(\tan(dx+c)-i)}{8}+\ln(\tan(dx+c))-\frac{\ln(\tan(dx+c)+i)}{8}}{d\,a^2}$

default	$\frac{-\frac{3i}{4(\tan(dx+c)-i)}-\frac{1}{4(\tan(dx+c)-i)^2}-\frac{7\ln(\tan(dx+c)-i)}{8}+\ln(\tan(dx+c))-\frac{\ln(\tan(dx+c)+i)}{8}}{d\,a^2}$
risch	$-\frac{7ix}{4a^2}+\frac{\mathrm{e}^{-2i(dx+c)}}{2a^2d}+\frac{\mathrm{e}^{-4i(dx+c)}}{16a^2d}-\frac{2ic}{a^2d}+\frac{\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{a^2d}$
norman	$\frac{\frac{1}{da}-\frac{3ix}{4a}+\frac{\tan^2(dx+c)}{2da}-\frac{3ix\left(\tan^2(dx+c)\right)}{2a}-\frac{3ix\left(\tan^4(dx+c)\right)}{4a}-\frac{5i\tan(dx+c)}{4da}-\frac{3i\left(\tan^3(dx+c)\right)}{4da}}{a(1+\tan^2(dx+c))^2}+\frac{\ln(\tan(dx+c))}{a^2d}-\frac{\ln(1+\mathrm{t}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(-3/4*I/(tan(d*x+c)-I)-1/4/(tan(d*x+c)-I)^2-7/8*ln(tan(d*x+c)-I)+ln
(tan(d*x+c))-1/8*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.37, size = 66, normalized size = 0.93

$$\frac{\left(-28i\,dxe^{(4i\,dx+4i\,c)}+16\,e^{(4i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}-1\right)+8\,e^{(2i\,dx+2i\,c)}+1\right)e^{(-4i\,dx-4i\,c)}}{16\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(-28*I*d*x*e^(4*I*d*x + 4*I*c) + 16*e^(4*I*d*x + 4*I*c)*log(e^(2*I*d*x
 + 2*I*c) - 1) + 8*e^(2*I*d*x + 2*I*c) + 1)*e^(-4*I*d*x - 4*I*c)/(a^2*d)
```

Sympy [A]
time = 0.25, size = 150, normalized size = 2.11

$$\begin{cases}\frac{\left(16a^2de^{4ic}e^{-2idx}+2a^2de^{2ic}e^{-4idx}\right)e^{-6ic}}{32a^4d^2} & \text{for } a^4d^2e^{6ic}\neq 0\\ x\left(\frac{\left(-7ie^{4ic}-4ie^{2ic}-i\right)e^{-4ic}}{4a^2}+\frac{7i}{4a^2}\right) & \text{otherwise}\end{cases}-\frac{7ix}{4a^2}+\frac{\log\left(e^{2idx}-e^{-2ic}\right)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise((((16*a**2*d*exp(4*I*c)*exp(-2*I*d*x) + 2*a**2*d*exp(2*I*c)*exp(-4
*I*d*x))*exp(-6*I*c)/(32*a**4*d**2), Ne(a**4*d**2*exp(6*I*c), 0)), (x*((-7*
I*exp(4*I*c) - 4*I*exp(2*I*c) - I)*exp(-4*I*c)/(4*a**2) + 7*I/(4*a**2)), Tr
ue)) - 7*I*x/(4*a**2) + log(exp(2*I*d*x) - exp(-2*I*c))/(a**2*d)
```

Giac [A]
time = 0.75, size = 81, normalized size = 1.14

$$-\frac{\frac{2\log(\tan(dx+c)+i)}{a^2}+\frac{14\log(\tan(dx+c)-i)}{a^2}-\frac{16\log(\tan(dx+c))}{a^2}-\frac{21\tan(dx+c)^2-54i\tan(dx+c)-37}{a^2(\tan(dx+c)-i)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/16*(2*log(tan(d*x + c) + I)/a^2 + 14*log(tan(d*x + c) - I)/a^2 - 16*log(
tan(d*x + c))/a^2 - (21*tan(d*x + c)^2 - 54*I*tan(d*x + c) - 37)/(a^2*(tan(
d*x + c) - I)^2))/d
```

Mupad [B]
time = 3.94, size = 97, normalized size = 1.37

$$\frac{\ln(\tan(c+d\,x))}{a^2\,d}-\frac{\ln(\tan(c+d\,x)+1\mathrm{i})}{8\,a^2\,d}-\frac{7\,\ln(\tan(c+d\,x)-\mathrm{i})}{8\,a^2\,d}+\frac{\frac{3\tan(c+d\,x)}{4\,a^2}-\frac{1\mathrm{i}}{a^2}}{d\,\left(\tan(c+d\,x)^2\,1\mathrm{i}+2\tan(c+d\,x)-\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] log(tan(c + d*x))/(a^2*d) - log(tan(c + d*x) + 1i)/(8*a^2*d) - (7*log(tan(c
 + d*x) - 1i))/(8*a^2*d) + ((3*tan(c + d*x))/(4*a^2) - 1i/a^2)/(d*(2*tan(c
+ d*x) + tan(c + d*x)^2*1i - 1i))
```

3.64 $\int \frac{\cot^2(c+dx)}{(a+ia\tan(c+dx))^2} dx$

Optimal. Leaf size=97

$$-\frac{9x}{4a^2}-\frac{9\cot(c+dx)}{4a^2d}-\frac{2i\log(\sin(c+dx))}{a^2d}+\frac{\cot(c+dx)}{a^2d(1+i\tan(c+dx))}+\frac{\cot(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

```
[Out] -9/4*x/a^2-9/4*cot(d*x+c)/a^2/d-2*I*ln(sin(d*x+c))/a^2/d+cot(d*x+c)/a^2/d/(
1+I*tan(d*x+c))+1/4*cot(d*x+c)/d/(a+I*a*tan(d*x+c))^2
```

Rubi [A]
time = 0.14, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3640, 3677, 3610, 3612, 3556}

$$-\frac{9\cot(c+dx)}{4a^2d}-\frac{2i\log(\sin(c+dx))}{a^2d}+\frac{\cot(c+dx)}{a^2d(1+i\tan(c+dx))}-\frac{9x}{4a^2}+\frac{\cot(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (-9*x)/(4*a^2) - (9*Cot[c + d*x])/(4*a^2*d) - ((2*I)*Log[Sin[c + d*x]])/(a^
2*d) + Cot[c + d*x]/(a^2*d*(1 + I*Tan[c + d*x])) + Cot[c + d*x]/(4*d*(a + I
*a*Tan[c + d*x])^2)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx &= \frac{\cot(c+dx)}{4d(a+ia\tan(c+dx))^2} + \frac{\int \frac{\cot^2(c+dx)(5a-3ia\tan(c+dx))}{a+ia\tan(c+dx)}\,dx}{4a^2} \\
&= \frac{\cot(c+dx)}{a^2d(1+i\tan(c+dx))} + \frac{\cot(c+dx)}{4d(a+ia\tan(c+dx))^2} + \frac{\int \cot^2(c+dx)\,(18a^2-16\ldots}{8a^4} \\
&= -\frac{9\cot(c+dx)}{4a^2d} + \frac{\cot(c+dx)}{a^2d(1+i\tan(c+dx))} + \frac{\cot(c+dx)}{4d(a+ia\tan(c+dx))^2} + \frac{\int \cot(c\ldots}{\ldots} \\
&= -\frac{9x}{4a^2} - \frac{9\cot(c+dx)}{4a^2d} + \frac{\cot(c+dx)}{a^2d(1+i\tan(c+dx))} + \frac{\cot(c+dx)}{4d(a+ia\tan(c+dx))^2} - \ldots \\
&= -\frac{9x}{4a^2} - \frac{9\cot(c+dx)}{4a^2d} - \frac{2i\log(\sin(c+dx))}{a^2d} + \frac{\cot(c+dx)}{a^2d(1+i\tan(c+dx))} + \frac{\ldots}{4d(a\ldots}
\end{aligned}
$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 276 vs. 2(97) = 194.
time = 1.98, size = 276, normalized size = 2.85

$$
-\frac{\sec^2(c+dx)(\cos(dx)+i\sin(dx))^2\left(-32dx+64dx\cos^2(c)-12i\cos(2dx)+32idx\cot(c)+8i\cos(2c-dx)\csc(c)\csc(c+dx)-8i\cos(2c+dx)\csc(c)\csc(c+dx)-32\text{ArcTan}(\tan(dx))(\cos(2c)+i\sin(2c))-36idx\sin(2c)-\cos(4dx)\sin(2c)+16\log\left(\sin^2(c+dx)\right)\sin(2c)-12\sin(2dx)+i\sin(2c)\sin(4dx)-i\cos(2c)\left(\cos(4dx)+32dx\cot(c)-i(36dx+16i\log\left(\sin^2(c+dx)\right)+\sin(4dx))\right)-8\csc(c)\csc(c+dx)\sin(2c-dx)+8\csc(c)\csc(c+dx)\sin(2c+dx)\right)}{16a^2d(-i+\tan(c+dx))^2}
$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^2/(a + I*a*Tan[c + d*x])^2,x]

[Out] -1/16*(Sec[c + d*x]^2*(Cos[d*x] + I*Sin[d*x])^2*(-32*d*x + 64*d*x*Cos[c]^2 - (12*I)*Cos[2*d*x] + (32*I)*d*x*Cot[c] + (8*I)*Cos[2*c - d*x]*Csc[c]*Csc[c + d*x] - (8*I)*Cos[2*c + d*x]*Csc[c]*Csc[c + d*x] - 32*ArcTan[Tan[d*x]]*(Cos[2*c] + I*Sin[2*c]) - (36*I)*d*x*Sin[2*c] - Cos[4*d*x]*Sin[2*c] + 16*Log[Sin[c + d*x]^2]*Sin[2*c] - 12*Sin[2*d*x] + I*Sin[2*c]*Sin[4*d*x] - I*Cos[2*c]*(Cos[4*d*x] + 32*d*x*Cot[c] - I*(36*d*x + (16*I)*Log[Sin[c + d*x]^2] + Sin[4*d*x])) - 8*Csc[c]*Csc[c + d*x]*Sin[2*c - d*x] + 8*Csc[c]*Csc[c + d*x]*Sin[2*c + d*x]))/(a^2*d*(-I + Tan[c + d*x])^2)

Maple [A]
time = 0.30, size = 82, normalized size = 0.85

method	result
derivativedivides	$\frac{\frac{i}{4(\tan(dx+c)-i)^2}+\frac{17i\ln(\tan(dx+c)-i)}{8}-\frac{5}{4(\tan(dx+c)-i)}-\frac{1}{\tan(dx+c)}-2i\ln(\tan(dx+c))-\frac{i\ln(\tan(dx+c)+i)}{8}}{d\,a^2}$
default	$\frac{\frac{i}{4(\tan(dx+c)-i)^2}+\frac{17i\ln(\tan(dx+c)-i)}{8}-\frac{5}{4(\tan(dx+c)-i)}-\frac{1}{\tan(dx+c)}-2i\ln(\tan(dx+c))-\frac{i\ln(\tan(dx+c)+i)}{8}}{d\,a^2}$
risch	$-\frac{17x}{4a^2}-\frac{3i\mathrm{e}^{-2i(dx+c)}}{4a^2d}-\frac{i\mathrm{e}^{-4i(dx+c)}}{16a^2d}-\frac{4c}{a^2d}-\frac{2i}{d\,a^2\left(\mathrm{e}^{2i(dx+c)}-1\right)}-\frac{2i\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{a^2d}$
norman	$\frac{-\frac{i\left(\tan^3(dx+c)\right)}{da}-\frac{1}{da}-\frac{9\left(\tan^4(dx+c)\right)}{4da}-\frac{9x\tan(dx+c)}{4a}-\frac{9x\left(\tan^3(dx+c)\right)}{2a}-\frac{9x\left(\tan^5(dx+c)\right)}{4a}-\frac{15\left(\tan^2(dx+c)\right)}{4da}-\frac{3i\tan(dx+c)}{2da}-}{\tan(dx+c)a(1+\tan^2(dx+c))^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d/a^2*(1/4*I/(tan(d*x+c)-I)^2+17/8*I*ln(tan(d*x+c)-I)-5/4/(tan(d*x+c)-I)-1/tan(d*x+c)-2*I*ln(tan(d*x+c))-1/8*I*ln(tan(d*x+c)+I))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

Fricas [A]
time = 0.38, size = 114, normalized size = 1.18

$$-\frac{68\,dxe^{(6i\,dx+6i\,c)}-4\,(17\,dx-11i)e^{(4i\,dx+4i\,c)}+32\,\left(i\,e^{(6i\,dx+6i\,c)}-i\,e^{(4i\,dx+4i\,c)}\right)\log\left(e^{(2i\,dx+2i\,c)}-1\right)-11i\,e^{(2i\,dx+2i\,c)}-i}{16\,\left(a^2de^{(6i\,dx+6i\,c)}-a^2de^{(4i\,dx+4i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/16*(68*d*x*e^(6*I*d*x + 6*I*c) - 4*(17*d*x - 11*I)*e^(4*I*d*x + 4*I*c) +
 32*(I*e^(6*I*d*x + 6*I*c) - I*e^(4*I*d*x + 4*I*c))*log(e^(2*I*d*x + 2*I*c)
 - 1) - 11*I*e^(2*I*d*x + 2*I*c) - I)/(a^2*d*e^(6*I*d*x + 6*I*c) - a^2*d*e^
(4*I*d*x + 4*I*c))
```

Sympy [A]
time = 0.28, size = 180, normalized size = 1.86

$$\begin{cases} \frac{\left(-48ia^2de^{4ic}e^{-2idx}-4ia^2de^{2ic}e^{-4idx}\right)e^{-6ic}}{64a^4d^2} & \text{for } a^4d^2e^{6ic} \neq 0 \\ x\left(\frac{\left(-17e^{4ic}-6e^{2ic}-1\right)e^{-4ic}}{4a^2}+\frac{17}{4a^2}\right) & \text{otherwise} \end{cases} - \frac{2i}{a^2de^{2ic}e^{2idx}-a^2d} - \frac{17x}{4a^2} - \frac{2i\log\left(e^{2idx}-e^{-2ic}\right)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise(((-48*I*a**2*d*exp(4*I*c)*exp(-2*I*d*x) - 4*I*a**2*d*exp(2*I*c)*e
xp(-4*I*d*x))*exp(-6*I*c)/(64*a**4*d**2), Ne(a**4*d**2*exp(6*I*c), 0)), (x*
((-17*exp(4*I*c) - 6*exp(2*I*c) - 1)*exp(-4*I*c)/(4*a**2) + 17/(4*a**2)), T
rue)) - 2*I/(a**2*d*exp(2*I*c)*exp(2*I*d*x) - a**2*d) - 17*x/(4*a**2) - 2*I
*log(exp(2*I*d*x) - exp(-2*I*c))/(a**2*d)
```

Giac [A]
time = 0.88, size = 109, normalized size = 1.12

$$-\frac{\frac{32i\log\left(i\tan\left(dx+c\right)\right)}{a^2}-\frac{34i\log\left(i\tan\left(dx+c\right)+1\right)}{a^2}+\frac{2i\log\left(-i\tan\left(dx+c\right)+1\right)}{a^2}+\frac{16\left(-2i\tan\left(dx+c\right)+1\right)}{a^2\tan\left(dx+c\right)}+\frac{51i\tan\left(dx+c\right)^2+122\tan\left(dx+c\right)-75i}{a^2\left(\tan\left(dx+c\right)-i\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/16*(32*I*log(I*tan(d*x + c))/a^2 - 34*I*log(I*tan(d*x + c) + 1)/a^2 + 2*
I*log(-I*tan(d*x + c) + 1)/a^2 + 16*(-2*I*tan(d*x + c) + 1)/(a^2*tan(d*x +
c)) + (51*I*tan(d*x + c)^2 + 122*tan(d*x + c) - 75*I)/(a^2*(tan(d*x + c) -
I)^2))/d
```

Mupad [B]
time = 3.96, size = 125, normalized size = 1.29

$$-\frac{\frac{7\tan\left(c+d\,x\right)}{2\,a^2}-\frac{1\mathrm{i}}{a^2}+\frac{{\tan\left(c+d\,x\right)}^2\,9\mathrm{i}}{4\,a^2}}{d\,\left({\tan\left(c+d\,x\right)}^3\,1\mathrm{i}+2\,{\tan\left(c+d\,x\right)}^2-\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}+\frac{\ln\left(\tan\left(c+d\,x\right)-\mathrm{i}\right)\,17\mathrm{i}}{8\,a^2\,d}-\frac{\ln\left(\tan\left(c+d\,x\right)+1\mathrm{i}\right)\,1\mathrm{i}}{8\,a^2\,d}-\frac{\ln\left(\tan\left(c+d\,x\right)\right)\,2\mathrm{i}}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] (log(tan(c + d*x) - 1i)*17i)/(8*a^2*d) - ((7*tan(c + d*x))/(2*a^2) - 1i/a^2
 + (tan(c + d*x)^2*9i)/(4*a^2))/(d*(2*tan(c + d*x)^2 - tan(c + d*x)*1i + ta
n(c + d*x)^3*1i)) - (log(tan(c + d*x) + 1i)*1i)/(8*a^2*d) - (log(tan(c + d*
x))*2i)/(a^2*d)
```

3.65 $\int \frac{\cot^3(c+dx)}{(a+ia\tan(c+dx))^2} dx$

Optimal. Leaf size=122

$$\frac{15ix}{4a^2}+\frac{15i\cot(c+dx)}{4a^2d}-\frac{2\cot^2(c+dx)}{a^2d}-\frac{4\log(\sin(c+dx))}{a^2d}+\frac{5\cot^2(c+dx)}{4a^2d(1+i\tan(c+dx))}+\frac{\cot^2(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

```
[Out] 15/4*I*x/a^2+15/4*I*cot(d*x+c)/a^2/d-2*cot(d*x+c)^2/a^2/d-4*ln(sin(d*x+c))/
a^2/d+5/4*cot(d*x+c)^2/a^2/d/(1+I*tan(d*x+c))+1/4*cot(d*x+c)^2/d/(a+I*a*tan
(d*x+c))^2
```

Rubi [A]
time = 0.16, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3640, 3677, 3610, 3612, 3556}

$$-\frac{2\cot^2(c+dx)}{a^2d}+\frac{15i\cot(c+dx)}{4a^2d}-\frac{4\log(\sin(c+dx))}{a^2d}+\frac{5\cot^2(c+dx)}{4a^2d(1+i\tan(c+dx))}+\frac{15ix}{4a^2}+\frac{\cot^2(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (((15*I)/4)*x)/a^2 + (((15*I)/4)*Cot[c + d*x])/(a^2*d) - (2*Cot[c + d*x]^2)
/(a^2*d) - (4*Log[Sin[c + d*x]])/(a^2*d) + (5*Cot[c + d*x]^2)/(4*a^2*d*(1 +
 I*Tan[c + d*x])) + Cot[c + d*x]^2/(4*d*(a + I*a*Tan[c + d*x])^2)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx &= \frac{\cot^2(c+dx)}{4d(a+ia\tan(c+dx))^2} + \frac{\int \frac{\cot^3(c+dx)(6a-4ia\tan(c+dx))}{a+ia\tan(c+dx)}\,dx}{4a^2} \\
&= \frac{5\cot^2(c+dx)}{4a^2d(1+i\tan(c+dx))} + \frac{\cot^2(c+dx)}{4d(a+ia\tan(c+dx))^2} + \frac{\int \cot^3(c+dx)\,(32a^2 -}{8a} \\
&= -\frac{2\cot^2(c+dx)}{a^2d} + \frac{5\cot^2(c+dx)}{4a^2d(1+i\tan(c+dx))} + \frac{\cot^2(c+dx)}{4d(a+ia\tan(c+dx))^2} + \frac{\int \cot}{} \\
&= \frac{15i\cot(c+dx)}{4a^2d} - \frac{2\cot^2(c+dx)}{a^2d} + \frac{5\cot^2(c+dx)}{4a^2d(1+i\tan(c+dx))} + \frac{\cot^2(c+}{4d(a+ia\tan(} \\
&= \frac{15ix}{4a^2} + \frac{15i\cot(c+dx)}{4a^2d} - \frac{2\cot^2(c+dx)}{a^2d} + \frac{5\cot^2(c+dx)}{4a^2d(1+i\tan(c+dx))} + \frac{}{4d(a+} \\
&= \frac{15ix}{4a^2} + \frac{15i\cot(c+dx)}{4a^2d} - \frac{2\cot^2(c+dx)}{a^2d} - \frac{4\log(\sin(c+dx))}{a^2d} + \frac{5\cot^2(}{4a^2d(1+i\,t}
\end{aligned}
$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 319 vs. 2(122) = 244.
time = 1.62, size = 319, normalized size = 2.61

$$
\frac{\sec^2(c+dx)(\cos(dx)+i\sin(dx))^2\left(-64idx+128idx\cos^2(c)-60idx\cos(2c)+16\cos(2dx)+\cos(2c)\cos(4dx)-64dx\cot(c)+64dx\cos(2c)\cot(c)-16\cos(2c-dx)\csc(c)\csc(c+dx)+16\cos(2c+dx)\csc(c)\csc(c+dx)+8\cos(2c)\csc^2(c+dx)+32\cos(2c)\log\left(\sin^2(c+dx)\right)+60dx\sin(2c)-i\cos(4dx)\sin(2c)+8i\csc^2(c+dx)\sin(2c)+32i\log\left(\sin^2(c+dx)\right)\sin(2c)+64\text{ArcTan}(\tan(dx))(-i\cos(2c)+\sin(2c))-16i\sin(2dx)-i\cos(2c)\sin(4dx)-\sin(2c)\sin(4dx)-16i\csc(c)\csc(c+dx)\sin(2c-dx)+16i\csc(c)\csc(c+dx)\sin(2c+dx)\right)}{16a^2d(-i+\tan(c+dx))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^2*(Cos[d*x] + I*Sin[d*x])^2*((-64*I)*d*x + (128*I)*d*x*Cos[c]
^2 - (60*I)*d*x*Cos[2*c] + 16*Cos[2*d*x] + Cos[2*c]*Cos[4*d*x] - 64*d*x*Cot
[c] + 64*d*x*Cos[2*c]*Cot[c] - 16*Cos[2*c - d*x]*Csc[c]*Csc[c + d*x] + 16*C
os[2*c + d*x]*Csc[c]*Csc[c + d*x] + 8*Cos[2*c]*Csc[c + d*x]^2 + 32*Cos[2*c]
*Log[Sin[c + d*x]^2] + 60*d*x*Sin[2*c] - I*Cos[4*d*x]*Sin[2*c] + (8*I)*Csc[
c + d*x]^2*Sin[2*c] + (32*I)*Log[Sin[c + d*x]^2]*Sin[2*c] + 64*ArcTan[Tan[d
*x]]*((-I)*Cos[2*c] + Sin[2*c]) - (16*I)*Sin[2*d*x] - I*Cos[2*c]*Sin[4*d*x]
 - Sin[2*c]*Sin[4*d*x] - (16*I)*Csc[c]*Csc[c + d*x]*Sin[2*c - d*x] + (16*I)
*Csc[c]*Csc[c + d*x]*Sin[2*c + d*x]))/(16*a^2*d*(-I + Tan[c + d*x])^2)
```

Maple [A]

time = 0.30, size = 90, normalized size = 0.74

method	result
derivativedivides	$\frac{\frac{7i}{4(\tan(dx+c)-i)}+\frac{1}{4(\tan(dx+c)-i)^2}+\frac{31\ln(\tan(dx+c)-i)}{8}-\frac{1}{2\tan(dx+c)^2}+\frac{2i}{\tan(dx+c)}-4\ln(\tan(dx+c))+\frac{\ln(\tan(dx+c)+i)}{8}}{d\,a^2}$
default	$\frac{\frac{7i}{4(\tan(dx+c)-i)}+\frac{1}{4(\tan(dx+c)-i)^2}+\frac{31\ln(\tan(dx+c)-i)}{8}-\frac{1}{2\tan(dx+c)^2}+\frac{2i}{\tan(dx+c)}-4\ln(\tan(dx+c))+\frac{\ln(\tan(dx+c)+i)}{8}}{d\,a^2}$
risch	$\frac{31ix}{4a^2}-\frac{e^{-2i(dx+c)}}{a^2d}-\frac{e^{-4i(dx+c)}}{16a^2d}+\frac{8ic}{a^2d}-\frac{2\left(e^{2i(dx+c)}-2\right)}{d\,a^2\left(e^{2i(dx+c)}-1\right)^2}-\frac{4\ln\left(e^{2i(dx+c)}-1\right)}{a^2d}$
norman	$\frac{-\frac{3\left(\tan^2(dx+c)\right)}{da}-\frac{1}{2da}-\frac{2\left(\tan^4(dx+c)\right)}{da}+\frac{15ix\left(\tan^2(dx+c)\right)}{4a}+\frac{15ix\left(\tan^4(dx+c)\right)}{2a}+\frac{15ix\left(\tan^6(dx+c)\right)}{4a}+\frac{2i\tan(dx+c)}{da}+\frac{25i\left(\tan^3(\right.}{4da}}{\tan(dx+c)^2a(1+\tan^2(dx+c))^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(7/4*I/(tan(d*x+c)-I)+1/4/(tan(d*x+c)-I)^2+31/8*ln(tan(d*x+c)-I)-1/
2/tan(d*x+c)^2+2*I/tan(d*x+c)-4*ln(tan(d*x+c))+1/8*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]

time = 0.38, size = 151, normalized size = 1.24

$$\frac{124i\,dxe^{(8i\,dx+8i\,c)}-8\,(31i\,dx+6)e^{(6i\,dx+6i\,c)}+(124i\,dx+95)e^{(4i\,dx+4i\,c)}-64\left(e^{(8i\,dx+8i\,c)}-2\,e^{(6i\,dx+6i\,c)}+e^{(4i\,dx+4i\,c)}\right)\log\left(e^{(2i\,dx+2i\,c)}-1\right)-14\,e^{(2i\,dx+2i\,c)}-1}{16\left(a^2de^{(8i\,dx+8i\,c)}-2\,a^2de^{(6i\,dx+6i\,c)}+a^2de^{(4i\,dx+4i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(124*I*d*x*e^(8*I*d*x + 8*I*c) - 8*(31*I*d*x + 6)*e^(6*I*d*x + 6*I*c)
+ (124*I*d*x + 95)*e^(4*I*d*x + 4*I*c) - 64*(e^(8*I*d*x + 8*I*c) - 2*e^(6*I
*d*x + 6*I*c) + e^(4*I*d*x + 4*I*c))*log(e^(2*I*d*x + 2*I*c) - 1) - 14*e^(2
*I*d*x + 2*I*c) - 1)/(a^2*d*e^(8*I*d*x + 8*I*c) - 2*a^2*d*e^(6*I*d*x + 6*I*
c) + a^2*d*e^(4*I*d*x + 4*I*c))
```

Sympy [A]
time = 0.46, size = 216, normalized size = 1.77

$$\frac{-2e^{2ic}e^{2idx}+4}{a^2de^{4ic}e^{4idx}-2a^2de^{2ic}e^{2idx}+a^2d}+\begin{cases}\frac{(-16a^2de^{4ic}e^{-2idx}-a^2de^{2ic}e^{-4idx})e^{-6ic}}{16a^4d^2} & \text{for } a^4d^2e^{6ic}\neq 0\\ x\left(\frac{(31ie^{4ic}+8ie^{2ic}+i)e^{-4ic}}{4a^2}-\frac{31i}{4a^2}\right) & \text{otherwise}\end{cases}+\frac{31ix}{4a^2}-\frac{4\log\left(e^{2idx}-e^{-2ic}\right)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] (-2*exp(2*I*c)*exp(2*I*d*x) + 4)/(a**2*d*exp(4*I*c)*exp(4*I*d*x) - 2*a**2*d
*exp(2*I*c)*exp(2*I*d*x) + a**2*d) + Piecewise(((-16*a**2*d*exp(4*I*c)*exp(
-2*I*d*x) - a**2*d*exp(2*I*c)*exp(-4*I*d*x))*exp(-6*I*c)/(16*a**4*d**2), Ne
(a**4*d**2*exp(6*I*c), 0)), (x*((31*I*exp(4*I*c) + 8*I*exp(2*I*c) + I)*exp(
-4*I*c)/(4*a**2) - 31*I/(4*a**2)), True)) + 31*I*x/(4*a**2) - 4*log(exp(2*I
*d*x) - exp(-2*I*c))/(a**2*d)
```

Giac [A]
time = 1.61, size = 109, normalized size = 0.89

$$\frac{\frac{4\log(\tan(dx+c)+i)}{a^2}+\frac{124\log(\tan(dx+c)-i)}{a^2}-\frac{128\log(\tan(dx+c))}{a^2}+\frac{3\tan(dx+c)^4+114i\tan(dx+c)^3+173\tan(dx+c)^2-32i\tan(dx+c)+16}{\left(\tan(dx+c)^2-i\tan(dx+c)\right)^2a^2}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/32*(4*log(tan(d*x + c) + I)/a^2 + 124*log(tan(d*x + c) - I)/a^2 - 128*log
(tan(d*x + c))/a^2 + (3*tan(d*x + c)^4 + 114*I*tan(d*x + c)^3 + 173*tan(d*x
 + c)^2 - 32*I*tan(d*x + c) + 16)/((tan(d*x + c)^2 - I*tan(d*x + c))^2*a^2)
)/d
```

Mupad [B]
time = 4.01, size = 135, normalized size = 1.11

$$\frac{31\,\ln\left(\tan(c+d\,x)-\mathrm{i}\right)}{8\,a^2\,d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)}{8\,a^2\,d}+\frac{\frac{\tan(c+d\,x)}{a^2}-\frac{15\tan(c+d\,x)^3}{4\,a^2}+\frac{1\mathrm{i}}{2\,a^2}+\frac{\tan(c+d\,x)^2\,11\mathrm{i}}{2\,a^2}}{d\,\left(\tan\left(c+d\,x\right)^4\,1\mathrm{i}+2\tan\left(c+d\,x\right)^3-\tan\left(c+d\,x\right)^2\,1\mathrm{i}\right)}-\frac{4\,\ln\left(\tan(c+d\,x)\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] (31*log(tan(c + d*x) - 1i))/(8*a^2*d) + log(tan(c + d*x) + 1i)/(8*a^2*d) +
(tan(c + d*x)/a^2 + 1i/(2*a^2) + (tan(c + d*x)^2*11i)/(2*a^2) - (15*tan(c +
 d*x)^3)/(4*a^2))/(d*(2*tan(c + d*x)^3 - tan(c + d*x)^2*1i + tan(c + d*x)^4
*1i)) - (4*log(tan(c + d*x)))/(a^2*d)
```

3.66 $\int \frac{\tan^6(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=161

$$\frac{55x}{8a^3}+\frac{7i\log(\cos(c+dx))}{a^3d}-\frac{55\tan(c+dx)}{8a^3d}+\frac{7i\tan^2(c+dx)}{2a^3d}-\frac{\tan^5(c+dx)}{6d(a+ia\tan(c+dx))^3}+\frac{13i\tan^4(c+dx}{24ad(a+ia\tan(c+$$

```
[Out] 55/8*x/a^3+7*I*ln(cos(d*x+c))/a^3/d-55/8*tan(d*x+c)/a^3/d+7/2*I*tan(d*x+c)^
2/a^3/d-1/6*tan(d*x+c)^5/d/(a+I*a*tan(d*x+c))^3+13/24*I*tan(d*x+c)^4/a/d/(a
+I*a*tan(d*x+c))^2+55/24*tan(d*x+c)^3/d/(a^3+I*a^3*tan(d*x+c))
```

Rubi [A]
time = 0.21, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3639, 3676, 3609, 3606, 3556}

$$\frac{55\tan^3(c+dx)}{24d\left(a^3+ia^3\tan(c+dx)\right)}+\frac{7i\tan^2(c+dx)}{2a^3d}-\frac{55\tan(c+dx)}{8a^3d}+\frac{7i\log(\cos(c+dx))}{a^3d}+\frac{55x}{8a^3}-\frac{\tan^5(c+dx)}{6d(a+ia\tan(c+dx))^3}+\frac{13i\tan^4(c+dx)}{24ad(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (55*x)/(8*a^3) + ((7*I)*Log[Cos[c + d*x]])/(a^3*d) - (55*Tan[c + d*x])/(8*a
^3*d) + (((7*I)/2)*Tan[c + d*x]^2)/(a^3*d) - Tan[c + d*x]^5/(6*d*(a + I*a*T
an[c + d*x])^3) + (((13*I)/24)*Tan[c + d*x]^4)/(a*d*(a + I*a*Tan[c + d*x])^
2) + (55*Tan[c + d*x]^3)/(24*d*(a^3 + I*a^3*Tan[c + d*x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^6(c+dx)}{(a+ia\tan(c+dx))^3}\,dx &= -\frac{\tan^5(c+dx)}{6d(a+ia\tan(c+dx))^3} - \frac{\int \frac{\tan^4(c+dx)(-5a+8ia\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx}{6a^2} \\
&= -\frac{\tan^5(c+dx)}{6d(a+ia\tan(c+dx))^3} + \frac{13i\tan^4(c+dx)}{24ad(a+ia\tan(c+dx))^2} + \frac{\int \frac{\tan^3(c+dx)(-52ia^2-58}{a+ia\tan(c+}}{24a^4} \\
&= -\frac{\tan^5(c+dx)}{6d(a+ia\tan(c+dx))^3} + \frac{13i\tan^4(c+dx)}{24ad(a+ia\tan(c+dx))^2} + \frac{55\tan^3(c+dx}{24d\left(a^3+ia^3\tan(c-\right.} \\
&= \frac{7i\tan^2(c+dx)}{2a^3d} - \frac{\tan^5(c+dx)}{6d(a+ia\tan(c+dx))^3} + \frac{13i\tan^4(c+dx)}{24ad(a+ia\tan(c+dx))^2} + \frac{}{24a} \\
&= \frac{55x}{8a^3} - \frac{55\tan(c+dx)}{8a^3d} + \frac{7i\tan^2(c+dx)}{2a^3d} - \frac{\tan^5(c+dx)}{6d(a+ia\tan(c+dx))^3} + \frac{13}{24ad(a} \\
&= \frac{55x}{8a^3} + \frac{7i\log(\cos(c+dx))}{a^3d} - \frac{55\tan(c+dx)}{8a^3d} + \frac{7i\tan^2(c+dx)}{2a^3d} - \frac{\tan^5(c}{6d(a+ia\tan}
\end{aligned}$$

Mathematica [A]
time = 4.26, size = 264, normalized size = 1.64

$$\frac{\sec^3(c+dx)(\cos(dx)+i\sin(dx))^3\left(234\cos(2dx)\sin(c)+27\cos(4dx)\sin(c)+660idx\sin(3c)-2\cos(6dx)\sin(3c)-672\log(\cos(c+dx))\sin(3c)-48\sec^2(c+dx)\sin(3c)-288i\sec(c)\sec(c+dx)\sin(3c)\sin(dx)-234i\sin(c)\sin(2dx)+9\cos(c)(-23i\cos(dx)+29\sin(dx))(\cos(3dx)-i\sin(3dx))-27i\sin(c)\sin(4dx)+\cos(3c)\left(660dx-2i\cos(6dx)+672i\log(\cos(c+dx))+48i\sec^2(c+dx)-288\sec(c)\sec(c+dx)\sin(dx)-2\sin(6dx)\right)+2i\sin(3c)\sin(6dx)\right)}{96d(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^6/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(Cos[d*x] + I*Sin[d*x])^3*(234*Cos[2*d*x]*Sin[c] + 27*Cos[4
*d*x]*Sin[c] + (660*I)*d*x*Sin[3*c] - 2*Cos[6*d*x]*Sin[3*c] - 672*Log[Cos[c
 + d*x]]*Sin[3*c] - 48*Sec[c + d*x]^2*Sin[3*c] - (288*I)*Sec[c]*Sec[c + d*x
]*Sin[3*c]*Sin[d*x] - (234*I)*Sin[c]*Sin[2*d*x] + 9*Cos[c]*((-23*I)*Cos[d*x
] + 29*Sin[d*x])*(Cos[3*d*x] - I*Sin[3*d*x]) - (27*I)*Sin[c]*Sin[4*d*x] + C
os[3*c]*(660*d*x - (2*I)*Cos[6*d*x] + (672*I)*Log[Cos[c + d*x]] + (48*I)*Se
c[c + d*x]^2 - 288*Sec[c]*Sec[c + d*x]*Sin[d*x] - 2*Sin[6*d*x]) + (2*I)*Sin
[3*c]*Sin[6*d*x]))/(96*d*(a + I*a*Tan[c + d*x])^3)
```

Maple [A]
time = 0.19, size = 94, normalized size = 0.58

method	result
derivativedivides	$\frac{-3\tan(dx+c)+\frac{i\left(\tan^2(dx+c)\right)}{2}-\frac{111i\ln(\tan(dx+c)-i)}{16}-\frac{11i}{8(\tan(dx+c)-i)^2}+\frac{1}{6(\tan(dx+c)-i)^3}-\frac{49}{8(\tan(dx+c)-i)}-\frac{i\ln(\tan(dx+c)}{16}}{d\,a^3}$
default	$\frac{-3\tan(dx+c)+\frac{i\left(\tan^2(dx+c)\right)}{2}-\frac{111i\ln(\tan(dx+c)-i)}{16}-\frac{11i}{8(\tan(dx+c)-i)^2}+\frac{1}{6(\tan(dx+c)-i)^3}-\frac{49}{8(\tan(dx+c)-i)}-\frac{i\ln(\tan(dx+c)}{16}}{d\,a^3}$
risch	$\frac{111x}{8a^3}-\frac{39i\mathrm{e}^{-2i(dx+c)}}{16a^3d}+\frac{9i\mathrm{e}^{-4i(dx+c)}}{32a^3d}-\frac{i\mathrm{e}^{-6i(dx+c)}}{48a^3d}+\frac{14c}{a^3d}-\frac{2i\left(2\,\mathrm{e}^{2i(dx+c)}+3\right)}{d\,a^3\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}+\frac{7i\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{a^3d}$
norman	$\frac{\frac{55x}{8a}-\frac{121\left(\tan^5(dx+c)\right)}{8da}-\frac{3\left(\tan^7(dx+c)\right)}{da}+\frac{165x\left(\tan^2(dx+c)\right)}{8a}+\frac{165x\left(\tan^4(dx+c)\right)}{8a}+\frac{55x\left(\tan^6(dx+c)\right)}{8a}-\frac{77i}{12da}-\frac{55\tan(dx+c)}{8da}-}{a^2(1+\tan^2(dx+c))^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-3*tan(d*x+c)+1/2*I*tan(d*x+c)^2-111/16*I*ln(tan(d*x+c)-I)-11/8*I/
(tan(d*x+c)-I)^2+1/6/(tan(d*x+c)-I)^3-49/8/(tan(d*x+c)-I)-1/16*I*ln(tan(d*x
+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.38, size = 167, normalized size = 1.04

$$\frac{1332\,dxe^{(10i\,dx+10i\,c)}+6\,(444\,dx-103i)e^{(8i\,dx+8i\,c)}+9\,(148\,dx-113i)e^{(6i\,dx+6i\,c)}-672\left(-i\,e^{(10i\,dx+10i\,c)}-2i\,e^{(8i\,dx+8i\,c)}-i\,e^{(6i\,dx+6i\,c)}\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-182i\,e^{(4i\,dx+4i\,c)}+23i\,e^{(2i\,dx+2i\,c)}-2i}{96\left(a^3de^{(10i\,dx+10i\,c)}+2\,a^3de^{(8i\,dx+8i\,c)}+a^3de^{(6i\,dx+6i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/96*(1332*d*x*e^(10*I*d*x + 10*I*c) + 6*(444*d*x - 103*I)*e^(8*I*d*x + 8*I
*c) + 9*(148*d*x - 113*I)*e^(6*I*d*x + 6*I*c) - 672*(-I*e^(10*I*d*x + 10*I*
c) - 2*I*e^(8*I*d*x + 8*I*c) - I*e^(6*I*d*x + 6*I*c))*log(e^(2*I*d*x + 2*I*
c) + 1) - 182*I*e^(4*I*d*x + 4*I*c) + 23*I*e^(2*I*d*x + 2*I*c) - 2*I)/(a^3*
d*e^(10*I*d*x + 10*I*c) + 2*a^3*d*e^(8*I*d*x + 8*I*c) + a^3*d*e^(6*I*d*x +
6*I*c))
```

Sympy [A]
time = 0.42, size = 257, normalized size = 1.60

$$\frac{-4ie^{2ic}e^{2idx} - 6i}{a^3de^{4ic}e^{4idx} + 2a^3de^{2ic}e^{2idx} + a^3d} + \begin{cases} \frac{\left(-59904ia^6d^2e^{10ic}e^{-2idx} + 6912ia^6d^2e^{8ic}e^{-4idx} - 512ia^6d^2e^{6ic}e^{-6idx}\right)e^{-12ic}}{24576a^9d^3} & \text{for } a^9d^3e^{12ic} \neq 0 \\ x\left(\frac{\left(111e^{6ic} - 39e^{4ic} + 9e^{2ic} - 1\right)e^{-6ic}}{8a^3} - \frac{111}{8a^3}\right) & \text{otherwise} \end{cases} + \frac{111x}{8a^3} + \frac{7i\log\left(e^{2idx} + e^{-2ic}\right)}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] (-4*I*exp(2*I*c)*exp(2*I*d*x) - 6*I)/(a**3*d*exp(4*I*c)*exp(4*I*d*x) + 2*a*
*3*d*exp(2*I*c)*exp(2*I*d*x) + a**3*d) + Piecewise(((-59904*I*a**6*d**2*exp
(10*I*c)*exp(-2*I*d*x) + 6912*I*a**6*d**2*exp(8*I*c)*exp(-4*I*d*x) - 512*I*
a**6*d**2*exp(6*I*c)*exp(-6*I*d*x))*exp(-12*I*c)/(24576*a**9*d**3), Ne(a**9
*d**3*exp(12*I*c), 0)), (x*((111*exp(6*I*c) - 39*exp(4*I*c) + 9*exp(2*I*c)
- 1)*exp(-6*I*c)/(8*a**3) - 111/(8*a**3)), True)) + 111*x/(8*a**3) + 7*I*lo
g(exp(2*I*d*x) + exp(-2*I*c))/(a**3*d)
```

Giac [A]
time = 2.66, size = 111, normalized size = 0.69

$$-\frac{\frac{666i\log(\tan(dx+c)-i)}{a^3} + \frac{6i\log(i\tan(dx+c)-1)}{a^3} + \frac{48\left(-ia^3\tan(dx+c)^2 + 6a^3\tan(dx+c)\right)}{a^6} - \frac{1221i\tan(dx+c)^3 + 3075\tan(dx+c)^2 - 2619i\tan(dx+c) - 749}{a^3(\tan(dx+c)-i)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/96*(666*I*log(tan(d*x + c) - I)/a^3 + 6*I*log(I*tan(d*x + c) - 1)/a^3 +
48*(-I*a^3*tan(d*x + c)^2 + 6*a^3*tan(d*x + c))/a^6 - (1221*I*tan(d*x + c)^
3 + 3075*tan(d*x + c)^2 - 2619*I*tan(d*x + c) - 749)/(a^3*(tan(d*x + c) - I
)^3))/d
```

Mupad [B]
time = 3.95, size = 140, normalized size = 0.87

$$\frac{\frac{87\tan(c+d\,x)}{8\,a^3} - \frac{59\mathrm{i}}{12\,a^3} + \frac{\tan(c+d\,x)^2\,49\mathrm{i}}{8\,a^3}}{d\,\left(-\tan\left(c+d\,x\right)^3\,1\mathrm{i} - 3\tan\left(c+d\,x\right)^2 + \tan\left(c+d\,x\right)\,3\mathrm{i} + 1\right)} - \frac{\ln\left(\tan(c+d\,x) - \mathrm{i}\right)\,111\mathrm{i}}{16\,a^3\,d} - \frac{\ln\left(\tan(c+d\,x) + 1\mathrm{i}\right)\,1\mathrm{i}}{16\,a^3\,d} - \frac{3\tan(c+d\,x)}{a^3\,d} + \frac{\tan(c+d\,x)^2\,1\mathrm{i}}{2\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6/(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] ((87*tan(c + d*x))/(8*a^3) - 59i/(12*a^3) + (tan(c + d*x)^2*49i)/(8*a^3))/(
d*(tan(c + d*x)*3i - 3*tan(c + d*x)^2 - tan(c + d*x)^3*1i + 1)) - (log(tan(
c + d*x) - 1i)*111i)/(16*a^3*d) - (log(tan(c + d*x) + 1i)*1i)/(16*a^3*d) -
(3*tan(c + d*x))/(a^3*d) + (tan(c + d*x)^2*1i)/(2*a^3*d)
```

3.67 $\int \frac{\tan^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=143

$$-\frac{25ix}{8a^3}+\frac{3\log(\cos(c+dx))}{a^3d}+\frac{25i\tan(c+dx)}{8a^3d}-\frac{\tan^4(c+dx)}{6d(a+ia\tan(c+dx))^3}+\frac{11i\tan^3(c+dx)}{24ad(a+ia\tan(c+dx))^2}+\frac{3\,\mathrm{t}}{2d\,(a^3+}$$

```
[Out] -25/8*I*x/a^3+3*ln(cos(d*x+c))/a^3/d+25/8*I*tan(d*x+c)/a^3/d-1/6*tan(d*x+c)
^4/d/(a+I*a*tan(d*x+c))^3+11/24*I*tan(d*x+c)^3/a/d/(a+I*a*tan(d*x+c))^2+3/2
*tan(d*x+c)^2/d/(a^3+I*a^3*tan(d*x+c))
```

Rubi [A]
time = 0.19, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3639, 3676, 3606, 3556}

$$\frac{3\tan^2(c+dx)}{2d\,(a^3+ia^3\tan(c+dx))}+\frac{25i\tan(c+dx)}{8a^3d}+\frac{3\log(\cos(c+dx))}{a^3d}-\frac{25ix}{8a^3}-\frac{\tan^4(c+dx)}{6d(a+ia\tan(c+dx))^3}+\frac{11i\tan^3(c+dx)}{24ad(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (((-25*I)/8)*x)/a^3 + (3*Log[Cos[c + d*x]])/(a^3*d) + (((25*I)/8)*Tan[c + d
*x])/(a^3*d) - Tan[c + d*x]^4/(6*d*(a + I*a*Tan[c + d*x])^3) + (((11*I)/24)
*Tan[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])^2) + (3*Tan[c + d*x]^2)/(2*d*(
a^3 + I*a^3*Tan[c + d*x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

```
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx &= -\frac{\tan^4(c+dx)}{6d(a+ia\tan(c+dx))^3} - \frac{\int \frac{\tan^3(c+dx)(-4a+7ia\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx}{6a^2} \\
&= -\frac{\tan^4(c+dx)}{6d(a+ia\tan(c+dx))^3} + \frac{11i\tan^3(c+dx)}{24ad(a+ia\tan(c+dx))^2} + \frac{\int \frac{\tan^2(c+dx)(-33ia^2-}{a+ia\tan(c}}{24a} \\
&= -\frac{\tan^4(c+dx)}{6d(a+ia\tan(c+dx))^3} + \frac{11i\tan^3(c+dx)}{24ad(a+ia\tan(c+dx))^2} + \frac{3\tan^2(c+dx}{2d\left(a^3+ia^3\tan(c\right.} \\
&= -\frac{25ix}{8a^3} + \frac{25i\tan(c+dx)}{8a^3d} - \frac{\tan^4(c+dx)}{6d(a+ia\tan(c+dx))^3} + \frac{11i\tan^3(c+dx)}{24ad(a+ia\tan(c+} \\
&= -\frac{25ix}{8a^3} + \frac{3\log(\cos(c+dx))}{a^3d} + \frac{25i\tan(c+dx)}{8a^3d} - \frac{\tan^4(c+dx)}{6d(a+ia\tan(c+dx))^3} +
\end{aligned}$$

Mathematica [A]
time = 2.70, size = 239, normalized size = 1.67

$$\frac{\sec^3(c+dx)(\cos(dx)+i\sin(dx))^3(-138i\cos(2dx)\sin(c)-21i\cos(4dx)\sin(c)+300dx\sin(3c)+2i\cos(6dx)\sin(3c)+288i\log(\cos(c+dx))\sin(3c)-96\sec(c)\sec(c+dx)\sin(3c)\sin(dx)-138\sin(c)\sin(2dx)+\cos(c)(39\cos(dx)+53i\sin(dx))(-3\cos(3dx)+3i\sin(3dx))-21\sin(c)\sin(4dx)+\cos(3c)(-300idx-2\cos(6dx)+288\log(\cos(c+dx))+96i\sec(c)\sec(c+dx)\sin(dx)+2i\sin(6dx))+2\sin(3c)\sin(6dx))}{96d(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(Cos[d*x] + I*Sin[d*x])^3*((-138*I)*Cos[2*d*x]*Sin[c] - (21
*I)*Cos[4*d*x]*Sin[c] + 300*d*x*Sin[3*c] + (2*I)*Cos[6*d*x]*Sin[3*c] + (288
*I)*Log[Cos[c + d*x]]*Sin[3*c] - 96*Sec[c]*Sec[c + d*x]*Sin[3*c]*Sin[d*x] -
 138*Sin[c]*Sin[2*d*x] + Cos[c]*(39*Cos[d*x] + (53*I)*Sin[d*x])*(-3*Cos[3*d
*x] + (3*I)*Sin[3*d*x]) - 21*Sin[c]*Sin[4*d*x] + Cos[3*c]*((-300*I)*d*x - 2
*Cos[6*d*x] + 288*Log[Cos[c + d*x]] + (96*I)*Sec[c]*Sec[c + d*x]*Sin[d*x] +
```

```
(2*I)*Sin[6*d*x]) + 2*Sin[3*c]*Sin[6*d*x]))/(96*d*(a + I*a*Tan[c + d*x])^3
)
```

Maple [A]
time = 0.16, size = 83, normalized size = 0.58

method	result
derivativedivides	$\frac{i\tan(dx+c)+\frac{31i}{8(\tan(dx+c)-i)}-\frac{i}{6(\tan(dx+c)-i)^3}-\frac{9}{8(\tan(dx+c)-i)^2}-\frac{49\ln(\tan(dx+c)-i)}{16}+\frac{\ln(\tan(dx+c)+i)}{16}}{d\,a^3}$
default	$\frac{i\tan(dx+c)+\frac{31i}{8(\tan(dx+c)-i)}-\frac{i}{6(\tan(dx+c)-i)^3}-\frac{9}{8(\tan(dx+c)-i)^2}-\frac{49\ln(\tan(dx+c)-i)}{16}+\frac{\ln(\tan(dx+c)+i)}{16}}{d\,a^3}$
risch	$-\frac{49ix}{8a^3}-\frac{23\,\mathrm{e}^{-2i(dx+c)}}{16a^3d}+\frac{7\,\mathrm{e}^{-4i(dx+c)}}{32a^3d}-\frac{\mathrm{e}^{-6i(dx+c)}}{48a^3d}-\frac{6ic}{a^3d}-\frac{2}{d\,a^3\left(\mathrm{e}^{2i(dx+c)}+1\right)}+\frac{3\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{a^3d}$
norman	$\frac{-\frac{5\left(\tan^4(dx+c)\right)}{da}+\frac{i\left(\tan^7(dx+c)\right)}{da}-\frac{35}{12da}-\frac{25ix}{8a}-\frac{29\left(\tan^2(dx+c)\right)}{4da}-\frac{75ix\left(\tan^2(dx+c)\right)}{8a}-\frac{75ix\left(\tan^4(dx+c)\right)}{8a}-\frac{25ix\left(\tan^6(dx+c)\right)}{8a}+}{a^2\left(1+\tan^2(dx+c)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(I*tan(d*x+c)+31/8*I/(tan(d*x+c)-I)-1/6*I/(tan(d*x+c)-I)^3-9/8/(tan
(d*x+c)-I)^2-49/16*ln(tan(d*x+c)-I)+1/16*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.37, size = 120, normalized size = 0.84

$$\frac{-588i\,dxe^{(8i\,dx+8i\,c)}-6\,(98i\,dx+55)e^{(6i\,dx+6i\,c)}+288\left(e^{(8i\,dx+8i\,c)}+e^{(6i\,dx+6i\,c)}\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-117\,e^{(4i\,dx+4i\,c)}+19\,e^{(2i\,dx+2i\,c)}-2}{96\left(a^3de^{(8i\,dx+8i\,c)}+a^3de^{(6i\,dx+6i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/96*(-588*I*d*x*e^(8*I*d*x + 8*I*c) - 6*(98*I*d*x + 55)*e^(6*I*d*x + 6*I*c
) + 288*(e^(8*I*d*x + 8*I*c) + e^(6*I*d*x + 6*I*c))*log(e^(2*I*d*x + 2*I*c)
 + 1) - 117*e^(4*I*d*x + 4*I*c) + 19*e^(2*I*d*x + 2*I*c) - 2)/(a^3*d*e^(8*I
*d*x + 8*I*c) + a^3*d*e^(6*I*d*x + 6*I*c))
```

Sympy [A]
time = 0.49, size = 214, normalized size = 1.50

$$\begin{cases} \frac{(-35328a^6d^2e^{10ic}e^{-2idx}+5376a^6d^2e^{8ic}e^{-4idx}-512a^6d^2e^{6ic}e^{-6idx})e^{-12ic}}{24576a^9d^3} & \text{for } a^9d^3e^{12ic} \neq 0 \\ x\left(\frac{(-49ie^{6ic}+23ie^{4ic}-7ie^{2ic}+i)e^{-6ic}}{8a^3}+\frac{49i}{8a^3}\right) & \text{otherwise} \end{cases} - \frac{2}{a^3de^{2ic}e^{2idx}+a^3d} - \frac{49ix}{8a^3} + \frac{3\log\left(e^{2idx}+e^{-2ic}\right)}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise(((-35328*a**6*d**2*exp(10*I*c)*exp(-2*I*d*x) + 5376*a**6*d**2*exp
(8*I*c)*exp(-4*I*d*x) - 512*a**6*d**2*exp(6*I*c)*exp(-6*I*d*x))*exp(-12*I*c
)/(24576*a**9*d**3), Ne(a**9*d**3*exp(12*I*c), 0)), (x*((-49*I*exp(6*I*c) +
 23*I*exp(4*I*c) - 7*I*exp(2*I*c) + I)*exp(-6*I*c)/(8*a**3) + 49*I/(8*a**3)
), True)) - 2/(a**3*d*exp(2*I*c)*exp(2*I*d*x) + a**3*d) - 49*I*x/(8*a**3) +
 3*log(exp(2*I*d*x) + exp(-2*I*c))/(a**3*d)
```

Giac [A]
time = 2.22, size = 91, normalized size = 0.64

$$\frac{\frac{6\log(\tan(dx+c)+i)}{a^3} - \frac{294\log(i\tan(dx+c)+1)}{a^3} + \frac{96i\tan(dx+c)}{a^3} + \frac{539\tan(dx+c)^3-1245i\tan(dx+c)^2-981\tan(dx+c)+259i}{a^3(\tan(dx+c)-i)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/96*(6*log(tan(d*x + c) + I)/a^3 - 294*log(I*tan(d*x + c) + 1)/a^3 + 96*I*
tan(d*x + c)/a^3 + (539*tan(d*x + c)^3 - 1245*I*tan(d*x + c)^2 - 981*tan(d*
x + c) + 259*I)/(a^3*(tan(d*x + c) - I)^3))/d
```

Mupad [B]
time = 3.81, size = 122, normalized size = 0.85

$$-\frac{\frac{35}{12\,a^3}-\frac{31\tan(c+d\,x)^2}{8\,a^3}+\frac{\tan(c+d\,x)\,53\mathrm{i}}{8\,a^3}}{d\,\left(-\tan\left(c+d\,x\right)^3\,1\mathrm{i}-3\tan\left(c+d\,x\right)^2+\tan\left(c+d\,x\right)\,3\mathrm{i}+1\right)} - \frac{49\ln\left(\tan(c+d\,x)-\mathrm{i}\right)}{16\,a^3\,d} + \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)}{16\,a^3\,d} + \frac{\tan(c+d\,x)\,1\mathrm{i}}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] log(tan(c + d*x) + 1i)/(16*a^3*d) - (49*log(tan(c + d*x) - 1i))/(16*a^3*d)
- ((tan(c + d*x)*53i)/(8*a^3) + 35/(12*a^3) - (31*tan(c + d*x)^2)/(8*a^3))/
(d*(tan(c + d*x)*3i - 3*tan(c + d*x)^2 - tan(c + d*x)^3*1i + 1)) + (tan(c +
 d*x)*1i)/(a^3*d)
```

3.68 $\int \frac{\tan^4(c+dx)}{(a+ia\tan(c+dx))^3} \, dx$

Optimal. Leaf size=119

$$-\frac{7x}{8a^3}-\frac{i\log(\cos(c+dx))}{a^3d}-\frac{\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3}+\frac{3i\tan^2(c+dx)}{8ad(a+ia\tan(c+dx))^2}+\frac{7i}{8d(a^3+ia^3\tan(c+dx))}$$

```
[Out] -7/8*x/a^3-I*ln(cos(d*x+c))/a^3/d-1/6*tan(d*x+c)^3/d/(a+I*a*tan(d*x+c))^3+3
/8*I*tan(d*x+c)^2/a/d/(a+I*a*tan(d*x+c))^2+7/8*I/d/(a^3+I*a^3*tan(d*x+c))
```

Rubi [A]
time = 0.16, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {3639, 3676, 3670, 3556, 12, 3607, 8}

$$\frac{7i}{8d(a^3+ia^3\tan(c+dx))}-\frac{i\log(\cos(c+dx))}{a^3d}-\frac{7x}{8a^3}-\frac{\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3}+\frac{3i\tan^2(c+dx)}{8ad(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (-7*x)/(8*a^3) - (I*Log[Cos[c + d*x]])/(a^3*d) - Tan[c + d*x]^3/(6*d*(a + I
*a*Tan[c + d*x])^3) + (((3*I)/8)*Tan[c + d*x]^2)/(a*d*(a + I*a*Tan[c + d*x]
)^2) + ((7*I)/8)/(d*(a^3 + I*a^3*Tan[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3670

```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)]))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[B*(d/
b), Int[Tan[e + f*x], x], x] + Dist[1/b, Int[Simp[A*b*c + (A*b*d + B*(b*c -
 a*d))*Tan[e + f*x], x]/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d,
e, f, A, B}, x] && NeQ[b*c - a*d, 0]
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3} - \frac{\int \frac{\tan^2(c+dx)(-3a+6ia\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx}{6a^2}$$

$$= -\frac{\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3} + \frac{3i\tan^2(c+dx)}{8ad(a+ia\tan(c+dx))^2} + \frac{\int \frac{\tan(c+dx)(-18ia^2-24a^2}{a+ia\tan(c+dx}}{24a^4}$$

$$= -\frac{\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3} + \frac{3i\tan^2(c+dx)}{8ad(a+ia\tan(c+dx))^2} - \frac{i\int \frac{42a^3\tan(c+dx)}{a+ia\tan(c+dx)}\,dx}{24a^5} +$$

$$= -\frac{i\log(\cos(c+dx))}{a^3d} - \frac{\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3} + \frac{3i\tan^2(c+dx)}{8ad(a+ia\tan(c+dx))^2} -$$

$$= -\frac{i\log(\cos(c+dx))}{a^3d} - \frac{\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3} + \frac{3i\tan^2(c+dx)}{8ad(a+ia\tan(c+dx))^2} +$$

$$= -\frac{7x}{8a^3} - \frac{i\log(\cos(c+dx))}{a^3d} - \frac{\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3} + \frac{3i\tan^2(c+dx)}{8ad(a+ia\tan(c+dx}$$

Mathematica [A]

time = 0.48, size = 118, normalized size = 0.99

$$\frac{\sec^3(c+dx)(-51\cos(c+dx)+\cos(3(c+dx))(-2-84idx+96\log(\cos(c+dx)))-81i\sin(c+dx)+2i\sin(3(c+dx))+84dx\sin(3(c+dx))+96i\log(\cos(c+dx))\sin(3(c+dx)))}{96a^3d(-i+\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(-51*Cos[c + d*x] + Cos[3*(c + d*x)]*(-2 - (84*I)*d*x + 96*
Log[Cos[c + d*x]]) - (81*I)*Sin[c + d*x] + (2*I)*Sin[3*(c + d*x)] + 84*d*x*
Sin[3*(c + d*x)] + (96*I)*Log[Cos[c + d*x]]*Sin[3*(c + d*x)]))/(96*a^3*d*(-
I + Tan[c + d*x])^3)
```

Maple [A]

time = 0.16, size = 75, normalized size = 0.63

method	result
derivativedivides	$\frac{\frac{7i}{8(\tan(dx+c)-i)^2}+\frac{15i\ln(\tan(dx+c)-i)}{16}-\frac{1}{6(\tan(dx+c)-i)^3}+\frac{17}{8(\tan(dx+c)-i)}+\frac{i\ln(\tan(dx+c)+i)}{16}}{d\,a^3}$
default	$\frac{\frac{7i}{8(\tan(dx+c)-i)^2}+\frac{15i\ln(\tan(dx+c)-i)}{16}-\frac{1}{6(\tan(dx+c)-i)^3}+\frac{17}{8(\tan(dx+c)-i)}+\frac{i\ln(\tan(dx+c)+i)}{16}}{d\,a^3}$
risch	$-\frac{15x}{8a^3}+\frac{11i e^{-2i(dx+c)}}{16a^3d}-\frac{5i e^{-4i(dx+c)}}{32a^3d}+\frac{i e^{-6i(dx+c)}}{48a^3d}-\frac{2c}{a^3d}-\frac{i\ln(e^{2i(dx+c)}+1)}{a^3d}$
norman	$\frac{-\frac{7x}{8a}+\frac{17\left(\tan^5(dx+c)\right)}{8da}-\frac{21x\left(\tan^2(dx+c)\right)}{8a}-\frac{21x\left(\tan^4(dx+c)\right)}{8a}-\frac{7x\left(\tan^6(dx+c)\right)}{8a}+\frac{17i}{12da}+\frac{7\tan(dx+c)}{8da}+\frac{7\left(\tan^3(dx+c)\right)}{3da}+\frac{3i(\tan^4}{}}{a^2(1+\tan^2(dx+c))^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(7/8*I/(tan(d*x+c)-I)^2+15/16*I*ln(tan(d*x+c)-I)-1/6/(tan(d*x+c)-I)
^3+17/8/(tan(d*x+c)-I)+1/16*I*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.36, size = 77, normalized size = 0.65

$$-\frac{\left(180\,dxe^{(6i\,dx+6i\,c)}+96i\,e^{(6i\,dx+6i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-66i\,e^{(4i\,dx+4i\,c)}+15i\,e^{(2i\,dx+2i\,c)}-2i\right)e^{(-6i\,dx-6i\,c)}}{96\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/96*(180*d*x*e^(6*I*d*x + 6*I*c) + 96*I*e^(6*I*d*x + 6*I*c)*log(e^(2*I*d*
x + 2*I*c) + 1) - 66*I*e^(4*I*d*x + 4*I*c) + 15*I*e^(2*I*d*x + 2*I*c) - 2*I
)*e^(-6*I*d*x - 6*I*c)/(a^3*d)
```

Sympy [A]
time = 0.33, size = 184, normalized size = 1.55

$$\begin{cases}\frac{\left(16896ia^6d^2e^{10ic}e^{-2idx}-3840ia^6d^2e^{8ic}e^{-4idx}+512ia^6d^2e^{6ic}e^{-6idx}\right)e^{-12ic}}{24576a^9d^3} & \text{for } a^9d^3e^{12ic}\neq 0\\ x\left(\frac{\left(-15e^{6ic}+11e^{4ic}-5e^{2ic}+1\right)e^{-6ic}}{8a^3}+\frac{15}{8a^3}\right) & \text{otherwise}\end{cases}-\frac{15x}{8a^3}-\frac{i\log\left(e^{2idx}+e^{-2ic}\right)}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise(((16896*I*a**6*d**2*exp(10*I*c)*exp(-2*I*d*x) - 3840*I*a**6*d**2*
exp(8*I*c)*exp(-4*I*d*x) + 512*I*a**6*d**2*exp(6*I*c)*exp(-6*I*d*x))*exp(-1
2*I*c)/(24576*a**9*d**3), Ne(a**9*d**3*exp(12*I*c), 0)), (x*((-15*exp(6*I*c
) + 11*exp(4*I*c) - 5*exp(2*I*c) + 1)*exp(-6*I*c)/(8*a**3) + 15/(8*a**3)),
True)) - 15*x/(8*a**3) - I*log(exp(2*I*d*x) + exp(-2*I*c))/(a**3*d)
```

Giac [A]
time = 1.39, size = 80, normalized size = 0.67

$$-\frac{-\frac{90i\log(\tan(dx+c)-i)}{a^3}-\frac{6i\log(-i\tan(dx+c)+1)}{a^3}+\frac{165i\tan(dx+c)^3+291\tan(dx+c)^2-171i\tan(dx+c)-29}{a^3(\tan(dx+c)-i)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/96*(-90*I*log(tan(d*x + c) - I)/a^3 - 6*I*log(-I*tan(d*x + c) + 1)/a^3 +
 (165*I*tan(d*x + c)^3 + 291*tan(d*x + c)^2 - 171*I*tan(d*x + c) - 29)/(a^3
*(tan(d*x + c) - I)^3))/d
```

Mupad [B]
time = 3.85, size = 110, normalized size = 0.92

$$-\frac{\frac{27\tan(c+d\,x)}{8\,a^3}-\frac{17\mathrm{i}}{12\,a^3}+\frac{\tan(c+d\,x)^2\,17\mathrm{i}}{8\,a^3}}{d\,\left(-\tan\left(c+d\,x\right)^3\,1\mathrm{i}-3\tan\left(c+d\,x\right)^2+\tan\left(c+d\,x\right)\,3\mathrm{i}+1\right)}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,15\mathrm{i}}{16\,a^3\,d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{16\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] (log(tan(c + d*x) - 1i)*15i)/(16*a^3*d) - ((27*tan(c + d*x))/(8*a^3) - 17i/
(12*a^3) + (tan(c + d*x)^2*17i)/(8*a^3))/(d*(tan(c + d*x)*3i - 3*tan(c + d*
x)^2 - tan(c + d*x)^3*1i + 1)) + (log(tan(c + d*x) + 1i)*1i)/(16*a^3*d)
```

3.69 $\int \frac{\tan^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=92

$$\frac{ix}{8a^3}+\frac{3}{8a^3d(1+i\tan(c+dx))}+\frac{i\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{1}{8ad(a+ia\tan(c+dx))^2}$$

```
[Out] 1/8*I*x/a^3+3/8/a^3/d/(1+I*tan(d*x+c))+1/6*I*tan(d*x+c)^3/d/(a+I*a*tan(d*x+
c))^3-1/8/a/d/(a+I*a*tan(d*x+c))^2
```

Rubi [A]
time = 0.09, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3627, 3621, 3607, 8}

$$\frac{3}{8a^3d(1+i\tan(c+dx))}+\frac{ix}{8a^3}+\frac{i\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{1}{8ad(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((I/8)*x)/a^3 + 3/(8*a^3*d*(1 + I*Tan[c + d*x])) + ((I/6)*Tan[c + d*x]^3)/(
d*(a + I*a*Tan[c + d*x])^3) - 1/(8*a*d*(a + I*a*Tan[c + d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rule 3621

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(a*c + b*d)^2*((a + b*Tan[e + f*x])^
m/(2*a^3*f*m)), x] + Dist[1/(2*a^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[
a*c^2 - 2*b*c*d + a*d^2 - 2*b*d^2*Tan[e + f*x], x], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LeQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Rule 3627

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^n/(2*b*f*m)), x] - Dist[(a*c - b*d)/(2*b^2), Int[(a + b*Tan[e + f*
x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && EqQ
[m + n, 0] && LeQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx &= \frac{i\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3} - \frac{i\int \frac{\tan^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{2a}\\ &= \frac{i\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3} - \frac{1}{8ad(a+ia\tan(c+dx))^2} - \frac{i\int \frac{a-2ia\tan(c+dx)}{a+ia\tan(c+dx)}\,dx}{4a^3}\\ &= \frac{3}{8a^3d(1+i\tan(c+dx))} + \frac{i\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3} - \frac{1}{8ad(a+ia\tan(c+dx))^2}\\ &= \frac{ix}{8a^3} + \frac{3}{8a^3d(1+i\tan(c+dx))} + \frac{i\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3} - \frac{1}{8ad(a+ia\tan(c+}\end{aligned}$$

Mathematica [A]
time = 0.28, size = 91, normalized size = 0.99

$$-\frac{\sec^3(c+dx)(-9i\cos(c+dx)+2(-i+6dx)\cos(3(c+dx))+27\sin(c+dx)-2\sin(3(c+dx))+12idx\sin(3(c+dx)))}{96a^3d(-i+\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] -1/96*(Sec[c + d*x]^3*((-9*I)*Cos[c + d*x] + 2*(-I + 6*d*x)*Cos[3*(c + d*x)
] + 27*Sin[c + d*x] - 2*Sin[3*(c + d*x)] + (12*I)*d*x*Sin[3*(c + d*x)]))/(a
^3*d*(-I + Tan[c + d*x])^3)
```

Maple [A]
time = 0.13, size = 74, normalized size = 0.80

method	result
risch	$\frac{ix}{8a^3}+\frac{3\,\mathrm{e}^{-2i(dx+c)}}{16a^3d}-\frac{3\,\mathrm{e}^{-4i(dx+c)}}{32a^3d}+\frac{\mathrm{e}^{-6i(dx+c)}}{48a^3d}$
derivativedivides	$\frac{\frac{i}{6(\tan(dx+c)-i)^3}-\frac{7i}{8(\tan(dx+c)-i)}+\frac{5}{8(\tan(dx+c)-i)^2}+\frac{\ln(\tan(dx+c)-i)}{16}-\frac{\ln(\tan(dx+c)+i)}{16}}{d\,a^3}$
default	$\frac{\frac{i}{6(\tan(dx+c)-i)^3}-\frac{7i}{8(\tan(dx+c)-i)}+\frac{5}{8(\tan(dx+c)-i)^2}+\frac{\ln(\tan(dx+c)-i)}{16}-\frac{\ln(\tan(dx+c)+i)}{16}}{d\,a^3}$

norman	$\frac{\frac{5}{12da}+\frac{ix}{8a}+\frac{5\left(\tan^2(dx+c)\right)}{4da}+\frac{3\left(\tan^4(dx+c)\right)}{2da}+\frac{3ix\left(\tan^2(dx+c)\right)}{8a}+\frac{3ix\left(\tan^4(dx+c)\right)}{8a}+\frac{ix\left(\tan^6(dx+c)\right)}{8a}-\frac{i\tan(dx+c)}{8da}-\frac{i\left(\tan^3(dx\right.}{3da}}{a^2(1+\tan^2(dx+c))^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(1/6*I/(tan(d*x+c)-I)^3-7/8*I/(tan(d*x+c)-I)+5/8/(tan(d*x+c)-I)^2+1
/16*ln(tan(d*x+c)-I)-1/16*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.36, size = 54, normalized size = 0.59

$$\frac{\left(12i\,dxe^{(6i\,dx+6i\,c)}+18\,e^{(4i\,dx+4i\,c)}-9\,e^{(2i\,dx+2i\,c)}+2\right)e^{(-6i\,dx-6i\,c)}}{96\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/96*(12*I*d*x*e^(6*I*d*x + 6*I*c) + 18*e^(4*I*d*x + 4*I*c) - 9*e^(2*I*d*x
+ 2*I*c) + 2)*e^(-6*I*d*x - 6*I*c)/(a^3*d)
```

Sympy [A]
time = 0.24, size = 156, normalized size = 1.70

$$\begin{cases}\frac{\left(4608a^6d^2e^{10ic}e^{-2idx}-2304a^6d^2e^{8ic}e^{-4idx}+512a^6d^2e^{6ic}e^{-6idx}\right)e^{-12ic}}{24576a^9d^3} & \text{for } a^9d^3e^{12ic}\neq 0\\ x\left(\frac{\left(ie^{6ic}-3ie^{4ic}+3ie^{2ic}-i\right)e^{-6ic}}{8a^3}-\frac{i}{8a^3}\right) & \text{otherwise}\end{cases}+\frac{ix}{8a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise(((4608*a**6*d**2*exp(10*I*c)*exp(-2*I*d*x) - 2304*a**6*d**2*exp(8
*I*c)*exp(-4*I*d*x) + 512*a**6*d**2*exp(6*I*c)*exp(-6*I*d*x))*exp(-12*I*c)/
(24576*a**9*d**3), Ne(a**9*d**3*exp(12*I*c), 0)), (x*((I*exp(6*I*c) - 3*I*e
```

```
xp(4*I*c) + 3*I*exp(2*I*c) - I)*exp(-6*I*c)/(8*a**3) - I/(8*a**3)), True))
+ I*x/(8*a**3)
```

Giac [A]
time = 1.07, size = 81, normalized size = 0.88

$$\frac{\frac{6 \log(\tan(dx+c)-i)}{a^3} - \frac{6 \log(i \tan(dx+c)-1)}{a^3} - \frac{11 \tan(dx+c)^3+51i \tan(dx+c)^2+75 \tan(dx+c)-29i}{a^3(\tan(dx+c)-i)^3}}{96\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/96*(6*log(tan(d*x + c) - I)/a^3 - 6*log(I*tan(d*x + c) - 1)/a^3 - (11*tan
(d*x + c)^3 + 51*I*tan(d*x + c)^2 + 75*tan(d*x + c) - 29*I)/(a^3*(tan(d*x +
 c) - I)^3))/d
```

Mupad [B]
time = 3.77, size = 49, normalized size = 0.53

$$\frac{x\,1\mathrm{i}}{8\,a^3}+\frac{-\frac{7\,\tan(c+d\,x)^2}{8}+\frac{\tan(c+d\,x)\,9\mathrm{i}}{8}+\frac{5}{12}}{a^3\,d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] (x*1i)/(8*a^3) + ((tan(c + d*x)*9i)/8 - (7*tan(c + d*x)^2)/8 + 5/12)/(a^3*d
*(tan(c + d*x)*1i + 1)^3)
```

3.70 $\int \frac{\tan^2(c+dx)}{(a+ia\tan(c+dx))^3} dx$

Optimal. Leaf size=88

$$-\frac{x}{8a^3}-\frac{i}{6d(a+ia\tan(c+dx))^3}+\frac{3i}{8ad(a+ia\tan(c+dx))^2}-\frac{i}{8d\left(a^3+ia^3\tan(c+dx)\right)}$$

```
[Out] -1/8*x/a^3-1/6*I/d/(a+I*a*tan(d*x+c))^3+3/8*I/a/d/(a+I*a*tan(d*x+c))^2-1/8*
I/d/(a^3+I*a^3*tan(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3621, 3607, 3560, 8}

$$-\frac{i}{8d\left(a^3+ia^3\tan(c+dx)\right)}-\frac{x}{8a^3}+\frac{3i}{8ad(a+ia\tan(c+dx))^2}-\frac{i}{6d(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] -1/8*x/a^3 - (I/6)/(d*(a + I*a*Tan[c + d*x])^3) + ((3*I)/8)/(a*d*(a + I*a*T
an[c + d*x])^2) - (I/8)/(d*(a^3 + I*a^3*Tan[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rule 3621

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(a*c + b*d)^2*((a + b*Tan[e + f*x])^
```

```
m/(2*a^3*f*m)), x] + Dist[1/(2*a^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[
a*c^2 - 2*b*c*d + a*d^2 - 2*b*d^2*Tan[e + f*x], x], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LeQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx &= -\frac{i}{6d(a+ia\tan(c+dx))^3}+\frac{\int \frac{a-2ia\tan(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{2a^2}\\ &= -\frac{i}{6d(a+ia\tan(c+dx))^3}+\frac{3i}{8ad(a+ia\tan(c+dx))^2}-\frac{\int \frac{1}{a+ia\tan(c+dx)}\,dx}{4a^2}\\ &= -\frac{i}{6d(a+ia\tan(c+dx))^3}+\frac{3i}{8ad(a+ia\tan(c+dx))^2}-\frac{i}{8d\left(a^3+ia^3\tan(c+d\right.}\\ &= -\frac{x}{8a^3}-\frac{i}{6d(a+ia\tan(c+dx))^3}+\frac{3i}{8ad(a+ia\tan(c+dx))^2}-\frac{i}{8d\left(a^3+ia^3\text{ta}\right.}\end{aligned}$$

Mathematica [A]

time = 0.45, size = 91, normalized size = 1.03

$$\frac{\sec^3(c+dx)(-9\cos(c+dx)+2(1-6idx)\cos(3(c+dx))-3i\sin(c+dx)-2i\sin(3(c+dx))+12dx\sin(3(c+dx)))}{96a^3d(-i+\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(-9*Cos[c + d*x] + 2*(1 - (6*I)*d*x)*Cos[3*(c + d*x)] - (3*
I)*Sin[c + d*x] - (2*I)*Sin[3*(c + d*x)] + 12*d*x*Sin[3*(c + d*x)]))/(96*a^
3*d*(-I + Tan[c + d*x])^3)
```

Maple [A]

time = 0.12, size = 75, normalized size = 0.85

method	result
risch	$-\frac{x}{8a^3}+\frac{i e^{-2i(dx+c)}}{16a^3d}+\frac{i e^{-4i(dx+c)}}{32a^3d}-\frac{i e^{-6i(dx+c)}}{48a^3d}$
derivativedivides	$\frac{\frac{i\ln(\tan(dx+c)-i)}{16}-\frac{3i}{8(\tan(dx+c)-i)^2}+\frac{1}{6(\tan(dx+c)-i)^3}-\frac{1}{8(\tan(dx+c)-i)}-\frac{i\ln(\tan(dx+c)+i)}{16}}{d\,a^3}$
default	$\frac{\frac{i\ln(\tan(dx+c)-i)}{16}-\frac{3i}{8(\tan(dx+c)-i)^2}+\frac{1}{6(\tan(dx+c)-i)^3}-\frac{1}{8(\tan(dx+c)-i)}-\frac{i\ln(\tan(dx+c)+i)}{16}}{d\,a^3}$
norman	$\frac{-\frac{x}{8a}+\frac{i}{12da}-\frac{\tan^5(dx+c)}{8da}-\frac{3x\left(\tan^2(dx+c)\right)}{8a}-\frac{3x\left(\tan^4(dx+c)\right)}{8a}-\frac{x\left(\tan^6(dx+c)\right)}{8a}+\frac{\tan(dx+c)}{8da}+\frac{2\left(\tan^3(dx+c)\right)}{3da}-\frac{i\left(\tan^4(dx+c)\right)}{2da}+}{a^2(1+\tan^2(dx+c))^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(1/16*I*ln(tan(d*x+c)-I)-3/8*I/(tan(d*x+c)-I)^2+1/6/(tan(d*x+c)-I)^
3-1/8/(tan(d*x+c)-I)-1/16*I*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.38, size = 54, normalized size = 0.61

$$-\frac{\left(12\,dxe^{(6i\,dx+6i\,c)}-6i\,e^{(4i\,dx+4i\,c)}-3i\,e^{(2i\,dx+2i\,c)}+2i\right)e^{(-6i\,dx-6i\,c)}}{96\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/96*(12*d*x*e^(6*I*d*x + 6*I*c) - 6*I*e^(4*I*d*x + 4*I*c) - 3*I*e^(2*I*d*
x + 2*I*c) + 2*I)*e^(-6*I*d*x - 6*I*c)/(a^3*d)
```

Sympy [A]
time = 0.21, size = 151, normalized size = 1.72

$$\begin{cases} \frac{(1536ia^6d^2e^{10ic}e^{-2idx}+768ia^6d^2e^{8ic}e^{-4idx}-512ia^6d^2e^{6ic}e^{-6idx})e^{-12ic}}{24576a^9d^3} & \text{for } a^9d^3e^{12ic}\neq 0 \\ x\left(\frac{(-e^{6ic}+e^{4ic}+e^{2ic}-1)e^{-6ic}}{8a^3}+\frac{1}{8a^3}\right) & \text{otherwise} \end{cases} - \frac{x}{8a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise(((1536*I*a**6*d**2*exp(10*I*c)*exp(-2*I*d*x) + 768*I*a**6*d**2*ex
p(8*I*c)*exp(-4*I*d*x) - 512*I*a**6*d**2*exp(6*I*c)*exp(-6*I*d*x))*exp(-12*
I*c)/(24576*a**9*d**3), Ne(a**9*d**3*exp(12*I*c), 0)), (x*((-exp(6*I*c) + e
xp(4*I*c) + exp(2*I*c) - 1)*exp(-6*I*c)/(8*a**3) + 1/(8*a**3)), True)) - x/
(8*a**3)
```

Giac [A]
time = 0.96, size = 80, normalized size = 0.91

$$-\frac{-\frac{6i\log(\tan(dx+c)-i)}{a^3}+\frac{6i\log(i\tan(dx+c)-1)}{a^3}+\frac{11i\tan(dx+c)^3+45\tan(dx+c)^2-21i\tan(dx+c)-3}{a^3(\tan(dx+c)-i)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/96*(-6*I*log(tan(d*x + c) - I)/a^3 + 6*I*log(I*tan(d*x + c) - 1)/a^3 + (
11*I*tan(d*x + c)^3 + 45*tan(d*x + c)^2 - 21*I*tan(d*x + c) - 3)/(a^3*(tan(
d*x + c) - I)^3))/d
```

Mupad [B]
time = 3.76, size = 49, normalized size = 0.56

$$-\frac{x}{8\,a^3}+\frac{\frac{\tan(c+d\,x)^2\,1\mathrm{i}}{8}-\frac{\tan(c+d\,x)}{8}+\frac{1}{12}\mathrm{i}}{a^3\,d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] ((tan(c + d*x)^2*1i)/8 - tan(c + d*x)/8 + 1i/12)/(a^3*d*(tan(c + d*x)*1i +
1)^3) - x/(8*a^3)
```

3.71 $\int \frac{\tan(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=84

$$-\frac{ix}{8a^3}-\frac{1}{6d(a+ia\tan(c+dx))^3}+\frac{1}{8ad(a+ia\tan(c+dx))^2}+\frac{1}{8d\left(a^3+ia^3\tan(c+dx)\right)}$$

```
[Out] -1/8*I*x/a^3-1/6/d/(a+I*a*tan(d*x+c))^3+1/8/a/d/(a+I*a*tan(d*x+c))^2+1/8/d/
(a^3+I*a^3*tan(d*x+c))
```

Rubi [A]

time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3607, 3560, 8}

$$\frac{1}{8d\left(a^3+ia^3\tan(c+dx)\right)}-\frac{ix}{8a^3}+\frac{1}{8ad(a+ia\tan(c+dx))^2}-\frac{1}{6d(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((-1/8*I)*x)/a^3 - 1/(6*d*(a + I*a*Tan[c + d*x])^3) + 1/(8*a*d*(a + I*a*Tan
[c + d*x])^2) + 1/(8*d*(a^3 + I*a^3*Tan[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{1}{6d(a+ia\tan(c+dx))^3} - \frac{i\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx}{2a}$$
$$= -\frac{1}{6d(a+ia\tan(c+dx))^3} + \frac{1}{8ad(a+ia\tan(c+dx))^2} - \frac{i\int \frac{1}{a+ia\tan(c+dx)}\,dx}{4a^2}$$
$$= -\frac{1}{6d(a+ia\tan(c+dx))^3} + \frac{1}{8ad(a+ia\tan(c+dx))^2} + \frac{1}{8d\left(a^3+ia^3\tan(c+\right.}$$
$$= -\frac{ix}{8a^3} - \frac{1}{6d(a+ia\tan(c+dx))^3} + \frac{1}{8ad(a+ia\tan(c+dx))^2} + \frac{1}{8d\left(a^3+ia^3\mathrm{ta}\right.}$$

Mathematica [A]
time = 0.38, size = 91, normalized size = 1.08

$$\frac{\sec^3(c+dx)(3i\cos(c+dx)+2(-i+6dx)\cos(3(c+dx))-9\sin(c+dx)-2\sin(3(c+dx))+12idx\sin(3(c+dx)))}{96a^3d(-i+\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*((3*I)*Cos[c + d*x] + 2*(-I + 6*d*x)*Cos[3*(c + d*x)] - 9*S
in[c + d*x] - 2*Sin[3*(c + d*x)] + (12*I)*d*x*Sin[3*(c + d*x)]))/(96*a^3*d*
(-I + Tan[c + d*x])^3)
```

Maple [A]
time = 0.12, size = 74, normalized size = 0.88

method	result
risch	$-\frac{ix}{8a^3}+\frac{e^{-2i(dx+c)}}{16a^3d}-\frac{e^{-4i(dx+c)}}{32a^3d}-\frac{e^{-6i(dx+c)}}{48a^3d}$
derivativedivides	$\frac{-\frac{i}{6(\tan(dx+c)-i)^3}-\frac{i}{8(\tan(dx+c)-i)}-\frac{1}{8(\tan(dx+c)-i)^2}-\frac{\ln(\tan(dx+c)-i)}{16}+\frac{\ln(\tan(dx+c)+i)}{16}}{d\,a^3}$
default	$\frac{-\frac{i}{6(\tan(dx+c)-i)^3}-\frac{i}{8(\tan(dx+c)-i)}-\frac{1}{8(\tan(dx+c)-i)^2}-\frac{\ln(\tan(dx+c)-i)}{16}+\frac{\ln(\tan(dx+c)+i)}{16}}{d\,a^3}$
norman	$\frac{\frac{1}{12da}-\frac{ix}{8a}+\frac{3\left(\tan^2(dx+c)\right)}{4da}-\frac{3ix\left(\tan^2(dx+c)\right)}{8a}-\frac{3ix\left(\tan^4(dx+c)\right)}{8a}-\frac{ix\left(\tan^6(dx+c)\right)}{8a}+\frac{i\tan(dx+c)}{8da}-\frac{2i\left(\tan^3(dx+c)\right)}{3da}-\frac{i\left(\tan^5(dx+\right.}{8da}}{a^2(1+\tan^2(dx+c))^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-1/6*I/(tan(d*x+c)-I)^3-1/8*I/(tan(d*x+c)-I)-1/8/(tan(d*x+c)-I)^2-
1/16*ln(tan(d*x+c)-I)+1/16*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.36, size = 54, normalized size = 0.64

$$\frac{\left(-12i\,dxe^{(6i\,dx+6i\,c)}+6\,e^{(4i\,dx+4i\,c)}-3\,e^{(2i\,dx+2i\,c)}-2\right)e^{(-6i\,dx-6i\,c)}}{96\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/96*(-12*I*d*x*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I*c) - 3*e^(2*I*d*x
+ 2*I*c) - 2)*e^(-6*I*d*x - 6*I*c)/(a^3*d)
```

Sympy [A]
time = 0.22, size = 153, normalized size = 1.82

$$\begin{cases}\frac{\left(1536a^6d^2e^{10ic}e^{-2idx}-768a^6d^2e^{8ic}e^{-4idx}-512a^6d^2e^{6ic}e^{-6idx}\right)e^{-12ic}}{24576a^9d^3} & \text{for } a^9d^3e^{12ic}\neq 0\\ x\left(\frac{\left(-ie^{6ic}-ie^{4ic}+ie^{2ic}+i\right)e^{-6ic}}{8a^3}+\frac{i}{8a^3}\right) & \text{otherwise}\end{cases}-\frac{ix}{8a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise(((1536*a**6*d**2*exp(10*I*c)*exp(-2*I*d*x) - 768*a**6*d**2*exp(8*
I*c)*exp(-4*I*d*x) - 512*a**6*d**2*exp(6*I*c)*exp(-6*I*d*x))*exp(-12*I*c)/(
24576*a**9*d**3), Ne(a**9*d**3*exp(12*I*c), 0)), (x*((-I*exp(6*I*c) - I*exp
(4*I*c) + I*exp(2*I*c) + I)*exp(-6*I*c)/(8*a**3) + I/(8*a**3)), True)) - I*
x/(8*a**3)
```

Giac [A]
time = 0.84, size = 81, normalized size = 0.96

$$-\frac{\frac{6\,\log(\tan(dx+c)-i)}{a^3}-\frac{6\,\log(i\,\tan(dx+c)-1)}{a^3}-\frac{11\,\tan(dx+c)^3-45i\,\tan(dx+c)^2-69\,\tan(dx+c)+19i}{a^3(\tan(dx+c)-i)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/96*(6*log(tan(d*x + c) - I)/a^3 - 6*log(I*tan(d*x + c) - 1)/a^3 - (11*ta
n(d*x + c)^3 - 45*I*tan(d*x + c)^2 - 69*tan(d*x + c) + 19*I)/(a^3*(tan(d*x
+ c) - I)^3))/d
```

Mupad [B]
time = 3.77, size = 49, normalized size = 0.58

$$-\frac{x\,1\mathrm{i}}{8\,a^3}+\frac{-\frac{{\mathrm{tan}\left(c+d\,x\right)}^2}{8}+\frac{\mathrm{tan}\left(c+d\,x\right)\,3\mathrm{i}}{8}+\frac{1}{12}}{a^3\,d\,{\left(1+\mathrm{tan}\left(c+d\,x\right)\,1\mathrm{i}\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] ((tan(c + d*x)*3i)/8 - tan(c + d*x)^2/8 + 1/12)/(a^3*d*(tan(c + d*x)*1i + 1
)^3) - (x*1i)/(8*a^3)
```

3.72 $\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=88

$$\frac{x}{8a^3}+\frac{i}{6d(a+ia\tan(c+dx))^3}+\frac{i}{8ad(a+ia\tan(c+dx))^2}+\frac{i}{8d\left(a^3+ia^3\tan(c+dx)\right)}$$

```
[Out] 1/8*x/a^3+1/6*I/d/(a+I*a*tan(d*x+c))^3+1/8*I/a/d/(a+I*a*tan(d*x+c))^2+1/8*I
/d/(a^3+I*a^3*tan(d*x+c))
```

Rubi [A]
time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3560, 8}

$$\frac{i}{8d\left(a^3+ia^3\tan(c+dx)\right)}+\frac{x}{8a^3}+\frac{i}{8ad(a+ia\tan(c+dx))^2}+\frac{i}{6d(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(-3),x]
```

```
[Out] x/(8*a^3) + (I/6)/(d*(a + I*a*Tan[c + d*x])^3) + (I/8)/(a*d*(a + I*a*Tan[c
+ d*x])^2) + (I/8)/(d*(a^3 + I*a^3*Tan[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx &= \frac{i}{6d(a+ia\tan(c+dx))^3}+\frac{\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx}{2a}\\
&= \frac{i}{6d(a+ia\tan(c+dx))^3}+\frac{i}{8ad(a+ia\tan(c+dx))^2}+\frac{\int \frac{1}{a+ia\tan(c+dx)}\,dx}{4a^2}\\
&= \frac{i}{6d(a+ia\tan(c+dx))^3}+\frac{i}{8ad(a+ia\tan(c+dx))^2}+\frac{i}{8d\left(a^3+ia^3\tan(c+d\right.}\\
&= \frac{x}{8a^3}+\frac{i}{6d(a+ia\tan(c+dx))^3}+\frac{i}{8ad(a+ia\tan(c+dx))^2}+\frac{i}{8d\left(a^3+ia^3\tan\right.}
\end{aligned}$$

Mathematica [A]
time = 0.25, size = 93, normalized size = 1.06

$$\frac{i\sec^3(c+dx)(27i\cos(c+dx)+2(i+6dx)\cos(3(c+dx))-9\sin(c+dx)+2\sin(3(c+dx))+12idx\sin(3(c+dx)))}{96a^3d(-i+\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(-3),x]
```

```
[Out] ((I/96)*Sec[c + d*x]^3*((27*I)*Cos[c + d*x] + 2*(I + 6*d*x)*Cos[3*(c + d*x)
] - 9*Sin[c + d*x] + 2*Sin[3*(c + d*x)] + (12*I)*d*x*Sin[3*(c + d*x)]))/(a^
3*d*(-I + Tan[c + d*x])^3)
```

Maple [A]
time = 0.12, size = 75, normalized size = 0.85

method	result	si
risch	$\frac{x}{8a^3}+\frac{3i\mathrm{e}^{-2i(dx+c)}}{16a^3d}+\frac{3i\mathrm{e}^{-4i(dx+c)}}{32a^3d}+\frac{i\mathrm{e}^{-6i(dx+c)}}{48a^3d}$	6
derivativedivides	$\frac{-\frac{i\ln(\tan(dx+c)-i)}{16}-\frac{i}{8(\tan(dx+c)-i)^2}-\frac{1}{6(\tan(dx+c)-i)^3}+\frac{1}{8\tan(dx+c)-8i}+\frac{i\ln(\tan(dx+c)+i)}{16}}{d\,a^3}$	7
default	$\frac{-\frac{i\ln(\tan(dx+c)-i)}{16}-\frac{i}{8(\tan(dx+c)-i)^2}-\frac{1}{6(\tan(dx+c)-i)^3}+\frac{1}{8\tan(dx+c)-8i}+\frac{i\ln(\tan(dx+c)+i)}{16}}{d\,a^3}$	7
norman	$\frac{\frac{x}{8a}+\frac{\tan^5(dx+c)}{8da}+\frac{3x\left(\tan^2(dx+c)\right)}{8a}+\frac{3x\left(\tan^4(dx+c)\right)}{8a}+\frac{x\left(\tan^6(dx+c)\right)}{8a}+\frac{5i}{12da}+\frac{7\tan(dx+c)}{8da}+\frac{\tan^3(dx+c)}{3da}-\frac{i\left(\tan^2(dx+c)\right)}{4da}}{a^2\left(1+\tan^2(dx+c)\right)^3}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-1/16*I*ln(tan(d*x+c)-I)-1/8*I/(tan(d*x+c)-I)^2-1/6/(tan(d*x+c)-I)
^3+1/8/(tan(d*x+c)-I)+1/16*I*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.36, size = 54, normalized size = 0.61

$$\frac{\left(12\,dxe^{(6i\,dx+6i\,c)}+18i\,e^{(4i\,dx+4i\,c)}+9i\,e^{(2i\,dx+2i\,c)}+2i\right)e^{(-6i\,dx-6i\,c)}}{96\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/96*(12*d*x*e^(6*I*d*x + 6*I*c) + 18*I*e^(4*I*d*x + 4*I*c) + 9*I*e^(2*I*d*
x + 2*I*c) + 2*I)*e^(-6*I*d*x - 6*I*c)/(a^3*d)
```

Sympy [A]
time = 0.18, size = 155, normalized size = 1.76

$$\begin{cases}\frac{\left(4608ia^6d^2e^{10ic}e^{-2idx}+2304ia^6d^2e^{8ic}e^{-4idx}+512ia^6d^2e^{6ic}e^{-6idx}\right)e^{-12ic}}{24576a^9d^3} & \text{for } a^9d^3e^{12ic}\neq 0\\ x\left(\frac{\left(e^{6ic}+3e^{4ic}+3e^{2ic}+1\right)e^{-6ic}}{8a^3}-\frac{1}{8a^3}\right) & \text{otherwise}\end{cases}+\frac{x}{8a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise(((4608*I*a**6*d**2*exp(10*I*c)*exp(-2*I*d*x) + 2304*I*a**6*d**2*e
xp(8*I*c)*exp(-4*I*d*x) + 512*I*a**6*d**2*exp(6*I*c)*exp(-6*I*d*x))*exp(-12
*I*c)/(24576*a**9*d**3), Ne(a**9*d**3*exp(12*I*c), 0)), (x*((exp(6*I*c) + 3
*exp(4*I*c) + 3*exp(2*I*c) + 1)*exp(-6*I*c)/(8*a**3) - 1/(8*a**3)), True))
+ x/(8*a**3)
```

Giac [A]
time = 0.60, size = 80, normalized size = 0.91

$$-\frac{\frac{6i\log\left(\tan\left(dx+c\right)-i\right)}{a^3}-\frac{6i\log\left(i\tan\left(dx+c\right)-1\right)}{a^3}+\frac{-11i\tan\left(dx+c\right)^3-45\tan\left(dx+c\right)^2+69i\tan\left(dx+c\right)+51}{a^3\left(\tan\left(dx+c\right)-i\right)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/96*(6*I*log(tan(d*x + c) - I)/a^3 - 6*I*log(I*tan(d*x + c) - 1)/a^3 + (-
11*I*tan(d*x + c)^3 - 45*tan(d*x + c)^2 + 69*I*tan(d*x + c) + 51)/(a^3*(tan
(d*x + c) - I)^3))/d
```

Mupad [B]
time = 3.75, size = 50, normalized size = 0.57

$$\frac{x}{8\,a^3}-\frac{\frac{{\mathrm{tan}\left(c+d\,x\right)}^2\,1{}\mathrm{i}}{8}+\frac{3\,\mathrm{tan}\left(c+d\,x\right)}{8}-\frac{5}{12}{}\mathrm{i}}{a^3\,d\,{\left(1+\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] x/(8*a^3) - ((3*tan(c + d*x))/8 + (tan(c + d*x)^2*1i)/8 - 5i/12)/(a^3*d*(ta
n(c + d*x)*1i + 1)^3)
```

3.73 $\int \frac{\cot(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=98

$$-\frac{7ix}{8a^3}+\frac{\log(\sin(c+dx))}{a^3d}+\frac{1}{6d(a+ia\tan(c+dx))^3}+\frac{3}{8ad(a+ia\tan(c+dx))^2}+\frac{7}{8d\,(a^3+ia^3\tan(c+dx))}$$

```
[Out] -7/8*I*x/a^3+ln(sin(d*x+c))/a^3/d+1/6/d/(a+I*a*tan(d*x+c))^3+3/8/a/d/(a+I*a
*tan(d*x+c))^2+7/8/d/(a^3+I*a^3*tan(d*x+c))
```

Rubi [A]
time = 0.15, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3640, 3677, 3612, 3556}

$$\frac{7}{8d\,(a^3+ia^3\tan(c+dx))}+\frac{\log(\sin(c+dx))}{a^3d}-\frac{7ix}{8a^3}+\frac{3}{8ad(a+ia\tan(c+dx))^2}+\frac{1}{6d(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (((-7*I)/8)*x)/a^3 + Log[Sin[c + d*x]]/(a^3*d) + 1/(6*d*(a + I*a*Tan[c + d*
x])^3) + 3/(8*a*d*(a + I*a*Tan[c + d*x])^2) + 7/(8*d*(a^3 + I*a^3*Tan[c + d
*x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

```
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot(c+dx)}{(a+ia\tan(c+dx))^3}\,dx &= \frac{1}{6d(a+ia\tan(c+dx))^3}+\frac{\int \frac{\cot(c+dx)(6a-3ia\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx}{6a^2}\\
&= \frac{1}{6d(a+ia\tan(c+dx))^3}+\frac{3}{8ad(a+ia\tan(c+dx))^2}+\frac{\int \frac{\cot(c+dx)\left(24a^2-18ia^2\tan \cdots\right.}{a+ia\tan(c+dx)}\cdots}{24a^4}\\
&= \frac{1}{6d(a+ia\tan(c+dx))^3}+\frac{3}{8ad(a+ia\tan(c+dx))^2}+\frac{7}{8d\left(a^3+ia^3\tan(c+d \cdots\right.}\\
&= -\frac{7ix}{8a^3}+\frac{1}{6d(a+ia\tan(c+dx))^3}+\frac{3}{8ad(a+ia\tan(c+dx))^2}+\frac{7}{8d\left(a^3+ia^3 \cdots\right.}\\
&= -\frac{7ix}{8a^3}+\frac{\log(\sin(c+dx))}{a^3d}+\frac{1}{6d(a+ia\tan(c+dx))^3}+\frac{3}{8ad(a+ia\tan(c+dx \cdots}
\end{aligned}
$$

Mathematica [A]
time = 0.47, size = 118, normalized size = 1.20

$$
\frac{\sec^3(c+dx)(81i\cos(c+dx)+\cos(3(c+dx))(2i+84dx+96i\log(\sin(c+dx)))-51\sin(c+dx)+2\sin(3(c+dx))+84idx\sin(3(c+dx))-96\log(\sin(c+dx))\sin(3(c+dx)))}{96a^3d(-i+\tan(c+dx))^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*((81*I)*Cos[c + d*x] + Cos[3*(c + d*x)]*(2*I + 84*d*x + (96
*I)*Log[Sin[c + d*x]]) - 51*Sin[c + d*x] + 2*Sin[3*(c + d*x)] + (84*I)*d*x*
Sin[3*(c + d*x)] - 96*Log[Sin[c + d*x]]*Sin[3*(c + d*x)]))/(96*a^3*d*(-I +
Tan[c + d*x])^3)
```

Maple [A]
time = 0.34, size = 81, normalized size = 0.83

method	result
derivativedivides	$\frac{\frac{i}{6(\tan(dx+c)-i)^3}-\frac{7i}{8(\tan(dx+c)-i)}-\frac{3}{8(\tan(dx+c)-i)^2}-\frac{15\ln(\tan(dx+c)-i)}{16}+\ln(\tan(dx+c))-\frac{\ln(\tan(dx+c)+i)}{16}}{d\,a^3}$
default	$\frac{\frac{i}{6(\tan(dx+c)-i)^3}-\frac{7i}{8(\tan(dx+c)-i)}-\frac{3}{8(\tan(dx+c)-i)^2}-\frac{15\ln(\tan(dx+c)-i)}{16}+\ln(\tan(dx+c))-\frac{\ln(\tan(dx+c)+i)}{16}}{d\,a^3}$
risch	$-\frac{15ix}{8a^3}+\frac{11\,\mathrm{e}^{-2i(dx+c)}}{16a^3d}+\frac{5\,\mathrm{e}^{-4i(dx+c)}}{32a^3d}+\frac{\mathrm{e}^{-6i(dx+c)}}{48a^3d}-\frac{2ic}{a^3d}+\frac{\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{a^3d}$
norman	$\frac{\frac{17}{12da}-\frac{7ix}{8a}+\frac{\tan^4(dx+c)}{2da}+\frac{5\left(\tan^2(dx+c)\right)}{4da}-\frac{21ix\left(\tan^2(dx+c)\right)}{8a}-\frac{21ix\left(\tan^4(dx+c)\right)}{8a}-\frac{7ix\left(\tan^6(dx+c)\right)}{8a}-\frac{17i\tan(dx+c)}{8da}-\frac{7i\left(\tan^3(\right.}{3d}}{a^2(1+\tan^2(dx+c))^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(1/6*I/(tan(d*x+c)-I)^3-7/8*I/(tan(d*x+c)-I)-3/8/(tan(d*x+c)-I)^2-1
5/16*ln(tan(d*x+c)-I)+ln(tan(d*x+c))-1/16*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.38, size = 77, normalized size = 0.79

$$\frac{\left(-180i\,dxe^{(6i\,dx+6i\,c)}+96\,e^{(6i\,dx+6i\,c)}\log\left(e^{(2i\,dx+2i\,c)}-1\right)+66\,e^{(4i\,dx+4i\,c)}+15\,e^{(2i\,dx+2i\,c)}+2\right)e^{(-6i\,dx-6i\,c)}}{96\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/96*(-180*I*d*x*e^(6*I*d*x + 6*I*c) + 96*e^(6*I*d*x + 6*I*c)*log(e^(2*I*d*
x + 2*I*c) - 1) + 66*e^(4*I*d*x + 4*I*c) + 15*e^(2*I*d*x + 2*I*c) + 2)*e^(-
6*I*d*x - 6*I*c)/(a^3*d)
```

Sympy [A]
time = 0.30, size = 187, normalized size = 1.91

$$\begin{cases}\frac{\left(16896a^6d^2e^{10ic}e^{-2idx}+3840a^6d^2e^{8ic}e^{-4idx}+512a^6d^2e^{6ic}e^{-6idx}\right)e^{-12ic}}{24576a^9d^3} & \text{for } a^9d^3e^{12ic}\neq 0\\ x\left(\frac{\left(-15ie^{6ic}-11ie^{4ic}-5ie^{2ic}-i\right)e^{-6ic}}{8a^3}+\frac{15i}{8a^3}\right) & \text{otherwise}\end{cases}-\frac{15ix}{8a^3}+\frac{\log\left(e^{2idx}-e^{-2ic}\right)}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise((((16896*a**6*d**2*exp(10*I*c)*exp(-2*I*d*x) + 3840*a**6*d**2*exp(
8*I*c)*exp(-4*I*d*x) + 512*a**6*d**2*exp(6*I*c)*exp(-6*I*d*x))*exp(-12*I*c)
/(24576*a**9*d**3), Ne(a**9*d**3*exp(12*I*c), 0)), (x*((-15*I*exp(6*I*c) -
11*I*exp(4*I*c) - 5*I*exp(2*I*c) - I)*exp(-6*I*c)/(8*a**3) + 15*I/(8*a**3))
, True)) - 15*I*x/(8*a**3) + log(exp(2*I*d*x) - exp(-2*I*c))/(a**3*d)
```

Giac [A]
time = 1.01, size = 93, normalized size = 0.95

$$-\frac{\frac{90\,\log(\tan(dx+c)-i)}{a^3}+\frac{6\,\log(i\,\tan(dx+c)-1)}{a^3}-\frac{96\,\log(\tan(dx+c))}{a^3}-\frac{165\,\tan(dx+c)^3-579i\,\tan(dx+c)^2-699\,\tan(dx+c)+301i}{a^3(\tan(dx+c)-i)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/96*(90*log(tan(d*x + c) - I)/a^3 + 6*log(I*tan(d*x + c) - 1)/a^3 - 96*lo
g(tan(d*x + c))/a^3 - (165*tan(d*x + c)^3 - 579*I*tan(d*x + c)^2 - 699*tan(
d*x + c) + 301*I)/(a^3*(tan(d*x + c) - I)^3))/d
```

Mupad [B]
time = 3.87, size = 120, normalized size = 1.22

$$\frac{\frac{17}{12\,a^3}-\frac{7\tan(c+d\,x)^2}{8\,a^3}+\frac{\tan(c+d\,x)\,17\mathrm{i}}{8\,a^3}}{d\,\left(-\tan\left(c+d\,x\right)^3\,1\mathrm{i}-3\tan\left(c+d\,x\right)^2+\tan\left(c+d\,x\right)\,3\mathrm{i}+1\right)}-\frac{15\,\ln\left(\tan(c+d\,x)-\mathrm{i}\right)}{16\,a^3\,d}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)}{16\,a^3\,d}+\frac{\ln\left(\tan(c+d\,x)\right)}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] ((tan(c + d*x)*17i)/(8*a^3) + 17/(12*a^3) - (7*tan(c + d*x)^2)/(8*a^3))/(d*
(tan(c + d*x)*3i - 3*tan(c + d*x)^2 - tan(c + d*x)^3*1i + 1)) - (15*log(tan
(c + d*x) - 1i))/(16*a^3*d) - log(tan(c + d*x) + 1i)/(16*a^3*d) + log(tan(c
 + d*x))/(a^3*d)
```

3.74 $\int \frac{\cot^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=133

$$-\frac{25x}{8a^3}-\frac{25\cot(c+dx)}{8a^3d}-\frac{3i\log(\sin(c+dx))}{a^3d}+\frac{\cot(c+dx)}{6d(a+ia\tan(c+dx))^3}+\frac{11\cot(c+dx)}{24ad(a+ia\tan(c+dx))^2}+\frac{3\cot(c+dx)}{2d(a^3+ia^3\tan(c+dx))}$$

```
[Out] -25/8*x/a^3-25/8*cot(d*x+c)/a^3/d-3*I*ln(sin(d*x+c))/a^3/d+1/6*cot(d*x+c)/d
/(a+I*a*tan(d*x+c))^3+11/24*cot(d*x+c)/a/d/(a+I*a*tan(d*x+c))^2+3/2*cot(d*x
+c)/d/(a^3+I*a^3*tan(d*x+c))
```

Rubi [A]
time = 0.21, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3640, 3677, 3610, 3612, 3556}

$$-\frac{25\cot(c+dx)}{8a^3d}-\frac{3i\log(\sin(c+dx))}{a^3d}+\frac{3\cot(c+dx)}{2d\left(a^3+ia^3\tan(c+dx)\right)}-\frac{25x}{8a^3}+\frac{11\cot(c+dx)}{24ad(a+ia\tan(c+dx))^2}+\frac{\cot(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (-25*x)/(8*a^3) - (25*Cot[c + d*x])/(8*a^3*d) - ((3*I)*Log[Sin[c + d*x]])/(
a^3*d) + Cot[c + d*x]/(6*d*(a + I*a*Tan[c + d*x])^3) + (11*Cot[c + d*x])/(2
4*a*d*(a + I*a*Tan[c + d*x])^2) + (3*Cot[c + d*x])/(2*d*(a^3 + I*a^3*Tan[c
+ d*x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
```

```
Q[a*c + b*d, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx &= \frac{\cot(c+dx)}{6d(a+ia\tan(c+dx))^3}+\frac{\int \frac{\cot^2(c+dx)(7a-4ia\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx}{6a^2}\\
&= \frac{\cot(c+dx)}{6d(a+ia\tan(c+dx))^3}+\frac{11\cot(c+dx)}{24ad(a+ia\tan(c+dx))^2}+\frac{\int \frac{\cot^2(c+dx)(39a^2-33ia^2}{a+ia\tan(c+dx}}{24a^4}\\
&= \frac{\cot(c+dx)}{6d(a+ia\tan(c+dx))^3}+\frac{11\cot(c+dx)}{24ad(a+ia\tan(c+dx))^2}+\frac{3\cot(c+dx)}{2d\,(a^3+ia^3\tan(c+}\\
&= -\frac{25\cot(c+dx)}{8a^3d}+\frac{\cot(c+dx)}{6d(a+ia\tan(c+dx))^3}+\frac{11\cot(c+dx)}{24ad(a+ia\tan(c+dx))^2}+\frac{}{2}\\
&= -\frac{25x}{8a^3}-\frac{25\cot(c+dx)}{8a^3d}+\frac{\cot(c+dx)}{6d(a+ia\tan(c+dx))^3}+\frac{11\cot(c+dx)}{24ad(a+ia\tan(c+dx}\\
&= -\frac{25x}{8a^3}-\frac{25\cot(c+dx)}{8a^3d}-\frac{3i\log(\sin(c+dx))}{a^3d}+\frac{\cot(c+dx)}{6d(a+ia\tan(c+dx))^3}+\frac{}{2}
\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 379 vs. $2(133) = 266$.

time = 4.99, size = 379, normalized size = 2.85

$$\frac{\sec^3(c+dx)(\cos(dx)+i\sin(dx))^3\left(-300idx\cos(3c)+2\cos(3c)\cos(6dx)+288dx\cos(3c)\cot(c)+144\cos(3c)\log\left(\sin^2(c+dx)\right)-24\cos(3c-dx)\csc\left(\frac{c}{2}\right)\csc(c+dx)\sec\left(\frac{c}{2}\right)+24\cos(3c+dx)\csc\left(\frac{c}{2}\right)\csc(c+dx)\sec\left(\frac{c}{2}\right)+288dx\sin(c)+138i\cos(2dx)\sin(c)-21i\cos(4dx)\sin(c)+300dx\sin(3c)-2i\cos(6dx)\sin(3c)+288idx\cot(c)\sin(3c)+144i\log\left(\sin^2(c+dx)\right)\sin(3c)+288\text{ArcTan}(\tan(dx))(-i\cos(3c)+\sin(3c))+138\sin(c)\sin(2dx)-3\cos(c)(192idx-46\cos(2dx)-7\cos(4dx)+96dx\cot(c)+46i\sin(2dx)+7i\sin(4dx))-21\sin(c)\sin(4dx)-2i\cos(3c)\sin(6dx)-2\sin(3c)\sin(6dx)-48i\csc(c)\csc(c+dx)\sin(3c-dx)+48i\csc(c)\csc(c+dx)\sin(3c+dx)\right)}{96a^3d(-i+\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(Cos[d*x] + I*Sin[d*x])^3*((-300*I)*d*x*Cos[3*c] + 2*Cos[3*
c]*Cos[6*d*x] + 288*d*x*Cos[3*c]*Cot[c] + 144*Cos[3*c]*Log[Sin[c + d*x]^2]
- 24*Cos[3*c - d*x]*Csc[c/2]*Csc[c + d*x]*Sec[c/2] + 24*Cos[3*c + d*x]*Csc[
c/2]*Csc[c + d*x]*Sec[c/2] + 288*d*x*Sin[c] + (138*I)*Cos[2*d*x]*Sin[c] - (
21*I)*Cos[4*d*x]*Sin[c] + 300*d*x*Sin[3*c] - (2*I)*Cos[6*d*x]*Sin[3*c] + (2
88*I)*d*x*Cot[c]*Sin[3*c] + (144*I)*Log[Sin[c + d*x]^2]*Sin[3*c] + 288*ArcT
an[Tan[d*x]]*((-I)*Cos[3*c] + Sin[3*c]) + 138*Sin[c]*Sin[2*d*x] - 3*Cos[c]*
((192*I)*d*x - 46*Cos[2*d*x] - 7*Cos[4*d*x] + 96*d*x*Cot[c] + (46*I)*Sin[2*
d*x] + (7*I)*Sin[4*d*x]) - 21*Sin[c]*Sin[4*d*x] - (2*I)*Cos[3*c]*Sin[6*d*x]
 - 2*Sin[3*c]*Sin[6*d*x] - (48*I)*Csc[c]*Csc[c + d*x]*Sin[3*c - d*x] + (48*
I)*Csc[c]*Csc[c + d*x]*Sin[3*c + d*x]))/(96*a^3*d*(-I + Tan[c + d*x])^3)
```

Maple [A]

time = 0.33, size = 95, normalized size = 0.71

method	result
derivativedivides	$\frac{\frac{5i}{8(\tan(dx+c)-i)^2}+\frac{49i\ln(\tan(dx+c)-i)}{16}+\frac{1}{6(\tan(dx+c)-i)^3}-\frac{17}{8(\tan(dx+c)-i)}-\frac{1}{\tan(dx+c)}-3i\ln(\tan(dx+c))-\frac{i\ln(\tan(dx+c)+i)}{16}}{d\,a^3}$
default	$\frac{\frac{5i}{8(\tan(dx+c)-i)^2}+\frac{49i\ln(\tan(dx+c)-i)}{16}+\frac{1}{6(\tan(dx+c)-i)^3}-\frac{17}{8(\tan(dx+c)-i)}-\frac{1}{\tan(dx+c)}-3i\ln(\tan(dx+c))-\frac{i\ln(\tan(dx+c)+i)}{16}}{d\,a^3}$
risch	$-\frac{49x}{8a^3}-\frac{23i\mathrm{e}^{-2i(dx+c)}}{16a^3d}-\frac{7i\mathrm{e}^{-4i(dx+c)}}{32a^3d}-\frac{i\mathrm{e}^{-6i(dx+c)}}{48a^3d}-\frac{6c}{a^3d}-\frac{2i}{d\,a^3\left(\mathrm{e}^{2i(dx+c)}-1\right)}-\frac{3i\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{a^3d}$
norman	$\frac{-\frac{1}{da}-\frac{25\left(\tan^4(dx+c)\right)}{3da}-\frac{25\left(\tan^6(dx+c)\right)}{8da}-\frac{25x\tan(dx+c)}{8a}-\frac{75x\left(\tan^3(dx+c)\right)}{8a}-\frac{75x\left(\tan^5(dx+c)\right)}{8a}-\frac{25x\left(\tan^7(dx+c)\right)}{8a}-\frac{55\left(\tan^2(d \cdots\right.}{8da}}{\tan(dx+c)a^2\left(1+\tan^2(dx+c)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(5/8*I/(tan(d*x+c)-I)^2+49/16*I*ln(tan(d*x+c)-I)+1/6/(tan(d*x+c)-I)
^3-17/8/(tan(d*x+c)-I)-1/tan(d*x+c)-3*I*ln(tan(d*x+c))-1/16*I*ln(tan(d*x+c)
+I))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.37, size = 125, normalized size = 0.94

$$-\frac{588\,dxe^{(8i\,dx+8i\,c)}-6\,(98\,dx-55i)e^{(6i\,dx+6i\,c)}+288\,\left(i\,e^{(8i\,dx+8i\,c)}-i\,e^{(6i\,dx+6i\,c)}\right)\log\left(e^{(2i\,dx+2i\,c)}-1\right)-117i\,e^{(4i\,dx+4i\,c)}-19i\,e^{(2i\,dx+2i\,c)}-2i}{96\,\left(a^3de^{(8i\,dx+8i\,c)}-a^3de^{(6i\,dx+6i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/96*(588*d*x*e^(8*I*d*x + 8*I*c) - 6*(98*d*x - 55*I)*e^(6*I*d*x + 6*I*c)
+ 288*(I*e^(8*I*d*x + 8*I*c) - I*e^(6*I*d*x + 6*I*c))*log(e^(2*I*d*x + 2*I*
c) - 1) - 117*I*e^(4*I*d*x + 4*I*c) - 19*I*e^(2*I*d*x + 2*I*c) - 2*I)/(a^3*
d*e^(8*I*d*x + 8*I*c) - a^3*d*e^(6*I*d*x + 6*I*c))
```

Sympy [A]
time = 0.36, size = 218, normalized size = 1.64

$$\begin{cases}\frac{\left(-35328ia^6d^2e^{10ic}e^{-2idx}-5376ia^6d^2e^{8ic}e^{-4idx}-512ia^6d^2e^{6ic}e^{-6idx}\right)e^{-12ic}}{24576a^9d^3} & \text{for } a^9d^3e^{12ic}\neq 0\\ x\left(\frac{\left(-49e^{6ic}-23e^{4ic}-7e^{2ic}-1\right)e^{-6ic}}{8a^3}+\frac{49}{8a^3}\right) & \text{otherwise}\end{cases}-\frac{2i}{a^3de^{2ic}e^{2idx}-a^3d}-\frac{49x}{8a^3}-\frac{3i\log\left(e^{2idx}-e^{-2ic}\right)}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise(((-35328*I*a**6*d**2*exp(10*I*c)*exp(-2*I*d*x) - 5376*I*a**6*d**2
*exp(8*I*c)*exp(-4*I*d*x) - 512*I*a**6*d**2*exp(6*I*c)*exp(-6*I*d*x))*exp(-
12*I*c)/(24576*a**9*d**3), Ne(a**9*d**3*exp(12*I*c), 0)), (x*((-49*exp(6*I*
c) - 23*exp(4*I*c) - 7*exp(2*I*c) - 1)*exp(-6*I*c)/(8*a**3) + 49/(8*a**3)),
 True)) - 2*I/(a**3*d*exp(2*I*c)*exp(2*I*d*x) - a**3*d) - 49*x/(8*a**3) - 3
*I*log(exp(2*I*d*x) - exp(-2*I*c))/(a**3*d)
```

Giac [A]
time = 0.95, size = 119, normalized size = 0.89

$$-\frac{-\frac{294i\,\log(i\,\tan(dx+c)+1)}{a^3}+\frac{6i\,\log(i\,\tan(dx+c)-1)}{a^3}+\frac{288i\,\log(\tan(dx+c))}{a^3}+\frac{96\,(-3i\,\tan(dx+c)+1)}{a^3\,\tan(dx+c)}+\frac{539\,\tan(dx+c)^3-1821i\,\tan(dx+c)^2-2085\,\tan(dx+c)+819i}{a^3(i\,\tan(dx+c)+1)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/96*(-294*I*log(I*tan(d*x + c) + 1)/a^3 + 6*I*log(I*tan(d*x + c) - 1)/a^3
 + 288*I*log(tan(d*x + c))/a^3 + 96*(-3*I*tan(d*x + c) + 1)/(a^3*tan(d*x +
c)) + (539*tan(d*x + c)^3 - 1821*I*tan(d*x + c)^2 - 2085*tan(d*x + c) + 819
*I)/(a^3*(I*tan(d*x + c) + 1)^3))/d
```

Mupad [B]
time = 4.05, size = 145, normalized size = 1.09

$$\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,49\mathrm{i}}{16\,a^3\,d}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{16\,a^3\,d}-\frac{\ln\left(\tan(c+d\,x)\right)\,3\mathrm{i}}{a^3\,d}+\frac{\frac{71\tan(c+d\,x)}{12\,a^3}-\frac{25\tan(c+d\,x)^3}{8\,a^3}-\frac{1\mathrm{i}}{a^3}+\frac{\tan(c+d\,x)^2\,63\mathrm{i}}{8\,a^3}}{d\,\left(\tan\left(c+d\,x\right)^4-\tan\left(c+d\,x\right)^3\,3\mathrm{i}-3\tan\left(c+d\,x\right)^2+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] (log(tan(c + d*x) - 1i)*49i)/(16*a^3*d) - (log(tan(c + d*x) + 1i)*1i)/(16*a
^3*d) - (log(tan(c + d*x))*3i)/(a^3*d) + ((71*tan(c + d*x))/(12*a^3) - 1i/a
^3 + (tan(c + d*x)^2*63i)/(8*a^3) - (25*tan(c + d*x)^3)/(8*a^3))/(d*(tan(c
+ d*x)*1i - 3*tan(c + d*x)^2 - tan(c + d*x)^3*3i + tan(c + d*x)^4))
```

3.75 $\int \frac{\tan^6(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=171

$$-\frac{65x}{16a^4}-\frac{4i\log(\cos(c+dx))}{a^4d}+\frac{65\tan(c+dx)}{16a^4d}-\frac{2i\tan^2(c+dx)}{a^4d(1+i\tan(c+dx))}+\frac{31\tan^3(c+dx)}{48a^4d(1+i\tan(c+dx))^2}-\frac{\tan}{8d(a+i}$$

```
[Out] -65/16*x/a^4-4*I*ln(cos(d*x+c))/a^4/d+65/16*tan(d*x+c)/a^4/d-2*I*tan(d*x+c)
^2/a^4/d/(1+I*tan(d*x+c))+31/48*tan(d*x+c)^3/a^4/d/(1+I*tan(d*x+c))^2-1/8*t
an(d*x+c)^5/d/(a+I*a*tan(d*x+c))^4+7/24*I*tan(d*x+c)^4/a/d/(a+I*a*tan(d*x+c
))^3
```

Rubi [A]
time = 0.26, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3639, 3676, 3606, 3556}

$$\frac{31\tan^3(c+dx)}{48a^4d(1+i\tan(c+dx))^2}-\frac{2i\tan^2(c+dx)}{a^4d(1+i\tan(c+dx))}+\frac{65\tan(c+dx)}{16a^4d}-\frac{4i\log(\cos(c+dx))}{a^4d}-\frac{65x}{16a^4}-\frac{\tan^5(c+dx)}{8d(a+ia\tan(c+dx))^4}+\frac{7i\tan^4(c+dx)}{24ad(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (-65*x)/(16*a^4) - ((4*I)*Log[Cos[c + d*x]])/(a^4*d) + (65*Tan[c + d*x])/(1
6*a^4*d) - ((2*I)*Tan[c + d*x]^2)/(a^4*d*(1 + I*Tan[c + d*x])) + (31*Tan[c
+ d*x]^3)/(48*a^4*d*(1 + I*Tan[c + d*x])^2) - Tan[c + d*x]^5/(8*d*(a + I*a*
Tan[c + d*x])^4) + (((7*I)/24)*Tan[c + d*x]^4)/(a*d*(a + I*a*Tan[c + d*x])^
3)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
```

```
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\tan^6(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{\tan^5(c+dx)}{8d(a+ia\tan(c+dx))^4} - \frac{\int \frac{\tan^4(c+dx)(-5a+9ia\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx}{8a^2}$$

$$= -\frac{\tan^5(c+dx)}{8d(a+ia\tan(c+dx))^4} + \frac{7i\tan^4(c+dx)}{24ad(a+ia\tan(c+dx))^3} + \frac{\int \frac{\tan^3(c+dx)(-56ia^2-68\ldots}{(a+ia\tan(c+\ldots}}{48a^4}$$

$$= \frac{31\tan^3(c+dx)}{48a^4d(1+i\tan(c+dx))^2} - \frac{\tan^5(c+dx)}{8d(a+ia\tan(c+dx))^4} + \frac{7i\tan^4(c+dx)}{24ad(a+ia\tan(c+d\ldots}$$

$$= \frac{31\tan^3(c+dx)}{48a^4d(1+i\tan(c+dx))^2} - \frac{\tan^5(c+dx)}{8d(a+ia\tan(c+dx))^4} + \frac{7i\tan^4(c+dx)}{24ad(a+ia\tan(c+d\ldots}$$

$$= -\frac{65x}{16a^4} + \frac{65\tan(c+dx)}{16a^4d} + \frac{31\tan^3(c+dx)}{48a^4d(1+i\tan(c+dx))^2} - \frac{\tan^5(c+dx)}{8d(a+ia\tan(c+dx\ldots}$$

$$= -\frac{65x}{16a^4} - \frac{4i\log(\cos(c+dx))}{a^4d} + \frac{65\tan(c+dx)}{16a^4d} + \frac{31\tan^3(c+dx)}{48a^4d(1+i\tan(c+dx))^2} - \ldots$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 429 vs. $2(171) = 342$.
time = 0.91, size = 429, normalized size = 2.51

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^6/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] -1/1536*(Sec[c]*Sec[c + d*x]^5*((-536*I)*Cos[d*x] - (536*I)*Cos[2*c + d*x]
 - (893*I)*Cos[2*c + 3*d*x] + 1560*d*x*Cos[2*c + 3*d*x] - (1661*I)*Cos[4*c +
 3*d*x] + 1560*d*x*Cos[4*c + 3*d*x] + (771*I)*Cos[4*c + 5*d*x] + 1560*d*x*C
os[4*c + 5*d*x] + (3*I)*Cos[6*c + 5*d*x] + 1560*d*x*Cos[6*c + 5*d*x] + (153
6*I)*Cos[2*c + 3*d*x]*Log[Cos[c + d*x]] + (1536*I)*Cos[4*c + 3*d*x]*Log[Cos
[c + d*x]] + (1536*I)*Cos[4*c + 5*d*x]*Log[Cos[c + d*x]] + (1536*I)*Cos[6*c
 + 5*d*x]*Log[Cos[c + d*x]] + 832*Sin[d*x] + 832*Sin[2*c + d*x] + 835*Sin[2
*c + 3*d*x] + (1560*I)*d*x*Sin[2*c + 3*d*x] - 1536*Log[Cos[c + d*x]]*Sin[2*
c + 3*d*x] + 1603*Sin[4*c + 3*d*x] + (1560*I)*d*x*Sin[4*c + 3*d*x] - 1536*L
og[Cos[c + d*x]]*Sin[4*c + 3*d*x] - 765*Sin[4*c + 5*d*x] + (1560*I)*d*x*Sin
[4*c + 5*d*x] - 1536*Log[Cos[c + d*x]]*Sin[4*c + 5*d*x] + 3*Sin[6*c + 5*d*x
] + (1560*I)*d*x*Sin[6*c + 5*d*x] - 1536*Log[Cos[c + d*x]]*Sin[6*c + 5*d*x]
))/(a^4*d*(-I + Tan[c + d*x])^4)
```

Maple [A]
time = 0.21, size = 95, normalized size = 0.56

method	result
derivativedivides	$\frac{\tan(dx+c)+\frac{49i}{16(\tan(dx+c)-i)^2}-\frac{i}{8(\tan(dx+c)-i)^4}+\frac{129i\ln(\tan(dx+c)-i)}{32}-\frac{11}{12(\tan(dx+c)-i)^3}+\frac{111}{16(\tan(dx+c)-i)}-\frac{i\ln(\tan(dx+}{32}}{d\,a^4}$
default	$\frac{\tan(dx+c)+\frac{49i}{16(\tan(dx+c)-i)^2}-\frac{i}{8(\tan(dx+c)-i)^4}+\frac{129i\ln(\tan(dx+c)-i)}{32}-\frac{11}{12(\tan(dx+c)-i)^3}+\frac{111}{16(\tan(dx+c)-i)}-\frac{i\ln(\tan(dx+}{32}}{d\,a^4}$
risch	$-\frac{129x}{16a^4}+\frac{9i\mathrm{e}^{-2i(dx+c)}}{4a^4d}-\frac{15i\mathrm{e}^{-4i(dx+c)}}{32a^4d}+\frac{i\mathrm{e}^{-6i(dx+c)}}{12a^4d}-\frac{i\mathrm{e}^{-8i(dx+c)}}{128a^4d}-\frac{8c}{a^4d}+\frac{2i}{d\,a^4\left(\mathrm{e}^{2i(dx+c)}+1\right)}-\frac{4i\ln(\mathrm{e}}{}$
norman	$\frac{\frac{\tan^9(dx+c)}{da}-\frac{65x}{16a}+\frac{949\left(\tan^5(dx+c)\right)}{48da}+\frac{175\left(\tan^7(dx+c)\right)}{16da}-\frac{65x\left(\tan^2(dx+c)\right)}{4a}-\frac{195x\left(\tan^4(dx+c)\right)}{8a}-\frac{65x\left(\tan^6(dx+c)\right)}{4a}-\frac{65x\left(\tan\right.}{1}}{\left(1+\tan^2(dx+c)\right)^4a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^4*(tan(d*x+c)+49/16*I/(tan(d*x+c)-I)^2-1/8*I/(tan(d*x+c)-I)^4+129/32*
I*ln(tan(d*x+c)-I)-11/12/(tan(d*x+c)-I)^3+111/16/(tan(d*x+c)-I)-1/32*I*ln(t
an(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.39, size = 135, normalized size = 0.79

$$-\frac{3096\,dxe^{(10i\,dx+10i\,c)}+24\,(129\,dx-68i)e^{(8i\,dx+8i\,c)}+1536\,\left(i\,e^{(10i\,dx+10i\,c)}+i\,e^{(8i\,dx+8i\,c)}\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-684i\,e^{(6i\,dx+6i\,c)}+148i\,e^{(4i\,dx+4i\,c)}-29i\,e^{(2i\,dx+2i\,c)}+3i}{384\,\left(a^4de^{(10i\,dx+10i\,c)}+a^4de^{(8i\,dx+8i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/384*(3096*d*x*e^(10*I*d*x + 10*I*c) + 24*(129*d*x - 68*I)*e^(8*I*d*x + 8
*I*c) + 1536*(I*e^(10*I*d*x + 10*I*c) + I*e^(8*I*d*x + 8*I*c))*log(e^(2*I*d
*x + 2*I*c) + 1) - 684*I*e^(6*I*d*x + 6*I*c) + 148*I*e^(4*I*d*x + 4*I*c) -
29*I*e^(2*I*d*x + 2*I*c) + 3*I)/(a^4*d*e^(10*I*d*x + 10*I*c) + a^4*d*e^(8*I
*d*x + 8*I*c))
```

Sympy [A]
time = 0.46, size = 248, normalized size = 1.45

$$\begin{cases} \frac{\left(442368ia^{12}d^3e^{18ic}e^{-2idx}-92160ia^{12}d^3e^{16ic}e^{-4idx}+16384ia^{12}d^3e^{14ic}e^{-6idx}-1536ia^{12}d^3e^{12ic}e^{-8idx}\right)e^{-20ic}}{196608a^{16}d^4} & \text{for } a^{16}d^4e^{20ic}\neq 0 \\ x\left(\frac{\left(-129e^{8ic}+72e^{6ic}-30e^{4ic}+8e^{2ic}-1\right)e^{-8ic}}{16a^4}+\frac{129}{16a^4}\right) & \text{otherwise} \end{cases} + \frac{2i}{a^4de^{2ic}e^{2idx}+a^4d} - \frac{129x}{16a^4} - \frac{4i\log\left(e^{2idx}+e^{-2ic}\right)}{a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((442368*I*a**12*d**3*exp(18*I*c)*exp(-2*I*d*x) - 92160*I*a**12*d
**3*exp(16*I*c)*exp(-4*I*d*x) + 16384*I*a**12*d**3*exp(14*I*c)*exp(-6*I*d*x
) - 1536*I*a**12*d**3*exp(12*I*c)*exp(-8*I*d*x))*exp(-20*I*c)/(196608*a**16
*d**4), Ne(a**16*d**4*exp(20*I*c), 0)), (x*((-129*exp(8*I*c) + 72*exp(6*I*c
) - 30*exp(4*I*c) + 8*exp(2*I*c) - 1)*exp(-8*I*c)/(16*a**4) + 129/(16*a**4)
), True)) + 2*I/(a**4*d*exp(2*I*c)*exp(2*I*d*x) + a**4*d) - 129*x/(16*a**4)
 - 4*I*log(exp(2*I*d*x) + exp(-2*I*c))/(a**4*d)
```

Giac [A]
time = 4.52, size = 100, normalized size = 0.58

$$-\frac{\frac{12i\,\log(\tan(dx+c)+i)}{a^4}-\frac{1548i\,\log(\tan(dx+c)-i)}{a^4}-\frac{384\,\tan(dx+c)}{a^4}-\frac{-3225i\,\tan(dx+c)^4-10236\,\tan(dx+c)^3+12534i\,\tan(dx+c)^2+6908\,\tan(dx+c)-1433i}{a^4(\tan(dx+c)-i)^4}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/384*(12*I*log(tan(d*x + c) + I)/a^4 - 1548*I*log(tan(d*x + c) - I)/a^4 -
 384*tan(d*x + c)/a^4 - (-3225*I*tan(d*x + c)^4 - 10236*tan(d*x + c)^3 + 12
534*I*tan(d*x + c)^2 + 6908*tan(d*x + c) - 1433*I)/(a^4*(tan(d*x + c) - I)^
4))/d
```

Mupad [B]
time = 3.91, size = 132, normalized size = 0.77

$$\frac{\tan(c+d\,x)}{a^4\,d}-\frac{65\,x}{16\,a^4}+\frac{\ln\left(\tan(c+d\,x)^2+1\right)\,2\mathrm{i}}{a^4\,d}-\frac{\frac{749\tan(c+d\,x)}{48\,a^4}-\frac{111\tan(c+d\,x)^3}{16\,a^4}-\frac{14\mathrm{i}}{3\,a^4}+\frac{\tan(c+d\,x)^2\,71\mathrm{i}}{4\,a^4}}{d\,\left(\tan\left(c+d\,x\right)^4-\tan\left(c+d\,x\right)^3\,4\mathrm{i}-6\tan\left(c+d\,x\right)^2+\tan\left(c+d\,x\right)\,4\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6/(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] tan(c + d*x)/(a^4*d) - (65*x)/(16*a^4) + (log(tan(c + d*x)^2 + 1)*2i)/(a^4*
d) - ((749*tan(c + d*x))/(48*a^4) - 14i/(3*a^4) + (tan(c + d*x)^2*71i)/(4*a
^4) - (111*tan(c + d*x)^3)/(16*a^4))/(d*(tan(c + d*x)*4i - 6*tan(c + d*x)^2
 - tan(c + d*x)^3*4i + tan(c + d*x)^4 + 1))
```

3.76 $\int \frac{\tan^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=147

$$\frac{15ix}{16a^4}-\frac{\log(\cos(c+dx))}{a^4d}+\frac{15}{16a^4d(1+i\tan(c+dx))}+\frac{7\tan^2(c+dx)}{16a^4d(1+i\tan(c+dx))^2}-\frac{\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4}+\frac{}{4a}$$

```
[Out] 15/16*I*x/a^4-ln(cos(d*x+c))/a^4/d+15/16/a^4/d/(1+I*tan(d*x+c))+7/16*tan(d*
x+c)^2/a^4/d/(1+I*tan(d*x+c))^2-1/8*tan(d*x+c)^4/d/(a+I*a*tan(d*x+c))^4+1/4
*I*tan(d*x+c)^3/a/d/(a+I*a*tan(d*x+c))^3
```

Rubi [A]
time = 0.24, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {3639, 3676, 3670, 3556, 12, 3607, 8}

$$\frac{7\tan^2(c+dx)}{16a^4d(1+i\tan(c+dx))^2}+\frac{15}{16a^4d(1+i\tan(c+dx))}-\frac{\log(\cos(c+dx))}{a^4d}+\frac{15ix}{16a^4}-\frac{\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4}+\frac{i\tan^3(c+dx)}{4ad(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (((15*I)/16)*x)/a^4 - Log[Cos[c + d*x]]/(a^4*d) + 15/(16*a^4*d*(1 + I*Tan[c
 + d*x])) + (7*Tan[c + d*x]^2)/(16*a^4*d*(1 + I*Tan[c + d*x])^2) - Tan[c +
d*x]^4/(8*d*(a + I*a*Tan[c + d*x])^4) + ((I/4)*Tan[c + d*x]^3)/(a*d*(a + I*
a*Tan[c + d*x])^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
```

```
0] && LtQ[m, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3670

```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)]))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[B*(d/
b), Int[Tan[e + f*x], x], x] + Dist[1/b, Int[Simp[A*b*c + (A*b*d + B*(b*c -
 a*d))*Tan[e + f*x], x]/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d,
e, f, A, B}, x] && NeQ[b*c - a*d, 0]
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4} - \frac{\int \frac{\tan^3(c+dx)(-4a+8ia\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx}{8a^2}$$
$$= -\frac{\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4} + \frac{i\tan^3(c+dx)}{4ad(a+ia\tan(c+dx))^3} + \frac{\int \frac{\tan^2(c+dx)(-36ia^2-48a}{(a+ia\tan(c+dx}}{48a^4}$$
$$= \frac{7\tan^2(c+dx)}{16a^4d(1+i\tan(c+dx))^2} - \frac{\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4} + \frac{i\tan^3(c+dx)}{4ad(a+ia\tan(c+dx}$$
$$= \frac{7\tan^2(c+dx)}{16a^4d(1+i\tan(c+dx))^2} - \frac{\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4} + \frac{i\tan^3(c+dx)}{4ad(a+ia\tan(c+dx}$$
$$= -\frac{\log(\cos(c+dx))}{a^4d} + \frac{7\tan^2(c+dx)}{16a^4d(1+i\tan(c+dx))^2} - \frac{\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4} +$$
$$= -\frac{\log(\cos(c+dx))}{a^4d} + \frac{7\tan^2(c+dx)}{16a^4d(1+i\tan(c+dx))^2} - \frac{\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4} +$$
$$= \frac{15ix}{16a^4} - \frac{\log(\cos(c+dx))}{a^4d} + \frac{7\tan^2(c+dx)}{16a^4d(1+i\tan(c+dx))^2} - \frac{\tan^4(c+dx)}{8d(a+ia\tan(c+dx)}$$

Mathematica [A]

time = 0.50, size = 126, normalized size = 0.86

$$\frac{\sec^4(c+dx)(112\cos(2(c+dx))+i(32i+\cos(4(c+dx))(i+120dx+128i\log(\cos(c+dx)))+96\sin(2(c+dx))+\sin(4(c+dx))+120idx\sin(4(c+dx))-128\log(\cos(c+dx))\sin(4(c+dx))))}{128a^4d(-i+\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^4*(112*Cos[2*(c + d*x)] + I*(32*I + Cos[4*(c + d*x)]*(I + 120
*d*x + (128*I)*Log[Cos[c + d*x]]) + 96*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]
+ (120*I)*d*x*Sin[4*(c + d*x)] - 128*Log[Cos[c + d*x]]*Sin[4*(c + d*x)])))/
(128*a^4*d*(-I + Tan[c + d*x])^4)
```

Maple [A]

time = 0.18, size = 87, normalized size = 0.59

method	result
derivativedivides	$\frac{\frac{3i}{4(\tan(dx+c)-i)^3}-\frac{49i}{16(\tan(dx+c)-i)}-\frac{1}{8(\tan(dx+c)-i)^4}+\frac{31}{16(\tan(dx+c)-i)^2}+\frac{31\ln(\tan(dx+c)-i)}{32}+\frac{\ln(\tan(dx+c)+i)}{32}}{d\,a^4}$
default	$\frac{\frac{3i}{4(\tan(dx+c)-i)^3}-\frac{49i}{16(\tan(dx+c)-i)}-\frac{1}{8(\tan(dx+c)-i)^4}+\frac{31}{16(\tan(dx+c)-i)^2}+\frac{31\ln(\tan(dx+c)-i)}{32}+\frac{\ln(\tan(dx+c)+i)}{32}}{d\,a^4}$
risch	$\frac{31ix}{16a^4}+\frac{13\,e^{-2i(dx+c)}}{16a^4d}-\frac{e^{-4i(dx+c)}}{4a^4d}+\frac{e^{-6i(dx+c)}}{16a^4d}-\frac{e^{-8i(dx+c)}}{128a^4d}+\frac{2ic}{a^4d}-\frac{\ln\left(e^{2i(dx+c)}+1\right)}{a^4d}$

norman	$\frac{\frac{5\left(\tan^6(dx+c)\right)}{da}+\frac{7}{4da}+\frac{15ix}{16a}+\frac{35\left(\tan^4(dx+c)\right)}{4da}+\frac{13\left(\tan^2(dx+c)\right)}{2da}+\frac{15ix\left(\tan^2(dx+c)\right)}{4a}+\frac{45ix\left(\tan^4(dx+c)\right)}{8a}+\frac{15ix\left(\tan^6(dx+c)\right)}{4a}}{\left(1+\tan^2(dx+c)\right)^4a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^4*(3/4*I/(tan(d*x+c)-I)^3-49/16*I/(tan(d*x+c)-I)-1/8/(tan(d*x+c)-I)^4
+31/16/(tan(d*x+c)-I)^2+31/32*ln(tan(d*x+c)-I)+1/32*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.39, size = 88, normalized size = 0.60

$$\frac{\left(248i\,dxe^{(8i\,dx+8i\,c)}-128\,e^{(8i\,dx+8i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+104\,e^{(6i\,dx+6i\,c)}-32\,e^{(4i\,dx+4i\,c)}+8\,e^{(2i\,dx+2i\,c)}-1\right)e^{(-8i\,dx-8i\,c)}}{128\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/128*(248*I*d*x*e^(8*I*d*x + 8*I*c) - 128*e^(8*I*d*x + 8*I*c)*log(e^(2*I*d
*x + 2*I*c) + 1) + 104*e^(6*I*d*x + 6*I*c) - 32*e^(4*I*d*x + 4*I*c) + 8*e^(
2*I*d*x + 2*I*c) - 1)*e^(-8*I*d*x - 8*I*c)/(a^4*d)
```

Sympy [A]
time = 1.83, size = 219, normalized size = 1.49

$$\begin{cases}\frac{\left(106496a^{12}d^3e^{18ic}e^{-2idx}-32768a^{12}d^3e^{16ic}e^{-4idx}+8192a^{12}d^3e^{14ic}e^{-6idx}-1024a^{12}d^3e^{12ic}e^{-8idx}\right)e^{-20ic}}{131072a^{16}d^4} & \text{for } a^{16}d^4e^{20ic}\neq 0\\ x\left(\frac{\left(31ie^{8ic}-26ie^{6ic}+16ie^{4ic}-6ie^{2ic}+i\right)e^{-8ic}}{16a^4}-\frac{31i}{16a^4}\right) & \text{otherwise}\end{cases}+\frac{31ix}{16a^4}-\frac{\log\left(e^{2idx}+e^{-2ic}\right)}{a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((106496*a**12*d**3*exp(18*I*c)*exp(-2*I*d*x) - 32768*a**12*d**3*
exp(16*I*c)*exp(-4*I*d*x) + 8192*a**12*d**3*exp(14*I*c)*exp(-6*I*d*x) - 102
4*a**12*d**3*exp(12*I*c)*exp(-8*I*d*x))*exp(-20*I*c)/(131072*a**16*d**4), N
```

```
e(a**16*d**4*exp(20*I*c), 0)), (x*((31*I*exp(8*I*c) - 26*I*exp(6*I*c) + 16*
I*exp(4*I*c) - 6*I*exp(2*I*c) + I)*exp(-8*I*c)/(16*a**4) - 31*I/(16*a**4)),
 True)) + 31*I*x/(16*a**4) - log(exp(2*I*d*x) + exp(-2*I*c))/(a**4*d)
```

Giac [A]
time = 2.08, size = 89, normalized size = 0.61

$$\frac{\frac{12 \log(\tan(dx+c)+i)}{a^4} + \frac{372 \log(\tan(dx+c)-i)}{a^4} - \frac{775 \tan(dx+c)^4 - 1924i \tan(dx+c)^3 - 1866 \tan(dx+c)^2 + 772i \tan(dx+c) + 103}{a^4(\tan(dx+c)-i)^4}}{384\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/384*(12*log(tan(d*x + c) + I)/a^4 + 372*log(tan(d*x + c) - I)/a^4 - (775*
tan(d*x + c)^4 - 1924*I*tan(d*x + c)^3 - 1866*tan(d*x + c)^2 + 772*I*tan(d*
x + c) + 103)/(a^4*(tan(d*x + c) - I)^4))/d
```

Mupad [B]
time = 3.98, size = 128, normalized size = 0.87

$$\frac{31\,\ln\left(\tan(c+d\,x)-\mathrm{i}\right)}{32\,a^4\,d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)}{32\,a^4\,d}+\frac{\frac{\tan(c+d\,x)\,97\mathrm{i}}{16\,a^4}+\frac{7}{4\,a^4}-\frac{29\tan(c+d\,x)^2}{4\,a^4}-\frac{\tan(c+d\,x)^3\,49\mathrm{i}}{16\,a^4}}{d\,\left(\tan\left(c+d\,x\right)^4-\tan\left(c+d\,x\right)^3\,4\mathrm{i}-6\tan\left(c+d\,x\right)^2+\tan\left(c+d\,x\right)\,4\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] (31*log(tan(c + d*x) - 1i))/(32*a^4*d) + log(tan(c + d*x) + 1i)/(32*a^4*d)
+ ((tan(c + d*x)*97i)/(16*a^4) + 7/(4*a^4) - (29*tan(c + d*x)^2)/(4*a^4) -
(tan(c + d*x)^3*49i)/(16*a^4))/(d*(tan(c + d*x)*4i - 6*tan(c + d*x)^2 - tan
(c + d*x)^3*4i + tan(c + d*x)^4 + 1))
```

3.77 $\int \frac{\tan^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=128

$$\frac{x}{16a^4}-\frac{3i}{16a^4d(1+i\tan(c+dx))}+\frac{i\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4}+\frac{\tan^3(c+dx)}{12ad(a+ia\tan(c+dx))^3}+\frac{i}{16d\left(a^2+ia^2\tan(c\right.}$$

```
[Out] 1/16*x/a^4-3/16*I/a^4/d/(1+I*tan(d*x+c))+1/8*I*tan(d*x+c)^4/d/(a+I*a*tan(d*
x+c))^4+1/12*tan(d*x+c)^3/a/d/(a+I*a*tan(d*x+c))^3+1/16*I/d/(a^2+I*a^2*tan(
d*x+c))^2
```

Rubi [A]

time = 0.12, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3627, 3621, 3607, 8}

$$-\frac{3i}{16a^4d(1+i\tan(c+dx))}+\frac{x}{16a^4}+\frac{i}{16d\left(a^2+ia^2\tan(c+dx)\right)^2}+\frac{i\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4}+\frac{\tan^3(c+dx)}{12ad(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] x/(16*a^4) - ((3*I)/16)/(a^4*d*(1 + I*Tan[c + d*x])) + ((I/8)*Tan[c + d*x]^
4)/(d*(a + I*a*Tan[c + d*x])^4) + Tan[c + d*x]^3/(12*a*d*(a + I*a*Tan[c + d
*x])^3) + (I/16)/(d*(a^2 + I*a^2*Tan[c + d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rule 3621

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(a*c + b*d)^2*((a + b*Tan[e + f*x])^
m/(2*a^3*f*m)), x] + Dist[1/(2*a^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[
a*c^2 - 2*b*c*d + a*d^2 - 2*b*d^2*Tan[e + f*x], x], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LeQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Rule 3627

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^n/(2*b*f*m)), x] - Dist[(a*c - b*d)/(2*b^2), Int[(a + b*Tan[e + f*
x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && EqQ
[m + n, 0] && LeQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{i\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4} - \frac{i\int \frac{\tan^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx}{2a}$$
$$= \frac{i\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4} + \frac{\tan^3(c+dx)}{12ad(a+ia\tan(c+dx))^3} - \frac{\int \frac{\tan^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{4a^2}$$
$$= \frac{i\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4} + \frac{\tan^3(c+dx)}{12ad(a+ia\tan(c+dx))^3} + \frac{i}{16d\,(a^2+ia^2\tan(c+$$
$$= -\frac{3i}{16a^4d(1+i\tan(c+dx))} + \frac{i\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4} + \frac{\tan^3(c+dx)}{12ad(a+ia\tan(c+$$
$$= \frac{x}{16a^4} - \frac{3i}{16a^4d(1+i\tan(c+dx))} + \frac{i\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4} + \frac{\tan^3(c+}{12ad(a+ia\tan}$$

Mathematica [A]
time = 0.27, size = 98, normalized size = 0.77

$$\frac{\sec^4(c+dx)(36i-64i\cos(2(c+dx))+3(i+8dx)\cos(4(c+dx))+32\sin(2(c+dx))+3\sin(4(c+dx))+24idx\sin(4(c+dx)))}{384a^4d(-i+\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^4*(36*I - (64*I)*Cos[2*(c + d*x)] + 3*(I + 8*d*x)*Cos[4*(c +
d*x)] + 32*Sin[2*(c + d*x)] + 3*Sin[4*(c + d*x)] + (24*I)*d*x*Sin[4*(c + d*
x)]))/(384*a^4*d*(-I + Tan[c + d*x])^4)
```

Maple [A]
time = 0.16, size = 89, normalized size = 0.70

method	result
risch	$\frac{x}{16a^4}-\frac{i e^{-2i(dx+c)}}{8a^4d}+\frac{3i e^{-4i(dx+c)}}{32a^4d}-\frac{i e^{-6i(dx+c)}}{24a^4d}+\frac{i e^{-8i(dx+c)}}{128a^4d}$

derivativedivides	$\frac{\frac{i}{8(\tan(dx+c)-i)^4}-\frac{i\ln(\tan(dx+c)-i)}{32}-\frac{17i}{16(\tan(dx+c)-i)^2}+\frac{7}{12(\tan(dx+c)-i)^3}-\frac{15}{16(\tan(dx+c)-i)}+\frac{i\ln(\tan(dx+c)+i)}{32}}{d\,a^4}$
default	$\frac{\frac{i}{8(\tan(dx+c)-i)^4}-\frac{i\ln(\tan(dx+c)-i)}{32}-\frac{17i}{16(\tan(dx+c)-i)^2}+\frac{7}{12(\tan(dx+c)-i)^3}-\frac{15}{16(\tan(dx+c)-i)}+\frac{i\ln(\tan(dx+c)+i)}{32}}{d\,a^4}$
norman	$\frac{\frac{x}{16a}-\frac{5\left(\tan^5(dx+c)\right)}{48da}-\frac{15\left(\tan^7(dx+c)\right)}{16da}+\frac{x\left(\tan^2(dx+c)\right)}{4a}+\frac{3x\left(\tan^4(dx+c)\right)}{8a}+\frac{x\left(\tan^6(dx+c)\right)}{4a}+\frac{x\left(\tan^8(dx+c)\right)}{16a}-\frac{i}{3da}-\frac{\tan(dx+}{16da}}{(1+\tan^2(dx+c))^4a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^4*(1/8*I/(tan(d*x+c)-I)^4-1/32*I*ln(tan(d*x+c)-I)-17/16*I/(tan(d*x+c)
-I)^2+7/12/(tan(d*x+c)-I)^3-15/16/(tan(d*x+c)-I)+1/32*I*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.37, size = 65, normalized size = 0.51

$$\frac{\left(24\,dxe^{(8i\,dx+8i\,c)}-48i\,e^{(6i\,dx+6i\,c)}+36i\,e^{(4i\,dx+4i\,c)}-16i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-8i\,dx-8i\,c)}}{384\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/384*(24*d*x*e^(8*I*d*x + 8*I*c) - 48*I*e^(6*I*d*x + 6*I*c) + 36*I*e^(4*I*
d*x + 4*I*c) - 16*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-8*I*d*x - 8*I*c)/(a^4*d)
```

Sympy [A]
time = 0.27, size = 189, normalized size = 1.48

$$\begin{cases} \frac{\left(-98304ia^{12}d^3e^{18ic}e^{-2idx}+73728ia^{12}d^3e^{16ic}e^{-4idx}-32768ia^{12}d^3e^{14ic}e^{-6idx}+6144ia^{12}d^3e^{12ic}e^{-8idx}\right)e^{-20ic}}{786432a^{16}d^4} & \text{for } a^{16}d^4e^{20ic}\neq 0 \\ x\left(\frac{\left(e^{8ic}-4e^{6ic}+6e^{4ic}-4e^{2ic}+1\right)e^{-8ic}}{16a^4}-\frac{1}{16a^4}\right) & \text{otherwise} \end{cases}+\frac{x}{16a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((-98304*I*a**12*d**3*exp(18*I*c)*exp(-2*I*d*x) + 73728*I*a**12*d
**3*exp(16*I*c)*exp(-4*I*d*x) - 32768*I*a**12*d**3*exp(14*I*c)*exp(-6*I*d*x
) + 6144*I*a**12*d**3*exp(12*I*c)*exp(-8*I*d*x))*exp(-20*I*c)/(786432*a**16
*d**4), Ne(a**16*d**4*exp(20*I*c), 0)), (x*((exp(8*I*c) - 4*exp(6*I*c) + 6*
exp(4*I*c) - 4*exp(2*I*c) + 1)*exp(-8*I*c)/(16*a**4) - 1/(16*a**4)), True))
 + x/(16*a**4)
```

Giac [A]
time = 1.39, size = 92, normalized size = 0.72

$$-\frac{-\frac{12i\log(-i\tan(dx+c)+1)}{a^4}+\frac{12i\log(-i\tan(dx+c)-1)}{a^4}+\frac{-25i\tan(dx+c)^4+260\tan(dx+c)^3-522i\tan(dx+c)^2-388\tan(dx+c)+103i}{a^4(\tan(dx+c)-i)^4}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/384*(-12*I*log(-I*tan(d*x + c) + 1)/a^4 + 12*I*log(-I*tan(d*x + c) - 1)/
a^4 + (-25*I*tan(d*x + c)^4 + 260*tan(d*x + c)^3 - 522*I*tan(d*x + c)^2 - 3
88*tan(d*x + c) + 103*I)/(a^4*(tan(d*x + c) - I)^4))/d
```

Mupad [B]
time = 3.94, size = 59, normalized size = 0.46

$$\frac{x}{16\,a^4}+\frac{-\frac{15\tan(c+d\,x)^3}{16}+\frac{\tan(c+d\,x)^2\,7\mathrm{i}}{4}+\frac{61\tan(c+d\,x)}{48}-\frac{1}{3}\mathrm{i}}{a^4\,d\,(1+\tan\left(c+d\,x\right)\,1\mathrm{i})^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] x/(16*a^4) + ((61*tan(c + d*x))/48 + (tan(c + d*x)^2*7i)/4 - (15*tan(c + d*
x)^3)/16 - 1i/3)/(a^4*d*(tan(c + d*x)*1i + 1)^4)
```

3.78 $\int \frac{\tan^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=126

$$\frac{ix}{16a^4}+\frac{3}{16a^4d(1+i\tan(c+dx))}+\frac{\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4}+\frac{i\tan^3(c+dx)}{12ad(a+ia\tan(c+dx))^3}-\frac{1}{16d\,(a^2+ia^2\tan(c$$

```
[Out] 1/16*I*x/a^4+3/16/a^4/d/(1+I*tan(d*x+c))+1/8*tan(d*x+c)^4/d/(a+I*a*tan(d*x+
c))^4+1/12*I*tan(d*x+c)^3/a/d/(a+I*a*tan(d*x+c))^3-1/16/d/(a^2+I*a^2*tan(d*
x+c))^2
```

Rubi [A]
time = 0.13, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3628, 3627, 3621, 3607, 8}

$$\frac{3}{16a^4d(1+i\tan(c+dx))}+\frac{ix}{16a^4}-\frac{1}{16d\,(a^2+ia^2\tan(c+dx))^2}+\frac{\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4}+\frac{i\tan^3(c+dx)}{12ad(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((I/16)*x)/a^4 + 3/(16*a^4*d*(1 + I*Tan[c + d*x])) + Tan[c + d*x]^4/(8*d*(a
 + I*a*Tan[c + d*x])^4) + ((I/12)*Tan[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x
])^3) - 1/(16*d*(a^2 + I*a^2*Tan[c + d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rule 3621

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(a*c + b*d)^2*((a + b*Tan[e + f*x])^
m/(2*a^3*f*m)), x] + Dist[1/(2*a^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[
a*c^2 - 2*b*c*d + a*d^2 - 2*b*d^2*Tan[e + f*x], x], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LeQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Rule 3627

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^n/(2*b*f*m)), x] - Dist[(a*c - b*d)/(2*b^2), Int[(a + b*Tan[e + f*
x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && EqQ
[m + n, 0] && LeQ[m, -2^(-1)]
```

Rule 3628

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a), Int[(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && EqQ[m
 + n + 1, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4} + \frac{\int \frac{\tan^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx}{2a}$$

$$= \frac{\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4} + \frac{i\tan^3(c+dx)}{12ad(a+ia\tan(c+dx))^3} - \frac{i\int \frac{\tan^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{4a^2}$$

$$= \frac{\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4} + \frac{i\tan^3(c+dx)}{12ad(a+ia\tan(c+dx))^3} - \frac{1}{16d\left(a^2+ia^2\tan(c+\right.}$$

$$= \frac{3}{16a^4d(1+i\tan(c+dx))} + \frac{\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4} + \frac{i\tan^3(c+dx)}{12ad(a+ia\tan(c+dx}$$

$$= \frac{ix}{16a^4} + \frac{3}{16a^4d(1+i\tan(c+dx))} + \frac{\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4} + \frac{i\tan^3(c+}{12ad(a+ia\tan}$$

Mathematica [A]
time = 0.49, size = 95, normalized size = 0.75

$$\frac{\sec^4(c+dx)(16\cos(2(c+dx))+3(1+8idx)\cos(4(c+dx))+32i\sin(2(c+dx))-3i\sin(4(c+dx))-24dx\sin(4(c+dx)))}{384a^4d(-i+\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^4*(16*Cos[2*(c + d*x)] + 3*(1 + (8*I)*d*x)*Cos[4*(c + d*x)] +
 (32*I)*Sin[2*(c + d*x)] - (3*I)*Sin[4*(c + d*x)] - 24*d*x*Sin[4*(c + d*x)]
))/(384*a^4*d*(-I + Tan[c + d*x])^4)
```

Maple [A]
time = 0.15, size = 87, normalized size = 0.69

method	result
risch	$\frac{ix}{16a^4}+\frac{e^{-2i(dx+c)}}{16a^4d}-\frac{e^{-6i(dx+c)}}{48a^4d}+\frac{e^{-8i(dx+c)}}{128a^4d}$
derivativedivides	$\frac{\frac{i}{16\tan(dx+c)-16i}-\frac{5i}{12(\tan(dx+c)-i)^3}+\frac{1}{8(\tan(dx+c)-i)^4}-\frac{7}{16(\tan(dx+c)-i)^2}+\frac{\ln(\tan(dx+c)-i)}{32}-\frac{\ln(\tan(dx+c)+i)}{32}}{d\,a^4}$
default	$\frac{\frac{i}{16\tan(dx+c)-16i}-\frac{5i}{12(\tan(dx+c)-i)^3}+\frac{1}{8(\tan(dx+c)-i)^4}-\frac{7}{16(\tan(dx+c)-i)^2}+\frac{\ln(\tan(dx+c)-i)}{32}-\frac{\ln(\tan(dx+c)+i)}{32}}{d\,a^4}$
norman	$\frac{\frac{1}{12da}+\frac{ix}{16a}-\frac{\tan^6(dx+c)}{2da}+\frac{3\left(\tan^4(dx+c)\right)}{4da}+\frac{\tan^2(dx+c)}{3da}+\frac{ix\left(\tan^2(dx+c)\right)}{4a}+\frac{3ix\left(\tan^4(dx+c)\right)}{8a}+\frac{ix\left(\tan^6(dx+c)\right)}{4a}+\frac{ix\left(\tan^8(dx+\right.}{16a}}{\left(1+\tan^2(dx+c)\right)^4a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^4*(1/16*I/(tan(d*x+c)-I)-5/12*I/(tan(d*x+c)-I)^3+1/8/(tan(d*x+c)-I)^4
-7/16/(tan(d*x+c)-I)^2+1/32*ln(tan(d*x+c)-I)-1/32*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.37, size = 54, normalized size = 0.43

$$\frac{\left(24i\,dxe^{(8i\,dx+8i\,c)}+24\,e^{(6i\,dx+6i\,c)}-8\,e^{(2i\,dx+2i\,c)}+3\right)e^{(-8i\,dx-8i\,c)}}{384\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/384*(24*I*d*x*e^(8*I*d*x + 8*I*c) + 24*e^(6*I*d*x + 6*I*c) - 8*e^(2*I*d*x
 + 2*I*c) + 3)*e^(-8*I*d*x - 8*I*c)/(a^4*d)
```

Sympy [A]
time = 0.61, size = 156, normalized size = 1.24

$$\begin{cases} \frac{(6144a^8d^2e^{14ic}e^{-2idx}-2048a^8d^2e^{10ic}e^{-6idx}+768a^8d^2e^{8ic}e^{-8idx})e^{-16ic}}{98304a^{12}d^3} & \text{for } a^{12}d^3e^{16ic}\neq 0 \\ x\left(\frac{(ie^{8ic}-2ie^{6ic}+2ie^{2ic}-i)e^{-8ic}}{16a^4}-\frac{i}{16a^4}\right) & \text{otherwise} \end{cases} + \frac{ix}{16a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((6144*a**8*d**2*exp(14*I*c)*exp(-2*I*d*x) - 2048*a**8*d**2*exp(1
0*I*c)*exp(-6*I*d*x) + 768*a**8*d**2*exp(8*I*c)*exp(-8*I*d*x))*exp(-16*I*c)
/(98304*a**12*d**3), Ne(a**12*d**3*exp(16*I*c), 0)), (x*((I*exp(8*I*c) - 2*
I*exp(6*I*c) + 2*I*exp(2*I*c) - I)*exp(-8*I*c)/(16*a**4) - I/(16*a**4)), Tr
ue)) + I*x/(16*a**4)
```

Giac [A]
time = 1.17, size = 88, normalized size = 0.70

$$-\frac{\frac{12\log(\tan(dx+c)+i)}{a^4}-\frac{12\log(\tan(dx+c)-i)}{a^4}+\frac{25\tan(dx+c)^4-124i\tan(dx+c)^3-54\tan(dx+c)^2-4i\tan(dx+c)-7}{a^4(\tan(dx+c)-i)^4}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/384*(12*log(tan(d*x + c) + I)/a^4 - 12*log(tan(d*x + c) - I)/a^4 + (25*t
an(d*x + c)^4 - 124*I*tan(d*x + c)^3 - 54*tan(d*x + c)^2 - 4*I*tan(d*x + c)
 - 7)/(a^4*(tan(d*x + c) - I)^4))/d
```

Mupad [B]
time = 3.90, size = 60, normalized size = 0.48

$$\frac{x\,1\mathrm{i}}{16\,a^4}+\frac{\frac{\tan(c+d\,x)^3\,1\mathrm{i}}{16}-\frac{\tan(c+d\,x)^2}{4}+\frac{\tan(c+d\,x)\,13\mathrm{i}}{48}+\frac{1}{12}}{a^4\,d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] (x*1i)/(16*a^4) + ((tan(c + d*x)*13i)/48 - tan(c + d*x)^2/4 + (tan(c + d*x)
^3*1i)/16 + 1/12)/(a^4*d*(tan(c + d*x)*1i + 1)^4)
```

3.79 $\int \frac{\tan^2(c+dx)}{(a+ia\tan(c+dx))^4} dx$

Optimal. Leaf size=116

$$-\frac{x}{16a^4}-\frac{i}{8d(a+ia\tan(c+dx))^4}+\frac{i}{4ad(a+ia\tan(c+dx))^3}-\frac{i}{16d\left(a^2+ia^2\tan(c+dx)\right)^2}-\frac{i}{16d\left(a^4+ia^4\tan(c+dx)\right)}$$

```
[Out] -1/16*x/a^4-1/8*I/d/(a+I*a*tan(d*x+c))^4+1/4*I/a/d/(a+I*a*tan(d*x+c))^3-1/1
6*I/d/(a^2+I*a^2*tan(d*x+c))^2-1/16*I/d/(a^4+I*a^4*tan(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3621, 3607, 3560, 8}

$$-\frac{i}{16d\left(a^4+ia^4\tan(c+dx)\right)}-\frac{x}{16a^4}-\frac{i}{16d\left(a^2+ia^2\tan(c+dx)\right)^2}+\frac{i}{4ad(a+ia\tan(c+dx))^3}-\frac{i}{8d(a+ia\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] -1/16*x/a^4 - (I/8)/(d*(a + I*a*Tan[c + d*x])^4) + (I/4)/(a*d*(a + I*a*Tan[
c + d*x])^3) - (I/16)/(d*(a^2 + I*a^2*Tan[c + d*x])^2) - (I/16)/(d*(a^4 + I
*a^4*Tan[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rule 3621

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(a*c + b*d)^2*((a + b*Tan[e + f*x])^
```

```
m/(2*a^3*f*m)), x] + Dist[1/(2*a^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[
a*c^2 - 2*b*c*d + a*d^2 - 2*b*d^2*Tan[e + f*x], x], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LeQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx &= -\frac{i}{8d(a+ia\tan(c+dx))^4} + \frac{\int \frac{a-2ia\tan(c+dx)}{(a+ia\tan(c+dx))^3}\,dx}{2a^2} \\
&= -\frac{i}{8d(a+ia\tan(c+dx))^4} + \frac{i}{4ad(a+ia\tan(c+dx))^3} - \frac{\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx}{4a^2} \\
&= -\frac{i}{8d(a+ia\tan(c+dx))^4} + \frac{i}{4ad(a+ia\tan(c+dx))^3} - \frac{i}{16d\left(a^2+ia^2\tan(c+\right.} \\
&= -\frac{i}{8d(a+ia\tan(c+dx))^4} + \frac{i}{4ad(a+ia\tan(c+dx))^3} - \frac{i}{16d\left(a^2+ia^2\tan(c+\right.} \\
&= -\frac{x}{16a^4} - \frac{i}{8d(a+ia\tan(c+dx))^4} + \frac{i}{4ad(a+ia\tan(c+dx))^3} - \frac{}{16d\left(a^2+ia^2\right.}
\end{aligned}$$

Mathematica [A]
time = 0.37, size = 69, normalized size = 0.59

$$-\frac{(\cos(4(c+dx))-i\sin(4(c+dx)))(-4i+(i+8dx)\cos(4(c+dx))+(1+8idx)\sin(4(c+dx)))}{128a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + I*a*Tan[c + d*x])^4,x]
[Out] -1/128*((Cos[4*(c + d*x)] - I*Sin[4*(c + d*x)])*(-4*I + (I + 8*d*x)*Cos[4*(
c + d*x)] + (1 + (8*I)*d*x)*Sin[4*(c + d*x)]))/(a^4*d)
```

Maple [A]
time = 0.15, size = 89, normalized size = 0.77

method	result
risch	$-\frac{x}{16a^4}+\frac{i e^{-4i(dx+c)}}{32a^4d}-\frac{i e^{-8i(dx+c)}}{128a^4d}$
derivativedivides	$\frac{\frac{i}{16(\tan(dx+c)-i)^2}-\frac{i}{8(\tan(dx+c)-i)^4}+\frac{i\ln(\tan(dx+c)-i)}{32}-\frac{1}{4(\tan(dx+c)-i)^3}-\frac{1}{16(\tan(dx+c)-i)}-\frac{i\ln(\tan(dx+c)+i)}{32}}{d\,a^4}$
default	$\frac{\frac{i}{16(\tan(dx+c)-i)^2}-\frac{i}{8(\tan(dx+c)-i)^4}+\frac{i\ln(\tan(dx+c)-i)}{32}-\frac{1}{4(\tan(dx+c)-i)^3}-\frac{1}{16(\tan(dx+c)-i)}-\frac{i\ln(\tan(dx+c)+i)}{32}}{d\,a^4}$

norman	$\frac{-\frac{i\left(\tan^4(dx+c)\right)}{da}-\frac{x}{16a}-\frac{9\left(\tan^5(dx+c)\right)}{16da}-\frac{\tan^7(dx+c)}{16da}-\frac{x\left(\tan^2(dx+c)\right)}{4a}-\frac{3x\left(\tan^4(dx+c)\right)}{8a}-\frac{x\left(\tan^6(dx+c)\right)}{4a}-\frac{x\left(\tan^8(dx+c)\right)}{16a}+}{\left(1+\tan^2(dx+c)\right)^4a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^4*(1/16*I/(tan(d*x+c)-I)^2-1/8*I/(tan(d*x+c)-I)^4+1/32*I*ln(tan(d*x+c
)-I)-1/4/(tan(d*x+c)-I)^3-1/16/(tan(d*x+c)-I)-1/32*I*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.37, size = 43, normalized size = 0.37

$$-\frac{\left(8\,dxe^{(8i\,dx+8i\,c)}-4i\,e^{(4i\,dx+4i\,c)}+i\right)e^{(-8i\,dx-8i\,c)}}{128\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/128*(8*d*x*e^(8*I*d*x + 8*I*c) - 4*I*e^(4*I*d*x + 4*I*c) + I)*e^(-8*I*d*
x - 8*I*c)/(a^4*d)
```

Sympy [A]
time = 0.20, size = 117, normalized size = 1.01

$$\begin{cases}\frac{\left(128ia^4de^{8ic}e^{-4idx}-32ia^4de^{4ic}e^{-8idx}\right)e^{-12ic}}{4096a^8d^2} & \text{for } a^8d^2e^{12ic}\neq 0\\ x\left(\frac{\left(-e^{8ic}+2e^{4ic}-1\right)e^{-8ic}}{16a^4}+\frac{1}{16a^4}\right) & \text{otherwise}\end{cases}-\frac{x}{16a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((128*I*a**4*d*exp(8*I*c)*exp(-4*I*d*x) - 32*I*a**4*d*exp(4*I*c)*
exp(-8*I*d*x))*exp(-12*I*c)/(4096*a**8*d**2), Ne(a**8*d**2*exp(12*I*c), 0))
```

```
, (x*((-exp(8*I*c) + 2*exp(4*I*c) - 1)*exp(-8*I*c)/(16*a**4) + 1/(16*a**4))
, True)) - x/(16*a**4)
```

Giac [A]
time = 1.10, size = 87, normalized size = 0.75

$$-\frac{\frac{2i\log(-i\tan(2dx+2c)+1)}{a^4}-\frac{2i\log(-i\tan(2dx+2c)-1)}{a^4}+\frac{3i\tan(2dx+2c)^2-6\tan(2dx+2c)+5i}{a^4(\tan(2dx+2c)-i)^2}}{128d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/128*(2*I*log(-I*tan(2*d*x + 2*c) + 1)/a^4 - 2*I*log(-I*tan(2*d*x + 2*c)
- 1)/a^4 + (3*I*tan(2*d*x + 2*c)^2 - 6*tan(2*d*x + 2*c) + 5*I)/(a^4*(tan(2*
d*x + 2*c) - I)^2))/d
```

Mupad [B]
time = 3.90, size = 92, normalized size = 0.79

$$-\frac{x}{16a^4}+\frac{\frac{\tan(c+dx)}{16a^4}-\frac{\tan(c+dx)^3}{16a^4}+\frac{\tan(c+dx)^2\,1\mathrm{i}}{4a^4}}{d\left(\tan(c+dx)^4-\tan(c+dx)^3\,4\mathrm{i}-6\tan(c+dx)^2+\tan(c+dx)\,4\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] (tan(c + d*x)/(16*a^4) + (tan(c + d*x)^2*1i)/(4*a^4) - tan(c + d*x)^3/(16*a
^4))/(d*(tan(c + d*x)*4i - 6*tan(c + d*x)^2 - tan(c + d*x)^3*4i + tan(c + d
*x)^4 + 1)) - x/(16*a^4)
```

3.80 $\int \frac{\tan(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=110

$$-\frac{ix}{16a^4}-\frac{1}{8d(a+ia\tan(c+dx))^4}+\frac{1}{12ad(a+ia\tan(c+dx))^3}+\frac{1}{16d\left(a^2+ia^2\tan(c+dx)\right)^2}+\frac{1}{16d\left(a^4+ia^4\tan\right.}$$

```
[Out] -1/16*I*x/a^4-1/8/d/(a+I*a*tan(d*x+c))^4+1/12/a/d/(a+I*a*tan(d*x+c))^3+1/16
/d/(a^2+I*a^2*tan(d*x+c))^2+1/16/d/(a^4+I*a^4*tan(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3607, 3560, 8}

$$\frac{1}{16d\left(a^4+ia^4\tan(c+dx)\right)}-\frac{ix}{16a^4}+\frac{1}{16d\left(a^2+ia^2\tan(c+dx)\right)^2}+\frac{1}{12ad(a+ia\tan(c+dx))^3}-\frac{1}{8d(a+ia\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((-1/16*I)*x)/a^4 - 1/(8*d*(a + I*a*Tan[c + d*x])^4) + 1/(12*a*d*(a + I*a*T
an[c + d*x])^3) + 1/(16*d*(a^2 + I*a^2*Tan[c + d*x])^2) + 1/(16*d*(a^4 + I*
a^4*Tan[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{1}{8d(a+ia\tan(c+dx))^4} - \frac{i\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx}{2a}$$
$$= -\frac{1}{8d(a+ia\tan(c+dx))^4} + \frac{1}{12ad(a+ia\tan(c+dx))^3} - \frac{i\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx}{4a^2}$$
$$= -\frac{1}{8d(a+ia\tan(c+dx))^4} + \frac{1}{12ad(a+ia\tan(c+dx))^3} + \frac{1}{16d\,(a^2+ia^2\tan(c\cdots}$$
$$= -\frac{1}{8d(a+ia\tan(c+dx))^4} + \frac{1}{12ad(a+ia\tan(c+dx))^3} + \frac{1}{16d\,(a^2+ia^2\tan(c\cdots}$$
$$= -\frac{ix}{16a^4} - \frac{1}{8d(a+ia\tan(c+dx))^4} + \frac{1}{12ad(a+ia\tan(c+dx))^3} + \frac{}{16d\,(a^2+ia\cdots}$$

Mathematica [A]

time = 0.36, size = 94, normalized size = 0.85

$$\frac{\sec^4(c+dx)(16\cos(2(c+dx)) + (-3-24idx)\cos(4(c+dx)) + 32i\sin(2(c+dx)) + 3i\sin(4(c+dx)) + 24dx\sin(4(c+dx)))}{384a^4d(-i+\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^4*(16*Cos[2*(c + d*x)] + (-3 - (24*I)*d*x)*Cos[4*(c + d*x)] +
 (32*I)*Sin[2*(c + d*x)] + (3*I)*Sin[4*(c + d*x)] + 24*d*x*Sin[4*(c + d*x)]
))/(384*a^4*d*(-I + Tan[c + d*x])^4)
```

Maple [A]

time = 0.16, size = 87, normalized size = 0.79

method	result
risch	$-\frac{ix}{16a^4} + \frac{e^{-2i(dx+c)}}{16a^4d} - \frac{e^{-6i(dx+c)}}{48a^4d} - \frac{e^{-8i(dx+c)}}{128a^4d}$
derivativedivides	$\frac{\frac{i}{12(\tan(dx+c)-i)^3} - \frac{i}{16(\tan(dx+c)-i)} - \frac{1}{8(\tan(dx+c)-i)^4} - \frac{1}{16(\tan(dx+c)-i)^2} - \frac{\ln(\tan(dx+c)-i)}{32} + \frac{\ln(\tan(dx+c)+i)}{32}}{d\,a^4}$
default	$\frac{\frac{i}{12(\tan(dx+c)-i)^3} - \frac{i}{16(\tan(dx+c)-i)} - \frac{1}{8(\tan(dx+c)-i)^4} - \frac{1}{16(\tan(dx+c)-i)^2} - \frac{\ln(\tan(dx+c)-i)}{32} + \frac{\ln(\tan(dx+c)+i)}{32}}{d\,a^4}$
norman	$\frac{\frac{1}{12da} - \frac{ix}{16a} - \frac{\tan^4(dx+c)}{4da} + \frac{5\left(\tan^2(dx+c)\right)}{6da} - \frac{ix\left(\tan^2(dx+c)\right)}{4a} - \frac{3ix\left(\tan^4(dx+c)\right)}{8a} - \frac{ix\left(\tan^6(dx+c)\right)}{4a} - \frac{ix\left(\tan^8(dx+c)\right)}{16a} + \frac{i\tan(dx+c\cdots}{16da}}{\left(1+\tan^2(dx+c)\right)^4a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^4*(1/12*I/(tan(d*x+c)-I)^3-1/16*I/(tan(d*x+c)-I)-1/8/(tan(d*x+c)-I)^4
-1/16/(tan(d*x+c)-I)^2-1/32*ln(tan(d*x+c)-I)+1/32*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.36, size = 54, normalized size = 0.49

$$\frac{\left(-24i\,dxe^{(8i\,dx+8i\,c)}+24\,e^{(6i\,dx+6i\,c)}-8\,e^{(2i\,dx+2i\,c)}-3\right)e^{(-8i\,dx-8i\,c)}}{384\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/384*(-24*I*d*x*e^(8*I*d*x + 8*I*c) + 24*e^(6*I*d*x + 6*I*c) - 8*e^(2*I*d*
x + 2*I*c) - 3)*e^(-8*I*d*x - 8*I*c)/(a^4*d)
```

Sympy [A]
time = 0.29, size = 156, normalized size = 1.42

$$\begin{cases}\frac{\left(6144a^8d^2e^{14ic}e^{-2idx}-2048a^8d^2e^{10ic}e^{-6idx}-768a^8d^2e^{8ic}e^{-8idx}\right)e^{-16ic}}{98304a^{12}d^3} & \text{for } a^{12}d^3e^{16ic}\neq 0\\ x\left(\frac{\left(-ie^{8ic}-2ie^{6ic}+2ie^{2ic}+i\right)e^{-8ic}}{16a^4}+\frac{i}{16a^4}\right) & \text{otherwise}\end{cases}-\frac{ix}{16a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((6144*a**8*d**2*exp(14*I*c)*exp(-2*I*d*x) - 2048*a**8*d**2*exp(1
0*I*c)*exp(-6*I*d*x) - 768*a**8*d**2*exp(8*I*c)*exp(-8*I*d*x))*exp(-16*I*c)
/(98304*a**12*d**3), Ne(a**12*d**3*exp(16*I*c), 0)), (x*((-I*exp(8*I*c) - 2
*I*exp(6*I*c) + 2*I*exp(2*I*c) + I)*exp(-8*I*c)/(16*a**4) + I/(16*a**4)), T
rue)) - I*x/(16*a**4)
```

Giac [A]
time = 1.02, size = 88, normalized size = 0.80

$$\frac{\frac{12\,\log(\tan(dx+c)+i)}{a^4}-\frac{12\,\log(\tan(dx+c)-i)}{a^4}+\frac{25\,\tan(dx+c)^4-124i\,\tan(dx+c)^3-246\,\tan(dx+c)^2+252i\,\tan(dx+c)+57}{a^4(\tan(dx+c)-i)^4}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/384*(12*log(tan(d*x + c) + I)/a^4 - 12*log(tan(d*x + c) - I)/a^4 + (25*ta
n(d*x + c)^4 - 124*I*tan(d*x + c)^3 - 246*tan(d*x + c)^2 + 252*I*tan(d*x +
c) + 57)/(a^4*(tan(d*x + c) - I)^4))/d
```

Mupad [B]
time = 3.90, size = 60, normalized size = 0.55

$$-\frac{x\,1{}\mathrm{i}}{16\,a^4}+\frac{-\frac{{\mathrm{tan}\left(c+d\,x\right)}^3\,1{}\mathrm{i}}{16}-\frac{{\mathrm{tan}\left(c+d\,x\right)}^2}{4}+\frac{\mathrm{tan}\left(c+d\,x\right)\,19{}\mathrm{i}}{48}+\frac{1}{12}}{a^4\,d\,{\left(1+\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] ((tan(c + d*x)*19i)/48 - tan(c + d*x)^2/4 - (tan(c + d*x)^3*1i)/16 + 1/12)/
(a^4*d*(tan(c + d*x)*1i + 1)^4) - (x*1i)/(16*a^4)
```

3.81 $\int \frac{1}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=116

$$\frac{x}{16a^4}+\frac{i}{8d(a+ia\tan(c+dx))^4}+\frac{i}{12ad(a+ia\tan(c+dx))^3}+\frac{i}{16d\left(a^2+ia^2\tan(c+dx)\right)^2}+\frac{i}{16d\left(a^4+ia^4\tan\right.}$$

```
[Out] 1/16*x/a^4+1/8*I/d/(a+I*a*tan(d*x+c))^4+1/12*I/a/d/(a+I*a*tan(d*x+c))^3+1/1
6*I/d/(a^2+I*a^2*tan(d*x+c))^2+1/16*I/d/(a^4+I*a^4*tan(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3560, 8}

$$\frac{i}{16d\left(a^4+ia^4\tan(c+dx)\right)}+\frac{x}{16a^4}+\frac{i}{16d\left(a^2+ia^2\tan(c+dx)\right)^2}+\frac{i}{12ad(a+ia\tan(c+dx))^3}+\frac{i}{8d(a+ia\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(-4),x]
```

```
[Out] x/(16*a^4) + (I/8)/(d*(a + I*a*Tan[c + d*x])^4) + (I/12)/(a*d*(a + I*a*Tan[
c + d*x])^3) + (I/16)/(d*(a^2 + I*a^2*Tan[c + d*x])^2) + (I/16)/(d*(a^4 + I
*a^4*Tan[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(c+dx))^4}\,dx = \frac{i}{8d(a+ia\tan(c+dx))^4} + \frac{\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx}{2a}$$
$$= \frac{i}{8d(a+ia\tan(c+dx))^4} + \frac{i}{12ad(a+ia\tan(c+dx))^3} + \frac{\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx}{4a^2}$$
$$= \frac{i}{8d(a+ia\tan(c+dx))^4} + \frac{i}{12ad(a+ia\tan(c+dx))^3} + \frac{i}{16d\,(a^2+ia^2\tan(c+$$
$$= \frac{i}{8d(a+ia\tan(c+dx))^4} + \frac{i}{12ad(a+ia\tan(c+dx))^3} + \frac{i}{16d\,(a^2+ia^2\tan(c+$$
$$= \frac{x}{16a^4} + \frac{i}{8d(a+ia\tan(c+dx))^4} + \frac{i}{12ad(a+ia\tan(c+dx))^3} + \frac{i}{16d\,(a^2+ia^2\,t$$

Mathematica [A]
time = 0.26, size = 98, normalized size = 0.84

$$\frac{\sec^4(c+dx)(36i+64i\cos(2(c+dx))+3(i+8dx)\cos(4(c+dx))-32\sin(2(c+dx))+3\sin(4(c+dx))+24idx\sin(4(c+dx)))}{384a^4d(-i+\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(-4),x]
```

```
[Out] (Sec[c + d*x]^4*(36*I + (64*I)*Cos[2*(c + d*x)] + 3*(I + 8*d*x)*Cos[4*(c +
d*x)] - 32*Sin[2*(c + d*x)] + 3*Sin[4*(c + d*x)] + (24*I)*d*x*Sin[4*(c + d*
x)]))/(384*a^4*d*(-I + Tan[c + d*x])^4)
```

Maple [A]
time = 0.00, size = 89, normalized size = 0.77

method	result
risch	$\frac{x}{16a^4}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8a^4d}+\frac{3i\mathrm{e}^{-4i(dx+c)}}{32a^4d}+\frac{i\mathrm{e}^{-6i(dx+c)}}{24a^4d}+\frac{i\mathrm{e}^{-8i(dx+c)}}{128a^4d}$
derivativedivides	$\frac{\frac{i}{8(\tan(dx+c)-i)^4}-\frac{i\ln(\tan(dx+c)-i)}{32}-\frac{i}{16(\tan(dx+c)-i)^2}-\frac{1}{12(\tan(dx+c)-i)^3}+\frac{1}{16\tan(dx+c)-16i}+\frac{i\ln(\tan(dx+c)+i)}{32}}{d\,a^4}$
default	$\frac{\frac{i}{8(\tan(dx+c)-i)^4}-\frac{i\ln(\tan(dx+c)-i)}{32}-\frac{i}{16(\tan(dx+c)-i)^2}-\frac{1}{12(\tan(dx+c)-i)^3}+\frac{1}{16\tan(dx+c)-16i}+\frac{i\ln(\tan(dx+c)+i)}{32}}{d\,a^4}$
norman	$\frac{\frac{x}{16a}+\frac{11\left(\tan^5(dx+c)\right)}{48da}+\frac{\tan^7(dx+c)}{16da}+\frac{x\left(\tan^2(dx+c)\right)}{4a}+\frac{3x\left(\tan^4(dx+c)\right)}{8a}+\frac{x\left(\tan^6(dx+c)\right)}{4a}+\frac{x\left(\tan^8(dx+c)\right)}{16a}+\frac{i}{3da}+\frac{15\tan(dx+c)}{16da}}{a^3(1+\tan^2(dx+c))^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^4*(1/8*I/(tan(d*x+c)-I)^4-1/32*I*ln(tan(d*x+c)-I)-1/16*I/(tan(d*x+c)-
I)^2-1/12/(tan(d*x+c)-I)^3+1/16/(tan(d*x+c)-I)+1/32*I*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.36, size = 65, normalized size = 0.56

$$\frac{\left(24\,dxe^{(8i\,dx+8i\,c)}+48i\,e^{(6i\,dx+6i\,c)}+36i\,e^{(4i\,dx+4i\,c)}+16i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-8i\,dx-8i\,c)}}{384\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/384*(24*d*x*e^(8*I*d*x + 8*I*c) + 48*I*e^(6*I*d*x + 6*I*c) + 36*I*e^(4*I*
d*x + 4*I*c) + 16*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-8*I*d*x - 8*I*c)/(a^4*d)
```

Sympy [A]
time = 0.22, size = 189, normalized size = 1.63

$$\begin{cases}\frac{(98304ia^{12}d^3e^{18ic}e^{-2idx}+73728ia^{12}d^3e^{16ic}e^{-4idx}+32768ia^{12}d^3e^{14ic}e^{-6idx}+6144ia^{12}d^3e^{12ic}e^{-8idx})e^{-20ic}}{786432a^{16}d^4} & \text{for } a^{16}d^4e^{20ic}\neq 0\\ x\left(\frac{(e^{8ic}+4e^{6ic}+6e^{4ic}+4e^{2ic}+1)e^{-8ic}}{16a^4}-\frac{1}{16a^4}\right) & \text{otherwise}\end{cases}+\frac{x}{16a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((98304*I*a**12*d**3*exp(18*I*c)*exp(-2*I*d*x) + 73728*I*a**12*d*
*3*exp(16*I*c)*exp(-4*I*d*x) + 32768*I*a**12*d**3*exp(14*I*c)*exp(-6*I*d*x)
 + 6144*I*a**12*d**3*exp(12*I*c)*exp(-8*I*d*x))*exp(-20*I*c)/(786432*a**16*
d**4), Ne(a**16*d**4*exp(20*I*c), 0)), (x*((exp(8*I*c) + 4*exp(6*I*c) + 6*e
xp(4*I*c) + 4*exp(2*I*c) + 1)*exp(-8*I*c)/(16*a**4) - 1/(16*a**4)), True))
+ x/(16*a**4)
```

Giac [A]
time = 0.78, size = 92, normalized size = 0.79

$$-\frac{-\frac{12i\,\log(-i\,\tan(dx+c)+1)}{a^4}+\frac{12i\,\log(-i\,\tan(dx+c)-1)}{a^4}+\frac{-25i\,\tan(dx+c)^4-124\,\tan(dx+c)^3+246i\,\tan(dx+c)^2+252\,\tan(dx+c)-153i}{a^4(\tan(dx+c)-i)^4}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/384*(-12*I*log(-I*tan(d*x + c) + 1)/a^4 + 12*I*log(-I*tan(d*x + c) - 1)/
a^4 + (-25*I*tan(d*x + c)^4 - 124*tan(d*x + c)^3 + 246*I*tan(d*x + c)^2 + 2
52*tan(d*x + c) - 153*I)/(a^4*(tan(d*x + c) - I)^4))/d
```

Mupad [B]
time = 3.89, size = 60, normalized size = 0.52

$$\frac{x}{16\,a^4}-\frac{-\frac{\tan(c+d\,x)^3}{16}+\frac{\tan(c+d\,x)^2\,1\mathrm{i}}{4}+\frac{19\tan(c+d\,x)}{48}-\frac{1}{3}\mathrm{i}}{a^4\,d\,(1+\tan\left(c+d\,x\right)\,1\mathrm{i})^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] x/(16*a^4) - ((19*tan(c + d*x))/48 + (tan(c + d*x)^2*1i)/4 - tan(c + d*x)^3
/16 - 1i/3)/(a^4*d*(tan(c + d*x)*1i + 1)^4)
```

3.82 $\int \frac{\cot(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=120

$$-\frac{15ix}{16a^4}+\frac{\log(\sin(c+dx))}{a^4d}+\frac{7}{16a^4d(1+i\tan(c+dx))^2}+\frac{15}{16a^4d(1+i\tan(c+dx))}+\frac{1}{8d(a+ia\tan(c+dx))^4}+$$

```
[Out] -15/16*I*x/a^4+ln(sin(d*x+c))/a^4/d+7/16/a^4/d/(1+I*tan(d*x+c))^2+15/16/a^4
/d/(1+I*tan(d*x+c))+1/8/d/(a+I*a*tan(d*x+c))^4+1/4/a/d/(a+I*a*tan(d*x+c))^3
```

Rubi [A]
time = 0.22, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3640, 3677, 3612, 3556}

$$\frac{15}{16a^4d(1+i\tan(c+dx))}+\frac{7}{16a^4d(1+i\tan(c+dx))^2}+\frac{\log(\sin(c+dx))}{a^4d}-\frac{15ix}{16a^4}+\frac{1}{4ad(a+ia\tan(c+dx))^3}+\frac{1}{8d(a+ia\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (((-15*I)/16)*x)/a^4 + Log[Sin[c + d*x]]/(a^4*d) + 7/(16*a^4*d*(1 + I*Tan[c
 + d*x])^2) + 15/(16*a^4*d*(1 + I*Tan[c + d*x])) + 1/(8*d*(a + I*a*Tan[c +
d*x])^4) + 1/(4*a*d*(a + I*a*Tan[c + d*x])^3)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

```
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{1}{8d(a+ia\tan(c+dx))^4} + \frac{\int \frac{\cot(c+dx)(8a-4ia\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx}{8a^2}$$

$$= \frac{1}{8d(a+ia\tan(c+dx))^4} + \frac{1}{4ad(a+ia\tan(c+dx))^3} + \frac{\int \frac{\cot(c+dx)(48a^2-36ia^2\tan(}{(a+ia\tan(c+dx))^2}}{48a^4}$$

$$= \frac{7}{16a^4d(1+i\tan(c+dx))^2} + \frac{1}{8d(a+ia\tan(c+dx))^4} + \frac{1}{4ad(a+ia\tan(c+dx}$$

$$= \frac{7}{16a^4d(1+i\tan(c+dx))^2} + \frac{1}{8d(a+ia\tan(c+dx))^4} + \frac{1}{4ad(a+ia\tan(c+dx}$$

$$= -\frac{15ix}{16a^4} + \frac{7}{16a^4d(1+i\tan(c+dx))^2} + \frac{1}{8d(a+ia\tan(c+dx))^4} + \frac{1}{4ad(a+ia\,\mathrm{ta}}$$

$$= -\frac{15ix}{16a^4} + \frac{\log(\sin(c+dx))}{a^4d} + \frac{7}{16a^4d(1+i\tan(c+dx))^2} + \frac{1}{8d(a+ia\tan(c+d}$$

Mathematica [A]

time = 0.45, size = 123, normalized size = 1.02

$$\frac{\sec^4(c+dx)(32+112\cos(2(c+dx))+\cos(4(c+dx))(1-120idx+128\log(\sin(c+dx)))+96i\sin(2(c+dx))-i\sin(4(c+dx))+120dx\sin(4(c+dx))+128i\log(\sin(c+dx))\sin(4(c+dx)))}{128a^4d(-i+\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^4*(32 + 112*Cos[2*(c + d*x)] + Cos[4*(c + d*x)]*(1 - (120*I)*
d*x + 128*Log[Sin[c + d*x]]) + (96*I)*Sin[2*(c + d*x)] - I*Sin[4*(c + d*x)]
 + 120*d*x*Sin[4*(c + d*x)] + (128*I)*Log[Sin[c + d*x]]*Sin[4*(c + d*x)]))/
(128*a^4*d*(-I + Tan[c + d*x])^4)
```

Maple [A]

time = 0.33, size = 94, normalized size = 0.78

method	result
derivativedivides	$\frac{\frac{i}{4(\tan(dx+c)-i)^3}-\frac{15i}{16(\tan(dx+c)-i)}+\frac{1}{8(\tan(dx+c)-i)^4}-\frac{7}{16(\tan(dx+c)-i)^2}-\frac{31\ln(\tan(dx+c)-i)}{32}+\ln(\tan(dx+c))-\frac{\ln(\tan(dx+c}{32}}{d\,a^4}$
default	$\frac{\frac{i}{4(\tan(dx+c)-i)^3}-\frac{15i}{16(\tan(dx+c)-i)}+\frac{1}{8(\tan(dx+c)-i)^4}-\frac{7}{16(\tan(dx+c)-i)^2}-\frac{31\ln(\tan(dx+c)-i)}{32}+\ln(\tan(dx+c))-\frac{\ln(\tan(dx+c}{32}}{d\,a^4}$
risch	$-\frac{31ix}{16a^4}+\frac{13\,e^{-2i(dx+c)}}{16a^4d}+\frac{e^{-4i(dx+c)}}{4a^4d}+\frac{e^{-6i(dx+c)}}{16a^4d}+\frac{e^{-8i(dx+c)}}{128a^4d}-\frac{2ic}{a^4d}+\frac{\ln\left(e^{2i(dx+c)}-1\right)}{a^4d}$
norman	$\frac{\frac{7}{4da}-\frac{15ix}{16a}+\frac{2\left(\tan^2(dx+c)\right)}{da}+\frac{\tan^6(dx+c)}{2da}+\frac{7\left(\tan^4(dx+c)\right)}{4da}-\frac{15ix\left(\tan^2(dx+c)\right)}{4a}-\frac{45ix\left(\tan^4(dx+c)\right)}{8a}-\frac{15ix\left(\tan^6(dx+c)\right)}{4a}-\frac{15ix}{}}{a^3\left(1+\tan^2(dx+c)\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^4*(1/4*I/(tan(d*x+c)-I)^3-15/16*I/(tan(d*x+c)-I)+1/8/(tan(d*x+c)-I)^4
-7/16/(tan(d*x+c)-I)^2-31/32*ln(tan(d*x+c)-I)+ln(tan(d*x+c))-1/32*ln(tan(d*
x+c)+I))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]

time = 0.39, size = 88, normalized size = 0.73

$$\frac{\left(-248i\,dxe^{(8i\,dx+8i\,c)}+128\,e^{(8i\,dx+8i\,c)}\log\left(e^{(2i\,dx+2i\,c)}-1\right)+104\,e^{(6i\,dx+6i\,c)}+32\,e^{(4i\,dx+4i\,c)}+8\,e^{(2i\,dx+2i\,c)}+1\right)e^{(-8i\,dx-8i\,c)}}{128\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/128*(-248*I*d*x*e^(8*I*d*x + 8*I*c) + 128*e^(8*I*d*x + 8*I*c)*log(e^(2*I*
d*x + 2*I*c) - 1) + 104*e^(6*I*d*x + 6*I*c) + 32*e^(4*I*d*x + 4*I*c) + 8*e^
(2*I*d*x + 2*I*c) + 1)*e^(-8*I*d*x - 8*I*c)/(a^4*d)
```

Sympy [A]

time = 0.44, size = 221, normalized size = 1.84

$$\begin{cases} \frac{\left(106496a^{12}d^3e^{18ic}e^{-2idx}+32768a^{12}d^3e^{16ic}e^{-4idx}+8192a^{12}d^3e^{14ic}e^{-6idx}+1024a^{12}d^3e^{12ic}e^{-8idx}\right)e^{-20ic}}{131072a^{16}d^4} & \text{for } a^{16}d^4e^{20ic}\neq 0 \\ x\left(\frac{\left(-31ie^{8ic}-26ie^{6ic}-16ie^{4ic}-6ie^{2ic}-i\right)e^{-8ic}}{16a^4}+\frac{31i}{16a^4}\right) & \text{otherwise} \end{cases} - \frac{31ix}{16a^4}+\frac{\log\left(e^{2idx}-e^{-2ic}\right)}{a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((106496*a**12*d**3*exp(18*I*c)*exp(-2*I*d*x) + 32768*a**12*d**3*
exp(16*I*c)*exp(-4*I*d*x) + 8192*a**12*d**3*exp(14*I*c)*exp(-6*I*d*x) + 102
4*a**12*d**3*exp(12*I*c)*exp(-8*I*d*x))*exp(-20*I*c)/(131072*a**16*d**4), N
e(a**16*d**4*exp(20*I*c), 0)), (x*((-31*I*exp(8*I*c) - 26*I*exp(6*I*c) - 16
*I*exp(4*I*c) - 6*I*exp(2*I*c) - I)*exp(-8*I*c)/(16*a**4) + 31*I/(16*a**4))
, True)) - 31*I*x/(16*a**4) + log(exp(2*I*d*x) - exp(-2*I*c))/(a**4*d)
```

Giac [A]
time = 1.27, size = 101, normalized size = 0.84

$$-\frac{\frac{12 \log(\tan(dx+c)+i)}{a^4}+\frac{372 \log(\tan(dx+c)-i)}{a^4}-\frac{384 \log(\tan(dx+c))}{a^4}-\frac{775 \tan(dx+c)^4-3460 i \tan(dx+c)^3-5898 \tan(dx+c)^2+4612 i \tan(dx+c)+1447}{a^4(\tan(dx+c)-i)^4}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/384*(12*log(tan(d*x + c) + I)/a^4 + 372*log(tan(d*x + c) - I)/a^4 - 384*
log(tan(d*x + c))/a^4 - (775*tan(d*x + c)^4 - 3460*I*tan(d*x + c)^3 - 5898*
tan(d*x + c)^2 + 4612*I*tan(d*x + c) + 1447)/(a^4*(tan(d*x + c) - I)^4))/d
```

Mupad [B]
time = 3.84, size = 142, normalized size = 1.18

$$\frac{\ln(\tan(c+d\,x))}{a^4\,d}-\frac{\ln(\tan(c+d\,x)+1\mathrm{i})}{32\,a^4\,d}-\frac{31\,\ln(\tan(c+d\,x)-\mathrm{i})}{32\,a^4\,d}+\frac{\frac{\tan(c+d\,x)\,63\mathrm{i}}{16\,a^4}+\frac{7}{4\,a^4}-\frac{13\tan(c+d\,x)^2}{4\,a^4}-\frac{\tan(c+d\,x)^3\,15\mathrm{i}}{16\,a^4}}{d\,\left(\tan(c+d\,x)^4-\tan(c+d\,x)^3\,4\mathrm{i}-6\tan(c+d\,x)^2+\tan(c+d\,x)\,4\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] log(tan(c + d*x))/(a^4*d) - log(tan(c + d*x) + 1i)/(32*a^4*d) - (31*log(tan
(c + d*x) - 1i))/(32*a^4*d) + ((tan(c + d*x)*63i)/(16*a^4) + 7/(4*a^4) - (1
3*tan(c + d*x)^2)/(4*a^4) - (tan(c + d*x)^3*15i)/(16*a^4))/(d*(tan(c + d*x)
*4i - 6*tan(c + d*x)^2 - tan(c + d*x)^3*4i + tan(c + d*x)^4 + 1))
```

3.83 $\int \frac{\cot^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=159

$$-\frac{65x}{16a^4}-\frac{65\cot(c+dx)}{16a^4d}-\frac{4i\log(\sin(c+dx))}{a^4d}+\frac{31\cot(c+dx)}{48a^4d(1+i\tan(c+dx))^2}+\frac{2\cot(c+dx)}{a^4d(1+i\tan(c+dx))}+\frac{\cot(c+dx)}{8d(a+i\ldots}$$

```
[Out] -65/16*x/a^4-65/16*cot(d*x+c)/a^4/d-4*I*ln(sin(d*x+c))/a^4/d+31/48*cot(d*x+
c)/a^4/d/(1+I*tan(d*x+c))^2+2*cot(d*x+c)/a^4/d/(1+I*tan(d*x+c))+1/8*cot(d*x
+c)/d/(a+I*a*tan(d*x+c))^4+7/24*cot(d*x+c)/a/d/(a+I*a*tan(d*x+c))^3
```

Rubi [A]
time = 0.30, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3640, 3677, 3610, 3612, 3556}

$$-\frac{65\cot(c+dx)}{16a^4d}-\frac{4i\log(\sin(c+dx))}{a^4d}+\frac{2\cot(c+dx)}{a^4d(1+i\tan(c+dx))}+\frac{31\cot(c+dx)}{48a^4d(1+i\tan(c+dx))^2}-\frac{65x}{16a^4}+\frac{7\cot(c+dx)}{24ad(a+ia\tan(c+dx))^3}+\frac{\cot(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (-65*x)/(16*a^4) - (65*Cot[c + d*x])/(16*a^4*d) - ((4*I)*Log[Sin[c + d*x]])
/(a^4*d) + (31*Cot[c + d*x])/(48*a^4*d*(1 + I*Tan[c + d*x])^2) + (2*Cot[c +
 d*x])/(a^4*d*(1 + I*Tan[c + d*x])) + Cot[c + d*x]/(8*d*(a + I*a*Tan[c + d*
x])^4) + (7*Cot[c + d*x])/(24*a*d*(a + I*a*Tan[c + d*x])^3)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
```

```
Q[a*c + b*d, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx &= \frac{\cot(c+dx)}{8d(a+ia\tan(c+dx))^4}+\frac{\int \frac{\cot^2(c+dx)(9a-5ia\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx}{8a^2}\\
&= \frac{\cot(c+dx)}{8d(a+ia\tan(c+dx))^4}+\frac{7\cot(c+dx)}{24ad(a+ia\tan(c+dx))^3}+\frac{\int \frac{\cot^2(c+dx)(68a^2-56ia^2\tan}{(a+ia\tan(c+dx))}}{48a^4}\\
&= \frac{31\cot(c+dx)}{48a^4d(1+i\tan(c+dx))^2}+\frac{\cot(c+dx)}{8d(a+ia\tan(c+dx))^4}+\frac{7\cot(c+dx)}{24ad(a+ia\tan(c+d}\\
&= \frac{31\cot(c+dx)}{48a^4d(1+i\tan(c+dx))^2}+\frac{\cot(c+dx)}{8d(a+ia\tan(c+dx))^4}+\frac{7\cot(c+dx)}{24ad(a+ia\tan(c+d}\\
&= -\frac{65\cot(c+dx)}{16a^4d}+\frac{31\cot(c+dx)}{48a^4d(1+i\tan(c+dx))^2}+\frac{\cot(c+dx)}{8d(a+ia\tan(c+dx))^4}+\frac{}{24}\\
&= -\frac{65x}{16a^4}-\frac{65\cot(c+dx)}{16a^4d}+\frac{31\cot(c+dx)}{48a^4d(1+i\tan(c+dx))^2}+\frac{\cot(c+dx)}{8d(a+ia\tan(c+dx)}\\
&= -\frac{65x}{16a^4}-\frac{65\cot(c+dx)}{16a^4d}-\frac{4i\log(\sin(c+dx))}{a^4d}+\frac{31\cot(c+dx)}{48a^4d(1+i\tan(c+dx))^2}+
\end{aligned}
$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 444 vs. $2(159) = 318$.
time = 2.96, size = 444, normalized size = 2.79

$$\frac{i\csc(c)\sec^4(c+dx)(\cos(dx)+i\sin(dx))^4\left(1536dx\cos^3(c)+4608idx\cos^2(c)\sin(c)+1536i\,\mathrm{ArcTan}(\tan(dx))\sin(c)(\cos(4c)+i\sin(4c))-64\cos(c)\left(24dx\cos(4c)+24idx\sin(4c)+\sin^2(c)(72dx-i\cos(6dx)-\sin(6dx))\right)+i\left(-192i\cos(4c-dx)\csc(c+dx)+192i\cos(4c+dx)\csc(c+dx)+1560dx\cos(4c)\sin(c)+864i\cos(2c)\cos(2dx)\sin(c)+180i\cos(4dx)\sin(c)+32i\cos(2c)\cos(6dx)\sin(c)+3i\cos(4c)\cos(8dx)\sin(c)+768i\cos(4c)\log(\sin^2(c+dx))\sin(c)-1536dx\sin^3(c)-864\cos(2dx)\sin(c)\sin(2c)+1560idx\sin(c)\sin(4c)+3\cos(8dx)\sin(c)\sin(4c)-768\log(\sin^2(c+dx))\sin(c)\sin(4c)+864\cos(2c)\sin(c)\sin(2dx)+864i\sin(c)\sin(2c)\sin(2dx)+180\sin(c)\sin(4dx)+32\cos(2c)\sin(c)\sin(6dx)+3\cos(4c)\sin(c)\sin(8dx)-3i\sin(c)\sin(4c)\sin(8dx)+192\csc(c+dx)\sin(4c-dx)-192\csc(c+dx)\sin(4c+dx)\right)\right)}{384a^4d(-i+\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((I/384)*Csc[c]*Sec[c + d*x]^4*(Cos[d*x] + I*Sin[d*x])^4*(1536*d*x*Cos[c]^3
 + (4608*I)*d*x*Cos[c]^2*Sin[c] + (1536*I)*ArcTan[Tan[d*x]]*Sin[c]*(Cos[4*c
] + I*Sin[4*c]) - 64*Cos[c]*(24*d*x*Cos[4*c] + (24*I)*d*x*Sin[4*c] + Sin[c]
^2*(72*d*x - I*Cos[6*d*x] - Sin[6*d*x])) + I*((-192*I)*Cos[4*c - d*x]*Csc[c
 + d*x] + (192*I)*Cos[4*c + d*x]*Csc[c + d*x] + 1560*d*x*Cos[4*c]*Sin[c] +
(864*I)*Cos[2*c]*Cos[2*d*x]*Sin[c] + (180*I)*Cos[4*d*x]*Sin[c] + (32*I)*Cos
[2*c]*Cos[6*d*x]*Sin[c] + (3*I)*Cos[4*c]*Cos[8*d*x]*Sin[c] + (768*I)*Cos[4*
c]*Log[Sin[c + d*x]^2]*Sin[c] - 1536*d*x*Sin[c]^3 - 864*Cos[2*d*x]*Sin[c]*S
in[2*c] + (1560*I)*d*x*Sin[c]*Sin[4*c] + 3*Cos[8*d*x]*Sin[c]*Sin[4*c] - 768
*Log[Sin[c + d*x]^2]*Sin[c]*Sin[4*c] + 864*Cos[2*c]*Sin[c]*Sin[2*d*x] + (86
4*I)*Sin[c]*Sin[2*c]*Sin[2*d*x] + 180*Sin[c]*Sin[4*d*x] + 32*Cos[2*c]*Sin[c
]*Sin[6*d*x] + 3*Cos[4*c]*Sin[c]*Sin[8*d*x] - (3*I)*Sin[c]*Sin[4*c]*Sin[8*d
*x] + 192*Csc[c + d*x]*Sin[4*c - d*x] - 192*Csc[c + d*x]*Sin[4*c + d*x]))))/
(a^4*d*(-I + Tan[c + d*x])^4)
```

Maple [**A**]
time = 0.39, size = 109, normalized size = 0.69

method	result
derivativedivides	$\frac{\frac{17i}{16(\tan(dx+c)-i)^2}-\frac{i}{8(\tan(dx+c)-i)^4}+\frac{129i\ln(\tan(dx+c)-i)}{32}+\frac{5}{12(\tan(dx+c)-i)^3}-\frac{49}{16(\tan(dx+c)-i)}-\frac{1}{\tan(dx+c)}-4i\ln(\tan(dx}{d\,a^4}$
default	$\frac{\frac{17i}{16(\tan(dx+c)-i)^2}-\frac{i}{8(\tan(dx+c)-i)^4}+\frac{129i\ln(\tan(dx+c)-i)}{32}+\frac{5}{12(\tan(dx+c)-i)^3}-\frac{49}{16(\tan(dx+c)-i)}-\frac{1}{\tan(dx+c)}-4i\ln(\tan(dx}{d\,a^4}$
risch	$-\frac{129x}{16a^4}-\frac{9i\mathrm{e}^{-2i(dx+c)}}{4a^4d}-\frac{15i\mathrm{e}^{-4i(dx+c)}}{32a^4d}-\frac{i\mathrm{e}^{-6i(dx+c)}}{12a^4d}-\frac{i\mathrm{e}^{-8i(dx+c)}}{128a^4d}-\frac{8c}{a^4d}-\frac{2i}{d\,a^4\left(\mathrm{e}^{2i(dx+c)}-1\right)}-\frac{4i\ln(\mathrm{e}}{}$
norman	$\frac{-\frac{7i\left(\tan^5(dx+c)\right)}{da}-\frac{1}{da}-\frac{949\left(\tan^4(dx+c)\right)}{48da}-\frac{715\left(\tan^6(dx+c)\right)}{48da}-\frac{65\left(\tan^8(dx+c)\right)}{16da}-\frac{65x\tan(dx+c)}{16a}-\frac{65x\left(\tan^3(dx+c)\right)}{4a}-\frac{195x\left(\tan\right.}{}}{\tan(dx+c)a^3(1+\mathrm{t}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^4*(17/16*I/(tan(d*x+c)-I)^2-1/8*I/(tan(d*x+c)-I)^4+129/32*I*ln(tan(d*
x+c)-I)+5/12/(tan(d*x+c)-I)^3-49/16/(tan(d*x+c)-I)-1/tan(d*x+c)-4*I*ln(tan(
d*x+c))-1/32*I*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.38, size = 136, normalized size = 0.86

$$-\frac{3096\,dxe^{(10i\,dx+10i\,c)}-24\,(129\,dx-68i)e^{(8i\,dx+8i\,c)}+1536\,\left(i\,e^{(10i\,dx+10i\,c)}-i\,e^{(8i\,dx+8i\,c)}\right)\log\left(e^{(2i\,dx+2i\,c)}-1\right)-684i\,e^{(6i\,dx+6i\,c)}-148i\,e^{(4i\,dx+4i\,c)}-29i\,e^{(2i\,dx+2i\,c)}-3i}{384\,(a^4de^{(10i\,dx+10i\,c)}-a^4de^{(8i\,dx+8i\,c)})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/384*(3096*d*x*e^(10*I*d*x + 10*I*c) - 24*(129*d*x - 68*I)*e^(8*I*d*x + 8
*I*c) + 1536*(I*e^(10*I*d*x + 10*I*c) - I*e^(8*I*d*x + 8*I*c))*log(e^(2*I*d
*x + 2*I*c) - 1) - 684*I*e^(6*I*d*x + 6*I*c) - 148*I*e^(4*I*d*x + 4*I*c) -
29*I*e^(2*I*d*x + 2*I*c) - 3*I)/(a^4*d*e^(10*I*d*x + 10*I*c) - a^4*d*e^(8*I
*d*x + 8*I*c))
```

Sympy [A]
time = 0.37, size = 252, normalized size = 1.58

$$\begin{cases}\frac{\left(-442368ia^{12}d^3e^{18ic}e^{-2idx}-92160ia^{12}d^3e^{16ic}e^{-4idx}-16384ia^{12}d^3e^{14ic}e^{-6idx}-1536ia^{12}d^3e^{12ic}e^{-8idx}\right)e^{-20ic}}{196608a^{16}d^4} & \text{for } a^{16}d^4e^{20ic}\neq 0\\ x\left(\frac{\left(-129e^{8ic}-72e^{6ic}-30e^{4ic}-8e^{2ic}-1\right)e^{-8ic}}{16a^4}+\frac{129}{16a^4}\right) & \text{otherwise}\end{cases}-\frac{2i}{a^4de^{2ic}e^{2idx}-a^4d}-\frac{129x}{16a^4}-\frac{4i\log\left(e^{2idx}-e^{-2ic}\right)}{a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((-442368*I*a**12*d**3*exp(18*I*c)*exp(-2*I*d*x) - 92160*I*a**12*
d**3*exp(16*I*c)*exp(-4*I*d*x) - 16384*I*a**12*d**3*exp(14*I*c)*exp(-6*I*d*
x) - 1536*I*a**12*d**3*exp(12*I*c)*exp(-8*I*d*x))*exp(-20*I*c)/(196608*a**1
6*d**4), Ne(a**16*d**4*exp(20*I*c), 0)), (x*((-129*exp(8*I*c) - 72*exp(6*I*
c) - 30*exp(4*I*c) - 8*exp(2*I*c) - 1)*exp(-8*I*c)/(16*a**4) + 129/(16*a**4
)), True)) - 2*I/(a**4*d*exp(2*I*c)*exp(2*I*d*x) - a**4*d) - 129*x/(16*a**4
) - 4*I*log(exp(2*I*d*x) - exp(-2*I*c))/(a**4*d)
```

Giac [A]
time = 1.24, size = 129, normalized size = 0.81

$$-\frac{\frac{1536i\,\log(-i\,\tan(dx+c))}{a^4}+\frac{12i\,\log(i\,\tan(dx+c)-1)}{a^4}-\frac{1548i\,\log(-i\,\tan(dx+c)-1)}{a^4}+\frac{384\,(-4i\,\tan(dx+c)+1)}{a^4\tan(dx+c)}+\frac{3225i\,\tan(dx+c)^4+14076\,\tan(dx+c)^3-23286i\,\tan(dx+c)^2-17404\,\tan(dx+c)+5017i}{a^4(\tan(dx+c)-i)^4}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/384*(1536*I*log(-I*tan(d*x + c))/a^4 + 12*I*log(I*tan(d*x + c) - 1)/a^4
- 1548*I*log(-I*tan(d*x + c) - 1)/a^4 + 384*(-4*I*tan(d*x + c) + 1)/(a^4*ta
n(d*x + c)) + (3225*I*tan(d*x + c)^4 + 14076*tan(d*x + c)^3 - 23286*I*tan(d
*x + c)^2 - 17404*tan(d*x + c) + 5017*I)/(a^4*(tan(d*x + c) - I)^4))/d
```

Mupad [B]
time = 4.16, size = 165, normalized size = 1.04

$$\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,129\mathrm{i}}{32\,a^4\,d}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{32\,a^4\,d}-\frac{\frac{1}{a^4}-\frac{851\,\tan(c+d\,x)^2}{48\,a^4}+\frac{65\,\tan(c+d\,x)^4}{16\,a^4}+\frac{\tan(c+d\,x)\,26\mathrm{i}}{3\,a^4}-\frac{\tan(c+d\,x)^3\,57\mathrm{i}}{4\,a^4}}{d\,\left(\tan\left(c+d\,x\right)^5-\tan\left(c+d\,x\right)^4\,4\mathrm{i}-6\,\tan\left(c+d\,x\right)^3+\tan\left(c+d\,x\right)^2\,4\mathrm{i}+\tan\left(c+d\,x\right)\right)}-\frac{\ln\left(\tan(c+d\,x)\right)\,4\mathrm{i}}{a^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] (log(tan(c + d*x) - 1i)*129i)/(32*a^4*d) - (log(tan(c + d*x) + 1i)*1i)/(32*
a^4*d) - ((tan(c + d*x)*26i)/(3*a^4) + 1/a^4 - (851*tan(c + d*x)^2)/(48*a^4
) - (tan(c + d*x)^3*57i)/(4*a^4) + (65*tan(c + d*x)^4)/(16*a^4))/(d*(tan(c
+ d*x) + tan(c + d*x)^2*4i - 6*tan(c + d*x)^3 - tan(c + d*x)^4*4i + tan(c +
 d*x)^5)) - (log(tan(c + d*x))*4i)/(a^4*d)
```

3.84 $\int \tan^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=168

$$-\frac{i\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}+\frac{8i\sqrt{a+ia\tan(c+dx)}}{35d}-\frac{2i\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{35d}$$

```
[Out] -I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)/d+
8/35*I*(a+I*a*tan(d*x+c))^(1/2)/d-2/35*I*(a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c
)^2/d+2/7*(a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^3/d+62/105*I*(a+I*a*tan(d*x+c
))^(3/2)/a/d
```

Rubi [A]
time = 0.23, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3641, 3678, 3673, 3608, 3561, 212}

$$\frac{2\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}-\frac{2i\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{35d}+\frac{62i(a+ia\tan(c+dx))^{3/2}}{105ad}+\frac{8i\sqrt{a+ia\tan(c+dx)}}{35d}-\frac{i\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-I)*Sqrt[2]*Sqrt[a]*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]
)/d + (((8*I)/35)*Sqrt[a + I*a*Tan[c + d*x]])/d - (((2*I)/35)*Tan[c + d*x]^
2*Sqrt[a + I*a*Tan[c + d*x]])/d + (2*Tan[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*
x]])/(7*d) + (((62*I)/105)*(a + I*a*Tan[c + d*x])^(3/2))/(a*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rule 3641

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[d*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n - 1)/(f*(m + n - 1))), x] - Dist[1/(a*(m + n - 1)), Int[(a + b*
Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 2)*Simp[d*(b*c*m + a*d*(-1 + n))
- a*c^2*(m + n - 1) + d*(b*d*m - a*c*(m + 2*n - 2))*Tan[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1] && NeQ[m + n - 1, 0] && (IntegerQ[n]
|| IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rule 3678

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Dist[
1/(a*(m + n)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Simp
[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \tan^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{2\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d} - \frac{2\int \tan^2(c+dx)\left(3a+\frac{1}{2}ia\right.}{}$$
$$= -\frac{2i\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{35d} + \frac{2\tan^3(c+dx)\sqrt{a+ia}}{7d}$$
$$= -\frac{2i\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{35d} + \frac{2\tan^3(c+dx)\sqrt{a+ia}}{7d}$$
$$= \frac{8i\sqrt{a+ia\tan(c+dx)}}{35d} - \frac{2i\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{35d}$$
$$= \frac{8i\sqrt{a+ia\tan(c+dx)}}{35d} - \frac{2i\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{35d}$$
$$= -\frac{i\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} + \frac{8i\sqrt{a+ia\tan(c}{35d}$$

Mathematica [A]
time = 2.38, size = 105, normalized size = 0.62

$$\frac{\sqrt{a+ia\tan(c+dx)}\left(-ie^{-i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right)+\frac{2}{105}(-46(-i+\tan(c+dx))+3\sec^2(c+dx)(-i+5\tan(c+dx)))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (Sqrt[a + I*a*Tan[c + d*x]]*(((-I)*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^
(I*(c + d*x))])/E^(I*(c + d*x)) + (2*(-46*(-I + Tan[c + d*x]) + 3*Sec[c + d
*x]^2*(-I + 5*Tan[c + d*x])))/105))/d
```

Maple [A]
time = 0.26, size = 94, normalized size = 0.56

method	result
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{2a(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{2a^2(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-\frac{a^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\sqrt{}}{2\sqrt{a}}\right)}{2}\right.}{d\,a^3}$

default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{2a(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{2a^{2}(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-\frac{a^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}}{2\sqrt{a}}\right.}{2}\right.}{d\,a^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/d/a^3*(1/7*(a+I*a*tan(d*x+c))^(7/2)-2/5*a*(a+I*a*tan(d*x+c))^(5/2)+2/3*
a^2*(a+I*a*tan(d*x+c))^(3/2)-1/2*a^(7/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x
+c))^(1/2)*2^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.49, size = 120, normalized size = 0.71

$$\frac{i\left(105\sqrt{2}\,a^{\frac{11}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+60\,(i\,a\tan(dx+c)+a)^{\frac{7}{2}}a^{2}-168\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}a^{3}+280\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a^{4}\right)}{210\,a^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/210*I*(105*sqrt(2)*a^(11/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c)
 + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 60*(I*a*tan(d*x +
c) + a)^(7/2)*a^2 - 168*(I*a*tan(d*x + c) + a)^(5/2)*a^3 + 280*(I*a*tan(d*x
 + c) + a)^(3/2)*a^4)/(a^5*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 330 vs. 2(125) = 250.
time = 0.49, size = 330, normalized size = 1.96

$$\frac{105\sqrt{2}\left(de^{(6idx+6ic)}+3de^{(4idx+4ic)}+3de^{(2idx+2ic)}+d\right)\sqrt{-\frac{a}{d^{2}}}\log\left(4\left(\left(ide^{(2idx+2ic)}+id\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{-\frac{a}{d^{2}}}+ae^{(idx+ic)}\right)e^{(-idx-ic)}\right)-105\sqrt{2}\left(de^{(6idx+6ic)}+3de^{(4idx+4ic)}+3de^{(2idx+2ic)}+d\right)\sqrt{-\frac{a}{d^{2}}}\log\left(4\left(\left(-ide^{(2idx+2ic)}-id\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{-\frac{a}{d^{2}}}+ae^{(idx+ic)}\right)e^{(-idx-ic)}\right)-16\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\left(-23ie^{(7idx+7ic)}-28ie^{(5idx+5ic)}-35ie^{(3idx+3ic)}\right)}{210\left(de^{(6idx+6ic)}+3de^{(4idx+4ic)}+3de^{(2idx+2ic)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/210*(105*sqrt(2)*(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e
^(2*I*d*x + 2*I*c) + d)*sqrt(-a/d^2)*log(4*((I*d*e^(2*I*d*x + 2*I*c) + I*d)
*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-a/d^2) + a*e^(I*d*x + I*c))*e^(-I*
d*x - I*c)) - 105*sqrt(2)*(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c)
+ 3*d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-a/d^2)*log(4*((-I*d*e^(2*I*d*x + 2*I*c
) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-a/d^2) + a*e^(I*d*x + I*c)
)*e^(-I*d*x - I*c)) - 16*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-23*I*e
^(7*I*d*x + 7*I*c) - 28*I*e^(5*I*d*x + 5*I*c) - 35*I*e^(3*I*d*x + 3*I*c)))/
(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c)
+ d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,\tan^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)*tan(d*x+c)**4,x)
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*tan(c + d*x)**4, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^4,x, algorithm="giac")
[Out] Timed out
```

Mupad [B]
time = 0.34, size = 109, normalized size = 0.65

$$\frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^{3/2}\,4\mathrm{i}}{3\,a\,d}-\frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^{5/2}\,4\mathrm{i}}{5\,a^2\,d}+\frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^{7/2}\,2\mathrm{i}}{7\,a^3\,d}-\frac{\sqrt{2}\,\sqrt{-a}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}{2\,\sqrt{-a}}\right)\,1\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] ((a + a*tan(c + d*x)*1i)^(3/2)*4i)/(3*a*d) - ((a + a*tan(c + d*x)*1i)^(5/2)
*4i)/(5*a^2*d) + ((a + a*tan(c + d*x)*1i)^(7/2)*2i)/(7*a^3*d) - (2^(1/2)*(-
a)^(1/2)*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*(-a)^(1/2)))*1i)/d
```

3.85 $\int \tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=127

$$\frac{\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{8\sqrt{a+ia\tan(c+dx)}}{5d}+\frac{2\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-$$

```
[Out] arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)/d-8/5
*(a+I*a*tan(d*x+c))^(1/2)/d+2/5*(a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^2/d-2/1
5*(a+I*a*tan(d*x+c))^(3/2)/a/d
```

Rubi [A]

time = 0.14, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3641, 3673, 3608, 3561, 212}

$$\frac{2\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{2(a+ia\tan(c+dx))^{3/2}}{15ad}-\frac{8\sqrt{a+ia\tan(c+dx)}}{5d}+\frac{\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (Sqrt[2]*Sqrt[a]*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d -
 (8*Sqrt[a + I*a*Tan[c + d*x]])/(5*d) + (2*Tan[c + d*x]^2*Sqrt[a + I*a*Tan[
c + d*x]])/(5*d) - (2*(a + I*a*Tan[c + d*x])^(3/2))/(15*a*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rule 3641

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[d*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n - 1)/(f*(m + n - 1))), x] - Dist[1/(a*(m + n - 1)), Int[(a + b*
Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 2)*Simp[d*(b*c*m + a*d*(-1 + n))
- a*c^2*(m + n - 1) + d*(b*d*m - a*c*(m + 2*n - 2))*Tan[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1] && NeQ[m + n - 1, 0] && (IntegerQ[n]
|| IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx &= \frac{2\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d} - \frac{2\int \tan(c+dx)\left(2a+\frac{1}{2}ia\right.}{}\\
&= \frac{2\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d} - \frac{2(a+ia\tan(c+dx))^{3/2}}{15ad}\\
&= -\frac{8\sqrt{a+ia\tan(c+dx)}}{5d} + \frac{2\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}\\
&= -\frac{8\sqrt{a+ia\tan(c+dx)}}{5d} + \frac{2\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}\\
&= \frac{\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{8\sqrt{a+ia\tan(c+d}}{5d}\end{aligned}$$

Mathematica [A]
time = 1.80, size = 95, normalized size = 0.75

$$\frac{\sec^2(c+dx)\left(-10+\frac{30\sinh^{-1}\left(e^{i(c+dx)}\right)\cos^3(c+dx)}{\sqrt{1+e^{2i(c+dx)}}}-16\cos(2(c+dx))-i\sin(2(c+dx))\right)\sqrt{a+ia\tan(c+dx)}}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]],x]
```

[Out] (Sec[c + d*x]^2*(-10 + (30*ArcSinh[E^(I*(c + d*x))]*Cos[c + d*x]^3)/Sqrt[1 + E^((2*I)*(c + d*x))] - 16*Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/(15*d)

Maple [A]
time = 0.20, size = 92, normalized size = 0.72

method	result
derivativedivides	$-\frac{2\left(\frac{(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{a(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+a^2\sqrt{a+ia\tan(dx+c)}-\frac{a^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\sqrt{2}}{2\sqrt{a}}\right)}{2}\right)}{d\,a^2}$
default	$-\frac{2\left(\frac{(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{a(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+a^2\sqrt{a+ia\tan(dx+c)}-\frac{a^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\sqrt{2}}{2\sqrt{a}}\right)}{2}\right)}{d\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^3,x,method=_RETURNVERBOSE)

[Out] -2/d/a^2*(1/5*(a+I*a*tan(d*x+c))^(5/2)-1/3*a*(a+I*a*tan(d*x+c))^(3/2)+a^2*(a+I*a*tan(d*x+c))^(1/2)-1/2*a^(5/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))

Maxima [A]
time = 0.48, size = 120, normalized size = 0.94

$$-\frac{15\sqrt{2}\,a^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+12\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}a^2-20\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a^3+60\sqrt{i\,a\tan(dx+c)+a}\,a^4}{30\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="maxima")

[Out] -1/30*(15*sqrt(2)*a^(9/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 12*(I*a*tan(d*x + c) + a)^(5/2)*a^2 - 20*(I*a*tan(d*x + c) + a)^(3/2)*a^3 + 60*sqrt(I*a*tan(d*x + c) + a)*a^4)/(a^4*d)

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 285 vs. 2(98) = 196.
time = 0.46, size = 285, normalized size = 2.24

$$\frac{15\sqrt{2}\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{d^2}}\log\left(4\left(\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{a}{d^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-15\sqrt{2}\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{d^2}}\log\left(-4\left(\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{a}{d^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-4\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(17\,e^{(5i\,dx+5i\,c)}+20\,e^{(3i\,dx+3i\,c)}+15\,e^{(i\,dx+i\,c)}\right)}{30\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/30*(15*sqrt(2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*sqrt
(a/d^2)*log(4*((d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
)*sqrt(a/d^2) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 15*sqrt(2)*(d*e^(4*I
*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/d^2)*log(-4*((d*e^(2*I*
d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(a/d^2) - a*e^(I*d*
x + I*c))*e^(-I*d*x - I*c)) - 4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(
17*e^(5*I*d*x + 5*I*c) + 20*e^(3*I*d*x + 3*I*c) + 15*e^(I*d*x + I*c)))/(d*e
^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,\tan^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)*tan(d*x+c)**3,x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*tan(c + d*x)**3, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 4.15, size = 100, normalized size = 0.79

$$-\frac{2\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{d}+\frac{2\,{\left(a+a\tan(c+d\,x)\,1{}\mathrm{i}\right)}^{3/2}}{3\,a\,d}-\frac{2\,{\left(a+a\tan(c+d\,x)\,1{}\mathrm{i}\right)}^{5/2}}{5\,a^2\,d}-\frac{\sqrt{2}\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2\,\sqrt{a}}\right)\,1{}\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] (2*(a + a*tan(c + d*x)*1i)^(3/2))/(3*a*d) - (2*(a + a*tan(c + d*x)*1i)^(1/2
))/d - (2*(a + a*tan(c + d*x)*1i)^(5/2))/(5*a^2*d) - (2^(1/2)*a^(1/2)*atan(
(2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)*1i)/(2*a^(1/2)))*1i)/d
```

3.86 $\int \tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=76

$$\frac{i\sqrt{2}\,\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{3ad}$$

```
[Out] I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)/d-2
/3*I*(a+I*a*tan(d*x+c))^(3/2)/a/d
```

Rubi [A]

time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3624, 3561, 212}

$$\frac{i\sqrt{2}\,\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (I*Sqrt[2]*Sqrt[a]*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d
 - (((2*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(a*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{2i(a+ia\tan(c+dx))^{3/2}}{3ad} - \int \sqrt{a+ia\tan(c+dx)}\,dx$$
$$= -\frac{2i(a+ia\tan(c+dx))^{3/2}}{3ad} + \frac{(2ia)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+ia}\right.}{d}$$
$$= \frac{i\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{2i(a+ia\tan(c+d}{3ad}$$

Mathematica [A]
time = 0.77, size = 97, normalized size = 1.28

$$-\frac{ie^{-i(c+dx)}\left(4e^{3i(c+dx)} - 3\left(1+e^{2i(c+dx)}\right)^{3/2}\sinh^{-1}\left(e^{i(c+dx)}\right)\right)\sqrt{a+ia\tan(c+dx)}}{3d\left(1+e^{2i(c+dx)}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-1/3*I)*(4*E^((3*I)*(c + d*x)) - 3*(1 + E^((2*I)*(c + d*x)))^(3/2)*ArcSin
h[E^(I*(c + d*x))])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^(I*(c + d*x))*(1 + E^(
(2*I)*(c + d*x))))
```

Maple [A]
time = 0.17, size = 58, normalized size = 0.76

method	result	size
derivativedivides	$-\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-\frac{a^{\frac{3}{2}}\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2}\right)}{da}$	58
default	$-\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-\frac{a^{\frac{3}{2}}\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2}\right)}{da}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/d/a*(1/3*(a+I*a*tan(d*x+c))^(3/2)-1/2*a^(3/2)*2^(1/2)*arctanh(1/2*(a+I
*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.49, size = 84, normalized size = 1.11

$$-\frac{i\left(3\sqrt{2}\,a^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)+4\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}a^{2}\right)}{6\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/6*I*(3*sqrt(2)*a^(7/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a
))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(I*a*tan(d*x + c) +
a)^(3/2)*a^2)/(a^3*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(55) = 110.
time = 0.41, size = 233, normalized size = 3.07

$$-\frac{3\sqrt{2}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{-\frac{a}{d^{2}}}\log\left(4\left(\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{a}{d^{2}}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-3\sqrt{2}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{-\frac{a}{d^{2}}}\log\left(4\left(\left(-i\,de^{(2i\,dx+2i\,c)}-i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{a}{d^{2}}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+8i\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{(3i\,dx+3i\,c)}}{6\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(3*sqrt(2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-a/d^2)*log(4*((I*d*e^(2*I
*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-a/d^2) + a*e^(
I*d*x + I*c))*e^(-I*d*x - I*c)) - 3*sqrt(2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqr
t(-a/d^2)*log(4*((-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*sqrt(-a/d^2) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 8*I*sqrt(2)*
sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(3*I*d*x + 3*I*c))/(d*e^(2*I*d*x + 2*I*
c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,\tan^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)*tan(d*x+c)**2,x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*tan(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="giac")
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*tan(d*x + c)^2, x)
```

Mupad [B]
time = 0.22, size = 63, normalized size = 0.83

$$-\frac{(a+a\tan(c+d\,x)\;1\mathrm{i})^{3/2}\,2\mathrm{i}}{3\,a\,d}+\frac{\sqrt{2}\;\sqrt{-a}\;\mathrm{atan}\left(\frac{\sqrt{2}\;\sqrt{a+a\tan\left(c+d\,x\right)\;1\mathrm{i}}}{2\,\sqrt{-a}}\right)\;1\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] (2^(1/2)*(-a)^(1/2)*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*(-a)^(1
/2)))*1i)/d - ((a + a*tan(c + d*x)*1i)^(3/2)*2i)/(3*a*d)
```

3.87 $\int \tan(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=67

$$-\frac{\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}+\frac{2\sqrt{a+ia\tan(c+dx)}}{d}$$

```
[Out] -arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)/d+2*
(a+I*a*tan(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3608, 3561, 212}

$$\frac{2\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*Sqrt[a]*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d
) + (2*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rubi steps

$$\int \tan(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{2\sqrt{a+ia\tan(c+dx)}}{d} - i\int \sqrt{a+ia\tan(c+dx)}\,dx$$

$$= \frac{2\sqrt{a+ia\tan(c+dx)}}{d} - \frac{(2a)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+ia\tan(c}\right.}{d}$$

$$= -\frac{\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} + \frac{2\sqrt{a+ia\tan(c+}}{d}$$

Mathematica [A]
time = 0.47, size = 77, normalized size = 1.15

$$\frac{e^{-i(c+dx)}\left(2e^{i(c+dx)} - \sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right)\right)\sqrt{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((2*E^(I*(c + d*x)) - Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))
])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^(I*(c + d*x)))
```

Maple [A]
time = 0.16, size = 53, normalized size = 0.79

method	result	size
derivativedivides	$\frac{2\sqrt{a+ia\tan(dx+c)}-\sqrt{a}\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\sqrt{2}}{2\sqrt{a}}\right)}{d}$	53
default	$\frac{2\sqrt{a+ia\tan(dx+c)}-\sqrt{a}\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\sqrt{2}}{2\sqrt{a}}\right)}{d}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(a+I*a*tan(d*x+c))^(1/2)-a^(1/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+
c))^(1/2)*2^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.50, size = 83, normalized size = 1.24

$$\frac{\sqrt{2}\,a^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+4\sqrt{i\,a\tan(dx+c)+a}\,a^2}{2\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(2)*a^(5/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(s
qrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*sqrt(I*a*tan(d*x + c) + a
)*a^2)/(a^2*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(52) = 104$.
time = 0.42, size = 187, normalized size = 2.79

$$-\frac{\sqrt{2}\, d\sqrt{\frac{a}{d^2}}\log\left(4\left(\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{a}{d^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-\sqrt{2}\, d\sqrt{\frac{a}{d^2}}\log\left(-4\left(\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{a}{d^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-4\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\, e^{(i\,dx+i\,c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*d*sqrt(a/d^2)*log(4*((d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*sqrt(a/d^2) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) -
sqrt(2)*d*sqrt(a/d^2)*log(-4*((d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*
x + 2*I*c) + 1))*sqrt(a/d^2) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 4*sqr
t(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,\tan\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)*tan(d*x+c),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*tan(c + d*x), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 4.04, size = 54, normalized size = 0.81

$$\frac{2\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{d}-\frac{\sqrt{2}\,\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{2\,\sqrt{a}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] (2*(a + a*tan(c + d*x)*1i)^(1/2))/d - (2^(1/2)*a^(1/2)*atanh((2^(1/2)*(a +
a*tan(c + d*x)*1i)^(1/2))/(2*a^(1/2))))/d
```

3.88 $\int \sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=46

$$-\frac{i\sqrt{2}\,\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}$$

```
[Out] -I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)/d
```

Rubi [A]

time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3561, 212}

$$-\frac{i\sqrt{2}\,\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-I)*Sqrt[2]*Sqrt[a]*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]
)/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+ia\tan(c+dx)}\,dx &= -\frac{(2ia)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+ia\tan(c+dx)}\right)}{d}\\ &= -\frac{i\sqrt{2}\,\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}\end{aligned}$$

Mathematica [A]
time = 0.20, size = 64, normalized size = 1.39

$$-\frac{ie^{-i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right)\sqrt{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-I)*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))]*Sqrt[a + I*a*T
an[c + d*x]])/(d*E^(I*(c + d*x)))
```

Maple [A]
time = 0.15, size = 36, normalized size = 0.78

method	result	size
derivativedivides	$-\frac{i\,\mathrm{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}\,\sqrt{a}}{d}$	36
default	$-\frac{i\,\mathrm{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}\,\sqrt{a}}{d}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)/d
```

Maxima [A]
time = 0.50, size = 60, normalized size = 1.30

$$\frac{i\,\sqrt{2}\,\sqrt{a}\,\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*I*sqrt(2)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(
sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(33) = 66.
time = 0.46, size = 159, normalized size = 3.46

$$\frac{1}{2}\sqrt{2}\sqrt{-\frac{a}{d^2}}\log\left(4\left(\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{a}{d^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-\frac{1}{2}\sqrt{2}\sqrt{-\frac{a}{d^2}}\log\left(4\left(\left(-i\,de^{(2i\,dx+2i\,c)}-i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{a}{d^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*sqrt(-a/d^2)*log(4*((I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(
2*I*d*x + 2*I*c) + 1))*sqrt(-a/d^2) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c))
- 1/2*sqrt(2)*sqrt(-a/d^2)*log(4*((-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1))*sqrt(-a/d^2) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c
))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\tan{(c+dx)}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*tan(c + d*x) + a), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [**B**]
time = 3.95, size = 39, normalized size = 0.85

$$-\frac{\sqrt{2}\,\sqrt{-a}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}{2\,\sqrt{-a}}\right)\,1\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] -(2^(1/2)*(-a)^(1/2)*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*(-a)^(
1/2)))*1i)/d
```

3.89 $\int \cot(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=78

$$-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{\sqrt{2}\,\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}$$

```
[Out] -2*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d+arctanh(1/2*(a+I*a*t
an(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)/d
```

Rubi [A]
time = 0.13, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3643, 3561, 212, 3680, 65, 214}

$$\frac{\sqrt{2}\,\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/d + (Sqrt[2]*Sqrt[
a]*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3643

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)/((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[a/(a*c - b*d), Int[(a + b*Tan[e + f*x])^m,
x], x] - Dist[d/(a*c - b*d), Int[(a + b*Tan[e + f*x])^m*((b + a*Tan[e + f*x
])/(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx &= i\int \sqrt{a+ia\tan(c+dx)}\,dx - \frac{i\int \cot(c+dx)\sqrt{a+ia\tan(c+d}{a}\\ &= \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+iax}}\,dx, x, \tan(c+dx)\right)}{d} + \frac{(2a)\text{Subst}\left(\int \frac{1}{2a-x^2}\right.}{}\\ &= \frac{\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{(2i)\text{Subst}\left(\int \frac{1}{i-\frac{ix^2}{a}}\right. a}{}\\ &= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{}}{}\right.}{d}\end{aligned}$$

Mathematica [F]
time = 0.71, size = 0, normalized size = 0.00

$$\int \cot(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] Integrate[Cot[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]], x]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(61) = 122$.
time = 4.58, size = 230, normalized size = 2.95

method	result
default	$-\frac{\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\left(i\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)-\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos}{1+\mathrm{co}}}}{}\right.\right.}{d(i\sin(dx+c)+\mathrm{c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(I*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^
(1/2))-2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/
cos(d*x+c)*2^(1/2))+I*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-ln(((-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c)))*si
n(d*x+c)/(I*sin(d*x+c)+cos(d*x+c)-1)
```

Maxima [A]
time = 0.49, size = 107, normalized size = 1.37

$$-\frac{\sqrt{2}\,\sqrt{a}\,\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)-2\,\sqrt{a}\,\log\left(\frac{\sqrt{i\,a\tan\left(dx+c\right)+a}-\sqrt{a}}{\sqrt{i\,a\tan\left(dx+c\right)+a}+\sqrt{a}}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(2)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(
sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - 2*sqrt(a)*log((sqrt(I*a*ta
n(d*x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x + c) + a) + sqrt(a))))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(59) = 118$.
time = 0.56, size = 336, normalized size = 4.31

$$\frac{1}{2}\sqrt{2}\sqrt{\frac{a}{d^2}}\log\left(4\left(\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{a}{d^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-\frac{1}{2}\sqrt{2}\sqrt{\frac{a}{d^2}}\log\left(-4\left(\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{a}{d^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-\frac{1}{2}\sqrt{\frac{a}{d^2}}\log\left(16\left(3\,a^2e^{(2i\,dx+2i\,c)}+2\sqrt{2}\left(ade^{(3i\,dx+3i\,c)}+ade^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{a}{d^2}}+a^2\right)e^{(-2i\,dx-2i\,c)}\right)+\frac{1}{2}\sqrt{\frac{a}{d^2}}\log\left(16\left(3\,a^2e^{(2i\,dx+2i\,c)}-2\sqrt{2}\left(ade^{(3i\,dx+3i\,c)}+ade^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{a}{d^2}}+a^2\right)e^{(-2i\,dx-2i\,c)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*sqrt(a/d^2)*log(4*((d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*sqrt(a/d^2) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 1/2*
sqrt(2)*sqrt(a/d^2)*log(-4*((d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*sqrt(a/d^2) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 1/2*sqr
t(a/d^2)*log(16*(3*a^2*e^(2*I*d*x + 2*I*c) + 2*sqrt(2)*(a*d*e^(3*I*d*x + 3*
I*c) + a*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(a/d^2) +
 a^2)*e^(-2*I*d*x - 2*I*c)) + 1/2*sqrt(a/d^2)*log(16*(3*a^2*e^(2*I*d*x + 2*
I*c) - 2*sqrt(2)*(a*d*e^(3*I*d*x + 3*I*c) + a*d*e^(I*d*x + I*c))*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1))*sqrt(a/d^2) + a^2)*e^(-2*I*d*x - 2*I*c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,\cot\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*cot(c + d*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*cot(d*x + c), x)
```

Mupad [**B**]
time = 0.17, size = 61, normalized size = 0.78

$$-\frac{2\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{\sqrt{a}}\right)}{d}+\frac{\sqrt{2}\,\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{2\sqrt{a}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] (2^(1/2)*a^(1/2)*atanh((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*a^(1/2)))
)/d - (2*a^(1/2)*atanh((a + a*tan(c + d*x)*1i)^(1/2)/a^(1/2)))/d
```

3.90 $\int \cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=111

$$-\frac{i\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{i\sqrt{2}\,\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}-\frac{\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

```
[Out] -I*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d+I*arctanh(1/2*(a+I*a
*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)/d-cot(d*x+c)*(a+I*a*tan
(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.17, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3642, 21, 3635, 3561, 212, 3680, 65, 214}

$$-\frac{i\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{i\sqrt{2}\,\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}-\frac{\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-I)*Sqrt[a]*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/d + (I*Sqrt[2]*S
qrt[a]*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d - (Cot[c +
d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3635

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[2*(a^2/(a*c - b*d)), Int[Sqrt[a + b*Tan[e
+ f*x]], x], x] - Dist[(2*b*c*d + a*(c^2 - d^2))/(a*(c^2 + d^2)), Int[(a -
b*Tan[e + f*x])*(Sqrt[a + b*Tan[e + f*x]]/(c + d*Tan[e + f*x])), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

Rule 3642

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[d*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(a*(c^2 + d^2)*(n +
1)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[b*d*m - a
*c*(n + 1) + a*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
 e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0
] && LtQ[n, -1] && (IntegerQ[n] || IntegersQ[2*m, 2*n])
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rubi steps

$$\begin{aligned}
\int \cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx &= -\frac{\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d} + \frac{\int \cot(c+dx)\left(\frac{ia}{2}-\frac{1}{2}a\tan(\right.}{} \\
&= -\frac{\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d} + \frac{i\int \cot(c+dx)(a+ia\tan(}{2a} \\
&= -\frac{\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d} + \frac{i\int \cot(c+dx)(a-ia\tan(}{} \\
&= -\frac{\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d} + \frac{(ia)\text{Subst}\left(\int \frac{1}{x\sqrt{a+iax}}\,d\right.}{2d} \\
&= \frac{i\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{\cot(c+dx)\sqrt{a+i}}{d} \\
&= -\frac{i\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{i\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{}}{}\right.}{d}
\end{aligned}$$

Mathematica [A]
time = 4.48, size = 197, normalized size = 1.77

$$\frac{\left(-4\cot(c+dx)+ie^{-i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(4\sinh^{-1}\left(e^{i(c+dx)}\right)+\sqrt{2}\left(\log\left(1-e^{i(c+dx)}\right)-\log\left(1+e^{i(c+dx)}\right)+\log\left(1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right)-\log\left(1+e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right)\right)\right)\right)\sqrt{a+ia\tan(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-4*Cot[c + d*x] + (I*Sqrt[1 + E^((2*I)*(c + d*x))]*(4*ArcSinh[E^(I*(c + d
*x))] + Sqrt[2]*(Log[1 - E^(I*(c + d*x))] - Log[1 + E^(I*(c + d*x))] + Log[
1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]] - Log[1 + E^(I
*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]])))/E^(I*(c + d*x)))*Sq
rt[a + I*a*Tan[c + d*x]])/(4*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 585 vs. 2(89) = 178.
time = 0.90, size = 586, normalized size = 5.28

method	result
default	$-\frac{\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(2i\sqrt{2}\left(\cos^2(dx+c)\right)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)+i(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(2*I*2^(1/2)*cos(d*x+
c)^2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+I*cos(d*x+c)^2*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)
-cos(d*x+c)+1)/sin(d*x+c))+2*2^(1/2)*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-2*I*2^
(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+cos(d*x+c)^2*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+2*I*c
os(d*x+c)*sin(d*x+c)-I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))-2*2^(1/2)*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*2^(1/2))+2*cos(d*x+c)^2-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan
(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-2*cos(d*x+c))/(I*sin(d*x+c)+cos(d*
x+c)-1)/sin(d*x+c)
```

Maxima [**A**]
time = 0.50, size = 134, normalized size = 1.21

$$-\frac{i\,a\left(\frac{\sqrt{2}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)}{\sqrt{a}}-\frac{\log\left(\frac{\sqrt{i\,a\tan\left(dx+c\right)+a}-\sqrt{a}}{\sqrt{i\,a\tan\left(dx+c\right)+a}+\sqrt{a}}\right)}{\sqrt{a}}-\frac{2i\,\sqrt{i\,a\tan\left(dx+c\right)+a}}{a\tan\left(dx+c\right)}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*I*a*(sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sqrt
(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/sqrt(a) - log((sqrt(I*a*tan(d*x
+ c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x + c) + a) + sqrt(a)))/sqrt(a) - 2*I*
sqrt(I*a*tan(d*x + c) + a)/(a*tan(d*x + c)))/d
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 476 vs. 2(84) = 168.
time = 0.45, size = 476, normalized size = 4.29

$$-\frac{2\sqrt{2}\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{-\frac{a}{d^2}}\log\left(4\left(\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{a}{d^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-2\sqrt{2}\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{-\frac{a}{d^2}}\log\left(4\left(\left(-i\,de^{(2i\,dx+2i\,c)}-i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{a}{d^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{-\frac{a}{d^2}}\log\left(16\left(3\,a^2e^{(2i\,dx+2i\,c)}-2\sqrt{2}\left(i\,ade^{(3i\,dx+3i\,c)}+i\,ade^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{a}{d^2}}+a^2\right)e^{(-2i\,dx-2i\,c)}\right)-\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{-\frac{a}{d^2}}\log\left(16\left(3\,a^2e^{(2i\,dx+2i\,c)}-2\sqrt{2}\left(-i\,ade^{(3i\,dx+3i\,c)}-i\,ade^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{a}{d^2}}+a^2\right)e^{(-2i\,dx-2i\,c)}\right)+4\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(i\,e^{(3i\,dx+3i\,c)}+i\,e^{(i\,dx+i\,c)}\right)}{4\left(de^{(2i\,dx+2i\,c)}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(-a/d^2)*log(4*((I*d*e^(2*I
*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-a/d^2) + a*e^(
I*d*x + I*c))*e^(-I*d*x - I*c)) - 2*sqrt(2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqr
t(-a/d^2)*log(4*((-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*sqrt(-a/d^2) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + (d*e^(2*I*d*
x + 2*I*c) - d)*sqrt(-a/d^2)*log(16*(3*a^2*e^(2*I*d*x + 2*I*c) - 2*sqrt(2)*
(I*a*d*e^(3*I*d*x + 3*I*c) + I*a*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*
I*c) + 1))*sqrt(-a/d^2) + a^2)*e^(-2*I*d*x - 2*I*c)) - (d*e^(2*I*d*x + 2*I*
c) - d)*sqrt(-a/d^2)*log(16*(3*a^2*e^(2*I*d*x + 2*I*c) - 2*sqrt(2)*(-I*a*d*
e^(3*I*d*x + 3*I*c) - I*a*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) +
1))*sqrt(-a/d^2) + a^2)*e^(-2*I*d*x - 2*I*c)) + 4*sqrt(2)*sqrt(a/(e^(2*I*d*
x + 2*I*c) + 1))*(I*e^(3*I*d*x + 3*I*c) + I*e^(I*d*x + I*c)))/(d*e^(2*I*d*x
 + 2*I*c) - d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,\cot^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*cot(c + d*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*cot(d*x + c)^2, x)
```

Mupad [**B**]
time = 4.02, size = 97, normalized size = 0.87

$$-\frac{\sqrt{-a}\,\mathrm{atan}\left(\frac{\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{\sqrt{-a}}\right)\,1{}\mathrm{i}}{d}-\frac{\cot\left(c+d\,x\right)\,\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{d}+\frac{\sqrt{2}\,\sqrt{-a}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{2\,\sqrt{-a}}\right)\,1{}\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] (2^(1/2)*(-a)^(1/2)*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*(-a)^(1
/2)))*1i)/d - (cot(c + d*x)*(a + a*tan(c + d*x)*1i)^(1/2))/d - ((-a)^(1/2)*
atan((a + a*tan(c + d*x)*1i)^(1/2)/(-a)^(1/2))*1i)/d
```

3.91 $\int \cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=145

$$\frac{7\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{4d}-\frac{\sqrt{2}\,\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}-\frac{i\cot(c+dx)\sqrt{a+ia}}{4d}$$

```
[Out] 7/4*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-arctanh(1/2*(a+I*a*
tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)/d-1/4*I*cot(d*x+c)*(a+I*
a*tan(d*x+c))^(1/2)/d-1/2*cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.29, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3642, 3679, 3681, 3561, 212, 3680, 65, 214}

$$\frac{7\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{4d}-\frac{\sqrt{2}\,\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}-\frac{\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{i\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (7*Sqrt[a]*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/(4*d) - (Sqrt[2]*Sq
rt[a]*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d - ((I/4)*Cot
[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d - (Cot[c + d*x]^2*Sqrt[a + I*a*Tan[
c + d*x]])/(2*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3642

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[d*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(a*(c^2 + d^2)*(n +
1)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[b*d*m - a
*c*(n + 1) + a*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
 e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0
] && LtQ[n, -1] && (IntegerQ[n] || IntegersQ[2*m, 2*n])
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3681

```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(
A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$
\begin{aligned}
\int \cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx &= -\frac{\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d} + \frac{\int \cot^2(c+dx)\left(\frac{ia}{2}-\frac{3}{2}a\,\mathrm{t}\right.}{} \\
&= -\frac{i\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d} - \frac{\cot^2(c+dx)\sqrt{a+ia\tan}}{2d} \\
&= -\frac{i\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d} - \frac{\cot^2(c+dx)\sqrt{a+ia\tan}}{2d} \\
&= -\frac{i\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d} - \frac{\cot^2(c+dx)\sqrt{a+ia\tan}}{2d} \\
&= -\frac{\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{i\cot(c+dx)\sqrt{a}}{4} \\
&= \frac{7\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{4d} - \frac{\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{}\right.}{d}
\end{aligned}
$$

Mathematica [A]
time = 2.16, size = 144, normalized size = 0.99

$$
\frac{\left(4+e^{-i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(-8\sinh^{-1}\left(e^{i(c+dx)}\right)+7\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}e^{i(c+dx)}}{\sqrt{1+e^{2i(c+dx)}}}\right)\right)-2i\cot(c)-4\csc^2(c+dx)+2i\csc(c)\csc(c+dx)\sin(dx)\right)\sqrt{a+ia\tan(c+dx)}}{8d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((4 + (Sqrt[1 + E^((2*I)*(c + d*x))]*(-8*ArcSinh[E^(I*(c + d*x))] + 7*Sqrt[
2]*ArcTanh[(Sqrt[2]*E^(I*(c + d*x)))/Sqrt[1 + E^((2*I)*(c + d*x))]]))/E^(I*
(c + d*x)) - (2*I)*Cot[c] - 4*Csc[c + d*x]^2 + (2*I)*Csc[c]*Csc[c + d*x]*Si
n[d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(8*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 903 vs. 2(115) = 230.
time = 0.90, size = 904, normalized size = 6.23

method	result	size
default	Expression too large to display	904

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(-14*I*cos(d*x+c)^2*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(1/2))-8*2^(1/2)*cos(d*x+c)^4*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(
1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+7*I
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(1/2))-16*I*2^(1/2)*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ar
ctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-7*cos(d*x+c)^4*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))+16*2^(1/2)*cos(d*x+c)^2*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d
*x+c)/cos(d*x+c)*2^(1/2))+7*I*cos(d*x+c)^4*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-2*I*cos(d*x+c)^3-6*I*co
s(d*x+c)^2+2*I*cos(d*x+c)-6*sin(d*x+c)*cos(d*x+c)^3+14*cos(d*x+c)^2*(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(
d*x+c)-cos(d*x+c)+1)/sin(d*x+c))-8*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)
*2^(1/2))+8*I*2^(1/2)*cos(d*x+c)^4*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arc
tan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+6*I*cos(d*x+c)^4+4*co
s(d*x+c)^2*sin(d*x+c)+8*I*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arct
an(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+2*sin(d*x+c)*cos(d*x+c
)-7*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c)))/(I*sin(d*x+c)+cos(d*x+c)-1)/si
n(d*x+c)^3
```

Maxima [A]
time = 0.49, size = 179, normalized size = 1.23

$$\frac{a^{2}\left(\frac{2\left((i a \tan(dx+c)+a)^{\frac{3}{2}}+\sqrt{i a \tan(dx+c)+a}\, a\right)}{(i a \tan(dx+c)+a)^{2} a-2\,(i a \tan(dx+c)+a) a^{2}+a^{3}}+\frac{4 \sqrt{2} \log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i a \tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i a \tan(dx+c)+a}}\right)}{a^{\frac{3}{2}}}-\frac{7 \log\left(\frac{\sqrt{i a \tan(dx+c)+a}-\sqrt{a}}{\sqrt{i a \tan(dx+c)+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right)}{8 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*a^2*(2*((I*a*tan(d*x + c) + a)^(3/2) + sqrt(I*a*tan(d*x + c) + a)*a)/((
I*a*tan(d*x + c) + a)^2*a - 2*(I*a*tan(d*x + c) + a)*a^2 + a^3) + 4*sqrt(2)
*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sqrt(2)*sqrt(a) + sqr
t(I*a*tan(d*x + c) + a)))/a^(3/2) - 7*log((sqrt(I*a*tan(d*x + c) + a) - sqr
t(a))/(sqrt(I*a*tan(d*x + c) + a) + sqrt(a)))/a^(3/2))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 519 vs. $2(110) = 220$.
time = 0.47, size = 519, normalized size = 3.58

$$-\frac{8\sqrt{2}\left(de^{(4i dx+4i c)}-2de^{(2i dx+2i c)}+d\right)\sqrt{\frac{a}{d^{2}}}\log\left(4\left(\left(de^{(2i dx+2i c)}+d\right)\sqrt{\frac{a}{e^{(2i dx+2i c)}+1}}\sqrt{\frac{a}{d^{2}}}+ae^{(i dx+i c)}\right)e^{(-i dx-i c)}\right)-8\sqrt{2}\left(de^{(4i dx+4i c)}-2de^{(2i dx+2i c)}+d\right)\sqrt{\frac{a}{d^{2}}}\log\left(-4\left(\left(de^{(2i dx+2i c)}+d\right)\sqrt{\frac{a}{e^{(2i dx+2i c)}+1}}\sqrt{\frac{a}{d^{2}}}-ae^{(i dx+i c)}\right)e^{(-i dx-i c)}\right)-7\left(de^{(4i dx+4i c)}-2de^{(2i dx+2i c)}+d\right)\sqrt{\frac{a}{d^{2}}}\log\left(16\left(3a^{2}e^{(2i dx+2i c)}+2\sqrt{2}\left(ade^{(3i dx+3i c)}+ade^{(i dx+i c)}\right)\sqrt{\frac{a}{e^{(2i dx+2i c)}+1}}\sqrt{\frac{a}{d^{2}}}+a^{2}\right)e^{(-2i dx-2i c)}\right)+7\left(de^{(4i dx+4i c)}-2de^{(2i dx+2i c)}+d\right)\sqrt{\frac{a}{d^{2}}}\log\left(16\left(3a^{2}e^{(2i dx+2i c)}-2\sqrt{2}\left(ade^{(3i dx+3i c)}+ade^{(i dx+i c)}\right)\sqrt{\frac{a}{e^{(2i dx+2i c)}+1}}\sqrt{\frac{a}{d^{2}}}+a^{2}\right)e^{(-2i dx-2i c)}\right)-4\sqrt{2}\sqrt{\frac{a}{e^{(2i dx+2i c)}+1}}\left(3e^{(5i dx+5i c)}+4e^{(3i dx+3i c)}+e^{(i dx+i c)}\right)}{16\left(de^{(4i dx+4i c)}-2de^{(2i dx+2i c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/16*(8*sqrt(2)*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*sqrt
(a/d^2)*log(4*((d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
)*sqrt(a/d^2) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 8*sqrt(2)*(d*e^(4*I*
d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/d^2)*log(-4*((d*e^(2*I*d
*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(a/d^2) - a*e^(I*d*x
 + I*c))*e^(-I*d*x - I*c)) - 7*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*
I*c) + d)*sqrt(a/d^2)*log(16*(3*a^2*e^(2*I*d*x + 2*I*c) + 2*sqrt(2)*(a*d*e^
(3*I*d*x + 3*I*c) + a*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*
sqrt(a/d^2) + a^2)*e^(-2*I*d*x - 2*I*c)) + 7*(d*e^(4*I*d*x + 4*I*c) - 2*d*e
^(2*I*d*x + 2*I*c) + d)*sqrt(a/d^2)*log(16*(3*a^2*e^(2*I*d*x + 2*I*c) - 2*s
qrt(2)*(a*d*e^(3*I*d*x + 3*I*c) + a*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*sqrt(a/d^2) + a^2)*e^(-2*I*d*x - 2*I*c)) - 4*sqrt(2)*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1))*(3*e^(5*I*d*x + 5*I*c) + 4*e^(3*I*d*x + 3*I*c) +
e^(I*d*x + I*c)))/(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,\cot^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*cot(c + d*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*cot(d*x + c)^3, x)
```

Mupad [B]
time = 4.07, size = 124, normalized size = 0.86

$$-\frac{\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{4\,d\,{\tan\left(c+d\,x\right)}^2}-\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,7{}\mathrm{i}}{4\,d}-\frac{{\left(a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}{4\,a\,d\,{\tan\left(c+d\,x\right)}^2}+\frac{\sqrt{2}\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2\,\sqrt{a}}\right)\,1{}\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] (2^(1/2)*a^(1/2)*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)*1i)/(2*a^(1/2)
))*1i)/d - (a^(1/2)*atan(((a + a*tan(c + d*x)*1i)^(1/2)*1i)/a^(1/2))*7i)/(4
*d) - (a + a*tan(c + d*x)*1i)^(3/2)/(4*a*d*tan(c + d*x)^2) - (a + a*tan(c +
 d*x)*1i)^(1/2)/(4*d*tan(c + d*x)^2)
```

3.92 $\int \tan^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=199

$$\frac{2\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}+\frac{2ia^2\tan^3(c+dx)}{7d\sqrt{a+ia\tan(c+dx)}}-\frac{2a^2\tan^4(c+dx)}{7d\sqrt{a+ia\tan(c+dx)}}-\frac{64a\sqrt{a+}}{}$$

```
[Out] 2*a^(3/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d-6
4/35*a*(a+I*a*tan(d*x+c))^(1/2)/d+16/35*a*(a+I*a*tan(d*x+c))^(1/2)*tan(d*x+
c)^2/d+2/7*I*a^2*tan(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(1/2)-2/7*a^2*tan(d*x+c)
^4/d/(a+I*a*tan(d*x+c))^(1/2)-76/105*(a+I*a*tan(d*x+c))^(3/2)/d
```

Rubi [A]

time = 0.33, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3637, 3676, 3678, 3673, 3608, 3561, 212}

$$\frac{2\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}-\frac{2a^2\tan^4(c+dx)}{7d\sqrt{a+ia\tan(c+dx)}}+\frac{2ia^2\tan^3(c+dx)}{7d\sqrt{a+ia\tan(c+dx)}}+\frac{16a\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{35d}-\frac{76(a+ia\tan(c+dx))^{3/2}}{105d}-\frac{64a\sqrt{a+ia\tan(c+dx)}}{35d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[2]*a^(3/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d
 + (((2*I)/7)*a^2*Tan[c + d*x]^3)/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (2*a^2*T
an[c + d*x]^4)/(7*d*Sqrt[a + I*a*Tan[c + d*x]]) - (64*a*Sqrt[a + I*a*Tan[c
+ d*x]])/(35*d) + (16*a*Tan[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/(35*d) -
 (76*(a + I*a*Tan[c + d*x])^(3/2))/(105*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rule 3678

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Dist[
1/(a*(m + n)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Simp
[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \tan^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx &= -\frac{2a^2\tan^4(c+dx)}{7d\sqrt{a+ia\tan(c+dx)}}+\frac{1}{7}(2a)\int\frac{\tan^3(c+dx)\left(\frac{15a}{2}+\frac{13}{2}ia \cdots\right.}{\sqrt{a+ia\tan(c+\cdots}} \\
&= \frac{2ia^2\tan^3(c+dx)}{7d\sqrt{a+ia\tan(c+dx)}}-\frac{2a^2\tan^4(c+dx)}{7d\sqrt{a+ia\tan(c+dx)}}-\frac{2\int\tan^2\cdots}{\cdots} \\
&= \frac{2ia^2\tan^3(c+dx)}{7d\sqrt{a+ia\tan(c+dx)}}-\frac{2a^2\tan^4(c+dx)}{7d\sqrt{a+ia\tan(c+dx)}}+\frac{16a\tan\cdots}{\cdots} \\
&= \frac{2ia^2\tan^3(c+dx)}{7d\sqrt{a+ia\tan(c+dx)}}-\frac{2a^2\tan^4(c+dx)}{7d\sqrt{a+ia\tan(c+dx)}}+\frac{16a\tan\cdots}{\cdots} \\
&= \frac{2ia^2\tan^3(c+dx)}{7d\sqrt{a+ia\tan(c+dx)}}-\frac{2a^2\tan^4(c+dx)}{7d\sqrt{a+ia\tan(c+dx)}}-\frac{64a\sqrt{a\cdots}}{\cdots} \\
&= \frac{2ia^2\tan^3(c+dx)}{7d\sqrt{a+ia\tan(c+dx)}}-\frac{2a^2\tan^4(c+dx)}{7d\sqrt{a+ia\tan(c+dx)}}-\frac{64a\sqrt{a\cdots}}{\cdots} \\
&= \frac{2\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}+\frac{2ia^2\tan^3(c+\cdots}{7d\sqrt{a+ia\tan(\cdots}}
\end{aligned}
$$

Mathematica [A]
time = 2.83, size = 166, normalized size = 0.83

$$
\frac{\left(\frac{2\sqrt{2}\sinh^{-1}\left(e^{i(c+dx)}\right)}{\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{3/2}\left(1+e^{2i(c+dx)}\right)^{3/2}}+\frac{1}{210}\sec^{\frac{5}{2}}(c+dx)(378\cos(c+dx)+158\cos(3(c+dx))-7i\sin(c+dx)+53i\sin(3(c+dx)))(-1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{3/2}}{d\sec^{\frac{3}{2}}(c+dx)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((2*Sqrt[2]*ArcSinh[E^(I*(c + d*x))])/((E^(I*(c + d*x))/(1 + E^((2*I)*(c +
 d*x))))^(3/2)*(1 + E^((2*I)*(c + d*x)))^(3/2)) + (Sec[c + d*x]^(5/2)*(378*
Cos[c + d*x] + 158*Cos[3*(c + d*x)] - (7*I)*Sin[c + d*x] + (53*I)*Sin[3*(c
+ d*x)])*(-1 + I*Tan[c + d*x]))/210)*(a + I*a*Tan[c + d*x])^(3/2))/(d*Sec[c
 + d*x]^(3/2))
```

Maple [A]
time = 0.20, size = 111, normalized size = 0.56

method	result

derivativedivides	$-\frac{2\left(\frac{(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{a(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{a^2(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+a^3\sqrt{a+ia\tan\left(dx+c\right)}-a^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\right.}{d\,a^2}$
default	$-\frac{2\left(\frac{(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{a(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{a^2(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+a^3\sqrt{a+ia\tan\left(dx+c\right)}-a^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\right.}{d\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a^2*(1/7*(a+I*a*tan(d*x+c))^(7/2)-1/5*a*(a+I*a*tan(d*x+c))^(5/2)+1/3*a
^2*(a+I*a*tan(d*x+c))^(3/2)+a^3*(a+I*a*tan(d*x+c))^(1/2)-a^(7/2)*2^(1/2)*ar
ctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.50, size = 138, normalized size = 0.69

$$-\frac{105\,\sqrt{2}\,a^{\frac{11}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)+30\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}a^2-42\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}a^3+70\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}a^4+210\,\sqrt{i\,a\tan\left(dx+c\right)+a}\,a^5}{105\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/105*(105*sqrt(2)*a^(11/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c)
+ a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 30*(I*a*tan(d*x + c
) + a)^(7/2)*a^2 - 42*(I*a*tan(d*x + c) + a)^(5/2)*a^3 + 70*(I*a*tan(d*x +
c) + a)^(3/2)*a^4 + 210*sqrt(I*a*tan(d*x + c) + a)*a^5)/(a^4*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(156) = 312$.
time = 0.39, size = 354, normalized size = 1.78

$$\frac{105\,\sqrt{2}\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a^3}{d^2}}\log\left(\frac{4\left(a^2e^{(i\,dx+i\,c)}+\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a}\right)-105\,\sqrt{2}\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a^3}{d^2}}\log\left(\frac{4\left(a^2e^{(i\,dx+i\,c)}-\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a}\right)-2\,\sqrt{2}\left(211\,ae^{(7i\,dx+7i\,c)}+371\,ae^{(5i\,dx+5i\,c)}+385\,ae^{(3i\,dx+3i\,c)}+105\,ae^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{105\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/105*(105*sqrt(2)*(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e
^(2*I*d*x + 2*I*c) + d)*sqrt(a^3/d^2)*log(4*(a^2*e^(I*d*x + I*c) + (d*e^(2*
I*d*x + 2*I*c) + d)*sqrt(a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*
d*x - I*c)/a) - 105*sqrt(2)*(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c
) + 3*d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a^3/d^2)*log(4*(a^2*e^(I*d*x + I*c) -
 (d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
))*e^(-I*d*x - I*c)/a) - 2*sqrt(2)*(211*a*e^(7*I*d*x + 7*I*c) + 371*a*e^(5*
```

```
I*d*x + 5*I*c) + 385*a*e^(3*I*d*x + 3*I*c) + 105*a*e^(I*d*x + I*c))*sqrt(a/
(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c
) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c+dx)-i))^{\frac{3}{2}}\tan^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)*tan(c + d*x)**3, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 4.23, size = 120, normalized size = 0.60

$$-\frac{2\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{3/2}}{3\,d}+\frac{2\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{5/2}}{5\,a\,d}-\frac{2\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{7/2}}{7\,a^2\,d}-\frac{2\,a\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{d}-\frac{\sqrt{2}\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}\,1\mathrm{i}}{2\,\sqrt{a}}\right)\,2\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] (2*(a + a*tan(c + d*x)*1i)^(5/2))/(5*a*d) - (2*(a + a*tan(c + d*x)*1i)^(3/2
))/(3*d) - (2*(a + a*tan(c + d*x)*1i)^(7/2))/(7*a^2*d) - (2*a*(a + a*tan(c
+ d*x)*1i)^(1/2))/d - (2^(1/2)*a^(3/2)*atan((2^(1/2)*(a + a*tan(c + d*x)*1i
)^(1/2)*1i)/(2*a^(1/2)))*2i)/d
```

3.93 $\int \tan^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=101

$$\frac{2i\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}-\frac{2ia\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{5ad}$$

```
[Out] 2*I*a^(3/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d
-2*I*a*(a+I*a*tan(d*x+c))^(1/2)/d-2/5*I*(a+I*a*tan(d*x+c))^(5/2)/a/d
```

Rubi [A]
time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3624, 3559, 3561, 212}

$$\frac{2i\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{5ad}-\frac{2ia\sqrt{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((2*I)*Sqrt[2]*a^(3/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])
])/d - ((2*I)*a*Sqrt[a + I*a*Tan[c + d*x]])/d - (((2*I)/5)*(a + I*a*Tan[c +
 d*x])^(5/2))/(a*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{2i(a+ia\tan(c+dx))^{5/2}}{5ad} - \int (a+ia\tan(c+dx))^{3/2}\,dx$$

$$= -\frac{2ia\sqrt{a+ia\tan(c+dx)}}{d} - \frac{2i(a+ia\tan(c+dx))^{5/2}}{5ad} - (2a)$$

$$= -\frac{2ia\sqrt{a+ia\tan(c+dx)}}{d} - \frac{2i(a+ia\tan(c+dx))^{5/2}}{5ad} + \frac{(4ia}{}$$

$$= \frac{2i\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{2ia\sqrt{a+ia\tan}}{d}$$

Mathematica [A]
time = 1.52, size = 162, normalized size = 1.60

$$\frac{ae^{-\frac{1}{2}i(2c+3dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(-i\cos\left(\frac{dx}{2}\right)+\sin\left(\frac{dx}{2}\right)\right)\left(-20\sinh^{-1}\left(e^{i(c+dx)}\right)+\sqrt{1+e^{2i(c+dx)}}\sec^3(c+dx)(5+7\cos(2(c+dx))+2i\sin(2(c+dx)))\right)}{5\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (a*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I
)*(c + d*x))]*((-I)*Cos[(d*x)/2] + Sin[(d*x)/2])*(-20*ArcSinh[E^(I*(c + d*x
))] + Sqrt[1 + E^((2*I)*(c + d*x))]*Sec[c + d*x]^3*(5 + 7*Cos[2*(c + d*x)]
+ (2*I)*Sin[2*(c + d*x)])))/(5*Sqrt[2]*d*E^((I/2)*(2*c + 3*d*x)))
```

Maple [A]
time = 0.19, size = 76, normalized size = 0.75

method	result
derivativedivides	$-\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+a^2\sqrt{a+ia\tan(dx+c)}-a^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)\right)}{da}$

default	$-\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+a^{2}\sqrt{a+ia\tan\left(dx+c\right)}-a^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan\left(dx+c\right)}\,\sqrt{2}}{2\sqrt{a}}\right)\right)}{da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/d/a*(1/5*(a+I*a*tan(d*x+c))^(5/2)+a^2*(a+I*a*tan(d*x+c))^(1/2)-a^(5/2)
*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.49, size = 102, normalized size = 1.01

$$-\frac{i\left(5\sqrt{2}\,a^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)+2\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}a^{2}+10\sqrt{i\,a\tan\left(dx+c\right)+a}\,a^{4}\right)}{5\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/5*I*(5*sqrt(2)*a^(9/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a
))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 2*(I*a*tan(d*x + c) +
a)^(5/2)*a^2 + 10*sqrt(I*a*tan(d*x + c) + a)*a^4)/(a^3*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 315 vs. 2(74) = 148.
time = 0.38, size = 315, normalized size = 3.12

$$-\frac{5\sqrt{2}\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)\sqrt{-\frac{a^{3}}{d^{2}}}\log\left(\frac{4\left(a^{2}e^{(i\,dx+i\,c)}+\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{-\frac{a^{3}}{d^{2}}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a}\right)-5\sqrt{2}\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)\sqrt{-\frac{a^{3}}{d^{2}}}\log\left(\frac{4\left(a^{2}e^{(i\,dx+i\,c)}+\left(-i\,de^{(2i\,dx+2i\,c)}-i\,d\right)\sqrt{-\frac{a^{3}}{d^{2}}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a}\right)+2\sqrt{2}\left(9i\,ae^{(5i\,dx+5i\,c)}+10i\,ae^{(3i\,dx+3i\,c)}+5i\,ae^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{5\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/5*(5*sqrt(2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*sqrt(
-a^3/d^2)*log(4*(a^2*e^(I*d*x + I*c) + (I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt
(-a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a) - 5*sqrt(
2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-a^3/d^2)*log
(4*(a^2*e^(I*d*x + I*c) + (-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(-a^3/d^2)*s
qrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a) + 2*sqrt(2)*(9*I*a*e^
(5*I*d*x + 5*I*c) + 10*I*a*e^(3*I*d*x + 3*I*c) + 5*I*a*e^(I*d*x + I*c))*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2
*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{3}{2}}\tan^{2}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)*tan(c + d*x)**2, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 0.30, size = 84, normalized size = 0.83

$$-\frac{\frac{(a+a\tan(c+d\,x)\,1\mathrm{i})^{5/2}\,2\mathrm{i}}{5}-\sqrt{2}\,\left(-a\right)^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}{2\,\sqrt{-a}}\right)\,2\mathrm{i}}{a\,d}-\frac{a\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,2\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] - (((a + a*tan(c + d*x)*1i)^(5/2)*2i)/5 - 2^(1/2)*(-a)^(5/2)*atan((2^(1/2)*
(a + a*tan(c + d*x)*1i)^(1/2))/(2*(-a)^(1/2)))*2i)/(a*d) - (a*(a + a*tan(c
+ d*x)*1i)^(1/2)*2i)/d
```

3.94 $\int \tan(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=92

$$-\frac{2\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}+\frac{2a\sqrt{a+ia\tan(c+dx)}}{d}+\frac{2(a+ia\tan(c+dx))^{3/2}}{3d}$$

```
[Out] -2*a^(3/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d+
2*a*(a+I*a*tan(d*x+c))^(1/2)/d+2/3*(a+I*a*tan(d*x+c))^(3/2)/d
```

Rubi [A]

time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3608, 3559, 3561, 212}

$$-\frac{2\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}+\frac{2a\sqrt{a+ia\tan(c+dx)}}{d}+\frac{2(a+ia\tan(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[2]*a^(3/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/
d + (2*a*Sqrt[a + I*a*Tan[c + d*x]])/d + (2*(a + I*a*Tan[c + d*x])^(3/2))/(
3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rubi steps

$$\int \tan(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{2(a+ia\tan(c+dx))^{3/2}}{3d} - i\int (a+ia\tan(c+dx))^{3/2}\,dx$$

$$= \frac{2a\sqrt{a+ia\tan(c+dx)}}{d} + \frac{2(a+ia\tan(c+dx))^{3/2}}{3d} - (2ia)\int$$

$$= \frac{2a\sqrt{a+ia\tan(c+dx)}}{d} + \frac{2(a+ia\tan(c+dx))^{3/2}}{3d} - \frac{(4a^2)\,\mathrm{Sul}}{}$$

$$= -\frac{2\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} + \frac{2a\sqrt{a+ia\tan(}}{d}$$

Mathematica [A]
time = 1.00, size = 148, normalized size = 1.61

$$\frac{\sqrt{2}\,ae^{-\frac{1}{2}i(2c+3dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(\cos\left(\frac{dx}{2}\right)+i\sin\left(\frac{dx}{2}\right)\right)\left(-6\sinh^{-1}\left(e^{i(c+dx)}\right)+\sqrt{1+e^{2i(c+dx)}}\sec(c+dx)(4+i\tan(c+dx))\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[2]*a*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 +
 E^((2*I)*(c + d*x))]*(Cos[(d*x)/2] + I*Sin[(d*x)/2])*(-6*ArcSinh[E^(I*(c +
 d*x))] + Sqrt[1 + E^((2*I)*(c + d*x))]*Sec[c + d*x]*(4 + I*Tan[c + d*x])))
/(3*d*E^((I/2)*(2*c + 3*d*x)))
```

Maple [A]
time = 0.17, size = 70, normalized size = 0.76

method	result
derivativedivides	$\frac{\frac{2(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+2a\sqrt{a+ia\tan(dx+c)}-2a^{\frac{3}{2}}\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{d}$

default	$\frac{\frac{2(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+2a\sqrt{a+ia\tan(dx+c)}-2a^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{d}$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/3*(a+I*a*tan(d*x+c))^(3/2)+2*a*(a+I*a*tan(d*x+c))^(1/2)-2*a^(3/2)*2^
(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.49, size = 102, normalized size = 1.11

$$\frac{3\sqrt{2}\,a^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+2\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a^2+6\,\sqrt{i\,a\tan(dx+c)+a}\,a^3}{3\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*sqrt(2)*a^(7/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/
(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 2*(I*a*tan(d*x + c) + a)^
(3/2)*a^2 + 6*sqrt(I*a*tan(d*x + c) + a)*a^3)/(a^2*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(71) = 142.
time = 0.39, size = 258, normalized size = 2.80

$$-\frac{3\sqrt{2}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a^3}{d^2}}\log\left(\frac{4\left(a^2e^{(i\,dx+i\,c)}+\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a}\right)-3\sqrt{2}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a^3}{d^2}}\log\left(\frac{4\left(a^2e^{(i\,dx+i\,c)}-\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a}\right)-2\sqrt{2}\left(5\,ae^{(3i\,dx+3i\,c)}+3\,ae^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{3\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*sqrt(2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a^3/d^2)*log(4*(a^2*e^(I*d
*x + I*c) + (d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a^3/d^2)*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1)))*e^(-I*d*x - I*c)/a) - 3*sqrt(2)*(d*e^(2*I*d*x + 2*I*c) + d)*s
qrt(a^3/d^2)*log(4*(a^2*e^(I*d*x + I*c) - (d*e^(2*I*d*x + 2*I*c) + d)*sqrt(
a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a) - 2*sqrt(2)
*(5*a*e^(3*I*d*x + 3*I*c) + 3*a*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1)))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(ia(\tan(c+dx)-i))^{\frac{3}{2}}\tan(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)*tan(c + d*x), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 4.06, size = 74, normalized size = 0.80

$$\frac{2\,{\left(a+a\,\tan(c+d\,x)\,1{}\mathrm{i}\right)}^{3/2}}{3\,d}+\frac{2\,a\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{d}-\frac{2\,\sqrt{2}\,a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{2\,\sqrt{a}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] (2*(a + a*tan(c + d*x)*1i)^(3/2))/(3*d) + (2*a*(a + a*tan(c + d*x)*1i)^(1/2
))/d - (2*2^(1/2)*a^(3/2)*atanh((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*
a^(1/2))))/d
```

3.95 $\int (a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=72

$$-\frac{2i\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}+\frac{2ia\sqrt{a+ia\tan(c+dx)}}{d}$$

```
[Out] -2*I*a^(3/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/
d+2*I*a*(a+I*a*tan(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3559, 3561, 212}

$$\frac{2ia\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2i\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-2*I)*Sqrt[2]*a^(3/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a]
)])/d + ((2*I)*a*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a+ia\tan(c+dx))^{3/2}\,dx = \frac{2ia\sqrt{a+ia\tan(c+dx)}}{d} + (2a)\int \sqrt{a+ia\tan(c+dx)}\,dx$$
$$= \frac{2ia\sqrt{a+ia\tan(c+dx)}}{d} - \frac{(4ia^2)\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+ia\tan(c+dx)}\right)}{d}$$
$$= -\frac{2i\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} + \frac{2ia\sqrt{a+ia\tan(c+dx)}}{d}$$

Mathematica [A]
time = 0.53, size = 79, normalized size = 1.10

$$\frac{2iae^{-i(c+dx)}\left(e^{i(c+dx)} - \sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right)\right)\sqrt{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(3/2),x]
[Out] ((2*I)*a*(E^(I*(c + d*x)) - Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c +
 d*x))])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^(I*(c + d*x)))
```

Maple [A]
time = 0.15, size = 54, normalized size = 0.75

method	result	size
derivativedivides	$\frac{2ia\left(\sqrt{a+ia\tan(dx+c)}-\sqrt{a}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)\right)}{d}$	54
default	$\frac{2ia\left(\sqrt{a+ia\tan(dx+c)}-\sqrt{a}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)\right)}{d}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2*I/d*a*((a+I*a*tan(d*x+c))^(1/2)-a^(1/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*
x+c))^(1/2)*2^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.48, size = 83, normalized size = 1.15

$$\frac{i\left(\sqrt{2}\,a^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+2\sqrt{i\,a\tan(dx+c)+a}\,a^2\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] I*(sqrt(2)*a^(5/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sqr
t(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 2*sqrt(I*a*tan(d*x + c) + a)*
a^2)/(a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(53) = 106$.
time = 0.47, size = 215, normalized size = 2.99

$$-\frac{-2i\sqrt{2}\,a\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{(i\,dx+i\,c)}-\sqrt{2}\sqrt{-\frac{a^3}{d^2}}\,d\log\left(\frac{4\left(a^2e^{(i\,dx+i\,c)}+\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{-\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a}\right)+\sqrt{2}\sqrt{-\frac{a^3}{d^2}}\,d\log\left(\frac{4\left(a^2e^{(i\,dx+i\,c)}+\left(-i\,de^{(2i\,dx+2i\,c)}-i\,d\right)\sqrt{-\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -(-2*I*sqrt(2)*a*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c) - sqrt(2
)*sqrt(-a^3/d^2)*d*log(4*(a^2*e^(I*d*x + I*c) + (I*d*e^(2*I*d*x + 2*I*c) +
I*d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a)
+ sqrt(2)*sqrt(-a^3/d^2)*d*log(4*(a^2*e^(I*d*x + I*c) + (-I*d*e^(2*I*d*x +
2*I*c) - I*d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x -
 I*c)/a))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia\tan(c+dx)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*a*tan(c + d*x) + a)**(3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 0.18, size = 61, normalized size = 0.85

$$\frac{a\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,2\mathrm{i}}{d}+\frac{\sqrt{2}\,\left(-a\right)^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}{2\sqrt{-a}}\right)\,2\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] (a*(a + a*tan(c + d*x)*1i)^(1/2)*2i)/d + (2^(1/2)*(-a)^(3/2)*atan((2^(1/2)*
(a + a*tan(c + d*x)*1i)^(1/2))/(2*(-a)^(1/2)))*2i)/d
```

3.96 $\int \cot(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=79

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{2\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}$$

```
[Out] -2*a^(3/2)*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/d+2*a^(3/2)*arctanh(1/
2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d
```

Rubi [A]
time = 0.12, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3635, 3561, 212, 3680, 65, 214}

$$\frac{2\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^(3/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/d + (2*Sqrt[2]*a^(
3/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3635

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[2*(a^2/(a*c - b*d)), Int[Sqrt[a + b*Tan[e
+ f*x]], x], x] - Dist[(2*b*c*d + a*(c^2 - d^2))/(a*(c^2 + d^2)), Int[(a -
b*Tan[e + f*x])*(Sqrt[a + b*Tan[e + f*x]]/(c + d*Tan[e + f*x])), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rubi steps

$$\int \cot(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = (2ia)\int \sqrt{a+ia\tan(c+dx)}\,dx + \int \cot(c+dx)(a-ia\tan(c\cdots$$

$$= \frac{a^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+iax}}\,dx, x, \tan(c+dx)\right)}{d} + \frac{(4a^2)\,\text{Subst}\left(\int \frac{\cdots}{2a\cdots}\cdots\right.}{\cdots}$$

$$= \frac{2\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{(2ia)\text{Subst}\left(\int \frac{1}{i-\cdots}\cdots\right.}{\cdots}$$

$$= -\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\cdots\right.}{\cdots}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 201 vs. 2(79) = 158.
time = 1.25, size = 201, normalized size = 2.54

$$\frac{e^{-3i(c+dx)}\left(\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{3/2}\left(1+e^{2i(c+dx)}\right)^{3/2}\left(4\sinh^{-1}\left(e^{i(c+dx)}\right)+\sqrt{2}\left(\log\left(1-e^{i(c+dx)}\right)-\log\left(1+e^{i(c+dx)}\right)+\log\left(1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right)-\log\left(1+e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right)\right)\right)}{\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))^(3/2)*(1 + E^((2*I)*(c
 + d*x)))^(3/2)*(4*ArcSinh[E^(I*(c + d*x))] + Sqrt[2]*(Log[1 - E^(I*(c + d*
x))] - Log[1 + E^(I*(c + d*x))] + Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1
+ E^((2*I)*(c + d*x))]] - Log[1 + E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^((2*
I)*(c + d*x))]])))/(Sqrt[2]*d*E^((3*I)*(c + d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(62) = 124$.
time = 0.77, size = 231, normalized size = 2.92

method	result
default	$-\frac{\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\left(2i\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)+i\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(dx+}{1+\cos(dx-}}}\right.\right.}{d(i\sin(dx+c)+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(2*I*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
2^(1/2))+I*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-2*2^(1/2)*arctanh
(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))-ln
(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))
)*sin(d*x+c)/(I*sin(d*x+c)+cos(d*x+c)-1)*a
```

Maple is above; **Maxima** [A]
time = 0.53, size = 107, normalized size = 1.35

$$-\frac{\sqrt{2}\,a^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)-a^{\frac{3}{2}}\log\left(\frac{\sqrt{i\,a\tan\left(dx+c\right)+a}-\sqrt{a}}{\sqrt{i\,a\tan\left(dx+c\right)+a}+\sqrt{a}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -(sqrt(2)*a^(3/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sqrt
(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - a^(3/2)*log((sqrt(I*a*tan(d*x
+ c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x + c) + a) + sqrt(a))))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(60) = 120$.
time = 0.47, size = 357, normalized size = 4.52

$$\sqrt{2}\,\sqrt{\frac{a^3}{d^2}}\log\left(\frac{4\left(a^2e^{(i\,dx+i\,c)}+\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a^3}{d^2}}\,\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a}\right)-\sqrt{2}\,\sqrt{\frac{a^3}{d^2}}\log\left(\frac{4\left(a^2e^{(i\,dx+i\,c)}-\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a^3}{d^2}}\,\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a}\right)-\frac{1}{2}\sqrt{\frac{a^3}{d^2}}\log\left(16\left(3\,a^2e^{(2i\,dx+2i\,c)}+2\sqrt{2}\left(de^{(3i\,dx+3i\,c)}+de^{(i\,dx+i\,c)}\right)\sqrt{\frac{a^3}{d^2}}\,\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}+a^2\right)e^{(-2i\,dx-2i\,c)}\right)+\frac{1}{2}\sqrt{\frac{a^3}{d^2}}\log\left(16\left(3\,a^2e^{(2i\,dx+2i\,c)}-2\sqrt{2}\left(de^{(3i\,dx+3i\,c)}+de^{(i\,dx+i\,c)}\right)\sqrt{\frac{a^3}{d^2}}\,\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}+a^2\right)e^{(-2i\,dx-2i\,c)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*sqrt(a^3/d^2)*log(4*(a^2*e^(I*d*x + I*c) + (d*e^(2*I*d*x + 2*I*c) +
 d)*sqrt(a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a) -
sqrt(2)*sqrt(a^3/d^2)*log(4*(a^2*e^(I*d*x + I*c) - (d*e^(2*I*d*x + 2*I*c) +
 d)*sqrt(a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a) -
1/2*sqrt(a^3/d^2)*log(16*(3*a^2*e^(2*I*d*x + 2*I*c) + 2*sqrt(2)*(d*e^(3*I*d
*x + 3*I*c) + d*e^(I*d*x + I*c))*sqrt(a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1)) + a^2)*e^(-2*I*d*x - 2*I*c)) + 1/2*sqrt(a^3/d^2)*log(16*(3*a^2*e^(2*I
*d*x + 2*I*c) - 2*sqrt(2)*(d*e^(3*I*d*x + 3*I*c) + d*e^(I*d*x + I*c))*sqrt(
a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)) + a^2)*e^(-2*I*d*x - 2*I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c+dx)-i))^{\frac{3}{2}}\cot(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)*cot(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*cot(d*x + c), x)
```

Mupad [B]
time = 4.09, size = 76, normalized size = 0.96

$$-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a^3}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{a^2}\right)\sqrt{a^3}}{d}+\frac{2\,\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{a^3}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{2\,a^2}\right)\sqrt{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] (2*2^(1/2)*atanh((2^(1/2)*(a^3)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*a^2
))*(a^3)^(1/2))/d - (2*atanh(((a^3)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/a^
2)*(a^3)^(1/2))/d
```

3.97 $\int \cot^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=141

$$-\frac{3ia^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{2i\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{ia^2}{d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] -3*I*a^(3/2)*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/d+2*I*a^(3/2)*arctan
h(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d-I*a^2/d/(a+I*a*ta
n(d*x+c))^(1/2)-a^2*cot(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3634, 3677, 3681, 3561, 212, 3680, 65, 214}

$$-\frac{3ia^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{2i\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{ia^2}{d\sqrt{a+ia\tan(c+dx)}}-\frac{a^2\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-3*I)*a^(3/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/d + ((2*I)*Sqr
t[2]*a^(3/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d - (I*
a^2)/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (a^2*Cot[c + d*x])/(d*Sqrt[a + I*a*Ta
n[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3634

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-a^2)*(b*c - a*d)*(a + b*Tan[e + f*x]
)^(m - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] + Di
st[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[b*(b*c*(m - 2) - a*d*(m - 2*n - 4)) + (a*b*c*(m - 2)
+ b^2*d*(n + 1) - a^2*d*(m + n - 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3681

```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(
A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{a^2\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \int \frac{\cot(c+dx)\left(-\frac{3ia^2}{2}+\frac{5}{2}a^2\tan(c+\right.}{\sqrt{a+ia\tan(c+dx)}}$$

$$= -\frac{ia^2}{d\sqrt{a+ia\tan(c+dx)}} - \frac{a^2\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{\int\cot(c+}{}$$

$$= -\frac{ia^2}{d\sqrt{a+ia\tan(c+dx)}} - \frac{a^2\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} + \frac{3}{2}i\int\cot($$

$$= -\frac{ia^2}{d\sqrt{a+ia\tan(c+dx)}} - \frac{a^2\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} + \frac{(3ia^2)\,\mathrm{Sub}}{}$$

$$= \frac{2i\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{ia^2}{d\sqrt{a+ia\tan(c+}}$$

$$= -\frac{3ia^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2i\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\right.}{}$$

Mathematica [A]
time = 1.45, size = 178, normalized size = 1.26

$$\frac{ae^{-\frac{1}{2}i(2c+3dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(-4\sinh^{-1}\left(e^{i(c+dx)}\right)+3\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,e^{i(c+dx)}}{\sqrt{1+e^{2i(c+dx)}}}\right)-i\sqrt{1+e^{2i(c+dx)}}\csc(c+dx)\right)\left(-i\cos\left(\frac{dx}{2}\right)+\sin\left(\frac{dx}{2}\right)\right)}{\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (a*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I
)*(c + d*x))]*(-4*ArcSinh[E^(I*(c + d*x))] + 3*Sqrt[2]*ArcTanh[(Sqrt[2]*E^(
I*(c + d*x)))/Sqrt[1 + E^((2*I)*(c + d*x))]] - I*Sqrt[1 + E^((2*I)*(c + d*x
))]*Csc[c + d*x])*((-I)*Cos[(d*x)/2] + Sin[(d*x)/2]))/(Sqrt[2]*d*E^((I/2)*(
2*c + 3*d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 630 vs. 2(115) = 230.
time = 0.84, size = 631, normalized size = 4.48

method	result

default	$\frac{\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(4i\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\cos(dx+c)\sin(dx+ \cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(4*I*2^(1/2)*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)*sin(d*x+c)+4*I*2^(1/2)*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+3*I*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1
)/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)+4*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)
*sin(d*x+c)+3*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)+4*2^(1/2
)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*2^(1/2))*sin(d*x+c)+3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arct
an(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)*sin(d*x+c)-2*I*cos(d*
x+c)^2+3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2))*sin(d*x+c)-2*I*cos(d*x+c)+2*sin(d*x+c)*cos(d*x+c))/(I*sin
(d*x+c)+cos(d*x+c)-1)/(1+cos(d*x+c))*a
```

Maxima [A]
time = 0.51, size = 132, normalized size = 0.94

$$-\frac{i\left(2\sqrt{2}\sqrt{a}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)-3\sqrt{a}\log\left(\frac{\sqrt{i\,a\tan\left(dx+c\right)+a}-\sqrt{a}}{\sqrt{i\,a\tan\left(dx+c\right)+a}+\sqrt{a}}\right)-\frac{2i\sqrt{i\,a\tan\left(dx+c\right)+a}}{\tan(dx+c)}\right)a}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*I*(2*sqrt(2)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a
))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - 3*sqrt(a)*log((sqrt(I*
a*tan(d*x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x + c) + a) + sqrt(a))) - 2*
I*sqrt(I*a*tan(d*x + c) + a)/tan(d*x + c))*a/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(108) = 216$.
time = 0.46, size = 501, normalized size = 3.55

$$-\frac{4\sqrt{2}\left(de^{(2idx+2ic)}-d\right)\sqrt{-\frac{a^3}{d^2}}\log\left(\frac{4\left(a^2e^{(idx+ic)}+\left(ide^{(2idx+2ic)}+id\right)\sqrt{-\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\right)e^{(-idx-ic)}}{a}\right)-4\sqrt{2}\left(de^{(2idx+2ic)}-d\right)\sqrt{-\frac{a^3}{d^2}}\log\left(\frac{4\left(a^2e^{(idx+ic)}+\left(-ide^{(2idx+2ic)}-id\right)\sqrt{-\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\right)e^{(-idx-ic)}}{a}\right)+3\left(de^{(2idx+2ic)}-d\right)\sqrt{-\frac{a^3}{d^2}}\log\left(16\left(3a^2e^{(2idx+2ic)}-2\sqrt{2}\left(ide^{(3idx+3ic)}+ide^{(idx+ic)}\right)\sqrt{-\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}+a^2\right)e^{(-2idx-2ic)}\right)-3\left(de^{(2idx+2ic)}-d\right)\sqrt{-\frac{a^3}{d^2}}\log\left(16\left(3a^2e^{(2idx+2ic)}-2\sqrt{2}\left(-ide^{(3idx+3ic)}-ide^{(idx+ic)}\right)\sqrt{-\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}+a^2\right)e^{(-2idx-2ic)}\right)+4\sqrt{2}\left(iae^{(3idx+3ic)}+iae^{(idx+ic)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}}{4\left(de^{(2idx+2ic)}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(-a^3/d^2)*log(4*(a^2*e^(I*
d*x + I*c) + (I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a) - 4*sqrt(2)*(d*e^(2*I*d*x + 2*I*c)
- d)*sqrt(-a^3/d^2)*log(4*(a^2*e^(I*d*x + I*c) + (-I*d*e^(2*I*d*x + 2*I*c)
- I*d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a
) + 3*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(-a^3/d^2)*log(16*(3*a^2*e^(2*I*d*x +
 2*I*c) - 2*sqrt(2)*(I*d*e^(3*I*d*x + 3*I*c) + I*d*e^(I*d*x + I*c))*sqrt(-a
^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)) + a^2)*e^(-2*I*d*x - 2*I*c)) - 3*
(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(-a^3/d^2)*log(16*(3*a^2*e^(2*I*d*x + 2*I*c
) - 2*sqrt(2)*(-I*d*e^(3*I*d*x + 3*I*c) - I*d*e^(I*d*x + I*c))*sqrt(-a^3/d^
2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)) + a^2)*e^(-2*I*d*x - 2*I*c)) + 4*sqrt(
2)*(I*a*e^(3*I*d*x + 3*I*c) + I*a*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1)))/(d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c+dx)-i))^{\frac{3}{2}} \cot^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)*cot(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^2, x)
```

Mupad [B]
time = 4.20, size = 112, normalized size = 0.79

$$\frac{\mathrm{atan}\left(\frac{\sqrt{-a^3}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{a^2}\right)\sqrt{-a^3}\,3\mathrm{i}}{d}-\frac{a\cot(c+d\,x)\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{d}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{-a^3}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{2\,a^2}\right)\sqrt{-a^3}\,2\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] (atan(((-a^3)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/a^2)*(-a^3)^(1/2)*3i)/d
- (a*cot(c + d*x)*(a + a*tan(c + d*x)*1i)^(1/2))/d - (2^(1/2)*atan((2^(1/2)
*(-a^3)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*a^2))*(-a^3)^(1/2)*2i)/d
```

3.98 $\int \cot^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=184

$$\frac{11a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{4d}-\frac{2\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}-\frac{ia^2\cot(c+dx)}{2d\sqrt{a+ia\tan(c+}}$$

```
[Out] 11/4*a^(3/2)*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/d-2*a^(3/2)*arctanh(
1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d-1/2*I*a^2*cot(d*x+c
)/d/(a+I*a*tan(d*x+c))^(1/2)-1/2*a^2*cot(d*x+c)^2/d/(a+I*a*tan(d*x+c))^(1/2
)-5/4*I*a*cot(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.39, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3634, 3677, 3679, 3681, 3561, 212, 3680, 65, 214}

$$\frac{11a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{4d}-\frac{2\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}-\frac{a^2\cot^2(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}-\frac{ia^2\cot(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}-\frac{5ia\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (11*a^(3/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/(4*d) - (2*Sqrt[2]
*a^(3/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d - ((I/2)*
a^2*Cot[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (a^2*Cot[c + d*x]^2)/(2*
d*Sqrt[a + I*a*Tan[c + d*x]]) - (((5*I)/4)*a*Cot[c + d*x]*Sqrt[a + I*a*Tan[
c + d*x]])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3634

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-a^2)*(b*c - a*d)*(a + b*Tan[e + f*x]
)^(m - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] + Di
st[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[b*(b*c*(m - 2) - a*d*(m - 2*n - 4)) + (a*b*c*(m - 2)
+ b^2*d*(n + 1) - a^2*d*(m + n - 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
```

```
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3681

```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(
A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{a^2\cot^2(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}} - \frac{1}{2}\int \frac{\cot^2(c+dx)\left(-\frac{7ia^2}{2}+\frac{9}{2}a^2\tan\right.}{\sqrt{a+ia\tan(c+dx}}$$

$$= -\frac{ia^2\cot(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}} - \frac{a^2\cot^2(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}} - \frac{\int\cot^2}{}$$

$$= -\frac{ia^2\cot(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}} - \frac{a^2\cot^2(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}} - \frac{5ia\,\mathrm{co}}{}$$

$$= -\frac{ia^2\cot(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}} - \frac{a^2\cot^2(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}} - \frac{5ia\,\mathrm{co}}{}$$

$$= -\frac{ia^2\cot(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}} - \frac{a^2\cot^2(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}} - \frac{5ia\,\mathrm{co}}{}$$

$$= -\frac{2\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{ia^2\cot(c+}{2d\sqrt{a+ia\tan}}$$

$$= \frac{11a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{4d} - \frac{2\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\right.}{}$$

Mathematica [A]

time = 2.08, size = 190, normalized size = 1.03

$$-\frac{ae^{-\frac{1}{2}i(2c+3dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(16\sinh^{-1}\left(e^{i(c+dx)}\right)-11\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,e^{i(c+dx)}}{\sqrt{1+e^{2i(c+dx)}}}\right)+\sqrt{1+e^{2i(c+dx)}}\,(5i+2\cot(c+dx))\csc(c+dx)\right)\left(\cos\left(\frac{dx}{2}\right)+i\sin\left(\frac{dx}{2}\right)\right)}{4\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] -1/4*(a*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^
((2*I)*(c + d*x))]*(16*ArcSinh[E^(I*(c + d*x))] - 11*Sqrt[2]*ArcTanh[(Sqrt[
2]*E^(I*(c + d*x)))/Sqrt[1 + E^((2*I)*(c + d*x))]] + Sqrt[1 + E^((2*I)*(c +
 d*x))]*(5*I + 2*Cot[c + d*x])*Csc[c + d*x])*(Cos[(d*x)/2] + I*Sin[(d*x)/2]
))/(Sqrt[2]*d*E^((I/2)*(2*c + 3*d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1141 vs. $2(148) = 296$.
time = 0.87, size = 1142, normalized size = 6.21

method	result	size
default	Expression too large to display	1142

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(11*I*cos(d*x+c)^2*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(1/2))-16*2^(1/2)*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh
(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))-11
*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2))+16*I*2^(1/2)*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-16*2^(1/2)*cos(d*x
+c)^2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+11*I*cos(d*x+c)^3*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-
11*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))+14*I*cos(d*x+c)^3+1
6*2^(1/2)*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))-16*I*2^(1/2)
*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*2^(1/2))+16*I*2^(1/2)*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))
-11*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))+4*I*cos(d*x+c)^2+1
6*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))-14*cos(d*x+c)^2*sin(d*x
+c)-10*I*cos(d*x+c)+11*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln((
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))-1
1*I*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2))-10*sin(d*x+c)*cos(d*x+c)-16*I*2^(1/2)*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(
```

```
1/2))+11*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c)))/(I*sin(d*x+c)+cos(d*x+c)-
1)/sin(d*x+c)/(1+cos(d*x+c))*a
```

Maxima [A]
time = 0.49, size = 178, normalized size = 0.97

$$\frac{a^2\left(\frac{8\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)}{\sqrt{a}}-\frac{11\log\left(\frac{\sqrt{ia\tan(dx+c)+a}-\sqrt{a}}{\sqrt{ia\tan(dx+c)+a}+\sqrt{a}}\right)}{\sqrt{a}}+\frac{2\left(5\,(ia\tan(dx+c)+a)^{\frac{3}{2}}-3\sqrt{ia\tan(dx+c)+a}\,a\right)}{(ia\tan(dx+c)+a)^2-2\,(ia\tan(dx+c)+a)a+a^2}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*a^2*(8*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sqr
t(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/sqrt(a) - 11*log((sqrt(I*a*tan(
d*x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x + c) + a) + sqrt(a)))/sqrt(a) +
2*(5*(I*a*tan(d*x + c) + a)^(3/2) - 3*sqrt(I*a*tan(d*x + c) + a)*a)/((I*a*t
an(d*x + c) + a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 546 vs. 2(141) = 282.
time = 0.46, size = 546, normalized size = 2.97

$$-\frac{16\sqrt{2}\left(de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a^3}{d^2}}\log\left(\frac{4\left(a^2e^{(i\,dx+i\,c)}+\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a}\right)-16\sqrt{2}\left(de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a^3}{d^2}}\log\left(\frac{4\left(a^2e^{(i\,dx+i\,c)}-\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a}\right)-11\left(de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a^3}{d^2}}\log\left(16\left(3\,a^2e^{(2i\,dx+2i\,c)}+2\sqrt{2}\left(de^{(3i\,dx+3i\,c)}+de^{(i\,dx+i\,c)}\right)\sqrt{\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}+a^2\right)e^{(-2i\,dx-2i\,c)}\right)+11\left(de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a^3}{d^2}}\log\left(16\left(3\,a^2e^{(2i\,dx+2i\,c)}-2\sqrt{2}\left(de^{(3i\,dx+3i\,c)}+de^{(i\,dx+i\,c)}\right)\sqrt{\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}+a^2\right)e^{(-2i\,dx-2i\,c)}\right)-4\sqrt{2}\left(7\,ae^{(5i\,dx+5i\,c)}+4\,ae^{(3i\,dx+3i\,c)}-3\,ae^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{16\left(de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/16*(16*sqrt(2)*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*sqr
t(a^3/d^2)*log(4*(a^2*e^(I*d*x + I*c) + (d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a^
3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a) - 16*sqrt(2)*
(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a^3/d^2)*log(4*(
a^2*e^(I*d*x + I*c) - (d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a^3/d^2)*sqrt(a/(e^(
2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a) - 11*(d*e^(4*I*d*x + 4*I*c) - 2
*d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a^3/d^2)*log(16*(3*a^2*e^(2*I*d*x + 2*I*c)
 + 2*sqrt(2)*(d*e^(3*I*d*x + 3*I*c) + d*e^(I*d*x + I*c))*sqrt(a^3/d^2)*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1)) + a^2)*e^(-2*I*d*x - 2*I*c)) + 11*(d*e^(4*I*d
*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a^3/d^2)*log(16*(3*a^2*e^(2
*I*d*x + 2*I*c) - 2*sqrt(2)*(d*e^(3*I*d*x + 3*I*c) + d*e^(I*d*x + I*c))*sqr
t(a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)) + a^2)*e^(-2*I*d*x - 2*I*c)) -
 4*sqrt(2)*(7*a*e^(5*I*d*x + 5*I*c) + 4*a*e^(3*I*d*x + 3*I*c) - 3*a*e^(I*d*
x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(4*I*d*x + 4*I*c) - 2*d*e
^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan (c+dx)-i))^{\frac{3}{2}} \cot^3 (c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)*cot(c + d*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^3, x)
```

Mupad [B]
time = 4.07, size = 136, normalized size = 0.74

$$-\frac{\mathrm{atan}\left(\frac{\sqrt{a^3}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}\,1\mathrm{i}}{a^2}\right)\sqrt{a^3}\,11\mathrm{i}}{4\,d}-\frac{5\,{\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)}^{3/2}}{4\,d\tan(c+d\,x)^2}+\frac{3\,a\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{4\,d\tan(c+d\,x)^2}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a^3}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}\,1\mathrm{i}}{2\,a^2}\right)\sqrt{a^3}\,2\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] (3*a*(a + a*tan(c + d*x)*1i)^(1/2))/(4*d*tan(c + d*x)^2) - (5*(a + a*tan(c
+ d*x)*1i)^(3/2))/(4*d*tan(c + d*x)^2) - (atan(((a^3)^(1/2)*(a + a*tan(c +
d*x)*1i)^(1/2)*1i)/a^2)*(a^3)^(1/2)*11i)/(4*d) + (2^(1/2)*atan((2^(1/2)*(a^
3)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)*1i)/(2*a^2))*(a^3)^(1/2)*2i)/d
```

3.99 $\int \tan^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=204

$$\frac{4\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}-\frac{368a^2\sqrt{a+ia\tan(c+dx)}}{105d}+\frac{92a^2\tan^2(c+dx)\sqrt{a+ia\tan(c}}{105d}$$

```
[Out] 4*a^(5/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d-3
68/105*a^2*(a+I*a*tan(d*x+c))^(1/2)/d+92/105*a^2*(a+I*a*tan(d*x+c))^(1/2)*t
an(d*x+c)^2/d+38/63*I*a^2*(a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^3/d-2/9*a^2*(
a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^4/d-472/315*a*(a+I*a*tan(d*x+c))^(3/2)/d
```

Rubi [A]
time = 0.33, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3637, 3678, 3673, 3608, 3561, 212}

$$\frac{4\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}-\frac{2a^2\tan^4(c+dx)\sqrt{a+ia\tan(c+dx)}}{9d}+\frac{38ia^2\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{63d}+\frac{92a^2\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{105d}-\frac{368a^2\sqrt{a+ia\tan(c+dx)}}{105d}-\frac{472a(a+ia\tan(c+dx))^{3/2}}{315d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (4*Sqrt[2]*a^(5/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d
 - (368*a^2*Sqrt[a + I*a*Tan[c + d*x]])/(105*d) + (92*a^2*Tan[c + d*x]^2*Sq
rt[a + I*a*Tan[c + d*x]])/(105*d) + (((38*I)/63)*a^2*Tan[c + d*x]^3*Sqrt[a
+ I*a*Tan[c + d*x]])/d - (2*a^2*Tan[c + d*x]^4*Sqrt[a + I*a*Tan[c + d*x]])/
(9*d) - (472*a*(a + I*a*Tan[c + d*x])^(3/2))/(315*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
```

```
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rule 3678

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Dist[
1/(a*(m + n)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Simp
[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \tan^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{2a^2\tan^4(c+dx)\sqrt{a+ia\tan(c+dx)}}{9d} + \frac{1}{9}(2a)\int \tan^3(c+d$$

$$= \frac{38ia^2\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{63d} - \frac{2a^2\tan^4(c+dx)\sqrt{}}{9}$$

$$= \frac{92a^2\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{105d} + \frac{38ia^2\tan^3(c+dx)\sqrt{}}{63}$$

$$= \frac{92a^2\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{105d} + \frac{38ia^2\tan^3(c+dx)\sqrt{}}{63}$$

$$= -\frac{368a^2\sqrt{a+ia\tan(c+dx)}}{105d} + \frac{92a^2\tan^2(c+dx)\sqrt{a+ia\tan}}{105d}$$

$$= -\frac{368a^2\sqrt{a+ia\tan(c+dx)}}{105d} + \frac{92a^2\tan^2(c+dx)\sqrt{a+ia\tan}}{105d}$$

$$= \frac{4\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{368a^2\sqrt{a+ia\tan}}{105d}$$

Mathematica [A]

time = 2.36, size = 176, normalized size = 0.86

$$-\frac{a^2e^{-i(c+2dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}(\cos(dx)+i\sin(dx))\left(-10080\sinh^{-1}\left(e^{i(c+dx)}\right)+\sqrt{1+e^{2i(c+dx)}}\sec^5(c+dx)(2331+3012\cos(2(c+dx))+961\cos(4(c+dx))+282i\sin(2(c+dx))+331i\sin(4(c+dx)))\right)}{1260\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] -1/1260*(a^2*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1
 + E^((2*I)*(c + d*x))]*(Cos[d*x] + I*Sin[d*x])*(-10080*ArcSinh[E^(I*(c + d
*x))] + Sqrt[1 + E^((2*I)*(c + d*x))]*Sec[c + d*x]^5*(2331 + 3012*Cos[2*(c
+ d*x)] + 961*Cos[4*(c + d*x)] + (282*I)*Sin[2*(c + d*x)] + (331*I)*Sin[4*(
c + d*x)])))/(Sqrt[2]*d*E^(I*(c + 2*d*x)))
```

Maple [A]

time = 0.19, size = 131, normalized size = 0.64

method	result
derivativedivides	$-\frac{2\left(\frac{(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}-\frac{a(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}+\frac{a^2(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{a^3(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+2a^4\sqrt{a+ia\tan(dx}\right.}{d\,a^2}$

default	$-\frac{2\left(\frac{(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}-\frac{a(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}+\frac{a^2(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{a^3(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+2a^4\sqrt{a+ia\tan(dx+\right.}{d\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a^2*(1/9*(a+I*a*tan(d*x+c))^(9/2)-1/7*a*(a+I*a*tan(d*x+c))^(7/2)+1/5*a
^2*(a+I*a*tan(d*x+c))^(5/2)+1/3*a^3*(a+I*a*tan(d*x+c))^(3/2)+2*a^4*(a+I*a*t
an(d*x+c))^(1/2)-2*a^(9/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(
1/2)/a^(1/2)))
```

Maxima [A]
time = 0.49, size = 156, normalized size = 0.76

$$-\frac{2\left(315\sqrt{2}\,a^{\frac{13}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)+35\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{9}{2}}a^2-45\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}a^3+63\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}a^4+105\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}a^5+630\sqrt{i\,a\tan\left(dx+c\right)+a}\,a^6\right)}{315\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/315*(315*sqrt(2)*a^(13/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c)
+ a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 35*(I*a*tan(d*x + c
) + a)^(9/2)*a^2 - 45*(I*a*tan(d*x + c) + a)^(7/2)*a^3 + 63*(I*a*tan(d*x +
c) + a)^(5/2)*a^4 + 105*(I*a*tan(d*x + c) + a)^(3/2)*a^5 + 630*sqrt(I*a*tan
(d*x + c) + a)*a^6)/(a^4*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 412 vs. 2(161) = 322.
time = 0.50, size = 412, normalized size = 2.02

$$\frac{2\left(315\sqrt{2}\sqrt{\frac{a^5}{d^2}}\left(de^{(8idx+8ic)}+4de^{(6idx+6ic)}+6de^{(4idx+4ic)}+4de^{(2idx+2ic)}+d\right)\log\left(\frac{4\left(a^3e^{(idx+ic)}+\sqrt{\frac{a^5}{d^2}}\left(de^{(2idx+2ic)}+d\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\right)e^{(-idx-ic)}}{a^2}\right)-315\sqrt{2}\sqrt{\frac{a^5}{d^2}}\left(de^{(8idx+8ic)}+4de^{(6idx+6ic)}+6de^{(4idx+4ic)}+4de^{(2idx+2ic)}+d\right)\log\left(\frac{4\left(a^3e^{(idx+ic)}-\sqrt{\frac{a^5}{d^2}}\left(de^{(2idx+2ic)}+d\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\right)e^{(-idx-ic)}}{a^2}\right)-2\sqrt{2}\left(646a^2e^{(9idx+9ic)}+1647a^2e^{(7idx+7ic)}+2331a^2e^{(5idx+5ic)}+1365a^2e^{(3idx+3ic)}+315a^2e^{(idx+ic)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\right)}{315\left(de^{(8idx+8ic)}+4de^{(6idx+6ic)}+6de^{(4idx+4ic)}+4de^{(2idx+2ic)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/315*(315*sqrt(2)*sqrt(a^5/d^2)*(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x +
6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)*log(4*(a^3*
e^(I*d*x + I*c) + sqrt(a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^2) - 315*sqrt(2)*sqrt(a^5/d^2)*(d*e^
(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d
*e^(2*I*d*x + 2*I*c) + d)*log(4*(a^3*e^(I*d*x + I*c) - sqrt(a^5/d^2)*(d*e^(
2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a
^2) - 2*sqrt(2)*(646*a^2*e^(9*I*d*x + 9*I*c) + 1647*a^2*e^(7*I*d*x + 7*I*c)
 + 2331*a^2*e^(5*I*d*x + 5*I*c) + 1365*a^2*e^(3*I*d*x + 3*I*c) + 315*a^2*e^
```

```
(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(8*I*d*x + 8*I*c) +
4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c)
 + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan{(c+dx)}-i))^{\frac{5}{2}}\tan^3{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(5/2)*tan(c + d*x)**3, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 4.29, size = 142, normalized size = 0.70

$$-\frac{2\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}}{5\,d}-\frac{4\,a^2\,\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}}{d}+\frac{2\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2}}{7\,a\,d}-\frac{2\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{9/2}}{9\,a^2\,d}-\frac{2\,a\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}{3\,d}-\frac{\sqrt{2}\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2\,\sqrt{a}}\right)\,4{}\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] (2*(a + a*tan(c + d*x)*1i)^(7/2))/(7*a*d) - (4*a^2*(a + a*tan(c + d*x)*1i)^
(1/2))/d - (2*(a + a*tan(c + d*x)*1i)^(5/2))/(5*d) - (2*(a + a*tan(c + d*x)
*1i)^(9/2))/(9*a^2*d) - (2*a*(a + a*tan(c + d*x)*1i)^(3/2))/(3*d) - (2^(1/2
)*a^(5/2)*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)*1i)/(2*a^(1/2)))*4i)/
d
```

3.100 $\int \tan^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=130

$$\frac{4i\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{4ia^2\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2ia(a+ia\tan(c+dx))^{3/2}}{3d}-\frac{2i(a+}{}$$

```
[Out] 4*I*a^(5/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d
-4*I*a^2*(a+I*a*tan(d*x+c))^(1/2)/d-2/3*I*a*(a+I*a*tan(d*x+c))^(3/2)/d-2/7*
I*(a+I*a*tan(d*x+c))^(7/2)/a/d
```

Rubi [A]
time = 0.08, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3624, 3559, 3561, 212}

$$\frac{4i\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{4ia^2\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2i(a+ia\tan(c+dx))^{7/2}}{7ad}-\frac{2ia(a+ia\tan(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((4*I)*Sqrt[2]*a^(5/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])
])/d - ((4*I)*a^2*Sqrt[a + I*a*Tan[c + d*x]])/d - (((2*I)/3)*a*(a + I*a*Tan
[c + d*x])^(3/2))/d - (((2*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \tan^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx &= -\frac{2i(a+ia\tan(c+dx))^{7/2}}{7ad} - \int (a+ia\tan(c+dx))^{5/2}\,dx \\ &= -\frac{2ia(a+ia\tan(c+dx))^{3/2}}{3d} - \frac{2i(a+ia\tan(c+dx))^{7/2}}{7ad} - (2a \\ &= -\frac{4ia^2\sqrt{a+ia\tan(c+dx)}}{d} - \frac{2ia(a+ia\tan(c+dx))^{3/2}}{3d} - \frac{2i \\ &= -\frac{4ia^2\sqrt{a+ia\tan(c+dx)}}{d} - \frac{2ia(a+ia\tan(c+dx))^{3/2}}{3d} - \frac{2i \\ &= \frac{4i\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{4ia^2\sqrt{a+ia\tan}}{d}\end{aligned}$$

Mathematica [A]
time = 2.82, size = 170, normalized size = 1.31

$$\frac{a^2e^{-i(c+2dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\,(-i\cos(dx)+\sin(dx))\left(-336\sinh^{-1}\left(e^{i(c+dx)}\right)+\sqrt{1+e^{2i(c+dx)}}\sec^3(c+dx)(86+122\cos(2(c+dx))+19i\sec(c+dx)\sin(3(c+dx))+7i\tan(c+dx))\right)}{42\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*((-I)*Cos[d*x] + Sin[d*x])*(-336*ArcSinh[E^(I*(c + d*x))] +
 Sqrt[1 + E^((2*I)*(c + d*x))]*Sec[c + d*x]^3*(86 + 122*Cos[2*(c + d*x)] +
(19*I)*Sec[c + d*x]*Sin[3*(c + d*x)] + (7*I)*Tan[c + d*x])))/(42*Sqrt[2]*d*
E^(I*(c + 2*d*x)))
```

Maple [A]
time = 0.19, size = 96, normalized size = 0.74

method	result

derivativedivides	$-\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}+\frac{a^2(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+2a^3\sqrt{a+ia\tan(dx+c)}-2a^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan}}{}\right.\right.}{da}$
default	$-\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}+\frac{a^2(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+2a^3\sqrt{a+ia\tan(dx+c)}-2a^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan}}{}\right.\right.}{da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/d/a*(1/7*(a+I*a*tan(d*x+c))^(7/2)+1/3*a^2*(a+I*a*tan(d*x+c))^(3/2)+2*a
^3*(a+I*a*tan(d*x+c))^(1/2)-2*a^(7/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c)
)^(1/2)*2^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.49, size = 120, normalized size = 0.92

$$-\frac{2i\left(21\sqrt{2}\,a^{\frac{11}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+3\,(i\,a\tan(dx+c)+a)^{\frac{7}{2}}a^2+7\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a^4+42\sqrt{i\,a\tan(dx+c)+a}\,a^5\right)}{21\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/21*I*(21*sqrt(2)*a^(11/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c)
+ a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 3*(I*a*tan(d*x + c)
 + a)^(7/2)*a^2 + 7*(I*a*tan(d*x + c) + a)^(3/2)*a^4 + 42*sqrt(I*a*tan(d*x
+ c) + a)*a^5)/(a^3*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 371 vs. 2(95) = 190.
time = 0.49, size = 371, normalized size = 2.85

$$-\frac{2\left(21\sqrt{2}\sqrt{-\frac{a^5}{d^2}}\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{4\left(a^3e^{(i\,dx+i\,c)}+\sqrt{-\frac{a^5}{d^2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^2}\right)-21\sqrt{2}\sqrt{-\frac{a^5}{d^2}}\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{4\left(a^3e^{(i\,dx+i\,c)}+\sqrt{-\frac{a^5}{d^2}}\left(-i\,de^{(2i\,dx+2i\,c)}-i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^2}\right)+2\sqrt{2}\left(40i\,a^2e^{(7i\,dx+7i\,c)}+77i\,a^2e^{(5i\,dx+5i\,c)}+70i\,a^2e^{(3i\,dx+3i\,c)}+21i\,a^2e^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)}{21\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/21*(21*sqrt(2)*sqrt(-a^5/d^2)*(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x +
4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)*log(4*(a^3*e^(I*d*x + I*c) + sqrt(-a^
5/d^2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e
^(-I*d*x - I*c)/a^2) - 21*sqrt(2)*sqrt(-a^5/d^2)*(d*e^(6*I*d*x + 6*I*c) + 3
*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)*log(4*(a^3*e^(I*d*x +
 I*c) + sqrt(-a^5/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^2) + 2*sqrt(2)*(40*I*a^2*e^(7*I*d*x + 7*
```

```
I*c) + 77*I*a^2*e^(5*I*d*x + 5*I*c) + 70*I*a^2*e^(3*I*d*x + 3*I*c) + 21*I*a
^2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(6*I*d*x + 6*I*
c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan (c+dx)-i))^{\frac{5}{2}} \tan^2 (c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(5/2)*tan(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*tan(d*x + c)^2, x)
```

Mupad [B]
time = 4.21, size = 107, normalized size = 0.82

$$-\frac{a^2\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}\,4{}\mathrm{i}}{d}-\frac{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2}\,2{}\mathrm{i}}{7\,a\,d}-\frac{a\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}\,2{}\mathrm{i}}{3\,d}+\frac{\sqrt{2}\,{\left(-a\right)}^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{2\,\sqrt{-a}}\right)\,4{}\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] (2^(1/2)*(-a)^(5/2)*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*(-a)^(1
/2)))*4i)/d - ((a + a*tan(c + d*x)*1i)^(7/2)*2i)/(7*a*d) - (a*(a + a*tan(c
+ d*x)*1i)^(3/2)*2i)/(3*d) - (a^2*(a + a*tan(c + d*x)*1i)^(1/2)*4i)/d
```

3.101 $\int \tan(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=119

$$-\frac{4\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}+\frac{4a^2\sqrt{a+ia\tan(c+dx)}}{d}+\frac{2a(a+ia\tan(c+dx))^{3/2}}{3d}+2(a+$$

```
[Out] -4*a^(5/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d+
4*a^2*(a+I*a*tan(d*x+c))^(1/2)/d+2/3*a*(a+I*a*tan(d*x+c))^(3/2)/d+2/5*(a+I*
a*tan(d*x+c))^(5/2)/d
```

Rubi [A]
time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3608, 3559, 3561, 212}

$$-\frac{4\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}+\frac{4a^2\sqrt{a+ia\tan(c+dx)}}{d}+\frac{2a(a+ia\tan(c+dx))^{3/2}}{3d}+\frac{2(a+ia\tan(c+dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (-4*Sqrt[2]*a^(5/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/
d + (4*a^2*Sqrt[a + I*a*Tan[c + d*x]])/d + (2*a*(a + I*a*Tan[c + d*x])^(3/2
))/(3*d) + (2*(a + I*a*Tan[c + d*x])^(5/2))/(5*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rubi steps

$$\int \tan(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{2(a+ia\tan(c+dx))^{5/2}}{5d} - i\int (a+ia\tan(c+dx))^{5/2}\,dx$$

$$= \frac{2a(a+ia\tan(c+dx))^{3/2}}{3d} + \frac{2(a+ia\tan(c+dx))^{5/2}}{5d} - (2ia)\int$$

$$= \frac{4a^2\sqrt{a+ia\tan(c+dx)}}{d} + \frac{2a(a+ia\tan(c+dx))^{3/2}}{3d} + \frac{2(a+$$

$$= \frac{4a^2\sqrt{a+ia\tan(c+dx)}}{d} + \frac{2a(a+ia\tan(c+dx))^{3/2}}{3d} + \frac{2(a+$$

$$= -\frac{4\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} + \frac{4a^2\sqrt{a+ia\tan}}{d}$$

Mathematica [A]
time = 1.42, size = 154, normalized size = 1.29

$$\frac{a^2e^{-i(c+2dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}(\cos(dx)+i\sin(dx))\left(-120\sinh^{-1}\left(e^{i(c+dx)}\right)+\sqrt{1+e^{2i(c+dx)}}\sec^3(c+dx)(35+41\cos(2(c+dx))+11i\sin(2(c+dx)))\right)}{15\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*(Cos[d*x] + I*Sin[d*x])*(-120*ArcSinh[E^(I*(c + d*x))] + Sq
rt[1 + E^((2*I)*(c + d*x))]*Sec[c + d*x]^3*(35 + 41*Cos[2*(c + d*x)] + (11*
I)*Sin[2*(c + d*x)])))/(15*Sqrt[2]*d*E^(I*(c + 2*d*x)))
```

Maple [A]
time = 0.18, size = 89, normalized size = 0.75

method	result

derivativedivides	$\frac{\frac{2(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{2a(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+4a^2\sqrt{a+ia\tan(dx+c)}-4a^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(d}}{2\sqrt{a}}\right.}{d}$
default	$\frac{\frac{2(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{2a(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+4a^2\sqrt{a+ia\tan(dx+c)}-4a^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(d}}{2\sqrt{a}}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/5*(a+I*a*tan(d*x+c))^(5/2)+2/3*a*(a+I*a*tan(d*x+c))^(3/2)+4*a^2*(a+I
*a*tan(d*x+c))^(1/2)-4*a^(5/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)
*2^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.51, size = 120, normalized size = 1.01

$$\frac{2\left(15\sqrt{2}\,a^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+3\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}a^2+5\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a^3+30\,\sqrt{i\,a\tan(dx+c)+a}\,a^4\right)}{15\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sqrt(2)*a^(9/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a)
)/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 3*(I*a*tan(d*x + c) + a
)^(5/2)*a^2 + 5*(I*a*tan(d*x + c) + a)^(3/2)*a^3 + 30*sqrt(I*a*tan(d*x + c)
 + a)*a^4)/(a^2*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 312 vs. 2(92) = 184.
time = 0.45, size = 312, normalized size = 2.62

$$-\frac{2\left(15\sqrt{2}\sqrt{\frac{a^5}{d^2}}\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{4\left(a^3e^{(i\,dx+i\,c)}+\sqrt{\frac{a^5}{d^2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^2}\right)-15\sqrt{2}\sqrt{\frac{a^5}{d^2}}\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{4\left(a^3e^{(i\,dx+i\,c)}-\sqrt{\frac{a^5}{d^2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^2}\right)-2\sqrt{2}\left(26\,a^2e^{(5i\,dx+5i\,c)}+35\,a^2e^{(3i\,dx+3i\,c)}+15\,a^2e^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)}{15\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*(15*sqrt(2)*sqrt(a^5/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2
*I*c) + d)*log(4*(a^3*e^(I*d*x + I*c) + sqrt(a^5/d^2)*(d*e^(2*I*d*x + 2*I*c
) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^2) - 15*sqrt(2
)*sqrt(a^5/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(4
*(a^3*e^(I*d*x + I*c) - sqrt(a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e
^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^2) - 2*sqrt(2)*(26*a^2*e^(5*I*
d*x + 5*I*c) + 35*a^2*e^(3*I*d*x + 3*I*c) + 15*a^2*e^(I*d*x + I*c))*sqrt(a/
```

```
(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c
) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c+dx)-i))^{\frac{5}{2}}\tan(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))**(5/2),x)
[Out] Integral((I*a*(tan(c + d*x) - I))**(5/2)*tan(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*tan(d*x + c), x)
```

Mupad [B]
time = 0.32, size = 98, normalized size = 0.82

$$\frac{2\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}}{5\,d}+\frac{4\,a^2\,\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}}{d}+\frac{2\,a\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}{3\,d}+\frac{\sqrt{2}\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2\,\sqrt{a}}\right)\,4{}\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + a*tan(c + d*x)*1i)^(5/2),x)
[Out] (2*(a + a*tan(c + d*x)*1i)^(5/2))/(5*d) + (4*a^2*(a + a*tan(c + d*x)*1i)^(1
/2))/d + (2*a*(a + a*tan(c + d*x)*1i)^(3/2))/(3*d) + (2^(1/2)*a^(5/2)*atan(
(2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)*1i)/(2*a^(1/2)))*4i)/d
```

3.102 $\int (a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=101

$$-\frac{4i\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}+\frac{4ia^2\sqrt{a+ia\tan(c+dx)}}{d}+\frac{2ia(a+ia\tan(c+dx))^{3/2}}{3d}$$

```
[Out] -4*I*a^(5/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/
d+4*I*a^2*(a+I*a*tan(d*x+c))^(1/2)/d+2/3*I*a*(a+I*a*tan(d*x+c))^(3/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3559, 3561, 212}

$$-\frac{4i\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}+\frac{4ia^2\sqrt{a+ia\tan(c+dx)}}{d}+\frac{2ia(a+ia\tan(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-4*I)*Sqrt[2]*a^(5/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a]
)])/d + ((4*I)*a^2*Sqrt[a + I*a*Tan[c + d*x]])/d + (((2*I)/3)*a*(a + I*a*Ta
n[c + d*x])^(3/2))/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a+ia\tan(c+dx))^{5/2}\,dx = \frac{2ia(a+ia\tan(c+dx))^{3/2}}{3d} + (2a)\int (a+ia\tan(c+dx))^{3/2}\,dx$$
$$= \frac{4ia^2\sqrt{a+ia\tan(c+dx)}}{d} + \frac{2ia(a+ia\tan(c+dx))^{3/2}}{3d} + \left(4a^2\right)\int \sqrt{a+ia$$
$$= \frac{4ia^2\sqrt{a+ia\tan(c+dx)}}{d} + \frac{2ia(a+ia\tan(c+dx))^{3/2}}{3d} - \frac{\left(8ia^3\right)\mathrm{Subst}\left(\int \frac{}{2a}\right.}{}$$
$$= -\frac{4i\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} + \frac{4ia^2\sqrt{a+ia\tan(c+dx)}}{d}$$

Mathematica [A]
time = 1.05, size = 140, normalized size = 1.39

$$-\frac{\sqrt{2}\,a^2e^{-i(c+2dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\,(\cos(dx)+i\sin(dx))\left(12i\sinh^{-1}\left(e^{i(c+dx)}\right)+\sqrt{1+e^{2i(c+dx)}}\sec(c+dx)(-7i+\tan(c+dx))\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] -1/3*(Sqrt[2]*a^2*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*S
qrt[1 + E^((2*I)*(c + d*x))]*(Cos[d*x] + I*Sin[d*x])*((12*I)*ArcSinh[E^(I*(
c + d*x))] + Sqrt[1 + E^((2*I)*(c + d*x))]*Sec[c + d*x]*(-7*I + Tan[c + d*x
])))/(d*E^(I*(c + 2*d*x)))
```

Maple [A]
time = 0.16, size = 73, normalized size = 0.72

method	result
derivativedivides	$\frac{2ia\left(\frac{(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+2a\sqrt{a+ia\tan(dx+c)}-2a^{\frac{3}{2}}\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)\right)}{d}$
default	$\frac{2ia\left(\frac{(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+2a\sqrt{a+ia\tan(dx+c)}-2a^{\frac{3}{2}}\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/d*a*(1/3*(a+I*a*tan(d*x+c))^(3/2)+2*a*(a+I*a*tan(d*x+c))^(1/2)-2*a^(3/2
)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.48, size = 101, normalized size = 1.00

$$\frac{2i\left(3\sqrt{2}\,a^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)+\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}a^{2}+6\,\sqrt{i\,a\tan\left(dx+c\right)+a}\,a^{3}\right)}{3\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*I*(3*sqrt(2)*a^(7/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a)
)/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + (I*a*tan(d*x + c) + a)^
(3/2)*a^2 + 6*sqrt(I*a*tan(d*x + c) + a)*a^3)/(a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(74) = 148$.
time = 0.44, size = 271, normalized size = 2.68

$$\frac{2\left(3\sqrt{2}\,\sqrt{-\frac{a^{5}}{d^{2}}}\left(de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{4\left(a^{3}e^{(i\,dx+i\,c)}+\sqrt{-\frac{a^{5}}{d^{2}}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^{2}}\right)-3\sqrt{2}\,\sqrt{-\frac{a^{5}}{d^{2}}}\left(de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{4\left(a^{3}e^{(i\,dx+i\,c)}+\sqrt{-\frac{a^{5}}{d^{2}}}\left(-i\,de^{(2i\,dx+2i\,c)}-i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^{2}}\right)-2\sqrt{2}\left(-4i\,a^{2}e^{(3i\,dx+3i\,c)}-3i\,a^{2}e^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)}{3\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*sqrt(2)*sqrt(-a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(4*(a^3*e^(I*d
*x + I*c) + sqrt(-a^5/d^2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^2) - 3*sqrt(2)*sqrt(-a^5/d^2)*(d*e^(2
*I*d*x + 2*I*c) + d)*log(4*(a^3*e^(I*d*x + I*c) + sqrt(-a^5/d^2)*(-I*d*e^(2
*I*d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/
a^2) - 2*sqrt(2)*(-4*I*a^2*e^(3*I*d*x + 3*I*c) - 3*I*a^2*e^(I*d*x + I*c))*s
qrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ia\tan\left(c+dx\right)+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral((I*a*tan(c + d*x) + a)**(5/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 4.08, size = 84, normalized size = 0.83

$$\frac{a^2\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}\,4{}\mathrm{i}}{d}+\frac{a\,{\left(a+a\,\tan(c+d\,x)\,1{}\mathrm{i}\right)}^{3/2}\,2{}\mathrm{i}}{3\,d}-\frac{\sqrt{2}\,{\left(-a\right)}^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{2\,\sqrt{-a}}\right)\,4{}\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] (a^2*(a + a*tan(c + d*x)*1i)^(1/2)*4i)/d + (a*(a + a*tan(c + d*x)*1i)^(3/2)
*2i)/(3*d) - (2^(1/2)*(-a)^(5/2)*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2
))/(2*(-a)^(1/2)))*4i)/d
```

3.103 $\int \cot(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=104

$$-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{4\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{2a^2\sqrt{a+ia\tan(c+dx)}}{d}$$

```
[Out] -2*a^(5/2)*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/d+4*a^(5/2)*arctanh(1/
2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d-2*a^2*(a+I*a*tan(d*x+
c))^(1/2)/d
```

Rubi [A]
time = 0.19, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {3637, 3681, 3561, 212, 3680, 65, 214}

$$-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{4\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{2a^2\sqrt{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^(5/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/d + (4*Sqrt[2]*a^(
5/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d - (2*a^2*Sqrt
[a + I*a*Tan[c + d*x]])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3681

```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(
A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \cot(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{2a^2\sqrt{a+ia\tan(c+dx)}}{d} + (2a)\int \cot(c+dx)\sqrt{a+ia\tan(c+$$

$$= -\frac{2a^2\sqrt{a+ia\tan(c+dx)}}{d} + a\int \cot(c+dx)(a-ia\tan(c+dx)$$

$$= -\frac{2a^2\sqrt{a+ia\tan(c+dx)}}{d} + \frac{a^3\text{Subst}\left(\int \frac{1}{x\sqrt{a+iax}}\,dx, x, \tan(c\right.}{d}$$

$$= \frac{4\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{2a^2\sqrt{a+ia\tan(c-}}{d}$$

$$= -\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{4\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{}}{}\right.}{d}$$

Mathematica [A]
time = 1.31, size = 148, normalized size = 1.42

$$-\frac{\sqrt{2}\,a^2e^{-i(c+dx)}\left(\sqrt{2}\,e^{i(c+dx)} - 2\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}\,\sinh^{-1}\left(e^{i(c+dx)}\right) + \sqrt{1+e^{2i(c+dx)}}\,\tanh^{-1}\left(\frac{\sqrt{2}\,e^{i(c+dx)}}{\sqrt{1+e^{2i(c+dx)}}}\right)\right)\sqrt{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] -((Sqrt[2]*a^2*(Sqrt[2]*E^(I*(c + d*x)) - 2*Sqrt[2]*Sqrt[1 + E^((2*I)*(c +
d*x))]*ArcSinh[E^(I*(c + d*x))] + Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(Sq
rt[2]*E^(I*(c + d*x)))/Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[a + I*a*Tan[c +
 d*x]])/(d*E^(I*(c + d*x))))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 328 vs. 2(84) = 168.
time = 0.97, size = 329, normalized size = 3.16

method	result
default	$-\frac{\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(4i\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)+i\sqrt{-\frac{2\cos(dx}{1+\cos(da}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(4*I*2^(1/2)*arctan(1/2
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*sin(d*x+c)+I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)-4*2^(1/2)*arctanh(1/2*(-2*cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*sin(d*x+c)-ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(
d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d
*x+c)+2*I*sin(d*x+c)+2*cos(d*x+c)-2)/(I*sin(d*x+c)+cos(d*x+c)-1)*a^2
```

Maxima [A]
time = 0.52, size = 126, normalized size = 1.21

$$-\frac{2\sqrt{2}\,a^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)-a^{\frac{5}{2}}\log\left(\frac{\sqrt{i\,a\tan\left(dx+c\right)+a}-\sqrt{a}}{\sqrt{i\,a\tan\left(dx+c\right)+a}+\sqrt{a}}\right)+2\sqrt{i\,a\tan\left(dx+c\right)+a}\,a^{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -(2*sqrt(2)*a^(5/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sq
rt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - a^(5/2)*log((sqrt(I*a*tan(d*
x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x + c) + a) + sqrt(a))) + 2*sqrt(I*a
*tan(d*x + c) + a)*a^2)/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(81) = 162.
time = 0.53, size = 406, normalized size = 3.90

$$-\frac{4\sqrt{2}\,a^{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{(i\,dx+i\,c)}-4\sqrt{2}\sqrt{\frac{a^{5}}{d^{2}}}\,d\log\left(\frac{4\left(a^{3}e^{(i\,dx+i\,c)}+\sqrt{\frac{a^{5}}{d^{2}}}\,(de^{(2i\,dx+2i\,c)}+d)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^{2}}\right)+4\sqrt{2}\sqrt{\frac{a^{5}}{d^{2}}}\,d\log\left(\frac{4\left(a^{3}e^{(i\,dx+i\,c)}-\sqrt{\frac{a^{5}}{d^{2}}}\,(de^{(2i\,dx+2i\,c)}+d)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^{2}}\right)+\sqrt{\frac{a^{5}}{d^{2}}}\,d\log\left(\frac{16\left(3\,a^{3}e^{(2i\,dx+2i\,c)}+a^{3}+2\sqrt{2}\sqrt{\frac{a^{5}}{d^{2}}}\,(de^{(3i\,dx+3i\,c)}+de^{(i\,dx+i\,c)})\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-2i\,dx-2i\,c)}}{a}\right)-\sqrt{\frac{a^{5}}{d^{2}}}\,d\log\left(\frac{16\left(3\,a^{3}e^{(2i\,dx+2i\,c)}+a^{3}-2\sqrt{2}\sqrt{\frac{a^{5}}{d^{2}}}\,(de^{(3i\,dx+3i\,c)}+de^{(i\,dx+i\,c)})\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-2i\,dx-2i\,c)}}{a}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/2*(4*sqrt(2)*a^2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c) - 4*s
qrt(2)*sqrt(a^5/d^2)*d*log(4*(a^3*e^(I*d*x + I*c) + sqrt(a^5/d^2)*(d*e^(2*I
*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^2)
 + 4*sqrt(2)*sqrt(a^5/d^2)*d*log(4*(a^3*e^(I*d*x + I*c) - sqrt(a^5/d^2)*(d*
e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c
)/a^2) + sqrt(a^5/d^2)*d*log(16*(3*a^3*e^(2*I*d*x + 2*I*c) + a^3 + 2*sqrt(2
)*sqrt(a^5/d^2)*(d*e^(3*I*d*x + 3*I*c) + d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/a) - sqrt(a^5/d^2)*d*log(16*(3*a^3
*e^(2*I*d*x + 2*I*c) + a^3 - 2*sqrt(2)*sqrt(a^5/d^2)*(d*e^(3*I*d*x + 3*I*c)
 + d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*
c)/a))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c+dx)-i))^{\frac{5}{2}}\cot(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(5/2)*cot(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*cot(d*x + c), x)
```

Mupad [B]
time = 0.23, size = 98, normalized size = 0.94

$$-\frac{2\,a^2\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{d}-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a^5}\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{a^3}\right)\,\sqrt{a^5}}{d}+\frac{4\,\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{a^5}\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{2\,a^3}\right)\,\sqrt{a^5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] (4*2^(1/2)*atanh((2^(1/2)*(a^5)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*a^3
))*(a^5)^(1/2))/d - (2*atanh(((a^5)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/a^
3)*(a^5)^(1/2))/d - (2*a^2*(a + a*tan(c + d*x)*1i)^(1/2))/d
```

3.104 $\int \cot^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=114

$$-\frac{5ia^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{4i\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}-\frac{a^2\cot(c+dx)\sqrt{a}}{c}$$

```
[Out] -5*I*a^(5/2)*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/d+4*I*a^(5/2)*arctan
h(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d-a^2*cot(d*x+c)*(a
+I*a*tan(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.19, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3634, 3681, 3561, 212, 3680, 65, 214}

$$-\frac{5ia^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{4i\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}-\frac{a^2\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-5*I)*a^(5/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/d + ((4*I)*Sqr
t[2]*a^(5/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d - (a^
2*Cot[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3634

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-a^2)*(b*c - a*d)*(a + b*Tan[e + f*x]
)^(m - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] + Di
st[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[b*(b*c*(m - 2) - a*d*(m - 2*n - 4)) + (a*b*c*(m - 2)
+ b^2*d*(n + 1) - a^2*d*(m + n - 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3681

```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(
A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{a^2\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d} - \int \cot(c+dx)\sqrt{a+ia}$$

$$= -\frac{a^2\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d} + \frac{1}{2}(5ia)\int \cot(c+dx)($$

$$= -\frac{a^2\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d} + \frac{(5ia^3)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a-}}\right.}{}$$

$$= \frac{4i\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{a^2\cot(c+dx)\sqrt{}}{}$$

$$= -\frac{5ia^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{4i\sqrt{2}\,a^{5/2}\tanh^{-1}}{}$$

Mathematica [A]
time = 1.54, size = 170, normalized size = 1.49

$$\frac{a^2e^{-i(c+2dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(-8\sinh^{-1}\left(e^{i(c+dx)}\right)+5\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,e^{i(c+dx)}}{\sqrt{1+e^{2i(c+dx)}}}\right)-i\sqrt{1+e^{2i(c+dx)}}\csc(c+dx)\right)(-i\cos(dx)+\sin(dx))}{\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*(-8*ArcSinh[E^(I*(c + d*x))] + 5*Sqrt[2]*ArcTanh[(Sqrt[2]*E
^(I*(c + d*x)))/Sqrt[1 + E^((2*I)*(c + d*x))]] - I*Sqrt[1 + E^((2*I)*(c + d
*x))]*Csc[c + d*x])*((-I)*Cos[d*x] + Sin[d*x]))/(Sqrt[2]*d*E^(I*(c + 2*d*x)
))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 607 vs. 2(92) = 184.
time = 0.82, size = 608, normalized size = 5.33

method	result
default	$\frac{\sin(dx+c)\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(8i\sqrt{2}\,(\cos^2(dx+c))\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{}}{2\cos(dx+c)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*sin(d*x+c)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(8*I*(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^2*2^(1/2)+5*I*cos(d*x+c)^2*(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))+8*2^(1/2)*cos(d*x+c)^2*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(
1/2))-8*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)+5*cos(d*x+c)^2*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2))-5*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))-8*2^(1/2)*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^
(1/2))+2*I*cos(d*x+c)*sin(d*x+c)-5*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arc
tan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+2*cos(d*x+c)^2-2*cos(d*x+c))/(1
+cos(d*x+c))/(I*sin(d*x+c)+cos(d*x+c)-1)/(-1+cos(d*x+c))*a^2
```

Maxima [A]
time = 0.50, size = 133, normalized size = 1.17

$$-\frac{i\left(4\sqrt{2}\,a^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)-5\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{i\,a\tan\left(dx+c\right)+a}-\sqrt{a}}{\sqrt{i\,a\tan\left(dx+c\right)+a}+\sqrt{a}}\right)-\frac{2i\,\sqrt{i\,a\tan\left(dx+c\right)+a}\,a}{\tan\left(dx+c\right)}\right)a}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/2*I*(4*sqrt(2)*a^(3/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a
))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - 5*a^(3/2)*log((sqrt(I*
a*tan(d*x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x + c) + a) + sqrt(a))) - 2*
I*sqrt(I*a*tan(d*x + c) + a)*a/tan(d*x + c))*a/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 511 vs. 2(87) = 174.
time = 0.50, size = 511, normalized size = 4.48

$$-\frac{8\sqrt{2}\sqrt{-\frac{a^5}{d^2}}\left(de^{(2i\,dx+2i\,c)}-d\right)\log\left(\frac{4\left(a^3e^{(i\,dx+i\,c)}+\sqrt{-\frac{a^5}{d^2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^2}\right)-8\sqrt{2}\sqrt{-\frac{a^5}{d^2}}\left(de^{(2i\,dx+2i\,c)}-d\right)\log\left(\frac{4\left(a^3e^{(i\,dx+i\,c)}+\sqrt{-\frac{a^5}{d^2}}\left(-i\,de^{(2i\,dx+2i\,c)}-i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^2}\right)+5\sqrt{-\frac{a^5}{d^2}}\left(de^{(2i\,dx+2i\,c)}-d\right)\log\left(\frac{16\left(3a^3e^{(2i\,dx+2i\,c)}+a^3-2\sqrt{2}\sqrt{-\frac{a^5}{d^2}}\left(i\,de^{(3i\,dx+3i\,c)}+i\,de^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-2i\,dx-2i\,c)}}{a}\right)-5\sqrt{-\frac{a^5}{d^2}}\left(de^{(2i\,dx+2i\,c)}-d\right)\log\left(\frac{16\left(3a^3e^{(2i\,dx+2i\,c)}+a^3-2\sqrt{2}\sqrt{-\frac{a^5}{d^2}}\left(-i\,de^{(3i\,dx+3i\,c)}-i\,de^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-2i\,dx-2i\,c)}}{a}\right)+4\sqrt{2}\left(i\,a^2e^{(3i\,dx+3i\,c)}+i\,a^2e^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{4\left(de^{(2i\,dx+2i\,c)}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/4*(8*sqrt(2)*sqrt(-a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*log(4*(a^3*e^(I*
d*x + I*c) + sqrt(-a^5/d^2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^2) - 8*sqrt(2)*sqrt(-a^5/d^2)*(d*e^(
2*I*d*x + 2*I*c) - d)*log(4*(a^3*e^(I*d*x + I*c) + sqrt(-a^5/d^2)*(-I*d*e^(
2*I*d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)
```

```
/a^2) + 5*sqrt(-a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*log(16*(3*a^3*e^(2*I*d
*x + 2*I*c) + a^3 - 2*sqrt(2)*sqrt(-a^5/d^2)*(I*d*e^(3*I*d*x + 3*I*c) + I*d
*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/a
) - 5*sqrt(-a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*log(16*(3*a^3*e^(2*I*d*x +
 2*I*c) + a^3 - 2*sqrt(2)*sqrt(-a^5/d^2)*(-I*d*e^(3*I*d*x + 3*I*c) - I*d*e^
(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/a) +
 4*sqrt(2)*(I*a^2*e^(3*I*d*x + 3*I*c) + I*a^2*e^(I*d*x + I*c))*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1)))/(d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^2, x)
```

Mupad [**B**]
time = 4.11, size = 114, normalized size = 1.00

$$\frac{\mathrm{atan}\left(\frac{\sqrt{-a^5}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}{a^3}\right)\,\sqrt{-a^5}\,5\mathrm{i}}{d}-\frac{a^2\cot\left(c+d\,x\right)\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}{d}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{-a^5}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}{2\,a^3}\right)\,\sqrt{-a^5}\,4\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] (atan(((-a^5)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/a^3)*(-a^5)^(1/2)*5i)/d
- (a^2*cot(c + d*x)*(a + a*tan(c + d*x)*1i)^(1/2))/d - (2^(1/2)*atan((2^(1/
2)*(-a^5)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*a^3))*(-a^5)^(1/2)*4i)/d
```

3.105 $\int \cot^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=151

$$\frac{23a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{4d}-\frac{4\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}-\frac{9ia^2\cot(c+dx)\sqrt{a+}}{4d}$$

```
[Out] 23/4*a^(5/2)*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/d-4*a^(5/2)*arctanh(
1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d-9/4*I*a^2*cot(d*x+c
)*(a+I*a*tan(d*x+c))^(1/2)/d-1/2*a^2*cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)/
d
```

Rubi [A]
time = 0.29, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3634, 3679, 3681, 3561, 212, 3680, 65, 214}

$$\frac{23a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{4d}-\frac{4\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}-\frac{a^2\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{9ia^2\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (23*a^(5/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/(4*d) - (4*Sqrt[2]
*a^(5/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d - (((9*I)
/4)*a^2*Cot[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d - (a^2*Cot[c + d*x]^2*Sq
rt[a + I*a*Tan[c + d*x]])/(2*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3634

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-a^2)*(b*c - a*d)*(a + b*Tan[e + f*x]
)^(m - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] + Di
st[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[b*(b*c*(m - 2) - a*d*(m - 2*n - 4)) + (a*b*c*(m - 2)
+ b^2*d*(n + 1) - a^2*d*(m + n - 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3681

```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(
A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{a^2\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d} - \frac{1}{2}\int \cot^2(c+dx)\sqrt{a+}$$

$$= -\frac{9ia^2\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d} - \frac{a^2\cot^2(c+dx)\sqrt{a+}}{2d}$$

$$= -\frac{9ia^2\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d} - \frac{a^2\cot^2(c+dx)\sqrt{a+}}{2d}$$

$$= -\frac{9ia^2\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d} - \frac{a^2\cot^2(c+dx)\sqrt{a+}}{2d}$$

$$= -\frac{4\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{9ia^2\cot(c+dx)}{}$$

$$= \frac{23a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{4d} - \frac{4\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{}}{}\right.}{d}$$

Mathematica [A]
time = 2.13, size = 182, normalized size = 1.21

$$-\frac{a^2e^{-i(c+2dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(32\sinh^{-1}\left(e^{i(c+dx)}\right)-23\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}e^{i(c+dx)}}{\sqrt{1+e^{2i(c+dx)}}}\right)+\sqrt{1+e^{2i(c+dx)}}\,(9i+2\cot(c+dx))\csc(c+dx)\right)(\cos(dx)+i\sin(dx))}{4\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] -1/4*(a^2*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 +
E^((2*I)*(c + d*x))]*(32*ArcSinh[E^(I*(c + d*x))] - 23*Sqrt[2]*ArcTanh[(Sqr
t[2]*E^(I*(c + d*x)))/Sqrt[1 + E^((2*I)*(c + d*x))]] + Sqrt[1 + E^((2*I)*(c
 + d*x))]*(9*I + 2*Cot[c + d*x])*Csc[c + d*x])*(Cos[d*x] + I*Sin[d*x]))/(Sq
rt[2]*d*E^(I*(c + 2*d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 676 vs. 2(121) = 242.
time = 0.83, size = 677, normalized size = 4.48

method	result
default	$\frac{\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(32i\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}}{2}\right)(\cos^2(dx+c))\sin(dx+c)\sqrt{2}+23i\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(32*I*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2
))*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)+23*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^2*sin(d*x+c)-32
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)-
32*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)*2^(1/2)-23*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/s
in(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-23*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)+22*I*cos(d*x+c)
^2*sin(d*x+c)+32*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*s
in(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+
c)-18*I*sin(d*x+c)*cos(d*x+c)+23*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*s
in(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)+22*cos(d*x+c)^3-4*cos(d*x+c)^2-18*cos(d*x+c))/(-1+cos(d*x+c))/(I*s
in(d*x+c)+cos(d*x+c)-1)/(1+cos(d*x+c))*a^2
```

Maxima [A]
time = 0.49, size = 181, normalized size = 1.20

$$\frac{\left(16\,\sqrt{2}\,\sqrt{a}\,\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)-23\,\sqrt{a}\,\log\left(\frac{\sqrt{i\,a\tan\left(dx+c\right)+a}-\sqrt{a}}{\sqrt{i\,a\tan\left(dx+c\right)+a}+\sqrt{a}}\right)+\frac{2\left(9\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}a-7\,\sqrt{i\,a\tan\left(dx+c\right)+a}\,a^{2}\right)}{\left(i\,a\tan\left(dx+c\right)+a\right)^{2}-2\left(i\,a\tan\left(dx+c\right)+a\right)a+a^{2}}\right)a^{2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/8*(16*sqrt(2)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))
/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - 23*sqrt(a)*log((sqrt(I*a
*tan(d*x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x + c) + a) + sqrt(a))) + 2*(
9*(I*a*tan(d*x + c) + a)^(3/2)*a - 7*sqrt(I*a*tan(d*x + c) + a)*a^2)/((I*a*
tan(d*x + c) + a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2))*a^2/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 558 vs. 2(116) = 232.
time = 0.45, size = 558, normalized size = 3.70

$$-\frac{32\,\sqrt{2}\,\sqrt{\frac{a^{5}}{d^{2}}}\left(de^{\left(4i\,dx+4i\,c\right)}-2\,de^{\left(2i\,dx+2i\,c\right)}+d\right)\log\left(\frac{4\left(a^{3}e^{\left(i\,dx+i\,c\right)}+\sqrt{\frac{a^{5}}{d^{2}}}\left(de^{\left(2i\,dx+2i\,c\right)}+d\right)\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}\right)e^{\left(-i\,dx-i\,c\right)}}{a^{2}}\right)-32\,\sqrt{2}\,\sqrt{\frac{a^{5}}{d^{2}}}\left(de^{\left(4i\,dx+4i\,c\right)}-2\,de^{\left(2i\,dx+2i\,c\right)}+d\right)\log\left(\frac{4\left(a^{3}e^{\left(i\,dx+i\,c\right)}-\sqrt{\frac{a^{5}}{d^{2}}}\left(de^{\left(2i\,dx+2i\,c\right)}+d\right)\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}\right)e^{\left(-i\,dx-i\,c\right)}}{a^{2}}\right)-23\,\sqrt{\frac{a^{5}}{d^{2}}}\left(de^{\left(4i\,dx+4i\,c\right)}-2\,de^{\left(2i\,dx+2i\,c\right)}+d\right)\log\left(\frac{16\left(3\,a^{3}e^{\left(2i\,dx+2i\,c\right)}+a^{3}+2\,\sqrt{2}\,\sqrt{\frac{a^{5}}{d^{2}}}\left(de^{\left(3i\,dx+3i\,c\right)}+de^{\left(i\,dx+i\,c\right)}\right)\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}\right)e^{\left(-2i\,dx-2i\,c\right)}}{a}\right)+23\,\sqrt{\frac{a^{5}}{d^{2}}}\left(de^{\left(4i\,dx+4i\,c\right)}-2\,de^{\left(2i\,dx+2i\,c\right)}+d\right)\log\left(\frac{16\left(3\,a^{3}e^{\left(2i\,dx+2i\,c\right)}+a^{3}-2\,\sqrt{2}\,\sqrt{\frac{a^{5}}{d^{2}}}\left(de^{\left(3i\,dx+3i\,c\right)}+de^{\left(i\,dx+i\,c\right)}\right)\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}\right)e^{\left(-2i\,dx-2i\,c\right)}}{a}\right)-4\,\sqrt{2}\left(11\,a^{2}e^{\left(5i\,dx+5i\,c\right)}+4\,a^{2}e^{\left(3i\,dx+3i\,c\right)}-7\,a^{2}e^{\left(i\,dx+i\,c\right)}\right)\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}}{16\left(de^{\left(4i\,dx+4i\,c\right)}-2\,de^{\left(2i\,dx+2i\,c\right)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/16*(32*sqrt(2)*sqrt(a^5/d^2)*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2
*I*c) + d)*log(4*(a^3*e^(I*d*x + I*c) + sqrt(a^5/d^2)*(d*e^(2*I*d*x + 2*I*c
) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^2) - 32*sqrt(2
)*sqrt(a^5/d^2)*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*log(4
*(a^3*e^(I*d*x + I*c) - sqrt(a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e
^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^2) - 23*sqrt(a^5/d^2)*(d*e^(4*
I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*log(16*(3*a^3*e^(2*I*d*x + 2*
I*c) + a^3 + 2*sqrt(2)*sqrt(a^5/d^2)*(d*e^(3*I*d*x + 3*I*c) + d*e^(I*d*x +
I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/a) + 23*sqrt(
a^5/d^2)*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*log(16*(3*a^
3*e^(2*I*d*x + 2*I*c) + a^3 - 2*sqrt(2)*sqrt(a^5/d^2)*(d*e^(3*I*d*x + 3*I*c
) + d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I
*c)/a) - 4*sqrt(2)*(11*a^2*e^(5*I*d*x + 5*I*c) + 4*a^2*e^(3*I*d*x + 3*I*c)
- 7*a^2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(4*I*d*x +
 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^3, x)
```

Mupad [**B**]
time = 4.00, size = 139, normalized size = 0.92

$$-\frac{\mathrm{atan}\left(\frac{\sqrt{a^5}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}\,1{}\mathrm{i}}{a^3}\right)\,\sqrt{a^5}\,23{}\mathrm{i}}{4\,d}+\frac{7\,a^2\,\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{4\,d\,{\tan\left(c+d\,x\right)}^2}-\frac{9\,a\,{\left(a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}{4\,d\,{\tan\left(c+d\,x\right)}^2}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a^5}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2\,a^3}\right)\,\sqrt{a^5}\,4{}\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] (7*a^2*(a + a*tan(c + d*x)*1i)^(1/2))/(4*d*tan(c + d*x)^2) - (atan(((a^5)^(
1/2)*(a + a*tan(c + d*x)*1i)^(1/2)*1i)/a^3)*(a^5)^(1/2)*23i)/(4*d) - (9*a*(
a + a*tan(c + d*x)*1i)^(3/2))/(4*d*tan(c + d*x)^2) + (2^(1/2)*atan((2^(1/2)
*(a^5)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)*1i)/(2*a^3))*(a^5)^(1/2)*4i)/d
```

3.106 $\int \cot^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=190

$$\frac{45ia^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{8d}-\frac{4i\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}+\frac{19a^2\cot(c+dx)\sqrt{\ldots}}{8\ldots}$$

```
[Out] 45/8*I*a^(5/2)*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/d-4*I*a^(5/2)*arct
anh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d+19/8*a^2*cot(d*
x+c)*(a+I*a*tan(d*x+c))^(1/2)/d-13/12*I*a^2*cot(d*x+c)^2*(a+I*a*tan(d*x+c))
^(1/2)/d-1/3*a^2*cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.39, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3634, 3679, 3681, 3561, 212, 3680, 65, 214}

$$\frac{45ia^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{8d}-\frac{4i\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{a^2\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}-\frac{13ia^2\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{12d}+\frac{19a^2\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((45*I)/8)*a^(5/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/d - ((4*I)
*Sqrt[2]*a^(5/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d +
 (19*a^2*Cot[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(8*d) - (((13*I)/12)*a^2*
Cot[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/d - (a^2*Cot[c + d*x]^3*Sqrt[a +
 I*a*Tan[c + d*x]])/(3*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3634

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-a^2)*(b*c - a*d)*(a + b*Tan[e + f*x]
)^(m - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] + Di
st[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[b*(b*c*(m - 2) - a*d*(m - 2*n - 4)) + (a*b*c*(m - 2)
+ b^2*d*(n + 1) - a^2*d*(m + n - 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3681

```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(
A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{a^2\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d} - \frac{1}{3}\int \cot^3(c+dx)\sqrt{a \cdots}$$

$$= -\frac{13ia^2\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{12d} - \frac{a^2\cot^3(c+dx)\sqrt{\cdots}}{3 \cdots}$$

$$= \frac{19a^2\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d} - \frac{13ia^2\cot^2(c+dx)\sqrt{\cdots}}{12 \cdots}$$

$$= \frac{19a^2\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d} - \frac{13ia^2\cot^2(c+dx)\sqrt{\cdots}}{12 \cdots}$$

$$= \frac{19a^2\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d} - \frac{13ia^2\cot^2(c+dx)\sqrt{\cdots}}{12 \cdots}$$

$$= -\frac{4i\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} + \frac{19a^2\cot(c+d \cdots}{\cdots}$$

$$= \frac{45ia^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{8d} - \frac{4i\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\cdots\right.}{\cdots}$$

Mathematica [A]
time = 2.65, size = 200, normalized size = 1.05

$$\frac{a^2e^{-i(c+2dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\,(\cos(dx)+i\sin(dx))\left(-384i\sinh^{-1}\left(e^{i(c+dx)}\right)+270i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,e^{i(c+dx)}}{\sqrt{1+e^{2i(c+dx)}}}\right)+\sqrt{1+e^{2i(c+dx)}}\csc^3(c+dx)(49-65\cos(2(c+dx))-26i\sin(2(c+dx)))\right)}{48\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*(Cos[d*x] + I*Sin[d*x])*((-384*I)*ArcSinh[E^(I*(c + d*x))]
+ (270*I)*Sqrt[2]*ArcTanh[(Sqrt[2]*E^(I*(c + d*x)))/Sqrt[1 + E^((2*I)*(c +
d*x))]] + Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c + d*x]^3*(49 - 65*Cos[2*(c +
d*x)] - (26*I)*Sin[2*(c + d*x)])))/(48*Sqrt[2]*d*E^(I*(c + 2*d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 925 vs. 2(153) = 306.
time = 0.86, size = 926, normalized size = 4.87

method	result	size
default	Expression too large to display	926

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(-114*I*cos(d*x+c)*si
n(d*x+c)-270*I*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))+192*cos
(d*x+c)^4*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+135*cos(d*x+c)^4*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+135*I*cos(d*
x+c)^4*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))+192*I*cos(d*x+c)^4*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)-52*I*cos(d*x+c)^2*sin(d*x+c)-384*2
^(1/2)*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+182*cos(d*x+c)^4+182*I*cos(d*x+c)^3*
sin(d*x+c)-270*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+135*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(
d*x+c))-130*cos(d*x+c)^3-384*I*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c
)*2^(1/2))*2^(1/2)+192*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)+
192*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*2^(1/2))-166*cos(d*x+c)^2+135*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+114*cos(d*x+c)
)/(-1+cos(d*x+c))/(I*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c)/(1+cos(d*x+c))*a^2
```

Maxima [A]
time = 0.50, size = 216, normalized size = 1.14

$$\frac{i\,a^3\left(\frac{96\,\sqrt{2}\,\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)}{\sqrt{a}}-\frac{135\,\log\left(\frac{\sqrt{i\,a\tan\left(dx+c\right)+a}-\sqrt{a}}{\sqrt{i\,a\tan\left(dx+c\right)+a}+\sqrt{a}}\right)}{\sqrt{a}}+\frac{2\left(57\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}-88\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}a+39\,\sqrt{i\,a\tan\left(dx+c\right)+a}\,a^2\right)}{\left(i\,a\tan\left(dx+c\right)+a\right)^3-3\left(i\,a\tan\left(dx+c\right)+a\right)^2a+3\left(i\,a\tan\left(dx+c\right)+a\right)a^2-a^3}\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/48*I*a^3*(96*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/
(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/sqrt(a) - 135*log((sqrt(I*a
*tan(d*x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x + c) + a) + sqrt(a)))/sqrt(
a) + 2*(57*(I*a*tan(d*x + c) + a)^(5/2) - 88*(I*a*tan(d*x + c) + a)^(3/2)*a
 + 39*sqrt(I*a*tan(d*x + c) + a)*a^2)/((I*a*tan(d*x + c) + a)^3 - 3*(I*a*ta
n(d*x + c) + a)^2*a + 3*(I*a*tan(d*x + c) + a)*a^2 - a^3))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 659 vs. $2(145) = 290$.
time = 0.44, size = 659, normalized size = 3.47

$$\frac{192\sqrt{2}\sqrt{-\frac{a^5}{d^2}}\left(de^{(6idx+6ic)}-3de^{(4idx+4ic)}+3de^{(2idx+2ic)}-d\right)\log\left(\frac{4\left(a^3e^{(idx+ic)}+\sqrt{-\frac{a^5}{d^2}}\left(ide^{(2idx+2ic)}+id\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\right)e^{(-idx-ic)}}{a^2}\right)-192\sqrt{2}\sqrt{-\frac{a^5}{d^2}}\left(de^{(6idx+6ic)}-3de^{(4idx+4ic)}+3de^{(2idx+2ic)}-d\right)\log\left(\frac{4\left(a^3e^{(idx+ic)}+\sqrt{-\frac{a^5}{d^2}}\left(-ide^{(2idx+2ic)}-id\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\right)e^{(-idx-ic)}}{a^2}\right)+135\sqrt{-\frac{a^5}{d^2}}\left(de^{(6idx+6ic)}-3de^{(4idx+4ic)}+3de^{(2idx+2ic)}-d\right)\log\left(\frac{16\left(3a^3e^{(2idx+2ic)}+a^3-2\sqrt{2}\sqrt{-\frac{a^5}{d^2}}\left(ide^{(3idx+3ic)}+ide^{(idx+ic)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\right)e^{(-2idx-2ic)}}{a}\right)-135\sqrt{-\frac{a^5}{d^2}}\left(de^{(6idx+6ic)}-3de^{(4idx+4ic)}+3de^{(2idx+2ic)}-d\right)\log\left(\frac{16\left(3a^3e^{(2idx+2ic)}+a^3-2\sqrt{2}\sqrt{-\frac{a^5}{d^2}}\left(-ide^{(3idx+3ic)}-ide^{(idx+ic)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\right)e^{(-2idx-2ic)}}{a}\right)+4\sqrt{2}\left(91ia^2e^{(7idx+7ic)}-7ia^2e^{(5idx+5ic)}-59ia^2e^{(3idx+3ic)}+39ia^2e^{(idx+ic)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}}{96\left(de^{(6idx+6ic)}-3de^{(4idx+4ic)}+3de^{(2idx+2ic)}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/96*(192*sqrt(2)*sqrt(-a^5/d^2)*(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x +
4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)*log(4*(a^3*e^(I*d*x + I*c) + sqrt(-a^
5/d^2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e
^(-I*d*x - I*c)/a^2) - 192*sqrt(2)*sqrt(-a^5/d^2)*(d*e^(6*I*d*x + 6*I*c) -
3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)*log(4*(a^3*e^(I*d*x
+ I*c) + sqrt(-a^5/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^2) + 135*sqrt(-a^5/d^2)*(d*e^(6*I*d*x +
 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)*log(16*(3*
a^3*e^(2*I*d*x + 2*I*c) + a^3 - 2*sqrt(2)*sqrt(-a^5/d^2)*(I*d*e^(3*I*d*x +
3*I*c) + I*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*
x - 2*I*c)/a) - 135*sqrt(-a^5/d^2)*(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x
+ 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)*log(16*(3*a^3*e^(2*I*d*x + 2*I*c) +
 a^3 - 2*sqrt(2)*sqrt(-a^5/d^2)*(-I*d*e^(3*I*d*x + 3*I*c) - I*d*e^(I*d*x +
I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/a) + 4*sqrt(2
)*(91*I*a^2*e^(7*I*d*x + 7*I*c) - 7*I*a^2*e^(5*I*d*x + 5*I*c) - 59*I*a^2*e^
(3*I*d*x + 3*I*c) + 39*I*a^2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) +
 1)))/(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2
*I*c) - d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^4, x)
```

Mupad [B]
time = 0.22, size = 171, normalized size = 0.90

$$-\frac{19\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}}{8\,d\,{\mathrm{tan}\left(c+d\,x\right)}^3}-\frac{\mathrm{atan}\left(\frac{\sqrt{-a^5}\,\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}}{a^3}\right)\,\sqrt{-a^5}\,45{}\mathrm{i}}{8\,d}-\frac{13\,a^2\,\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}}{8\,d\,{\mathrm{tan}\left(c+d\,x\right)}^3}+\frac{11\,a\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}{3\,d\,{\mathrm{tan}\left(c+d\,x\right)}^3}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{-a^5}\,\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}}{2\,a^3}\right)\,\sqrt{-a^5}\,4{}\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] (11*a*(a + a*tan(c + d*x)*1i)^(3/2))/(3*d*tan(c + d*x)^3) - (atan(((-a^5)^(
1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/a^3)*(-a^5)^(1/2)*45i)/(8*d) - (13*a^2*
(a + a*tan(c + d*x)*1i)^(1/2))/(8*d*tan(c + d*x)^3) - (19*(a + a*tan(c + d*
x)*1i)^(5/2))/(8*d*tan(c + d*x)^3) + (2^(1/2)*atan((2^(1/2)*(-a^5)^(1/2)*(a
 + a*tan(c + d*x)*1i)^(1/2))/(2*a^3))*(-a^5)^(1/2)*4i)/d
```

3.107 $\int (a+ia\tan (c+dx))^{7/2}\,dx$

Optimal. Leaf size=130

$$-\frac{8i\sqrt{2}\,a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}+\frac{8ia^3\sqrt{a+ia\tan(c+dx)}}{d}+\frac{4ia^2(a+ia\tan(c+dx))^{3/2}}{3d}+\frac{2i\ldots}{}$$

```
[Out] -8*I*a^(7/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/
d+8*I*a^3*(a+I*a*tan(d*x+c))^(1/2)/d+4/3*I*a^2*(a+I*a*tan(d*x+c))^(3/2)/d+2
/5*I*a*(a+I*a*tan(d*x+c))^(5/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3559, 3561, 212}

$$-\frac{8i\sqrt{2}\,a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}+\frac{8ia^3\sqrt{a+ia\tan(c+dx)}}{d}+\frac{4ia^2(a+ia\tan(c+dx))^{3/2}}{3d}+\frac{2ia(a+ia\tan(c+dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((-8*I)*Sqrt[2]*a^(7/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a]
)])/d + ((8*I)*a^3*Sqrt[a + I*a*Tan[c + d*x]])/d + (((4*I)/3)*a^2*(a + I*a*
Tan[c + d*x])^(3/2))/d + (((2*I)/5)*a*(a + I*a*Tan[c + d*x])^(5/2))/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int (a+ia\tan(c+dx))^{7/2}\,dx &= \frac{2ia(a+ia\tan(c+dx))^{5/2}}{5d} + (2a)\int (a+ia\tan(c+dx))^{5/2}\,dx \\
&= \frac{4ia^2(a+ia\tan(c+dx))^{3/2}}{3d} + \frac{2ia(a+ia\tan(c+dx))^{5/2}}{5d} + (4a^2)\int (a+ia\,\mathrm{ta} \\
&= \frac{8ia^3\sqrt{a+ia\tan(c+dx)}}{d} + \frac{4ia^2(a+ia\tan(c+dx))^{3/2}}{3d} + \frac{2ia(a+ia\tan(c+}{5d} \\
&= \frac{8ia^3\sqrt{a+ia\tan(c+dx)}}{d} + \frac{4ia^2(a+ia\tan(c+dx))^{3/2}}{3d} + \frac{2ia(a+ia\tan(c+}{5d} \\
&= -\frac{8i\sqrt{2}\,a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} + \frac{8ia^3\sqrt{a+ia\tan(c+dx)}}{d} +
\end{aligned}
$$

Mathematica [A]

time = 1.85, size = 166, normalized size = 1.28

$$\frac{i\sqrt{2}\,a^3e^{-\frac{1}{2}i(2c+5dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(\cos\left(\frac{3dx}{2}\right)+i\sin\left(\frac{3dx}{2}\right)\right)\left(-120\sinh^{-1}\left(e^{i(c+dx)}\right)+\sqrt{1+e^{2i(c+dx)}}\sec^3(c+dx)(35+38\cos(2(c+dx))+8i\sin(2(c+dx)))\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((I/15)*Sqrt[2]*a^3*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]
*Sqrt[1 + E^((2*I)*(c + d*x))]*(Cos[(3*d*x)/2] + I*Sin[(3*d*x)/2])*(-120*Ar
cSinh[E^(I*(c + d*x))] + Sqrt[1 + E^((2*I)*(c + d*x))]*Sec[c + d*x]^3*(35 +
 38*Cos[2*(c + d*x)] + (8*I)*Sin[2*(c + d*x)])))/(d*E^((I/2)*(2*c + 5*d*x))
)
```

Maple [A]

time = 0.17, size = 92, normalized size = 0.71

method	result
derivativedivides	$\frac{2ia\left(\frac{(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{2a(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+4a^2\sqrt{a+ia\tan(dx+c)}-4a^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan}}{2}\right.\right.}{d}$
default	$\frac{2ia\left(\frac{(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{2a(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+4a^2\sqrt{a+ia\tan(dx+c)}-4a^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan}}{2}\right.\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/d*a*(1/5*(a+I*a*tan(d*x+c))^(5/2)+2/3*a*(a+I*a*tan(d*x+c))^(3/2)+4*a^2*
(a+I*a*tan(d*x+c))^(1/2)-4*a^(5/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(
1/2)*2^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.50, size = 120, normalized size = 0.92

$$\frac{2i\left(30\sqrt{2}\,a^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)+3\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}a^{2}+10\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}a^{3}+60\sqrt{i\,a\tan\left(dx+c\right)+a}\,a^{4}\right)}{15\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*I*(30*sqrt(2)*a^(9/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) +
a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 3*(I*a*tan(d*x + c) +
 a)^(5/2)*a^2 + 10*(I*a*tan(d*x + c) + a)^(3/2)*a^3 + 60*sqrt(I*a*tan(d*x +
 c) + a)*a^4)/(a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(95) = 190.
time = 0.43, size = 321, normalized size = 2.47

$$\frac{4\left(15\sqrt{2}\sqrt{-\frac{a^{7}}{d^{2}}}\left(de^{\left(4i\,dx+4i\,c\right)}+2\,de^{\left(2i\,dx+2i\,c\right)}+d\right)\log\left(\frac{4\left(a^{4}e^{\left(i\,dx+i\,c\right)}+\sqrt{-\frac{a^{7}}{d^{2}}}\left(i\,de^{\left(2i\,dx+2i\,c\right)}+i\,d\right)\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}\right)e^{\left(-i\,dx-i\,c\right)}}{a^{3}}\right)-15\sqrt{2}\sqrt{-\frac{a^{7}}{d^{2}}}\left(de^{\left(4i\,dx+4i\,c\right)}+2\,de^{\left(2i\,dx+2i\,c\right)}+d\right)\log\left(\frac{4\left(a^{4}e^{\left(i\,dx+i\,c\right)}+\sqrt{-\frac{a^{7}}{d^{2}}}\left(-i\,de^{\left(2i\,dx+2i\,c\right)}-i\,d\right)\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}\right)e^{\left(-i\,dx-i\,c\right)}}{a^{3}}\right)-2\sqrt{2}\left(-23i\,a^{3}e^{\left(5i\,dx+5i\,c\right)}-35i\,a^{3}e^{\left(3i\,dx+3i\,c\right)}-15i\,a^{3}e^{\left(i\,dx+i\,c\right)}\right)\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}\right)}{15\left(de^{\left(4i\,dx+4i\,c\right)}+2\,de^{\left(2i\,dx+2i\,c\right)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 4/15*(15*sqrt(2)*sqrt(-a^7/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2
*I*c) + d)*log(4*(a^4*e^(I*d*x + I*c) + sqrt(-a^7/d^2)*(I*d*e^(2*I*d*x + 2*
I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^3) - 15*s
qrt(2)*sqrt(-a^7/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
*log(4*(a^4*e^(I*d*x + I*c) + sqrt(-a^7/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) - I*
d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^3) - 2*sqrt(2)*(-2
3*I*a^3*e^(5*I*d*x + 5*I*c) - 35*I*a^3*e^(3*I*d*x + 3*I*c) - 15*I*a^3*e^(I*
d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(4*I*d*x + 4*I*c) + 2*d
*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ia\tan\left(c+dx\right)+a\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Integral((I*a*tan(c + d*x) + a)**(7/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 4.14, size = 107, normalized size = 0.82

$$\frac{a^3\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}\,8{}\mathrm{i}}{d}+\frac{a^2\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}\,4{}\mathrm{i}}{3\,d}+\frac{a\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}\,2{}\mathrm{i}}{5\,d}+\frac{\sqrt{2}\,{\left(-a\right)}^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{2\,\sqrt{-a}}\right)\,8{}\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(7/2),x)
```

```
[Out] (a^3*(a + a*tan(c + d*x)*1i)^(1/2)*8i)/d + (a^2*(a + a*tan(c + d*x)*1i)^(3/
2)*4i)/(3*d) + (a*(a + a*tan(c + d*x)*1i)^(5/2)*2i)/(5*d) + (2^(1/2)*(-a)^(
7/2)*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*(-a)^(1/2)))*8i)/d
```

3.108 $\int \frac{\tan^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=201

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d}-\frac{\tan^4(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{188\sqrt{a+ia\tan(c+dx)}}{35ad}+\frac{47\tan^2(c+dx)\sqrt{\ }}{3}$$

```
[Out] -1/2*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/2)/a^(1/2
)-188/35*(a+I*a*tan(d*x+c))^(1/2)/a/d+47/35*(a+I*a*tan(d*x+c))^(1/2)*tan(d*
x+c)^2/a/d-9/7*I*(a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^3/a/d-tan(d*x+c)^4/d/(
a+I*a*tan(d*x+c))^(1/2)+223/105*(a+I*a*tan(d*x+c))^(3/2)/a^2/d
```

Rubi [A]
time = 0.32, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3639, 3678, 3673, 3608, 3561, 212}

$$\frac{223(a+ia\tan(c+dx))^{3/2}}{105a^2d}-\frac{\tan^4(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{9i\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7ad}+\frac{47\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{35ad}-\frac{188\sqrt{a+ia\tan(c+dx)}}{35ad}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] -(ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(Sqrt[2]*Sqrt[a]*d)
) - Tan[c + d*x]^4/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (188*Sqrt[a + I*a*Tan[c
 + d*x]])/(35*a*d) + (47*Tan[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/(35*a*d
) - (((9*I)/7)*Tan[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/(a*d) + (223*(a +
 I*a*Tan[c + d*x])^(3/2))/(105*a^2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
```

```
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rule 3678

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Dist[
1/(a*(m + n)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Simp
[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{\tan^4(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{\int \tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}\left(-4a+\frac{9}{2}ia\right.}{a^2}$$

$$= -\frac{\tan^4(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{9i\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7ad} - \frac{2\int \tan^2}{}$$

$$= -\frac{\tan^4(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} + \frac{47\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{35ad} - \frac{9i\tan^3(}{}$$

$$= -\frac{\tan^4(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} + \frac{47\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{35ad} - \frac{9i\tan^3(}{}$$

$$= -\frac{\tan^4(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{188\sqrt{a+ia\tan(c+dx)}}{35ad} + \frac{47\tan^2(c+dx)\sqrt{a}}{35a}$$

$$= -\frac{\tan^4(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{188\sqrt{a+ia\tan(c+dx)}}{35ad} + \frac{47\tan^2(c+dx)\sqrt{a}}{35a}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d} - \frac{\tan^4(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{188\sqrt{a+i}}{3}$$

Mathematica [A]
time = 1.80, size = 123, normalized size = 0.61

$$\frac{-\frac{840e^{i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} - \sec^4(c+dx)(1015+1484\cos(2(c+dx))+229\cos(4(c+dx))+224i\sin(2(c+dx))+124i\sin(4(c+dx)))}{840d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-840*E^(I*(c + d*x))*ArcSinh[E^(I*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x
))] - Sec[c + d*x]^4*(1015 + 1484*Cos[2*(c + d*x)] + 229*Cos[4*(c + d*x)] +
 (224*I)*Sin[2*(c + d*x)] + (124*I)*Sin[4*(c + d*x)]))/(840*d*Sqrt[a + I*a*
Tan[c + d*x]])
```

Maple [A]
time = 0.19, size = 131, normalized size = 0.65

method	result

derivativedivides	$\frac{\frac{2(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{6a(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{8a^2(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-4a^3\sqrt{a+ia\tan(dx+c)}-\frac{a^4}{\sqrt{a+ia\tan}}}{a^4d}$
default	$\frac{\frac{2(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{6a(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{8a^2(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-4a^3\sqrt{a+ia\tan(dx+c)}-\frac{a^4}{\sqrt{a+ia\tan}}}{a^4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^4*(1/7*(a+I*a*tan(d*x+c))^(7/2)-3/5*a*(a+I*a*tan(d*x+c))^(5/2)+4/3*a^
2*(a+I*a*tan(d*x+c))^(3/2)-2*a^3*(a+I*a*tan(d*x+c))^(1/2)-1/2*a^4/(a+I*a*ta
n(d*x+c))^(1/2)-1/4*a^(7/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^
(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.49, size = 156, normalized size = 0.78

$$\frac{105\sqrt{2}\,a^{\frac{11}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+120\,(i\,a\tan(dx+c)+a)^{\frac{7}{2}}a^2-504\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}a^3+1120\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a^4-1680\,\sqrt{i\,a\tan(dx+c)+a}\,a^5-\frac{420\,a^6}{\sqrt{i\,a\tan(dx+c)+a}}}{420\,a^6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/420*(105*sqrt(2)*a^(11/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) +
 a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 120*(I*a*tan(d*x + c
) + a)^(7/2)*a^2 - 504*(I*a*tan(d*x + c) + a)^(5/2)*a^3 + 1120*(I*a*tan(d*x
 + c) + a)^(3/2)*a^4 - 1680*sqrt(I*a*tan(d*x + c) + a)*a^5 - 420*a^6/sqrt(I
*a*tan(d*x + c) + a))/(a^6*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 389 vs. 2(160) = 320.
time = 0.44, size = 389, normalized size = 1.94

$$-\frac{105\sqrt{2}\left(ade^{(7i\,dx+7i\,c)}+3\,ade^{(5i\,dx+5i\,c)}+3\,ade^{(3i\,dx+3i\,c)}+ade^{(i\,dx+i\,c)}\right)\sqrt{\frac{1}{ad^2}}\log\left(4\left(\left(ade^{(2i\,dx+2i\,c)}+ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-105\sqrt{2}\left(ade^{(7i\,dx+7i\,c)}+3\,ade^{(5i\,dx+5i\,c)}+3\,ade^{(3i\,dx+3i\,c)}+ade^{(i\,dx+i\,c)}\right)\sqrt{\frac{1}{ad^2}}\log\left(-4\left(\left(ade^{(2i\,dx+2i\,c)}+ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+2\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(353\,e^{(8i\,dx+8i\,c)}+1708\,e^{(6i\,dx+6i\,c)}+2030\,e^{(4i\,dx+4i\,c)}+1260\,e^{(2i\,dx+2i\,c)}+105\right)}{420\left(ade^{(7i\,dx+7i\,c)}+3\,ade^{(5i\,dx+5i\,c)}+3\,ade^{(3i\,dx+3i\,c)}+ade^{(i\,dx+i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/420*(105*sqrt(2)*(a*d*e^(7*I*d*x + 7*I*c) + 3*a*d*e^(5*I*d*x + 5*I*c) +
3*a*d*e^(3*I*d*x + 3*I*c) + a*d*e^(I*d*x + I*c))*sqrt(1/(a*d^2))*log(4*((a*
d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^
2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 105*sqrt(2)*(a*d*e^(7*I*d*x +
```

```
7*I*c) + 3*a*d*e^(5*I*d*x + 5*I*c) + 3*a*d*e^(3*I*d*x + 3*I*c) + a*d*e^(I*d
*x + I*c))*sqrt(1/(a*d^2))*log(-4*((a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x -
I*c)) + 2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(353*e^(8*I*d*x + 8*I*c
) + 1708*e^(6*I*d*x + 6*I*c) + 2030*e^(4*I*d*x + 4*I*c) + 1260*e^(2*I*d*x +
 2*I*c) + 105))/(a*d*e^(7*I*d*x + 7*I*c) + 3*a*d*e^(5*I*d*x + 5*I*c) + 3*a*
d*e^(3*I*d*x + 3*I*c) + a*d*e^(I*d*x + I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5(c+dx)}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**5/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^5/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [B]
time = 0.45, size = 144, normalized size = 0.72

$$-\frac{1}{d\sqrt{a+a\tan(c+dx)\,1i}}-\frac{4\sqrt{a+a\tan(c+dx)\,1i}}{a\,d}+\frac{8\,(a+a\tan(c+dx)\,1i)^{3/2}}{3\,a^2\,d}-\frac{6\,(a+a\tan(c+dx)\,1i)^{5/2}}{5\,a^3\,d}+\frac{2\,(a+a\tan(c+dx)\,1i)^{7/2}}{7\,a^4\,d}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+dx)\,1i}\,1i}{2\sqrt{a}}\right)\,1i}{2\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] (8*(a + a*tan(c + d*x)*1i)^(3/2))/(3*a^2*d) - (4*(a + a*tan(c + d*x)*1i)^(1
/2))/(a*d) - 1/(d*(a + a*tan(c + d*x)*1i)^(1/2)) - (6*(a + a*tan(c + d*x)*1
i)^(5/2))/(5*a^3*d) + (2*(a + a*tan(c + d*x)*1i)^(7/2))/(7*a^4*d) + (2^(1/2
)*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)*1i)/(2*a^(1/2)))*1i)/(2*a^(1/
2)*d)
```

3.109 $\int \frac{\tan^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=172

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d}-\frac{\tan^3(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}+\frac{28i\sqrt{a+ia\tan(c+dx)}}{5ad}-\frac{7i\tan^2(c+dx)\sqrt{}}{5}$$

```
[Out] -1/2*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/2)/a^(1
/2)+28/5*I*(a+I*a*tan(d*x+c))^(1/2)/a/d-7/5*I*(a+I*a*tan(d*x+c))^(1/2)*tan(
d*x+c)^2/a/d-tan(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(1/2)-23/15*I*(a+I*a*tan(d*x
+c))^(3/2)/a^2/d
```

Rubi [A]

time = 0.23, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3639, 3678, 3673, 3608, 3561, 212}

$$-\frac{23i(a+ia\tan(c+dx))^{3/2}}{15a^2d}-\frac{\tan^3(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{7i\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5ad}+\frac{28i\sqrt{a+ia\tan(c+dx)}}{5ad}-\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-I)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]*Sqrt[
a]*d) - Tan[c + d*x]^3/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((28*I)/5)*Sqrt[a
+ I*a*Tan[c + d*x]])/(a*d) - (((7*I)/5)*Tan[c + d*x]^2*Sqrt[a + I*a*Tan[c +
 d*x]])/(a*d) - (((23*I)/15)*(a + I*a*Tan[c + d*x])^(3/2))/(a^2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
```

```
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rule 3678

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Dist[
1/(a*(m + n)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Simp
[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{\tan^3(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{\int \tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}\left(-3a+\frac{7}{2}ia\,\mathrm{t}\right.}{a^2}$$

$$= -\frac{\tan^3(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{7i\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5ad} - \frac{2\int \tan(c}{}$$

$$= -\frac{\tan^3(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{7i\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5ad} - \frac{23i(a+i}{}$$

$$= -\frac{\tan^3(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} + \frac{28i\sqrt{a+ia\tan(c+dx)}}{5ad} - \frac{7i\tan^2(c+dx)\sqrt{a+}}{5ad}$$

$$= -\frac{\tan^3(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} + \frac{28i\sqrt{a+ia\tan(c+dx)}}{5ad} - \frac{7i\tan^2(c+dx)\sqrt{a+}}{5ad}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d} - \frac{\tan^3(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} + \frac{28i\sqrt{a+ia}}{5}$$

Mathematica [A]
time = 1.90, size = 122, normalized size = 0.71

$$\frac{-\frac{60ie^{i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} + i\sec^3(c+dx)(185\cos(c+dx)+59\cos(3(c+dx))+20i\sin(c+dx)+44i\sin(3(c+dx)))}{60d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] (((-60*I)*E^(I*(c + d*x))*ArcSinh[E^(I*(c + d*x))])/Sqrt[1 + E^((2*I)*(c +
d*x))] + I*Sec[c + d*x]^3*(185*Cos[c + d*x] + 59*Cos[3*(c + d*x)] + (20*I)*
Sin[c + d*x] + (44*I)*Sin[3*(c + d*x)]))/(60*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.20, size = 113, normalized size = 0.66

method	result
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{2a(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+2a^2\sqrt{a+ia\tan(dx+c)}+\frac{a^3}{2\sqrt{a+ia\tan(dx+c)}}-\frac{a^{\frac{5}{2}}\sqrt{}}{}\right.}{d\,a^3}$

default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{2a(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+2a^2\sqrt{a+ia\tan(dx+c)}+\frac{a^3}{2\sqrt{a+ia\tan(dx+c)}}-\frac{a^{\frac{5}{2}} \cdots}{\cdots}\right.}{d\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/d/a^3*(1/5*(a+I*a*tan(d*x+c))^(5/2)-2/3*a*(a+I*a*tan(d*x+c))^(3/2)+2*a^
2*(a+I*a*tan(d*x+c))^(1/2)+1/2*a^3/(a+I*a*tan(d*x+c))^(1/2)-1/4*a^(5/2)*2^(
1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.48, size = 138, normalized size = 0.80

$$\frac{i\left(15\sqrt{2}\,a^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)+24\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}a^2-80\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}a^3+240\sqrt{i\,a\tan\left(dx+c\right)+a}\,a^4+\frac{60\,a^5}{\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)}{60\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/60*I*(15*sqrt(2)*a^(9/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) +
a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 24*(I*a*tan(d*x + c)
+ a)^(5/2)*a^2 - 80*(I*a*tan(d*x + c) + a)^(3/2)*a^3 + 240*sqrt(I*a*tan(d*x
 + c) + a)*a^4 + 60*a^5/sqrt(I*a*tan(d*x + c) + a))/(a^5*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(131) = 262$.
time = 0.45, size = 343, normalized size = 1.99

$$-\frac{15\sqrt{2}\left(i\,ade^{(5i\,dx+5i\,c)}+2i\,ade^{(3i\,dx+3i\,c)}+i\,ade^{(i\,dx+i\,c)}\right)\sqrt{\frac{1}{ad^2}}\log\left(4\left(\left(ade^{(2i\,dx+2i\,c)}+ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+15\sqrt{2}\left(-i\,ade^{(5i\,dx+5i\,c)}-2i\,ade^{(3i\,dx+3i\,c)}-i\,ade^{(i\,dx+i\,c)}\right)\sqrt{\frac{1}{ad^2}}\log\left(-4\left(\left(ade^{(2i\,dx+2i\,c)}+ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+2\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-103i\,e^{(6i\,dx+6i\,c)}-205i\,e^{(4i\,dx+4i\,c)}-165i\,e^{(2i\,dx+2i\,c)}-15i\right)}{60\left(ade^{(5i\,dx+5i\,c)}+2\,ade^{(3i\,dx+3i\,c)}+ade^{(i\,dx+i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/60*(15*sqrt(2)*(I*a*d*e^(5*I*d*x + 5*I*c) + 2*I*a*d*e^(3*I*d*x + 3*I*c)
+ I*a*d*e^(I*d*x + I*c))*sqrt(1/(a*d^2))*log(4*((a*d*e^(2*I*d*x + 2*I*c) +
a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)) + a*e^(I*d*x + I*c))
*e^(-I*d*x - I*c)) + 15*sqrt(2)*(-I*a*d*e^(5*I*d*x + 5*I*c) - 2*I*a*d*e^(3*
I*d*x + 3*I*c) - I*a*d*e^(I*d*x + I*c))*sqrt(1/(a*d^2))*log(-4*((a*d*e^(2*I
*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)) - a*
e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1))*(-103*I*e^(6*I*d*x + 6*I*c) - 205*I*e^(4*I*d*x + 4*I*c) - 165*I*e^(2*
I*d*x + 2*I*c) - 15*I))/(a*d*e^(5*I*d*x + 5*I*c) + 2*a*d*e^(3*I*d*x + 3*I*c
) + a*d*e^(I*d*x + I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4{(c+dx)}}{\sqrt{ia(\tan{(c+dx)}-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**4/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^4/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [B]
time = 4.18, size = 129, normalized size = 0.75

$$\frac{1\mathrm{i}}{d\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}+\frac{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,4\mathrm{i}}{a\,d}-\frac{{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}^{3/2}\,4\mathrm{i}}{3\,a^2\,d}+\frac{{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}^{5/2}\,2\mathrm{i}}{5\,a^3\,d}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}{2\,\sqrt{-a}}\right)\,1\mathrm{i}}{2\,\sqrt{-a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] 1i/(d*(a + a*tan(c + d*x)*1i)^(1/2)) + ((a + a*tan(c + d*x)*1i)^(1/2)*4i)/(
a*d) - ((a + a*tan(c + d*x)*1i)^(3/2)*4i)/(3*a^2*d) + ((a + a*tan(c + d*x)*
1i)^(5/2)*2i)/(5*a^3*d) + (2^(1/2)*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1
/2))/(2*(-a)^(1/2)))*1i)/(2*(-a)^(1/2)*d)
```

3.110 $\int \frac{\tan^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=126

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d}-\frac{\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}+\frac{4\sqrt{a+ia\tan(c+dx)}}{ad}-\frac{5(a+ia\tan(c+dx))}{3a^2d}$$

```
[Out] 1/2*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/2)/a^(1/2)
+4*(a+I*a*tan(d*x+c))^(1/2)/a/d-tan(d*x+c)^2/d/(a+I*a*tan(d*x+c))^(1/2)-5/3
*(a+I*a*tan(d*x+c))^(3/2)/a^2/d
```

Rubi [A]
time = 0.13, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3639, 3673, 3608, 3561, 212}

$$-\frac{5(a+ia\tan(c+dx))^{3/2}}{3a^2d}-\frac{\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}+\frac{4\sqrt{a+ia\tan(c+dx)}}{ad}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(Sqrt[2]*Sqrt[a]*d) -
 Tan[c + d*x]^2/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (4*Sqrt[a + I*a*Tan[c + d*
x]])/(a*d) - (5*(a + I*a*Tan[c + d*x])^(3/2))/(3*a^2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && !LtQ[m, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{\int \tan(c+dx)\sqrt{a+ia\tan(c+dx)}\left(-2a+\frac{5}{2}ia\,\mathrm{ta}\right.}{a^2}$$

$$= -\frac{\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{5(a+ia\tan(c+dx))^{3/2}}{3a^2d} - \frac{\int\left(-\frac{5ia}{2}-2a\tan(c+\right.}{}$$

$$= -\frac{\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} + \frac{4\sqrt{a+ia\tan(c+dx)}}{ad} - \frac{5(a+ia\tan(c+dx))^{3/}}{3a^2d}$$

$$= -\frac{\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} + \frac{4\sqrt{a+ia\tan(c+dx)}}{ad} - \frac{5(a+ia\tan(c+dx))^{3/}}{3a^2d}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d} - \frac{\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} + \frac{4\sqrt{a+ia\tan(}}{ad}$$

Mathematica [A]

time = 1.13, size = 129, normalized size = 1.02

$$\frac{3+18e^{2i(c+dx)}+7e^{4i(c+dx)}+3e^{i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{3/2}\sinh^{-1}\left(e^{i(c+dx)}\right)}{3\sqrt{2}\,d\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\,\left(1+e^{2i(c+dx)}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (3 + 18*E^((2*I)*(c + d*x)) + 7*E^((4*I)*(c + d*x)) + 3*E^(I*(c + d*x))*(1
+ E^((2*I)*(c + d*x)))^(3/2)*ArcSinh[E^(I*(c + d*x))])/(3*Sqrt[2]*d*Sqrt[(a
*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))^
2)
```

Maple [A]
time = 0.19, size = 93, normalized size = 0.74

method	result
derivativedivides	$-\frac{2\left(\frac{(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-a\sqrt{a+ia\tan(dx+c)}-\frac{a^2}{2\sqrt{a+ia\tan(dx+c)}}-\frac{a^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+}}{}\right.}{}\right.}{d\,a^2}$
default	$-\frac{2\left(\frac{(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-a\sqrt{a+ia\tan(dx+c)}-\frac{a^2}{2\sqrt{a+ia\tan(dx+c)}}-\frac{a^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+}}{}\right.}{}\right.}{d\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a^2*(1/3*(a+I*a*tan(d*x+c))^(3/2)-a*(a+I*a*tan(d*x+c))^(1/2)-1/2*a^2/(
a+I*a*tan(d*x+c))^(1/2)-1/4*a^(3/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^
(1/2)*2^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.50, size = 120, normalized size = 0.95

$$-\frac{3\sqrt{2}\,a^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+8\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a^2-24\,\sqrt{i\,a\tan(dx+c)+a}\,a^3-\frac{12\,a^4}{\sqrt{i\,a\tan(dx+c)+a}}}{12\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*(3*sqrt(2)*a^(7/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a)
)/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 8*(I*a*tan(d*x + c) + a
)^(3/2)*a^2 - 24*sqrt(I*a*tan(d*x + c) + a)*a^3 - 12*a^4/sqrt(I*a*tan(d*x +
 c) + a))/(a^4*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(102) = 204$.
time = 0.47, size = 289, normalized size = 2.29

$$\frac{3\sqrt{2}\left(ade^{(3i\,dx+3i\,c)}+ade^{(i\,dx+i\,c)}\right)\sqrt{\frac{1}{ad^2}}\log\left(4\left(\left(ade^{(2i\,dx+2i\,c)}+ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-3\sqrt{2}\left(ade^{(3i\,dx+3i\,c)}+ade^{(i\,dx+i\,c)}\right)\sqrt{\frac{1}{ad^2}}\log\left(-4\left(\left(ade^{(2i\,dx+2i\,c)}+ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+2\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(7\,e^{(4i\,dx+4i\,c)}+18\,e^{(2i\,dx+2i\,c)}+3\right)}{12\left(ade^{(3i\,dx+3i\,c)}+ade^{(i\,dx+i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*sqrt(2)*(a*d*e^(3*I*d*x + 3*I*c) + a*d*e^(I*d*x + I*c))*sqrt(1/(a*d
^2))*log(4*((a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
))*sqrt(1/(a*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 3*sqrt(2)*(a*d*
e^(3*I*d*x + 3*I*c) + a*d*e^(I*d*x + I*c))*sqrt(1/(a*d^2))*log(-4*((a*d*e^(
2*I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)) -
 a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*(7*e^(4*I*d*x + 4*I*c) + 18*e^(2*I*d*x + 2*I*c) + 3))/(a*d*e^(3*I*
d*x + 3*I*c) + a*d*e^(I*d*x + I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(c+dx)}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**3/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^3/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [B]
time = 4.17, size = 99, normalized size = 0.79

$$\frac{1}{d\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}+\frac{2\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{a\,d}-\frac{2\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{3/2}}{3\,a^2\,d}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}\,1\mathrm{i}}{2\sqrt{a}}\right)\,1\mathrm{i}}{2\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] 1/(d*(a + a*tan(c + d*x)*1i)^(1/2)) + (2*(a + a*tan(c + d*x)*1i)^(1/2))/(a*
d) - (2*(a + a*tan(c + d*x)*1i)^(3/2))/(3*a^2*d) - (2^(1/2)*atan((2^(1/2)*(
a + a*tan(c + d*x)*1i)^(1/2)*1i)/(2*a^(1/2)))*1i)/(2*a^(1/2)*d)
```

3.111 $\int \frac{\tan^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=98

$$\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d}-\frac{i}{d\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{ad}$$

```
[Out] 1/2*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/2)/a^(1/
2)-I/d/(a+I*a*tan(d*x+c))^(1/2)-2*I*(a+I*a*tan(d*x+c))^(1/2)/a/d
```

Rubi [A]
time = 0.07, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3624, 3560, 3561, 212}

$$-\frac{2i\sqrt{a+ia\tan(c+dx)}}{ad}-\frac{i}{d\sqrt{a+ia\tan(c+dx)}}+\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (I*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]*Sqrt[a]*
d) - I/(d*Sqrt[a + I*a*Tan[c + d*x]]) - ((2*I)*Sqrt[a + I*a*Tan[c + d*x]])/
(a*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2i\sqrt{a+ia\tan(c+dx)}}{ad} - \int \frac{1}{\sqrt{a+ia\tan(c+dx)}}\,dx$$

$$= -\frac{i}{d\sqrt{a+ia\tan(c+dx)}} - \frac{2i\sqrt{a+ia\tan(c+dx)}}{ad} - \frac{\int \sqrt{a+ia\tan(c+d}}{2a}$$

$$= -\frac{i}{d\sqrt{a+ia\tan(c+dx)}} - \frac{2i\sqrt{a+ia\tan(c+dx)}}{ad} + \frac{i\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x\right.}{}$$

$$= \frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d} - \frac{i}{d\sqrt{a+ia\tan(c+dx)}} - \frac{2i\sqrt{a+ia\,\mathrm{t}}}{a}$$

Mathematica [A]
time = 0.85, size = 113, normalized size = 1.15

$$-\frac{i\left(\sqrt{1+e^{2i(c+dx)}}\left(1+5e^{2i(c+dx)}\right)-e^{i(c+dx)}\left(1+e^{2i(c+dx)}\right)\sinh^{-1}\left(e^{i(c+dx)}\right)\right)}{d\left(1+e^{2i(c+dx)}\right)^{3/2}\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-I)*(Sqrt[1 + E^((2*I)*(c + d*x))]*(1 + 5*E^((2*I)*(c + d*x))) - E^(I*(c
+ d*x))*(1 + E^((2*I)*(c + d*x)))*ArcSinh[E^(I*(c + d*x))]))/(d*(1 + E^((2*
I)*(c + d*x)))^(3/2)*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.17, size = 73, normalized size = 0.74

method	result

derivativedivides	$-\frac{2i\left(\sqrt{a+ia\tan(dx+c)}+\frac{a}{2\sqrt{a+ia\tan(dx+c)}}-\frac{\sqrt{a}\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+}}{2\sqrt{a}}\right.}{4}\right.}{da}$
default	$-\frac{2i\left(\sqrt{a+ia\tan(dx+c)}+\frac{a}{2\sqrt{a+ia\tan(dx+c)}}-\frac{\sqrt{a}\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+}}{2\sqrt{a}}\right.}{4}\right.}{da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/d/a*((a+I*a*tan(d*x+c))^(1/2)+1/2*a/(a+I*a*tan(d*x+c))^(1/2)-1/4*a^(1/
2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.84, size = 101, normalized size = 1.03

$$-\frac{i\left(\sqrt{2}\,a^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+8\,\sqrt{i\,a\tan(dx+c)+a}\,a^2+\frac{4\,a^3}{\sqrt{i\,a\tan(dx+c)+a}}\right)}{4\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*I*(sqrt(2)*a^(5/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))
/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 8*sqrt(I*a*tan(d*x + c)
+ a)*a^2 + 4*a^3/sqrt(I*a*tan(d*x + c) + a))/(a^3*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(73) = 146.
time = 0.45, size = 238, normalized size = 2.43

$$\frac{\left(i\sqrt{2}\,ad\sqrt{\frac{1}{ad^2}}\,e^{(i\,dx+i\,c)}\log\left(4\left(\left(ade^{(2i\,dx+2i\,c)}+ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,\sqrt{\frac{1}{ad^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-i\sqrt{2}\,ad\sqrt{\frac{1}{ad^2}}\,e^{(i\,dx+i\,c)}\log\left(-4\left(\left(ade^{(2i\,dx+2i\,c)}+ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,\sqrt{\frac{1}{ad^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-2\sqrt{2}\,\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,\left(5i\,e^{(2i\,dx+2i\,c)}+i\right)\right)e^{(-i\,dx-i\,c)}}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(I*sqrt(2)*a*d*sqrt(1/(a*d^2))*e^(I*d*x + I*c)*log(4*((a*d*e^(2*I*d*x +
 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)) + a*e^(I*d
*x + I*c))*e^(-I*d*x - I*c)) - I*sqrt(2)*a*d*sqrt(1/(a*d^2))*e^(I*d*x + I*c
)*log(-4*((a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
```

```
*sqrt(1/(a*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 2*sqrt(2)*sqrt(a/
(e^(2*I*d*x + 2*I*c) + 1))*(5*I*e^(2*I*d*x + 2*I*c) + I))*e^(-I*d*x - I*c)/
(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(c+dx)}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral(tan(c + d*x)**2/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(tan(d*x + c)^2/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [B]
time = 0.26, size = 83, normalized size = 0.85

$$-\frac{1\mathrm{i}}{d\sqrt{a+a\tan(c+dx)\,1\mathrm{i}}}-\frac{\sqrt{a+a\tan(c+dx)\,1\mathrm{i}}\,2\mathrm{i}}{a\,d}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+dx)\,1\mathrm{i}}}{2\sqrt{-a}}\right)1\mathrm{i}}{2\sqrt{-a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] - 1i/(d*(a + a*tan(c + d*x)*1i)^(1/2)) - ((a + a*tan(c + d*x)*1i)^(1/2)*2i)
/(a*d) - (2^(1/2)*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*(-a)^(1/2
)))*1i)/(2*(-a)^(1/2)*d)
```

3.112 $\int \frac{\tan(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=67

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d}-\frac{1}{d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] -1/2*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/2)/a^(1/2
)-1/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3607, 3561, 212}

$$-\frac{1}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] -(ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(Sqrt[2]*Sqrt[a]*d)
) - 1/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
```

```
0] && LtQ[m, 0]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{1}{d\sqrt{a+ia\tan(c+dx)}} - \frac{i\int \sqrt{a+ia\tan(c+dx)}\,dx}{2a}$$
$$= -\frac{1}{d\sqrt{a+ia\tan(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+ia\tan(c+dx)}\right)}{d}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d} - \frac{1}{d\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A]
time = 0.27, size = 83, normalized size = 1.24

$$\frac{-\sqrt{1+e^{2i(c+dx)}} - e^{i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)}{d\sqrt{1+e^{2i(c+dx)}}\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (-Sqrt[1 + E^((2*I)*(c + d*x))] - E^(I*(c + d*x))*ArcSinh[E^(I*(c + d*x))])
/(d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.20, size = 53, normalized size = 0.79

method	result	size
derivativedivides	$\frac{-\frac{1}{\sqrt{a+ia\tan(dx+c)}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2\sqrt{a}}}{d}$	53
default	$\frac{-\frac{1}{\sqrt{a+ia\tan(dx+c)}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2\sqrt{a}}}{d}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/(a+I*a*tan(d*x+c))^(1/2)-1/2*2^(1/2)/a^(1/2)*arctanh(1/2*(a+I*a*tan
(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.53, size = 83, normalized size = 1.24

$$\frac{\sqrt{2}\,a^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)-\frac{4\,a^{2}}{\sqrt{i\,a\tan\left(dx+c\right)+a}}}{4\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(sqrt(2)*a^(3/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(s
qrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - 4*a^2/sqrt(I*a*tan(d*x + c)
 + a))/(a^2*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(52) = 104.
time = 0.46, size = 235, normalized size = 3.51

$$-\frac{\left(\sqrt{2}\,ad\sqrt{\frac{1}{ad^{2}}}\,e^{\left(i\,dx+i\,c\right)}\log\left(4\left(\left(ade^{\left(2i\,dx+2i\,c\right)}+ad\right)\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}\,\sqrt{\frac{1}{ad^{2}}}+ae^{\left(i\,dx+i\,c\right)}\right)e^{\left(-i\,dx-i\,c\right)}\right)-\sqrt{2}\,ad\sqrt{\frac{1}{ad^{2}}}\,e^{\left(i\,dx+i\,c\right)}\log\left(-4\left(\left(ade^{\left(2i\,dx+2i\,c\right)}+ad\right)\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}\,\sqrt{\frac{1}{ad^{2}}}-ae^{\left(i\,dx+i\,c\right)}\right)e^{\left(-i\,dx-i\,c\right)}\right)+2\,\sqrt{2}\,\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}\left(e^{\left(2i\,dx+2i\,c\right)}+1\right)\right)e^{\left(-i\,dx-i\,c\right)}}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*a*d*sqrt(1/(a*d^2))*e^(I*d*x + I*c)*log(4*((a*d*e^(2*I*d*x +
2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)) + a*e^(I*d*
x + I*c))*e^(-I*d*x - I*c)) - sqrt(2)*a*d*sqrt(1/(a*d^2))*e^(I*d*x + I*c)*l
og(-4*((a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sq
rt(1/(a*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 2*sqrt(2)*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1))*e^(-I*d*x - I*c)/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan\left(c+dx\right)}{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [B]
time = 4.09, size = 54, normalized size = 0.81

$$-\frac{1}{d\,\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{2\,\sqrt{a}}\right)}{2\,\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] - 1/(d*(a + a*tan(c + d*x)*1i)^(1/2)) - (2^(1/2)*atanh((2^(1/2)*(a + a*tan(
c + d*x)*1i)^(1/2))/(2*a^(1/2))))/(2*a^(1/2)*d)
```

3.113 $\int \frac{1}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=71

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d}+\frac{i}{d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] -1/2*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/2)/a^(1
/2)+I/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3560, 3561, 212}

$$\frac{i}{d\sqrt{a+ia\tan(c+dx)}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-I)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]*Sqrt[
a]*d) + I/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{i}{d\sqrt{a+ia\tan(c+dx)}} + \frac{\int \sqrt{a+ia\tan(c+dx)}\,dx}{2a}$$
$$= \frac{i}{d\sqrt{a+ia\tan(c+dx)}} - \frac{i\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+ia\tan(c+dx)}\right)}{d}$$
$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d} + \frac{i}{d\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A]
time = 0.24, size = 84, normalized size = 1.18

$$\frac{i\left(\sqrt{1+e^{2i(c+dx)}} - e^{i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)\right)}{d\sqrt{1+e^{2i(c+dx)}}\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] (I*(Sqrt[1 + E^((2*I)*(c + d*x))] - E^(I*(c + d*x))*ArcSinh[E^(I*(c + d*x))
]))/(d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.16, size = 59, normalized size = 0.83

method	result	size
derivativedivides	$\frac{2ia\left(\frac{1}{2a\sqrt{a+ia\tan(dx+c)}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{4a^{\frac{3}{2}}}\right)}{d}$	59
default	$\frac{2ia\left(\frac{1}{2a\sqrt{a+ia\tan(dx+c)}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{4a^{\frac{3}{2}}}\right)}{d}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*I/d*a*(1/2/a/(a+I*a*tan(d*x+c))^(1/2)-1/4/a^(3/2)*2^(1/2)*arctanh(1/2*(a+
I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.50, size = 81, normalized size = 1.14

$$\frac{i\left(\sqrt{2}\sqrt{a}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)+\frac{4\,a}{\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*I*(sqrt(2)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/
(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*a/sqrt(I*a*tan(d*x + c)
 + a))/(a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(52) = 104.
time = 0.44, size = 238, normalized size = 3.35

$$\frac{\left(-i\sqrt{2}\,ad\sqrt{\frac{1}{ad^2}}\,e^{(i\,dx+i\,c)}\log\left(4\left(\left(ade^{(2i\,dx+2i\,c)}+ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+i\sqrt{2}\,ad\sqrt{\frac{1}{ad^2}}\,e^{(i\,dx+i\,c)}\log\left(-4\left(\left(ade^{(2i\,dx+2i\,c)}+ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-2\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-i\,e^{(2i\,dx+2i\,c)}-i\right)\right)e^{(-i\,dx-i\,c)}}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(-I*sqrt(2)*a*d*sqrt(1/(a*d^2))*e^(I*d*x + I*c)*log(4*((a*d*e^(2*I*d*x
+ 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)) + a*e^(I*
d*x + I*c))*e^(-I*d*x - I*c)) + I*sqrt(2)*a*d*sqrt(1/(a*d^2))*e^(I*d*x + I*
c)*log(-4*((a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
)*sqrt(1/(a*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 2*sqrt(2)*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1))*(-I*e^(2*I*d*x + 2*I*c) - I))*e^(-I*d*x - I*c)/
(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ia\tan\left(c+dx\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(I*a*tan(c + d*x) + a), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [B]
time = 0.23, size = 60, normalized size = 0.85

$$\frac{1\mathrm{i}}{d\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}{2\,\sqrt{-a}}\right)\,1\mathrm{i}}{2\,\sqrt{-a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] 1i/(d*(a + a*tan(c + d*x)*1i)^(1/2)) + (2^(1/2)*atan((2^(1/2)*(a + a*tan(c
+ d*x)*1i)^(1/2))/(2*(-a)^(1/2)))*1i)/(2*(-a)^(1/2)*d)
```

3.114 $\int \frac{\cot(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=99

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d}+\frac{1}{d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] -2*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)+1/2*arctanh(1/2*(a+I
*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/2)/a^(1/2)+1/d/(a+I*a*tan(d*x+
c))^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {3640, 3681, 3561, 212, 3680, 65, 214}

$$\frac{1}{d\sqrt{a+ia\tan(c+dx)}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) + ArcTanh[Sqrt
[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(Sqrt[2]*Sqrt[a]*d) + 1/(d*Sqrt[a
 + I*a*Tan[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3681

```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(
A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{1}{d\sqrt{a+ia\tan(c+dx)}} + \frac{\int \cot(c+dx)\left(a-\frac{1}{2}ia\tan(c+dx)\right)\sqrt{a+ia\tan(c$$

$$= \frac{1}{d\sqrt{a+ia\tan(c+dx)}} + \frac{\int \cot(c+dx)(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+$$

$$= \frac{1}{d\sqrt{a+ia\tan(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+iax}}\,dx, x, \tan(c+dx)\right)}{d} + \frac{\text{Subst}(}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d} + \frac{1}{d\sqrt{a+ia\tan(c+dx)}} - \frac{(2i)\text{Subst}\left(\int \frac{}{i}\right.}{}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d} + \frac{}{d}$$

Mathematica [A]
time = 1.14, size = 133, normalized size = 1.34

$$\frac{\sqrt{1+e^{2i(c+dx)}} + e^{i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right) - 2\sqrt{2}\,e^{i(c+dx)}\tanh^{-1}\left(\frac{\sqrt{2}\,e^{i(c+dx)}}{\sqrt{1+e^{2i(c+dx)}}}\right)}{d\sqrt{1+e^{2i(c+dx)}}\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] (Sqrt[1 + E^((2*I)*(c + d*x))] + E^(I*(c + d*x))*ArcSinh[E^(I*(c + d*x))] -
 2*Sqrt[2]*E^(I*(c + d*x))*ArcTanh[(Sqrt[2]*E^(I*(c + d*x)))/Sqrt[1 + E^((2
*I)*(c + d*x))]])/(d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[a + I*a*Tan[c + d*x
]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 667 vs. 2(80) = 160.
time = 5.85, size = 668, normalized size = 6.75

method	result
default	$\frac{\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(-i\sqrt{2}\,\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{(-i\cos(dx+c)+\sin(dx+c)+i)\sqrt{2}}{2\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}\right)+\sqrt{2}\,\cos(dx-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(-I*2^(1/2)*sin(d*x+c)
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+
I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+2^(1/2)*cos(d*x
+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+
c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-2*I*cos(d*x+
c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))+2*I*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)+2^(1/2)*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I
)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-4*I*cos(d*x+c)*s
in(d*x+c)-2*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(1/2))-2*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*s
in(d*x+c)-2*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))+4*cos(d*x+c)^2-2*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)))/a
```

Maxima [A]
time = 0.50, size = 122, normalized size = 1.23

$$-\frac{\frac{\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)}{\sqrt{a}}-\frac{4\log\left(\frac{\sqrt{i\,a\tan\left(dx+c\right)+a}-\sqrt{a}}{\sqrt{i\,a\tan\left(dx+c\right)+a}+\sqrt{a}}\right)}{\sqrt{a}}-\frac{4}{\sqrt{i\,a\tan\left(dx+c\right)+a}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sqrt(2)*
sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/sqrt(a) - 4*log((sqrt(I*a*tan(d*x +
c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x + c) + a) + sqrt(a)))/sqrt(a) - 4/sqrt
(I*a*tan(d*x + c) + a))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 459 vs. 2(77) = 154.
time = 0.45, size = 459, normalized size = 4.64

$$\frac{\left(\sqrt{2}\,ad\sqrt{\frac{1}{ad^2}}\,e^{(i\,dx+i\,c)}\log\left(4\left(\left(ade^{(2i\,dx+2i\,c)}+ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-\sqrt{2}\,ad\sqrt{\frac{1}{ad^2}}\,e^{(i\,dx+i\,c)}\log\left(-4\left(\left(ade^{(2i\,dx+2i\,c)}+ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-2\,ad\sqrt{\frac{1}{ad^2}}\,e^{(i\,dx+i\,c)}\log\left(16\left(3\,a^2e^{(2i\,dx+2i\,c)}+2\sqrt{2}\left(a^2de^{(3i\,dx+3i\,c)}+a^2de^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}+a^2\right)e^{(-2i\,dx-2i\,c)}\right)+2\,ad\sqrt{\frac{1}{ad^2}}\,e^{(i\,dx+i\,c)}\log\left(16\left(3\,a^2e^{(2i\,dx+2i\,c)}-2\sqrt{2}\left(a^2de^{(3i\,dx+3i\,c)}+a^2de^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}+a^2\right)e^{(-2i\,dx-2i\,c)}\right)+2\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)\right)e^{(-i\,dx-i\,c)}}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*a*d*sqrt(1/(a*d^2))*e^(I*d*x + I*c)*log(4*((a*d*e^(2*I*d*x + 2
*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)) + a*e^(I*d*x
 + I*c))*e^(-I*d*x - I*c)) - sqrt(2)*a*d*sqrt(1/(a*d^2))*e^(I*d*x + I*c)*lo
g(-4*((a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqr
t(1/(a*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 2*a*d*sqrt(1/(a*d^2))
*e^(I*d*x + I*c)*log(16*(3*a^2*e^(2*I*d*x + 2*I*c) + 2*sqrt(2)*(a^2*d*e^(3*
I*d*x + 3*I*c) + a^2*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*s
qrt(1/(a*d^2)) + a^2)*e^(-2*I*d*x - 2*I*c)) + 2*a*d*sqrt(1/(a*d^2))*e^(I*d*
x + I*c)*log(16*(3*a^2*e^(2*I*d*x + 2*I*c) - 2*sqrt(2)*(a^2*d*e^(3*I*d*x +
3*I*c) + a^2*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a
*d^2)) + a^2)*e^(-2*I*d*x - 2*I*c)) + 2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c)
 + 1))*(e^(2*I*d*x + 2*I*c) + 1))*e^(-I*d*x - I*c)/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(c+dx)}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [B]
time = 0.19, size = 80, normalized size = 0.81

$$\frac{1}{d\sqrt{a+a\tan(c+dx)\,1i}}-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+a\tan(c+dx)\,1i}}{\sqrt{a}}\right)}{\sqrt{a}\,d}+\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+dx)\,1i}}{2\sqrt{a}}\right)}{2\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] 1/(d*(a + a*tan(c + d*x)*1i)^(1/2)) - (2*atanh((a + a*tan(c + d*x)*1i)^(1/2
)/a^(1/2)))/(a^(1/2)*d) + (2^(1/2)*atanh((2^(1/2)*(a + a*tan(c + d*x)*1i)^(
1/2))/(2*a^(1/2))))/(2*a^(1/2)*d)
```

3.115 $\int \frac{\cot^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=141

$$\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}+\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{\sqrt{2}\,\sqrt{a}\,d}+\frac{\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{2\cot(c+$$

```
[Out] I*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)+1/2*I*arctanh(1/2*(a+
I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/2)/a^(1/2)+cot(d*x+c)/d/(a+I*
a*tan(d*x+c))^(1/2)-2*cot(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/a/d
```

Rubi [A]
time = 0.28, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3640, 3679, 3681, 3561, 212, 3680, 65, 214}

$$\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}+\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{\sqrt{2}\,\sqrt{a}\,d}-\frac{2\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{ad}+\frac{\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (I*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) + (I*ArcTanh[Sq
rt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]*Sqrt[a]*d) + Cot[c +
d*x]/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (2*Cot[c + d*x]*Sqrt[a + I*a*Tan[c +
d*x]])/(a*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3681

```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(
A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
```

```
*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} + \frac{\int \cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}\left(2a-\frac{3}{2}ia\tan\right.}{a^2}$$

$$= \frac{\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{2\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{ad} + \frac{\int \cot(c+d}{}$$

$$= \frac{\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{2\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{ad} - \frac{i\int \cot(c+}{}$$

$$= \frac{\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{2\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{ad} - \frac{i\text{Subst}\left(\int \frac{}{x}\right.}{}$$

$$= \frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d} + \frac{\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{2\cot(c+dx}{}$$

$$= \frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d} + \frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d} + \frac{}{d}$$

Mathematica [A]
time = 2.33, size = 153, normalized size = 1.09

$$\frac{i\sec(c+dx)\left(\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right)+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{\sqrt{2}\,e^{i(c+dx)}}{\sqrt{1+e^{2i(c+dx)}}}\right)+i\csc(c+dx)(1+\cos(2(c+dx))+2i\sin(2(c+dx)))\right)}{2d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((I/2)*Sec[c + d*x]*(Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))]
 + Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(Sqrt[2]*E^(I*(c + d*x)))/
Sqrt[1 + E^((2*I)*(c + d*x))]] + I*Csc[c + d*x]*(1 + Cos[2*(c + d*x)] + (2*
I)*Sin[2*(c + d*x)])))/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1367 vs. 2(116) = 232.
time = 1.08, size = 1368, normalized size = 9.70

method	result	size
default	Expression too large to display	1368

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(-I*cos(d*x+c)^2*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2))+I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2))+I*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a
rctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*2^(1/2))-cos(d*x+c)^2*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+I*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-4*I*cos(d*x+c)^2
+cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))-cos(d*x+c)^2*sin(d*x+
c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2))-I*cos(d*x+c)^3*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arc
tan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*2^(1/2))+I*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/
2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*2^(1/2))+4*sin(d*x+c)*cos(d*x+c)^3+cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c
)+1)/sin(d*x+c))-I*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*2^(1/2))+I*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*l
n(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c)
)+sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-I*co
s(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1
/2))-I*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))-I*co
s(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2))-cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))+(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2))*sin(d*x+c)+4*I*cos(d*x+c)^4-8*sin(d*x+c)*cos(d*x+c)-(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(
d*x+c)+1)/sin(d*x+c)))/(cos(d*x+c)^2-1)/a
```

<u>Maxima</u> [A]

time = 0.53, size = 160, normalized size = 1.13

$$\frac{ia\left(\frac{4(-2ia\tan(dx+c)-a)}{(ia\tan(dx+c)+a)^{\frac{3}{2}}a-\sqrt{ia\tan(dx+c)+a}\,a^2}-\frac{\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)}{a^{\frac{3}{2}}}-\frac{2\log\left(\frac{\sqrt{ia\tan(dx+c)+a}-\sqrt{a}}{\sqrt{ia\tan(dx+c)+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*I*a*(4*(-2*I*a*tan(d*x + c) - a)/((I*a*tan(d*x + c) + a)^(3/2)*a - sqrt
(I*a*tan(d*x + c) + a)*a^2) - sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(
d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/a^(3/2) - 2*
log((sqrt(I*a*tan(d*x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x + c) + a) + sq
rt(a)))/a^(3/2))/d
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 546 vs. 2(110) = 220.
time = 0.47, size = 546, normalized size = 3.87

$$\frac{\sqrt{2}\left(iade^{(3idx+3ic)}-iade^{(idx+ic)}\right)\sqrt{\frac{1}{ad^2}}\log\left(4\left(\left(ade^{(2idx+2ic)}+ad\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{1}{ad^2}}+ae^{(idx+ic)}\right)e^{(-idx-ic)}\right)+\sqrt{2}\left(-iade^{(3idx+3ic)}+iade^{(idx+ic)}\right)\sqrt{\frac{1}{ad^2}}\log\left(-4\left(\left(ade^{(2idx+2ic)}+ad\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{1}{ad^2}}-ae^{(idx+ic)}\right)e^{(-idx-ic)}\right)+\left(iade^{(3idx+3ic)}-iade^{(idx+ic)}\right)\sqrt{\frac{1}{ad^2}}\log\left(16\left(3a^2e^{(2idx+2ic)}+2\sqrt{2}\left(a^2de^{(3idx+3ic)}+a^2de^{(idx+ic)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{1}{ad^2}}+a^2\right)e^{(-2idx-2ic)}\right)+\left(-iade^{(3idx+3ic)}+iade^{(idx+ic)}\right)\sqrt{\frac{1}{ad^2}}\log\left(16\left(3a^2e^{(2idx+2ic)}-2\sqrt{2}\left(a^2de^{(3idx+3ic)}+a^2de^{(idx+ic)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{1}{ad^2}}+a^2\right)e^{(-2idx-2ic)}\right)-2\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\left(3ie^{(4idx+4ic)}+2ie^{(2idx+2ic)}-i\right)}{4\left(ade^{(3idx+3ic)}-ade^{(idx+ic)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*(I*a*d*e^(3*I*d*x + 3*I*c) - I*a*d*e^(I*d*x + I*c))*sqrt(1/(a*
d^2))*log(4*((a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
1))*sqrt(1/(a*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*(-I*a*
d*e^(3*I*d*x + 3*I*c) + I*a*d*e^(I*d*x + I*c))*sqrt(1/(a*d^2))*log(-4*((a*d
*e^(2*I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2
)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + (I*a*d*e^(3*I*d*x + 3*I*c) - I*
a*d*e^(I*d*x + I*c))*sqrt(1/(a*d^2))*log(16*(3*a^2*e^(2*I*d*x + 2*I*c) + 2*
sqrt(2)*(a^2*d*e^(3*I*d*x + 3*I*c) + a^2*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)) + a^2)*e^(-2*I*d*x - 2*I*c)) + (-I*a*d*e
^(3*I*d*x + 3*I*c) + I*a*d*e^(I*d*x + I*c))*sqrt(1/(a*d^2))*log(16*(3*a^2*e
^(2*I*d*x + 2*I*c) - 2*sqrt(2)*(a^2*d*e^(3*I*d*x + 3*I*c) + a^2*d*e^(I*d*x
+ I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)) + a^2)*e^(-2*I*d*
x - 2*I*c)) - 2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(3*I*e^(4*I*d*x +
 4*I*c) + 2*I*e^(2*I*d*x + 2*I*c) - I))/(a*d*e^(3*I*d*x + 3*I*c) - a*d*e^(I
*d*x + I*c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot^2(c+dx)}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)**2/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)^2/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [B]
time = 4.09, size = 134, normalized size = 0.95

$$\frac{\frac{(a+a\tan(c+d\,x)\,1\mathrm{i})\,2\mathrm{i}}{d}-\frac{a\,1\mathrm{i}}{d}}{a\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}-(a+a\tan(c+d\,x)\,1\mathrm{i})^{3/2}}-\frac{\mathrm{atan}\left(\frac{\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{\sqrt{-a}}\right)\,1\mathrm{i}}{\sqrt{-a}\,d}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{2\,\sqrt{-a}}\right)\,1\mathrm{i}}{2\,\sqrt{-a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] (((a + a*tan(c + d*x)*1i)*2i)/d - (a*1i)/d)/(a*(a + a*tan(c + d*x)*1i)^(1/2
) - (a + a*tan(c + d*x)*1i)^(3/2)) - (atan((a + a*tan(c + d*x)*1i)^(1/2)/(-
a)^(1/2))*1i)/((-a)^(1/2)*d) - (2^(1/2)*atan((2^(1/2)*(a + a*tan(c + d*x)*1
i)^(1/2))/(2*(-a)^(1/2)))*1i)/(2*(-a)^(1/2)*d)
```

3.116 $\int \frac{\cot^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=180

$$\frac{11\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{4\sqrt{a}\,d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d}+\frac{\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}+\frac{7i\cot(c$$

```
[Out] 11/4*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)-1/2*arctanh(1/2*(a
+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/2)/a^(1/2)+cot(d*x+c)^2/d/(a
+I*a*tan(d*x+c))^(1/2)+7/4*I*cot(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/a/d-3/2*co
t(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)/a/d
```

Rubi [A]

time = 0.38, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3640, 3679, 3681, 3561, 212, 3680, 65, 214}

$$\frac{11\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{4\sqrt{a}\,d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d}-\frac{3\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2ad}+\frac{\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}+\frac{7i\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (11*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/(4*Sqrt[a]*d) - ArcTanh[Sq
rt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(Sqrt[2]*Sqrt[a]*d) + Cot[c + d
*x]^2/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((7*I)/4)*Cot[c + d*x]*Sqrt[a + I*a
*Tan[c + d*x]])/(a*d) - (3*Cot[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/(2*a*
d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3681

```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(
A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
```

```
*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} + \frac{\int \cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,\left(3a-\frac{5}{2}ia\tan\right.}{a^2}$$

$$= \frac{\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{3\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2ad} + \frac{\int \cot^2(c+}{}$$

$$= \frac{\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} + \frac{7i\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{4ad} - \frac{3\cot^2(c+}{}$$

$$= \frac{\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} + \frac{7i\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{4ad} - \frac{3\cot^2(c+}{}$$

$$= \frac{\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} + \frac{7i\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{4ad} - \frac{3\cot^2(c+}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d} + \frac{\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} + \frac{7i\cot(c+d}{}$$

$$= \frac{11\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{4\sqrt{a}\,d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d} + $$

Mathematica [A]
time = 2.77, size = 170, normalized size = 0.94

$$\frac{-8e^{i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)+22\sqrt{2}\,e^{i(c+dx)}\tanh^{-1}\left(\frac{\sqrt{2}\,e^{i(c+dx)}}{\sqrt{1+e^{2i(c+dx)}}}\right)+\sqrt{1+e^{2i(c+dx)}}\csc^2(c+dx)(-9+5\cos(2(c+dx))+i\sin(2(c+dx)))}{8d\sqrt{1+e^{2i(c+dx)}}\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (-8*E^(I*(c + d*x))*ArcSinh[E^(I*(c + d*x))] + 22*Sqrt[2]*E^(I*(c + d*x))*A
rcTanh[(Sqrt[2]*E^(I*(c + d*x)))/Sqrt[1 + E^((2*I)*(c + d*x))]] + Sqrt[1 +
E^((2*I)*(c + d*x))]*Csc[c + d*x]^2*(-9 + 5*Cos[2*(c + d*x)] + I*Sin[2*(c +
 d*x)]))/(8*d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1369 vs. 2(147) = 294.
time = 1.12, size = 1370, normalized size = 7.61

method	result	size
default	Expression too large to display	1370

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(-28*I*sin(d*x+c)*cos
(d*x+c)+11*I*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))+11*I*cos(
d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))-4*cos(d*x+c)^3*2^(1/2)*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/s
in(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-11*I*cos(d*x+c)*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))+11*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)-11*I*cos(d*x+
c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2))+11*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+11*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+
c)+1)/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-4*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+
c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+16*I*cos(d*x+c)^3*sin(d*x+
c)-4*I*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(
-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*2^(1/2))-16*cos(d*x+c)^4+11*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+4*2^(1/2)*cos(d*x+c)*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/s
in(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-11*I*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-
cos(d*x+c)+1)/sin(d*x+c))+4*I*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-11*cos(d*x+c)*(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-11*ln(
((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+4*2^(1/2)*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+24*cos(d*x+c)^2-11*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))/(cos
(d*x+c)^2-1)/a
```

Maxima [A]

time = 0.49, size = 203, normalized size = 1.13

$$-\frac{a^2\left(\frac{2\left(7\,(i\,a\tan(dx+c)+a)^2-13\,(i\,a\tan(dx+c)+a)a+4\,a^2\right)}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}a^2-2\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a^3+\sqrt{i\,a\tan(dx+c)+a}\,a^4}-\frac{2\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)}{a^{\frac{5}{2}}}+\frac{11\log\left(\frac{\sqrt{i\,a\tan(dx+c)+a}-\sqrt{a}}{\sqrt{i\,a\tan(dx+c)+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*a^2*(2*(7*(I*a*tan(d*x + c) + a)^2 - 13*(I*a*tan(d*x + c) + a)*a + 4*a
^2)/((I*a*tan(d*x + c) + a)^(5/2)*a^2 - 2*(I*a*tan(d*x + c) + a)^(3/2)*a^3
+ sqrt(I*a*tan(d*x + c) + a)*a^4) - 2*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(
I*a*tan(d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/a^(5
/2) + 11*log((sqrt(I*a*tan(d*x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x + c)
+ a) + sqrt(a)))/a^(5/2))/d
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 617 vs. 2(141) = 282.
time = 0.46, size = 617, normalized size = 3.43

$$-\frac{4\sqrt{2}\left(ade^{(5idx+5ic)}-2ade^{(3idx+3ic)}+ade^{(idx+ic)}\right)\sqrt{\frac{1}{ad^2}}\log\left(4\left(\left(ade^{(2idx+2ic)}+ad\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{1}{ad^2}}+ae^{(idx+ic)}\right)e^{(-idx-ic)}\right)-4\sqrt{2}\left(ade^{(5idx+5ic)}-2ade^{(3idx+3ic)}+ade^{(idx+ic)}\right)\sqrt{\frac{1}{ad^2}}\log\left(-4\left(\left(ade^{(2idx+2ic)}+ad\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{1}{ad^2}}-ae^{(idx+ic)}\right)e^{(-idx-ic)}\right)-11\left(ade^{(5idx+5ic)}-2ade^{(3idx+3ic)}+ade^{(idx+ic)}\right)\sqrt{\frac{1}{ad^2}}\log\left(16\left(3a^2e^{(2idx+2ic)}+2\sqrt{2}\left(a^2de^{(3idx+3ic)}+a^2de^{(idx+ic)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{1}{ad^2}}+a^2\right)e^{(-2idx-2ic)}\right)+11\left(ade^{(5idx+5ic)}-2ade^{(3idx+3ic)}+ade^{(idx+ic)}\right)\sqrt{\frac{1}{ad^2}}\log\left(16\left(3a^2e^{(2idx+2ic)}-2\sqrt{2}\left(a^2de^{(3idx+3ic)}+a^2de^{(idx+ic)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{1}{ad^2}}+a^2\right)e^{(-2idx-2ic)}\right)+4\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\left(3e^{(6idx+6ic)}-6e^{(4idx+4ic)}-7e^{(2idx+2ic)}+2\right)}{16\left(ade^{(5idx+5ic)}-2ade^{(3idx+3ic)}+ade^{(idx+ic)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/16*(4*sqrt(2)*(a*d*e^(5*I*d*x + 5*I*c) - 2*a*d*e^(3*I*d*x + 3*I*c) + a*d
*e^(I*d*x + I*c))*sqrt(1/(a*d^2))*log(4*((a*d*e^(2*I*d*x + 2*I*c) + a*d)*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)) + a*e^(I*d*x + I*c))*e^(-I*
d*x - I*c)) - 4*sqrt(2)*(a*d*e^(5*I*d*x + 5*I*c) - 2*a*d*e^(3*I*d*x + 3*I*c
) + a*d*e^(I*d*x + I*c))*sqrt(1/(a*d^2))*log(-4*((a*d*e^(2*I*d*x + 2*I*c) +
 a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)) - a*e^(I*d*x + I*c)
)*e^(-I*d*x - I*c)) - 11*(a*d*e^(5*I*d*x + 5*I*c) - 2*a*d*e^(3*I*d*x + 3*I*
c) + a*d*e^(I*d*x + I*c))*sqrt(1/(a*d^2))*log(16*(3*a^2*e^(2*I*d*x + 2*I*c)
 + 2*sqrt(2)*(a^2*d*e^(3*I*d*x + 3*I*c) + a^2*d*e^(I*d*x + I*c))*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)) + a^2)*e^(-2*I*d*x - 2*I*c)) + 11*(
a*d*e^(5*I*d*x + 5*I*c) - 2*a*d*e^(3*I*d*x + 3*I*c) + a*d*e^(I*d*x + I*c))*
sqrt(1/(a*d^2))*log(16*(3*a^2*e^(2*I*d*x + 2*I*c) - 2*sqrt(2)*(a^2*d*e^(3*I
*d*x + 3*I*c) + a^2*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sq
rt(1/(a*d^2)) + a^2)*e^(-2*I*d*x - 2*I*c)) + 4*sqrt(2)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*(3*e^(6*I*d*x + 6*I*c) - 6*e^(4*I*d*x + 4*I*c) - 7*e^(2*I*d*x
 + 2*I*c) + 2))/(a*d*e^(5*I*d*x + 5*I*c) - 2*a*d*e^(3*I*d*x + 3*I*c) + a*d*
e^(I*d*x + I*c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot^3(c+dx)}{\sqrt{ia\,(\tan(c+dx)-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)**3/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)^3/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [B]
time = 4.12, size = 155, normalized size = 0.86

$$-\frac{\frac{7\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2}{4}-\frac{13\,a\,\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}{4}+a^2}{d\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}-2\,a\,d\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}+a^2\,d\,\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}}+\frac{11\,\mathrm{atanh}\left(\frac{\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}}{\sqrt{a}}\right)}{4\,\sqrt{a}\,d}-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}}{2\,\sqrt{a}}\right)}{2\,\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] (11*atanh((a + a*tan(c + d*x)*1i)^(1/2)/a^(1/2)))/(4*a^(1/2)*d) - ((7*(a +
a*tan(c + d*x)*1i)^2)/4 - (13*a*(a + a*tan(c + d*x)*1i))/4 + a^2)/(d*(a + a
*tan(c + d*x)*1i)^(5/2) - 2*a*d*(a + a*tan(c + d*x)*1i)^(3/2) + a^2*d*(a +
a*tan(c + d*x)*1i)^(1/2)) - (2^(1/2)*atanh((2^(1/2)*(a + a*tan(c + d*x)*1i)
^(1/2))/(2*a^(1/2))))/(2*a^(1/2)*d)
```

3.117 $\int \frac{\tan^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=205

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\tan^4(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}+\frac{19i\tan^3(c+dx)}{6ad\sqrt{a+ia\tan(c+dx)}}+\frac{78\sqrt{a+ia\tan}}{5a^2d}$$

```
[Out] -1/4*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(3/2)/d*2^(1/2
)+78/5*(a+I*a*tan(d*x+c))^(1/2)/a^2/d-39/10*(a+I*a*tan(d*x+c))^(1/2)*tan(d*
x+c)^2/a^2/d+19/6*I*tan(d*x+c)^3/a/d/(a+I*a*tan(d*x+c))^(1/2)-1/3*tan(d*x+c
)^4/d/(a+I*a*tan(d*x+c))^(3/2)-151/30*(a+I*a*tan(d*x+c))^(3/2)/a^3/d
```

Rubi [A]
time = 0.32, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3639, 3676, 3678, 3673, 3608, 3561, 212}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{151(a+ia\tan(c+dx))^{3/2}}{30a^3d}-\frac{39\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{10a^2d}+\frac{78\sqrt{a+ia\tan(c+dx)}}{5a^2d}-\frac{\tan^4(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}+\frac{19i\tan^3(c+dx)}{6ad\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] -1/2*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(Sqrt[2]*a^(3/2)
*d) - Tan[c + d*x]^4/(3*d*(a + I*a*Tan[c + d*x])^(3/2)) + (((19*I)/6)*Tan[c
 + d*x]^3)/(a*d*Sqrt[a + I*a*Tan[c + d*x]]) + (78*Sqrt[a + I*a*Tan[c + d*x]
])/(5*a^2*d) - (39*Tan[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/(10*a^2*d) -
(151*(a + I*a*Tan[c + d*x])^(3/2))/(30*a^3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
```

```
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rule 3678

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Dist[
1/(a*(m + n)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Simp
[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{\tan^4(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} - \frac{\int \frac{\tan^3(c+dx)\left(-4a+\frac{11}{2}ia\tan(c+dx)\right)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{3a^2}$$

$$= -\frac{\tan^4(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{19i\tan^3(c+dx)}{6ad\sqrt{a+ia\tan(c+dx)}} + \frac{\int \tan^2(c+dx)\,\sqrt{}}{}$$

$$= -\frac{\tan^4(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{19i\tan^3(c+dx)}{6ad\sqrt{a+ia\tan(c+dx)}} - \frac{39\tan^2(c+dx)}{}$$

$$= -\frac{\tan^4(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{19i\tan^3(c+dx)}{6ad\sqrt{a+ia\tan(c+dx)}} - \frac{39\tan^2(c+dx)}{}$$

$$= -\frac{\tan^4(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{19i\tan^3(c+dx)}{6ad\sqrt{a+ia\tan(c+dx)}} + \frac{78\sqrt{a+ia\tan(}}{5a^2d}$$

$$= -\frac{\tan^4(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{19i\tan^3(c+dx)}{6ad\sqrt{a+ia\tan(c+dx)}} + \frac{78\sqrt{a+ia\tan(}}{5a^2d}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{\tan^4(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{19i}{6ad\sqrt{a}}$$

Mathematica [A]
time = 1.87, size = 149, normalized size = 0.73

$$\frac{e^{-2i(c+dx)}\left(-5+115e^{2i(c+dx)}+855e^{4i(c+dx)}+1105e^{6i(c+dx)}+466e^{8i(c+dx)}-15e^{3i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}\sinh^{-1}\left(e^{i(c+dx)}\right)\right)}{30ad\left(1+e^{2i(c+dx)}\right)^3\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (-5 + 115*E^((2*I)*(c + d*x)) + 855*E^((4*I)*(c + d*x)) + 1105*E^((6*I)*(c
+ d*x)) + 466*E^((8*I)*(c + d*x)) - 15*E^((3*I)*(c + d*x))*(1 + E^((2*I)*(c
 + d*x)))^(5/2)*ArcSinh[E^(I*(c + d*x))])/(30*a*d*E^((2*I)*(c + d*x))*(1 +
E^((2*I)*(c + d*x)))^3*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.18, size = 131, normalized size = 0.64

method	result

derivativedivides	$\frac{\frac{2(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-2a(a+ia\tan(dx+c))^{\frac{3}{2}}+8a^{2}\sqrt{a+ia\tan\left(dx+c\right)}+\frac{9a^{3}}{2\sqrt{a+ia\tan\left(dx+c\right)}}-\frac{}{3(a+ia}}{a^{4}d}$
default	$\frac{\frac{2(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-2a(a+ia\tan(dx+c))^{\frac{3}{2}}+8a^{2}\sqrt{a+ia\tan\left(dx+c\right)}+\frac{9a^{3}}{2\sqrt{a+ia\tan\left(dx+c\right)}}-\frac{}{3(a+ia}}{a^{4}d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^4*(1/5*(a+I*a*tan(d*x+c))^(5/2)-a*(a+I*a*tan(d*x+c))^(3/2)+4*a^2*(a+I
*a*tan(d*x+c))^(1/2)+9/4*a^3/(a+I*a*tan(d*x+c))^(1/2)-1/6*a^4/(a+I*a*tan(d*
x+c))^(3/2)-1/8*a^(5/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2
)/a^(1/2)))
```

Maxima [A]
time = 0.51, size = 157, normalized size = 0.77

$$\frac{15\sqrt{2}\,a^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)+48\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}a^{2}-240\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}a^{3}+960\sqrt{i\,a\tan\left(dx+c\right)+a}\;a^{4}+\frac{20\left(27\left(i\,a\tan(dx+c)+a\right)a^{5}-2\,a^{6}\right)}{\left(i\,a\tan(dx+c)+a\right)^{\frac{3}{2}}}}{120\,a^{6}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/120*(15*sqrt(2)*a^(9/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a
))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 48*(I*a*tan(d*x + c) +
 a)^(5/2)*a^2 - 240*(I*a*tan(d*x + c) + a)^(3/2)*a^3 + 960*sqrt(I*a*tan(d*x
 + c) + a)*a^4 + 20*(27*(I*a*tan(d*x + c) + a)*a^5 - 2*a^6)/(I*a*tan(d*x +
c) + a)^(3/2))/(a^6*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 388 vs. 2(160) = 320.
time = 0.48, size = 388, normalized size = 1.89

$$-\frac{15\sqrt{\frac{1}{2}}\left(a^{2}de^{(7i\,dx+7i\,c)}+2\,a^{2}de^{(5i\,dx+5i\,c)}+a^{2}de^{(3i\,dx+3i\,c)}\right)\sqrt{\frac{1}{a^{3}d^{2}}}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^{2}de^{(2i\,dx+2i\,c)}+a^{2}d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^{3}d^{2}}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-15\sqrt{\frac{1}{2}}\left(a^{2}de^{(7i\,dx+7i\,c)}+2\,a^{2}de^{(5i\,dx+5i\,c)}+a^{2}de^{(3i\,dx+3i\,c)}\right)\sqrt{\frac{1}{a^{3}d^{2}}}\log\left(-4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^{2}de^{(2i\,dx+2i\,c)}+a^{2}d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^{3}d^{2}}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(466\,e^{(8i\,dx+8i\,c)}+1105\,e^{(6i\,dx+6i\,c)}+855\,e^{(4i\,dx+4i\,c)}+115\,e^{(2i\,dx+2i\,c)}-5\right)}{60\left(a^{2}de^{(7i\,dx+7i\,c)}+2\,a^{2}de^{(5i\,dx+5i\,c)}+a^{2}de^{(3i\,dx+3i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/60*(15*sqrt(1/2)*(a^2*d*e^(7*I*d*x + 7*I*c) + 2*a^2*d*e^(5*I*d*x + 5*I*c
) + a^2*d*e^(3*I*d*x + 3*I*c))*sqrt(1/(a^3*d^2))*log(4*(sqrt(2)*sqrt(1/2)*(
a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1
```

```
/(a^3*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 15*sqrt(1/2)*(a^2*d*e^
(7*I*d*x + 7*I*c) + 2*a^2*d*e^(5*I*d*x + 5*I*c) + a^2*d*e^(3*I*d*x + 3*I*c)
)*sqrt(1/(a^3*d^2))*log(-4*(sqrt(2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) +
a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) - a*e^(I*d*x + I
*c))*e^(-I*d*x - I*c)) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(466*e^(
8*I*d*x + 8*I*c) + 1105*e^(6*I*d*x + 6*I*c) + 855*e^(4*I*d*x + 4*I*c) + 115
*e^(2*I*d*x + 2*I*c) - 5))/(a^2*d*e^(7*I*d*x + 7*I*c) + 2*a^2*d*e^(5*I*d*x
+ 5*I*c) + a^2*d*e^(3*I*d*x + 3*I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**5/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^5/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [B]
time = 4.20, size = 138, normalized size = 0.67

$$\frac{8\sqrt{a+a\tan(c+dx)\,1i}}{a^2\,d}-\frac{2\,(a+a\tan(c+dx)\,1i)^{3/2}}{a^3\,d}+\frac{2\,(a+a\tan(c+dx)\,1i)^{5/2}}{5\,a^4\,d}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+dx)\,1i}\,1i}{2\sqrt{a}}\right)\,1i}{4\,a^{3/2}\,d}+\frac{\frac{25\,a}{6}+\frac{a\tan(c+dx)\,9i}{2}}{a\,d\,(a+a\tan(c+dx)\,1i)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] (8*(a + a*tan(c + d*x)*1i)^(1/2))/(a^2*d) - (2*(a + a*tan(c + d*x)*1i)^(3/2
))/(a^3*d) + (2*(a + a*tan(c + d*x)*1i)^(5/2))/(5*a^4*d) + (2^(1/2)*atan((2
^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)*1i)/(2*a^(1/2)))*1i)/(4*a^(3/2)*d) + (
(25*a)/6 + (a*tan(c + d*x)*9i)/2)/(a*d*(a + a*tan(c + d*x)*1i)^(3/2))
```

3.118 $\int \frac{\tan^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=174

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\tan^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}+\frac{5i\tan^2(c+dx)}{2ad\sqrt{a+ia\tan(c+dx)}}-\frac{10i\sqrt{a+ia\tan}}{a^2d}$$

```
[Out] -1/4*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(3/2)/d*2^(1
/2)-10*I*(a+I*a*tan(d*x+c))^(1/2)/a^2/d+5/2*I*tan(d*x+c)^2/a/d/(a+I*a*tan(d
*x+c))^(1/2)-1/3*tan(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(3/2)+7/2*I*(a+I*a*tan(d
*x+c))^(3/2)/a^3/d
```

Rubi [A]
time = 0.24, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3639, 3676, 3673, 3608, 3561, 212}

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{7i(a+ia\tan(c+dx))^{3/2}}{2a^3d}-\frac{10i\sqrt{a+ia\tan(c+dx)}}{a^2d}-\frac{\tan^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}+\frac{5i\tan^2(c+dx)}{2ad\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-1/2*I)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]*a
^(3/2)*d) - Tan[c + d*x]^3/(3*d*(a + I*a*Tan[c + d*x])^(3/2)) + (((5*I)/2)*
Tan[c + d*x]^2)/(a*d*Sqrt[a + I*a*Tan[c + d*x]]) - ((10*I)*Sqrt[a + I*a*Tan
[c + d*x]])/(a^2*d) + (((7*I)/2)*(a + I*a*Tan[c + d*x])^(3/2))/(a^3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
```

```
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{\tan^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} - \frac{\int \frac{\tan^2(c+dx)\left(-3a+\frac{9}{2}ia\tan(c+dx)\right)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{3a^2}$$

$$= -\frac{\tan^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{5i\tan^2(c+dx)}{2ad\sqrt{a+ia\tan(c+dx)}} + \frac{\int \tan(c+dx)\sqrt{a}$$

$$= -\frac{\tan^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{5i\tan^2(c+dx)}{2ad\sqrt{a+ia\tan(c+dx)}} + \frac{7i(a+ia\tan(c+}{2a^3d}$$

$$= -\frac{\tan^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{5i\tan^2(c+dx)}{2ad\sqrt{a+ia\tan(c+dx)}} - \frac{10i\sqrt{a+ia\tan(c}}{a^2d}$$

$$= -\frac{\tan^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{5i\tan^2(c+dx)}{2ad\sqrt{a+ia\tan(c+dx)}} - \frac{10i\sqrt{a+ia\tan(c}}{a^2d}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{\tan^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{5i\,\mathrm{t}}{2ad\sqrt{a}}$$

Mathematica [A]

time = 1.35, size = 165, normalized size = 0.95

$$-\frac{ie^{-4i(c+dx)}\left(\sqrt{1+e^{2i(c+dx)}}\left(-1+18e^{2i(c+dx)}+87e^{4i(c+dx)}+52e^{6i(c+dx)}\right)+3e^{3i(c+dx)}\left(1+e^{2i(c+dx)}\right)^2\sinh^{-1}\left(e^{i(c+dx)}\right)\right)\sec^2(c+dx)}{24ad\sqrt{1+e^{2i(c+dx)}}\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-1/24*I)*(Sqrt[1 + E^((2*I)*(c + d*x))]*(-1 + 18*E^((2*I)*(c + d*x)) + 87
*E^((4*I)*(c + d*x)) + 52*E^((6*I)*(c + d*x))) + 3*E^((3*I)*(c + d*x))*(1 +
 E^((2*I)*(c + d*x)))^2*ArcSinh[E^(I*(c + d*x))])*Sec[c + d*x]^2)/(a*d*E^((
4*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]

time = 0.17, size = 113, normalized size = 0.65

method	result
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-2a\sqrt{a+ia\tan(dx+c)}-\frac{7a^2}{4\sqrt{a+ia\tan(dx+c)}}+\frac{a^3}{6(a+ia\tan(dx+c))^{\frac{3}{2}}}-\frac{a^{\frac{3}{2}}\sqrt{2}}{}\right.}{d\,a^3}$

default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-2a\sqrt{a+ia\tan\left(dx+c\right)}-\frac{7a^{2}}{4\sqrt{a+ia\tan\left(dx+c\right)}}+\frac{a^{3}}{6(a+ia\tan(dx+c))^{\frac{3}{2}}}-\frac{a^{\frac{3}{2}}\sqrt{\ldots}}{\ldots}\right.}{d\,a^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/d/a^3*(1/3*(a+I*a*tan(d*x+c))^(3/2)-2*a*(a+I*a*tan(d*x+c))^(1/2)-7/4*a^
2/(a+I*a*tan(d*x+c))^(1/2)+1/6*a^3/(a+I*a*tan(d*x+c))^(3/2)-1/8*a^(3/2)*2^(
1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.50, size = 139, normalized size = 0.80

$$\frac{i\left(3\sqrt{2}\,a^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)+16\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}a^{2}-96\sqrt{i\,a\tan\left(dx+c\right)+a}\,a^{3}-\frac{4\left(21\left(i\,a\tan(dx+c)+a\right)a^{4}-2\,a^{5}\right)}{\left(i\,a\tan(dx+c)+a\right)^{\frac{3}{2}}}\right)}{24\,a^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/24*I*(3*sqrt(2)*a^(7/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a
))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 16*(I*a*tan(d*x + c) +
 a)^(3/2)*a^2 - 96*sqrt(I*a*tan(d*x + c) + a)*a^3 - 4*(21*(I*a*tan(d*x + c)
 + a)*a^4 - 2*a^5)/(I*a*tan(d*x + c) + a)^(3/2))/(a^5*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 336 vs. 2(131) = 262.
time = 0.43, size = 336, normalized size = 1.93

$$-\frac{3\sqrt{\frac{1}{2}}\left(i\,a^{2}de^{\left(5i\,dx+5i\,c\right)}+i\,a^{2}de^{\left(3i\,dx+3i\,c\right)}\right)\sqrt{\frac{1}{a^{3}d^{2}}}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^{2}de^{\left(2i\,dx+2i\,c\right)}+a^{2}d\right)\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}\sqrt{\frac{1}{a^{3}d^{2}}}+ae^{\left(i\,dx+i\,c\right)}\right)e^{\left(-i\,dx-i\,c\right)}\right)+3\sqrt{\frac{1}{2}}\left(-i\,a^{2}de^{\left(5i\,dx+5i\,c\right)}-i\,a^{2}de^{\left(3i\,dx+3i\,c\right)}\right)\sqrt{\frac{1}{a^{3}d^{2}}}\log\left(-4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^{2}de^{\left(2i\,dx+2i\,c\right)}+a^{2}d\right)\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}\sqrt{\frac{1}{a^{3}d^{2}}}-ae^{\left(i\,dx+i\,c\right)}\right)e^{\left(-i\,dx-i\,c\right)}\right)-\sqrt{2}\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}\left(-52i\,e^{\left(6i\,dx+6i\,c\right)}-87i\,e^{\left(4i\,dx+4i\,c\right)}-18i\,e^{\left(2i\,dx+2i\,c\right)}+i\right)}{12\left(a^{2}de^{\left(5i\,dx+5i\,c\right)}+a^{2}de^{\left(3i\,dx+3i\,c\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/12*(3*sqrt(1/2)*(I*a^2*d*e^(5*I*d*x + 5*I*c) + I*a^2*d*e^(3*I*d*x + 3*I*
c))*sqrt(1/(a^3*d^2))*log(4*(sqrt(2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) +
 a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) + a*e^(I*d*x +
I*c))*e^(-I*d*x - I*c)) + 3*sqrt(1/2)*(-I*a^2*d*e^(5*I*d*x + 5*I*c) - I*a^2
*d*e^(3*I*d*x + 3*I*c))*sqrt(1/(a^3*d^2))*log(-4*(sqrt(2)*sqrt(1/2)*(a^2*d*
e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^3*
d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - sqrt(2)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*(-52*I*e^(6*I*d*x + 6*I*c) - 87*I*e^(4*I*d*x + 4*I*c) - 18*I*
e^(2*I*d*x + 2*I*c) + I))/(a^2*d*e^(5*I*d*x + 5*I*c) + a^2*d*e^(3*I*d*x + 3
*I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**4/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^4/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [B]
time = 0.28, size = 129, normalized size = 0.74

$$\frac{\frac{1\mathrm{i}}{3d}-\frac{(a+a\tan(c+dx)\,1\mathrm{i})\,7\mathrm{i}}{2\,a\,d}}{(a+a\tan(c+dx)\,1\mathrm{i})^{3/2}}-\frac{\sqrt{a+a\tan(c+dx)\,1\mathrm{i}}\;4\mathrm{i}}{a^2\,d}+\frac{(a+a\tan(c+dx)\,1\mathrm{i})^{3/2}\,2\mathrm{i}}{3\,a^3\,d}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+dx)\,1\mathrm{i}}}{2\,\sqrt{-a}}\right)\,1\mathrm{i}}{4\,(-a)^{3/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] (1i/(3*d) - ((a + a*tan(c + d*x)*1i)*7i)/(2*a*d))/(a + a*tan(c + d*x)*1i)^(
3/2) - ((a + a*tan(c + d*x)*1i)^(1/2)*4i)/(a^2*d) + ((a + a*tan(c + d*x)*1i
)^(3/2)*2i)/(3*a^3*d) - (2^(1/2)*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2
))/(2*(-a)^(1/2)))*1i)/(4*(-a)^(3/2)*d)
```

3.119 $\int \frac{\tan^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=133

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\tan^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{11}{6ad\sqrt{a+ia\tan(c+dx)}}-\frac{7\sqrt{a+ia\tan(c+dx)}}{3a^2d}$$

```
[Out] 1/4*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(3/2)/d*2^(1/2)
-11/6/a/d/(a+I*a*tan(d*x+c))^(1/2)-7/3*(a+I*a*tan(d*x+c))^(1/2)/a^2/d-1/3*t
an(d*x+c)^2/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.14, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3639, 3673, 3607, 3561, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{7\sqrt{a+ia\tan(c+dx)}}{3a^2d}-\frac{\tan^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{11}{6ad\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(2*Sqrt[2]*a^(3/2)*d)
 - Tan[c + d*x]^2/(3*d*(a + I*a*Tan[c + d*x])^(3/2)) - 11/(6*a*d*Sqrt[a + I
*a*Tan[c + d*x]]) - (7*Sqrt[a + I*a*Tan[c + d*x]])/(3*a^2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
```

```
0] && LtQ[m, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx &= -\frac{\tan^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} - \frac{\int \frac{\tan(c+dx)\left(-2a+\frac{7}{2}ia\tan(c+dx)\right)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{3a^2} \\
&= -\frac{\tan^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} - \frac{7\sqrt{a+ia\tan(c+dx)}}{3a^2d} - \frac{\int \frac{-\frac{7ia}{2}-2a\tan(c+dx}{\sqrt{a+ia\tan(c+}}}{3a^2} \\
&= -\frac{\tan^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} - \frac{11}{6ad\sqrt{a+ia\tan(c+dx)}} - \frac{7\sqrt{a+ia\tan(c-}}{3a^2d} \\
&= -\frac{\tan^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} - \frac{11}{6ad\sqrt{a+ia\tan(c+dx)}} - \frac{7\sqrt{a+ia\tan(c-}}{3a^2d} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{\tan^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} - \frac{}{6ad\sqrt{a+i}}
\end{aligned}
$$

Mathematica [A]

time = 1.27, size = 123, normalized size = 0.92

$$\frac{e^{-2i(c+dx)}\left(1-13e^{2i(c+dx)}-38e^{4i(c+dx)}+3e^{3i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right)\right)}{6ad\left(1+e^{2i(c+dx)}\right)\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (1 - 13*E^((2*I)*(c + d*x)) - 38*E^((4*I)*(c + d*x)) + 3*E^((3*I)*(c + d*x)
)*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))])/(6*a*d*E^((2*I)*(
c + d*x))*(1 + E^((2*I)*(c + d*x)))*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.19, size = 91, normalized size = 0.68

method	result
derivativedivides	$-\frac{2\left(\sqrt{a+ia\tan(dx+c)}+\frac{5a}{4\sqrt{a+ia\tan(dx+c)}}-\frac{a^2}{6(a+ia\tan(dx+c))^{\frac{3}{2}}}-\frac{\sqrt{a}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c}}{}\right.}{}\right.}{d\,a^2}$
default	$-\frac{2\left(\sqrt{a+ia\tan(dx+c)}+\frac{5a}{4\sqrt{a+ia\tan(dx+c)}}-\frac{a^2}{6(a+ia\tan(dx+c))^{\frac{3}{2}}}-\frac{\sqrt{a}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c}}{}\right.}{}\right.}{d\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a^2*((a+I*a*tan(d*x+c))^(1/2)+5/4*a/(a+I*a*tan(d*x+c))^(1/2)-1/6*a^2/(
a+I*a*tan(d*x+c))^(3/2)-1/8*a^(1/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^
(1/2)*2^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.51, size = 121, normalized size = 0.91

$$-\frac{3\sqrt{2}\,a^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+48\sqrt{i\,a\tan(dx+c)+a}\,a^2+\frac{4\left(15\,(i\,a\tan(dx+c)+a)a^3-2\,a^4\right)}{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}}{24\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/24*(3*sqrt(2)*a^(5/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a)
)/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 48*sqrt(I*a*tan(d*x + c
```

```
) + a)*a^2 + 4*(15*(I*a*tan(d*x + c) + a)*a^3 - 2*a^4)/(I*a*tan(d*x + c) +
a)^(3/2))/(a^4*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(102) = 204$.
time = 0.41, size = 273, normalized size = 2.05

$$\frac{\left(3\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(3i\,dx+3i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^2de^{(2i\,dx+2i\,c)}+a^2d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^3d^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-3\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(3i\,dx+3i\,c)}\log\left(-4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^2de^{(2i\,dx+2i\,c)}+a^2d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^3d^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(38\,e^{(4i\,dx+4i\,c)}+13\,e^{(2i\,dx+2i\,c)}-1\right)\right)e^{(-3i\,dx-3i\,c)}}{12\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(4*(sqrt(2
)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c)
 + 1))*sqrt(1/(a^3*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 3*sqrt(1/
2)*a^2*d*sqrt(1/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(a
^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/
(a^3*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - sqrt(2)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1))*(38*e^(4*I*d*x + 4*I*c) + 13*e^(2*I*d*x + 2*I*c) - 1))*e
^(-3*I*d*x - 3*I*c)/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**3/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^3/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [B]
time = 0.20, size = 93, normalized size = 0.70

$$-\frac{2\sqrt{a+a\tan(c+d\,x)\,1i}}{a^2\,d}+\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1i}}{2\sqrt{a}}\right)}{4\,a^{3/2}\,d}-\frac{\frac{13\,a}{6}+\frac{a\tan(c+d\,x)\,5i}{2}}{a\,d\,(a+a\tan(c+d\,x)\,1i)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*a^(1/2))))/(4*a^(
3/2)*d) - (2*(a + a*tan(c + d*x)*1i)^(1/2))/(a^2*d) - ((13*a)/6 + (a*tan(c
+ d*x)*5i)/2)/(a*d*(a + a*tan(c + d*x)*1i)^(3/2))
```

3.120 $\int \frac{\tan^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=104

$$\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{i}{3d(a+ia\tan(c+dx))^{3/2}}+\frac{3i}{2ad\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 1/4*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(3/2)/d*2^(1/
2)+3/2*I/a/d/(a+I*a*tan(d*x+c))^(1/2)-1/3*I/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3621, 3607, 3561, 212}

$$\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{3i}{2ad\sqrt{a+ia\tan(c+dx)}}-\frac{i}{3d(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((I/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]*a^(3
/2)*d) - (I/3)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + ((3*I)/2)/(a*d*Sqrt[a + I
*a*Tan[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
```

```
0] && LtQ[m, 0]
```

Rule 3621

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(a*c + b*d)^2*((a + b*Tan[e + f*x])^
m/(2*a^3*f*m)), x] + Dist[1/(2*a^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[
a*c^2 - 2*b*c*d + a*d^2 - 2*b*d^2*Tan[e + f*x], x], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LeQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{i}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{\int \frac{a-2ia\tan(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{2a^2}$$
$$= -\frac{i}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{3i}{2ad\sqrt{a+ia\tan(c+dx)}} - \frac{\int \sqrt{a+ia\tan(}}{4a^2}$$
$$= -\frac{i}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{3i}{2ad\sqrt{a+ia\tan(c+dx)}} + \frac{i\text{Subst}\left(\int \frac{1}{2a-x^2}\right.}{}$$
$$= \frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{i}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{}{2ad\sqrt{a-}}$$

Mathematica [A]
time = 1.09, size = 124, normalized size = 1.19

$$\frac{-1+7e^{2i(c+dx)}+8e^{4i(c+dx)}+3e^{3i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,\sinh^{-1}\left(e^{i(c+dx)}\right)}{3ad\left(1+e^{2i(c+dx)}\right)^2(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (-1 + 7*E^((2*I)*(c + d*x)) + 8*E^((4*I)*(c + d*x)) + 3*E^((3*I)*(c + d*x))
*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))])/(3*a*d*(1 + E^((2*
I)*(c + d*x)))^2*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.18, size = 75, normalized size = 0.72

method	result

derivativedivides	$-\frac{2i\left(-\frac{3}{4\sqrt{a+ia\tan(dx+c)}}+\frac{a}{6(a+ia\tan(dx+c))^{\frac{3}{2}}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{8\sqrt{a}}\right)}{da}$
default	$-\frac{2i\left(-\frac{3}{4\sqrt{a+ia\tan(dx+c)}}+\frac{a}{6(a+ia\tan(dx+c))^{\frac{3}{2}}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{8\sqrt{a}}\right)}{da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/d/a*(-3/4/(a+I*a*tan(d*x+c))^(1/2)+1/6*a/(a+I*a*tan(d*x+c))^(3/2)-1/8*
2^(1/2)/a^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.51, size = 103, normalized size = 0.99

$$-\frac{i\left(3\sqrt{2}\,a^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)-\frac{4\left(9\left(i\,a\tan(dx+c)+a\right)a^2-2\,a^3\right)}{\left(i\,a\tan(dx+c)+a\right)^{\frac{3}{2}}}\right)}{24\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/24*I*(3*sqrt(2)*a^(3/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) +
a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - 4*(9*(I*a*tan(d*x + c
) + a)*a^2 - 2*a^3)/(I*a*tan(d*x + c) + a)^(3/2))/(a^3*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(73) = 146.
time = 0.39, size = 272, normalized size = 2.62

$$\frac{\left(3i\sqrt{\frac{1}{2}}\,a^2d\sqrt{\frac{1}{a^3d^2}}\,e^{(3i\,dx+3i\,c)}\log\left(4\left(\sqrt{2}\,\sqrt{\frac{1}{2}}\,(a^2de^{(2i\,dx+2i\,c)}+a^2d)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,\sqrt{\frac{1}{a^3d^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-3i\sqrt{\frac{1}{2}}\,a^2d\sqrt{\frac{1}{a^3d^2}}\,e^{(3i\,dx+3i\,c)}\log\left(-4\left(\sqrt{2}\,\sqrt{\frac{1}{2}}\,(a^2de^{(2i\,dx+2i\,c)}+a^2d)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,\sqrt{\frac{1}{a^3d^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+\sqrt{2}\,\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,(8i\,e^{(4i\,dx+4i\,c)}+7i\,e^{(2i\,dx+2i\,c)}-i)\right)e^{(-3i\,dx-3i\,c)}}{12\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(4*(sqrt
(2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*sqrt(1/(a^3*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 3*I*sqr
t(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(-4*(sqrt(2)*sqrt(1/2
```

```
)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqr
t(1/(a^3*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*sqrt(a/(e^(
2*I*d*x + 2*I*c) + 1))*(8*I*e^(4*I*d*x + 4*I*c) + 7*I*e^(2*I*d*x + 2*I*c) -
 I))*e^(-3*I*d*x - 3*I*c)/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**2/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^2/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [B]
time = 4.05, size = 84, normalized size = 0.81

$$-\frac{\frac{1\mathrm{i}}{3\,d}-\frac{(a+a\tan(c+d\,x)\,1\mathrm{i})\,3\mathrm{i}}{2\,a\,d}}{(a+a\tan(c+d\,x)\,1\mathrm{i})^{3/2}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{2\,\sqrt{-a}}\right)\,1\mathrm{i}}{4\,(-a)^{3/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*(-a)^(1/2)))*1i)/(
4*(-a)^(3/2)*d) - (1i/(3*d) - ((a + a*tan(c + d*x)*1i)*3i)/(2*a*d))/(a + a*
tan(c + d*x)*1i)^(3/2)
```

3.121 $\int \frac{\tan(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{1}{3d(a+ia\tan(c+dx))^{3/2}}+\frac{1}{2ad\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] -1/4*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(3/2)/d*2^(1/2
)+1/2/a/d/(a+I*a*tan(d*x+c))^(1/2)-1/3/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.06, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3607, 3560, 3561, 212}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{1}{2ad\sqrt{a+ia\tan(c+dx)}}-\frac{1}{3d(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] -1/2*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(Sqrt[2]*a^(3/2)
*d) - 1/(3*d*(a + I*a*Tan[c + d*x])^(3/2)) + 1/(2*a*d*Sqrt[a + I*a*Tan[c +
d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{1}{3d(a+ia\tan(c+dx))^{3/2}} - \frac{i\int \frac{1}{\sqrt{a+ia\tan(c+dx)}}\,dx}{2a}$$

$$= -\frac{1}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{1}{2ad\sqrt{a+ia\tan(c+dx)}} - \frac{i\int \sqrt{a+ia\tan}}{4a^2}$$

$$= -\frac{1}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{1}{2ad\sqrt{a+ia\tan(c+dx)}} - \frac{\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\right.}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{1}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{}{2ad\sqrt{a}}$$

Mathematica [A]
time = 0.64, size = 124, normalized size = 1.27

$$-\frac{i\left(-1+e^{2i(c+dx)}+2e^{4i(c+dx)}-3e^{3i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,\sinh^{-1}\left(e^{i(c+dx)}\right)\right)}{3ad\left(1+e^{2i(c+dx)}\right)^2(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-1/3*I)*(-1 + E^((2*I)*(c + d*x)) + 2*E^((4*I)*(c + d*x)) - 3*E^((3*I)*(c
 + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))]))/(a*d*(1 +
 E^((2*I)*(c + d*x)))^2*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.17, size = 72, normalized size = 0.73

method	result	siz

derivativedivides	$\frac{-\frac{1}{3(a+ia\tan(dx+c))^{\frac{3}{2}}}+\frac{1}{2a\sqrt{a+ia\tan(dx+c)}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{4a^{\frac{3}{2}}}}{d}$	72
default	$\frac{-\frac{1}{3(a+ia\tan(dx+c))^{\frac{3}{2}}}+\frac{1}{2a\sqrt{a+ia\tan(dx+c)}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{4a^{\frac{3}{2}}}}{d}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3/(a+I*a*tan(d*x+c))^(3/2)+1/2/a/(a+I*a*tan(d*x+c))^(1/2)-1/4/a^(3/
2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.50, size = 101, normalized size = 1.03

$$\frac{3\sqrt{2}\,\sqrt{a}\,\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+\frac{4\,(3\,(i\,a\tan(dx+c)+a)a-2\,a^2)}{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}}{24\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/24*(3*sqrt(2)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))
/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(3*(I*a*tan(d*x + c) +
 a)*a - 2*a^2)/(I*a*tan(d*x + c) + a)^(3/2))/(a^2*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(73) = 146$.
time = 0.37, size = 271, normalized size = 2.77

$$-\frac{\left(3\sqrt{\frac{1}{2}}\,a^2d\sqrt{\frac{1}{a^3d^2}}\,e^{(3i\,dx+3i\,c)}\log\left(4\left(\sqrt{2}\,\sqrt{\frac{1}{2}}\,(a^2de^{(2i\,dx+2i\,c)}+a^2d)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,\sqrt{\frac{1}{a^3d^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-3\sqrt{\frac{1}{2}}\,a^2d\sqrt{\frac{1}{a^3d^2}}\,e^{(3i\,dx+3i\,c)}\log\left(-4\left(\sqrt{2}\,\sqrt{\frac{1}{2}}\,(a^2de^{(2i\,dx+2i\,c)}+a^2d)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,\sqrt{\frac{1}{a^3d^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-\sqrt{2}\,\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,(2\,e^{(4i\,dx+4i\,c)}+e^{(2i\,dx+2i\,c)}-1)\right)e^{(-3i\,dx-3i\,c)}}{12\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/12*(3*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(4*(sqrt(
2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*sqrt(1/(a^3*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 3*sqrt(1
/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(
a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1
/(a^3*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - sqrt(2)*sqrt(a/(e^(2*I
```

```
*d*x + 2*I*c) + 1))*(2*e^(4*I*d*x + 4*I*c) + e^(2*I*d*x + 2*I*c) - 1))*e^(-
3*I*d*x - 3*I*c)/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [B]
time = 3.98, size = 72, normalized size = 0.73

$$\frac{\frac{a+a\tan(c+d\,x)\,1\mathrm{i}}{2\,a}-\frac{1}{3}}{d\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{3/2}}-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{2\,\sqrt{a}}\right)}{4\,a^{3/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] ((a + a*tan(c + d*x)*1i)/(2*a) - 1/3)/(d*(a + a*tan(c + d*x)*1i)^(3/2)) - (
2^(1/2)*atanh((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*a^(1/2))))/(4*a^(3
/2)*d)
```

3.122 $\int \frac{1}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{i}{3d(a+ia\tan(c+dx))^{3/2}}+\frac{i}{2ad\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] -1/4*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(3/2)/d*2^(1
/2)+1/2*I/a/d/(a+I*a*tan(d*x+c))^(1/2)+1/3*I/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.04, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3560, 3561, 212}

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{i}{2ad\sqrt{a+ia\tan(c+dx)}}+\frac{i}{3d(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(-3/2),x]
```

```
[Out] ((-1/2*I)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]*a
^(3/2)*d) + (I/3)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (I/2)/(a*d*Sqrt[a + I*
a*Tan[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{i}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{\int \frac{1}{\sqrt{a+ia\tan(c+dx)}}\,dx}{2a}$$
$$= \frac{i}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{i}{2ad\sqrt{a+ia\tan(c+dx)}} + \frac{\int \sqrt{a+ia\tan(c-}{4a^2}$$
$$= \frac{i}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{i}{2ad\sqrt{a+ia\tan(c+dx)}} - \frac{i\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx\right.}{}$$
$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d} + \frac{i}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{}{2ad\sqrt{a}}$$

Mathematica [A]
time = 0.61, size = 124, normalized size = 1.19

$$\frac{1+5e^{2i(c+dx)}+4e^{4i(c+dx)}-3e^{3i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right)}{3ad\left(1+e^{2i(c+dx)}\right)^2(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(-3/2),x]
```

```
[Out] (1 + 5*E^((2*I)*(c + d*x)) + 4*E^((4*I)*(c + d*x)) - 3*E^((3*I)*(c + d*x))*
Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))])/(3*a*d*(1 + E^((2*I
)*(c + d*x)))^2*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.15, size = 78, normalized size = 0.75

method	result
derivativedivides	$\frac{2ia\left(\frac{1}{4a^2\sqrt{a+ia\tan(dx+c)}}+\frac{1}{6a(a+ia\tan(dx+c))^{\frac{3}{2}}}-\frac{\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{8a^{\frac{5}{2}}}\right)}{d}$
default	$\frac{2ia\left(\frac{1}{4a^2\sqrt{a+ia\tan(dx+c)}}+\frac{1}{6a(a+ia\tan(dx+c))^{\frac{3}{2}}}-\frac{\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{8a^{\frac{5}{2}}}\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/d*a*(1/4/a^2/(a+I*a*tan(d*x+c))^(1/2)+1/6/a/(a+I*a*tan(d*x+c))^(3/2)-1/
8/a^(5/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.50, size = 94, normalized size = 0.90

$$\frac{i\left(\frac{3\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)}{\sqrt{a}}+\frac{4\left(3i\,a\tan(dx+c)+5\,a\right)}{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}\right)}{24\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/24*I*(3*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sqrt
(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/sqrt(a) + 4*(3*I*a*tan(d*x + c)
+ 5*a)/(I*a*tan(d*x + c) + a)^(3/2))/(a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(73) = 146.
time = 0.37, size = 272, normalized size = 2.62

$$\frac{\left(-3i\sqrt{\frac{1}{2}}\,a^2d\sqrt{\frac{1}{a^3d^2}}\,e^{(3i\,dx+3i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^2de^{(2i\,dx+2i\,c)}+a^2d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^3d^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+3i\sqrt{\frac{1}{2}}\,a^2d\sqrt{\frac{1}{a^3d^2}}\,e^{(3i\,dx+3i\,c)}\log\left(-4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^2de^{(2i\,dx+2i\,c)}+a^2d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^3d^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(4i\,e^{(4i\,dx+4i\,c)}+5i\,e^{(2i\,dx+2i\,c)}+i\right)\right)e^{(-3i\,dx-3i\,c)}}{12\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*(-3*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(4*(sqr
t(2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*sqrt(1/(a^3*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 3*I*sq
rt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(-4*(sqrt(2)*sqrt(1/
2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sq
rt(1/(a^3*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1))*(4*I*e^(4*I*d*x + 4*I*c) + 5*I*e^(2*I*d*x + 2*I*c)
+ I))*e^(-3*I*d*x - 3*I*c)/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(ia\tan\left(c+dx\right)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*a*tan(c + d*x) + a)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(-3/2), x)
```

Mupad [B]
time = 3.99, size = 83, normalized size = 0.80

$$\frac{\frac{1\mathrm{i}}{3\,d}+\frac{(a+a\tan(c+d\,x)\;1\mathrm{i})\;1\mathrm{i}}{2\,a\,d}}{(a+a\tan{(c+d\,x)}\;1\mathrm{i})^{3/2}}-\frac{\sqrt{2}\;\mathrm{atan}\left(\frac{\sqrt{2}\;\sqrt{a+a\tan{(c+d\,x)}\;1\mathrm{i}}}{2\,\sqrt{-a}}\right)\;1\mathrm{i}}{4\,(-a)^{3/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] (1i/(3*d) + ((a + a*tan(c + d*x)*1i)*1i)/(2*a*d))/(a + a*tan(c + d*x)*1i)^(
3/2) - (2^(1/2)*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*(-a)^(1/2))
)*1i)/(4*(-a)^(3/2)*d)
```

3.123 $\int \frac{\cot(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=132

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{1}{3d(a+ia\tan(c+dx))^{3/2}}+\frac{}{2ad\sqrt{a}}$$

```
[Out] -2*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+1/4*arctanh(1/2*(a+I
*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(3/2)/d*2^(1/2)+3/2/a/d/(a+I*a*tan(
d*x+c))^(1/2)+1/3/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.26, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3640, 3677, 3681, 3561, 212, 3680, 65, 214}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{3}{2ad\sqrt{a+ia\tan(c+dx)}}+\frac{1}{3d(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + ArcTanh[Sqrt
[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(2*Sqrt[2]*a^(3/2)*d) + 1/(3*d*(a
 + I*a*Tan[c + d*x])^(3/2)) + 3/(2*a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3681

```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(
A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{1}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{\int \frac{\cot(c+dx)\left(3a-\frac{3}{2}ia\tan(c+dx)\right)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{3a^2}$$

$$= \frac{1}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{3}{2ad\sqrt{a+ia\tan(c+dx)}} + \frac{\int \cot(c+dx)\sqrt{a+\cdots}}{}$$

$$= \frac{1}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{3}{2ad\sqrt{a+ia\tan(c+dx)}} + \frac{\int \cot(c+dx)(a-\cdots}{}$$

$$= \frac{1}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{3}{2ad\sqrt{a+ia\tan(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,\cdots\right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d} + \frac{1}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{}{2ad\sqrt{a+i\cdots}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d} + \cdots$$

Mathematica [A]
time = 2.05, size = 196, normalized size = 1.48

$$-\frac{i\left(1+11e^{2i(c+dx)}+10e^{4i(c+dx)}+3e^{3i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right)-12\sqrt{2}\,e^{3i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{\sqrt{2}\,e^{i(c+dx)}}{\sqrt{1+e^{2i(c+dx)}}}\right)\right)}{3ad\left(1+e^{2i(c+dx)}\right)^2(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-1/3*I)*(1 + 11*E^((2*I)*(c + d*x)) + 10*E^((4*I)*(c + d*x)) + 3*E^((3*I)
*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))] - 12*Sqr
t[2]*E^((3*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(Sqrt[2]*E^(
I*(c + d*x)))/Sqrt[1 + E^((2*I)*(c + d*x))]]))/(a*d*(1 + E^((2*I)*(c + d*x)
))^2*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 696 vs. 2(103) = 206.
time = 1.24, size = 697, normalized size = 5.28

method	result

default	$-\frac{\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(16i(\cos^3(dx+c))\sin(dx+c)+3i\sqrt{2}\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{(-i\cos(dx+c)+\sin}{2\sin(dx+c)\sqrt{-}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(16*I*cos(d*x+c)^3*s
in(d*x+c)+3*I*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-
I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
2^(1/2))*2^(1/2)+12*I*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))-12
*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2))*sin(d*x+c)-16*cos(d*x+c)^4-3*2^(1/2)*cos(d*x+c)*(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+36*I*sin(d*x+c)*cos(d*x+c)+12*I
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))+12*cos(d*x+c)*(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+12*ln(((-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-3*2^(1/2)*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-28*cos(d*x+c)^2+12*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))/a^2
```

Maxima [A]
time = 0.53, size = 139, normalized size = 1.05

$$-\frac{\frac{3\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)}{a^{\frac{3}{2}}}-\frac{24\log\left(\frac{\sqrt{i\,a\tan\left(dx+c\right)+a}-\sqrt{a}}{\sqrt{i\,a\tan\left(dx+c\right)+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{4\,(9i\,a\tan(dx+c)+11\,a)}{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/24*(3*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sqrt(
2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/a^(3/2) - 24*log((sqrt(I*a*tan(d*
x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x + c) + a) + sqrt(a)))/a^(3/2) - 4*
(9*I*a*tan(d*x + c) + 11*a)/((I*a*tan(d*x + c) + a)^(3/2)*a))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 500 vs. 2(99) = 198.
time = 0.38, size = 500, normalized size = 3.79

$$\frac{\left(3\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(3idx+3ic)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^2de^{(2idx+2ic)}+a^2d\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{1}{a^3d^2}}+ae^{(idx+ic)}\right)e^{(-idx-ic)}\right)-3\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(3idx+3ic)}\log\left(-4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^2de^{(2idx+2ic)}+a^2d\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{1}{a^3d^2}}-ae^{(idx+ic)}\right)e^{(-idx-ic)}\right)-6a^2d\sqrt{\frac{1}{a^3d^2}}e^{(3idx+3ic)}\log\left(16\left(3a^2e^{(2idx+2ic)}+2\sqrt{2}\left(a^3de^{(3idx+3ic)}+a^3de^{(idx+ic)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{1}{a^3d^2}}+a^2\right)e^{(-2idx-2ic)}\right)+6a^2d\sqrt{\frac{1}{a^3d^2}}e^{(3idx+3ic)}\log\left(16\left(3a^2e^{(2idx+2ic)}-2\sqrt{2}\left(a^3de^{(3idx+3ic)}+a^3de^{(idx+ic)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{1}{a^3d^2}}+a^2\right)e^{(-2idx-2ic)}\right)+\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\left(10e^{(4idx+4ic)}+11e^{(2idx+2ic)}+1\right)\right)e^{(-3idx-3ic)}}{12a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")

[Out] 1/12*(3*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(4*(sqrt(2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 3*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 6*a^2*d*sqrt(1/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(16*(3*a^2*e^(2*I*d*x + 2*I*c) + 2*sqrt(2)*(a^3*d*e^(3*I*d*x + 3*I*c) + a^3*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) + a^2)*e^(-2*I*d*x - 2*I*c)) + 6*a^2*d*sqrt(1/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(16*(3*a^2*e^(2*I*d*x + 2*I*c) - 2*sqrt(2)*(a^3*d*e^(3*I*d*x + 3*I*c) + a^3*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) + a^2)*e^(-2*I*d*x - 2*I*c)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(10*e^(4*I*d*x + 4*I*c) + 11*e^(2*I*d*x + 2*I*c) + 1))*e^(-3*I*d*x - 3*I*c)/(a^2*d)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c))**(3/2),x)

[Out] Integral(cot(c + d*x)/(I*a*(tan(c + d*x) - I))**(3/2), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate(cot(d*x + c)/(I*a*tan(d*x + c) + a)^(3/2), x)

Mupad [B]
time = 4.05, size = 109, normalized size = 0.83

$$-\frac{2\,\mathrm{atanh}\left(\frac{a\,\sqrt{a+a\tan(c+d\,x)\ 1\mathrm{i}}}{\sqrt{a^3}}\right)}{d\,\sqrt{a^3}}+\frac{\frac{3\,(a+a\tan(c+d\,x)\ 1\mathrm{i})}{2\,a}+\frac{1}{3}}{d\,(a+a\tan(c+d\,x)\ 1\mathrm{i})^{3/2}}+\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,a\,\sqrt{a+a\tan(c+d\,x)\ 1\mathrm{i}}}{2\,\sqrt{a^3}}\right)}{4\,d\,\sqrt{a^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] ((3*(a + a*tan(c + d*x)*1i))/(2*a) + 1/3)/(d*(a + a*tan(c + d*x)*1i)^(3/2))
 - (2*atanh((a*(a + a*tan(c + d*x)*1i)^(1/2))/(a^3)^(1/2)))/(d*(a^3)^(1/2))
 + (2^(1/2)*atanh((2^(1/2)*a*(a + a*tan(c + d*x)*1i)^(1/2))/(2*(a^3)^(1/2))
))/(4*d*(a^3)^(1/2))
```

3.124 $\int \frac{\cot^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=181

$$\frac{3i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{\cot(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}+\frac{1\cdots}{6ad\sqrt{\cdots}}$$

```
[Out] 3*I*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+1/4*I*arctanh(1/2*(
a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(3/2)/d*2^(1/2)+13/6*cot(d*x+c)/
a/d/(a+I*a*tan(d*x+c))^(1/2)-7/2*cot(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/a^2/d+
1/3*cot(d*x+c)/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.38, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3640, 3677, 3679, 3681, 3561, 212, 3680, 65, 214}

$$\frac{3i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{7\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{2a^2d}+\frac{13\cot(c+dx)}{6ad\sqrt{a+ia\tan(c+dx)}}+\frac{\cot(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((3*I)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + ((I/2)*Ar
cTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]*a^(3/2)*d) +
Cot[c + d*x]/(3*d*(a + I*a*Tan[c + d*x])^(3/2)) + (13*Cot[c + d*x])/(6*a*d*
Sqrt[a + I*a*Tan[c + d*x]]) - (7*Cot[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(
2*a^2*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
```

```
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3681

```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(
A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\cot(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{\int \frac{\cot^2(c+dx)\left(4a-\frac{5}{2}ia\tan(c+dx)\right)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{3a^2}$$

$$= \frac{\cot(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{13\cot(c+dx)}{6ad\sqrt{a+ia\tan(c+dx)}} + \frac{\int \cot^2(c+dx)\sqrt{a\cdots}}{\cdots}$$

$$= \frac{\cot(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{13\cot(c+dx)}{6ad\sqrt{a+ia\tan(c+dx)}} - \frac{7\cot(c+dx)\sqrt{a+\cdots}}{2a^2\cdots}$$

$$= \frac{\cot(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{13\cot(c+dx)}{6ad\sqrt{a+ia\tan(c+dx)}} - \frac{7\cot(c+dx)\sqrt{a+\cdots}}{2a^2\cdots}$$

$$= \frac{\cot(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{13\cot(c+dx)}{6ad\sqrt{a+ia\tan(c+dx)}} - \frac{7\cot(c+dx)\sqrt{a+\cdots}}{2a^2\cdots}$$

$$= \frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{\cot(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{13\cot\cdots}{6ad\sqrt{a+\cdots}}$$

$$= \frac{3i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}d} + \cdots$$

Mathematica [A]
time = 2.01, size = 214, normalized size = 1.18

$$-\frac{e^{-4i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(\sqrt{1+e^{2i(c+dx)}}\left(-1-15e^{2i(c+dx)}+28e^{4i(c+dx)}\right)-3e^{3i(c+dx)}\left(-1+e^{2i(c+dx)}\right)\sinh^{-1}\left(e^{i(c+dx)}\right)-18\sqrt{2}e^{3i(c+dx)}\left(-1+e^{2i(c+dx)}\right)\tanh^{-1}\left(\frac{\sqrt{2}e^{i(c+dx)}}{\sqrt{1+e^{2i(c+dx)}}}\right)\right)\csc(2(c+dx))}{12ad\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] -1/12*(Sqrt[1 + E^((2*I)*(c + d*x))]*(Sqrt[1 + E^((2*I)*(c + d*x))]*(-1 - 1
5*E^((2*I)*(c + d*x)) + 28*E^((4*I)*(c + d*x))) - 3*E^((3*I)*(c + d*x))*(-1
 + E^((2*I)*(c + d*x)))*ArcSinh[E^(I*(c + d*x))] - 18*Sqrt[2]*E^((3*I)*(c +
 d*x))*(-1 + E^((2*I)*(c + d*x)))*ArcTanh[(Sqrt[2]*E^(I*(c + d*x)))/Sqrt[1
+ E^((2*I)*(c + d*x))]])*Csc[2*(c + d*x)])/(a*d*E^((4*I)*(c + d*x))*Sqrt[a
+ I*a*Tan[c + d*x]])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1398 vs. $2(145) = 290$.
time = 0.91, size = 1399, normalized size = 7.73

method	result	size
default	Expression too large to display	1399

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(-18*I*cos(d*x+c)^2*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2))-3*I*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(
-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*2^(1/2))*2^(1/2)+16*sin(d*x+c)*cos(d*x+c)^5+18*I*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-3*cos(d*x+c)^2*s
in(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-I*cos(d
*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)
)-18*I*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2))-3*I*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+3*I*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+18*cos(d*x+c)^3*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-
cos(d*x+c)+1)/sin(d*x+c))-18*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+18*I*cos(d*x+
c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2))-52*I*cos(d*x+c)^2+44*sin(d*x+c)*cos(d*x+c)^3+18*cos(d*x+c)^2*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))+3*I*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+3*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+18*I*sin(d*x+c)*(-2*cos(d*x+c)/(1+co
```

```
s(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*
x+c)+1)/sin(d*x+c))-18*I*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1
)/sin(d*x+c))+36*I*cos(d*x+c)^4-18*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/s
in(d*x+c))+18*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(1/2))*sin(d*x+c)+16*I*cos(d*x+c)^6-84*sin(d*x+c)*cos(d*x+c
)-18*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c)))/(cos(d*x+c)^2-1)/a^2
```

Maxima [A]
time = 0.50, size = 184, normalized size = 1.02

$$-\frac{i\,a\left(\frac{4\left(21\,(i\,a\tan\left(dx+c\right)+a)^{2}-13\,(i\,a\tan\left(dx+c\right)+a)a-2\,a^{2}\right)}{(i\,a\tan\left(dx+c\right)+a)^{\frac{5}{2}}a^{2}-(i\,a\tan\left(dx+c\right)+a)^{\frac{3}{2}}a^{3}}+\frac{3\,\sqrt{2}\,\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)}{a^{\frac{5}{2}}}+\frac{36\,\log\left(\frac{\sqrt{i\,a\tan\left(dx+c\right)+a}-\sqrt{a}}{\sqrt{i\,a\tan\left(dx+c\right)+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/24*I*a*(4*(21*(I*a*tan(d*x + c) + a)^2 - 13*(I*a*tan(d*x + c) + a)*a - 2
*a^2)/((I*a*tan(d*x + c) + a)^(5/2)*a^2 - (I*a*tan(d*x + c) + a)^(3/2)*a^3)
 + 3*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sqrt(2)*s
qrt(a) + sqrt(I*a*tan(d*x + c) + a)))/a^(5/2) + 36*log((sqrt(I*a*tan(d*x +
c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x + c) + a) + sqrt(a)))/a^(5/2))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(138) = 276$.
time = 0.38, size = 601, normalized size = 3.32

$$-\frac{3\sqrt{\frac{1}{2}}\left(-i\,a^{2}de^{(5i\,dx+5i\,c)}+i\,a^{2}de^{(3i\,dx+3i\,c)}\right)\sqrt{\frac{1}{a^{3}d^{2}}}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^{2}de^{(2i\,dx+2i\,c)}+a^{2}d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^{3}d^{2}}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+3\sqrt{\frac{1}{2}}\left(i\,a^{2}de^{(5i\,dx+5i\,c)}-i\,a^{2}de^{(3i\,dx+3i\,c)}\right)\sqrt{\frac{1}{a^{3}d^{2}}}\log\left(-4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^{2}de^{(2i\,dx+2i\,c)}+a^{2}d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^{3}d^{2}}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+9\left(-i\,a^{2}de^{(5i\,dx+5i\,c)}+i\,a^{2}de^{(3i\,dx+3i\,c)}\right)\sqrt{\frac{1}{a^{3}d^{2}}}\log\left(16\left(3\,a^{2}e^{(2i\,dx+2i\,c)}+2\sqrt{2}\left(a^{3}de^{(3i\,dx+3i\,c)}+a^{3}de^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^{3}d^{2}}}+a^{2}\right)e^{(-2i\,dx-2i\,c)}\right)+9\left(i\,a^{2}de^{(5i\,dx+5i\,c)}-i\,a^{2}de^{(3i\,dx+3i\,c)}\right)\sqrt{\frac{1}{a^{3}d^{2}}}\log\left(16\left(3\,a^{2}e^{(2i\,dx+2i\,c)}-2\sqrt{2}\left(a^{3}de^{(3i\,dx+3i\,c)}+a^{3}de^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^{3}d^{2}}}+a^{2}\right)e^{(-2i\,dx-2i\,c)}\right)-\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-28i\,e^{(6i\,dx+6i\,c)}-13i\,e^{(4i\,dx+4i\,c)}+16i\,e^{(2i\,dx+2i\,c)}+i\right)}{12\left(a^{2}de^{(5i\,dx+5i\,c)}-a^{2}de^{(3i\,dx+3i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/12*(3*sqrt(1/2)*(-I*a^2*d*e^(5*I*d*x + 5*I*c) + I*a^2*d*e^(3*I*d*x + 3*I
*c))*sqrt(1/(a^3*d^2))*log(4*(sqrt(2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c)
+ a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) + a*e^(I*d*x +
 I*c))*e^(-I*d*x - I*c)) + 3*sqrt(1/2)*(I*a^2*d*e^(5*I*d*x + 5*I*c) - I*a^2
*d*e^(3*I*d*x + 3*I*c))*sqrt(1/(a^3*d^2))*log(-4*(sqrt(2)*sqrt(1/2)*(a^2*d*
e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^3*
d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 9*(-I*a^2*d*e^(5*I*d*x + 5*I
*c) + I*a^2*d*e^(3*I*d*x + 3*I*c))*sqrt(1/(a^3*d^2))*log(16*(3*a^2*e^(2*I*d
*x + 2*I*c) + 2*sqrt(2)*(a^3*d*e^(3*I*d*x + 3*I*c) + a^3*d*e^(I*d*x + I*c))
*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) + a^2)*e^(-2*I*d*x - 2
```

```
*I*c)) + 9*(I*a^2*d*e^(5*I*d*x + 5*I*c) - I*a^2*d*e^(3*I*d*x + 3*I*c))*sqrt
(1/(a^3*d^2))*log(16*(3*a^2*e^(2*I*d*x + 2*I*c) - 2*sqrt(2)*(a^3*d*e^(3*I*d
*x + 3*I*c) + a^3*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt
(1/(a^3*d^2)) + a^2)*e^(-2*I*d*x - 2*I*c)) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*(-28*I*e^(6*I*d*x + 6*I*c) - 13*I*e^(4*I*d*x + 4*I*c) + 16*I*e^
(2*I*d*x + 2*I*c) + I))/(a^2*d*e^(5*I*d*x + 5*I*c) - a^2*d*e^(3*I*d*x + 3*I
*c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(c + dx)}{(ia(\tan(c + dx) - i))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(cot(c + d*x)**2/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)^2/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [**B**]
time = 4.02, size = 178, normalized size = 0.98

$$-\frac{\frac{(a+a\tan(c+d\,x)\,1\mathrm{i})\,13\mathrm{i}}{6\,d}+\frac{a\,1\mathrm{i}}{3\,d}-\frac{(a+a\tan(c+d\,x)\,1\mathrm{i})^2\,7\mathrm{i}}{2\,a\,d}}{a\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{3/2}-(a+a\tan(c+d\,x)\,1\mathrm{i})^{5/2}}-\frac{\mathrm{atan}\left(\frac{\sqrt{-a^3}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{a^2}\right)\sqrt{-a^3}\,3\mathrm{i}}{a^3\,d}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{-a^3}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{2\,a^2}\right)\sqrt{-a^3}\,1\mathrm{i}}{4\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] - (((a + a*tan(c + d*x)*1i)*13i)/(6*d) + (a*1i)/(3*d) - ((a + a*tan(c + d*x
)*1i)^2*7i)/(2*a*d))/(a*(a + a*tan(c + d*x)*1i)^(3/2) - (a + a*tan(c + d*x)
*1i)^(5/2)) - (atan(((-a^3)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/a^2)*(-a^3
)^(1/2)*3i)/(a^3*d) - (2^(1/2)*atan((2^(1/2)*(-a^3)^(1/2)*(a + a*tan(c + d*
x)*1i)^(1/2))/(2*a^2))*(-a^3)^(1/2)*1i)/(4*a^3*d)
```

3.125 $\int \frac{\cot^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=220

$$\frac{23\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{3/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}+\frac{17}{6ad\sqrt{a\cdots}}$$

```
[Out] 23/4*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-1/4*arctanh(1/2*(a
+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(3/2)/d*2^(1/2)+17/6*cot(d*x+c)^2
/a/d/(a+I*a*tan(d*x+c))^(1/2)+21/4*I*cot(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/a^
2/d-11/3*cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)/a^2/d+1/3*cot(d*x+c)^2/d/(a+
I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.48, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3640, 3677, 3679, 3681, 3561, 212, 3680, 65, 214}

$$\frac{23\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{3/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{11\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{3a^2d}+\frac{21i\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{4a^2d}+\frac{17\cot^2(c+dx)}{6ad\sqrt{a+ia\tan(c+dx)}}+\frac{\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (23*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/(4*a^(3/2)*d) - ArcTanh[Sq
rt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(2*Sqrt[2]*a^(3/2)*d) + Cot[c +
 d*x]^2/(3*d*(a + I*a*Tan[c + d*x])^(3/2)) + (17*Cot[c + d*x]^2)/(6*a*d*Sqr
t[a + I*a*Tan[c + d*x]]) + (((21*I)/4)*Cot[c + d*x]*Sqrt[a + I*a*Tan[c + d*
x]])/(a^2*d) - (11*Cot[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/(3*a^2*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
```

```
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3681

```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(
A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{\int \frac{\cot^3(c+dx)\left(5a-\frac{7}{2}ia\tan(c+dx)\right)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{3a^2}$$

$$= \frac{\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{17\cot^2(c+dx)}{6ad\sqrt{a+ia\tan(c+dx)}} + \frac{\int \cot^3(c+dx)\sqrt{a}\ldots}{\ldots}$$

$$= \frac{\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{17\cot^2(c+dx)}{6ad\sqrt{a+ia\tan(c+dx)}} - \frac{11\cot^2(c+dx)\sqrt{a}\ldots}{3a^{\ldots}}$$

$$= \frac{\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{17\cot^2(c+dx)}{6ad\sqrt{a+ia\tan(c+dx)}} + \frac{21i\cot(c+dx)\sqrt{a}\ldots}{4a^{\ldots}}$$

$$= \frac{\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{17\cot^2(c+dx)}{6ad\sqrt{a+ia\tan(c+dx)}} + \frac{21i\cot(c+dx)\sqrt{a}\ldots}{4a^{\ldots}}$$

$$= \frac{\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{17\cot^2(c+dx)}{6ad\sqrt{a+ia\tan(c+dx)}} + \frac{21i\cot(c+dx)\sqrt{a}\ldots}{4a^{\ldots}}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d} + \frac{\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{17\,\mathrm{co}\ldots}{6ad\sqrt{a+\ldots}}$$

$$= \frac{23\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{3/2}d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d} + \frac{\ldots}{3\ldots}$$

Mathematica [A]
time = 4.33, size = 214, normalized size = 0.97

$$\frac{\sec^{\frac{3}{2}}(c+dx)\left(\sqrt{2}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{3/2}\left(1+e^{2i(c+dx)}\right)^{3/2}\left(-2\sinh^{-1}\left(e^{i(c+dx)}\right)+23\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,e^{i(c+dx)}}{\sqrt{1+e^{2i(c+dx)}}}\right)\right)-\frac{1}{3}\csc^2(c+dx)\sqrt{\sec(c+dx)}\,(25+6\cos(2(c+dx))-19\cos(4(c+dx))+27i\sin(2(c+dx))-18i\sin(4(c+dx)))\right)}{8d(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (Sec[c + d*x]^(3/2)*(Sqrt[2]*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(3
/2)*(1 + E^((2*I)*(c + d*x)))^(3/2)*(-2*ArcSinh[E^(I*(c + d*x))] + 23*Sqrt[
2]*ArcTanh[(Sqrt[2]*E^(I*(c + d*x)))/Sqrt[1 + E^((2*I)*(c + d*x))]]) - (Csc
[c + d*x]^2*Sqrt[Sec[c + d*x]]*(25 + 6*Cos[2*(c + d*x)] - 19*Cos[4*(c + d*x
)] + (27*I)*Sin[2*(c + d*x)] - (18*I)*Sin[4*(c + d*x)]))/3))/(8*d*(a + I*a*
Tan[c + d*x])^(3/2))
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1396 vs. 2(178) = 356.
time = 1.01, size = 1397, normalized size = 6.35

method	result	size
default	Expression too large to display	1397

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(-252*I*cos(d*x+c)*si
n(d*x+c)+69*I*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))-6*cos(d*
x+c)^3*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-I*cos(d*x+
c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+6
9*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*cos(d*x+c)^3-69*I*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)
-cos(d*x+c)+1)/sin(d*x+c))*cos(d*x+c)-32*cos(d*x+c)^6+69*I*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*sin(d*x
+c)-6*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*
(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*2^(1/2))+69*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+32*I*cos(d*x+c)^5*sin(d*x+c)+69*(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(
d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-69*I*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(
d*x+c)^2*sin(d*x+c)+6*2^(1/2)*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*2^(1/2))+136*I*cos(d*x+c)^3*sin(d*x+c)+69*cos(d*x+c)^2*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2))-6*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-I*cos(d*x+c)+sin
(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)
*sin(d*x+c)-120*cos(d*x+c)^4+6*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d
```

```
*x+c)))^(1/2)*2^(1/2))-69*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a
rctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-69*I*(-2*cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)
+1)/sin(d*x+c))-69*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(
d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+6*I*(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)
/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)*cos(d*x+c
)^2*sin(d*x+c)-69*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x
+c)/(1+cos(d*x+c)))^(1/2))+176*cos(d*x+c)^2)/(cos(d*x+c)^2-1)/a^2
```

Maxima [A]
time = 0.55, size = 221, normalized size = 1.00

$$-\frac{a^2\left(\frac{2\left(63\,(i\,a\tan(dx+c)+a)^3-107\,(i\,a\tan(dx+c)+a)^2a+34\,(i\,a\tan(dx+c)+a)a^2+4\,a^3\right)}{(i\,a\tan(dx+c)+a)^{\frac{7}{2}}a^3-2\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}a^4+(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a^5}-\frac{3\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)}{a^{\frac{7}{2}}}+\frac{69\log\left(\frac{\sqrt{i\,a\tan(dx+c)+a}-\sqrt{a}}{\sqrt{i\,a\tan(dx+c)+a}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/24*a^2*(2*(63*(I*a*tan(d*x + c) + a)^3 - 107*(I*a*tan(d*x + c) + a)^2*a
+ 34*(I*a*tan(d*x + c) + a)*a^2 + 4*a^3)/((I*a*tan(d*x + c) + a)^(7/2)*a^3
- 2*(I*a*tan(d*x + c) + a)^(5/2)*a^4 + (I*a*tan(d*x + c) + a)^(3/2)*a^5) -
3*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sqrt(2)*sqrt
(a) + sqrt(I*a*tan(d*x + c) + a)))/a^(7/2) + 69*log((sqrt(I*a*tan(d*x + c)
+ a) - sqrt(a))/(sqrt(I*a*tan(d*x + c) + a) + sqrt(a)))/a^(7/2))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 678 vs. $2(171) = 342$.
time = 0.39, size = 678, normalized size = 3.08

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/48*(12*sqrt(1/2)*(a^2*d*e^(7*I*d*x + 7*I*c) - 2*a^2*d*e^(5*I*d*x + 5*I*c
) + a^2*d*e^(3*I*d*x + 3*I*c))*sqrt(1/(a^3*d^2))*log(4*(sqrt(2)*sqrt(1/2)*(
a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1
/(a^3*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 12*sqrt(1/2)*(a^2*d*e^
(7*I*d*x + 7*I*c) - 2*a^2*d*e^(5*I*d*x + 5*I*c) + a^2*d*e^(3*I*d*x + 3*I*c)
)*sqrt(1/(a^3*d^2))*log(-4*(sqrt(2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) +
a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) - a*e^(I*d*x + I
*c))*e^(-I*d*x - I*c)) - 69*(a^2*d*e^(7*I*d*x + 7*I*c) - 2*a^2*d*e^(5*I*d*x
 + 5*I*c) + a^2*d*e^(3*I*d*x + 3*I*c))*sqrt(1/(a^3*d^2))*log(16*(3*a^2*e^(2
*I*d*x + 2*I*c) + 2*sqrt(2)*(a^3*d*e^(3*I*d*x + 3*I*c) + a^3*d*e^(I*d*x + I
```

```
*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) + a^2)*e^(-2*I*d*x
 - 2*I*c)) + 69*(a^2*d*e^(7*I*d*x + 7*I*c) - 2*a^2*d*e^(5*I*d*x + 5*I*c) +
a^2*d*e^(3*I*d*x + 3*I*c))*sqrt(1/(a^3*d^2))*log(16*(3*a^2*e^(2*I*d*x + 2*I
*c) - 2*sqrt(2)*(a^3*d*e^(3*I*d*x + 3*I*c) + a^3*d*e^(I*d*x + I*c))*sqrt(a/
(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) + a^2)*e^(-2*I*d*x - 2*I*c)) +
 4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(37*e^(8*I*d*x + 8*I*c) - 33*e
^(6*I*d*x + 6*I*c) - 50*e^(4*I*d*x + 4*I*c) + 21*e^(2*I*d*x + 2*I*c) + 1))/
(a^2*d*e^(7*I*d*x + 7*I*c) - 2*a^2*d*e^(5*I*d*x + 5*I*c) + a^2*d*e^(3*I*d*x
 + 3*I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(c+dx)}{\left(ia\left(\tan(c+dx)-i\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+I*a*tan(d*x+c))**(3/2),x)
[Out] Integral(cot(c + d*x)**3/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(cot(d*x + c)^3/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [B]
time = 3.94, size = 186, normalized size = 0.85

$$-\frac{\frac{a^2}{3}+\frac{21\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^3}{4\,a}+\frac{17\,a\,\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}{6}-\frac{107\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2}{12}}{d\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2}-2\,a\,d\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}+a^2\,d\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}+\frac{23\,\mathrm{atanh}\left(\frac{a\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{\sqrt{a^3}}\right)}{4\,d\,\sqrt{a^3}}-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,a\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{2\,\sqrt{a^3}}\right)}{4\,d\,\sqrt{a^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] (23*atanh((a*(a + a*tan(c + d*x)*1i)^(1/2))/(a^3)^(1/2)))/(4*d*(a^3)^(1/2))
 - ((21*(a + a*tan(c + d*x)*1i)^3)/(4*a) - (107*(a + a*tan(c + d*x)*1i)^2)/
12 + (17*a*(a + a*tan(c + d*x)*1i))/6 + a^2/3)/(d*(a + a*tan(c + d*x)*1i)^(
7/2) - 2*a*d*(a + a*tan(c + d*x)*1i)^(5/2) + a^2*d*(a + a*tan(c + d*x)*1i)^
(3/2)) - (2^(1/2)*atanh((2^(1/2)*a*(a + a*tan(c + d*x)*1i)^(1/2))/(2*(a^3)^
(1/2))))/(4*d*(a^3)^(1/2))
```

3.126 $\int \frac{\tan^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=205

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,a^{5/2}d}-\frac{\tan^4(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}+\frac{7i\tan^3(c+dx)}{10ad(a+ia\tan(c+dx))^{3/2}}+\frac{89\tan^2(}{20a^2d\sqrt{a+ia}}$$

```
[Out] -1/8*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(5/2)/d*2^(1/2
)-89/5*(a+I*a*tan(d*x+c))^(1/2)/a^3/d+89/20*tan(d*x+c)^2/a^2/d/(a+I*a*tan(d
*x+c))^(1/2)-1/5*tan(d*x+c)^4/d/(a+I*a*tan(d*x+c))^(5/2)+7/10*I*tan(d*x+c)^
3/a/d/(a+I*a*tan(d*x+c))^(3/2)+361/60*(a+I*a*tan(d*x+c))^(3/2)/a^4/d
```

Rubi [A]

time = 0.34, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3639, 3676, 3673, 3608, 3561, 212}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,a^{5/2}d}+\frac{361(a+ia\tan(c+dx))^{3/2}}{60a^4d}-\frac{89\sqrt{a+ia\tan(c+dx)}}{5a^3d}+\frac{89\tan^2(c+dx)}{20a^2d\sqrt{a+ia\tan(c+dx)}}-\frac{\tan^4(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}+\frac{7i\tan^3(c+dx)}{10ad(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] -1/4*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(Sqrt[2]*a^(5/2)
*d) - Tan[c + d*x]^4/(5*d*(a + I*a*Tan[c + d*x])^(5/2)) + (((7*I)/10)*Tan[c
 + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])^(3/2)) + (89*Tan[c + d*x]^2)/(20*a^2
*d*Sqrt[a + I*a*Tan[c + d*x]]) - (89*Sqrt[a + I*a*Tan[c + d*x]])/(5*a^3*d)
+ (361*(a + I*a*Tan[c + d*x])^(3/2))/(60*a^4*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
```

```
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{\tan^4(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} - \frac{\int \frac{\tan^3(c+dx)\left(-4a+\frac{13}{2}ia\tan(c+dx)\right)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{5a^2}$$

$$= -\frac{\tan^4(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{7i\tan^3(c+dx)}{10ad(a+ia\tan(c+dx))^{3/2}} + \frac{\int \frac{\tan^2(c+dx)\left(-63\right.}{\sqrt{a+ia}}}{}$$

$$= -\frac{\tan^4(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{7i\tan^3(c+dx)}{10ad(a+ia\tan(c+dx))^{3/2}} + \frac{89\tan^2(c}{20a^2d\sqrt{a+ia}}$$

$$= -\frac{\tan^4(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{7i\tan^3(c+dx)}{10ad(a+ia\tan(c+dx))^{3/2}} + \frac{89\tan^2(c}{20a^2d\sqrt{a+ia}}$$

$$= -\frac{\tan^4(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{7i\tan^3(c+dx)}{10ad(a+ia\tan(c+dx))^{3/2}} + \frac{89\tan^2(c}{20a^2d\sqrt{a+ia}}$$

$$= -\frac{\tan^4(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{7i\tan^3(c+dx)}{10ad(a+ia\tan(c+dx))^{3/2}} + \frac{89\tan^2(c}{20a^2d\sqrt{a+ia}}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,a^{5/2}d} - \frac{\tan^4(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{7i\,\mathrm{t}}{10ad(a+}$$

Mathematica [A]
time = 2.01, size = 149, normalized size = 0.73

$$-\frac{e^{-4i(c+dx)}\left(3-33e^{2i(c+dx)}+348e^{4i(c+dx)}+1527e^{6i(c+dx)}+983e^{8i(c+dx)}+15e^{5i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{3/2}\sinh^{-1}\left(e^{i(c+dx)}\right)\right)}{60a^2d\left(1+e^{2i(c+dx)}\right)^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] -1/60*(3 - 33*E^((2*I)*(c + d*x)) + 348*E^((4*I)*(c + d*x)) + 1527*E^((6*I)
*(c + d*x)) + 983*E^((8*I)*(c + d*x)) + 15*E^((5*I)*(c + d*x))*(1 + E^((2*I
)*(c + d*x)))^(3/2)*ArcSinh[E^(I*(c + d*x))])/(a^2*d*E^((4*I)*(c + d*x))*(1
 + E^((2*I)*(c + d*x)))^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.22, size = 131, normalized size = 0.64

method	result

derivativedivides	$\frac{\frac{2(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-6a\sqrt{a+ia\tan(dx+c)}-\frac{31a^2}{4\sqrt{a+ia\tan(dx+c)}}+\frac{3a^3}{2(a+ia\tan(dx+c))^{\frac{3}{2}}}-\frac{}{5(a+ia\,\mathrm{ta}}}{a^4d}$
default	$\frac{\frac{2(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-6a\sqrt{a+ia\tan(dx+c)}-\frac{31a^2}{4\sqrt{a+ia\tan(dx+c)}}+\frac{3a^3}{2(a+ia\tan(dx+c))^{\frac{3}{2}}}-\frac{}{5(a+ia\,\mathrm{ta}}}{a^4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^4*(1/3*(a+I*a*tan(d*x+c))^(3/2)-3*a*(a+I*a*tan(d*x+c))^(1/2)-31/8*a^2
/(a+I*a*tan(d*x+c))^(1/2)+3/4*a^3/(a+I*a*tan(d*x+c))^(3/2)-1/10*a^4/(a+I*a*
tan(d*x+c))^(5/2)-1/16*a^(3/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)
*2^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.59, size = 157, normalized size = 0.77

$$\frac{15\sqrt{2}\,a^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+160\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a^2-1440\,\sqrt{i\,a\tan(dx+c)+a}\,a^3-\frac{12\left(155\,(i\,a\tan(dx+c)+a)^2a^4-30\,(i\,a\tan(dx+c)+a)a^5+4\,a^6\right)}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}}{240\,a^6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/240*(15*sqrt(2)*a^(7/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a
))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 160*(I*a*tan(d*x + c)
+ a)^(3/2)*a^2 - 1440*sqrt(I*a*tan(d*x + c) + a)*a^3 - 12*(155*(I*a*tan(d*x
 + c) + a)^2*a^4 - 30*(I*a*tan(d*x + c) + a)*a^5 + 4*a^6)/(I*a*tan(d*x + c)
 + a)^(5/2))/(a^6*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 342 vs. 2(160) = 320.
time = 0.38, size = 342, normalized size = 1.67

$$-\frac{15\sqrt{\frac{1}{2}}\left(a^3de^{(7i\,dx+7i\,c)}+a^3de^{(5i\,dx+5i\,c)}\right)\sqrt{\frac{1}{a^5d^2}}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^3de^{(2i\,dx+2i\,c)}+a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^5d^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-15\sqrt{\frac{1}{2}}\left(a^3de^{(7i\,dx+7i\,c)}+a^3de^{(5i\,dx+5i\,c)}\right)\sqrt{\frac{1}{a^5d^2}}\log\left(-4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^3de^{(2i\,dx+2i\,c)}+a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^5d^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(983\,e^{(8i\,dx+8i\,c)}+1527\,e^{(6i\,dx+6i\,c)}+348\,e^{(4i\,dx+4i\,c)}-33\,e^{(2i\,dx+2i\,c)}+3\right)}{120\left(a^3de^{(7i\,dx+7i\,c)}+a^3de^{(5i\,dx+5i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/120*(15*sqrt(1/2)*(a^3*d*e^(7*I*d*x + 7*I*c) + a^3*d*e^(5*I*d*x + 5*I*c)
)*sqrt(1/(a^5*d^2))*log(4*(sqrt(2)*sqrt(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) + a
^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^5*d^2)) + a*e^(I*d*x + I*
```

```
c))*e^(-I*d*x - I*c)) - 15*sqrt(1/2)*(a^3*d*e^(7*I*d*x + 7*I*c) + a^3*d*e^(
5*I*d*x + 5*I*c))*sqrt(1/(a^5*d^2))*log(-4*(sqrt(2)*sqrt(1/2)*(a^3*d*e^(2*I
*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^5*d^2))
- a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*(983*e^(8*I*d*x + 8*I*c) + 1527*e^(6*I*d*x + 6*I*c) + 348*e^(4*I*d*
x + 4*I*c) - 33*e^(2*I*d*x + 2*I*c) + 3))/(a^3*d*e^(7*I*d*x + 7*I*c) + a^3*
d*e^(5*I*d*x + 5*I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{5}(c + dx)}{(ia(\tan(c + dx) - i))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(tan(c + d*x)**5/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^5/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [B]
time = 3.91, size = 140, normalized size = 0.68

$$-\frac{6\sqrt{a + a\tan(c + dx)\,1i}}{a^3\,d} + \frac{2\,(a + a\tan(c + dx)\,1i)^{3/2}}{3\,a^4\,d} - \frac{\frac{31\,(a + a\tan(c + dx)\,1i)^2}{4} - \frac{3\,a\,(a + a\tan(c + dx)\,1i)}{2} + \frac{a^2}{5}}{a^2\,d\,(a + a\tan(c + dx)\,1i)^{5/2}} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a + a\tan(c + dx)\,1i}\,1i}{2\sqrt{a}}\right)\,1i}{8\,a^{5/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] (2*(a + a*tan(c + d*x)*1i)^(3/2))/(3*a^4*d) - (6*(a + a*tan(c + d*x)*1i)^(1
/2))/(a^3*d) - ((31*(a + a*tan(c + d*x)*1i)^2)/4 - (3*a*(a + a*tan(c + d*x)
*1i))/2 + a^2/5)/(a^2*d*(a + a*tan(c + d*x)*1i)^(5/2)) + (2^(1/2)*atan((2^(
1/2)*(a + a*tan(c + d*x)*1i)^(1/2)*1i)/(2*a^(1/2)))*1i)/(8*a^(5/2)*d)
```

3.127 $\int \frac{\tan^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=176

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,a^{5/2}d}-\frac{\tan^3(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}+\frac{17i\tan^2(c+dx)}{30ad(a+ia\tan(c+dx))^{3/2}}+\frac{}{60a^2d\sqrt{a+}}$$

```
[Out] -1/8*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(5/2)/d*2^(1
/2)+151/60*I/a^2/d/(a+I*a*tan(d*x+c))^(1/2)+83/30*I*(a+I*a*tan(d*x+c))^(1/2
)/a^3/d-1/5*tan(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(5/2)+17/30*I*tan(d*x+c)^2/a/
d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.25, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3639, 3676, 3673, 3607, 3561, 212}

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,a^{5/2}d}+\frac{83i\sqrt{a+ia\tan(c+dx)}}{30a^3d}+\frac{151i}{60a^2d\sqrt{a+ia\tan(c+dx)}}-\frac{\tan^3(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}+\frac{17i\tan^2(c+dx)}{30ad(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-1/4*I)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]*a
^(5/2)*d) - Tan[c + d*x]^3/(5*d*(a + I*a*Tan[c + d*x])^(5/2)) + (((17*I)/30
)*Tan[c + d*x]^2)/(a*d*(a + I*a*Tan[c + d*x])^(3/2)) + ((151*I)/60)/(a^2*d*
Sqrt[a + I*a*Tan[c + d*x]]) + (((83*I)/30)*Sqrt[a + I*a*Tan[c + d*x]])/(a^3
*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
```

```
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{\tan^3(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} - \frac{\int \frac{\tan^2(c+dx)\left(-3a+\frac{11}{2}ia\tan(c+dx)\right)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{5a^2}$$

$$= -\frac{\tan^3(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{17i\tan^2(c+dx)}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{\int \frac{\tan(c+dx)(-17}{\sqrt{a+ia}}$$

$$= -\frac{\tan^3(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{17i\tan^2(c+dx)}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{83i\sqrt{a+ia\,\mathrm{t}}}{30a}$$

$$= -\frac{\tan^3(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{17i\tan^2(c+dx)}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{15}{60a^2d\sqrt{a+i}}$$

$$= -\frac{\tan^3(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{17i\tan^2(c+dx)}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{15}{60a^2d\sqrt{a+i}}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,a^{5/2}d} - \frac{\tan^3(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{1}{30ad(a}$$

Mathematica [A]
time = 1.61, size = 149, normalized size = 0.85

$$\frac{e^{-6i(c+dx)}\left(-\frac{60ie^{7i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} + i\left(1+e^{2i(c+dx)}\right)\left(3-26e^{2i(c+dx)}+194e^{4i(c+dx)}+463e^{6i(c+dx)}\right)\sec^2(c+dx)\right)}{240a^2d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((-60*I)*E^((7*I)*(c + d*x))*ArcSinh[E^(I*(c + d*x))])/Sqrt[1 + E^((2*I)*(
c + d*x))] + I*(1 + E^((2*I)*(c + d*x)))*(3 - 26*E^((2*I)*(c + d*x)) + 194*
E^((4*I)*(c + d*x)) + 463*E^((6*I)*(c + d*x)))*Sec[c + d*x]^2)/(240*a^2*d*E
^((6*I)*(c + d*x))*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.17, size = 111, normalized size = 0.63

method	result

derivativedivides	$\frac{2i\left(\sqrt{a+ia\tan(dx+c)}+\frac{17a}{8\sqrt{a+ia\tan(dx+c)}}-\frac{7a^2}{12(a+ia\tan(dx+c))^{\frac{3}{2}}}+\frac{a^3}{10(a+ia\tan(dx+c))^{\frac{5}{2}}}-\frac{\sqrt{a}}{}\right.}{d\,a^3}$
default	$\frac{2i\left(\sqrt{a+ia\tan(dx+c)}+\frac{17a}{8\sqrt{a+ia\tan(dx+c)}}-\frac{7a^2}{12(a+ia\tan(dx+c))^{\frac{3}{2}}}+\frac{a^3}{10(a+ia\tan(dx+c))^{\frac{5}{2}}}-\frac{\sqrt{a}}{}\right.}{d\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/d/a^3*((a+I*a*tan(d*x+c))^(1/2)+17/8*a/(a+I*a*tan(d*x+c))^(1/2)-7/12*a^
2/(a+I*a*tan(d*x+c))^(3/2)+1/10*a^3/(a+I*a*tan(d*x+c))^(5/2)-1/16*a^(1/2)*2
^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.53, size = 139, normalized size = 0.79

$$\frac{i\left(15\sqrt{2}\,a^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+480\sqrt{i\,a\tan(dx+c)+a}\,a^2+\frac{4\left(255\,(i\,a\tan(dx+c)+a)^2a^3-70\,(i\,a\tan(dx+c)+a)a^4+12\,a^5\right)}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\right)}{240\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/240*I*(15*sqrt(2)*a^(5/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) +
 a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 480*sqrt(I*a*tan(d*x
 + c) + a)*a^2 + 4*(255*(I*a*tan(d*x + c) + a)^2*a^3 - 70*(I*a*tan(d*x + c)
 + a)*a^4 + 12*a^5)/(I*a*tan(d*x + c) + a)^(5/2))/(a^5*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(131) = 262$.
time = 0.38, size = 283, normalized size = 1.61

$$\frac{\left(-15i\sqrt{\frac{1}{2}}\,a^3d\sqrt{\frac{1}{a^5d^2}}\,e^{(5i\,dx+5i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^3de^{(2i\,dx+2i\,c)}+a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^5d^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+15i\sqrt{\frac{1}{2}}\,a^3d\sqrt{\frac{1}{a^5d^2}}\,e^{(5i\,dx+5i\,c)}\log\left(-4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^3de^{(2i\,dx+2i\,c)}+a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^5d^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(463i\,e^{(6i\,dx+6i\,c)}+194i\,e^{(4i\,dx+4i\,c)}-26i\,e^{(2i\,dx+2i\,c)}+3i\right)\right)e^{(-5i\,dx-5i\,c)}}{120\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/120*(-15*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(4*(s
qrt(2)*sqrt(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*sqrt(1/(a^5*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 15*I
```

```
*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(-4*(sqrt(2)*sqrt
(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
*sqrt(1/(a^5*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*sqrt(a/
(e^(2*I*d*x + 2*I*c) + 1))*(463*I*e^(6*I*d*x + 6*I*c) + 194*I*e^(4*I*d*x +
4*I*c) - 26*I*e^(2*I*d*x + 2*I*c) + 3*I))*e^(-5*I*d*x - 5*I*c)/(a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(tan(c + d*x)**4/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^4/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [B]
time = 0.25, size = 129, normalized size = 0.73

$$\frac{\frac{1\mathrm{i}}{5\,d}+\frac{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1\mathrm{i}\right)}^2\,17\mathrm{i}}{4\,a^2\,d}-\frac{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1\mathrm{i}\right)\,7\mathrm{i}}{6\,a\,d}}{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1\mathrm{i}\right)}^{5/2}}+\frac{\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1\mathrm{i}}\,2\mathrm{i}}{a^3\,d}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1\mathrm{i}}}{2\,\sqrt{-a}}\right)\,1\mathrm{i}}{8\,{\left(-a\right)}^{5/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] (1i/(5*d) + ((a + a*tan(c + d*x)*1i)^2*17i)/(4*a^2*d) - ((a + a*tan(c + d*x
)*1i)*7i)/(6*a*d))/(a + a*tan(c + d*x)*1i)^(5/2) + ((a + a*tan(c + d*x)*1i)
^(1/2)*2i)/(a^3*d) + (2^(1/2)*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/
(2*(-a)^(1/2)))*1i)/(8*(-a)^(5/2)*d)
```

3.128 $\int \frac{\tan^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=133

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,a^{5/2}d}-\frac{\tan^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{13}{30ad(a+ia\tan(c+dx))^{3/2}}+\frac{31}{20a^2d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 1/8*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(5/2)/d*2^(1/2)
+31/20/a^2/d/(a+I*a*tan(d*x+c))^(1/2)-1/5*tan(d*x+c)^2/d/(a+I*a*tan(d*x+c))
^(5/2)-13/30/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.16, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3639, 3671, 3607, 3561, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,a^{5/2}d}+\frac{31}{20a^2d\sqrt{a+ia\tan(c+dx)}}-\frac{\tan^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{13}{30ad(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(4*Sqrt[2]*a^(5/2)*d)
 - Tan[c + d*x]^2/(5*d*(a + I*a*Tan[c + d*x])^(5/2)) - 13/(30*a*d*(a + I*a*
Tan[c + d*x])^(3/2)) + 31/(20*a^2*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
```

```
0] && LtQ[m, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3671

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(
A*b - a*B))*(a*c + b*d)*((a + b*Tan[e + f*x])^m/(2*a^2*f*m)), x] + Dist[1/(
2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[A*b*c + a*B*c + a*A*d + b*B*d
 + 2*a*B*d*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{\tan^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} - \frac{\int \frac{\tan(c+dx)\left(-2a+\frac{9}{2}ia\tan(c+dx)\right)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{5a^2}$$

$$= -\frac{\tan^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} - \frac{13}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{i\int \frac{-\frac{13a^2}{2}+9i}{\sqrt{a+ia\,\mathrm{t}}}}{1}$$

$$= -\frac{\tan^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} - \frac{13}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{3}{20a^2d\sqrt{a+i}}$$

$$= -\frac{\tan^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} - \frac{13}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{3}{20a^2d\sqrt{a+i}}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,a^{5/2}d} - \frac{\tan^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} - \frac{}{30ad(a+}$$

Mathematica [A]

time = 1.20, size = 135, normalized size = 1.02

$$\frac{e^{-6i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{3/2}\left(\sqrt{1+e^{2i(c+dx)}}\left(3-19e^{2i(c+dx)}+83e^{4i(c+dx)}\right)+15e^{5i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)\right)\sec^2(c+dx)}{240a^2d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((1 + E^((2*I)*(c + d*x)))^(3/2)*(Sqrt[1 + E^((2*I)*(c + d*x))]*(3 - 19*E^(
(2*I)*(c + d*x)) + 83*E^((4*I)*(c + d*x))) + 15*E^((5*I)*(c + d*x))*ArcSinh
[E^(I*(c + d*x))])*Sec[c + d*x]^2)/(240*a^2*d*E^((6*I)*(c + d*x))*Sqrt[a +
I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.19, size = 93, normalized size = 0.70

method	result
derivativedivides	$-\frac{2\left(-\frac{7}{8\sqrt{a+ia\tan(dx+c)}}+\frac{5a}{12(a+ia\tan(dx+c))^{\frac{3}{2}}}-\frac{a^2}{10(a+ia\tan(dx+c))^{\frac{5}{2}}}-\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\sqrt{2}}{2\sqrt{a}}\right)}{16\sqrt{a}}\right)}{d\,a^2}$
default	$-\frac{2\left(-\frac{7}{8\sqrt{a+ia\tan(dx+c)}}+\frac{5a}{12(a+ia\tan(dx+c))^{\frac{3}{2}}}-\frac{a^2}{10(a+ia\tan(dx+c))^{\frac{5}{2}}}-\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\sqrt{2}}{2\sqrt{a}}\right)}{16\sqrt{a}}\right)}{d\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a^2*(-7/8/(a+I*a*tan(d*x+c))^(1/2)+5/12*a/(a+I*a*tan(d*x+c))^(3/2)-1/1
0*a^2/(a+I*a*tan(d*x+c))^(5/2)-1/16*2^(1/2)/a^(1/2)*arctanh(1/2*(a+I*a*tan(
d*x+c))^(1/2)*2^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.51, size = 121, normalized size = 0.91

$$-\frac{15\sqrt{2}\,a^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)-\frac{4\left(105\,(i\,a\tan(dx+c)+a)^2a^2-50\,(i\,a\tan(dx+c)+a)a^3+12\,a^4\right)}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}}{240\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/240*(15*sqrt(2)*a^(3/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) +
a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - 4*(105*(I*a*tan(d*x +
 c) + a)^2*a^2 - 50*(I*a*tan(d*x + c) + a)*a^3 + 12*a^4)/(I*a*tan(d*x + c)
+ a)^(5/2))/(a^4*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(102) = 204$.
time = 0.37, size = 283, normalized size = 2.13

$$\frac{\left(15\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{1}{a^5d^2}}e^{(5i\,dx+5i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^3de^{(2i\,dx+2i\,c)}+a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^5d^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-15\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{1}{a^5d^2}}e^{(5i\,dx+5i\,c)}\log\left(-4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^3de^{(2i\,dx+2i\,c)}+a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^5d^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(83\,e^{(6i\,dx+6i\,c)}+64\,e^{(4i\,dx+4i\,c)}-16\,e^{(2i\,dx+2i\,c)}+3\right)\right)e^{(-5i\,dx-5i\,c)}}{120\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/120*(15*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(4*(sqrt
(2)*sqrt(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*sqrt(1/(a^5*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 15*sqrt
(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(-4*(sqrt(2)*sqrt(1/2)
*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt
(1/(a^5*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*(83*e^(6*I*d*x + 6*I*c) + 64*e^(4*I*d*x + 4*I*c) - 16
*e^(2*I*d*x + 2*I*c) + 3))*e^(-5*I*d*x - 5*I*c)/(a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(tan(c + d*x)**3/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^3/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [B]
time = 3.86, size = 93, normalized size = 0.70

$$\frac{\frac{7\,(a+a\tan(c+d\,x)\,1\mathrm{i})^2}{4}-\frac{5\,a\,(a+a\tan(c+d\,x)\,1\mathrm{i})}{6}+\frac{a^2}{5}}{a^2\,d\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{5/2}}+\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{2\,\sqrt{a}}\right)}{8\,a^{5/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] ((7*(a + a*tan(c + d*x)*1i)^2)/4 - (5*a*(a + a*tan(c + d*x)*1i))/6 + a^2/5)
/(a^2*d*(a + a*tan(c + d*x)*1i)^(5/2)) + (2^(1/2)*atanh((2^(1/2)*(a + a*tan
(c + d*x)*1i)^(1/2))/(2*a^(1/2))))/(8*a^(5/2)*d)
```

3.129 $\int \frac{\tan^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=133

$$\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,a^{5/2}d}-\frac{i}{5d(a+ia\tan(c+dx))^{5/2}}+\frac{i}{2ad(a+ia\tan(c+dx))^{3/2}}-\frac{i}{4a^2d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 1/8*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(5/2)/d*2^(1/
2)-1/4*I/a^2/d/(a+I*a*tan(d*x+c))^(1/2)-1/5*I/d/(a+I*a*tan(d*x+c))^(5/2)+1/
2*I/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.11, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3621, 3607, 3560, 3561, 212}

$$\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,a^{5/2}d}-\frac{i}{4a^2d\sqrt{a+ia\tan(c+dx)}}+\frac{i}{2ad(a+ia\tan(c+dx))^{3/2}}-\frac{i}{5d(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((I/4)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]*a^(5
/2)*d) - (I/5)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (I/2)/(a*d*(a + I*a*Tan[c
 + d*x])^(3/2)) - (I/4)/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rule 3621

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(a*c + b*d)^2*((a + b*Tan[e + f*x])^
m/(2*a^3*f*m)), x] + Dist[1/(2*a^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[
a*c^2 - 2*b*c*d + a*d^2 - 2*b*d^2*Tan[e + f*x], x], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LeQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{i}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{\int \frac{a-2ia\tan(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{2a^2}$$

$$= -\frac{i}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{i}{2ad(a+ia\tan(c+dx))^{3/2}} - \frac{\int \frac{1}{\sqrt{a+ia\tan(}}}{4a^2}$$

$$= -\frac{i}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{i}{2ad(a+ia\tan(c+dx))^{3/2}} - \frac{i}{4a^2d\sqrt{a+ia\tan}}$$

$$= -\frac{i}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{i}{2ad(a+ia\tan(c+dx))^{3/2}} - \frac{i}{4a^2d\sqrt{a+ia\tan}}$$

$$= \frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,a^{5/2}d} - \frac{i}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{}{2ad(a+i}$$

Mathematica [A]
time = 1.11, size = 135, normalized size = 1.02

$$-\frac{ie^{-6i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{3/2}\left(\sqrt{1+e^{2i(c+dx)}}\left(1-3e^{2i(c+dx)}+e^{4i(c+dx)}\right)-5e^{5i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)\right)\sec^2(c+dx)}{80a^2d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-1/80*I)*(1 + E^((2*I)*(c + d*x)))^(3/2)*(Sqrt[1 + E^((2*I)*(c + d*x))]*(
1 - 3*E^((2*I)*(c + d*x)) + E^((4*I)*(c + d*x))) - 5*E^((5*I)*(c + d*x))*Ar
cSinh[E^(I*(c + d*x))])*Sec[c + d*x]^2)/(a^2*d*E^((6*I)*(c + d*x))*Sqrt[a +
 I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.18, size = 94, normalized size = 0.71

method	result
derivativedivides	$-\frac{2i\left(-\frac{1}{4(a+ia\tan(dx+c))^{\frac{3}{2}}}+\frac{1}{8a\sqrt{a+ia\tan(dx+c)}}+\frac{a}{10(a+ia\tan(dx+c))^{\frac{5}{2}}}-\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{a+ia\tan}}{2}\right.}{16a^{\frac{3}{2}}}\right.}{da}$
default	$-\frac{2i\left(-\frac{1}{4(a+ia\tan(dx+c))^{\frac{3}{2}}}+\frac{1}{8a\sqrt{a+ia\tan(dx+c)}}+\frac{a}{10(a+ia\tan(dx+c))^{\frac{5}{2}}}-\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{a+ia\tan}}{2}\right.}{16a^{\frac{3}{2}}}\right.}{da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/d/a*(-1/4/(a+I*a*tan(d*x+c))^(3/2)+1/8/a/(a+I*a*tan(d*x+c))^(1/2)+1/10
*a/(a+I*a*tan(d*x+c))^(5/2)-1/16/a^(3/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x
+c))^(1/2)*2^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.52, size = 119, normalized size = 0.89

$$-\frac{i\left(5\sqrt{2}\sqrt{a}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)+\frac{4\left(5\left(i\,a\tan(dx+c)+a\right)^2a-10\left(i\,a\tan(dx+c)+a\right)a^2+4\,a^3\right)}{\left(i\,a\tan(dx+c)+a\right)^{\frac{5}{2}}}\right)}{80\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/80*I*(5*sqrt(2)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) +
a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(5*(I*a*tan(d*x + c
) + a)^2*a - 10*(I*a*tan(d*x + c) + a)*a^2 + 4*a^3)/(I*a*tan(d*x + c) + a)^
(5/2))/(a^3*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(94) = 188$.
time = 0.37, size = 283, normalized size = 2.13

$$\frac{\left(5i\sqrt{\frac{1}{2}}\,a^3d\sqrt{\frac{1}{a^5d^2}}\,e^{(5i\,dx+5i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^3de^{(2i\,dx+2i\,c)}+a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^5d^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-5i\sqrt{\frac{1}{2}}\,a^3d\sqrt{\frac{1}{a^5d^2}}\,e^{(5i\,dx+5i\,c)}\log\left(-4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^3de^{(2i\,dx+2i\,c)}+a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^5d^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-i\,e^{(6i\,dx+6i\,c)}+2i\,e^{(4i\,dx+4i\,c)}+2i\,e^{(2i\,dx+2i\,c)}-i\right)\right)e^{(-5i\,dx-5i\,c)}}{40\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/40*(5*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(4*(sqrt
(2)*sqrt(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*sqrt(1/(a^5*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 5*I*sqr
t(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(-4*(sqrt(2)*sqrt(1/2
)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqr
t(1/(a^5*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*sqrt(a/(e^(
2*I*d*x + 2*I*c) + 1))*(-I*e^(6*I*d*x + 6*I*c) + 2*I*e^(4*I*d*x + 4*I*c) +
2*I*e^(2*I*d*x + 2*I*c) - I))*e^(-5*I*d*x - 5*I*c)/(a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(tan(c + d*x)**2/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^2/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [B]
time = 0.20, size = 88, normalized size = 0.66

$$\frac{1\mathrm{i}}{20\,d\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{5/2}}+\frac{\tan(c+d\,x)^2\,1\mathrm{i}}{4\,d\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{5/2}}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{2\,\sqrt{-a}}\right)\,1\mathrm{i}}{8\,(-a)^{5/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] 1i/(20*d*(a + a*tan(c + d*x)*1i)^(5/2)) + (tan(c + d*x)^2*1i)/(4*d*(a + a*t
an(c + d*x)*1i)^(5/2)) - (2^(1/2)*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/
2))/(2*(-a)^(1/2)))*1i)/(8*(-a)^(5/2)*d)
```

3.130 $\int \frac{\tan(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=125

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,a^{5/2}d}-\frac{1}{5d(a+ia\tan(c+dx))^{5/2}}+\frac{1}{6ad(a+ia\tan(c+dx))^{3/2}}+\frac{1}{4a^2d\sqrt{a+ia}}$$

```
[Out] -1/8*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(5/2)/d*2^(1/2
)+1/4/a^2/d/(a+I*a*tan(d*x+c))^(1/2)-1/5/d/(a+I*a*tan(d*x+c))^(5/2)+1/6/a/d
/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3607, 3560, 3561, 212}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,a^{5/2}d}+\frac{1}{4a^2d\sqrt{a+ia\tan(c+dx)}}+\frac{1}{6ad(a+ia\tan(c+dx))^{3/2}}-\frac{1}{5d(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] -1/4*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(Sqrt[2]*a^(5/2)
*d) - 1/(5*d*(a + I*a*Tan[c + d*x])^(5/2)) + 1/(6*a*d*(a + I*a*Tan[c + d*x]
)^(3/2)) + 1/(4*a^2*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{1}{5d(a+ia\tan(c+dx))^{5/2}} - \frac{i\int \frac{1}{(a+ia\tan(c+dx))^{3/2}}\,dx}{2a}$$

$$= -\frac{1}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{1}{6ad(a+ia\tan(c+dx))^{3/2}} - \frac{i\int \frac{1}{\sqrt{a+ia\tan}}}{4a^2}$$

$$= -\frac{1}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{1}{6ad(a+ia\tan(c+dx))^{3/2}} + \frac{1}{4a^2d\sqrt{a+ia\tan}}$$

$$= -\frac{1}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{1}{6ad(a+ia\tan(c+dx))^{3/2}} + \frac{1}{4a^2d\sqrt{a+ia\tan}}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,a^{5/2}d} - \frac{1}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{1}{6ad(a+i}$$

Mathematica [A]
time = 0.78, size = 135, normalized size = 1.08

$$\frac{e^{-6i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{3/2}\left(\sqrt{1+e^{2i(c+dx)}}\left(-3-e^{2i(c+dx)}+17e^{4i(c+dx)}\right)-15e^{5i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)\right)\sec^2(c+dx)}{240a^2d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((1 + E^((2*I)*(c + d*x)))^(3/2)*(Sqrt[1 + E^((2*I)*(c + d*x))]*(-3 - E^((2
*I)*(c + d*x)) + 17*E^((4*I)*(c + d*x))) - 15*E^((5*I)*(c + d*x))*ArcSinh[E
^(I*(c + d*x))])*Sec[c + d*x]^2)/(240*a^2*d*E^((6*I)*(c + d*x))*Sqrt[a + I*
a*Tan[c + d*x]])
```

Maple [A]
time = 0.18, size = 91, normalized size = 0.73

method	result
derivativedivides	$\frac{-\frac{1}{5(a+ia\tan(dx+c))^{\frac{5}{2}}}+\frac{1}{4a^2\sqrt{a+ia\tan(dx+c)}}+\frac{1}{6a(a+ia\tan(dx+c))^{\frac{3}{2}}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx}{2\sqrt{a}}\right.}{8a^{\frac{5}{2}}}}{d}$
default	$\frac{-\frac{1}{5(a+ia\tan(dx+c))^{\frac{5}{2}}}+\frac{1}{4a^2\sqrt{a+ia\tan(dx+c)}}+\frac{1}{6a(a+ia\tan(dx+c))^{\frac{3}{2}}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx}{2\sqrt{a}}\right.}{8a^{\frac{5}{2}}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5/(a+I*a*tan(d*x+c))^(5/2)+1/4/a^2/(a+I*a*tan(d*x+c))^(1/2)+1/6/a/(
a+I*a*tan(d*x+c))^(3/2)-1/8/a^(5/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^
(1/2)*2^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.51, size = 116, normalized size = 0.93

$$\frac{\frac{15\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)}{\sqrt{a}}+\frac{4\left(15\,(i\,a\tan(dx+c)+a)^2+10\,(i\,a\tan(dx+c)+a)a-12\,a^2\right)}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}}{240\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/240*(15*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sqrt
(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/sqrt(a) + 4*(15*(I*a*tan(d*x + c
) + a)^2 + 10*(I*a*tan(d*x + c) + a)*a - 12*a^2)/(I*a*tan(d*x + c) + a)^(5/
2))/(a^2*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(94) = 188$.
time = 0.38, size = 284, normalized size = 2.27

$$-\frac{\left(15\sqrt{\frac{1}{2}}\,a^3d\sqrt{\frac{1}{a^5d^2}}\,e^{(5i\,dx+5i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^3de^{(2i\,dx+2i\,c)}+a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^5d^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-15\sqrt{\frac{1}{2}}\,a^3d\sqrt{\frac{1}{a^5d^2}}\,e^{(5i\,dx+5i\,c)}\log\left(-4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^3de^{(2i\,dx+2i\,c)}+a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^5d^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(17\,e^{(6i\,dx+6i\,c)}+16\,e^{(4i\,dx+4i\,c)}-4\,e^{(2i\,dx+2i\,c)}-3\right)\right)e^{(-5i\,dx-5i\,c)}}{120\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/120*(15*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(4*(sqr
t(2)*sqrt(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I
```

```
*c) + 1))*sqrt(1/(a^5*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 15*sqr
t(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(-4*(sqrt(2)*sqrt(1/2
)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqr
t(1/(a^5*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - sqrt(2)*sqrt(a/(e^(
2*I*d*x + 2*I*c) + 1))*(17*e^(6*I*d*x + 6*I*c) + 16*e^(4*I*d*x + 4*I*c) - 4
*e^(2*I*d*x + 2*I*c) - 3))*e^(-5*I*d*x - 5*I*c)/(a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan (c + dx)}{(ia (\tan (c + dx) - i))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(tan(c + d*x)/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [B]
time = 0.20, size = 91, normalized size = 0.73

$$\frac{\frac{a+a\tan(c+d\,x)\,1\mathrm{i}}{6\,a}+\frac{(a+a\tan(c+d\,x)\,1\mathrm{i})^2}{4\,a^2}-\frac{1}{5}}{d\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{5/2}}-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{2\,\sqrt{a}}\right)}{8\,a^{5/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] ((a + a*tan(c + d*x)*1i)/(6*a) + (a + a*tan(c + d*x)*1i)^2/(4*a^2) - 1/5)/(
d*(a + a*tan(c + d*x)*1i)^(5/2)) - (2^(1/2)*atanh((2^(1/2)*(a + a*tan(c + d
*x)*1i)^(1/2))/(2*a^(1/2))))/(8*a^(5/2)*d)
```

3.131 $\int \frac{1}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=133

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,a^{5/2}d}+\frac{i}{5d(a+ia\tan(c+dx))^{5/2}}+\frac{i}{6ad(a+ia\tan(c+dx))^{3/2}}+\frac{i}{4a^2d\sqrt{a+i}}$$

```
[Out] -1/8*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(5/2)/d*2^(1
/2)+1/4*I/a^2/d/(a+I*a*tan(d*x+c))^(1/2)+1/5*I/d/(a+I*a*tan(d*x+c))^(5/2)+1
/6*I/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.06, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3560, 3561, 212}

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,a^{5/2}d}+\frac{i}{4a^2d\sqrt{a+ia\tan(c+dx)}}+\frac{i}{6ad(a+ia\tan(c+dx))^{3/2}}+\frac{i}{5d(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(-5/2),x]
```

```
[Out] ((-1/4*I)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]*a
^(5/2)*d) + (I/5)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (I/6)/(a*d*(a + I*a*Ta
n[c + d*x])^(3/2)) + (I/4)/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{i}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{\int \frac{1}{(a+ia\tan(c+dx))^{3/2}}\,dx}{2a}$$

$$= \frac{i}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{i}{6ad(a+ia\tan(c+dx))^{3/2}} + \frac{\int \frac{1}{\sqrt{a+ia\tan(c-}}}{4a^2}$$

$$= \frac{i}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{i}{6ad(a+ia\tan(c+dx))^{3/2}} + \frac{i}{4a^2d\sqrt{a+ia\tan(}}$$

$$= \frac{i}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{i}{6ad(a+ia\tan(c+dx))^{3/2}} + \frac{i}{4a^2d\sqrt{a+ia\tan(}}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,a^{5/2}d} + \frac{i}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{}{6ad(a+}$$

Mathematica [A]
time = 0.75, size = 137, normalized size = 1.03

$$\frac{ie^{-6i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{3/2}\left(\sqrt{1+e^{2i(c+dx)}}\left(3+11e^{2i(c+dx)}+23e^{4i(c+dx)}\right)-15e^{5i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)\right)\sec^2(c+dx)}{240a^2d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(-5/2),x]
```

```
[Out] ((I/240)*(1 + E^((2*I)*(c + d*x)))^(3/2)*(Sqrt[1 + E^((2*I)*(c + d*x))]*(3
+ 11*E^((2*I)*(c + d*x)) + 23*E^((4*I)*(c + d*x))) - 15*E^((5*I)*(c + d*x))
*ArcSinh[E^(I*(c + d*x))])*Sec[c + d*x]^2)/(a^2*d*E^((6*I)*(c + d*x))*Sqrt[
a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.14, size = 97, normalized size = 0.73

method	result
derivativedivides	$\frac{2ia\left(\frac{1}{8a^3\sqrt{a+ia\tan(dx+c)}}+\frac{1}{12a^2(a+ia\tan(dx+c))^{\frac{3}{2}}}+\frac{1}{10a(a+ia\tan(dx+c))^{\frac{5}{2}}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan}}{2}\right.}{16a^{\frac{7}{2}}}\right.}{d}$

default	$\frac{2ia\left(\frac{1}{8a^{3}\sqrt{a+ia\tan\left(dx+c\right)}}+\frac{1}{12a^{2}\left(a+ia\tan\left(dx+c\right)\right)^{\frac{3}{2}}}+\frac{1}{10a\left(a+ia\tan\left(dx+c\right)\right)^{\frac{5}{2}}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan}}{}\right.}{16a^{\frac{7}{2}}}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/d*a*(1/8/a^3/(a+I*a*tan(d*x+c))^(1/2)+1/12/a^2/(a+I*a*tan(d*x+c))^(3/2)
+1/10/a/(a+I*a*tan(d*x+c))^(5/2)-1/16/a^(7/2)*2^(1/2)*arctanh(1/2*(a+I*a*ta
n(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.55, size = 119, normalized size = 0.89

$$\frac{i\left(\frac{15\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)}{a^{\frac{3}{2}}}+\frac{4\left(15\left(i\,a\tan\left(dx+c\right)+a\right)^{2}+10\left(i\,a\tan\left(dx+c\right)+a\right)a+12\,a^{2}\right)}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}a}\right)}{240\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/240*I*(15*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sq
rt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/a^(3/2) + 4*(15*(I*a*tan(d*x +
 c) + a)^2 + 10*(I*a*tan(d*x + c) + a)*a + 12*a^2)/((I*a*tan(d*x + c) + a)^
(5/2)*a))/(a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(94) = 188$.
time = 0.36, size = 283, normalized size = 2.13

$$\frac{\left(-15i\sqrt{\frac{1}{2}}\,a^{3}d\sqrt{\frac{1}{a^{5}d^{2}}}\,e^{\left(5i\,dx+5i\,c\right)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^{3}de^{\left(2i\,dx+2i\,c\right)}+a^{3}d\right)\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}\sqrt{\frac{1}{a^{5}d^{2}}}+ae^{\left(i\,dx+i\,c\right)}\right)e^{\left(-i\,dx-i\,c\right)}\right)+15i\sqrt{\frac{1}{2}}\,a^{3}d\sqrt{\frac{1}{a^{5}d^{2}}}\,e^{\left(5i\,dx+5i\,c\right)}\log\left(-4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^{3}de^{\left(2i\,dx+2i\,c\right)}+a^{3}d\right)\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}\sqrt{\frac{1}{a^{5}d^{2}}}-ae^{\left(i\,dx+i\,c\right)}\right)e^{\left(-i\,dx-i\,c\right)}\right)+\sqrt{2}\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}\left(23i\,e^{\left(6i\,dx+6i\,c\right)}+34i\,e^{\left(4i\,dx+4i\,c\right)}+14i\,e^{\left(2i\,dx+2i\,c\right)}+3i\right)\right)e^{\left(-5i\,dx-5i\,c\right)}}{120\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/120*(-15*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(4*(s
qrt(2)*sqrt(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*sqrt(1/(a^5*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 15*I
*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(-4*(sqrt(2)*sqrt
(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
*sqrt(1/(a^5*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*sqrt(a/
```

```
(e^(2*I*d*x + 2*I*c) + 1))*(23*I*e^(6*I*d*x + 6*I*c) + 34*I*e^(4*I*d*x + 4*
I*c) + 14*I*e^(2*I*d*x + 2*I*c) + 3*I))*e^(-5*I*d*x - 5*I*c)/(a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia\tan{(c+dx)}+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral((I*a*tan(c + d*x) + a)**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(-5/2), x)
```

Mupad [B]
time = 3.81, size = 106, normalized size = 0.80

$$\frac{\frac{1\mathrm{i}}{5\,d}+\frac{(a+a\tan(c+d\,x)\,1\mathrm{i})^2\,1\mathrm{i}}{4\,a^2\,d}+\frac{(a+a\tan(c+d\,x)\,1\mathrm{i})\,1\mathrm{i}}{6\,a\,d}}{(a+a\tan{(c+d\,x)}\,1\mathrm{i})^{5/2}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan{(c+d\,x)}\,1\mathrm{i}}}{2\,\sqrt{-a}}\right)\,1\mathrm{i}}{8\,(-a)^{5/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] (1i/(5*d) + ((a + a*tan(c + d*x)*1i)^2*1i)/(4*a^2*d) + ((a + a*tan(c + d*x)
*1i)*1i)/(6*a*d))/(a + a*tan(c + d*x)*1i)^(5/2) + (2^(1/2)*atan((2^(1/2)*(a
 + a*tan(c + d*x)*1i)^(1/2))/(2*(-a)^(1/2)))*1i)/(8*(-a)^(5/2)*d)
```

3.132 $\int \frac{\cot(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=159

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,a^{5/2}d}+\frac{1}{5d(a+ia\tan(c+dx))^{5/2}}+\frac{}{2ad(a}$$

```
[Out] -2*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d+1/8*arctanh(1/2*(a+I
*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(5/2)/d*2^(1/2)+7/4/a^2/d/(a+I*a*ta
n(d*x+c))^(1/2)+1/5/d/(a+I*a*tan(d*x+c))^(5/2)+1/2/a/d/(a+I*a*tan(d*x+c))^(
3/2)
```

Rubi [A]
time = 0.35, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3640, 3677, 3681, 3561, 212, 3680, 65, 214}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,a^{5/2}d}+\frac{7}{4a^2d\sqrt{a+ia\tan(c+dx)}}+\frac{1}{2ad(a+ia\tan(c+dx))^{3/2}}+\frac{1}{5d(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/(a^(5/2)*d) + ArcTanh[Sqrt
[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(4*Sqrt[2]*a^(5/2)*d) + 1/(5*d*(a
 + I*a*Tan[c + d*x])^(5/2)) + 1/(2*a*d*(a + I*a*Tan[c + d*x])^(3/2)) + 7/(4
*a^2*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3681

```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(
A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
```

*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{1}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{\int \frac{\cot(c+dx)\left(5a-\frac{5}{2}ia\tan(c+dx)\right)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{5a^2}$$

$$= \frac{1}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{1}{2ad(a+ia\tan(c+dx))^{3/2}} + \frac{\int \frac{\cot(c+dx)\left(15a^2-\frac{4}{\ldots}\right.}{\sqrt{a+ia\tan\ldots}}}{15a\ldots}$$

$$= \frac{1}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{1}{2ad(a+ia\tan(c+dx))^{3/2}} + \frac{7}{4a^2d\sqrt{a+ia\tan\ldots}}$$

$$= \frac{1}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{1}{2ad(a+ia\tan(c+dx))^{3/2}} + \frac{7}{4a^2d\sqrt{a+ia\tan\ldots}}$$

$$= \frac{1}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{1}{2ad(a+ia\tan(c+dx))^{3/2}} + \frac{7}{4a^2d\sqrt{a+ia\tan\ldots}}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,a^{5/2}d} + \frac{1}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{\ldots}{2ad(a+i\ldots}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,a^{5/2}d} + \ldots$$

Mathematica [A]
time = 2.07, size = 188, normalized size = 1.18

$$\frac{e^{-6i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{3/2}\left(\sqrt{1+e^{2i(c+dx)}}\left(1+7e^{2i(c+dx)}+41e^{4i(c+dx)}\right)+5e^{5i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)-40\sqrt{2}\,e^{5i(c+dx)}\tanh^{-1}\left(\frac{\sqrt{2}\,e^{i(c+dx)}}{\sqrt{1+e^{2i(c+dx)}}}\right)\right)\sec^2(c+dx)}{80a^2d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((1 + E^((2*I)*(c + d*x)))^(3/2)*(Sqrt[1 + E^((2*I)*(c + d*x))]*(1 + 7*E^((
2*I)*(c + d*x)) + 41*E^((4*I)*(c + d*x))) + 5*E^((5*I)*(c + d*x))*ArcSinh[E
^(I*(c + d*x))] - 40*Sqrt[2]*E^((5*I)*(c + d*x))*ArcTanh[(Sqrt[2]*E^(I*(c +
 d*x)))/Sqrt[1 + E^((2*I)*(c + d*x))]])*Sec[c + d*x]^2)/(80*a^2*d*E^((6*I)*
(c + d*x))*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 723 vs. $2(125) = 250$.
time = 0.93, size = 724, normalized size = 4.55

method	result
default	$\frac{\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(-64i\left(\cos^5(dx+c)\right)\sin(dx+c)+64\left(\cos^6(dx+c)\right)-5i\sqrt{2}\,\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\ldots\right.\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/80/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(-64*I*cos(d*x+c)^5*s
in(d*x+c)+64*cos(d*x+c)^6-5*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1
/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*2^(1/2))*sin(d*x+c)*2^(1/2)-64*I*cos(d*x+c)^3*sin(d*x+c)+40*I*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))
*sin(d*x+c)-40*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*cos(d*x+c)+5*2^(1/
2)*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-I*cos(d*x+c
)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+32
*cos(d*x+c)^4-40*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))-140*I*cos(d*x+c)
*sin(d*x+c)-40*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+5*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*2^(1/2))-40*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin
(d*x+c)-40*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2))+100*cos(d*x+c)^2)/a^3
```

Maxima [A]
time = 0.50, size = 161, normalized size = 1.01

$$-\frac{\frac{5\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan\left(dx+c\right)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan\left(dx+c\right)+a}}\right)}{a^{\frac{5}{2}}}-\frac{80\log\left(\frac{\sqrt{ia\tan\left(dx+c\right)+a}-\sqrt{a}}{\sqrt{ia\tan\left(dx+c\right)+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}-\frac{4\left(35\left(ia\tan(dx+c)+a\right)^2+10\left(ia\tan(dx+c)+a\right)a+4a^2\right)}{\left(ia\tan(dx+c)+a\right)^{\frac{5}{2}}a^2}}{80\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/80*(5*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sqrt(
2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/a^(5/2) - 80*log((sqrt(I*a*tan(d*
x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x + c) + a) + sqrt(a)))/a^(5/2) - 4*
```

```
(35*(I*a*tan(d*x + c) + a)^2 + 10*(I*a*tan(d*x + c) + a)*a + 4*a^2)/((I*a*t
an(d*x + c) + a)^(5/2)*a^2))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(120) = 240$.
time = 0.37, size = 511, normalized size = 3.21

$$\frac{5\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{1}{a^5d^2}}e^{(5idx+5ic)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^3de^{(2idx+2ic)}+a^3d\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{1}{a^5d^2}}+ae^{(idx+ic)}\right)e^{(-idx-ic)}\right)-5\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{1}{a^5d^2}}e^{(5idx+5ic)}\log\left(-4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^3de^{(2idx+2ic)}+a^3d\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{1}{a^5d^2}}-ae^{(idx+ic)}\right)e^{(-idx-ic)}\right)-20a^3d\sqrt{\frac{1}{a^5d^2}}e^{(5idx+5ic)}\log\left(16\left(3a^2e^{(2idx+2ic)}+2\sqrt{2}\left(a^4de^{(3idx+3ic)}+a^4de^{(idx+ic)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{1}{a^5d^2}}+a^2\right)e^{(-2idx-2ic)}\right)+20a^3d\sqrt{\frac{1}{a^5d^2}}e^{(5idx+5ic)}\log\left(16\left(3a^2e^{(2idx+2ic)}-2\sqrt{2}\left(a^4de^{(3idx+3ic)}+a^4de^{(idx+ic)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{1}{a^5d^2}}+a^2\right)e^{(-2idx-2ic)}\right)+\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\left(41e^{(6idx+6ic)}+48e^{(4idx+4ic)}+8e^{(2idx+2ic)}+1\right)e^{(-5idx-5ic)}}{40a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/40*(5*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(4*(sqrt(2
)*sqrt(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c)
 + 1))*sqrt(1/(a^5*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 5*sqrt(1/
2)*a^3*d*sqrt(1/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(a
^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/
(a^5*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 20*a^3*d*sqrt(1/(a^5*d^
2))*e^(5*I*d*x + 5*I*c)*log(16*(3*a^2*e^(2*I*d*x + 2*I*c) + 2*sqrt(2)*(a^4*
d*e^(3*I*d*x + 3*I*c) + a^4*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1))*sqrt(1/(a^5*d^2)) + a^2)*e^(-2*I*d*x - 2*I*c)) + 20*a^3*d*sqrt(1/(a^5
*d^2))*e^(5*I*d*x + 5*I*c)*log(16*(3*a^2*e^(2*I*d*x + 2*I*c) - 2*sqrt(2)*(a
^4*d*e^(3*I*d*x + 3*I*c) + a^4*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*sqrt(1/(a^5*d^2)) + a^2)*e^(-2*I*d*x - 2*I*c)) + sqrt(2)*sqrt(a/(e
^(2*I*d*x + 2*I*c) + 1))*(41*e^(6*I*d*x + 6*I*c) + 48*e^(4*I*d*x + 4*I*c) +
 8*e^(2*I*d*x + 2*I*c) + 1))*e^(-5*I*d*x - 5*I*c)/(a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(cot(c + d*x)/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [B]
time = 0.25, size = 132, normalized size = 0.83

$$-\frac{2\,\mathrm{atanh}\left(\frac{a^2\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{\sqrt{a^5}}\right)}{d\,\sqrt{a^5}}+\frac{\frac{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}{2\,a}+\frac{7\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2}{4\,a^2}+\frac{1}{5}}{d\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}}+\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,a^2\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{2\,\sqrt{a^5}}\right)}{8\,d\,\sqrt{a^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] ((a + a*tan(c + d*x)*1i)/(2*a) + (7*(a + a*tan(c + d*x)*1i)^2)/(4*a^2) + 1/
5)/(d*(a + a*tan(c + d*x)*1i)^(5/2)) - (2*atanh((a^2*(a + a*tan(c + d*x)*1i
)^(1/2))/(a^5)^(1/2)))/(d*(a^5)^(1/2)) + (2^(1/2)*atanh((2^(1/2)*a^2*(a + a
*tan(c + d*x)*1i)^(1/2))/(2*(a^5)^(1/2))))/(8*d*(a^5)^(1/2))
```

3.133 $\int \frac{\cot^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=214

$$\frac{5i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d}+\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,a^{5/2}d}+\frac{\cot(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}+\frac{}{30ad(}$$

```
[Out] 5*I*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d+1/8*I*arctanh(1/2*(
a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(5/2)/d*2^(1/2)+41/12*cot(d*x+c)
/a^2/d/(a+I*a*tan(d*x+c))^(1/2)-21/4*cot(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/a^
3/d+1/5*cot(d*x+c)/d/(a+I*a*tan(d*x+c))^(5/2)+19/30*cot(d*x+c)/a/d/(a+I*a*t
an(d*x+c))^(3/2)
```

Rubi [A]

time = 0.48, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3640, 3677, 3679, 3681, 3561, 212, 3680, 65, 214}

$$\frac{5i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d}+\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,a^{5/2}d}-\frac{21\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{4a^3d}+\frac{41\cot(c+dx)}{12a^2d\sqrt{a+ia\tan(c+dx)}}+\frac{19\cot(c+dx)}{30ad(a+ia\tan(c+dx))^{3/2}}+\frac{\cot(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((5*I)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/(a^(5/2)*d) + ((I/4)*Ar
cTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]*a^(5/2)*d) +
Cot[c + d*x]/(5*d*(a + I*a*Tan[c + d*x])^(5/2)) + (19*Cot[c + d*x])/(30*a*d
*(a + I*a*Tan[c + d*x])^(3/2)) + (41*Cot[c + d*x])/(12*a^2*d*Sqrt[a + I*a*T
an[c + d*x]]) - (21*Cot[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(4*a^3*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
```

```
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3681

```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(
A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\cot(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{\int \frac{\cot^2(c+dx)\left(6a-\frac{7}{2}ia\tan(c+dx)\right)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{5a^2}$$

$$= \frac{\cot(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{19\cot(c+dx)}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{\int \frac{\cot^2(c+dx)\left(\frac{55a^2}{2}\right.}{\sqrt{a+ia\,\mathrm{t}}}}{1}$$

$$= \frac{\cot(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{19\cot(c+dx)}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{41\cot(c}{12a^2d\sqrt{a+ia\,\mathrm{t}}}$$

$$= \frac{\cot(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{19\cot(c+dx)}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{41\cot(c}{12a^2d\sqrt{a+ia\,\mathrm{t}}}$$

$$= \frac{\cot(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{19\cot(c+dx)}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{41\cot(c}{12a^2d\sqrt{a+ia\,\mathrm{t}}}$$

$$= \frac{\cot(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{19\cot(c+dx)}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{41\cot(c}{12a^2d\sqrt{a+ia\,\mathrm{t}}}$$

$$= \frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,a^{5/2}d} + \frac{\cot(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{19}{30ad(a+}$$

$$= \frac{5i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,a^{5/2}d} +$$

Mathematica [A]

time = 3.60, size = 263, normalized size = 1.23

$$-\frac{i\left(3+31e^{2i(c+dx)}+280e^{4i(c+dx)}-151e^{6i(c+dx)}-403e^{8i(c+dx)}+15e^{5i(c+dx)}\left(-1+e^{2i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\,\sinh^{-1}\left(e^{i(c+dx)}\right)+300\sqrt{2}\,e^{5i(c+dx)}\left(-1+e^{2i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\,\tanh^{-1}\left(\frac{\sqrt{2}\,e^{i(c+dx)}}{\sqrt{1+e^{2i(c+dx)}}}\right)\right)}{15a^2d\left(-1+e^{2i(c+dx)}\right)\left(1+e^{2i(c+dx)}\right)^3(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-1/15*I)*(3 + 31*E^((2*I)*(c + d*x)) + 280*E^((4*I)*(c + d*x)) - 151*E^((
6*I)*(c + d*x)) - 403*E^((8*I)*(c + d*x)) + 15*E^((5*I)*(c + d*x))*(-1 + E^
((2*I)*(c + d*x)))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))] +
 300*Sqrt[2]*E^((5*I)*(c + d*x))*(-1 + E^((2*I)*(c + d*x)))*Sqrt[1 + E^((2*
I)*(c + d*x))]*ArcTanh[(Sqrt[2]*E^(I*(c + d*x)))/Sqrt[1 + E^((2*I)*(c + d*x
))]]))/(a^2*d*(-1 + E^((2*I)*(c + d*x)))*(1 + E^((2*I)*(c + d*x)))^3*(-I +
Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1425 vs. 2(173) = 346.
time = 0.91, size = 1426, normalized size = 6.66

method	result	size
default	Expression too large to display	1426

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/240/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(-160*sin(d*x+c)*cos
(d*x+c)^5-300*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))+300*cos(
d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))+1260*sin(d*x+c)*cos(d*x+c)-3
00*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))-300*I*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+300*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))-192*I*cos(d*x+c)^8-64*I*cos(d*x+c)^
6-564*I*cos(d*x+c)^4+820*I*cos(d*x+c)^2-300*(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)+300*I*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)+15*I*(-2*cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c
)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)*cos(d*x+c)^3+15*I*(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)
/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)*cos(d*x+c
)^2-15*I*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2
```

```
*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*2^(1/2))+300*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-15*sin(d*x+c)*2^(1/2)*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/si
n(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-300*I*sin(d*x+c)*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))+300*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^3+300*I*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2))*cos(d*x+c)^2-300*I*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arc
tan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-15*I*2^(1/2)*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-668*sin(d*x+c)*cos(d*x+c)^3+15*cos
(d*x+c)^2*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/
2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*2^(1/2))-192*sin(d*x+c)*cos(d*x+c)^7)/(cos(d*x+c)^2-1)/a^3
```

Maxima [**A**]
time = 0.53, size = 202, normalized size = 0.94

$$-\frac{i a \left(\frac{4\left(315\,(i a\tan(dx+c)+a)^3-205\,(i a\tan(dx+c)+a)^2 a-38\,(i a\tan(dx+c)+a)a^2-12a^3\right)}{(i a\tan(dx+c)+a)^{\frac{7}{2}}a^3-(i a\tan(dx+c)+a)^{\frac{5}{2}}a^4}+\frac{15\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i a\tan(dx+c)+a}}\right)}{a^{\frac{7}{2}}}+\frac{600\log\left(\frac{\sqrt{i a\tan(dx+c)+a}-\sqrt{a}}{\sqrt{i a\tan(dx+c)+a}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/240*I*a*(4*(315*(I*a*tan(d*x + c) + a)^3 - 205*(I*a*tan(d*x + c) + a)^2*
a - 38*(I*a*tan(d*x + c) + a)*a^2 - 12*a^3)/((I*a*tan(d*x + c) + a)^(7/2)*a
^3 - (I*a*tan(d*x + c) + a)^(5/2)*a^4) + 15*sqrt(2)*log(-(sqrt(2)*sqrt(a) -
 sqrt(I*a*tan(d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))
)/a^(7/2) + 600*log((sqrt(I*a*tan(d*x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*
x + c) + a) + sqrt(a)))/a^(7/2))/d
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 612 vs. 2(165) = 330.
time = 0.38, size = 612, normalized size = 2.86

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/120*(15*sqrt(1/2)*(-I*a^3*d*e^(7*I*d*x + 7*I*c) + I*a^3*d*e^(5*I*d*x + 5
*I*c))*sqrt(1/(a^5*d^2))*log(4*(sqrt(2)*sqrt(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c
) + a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^5*d^2)) + a*e^(I*d*x
```

```
 + I*c))*e^(-I*d*x - I*c)) + 15*sqrt(1/2)*(I*a^3*d*e^(7*I*d*x + 7*I*c) - I*
a^3*d*e^(5*I*d*x + 5*I*c))*sqrt(1/(a^5*d^2))*log(-4*(sqrt(2)*sqrt(1/2)*(a^3
*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a
^5*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 150*(-I*a^3*d*e^(7*I*d*x
+ 7*I*c) + I*a^3*d*e^(5*I*d*x + 5*I*c))*sqrt(1/(a^5*d^2))*log(16*(3*a^2*e^(
2*I*d*x + 2*I*c) + 2*sqrt(2)*(a^4*d*e^(3*I*d*x + 3*I*c) + a^4*d*e^(I*d*x +
I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^5*d^2)) + a^2)*e^(-2*I*d*
x - 2*I*c)) + 150*(I*a^3*d*e^(7*I*d*x + 7*I*c) - I*a^3*d*e^(5*I*d*x + 5*I*c
))*sqrt(1/(a^5*d^2))*log(16*(3*a^2*e^(2*I*d*x + 2*I*c) - 2*sqrt(2)*(a^4*d*e
^(3*I*d*x + 3*I*c) + a^4*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
))*sqrt(1/(a^5*d^2)) + a^2)*e^(-2*I*d*x - 2*I*c)) - sqrt(2)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1))*(-403*I*e^(8*I*d*x + 8*I*c) - 151*I*e^(6*I*d*x + 6*I*c)
+ 280*I*e^(4*I*d*x + 4*I*c) + 31*I*e^(2*I*d*x + 2*I*c) + 3*I))/(a^3*d*e^(7*
I*d*x + 7*I*c) - a^3*d*e^(5*I*d*x + 5*I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(cot(c + d*x)**2/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)^2/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [B]
time = 4.06, size = 201, normalized size = 0.94

$$-\frac{\frac{(a+a\tan(c+dx)\,1\mathrm{i})\,19\mathrm{i}}{30\,d}+\frac{a\,1\mathrm{i}}{5\,d}+\frac{(a+a\tan(c+dx)\,1\mathrm{i})^2\,41\mathrm{i}}{12\,a\,d}-\frac{(a+a\tan(c+dx)\,1\mathrm{i})^3\,21\mathrm{i}}{4\,a^2\,d}}{a\,(a+a\tan(c+dx)\,1\mathrm{i})^{5/2}-(a+a\tan(c+dx)\,1\mathrm{i})^{7/2}}-\frac{\mathrm{atan}\left(\frac{\sqrt{-a^5}\,\sqrt{a+a\tan(c+dx)\,1\mathrm{i}}}{a^3}\right)\sqrt{-a^5}\,5\mathrm{i}}{a^5\,d}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{-a^5}\,\sqrt{a+a\tan(c+dx)\,1\mathrm{i}}}{2\,a^3}\right)\sqrt{-a^5}\,1\mathrm{i}}{8\,a^5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] - (((a + a*tan(c + d*x)*1i)*19i)/(30*d) + (a*1i)/(5*d) + ((a + a*tan(c + d*
x)*1i)^2*41i)/(12*a*d) - ((a + a*tan(c + d*x)*1i)^3*21i)/(4*a^2*d))/(a*(a +
```

```
a*tan(c + d*x)*1i)^(5/2) - (a + a*tan(c + d*x)*1i)^(7/2)) - (atan(((-a^5)^
(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/a^3)*(-a^5)^(1/2)*5i)/(a^5*d) - (2^(1/
2)*atan((2^(1/2)*(-a^5)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*a^3))*(-a^5
)^(1/2)*1i)/(8*a^5*d)
```

3.134 $\int \frac{1}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=162

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}\,a^{7/2}d}+\frac{i}{7d(a+ia\tan(c+dx))^{7/2}}+\frac{i}{10ad(a+ia\tan(c+dx))^{5/2}}+\frac{}{12a^2d(a+ia}$$

```
[Out] -1/16*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(7/2)/d*2^(
1/2)+1/8*I/a^3/d/(a+I*a*tan(d*x+c))^(1/2)+1/7*I/d/(a+I*a*tan(d*x+c))^(7/2)+
1/10*I/a/d/(a+I*a*tan(d*x+c))^(5/2)+1/12*I/a^2/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3560, 3561, 212}

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}\,a^{7/2}d}+\frac{i}{8a^3d\sqrt{a+ia\tan(c+dx)}}+\frac{i}{12a^2d(a+ia\tan(c+dx))^{3/2}}+\frac{i}{10ad(a+ia\tan(c+dx))^{5/2}}+\frac{i}{7d(a+ia\tan(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(-7/2),x]
```

```
[Out] ((-1/8*I)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]*a
^(7/2)*d) + (I/7)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (I/10)/(a*d*(a + I*a*T
an[c + d*x])^(5/2)) + (I/12)/(a^2*d*(a + I*a*Tan[c + d*x])^(3/2)) + (I/8)/(
a^3*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+ia\tan(c+dx))^{7/2}}\,dx &= \frac{i}{7d(a+ia\tan(c+dx))^{7/2}}+\frac{\int \frac{1}{(a+ia\tan(c+dx))^{5/2}}\,dx}{2a}\\
&= \frac{i}{7d(a+ia\tan(c+dx))^{7/2}}+\frac{i}{10ad(a+ia\tan(c+dx))^{5/2}}+\frac{\int \frac{1}{(a+ia\tan(c+dx))}}{4a^2}\\
&= \frac{i}{7d(a+ia\tan(c+dx))^{7/2}}+\frac{i}{10ad(a+ia\tan(c+dx))^{5/2}}+\frac{i}{12a^2d(a+ia\,\mathrm{ta}}\\
&= \frac{i}{7d(a+ia\tan(c+dx))^{7/2}}+\frac{i}{10ad(a+ia\tan(c+dx))^{5/2}}+\frac{i}{12a^2d(a+ia\,\mathrm{ta}}\\
&= \frac{i}{7d(a+ia\tan(c+dx))^{7/2}}+\frac{i}{10ad(a+ia\tan(c+dx))^{5/2}}+\frac{i}{12a^2d(a+ia\,\mathrm{ta}}\\
&= -\frac{i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}\,a^{7/2}d}+\frac{i}{7d(a+ia\tan(c+dx))^{7/2}}+\frac{}{10ad(a}
\end{aligned}
$$

Mathematica [A]
time = 2.26, size = 150, normalized size = 0.93

$$
-\frac{15+81e^{2i(c+dx)}+188e^{4i(c+dx)}+298e^{6i(c+dx)}+176e^{8i(c+dx)}-105e^{7i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,\sinh^{-1}\left(e^{i(c+dx)}\right)}{105a^3d\left(1+e^{2i(c+dx)}\right)^4(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(-7/2),x]
```

```
[Out] -1/105*(15 + 81*E^((2*I)*(c + d*x)) + 188*E^((4*I)*(c + d*x)) + 298*E^((6*I
)*(c + d*x)) + 176*E^((8*I)*(c + d*x)) - 105*E^((7*I)*(c + d*x))*Sqrt[1 + E
^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))])/(a^3*d*(1 + E^((2*I)*(c + d*x
)))^4*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.18, size = 116, normalized size = 0.72

method	result
derivativedivides	$\frac{2ia\left(\frac{1}{16a^4\sqrt{a+ia\tan(dx+c)}}+\frac{1}{24a^3(a+ia\tan(dx+c))^{\frac{3}{2}}}+\frac{1}{20a^2(a+ia\tan(dx+c))^{\frac{5}{2}}}+\frac{1}{14a(a+ia\tan(dx+c))^{\frac{7}{2}}}-\frac{\sqrt{2}}{}\right.}{d}$

default	$\frac{2ia\left(\frac{1}{16a^4\sqrt{a+ia\tan(dx+c)}}+\frac{1}{24a^3(a+ia\tan(dx+c))^{\frac{3}{2}}}+\frac{1}{20a^2(a+ia\tan(dx+c))^{\frac{5}{2}}}+\frac{1}{14a(a+ia\tan(dx+c))^{\frac{7}{2}}}-\frac{\sqrt{2}}{\ldots}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/d*a*(1/16/a^4/(a+I*a*tan(d*x+c))^(1/2)+1/24/a^3/(a+I*a*tan(d*x+c))^(3/2
)+1/20/a^2/(a+I*a*tan(d*x+c))^(5/2)+1/14/a/(a+I*a*tan(d*x+c))^(7/2)-1/32/a^
(9/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.51, size = 137, normalized size = 0.85

$$\frac{i\left(\frac{105\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)}{a^{\frac{5}{2}}}+\frac{4\left(105\,(ia\tan(dx+c)+a)^3+70\,(ia\tan(dx+c)+a)^2a+84\,(ia\tan(dx+c)+a)a^2+120\,a^3\right)}{(ia\tan(dx+c)+a)^{\frac{7}{2}}a^2}\right)}{3360\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 1/3360*I*(105*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(
sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/a^(5/2) + 4*(105*(I*a*tan(d*
x + c) + a)^3 + 70*(I*a*tan(d*x + c) + a)^2*a + 84*(I*a*tan(d*x + c) + a)*a
^2 + 120*a^3)/((I*a*tan(d*x + c) + a)^(7/2)*a^2))/(a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(115) = 230$.
time = 0.37, size = 294, normalized size = 1.81

$$\frac{\left(-105i\sqrt{\frac{1}{2}}\,a^4d\sqrt{\frac{1}{a^7d^2}}\,e^{(7i\,dx+7i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^4de^{(2i\,dx+2i\,c)}+a^4d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^7d^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+105i\sqrt{\frac{1}{2}}\,a^4d\sqrt{\frac{1}{a^7d^2}}\,e^{(7i\,dx+7i\,c)}\log\left(-4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^4de^{(2i\,dx+2i\,c)}+a^4d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^7d^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(176i\,e^{(8i\,dx+8i\,c)}+298i\,e^{(6i\,dx+6i\,c)}+188i\,e^{(4i\,dx+4i\,c)}+81i\,e^{(2i\,dx+2i\,c)}+15i\right)\right)e^{(-7i\,dx-7i\,c)}}{1680\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/1680*(-105*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(7*I*d*x + 7*I*c)*log(4*
(sqrt(2)*sqrt(1/2)*(a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*sqrt(1/(a^7*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 10
5*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(7*I*d*x + 7*I*c)*log(-4*(sqrt(2)*s
qrt(1/2)*(a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
1))*sqrt(1/(a^7*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*(176*I*e^(8*I*d*x + 8*I*c) + 298*I*e^(6*I*d*x
```

```
 + 6*I*c) + 188*I*e^(4*I*d*x + 4*I*c) + 81*I*e^(2*I*d*x + 2*I*c) + 15*I))*e
^(-7*I*d*x - 7*I*c)/(a^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia\tan(c+dx)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))**(7/2),x)
[Out] Integral((I*a*tan(c + d*x) + a)**(-7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(-7/2), x)
```

Mupad [B]
time = 3.98, size = 129, normalized size = 0.80

$$\frac{\frac{1\mathrm{i}}{7\,d}+\frac{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1\mathrm{i}\right)}^2\,1\mathrm{i}}{12\,a^2\,d}+\frac{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1\mathrm{i}\right)}^3\,1\mathrm{i}}{8\,a^3\,d}+\frac{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1\mathrm{i}\right)\,1\mathrm{i}}{10\,a\,d}}{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1\mathrm{i}\right)}^{7/2}}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1\mathrm{i}}}{2\,\sqrt{-a}}\right)\,1\mathrm{i}}{16\,{\left(-a\right)}^{7/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*tan(c + d*x)*1i)^(7/2),x)
[Out] (1i/(7*d) + ((a + a*tan(c + d*x)*1i)^2*1i)/(12*a^2*d) + ((a + a*tan(c + d*x
)*1i)^3*1i)/(8*a^3*d) + ((a + a*tan(c + d*x)*1i)*1i)/(10*a*d))/(a + a*tan(c
 + d*x)*1i)^(7/2) - (2^(1/2)*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(
2*(-a)^(1/2)))*1i)/(16*(-a)^(7/2)*d)
```

3.135 $\int (d\tan(e+fx))^{5/2}(a+ia\tan(e+fx))\,dx$

Optimal. Leaf size=107

$$-\frac{2(-1)^{3/4}ad^{5/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}-\frac{2iad^2\sqrt{d\tan(e+fx)}}{f}+\frac{2ad(d\tan(e+fx))^{3/2}}{3f}+\frac{2ia(d\ta$$

```
[Out] -2*(-1)^(3/4)*a*d^(5/2)*arctan((-1)^(3/4)*(d*tan(f*x+e))^(1/2)/d^(1/2))/f-2
*I*a*d^2*(d*tan(f*x+e))^(1/2)/f+2/3*a*d*(d*tan(f*x+e))^(3/2)/f+2/5*I*a*(d*t
an(f*x+e))^(5/2)/f
```

Rubi [A]
time = 0.11, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3609, 3614, 211}

$$-\frac{2(-1)^{3/4}ad^{5/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}-\frac{2iad^2\sqrt{d\tan(e+fx)}}{f}+\frac{2ad(d\tan(e+fx))^{3/2}}{3f}+\frac{2ia(d\tan(e+fx))^{5/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(5/2)*(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (-2*(-1)^(3/4)*a*d^(5/2)*ArcTan[((-1)^(3/4)*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])
/f - ((2*I)*a*d^2*Sqrt[d*Tan[e + f*x]])/f + (2*a*d*(d*Tan[e + f*x])^(3/2))/
(3*f) + (((2*I)/5)*a*(d*Tan[e + f*x])^(5/2))/f
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rubi steps

$$\int (d\tan(e+fx))^{5/2}(a+ia\tan(e+fx))\,dx = \frac{2ia(d\tan(e+fx))^{5/2}}{5f} + \int (d\tan(e+fx))^{3/2}(-iad+ad\tan$$
$$= \frac{2ad(d\tan(e+fx))^{3/2}}{3f} + \frac{2ia(d\tan(e+fx))^{5/2}}{5f} + \int \sqrt{d\tan}$$
$$= -\frac{2iad^2\sqrt{d\tan(e+fx)}}{f} + \frac{2ad(d\tan(e+fx))^{3/2}}{3f} + \frac{2ia(d\tan}{}$$
$$= -\frac{2iad^2\sqrt{d\tan(e+fx)}}{f} + \frac{2ad(d\tan(e+fx))^{3/2}}{3f} + \frac{2ia(d\tan}{}$$
$$= -\frac{2(-1)^{3/4}ad^{5/2}\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f} - \frac{2iad^2\sqrt{}}{}$$

Mathematica [A]
time = 2.40, size = 125, normalized size = 1.17

$$\frac{ad^2\left(30i\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\right)+\sec^2(e+fx)(-12i-18i\cos(2(e+fx))+5\sin(2(e+fx)))\sqrt{i\tan(e+fx)}\right)\sqrt{d\tan(e+fx)}}{15f\sqrt{i\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(5/2)*(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (a*d^2*((30*I)*ArcTanh[Sqrt[(-1 + E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f
*x)))]] + Sec[e + f*x]^2*(-12*I - (18*I)*Cos[2*(e + f*x)] + 5*Sin[2*(e + f*
x)])*Sqrt[I*Tan[e + f*x]])*Sqrt[d*Tan[e + f*x]])/(15*f*Sqrt[I*Tan[e + f*x]]
)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(85) = 170.
time = 0.24, size = 321, normalized size = 3.00

method	result
derivativedivides	$a\left(\frac{2i(d\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{2d(d\tan(fx+e))^{\frac{3}{2}}}{3}-2id^2\sqrt{d\tan(fx+e)}+2d^3\left(\frac{i(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d}}\right.\right.}{}\right.\right.$

default	$\frac{a\left(\frac{2i(d\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{2d(d\tan(fx+e))^{\frac{3}{2}}}{3}-2id^2\sqrt{d\tan(fx+e)}+2d^3\left(\frac{i(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\ldots}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\ldots}\right)\ldots\right.}{\ldots}\right.\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(5/2)*(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a*(2/5*I*(d*tan(f*x+e))^(5/2)+2/3*d*(d*tan(f*x+e))^(3/2)-2*I*d^2*(d*tan
(f*x+e))^(1/2)+2*d^3*(1/8*I/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(
1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d
*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*ta
n(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))-1
/8/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2
^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+
(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan
(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(87) = 174$.
time = 0.51, size = 218, normalized size = 2.04

$$\frac{15\,ad^4\left(\frac{(2i-2)\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{(2i-2)\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{(i+1)\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}-\frac{(i+1)\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)+24i\,(d\tan(fx+e))^{\frac{5}{2}}ad+40\,(d\tan(fx+e))^{\frac{3}{2}}ad^2-120i\sqrt{d\tan(fx+e)}\,ad^3}{60\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)*(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/60*(15*a*d^4*((2*I - 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*s
qrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) + (2*I - 2)*sqrt(2)*arctan(-1/2*sqrt(
2)*(sqrt(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) + (I + 1)*sq
rt(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d
) - (I + 1)*sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(
d) + d)/sqrt(d)) + 24*I*(d*tan(f*x + e))^(5/2)*a*d + 40*(d*tan(f*x + e))^(3
/2)*a*d^2 - 120*I*sqrt(d*tan(f*x + e))*a*d^3)/(d*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(87) = 174$.
time = 0.39, size = 391, normalized size = 3.65

$$-\frac{15\sqrt{\frac{4i\,a^2d^5}{f^2}}\left(fe^{(4i\,fx+4i\,e)}+2\,fe^{(2i\,fx+2i\,e)}+f\right)\log\left(\frac{\left(-2i\,ad^3e^{(2i\,fx+2i\,e)}+\sqrt{\frac{4i\,a^2d^5}{f^2}}\left(i\,fe^{(2i\,fx+2i\,e)}+i\,f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-2i\,fx-2i\,e)}}{ad^2}\right)-15\sqrt{\frac{4i\,a^2d^5}{f^2}}\left(fe^{(4i\,fx+4i\,e)}+2\,fe^{(2i\,fx+2i\,e)}+f\right)\log\left(\frac{\left(-2i\,ad^3e^{(2i\,fx+2i\,e)}+\sqrt{\frac{4i\,a^2d^5}{f^2}}\left(-i\,fe^{(2i\,fx+2i\,e)}-i\,f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-2i\,fx-2i\,e)}}{ad^2}\right)+8\left(23i\,ad^2e^{(4i\,fx+4i\,e)}+24i\,ad^2e^{(2i\,fx+2i\,e)}+13i\,ad^2\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}}{60\left(fe^{(4i\,fx+4i\,e)}+2\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)*(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/60*(15*sqrt(4*I*a^2*d^5/f^2)*(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2
*I*e) + f)*log((-2*I*a*d^3*e^(2*I*f*x + 2*I*e) + sqrt(4*I*a^2*d^5/f^2)*(I*f
*e^(2*I*f*x + 2*I*e) + I*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f
*x + 2*I*e) + 1)))*e^(-2*I*f*x - 2*I*e)/(a*d^2)) - 15*sqrt(4*I*a^2*d^5/f^2)
*(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*e) + f)*log((-2*I*a*d^3*e^(2
*I*f*x + 2*I*e) + sqrt(4*I*a^2*d^5/f^2)*(-I*f*e^(2*I*f*x + 2*I*e) - I*f)*sq
rt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-2*I*f*x
 - 2*I*e)/(a*d^2)) + 8*(23*I*a*d^2*e^(4*I*f*x + 4*I*e) + 24*I*a*d^2*e^(2*I*
f*x + 2*I*e) + 13*I*a*d^2)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*
x + 2*I*e) + 1)))/(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int \left(-i(d\tan(e+fx))^{\frac{5}{2}}\right)\,dx + \int (d\tan(e+fx))^{\frac{5}{2}}\tan(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(5/2)*(a+I*a*tan(f*x+e)),x)
```

```
[Out] I*a*(Integral(-I*(d*tan(e + f*x))**(5/2), x) + Integral((d*tan(e + f*x))**(
5/2)*tan(e + f*x), x))
```

Giac [A]
time = 0.65, size = 154, normalized size = 1.44

$$-\frac{2}{15}\,ad^2\left(\frac{15\,\sqrt{2}\,\sqrt{d}\,\arctan\left(\frac{8\,\sqrt{d^2}\,\sqrt{d\tan(fx+e)}}{4i\,\sqrt{2}\,d^{\frac{3}{2}}+4\,\sqrt{2}\,\sqrt{d^2}\,\sqrt{d}}\right)}{f\left(\frac{i\,d}{\sqrt{d^2}}+1\right)}+\frac{-3i\,\sqrt{d\tan(fx+e)}\,d^{10}f^4\tan(fx+e)^2-5\,\sqrt{d\tan(fx+e)}\,d^{10}f^4\tan(fx+e)+15i\,\sqrt{d\tan(fx+e)}\,d^{10}f^4}{d^{10}f^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)*(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] -2/15*a*d^2*(15*sqrt(2)*sqrt(d)*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*
I*sqrt(2)*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(f*(I*d/sqrt(d^2) + 1)) +
 (-3*I*sqrt(d*tan(f*x + e))*d^10*f^4*tan(f*x + e)^2 - 5*sqrt(d*tan(f*x + e)
)*d^10*f^4*tan(f*x + e) + 15*I*sqrt(d*tan(f*x + e))*d^10*f^4)/(d^10*f^5))
```

Mupad [B]
time = 4.93, size = 114, normalized size = 1.07

$$\frac{2\,a\,d\,(d\tan(e+f\,x))^{3/2}}{3\,f}+\frac{(-1)^{1/4}\,a\,d^{5/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan(e+f\,x)}}{\sqrt{d}}\right)}{f}+\frac{a\,(d\tan(e+f\,x))^{5/2}\,2\mathrm{i}}{5\,f}-\frac{a\,d^2\,\sqrt{d\tan(e+f\,x)}\,2\mathrm{i}}{f}-\frac{(-1)^{1/4}\,a\,d^{5/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan(e+f\,x)}\,1\mathrm{i}}{\sqrt{d}}\right)\,1\mathrm{i}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(5/2)*(a + a*tan(e + f*x)*1i),x)
```

```
[Out] (a*(d*tan(e + f*x))^(5/2)*2i)/(5*f) + (2*a*d*(d*tan(e + f*x))^(3/2))/(3*f)
- (a*d^2*(d*tan(e + f*x))^(1/2)*2i)/f - ((-1)^(1/4)*a*d^(5/2)*atan(((-1)^(1
/4)*(d*tan(e + f*x))^(1/2)*1i)/d^(1/2))*1i)/f + ((-1)^(1/4)*a*d^(5/2)*atanh
(((-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2)))/f
```

3.136 $\int (d\tan(e+fx))^{3/2}(a+ia\tan(e+fx))\,dx$

Optimal. Leaf size=82

$$\frac{2\sqrt[4]{-1}\,ad^{3/2}\mathrm{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}+\frac{2ad\sqrt{d\tan(e+fx)}}{f}+\frac{2ia(d\tan(e+fx))^{3/2}}{3f}$$

```
[Out] 2*(-1)^(1/4)*a*d^(3/2)*arctan((-1)^(3/4)*(d*tan(f*x+e))^(1/2)/d^(1/2))/f+2*
a*d*(d*tan(f*x+e))^(1/2)/f+2/3*I*a*(d*tan(f*x+e))^(3/2)/f
```

Rubi [A]
time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3609, 3614, 211}

$$\frac{2\sqrt[4]{-1}\,ad^{3/2}\mathrm{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}+\frac{2ad\sqrt{d\tan(e+fx)}}{f}+\frac{2ia(d\tan(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(3/2)*(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (2*(-1)^(1/4)*a*d^(3/2)*ArcTan[((-1)^(3/4)*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/
f + (2*a*d*Sqrt[d*Tan[e + f*x]])/f + (((2*I)/3)*a*(d*Tan[e + f*x])^(3/2))/f
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rubi steps

$$\int (d\tan(e+fx))^{3/2}(a+ia\tan(e+fx))\,dx = \frac{2ia(d\tan(e+fx))^{3/2}}{3f} + \int \sqrt{d\tan(e+fx)}\,(-iad+ad\tan(e$$
$$= \frac{2ad\sqrt{d\tan(e+fx)}}{f} + \frac{2ia(d\tan(e+fx))^{3/2}}{3f} + \int \frac{-ad^2-ia}{\sqrt{d\,\mathrm{ta}}}$$
$$= \frac{2ad\sqrt{d\tan(e+fx)}}{f} + \frac{2ia(d\tan(e+fx))^{3/2}}{3f} + \frac{(2a^2d^4)\,\mathrm{Subs}}{}$$
$$= \frac{2\sqrt[4]{-1}\,ad^{3/2}\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f} + \frac{2ad\sqrt{d\tan(}}{f}$$

Mathematica [A]
time = 0.91, size = 148, normalized size = 1.80

$$-\frac{2ad\left(2+2e^{2i(e+fx)}-4e^{4i(e+fx)}+3\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\left(1+e^{2i(e+fx)}\right)^2\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\right)\right)\sqrt{d\tan(e+fx)}}{3\left(-1+e^{4i(e+fx)}\right)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(3/2)*(a + I*a*Tan[e + f*x]),x]
[Out] (-2*a*d*(2 + 2*E^((2*I)*(e + f*x)) - 4*E^((4*I)*(e + f*x)) + 3*Sqrt[(-1 + E
^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f*x)))]*(1 + E^((2*I)*(e + f*x)))^2*
ArcTanh[Sqrt[(-1 + E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f*x)))]])*Sqrt[d
*Tan[e + f*x]])/(3*(-1 + E^((4*I)*(e + f*x)))*f)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(65) = 130.
time = 0.10, size = 305, normalized size = 3.72

method	result
derivativedivides	$a\left(\frac{2i(d\tan(fx+e))^{\frac{3}{2}}}{3}+2d\sqrt{d\tan(fx+e)}-2d^2\left(\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}}\right.\right.}{}\right.\right.$

default	$\frac{a\left(\frac{2i(d\tan(fx+e))^{\frac{3}{2}}}{3}+2d\sqrt{d\tan\left(fx+e\right)}-2d^2\left(\frac{\left(d^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+\left(d^2\right)^{\frac{1}{4}}\sqrt{d\tan\left(fx+e\right)}\sqrt{}}{d\tan(fx+e)-\left(d^2\right)^{\frac{1}{4}}\sqrt{d\tan\left(fx+e\right)}\sqrt{}}\right.\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(3/2)*(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a*(2/3*I*(d*tan(f*x+e))^(3/2)+2*d*(d*tan(f*x+e))^(1/2)-2*d^2*(1/8/d*(d^
2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)
+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^
(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1
/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))+1/8*I/(d^2)^(1/4)*2^(1/2)*(ln((d*t
an(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+
e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/
(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*
x+e))^(1/2)+1))))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(67) = 134$.
time = 0.55, size = 201, normalized size = 2.45

$$\frac{3\,ad^3\left(-\frac{(2i+2)\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(2i+2)\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{(i-1)\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{d}+d\right)}{\sqrt{d}}-\frac{(i-1)\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{d}+d\right)}{\sqrt{d}}\right)+8i\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}ad+24\sqrt{d\tan\left(fx+e\right)}ad^2}{12\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)*(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*a*d^3*(-(2*I + 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*s
qrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - (2*I + 2)*sqrt(2)*arctan(-1/2*sqrt(
2)*(sqrt(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) + (I - 1)*sq
rt(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d
) - (I - 1)*sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(
d) + d)/sqrt(d)) + 8*I*(d*tan(f*x + e))^(3/2)*a*d + 24*sqrt(d*tan(f*x + e))
*a*d^2)/(d*f)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(67) = 134$.
time = 0.40, size = 326, normalized size = 3.98

$$-\frac{3\sqrt{-\frac{4i\,a^2d^3}{f^2}}\left(fe^{(2i\,fx+2i\,e)}+f\right)\log\left(\frac{\left(-2i\,ad^2e^{(2i\,fx+2i\,e)}+\sqrt{-\frac{4i\,a^2d^3}{f^2}}\left(fe^{(2i\,fx+2i\,e)}+f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-2i\,fx-2i\,e)}}{ad}\right)-3\sqrt{-\frac{4i\,a^2d^3}{f^2}}\left(fe^{(2i\,fx+2i\,e)}+f\right)\log\left(\frac{\left(-2i\,ad^2e^{(2i\,fx+2i\,e)}-\sqrt{-\frac{4i\,a^2d^3}{f^2}}\left(fe^{(2i\,fx+2i\,e)}+f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-2i\,fx-2i\,e)}}{ad}\right)-16\left(2\,ade^{(2i\,fx+2i\,e)}+ad\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}}{12\left(fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)*(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/12*(3*sqrt(-4*I*a^2*d^3/f^2)*(f*e^(2*I*f*x + 2*I*e) + f)*log((-2*I*a*d^2
*e^(2*I*f*x + 2*I*e) + sqrt(-4*I*a^2*d^3/f^2)*(f*e^(2*I*f*x + 2*I*e) + f)*s
qrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-2*I*f*
x - 2*I*e)/(a*d)) - 3*sqrt(-4*I*a^2*d^3/f^2)*(f*e^(2*I*f*x + 2*I*e) + f)*lo
g((-2*I*a*d^2*e^(2*I*f*x + 2*I*e) - sqrt(-4*I*a^2*d^3/f^2)*(f*e^(2*I*f*x +
2*I*e) + f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)
))*e^(-2*I*f*x - 2*I*e)/(a*d)) - 16*(2*a*d*e^(2*I*f*x + 2*I*e) + a*d)*sqrt(
(-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))/(f*e^(2*I*f*x
+ 2*I*e) + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-i(d\tan(e+fx))^{\frac{3}{2}}\right)dx+\int(d\tan(e+fx))^{\frac{3}{2}}\tan(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(3/2)*(a+I*a*tan(f*x+e)),x)
```

```
[Out] I*a*(Integral(-I*(d*tan(e + f*x))**(3/2), x) + Integral((d*tan(e + f*x))**(
3/2)*tan(e + f*x), x))
```

Giac [A]
time = 0.52, size = 124, normalized size = 1.51

$$-\frac{2}{3}ad\left(\frac{3i\sqrt{2}\sqrt{d}\arctan\left(\frac{8\sqrt{d^2}\sqrt{d\tan(fx+e)}}{4i\sqrt{2}d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{f\left(\frac{id}{\sqrt{d^2}}+1\right)}+\frac{-i\sqrt{d\tan(fx+e)}\,d^3f^2\tan(fx+e)-3\sqrt{d\tan(fx+e)}\,d^3f^2}{d^3f^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)*(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] -2/3*a*d*(3*I*sqrt(2)*sqrt(d)*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*I*
sqrt(2)*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(f*(I*d/sqrt(d^2) + 1)) + (
-I*sqrt(d*tan(f*x + e))*d^3*f^2*tan(f*x + e) - 3*sqrt(d*tan(f*x + e))*d^3*f
^2)/(d^3*f^3))
```

Mupad [B]
time = 4.60, size = 65, normalized size = 0.79

$$\frac{a\,(d\tan(e+f\,x))^{3/2}\,2\mathrm{i}}{3\,f}+\frac{2\,a\,d\,\sqrt{d\tan(e+f\,x)}}{f}+\frac{(-1)^{1/4}\,a\,d^{3/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan(e+f\,x)}}{\sqrt{d}}\right)\,2\mathrm{i}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(3/2)*(a + a*tan(e + f*x)*1i),x)
```

```
[Out] (a*(d*tan(e + f*x))^(3/2)*2i)/(3*f) + (2*a*d*(d*tan(e + f*x))^(1/2))/f + ((
-1)^(1/4)*a*d^(3/2)*atanh(((-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2))*2i)/
f
```

3.137 $\int \sqrt{d\tan(e+fx)}\,(a+ia\tan(e+fx))\,dx$

Optimal. Leaf size=61

$$\frac{2(-1)^{3/4}a\sqrt{d}\,\mathrm{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}+\frac{2ia\sqrt{d\tan(e+fx)}}{f}$$

```
[Out] 2*(-1)^(3/4)*a*arctan((-1)^(3/4)*(d*tan(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f+2*
I*a*(d*tan(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3609, 3614, 211}

$$\frac{2(-1)^{3/4}a\sqrt{d}\,\mathrm{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}+\frac{2ia\sqrt{d\tan(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Tan[e + f*x]]*(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (2*(-1)^(3/4)*a*Sqrt[d]*ArcTan[((-1)^(3/4)*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/
f + ((2*I)*a*Sqrt[d*Tan[e + f*x]])/f
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \sqrt{d\tan(e+fx)}\,(a+ia\tan(e+fx))\,dx = \frac{2ia\sqrt{d\tan(e+fx)}}{f} + \int \frac{-iad+ad\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$$

$$= \frac{2ia\sqrt{d\tan(e+fx)}}{f} - \frac{(2a^2d^2)\,\mathrm{Subst}\left(\int \frac{1}{-iad^2-adx^2}\,dx, x, \sqrt{d\right.}{f}$$

$$= \frac{2(-1)^{3/4}a\sqrt{d}\,\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f} + \frac{2ia\sqrt{d\tan}}{}$$

Mathematica [A]
time = 0.71, size = 85, normalized size = 1.39

$$\frac{2ia\left(-\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\right)+\sqrt{i\tan(e+fx)}\right)\sqrt{d\tan(e+fx)}}{f\sqrt{i\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Tan[e + f*x]]*(a + I*a*Tan[e + f*x]),x]
```

```
[Out] ((2*I)*a*(-ArcTanh[Sqrt[(-1 + E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f*x))
)]] + Sqrt[I*Tan[e + f*x]])*Sqrt[d*Tan[e + f*x]])/(f*Sqrt[I*Tan[e + f*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(48) = 96.
time = 0.10, size = 290, normalized size = 4.75

method	result
derivativedivides	$a\left(2i\sqrt{d\tan(fx+e)}-2d\left(\frac{i(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\,\mathrm{arc}\right.}{}\right.\right.$
default	$a\left(2i\sqrt{d\tan(fx+e)}-2d\left(\frac{i(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\,\mathrm{arc}\right.}{}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a*(2*I*(d*tan(f*x+e))^(1/2)-2*d*(1/8*I/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan
(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)
-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d
^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+
e))^(1/2)+1))-1/8/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(
f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))
^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1
/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(49) = 98$.
time = 0.51, size = 183, normalized size = 3.00

$$-\frac{ad^2\left(\frac{(2i-2)\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{(2i-2)\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{(i+1)\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{d}+d\right)}{\sqrt{d}}-\frac{(i+1)\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{d}+d\right)}{\sqrt{d}}\right)-8i\sqrt{d\tan\left(fx+e\right)}ad}{4df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/4*(a*d^2*((2*I - 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt
(d*tan(f*x + e)))/sqrt(d))/sqrt(d) + (2*I - 2)*sqrt(2)*arctan(-1/2*sqrt(2)*
(sqrt(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) + (I + 1)*sqrt(
2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) -
 (I + 1)*sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d)
+ d)/sqrt(d)) - 8*I*sqrt(d*tan(f*x + e))*a*d)/(d*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(49) = 98$.
time = 0.39, size = 256, normalized size = 4.20

$$\frac{\sqrt{\frac{4i\,a^2d}{f^2}}\,f\log\left(\frac{\left(-2i\,ade^{(2i\,fx+2i\,e)}+\left(i\,fe^{(2i\,fx+2i\,e)}+i\,f\right)\sqrt{\frac{4i\,a^2d}{f^2}}\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-2i\,fx-2i\,e)}}{a}\right)-\sqrt{\frac{4i\,a^2d}{f^2}}\,f\log\left(\frac{\left(-2i\,ade^{(2i\,fx+2i\,e)}+\left(-i\,fe^{(2i\,fx+2i\,e)}-i\,f\right)\sqrt{\frac{4i\,a^2d}{f^2}}\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-2i\,fx-2i\,e)}}{a}\right)+8i\,a\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(4*I*a^2*d/f^2)*f*log((-2*I*a*d*e^(2*I*f*x + 2*I*e) + (I*f*e^(2*I*
f*x + 2*I*e) + I*f)*sqrt(4*I*a^2*d/f^2)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*
d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-2*I*f*x - 2*I*e)/a) - sqrt(4*I*a^2*d/f^2
)*f*log((-2*I*a*d*e^(2*I*f*x + 2*I*e) + (-I*f*e^(2*I*f*x + 2*I*e) - I*f)*sq
rt(4*I*a^2*d/f^2)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e
) + 1)))*e^(-2*I*f*x - 2*I*e)/a) + 8*I*a*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I
*d)/(e^(2*I*f*x + 2*I*e) + 1)))/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-i\sqrt{d\tan\left(e+fx\right)}\right)dx+\int\sqrt{d\tan\left(e+fx\right)}\tan\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(1/2)*(a+I*a*tan(f*x+e)),x)
```

```
[Out] I*a*(Integral(-I*sqrt(d*tan(e + f*x)), x) + Integral(sqrt(d*tan(e + f*x))*t
an(e + f*x), x))
```

Giac [A]
time = 0.51, size = 89, normalized size = 1.46

$$\frac{2\,a\left(\frac{\sqrt{2}\,d^{\frac{3}{2}}\arctan\left(\frac{8\,\sqrt{d^2}\,\sqrt{d\tan\left(fx+e\right)}}{4i\,\sqrt{2}\,d^{\frac{3}{2}}+4\,\sqrt{2}\,\sqrt{d^2}\,\sqrt{d}}\right)}{f\left(\frac{i\,d}{\sqrt{d^2}}+1\right)}+\frac{i\,\sqrt{d\tan\left(fx+e\right)}\,d}{f}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] 2*a*(sqrt(2)*d^(3/2)*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*I*sqrt(2)*d
^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(f*(I*d/sqrt(d^2) + 1)) + I*sqrt(d*t
an(f*x + e))*d/f)/d
```

Mupad [B]
time = 4.26, size = 128, normalized size = 2.10

$$\frac{(-1)^{1/4}\,a\,\sqrt{d}\,\left(\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)-\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)\right)}{f}-\frac{(-1)^{1/4}\,a\,\sqrt{d}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)}{f}-\frac{(-1)^{1/4}\,a\,\sqrt{d}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)}{f}+\frac{a\,\sqrt{d\tan\left(e+f\,x\right)}\,2\mathrm{i}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(1/2)*(a + a*tan(e + f*x)*1i),x)
```

```
[Out] (a*(d*tan(e + f*x))^(1/2)*2i)/f - ((-1)^(1/4)*a*d^(1/2)*atan(((-1)^(1/4)*(d
*tan(e + f*x))^(1/2))/d^(1/2)))/f - ((-1)^(1/4)*a*d^(1/2)*atanh(((-1)^(1/4)
*(d*tan(e + f*x))^(1/2))/d^(1/2)))/f + ((-1)^(1/4)*a*d^(1/2)*(atan(((-1)^(1
/4)*(d*tan(e + f*x))^(1/2))/d^(1/2)) - atanh(((-1)^(1/4)*(d*tan(e + f*x))^(
1/2))/d^(1/2))))/f
```

3.138 $\int \frac{a+ia\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$

Optimal. Leaf size=40

$$-\frac{2\sqrt[4]{-1}\,a\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{d}\,f}$$

```
[Out] -2*(-1)^(1/4)*a*arctan((-1)^(3/4)*(d*tan(f*x+e))^(1/2)/d^(1/2))/f/d^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3614, 211}

$$-\frac{2\sqrt[4]{-1}\,a\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{d}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (-2*(-1)^(1/4)*a*ArcTan[((-1)^(3/4)*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(Sqrt[d
]*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+ia\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx &= \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{ad-iax^2}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{f}\\ &= -\frac{2\sqrt[4]{-1}\,a\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{d}\,f}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.63, size = 87, normalized size = 2.18

$$-\frac{2ia\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\right)}{f\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] ((-2*I)*a*Sqrt[(-1 + E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f*x)))]*ArcTan
h[Sqrt[(-1 + E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f*x)))]])/(f*Sqrt[d*Ta
n[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.15, size = 273, normalized size = 6.82

method	result
derivativedivides	$a\left(\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+e}}{(d^2)^{\frac{1}{4}}}\right.\right.}{4d}\right.$
default	$a\left(\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+e}}{(d^2)^{\frac{1}{4}}}\right.\right.}{4d}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))/(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a*(1/4/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e
))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2
)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1
)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))+1/4*I/(d^2)^(1/4)*
2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1
/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2
*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)
^(1/4)*(d*tan(f*x+e))^(1/2)+1)))
```

Maxima [C] Result contains complex when optimal does not.

time = 0.53, size = 160, normalized size = 4.00

$$-\frac{a\left(-\frac{(2i+2)\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(2i+2)\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{(i-1)\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}-\frac{(i-1)\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*a*(-(2*I + 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*t
an(f*x + e)))/sqrt(d))/sqrt(d) - (2*I + 2)*sqrt(2)*arctan(-1/2*sqrt(2)*(sqr
t(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) + (I - 1)*sqrt(2)*l
og(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) - (I
- 1)*sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)
/sqrt(d))/f
```

Fricas [C] Result contains complex when optimal does not.
time = 0.36, size = 222, normalized size = 5.55

$$\frac{1}{4}\sqrt{-\frac{4i\,a^2}{df^2}}\log\left(\frac{\left(-2i\,ade^{(2i\,fx+2i\,e)}+\left(dfe^{(2i\,fx+2i\,e)}+df\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{-\frac{4i\,a^2}{df^2}}\right)e^{(-2i\,fx-2i\,e)}}{a}\right)-\frac{1}{4}\sqrt{-\frac{4i\,a^2}{df^2}}\log\left(\frac{\left(-2i\,ade^{(2i\,fx+2i\,e)}-\left(dfe^{(2i\,fx+2i\,e)}+df\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{-\frac{4i\,a^2}{df^2}}\right)e^{(-2i\,fx-2i\,e)}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-4*I*a^2/(d*f^2))*log((-2*I*a*d*e^(2*I*f*x + 2*I*e) + (d*f*e^(2*I*
f*x + 2*I*e) + d*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I
*e) + 1))*sqrt(-4*I*a^2/(d*f^2)))*e^(-2*I*f*x - 2*I*e)/a) - 1/4*sqrt(-4*I*a
^2/(d*f^2))*log((-2*I*a*d*e^(2*I*f*x + 2*I*e) - (d*f*e^(2*I*f*x + 2*I*e) +
d*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(
-4*I*a^2/(d*f^2)))*e^(-2*I*f*x - 2*I*e)/a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-\frac{i}{\sqrt{d\tan(e+fx)}}\right)dx+\int\frac{\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(d*tan(f*x+e))**(1/2),x)
```

```
[Out] I*a*(Integral(-I/sqrt(d*tan(e + f*x)), x) + Integral(tan(e + f*x)/sqrt(d*ta
n(e + f*x)), x))
```

Giac [C] Result contains complex when optimal does not.
time = 0.53, size = 67, normalized size = 1.68

$$\frac{2\sqrt{2}\,a\arctan\left(-\frac{8i\sqrt{d^2}\sqrt{d\tan\left(fx+e\right)}}{-4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{\sqrt{d}\,f\left(-\frac{i\,d}{\sqrt{d^2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*a*arctan(-8*I*sqrt(d^2)*sqrt(d*tan(f*x + e))/(-4*I*sqrt(2)*d^(3/2
) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(sqrt(d)*f*(-I*d/sqrt(d^2) + 1))
```

Mupad [B]
time = 4.29, size = 30, normalized size = 0.75

$$-\frac{(-1)^{1/4}\,a\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)\,2\mathrm{i}}{\sqrt{d}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)/(d*tan(e + f*x))^(1/2),x)
```

```
[Out] -((-1)^(1/4)*a*atanh(((-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2))*2i)/(d^(1
/2)*f)
```

3.139 $\int \frac{a+ia\tan(e+fx)}{(d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{2(-1)^{3/4}a\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{3/2}f}-\frac{2a}{df\sqrt{d\tan(e+fx)}}$$

```
[Out] -2*(-1)^(3/4)*a*arctan((-1)^(3/4)*(d*tan(f*x+e))^(1/2)/d^(1/2))/d^(3/2)/f-2
*a/d/f/(d*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3610, 3614, 211}

$$-\frac{2(-1)^{3/4}a\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{3/2}f}-\frac{2a}{df\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])/(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*(-1)^(3/4)*a*ArcTan[((-1)^(3/4)*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(d^(3/2
)*f) - (2*a)/(d*f*Sqrt[d*Tan[e + f*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{a+ia\tan(e+fx)}{(d\tan(e+fx))^{3/2}}\,dx = -\frac{2a}{df\sqrt{d\tan(e+fx)}} + \frac{\int \frac{iad-ad\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx}{d^2}$$

$$= -\frac{2a}{df\sqrt{d\tan(e+fx)}} - \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{iad^2+adx^2}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{f}$$

$$= -\frac{2(-1)^{3/4}a\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{3/2}f} - \frac{2a}{df\sqrt{d\tan(e+fx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.08, size = 138, normalized size = 2.23

$$-\frac{2ae^{-i(e+fx)}\sin(e+fx)(-i+\tan(e+fx))\left(i\sqrt{i\tan(e+fx)}+\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\right)\tan(e+fx)\right)}{\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\,f(d\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])/(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*a*Sin[e + f*x]*(-I + Tan[e + f*x])*(I*Sqrt[I*Tan[e + f*x]] + ArcTanh[Sq
rt[(-1 + E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f*x)))]]*Tan[e + f*x]))/(E
^(I*(e + f*x))*Sqrt[(-1 + E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f*x)))]*f
*(d*Tan[e + f*x])^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 294, normalized size = 4.74

method	result
derivativedivides	$a\left(\frac{i(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+e}}{(d^2)^{\frac{1}{4}}}\right.\right.}{4d}\right.$

default	$a\left(\frac{\frac{i(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+e)}}{(d^2)^{\frac{1}{4}}}\right.\right.}{4d}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))/(d*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a*(2/d*(1/8*I/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*ta
n(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e
))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^
(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))-1/8/(d^2)^(
1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^
2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2
)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/
(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)))-2/d/(d*tan(f*x+e))^(1/2))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.52, size = 179, normalized size = 2.89

$$\frac{a\left(\frac{(2i-2)\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{(2i-2)\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{(i+1)\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}-\frac{(i+1)\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)-\frac{8a}{\sqrt{d\tan(fx+e)}}}{4df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(a*((2*I - 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*ta
n(f*x + e)))/sqrt(d))/sqrt(d) + (2*I - 2)*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt
(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) + (I + 1)*sqrt(2)*lo
g(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) - (I +
 1)*sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/
sqrt(d)) - 8*a/sqrt(d*tan(f*x + e)))/(d*f)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.39, size = 355, normalized size = 5.73

$$-\frac{(d^2fe^{(2ifx+2ie)}-d^2f)\sqrt{\frac{4ia^2}{d^3f^2}}\log\left(\frac{\left(-2iade^{(2ifx+2ie)}+(id^2fe^{(2ifx+2ie)}+id^2f)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{4ia^2}{d^3f^2}}\right)e^{(-2ifx-2ie)}}{a}\right)-(d^2fe^{(2ifx+2ie)}-d^2f)\sqrt{\frac{4ia^2}{d^3f^2}}\log\left(\frac{\left(-2iade^{(2ifx+2ie)}+(-id^2fe^{(2ifx+2ie)}-id^2f)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{4ia^2}{d^3f^2}}\right)e^{(-2ifx-2ie)}}{a}\right)+8(iae^{(2ifx+2ie)}+ia)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}}{4(d^2fe^{(2ifx+2ie)}-d^2f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*((d^2*f*e^(2*I*f*x + 2*I*e) - d^2*f)*sqrt(4*I*a^2/(d^3*f^2))*log((-2*I
*a*d*e^(2*I*f*x + 2*I*e) + (I*d^2*f*e^(2*I*f*x + 2*I*e) + I*d^2*f)*sqrt((-I
*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(4*I*a^2/(d^3*
f^2)))*e^(-2*I*f*x - 2*I*e)/a) - (d^2*f*e^(2*I*f*x + 2*I*e) - d^2*f)*sqrt(4
*I*a^2/(d^3*f^2))*log((-2*I*a*d*e^(2*I*f*x + 2*I*e) + (-I*d^2*f*e^(2*I*f*x
+ 2*I*e) - I*d^2*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I
*e) + 1))*sqrt(4*I*a^2/(d^3*f^2)))*e^(-2*I*f*x - 2*I*e)/a) + 8*(I*a*e^(2*I*
f*x + 2*I*e) + I*a)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I
*e) + 1)))/(d^2*f*e^(2*I*f*x + 2*I*e) - d^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-\frac{i}{(d\tan{(e+fx)})^{\frac{3}{2}}}\right)dx+\int\frac{\tan{(e+fx)}}{(d\tan{(e+fx)})^{\frac{3}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(d*tan(f*x+e))**(3/2),x)
```

```
[Out] I*a*(Integral(-I/(d*tan(e + f*x))**(3/2), x) + Integral(tan(e + f*x)/(d*tan
(e + f*x))**(3/2), x))
```

Giac [C] Result contains complex when optimal does not.
time = 0.58, size = 89, normalized size = 1.44

$$\frac{2\,a\left(\frac{i\sqrt{2}\arctan\left(-\frac{8i\sqrt{d^2}\sqrt{d\tan{(fx+e)}}}{-4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{\sqrt{d}\,f\left(-\frac{id}{\sqrt{d^2}}+1\right)}-\frac{1}{\sqrt{d\tan{(fx+e)}}\,f}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] 2*a*(I*sqrt(2)*arctan(-8*I*sqrt(d^2)*sqrt(d*tan(f*x + e))/(-4*I*sqrt(2)*d^(
3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(sqrt(d)*f*(-I*d/sqrt(d^2) + 1)) - 1/(
sqrt(d*tan(f*x + e))*f))/d
```

Mupad [B]
time = 4.36, size = 50, normalized size = 0.81

$$\frac{2\,{(-1)}^{1/4}\,a\,\mathrm{atanh}\left(\frac{{(-1)}^{1/4}\,\sqrt{d\,\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)}{d^{3/2}\,f}-\frac{2\,a}{d\,f\,\sqrt{d\,\tan\left(e+f\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)/(d*tan(e + f*x))^(3/2),x)
```

```
[Out] (2*(-1)^(1/4)*a*atanh(((-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2)))/(d^(3/2
)*f) - (2*a)/(d*f*(d*tan(e + f*x))^(1/2))
```

3.140 $\int \frac{a+ia\tan(e+fx)}{(d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=87

$$\frac{2\sqrt[4]{-1}\,a\mathrm{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{5/2}f}-\frac{2a}{3df(d\tan(e+fx))^{3/2}}-\frac{2ia}{d^2f\sqrt{d\tan(e+fx)}}$$

```
[Out] 2*(-1)^(1/4)*a*arctan((-1)^(3/4)*(d*tan(f*x+e))^(1/2)/d^(1/2))/d^(5/2)/f-2*
I*a/d^2/f/(d*tan(f*x+e))^(1/2)-2/3*a/d/f/(d*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.09, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3610, 3614, 211}

$$\frac{2\sqrt[4]{-1}\,a\mathrm{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{5/2}f}-\frac{2ia}{d^2f\sqrt{d\tan(e+fx)}}-\frac{2a}{3df(d\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])/(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (2*(-1)^(1/4)*a*ArcTan[((-1)^(3/4)*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(d^(5/2)
*f) - (2*a)/(3*d*f*(d*Tan[e + f*x])^(3/2)) - ((2*I)*a)/(d^2*f*Sqrt[d*Tan[e
+ f*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{a+ia\tan(e+fx)}{(d\tan(e+fx))^{5/2}}\,dx = -\frac{2a}{3df(d\tan(e+fx))^{3/2}} + \frac{\int \frac{iad-ad\tan(e+fx)}{(d\tan(e+fx))^{3/2}}\,dx}{d^2}$$

$$= -\frac{2a}{3df(d\tan(e+fx))^{3/2}} - \frac{2ia}{d^2f\sqrt{d\tan(e+fx)}} + \frac{\int \frac{-ad^2-iad^2\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx}{d^4}$$

$$= -\frac{2a}{3df(d\tan(e+fx))^{3/2}} - \frac{2ia}{d^2f\sqrt{d\tan(e+fx)}} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{-ad^3+iad^2x^2}\,dx,\right.}{f}$$

$$= \frac{2\sqrt[4]{-1}\,a\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{5/2}f} - \frac{2a}{3df(d\tan(e+fx))^{3/2}} - \frac{2i}{d^2f\sqrt{d\tan}}$$

Mathematica [A]
time = 1.20, size = 140, normalized size = 1.61

$$\frac{2ae^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)\cos(e+fx)\left(1-3\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\right)(i\tan(e+fx))^{3/2}+3i\tan(e+fx)\right)(-i+\tan(e+fx))}{3d^2\left(-1+e^{2i(e+fx)}\right)f\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])/(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (2*a*(1 + E^((2*I)*(e + f*x)))*Cos[e + f*x]*(1 - 3*ArcTanh[Sqrt[(-1 + E^((2
*I)*(e + f*x)))/(1 + E^((2*I)*(e + f*x)))]]*(I*Tan[e + f*x])^(3/2) + (3*I)*
Tan[e + f*x])*(-I + Tan[e + f*x]))/(3*d^2*E^(I*(e + f*x))*(-1 + E^((2*I)*(e
 + f*x)))*f*Sqrt[d*Tan[e + f*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 309 vs. 2(70) = 140.
time = 0.10, size = 310, normalized size = 3.56

method	result
derivativedivides	$a\left(-\frac{\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+e}}{(d^2)^{\frac{1}{4}}}\right.\right.}{4d}}{}\right.$

default	$a\left(\frac{-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+}{(d^2)^{\frac{1}{4}}}\right.\right.}{4d}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))/(d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a*(2/d^2*(-1/8/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*t
an(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+
e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))
^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))-1/8*I/(d^2
)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+
(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(
1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/
2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)))-2/3/d/(d*tan(f*x+e))^(3/2)-2*I/d^2
/(d*tan(f*x+e))^(1/2))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(72) = 144$.
time $= 0.51$, size $= 200$, normalized size $= 2.30$

$$\frac{\frac{3a\left(-\frac{(2i+2)\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(2i+2)\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{(i-1)\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}-\frac{(i-1)\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)}{d}-\frac{8(3iad\tan(fx+e)+ad)}{(d\tan(fx+e))^{\frac{3}{2}}d}}{12df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*a*(-(2*I + 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(
d*tan(f*x + e)))/sqrt(d))/sqrt(d) - (2*I + 2)*sqrt(2)*arctan(-1/2*sqrt(2)*(
sqrt(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) + (I - 1)*sqrt(2
)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) -
(I - 1)*sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) +
 d)/sqrt(d))/d - 8*(3*I*a*d*tan(f*x + e) + a*d)/((d*tan(f*x + e))^(3/2)*d))
/(d*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(72) = 144$.
time $= 0.37$, size $= 410$, normalized size $= 4.71$

$$-\frac{3\left(d^3fe^{(4ifx+4ie)}-2d^3fe^{(2ifx+2ie)}+d^3f\right)\sqrt{-\frac{4ia^2}{d^5f^2}}\log\left(\frac{\left(-2iade^{(2ifx+2ie)}+\left(d^3fe^{(2ifx+2ie)}+d^3f\right)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\sqrt{-\frac{4ia^2}{d^5f^2}}\right)e^{(-2ifx-2ie)}}{a}\right)-3\left(d^3fe^{(4ifx+4ie)}-2d^3fe^{(2ifx+2ie)}+d^3f\right)\sqrt{-\frac{4ia^2}{d^5f^2}}\log\left(\frac{\left(-2iade^{(2ifx+2ie)}-\left(d^3fe^{(2ifx+2ie)}+d^3f\right)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\sqrt{-\frac{4ia^2}{d^5f^2}}\right)e^{(-2ifx-2ie)}}{a}\right)-16\left(2ae^{(4ifx+4ie)}+ae^{(2ifx+2ie)}-a\right)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}}{12\left(d^3fe^{(4ifx+4ie)}-2d^3fe^{(2ifx+2ie)}+d^3f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(3*(d^3*f*e^(4*I*f*x + 4*I*e) - 2*d^3*f*e^(2*I*f*x + 2*I*e) + d^3*f)*
sqrt(-4*I*a^2/(d^5*f^2))*log((-2*I*a*d*e^(2*I*f*x + 2*I*e) + (d^3*f*e^(2*I*
f*x + 2*I*e) + d^3*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2
*I*e) + 1))*sqrt(-4*I*a^2/(d^5*f^2)))*e^(-2*I*f*x - 2*I*e)/a) - 3*(d^3*f*e^
(4*I*f*x + 4*I*e) - 2*d^3*f*e^(2*I*f*x + 2*I*e) + d^3*f)*sqrt(-4*I*a^2/(d^5
*f^2))*log((-2*I*a*d*e^(2*I*f*x + 2*I*e) - (d^3*f*e^(2*I*f*x + 2*I*e) + d^3
*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-
4*I*a^2/(d^5*f^2)))*e^(-2*I*f*x - 2*I*e)/a) - 16*(2*a*e^(4*I*f*x + 4*I*e) +
 a*e^(2*I*f*x + 2*I*e) - a)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f
*x + 2*I*e) + 1)))/(d^3*f*e^(4*I*f*x + 4*I*e) - 2*d^3*f*e^(2*I*f*x + 2*I*e)
 + d^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-\frac{i}{(d\tan(e+fx))^{\frac{5}{2}}}\right)dx+\int\frac{\tan(e+fx)}{(d\tan(e+fx))^{\frac{5}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(d*tan(f*x+e))**(5/2),x)
```

```
[Out] I*a*(Integral(-I/(d*tan(e + f*x))**(5/2), x) + Integral(tan(e + f*x)/(d*tan
(e + f*x))**(5/2), x))
```

Giac [A]
time = 0.71, size = 109, normalized size = 1.25

$$-\frac{2}{3}\,a\left(\frac{3i\,\sqrt{2}\,\arctan\left(\frac{8\sqrt{d^2}\,\sqrt{d\tan(fx+e)}}{4i\,\sqrt{2}\,d^{\frac{3}{2}}+4\,\sqrt{2}\,\sqrt{d^2}\,\sqrt{d}}\right)}{d^{\frac{5}{2}}f\left(\frac{id}{\sqrt{d^2}}+1\right)}+\frac{3i\,d\tan(fx+e)+d}{\sqrt{d\tan(fx+e)}\,d^3f\tan(fx+e)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*a*(3*I*sqrt(2)*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*I*sqrt(2)*d^
(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(d^(5/2)*f*(I*d/sqrt(d^2) + 1)) + (3*
I*d*tan(f*x + e) + d)/(sqrt(d*tan(f*x + e))*d^3*f*tan(f*x + e)))
```

Mupad [B]
time = 4.94, size = 70, normalized size = 0.80

$$-\frac{a\,2\mathrm{i}}{d^2\,f\,\sqrt{d\tan(e+f\,x)}}-\frac{2\,a}{3\,d\,f\,(d\tan(e+f\,x))^{3/2}}+\frac{(-1)^{1/4}\,a\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan(e+f\,x)}}{\sqrt{d}}\right)\,2\mathrm{i}}{d^{5/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)/(d*tan(e + f*x))^(5/2),x)
```

```
[Out] ((-1)^(1/4)*a*atanh(((-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2))*2i)/(d^(5/
2)*f) - (2*a)/(3*d*f*(d*tan(e + f*x))^(3/2)) - (a*2i)/(d^2*f*(d*tan(e + f*x
))^(1/2))
```

3.141 $\int \frac{a+ia\tan(e+fx)}{(d\tan(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=110

$$\frac{2(-1)^{3/4}a\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{7/2}f}-\frac{2a}{5df(d\tan(e+fx))^{5/2}}-\frac{2ia}{3d^2f(d\tan(e+fx))^{3/2}}+\frac{2a}{d^3f\sqrt{d\tan(e+fx)}}$$

```
[Out] 2*(-1)^(3/4)*a*arctan((-1)^(3/4)*(d*tan(f*x+e))^(1/2)/d^(1/2))/d^(7/2)/f+2*
a/d^3/f/(d*tan(f*x+e))^(1/2)-2/5*a/d/f/(d*tan(f*x+e))^(5/2)-2/3*I*a/d^2/f/(
d*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.12, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3610, 3614, 211}

$$\frac{2(-1)^{3/4}a\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{7/2}f}+\frac{2a}{d^3f\sqrt{d\tan(e+fx)}}-\frac{2ia}{3d^2f(d\tan(e+fx))^{3/2}}-\frac{2a}{5df(d\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])/(d*Tan[e + f*x])^(7/2),x]
```

```
[Out] (2*(-1)^(3/4)*a*ArcTan[((-1)^(3/4)*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(d^(7/2)
*f) - (2*a)/(5*d*f*(d*Tan[e + f*x])^(5/2)) - (((2*I)/3)*a)/(d^2*f*(d*Tan[e
+ f*x])^(3/2)) + (2*a)/(d^3*f*Sqrt[d*Tan[e + f*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{a+ia\tan(e+fx)}{(d\tan(e+fx))^{7/2}}\,dx = -\frac{2a}{5df(d\tan(e+fx))^{5/2}} + \frac{\int \frac{iad-ad\tan(e+fx)}{(d\tan(e+fx))^{5/2}}\,dx}{d^2}$$

$$= -\frac{2a}{5df(d\tan(e+fx))^{5/2}} - \frac{2ia}{3d^2f(d\tan(e+fx))^{3/2}} + \frac{\int \frac{-ad^2-iad^2\tan(e+fx)}{(d\tan(e+fx))^{3/2}}\,dx}{d^4}$$

$$= -\frac{2a}{5df(d\tan(e+fx))^{5/2}} - \frac{2ia}{3d^2f(d\tan(e+fx))^{3/2}} + \frac{2a}{d^3f\sqrt{d\tan(e+fx)}} + \frac{\int}{}$$

$$= -\frac{2a}{5df(d\tan(e+fx))^{5/2}} - \frac{2ia}{3d^2f(d\tan(e+fx))^{3/2}} + \frac{2a}{d^3f\sqrt{d\tan(e+fx)}} - \frac{(2}{}$$

$$= \frac{2(-1)^{3/4}a\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{7/2}f} - \frac{2a}{5df(d\tan(e+fx))^{5/2}} - \frac{}{3d^2f(d}$$

Mathematica [**A**]
time = 4.14, size = 105, normalized size = 0.95

$$-\frac{2a\left(\cot(e+fx)(5i+18\cot(2(e+fx)))-6\csc^2(e+fx)+15\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\right)\sqrt{i\tan(e+fx)}\right)}{15d^3f\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])/(d*Tan[e + f*x])^(7/2),x]
```

```
[Out] (-2*a*(Cot[e + f*x]*(5*I + 18*Cot[2*(e + f*x)]) - 6*Csc[e + f*x]^2 + 15*Arc
Tanh[Sqrt[(-1 + E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f*x)))]]*Sqrt[I*Tan
[e + f*x]]))/(15*d^3*f*Sqrt[d*Tan[e + f*x]])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 324 vs. 2(89) = 178.
time = 0.10, size = 325, normalized size = 2.95

method	result

derivativedivides	$a\left(-\frac{i\left(d^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+\left(d^{2}\right)^{\frac{1}{4}}\sqrt{d\tan\left(fx+e\right)}\sqrt{2}+\sqrt{d^{2}}}{d\tan(fx+e)-\left(d^{2}\right)^{\frac{1}{4}}\sqrt{d\tan\left(fx+e\right)}\sqrt{2}+\sqrt{d^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan\left(fx+e\right.}}{\left(d^{2}\right)^{\frac{1}{4}}}\right.\right.}{4d}\right.$
default	$a\left(-\frac{i\left(d^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+\left(d^{2}\right)^{\frac{1}{4}}\sqrt{d\tan\left(fx+e\right)}\sqrt{2}+\sqrt{d^{2}}}{d\tan(fx+e)-\left(d^{2}\right)^{\frac{1}{4}}\sqrt{d\tan\left(fx+e\right)}\sqrt{2}+\sqrt{d^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan\left(fx+e\right.}}{\left(d^{2}\right)^{\frac{1}{4}}}\right.\right.}{4d}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))/(d*tan(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a*(2/d^3*(-1/8*I/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d
*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*
x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e
))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))+1/8/(d^2
)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+
(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(
1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/
2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)))-2/5/d/(d*tan(f*x+e))^(5/2)-2/3*I/d
^2/(d*tan(f*x+e))^(3/2)+2/d^3/(d*tan(f*x+e))^(1/2))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(92) = 184$.
time = 0.51, size = 220, normalized size = 2.00

$$-\frac{\frac{15\,a\left(\frac{(2i-2)\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{(2i-2)\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{(i+1)\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{d}+d\right)}{\sqrt{d}}-\frac{(i+1)\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{d}+d\right)}{\sqrt{d}}\right)}{d^{2}}-\frac{8\left(15\,ad^{2}\tan(fx+e)^{2}-5i\,ad^{2}\tan(fx+e)-3\,ad^{2}\right)}{\left(d\tan(fx+e)\right)^{\frac{5}{2}}d^{2}}}{60\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(d*tan(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] -1/60*(15*a*((2*I - 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt
(d*tan(f*x + e)))/sqrt(d))/sqrt(d) + (2*I - 2)*sqrt(2)*arctan(-1/2*sqrt(2)*
(sqrt(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) + (I + 1)*sqrt(
2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) -
```

```
(I + 1)*sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d)
+ d)/sqrt(d))/d^2 - 8*(15*a*d^2*tan(f*x + e)^2 - 5*I*a*d^2*tan(f*x + e) - 3
*a*d^2)/((d*tan(f*x + e))^(5/2)*d^2))/(d*f)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 478 vs. 2(92) = 184.
time = 0.40, size = 478, normalized size = 4.35

$$\frac{15\left(d^4 f e^{(6i fx+6i e)}-3 d^4 f e^{(4i fx+4i e)}+3 d^4 f e^{(2i fx+2i e)}-d^4 f\right)\sqrt{\frac{4i a^2}{d^7 f^2}}\log\left(\frac{\left(-2i a d e^{(2i fx+2i e)}+\left(i d^4 f e^{(2i fx+2i e)}+i d^4 f\right)\sqrt{\frac{-i d e^{(2i fx+2i e)}+i d}{e^{(2i fx+2i e)}+1}}\sqrt{\frac{4i a^2}{d^7 f^2}}\right)e^{(-2i fx-2i e)}}{a}\right)-15\left(d^4 f e^{(6i fx+6i e)}-3 d^4 f e^{(4i fx+4i e)}+3 d^4 f e^{(2i fx+2i e)}-d^4 f\right)\sqrt{\frac{4i a^2}{d^7 f^2}}\log\left(\frac{\left(-2i a d e^{(2i fx+2i e)}+\left(-i d^4 f e^{(2i fx+2i e)}-i d^4 f\right)\sqrt{\frac{-i d e^{(2i fx+2i e)}+i d}{e^{(2i fx+2i e)}+1}}\sqrt{\frac{4i a^2}{d^7 f^2}}\right)e^{(-2i fx-2i e)}}{a}\right)-8\left(-23i a e^{(6i fx+6i e)}+i a e^{(4i fx+4i e)}+11i a e^{(2i fx+2i e)}-13i a\right)\sqrt{\frac{-i d e^{(2i fx+2i e)}+i d}{e^{(2i fx+2i e)}+1}}}{60\left(d^4 f e^{(6i fx+6i e)}-3 d^4 f e^{(4i fx+4i e)}+3 d^4 f e^{(2i fx+2i e)}-d^4 f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(d*tan(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/60*(15*(d^4*f*e^(6*I*f*x + 6*I*e) - 3*d^4*f*e^(4*I*f*x + 4*I*e) + 3*d^4*f
*e^(2*I*f*x + 2*I*e) - d^4*f)*sqrt(4*I*a^2/(d^7*f^2))*log((-2*I*a*d*e^(2*I*
f*x + 2*I*e) + (I*d^4*f*e^(2*I*f*x + 2*I*e) + I*d^4*f)*sqrt((-I*d*e^(2*I*f*
x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(4*I*a^2/(d^7*f^2)))*e^(-2
*I*f*x - 2*I*e)/a) - 15*(d^4*f*e^(6*I*f*x + 6*I*e) - 3*d^4*f*e^(4*I*f*x + 4
*I*e) + 3*d^4*f*e^(2*I*f*x + 2*I*e) - d^4*f)*sqrt(4*I*a^2/(d^7*f^2))*log((-
2*I*a*d*e^(2*I*f*x + 2*I*e) + (-I*d^4*f*e^(2*I*f*x + 2*I*e) - I*d^4*f)*sqrt
((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(4*I*a^2/(
d^7*f^2)))*e^(-2*I*f*x - 2*I*e)/a) - 8*(-23*I*a*e^(6*I*f*x + 6*I*e) + I*a*e
^(4*I*f*x + 4*I*e) + 11*I*a*e^(2*I*f*x + 2*I*e) - 13*I*a)*sqrt((-I*d*e^(2*I
*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))/(d^4*f*e^(6*I*f*x + 6*I*e)
 - 3*d^4*f*e^(4*I*f*x + 4*I*e) + 3*d^4*f*e^(2*I*f*x + 2*I*e) - d^4*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-\frac{i}{(d\tan(e+fx))^{\frac{7}{2}}}\right)dx+\int\frac{\tan(e+fx)}{(d\tan(e+fx))^{\frac{7}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(d*tan(f*x+e))**(7/2),x)
```

```
[Out] I*a*(Integral(-I/(d*tan(e + f*x))**(7/2), x) + Integral(tan(e + f*x)/(d*tan
(e + f*x))**(7/2), x))
```

Giac [**A**]
time = 0.93, size = 130, normalized size = 1.18

$$-\frac{2}{15}\,a\left(\frac{15i\sqrt{2}\arctan\left(-\frac{8i\sqrt{d^2}\sqrt{d\tan(fx+e)}}{-4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{d^{\frac{7}{2}}f\left(-\frac{i\,d}{\sqrt{d^2}}+1\right)}-\frac{15\,d^2\tan(fx+e)^2-5i\,d^2\tan(fx+e)-3\,d^2}{\sqrt{d\tan(fx+e)}\,d^5 f\tan(fx+e)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(d*tan(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] -2/15*a*(15*I*sqrt(2)*arctan(-8*I*sqrt(d^2)*sqrt(d*tan(f*x + e))/(-4*I*sqrt
(2)*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(d^(7/2)*f*(-I*d/sqrt(d^2) + 1)
) - (15*d^2*tan(f*x + e)^2 - 5*I*d^2*tan(f*x + e) - 3*d^2)/(sqrt(d*tan(f*x
+ e))*d^5*f*tan(f*x + e)^2))
```

Mupad [B]
time = 5.28, size = 87, normalized size = 0.79

$$-\frac{\frac{2\,a}{5\,d}-\frac{2\,a\,\tan(e+f\,x)^2}{d}}{f\,{\left(d\tan\left(e+f\,x\right)\right)}^{5/2}}-\frac{2\,{\left(-1\right)}^{1/4}\,a\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{d\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)}{d^{7/2}\,f}-\frac{a\,2\mathrm{i}}{3\,d^2\,f\,{\left(d\tan\left(e+f\,x\right)\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)/(d*tan(e + f*x))^(7/2),x)
```

```
[Out] - ((2*a)/(5*d) - (2*a*tan(e + f*x)^2)/d)/(f*(d*tan(e + f*x))^(5/2)) - (a*2i
)/(3*d^2*f*(d*tan(e + f*x))^(3/2)) - (2*(-1)^(1/4)*a*atanh(((-1)^(1/4)*(d*t
an(e + f*x))^(1/2))/d^(1/2)))/(d^(7/2)*f)
```

3.142 $\int (d\tan(e+fx))^{5/2}(a-ia\tan(e+fx))\,dx$

Optimal. Leaf size=107

$$\frac{2(-1)^{3/4}ad^{5/2}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}+\frac{2iad^2\sqrt{d\tan(e+fx)}}{f}+\frac{2ad(d\tan(e+fx))^{3/2}}{3f}-\frac{2ia(d\ta$$

```
[Out] 2*(-1)^(3/4)*a*d^(5/2)*arctanh((-1)^(3/4)*(d*tan(f*x+e))^(1/2)/d^(1/2))/f+2
*I*a*d^2*(d*tan(f*x+e))^(1/2)/f+2/3*a*d*(d*tan(f*x+e))^(3/2)/f-2/5*I*a*(d*t
an(f*x+e))^(5/2)/f
```

Rubi [A]
time = 0.11, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3609, 3614, 214}

$$\frac{2(-1)^{3/4}ad^{5/2}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}+\frac{2iad^2\sqrt{d\tan(e+fx)}}{f}+\frac{2ad(d\tan(e+fx))^{3/2}}{3f}-\frac{2ia(d\tan(e+fx))^{5/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(5/2)*(a - I*a*Tan[e + f*x]),x]
```

```
[Out] (2*(-1)^(3/4)*a*d^(5/2)*ArcTanh[((-1)^(3/4)*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])
/f + ((2*I)*a*d^2*Sqrt[d*Tan[e + f*x]])/f + (2*a*d*(d*Tan[e + f*x])^(3/2))/
(3*f) - (((2*I)/5)*a*(d*Tan[e + f*x])^(5/2))/f
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rubi steps

$$\int (d\tan(e+fx))^{5/2}(a-ia\tan(e+fx))\,dx = -\frac{2ia(d\tan(e+fx))^{5/2}}{5f} + \int (d\tan(e+fx))^{3/2}(iad+ad\tan($$
$$= \frac{2ad(d\tan(e+fx))^{3/2}}{3f} - \frac{2ia(d\tan(e+fx))^{5/2}}{5f} + \int \sqrt{d\tan($$
$$= \frac{2iad^2\sqrt{d\tan(e+fx)}}{f} + \frac{2ad(d\tan(e+fx))^{3/2}}{3f} - \frac{2ia(d\tan(}{5}$$
$$= \frac{2iad^2\sqrt{d\tan(e+fx)}}{f} + \frac{2ad(d\tan(e+fx))^{3/2}}{3f} - \frac{2ia(d\tan(}{5}$$
$$= \frac{2(-1)^{3/4}ad^{5/2}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f} + \frac{2iad^2\sqrt{d}}{}$$

Mathematica [A]
time = 0.53, size = 90, normalized size = 0.84

$$\frac{2a(d\tan(e+fx))^{5/2}\left(15(-1)^{3/4}\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(e+fx)}\right)+\sqrt{\tan(e+fx)}\,(15i+5\tan(e+fx)-3i\tan^2(e+fx))\right)}{15f\tan^{\frac{5}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(5/2)*(a - I*a*Tan[e + f*x]),x]
```

```
[Out] (2*a*(d*Tan[e + f*x])^(5/2)*(15*(-1)^(3/4)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[e +
f*x]]] + Sqrt[Tan[e + f*x]]*(15*I + 5*Tan[e + f*x] - (3*I)*Tan[e + f*x]^2))
)/(15*f*Tan[e + f*x]^(5/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(85) = 170.
time = 0.11, size = 322, normalized size = 3.01

method	result
derivativedivides	$-\frac{a\left(\frac{2i(d\tan(fx+e))^{\frac{5}{2}}}{5}-\frac{2d(d\tan(fx+e))^{\frac{3}{2}}}{3}-2id^2\sqrt{d\tan(fx+e)}+2d^3\left(\frac{i(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d}}\right.\right.}{}\right.\right.}{}$

default	$-\frac{a\left(\frac{2i(d\tan(fx+e))^{\frac{5}{2}}}{5}-\frac{2d(d\tan(fx+e))^{\frac{3}{2}}}{3}-2id^{2}\sqrt{d\tan(fx+e)}+2d^{3}\left(\frac{i(d^{2})^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^{2})^{\frac{1}{4}}\sqrt{\cdots}}{d\tan(fx+e)-(d^{2})^{\frac{1}{4}}\sqrt{\cdots}}\right.\right.}{\cdots}\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(5/2)*(a-I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*a*(2/5*I*(d*tan(f*x+e))^(5/2)-2/3*d*(d*tan(f*x+e))^(3/2)-2*I*d^2*(d*ta
n(f*x+e))^(1/2)+2*d^3*(1/8*I/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^
(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(
d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*t
an(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))+
1/8/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*
2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)
+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arcta
n(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(87) = 174$.
time = 0.52, size = 218, normalized size = 2.04

$$\frac{15\,ad^{4}\left(-\frac{(2i+2)\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(2i+2)\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(i-1)\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{(i-1)\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)-24i\,(d\tan(fx+e))^{\frac{5}{2}}ad+40\,(d\tan(fx+e))^{\frac{3}{2}}ad^{2}+120i\sqrt{d\tan(fx+e)}\,ad^{3}}{60\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)*(a-I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/60*(15*a*d^4*(-(2*I + 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*
sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - (2*I + 2)*sqrt(2)*arctan(-1/2*sqrt
(2)*(sqrt(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - (I - 1)*s
qrt(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(
d) + (I - 1)*sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt
(d) + d)/sqrt(d)) - 24*I*(d*tan(f*x + e))^(5/2)*a*d + 40*(d*tan(f*x + e))^(
3/2)*a*d^2 + 120*I*sqrt(d*tan(f*x + e))*a*d^3)/(d*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(87) = 174$.
time = 0.38, size = 360, normalized size = 3.36

$$\frac{15\sqrt{-\frac{4i\,a^{2}d^{5}}{f^{2}}}\left(fe^{(4i\,fx+4i\,e)}+2\,fe^{(2i\,fx+2i\,e)}+f\right)\log\left(\frac{\left(2\,ad^{3}+\sqrt{-\frac{4i\,a^{2}d^{5}}{f^{2}}}\left(fe^{(2i\,fx+2i\,e)}+f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-2i\,fx-2i\,e)}}{f}\right)-15\sqrt{-\frac{4i\,a^{2}d^{5}}{f^{2}}}\left(fe^{(4i\,fx+4i\,e)}+2\,fe^{(2i\,fx+2i\,e)}+f\right)\log\left(\frac{\left(2\,ad^{3}-\sqrt{-\frac{4i\,a^{2}d^{5}}{f^{2}}}\left(fe^{(2i\,fx+2i\,e)}+f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-2i\,fx-2i\,e)}}{f}\right)-8\left(-13i\,ad^{2}e^{(4i\,fx+4i\,e)}-24i\,ad^{2}e^{(2i\,fx+2i\,e)}-23i\,ad^{2}\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}}{60\left(fe^{(4i\,fx+4i\,e)}+2\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)*(a-I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/60*(15*sqrt(-4*I*a^2*d^5/f^2)*(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2
*I*e) + f)*log((2*a*d^3 + sqrt(-4*I*a^2*d^5/f^2)*(f*e^(2*I*f*x + 2*I*e) + f
)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-2*I
*f*x - 2*I*e)/f) - 15*sqrt(-4*I*a^2*d^5/f^2)*(f*e^(4*I*f*x + 4*I*e) + 2*f*e
^(2*I*f*x + 2*I*e) + f)*log((2*a*d^3 - sqrt(-4*I*a^2*d^5/f^2)*(f*e^(2*I*f*x
 + 2*I*e) + f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) +
 1)))*e^(-2*I*f*x - 2*I*e)/f) - 8*(-13*I*a*d^2*e^(4*I*f*x + 4*I*e) - 24*I*a
*d^2*e^(2*I*f*x + 2*I*e) - 23*I*a*d^2)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d
)/(e^(2*I*f*x + 2*I*e) + 1)))/(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I
*e) + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia\left(\int i(d\tan(e+fx))^{\frac{5}{2}}\,dx + \int (d\tan(e+fx))^{\frac{5}{2}}\tan(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(5/2)*(a-I*a*tan(f*x+e)),x)
```

```
[Out] -I*a*(Integral(I*(d*tan(e + f*x))**(5/2), x) + Integral((d*tan(e + f*x))**(
5/2)*tan(e + f*x), x))
```

Giac [A]
time = 0.58, size = 154, normalized size = 1.44

$$-\frac{2}{15}\,ad^2\left(\frac{15\sqrt{2}\sqrt{d}\arctan\left(\frac{8\sqrt{d^2}\sqrt{d\tan(fx+e)}}{-4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{f\left(-\frac{id}{\sqrt{d^2}}+1\right)}+\frac{3i\sqrt{d\tan(fx+e)}\,d^{10}f^4\tan(fx+e)^2-5\sqrt{d\tan(fx+e)}\,d^{10}f^4\tan(fx+e)-15i\sqrt{d\tan(fx+e)}\,d^{10}f^4}{d^{10}f^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)*(a-I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] -2/15*a*d^2*(15*sqrt(2)*sqrt(d)*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(-4
*I*sqrt(2)*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(f*(-I*d/sqrt(d^2) + 1))
 + (3*I*sqrt(d*tan(f*x + e))*d^10*f^4*tan(f*x + e)^2 - 5*sqrt(d*tan(f*x + e
))*d^10*f^4*tan(f*x + e) - 15*I*sqrt(d*tan(f*x + e))*d^10*f^4)/(d^10*f^5))
```

Mupad [B]
time = 4.58, size = 143, normalized size = 1.34

$$\frac{2\,a\,d\,(d\tan(e+f\,x))^{3/2}}{3\,f}-\frac{2\,(-1)^{1/4}\,a\,d^{5/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{d\tan(e+f\,x)}}{\sqrt{d}}\right)}{f}+\frac{(-1)^{1/4}\,a\,d^{5/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\tan(e+f\,x)}}{\sqrt{d}}\right)}{f}-\frac{a\,(d\tan(e+f\,x))^{5/2}\,2\mathrm{i}}{5\,f}+\frac{a\,d^2\sqrt{d\tan(e+f\,x)}\,2\mathrm{i}}{f}+\frac{(-1)^{1/4}\,a\,d^{5/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{d\tan(e+f\,x)}\,1\mathrm{i}}{\sqrt{d}}\right)\,1\mathrm{i}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(5/2)*(a - a*tan(e + f*x)*1i),x)
```

```
[Out] (2*a*d*(d*tan(e + f*x))^(3/2))/(3*f) - (a*(d*tan(e + f*x))^(5/2)*2i)/(5*f)
+ (a*d^2*(d*tan(e + f*x))^(1/2)*2i)/f - (2*(-1)^(1/4)*a*d^(5/2)*atan(((-1)^
(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2)))/f + ((-1)^(1/4)*a*d^(5/2)*atan(((-1
)^(1/4)*(d*tan(e + f*x))^(1/2)*1i)/d^(1/2))*1i)/f + ((-1)^(1/4)*a*d^(5/2)*a
tanh(((-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2)))/f
```

3.143 $\int (d\tan(e+fx))^{3/2}(a-ia\tan(e+fx))\,dx$

Optimal. Leaf size=82

$$\frac{2\sqrt[4]{-1}\,ad^{3/2}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}+\frac{2ad\sqrt{d\tan(e+fx)}}{f}-\frac{2ia(d\tan(e+fx))^{3/2}}{3f}$$

```
[Out] 2*(-1)^(1/4)*a*d^(3/2)*arctanh((-1)^(3/4)*(d*tan(f*x+e))^(1/2)/d^(1/2))/f+2
*a*d*(d*tan(f*x+e))^(1/2)/f-2/3*I*a*(d*tan(f*x+e))^(3/2)/f
```

Rubi [A]
time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3609, 3614, 214}

$$\frac{2\sqrt[4]{-1}\,ad^{3/2}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}+\frac{2ad\sqrt{d\tan(e+fx)}}{f}-\frac{2ia(d\tan(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(3/2)*(a - I*a*Tan[e + f*x]),x]
```

```
[Out] (2*(-1)^(1/4)*a*d^(3/2)*ArcTanh[((-1)^(3/4)*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])
/f + (2*a*d*Sqrt[d*Tan[e + f*x]])/f - (((2*I)/3)*a*(d*Tan[e + f*x])^(3/2))/
f
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rubi steps

$$\int (d\tan(e+fx))^{3/2}(a-ia\tan(e+fx))\,dx = -\frac{2ia(d\tan(e+fx))^{3/2}}{3f} + \int \sqrt{d\tan(e+fx)}\,(iad+ad\tan($$

$$= \frac{2ad\sqrt{d\tan(e+fx)}}{f} - \frac{2ia(d\tan(e+fx))^{3/2}}{3f} + \int \frac{-ad^2+}{\sqrt{d}}$$

$$= \frac{2ad\sqrt{d\tan(e+fx)}}{f} - \frac{2ia(d\tan(e+fx))^{3/2}}{3f} + \frac{(2a^2d^4)\,\mathrm{Sub}}{}$$

$$= \frac{2\sqrt[4]{-1}\,ad^{3/2}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f} + \frac{2ad\sqrt{d\,\mathrm{ta}}}{}$$

Mathematica [A]
time = 0.24, size = 78, normalized size = 0.95

$$\frac{2a\left(3\sqrt[4]{-1}\,\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(e+fx)}\right)+(3-i\tan(e+fx))\sqrt{\tan(e+fx)}\right)(d\tan(e+fx))^{3/2}}{3f\tan^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(3/2)*(a - I*a*Tan[e + f*x]),x]
```

```
[Out] (2*a*(3*(-1)^(1/4)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[e + f*x]]] + (3 - I*Tan[e +
f*x])*Sqrt[Tan[e + f*x]])*(d*Tan[e + f*x])^(3/2))/(3*f*Tan[e + f*x]^(3/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(65) = 130.
time = 0.11, size = 306, normalized size = 3.73

method	result
derivativedivides	$-\frac{a\left(\frac{2i(d\tan(fx+e))^{\frac{3}{2}}}{3}-2d\sqrt{d\tan(fx+e)}-2d^2\left(-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}}\right.\right.}{}\right.\right.}{}$
default	$-\frac{a\left(\frac{2i(d\tan(fx+e))^{\frac{3}{2}}}{3}-2d\sqrt{d\tan(fx+e)}-2d^2\left(-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}}\right.\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(3/2)*(a-I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*a*(2/3*I*(d*tan(f*x+e))^(3/2)-2*d*(d*tan(f*x+e))^(1/2)-2*d^2*(-1/8/d*(
d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/
2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2
)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^
(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))+1/8*I/(d^2)^(1/4)*2^(1/2)*(ln((d
*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*
x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2
)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(
f*x+e))^(1/2)+1))))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(67) = 134$.
time = 0.52, size = 201, normalized size = 2.45

$$-\frac{3\,ad^3\left(-\frac{(2i-2)\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(2i-2)\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{(i+1)\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}-\frac{(i+1)\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)+8i\,(d\tan(fx+e))^{\frac{3}{2}}ad-24\sqrt{d\tan(fx+e)}\,ad^2}{12\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)*(a-I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/12*(3*a*d^3*(-(2*I - 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*
sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - (2*I - 2)*sqrt(2)*arctan(-1/2*sqrt
(2)*(sqrt(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) + (I + 1)*s
qrt(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(
d) - (I + 1)*sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt
(d) + d)/sqrt(d)) + 8*I*(d*tan(f*x + e))^(3/2)*a*d - 24*sqrt(d*tan(f*x + e)
)*a*d^2)/(d*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(67) = 134$.
time = 0.37, size = 300, normalized size = 3.66

$$\frac{3\sqrt{\frac{4i\,a^2d^3}{f^2}}\left(fe^{(2i\,fx+2i\,e)}+f\right)\log\left(\frac{\left(-2i\,ad^2+\sqrt{\frac{4i\,a^2d^3}{f^2}}\left(fe^{(2i\,fx+2i\,e)}+f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-2i\,fx-2i\,e)}}{f}\right)-3\sqrt{\frac{4i\,a^2d^3}{f^2}}\left(fe^{(2i\,fx+2i\,e)}+f\right)\log\left(\frac{\left(-2i\,ad^2-\sqrt{\frac{4i\,a^2d^3}{f^2}}\left(fe^{(2i\,fx+2i\,e)}+f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-2i\,fx-2i\,e)}}{f}\right)+16\left(ade^{(2i\,fx+2i\,e)}+2\,ad\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}}{12\left(fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)*(a-I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*sqrt(4*I*a^2*d^3/f^2)*(f*e^(2*I*f*x + 2*I*e) + f)*log((-2*I*a*d^2 +
 sqrt(4*I*a^2*d^3/f^2)*(f*e^(2*I*f*x + 2*I*e) + f)*sqrt((-I*d*e^(2*I*f*x +
```

```
2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-2*I*f*x - 2*I*e)/f) - 3*sqrt(
4*I*a^2*d^3/f^2)*(f*e^(2*I*f*x + 2*I*e) + f)*log((-2*I*a*d^2 - sqrt(4*I*a^2
*d^3/f^2)*(f*e^(2*I*f*x + 2*I*e) + f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)
/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-2*I*f*x - 2*I*e)/f) + 16*(a*d*e^(2*I*f*x +
 2*I*e) + 2*a*d)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e)
 + 1)))/(f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-ia\left(\int i(d\tan{(e+fx)})^{\frac{3}{2}}\,dx+\int(d\tan{(e+fx)})^{\frac{3}{2}}\tan{(e+fx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(3/2)*(a-I*a*tan(f*x+e)),x)
```

```
[Out] -I*a*(Integral(I*(d*tan(e + f*x))**(3/2), x) + Integral((d*tan(e + f*x))**(
3/2)*tan(e + f*x), x))
```

Giac [**A**]
time = 0.53, size = 124, normalized size = 1.51

$$-\frac{2}{3}\,ad\left(-\frac{3i\sqrt{2}\,\sqrt{d}\,\arctan\left(\frac{8\sqrt{d^2}\,\sqrt{d\tan{(fx+e)}}}{-4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\,\sqrt{d^2}\,\sqrt{d}}\right)}{f\left(-\frac{id}{\sqrt{d^2}}+1\right)}+\frac{i\sqrt{d\tan{(fx+e)}}\,d^3f^2\tan{(fx+e)}-3\sqrt{d\tan{(fx+e)}}\,d^3f^2}{d^3f^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)*(a-I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] -2/3*a*d*(-3*I*sqrt(2)*sqrt(d)*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(-4*
I*sqrt(2)*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(f*(-I*d/sqrt(d^2) + 1))
+ (I*sqrt(d*tan(f*x + e))*d^3*f^2*tan(f*x + e) - 3*sqrt(d*tan(f*x + e))*d^3
*f^2)/(d^3*f^3))
```

Mupad [**B**]
time = 4.26, size = 65, normalized size = 0.79

$$\frac{2\,a\,d\,\sqrt{d\tan{(e+f\,x)}}}{f}-\frac{a\,(d\tan(e+f\,x))^{3/2}\,2\mathrm{i}}{3\,f}+\frac{(-1)^{1/4}\,a\,d^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan{(e+f\,x)}}}{\sqrt{d}}\right)\,2\mathrm{i}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(3/2)*(a - a*tan(e + f*x)*1i),x)
```

```
[Out] (2*a*d*(d*tan(e + f*x))^(1/2))/f - (a*(d*tan(e + f*x))^(3/2)*2i)/(3*f) + ((
-1)^(1/4)*a*d^(3/2)*atan(((-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2))*2i)/f
```

3.144 $\int \sqrt{d\tan(e+fx)}\,(a-ia\tan(e+fx))\,dx$

Optimal. Leaf size=61

$$-\frac{2(-1)^{3/4}a\sqrt{d}\,\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}-\frac{2ia\sqrt{d\tan(e+fx)}}{f}$$

```
[Out] -2*(-1)^(3/4)*a*arctanh((-1)^(3/4)*(d*tan(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f-
2*I*a*(d*tan(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3609, 3614, 214}

$$-\frac{2(-1)^{3/4}a\sqrt{d}\,\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}-\frac{2ia\sqrt{d\tan(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Tan[e + f*x]]*(a - I*a*Tan[e + f*x]),x]
```

```
[Out] (-2*(-1)^(3/4)*a*Sqrt[d]*ArcTanh[((-1)^(3/4)*Sqrt[d*Tan[e + f*x]])/Sqrt[d]]
)/f - ((2*I)*a*Sqrt[d*Tan[e + f*x]])/f
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \sqrt{d\tan(e+fx)}\,(a-ia\tan(e+fx))\,dx = -\frac{2ia\sqrt{d\tan(e+fx)}}{f} + \int \frac{iad+ad\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$$
$$= -\frac{2ia\sqrt{d\tan(e+fx)}}{f} - \frac{(2a^2d^2)\,\text{Subst}\left(\int \frac{1}{iad^2-adx^2}\,dx, x, \sqrt{c\right.}}{f}$$
$$= -\frac{2(-1)^{3/4}a\sqrt{d}\,\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f} - \frac{2ia\sqrt{c}}{}$$

Mathematica [A]
time = 0.07, size = 64, normalized size = 1.05

$$-\frac{2ia\left(\sqrt[4]{-1}\,\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(e+fx)}\right)+\sqrt{\tan(e+fx)}\right)\sqrt{d\tan(e+fx)}}{f\sqrt{\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Tan[e + f*x]]*(a - I*a*Tan[e + f*x]),x]
```

```
[Out] ((-2*I)*a*((-1)^(1/4)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[e + f*x]]] + Sqrt[Tan[e +
 f*x]])*Sqrt[d*Tan[e + f*x]])/(f*Sqrt[Tan[e + f*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 290 vs. 2(48) = 96.
time = 0.11, size = 291, normalized size = 4.77

method	result
derivativedivides	$-\frac{a\left(2i\sqrt{d\tan(fx+e)}-2d\left(\frac{i(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\right.}{}\right.\right.}{}$
default	$-\frac{a\left(2i\sqrt{d\tan(fx+e)}-2d\left(\frac{i(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(1/2)*(a-I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*a*(2*I*(d*tan(f*x+e))^(1/2)-2*d*(1/8*I/d*(d^2)^(1/4)*2^(1/2)*(ln((d*ta
n(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e
)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(
d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x
+e))^(1/2)+1))+1/8/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan
(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e)
)^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(
1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(49) = 98$.
time = 0.52, size = 183, normalized size = 3.00

$$-\frac{ad^2\left(-\frac{(2i+2)\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(2i+2)\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(i-1)\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{(i-1)\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)+8i\sqrt{d\tan(fx+e)}\,ad}{4\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)*(a-I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/4*(a*d^2*(-(2*I + 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqr
t(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - (2*I + 2)*sqrt(2)*arctan(-1/2*sqrt(2)
*(sqrt(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - (I - 1)*sqrt
(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d)
+ (I - 1)*sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d)
 + d)/sqrt(d)) + 8*I*sqrt(d*tan(f*x + e))*a*d)/(d*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(49) = 98$.
time = 0.38, size = 233, normalized size = 3.82

$$-\frac{\sqrt{-\frac{4i\,a^2d}{f^2}}\,f\log\left(-\frac{\left(2\,ad+\left(fe^{(2i\,fx+2i\,e)}+f\right)\sqrt{-\frac{4i\,a^2d}{f^2}}\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-2i\,fx-2i\,e)}}{f}\right)-\sqrt{-\frac{4i\,a^2d}{f^2}}\,f\log\left(-\frac{\left(2\,ad-\left(fe^{(2i\,fx+2i\,e)}+f\right)\sqrt{-\frac{4i\,a^2d}{f^2}}\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-2i\,fx-2i\,e)}}{f}\right)+8i\,a\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)*(a-I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(-4*I*a^2*d/f^2)*f*log(-(2*a*d + (f*e^(2*I*f*x + 2*I*e) + f)*sqrt
(-4*I*a^2*d/f^2)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e)
 + 1)))*e^(-2*I*f*x - 2*I*e)/f) - sqrt(-4*I*a^2*d/f^2)*f*log(-(2*a*d - (f*e
^(2*I*f*x + 2*I*e) + f)*sqrt(-4*I*a^2*d/f^2)*sqrt((-I*d*e^(2*I*f*x + 2*I*e)
 + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-2*I*f*x - 2*I*e)/f) + 8*I*a*sqrt((-
I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia\left(\int i\sqrt{d\tan(e+fx)}\,dx+\int\sqrt{d\tan(e+fx)}\tan(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(1/2)*(a-I*a*tan(f*x+e)),x)
```

```
[Out] -I*a*(Integral(I*sqrt(d*tan(e + f*x)), x) + Integral(sqrt(d*tan(e + f*x))*t
an(e + f*x), x))
```

Giac [A]
time = 0.50, size = 89, normalized size = 1.46

$$\frac{2\,a\left(\frac{\sqrt{2}\,d^{\frac{3}{2}}\arctan\left(\frac{8\,\sqrt{d^2}\,\sqrt{d\tan\left(fx+e\right)}}{-4i\,\sqrt{2}\,d^{\frac{3}{2}}+4\,\sqrt{2}\,\sqrt{d^2}\,\sqrt{d}}\right)}{f\left(-\frac{i\,d}{\sqrt{d^2}}+1\right)}-\frac{i\,\sqrt{d\tan\left(fx+e\right)}\,d}{f}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)*(a-I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] 2*a*(sqrt(2)*d^(3/2)*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(-4*I*sqrt(2)*
d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(f*(-I*d/sqrt(d^2) + 1)) - I*sqrt(d
*tan(f*x + e))*d/f)/d
```

Mupad [B]
time = 3.94, size = 126, normalized size = 2.07

$$\frac{(-1)^{1/4}\,a\,\sqrt{d}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\,\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)}{f}+\frac{(-1)^{1/4}\,a\,\sqrt{d}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\,\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)}{f}+\frac{(-1)^{1/4}\,a\,\sqrt{d}\,\left(\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\,\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)-\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\,\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)\right)}{f}-\frac{a\,\sqrt{d\,\tan\left(e+f\,x\right)}\,2\mathrm{i}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(1/2)*(a - a*tan(e + f*x)*1i),x)
```

```
[Out] ((-1)^(1/4)*a*d^(1/2)*atan(((-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2)))/f
- (a*(d*tan(e + f*x))^(1/2)*2i)/f + ((-1)^(1/4)*a*d^(1/2)*atanh(((-1)^(1/4)
*(d*tan(e + f*x))^(1/2))/d^(1/2)))/f + ((-1)^(1/4)*a*d^(1/2)*(atan(((-1)^(1
/4)*(d*tan(e + f*x))^(1/2))/d^(1/2)) - atanh(((-1)^(1/4)*(d*tan(e + f*x))^(
1/2))/d^(1/2))))/f
```

3.145 $\int \frac{a-ia\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$

Optimal. Leaf size=40

$$-\frac{2\sqrt[4]{-1}\,a\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{d}\,f}$$

```
[Out] -2*(-1)^(1/4)*a*arctanh((-1)^(3/4)*(d*tan(f*x+e))^(1/2)/d^(1/2))/f/d^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3614, 214}

$$-\frac{2\sqrt[4]{-1}\,a\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{d}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*Tan[e + f*x])/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (-2*(-1)^(1/4)*a*ArcTanh[((-1)^(3/4)*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(Sqrt[
d]*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a-ia\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx &= \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{ad+iax^2}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{f}\\
&= -\frac{2\sqrt[4]{-1}\,a\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{d}\,f}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 50, normalized size = 1.25

$$-\frac{2\sqrt[4]{-1}\,a\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(e+fx)}\right)\sqrt{\tan(e+fx)}}{f\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*Tan[e + f*x])/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (-2*(-1)^(1/4)*a*ArcTanh[(-1)^(3/4)*Sqrt[Tan[e + f*x]]]*Sqrt[Tan[e + f*x]])
/(f*Sqrt[d*Tan[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 274, normalized size = 6.85

method	result
derivativedivides	$-\frac{a\left(-\frac{\left(d^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+\left(d^{2}\right)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\,\sqrt{2}+\sqrt{d^{2}}}{d\tan(fx+e)-\left(d^{2}\right)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\,\sqrt{2}+\sqrt{d^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\tan(fx-}{\left(d^{2}\right)^{\frac{1}{4}}}\right.\right.}{4d}\right.}{}$
default	$-\frac{a\left(-\frac{\left(d^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+\left(d^{2}\right)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\,\sqrt{2}+\sqrt{d^{2}}}{d\tan(fx+e)-\left(d^{2}\right)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\,\sqrt{2}+\sqrt{d^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\tan(fx-}{\left(d^{2}\right)^{\frac{1}{4}}}\right.\right.}{4d}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*tan(f*x+e))/(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*a*(-1/4/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x
+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1
/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)
+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))+1/4*I/(d^2)^(1/4
)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^
(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))
+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^
2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.51, size = 160, normalized size = 4.00

$$\frac{a\left(-\frac{(2i-2)\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(2i-2)\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{(i+1)\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}-\frac{(i+1)\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*tan(f*x+e))/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*a*(-(2*I - 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*ta
n(f*x + e)))/sqrt(d))/sqrt(d) - (2*I - 2)*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt
(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) + (I + 1)*sqrt(2)*lo
g(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) - (I +
 1)*sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/
sqrt(d))/f
```

Fricas [C] Result contains complex when optimal does not.
time = 0.37, size = 175, normalized size = 4.38

$$\frac{1}{2}\sqrt{\frac{i\,a^2}{df^2}}\log\left(\frac{df\sqrt{\frac{i\,a^2}{df^2}}+\left(ae^{(2i\,fx+2i\,e)}+a\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}}{a}\right)-\frac{1}{2}\sqrt{\frac{i\,a^2}{df^2}}\log\left(-\frac{df\sqrt{\frac{i\,a^2}{df^2}}-\left(ae^{(2i\,fx+2i\,e)}+a\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*tan(f*x+e))/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(I*a^2/(d*f^2))*log((d*f*sqrt(I*a^2/(d*f^2)) + (a*e^(2*I*f*x + 2*I*
e) + a)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))/a
) - 1/2*sqrt(I*a^2/(d*f^2))*log(-(d*f*sqrt(I*a^2/(d*f^2)) - (a*e^(2*I*f*x +
 2*I*e) + a)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1
)))/a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia\left(\int\frac{i}{\sqrt{d\tan\left(e+fx\right)}}\,dx+\int\frac{\tan\left(e+fx\right)}{\sqrt{d\tan\left(e+fx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*tan(f*x+e))/(d*tan(f*x+e))**(1/2),x)
```

```
[Out] -I*a*(Integral(I/sqrt(d*tan(e + f*x)), x) + Integral(tan(e + f*x)/sqrt(d*ta
n(e + f*x)), x))
```

Giac [C] Result contains complex when optimal does not.
time = 0.54, size = 67, normalized size = 1.68

$$\frac{2\sqrt{2}\,a\arctan\left(\frac{8i\sqrt{d^2}\sqrt{d\tan\left(fx+e\right)}}{4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{\sqrt{d}\,f\left(\frac{i\,d}{\sqrt{d^2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-I*a*tan(f*x+e))/(d*tan(f*x+e))^(1/2),x, algorithm="giac")

[Out] 2*sqrt(2)*a*arctan(8*I*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*I*sqrt(2)*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(sqrt(d)*f*(I*d/sqrt(d^2) + 1))

Mupad [B]
time = 4.22, size = 30, normalized size = 0.75

$$-\frac{(-1)^{1/4}\,a\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)\,2\mathrm{i}}{\sqrt{d}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - a*tan(e + f*x)*1i)/(d*tan(e + f*x))^(1/2),x)

[Out] -((-1)^(1/4)*a*atan(((-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2))*2i)/(d^(1/2)*f)

3.146 $\int \frac{a-ia\tan(e+fx)}{(d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=62

$$\frac{2(-1)^{3/4}a\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{3/2}f}-\frac{2a}{df\sqrt{d\tan(e+fx)}}$$

```
[Out] 2*(-1)^(3/4)*a*arctanh((-1)^(3/4)*(d*tan(f*x+e))^(1/2)/d^(1/2))/d^(3/2)/f-2
*a/d/f/(d*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3610, 3614, 214}

$$\frac{2(-1)^{3/4}a\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{3/2}f}-\frac{2a}{df\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*Tan[e + f*x])/(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*(-1)^(3/4)*a*ArcTanh[((-1)^(3/4)*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(d^(3/2
)*f) - (2*a)/(d*f*Sqrt[d*Tan[e + f*x]])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{a-ia\tan(e+fx)}{(d\tan(e+fx))^{3/2}}\,dx = -\frac{2a}{df\sqrt{d\tan(e+fx)}}+\frac{\int \frac{-iad-ad\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx}{d^2}$$
$$= -\frac{2a}{df\sqrt{d\tan(e+fx)}}-\frac{(2a^2)\,\mathrm{Subst}\left(\int \frac{1}{-iad^2+adx^2}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{f}$$
$$= \frac{2(-1)^{3/4}a\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{3/2}f}-\frac{2a}{df\sqrt{d\tan(e+fx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.12, size = 39, normalized size = 0.63

$$-\frac{2a\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-i\tan(e+fx)\right)}{df\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*Tan[e + f*x])/(d*Tan[e + f*x])^(3/2),x]
[Out] (-2*a*Hypergeometric2F1[-1/2, 1, 1/2, (-I)*Tan[e + f*x]])/(d*f*Sqrt[d*Tan[e
 + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 295, normalized size = 4.76

method	result
derivativedivides	$-\frac{a\left(\frac{i(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+\ldots}}{(d^2)^{\frac{1}{4}}}\right.\right.}{4d}\right.}{}$

default	$-\frac{a\left(\frac{i(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\,\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\tan(fx+}}{(d^2)^{\frac{1}{4}}}\right.\right.}{4d}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*tan(f*x+e))/(d*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*a*(2/d*(1/8*I/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*t
an(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+
e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))
^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))+1/8/(d^2)^
(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d
^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/
2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)
/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)))+2/d/(d*tan(f*x+e))^(1/2))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.52, size = 179, normalized size = 2.89

$$\frac{a\left(-\frac{(2i+2)\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(2i+2)\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(i-1)\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{(i-1)\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)-\frac{8a}{\sqrt{d\tan(fx+e)}}}{4df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*tan(f*x+e))/(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(a*(-(2*I + 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*t
an(f*x + e)))/sqrt(d))/sqrt(d) - (2*I + 2)*sqrt(2)*arctan(-1/2*sqrt(2)*(sqr
t(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - (I - 1)*sqrt(2)*l
og(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) + (I
- 1)*sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)
/sqrt(d)) - 8*a/sqrt(d*tan(f*x + e)))/(d*f)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.38, size = 328, normalized size = 5.29

$$\frac{(d^2fe^{(2ifx+2ie)}-d^2f)\sqrt{-\frac{4ia^2}{d^3f^2}}\log\left(\frac{\left((dfe^{(2ifx+2ie)}+df)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\sqrt{-\frac{4ia^2}{d^3f^2}}+2a\right)e^{(-2ifx-2ie)}}{df}\right)-(d^2fe^{(2ifx+2ie)}-d^2f)\sqrt{-\frac{4ia^2}{d^3f^2}}\log\left(-\frac{\left((dfe^{(2ifx+2ie)}+df)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\sqrt{-\frac{4ia^2}{d^3f^2}}-2a\right)e^{(-2ifx-2ie)}}{df}\right)-8(iae^{(2ifx+2ie)}+ia)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}}{4(d^2fe^{(2ifx+2ie)}-d^2f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*tan(f*x+e))/(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*((d^2*f*e^(2*I*f*x + 2*I*e) - d^2*f)*sqrt(-4*I*a^2/(d^3*f^2))*log(((d*f
*e^(2*I*f*x + 2*I*e) + d*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f
*x + 2*I*e) + 1))*sqrt(-4*I*a^2/(d^3*f^2)) + 2*a)*e^(-2*I*f*x - 2*I*e)/(d*f
)) - (d^2*f*e^(2*I*f*x + 2*I*e) - d^2*f)*sqrt(-4*I*a^2/(d^3*f^2))*log(-((d*
f*e^(2*I*f*x + 2*I*e) + d*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*
f*x + 2*I*e) + 1))*sqrt(-4*I*a^2/(d^3*f^2)) - 2*a)*e^(-2*I*f*x - 2*I*e)/(d*
f)) - 8*(I*a*e^(2*I*f*x + 2*I*e) + I*a)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*
d)/(e^(2*I*f*x + 2*I*e) + 1)))/(d^2*f*e^(2*I*f*x + 2*I*e) - d^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia\left(\int \frac{i}{(d\tan(e+fx))^{\frac{3}{2}}}\,dx + \int \frac{\tan(e+fx)}{(d\tan(e+fx))^{\frac{3}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*tan(f*x+e))/(d*tan(f*x+e))**(3/2),x)
```

```
[Out] -I*a*(Integral(I/(d*tan(e + f*x))**(3/2), x) + Integral(tan(e + f*x)/(d*tan
(e + f*x))**(3/2), x))
```

Giac [C] Result contains complex when optimal does not.
time = 0.63, size = 89, normalized size = 1.44

$$\frac{2\,a\left(-\frac{i\sqrt{2}\,\arctan\left(\frac{8i\sqrt{d^2}\,\sqrt{d\tan(fx+e)}}{4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\,\sqrt{d^2}\,\sqrt{d}}\right)}{\sqrt{d}\,f\left(\frac{i\,d}{\sqrt{d^2}}+1\right)} - \frac{1}{\sqrt{d\tan(fx+e)}\,f}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*tan(f*x+e))/(d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] 2*a*(-I*sqrt(2)*arctan(8*I*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*I*sqrt(2)*d^(3
/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(sqrt(d)*f*(I*d/sqrt(d^2) + 1)) - 1/(sq
rt(d*tan(f*x + e))*f))/d
```

Mupad [B]
time = 4.30, size = 50, normalized size = 0.81

$$-\frac{2\,a}{d\,f\,\sqrt{d\tan(e+f\,x)}} - \frac{2\,(-1)^{1/4}\,a\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan(e+f\,x)}}{\sqrt{d}}\right)}{d^{3/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*tan(e + f*x)*1i)/(d*tan(e + f*x))^(3/2),x)
```

```
[Out] - (2*a)/(d*f*(d*tan(e + f*x))^(1/2)) - (2*(-1)^(1/4)*a*atan(((-1)^(1/4)*(d*
tan(e + f*x))^(1/2))/d^(1/2)))/(d^(3/2)*f)
```

3.147 $\int \frac{a-ia\tan(e+fx)}{(d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=87

$$\frac{2\sqrt[4]{-1}\,a\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{5/2}f}-\frac{2a}{3df(d\tan(e+fx))^{3/2}}+\frac{2ia}{d^2f\sqrt{d\tan(e+fx)}}$$

```
[Out] 2*(-1)^(1/4)*a*arctanh((-1)^(3/4)*(d*tan(f*x+e))^(1/2)/d^(1/2))/d^(5/2)/f+2
*I*a/d^2/f/(d*tan(f*x+e))^(1/2)-2/3*a/d/f/(d*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3610, 3614, 214}

$$\frac{2\sqrt[4]{-1}\,a\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{5/2}f}+\frac{2ia}{d^2f\sqrt{d\tan(e+fx)}}-\frac{2a}{3df(d\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*Tan[e + f*x])/(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (2*(-1)^(1/4)*a*ArcTanh[((-1)^(3/4)*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(d^(5/2
)*f) - (2*a)/(3*d*f*(d*Tan[e + f*x])^(3/2)) + ((2*I)*a)/(d^2*f*Sqrt[d*Tan[e
 + f*x]])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{a-ia\tan(e+fx)}{(d\tan(e+fx))^{5/2}}\,dx = -\frac{2a}{3df(d\tan(e+fx))^{3/2}} + \frac{\int \frac{-iad-ad\tan(e+fx)}{(d\tan(e+fx))^{3/2}}\,dx}{d^2}$$

$$= -\frac{2a}{3df(d\tan(e+fx))^{3/2}} + \frac{2ia}{d^2f\sqrt{d\tan(e+fx)}} + \frac{\int \frac{-ad^2+iad^2\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx}{d^4}$$

$$= -\frac{2a}{3df(d\tan(e+fx))^{3/2}} + \frac{2ia}{d^2f\sqrt{d\tan(e+fx)}} + \frac{(2a^2)\,\mathrm{Subst}\left(\int \frac{1}{-ad^3-iad^2x^2}\,dx,\right.}{f}$$

$$= \frac{2\sqrt[4]{-1}\,a\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{5/2}f} - \frac{2a}{3df(d\tan(e+fx))^{3/2}} + \frac{2}{d^2f\sqrt{d\,\mathrm{ta}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.12, size = 41, normalized size = 0.47

$$-\frac{2a\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-i\tan(e+fx)\right)}{3df(d\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*Tan[e + f*x])/(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (-2*a*Hypergeometric2F1[-3/2, 1, -1/2, (-I)*Tan[e + f*x]])/(3*d*f*(d*Tan[e
+ f*x])^(3/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(70) = 140.
time = 0.10, size = 311, normalized size = 3.57

method	result
derivativedivides	$-\frac{a\left(\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+e}}{(d^2)^{\frac{1}{4}}}\right.\right.}{4d}\right.}{}$

default	$-\frac{a\left(\frac{\left(d^{2}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{d\tan(fx+e)+\left(d^{2}\right)^{\frac{1}{4}}\sqrt{d\tan\left(fx+e\right)}\,\sqrt{2}+\sqrt{d^{2}}}{d\tan(fx+e)-\left(d^{2}\right)^{\frac{1}{4}}\sqrt{d\tan\left(fx+e\right)}\,\sqrt{2}+\sqrt{d^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\tan\left(fx+\right.}}{\left(d^{2}\right)^{\frac{1}{4}}}\right.\right.}{4d}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a-I*a*tan(f*x+e))/(d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/f*a*(2/d^2*(1/8/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))-1/8*I/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)))-2*I/d^2/(d*tan(f*x+e))^(1/2)+2/3/d/(d*tan(f*x+e))^(3/2))

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(72) = 144$.
time = 0.49, size = 201, normalized size = 2.31

$$-\frac{\frac{3\,a\left(-\frac{\left(2i-2\right)\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\left(2i-2\right)\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{\left(i+1\right)\sqrt{2}\log\left(d\tan\left(fx+e\right)+\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{d}+d\right)}{\sqrt{d}}-\frac{\left(i+1\right)\sqrt{2}\log\left(d\tan\left(fx+e\right)-\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{d}+d\right)}{\sqrt{d}}\right)}{d}-\frac{8\left(3i\,ad\tan\left(fx+e\right)-ad\right)}{\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}d}}{12\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-I*a*tan(f*x+e))/(d*tan(f*x+e))^(5/2),x, algorithm="maxima")

[Out] -1/12*(3*a*(-(2*I - 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - (2*I - 2)*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) + (I + 1)*sqrt(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) - (I + 1)*sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d))/d - 8*(3*I*a*d*tan(f*x + e) - a*d)/((d*tan(f*x + e))^(3/2)*d))/(d*f)

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(72) = 144$.
time = 0.37, size = 394, normalized size = 4.53

$$-\frac{3\left(d^{3}fe^{(4i\,fx+4i\,e)}-2\,d^{3}fe^{(2i\,fx+2i\,e)}+d^{3}f\right)\sqrt{\frac{4i\,a^{2}}{d^{5}f^{2}}}\log\left(-\frac{\left(\left(d^{2}fe^{(2i\,fx+2i\,e)}+d^{2}f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{4i\,a^{2}}{d^{5}f^{2}}}+2i\,a\right)e^{(-2i\,fx-2i\,e)}}{d^{2}f}\right)-3\left(d^{3}fe^{(4i\,fx+4i\,e)}-2\,d^{3}fe^{(2i\,fx+2i\,e)}+d^{3}f\right)\sqrt{\frac{4i\,a^{2}}{d^{5}f^{2}}}\log\left(\frac{\left(\left(d^{2}fe^{(2i\,fx+2i\,e)}+d^{2}f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{4i\,a^{2}}{d^{5}f^{2}}}-2i\,a\right)e^{(-2i\,fx-2i\,e)}}{d^{2}f}\right)+16\left(ae^{(4i\,fx+4i\,e)}-ae^{(2i\,fx+2i\,e)}-2\,a\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}}{12\left(d^{3}fe^{(4i\,fx+4i\,e)}-2\,d^{3}fe^{(2i\,fx+2i\,e)}+d^{3}f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*tan(f*x+e))/(d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(3*(d^3*f*e^(4*I*f*x + 4*I*e) - 2*d^3*f*e^(2*I*f*x + 2*I*e) + d^3*f)*
sqrt(4*I*a^2/(d^5*f^2))*log(-((d^2*f*e^(2*I*f*x + 2*I*e) + d^2*f)*sqrt((-I*
d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(4*I*a^2/(d^5*f
^2)) + 2*I*a)*e^(-2*I*f*x - 2*I*e)/(d^2*f)) - 3*(d^3*f*e^(4*I*f*x + 4*I*e)
- 2*d^3*f*e^(2*I*f*x + 2*I*e) + d^3*f)*sqrt(4*I*a^2/(d^5*f^2))*log(((d^2*f*
e^(2*I*f*x + 2*I*e) + d^2*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*
f*x + 2*I*e) + 1))*sqrt(4*I*a^2/(d^5*f^2)) - 2*I*a)*e^(-2*I*f*x - 2*I*e)/(d
^2*f)) + 16*(a*e^(4*I*f*x + 4*I*e) - a*e^(2*I*f*x + 2*I*e) - 2*a)*sqrt((-I*
d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))/(d^3*f*e^(4*I*f*x
+ 4*I*e) - 2*d^3*f*e^(2*I*f*x + 2*I*e) + d^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia\left(\int \frac{i}{(d\tan(e+fx))^{\frac{5}{2}}}\,dx + \int \frac{\tan(e+fx)}{(d\tan(e+fx))^{\frac{5}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*tan(f*x+e))/(d*tan(f*x+e))**(5/2),x)
```

```
[Out] -I*a*(Integral(I/(d*tan(e + f*x))**(5/2), x) + Integral(tan(e + f*x)/(d*tan
(e + f*x))**(5/2), x))
```

Giac [A]
time = 0.62, size = 109, normalized size = 1.25

$$-\frac{2}{3}a\left(-\frac{3i\sqrt{2}\arctan\left(\frac{8\sqrt{d^2}\sqrt{d\tan(fx+e)}}{-4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{d^{\frac{5}{2}}f\left(-\frac{id}{\sqrt{d^2}}+1\right)} + \frac{-3i\,d\tan(fx+e)+d}{\sqrt{d\tan(fx+e)}\,d^3f\tan(fx+e)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*tan(f*x+e))/(d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*a*(-3*I*sqrt(2)*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(-4*I*sqrt(2)*
d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(d^(5/2)*f*(-I*d/sqrt(d^2) + 1)) +
(-3*I*d*tan(f*x + e) + d)/(sqrt(d*tan(f*x + e))*d^3*f*tan(f*x + e)))
```

Mupad [B]
time = 4.62, size = 70, normalized size = 0.80

$$\frac{a\,2\mathrm{i}}{d^2\,f\,\sqrt{d\tan(e+f\,x)}} - \frac{2\,a}{3\,d\,f\,(d\tan(e+f\,x))^{3/2}} + \frac{(-1)^{1/4}\,a\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan(e+f\,x)}}{\sqrt{d}}\right)\,2\mathrm{i}}{d^{5/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*tan(e + f*x)*1i)/(d*tan(e + f*x))^(5/2),x)
```

```
[Out] (a*2i)/(d^2*f*(d*tan(e + f*x))^(1/2)) - (2*a)/(3*d*f*(d*tan(e + f*x))^(3/2)
) + ((-1)^(1/4)*a*atan(((-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2))*2i)/(d^
(5/2)*f)
```

3.148 $\int \frac{a-ia\tan(e+fx)}{(d\tan(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=110

$$-\frac{2(-1)^{3/4}a\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{7/2}f}-\frac{2a}{5df(d\tan(e+fx))^{5/2}}+\frac{2ia}{3d^2f(d\tan(e+fx))^{3/2}}+\frac{2a}{d^3f\sqrt{d\tan(e+fx)}}$$

```
[Out] -2*(-1)^(3/4)*a*arctanh((-1)^(3/4)*(d*tan(f*x+e))^(1/2)/d^(1/2))/d^(7/2)/f+
2*a/d^3/f/(d*tan(f*x+e))^(1/2)-2/5*a/d/f/(d*tan(f*x+e))^(5/2)+2/3*I*a/d^2/f
/(d*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.11, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3610, 3614, 214}

$$-\frac{2(-1)^{3/4}a\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{7/2}f}+\frac{2a}{d^3f\sqrt{d\tan(e+fx)}}+\frac{2ia}{3d^2f(d\tan(e+fx))^{3/2}}-\frac{2a}{5df(d\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*Tan[e + f*x])/(d*Tan[e + f*x])^(7/2),x]
```

```
[Out] (-2*(-1)^(3/4)*a*ArcTanh[((-1)^(3/4)*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(d^(7/
2)*f) - (2*a)/(5*d*f*(d*Tan[e + f*x])^(5/2)) + (((2*I)/3)*a)/(d^2*f*(d*Tan[
e + f*x])^(3/2)) + (2*a)/(d^3*f*Sqrt[d*Tan[e + f*x]])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{a-ia\tan(e+fx)}{(d\tan(e+fx))^{7/2}}\,dx = -\frac{2a}{5df(d\tan(e+fx))^{5/2}} + \frac{\int \frac{-iad-ad\tan(e+fx)}{(d\tan(e+fx))^{5/2}}\,dx}{d^2}$$

$$= -\frac{2a}{5df(d\tan(e+fx))^{5/2}} + \frac{2ia}{3d^2f(d\tan(e+fx))^{3/2}} + \frac{\int \frac{-ad^2+iad^2\tan(e+fx)}{(d\tan(e+fx))^{3/2}}\,dx}{d^4}$$

$$= -\frac{2a}{5df(d\tan(e+fx))^{5/2}} + \frac{2ia}{3d^2f(d\tan(e+fx))^{3/2}} + \frac{2a}{d^3f\sqrt{d\tan(e+fx)}} + \frac{\int}{}$$

$$= -\frac{2a}{5df(d\tan(e+fx))^{5/2}} + \frac{2ia}{3d^2f(d\tan(e+fx))^{3/2}} + \frac{2a}{d^3f\sqrt{d\tan(e+fx)}} - \frac{(2}{}$$

$$= -\frac{2(-1)^{3/4}a\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{7/2}f} - \frac{2a}{5df(d\tan(e+fx))^{5/2}} + \frac{}{3d^2f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.17, size = 41, normalized size = 0.37

$$-\frac{2a\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-i\tan(e+fx)\right)}{5df(d\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*Tan[e + f*x])/(d*Tan[e + f*x])^(7/2),x]
```

```
[Out] (-2*a*Hypergeometric2F1[-5/2, 1, -3/2, (-I)*Tan[e + f*x]])/(5*d*f*(d*Tan[e
+ f*x])^(5/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 325 vs. 2(89) = 178.
time = 0.10, size = 326, normalized size = 2.96

method	result

derivativedivides	$-\frac{a\left(-\frac{i\left(d^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+\left(d^{2}\right)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^{2}}}{d\tan(fx+e)-\left(d^{2}\right)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+}{\left(d^{2}\right)^{\frac{1}{4}}}\right.\right.}{4d}\right.}{}$
default	$-\frac{a\left(-\frac{i\left(d^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+\left(d^{2}\right)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^{2}}}{d\tan(fx+e)-\left(d^{2}\right)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+}{\left(d^{2}\right)^{\frac{1}{4}}}\right.\right.}{4d}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*tan(f*x+e))/(d*tan(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*a*(2/d^3*(-1/8*I/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(
d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f
*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+
e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))-1/8/(d^
2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)
+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^
(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1
/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)))-2/d^3/(d*tan(f*x+e))^(1/2)-2/3*I/
d^2/(d*tan(f*x+e))^(3/2)+2/5/d/(d*tan(f*x+e))^(5/2))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(92) = 184$.
time = 0.51, size = 220, normalized size = 2.00

$$-\frac{\frac{15\,a\left(-\frac{(2i+2)\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(2i+2)\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(i-1)\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{(i-1)\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)}{d^{2}}-\frac{8\left(15\,ad^{2}\tan(fx+e)^{2}+5i\,ad^{2}\tan(fx+e)-3\,ad^{2}\right)}{(d\tan(fx+e))^{\frac{5}{2}}d^{2}}}{60\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*tan(f*x+e))/(d*tan(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] -1/60*(15*a*(-(2*I + 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqr
t(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - (2*I + 2)*sqrt(2)*arctan(-1/2*sqrt(2)
*(sqrt(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - (I - 1)*sqrt
(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d)
```

```
+ (I - 1)*sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d)
 + d)/sqrt(d))/d^2 - 8*(15*a*d^2*tan(f*x + e)^2 + 5*I*a*d^2*tan(f*x + e) -
3*a*d^2)/((d*tan(f*x + e))^(5/2)*d^2))/(d*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(92) = 184$.
time = 0.40, size = 459, normalized size = 4.17

$$-\frac{15\left(d^{4} f e^{(6 i f x+6 i e)}-3 d^{4} f e^{(4 i f x+4 i e)}+3 d^{4} f e^{(2 i f x+2 i e)}-d^{4} f\right) \sqrt{-\frac{4 i a^{2}}{d^{7} f^{2}}} \log\left(-\frac{\left(\left(d^{3} f e^{(2 i f x+2 i e)}+d^{3} f\right) \sqrt{\frac{-i d e^{(2 i f x+2 i e)}+i d}{e^{(2 i f x+2 i e)}+1}} \sqrt{-\frac{4 i a^{2}}{d^{7} f^{2}}}+2 a\right) e^{(-2 i f x-2 i e)}}{d^{3} f}\right)-15\left(d^{4} f e^{(6 i f x+6 i e)}-3 d^{4} f e^{(4 i f x+4 i e)}+3 d^{4} f e^{(2 i f x+2 i e)}-d^{4} f\right) \sqrt{-\frac{4 i a^{2}}{d^{7} f^{2}}} \log\left(\frac{\left(\left(d^{3} f e^{(2 i f x+2 i e)}+d^{3} f\right) \sqrt{\frac{-i d e^{(2 i f x+2 i e)}+i d}{e^{(2 i f x+2 i e)}+1}} \sqrt{-\frac{4 i a^{2}}{d^{7} f^{2}}}-2 a\right) e^{(-2 i f x-2 i e)}}{d^{3} f}\right)+8\left(-13 i a e^{(6 i f x+6 i e)}+11 i a e^{(4 i f x+4 i e)}+i a e^{(2 i f x+2 i e)}-23 i a\right) \sqrt{\frac{-i d e^{(2 i f x+2 i e)}+i d}{e^{(2 i f x+2 i e)}+1}}}{60\left(d^{4} f e^{(6 i f x+6 i e)}-3 d^{4} f e^{(4 i f x+4 i e)}+3 d^{4} f e^{(2 i f x+2 i e)}-d^{4} f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*tan(f*x+e))/(d*tan(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/60*(15*(d^4*f*e^(6*I*f*x + 6*I*e) - 3*d^4*f*e^(4*I*f*x + 4*I*e) + 3*d^4*
f*e^(2*I*f*x + 2*I*e) - d^4*f)*sqrt(-4*I*a^2/(d^7*f^2))*log(-((d^3*f*e^(2*I
*f*x + 2*I*e) + d^3*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x +
2*I*e) + 1))*sqrt(-4*I*a^2/(d^7*f^2)) + 2*a)*e^(-2*I*f*x - 2*I*e)/(d^3*f))
- 15*(d^4*f*e^(6*I*f*x + 6*I*e) - 3*d^4*f*e^(4*I*f*x + 4*I*e) + 3*d^4*f*e^(
2*I*f*x + 2*I*e) - d^4*f)*sqrt(-4*I*a^2/(d^7*f^2))*log(((d^3*f*e^(2*I*f*x +
 2*I*e) + d^3*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e)
 + 1))*sqrt(-4*I*a^2/(d^7*f^2)) - 2*a)*e^(-2*I*f*x - 2*I*e)/(d^3*f)) + 8*(-
13*I*a*e^(6*I*f*x + 6*I*e) + 11*I*a*e^(4*I*f*x + 4*I*e) + I*a*e^(2*I*f*x +
2*I*e) - 23*I*a)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e)
 + 1)))/(d^4*f*e^(6*I*f*x + 6*I*e) - 3*d^4*f*e^(4*I*f*x + 4*I*e) + 3*d^4*f*
e^(2*I*f*x + 2*I*e) - d^4*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia\left(\int\frac{i}{(d\tan(e+fx))^{\frac{7}{2}}}\,dx+\int\frac{\tan(e+fx)}{(d\tan(e+fx))^{\frac{7}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*tan(f*x+e))/(d*tan(f*x+e))**(7/2),x)
```

```
[Out] -I*a*(Integral(I/(d*tan(e + f*x))**(7/2), x) + Integral(tan(e + f*x)/(d*tan
(e + f*x))**(7/2), x))
```

Giac [A]
time = 0.66, size = 130, normalized size = 1.18

$$-\frac{2}{15}\,a\left(-\frac{15 i\sqrt{2}\,\arctan\left(\frac{8 i\sqrt{d^{2}}\,\sqrt{d\tan(fx+e)}}{4 i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\,\sqrt{d^{2}}\,\sqrt{d}}\right)}{d^{\frac{7}{2}}f\left(\frac{i d}{\sqrt{d^{2}}}+1\right)}-\frac{15\,d^{2}\tan(fx+e)^{2}+5 i\,d^{2}\tan(fx+e)-3\,d^{2}}{\sqrt{d\tan(fx+e)}\,d^{5}f\tan(fx+e)^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*tan(f*x+e))/(d*tan(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] -2/15*a*(-15*I*sqrt(2)*arctan(8*I*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*I*sqrt(
2)*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(d^(7/2)*f*(I*d/sqrt(d^2) + 1))
- (15*d^2*tan(f*x + e)^2 + 5*I*d^2*tan(f*x + e) - 3*d^2)/(sqrt(d*tan(f*x +
e))*d^5*f*tan(f*x + e)^2))
```

Mupad [B]
time = 4.91, size = 87, normalized size = 0.79

$$\frac{2\,{\left(-1\right)}^{1/4}\,a\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{d\,\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)}{d^{7/2}\,f}-\frac{\frac{2\,a}{5\,d}-\frac{2\,a\,{\tan\left(e+f\,x\right)}^2}{d}}{f\,{\left(d\,\tan\left(e+f\,x\right)\right)}^{5/2}}+\frac{a\,2{}\mathrm{i}}{3\,d^2\,f\,{\left(d\,\tan\left(e+f\,x\right)\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*tan(e + f*x)*1i)/(d*tan(e + f*x))^(7/2),x)
```

```
[Out] (a*2i)/(3*d^2*f*(d*tan(e + f*x))^(3/2)) - ((2*a)/(5*d) - (2*a*tan(e + f*x)^
2)/d)/(f*(d*tan(e + f*x))^(5/2)) + (2*(-1)^(1/4)*a*atan(((-1)^(1/4)*(d*tan(
e + f*x))^(1/2))/d^(1/2)))/(d^(7/2)*f)
```

3.149 $\int (d\tan(e+fx))^{5/2}(a+ia\tan(e+fx))^2\,dx$

Optimal. Leaf size=140

$$-\frac{4(-1)^{3/4}a^2d^{5/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}-\frac{4ia^2d^2\sqrt{d\tan(e+fx)}}{f}+\frac{4a^2d(d\tan(e+fx))^{3/2}}{3f}+\frac{4ia}{}$$

```
[Out] -4*(-1)^(3/4)*a^2*d^(5/2)*arctan((-1)^(3/4)*(d*tan(f*x+e))^(1/2)/d^(1/2))/f
-4*I*a^2*d^2*(d*tan(f*x+e))^(1/2)/f+4/3*a^2*d*(d*tan(f*x+e))^(3/2)/f+4/5*I*
a^2*(d*tan(f*x+e))^(5/2)/f-2/7*a^2*(d*tan(f*x+e))^(7/2)/d/f
```

Rubi [A]

time = 0.15, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3624, 3609, 3614, 211}

$$-\frac{4(-1)^{3/4}a^2d^{5/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}-\frac{4ia^2d^2\sqrt{d\tan(e+fx)}}{f}-\frac{2a^2(d\tan(e+fx))^{7/2}}{7df}+\frac{4ia^2(d\tan(e+fx))^{5/2}}{5f}+\frac{4a^2d(d\tan(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(5/2)*(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (-4*(-1)^(3/4)*a^2*d^(5/2)*ArcTan[((-1)^(3/4)*Sqrt[d*Tan[e + f*x]])/Sqrt[d]
])/f - ((4*I)*a^2*d^2*Sqrt[d*Tan[e + f*x]])/f + (4*a^2*d*(d*Tan[e + f*x])^(
3/2))/(3*f) + (((4*I)/5)*a^2*(d*Tan[e + f*x])^(5/2))/f - (2*a^2*(d*Tan[e +
f*x])^(7/2))/(7*d*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && !LeQ
[m, -1] && !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int (d\tan(e+fx))^{5/2}(a+ia\tan(e+fx))^2\,dx = -\frac{2a^2(d\tan(e+fx))^{7/2}}{7df} + \int (d\tan(e+fx))^{5/2}\left(2a^2+2ia^2\,\mathrm{t}\right.$$
$$= \frac{4ia^2(d\tan(e+fx))^{5/2}}{5f} - \frac{2a^2(d\tan(e+fx))^{7/2}}{7df} + \int (d\tan(e$$
$$= \frac{4a^2d(d\tan(e+fx))^{3/2}}{3f} + \frac{4ia^2(d\tan(e+fx))^{5/2}}{5f} - \frac{2a^2(d\tan}{}$$
$$= -\frac{4ia^2d^2\sqrt{d\tan(e+fx)}}{f} + \frac{4a^2d(d\tan(e+fx))^{3/2}}{3f} + \frac{4ia^2(}{}$$
$$= -\frac{4ia^2d^2\sqrt{d\tan(e+fx)}}{f} + \frac{4a^2d(d\tan(e+fx))^{3/2}}{3f} + \frac{4ia^2(}{}$$
$$= -\frac{4(-1)^{3/4}a^2d^{5/2}\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f} - \frac{4ia^2d^2}{}$$

Mathematica [A]
time = 2.58, size = 145, normalized size = 1.04

$$\frac{a^2d^2\left(840i\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\right)-i\sec^3(e+fx)(588\cos(e+fx)+252\cos(3(e+fx))+25i\sin(e+fx)+85i\sin(3(e+fx)))\sqrt{i\tan(e+fx)}\right)\sqrt{d\tan(e+fx)}}{210f\sqrt{i\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(5/2)*(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (a^2*d^2*((840*I)*ArcTanh[Sqrt[(-1 + E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e
 + f*x)))]] - I*Sec[e + f*x]^3*(588*Cos[e + f*x] + 252*Cos[3*(e + f*x)] + (2
5*I)*Sin[e + f*x] + (85*I)*Sin[3*(e + f*x)])*Sqrt[I*Tan[e + f*x]])*Sqrt[d*T
an[e + f*x]])/(210*f*Sqrt[I*Tan[e + f*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(114) = 228.
time = 0.12, size = 342, normalized size = 2.44

method	result
derivativedivides	$\frac{2a^2\left(-\frac{(d\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{2id(d\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{2d^2(d\tan(fx+e))^{\frac{3}{2}}}{3}-2id^3\sqrt{d\tan(fx+e)}+2d^4\left(\frac{i(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\right.\right.}{}\right.\right.}{}$
default	$\frac{2a^2\left(-\frac{(d\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{2id(d\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{2d^2(d\tan(fx+e))^{\frac{3}{2}}}{3}-2id^3\sqrt{d\tan(fx+e)}+2d^4\left(\frac{i(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\right.\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(5/2)*(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^2/d*(-1/7*(d*tan(f*x+e))^(7/2)+2/5*I*d*(d*tan(f*x+e))^(5/2)+2/3*d^2*(
d*tan(f*x+e))^(3/2)-2*I*d^3*(d*tan(f*x+e))^(1/2)+2*d^4*(1/8*I/d*(d^2)^(1/4)
*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(
1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+
2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2
)^(1/4)*(d*tan(f*x+e))^(1/2)+1))-1/8/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-
(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(
1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4
)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2
)+1))))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(117) = 234$.
time = 0.51, size = 242, normalized size = 1.73

$$-\frac{105\,a^2d^4\left(-\frac{(2i-2)\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(2i-2)\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(i+1)\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{(i+1)\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)+60\,(d\tan(fx+e))^{\frac{7}{2}}a^2-168i\,(d\tan(fx+e))^{\frac{5}{2}}a^2d-280\,(d\tan(fx+e))^{\frac{3}{2}}a^2d^2+840i\sqrt{d\tan(fx+e)}\,a^2d^3}{210\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)*(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/210*(105*a^2*d^4*(-(2*I - 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d)
 + 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - (2*I - 2)*sqrt(2)*arctan(-1/2
*sqrt(2)*(sqrt(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - (I +
 1)*sqrt(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/
sqrt(d) + (I + 1)*sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))
*sqrt(d) + d)/sqrt(d)) + 60*(d*tan(f*x + e))^(7/2)*a^2 - 168*I*(d*tan(f*x +
 e))^(5/2)*a^2*d - 280*(d*tan(f*x + e))^(3/2)*a^2*d^2 + 840*I*sqrt(d*tan(f*
x + e))*a^2*d^3)/(d*f)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 460 vs. $2(117) = 234$.
time = 0.40, size = 460, normalized size = 3.29

$$-\frac{105 \sqrt{\frac{16 i a^4 d^5}{f^2}} \left(f e^{(6 i f x+6 i e)}+3 f e^{(4 i f x+4 i e)}+3 f e^{(2 i f x+2 i e)}+f\right) \log\left(\frac{\left(-4 i a^2 d^3 e^{(2 i f x+2 i e)}+\sqrt{\frac{16 i a^4 d^5}{f^2}} \left(i f e^{(2 i f x+2 i e)}+i f\right) \sqrt{\frac{-i d e^{(2 i f x+2 i e)}+i d}{e^{(2 i f x+2 i e)}+1}}\right) e^{(-2 i f x-2 i e)}}{2 a^2 d^2}\right)-105 \sqrt{\frac{16 i a^4 d^5}{f^2}} \left(f e^{(6 i f x+6 i e)}+3 f e^{(4 i f x+4 i e)}+3 f e^{(2 i f x+2 i e)}+f\right) \log\left(\frac{\left(-4 i a^2 d^3 e^{(2 i f x+2 i e)}+\sqrt{\frac{16 i a^4 d^5}{f^2}} \left(-i f e^{(2 i f x+2 i e)}-i f\right) \sqrt{\frac{-i d e^{(2 i f x+2 i e)}+i d}{e^{(2 i f x+2 i e)}+1}}\right) e^{(-2 i f x-2 i e)}}{2 a^2 d^2}\right)+8 \left(337 i a^2 d^2 e^{(6 i f x+6 i e)}+613 i a^2 d^2 e^{(4 i f x+4 i e)}+563 i a^2 d^2 e^{(2 i f x+2 i e)}+167 i a^2 d^2\right) \sqrt{\frac{-i d e^{(2 i f x+2 i e)}+i d}{e^{(2 i f x+2 i e)}+1}}}{420 \left(f e^{(6 i f x+6 i e)}+3 f e^{(4 i f x+4 i e)}+3 f e^{(2 i f x+2 i e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)*(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/420*(105*sqrt(16*I*a^4*d^5/f^2)*(f*e^(6*I*f*x + 6*I*e) + 3*f*e^(4*I*f*x
+ 4*I*e) + 3*f*e^(2*I*f*x + 2*I*e) + f)*log(1/2*(-4*I*a^2*d^3*e^(2*I*f*x +
2*I*e) + sqrt(16*I*a^4*d^5/f^2)*(I*f*e^(2*I*f*x + 2*I*e) + I*f)*sqrt((-I*d*
e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-2*I*f*x - 2*I*e)
/(a^2*d^2)) - 105*sqrt(16*I*a^4*d^5/f^2)*(f*e^(6*I*f*x + 6*I*e) + 3*f*e^(4*
I*f*x + 4*I*e) + 3*f*e^(2*I*f*x + 2*I*e) + f)*log(1/2*(-4*I*a^2*d^3*e^(2*I*
f*x + 2*I*e) + sqrt(16*I*a^4*d^5/f^2)*(-I*f*e^(2*I*f*x + 2*I*e) - I*f)*sqrt
((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-2*I*f*x -
 2*I*e)/(a^2*d^2)) + 8*(337*I*a^2*d^2*e^(6*I*f*x + 6*I*e) + 613*I*a^2*d^2*e
^(4*I*f*x + 4*I*e) + 563*I*a^2*d^2*e^(2*I*f*x + 2*I*e) + 167*I*a^2*d^2)*sqr
t((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))/(f*e^(6*I*f*
x + 6*I*e) + 3*f*e^(4*I*f*x + 4*I*e) + 3*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2 \left(\int \left(-(d \tan(e+fx))^{\frac{5}{2}}\right) dx + \int (d \tan(e+fx))^{\frac{5}{2}} \tan^2(e+fx)\, dx + \int \left(-2i(d \tan(e+fx))^{\frac{5}{2}} \tan(e+fx)\right) dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(5/2)*(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -a**2*(Integral(-(d*tan(e + f*x))**(5/2), x) + Integral((d*tan(e + f*x))**(
5/2)*tan(e + f*x)**2, x) + Integral(-2*I*(d*tan(e + f*x))**(5/2)*tan(e + f*
x), x))
```

Giac [**A**]
time = 0.91, size = 192, normalized size = 1.37

$$-\frac{4 \sqrt{2} a^2 d^{\frac{5}{2}} \arctan\left(\frac{8 \sqrt{d^2} \sqrt{d \tan(fx+e)}}{4i \sqrt{2} d^{\frac{3}{2}}+4 \sqrt{2} \sqrt{d^2} \sqrt{d}}\right)}{f\left(\frac{i d}{\sqrt{d^2}}+1\right)}-\frac{2\left(15 \sqrt{d \tan(fx+e)} a^2 d^9 f^6 \tan(fx+e)^3-42 i \sqrt{d \tan(fx+e)} a^2 d^9 f^6 \tan(fx+e)^2-70 \sqrt{d \tan(fx+e)} a^2 d^9 f^6 \tan(fx+e)+210 i \sqrt{d \tan(fx+e)} a^2 d^9 f^6\right)}{105 d^7 f^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)*(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*a^2*d^(5/2)*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*I*sqrt(2)
*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(f*(I*d/sqrt(d^2) + 1)) - 2/105*(1
```

```
5*sqrt(d*tan(f*x + e))*a^2*d^9*f^6*tan(f*x + e)^3 - 42*I*sqrt(d*tan(f*x + e
))*a^2*d^9*f^6*tan(f*x + e)^2 - 70*sqrt(d*tan(f*x + e))*a^2*d^9*f^6*tan(f*x
 + e) + 210*I*sqrt(d*tan(f*x + e))*a^2*d^9*f^6)/(d^7*f^7)
```

Mupad [B]
time = 4.64, size = 115, normalized size = 0.82

$$\frac{a^2\,{\left(d\,\tan\left(e+f\,x\right)\right)}^{5/2}\,4{}\mathrm{i}}{5\,f}-\frac{a^2\,d^2\,\sqrt{d\,\tan\left(e+f\,x\right)}\,4{}\mathrm{i}}{f}-\frac{2\,a^2\,{\left(d\,\tan\left(e+f\,x\right)\right)}^{7/2}}{7\,d\,f}+\frac{4\,a^2\,d\,{\left(d\,\tan\left(e+f\,x\right)\right)}^{3/2}}{3\,f}-\frac{\sqrt{4{}\mathrm{i}}\,a^2\,d^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{4{}\mathrm{i}}\,\sqrt{d\,\tan\left(e+f\,x\right)}\,1{}\mathrm{i}}{2\,\sqrt{d}}\right)\,2{}\mathrm{i}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(5/2)*(a + a*tan(e + f*x)*1i)^2,x)
```

```
[Out] (a^2*(d*tan(e + f*x))^(5/2)*4i)/(5*f) - (a^2*d^2*(d*tan(e + f*x))^(1/2)*4i)
/f - (2*a^2*(d*tan(e + f*x))^(7/2))/(7*d*f) + (4*a^2*d*(d*tan(e + f*x))^(3/
2))/(3*f) - (4i^(1/2)*a^2*d^(5/2)*atan((4i^(1/2)*(d*tan(e + f*x))^(1/2)*1i)
/(2*d^(1/2)))*2i)/f
```

3.150 $\int (d\tan(e+fx))^{3/2}(a+ia\tan(e+fx))^2\,dx$

Optimal. Leaf size=113

$$\frac{4\sqrt[4]{-1}\,a^2d^{3/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}+\frac{4a^2d\sqrt{d\tan(e+fx)}}{f}+\frac{4ia^2(d\tan(e+fx))^{3/2}}{3f}-\frac{2a^2(d\tan(\cdot}{5\cdot}$$

```
[Out] 4*(-1)^(1/4)*a^2*d^(3/2)*arctan((-1)^(3/4)*(d*tan(f*x+e))^(1/2)/d^(1/2))/f+
4*a^2*d*(d*tan(f*x+e))^(1/2)/f+4/3*I*a^2*(d*tan(f*x+e))^(3/2)/f-2/5*a^2*(d*
tan(f*x+e))^(5/2)/d/f
```

Rubi [A]
time = 0.12, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3624, 3609, 3614, 211}

$$\frac{4\sqrt[4]{-1}\,a^2d^{3/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}-\frac{2a^2(d\tan(e+fx))^{5/2}}{5df}+\frac{4ia^2(d\tan(e+fx))^{3/2}}{3f}+\frac{4a^2d\sqrt{d\tan(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(3/2)*(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (4*(-1)^(1/4)*a^2*d^(3/2)*ArcTan[((-1)^(3/4)*Sqrt[d*Tan[e + f*x]])/Sqrt[d]]
)/f + (4*a^2*d*Sqrt[d*Tan[e + f*x]])/f + (((4*I)/3)*a^2*(d*Tan[e + f*x])^(3
/2))/f - (2*a^2*(d*Tan[e + f*x])^(5/2))/(5*d*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int (d\tan(e+fx))^{3/2}(a+ia\tan(e+fx))^2\,dx = -\frac{2a^2(d\tan(e+fx))^{5/2}}{5df} + \int (d\tan(e+fx))^{3/2}\left(2a^2+2ia^2\right.$$
$$= \frac{4ia^2(d\tan(e+fx))^{3/2}}{3f} - \frac{2a^2(d\tan(e+fx))^{5/2}}{5df} + \int \sqrt{d\,\mathrm{ta}}$$
$$= \frac{4a^2d\sqrt{d\tan(e+fx)}}{f} + \frac{4ia^2(d\tan(e+fx))^{3/2}}{3f} - \frac{2a^2(d\,\mathrm{tan}}{}$$
$$= \frac{4a^2d\sqrt{d\tan(e+fx)}}{f} + \frac{4ia^2(d\tan(e+fx))^{3/2}}{3f} - \frac{2a^2(d\,\mathrm{tan}}{}$$
$$= \frac{4\sqrt[4]{-1}\,a^2d^{3/2}\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f} + \frac{4a^2d\sqrt{d}}{}$$

Mathematica [A]
time = 3.04, size = 127, normalized size = 1.12

$$\frac{a^2d^2\left(120i\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\right)\sqrt{i\tan(e+fx)}+\sec^2(e+fx)(20i-20i\cos(2(e+fx))+33\sec(e+fx)\sin(3(e+fx))+21\tan(e+fx))\right)}{30f\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(3/2)*(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (a^2*d^2*((120*I)*ArcTanh[Sqrt[(-1 + E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e
+ f*x)))]]*Sqrt[I*Tan[e + f*x]] + Sec[e + f*x]^2*(20*I - (20*I)*Cos[2*(e +
f*x)] + 33*Sec[e + f*x]*Sin[3*(e + f*x)] + 21*Tan[e + f*x])))/(30*f*Sqrt[d*
Tan[e + f*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 325 vs. 2(92) = 184.
time = 0.11, size = 326, normalized size = 2.88

method	result
derivativedivides	$2a^2\left(-\frac{(d\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{2id(d\tan(fx+e))^{\frac{3}{2}}}{3}+2d^2\sqrt{d\tan(fx+e)}-2d^3\left(\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d}}\right.\right.}{}\right.\right.$
default	$2a^2\left(-\frac{(d\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{2id(d\tan(fx+e))^{\frac{3}{2}}}{3}+2d^2\sqrt{d\tan(fx+e)}-2d^3\left(\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d}}\right.\right.}{}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(3/2)*(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^2/d*(-1/5*(d*tan(f*x+e))^(5/2)+2/3*I*d*(d*tan(f*x+e))^(3/2)+2*d^2*(d*
tan(f*x+e))^(1/2)-2*d^3*(1/8/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^
(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(
d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*t
an(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))+
1/8*I/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2
)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/
2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arc
tan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(95) = 190$.
time = 0.51, size = 223, normalized size = 1.97

$$\frac{15\,a^2d^3\left(-\frac{(2i+2)\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(2i+2)\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{(i-1)\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}-\frac{(i-1)\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)-12\,(d\tan(fx+e))^{\frac{5}{2}}a^2+40i\,(d\tan(fx+e))^{\frac{3}{2}}a^2d+120\sqrt{d\tan(fx+e)}\,a^2d^2}{30\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)*(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/30*(15*a^2*d^3*(-(2*I + 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) +
2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - (2*I + 2)*sqrt(2)*arctan(-1/2*sq
rt(2)*(sqrt(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) + (I - 1)
*sqrt(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqr
t(d) - (I - 1)*sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sq
rt(d) + d)/sqrt(d)) - 12*(d*tan(f*x + e))^(5/2)*a^2 + 40*I*(d*tan(f*x + e))
^(3/2)*a^2*d + 120*sqrt(d*tan(f*x + e))*a^2*d^2)/(d*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 392 vs. 2(95) = 190.
time = 0.38, size = 392, normalized size = 3.47

$$-\frac{15 \sqrt{-\frac{16 i a^4 d^3}{f^2}} \left(f e^{(4 i f x+4 i e)}+2 f e^{(2 i f x+2 i e)}+f\right) \log\left(\frac{\left(-4 i a^2 d^2 e^{(2 i f x+2 i e)}+\sqrt{-\frac{16 i a^4 d^3}{f^2}} \left(f e^{(2 i f x+2 i e)}+f\right) \sqrt{\frac{-i d e^{(2 i f x+2 i e)}+i d}{e^{(2 i f x+2 i e)}+1}}\right) e^{(-2 i f x-2 i e)}}{2 a^2 d}\right)-15 \sqrt{-\frac{16 i a^4 d^3}{f^2}} \left(f e^{(4 i f x+4 i e)}+2 f e^{(2 i f x+2 i e)}+f\right) \log\left(\frac{\left(-4 i a^2 d^2 e^{(2 i f x+2 i e)}-\sqrt{-\frac{16 i a^4 d^3}{f^2}} \left(f e^{(2 i f x+2 i e)}+f\right) \sqrt{\frac{-i d e^{(2 i f x+2 i e)}+i d}{e^{(2 i f x+2 i e)}+1}}\right) e^{(-2 i f x-2 i e)}}{2 a^2 d}\right)-8 \left(43 a^2 d e^{(4 i f x+4 i e)}+54 a^2 d e^{(2 i f x+2 i e)}+23 a^2 d\right) \sqrt{\frac{-i d e^{(2 i f x+2 i e)}+i d}{e^{(2 i f x+2 i e)}+1}}}{60 \left(f e^{(4 i f x+4 i e)}+2 f e^{(2 i f x+2 i e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)*(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/60*(15*sqrt(-16*I*a^4*d^3/f^2)*(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x +
 2*I*e) + f)*log(1/2*(-4*I*a^2*d^2*e^(2*I*f*x + 2*I*e) + sqrt(-16*I*a^4*d^3
/f^2)*(f*e^(2*I*f*x + 2*I*e) + f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^
(2*I*f*x + 2*I*e) + 1)))*e^(-2*I*f*x - 2*I*e)/(a^2*d)) - 15*sqrt(-16*I*a^4*
d^3/f^2)*(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*e) + f)*log(1/2*(-4*
I*a^2*d^2*e^(2*I*f*x + 2*I*e) - sqrt(-16*I*a^4*d^3/f^2)*(f*e^(2*I*f*x + 2*I
*e) + f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*
e^(-2*I*f*x - 2*I*e)/(a^2*d)) - 8*(43*a^2*d*e^(4*I*f*x + 4*I*e) + 54*a^2*d*
e^(2*I*f*x + 2*I*e) + 23*a^2*d)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2
*I*f*x + 2*I*e) + 1)))/(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*e) + f
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int\left(-(d\tan(e+fx))^{\frac{3}{2}}\right)dx+\int(d\tan(e+fx))^{\frac{3}{2}}\tan^2(e+fx)\,dx+\int\left(-2i(d\tan(e+fx))^{\frac{3}{2}}\tan(e+fx)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(3/2)*(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -a**2*(Integral(-(d*tan(e + f*x))**(3/2), x) + Integral((d*tan(e + f*x))**(
3/2)*tan(e + f*x)**2, x) + Integral(-2*I*(d*tan(e + f*x))**(3/2)*tan(e + f*
x), x))
```

Giac [A]
time = 0.77, size = 163, normalized size = 1.44

$$-\frac{2}{15}\left(\frac{30 i \sqrt{2} a^2 \sqrt{d} \arctan\left(\frac{8 \sqrt{d^2} \sqrt{d\tan(fx+e)}}{4 i \sqrt{2} d^{\frac{3}{2}}+4 \sqrt{2} \sqrt{d^2} \sqrt{d}}\right)}{f\left(\frac{i d}{\sqrt{d^2}}+1\right)}+\frac{3 \sqrt{d\tan(fx+e)}\, a^2 d^{10} f^4 \tan(fx+e)^2-10 i \sqrt{d\tan(fx+e)}\, a^2 d^{10} f^4 \tan(fx+e)-30 \sqrt{d\tan(fx+e)}\, a^2 d^{10} f^4}{d^{10} f^5}\right) d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)*(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -2/15*(30*I*sqrt(2)*a^2*sqrt(d)*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*
I*sqrt(2)*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(f*(I*d/sqrt(d^2) + 1)) +
```

```
(3*sqrt(d*tan(f*x + e))*a^2*d^10*f^4*tan(f*x + e)^2 - 10*I*sqrt(d*tan(f*x
+ e))*a^2*d^10*f^4*tan(f*x + e) - 30*sqrt(d*tan(f*x + e))*a^2*d^10*f^4)/(d^
10*f^5))*d
```

Mupad [B]
time = 4.54, size = 97, normalized size = 0.86

$$\frac{a^2\,{\left(d\tan(e+f\,x)\right)}^{3/2}\,4\mathrm{i}}{3\,f}-\frac{2\,a^2\,{\left(d\tan(e+f\,x)\right)}^{5/2}}{5\,d\,f}+\frac{4\,a^2\,d\,\sqrt{d\tan\left(e+f\,x\right)}}{f}-\frac{\sqrt{4\mathrm{i}}\,a^2\,{\left(-d\right)}^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{4\mathrm{i}}\,\sqrt{d\tan\left(e+f\,x\right)}}{2\,\sqrt{-d}}\right)\,2\mathrm{i}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(3/2)*(a + a*tan(e + f*x)*1i)^2,x)
```

```
[Out] (a^2*(d*tan(e + f*x))^(3/2)*4i)/(3*f) - (2*a^2*(d*tan(e + f*x))^(5/2))/(5*d
*f) + (4*a^2*d*(d*tan(e + f*x))^(1/2))/f - (4i^(1/2)*a^2*(-d)^(3/2)*atan((4
i^(1/2)*(d*tan(e + f*x))^(1/2))/(2*(-d)^(1/2)))*2i)/f
```

3.151 $\int \sqrt{d\tan(e+fx)}\,(a+ia\tan(e+fx))^2\,dx$

Optimal. Leaf size=90

$$\frac{4(-1)^{3/4}a^2\sqrt{d}\,\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}+\frac{4ia^2\sqrt{d\tan(e+fx)}}{f}-\frac{2a^2(d\tan(e+fx))^{3/2}}{3df}$$

```
[Out] 4*(-1)^(3/4)*a^2*arctan((-1)^(3/4)*(d*tan(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f+
4*I*a^2*(d*tan(f*x+e))^(1/2)/f-2/3*a^2*(d*tan(f*x+e))^(3/2)/d/f
```

Rubi [A]
time = 0.09, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3624, 3609, 3614, 211}

$$\frac{4(-1)^{3/4}a^2\sqrt{d}\,\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}-\frac{2a^2(d\tan(e+fx))^{3/2}}{3df}+\frac{4ia^2\sqrt{d\tan(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Tan[e + f*x]]*(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (4*(-1)^(3/4)*a^2*Sqrt[d]*ArcTan[((-1)^(3/4)*Sqrt[d*Tan[e + f*x]])/Sqrt[d]]
)/f + ((4*I)*a^2*Sqrt[d*Tan[e + f*x]])/f - (2*a^2*(d*Tan[e + f*x])^(3/2))/(
3*d*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \sqrt{d\tan(e+fx)}\,(a+ia\tan(e+fx))^2\,dx = -\frac{2a^2(d\tan(e+fx))^{3/2}}{3df} + \int \sqrt{d\tan(e+fx)}\,(2a^2+2ia^2\tan$$

$$= \frac{4ia^2\sqrt{d\tan(e+fx)}}{f} - \frac{2a^2(d\tan(e+fx))^{3/2}}{3df} + \int \frac{-2ia^2d+}{\sqrt{d}}$$

$$= \frac{4ia^2\sqrt{d\tan(e+fx)}}{f} - \frac{2a^2(d\tan(e+fx))^{3/2}}{3df} - \frac{(8a^4d^2)\,\mathrm{Sub}}{}$$

$$= \frac{4(-1)^{3/4}a^2\sqrt{d}\,\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f} + \frac{4ia^2\sqrt{d\,t}}{}$$

Mathematica [A]
time = 1.63, size = 102, normalized size = 1.13

$$\frac{2ia^2\left(-6\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\right)+(6+i\tan(e+fx))\sqrt{i\tan(e+fx)}\right)\sqrt{d\tan(e+fx)}}{3f\sqrt{i\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Tan[e + f*x]]*(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (((2*I)/3)*a^2*(-6*ArcTanh[Sqrt[(-1 + E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e
 + f*x)))]] + (6 + I*Tan[e + f*x])*Sqrt[I*Tan[e + f*x]])*Sqrt[d*Tan[e + f*x
]])/(f*Sqrt[I*Tan[e + f*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(73) = 146.
time = 0.12, size = 311, normalized size = 3.46

method	result

derivativedivides	$\frac{2a^2\left(-\frac{(d\tan(fx+e))^{\frac{3}{2}}}{3}+2id\sqrt{d\tan(fx+e)}-2d^2\left(\frac{i(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}}\right.\right.}{}\right.\right.}{}$
default	$\frac{2a^2\left(-\frac{(d\tan(fx+e))^{\frac{3}{2}}}{3}+2id\sqrt{d\tan(fx+e)}-2d^2\left(\frac{i(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}}\right.\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^2/d*(-1/3*(d*tan(f*x+e))^(3/2)+2*I*d*(d*tan(f*x+e))^(1/2)-2*d^2*(1/8*
I/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*
2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)
+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arcta
n(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))-1/8/(d^2)^(1/4)*2^(1/2)*(ln
((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan
(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(
1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*t
an(f*x+e))^(1/2)+1))))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(75) = 150$.
time = 0.51, size = 204, normalized size = 2.27

$$\frac{3a^2d^2\left(-\frac{(2i-2)\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(2i-2)\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(i+1)\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{(i+1)\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)-4(d\tan(fx+e))^{\frac{3}{2}}a^2+24i\sqrt{d\tan(fx+e)}a^2d}{6df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*a^2*d^2*(-(2*I - 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*
sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - (2*I - 2)*sqrt(2)*arctan(-1/2*sqrt
(2)*(sqrt(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - (I + 1)*s
qrt(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(
d) + (I + 1)*sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt
(d) + d)/sqrt(d)) - 4*(d*tan(f*x + e))^(3/2)*a^2 + 24*I*sqrt(d*tan(f*x + e)
)*a^2*d)/(d*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(75) = 150$.

time = 0.40, size = 322, normalized size = 3.58

$$\frac{3\sqrt{\frac{16i\,a^4d}{f^2}}\left(fe^{(2i\,fx+2i\,e)}+f\right)\log\left(\frac{\left(-4i\,a^2de^{(2i\,fx+2i\,e)}+\sqrt{\frac{16i\,a^4d}{f^2}}\left(i\,fe^{(2i\,fx+2i\,e)}+i\,f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-2i\,fx-2i\,e)}}{2\,a^2}\right)-3\sqrt{\frac{16i\,a^4d}{f^2}}\left(fe^{(2i\,fx+2i\,e)}+f\right)\log\left(\frac{\left(-4i\,a^2de^{(2i\,fx+2i\,e)}+\sqrt{\frac{16i\,a^4d}{f^2}}\left(-i\,fe^{(2i\,fx+2i\,e)}-i\,f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-2i\,fx-2i\,e)}}{2\,a^2}\right)-8\left(-7i\,a^2e^{(2i\,fx+2i\,e)}-5i\,a^2\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}}{12\left(fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*sqrt(16*I*a^4*d/f^2)*(f*e^(2*I*f*x + 2*I*e) + f)*log(1/2*(-4*I*a^2*
d*e^(2*I*f*x + 2*I*e) + sqrt(16*I*a^4*d/f^2)*(I*f*e^(2*I*f*x + 2*I*e) + I*f
)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-2*I
*f*x - 2*I*e)/a^2) - 3*sqrt(16*I*a^4*d/f^2)*(f*e^(2*I*f*x + 2*I*e) + f)*log
(1/2*(-4*I*a^2*d*e^(2*I*f*x + 2*I*e) + sqrt(16*I*a^4*d/f^2)*(-I*f*e^(2*I*f*
x + 2*I*e) - I*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e
) + 1)))*e^(-2*I*f*x - 2*I*e)/a^2) - 8*(-7*I*a^2*e^(2*I*f*x + 2*I*e) - 5*I*
a^2)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))/(f*e
^(2*I*f*x + 2*I*e) + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int\left(-\sqrt{d\tan\left(e+fx\right)}\right)dx+\int\sqrt{d\tan\left(e+fx\right)}\tan^2\left(e+fx\right)dx+\int\left(-2i\sqrt{d\tan\left(e+fx\right)}\tan\left(e+fx\right)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(1/2)*(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -a**2*(Integral(-sqrt(d*tan(e + f*x)), x) + Integral(sqrt(d*tan(e + f*x))*t
an(e + f*x)**2, x) + Integral(-2*I*sqrt(d*tan(e + f*x))*tan(e + f*x), x))
```

Giac [A]
time = 0.59, size = 129, normalized size = 1.43

$$\frac{4\sqrt{2}\,a^2\sqrt{d}\,\arctan\left(\frac{8\sqrt{d^2}\sqrt{d\tan\left(fx+e\right)}}{4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{f\left(\frac{i\,d}{\sqrt{d^2}}+1\right)}-\frac{2\left(\sqrt{d\tan\left(fx+e\right)}\,a^2d^3f^2\tan\left(fx+e\right)-6i\sqrt{d\tan\left(fx+e\right)}\,a^2d^3f^2\right)}{3\,d^3f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*a^2*sqrt(d)*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*I*sqrt(2)*
d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(f*(I*d/sqrt(d^2) + 1)) - 2/3*(sqrt
(d*tan(f*x + e))*a^2*d^3*f^2*tan(f*x + e) - 6*I*sqrt(d*tan(f*x + e))*a^2*d^
3*f^2)/(d^3*f^3)
```

Mupad [B]
time = 4.28, size = 74, normalized size = 0.82

$$\frac{a^2\sqrt{d\tan(e+fx)}\,4\mathrm{i}}{f}-\frac{2a^2(d\tan(e+fx))^{3/2}}{3df}-\frac{2\sqrt{4\mathrm{i}}\,a^2\sqrt{d}\,\mathrm{atanh}\left(\frac{\sqrt{4\mathrm{i}}\,\sqrt{d\tan(e+fx)}}{2\sqrt{d}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(1/2)*(a + a*tan(e + f*x)*1i)^2,x)
```

```
[Out] (a^2*(d*tan(e + f*x))^(1/2)*4i)/f - (2*a^2*(d*tan(e + f*x))^(3/2))/(3*d*f)
- (2*4i^(1/2)*a^2*d^(1/2)*atanh((4i^(1/2)*(d*tan(e + f*x))^(1/2))/(2*d^(1/2
))))/f
```

3.152 $\int \frac{(a+ia\tan(e+fx))^2}{\sqrt{d\tan(e+fx)}}\,dx$

Optimal. Leaf size=66

$$-\frac{4\sqrt[4]{-1}\,a^2\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{d}\,f}-\frac{2a^2\sqrt{d\tan(e+fx)}}{df}$$

```
[Out] -4*(-1)^(1/4)*a^2*arctan((-1)^(3/4)*(d*tan(f*x+e))^(1/2)/d^(1/2))/f/d^(1/2)
-2*a^2*(d*tan(f*x+e))^(1/2)/d/f
```

Rubi [A]
time = 0.07, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3624, 3614, 211}

$$-\frac{4\sqrt[4]{-1}\,a^2\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{d}\,f}-\frac{2a^2\sqrt{d\tan(e+fx)}}{df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (-4*(-1)^(1/4)*a^2*ArcTan[((-1)^(3/4)*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(Sqrt
[d]*f) - (2*a^2*Sqrt[d*Tan[e + f*x]])/(d*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^2}{\sqrt{d\tan(e+fx)}}\,dx = -\frac{2a^2\sqrt{d\tan(e+fx)}}{df}+\int\frac{2a^2+2ia^2\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$$

$$= -\frac{2a^2\sqrt{d\tan(e+fx)}}{df}+\frac{(8a^4)\,\mathrm{Subst}\left(\int\frac{1}{2a^2d-2ia^2x^2}\,dx,x,\sqrt{d\tan(e+fx)}\right)}{f}$$

$$= -\frac{4\sqrt[4]{-1}\,a^2\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{d}\,f}-\frac{2a^2\sqrt{d\tan(e+fx)}}{df}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.51, size = 157, normalized size = 2.38

$$-\frac{2a^2e^{-2i(e+fx)}(\cos(2(e+fx))+i\sin(2(e+fx)))\sqrt{\tan(e+fx)}\left(2i\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\right)\sqrt{i\tan(e+fx)}+\tan(e+fx)\right)}{\sqrt{-\frac{i\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\,f\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (-2*a^2*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)])*Sqrt[Tan[e + f*x]]*((2*I)*A
rcTanh[Sqrt[(-1 + E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f*x)))]]*Sqrt[I*T
an[e + f*x]] + Tan[e + f*x]))/(E^((2*I)*(e + f*x))*Sqrt[((-I)*(-1 + E^((2*I
)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))]*f*Sqrt[d*Tan[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 295, normalized size = 4.47

method	result
derivativedivides	$2a^2\left(-\sqrt{d\tan(fx+e)}+2d\left(\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\,a\right.}{}\right.\right.$
default	$2a^2\left(-\sqrt{d\tan(fx+e)}+2d\left(\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\,a\right.}{}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2/(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^2/d*(-(d*tan(f*x+e))^(1/2)+2*d*(1/8/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(
f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-
(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^
2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e
))^(1/2)+1))+1/8*I/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan
(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e)
)^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(
1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.52, size = 184, normalized size = 2.79

$$-\frac{a^2 d \left(-\frac{(2i+2)\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(2i+2)\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{(i-1)\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{d}+d\right)}{\sqrt{d}}-\frac{(i-1)\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{d}+d\right)}{\sqrt{d}}\right)+4\sqrt{d\tan\left(fx+e\right)}\,a^2}{2\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(a^2*d*(-(2*I + 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqr
t(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - (2*I + 2)*sqrt(2)*arctan(-1/2*sqrt(2)
*(sqrt(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) + (I - 1)*sqrt
(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d)
- (I - 1)*sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d)
 + d)/sqrt(d)) + 4*sqrt(d*tan(f*x + e))*a^2)/(d*f)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.38, size = 278, normalized size = 4.21

$$\frac{d\sqrt{-\frac{16i\,a^4}{df^2}}\,f\log\left(\frac{\left(-4i\,a^2de^{(2i\,fx+2i\,e)}+\left(dfe^{(2i\,fx+2i\,e)}+df\right)\sqrt{-\frac{16i\,a^4}{df^2}}\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-2i\,fx-2i\,e)}}{2\,a^2}\right)-d\sqrt{-\frac{16i\,a^4}{df^2}}\,f\log\left(\frac{\left(-4i\,a^2de^{(2i\,fx+2i\,e)}-\left(dfe^{(2i\,fx+2i\,e)}+df\right)\sqrt{-\frac{16i\,a^4}{df^2}}\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-2i\,fx-2i\,e)}}{2\,a^2}\right)-8\,a^2\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}}{4\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(d*sqrt(-16*I*a^4/(d*f^2))*f*log(1/2*(-4*I*a^2*d*e^(2*I*f*x + 2*I*e) +
(d*f*e^(2*I*f*x + 2*I*e) + d*f)*sqrt(-16*I*a^4/(d*f^2))*sqrt((-I*d*e^(2*I*f
*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-2*I*f*x - 2*I*e)/a^2) -
d*sqrt(-16*I*a^4/(d*f^2))*f*log(1/2*(-4*I*a^2*d*e^(2*I*f*x + 2*I*e) - (d*f*
e^(2*I*f*x + 2*I*e) + d*f)*sqrt(-16*I*a^4/(d*f^2))*sqrt((-I*d*e^(2*I*f*x +
```

```
2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-2*I*f*x - 2*I*e)/a^2) - 8*a^2
*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))/(d*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int\left(-\frac{1}{\sqrt{d\tan(e+fx)}}\right)dx+\int\frac{\tan^2(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx+\int\left(-\frac{2i\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2/(d*tan(f*x+e))**(1/2),x)
```

```
[Out] -a**2*(Integral(-1/sqrt(d*tan(e + f*x)), x) + Integral(tan(e + f*x)**2/sqrt
(d*tan(e + f*x)), x) + Integral(-2*I*tan(e + f*x)/sqrt(d*tan(e + f*x)), x))
```

Giac [C] Result contains complex when optimal does not.
time = 0.60, size = 92, normalized size = 1.39

$$\frac{4i\sqrt{2}\,a^2\arctan\left(\frac{8\sqrt{d^2}\,\sqrt{d\tan(fx+e)}}{4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\,\sqrt{d^2}\,\sqrt{d}}\right)}{\sqrt{d}\,f\left(\frac{id}{\sqrt{d^2}}+1\right)}-\frac{2\sqrt{d\tan(fx+e)}\,a^2}{df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 4*I*sqrt(2)*a^2*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*I*sqrt(2)*d^(3/2
) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(sqrt(d)*f*(I*d/sqrt(d^2) + 1)) - 2*sqrt(
d*tan(f*x + e))*a^2/(d*f)
```

Mupad [B]
time = 4.15, size = 59, normalized size = 0.89

$$-\frac{2\,a^2\,\sqrt{d\tan(e+f\,x)}}{d\,f}+\frac{\sqrt{4\mathrm{i}}\,a^2\,\mathrm{atan}\left(\frac{\sqrt{4\mathrm{i}}\,\sqrt{d\tan(e+f\,x)}}{2\sqrt{-d}}\right)2\mathrm{i}}{\sqrt{-d}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^2/(d*tan(e + f*x))^(1/2),x)
```

```
[Out] (4i^(1/2)*a^2*atan((4i^(1/2)*(d*tan(e + f*x))^(1/2))/(2*(-d)^(1/2)))*2i)/((
-d)^(1/2)*f) - (2*a^2*(d*tan(e + f*x))^(1/2))/(d*f)
```

3.153 $\int \frac{(a+ia\tan(e+fx))^2}{(d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{4(-1)^{3/4}a^2\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{3/2}f}-\frac{2a^2}{df\sqrt{d\tan(e+fx)}}$$

```
[Out] -4*(-1)^(3/4)*a^2*arctan((-1)^(3/4)*(d*tan(f*x+e))^(1/2)/d^(1/2))/d^(3/2)/f
-2*a^2/d/f/(d*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3623, 3614, 211}

$$-\frac{4(-1)^{3/4}a^2\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{3/2}f}-\frac{2a^2}{df\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2/(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-4*(-1)^(3/4)*a^2*ArcTan[((-1)^(3/4)*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(d^(3
/2)*f) - (2*a^2)/(d*f*Sqrt[d*Tan[e + f*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^2}{(d\tan(e+fx))^{3/2}}\,dx = -\frac{2a^2}{df\sqrt{d\tan(e+fx)}}+\frac{\int \frac{2ia^2d-2a^2d\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx}{d^2}$$

$$= -\frac{2a^2}{df\sqrt{d\tan(e+fx)}}-\frac{(8a^4)\,\text{Subst}\left(\int \frac{1}{2ia^2d^2+2a^2dx^2}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{f}$$

$$= -\frac{4(-1)^{3/4}a^2\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{3/2}f}-\frac{2a^2}{df\sqrt{d\tan(e+fx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.06, size = 147, normalized size = 2.23

$$-\frac{2a^2e^{-2i(e+fx)}(\cos(2(e+fx))+i\sin(2(e+fx)))\left(\sqrt{i\tan(e+fx)}-2i\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\right)\tan(e+fx)\right)}{d\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\,f\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2/(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*a^2*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)])*(Sqrt[I*Tan[e + f*x]] - (2*
I)*ArcTanh[Sqrt[(-1 + E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f*x)))]]*Tan[
e + f*x]))/(d*E^((2*I)*(e + f*x))*Sqrt[(-1 + E^((2*I)*(e + f*x)))/(1 + E^((
2*I)*(e + f*x)))]*f*Sqrt[d*Tan[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.10, size = 291, normalized size = 4.41

method	result
derivativedivides	$2a^2\left(\frac{i(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+\right.\right.}{(d^2)^{\frac{1}{4}}}\right.}{4d}\right.$

default	$\frac{2a^2\left(\frac{i(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+e}}{(d^2)^{\frac{1}{4}}}\right.\right.}{4d}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2/(d*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^2/d*(1/4*I/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*tan
(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e)
)^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(
1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))-1/4/(d^2)^(1
/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2
)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)
))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(
d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))-1/(d*tan(f*x+e))^(1/2))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.52, size = 183, normalized size = 2.77

$$-\frac{a^2\left(-\frac{(2i-2)\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(2i-2)\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(i+1)\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{(i+1)\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)+\frac{4a^2}{\sqrt{d\tan(fx+e)}}}{2df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*(a^2*(-(2*I - 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(
d*tan(f*x + e)))/sqrt(d))/sqrt(d) - (2*I - 2)*sqrt(2)*arctan(-1/2*sqrt(2)*(
sqrt(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - (I + 1)*sqrt(2
)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) +
(I + 1)*sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) +
 d)/sqrt(d)) + 4*a^2/sqrt(d*tan(f*x + e)))/(d*f)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.38, size = 365, normalized size = 5.53

$$-\frac{(d^2fe^{(2ifx+2ie)}-d^2f)\sqrt{\frac{16ia^4}{d^3f^2}}\log\left(\frac{\left(-4ia^2de^{(2ifx+2ie)}+(id^2fe^{(2ifx+2ie)}+id^2f)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{16ia^4}{d^3f^2}}\right)e^{(-2ifx-2ie)}}{2a^2}\right)-(d^2fe^{(2ifx+2ie)}-d^2f)\sqrt{\frac{16ia^4}{d^3f^2}}\log\left(\frac{\left(-4ia^2de^{(2ifx+2ie)}+(-id^2fe^{(2ifx+2ie)}-id^2f)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{16ia^4}{d^3f^2}}\right)e^{(-2ifx-2ie)}}{2a^2}\right)+8(ia^2e^{(2ifx+2ie)}+ia^2)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}}{4(d^2fe^{(2ifx+2ie)}-d^2f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*((d^2*f*e^(2*I*f*x + 2*I*e) - d^2*f)*sqrt(16*I*a^4/(d^3*f^2))*log(1/2*
(-4*I*a^2*d*e^(2*I*f*x + 2*I*e) + (I*d^2*f*e^(2*I*f*x + 2*I*e) + I*d^2*f)*s
```

```
qrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(16*I*a
^4/(d^3*f^2)))*e^(-2*I*f*x - 2*I*e)/a^2) - (d^2*f*e^(2*I*f*x + 2*I*e) - d^2
*f)*sqrt(16*I*a^4/(d^3*f^2))*log(1/2*(-4*I*a^2*d*e^(2*I*f*x + 2*I*e) + (-I*
d^2*f*e^(2*I*f*x + 2*I*e) - I*d^2*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/
(e^(2*I*f*x + 2*I*e) + 1))*sqrt(16*I*a^4/(d^3*f^2)))*e^(-2*I*f*x - 2*I*e)/a
^2) + 8*(I*a^2*e^(2*I*f*x + 2*I*e) + I*a^2)*sqrt((-I*d*e^(2*I*f*x + 2*I*e)
+ I*d)/(e^(2*I*f*x + 2*I*e) + 1)))/(d^2*f*e^(2*I*f*x + 2*I*e) - d^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int\left(-\frac{1}{(d\tan(e+fx))^{\frac{3}{2}}}\right)dx+\int\frac{\tan^2(e+fx)}{(d\tan(e+fx))^{\frac{3}{2}}}\,dx+\int\left(-\frac{2i\tan(e+fx)}{(d\tan(e+fx))^{\frac{3}{2}}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2/(d*tan(f*x+e))**(3/2),x)
```

```
[Out] -a**2*(Integral(-1/(d*tan(e + f*x))**(3/2), x) + Integral(tan(e + f*x)**2/(
d*tan(e + f*x))**(3/2), x) + Integral(-2*I*tan(e + f*x)/(d*tan(e + f*x))**(
3/2), x))
```

Giac [C] Result contains complex when optimal does not.
time = 0.68, size = 93, normalized size = 1.41

$$-\frac{2\left(-\frac{2i\sqrt{2}\,a^2\arctan\left(-\frac{8i\sqrt{d^2}\sqrt{d\tan(fx+e)}}{-4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{\sqrt{d}\,f\left(-\frac{id}{\sqrt{d^2}}+1\right)}+\frac{a^2}{\sqrt{d\tan(fx+e)}\,f}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] -2*(-2*I*sqrt(2)*a^2*arctan(-8*I*sqrt(d^2)*sqrt(d*tan(f*x + e))/(-4*I*sqrt(
2)*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(sqrt(d)*f*(-I*d/sqrt(d^2) + 1))
 + a^2/(sqrt(d*tan(f*x + e))*f))/d
```

Mupad [B]
time = 4.29, size = 55, normalized size = 0.83

$$-\frac{2\,a^2}{d\,f\,\sqrt{d\tan(e+f\,x)}}+\frac{2\,\sqrt{4\mathrm{i}}\,a^2\,\mathrm{atanh}\left(\frac{\sqrt{4\mathrm{i}}\,\sqrt{d\tan(e+f\,x)}}{2\,\sqrt{d}}\right)}{d^{3/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^2/(d*tan(e + f*x))^(3/2),x)
```

```
[Out] (2*4i^(1/2)*a^2*atanh((4i^(1/2)*(d*tan(e + f*x))^(1/2))/(2*d^(1/2))))/(d^(3
/2)*f) - (2*a^2)/(d*f*(d*tan(e + f*x))^(1/2))
```

3.154 $\int \frac{(a+ia\tan(e+fx))^2}{(d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=93

$$\frac{4\sqrt[4]{-1}\,a^2\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{5/2}f}-\frac{2a^2}{3df(d\tan(e+fx))^{3/2}}-\frac{4ia^2}{d^2f\sqrt{d\tan(e+fx)}}$$

```
[Out] 4*(-1)^(1/4)*a^2*arctan((-1)^(3/4)*(d*tan(f*x+e))^(1/2)/d^(1/2))/d^(5/2)/f-
4*I*a^2/d^2/f/(d*tan(f*x+e))^(1/2)-2/3*a^2/d/f/(d*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.11, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3623, 3610, 3614, 211}

$$\frac{4\sqrt[4]{-1}\,a^2\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{5/2}f}-\frac{4ia^2}{d^2f\sqrt{d\tan(e+fx)}}-\frac{2a^2}{3df(d\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2/(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (4*(-1)^(1/4)*a^2*ArcTan[((-1)^(3/4)*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(d^(5/
2)*f) - (2*a^2)/(3*d*f*(d*Tan[e + f*x])^(3/2)) - ((4*I)*a^2)/(d^2*f*Sqrt[d*
Tan[e + f*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^2}{(d\tan(e+fx))^{5/2}}\,dx = -\frac{2a^2}{3df(d\tan(e+fx))^{3/2}} + \frac{\int \frac{2ia^2d-2a^2d\tan(e+fx)}{(d\tan(e+fx))^{3/2}}\,dx}{d^2}$$

$$= -\frac{2a^2}{3df(d\tan(e+fx))^{3/2}} - \frac{4ia^2}{d^2f\sqrt{d\tan(e+fx)}} + \frac{\int \frac{-2a^2d^2-2ia^2d^2\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx}{d^4}$$

$$= -\frac{2a^2}{3df(d\tan(e+fx))^{3/2}} - \frac{4ia^2}{d^2f\sqrt{d\tan(e+fx)}} + \frac{(8a^4)\,\mathrm{Subst}\left(\int \frac{1}{-2a^2d^3+2ia^2\ldots}\right.}{\ldots}$$

$$= \frac{4\sqrt[4]{-1}\,a^2\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{5/2}f} - \frac{2a^2}{3df(d\tan(e+fx))^{3/2}} - \frac{\ldots}{d^2f\sqrt{\ldots}}$$

Mathematica [A]
time = 1.42, size = 87, normalized size = 0.94

$$-\frac{2a^2\left(6i+\cot(e+fx)-6i\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\right)\sqrt{i\tan(e+fx)}\right)}{3d^2f\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2/(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (-2*a^2*(6*I + Cot[e + f*x] - (6*I)*ArcTanh[Sqrt[(-1 + E^((2*I)*(e + f*x)))
/(1 + E^((2*I)*(e + f*x)))]]*Sqrt[I*Tan[e + f*x]]))/(3*d^2*f*Sqrt[d*Tan[e +
 f*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(76) = 152.
time = 0.10, size = 312, normalized size = 3.35

method	result
derivativedivides	$\frac{2a^2\left(-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+}{(d^2)^{\frac{1}{4}}}\right.\right.}{4d}\right.}{}$
default	$\frac{2a^2\left(-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+}{(d^2)^{\frac{1}{4}}}\right.\right.}{4d}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2/(d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^2/d*(1/d*(-1/4/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d
*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*
x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e
))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))-1/4*I/(d
^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2
)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)
^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(
1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)))-1/3/(d*tan(f*x+e))^(3/2)-2*I/d/(
d*tan(f*x+e))^(1/2))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(78) = 156$.
time $= 0.52$, size $= 206$, normalized size $= 2.22$

$$\frac{\frac{3a^2\left(-\frac{(2i+2)\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(2i+2)\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{(i-1)\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}-\frac{(i-1)\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)}{d}-\frac{4(6ia^2d\tan(fx+e)+a^2d)}{(d\tan(fx+e))^{\frac{3}{2}}d}}{6df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(3*a^2*(-(2*I + 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt
(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - (2*I + 2)*sqrt(2)*arctan(-1/2*sqrt(2)*
(sqrt(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) + (I - 1)*sqrt(
```

```
2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) -
 (I - 1)*sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d)
+ d)/sqrt(d))/d - 4*(6*I*a^2*d*tan(f*x + e) + a^2*d)/((d*tan(f*x + e))^(3/2
)*d))/(d*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(78) = 156$.
time = 0.38, size = 423, normalized size = 4.55

$$-\frac{3\left(d^3fe^{(4i\,fx+4i\,e)}-2\,d^3fe^{(2i\,fx+2i\,e)}+d^3f\right)\sqrt{-\frac{16i\,a^4}{d^5f^2}}\log\left(\frac{\left(-4i\,a^2de^{(2i\,fx+2i\,e)}+\left(d^3fe^{(2i\,fx+2i\,e)}+d^3f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{-\frac{16i\,a^4}{d^5f^2}}\right)e^{(-2i\,fx-2i\,e)}}{2\,a^2}\right)-3\left(d^3fe^{(4i\,fx+4i\,e)}-2\,d^3fe^{(2i\,fx+2i\,e)}+d^3f\right)\sqrt{-\frac{16i\,a^4}{d^5f^2}}\log\left(\frac{\left(-4i\,a^2de^{(2i\,fx+2i\,e)}-\left(d^3fe^{(2i\,fx+2i\,e)}+d^3f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{-\frac{16i\,a^4}{d^5f^2}}\right)e^{(-2i\,fx-2i\,e)}}{2\,a^2}\right)-8\left(7\,a^2e^{(4i\,fx+4i\,e)}+2\,a^2e^{(2i\,fx+2i\,e)}-5\,a^2\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}}{12\left(d^3fe^{(4i\,fx+4i\,e)}-2\,d^3fe^{(2i\,fx+2i\,e)}+d^3f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(3*(d^3*f*e^(4*I*f*x + 4*I*e) - 2*d^3*f*e^(2*I*f*x + 2*I*e) + d^3*f)*
sqrt(-16*I*a^4/(d^5*f^2))*log(1/2*(-4*I*a^2*d*e^(2*I*f*x + 2*I*e) + (d^3*f*
e^(2*I*f*x + 2*I*e) + d^3*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*
f*x + 2*I*e) + 1))*sqrt(-16*I*a^4/(d^5*f^2)))*e^(-2*I*f*x - 2*I*e)/a^2) - 3
*(d^3*f*e^(4*I*f*x + 4*I*e) - 2*d^3*f*e^(2*I*f*x + 2*I*e) + d^3*f)*sqrt(-16
*I*a^4/(d^5*f^2))*log(1/2*(-4*I*a^2*d*e^(2*I*f*x + 2*I*e) - (d^3*f*e^(2*I*f
*x + 2*I*e) + d^3*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*
I*e) + 1))*sqrt(-16*I*a^4/(d^5*f^2)))*e^(-2*I*f*x - 2*I*e)/a^2) - 8*(7*a^2*
e^(4*I*f*x + 4*I*e) + 2*a^2*e^(2*I*f*x + 2*I*e) - 5*a^2)*sqrt((-I*d*e^(2*I*
f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))/(d^3*f*e^(4*I*f*x + 4*I*e)
- 2*d^3*f*e^(2*I*f*x + 2*I*e) + d^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int\left(-\frac{1}{(d\tan(e+fx))^{\frac{5}{2}}}\right)dx+\int\frac{\tan^2(e+fx)}{(d\tan(e+fx))^{\frac{5}{2}}}\,dx+\int\left(-\frac{2i\tan(e+fx)}{(d\tan(e+fx))^{\frac{5}{2}}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2/(d*tan(f*x+e))**(5/2),x)
```

```
[Out] -a**2*(Integral(-1/(d*tan(e + f*x))**(5/2), x) + Integral(tan(e + f*x)**2/(
d*tan(e + f*x))**(5/2), x) + Integral(-2*I*tan(e + f*x)/(d*tan(e + f*x))**(
5/2), x))
```

Giac [A]
time = 0.66, size = 117, normalized size = 1.26

$$-\frac{4i\sqrt{2}\,a^2\arctan\left(\frac{8\sqrt{d^2}\sqrt{d\tan(fx+e)}}{4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{d^{\frac{5}{2}}f\left(\frac{i\,d}{\sqrt{d^2}}+1\right)}-\frac{2\left(6i\,a^2d\tan(fx+e)+a^2d\right)}{3\sqrt{d\tan(fx+e)}\,d^3f\tan(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] -4*I*sqrt(2)*a^2*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*I*sqrt(2)*d^(3/
2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(d^(5/2)*f*(I*d/sqrt(d^2) + 1)) - 2/3*(6
*I*a^2*d*tan(f*x + e) + a^2*d)/(sqrt(d*tan(f*x + e))*d^3*f*tan(f*x + e))
```

Mupad [B]
time = 4.38, size = 80, normalized size = 0.86

$$-\frac{\frac{2\,a^2}{3\,d\,f}+\frac{a^2\,\tan(e+f\,x)\,4\mathrm{i}}{d\,f}}{\left(d\tan\left(e+f\,x\right)\right)^{3/2}}-\frac{\sqrt{4\mathrm{i}}\;a^2\,\mathrm{atan}\left(\frac{\sqrt{4\mathrm{i}}\;\sqrt{d\tan\left(e+f\,x\right)}}{2\,\sqrt{-d}}\right)\,2\mathrm{i}}{{\left(-d\right)}^{5/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^2/(d*tan(e + f*x))^(5/2),x)
```

```
[Out] - ((2*a^2)/(3*d*f) + (a^2*tan(e + f*x)*4i)/(d*f))/(d*tan(e + f*x))^(3/2) -
(4i^(1/2)*a^2*atan((4i^(1/2)*(d*tan(e + f*x))^(1/2))/(2*(-d)^(1/2)))*2i)/((
-d)^(5/2)*f)
```

3.155 $\int \frac{(a+ia\tan(e+fx))^2}{(d\tan(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=118

$$\frac{4(-1)^{3/4}a^2\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{7/2}f}-\frac{2a^2}{5df(d\tan(e+fx))^{5/2}}-\frac{4ia^2}{3d^2f(d\tan(e+fx))^{3/2}}+\frac{4a^2}{d^3f\sqrt{d\tan(e+fx)}}$$

```
[Out] 4*(-1)^(3/4)*a^2*arctan((-1)^(3/4)*(d*tan(f*x+e))^(1/2)/d^(1/2))/d^(7/2)/f+
4*a^2/d^3/f/(d*tan(f*x+e))^(1/2)-2/5*a^2/d/f/(d*tan(f*x+e))^(5/2)-4/3*I*a^2
/d^2/f/(d*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.14, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3623, 3610, 3614, 211}

$$\frac{4(-1)^{3/4}a^2\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{7/2}f}+\frac{4a^2}{d^3f\sqrt{d\tan(e+fx)}}-\frac{4ia^2}{3d^2f(d\tan(e+fx))^{3/2}}-\frac{2a^2}{5df(d\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2/(d*Tan[e + f*x])^(7/2),x]
```

```
[Out] (4*(-1)^(3/4)*a^2*ArcTan[((-1)^(3/4)*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(d^(7/
2)*f) - (2*a^2)/(5*d*f*(d*Tan[e + f*x])^(5/2)) - (((4*I)/3)*a^2)/(d^2*f*(d*
Tan[e + f*x])^(3/2)) + (4*a^2)/(d^3*f*Sqrt[d*Tan[e + f*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^2}{(d\tan(e+fx))^{7/2}}\,dx = -\frac{2a^2}{5df(d\tan(e+fx))^{5/2}} + \frac{\int \frac{2ia^2d-2a^2d\tan(e+fx)}{(d\tan(e+fx))^{5/2}}\,dx}{d^2}$$

$$= -\frac{2a^2}{5df(d\tan(e+fx))^{5/2}} - \frac{4ia^2}{3d^2f(d\tan(e+fx))^{3/2}} + \frac{\int \frac{-2a^2d^2-2ia^2d^2\tan(e+fx)}{(d\tan(e+fx))^{3/2}}\,d}{d^4}$$

$$= -\frac{2a^2}{5df(d\tan(e+fx))^{5/2}} - \frac{4ia^2}{3d^2f(d\tan(e+fx))^{3/2}} + \frac{4a^2}{d^3f\sqrt{d\tan(e+fx)}} + $$

$$= -\frac{2a^2}{5df(d\tan(e+fx))^{5/2}} - \frac{4ia^2}{3d^2f(d\tan(e+fx))^{3/2}} + \frac{4a^2}{d^3f\sqrt{d\tan(e+fx)}} - $$

$$= \frac{4(-1)^{3/4}a^2\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{7/2}f} - \frac{2a^2}{5df(d\tan(e+fx))^{5/2}} - \frac{}{3d^2f}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 381 vs. 2(118) = 236.
time = 7.02, size = 381, normalized size = 3.23

$$\frac{(\csc(e)(33\cos(e)+10i\sin(e))\left(\frac{2}{15}\cos(2e)-\frac{2}{15}i\sin(2e)\right)+\csc(e)\csc^2(e+fx)(3\cos(e)+10i\sin(e))\left(-\frac{2}{15}\cos(2e)+\frac{2}{15}i\sin(2e)\right)+\csc(e)\csc^3(e+fx)\left(\frac{2}{5}\cos(2e)-\frac{2}{5}i\sin(2e)\right)\sin(fx)+\csc(e)\csc(e+fx)\left(-\frac{22}{5}\cos(2e)+\frac{22}{5}i\sin(2e)\right)\sin(fx))\sin^2(e+fx)\tan^2(e+fx)(a+ia\tan(e+fx))^2}{f(\cos(fx)+i\sin(fx))^2(d\tan(e+fx))^{7/2}} - \frac{4ie^{-2ie}\sqrt{-\frac{i\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\right)\cos^2(e+fx)\tan^{\frac{7}{2}}(e+fx)(a+ia\tan(e+fx))^2}{\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}f(\cos(fx)+i\sin(fx))^2(d\tan(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2/(d*Tan[e + f*x])^(7/2),x]
```

```
[Out] ((Csc[e]*(33*Cos[e] + (10*I)*Sin[e])*((2*Cos[2*e])/15 - ((2*I)/15)*Sin[2*e]
) + Csc[e]*Csc[e + f*x]^2*(3*Cos[e] + (10*I)*Sin[e])*((-2*Cos[2*e])/15 + ((
2*I)/15)*Sin[2*e]) + Csc[e]*Csc[e + f*x]^3*((2*Cos[2*e])/5 - ((2*I)/5)*Sin[
2*e])*Sin[f*x] + Csc[e]*Csc[e + f*x]*((-22*Cos[2*e])/5 + ((22*I)/5)*Sin[2*e
])*Sin[f*x])*Sin[e + f*x]^2*Tan[e + f*x]^2*(a + I*a*Tan[e + f*x])^2)/(f*(Co
s[f*x] + I*Sin[f*x])^2*(d*Tan[e + f*x])^(7/2)) - ((4*I)*Sqrt[((-I)*(-1 + E^
```

```
((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))]*ArcTanh[Sqrt[(-1 + E^((2*I)*
(e + f*x)))/(1 + E^((2*I)*(e + f*x)))]]*Cos[e + f*x]^2*Tan[e + f*x]^(7/2)*(
a + I*a*Tan[e + f*x])^2)/(E^((2*I)*e)*Sqrt[(-1 + E^((2*I)*(e + f*x)))/(1 +
E^((2*I)*(e + f*x)))]*f*(Cos[f*x] + I*Sin[f*x])^2*(d*Tan[e + f*x])^(7/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(97) = 194$.
time = 0.10, size = 327, normalized size = 2.77

method	result
derivativedivides	$2a^2\left(-\frac{i(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx}{(d^2)^{\frac{1}{4}}}\right.\right.}{4d}\right.$
default	$2a^2\left(-\frac{i(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx}{(d^2)^{\frac{1}{4}}}\right.\right.}{4d}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2/(d*tan(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^2/d*(1/d^2*(-1/4*I/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4
)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*ta
n(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f
*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))+1/4/
(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1
/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^
2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2
^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)))-1/5/(d*tan(f*x+e))^(5/2)-2/3*I
/d/(d*tan(f*x+e))^(3/2)+2/d^2/(d*tan(f*x+e))^(1/2))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(100) = 200$.
time = 0.52, size = 228, normalized size = 1.93

$$\frac{\frac{15a^2\left(-\frac{(2i-2)\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(2i-2)\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(i+1)\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{(i+1)\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)}{d^2}+\frac{4\left(30a^2d^2\tan(fx+e)^2-10ia^2d^2\tan(fx+e)-3a^2d^2\right)}{(d\tan(fx+e))^{\frac{5}{2}}d^2}}{30df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(d*tan(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] 1/30*(15*a^2*(-(2*I - 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sq
rt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - (2*I - 2)*sqrt(2)*arctan(-1/2*sqrt(2
)*(sqrt(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - (I + 1)*sqr
t(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d)
 + (I + 1)*sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d
) + d)/sqrt(d))/d^2 + 4*(30*a^2*d^2*tan(f*x + e)^2 - 10*I*a^2*d^2*tan(f*x +
 e) - 3*a^2*d^2)/((d*tan(f*x + e))^(5/2)*d^2))/(d*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 492 vs. $2(100) = 200$.
time = 0.39, size = 492, normalized size = 4.17

$$\frac{15\left(d^4 f e^{(6i fx+6ie)} - 3d^4 f e^{(4i fx+4ie)} + 3d^4 f e^{(2i fx+2ie)} - d^4 f\right)\sqrt{\frac{16i a^4}{d^7 f^2}}\log\left(\frac{\left(-4i a^2 d e^{(2i fx+2ie)} + \left(i d^4 f e^{(2i fx+2ie)} + i d^4 f\right)\sqrt{\frac{-i d e^{(2i fx+2ie)} + i d}{e^{(2i fx+2ie)}+1}}\sqrt{\frac{16i a^4}{d^7 f^2}}\right)e^{(-2i fx-2ie)}}{2a^2}\right) - 15\left(d^4 f e^{(6i fx+6ie)} - 3d^4 f e^{(4i fx+4ie)} + 3d^4 f e^{(2i fx+2ie)} - d^4 f\right)\sqrt{\frac{16i a^4}{d^7 f^2}}\log\left(\frac{\left(-4i a^2 d e^{(2i fx+2ie)} + \left(-i d^4 f e^{(2i fx+2ie)} - i d^4 f\right)\sqrt{\frac{-i d e^{(2i fx+2ie)} + i d}{e^{(2i fx+2ie)}+1}}\sqrt{\frac{16i a^4}{d^7 f^2}}\right)e^{(-2i fx-2ie)}}{2a^2}\right) - 8\left(-43i a^2 e^{(6i fx+6ie)} + 11i a^2 e^{(4i fx+4ie)} + 31i a^2 e^{(2i fx+2ie)} - 23i a^2\right)\sqrt{\frac{-i d e^{(2i fx+2ie)} + i d}{e^{(2i fx+2ie)}+1}}}{60\left(d^4 f e^{(6i fx+6ie)} - 3d^4 f e^{(4i fx+4ie)} + 3d^4 f e^{(2i fx+2ie)} - d^4 f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(d*tan(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/60*(15*(d^4*f*e^(6*I*f*x + 6*I*e) - 3*d^4*f*e^(4*I*f*x + 4*I*e) + 3*d^4*f
*e^(2*I*f*x + 2*I*e) - d^4*f)*sqrt(16*I*a^4/(d^7*f^2))*log(1/2*(-4*I*a^2*d*
e^(2*I*f*x + 2*I*e) + (I*d^4*f*e^(2*I*f*x + 2*I*e) + I*d^4*f)*sqrt((-I*d*e^
(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(16*I*a^4/(d^7*f^2)
))*e^(-2*I*f*x - 2*I*e)/a^2) - 15*(d^4*f*e^(6*I*f*x + 6*I*e) - 3*d^4*f*e^(4
*I*f*x + 4*I*e) + 3*d^4*f*e^(2*I*f*x + 2*I*e) - d^4*f)*sqrt(16*I*a^4/(d^7*f
^2))*log(1/2*(-4*I*a^2*d*e^(2*I*f*x + 2*I*e) + (-I*d^4*f*e^(2*I*f*x + 2*I*e
) - I*d^4*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1
))*sqrt(16*I*a^4/(d^7*f^2)))*e^(-2*I*f*x - 2*I*e)/a^2) - 8*(-43*I*a^2*e^(6*
I*f*x + 6*I*e) + 11*I*a^2*e^(4*I*f*x + 4*I*e) + 31*I*a^2*e^(2*I*f*x + 2*I*e
) - 23*I*a^2)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) +
1)))/(d^4*f*e^(6*I*f*x + 6*I*e) - 3*d^4*f*e^(4*I*f*x + 4*I*e) + 3*d^4*f*e^(
2*I*f*x + 2*I*e) - d^4*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int\left(-\frac{1}{(d\tan(e+fx))^{\frac{7}{2}}}\right)dx + \int\frac{\tan^2(e+fx)}{(d\tan(e+fx))^{\frac{7}{2}}}\,dx + \int\left(-\frac{2i\tan(e+fx)}{(d\tan(e+fx))^{\frac{7}{2}}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2/(d*tan(f*x+e))**(7/2),x)
```

```
[Out] -a**2*(Integral(-1/(d*tan(e + f*x))**(7/2), x) + Integral(tan(e + f*x)**2/(
d*tan(e + f*x))**(7/2), x) + Integral(-2*I*tan(e + f*x)/(d*tan(e + f*x))**(
7/2), x))
```

Giac [A]
time = 0.79, size = 139, normalized size = 1.18

$$-\frac{4i\sqrt{2}\,a^2\arctan\left(-\frac{8i\sqrt{d^2}\sqrt{d\tan(fx+e)}}{-4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{d^{\frac{7}{2}}f\left(-\frac{id}{\sqrt{d^2}}+1\right)}+\frac{2\left(30\,a^2d^2\tan(fx+e)^2-10i\,a^2d^2\tan(fx+e)-3\,a^2d^2\right)}{15\sqrt{d\tan(fx+e)}\,d^5f\tan(fx+e)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(d*tan(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] -4*I*sqrt(2)*a^2*arctan(-8*I*sqrt(d^2)*sqrt(d*tan(f*x + e))/(-4*I*sqrt(2)*d
^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(d^(7/2)*f*(-I*d/sqrt(d^2) + 1)) + 2
/15*(30*a^2*d^2*tan(f*x + e)^2 - 10*I*a^2*d^2*tan(f*x + e) - 3*a^2*d^2)/(sq
rt(d*tan(f*x + e))*d^5*f*tan(f*x + e)^2)
```

Mupad [B]
time = 4.66, size = 95, normalized size = 0.81

$$-\frac{\frac{2\,a^2}{5\,d\,f}-\frac{4\,a^2\tan(e+f\,x)^2}{d\,f}+\frac{a^2\tan(e+f\,x)\,4\mathrm{i}}{3\,d\,f}}{(d\tan(e+f\,x))^{5/2}}-\frac{2\sqrt{4\mathrm{i}}\;a^2\,\mathrm{atanh}\left(\frac{\sqrt{4\mathrm{i}}\;\sqrt{d\tan(e+f\,x)}}{2\sqrt{d}}\right)}{d^{7/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^2/(d*tan(e + f*x))^(7/2),x)
```

```
[Out] - ((2*a^2)/(5*d*f) + (a^2*tan(e + f*x)*4i)/(3*d*f) - (4*a^2*tan(e + f*x)^2)
/(d*f))/(d*tan(e + f*x))^(5/2) - (2*4i^(1/2)*a^2*atanh((4i^(1/2)*(d*tan(e +
 f*x))^(1/2))/(2*d^(1/2))))/(d^(7/2)*f)
```

3.156 $\int (d\tan(e+fx))^{5/2}(a+ia\tan(e+fx))^3\,dx$

Optimal. Leaf size=179

$$-\frac{8(-1)^{3/4}a^3d^{5/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}-\frac{8ia^3d^2\sqrt{d\tan(e+fx)}}{f}+\frac{8a^3d(d\tan(e+fx))^{3/2}}{3f}+\frac{8ia^3}{}$$

```
[Out] -8*(-1)^(3/4)*a^3*d^(5/2)*arctan((-1)^(3/4)*(d*tan(f*x+e))^(1/2)/d^(1/2))/f
-8*I*a^3*d^2*(d*tan(f*x+e))^(1/2)/f+8/3*a^3*d*(d*tan(f*x+e))^(3/2)/f+8/5*I*
a^3*(d*tan(f*x+e))^(5/2)/f-40/63*a^3*(d*tan(f*x+e))^(7/2)/d/f-2/9*(d*tan(f*
x+e))^(7/2)*(a^3+I*a^3*tan(f*x+e))/d/f
```

Rubi [A]
time = 0.21, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3637, 3673, 3609, 3614, 211}

$$-\frac{8(-1)^{3/4}a^3d^{5/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}-\frac{8ia^3d^2\sqrt{d\tan(e+fx)}}{f}-\frac{2(a^3+ia^3\tan(e+fx))(d\tan(e+fx))^{7/2}}{9df}-\frac{40a^3(d\tan(e+fx))^{7/2}}{63df}+\frac{8ia^3(d\tan(e+fx))^{5/2}}{5f}+\frac{8a^3d(d\tan(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(5/2)*(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] (-8*(-1)^(3/4)*a^3*d^(5/2)*ArcTan[((-1)^(3/4)*Sqrt[d*Tan[e + f*x]])/Sqrt[d]
])/f - ((8*I)*a^3*d^2*Sqrt[d*Tan[e + f*x]])/f + (8*a^3*d*(d*Tan[e + f*x])^(
3/2))/(3*f) + (((8*I)/5)*a^3*(d*Tan[e + f*x])^(5/2))/f - (40*a^3*(d*Tan[e +
 f*x])^(7/2))/(63*d*f) - (2*(d*Tan[e + f*x])^(7/2)*(a^3 + I*a^3*Tan[e + f*x
]))/(9*d*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
```

```
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (d\tan(e+fx))^{5/2}(a+ia\tan(e+fx))^3\,dx &= -\frac{2(d\tan(e+fx))^{7/2}\left(a^3+ia^3\tan(e+fx)\right)}{9df}+\frac{(2a)\int(d\tan\\
&= -\frac{40a^3(d\tan(e+fx))^{7/2}}{63df}-\frac{2(d\tan(e+fx))^{7/2}\left(a^3+ia^3\tan\right.}{9df}\\
&= \frac{8ia^3(d\tan(e+fx))^{5/2}}{5f}-\frac{40a^3(d\tan(e+fx))^{7/2}}{63df}-\frac{2(d\tan}{}\\
&= \frac{8a^3d(d\tan(e+fx))^{3/2}}{3f}+\frac{8ia^3(d\tan(e+fx))^{5/2}}{5f}-\frac{40a^3(d}{}\\
&= -\frac{8ia^3d^2\sqrt{d\tan(e+fx)}}{f}+\frac{8a^3d(d\tan(e+fx))^{3/2}}{3f}+\frac{8ia^3}{}\\
&= -\frac{8ia^3d^2\sqrt{d\tan(e+fx)}}{f}+\frac{8a^3d(d\tan(e+fx))^{3/2}}{3f}+\frac{8ia^3}{}\\
&= -\frac{8(-1)^{3/4}a^3d^{5/2}\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}-\frac{8ia^3d}{}
\end{aligned}$$

Mathematica [A]

time = 3.02, size = 150, normalized size = 0.84

$$\frac{a^3d^2\left(10080i\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\right)-i\sec^4(e+fx)(3633+4900\cos(2(e+fx))+1547\cos(4(e+fx))+570i\sin(2(e+fx))+555i\sin(4(e+fx)))\sqrt{i\tan(e+fx)}\right)\sqrt{d\tan(e+fx)}}{1260f\sqrt{i\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(5/2)*(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] (a^3*d^2*((10080*I)*ArcTanh[Sqrt[(-1 + E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(
e + f*x)))]] - I*Sec[e + f*x]^4*(3633 + 4900*Cos[2*(e + f*x)] + 1547*Cos[4*
(e + f*x)] + (570*I)*Sin[2*(e + f*x)] + (555*I)*Sin[4*(e + f*x)])*Sqrt[I*Ta
n[e + f*x]])*Sqrt[d*Tan[e + f*x]])/(1260*f*Sqrt[I*Tan[e + f*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 357 vs. 2(148) = 296.
time = 0.14, size = 358, normalized size = 2.00

method	result
derivativedivides	$2a^3\left(-\frac{i(d\tan(fx+e))^{\frac{9}{2}}}{9}-\frac{3d(d\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{4id^2(d\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{4d^3(d\tan(fx+e))^{\frac{3}{2}}}{3}-4id^4\sqrt{d\tan(fx+e)}+4d^5\left(\right.\right.$
default	$2a^3\left(-\frac{i(d\tan(fx+e))^{\frac{9}{2}}}{9}-\frac{3d(d\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{4id^2(d\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{4d^3(d\tan(fx+e))^{\frac{3}{2}}}{3}-4id^4\sqrt{d\tan(fx+e)}+4d^5\left(\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(5/2)*(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3/d^2*(-1/9*I*(d*tan(f*x+e))^(9/2)-3/7*d*(d*tan(f*x+e))^(7/2)+4/5*I*d
^2*(d*tan(f*x+e))^(5/2)+4/3*d^3*(d*tan(f*x+e))^(3/2)-4*I*d^4*(d*tan(f*x+e))
^(1/2)+4*d^5*(1/8*I/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*
tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x
+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e)
)^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))-1/8/(d^2)
^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(
d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1
/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2
)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))))
```

Maxima [A]
time = 0.53, size = 267, normalized size = 1.49

$$-\frac{315\,a^3d^4\left(-\frac{(2i-2)\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(2i-2)\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(i+1)\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{(i+1)\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)-\frac{2\left(-35i\,(d\tan(fx+e))^{\frac{9}{2}}a^3-135\,(d\tan(fx+e))^{\frac{7}{2}}a^3d+252i\,(d\tan(fx+e))^{\frac{5}{2}}a^3d^2+420\,(d\tan(fx+e))^{\frac{3}{2}}a^3d^3-1260i\sqrt{d\tan(fx+e)}\,a^3d^4\right)}{d}}{315\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)*(a+I*a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/315*(315*a^3*d^4*(-(2*I - 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d)
 + 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - (2*I - 2)*sqrt(2)*arctan(-1/2
*sqrt(2)*(sqrt(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - (I +
 1)*sqrt(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/
sqrt(d) + (I + 1)*sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))
*sqrt(d) + d)/sqrt(d)) - 2*(-35*I*(d*tan(f*x + e))^(9/2)*a^3 - 135*(d*tan(f
*x + e))^(7/2)*a^3*d + 252*I*(d*tan(f*x + e))^(5/2)*a^3*d^2 + 420*(d*tan(f*
x + e))^(3/2)*a^3*d^3 - 1260*I*sqrt(d*tan(f*x + e))*a^3*d^4)/d)/(d*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(152) = 304$.
time = 0.42, size = 517, normalized size = 2.89

$$-\frac{315\sqrt{\frac{64i\,a^6d^5}{f^2}}\left(fe^{(8ifx+8ie)}+4fe^{(6ifx+6ie)}+6fe^{(4ifx+4ie)}+4fe^{(2ifx+2ie)}+f\right)\log\left(\frac{\left(-8i\,a^3d^3e^{(2ifx+2ie)}+\sqrt{\frac{64i\,a^6d^5}{f^2}}\left(ife^{(2ifx+2ie)}+if\right)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\right)e^{(-2ifx-2ie)}}{4\,a^3d^2}\right)-315\sqrt{\frac{64i\,a^6d^5}{f^2}}\left(fe^{(8ifx+8ie)}+4fe^{(6ifx+6ie)}+6fe^{(4ifx+4ie)}+4fe^{(2ifx+2ie)}+f\right)\log\left(\frac{\left(-8i\,a^3d^3e^{(2ifx+2ie)}+\sqrt{\frac{64i\,a^6d^5}{f^2}}\left(-ife^{(2ifx+2ie)}-if\right)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\right)e^{(-2ifx-2ie)}}{4\,a^3d^2}\right)+16\left(1051i\,a^3d^2e^{(8ifx+8ie)}+2735i\,a^3d^2e^{(6ifx+6ie)}+3633i\,a^3d^2e^{(4ifx+4ie)}+2165i\,a^3d^2e^{(2ifx+2ie)}+496i\,a^3d^2\right)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}}{1260\left(fe^{(8ifx+8ie)}+4fe^{(6ifx+6ie)}+6fe^{(4ifx+4ie)}+4fe^{(2ifx+2ie)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)*(a+I*a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/1260*(315*sqrt(64*I*a^6*d^5/f^2)*(f*e^(8*I*f*x + 8*I*e) + 4*f*e^(6*I*f*x
 + 6*I*e) + 6*f*e^(4*I*f*x + 4*I*e) + 4*f*e^(2*I*f*x + 2*I*e) + f)*log(1/4*
(-8*I*a^3*d^3*e^(2*I*f*x + 2*I*e) + sqrt(64*I*a^6*d^5/f^2)*(I*f*e^(2*I*f*x
+ 2*I*e) + I*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e)
+ 1)))*e^(-2*I*f*x - 2*I*e)/(a^3*d^2)) - 315*sqrt(64*I*a^6*d^5/f^2)*(f*e^(8
*I*f*x + 8*I*e) + 4*f*e^(6*I*f*x + 6*I*e) + 6*f*e^(4*I*f*x + 4*I*e) + 4*f*e
^(2*I*f*x + 2*I*e) + f)*log(1/4*(-8*I*a^3*d^3*e^(2*I*f*x + 2*I*e) + sqrt(64
*I*a^6*d^5/f^2)*(-I*f*e^(2*I*f*x + 2*I*e) - I*f)*sqrt((-I*d*e^(2*I*f*x + 2*
I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-2*I*f*x - 2*I*e)/(a^3*d^2)) + 1
6*(1051*I*a^3*d^2*e^(8*I*f*x + 8*I*e) + 2735*I*a^3*d^2*e^(6*I*f*x + 6*I*e)
+ 3633*I*a^3*d^2*e^(4*I*f*x + 4*I*e) + 2165*I*a^3*d^2*e^(2*I*f*x + 2*I*e) +
 496*I*a^3*d^2)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e)
+ 1)))/(f*e^(8*I*f*x + 8*I*e) + 4*f*e^(6*I*f*x + 6*I*e) + 6*f*e^(4*I*f*x +
4*I*e) + 4*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia^3\left(\int i(d\tan(e+fx))^{\frac{5}{2}}\,dx+\int\left(-3(d\tan(e+fx))^{\frac{5}{2}}\tan(e+fx)\right)dx+\int(d\tan(e+fx))^{\frac{5}{2}}\tan^3(e+fx)\,dx+\int\left(-3i(d\tan(e+fx))^{\frac{5}{2}}\tan^2(e+fx)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(5/2)*(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] -I*a**3*(Integral(I*(d*tan(e + f*x))**(5/2), x) + Integral(-3*(d*tan(e + f*
x))**(5/2)*tan(e + f*x), x) + Integral((d*tan(e + f*x))**(5/2)*tan(e + f*x)
**3, x) + Integral(-3*I*(d*tan(e + f*x))**(5/2)*tan(e + f*x)**2, x))
```

Giac [A]
time = 0.87, size = 223, normalized size = 1.25

$$-\frac{8 \sqrt{2} a^3 d^{\frac{5}{2}} \arctan\left(\frac{8 \sqrt{d^2} \sqrt{d \tan(fx+e)}}{4i \sqrt{2} d^{\frac{3}{2}} + 4 \sqrt{2} \sqrt{d^2} \sqrt{d}}\right)}{f\left(\frac{id}{\sqrt{d^2}} + 1\right)} - \frac{2\left(35i \sqrt{d \tan(fx+e)} a^3 d^{20} f^8 \tan(fx+e)^4 + 135 \sqrt{d \tan(fx+e)} a^3 d^{20} f^8 \tan(fx+e)^3 - 252i \sqrt{d \tan(fx+e)} a^3 d^{20} f^8 \tan(fx+e)^2 - 420 \sqrt{d \tan(fx+e)} a^3 d^{20} f^8 \tan(fx+e) + 1260i \sqrt{d \tan(fx+e)} a^3 d^{20} f^8\right)}{315 d^{18} f^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)*(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -8*sqrt(2)*a^3*d^(5/2)*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*I*sqrt(2)
*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(f*(I*d/sqrt(d^2) + 1)) - 2/315*(3
5*I*sqrt(d*tan(f*x + e))*a^3*d^20*f^8*tan(f*x + e)^4 + 135*sqrt(d*tan(f*x +
 e))*a^3*d^20*f^8*tan(f*x + e)^3 - 252*I*sqrt(d*tan(f*x + e))*a^3*d^20*f^8*
tan(f*x + e)^2 - 420*sqrt(d*tan(f*x + e))*a^3*d^20*f^8*tan(f*x + e) + 1260*
I*sqrt(d*tan(f*x + e))*a^3*d^20*f^8)/(d^18*f^9)
```

Mupad [B]
time = 4.97, size = 137, normalized size = 0.77

$$\frac{a^3 (d \tan(e+fx))^{5/2} 8\mathrm{i}}{5f} - \frac{a^3 d^2 \sqrt{d \tan(e+fx)}\, 8\mathrm{i}}{f} - \frac{6 a^3 (d \tan(e+fx))^{7/2}}{7df} - \frac{a^3 (d \tan(e+fx))^{9/2} 2\mathrm{i}}{9 d^2 f} + \frac{8 a^3 d (d \tan(e+fx))^{3/2}}{3f} - \frac{\sqrt{16\mathrm{i}}\, a^3 d^{5/2} \operatorname{atan}\left(\frac{\sqrt{16\mathrm{i}} \sqrt{d \tan(e+fx)}\, 1\mathrm{i}}{4\sqrt{d}}\right) 2\mathrm{i}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(5/2)*(a + a*tan(e + f*x)*1i)^3,x)
```

```
[Out] (a^3*(d*tan(e + f*x))^(5/2)*8i)/(5*f) - (a^3*d^2*(d*tan(e + f*x))^(1/2)*8i)
/f - (6*a^3*(d*tan(e + f*x))^(7/2))/(7*d*f) - (a^3*(d*tan(e + f*x))^(9/2)*2
i)/(9*d^2*f) + (8*a^3*d*(d*tan(e + f*x))^(3/2))/(3*f) - (16i^(1/2)*a^3*d^(5
/2)*atan((16i^(1/2)*(d*tan(e + f*x))^(1/2)*1i)/(4*d^(1/2)))*2i)/f
```

3.157 $\int (d\tan(e+fx))^{3/2}(a+ia\tan(e+fx))^3\,dx$

Optimal. Leaf size=152

$$\frac{8\sqrt[4]{-1}\,a^3d^{3/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}+\frac{8a^3d\sqrt{d\tan(e+fx)}}{f}+\frac{8ia^3(d\tan(e+fx))^{3/2}}{3f}-\frac{32a^3(d\ta$$

```
[Out] 8*(-1)^(1/4)*a^3*d^(3/2)*arctan((-1)^(3/4)*(d*tan(f*x+e))^(1/2)/d^(1/2))/f+
8*a^3*d*(d*tan(f*x+e))^(1/2)/f+8/3*I*a^3*(d*tan(f*x+e))^(3/2)/f-32/35*a^3*(
d*tan(f*x+e))^(5/2)/d/f-2/7*(d*tan(f*x+e))^(5/2)*(a^3+I*a^3*tan(f*x+e))/d/f
```

Rubi [A]
time = 0.18, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3637, 3673, 3609, 3614, 211}

$$\frac{8\sqrt[4]{-1}\,a^3d^{3/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}-\frac{32a^3(d\tan(e+fx))^{5/2}}{35df}+\frac{8ia^3(d\tan(e+fx))^{3/2}}{3f}+\frac{8a^3d\sqrt{d\tan(e+fx)}}{f}-\frac{2(a^3+ia^3\tan(e+fx))(d\tan(e+fx))^{5/2}}{7df}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(3/2)*(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] (8*(-1)^(1/4)*a^3*d^(3/2)*ArcTan[((-1)^(3/4)*Sqrt[d*Tan[e + f*x]])/Sqrt[d]]
)/f + (8*a^3*d*Sqrt[d*Tan[e + f*x]])/f + (((8*I)/3)*a^3*(d*Tan[e + f*x])^(3
/2))/f - (32*a^3*(d*Tan[e + f*x])^(5/2))/(35*d*f) - (2*(d*Tan[e + f*x])^(5/
2)*(a^3 + I*a^3*Tan[e + f*x]))/(7*d*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (d\tan(e+fx))^{3/2}(a+ia\tan(e+fx))^3\,dx = -\frac{2(d\tan(e+fx))^{5/2}\left(a^3+ia^3\tan(e+fx)\right)}{7df}+\frac{(2a)\int(d\tan($$

$$= -\frac{32a^3(d\tan(e+fx))^{5/2}}{35df}-\frac{2(d\tan(e+fx))^{5/2}\left(a^3+ia^3\tan\right.}{7df}$$

$$= \frac{8ia^3(d\tan(e+fx))^{3/2}}{3f}-\frac{32a^3(d\tan(e+fx))^{5/2}}{35df}-\frac{2(d\tan(e}{}$$

$$= \frac{8a^3d\sqrt{d\tan(e+fx)}}{f}+\frac{8ia^3(d\tan(e+fx))^{3/2}}{3f}-\frac{32a^3(d\tan}{3}$$

$$= \frac{8a^3d\sqrt{d\tan(e+fx)}}{f}+\frac{8ia^3(d\tan(e+fx))^{3/2}}{3f}-\frac{32a^3(d\tan}{3}$$

$$= \frac{8\sqrt[4]{-1}\,a^3d^{3/2}\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}+\frac{8a^3d\sqrt{d\tan}}{}$$

Mathematica [A]
time = 2.47, size = 138, normalized size = 0.91

$$\frac{a^3d\left(-1680\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\right)+\sec^3(e+fx)(1197\cos(e+fx)+483\cos(3(e+fx))+95i\sin(e+fx)+155i\sin(3(e+fx)))\sqrt{i\tan(e+fx)}\right)\sqrt{d\tan(e+fx)}}{210f\sqrt{i\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(3/2)*(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] (a^3*d*(-1680*ArcTanh[Sqrt[(-1 + E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f*
x)))]] + Sec[e + f*x]^3*(1197*Cos[e + f*x] + 483*Cos[3*(e + f*x)] + (95*I)*
Sin[e + f*x] + (155*I)*Sin[3*(e + f*x)])*Sqrt[I*Tan[e + f*x]])*Sqrt[d*Tan[e
 + f*x]])/(210*f*Sqrt[I*Tan[e + f*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(126) = 252$.
time = 0.16, size = 342, normalized size = 2.25

method	result
derivativedivides	$\frac{2a^3\left(-\frac{i(d\tan(fx+e))^{\frac{7}{2}}}{7}-\frac{3d(d\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{4id^2(d\tan(fx+e))^{\frac{3}{2}}}{3}+4d^3\sqrt{d\tan(fx+e)}-4d^4\left(\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d}{d}\right.\right.}{}\right.\right.}{}$
default	$\frac{2a^3\left(-\frac{i(d\tan(fx+e))^{\frac{7}{2}}}{7}-\frac{3d(d\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{4id^2(d\tan(fx+e))^{\frac{3}{2}}}{3}+4d^3\sqrt{d\tan(fx+e)}-4d^4\left(\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d}{d}\right.\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(3/2)*(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3/d^2*(-1/7*I*(d*tan(f*x+e))^(7/2)-3/5*d*(d*tan(f*x+e))^(5/2)+4/3*I*d
^2*(d*tan(f*x+e))^(3/2)+4*d^3*(d*tan(f*x+e))^(1/2)-4*d^4*(1/8/d*(d^2)^(1/4)
*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(
1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+
2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2
)^(1/4)*(d*tan(f*x+e))^(1/2)+1))+1/8*I/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e
)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)
^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1
/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1
/2)+1))))
```

Maxima [A]
time = 0.56, size = 248, normalized size = 1.63

$$-\frac{105\,a^3d^3\left(\frac{(2i+2)\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{(2i+2)\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(i-1)\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{(i-1)\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)-\frac{2\left(-15i(d\tan(fx+e))^{\frac{7}{2}}a^3-63(d\tan(fx+e))^{\frac{5}{2}}a^3d+140i(d\tan(fx+e))^{\frac{3}{2}}a^3d^2+420\sqrt{d\tan(fx+e)}a^3d^3\right)}{d}}{105\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)*(a+I*a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/105*(105*a^3*d^3*((2*I + 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d)
+ 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) + (2*I + 2)*sqrt(2)*arctan(-1/2*
sqrt(2)*(sqrt(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - (I -
1)*sqrt(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/s
qrt(d) + (I - 1)*sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*
sqrt(d) + d)/sqrt(d)) - 2*(-15*I*(d*tan(f*x + e))^(7/2)*a^3 - 63*(d*tan(f*x
 + e))^(5/2)*a^3*d + 140*I*(d*tan(f*x + e))^(3/2)*a^3*d^2 + 420*sqrt(d*tan(
f*x + e))*a^3*d^3)/d)/(d*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(130) = 260$.
time = 0.38, size = 447, normalized size = 2.94

$$-\frac{105\,\sqrt{-\frac{64i\,a^6d^3}{f^2}}\left(fe^{(6i\,fx+6i\,e)}+3\,fe^{(4i\,fx+4i\,e)}+3\,fe^{(2i\,fx+2i\,e)}+f\right)\log\left(\frac{\left(-8i\,a^3d^2e^{(2i\,fx+2i\,e)}+\sqrt{-\frac{64i\,a^6d^3}{f^2}}\left(fe^{(2i\,fx+2i\,e)}+f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-2i\,fx-2i\,e)}}{4\,a^3d}\right)-105\,\sqrt{-\frac{64i\,a^6d^3}{f^2}}\left(fe^{(6i\,fx+6i\,e)}+3\,fe^{(4i\,fx+4i\,e)}+3\,fe^{(2i\,fx+2i\,e)}+f\right)\log\left(\frac{\left(-8i\,a^3d^2e^{(2i\,fx+2i\,e)}-\sqrt{-\frac{64i\,a^6d^3}{f^2}}\left(fe^{(2i\,fx+2i\,e)}+f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-2i\,fx-2i\,e)}}{4\,a^3d}\right)-16\left(319\,a^3de^{(6i\,fx+6i\,e)}+646\,a^3de^{(4i\,fx+4i\,e)}+551\,a^3de^{(2i\,fx+2i\,e)}+164\,a^3d\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}}{420\left(fe^{(6i\,fx+6i\,e)}+3\,fe^{(4i\,fx+4i\,e)}+3\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)*(a+I*a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/420*(105*sqrt(-64*I*a^6*d^3/f^2)*(f*e^(6*I*f*x + 6*I*e) + 3*f*e^(4*I*f*x
 + 4*I*e) + 3*f*e^(2*I*f*x + 2*I*e) + f)*log(1/4*(-8*I*a^3*d^2*e^(2*I*f*x +
 2*I*e) + sqrt(-64*I*a^6*d^3/f^2)*(f*e^(2*I*f*x + 2*I*e) + f)*sqrt((-I*d*e^
(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-2*I*f*x - 2*I*e)/(
a^3*d)) - 105*sqrt(-64*I*a^6*d^3/f^2)*(f*e^(6*I*f*x + 6*I*e) + 3*f*e^(4*I*f
*x + 4*I*e) + 3*f*e^(2*I*f*x + 2*I*e) + f)*log(1/4*(-8*I*a^3*d^2*e^(2*I*f*x
 + 2*I*e) - sqrt(-64*I*a^6*d^3/f^2)*(f*e^(2*I*f*x + 2*I*e) + f)*sqrt((-I*d*
e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-2*I*f*x - 2*I*e)
/(a^3*d)) - 16*(319*a^3*d*e^(6*I*f*x + 6*I*e) + 646*a^3*d*e^(4*I*f*x + 4*I*
e) + 551*a^3*d*e^(2*I*f*x + 2*I*e) + 164*a^3*d)*sqrt((-I*d*e^(2*I*f*x + 2*I
*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))/(f*e^(6*I*f*x + 6*I*e) + 3*f*e^(4*I*
f*x + 4*I*e) + 3*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia^3\left(\int i(d\tan(e+fx))^{\frac{3}{2}}\,dx+\int\left(-3(d\tan(e+fx))^{\frac{3}{2}}\tan(e+fx)\right)dx+\int(d\tan(e+fx))^{\frac{3}{2}}\tan^3(e+fx)\,dx+\int\left(-3i(d\tan(e+fx))^{\frac{3}{2}}\tan^2(e+fx)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(3/2)*(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] -I*a**3*(Integral(I*(d*tan(e + f*x))**(3/2), x) + Integral(-3*(d*tan(e + f*
x))**(3/2)*tan(e + f*x), x) + Integral((d*tan(e + f*x))**(3/2)*tan(e + f*x)
**3, x) + Integral(-3*I*(d*tan(e + f*x))**(3/2)*tan(e + f*x)**2, x))
```

Giac [A]
time = 0.77, size = 194, normalized size = 1.28

$$-\frac{2}{105}\left(\frac{420i\sqrt{2}\,a^3\sqrt{d}\,\arctan\left(\frac{8\sqrt{d^2}\,\sqrt{d\tan(fx+e)}}{4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\,\sqrt{d^2}\,\sqrt{d}}\right)}{f\left(\frac{id}{\sqrt{d^2}}+1\right)}+\frac{15i\sqrt{d\tan(fx+e)}\,a^3d^{21}f^6\tan(fx+e)^3+63\sqrt{d\tan(fx+e)}\,a^3d^{21}f^6\tan(fx+e)^2-140i\sqrt{d\tan(fx+e)}\,a^3d^{21}f^6\tan(fx+e)-420\sqrt{d\tan(fx+e)}\,a^3d^{21}f^6}{d^{21}f^7}\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)*(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/105*(420*I*sqrt(2)*a^3*sqrt(d)*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(
4*I*sqrt(2)*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(f*(I*d/sqrt(d^2) + 1))
 + (15*I*sqrt(d*tan(f*x + e))*a^3*d^21*f^6*tan(f*x + e)^3 + 63*sqrt(d*tan(f
*x + e))*a^3*d^21*f^6*tan(f*x + e)^2 - 140*I*sqrt(d*tan(f*x + e))*a^3*d^21*
f^6*tan(f*x + e) - 420*sqrt(d*tan(f*x + e))*a^3*d^21*f^6)/(d^21*f^7))*d
```

Mupad [B]
time = 4.77, size = 119, normalized size = 0.78

$$\frac{a^3\,{\left(d\,\mathrm{tan}\left(e+f\,x\right)\right)}^{3/2}\,8{}\mathrm{i}}{3\,f}-\frac{6\,a^3\,{\left(d\,\mathrm{tan}\left(e+f\,x\right)\right)}^{5/2}}{5\,d\,f}-\frac{a^3\,{\left(d\,\mathrm{tan}\left(e+f\,x\right)\right)}^{7/2}\,2{}\mathrm{i}}{7\,d^2\,f}+\frac{8\,a^3\,d\,\sqrt{d\,\mathrm{tan}\left(e+f\,x\right)}}{f}-\frac{\sqrt{16{}\mathrm{i}}\,a^3\,{\left(-d\right)}^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{16{}\mathrm{i}}\,\sqrt{d\,\mathrm{tan}\left(e+f\,x\right)}}{4\,\sqrt{-d}}\right)\,2{}\mathrm{i}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(3/2)*(a + a*tan(e + f*x)*1i)^3,x)
```

```
[Out] (a^3*(d*tan(e + f*x))^(3/2)*8i)/(3*f) - (6*a^3*(d*tan(e + f*x))^(5/2))/(5*d
*f) - (a^3*(d*tan(e + f*x))^(7/2)*2i)/(7*d^2*f) + (8*a^3*d*(d*tan(e + f*x))
^(1/2))/f - (16i^(1/2)*a^3*(-d)^(3/2)*atan((16i^(1/2)*(d*tan(e + f*x))^(1/2
))/(4*(-d)^(1/2)))*2i)/f
```

3.158 $\int \sqrt{d\tan(e+fx)}\,(a+ia\tan(e+fx))^3\,dx$

Optimal. Leaf size=129

$$\frac{8(-1)^{3/4}a^3\sqrt{d}\,\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}+\frac{8ia^3\sqrt{d\tan(e+fx)}}{f}-\frac{8a^3(d\tan(e+fx))^{3/2}}{5df}-\frac{2(d\tan(e$$

[Out] 8*(-1)^(3/4)*a^3*arctan((-1)^(3/4)*(d*tan(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f+8*I*a^3*(d*tan(f*x+e))^(1/2)/f-8/5*a^3*(d*tan(f*x+e))^(3/2)/d/f-2/5*(d*tan(f*x+e))^(3/2)*(a^3+I*a^3*tan(f*x+e))/d/f

Rubi [A]
time = 0.14, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3637, 3673, 3609, 3614, 211}

$$\frac{8(-1)^{3/4}a^3\sqrt{d}\,\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}-\frac{8a^3(d\tan(e+fx))^{3/2}}{5df}+\frac{8ia^3\sqrt{d\tan(e+fx)}}{f}-\frac{2(a^3+ia^3\tan(e+fx))\,(d\tan(e+fx))^{3/2}}{5df}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d*Tan[e + f*x]]*(a + I*a*Tan[e + f*x])^3,x]

[Out] (8*(-1)^(3/4)*a^3*Sqrt[d]*ArcTan[((-1)^(3/4)*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/f + ((8*I)*a^3*Sqrt[d*Tan[e + f*x]])/f - (8*a^3*(d*Tan[e + f*x])^(3/2))/(5*d*f) - (2*(d*Tan[e + f*x])^(3/2)*(a^3 + I*a^3*Tan[e + f*x]))/(5*d*f)

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \sqrt{d\tan(e+fx)}\,(a+ia\tan(e+fx))^3\,dx = -\frac{2(d\tan(e+fx))^{3/2}\,(a^3+ia^3\tan(e+fx))}{5df} + \frac{(2a)\int \sqrt{d\,\mathrm{ta}}$$

$$= -\frac{8a^3(d\tan(e+fx))^{3/2}}{5df} - \frac{2(d\tan(e+fx))^{3/2}\,(a^3+ia^3\tan}{5df}$$

$$= \frac{8ia^3\sqrt{d\tan(e+fx)}}{f} - \frac{8a^3(d\tan(e+fx))^{3/2}}{5df} - \frac{2(d\tan(e}{}$$

$$= \frac{8ia^3\sqrt{d\tan(e+fx)}}{f} - \frac{8a^3(d\tan(e+fx))^{3/2}}{5df} - \frac{2(d\tan(e}{}$$

$$= \frac{8(-1)^{3/4}a^3\sqrt{d}\,\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f} + \frac{8ia^3\sqrt{d}}{}$$

Mathematica [A]
time = 3.32, size = 122, normalized size = 0.95

$$\frac{ia^3\left(-40\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\right)+\sec^2(e+fx)(19+21\cos(2(e+fx))+5i\sin(2(e+fx)))\sqrt{i\tan(e+fx)}\right)\sqrt{d\tan(e+fx)}}{5f\sqrt{i\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Tan[e + f*x]]*(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] ((I/5)*a^3*(-40*ArcTanh[Sqrt[(-1 + E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e +
f*x)))]] + Sec[e + f*x]^2*(19 + 21*Cos[2*(e + f*x)] + (5*I)*Sin[2*(e + f*x)
])*Sqrt[I*Tan[e + f*x]])*Sqrt[d*Tan[e + f*x]])/(f*Sqrt[I*Tan[e + f*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(107) = 214$.
time = 0.16, size = 327, normalized size = 2.53

method	result
derivativedivides	$\frac{2a^3\left(-\frac{i(d\tan(fx+e))^{\frac{5}{2}}}{5}-d(d\tan(fx+e))^{\frac{3}{2}}+4id^2\sqrt{d\tan(fx+e)}-4d^3\left(\frac{i(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{}}\right.\right.}{}\right.\right.}{}$
default	$\frac{2a^3\left(-\frac{i(d\tan(fx+e))^{\frac{5}{2}}}{5}-d(d\tan(fx+e))^{\frac{3}{2}}+4id^2\sqrt{d\tan(fx+e)}-4d^3\left(\frac{i(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{}}\right.\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3/d^2*(-1/5*I*(d*tan(f*x+e))^(5/2)-d*(d*tan(f*x+e))^(3/2)+4*I*d^2*(d*
tan(f*x+e))^(1/2)-4*d^3*(1/8*I/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2
)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)
*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d
*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)
)-1/8/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2
)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/
2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arc
tan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(110) = 220$.
time = 0.58, size = 229, normalized size = 1.78

$$\frac{5a^3d^2\left(-\frac{(2i-2)\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(2i-2)\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(i+1)\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{(i+1)\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)+\frac{2\left(-i(d\tan(fx+e))^{\frac{5}{2}}a^3-5(d\tan(fx+e))^{\frac{3}{2}}a^3d+20i\sqrt{d\tan(fx+e)}a^3d^2\right)}{d}}{5df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/5*(5*a^3*d^2*(-(2*I - 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*
sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - (2*I - 2)*sqrt(2)*arctan(-1/2*sqrt
(2)*(sqrt(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - (I + 1)*s
qrt(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(
d) + (I + 1)*sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt
(d) + d)/sqrt(d)) + 2*(-I*(d*tan(f*x + e))^(5/2)*a^3 - 5*(d*tan(f*x + e))^(
3/2)*a^3*d + 20*I*sqrt(d*tan(f*x + e))*a^3*d^2)/d)/(d*f)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(110) = 220$.
time = 0.38, size = 376, normalized size = 2.91

$$\frac{5\sqrt{\frac{64i\,a^6d}{f^2}}\left(fe^{(4i\,fx+4i\,e)}+2\,fe^{(2i\,fx+2i\,e)}+f\right)\log\left(\frac{\left(-8i\,a^3de^{(2i\,fx+2i\,e)}+\sqrt{\frac{64i\,a^6d}{f^2}}\left(i\,fe^{(2i\,fx+2i\,e)}+i\,f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-2i\,fx-2i\,e)}}{4\,a^3}\right)-5\sqrt{\frac{64i\,a^6d}{f^2}}\left(fe^{(4i\,fx+4i\,e)}+2\,fe^{(2i\,fx+2i\,e)}+f\right)\log\left(\frac{\left(-8i\,a^3de^{(2i\,fx+2i\,e)}+\sqrt{\frac{64i\,a^6d}{f^2}}\left(-i\,fe^{(2i\,fx+2i\,e)}-i\,f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-2i\,fx-2i\,e)}}{4\,a^3}\right)-16\left(-13i\,a^3e^{(4i\,fx+4i\,e)}-19i\,a^3e^{(2i\,fx+2i\,e)}-8i\,a^3\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}}{20\left(fe^{(4i\,fx+4i\,e)}+2\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/20*(5*sqrt(64*I*a^6*d/f^2)*(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*
e) + f)*log(1/4*(-8*I*a^3*d*e^(2*I*f*x + 2*I*e) + sqrt(64*I*a^6*d/f^2)*(I*f
*e^(2*I*f*x + 2*I*e) + I*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f
*x + 2*I*e) + 1)))*e^(-2*I*f*x - 2*I*e)/a^3) - 5*sqrt(64*I*a^6*d/f^2)*(f*e^
(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*e) + f)*log(1/4*(-8*I*a^3*d*e^(2*I
*f*x + 2*I*e) + sqrt(64*I*a^6*d/f^2)*(-I*f*e^(2*I*f*x + 2*I*e) - I*f)*sqrt(
(-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-2*I*f*x -
2*I*e)/a^3) - 16*(-13*I*a^3*e^(4*I*f*x + 4*I*e) - 19*I*a^3*e^(2*I*f*x + 2*I
*e) - 8*I*a^3)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) +
 1)))/(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-ia^3\left(\int i\sqrt{d\tan(e+fx)}\,dx+\int\left(-3\sqrt{d\tan(e+fx)}\tan(e+fx)\right)dx+\int\sqrt{d\tan(e+fx)}\tan^3(e+fx)\,dx+\int\left(-3i\sqrt{d\tan(e+fx)}\tan^2(e+fx)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(1/2)*(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] -I*a**3*(Integral(I*sqrt(d*tan(e + f*x)), x) + Integral(-3*sqrt(d*tan(e + f
*x))*tan(e + f*x), x) + Integral(sqrt(d*tan(e + f*x))*tan(e + f*x)**3, x) +
 Integral(-3*I*sqrt(d*tan(e + f*x))*tan(e + f*x)**2, x))
```

Giac [**A**]
time = 0.66, size = 161, normalized size = 1.25

$$\frac{8\sqrt{2}\,a^3\sqrt{d}\,\arctan\left(\frac{8\sqrt{d^2}\sqrt{d\tan(fx+e)}}{4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{f\left(\frac{i\,d}{\sqrt{d^2}}+1\right)}-\frac{2\left(i\sqrt{d\tan(fx+e)}\,a^3d^{10}f^4\tan(fx+e)^2+5\sqrt{d\tan(fx+e)}\,a^3d^{10}f^4\tan(fx+e)-20i\sqrt{d\tan(fx+e)}\,a^3d^{10}f^4\right)}{5\,d^{10}f^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 8*sqrt(2)*a^3*sqrt(d)*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*I*sqrt(2)*
d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(f*(I*d/sqrt(d^2) + 1)) - 2/5*(I*sq
rt(d*tan(f*x + e))*a^3*d^10*f^4*tan(f*x + e)^2 + 5*sqrt(d*tan(f*x + e))*a^3
*d^10*f^4*tan(f*x + e) - 20*I*sqrt(d*tan(f*x + e))*a^3*d^10*f^4)/(d^10*f^5)
```

Mupad [B]
time = 4.44, size = 96, normalized size = 0.74

$$\frac{a^3\sqrt{d\tan(e+fx)}\,8\mathrm{i}}{f}-\frac{2a^3(d\tan(e+fx))^{3/2}}{df}-\frac{a^3(d\tan(e+fx))^{5/2}\,2\mathrm{i}}{5d^2f}+\frac{\sqrt{16\mathrm{i}}\,a^3\sqrt{d}\,\mathrm{atan}\left(\frac{\sqrt{16\mathrm{i}}\,\sqrt{d\tan(e+fx)}\,1\mathrm{i}}{4\sqrt{d}}\right)2\mathrm{i}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(1/2)*(a + a*tan(e + f*x)*1i)^3,x)
```

```
[Out] (a^3*(d*tan(e + f*x))^(1/2)*8i)/f - (2*a^3*(d*tan(e + f*x))^(3/2))/(d*f) -
(a^3*(d*tan(e + f*x))^(5/2)*2i)/(5*d^2*f) + (16i^(1/2)*a^3*d^(1/2)*atan((16
i^(1/2)*(d*tan(e + f*x))^(1/2)*1i)/(4*d^(1/2)))*2i)/f
```

3.159 $\int \frac{(a+ia\tan(e+fx))^3}{\sqrt{d\tan(e+fx)}}\,dx$

Optimal. Leaf size=107

$$-\frac{8\sqrt[4]{-1}\,a^3\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{d}\,f}-\frac{16a^3\sqrt{d\tan(e+fx)}}{3df}-\frac{2\sqrt{d\tan(e+fx)}\,(a^3+ia^3\tan(e+$$

```
[Out] -8*(-1)^(1/4)*a^3*arctan((-1)^(3/4)*(d*tan(f*x+e))^(1/2)/d^(1/2))/f/d^(1/2)
-16/3*a^3*(d*tan(f*x+e))^(1/2)/d/f-2/3*(d*tan(f*x+e))^(1/2)*(a^3+I*a^3*tan(
f*x+e))/d/f
```

Rubi [A]
time = 0.12, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3637, 3673, 3614, 211}

$$-\frac{8\sqrt[4]{-1}\,a^3\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{d}\,f}-\frac{16a^3\sqrt{d\tan(e+fx)}}{3df}-\frac{2(a^3+ia^3\tan(e+fx))\sqrt{d\tan(e+fx)}}{3df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^3/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (-8*(-1)^(1/4)*a^3*ArcTan[((-1)^(3/4)*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(Sqrt
[d]*f) - (16*a^3*Sqrt[d*Tan[e + f*x]])/(3*d*f) - (2*Sqrt[d*Tan[e + f*x]]*(a
^3 + I*a^3*Tan[e + f*x]))/(3*d*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
```

```
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^3}{\sqrt{d\tan(e+fx)}}\,dx = -\frac{2\sqrt{d\tan(e+fx)}\,(a^3+ia^3\tan(e+fx))}{3df} + \frac{(2a)\int \frac{(a+ia\tan(e+fx))(2ad+4iad\tan(e}{\sqrt{d\tan(e+fx)}}}{3d}$$

$$= -\frac{16a^3\sqrt{d\tan(e+fx)}}{3df} - \frac{2\sqrt{d\tan(e+fx)}\,(a^3+ia^3\tan(e+fx))}{3df} + \frac{(2a)\int}{}$$

$$= -\frac{16a^3\sqrt{d\tan(e+fx)}}{3df} - \frac{2\sqrt{d\tan(e+fx)}\,(a^3+ia^3\tan(e+fx))}{3df} + \frac{(48a^5d}{}$$

$$= -\frac{8\sqrt[4]{-1}\,a^3\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{d}\,f} - \frac{16a^3\sqrt{d\tan(e+fx)}}{3df} - \frac{2\sqrt{d}}{}$$

Mathematica [A]
time = 2.59, size = 154, normalized size = 1.44

$$-\frac{2a^3e^{-3i(e+fx)}(\cos(3(e+fx))+i\sin(3(e+fx)))\left(-12\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\right)+(9+i\tan(e+fx))\sqrt{i\tan(e+fx)}\right)\sqrt{d\tan(e+fx)}}{3d\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (-2*a^3*(Cos[3*(e + f*x)] + I*Sin[3*(e + f*x)])*(-12*ArcTanh[Sqrt[(-1 + E^(
(2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f*x)))]] + (9 + I*Tan[e + f*x])*Sqrt[I
*Tan[e + f*x]])*Sqrt[d*Tan[e + f*x]])/(3*d*E^((3*I)*(e + f*x))*Sqrt[(-1 + E
^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f*x)))]*f)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(88) = 176.
time = 0.12, size = 311, normalized size = 2.91

method	result
derivativedivides	$\frac{2a^3\left(-\frac{i(d\tan(fx+e))^{\frac{3}{2}}}{3}-3d\sqrt{d\tan(fx+e)}+4d^2\left(\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}}\right.\right.}{}\right.\right.}{}$
default	$\frac{2a^3\left(-\frac{i(d\tan(fx+e))^{\frac{3}{2}}}{3}-3d\sqrt{d\tan(fx+e)}+4d^2\left(\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}}\right.\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3/(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3/d^2*(-1/3*I*(d*tan(f*x+e))^(3/2)-3*d*(d*tan(f*x+e))^(1/2)+4*d^2*(1/
8/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*
2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)
+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arcta
n(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))+1/8*I/(d^2)^(1/4)*2^(1/2)*(
ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*t
an(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2
^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d
*tan(f*x+e))^(1/2)+1))))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(91) = 182.
time = 0.56, size = 208, normalized size = 1.94

$$\frac{3a^3d\left(\frac{(2i+2)\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{(2i+2)\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(i-1)\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{(i-1)\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)+\frac{2\left(-i(d\tan(fx+e))^{\frac{3}{2}}a^3-9\sqrt{d\tan(fx+e)}a^3d\right)}{d}}{3df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*a^3*d*((2*I + 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqr
t(d*tan(f*x + e)))/sqrt(d))/sqrt(d) + (2*I + 2)*sqrt(2)*arctan(-1/2*sqrt(2)
*(sqrt(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - (I - 1)*sqrt
(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d)
+ (I - 1)*sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d)
```

```
 + d)/sqrt(d)) + 2*(-I*(d*tan(f*x + e))^(3/2)*a^3 - 9*sqrt(d*tan(f*x + e))*
a^3*d)/d)/(d*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(91) = 182$.
time = 0.37, size = 340, normalized size = 3.18

$$\frac{3\sqrt{-\frac{64i\,a^6}{df^2}}\left(dfe^{(2i\,fx+2i\,e)}+df\right)\log\left(\frac{\left(-8i\,a^3de^{(2i\,fx+2i\,e)}+\sqrt{-\frac{64i\,a^6}{df^2}}\left(dfe^{(2i\,fx+2i\,e)}+df\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-2i\,fx-2i\,e)}}{4\,a^3}\right)-3\sqrt{-\frac{64i\,a^6}{df^2}}\left(dfe^{(2i\,fx+2i\,e)}+df\right)\log\left(\frac{\left(-8i\,a^3de^{(2i\,fx+2i\,e)}-\sqrt{-\frac{64i\,a^6}{df^2}}\left(dfe^{(2i\,fx+2i\,e)}+df\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-2i\,fx-2i\,e)}}{4\,a^3}\right)-16\left(5\,a^3e^{(2i\,fx+2i\,e)}+4\,a^3\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}}{12\left(dfe^{(2i\,fx+2i\,e)}+df\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*sqrt(-64*I*a^6/(d*f^2))*(d*f*e^(2*I*f*x + 2*I*e) + d*f)*log(1/4*(-8
*I*a^3*d*e^(2*I*f*x + 2*I*e) + sqrt(-64*I*a^6/(d*f^2))*(d*f*e^(2*I*f*x + 2*
I*e) + d*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)
))*e^(-2*I*f*x - 2*I*e)/a^3) - 3*sqrt(-64*I*a^6/(d*f^2))*(d*f*e^(2*I*f*x +
2*I*e) + d*f)*log(1/4*(-8*I*a^3*d*e^(2*I*f*x + 2*I*e) - sqrt(-64*I*a^6/(d*f
^2))*(d*f*e^(2*I*f*x + 2*I*e) + d*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/
(e^(2*I*f*x + 2*I*e) + 1)))*e^(-2*I*f*x - 2*I*e)/a^3) - 16*(5*a^3*e^(2*I*f*
x + 2*I*e) + 4*a^3)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I
*e) + 1)))/(d*f*e^(2*I*f*x + 2*I*e) + d*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia^3\left(\int\frac{i}{\sqrt{d\tan\left(e+fx\right)}}\,dx+\int\left(-\frac{3\tan\left(e+fx\right)}{\sqrt{d\tan\left(e+fx\right)}}\right)dx+\int\frac{\tan^3\left(e+fx\right)}{\sqrt{d\tan\left(e+fx\right)}}\,dx+\int\left(-\frac{3i\tan^2\left(e+fx\right)}{\sqrt{d\tan\left(e+fx\right)}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3/(d*tan(f*x+e))**(1/2),x)
```

```
[Out] -I*a**3*(Integral(I/sqrt(d*tan(e + f*x)), x) + Integral(-3*tan(e + f*x)/sqr
t(d*tan(e + f*x)), x) + Integral(tan(e + f*x)**3/sqrt(d*tan(e + f*x)), x) +
 Integral(-3*I*tan(e + f*x)**2/sqrt(d*tan(e + f*x)), x))
```

Giac [A]
time = 0.62, size = 130, normalized size = 1.21

$$\frac{8i\,\sqrt{2}\,a^3\arctan\left(\frac{8\,\sqrt{d^2}\,\sqrt{d\tan\left(fx+e\right)}}{4i\,\sqrt{2}\,d^{\frac{3}{2}}+4\,\sqrt{2}\,\sqrt{d^2}\,\sqrt{d}}\right)}{\sqrt{d}\,f\left(\frac{i\,d}{\sqrt{d^2}}+1\right)}-\frac{2\left(i\,\sqrt{d\tan\left(fx+e\right)}\,a^3d^5f^2\tan\left(fx+e\right)+9\,\sqrt{d\tan\left(fx+e\right)}\,a^3d^5f^2\right)}{3\,d^6f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 8*I*sqrt(2)*a^3*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*I*sqrt(2)*d^(3/2
) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(sqrt(d)*f*(I*d/sqrt(d^2) + 1)) - 2/3*(I*
sqrt(d*tan(f*x + e))*a^3*d^5*f^2*tan(f*x + e) + 9*sqrt(d*tan(f*x + e))*a^3*
d^5*f^2)/(d^6*f^3)
```

Mupad [B]
time = 4.38, size = 81, normalized size = 0.76

$$-\frac{6\,a^3\,\sqrt{d\tan\left(e+f\,x\right)}}{d\,f}-\frac{a^3\,{\left(d\tan(e+f\,x)\right)}^{3/2}\,2{}\mathrm{i}}{3\,d^2\,f}+\frac{\sqrt{16{}\mathrm{i}}\,a^3\,\mathrm{atan}\left(\frac{\sqrt{16{}\mathrm{i}}\,\sqrt{d\tan\left(e+f\,x\right)}}{4\,\sqrt{-d}}\right)\,2{}\mathrm{i}}{\sqrt{-d}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^3/(d*tan(e + f*x))^(1/2),x)
```

```
[Out] (16i^(1/2)*a^3*atan((16i^(1/2)*(d*tan(e + f*x))^(1/2))/(4*(-d)^(1/2)))*2i)/
((-d)^(1/2)*f) - (a^3*(d*tan(e + f*x))^(3/2)*2i)/(3*d^2*f) - (6*a^3*(d*tan(
e + f*x))^(1/2))/(d*f)
```

3.160 $\int \frac{(a+ia\tan(e+fx))^3}{(d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=80

$$-\frac{8(-1)^{3/4}a^3\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{3/2}f}-\frac{2(a^3+ia^3\tan(e+fx))}{df\sqrt{d\tan(e+fx)}}$$

```
[Out] -8*(-1)^(3/4)*a^3*arctan((-1)^(3/4)*(d*tan(f*x+e))^(1/2)/d^(1/2))/d^(3/2)/f
-2*(a^3+I*a^3*tan(f*x+e))/d/f/(d*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3634, 12, 3614, 211}

$$-\frac{8(-1)^{3/4}a^3\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{3/2}f}-\frac{2(a^3+ia^3\tan(e+fx))}{df\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^3/(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-8*(-1)^(3/4)*a^3*ArcTan[((-1)^(3/4)*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(d^(3
/2)*f) - (2*(a^3 + I*a^3*Tan[e + f*x]))/(d*f*Sqrt[d*Tan[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3634

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-a^2)*(b*c - a*d)*(a + b*Tan[e + f*x]
```

```
)^(m - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] + Di
st[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[b*(b*c*(m - 2) - a*d*(m - 2*n - 4)) + (a*b*c*(m - 2)
+ b^2*d*(n + 1) - a^2*d*(m + n - 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+ia\tan(e+fx))^3}{(d\tan(e+fx))^{3/2}}\,dx &= -\frac{2(a^3+ia^3\tan(e+fx))}{df\sqrt{d\tan(e+fx)}}-\frac{2\int -\frac{2ia^2d(a+ia\tan(e+fx))}{\sqrt{d\tan(e+fx)}}\,dx}{d^2}\\
&= -\frac{2(a^3+ia^3\tan(e+fx))}{df\sqrt{d\tan(e+fx)}}+\frac{(4ia^2)\int \frac{a+ia\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx}{d}\\
&= -\frac{2(a^3+ia^3\tan(e+fx))}{df\sqrt{d\tan(e+fx)}}+\frac{(8ia^4)\,\mathrm{Subst}\left(\int \frac{1}{ad-iax^2}\,dx,x,\sqrt{d\tan(e+fx)}\right)}{df}\\
&= -\frac{8(-1)^{3/4}a^3\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{3/2}f}-\frac{2(a^3+ia^3\tan(e+fx))}{df\sqrt{d\tan(e+fx)}}
\end{aligned}$$

Mathematica [A]
time = 1.69, size = 156, normalized size = 1.95

$$\frac{2a^3e^{-3i(e+fx)}(-i\cos(3(e+fx))+\sin(3(e+fx)))\left(-4\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\right)\tan(e+fx)+\sqrt{i\tan(e+fx)}\,(-i+\tan(e+fx))\right)}{d\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\,f\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3/(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*a^3*((-I)*Cos[3*(e + f*x)] + Sin[3*(e + f*x)])*(-4*ArcTanh[Sqrt[(-1 + E^
((2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f*x)))]]*Tan[e + f*x] + Sqrt[I*Tan[e
+ f*x]]*(-I + Tan[e + f*x])))/(d*E^((3*I)*(e + f*x))*Sqrt[(-1 + E^((2*I)*(e
 + f*x)))/(1 + E^((2*I)*(e + f*x)))]*f*Sqrt[d*Tan[e + f*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(67) = 134.
time = 0.11, size = 309, normalized size = 3.86

method	result
derivativedivides	$\frac{2a^3\left(-i\sqrt{d\tan(fx+e)}+4d\left(\frac{i(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\,\mathrm{ar}\right.}{}\right.\right.}{}$
default	$\frac{2a^3\left(-i\sqrt{d\tan(fx+e)}+4d\left(\frac{i(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\,\mathrm{ar}\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3/(d*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3/d^2*(-I*(d*tan(f*x+e))^(1/2)+4*d*(1/8*I/d*(d^2)^(1/4)*2^(1/2)*(ln((
d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f
*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/
2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan
(f*x+e))^(1/2)+1))-1/8/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d
*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*
x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e
))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)))-d/(d*ta
n(f*x+e))^(1/2))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(71) = 142$.
time = 0.55, size = 202, normalized size = 2.52

$$-\frac{a^3\left(-\frac{(2i-2)\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(2i-2)\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(i+1)\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{(i+1)\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)+\frac{2a^3}{\sqrt{d\tan(fx+e)}}+\frac{2i\sqrt{d\tan(fx+e)}a^3}{d}}{df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] -(a^3*(-(2*I - 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*ta
n(f*x + e)))/sqrt(d))/sqrt(d) - (2*I - 2)*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt
(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - (I + 1)*sqrt(2)*lo
g(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) + (I +
 1)*sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/
sqrt(d)) + 2*a^3/sqrt(d*tan(f*x + e)) + 2*I*sqrt(d*tan(f*x + e))*a^3/d)/(d*
f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 357 vs. 2(71) = 142.
time = 0.36, size = 357, normalized size = 4.46

$$\frac{-16 i a^3 \sqrt{\frac{-i d e^{(2 i f x+2 i e)}+i d}{e^{(2 i f x+2 i e)}+1}}\, e^{(2 i f x+2 i e)} - \left(d^2 f e^{(2 i f x+2 i e)} - d^2 f\right)\sqrt{\frac{64 i a^6}{d^3 f^2}} \log\left(\frac{\left(-8 i a^3 d e^{(2 i f x+2 i e)} + \left(i d^2 f e^{(2 i f x+2 i e)} + i d^2 f\right)\sqrt{\frac{64 i a^6}{d^3 f^2}} \sqrt{\frac{-i d e^{(2 i f x+2 i e)}+i d}{e^{(2 i f x+2 i e)}+1}}\right) e^{(-2 i f x-2 i e)}}{4 a^3}\right) + \left(d^2 f e^{(2 i f x+2 i e)} - d^2 f\right)\sqrt{\frac{64 i a^6}{d^3 f^2}} \log\left(\frac{\left(-8 i a^3 d e^{(2 i f x+2 i e)} + \left(-i d^2 f e^{(2 i f x+2 i e)} - i d^2 f\right)\sqrt{\frac{64 i a^6}{d^3 f^2}} \sqrt{\frac{-i d e^{(2 i f x+2 i e)}+i d}{e^{(2 i f x+2 i e)}+1}}\right) e^{(-2 i f x-2 i e)}}{4 a^3}\right)}{4 \left(d^2 f e^{(2 i f x+2 i e)} - d^2 f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(-16*I*a^3*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) +
 1))*e^(2*I*f*x + 2*I*e) - (d^2*f*e^(2*I*f*x + 2*I*e) - d^2*f)*sqrt(64*I*a^
6/(d^3*f^2))*log(1/4*(-8*I*a^3*d*e^(2*I*f*x + 2*I*e) + (I*d^2*f*e^(2*I*f*x
+ 2*I*e) + I*d^2*f)*sqrt(64*I*a^6/(d^3*f^2))*sqrt((-I*d*e^(2*I*f*x + 2*I*e)
 + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-2*I*f*x - 2*I*e)/a^3) + (d^2*f*e^(2
*I*f*x + 2*I*e) - d^2*f)*sqrt(64*I*a^6/(d^3*f^2))*log(1/4*(-8*I*a^3*d*e^(2*
I*f*x + 2*I*e) + (-I*d^2*f*e^(2*I*f*x + 2*I*e) - I*d^2*f)*sqrt(64*I*a^6/(d^
3*f^2))*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e
^(-2*I*f*x - 2*I*e)/a^3))/(d^2*f*e^(2*I*f*x + 2*I*e) - d^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia^3\left(\int \frac{i}{(d\tan(e+fx))^{\frac{3}{2}}}\,dx + \int \left(-\frac{3\tan(e+fx)}{(d\tan(e+fx))^{\frac{3}{2}}}\right)dx + \int \frac{\tan^3(e+fx)}{(d\tan(e+fx))^{\frac{3}{2}}}\,dx + \int \left(-\frac{3i\tan^2(e+fx)}{(d\tan(e+fx))^{\frac{3}{2}}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3/(d*tan(f*x+e))**(3/2),x)
```

```
[Out] -I*a**3*(Integral(I/(d*tan(e + f*x))**(3/2), x) + Integral(-3*tan(e + f*x)/
(d*tan(e + f*x))**(3/2), x) + Integral(tan(e + f*x)**3/(d*tan(e + f*x))**(3
/2), x) + Integral(-3*I*tan(e + f*x)**2/(d*tan(e + f*x))**(3/2), x))
```

Giac [A]
time = 0.72, size = 115, normalized size = 1.44

$$-\frac{2\left(-\frac{4i\sqrt{2}\,a^3\arctan\left(-\frac{8i\sqrt{d^2}\sqrt{d\tan(fx+e)}}{-4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{\sqrt{d}\,f\left(-\frac{id}{\sqrt{d^2}}+1\right)} + \frac{a^3}{\sqrt{d\tan(fx+e)}\,f} + \frac{i\sqrt{d\tan(fx+e)}\,a^3}{df}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] -2*(-4*I*sqrt(2)*a^3*arctan(-8*I*sqrt(d^2)*sqrt(d*tan(f*x + e))/(-4*I*sqrt(
2)*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(sqrt(d)*f*(-I*d/sqrt(d^2) + 1))
 + a^3/(sqrt(d*tan(f*x + e))*f) + I*sqrt(d*tan(f*x + e))*a^3/(d*f))/d
```

Mupad [B]
time = 4.33, size = 77, normalized size = 0.96

$$-\frac{2\,a^3}{d\,f\,\sqrt{d\tan\left(e+f\,x\right)}}-\frac{a^3\,\sqrt{d\tan\left(e+f\,x\right)}\,2\mathrm{i}}{d^2\,f}+\frac{2\,\sqrt{16\mathrm{i}}\,a^3\,\mathrm{atanh}\left(\frac{\sqrt{16\mathrm{i}}\,\sqrt{d\tan\left(e+f\,x\right)}}{4\,\sqrt{d}}\right)}{d^{3/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^3/(d*tan(e + f*x))^(3/2),x)
```

```
[Out] (2*16i^(1/2)*a^3*atanh((16i^(1/2)*(d*tan(e + f*x))^(1/2))/(4*d^(1/2))))/(d^
(3/2)*f) - (a^3*(d*tan(e + f*x))^(1/2)*2i)/(d^2*f) - (2*a^3)/(d*f*(d*tan(e
+ f*x))^(1/2))
```

3.161 $\int \frac{(a+ia\tan(e+fx))^3}{(d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=109

$$\frac{8\sqrt[4]{-1}\,a^3\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{5/2}f}-\frac{16ia^3}{3d^2f\sqrt{d\tan(e+fx)}}-\frac{2(a^3+ia^3\tan(e+fx))}{3df(d\tan(e+fx))^{3/2}}$$

```
[Out] 8*(-1)^(1/4)*a^3*arctan((-1)^(3/4)*(d*tan(f*x+e))^(1/2)/d^(1/2))/d^(5/2)/f-
16/3*I*a^3/d^2/f/(d*tan(f*x+e))^(1/2)-2/3*(a^3+I*a^3*tan(f*x+e))/d/f/(d*tan
(f*x+e))^(3/2)
```

Rubi [A]
time = 0.14, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3634, 3672, 3614, 211}

$$\frac{8\sqrt[4]{-1}\,a^3\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{5/2}f}-\frac{16ia^3}{3d^2f\sqrt{d\tan(e+fx)}}-\frac{2(a^3+ia^3\tan(e+fx))}{3df(d\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^3/(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (8*(-1)^(1/4)*a^3*ArcTan[((-1)^(3/4)*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(d^(5/
2)*f) - (((16*I)/3)*a^3)/(d^2*f*Sqrt[d*Tan[e + f*x]]) - (2*(a^3 + I*a^3*Tan
[e + f*x]))/(3*d*f*(d*Tan[e + f*x])^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3634

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-a^2)*(b*c - a*d)*(a + b*Tan[e + f*x]
)^(m - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] + Di
st[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e
```

```
 + f*x])^(n + 1)*Simp[b*(b*c*(m - 2) - a*d*(m - 2*n - 4)) + (a*b*c*(m - 2)
+ b^2*d*(n + 1) - a^2*d*(m + n - 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3672

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
 b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
 -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^3}{(d\tan(e+fx))^{5/2}}\,dx = -\frac{2(a^3+ia^3\tan(e+fx))}{3df(d\tan(e+fx))^{3/2}} - \frac{2\int \frac{(a+ia\tan(e+fx))(-4ia^2d+2a^2d\tan(e+fx))}{(d\tan(e+fx))^{3/2}}\,dx}{3d^2}$$

$$= -\frac{16ia^3}{3d^2f\sqrt{d\tan(e+fx)}} - \frac{2(a^3+ia^3\tan(e+fx))}{3df(d\tan(e+fx))^{3/2}} - \frac{2\int \frac{6a^3d^2+6ia^3d^2\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,d}{3d^4}$$

$$= -\frac{16ia^3}{3d^2f\sqrt{d\tan(e+fx)}} - \frac{2(a^3+ia^3\tan(e+fx))}{3df(d\tan(e+fx))^{3/2}} - \frac{(48a^6)\,\mathrm{Subst}\left(\int \frac{1}{6a^3d^3-6i}\right.}{}$$

$$= \frac{8\sqrt[4]{-1}\,a^3\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{5/2}f} - \frac{16ia^3}{3d^2f\sqrt{d\tan(e+fx)}} - \frac{2(a^3+}{3df(d}$$

Mathematica [A]
time = 2.58, size = 87, normalized size = 0.80

$$-\frac{2a^3\left(9i+\cot(e+fx)-12i\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\right)\sqrt{i\tan(e+fx)}\right)}{3d^2f\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3/(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (-2*a^3*(9*I + Cot[e + f*x] - (12*I)*ArcTanh[Sqrt[(-1 + E^((2*I)*(e + f*x))
)/(1 + E^((2*I)*(e + f*x)))]]*Sqrt[I*Tan[e + f*x]]))/(3*d^2*f*Sqrt[d*Tan[e
+ f*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(89) = 178$.
time = 0.11, size = 305, normalized size = 2.80

method	result
derivativedivides	$\frac{2a^3\left(-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx-\cdots}}{(d^2)^{\frac{1}{4}}}\cdots\right.\right.}{2d}\cdots\right.}{\cdots}$
default	$\frac{2a^3\left(-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx-\cdots}}{(d^2)^{\frac{1}{4}}}\cdots\right.\right.}{2d}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3/(d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3/d^2*(-1/2/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*ta
n(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e
))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^
(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))-1/2*I/(d^2)
^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(
d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1
/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2
)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))-3*I/(d*tan(f*x+e))^(1/2)-1/3*d/(d*ta
n(f*x+e))^(3/2))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(93) = 186$.
time = 0.53, size = 206, normalized size = 1.89

$$-\frac{\frac{3a^3\left(\frac{(2i+2)\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{(2i+2)\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(i-1)\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{(i-1)\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)}{d}+\frac{2\left(9ia^3d\tan(fx+e)+a^3d\right)}{(d\tan(fx+e))^{\frac{3}{2}}d}}{3df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*a^3*((2*I + 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt
(d*tan(f*x + e)))/sqrt(d))/sqrt(d) + (2*I + 2)*sqrt(2)*arctan(-1/2*sqrt(2)*
(sqrt(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - (I - 1)*sqrt(
2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) +
 (I - 1)*sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d)
+ d)/sqrt(d))/d + 2*(9*I*a^3*d*tan(f*x + e) + a^3*d)/((d*tan(f*x + e))^(3/2
)*d))/(d*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(93) = 186$.
time = 0.38, size = 422, normalized size = 3.87

$$-\frac{3\left(d^{3}fe^{(4ifx+4ie)}-2\,d^{3}fe^{(2ifx+2ie)}+d^{3}f\right)\sqrt{-\frac{64i\,a^{6}}{d^{5}f^{2}}}\log\left(\frac{\left(-8i\,a^{3}de^{(2ifx+2ie)}+\left(d^{3}fe^{(2ifx+2ie)}+d^{3}f\right)\sqrt{\frac{-i\,de^{(2ifx+2ie)}+i\,d}{e^{(2ifx+2ie)}+1}}\sqrt{-\frac{64i\,a^{6}}{d^{5}f^{2}}}\right)e^{(-2ifx-2ie)}}{4\,a^{3}}\right)-3\left(d^{3}fe^{(4ifx+4ie)}-2\,d^{3}fe^{(2ifx+2ie)}+d^{3}f\right)\sqrt{-\frac{64i\,a^{6}}{d^{5}f^{2}}}\log\left(\frac{\left(-8i\,a^{3}de^{(2ifx+2ie)}-\left(d^{3}fe^{(2ifx+2ie)}+d^{3}f\right)\sqrt{\frac{-i\,de^{(2ifx+2ie)}+i\,d}{e^{(2ifx+2ie)}+1}}\sqrt{-\frac{64i\,a^{6}}{d^{5}f^{2}}}\right)e^{(-2ifx-2ie)}}{4\,a^{3}}\right)-16\left(5\,a^{3}e^{(4ifx+4ie)}+a^{3}e^{(2ifx+2ie)}-4\,a^{3}\right)\sqrt{\frac{-i\,de^{(2ifx+2ie)}+i\,d}{e^{(2ifx+2ie)}+1}}}{12\left(d^{3}fe^{(4ifx+4ie)}-2\,d^{3}fe^{(2ifx+2ie)}+d^{3}f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(3*(d^3*f*e^(4*I*f*x + 4*I*e) - 2*d^3*f*e^(2*I*f*x + 2*I*e) + d^3*f)*
sqrt(-64*I*a^6/(d^5*f^2))*log(1/4*(-8*I*a^3*d*e^(2*I*f*x + 2*I*e) + (d^3*f*
e^(2*I*f*x + 2*I*e) + d^3*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*
f*x + 2*I*e) + 1))*sqrt(-64*I*a^6/(d^5*f^2)))*e^(-2*I*f*x - 2*I*e)/a^3) - 3
*(d^3*f*e^(4*I*f*x + 4*I*e) - 2*d^3*f*e^(2*I*f*x + 2*I*e) + d^3*f)*sqrt(-64
*I*a^6/(d^5*f^2))*log(1/4*(-8*I*a^3*d*e^(2*I*f*x + 2*I*e) - (d^3*f*e^(2*I*f
*x + 2*I*e) + d^3*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*
I*e) + 1))*sqrt(-64*I*a^6/(d^5*f^2)))*e^(-2*I*f*x - 2*I*e)/a^3) - 16*(5*a^3
*e^(4*I*f*x + 4*I*e) + a^3*e^(2*I*f*x + 2*I*e) - 4*a^3)*sqrt((-I*d*e^(2*I*f
*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))/(d^3*f*e^(4*I*f*x + 4*I*e) -
 2*d^3*f*e^(2*I*f*x + 2*I*e) + d^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia^{3}\left(\int\frac{i}{\left(d\tan\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx+\int\left(-\frac{3\tan\left(e+fx\right)}{\left(d\tan\left(e+fx\right)\right)^{\frac{5}{2}}}\right)dx+\int\frac{\tan^{3}\left(e+fx\right)}{\left(d\tan\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx+\int\left(-\frac{3i\tan^{2}\left(e+fx\right)}{\left(d\tan\left(e+fx\right)\right)^{\frac{5}{2}}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3/(d*tan(f*x+e))**(5/2),x)
```

```
[Out] -I*a**3*(Integral(I/(d*tan(e + f*x))**(5/2), x) + Integral(-3*tan(e + f*x)/
(d*tan(e + f*x))**(5/2), x) + Integral(tan(e + f*x)**3/(d*tan(e + f*x))**(5
/2), x) + Integral(-3*I*tan(e + f*x)**2/(d*tan(e + f*x))**(5/2), x))
```

Giac [A]

time = 0.69, size = 117, normalized size = 1.07

$$-\frac{8i\sqrt{2}\,a^3\arctan\left(\frac{8\sqrt{d^2}\,\sqrt{d\tan\left(fx+e\right)}}{4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\,\sqrt{d^2}\,\sqrt{d}}\right)}{d^{\frac{5}{2}}f\left(\frac{id}{\sqrt{d^2}}+1\right)}-\frac{2\left(9i\,a^3d\tan\left(fx+e\right)+a^3d\right)}{3\sqrt{d\tan\left(fx+e\right)}\,d^3f\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] -8*I*sqrt(2)*a^3*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*I*sqrt(2)*d^(3/
2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(d^(5/2)*f*(I*d/sqrt(d^2) + 1)) - 2/3*(9
*I*a^3*d*tan(f*x + e) + a^3*d)/(sqrt(d*tan(f*x + e))*d^3*f*tan(f*x + e))
```

Mupad [B]
time = 4.44, size = 80, normalized size = 0.73

$$-\frac{\frac{2\,a^3}{3\,d\,f}+\frac{a^3\tan(e+f\,x)\,6\mathrm{i}}{d\,f}}{\left(d\tan\left(e+f\,x\right)\right)^{3/2}}-\frac{\sqrt{16\mathrm{i}}\,a^3\,\mathrm{atan}\left(\frac{\sqrt{16\mathrm{i}}\,\sqrt{d\tan\left(e+f\,x\right)}}{4\sqrt{-d}}\right)\,2\mathrm{i}}{\left(-d\right)^{5/2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^3/(d*tan(e + f*x))^(5/2),x)
```

```
[Out] - ((2*a^3)/(3*d*f) + (a^3*tan(e + f*x)*6i)/(d*f))/(d*tan(e + f*x))^(3/2) -
(16i^(1/2)*a^3*atan((16i^(1/2)*(d*tan(e + f*x))^(1/2))/(4*(-d)^(1/2)))*2i)/
((-d)^(5/2)*f)
```

3.162 $\int \frac{(a+ia\tan(e+fx))^3}{(d\tan(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=132

$$\frac{8(-1)^{3/4}a^3\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{7/2}f}-\frac{8ia^3}{5d^2f(d\tan(e+fx))^{3/2}}+\frac{8a^3}{d^3f\sqrt{d\tan(e+fx)}}-\frac{2(a^3+ia^3\tan(e+fx))}{5df(d\tan(e+fx))^{5/2}}$$

```
[Out] 8*(-1)^(3/4)*a^3*arctan((-1)^(3/4)*(d*tan(f*x+e))^(1/2)/d^(1/2))/d^(7/2)/f+
8*a^3/d^3/f/(d*tan(f*x+e))^(1/2)-8/5*I*a^3/d^2/f/(d*tan(f*x+e))^(3/2)-2/5*(
a^3+I*a^3*tan(f*x+e))/d/f/(d*tan(f*x+e))^(5/2)
```

Rubi [A]
time = 0.17, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3634, 3672, 3610, 3614, 211}

$$\frac{8(-1)^{3/4}a^3\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{7/2}f}+\frac{8a^3}{d^3f\sqrt{d\tan(e+fx)}}-\frac{8ia^3}{5d^2f(d\tan(e+fx))^{3/2}}-\frac{2(a^3+ia^3\tan(e+fx))}{5df(d\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^3/(d*Tan[e + f*x])^(7/2),x]
```

```
[Out] (8*(-1)^(3/4)*a^3*ArcTan[((-1)^(3/4)*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(d^(7/
2)*f) - (((8*I)/5)*a^3)/(d^2*f*(d*Tan[e + f*x])^(3/2)) + (8*a^3)/(d^3*f*Sqr
t[d*Tan[e + f*x]]) - (2*(a^3 + I*a^3*Tan[e + f*x]))/(5*d*f*(d*Tan[e + f*x])
^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
```

```
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3634

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-a^2)*(b*c - a*d)*(a + b*Tan[e + f*x]
)^(m - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] + Di
st[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[b*(b*c*(m - 2) - a*d*(m - 2*n - 4)) + (a*b*c*(m - 2)
+ b^2*d*(n + 1) - a^2*d*(m + n - 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3672

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
 b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
 -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+ia\tan(e+fx))^3}{(d\tan(e+fx))^{7/2}}\,dx &= -\frac{2(a^3+ia^3\tan(e+fx))}{5df(d\tan(e+fx))^{5/2}}-\frac{2\int \frac{(a+ia\tan(e+fx))(-6ia^2d+4a^2d\tan(e+fx))}{(d\tan(e+fx))^{5/2}}\,dx}{5d^2}\\
&= -\frac{8ia^3}{5d^2f(d\tan(e+fx))^{3/2}}-\frac{2(a^3+ia^3\tan(e+fx))}{5df(d\tan(e+fx))^{5/2}}-\frac{2\int \frac{10a^3d^2+10ia^3d^2\tan(e+}{(d\tan(e+fx))^{3/2}}}{5d^4}\\
&= -\frac{8ia^3}{5d^2f(d\tan(e+fx))^{3/2}}+\frac{8a^3}{d^3f\sqrt{d\tan(e+fx)}}-\frac{2(a^3+ia^3\tan(e+fx))}{5df(d\tan(e+fx))^{5/2}}\\
&= -\frac{8ia^3}{5d^2f(d\tan(e+fx))^{3/2}}+\frac{8a^3}{d^3f\sqrt{d\tan(e+fx)}}-\frac{2(a^3+ia^3\tan(e+fx))}{5df(d\tan(e+fx))^{5/2}}\\
&= \frac{8(-1)^{3/4}a^3\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{7/2}f}-\frac{8ia^3}{5d^2f(d\tan(e+fx))^{3/2}}+\frac{}{d^3}
\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 377 vs. 2(132) = 264.

time = 7.16, size = 377, normalized size = 2.86

$$\frac{(\csc(e)(21\cos(e)+5i\sin(e))\left(\frac{2}{5}\cos(3e)-\frac{2}{5}i\sin(3e)\right)+\csc(e)\csc^2(e+fx)(\cos(e)+5i\sin(e))\left(-\frac{2}{5}\cos(3e)+\frac{2}{5}i\sin(3e)\right)+\csc(e)\csc^3(e+fx)\left(\frac{2}{5}\cos(3e)-\frac{2}{5}i\sin(3e)\right)\sin(fx)+\csc(e)\csc(e+fx)\left(-\frac{42}{5}\cos(3e)+\frac{42}{5}i\sin(3e)\right)\sin(fx))\sin^3(e+fx)\tan(e+fx)(a+ia\tan(e+fx))^3}{f(\cos(fx)+i\sin(fx))^3(d\tan(e+fx))^{7/2}}-\frac{8ie^{-3ie}\sqrt{-\frac{i(-1+e^{2i(e+fx)})}{1+e^{2i(e+fx)}}}\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\right)\cos^3(e+fx)\tan^{\frac{7}{2}}(e+fx)(a+ia\tan(e+fx))^3}{\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}f(\cos(fx)+i\sin(fx))^3(d\tan(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3/(d*Tan[e + f*x])^(7/2),x]
```

```
[Out] ((Csc[e]*(21*Cos[e] + (5*I)*Sin[e])*((2*Cos[3*e])/5 - ((2*I)/5)*Sin[3*e]) +
 Csc[e]*Csc[e + f*x]^2*(Cos[e] + (5*I)*Sin[e])*((-2*Cos[3*e])/5 + ((2*I)/5)
*Sin[3*e]) + Csc[e]*Csc[e + f*x]^3*((2*Cos[3*e])/5 - ((2*I)/5)*Sin[3*e])*Si
n[f*x] + Csc[e]*Csc[e + f*x]*((-42*Cos[3*e])/5 + ((42*I)/5)*Sin[3*e])*Sin[f
*x])*Sin[e + f*x]^3*Tan[e + f*x]*(a + I*a*Tan[e + f*x])^3)/(f*(Cos[f*x] + I
*Sin[f*x])^3*(d*Tan[e + f*x])^(7/2)) - ((8*I)*Sqrt[((-I)*(-1 + E^((2*I)*(e
+ f*x))))/(1 + E^((2*I)*(e + f*x)))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(e + f*x))
)/(1 + E^((2*I)*(e + f*x)))]]*Cos[e + f*x]^3*Tan[e + f*x]^(7/2)*(a + I*a*Ta
n[e + f*x])^3)/(E^((3*I)*e)*Sqrt[(-1 + E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(
e + f*x)))]*f*(Cos[f*x] + I*Sin[f*x])^3*(d*Tan[e + f*x])^(7/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(110) = 220$.
time = 0.12, size = 325, normalized size = 2.46

method	result
derivativedivides	$2a^3\left(-\frac{i(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+\ldots}{(d^2)^{\frac{1}{4}}}\right.\right.}{2d}\ldots$
default	$2a^3\left(-\frac{i(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+\ldots}{(d^2)^{\frac{1}{4}}}\right.\right.}{2d}\ldots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3/(d*tan(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3/d^2*(1/d*(-1/2*I/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4
)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*ta
n(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f
```

```
*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))+1/2/
(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1
/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^
2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2
^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)))-I/(d*tan(f*x+e))^(3/2)-1/5*d/(
d*tan(f*x+e))^(5/2)+4/d/(d*tan(f*x+e))^(1/2))
```

Maxima [A]
time = 0.57, size = 228, normalized size = 1.73

$$\frac{\frac{5a^3\left(-\frac{(2i-2)\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(2i-2)\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(i+1)\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{(i+1)\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)}{d^2}+\frac{2\left(20a^3d^2\tan(fx+e)^2-5i\,a^3d^2\tan(fx+e)-a^3d^2\right)}{(d\tan(fx+e))^{\frac{5}{2}}d^2}}{5\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(d*tan(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] 1/5*(5*a^3*(-(2*I - 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt
(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - (2*I - 2)*sqrt(2)*arctan(-1/2*sqrt(2)*
(sqrt(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - (I + 1)*sqrt(
2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) +
 (I + 1)*sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d)
+ d)/sqrt(d))/d^2 + 2*(20*a^3*d^2*tan(f*x + e)^2 - 5*I*a^3*d^2*tan(f*x + e)
 - a^3*d^2)/((d*tan(f*x + e))^(5/2)*d^2))/(d*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 492 vs. 2(115) = 230.
time = 0.38, size = 492, normalized size = 3.73

$$\frac{5\left(d^4fe^{(6i\,fx+6i\,e)}-3\,d^4fe^{(4i\,fx+4i\,e)}+3\,d^4fe^{(2i\,fx+2i\,e)}-d^4f\right)\sqrt{\frac{64i\,a^6}{d^7f^2}}\log\left(\frac{\left(-8i\,a^3de^{(2i\,fx+2i\,e)}+\left(i\,d^4fe^{(2i\,fx+2i\,e)}+i\,d^4f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{64i\,a^6}{d^7f^2}}\right)e^{(-2i\,fx-2i\,e)}}{4\,a^3}\right)-5\left(d^4fe^{(6i\,fx+6i\,e)}-3\,d^4fe^{(4i\,fx+4i\,e)}+3\,d^4fe^{(2i\,fx+2i\,e)}-d^4f\right)\sqrt{\frac{64i\,a^6}{d^7f^2}}\log\left(\frac{\left(-8i\,a^3de^{(2i\,fx+2i\,e)}+\left(-i\,d^4fe^{(2i\,fx+2i\,e)}-i\,d^4f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{64i\,a^6}{d^7f^2}}\right)e^{(-2i\,fx-2i\,e)}}{4\,a^3}\right)-16\left(-13i\,a^3e^{(6i\,fx+6i\,e)}+6i\,a^3e^{(4i\,fx+4i\,e)}+11i\,a^3e^{(2i\,fx+2i\,e)}-8i\,a^3\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}}{20\left(d^4fe^{(6i\,fx+6i\,e)}-3\,d^4fe^{(4i\,fx+4i\,e)}+3\,d^4fe^{(2i\,fx+2i\,e)}-d^4f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(d*tan(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/20*(5*(d^4*f*e^(6*I*f*x + 6*I*e) - 3*d^4*f*e^(4*I*f*x + 4*I*e) + 3*d^4*f*
e^(2*I*f*x + 2*I*e) - d^4*f)*sqrt(64*I*a^6/(d^7*f^2))*log(1/4*(-8*I*a^3*d*e
^(2*I*f*x + 2*I*e) + (I*d^4*f*e^(2*I*f*x + 2*I*e) + I*d^4*f)*sqrt((-I*d*e^(
2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(64*I*a^6/(d^7*f^2))
)*e^(-2*I*f*x - 2*I*e)/a^3) - 5*(d^4*f*e^(6*I*f*x + 6*I*e) - 3*d^4*f*e^(4*I
*f*x + 4*I*e) + 3*d^4*f*e^(2*I*f*x + 2*I*e) - d^4*f)*sqrt(64*I*a^6/(d^7*f^2
))*log(1/4*(-8*I*a^3*d*e^(2*I*f*x + 2*I*e) + (-I*d^4*f*e^(2*I*f*x + 2*I*e)
- I*d^4*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))
*sqrt(64*I*a^6/(d^7*f^2)))*e^(-2*I*f*x - 2*I*e)/a^3) - 16*(-13*I*a^3*e^(6*I
*f*x + 6*I*e) + 6*I*a^3*e^(4*I*f*x + 4*I*e) + 11*I*a^3*e^(2*I*f*x + 2*I*e)
- 8*I*a^3)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))
```

```
)/(d^4*f*e^(6*I*f*x + 6*I*e) - 3*d^4*f*e^(4*I*f*x + 4*I*e) + 3*d^4*f*e^(2*I
*f*x + 2*I*e) - d^4*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia^3\left(\int\frac{i}{(d\tan(e+fx))^{\frac{7}{2}}}\,dx+\int\left(-\frac{3\tan(e+fx)}{(d\tan(e+fx))^{\frac{7}{2}}}\right)dx+\int\frac{\tan^3(e+fx)}{(d\tan(e+fx))^{\frac{7}{2}}}\,dx+\int\left(-\frac{3i\tan^2(e+fx)}{(d\tan(e+fx))^{\frac{7}{2}}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3/(d*tan(f*x+e))**(7/2),x)
```

```
[Out] -I*a**3*(Integral(I/(d*tan(e + f*x))**(7/2), x) + Integral(-3*tan(e + f*x)/
(d*tan(e + f*x))**(7/2), x) + Integral(tan(e + f*x)**3/(d*tan(e + f*x))**(7
/2), x) + Integral(-3*I*tan(e + f*x)**2/(d*tan(e + f*x))**(7/2), x))
```

Giac [A]
time = 0.85, size = 139, normalized size = 1.05

$$\frac{8i\sqrt{2}\,a^3\arctan\left(\frac{8i\sqrt{d^2}\sqrt{d\tan(fx+e)}}{-4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{d^{\frac{7}{2}}f\left(-\frac{id}{\sqrt{d^2}}+1\right)}+\frac{2\left(20a^3d^2\tan(fx+e)^2-5ia^3d^2\tan(fx+e)-a^3d^2\right)}{5\sqrt{d\tan(fx+e)}\,d^5f\tan(fx+e)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(d*tan(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] 8*I*sqrt(2)*a^3*arctan(8*I*sqrt(d^2)*sqrt(d*tan(f*x + e))/(-4*I*sqrt(2)*d^(
3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(d^(7/2)*f*(-I*d/sqrt(d^2) + 1)) + 2/5
*(20*a^3*d^2*tan(f*x + e)^2 - 5*I*a^3*d^2*tan(f*x + e) - a^3*d^2)/(sqrt(d*t
an(f*x + e))*d^5*f*tan(f*x + e)^2)
```

Mupad [B]
time = 4.66, size = 95, normalized size = 0.72

$$-\frac{\frac{2a^3}{5df}-\frac{8a^3\tan(e+fx)^2}{df}+\frac{a^3\tan(e+fx)\,2\mathrm{i}}{df}}{(d\tan(e+fx))^{5/2}}-\frac{2\sqrt{16\mathrm{i}}\,a^3\,\mathrm{atanh}\left(\frac{\sqrt{16\mathrm{i}}\,\sqrt{d\tan(e+fx)}}{4\sqrt{d}}\right)}{d^{7/2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^3/(d*tan(e + f*x))^(7/2),x)
```

```
[Out] - ((2*a^3)/(5*d*f) + (a^3*tan(e + f*x)*2i)/(d*f) - (8*a^3*tan(e + f*x)^2)/(
d*f))/(d*tan(e + f*x))^(5/2) - (2*16i^(1/2)*a^3*atanh((16i^(1/2)*(d*tan(e +
 f*x))^(1/2))/(4*d^(1/2))))/(d^(7/2)*f)
```

3.163 $\int \frac{(a+ia\tan(e+fx))^3}{(d\tan(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=159

$$-\frac{8\sqrt[4]{-1}\,a^3\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{9/2}f}-\frac{32ia^3}{35d^2f(d\tan(e+fx))^{5/2}}+\frac{8a^3}{3d^3f(d\tan(e+fx))^{3/2}}+\frac{}{d^4f\sqrt{d}}$$

```
[Out] -8*(-1)^(1/4)*a^3*arctan((-1)^(3/4)*(d*tan(f*x+e))^(1/2)/d^(1/2))/d^(9/2)/f
+8*I*a^3/d^4/f/(d*tan(f*x+e))^(1/2)-32/35*I*a^3/d^2/f/(d*tan(f*x+e))^(5/2)+
8/3*a^3/d^3/f/(d*tan(f*x+e))^(3/2)-2/7*(a^3+I*a^3*tan(f*x+e))/d/f/(d*tan(f*
x+e))^(7/2)
```

Rubi [A]

time = 0.21, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3634, 3672, 3610, 3614, 211}

$$-\frac{8\sqrt[4]{-1}\,a^3\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{9/2}f}+\frac{8ia^3}{d^4f\sqrt{d\tan(e+fx)}}+\frac{8a^3}{3d^3f(d\tan(e+fx))^{3/2}}-\frac{32ia^3}{35d^2f(d\tan(e+fx))^{5/2}}-\frac{2(a^3+ia^3\tan(e+fx))}{7df(d\tan(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^3/(d*Tan[e + f*x])^(9/2),x]
```

```
[Out] (-8*(-1)^(1/4)*a^3*ArcTan[((-1)^(3/4)*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(d^(9
/2)*f) - (((32*I)/35)*a^3)/(d^2*f*(d*Tan[e + f*x])^(5/2)) + (8*a^3)/(3*d^3*
f*(d*Tan[e + f*x])^(3/2)) + ((8*I)*a^3)/(d^4*f*Sqrt[d*Tan[e + f*x]]) - (2*(
a^3 + I*a^3*Tan[e + f*x]))/(7*d*f*(d*Tan[e + f*x])^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
```

```
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3634

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-a^2)*(b*c - a*d)*(a + b*Tan[e + f*x]
)^(m - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] + Di
st[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[b*(b*c*(m - 2) - a*d*(m - 2*n - 4)) + (a*b*c*(m - 2)
+ b^2*d*(n + 1) - a^2*d*(m + n - 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3672

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
 b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
 -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^3}{(d\tan(e+fx))^{9/2}}\,dx = -\frac{2(a^3+ia^3\tan(e+fx))}{7df(d\tan(e+fx))^{7/2}} - \frac{2\int \frac{(a+ia\tan(e+fx))(-8ia^2d+6a^2d\tan(e+fx))}{(d\tan(e+fx))^{7/2}}\,dx}{7d^2}$$

$$= -\frac{32ia^3}{35d^2f(d\tan(e+fx))^{5/2}} - \frac{2(a^3+ia^3\tan(e+fx))}{7df(d\tan(e+fx))^{7/2}} - \frac{2\int \frac{14a^3d^2+14ia^3d^2\tan(e+}{(d\tan(e+fx))^{5/2}}}{7d^4}$$

$$= -\frac{32ia^3}{35d^2f(d\tan(e+fx))^{5/2}} + \frac{8a^3}{3d^3f(d\tan(e+fx))^{3/2}} - \frac{2(a^3+ia^3\tan(e+fx)}{7df(d\tan(e+fx))^{7/2}}$$

$$= -\frac{32ia^3}{35d^2f(d\tan(e+fx))^{5/2}} + \frac{8a^3}{3d^3f(d\tan(e+fx))^{3/2}} + \frac{8ia^3}{d^4f\sqrt{d\tan(e+fx)}} -$$

$$= -\frac{32ia^3}{35d^2f(d\tan(e+fx))^{5/2}} + \frac{8a^3}{3d^3f(d\tan(e+fx))^{3/2}} + \frac{8ia^3}{d^4f\sqrt{d\tan(e+fx)}} -$$

$$= -\frac{8\sqrt[4]{-1}\,a^3\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{9/2}f} - \frac{32ia^3}{35d^2f(d\tan(e+fx))^{5/2}} + \frac{}{3d}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 416 vs. 2(159) = 318.
time = 7.70, size = 416, normalized size = 2.62

$$\frac{\left(\csc(e)\csc^2(e+fx)(63\cos(e)+170i\sin(e))\left(-\frac{2}{105}i\cos(3e)-\frac{2}{105}\sin(3e)\right)+i\csc(e)(483\cos(e)+155i\sin(e))\left(\frac{2}{105}\cos(3e)-\frac{2}{105}i\sin(3e)\right)+\csc^4(e+fx)\left(-\frac{2}{7}\cos(3e)+\frac{2}{7}i\sin(3e)\right)+i\csc(e)\csc^3(e+fx)\left(\frac{6}{5}\cos(3e)-\frac{6}{5}i\sin(3e)\right)\sin(fx)-i\csc(e)\csc(e+fx)\left(\frac{46}{5}\cos(3e)-\frac{46}{5}i\sin(3e)\right)\sin(fx)\right)\sin^3(e+fx)\tan^2(e+fx)(a+ia\tan(e+fx))^3}{f(\cos(fx)+i\sin(fx))^3(d\tan(e+fx))^{9/2}}+\frac{8e^{-3ie}\sqrt{-\frac{i\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\right)\cos^3(e+fx)\tan^{\frac{9}{2}}(e+fx)(a+ia\tan(e+fx))^3}{\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}f(\cos(fx)+i\sin(fx))^3(d\tan(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3/(d*Tan[e + f*x])^(9/2),x]
```

```
[Out] ((Csc[e]*Csc[e + f*x]^2*(63*Cos[e] + (170*I)*Sin[e])*(((-2*I)/105)*Cos[3*e]
 - (2*Sin[3*e])/105) + I*Csc[e]*(483*Cos[e] + (155*I)*Sin[e])*((2*Cos[3*e])
/105 - ((2*I)/105)*Sin[3*e]) + Csc[e + f*x]^4*((-2*Cos[3*e])/7 + ((2*I)/7)*
Sin[3*e]) + I*Csc[e]*Csc[e + f*x]^3*((6*Cos[3*e])/5 - ((6*I)/5)*Sin[3*e])*S
in[f*x] - I*Csc[e]*Csc[e + f*x]*((46*Cos[3*e])/5 - ((46*I)/5)*Sin[3*e])*Sin
[f*x])*Sin[e + f*x]^3*Tan[e + f*x]^2*(a + I*a*Tan[e + f*x])^3)/(f*(Cos[f*x]
 + I*Sin[f*x])^3*(d*Tan[e + f*x])^(9/2)) + (8*Sqrt[((-I)*(-1 + E^((2*I)*(e
+ f*x))))/(1 + E^((2*I)*(e + f*x)))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(e + f*x))
)/(1 + E^((2*I)*(e + f*x)))]]*Cos[e + f*x]^3*Tan[e + f*x]^(9/2)*(a + I*a*Ta
n[e + f*x])^3)/(E^((3*I)*e)*Sqrt[(-1 + E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(
e + f*x)))]*f*(Cos[f*x] + I*Sin[f*x])^3*(d*Tan[e + f*x])^(9/2))
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 340 vs. 2(132) = 264.
time = 0.12, size = 341, normalized size = 2.14

method	result
derivativedivides	$2a^3\left(\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+}{(d^2)^{\frac{1}{4}}}\right.\right.}{2d}\right.$
default	$2a^3\left(\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+}{(d^2)^{\frac{1}{4}}}\right.\right.}{2d}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3/(d*tan(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

[Out] 2/f*a^3/d^2*(1/d^2*(1/2/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))+1/2*I/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)))-3/5*I/(d*tan(f*x+e))^(5/2)-1/7*d/(d*tan(f*x+e))^(7/2)+4*I/d^2/(d*tan(f*x+e))^(1/2)+4/3/d/(d*tan(f*x+e))^(3/2))

Maxima [A]
time = 0.54, size = 245, normalized size = 1.54

$$\frac{105\,a^3\left(\frac{\frac{(2i+2)\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{(2i+2)\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{(i-1)\sqrt{2}\log\left(d\tan\left(fx+e\right)+\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{(i-1)\sqrt{2}\log\left(d\tan\left(fx+e\right)-\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{d}+d\right)}{\sqrt{d}}}{d^3}\right)-\frac{2\left(-420i\,a^3d^3\tan\left(fx+e\right)^3-140\,a^3d^3\tan\left(fx+e\right)^2+63i\,a^3d^3\tan\left(fx+e\right)+15\,a^3d^3\right)}{\left(d\tan\left(fx+e\right)\right)^{\frac{7}{2}}d^3}}{105\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I*a*tan(f*x+e))^3/(d*tan(f*x+e))^(9/2),x, algorithm="maxima")

[Out] 1/105*(105*a^3*((2*I + 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) + (2*I + 2)*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - (I - 1)*sqrt(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) + (I - 1)*sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d))/d^3 - 2*(-420*I*a^3*d^3*tan(f*x + e)^3 - 140*a^3*d^3*tan(f*x + e)^2 + 63*I*a^3*d^3*tan(f*x + e) + 15*a^3*d^3)/((d*tan(f*x + e))^(7/2)*d^3))/(d*f)

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 549 vs. 2(137) = 274.
time = 0.41, size = 549, normalized size = 3.45

$$\frac{105\left(d^5fe^{(8i\,fx+8i\,e)}-4\,d^5fe^{(6i\,fx+6i\,e)}+6\,d^5fe^{(4i\,fx+4i\,e)}-4\,d^5fe^{(2i\,fx+2i\,e)}+d^5f\right)\sqrt{-\frac{64i\,a^6}{d^9f^2}}\log\left(\frac{\left(-8i\,a^3de^{(2i\,fx+2i\,e)}+\left(d^5fe^{(2i\,fx+2i\,e)}+d^5f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{-\frac{64i\,a^6}{d^9f^2}}\right)e^{(-2i\,fx-2i\,e)}}{4\,a^3}\right)-105\left(d^5fe^{(8i\,fx+8i\,e)}-4\,d^5fe^{(6i\,fx+6i\,e)}+6\,d^5fe^{(4i\,fx+4i\,e)}-4\,d^5fe^{(2i\,fx+2i\,e)}+d^5f\right)\sqrt{-\frac{64i\,a^6}{d^9f^2}}\log\left(\frac{\left(-8i\,a^3de^{(2i\,fx+2i\,e)}-\left(d^5fe^{(2i\,fx+2i\,e)}+d^5f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{-\frac{64i\,a^6}{d^9f^2}}\right)e^{(-2i\,fx-2i\,e)}}{4\,a^3}\right)-16\left(319\,a^3e^{(8i\,fx+8i\,e)}-327\,a^3e^{(6i\,fx+6i\,e)}-95\,a^3e^{(4i\,fx+4i\,e)}+387\,a^3e^{(2i\,fx+2i\,e)}-164\,a^3\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}}{420\left(d^5fe^{(8i\,fx+8i\,e)}-4\,d^5fe^{(6i\,fx+6i\,e)}+6\,d^5fe^{(4i\,fx+4i\,e)}-4\,d^5fe^{(2i\,fx+2i\,e)}+d^5f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I*a*tan(f*x+e))^3/(d*tan(f*x+e))^(9/2),x, algorithm="fricas")

[Out] 1/420*(105*(d^5*f*e^(8*I*f*x + 8*I*e) - 4*d^5*f*e^(6*I*f*x + 6*I*e) + 6*d^5*f*e^(4*I*f*x + 4*I*e) - 4*d^5*f*e^(2*I*f*x + 2*I*e) + d^5*f)*sqrt(-64*I*a^6/(d^9*f^2))*log(1/4*(-8*I*a^3*d*e^(2*I*f*x + 2*I*e) + (d^5*f*e^(2*I*f*x + 2*I*e) + d^5*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-64*I*a^6/(d^9*f^2)))*e^(-2*I*f*x - 2*I*e)/a^3) - 105*(d^5*f*e^(8*I*f*x + 8*I*e) - 4*d^5*f*e^(6*I*f*x + 6*I*e) + 6*d^5*f*e^(4*I*f*x + 4*I*e

```
) - 4*d^5*f*e^(2*I*f*x + 2*I*e) + d^5*f)*sqrt(-64*I*a^6/(d^9*f^2))*log(1/4*
(-8*I*a^3*d*e^(2*I*f*x + 2*I*e) - (d^5*f*e^(2*I*f*x + 2*I*e) + d^5*f)*sqrt(
(-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-64*I*a^6/
(d^9*f^2)))*e^(-2*I*f*x - 2*I*e)/a^3) - 16*(319*a^3*e^(8*I*f*x + 8*I*e) - 3
27*a^3*e^(6*I*f*x + 6*I*e) - 95*a^3*e^(4*I*f*x + 4*I*e) + 387*a^3*e^(2*I*f*
x + 2*I*e) - 164*a^3)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2
*I*e) + 1)))/(d^5*f*e^(8*I*f*x + 8*I*e) - 4*d^5*f*e^(6*I*f*x + 6*I*e) + 6*d
^5*f*e^(4*I*f*x + 4*I*e) - 4*d^5*f*e^(2*I*f*x + 2*I*e) + d^5*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia^3\left(\int \frac{i}{(d\tan(e+fx))^{\frac{9}{2}}}\,dx + \int \left(-\frac{3\tan(e+fx)}{(d\tan(e+fx))^{\frac{9}{2}}}\right)dx + \int \frac{\tan^3(e+fx)}{(d\tan(e+fx))^{\frac{9}{2}}}\,dx + \int \left(-\frac{3i\tan^2(e+fx)}{(d\tan(e+fx))^{\frac{9}{2}}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3/(d*tan(f*x+e))**(9/2),x)
```

```
[Out] -I*a**3*(Integral(I/(d*tan(e + f*x))**(9/2), x) + Integral(-3*tan(e + f*x)/
(d*tan(e + f*x))**(9/2), x) + Integral(tan(e + f*x)**3/(d*tan(e + f*x))**(9
/2), x) + Integral(-3*I*tan(e + f*x)**2/(d*tan(e + f*x))**(9/2), x))
```

Giac [A]
time = 0.91, size = 156, normalized size = 0.98

$$\frac{8i\sqrt{2}\,a^3\arctan\left(\frac{8\sqrt{d^2}\sqrt{d\tan(fx+e)}}{4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{d^{\frac{9}{2}}f\left(\frac{id}{\sqrt{d^2}}+1\right)} - \frac{2\left(-420i\,a^3d^3\tan(fx+e)^3 - 140\,a^3d^3\tan(fx+e)^2 + 63i\,a^3d^3\tan(fx+e) + 15\,a^3d^3\right)}{105\sqrt{d\tan(fx+e)}\,d^7f\tan(fx+e)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(d*tan(f*x+e))^(9/2),x, algorithm="giac")
```

```
[Out] 8*I*sqrt(2)*a^3*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*I*sqrt(2)*d^(3/2
) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(d^(9/2)*f*(I*d/sqrt(d^2) + 1)) - 2/105*(
-420*I*a^3*d^3*tan(f*x + e)^3 - 140*a^3*d^3*tan(f*x + e)^2 + 63*I*a^3*d^3*t
an(f*x + e) + 15*a^3*d^3)/(sqrt(d*tan(f*x + e))*d^7*f*tan(f*x + e)^3)
```

Mupad [B]
time = 5.01, size = 119, normalized size = 0.75

$$-\frac{\frac{2\,a^3}{7\,d\,f} + \frac{a^3\tan(e+f\,x)\,6\mathrm{i}}{5\,d\,f} - \frac{8\,a^3\tan(e+f\,x)^2}{3\,d\,f} - \frac{a^3\tan(e+f\,x)^3\,8\mathrm{i}}{d\,f}}{(d\tan(e+f\,x))^{7/2}} + \frac{\sqrt{16\mathrm{i}}\,a^3\,\mathrm{atan}\left(\frac{\sqrt{16\mathrm{i}}\,\sqrt{d\tan(e+f\,x)}}{4\sqrt{-d}}\right)2\mathrm{i}}{(-d)^{9/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^3/(d*tan(e + f*x))^(9/2),x)
```

```
[Out] (16i^(1/2)*a^3*atan((16i^(1/2)*(d*tan(e + f*x))^(1/2))/(4*(-d)^(1/2)))*2i)/
((-d)^(9/2)*f) - ((2*a^3)/(7*d*f) + (a^3*tan(e + f*x)*6i)/(5*d*f) - (8*a^3*
tan(e + f*x)^2)/(3*d*f) - (a^3*tan(e + f*x)^3*8i)/(d*f))/(d*tan(e + f*x))^(
7/2)
```

3.164 $\int \frac{(d\tan(e+fx))^{7/2}}{a+ia\tan(e+fx)}\,dx$

Optimal. Leaf size=312

$$\frac{\left(\frac{5}{4}-\frac{7i}{4}\right)d^{7/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,af}-\frac{\left(\frac{5}{4}-\frac{7i}{4}\right)d^{7/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,af}+\frac{\left(\frac{5}{8}\right.}{}$$

```
[Out] (5/8-7/8*I)*d^(7/2)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a/f*2^(1
/2)+(-5/8+7/8*I)*d^(7/2)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a/f
*2^(1/2)+(5/16+7/16*I)*d^(7/2)*ln(d^(1/2)-2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1
/2)*tan(f*x+e))/a/f*2^(1/2)-(5/16+7/16*I)*d^(7/2)*ln(d^(1/2)+2^(1/2)*(d*tan
(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/a/f*2^(1/2)+5/2*d^3*(d*tan(f*x+e))^(1/2)
/a/f-7/6*I*d^2*(d*tan(f*x+e))^(3/2)/a/f-1/2*d*(d*tan(f*x+e))^(5/2)/f/(a+I*a
*tan(f*x+e))
```

Rubi [A]

time = 0.23, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {3631, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\left(\frac{5}{4}-\frac{7i}{4}\right)d^{7/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,af}-\frac{\left(\frac{5}{4}-\frac{7i}{4}\right)d^{7/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,af}+\frac{\left(\frac{5}{8}+\frac{7i}{8}\right)d^{7/2}\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,af}-\frac{\left(\frac{5}{8}+\frac{7i}{8}\right)d^{7/2}\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,af}+\frac{5d^3\sqrt{d\tan(e+fx)}}{2af}-\frac{7id^2(d\tan(e+fx))^{3/2}}{6af}-\frac{d(d\tan(e+fx))^{5/2}}{2f(a+ia\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(7/2)/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] ((5/4 - (7*I)/4)*d^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]]
)/(Sqrt[2]*a*f) - ((5/4 - (7*I)/4)*d^(7/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e
 + f*x]])/Sqrt[d]])/(Sqrt[2]*a*f) + ((5/8 + (7*I)/8)*d^(7/2)*Log[Sqrt[d] +
Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e + f*x]]])/(Sqrt[2]*a*f) - ((5/8
 + (7*I)/8)*d^(7/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqrt[2]*Sqrt[d*Tan
[e + f*x]]])/(Sqrt[2]*a*f) + (5*d^3*Sqrt[d*Tan[e + f*x]])/(2*a*f) - (((7*I)
/6)*d^2*(d*Tan[e + f*x])^(3/2))/(a*f) - (d*(d*Tan[e + f*x])^(5/2))/(2*f*(a
+ I*a*Tan[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3631

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((c + d*Tan[e + f*x])^(n - 1)/(
2*a*f*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a^2), Int[(c + d*Tan[e + f*x])
^(n - 2)*Simp[a*c^2 + a*d^2*(n - 1) - b*c*d*n - d*(a*c*(n - 2) + b*d*n)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^{7/2}}{a+ia\tan(e+fx)}\,dx = -\frac{d(d\tan(e+fx))^{5/2}}{2f(a+ia\tan(e+fx))} + \frac{\int (d\tan(e+fx))^{3/2}\left(\frac{5ad^2}{2}-\frac{7}{2}iad^2\tan(e+fx)\right)\,dx}{2a^2}$$

$$= -\frac{7id^2(d\tan(e+fx))^{3/2}}{6af} - \frac{d(d\tan(e+fx))^{5/2}}{2f(a+ia\tan(e+fx))} + \frac{\int \sqrt{d\tan(e+fx)}\,\left(\frac{7}{2}iad^3 \right.}{2a}$$

$$= \frac{5d^3\sqrt{d\tan(e+fx)}}{2af} - \frac{7id^2(d\tan(e+fx))^{3/2}}{6af} - \frac{d(d\tan(e+fx))^{5/2}}{2f(a+ia\tan(e+fx))} + \frac{\int}{}$$

$$= \frac{5d^3\sqrt{d\tan(e+fx)}}{2af} - \frac{7id^2(d\tan(e+fx))^{3/2}}{6af} - \frac{d(d\tan(e+fx))^{5/2}}{2f(a+ia\tan(e+fx))} + \frac{\text{Sub}}{}$$

$$= \frac{5d^3\sqrt{d\tan(e+fx)}}{2af} - \frac{7id^2(d\tan(e+fx))^{3/2}}{6af} - \frac{d(d\tan(e+fx))^{5/2}}{2f(a+ia\tan(e+fx))} + -\frac{(}{}$$

$$= \frac{5d^3\sqrt{d\tan(e+fx)}}{2af} - \frac{7id^2(d\tan(e+fx))^{3/2}}{6af} - \frac{d(d\tan(e+fx))^{5/2}}{2f(a+ia\tan(e+fx))} + \frac{((\frac{5}{8}}{}$$

$$= \frac{\left(\frac{5}{8}+\frac{7i}{8}\right)d^{7/2}\log\left(\sqrt{d}+\sqrt{d}\,\tan(e+fx)-\sqrt{2}\,\sqrt{d\tan(e+fx)}\right)}{\sqrt{2}\,af} - \frac{\left(\frac{5}{8}+\frac{7i}{8}\right)}{}$$

$$= \frac{\left(\frac{5}{4}-\frac{7i}{4}\right)d^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,af} - \frac{\left(\frac{5}{4}-\frac{7i}{4}\right)d^{7/2}\tan^{-1}\left(1+\frac{\sqrt{}}{}\right.}{\sqrt{2}\,af}$$

Mathematica [A]
time = 1.91, size = 275, normalized size = 0.88

$$-\frac{d^4\sec^3(e+fx)\left(-16\cos(e+fx)+16\cos(3(e+fx))+54i\sin(e+fx)+(21-15i)\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sqrt{\sin(2(e+fx))}+(21-15i)\cos(2(e+fx))\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sqrt{\sin(2(e+fx))}+(42+30i)\text{ArcSin}(\cos(e+fx)-\sin(e+fx))\cos(e+fx)(\cos(e+fx)+i\sin(e+fx))\sqrt{\sin(2(e+fx))}+(15+21i)\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sin^{\frac{3}{2}}(2(e+fx))+22i\sin(3(e+fx))\right)}{48af\sqrt{d\tan(e+fx)}\,(-i+\tan(e+fx))}$$

Antiderivative was successfully verified.

[In] Integrate[(d*Tan[e + f*x])^(7/2)/(a + I*a*Tan[e + f*x]),x]

[Out] -1/48*(d^4*Sec[e + f*x]^3*(-16*Cos[e + f*x] + 16*Cos[3*(e + f*x)] + (54*I)*Sin[e + f*x] + (21 - 15*I)*Log[Cos[e + f*x] + Sin[e + f*x] + Sqrt[Sin[2*(e + f*x)]]]*Sqrt[Sin[2*(e + f*x)]] + (21 - 15*I)*Cos[2*(e + f*x)]*Log[Cos[e + f*x] + Sin[e + f*x] + Sqrt[Sin[2*(e + f*x)]]]*Sqrt[Sin[2*(e + f*x)]] + (42 + 30*I)*ArcSin[Cos[e + f*x] - Sin[e + f*x]]*Cos[e + f*x]*(Cos[e + f*x] + I*Sin[e + f*x])*Sqrt[Sin[2*(e + f*x)]] + (15 + 21*I)*Log[Cos[e + f*x] + Sin[e + f*x] + Sqrt[Sin[2*(e + f*x)]]]*Sin[2*(e + f*x)]^(3/2) + (22*I)*Sin[3*(e + f*x)]))/(a*f*Sqrt[d*Tan[e + f*x]]*(-I + Tan[e + f*x]))

Maple [A]
time = 0.21, size = 126, normalized size = 0.40

method	result
derivativedivides	$\frac{2d^2\left(-\frac{i(d\tan(fx+e))^{\frac{3}{2}}}{3}+d\sqrt{d\tan(fx+e)}+\frac{d^2\left(\frac{\sqrt{d\tan(fx+e)}}{id\tan(fx+e)+d}+\frac{6i\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{-id}}\right)}{\sqrt{-id}}\right)}{4}\right.}{fa}$
default	$\frac{2d^2\left(-\frac{i(d\tan(fx+e))^{\frac{3}{2}}}{3}+d\sqrt{d\tan(fx+e)}+\frac{d^2\left(\frac{\sqrt{d\tan(fx+e)}}{id\tan(fx+e)+d}+\frac{6i\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{-id}}\right)}{\sqrt{-id}}\right)}{4}\right.}{fa}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)

[Out] 2/f/a*d^2*(-1/3*I*(d*tan(f*x+e))^(3/2)+d*(d*tan(f*x+e))^(1/2)+1/4*d^2*((d*tan(f*x+e))^(1/2)/(I*d*tan(f*x+e)+d)+6*I/(-I*d)^(1/2)*arctan((d*tan(f*x+e))^(1/2)/(-I*d)^(1/2)))+1/4*I*d^2/(I*d)^(1/2)*arctan((d*tan(f*x+e))^(1/2)/(I*d)^(1/2)))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 646 vs. 2(238) = 476.
time = 0.38, size = 646, normalized size = 2.07

$$\frac{3\sqrt{\frac{9id^7}{a^2f^2}}\left(afe^{(4ifx+4ie)}+afe^{(2ifx+2ie)}\right)\log\left(\frac{\left(-3id^4+\sqrt{\frac{9id^7}{a^2f^2}}\left(afe^{(2ifx+2ie)}+af\right)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\right)e^{(-2ifx-2ie)}}{af}\right)-3\sqrt{\frac{9id^7}{a^2f^2}}\left(afe^{(4ifx+4ie)}+afe^{(2ifx+2ie)}\right)\log\left(\frac{\left(-3id^4-\sqrt{\frac{9id^7}{a^2f^2}}\left(afe^{(2ifx+2ie)}+af\right)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\right)e^{(-2ifx-2ie)}}{af}\right)-3\sqrt{-\frac{id^7}{4a^2f^2}}\left(afe^{(4ifx+4ie)}+afe^{(2ifx+2ie)}\right)\log\left(-\frac{2\left(id^4e^{(2ifx+2ie)}+2\sqrt{-\frac{id^7}{4a^2f^2}}\left(afe^{(2ifx+2ie)}+af\right)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\right)e^{(-2ifx-2ie)}}{d^3}\right)+3\sqrt{-\frac{id^7}{4a^2f^2}}\left(afe^{(4ifx+4ie)}+afe^{(2ifx+2ie)}\right)\log\left(-\frac{2\left(id^4e^{(2ifx+2ie)}-2\sqrt{-\frac{id^7}{4a^2f^2}}\left(afe^{(2ifx+2ie)}+af\right)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\right)e^{(-2ifx-2ie)}}{d^3}\right)+\left(19d^3e^{(4ifx+4ie)}+38d^3e^{(2ifx+2ie)}+3d^3\right)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}}{12\left(afe^{(4ifx+4ie)}+afe^{(2ifx+2ie)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*sqrt(9*I*d^7/(a^2*f^2))*(a*f*e^(4*I*f*x + 4*I*e) + a*f*e^(2*I*f*x +
 2*I*e))*log((-3*I*d^4 + sqrt(9*I*d^7/(a^2*f^2))*(a*f*e^(2*I*f*x + 2*I*e) +
 a*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(
-2*I*f*x - 2*I*e)/(a*f)) - 3*sqrt(9*I*d^7/(a^2*f^2))*(a*f*e^(4*I*f*x + 4*I*
e) + a*f*e^(2*I*f*x + 2*I*e))*log((-3*I*d^4 - sqrt(9*I*d^7/(a^2*f^2))*(a*f*
e^(2*I*f*x + 2*I*e) + a*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*
x + 2*I*e) + 1)))*e^(-2*I*f*x - 2*I*e)/(a*f)) - 3*sqrt(-1/4*I*d^7/(a^2*f^2)
)*(a*f*e^(4*I*f*x + 4*I*e) + a*f*e^(2*I*f*x + 2*I*e))*log(-2*(I*d^4*e^(2*I*
f*x + 2*I*e) + 2*sqrt(-1/4*I*d^7/(a^2*f^2))*(a*f*e^(2*I*f*x + 2*I*e) + a*f)
*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-2*I*
f*x - 2*I*e)/d^3) + 3*sqrt(-1/4*I*d^7/(a^2*f^2))*(a*f*e^(4*I*f*x + 4*I*e) +
 a*f*e^(2*I*f*x + 2*I*e))*log(-2*(I*d^4*e^(2*I*f*x + 2*I*e) - 2*sqrt(-1/4*I
*d^7/(a^2*f^2))*(a*f*e^(2*I*f*x + 2*I*e) + a*f)*sqrt((-I*d*e^(2*I*f*x + 2*I
*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-2*I*f*x - 2*I*e)/d^3) + (19*d^3*
e^(4*I*f*x + 4*I*e) + 38*d^3*e^(2*I*f*x + 2*I*e) + 3*d^3)*sqrt((-I*d*e^(2*I
*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))/(a*f*e^(4*I*f*x + 4*I*e) +
 a*f*e^(2*I*f*x + 2*I*e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{(d\tan(e+fx))^{\frac{7}{2}}}{\tan(e+fx)-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(7/2)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] -I*Integral((d*tan(e + f*x))**(7/2)/(tan(e + f*x) - I), x)/a
```

Giac [A]
time = 0.65, size = 242, normalized size = 0.78

$$-\frac{1}{6} d^3 \left(\frac{18\sqrt{2}\sqrt{d}\arctan\left(\frac{8i\sqrt{d^2}\sqrt{d\tan(fx+e)}}{4i\sqrt{2}d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{af\left(\frac{id}{\sqrt{d^2}}+1\right)} + \frac{3\sqrt{2}\sqrt{d}\arctan\left(\frac{8i\sqrt{d^2}\sqrt{d\tan(fx+e)}}{-4i\sqrt{2}d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{af\left(-\frac{id}{\sqrt{d^2}}+1\right)} + \frac{3i\sqrt{d\tan(fx+e)}\,d}{(d\tan(fx+e)-id)af} + \frac{4\left(i\sqrt{d\tan(fx+e)}\,a^2d^3f^2\tan(fx+e)-3\sqrt{d\tan(fx+e)}\,a^2d^3f^2\right)}{a^3d^3f^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/6*d^3*(18*sqrt(2)*sqrt(d)*arctan(8*I*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*I
*sqrt(2)*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a*f*(I*d/sqrt(d^2) + 1))
+ 3*sqrt(2)*sqrt(d)*arctan(8*I*sqrt(d^2)*sqrt(d*tan(f*x + e))/(-4*I*sqrt(2)
*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a*f*(-I*d/sqrt(d^2) + 1)) + 3*I*s
qrt(d*tan(f*x + e))*d/((d*tan(f*x + e) - I*d)*a*f) + 4*(I*sqrt(d*tan(f*x +
e))*a^2*d^3*f^2*tan(f*x + e) - 3*sqrt(d*tan(f*x + e))*a^2*d^3*f^2)/(a^3*d^3
*f^3))
```

Mupad [B]
time = 6.21, size = 180, normalized size = 0.58

$$\mathrm{atan}\left(\frac{2\,a\,f\,\sqrt{d\tan(e+f\,x)}\,\sqrt{\frac{d^7\,9\mathrm{i}}{4\,a^2\,f^2}}}{3\,d^4}\right)\sqrt{\frac{d^7\,9\mathrm{i}}{4\,a^2\,f^2}}\,2\mathrm{i}+\mathrm{atan}\left(\frac{4\,a\,f\,\sqrt{d\tan(e+f\,x)}\,\sqrt{-\frac{d^7\,1\mathrm{i}}{16\,a^2\,f^2}}}{d^4}\right)\sqrt{-\frac{d^7\,1\mathrm{i}}{16\,a^2\,f^2}}\,2\mathrm{i}+\frac{2\,d^3\,\sqrt{d\tan(e+f\,x)}}{a\,f}-\frac{d^2\,(d\tan(e+f\,x))^{3/2}\,2\mathrm{i}}{3\,a\,f}+\frac{d^4\,\sqrt{d\tan(e+f\,x)}\,1\mathrm{i}}{2\,a\,f\,(-d\tan(e+f\,x)+d\,1\mathrm{i})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(7/2)/(a + a*tan(e + f*x)*1i),x)
```

```
[Out] atan((2*a*f*(d*tan(e + f*x))^(1/2)*((d^7*9i)/(4*a^2*f^2))^(1/2))/(3*d^4))*(
(d^7*9i)/(4*a^2*f^2))^(1/2)*2i + atan((4*a*f*(d*tan(e + f*x))^(1/2)*(-(d^7*
1i)/(16*a^2*f^2))^(1/2))/d^4)*(-(d^7*1i)/(16*a^2*f^2))^(1/2)*2i + (2*d^3*(d
*tan(e + f*x))^(1/2))/(a*f) - (d^2*(d*tan(e + f*x))^(3/2)*2i)/(3*a*f) + (d^
4*(d*tan(e + f*x))^(1/2)*1i)/(2*a*f*(d*1i - d*tan(e + f*x)))
```

3.165 $\int \frac{(d\tan(e+fx))^{5/2}}{a+ia\tan(e+fx)}\,dx$

Optimal. Leaf size=287

$$-\frac{\left(\frac{3}{4}+\frac{5i}{4}\right)d^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,af}+\frac{\left(\frac{3}{4}+\frac{5i}{4}\right)d^{5/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,af}+\frac{(\frac{3}{8}$$

```
[Out] (-3/8-5/8*I)*d^(5/2)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a/f*2^(
1/2)+(3/8+5/8*I)*d^(5/2)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a/f
*2^(1/2)+(3/16-5/16*I)*d^(5/2)*ln(d^(1/2)-2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1
/2)*tan(f*x+e))/a/f*2^(1/2)+(-3/16+5/16*I)*d^(5/2)*ln(d^(1/2)+2^(1/2)*(d*ta
n(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/a/f*2^(1/2)-5/2*I*d^2*(d*tan(f*x+e))^(1
/2)/a/f-1/2*d*(d*tan(f*x+e))^(3/2)/f/(a+I*a*tan(f*x+e))
```

Rubi [A]
time = 0.19, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {3631, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\left(\frac{3}{4}+\frac{5i}{4}\right)d^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,af}+\frac{\left(\frac{3}{4}+\frac{5i}{4}\right)d^{5/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,af}+\frac{\left(\frac{3}{8}-\frac{5i}{8}\right)d^{5/2}\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,af}-\frac{\left(\frac{3}{8}-\frac{5i}{8}\right)d^{5/2}\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,af}-\frac{5id^2\sqrt{d\tan(e+fx)}}{2af}-\frac{d(d\tan(e+fx))^{3/2}}{2f(a+ia\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(5/2)/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] ((-3/4 - (5*I)/4)*d^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]
])/(Sqrt[2]*a*f) + ((3/4 + (5*I)/4)*d^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[
e + f*x]])/Sqrt[d]])/(Sqrt[2]*a*f) + ((3/8 - (5*I)/8)*d^(5/2)*Log[Sqrt[d] +
 Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e + f*x]]])/(Sqrt[2]*a*f) - ((3/
8 - (5*I)/8)*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqrt[2]*Sqrt[d*Ta
n[e + f*x]]])/(Sqrt[2]*a*f) - (((5*I)/2)*d^2*Sqrt[d*Tan[e + f*x]])/(a*f) -
(d*(d*Tan[e + f*x])^(3/2))/(2*f*(a + I*a*Tan[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3631

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((c + d*Tan[e + f*x])^(n - 1)/(
```

```
2*a*f*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a^2), Int[(c + d*Tan[e + f*x])
^(n - 2)*Simp[a*c^2 + a*d^2*(n - 1) - b*c*d*n - d*(a*c*(n - 2) + b*d*n)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d\tan(e+fx))^{5/2}}{a+ia\tan(e+fx)}\,dx &= -\frac{d(d\tan(e+fx))^{3/2}}{2f(a+ia\tan(e+fx))}+\frac{\int \sqrt{d\tan(e+fx)}\left(\frac{3ad^2}{2}-\frac{5}{2}iad^2\tan(e+fx)\right)dx}{2a^2}\\
&= -\frac{5id^2\sqrt{d\tan(e+fx)}}{2af}-\frac{d(d\tan(e+fx))^{3/2}}{2f(a+ia\tan(e+fx))}+\frac{\int \frac{\frac{5}{2}iad^3+\frac{3}{2}ad^3\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx}{2a^2}\\
&= -\frac{5id^2\sqrt{d\tan(e+fx)}}{2af}-\frac{d(d\tan(e+fx))^{3/2}}{2f(a+ia\tan(e+fx))}+\frac{\operatorname{Subst}\left(\int \frac{\frac{5}{2}iad^4+\frac{3}{2}ad^3x^2}{d^2+x^4}\,dx, x \ldots\right.}{a^2f}\\
&= -\frac{5id^2\sqrt{d\tan(e+fx)}}{2af}-\frac{d(d\tan(e+fx))^{3/2}}{2f(a+ia\tan(e+fx))}+-\frac{\left(\left(\frac{3}{4}-\frac{5i}{4}\right)d^3\right)\operatorname{Subst}\left(\int \frac{d \ldots}{d^2 \ldots}\right.}{\ldots}\\
&= -\frac{5id^2\sqrt{d\tan(e+fx)}}{2af}-\frac{d(d\tan(e+fx))^{3/2}}{2f(a+ia\tan(e+fx))}+\frac{\left(\left(\frac{3}{8}-\frac{5i}{8}\right)d^{5/2}\right)\operatorname{Subst}\left(\int - \ldots\right.}{\ldots}\\
&= \frac{\left(\frac{3}{8}-\frac{5i}{8}\right)d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{\sqrt{2}af}-\frac{\left(\frac{3}{8}-\frac{5i}{8}\right) \ldots}{\ldots}\\
&= -\frac{\left(\frac{3}{4}+\frac{5i}{4}\right)d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}af}+\frac{\left(\frac{3}{4}+\frac{5i}{4}\right)d^{5/2}\tan^{-1}\left(1+ \ldots\right.}{\sqrt{2}\, a \ldots}
\end{aligned}$$

Mathematica [A]
time = 1.98, size = 164, normalized size = 0.57

$$\frac{\left(\frac{1}{8}+\frac{i}{8}\right)d^2\csc(e+fx)\sqrt{d\tan(e+fx)}\left((-4-i)\text{ArcSin}(\cos(e+fx)-\sin(e+fx))\sqrt{\sin(2(e+fx))}(-i+\tan(e+fx))+(1+4i)\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sqrt{\sin(2(e+fx))}(-i+\tan(e+fx))-(2+2i)\sin(e+fx)(-5i+4\tan(e+fx))\right)}{af(-i+\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(5/2)/(a + I*a*Tan[e + f*x]),x]
[Out] ((1/8 + I/8)*d^2*Csc[e + f*x]*Sqrt[d*Tan[e + f*x]]*((-4 - I)*ArcSin[Cos[e +
 f*x] - Sin[e + f*x]]*Sqrt[Sin[2*(e + f*x)]]*(-I + Tan[e + f*x]) + (1 + 4*I
)*Log[Cos[e + f*x] + Sin[e + f*x] + Sqrt[Sin[2*(e + f*x)]]]*Sqrt[Sin[2*(e +
```

```
 f*x)]]*(-I + Tan[e + f*x]) - (2 + 2*I)*Sin[e + f*x]*(-5*I + 4*Tan[e + f*x]
)))/(a*f*(-I + Tan[e + f*x]))
```

Maple [A]
time = 0.16, size = 109, normalized size = 0.38

method	result
derivativedivides	$\frac{2d^2\left(-i\sqrt{d\tan(fx+e)}-\frac{d\left(\frac{\sqrt{d\tan(fx+e)}}{-id+d\tan(fx+e)}-\frac{4\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{-id}}\right)}{\sqrt{-id}}\right)}{4}-\frac{d\arctan\left(\frac{\sqrt{d\tan}}{}\right.}{4\sqrt{}}\right.}{fa}$
default	$\frac{2d^2\left(-i\sqrt{d\tan(fx+e)}-\frac{d\left(\frac{\sqrt{d\tan(fx+e)}}{-id+d\tan(fx+e)}-\frac{4\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{-id}}\right)}{\sqrt{-id}}\right)}{4}-\frac{d\arctan\left(\frac{\sqrt{d\tan}}{}\right.}{4\sqrt{}}\right.}{fa}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a*d^2*(-I*(d*tan(f*x+e))^(1/2)-1/4*d*((d*tan(f*x+e))^(1/2)/(-I*d+d*tan(
f*x+e))-4/(-I*d)^(1/2)*arctan((d*tan(f*x+e))^(1/2)/(-I*d)^(1/2)))-1/4*d/(I*
d)^(1/2)*arctan((d*tan(f*x+e))^(1/2)/(I*d)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 562 vs. 2(216) = 432.

time = 0.39, size = 562, normalized size = 1.96

$$\frac{a\sqrt{\frac{i\,d^5}{4\,a^2f^2}}fe^{(2i\,fx+2i\,e)}\log\left(-\frac{2\left(i\,d^3e^{(2i\,fx+2i\,e)}+2\,(i\,afe^{(2i\,fx+2i\,e)}+i\,af)\sqrt{\frac{i\,d^5}{4\,a^2f^2}}\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-2i\,fx-2i\,e)}}{d^2}\right)-a\sqrt{\frac{i\,d^5}{4\,a^2f^2}}fe^{(2i\,fx+2i\,e)}\log\left(-\frac{2\left(i\,d^3e^{(2i\,fx+2i\,e)}+2\,(-i\,afe^{(2i\,fx+2i\,e)}-i\,af)\sqrt{\frac{i\,d^5}{4\,a^2f^2}}\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-2i\,fx-2i\,e)}}{d^2}\right)-a\sqrt{-\frac{4i\,d^5}{a^2f^2}}fe^{(2i\,fx+2i\,e)}\log\left(-\frac{\left(2\,d^3+(afe^{(2i\,fx+2i\,e)}+af)\sqrt{-\frac{4i\,d^5}{a^2f^2}}\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-2i\,fx-2i\,e)}}{af}\right)+a\sqrt{-\frac{4i\,d^5}{a^2f^2}}fe^{(2i\,fx+2i\,e)}\log\left(-\frac{\left(2\,d^3-(afe^{(2i\,fx+2i\,e)}+af)\sqrt{-\frac{4i\,d^5}{a^2f^2}}\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-2i\,fx-2i\,e)}}{af}\right)+\left(-9i\,d^2e^{(2i\,fx+2i\,e)}-i\,d^2\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-2i\,fx-2i\,e)}}{4\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(a*sqrt(1/4*I*d^5/(a^2*f^2))*f*e^(2*I*f*x + 2*I*e)*log(-2*(I*d^3*e^(2*I
*f*x + 2*I*e) + 2*(I*a*f*e^(2*I*f*x + 2*I*e) + I*a*f)*sqrt(1/4*I*d^5/(a^2*f
^2))*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-
2*I*f*x - 2*I*e)/d^2) - a*sqrt(1/4*I*d^5/(a^2*f^2))*f*e^(2*I*f*x + 2*I*e)*l
og(-2*(I*d^3*e^(2*I*f*x + 2*I*e) + 2*(-I*a*f*e^(2*I*f*x + 2*I*e) - I*a*f)*s
qrt(1/4*I*d^5/(a^2*f^2))*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x
+ 2*I*e) + 1)))*e^(-2*I*f*x - 2*I*e)/d^2) - a*sqrt(-4*I*d^5/(a^2*f^2))*f*e^
(2*I*f*x + 2*I*e)*log(-(2*d^3 + (a*f*e^(2*I*f*x + 2*I*e) + a*f)*sqrt(-4*I*d
^5/(a^2*f^2))*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) +
1)))*e^(-2*I*f*x - 2*I*e)/(a*f)) + a*sqrt(-4*I*d^5/(a^2*f^2))*f*e^(2*I*f*x
+ 2*I*e)*log(-(2*d^3 - (a*f*e^(2*I*f*x + 2*I*e) + a*f)*sqrt(-4*I*d^5/(a^2*f
^2))*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-
2*I*f*x - 2*I*e)/(a*f)) + (-9*I*d^2*e^(2*I*f*x + 2*I*e) - I*d^2)*sqrt((-I*d
*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-2*I*f*x - 2*I*e
)/(a*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{(d\tan{(e+fx)})^{\frac{5}{2}}}{\tan{(e+fx)}-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(5/2)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] -I*Integral((d*tan(e + f*x))**(5/2)/(tan(e + f*x) - I), x)/a
```

Giac [A]
time = 0.60, size = 197, normalized size = 0.69

$$-\frac{1}{2}\,d^2\left(\frac{4i\,\sqrt{2}\,\sqrt{d}\,\arctan\left(-\frac{8i\,\sqrt{d^2}\,\sqrt{d\tan{(fx+e)}}}{4i\,\sqrt{2}\,d^{\frac{3}{2}}+4\,\sqrt{2}\,\sqrt{d^2}\,\sqrt{d}}\right)}{af\left(\frac{i\,d}{\sqrt{d^2}}+1\right)}-\frac{i\,\sqrt{2}\,\sqrt{d}\,\arctan\left(-\frac{8i\,\sqrt{d^2}\,\sqrt{d\tan{(fx+e)}}}{-4i\,\sqrt{2}\,d^{\frac{3}{2}}+4\,\sqrt{2}\,\sqrt{d^2}\,\sqrt{d}}\right)}{af\left(-\frac{i\,d}{\sqrt{d^2}}+1\right)}+\frac{4i\,\sqrt{d\tan{(fx+e)}}}{af}+\frac{\sqrt{d\tan{(fx+e)}}\,d}{(d\tan{(fx+e)}-i\,d)af}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/2*d^2*(4*I*sqrt(2)*sqrt(d)*arctan(-8*I*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4
*I*sqrt(2)*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a*f*(I*d/sqrt(d^2) + 1)
```

```
) - I*sqrt(2)*sqrt(d)*arctan(-8*I*sqrt(d^2)*sqrt(d*tan(f*x + e))/(-4*I*sqrt
(2)*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a*f*(-I*d/sqrt(d^2) + 1)) + 4*
I*sqrt(d*tan(f*x + e))/(a*f) + sqrt(d*tan(f*x + e))*d/((d*tan(f*x + e) - I*
d)*a*f))
```

Mupad [B]
time = 6.16, size = 160, normalized size = 0.56

$$\mathrm{atan}\left(\frac{a\,f\,\sqrt{d\,\tan\left(e+f\,x\right)}\,\sqrt{-\frac{d^5\,1{}\mathrm{i}}{a^2\,f^2}}\,1{}\mathrm{i}}{d^3}\right)\,\sqrt{-\frac{d^5\,1{}\mathrm{i}}{a^2\,f^2}}\,2{}\mathrm{i}-\mathrm{atan}\left(\frac{a\,f\,\sqrt{d\,\tan\left(e+f\,x\right)}\,\sqrt{\frac{d^5\,1{}\mathrm{i}}{16\,a^2\,f^2}}\,4{}\mathrm{i}}{d^3}\right)\,\sqrt{\frac{d^5\,1{}\mathrm{i}}{16\,a^2\,f^2}}\,2{}\mathrm{i}-\frac{d^2\,\sqrt{d\,\tan\left(e+f\,x\right)}\,2{}\mathrm{i}}{a\,f}+\frac{d^3\,\sqrt{d\,\tan\left(e+f\,x\right)}}{2\,a\,f\,\left(-d\,\tan\left(e+f\,x\right)+d\,1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(5/2)/(a + a*tan(e + f*x)*1i),x)
```

```
[Out] atan((a*f*(d*tan(e + f*x))^(1/2)*(-(d^5*1i)/(a^2*f^2))^(1/2)*1i)/d^3)*(-(d^
5*1i)/(a^2*f^2))^(1/2)*2i - atan((a*f*(d*tan(e + f*x))^(1/2)*((d^5*1i)/(16*
a^2*f^2))^(1/2)*4i)/d^3)*((d^5*1i)/(16*a^2*f^2))^(1/2)*2i - (d^2*(d*tan(e +
 f*x))^(1/2)*2i)/(a*f) + (d^3*(d*tan(e + f*x))^(1/2))/(2*a*f*(d*1i - d*tan(
e + f*x)))
```

3.166 $\int \frac{(d\tan(e+fx))^{3/2}}{a+ia\tan(e+fx)}\,dx$

Optimal. Leaf size=260

$$-\frac{\left(\frac{1}{4}-\frac{3i}{4}\right)d^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,af}+\frac{\left(\frac{1}{4}-\frac{3i}{4}\right)d^{3/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,af}-\cdots$$

```
[Out] (-1/8+3/8*I)*d^(3/2)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a/f*2^(
1/2)+(1/8-3/8*I)*d^(3/2)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a/f
*2^(1/2)-(1/16+3/16*I)*d^(3/2)*ln(d^(1/2)-2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1
/2)*tan(f*x+e))/a/f*2^(1/2)+(1/16+3/16*I)*d^(3/2)*ln(d^(1/2)+2^(1/2)*(d*tan
(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/a/f*2^(1/2)-1/2*d*(d*tan(f*x+e))^(1/2)/f
/(a+I*a*tan(f*x+e))
```

Rubi [A]

time = 0.16, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3631, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\left(\frac{1}{4}-\frac{3i}{4}\right)d^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,af}+\frac{\left(\frac{1}{4}-\frac{3i}{4}\right)d^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,af}-\frac{\left(\frac{1}{8}+\frac{3i}{8}\right)d^{3/2}\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,af}+\frac{\left(\frac{1}{8}+\frac{3i}{8}\right)d^{3/2}\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,af}-\frac{d\sqrt{d\tan(e+fx)}}{2f(a+ia\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(3/2)/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] ((-1/4 + (3*I)/4)*d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]
])/(Sqrt[2]*a*f) + ((1/4 - (3*I)/4)*d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[
e + f*x]])/Sqrt[d]])/(Sqrt[2]*a*f) - ((1/8 + (3*I)/8)*d^(3/2)*Log[Sqrt[d] +
 Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e + f*x]]])/(Sqrt[2]*a*f) + ((1/
8 + (3*I)/8)*d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqrt[2]*Sqrt[d*Ta
n[e + f*x]]])/(Sqrt[2]*a*f) - (d*Sqrt[d*Tan[e + f*x]])/(2*f*(a + I*a*Tan[e
+ f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3631

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((c + d*Tan[e + f*x])^(n - 1)/(
2*a*f*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a^2), Int[(c + d*Tan[e + f*x])
^(n - 2)*Simp[a*c^2 + a*d^2*(n - 1) - b*c*d*n - d*(a*c*(n - 2) + b*d*n)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^{3/2}}{a+ia\tan(e+fx)}\,dx = -\frac{d\sqrt{d\tan(e+fx)}}{2f(a+ia\tan(e+fx))} + \frac{\int \frac{\frac{ad^2}{2}-\frac{3}{2}iad^2\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx}{2a^2}$$

$$= -\frac{d\sqrt{d\tan(e+fx)}}{2f(a+ia\tan(e+fx))} + \frac{\text{Subst}\left(\int \frac{\frac{ad^3}{2}-\frac{3}{2}iad^2x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{a^2 f}$$

$$= -\frac{d\sqrt{d\tan(e+fx)}}{2f(a+ia\tan(e+fx))} + \frac{\left(\left(\frac{1}{4}-\frac{3i}{4}\right)d^2\right)\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(e+fx)}\right.}{af}$$

$$= -\frac{d\sqrt{d\tan(e+fx)}}{2f(a+ia\tan(e+fx))} + -\frac{\left(\left(\frac{1}{8}+\frac{3i}{8}\right)d^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x,\right.}{\sqrt{2}af}$$

$$= -\frac{\left(\frac{1}{8}+\frac{3i}{8}\right)d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{\sqrt{2}af} + \frac{\left(\frac{1}{8}+\frac{3}{8}\right.}{}$$

$$= -\frac{\left(\frac{1}{4}-\frac{3i}{4}\right)d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}af} + \frac{\left(\frac{1}{4}-\frac{3i}{4}\right)d^{3/2}\tan^{-1}\left(1+\right.}{\sqrt{2}a}$$

Mathematica [A]
time = 1.03, size = 150, normalized size = 0.58

$$\frac{\left(\frac{1}{8}+\frac{i}{8}\right)d\csc(e+fx)\sqrt{\sin(2(e+fx))}\sqrt{d\tan(e+fx)}\left((1+i)\sec(e+fx)\sqrt{\sin(2(e+fx))}+(1+2i)\text{ArcSin}(\cos(e+fx)-\sin(e+fx))(-i+\tan(e+fx))+(2+i)\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)(-i+\tan(e+fx))\right)}{af(-i+\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(3/2)/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] ((1/8 + I/8)*d*Csc[e + f*x]*Sqrt[Sin[2*(e + f*x)]]*Sqrt[d*Tan[e + f*x]]*((1
 + I)*Sec[e + f*x]*Sqrt[Sin[2*(e + f*x)]] + (1 + 2*I)*ArcSin[Cos[e + f*x] -
 Sin[e + f*x]]*(-I + Tan[e + f*x]) + (2 + I)*Log[Cos[e + f*x] + Sin[e + f*x
] + Sqrt[Sin[2*(e + f*x)]]]*(-I + Tan[e + f*x])))/(a*f*(-I + Tan[e + f*x]))
```

Maple [A]
time = 0.16, size = 93, normalized size = 0.36

method	result	size

derivativedivides	$\frac{2d^2\left(-\frac{\sqrt{d\tan(fx+e)}}{4(id\tan(fx+e)+d)}-\frac{i\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{-id}}\right)}{2\sqrt{-id}}-\frac{i\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{id}}\right)}{4\sqrt{id}}\right)}{fa}$	93
default	$\frac{2d^2\left(-\frac{\sqrt{d\tan(fx+e)}}{4(id\tan(fx+e)+d)}-\frac{i\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{-id}}\right)}{2\sqrt{-id}}-\frac{i\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{id}}\right)}{4\sqrt{id}}\right)}{fa}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a*d^2*(-1/4*(d*tan(f*x+e))^(1/2)/(I*d*tan(f*x+e)+d)-1/2*I/(-I*d)^(1/2)*
arctan((d*tan(f*x+e))^(1/2)/(-I*d)^(1/2))-1/4*I/(I*d)^(1/2)*arctan((d*tan(f
*x+e))^(1/2)/(I*d)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(194) = 388$.
time = 0.38, size = 550, normalized size = 2.12

$$\frac{af\sqrt{-\frac{id^3}{4a^2f^2}}e^{(2ifx+2ie)}\log\left(-\frac{2\left(id^2e^{(2ifx+2ie)}+2\left(afe^{(2ifx+2ie)}+af\right)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\sqrt{-\frac{id^3}{4a^2f^2}}\right)e^{(-2ifx-2ie)}}{d}\right)-af\sqrt{-\frac{id^3}{4a^2f^2}}e^{(2ifx+2ie)}\log\left(-\frac{2\left(id^2e^{(2ifx+2ie)}-2\left(afe^{(2ifx+2ie)}+af\right)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\sqrt{-\frac{id^3}{4a^2f^2}}\right)e^{(-2ifx-2ie)}}{d}\right)+af\sqrt{\frac{id^3}{a^2f^2}}e^{(2ifx+2ie)}\log\left(\frac{\left(id^2+\left(afe^{(2ifx+2ie)}+af\right)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{id^3}{a^2f^2}}\right)e^{(-2ifx-2ie)}}{af}\right)-af\sqrt{\frac{id^3}{a^2f^2}}e^{(2ifx+2ie)}\log\left(\frac{\left(id^2-\left(afe^{(2ifx+2ie)}+af\right)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{id^3}{a^2f^2}}\right)e^{(-2ifx-2ie)}}{af}\right)-\left(de^{(2ifx+2ie)}+d\right)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}e^{(-2ifx-2ie)}}{4af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(a*f*sqrt(-1/4*I*d^3/(a^2*f^2))*e^(2*I*f*x + 2*I*e)*log(-2*(I*d^2*e^(2*
I*f*x + 2*I*e) + 2*(a*f*e^(2*I*f*x + 2*I*e) + a*f)*sqrt((-I*d*e^(2*I*f*x +
2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-1/4*I*d^3/(a^2*f^2)))*e^(-2*
I*f*x - 2*I*e)/d) - a*f*sqrt(-1/4*I*d^3/(a^2*f^2))*e^(2*I*f*x + 2*I*e)*log(
-2*(I*d^2*e^(2*I*f*x + 2*I*e) - 2*(a*f*e^(2*I*f*x + 2*I*e) + a*f)*sqrt((-I*
```

```
d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-1/4*I*d^3/(a^
2*f^2)))*e^(-2*I*f*x - 2*I*e)/d) + a*f*sqrt(I*d^3/(a^2*f^2))*e^(2*I*f*x + 2
*I*e)*log((I*d^2 + (a*f*e^(2*I*f*x + 2*I*e) + a*f)*sqrt((-I*d*e^(2*I*f*x +
2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(I*d^3/(a^2*f^2)))*e^(-2*I*f*x
 - 2*I*e)/(a*f)) - a*f*sqrt(I*d^3/(a^2*f^2))*e^(2*I*f*x + 2*I*e)*log((I*d^2
 - (a*f*e^(2*I*f*x + 2*I*e) + a*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e
^(2*I*f*x + 2*I*e) + 1))*sqrt(I*d^3/(a^2*f^2)))*e^(-2*I*f*x - 2*I*e)/(a*f))
 - (d*e^(2*I*f*x + 2*I*e) + d)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*
I*f*x + 2*I*e) + 1)))*e^(-2*I*f*x - 2*I*e)/(a*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{(d\tan{(e+fx)})^{\frac{3}{2}}}{\tan{(e+fx)}-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(3/2)/(a+I*a*tan(f*x+e)),x)
[Out] -I*Integral((d*tan(e + f*x))**(3/2)/(tan(e + f*x) - I), x)/a
```

Giac [A]
time = 0.52, size = 177, normalized size = 0.68

$$-\frac{1}{2}d\left(\frac{i\sqrt{2}\sqrt{d}\arctan\left(\frac{8\sqrt{d^2}\sqrt{d\tan(fx+e)}}{4i\sqrt{2}d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{af\left(\frac{id}{\sqrt{d^2}}+1\right)}+\frac{2i\sqrt{2}\sqrt{d}\arctan\left(\frac{8\sqrt{d^2}\sqrt{d\tan(fx+e)}}{-4i\sqrt{2}d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{af\left(-\frac{id}{\sqrt{d^2}}+1\right)}-\frac{i\sqrt{d\tan(fx+e)}\,d}{(d\tan(fx+e)-id)af}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
[Out] -1/2*d*(I*sqrt(2)*sqrt(d)*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*I*sqrt
(2)*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a*f*(I*d/sqrt(d^2) + 1)) + 2*I
*sqrt(2)*sqrt(d)*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(-4*I*sqrt(2)*d^(3
/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a*f*(-I*d/sqrt(d^2) + 1)) - I*sqrt(d*t
an(f*x + e))*d/((d*tan(f*x + e) - I*d)*a*f))
```

Mupad [B]
time = 5.89, size = 137, normalized size = 0.53

$$-\mathrm{atan}\left(\frac{2\,a\,f\,\sqrt{d\tan(e+f\,x)}\,\sqrt{\frac{d^3\,1\mathrm{i}}{4\,a^2\,f^2}}}{d^2}\right)\sqrt{\frac{d^3\,1\mathrm{i}}{4\,a^2\,f^2}}\,2\mathrm{i}-\mathrm{atan}\left(\frac{4\,a\,f\,\sqrt{d\tan(e+f\,x)}\,\sqrt{-\frac{d^3\,1\mathrm{i}}{16\,a^2\,f^2}}}{d^2}\right)\sqrt{-\frac{d^3\,1\mathrm{i}}{16\,a^2\,f^2}}\,2\mathrm{i}-\frac{d^2\,\sqrt{d\tan(e+f\,x)}\,1\mathrm{i}}{2\,a\,f\,(-d\tan(e+f\,x)+d\,1\mathrm{i})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(3/2)/(a + a*tan(e + f*x)*1i),x)
```

```
[Out] - atan((2*a*f*(d*tan(e + f*x))^(1/2)*((d^3*1i)/(4*a^2*f^2))^(1/2))/d^2)*((d
^3*1i)/(4*a^2*f^2))^(1/2)*2i - atan((4*a*f*(d*tan(e + f*x))^(1/2)*(-(d^3*1i
)/(16*a^2*f^2))^(1/2))/d^2)*(-(d^3*1i)/(16*a^2*f^2))^(1/2)*2i - (d^2*(d*tan
(e + f*x))^(1/2)*1i)/(2*a*f*(d*1i - d*tan(e + f*x)))
```

3.167 $\int \frac{\sqrt{d\tan(e+fx)}}{a+ia\tan(e+fx)}\,dx$

Optimal. Leaf size=81

$$\frac{(-1)^{3/4}\sqrt{d}\,\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2af}+\frac{i\sqrt{d\tan(e+fx)}}{2f(a+ia\tan(e+fx))}$$

```
[Out] 1/2*(-1)^(3/4)*arctan((-1)^(3/4)*(d*tan(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/a/f+
1/2*I*(d*tan(f*x+e))^(1/2)/f/(a+I*a*tan(f*x+e))
```

Rubi [A]
time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3630, 3614, 211}

$$\frac{(-1)^{3/4}\sqrt{d}\,\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2af}+\frac{i\sqrt{d\tan(e+fx)}}{2f(a+ia\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Tan[e + f*x]]/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] ((-1)^(3/4)*Sqrt[d]*ArcTan[((-1)^(3/4)*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(2*a
*f) + ((I/2)*Sqrt[d*Tan[e + f*x]])/(f*(a + I*a*Tan[e + f*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3630

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(a*c + b*d))*((c + d*Tan[e + f*x])^n/(2*(
b*c - a*d)*f*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c
+ d*Tan[e + f*x])^(n - 1)*Simp[a*c*d*(n - 1) + b*c^2 + b*d^2*n - d*(b*c - a
*d)*(n - 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[0, n, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{d\tan(e+fx)}}{a+ia\tan(e+fx)}\,dx &= \frac{i\sqrt{d\tan(e+fx)}}{2f(a+ia\tan(e+fx))} - \frac{\int \frac{\frac{1}{2}iad^2-\frac{1}{2}ad^2\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx}{2a^2d}\\ &= \frac{i\sqrt{d\tan(e+fx)}}{2f(a+ia\tan(e+fx))} + \frac{d^3\mathrm{Subst}\left(\int \frac{1}{\frac{1}{2}iad^3+\frac{1}{2}ad^2x^2}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{4f}\\ &= \frac{(-1)^{3/4}\sqrt{d}\,\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2af} + \frac{i\sqrt{d\tan(e+fx)}}{2f(a+ia\tan(e+fx))}\end{aligned}$$

Mathematica [A]
time = 1.15, size = 129, normalized size = 1.59

$$\frac{\left(\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\right)(-1-i\tan(e+fx))+\sqrt{i\tan(e+fx)}\right)\sqrt{d\tan(e+fx)}}{2a\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\,f(-i+\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Tan[e + f*x]]/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] ((ArcTanh[Sqrt[(-1 + E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f*x)))]]*(-1 -
 I*Tan[e + f*x]) + Sqrt[I*Tan[e + f*x]])*Sqrt[d*Tan[e + f*x]])/(2*a*Sqrt[(-
1 + E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f*x)))]*f*(-I + Tan[e + f*x]))
```

Maple [A]
time = 0.18, size = 72, normalized size = 0.89

method	result	size
derivativedivides	$\frac{2d^2\left(\frac{\sqrt{d\tan(fx+e)}}{4d(-id+d\tan(fx+e))}+\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{id}}\right)}{4d\sqrt{id}}\right)}{fa}$	72

default	$\frac{2d^2\left(\frac{\sqrt{d\tan(fx+e)}}{4d(-id+d\tan(fx+e))}+\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{id}}\right)}{4d\sqrt{id}}\right)}{fa}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a*d^2*(1/4/d*(d*tan(f*x+e))^(1/2)/(-I*d+d*tan(f*x+e))+1/4/d/(I*d)^(1/2)
*arctan((d*tan(f*x+e))^(1/2)/(I*d)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(64) = 128$.
time = 0.39, size = 304, normalized size = 3.75

$$-\frac{\left(af\sqrt{\frac{id}{4a^2f^2}}e^{(2ifx+2ie)}\log\left(-2\left(2\left(iafe^{(2ifx+2ie)}+iaf\right)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{id}{4a^2f^2}}+ide^{(2ifx+2ie)}\right)e^{(-2ifx-2ie)}\right)-af\sqrt{\frac{id}{4a^2f^2}}e^{(2ifx+2ie)}\log\left(-2\left(2\left(-iafe^{(2ifx+2ie)}-iaf\right)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{id}{4a^2f^2}}+ide^{(2ifx+2ie)}\right)e^{(-2ifx-2ie)}\right)-\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\left(ie^{(2ifx+2ie)}+i\right)\right)e^{(-2ifx-2ie)}}{4af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/4*(a*f*sqrt(1/4*I*d/(a^2*f^2))*e^(2*I*f*x + 2*I*e)*log(-2*(2*(I*a*f*e^(2
*I*f*x + 2*I*e) + I*a*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x
+ 2*I*e) + 1))*sqrt(1/4*I*d/(a^2*f^2)) + I*d*e^(2*I*f*x + 2*I*e))*e^(-2*I*f
*x - 2*I*e)) - a*f*sqrt(1/4*I*d/(a^2*f^2))*e^(2*I*f*x + 2*I*e)*log(-2*(2*(-
I*a*f*e^(2*I*f*x + 2*I*e) - I*a*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e
^(2*I*f*x + 2*I*e) + 1))*sqrt(1/4*I*d/(a^2*f^2)) + I*d*e^(2*I*f*x + 2*I*e))
*e^(-2*I*f*x - 2*I*e)) - sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x
+ 2*I*e) + 1))*(I*e^(2*I*f*x + 2*I*e) + I))*e^(-2*I*f*x - 2*I*e)/(a*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{\sqrt{d\tan(e+fx)}}{\tan(e+fx)-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] -I*Integral(sqrt(d*tan(e + f*x))/(tan(e + f*x) - I), x)/a
```

Giac [A]
time = 0.57, size = 110, normalized size = 1.36

$$\frac{\frac{\sqrt{2}\, d^{\frac{3}{2}} \arctan\left(\frac{8\sqrt{d^2}\sqrt{d\tan(fx+e)}}{4i\sqrt{2}\, d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{af\left(\frac{id}{\sqrt{d^2}}+1\right)}+\frac{\sqrt{d\tan(fx+e)}\, d^2}{(d\tan(fx+e)-id)af}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2)*d^(3/2)*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*I*sqrt(2)*d
^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a*f*(I*d/sqrt(d^2) + 1)) + sqrt(d*t
an(f*x + e))*d^2/((d*tan(f*x + e) - I*d)*a*f))/d
```

Mupad [B]
time = 4.23, size = 71, normalized size = 0.88

$$-\frac{2\sqrt{\frac{1}{16}\mathrm{i}}\,\sqrt{d}\,\mathrm{atanh}\left(\frac{4\sqrt{\frac{1}{16}\mathrm{i}}\,\sqrt{d\tan(e+f\,x)}}{\sqrt{d}}\right)}{a\,f}-\frac{d\,\sqrt{d\tan(e+f\,x)}}{2\,a\,f\,\left(-d\tan(e+f\,x)+d\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(1/2)/(a + a*tan(e + f*x)*1i),x)
```

```
[Out] - (2*(1i/16)^(1/2)*d^(1/2)*atanh((4*(1i/16)^(1/2)*(d*tan(e + f*x))^(1/2))/d
^(1/2)))/(a*f) - (d*(d*tan(e + f*x))^(1/2))/(2*a*f*(d*1i - d*tan(e + f*x)))
```

3.168 $\int \frac{1}{\sqrt{d\tan(e+fx)}\,(a+ia\tan(e+fx))}\,dx$

Optimal. Leaf size=262

$$-\frac{\left(\frac{3}{4}-\frac{i}{4}\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a\sqrt{d}\,f}+\frac{\left(\frac{3}{4}-\frac{i}{4}\right)\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a\sqrt{d}\,f}-\frac{\left(\frac{3}{8}+\frac{i}{8}\right)\log}{}$$

```
[Out] (-3/8+1/8*I)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a/f*2^(1/2)/d^(
1/2)+(3/8-1/8*I)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a/f*2^(1/2)
/d^(1/2)-(3/16+1/16*I)*ln(d^(1/2)-2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(
f*x+e))/a/f*2^(1/2)/d^(1/2)+(3/16+1/16*I)*ln(d^(1/2)+2^(1/2)*(d*tan(f*x+e))
^(1/2)+d^(1/2)*tan(f*x+e))/a/f*2^(1/2)/d^(1/2)+1/2*(d*tan(f*x+e))^(1/2)/d/f
/(a+I*a*tan(f*x+e))
```

Rubi [A]

time = 0.15, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3633, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\left(\frac{3}{4}-\frac{i}{4}\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a\sqrt{d}\,f}+\frac{\left(\frac{3}{4}-\frac{i}{4}\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,a\sqrt{d}\,f}+\frac{\sqrt{d\tan(e+fx)}}{2df(a+ia\tan(e+fx))}-\frac{\left(\frac{3}{8}+\frac{i}{8}\right)\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,a\sqrt{d}\,f}+\frac{\left(\frac{3}{8}+\frac{i}{8}\right)\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,a\sqrt{d}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*Tan[e + f*x]]*(a + I*a*Tan[e + f*x])),x]
```

```
[Out] ((-3/4 + I/4)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(Sqrt[2]*
a*Sqrt[d]*f) + ((3/4 - I/4)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[
d]])/(Sqrt[2]*a*Sqrt[d]*f) - ((3/8 + I/8)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x
] - Sqrt[2]*Sqrt[d*Tan[e + f*x]]])/(Sqrt[2]*a*Sqrt[d]*f) + ((3/8 + I/8)*Log
[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqrt[2]*Sqrt[d*Tan[e + f*x]]])/(Sqrt[2]*a
*Sqrt[d]*f) + Sqrt[d*Tan[e + f*x]]/(2*d*f*(a + I*a*Tan[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3633

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-a)*((c + d*Tan[e + f*x])^(n + 1)/(2*f*(b*
c - a*d)*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*
Tan[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &
& NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{d\tan(e+fx)}\,(a+ia\tan(e+fx))}\,dx = \frac{\sqrt{d\tan(e+fx)}}{2df(a+ia\tan(e+fx))} - \frac{\int \frac{-\frac{3ad}{2}+\frac{1}{2}iad\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx}{2a^2d}$$

$$= \frac{\sqrt{d\tan(e+fx)}}{2df(a+ia\tan(e+fx))} - \frac{\text{Subst}\left(\int \frac{-\frac{3ad^2}{2}+\frac{1}{2}iadx^2}{d^2+x^4}\,dx, x, \sqrt{d\,\text{t}}\right.}{a^2df}$$

$$= \frac{\sqrt{d\tan(e+fx)}}{2df(a+ia\tan(e+fx))} - -\frac{\left(\frac{3}{4}+\frac{i}{4}\right)\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{}\right.}{af}$$

$$= \frac{\sqrt{d\tan(e+fx)}}{2df(a+ia\tan(e+fx))} - -\frac{\left(\frac{3}{8}-\frac{i}{8}\right)\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\,\sqrt{d}\,x+}\right.}{af}$$

$$= -\frac{\left(\frac{3}{8}+\frac{i}{8}\right)\log\left(\sqrt{d}+\sqrt{d}\,\tan(e+fx)-\sqrt{2}\,\sqrt{d\tan(e+f}\right.}{\sqrt{2}\,a\sqrt{d}\,f}$$

$$= -\frac{\left(\frac{3}{4}-\frac{i}{4}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a\sqrt{d}\,f} + \frac{\left(\frac{3}{4}-\frac{i}{4}\right)\tan}{}$$

Mathematica [A]

time = 0.87, size = 147, normalized size = 0.56

$$\frac{\sec(e+fx)\sqrt{\sin(2(e+fx))}\left(-2i\sec(e+fx)\sqrt{\sin(2(e+fx))}+(1+3i)\text{ArcSin}(\cos(e+fx)-\sin(e+fx))(1+i\tan(e+fx))+(3+i)\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)(-i+\tan(e+fx))\right)}{8af\sqrt{d\tan(e+fx)}\,(-i+\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*Tan[e + f*x]]*(a + I*a*Tan[e + f*x])),x]
```

```
[Out] (Sec[e + f*x]*Sqrt[Sin[2*(e + f*x)]]*((-2*I)*Sec[e + f*x]*Sqrt[Sin[2*(e + f
*x)]] + (1 + 3*I)*ArcSin[Cos[e + f*x] - Sin[e + f*x]]*(1 + I*Tan[e + f*x])
+ (3 + I)*Log[Cos[e + f*x] + Sin[e + f*x] + Sqrt[Sin[2*(e + f*x)]]]*(-I + T
an[e + f*x])))/(8*a*f*Sqrt[d*Tan[e + f*x]]*(-I + Tan[e + f*x]))
```

Maple [A]

time = 0.17, size = 102, normalized size = 0.39

method	result	size

derivativedivides	$\frac{2d^2\left(-\frac{-\frac{\sqrt{d\tan(fx+e)}}{id\tan(fx+e)+d}+\frac{2i\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{-id}}\right)}{\sqrt{-id}}}{4d^2}+\frac{i\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{id}}\right)}{4d^2\sqrt{id}}\right)}{fa}$	102
default	$\frac{2d^2\left(-\frac{-\frac{\sqrt{d\tan(fx+e)}}{id\tan(fx+e)+d}+\frac{2i\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{-id}}\right)}{\sqrt{-id}}}{4d^2}+\frac{i\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{id}}\right)}{4d^2\sqrt{id}}\right)}{fa}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a*d^2*(-1/4/d^2*(-(d*tan(f*x+e))^(1/2)/(I*d*tan(f*x+e)+d)+2*I/(-I*d)^(1
/2)*arctan((d*tan(f*x+e))^(1/2)/(-I*d)^(1/2)))+1/4*I/d^2/(I*d)^(1/2)*arctan
((d*tan(f*x+e))^(1/2)/(I*d)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 541 vs. 2(196) = 392.
time = 0.40, size = 541, normalized size = 2.06

$$-\frac{\left(adf\sqrt{-\frac{i}{4a^2df^2}}e^{(2ifx+2ie)}\log\left(-2\left(2\left(adfe^{(2ifx+2ie)}+adf\right)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\sqrt{-\frac{i}{4a^2df^2}}+ide^{(2ifx+2ie)}\right)e^{(-2ifx-2ie)}\right)-adf\sqrt{-\frac{i}{4a^2df^2}}e^{(2ifx+2ie)}\log\left(2\left(2\left(adfe^{(2ifx+2ie)}+adf\right)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\sqrt{-\frac{i}{4a^2df^2}}-ide^{(2ifx+2ie)}\right)e^{(-2ifx-2ie)}\right)-adf\sqrt{\frac{i}{a^2df^2}}e^{(2ifx+2ie)}\log\left(\frac{\left((afe^{(2ifx+2ie)}+af)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{i}{a^2df^2}}+i\right)e^{(-2ifx-2ie)}}{af}\right)+adf\sqrt{\frac{i}{a^2df^2}}e^{(2ifx+2ie)}\log\left(-\frac{\left((afe^{(2ifx+2ie)}+af)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{i}{a^2df^2}}-i\right)e^{(-2ifx-2ie)}}{af}\right)-\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\left(e^{(2ifx+2ie)}+1\right)\right)e^{(-2ifx-2ie)}}{4adf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/4*(a*d*f*sqrt(-1/4*I/(a^2*d*f^2))*e^(2*I*f*x + 2*I*e)*log(-2*(2*(a*d*f*e
^(2*I*f*x + 2*I*e) + a*d*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f
```

```
*x + 2*I*e) + 1))*sqrt(-1/4*I/(a^2*d*f^2)) + I*d*e^(2*I*f*x + 2*I*e))*e^(-2
*I*f*x - 2*I*e)) - a*d*f*sqrt(-1/4*I/(a^2*d*f^2))*e^(2*I*f*x + 2*I*e)*log(2
*(2*(a*d*f*e^(2*I*f*x + 2*I*e) + a*d*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*
d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-1/4*I/(a^2*d*f^2)) - I*d*e^(2*I*f*x + 2
*I*e))*e^(-2*I*f*x - 2*I*e)) - a*d*f*sqrt(I/(a^2*d*f^2))*e^(2*I*f*x + 2*I*e
)*log(((a*f*e^(2*I*f*x + 2*I*e) + a*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d
)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(I/(a^2*d*f^2)) + I)*e^(-2*I*f*x - 2*I*e)/
(a*f)) + a*d*f*sqrt(I/(a^2*d*f^2))*e^(2*I*f*x + 2*I*e)*log(-((a*f*e^(2*I*f*
x + 2*I*e) + a*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e
) + 1))*sqrt(I/(a^2*d*f^2)) - I)*e^(-2*I*f*x - 2*I*e)/(a*f)) - sqrt((-I*d*e
^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*(e^(2*I*f*x + 2*I*e) +
 1))*e^(-2*I*f*x - 2*I*e)/(a*d*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i \int \frac{1}{\sqrt{d \tan{\left(e + f x \right)}} \tan{\left(e + f x \right)} - i \sqrt{d \tan{\left(e + f x \right)}}}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] -I*Integral(1/(sqrt(d*tan(e + f*x))*tan(e + f*x) - I*sqrt(d*tan(e + f*x))),
 x)/a
```

Giac [A]
time = 0.60, size = 173, normalized size = 0.66

$$\frac{i \sqrt{2} \arctan\left(\frac{8 \sqrt{d^2} \sqrt{d \tan\left(fx+e\right)}}{4i \sqrt{2} d^{\frac{3}{2}}+4 \sqrt{2} \sqrt{d^2} \sqrt{d}}\right)}{2 a \sqrt{d} f \left(\frac{i d}{\sqrt{d^2}}+1\right)} - \frac{i \sqrt{2} \arctan\left(\frac{8 \sqrt{d^2} \sqrt{d \tan\left(fx+e\right)}}{-4i \sqrt{2} d^{\frac{3}{2}}+4 \sqrt{2} \sqrt{d^2} \sqrt{d}}\right)}{a \sqrt{d} f \left(-\frac{i d}{\sqrt{d^2}}+1\right)} - \frac{i \sqrt{d \tan\left(fx+e\right)}}{2 \left(d \tan\left(fx+e\right)-i d\right) a f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*I*sqrt(2)*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*I*sqrt(2)*d^(3/2)
+ 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a*sqrt(d)*f*(I*d/sqrt(d^2) + 1)) - I*sqrt(
2)*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(-4*I*sqrt(2)*d^(3/2) + 4*sqrt(2
)*sqrt(d^2)*sqrt(d)))/(a*sqrt(d)*f*(-I*d/sqrt(d^2) + 1)) - 1/2*I*sqrt(d*tan
(f*x + e))/((d*tan(f*x + e) - I*d)*a*f)
```

Mupad [B]
time = 5.92, size = 128, normalized size = 0.49

$$-\mathrm{atan}\left(2\,a\,f\,\sqrt{d\,\tan\left(e+f\,x\right)}\,\sqrt{\frac{1{}\mathrm{i}}{4\,a^2\,d\,f^2}}\right)\,\sqrt{\frac{1{}\mathrm{i}}{4\,a^2\,d\,f^2}}\,2{}\mathrm{i}+\mathrm{atan}\left(4\,a\,f\,\sqrt{d\,\tan\left(e+f\,x\right)}\,\sqrt{-\frac{1{}\mathrm{i}}{16\,a^2\,d\,f^2}}\right)\,\sqrt{-\frac{1{}\mathrm{i}}{16\,a^2\,d\,f^2}}\,2{}\mathrm{i}+\frac{\sqrt{d\,\tan\left(e+f\,x\right)}\,1{}\mathrm{i}}{2\,a\,f\,\left(-d\,\tan\left(e+f\,x\right)+d\,1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*tan(e + f*x))^(1/2)*(a + a*tan(e + f*x)*1i)),x)
```

```
[Out] atan(4*a*f*(d*tan(e + f*x))^(1/2)*(-1i/(16*a^2*d*f^2))^(1/2))*(-1i/(16*a^2*
d*f^2))^(1/2)*2i - atan(2*a*f*(d*tan(e + f*x))^(1/2)*(1i/(4*a^2*d*f^2))^(1/
2))*(1i/(4*a^2*d*f^2))^(1/2)*2i + ((d*tan(e + f*x))^(1/2)*1i)/(2*a*f*(d*1i
- d*tan(e + f*x)))
```

3.169 $\int \frac{1}{(d\tan(e+fx))^{3/2}(a+ia\tan(e+fx))}\,dx$

Optimal. Leaf size=287

$$\frac{\left(\frac{5}{4}+\frac{3i}{4}\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,ad^{3/2}f}-\frac{\left(\frac{5}{4}+\frac{3i}{4}\right)\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,ad^{3/2}f}-\frac{\left(\frac{5}{8}-\frac{3i}{8}\right)\log}{}$$

```
[Out] (5/8+3/8*I)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a/d^(3/2)/f*2^(1
/2)-(5/8+3/8*I)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a/d^(3/2)/f*
2^(1/2)+(-5/16+3/16*I)*ln(d^(1/2)-2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(
f*x+e))/a/d^(3/2)/f*2^(1/2)+(5/16-3/16*I)*ln(d^(1/2)+2^(1/2)*(d*tan(f*x+e))
^(1/2)+d^(1/2)*tan(f*x+e))/a/d^(3/2)/f*2^(1/2)-5/2/a/d/f/(d*tan(f*x+e))^(1/
2)+1/2/d/f/(d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))
```

Rubi [A]
time = 0.21, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {3633, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\left(\frac{5}{4}+\frac{3i}{4}\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,ad^{3/2}f}-\frac{\left(\frac{5}{4}+\frac{3i}{4}\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,ad^{3/2}f}-\frac{\left(\frac{5}{8}-\frac{3i}{8}\right)\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,ad^{3/2}f}+\frac{\left(\frac{5}{8}-\frac{3i}{8}\right)\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,ad^{3/2}f}-\frac{5}{2adf\sqrt{d\tan(e+fx)}}+\frac{1}{2df(a+ia\tan(e+fx))\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Tan[e + f*x])^(3/2)*(a + I*a*Tan[e + f*x])),x]
```

```
[Out] ((5/4 + (3*I)/4)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(Sqrt[
2]*a*d^(3/2)*f) - ((5/4 + (3*I)/4)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]]
)/Sqrt[d]])/(Sqrt[2]*a*d^(3/2)*f) - ((5/8 - (3*I)/8)*Log[Sqrt[d] + Sqrt[d]*
Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e + f*x]]])/(Sqrt[2]*a*d^(3/2)*f) + ((5/8
 - (3*I)/8)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqrt[2]*Sqrt[d*Tan[e + f*x
]]])/(Sqrt[2]*a*d^(3/2)*f) - 5/(2*a*d*f*Sqrt[d*Tan[e + f*x]]) + 1/(2*d*f*Sq
rt[d*Tan[e + f*x]]*(a + I*a*Tan[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3633

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-a)*((c + d*Tan[e + f*x])^(n + 1)/(2*f*(b*
```

```
c - a*d)*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*
Tan[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &
& NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(d\tan(e+fx))^{3/2}(a+ia\tan(e+fx))}\,dx = \frac{1}{2df\sqrt{d\tan(e+fx)}\,(a+ia\tan(e+fx))} - \frac{\int \frac{-\frac{5ad}{2}+\frac{3}{2}iad\tan(}{(d\tan(e+fx))}}{2a^2d}$$

$$= -\frac{5}{2adf\sqrt{d\tan(e+fx)}} + \frac{1}{2df\sqrt{d\tan(e+fx)}\,(a+ia\tan(}$$

$$= -\frac{5}{2adf\sqrt{d\tan(e+fx)}} + \frac{1}{2df\sqrt{d\tan(e+fx)}\,(a+ia\tan(}$$

$$= -\frac{5}{2adf\sqrt{d\tan(e+fx)}} + \frac{1}{2df\sqrt{d\tan(e+fx)}\,(a+ia\tan(}$$

$$= -\frac{5}{2adf\sqrt{d\tan(e+fx)}} + \frac{1}{2df\sqrt{d\tan(e+fx)}\,(a+ia\tan(}$$

$$= -\frac{\left(\frac{5}{8}-\frac{3i}{8}\right)\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+}\right.}{\sqrt{2}\,ad^{3/2}f}$$

$$= \frac{\left(\frac{5}{4}+\frac{3i}{4}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,ad^{3/2}f} - \frac{\left(\frac{5}{4}+\frac{3i}{4}\right)\tan}{}$$

Mathematica [A]
time = 1.37, size = 155, normalized size = 0.54

$$\frac{16i-20\tan(e+fx)+(5+3i)\text{ArcSin}(\cos(e+fx)-\sin(e+fx))\sec(e+fx)\sqrt{\sin(2(e+fx))}\,(-i+\tan(e+fx))+(5-3i)\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sec(e+fx)\sqrt{\sin(2(e+fx))}\,(-i+\tan(e+fx))}{8adf\sqrt{d\tan(e+fx)}\,(-i+\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Tan[e + f*x])^(3/2)*(a + I*a*Tan[e + f*x])),x]
```

```
[Out] (16*I - 20*Tan[e + f*x] + (5 + 3*I)*ArcSin[Cos[e + f*x] - Sin[e + f*x]]*Sec
[e + f*x]*Sqrt[Sin[2*(e + f*x)]]*(-I + Tan[e + f*x]) + (5 - 3*I)*Log[Cos[e
+ f*x] + Sin[e + f*x] + Sqrt[Sin[2*(e + f*x)]]]*Sec[e + f*x]*Sqrt[Sin[2*(e
```

```
+ f*x)]]*(-I + Tan[e + f*x]))/(8*a*d*f*Sqrt[d*Tan[e + f*x]]*(-I + Tan[e + f
*x]))
```

Maple [A]
time = 0.14, size = 116, normalized size = 0.40

method	result
derivativedivides	$\frac{2d^2\left(-\frac{1}{d^3\sqrt{d\tan(fx+e)}}+\frac{-\frac{\sqrt{d\tan(fx+e)}}{-id+d\tan(fx+e)}-\frac{4\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{-id}}\right)}{\sqrt{-id}}}{4d^3}-\frac{\arctan\left(\frac{\sqrt{d\tan}}{\sqrt{}}\right.}{4d^3\sqrt{i}}\right.}{fa}$
default	$\frac{2d^2\left(-\frac{1}{d^3\sqrt{d\tan(fx+e)}}+\frac{-\frac{\sqrt{d\tan(fx+e)}}{-id+d\tan(fx+e)}-\frac{4\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{-id}}\right)}{\sqrt{-id}}}{4d^3}-\frac{\arctan\left(\frac{\sqrt{d\tan}}{\sqrt{}}\right.}{4d^3\sqrt{i}}\right.}{fa}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a*d^2*(-1/d^3/(d*tan(f*x+e))^(1/2)+1/4/d^3*(-(d*tan(f*x+e))^(1/2)/(-I*d
+d*tan(f*x+e))-4/(-I*d)^(1/2)*arctan((d*tan(f*x+e))^(1/2)/(-I*d)^(1/2)))-1/
4/d^3/(I*d)^(1/2)*arctan((d*tan(f*x+e))^(1/2)/(I*d)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 677 vs. 2(218) = 436.
time = 0.38, size = 677, normalized size = 2.36

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*((a*d^2*f*e^(4*I*f*x + 4*I*e) - a*d^2*f*e^(2*I*f*x + 2*I*e))*sqrt(1/4*I
/(a^2*d^3*f^2))*log(-2*(2*(I*a*d^2*f*e^(2*I*f*x + 2*I*e) + I*a*d^2*f)*sqrt(
(-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(1/4*I/(a^2
*d^3*f^2)) + I*d*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)) - (a*d^2*f*e^(4
*I*f*x + 4*I*e) - a*d^2*f*e^(2*I*f*x + 2*I*e))*sqrt(1/4*I/(a^2*d^3*f^2))*lo
g(-2*(2*(-I*a*d^2*f*e^(2*I*f*x + 2*I*e) - I*a*d^2*f)*sqrt((-I*d*e^(2*I*f*x
 + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(1/4*I/(a^2*d^3*f^2)) + I*d*
e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)) + (a*d^2*f*e^(4*I*f*x + 4*I*e) -
 a*d^2*f*e^(2*I*f*x + 2*I*e))*sqrt(-4*I/(a^2*d^3*f^2))*log(((a*d*f*e^(2*I*f
*x + 2*I*e) + a*d*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*
I*e) + 1))*sqrt(-4*I/(a^2*d^3*f^2)) + 2)*e^(-2*I*f*x - 2*I*e)/(a*d*f)) - (a
*d^2*f*e^(4*I*f*x + 4*I*e) - a*d^2*f*e^(2*I*f*x + 2*I*e))*sqrt(-4*I/(a^2*d^
3*f^2))*log(-((a*d*f*e^(2*I*f*x + 2*I*e) + a*d*f)*sqrt((-I*d*e^(2*I*f*x + 2
*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-4*I/(a^2*d^3*f^2)) - 2)*e^(-2
*I*f*x - 2*I*e)/(a*d*f)) + sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*
x + 2*I*e) + 1))*(-9*I*e^(4*I*f*x + 4*I*e) - 8*I*e^(2*I*f*x + 2*I*e) + I))/
(a*d^2*f*e^(4*I*f*x + 4*I*e) - a*d^2*f*e^(2*I*f*x + 2*I*e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int \frac{1}{\left(d\tan\left(e+fx\right)\right)^{\frac{3}{2}}\tan\left(e+fx\right)-i\left(d\tan\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))**(3/2)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] -I*Integral(1/((d*tan(e + f*x))**(3/2)*tan(e + f*x) - I*(d*tan(e + f*x))**(
3/2)), x)/a
```

Giac [A]
time = 0.59, size = 202, normalized size = 0.70

$$-\frac{\frac{5i\,d\tan\left(fx+e\right)+4\,d}{\left(i\sqrt{d\tan\left(fx+e\right)}\,d\tan\left(fx+e\right)+\sqrt{d\tan\left(fx+e\right)}\,d\right)af}+\frac{4i\sqrt{2}\arctan\left(\frac{8i\sqrt{d^2}\sqrt{d\tan\left(fx+e\right)}}{4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{a\sqrt{d}\,f\left(\frac{i\,d}{\sqrt{d^2}}+1\right)}+\frac{i\sqrt{2}\arctan\left(\frac{8i\sqrt{d^2}\sqrt{d\tan\left(fx+e\right)}}{-4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{a\sqrt{d}\,f\left(-\frac{i\,d}{\sqrt{d^2}}+1\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/2*((5*I*d*tan(f*x + e) + 4*d)/((I*sqrt(d*tan(f*x + e))*d*tan(f*x + e) +
sqrt(d*tan(f*x + e))*d)*a*f) + 4*I*sqrt(2)*arctan(8*I*sqrt(d^2)*sqrt(d*tan(
```

```
f*x + e))/(4*I*sqrt(2)*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a*sqrt(d)*f
*(I*d/sqrt(d^2) + 1)) + I*sqrt(2)*arctan(8*I*sqrt(d^2)*sqrt(d*tan(f*x + e))
/(-4*I*sqrt(2)*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a*sqrt(d)*f*(-I*d/s
qrt(d^2) + 1)))/d
```

Mupad [B]
time = 6.06, size = 147, normalized size = 0.51

$$-\frac{-\frac{5\tan(e+fx)}{2af}+\frac{2i}{af}}{-(d\tan(e+fx))^{3/2}+d\sqrt{d\tan(e+fx)}\,1i}+2\,\mathrm{atanh}\left(a\,d\,f\,\sqrt{d\tan(e+fx)}\,\sqrt{-\frac{1i}{a^2\,d^3\,f^2}}\right)\sqrt{-\frac{1i}{a^2\,d^3\,f^2}}+2\,\mathrm{atanh}\left(4\,a\,d\,f\,\sqrt{d\tan(e+fx)}\,\sqrt{\frac{1i}{16\,a^2\,d^3\,f^2}}\right)\sqrt{\frac{1i}{16\,a^2\,d^3\,f^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*tan(e + f*x))^(3/2)*(a + a*tan(e + f*x)*1i)),x)
```

```
[Out] 2*atanh(a*d*f*(d*tan(e + f*x))^(1/2)*(-1i/(a^2*d^3*f^2))^(1/2))*(-1i/(a^2*d
^3*f^2))^(1/2) - (2i/(a*f) - (5*tan(e + f*x))/(2*a*f))/(d*(d*tan(e + f*x))^
(1/2)*1i - (d*tan(e + f*x))^(3/2)) + 2*atanh(4*a*d*f*(d*tan(e + f*x))^(1/2)
*(1i/(16*a^2*d^3*f^2))^(1/2))*(1i/(16*a^2*d^3*f^2))^(1/2)
```

3.170 $\int \frac{1}{(d\tan(e+fx))^{5/2}(a+ia\tan(e+fx))}\,dx$

Optimal. Leaf size=314

$$\frac{\left(\frac{7}{4}-\frac{5i}{4}\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,ad^{5/2}f}-\frac{\left(\frac{7}{4}-\frac{5i}{4}\right)\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,ad^{5/2}f}+\frac{\left(\frac{7}{8}+\frac{5i}{8}\right)\log}{}$$

```
[Out] (7/8-5/8*I)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a/d^(5/2)/f*2^(1
/2)+(-7/8+5/8*I)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a/d^(5/2)/f
*2^(1/2)+(7/16+5/16*I)*ln(d^(1/2)-2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(
f*x+e))/a/d^(5/2)/f*2^(1/2)-(7/16+5/16*I)*ln(d^(1/2)+2^(1/2)*(d*tan(f*x+e))
^(1/2)+d^(1/2)*tan(f*x+e))/a/d^(5/2)/f*2^(1/2)+5/2*I/a/d^2/f/(d*tan(f*x+e))
^(1/2)-7/6/a/d/f/(d*tan(f*x+e))^(3/2)+1/2/d/f/(d*tan(f*x+e))^(3/2)/(a+I*a*t
an(f*x+e))
```

Rubi [A]
time = 0.26, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {3633, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\left(\frac{7}{4}-\frac{5i}{4}\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,ad^{5/2}f}-\frac{\left(\frac{7}{4}-\frac{5i}{4}\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,ad^{5/2}f}+\frac{\left(\frac{7}{8}+\frac{5i}{8}\right)\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,ad^{5/2}f}-\frac{\left(\frac{7}{8}+\frac{5i}{8}\right)\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,ad^{5/2}f}+\frac{5i}{2ad^2f\sqrt{d\tan(e+fx)}}+\frac{1}{2df(a+ia\tan(e+fx))(d\tan(e+fx))^{3/2}}-\frac{7}{6adf(d\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Tan[e + f*x])^(5/2)*(a + I*a*Tan[e + f*x])),x]
```

```
[Out] ((7/4 - (5*I)/4)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(Sqrt[
2]*a*d^(5/2)*f) - ((7/4 - (5*I)/4)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]]
)/Sqrt[d]])/(Sqrt[2]*a*d^(5/2)*f) + ((7/8 + (5*I)/8)*Log[Sqrt[d] + Sqrt[d]*
Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e + f*x]]])/(Sqrt[2]*a*d^(5/2)*f) - ((7/8
 + (5*I)/8)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqrt[2]*Sqrt[d*Tan[e + f*x
]]])/(Sqrt[2]*a*d^(5/2)*f) - 7/(6*a*d*f*(d*Tan[e + f*x])^(3/2)) + ((5*I)/2)
/(a*d^2*f*Sqrt[d*Tan[e + f*x]]) + 1/(2*d*f*(d*Tan[e + f*x])^(3/2)*(a + I*a*
Tan[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3633

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-a)*((c + d*Tan[e + f*x])^(n + 1)/(2*f*(b*
c - a*d)*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*
Tan[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &
& NeQ[c^2 + d^2, 0] && !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(d\tan(e+fx))^{5/2}(a+ia\tan(e+fx))}\,dx = \frac{1}{2df(d\tan(e+fx))^{3/2}(a+ia\tan(e+fx))} - \frac{\int \frac{-\frac{7ad}{2}+\frac{5}{2}iad\tan}{(d\tan(e+fx)}}{2a^2d}$$

$$= -\frac{7}{6adf(d\tan(e+fx))^{3/2}} + \frac{1}{2df(d\tan(e+fx))^{3/2}(a+ia\tan}$$

$$= -\frac{7}{6adf(d\tan(e+fx))^{3/2}} + \frac{5i}{2ad^2f\sqrt{d\tan(e+fx)}} + \frac{}{2df(}$$

$$= -\frac{7}{6adf(d\tan(e+fx))^{3/2}} + \frac{5i}{2ad^2f\sqrt{d\tan(e+fx)}} + \frac{}{2df(}$$

$$= -\frac{7}{6adf(d\tan(e+fx))^{3/2}} + \frac{5i}{2ad^2f\sqrt{d\tan(e+fx)}} + \frac{}{2df(}$$

$$= -\frac{7}{6adf(d\tan(e+fx))^{3/2}} + \frac{5i}{2ad^2f\sqrt{d\tan(e+fx)}} + \frac{}{2df(}$$

$$= \frac{\left(\frac{7}{8}+\frac{5i}{8}\right)\log\left(\sqrt{d}+\sqrt{d}\,\tan(e+fx)-\sqrt{2}\,\sqrt{d\tan(e+fx}\right.}{\sqrt{2}\,ad^{5/2}f}$$

$$= \frac{\left(\frac{7}{4}-\frac{5i}{4}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,ad^{5/2}f} - \frac{\left(\frac{7}{4}-\frac{5i}{4}\right)\tan}{}$$

Mathematica [A]
time = 2.52, size = 273, normalized size = 0.87

$$\frac{\sec^3(e+fx)\left(54i\cos(e+fx)-22i\cos(3(e+fx))+16\sin(e+fx)-(21+15i)\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sqrt{\sin(2(e+fx))}+(21+15i)\cos(2(e+fx))\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sqrt{\sin(2(e+fx))}-(15-21i)\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sin^{\frac{3}{2}}(2(e+fx))-(15+21i)\mathrm{ArcSin}(\cos(e+fx)-\sin(e+fx))\sqrt{\sin(2(e+fx))}\left(2i\sin^2(e+fx)+\sin(2(e+fx))\right)+16\sin(3(e+fx))\right)}{48adf(d\tan(e+fx))^{3/2}(-i+\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Tan[e + f*x])^(5/2)*(a + I*a*Tan[e + f*x])),x]
```

```
[Out] (Sec[e + f*x]^3*((54*I)*Cos[e + f*x] - (22*I)*Cos[3*(e + f*x)] + 16*Sin[e +
 f*x] - (21 + 15*I)*Log[Cos[e + f*x] + Sin[e + f*x] + Sqrt[Sin[2*(e + f*x)]
]]*Sqrt[Sin[2*(e + f*x)]] + (21 + 15*I)*Cos[2*(e + f*x)]*Log[Cos[e + f*x] +
 Sin[e + f*x] + Sqrt[Sin[2*(e + f*x)]]]*Sqrt[Sin[2*(e + f*x)]] - (15 - 21*I
)*Log[Cos[e + f*x] + Sin[e + f*x] + Sqrt[Sin[2*(e + f*x)]]]*Sin[2*(e + f*x)
]^(3/2) - (15 + 21*I)*ArcSin[Cos[e + f*x] - Sin[e + f*x]]*Sqrt[Sin[2*(e + f
*x)]]*((2*I)*Sin[e + f*x]^2 + Sin[2*(e + f*x)]) + 16*Sin[3*(e + f*x)]))/(48
*a*d*f*(d*Tan[e + f*x])^(3/2)*(-I + Tan[e + f*x]))
```

Maple [**A**]
time = 0.15, size = 133, normalized size = 0.42

method	result
derivativedivides	$\frac{2d^2\left(-\frac{1}{3d^3(d\tan(fx+e))^{\frac{3}{2}}}+\frac{i}{d^4\sqrt{d\tan(fx+e)}}+\frac{-\frac{\sqrt{d\tan(fx+e)}}{id\tan(fx+e)+d}+\frac{6i\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{-id}}\right)}{\sqrt{-id}}}{4d^4}\right.}{fa}$
default	$\frac{2d^2\left(-\frac{1}{3d^3(d\tan(fx+e))^{\frac{3}{2}}}+\frac{i}{d^4\sqrt{d\tan(fx+e)}}+\frac{-\frac{\sqrt{d\tan(fx+e)}}{id\tan(fx+e)+d}+\frac{6i\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{-id}}\right)}{\sqrt{-id}}}{4d^4}\right.}{fa}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a*d^2*(-1/3/d^3/(d*tan(f*x+e))^(3/2)+I/d^4/(d*tan(f*x+e))^(1/2)+1/4/d^4
*(-(d*tan(f*x+e))^(1/2)/(I*d*tan(f*x+e)+d)+6*I/(-I*d)^(1/2)*arctan((d*tan(f
*x+e))^(1/2)/(-I*d)^(1/2)))-1/4*I/d^4/(I*d)^(1/2)*arctan((d*tan(f*x+e))^(1/
2)/(I*d)^(1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 776 vs. $2(240) = 480$.
time = 0.40, size = 776, normalized size = 2.47

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*(a*d^3*f*e^(6*I*f*x + 6*I*e) - 2*a*d^3*f*e^(4*I*f*x + 4*I*e) + a*d^
3*f*e^(2*I*f*x + 2*I*e))*sqrt(-1/4*I/(a^2*d^5*f^2))*log(-2*(2*(a*d^3*f*e^(2
*I*f*x + 2*I*e) + a*d^3*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*
x + 2*I*e) + 1))*sqrt(-1/4*I/(a^2*d^5*f^2)) + I*d*e^(2*I*f*x + 2*I*e))*e^(-
2*I*f*x - 2*I*e)) - 3*(a*d^3*f*e^(6*I*f*x + 6*I*e) - 2*a*d^3*f*e^(4*I*f*x +
 4*I*e) + a*d^3*f*e^(2*I*f*x + 2*I*e))*sqrt(-1/4*I/(a^2*d^5*f^2))*log(2*(2*
(a*d^3*f*e^(2*I*f*x + 2*I*e) + a*d^3*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*
d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-1/4*I/(a^2*d^5*f^2)) - I*d*e^(2*I*f*x +
 2*I*e))*e^(-2*I*f*x - 2*I*e)) - 3*(a*d^3*f*e^(6*I*f*x + 6*I*e) - 2*a*d^3*f
*e^(4*I*f*x + 4*I*e) + a*d^3*f*e^(2*I*f*x + 2*I*e))*sqrt(9*I/(a^2*d^5*f^2))
*log(-((a*d^2*f*e^(2*I*f*x + 2*I*e) + a*d^2*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*
e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(9*I/(a^2*d^5*f^2)) + 3*I)*e^(-2*I
*f*x - 2*I*e)/(a*d^2*f)) + 3*(a*d^3*f*e^(6*I*f*x + 6*I*e) - 2*a*d^3*f*e^(4*
I*f*x + 4*I*e) + a*d^3*f*e^(2*I*f*x + 2*I*e))*sqrt(9*I/(a^2*d^5*f^2))*log((
(a*d^2*f*e^(2*I*f*x + 2*I*e) + a*d^2*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*
d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(9*I/(a^2*d^5*f^2)) - 3*I)*e^(-2*I*f*x -
2*I*e)/(a*d^2*f)) - sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I
*e) + 1))*(19*e^(6*I*f*x + 6*I*e) - 19*e^(4*I*f*x + 4*I*e) - 35*e^(2*I*f*x
+ 2*I*e) + 3))/(a*d^3*f*e^(6*I*f*x + 6*I*e) - 2*a*d^3*f*e^(4*I*f*x + 4*I*e)
 + a*d^3*f*e^(2*I*f*x + 2*I*e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i \int \frac{1}{(d\tan(e+fx))^{\frac{5}{2}}\tan(e+fx) - i(d\tan(e+fx))^{\frac{5}{2}}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))**(5/2)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] -I*Integral(1/((d*tan(e + f*x))**(5/2)*tan(e + f*x) - I*(d*tan(e + f*x))**(
5/2)), x)/a
```

Giac [A]
time = 0.64, size = 220, normalized size = 0.70

$$-\frac{i\sqrt{2}\arctan\left(\frac{8\sqrt{d^2}\sqrt{d\tan(fx+e)}}{4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{2ad^{\frac{5}{2}}f\left(\frac{id}{\sqrt{d^2}}+1\right)}+\frac{3i\sqrt{2}\arctan\left(\frac{8\sqrt{d^2}\sqrt{d\tan(fx+e)}}{-4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{ad^{\frac{5}{2}}f\left(-\frac{id}{\sqrt{d^2}}+1\right)}-\frac{\sqrt{d\tan(fx+e)}}{2(id\tan(fx+e)+d)ad^2f}+\frac{2(3id\tan(fx+e)-d)}{3\sqrt{d\tan(fx+e)}\,ad^3f\tan(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/2*I*sqrt(2)*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*I*sqrt(2)*d^(3/2)
 + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a*d^(5/2)*f*(I*d/sqrt(d^2) + 1)) + 3*I*sq
rt(2)*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(-4*I*sqrt(2)*d^(3/2) + 4*sqr
t(2)*sqrt(d^2)*sqrt(d)))/(a*d^(5/2)*f*(-I*d/sqrt(d^2) + 1)) - 1/2*sqrt(d*ta
n(f*x + e))/((I*d*tan(f*x + e) + d)*a*d^2*f) + 2/3*(3*I*d*tan(f*x + e) - d)
/(sqrt(d*tan(f*x + e))*a*d^3*f*tan(f*x + e))
```

Mupad [B]
time = 6.66, size = 171, normalized size = 0.54

$$\mathrm{atan}\left(\frac{2\,a\,d^2\,f\,\sqrt{d\tan(e+f\,x)}\,\sqrt{\frac{9\mathrm{i}}{4\,a^2\,d^5\,f^2}}}{3}\right)\sqrt{\frac{9\mathrm{i}}{4\,a^2\,d^5\,f^2}}\,2\mathrm{i}-\mathrm{atan}\left(4\,a\,d^2\,f\,\sqrt{d\tan(e+f\,x)}\,\sqrt{-\frac{1\mathrm{i}}{16\,a^2\,d^5\,f^2}}\right)\sqrt{-\frac{1\mathrm{i}}{16\,a^2\,d^5\,f^2}}\,2\mathrm{i}-\frac{\frac{2\mathrm{i}}{3\,a\,f}+\frac{4\tan(e+f\,x)}{3\,a\,f}+\frac{\tan(e+f\,x)^2\,5\mathrm{i}}{2\,a\,f}}{-(d\tan(e+f\,x))^{5/2}+d\,(d\tan(e+f\,x))^{3/2}\,1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*tan(e + f*x))^(5/2)*(a + a*tan(e + f*x)*1i)),x)
```

```
[Out] atan((2*a*d^2*f*(d*tan(e + f*x))^(1/2)*(9i/(4*a^2*d^5*f^2))^(1/2))/3)*(9i/(
4*a^2*d^5*f^2))^(1/2)*2i - atan(4*a*d^2*f*(d*tan(e + f*x))^(1/2)*(-1i/(16*a
^2*d^5*f^2))^(1/2))*(-1i/(16*a^2*d^5*f^2))^(1/2)*2i - (2i/(3*a*f) + (4*tan(
e + f*x))/(3*a*f) + (tan(e + f*x)^2*5i)/(2*a*f))/(d*(d*tan(e + f*x))^(3/2)*
1i - (d*tan(e + f*x))^(5/2))
```

3.171 $\int \frac{(d\tan(e+fx))^{9/2}}{(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=353

$$-\frac{\left(\frac{49}{16}+\frac{45i}{16}\right)d^{9/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^2f}+\frac{\left(\frac{49}{16}+\frac{45i}{16}\right)d^{9/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^2f}$$

```
[Out] (-49/32-45/32*I)*d^(9/2)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a^2
/f*2^(1/2)+(49/32+45/32*I)*d^(9/2)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^
(1/2))/a^2/f*2^(1/2)+(49/64-45/64*I)*d^(9/2)*ln(d^(1/2)-2^(1/2)*(d*tan(f*x+
e))^(1/2)+d^(1/2)*tan(f*x+e))/a^2/f*2^(1/2)+(-49/64+45/64*I)*d^(9/2)*ln(d^(
1/2)+2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/a^2/f*2^(1/2)-45/8*I*
d^4*(d*tan(f*x+e))^(1/2)/a^2/f-49/24*d^3*(d*tan(f*x+e))^(3/2)/a^2/f+9/8*I*d
^2*(d*tan(f*x+e))^(5/2)/a^2/f/(1+I*tan(f*x+e))-1/4*d*(d*tan(f*x+e))^(7/2)/f
/(a+I*a*tan(f*x+e))^2
```

Rubi [A]
time = 0.34, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3639, 3676, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\left(\frac{49}{16}+\frac{45i}{16}\right)d^{9/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^2f}+\frac{\left(\frac{49}{16}+\frac{45i}{16}\right)d^{9/2}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,a^2f}+\frac{\left(\frac{49}{32}-\frac{45i}{32}\right)d^{9/2}\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\,\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,a^2f}-\frac{\left(\frac{49}{32}-\frac{45i}{32}\right)d^{9/2}\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\,\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,a^2f}-\frac{45id^4\sqrt{d\tan(e+fx)}}{8a^2f}-\frac{49d^3(d\tan(e+fx))^{3/2}}{24a^2f}+\frac{9id^2(d\tan(e+fx))^{5/2}}{8a^2f(1+i\tan(e+fx))}-\frac{d(d\tan(e+fx))^{7/2}}{4f(a+ia\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(9/2)/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] ((-49/16 - (45*I)/16)*d^(9/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqr
t[d]])/(Sqrt[2]*a^2*f) + ((49/16 + (45*I)/16)*d^(9/2)*ArcTan[1 + (Sqrt[2]*S
qrt[d*Tan[e + f*x]])/Sqrt[d]])/(Sqrt[2]*a^2*f) + ((49/32 - (45*I)/32)*d^(9/
2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e + f*x]]])/(Sqr
t[2]*a^2*f) - ((49/32 - (45*I)/32)*d^(9/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*
x] + Sqrt[2]*Sqrt[d*Tan[e + f*x]]])/(Sqrt[2]*a^2*f) - (((45*I)/8)*d^4*Sqrt[
d*Tan[e + f*x]])/(a^2*f) - (49*d^3*(d*Tan[e + f*x])^(3/2))/(24*a^2*f) + (((
9*I)/8)*d^2*(d*Tan[e + f*x])^(5/2))/(a^2*f*(1 + I*Tan[e + f*x])) - (d*(d*Ta
n[e + f*x])^(7/2))/(4*f*(a + I*a*Tan[e + f*x])^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
```

```
NeQ[c^2 + d^2, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^{9/2}}{(a+ia\tan(e+fx))^2}\,dx = -\frac{d(d\tan(e+fx))^{7/2}}{4f(a+ia\tan(e+fx))^2} - \frac{\int \frac{(d\tan(e+fx))^{5/2}\left(-\frac{7ad^2}{2}+\frac{11}{2}iad^2\tan(e+fx)\right)}{a+ia\tan(e+fx)}\,dx}{4a^2}$$

$$= \frac{9id^2(d\tan(e+fx))^{5/2}}{8a^2f(1+i\tan(e+fx))} - \frac{d(d\tan(e+fx))^{7/2}}{4f(a+ia\tan(e+fx))^2} + \frac{\int(d\tan(e+fx))^{3/2}\,(-$$

$$= -\frac{49d^3(d\tan(e+fx))^{3/2}}{24a^2f} + \frac{9id^2(d\tan(e+fx))^{5/2}}{8a^2f(1+i\tan(e+fx))} - \frac{d(d\tan(e+fx))^{7/2}}{4f(a+ia\tan(e+fx)}$$

$$= -\frac{45id^4\sqrt{d\tan(e+fx)}}{8a^2f} - \frac{49d^3(d\tan(e+fx))^{3/2}}{24a^2f} + \frac{9id^2(d\tan(e+fx))^{5/2}}{8a^2f(1+i\tan(e+fx))}$$

$$= -\frac{45id^4\sqrt{d\tan(e+fx)}}{8a^2f} - \frac{49d^3(d\tan(e+fx))^{3/2}}{24a^2f} + \frac{9id^2(d\tan(e+fx))^{5/2}}{8a^2f(1+i\tan(e+fx))}$$

$$= -\frac{45id^4\sqrt{d\tan(e+fx)}}{8a^2f} - \frac{49d^3(d\tan(e+fx))^{3/2}}{24a^2f} + \frac{9id^2(d\tan(e+fx))^{5/2}}{8a^2f(1+i\tan(e+fx))}$$

$$= -\frac{45id^4\sqrt{d\tan(e+fx)}}{8a^2f} - \frac{49d^3(d\tan(e+fx))^{3/2}}{24a^2f} + \frac{9id^2(d\tan(e+fx))^{5/2}}{8a^2f(1+i\tan(e+fx))}$$

$$= \frac{\left(\frac{49}{32}-\frac{45i}{32}\right)d^{9/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{\sqrt{2}\,a^2f} - \frac{\left(\frac{49}{32}-\right.}{}$$

$$= -\frac{\left(\frac{49}{16}+\frac{45i}{16}\right)d^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^2f} + \frac{\left(\frac{49}{16}+\frac{45i}{16}\right)d^{9/2}\tan^{-1}\left(\right.}{\surd}$$

Mathematica [A]
time = 2.46, size = 346, normalized size = 0.98

$$\frac{d^5\sec^4(e+fx)\left(-269+64\cos(2(e+fx))+205\cos(4(e+fx))+(147-135i)\cos(e+fx)\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sqrt{\sin(2(e+fx))}+(147-135i)\cos(3(e+fx))\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sqrt{\sin(2(e+fx))}+(135+147i)\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sin(e+fx)\sqrt{\sin(2(e+fx))}+(294+270i)\text{ArcSin}(\cos(e+fx)-\sin(e+fx))\cos(e+fx)(\cos(2(e+fx))+i\sin(2(e+fx)))\sqrt{\sin(2(e+fx))}+142i\sin(2(e+fx))+(135+147i)\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sqrt{\sin(2(e+fx))}\sin(3(e+fx))+199i\sin(4(e+fx))\right)}{192a^2f\sqrt{d\tan(e+fx)}(-i+\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(9/2)/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (d^5*Sec[e + f*x]^4*(-269 + 64*Cos[2*(e + f*x)] + 205*Cos[4*(e + f*x)] + (1
47 - 135*I)*Cos[e + f*x]*Log[Cos[e + f*x] + Sin[e + f*x] + Sqrt[Sin[2*(e +
f*x)]]]*Sqrt[Sin[2*(e + f*x)]] + (147 - 135*I)*Cos[3*(e + f*x)]*Log[Cos[e +
 f*x] + Sin[e + f*x] + Sqrt[Sin[2*(e + f*x)]]]*Sqrt[Sin[2*(e + f*x)]] + (13
5 + 147*I)*Log[Cos[e + f*x] + Sin[e + f*x] + Sqrt[Sin[2*(e + f*x)]]]*Sin[e
```

```
+ f*x]*Sqrt[Sin[2*(e + f*x)]] + (294 + 270*I)*ArcSin[Cos[e + f*x] - Sin[e +
 f*x]]*Cos[e + f*x]*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)])*Sqrt[Sin[2*(e +
 f*x)]] + (142*I)*Sin[2*(e + f*x)] + (135 + 147*I)*Log[Cos[e + f*x] + Sin[e
 + f*x] + Sqrt[Sin[2*(e + f*x)]]]*Sqrt[Sin[2*(e + f*x)]]*Sin[3*(e + f*x)] +
 (199*I)*Sin[4*(e + f*x)]))/(192*a^2*f*Sqrt[d*Tan[e + f*x]]*(-I + Tan[e + f
*x])^2)
```

Maple [A]
time = 0.18, size = 142, normalized size = 0.40

method	result
derivativedivides	$\frac{2d^3\left(-\frac{(d\tan(fx+e))^{\frac{3}{2}}}{3}-2id\sqrt{d\tan(fx+e)}+\frac{d^2\left(\frac{\frac{15(d\tan(fx+e))^{\frac{3}{2}}}{2}-\frac{13id\sqrt{d\tan(fx+e)}}{2}}{(id\tan(fx+e)+d)^2}+\frac{47\arctan\left(\frac{\sqrt{}}{}\right.}{}\right.}{8}\right.}{f\,a^2}$
default	$\frac{2d^3\left(-\frac{(d\tan(fx+e))^{\frac{3}{2}}}{3}-2id\sqrt{d\tan(fx+e)}+\frac{d^2\left(\frac{\frac{15(d\tan(fx+e))^{\frac{3}{2}}}{2}-\frac{13id\sqrt{d\tan(fx+e)}}{2}}{(id\tan(fx+e)+d)^2}+\frac{47\arctan\left(\frac{\sqrt{}}{}\right.}{}\right.}{8}\right.}{f\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^2*d^3*(-1/3*(d*tan(f*x+e))^(3/2)-2*I*d*(d*tan(f*x+e))^(1/2)+1/8*d^2*(
(15/2*(d*tan(f*x+e))^(3/2)-13/2*I*d*(d*tan(f*x+e))^(1/2))/(I*d*tan(f*x+e)+d
)^2+47/2/(-I*d)^(1/2)*arctan((d*tan(f*x+e))^(1/2)/(-I*d)^(1/2)))+1/8*d^2/(I
*d)^(1/2)*arctan((d*tan(f*x+e))^(1/2)/(I*d)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 705 vs. $2(273) = 546$.
time = 0.41, size = 705, normalized size = 2.00

$$-\frac{12\left(a^{2} f e^{\left(6 i f x+6 i e\right)}+a^{2} f e^{\left(4 i f x+4 i e\right)}\right) \sqrt{-\frac{2209 i d^{9}}{64 a^{4} f^{2}}} \log\left(-\frac{\left(47 d^{5}+8\left(a^{2} f e^{\left(2 i f x+2 i e\right)}+a^{2} f\right) \sqrt{-\frac{2209 i d^{9}}{64 a^{4} f^{2}}} \sqrt{\frac{-i d e^{\left(2 i f x+2 i e\right)}+i d}{e^{\left(2 i f x+2 i e\right)}+1}}\right) e^{\left(-2 i f x-2 i e\right)}}{8 a^{2} f}\right)-12\left(a^{2} f e^{\left(6 i f x+6 i e\right)}+a^{2} f e^{\left(4 i f x+4 i e\right)}\right) \sqrt{-\frac{2209 i d^{9}}{64 a^{4} f^{2}}} \log\left(-\frac{\left(47 d^{5}-8\left(a^{2} f e^{\left(2 i f x+2 i e\right)}+a^{2} f\right) \sqrt{-\frac{2209 i d^{9}}{64 a^{4} f^{2}}} \sqrt{\frac{-i d e^{\left(2 i f x+2 i e\right)}+i d}{e^{\left(2 i f x+2 i e\right)}+1}}\right) e^{\left(-2 i f x-2 i e\right)}}{8 a^{2} f}\right)+12\left(a^{2} f e^{\left(6 i f x+6 i e\right)}+a^{2} f e^{\left(4 i f x+4 i e\right)}\right) \sqrt{\frac{i d^{9}}{16 a^{4} f^{2}}} \log\left(-\frac{2\left(i d^{5} e^{\left(2 i f x+2 i e\right)}+4\left(i a^{2} f e^{\left(2 i f x+2 i e\right)}+i a^{2} f\right) \sqrt{\frac{i d^{9}}{16 a^{4} f^{2}}} \sqrt{\frac{-i d e^{\left(2 i f x+2 i e\right)}+i d}{e^{\left(2 i f x+2 i e\right)}+1}}\right) e^{\left(-2 i f x-2 i e\right)}}{d^{4}}\right)-12\left(a^{2} f e^{\left(6 i f x+6 i e\right)}+a^{2} f e^{\left(4 i f x+4 i e\right)}\right) \sqrt{\frac{i d^{9}}{16 a^{4} f^{2}}} \log\left(-\frac{2\left(i d^{5} e^{\left(2 i f x+2 i e\right)}+4\left(-i a^{2} f e^{\left(2 i f x+2 i e\right)}-i a^{2} f\right) \sqrt{\frac{i d^{9}}{16 a^{4} f^{2}}} \sqrt{\frac{-i d e^{\left(2 i f x+2 i e\right)}+i d}{e^{\left(2 i f x+2 i e\right)}+1}}\right) e^{\left(-2 i f x-2 i e\right)}}{d^{4}}\right)-\left(-202 i d^{4} e^{\left(6 i f x+6 i e\right)}-305 i d^{4} e^{\left(4 i f x+4 i e\right)}-36 i d^{4} e^{\left(2 i f x+2 i e\right)}+3 i d^{4}\right) \sqrt{\frac{-i d e^{\left(2 i f x+2 i e\right)}+i d}{e^{\left(2 i f x+2 i e\right)}+1}}}{48\left(a^{2} f e^{\left(6 i f x+6 i e\right)}+a^{2} f e^{\left(4 i f x+4 i e\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/48*(12*(a^2*f*e^(6*I*f*x + 6*I*e) + a^2*f*e^(4*I*f*x + 4*I*e))*sqrt(-220
9/64*I*d^9/(a^4*f^2))*log(-1/8*(47*d^5 + 8*(a^2*f*e^(2*I*f*x + 2*I*e) + a^2
*f)*sqrt(-2209/64*I*d^9/(a^4*f^2))*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e
^(2*I*f*x + 2*I*e) + 1)))*e^(-2*I*f*x - 2*I*e)/(a^2*f)) - 12*(a^2*f*e^(6*I*
f*x + 6*I*e) + a^2*f*e^(4*I*f*x + 4*I*e))*sqrt(-2209/64*I*d^9/(a^4*f^2))*lo
g(-1/8*(47*d^5 - 8*(a^2*f*e^(2*I*f*x + 2*I*e) + a^2*f)*sqrt(-2209/64*I*d^9/
(a^4*f^2))*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))
)*e^(-2*I*f*x - 2*I*e)/(a^2*f)) + 12*(a^2*f*e^(6*I*f*x + 6*I*e) + a^2*f*e^(
4*I*f*x + 4*I*e))*sqrt(1/16*I*d^9/(a^4*f^2))*log(-2*(I*d^5*e^(2*I*f*x + 2*I
*e) + 4*(I*a^2*f*e^(2*I*f*x + 2*I*e) + I*a^2*f)*sqrt(1/16*I*d^9/(a^4*f^2))*
sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-2*I*f
*x - 2*I*e)/d^4) - 12*(a^2*f*e^(6*I*f*x + 6*I*e) + a^2*f*e^(4*I*f*x + 4*I*e
))*sqrt(1/16*I*d^9/(a^4*f^2))*log(-2*(I*d^5*e^(2*I*f*x + 2*I*e) + 4*(-I*a^2
*f*e^(2*I*f*x + 2*I*e) - I*a^2*f)*sqrt(1/16*I*d^9/(a^4*f^2))*sqrt((-I*d*e^(
2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-2*I*f*x - 2*I*e)/d^
4) - (-202*I*d^4*e^(6*I*f*x + 6*I*e) - 305*I*d^4*e^(4*I*f*x + 4*I*e) - 36*I
*d^4*e^(2*I*f*x + 2*I*e) + 3*I*d^4)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(
e^(2*I*f*x + 2*I*e) + 1)))/(a^2*f*e^(6*I*f*x + 6*I*e) + a^2*f*e^(4*I*f*x +
4*I*e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(9/2)/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.65, size = 269, normalized size = 0.76

$$\frac{1}{24} d^{4}\left(-\frac{141 i \sqrt{2} \sqrt{d} \arctan\left(-\frac{8 i \sqrt{d^{2}} \sqrt{d \tan\left(f x+e\right)}}{4 i \sqrt{2} d^{\frac{3}{2}}+4 \sqrt{2} \sqrt{d^{2}} \sqrt{d}}\right)}{a^{2} f\left(\frac{i d}{\sqrt{d^{2}}}+1\right)}-\frac{6 i \sqrt{2} \sqrt{d} \arctan\left(-\frac{8 i \sqrt{d^{2}} \sqrt{d \tan\left(f x+e\right)}}{-4 i \sqrt{2} d^{\frac{3}{2}}+4 \sqrt{2} \sqrt{d^{2}} \sqrt{d}}\right)}{a^{2} f\left(-\frac{i d}{\sqrt{d^{2}}}+1\right)}-\frac{3\left(15 \sqrt{d \tan\left(f x+e\right)} d^{2} \tan\left(f x+e\right)-13 i \sqrt{d \tan\left(f x+e\right)} d^{2}\right)}{\left(d \tan\left(f x+e\right)-i d\right)^{2} a^{2} f}-\frac{16\left(\sqrt{d \tan\left(f x+e\right)} a^{4} d^{3} f^{2} \tan\left(f x+e\right)+6 i \sqrt{d \tan\left(f x+e\right)} a^{4} d^{3} f^{2}\right)}{a^{6} d^{3} f^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/24*d^4*(-141*I*sqrt(2)*sqrt(d)*arctan(-8*I*sqrt(d^2)*sqrt(d*tan(f*x + e))
/(4*I*sqrt(2)*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a^2*f*(I*d/sqrt(d^2)
 + 1)) - 6*I*sqrt(2)*sqrt(d)*arctan(-8*I*sqrt(d^2)*sqrt(d*tan(f*x + e))/(-4
*I*sqrt(2)*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a^2*f*(-I*d/sqrt(d^2) +
 1)) - 3*(15*sqrt(d*tan(f*x + e))*d^2*tan(f*x + e) - 13*I*sqrt(d*tan(f*x +
e))*d^2)/((d*tan(f*x + e) - I*d)^2*a^2*f) - 16*(sqrt(d*tan(f*x + e))*a^4*d^
3*f^2*tan(f*x + e) + 6*I*sqrt(d*tan(f*x + e))*a^4*d^3*f^2)/(a^6*d^3*f^3))
```

Mupad [B]
time = 5.64, size = 225, normalized size = 0.64

$$-\frac{-\frac{15\,d^5\,{\left(d\,\mathrm{tan}\left(e+f\,x\right)\right)}^{3/2}}{8\,a^2\,f}+\frac{d^6\,\sqrt{d\,\mathrm{tan}\left(e+f\,x\right)}\,13{}\mathrm{i}}{8\,a^2\,f}}{-d^2\,{\mathrm{tan}\left(e+f\,x\right)}^2+d^2\,\mathrm{tan}\left(e+f\,x\right)\,2{}\mathrm{i}+d^2}-\frac{2\,d^3\,{\left(d\,\mathrm{tan}\left(e+f\,x\right)\right)}^{3/2}}{3\,a^2\,f}+\mathrm{atan}\left(\frac{a^2\,f\,\sqrt{d\,\mathrm{tan}\left(e+f\,x\right)}\,\sqrt{\frac{d^9\,1{}\mathrm{i}}{64\,a^4\,f^2}}\,8{}\mathrm{i}}{d^5}\right)\,\sqrt{\frac{d^9\,1{}\mathrm{i}}{64\,a^4\,f^2}}\,2{}\mathrm{i}+\mathrm{atan}\left(\frac{a^2\,f\,\sqrt{d\,\mathrm{tan}\left(e+f\,x\right)}\,\sqrt{-\frac{d^9\,2209{}\mathrm{i}}{256\,a^4\,f^2}}\,16{}\mathrm{i}}{47\,d^5}\right)\,\sqrt{-\frac{d^9\,2209{}\mathrm{i}}{256\,a^4\,f^2}}\,2{}\mathrm{i}-\frac{d^4\,\sqrt{d\,\mathrm{tan}\left(e+f\,x\right)}\,4{}\mathrm{i}}{a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(9/2)/(a + a*tan(e + f*x)*1i)^2,x)
```

```
[Out] atan((a^2*f*(d*tan(e + f*x))^(1/2)*((d^9*1i)/(64*a^4*f^2))^(1/2)*8i)/d^5)*(
(d^9*1i)/(64*a^4*f^2))^(1/2)*2i - ((d^6*(d*tan(e + f*x))^(1/2)*13i)/(8*a^2*
f) - (15*d^5*(d*tan(e + f*x))^(3/2))/(8*a^2*f))/(d^2*tan(e + f*x)*2i + d^2
- d^2*tan(e + f*x)^2) + atan((a^2*f*(d*tan(e + f*x))^(1/2)*(-(d^9*2209i)/(2
56*a^4*f^2))^(1/2)*16i)/(47*d^5))*(-(d^9*2209i)/(256*a^4*f^2))^(1/2)*2i - (
d^4*(d*tan(e + f*x))^(1/2)*4i)/(a^2*f) - (2*d^3*(d*tan(e + f*x))^(3/2))/(3*
a^2*f)
```

3.172 $\int \frac{(d\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=326

$$-\frac{\left(\frac{25}{16}-\frac{21i}{16}\right)d^{7/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^2f}+\frac{\left(\frac{25}{16}-\frac{21i}{16}\right)d^{7/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^2f}-$$

```
[Out] (-25/32+21/32*I)*d^(7/2)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a^2
/f*2^(1/2)+(25/32-21/32*I)*d^(7/2)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^
(1/2))/a^2/f*2^(1/2)-(25/64+21/64*I)*d^(7/2)*ln(d^(1/2)-2^(1/2)*(d*tan(f*x+
e))^(1/2)+d^(1/2)*tan(f*x+e))/a^2/f*2^(1/2)+(25/64+21/64*I)*d^(7/2)*ln(d^(1
/2)+2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/a^2/f*2^(1/2)-25/8*d^3
*(d*tan(f*x+e))^(1/2)/a^2/f+7/8*I*d^2*(d*tan(f*x+e))^(3/2)/a^2/f/(1+I*tan(f
*x+e))-1/4*d*(d*tan(f*x+e))^(5/2)/f/(a+I*a*tan(f*x+e))^2
```

Rubi [A]

time = 0.31, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3639, 3676, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\left(\frac{25}{16}-\frac{21i}{16}\right)d^{7/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^2f}+\frac{\left(\frac{25}{16}-\frac{21i}{16}\right)d^{7/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,a^2f}-\frac{\left(\frac{25}{32}+\frac{21i}{32}\right)d^{7/2}\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,a^2f}+\frac{\left(\frac{25}{32}+\frac{21i}{32}\right)d^{7/2}\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,a^2f}-\frac{25d^3\sqrt{d\tan(e+fx)}}{8a^2f}+\frac{7id^2(d\tan(e+fx))^{3/2}}{8a^2f(1+i\tan(e+fx))}-\frac{d(d\tan(e+fx))^{5/2}}{4f(a+ia\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(7/2)/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] ((-25/16 + (21*I)/16)*d^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqr
t[d]])/(Sqrt[2]*a^2*f) + ((25/16 - (21*I)/16)*d^(7/2)*ArcTan[1 + (Sqrt[2]*S
qrt[d*Tan[e + f*x]])/Sqrt[d]])/(Sqrt[2]*a^2*f) - ((25/32 + (21*I)/32)*d^(7/
2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e + f*x]]])/(Sqr
t[2]*a^2*f) + ((25/32 + (21*I)/32)*d^(7/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*
x] + Sqrt[2]*Sqrt[d*Tan[e + f*x]]])/(Sqrt[2]*a^2*f) - (25*d^3*Sqrt[d*Tan[e
+ f*x]])/(8*a^2*f) + (((7*I)/8)*d^2*(d*Tan[e + f*x])^(3/2))/(a^2*f*(1 + I*T
an[e + f*x])) - (d*(d*Tan[e + f*x])^(5/2))/(4*f*(a + I*a*Tan[e + f*x])^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^2}\,dx = -\frac{d(d\tan(e+fx))^{5/2}}{4f(a+ia\tan(e+fx))^2} - \frac{\int \frac{(d\tan(e+fx))^{3/2}\left(-\frac{5ad^2}{2}+\frac{9}{2}iad^2\tan(e+fx)\right)}{a+ia\tan(e+fx)}\,dx}{4a^2}$$

$$= \frac{7id^2(d\tan(e+fx))^{3/2}}{8a^2f(1+i\tan(e+fx))} - \frac{d(d\tan(e+fx))^{5/2}}{4f(a+ia\tan(e+fx))^2} + \frac{\int\sqrt{d\tan(e+fx)}\,(-$$

$$= -\frac{25d^3\sqrt{d\tan(e+fx)}}{8a^2f} + \frac{7id^2(d\tan(e+fx))^{3/2}}{8a^2f(1+i\tan(e+fx))} - \frac{d(d\tan(e+fx))^{5/2}}{4f(a+ia\tan(e+fx)}$$

$$= -\frac{25d^3\sqrt{d\tan(e+fx)}}{8a^2f} + \frac{7id^2(d\tan(e+fx))^{3/2}}{8a^2f(1+i\tan(e+fx))} - \frac{d(d\tan(e+fx))^{5/2}}{4f(a+ia\tan(e+fx)}$$

$$= -\frac{25d^3\sqrt{d\tan(e+fx)}}{8a^2f} + \frac{7id^2(d\tan(e+fx))^{3/2}}{8a^2f(1+i\tan(e+fx))} - \frac{d(d\tan(e+fx))^{5/2}}{4f(a+ia\tan(e+fx)}$$

$$= -\frac{25d^3\sqrt{d\tan(e+fx)}}{8a^2f} + \frac{7id^2(d\tan(e+fx))^{3/2}}{8a^2f(1+i\tan(e+fx))} - \frac{d(d\tan(e+fx))^{5/2}}{4f(a+ia\tan(e+fx)}$$

$$= -\frac{\left(\frac{25}{32}+\frac{21i}{32}\right)d^{7/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{\sqrt{2}\,a^2f} + \frac{\left(\frac{2}{3}\right.}{}$$

$$= -\frac{\left(\frac{25}{16}-\frac{21i}{16}\right)d^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^2f} + \frac{\left(\frac{25}{16}-\frac{21i}{16}\right)d^{7/2}\tan^{-1}}{}$$

Mathematica [A]

time = 1.46, size = 233, normalized size = 0.71

$$-\frac{id^4\sec^3(e+fx)\left(-43\cos(e+fx)+43\cos(3(e+fx))-23i\sin(e+fx)+(21-25i)\cos(2(e+fx))\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sqrt{\sin(2(e+fx))}+(21+25i)\text{ArcSin}(\cos(e+fx)-\sin(e+fx))(\cos(2(e+fx))+i\sin(2(e+fx)))\sqrt{\sin(2(e+fx))}+(25+21i)\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sin^{\frac{3}{2}}(2(e+fx))+41i\sin(3(e+fx))\right)}{32a^2f\sqrt{d\tan(e+fx)}\,(-i+\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(7/2)/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] ((-1/32*I)*d^4*Sec[e + f*x]^3*(-43*Cos[e + f*x] + 43*Cos[3*(e + f*x)] - (23
*I)*Sin[e + f*x] + (21 - 25*I)*Cos[2*(e + f*x)]*Log[Cos[e + f*x] + Sin[e +
f*x] + Sqrt[Sin[2*(e + f*x)]]]*Sqrt[Sin[2*(e + f*x)]] + (21 + 25*I)*ArcSin[
Cos[e + f*x] - Sin[e + f*x]]*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)])*Sqrt[S
in[2*(e + f*x)]] + (25 + 21*I)*Log[Cos[e + f*x] + Sin[e + f*x] + Sqrt[Sin[2
*(e + f*x)]]]*Sin[2*(e + f*x)]^(3/2) + (41*I)*Sin[3*(e + f*x)]))/(a^2*f*Sqr
t[d*Tan[e + f*x]]*(-I + Tan[e + f*x])^2)
```

Maple [A]
time = 0.17, size = 126, normalized size = 0.39

method	result
derivativedivides	$\frac{2d^3\left(-\sqrt{d\tan(fx+e)}-\frac{d\left(\frac{\frac{11i(d\tan(fx+e))^{\frac{3}{2}}}{2}+\frac{9d\sqrt{d\tan(fx+e)}}{2}}{(id\tan(fx+e)+d)^2}+\frac{23i\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{-id}}\right)}{2\sqrt{-id}}\right.}{8}\right.}{f\,a^2}$
default	$\frac{2d^3\left(-\sqrt{d\tan(fx+e)}-\frac{d\left(\frac{\frac{11i(d\tan(fx+e))^{\frac{3}{2}}}{2}+\frac{9d\sqrt{d\tan(fx+e)}}{2}}{(id\tan(fx+e)+d)^2}+\frac{23i\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{-id}}\right)}{2\sqrt{-id}}\right.}{8}\right.}{f\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^2*d^3*(-(d*tan(f*x+e))^(1/2)-1/8*d*((11/2*I*(d*tan(f*x+e))^(3/2)+9/2*
d*(d*tan(f*x+e))^(1/2))/(I*d*tan(f*x+e)+d)^2+23/2*I/(-I*d)^(1/2)*arctan((d*
tan(f*x+e))^(1/2)/(-I*d)^(1/2)))+1/8*I*d/(I*d)^(1/2)*arctan((d*tan(f*x+e))^
(1/2)/(I*d)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 600 vs. 2(251) = 502.
time = 0.39, size = 600, normalized size = 1.84

$$-\frac{\left(4a^2\sqrt{-\frac{id^7}{16a^4f^2}}fe^{(4ifx+4ie)}\log\left(-\frac{2\left(id^4e^{(2ifx+2ie)}+4\left(a^2fe^{(2ifx+2ie)}+a^2f\right)\sqrt{-\frac{id^7}{16a^4f^2}}\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\right)e^{(-2ifx-2ie)}}{d^3}\right)-4a^2\sqrt{-\frac{id^7}{16a^4f^2}}fe^{(4ifx+4ie)}\log\left(-\frac{2\left(id^4e^{(2ifx+2ie)}-4\left(a^2fe^{(2ifx+2ie)}+a^2f\right)\sqrt{-\frac{id^7}{16a^4f^2}}\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\right)e^{(-2ifx-2ie)}}{d^3}\right)-4a^2\sqrt{\frac{529id^7}{64a^4f^2}}fe^{(4ifx+4ie)}\log\left(\frac{\left(23id^4+8\left(a^2fe^{(2ifx+2ie)}+a^2f\right)\sqrt{\frac{529id^7}{64a^4f^2}}\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\right)e^{(-2ifx-2ie)}}{8a^2f}\right)+4a^2\sqrt{\frac{529id^7}{64a^4f^2}}fe^{(4ifx+4ie)}\log\left(\frac{\left(23id^4-8\left(a^2fe^{(2ifx+2ie)}+a^2f\right)\sqrt{\frac{529id^7}{64a^4f^2}}\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\right)e^{(-2ifx-2ie)}}{8a^2f}\right)+\left(42d^3e^{(4ifx+4ie)}+9d^3e^{(2ifx+2ie)}-d^3\right)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\right)e^{(-4ifx-4ie)}}{16a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")

[Out] -1/16*(4*a^2*sqrt(-1/16*I*d^7/(a^4*f^2))*f*e^(4*I*f*x + 4*I*e)*log(-2*(I*d^4*e^(2*I*f*x + 2*I*e) + 4*(a^2*f*e^(2*I*f*x + 2*I*e) + a^2*f)*sqrt(-1/16*I*d^7/(a^4*f^2))*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-2*I*f*x - 2*I*e)/d^3) - 4*a^2*sqrt(-1/16*I*d^7/(a^4*f^2))*f*e^(4*I*f*x + 4*I*e)*log(-2*(I*d^4*e^(2*I*f*x + 2*I*e) - 4*(a^2*f*e^(2*I*f*x + 2*I*e) + a^2*f)*sqrt(-1/16*I*d^7/(a^4*f^2))*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-2*I*f*x - 2*I*e)/d^3) - 4*a^2*sqrt(529/64*I*d^7/(a^4*f^2))*f*e^(4*I*f*x + 4*I*e)*log(1/8*(23*I*d^4 + 8*(a^2*f*e^(2*I*f*x + 2*I*e) + a^2*f)*sqrt(529/64*I*d^7/(a^4*f^2))*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-2*I*f*x - 2*I*e)/(a^2*f)) + 4*a^2*sqrt(529/64*I*d^7/(a^4*f^2))*f*e^(4*I*f*x + 4*I*e)*log(1/8*(23*I*d^4 - 8*(a^2*f*e^(2*I*f*x + 2*I*e) + a^2*f)*sqrt(529/64*I*d^7/(a^4*f^2))*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-2*I*f*x - 2*I*e)/(a^2*f)) + (42*d^3*e^(4*I*f*x + 4*I*e) + 9*d^3*e^(2*I*f*x + 2*I*e) - d^3)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-4*I*f*x - 4*I*e)/(a^2*f)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{(d\tan{(e+fx)})^{\frac{7}{2}}}{\tan^2{(e+fx)}-2i\tan{(e+fx)}-1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*tan(f*x+e))**(7/2)/(a+I*a*tan(f*x+e))**2,x)

[Out] -Integral((d*tan(e + f*x))**(7/2)/(tan(e + f*x)**2 - 2*I*tan(e + f*x) - 1), x)/a**2

Giac [A]
time = 0.67, size = 226, normalized size = 0.69

$$-\frac{1}{8}\,d^3\left(\frac{23\sqrt{2}\sqrt{d}\arctan\left(-\frac{8i\sqrt{d^2}\sqrt{d\tan(fx+e)}}{4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{a^2f\left(\frac{id}{\sqrt{d^2}}+1\right)}-\frac{2\sqrt{2}\sqrt{d}\arctan\left(-\frac{8i\sqrt{d^2}\sqrt{d\tan(fx+e)}}{-4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{a^2f\left(-\frac{id}{\sqrt{d^2}}+1\right)}+\frac{16\sqrt{d\tan(fx+e)}}{a^2f}-\frac{11i\sqrt{d\tan(fx+e)}\,d^2\tan(fx+e)+9\sqrt{d\tan(fx+e)}\,d^2}{(d\tan(fx+e)-i\,d)^2a^2f}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")

[Out] -1/8*d^3*(23*sqrt(2)*sqrt(d)*arctan(-8*I*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*I*sqrt(2)*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a^2*f*(I*d/sqrt(d^2) + 1)) - 2*sqrt(2)*sqrt(d)*arctan(-8*I*sqrt(d^2)*sqrt(d*tan(f*x + e))/(-4*I*sqrt(2)*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a^2*f*(-I*d/sqrt(d^2) + 1)) +

```
16*sqrt(d*tan(f*x + e))/(a^2*f) - (11*I*sqrt(d*tan(f*x + e))*d^2*tan(f*x +
e) + 9*sqrt(d*tan(f*x + e))*d^2)/((d*tan(f*x + e) - I*d)^2*a^2*f))
```

Mupad [B]
time = 5.56, size = 201, normalized size = 0.62

$$-\frac{\frac{9\,d^5\,\sqrt{d\tan\left(e+f\,x\right)}}{8\,a^2\,f}+\frac{d^4\,{\left(d\tan\left(e+f\,x\right)\right)}^{3/2}\,11\mathrm{i}}{8\,a^2\,f}}{-d^2\,{\tan\left(e+f\,x\right)}^2+d^2\,\tan\left(e+f\,x\right)\,2\mathrm{i}+d^2}-\frac{2\,d^3\,\sqrt{d\tan\left(e+f\,x\right)}}{a^2\,f}+\mathrm{atan}\left(\frac{8\,a^2\,f\,\sqrt{d\tan\left(e+f\,x\right)}\,\sqrt{-\frac{d^7\,1\mathrm{i}}{64\,a^4\,f^2}}}{d^4}\right)\,\sqrt{-\frac{d^7\,1\mathrm{i}}{64\,a^4\,f^2}}\,2\mathrm{i}-\mathrm{atan}\left(\frac{16\,a^2\,f\,\sqrt{d\tan\left(e+f\,x\right)}\,\sqrt{\frac{d^7\,529\mathrm{i}}{256\,a^4\,f^2}}}{23\,d^4}\right)\,\sqrt{\frac{d^7\,529\mathrm{i}}{256\,a^4\,f^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(7/2)/(a + a*tan(e + f*x)*1i)^2,x)
```

```
[Out] atan((8*a^2*f*(d*tan(e + f*x))^(1/2)*(-(d^7*1i)/(64*a^4*f^2))^(1/2))/d^4)*(
-(d^7*1i)/(64*a^4*f^2))^(1/2)*2i - ((9*d^5*(d*tan(e + f*x))^(1/2))/(8*a^2*f
) + (d^4*(d*tan(e + f*x))^(3/2)*11i)/(8*a^2*f))/(d^2*tan(e + f*x)*2i + d^2
- d^2*tan(e + f*x)^2) - atan((16*a^2*f*(d*tan(e + f*x))^(1/2)*((d^7*529i)/(
256*a^4*f^2))^(1/2))/(23*d^4))*((d^7*529i)/(256*a^4*f^2))^(1/2)*2i - (2*d^3
*(d*tan(e + f*x))^(1/2))/(a^2*f)
```

3.173 $\int \frac{(d\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=301

$$\frac{\left(\frac{9}{16}+\frac{5i}{16}\right)d^{5/2}\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^2f}-\frac{\left(\frac{9}{16}+\frac{5i}{16}\right)d^{5/2}\mathrm{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^2f}-\cdots$$

```
[Out] (9/32+5/32*I)*d^(5/2)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a^2/f*
2^(1/2)-(9/32+5/32*I)*d^(5/2)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2)
)/a^2/f*2^(1/2)+(-9/64+5/64*I)*d^(5/2)*ln(d^(1/2)-2^(1/2)*(d*tan(f*x+e))^(1
/2)+d^(1/2)*tan(f*x+e))/a^2/f*2^(1/2)+(9/64-5/64*I)*d^(5/2)*ln(d^(1/2)+2^(1
/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/a^2/f*2^(1/2)+5/8*I*d^2*(d*tan
(f*x+e))^(1/2)/a^2/f/(1+I*tan(f*x+e))-1/4*d*(d*tan(f*x+e))^(3/2)/f/(a+I*a*t
an(f*x+e))^2
```

Rubi [A]

time = 0.28, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {3639, 3676, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\left(\frac{9}{16}+\frac{5i}{16}\right)d^{5/2}\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^2f}-\frac{\left(\frac{9}{16}+\frac{5i}{16}\right)d^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,a^2f}-\frac{\left(\frac{9}{32}-\frac{5i}{32}\right)d^{5/2}\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,a^2f}+\frac{\left(\frac{9}{32}-\frac{5i}{32}\right)d^{5/2}\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,a^2f}+\frac{5id^2\sqrt{d\tan(e+fx)}}{8a^2f(1+i\tan(e+fx))}-\frac{d(d\tan(e+fx))^{3/2}}{4f(a+ia\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(5/2)/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] ((9/16 + (5*I)/16)*d^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d
]])/(Sqrt[2]*a^2*f) - ((9/16 + (5*I)/16)*d^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d
*Tan[e + f*x]])/Sqrt[d]])/(Sqrt[2]*a^2*f) - ((9/32 - (5*I)/32)*d^(5/2)*Log[
Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e + f*x]]])/(Sqrt[2]*a^
2*f) + ((9/32 - (5*I)/32)*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqrt
[2]*Sqrt[d*Tan[e + f*x]]])/(Sqrt[2]*a^2*f) + (((5*I)/8)*d^2*Sqrt[d*Tan[e +
f*x]])/(a^2*f*(1 + I*Tan[e + f*x])) - (d*(d*Tan[e + f*x])^(3/2))/(4*f*(a +
I*a*Tan[e + f*x])^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

```
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^2}\,dx = -\frac{d(d\tan(e+fx))^{3/2}}{4f(a+ia\tan(e+fx))^2} - \frac{\int \frac{\sqrt{d\tan(e+fx)}\left(-\frac{3ad^2}{2}+\frac{7}{2}iad^2\tan(e+fx)\right)}{a+ia\tan(e+fx)}\,dx}{4a^2}$$

$$= \frac{5id^2\sqrt{d\tan(e+fx)}}{8a^2f(1+i\tan(e+fx))} - \frac{d(d\tan(e+fx))^{3/2}}{4f(a+ia\tan(e+fx))^2} + \frac{\int \frac{-\frac{5}{2}ia^2d^3-\frac{9}{2}a^2d^3\tan(e+f}{\sqrt{d\tan(e+fx)}}}{8a^4}$$

$$= \frac{5id^2\sqrt{d\tan(e+fx)}}{8a^2f(1+i\tan(e+fx))} - \frac{d(d\tan(e+fx))^{3/2}}{4f(a+ia\tan(e+fx))^2} + \frac{\text{Subst}\left(\int \frac{-\frac{5}{2}ia^2d^4-\frac{9}{2}a^2d^3}{d^2+x^4}\right.}{}$$

$$= \frac{5id^2\sqrt{d\tan(e+fx)}}{8a^2f(1+i\tan(e+fx))} - \frac{d(d\tan(e+fx))^{3/2}}{4f(a+ia\tan(e+fx))^2} + -\frac{\left(\left(\frac{9}{16}+\frac{5i}{16}\right)d^3\right)\text{Subs}}{}$$

$$= \frac{5id^2\sqrt{d\tan(e+fx)}}{8a^2f(1+i\tan(e+fx))} - \frac{d(d\tan(e+fx))^{3/2}}{4f(a+ia\tan(e+fx))^2} + -\frac{\left(\left(\frac{9}{32}-\frac{5i}{32}\right)d^{5/2}\right)\text{Sub}}{}$$

$$= -\frac{\left(\frac{9}{32}-\frac{5i}{32}\right)d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{\sqrt{2}a^2f} + \frac{\left(\frac{9}{32}\right.}{}$$

$$= \frac{\left(\frac{9}{16}+\frac{5i}{16}\right)d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}a^2f} - \frac{\left(\frac{9}{16}+\frac{5i}{16}\right)d^{5/2}\tan^{-1}\left(1\right.}{\sqrt{2}}$$

Mathematica [A]
time = 1.07, size = 231, normalized size = 0.77

$$-\frac{d^3\sec^3(e+fx)\left(-7\cos(e+fx)+7\cos(3(e+fx))+5i\sin(e+fx)+(9-5i)\cos(2(e+fx))\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sqrt{\sin(2(e+fx))}+(9+5i)\text{ArcSin}(\cos(e+fx)-\sin(e+fx))(\cos(2(e+fx))+i\sin(2(e+fx)))\sqrt{\sin(2(e+fx))}+(5+9i)\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sin^{\frac{3}{2}}(2(e+fx))+5i\sin(3(e+fx))\right)}{32a^2f\sqrt{d\tan(e+fx)}(-i+\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(5/2)/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] -1/32*(d^3*Sec[e + f*x]^3*(-7*Cos[e + f*x] + 7*Cos[3*(e + f*x)] + (5*I)*Sin
[e + f*x] + (9 - 5*I)*Cos[2*(e + f*x)]*Log[Cos[e + f*x] + Sin[e + f*x] + Sq
rt[Sin[2*(e + f*x)]]]*Sqrt[Sin[2*(e + f*x)]] + (9 + 5*I)*ArcSin[Cos[e + f*x
] - Sin[e + f*x]]*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)])*Sqrt[Sin[2*(e + f
*x)]] + (5 + 9*I)*Log[Cos[e + f*x] + Sin[e + f*x] + Sqrt[Sin[2*(e + f*x)]]]
*Sin[2*(e + f*x)]^(3/2) + (5*I)*Sin[3*(e + f*x)]))/(a^2*f*Sqrt[d*Tan[e + f*
x]]*(-I + Tan[e + f*x])^2)
```

Maple [A]
time = 0.17, size = 108, normalized size = 0.36

method	result
derivativedivides	$\frac{2d^3\left(-\frac{\frac{7(d\tan(fx+e))^{\frac{3}{2}}}{2}-\frac{5id\sqrt{d\tan(fx+e)}}{2}}{8(id\tan(fx+e)+d)^2}-\frac{7\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{-id}}\right)}{16\sqrt{-id}}-\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e}}{\sqrt{id}}\right.}{8\sqrt{id}}\right.}{f\,a^2}$
default	$\frac{2d^3\left(-\frac{\frac{7(d\tan(fx+e))^{\frac{3}{2}}}{2}-\frac{5id\sqrt{d\tan(fx+e)}}{2}}{8(id\tan(fx+e)+d)^2}-\frac{7\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{-id}}\right)}{16\sqrt{-id}}-\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e}}{\sqrt{id}}\right.}{8\sqrt{id}}\right.}{f\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^2*d^3*(-1/8*(7/2*(d*tan(f*x+e))^(3/2)-5/2*I*d*(d*tan(f*x+e))^(1/2))/(
I*d*tan(f*x+e)+d)^2-7/16/(-I*d)^(1/2)*arctan((d*tan(f*x+e))^(1/2)/(-I*d)^(1
/2))-1/8/(I*d)^(1/2)*arctan((d*tan(f*x+e))^(1/2)/(I*d)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 604 vs. $2(229) = 458$.
time = 0.38, size = 604, normalized size = 2.01

$$\frac{4\,a^{2}f\sqrt{\frac{i\,d^{5}}{16\,a^{4}f^{2}}}\,e^{(4i\,fx+4i\,e)}\log\left(-\frac{2\left(i\,d^{3}e^{(2i\,fx+2i\,e)}+4\left(i\,a^{2}fe^{(2i\,fx+2i\,e)}+i\,a^{2}f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{i\,d^{5}}{16\,a^{4}f^{2}}}\right)e^{(-2i\,fx-2i\,e)}}{d^{2}}\right)-4\,a^{2}f\sqrt{\frac{i\,d^{5}}{16\,a^{4}f^{2}}}\,e^{(4i\,fx+4i\,e)}\log\left(-\frac{2\left(i\,d^{3}e^{(2i\,fx+2i\,e)}+4\left(-i\,a^{2}fe^{(2i\,fx+2i\,e)}-i\,a^{2}f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{i\,d^{5}}{16\,a^{4}f^{2}}}\right)e^{(-2i\,fx-2i\,e)}}{d^{2}}\right)+4\,a^{2}f\sqrt{-\frac{49i\,d^{5}}{64\,a^{4}f^{2}}}\,e^{(4i\,fx+4i\,e)}\log\left(\frac{\left(7\,d^{3}+8\left(a^{2}fe^{(2i\,fx+2i\,e)}+a^{2}f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{-\frac{49i\,d^{5}}{64\,a^{4}f^{2}}}\right)e^{(-2i\,fx-2i\,e)}}{8\,a^{2}f}\right)-4\,a^{2}f\sqrt{-\frac{49i\,d^{5}}{64\,a^{4}f^{2}}}\,e^{(4i\,fx+4i\,e)}\log\left(\frac{\left(7\,d^{3}-8\left(a^{2}fe^{(2i\,fx+2i\,e)}+a^{2}f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{-\frac{49i\,d^{5}}{64\,a^{4}f^{2}}}\right)e^{(-2i\,fx-2i\,e)}}{8\,a^{2}f}\right)+\left(6i\,d^{2}e^{(4i\,fx+4i\,e)}+5i\,d^{2}e^{(2i\,fx+2i\,e)}-i\,d^{2}\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\,e^{(-4i\,fx-4i\,e)}}{16\,a^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*a^2*f*sqrt(1/16*I*d^5/(a^4*f^2))*e^(4*I*f*x + 4*I*e)*log(-2*(I*d^3*
e^(2*I*f*x + 2*I*e) + 4*(I*a^2*f*e^(2*I*f*x + 2*I*e) + I*a^2*f)*sqrt((-I*d*
e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(1/16*I*d^5/(a^4*
f^2)))*e^(-2*I*f*x - 2*I*e)/d^2) - 4*a^2*f*sqrt(1/16*I*d^5/(a^4*f^2))*e^(4*
I*f*x + 4*I*e)*log(-2*(I*d^3*e^(2*I*f*x + 2*I*e) + 4*(-I*a^2*f*e^(2*I*f*x +
 2*I*e) - I*a^2*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*
e) + 1))*sqrt(1/16*I*d^5/(a^4*f^2)))*e^(-2*I*f*x - 2*I*e)/d^2) + 4*a^2*f*sq
rt(-49/64*I*d^5/(a^4*f^2))*e^(4*I*f*x + 4*I*e)*log(1/8*(7*d^3 + 8*(a^2*f*e^
(2*I*f*x + 2*I*e) + a^2*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*
x + 2*I*e) + 1))*sqrt(-49/64*I*d^5/(a^4*f^2)))*e^(-2*I*f*x - 2*I*e)/(a^2*f)
) - 4*a^2*f*sqrt(-49/64*I*d^5/(a^4*f^2))*e^(4*I*f*x + 4*I*e)*log(1/8*(7*d^3
 - 8*(a^2*f*e^(2*I*f*x + 2*I*e) + a^2*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I
*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-49/64*I*d^5/(a^4*f^2)))*e^(-2*I*f*x -
2*I*e)/(a^2*f)) + (6*I*d^2*e^(4*I*f*x + 4*I*e) + 5*I*d^2*e^(2*I*f*x + 2*I*e
) - I*d^2)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))
)*e^(-4*I*f*x - 4*I*e)/(a^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{(d\tan(e+fx))^{\frac{5}{2}}}{\tan^{2}(e+fx)-2i\tan(e+fx)-1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(5/2)/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -Integral((d*tan(e + f*x))**(5/2)/(tan(e + f*x)**2 - 2*I*tan(e + f*x) - 1),
 x)/a**2
```

Giac [A]
time = 0.75, size = 206, normalized size = 0.68

$$\frac{1}{8}\,d^{2}\left(\frac{7i\,\sqrt{2}\,\sqrt{d}\,\arctan\left(-\frac{8i\,\sqrt{d^{2}}\,\sqrt{d\tan(fx+e)}}{4i\,\sqrt{2}\,d^{\frac{3}{2}}+4\,\sqrt{2}\,\sqrt{d^{2}}\,\sqrt{d}}\right)}{a^{2}f\left(\frac{i\,d}{\sqrt{d^{2}}}+1\right)}+\frac{2i\,\sqrt{2}\,\sqrt{d}\,\arctan\left(-\frac{8i\,\sqrt{d^{2}}\,\sqrt{d\tan(fx+e)}}{-4i\,\sqrt{2}\,d^{\frac{3}{2}}+4\,\sqrt{2}\,\sqrt{d^{2}}\,\sqrt{d}}\right)}{a^{2}f\left(-\frac{i\,d}{\sqrt{d^{2}}}+1\right)}+\frac{7\,\sqrt{d\tan(fx+e)}\,d^{2}\tan(fx+e)-5i\,\sqrt{d\tan(fx+e)}\,d^{2}}{(d\tan(fx+e)-i\,d)^{2}a^{2}f}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/8*d^2*(7*I*sqrt(2)*sqrt(d)*arctan(-8*I*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*
I*sqrt(2)*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a^2*f*(I*d/sqrt(d^2) + 1
)) + 2*I*sqrt(2)*sqrt(d)*arctan(-8*I*sqrt(d^2)*sqrt(d*tan(f*x + e))/(-4*I*s
qrt(2)*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a^2*f*(-I*d/sqrt(d^2) + 1))
 + (7*sqrt(d*tan(f*x + e))*d^2*tan(f*x + e) - 5*I*sqrt(d*tan(f*x + e))*d^2)
/((d*tan(f*x + e) - I*d)^2*a^2*f))
```

Mupad [B]
time = 5.67, size = 177, normalized size = 0.59

$$\frac{-\frac{7\,d^3\,{\left(d\,\tan\left(e+f\,x\right)\right)}^{3/2}}{8\,a^2\,f}+\frac{d^4\,\sqrt{d\,\tan\left(e+f\,x\right)}\,5{}\mathrm{i}}{8\,a^2\,f}}{-d^2\,{\tan\left(e+f\,x\right)}^2+d^2\,\tan\left(e+f\,x\right)\,2{}\mathrm{i}+d^2}+2\,\mathrm{atanh}\left(\frac{8\,a^2\,f\,\sqrt{d\,\tan\left(e+f\,x\right)}\,\sqrt{\frac{d^5\,1{}\mathrm{i}}{64\,a^4\,f^2}}}{d^3}\right)\,\sqrt{\frac{d^5\,1{}\mathrm{i}}{64\,a^4\,f^2}}+2\,\mathrm{atanh}\left(\frac{16\,a^2\,f\,\sqrt{d\,\tan\left(e+f\,x\right)}\,\sqrt{-\frac{d^5\,49{}\mathrm{i}}{256\,a^4\,f^2}}}{7\,d^3}\right)\,\sqrt{-\frac{d^5\,49{}\mathrm{i}}{256\,a^4\,f^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(5/2)/(a + a*tan(e + f*x)*1i)^2,x)
```

```
[Out] ((d^4*(d*tan(e + f*x))^(1/2)*5i)/(8*a^2*f) - (7*d^3*(d*tan(e + f*x))^(3/2))
/(8*a^2*f))/(d^2*tan(e + f*x)*2i + d^2 - d^2*tan(e + f*x)^2) + 2*atanh((8*a
^2*f*(d*tan(e + f*x))^(1/2)*((d^5*1i)/(64*a^4*f^2))^(1/2))/d^3)*((d^5*1i)/(
64*a^4*f^2))^(1/2) + 2*atanh((16*a^2*f*(d*tan(e + f*x))^(1/2)*(-(d^5*49i)/(
256*a^4*f^2))^(1/2))/(7*d^3))*(-(d^5*49i)/(256*a^4*f^2))^(1/2)
```

3.174 $\int \frac{(d\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=297

$$\frac{\left(\frac{1}{16}+\frac{3i}{16}\right)d^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^2f}-\frac{\left(\frac{1}{16}+\frac{3i}{16}\right)d^{3/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^2f}+\cdots$$

```
[Out] (1/32+3/32*I)*d^(3/2)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a^2/f*
2^(1/2)-(1/32+3/32*I)*d^(3/2)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2)
)/a^2/f*2^(1/2)+(1/64-3/64*I)*d^(3/2)*ln(d^(1/2)-2^(1/2)*(d*tan(f*x+e))^(1/
2)+d^(1/2)*tan(f*x+e))/a^2/f*2^(1/2)+(-1/64+3/64*I)*d^(3/2)*ln(d^(1/2)+2^(1
/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/a^2/f*2^(1/2)+3/8*d*(d*tan(f*x
+e))^(1/2)/a^2/f/(1+I*tan(f*x+e))-1/4*d*(d*tan(f*x+e))^(1/2)/f/(a+I*a*tan(f
*x+e))^2
```

Rubi [A]
time = 0.28, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$,
Rules used = {3639, 3677, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\left(\frac{1}{16}+\frac{3i}{16}\right)d^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^2f}-\frac{\left(\frac{1}{16}+\frac{3i}{16}\right)d^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,a^2f}+\frac{\left(\frac{1}{32}-\frac{3i}{32}\right)d^{3/2}\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,a^2f}-\frac{\left(\frac{1}{32}-\frac{3i}{32}\right)d^{3/2}\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,a^2f}+\frac{3d\sqrt{d\tan(e+fx)}}{8a^2f(1+i\tan(e+fx))}-\frac{d\sqrt{d\tan(e+fx)}}{4f(a+ia\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(3/2)/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] ((1/16 + (3*I)/16)*d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d
]])/(Sqrt[2]*a^2*f) - ((1/16 + (3*I)/16)*d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d
*Tan[e + f*x]])/Sqrt[d]])/(Sqrt[2]*a^2*f) + ((1/32 - (3*I)/32)*d^(3/2)*Log[
Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e + f*x]]])/(Sqrt[2]*a^
2*f) - ((1/32 - (3*I)/32)*d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqrt
[2]*Sqrt[d*Tan[e + f*x]]])/(Sqrt[2]*a^2*f) + (3*d*Sqrt[d*Tan[e + f*x]])/(8*
a^2*f*(1 + I*Tan[e + f*x])) - (d*Sqrt[d*Tan[e + f*x]])/(4*f*(a + I*a*Tan[e
+ f*x])^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

```
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^2}\,dx = -\frac{d\sqrt{d\tan(e+fx)}}{4f(a+ia\tan(e+fx))^2} - \frac{\int \frac{-\frac{ad^2}{2}+\frac{5}{2}iad^2\tan(e+fx)}{\sqrt{d\tan(e+fx)}\,(a+ia\tan(e+fx))}\,dx}{4a^2}$$

$$= \frac{3d\sqrt{d\tan(e+fx)}}{8a^2f(1+i\tan(e+fx))} - \frac{d\sqrt{d\tan(e+fx)}}{4f(a+ia\tan(e+fx))^2} - \frac{\int \frac{\frac{a^2d^3}{2}+\frac{3}{2}ia^2d^3\tan(e+fx)}{\sqrt{d\tan(e+fx)}} \cdots}{8a^4d}$$

$$= \frac{3d\sqrt{d\tan(e+fx)}}{8a^2f(1+i\tan(e+fx))} - \frac{d\sqrt{d\tan(e+fx)}}{4f(a+ia\tan(e+fx))^2} - \frac{\text{Subst}\left(\int \frac{\frac{a^2d^4}{2}+\frac{3}{2}ia^2d^3x^2 \cdots}{d^2+x^4} \cdots\right.}{4 \cdots}$$

$$= \frac{3d\sqrt{d\tan(e+fx)}}{8a^2f(1+i\tan(e+fx))} - \frac{d\sqrt{d\tan(e+fx)}}{4f(a+ia\tan(e+fx))^2} - \frac{\left(\left(\frac{1}{16}-\frac{3i}{16}\right)d^2\right)\text{Subst}\left(\cdots\right.}{\cdots}$$

$$= \frac{3d\sqrt{d\tan(e+fx)}}{8a^2f(1+i\tan(e+fx))} - \frac{d\sqrt{d\tan(e+fx)}}{4f(a+ia\tan(e+fx))^2} - -\frac{\left(\left(\frac{1}{32}-\frac{3i}{32}\right)d^{3/2}\right)\text{Sub}\cdots}{\cdots}$$

$$= \frac{\left(\frac{1}{32}-\frac{3i}{32}\right)d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{\sqrt{2}\,a^2f} - \frac{\left(\frac{1}{32} \cdots\right.}{\cdots}$$

$$= \frac{\left(\frac{1}{16}+\frac{3i}{16}\right)d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^2f} - \frac{\left(\frac{1}{16}+\frac{3i}{16}\right)d^{3/2}\tan^{-1}\left(1 \cdots\right.}{\sqrt{2} \cdots}$$

Mathematica [A]
time = 1.08, size = 231, normalized size = 0.78

$$\frac{d^2\sec^3(e+fx)\left(-3i\cos(e+fx)+3i\cos(3(e+fx))-\sin(e+fx)+(1-3i)\cos(2(e+fx))\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sqrt{\sin(2(e+fx))}+(3+i)\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sin^{\frac{3}{2}}(2(e+fx))+(3-i)\text{ArcSin}(\cos(e+fx)-\sin(e+fx))\sqrt{\sin(2(e+fx))}\,(-i\cos(2(e+fx))+\sin(2(e+fx)))-\sin(3(e+fx))\right)}{32a^2f\sqrt{d\tan(e+fx)}\,(-i+\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(3/2)/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (d^2*Sec[e + f*x]^3*((-3*I)*Cos[e + f*x] + (3*I)*Cos[3*(e + f*x)] - Sin[e +
 f*x] + (1 - 3*I)*Cos[2*(e + f*x)]*Log[Cos[e + f*x] + Sin[e + f*x] + Sqrt[S
in[2*(e + f*x)]]]*Sqrt[Sin[2*(e + f*x)]] + (3 + I)*Log[Cos[e + f*x] + Sin[e
 + f*x] + Sqrt[Sin[2*(e + f*x)]]]*Sin[2*(e + f*x)]^(3/2) + (3 - I)*ArcSin[C
os[e + f*x] - Sin[e + f*x]]*Sqrt[Sin[2*(e + f*x)]]*((-I)*Cos[2*(e + f*x)] +
 Sin[2*(e + f*x)]) - Sin[3*(e + f*x)]))/(32*a^2*f*Sqrt[d*Tan[e + f*x]]*(-I
+ Tan[e + f*x])^2)
```

Maple [A]
time = 0.17, size = 118, normalized size = 0.40

method	result
derivativedivides	$\frac{2d^3\left(-\frac{\frac{-\frac{3i(d\tan(fx+e))^{\frac{3}{2}}}{2}-\frac{d\sqrt{d\tan(fx+e)}}{2}}{(id\tan(fx+e)+d)^2}+\frac{i\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{-id}}\right)}{2\sqrt{-id}}}{8d}-\frac{i\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{id}}\right)}{8d\sqrt{id}}\right)}{f\,a^2}$
default	$\frac{2d^3\left(-\frac{\frac{-\frac{3i(d\tan(fx+e))^{\frac{3}{2}}}{2}-\frac{d\sqrt{d\tan(fx+e)}}{2}}{(id\tan(fx+e)+d)^2}+\frac{i\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{-id}}\right)}{2\sqrt{-id}}}{8d}-\frac{i\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{id}}\right)}{8d\sqrt{id}}\right)}{f\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^2*d^3*(-1/8/d*((-3/2*I*(d*tan(f*x+e))^(3/2)-1/2*d*(d*tan(f*x+e))^(1/2
))/(I*d*tan(f*x+e)+d)^2+1/2*I/(-I*d)^(1/2)*arctan((d*tan(f*x+e))^(1/2)/(-I*
d)^(1/2)))-1/8*I/d/(I*d)^(1/2)*arctan((d*tan(f*x+e))^(1/2)/(I*d)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 593 vs. $2(227) = 454$.
time = 0.38, size = 593, normalized size = 2.00

$$\frac{4\,a^2 f\sqrt{-\frac{i\,d^3}{16\,a^4 f^2}}\,e^{(4i\,fx+4i\,e)}\log\left(-\frac{2\left(i\,d^2 e^{(2i\,fx+2i\,e)}+4\left(a^2 f e^{(2i\,fx+2i\,e)}+a^2 f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{-\frac{i\,d^3}{16\,a^4 f^2}}\right)e^{(-2i\,fx-2i\,e)}}{d}\right)-4\,a^2 f\sqrt{-\frac{i\,d^3}{16\,a^4 f^2}}\,e^{(4i\,fx+4i\,e)}\log\left(-\frac{2\left(i\,d^2 e^{(2i\,fx+2i\,e)}-4\left(a^2 f e^{(2i\,fx+2i\,e)}+a^2 f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{-\frac{i\,d^3}{16\,a^4 f^2}}\right)e^{(-2i\,fx-2i\,e)}}{d}\right)+4\,a^2 f\sqrt{\frac{i\,d^3}{64\,a^4 f^2}}\,e^{(4i\,fx+4i\,e)}\log\left(\frac{\left(i\,d^2+8\left(a^2 f e^{(2i\,fx+2i\,e)}+a^2 f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{i\,d^3}{64\,a^4 f^2}}\right)e^{(-2i\,fx-2i\,e)}}{8\,a^2 f}\right)-4\,a^2 f\sqrt{\frac{i\,d^3}{64\,a^4 f^2}}\,e^{(4i\,fx+4i\,e)}\log\left(\frac{\left(i\,d^2-8\left(a^2 f e^{(2i\,fx+2i\,e)}+a^2 f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{i\,d^3}{64\,a^4 f^2}}\right)e^{(-2i\,fx-2i\,e)}}{8\,a^2 f}\right)+\left(2\,de^{(4i\,fx+4i\,e)}+de^{(2i\,fx+2i\,e)}-d\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\,e^{(-4i\,fx-4i\,e)}}{16\,a^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*a^2*f*sqrt(-1/16*I*d^3/(a^4*f^2))*e^(4*I*f*x + 4*I*e)*log(-2*(I*d^2
*e^(2*I*f*x + 2*I*e) + 4*(a^2*f*e^(2*I*f*x + 2*I*e) + a^2*f)*sqrt((-I*d*e^(
2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-1/16*I*d^3/(a^4*f^
2)))*e^(-2*I*f*x - 2*I*e)/d) - 4*a^2*f*sqrt(-1/16*I*d^3/(a^4*f^2))*e^(4*I*f
*x + 4*I*e)*log(-2*(I*d^2*e^(2*I*f*x + 2*I*e) - 4*(a^2*f*e^(2*I*f*x + 2*I*e
) + a^2*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))
*sqrt(-1/16*I*d^3/(a^4*f^2)))*e^(-2*I*f*x - 2*I*e)/d) + 4*a^2*f*sqrt(1/64*I
*d^3/(a^4*f^2))*e^(4*I*f*x + 4*I*e)*log(1/8*(I*d^2 + 8*(a^2*f*e^(2*I*f*x +
2*I*e) + a^2*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e)
+ 1))*sqrt(1/64*I*d^3/(a^4*f^2)))*e^(-2*I*f*x - 2*I*e)/(a^2*f)) - 4*a^2*f*s
qrt(1/64*I*d^3/(a^4*f^2))*e^(4*I*f*x + 4*I*e)*log(1/8*(I*d^2 - 8*(a^2*f*e^(
2*I*f*x + 2*I*e) + a^2*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x
 + 2*I*e) + 1))*sqrt(1/64*I*d^3/(a^4*f^2)))*e^(-2*I*f*x - 2*I*e)/(a^2*f)) +
 (2*d*e^(4*I*f*x + 4*I*e) + d*e^(2*I*f*x + 2*I*e) - d)*sqrt((-I*d*e^(2*I*f*
x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-4*I*f*x - 4*I*e)/(a^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{(d\tan(e+fx))^{\frac{3}{2}}}{\tan^2(e+fx)-2i\tan(e+fx)-1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(3/2)/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -Integral((d*tan(e + f*x))**(3/2)/(tan(e + f*x)**2 - 2*I*tan(e + f*x) - 1),
 x)/a**2
```

Giac [A]
time = 0.67, size = 203, normalized size = 0.68

$$\frac{1}{8}\,d\left(\frac{\sqrt{2}\,\sqrt{d}\,\arctan\left(\frac{8i\,\sqrt{d^2}\,\sqrt{d\tan(fx+e)}}{4i\,\sqrt{2}\,d^{\frac{3}{2}}+4\,\sqrt{2}\,\sqrt{d^2}\,\sqrt{d}}\right)}{a^2 f\left(\frac{i\,d}{\sqrt{d^2}}+1\right)}+\frac{2\,\sqrt{2}\,\sqrt{d}\,\arctan\left(\frac{8i\,\sqrt{d^2}\,\sqrt{d\tan(fx+e)}}{-4i\,\sqrt{2}\,d^{\frac{3}{2}}+4\,\sqrt{2}\,\sqrt{d^2}\,\sqrt{d}}\right)}{a^2 f\left(-\frac{i\,d}{\sqrt{d^2}}+1\right)}+\frac{-3i\,\sqrt{d\tan(fx+e)}\,d^2\tan(fx+e)-\sqrt{d\tan(fx+e)}\,d^2}{\left(d\tan(fx+e)-i\,d\right)^2 a^2 f}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/8*d*(sqrt(2)*sqrt(d)*arctan(8*I*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*I*sqrt(
2)*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a^2*f*(I*d/sqrt(d^2) + 1)) + 2*
sqrt(2)*sqrt(d)*arctan(8*I*sqrt(d^2)*sqrt(d*tan(f*x + e))/(-4*I*sqrt(2)*d^(
3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a^2*f*(-I*d/sqrt(d^2) + 1)) + (-3*I*s
qrt(d*tan(f*x + e))*d^2*tan(f*x + e) - sqrt(d*tan(f*x + e))*d^2)/((d*tan(f*
x + e) - I*d)^2*a^2*f))
```

Mupad [B]
time = 5.55, size = 179, normalized size = 0.60

$$\frac{\frac{d^3\sqrt{d\tan(e+fx)}}{8a^2f}+\frac{d^2(d\tan(e+fx))^{3/2}3\mathrm{i}}{8a^2f}}{-d^2\tan(e+fx)^2+d^2\tan(e+fx)\,2\mathrm{i}+d^2}-\mathrm{atan}\left(\frac{8a^2f\sqrt{d\tan(e+fx)}\sqrt{-\frac{d^3\,1\mathrm{i}}{64a^4f^2}}}{d^2}\right)\sqrt{-\frac{d^3\,1\mathrm{i}}{64a^4f^2}}\,2\mathrm{i}-\mathrm{atan}\left(\frac{16a^2f\sqrt{d\tan(e+fx)}\sqrt{\frac{d^3\,1\mathrm{i}}{256a^4f^2}}}{d^2}\right)\sqrt{\frac{d^3\,1\mathrm{i}}{256a^4f^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(3/2)/(a + a*tan(e + f*x)*1i)^2,x)
```

```
[Out] ((d^3*(d*tan(e + f*x))^(1/2))/(8*a^2*f) + (d^2*(d*tan(e + f*x))^(3/2)*3i)/(
8*a^2*f))/(d^2*tan(e + f*x)*2i + d^2 - d^2*tan(e + f*x)^2) - atan((8*a^2*f*
(d*tan(e + f*x))^(1/2)*(-(d^3*1i)/(64*a^4*f^2))^(1/2))/d^2)*(-(d^3*1i)/(64*
a^4*f^2))^(1/2)*2i - atan((16*a^2*f*(d*tan(e + f*x))^(1/2)*((d^3*1i)/(256*a
^4*f^2))^(1/2))/d^2)*((d^3*1i)/(256*a^4*f^2))^(1/2)*2i
```

3.175 $\int \frac{\sqrt{d\tan(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=299

$$-\frac{\left(\frac{1}{16}-\frac{3i}{16}\right)\sqrt{d}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^2f}+\frac{\left(\frac{1}{16}-\frac{3i}{16}\right)\sqrt{d}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^2f}-$$

```
[Out] (-1/32+3/32*I)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/a^2/f
*2^(1/2)+(1/32-3/32*I)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))*d^(1/
2)/a^2/f*2^(1/2)+(1/64+3/64*I)*ln(d^(1/2)-2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1
/2)*tan(f*x+e))*d^(1/2)/a^2/f*2^(1/2)-(1/64+3/64*I)*ln(d^(1/2)+2^(1/2)*(d*t
an(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))*d^(1/2)/a^2/f*2^(1/2)+1/8*I*(d*tan(f*x
+e))^(1/2)/a^2/f/(1+I*tan(f*x+e))+1/4*I*(d*tan(f*x+e))^(1/2)/f/(a+I*a*tan(f
*x+e))^2
```

Rubi [A]
time = 0.25, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {3638, 3677, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\left(\frac{1}{16}-\frac{3i}{16}\right)\sqrt{d}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^2f}+\frac{\left(\frac{1}{16}-\frac{3i}{16}\right)\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,a^2f}+\frac{i\sqrt{d\tan(e+fx)}}{8a^2f(1+i\tan(e+fx))}+\frac{\left(\frac{1}{32}+\frac{3i}{32}\right)\sqrt{d}\,\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,a^2f}-\frac{\left(\frac{1}{32}+\frac{3i}{32}\right)\sqrt{d}\,\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,a^2f}+\frac{i\sqrt{d\tan(e+fx)}}{4f(a+ia\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Tan[e + f*x]]/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] ((-1/16 + (3*I)/16)*Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[
d]])/(Sqrt[2]*a^2*f) + ((1/16 - (3*I)/16)*Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[
d*Tan[e + f*x]])/Sqrt[d]])/(Sqrt[2]*a^2*f) + ((1/32 + (3*I)/32)*Sqrt[d]*Log
[Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e + f*x]]])/(Sqrt[2]*a
^2*f) - ((1/32 + (3*I)/32)*Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqr
t[2]*Sqrt[d*Tan[e + f*x]]])/(Sqrt[2]*a^2*f) + ((I/8)*Sqrt[d*Tan[e + f*x]])/
(a^2*f*(1 + I*Tan[e + f*x])) + ((I/4)*Sqrt[d*Tan[e + f*x]])/(f*(a + I*a*Tan
[e + f*x])^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3638

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*Sqrt[(c_.) + (d_.)*tan[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-b)*(a + b*Tan[e + f*x])^m*(Sqrt[c + d
*Tan[e + f*x]]/(2*a*f*m)), x] + Dist[1/(4*a^2*m), Int[(a + b*Tan[e + f*x])^
(m + 1)*(Simp[2*a*c*m + b*d + a*d*(2*m + 1)*Tan[e + f*x], x]/Sqrt[c + d*Tan
[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && IntegersQ[2*m]
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{d\tan(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx = \frac{i\sqrt{d\tan(e+fx)}}{4f(a+ia\tan(e+fx))^2} - \frac{\int \frac{iad-3ad\tan(e+fx)}{\sqrt{d\tan(e+fx)}\,(a+ia\tan(e+fx))}\,dx}{8a^2}$$

$$= \frac{i\sqrt{d\tan(e+fx)}}{8a^2f(1+i\tan(e+fx))} + \frac{i\sqrt{d\tan(e+fx)}}{4f(a+ia\tan(e+fx))^2} - \frac{\int \frac{3ia^2d^2-a^2d^2\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,d\cdots}{16a^4d}$$

$$= \frac{i\sqrt{d\tan(e+fx)}}{8a^2f(1+i\tan(e+fx))} + \frac{i\sqrt{d\tan(e+fx)}}{4f(a+ia\tan(e+fx))^2} - \frac{\text{Subst}\left(\int \frac{3ia^2d^3-a^2d^2x^2}{d^2+x^4}\cdots\right.}{8\cdots}$$

$$= \frac{i\sqrt{d\tan(e+fx)}}{8a^2f(1+i\tan(e+fx))} + \frac{i\sqrt{d\tan(e+fx)}}{4f(a+ia\tan(e+fx))^2} - -\frac{\left(\left(\frac{1}{16}-\frac{3i}{16}\right)d\right)\text{Subst}\cdots}{\cdots}$$

$$= \frac{i\sqrt{d\tan(e+fx)}}{8a^2f(1+i\tan(e+fx))} + \frac{i\sqrt{d\tan(e+fx)}}{4f(a+ia\tan(e+fx))^2} - -\frac{\left(\left(\frac{1}{32}+\frac{3i}{32}\right)\sqrt{d}\right)\text{Su}\cdots}{\cdots}$$

$$= \frac{\left(\frac{1}{32}+\frac{3i}{32}\right)\sqrt{d}\,\log\left(\sqrt{d}+\sqrt{d}\,\tan(e+fx)-\sqrt{2}\,\sqrt{d\tan(e+fx)}\right)}{\sqrt{2}\,a^2f} - \frac{\left(\frac{1}{32}\cdots\right.}{\cdots}$$

$$= -\frac{\left(\frac{1}{16}-\frac{3i}{16}\right)\sqrt{d}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^2f} + \frac{\left(\frac{1}{16}-\frac{3i}{16}\right)\sqrt{d}\,\tan^{-1}\left(\cdots\right.}{\cdots}$$

Mathematica [A]
time = 0.95, size = 227, normalized size = 0.76

$$-\frac{d\sec^3(e+fx)\left(-\cos(e+fx)+\cos(3(e+fx))+3i\sin(e+fx)-(1+3i)\cos(2(e+fx))\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sqrt{\sin(2(e+fx))}+(3-i)\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sin^{\frac{3}{2}}(2(e+fx))-(3+i)\text{ArcSin}(\cos(e+fx)-\sin(e+fx))\sqrt{\sin(2(e+fx))}\,(-i\cos(2(e+fx))+\sin(2(e+fx)))+3i\sin(3(e+fx))\right)}{32a^2f\sqrt{d\tan(e+fx)}\,(-i+\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Tan[e + f*x]]/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] -1/32*(d*Sec[e + f*x]^3*(-Cos[e + f*x] + Cos[3*(e + f*x)] + (3*I)*Sin[e + f
*x] - (1 + 3*I)*Cos[2*(e + f*x)]*Log[Cos[e + f*x] + Sin[e + f*x] + Sqrt[Sin
[2*(e + f*x)]]]*Sqrt[Sin[2*(e + f*x)]] + (3 - I)*Log[Cos[e + f*x] + Sin[e +
 f*x] + Sqrt[Sin[2*(e + f*x)]]]*Sin[2*(e + f*x)]^(3/2) - (3 + I)*ArcSin[Cos
[e + f*x] - Sin[e + f*x]]*Sqrt[Sin[2*(e + f*x)]]*((-I)*Cos[2*(e + f*x)] + S
in[2*(e + f*x)]) + (3*I)*Sin[3*(e + f*x)]))/(a^2*f*Sqrt[d*Tan[e + f*x]]*(-I
 + Tan[e + f*x])^2)
```

Maple [A]
time = 0.19, size = 116, normalized size = 0.39

method	result
derivativedivides	$\frac{2d^3\left(-\frac{\frac{\frac{(d\tan(fx+e))^{\frac{3}{2}}}{2}-\frac{3id\sqrt{d\tan(fx+e)}}{2}}{(id\tan(fx+e)+d)^2}+\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{-id}}\right)}{2\sqrt{-id}}}{8d^2}+\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{id}}\right)}{8d^2\sqrt{id}}\right)}{f\,a^2}$
default	$\frac{2d^3\left(-\frac{\frac{\frac{(d\tan(fx+e))^{\frac{3}{2}}}{2}-\frac{3id\sqrt{d\tan(fx+e)}}{2}}{(id\tan(fx+e)+d)^2}+\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{-id}}\right)}{2\sqrt{-id}}}{8d^2}+\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{id}}\right)}{8d^2\sqrt{id}}\right)}{f\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^2*d^3*(-1/8/d^2*((1/2*(d*tan(f*x+e))^(3/2)-3/2*I*d*(d*tan(f*x+e))^(1/
2))/(I*d*tan(f*x+e)+d)^2+1/2/(-I*d)^(1/2)*arctan((d*tan(f*x+e))^(1/2)/(-I*d
)^(1/2)))+1/8/d^2/(I*d)^(1/2)*arctan((d*tan(f*x+e))^(1/2)/(I*d)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 563 vs. 2(225) = 450.
time = 0.39, size = 563, normalized size = 1.88

$$\frac{\left(4a^2f\sqrt{\frac{id}{16a^4f^2}}e^{(4ifx+4ie)}\log\left(-2\left(4\left(ia^2fe^{(2ifx+2ie)}+ia^2f\right)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{id}{16a^4f^2}}+ide^{(2ifx+2ie)}\right)e^{(-2ifx-2ie)}\right)-4a^2f\sqrt{\frac{id}{16a^4f^2}}e^{(4ifx+4ie)}\log\left(-2\left(4\left(-ia^2fe^{(2ifx+2ie)}-ia^2f\right)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{id}{16a^4f^2}}+ide^{(2ifx+2ie)}\right)e^{(-2ifx-2ie)}\right)-4a^2f\sqrt{-\frac{id}{64a^4f^2}}e^{(4ifx+4ie)}\log\left(\frac{\left(8\left(a^2fe^{(2ifx+2ie)}+a^2f\right)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\sqrt{-\frac{id}{64a^4f^2}}+d\right)e^{(-2ifx-2ie)}}{8a^2f}\right)+4a^2f\sqrt{-\frac{id}{64a^4f^2}}e^{(4ifx+4ie)}\log\left(-\frac{\left(8\left(a^2fe^{(2ifx+2ie)}+a^2f\right)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\sqrt{-\frac{id}{64a^4f^2}}-d\right)e^{(-2ifx-2ie)}}{8a^2f}\right)-\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\left(2ie^{(4ifx+4ie)}+3ie^{(2ifx+2ie)}+i\right)\right)e^{(-4ifx-4ie)}}{16a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*a^2*f*sqrt(1/16*I*d/(a^4*f^2))*e^(4*I*f*x + 4*I*e)*log(-2*(4*(I*a^
2*f*e^(2*I*f*x + 2*I*e) + I*a^2*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e
^(2*I*f*x + 2*I*e) + 1))*sqrt(1/16*I*d/(a^4*f^2)) + I*d*e^(2*I*f*x + 2*I*e)
)*e^(-2*I*f*x - 2*I*e)) - 4*a^2*f*sqrt(1/16*I*d/(a^4*f^2))*e^(4*I*f*x + 4*I
*e)*log(-2*(4*(-I*a^2*f*e^(2*I*f*x + 2*I*e) - I*a^2*f)*sqrt((-I*d*e^(2*I*f*
x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(1/16*I*d/(a^4*f^2)) + I*d
*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)) - 4*a^2*f*sqrt(-1/64*I*d/(a^4*f
^2))*e^(4*I*f*x + 4*I*e)*log(1/8*(8*(a^2*f*e^(2*I*f*x + 2*I*e) + a^2*f)*sqr
t((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-1/64*I*
d/(a^4*f^2)) + d)*e^(-2*I*f*x - 2*I*e)/(a^2*f)) + 4*a^2*f*sqrt(-1/64*I*d/(a
^4*f^2))*e^(4*I*f*x + 4*I*e)*log(-1/8*(8*(a^2*f*e^(2*I*f*x + 2*I*e) + a^2*f
)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-1/
64*I*d/(a^4*f^2)) - d)*e^(-2*I*f*x - 2*I*e)/(a^2*f)) - sqrt((-I*d*e^(2*I*f*
x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*(2*I*e^(4*I*f*x + 4*I*e) + 3*I
*e^(2*I*f*x + 2*I*e) + I))*e^(-4*I*f*x - 4*I*e)/(a^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt{d\tan(e+fx)}}{\tan^2(e+fx)-2i\tan(e+fx)-1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -Integral(sqrt(d*tan(e + f*x))/(tan(e + f*x)**2 - 2*I*tan(e + f*x) - 1), x)
/a**2
```

Giac [A]
time = 0.61, size = 205, normalized size = 0.69

$$\frac{\frac{i\sqrt{2}\,d^{\frac{3}{2}}\arctan\left(-\frac{8i\sqrt{d^2}\sqrt{d\tan(fx+e)}}{4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{a^2f\left(\frac{id}{\sqrt{d^2}}+1\right)}-\frac{2i\sqrt{2}\,d^{\frac{3}{2}}\arctan\left(-\frac{8i\sqrt{d^2}\sqrt{d\tan(fx+e)}}{-4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{a^2f\left(-\frac{id}{\sqrt{d^2}}+1\right)}+\frac{\sqrt{d\tan(fx+e)}\,d^3\tan(fx+e)-3i\sqrt{d\tan(fx+e)}\,d^3}{(d\tan(fx+e)-id)^2a^2f}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/8*(I*sqrt(2)*d^(3/2)*arctan(-8*I*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*I*sqrt
(2)*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a^2*f*(I*d/sqrt(d^2) + 1)) - 2
*I*sqrt(2)*d^(3/2)*arctan(-8*I*sqrt(d^2)*sqrt(d*tan(f*x + e))/(-4*I*sqrt(2)
*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a^2*f*(-I*d/sqrt(d^2) + 1)) + (sq
rt(d*tan(f*x + e))*d^3*tan(f*x + e) - 3*I*sqrt(d*tan(f*x + e))*d^3)/((d*tan
(f*x + e) - I*d)^2*a^2*f))/d
```

Mupad [B]
time = 5.20, size = 147, normalized size = 0.49

$$\frac{-\frac{d\,(d\tan(e+f\,x))^{3/2}}{8\,a^2\,f}+\frac{d^2\,\sqrt{d\tan\left(e+f\,x\right)}\,3\mathrm{i}}{8\,a^2\,f}}{-d^2\tan\left(e+f\,x\right)^2+d^2\tan\left(e+f\,x\right)\,2\mathrm{i}+d^2}+\frac{\mathrm{atanh}\left(\frac{2\,\sqrt{d\tan\left(e+f\,x\right)}\,\sqrt{-\frac{d\,1\mathrm{i}}{4}}}{d}\right)\,\sqrt{-\frac{d\,1\mathrm{i}}{4}}}{4\,a^2\,f}-\frac{\mathrm{atanh}\left(\frac{4\,\sqrt{d\tan\left(e+f\,x\right)}\,\sqrt{\frac{d\,1\mathrm{i}}{16}}}{d}\right)\,\sqrt{\frac{d\,1\mathrm{i}}{16}}}{a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(1/2)/(a + a*tan(e + f*x)*1i)^2,x)
```

```
[Out] ((d^2*(d*tan(e + f*x))^(1/2)*3i)/(8*a^2*f) - (d*(d*tan(e + f*x))^(3/2))/(8*
a^2*f))/(d^2*tan(e + f*x)*2i + d^2 - d^2*tan(e + f*x)^2) + (atanh((2*(d*tan
(e + f*x))^(1/2)*(-(d*1i)/4)^(1/2))/d)*(-(d*1i)/4)^(1/2))/(4*a^2*f) - (atan
h((4*(d*tan(e + f*x))^(1/2)*((d*1i)/16)^(1/2))/d)*((d*1i)/16)^(1/2))/(a^2*f
)
```

3.176 $\int \frac{1}{\sqrt{d\tan(e+fx)}\,(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=301

$$-\frac{\left(\frac{9}{16}-\frac{5i}{16}\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^2\sqrt{d}\,f}+\frac{\left(\frac{9}{16}-\frac{5i}{16}\right)\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^2\sqrt{d}\,f}-\frac{\left(\frac{9}{32}+\right.}{}$$

```
[Out] (-9/32+5/32*I)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a^2/f*2^(1/2)
/d^(1/2)+(9/32-5/32*I)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a^2/f
*2^(1/2)/d^(1/2)-(9/64+5/64*I)*ln(d^(1/2)-2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1
/2)*tan(f*x+e))/a^2/f*2^(1/2)/d^(1/2)+(9/64+5/64*I)*ln(d^(1/2)+2^(1/2)*(d*t
an(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/a^2/f*2^(1/2)/d^(1/2)+5/8*(d*tan(f*x+e
))^(1/2)/a^2/d/f/(1+I*tan(f*x+e))+1/4*(d*tan(f*x+e))^(1/2)/d/f/(a+I*a*tan(f
*x+e))^2
```

Rubi [A]
time = 0.27, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {3640, 3677, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\left(\frac{9}{16}-\frac{5i}{16}\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^2\sqrt{d}\,f}+\frac{\left(\frac{9}{16}-\frac{5i}{16}\right)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,a^2\sqrt{d}\,f}+\frac{5\sqrt{d\tan(e+fx)}}{8a^2df(1+i\tan(e+fx))}-\frac{\left(\frac{9}{32}+\frac{5i}{32}\right)\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\,\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,a^2\sqrt{d}\,f}+\frac{\left(\frac{9}{32}+\frac{5i}{32}\right)\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\,\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,a^2\sqrt{d}\,f}+\frac{\sqrt{d\tan(e+fx)}}{4df(a+ia\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*Tan[e + f*x]]*(a + I*a*Tan[e + f*x])^2),x]
```

```
[Out] ((-9/16 + (5*I)/16)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(Sq
rt[2]*a^2*Sqrt[d]*f) + ((9/16 - (5*I)/16)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e
+ f*x]])/Sqrt[d]])/(Sqrt[2]*a^2*Sqrt[d]*f) - ((9/32 + (5*I)/32)*Log[Sqrt[d]
 + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e + f*x]]])/(Sqrt[2]*a^2*Sqrt[
d]*f) + ((9/32 + (5*I)/32)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqrt[2]*Sqr
t[d*Tan[e + f*x]]])/(Sqrt[2]*a^2*Sqrt[d]*f) + (5*Sqrt[d*Tan[e + f*x]])/(8*a
^2*d*f*(1 + I*Tan[e + f*x])) + Sqrt[d*Tan[e + f*x]]/(4*d*f*(a + I*a*Tan[e +
 f*x])^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

```
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{d\tan(e+fx)}\,(a+ia\tan(e+fx))^2}\,dx = \frac{\sqrt{d\tan(e+fx)}}{4df(a+ia\tan(e+fx))^2} + \frac{\int \frac{\frac{7ad}{2}-\frac{3}{2}iad\tan(e+fx)}{\sqrt{d\tan(e+fx)}\,(a+ia\tan(e+fx}}{4a^2d}$$

$$= \frac{5\sqrt{d\tan(e+fx)}}{8a^2df(1+i\tan(e+fx))} + \frac{\sqrt{d\tan(e+fx)}}{4df(a+ia\tan(e+fx))^2} + \int$$

$$= \frac{5\sqrt{d\tan(e+fx)}}{8a^2df(1+i\tan(e+fx))} + \frac{\sqrt{d\tan(e+fx)}}{4df(a+ia\tan(e+fx))^2} + \text{Sul}$$

$$= \frac{5\sqrt{d\tan(e+fx)}}{8a^2df(1+i\tan(e+fx))} + \frac{\sqrt{d\tan(e+fx)}}{4df(a+ia\tan(e+fx))^2} + \frac{(\frac{9}{16}}{}$$

$$= \frac{5\sqrt{d\tan(e+fx)}}{8a^2df(1+i\tan(e+fx))} + \frac{\sqrt{d\tan(e+fx)}}{4df(a+ia\tan(e+fx))^2} + \frac{(\frac{9}{32}}{}$$

$$= -\frac{\left(\frac{9}{32}+\frac{5i}{32}\right)\log\left(\sqrt{d}+\sqrt{d}\,\tan(e+fx)-\sqrt{2}\,\sqrt{d\tan(e+}\right.}{\sqrt{2}\,a^2\sqrt{d}\,f}$$

$$= -\frac{\left(\frac{9}{16}-\frac{5i}{16}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^2\sqrt{d}\,f} + \frac{\left(\frac{9}{16}-\frac{5i}{16}\right.}{}$$

<u>**Mathematica**</u> [A]

time = 1.05, size = 228, normalized size = 0.76

$$\frac{\sec^3(e+fx)\left(-5i\cos(e+fx)+5i\cos(3(e+fx))-7\sin(e+fx)-(9+5i)\cos(2(e+fx))\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sqrt{\sin(2(e+fx))}+(5-9i)\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sin^{\frac{3}{2}}(2(e+fx))+(5+9i)\text{ArcSin}(\cos(e+fx)-\sin(e+fx))\sqrt{\sin(2(e+fx))}\,(-i\cos(2(e+fx))+\sin(2(e+fx)))-7\sin(3(e+fx))\right)}{32a^2f\sqrt{d\tan(e+fx)}\,(-i+\tan(e+fx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[d*Tan[e + f*x]]*(a + I*a*Tan[e + f*x])^2),x]

[Out] (Sec[e + f*x]^3*((-5*I)*Cos[e + f*x] + (5*I)*Cos[3*(e + f*x)] - 7*Sin[e + f*x] - (9 + 5*I)*Cos[2*(e + f*x)]*Log[Cos[e + f*x] + Sin[e + f*x] + Sqrt[Sin[2*(e + f*x)]]]*Sqrt[Sin[2*(e + f*x)]] + (5 - 9*I)*Log[Cos[e + f*x] + Sin[e + f*x] + Sqrt[Sin[2*(e + f*x)]]]*Sin[2*(e + f*x)]^(3/2) + (5 + 9*I)*ArcSin[Cos[e + f*x] - Sin[e + f*x]]*Sqrt[Sin[2*(e + f*x)]]*((-I)*Cos[2*(e + f*x)] + Sin[2*(e + f*x)]) - 7*Sin[3*(e + f*x)]))/(32*a^2*f*Sqrt[d*Tan[e + f*x]]*(-I + Tan[e + f*x])^2)

Maple [A]
time = 0.18, size = 118, normalized size = 0.39

method	result
derivativedivides	$\frac{2d^3\left(-\frac{\frac{-\frac{5i(d\tan(fx+e))^{\frac{3}{2}}}{2}-\frac{7d\sqrt{d\tan(fx+e)}}{2}}{(id\tan(fx+e)+d)^2}+\frac{7i\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{-id}}\right)}{2\sqrt{-id}}}{8d^3}+\frac{i\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{id}}\right)}{8d^3\sqrt{id}}\right)}{f\,a^2}$
default	$\frac{2d^3\left(-\frac{\frac{-\frac{5i(d\tan(fx+e))^{\frac{3}{2}}}{2}-\frac{7d\sqrt{d\tan(fx+e)}}{2}}{(id\tan(fx+e)+d)^2}+\frac{7i\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{-id}}\right)}{2\sqrt{-id}}}{8d^3}+\frac{i\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{id}}\right)}{8d^3\sqrt{id}}\right)}{f\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)

[Out] 2/f/a^2*d^3*(-1/8/d^3*((-5/2*I*(d*tan(f*x+e))^(3/2)-7/2*d*(d*tan(f*x+e))^(1/2))/(I*d*tan(f*x+e)+d)^2+7/2*I/(-I*d)^(1/2)*arctan((d*tan(f*x+e))^(1/2)/(-I*d)^(1/2)))+1/8*I/d^3/(I*d)^(1/2)*arctan((d*tan(f*x+e))^(1/2)/(I*d)^(1/2)))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 584 vs. 2(231) = 462.
time = 0.39, size = 584, normalized size = 1.94

$$-\frac{4\,a^2df\sqrt{-\frac{i}{16\,a^4df^2}}\,e^{(4i\,fx+4i\,e)}\log\left(-2\left(4\left(a^2dfe^{(2i\,fx+2i\,e)}+a^2df\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{-\frac{i}{16\,a^4df^2}}+i\,de^{(2i\,fx+2i\,e)}\right)e^{(-2i\,fx-2i\,e)}\right)-4\,a^2df\sqrt{-\frac{i}{16\,a^4df^2}}\,e^{(4i\,fx+4i\,e)}\log\left(2\left(4\left(a^2dfe^{(2i\,fx+2i\,e)}+a^2df\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{-\frac{i}{16\,a^4df^2}}-i\,de^{(2i\,fx+2i\,e)}\right)e^{(-2i\,fx-2i\,e)}\right)-4\,a^2df\sqrt{\frac{49i}{64\,a^4df^2}}\,e^{(4i\,fx+4i\,e)}\log\left(\frac{\left(8\left(a^2fe^{(2i\,fx+2i\,e)}+a^2f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{49i}{64\,a^4df^2}}+7i\right)e^{(-2i\,fx-2i\,e)}}{8\,a^2f}\right)+4\,a^2df\sqrt{\frac{49i}{64\,a^4df^2}}\,e^{(4i\,fx+4i\,e)}\log\left(-\frac{\left(8\left(a^2fe^{(2i\,fx+2i\,e)}+a^2f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{49i}{64\,a^4df^2}}-7i\right)e^{(-2i\,fx-2i\,e)}}{8\,a^2f}\right)-\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\left(6\,e^{(4i\,fx+4i\,e)}+7\,e^{(2i\,fx+2i\,e)}+1\right)}{16\,a^2df}e^{(-4i\,fx-4i\,e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] -1/16*(4*a^2*d*f*sqrt(-1/16*I/(a^4*d*f^2))*e^(4*I*f*x + 4*I*e)*log(-2*(4*(a
^2*d*f*e^(2*I*f*x + 2*I*e) + a^2*d*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)
/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-1/16*I/(a^4*d*f^2)) + I*d*e^(2*I*f*x + 2*
I*e))*e^(-2*I*f*x - 2*I*e)) - 4*a^2*d*f*sqrt(-1/16*I/(a^4*d*f^2))*e^(4*I*f*
x + 4*I*e)*log(2*(4*(a^2*d*f*e^(2*I*f*x + 2*I*e) + a^2*d*f)*sqrt((-I*d*e^(2
*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-1/16*I/(a^4*d*f^2))
 - I*d*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)) - 4*a^2*d*f*sqrt(49/64*I/
(a^4*d*f^2))*e^(4*I*f*x + 4*I*e)*log(1/8*(8*(a^2*f*e^(2*I*f*x + 2*I*e) + a^
2*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(
49/64*I/(a^4*d*f^2)) + 7*I)*e^(-2*I*f*x - 2*I*e)/(a^2*f)) + 4*a^2*d*f*sqrt(
49/64*I/(a^4*d*f^2))*e^(4*I*f*x + 4*I*e)*log(-1/8*(8*(a^2*f*e^(2*I*f*x + 2*
I*e) + a^2*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) +
1))*sqrt(49/64*I/(a^4*d*f^2)) - 7*I)*e^(-2*I*f*x - 2*I*e)/(a^2*f)) - sqrt((
-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*(6*e^(4*I*f*x +
4*I*e) + 7*e^(2*I*f*x + 2*I*e) + 1))*e^(-4*I*f*x - 4*I*e)/(a^2*d*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{\sqrt{d\tan\left(e+fx\right)}\tan^{2}\left(e+fx\right)-2i\sqrt{d\tan\left(e+fx\right)}\tan\left(e+fx\right)-\sqrt{d\tan\left(e+fx\right)}}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -Integral(1/(sqrt(d*tan(e + f*x))*tan(e + f*x)**2 - 2*I*sqrt(d*tan(e + f*x)
)*tan(e + f*x) - sqrt(d*tan(e + f*x))), x)/a**2
```

Giac [A]
time = 0.62, size = 198, normalized size = 0.66

$$\frac{7\sqrt{2}\arctan\left(\frac{8i\sqrt{d^2}\sqrt{d\tan\left(fx+e\right)}}{4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{8\,a^2\sqrt{d}\,f\left(\frac{id}{\sqrt{d^2}}+1\right)}-\frac{\sqrt{2}\arctan\left(\frac{8i\sqrt{d^2}\sqrt{d\tan\left(fx+e\right)}}{-4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{4\,a^2\sqrt{d}\,f\left(-\frac{id}{\sqrt{d^2}}+1\right)}+\frac{-5i\sqrt{d\tan\left(fx+e\right)}\,d\tan\left(fx+e\right)-7\sqrt{d\tan\left(fx+e\right)}\,d}{8\left(d\tan\left(fx+e\right)-i\,d\right)^2a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 7/8*sqrt(2)*arctan(8*I*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*I*sqrt(2)*d^(3/2)
+ 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a^2*sqrt(d)*f*(I*d/sqrt(d^2) + 1)) - 1/4*s
qrt(2)*arctan(8*I*sqrt(d^2)*sqrt(d*tan(f*x + e))/(-4*I*sqrt(2)*d^(3/2) + 4*
sqrt(2)*sqrt(d^2)*sqrt(d)))/(a^2*sqrt(d)*f*(-I*d/sqrt(d^2) + 1)) + 1/8*(-5*
I*sqrt(d*tan(f*x + e))*d*tan(f*x + e) - 7*sqrt(d*tan(f*x + e))*d)/((d*tan(f
*x + e) - I*d)^2*a^2*f)
```

Mupad [B]
time = 5.48, size = 168, normalized size = 0.56

$$\frac{\frac{7\,d\,\sqrt{d\,\tan\left(e+f\,x\right)}}{8\,a^2\,f}+\frac{{\left(d\,\tan\left(e+f\,x\right)\right)}^{3/2}\,5{}\mathrm{i}}{8\,a^2\,f}}{-d^2\,{\tan\left(e+f\,x\right)}^2+d^2\,\tan\left(e+f\,x\right)\,2{}\mathrm{i}+d^2}+\mathrm{atan}\left(8\,a^2\,f\,\sqrt{d\,\tan\left(e+f\,x\right)}\,\sqrt{-\frac{1{}\mathrm{i}}{64\,a^4\,d\,f^2}}\right)\,\sqrt{-\frac{1{}\mathrm{i}}{64\,a^4\,d\,f^2}}\,2{}\mathrm{i}-\mathrm{atan}\left(\frac{16\,a^2\,f\,\sqrt{d\,\tan\left(e+f\,x\right)}\,\sqrt{\frac{49{}\mathrm{i}}{256\,a^4\,d\,f^2}}}{7}\right)\,\sqrt{\frac{49{}\mathrm{i}}{256\,a^4\,d\,f^2}}\,2{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*tan(e + f*x))^(1/2)*(a + a*tan(e + f*x)*1i)^2),x)
```

```
[Out] (((d*tan(e + f*x))^(3/2)*5i)/(8*a^2*f) + (7*d*(d*tan(e + f*x))^(1/2))/(8*a^
2*f))/(d^2*tan(e + f*x)*2i + d^2 - d^2*tan(e + f*x)^2) + atan(8*a^2*f*(d*ta
n(e + f*x))^(1/2)*(-1i/(64*a^4*d*f^2))^(1/2))*(-1i/(64*a^4*d*f^2))^(1/2)*2i
 - atan((16*a^2*f*(d*tan(e + f*x))^(1/2)*(49i/(256*a^4*d*f^2))^(1/2))/7)*(4
9i/(256*a^4*d*f^2))^(1/2)*2i
```

3.177 $\int \frac{1}{(d\tan(e+fx))^{3/2}(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=326

$$\frac{\left(\frac{25}{16}+\frac{21i}{16}\right)\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^2d^{3/2}f}-\frac{\left(\frac{25}{16}+\frac{21i}{16}\right)\mathrm{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^2d^{3/2}f}-\frac{\left(\frac{25}{32}-\frac{21}{3\ldots}\right.}{}$$

```
[Out] (25/32+21/32*I)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a^2/d^(3/2)/
f*2^(1/2)-(25/32+21/32*I)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a^
2/d^(3/2)/f*2^(1/2)+(-25/64+21/64*I)*ln(d^(1/2)-2^(1/2)*(d*tan(f*x+e))^(1/2
)+d^(1/2)*tan(f*x+e))/a^2/d^(3/2)/f*2^(1/2)+(25/64-21/64*I)*ln(d^(1/2)+2^(1
/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/a^2/d^(3/2)/f*2^(1/2)-25/8/a^2
/d/f/(d*tan(f*x+e))^(1/2)+7/8/a^2/d/f/(d*tan(f*x+e))^(1/2)/(1+I*tan(f*x+e))
+1/4/d/f/(d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2
```

Rubi [A]

time = 0.34, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3640, 3677, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\left(\frac{25}{16}+\frac{21i}{16}\right)\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^2d^{3/2}f}-\frac{\left(\frac{25}{16}+\frac{21i}{16}\right)\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,a^2d^{3/2}f}-\frac{\left(\frac{25}{32}-\frac{21i}{32}\right)\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,a^2d^{3/2}f}+\frac{\left(\frac{25}{32}-\frac{21i}{32}\right)\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,a^2d^{3/2}f}-\frac{25}{8a^2df\sqrt{d\tan(e+fx)}}+\frac{7}{8a^2df(1+i\tan(e+fx))\sqrt{d\tan(e+fx)}}+\frac{1}{4df(a+ia\tan(e+fx))^2\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Tan[e + f*x])^(3/2)*(a + I*a*Tan[e + f*x])^2),x]
```

```
[Out] ((25/16 + (21*I)/16)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(S
qrt[2]*a^2*d^(3/2)*f) - ((25/16 + (21*I)/16)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan
[e + f*x]])/Sqrt[d]])/(Sqrt[2]*a^2*d^(3/2)*f) - ((25/32 - (21*I)/32)*Log[Sq
rt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e + f*x]]])/(Sqrt[2]*a^2*
d^(3/2)*f) + ((25/32 - (21*I)/32)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqrt
[2]*Sqrt[d*Tan[e + f*x]]])/(Sqrt[2]*a^2*d^(3/2)*f) - 25/(8*a^2*d*f*Sqrt[d*T
an[e + f*x]]) + 7/(8*a^2*d*f*(1 + I*Tan[e + f*x])*Sqrt[d*Tan[e + f*x]]) + 1
/(4*d*f*Sqrt[d*Tan[e + f*x]]*(a + I*a*Tan[e + f*x])^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(d\tan(e+fx))^{3/2}(a+ia\tan(e+fx))^2}\,dx = \frac{1}{4df\sqrt{d\tan(e+fx)}\,(a+ia\tan(e+fx))^2} + \frac{\int \frac{\frac{9ad}{2}-\frac{5}{2}iad\ldots}{(d\tan(e+fx))^{3/\ldots}}}{4\ldots}$$

$$= \frac{7}{8a^2df(1+i\tan(e+fx))\sqrt{d\tan(e+fx)}} + \frac{\ldots}{4df\sqrt{d\tan(e+\ldots}}$$

$$= -\frac{25}{8a^2df\sqrt{d\tan(e+fx)}} + \frac{7}{8a^2df(1+i\tan(e+fx))\sqrt{d\tan\ldots}}$$

$$= -\frac{25}{8a^2df\sqrt{d\tan(e+fx)}} + \frac{7}{8a^2df(1+i\tan(e+fx))\sqrt{d\tan\ldots}}$$

$$= -\frac{25}{8a^2df\sqrt{d\tan(e+fx)}} + \frac{7}{8a^2df(1+i\tan(e+fx))\sqrt{d\tan\ldots}}$$

$$= -\frac{25}{8a^2df\sqrt{d\tan(e+fx)}} + \frac{7}{8a^2df(1+i\tan(e+fx))\sqrt{d\tan\ldots}}$$

$$= -\frac{\left(\frac{25}{32}-\frac{21i}{32}\right)\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+\ldots}\right.}{\sqrt{2}\,a^2d^{3/2}f}$$

$$= \frac{\left(\frac{25}{16}+\frac{21i}{16}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^2d^{3/2}f} - \frac{\left(\frac{25}{16}+\frac{21i}{16}\right)\ldots}{\ldots}$$

Mathematica [A]
time = 1.16, size = 231, normalized size = 0.71

$$\frac{\sec^3(e+fx)\left(23\cos(e+fx)+41\cos(3(e+fx))+43i\sin(e+fx)-(25-21i)\cos(2(e+fx))\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sqrt{\sin(2(e+fx))}-(21+25i)\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sin^{\frac{3}{2}}(2(e+fx))+(21-25i)\mathrm{ArcSin}(\cos(e+fx)-\sin(e+fx))\sqrt{\sin(2(e+fx))}\,(-i\cos(2(e+fx))+\sin(2(e+fx)))+43i\sin(3(e+fx))\right)}{32a^2df\sqrt{d\tan(e+fx)}\,(-i+\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Tan[e + f*x])^(3/2)*(a + I*a*Tan[e + f*x])^2),x]
```

```
[Out] (Sec[e + f*x]^3*(23*Cos[e + f*x] + 41*Cos[3*(e + f*x)] + (43*I)*Sin[e + f*x
] - (25 - 21*I)*Cos[2*(e + f*x)]*Log[Cos[e + f*x] + Sin[e + f*x] + Sqrt[Sin
[2*(e + f*x)]]]*Sqrt[Sin[2*(e + f*x)]] - (21 + 25*I)*Log[Cos[e + f*x] + Sin
[e + f*x] + Sqrt[Sin[2*(e + f*x)]]]*Sin[2*(e + f*x)]^(3/2) + (21 - 25*I)*Ar
cSin[Cos[e + f*x] - Sin[e + f*x]]*Sqrt[Sin[2*(e + f*x)]]*((-I)*Cos[2*(e + f
*x)] + Sin[2*(e + f*x)]) + (43*I)*Sin[3*(e + f*x)]))/(32*a^2*d*f*Sqrt[d*Tan
[e + f*x]]*(-I + Tan[e + f*x])^2)
```

Maple [A]
time = 0.16, size = 131, normalized size = 0.40

method	result
derivativedivides	$\frac{2d^3\left(-\frac{1}{d^4\sqrt{d\tan(fx+e)}}-\frac{\frac{-\frac{9(d\tan(fx+e))^{\frac{3}{2}}}{2}+\frac{11id\sqrt{d\tan(fx+e)}}{2}}{(id\tan(fx+e)+d)^2}+\frac{23\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{-id}}\right.}{2\sqrt{-id}}}{8d^4}\right.}{f\,a^2}$
default	$\frac{2d^3\left(-\frac{1}{d^4\sqrt{d\tan(fx+e)}}-\frac{\frac{-\frac{9(d\tan(fx+e))^{\frac{3}{2}}}{2}+\frac{11id\sqrt{d\tan(fx+e)}}{2}}{(id\tan(fx+e)+d)^2}+\frac{23\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{-id}}\right.}{2\sqrt{-id}}}{8d^4}\right.}{f\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^2*d^3*(-1/d^4/(d*tan(f*x+e))^(1/2)-1/8/d^4*((-9/2*(d*tan(f*x+e))^(3/2
)+11/2*I*d*(d*tan(f*x+e))^(1/2))/(I*d*tan(f*x+e)+d)^2+23/2/(-I*d)^(1/2)*arc
tan((d*tan(f*x+e))^(1/2)/(-I*d)^(1/2)))-1/8/d^4/(I*d)^(1/2)*arctan((d*tan(f
*x+e))^(1/2)/(I*d)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 730 vs. 2(253) = 506.
time = 0.40, size = 730, normalized size = 2.24

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] 1/16*(4*(a^2*d^2*f*e^(6*I*f*x + 6*I*e) - a^2*d^2*f*e^(4*I*f*x + 4*I*e))*sqr
t(1/16*I/(a^4*d^3*f^2))*log(-2*(4*(I*a^2*d^2*f*e^(2*I*f*x + 2*I*e) + I*a^2*
d^2*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqr
t(1/16*I/(a^4*d^3*f^2)) + I*d*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)) -
4*(a^2*d^2*f*e^(6*I*f*x + 6*I*e) - a^2*d^2*f*e^(4*I*f*x + 4*I*e))*sqrt(1/16
*I/(a^4*d^3*f^2))*log(-2*(4*(-I*a^2*d^2*f*e^(2*I*f*x + 2*I*e) - I*a^2*d^2*f
)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(1/1
6*I/(a^4*d^3*f^2)) + I*d*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)) + 4*(a^
2*d^2*f*e^(6*I*f*x + 6*I*e) - a^2*d^2*f*e^(4*I*f*x + 4*I*e))*sqrt(-529/64*I
/(a^4*d^3*f^2))*log(1/8*(8*(a^2*d*f*e^(2*I*f*x + 2*I*e) + a^2*d*f)*sqrt((-I
*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-529/64*I/(a^
4*d^3*f^2)) + 23)*e^(-2*I*f*x - 2*I*e)/(a^2*d*f)) - 4*(a^2*d^2*f*e^(6*I*f*x
 + 6*I*e) - a^2*d^2*f*e^(4*I*f*x + 4*I*e))*sqrt(-529/64*I/(a^4*d^3*f^2))*lo
g(-1/8*(8*(a^2*d*f*e^(2*I*f*x + 2*I*e) + a^2*d*f)*sqrt((-I*d*e^(2*I*f*x + 2
*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-529/64*I/(a^4*d^3*f^2)) - 23)
*e^(-2*I*f*x - 2*I*e)/(a^2*d*f)) + sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e
^(2*I*f*x + 2*I*e) + 1))*(-42*I*e^(6*I*f*x + 6*I*e) - 33*I*e^(4*I*f*x + 4*I
*e) + 10*I*e^(2*I*f*x + 2*I*e) + I))/(a^2*d^2*f*e^(6*I*f*x + 6*I*e) - a^2*d
^2*f*e^(4*I*f*x + 4*I*e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{\left(d\tan\left(e+fx\right)\right)^{\frac{3}{2}}\tan^{2}\left(e+fx\right)-2i\left(d\tan\left(e+fx\right)\right)^{\frac{3}{2}}\tan\left(e+fx\right)-\left(d\tan\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))**(3/2)/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -Integral(1/((d*tan(e + f*x))**(3/2)*tan(e + f*x)**2 - 2*I*(d*tan(e + f*x))
**(3/2)*tan(e + f*x) - (d*tan(e + f*x))**(3/2)), x)/a**2
```

Giac [A]
time = 0.67, size = 221, normalized size = 0.68

$$-\frac{\frac{2\sqrt{2}\arctan\left(\frac{8\sqrt{d^{2}}\sqrt{d\tan\left(fx+e\right)}}{4i\sqrt{2}d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^{2}}\sqrt{d}}\right)}{a^{2}\sqrt{d}f\left(\frac{id}{\sqrt{d^{2}}}+1\right)}-\frac{23i\sqrt{2}\arctan\left(-\frac{8i\sqrt{d^{2}}\sqrt{d\tan\left(fx+e\right)}}{4i\sqrt{2}d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^{2}}\sqrt{d}}\right)}{a^{2}\sqrt{d}f\left(\frac{id}{\sqrt{d^{2}}}+1\right)}+\frac{16}{\sqrt{d\tan\left(fx+e\right)}a^{2}f}+\frac{9\sqrt{d\tan\left(fx+e\right)}d\tan\left(fx+e\right)-11i\sqrt{d\tan\left(fx+e\right)}d}{\left(d\tan\left(fx+e\right)-id\right)^{2}a^{2}f}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/8*(2*sqrt(2)*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*I*sqrt(2)*d^(3/2
) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a^2*sqrt(d)*f*(I*d/sqrt(d^2) + 1)) - 23*
```

```
I*sqrt(2)*arctan(-8*I*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*I*sqrt(2)*d^(3/2) +
 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a^2*sqrt(d)*f*(I*d/sqrt(d^2) + 1)) + 16/(sq
rt(d*tan(f*x + e))*a^2*f) + (9*sqrt(d*tan(f*x + e))*d*tan(f*x + e) - 11*I*s
qrt(d*tan(f*x + e))*d)/((d*tan(f*x + e) - I*d)^2*a^2*f))/d
```

Mupad [B]
time = 5.96, size = 185, normalized size = 0.57

$$2\,\mathrm{atanh}\left(8\,a^2\,d\,f\,\sqrt{d\,\tan\left(e+f\,x\right)}\,\sqrt{\frac{1\mathrm{i}}{64\,a^4\,d^3\,f^2}}\right)\,\sqrt{\frac{1\mathrm{i}}{64\,a^4\,d^3\,f^2}}+2\,\mathrm{atanh}\left(\frac{16\,a^2\,d\,f\,\sqrt{d\,\tan\left(e+f\,x\right)}\,\sqrt{-\frac{529\mathrm{i}}{256\,a^4\,d^3\,f^2}}}{23}\right)\,\sqrt{-\frac{529\mathrm{i}}{256\,a^4\,d^3\,f^2}}-\frac{\frac{2\,d}{a^2\,f}-\frac{25\,d\,{\tan\left(e+f\,x\right)}^2}{8\,a^2\,f}+\frac{d\,\tan\left(e+f\,x\right)\,43\mathrm{i}}{8\,a^2\,f}}{d^2\,\sqrt{d\,\tan\left(e+f\,x\right)}-{\left(d\,\tan\left(e+f\,x\right)\right)}^{5/2}+d\,{\left(d\,\tan\left(e+f\,x\right)\right)}^{3/2}\,2\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*tan(e + f*x))^(3/2)*(a + a*tan(e + f*x)*1i)^2),x)
```

```
[Out] 2*atanh(8*a^2*d*f*(d*tan(e + f*x))^(1/2)*(1i/(64*a^4*d^3*f^2))^(1/2))*(1i/(
64*a^4*d^3*f^2))^(1/2) + 2*atanh((16*a^2*d*f*(d*tan(e + f*x))^(1/2)*(-529i/
(256*a^4*d^3*f^2))^(1/2))/23)*(-529i/(256*a^4*d^3*f^2))^(1/2) - ((2*d)/(a^2
*f) - (25*d*tan(e + f*x)^2)/(8*a^2*f) + (d*tan(e + f*x)*43i)/(8*a^2*f))/(d*
(d*tan(e + f*x))^(3/2)*2i - (d*tan(e + f*x))^(5/2) + d^2*(d*tan(e + f*x))^(
1/2))
```

3.178 $\int \frac{1}{(d\tan(e+fx))^{5/2}(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=353

$$\frac{\left(\frac{49}{16}-\frac{45i}{16}\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^2d^{5/2}f}-\frac{\left(\frac{49}{16}-\frac{45i}{16}\right)\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^2d^{5/2}f}+\frac{\left(\frac{49}{32}+\frac{45i}{32}\right.}{}$$

```
[Out] (49/32-45/32*I)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a^2/d^(5/2)/
f*2^(1/2)+(-49/32+45/32*I)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a
^2/d^(5/2)/f*2^(1/2)+(49/64+45/64*I)*ln(d^(1/2)-2^(1/2)*(d*tan(f*x+e))^(1/2
)+d^(1/2)*tan(f*x+e))/a^2/d^(5/2)/f*2^(1/2)-(49/64+45/64*I)*ln(d^(1/2)+2^(1
/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/a^2/d^(5/2)/f*2^(1/2)+45/8*I/a
^2/d^2/f/(d*tan(f*x+e))^(1/2)-49/24/a^2/d/f/(d*tan(f*x+e))^(3/2)+9/8/a^2/d/
f/(1+I*tan(f*x+e))/(d*tan(f*x+e))^(3/2)+1/4/d/f/(d*tan(f*x+e))^(3/2)/(a+I*a
*tan(f*x+e))^2
```

Rubi [A]
time = 0.38, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3640, 3677, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\left(\frac{49}{16}-\frac{45i}{16}\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^2d^{5/2}f}-\frac{\left(\frac{49}{16}-\frac{45i}{16}\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,a^2d^{5/2}f}+\frac{\left(\frac{49}{32}+\frac{45i}{32}\right)\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,a^2d^{5/2}f}-\frac{\left(\frac{49}{32}+\frac{45i}{32}\right)\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,a^2d^{5/2}f}+\frac{45i}{8a^2d^2f\sqrt{d\tan(e+fx)}}-\frac{49}{24a^2df(d\tan(e+fx))^{3/2}}+\frac{9}{8a^2df(1+i\tan(e+fx))(d\tan(e+fx))^{3/2}}+\frac{1}{4df(a+ia\tan(e+fx))^2(d\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Tan[e + f*x])^(5/2)*(a + I*a*Tan[e + f*x])^2),x]
```

```
[Out] ((49/16 - (45*I)/16)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(S
qrt[2]*a^2*d^(5/2)*f) - ((49/16 - (45*I)/16)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan
[e + f*x]])/Sqrt[d]])/(Sqrt[2]*a^2*d^(5/2)*f) + ((49/32 + (45*I)/32)*Log[Sq
rt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e + f*x]]])/(Sqrt[2]*a^2*
d^(5/2)*f) - ((49/32 + (45*I)/32)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqrt
[2]*Sqrt[d*Tan[e + f*x]]])/(Sqrt[2]*a^2*d^(5/2)*f) - 49/(24*a^2*d*f*(d*Tan[
e + f*x])^(3/2)) + 9/(8*a^2*d*f*(1 + I*Tan[e + f*x])*(d*Tan[e + f*x])^(3/2)
) + ((45*I)/8)/(a^2*d^2*f*Sqrt[d*Tan[e + f*x]]) + 1/(4*d*f*(d*Tan[e + f*x])
^(3/2)*(a + I*a*Tan[e + f*x])^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
```

```
NeQ[c^2 + d^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(d\tan(e+fx))^{5/2}(a+ia\tan(e+fx))^2}\,dx = \frac{1}{4df(d\tan(e+fx))^{3/2}(a+ia\tan(e+fx))^2} + \frac{\int \frac{\frac{11ad}{2} -}{(d\tan(e+fx)}}{}$$

$$= \frac{9}{8a^2df(1+i\tan(e+fx))(d\tan(e+fx))^{3/2}} + \frac{}{4df(d\tan(e}$$

$$= -\frac{49}{24a^2df(d\tan(e+fx))^{3/2}} + \frac{9}{8a^2df(1+i\tan(e+fx))(d\,\mathrm{t}}$$

$$= -\frac{49}{24a^2df(d\tan(e+fx))^{3/2}} + \frac{9}{8a^2df(1+i\tan(e+fx))(d\,\mathrm{t}}$$

$$= -\frac{49}{24a^2df(d\tan(e+fx))^{3/2}} + \frac{9}{8a^2df(1+i\tan(e+fx))(d\,\mathrm{t}}$$

$$= -\frac{49}{24a^2df(d\tan(e+fx))^{3/2}} + \frac{9}{8a^2df(1+i\tan(e+fx))(d\,\mathrm{t}}$$

$$= -\frac{49}{24a^2df(d\tan(e+fx))^{3/2}} + \frac{9}{8a^2df(1+i\tan(e+fx))(d\,\mathrm{t}}$$

$$= \frac{\left(\frac{49}{32}+\frac{45i}{32}\right)\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+\right.}{\sqrt{2}\,a^2d^{5/2}f}$$

$$= \frac{\left(\frac{49}{16}-\frac{45i}{16}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^2d^{5/2}f} - \frac{\left(\frac{49}{16}-\frac{45i}{16}\right.}{}$$

Mathematica [A]
time = 2.36, size = 346, normalized size = 0.98

$$-\frac{\sec^4(e+fx)\left(-269-64\cos(2(e+fx))+205\cos(4(e+fx))+(135-147i)\cos(e+fx)\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sqrt{\sin(2(e+fx))}-(135-147i)\cos(3(e+fx))\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sqrt{\sin(2(e+fx))}+(147+135i)\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sin(e+fx)\sqrt{\sin(2(e+fx))}-142i\sin(2(e+fx))+(270+294i)\mathrm{ArcSin}(\cos(e+fx)-\sin(e+fx))\sin(e+fx)\sqrt{\sin(2(e+fx))}(-i\cos(2(e+fx))+\sin(2(e+fx)))-(147+135i)\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sqrt{\sin(2(e+fx))}\sin(3(e+fx))+199i\sin(4(e+fx))\right)}{192a^2df(d\tan(e+fx))^{3/2}(-i+\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Tan[e + f*x])^(5/2)*(a + I*a*Tan[e + f*x])^2),x]
```

```
[Out] -1/192*(Sec[e + f*x]^4*(-269 - 64*Cos[2*(e + f*x)] + 205*Cos[4*(e + f*x)] +
 (135 - 147*I)*Cos[e + f*x]*Log[Cos[e + f*x] + Sin[e + f*x] + Sqrt[Sin[2*(e
 + f*x)]]]*Sqrt[Sin[2*(e + f*x)]] - (135 - 147*I)*Cos[3*(e + f*x)]*Log[Cos[
e + f*x] + Sin[e + f*x] + Sqrt[Sin[2*(e + f*x)]]]*Sqrt[Sin[2*(e + f*x)]] +
```

```
(147 + 135*I)*Log[Cos[e + f*x] + Sin[e + f*x] + Sqrt[Sin[2*(e + f*x)]]]*Sin
[e + f*x]*Sqrt[Sin[2*(e + f*x)]] - (142*I)*Sin[2*(e + f*x)] + (270 + 294*I)
*ArcSin[Cos[e + f*x] - Sin[e + f*x]]*Sin[e + f*x]*Sqrt[Sin[2*(e + f*x)]]*((
-I)*Cos[2*(e + f*x)] + Sin[2*(e + f*x)]) - (147 + 135*I)*Log[Cos[e + f*x] +
 Sin[e + f*x] + Sqrt[Sin[2*(e + f*x)]]]*Sqrt[Sin[2*(e + f*x)]]*Sin[3*(e + f
*x)] + (199*I)*Sin[4*(e + f*x)]))/(a^2*d*f*(d*Tan[e + f*x])^(3/2)*(-I + Tan
[e + f*x])^2)
```

Maple [A]
time = 0.15, size = 149, normalized size = 0.42

method	result
derivativedivides	$\frac{2d^3\left(-\frac{1}{3d^4(d\tan(fx+e))^{\frac{3}{2}}}+\frac{2i}{d^5\sqrt{d\tan(fx+e)}}+\frac{\frac{-\frac{13i(d\tan(fx+e))^{\frac{3}{2}}}{2}-\frac{15d\sqrt{d\tan(fx+e)}}{2}}{(id\tan(fx+e)+d)^2}+\frac{47i\arctan\left(\frac{\sqrt{}}{}\right.}{}}{8d^5}\right.}{f\,a^2}$
default	$\frac{2d^3\left(-\frac{1}{3d^4(d\tan(fx+e))^{\frac{3}{2}}}+\frac{2i}{d^5\sqrt{d\tan(fx+e)}}+\frac{\frac{-\frac{13i(d\tan(fx+e))^{\frac{3}{2}}}{2}-\frac{15d\sqrt{d\tan(fx+e)}}{2}}{(id\tan(fx+e)+d)^2}+\frac{47i\arctan\left(\frac{\sqrt{}}{}\right.}{}}{8d^5}\right.}{f\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^2*d^3*(-1/3/d^4/(d*tan(f*x+e))^(3/2)+2*I/d^5/(d*tan(f*x+e))^(1/2)+1/8
/d^5*((-13/2*I*(d*tan(f*x+e))^(3/2)-15/2*d*(d*tan(f*x+e))^(1/2))/(I*d*tan(f
*x+e)+d)^2+47/2*I/(-I*d)^(1/2)*arctan((d*tan(f*x+e))^(1/2)/(-I*d)^(1/2)))-1
/8*I/d^5/(I*d)^(1/2)*arctan((d*tan(f*x+e))^(1/2)/(I*d)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 837 vs. $2(275) = 550$.
time = 0.40, size = 837, normalized size = 2.37

$$\frac{12\left(a^{2}d^{3}fe^{(8ifx+8ie)}-2a^{2}d^{3}fe^{(6ifx+6ie)}+a^{2}d^{3}fe^{(4ifx+4ie)}\right)\sqrt{-\frac{i}{16a^{4}d^{5}f^{2}}}\log\left(-2\left(4\left(a^{2}d^{3}fe^{(2ifx+2ie)}+a^{2}d^{3}f\right)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\sqrt{-\frac{i}{16a^{4}d^{5}f^{2}}}+ide^{(2ifx+2ie)}\right)e^{(-2ifx-2ie)}\right)-12\left(a^{2}d^{3}fe^{(8ifx+8ie)}-2a^{2}d^{3}fe^{(6ifx+6ie)}+a^{2}d^{3}fe^{(4ifx+4ie)}\right)\sqrt{-\frac{i}{16a^{4}d^{5}f^{2}}}\log\left(2\left(4\left(a^{2}d^{3}fe^{(2ifx+2ie)}+a^{2}d^{3}f\right)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\sqrt{-\frac{i}{16a^{4}d^{5}f^{2}}}-ide^{(2ifx+2ie)}\right)e^{(-2ifx-2ie)}\right)-12\left(a^{2}d^{3}fe^{(8ifx+8ie)}-2a^{2}d^{3}fe^{(6ifx+6ie)}+a^{2}d^{3}fe^{(4ifx+4ie)}\right)\sqrt{\frac{2209i}{64a^{4}d^{5}f^{2}}}\log\left(-\frac{\left(8\left(a^{2}d^{2}fe^{(2ifx+2ie)}+a^{2}d^{2}f\right)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{2209i}{64a^{4}d^{5}f^{2}}}+47i\right)e^{(-2ifx-2ie)}}{8a^{2}d^{2}f}\right)+12\left(a^{2}d^{3}fe^{(8ifx+8ie)}-2a^{2}d^{3}fe^{(6ifx+6ie)}+a^{2}d^{3}fe^{(4ifx+4ie)}\right)\sqrt{\frac{2209i}{64a^{4}d^{5}f^{2}}}\log\left(\frac{\left(8\left(a^{2}d^{2}fe^{(2ifx+2ie)}+a^{2}d^{2}f\right)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{2209i}{64a^{4}d^{5}f^{2}}}-47i\right)e^{(-2ifx-2ie)}}{8a^{2}d^{2}f}\right)-\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\left(202e^{(8ifx+8ie)}-103e^{(6ifx+6ie)}-269e^{(4ifx+4ie)}+39e^{(2ifx+2ie)}+3\right)}{48\left(a^{2}d^{3}fe^{(8ifx+8ie)}-2a^{2}d^{3}fe^{(6ifx+6ie)}+a^{2}d^{3}fe^{(4ifx+4ie)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] 1/48*(12*(a^2*d^3*f*e^(8*I*f*x + 8*I*e) - 2*a^2*d^3*f*e^(6*I*f*x + 6*I*e) +
 a^2*d^3*f*e^(4*I*f*x + 4*I*e))*sqrt(-1/16*I/(a^4*d^5*f^2))*log(-2*(4*(a^2*
d^3*f*e^(2*I*f*x + 2*I*e) + a^2*d^3*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d
)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-1/16*I/(a^4*d^5*f^2)) + I*d*e^(2*I*f*x +
 2*I*e))*e^(-2*I*f*x - 2*I*e)) - 12*(a^2*d^3*f*e^(8*I*f*x + 8*I*e) - 2*a^2*
d^3*f*e^(6*I*f*x + 6*I*e) + a^2*d^3*f*e^(4*I*f*x + 4*I*e))*sqrt(-1/16*I/(a^
4*d^5*f^2))*log(2*(4*(a^2*d^3*f*e^(2*I*f*x + 2*I*e) + a^2*d^3*f)*sqrt((-I*d
*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-1/16*I/(a^4*d^
5*f^2)) - I*d*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)) - 12*(a^2*d^3*f*e^
(8*I*f*x + 8*I*e) - 2*a^2*d^3*f*e^(6*I*f*x + 6*I*e) + a^2*d^3*f*e^(4*I*f*x
+ 4*I*e))*sqrt(2209/64*I/(a^4*d^5*f^2))*log(-1/8*(8*(a^2*d^2*f*e^(2*I*f*x +
 2*I*e) + a^2*d^2*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*
I*e) + 1))*sqrt(2209/64*I/(a^4*d^5*f^2)) + 47*I)*e^(-2*I*f*x - 2*I*e)/(a^2*
d^2*f)) + 12*(a^2*d^3*f*e^(8*I*f*x + 8*I*e) - 2*a^2*d^3*f*e^(6*I*f*x + 6*I*
e) + a^2*d^3*f*e^(4*I*f*x + 4*I*e))*sqrt(2209/64*I/(a^4*d^5*f^2))*log(1/8*(
8*(a^2*d^2*f*e^(2*I*f*x + 2*I*e) + a^2*d^2*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e
) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(2209/64*I/(a^4*d^5*f^2)) - 47*I)*e
^(-2*I*f*x - 2*I*e)/(a^2*d^2*f)) - sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e
^(2*I*f*x + 2*I*e) + 1))*(202*e^(8*I*f*x + 8*I*e) - 103*e^(6*I*f*x + 6*I*e)
 - 269*e^(4*I*f*x + 4*I*e) + 39*e^(2*I*f*x + 2*I*e) + 3))/(a^2*d^3*f*e^(8*I
*f*x + 8*I*e) - 2*a^2*d^3*f*e^(6*I*f*x + 6*I*e) + a^2*d^3*f*e^(4*I*f*x + 4*
I*e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{\left(d\tan\left(e+fx\right)\right)^{\frac{5}{2}}\tan^{2}\left(e+fx\right)-2i\left(d\tan\left(e+fx\right)\right)^{\frac{5}{2}}\tan\left(e+fx\right)-\left(d\tan\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))**(5/2)/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -Integral(1/((d*tan(e + f*x))**(5/2)*tan(e + f*x)**2 - 2*I*(d*tan(e + f*x))
**(5/2)*tan(e + f*x) - (d*tan(e + f*x))**(5/2)), x)/a**2
```

Giac [A]
time = 0.69, size = 246, normalized size = 0.70

$$\frac{47\sqrt{2}\arctan\left(-\frac{8i\sqrt{d^2}\sqrt{d\tan(fx+e)}}{4i\sqrt{2}d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{8a^2d^{\frac{5}{2}}f\left(\frac{id}{\sqrt{d^2}}+1\right)}-\frac{\sqrt{2}\arctan\left(-\frac{8i\sqrt{d^2}\sqrt{d\tan(fx+e)}}{-4i\sqrt{2}d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{4a^2d^{\frac{5}{2}}f\left(-\frac{id}{\sqrt{d^2}}+1\right)}+\frac{13i\sqrt{d\tan(fx+e)}\,d\tan(fx+e)+15\sqrt{d\tan(fx+e)}\,d}{8(d\tan(fx+e)-id)^2a^2d^2f}+\frac{2(6id\tan(fx+e)-d)}{3\sqrt{d\tan(fx+e)}\,a^2d^3f\tan(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 47/8*sqrt(2)*arctan(-8*I*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*I*sqrt(2)*d^(3/2
) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a^2*d^(5/2)*f*(I*d/sqrt(d^2) + 1)) - 1/4
*sqrt(2)*arctan(-8*I*sqrt(d^2)*sqrt(d*tan(f*x + e))/(-4*I*sqrt(2)*d^(3/2) +
 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a^2*d^(5/2)*f*(-I*d/sqrt(d^2) + 1)) + 1/8*(
13*I*sqrt(d*tan(f*x + e))*d*tan(f*x + e) + 15*sqrt(d*tan(f*x + e))*d)/((d*t
an(f*x + e) - I*d)^2*a^2*d^2*f) + 2/3*(6*I*d*tan(f*x + e) - d)/(sqrt(d*tan(
f*x + e))*a^2*d^3*f*tan(f*x + e))
```

Mupad [B]
time = 6.28, size = 209, normalized size = 0.59

$$-\mathrm{atan}\left(8a^2d^2f\sqrt{d\tan(e+fx)}\sqrt{-\frac{1\mathrm{i}}{64a^4d^5f^2}}\right)\sqrt{-\frac{1\mathrm{i}}{64a^4d^5f^2}}\,2\mathrm{i}+\mathrm{atan}\left(\frac{16a^2d^2f\sqrt{d\tan(e+fx)}\sqrt{\frac{2209\mathrm{i}}{256a^4d^5f^2}}}{47}\right)\sqrt{\frac{2209\mathrm{i}}{256a^4d^5f^2}}\,2\mathrm{i}-\frac{\frac{2d}{3a^2f}+\frac{221d\tan(e+fx)^2}{24a^2f}+\frac{d\tan(e+fx)^3\,45\mathrm{i}}{8a^2f}-\frac{d\tan(e+fx)\,8\mathrm{i}}{3a^2f}}{d^2(d\tan(e+fx))^{3/2}-(d\tan(e+fx))^{7/2}+d(d\tan(e+fx))^{5/2}\,2\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*tan(e + f*x))^(5/2)*(a + a*tan(e + f*x)*1i)^2),x)
```

```
[Out] atan((16*a^2*d^2*f*(d*tan(e + f*x))^(1/2)*(2209i/(256*a^4*d^5*f^2))^(1/2))/
47)*(2209i/(256*a^4*d^5*f^2))^(1/2)*2i - atan(8*a^2*d^2*f*(d*tan(e + f*x))^
(1/2)*(-1i/(64*a^4*d^5*f^2))^(1/2))*(-1i/(64*a^4*d^5*f^2))^(1/2)*2i - ((2*d
)/(3*a^2*f) + (221*d*tan(e + f*x)^2)/(24*a^2*f) + (d*tan(e + f*x)^3*45i)/(8
*a^2*f) - (d*tan(e + f*x)*8i)/(3*a^2*f))/(d*(d*tan(e + f*x))^(5/2)*2i - (d*
tan(e + f*x))^(7/2) + d^2*(d*tan(e + f*x))^(3/2))
```

3.179 $\int \frac{(d\tan(e+fx))^{9/2}}{(a+ia\tan(e+fx))^3}\,dx$

Optimal. Leaf size=370

$$\frac{\left(\frac{7}{4}+\frac{15i}{8}\right)d^{9/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}a^3f}-\frac{\left(\frac{7}{4}+\frac{15i}{8}\right)d^{9/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}a^3f}-\left(\frac{7}{8}\right.$$

```
[Out] (7/8+15/16*I)*d^(9/2)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a^3/f*
2^(1/2)-(7/8+15/16*I)*d^(9/2)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2)
)/a^3/f*2^(1/2)+(-7/16+15/32*I)*d^(9/2)*ln(d^(1/2)-2^(1/2)*(d*tan(f*x+e))^(
1/2)+d^(1/2)*tan(f*x+e))/a^3/f*2^(1/2)+(7/16-15/32*I)*d^(9/2)*ln(d^(1/2)+2^
(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/a^3/f*2^(1/2)+15/4*I*d^4*(d*
tan(f*x+e))^(1/2)/a^3/f-1/6*d*(d*tan(f*x+e))^(7/2)/f/(a+I*a*tan(f*x+e))^3+5
/12*I*d^2*(d*tan(f*x+e))^(5/2)/a/f/(a+I*a*tan(f*x+e))^2+7/6*d^3*(d*tan(f*x+
e))^(3/2)/f/(a^3+I*a^3*tan(f*x+e))
```

Rubi [A]
time = 0.43, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3639, 3676, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\left(\frac{7}{4}+\frac{15i}{8}\right)d^{9/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}a^3f}-\frac{\left(\frac{7}{4}+\frac{15i}{8}\right)d^{9/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}a^3f}-\frac{\left(\frac{7}{8}-\frac{15i}{16}\right)d^{9/2}\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}a^3f}+\frac{\left(\frac{7}{8}-\frac{15i}{16}\right)d^{9/2}\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}a^3f}+\frac{15id^4\sqrt{d\tan(e+fx)}}{4a^3f}+\frac{7d^3(d\tan(e+fx))^{3/2}}{6f\left(a^3+ia^3\tan(e+fx)\right)}+\frac{5id^2(d\tan(e+fx))^{5/2}}{12af(a+ia\tan(e+fx))^2}-\frac{d(d\tan(e+fx))^{7/2}}{6f(a+ia\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(9/2)/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] ((7/4 + (15*I)/8)*d^(9/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]
])/(Sqrt[2]*a^3*f) - ((7/4 + (15*I)/8)*d^(9/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*T
an[e + f*x]])/Sqrt[d]])/(Sqrt[2]*a^3*f) - ((7/8 - (15*I)/16)*d^(9/2)*Log[Sq
rt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e + f*x]]])/(Sqrt[2]*a^3*
f) + ((7/8 - (15*I)/16)*d^(9/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqrt[2
]*Sqrt[d*Tan[e + f*x]]])/(Sqrt[2]*a^3*f) + (((15*I)/4)*d^4*Sqrt[d*Tan[e + f
*x]])/(a^3*f) - (d*(d*Tan[e + f*x])^(7/2))/(6*f*(a + I*a*Tan[e + f*x])^3) +
 (((5*I)/12)*d^2*(d*Tan[e + f*x])^(5/2))/(a*f*(a + I*a*Tan[e + f*x])^2) + (
7*d^3*(d*Tan[e + f*x])^(3/2))/(6*f*(a^3 + I*a^3*Tan[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
```

```
NeQ[c^2 + d^2, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^{9/2}}{(a+ia\tan(e+fx))^3}\,dx = -\frac{d(d\tan(e+fx))^{7/2}}{6f(a+ia\tan(e+fx))^3} - \frac{\int \frac{(d\tan(e+fx))^{5/2}\left(-\frac{7ad^2}{2}+\frac{13}{2}iad^2\tan(e+fx)\right)}{(a+ia\tan(e+fx))^2}\,dx}{6a^2}$$

$$= -\frac{d(d\tan(e+fx))^{7/2}}{6f(a+ia\tan(e+fx))^3} + \frac{5id^2(d\tan(e+fx))^{5/2}}{12af(a+ia\tan(e+fx))^2} + \frac{\int \frac{(d\tan(e+fx))^{3/2}(-25}{a+ia\,\mathrm{t}}}{2}$$

$$= -\frac{d(d\tan(e+fx))^{7/2}}{6f(a+ia\tan(e+fx))^3} + \frac{5id^2(d\tan(e+fx))^{5/2}}{12af(a+ia\tan(e+fx))^2} + \frac{7d^3(d\tan(e+fx}{6f\left(a^3+ia^3\tan(e+\right.}$$

$$= \frac{15id^4\sqrt{d\tan(e+fx)}}{4a^3f} - \frac{d(d\tan(e+fx))^{7/2}}{6f(a+ia\tan(e+fx))^3} + \frac{5id^2(d\tan(e+fx))^{5/2}}{12af(a+ia\tan(e+fx)}$$

$$= \frac{15id^4\sqrt{d\tan(e+fx)}}{4a^3f} - \frac{d(d\tan(e+fx))^{7/2}}{6f(a+ia\tan(e+fx))^3} + \frac{5id^2(d\tan(e+fx))^{5/2}}{12af(a+ia\tan(e+fx)}$$

$$= \frac{15id^4\sqrt{d\tan(e+fx)}}{4a^3f} - \frac{d(d\tan(e+fx))^{7/2}}{6f(a+ia\tan(e+fx))^3} + \frac{5id^2(d\tan(e+fx))^{5/2}}{12af(a+ia\tan(e+fx)}$$

$$= \frac{15id^4\sqrt{d\tan(e+fx)}}{4a^3f} - \frac{d(d\tan(e+fx))^{7/2}}{6f(a+ia\tan(e+fx))^3} + \frac{5id^2(d\tan(e+fx))^{5/2}}{12af(a+ia\tan(e+fx)}$$

$$= -\frac{\left(\frac{7}{8}-\frac{15i}{16}\right)d^{9/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{\sqrt{2}\,a^3f} + \frac{\left(\frac{7}{8}-\right.}{}$$

$$= \frac{\left(\frac{7}{4}+\frac{15i}{8}\right)d^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^3f} - \frac{\left(\frac{7}{4}+\frac{15i}{8}\right)d^{9/2}\tan^{-1}\left(1+\right.}{\sqrt{2}}$$

Mathematica [A]
time = 3.69, size = 236, normalized size = 0.64

$$\frac{id^4e^{-6i(e+fx)}\left(-1+9e^{2i(e+fx)}-49e^{4i(e+fx)}-105e^{6i(e+fx)}+146e^{8i(e+fx)}-87e^{6i(e+fx)}\sqrt{-1+e^{4i(e+fx)}}\,\mathrm{ArcTan}\left(\sqrt{-1+e^{4i(e+fx)}}\right)-6e^{6i(e+fx)}\sqrt{-1+e^{2i(e+fx)}}\sqrt{1+e^{2i(e+fx)}}\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\right)\right)\sqrt{d\tan(e+fx)}}{48a^3\left(-1+e^{2i(e+fx)}\right)f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Tan[e + f*x])^(9/2)/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] ((I/48)*d^4*(-1 + 9*E^((2*I)*(e + f*x)) - 49*E^((4*I)*(e + f*x)) - 105*E^((
6*I)*(e + f*x)) + 146*E^((8*I)*(e + f*x)) - 87*E^((6*I)*(e + f*x))*Sqrt[-1
+ E^((4*I)*(e + f*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(e + f*x))]] - 6*E^((6*I)*
(e + f*x))*Sqrt[-1 + E^((2*I)*(e + f*x))]*Sqrt[1 + E^((2*I)*(e + f*x))]*Arc
```

```
Tanh[Sqrt[(-1 + E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f*x)))]])*Sqrt[d*Ta
n[e + f*x]])/(a^3*E^((6*I)*(e + f*x))*(-1 + E^((2*I)*(e + f*x)))*f)
```

Maple [A]
time = 0.19, size = 141, normalized size = 0.38

method	result
derivativedivides	$\frac{2d^4\left(i\sqrt{d\tan(fx+e)}-\frac{d\left(\frac{-20(d\tan(fx+e))^{\frac{5}{2}}+\frac{98id(d\tan(fx+e))^{\frac{3}{2}}}{3}+14d^2\sqrt{d\tan(fx+e)}}{(-id+d\tan(fx+e))^3}+\frac{29\arctan\left(\frac{\sqrt{}}{}\right.}{}\right.}{16}\right.}{f\,a^3}$
default	$\frac{2d^4\left(i\sqrt{d\tan(fx+e)}-\frac{d\left(\frac{-20(d\tan(fx+e))^{\frac{5}{2}}+\frac{98id(d\tan(fx+e))^{\frac{3}{2}}}{3}+14d^2\sqrt{d\tan(fx+e)}}{(-id+d\tan(fx+e))^3}+\frac{29\arctan\left(\frac{\sqrt{}}{}\right.}{}\right.}{16}\right.}{f\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3*d^4*(I*(d*tan(f*x+e))^(1/2)-1/16*d*((-20*(d*tan(f*x+e))^(5/2)+98/3*
I*d*(d*tan(f*x+e))^(3/2)+14*d^2*(d*tan(f*x+e))^(1/2))/(-I*d+d*tan(f*x+e))^3
+29/(-I*d)^(1/2)*arctan((d*tan(f*x+e))^(1/2)/(-I*d)^(1/2)))+1/16*d/(I*d)^(1
/2)*arctan((d*tan(f*x+e))^(1/2)/(I*d)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 620 vs. 2(289) = 578.
time = 0.40, size = 620, normalized size = 1.68

$$-\frac{12\,a^{3}\sqrt{\frac{i\,d^{9}}{64\,a^{6}f^{2}}}\,fe^{(6i\,fx+6i\,e)}\log\left(-\frac{2\left(i\,d^{5}e^{(2i\,fx+2i\,e)}+8\left(i\,a^{3}fe^{(2i\,fx+2i\,e)}+i\,a^{3}f\right)\sqrt{\frac{i\,d^{9}}{64\,a^{6}f^{2}}}\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-2i\,fx-2i\,e)}}{d^{4}}\right)-12\,a^{3}\sqrt{\frac{i\,d^{9}}{64\,a^{6}f^{2}}}\,fe^{(6i\,fx+6i\,e)}\log\left(-\frac{2\left(i\,d^{5}e^{(2i\,fx+2i\,e)}+8\left(-i\,a^{3}fe^{(2i\,fx+2i\,e)}-i\,a^{3}f\right)\sqrt{\frac{i\,d^{9}}{64\,a^{6}f^{2}}}\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-2i\,fx-2i\,e)}}{d^{4}}\right)-12\,a^{3}\sqrt{-\frac{841i\,d^{9}}{64\,a^{6}f^{2}}}\,fe^{(6i\,fx+6i\,e)}\log\left(\frac{\left(29\,d^{5}+8\left(a^{3}fe^{(2i\,fx+2i\,e)}+a^{3}f\right)\sqrt{-\frac{841i\,d^{9}}{64\,a^{6}f^{2}}}\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-2i\,fx-2i\,e)}}{8\,a^{3}f}\right)+12\,a^{3}\sqrt{-\frac{841i\,d^{9}}{64\,a^{6}f^{2}}}\,fe^{(6i\,fx+6i\,e)}\log\left(\frac{\left(29\,d^{5}-8\left(a^{3}fe^{(2i\,fx+2i\,e)}+a^{3}f\right)\sqrt{-\frac{841i\,d^{9}}{64\,a^{6}f^{2}}}\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-2i\,fx-2i\,e)}}{8\,a^{3}f}\right)-\left(146i\,d^{4}e^{(6i\,fx+6i\,e)}+41i\,d^{4}e^{(4i\,fx+4i\,e)}-8i\,d^{4}e^{(2i\,fx+2i\,e)}+i\,d^{4}\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\,e^{(-6i\,fx-6i\,e)}}{48\,a^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/48*(12*a^3*sqrt(1/64*I*d^9/(a^6*f^2))*f*e^(6*I*f*x + 6*I*e)*log(-2*(I*d^
5*e^(2*I*f*x + 2*I*e) + 8*(I*a^3*f*e^(2*I*f*x + 2*I*e) + I*a^3*f)*sqrt(1/64
*I*d^9/(a^6*f^2))*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e
) + 1)))*e^(-2*I*f*x - 2*I*e)/d^4) - 12*a^3*sqrt(1/64*I*d^9/(a^6*f^2))*f*e^
(6*I*f*x + 6*I*e)*log(-2*(I*d^5*e^(2*I*f*x + 2*I*e) + 8*(-I*a^3*f*e^(2*I*f*
x + 2*I*e) - I*a^3*f)*sqrt(1/64*I*d^9/(a^6*f^2))*sqrt((-I*d*e^(2*I*f*x + 2*
I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-2*I*f*x - 2*I*e)/d^4) - 12*a^3*
sqrt(-841/64*I*d^9/(a^6*f^2))*f*e^(6*I*f*x + 6*I*e)*log(1/8*(29*d^5 + 8*(a^
3*f*e^(2*I*f*x + 2*I*e) + a^3*f)*sqrt(-841/64*I*d^9/(a^6*f^2))*sqrt((-I*d*e
^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-2*I*f*x - 2*I*e)/
(a^3*f)) + 12*a^3*sqrt(-841/64*I*d^9/(a^6*f^2))*f*e^(6*I*f*x + 6*I*e)*log(1
/8*(29*d^5 - 8*(a^3*f*e^(2*I*f*x + 2*I*e) + a^3*f)*sqrt(-841/64*I*d^9/(a^6*
f^2))*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(
-2*I*f*x - 2*I*e)/(a^3*f)) - (146*I*d^4*e^(6*I*f*x + 6*I*e) + 41*I*d^4*e^(4
*I*f*x + 4*I*e) - 8*I*d^4*e^(2*I*f*x + 2*I*e) + I*d^4)*sqrt((-I*d*e^(2*I*f*
x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-6*I*f*x - 6*I*e)/(a^3*f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(9/2)/(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.93, size = 251, normalized size = 0.68

$$-\frac{1}{24}\,d^{4}\left(\frac{87i\,\sqrt{2}\,\sqrt{d}\,\arctan\left(\frac{8i\,\sqrt{d^{2}}\,\sqrt{d\tan\left(fx+e\right)}}{4i\,\sqrt{2}\,d^{\frac{3}{2}}+4\,\sqrt{2}\,\sqrt{d^{2}}\,\sqrt{d}}\right)}{a^{3}f\left(\frac{i\,d}{\sqrt{d^{2}}}+1\right)}+\frac{3i\,\sqrt{2}\,\sqrt{d}\,\arctan\left(-\frac{8i\,\sqrt{d^{2}}\,\sqrt{d\tan\left(fx+e\right)}}{-4i\,\sqrt{2}\,d^{\frac{3}{2}}+4\,\sqrt{2}\,\sqrt{d^{2}}\,\sqrt{d}}\right)}{a^{3}f\left(-\frac{i\,d}{\sqrt{d^{2}}}+1\right)}-\frac{48i\,\sqrt{d\tan\left(fx+e\right)}}{a^{3}f}-\frac{2\left(30\,\sqrt{d\tan\left(fx+e\right)}\,d^{3}\tan\left(fx+e\right)^{2}-49i\,\sqrt{d\tan\left(fx+e\right)}\,d^{3}\tan\left(fx+e\right)-21\,\sqrt{d\tan\left(fx+e\right)}\,d^{3}\right)}{\left(d\tan\left(fx+e\right)-i\,d\right)^{3}a^{3}f}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/24*d^4*(87*I*sqrt(2)*sqrt(d)*arctan(8*I*sqrt(d^2)*sqrt(d*tan(f*x + e))/(
4*I*sqrt(2)*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a^3*f*(I*d/sqrt(d^2) +
 1)) + 3*I*sqrt(2)*sqrt(d)*arctan(-8*I*sqrt(d^2)*sqrt(d*tan(f*x + e))/(-4*I
*sqrt(2)*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a^3*f*(-I*d/sqrt(d^2) + 1
)) - 48*I*sqrt(d*tan(f*x + e))/(a^3*f) - 2*(30*sqrt(d*tan(f*x + e))*d^3*tan
(f*x + e)^2 - 49*I*sqrt(d*tan(f*x + e))*d^3*tan(f*x + e) - 21*sqrt(d*tan(f*
x + e))*d^3)/((d*tan(f*x + e) - I*d)^3*a^3*f))
```

Mupad [B]
time = 5.48, size = 240, normalized size = 0.65

$$\mathrm{atan}\left(\frac{a^3\,f\,\sqrt{d\,\tan\left(e+f\,x\right)}\,\sqrt{\frac{d^9\,1{}\mathrm{i}}{256\,a^6\,f^2}}\,16{}\mathrm{i}}{d^5}\right)\,\sqrt{\frac{d^9\,1{}\mathrm{i}}{256\,a^6\,f^2}}\,2{}\mathrm{i}-\mathrm{atan}\left(\frac{a^3\,f\,\sqrt{d\,\tan\left(e+f\,x\right)}\,\sqrt{-\frac{d^9\,841{}\mathrm{i}}{256\,a^6\,f^2}}\,16{}\mathrm{i}}{29\,d^5}\right)\,\sqrt{-\frac{d^9\,841{}\mathrm{i}}{256\,a^6\,f^2}}\,2{}\mathrm{i}+\frac{\frac{7\,d^7\,\sqrt{d\,\tan\left(e+f\,x\right)}}{4\,a^3\,f}-\frac{5\,d^5\,{\left(d\,\tan\left(e+f\,x\right)\right)}^{5/2}}{2\,a^3\,f}+\frac{d^6\,{\left(d\,\tan\left(e+f\,x\right)\right)}^{3/2}\,49{}\mathrm{i}}{12\,a^3\,f}}{-d^3\,{\tan\left(e+f\,x\right)}^3+d^3\,{\tan\left(e+f\,x\right)}^2\,3{}\mathrm{i}+3\,d^3\,\tan\left(e+f\,x\right)-d^3\,1{}\mathrm{i}}+\frac{d^4\,\sqrt{d\,\tan\left(e+f\,x\right)}\,2{}\mathrm{i}}{a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(9/2)/(a + a*tan(e + f*x)*1i)^3,x)
```

```
[Out] atan((a^3*f*(d*tan(e + f*x))^(1/2)*((d^9*1i)/(256*a^6*f^2))^(1/2)*16i)/d^5)
*((d^9*1i)/(256*a^6*f^2))^(1/2)*2i - atan((a^3*f*(d*tan(e + f*x))^(1/2)*(-(
d^9*841i)/(256*a^6*f^2))^(1/2)*16i)/(29*d^5))*(-(d^9*841i)/(256*a^6*f^2))^(
1/2)*2i + ((7*d^7*(d*tan(e + f*x))^(1/2))/(4*a^3*f) + (d^6*(d*tan(e + f*x))
^(3/2)*49i)/(12*a^3*f) - (5*d^5*(d*tan(e + f*x))^(5/2))/(2*a^3*f))/(3*d^3*t
an(e + f*x) - d^3*1i + d^3*tan(e + f*x)^2*3i - d^3*tan(e + f*x)^3) + (d^4*(
d*tan(e + f*x))^(1/2)*2i)/(a^3*f)
```

3.180 $\int \frac{(d\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^3}\,dx$

Optimal. Leaf size=343

$$\frac{\left(\frac{5}{16}-\frac{7i}{16}\right)d^{7/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^3f}-\frac{\left(\frac{5}{16}-\frac{7i}{16}\right)d^{7/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^3f}+\frac{\left(\frac{5}{32}\right.}{}$$

```
[Out] (5/32-7/32*I)*d^(7/2)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a^3/f*
2^(1/2)+(-5/32+7/32*I)*d^(7/2)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2
))/a^3/f*2^(1/2)+(5/64+7/64*I)*d^(7/2)*ln(d^(1/2)-2^(1/2)*(d*tan(f*x+e))^(1
/2)+d^(1/2)*tan(f*x+e))/a^3/f*2^(1/2)-(5/64+7/64*I)*d^(7/2)*ln(d^(1/2)+2^(1
/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/a^3/f*2^(1/2)-1/6*d*(d*tan(f*x
+e))^(5/2)/f/(a+I*a*tan(f*x+e))^3+1/3*I*d^2*(d*tan(f*x+e))^(3/2)/a/f/(a+I*a
*tan(f*x+e))^2+5/8*d^3*(d*tan(f*x+e))^(1/2)/f/(a^3+I*a^3*tan(f*x+e))
```

Rubi [A]
time = 0.39, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {3639, 3676, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\left(\frac{5}{16}-\frac{7i}{16}\right)d^{7/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^3f}-\frac{\left(\frac{5}{16}-\frac{7i}{16}\right)d^{7/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,a^3f}+\frac{\left(\frac{5}{32}+\frac{7i}{32}\right)d^{7/2}\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,a^3f}-\frac{\left(\frac{5}{32}+\frac{7i}{32}\right)d^{7/2}\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,a^3f}+\frac{5d^3\sqrt{d\tan(e+fx)}}{8f\left(a^3+ia^3\tan(e+fx)\right)}+\frac{id^2(d\tan(e+fx))^{3/2}}{3af(a+ia\tan(e+fx))^2}-\frac{d(d\tan(e+fx))^{5/2}}{6f(a+ia\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(7/2)/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] ((5/16 - (7*I)/16)*d^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d
]])/(Sqrt[2]*a^3*f) - ((5/16 - (7*I)/16)*d^(7/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d
*Tan[e + f*x]])/Sqrt[d]])/(Sqrt[2]*a^3*f) + ((5/32 + (7*I)/32)*d^(7/2)*Log[
Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e + f*x]]])/(Sqrt[2]*a^
3*f) - ((5/32 + (7*I)/32)*d^(7/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqrt
[2]*Sqrt[d*Tan[e + f*x]]])/(Sqrt[2]*a^3*f) - (d*(d*Tan[e + f*x])^(5/2))/(6*
f*(a + I*a*Tan[e + f*x])^3) + ((I/3)*d^2*(d*Tan[e + f*x])^(3/2))/(a*f*(a +
I*a*Tan[e + f*x])^2) + (5*d^3*Sqrt[d*Tan[e + f*x]])/(8*f*(a^3 + I*a^3*Tan[e
 + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

```
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^3}\,dx = -\frac{d(d\tan(e+fx))^{5/2}}{6f(a+ia\tan(e+fx))^3} - \frac{\int \frac{(d\tan(e+fx))^{3/2}\left(-\frac{5ad^2}{2}+\frac{11}{2}iad^2\tan(e+fx)\right)}{(a+ia\tan(e+fx))^2}\,dx}{6a^2}$$

$$= -\frac{d(d\tan(e+fx))^{5/2}}{6f(a+ia\tan(e+fx))^3} + \frac{id^2(d\tan(e+fx))^{3/2}}{3af(a+ia\tan(e+fx))^2} + \frac{\int \frac{\sqrt{d\tan(e+fx)}\,(}{a+ia}}{}$$

$$= -\frac{d(d\tan(e+fx))^{5/2}}{6f(a+ia\tan(e+fx))^3} + \frac{id^2(d\tan(e+fx))^{3/2}}{3af(a+ia\tan(e+fx))^2} + \frac{5d^3\sqrt{d\tan(e+f}}{8f\left(a^3+ia^3\tan(e+\right.}$$

$$= -\frac{d(d\tan(e+fx))^{5/2}}{6f(a+ia\tan(e+fx))^3} + \frac{id^2(d\tan(e+fx))^{3/2}}{3af(a+ia\tan(e+fx))^2} + \frac{5d^3\sqrt{d\tan(e+f}}{8f\left(a^3+ia^3\tan(e+\right.}$$

$$= -\frac{d(d\tan(e+fx))^{5/2}}{6f(a+ia\tan(e+fx))^3} + \frac{id^2(d\tan(e+fx))^{3/2}}{3af(a+ia\tan(e+fx))^2} + \frac{5d^3\sqrt{d\tan(e+f}}{8f\left(a^3+ia^3\tan(e+\right.}$$

$$= -\frac{d(d\tan(e+fx))^{5/2}}{6f(a+ia\tan(e+fx))^3} + \frac{id^2(d\tan(e+fx))^{3/2}}{3af(a+ia\tan(e+fx))^2} + \frac{5d^3\sqrt{d\tan(e+f}}{8f\left(a^3+ia^3\tan(e+\right.}$$

$$= \frac{\left(\frac{5}{32}+\frac{7i}{32}\right)d^{7/2}\log\left(\sqrt{d}+\sqrt{d}\,\tan(e+fx)-\sqrt{2}\,\sqrt{d\tan(e+fx)}\right)}{\sqrt{2}\,a^3f} - \frac{\left(\frac{5}{32}+\right.}{}$$

$$= \frac{\left(\frac{5}{16}-\frac{7i}{16}\right)d^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^3f} - \frac{\left(\frac{5}{16}-\frac{7i}{16}\right)d^{7/2}\tan^{-1}\left(1+\right.}{\sqrt{2}}$$

Mathematica [A]

time = 1.56, size = 234, normalized size = 0.68

$$\frac{d^4\sec^4(e+fx)\left(-19+19\cos(4(e+fx))+(21-15i)\cos(3(e+fx))\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sqrt{\sin(2(e+fx))}-12i\sin(2(e+fx))+(21+15i)\text{ArcSin}(\cos(e+fx)-\sin(e+fx))\sqrt{\sin(2(e+fx))}\,(\cos(3(e+fx))+i\sin(3(e+fx)))+(15+21i)\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sqrt{\sin(2(e+fx))}\,\sin(3(e+fx))+21i\sin(4(e+fx))\right)}{96a^3f\sqrt{d\tan(e+fx)}\,(-i+\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(7/2)/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] (d^4*Sec[e + f*x]^4*(-19 + 19*Cos[4*(e + f*x)] + (21 - 15*I)*Cos[3*(e + f*x
)]*Log[Cos[e + f*x] + Sin[e + f*x] + Sqrt[Sin[2*(e + f*x)]]]*Sqrt[Sin[2*(e
+ f*x)]] - (12*I)*Sin[2*(e + f*x)] + (21 + 15*I)*ArcSin[Cos[e + f*x] - Sin[
e + f*x]]*Sqrt[Sin[2*(e + f*x)]]*(Cos[3*(e + f*x)] + I*Sin[3*(e + f*x)]) +
(15 + 21*I)*Log[Cos[e + f*x] + Sin[e + f*x] + Sqrt[Sin[2*(e + f*x)]]]*Sqrt[
Sin[2*(e + f*x)]]*Sin[3*(e + f*x)] + (21*I)*Sin[4*(e + f*x)]))/(96*a^3*f*Sq
rt[d*Tan[e + f*x]]*(-I + Tan[e + f*x])^3)
```

Maple [A]

time = 0.18, size = 125, normalized size = 0.36

method	result
derivativedivides	$\frac{2d^4\left(-\frac{9(d\tan(fx+e))^{\frac{5}{2}}-\frac{38id(d\tan(fx+e))^{\frac{3}{2}}}{3}-5d^2\sqrt{d\tan(fx+e)}}{16(id\tan(fx+e)+d)^3}+\frac{3i\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{-id}}\right)}{8\sqrt{-id}}+\frac{i\arctan \ldots}{}\right)}{f\,a^3}$
default	$\frac{2d^4\left(-\frac{9(d\tan(fx+e))^{\frac{5}{2}}-\frac{38id(d\tan(fx+e))^{\frac{3}{2}}}{3}-5d^2\sqrt{d\tan(fx+e)}}{16(id\tan(fx+e)+d)^3}+\frac{3i\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{-id}}\right)}{8\sqrt{-id}}+\frac{i\arctan \ldots}{}\right)}{f\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3*d^4*(-1/16*(9*(d*tan(f*x+e))^(5/2)-38/3*I*d*(d*tan(f*x+e))^(3/2)-5*
d^2*(d*tan(f*x+e))^(1/2))/(I*d*tan(f*x+e)+d)^3+3/8*I/(-I*d)^(1/2)*arctan((d
*tan(f*x+e))^(1/2)/(-I*d)^(1/2))+1/16*I/(I*d)^(1/2)*arctan((d*tan(f*x+e))^(
1/2)/(I*d)^(1/2)))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 614 vs. $2(267) = 534$.
time = 0.40, size = 614, normalized size = 1.79

$$-\frac{12\,a^{3}f\sqrt{-\frac{i\,d^{7}}{64\,a^{6}f^{2}}}\,e^{(6i\,fx+6i\,e)}\log\left(-\frac{2\left(i\,d^{4}e^{(2i\,fx+2i\,e)}+8\,(a^{3}fe^{(2i\,fx+2i\,e)}+a^{3}f)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{-\frac{i\,d^{7}}{64\,a^{6}f^{2}}}\right)e^{(-2i\,fx-2i\,e)}}{d^{3}}\right)-12\,a^{3}f\sqrt{-\frac{i\,d^{7}}{64\,a^{6}f^{2}}}\,e^{(6i\,fx+6i\,e)}\log\left(-\frac{2\left(i\,d^{4}e^{(2i\,fx+2i\,e)}-8\,(a^{3}fe^{(2i\,fx+2i\,e)}+a^{3}f)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{-\frac{i\,d^{7}}{64\,a^{6}f^{2}}}\right)e^{(-2i\,fx-2i\,e)}}{d^{3}}\right)-12\,a^{3}f\sqrt{\frac{9i\,d^{7}}{16\,a^{6}f^{2}}}\,e^{(6i\,fx+6i\,e)}\log\left(\frac{\left(-3i\,d^{4}+4\,(a^{3}fe^{(2i\,fx+2i\,e)}+a^{3}f)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{9i\,d^{7}}{16\,a^{6}f^{2}}}\right)e^{(-2i\,fx-2i\,e)}}{4\,a^{3}f}\right)+12\,a^{3}f\sqrt{\frac{9i\,d^{7}}{16\,a^{6}f^{2}}}\,e^{(6i\,fx+6i\,e)}\log\left(\frac{\left(-3i\,d^{4}-4\,(a^{3}fe^{(2i\,fx+2i\,e)}+a^{3}f)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{9i\,d^{7}}{16\,a^{6}f^{2}}}\right)e^{(-2i\,fx-2i\,e)}}{4\,a^{3}f}\right)-\left(20\,d^{3}e^{(6i\,fx+6i\,e)}+14\,d^{3}e^{(4i\,fx+4i\,e)}-5\,d^{3}e^{(2i\,fx+2i\,e)}+d^{3}\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\,e^{(-6i\,fx-6i\,e)}}{48\,a^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/48*(12*a^3*f*sqrt(-1/64*I*d^7/(a^6*f^2))*e^(6*I*f*x + 6*I*e)*log(-2*(I*d
^4*e^(2*I*f*x + 2*I*e) + 8*(a^3*f*e^(2*I*f*x + 2*I*e) + a^3*f)*sqrt((-I*d*e
^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-1/64*I*d^7/(a^6*
f^2)))*e^(-2*I*f*x - 2*I*e)/d^3) - 12*a^3*f*sqrt(-1/64*I*d^7/(a^6*f^2))*e^(
6*I*f*x + 6*I*e)*log(-2*(I*d^4*e^(2*I*f*x + 2*I*e) - 8*(a^3*f*e^(2*I*f*x +
2*I*e) + a^3*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e)
+ 1))*sqrt(-1/64*I*d^7/(a^6*f^2)))*e^(-2*I*f*x - 2*I*e)/d^3) - 12*a^3*f*sqr
t(9/16*I*d^7/(a^6*f^2))*e^(6*I*f*x + 6*I*e)*log(1/4*(-3*I*d^4 + 4*(a^3*f*e^
(2*I*f*x + 2*I*e) + a^3*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*
x + 2*I*e) + 1))*sqrt(9/16*I*d^7/(a^6*f^2)))*e^(-2*I*f*x - 2*I*e)/(a^3*f))
+ 12*a^3*f*sqrt(9/16*I*d^7/(a^6*f^2))*e^(6*I*f*x + 6*I*e)*log(1/4*(-3*I*d^4
 - 4*(a^3*f*e^(2*I*f*x + 2*I*e) + a^3*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I
*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(9/16*I*d^7/(a^6*f^2)))*e^(-2*I*f*x - 2*
I*e)/(a^3*f)) - (20*d^3*e^(6*I*f*x + 6*I*e) + 14*d^3*e^(4*I*f*x + 4*I*e) -
5*d^3*e^(2*I*f*x + 2*I*e) + d^3)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(
2*I*f*x + 2*I*e) + 1)))*e^(-6*I*f*x - 6*I*e)/(a^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int \frac{(d\tan(e+fx))^{\frac{7}{2}}}{\tan^3(e+fx)-3i\tan^2(e+fx)-3\tan(e+fx)+i}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(7/2)/(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] I*Integral((d*tan(e + f*x))**(7/2)/(tan(e + f*x)**3 - 3*I*tan(e + f*x)**2 -
 3*tan(e + f*x) + I), x)/a**3
```

Giac [A]
time = 0.77, size = 231, normalized size = 0.67

$$-\frac{1}{24}\,d^{3}\left(-\frac{3i\,\sqrt{2}\,\sqrt{d}\,\arctan\left(\frac{8\sqrt{d^{2}}\,\sqrt{d\tan(fx+e)}}{4i\,\sqrt{2}\,d^{\frac{3}{2}}+4\,\sqrt{2}\,\sqrt{d^{2}}\,\sqrt{d}}\right)}{a^{3}f\left(\frac{i\,d}{\sqrt{d^{2}}}+1\right)}-\frac{18i\,\sqrt{2}\,\sqrt{d}\,\arctan\left(\frac{8\sqrt{d^{2}}\,\sqrt{d\tan(fx+e)}}{-4i\,\sqrt{2}\,d^{\frac{3}{2}}+4\,\sqrt{2}\,\sqrt{d^{2}}\,\sqrt{d}}\right)}{a^{3}f\left(-\frac{i\,d}{\sqrt{d^{2}}}+1\right)}+\frac{27i\,\sqrt{d\tan(fx+e)}\,d^{3}\tan(fx+e)^{2}+38\,\sqrt{d\tan(fx+e)}\,d^{3}\tan(fx+e)-15i\,\sqrt{d\tan(fx+e)}\,d^{3}}{(d\tan(fx+e)-i\,d)^{3}a^{3}f}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/24*d^3*(-3*I*sqrt(2)*sqrt(d)*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*
I*sqrt(2)*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a^3*f*(I*d/sqrt(d^2) + 1
)) - 18*I*sqrt(2)*sqrt(d)*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(-4*I*sqr
t(2)*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a^3*f*(-I*d/sqrt(d^2) + 1)) +
 (27*I*sqrt(d*tan(f*x + e))*d^3*tan(f*x + e)^2 + 38*sqrt(d*tan(f*x + e))*d^
3*tan(f*x + e) - 15*I*sqrt(d*tan(f*x + e))*d^3)/((d*tan(f*x + e) - I*d)^3*a
^3*f))
```

Mupad [B]
time = 5.78, size = 217, normalized size = 0.63

$$\mathrm{atan}\left(\frac{8\,a^3\,f\,\sqrt{d\,\tan\left(e+f\,x\right)}\,\sqrt{\frac{d^7\,9\mathrm{i}}{64\,a^6\,f^2}}}{3\,d^4}\right)\,\sqrt{\frac{d^7\,9\mathrm{i}}{64\,a^6\,f^2}}\,2\mathrm{i}+\mathrm{atan}\left(\frac{16\,a^3\,f\,\sqrt{d\,\tan\left(e+f\,x\right)}\,\sqrt{-\frac{d^7\,1\mathrm{i}}{256\,a^6\,f^2}}}{d^4}\right)\,\sqrt{-\frac{d^7\,1\mathrm{i}}{256\,a^6\,f^2}}\,2\mathrm{i}+\frac{\frac{19\,d^5\,{\left(d\,\tan\left(e+f\,x\right)\right)}^{3/2}}{12\,a^3\,f}-\frac{d^6\,\sqrt{d\,\tan\left(e+f\,x\right)}\,5\mathrm{i}}{8\,a^3\,f}+\frac{d^4\,{\left(d\,\tan\left(e+f\,x\right)\right)}^{5/2}\,9\mathrm{i}}{8\,a^3\,f}}{-d^3\,{\tan\left(e+f\,x\right)}^3+d^3\,{\tan\left(e+f\,x\right)}^2\,3\mathrm{i}+3\,d^3\,\tan\left(e+f\,x\right)-d^3\,1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(7/2)/(a + a*tan(e + f*x)*1i)^3,x)
```

```
[Out] atan((8*a^3*f*(d*tan(e + f*x))^(1/2)*((d^7*9i)/(64*a^6*f^2))^(1/2))/(3*d^4)
)*((d^7*9i)/(64*a^6*f^2))^(1/2)*2i + atan((16*a^3*f*(d*tan(e + f*x))^(1/2)*
(-(d^7*1i)/(256*a^6*f^2))^(1/2))/d^4)*(-(d^7*1i)/(256*a^6*f^2))^(1/2)*2i +
((19*d^5*(d*tan(e + f*x))^(3/2))/(12*a^3*f) - (d^6*(d*tan(e + f*x))^(1/2)*5
i)/(8*a^3*f) + (d^4*(d*tan(e + f*x))^(5/2)*9i)/(8*a^3*f))/(3*d^3*tan(e + f*
x) - d^3*1i + d^3*tan(e + f*x)^2*3i - d^3*tan(e + f*x)^3)
```

3.181 $\int \frac{(d\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^3}\,dx$

Optimal. Leaf size=329

$$\frac{d^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8\sqrt{2}\,a^3f}-\frac{d^{5/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8\sqrt{2}\,a^3f}-\frac{d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\,\text{ta}\right.}{}$$

```
[Out] 1/16*d^(5/2)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a^3/f*2^(1/2)-1
/16*d^(5/2)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a^3/f*2^(1/2)-1/
32*d^(5/2)*ln(d^(1/2)-2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/a^3/
f*2^(1/2)+1/32*d^(5/2)*ln(d^(1/2)+2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(
f*x+e))/a^3/f*2^(1/2)-1/6*d*(d*tan(f*x+e))^(3/2)/f/(a+I*a*tan(f*x+e))^3+1/4
*I*d^2*(d*tan(f*x+e))^(1/2)/a/f/(a+I*a*tan(f*x+e))^2-1/4*I*d^2*(d*tan(f*x+e
))^(1/2)/f/(a^3+I*a^3*tan(f*x+e))
```

Rubi [A]
time = 0.40, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {3639, 3676, 3677, 12, 16, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{d^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8\sqrt{2}\,a^3f}-\frac{d^{5/2}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{8\sqrt{2}\,a^3f}-\frac{d^{5/2}\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\,\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{16\sqrt{2}\,a^3f}+\frac{d^{5/2}\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\,\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{16\sqrt{2}\,a^3f}-\frac{id^2\sqrt{d\tan(e+fx)}}{4f\left(a^3+ia^3\tan(e+fx)\right)}+\frac{id^2\sqrt{d\tan(e+fx)}}{4af(a+ia\tan(e+fx))^2}-\frac{d(d\tan(e+fx))^{3/2}}{6f(a+ia\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(5/2)/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] (d^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(8*Sqrt[2]*a^3
*f) - (d^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(8*Sqrt[
2]*a^3*f) - (d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Ta
n[e + f*x]]])/(16*Sqrt[2]*a^3*f) + (d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f
*x] + Sqrt[2]*Sqrt[d*Tan[e + f*x]]])/(16*Sqrt[2]*a^3*f) - (d*(d*Tan[e + f*x
])^(3/2))/(6*f*(a + I*a*Tan[e + f*x])^3) + ((I/4)*d^2*Sqrt[d*Tan[e + f*x]])
/(a*f*(a + I*a*Tan[e + f*x])^2) - ((I/4)*d^2*Sqrt[d*Tan[e + f*x]])/(f*(a^3
+ I*a^3*Tan[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^3}\,dx = -\frac{d(d\tan(e+fx))^{3/2}}{6f(a+ia\tan(e+fx))^3} - \frac{\int \frac{\sqrt{d\tan(e+fx)}\left(-\frac{3ad^2}{2}+\frac{9}{2}iad^2\tan(e+fx)\right)}{(a+ia\tan(e+fx))^2}\,dx}{6a^2}$$

$$= -\frac{d(d\tan(e+fx))^{3/2}}{6f(a+ia\tan(e+fx))^3} + \frac{id^2\sqrt{d\tan(e+fx)}}{4af(a+ia\tan(e+fx))^2} + \frac{\int \frac{-3ia^2d^3-9a^2d}{\sqrt{d\tan(e+fx)}}}{24a}$$

$$= -\frac{d(d\tan(e+fx))^{3/2}}{6f(a+ia\tan(e+fx))^3} + \frac{id^2\sqrt{d\tan(e+fx)}}{4af(a+ia\tan(e+fx))^2} - \frac{id^2\sqrt{d\tan(e+}}{4f\left(a^3+ia^3\tan(e\right.}$$

$$= -\frac{d(d\tan(e+fx))^{3/2}}{6f(a+ia\tan(e+fx))^3} + \frac{id^2\sqrt{d\tan(e+fx)}}{4af(a+ia\tan(e+fx))^2} - \frac{id^2\sqrt{d\tan(e+}}{4f\left(a^3+ia^3\tan(e\right.}$$

$$= -\frac{d(d\tan(e+fx))^{3/2}}{6f(a+ia\tan(e+fx))^3} + \frac{id^2\sqrt{d\tan(e+fx)}}{4af(a+ia\tan(e+fx))^2} - \frac{id^2\sqrt{d\tan(e+}}{4f\left(a^3+ia^3\tan(e\right.}$$

$$= -\frac{d(d\tan(e+fx))^{3/2}}{6f(a+ia\tan(e+fx))^3} + \frac{id^2\sqrt{d\tan(e+fx)}}{4af(a+ia\tan(e+fx))^2} - \frac{id^2\sqrt{d\tan(e+}}{4f\left(a^3+ia^3\tan(e\right.}$$

$$= -\frac{d(d\tan(e+fx))^{3/2}}{6f(a+ia\tan(e+fx))^3} + \frac{id^2\sqrt{d\tan(e+fx)}}{4af(a+ia\tan(e+fx))^2} - \frac{id^2\sqrt{d\tan(e+}}{4f\left(a^3+ia^3\tan(e\right.}$$

$$= -\frac{d(d\tan(e+fx))^{3/2}}{6f(a+ia\tan(e+fx))^3} + \frac{id^2\sqrt{d\tan(e+fx)}}{4af(a+ia\tan(e+fx))^2} - \frac{id^2\sqrt{d\tan(e+}}{4f\left(a^3+ia^3\tan(e\right.}$$

$$= -\frac{d(d\tan(e+fx))^{3/2}}{6f(a+ia\tan(e+fx))^3} + \frac{id^2\sqrt{d\tan(e+fx)}}{4af(a+ia\tan(e+fx))^2} - \frac{id^2\sqrt{d\tan(e+}}{4f\left(a^3+ia^3\tan(e\right.}$$

$$= -\frac{d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{16\sqrt{2}a^3f} + \frac{d^{5/2}\log\left(\sqrt{d}\right.}{}$$

$$= \frac{d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8\sqrt{2}a^3f} - \frac{d^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+}}{\sqrt{d}}\right.}{8\sqrt{2}a^3f}$$

Mathematica [A]
time = 1.15, size = 232, normalized size = 0.71

$$\frac{d^3\sec^4(e+fx)\left(i-i\cos(4(e+fx))+6i\cos(3(e+fx))\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sqrt{\sin(2(e+fx))}-6\sin(2(e+fx))+6i\text{ArcSin}(\cos(e+fx)-\sin(e+fx))\sqrt{\sin(2(e+fx))}(\cos(3(e+fx))+i\sin(3(e+fx)))-6\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sqrt{\sin(2(e+fx))}\sin(3(e+fx))+3\sin(4(e+fx))\right)}{96a^3f\sqrt{d\tan(e+fx)}(-i+\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(5/2)/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] (d^3*Sec[e + f*x]^4*(I - I*Cos[4*(e + f*x)] + (6*I)*Cos[3*(e + f*x)]*Log[Co
s[e + f*x] + Sin[e + f*x] + Sqrt[Sin[2*(e + f*x)]]]*Sqrt[Sin[2*(e + f*x)]]
- 6*Sin[2*(e + f*x)] + (6*I)*ArcSin[Cos[e + f*x] - Sin[e + f*x]]*Sqrt[Sin[2
*(e + f*x)]]*(Cos[3*(e + f*x)] + I*Sin[3*(e + f*x)]) - 6*Log[Cos[e + f*x] +
 Sin[e + f*x] + Sqrt[Sin[2*(e + f*x)]]]*Sqrt[Sin[2*(e + f*x)]]*Sin[3*(e + f
*x)] + 3*Sin[4*(e + f*x)]))/(96*a^3*f*Sqrt[d*Tan[e + f*x]]*(-I + Tan[e + f*
x])^3)
```

Maple [A]
time = 0.19, size = 116, normalized size = 0.35

method	result
derivativedivides	$\frac{2d^4\left(-\frac{\frac{2(d\tan(fx+e))^{\frac{5}{2}}-\frac{2id(d\tan(fx+e))^{\frac{3}{2}}}{3}}{(-id+d\tan(fx+e))^3}+\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{-id}}\right)}{\sqrt{-id}}}{16d}-\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{id}}\right)}{16d\sqrt{id}}\right)}{f\,a^3}$
default	$\frac{2d^4\left(-\frac{\frac{2(d\tan(fx+e))^{\frac{5}{2}}-\frac{2id(d\tan(fx+e))^{\frac{3}{2}}}{3}}{(-id+d\tan(fx+e))^3}+\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{-id}}\right)}{\sqrt{-id}}}{16d}-\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{id}}\right)}{16d\sqrt{id}}\right)}{f\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3*d^4*(-1/16/d*((2*(d*tan(f*x+e))^(5/2)-2/3*I*d*(d*tan(f*x+e))^(3/2))
/(-I*d+d*tan(f*x+e))^3+1/(-I*d)^(1/2)*arctan((d*tan(f*x+e))^(1/2)/(-I*d)^(1
/2)))-1/16/d/(I*d)^(1/2)*arctan((d*tan(f*x+e))^(1/2)/(I*d)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 615 vs. $2(267) = 534$.
time = 0.38, size = 615, normalized size = 1.87

$$\frac{12\,a^3 f\sqrt{\frac{i\,d^5}{64\,a^6 f^2}}\,e^{(6i\,fx+6i\,e)}\log\left(-\frac{2\left(i\,d^3e^{(2i\,fx+2i\,e)}+8\left(i\,a^3fe^{(2i\,fx+2i\,e)}+i\,a^3f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{i\,d^5}{64\,a^6f^2}}\right)e^{(-2i\,fx-2i\,e)}}{d^2}\right)-12\,a^3 f\sqrt{\frac{i\,d^5}{64\,a^6 f^2}}\,e^{(6i\,fx+6i\,e)}\log\left(-\frac{2\left(i\,d^3e^{(2i\,fx+2i\,e)}+8\left(-i\,a^3fe^{(2i\,fx+2i\,e)}-i\,a^3f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{i\,d^5}{64\,a^6f^2}}\right)e^{(-2i\,fx-2i\,e)}}{d^2}\right)+12\,a^3 f\sqrt{-\frac{i\,d^5}{64\,a^6 f^2}}\,e^{(6i\,fx+6i\,e)}\log\left(\frac{\left(d^3+8\left(a^3fe^{(2i\,fx+2i\,e)}+a^3f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{-\frac{i\,d^5}{64\,a^6f^2}}\right)e^{(-2i\,fx-2i\,e)}}{8\,a^3f}\right)-12\,a^3 f\sqrt{-\frac{i\,d^5}{64\,a^6 f^2}}\,e^{(6i\,fx+6i\,e)}\log\left(\frac{\left(d^3-8\left(a^3fe^{(2i\,fx+2i\,e)}+a^3f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{-\frac{i\,d^5}{64\,a^6f^2}}\right)e^{(-2i\,fx-2i\,e)}}{8\,a^3f}\right)+\left(-2i\,d^2e^{(6i\,fx+6i\,e)}+i\,d^2e^{(4i\,fx+4i\,e)}+2i\,d^2e^{(2i\,fx+2i\,e)}-i\,d^2\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\,e^{(-6i\,fx-6i\,e)}}{48\,a^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/48*(12*a^3*f*sqrt(1/64*I*d^5/(a^6*f^2))*e^(6*I*f*x + 6*I*e)*log(-2*(I*d^3
*e^(2*I*f*x + 2*I*e) + 8*(I*a^3*f*e^(2*I*f*x + 2*I*e) + I*a^3*f)*sqrt((-I*d
*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(1/64*I*d^5/(a^6
*f^2)))*e^(-2*I*f*x - 2*I*e)/d^2) - 12*a^3*f*sqrt(1/64*I*d^5/(a^6*f^2))*e^(
6*I*f*x + 6*I*e)*log(-2*(I*d^3*e^(2*I*f*x + 2*I*e) + 8*(-I*a^3*f*e^(2*I*f*x
 + 2*I*e) - I*a^3*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*
I*e) + 1))*sqrt(1/64*I*d^5/(a^6*f^2)))*e^(-2*I*f*x - 2*I*e)/d^2) + 12*a^3*f
*sqrt(-1/64*I*d^5/(a^6*f^2))*e^(6*I*f*x + 6*I*e)*log(1/8*(d^3 + 8*(a^3*f*e^
(2*I*f*x + 2*I*e) + a^3*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*
x + 2*I*e) + 1))*sqrt(-1/64*I*d^5/(a^6*f^2)))*e^(-2*I*f*x - 2*I*e)/(a^3*f))
 - 12*a^3*f*sqrt(-1/64*I*d^5/(a^6*f^2))*e^(6*I*f*x + 6*I*e)*log(1/8*(d^3 -
8*(a^3*f*e^(2*I*f*x + 2*I*e) + a^3*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)
/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-1/64*I*d^5/(a^6*f^2)))*e^(-2*I*f*x - 2*I*
e)/(a^3*f)) + (-2*I*d^2*e^(6*I*f*x + 6*I*e) + I*d^2*e^(4*I*f*x + 4*I*e) + 2
*I*d^2*e^(2*I*f*x + 2*I*e) - I*d^2)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(
e^(2*I*f*x + 2*I*e) + 1)))*e^(-6*I*f*x - 6*I*e)/(a^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int\frac{(d\tan(e+fx))^{\frac{5}{2}}}{\tan^3(e+fx)-3i\tan^2(e+fx)-3\tan(e+fx)+i}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(5/2)/(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] I*Integral((d*tan(e + f*x))**(5/2)/(tan(e + f*x)**3 - 3*I*tan(e + f*x)**2 -
 3*tan(e + f*x) + I), x)/a**3
```

Giac [A]
time = 0.81, size = 216, normalized size = 0.66

$$-\frac{1}{24}\,d^2\left(\frac{3\sqrt{2}\sqrt{d}\arctan\left(\frac{8\sqrt{d^2}\sqrt{d\tan(fx+e)}}{4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{a^3f\left(\frac{i\,d}{\sqrt{d^2}}+1\right)}+\frac{3\sqrt{2}\sqrt{d}\arctan\left(\frac{8\sqrt{d^2}\sqrt{d\tan(fx+e)}}{-4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{a^3f\left(-\frac{i\,d}{\sqrt{d^2}}+1\right)}+\frac{2\left(3\sqrt{d\tan(fx+e)}\,d^3\tan(fx+e)^2-i\sqrt{d\tan(fx+e)}\,d^3\tan(fx+e)\right)}{(d\tan(fx+e)-i\,d)^3a^3f}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/24*d^2*(3*sqrt(2)*sqrt(d)*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*I*s
qrt(2)*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a^3*f*(I*d/sqrt(d^2) + 1))
+ 3*sqrt(2)*sqrt(d)*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(-4*I*sqrt(2)*d
^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a^3*f*(-I*d/sqrt(d^2) + 1)) + 2*(3*
sqrt(d*tan(f*x + e))*d^3*tan(f*x + e)^2 - I*sqrt(d*tan(f*x + e))*d^3*tan(f*
x + e))/((d*tan(f*x + e) - I*d)^3*a^3*f))
```

Mupad [B]
time = 4.16, size = 158, normalized size = 0.48

$$\frac{(-1)^{1/4}\,d^{5/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)}{8\,a^3\,f}-\frac{(-1)^{1/4}\,d^{5/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)}{8\,a^3\,f}-\frac{-\frac{d^3\,{\left(d\tan(e+f\,x)\right)}^{5/2}}{4\,a^3\,f}+\frac{d^4\,{\left(d\tan(e+f\,x)\right)}^{3/2}\,1\mathrm{i}}{12\,a^3\,f}}{-d^3\,{\tan\left(e+f\,x\right)}^3+d^3\,{\tan\left(e+f\,x\right)}^2\,3\mathrm{i}+3\,d^3\tan\left(e+f\,x\right)-d^3\,1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(5/2)/(a + a*tan(e + f*x)*1i)^3,x)
```

```
[Out] ((-1)^(1/4)*d^(5/2)*atanh(((-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2)))/(8*
a^3*f) - ((-1)^(1/4)*d^(5/2)*atan(((-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/
2)))/(8*a^3*f) - ((d^4*(d*tan(e + f*x))^(3/2)*1i)/(12*a^3*f) - (d^3*(d*tan(
e + f*x))^(5/2))/(4*a^3*f))/(3*d^3*tan(e + f*x) - d^3*1i + d^3*tan(e + f*x)
^2*3i - d^3*tan(e + f*x)^3)
```

3.182 $\int \frac{(d\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^3}\,dx$

Optimal. Leaf size=157

$$\frac{\sqrt[4]{-1}\,d^{3/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8a^3f}-\frac{d\sqrt{d\tan(e+fx)}}{6f(a+ia\tan(e+fx))^3}+\frac{d\sqrt{d\tan(e+fx)}}{6af(a+ia\tan(e+fx))^2}+\frac{d\sqrt{\ }}{8f\,(a^3}$$

```
[Out] 1/8*(-1)^(1/4)*d^(3/2)*arctan((-1)^(3/4)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a^3/
f-1/6*d*(d*tan(f*x+e))^(1/2)/f/(a+I*a*tan(f*x+e))^3+1/6*d*(d*tan(f*x+e))^(1
/2)/a/f/(a+I*a*tan(f*x+e))^2+1/8*d*(d*tan(f*x+e))^(1/2)/f/(a^3+I*a^3*tan(f*
x+e))
```

Rubi [A]
time = 0.24, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3639, 3677, 12, 16, 3630, 3614, 211}

$$\frac{\sqrt[4]{-1}\,d^{3/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8a^3f}+\frac{d\sqrt{d\tan(e+fx)}}{8f\,(a^3+ia^3\tan(e+fx))}+\frac{d\sqrt{d\tan(e+fx)}}{6af(a+ia\tan(e+fx))^2}-\frac{d\sqrt{d\tan(e+fx)}}{6f(a+ia\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(3/2)/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] ((-1)^(1/4)*d^(3/2)*ArcTan[((-1)^(3/4)*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(8*a
^3*f) - (d*Sqrt[d*Tan[e + f*x]])/(6*f*(a + I*a*Tan[e + f*x])^3) + (d*Sqrt[d
*Tan[e + f*x]])/(6*a*f*(a + I*a*Tan[e + f*x])^2) + (d*Sqrt[d*Tan[e + f*x]])
/(8*f*(a^3 + I*a^3*Tan[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3630

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(a*c + b*d))*((c + d*Tan[e + f*x])^n/(2*(
b*c - a*d)*f*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c
+ d*Tan[e + f*x])^(n - 1)*Simp[a*c*d*(n - 1) + b*c^2 + b*d^2*n - d*(b*c - a
*d)*(n - 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[0, n, 1]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^3}\,dx = -\frac{d\sqrt{d\tan(e+fx)}}{6f(a+ia\tan(e+fx))^3} - \frac{\int \frac{-\frac{ad^2}{2}+\frac{7}{2}iad^2\tan(e+fx)}{\sqrt{d\tan(e+fx)}\,(a+ia\tan(e+fx))^2}\,dx}{6a^2}$$

$$= -\frac{d\sqrt{d\tan(e+fx)}}{6f(a+ia\tan(e+fx))^3} + \frac{d\sqrt{d\tan(e+fx)}}{6af(a+ia\tan(e+fx))^2} - \frac{\int \frac{6ia^2d^3\tan}{\sqrt{d\tan(e+fx)}}}{24a}$$

$$= -\frac{d\sqrt{d\tan(e+fx)}}{6f(a+ia\tan(e+fx))^3} + \frac{d\sqrt{d\tan(e+fx)}}{6af(a+ia\tan(e+fx))^2} - \frac{(id^2)\int \frac{}{\sqrt{d\tan(e-}}}{}$$

$$= -\frac{d\sqrt{d\tan(e+fx)}}{6f(a+ia\tan(e+fx))^3} + \frac{d\sqrt{d\tan(e+fx)}}{6af(a+ia\tan(e+fx))^2} - \frac{(id)\int \frac{\sqrt{d\tan(e+}}{a+ia\tan(e+}}{4a^2}$$

$$= -\frac{d\sqrt{d\tan(e+fx)}}{6f(a+ia\tan(e+fx))^3} + \frac{d\sqrt{d\tan(e+fx)}}{6af(a+ia\tan(e+fx))^2} + \frac{d\sqrt{d\tan(e+f}}{8f(a^3+ia^3\tan(e}$$

$$= -\frac{d\sqrt{d\tan(e+fx)}}{6f(a+ia\tan(e+fx))^3} + \frac{d\sqrt{d\tan(e+fx)}}{6af(a+ia\tan(e+fx))^2} + \frac{d\sqrt{d\tan(e+f}}{8f(a^3+ia^3\tan(e}$$

$$= \frac{\sqrt[4]{-1}\,d^{3/2}\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8a^3f} - \frac{d\sqrt{d\tan(e+fx)}}{6f(a+ia\tan(e+fx))^3} + \frac{}{6a}$$

Mathematica [A]
time = 2.38, size = 158, normalized size = 1.01

$$\frac{d^2(i\cos(3(e+fx))+\sin(3(e+fx)))\left(5\cos(e+fx)-5\cos(3(e+fx))+3i\sin(e+fx)-3i\sin(3(e+fx))+6\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\right)(\cos(3(e+fx))+i\sin(3(e+fx)))\sqrt{i\tan(e+fx)}\right)}{48a^3f\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(3/2)/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] (d^2*(I*Cos[3*(e + f*x)] + Sin[3*(e + f*x)])*(5*Cos[e + f*x] - 5*Cos[3*(e +
 f*x)] + (3*I)*Sin[e + f*x] - (3*I)*Sin[3*(e + f*x)] + 6*ArcTanh[Sqrt[(-1 +
 E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f*x)))]]*(Cos[3*(e + f*x)] + I*Sin
[3*(e + f*x)])*Sqrt[I*Tan[e + f*x]]))/(48*a^3*f*Sqrt[d*Tan[e + f*x]])
```

Maple [A]
time = 0.20, size = 103, normalized size = 0.66

method	result	size

derivativedivides	$\frac{2d^4\left(\frac{-(d\tan(fx+e))^{\frac{5}{2}}+\frac{10id(d\tan(fx+e))^{\frac{3}{2}}}{3}+d^2\sqrt{d\tan(fx+e)}}{16d^2(id\tan(fx+e)+d)^3}-\frac{i\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{id}}\right)}{16d^2\sqrt{id}}\right)}{f\,a^3}$	103
default	$\frac{2d^4\left(\frac{-(d\tan(fx+e))^{\frac{5}{2}}+\frac{10id(d\tan(fx+e))^{\frac{3}{2}}}{3}+d^2\sqrt{d\tan(fx+e)}}{16d^2(id\tan(fx+e)+d)^3}-\frac{i\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{id}}\right)}{16d^2\sqrt{id}}\right)}{f\,a^3}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3*d^4*(1/16/d^2*(-(d*tan(f*x+e))^(5/2)+10/3*I*d*(d*tan(f*x+e))^(3/2)+
d^2*(d*tan(f*x+e))^(1/2))/(I*d*tan(f*x+e)+d)^3-1/16*I/d^2/(I*d)^(1/2)*arcta
n((d*tan(f*x+e))^(1/2)/(I*d)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(134) = 268$.
time = 0.38, size = 359, normalized size = 2.29

$$\frac{\left(12\,a^3f\sqrt{-\frac{i\,d^3}{64\,a^6f^2}}\,e^{(6i\,fx+6i\,e)}\log\left(-\frac{2\left(i\,d^2e^{(2i\,fx+2i\,e)}+8\,(a^3fe^{(2i\,fx+2i\,e)}+a^3f)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{-\frac{i\,d^3}{64\,a^6f^2}}\right)e^{(-2i\,fx-2i\,e)}}{d}\right)-12\,a^3f\sqrt{-\frac{i\,d^3}{64\,a^6f^2}}\,e^{(6i\,fx+6i\,e)}\log\left(-\frac{2\left(i\,d^2e^{(2i\,fx+2i\,e)}-8\,(a^3fe^{(2i\,fx+2i\,e)}+a^3f)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{-\frac{i\,d^3}{64\,a^6f^2}}\right)e^{(-2i\,fx-2i\,e)}}{d}\right)+\left(4\,de^{(6i\,fx+6i\,e)}+4\,de^{(4i\,fx+4i\,e)}-de^{(2i\,fx+2i\,e)}-d\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-6i\,fx-6i\,e)}}{48\,a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/48*(12*a^3*f*sqrt(-1/64*I*d^3/(a^6*f^2))*e^(6*I*f*x + 6*I*e)*log(-2*(I*d^
2*e^(2*I*f*x + 2*I*e) + 8*(a^3*f*e^(2*I*f*x + 2*I*e) + a^3*f)*sqrt((-I*d*e^
(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-1/64*I*d^3/(a^6*f
^2)))*e^(-2*I*f*x - 2*I*e)/d) - 12*a^3*f*sqrt(-1/64*I*d^3/(a^6*f^2))*e^(6*I
*f*x + 6*I*e)*log(-2*(I*d^2*e^(2*I*f*x + 2*I*e) - 8*(a^3*f*e^(2*I*f*x + 2*I
```

```
*e) + a^3*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1
))*sqrt(-1/64*I*d^3/(a^6*f^2)))*e^(-2*I*f*x - 2*I*e)/d) + (4*d*e^(6*I*f*x +
 6*I*e) + 4*d*e^(4*I*f*x + 4*I*e) - d*e^(2*I*f*x + 2*I*e) - d)*sqrt((-I*d*e
^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-6*I*f*x - 6*I*e)/
(a^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int\frac{(d\tan(e+fx))^{\frac{3}{2}}}{\tan^3(e+fx)-3i\tan^2(e+fx)-3\tan(e+fx)+i}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(3/2)/(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] I*Integral((d*tan(e + f*x))**(3/2)/(tan(e + f*x)**3 - 3*I*tan(e + f*x)**2 -
 3*tan(e + f*x) + I), x)/a**3
```

Giac [A]
time = 0.73, size = 160, normalized size = 1.02

$$-\frac{1}{24}\,d\left(\frac{3i\,\sqrt{2}\,\sqrt{d}\,\arctan\left(\frac{8\,\sqrt{d^2}\,\sqrt{d\tan(fx+e)}}{4i\,\sqrt{2}\,d^{\frac{3}{2}}+4\,\sqrt{2}\,\sqrt{d^2}\,\sqrt{d}}\right)}{a^3f\left(\frac{i\,d}{\sqrt{d^2}}+1\right)}+\frac{3i\,\sqrt{d\tan(fx+e)}\,d^3\tan(fx+e)^2+10\,\sqrt{d\tan(fx+e)}\,d^3\tan(fx+e)-3i\,\sqrt{d\tan(fx+e)}\,d^3}{(d\tan(fx+e)-i\,d)^3a^3f}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/24*d*(3*I*sqrt(2)*sqrt(d)*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*I*s
qrt(2)*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a^3*f*(I*d/sqrt(d^2) + 1))
+ (3*I*sqrt(d*tan(f*x + e))*d^3*tan(f*x + e)^2 + 10*sqrt(d*tan(f*x + e))*d^
3*tan(f*x + e) - 3*I*sqrt(d*tan(f*x + e))*d^3)/((d*tan(f*x + e) - I*d)^3*a^
3*f))
```

Mupad [B]
time = 4.22, size = 155, normalized size = 0.99

$$\frac{\frac{5\,d^3\,(d\tan(e+f\,x))^{3/2}}{12\,a^3\,f}-\frac{d^4\,\sqrt{d\tan(e+f\,x)}\,1\mathrm{i}}{8\,a^3\,f}+\frac{d^2\,(d\tan(e+f\,x))^{5/2}\,1\mathrm{i}}{8\,a^3\,f}}{-d^3\tan(e+f\,x)^3+d^3\tan(e+f\,x)^2\,3\mathrm{i}+3\,d^3\tan(e+f\,x)-d^3\,1\mathrm{i}}-\frac{\sqrt{\frac{1}{256}\mathrm{i}}\,(-d)^{3/2}\,\mathrm{atan}\left(\frac{16\,\sqrt{\frac{1}{256}\mathrm{i}}\,\sqrt{d\tan(e+f\,x)}}{\sqrt{-d}}\right)\,2\mathrm{i}}{a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(3/2)/(a + a*tan(e + f*x)*1i)^3,x)
```

```
[Out] ((5*d^3*(d*tan(e + f*x))^(3/2))/(12*a^3*f) - (d^4*(d*tan(e + f*x))^(1/2)*1i
)/(8*a^3*f) + (d^2*(d*tan(e + f*x))^(5/2)*1i)/(8*a^3*f))/(3*d^3*tan(e + f*x
) - d^3*1i + d^3*tan(e + f*x)^2*3i - d^3*tan(e + f*x)^3) - ((1i/256)^(1/2)*
(-d)^(3/2)*atan((16*(1i/256)^(1/2)*(d*tan(e + f*x))^(1/2))/(-d)^(1/2))*2i)/
(a^3*f)
```

3.183 $\int \frac{\sqrt{d\tan(e+fx)}}{(a+ia\tan(e+fx))^3}\,dx$

Optimal. Leaf size=292

$$\frac{i\sqrt{d}\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8\sqrt{2}\,a^3f}-\frac{i\sqrt{d}\,\mathrm{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8\sqrt{2}\,a^3f}+\frac{i\sqrt{d}\,\log\left(\sqrt{d}+\sqrt{d}\right.}{}$$

```
[Out] 1/16*I*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/a^3/f*2^(1/2)
-1/16*I*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/a^3/f*2^(1/2
)+1/32*I*ln(d^(1/2)-2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))*d^(1/2
)/a^3/f*2^(1/2)-1/32*I*ln(d^(1/2)+2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(
f*x+e))*d^(1/2)/a^3/f*2^(1/2)+1/6*I*(d*tan(f*x+e))^(1/2)/f/(a+I*a*tan(f*x+e
))^3+1/12*I*(d*tan(f*x+e))^(1/2)/a/f/(a+I*a*tan(f*x+e))^2
```

Rubi [A]
time = 0.25, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {3638, 3677, 21, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{i\sqrt{d}\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8\sqrt{2}\,a^3f}-\frac{i\sqrt{d}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{8\sqrt{2}\,a^3f}+\frac{i\sqrt{d}\,\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\,\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{16\sqrt{2}\,a^3f}-\frac{i\sqrt{d}\,\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\,\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{16\sqrt{2}\,a^3f}+\frac{i\sqrt{d\tan(e+fx)}}{12af(a+ia\tan(e+fx))^2}+\frac{i\sqrt{d\tan(e+fx)}}{6f(a+ia\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Tan[e + f*x]]/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] ((I/8)*Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(Sqrt[2]
*a^3*f) - ((I/8)*Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]]
)/(Sqrt[2]*a^3*f) + ((I/16)*Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sq
rt[2]*Sqrt[d*Tan[e + f*x]]])/(Sqrt[2]*a^3*f) - ((I/16)*Sqrt[d]*Log[Sqrt[d]
+ Sqrt[d]*Tan[e + f*x] + Sqrt[2]*Sqrt[d*Tan[e + f*x]]])/(Sqrt[2]*a^3*f) + (
(I/6)*Sqrt[d*Tan[e + f*x]])/(f*(a + I*a*Tan[e + f*x])^3) + ((I/12)*Sqrt[d*T
an[e + f*x]])/(a*f*(a + I*a*Tan[e + f*x])^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3638

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*Sqrt[(c_.) + (d_.)*tan[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-b)*(a + b*Tan[e + f*x])^m*(Sqrt[c + d
*Tan[e + f*x]]/(2*a*f*m)), x] + Dist[1/(4*a^2*m), Int[(a + b*Tan[e + f*x])^
(m + 1)*(Simp[2*a*c*m + b*d + a*d*(2*m + 1)*Tan[e + f*x], x]/Sqrt[c + d*Tan
[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && IntegersQ[2*m]
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{d\tan(e+fx)}}{(a+ia\tan(e+fx))^3}\,dx = \frac{i\sqrt{d\tan(e+fx)}}{6f(a+ia\tan(e+fx))^3} - \frac{\int \frac{iad-5ad\tan(e+fx)}{\sqrt{d\tan(e+fx)}\,(a+ia\tan(e+fx))^2}\,dx}{12a^2}$$

$$= \frac{i\sqrt{d\tan(e+fx)}}{6f(a+ia\tan(e+fx))^3} + \frac{i\sqrt{d\tan(e+fx)}}{12af(a+ia\tan(e+fx))^2} - \frac{\int \frac{6ia^2d^2-6a^2d^2\,t}{\sqrt{d\tan(e+fx)}}}{48a^4}$$

$$= \frac{i\sqrt{d\tan(e+fx)}}{6f(a+ia\tan(e+fx))^3} + \frac{i\sqrt{d\tan(e+fx)}}{12af(a+ia\tan(e+fx))^2} - \frac{(id)\int \frac{1}{\sqrt{d\tan(e+}}}{8a^3}$$

$$= \frac{i\sqrt{d\tan(e+fx)}}{6f(a+ia\tan(e+fx))^3} + \frac{i\sqrt{d\tan(e+fx)}}{12af(a+ia\tan(e+fx))^2} - \frac{(id^2)\,\mathrm{Subst}\Big(\int \frac{}{\sqrt{x}}}{}$$

$$= \frac{i\sqrt{d\tan(e+fx)}}{6f(a+ia\tan(e+fx))^3} + \frac{i\sqrt{d\tan(e+fx)}}{12af(a+ia\tan(e+fx))^2} - \frac{(id^2)\,\mathrm{Subst}\Big(\int \frac{1}{d^2+x^4}}{}$$

$$= \frac{i\sqrt{d\tan(e+fx)}}{6f(a+ia\tan(e+fx))^3} + \frac{i\sqrt{d\tan(e+fx)}}{12af(a+ia\tan(e+fx))^2} - \frac{(id)\mathrm{Subst}\Big(\int \frac{d-x^2}{d^2+x^4}}{}$$

$$= \frac{i\sqrt{d\tan(e+fx)}}{6f(a+ia\tan(e+fx))^3} + \frac{i\sqrt{d\tan(e+fx)}}{12af(a+ia\tan(e+fx))^2} + \frac{\left(i\sqrt{d}\right)\mathrm{Subst}\Big(\int}{}$$

$$= \frac{i\sqrt{d}\,\log\left(\sqrt{d}+\sqrt{d}\,\tan(e+fx)-\sqrt{2}\,\sqrt{d\tan(e+fx)}\right)}{16\sqrt{2}\,a^3f} - \frac{i\sqrt{d}\,\log\Big(\sqrt{}}{}$$

$$= \frac{i\sqrt{d}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8\sqrt{2}\,a^3f} - \frac{i\sqrt{d}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt{d\tan(e}}{\sqrt{d}}\right.}{8\sqrt{2}\,a^3f}$$

Mathematica [A]

time = 2.63, size = 225, normalized size = 0.77

$$-\frac{d(\cos(3(e+fx))-i\sin(3(e+fx)))\left(\cos(e+fx)-\cos(3(e+fx))-3i\sin(e+fx)-3i\sin(3(e+fx))+6\mathrm{ArcTan}\left(\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\right)(\cos(3(e+fx))+i\sin(3(e+fx)))\sqrt{i\tan(e+fx)}+6\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\right)(\cos(3(e+fx))+i\sin(3(e+fx)))\sqrt{i\tan(e+fx)}\right)}{48a^3f\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Tan[e + f*x]]/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] -1/48*(d*(Cos[3*(e + f*x)] - I*Sin[3*(e + f*x)])*(Cos[e + f*x] - Cos[3*(e +
 f*x)] - (3*I)*Sin[e + f*x] - (3*I)*Sin[3*(e + f*x)] + 6*ArcTan[Sqrt[(-1 +
E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f*x)))]]*(Cos[3*(e + f*x)] + I*Sin[
3*(e + f*x)])*Sqrt[I*Tan[e + f*x]] + 6*ArcTanh[Sqrt[(-1 + E^((2*I)*(e + f*x
```

```
)))/(1 + E^((2*I)*(e + f*x)))]]*(Cos[3*(e + f*x)] + I*Sin[3*(e + f*x)])*Sqr
t[I*Tan[e + f*x]]))/(a^3*f*Sqrt[d*Tan[e + f*x]])
```

Maple [A]
time = 0.21, size = 120, normalized size = 0.41

method	result
derivativedivides	$\frac{2d^4\left(\frac{\frac{-\frac{2id(d\tan(fx+e))^{\frac{3}{2}}}{3}-2d^2\sqrt{d\tan(fx+e)}}{(-id+d\tan(fx+e))^3}-\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{-id}}\right)}{\sqrt{-id}}}{16d^3}+\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e}}{\sqrt{id}}\right.}{16d^3\sqrt{id}}\right.}{f\,a^3}$
default	$\frac{2d^4\left(\frac{\frac{-\frac{2id(d\tan(fx+e))^{\frac{3}{2}}}{3}-2d^2\sqrt{d\tan(fx+e)}}{(-id+d\tan(fx+e))^3}-\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{-id}}\right)}{\sqrt{-id}}}{16d^3}+\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e}}{\sqrt{id}}\right.}{16d^3\sqrt{id}}\right.}{f\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3*d^4*(1/16/d^3*((-2/3*I*d*(d*tan(f*x+e))^(3/2)-2*d^2*(d*tan(f*x+e))^
(1/2))/(-I*d+d*tan(f*x+e))^3-1/(-I*d)^(1/2)*arctan((d*tan(f*x+e))^(1/2)/(-I
*d)^(1/2)))+1/16/d^3/(I*d)^(1/2)*arctan((d*tan(f*x+e))^(1/2)/(I*d)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 575 vs. 2(226) = 452.
time = 0.38, size = 575, normalized size = 1.97

$$-\frac{\left(12a^3f\sqrt{\frac{id}{64a^6f^2}}e^{(6ifx+6ie)}\log\left(-2\left(8\left(ia^3fe^{(2ifx+2ie)}+ia^3f\right)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{id}{64a^6f^2}}+ide^{(2ifx+2ie)}\right)e^{(-2ifx-2ie)}\right)-12a^3f\sqrt{\frac{id}{64a^6f^2}}e^{(6ifx+6ie)}\log\left(-2\left(8\left(-ia^3fe^{(2ifx+2ie)}-ia^3f\right)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{id}{64a^6f^2}}+ide^{(2ifx+2ie)}\right)e^{(-2ifx-2ie)}\right)-12a^3f\sqrt{-\frac{id}{64a^6f^2}}e^{(6ifx+6ie)}\log\left(\frac{\left(8\left(a^3fe^{(2ifx+2ie)}+a^3f\right)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\sqrt{-\frac{id}{64a^6f^2}}+d\right)e^{(-2ifx-2ie)}}{8a^3f}\right)+12a^3f\sqrt{-\frac{id}{64a^6f^2}}e^{(6ifx+6ie)}\log\left(-\frac{\left(8\left(a^3fe^{(2ifx+2ie)}+a^3f\right)\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\sqrt{-\frac{id}{64a^6f^2}}-d\right)e^{(-2ifx-2ie)}}{8a^3f}\right)-\sqrt{\frac{-ide^{(2ifx+2ie)}+id}{e^{(2ifx+2ie)}+1}}\left(2ie^{(6ifx+6ie)}+5ie^{(4ifx+4ie)}+4ie^{(2ifx+2ie)}+i\right)\right)e^{(-6ifx-6ie)}}{48a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/48*(12*a^3*f*sqrt(1/64*I*d/(a^6*f^2))*e^(6*I*f*x + 6*I*e)*log(-2*(8*(I*a
^3*f*e^(2*I*f*x + 2*I*e) + I*a^3*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(
e^(2*I*f*x + 2*I*e) + 1))*sqrt(1/64*I*d/(a^6*f^2)) + I*d*e^(2*I*f*x + 2*I*e
))*e^(-2*I*f*x - 2*I*e)) - 12*a^3*f*sqrt(1/64*I*d/(a^6*f^2))*e^(6*I*f*x + 6
*I*e)*log(-2*(8*(-I*a^3*f*e^(2*I*f*x + 2*I*e) - I*a^3*f)*sqrt((-I*d*e^(2*I*
f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(1/64*I*d/(a^6*f^2)) + I
*d*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)) - 12*a^3*f*sqrt(-1/64*I*d/(a^
6*f^2))*e^(6*I*f*x + 6*I*e)*log(1/8*(8*(a^3*f*e^(2*I*f*x + 2*I*e) + a^3*f)*
sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-1/64
*I*d/(a^6*f^2)) + d)*e^(-2*I*f*x - 2*I*e)/(a^3*f)) + 12*a^3*f*sqrt(-1/64*I*
d/(a^6*f^2))*e^(6*I*f*x + 6*I*e)*log(-1/8*(8*(a^3*f*e^(2*I*f*x + 2*I*e) + a
^3*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt
(-1/64*I*d/(a^6*f^2)) - d)*e^(-2*I*f*x - 2*I*e)/(a^3*f)) - sqrt((-I*d*e^(2*
I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*(2*I*e^(6*I*f*x + 6*I*e) +
 5*I*e^(4*I*f*x + 4*I*e) + 4*I*e^(2*I*f*x + 2*I*e) + I))*e^(-6*I*f*x - 6*I*
e)/(a^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int\frac{\sqrt{d\tan\left(e+fx\right)}}{\tan^{3}\left(e+fx\right)-3i\tan^{2}\left(e+fx\right)-3\tan\left(e+fx\right)+i}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] I*Integral(sqrt(d*tan(e + f*x))/(tan(e + f*x)**3 - 3*I*tan(e + f*x)**2 - 3*
tan(e + f*x) + I), x)/a**3
```

Giac [A]
time = 0.71, size = 207, normalized size = 0.71

$$\frac{\frac{3\sqrt{2}\,d^{\frac{3}{2}}\arctan\left(\frac{8\sqrt{d^{2}}\sqrt{d\tan\left(fx+e\right)}}{4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^{2}}\sqrt{d}}\right)}{a^{3}f\left(\frac{id}{\sqrt{d^{2}}}+1\right)}-\frac{3\sqrt{2}\,d^{\frac{3}{2}}\arctan\left(\frac{8\sqrt{d^{2}}\sqrt{d\tan\left(fx+e\right)}}{-4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^{2}}\sqrt{d}}\right)}{a^{3}f\left(-\frac{id}{\sqrt{d^{2}}}+1\right)}-\frac{2\left(i\sqrt{d\tan\left(fx+e\right)}\,d^{4}\tan\left(fx+e\right)+3\sqrt{d\tan\left(fx+e\right)}\,d^{4}\right)}{\left(d\tan\left(fx+e\right)-id\right)^{3}a^{3}f}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/24*(3*sqrt(2)*d^(3/2)*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*I*sqrt(2
)*d^(3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a^3*f*(I*d/sqrt(d^2) + 1)) - 3*s
qrt(2)*d^(3/2)*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(-4*I*sqrt(2)*d^(3/2
```

```
) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a^3*f*(-I*d/sqrt(d^2) + 1)) - 2*(I*sqrt(
d*tan(f*x + e))*d^4*tan(f*x + e) + 3*sqrt(d*tan(f*x + e))*d^4)/((d*tan(f*x
+ e) - I*d)^3*a^3*f))/d
```

Mupad [B]
time = 4.18, size = 157, normalized size = 0.54

$$\frac{\frac{d^3\,\sqrt{d\,\tan\left(e+f\,x\right)}}{4\,a^3\,f}+\frac{d^2\,{\left(d\,\tan\left(e+f\,x\right)\right)}^{3/2}\,1{}\mathrm{i}}{12\,a^3\,f}}{-d^3\,{\tan\left(e+f\,x\right)}^3+d^3\,{\tan\left(e+f\,x\right)}^2\,3{}\mathrm{i}+3\,d^3\,\tan\left(e+f\,x\right)-d^3\,1{}\mathrm{i}}-\frac{{\left(-1\right)}^{1/4}\,\sqrt{d}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{d\,\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)}{8\,a^3\,f}-\frac{{\left(-1\right)}^{1/4}\,\sqrt{d}\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{d\,\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)}{8\,a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(1/2)/(a + a*tan(e + f*x)*1i)^3,x)
```

```
[Out] ((d^3*(d*tan(e + f*x))^(1/2))/(4*a^3*f) + (d^2*(d*tan(e + f*x))^(3/2)*1i)/(
12*a^3*f))/(3*d^3*tan(e + f*x) - d^3*1i + d^3*tan(e + f*x)^2*3i - d^3*tan(e
 + f*x)^3) - ((-1)^(1/4)*d^(1/2)*atan(((-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d
^(1/2)))/(8*a^3*f) - ((-1)^(1/4)*d^(1/2)*atanh(((-1)^(1/4)*(d*tan(e + f*x))
^(1/2))/d^(1/2)))/(8*a^3*f)
```

3.184 $\int \frac{1}{\sqrt{d\tan(e+fx)}\,(a+ia\tan(e+fx))^3}\,dx$

Optimal. Leaf size=343

$$-\frac{\left(\frac{7}{16}-\frac{5i}{16}\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^3\sqrt{d}\,f}+\frac{\left(\frac{7}{16}-\frac{5i}{16}\right)\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^3\sqrt{d}\,f}-\frac{\left(\frac{7}{32}+\right.}{}$$

```
[Out] (-7/32+5/32*I)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a^3/f*2^(1/2)
/d^(1/2)+(7/32-5/32*I)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a^3/f
*2^(1/2)/d^(1/2)-(7/64+5/64*I)*ln(d^(1/2)-2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1
/2)*tan(f*x+e))/a^3/f*2^(1/2)/d^(1/2)+(7/64+5/64*I)*ln(d^(1/2)+2^(1/2)*(d*t
an(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/a^3/f*2^(1/2)/d^(1/2)+1/6*(d*tan(f*x+e
))^(1/2)/d/f/(a+I*a*tan(f*x+e))^3+1/3*(d*tan(f*x+e))^(1/2)/a/d/f/(a+I*a*tan
(f*x+e))^2+5/8*(d*tan(f*x+e))^(1/2)/d/f/(a^3+I*a^3*tan(f*x+e))
```

Rubi [A]
time = 0.38, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {3640, 3677, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\left(\frac{7}{16}-\frac{5i}{16}\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^3\sqrt{d}\,f}+\frac{\left(\frac{7}{16}-\frac{5i}{16}\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,a^3\sqrt{d}\,f}+\frac{5\sqrt{d\tan(e+fx)}}{8df\left(a^3+ia^3\tan(e+fx)\right)}-\frac{\left(\frac{7}{32}+\frac{5i}{32}\right)\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,a^3\sqrt{d}\,f}+\frac{\left(\frac{7}{32}+\frac{5i}{32}\right)\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,a^3\sqrt{d}\,f}+\frac{\sqrt{d\tan(e+fx)}}{3adf(a+ia\tan(e+fx))^2}+\frac{\sqrt{d\tan(e+fx)}}{6df(a+ia\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*Tan[e + f*x]]*(a + I*a*Tan[e + f*x])^3),x]
```

```
[Out] ((-7/16 + (5*I)/16)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(Sq
rt[2]*a^3*Sqrt[d]*f) + ((7/16 - (5*I)/16)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e
+ f*x]])/Sqrt[d]])/(Sqrt[2]*a^3*Sqrt[d]*f) - ((7/32 + (5*I)/32)*Log[Sqrt[d]
 + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e + f*x]]])/(Sqrt[2]*a^3*Sqrt[
d]*f) + ((7/32 + (5*I)/32)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqrt[2]*Sqr
t[d*Tan[e + f*x]]])/(Sqrt[2]*a^3*Sqrt[d]*f) + Sqrt[d*Tan[e + f*x]]/(6*d*f*(
a + I*a*Tan[e + f*x])^3) + Sqrt[d*Tan[e + f*x]]/(3*a*d*f*(a + I*a*Tan[e + f
*x])^2) + (5*Sqrt[d*Tan[e + f*x]])/(8*d*f*(a^3 + I*a^3*Tan[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

```
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{d\tan(e+fx)}\,(a+ia\tan(e+fx))^3}\,dx = \frac{\sqrt{d\tan(e+fx)}}{6df(a+ia\tan(e+fx))^3} + \frac{\int \frac{\frac{11ad}{2}-\frac{5}{2}iad\tan(e+fx)}{\sqrt{d\tan(e+fx)}\,(a+ia\tan(e+fx}}{6a^2d}$$

$$= \frac{\sqrt{d\tan(e+fx)}}{6df(a+ia\tan(e+fx))^3} + \frac{\sqrt{d\tan(e+fx)}}{3adf(a+ia\tan(e+fx))^2} + \frac{\int}{}$$

$$= \frac{\sqrt{d\tan(e+fx)}}{6df(a+ia\tan(e+fx))^3} + \frac{\sqrt{d\tan(e+fx)}}{3adf(a+ia\tan(e+fx))^2} + \frac{}{8a}$$

$$= \frac{\sqrt{d\tan(e+fx)}}{6df(a+ia\tan(e+fx))^3} + \frac{\sqrt{d\tan(e+fx)}}{3adf(a+ia\tan(e+fx))^2} + \frac{}{8a}$$

$$= \frac{\sqrt{d\tan(e+fx)}}{6df(a+ia\tan(e+fx))^3} + \frac{\sqrt{d\tan(e+fx)}}{3adf(a+ia\tan(e+fx))^2} + \frac{}{8a}$$

$$= \frac{\sqrt{d\tan(e+fx)}}{6df(a+ia\tan(e+fx))^3} + \frac{\sqrt{d\tan(e+fx)}}{3adf(a+ia\tan(e+fx))^2} + \frac{}{8a}$$

$$= -\frac{\left(\frac{7}{32}+\frac{5i}{32}\right)\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+}\right.}{\sqrt{2}\,a^3\sqrt{d}\,f}$$

$$= -\frac{\left(\frac{7}{16}-\frac{5i}{16}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^3\sqrt{d}\,f} + \frac{\left(\frac{7}{16}-\frac{5i}{16}\right.}{}$$

Mathematica [A]

time = 1.31, size = 234, normalized size = 0.68

$$\frac{\sec^4(e+fx)\left(19\cos(4(e+fx))-(15+21i)\mathrm{ArcSin}(\cos(e+fx)-\sin(e+fx))\sqrt{\sin(2(e+fx))}\,(\cos(3(e+fx))+i\sin(3(e+fx)))+i\left(19i+(21+15i)\cos(3(e+fx))\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sqrt{\sin(2(e+fx))}+12\sin(2(e+fx))-(15-21i)\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sqrt{\sin(2(e+fx))}\,\sin(3(e+fx))+21\sin(4(e+fx))\right)\right)}{96a^3f\sqrt{d\tan(e+fx)}\,(-i+\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*Tan[e + f*x]]*(a + I*a*Tan[e + f*x])^3),x]
```

```
[Out] (Sec[e + f*x]^4*(19*Cos[4*(e + f*x)] - (15 + 21*I)*ArcSin[Cos[e + f*x] - Si
n[e + f*x]]*Sqrt[Sin[2*(e + f*x)]]*(Cos[3*(e + f*x)] + I*Sin[3*(e + f*x)])
+ I*(19*I + (21 + 15*I)*Cos[3*(e + f*x)]*Log[Cos[e + f*x] + Sin[e + f*x] +
Sqrt[Sin[2*(e + f*x)]]]*Sqrt[Sin[2*(e + f*x)]] + 12*Sin[2*(e + f*x)] - (15
- 21*I)*Log[Cos[e + f*x] + Sin[e + f*x] + Sqrt[Sin[2*(e + f*x)]]]*Sqrt[Sin[
2*(e + f*x)]]*Sin[3*(e + f*x)] + 21*Sin[4*(e + f*x)])))/(96*a^3*f*Sqrt[d*Ta
n[e + f*x]]*(-I + Tan[e + f*x])^3)
```

Maple [A]
time = 0.25, size = 133, normalized size = 0.39

method	result
derivativedivides	$\frac{2d^4\left(\frac{\frac{-5(d\tan(fx+e))^{\frac{5}{2}}+\frac{38id(d\tan(fx+e))^{\frac{3}{2}}}{3}+9d^2\sqrt{d\tan(fx+e)}}{(id\tan(fx+e)+d)^3}-\frac{6i\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{-id}}\right)}{\sqrt{-id}}}{16d^4}+\frac{i\arctan\left(\right.}{}\right.}{f\,a^3}$
default	$\frac{2d^4\left(\frac{\frac{-5(d\tan(fx+e))^{\frac{5}{2}}+\frac{38id(d\tan(fx+e))^{\frac{3}{2}}}{3}+9d^2\sqrt{d\tan(fx+e)}}{(id\tan(fx+e)+d)^3}-\frac{6i\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{-id}}\right)}{\sqrt{-id}}}{16d^4}+\frac{i\arctan\left(\right.}{}\right.}{f\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3*d^4*(1/16/d^4*((-5*(d*tan(f*x+e))^(5/2)+38/3*I*d*(d*tan(f*x+e))^(3/
2)+9*d^2*(d*tan(f*x+e))^(1/2))/(I*d*tan(f*x+e)+d)^3-6*I/(-I*d)^(1/2)*arctan
((d*tan(f*x+e))^(1/2)/(-I*d)^(1/2)))+1/16*I/d^4/(I*d)^(1/2)*arctan((d*tan(f
*x+e))^(1/2)/(I*d)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 596 vs. $2(269) = 538$.
time = 0.39, size = 596, normalized size = 1.74

$$-\frac{12\,a^3 df\sqrt{-\frac{i}{64\,a^6 df^2}}\,e^{(6i\,fx+6i\,e)}\log\left(-2\left(8\left(a^3 df e^{(2i\,fx+2i\,e)}+a^3 df\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{-\frac{i}{64\,a^6 df^2}}+i\,de^{(2i\,fx+2i\,e)}\right)e^{(-2i\,fx-2i\,e)}\right)-12\,a^3 df\sqrt{-\frac{i}{64\,a^6 df^2}}\,e^{(6i\,fx+6i\,e)}\log\left(2\left(8\left(a^3 df e^{(2i\,fx+2i\,e)}+a^3 df\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{-\frac{i}{64\,a^6 df^2}}-i\,de^{(2i\,fx+2i\,e)}\right)e^{(-2i\,fx-2i\,e)}\right)-12\,a^3 df\sqrt{\frac{9i}{16\,a^6 df^2}}\,e^{(6i\,fx+6i\,e)}\log\left(\frac{\left(4\left(a^3 fe^{(2i\,fx+2i\,e)}+a^3 f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{9i}{16\,a^6 df^2}}+3i\right)e^{(-2i\,fx-2i\,e)}}{4\,a^3 f}\right)+12\,a^3 df\sqrt{\frac{9i}{16\,a^6 df^2}}\,e^{(6i\,fx+6i\,e)}\log\left(-\frac{\left(4\left(a^3 fe^{(2i\,fx+2i\,e)}+a^3 f\right)\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{9i}{16\,a^6 df^2}}-3i\right)e^{(-2i\,fx-2i\,e)}}{4\,a^3 f}\right)-\sqrt{\frac{-i\,de^{(2i\,fx+2i\,e)}+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\left(20\,e^{(6i\,fx+6i\,e)}+26\,e^{(4i\,fx+4i\,e)}+7\,e^{(2i\,fx+2i\,e)}+1\right)}{48\,a^3 df}e^{(-6i\,fx-6i\,e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="fricas"
)
```

```
[Out] -1/48*(12*a^3*d*f*sqrt(-1/64*I/(a^6*d*f^2))*e^(6*I*f*x + 6*I*e)*log(-2*(8*(
a^3*d*f*e^(2*I*f*x + 2*I*e) + a^3*d*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d
)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-1/64*I/(a^6*d*f^2)) + I*d*e^(2*I*f*x + 2
*I*e))*e^(-2*I*f*x - 2*I*e)) - 12*a^3*d*f*sqrt(-1/64*I/(a^6*d*f^2))*e^(6*I*
f*x + 6*I*e)*log(2*(8*(a^3*d*f*e^(2*I*f*x + 2*I*e) + a^3*d*f)*sqrt((-I*d*e^
(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-1/64*I/(a^6*d*f^2
)) - I*d*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)) - 12*a^3*d*f*sqrt(9/16*
I/(a^6*d*f^2))*e^(6*I*f*x + 6*I*e)*log(1/4*(4*(a^3*f*e^(2*I*f*x + 2*I*e) +
a^3*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqr
t(9/16*I/(a^6*d*f^2)) + 3*I)*e^(-2*I*f*x - 2*I*e)/(a^3*f)) + 12*a^3*d*f*sqr
t(9/16*I/(a^6*d*f^2))*e^(6*I*f*x + 6*I*e)*log(-1/4*(4*(a^3*f*e^(2*I*f*x + 2
*I*e) + a^3*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) +
 1))*sqrt(9/16*I/(a^6*d*f^2)) - 3*I)*e^(-2*I*f*x - 2*I*e)/(a^3*f)) - sqrt((
-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*(20*e^(6*I*f*x +
 6*I*e) + 26*e^(4*I*f*x + 4*I*e) + 7*e^(2*I*f*x + 2*I*e) + 1))*e^(-6*I*f*x
- 6*I*e)/(a^3*d*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int\frac{1}{\sqrt{d\tan(e+fx)}\tan^3(e+fx)-3i\sqrt{d\tan(e+fx)}\tan^2(e+fx)-3\sqrt{d\tan(e+fx)}\tan(e+fx)+i\sqrt{d\tan(e+fx)}}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] I*Integral(1/(sqrt(d*tan(e + f*x))*tan(e + f*x)**3 - 3*I*sqrt(d*tan(e + f*x
))*tan(e + f*x)**2 - 3*sqrt(d*tan(e + f*x))*tan(e + f*x) + I*sqrt(d*tan(e +
 f*x))), x)/a**3
```

Giac [A]
time = 0.73, size = 227, normalized size = 0.66

$$\frac{i\sqrt{2}\arctan\left(\frac{8\sqrt{d^2}\sqrt{d\tan(fx+e)}}{4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{8a^3\sqrt{d}\,f\left(\frac{id}{\sqrt{d^2}}+1\right)}-\frac{3i\sqrt{2}\arctan\left(\frac{8\sqrt{d^2}\sqrt{d\tan(fx+e)}}{-4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{4a^3\sqrt{d}\,f\left(-\frac{id}{\sqrt{d^2}}+1\right)}-\frac{15i\sqrt{d\tan(fx+e)}\,d^2\tan(fx+e)^2+38\sqrt{d\tan(fx+e)}\,d^2\tan(fx+e)-27i\sqrt{d\tan(fx+e)}\,d^2}{24\,(d\tan(fx+e)-id)^3a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/8*I*sqrt(2)*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(4*I*sqrt(2)*d^(3/2)
+ 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a^3*sqrt(d)*f*(I*d/sqrt(d^2) + 1)) - 3/4*I
*sqrt(2)*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(-4*I*sqrt(2)*d^(3/2) + 4*
sqrt(2)*sqrt(d^2)*sqrt(d)))/(a^3*sqrt(d)*f*(-I*d/sqrt(d^2) + 1)) - 1/24*(15
*I*sqrt(d*tan(f*x + e))*d^2*tan(f*x + e)^2 + 38*sqrt(d*tan(f*x + e))*d^2*ta
n(f*x + e) - 27*I*sqrt(d*tan(f*x + e))*d^2)/((d*tan(f*x + e) - I*d)^3*a^3*f
)
```

Mupad [B]
time = 5.68, size = 206, normalized size = 0.60

$$\frac{\frac{19\,d\,(d\tan(e+f\,x))^{3/2}}{12\,a^3\,f}+\frac{(d\tan(e+f\,x))^{5/2}\,5\mathrm{i}}{8\,a^3\,f}-\frac{d^2\sqrt{d\tan(e+f\,x)}\,9\mathrm{i}}{8\,a^3\,f}}{-d^3\tan(e+f\,x)^3+d^3\tan(e+f\,x)^2\,3\mathrm{i}+3\,d^3\tan(e+f\,x)-d^3\,1\mathrm{i}}-\mathrm{atan}\left(\frac{8\,a^3\,f\sqrt{d\tan(e+f\,x)}\sqrt{\frac{9\mathrm{i}}{64\,a^6\,d\,f^2}}}{3}\right)\sqrt{\frac{9\mathrm{i}}{64\,a^6\,d\,f^2}}\,2\mathrm{i}+\mathrm{atan}\left(16\,a^3\,f\sqrt{d\tan(e+f\,x)}\sqrt{-\frac{1\mathrm{i}}{256\,a^6\,d\,f^2}}\right)\sqrt{-\frac{1\mathrm{i}}{256\,a^6\,d\,f^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*tan(e + f*x))^(1/2)*(a + a*tan(e + f*x)*1i)^3),x)
```

```
[Out] (((d*tan(e + f*x))^(5/2)*5i)/(8*a^3*f) - (d^2*(d*tan(e + f*x))^(1/2)*9i)/(8
*a^3*f) + (19*d*(d*tan(e + f*x))^(3/2))/(12*a^3*f))/(3*d^3*tan(e + f*x) - d
^3*1i + d^3*tan(e + f*x)^2*3i - d^3*tan(e + f*x)^3) - atan((8*a^3*f*(d*tan(
e + f*x))^(1/2)*(9i/(64*a^6*d*f^2))^(1/2))/3)*(9i/(64*a^6*d*f^2))^(1/2)*2i
+ atan(16*a^3*f*(d*tan(e + f*x))^(1/2)*(-1i/(256*a^6*d*f^2))^(1/2))*(-1i/(2
56*a^6*d*f^2))^(1/2)*2i
```

3.185 $\int \frac{1}{(d\tan(e+fx))^{3/2}(a+ia\tan(e+fx))^3}\,dx$

Optimal. Leaf size=368

$$\frac{\left(\frac{15}{8}+\frac{7i}{4}\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^3d^{3/2}f}-\frac{\left(\frac{15}{8}+\frac{7i}{4}\right)\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^3d^{3/2}f}-\frac{\left(\frac{15}{16}-\frac{7i}{8}\right)}{}$$

```
[Out] (15/16+7/8*I)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a^3/d^(3/2)/f*
2^(1/2)-(15/16+7/8*I)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a^3/d^
(3/2)/f*2^(1/2)+(-15/32+7/16*I)*ln(d^(1/2)-2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(
1/2)*tan(f*x+e))/a^3/d^(3/2)/f*2^(1/2)+(15/32-7/16*I)*ln(d^(1/2)+2^(1/2)*(d
*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/a^3/d^(3/2)/f*2^(1/2)-15/4/a^3/d/f/(
d*tan(f*x+e))^(1/2)+1/6/d/f/(d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^3+5/12/
a/d/f/(d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2+7/6/d/f/(d*tan(f*x+e))^(1/2
)/(a^3+I*a^3*tan(f*x+e))
```

Rubi [A]
time = 0.45, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3640, 3677, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\left(\frac{15}{8}+\frac{7i}{4}\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^3d^{3/2}f}-\frac{\left(\frac{15}{8}+\frac{7i}{4}\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,a^3d^{3/2}f}-\frac{\left(\frac{15}{16}-\frac{7i}{8}\right)\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,a^3d^{3/2}f}+\frac{\left(\frac{15}{16}-\frac{7i}{8}\right)\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,a^3d^{3/2}f}-\frac{15}{4a^3df\sqrt{d\tan(e+fx)}}+\frac{7}{6df\left(a^3+ia^3\tan(e+fx)\right)\sqrt{d\tan(e+fx)}}+\frac{5}{12adf(a+ia\tan(e+fx))^2\sqrt{d\tan(e+fx)}}+\frac{1}{6df(a+ia\tan(e+fx))^3\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Tan[e + f*x])^(3/2)*(a + I*a*Tan[e + f*x])^3),x]
```

```
[Out] ((15/8 + (7*I)/4)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(Sqrt
[2]*a^3*d^(3/2)*f) - ((15/8 + (7*I)/4)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f
*x]])/Sqrt[d]])/(Sqrt[2]*a^3*d^(3/2)*f) - ((15/16 - (7*I)/8)*Log[Sqrt[d] +
Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e + f*x]]])/(Sqrt[2]*a^3*d^(3/2)*
f) + ((15/16 - (7*I)/8)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqrt[2]*Sqrt[d
*Tan[e + f*x]]])/(Sqrt[2]*a^3*d^(3/2)*f) - 15/(4*a^3*d*f*Sqrt[d*Tan[e + f*x
]]) + 1/(6*d*f*Sqrt[d*Tan[e + f*x]]*(a + I*a*Tan[e + f*x])^3) + 5/(12*a*d*f
*Sqrt[d*Tan[e + f*x]]*(a + I*a*Tan[e + f*x])^2) + 7/(6*d*f*Sqrt[d*Tan[e + f
*x]]*(a^3 + I*a^3*Tan[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
```

```
NeQ[c^2 + d^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(d\tan(e+fx))^{3/2}(a+ia\tan(e+fx))^3}\,dx = \frac{1}{6df\sqrt{d\tan(e+fx)}\,(a+ia\tan(e+fx))^3} + \frac{\int \frac{\frac{13ad}{2}-\frac{7}{2}i\cdots}{(d\tan(e+fx))^{3/\cdots}}}{6\cdots}$$

$$= \frac{1}{6df\sqrt{d\tan(e+fx)}\,(a+ia\tan(e+fx))^3} + \frac{\cdots}{12adf\sqrt{d\tan(\cdots}}$$

$$= \frac{1}{6df\sqrt{d\tan(e+fx)}\,(a+ia\tan(e+fx))^3} + \frac{\cdots}{12adf\sqrt{d\tan(\cdots}}$$

$$= -\frac{15}{4a^3df\sqrt{d\tan(e+fx)}} + \frac{1}{6df\sqrt{d\tan(e+fx)}\,(a+ia\tan\cdots}$$

$$= -\frac{15}{4a^3df\sqrt{d\tan(e+fx)}} + \frac{1}{6df\sqrt{d\tan(e+fx)}\,(a+ia\tan\cdots}$$

$$= -\frac{15}{4a^3df\sqrt{d\tan(e+fx)}} + \frac{1}{6df\sqrt{d\tan(e+fx)}\,(a+ia\tan\cdots}$$

$$= -\frac{15}{4a^3df\sqrt{d\tan(e+fx)}} + \frac{1}{6df\sqrt{d\tan(e+fx)}\,(a+ia\tan\cdots}$$

$$= -\frac{\left(\frac{15}{16}-\frac{7i}{8}\right)\log\left(\sqrt{d}+\sqrt{d}\,\tan(e+fx)-\sqrt{2}\,\sqrt{d\tan(e+\cdots}\right.}{\sqrt{2}\,a^3d^{3/2}f}$$

$$= \frac{\left(\frac{15}{8}+\frac{7i}{4}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,a^3d^{3/2}f} - \frac{\left(\frac{15}{8}+\frac{7i}{4}\right)\mathrm{ta}\cdots}{\cdots}$$

Mathematica [A]

time = 2.23, size = 234, normalized size = 0.64

$$\frac{e^{-6i(e+fx)}\left(1+9e^{2i(e+fx)}+49e^{4i(e+fx)}-105e^{6i(e+fx)}-146e^{8i(e+fx)}-87e^{6i(e+fx)}\sqrt{-1+e^{4i(e+fx)}}\,\mathrm{ArcTan}\left(\sqrt{-1+e^{4i(e+fx)}}\right)+6e^{6i(e+fx)}\sqrt{-1+e^{2i(e+fx)}}\,\sqrt{1+e^{2i(e+fx)}}\,\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\right)\right)}{48a^3d\left(1+e^{2i(e+fx)}\right)f\sqrt{d\tan(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d*Tan[e + f*x])^(3/2)*(a + I*a*Tan[e + f*x])^3),x]
```

```
[Out] (1 + 9*E^((2*I)*(e + f*x)) + 49*E^((4*I)*(e + f*x)) - 105*E^((6*I)*(e + f*x
)) - 146*E^((8*I)*(e + f*x)) - 87*E^((6*I)*(e + f*x))*Sqrt[-1 + E^((4*I)*(e
 + f*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(e + f*x))]] + 6*E^((6*I)*(e + f*x))*Sq
rt[-1 + E^((2*I)*(e + f*x))]*Sqrt[1 + E^((2*I)*(e + f*x))]*ArcTanh[Sqrt[(-1
```

```
+ E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f*x)))]])/(48*a^3*d*E^((6*I)*(e
+ f*x))*(1 + E^((2*I)*(e + f*x)))*f*Sqrt[d*Tan[e + f*x]])
```

Maple [A]
time = 0.20, size = 147, normalized size = 0.40

method	result
derivativedivides	$\frac{2d^4\left(-\frac{1}{d^5\sqrt{d\tan(fx+e)}}-\frac{\frac{14(d\tan(fx+e))^{\frac{5}{2}}-\frac{98id(d\tan(fx+e))^{\frac{3}{2}}}{3}-20d^2\sqrt{d\tan(fx+e)}}{(-id+d\tan(fx+e))^3}+\frac{29\arctan\left(\sqrt{\cdots}\right.}{\cdots}}{16d^5}\cdots\right.}{f\,a^3}$
default	$\frac{2d^4\left(-\frac{1}{d^5\sqrt{d\tan(fx+e)}}-\frac{\frac{14(d\tan(fx+e))^{\frac{5}{2}}-\frac{98id(d\tan(fx+e))^{\frac{3}{2}}}{3}-20d^2\sqrt{d\tan(fx+e)}}{(-id+d\tan(fx+e))^3}+\frac{29\arctan\left(\sqrt{\cdots}\right.}{\cdots}}{16d^5}\cdots\right.}{f\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3*d^4*(-1/d^5/(d*tan(f*x+e))^(1/2)-1/16/d^5*((14*(d*tan(f*x+e))^(5/2)
-98/3*I*d*(d*tan(f*x+e))^(3/2)-20*d^2*(d*tan(f*x+e))^(1/2))/(-I*d+d*tan(f*x
+e))^3+29/(-I*d)^(1/2)*arctan((d*tan(f*x+e))^(1/2)/(-I*d)^(1/2)))-1/16/d^5/
(I*d)^(1/2)*arctan((d*tan(f*x+e))^(1/2)/(I*d)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 742 vs. 2(291) = 582.
time = 0.39, size = 742, normalized size = 2.02

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="fricas"
)
```

```
[Out] 1/48*(12*(a^3*d^2*f*e^(8*I*f*x + 8*I*e) - a^3*d^2*f*e^(6*I*f*x + 6*I*e))*sq
rt(1/64*I/(a^6*d^3*f^2))*log(-2*(8*(I*a^3*d^2*f*e^(2*I*f*x + 2*I*e) + I*a^3
*d^2*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sq
rt(1/64*I/(a^6*d^3*f^2)) + I*d*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)) -
 12*(a^3*d^2*f*e^(8*I*f*x + 8*I*e) - a^3*d^2*f*e^(6*I*f*x + 6*I*e))*sqrt(1/
64*I/(a^6*d^3*f^2))*log(-2*(8*(-I*a^3*d^2*f*e^(2*I*f*x + 2*I*e) - I*a^3*d^2
*f)*sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(1
/64*I/(a^6*d^3*f^2)) + I*d*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)) + 12*
(a^3*d^2*f*e^(8*I*f*x + 8*I*e) - a^3*d^2*f*e^(6*I*f*x + 6*I*e))*sqrt(-841/6
4*I/(a^6*d^3*f^2))*log(1/8*(8*(a^3*d*f*e^(2*I*f*x + 2*I*e) + a^3*d*f)*sqrt(
(-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-841/64*I/
(a^6*d^3*f^2)) + 29)*e^(-2*I*f*x - 2*I*e)/(a^3*d*f)) - 12*(a^3*d^2*f*e^(8*I
*f*x + 8*I*e) - a^3*d^2*f*e^(6*I*f*x + 6*I*e))*sqrt(-841/64*I/(a^6*d^3*f^2)
)*log(-1/8*(8*(a^3*d*f*e^(2*I*f*x + 2*I*e) + a^3*d*f)*sqrt((-I*d*e^(2*I*f*x
 + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-841/64*I/(a^6*d^3*f^2)) -
 29)*e^(-2*I*f*x - 2*I*e)/(a^3*d*f)) + sqrt((-I*d*e^(2*I*f*x + 2*I*e) + I*d
)/(e^(2*I*f*x + 2*I*e) + 1))*(-146*I*e^(8*I*f*x + 8*I*e) - 105*I*e^(6*I*f*x
 + 6*I*e) + 49*I*e^(4*I*f*x + 4*I*e) + 9*I*e^(2*I*f*x + 2*I*e) + I))/(a^3*d
^2*f*e^(8*I*f*x + 8*I*e) - a^3*d^2*f*e^(6*I*f*x + 6*I*e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int\frac{1}{(d\tan(e+fx))^{\frac{3}{2}}\tan^3(e+fx)-3i(d\tan(e+fx))^{\frac{3}{2}}\tan^2(e+fx)-3(d\tan(e+fx))^{\frac{3}{2}}\tan(e+fx)+i(d\tan(e+fx))^{\frac{3}{2}}}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))**(3/2)/(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] I*Integral(1/((d*tan(e + f*x))**(3/2)*tan(e + f*x)**3 - 3*I*(d*tan(e + f*x)
)**(3/2)*tan(e + f*x)**2 - 3*(d*tan(e + f*x))**(3/2)*tan(e + f*x) + I*(d*ta
n(e + f*x))**(3/2)), x)/a**3
```

Giac [A]
time = 0.82, size = 252, normalized size = 0.68

$$-\frac{\frac{87\sqrt{2}\arctan\left(\frac{8\sqrt{d^2}\sqrt{d\tan(fx+e)}}{-4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{a^3\sqrt{d}\,f\left(-\frac{id}{\sqrt{d^2}}+1\right)}+\frac{3i\sqrt{2}\arctan\left(\frac{8i\sqrt{d^2}\sqrt{d\tan(fx+e)}}{-4i\sqrt{2}\,d^{\frac{3}{2}}+4\sqrt{2}\sqrt{d^2}\sqrt{d}}\right)}{a^3\sqrt{d}\,f\left(-\frac{id}{\sqrt{d^2}}+1\right)}+\frac{48}{\sqrt{d\tan(fx+e)}\,a^3f}+\frac{2\left(21i\sqrt{d\tan(fx+e)}\,d^2\tan(fx+e)^2+49\sqrt{d\tan(fx+e)}\,d^2\tan(fx+e)-30i\sqrt{d\tan(fx+e)}\,d^2\right)}{(-id\tan(fx+e)-d)^3a^3f}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/24*(87*sqrt(2)*arctan(8*sqrt(d^2)*sqrt(d*tan(f*x + e))/(-4*I*sqrt(2)*d^(
3/2) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a^3*sqrt(d)*f*(-I*d/sqrt(d^2) + 1)) +
 3*I*sqrt(2)*arctan(8*I*sqrt(d^2)*sqrt(d*tan(f*x + e))/(-4*I*sqrt(2)*d^(3/2
) + 4*sqrt(2)*sqrt(d^2)*sqrt(d)))/(a^3*sqrt(d)*f*(-I*d/sqrt(d^2) + 1)) + 48
/(sqrt(d*tan(f*x + e))*a^3*f) + 2*(21*I*sqrt(d*tan(f*x + e))*d^2*tan(f*x +
e)^2 + 49*sqrt(d*tan(f*x + e))*d^2*tan(f*x + e) - 30*I*sqrt(d*tan(f*x + e))
*d^2)/((-I*d*tan(f*x + e) - d)^3*a^3*f))/d
```

Mupad [B]
time = 6.12, size = 227, normalized size = 0.62

$$\frac{\frac{15\,d^2\,\tan(e+f\,x)^3}{4\,a^3\,f}-\frac{17\,d^2\,\tan(e+f\,x)}{2\,a^3\,f}+\frac{d^2\,2\mathrm{i}}{a^3\,f}-\frac{d^2\,\tan(e+f\,x)^2\,121\mathrm{i}}{12\,a^3\,f}}{3\,d^2\,{\left(d\,\tan\left(e+f\,x\right)\right)}^{3/2}-{\left(d\,\tan\left(e+f\,x\right)\right)}^{7/2}+d\,{\left(d\,\tan\left(e+f\,x\right)\right)}^{5/2}\,3\mathrm{i}-d^3\,\sqrt{d\,\tan\left(e+f\,x\right)}\,1\mathrm{i}}+2\,\mathrm{atanh}\left(16\,a^3\,d\,f\,\sqrt{d\,\tan\left(e+f\,x\right)}\,\sqrt{\frac{1\mathrm{i}}{256\,a^6\,d^3\,f^2}}\right)\,\sqrt{\frac{1\mathrm{i}}{256\,a^6\,d^3\,f^2}}+2\,\mathrm{atanh}\left(\frac{16\,a^3\,d\,f\,\sqrt{d\,\tan\left(e+f\,x\right)}\,\sqrt{-\frac{841\mathrm{i}}{256\,a^6\,d^3\,f^2}}}{29}\right)\,\sqrt{-\frac{841\mathrm{i}}{256\,a^6\,d^3\,f^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*tan(e + f*x))^(3/2)*(a + a*tan(e + f*x)*1i)^3),x)
```

```
[Out] ((d^2*2i)/(a^3*f) - (17*d^2*tan(e + f*x))/(2*a^3*f) - (d^2*tan(e + f*x)^2*1
21i)/(12*a^3*f) + (15*d^2*tan(e + f*x)^3)/(4*a^3*f))/(d*(d*tan(e + f*x))^(5
/2)*3i - (d*tan(e + f*x))^(7/2) - d^3*(d*tan(e + f*x))^(1/2)*1i + 3*d^2*(d*
tan(e + f*x))^(3/2)) + 2*atanh(16*a^3*d*f*(d*tan(e + f*x))^(1/2)*(1i/(256*a
^6*d^3*f^2))^(1/2))*(1i/(256*a^6*d^3*f^2))^(1/2) + 2*atanh((16*a^3*d*f*(d*t
an(e + f*x))^(1/2)*(-841i/(256*a^6*d^3*f^2))^(1/2))/29)*(-841i/(256*a^6*d^3
*f^2))^(1/2)
```

3.186 $\int \tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=176

$$\frac{7(-1)^{3/4}\sqrt{a}\,\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{4d}+\frac{(1+i)\sqrt{a}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{i\sqrt{\tan(c}$$

```
[Out] 7/4*(-1)^(3/4)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c)
)^(1/2))*a^(1/2)/d+(1+I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(
d*x+c))^(1/2))*a^(1/2)/d-1/4*I*tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/d+
1/2*(a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^(3/2)/d
```

Rubi [A]
time = 0.35, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {3641, 3678, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{7(-1)^{3/4}\sqrt{a}\,\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{4d}+\frac{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{i\sqrt{\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{4d}+\frac{(1+i)\sqrt{a}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(5/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (7*(-1)^(3/4)*Sqrt[a]*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a
 + I*a*Tan[c + d*x]]])/(4*d) + ((1 + I)*Sqrt[a]*ArcTanh[((1 + I)*Sqrt[a]*Sq
rt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d - ((I/4)*Sqrt[Tan[c + d*x]
]*Sqrt[a + I*a*Tan[c + d*x]])/d + (Tan[c + d*x]^(3/2)*Sqrt[a + I*a*Tan[c +
d*x]])/(2*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3641

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[d*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n - 1)/(f*(m + n - 1))), x] - Dist[1/(a*(m + n - 1)), Int[(a + b*
Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 2)*Simp[d*(b*c*m + a*d*(-1 + n))
- a*c^2*(m + n - 1) + d*(b*d*m - a*c*(m + 2*n - 2))*Tan[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1] && NeQ[m + n - 1, 0] && (IntegerQ[n]
|| IntegersQ[2*m, 2*n])
```

Rule 3678

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Dist[
1/(a*(m + n)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Simp
[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d} - \frac{\int \sqrt{\tan(c+dx)}\left(\frac{3a}{2}+\frac{1}{2}ia\right.}{}$$

$$= -\frac{i\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{4d} + \frac{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia}}{2d}$$

$$= -\frac{i\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{4d} + \frac{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia}}{2d}$$

$$= -\frac{i\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{4d} + \frac{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia}}{2d}$$

$$= \frac{(1+i)\sqrt{a}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{i\sqrt{\tan(c+dx)}}{}$$

$$= \frac{(1+i)\sqrt{a}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{i\sqrt{\tan(c+dx)}}{}$$

$$= \frac{7(-1)^{3/4}\sqrt{a}\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{4d} + \frac{(1+i)\sqrt{a}\ \mathrm{ta}}{}$$

Mathematica [A]
time = 4.11, size = 267, normalized size = 1.52

$$\frac{ie^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\left(4\sqrt{2}\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)+\frac{\sqrt{2}\,e^{i(c+dx)}\left(1-3e^{2i(c+dx)}\right)\sqrt{-1+e^{2i(c+dx)}}\sqrt{\tan(c+dx)}}{\left(1+e^{2i(c+dx)}\right)^2}-7\tanh^{-1}\left(\frac{\sqrt{2}\,e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\sqrt{\tan(c+dx)}\right)\sqrt{a+ia\tan(c+dx)}}{4\sqrt{2}\,d\sqrt{-1+e^{2i(c+dx)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(5/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] ((I/4)*(1 + E^((2*I)*(c + d*x)))*(4*Sqrt[2]*Sqrt[((-I)*(-1 + E^((2*I)*(c +
d*x))))/(1 + E^((2*I)*(c + d*x)))]*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*
```

```
I)*(c + d*x))]] + (Sqrt[2]*E^(I*(c + d*x))*(1 - 3*E^((2*I)*(c + d*x)))*Sqrt
[-1 + E^((2*I)*(c + d*x))]*Sqrt[Tan[c + d*x]])/(1 + E^((2*I)*(c + d*x)))^2
- 7*ArcTanh[(Sqrt[2]*E^(I*(c + d*x)))/Sqrt[-1 + E^((2*I)*(c + d*x))]]*Sqrt[
Tan[c + d*x]])*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[2]*d*E^(I*(c + d*x))*Sqrt[
-1 + E^((2*I)*(c + d*x))])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 471 vs. $2(137) = 274$.
time = 0.35, size = 472, normalized size = 2.68

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(dx+c))}\,\left(\sqrt{\tan}\,(dx+c)\right)\left(6i\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}\,\sqrt{}\right.}{}$
default	$\frac{\sqrt{a(1+i\tan(dx+c))}\,\left(\sqrt{\tan}\,(dx+c)\right)\left(6i\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}\,\sqrt{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d*(a*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^(1/2)*(6*I*(a*tan(d*x+c)*(1+I*t
an(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)+7*I*ln(1/2*(2*I*a*tan
(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))
*a*(-I*a)^(1/2)*tan(d*x+c)+4*I*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(
d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*(I*a)^(1
/2)*a-4*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)*tan(
d*x+c)^2-4*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/
2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*(I*a)^(1/2)*2^(1/2)*a*tan(d*x+c)+2*(
a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)+7*ln(1/2*(2*I
*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^
(1/2))*a*(-I*a)^(1/2))/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2)/(-
tan(d*x+c)+I)/(-I*a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*tan(d*x + c)^(5/2), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(128) = 256$.
time = 0.37, size = 559, normalized size = 3.18

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}\left(-3ie^{(3idx+3ic)}+ie^{(idx+ic)}\right)+2\left(de^{(2idx+2ic)}+d\right)\sqrt{\frac{49ia}{16d^2}}\log\left(\frac{1}{7}\left(7\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}\left(e^{(2idx+2ic)}+1\right)+8id\sqrt{\frac{49ia}{16d^2}}e^{(idx+ic)}\right)e^{(-idx-ic)}\right)-2\left(de^{(2idx+2ic)}+d\right)\sqrt{\frac{49ia}{16d^2}}\log\left(\frac{1}{7}\left(7\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}\left(e^{(2idx+2ic)}+1\right)-8id\sqrt{\frac{49ia}{16d^2}}e^{(idx+ic)}\right)e^{(-idx-ic)}\right)-2\left(de^{(2idx+2ic)}+d\right)\sqrt{\frac{2ia}{d^2}}\log\left(\left(\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}\left(e^{(2idx+2ic)}+1\right)+id\sqrt{\frac{2ia}{d^2}}e^{(idx+ic)}\right)e^{(-idx-ic)}\right)+2\left(de^{(2idx+2ic)}+d\right)\sqrt{\frac{2ia}{d^2}}\log\left(\left(\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}\left(e^{(2idx+2ic)}+1\right)-id\sqrt{\frac{2ia}{d^2}}e^{(idx+ic)}\right)e^{(-idx-ic)}\right)}{4\left(de^{(2idx+2ic)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c)
 + I)/(e^(2*I*d*x + 2*I*c) + 1))*(-3*I*e^(3*I*d*x + 3*I*c) + I*e^(I*d*x + I
*c)) + 2*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(49/16*I*a/d^2)*log(1/7*(7*sqrt(2)
*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*
I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1) + 8*I*d*sqrt(49/16*I*a/d^2)*
e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 2*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(49/
16*I*a/d^2)*log(1/7*(7*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e
^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1
) - 8*I*d*sqrt(49/16*I*a/d^2)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 2*(d*e^(
2*I*d*x + 2*I*c) + d)*sqrt(2*I*a/d^2)*log((sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(
2*I*d*x + 2*I*c) + 1) + I*d*sqrt(2*I*a/d^2)*e^(I*d*x + I*c))*e^(-I*d*x - I*
c)) + 2*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(2*I*a/d^2)*log((sqrt(2)*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I
*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1) - I*d*sqrt(2*I*a/d^2)*e^(I*d*x + I*c))*
e^(-I*d*x - I*c)))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,\tan^{\frac{5}{2}}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)*tan(d*x+c)**(5/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*tan(c + d*x)**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 2.38Unable to divide
, perhaps due to rounding error%%%{%%{[%%%{%%{poly1[-16*i,0]:[1,0,-2]%%},[0
]%%%},
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^{5/2}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int(tan(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.187 $\int \tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=135

$$-\frac{\sqrt[4]{-1}\sqrt{a}\,\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{(1-i)\sqrt{a}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{\sqrt{\tan(c+dx)}\cdots}{}$$

```
[Out] -(-1)^(1/4)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(
1/2))*a^(1/2)/d+(-1+I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*
x+c))^(1/2))*a^(1/2)/d+tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.23, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {3641, 21, 3636, 3625, 211, 3680, 65, 223, 209}

$$-\frac{\sqrt[4]{-1}\sqrt{a}\,\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{(1-i)\sqrt{a}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(3/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] -(((-1)^(1/4)*Sqrt[a]*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a
 + I*a*Tan[c + d*x]]])/d) - ((1 - I)*Sqrt[a]*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[
Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d + (Sqrt[Tan[c + d*x]]*Sqrt[a
+ I*a*Tan[c + d*x]])/d
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3636

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(c_.) + (d_.)*tan[(e_
.) + (f_.)*(x_)]], x_Symbol] :> Dist[2*a, Int[Sqrt[a + b*Tan[e + f*x]]/Sqrt
[c + d*Tan[e + f*x]], x], x] + Dist[b/a, Int[(b + a*Tan[e + f*x])*(Sqrt[a +
 b*Tan[e + f*x]]/Sqrt[c + d*Tan[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3641

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[d*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n - 1)/(f*(m + n - 1))), x] - Dist[1/(a*(m + n - 1)), Int[(a + b*
Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 2)*Simp[d*(b*c*m + a*d*(-1 + n))
- a*c^2*(m + n - 1) + d*(b*d*m - a*c*(m + 2*n - 2))*Tan[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1] && NeQ[m + n - 1, 0] && (IntegerQ[n]
|| IntegersQ[2*m, 2*n])
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rubi steps

$$\int \tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d} - \frac{\int \frac{\left(\frac{a}{2}+\frac{1}{2}ia\tan(c+dx)\right)\sqrt{a+i}}{\sqrt{\tan(c+}}}{a}$$

$$= \frac{\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d} - \frac{\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{\tan(c+dx)}}\,dx}{2a}$$

$$= \frac{\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d} - \frac{i\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{\tan(c+}}}{2a}$$

$$= \frac{\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d} + \frac{a\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{a+iax}}\right.}{2d}$$

$$= -\frac{(1-i)\sqrt{a}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} + \frac{\sqrt{\tan(c+dx}}{}$$

$$= -\frac{(1-i)\sqrt{a}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} + \frac{\sqrt{\tan(c+dx}}{}$$

$$= -\frac{\sqrt[4]{-1}\sqrt{a}\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{(1-i)\sqrt{a}\tan}{}$$

Mathematica [A]
time = 3.96, size = 249, normalized size = 1.84

$$\frac{\left(\frac{ie^{-i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\left(8\log\left(e^{i(c+dx)}+\sqrt{-1+e^{2i(c+dx)}}\right)+\sqrt{2}\left(-\log\left(1-3e^{2i(c+dx)}-2\sqrt{2}e^{i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\right)+\log\left(1-3e^{2i(c+dx)}+2\sqrt{2}e^{i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\right)\right)\right)}{\sqrt{-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}}+8\sqrt{\tan(c+dx)}\right)\sqrt{a+ia\tan(c+dx)}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(3/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((I*Sqrt[-1 + E^((2*I)*(c + d*x))]*(8*Log[E^(I*(c + d*x)) + Sqrt[-1 + E^((
2*I)*(c + d*x))]] + Sqrt[2]*(-Log[1 - 3*E^((2*I)*(c + d*x)) - 2*Sqrt[2]*E^(
I*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]] + Log[1 - 3*E^((2*I)*(c + d*x)
) + 2*Sqrt[2]*E^(I*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]])))/(E^(I*(c +
 d*x))*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]) +
 8*Sqrt[Tan[c + d*x]])*Sqrt[a + I*a*Tan[c + d*x]])/(8*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(108) = 216.
time = 0.19, size = 229, normalized size = 1.70

method	result
derivativedivides	$\frac{\left(\ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a\tan(dx+c)\,(1+i\tan(dx+c))}\,\sqrt{ia}+a}{2\sqrt{ia}}\right)a\sqrt{-ia}+2\sqrt{a\tan(dx+c}\right.}{}$
default	$\frac{\left(\ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a\tan(dx+c)\,(1+i\tan(dx+c))}\,\sqrt{ia}+a}{2\sqrt{ia}}\right)a\sqrt{-ia}+2\sqrt{a\tan(dx+c}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*
a)^(1/2)+a)/(I*a)^(1/2))*a*(-I*a)^(1/2)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(
1/2)*(I*a)^(1/2)*(-I*a)^(1/2)+I*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan
(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*(I*a)^(
1/2)*a)*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)/(-I*a)^(1/2)/(I*a)^(1/2
)/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*tan(d*x + c)^(3/2), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 479 vs. 2(101) = 202.
time = 0.38, size = 479, normalized size = 3.55

$$\frac{2\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}e^{(idx+ic)}+d\sqrt{-\frac{ia}{d^2}}\log\left(\left(\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}\left(e^{(2idx+2ic)}+1\right)+2d\sqrt{-\frac{ia}{d^2}}e^{(idx+ic)}\right)e^{(-idx-ic)}\right)-d\sqrt{-\frac{ia}{d^2}}\log\left(\left(\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}\left(e^{(2idx+2ic)}+1\right)-2d\sqrt{-\frac{ia}{d^2}}e^{(idx+ic)}\right)e^{(-idx-ic)}\right)-d\sqrt{-\frac{2ia}{d^2}}\log\left(\left(\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}\left(e^{(2idx+2ic)}+1\right)+d\sqrt{-\frac{2ia}{d^2}}e^{(idx+ic)}\right)e^{(-idx-ic)}\right)+d\sqrt{-\frac{2ia}{d^2}}\log\left(\left(\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}\left(e^{(2idx+2ic)}+1\right)-d\sqrt{-\frac{2ia}{d^2}}e^{(idx+ic)}\right)e^{(-idx-ic)}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*
c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c) + d*sqrt(-I*a/d^2)*log((
sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)
```

```
/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1) + 2*d*sqrt(-I*a/d^2)*
e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - d*sqrt(-I*a/d^2)*log((sqrt(2)*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2
*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1) - 2*d*sqrt(-I*a/d^2)*e^(I*d*x + I*c))
*e^(-I*d*x - I*c)) - d*sqrt(-2*I*a/d^2)*log((sqrt(2)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e
^(2*I*d*x + 2*I*c) + 1) + d*sqrt(-2*I*a/d^2)*e^(I*d*x + I*c))*e^(-I*d*x - I
*c)) + d*sqrt(-2*I*a/d^2)*log((sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sq
rt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*
I*c) + 1) - d*sqrt(-2*I*a/d^2)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,\tan^{\frac{3}{2}}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)*tan(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*tan(c + d*x)**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 1.59Unable to divide
, perhaps due to rounding error%%%{%%{[%%%{%%{poly1[-4*i,0]:[1,0,-2]%%},[0]
%%%},0
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{tan}\left(c+d\,x\right)}^{3/2}\,\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] int(tan(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.188 $\int \sqrt{\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=104

$$-\frac{2(-1)^{3/4}\sqrt{a}\,\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{(1+i)\sqrt{a}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

```
[Out] -2*(-1)^(3/4)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))
^(1/2))*a^(1/2)/d-(1+I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d
*x+c))^(1/2))*a^(1/2)/d
```

Rubi [A]
time = 0.17, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3644, 3625, 211, 3680, 65, 223, 209}

$$-\frac{2(-1)^{3/4}\sqrt{a}\,\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{(1+i)\sqrt{a}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (-2*(-1)^(3/4)*Sqrt[a]*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[
a + I*a*Tan[c + d*x]]])/d - ((1 + I)*Sqrt[a]*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[
Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3644

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(a*c - b*d)/a, Int[Sqrt[a + b*Tan[e + f*
x]]/Sqrt[c + d*Tan[e + f*x]], x], x] + Dist[d/a, Int[Sqrt[a + b*Tan[e + f*x
]]*((b + a*Tan[e + f*x])/Sqrt[c + d*Tan[e + f*x]]), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rubi steps

$$\int \sqrt{\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}\,dx = -\left(i\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{\tan(c+dx)}}\,dx\right) + \frac{\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{\tan(c+}}}{a}$$

$$= \frac{(ia)\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\,\sqrt{a+iax}}\,dx, x, \tan(c+dx)\right)}{d} - \frac{(2a^2)\,\mathrm{Sub}}{}$$

$$= -\frac{(1+i)\sqrt{a}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} + \frac{(2ia)\mathrm{Subs}}{}$$

$$= -\frac{(1+i)\sqrt{a}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} + \frac{(2ia)\mathrm{Subs}}{}$$

$$= -\frac{2(-1)^{3/4}\sqrt{a}\,\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{(1+i)}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 229 vs. 2(104) = 208.
time = 1.23, size = 229, normalized size = 2.20

$$\frac{i\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}\cos(c+dx)\left(-4\log\left(e^{i(c+dx)}+\sqrt{-1+e^{2i(c+dx)}}\right)+\sqrt{2}\left(\log\left(1-3e^{2i(c+dx)}-2\sqrt{2}\,e^{i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\right)-\log\left(1-3e^{2i(c+dx)}+2\sqrt{2}\,e^{i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\right)\right)\right)\sqrt{a+ia\tan(c+dx)}}{2d\sqrt{-1+e^{2i(c+dx)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((I/2)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Co
s[c + d*x]*(-4*Log[E^(I*(c + d*x)) + Sqrt[-1 + E^((2*I)*(c + d*x))]] + Sqrt
[2]*(Log[1 - 3*E^((2*I)*(c + d*x)) - 2*Sqrt[2]*E^(I*(c + d*x))*Sqrt[-1 + E^
((2*I)*(c + d*x))]] - Log[1 - 3*E^((2*I)*(c + d*x)) + 2*Sqrt[2]*E^(I*(c + d
*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]]))*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[
-1 + E^((2*I)*(c + d*x))])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(82) = 164.
time = 0.19, size = 349, normalized size = 3.36

method	result

derivativedivides	$-\frac{\left(\sqrt{\tan}\left(dx+c\right)\right)\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\,a\left(i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1+\right.}}{\tan(dx+c)+i}\right.\right.}{}$
default	$-\frac{\left(\sqrt{\tan}\left(dx+c\right)\right)\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\,a\left(i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1+\right.}}{\tan(dx+c)+i}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)*a*(I*2^(1/2)*ln(-(-2*2^(
1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))
/(tan(d*x+c)+I))*(I*a)^(1/2)+2*I*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a
*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*tan(d*x+c)-
2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+
I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*(I*a)^(1/2)*tan(d*x+c)+2*(-I*a)^(1/2)*l
n(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)
+a)/(I*a)^(1/2)))/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2)/(-tan(d
*x+c)+I)/(-I*a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*sqrt(tan(d*x + c)), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(76) = 152$.
time = 0.39, size = 413, normalized size = 3.97

$$-\frac{1}{2}\sqrt{\frac{4ia}{d^2}}\log\left(\left(\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}\left(e^{(2idx+2ic)}+1\right)+id\sqrt{\frac{4ia}{d^2}}e^{(idx+ic)}\right)e^{(-idx-ic)}\right)+\frac{1}{2}\sqrt{\frac{4ia}{d^2}}\log\left(\left(\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}\left(e^{(2idx+2ic)}+1\right)-id\sqrt{\frac{4ia}{d^2}}e^{(idx+ic)}\right)e^{(-idx-ic)}\right)+\frac{1}{2}\sqrt{\frac{2ia}{d^2}}\log\left(\left(\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}\left(e^{(2idx+2ic)}+1\right)+id\sqrt{\frac{2ia}{d^2}}e^{(idx+ic)}\right)e^{(-idx-ic)}\right)-\frac{1}{2}\sqrt{\frac{2ia}{d^2}}\log\left(\left(\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}\left(e^{(2idx+2ic)}+1\right)-id\sqrt{\frac{2ia}{d^2}}e^{(idx+ic)}\right)e^{(-idx-ic)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(4*I*a/d^2)*log((sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-
I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c)
+ 1) + I*d*sqrt(4*I*a/d^2)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 1/2*sqrt(4*
I*a/d^2)*log((sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x
```

```
 + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1) - I*d*s
qrt(4*I*a/d^2)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 1/2*sqrt(2*I*a/d^2)*log
((sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) +
I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1) + I*d*sqrt(2*I*a/d^
2)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 1/2*sqrt(2*I*a/d^2)*log((sqrt(2)*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d
*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1) - I*d*sqrt(2*I*a/d^2)*e^(I*d*x
+ I*c))*e^(-I*d*x - I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,\sqrt{\tan\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/2)*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*sqrt(tan(c + d*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 1.09Unable to divide
, perhaps due to rounding error%%%{%%{[%%%{%%{poly1[-2*i,0]:[1,0,-2]%%},[0]
%%%},0
```

Mupad [B]
time = 7.31, size = 278, normalized size = 2.67

$$\frac{\sqrt{a}\,\ln\left(\frac{\sqrt{a}\,\sqrt{\tan\left(c+d\,x\right)}\,\left(2-2{}\mathrm{i}\right)}{\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}-\sqrt{a}}-\frac{a\,\tan\left(c+d\,x\right)}{{\left(\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}-\sqrt{a}\right)}^2}+1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{1}{2}{}\mathrm{i}\right)}{d}-\frac{\sqrt{\frac{1}{2}{}\mathrm{i}}\,\sqrt{a}\,\ln\left(-\frac{a\,\tan\left(c+d\,x\right)}{{\left(\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}-\sqrt{a}\right)}^2}+\frac{2\,{\left(-1\right)}^{3/4}\,\sqrt{2}\,\sqrt{a}\,\sqrt{\tan\left(c+d\,x\right)}}{\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}-\sqrt{a}}+1{}\mathrm{i}\right)}{d}-\frac{\sqrt{4{}\mathrm{i}}\,\sqrt{a}\,\ln\left({\left(-1\right)}^{3/4}+\frac{\sqrt{a}\,\sqrt{\tan\left(c+d\,x\right)}}{\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}-\sqrt{a}}\right)}{d}+\frac{\sqrt{2}\,\sqrt{a}\,\ln\left(\sqrt{2}\,\left(1-\mathrm{i}\right)+\frac{2\,\sqrt{a}\,\sqrt{\tan\left(c+d\,x\right)}}{\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}-\sqrt{a}}\right)\,\left(1+1{}\mathrm{i}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] (a^(1/2)*log((a^(1/2)*tan(c + d*x)^(1/2)*(2 - 2i))/((a + a*tan(c + d*x)*1i)
^(1/2) - a^(1/2)) - (a*tan(c + d*x))/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/
2))^2 + 1i)*(1/2 + 1i/2))/d - ((1i/2)^(1/2)*a^(1/2)*log((2*(-1)^(3/4)*2^(1/
2)*a^(1/2)*tan(c + d*x)^(1/2))/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2)) -
(a*tan(c + d*x))/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))^2 + 1i))/d - (4i
^(1/2)*a^(1/2)*log((-1)^(3/4) + (a^(1/2)*tan(c + d*x)^(1/2))/((a + a*tan(c
+ d*x)*1i)^(1/2) - a^(1/2))))/d + (2^(1/2)*a^(1/2)*log(2^(1/2)*(1 - 1i) + (
2*a^(1/2)*tan(c + d*x)^(1/2))/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2)))*(1
 + 1i))/d
```

3.189 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{\tan(c+dx)}}\,dx$

Optimal. Leaf size=49

$$\frac{(1-i)\sqrt{a}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

```
[Out] (1-I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*a^(1
/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3625, 211}

$$\frac{(1-i)\sqrt{a}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[Tan[c + d*x]],x]
```

```
[Out] ((1 - I)*Sqrt[a]*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*
Tan[c + d*x]]])/d
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{\tan(c+dx)}}\,dx = -\frac{(2ia^2)\,\mathrm{Subst}\left(\int \frac{1}{-ia-2a^2x^2}\,dx, x, \frac{\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

$$= \frac{(1-i)\sqrt{a}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

Mathematica [F]
time = 1.12, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{\tan(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[Tan[c + d*x]],x]
```

```
[Out] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[Tan[c + d*x]], x]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(40) = 80.
time = 0.20, size = 121, normalized size = 2.47

method	result
derivativedivides	$-\frac{i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\left(\sqrt{\tan}(dx\right.}{2d\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}}$
default	$-\frac{i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\left(\sqrt{\tan}(dx\right.}{2d\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/d*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)
))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^(1/2)*(a*(1+I*tan
(d*x+c)))^(1/2)/(-I*a)^(1/2)/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(35) = 70.
time = 0.63, size = 379, normalized size = 7.73

$$\frac{\sqrt{a}\left((2i+2)\arctan\left(\left(\cos(2dx+2c)^2+\sin(2dx+2c)^2-2\cos(2dx+2c)+1\right)^{\frac14}\sin\left(\frac12\arctan\left(\sin(2dx+2c),-\cos(2dx+2c)+1\right)\right)-\cos(dx+c),\left(\cos(2dx+2c)^2+\sin(2dx+2c)^2-2\cos(2dx+2c)+1\right)^{\frac14}\cos\left(\frac12\arctan\left(\sin(2dx+2c),-\cos(2dx+2c)+1\right)\right)-\sin(dx+c)\right)+(i-1)\log\left(\cos(dx+c)^2+\sin(dx+c)^2+\sqrt{\cos(2dx+2c)^2+\sin(2dx+2c)^2-2\cos(2dx+2c)+1}\left(\cos\left(\frac12\arctan\left(\sin(2dx+2c),-\cos(2dx+2c)+1\right)\right)^2+\sin\left(\frac12\arctan\left(\sin(2dx+2c),-\cos(2dx+2c)+1\right)\right)^2\right)-2\left(\cos(2dx+2c)^2+\sin(2dx+2c)^2-2\cos(2dx+2c)+1\right)^{\frac14}\left(\cos\left(\frac12\arctan\left(\sin(2dx+2c),-\cos(2dx+2c)+1\right)\right)\sin(dx+c)+\cos(dx+c)\sin\left(\frac12\arctan\left(\sin(2dx+2c),-\cos(2dx+2c)+1\right)\right)\right)\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*((2*I + 2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2
*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x +
 2*c) + 1)) - cos(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*co
s(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*
c) + 1)) - sin(d*x + c)) + (I - 1)*log(cos(d*x + c)^2 + sin(d*x + c)^2 + sq
rt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)*(cos(1
/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1))^2 + sin(1/2*arctan2(si
n(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1))^2) - 2*(cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2
*c), -cos(2*d*x + 2*c) + 1))*sin(d*x + c) + cos(d*x + c)*sin(1/2*arctan2(si
n(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)))))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(35) = 70$.
time = 0.36, size = 206, normalized size = 4.20

$$\frac{1}{2} \sqrt{-\frac{2i a}{d^2}} \log \left(\left(\sqrt{2} \sqrt{\frac{a}{e^{(2i dx+2i c)}+1}} \sqrt{\frac{-i e^{(2i dx+2i c)}+i}{e^{(2i dx+2i c)}+1}} \left(e^{(2i dx+2i c)}+1\right) + d\sqrt{-\frac{2i a}{d^2}} e^{(i dx+i c)} \right) e^{(-i dx-i c)} \right) - \frac{1}{2} \sqrt{-\frac{2i a}{d^2}} \log \left(\left(\sqrt{2} \sqrt{\frac{a}{e^{(2i dx+2i c)}+1}} \sqrt{\frac{-i e^{(2i dx+2i c)}+i}{e^{(2i dx+2i c)}+1}} \left(e^{(2i dx+2i c)}+1\right) - d\sqrt{-\frac{2i a}{d^2}} e^{(i dx+i c)} \right) e^{(-i dx-i c)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-2*I*a/d^2)*log((sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-
I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c)
+ 1) + d*sqrt(-2*I*a/d^2)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 1/2*sqrt(-2*
I*a/d^2)*log((sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x
 + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1) - d*sqr
t(-2*I*a/d^2)*e^(I*d*x + I*c))*e^(-I*d*x - I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ia \left(\tan \left(c + dx\right) - i\right)}}{\sqrt{\tan \left(c + dx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)/tan(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))/sqrt(tan(c + d*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

Mupad [B]
time = 5.12, size = 89, normalized size = 1.82

$$\frac{2\sqrt{\frac{1}{2}\mathrm{i}}\,\sqrt{-a}\,\mathrm{atanh}\left(\frac{2\sqrt{\frac{1}{2}\mathrm{i}}\,\sqrt{-a}\,\sqrt{\tan\left(c+d\,x\right)}\,\left(\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}-\sqrt{a}\right)}{a\tan(c+d\,x)-a\,1\mathrm{i}+\sqrt{a}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,1\mathrm{i}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/tan(c + d*x)^(1/2),x)
```

```
[Out] (2*(1i/2)^(1/2)*(-a)^(1/2)*atanh((2*(1i/2)^(1/2)*(-a)^(1/2)*tan(c + d*x)^(1
/2)*((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2)))/(a*tan(c + d*x) - a*1i + a^(
1/2)*(a + a*tan(c + d*x)*1i)^(1/2)*1i)))/d
```

3.190 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=82

$$\frac{(1+i)\sqrt{a}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

```
[Out] (1+I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*a^(1
/2)/d-2*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3629, 3625, 211}

$$\frac{(1+i)\sqrt{a}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[c + d*x]]/Tan[c + d*x]^(3/2),x]
```

```
[Out] ((1 + I)*Sqrt[a]*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*
Tan[c + d*x]]])/d - (2*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3629

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-d)*(a + b*Tan[e + f*x])^m*((c + d*Ta
n[e + f*x])^(n + 1)/(f*m*(c^2 + d^2))), x] + Dist[a/(a*c - b*d), Int[(a + b
```

```
*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+ia\tan(c+dx)}}{\tan^{\frac{3}{2}}(c+dx)}\,dx &= -\frac{2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}+i\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{\tan(c+dx)}}\,dx \\
&= -\frac{2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}+\frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{-ia-2a^2x^2}\,dx, x, \frac{\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} \\
&= \frac{(1+i)\sqrt{a}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}
\end{aligned}$$

Mathematica [F]
time = 2.44, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/Tan[c + d*x]^(3/2),x]
[Out] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/Tan[c + d*x]^(3/2), x]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(68) = 136.
time = 0.20, size = 157, normalized size = 1.91

method	result
derivativedivides	$\frac{\left(\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)\,(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\tan(dx+c)-4\right.}{2d\sqrt{-ia}\,\sqrt{a\tan(dx+c)\,(1+i\tan(dx}$
default	$\frac{\left(\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)\,(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\tan(dx+c)-4\right.}{2d\sqrt{-ia}\,\sqrt{a\tan(dx+c)\,(1+i\tan(dx}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))
^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)-4*(a*tan(d*x+c)*(1+
I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2))*(a*(1+I*tan(d*x+c)))^(1/2)/(-I*a)^(1/2)/
(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/tan(d*x+c)^(1/2)
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(62) = 124$.
time $= 0.63$, size $= 549$, normalized size $= 6.70$

$$-\frac{\left(\cos(2dx+2c)^2+\sin(2dx+2c)^2-2\cos(2dx+2c)+1\right)^{\frac{1}{4}}\sqrt{a}\left(-(2i-2)\arctan\left(\left(\cos(2dx+2c)^2+\sin(2dx+2c)^2-2\cos(2dx+2c)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\arctan\left(\sin(2dx+2c),-\cos(2dx+2c)+1\right)\right)-\cos(dx+c),\left(\cos(2dx+2c)^2+\sin(2dx+2c)^2-2\cos(2dx+2c)+1\right)^{\frac{1}{4}}\cos\left(\frac{1}{2}\arctan\left(\sin(2dx+2c),-\cos(2dx+2c)+1\right)\right)-\sin(dx+c)\right)+(i+1)\log\left(\cos(dx+c)^2+\sin(dx+c)^2+\sqrt{\cos(2dx+2c)^2+\sin(2dx+2c)^2-2\cos(2dx+2c)+1}\left(\cos\left(\frac{1}{2}\arctan\left(\sin(2dx+2c),-\cos(2dx+2c)+1\right)\right)^2+\sin\left(\frac{1}{2}\arctan\left(\sin(2dx+2c),-\cos(2dx+2c)+1\right)\right)^2\right)-2\left(\cos(2dx+2c)^2+\sin(2dx+2c)^2-2\cos(2dx+2c)+1\right)^{\frac{1}{4}}\left(\cos\left(\frac{1}{2}\arctan\left(\sin(2dx+2c),-\cos(2dx+2c)+1\right)\right)\sin(dx+c)+\cos(dx+c)\sin\left(\frac{1}{2}\arctan\left(\sin(2dx+2c),-\cos(2dx+2c)+1\right)\right)\right)\right)\right)-4\left(\left((i-1)\cos(dx+c)-(i+1)\sin(dx+c)\right)\cos\left(\frac{1}{2}\arctan\left(\sin(2dx+2c),-\cos(2dx+2c)+1\right)\right)+\left(-(i+1)\cos(dx+c)-(i-1)\sin(dx+c)\right)\sin\left(\frac{1}{2}\arctan\left(\sin(2dx+2c),-\cos(2dx+2c)+1\right)\right)\right)\sqrt{a}}{2\left(\cos(2dx+2c)^2+\sin(2dx+2c)^2-2\cos(2dx+2c)+1\right)^{\frac{1}{4}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1
/4)*sqrt(a)*(-(2*I - 2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 -
2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x
+ 2*c) + 1)) - cos(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*c
os(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2
*c) + 1)) - sin(d*x + c)) + (I + 1)*log(cos(d*x + c)^2 + sin(d*x + c)^2 + s
qrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)*(cos(
1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1))^2 + sin(1/2*arctan2(s
in(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1))^2) - 2*(cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
2*c), -cos(2*d*x + 2*c) + 1))*sin(d*x + c) + cos(d*x + c)*sin(1/2*arctan2(s
in(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1))))) - 4*(((I - 1)*cos(d*x + c) - (I
 + 1)*sin(d*x + c))*cos(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1
)) + (-(I + 1)*cos(d*x + c) - (I - 1)*sin(d*x + c))*sin(1/2*arctan2(sin(2*d
*x + 2*c), -cos(2*d*x + 2*c) + 1)))*sqrt(a))/((cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*d)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(62) = 124$.
time $= 0.37$, size $= 329$, normalized size $= 4.01$

$$-\frac{4\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(i\,e^{(3i\,dx+3i\,c)}+i\,e^{(i\,dx+i\,c)}\right)+\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{\frac{2i\,a}{d^2}}\log\left(\left(\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)+i\,d\sqrt{\frac{2i\,a}{d^2}}e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{\frac{2i\,a}{d^2}}\log\left(\left(\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)-i\,d\sqrt{\frac{2i\,a}{d^2}}e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)}{2\left(de^{(2i\,dx+2i\,c)}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I
*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^(3*I*d*x + 3*I*c) + I*e^(I*d*x + I
*c)) + (d*e^(2*I*d*x + 2*I*c) - d)*sqrt(2*I*a/d^2)*log((sqrt(2)*sqrt(a/(e^(
2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*
```

```
c) + 1))*(e^(2*I*d*x + 2*I*c) + 1) + I*d*sqrt(2*I*a/d^2)*e^(I*d*x + I*c))*e
^(-I*d*x - I*c)) - (d*e^(2*I*d*x + 2*I*c) - d)*sqrt(2*I*a/d^2)*log((sqrt(2)
*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*
I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1) - I*d*sqrt(2*I*a/d^2)*e^(I*d
*x + I*c))*e^(-I*d*x - I*c)))/(d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}{\tan^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)/tan(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))/tan(c + d*x)**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{{\tan\left(c+d\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/tan(c + d*x)^(3/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^(1/2)/tan(c + d*x)^(3/2), x)
```

3.191 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{\tan^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=120

$$-\frac{(1-i)\sqrt{a}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d\sqrt{\tan(c+dx)}}$$

```
[Out] (-1+I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*a^(
1/2)/d-2/3*I*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(1/2)-2/3*(a+I*a*tan(d*x
+c))^(1/2)/d/tan(d*x+c)^(3/2)
```

Rubi [A]
time = 0.17, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3642, 3679, 12, 3625, 211}

$$-\frac{2\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d\sqrt{\tan(c+dx)}}-\frac{(1-i)\sqrt{a}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[c + d*x]]/Tan[c + d*x]^(5/2),x]
```

```
[Out] ((-1 + I)*Sqrt[a]*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a
*Tan[c + d*x]]])/d - (2*Sqrt[a + I*a*Tan[c + d*x]])/(3*d*Tan[c + d*x]^(3/2)
) - (((2*I)/3)*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
```

```
Q[c^2 + d^2, 0]
```

Rule 3642

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[d*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(a*(c^2 + d^2)*(n +
1)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[b*d*m - a
*c*(n + 1) + a*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
 e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0
] && LtQ[n, -1] && (IntegerQ[n] || IntegersQ[2*m, 2*n])
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\tan^{\frac{5}{2}}(c+dx)}\,dx = -\frac{2\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{\left(\frac{ia}{2}-a\tan(c+dx)\right)\sqrt{a+ia\tan(c+dx)}}{\tan^{\frac{3}{2}}(c+dx)}\,dx}{3a}$$

$$= -\frac{2\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{2i\sqrt{a+ia\tan(c+dx)}}{3d\sqrt{\tan(c+dx)}} + \frac{4\int -\frac{3a^2\sqrt{a+ia\tan}}{4\sqrt{\tan(c+}}}{3a^2}$$

$$= -\frac{2\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{2i\sqrt{a+ia\tan(c+dx)}}{3d\sqrt{\tan(c+dx)}} - \int \frac{\sqrt{a+ia\tan(c+}}{\sqrt{\tan(c+dx)}}$$

$$= -\frac{2\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{2i\sqrt{a+ia\tan(c+dx)}}{3d\sqrt{\tan(c+dx)}} + \frac{(2ia^2)\,\mathrm{Subst}\left(\int \frac{1}{-ia-}\right.}{}$$

$$= -\frac{(1-i)\sqrt{a}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{2\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)} -$$

Mathematica [F]
time = 2.70, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + ia\tan(c + dx)}}{\tan^{\frac{5}{2}}(c + dx)}\, dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/Tan[c + d*x]^(5/2),x]
```

```
[Out] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/Tan[c + d*x]^(5/2), x]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(96) = 192.
time = 0.19, size = 197, normalized size = 1.64

method	result
derivativedivides	$\frac{\left(3i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)a(\tan^2(dx+c))-}{6d\tan(dx+}$
default	$\frac{\left(3i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)a(\tan^2(dx+c))-}{6d\tan(dx+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/d/tan(d*x+c)^(3/2)*(3*I*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x
+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c
)^2-4*I*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)-4*(a*
tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2))*(a*(1+I*tan(d*x+c)))^(1/2)
/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(-I*a)^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 974 vs. 2(88) = 176.
time = 0.67, size = 974, normalized size = 8.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(2*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) +
1)*(((3*I + 3)*cos(3*d*x + 3*c) + (I + 1)*cos(d*x + c) + (3*I - 3)*sin(3*d*
x + 3*c) + (I - 1)*sin(d*x + c))*cos(3/2*arctan2(sin(2*d*x + 2*c), -cos(2*d
```

```
*x + 2*c) + 1)) + ((3*I - 3)*cos(3*d*x + 3*c) + (I - 1)*cos(d*x + c) - (3*I
 + 3)*sin(3*d*x + 3*c) - (I + 1)*sin(d*x + c))*sin(3/2*arctan2(sin(2*d*x +
2*c), -cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3*(2*(-(I + 1)*cos(2*d*x + 2*c)^2
- (I + 1)*sin(2*d*x + 2*c)^2 + (2*I + 2)*cos(2*d*x + 2*c) - I - 1)*arctan2(
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*si
n(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)) - cos(d*x + c), (co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1
/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)) - sin(d*x + c)) + (-(I
 - 1)*cos(2*d*x + 2*c)^2 - (I - 1)*sin(2*d*x + 2*c)^2 + (2*I - 2)*cos(2*d*x
 + 2*c) - I + 1)*log(cos(d*x + c)^2 + sin(d*x + c)^2 + sqrt(cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)*(cos(1/2*arctan2(sin(2*d
*x + 2*c), -cos(2*d*x + 2*c) + 1))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), -c
os(2*d*x + 2*c) + 1))^2) - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*c
os(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x +
2*c) + 1))*sin(d*x + c) + cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), -c
os(2*d*x + 2*c) + 1)))))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2
*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 2*(((-(I + 1)*cos(d*x + c) - (I - 1)*sin(d
*x + c))*cos(2*d*x + 2*c)^2 + (-(I + 1)*cos(d*x + c) - (I - 1)*sin(d*x + c)
)*sin(2*d*x + 2*c)^2 + 2*((I + 1)*cos(d*x + c) + (I - 1)*sin(d*x + c))*cos(
2*d*x + 2*c) - (I + 1)*cos(d*x + c) - (I - 1)*sin(d*x + c))*cos(1/2*arctan2
(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)) + ((-(I - 1)*cos(d*x + c) + (I +
 1)*sin(d*x + c))*cos(2*d*x + 2*c)^2 + (-(I - 1)*cos(d*x + c) + (I + 1)*sin
(d*x + c))*sin(2*d*x + 2*c)^2 + 2*((I - 1)*cos(d*x + c) - (I + 1)*sin(d*x +
 c))*cos(2*d*x + 2*c) - (I - 1)*cos(d*x + c) + (I + 1)*sin(d*x + c))*sin(1/
2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)))*sqrt(a))/((cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(5/4)*d)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(88) = 176$.
time = 0.37, size = 355, normalized size = 2.96

$$\frac{8\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(5i\,dx+5i\,c)}+e^{(3i\,dx+3i\,c)}\right)-3\left(de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}+d\right)\sqrt{-\frac{2i\,a}{d^2}}\log\left(\left(\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)+d\sqrt{-\frac{2i\,a}{d^2}}\,e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+3\left(de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}+d\right)\sqrt{-\frac{2i\,a}{d^2}}\log\left(\left(\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)-d\sqrt{-\frac{2i\,a}{d^2}}\,e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)}{6\left(de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(8*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*
c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(5*I*d*x + 5*I*c) + e^(3*I*d*x + 3*I*
c)) - 3*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-2*I*a/d
^2)*log((sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*
I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1) + d*sqrt(-2*
I*a/d^2)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 3*(d*e^(4*I*d*x + 4*I*c) - 2*
d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-2*I*a/d^2)*log((sqrt(2)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)
)*(e^(2*I*d*x + 2*I*c) + 1) - d*sqrt(-2*I*a/d^2)*e^(I*d*x + I*c))*e^(-I*d*x
 - I*c)))/(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}{\tan^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)/tan(d*x+c)**(5/2),x)
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))/tan(c + d*x)**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(5/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{{\tan\left(c+d\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/tan(c + d*x)^(5/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^(1/2)/tan(c + d*x)^(5/2), x)
```

3.192 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{\tan^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=154

$$-\frac{(1+i)\sqrt{a}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{15d\tan^{\frac{3}{2}}(c+dx)}+\frac{26\sqrt{a}}{15a}$$

```
[Out] (-1-I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*a^(
1/2)/d+26/15*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(1/2)-2/5*(a+I*a*tan(d*x
+c))^(1/2)/d/tan(d*x+c)^(5/2)-2/15*I*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^
(3/2)
```

Rubi [A]
time = 0.26, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3642, 3679, 12, 3625, 211}

$$-\frac{2i\sqrt{a+ia\tan(c+dx)}}{15d\tan^{\frac{3}{2}}(c+dx)}-\frac{2\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{26\sqrt{a+ia\tan(c+dx)}}{15d\sqrt{\tan(c+dx)}}-\frac{(1+i)\sqrt{a}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[c + d*x]]/Tan[c + d*x]^(7/2),x]
```

```
[Out] ((-1 - I)*Sqrt[a]*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a
*Tan[c + d*x]]])/d - (2*Sqrt[a + I*a*Tan[c + d*x]])/(5*d*Tan[c + d*x]^(5/2)
) - (((2*I)/15)*Sqrt[a + I*a*Tan[c + d*x]])/(d*Tan[c + d*x]^(3/2)) + (26*Sq
rt[a + I*a*Tan[c + d*x]])/(15*d*Sqrt[Tan[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
```

```
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3642

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[d*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(a*(c^2 + d^2)*(n +
1)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[b*d*m - a
*c*(n + 1) + a*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
 e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0
] && LtQ[n, -1] && (IntegerQ[n] || IntegersQ[2*m, 2*n])
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\tan^{\frac{7}{2}}(c+dx)}\,dx = -\frac{2\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)} + \frac{2\int \frac{\left(\frac{ia}{2}-2a\tan(c+dx)\right)\sqrt{a+ia\tan(c+dx)}}{\tan^{\frac{5}{2}}(c+dx)}\,dx}{5a}$$

$$= -\frac{2\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{2i\sqrt{a+ia\tan(c+dx)}}{15d\tan^{\frac{3}{2}}(c+dx)} + \frac{4\int \frac{\sqrt{a+ia\tan(c+}}{\mathrm{t}}}{}$$

$$= -\frac{2\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{2i\sqrt{a+ia\tan(c+dx)}}{15d\tan^{\frac{3}{2}}(c+dx)} + \frac{26\sqrt{a+ia\tan(c+d}}{15d\sqrt{\tan(c+dx)}}$$

$$= -\frac{2\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{2i\sqrt{a+ia\tan(c+dx)}}{15d\tan^{\frac{3}{2}}(c+dx)} + \frac{26\sqrt{a+ia\tan(c+d}}{15d\sqrt{\tan(c+dx)}}$$

$$= -\frac{2\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{2i\sqrt{a+ia\tan(c+dx)}}{15d\tan^{\frac{3}{2}}(c+dx)} + \frac{26\sqrt{a+ia\tan(c+d}}{15d\sqrt{\tan(c+dx)}}$$

$$= -\frac{(1+i)\sqrt{a}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{2\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)} -$$

Mathematica [F]
time = 3.67, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\tan^{\frac{7}{2}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/Tan[c + d*x]^(7/2),x]
```

```
[Out] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/Tan[c + d*x]^(7/2), x]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 358 vs. $2(123) = 246$.
time = 0.19, size = 359, normalized size = 2.33

method	result
derivativedivides	$-\frac{\sqrt{a(1+i\tan(dx+c))}\left(15i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c}}{\tan(dx+c)+i}\right.\right.}{}$

default	$-\frac{\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\,\left(15i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1+i\tan\left(dx+c\right)\right)}}{\tan(dx+c)+i}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/30/d*(a*(1+I*tan(d*x+c)))^(1/2)*(15*I*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/
2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I)
)*a*tan(d*x+c)^3-15*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I
*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4-56*I
*tan(d*x+c)^2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)+52*(a*tan(
d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^3+12*I*(a*tan(d*x+c)
*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)-16*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan(
d*x+c)))^(1/2)*(-I*a)^(1/2))/tan(d*x+c)^(5/2)/(a*tan(d*x+c)*(1+I*tan(d*x+c)
))^(1/2)/(-tan(d*x+c)+I)/(-I*a)^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1151 vs. $2(114) = 228$.
time = 0.73, size = 1151, normalized size = 7.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/30*(3*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) +
 1)*(((10*I - 10)*cos(3*d*x + 3*c) - (13*I - 13)*cos(d*x + c) - (10*I + 10)
*sin(3*d*x + 3*c) + (13*I + 13)*sin(d*x + c))*cos(3/2*arctan2(sin(2*d*x + 2
*c), -cos(2*d*x + 2*c) + 1)) + (-(10*I + 10)*cos(3*d*x + 3*c) + (13*I + 13)
*cos(d*x + c) - (10*I - 10)*sin(3*d*x + 3*c) + (13*I - 13)*sin(d*x + c))*si
n(3/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)))*sqrt(a) + 15*(2*(-
(I - 1)*cos(2*d*x + 2*c)^2 - (I - 1)*sin(2*d*x + 2*c)^2 + (2*I - 2)*cos(2*d
*x + 2*c) - I + 1)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos
(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c
) + 1)) - cos(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*
d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) +
 1)) - sin(d*x + c)) + ((I + 1)*cos(2*d*x + 2*c)^2 + (I + 1)*sin(2*d*x + 2*
c)^2 - (2*I + 2)*cos(2*d*x + 2*c) + I + 1)*log(cos(d*x + c)^2 + sin(d*x + c
)^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1
)*(cos(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1))^2 + sin(1/2*ar
ctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1))^2) - 2*(cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2
*d*x + 2*c), -cos(2*d*x + 2*c) + 1))*sin(d*x + c) + cos(d*x + c)*sin(1/2*ar
```

```
ctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)))))*(cos(2*d*x + 2*c)^2 + si
n(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + ((-(30*I - 30)*c
os(5*d*x + 5*c) - (5*I - 5)*cos(3*d*x + 3*c) - (13*I - 13)*cos(d*x + c) + (
30*I + 30)*sin(5*d*x + 5*c) + (5*I + 5)*sin(3*d*x + 3*c) + (13*I + 13)*sin(
d*x + c))*cos(5/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)) + 8*((-
(I - 1)*cos(d*x + c) + (I + 1)*sin(d*x + c))*cos(2*d*x + 2*c)^2 + (-(I - 1)
*cos(d*x + c) + (I + 1)*sin(d*x + c))*sin(2*d*x + 2*c)^2 + 2*((I - 1)*cos(d
*x + c) - (I + 1)*sin(d*x + c))*cos(2*d*x + 2*c) - (I - 1)*cos(d*x + c) + (
I + 1)*sin(d*x + c))*cos(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) +
1)) + ((30*I + 30)*cos(5*d*x + 5*c) + (5*I + 5)*cos(3*d*x + 3*c) + (13*I +
13)*cos(d*x + c) + (30*I - 30)*sin(5*d*x + 5*c) + (5*I - 5)*sin(3*d*x + 3*c
) + (13*I - 13)*sin(d*x + c))*sin(5/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x
+ 2*c) + 1)) + 8*(((I + 1)*cos(d*x + c) + (I - 1)*sin(d*x + c))*cos(2*d*x +
 2*c)^2 + ((I + 1)*cos(d*x + c) + (I - 1)*sin(d*x + c))*sin(2*d*x + 2*c)^2
+ 2*(-(I + 1)*cos(d*x + c) - (I - 1)*sin(d*x + c))*cos(2*d*x + 2*c) + (I +
1)*cos(d*x + c) + (I - 1)*sin(d*x + c))*sin(1/2*arctan2(sin(2*d*x + 2*c), -
cos(2*d*x + 2*c) + 1)))*sqrt(a))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
- 2*cos(2*d*x + 2*c) + 1)^(5/4)*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(114) = 228$.
time = 0.39, size = 424, normalized size = 2.75

$$-\frac{4\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(-17i\,e^{(7i\,dx+7i\,c)}+3i\,e^{(5i\,dx+5i\,c)}+5i\,e^{(3i\,dx+3i\,c)}-15i\,e^{(i\,dx+i\,c)}\right)-15\left(de^{(6i\,dx+6i\,c)}-3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}-d\right)\sqrt{\frac{2i\,a}{d^2}}\log\left(\left(\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)+i\,d\sqrt{\frac{2i\,a}{d^2}}\,e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+15\left(de^{(6i\,dx+6i\,c)}-3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}-d\right)\sqrt{\frac{2i\,a}{d^2}}\log\left(\left(\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)-i\,d\sqrt{\frac{2i\,a}{d^2}}\,e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)}{30\left(de^{(6i\,dx+6i\,c)}-3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/30*(4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*
I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(-17*I*e^(7*I*d*x + 7*I*c) + 3*I*e^(5*
I*d*x + 5*I*c) + 5*I*e^(3*I*d*x + 3*I*c) - 15*I*e^(I*d*x + I*c)) - 15*(d*e^
(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)*
sqrt(2*I*a/d^2)*log((sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(
2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)
+ I*d*sqrt(2*I*a/d^2)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 15*(d*e^(6*I*d*x
 + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)*sqrt(2*I
*a/d^2)*log((sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x
+ 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1) - I*d*sq
rt(2*I*a/d^2)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)))/(d*e^(6*I*d*x + 6*I*c) -
3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}{\tan^{\frac{7}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)/tan(d*x+c)**(7/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))/tan(c + d*x)**(7/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{{\tan\left(c+d\,x\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/tan(c + d*x)^(7/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^(1/2)/tan(c + d*x)^(7/2), x)
```

3.193 $\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=254

$$\frac{23(-1)^{3/4}a^{3/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{8d}+\frac{(2+2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{ia}{3d\sqrt{}}$$

```
[Out] 23/8*(-1)^(3/4)*a^(3/2)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*t
an(d*x+c))^(1/2))/d+(2+2*I)*a^(3/2)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/
(a+I*a*tan(d*x+c))^(1/2))/d-9/8*I*a*tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/
2)/d+7/12*a*(a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^(3/2)/d+1/3*I*a^2*tan(d*x+c
)^(5/2)/d/(a+I*a*tan(d*x+c))^(1/2)-1/3*a^2*tan(d*x+c)^(7/2)/d/(a+I*a*tan(d*
x+c))^(1/2)
```

Rubi [A]

time = 0.53, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3637, 3676, 3678, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{23(-1)^{3/4}a^{3/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{8d}+\frac{(2+2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{a^2\tan^{\frac{7}{2}}(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}}+\frac{ia^2\tan^{\frac{5}{2}}(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}}+\frac{7a\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{12d}-\frac{9ia\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (23*(-1)^(3/4)*a^(3/2)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[
a + I*a*Tan[c + d*x]]])/(8*d) + ((2 + 2*I)*a^(3/2)*ArcTanh[((1 + I)*Sqrt[a]
*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d + ((I/3)*a^2*Tan[c + d*
x]^(5/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (a^2*Tan[c + d*x]^(7/2))/(3*d*Sq
rt[a + I*a*Tan[c + d*x]]) - (((9*I)/8)*a*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Ta
n[c + d*x]])/d + (7*a*Tan[c + d*x]^(3/2)*Sqrt[a + I*a*Tan[c + d*x]])/(12*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rule 3678

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Dist[
1/(a*(m + n)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Simp
[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*Ta
```

```
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{a^2\tan^{\frac{7}{2}}(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}} + \frac{1}{3}a\int \frac{\tan^{\frac{5}{2}}(c+dx)\left(\frac{13a}{2}+\frac{11}{2}ia\tan\right.}{\sqrt{a+ia\tan(c+dx}}$$

$$= \frac{ia^2\tan^{\frac{5}{2}}(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}} - \frac{a^2\tan^{\frac{7}{2}}(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}} - \frac{\int \tan^{\frac{3}{2}}(c}{}$$

$$= \frac{ia^2\tan^{\frac{5}{2}}(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}} - \frac{a^2\tan^{\frac{7}{2}}(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}} + \frac{7a\tan^{\frac{3}{2}}(}{}$$

$$= \frac{ia^2\tan^{\frac{5}{2}}(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}} - \frac{a^2\tan^{\frac{7}{2}}(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}} - \frac{9ia\sqrt{\tan}}{}$$

$$= \frac{ia^2\tan^{\frac{5}{2}}(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}} - \frac{a^2\tan^{\frac{7}{2}}(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}} - \frac{9ia\sqrt{\tan}}{}$$

$$= \frac{ia^2\tan^{\frac{5}{2}}(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}} - \frac{a^2\tan^{\frac{7}{2}}(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}} - \frac{9ia\sqrt{\tan}}{}$$

$$= \frac{(2+2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} + \frac{ia^2\tan^{\frac{5}{2}}(c}{3d\sqrt{a+ia\tan}}$$

$$= \frac{(2+2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} + \frac{ia^2\tan^{\frac{5}{2}}(c}{3d\sqrt{a+ia\tan}}$$

$$= \frac{23(-1)^{3/4}a^{3/2}\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{8d} + \frac{(2+2i)a^{3/}}{}$$

Mathematica [A]
time = 3.10, size = 213, normalized size = 0.84

$$\frac{a\left(\frac{6e^{-i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\left(32\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)-23\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)}{\sqrt{-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}}+2\sec^2(c+dx)(-19i-35i\cos(2(c+dx))+14\sin(2(c+dx)))\sqrt{\tan(c+dx)}\right)\sqrt{a+ia\tan(c+dx)}}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (a*((6*Sqrt[-1 + E^((2*I)*(c + d*x))]*(32*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 +
 E^((2*I)*(c + d*x))]] - 23*Sqrt[2]*ArcTanh[(Sqrt[2]*E^(I*(c + d*x)))/Sqrt[
```

```
-1 + E^((2*I)*(c + d*x))]]))/(E^(I*(c + d*x))*Sqrt[((-I)*(-1 + E^((2*I)*(c
+ d*x))))/(1 + E^((2*I)*(c + d*x)))]) + 2*Sec[c + d*x]^2*(-19*I - (35*I)*Co
s[2*(c + d*x)] + 14*Sin[2*(c + d*x)])*Sqrt[Tan[c + d*x]])*Sqrt[a + I*a*Tan[
c + d*x]])/(96*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 448 vs. 2(200) = 400.
time = 0.22, size = 449, normalized size = 1.77

method	result
derivativedivides	$-\frac{\left(\sqrt{\tan}\,(dx+c)\right)\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\,a\left(-16i\sqrt{a\tan\left(dx+c\right)\left(1+i\tan\left(dx+c\right)\right)}\,\sqrt{i}\right.}{}$
default	$-\frac{\left(\sqrt{\tan}\,(dx+c)\right)\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\,a\left(-16i\sqrt{a\tan\left(dx+c\right)\left(1+i\tan\left(dx+c\right)\right)}\,\sqrt{i}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)*a*(-16*I*(a*tan(d*x+c)*
(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^2+24*I*(I*a)^(1
/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1
/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a+69*I*ln(1/2*(2*I*a*tan(d*x+c)+2*(
a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/
2)*a+54*I*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)+24
*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*
x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a-28*(a*tan(d*x+c)*(1+I*ta
n(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)+96*ln(1/2*(2*I*a*tan(d
*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a
*(-I*a)^(1/2))/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2)/(-I*a)^(1/
2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*tan(d*x + c)^(5/2), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 673 vs. 2(188) = 376.

time = 0.38, size = 673, normalized size = 2.65

$$\frac{\sqrt{2}\left(-49 i a e^{\left(5 i d x+5 i c\right)}-38 i a e^{\left(3 i d x+3 i c\right)}-21 i a e^{\left(i d x+i c\right)}\right)\sqrt{\frac{a}{e^{\left(2 i d x+2 i c\right)}+1}}\sqrt{\frac{-i e^{\left(2 i d x+2 i c\right)}+i}{e^{\left(2 i d x+2 i c\right)}+1}}+12\left(d e^{\left(4 i d x+4 i c\right)}+2 d e^{\left(2 i d x+2 i c\right)}+d\right)\sqrt{\frac{529 i a^{3}}{64 d^{2}}}\log\left(\frac{\left(23\sqrt{2}\left(a e^{\left(2 i d x+2 i c\right)}+a\right)\sqrt{\frac{a}{e^{\left(2 i d x+2 i c\right)}+1}}\sqrt{\frac{-i e^{\left(2 i d x+2 i c\right)}+i}{e^{\left(2 i d x+2 i c\right)}+1}}+16 i\sqrt{\frac{529 i a^{3}}{64 d^{2}}} d e^{\left(i d x+i c\right)}\right)e^{\left(-i d x-i c\right)}}{23 a}\right)-12\left(d e^{\left(4 i d x+4 i c\right)}+2 d e^{\left(2 i d x+2 i c\right)}+d\right)\sqrt{\frac{529 i a^{3}}{64 d^{2}}}\log\left(\frac{\left(23\sqrt{2}\left(a e^{\left(2 i d x+2 i c\right)}+a\right)\sqrt{\frac{a}{e^{\left(2 i d x+2 i c\right)}+1}}\sqrt{\frac{-i e^{\left(2 i d x+2 i c\right)}+i}{e^{\left(2 i d x+2 i c\right)}+1}}-16 i\sqrt{\frac{529 i a^{3}}{64 d^{2}}} d e^{\left(i d x+i c\right)}\right)e^{\left(-i d x-i c\right)}}{23 a}\right)-12\left(d e^{\left(4 i d x+4 i c\right)}+2 d e^{\left(2 i d x+2 i c\right)}+d\right)\sqrt{\frac{8 i a^{3}}{d^{2}}}\log\left(\frac{\left(2\sqrt{2}\left(a e^{\left(2 i d x+2 i c\right)}+a\right)\sqrt{\frac{a}{e^{\left(2 i d x+2 i c\right)}+1}}\sqrt{\frac{-i e^{\left(2 i d x+2 i c\right)}+i}{e^{\left(2 i d x+2 i c\right)}+1}}+i\sqrt{\frac{8 i a^{3}}{d^{2}}} d e^{\left(i d x+i c\right)}\right)e^{\left(-i d x-i c\right)}}{2 a}\right)+12\left(d e^{\left(4 i d x+4 i c\right)}+2 d e^{\left(2 i d x+2 i c\right)}+d\right)\sqrt{\frac{8 i a^{3}}{d^{2}}}\log\left(\frac{\left(2\sqrt{2}\left(a e^{\left(2 i d x+2 i c\right)}+a\right)\sqrt{\frac{a}{e^{\left(2 i d x+2 i c\right)}+1}}\sqrt{\frac{-i e^{\left(2 i d x+2 i c\right)}+i}{e^{\left(2 i d x+2 i c\right)}+1}}-i\sqrt{\frac{8 i a^{3}}{d^{2}}} d e^{\left(i d x+i c\right)}\right)e^{\left(-i d x-i c\right)}}{2 a}\right)}{24\left(d e^{\left(4 i d x+4 i c\right)}+2 d e^{\left(2 i d x+2 i c\right)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(sqrt(2)*(-49*I*a*e^(5*I*d*x + 5*I*c) - 38*I*a*e^(3*I*d*x + 3*I*c) - 2
1*I*a*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*
x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) + 12*(d*e^(4*I*d*x + 4*I*c) + 2*
d*e^(2*I*d*x + 2*I*c) + d)*sqrt(529/64*I*a^3/d^2)*log(1/23*(23*sqrt(2)*(a*e
^(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d
*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) + 16*I*sqrt(529/64*I*a^3/d^2)*d
*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a) - 12*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(
2*I*d*x + 2*I*c) + d)*sqrt(529/64*I*a^3/d^2)*log(1/23*(23*sqrt(2)*(a*e^(2*I
*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x +
2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) - 16*I*sqrt(529/64*I*a^3/d^2)*d*e^(I
*d*x + I*c))*e^(-I*d*x - I*c)/a) - 12*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d
*x + 2*I*c) + d)*sqrt(8*I*a^3/d^2)*log(1/2*(2*sqrt(2)*(a*e^(2*I*d*x + 2*I*c
) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/
(e^(2*I*d*x + 2*I*c) + 1)) + I*sqrt(8*I*a^3/d^2)*d*e^(I*d*x + I*c))*e^(-I*d
*x - I*c)/a) + 12*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*sqr
t(8*I*a^3/d^2)*log(1/2*(2*sqrt(2)*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*
I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c)
 + 1)) - I*sqrt(8*I*a^3/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a))/(d*e^(
4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(5/2)*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 0.45Unable to divide
, perhaps due to rounding error%%%{%%{[%%%{%%{poly1[-32*i,0]:[1,0,-2]%%},[0
]%%%},
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c+d\,x)^{5/2}\,(a+a\tan(c+d\,x)\ 1\mathrm{i})^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int(tan(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.194 $\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=217

$$-\frac{11\sqrt[4]{-1}\,a^{3/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{4d}-\frac{(2-2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{ia}{2d\sqrt{a}}$$

```
[Out] -11/4*(-1)^(1/4)*a^(3/2)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*
tan(d*x+c))^(1/2))/d+(-2+2*I)*a^(3/2)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2
)/(a+I*a*tan(d*x+c))^(1/2))/d+5/4*a*tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/
2)/d+1/2*I*a^2*tan(d*x+c)^(3/2)/d/(a+I*a*tan(d*x+c))^(1/2)-1/2*a^2*tan(d*x+
c)^(5/2)/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3637, 3676, 3678, 3682, 3625, 211, 3680, 65, 223, 209}

$$-\frac{11\sqrt[4]{-1}\,a^{3/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{4d}-\frac{(2-2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{a^2\tan^{\frac{5}{2}}(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}+\frac{ia^2\tan^{\frac{3}{2}}(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}+\frac{5a\sqrt{\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (-11*(-1)^(1/4)*a^(3/2)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt
[a + I*a*Tan[c + d*x]]])/(4*d) - ((2 - 2*I)*a^(3/2)*ArcTanh[((1 + I)*Sqrt[a
]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d + ((I/2)*a^2*Tan[c + d
*x]^(3/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (a^2*Tan[c + d*x]^(5/2))/(2*d*S
qrt[a + I*a*Tan[c + d*x]]) + (5*a*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d
*x]])/(4*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rule 3678

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Dist[
1/(a*(m + n)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Simp
[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*Ta
```

```
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{a^2\tan^{\frac{5}{2}}(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}} + \frac{1}{2}a\int \frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{9a}{2}+\frac{7}{2}ia\tan\right.}{\sqrt{a+ia\tan(c+dx}}$$

$$= \frac{ia^2\tan^{\frac{3}{2}}(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}} - \frac{a^2\tan^{\frac{5}{2}}(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}} - \frac{\int \sqrt{\tan}}{}$$

$$= \frac{ia^2\tan^{\frac{3}{2}}(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}} - \frac{a^2\tan^{\frac{5}{2}}(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}} + \frac{5a\sqrt{\tan}}{}$$

$$= \frac{ia^2\tan^{\frac{3}{2}}(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}} - \frac{a^2\tan^{\frac{5}{2}}(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}} + \frac{5a\sqrt{\tan}}{}$$

$$= \frac{ia^2\tan^{\frac{3}{2}}(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}} - \frac{a^2\tan^{\frac{5}{2}}(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}} + \frac{5a\sqrt{\tan}}{}$$

$$= -\frac{(2-2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} + \frac{ia^2\tan}{2d\sqrt{a+i}}$$

$$= -\frac{(2-2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} + \frac{ia^2\tan}{2d\sqrt{a+i}}$$

$$= -\frac{11\sqrt[4]{-1}\,a^{3/2}\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{4d} - \frac{(2-2i)a}{}$$

Mathematica [A]
time = 3.34, size = 234, normalized size = 1.08

$$\frac{iae^{-i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(16\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)-11\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)}{4\sqrt{2}d\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}}+\frac{a(5+2i\tan(c+dx))\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((I/4)*a*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E
^((2*I)*(c + d*x)))]*(16*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*
x))]] - 11*Sqrt[2]*ArcTanh[(Sqrt[2]*E^(I*(c + d*x)))/Sqrt[-1 + E^((2*I)*(c
+ d*x))]]))/(Sqrt[2]*d*E^(I*(c + d*x))*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))
```

```
))/(1 + E^((2*I)*(c + d*x)))]) + (a*(5 + (2*I)*Tan[c + d*x])*Sqrt[Tan[c + d
*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(4*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(171) = 342$.
time = 0.17, size = 405, normalized size = 1.87

method	result
derivativedivides	$-\frac{\left(\sqrt{\tan}\,(dx+c)\right)\sqrt{a\,(1+i\tan(dx+c))}\;a\left(4i\sqrt{2}\;\ln\left(-\frac{-2\sqrt{2}\;\sqrt{-ia}\;\sqrt{a\tan(dx+c)\,(1+}}{\tan(dx+c)+}\right.\right.}{}$
default	$-\frac{\left(\sqrt{\tan}\,(dx+c)\right)\sqrt{a\,(1+i\tan(dx+c))}\;a\left(4i\sqrt{2}\;\ln\left(-\frac{-2\sqrt{2}\;\sqrt{-ia}\;\sqrt{a\tan(dx+c)\,(1+}}{\tan(dx+c)+}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)*a*(4*I*2^(1/2)*ln(-(-2*2
^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c
))/(tan(d*x+c)+I))*(I*a)^(1/2)*a-4*I*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*
(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)+16*I*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(
d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*(-I*a)^(1/2)-4
*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*
x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a-10*(a*tan(d*x+c)*(1+I*ta
n(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)-11*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*
tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*(-I*a)^(1/
2))/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2)/(-I*a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*tan(d*x + c)^(3/2), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(161) = 322$.
time = 0.38, size = 600, normalized size = 2.76

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")

[Out] 1/4*(sqrt(2)*(7*a*e^(3*I*d*x + 3*I*c) + 3*a*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) + 2*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-121/16*I*a^3/d^2)*log(1/11*(11*sqrt(2)*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) + 8*sqrt(-121/16*I*a^3/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a) - 2*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-121/16*I*a^3/d^2)*log(1/11*(11*sqrt(2)*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) - 8*sqrt(-121/16*I*a^3/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a) - 2*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-8*I*a^3/d^2)*log(1/2*(2*sqrt(2)*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) + sqrt(-8*I*a^3/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a) + 2*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-8*I*a^3/d^2)*log(1/2*(2*sqrt(2)*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) - sqrt(-8*I*a^3/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a))/(d*e^(2*I*d*x + 2*I*c) + d)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan (c+dx)-i))^{\frac{3}{2}} \tan^{\frac{3}{2}}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)**(3/2)*(a+I*a*tan(d*x+c))**(3/2),x)

[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)*tan(c + d*x)**(3/2), x)

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to rounding error%%%{%%{[%%%{%%{poly1[-16*i,0]:[1,0,-2]%%},[0]%%%},0]:[1,0,%%%{-1,[1]%%%

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c+d\,x)^{3/2}\,(a+a\tan(c+d\,x)\ 1\mathrm{i})^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int(tan(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.195 $\int \sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=176

$$-\frac{3(-1)^{3/4}a^{3/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{(2+2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{ia}{d\sqrt{a}}$$

[Out] -3*(-1)^(3/4)*a^(3/2)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/d-(2+2*I)*a^(3/2)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/d+I*a^2*tan(d*x+c)^(1/2)/d/(a+I*a*tan(d*x+c))^(1/2)-a^2*tan(d*x+c)^(3/2)/d/(a+I*a*tan(d*x+c))^(1/2)

Rubi [A]
time = 0.34, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {3637, 3676, 3682, 3625, 211, 3680, 65, 223, 209}

$$-\frac{3(-1)^{3/4}a^{3/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{(2+2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{a^2\tan^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}+\frac{ia^2\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2),x]

[Out] (-3*(-1)^(3/4)*a^(3/2)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d - ((2 + 2*I)*a^(3/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d + (I*a^2*Sqrt[Tan[c + d*x]])/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (a^2*Tan[c + d*x]^(3/2))/(d*Sqrt[a + I*a*Tan[c + d*x]])

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{a^2\tan^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} + a\int \frac{\sqrt{\tan(c+dx)}\,\left(\frac{5a}{2}+\frac{3}{2}ia\tan\ldots\right.}{\sqrt{a+ia\tan(c+d\ldots}}$$

$$= \frac{ia^2\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}} - \frac{a^2\tan^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{\int \frac{\sqrt{a+\ldots}}{\ldots}}{\ldots}$$

$$= \frac{ia^2\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}} - \frac{a^2\tan^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} + \frac{3}{2}i\int \frac{(a\ldots}{\ldots}$$

$$= \frac{ia^2\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}} - \frac{a^2\tan^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} + \frac{(3ia^2)\,\mathrm{S}\ldots}{\ldots}$$

$$= -\frac{(2+2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} + \frac{ia^2\sqrt{\ldots}}{d\sqrt{a+\ldots}}$$

$$= -\frac{(2+2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} + \frac{ia^2\sqrt{\ldots}}{d\sqrt{a+\ldots}}$$

$$= -\frac{3(-1)^{3/4}a^{3/2}\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{(2+2i\ldots}{\ldots}$$

Mathematica [A]
time = 1.83, size = 203, normalized size = 1.15

$$\frac{iae^{-i(c+dx)}\left(\sqrt{2}\,e^{i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}-2\sqrt{2}\left(1+e^{2i(c+dx)}\right)\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)+3\left(1+e^{2i(c+dx)}\right)\tanh^{-1}\left(\frac{\sqrt{2}\,e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,d\sqrt{-1+e^{2i(c+dx)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (I*a*(Sqrt[2]*E^(I*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))] - 2*Sqrt[2]*(1
 + E^((2*I)*(c + d*x)))*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x
))]] + 3*(1 + E^((2*I)*(c + d*x)))*ArcTanh[(Sqrt[2]*E^(I*(c + d*x)))/Sqrt[-
1 + E^((2*I)*(c + d*x))]])*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(
Sqrt[2]*d*E^(I*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(143) = 286$.
time = 0.17, size = 363, normalized size = 2.06

method	result
derivativedivides	$\frac{\left(\sqrt{\tan}\,(dx+c)\right)\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\,a\left(3i\ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a\tan\left(dx+c\right)\left(1+i\tan\left(dx\right.\right.}}{2\sqrt{ia}}\right.\right.}{}$
default	$\frac{\left(\sqrt{\tan}\,(dx+c)\right)\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\,a\left(3i\ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a\tan\left(dx+c\right)\left(1+i\tan\left(dx\right.\right.}}{2\sqrt{ia}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)*a*(3*I*ln(1/2*(2*I*a*tan(
d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*
(-I*a)^(1/2)*a+2*I*(I*a)^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))
^(1/2)+4*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(
I*a)^(1/2)+a)/(I*a)^(1/2))*a*(-I*a)^(1/2)+I*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^
(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)
+I))*(I*a)^(1/2)*a+(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(
d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a)/(a*ta
n(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2)/(-I*a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*sqrt(tan(d*x + c)), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(134) = 268$.
time = 0.38, size = 525, normalized size = 2.98

$$\frac{2i\sqrt{2}\,a\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\,e^{(i\,dx+i\,c)}-\sqrt{\frac{9i\,a^3}{d^2}}\,d\log\left(\frac{\left(3\sqrt{2}\,(ae^{(2i\,dx+2i\,c)}+a)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}+2i\sqrt{\frac{9i\,a^3}{d^2}}\,de^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{3a}\right)+\sqrt{\frac{9i\,a^3}{d^2}}\,d\log\left(\frac{\left(3\sqrt{2}\,(ae^{(2i\,dx+2i\,c)}+a)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}-2i\sqrt{\frac{9i\,a^3}{d^2}}\,de^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{3a}\right)+\sqrt{\frac{8i\,a^3}{d^2}}\,d\log\left(\frac{\left(2\sqrt{2}\,(ae^{(2i\,dx+2i\,c)}+a)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}+i\sqrt{\frac{8i\,a^3}{d^2}}\,de^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{2a}\right)-\sqrt{\frac{8i\,a^3}{d^2}}\,d\log\left(\frac{\left(2\sqrt{2}\,(ae^{(2i\,dx+2i\,c)}+a)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}-i\sqrt{\frac{8i\,a^3}{d^2}}\,de^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{2a}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*I*sqrt(2)*a*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x +
2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c) - sqrt(9*I*a^3/d^2)*
d*log(1/3*(3*sqrt(2)*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) + 2*I*
sqrt(9*I*a^3/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a) + sqrt(9*I*a^3/d^2
)*d*log(1/3*(3*sqrt(2)*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) - 2*
I*sqrt(9*I*a^3/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a) + sqrt(8*I*a^3/d
^2)*d*log(1/2*(2*sqrt(2)*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) +
I*sqrt(8*I*a^3/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a) - sqrt(8*I*a^3/d
^2)*d*log(1/2*(2*sqrt(2)*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) -
I*sqrt(8*I*a^3/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan{(c+dx)} - i))^{\frac{3}{2}} \sqrt{\tan{(c+dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/2)*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)*sqrt(tan(c + d*x)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%{[%%%{%%{poly1[-4*i,0]:[1,0,-2]%%},[0]%%%},0]:[1,0,%%%{-1,[
1]%%%}
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\tan{(c+d\,x)}}\,\left(a+a\,\tan(c+d\,x)\,1\mathrm{i}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int(tan(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.196 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{\tan(c+dx)}}\,dx$

Optimal. Leaf size=104

$$\frac{2\sqrt[4]{-1}\,a^{3/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{(2-2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

```
[Out] 2*(-1)^(1/4)*a^(3/2)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(
d*x+c))^(1/2))/d+(2-2*I)*a^(3/2)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+
I*a*tan(d*x+c))^(1/2))/d
```

Rubi [A]

time = 0.17, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3636, 3625, 211, 3680, 65, 223, 209}

$$\frac{2\sqrt[4]{-1}\,a^{3/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{(2-2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(3/2)/Sqrt[Tan[c + d*x]],x]
```

```
[Out] (2*(-1)^(1/4)*a^(3/2)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a
 + I*a*Tan[c + d*x]]])/d + ((2 - 2*I)*a^(3/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt
[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3636

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(c_.) + (d_.)*tan[(e_
.) + (f_.)*(x_)]], x_Symbol] :> Dist[2*a, Int[Sqrt[a + b*Tan[e + f*x]]/Sqrt
[c + d*Tan[e + f*x]], x], x] + Dist[b/a, Int[(b + a*Tan[e + f*x])*(Sqrt[a +
 b*Tan[e + f*x]]/Sqrt[c + d*Tan[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{\tan(c+dx)}}\,dx = i\int \frac{\sqrt{a+ia\tan(c+dx)}\,(ia+a\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx + (2a)\int \frac{\sqrt{a+ia\tan(c}{\sqrt{\tan(c+d}}$$

$$= -\frac{a^2\text{Subst}\left(\int \frac{1}{\sqrt{x}\,\sqrt{a+iax}}\,dx, x, \tan(c+dx)\right)}{d} - \frac{(4ia^3)\,\text{Subst}\left(\int \frac{1}{-ia-2a^2}\right.}{}$$

$$= \frac{(2-2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+iax}}\right.}{}$$

$$= \frac{(2-2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{1-iax^2}\,dx,\right.}{}$$

$$= \frac{2\sqrt[4]{-1}\,a^{3/2}\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} + \frac{(2-2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i}{\sqrt{}}\right.}{d}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 255 vs. $2(104) = 208$.
time = 2.04, size = 255, normalized size = 2.45

$$-\frac{iae^{-i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(8\log\left(e^{i(c+dx)}+\sqrt{-1+e^{2i(c+dx)}}\right)+\sqrt{2}\left(-\log\left(1-3e^{2i(c+dx)}-2\sqrt{2}\,e^{i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\right)+\log\left(1-3e^{2i(c+dx)}+2\sqrt{2}\,e^{i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\right)\right)\right)}{2\sqrt{2}\,d\sqrt{-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(3/2)/Sqrt[Tan[c + d*x]],x]
```

```
[Out] ((-1/2*I)*a*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[(a*E^((2*I)*(c + d*x)))/(1
+ E^((2*I)*(c + d*x)))]*(8*Log[E^(I*(c + d*x)) + Sqrt[-1 + E^((2*I)*(c + d*
x))]] + Sqrt[2]*(-Log[1 - 3*E^((2*I)*(c + d*x)) - 2*Sqrt[2]*E^(I*(c + d*x))
*Sqrt[-1 + E^((2*I)*(c + d*x))]] + Log[1 - 3*E^((2*I)*(c + d*x)) + 2*Sqrt[2
]*E^(I*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]])))/(Sqrt[2]*d*E^(I*(c + d
*x))*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(82) = 164$.
time = 0.18, size = 326, normalized size = 3.13

method	result

derivativedivides	$\frac{\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\,\left(\sqrt{\tan}\left(dx+c\right)\right)a^{2}\left(i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1+i\right.}}{\tan\left(dx+c\right)+i}\right.\right.}{}$
default	$\frac{\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\,\left(\sqrt{\tan}\left(dx+c\right)\right)a^{2}\left(i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1+i\right.}}{\tan\left(dx+c\right)+i}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(a*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^(1/2)*a^2*(I*2^(1/2)*ln(-(-2*2^
(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c)
)/(tan(d*x+c)+I))*(I*a)^(1/2)+4*I*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*
(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)-2^(1/2)*ln
(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*ta
n(d*x+c))/(tan(d*x+c)+I))*(I*a)^(1/2)-2*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+
c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2)))/(a*
tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2)/(-I*a)^(1/2)
```

<u>Maxima</u> [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)/sqrt(tan(d*x + c)), x)
```

<u>Fricas</u> [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(76) = 152$.
time = 0.38, size = 451, normalized size = 4.34

$$\frac{1}{2}\sqrt{-\frac{8i\,a^{3}}{d^{2}}}\log\left(\frac{\left(2\sqrt{2}\left(ae^{(2i\,dx+2i\,c)}+a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}+\sqrt{-\frac{8i\,a^{3}}{d^{2}}}\,de^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{2\,a}\right)-\frac{1}{2}\sqrt{-\frac{8i\,a^{3}}{d^{2}}}\log\left(\frac{\left(2\sqrt{2}\left(ae^{(2i\,dx+2i\,c)}+a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}-\sqrt{-\frac{8i\,a^{3}}{d^{2}}}\,de^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{2\,a}\right)-\frac{1}{2}\sqrt{-\frac{4i\,a^{3}}{d^{2}}}\log\left(\frac{\left(\sqrt{2}\left(ae^{(2i\,dx+2i\,c)}+a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}+\sqrt{-\frac{4i\,a^{3}}{d^{2}}}\,de^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{a}\right)+\frac{1}{2}\sqrt{-\frac{4i\,a^{3}}{d^{2}}}\log\left(\frac{\left(\sqrt{2}\left(ae^{(2i\,dx+2i\,c)}+a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}-\sqrt{-\frac{4i\,a^{3}}{d^{2}}}\,de^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-8*I*a^3/d^2)*log(1/2*(2*sqrt(2)*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(
a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x
+ 2*I*c) + 1)) + sqrt(-8*I*a^3/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a)
- 1/2*sqrt(-8*I*a^3/d^2)*log(1/2*(2*sqrt(2)*(a*e^(2*I*d*x + 2*I*c) + a)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*
```

```
x + 2*I*c) + 1)) - sqrt(-8*I*a^3/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a
) - 1/2*sqrt(-4*I*a^3/d^2)*log((sqrt(2)*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/
(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
2*I*c) + 1)) + sqrt(-4*I*a^3/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a) +
1/2*sqrt(-4*I*a^3/d^2)*log((sqrt(2)*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(
2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*
c) + 1)) - sqrt(-4*I*a^3/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}{\sqrt{\tan(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(3/2)/tan(d*x+c)**(1/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)/sqrt(tan(c + d*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\tan(c+d\,x)\,1\mathrm{i})^{3/2}}{\sqrt{\tan(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(3/2)/tan(c + d*x)^(1/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^(3/2)/tan(c + d*x)^(1/2), x)
```

3.197 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=83

$$\frac{(2+2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

```
[Out] (2+2*I)*a^(3/2)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(
1/2))/d-2*a*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3626, 3625, 211}

$$\frac{(2+2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(3/2)/Tan[c + d*x]^(3/2),x]
```

```
[Out] ((2 + 2*I)*a^(3/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*
a*Tan[c + d*x]]])/d - (2*a*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]
])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3626

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*b*(a + b*Tan[e + f*x])^(m - 1)*((c +
 d*Tan[e + f*x])^(n + 1)/(f*(m - 1)*(a*c - b*d))), x] + Dist[2*(a^2/(a*c -
```

```
b*d)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\tan^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2a\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}} + (2ia)\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{\tan(c+dx)}}\,dx$$

$$= -\frac{2a\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}} + \frac{(4a^3)\,\text{Subst}\left(\int \frac{1}{-ia-2a^2x^2}\,dx, x, \frac{\sqrt{\tan(c+}}{\sqrt{a+ia\tan(c}}\right.}{d}$$

$$= \frac{(2+2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{2a\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

Mathematica [A]
time = 1.75, size = 160, normalized size = 1.93

$$-\frac{2i\sqrt{2}\,a^2e^{i(c+dx)}\left(e^{i(c+dx)}\sqrt{-1+e^{2i(c+dx)}} - \left(-1+e^{2i(c+dx)}\right)\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\tan(c+dx)}}{d\left(-1+e^{2i(c+dx)}\right)^{3/2}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(3/2)/Tan[c + d*x]^(3/2),x]
```

```
[Out] ((-2*I)*Sqrt[2]*a^2*E^(I*(c + d*x))*(E^(I*(c + d*x))*Sqrt[-1 + E^((2*I)*(c
+ d*x))] - (-1 + E^((2*I)*(c + d*x)))*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^(
(2*I)*(c + d*x))]])*Sqrt[Tan[c + d*x]])/(d*(-1 + E^((2*I)*(c + d*x)))^(3/2)
*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 319 vs. 2(69) = 138.
time = 0.19, size = 320, normalized size = 3.86

method	result
derivativedivides	$-\frac{\sqrt{a\,(1+i\tan(dx+c))}\,a\left(i\sqrt{ia}\,\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)\,(1+i\tan(d}}{\tan(dx+c)+i}\right.\right.}{}$

default	$-\frac{\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\, a\left(i\sqrt{ia}\,\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1+i\tan\left(dx\right.\right.}}{\tan(dx+c)+i}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(a*(1+I*tan(d*x+c)))^(1/2)*a*(I*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*
(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan
(d*x+c)+I))*a*tan(d*x+c)+ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*ta
n(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*(I*a)^(1/2)*2^(1/2)*a*
tan(d*x+c)+4*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/
2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)+4*(a*tan(d*x+c)*(1
+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2))/tan(d*x+c)^(1/2)/(a*tan(d*x
+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2)/(-I*a)^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 555 vs. 2(63) = 126.
time = 0.63, size = 555, normalized size = 6.69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -((-(2*I - 2)*a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*
d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) +
 1)) - cos(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)
) - sin(d*x + c)) + (I + 1)*a*log(cos(d*x + c)^2 + sin(d*x + c)^2 + sqrt(co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)*(cos(1/2*ar
ctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1))^2 + sin(1/2*arctan2(sin(2*d
*x + 2*c), -cos(2*d*x + 2*c) + 1))^2) - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c),
-cos(2*d*x + 2*c) + 1))*sin(d*x + c) + cos(d*x + c)*sin(1/2*arctan2(sin(2*d
*x + 2*c), -cos(2*d*x + 2*c) + 1)))))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) - 2*(((I - 1)*a*cos(d*x + c) -
(I + 1)*a*sin(d*x + c))*cos(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c)
 + 1)) + (-(I + 1)*a*cos(d*x + c) - (I - 1)*a*sin(d*x + c))*sin(1/2*arctan2
(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)))*sqrt(a))/((cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 353 vs. 2(63) = 126.

time = 0.36, size = 353, normalized size = 4.25

$$-\frac{4\sqrt{2}\left(iae^{(3i\,dx+3i\,c)}+iae^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}+\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{\frac{8i\,a^3}{d^2}}\log\left(\frac{\left(2\sqrt{2}\left(ae^{(2i\,dx+2i\,c)}+a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}+i\sqrt{\frac{8i\,a^3}{d^2}}de^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{2\,a}\right)-\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{\frac{8i\,a^3}{d^2}}\log\left(\frac{\left(2\sqrt{2}\left(ae^{(2i\,dx+2i\,c)}+a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}-i\sqrt{\frac{8i\,a^3}{d^2}}de^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{2\,a}\right)}{2\left(de^{(2i\,dx+2i\,c)}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(4*sqrt(2)*(I*a*e^(3*I*d*x + 3*I*c) + I*a*e^(I*d*x + I*c))*sqrt(a/(e^(
2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*
c) + 1)) + (d*e^(2*I*d*x + 2*I*c) - d)*sqrt(8*I*a^3/d^2)*log(1/2*(2*sqrt(2)
*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(
2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) + I*sqrt(8*I*a^3/d^2)*d*e^
(I*d*x + I*c))*e^(-I*d*x - I*c)/a) - (d*e^(2*I*d*x + 2*I*c) - d)*sqrt(8*I*a
^3/d^2)*log(1/2*(2*sqrt(2)*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))
- I*sqrt(8*I*a^3/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a))/(d*e^(2*I*d*x
 + 2*I*c) - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan{(c+dx)}-i))^{\frac{3}{2}}}{\tan^{\frac{3}{2}}{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(3/2)/tan(d*x+c)**(3/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)/tan(c + d*x)**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}{{\mathrm{tan}\left(c+d\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(3/2)/tan(c + d*x)^(3/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^(3/2)/tan(c + d*x)^(3/2), x)
```

3.198 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{\tan^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=119

$$-\frac{(2-2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{2(a+ia\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

```
[Out] (-2+2*I)*a^(3/2)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^
(1/2))/d-2*I*a*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(1/2)-2/3*(a+I*a*tan(d
*x+c))^(3/2)/d/tan(d*x+c)^(3/2)
```

Rubi [A]
time = 0.14, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3629, 3626, 3625, 211}

$$-\frac{(2-2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2(a+ia\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(3/2)/Tan[c + d*x]^(5/2),x]
```

```
[Out] ((-2 + 2*I)*a^(3/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I
*a*Tan[c + d*x]]])/d - ((2*I)*a*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[Tan[c +
 d*x]]) - (2*(a + I*a*Tan[c + d*x])^(3/2))/(3*d*Tan[c + d*x]^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3626

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*b*(a + b*Tan[e + f*x])^(m - 1)*((c +
```

```
 d*Tan[e + f*x])^(n + 1)/(f*(m - 1)*(a*c - b*d))), x] + Dist[2*(a^2/(a*c -
b*d)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2]
```

Rule 3629

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-d)*(a + b*Tan[e + f*x])^m*((c + d*Ta
n[e + f*x])^(n + 1)/(f*m*(c^2 + d^2))), x] + Dist[a/(a*c - b*d), Int[(a + b
*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\tan^{\frac{5}{2}}(c+dx)}\,dx = -\frac{2(a+ia\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)} + i\int \frac{(a+ia\tan(c+dx))^{3/2}}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

$$= -\frac{2ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}} - \frac{2(a+ia\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)} - (2a)\int \frac{\sqrt{a+ia\tan}}{\sqrt{\tan(c}}$$

$$= -\frac{2ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}} - \frac{2(a+ia\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)} + \frac{(4ia^3)\,\mathrm{Subst}\left(\int \frac{}{-ia}\right.}{}$$

$$= -\frac{(2-2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{2ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

Mathematica [A]
time = 1.87, size = 160, normalized size = 1.34

$$-\frac{2i\sqrt{2}\,ae^{-i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(e^{i(c+dx)}\left(-3+5e^{2i(c+dx)}\right)-3\left(-1+e^{2i(c+dx)}\right)^{3/2}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)}{3d\left(-1+e^{2i(c+dx)}\right)\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(3/2)/Tan[c + d*x]^(5/2),x]
```

```
[Out] (((-2*I)/3)*Sqrt[2]*a*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))
)]*(E^(I*(c + d*x))*(-3 + 5*E^((2*I)*(c + d*x))) - 3*(-1 + E^((2*I)*(c + d*
x)))^(3/2)*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]]))/(d*E^(
I*(c + d*x))*(-1 + E^((2*I)*(c + d*x)))*Sqrt[Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(97) = 194$.
time = 0.17, size = 370, normalized size = 3.11

method	result
derivativedivides	$-\frac{\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\, a\left(3i\sqrt{ia}\,\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1+i\tan\left(d\right.\right.}}{\tan(dx+c)+i}\right.\right.}{}$
default	$-\frac{\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\, a\left(3i\sqrt{ia}\,\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1+i\tan\left(d\right.\right.}}{\tan(dx+c)+i}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/d*(a*(1+I*tan(d*x+c)))^(1/2)*a/tan(d*x+c)^(3/2)*(3*I*(I*a)^(1/2)*2^(1/
2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3
*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2+12*I*ln(1/2*(2*I*a*tan(d*x+c)
+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)
^(1/2)*a*tan(d*x+c)^2-3*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a
*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*t
an(d*x+c)^2+16*I*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/
2)*(-I*a)^(1/2)+4*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^
(1/2))/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2)/(-I*a)^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1008 vs. $2(89) = 178$.
time = 0.73, size = 1008, normalized size = 8.47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) +
1)*(((3*I + 3)*a*cos(3*d*x + 3*c) - (I + 1)*a*cos(d*x + c) + (3*I - 3)*a*si
n(3*d*x + 3*c) - (I - 1)*a*sin(d*x + c))*cos(3/2*arctan2(sin(2*d*x + 2*c),
-cos(2*d*x + 2*c) + 1)) + ((3*I - 3)*a*cos(3*d*x + 3*c) - (I - 1)*a*cos(d*x
 + c) - (3*I + 3)*a*sin(3*d*x + 3*c) + (I + 1)*a*sin(d*x + c))*sin(3/2*arct
an2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3*(2*(-(I + 1)*a*c
os(2*d*x + 2*c)^2 - (I + 1)*a*sin(2*d*x + 2*c)^2 + (2*I + 2)*a*cos(2*d*x +
2*c) - (I + 1)*a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(
2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c)
 + 1)) - cos(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d
```

```
*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) +
1)) - sin(d*x + c)) + (-(I - 1)*a*cos(2*d*x + 2*c)^2 - (I - 1)*a*sin(2*d*x
+ 2*c)^2 + (2*I - 2)*a*cos(2*d*x + 2*c) - (I - 1)*a)*log(cos(d*x + c)^2 + s
in(d*x + c)^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x
+ 2*c) + 1)*(cos(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1))^2 +
sin(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1))^2) - 2*(cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arc
tan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1))*sin(d*x + c) + cos(d*x + c)*
sin(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)))))*(cos(2*d*x + 2
*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 4*(((-
(I + 1)*a*cos(d*x + c) - (I - 1)*a*sin(d*x + c))*cos(2*d*x + 2*c)^2 + (-(I
+ 1)*a*cos(d*x + c) - (I - 1)*a*sin(d*x + c))*sin(2*d*x + 2*c)^2 + 2*((I +
1)*a*cos(d*x + c) + (I - 1)*a*sin(d*x + c))*cos(2*d*x + 2*c) - (I + 1)*a*co
s(d*x + c) - (I - 1)*a*sin(d*x + c))*cos(1/2*arctan2(sin(2*d*x + 2*c), -cos
(2*d*x + 2*c) + 1)) + ((-(I - 1)*a*cos(d*x + c) + (I + 1)*a*sin(d*x + c))*c
os(2*d*x + 2*c)^2 + (-(I - 1)*a*cos(d*x + c) + (I + 1)*a*sin(d*x + c))*sin(
2*d*x + 2*c)^2 + 2*((I - 1)*a*cos(d*x + c) - (I + 1)*a*sin(d*x + c))*cos(2*
d*x + 2*c) - (I - 1)*a*cos(d*x + c) + (I + 1)*a*sin(d*x + c))*sin(1/2*arcta
n2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)))*sqrt(a))/((cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(5/4)*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(89) = 178$.
time = 0.36, size = 395, normalized size = 3.32

$$\frac{4\sqrt{2}\left(5\,ae^{(5i\,dx+5i\,c)}+2\,ae^{(3i\,dx+3i\,c)}-3\,ae^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}-3\left(de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}+d\right)\sqrt{-\frac{8i\,a^3}{d^2}}\log\left(\frac{\left(2\sqrt{2}\left(ae^{(2i\,dx+2i\,c)}+a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}+\sqrt{-\frac{8i\,a^3}{d^2}}\,de^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{2\,a}\right)+3\left(de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}+d\right)\sqrt{-\frac{8i\,a^3}{d^2}}\log\left(\frac{\left(2\sqrt{2}\left(ae^{(2i\,dx+2i\,c)}+a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}-\sqrt{-\frac{8i\,a^3}{d^2}}\,de^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{2\,a}\right)}{6\left(de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(2)*(5*a*e^(5*I*d*x + 5*I*c) + 2*a*e^(3*I*d*x + 3*I*c) - 3*a*e^(
I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c
) + I)/(e^(2*I*d*x + 2*I*c) + 1)) - 3*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d
*x + 2*I*c) + d)*sqrt(-8*I*a^3/d^2)*log(1/2*(2*sqrt(2)*(a*e^(2*I*d*x + 2*I*
c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)
/(e^(2*I*d*x + 2*I*c) + 1)) + sqrt(-8*I*a^3/d^2)*d*e^(I*d*x + I*c))*e^(-I*d
*x - I*c)/a) + 3*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*sqrt
(-8*I*a^3/d^2)*log(1/2*(2*sqrt(2)*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*
I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c)
 + 1)) - sqrt(-8*I*a^3/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a))/(d*e^(4
*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan{(c+dx)}-i))^{\frac{3}{2}}}{\tan^{\frac{5}{2}}{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(3/2)/tan(d*x+c)**(5/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)/tan(c + d*x)**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}{{\mathrm{tan}\left(c+d\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(3/2)/tan(c + d*x)^(5/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^(3/2)/tan(c + d*x)^(5/2), x)
```

3.199 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{\tan^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=198

$$-\frac{(2+2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^2}{5d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}-\frac{2}{5d\tan^{\frac{3}{2}}(c+dx)\sqrt{}}$$

```
[Out] (-2-2*I)*a^(3/2)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^
(1/2))/d+12/5*a*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(1/2)-2/5*a^2/d/(a+I*
a*tan(d*x+c))^(1/2)/tan(d*x+c)^(5/2)-2/5*I*a^2/d/(a+I*a*tan(d*x+c))^(1/2)/t
an(d*x+c)^(3/2)-4/5*I*a*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(3/2)
```

Rubi [A]
time = 0.36, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3634, 3677, 3679, 12, 3625, 211}

$$-\frac{(2+2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2ia^2}{5d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}-\frac{2a^2}{5d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{3}{2}}(c+dx)}+\frac{12a\sqrt{a+ia\tan(c+dx)}}{5d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(3/2)/Tan[c + d*x]^(7/2),x]
```

```
[Out] ((-2 - 2*I)*a^(3/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I
*a*Tan[c + d*x]]])/d - (2*a^2)/(5*d*Tan[c + d*x]^(5/2)*Sqrt[a + I*a*Tan[c +
 d*x]]) - (((2*I)/5)*a^2)/(d*Tan[c + d*x]^(3/2)*Sqrt[a + I*a*Tan[c + d*x]])
 - (((4*I)/5)*a*Sqrt[a + I*a*Tan[c + d*x]])/(d*Tan[c + d*x]^(3/2)) + (12*a*
Sqrt[a + I*a*Tan[c + d*x]])/(5*d*Sqrt[Tan[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
```

```
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3634

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-a^2)*(b*c - a*d)*(a + b*Tan[e + f*x]
)^(m - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] + Di
st[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[b*(b*c*(m - 2) - a*d*(m - 2*n - 4)) + (a*b*c*(m - 2)
+ b^2*d*(n + 1) - a^2*d*(m + n - 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\tan^{\frac{7}{2}}(c+dx)}\,dx = -\frac{2a^2}{5d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} - \frac{2}{5}\int \frac{-\frac{9ia^2}{2}+\frac{11}{2}a^2\tan(c+dx}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+}}$$

$$= -\frac{2a^2}{5d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} - \frac{2ia^2}{5d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+}}$$

$$= -\frac{2a^2}{5d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} - \frac{2ia^2}{5d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+}}$$

$$= -\frac{2a^2}{5d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} - \frac{2ia^2}{5d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+}}$$

$$= -\frac{2a^2}{5d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} - \frac{2ia^2}{5d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+}}$$

$$= -\frac{2a^2}{5d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} - \frac{2ia^2}{5d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+}}$$

$$= -\frac{(2+2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{2a^2}{5d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia}}$$

Mathematica [**A**]
time = 2.79, size = 166, normalized size = 0.84

$$\frac{2iae^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\left(e^{i(c+dx)}\left(5-10e^{2i(c+dx)}+9e^{4i(c+dx)}\right)-5\left(-1+e^{2i(c+dx)}\right)^{5/2}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{5d\left(-1+e^{2i(c+dx)}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(3/2)/Tan[c + d*x]^(7/2),x]
```

```
[Out] (((2*I)/5)*a*(1 + E^((2*I)*(c + d*x)))*(E^(I*(c + d*x))*(5 - 10*E^((2*I)*(c
 + d*x)) + 9*E^((4*I)*(c + d*x))) - 5*(-1 + E^((2*I)*(c + d*x)))^(5/2)*ArcT
anh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]])*Sqrt[Tan[c + d*x]]*Sqr
t[a + I*a*Tan[c + d*x]])/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*(c + d*x)))^3)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 411 vs. 2(159) = 318.
time = 0.17, size = 412, normalized size = 2.08

method	result
derivativedivides	$\frac{\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\,a\left(5i\sqrt{ia}\,\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1+i\tan\left(dx\right.\right.}}{\tan(dx+c)+i}\right.\right.}{}$
default	$\frac{\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\,a\left(5i\sqrt{ia}\,\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1+i\tan\left(dx\right.\right.}}{\tan(dx+c)+i}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10/d*(a*(1+I*tan(d*x+c)))^(1/2)*a*(5*I*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2
)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(t
an(d*x+c)+I))*a*tan(d*x+c)^3+5*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(
1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+
I))*a*tan(d*x+c)^3+20*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x
+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^3-8*I*tan
(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)+24*(
a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^2-
4*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2))/tan(d*x+c
)^(5/2)/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2)/(-I*a)^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1198 vs. 2(148) = 296.
time = 0.78, size = 1198, normalized size = 6.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*(2*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) +
 1)*(((15*I - 15)*a*cos(3*d*x + 3*c) - (16*I - 16)*a*cos(d*x + c) - (15*I +
 15)*a*sin(3*d*x + 3*c) + (16*I + 16)*a*sin(d*x + c))*cos(3/2*arctan2(sin(2
*d*x + 2*c), -cos(2*d*x + 2*c) + 1)) + (-(15*I + 15)*a*cos(3*d*x + 3*c) + (
16*I + 16)*a*cos(d*x + c) - (15*I - 15)*a*sin(3*d*x + 3*c) + (16*I - 16)*a*
sin(d*x + c))*sin(3/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)))*sq
rt(a) + 15*(2*(-(I - 1)*a*cos(2*d*x + 2*c)^2 - (I - 1)*a*sin(2*d*x + 2*c)^2
 + (2*I - 2)*a*cos(2*d*x + 2*c) - (I - 1)*a)*arctan2((cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*
x + 2*c), -cos(2*d*x + 2*c) + 1)) - cos(d*x + c), (cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x +
 2*c), -cos(2*d*x + 2*c) + 1)) - sin(d*x + c)) + ((I + 1)*a*cos(2*d*x + 2*c
```

)^2 + (I + 1)*a*sin(2*d*x + 2*c)^2 - (2*I + 2)*a*cos(2*d*x + 2*c) + (I + 1)
*a)*log(cos(d*x + c)^2 + sin(d*x + c)^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)*(cos(1/2*arctan2(sin(2*d*x + 2*c), -c
os(2*d*x + 2*c) + 1))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*
c) + 1))^2) - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*
c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1))*si
n(d*x + c) + cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*
c) + 1)))))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) +
1)^(1/4)*sqrt(a) + 2*((-(15*I - 15)*a*cos(5*d*x + 5*c) + (5*I - 5)*a*cos(3
*d*x + 3*c) - (2*I - 2)*a*cos(d*x + c) + (15*I + 15)*a*sin(5*d*x + 5*c) - (
5*I + 5)*a*sin(3*d*x + 3*c) + (2*I + 2)*a*sin(d*x + c))*cos(5/2*arctan2(sin
(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)) + 3*(((I - 1)*a*cos(d*x + c) - (I +
1)*a*sin(d*x + c))*cos(2*d*x + 2*c)^2 + ((I - 1)*a*cos(d*x + c) - (I + 1)*a
*sin(d*x + c))*sin(2*d*x + 2*c)^2 + 2*(-(I - 1)*a*cos(d*x + c) + (I + 1)*a*
sin(d*x + c))*cos(2*d*x + 2*c) + (I - 1)*a*cos(d*x + c) - (I + 1)*a*sin(d*x
+ c))*cos(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)) + ((15*I +
15)*a*cos(5*d*x + 5*c) - (5*I + 5)*a*cos(3*d*x + 3*c) + (2*I + 2)*a*cos(d*
x + c) + (15*I - 15)*a*sin(5*d*x + 5*c) - (5*I - 5)*a*sin(3*d*x + 3*c) + (2
*I - 2)*a*sin(d*x + c))*sin(5/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c)
+ 1)) + 3*((-(I + 1)*a*cos(d*x + c) - (I - 1)*a*sin(d*x + c))*cos(2*d*x +
2*c)^2 + (-(I + 1)*a*cos(d*x + c) - (I - 1)*a*sin(d*x + c))*sin(2*d*x + 2*c
)^2 + 2*((I + 1)*a*cos(d*x + c) + (I - 1)*a*sin(d*x + c))*cos(2*d*x + 2*c)
- (I + 1)*a*cos(d*x + c) - (I - 1)*a*sin(d*x + c))*sin(1/2*arctan2(sin(2*d*
x + 2*c), -cos(2*d*x + 2*c) + 1)))*sqrt(a))/((cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(5/4)*d)

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(148) = 296$.
time = 0.39, size = 450, normalized size = 2.27

$$-\frac{4\sqrt{2}\left(-9i\,ae^{(7i\,dx+7i\,c)}+i\,ae^{(5i\,dx+5i\,c)}+5i\,ae^{(3i\,dx+3i\,c)}-5i\,ae^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}-5\left(de^{(6i\,dx+6i\,c)}-3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}-d\right)\sqrt{\frac{8i\,a^3}{d^2}}\log\left(\frac{\left(2\sqrt{2}\left(ae^{(2i\,dx+2i\,c)}+a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}+i\sqrt{\frac{8i\,a^3}{d^2}}\,de^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{2\,a}\right)+5\left(de^{(6i\,dx+6i\,c)}-3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}-d\right)\sqrt{\frac{8i\,a^3}{d^2}}\log\left(\frac{\left(2\sqrt{2}\left(ae^{(2i\,dx+2i\,c)}+a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}-i\sqrt{\frac{8i\,a^3}{d^2}}\,de^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{2\,a}\right)}{10\left(de^{(6i\,dx+6i\,c)}-3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(7/2),x, algorithm="fricas")

[Out] -1/10*(4*sqrt(2)*(-9*I*a*e^(7*I*d*x + 7*I*c) + I*a*e^(5*I*d*x + 5*I*c) + 5*
I*a*e^(3*I*d*x + 3*I*c) - 5*I*a*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) - 5*(d
*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) -
d)*sqrt(8*I*a^3/d^2)*log(1/2*(2*sqrt(2)*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/
(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
2*I*c) + 1)) + I*sqrt(8*I*a^3/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a) +
5*(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*
c) - d)*sqrt(8*I*a^3/d^2)*log(1/2*(2*sqrt(2)*(a*e^(2*I*d*x + 2*I*c) + a)*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d

```
*x + 2*I*c) + 1)) - I*sqrt(8*I*a^3/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)
/a))/(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*
I*c) - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan{(c+dx)}-i))^{\frac{3}{2}}}{\tan^{\frac{7}{2}}{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(3/2)/tan(d*x+c)**(7/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)/tan(c + d*x)**(7/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}{{\tan\left(c+d\,x\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(3/2)/tan(c + d*x)^(7/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^(3/2)/tan(c + d*x)^(7/2), x)
```

3.200 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{\tan^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=235

$$\frac{(2-2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^2}{7d\tan^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}-\frac{2ia}{7d\tan^{\frac{5}{2}}(c+dx)\sqrt{a}}$$

```
[Out] (2-2*I)*a^(3/2)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(
1/2))/d+268/105*I*a*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(1/2)-2/7*a^2/d/(
a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(7/2)-2/7*I*a^2/d/(a+I*a*tan(d*x+c))^(1/
2)/tan(d*x+c)^(5/2)-16/35*I*a*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(5/2)+7
6/105*a*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(3/2)
```

Rubi [A]
time = 0.45, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3634, 3677, 3679, 12, 3625, 211}

$$\frac{(2-2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2ia^2}{7d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}-\frac{2a^2}{7d\tan^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}+\frac{76a\sqrt{a+ia\tan(c+dx)}}{105d\tan^{\frac{3}{2}}(c+dx)}-\frac{16ia\sqrt{a+ia\tan(c+dx)}}{35d\tan^{\frac{5}{2}}(c+dx)}+\frac{268ia\sqrt{a+ia\tan(c+dx)}}{105d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(3/2)/Tan[c + d*x]^(9/2),x]
```

```
[Out] ((2 - 2*I)*a^(3/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*
a*Tan[c + d*x]]])/d - (2*a^2)/(7*d*Tan[c + d*x]^(7/2)*Sqrt[a + I*a*Tan[c +
d*x]]) - (((2*I)/7)*a^2)/(d*Tan[c + d*x]^(5/2)*Sqrt[a + I*a*Tan[c + d*x]])
- (((16*I)/35)*a*Sqrt[a + I*a*Tan[c + d*x]])/(d*Tan[c + d*x]^(5/2)) + (76*a
*Sqrt[a + I*a*Tan[c + d*x]])/(105*d*Tan[c + d*x]^(3/2)) + (((268*I)/105)*a*
Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
```

```
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3634

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-a^2)*(b*c - a*d)*(a + b*Tan[e + f*x]
)^(m - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] + Di
st[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[b*(b*c*(m - 2) - a*d*(m - 2*n - 4)) + (a*b*c*(m - 2)
+ b^2*d*(n + 1) - a^2*d*(m + n - 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\tan^{\frac{9}{2}}(c+dx)}\,dx = -\frac{2a^2}{7d\tan^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} - \frac{2}{7}\int \frac{-\frac{13ia^2}{2}+\frac{15}{2}a^2\tan(c+dx}{\tan^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c}}$$

$$= -\frac{2a^2}{7d\tan^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} - \frac{2ia^2}{7d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+}}$$

$$= -\frac{2a^2}{7d\tan^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} - \frac{2ia^2}{7d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+}}$$

$$= -\frac{2a^2}{7d\tan^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} - \frac{2ia^2}{7d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+}}$$

$$= -\frac{2a^2}{7d\tan^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} - \frac{2ia^2}{7d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+}}$$

$$= -\frac{2a^2}{7d\tan^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} - \frac{2ia^2}{7d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+}}$$

$$= -\frac{2a^2}{7d\tan^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} - \frac{2ia^2}{7d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+}}$$

$$= \frac{(2-2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{2a^2}{7d\tan^{\frac{7}{2}}(c+dx)\sqrt{a+ia\,\mathrm{t}}}$$

Mathematica [A]
time = 3.26, size = 224, normalized size = 0.95

$$-\frac{2i\sqrt{2}\,ae^{-i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)}{d\sqrt{-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}} - \frac{a\csc^3(c+dx)(7\cos(c+dx)+53\cos(3(c+dx))-378i\sin(c+dx)+158i\sin(3(c+dx)))\sqrt{a+ia\tan(c+dx)}}{210d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(3/2)/Tan[c + d*x]^(9/2),x]
```

```
[Out] ((-2*I)*Sqrt[2]*a*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[(a*E^((2*I)*(c + d*x)
))/(1 + E^((2*I)*(c + d*x)))]*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c
 + d*x))]])/(d*E^(I*(c + d*x))*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 +
E^((2*I)*(c + d*x)))]) - (a*Csc[c + d*x]^3*(7*Cos[c + d*x] + 53*Cos[3*(c +
```

```
d*x)] - (378*I)*Sin[c + d*x] + (158*I)*Sin[3*(c + d*x)])*Sqrt[a + I*a*Tan[c
 + d*x]])/(210*d*Sqrt[Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(188) = 376$.
time = 0.19, size = 457, normalized size = 1.94

method	result
derivativedivides	$\frac{\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\, a\left(105i\sqrt{ia}\,\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1+i\tan\left(d\right.\right.}}{\tan(dx+c)+i}\right.\right.}{}$
default	$\frac{\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\, a\left(105i\sqrt{ia}\,\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1+i\tan\left(d\right.\right.}}{\tan(dx+c)+i}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/210/d*(a*(1+I*tan(d*x+c)))^(1/2)*a/tan(d*x+c)^(7/2)*(105*I*(I*a)^(1/2)*2^
(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*
a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4+420*I*ln(1/2*(2*I*a*tan(d*
x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-
I*a)^(1/2)*a*tan(d*x+c)^4-105*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1
/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I
))*a*tan(d*x+c)^4+152*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I
*a)^(1/2)*tan(d*x+c)^2+536*I*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^3*(a*tan(d
*x+c)*(1+I*tan(d*x+c)))^(1/2)-96*I*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)
))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)-60*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*
(I*a)^(1/2)*(-I*a)^(1/2))/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2)
/(-I*a)^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2883 vs. $2(175) = 350$.
time = 0.99, size = 2883, normalized size = 12.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/420*(3*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c)
 + 1)*(((280*I + 280)*a*cos(7*d*x + 7*c) - (140*I + 140)*a*cos(5*d*x + 5*c)
 + (133*I + 133)*a*cos(3*d*x + 3*c) + (47*I + 47)*a*cos(d*x + c) + (280*I -
 280)*a*sin(7*d*x + 7*c) - (140*I - 140)*a*sin(5*d*x + 5*c) + (133*I - 133)
```

```
*a*sin(3*d*x + 3*c) + (47*I - 47)*a*sin(d*x + c))*cos(7/2*arctan2(sin(2*d*x
 + 2*c), -cos(2*d*x + 2*c) + 1)) + 4*(47*(-(I + 1)*a*cos(d*x + c) - (I - 1)
*a*sin(d*x + c))*cos(2*d*x + 2*c)^2 + 47*(-(I + 1)*a*cos(d*x + c) - (I - 1)
*a*sin(d*x + c))*sin(2*d*x + 2*c)^2 + 70*((I + 1)*a*cos(2*d*x + 2*c)^2 + (I
 + 1)*a*sin(2*d*x + 2*c)^2 - (2*I + 2)*a*cos(2*d*x + 2*c) + (I + 1)*a)*cos(
3*d*x + 3*c) + 94*((I + 1)*a*cos(d*x + c) + (I - 1)*a*sin(d*x + c))*cos(2*d
*x + 2*c) - (47*I + 47)*a*cos(d*x + c) + 70*((I - 1)*a*cos(2*d*x + 2*c)^2 +
 (I - 1)*a*sin(2*d*x + 2*c)^2 - (2*I - 2)*a*cos(2*d*x + 2*c) + (I - 1)*a)*s
in(3*d*x + 3*c) - (47*I - 47)*a*sin(d*x + c))*cos(3/2*arctan2(sin(2*d*x + 2
*c), -cos(2*d*x + 2*c) + 1)) + ((280*I - 280)*a*cos(7*d*x + 7*c) - (140*I -
 140)*a*cos(5*d*x + 5*c) + (133*I - 133)*a*cos(3*d*x + 3*c) + (47*I - 47)*a
*cos(d*x + c) - (280*I + 280)*a*sin(7*d*x + 7*c) + (140*I + 140)*a*sin(5*d*
x + 5*c) - (133*I + 133)*a*sin(3*d*x + 3*c) - (47*I + 47)*a*sin(d*x + c))*s
in(7/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)) + 4*(47*(-(I - 1)*
a*cos(d*x + c) + (I + 1)*a*sin(d*x + c))*cos(2*d*x + 2*c)^2 + 47*(-(I - 1)*
a*cos(d*x + c) + (I + 1)*a*sin(d*x + c))*sin(2*d*x + 2*c)^2 + 70*((I - 1)*a
*cos(2*d*x + 2*c)^2 + (I - 1)*a*sin(2*d*x + 2*c)^2 - (2*I - 2)*a*cos(2*d*x
 + 2*c) + (I - 1)*a)*cos(3*d*x + 3*c) + 94*((I - 1)*a*cos(d*x + c) - (I + 1)
*a*sin(d*x + c))*cos(2*d*x + 2*c) - (47*I - 47)*a*cos(d*x + c) + 70*(-(I +
1)*a*cos(2*d*x + 2*c)^2 - (I + 1)*a*sin(2*d*x + 2*c)^2 + (2*I + 2)*a*cos(2*
d*x + 2*c) - (I + 1)*a)*sin(3*d*x + 3*c) + (47*I + 47)*a*sin(d*x + c))*sin(
3/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)))*sqrt(a) + 420*(2*(-(
I + 1)*a*cos(2*d*x + 2*c)^4 - (I + 1)*a*sin(2*d*x + 2*c)^4 + (4*I + 4)*a*co
s(2*d*x + 2*c)^3 - (6*I + 6)*a*cos(2*d*x + 2*c)^2 + 2*(-(I + 1)*a*cos(2*d*x
 + 2*c)^2 + (2*I + 2)*a*cos(2*d*x + 2*c) - (I + 1)*a)*sin(2*d*x + 2*c)^2 +
(4*I + 4)*a*cos(2*d*x + 2*c) - (I + 1)*a)*arctan2((cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x +
 2*c), -cos(2*d*x + 2*c) + 1)) - cos(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*
c), -cos(2*d*x + 2*c) + 1)) - sin(d*x + c)) + (-(I - 1)*a*cos(2*d*x + 2*c)^
4 - (I - 1)*a*sin(2*d*x + 2*c)^4 + (4*I - 4)*a*cos(2*d*x + 2*c)^3 - (6*I -
6)*a*cos(2*d*x + 2*c)^2 + 2*(-(I - 1)*a*cos(2*d*x + 2*c)^2 + (2*I - 2)*a*co
s(2*d*x + 2*c) - (I - 1)*a)*sin(2*d*x + 2*c)^2 + (4*I - 4)*a*cos(2*d*x + 2*
c) - (I - 1)*a)*log(cos(d*x + c)^2 + sin(d*x + c)^2 + sqrt(cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)*(cos(1/2*arctan2(sin(2*d*
x + 2*c), -cos(2*d*x + 2*c) + 1))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), -co
s(2*d*x + 2*c) + 1))^2) - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2
*c) + 1))*sin(d*x + c) + cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), -co
s(2*d*x + 2*c) + 1)))))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*
d*x + 2*c) + 1)^(1/4)*sqrt(a) + ((1249*(-(I + 1)*a*cos(d*x + c) - (I - 1)*a
*sin(d*x + c))*cos(2*d*x + 2*c)^2 + 1249*(-(I + 1)*a*cos(d*x + c) - (I - 1)
*a*sin(d*x + c))*sin(2*d*x + 2*c)^2 + 840*(-(I + 1)*a*cos(2*d*x + 2*c)^2 -
(I + 1)*a*sin(2*d*x + 2*c)^2 + (2*I + 2)*a*cos(2*d*x + 2*c) - (I + 1)*a)*co
s(5*d*x + 5*c) + 1960*((I + 1)*a*cos(2*d*x + 2*c)^2 + (I + 1)*a*sin(2*d*x +
```

```
 2*c)^2 - (2*I + 2)*a*cos(2*d*x + 2*c) + (I + 1)*a)*cos(3*d*x + 3*c) + 2498
*((I + 1)*a*cos(d*x + c) + (I - 1)*a*sin(d*x + c))*cos(2*d*x + 2*c) - (1249
*I + 1249)*a*cos(d*x + c) + 840*(-(I - 1)*a*cos(2*d*x + 2*c)^2 - (I - 1)*a*
sin(2*d*x + 2*c)^2 + (2*I - 2)*a*cos(2*d*x + 2*c) - (I - 1)*a)*sin(5*d*x +
5*c) + 1960*((I - 1)*a*cos(2*d*x + 2*c)^2 + (I - 1)*a*sin(2*d*x + 2*c)^2 -
(2*I - 2)*a*cos(2*d*x + 2*c) + (I - 1)*a)*sin(3*d*x + 3*c) - (1249*I - 1249
)*a*sin(d*x + c))*cos(5/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1))
 + 832*(((I + 1)*a*cos(d*x + c) + (I - 1)*a*sin(d*x + c))*cos(2*d*x + 2*c)^
4 + ((I + 1)*a*cos(d*x + c) + (I - 1)*a*sin(d*x + c))*sin(2*d*x + 2*c)^4 +
4*(-(I + 1)*a*cos(d*x + c) - (I - 1)*a*sin(d*x + c))*cos(2*d*x + 2*c)^3 + 6
*((I + 1)*a*cos(d*x + c) + (I - 1)*a*sin(d*x + c))*cos(2*d*x + 2*c)^2 + 2*(
((I + 1)*a*cos(d*x + c) + (I - 1)*a*sin(d*x + c))*cos(2*d*x + 2*c)^2 + 2*(-
(I + 1)*a*cos(d*x + c) - (I - 1)*a*sin(d*x + c))*cos(2*d*x + 2*c) + (I + 1)
*a*cos(d*x + c) + (I - 1)*a*sin(d*x + c))*sin(2*d*x + 2*c)^2 + 4*(-(I + 1)*
a*cos(d*x + c) - (I - 1)*a*sin(d*x + c))*cos(2*d*x + 2*c) + (I + 1)*a*cos(d
*x + c) + (I - 1)*a*sin(d*x + c))*cos(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*
d*x + 2*c) + 1)) + (1249*(-(I - 1)*a*cos(d*x + c) + (I + 1)*a*sin(d*x + c))
*cos(2*d*x + 2*c)^2 + 1249*(-(I - 1)*a*cos(d*x ...
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(175) = 350$.
time = 0.38, size = 491, normalized size = 2.09

$$-\frac{4\sqrt{2}\left(211\,ae^{(9i\,dx+9i\,c)}-160\,ae^{(7i\,dx+7i\,c)}+14\,ae^{(5i\,dx+5i\,c)}+280\,ae^{(3i\,dx+3i\,c)}-105\,ae^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}-105\left(de^{(8i\,dx+8i\,c)}-4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}-4\,de^{(2i\,dx+2i\,c)}+d\right)\sqrt{-\frac{8i\,a^3}{d^2}}\log\left(\frac{\left(2\sqrt{2}\left(ae^{(2i\,dx+2i\,c)}+a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}+\sqrt{-\frac{8i\,a^3}{d^2}}\,de^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{2\,a}\right)+105\left(de^{(8i\,dx+8i\,c)}-4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}-4\,de^{(2i\,dx+2i\,c)}+d\right)\sqrt{-\frac{8i\,a^3}{d^2}}\log\left(\frac{\left(2\sqrt{2}\left(ae^{(2i\,dx+2i\,c)}+a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}-\sqrt{-\frac{8i\,a^3}{d^2}}\,de^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{2\,a}\right)}{210\left(de^{(8i\,dx+8i\,c)}-4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}-4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/210*(4*sqrt(2)*(211*a*e^(9*I*d*x + 9*I*c) - 160*a*e^(7*I*d*x + 7*I*c) +
14*a*e^(5*I*d*x + 5*I*c) + 280*a*e^(3*I*d*x + 3*I*c) - 105*a*e^(I*d*x + I*c
))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(
2*I*d*x + 2*I*c) + 1)) - 105*(d*e^(8*I*d*x + 8*I*c) - 4*d*e^(6*I*d*x + 6*I*
c) + 6*d*e^(4*I*d*x + 4*I*c) - 4*d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-8*I*a^3/d
^2)*log(1/2*(2*sqrt(2)*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) + sq
rt(-8*I*a^3/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a) + 105*(d*e^(8*I*d*x
 + 8*I*c) - 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) - 4*d*e^(2*I*
d*x + 2*I*c) + d)*sqrt(-8*I*a^3/d^2)*log(1/2*(2*sqrt(2)*(a*e^(2*I*d*x + 2*I
*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I
)/(e^(2*I*d*x + 2*I*c) + 1)) - sqrt(-8*I*a^3/d^2)*d*e^(I*d*x + I*c))*e^(-I*
d*x - I*c)/a))/(d*e^(8*I*d*x + 8*I*c) - 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*
I*d*x + 4*I*c) - 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(3/2)/tan(d*x+c)**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^{3/2}}{\tan\left(c+d\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(3/2)/tan(c + d*x)^(9/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^(3/2)/tan(c + d*x)^(9/2), x)
```

3.201 $\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=258

$$\frac{363(-1)^{3/4}a^{5/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{64d}+\frac{(4+4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{149\cdots}{}$$

```
[Out] 363/64*(-1)^(3/4)*a^(5/2)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a
*tan(d*x+c))^(1/2))/d+(4+4*I)*a^(5/2)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2
)/(a+I*a*tan(d*x+c))^(1/2))/d-149/64*I*a^2*tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+
c))^(1/2)/d+107/96*a^2*(a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^(3/2)/d+17/24*I*
a^2*(a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^(5/2)/d-1/4*a^2*(a+I*a*tan(d*x+c))^
(1/2)*tan(d*x+c)^(7/2)/d
```

Rubi [A]
time = 0.52, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {3637, 3678, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{363(-1)^{3/4}a^{5/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{64d}+\frac{(4+4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{a^2\tan^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d}+\frac{17ia^2\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{24d}+\frac{107a^2\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{96d}-\frac{149ia^2\sqrt{\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{64d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (363*(-1)^(3/4)*a^(5/2)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt
[a + I*a*Tan[c + d*x]]])/(64*d) + ((4 + 4*I)*a^(5/2)*ArcTanh[((1 + I)*Sqrt[
a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d - (((149*I)/64)*a^2*S
qrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d + (107*a^2*Tan[c + d*x]^(3/
2)*Sqrt[a + I*a*Tan[c + d*x]])/(96*d) + (((17*I)/24)*a^2*Tan[c + d*x]^(5/2)
*Sqrt[a + I*a*Tan[c + d*x]])/d - (a^2*Tan[c + d*x]^(7/2)*Sqrt[a + I*a*Tan[c
 + d*x]])/(4*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3678

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Dist[
1/(a*(m + n)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Simp
[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{a^2\tan^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d} + \frac{1}{4}a\int \tan^{\frac{5}{2}}(c+dx)\cdots$$

$$= \frac{17ia^2\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{24d} - \frac{a^2\tan^{\frac{7}{2}}(c+dx)\sqrt{\cdots}}{4\cdots}$$

$$= \frac{107a^2\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{96d} + \frac{17ia^2\tan^{\frac{5}{2}}(c+dx\cdots}{\cdots}$$

$$= -\frac{149ia^2\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{64d} + \frac{107a^2\tan^{\frac{3}{2}}(c\cdots}{\cdots}$$

$$= -\frac{149ia^2\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{64d} + \frac{107a^2\tan^{\frac{3}{2}}(c\cdots}{\cdots}$$

$$= -\frac{149ia^2\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{64d} + \frac{107a^2\tan^{\frac{3}{2}}(c\cdots}{\cdots}$$

$$= \frac{(4+4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{149ia^2\sqrt{\tan\cdots}}{\cdots}$$

$$= \frac{(4+4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{149ia^2\sqrt{\tan\cdots}}{\cdots}$$

$$= \frac{363(-1)^{3/4}a^{5/2}\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{64d} + \frac{(4+4i)\cdots}{\cdots}$$

Mathematica [A]

time = 4.39, size = 232, normalized size = 0.90

$$\frac{a^2\left(\frac{6e^{-i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\left(512\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)-363\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)}{\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}}-i\sec^3(c+dx)(1205\cos(c+dx)+583\cos(3(c+dx))+70i\sin(c+dx)+262i\sin(3(c+dx)))\sqrt{\tan(c+dx)}\right)\sqrt{a+ia\tan(c+dx)}}{768d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (a^2*((6*Sqrt[-1 + E^((2*I)*(c + d*x))]*(512*ArcTanh[E^(I*(c + d*x))/Sqrt[-
1 + E^((2*I)*(c + d*x))]] - 363*Sqrt[2]*ArcTanh[(Sqrt[2]*E^(I*(c + d*x)))/S
qrt[-1 + E^((2*I)*(c + d*x))]]))/(E^(I*(c + d*x))*Sqrt[((-I)*(-1 + E^((2*I)
*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]) - I*Sec[c + d*x]^3*(1205*Cos[c +
d*x] + 583*Cos[3*(c + d*x)] + (70*I)*Sin[c + d*x] + (262*I)*Sin[3*(c + d*x)
])*Sqrt[Tan[c + d*x]])*Sqrt[a + I*a*Tan[c + d*x]])/(768*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(204) = 408$.
time = 0.20, size = 494, normalized size = 1.91

method	result
derivativedivides	$-\frac{\left(\sqrt{\tan}(dx+c)\right)\sqrt{a(1+i\tan(dx+c))}\,a^2\left(-272i\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{\ldots}\right.}{\ldots}$
default	$-\frac{\left(\sqrt{\tan}(dx+c)\right)\sqrt{a(1+i\tan(dx+c))}\,a^2\left(-272i\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{\ldots}\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/384/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)*a^2*(-272*I*tan(d*x+c)
^2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)+96*(a*tan
(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^3+384*I
*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*
x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a+1089*I*a*ln(1/2*(2*I*a*t
an(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2
))*(-I*a)^(1/2)+894*I*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I
*a)^(1/2)+384*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c
)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a-428*(a*tan(
d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)+1536*ln(
1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a
)/(I*a)^(1/2))*a*(-I*a)^(1/2))/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^
(1/2)/(-I*a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*tan(d*x + c)^(5/2), x)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 769 vs. $2(192) = 384$.
time = 0.41, size = 769, normalized size = 2.98

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/192*(sqrt(2)*(-845*I*a^2*e^(7*I*d*x + 7*I*c) - 1275*I*a^2*e^(5*I*d*x + 5*
I*c) - 1135*I*a^2*e^(3*I*d*x + 3*I*c) - 321*I*a^2*e^(I*d*x + I*c))*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2
*I*c) + 1)) + 96*sqrt(131769/4096*I*a^5/d^2)*(d*e^(6*I*d*x + 6*I*c) + 3*d*e
^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)*log(1/363*(363*sqrt(2)*(a
^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^
(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) + 128*I*sqrt(131769/4096*
I*a^5/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a^2) - 96*sqrt(131769/4096*I
*a^5/d^2)*(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x
 + 2*I*c) + d)*log(1/363*(363*sqrt(2)*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(
a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x
+ 2*I*c) + 1)) - 128*I*sqrt(131769/4096*I*a^5/d^2)*d*e^(I*d*x + I*c))*e^(-I
*d*x - I*c)/a^2) - 96*sqrt(32*I*a^5/d^2)*(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*
I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)*log(1/4*(4*sqrt(2)*(a^2*e^(2*
I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x
 + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) + I*sqrt(32*I*a^5/d^2)*d*e^(I*d*x
 + I*c))*e^(-I*d*x - I*c)/a^2) + 96*sqrt(32*I*a^5/d^2)*(d*e^(6*I*d*x + 6*I*
c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)*log(1/4*(4*sqrt
(2)*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(
(-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) - I*sqrt(32*I*a^5/d
^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a^2))/(d*e^(6*I*d*x + 6*I*c) + 3*d*
e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(5/2)*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 1.26Unable to divide
, perhaps due to rounding error%%%{%%{[%%%{%%{poly1[-128*i,0]:[1,0,-2]%%},[
0]%%%}
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c+d\,x)^{5/2}\,(a+a\tan(c+d\,x)\;1\mathrm{i})^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] int(tan(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.202 $\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=219

$$-\frac{45\sqrt[4]{-1}\,a^{5/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{8d}-\frac{(4-4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{19a^2}{}$$

```
[Out] -45/8*(-1)^(1/4)*a^(5/2)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*
tan(d*x+c))^(1/2))/d+(-4+4*I)*a^(5/2)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2
)/(a+I*a*tan(d*x+c))^(1/2))/d+19/8*a^2*tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^
(1/2)/d+13/12*I*a^2*(a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^(3/2)/d-1/3*a^2*(a+
I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^(5/2)/d
```

Rubi [A]
time = 0.45, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {3637, 3678, 3682, 3625, 211, 3680, 65, 223, 209}

$$-\frac{45\sqrt[4]{-1}\,a^{5/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{8d}-\frac{(4-4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{a^2\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}+\frac{13ia^2\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{12d}+\frac{19a^2\sqrt{\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (-45*(-1)^(1/4)*a^(5/2)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt
[a + I*a*Tan[c + d*x]]])/(8*d) - ((4 - 4*I)*a^(5/2)*ArcTanh[((1 + I)*Sqrt[a
]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d + (19*a^2*Sqrt[Tan[c +
 d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(8*d) + (((13*I)/12)*a^2*Tan[c + d*x]^(3
/2)*Sqrt[a + I*a*Tan[c + d*x]])/d - (a^2*Tan[c + d*x]^(5/2)*Sqrt[a + I*a*Ta
n[c + d*x]])/(3*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3678

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Dist[
1/(a*(m + n)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Simp
[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{a^2\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}+\frac{1}{3}a\int \tan^{\frac{3}{2}}(c+dx)$$

$$= \frac{13ia^2\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{12d}-\frac{a^2\tan^{\frac{5}{2}}(c+dx)\sqrt{}}{3}$$

$$= \frac{19a^2\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{8d}+\frac{13ia^2\tan^{\frac{3}{2}}(c+dx}{}$$

$$= \frac{19a^2\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{8d}+\frac{13ia^2\tan^{\frac{3}{2}}(c+dx}{}$$

$$= \frac{19a^2\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{8d}+\frac{13ia^2\tan^{\frac{3}{2}}(c+dx}{}$$

$$= -\frac{(4-4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{19a^2\sqrt{\tan}}{}$$

$$= -\frac{(4-4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{19a^2\sqrt{\tan}}{}$$

$$= -\frac{45\sqrt[4]{-1}\,a^{5/2}\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{8d}-\frac{(4-4i)a}{}$$

Mathematica [A]
time = 3.29, size = 258, normalized size = 1.18

$$\frac{ia^2e^{-i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(64\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)-45\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)}{8\sqrt{2}d\sqrt{-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}}+\frac{a^2\sec^2(c+dx)(49+65\cos(2(c+dx))+26i\sin(2(c+dx)))\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((I/8)*a^2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 +
 E^((2*I)*(c + d*x)))]*(64*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c +
d*x))]] - 45*Sqrt[2]*ArcTanh[(Sqrt[2]*E^(I*(c + d*x)))/Sqrt[-1 + E^((2*I)*(
c + d*x))]]))/(Sqrt[2]*d*E^(I*(c + d*x))*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x
))))/(1 + E^((2*I)*(c + d*x)))]) + (a^2*Sec[c + d*x]^2*(49 + 65*Cos[2*(c +
d*x)] + (26*I)*Sin[2*(c + d*x)])*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*
x]])/(48*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(173) = 346$.
time = 0.17, size = 450, normalized size = 2.05

method	result
derivativedivides	$-\frac{\left(\sqrt{\tan}(dx+c)\right)\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\,a^{2}\left(48i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1\right.}}{\tan(dx+c}\right.\right.}{}$
default	$-\frac{\left(\sqrt{\tan}(dx+c)\right)\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\,a^{2}\left(48i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1\right.}}{\tan(dx+c}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)*a^2*(48*I*(I*a)^(1/2)*2
^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I
*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a-52*I*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(
1/2)*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)+16*(a*tan(d*x+c)*(1+I*tan(d*x+c)))
^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^2+192*I*ln(1/2*(2*I*a*tan(d*x+c)
+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)
^(1/2)*a-48*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*
(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a-135*ln(1/2*(2
*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a
)^(1/2))*a*(-I*a)^(1/2)-114*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/
2)*(-I*a)^(1/2))/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2)/(-I*a)^(
1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*tan(d*x + c)^(3/2), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 694 vs. $2(163) = 326$.
time = 0.38, size = 694, normalized size = 3.17

$$\frac{\sqrt{2}\left(91\,a^{2}e^{\left(5i\,dx+5i\,c\right)}+98\,a^{2}e^{\left(3i\,dx+3i\,c\right)}+39\,a^{2}e^{\left(i\,dx+i\,c\right)}\right)\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}\sqrt{\frac{-i\,e^{\left(2i\,dx+2i\,c\right)}+i}{e^{\left(2i\,dx+2i\,c\right)}+1}}+12\sqrt{-\frac{2025i\,a^{5}}{64\,d^{2}}}\left(de^{\left(4i\,dx+4i\,c\right)}+2\,de^{\left(2i\,dx+2i\,c\right)}+d\right)\log\left(\frac{\left(45\sqrt{2}\left(a^{2}e^{\left(2i\,dx+2i\,c\right)}+a^{2}\right)\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}\sqrt{\frac{-i\,e^{\left(2i\,dx+2i\,c\right)}+i}{e^{\left(2i\,dx+2i\,c\right)}+1}}+16\sqrt{-\frac{2025i\,a^{5}}{64\,d^{2}}}\,de^{\left(i\,dx+i\,c\right)}\right)e^{\left(-i\,dx-i\,c\right)}}{45\,a^{2}}\right)-12\sqrt{-\frac{2025i\,a^{5}}{64\,d^{2}}}\left(de^{\left(4i\,dx+4i\,c\right)}+2\,de^{\left(2i\,dx+2i\,c\right)}+d\right)\log\left(\frac{\left(45\sqrt{2}\left(a^{2}e^{\left(2i\,dx+2i\,c\right)}+a^{2}\right)\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}\sqrt{\frac{-i\,e^{\left(2i\,dx+2i\,c\right)}+i}{e^{\left(2i\,dx+2i\,c\right)}+1}}-16\sqrt{-\frac{2025i\,a^{5}}{64\,d^{2}}}\,de^{\left(i\,dx+i\,c\right)}\right)e^{\left(-i\,dx-i\,c\right)}}{45\,a^{2}}\right)-12\sqrt{-\frac{32i\,a^{5}}{d^{2}}}\left(de^{\left(4i\,dx+4i\,c\right)}+2\,de^{\left(2i\,dx+2i\,c\right)}+d\right)\log\left(\frac{\left(4\sqrt{2}\left(a^{2}e^{\left(2i\,dx+2i\,c\right)}+a^{2}\right)\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}\sqrt{\frac{-i\,e^{\left(2i\,dx+2i\,c\right)}+i}{e^{\left(2i\,dx+2i\,c\right)}+1}}+\sqrt{-\frac{32i\,a^{5}}{d^{2}}}\,de^{\left(i\,dx+i\,c\right)}\right)e^{\left(-i\,dx-i\,c\right)}}{4\,a^{2}}\right)+12\sqrt{-\frac{32i\,a^{5}}{d^{2}}}\left(de^{\left(4i\,dx+4i\,c\right)}+2\,de^{\left(2i\,dx+2i\,c\right)}+d\right)\log\left(\frac{\left(4\sqrt{2}\left(a^{2}e^{\left(2i\,dx+2i\,c\right)}+a^{2}\right)\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}\sqrt{\frac{-i\,e^{\left(2i\,dx+2i\,c\right)}+i}{e^{\left(2i\,dx+2i\,c\right)}+1}}-\sqrt{-\frac{32i\,a^{5}}{d^{2}}}\,de^{\left(i\,dx+i\,c\right)}\right)e^{\left(-i\,dx-i\,c\right)}}{4\,a^{2}}\right)}{24\left(de^{\left(4i\,dx+4i\,c\right)}+2\,de^{\left(2i\,dx+2i\,c\right)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(sqrt(2)*(91*a^2*e^(5*I*d*x + 5*I*c) + 98*a^2*e^(3*I*d*x + 3*I*c) + 39
*a^2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x
 + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) + 12*sqrt(-2025/64*I*a^5/d^2)*(d*
e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(1/45*(45*sqrt(2)*(a^
2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(
2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) + 16*sqrt(-2025/64*I*a^5/d
^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a^2) - 12*sqrt(-2025/64*I*a^5/d^2)*
(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(1/45*(45*sqrt(2)*
(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*
e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) - 16*sqrt(-2025/64*I*a^
5/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a^2) - 12*sqrt(-32*I*a^5/d^2)*(d
*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(1/4*(4*sqrt(2)*(a^2
*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2
*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) + sqrt(-32*I*a^5/d^2)*d*e^(
I*d*x + I*c))*e^(-I*d*x - I*c)/a^2) + 12*sqrt(-32*I*a^5/d^2)*(d*e^(4*I*d*x
+ 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(1/4*(4*sqrt(2)*(a^2*e^(2*I*d*x
+ 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I
*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) - sqrt(-32*I*a^5/d^2)*d*e^(I*d*x + I*c)
)*e^(-I*d*x - I*c)/a^2))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) +
 d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(3/2)*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 0.92Unable to divide
, perhaps due to rounding error%%%{%%{[%%%{%%{poly1[-16,0]:[1,0,-2]%%},[0]%
%%},0]
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c+d\,x)^{3/2}\,(a+a\tan(c+d\,x)\ 1\mathrm{i})^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] int(tan(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.203 $\int \sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=182

$$-\frac{23(-1)^{3/4}a^{5/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{4d}-\frac{(4+4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{9ia}{}$$

```
[Out] -23/4*(-1)^(3/4)*a^(5/2)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*
tan(d*x+c))^(1/2))/d-(4+4*I)*a^(5/2)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)
/(a+I*a*tan(d*x+c))^(1/2))/d+9/4*I*a^2*tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^
(1/2)/d-1/2*a^2*(a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^(3/2)/d
```

Rubi [A]

time = 0.34, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {3637, 3678, 3682, 3625, 211, 3680, 65, 223, 209}

$$-\frac{23(-1)^{3/4}a^{5/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{4d}-\frac{(4+4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{a^2\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}+\frac{9ia^2\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (-23*(-1)^(3/4)*a^(5/2)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt
[a + I*a*Tan[c + d*x]]])/(4*d) - ((4 + 4*I)*a^(5/2)*ArcTanh[((1 + I)*Sqrt[a
]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d + (((9*I)/4)*a^2*Sqrt[
Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d - (a^2*Tan[c + d*x]^(3/2)*Sqrt[
a + I*a*Tan[c + d*x]])/(2*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3678

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Dist[
1/(a*(m + n)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Simp
[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^{5/2}\,dx &= -\frac{a^2\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}+\frac{1}{2}a\int \sqrt{\tan(c+a\cdots} \\
&= \frac{9ia^2\sqrt{\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{4d}-\frac{a^2\tan^{\frac{3}{2}}(c+dx)\cdots}{\cdots} \\
&= \frac{9ia^2\sqrt{\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{4d}-\frac{a^2\tan^{\frac{3}{2}}(c+dx)\cdots}{\cdots} \\
&= \frac{9ia^2\sqrt{\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{4d}-\frac{a^2\tan^{\frac{3}{2}}(c+dx)\cdots}{\cdots} \\
&= -\frac{(4+4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{9ia^2\sqrt{\mathrm{ta}\cdots}}{\cdots} \\
&= -\frac{(4+4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{9ia^2\sqrt{\mathrm{ta}\cdots}}{\cdots} \\
&= -\frac{23(-1)^{3/4}a^{5/2}\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{4d}-\frac{(4+\cdots}{\cdots}
\end{aligned}
$$

Mathematica [A]
time = 3.41, size = 192, normalized size = 1.05

$$
\frac{a^2\sqrt{a+ia\tan(c+dx)}\left(\frac{e^{-i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\left(-32\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)+23\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)}{\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}}-2\sqrt{\tan(c+dx)}\,(-9i+2\tan(c+dx))\right)}{8d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Sqrt[a + I*a*Tan[c + d*x]]*((Sqrt[-1 + E^((2*I)*(c + d*x))]*(-32*ArcTa
nh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]] + 23*Sqrt[2]*ArcTanh[(Sq
rt[2]*E^(I*(c + d*x)))/Sqrt[-1 + E^((2*I)*(c + d*x))]]))/(E^(I*(c + d*x))*S
qrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]) - 2*Sqrt[
Tan[c + d*x]]*(-9*I + 2*Tan[c + d*x])))/(8*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(143) = 286$.
time = 0.18, size = 407, normalized size = 2.24

method	result
derivativedivides	$\frac{\left(\sqrt{\tan}(dx+c)\right)\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\,a^2\left(8i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1+\right.}}{\tan(dx+c)+i}\right.\right.}{}$
default	$\frac{\left(\sqrt{\tan}(dx+c)\right)\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\,a^2\left(8i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1+\right.}}{\tan(dx+c)+i}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)*a^2*(8*I*(I*a)^(1/2)*2^(1
/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-
3*a*tan(d*x+c))/(tan(d*x+c)+I))*a+18*I*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2
)*(I*a)^(1/2)*(-I*a)^(1/2)+23*I*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1
+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*(-I*a)^(1/2)-4*(a*tan(d
*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)+8*(I*a)^(
1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(
1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a+32*ln(1/2*(2*I*a*tan(d*x+c)+2*(a
*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*(-I*a)^(1
/2))/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2)/(-I*a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*sqrt(tan(d*x + c)), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 621 vs. $2(134) = 268$.

time = 0.38, size = 621, normalized size = 3.41

$$\frac{\sqrt{2}\left(11i\,a^{2}e^{(3i\,dx+3i\,c)}+7i\,a^{2}e^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}-2\sqrt{\frac{529i\,a^{5}}{16\,d^{2}}}\left(de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{\left(23\sqrt{2}\left(a^{2}e^{(2i\,dx+2i\,c)}+a^{2}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}+8i\sqrt{\frac{529i\,a^{5}}{16\,d^{2}}}\,de^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{23\,a^{2}}\right)+2\sqrt{\frac{529i\,a^{5}}{16\,d^{2}}}\left(de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{\left(23\sqrt{2}\left(a^{2}e^{(2i\,dx+2i\,c)}+a^{2}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}-8i\sqrt{\frac{529i\,a^{5}}{16\,d^{2}}}\,de^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{23\,a^{2}}\right)+2\sqrt{\frac{32i\,a^{5}}{d^{2}}}\left(de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{\left(4\sqrt{2}\left(a^{2}e^{(2i\,dx+2i\,c)}+a^{2}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}+i\sqrt{\frac{32i\,a^{5}}{d^{2}}}\,de^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{4\,a^{2}}\right)-2\sqrt{\frac{32i\,a^{5}}{d^{2}}}\left(de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{\left(4\sqrt{2}\left(a^{2}e^{(2i\,dx+2i\,c)}+a^{2}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}-i\sqrt{\frac{32i\,a^{5}}{d^{2}}}\,de^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{4\,a^{2}}\right)}{4\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*(11*I*a^2*e^(3*I*d*x + 3*I*c) + 7*I*a^2*e^(I*d*x + I*c))*sqrt(
a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x
+ 2*I*c) + 1)) - 2*sqrt(529/16*I*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(1
/23*(23*sqrt(2)*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c)
 + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) + 8*I*s
qrt(529/16*I*a^5/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a^2) + 2*sqrt(529
/16*I*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(1/23*(23*sqrt(2)*(a^2*e^(2*I
*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x
+ 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) - 8*I*sqrt(529/16*I*a^5/d^2)*d*e^(
I*d*x + I*c))*e^(-I*d*x - I*c)/a^2) + 2*sqrt(32*I*a^5/d^2)*(d*e^(2*I*d*x +
2*I*c) + d)*log(1/4*(4*sqrt(2)*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c
) + 1)) + I*sqrt(32*I*a^5/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a^2) - 2
*sqrt(32*I*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(1/4*(4*sqrt(2)*(a^2*e^(
2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d
*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) - I*sqrt(32*I*a^5/d^2)*d*e^(I*d
*x + I*c))*e^(-I*d*x - I*c)/a^2))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c+dx)-i))^{\frac{5}{2}}\sqrt{\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/2)*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(5/2)*sqrt(tan(c + d*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 0.67Unable to divide
```

```
, perhaps due to rounding error%%%{%%{[%%%{%%{poly1[-8*i,0]:[1,0,-2]%%},[0]
%%%},0
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\tan\left(c+d\,x\right)}\,\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(5/2),x)
[Out] int(tan(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.204 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{\tan(c+dx)}}\,dx$

Optimal. Leaf size=139

$$\frac{5\sqrt[4]{-1}\,a^{5/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{(4-4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{a^2\sqrt{\tan(c+dx)}\ldots}{}$$

```
[Out] 5*(-1)^(1/4)*a^(5/2)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(
d*x+c))^(1/2))/d+(4-4*I)*a^(5/2)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+
I*a*tan(d*x+c))^(1/2))/d-a^2*tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.26, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3637, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{5\sqrt[4]{-1}\,a^{5/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{(4-4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{a^2\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(5/2)/Sqrt[Tan[c + d*x]],x]
```

```
[Out] (5*(-1)^(1/4)*a^(5/2)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a
 + I*a*Tan[c + d*x]]])/d + ((4 - 4*I)*a^(5/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt
[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d - (a^2*Sqrt[Tan[c + d*x]]*Sq
rt[a + I*a*Tan[c + d*x]])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{\tan(c+dx)}}\,dx = -\frac{a^2\sqrt{\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{d} + a\int \frac{\sqrt{a+ia\tan(c+dx)}\,\left(\frac{3a}{2}\right.}{\sqrt{\tan(c+}}$$

$$= -\frac{a^2\sqrt{\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{d} - \frac{1}{2}(5a)\int \frac{(a-ia\tan(c+dx))\sqrt{}}{\sqrt{\tan(c}}$$

$$= -\frac{a^2\sqrt{\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{d} - \frac{(5a^3)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\,\sqrt{a+iax}}\right.}{2d}$$

$$= \frac{(4-4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{a^2\sqrt{\tan(c+dx)}\,\sqrt{a+i}}{d}$$

$$= \frac{(4-4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{a^2\sqrt{\tan(c+dx)}\,\sqrt{a+i}}{d}$$

$$= \frac{5\sqrt[4]{-1}\,a^{5/2}\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} + \frac{(4-4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i}{\sqrt{}}\right.}{d}$$

Mathematica [A]
time = 2.61, size = 267, normalized size = 1.92

$$\frac{2i\sqrt{2}\,a^2e^{i(c+dx)}\left(\sqrt{2}\,e^{i(c+dx)}\left(-1+e^{2i(c+dx)}\right)-4\sqrt{2}\,\sqrt{-1+e^{2i(c+dx)}}\left(1+e^{2i(c+dx)}\right)\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)+5\sqrt{-1+e^{2i(c+dx)}}\left(1+e^{2i(c+dx)}\right)\tanh^{-1}\left(\frac{\sqrt{2}\,e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}\left(1+e^{2i(c+dx)}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(5/2)/Sqrt[Tan[c + d*x]],x]
```

```
[Out] ((2*I)*Sqrt[2]*a^2*E^(I*(c + d*x))*(Sqrt[2]*E^(I*(c + d*x))*(-1 + E^((2*I)*
(c + d*x))) - 4*Sqrt[2]*Sqrt[-1 + E^((2*I)*(c + d*x))]*(1 + E^((2*I)*(c + d
*x)))*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]] + 5*Sqrt[-1 +
 E^((2*I)*(c + d*x))]*(1 + E^((2*I)*(c + d*x)))*ArcTanh[(Sqrt[2]*E^(I*(c +
d*x)))/Sqrt[-1 + E^((2*I)*(c + d*x))]])*Cos[c + d*x]^2*Sqrt[a + I*a*Tan[c +
 d*x]])/(d*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))
]*(1 + E^((2*I)*(c + d*x)))^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(112) = 224.
time = 0.20, size = 365, normalized size = 2.63

method	result

derivativedivides	$\frac{\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\,\left(\sqrt{\tan}\left(dx+c\right)\right)a^{2}\left(-2\sqrt{a\tan\left(dx+c\right)\left(1+i\tan\left(dx+c\right)\right)}\,\sqrt{ia}\,\sqrt{\right.}$
default	$\frac{\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\,\left(\sqrt{\tan}\left(dx+c\right)\right)a^{2}\left(-2\sqrt{a\tan\left(dx+c\right)\left(1+i\tan\left(dx+c\right)\right)}\,\sqrt{ia}\,\sqrt{\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(5/2)/tan(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(a*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^(1/2)*a^2*(-2*(a*tan(d*x+c)*(1+
I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)-5*ln(1/2*(2*I*a*tan(d*x+c)+2*
(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*(-I*a)^
(1/2)+2*I*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1
+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a+8*I*ln(1/2*(2*I
*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^
(1/2))*a*(-I*a)^(1/2)-2*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a
*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a)/
(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2)/(-I*a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/tan(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)/sqrt(tan(d*x + c)), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 542 vs. 2(105) = 210.
time = 0.40, size = 542, normalized size = 3.90

$$-\frac{2\sqrt{2}a^{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}e^{(idx+ic)}+\sqrt{-\frac{25ia^{5}}{d^{2}}}d\log\left(\frac{\left(5\sqrt{2}\left(a^{2}e^{(2idx+2ic)}+a^{2}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}+2\sqrt{-\frac{25ia^{5}}{d^{2}}}de^{(idx+ic)}\right)e^{(-idx-ic)}}{5a^{2}}\right)-\sqrt{-\frac{25ia^{5}}{d^{2}}}d\log\left(\frac{\left(5\sqrt{2}\left(a^{2}e^{(2idx+2ic)}+a^{2}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}-2\sqrt{-\frac{25ia^{5}}{d^{2}}}de^{(idx+ic)}\right)e^{(-idx-ic)}}{5a^{2}}\right)-\sqrt{-\frac{32ia^{5}}{d^{2}}}d\log\left(\frac{\left(4\sqrt{2}\left(a^{2}e^{(2idx+2ic)}+a^{2}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}+\sqrt{-\frac{32ia^{5}}{d^{2}}}de^{(idx+ic)}\right)e^{(-idx-ic)}}{4a^{2}}\right)+\sqrt{-\frac{32ia^{5}}{d^{2}}}d\log\left(\frac{\left(4\sqrt{2}\left(a^{2}e^{(2idx+2ic)}+a^{2}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}-\sqrt{-\frac{32ia^{5}}{d^{2}}}de^{(idx+ic)}\right)e^{(-idx-ic)}}{4a^{2}}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/tan(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(2)*a^2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x +
 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c) + sqrt(-25*I*a^5/d^
2)*d*log(1/5*(5*sqrt(2)*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))
```

```
 + 2*sqrt(-25*I*a^5/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a^2) - sqrt(-2
5*I*a^5/d^2)*d*log(1/5*(5*sqrt(2)*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e
^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*
I*c) + 1)) - 2*sqrt(-25*I*a^5/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a^2)
 - sqrt(-32*I*a^5/d^2)*d*log(1/4*(4*sqrt(2)*(a^2*e^(2*I*d*x + 2*I*c) + a^2)
*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*
I*d*x + 2*I*c) + 1)) + sqrt(-32*I*a^5/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I
*c)/a^2) + sqrt(-32*I*a^5/d^2)*d*log(1/4*(4*sqrt(2)*(a^2*e^(2*I*d*x + 2*I*c
) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I
)/(e^(2*I*d*x + 2*I*c) + 1)) - sqrt(-32*I*a^5/d^2)*d*e^(I*d*x + I*c))*e^(-I
*d*x - I*c)/a^2))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan{(c+dx)}-i))^{\frac{5}{2}}}{\sqrt{\tan{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(5/2)/tan(d*x+c)**(1/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(5/2)/sqrt(tan(c + d*x)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/tan(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}}{\sqrt{\mathrm{tan}\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(5/2)/tan(c + d*x)^(1/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^(5/2)/tan(c + d*x)^(1/2), x)
```

3.205 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=139

$$\frac{2(-1)^{3/4}a^{5/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{(4+4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^2\sqrt{a}}{d\sqrt{}}$$

[Out] 2*(-1)^(3/4)*a^(5/2)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/d+(4+4*I)*a^(5/2)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/d-2*a^2*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(1/2)

Rubi [A]
time = 0.26, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3634, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{2(-1)^{3/4}a^{5/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{(4+4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(5/2)/Tan[c + d*x]^(3/2),x]
```

```
[Out] (2*(-1)^(3/4)*a^(5/2)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a
 + I*a*Tan[c + d*x]]])/d + ((4 + 4*I)*a^(5/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt
[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d - (2*a^2*Sqrt[a + I*a*Tan[c
+ d*x]])/(d*Sqrt[Tan[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3634

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-a^2)*(b*c - a*d)*(a + b*Tan[e + f*x]
)^(m - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] + Di
st[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[b*(b*c*(m - 2) - a*d*(m - 2*n - 4)) + (a*b*c*(m - 2)
+ b^2*d*(n + 1) - a^2*d*(m + n - 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\tan^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2a^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}} - 2\int \frac{\sqrt{a+ia\tan(c+dx)}\left(-\frac{3ia^2}{2}+\frac{1}{2}a^2\tan(c\right.}{\sqrt{\tan(c+dx)}}$$

$$= -\frac{2a^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}} - (ia)\int \frac{(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+d}}{\sqrt{\tan(c+dx)}}$$

$$= -\frac{2a^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}} - \frac{(ia^3)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{a+iax}}\,dx, x, \tan(c+d\right.}{d}$$

$$= \frac{(4+4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{2a^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$= \frac{(4+4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{2a^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$= \frac{2(-1)^{3/4}a^{5/2}\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} + \frac{(4+4i)a^{5/2}\tanh^{-1}\left(\frac{(1+\right.}{\sqrt{}}}{d}$$

Mathematica [**A**]
time = 2.38, size = 215, normalized size = 1.55

$$-\frac{i\sqrt{2}\,a^3e^{i(c+dx)}\left(2e^{i(c+dx)}\sqrt{-1+e^{2i(c+dx)}} - 4\left(-1+e^{2i(c+dx)}\right)\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right) + \sqrt{2}\left(-1+e^{2i(c+dx)}\right)\tanh^{-1}\left(\frac{\sqrt{2}\,e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\tan(c+dx)}}{d\left(-1+e^{2i(c+dx)}\right)^{3/2}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(5/2)/Tan[c + d*x]^(3/2),x]
```

```
[Out] ((-I)*Sqrt[2]*a^3*E^(I*(c + d*x))*(2*E^(I*(c + d*x))*Sqrt[-1 + E^((2*I)*(c
+ d*x))] - 4*(-1 + E^((2*I)*(c + d*x)))*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E
^((2*I)*(c + d*x))]] + Sqrt[2]*(-1 + E^((2*I)*(c + d*x)))*ArcTanh[(Sqrt[2]*
E^(I*(c + d*x)))/Sqrt[-1 + E^((2*I)*(c + d*x))]])*Sqrt[Tan[c + d*x]])/(d*(-
1 + E^((2*I)*(c + d*x)))^(3/2)*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(
c + d*x)))])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 387 vs. 2(112) = 224.
time = 0.19, size = 388, normalized size = 2.79

method	result
derivativedivides	$\frac{\left(-i\sqrt{ia}\,\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)\,(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\tan(\cdots}{}$
default	$\frac{\left(-i\sqrt{ia}\,\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)\,(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\tan(\cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(5/2)/tan(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-I*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I
*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)-I*ln(1
/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)
/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)-ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan
(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*(I*a)^(
1/2)*2^(1/2)*a*tan(d*x+c)-4*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*t
an(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)-2*(
a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2))*(a*(1+I*tan(
d*x+c)))^(1/2)*a^2/tan(d*x+c)^(1/2)/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(
I*a)^(1/2)/(-I*a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/tan(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)/tan(d*x + c)^(3/2), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 626 vs. 2(105) = 210.
time = 0.38, size = 626, normalized size = 4.50

$$-\frac{4\sqrt{2}\left(ia^{2}e^{(3idx+3ic)}+ia^{2}e^{(idx+ic)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}+\sqrt{\frac{32ia^{5}}{d^{2}}}\left(de^{(2idx+2ic)}-d\right)\log\left(\frac{\left(4\sqrt{2}\left(a^{2}e^{(2idx+2ic)}+a^{2}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}+i\sqrt{\frac{32ia^{5}}{d^{2}}}de^{(idx+ic)}\right)e^{(-idx-ic)}}{4a^{2}}\right)-\sqrt{\frac{32ia^{5}}{d^{2}}}\left(de^{(2idx+2ic)}-d\right)\log\left(\frac{\left(4\sqrt{2}\left(a^{2}e^{(2idx+2ic)}+a^{2}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}-i\sqrt{\frac{32ia^{5}}{d^{2}}}de^{(idx+ic)}\right)e^{(-idx-ic)}}{4a^{2}}\right)-\sqrt{\frac{4ia^{5}}{d^{2}}}\left(de^{(2idx+2ic)}-d\right)\log\left(\frac{\left(\sqrt{2}\left(a^{2}e^{(2idx+2ic)}+a^{2}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}+i\sqrt{\frac{4ia^{5}}{d^{2}}}de^{(idx+ic)}\right)e^{(-idx-ic)}}{a^{2}}\right)+\sqrt{\frac{4ia^{5}}{d^{2}}}\left(de^{(2idx+2ic)}-d\right)\log\left(\frac{\left(\sqrt{2}\left(a^{2}e^{(2idx+2ic)}+a^{2}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}-i\sqrt{\frac{4ia^{5}}{d^{2}}}de^{(idx+ic)}\right)e^{(-idx-ic)}}{a^{2}}\right)}{2\left(de^{(2idx+2ic)}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/tan(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(4*sqrt(2)*(I*a^2*e^(3*I*d*x + 3*I*c) + I*a^2*e^(I*d*x + I*c))*sqrt(a/
(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
```

```
2*I*c) + 1)) + sqrt(32*I*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*log(1/4*(4*sq
rt(2)*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqr
t((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) + I*sqrt(32*I*a^5
/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a^2) - sqrt(32*I*a^5/d^2)*(d*e^(2
*I*d*x + 2*I*c) - d)*log(1/4*(4*sqrt(2)*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*
x + 2*I*c) + 1)) - I*sqrt(32*I*a^5/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)
/a^2) - sqrt(4*I*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*log((sqrt(2)*(a^2*e^(
2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d
*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) + I*sqrt(4*I*a^5/d^2)*d*e^(I*d*
x + I*c))*e^(-I*d*x - I*c)/a^2) + sqrt(4*I*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c)
- d)*log((sqrt(2)*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) - I*s
qrt(4*I*a^5/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a^2))/(d*e^(2*I*d*x +
2*I*c) - d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan{(c + dx)} - i))^{\frac{5}{2}}}{\tan^{\frac{3}{2}}{(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(5/2)/tan(d*x+c)**(3/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(5/2)/tan(c + d*x)**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/tan(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}}{{\mathrm{tan}\left(c+d\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(5/2)/tan(c + d*x)^(3/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^(5/2)/tan(c + d*x)^(3/2), x)
```

3.206 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{\tan^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=122

$$-\frac{(4-4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{4ia^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{2a(a+ia\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

```
[Out] (-4+4*I)*a^(5/2)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^
(1/2))/d-4*I*a^2*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(1/2)-2/3*a*(a+I*a*t
an(d*x+c))^(3/2)/d/tan(d*x+c)^(3/2)
```

Rubi [A]
time = 0.13, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3626, 3625, 211}

$$-\frac{(4-4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{4ia^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{2a(a+ia\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(5/2)/Tan[c + d*x]^(5/2),x]
```

```
[Out] ((-4 + 4*I)*a^(5/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I
*a*Tan[c + d*x]]])/d - ((4*I)*a^2*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[Tan[c
 + d*x]]) - (2*a*(a + I*a*Tan[c + d*x])^(3/2))/(3*d*Tan[c + d*x]^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3626

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*b*(a + b*Tan[e + f*x])^(m - 1)*((c +
```

```
d*Tan[e + f*x])^(n + 1)/(f*(m - 1)*(a*c - b*d))), x] + Dist[2*(a^2/(a*c -
b*d)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\tan^{\frac{5}{2}}(c+dx)}\,dx = -\frac{2a(a+ia\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)} + (2ia)\int \frac{(a+ia\tan(c+dx))^{3/2}}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

$$= -\frac{4ia^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}} - \frac{2a(a+ia\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)} - (4a^2)\int \frac{\sqrt{a+}}{\sqrt{t}}$$

$$= -\frac{4ia^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}} - \frac{2a(a+ia\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)} + \frac{(8ia^4)\,\mathrm{Subst}\big(\int}{}$$

$$= -\frac{(4-4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{4ia^2\sqrt{a+ia\tan(c+d}}{d\sqrt{\tan(c+dx)}}$$

Mathematica [A]
time = 2.01, size = 162, normalized size = 1.33

$$-\frac{4i\sqrt{2}\,a^2e^{-i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(e^{i(c+dx)}\left(-3+4e^{2i(c+dx)}\right)-3\left(-1+e^{2i(c+dx)}\right)^{3/2}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)}{3d\left(-1+e^{2i(c+dx)}\right)\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(5/2)/Tan[c + d*x]^(5/2),x]
```

```
[Out] (((-4*I)/3)*Sqrt[2]*a^2*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x
)))]*(E^(I*(c + d*x))*(-3 + 4*E^((2*I)*(c + d*x))) - 3*(-1 + E^((2*I)*(c +
d*x)))^(3/2)*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]]))/(d*E
^(I*(c + d*x))*(-1 + E^((2*I)*(c + d*x)))*Sqrt[Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 371 vs. 2(100) = 200.
time = 0.17, size = 372, normalized size = 3.05

method	result

derivativedivides	$-\frac{\sqrt{a(1+i\tan(dx+c))}\,a^2\left(3i\sqrt{ia}\,\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(d}{\tan(dx+c)+i}\right.\right.}{}$
default	$-\frac{\sqrt{a(1+i\tan(dx+c))}\,a^2\left(3i\sqrt{ia}\,\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(d}{\tan(dx+c)+i}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(5/2)/tan(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/d*(a*(1+I*tan(d*x+c)))^(1/2)*a^2/tan(d*x+c)^(3/2)*(3*I*(I*a)^(1/2)*2^(
1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a
-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2+12*I*ln(1/2*(2*I*a*tan(d*x+
c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*
a)^(1/2)*a*tan(d*x+c)^2-3*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*
(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a
*tan(d*x+c)^2+14*I*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(
1/2)*(-I*a)^(1/2)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a
)^(1/2))/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2)/(-I*a)^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1072 vs. $2(92) = 184$.
time = 0.68, size = 1072, normalized size = 8.79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/tan(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(2*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) +
1)*(((3*I + 3)*a^2*cos(3*d*x + 3*c) - (2*I + 2)*a^2*cos(d*x + c) + (3*I - 3
)*a^2*sin(3*d*x + 3*c) - (2*I - 2)*a^2*sin(d*x + c))*cos(3/2*arctan2(sin(2*
d*x + 2*c), -cos(2*d*x + 2*c) + 1)) + ((3*I - 3)*a^2*cos(3*d*x + 3*c) - (2*
I - 2)*a^2*cos(d*x + c) - (3*I + 3)*a^2*sin(3*d*x + 3*c) + (2*I + 2)*a^2*si
n(d*x + c))*sin(3/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)))*sqrt
(a) + 3*(2*(-(I + 1)*a^2*cos(2*d*x + 2*c)^2 - (I + 1)*a^2*sin(2*d*x + 2*c)^
2 + (2*I + 2)*a^2*cos(2*d*x + 2*c) - (I + 1)*a^2)*arctan2((cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin
(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)) - cos(d*x + c), (cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*
d*x + 2*c), -cos(2*d*x + 2*c) + 1)) - sin(d*x + c)) + (-(I - 1)*a^2*cos(2*d
*x + 2*c)^2 - (I - 1)*a^2*sin(2*d*x + 2*c)^2 + (2*I - 2)*a^2*cos(2*d*x + 2*
c) - (I - 1)*a^2)*log(cos(d*x + c)^2 + sin(d*x + c)^2 + sqrt(cos(2*d*x + 2*
```

```
c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)*(cos(1/2*arctan2(sin(2*
d*x + 2*c), -cos(2*d*x + 2*c) + 1))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), -
cos(2*d*x + 2*c) + 1))^2) - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x +
 2*c) + 1))*sin(d*x + c) + cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), -
cos(2*d*x + 2*c) + 1)))))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(
2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 2*(((-(I + 1)*a^2*cos(d*x + c) - (I - 1)*
a^2*sin(d*x + c))*cos(2*d*x + 2*c)^2 - (I + 1)*a^2*cos(d*x + c) + (-(I + 1)
*a^2*cos(d*x + c) - (I - 1)*a^2*sin(d*x + c))*sin(2*d*x + 2*c)^2 - (I - 1)*
a^2*sin(d*x + c) + 2*((I + 1)*a^2*cos(d*x + c) + (I - 1)*a^2*sin(d*x + c))*
cos(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1))
 + ((-(I - 1)*a^2*cos(d*x + c) + (I + 1)*a^2*sin(d*x + c))*cos(2*d*x + 2*c)
^2 - (I - 1)*a^2*cos(d*x + c) + (-(I - 1)*a^2*cos(d*x + c) + (I + 1)*a^2*si
n(d*x + c))*sin(2*d*x + 2*c)^2 + (I + 1)*a^2*sin(d*x + c) + 2*((I - 1)*a^2*
cos(d*x + c) - (I + 1)*a^2*sin(d*x + c))*cos(2*d*x + 2*c))*sin(1/2*arctan2(
sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)))*sqrt(a))/((cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(5/4)*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(92) = 184$.
time = 0.37, size = 408, normalized size = 3.34

$$\frac{8\sqrt{2}\left(4a^2e^{(5idx+5ic)}+a^2e^{(3idx+3ic)}-3a^2e^{(idx+ic)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}-3\sqrt{-\frac{32ia^5}{d^2}}\left(de^{(4idx+4ic)}-2de^{(2idx+2ic)}+d\right)\log\left(\frac{\left(4\sqrt{2}\left(a^2e^{(2idx+2ic)}+a^2\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}+\sqrt{-\frac{32ia^5}{d^2}}de^{(idx+ic)}\right)e^{(-idx-ic)}}{4a^2}\right)+3\sqrt{-\frac{32ia^5}{d^2}}\left(de^{(4idx+4ic)}-2de^{(2idx+2ic)}+d\right)\log\left(\frac{\left(4\sqrt{2}\left(a^2e^{(2idx+2ic)}+a^2\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}-\sqrt{-\frac{32ia^5}{d^2}}de^{(idx+ic)}\right)e^{(-idx-ic)}}{4a^2}\right)}{6\left(de^{(4idx+4ic)}-2de^{(2idx+2ic)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/tan(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(8*sqrt(2)*(4*a^2*e^(5*I*d*x + 5*I*c) + a^2*e^(3*I*d*x + 3*I*c) - 3*a^2
*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2
*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) - 3*sqrt(-32*I*a^5/d^2)*(d*e^(4*I*d*x
 + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*log(1/4*(4*sqrt(2)*(a^2*e^(2*I*d*x
 + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*
I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) + sqrt(-32*I*a^5/d^2)*d*e^(I*d*x + I*c
))*e^(-I*d*x - I*c)/a^2) + 3*sqrt(-32*I*a^5/d^2)*(d*e^(4*I*d*x + 4*I*c) - 2
*d*e^(2*I*d*x + 2*I*c) + d)*log(1/4*(4*sqrt(2)*(a^2*e^(2*I*d*x + 2*I*c) + a
^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^
(2*I*d*x + 2*I*c) + 1)) - sqrt(-32*I*a^5/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x
- I*c)/a^2))/(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(5/2)/tan(d*x+c)**(5/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(5/2)/tan(c + d*x)**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/tan(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\tan(c + d\,x)\ 1\mathrm{i})^{5/2}}{\tan{(c + d\,x)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(5/2)/tan(c + d*x)^(5/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^(5/2)/tan(c + d*x)^(5/2), x)
```

3.207 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{\tan^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=156

$$-\frac{(4+4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{4a^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{2ia(a+ia\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)}-$$

```
[Out] (-4-4*I)*a^(5/2)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^
(1/2))/d+4*a^2*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(1/2)-2/3*I*a*(a+I*a*t
an(d*x+c))^(3/2)/d/tan(d*x+c)^(3/2)-2/5*(a+I*a*tan(d*x+c))^(5/2)/d/tan(d*x+
c)^(5/2)
```

Rubi [A]

time = 0.18, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3629, 3626, 3625, 211}

$$-\frac{(4+4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{4a^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{2ia(a+ia\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2(a+ia\tan(c+dx))^{5/2}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(5/2)/Tan[c + d*x]^(7/2),x]
```

```
[Out] ((-4 - 4*I)*a^(5/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I
*a*Tan[c + d*x]]])/d + (4*a^2*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[Tan[c + d
*x]]) - (((2*I)/3)*a*(a + I*a*Tan[c + d*x])^(3/2))/(d*Tan[c + d*x]^(3/2)) -
 (2*(a + I*a*Tan[c + d*x])^(5/2))/(5*d*Tan[c + d*x]^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3626

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*b*(a + b*Tan[e + f*x])^(m - 1)*((c +
 d*Tan[e + f*x])^(n + 1)/(f*(m - 1)*(a*c - b*d))), x] + Dist[2*(a^2/(a*c -
b*d)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2]
```

Rule 3629

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-d)*(a + b*Tan[e + f*x])^m*((c + d*Ta
n[e + f*x])^(n + 1)/(f*m*(c^2 + d^2))), x] + Dist[a/(a*c - b*d), Int[(a + b
*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\tan^{\frac{7}{2}}(c+dx)}\,dx = -\frac{2(a+ia\tan(c+dx))^{5/2}}{5d\tan^{\frac{5}{2}}(c+dx)} + i\int \frac{(a+ia\tan(c+dx))^{5/2}}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$

$$= -\frac{2ia(a+ia\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{2(a+ia\tan(c+dx))^{5/2}}{5d\tan^{\frac{5}{2}}(c+dx)} - (2a)\int \frac{(a+ia\tan}{\tan^{\frac{3}{2}}(}$$

$$= \frac{4a^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}} - \frac{2ia(a+ia\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{2(a+ia\tan(c+d}{5d\tan^{\frac{5}{2}}(c+dx}$$

$$= \frac{4a^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}} - \frac{2ia(a+ia\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{2(a+ia\tan(c+d}{5d\tan^{\frac{5}{2}}(c+dx}$$

$$= -\frac{(4+4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} + \frac{4a^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

Mathematica [A]
time = 3.72, size = 187, normalized size = 1.20

$$-\frac{4ia^2e^{-i(c+dx)}\left(e^{i(c+dx)}\left(15-35e^{2i(c+dx)}+26e^{4i(c+dx)}\right)-15\left(-1+e^{2i(c+dx)}\right)^{5/2}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{15d\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}\left(-1+e^{4i(c+dx)}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(5/2)/Tan[c + d*x]^(7/2),x]
```

```
[Out] (((-4*I)/15)*a^2*(E^(I*(c + d*x))*(15 - 35*E^((2*I)*(c + d*x)) + 26*E^((4*I
)*(c + d*x))) - 15*(-1 + E^((2*I)*(c + d*x)))^(5/2)*ArcTanh[E^(I*(c + d*x))
/Sqrt[-1 + E^((2*I)*(c + d*x))]])*Cot[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/
(d*E^(I*(c + d*x))*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c
+ d*x)))]*(-1 + E^((4*I)*(c + d*x))))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 413 vs. 2(127) = 254.
time = 0.17, size = 414, normalized size = 2.65

method	result
derivativedivides	$\frac{\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\, a^{2}\left(15i\sqrt{ia}\,\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1+i\tan\left(d\right.\right.}}{\tan(dx+c)+i}\right.\right.}{}$
default	$\frac{\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\, a^{2}\left(15i\sqrt{ia}\,\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1+i\tan\left(d\right.\right.}}{\tan(dx+c)+i}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(5/2)/tan(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15/d*(a*(1+I*tan(d*x+c)))^(1/2)*a^2/tan(d*x+c)^(5/2)*(15*I*(I*a)^(1/2)*2^
(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*
a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3+15*(I*a)^(1/2)*2^(1/2)*ln(
-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan
(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3+76*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^
(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^2+60*ln(1/2*(2*I*a*tan(d*x+c)+2*(
a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/
2)*a*tan(d*x+c)^3-22*I*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*
a)^(1/2)*(-I*a)^(1/2)-6*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(
-I*a)^(1/2))/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2)/(-I*a)^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1284 vs. 2(118) = 236.
time = 0.68, size = 1284, normalized size = 8.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/tan(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1
)*(((30*I - 30)*a^2*cos(3*d*x + 3*c) - (31*I - 31)*a^2*cos(d*x + c) - (30*I
```

```
 + 30)*a^2*sin(3*d*x + 3*c) + (31*I + 31)*a^2*sin(d*x + c))*cos(3/2*arctan2
(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)) + (-(30*I + 30)*a^2*cos(3*d*x +
3*c) + (31*I + 31)*a^2*cos(d*x + c) - (30*I - 30)*a^2*sin(3*d*x + 3*c) + (3
1*I - 31)*a^2*sin(d*x + c))*sin(3/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x +
2*c) + 1)))*sqrt(a) + 15*(2*(-(I - 1)*a^2*cos(2*d*x + 2*c)^2 - (I - 1)*a^2*
sin(2*d*x + 2*c)^2 + (2*I - 2)*a^2*cos(2*d*x + 2*c) - (I - 1)*a^2)*arctan2(
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*si
n(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)) - cos(d*x + c), (co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1
/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)) - sin(d*x + c)) + ((I
+ 1)*a^2*cos(2*d*x + 2*c)^2 + (I + 1)*a^2*sin(2*d*x + 2*c)^2 - (2*I + 2)*a^
2*cos(2*d*x + 2*c) + (I + 1)*a^2)*log(cos(d*x + c)^2 + sin(d*x + c)^2 + sqr
t(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)*(cos(1/
2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1))^2 + sin(1/2*arctan2(sin
(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1))^2) - 2*(cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*
c), -cos(2*d*x + 2*c) + 1))*sin(d*x + c) + cos(d*x + c)*sin(1/2*arctan2(sin
(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)))))*(cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + ((-(30*I - 30)*a^2*cos(5*
d*x + 5*c) + (25*I - 25)*a^2*cos(3*d*x + 3*c) - (7*I - 7)*a^2*cos(d*x + c)
+ (30*I + 30)*a^2*sin(5*d*x + 5*c) - (25*I + 25)*a^2*sin(3*d*x + 3*c) + (7*
I + 7)*a^2*sin(d*x + c))*cos(5/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c
) + 1)) + 8*(((I - 1)*a^2*cos(d*x + c) - (I + 1)*a^2*sin(d*x + c))*cos(2*d*
x + 2*c)^2 + (I - 1)*a^2*cos(d*x + c) + ((I - 1)*a^2*cos(d*x + c) - (I + 1)
*a^2*sin(d*x + c))*sin(2*d*x + 2*c)^2 - (I + 1)*a^2*sin(d*x + c) + 2*(-(I -
 1)*a^2*cos(d*x + c) + (I + 1)*a^2*sin(d*x + c))*cos(2*d*x + 2*c))*cos(1/2*
arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)) + ((30*I + 30)*a^2*cos(5*
d*x + 5*c) - (25*I + 25)*a^2*cos(3*d*x + 3*c) + (7*I + 7)*a^2*cos(d*x + c)
+ (30*I - 30)*a^2*sin(5*d*x + 5*c) - (25*I - 25)*a^2*sin(3*d*x + 3*c) + (7*
I - 7)*a^2*sin(d*x + c))*sin(5/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c
) + 1)) + 8*((-(I + 1)*a^2*cos(d*x + c) - (I - 1)*a^2*sin(d*x + c))*cos(2*d
*x + 2*c)^2 - (I + 1)*a^2*cos(d*x + c) + (-(I + 1)*a^2*cos(d*x + c) - (I -
1)*a^2*sin(d*x + c))*sin(2*d*x + 2*c)^2 - (I - 1)*a^2*sin(d*x + c) + 2*((I
+ 1)*a^2*cos(d*x + c) + (I - 1)*a^2*sin(d*x + c))*cos(2*d*x + 2*c))*sin(1/2
*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)))*sqrt(a))/((cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(5/4)*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(118) = 236$.
time = 0.39, size = 466, normalized size = 2.99

$$-\frac{8\sqrt{2}\left(-26i\,a^{2}e^{(7i\,dx+7i\,c)}+9i\,a^{2}e^{(5i\,dx+5i\,c)}+20i\,a^{2}e^{(3i\,dx+3i\,c)}-15i\,a^{2}e^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}-15\sqrt{\frac{32i\,a^{5}}{d^{2}}}\left(de^{(6i\,dx+6i\,c)}-3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}-d\right)\log\left(\frac{\left(4\sqrt{2}\left(a^{2}e^{(2i\,dx+2i\,c)}+a^{2}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}+i\sqrt{\frac{32i\,a^{5}}{d^{2}}}\,de^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{4\,a^{2}}\right)+15\sqrt{\frac{32i\,a^{5}}{d^{2}}}\left(de^{(6i\,dx+6i\,c)}-3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}-d\right)\log\left(\frac{\left(4\sqrt{2}\left(a^{2}e^{(2i\,dx+2i\,c)}+a^{2}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}-i\sqrt{\frac{32i\,a^{5}}{d^{2}}}\,de^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{4\,a^{2}}\right)}{30\left(de^{(6i\,dx+6i\,c)}-3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/tan(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/30*(8*sqrt(2)*(-26*I*a^2*e^(7*I*d*x + 7*I*c) + 9*I*a^2*e^(5*I*d*x + 5*I*
c) + 20*I*a^2*e^(3*I*d*x + 3*I*c) - 15*I*a^2*e^(I*d*x + I*c))*sqrt(a/(e^(2*
I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c)
 + 1)) - 15*sqrt(32*I*a^5/d^2)*(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*
I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)*log(1/4*(4*sqrt(2)*(a^2*e^(2*I*d*x + 2*
I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c)
+ I)/(e^(2*I*d*x + 2*I*c) + 1)) + I*sqrt(32*I*a^5/d^2)*d*e^(I*d*x + I*c))*e
^(-I*d*x - I*c)/a^2) + 15*sqrt(32*I*a^5/d^2)*(d*e^(6*I*d*x + 6*I*c) - 3*d*e
^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)*log(1/4*(4*sqrt(2)*(a^2*e
^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I
*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) - I*sqrt(32*I*a^5/d^2)*d*e^(I
*d*x + I*c))*e^(-I*d*x - I*c)/a^2))/(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x
 + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(5/2)/tan(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/tan(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}}{{\mathrm{tan}\left(c+d\,x\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(5/2)/tan(c + d*x)^(7/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^(5/2)/tan(c + d*x)^(7/2), x)
```

3.208 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{\tan^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=202

$$\frac{(4-4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^2\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{6ia^2\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{5}{2}}(c+dx)}+\frac{32a}{}$$

```
[Out] (4-4*I)*a^(5/2)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(
1/2))/d+104/21*I*a^2*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(1/2)-2/7*a^2*(a
+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(7/2)-6/7*I*a^2*(a+I*a*tan(d*x+c))^(1/2
)/d/tan(d*x+c)^(5/2)+32/21*a^2*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(3/2)
```

Rubi [A]
time = 0.36, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3634, 3679, 12, 3625, 211}

$$\frac{(4-4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{32a^2\sqrt{a+ia\tan(c+dx)}}{21d\tan^{\frac{3}{2}}(c+dx)}-\frac{6ia^2\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{5}{2}}(c+dx)}-\frac{2a^2\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+\frac{104ia^2\sqrt{a+ia\tan(c+dx)}}{21d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(5/2)/Tan[c + d*x]^(9/2),x]
```

```
[Out] ((4 - 4*I)*a^(5/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*
a*Tan[c + d*x]]])/d - (2*a^2*Sqrt[a + I*a*Tan[c + d*x]])/(7*d*Tan[c + d*x]^
(7/2)) - (((6*I)/7)*a^2*Sqrt[a + I*a*Tan[c + d*x]])/(d*Tan[c + d*x]^(5/2))
+ (32*a^2*Sqrt[a + I*a*Tan[c + d*x]])/(21*d*Tan[c + d*x]^(3/2)) + (((104*I)
/21)*a^2*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
```

```
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3634

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-a^2)*(b*c - a*d)*(a + b*Tan[e + f*x]
)^(m - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] + Di
st[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[b*(b*c*(m - 2) - a*d*(m - 2*n - 4)) + (a*b*c*(m - 2)
+ b^2*d*(n + 1) - a^2*d*(m + n - 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\tan^{\frac{9}{2}}(c+dx)}\,dx = -\frac{2a^2\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)} - \frac{2}{7}\int \frac{\sqrt{a+ia\tan(c+dx)}\left(-\frac{15ia^2}{2}+\frac{13}{2}a^2\tan(\right.}{\tan^{\frac{7}{2}}(c+dx)}$$

$$= -\frac{2a^2\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)} - \frac{6ia^2\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{5}{2}}(c+dx)} - \frac{4\int \frac{\sqrt{a+ia\tan}}{}}{}$$

$$= -\frac{2a^2\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)} - \frac{6ia^2\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{5}{2}}(c+dx)} + \frac{32a^2\sqrt{a+ia\tan}}{21d\tan^{\frac{3}{2}}(c-}$$

$$= -\frac{2a^2\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)} - \frac{6ia^2\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{5}{2}}(c+dx)} + \frac{32a^2\sqrt{a+ia\tan}}{21d\tan^{\frac{3}{2}}(c-}$$

$$= -\frac{2a^2\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)} - \frac{6ia^2\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{5}{2}}(c+dx)} + \frac{32a^2\sqrt{a+ia\tan}}{21d\tan^{\frac{3}{2}}(c-}$$

$$= -\frac{2a^2\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)} - \frac{6ia^2\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{5}{2}}(c+dx)} + \frac{32a^2\sqrt{a+ia\tan}}{21d\tan^{\frac{3}{2}}(c-}$$

$$= \frac{(4-4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{2a^2\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

Mathematica [A]
time = 3.09, size = 188, normalized size = 0.93

$$\frac{4i\sqrt{2}\,a^2e^{-i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(e^{i(c+dx)}\left(-21+70e^{2i(c+dx)}-77e^{4i(c+dx)}+40e^{6i(c+dx)}\right)-21\left(-1+e^{2i(c+dx)}\right)^{7/2}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)}{21d\left(-1+e^{2i(c+dx)}\right)^3\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(5/2)/Tan[c + d*x]^(9/2),x]
```

```
[Out] (((4*I)/21)*Sqrt[2]*a^2*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x
)))]*(E^(I*(c + d*x))*(-21 + 70*E^((2*I)*(c + d*x)) - 77*E^((4*I)*(c + d*x)
) + 40*E^((6*I)*(c + d*x))) - 21*(-1 + E^((2*I)*(c + d*x)))^(7/2)*ArcTanh[E
^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]]))/(d*E^(I*(c + d*x))*(-1 + E
^((2*I)*(c + d*x)))^3*Sqrt[Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 458 vs. 2(163) = 326.
time = 0.17, size = 459, normalized size = 2.27

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(dx+c))}\, a^2 \left(21i\sqrt{ia}\,\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(d}{\tan(dx+c)+i}\right.\right.}{}$
default	$\frac{\sqrt{a(1+i\tan(dx+c))}\, a^2 \left(21i\sqrt{ia}\,\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(d}{\tan(dx+c)+i}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(5/2)/tan(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/21/d*(a*(1+I*tan(d*x+c)))^(1/2)*a^2/tan(d*x+c)^(7/2)*(21*I*(I*a)^(1/2)*2^
(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*
a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4+84*I*ln(1/2*(2*I*a*tan(d*x
+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I
*a)^(1/2)*a*tan(d*x+c)^4-21*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2
)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))
*a*tan(d*x+c)^4+32*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)
^(1/2)*tan(d*x+c)^2+104*I*tan(d*x+c)^3*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2
)*(I*a)^(1/2)*(-I*a)^(1/2)-18*I*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^
(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)-6*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a
)^(1/2)*(-I*a)^(1/2))/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2)/(-I
*a)^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3186 vs. $2(152) = 304$.
time = 0.96, size = 3186, normalized size = 15.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/tan(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] -2/105*(2*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c)
 + 1)*(3*((35*I + 35)*a^2*cos(7*d*x + 7*c) - (35*I + 35)*a^2*cos(5*d*x + 5*
c) + (21*I + 21)*a^2*cos(3*d*x + 3*c) - (I + 1)*a^2*cos(d*x + c) + (35*I -
35)*a^2*sin(7*d*x + 7*c) - (35*I - 35)*a^2*sin(5*d*x + 5*c) + (21*I - 21)*a
^2*sin(3*d*x + 3*c) - (I - 1)*a^2*sin(d*x + c))*cos(7/2*arctan2(sin(2*d*x +
 2*c), -cos(2*d*x + 2*c) + 1)) + 5*(13*(-(I + 1)*a^2*cos(d*x + c) - (I - 1)
*a^2*sin(d*x + c))*cos(2*d*x + 2*c)^2 - (13*I + 13)*a^2*cos(d*x + c) + 13*(
-(I + 1)*a^2*cos(d*x + c) - (I - 1)*a^2*sin(d*x + c))*sin(2*d*x + 2*c)^2 -
(13*I - 13)*a^2*sin(d*x + c) + 21*((I + 1)*a^2*cos(2*d*x + 2*c)^2 + (I + 1)
*a^2*sin(2*d*x + 2*c)^2 - (2*I + 2)*a^2*cos(2*d*x + 2*c) + (I + 1)*a^2)*cos
```

```
(3*d*x + 3*c) + 26*((I + 1)*a^2*cos(d*x + c) + (I - 1)*a^2*sin(d*x + c))*co
s(2*d*x + 2*c) + 21*((I - 1)*a^2*cos(2*d*x + 2*c)^2 + (I - 1)*a^2*sin(2*d*x
 + 2*c)^2 - (2*I - 2)*a^2*cos(2*d*x + 2*c) + (I - 1)*a^2)*sin(3*d*x + 3*c))
*cos(3/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)) + 3*((35*I - 35)
*a^2*cos(7*d*x + 7*c) - (35*I - 35)*a^2*cos(5*d*x + 5*c) + (21*I - 21)*a^2*
cos(3*d*x + 3*c) - (I - 1)*a^2*cos(d*x + c) - (35*I + 35)*a^2*sin(7*d*x + 7
*c) + (35*I + 35)*a^2*sin(5*d*x + 5*c) - (21*I + 21)*a^2*sin(3*d*x + 3*c) +
 (I + 1)*a^2*sin(d*x + c))*sin(7/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2
*c) + 1)) + 5*(13*(-(I - 1)*a^2*cos(d*x + c) + (I + 1)*a^2*sin(d*x + c))*co
s(2*d*x + 2*c)^2 - (13*I - 13)*a^2*cos(d*x + c) + 13*(-(I - 1)*a^2*cos(d*x
+ c) + (I + 1)*a^2*sin(d*x + c))*sin(2*d*x + 2*c)^2 + (13*I + 13)*a^2*sin(d
*x + c) + 21*((I - 1)*a^2*cos(2*d*x + 2*c)^2 + (I - 1)*a^2*sin(2*d*x + 2*c)
^2 - (2*I - 2)*a^2*cos(2*d*x + 2*c) + (I - 1)*a^2)*cos(3*d*x + 3*c) + 26*((
I - 1)*a^2*cos(d*x + c) - (I + 1)*a^2*sin(d*x + c))*cos(2*d*x + 2*c) + 21*(
-(I + 1)*a^2*cos(2*d*x + 2*c)^2 - (I + 1)*a^2*sin(2*d*x + 2*c)^2 + (2*I + 2
)*a^2*cos(2*d*x + 2*c) - (I + 1)*a^2)*sin(3*d*x + 3*c))*sin(3/2*arctan2(sin
(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)))*sqrt(a) + 105*(2*(-(I + 1)*a^2*cos(
2*d*x + 2*c)^4 - (I + 1)*a^2*sin(2*d*x + 2*c)^4 + (4*I + 4)*a^2*cos(2*d*x +
 2*c)^3 - (6*I + 6)*a^2*cos(2*d*x + 2*c)^2 + (4*I + 4)*a^2*cos(2*d*x + 2*c)
 + 2*(-(I + 1)*a^2*cos(2*d*x + 2*c)^2 + (2*I + 2)*a^2*cos(2*d*x + 2*c) - (I
 + 1)*a^2)*sin(2*d*x + 2*c)^2 - (I + 1)*a^2)*arctan2((cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*
x + 2*c), -cos(2*d*x + 2*c) + 1)) - cos(d*x + c), (cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x +
 2*c), -cos(2*d*x + 2*c) + 1)) - sin(d*x + c)) + (-(I - 1)*a^2*cos(2*d*x +
2*c)^4 - (I - 1)*a^2*sin(2*d*x + 2*c)^4 + (4*I - 4)*a^2*cos(2*d*x + 2*c)^3
- (6*I - 6)*a^2*cos(2*d*x + 2*c)^2 + (4*I - 4)*a^2*cos(2*d*x + 2*c) + 2*(-(
I - 1)*a^2*cos(2*d*x + 2*c)^2 + (2*I - 2)*a^2*cos(2*d*x + 2*c) - (I - 1)*a^
2)*sin(2*d*x + 2*c)^2 - (I - 1)*a^2)*log(cos(d*x + c)^2 + sin(d*x + c)^2 +
sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)*(cos
(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1))^2 + sin(1/2*arctan2(
sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1))^2) - 2*(cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
 2*c), -cos(2*d*x + 2*c) + 1))*sin(d*x + c) + cos(d*x + c)*sin(1/2*arctan2(
sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)))))*(cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 2*((152*(-(I + 1)*a^2*
cos(d*x + c) - (I - 1)*a^2*sin(d*x + c))*cos(2*d*x + 2*c)^2 - (152*I + 152)
*a^2*cos(d*x + c) + 152*(-(I + 1)*a^2*cos(d*x + c) - (I - 1)*a^2*sin(d*x +
c))*sin(2*d*x + 2*c)^2 - (152*I - 152)*a^2*sin(d*x + c) + 105*(-(I + 1)*a^2
*cos(2*d*x + 2*c)^2 - (I + 1)*a^2*sin(2*d*x + 2*c)^2 + (2*I + 2)*a^2*cos(2*
d*x + 2*c) - (I + 1)*a^2)*cos(5*d*x + 5*c) + 245*((I + 1)*a^2*cos(2*d*x + 2
*c)^2 + (I + 1)*a^2*sin(2*d*x + 2*c)^2 - (2*I + 2)*a^2*cos(2*d*x + 2*c) + (
I + 1)*a^2)*cos(3*d*x + 3*c) + 304*((I + 1)*a^2*cos(d*x + c) + (I - 1)*a^2*
sin(d*x + c))*cos(2*d*x + 2*c) + 105*(-(I - 1)*a^2*cos(2*d*x + 2*c)^2 - (I
- 1)*a^2*sin(2*d*x + 2*c)^2 + (2*I - 2)*a^2*cos(2*d*x + 2*c) - (I - 1)*a^2)
```

```
*sin(5*d*x + 5*c) + 245*((I - 1)*a^2*cos(2*d*x + 2*c)^2 + (I - 1)*a^2*sin(2
*d*x + 2*c)^2 - (2*I - 2)*a^2*cos(2*d*x + 2*c) + (I - 1)*a^2)*sin(3*d*x + 3
*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)) + 115*(((I +
 1)*a^2*cos(d*x + c) + (I - 1)*a^2*sin(d*x + c))*cos(2*d*x + 2*c)^4 + ((I +
 1)*a^2*cos(d*x + c) + (I - 1)*a^2*sin(d*x + c))*sin(2*d*x + 2*c)^4 + 4*(-(
I + 1)*a^2*cos(d*x + c) - (I - 1)*a^2*sin(d*x + c))*cos(2*d*x + 2*c)^3 + 6*
((I + 1)*a^2*cos(d*x + c) + (I - 1)*a^2*sin(d*x + c))*cos(2*d*x + 2*c)^2 +
(I + 1)*a^2*cos(d*x + c) + 2*(((I + 1)*a^2*cos(d*x + c) + (I - 1)*a^2*sin(d
*x + c))*cos(2*d*x + 2*c)^2 + (I + 1)*a^2*cos(d*x + c) + (I - 1)*a^2*sin(d*
x + c) + 2*(-(I + 1)*a^2*cos(d*x + c) - (I - 1)*a^2*sin(d*x + c))*cos(2*d*x
 + 2*c))*sin(2*d*x + 2*c)^2 + (I - 1)*a^2*sin(d*x + c) + 4*(-(I + 1)*a^2*co
s(d*x + c) - (I - 1)*a^2*sin(d*x + c))*cos(2*d*...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 509 vs. 2(152) = 304.
time = 0.37, size = 509, normalized size = 2.52

$$-\frac{8\sqrt{2}\left(40\,a^{2}e^{(9i\,dx+9i\,c)}-37\,a^{2}e^{(7i\,dx+7i\,c)}-7\,a^{2}e^{(5i\,dx+5i\,c)}+49\,a^{2}e^{(3i\,dx+3i\,c)}-21\,a^{2}e^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}-21\sqrt{-\frac{32i\,a^{5}}{d^{2}}}\left(de^{(8i\,dx+8i\,c)}-4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}-4\,de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{\left(4\sqrt{2}\left(a^{2}e^{(2i\,dx+2i\,c)}+a^{2}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}+\sqrt{-\frac{32i\,a^{5}}{d^{2}}}\,de^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{4\,a^{2}}\right)+21\sqrt{-\frac{32i\,a^{5}}{d^{2}}}\left(de^{(8i\,dx+8i\,c)}-4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}-4\,de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{\left(4\sqrt{2}\left(a^{2}e^{(2i\,dx+2i\,c)}+a^{2}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}-\sqrt{-\frac{32i\,a^{5}}{d^{2}}}\,de^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{4\,a^{2}}\right)}{42\left(de^{(8i\,dx+8i\,c)}-4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}-4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/tan(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/42*(8*sqrt(2)*(40*a^2*e^(9*I*d*x + 9*I*c) - 37*a^2*e^(7*I*d*x + 7*I*c) -
 7*a^2*e^(5*I*d*x + 5*I*c) + 49*a^2*e^(3*I*d*x + 3*I*c) - 21*a^2*e^(I*d*x +
 I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/
(e^(2*I*d*x + 2*I*c) + 1)) - 21*sqrt(-32*I*a^5/d^2)*(d*e^(8*I*d*x + 8*I*c)
- 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) - 4*d*e^(2*I*d*x + 2*I*
c) + d)*log(1/4*(4*sqrt(2)*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) +
1)) + sqrt(-32*I*a^5/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a^2) + 21*sqr
t(-32*I*a^5/d^2)*(d*e^(8*I*d*x + 8*I*c) - 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(
4*I*d*x + 4*I*c) - 4*d*e^(2*I*d*x + 2*I*c) + d)*log(1/4*(4*sqrt(2)*(a^2*e^(
2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d
*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) - sqrt(-32*I*a^5/d^2)*d*e^(I*d*
x + I*c))*e^(-I*d*x - I*c)/a^2))/(d*e^(8*I*d*x + 8*I*c) - 4*d*e^(6*I*d*x +
6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) - 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(5/2)/tan(d*x+c)**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/tan(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^{5/2}}{\tan\left(c+d\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(5/2)/tan(c + d*x)^(9/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^(5/2)/tan(c + d*x)^(9/2), x)
```

3.209 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{\tan^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=239

$$\frac{(4+4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^2\sqrt{a+ia\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}-\frac{38ia^2\sqrt{a+ia\tan(c+dx)}}{63d\tan^{\frac{7}{2}}(c+dx)}+\frac{9}{}$$

```
[Out] (4+4*I)*a^(5/2)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(
1/2))/d-1576/315*a^2*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(1/2)-2/9*a^2*(a
+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(9/2)-38/63*I*a^2*(a+I*a*tan(d*x+c))^(1
/2)/d/tan(d*x+c)^(7/2)+92/105*a^2*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(5/
2)+472/315*I*a^2*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(3/2)
```

Rubi [A]
time = 0.47, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3634, 3679, 12, 3625, 211}

$$\frac{(4+4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{472ia^2\sqrt{a+ia\tan(c+dx)}}{315d\tan^{\frac{3}{2}}(c+dx)}+\frac{92a^2\sqrt{a+ia\tan(c+dx)}}{105d\tan^{\frac{5}{2}}(c+dx)}-\frac{38ia^2\sqrt{a+ia\tan(c+dx)}}{63d\tan^{\frac{7}{2}}(c+dx)}-\frac{2a^2\sqrt{a+ia\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}-\frac{1576a^2\sqrt{a+ia\tan(c+dx)}}{315d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(5/2)/Tan[c + d*x]^(11/2),x]
```

```
[Out] ((4 + 4*I)*a^(5/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*
a*Tan[c + d*x]]])/d - (2*a^2*Sqrt[a + I*a*Tan[c + d*x]])/(9*d*Tan[c + d*x]^
(9/2)) - (((38*I)/63)*a^2*Sqrt[a + I*a*Tan[c + d*x]])/(d*Tan[c + d*x]^(7/2)
) + (92*a^2*Sqrt[a + I*a*Tan[c + d*x]])/(105*d*Tan[c + d*x]^(5/2)) + (((472
*I)/315)*a^2*Sqrt[a + I*a*Tan[c + d*x]])/(d*Tan[c + d*x]^(3/2)) - (1576*a^2
*Sqrt[a + I*a*Tan[c + d*x]])/(315*d*Sqrt[Tan[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
```

```
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3634

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-a^2)*(b*c - a*d)*(a + b*Tan[e + f*x]
)^(m - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] + Di
st[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[b*(b*c*(m - 2) - a*d*(m - 2*n - 4)) + (a*b*c*(m - 2)
+ b^2*d*(n + 1) - a^2*d*(m + n - 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\tan^{\frac{11}{2}}(c+dx)}\,dx = -\frac{2a^2\sqrt{a+ia\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)} - \frac{2}{9}\int \frac{\sqrt{a+ia\tan(c+dx)}\left(-\frac{19ia^2}{2}+\frac{17}{2}a^2\tan\right.}{\tan^{\frac{9}{2}}(c+dx)}$$

$$= -\frac{2a^2\sqrt{a+ia\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)} - \frac{38ia^2\sqrt{a+ia\tan(c+dx)}}{63d\tan^{\frac{7}{2}}(c+dx)} - \frac{4\int \sqrt{a+ia\tan}}{}$$

$$= -\frac{2a^2\sqrt{a+ia\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)} - \frac{38ia^2\sqrt{a+ia\tan(c+dx)}}{63d\tan^{\frac{7}{2}}(c+dx)} + \frac{92a^2\sqrt{a+ia}}{105d\tan^{\frac{5}{2}}}$$

$$= -\frac{2a^2\sqrt{a+ia\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)} - \frac{38ia^2\sqrt{a+ia\tan(c+dx)}}{63d\tan^{\frac{7}{2}}(c+dx)} + \frac{92a^2\sqrt{a+ia}}{105d\tan^{\frac{5}{2}}}$$

$$= -\frac{2a^2\sqrt{a+ia\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)} - \frac{38ia^2\sqrt{a+ia\tan(c+dx)}}{63d\tan^{\frac{7}{2}}(c+dx)} + \frac{92a^2\sqrt{a+ia}}{105d\tan^{\frac{5}{2}}}$$

$$= -\frac{2a^2\sqrt{a+ia\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)} - \frac{38ia^2\sqrt{a+ia\tan(c+dx)}}{63d\tan^{\frac{7}{2}}(c+dx)} + \frac{92a^2\sqrt{a+ia}}{105d\tan^{\frac{5}{2}}}$$

$$= -\frac{2a^2\sqrt{a+ia\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)} - \frac{38ia^2\sqrt{a+ia\tan(c+dx)}}{63d\tan^{\frac{7}{2}}(c+dx)} + \frac{92a^2\sqrt{a+ia}}{105d\tan^{\frac{5}{2}}}$$

$$= \frac{(4+4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{2a^2\sqrt{a+ia\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

Mathematica [A]

time = 4.99, size = 207, normalized size = 0.87

$$\frac{a^2\left(\frac{10080e^{-i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)}{\sqrt{-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}} - \csc^5(c+dx)(1650\cos(c+dx)-2051\cos(3(c+dx))+961\cos(5(c+dx))-282i\sin(c+dx)+49i\sin(3(c+dx))+331i\sin(5(c+dx)))\sqrt{\tan(c+dx)}\right)\sqrt{a+ia\tan(c+dx)}}{2520d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(5/2)/Tan[c + d*x]^(11/2),x]
```

```
[Out] (a^2*((10080*Sqrt[-1 + E^((2*I)*(c + d*x))]*ArcTanh[E^(I*(c + d*x))/Sqrt[-1
 + E^((2*I)*(c + d*x))]])/(E^(I*(c + d*x))*Sqrt[((-I)*(-1 + E^((2*I)*(c + d
*x))))/(1 + E^((2*I)*(c + d*x)))]) - Csc[c + d*x]^5*(1650*Cos[c + d*x] - 20
51*Cos[3*(c + d*x)] + 961*Cos[5*(c + d*x)] - (282*I)*Sin[c + d*x] + (49*I)*
```

```
Sin[3*(c + d*x)] + (331*I)*Sin[5*(c + d*x)])*Sqrt[Tan[c + d*x]])*Sqrt[a + I
*a*Tan[c + d*x]])/(2520*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 500 vs. 2(193) = 386.
time = 0.19, size = 501, normalized size = 2.10

method	result
derivativedivides	$-\frac{\sqrt{a(1+i\tan(dx+c))}\,a^2\left(315i\sqrt{ia}\,\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(}{\tan(dx+c)+i}\right.\right.}{}$
default	$-\frac{\sqrt{a(1+i\tan(dx+c))}\,a^2\left(315i\sqrt{ia}\,\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(}{\tan(dx+c)+i}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(5/2)/tan(d*x+c)^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/315/d*(a*(1+I*tan(d*x+c)))^(1/2)*a^2*(315*I*(I*a)^(1/2)*2^(1/2)*ln(-(-2*
2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+
c))/(tan(d*x+c)+I))*a*tan(d*x+c)^5+315*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*
(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan
(d*x+c)+I))*a*tan(d*x+c)^5+1260*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1
+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)
^5-472*I*tan(d*x+c)^3*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I
*a)^(1/2)+1576*tan(d*x+c)^4*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/
2)*(-I*a)^(1/2)+190*I*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a
)^(1/2)*(-I*a)^(1/2)-276*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*
(-I*a)^(1/2)*tan(d*x+c)^2+70*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1
/2)*(-I*a)^(1/2))/tan(d*x+c)^(9/2)/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I
*a)^(1/2)/(-I*a)^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3508 vs. 2(181) = 362.
time = 1.70, size = 3508, normalized size = 14.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/tan(d*x+c)^(11/2),x, algorithm="maxima")
```

```
[Out] -1/1260*(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c)
+ 1)*(((5040*I - 5040)*a^2*cos(7*d*x + 7*c) - (16800*I - 16800)*a^2*cos(5*d
*x + 5*c) + (20496*I - 20496)*a^2*cos(3*d*x + 3*c) - (9071*I - 9071)*a^2*co
```

s(d*x + c) - (5040*I + 5040)*a^2*sin(7*d*x + 7*c) + (16800*I + 16800)*a^2*s
in(5*d*x + 5*c) - (20496*I + 20496)*a^2*sin(3*d*x + 3*c) + (9071*I + 9071)*
a^2*sin(d*x + c))*cos(7/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1))
 + 8*(121*(-(I - 1)*a^2*cos(d*x + c) + (I + 1)*a^2*sin(d*x + c))*cos(2*d*x
+ 2*c)^2 - (121*I - 121)*a^2*cos(d*x + c) + 121*(-(I - 1)*a^2*cos(d*x + c)
+ (I + 1)*a^2*sin(d*x + c))*sin(2*d*x + 2*c)^2 + (121*I + 121)*a^2*sin(d*x
+ c) + 630*((I - 1)*a^2*cos(2*d*x + 2*c)^2 + (I - 1)*a^2*sin(2*d*x + 2*c)^2
 - (2*I - 2)*a^2*cos(2*d*x + 2*c) + (I - 1)*a^2)*cos(3*d*x + 3*c) + 242*((I
 - 1)*a^2*cos(d*x + c) - (I + 1)*a^2*sin(d*x + c))*cos(2*d*x + 2*c) + 630*(
-(I + 1)*a^2*cos(2*d*x + 2*c)^2 - (I + 1)*a^2*sin(2*d*x + 2*c)^2 + (2*I + 2
)*a^2*cos(2*d*x + 2*c) - (I + 1)*a^2)*sin(3*d*x + 3*c))*cos(3/2*arctan2(sin
(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)) + (-(5040*I + 5040)*a^2*cos(7*d*x +
7*c) + (16800*I + 16800)*a^2*cos(5*d*x + 5*c) - (20496*I + 20496)*a^2*cos(3
*d*x + 3*c) + (9071*I + 9071)*a^2*cos(d*x + c) - (5040*I - 5040)*a^2*sin(7*
d*x + 7*c) + (16800*I - 16800)*a^2*sin(5*d*x + 5*c) - (20496*I - 20496)*a^2
*sin(3*d*x + 3*c) + (9071*I - 9071)*a^2*sin(d*x + c))*sin(7/2*arctan2(sin(2
*d*x + 2*c), -cos(2*d*x + 2*c) + 1)) + 8*(121*((I + 1)*a^2*cos(d*x + c) + (
I - 1)*a^2*sin(d*x + c))*cos(2*d*x + 2*c)^2 + (121*I + 121)*a^2*cos(d*x + c
) + 121*((I + 1)*a^2*cos(d*x + c) + (I - 1)*a^2*sin(d*x + c))*sin(2*d*x + 2
*c)^2 + (121*I - 121)*a^2*sin(d*x + c) + 630*(-(I + 1)*a^2*cos(2*d*x + 2*c)
^2 - (I + 1)*a^2*sin(2*d*x + 2*c)^2 + (2*I + 2)*a^2*cos(2*d*x + 2*c) - (I +
 1)*a^2)*cos(3*d*x + 3*c) + 242*(-(I + 1)*a^2*cos(d*x + c) - (I - 1)*a^2*si
n(d*x + c))*cos(2*d*x + 2*c) + 630*(-(I - 1)*a^2*cos(2*d*x + 2*c)^2 - (I -
1)*a^2*sin(2*d*x + 2*c)^2 + (2*I - 2)*a^2*cos(2*d*x + 2*c) - (I - 1)*a^2)*s
in(3*d*x + 3*c))*sin(3/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)))
sqrt(a) + 2520(2*(-(I - 1)*a^2*cos(2*d*x + 2*c)^4 - (I - 1)*a^2*sin(2*d*x
 + 2*c)^4 + (4*I - 4)*a^2*cos(2*d*x + 2*c)^3 - (6*I - 6)*a^2*cos(2*d*x + 2*
c)^2 + (4*I - 4)*a^2*cos(2*d*x + 2*c) + 2*(-(I - 1)*a^2*cos(2*d*x + 2*c)^2
+ (2*I - 2)*a^2*cos(2*d*x + 2*c) - (I - 1)*a^2)*sin(2*d*x + 2*c)^2 - (I - 1
)*a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c
) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)) - co
s(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) +
 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)) - sin(d
*x + c)) + ((I + 1)*a^2*cos(2*d*x + 2*c)^4 + (I + 1)*a^2*sin(2*d*x + 2*c)^4
 - (4*I + 4)*a^2*cos(2*d*x + 2*c)^3 + (6*I + 6)*a^2*cos(2*d*x + 2*c)^2 - (4
*I + 4)*a^2*cos(2*d*x + 2*c) + 2*((I + 1)*a^2*cos(2*d*x + 2*c)^2 - (2*I + 2
)*a^2*cos(2*d*x + 2*c) + (I + 1)*a^2)*sin(2*d*x + 2*c)^2 + (I + 1)*a^2)*log
(cos(d*x + c)^2 + sin(d*x + c)^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 - 2*cos(2*d*x + 2*c) + 1)*(cos(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*
x + 2*c) + 1))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)
)^2) - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)
^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1))*sin(d*x +
 c) + cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)
))))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/
4)*sqrt(a) + ((-(5040*I - 5040)*a^2*cos(9*d*x + 9*c) + (5880*I - 5880)*a^2*

```
cos(7*d*x + 7*c) - (6678*I - 6678)*a^2*cos(5*d*x + 5*c) + (501*I - 501)*a^2
*cos(3*d*x + 3*c) + (857*I - 857)*a^2*cos(d*x + c) + (5040*I + 5040)*a^2*si
n(9*d*x + 9*c) - (5880*I + 5880)*a^2*sin(7*d*x + 7*c) + (6678*I + 6678)*a^2
*sin(5*d*x + 5*c) - (501*I + 501)*a^2*sin(3*d*x + 3*c) - (857*I + 857)*a^2*
sin(d*x + c))*cos(9/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x + 2*c) + 1)) + 6
*(947*(-(I - 1)*a^2*cos(d*x + c) + (I + 1)*a^2*sin(d*x + c))*cos(2*d*x + 2*
c)^2 - (947*I - 947)*a^2*cos(d*x + c) + 947*(-(I - 1)*a^2*cos(d*x + c) + (I
 + 1)*a^2*sin(d*x + c))*sin(2*d*x + 2*c)^2 + (947*I + 947)*a^2*sin(d*x + c)
 + 840*(-(I - 1)*a^2*cos(2*d*x + 2*c)^2 - (I - 1)*a^2*sin(2*d*x + 2*c)^2 +
(2*I - 2)*a^2*cos(2*d*x + 2*c) - (I - 1)*a^2)*cos(5*d*x + 5*c) + 1960*((I -
 1)*a^2*cos(2*d*x + 2*c)^2 + (I - 1)*a^2*sin(2*d*x + 2*c)^2 - (2*I - 2)*a^2
*cos(2*d*x + 2*c) + (I - 1)*a^2)*cos(3*d*x + 3*c) + 1894*((I - 1)*a^2*cos(d
*x + c) - (I + 1)*a^2*sin(d*x + c))*cos(2*d*x + 2*c) + 840*((I + 1)*a^2*cos
(2*d*x + 2*c)^2 + (I + 1)*a^2*sin(2*d*x + 2*c)^2 - (2*I + 2)*a^2*cos(2*d*x
+ 2*c) + (I + 1)*a^2)*sin(5*d*x + 5*c) + 1960*(-(I + 1)*a^2*cos(2*d*x + 2*c
)^2 - (I + 1)*a^2*sin(2*d*x + 2*c)^2 + (2*I + 2)*a^2*cos(2*d*x + 2*c) - (I
+ 1)*a^2)*sin(3*d*x + 3*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), -cos(2*d*x +
2*c) + 1)) + 9824*(((I - 1)*a^2*cos(d*x + c) - (I + 1)*a^2*sin(d*x + c))*co
s(2*d*x + 2*c)^4 + ((I - 1)*a^2*cos(d*x + c) - ...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(181) = 362$.
time = 0.38, size = 566, normalized size = 2.37

$$-\frac{8\sqrt{2}\left(646i\,a^{2}e^{(11i\,dx+11i\,c)}-1001i\,a^{2}e^{(9i\,dx+9i\,c)}+684i\,a^{2}e^{(7i\,dx+7i\,c)}+966i\,a^{2}e^{(5i\,dx+5i\,c)}-1050i\,a^{2}e^{(3i\,dx+3i\,c)}+315i\,a^{2}e^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}+315\sqrt{\frac{32i\,a^{5}}{d^{2}}}\left(de^{(10i\,dx+10i\,c)}-5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}-10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}-d\right)\log\left(\frac{\left(4\sqrt{2}\left(a^{2}e^{(2i\,dx+2i\,c)}+a^{2}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}+i\sqrt{\frac{32i\,a^{5}}{d^{2}}}\,de^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{4\,a^{2}}\right)-315\sqrt{\frac{32i\,a^{5}}{d^{2}}}\left(de^{(10i\,dx+10i\,c)}-5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}-10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}-d\right)\log\left(\frac{\left(4\sqrt{2}\left(a^{2}e^{(2i\,dx+2i\,c)}+a^{2}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}-i\sqrt{\frac{32i\,a^{5}}{d^{2}}}\,de^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{4\,a^{2}}\right)}{630\left(de^{(10i\,dx+10i\,c)}-5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}-10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/tan(d*x+c)^(11/2),x, algorithm="fricas")
```

```
[Out] -1/630*(8*sqrt(2)*(646*I*a^2*e^(11*I*d*x + 11*I*c) - 1001*I*a^2*e^(9*I*d*x
+ 9*I*c) + 684*I*a^2*e^(7*I*d*x + 7*I*c) + 966*I*a^2*e^(5*I*d*x + 5*I*c) -
1050*I*a^2*e^(3*I*d*x + 3*I*c) + 315*I*a^2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) +
 1)) + 315*sqrt(32*I*a^5/d^2)*(d*e^(10*I*d*x + 10*I*c) - 5*d*e^(8*I*d*x + 8
*I*c) + 10*d*e^(6*I*d*x + 6*I*c) - 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*
x + 2*I*c) - d)*log(1/4*(4*sqrt(2)*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2
*I*c) + 1)) + I*sqrt(32*I*a^5/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a^2)
 - 315*sqrt(32*I*a^5/d^2)*(d*e^(10*I*d*x + 10*I*c) - 5*d*e^(8*I*d*x + 8*I*c
) + 10*d*e^(6*I*d*x + 6*I*c) - 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x +
2*I*c) - d)*log(1/4*(4*sqrt(2)*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c
) + 1)) - I*sqrt(32*I*a^5/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a^2))/(d
*e^(10*I*d*x + 10*I*c) - 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c)
 - 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(5/2)/tan(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/tan(d*x+c)^(11/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^{5/2}}{\tan\left(c+d\,x\right)^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(5/2)/tan(c + d*x)^(11/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^(5/2)/tan(c + d*x)^(11/2), x)
```

3.210 $\int \frac{\tan^{\frac{7}{2}}(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=218

$$\frac{11\sqrt[4]{-1}\,\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{4\sqrt{a}\,d}+\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\tan^{\frac{5}{2}}(c+d}{d\sqrt{a+ia\tan(c}$$

```
[Out] 11/4*(-1)^(1/4)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c
))^(1/2))/d/a^(1/2)+(1/2-1/2*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I
*a*tan(d*x+c))^(1/2))/d/a^(1/2)+7/4*tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/
2)/a/d-3/2*I*(a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^(3/2)/a/d-tan(d*x+c)^(5/2)
/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {3639, 3678, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{11\sqrt[4]{-1}\,\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{4\sqrt{a}\,d}-\frac{\tan^{\frac{5}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{3i\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2ad}+\frac{7\sqrt{\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{4ad}+\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(7/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (11*(-1)^(1/4)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*
Tan[c + d*x]]])/(4*Sqrt[a]*d) + ((1/2 - I/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[
Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/(Sqrt[a]*d) - Tan[c + d*x]^(5/2
)/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (7*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c
 + d*x]])/(4*a*d) - (((3*I)/2)*Tan[c + d*x]^(3/2)*Sqrt[a + I*a*Tan[c + d*x]
])/(a*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3678

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Dist[
1/(a*(m + n)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Simp
[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
```

```
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{\tan^{\frac{7}{2}}(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{\tan^{\frac{5}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{\int \tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}\left(-\frac{5a}{2}+3ia\,\mathrm{t}\right.}{a^2}$$

$$= -\frac{\tan^{\frac{5}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{3i\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2ad} - \frac{\int\sqrt{\tan(}}{}$$

$$= -\frac{\tan^{\frac{5}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} + \frac{7\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{4ad} - \frac{3i\tan^{\frac{3}{2}}(}{}$$

$$= -\frac{\tan^{\frac{5}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} + \frac{7\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{4ad} - \frac{3i\tan^{\frac{3}{2}}(}{}$$

$$= -\frac{\tan^{\frac{5}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} + \frac{7\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{4ad} - \frac{3i\tan^{\frac{3}{2}}(}{}$$

$$= \frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\tan^{\frac{5}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} + \frac{7\sqrt{\mathrm{t}}}{}$$

$$= \frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\tan^{\frac{5}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} + \frac{7\sqrt{\mathrm{t}}}{}$$

$$= \frac{11\sqrt[4]{-1}\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{4\sqrt{a}\,d} + \frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\mathrm{ta}}}{\sqrt{a+ia\,\mathrm{tar}}}\right.}{\sqrt{a}\,d}$$

Mathematica [A]
time = 3.48, size = 224, normalized size = 1.03

$$\frac{-\frac{2e^{i(c+dx)}\left(-1+e^{2i(c+dx)}\right)^{3/2}\left(4\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)-11\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)}{\left(-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}\right)^{3/2}\left(1+e^{2i(c+dx)}\right)^2}+\sec^2(c+dx)(9+5\cos(2(c+dx))+i\sin(2(c+dx)))\sqrt{\tan(c+dx)}}{8d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(7/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-2*E^(I*(c + d*x))*(-1 + E^((2*I)*(c + d*x)))^(3/2)*(4*ArcTanh[E^(I*(c +
d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]] - 11*Sqrt[2]*ArcTanh[(Sqrt[2]*E^(I*(c
 + d*x)))/Sqrt[-1 + E^((2*I)*(c + d*x))]]))/((((-I)*(-1 + E^((2*I)*(c + d*x
))))/(1 + E^((2*I)*(c + d*x))))^(3/2)*(1 + E^((2*I)*(c + d*x)))^2) + Sec[c
+ d*x]^2*(9 + 5*Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)])*Sqrt[Tan[c + d*x]])/
(8*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 666 vs. $2(170) = 340$.
time = 0.20, size = 667, normalized size = 3.06

method	result
derivativedivides	$-\frac{\left(\sqrt{\tan}\,(dx+c)\right)\sqrt{a\,(1+i\tan(dx+c))}\,\left(2i\sqrt{ia}\,\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c}}{\tan}\right.\right.}{}$
default	$-\frac{\left(\sqrt{\tan}\,(dx+c)\right)\sqrt{a\,(1+i\tan(dx+c))}\,\left(2i\sqrt{ia}\,\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c}}{\tan}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)*(2*I*(I*a)^(1/2)*2^(1/2)
*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a
*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2+4*I*(I*a)^(1/2)*(-I*a)^(1/2)*(a
*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^3-2*I*a*2^(1/2)*ln(-(-2*2^(1
/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/
(tan(d*x+c)+I))*(I*a)^(1/2)-22*I*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(
1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c
)+16*I*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^
(1/2)+4*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+
I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*(I*a)^(1/2)*2^(1/2)*a*tan(d*x+c)+11*ln(
1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a
)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^2+2*(a*tan(d*x+c)*(1+I*tan(d*x+c))
```

)^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^2-11*ln(1/2*(2*I*a*tan(d*x+c)+2
*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*(-I*a)
^(1/2)+14*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2))/a
/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2)/(-I*a)^(1/2)/(-tan(d*x+c
)+I)^2

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(tan(d*x + c)^(7/2)/sqrt(I*a*tan(d*x + c) + a), x)

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 658 vs. 2(160) = 320.
time = 0.44, size = 658, normalized size = 3.02

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}\left(3e^{(4idx+4ic)}+9e^{(2idx+2ic)}+2\right)+\left(ade^{(3idx+3ic)}+ade^{(idx+ic)}\right)\sqrt{-\frac{2i}{ad^2}}\log\left(\frac{1}{4}ad\sqrt{-\frac{2i}{ad^2}}e^{(idx+ic)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}\left(e^{(2idx+2ic)}+1\right)\right)-\left(ade^{(3idx+3ic)}+ade^{(idx+ic)}\right)\sqrt{-\frac{2i}{ad^2}}\log\left(-\frac{1}{4}ad\sqrt{-\frac{2i}{ad^2}}e^{(idx+ic)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}\left(e^{(2idx+2ic)}+1\right)\right)-\left(ade^{(3idx+3ic)}+ade^{(idx+ic)}\right)\sqrt{-\frac{121i}{16ad^2}}\log\left(\frac{208\left(11\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}\left(e^{(3idx+3ic)}+e^{(idx+ic)}\right)+2\left(3ade^{(2idx+2ic)}-ad\right)\sqrt{-\frac{121i}{16ad^2}}\right)}{6655\left(e^{(2idx+2ic)}+1\right)}\right)+\left(ade^{(3idx+3ic)}+ade^{(idx+ic)}\right)\sqrt{-\frac{121i}{16ad^2}}\log\left(\frac{208\left(11\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}\left(e^{(3idx+3ic)}+e^{(idx+ic)}\right)-2\left(3ade^{(2idx+2ic)}-ad\right)\sqrt{-\frac{121i}{16ad^2}}\right)}{6655\left(e^{(2idx+2ic)}+1\right)}\right)}{4\left(ade^{(3idx+3ic)}+ade^{(idx+ic)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 1/4*(sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c)
 + I)/(e^(2*I*d*x + 2*I*c) + 1))*(3*e^(4*I*d*x + 4*I*c) + 9*e^(2*I*d*x + 2*
I*c) + 2) + (a*d*e^(3*I*d*x + 3*I*c) + a*d*e^(I*d*x + I*c))*sqrt(-2*I/(a*d^
2))*log(1/4*a*d*sqrt(-2*I/(a*d^2))*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I
c) + 1))(e^(2*I*d*x + 2*I*c) + 1)) - (a*d*e^(3*I*d*x + 3*I*c) + a*d*e^(I*
d*x + I*c))*sqrt(-2*I/(a*d^2))*log(-1/4*a*d*sqrt(-2*I/(a*d^2))*e^(I*d*x + I
*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2
*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)) - (a*d*e^(
3*I*d*x + 3*I*c) + a*d*e^(I*d*x + I*c))*sqrt(-121/16*I/(a*d^2))*log(208/665
5*(11*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c
) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(3*I*d*x + 3*I*c) + e^(I*d*x + I*c)) +
 2*(3*a*d*e^(2*I*d*x + 2*I*c) - a*d)*sqrt(-121/16*I/(a*d^2)))/(e^(2*I*d*x +
 2*I*c) + 1)) + (a*d*e^(3*I*d*x + 3*I*c) + a*d*e^(I*d*x + I*c))*sqrt(-121/1
6*I/(a*d^2))*log(208/6655*(11*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqr
t((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(3*I*d*x + 3*I
*c) + e^(I*d*x + I*c)) - 2*(3*a*d*e^(2*I*d*x + 2*I*c) - a*d)*sqrt(-121/16*I
/(a*d^2)))/(e^(2*I*d*x + 2*I*c) + 1)))/(a*d*e^(3*I*d*x + 3*I*c) + a*d*e^(I*
d*x + I*c))

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(7/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(c+d\,x)^{7/2}}{\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(7/2)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] int(tan(c + d*x)^(7/2)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.211 $\int \frac{\tan^{\frac{5}{2}}(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=177

$$-\frac{(-1)^{3/4}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\tan^{\frac{3}{2}}(c+ \ldots}{d\sqrt{a+ia\tan(\ldots}}$$

```
[Out] -(-1)^(3/4)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(
1/2))/d/a^(1/2)+(1/2+1/2*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*t
an(d*x+c))^(1/2))/d/a^(1/2)-2*I*tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a
/d-tan(d*x+c)^(3/2)/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]

time = 0.34, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {3639, 3678, 3682, 3625, 211, 3680, 65, 223, 209}

$$-\frac{(-1)^{3/4}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\tan^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{ad}+\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(5/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] -(((-1)^(3/4)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*T
an[c + d*x]]])/(Sqrt[a]*d)) + ((1/2 + I/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Ta
n[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/(Sqrt[a]*d) - Tan[c + d*x]^(3/2)/
(d*Sqrt[a + I*a*Tan[c + d*x]]) - ((2*I)*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan
[c + d*x]])/(a*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3678

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Dist[
1/(a*(m + n)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Simp
[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{\tan^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{\int \sqrt{\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}\,\left(-\frac{3a}{2}+2i\right.}{a^2}$$

$$= -\frac{\tan^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{2i\sqrt{\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{ad} - \frac{\int \frac{\sqrt{a}}{}}{}$$

$$= -\frac{\tan^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{2i\sqrt{\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{ad} + \frac{i\int \frac{(a-i}{}}{}$$

$$= -\frac{\tan^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{2i\sqrt{\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{ad} + \frac{i\text{Subst}}{}$$

$$= \frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\tan^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{2i\sqrt{}}{}$$

$$= \frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\tan^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{2i\sqrt{}}{}$$

$$= -\frac{(-1)^{3/4}\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{t}}{\sqrt{a+ia\,ta}}\right.}{\sqrt{a}\,d}$$

Mathematica [A]
time = 2.28, size = 222, normalized size = 1.25

$$\frac{ie^{i(c+dx)}\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}\left(\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)+\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)}{\sqrt{2}\,d\sqrt{-1+e^{2i(c+dx)}}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}}+\frac{\sqrt{\tan(c+dx)}\,(-2i+\tan(c+dx))}{d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(5/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (I*E^(I*(c + d*x))*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c
+ d*x)))]*(ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]] + Sqrt[2
]*ArcTanh[(Sqrt[2]*E^(I*(c + d*x)))/Sqrt[-1 + E^((2*I)*(c + d*x))]]))/(Sqrt
[2]*d*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((
2*I)*(c + d*x)))]) + (Sqrt[Tan[c + d*x]]*(-2*I + Tan[c + d*x]))/(d*Sqrt[a +
 I*a*Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 622 vs. $2(140) = 280$.
time = 0.18, size = 623, normalized size = 3.52

method	result
derivativedivides	$-\frac{\left(\sqrt{\tan}\left(dx+c\right)\right)\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\left(2i\sqrt{ia}\,\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right.}}{\tan}\right.\right.}{}$
default	$-\frac{\left(\sqrt{\tan}\left(dx+c\right)\right)\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\left(2i\sqrt{ia}\,\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right.}}{\tan}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)/a*(2*I*(I*a)^(1/2)*2^(1/
2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3
*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)-2*I*ln(1/2*(2*I*a*tan(d*x+c)+2*
(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1
/2)*a*tan(d*x+c)^2+4*I*(I*a)^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+
c)))^(1/2)*tan(d*x+c)^2-(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a
*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*t
an(d*x+c)^2+2*I*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^
(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a-8*I*(I*a)^(1/2)*(-I*a)^(1/
2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2
)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(t
an(d*x+c)+I))*a-4*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c))
)^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)+12*(a*tan(d*x
+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c))/(a*tan(d*x
+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2)/(-I*a)^(1/2)/(-tan(d*x+c)+I)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^(5/2)/sqrt(I*a*tan(d*x + c) + a), x)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(131) = 262$.
time = 0.44, size = 580, normalized size = 3.28

$$-\frac{\left(ad\sqrt{\frac{2i}{ad^2}}e^{(idx+ic)}\log\left(\frac{1}{4}iad\sqrt{\frac{2i}{ad^2}}e^{(idx+ic)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}\left(e^{(2idx+2ic)}+1\right)\right)-ad\sqrt{\frac{2i}{ad^2}}e^{(idx+ic)}\log\left(-\frac{1}{4}iad\sqrt{\frac{2i}{ad^2}}e^{(idx+ic)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}\left(e^{(2idx+2ic)}+1\right)\right)-ad\sqrt{\frac{i}{ad^2}}e^{(idx+ic)}\log\left(\frac{104\left(2\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}\left(e^{(3idx+3ic)}+e^{(idx+ic)}\right)-\left(3iade^{(2idx+2ic)}-iad\right)\sqrt{\frac{i}{ad^2}}\right)}{605\left(e^{(2idx+2ic)}+1\right)}\right)+ad\sqrt{\frac{i}{ad^2}}e^{(idx+ic)}\log\left(\frac{104\left(2\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}\left(e^{(3idx+3ic)}+e^{(idx+ic)}\right)-\left(-3iade^{(2idx+2ic)}+iad\right)\sqrt{\frac{i}{ad^2}}\right)}{605\left(e^{(2idx+2ic)}+1\right)}\right)+2\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}\left(3ie^{(2idx+2ic)}+i\right)\right)e^{(-idx-ic)}}{4ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(a*d*sqrt(2*I/(a*d^2))*e^(I*d*x + I*c)*log(1/4*I*a*d*sqrt(2*I/(a*d^2))
*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e
^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1
)) - a*d*sqrt(2*I/(a*d^2))*e^(I*d*x + I*c)*log(-1/4*I*a*d*sqrt(2*I/(a*d^2))
*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e
^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1
)) - a*d*sqrt(I/(a*d^2))*e^(I*d*x + I*c)*log(104/605*(2*sqrt(2)*sqrt(a/(e^(
2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*
c) + 1))*(e^(3*I*d*x + 3*I*c) + e^(I*d*x + I*c)) - (3*I*a*d*e^(2*I*d*x + 2*
I*c) - I*a*d)*sqrt(I/(a*d^2)))/(e^(2*I*d*x + 2*I*c) + 1)) + a*d*sqrt(I/(a*d
^2))*e^(I*d*x + I*c)*log(104/605*(2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(3*I*d*x
 + 3*I*c) + e^(I*d*x + I*c)) - (-3*I*a*d*e^(2*I*d*x + 2*I*c) + I*a*d)*sqrt(
I/(a*d^2)))/(e^(2*I*d*x + 2*I*c) + 1)) + 2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(3*I
*e^(2*I*d*x + 2*I*c) + I))*e^(-I*d*x - I*c)/(a*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{5}{2}}\left(c+dx\right)}{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(5/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**(5/2)/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^{5/2}}{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(5/2)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] int(tan(c + d*x)^(5/2)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.212 $\int \frac{\tan^{\frac{3}{2}}(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=140

$$-\frac{2\sqrt[4]{-1}\,\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] -2*(-1)^(1/4)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))
^(1/2))/d/a^(1/2)+(-1/2+1/2*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*
a*tan(d*x+c))^(1/2))/d/a^(1/2)-tan(d*x+c)^(1/2)/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3639, 3682, 3625, 211, 3680, 65, 223, 209}

$$-\frac{2\sqrt[4]{-1}\,\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(3/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (-2*(-1)^(1/4)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*
Tan[c + d*x]]])/(Sqrt[a]*d) - ((1/2 - I/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Ta
n[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/(Sqrt[a]*d) - Sqrt[Tan[c + d*x]]/
(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}} - \frac{\int \frac{\left(-\frac{a}{2}+ia\tan(c+dx)\right)\sqrt{a+ia\tan(c+dx)}}{\sqrt{\tan(c+dx)}}\,dx}{a^2}$$

$$= -\frac{\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}} + \frac{\int \frac{(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{\sqrt{\tan(c+dx)}}\,dx}{a^2} - \frac{\int \sqrt{a}}{}$$

$$= -\frac{\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{a+iax}}\,dx, x, \tan(c+dx)\right)}{d} + -$$

$$= -\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}} + \frac{2\text{Su}}{}$$

$$= -\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}} + \frac{2\text{Su}}{}$$

$$= -\frac{2\sqrt[4]{-1}\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\text{t}}}{\sqrt{a+ia\,\text{ta}}}\right)}{\sqrt{a}\,d}$$

Mathematica [A]
time = 2.01, size = 210, normalized size = 1.50

$$\frac{ie^{-2i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(-1+e^{2i(c+dx)}+e^{i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)-2\sqrt{2}\,e^{i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{\sqrt{2}\,e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)}{\sqrt{2}\,ad\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(3/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (I*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(-1 + E^((2*I)*(
c + d*x)) + E^(I*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]*ArcTanh[E^(I*(c
+ d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]] - 2*Sqrt[2]*E^(I*(c + d*x))*Sqrt[-1
 + E^((2*I)*(c + d*x))]*ArcTanh[(Sqrt[2]*E^(I*(c + d*x)))/Sqrt[-1 + E^((2*I
)*(c + d*x))]]))/(Sqrt[2]*a*d*E^((2*I)*(c + d*x))*Sqrt[Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 579 vs. 2(109) = 218.
time = 0.20, size = 580, normalized size = 4.14

method	result
derivativedivides	$-\frac{\left(\sqrt{\tan}(dx+c)\right)\sqrt{a\,(1+i\tan\,(dx+c))}\,\left(-i\sqrt{ia}\,\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\,(dx+c}}{\text{ta}}\right.\right.}{}$
default	$-\frac{\left(\sqrt{\tan}(dx+c)\right)\sqrt{a\,(1+i\tan\,(dx+c))}\,\left(-i\sqrt{ia}\,\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\,(dx+c}}{\text{ta}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)/a*(-I*(I*a)^(1/2)*2^(1/2
)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*
a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2+8*I*ln(1/2*(2*I*a*tan(d*x+c)+2
*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(
1/2)*a*tan(d*x+c)+I*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I
*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*(I*a)^(1/2)*a-4*I*(
a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)-4*
ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2
)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^2-2*ln(-(-2*2^(1/2)*(-I*a)^(1/2
)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))
*(I*a)^(1/2)*2^(1/2)*a*tan(d*x+c)+4*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c
)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*(-I*a)^(1/2)-4*(a*t
an(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2))/(a*tan(d*x+c)*(
1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2)/(-I*a)^(1/2)/(-tan(d*x+c)+I)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^(3/2)/sqrt(I*a*tan(d*x + c) + a), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 577 vs. 2(102) = 204.
time = 0.44, size = 577, normalized size = 4.12

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(a*d*sqrt(-2*I/(a*d^2))*e^(I*d*x + I*c)*log(1/4*a*d*sqrt(-2*I/(a*d^2))
*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e
^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1
)) - a*d*sqrt(-2*I/(a*d^2))*e^(I*d*x + I*c)*log(-1/4*a*d*sqrt(-2*I/(a*d^2))
*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e
^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1
)) - a*d*sqrt(-4*I/(a*d^2))*e^(I*d*x + I*c)*log(52/605*(4*sqrt(2)*sqrt(a/(e
^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*
I*c) + 1))*(e^(3*I*d*x + 3*I*c) + e^(I*d*x + I*c)) + (3*a*d*e^(2*I*d*x + 2*
I*c) - a*d)*sqrt(-4*I/(a*d^2)))/(e^(2*I*d*x + 2*I*c) + 1)) + a*d*sqrt(-4*I/
(a*d^2))*e^(I*d*x + I*c)*log(52/605*(4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(3*I*
d*x + 3*I*c) + e^(I*d*x + I*c)) - (3*a*d*e^(2*I*d*x + 2*I*c) - a*d)*sqrt(-4
*I/(a*d^2)))/(e^(2*I*d*x + 2*I*c) + 1)) + 2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*
I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^
(2*I*d*x + 2*I*c) + 1))*e^(-I*d*x - I*c)/(a*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(3/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**(3/2)/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^{3/2}}{\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(3/2)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] int(tan(c + d*x)^(3/2)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.213 $\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=88

$$-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{i\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] (-1/2-1/2*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2
))/d/a^(1/2)+I*tan(d*x+c)^(1/2)/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3627, 3625, 211}

$$\frac{i\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[c + d*x]]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-1/2 - I/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan
[c + d*x]]])/(Sqrt[a]*d) + (I*Sqrt[Tan[c + d*x]])/(d*Sqrt[a + I*a*Tan[c + d
*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3627

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
```

```
 + f*x])^n/(2*b*f*m)), x] - Dist[(a*c - b*d)/(2*b^2), Int[(a + b*Tan[e + f*
x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && EqQ
[m + n, 0] && LeQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx &= \frac{i\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}} - \frac{i\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{\tan(c+dx)}}\,dx}{2a}\\
&= \frac{i\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}} - \frac{a\mathrm{Subst}\left(\int \frac{1}{-ia-2a^2x^2}\,dx, x, \frac{\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}\\
&= -\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{i\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}
\end{aligned}$$

Mathematica [A]
time = 1.41, size = 132, normalized size = 1.50

$$\frac{i\left(\sqrt{-1+e^{2i(c+dx)}} - e^{i(c+dx)}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\tan(c+dx)}}{\sqrt{2}\,d\sqrt{-1+e^{2i(c+dx)}}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tan[c + d*x]]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (I*(Sqrt[-1 + E^((2*I)*(c + d*x))] - E^(I*(c + d*x))*ArcTanh[E^(I*(c + d*x)
)/Sqrt[-1 + E^((2*I)*(c + d*x))]])*Sqrt[Tan[c + d*x]])/(Sqrt[2]*d*Sqrt[-1 +
 E^((2*I)*(c + d*x))]*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))
)])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(69) = 138.
time = 0.20, size = 350, normalized size = 3.98

method	result

derivativedivides	$\frac{\left(\sqrt{\tan}\,(dx+c)\right)\sqrt{a\,(1+i\tan\,(dx+c))}\,\left(2i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\,(dx+c)\,(1+i}}{\tan(dx+c)+i}\right.\right.}{}$
default	$\frac{\left(\sqrt{\tan}\,(dx+c)\right)\sqrt{a\,(1+i\tan\,(dx+c))}\,\left(2i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\,(dx+c)\,(1+i}}{\tan(dx+c)+i}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)/a*(2*I*2^(1/2)*ln(-(-2*2^
(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c)
)/(tan(d*x+c)+I))*a*tan(d*x+c)-2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(
d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*
x+c)^2-4*I*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+2^(1/2)*ln(-(
-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d
*x+c))/(tan(d*x+c)+I))*a+4*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)
*(-I*a)^(1/2))/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(-tan(d*x+c)+I)^2/(-I*
a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(tan(d*x + c))/sqrt(I*a*tan(d*x + c) + a), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(62) = 124.
time = 0.39, size = 301, normalized size = 3.42

$$\frac{\left(ad\sqrt{\frac{2i}{ad^2}}\,e^{(i\,dx+i\,c)}\log\left(\frac{1}{4}i\,ad\sqrt{\frac{2i}{ad^2}}\,e^{(i\,dx+i\,c)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)\right)-ad\sqrt{\frac{2i}{ad^2}}\,e^{(i\,dx+i\,c)}\log\left(-\frac{1}{4}i\,ad\sqrt{\frac{2i}{ad^2}}\,e^{(i\,dx+i\,c)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)\right)-2\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(-i\,e^{(2i\,dx+2i\,c)}-i\right)\right)e^{(-i\,dx-i\,c)}}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(a*d*sqrt(2*I/(a*d^2))*e^(I*d*x + I*c)*log(1/4*I*a*d*sqrt(2*I/(a*d^2))*
e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^
(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)
) - a*d*sqrt(2*I/(a*d^2))*e^(I*d*x + I*c)*log(-1/4*I*a*d*sqrt(2*I/(a*d^2))*
```

```
e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^
(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)
) - 2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c
) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(-I*e^(2*I*d*x + 2*I*c) - I))*e^(-I*d*x -
 I*c)/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(tan(c + d*x))/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 6.71, size = 242, normalized size = 2.75

$$\frac{\ln\left(2+\frac{\sqrt{a}\,\sqrt{\tan(c+dx)}\,(-4-4\mathrm{i})}{\sqrt{a+a\tan(c+dx)\,1\mathrm{i}}-\sqrt{a}}+\frac{a\tan(c+dx)\,2\mathrm{i}}{\left(\sqrt{a+a\tan(c+dx)\,1\mathrm{i}}-\sqrt{a}\right)^2}\right)\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)}{\sqrt{a}\,d}-\frac{2\sqrt{\tan(c+dx)}}{\left(\sqrt{a+a\tan(c+dx)\,1\mathrm{i}}-\sqrt{a}\right)\left(d\,1\mathrm{i}-\frac{a\,d\tan(c+dx)}{\left(\sqrt{a+a\tan(c+dx)\,1\mathrm{i}}-\sqrt{a}\right)^2}\right)}-\frac{\sqrt{\frac{1}{8}\mathrm{i}}\,\ln\left(-\frac{a\tan(c+dx)}{\left(\sqrt{a+a\tan(c+dx)\,1\mathrm{i}}-\sqrt{a}\right)^2}+\frac{2\,(-1)^{3/4}\sqrt{2}\,\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+a\tan(c+dx)\,1\mathrm{i}}-\sqrt{a}}+1\mathrm{i}\right)}{\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] (log((a*tan(c + d*x)*2i)/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))^2 - (a^(
1/2)*tan(c + d*x)^(1/2)*(4 + 4i))/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))
 + 2)*(1/4 + 1i/4))/(a^(1/2)*d) - (2*tan(c + d*x)^(1/2))/(((a + a*tan(c + d
*x)*1i)^(1/2) - a^(1/2))*(d*1i - (a*d*tan(c + d*x))/((a + a*tan(c + d*x)*1i
)^(1/2) - a^(1/2))^2)) - ((1i/8)^(1/2)*log((2*(-1)^(3/4)*2^(1/2)*a^(1/2)*ta
n(c + d*x)^(1/2))/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2)) - (a*tan(c + d*
x))/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))^2 + 1i))/(a^(1/2)*d)
```

3.214 $\int \frac{1}{\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=85

$$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] (1/2-1/2*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
)/d/a^(1/2)+tan(d*x+c)^(1/2)/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3629, 3627, 3625, 211}

$$\frac{\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}+\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] ((1/2 - I/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[
c + d*x]]])/(Sqrt[a]*d) + Sqrt[Tan[c + d*x]]/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3627

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^n/(2*b*f*m)), x] - Dist[(a*c - b*d)/(2*b^2), Int[(a + b*Tan[e + f*
x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
```

```
}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && EqQ
[m + n, 0] && LeQ[m, -2^(-1)]
```

Rule 3629

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-d)*(a + b*Tan[e + f*x])^m*((c + d*Ta
n[e + f*x])^(n + 1)/(f*m*(c^2 + d^2))), x] + Dist[a/(a*c - b*d), Int[(a + b
*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}} + i\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx$$

$$= \frac{\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}} + \frac{\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{\tan(c+dx)}}\,dx}{2a}$$

$$= \frac{\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}} - \frac{(ia)\text{Subst}\left(\int \frac{1}{-ia-2a^2x^2}\,dx, x, \frac{\sqrt{t}}{\sqrt{a+}}\right.}{d}$$

$$= \frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{\sqrt{\tan(c+d}}{d\sqrt{a+ia\tan(c}}$$

Mathematica [A]
time = 1.64, size = 140, normalized size = 1.65

$$-\frac{ie^{-2i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(-1+e^{2i(c+dx)}+e^{i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)}{\sqrt{2}\,ad\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] ((-I)*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(-1 + E^((2*I
)*(c + d*x)) + E^(I*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]*ArcTanh[E^(I*
(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]]))/(Sqrt[2]*a*d*E^((2*I)*(c + d*x
))*Sqrt[Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(67) = 134$.
time = 0.19, size = 352, normalized size = 4.14

method	result
derivativedivides	$-\frac{\sqrt{a(1+i\tan(dx+c))}\,\left(\sqrt{\tan}(dx+c)\right)\left(i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+}}{\tan(dx+c)+}\right.\right.}{}$
default	$-\frac{\sqrt{a(1+i\tan(dx+c))}\,\left(\sqrt{\tan}(dx+c)\right)\left(i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+}}{\tan(dx+c)+}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*(a*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^(1/2)*(I*2^(1/2)*ln(-(-2*2^(1/
2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(
tan(d*x+c)+I))*a*tan(d*x+c)^2-I*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan
(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a+4*I*(
-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)+2*2^(1/2)*ln(-
(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(
d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)+4*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2
)*(-I*a)^(1/2))/a/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(-I*a)^(1/2)/(-tan(
d*x+c)+I)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(I*a*tan(d*x + c) + a)*sqrt(tan(d*x + c))), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(61) = 122$.
time = 0.39, size = 299, normalized size = 3.52

$$\frac{\left(ad\sqrt{-\frac{2i}{ad^2}}e^{(idx+ic)}\log\left(\frac{1}{4}ad\sqrt{-\frac{2i}{ad^2}}e^{(idx+ic)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}\left(e^{(2idx+2ic)}+1\right)\right)-ad\sqrt{-\frac{2i}{ad^2}}e^{(idx+ic)}\log\left(-\frac{1}{4}ad\sqrt{-\frac{2i}{ad^2}}e^{(idx+ic)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}\left(e^{(2idx+2ic)}+1\right)\right)+2\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}\left(e^{(2idx+2ic)}+1\right)\right)e^{(-idx-ic)}}{4ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/2),x, algorithm="fricas"
)

[Out] 1/4*(a*d*sqrt(-2*I/(a*d^2))*e^(I*d*x + I*c)*log(1/4*a*d*sqrt(-2*I/(a*d^2))*
e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^
(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)
) - a*d*sqrt(-2*I/(a*d^2))*e^(I*d*x + I*c)*log(-1/4*a*d*sqrt(-2*I/(a*d^2))*
e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^
(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)
) + 2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c
) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1))*e^(-I*d*x - I*
c)/(a*d)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,\sqrt{\tan\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+I*a*tan(d*x+c))**(1/2)/tan(d*x+c)**(1/2),x)

[Out] Integral(1/(sqrt(I*a*(tan(c + d*x) - I))*sqrt(tan(c + d*x))), x)

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 407 vs. 2(61) = 122.
time = 0.78, size = 407, normalized size = 4.79

$$-\frac{\left(i a\sqrt{|a|}-|a|^{\frac{3}{2}}\right)\log\left(-\frac{\left(-i\left(\frac{\sqrt{2}\sqrt{i a\tan\left(dx+c\right)+a}\left(-\frac{i|a|}{a}+1\right)|a|^{\frac{3}{2}}}{a^2}-\frac{\sqrt{-2\left(i a\tan\left(dx+c\right)+a\right)a+2a^2}\left(\frac{\tan\left(dx+c\right)}{\sqrt{\frac{\left(i a\tan\left(dx+c\right)+a\right)^2-2\left(i a\tan\left(dx+c\right)+a\right)a+a^2}{a^2}}}+1\right)|a|}{a^2}\right)\right)^2+8\sqrt{2}+12}{2\left(\left(\frac{1}{2}i\left(\frac{\sqrt{2}\sqrt{i a\tan\left(dx+c\right)+a}\left(-\frac{i|a|}{a}+1\right)|a|^{\frac{3}{2}}}{a^2}-\frac{\sqrt{-2\left(i a\tan\left(dx+c\right)+a\right)a+2a^2}\left(\frac{\tan\left(dx+c\right)}{\sqrt{\frac{\left(i a\tan\left(dx+c\right)+a\right)^2-2\left(i a\tan\left(dx+c\right)+a\right)a+a^2}{a^2}}}+1\right)|a|}{a^2}\right)\right)^2+4\sqrt{2}-6\right)}\right)}{4a^2d}-\frac{4\sqrt{2}\left(a\sqrt{|a|}+i|a|^{\frac{3}{2}}\right)}{\left(\left(\frac{\sqrt{2}\sqrt{i a\tan\left(dx+c\right)+a}\left(-\frac{i|a|}{a}+1\right)|a|^{\frac{3}{2}}}{a^2}-\frac{\sqrt{-2\left(i a\tan\left(dx+c\right)+a\right)a+2a^2}\left(\frac{\tan\left(dx+c\right)}{\sqrt{\frac{\left(i a\tan\left(dx+c\right)+a\right)^2-2\left(i a\tan\left(dx+c\right)+a\right)a+a^2}{a^2}}}+1\right)|a|}{a^2}\right)^2-4i\right)a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/2),x, algorithm="giac")

[Out] -1/4*(I*a*sqrt(abs(a)) - abs(a)^(3/2))*log(-1/2*(-I*(sqrt(2)*sqrt(I*a*tan(d
x + c) + a)(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*tan(d*x + c)
 + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(
d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^2 + 8*sqrt(2) + 12)/(1/2*I*(sq
rt(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(
-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c)

```
+ a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^2 + 4*sqrt
(2) - 6))/(a^2*d) - 4*sqrt(2)*(a*sqrt(abs(a)) + I*abs(a)^(3/2))/(((sqrt(2)*
sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*
a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2
 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^2 - 4*I)*a^2*d)
```

Mupad [B]
time = 6.09, size = 171, normalized size = 2.01

$$\frac{\sqrt{\tan\left(c+d\,x\right)}\,2{}\mathrm{i}}{\left(\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}-\sqrt{a}\right)\,\left(d\,1{}\mathrm{i}-\frac{a\,d\,\tan\left(c+d\,x\right)}{{\left(\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}-\sqrt{a}\right)}^2}\right)}+\frac{2\,\sqrt{\frac{1}{8}{}\mathrm{i}}\,\mathrm{atanh}\left(\frac{32\,\sqrt{\frac{1}{8}{}\mathrm{i}}\,{\left(-a\right)}^{9/2}\,\sqrt{\tan\left(c+d\,x\right)}}{\left(a^4\,4{}\mathrm{i}-\frac{4\,a^5\,\tan\left(c+d\,x\right)}{{\left(\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}-\sqrt{a}\right)}^2}\right)\,\left(\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}-\sqrt{a}\right)}\right)}{\sqrt{-a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

```
[Out] (tan(c + d*x)^(1/2)*2i)/(((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))*(d*1i -
(a*d*tan(c + d*x))/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))^2)) + (2*(1i/8
)^(1/2)*atanh((32*(1i/8)^(1/2)*(-a)^(9/2)*tan(c + d*x)^(1/2))/((a^4*4i - (4
*a^5*tan(c + d*x))/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))^2)*((a + a*tan
(c + d*x)*1i)^(1/2) - a^(1/2)))))/((-a)^(1/2)*d)
```

3.215 $\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=120

$$\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{1}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{3\sqrt{a+ia\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}$$

```
[Out] (1/2+1/2*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
)/d/a^(1/2)+1/d/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-3*(a+I*a*tan(d*x+
c))^(1/2)/a/d/tan(d*x+c)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3640, 3679, 12, 3625, 211}

$$-\frac{3\sqrt{a+ia\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{1}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] ((1/2 + I/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[
c + d*x]]])/(Sqrt[a]*d) + 1/(d*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]
]) - (3*Sqrt[a + I*a*Tan[c + d*x]])/(a*d*Sqrt[Tan[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{1}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{\int \frac{\left(\frac{3a}{2}-ia\tan(c+dx)\right)\sqrt{a}}{\tan^{\frac{3}{2}}(c}}{a^2}$$

$$= \frac{1}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{3\sqrt{a+ia\tan(c+dx}}{ad\sqrt{\tan(c+dx)}}$$

$$= \frac{1}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{3\sqrt{a+ia\tan(c+dx}}{ad\sqrt{\tan(c+dx)}}$$

$$= \frac{1}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{3\sqrt{a+ia\tan(c+dx}}{ad\sqrt{\tan(c+dx)}}$$

$$= \frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{}{d\sqrt{\tan(c+dx)}}$$

Mathematica [A]

time = 1.64, size = 160, normalized size = 1.33

$$\frac{i\left(\left(1-5e^{2i(c+dx)}\right)\sqrt{-1+e^{2i(c+dx)}}+e^{i(c+dx)}\left(-1+e^{2i(c+dx)}\right)\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\tan(c+dx)}}{\sqrt{2}\,d\left(-1+e^{2i(c+dx)}\right)^{3/2}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Tan[c + d*x]^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] (I*((1 - 5*E^((2*I)*(c + d*x)))*Sqrt[-1 + E^((2*I)*(c + d*x))] + E^(I*(c +
d*x))*(-1 + E^((2*I)*(c + d*x)))*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)
*(c + d*x))]])*Sqrt[Tan[c + d*x]])/(Sqrt[2]*d*(-1 + E^((2*I)*(c + d*x)))^(3
/2)*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 394 vs. 2(97) = 194.
time = 0.20, size = 395, normalized size = 3.29

method	result
derivativedivides	$-\frac{\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\left(2i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1+i\tan\left(dx+c\right)\right)}}{\tan(dx+c)+i}\right.\right.}{}$
default	$-\frac{\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\left(2i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1+i\tan\left(dx+c\right)\right)}}{\tan(dx+c)+i}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*(a*(1+I*tan(d*x+c)))^(1/2)*(2*I*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)
*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*
a*tan(d*x+c)^2-2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(
d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3-20*I*(-I*
a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)+2^(1/2)*ln(-(-2*2
^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c
))/(tan(d*x+c)+I))*a*tan(d*x+c)+12*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+
c)))^(1/2)*tan(d*x+c)^2-8*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2
))/a/tan(d*x+c)^(1/2)/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(-tan(d*x+c)+I)
^2/(-I*a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(I*a*tan(d*x + c) + a)*tan(d*x + c)^(3/2)), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(90) = 180$.
time = 0.39, size = 355, normalized size = 2.96

$$-\frac{2\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(5i\,e^{(4i\,dx+4i\,c)}+4i\,e^{(2i\,dx+2i\,c)}-i\right)+\left(ade^{(3i\,dx+3i\,c)}-ade^{(i\,dx+i\,c)}\right)\sqrt{\frac{2i}{ad^2}}\log\left(\frac{1}{4}i\,ad\sqrt{\frac{2i}{ad^2}}e^{(i\,dx+i\,c)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)\right)-\left(ade^{(3i\,dx+3i\,c)}-ade^{(i\,dx+i\,c)}\right)\sqrt{\frac{2i}{ad^2}}\log\left(-\frac{1}{4}i\,ad\sqrt{\frac{2i}{ad^2}}e^{(i\,dx+i\,c)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)\right)}{4\left(ade^{(3i\,dx+3i\,c)}-ade^{(i\,dx+i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(3/2),x, algorithm="fricas"
)
```

```
[Out] -1/4*(2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I
*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(5*I*e^(4*I*d*x + 4*I*c) + 4*I*e^(2*I*d
*x + 2*I*c) - I) + (a*d*e^(3*I*d*x + 3*I*c) - a*d*e^(I*d*x + I*c))*sqrt(2*I
/(a*d^2))*log(1/4*I*a*d*sqrt(2*I/(a*d^2))*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*
x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)) - (a*d*e^(3*I*d*x + 3*I*c) - a*
d*e^(I*d*x + I*c))*sqrt(2*I/(a*d^2))*log(-1/4*I*a*d*sqrt(2*I/(a*d^2))*e^(I*
d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*
d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)))/(a
*d*e^(3*I*d*x + 3*I*c) - a*d*e^(I*d*x + I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\tan^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))**(1/2)/tan(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/(sqrt(I*a*(tan(c + d*x) - I))*tan(c + d*x)**(3/2)), x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(90) = 180$.
time = 1.42, size = 504, normalized size = 4.20

$$\frac{\left(a\sqrt{|a|}+i\,|a|^{\frac{3}{2}}\right)\log\left(-\frac{-i\left(\frac{\sqrt{2}\sqrt{i\,a\tan\left(dx+c\right)+a}\left(-\frac{i\,|a|}{a}+1\right)|a|^{\frac{3}{2}}}{a^2}-\frac{\sqrt{-2\left(i\,a\tan\left(dx+c\right)+a\right)a+2\,a^2}\left(\frac{\tan\left(dx+c\right)}{\sqrt{\frac{\left(i\,a\tan\left(dx+c\right)+a\right)^2-2\left(i\,a\tan\left(dx+c\right)+a\right)a+a^2}{a^2}}}+1\right)|a|}{a^2}\right)^2+8\sqrt{2}+12}{2\left(\frac{1}{2}i\left(\frac{\sqrt{2}\sqrt{i\,a\tan\left(dx+c\right)+a}\left(-\frac{i\,|a|}{a}+1\right)|a|^{\frac{3}{2}}}{a^2}-\frac{\sqrt{-2\left(i\,a\tan\left(dx+c\right)+a\right)a+2\,a^2}\left(\frac{\tan\left(dx+c\right)}{\sqrt{\frac{\left(i\,a\tan\left(dx+c\right)+a\right)^2-2\left(i\,a\tan\left(dx+c\right)+a\right)a+a^2}{a^2}}}+1\right)|a|}{a^2}\right)^2+4\sqrt{2}-6\right)}\right)}{4\,a^2d}-\frac{\sqrt{-2\left(i\,a\tan\left(dx+c\right)+a\right)a+2\,a^2}\sqrt{i\,a\tan\left(dx+c\right)+a}\left(\frac{\tan\left(dx+c\right)}{\sqrt{\frac{\left(i\,a\tan\left(dx+c\right)+a\right)^2-2\left(i\,a\tan\left(dx+c\right)+a\right)a+a^2}{a^2}}}+1\right)|a|}{a^3d\tan\left(dx+c\right)}-\frac{4\sqrt{2}\left(-i\,a\sqrt{|a|}+|a|^{\frac{3}{2}}\right)}{\left(\left(\frac{\sqrt{2}\sqrt{i\,a\tan\left(dx+c\right)+a}\left(-\frac{i\,|a|}{a}+1\right)|a|^{\frac{3}{2}}}{a^2}-\frac{\sqrt{-2\left(i\,a\tan\left(dx+c\right)+a\right)a+2\,a^2}\left(\frac{\tan\left(dx+c\right)}{\sqrt{\frac{\left(i\,a\tan\left(dx+c\right)+a\right)^2-2\left(i\,a\tan\left(dx+c\right)+a\right)a+a^2}{a^2}}}+1\right)|a|}{a^2}\right)^2-4i\right)a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(a*sqrt(abs(a)) + I*abs(a)^(3/2))*log(-1/2*(-I*(sqrt(2)*sqrt(I*a*tan(d*
x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*tan(d*x + c)
+ a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(d
*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^2 + 8*sqrt(2) + 12)/(1/2*I*(sqr
t(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-
2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) +
 a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^2 + 4*sqrt(
2) - 6))/(a^2*d) - sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*sqrt(I*a*tan(d
*x + c) + a)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(d*x
+ c) + a)*a + a^2)/a^2) + 1)*abs(a)/(a^3*d*tan(d*x + c)) - 4*sqrt(2)*(-I*a*
sqrt(abs(a)) + abs(a)^(3/2))/(((sqrt(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(
a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan
(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2
)/a^2) + 1)*abs(a)/a^2)^2 - 4*I)*a^2*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\tan(c+d\,x)^{3/2}\,\sqrt{a+a\tan(c+d\,x)\ 1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

```
[Out] int(1/(tan(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

3.216 $\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=161

$$-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{1}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}-\frac{5\sqrt{a+ia\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}$$

```
[Out] (-1/2+1/2*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2
))/d/a^(1/2)+7/3*I*(a+I*a*tan(d*x+c))^(1/2)/a/d/tan(d*x+c)^(1/2)+1/d/(a+I*a
*tan(d*x+c))^(1/2)/tan(d*x+c)^(3/2)-5/3*(a+I*a*tan(d*x+c))^(1/2)/a/d/tan(d*
x+c)^(3/2)
```

Rubi [A]
time = 0.27, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3640, 3679, 12, 3625, 211}

$$-\frac{5\sqrt{a+ia\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}+\frac{1}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}+\frac{7i\sqrt{a+ia\tan(c+dx)}}{3ad\sqrt{\tan(c+dx)}}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] ((-1/2 + I/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan
[c + d*x]]])/(Sqrt[a]*d) + 1/(d*Tan[c + d*x]^(3/2)*Sqrt[a + I*a*Tan[c + d*x
]]) - (5*Sqrt[a + I*a*Tan[c + d*x]])/(3*a*d*Tan[c + d*x]^(3/2)) + (((7*I)/3
)*Sqrt[a + I*a*Tan[c + d*x]])/(a*d*Sqrt[Tan[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
```

```
Q[c^2 + d^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{1}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} + \frac{\int \frac{\sqrt{a+ia\tan(c+dx)}}{\tan^{\frac{5}{2}}(c+}}{a^2}$$

$$= \frac{1}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} - \frac{5\sqrt{a+ia\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{1}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} - \frac{5\sqrt{a+ia\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{1}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} - \frac{5\sqrt{a+ia\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{1}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} - \frac{5\sqrt{a+ia\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{}{d\tan^{\frac{3}{2}}(c+dx)}$$

Mathematica [A]
time = 1.85, size = 159, normalized size = 0.99

$$\frac{i\left(3-18e^{2i(c+dx)}+7e^{4i(c+dx)}+3e^{i(c+dx)}\left(-1+e^{2i(c+dx)}\right)^{3/2}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)}{3\sqrt{2}\,d\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(-1+e^{4i(c+dx)}\right)\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Tan[c + d*x]^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] ((I/3)*(3 - 18*E^((2*I)*(c + d*x)) + 7*E^((4*I)*(c + d*x)) + 3*E^(I*(c + d*
x))*(-1 + E^((2*I)*(c + d*x)))^(3/2)*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((
2*I)*(c + d*x))]]))/(Sqrt[2]*d*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(
c + d*x)))]*(-1 + E^((4*I)*(c + d*x)))*Sqrt[Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 400 vs. 2(128) = 256.
time = 0.21, size = 401, normalized size = 2.49

method	result
derivativedivides	$\dfrac{\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\,\left(3i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1+i\tan\left(dx+c\right)\right)}+}{\tan(dx+c)+i}\right.\right.}{}$
default	$\dfrac{\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\,\left(3i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1+i\tan\left(dx+c\right)\right)}+}{\tan(dx+c)+i}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/d*(a*(1+I*tan(d*x+c)))^(1/2)/a/tan(d*x+c)^(3/2)*(3*I*2^(1/2)*ln(-(-2*2
^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c
))/(tan(d*x+c)+I))*a*tan(d*x+c)^4+36*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*
x+c)))^(1/2)*tan(d*x+c)^2-3*I*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d
*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x
+c)^2+28*I*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^3+
6*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2
)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3+8*(a*tan(d*x+c)*(1+I*t
an(d*x+c)))^(1/2)*(-I*a)^(1/2))/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(-I*a
)^(1/2)/(-tan(d*x+c)+I)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(I*a*tan(d*x + c) + a)*tan(d*x + c)^(5/2)), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 403 vs. 2(119) = 238.
time = 0.40, size = 403, normalized size = 2.50

$$-\frac{2\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}\left(7e^{(6idx+6ic)}-11e^{(4idx+4ic)}-15e^{(2idx+2ic)}+3\right)+3\left(ade^{(5idx+5ic)}-2ade^{(3idx+3ic)}+ade^{(idx+ic)}\right)\sqrt{-\frac{2i}{ad^2}}\log\left(\frac{1}{4}ad\sqrt{-\frac{2i}{ad^2}}e^{(idx+ic)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}\left(e^{(2idx+2ic)}+1\right)\right)-3\left(ade^{(5idx+5ic)}-2ade^{(3idx+3ic)}+ade^{(idx+ic)}\right)\sqrt{-\frac{2i}{ad^2}}\log\left(-\frac{1}{4}ad\sqrt{-\frac{2i}{ad^2}}e^{(idx+ic)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}}\left(e^{(2idx+2ic)}+1\right)\right)}{12\left(ade^{(5idx+5ic)}-2ade^{(3idx+3ic)}+ade^{(idx+ic)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(5/2),x, algorithm="fricas"
)
```

```
[Out] -1/12*(2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*
I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(7*e^(6*I*d*x + 6*I*c) - 11*e^(4*I*d*x
 + 4*I*c) - 15*e^(2*I*d*x + 2*I*c) + 3) + 3*(a*d*e^(5*I*d*x + 5*I*c) - 2*a*
d*e^(3*I*d*x + 3*I*c) + a*d*e^(I*d*x + I*c))*sqrt(-2*I/(a*d^2))*log(1/4*a*d
*sqrt(-2*I/(a*d^2))*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2
*I*d*x + 2*I*c) + 1)) - 3*(a*d*e^(5*I*d*x + 5*I*c) - 2*a*d*e^(3*I*d*x + 3*I
*c) + a*d*e^(I*d*x + I*c))*sqrt(-2*I/(a*d^2))*log(-1/4*a*d*sqrt(-2*I/(a*d^2
))*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I
*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) +
 1)))/(a*d*e^(5*I*d*x + 5*I*c) - 2*a*d*e^(3*I*d*x + 3*I*c) + a*d*e^(I*d*x +
 I*c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,\tan^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))**(1/2)/tan(d*x+c)**(5/2),x)
```

```
[Out] Integral(1/(sqrt(I*a*(tan(c + d*x) - I))*tan(c + d*x)**(5/2)), x)
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 529 vs. 2(119) = 238.
time = 1.57, size = 529, normalized size = 3.29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*sqrt(I*a*tan(d*x + c) + a)*(t
an(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a
^2)/a^2) + 1)*(2*(I*a*tan(d*x + c) + a)/(a^2*d) - 3/(a*d))*abs(a)/(a^2*tan(
d*x + c)^2) - 1/4*(-I*a*sqrt(abs(a)) + abs(a)^(3/2))*log(-1/2*(-I*(sqrt(2)*
sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*
a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2
 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^2 + 8*sqrt(2) +
12)/(1/2*I*(sqrt(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/
```

```
2)/a^2 - sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a
*tan(d*x + c) + a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a
^2)^2 + 4*sqrt(2) - 6))/(a^2*d) + 4*sqrt(2)*(a*sqrt(abs(a)) + I*abs(a)^(3/2
))/(((sqrt(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2
 - sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d
*x + c) + a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^2
- 4*I)*a^2*d)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\tan\left(c+d\,x\right)^{5/2}\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
[Out] int(1/(tan(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

3.217 $\int \frac{1}{\tan^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=198

$$-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{1}{d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}-\frac{7\sqrt{a+ia\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}$$

```
[Out] (-1/2-1/2*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2
))/d/a^(1/2)+61/15*(a+I*a*tan(d*x+c))^(1/2)/a/d/tan(d*x+c)^(1/2)+1/d/(a+I*a
*tan(d*x+c))^(1/2)/tan(d*x+c)^(5/2)-7/5*(a+I*a*tan(d*x+c))^(1/2)/a/d/tan(d*
x+c)^(5/2)+23/15*I*(a+I*a*tan(d*x+c))^(1/2)/a/d/tan(d*x+c)^(3/2)
```

Rubi [A]
time = 0.35, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3640, 3679, 12, 3625, 211}

$$\frac{23i\sqrt{a+ia\tan(c+dx)}}{15ad\tan^{\frac{3}{2}}(c+dx)}-\frac{7\sqrt{a+ia\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}+\frac{1}{d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}+\frac{61\sqrt{a+ia\tan(c+dx)}}{15ad\sqrt{\tan(c+dx)}}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] ((-1/2 - I/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan
[c + d*x]]])/(Sqrt[a]*d) + 1/(d*Tan[c + d*x]^(5/2)*Sqrt[a + I*a*Tan[c + d*x
]]) - (7*Sqrt[a + I*a*Tan[c + d*x]])/(5*a*d*Tan[c + d*x]^(5/2)) + (((23*I)/
15)*Sqrt[a + I*a*Tan[c + d*x]])/(a*d*Tan[c + d*x]^(3/2)) + (61*Sqrt[a + I*a
*Tan[c + d*x]])/(15*a*d*Sqrt[Tan[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
```

```
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\tan^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{1}{d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} + \frac{\int \frac{\sqrt{a+ia\tan(c+dx)}}{\tan^{\frac{7}{2}}(c+}}{a^2}$$

$$= \frac{1}{d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} - \frac{7\sqrt{a+ia\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}$$

$$= \frac{1}{d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} - \frac{7\sqrt{a+ia\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}$$

$$= \frac{1}{d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} - \frac{7\sqrt{a+ia\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}$$

$$= \frac{1}{d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} - \frac{7\sqrt{a+ia\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}$$

$$= \frac{1}{d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} - \frac{7\sqrt{a+ia\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}$$

$$= -\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{}{d\tan^{\frac{5}{2}}(c+dx)}$$

Mathematica [A]
time = 1.41, size = 191, normalized size = 0.96

$$\frac{i\left(\sqrt{-1+e^{2i(c+dx)}}\left(-15+165e^{2i(c+dx)}-205e^{4i(c+dx)}+103e^{6i(c+dx)}\right)-15e^{i(c+dx)}\left(-1+e^{2i(c+dx)}\right)^3\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\tan(c+dx)}}{15\sqrt{2}\,d\left(-1+e^{2i(c+dx)}\right)^{7/2}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Tan[c + d*x]^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
[Out] ((I/15)*(Sqrt[-1 + E^((2*I)*(c + d*x))]*(-15 + 165*E^((2*I)*(c + d*x)) - 20
5*E^((4*I)*(c + d*x)) + 103*E^((6*I)*(c + d*x))) - 15*E^(I*(c + d*x))*(-1 +
 E^((2*I)*(c + d*x)))^3*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x
))]])*Sqrt[Tan[c + d*x]])/(Sqrt[2]*d*(-1 + E^((2*I)*(c + d*x)))^(7/2)*Sqrt[
(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(158) = 316$.
time = 0.19, size = 473, normalized size = 2.39

method	result
derivativedivides	$\frac{\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\,\left(30i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1+i\tan\left(dx+c\right)\right)}}{\tan(dx+c)+i}\right.\right.}{}$
default	$\frac{\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\,\left(30i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1+i\tan\left(dx+c\right)\right)}}{\tan(dx+c)+i}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/60/d*(a*(1+I*tan(d*x+c)))^(1/2)*(30*I*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2
)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))
*a*tan(d*x+c)^4-15*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*
tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^5-396*I
*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^3+15*2^(1/2)
*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a
*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3+244*(-I*a)^(1/2)*(a*tan(d*x+c)*
(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^4+16*I*(-I*a)^(1/2)*tan(d*x+c)*(a*tan(d*
x+c)*(1+I*tan(d*x+c)))^(1/2)-144*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)
))^(1/2)*tan(d*x+c)^2+24*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)
)/a/tan(d*x+c)^(5/2)/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(-tan(d*x+c)+I)^
2/(-I*a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(7/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(I*a*tan(d*x + c) + a)*tan(d*x + c)^(7/2)), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(148) = 296$.
time = 0.49, size = 456, normalized size = 2.30

$$-\frac{2\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(-103i\,e^{(8i\,dx+8i\,c)}+102i\,e^{(6i\,dx+6i\,c)}+40i\,e^{(4i\,dx+4i\,c)}-150i\,e^{(2i\,dx+2i\,c)}+15i\right)-15\left(ade^{(7i\,dx+7i\,c)}-3\,ade^{(5i\,dx+5i\,c)}+3\,ade^{(3i\,dx+3i\,c)}-ade^{(i\,dx+i\,c)}\right)\sqrt{\frac{2i}{ad^2}}\log\left(\frac{1}{4}i\,ad\sqrt{\frac{2i}{ad^2}}e^{(i\,dx+i\,c)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)\right)+15\left(ade^{(7i\,dx+7i\,c)}-3\,ade^{(5i\,dx+5i\,c)}+3\,ade^{(3i\,dx+3i\,c)}-ade^{(i\,dx+i\,c)}\right)\sqrt{\frac{2i}{ad^2}}\log\left(-\frac{1}{4}i\,ad\sqrt{\frac{2i}{ad^2}}e^{(i\,dx+i\,c)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)\right)}{60\left(ade^{(7i\,dx+7i\,c)}-3\,ade^{(5i\,dx+5i\,c)}+3\,ade^{(3i\,dx+3i\,c)}-ade^{(i\,dx+i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(7/2),x, algorithm="fricas"
)
```

```
[Out] -1/60*(2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*
I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(-103*I*e^(8*I*d*x + 8*I*c) + 102*I*e^
(6*I*d*x + 6*I*c) + 40*I*e^(4*I*d*x + 4*I*c) - 150*I*e^(2*I*d*x + 2*I*c) +
15*I) - 15*(a*d*e^(7*I*d*x + 7*I*c) - 3*a*d*e^(5*I*d*x + 5*I*c) + 3*a*d*e^(
3*I*d*x + 3*I*c) - a*d*e^(I*d*x + I*c))*sqrt(2*I/(a*d^2))*log(1/4*I*a*d*sqr
t(2*I/(a*d^2))*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*
x + 2*I*c) + 1)) + 15*(a*d*e^(7*I*d*x + 7*I*c) - 3*a*d*e^(5*I*d*x + 5*I*c)
+ 3*a*d*e^(3*I*d*x + 3*I*c) - a*d*e^(I*d*x + I*c))*sqrt(2*I/(a*d^2))*log(-1
/4*I*a*d*sqrt(2*I/(a*d^2))*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)
)*(e^(2*I*d*x + 2*I*c) + 1)))/(a*d*e^(7*I*d*x + 7*I*c) - 3*a*d*e^(5*I*d*x +
 5*I*c) + 3*a*d*e^(3*I*d*x + 3*I*c) - a*d*e^(I*d*x + I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,\tan^{\frac{7}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))**(1/2)/tan(d*x+c)**(7/2),x)
```

```
[Out] Integral(1/(sqrt(I*a*(tan(c + d*x) - I))*tan(c + d*x)**(7/2)), x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 552 vs. 2(148) = 296.
time = 2.05, size = 552, normalized size = 2.79

$$\frac{\left(a\sqrt{|a|}+i|a|^{\frac{3}{2}}\right)\log\left(-\frac{i\left(\frac{\sqrt{2}\sqrt{ia\tan(dx+c)+a}\left(-\frac{i|a|}{a}+1\right)|a|^{\frac{3}{2}}}{a^2}-\frac{\sqrt{-2(ia\tan(dx+c)+a)a+2a^2}\left(\frac{\tan(dx+c)}{\sqrt{\frac{(ia\tan(dx+c)+a)^2-2(ia\tan(dx+c)+a)a+a^2}{a^2}}}+1\right)|a|}{a^2}\right)^2+8\sqrt{2}-12}{2\left(-\frac{1}{2}i\left(\frac{\sqrt{2}\sqrt{ia\tan(dx+c)+a}\left(-\frac{i|a|}{a}+1\right)|a|^{\frac{3}{2}}}{a^2}-\frac{\sqrt{-2(ia\tan(dx+c)+a)a+2a^2}\left(\frac{\tan(dx+c)}{\sqrt{\frac{(ia\tan(dx+c)+a)^2-2(ia\tan(dx+c)+a)a+a^2}{a^2}}}+1\right)|a|}{a^2}\right)^2+4\sqrt{2}+6\right)}\right)}{4a^2d}-\frac{\sqrt{-2(ia\tan(dx+c)+a)a+2a^2}\left((ia\tan(dx+c)+a)\left(\frac{23(ia\tan(dx+c)+a)}{a^3d}-\frac{50}{a^2d}\right)+\frac{30}{ad}\right)\sqrt{ia\tan(dx+c)+a}\left(\frac{\tan(dx+c)}{\sqrt{\frac{(ia\tan(dx+c)+a)^2-2(ia\tan(dx+c)+a)a+a^2}{a^2}}}+1\right)|a|}{15a^2\tan(dx+c)^3}-\frac{4\sqrt{2}\left(ia\sqrt{|a|}-|a|^{\frac{3}{2}}\right)}{\left(\left(\frac{\sqrt{2}\sqrt{ia\tan(dx+c)+a}\left(-\frac{i|a|}{a}+1\right)|a|^{\frac{3}{2}}}{a^2}-\frac{\sqrt{-2(ia\tan(dx+c)+a)a+2a^2}\left(\frac{\tan(dx+c)}{\sqrt{\frac{(ia\tan(dx+c)+a)^2-2(ia\tan(dx+c)+a)a+a^2}{a^2}}}+1\right)|a|}{a^2}\right)^2-4i\right)a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] 1/4*(a*sqrt(abs(a)) + I*abs(a)^(3/2))*log(-1/2*(I*(sqrt(2)*sqrt(I*a*tan(d*x
 + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*tan(d*x + c) +
 a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(d*
```

```
x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^2 + 8*sqrt(2) - 12)/(-1/2*I*(sqr
t(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-
2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) +
 a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^2 + 4*sqrt(
2) + 6))/(a^2*d) - 1/15*sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*((I*a*tan
(d*x + c) + a)*(23*(I*a*tan(d*x + c) + a)/(a^3*d) - 50/(a^2*d)) + 30/(a*d))
*sqrt(I*a*tan(d*x + c) + a)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2 -
2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/(a^2*tan(d*x + c)^3) - 4
*sqrt(2)*(I*a*sqrt(abs(a)) - abs(a)^(3/2))/(((sqrt(2)*sqrt(I*a*tan(d*x + c)
 + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*tan(d*x + c) + a)*a
 + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(d*x + c
) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^2 - 4*I)*a^2*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\tan\left(c+d\,x\right)^{7/2}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(7/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
[Out] int(1/(tan(c + d*x)^(7/2)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

3.218 $\int \frac{\tan^{\frac{7}{2}}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=218

$$-\frac{3\sqrt[4]{-1}\,\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}+\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}-\frac{\tan^{\frac{5}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] -3*(-1)^(1/4)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))
^(1/2))/a^(3/2)/d+(1/4-1/4*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a
*tan(d*x+c))^(1/2))/a^(3/2)/d-7/2*tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)
/a^2/d+13/6*I*tan(d*x+c)^(3/2)/a/d/(a+I*a*tan(d*x+c))^(1/2)-1/3*tan(d*x+c)^
(5/2)/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.45, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3639, 3676, 3678, 3682, 3625, 211, 3680, 65, 223, 209}

$$-\frac{3\sqrt[4]{-1}\,\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}+\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}-\frac{7\sqrt{\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{2a^2d}-\frac{\tan^{\frac{5}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}+\frac{13i\tan^{\frac{3}{2}}(c+dx)}{6ad\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(7/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (-3*(-1)^(1/4)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*
Tan[c + d*x]]])/(a^(3/2)*d) + ((1/4 - I/4)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Ta
n[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/(a^(3/2)*d) - Tan[c + d*x]^(5/2)/
(3*d*(a + I*a*Tan[c + d*x])^(3/2)) + (((13*I)/6)*Tan[c + d*x]^(3/2))/(a*d*S
qrt[a + I*a*Tan[c + d*x]]) - (7*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x
]])/(2*a^2*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rule 3678

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Dist[
1/(a*(m + n)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Simp
[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*Ta
```

```
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{\tan^{\frac{7}{2}}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{\tan^{\frac{5}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} - \frac{\int \frac{\tan^{\frac{3}{2}}(c+dx)\left(-\frac{5a}{2}+4ia\tan(c+dx)\right)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{3a^2}$$

$$= -\frac{\tan^{\frac{5}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{13i\tan^{\frac{3}{2}}(c+dx)}{6ad\sqrt{a+ia\tan(c+dx)}} + \frac{\int \sqrt{\tan(c+dx)}}{}$$

$$= -\frac{\tan^{\frac{5}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{13i\tan^{\frac{3}{2}}(c+dx)}{6ad\sqrt{a+ia\tan(c+dx)}} - \frac{7\sqrt{\tan(c+dx)}}{2}$$

$$= -\frac{\tan^{\frac{5}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{13i\tan^{\frac{3}{2}}(c+dx)}{6ad\sqrt{a+ia\tan(c+dx)}} - \frac{7\sqrt{\tan(c+dx)}}{2}$$

$$= -\frac{\tan^{\frac{5}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{13i\tan^{\frac{3}{2}}(c+dx)}{6ad\sqrt{a+ia\tan(c+dx)}} - \frac{7\sqrt{\tan(c+dx)}}{2}$$

$$= \frac{\left(\frac{1}{4}-\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d} - \frac{\tan^{\frac{5}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{}{6}$$

$$= \frac{\left(\frac{1}{4}-\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d} - \frac{\tan^{\frac{5}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{}{6}$$

$$= -\frac{3\sqrt[4]{-1}\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d} + \frac{\left(\frac{1}{4}-\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{}}{\sqrt{a+ia\,\mathrm{t}}}\right)}{a^{3/2}d}$$

Mathematica [A]
time = 4.28, size = 240, normalized size = 1.10

$$\frac{-\frac{12e^{3i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\left(\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)+6\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)}{\sqrt{-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}\,(1+e^{2i(c+dx)})^2} + i\sec^2(c+dx)(15+27\cos(2(c+dx))+29i\sin(2(c+dx)))\sqrt{\tan(c+dx)}}{12ad(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(7/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-12*E^((3*I)*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]*(ArcTanh[E^(I*(c +
 d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]] + 6*Sqrt[2]*ArcTanh[(Sqrt[2]*E^(I*(c
```

```
 + d*x)))/Sqrt[-1 + E^((2*I)*(c + d*x))]]))/(Sqrt[((-I)*(-1 + E^((2*I)*(c +
 d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))^2) + I*Sec[c
+ d*x]^2*(15 + 27*Cos[2*(c + d*x)] + (29*I)*Sin[2*(c + d*x)])*Sqrt[Tan[c +
d*x]])/(12*a*d*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 814 vs. $2(170) = 340$.
time = 0.18, size = 815, normalized size = 3.74 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)/a^2*(24*(a*tan(d*x+c)*(1
+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^3+3*I*(I*a)^(1/2)
*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)
+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3-9*I*(I*a)^(1/2)*2^(1/2)
*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a
*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)+108*I*ln(1/2*(2*I*a*tan(d*x+c)+2*
(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1
/2)*a*tan(d*x+c)^2-140*I*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*
(-I*a)^(1/2)*tan(d*x+c)^2+9*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2
)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))
*a*tan(d*x+c)^2-36*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)
))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^3-36*I*ln(1/
2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/
(I*a)^(1/2))*(-I*a)^(1/2)*a+84*I*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a
)^(1/2)*(-I*a)^(1/2)-3*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*
tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a+10
8*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1
/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)-200*(a*tan(d*x+c)*(1+I*tan(d*
x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c))/(a*tan(d*x+c)*(1+I*tan(d*
x+c)))^(1/2)/(-tan(d*x+c)+I)^3/(I*a)^(1/2)/(-I*a)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 612 vs. $2(160) = 320$.

time = 0.62, size = 612, normalized size = 2.81

$$\frac{3 a^{2} d \sqrt{-\frac{i}{2 a^{3} d^{2}}} e^{(3 i d x + 3 i c)} \log\left(\frac{1}{2} a^{2} d \sqrt{-\frac{i}{2 a^{3} d^{2}}} e^{(i d x + i c)} + \frac{1}{4} \sqrt{2} \sqrt{\frac{a}{e^{(2 i d x + 2 i c)} + 1}} \sqrt{\frac{-i e^{(2 i d x + 2 i c)} + i}{e^{(2 i d x + 2 i c)} + 1}} \left(e^{(2 i d x + 2 i c)} + 1\right)\right) - 3 a^{2} d \sqrt{-\frac{i}{2 a^{3} d^{2}}} e^{(3 i d x + 3 i c)} \log\left(-\frac{1}{2} a^{2} d \sqrt{-\frac{i}{2 a^{3} d^{2}}} e^{(i d x + i c)} + \frac{1}{4} \sqrt{2} \sqrt{\frac{a}{e^{(2 i d x + 2 i c)} + 1}} \sqrt{\frac{-i e^{(2 i d x + 2 i c)} + i}{e^{(2 i d x + 2 i c)} + 1}} \left(e^{(2 i d x + 2 i c)} + 1\right)\right) + 3 a^{2} d \sqrt{-\frac{9 i}{a^{3} d^{2}}} e^{(3 i d x + 3 i c)} \log\left(\frac{104 \left(6 \sqrt{2} \sqrt{\frac{a}{e^{(2 i d x + 2 i c)} + 1}} \sqrt{\frac{-i e^{(2 i d x + 2 i c)} + i}{e^{(2 i d x + 2 i c)} + 1}} \left(e^{(3 i d x + 3 i c)} + e^{(i d x + i c)}\right) + \left(3 a^{2} d e^{(2 i d x + 2 i c)} - a^{2} d\right) \sqrt{-\frac{9 i}{a^{3} d^{2}}}\right)}{1815 \left(e^{(2 i d x + 2 i c)} + 1\right)}\right) - 3 a^{2} d \sqrt{-\frac{9 i}{a^{3} d^{2}}} e^{(3 i d x + 3 i c)} \log\left(\frac{104 \left(6 \sqrt{2} \sqrt{\frac{a}{e^{(2 i d x + 2 i c)} + 1}} \sqrt{\frac{-i e^{(2 i d x + 2 i c)} + i}{e^{(2 i d x + 2 i c)} + 1}} \left(e^{(3 i d x + 3 i c)} + e^{(i d x + i c)}\right) - \left(3 a^{2} d e^{(2 i d x + 2 i c)} - a^{2} d\right) \sqrt{-\frac{9 i}{a^{3} d^{2}}}\right)}{1815 \left(e^{(2 i d x + 2 i c)} + 1\right)}\right) - \sqrt{2} \sqrt{\frac{a}{e^{(2 i d x + 2 i c)} + 1}} \sqrt{\frac{-i e^{(2 i d x + 2 i c)} + i}{e^{(2 i d x + 2 i c)} + 1}} \left(28 e^{(4 i d x + 4 i c)} + 15 e^{(2 i d x + 2 i c)} - 1\right)\right) e^{(-3 i d x - 3 i c)}}{12 a^{2} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^2*d*sqrt(-1/2*I/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(1/2*a^2*d*sqrt
(-1/2*I/(a^3*d^2))*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*
I*d*x + 2*I*c) + 1)) - 3*a^2*d*sqrt(-1/2*I/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*l
og(-1/2*a^2*d*sqrt(-1/2*I/(a^3*d^2))*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2
*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)) + 3*a^2*d*sqrt(-9*I/(a^3*d^2))*e^(3*
I*d*x + 3*I*c)*log(104/1815*(6*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sq
rt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(3*I*d*x + 3*
I*c) + e^(I*d*x + I*c)) + (3*a^2*d*e^(2*I*d*x + 2*I*c) - a^2*d)*sqrt(-9*I/(
a^3*d^2)))/(e^(2*I*d*x + 2*I*c) + 1)) - 3*a^2*d*sqrt(-9*I/(a^3*d^2))*e^(3*I
*d*x + 3*I*c)*log(104/1815*(6*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqr
t((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(3*I*d*x + 3*I
*c) + e^(I*d*x + I*c)) - (3*a^2*d*e^(2*I*d*x + 2*I*c) - a^2*d)*sqrt(-9*I/(a
^3*d^2)))/(e^(2*I*d*x + 2*I*c) + 1)) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(28*e^(4
*I*d*x + 4*I*c) + 15*e^(2*I*d*x + 2*I*c) - 1))*e^(-3*I*d*x - 3*I*c)/(a^2*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(7/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(c+d\,x)^{7/2}}{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(7/2)/(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int(tan(c + d*x)^(7/2)/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.219 $\int \frac{\tan^{\frac{5}{2}}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=181

$$\frac{2(-1)^{3/4}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}+\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}-\frac{\tan^{\frac{3}{2}}(c+}{3d(a+ia\tan(c}$$

[Out] 2*(-1)^(3/4)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/a^(3/2)/d+(1/4+1/4*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/a^(3/2)/d+3/2*I*tan(d*x+c)^(1/2)/a/d/(a+I*a*tan(d*x+c))^(1/2)-1/3*tan(d*x+c)^(3/2)/d/(a+I*a*tan(d*x+c))^(3/2)

Rubi [A]

time = 0.36, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {3639, 3676, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{2(-1)^{3/4}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}+\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}-\frac{\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}+\frac{3i\sqrt{\tan(c+dx)}}{2ad\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d*x]^(5/2)/(a + I*a*Tan[c + d*x])^(3/2),x]

[Out] (2*(-1)^(3/4)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/(a^(3/2)*d) + ((1/4 + I/4)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/(a^(3/2)*d) - Tan[c + d*x]^(3/2)/(3*d*(a + I*a*Tan[c + d*x])^(3/2)) + (((3*I)/2)*Sqrt[Tan[c + d*x]])/(a*d*Sqrt[a + I*a*Tan[c + d*x]])

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} - \frac{\int \frac{\sqrt{\tan(c+dx)}\left(-\frac{3a}{2}+3ia\tan(c+dx)\right)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{3a^2}$$

$$= -\frac{\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{3i\sqrt{\tan(c+dx)}}{2ad\sqrt{a+ia\tan(c+dx)}} + \frac{\int \frac{\sqrt{a+ia\tan(c}}{\sqrt{}}}{}$$

$$= -\frac{\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{3i\sqrt{\tan(c+dx)}}{2ad\sqrt{a+ia\tan(c+dx)}} - \frac{i\int \frac{(a-ia\tan(c+dx))}{\sqrt{ta}}}{}$$

$$= -\frac{\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{3i\sqrt{\tan(c+dx)}}{2ad\sqrt{a+ia\tan(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{-ia-2a^2x}\right.}{}$$

$$= \frac{\left(\frac{1}{4}+\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d} - \frac{\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{}{2a}$$

$$= \frac{\left(\frac{1}{4}+\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d} - \frac{\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{}{2a}$$

$$= \frac{2(-1)^{3/4}\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d} + \frac{\left(\frac{1}{4}+\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{t}}{\sqrt{a+ia\,ta}}\right.}{a^{3/2}d}$$

Mathematica [A]
time = 2.13, size = 217, normalized size = 1.20

$$-\frac{ie^{-2i(c+dx)}\left(\left(1-10e^{2i(c+dx)}\right)\sqrt{-1+e^{2i(c+dx)}}-3e^{3i(c+dx)}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)+12\sqrt{2}\,e^{3i(c+dx)}\tanh^{-1}\left(\frac{\sqrt{2}\,e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\tan(c+dx)}}{6\sqrt{2}\,ad\sqrt{-1+e^{2i(c+dx)}}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(5/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-1/6*I)*((1 - 10*E^((2*I)*(c + d*x)))*Sqrt[-1 + E^((2*I)*(c + d*x))] - 3*
E^((3*I)*(c + d*x))*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]]
 + 12*Sqrt[2]*E^((3*I)*(c + d*x))*ArcTanh[(Sqrt[2]*E^(I*(c + d*x)))/Sqrt[-1
 + E^((2*I)*(c + d*x))]])*Sqrt[Tan[c + d*x]])/(Sqrt[2]*a*d*E^((2*I)*(c + d*
x))*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*
I)*(c + d*x)))]])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 770 vs. $2(140) = 280$.
time = 0.18, size = 771, normalized size = 4.26 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)*(9*I*(I*a)^(1/2)*2^(1/2)
*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a
*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2+24*I*ln(1/2*(2*I*a*tan(d*x+c)+2
*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(
1/2)*a*tan(d*x+c)^3-3*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*t
an(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan
(d*x+c)^3-3*I*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c
)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a-72*I*ln(1/2
*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(
I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)+80*I*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^
(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)+9*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a
*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*(I*
a)^(1/2)*2^(1/2)*a*tan(d*x+c)+72*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(
1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c
)^2-44*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d
*x+c)^2-24*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)
*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*(-I*a)^(1/2)+36*(a*tan(d*x+c)*(1+I*tan(d*x+c
)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2))/a^2/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/
2)/(-tan(d*x+c)+I)^3/(I*a)^(1/2)/(-I*a)^(1/2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(131) = 262$.
time = 0.52, size = 613, normalized size = 3.39

$$-\frac{\left(3\,a^{2} d \sqrt{\frac{i}{2\,a^{3} d^{2}}} e^{\left(3 i\,d x + 3 i\,c\right)} \log\left(\frac{1}{2} i\,a^{2} d \sqrt{\frac{i}{2\,a^{3} d^{2}}} e^{\left(i\,d x + i\,c\right)} + \frac{1}{4}\,\sqrt{2} \sqrt{\frac{a}{e^{\left(2 i\,d x + 2 i\,c\right)} + 1}} \sqrt{\frac{-i\,e^{\left(2 i\,d x + 2 i\,c\right)} + i}{e^{\left(2 i\,d x + 2 i\,c\right)} + 1}} \left(e^{\left(2 i\,d x + 2 i\,c\right)} + 1\right)\right) - 3\,a^{2} d \sqrt{\frac{i}{2\,a^{3} d^{2}}} e^{\left(3 i\,d x + 3 i\,c\right)} \log\left(-\frac{1}{2} i\,a^{2} d \sqrt{\frac{i}{2\,a^{3} d^{2}}} e^{\left(i\,d x + i\,c\right)} + \frac{1}{4}\,\sqrt{2} \sqrt{\frac{a}{e^{\left(2 i\,d x + 2 i\,c\right)} + 1}} \sqrt{\frac{-i\,e^{\left(2 i\,d x + 2 i\,c\right)} + i}{e^{\left(2 i\,d x + 2 i\,c\right)} + 1}} \left(e^{\left(2 i\,d x + 2 i\,c\right)} + 1\right)\right) + 3\,a^{2} d \sqrt{\frac{4 i}{a^{3} d^{2}}} e^{\left(3 i\,d x + 3 i\,c\right)} \log\left(\frac{52 \left(4 \sqrt{2} \sqrt{\frac{a}{e^{\left(2 i\,d x + 2 i\,c\right)} + 1}} \sqrt{\frac{-i\,e^{\left(2 i\,d x + 2 i\,c\right)} + i}{e^{\left(2 i\,d x + 2 i\,c\right)} + 1}} \left(e^{\left(3 i\,d x + 3 i\,c\right)} + e^{\left(i\,d x + i\,c\right)}\right) - \left(3 i\,a^{2} d e^{\left(2 i\,d x + 2 i\,c\right)} - i\,a^{2} d\right) \sqrt{\frac{4 i}{a^{3} d^{2}}}\right)}{605 \left(e^{\left(2 i\,d x + 2 i\,c\right)} + 1\right)}\right) - 3\,a^{2} d \sqrt{\frac{4 i}{a^{3} d^{2}}} e^{\left(3 i\,d x + 3 i\,c\right)} \log\left(\frac{52 \left(4 \sqrt{2} \sqrt{\frac{a}{e^{\left(2 i\,d x + 2 i\,c\right)} + 1}} \sqrt{\frac{-i\,e^{\left(2 i\,d x + 2 i\,c\right)} + i}{e^{\left(2 i\,d x + 2 i\,c\right)} + 1}} \left(e^{\left(3 i\,d x + 3 i\,c\right)} + e^{\left(i\,d x + i\,c\right)}\right) - \left(-3 i\,a^{2} d e^{\left(2 i\,d x + 2 i\,c\right)} + i\,a^{2} d\right) \sqrt{\frac{4 i}{a^{3} d^{2}}}\right)}{605 \left(e^{\left(2 i\,d x + 2 i\,c\right)} + 1\right)}\right) - \sqrt{2} \sqrt{\frac{a}{e^{\left(2 i\,d x + 2 i\,c\right)} + 1}} \sqrt{\frac{-i\,e^{\left(2 i\,d x + 2 i\,c\right)} + i}{e^{\left(2 i\,d x + 2 i\,c\right)} + 1}} \left(10 i\,e^{\left(4 i\,d x + 4 i\,c\right)} + 9 i\,e^{\left(2 i\,d x + 2 i\,c\right)} - i\right)\right)} e^{\left(-3 i\,d x - 3 i\,c\right)}}{12\,a^{2} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/12*(3*a^2*d*sqrt(1/2*I/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(1/2*I*a^2*d*sq
rt(1/2*I/(a^3*d^2))*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2
*I*d*x + 2*I*c) + 1)) - 3*a^2*d*sqrt(1/2*I/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*l
og(-1/2*I*a^2*d*sqrt(1/2*I/(a^3*d^2))*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/
(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)) + 3*a^2*d*sqrt(4*I/(a^3*d^2))*e^(3*
I*d*x + 3*I*c)*log(52/605*(4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt
((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(3*I*d*x + 3*I*
c) + e^(I*d*x + I*c)) - (3*I*a^2*d*e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt(4*I/
(a^3*d^2)))/(e^(2*I*d*x + 2*I*c) + 1)) - 3*a^2*d*sqrt(4*I/(a^3*d^2))*e^(3*I
*d*x + 3*I*c)*log(52/605*(4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(
(-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(3*I*d*x + 3*I*c
) + e^(I*d*x + I*c)) - (-3*I*a^2*d*e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt(4*I/
(a^3*d^2)))/(e^(2*I*d*x + 2*I*c) + 1)) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(10*I*
e^(4*I*d*x + 4*I*c) + 9*I*e^(2*I*d*x + 2*I*c) - I))*e^(-3*I*d*x - 3*I*c)/(a
^2*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{5}{2}}{\left(c + dx \right)}}{\left(ia \left(\tan{\left(c + dx \right)} - i\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(5/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**(5/2)/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^{5/2}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(5/2)/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] int(tan(c + d*x)^(5/2)/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.220 $\int \frac{\tan^{\frac{3}{2}}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}+\frac{i\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}+\frac{\sqrt{\tan(c+dx)}}{2ad\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] (-1/4+1/4*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2
))/a^(3/2)/d+1/2*tan(d*x+c)^(1/2)/a/d/(a+I*a*tan(d*x+c))^(1/2)+1/3*I*tan(d*
x+c)^(3/2)/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.13, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3627, 3625, 211}

$$-\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}+\frac{i\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}+\frac{\sqrt{\tan(c+dx)}}{2ad\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(3/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-1/4 + I/4)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan
[c + d*x]]])/(a^(3/2)*d) + ((I/3)*Tan[c + d*x]^(3/2))/(d*(a + I*a*Tan[c + d
*x])^(3/2)) + Sqrt[Tan[c + d*x]]/(2*a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3627

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
```

```
 + f*x])^n/(2*b*f*m)), x] - Dist[(a*c - b*d)/(2*b^2), Int[(a + b*Tan[e + f*
x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && EqQ
[m + n, 0] && LeQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{i\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} - \frac{i\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{2a}$$

$$= \frac{i\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{\sqrt{\tan(c+dx)}}{2ad\sqrt{a+ia\tan(c+dx)}} - \frac{\int \frac{\sqrt{a+ia\tan(c}{\sqrt{\tan(c+d}}}{4a^2}$$

$$= \frac{i\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{\sqrt{\tan(c+dx)}}{2ad\sqrt{a+ia\tan(c+dx)}} + \frac{i\text{Subst}\left(\int \frac{1}{-ia-2a^2}\right.}{}$$

$$= -\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d} + \frac{i\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

Mathematica [A]
time = 1.99, size = 158, normalized size = 1.24

$$-\frac{ie^{-4i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(1-5e^{2i(c+dx)}+4e^{4i(c+dx)}-3e^{3i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)}{6\sqrt{2}\,a^2d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(3/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-1/6*I)*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(1 - 5*E^
((2*I)*(c + d*x)) + 4*E^((4*I)*(c + d*x)) - 3*E^((3*I)*(c + d*x))*Sqrt[-1 +
 E^((2*I)*(c + d*x))]*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))
]]))/(Sqrt[2]*a^2*d*E^((4*I)*(c + d*x))*Sqrt[Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 463 vs. 2(99) = 198.
time = 0.17, size = 464, normalized size = 3.65

method	result
derivativedivides	$-\frac{\left(\sqrt{\tan}(dx+c)\right)\sqrt{a(1+i\tan(dx+c))}\left(3i\sqrt{2}\ \ln\left(-\frac{-2\sqrt{2}\sqrt{-ia}\ \sqrt{a\tan(dx+c)(1+}}{\tan(dx+c)+i}\right.\right.}{}$
default	$-\frac{\left(\sqrt{\tan}(dx+c)\right)\sqrt{a(1+i\tan(dx+c))}\left(3i\sqrt{2}\ \ln\left(-\frac{-2\sqrt{2}\sqrt{-ia}\ \sqrt{a\tan(dx+c)(1+}}{\tan(dx+c)+i}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)/a^2*(3*I*2^(1/2)*ln(-(-
2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*
x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3-9*I*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/
2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I)
)*a*tan(d*x+c)-20*I*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(
d*x+c)^2+9*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+
c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2+12*I*(a*tan(d
*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)-3*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)
^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c
)+I))*a-32*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2))/(
a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(-tan(d*x+c)+I)^3/(-I*a)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(91) = 182$.
time = 0.49, size = 321, normalized size = 2.53

$$-\frac{\left(3\,a^{2}d\sqrt{-\frac{i}{2\,a^{3}d^{2}}}\,e^{(3i\,dx+3i\,c)}\log\left(\frac{1}{2}\,a^{2}d\sqrt{-\frac{i}{2\,a^{3}d^{2}}}\,e^{(i\,dx+i\,c)}+\frac{1}{4}\,\sqrt{2}\,\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\,\left(e^{(2i\,dx+2i\,c)}+1\right)\right)-3\,a^{2}d\sqrt{-\frac{i}{2\,a^{3}d^{2}}}\,e^{(3i\,dx+3i\,c)}\log\left(-\frac{1}{2}\,a^{2}d\sqrt{-\frac{i}{2\,a^{3}d^{2}}}\,e^{(i\,dx+i\,c)}+\frac{1}{4}\,\sqrt{2}\,\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\,\left(e^{(2i\,dx+2i\,c)}+1\right)\right)-\sqrt{2}\,\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\,\left(4\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i\,c)}-1\right)\right)e^{(-3i\,dx-3i\,c)}}{12\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/12*(3*a^2*d*sqrt(-1/2*I/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(1/2*a^2*d*sqr
t(-1/2*I/(a^3*d^2))*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2
*I*d*x + 2*I*c) + 1)) - 3*a^2*d*sqrt(-1/2*I/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*
log(-1/2*a^2*d*sqrt(-1/2*I/(a^3*d^2))*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/
(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(4*e^
(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c) - 1))*e^(-3*I*d*x - 3*I*c)/(a^2*d
)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(3/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**(3/2)/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^{3/2}}{(a+a\tan(c+d\,x)\,1\mathrm{i})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(3/2)/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] int(tan(c + d*x)^(3/2)/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.221 $\int \frac{\sqrt{\tan(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}+\frac{\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}+\frac{i\sqrt{\tan(c+dx)}}{2ad\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] (-1/4-1/4*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2
))/a^(3/2)/d+1/2*I*tan(d*x+c)^(1/2)/a/d/(a+I*a*tan(d*x+c))^(1/2)+1/3*tan(d*
x+c)^(3/2)/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.13, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3628, 3627, 3625, 211}

$$-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}+\frac{\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}+\frac{i\sqrt{\tan(c+dx)}}{2ad\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[c + d*x]]/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-1/4 - I/4)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan
[c + d*x]]])/(a^(3/2)*d) + Tan[c + d*x]^(3/2)/(3*d*(a + I*a*Tan[c + d*x])^(
3/2)) + ((I/2)*Sqrt[Tan[c + d*x]])/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3627

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
```

```
 + f*x])^n/(2*b*f*m)), x] - Dist[(a*c - b*d)/(2*b^2), Int[(a + b*Tan[e + f*
x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && EqQ
[m + n, 0] && LeQ[m, -2^(-1)]
```

Rule 3628

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a), Int[(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && EqQ[m
 + n + 1, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{\tan(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{2a}$$

$$= \frac{\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{i\sqrt{\tan(c+dx)}}{2ad\sqrt{a+ia\tan(c+dx)}} - \frac{i\int \frac{\sqrt{a+ia\tan(c}}{\sqrt{\tan(c+}}}{4a^2}$$

$$= \frac{\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{i\sqrt{\tan(c+dx)}}{2ad\sqrt{a+ia\tan(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{-ia-2a^2x}\right.}{}$$

$$= -\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d} + \frac{\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

Mathematica [A]
time = 1.63, size = 164, normalized size = 1.29

$$\frac{ie^{-2i(c+dx)}\left(\sqrt{-1+e^{2i(c+dx)}}\left(1+2e^{2i(c+dx)}\right)-3e^{3i(c+dx)}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\tan(c+dx)}}{6\sqrt{2}\,ad\sqrt{-1+e^{2i(c+dx)}}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tan[c + d*x]]/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((I/6)*(Sqrt[-1 + E^((2*I)*(c + d*x))]*(1 + 2*E^((2*I)*(c + d*x))) - 3*E^((
3*I)*(c + d*x))*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]])*Sq
rt[Tan[c + d*x]])/(Sqrt[2]*a*d*E^((2*I)*(c + d*x))*Sqrt[-1 + E^((2*I)*(c +
d*x))]*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 462 vs. 2(99) = 198.
time = 0.18, size = 463, normalized size = 3.65

method	result
derivativedivides	$-\frac{\left(\sqrt{\tan}\left(dx+c\right)\right)\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\left(4\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1+i\tan\left(dx+c\right)\right)}\,(\right.}{}$
default	$-\frac{\left(\sqrt{\tan}\left(dx+c\right)\right)\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\left(4\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1+i\tan\left(dx+c\right)\right)}\,(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)/a^2*(4*(-I*a)^(1/2)*(a*
tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^2+9*I*2^(1/2)*ln(-(-2*2^(1/2)
*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(ta
n(d*x+c)+I))*a*tan(d*x+c)^2-3*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d
*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x
+c)^3-3*I*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c
)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a-16*I*(-I*a)^(1/2)*(a*tan(d*
x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)+9*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1
/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I
))*a*tan(d*x+c)-12*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2))/(a*t
an(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(-tan(d*x+c)+I)^3/(-I*a)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(91) = 182.

time = 0.45, size = 320, normalized size = 2.52

$$\frac{\left(3\,a^{2}d\sqrt{\frac{i}{2\,a^{3}d^{2}}}\,e^{(3i\,dx+3i\,c)}\log\left(\frac{1}{2}i\,a^{2}d\sqrt{\frac{i}{2\,a^{3}d^{2}}}\,e^{(i\,dx+i\,c)}+\frac{1}{4}\,\sqrt{2}\,\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\,(e^{(2i\,dx+2i\,c)}+1)\right)-3\,a^{2}d\sqrt{\frac{i}{2\,a^{3}d^{2}}}\,e^{(3i\,dx+3i\,c)}\log\left(-\frac{1}{2}i\,a^{2}d\sqrt{\frac{i}{2\,a^{3}d^{2}}}\,e^{(i\,dx+i\,c)}+\frac{1}{4}\,\sqrt{2}\,\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\,(e^{(2i\,dx+2i\,c)}+1)\right)+\sqrt{2}\,\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\,(2i\,e^{(4i\,dx+4i\,c)}+3i\,e^{(2i\,dx+2i\,c)}+i)\right)e^{(-3i\,dx-3i\,c)}}{12\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^2*d*sqrt(1/2*I/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(1/2*I*a^2*d*sqr
t(1/2*I/(a^3*d^2))*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*
I*d*x + 2*I*c) + 1)) - 3*a^2*d*sqrt(1/2*I/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*lo
g(-1/2*I*a^2*d*sqrt(1/2*I/(a^3*d^2))*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2
*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(2*I*e
^(4*I*d*x + 4*I*c) + 3*I*e^(2*I*d*x + 2*I*c) + I))*e^(-3*I*d*x - 3*I*c)/(a^
2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\tan{\left(c+dx \right)}}}{\left(ia\left(\tan{\left(c+dx \right)}-i\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(tan(c + d*x))/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\mathrm{tan}\left(c+d\,x\right)}}{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] int(tan(c + d*x)^(1/2)/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.222 $\int \frac{1}{\sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=125

$$\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}+\frac{\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}+\frac{7\sqrt{\tan(c+dx)}}{6ad\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] (1/4-1/4*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
)/a^(3/2)/d+7/6*tan(d*x+c)^(1/2)/a/d/(a+I*a*tan(d*x+c))^(1/2)+1/3*tan(d*x+c
)^(1/2)/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.18, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3640, 3677, 12, 3625, 211}

$$\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}+\frac{7\sqrt{\tan(c+dx)}}{6ad\sqrt{a+ia\tan(c+dx)}}+\frac{\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

```
[Out] ((1/4 - I/4)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[
c + d*x]]])/(a^(3/2)*d) + Sqrt[Tan[c + d*x]]/(3*d*(a + I*a*Tan[c + d*x])^(3
/2)) + (7*Sqrt[Tan[c + d*x]])/(6*a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{\int \frac{\frac{5a}{2}-ia\tan(c+dx)}{\sqrt{\tan(c+dx)}\,\sqrt{a+ia\tan(\ldots}}}{3a^2}$$

$$= \frac{\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{7\sqrt{\tan(c+dx)}}{6ad\sqrt{a+ia\tan(c+dx)}} + \frac{\int \ldots}{\ldots}$$

$$= \frac{\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{7\sqrt{\tan(c+dx)}}{6ad\sqrt{a+ia\tan(c+dx)}} + \frac{\int \ldots}{\ldots}$$

$$= \frac{\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{7\sqrt{\tan(c+dx)}}{6ad\sqrt{a+ia\tan(c+dx)}} - \frac{i\mathrm{S}\ldots}{\ldots}$$

$$= \frac{\left(\frac{1}{4}-\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d} + \frac{\sqrt{\tan(c\ldots}}{3d(a+ia\tan\ldots}$$

Mathematica [A]

time = 1.91, size = 158, normalized size = 1.26

$$-\frac{ie^{-4i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(-1-7e^{2i(c+dx)}+8e^{4i(c+dx)}+3e^{3i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)}{6\sqrt{2}\,a^2d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

```
[Out] ((-1/6*I)*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(-1 - 7*E
^((2*I)*(c + d*x)) + 8*E^((4*I)*(c + d*x)) + 3*E^((3*I)*(c + d*x))*Sqrt[-1
+ E^((2*I)*(c + d*x))]*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x)
)]]))/(Sqrt[2]*a^2*d*E^((4*I)*(c + d*x))*Sqrt[Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 463 vs. 2(98) = 196.
time = 0.18, size = 464, normalized size = 3.71

method	result
derivativedivides	$\frac{\left(\sqrt{\tan}\,(dx+c)\right)\sqrt{a\,(1+i\tan\,(dx+c))}\,\left(3i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\,(dx+c)\,(1+i\,t}{\tan(dx+c)+i}\right.\right.}{}$
default	$\frac{\left(\sqrt{\tan}\,(dx+c)\right)\sqrt{a\,(1+i\tan\,(dx+c))}\,\left(3i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\,(dx+c)\,(1+i\,t}{\tan(dx+c)+i}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)*(3*I*2^(1/2)*ln(-(-2*2^(
1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))
/(tan(d*x+c)+I))*a*tan(d*x+c)^3-9*I*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a
*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*t
an(d*x+c)+28*I*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c
)^2+9*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^
(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2-36*I*(-I*a)^(1/2)*
(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-3*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2
)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))
*a+64*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2))/a^2/(a
*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(-tan(d*x+c)+I)^3/(-I*a)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(91) = 182$.
time = 0.45, size = 320, normalized size = 2.56

$$\frac{\left(3a^2d\sqrt{-\frac{i}{2a^3d^2}}e^{(3i\,dx+3i\,c)}\log\left(\frac{1}{2}a^2d\sqrt{-\frac{i}{2a^3d^2}}e^{(i\,dx+i\,c)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)\right)-3a^2d\sqrt{-\frac{i}{2a^3d^2}}e^{(3i\,dx+3i\,c)}\log\left(-\frac{1}{2}a^2d\sqrt{-\frac{i}{2a^3d^2}}e^{(i\,dx+i\,c)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)\right)+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(8e^{(4i\,dx+4i\,c)}+9e^{(2i\,dx+2i\,c)}+1\right)\right)e^{(-3i\,dx-3i\,c)}}{12a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/12*(3*a^2*d*sqrt(-1/2*I/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(1/2*a^2*d*sqrt
(-1/2*I/(a^3*d^2))*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*
I*d*x + 2*I*c) + 1)) - 3*a^2*d*sqrt(-1/2*I/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*l
og(-1/2*a^2*d*sqrt(-1/2*I/(a^3*d^2))*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2
*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(8*e^(
4*I*d*x + 4*I*c) + 9*e^(2*I*d*x + 2*I*c) + 1))*e^(-3*I*d*x - 3*I*c)/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{3}{2}}\sqrt{\tan\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(1/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/((I*a*(tan(c + d*x) - I))**(3/2)*sqrt(tan(c + d*x))), x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 878 vs. $2(91) = 182$.
time = 1.70, size = 878, normalized size = 7.02

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(I*a*sqrt(abs(a)) - abs(a)^(3/2))*log(-1/2*(I*(sqrt(2)*sqrt(I*a*tan(d*x
 + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*tan(d*x + c) +
 a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(d*
x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^2 + 8*sqrt(2) - 12)/(-1/2*I*(sqr
t(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-
2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) +
 a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^2 + 4*sqrt(
2) + 6))/(a^3*d) - 2/3*sqrt(2)*(3*(sqrt(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*a
bs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(
tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(d*x + c) + a)*a +
a^2)/a^2) + 1)*abs(a)/a^2)^4*a*sqrt(abs(a)) + 3*I*(sqrt(2)*sqrt(I*a*tan(d*x
 + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*tan(d*x + c) +
 a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(d*
x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^4*abs(a)^(3/2) - 72*I*(sqrt(2)*s
qrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a
*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2
- 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^2*a*sqrt(abs(a))
+ 72*(sqrt(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2
 - sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d
*x + c) + a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^2*
abs(a)^(3/2) - 112*a*sqrt(abs(a)) - 112*I*abs(a)^(3/2))/(((sqrt(2)*sqrt(I*a
*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*tan(d*
x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*
a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^2 - 4*I)^3*a^3*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\tan\left(c+d\,x\right)}\,\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

```
[Out] int(1/(tan(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(3/2)), x)
```

3.223 $\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=162

$$\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}+\frac{1}{3d\sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^{3/2}}+\frac{}{6ad\sqrt{\tan(c+dx)}}$$

```
[Out] (1/4+1/4*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
)/a^(3/2)/d+11/6/a/d/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-25/6*(a+I*a*
tan(d*x+c))^(1/2)/a^2/d/tan(d*x+c)^(1/2)+1/3/d/tan(d*x+c)^(1/2)/(a+I*a*tan(
d*x+c))^(3/2)
```

Rubi [A]
time = 0.27, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3640, 3677, 3679, 12, 3625, 211}

$$\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}-\frac{25\sqrt{a+ia\tan(c+dx)}}{6a^2d\sqrt{\tan(c+dx)}}+\frac{11}{6ad\sqrt{\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}+\frac{1}{3d\sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

```
[Out] ((1/4 + I/4)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[
c + d*x]]])/(a^(3/2)*d) + 1/(3*d*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^
(3/2)) + 11/(6*a*d*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (25*Sqr
t[a + I*a*Tan[c + d*x]])/(6*a^2*d*Sqrt[Tan[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
```

```
Q[c^2 + d^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{1}{3d\sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^{3/2}} + \frac{\int \frac{\frac{7a}{2}-2ia\tan\ldots}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+\ldots}}}{3\ldots}$$

$$= \frac{1}{3d\sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^{3/2}} + \frac{\ldots}{6ad\sqrt{\tan(c+dx)}\ldots}$$

$$= \frac{1}{3d\sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^{3/2}} + \frac{\ldots}{6ad\sqrt{\tan(c+dx)}\ldots}$$

$$= \frac{1}{3d\sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^{3/2}} + \frac{\ldots}{6ad\sqrt{\tan(c+dx)}\ldots}$$

$$= \frac{1}{3d\sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^{3/2}} + \frac{\ldots}{6ad\sqrt{\tan(c+dx)}\ldots}$$

$$= \frac{\left(\frac{1}{4}+\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d} + \frac{\ldots}{3d\sqrt{\tan(c+dx\ldots}}$$

Mathematica [A]
time = 2.03, size = 190, normalized size = 1.17

$$\frac{ie^{-2i(c+dx)}\left(\sqrt{-1+e^{2i(c+dx)}}\left(1+13e^{2i(c+dx)}-38e^{4i(c+dx)}\right)+3e^{3i(c+dx)}\left(-1+e^{2i(c+dx)}\right)\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\tan(c+dx)}}{6\sqrt{2}\,ad\left(-1+e^{2i(c+dx)}\right)^{3/2}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

```
[Out] ((I/6)*(Sqrt[-1 + E^((2*I)*(c + d*x))]*(1 + 13*E^((2*I)*(c + d*x)) - 38*E^(
(4*I)*(c + d*x))) + 3*E^((3*I)*(c + d*x))*(-1 + E^((2*I)*(c + d*x)))*ArcTan
h[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]])*Sqrt[Tan[c + d*x]])/(Sqr
t[2]*a*d*E^((2*I)*(c + d*x))*(-1 + E^((2*I)*(c + d*x)))^(3/2)*Sqrt[(a*E^((2
*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 508 vs. 2(128) = 256.
time = 0.19, size = 509, normalized size = 3.14

method	result
derivativedivides	$\frac{\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\,\left(9i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1+i\tan\left(dx+c\right)\right)}+}{\tan(dx+c)+i}\right.\right.}{}$
default	$\frac{\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\,\left(9i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1+i\tan\left(dx+c\right)\right)}+}{\tan(dx+c)+i}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/d*(a*(1+I*tan(d*x+c)))^(1/2)*(9*I*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)
*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*
a*tan(d*x+c)^3-3*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*ta
n(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4-3*I*2^(
1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a
-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)-256*I*(-I*a)^(1/2)*(a*tan(d*x
+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^2+9*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(
1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+
I))*a*tan(d*x+c)^2+100*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*t
an(d*x+c)^3+48*I*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-204*tan
(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2))/a^2/tan(d*x+c)^
(1/2)/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(-tan(d*x+c)+I)^3/(-I*a)^(1/2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 382 vs. 2(120) = 240.
time = 0.47, size = 382, normalized size = 2.36

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(-38i\,e^{(6i\,dx+6i\,c)}-25i\,e^{(4i\,dx+4i\,c)}+14i\,e^{(2i\,dx+2i\,c)}+i\right)-3\left(a^2de^{(5i\,dx+5i\,c)}-a^2de^{(3i\,dx+3i\,c)}\right)\sqrt{\frac{i}{2\,a^3d^2}}\log\left(\frac{1}{2}i\,a^2d\sqrt{\frac{i}{2\,a^3d^2}}\,e^{(i\,dx+i\,c)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)\right)+3\left(a^2de^{(5i\,dx+5i\,c)}-a^2de^{(3i\,dx+3i\,c)}\right)\sqrt{\frac{i}{2\,a^3d^2}}\log\left(-\frac{1}{2}i\,a^2d\sqrt{\frac{i}{2\,a^3d^2}}\,e^{(i\,dx+i\,c)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)\right)}{12\left(a^2de^{(5i\,dx+5i\,c)}-a^2de^{(3i\,dx+3i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/12*(sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c
) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(-38*I*e^(6*I*d*x + 6*I*c) - 25*I*e^(4*I*
d*x + 4*I*c) + 14*I*e^(2*I*d*x + 2*I*c) + I) - 3*(a^2*d*e^(5*I*d*x + 5*I*c)
 - a^2*d*e^(3*I*d*x + 3*I*c))*sqrt(1/2*I/(a^3*d^2))*log(1/2*I*a^2*d*sqrt(1/
2*I/(a^3*d^2))*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*
x + 2*I*c) + 1)) + 3*(a^2*d*e^(5*I*d*x + 5*I*c) - a^2*d*e^(3*I*d*x + 3*I*c)
)*sqrt(1/2*I/(a^3*d^2))*log(-1/2*I*a^2*d*sqrt(1/2*I/(a^3*d^2))*e^(I*d*x + I
*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2
*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)))/(a^2*d*e^
(5*I*d*x + 5*I*c) - a^2*d*e^(3*I*d*x + 3*I*c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

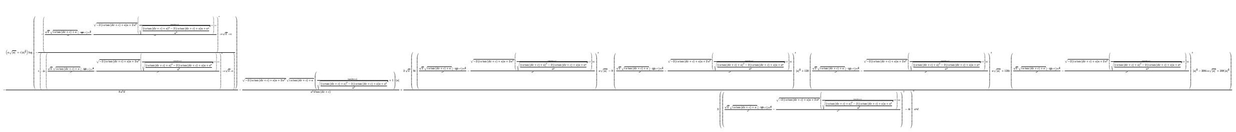
$$\int \frac{1}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{3}{2}}\tan^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(3/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/((I*a*(tan(c + d*x) - I))**(3/2)*tan(c + d*x)**(3/2)), x)
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 976 vs. 2(120) = 240.
time = 6.10, size = 976, normalized size = 6.02



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*(a*sqrt(abs(a)) + I*abs(a)^(3/2))*log(-1/2*(I*(sqrt(2)*sqrt(I*a*tan(d*
x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*tan(d*x + c)
+ a)*a + 2*a^2)*(tan(d*x + c)/sqrt((((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(d
*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^2 + 8*sqrt(2) - 12)/(-1/2*I*(sq
rt(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(
-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt((((I*a*tan(d*x + c)
+ a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^2 + 4*sqrt
(2) + 6))/(a^3*d) - sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*sqrt(I*a*tan(
d*x + c) + a)*(tan(d*x + c)/sqrt((((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(d*x
```

```
 + c) + a)*a + a^2)/a^2) + 1)*abs(a)/(a^4*d*tan(d*x + c)) + 2/3*sqrt(2)*(9*
I*(sqrt(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 -
sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x
+ c) + a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^4*a*s
qrt(abs(a)) - 9*(sqrt(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a
)^(3/2)/a^2 - sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(
((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs
(a)/a^2)^4*abs(a)^(3/2) + 120*(sqrt(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a
)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(
d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)
/a^2) + 1)*abs(a)/a^2)^2*a*sqrt(abs(a)) + 120*I*(sqrt(2)*sqrt(I*a*tan(d*x +
 c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*tan(d*x + c) + a
)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(d*x
+ c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^2*abs(a)^(3/2) - 208*I*a*sqrt(abs(
a)) + 208*abs(a)^(3/2))/(((sqrt(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a
+ 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x
+ c)/sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2
) + 1)*abs(a)/a^2)^2 - 4*I)^3*a^3*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\tan\left(c+d\,x\right)^{3/2}\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(3/2)),x)
[Out] int(1/(tan(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(3/2)), x)
```

3.224 $\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=201

$$-\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}+\frac{1}{3d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}+\frac{}{2ad\tan^{\frac{3}{2}}(c+dx)\sqrt{}}$$

```
[Out] (-1/4+1/4*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2
))/a^(3/2)/d+13/2*I*(a+I*a*tan(d*x+c))^(1/2)/a^2/d/tan(d*x+c)^(1/2)+5/2/a/d
/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(3/2)-7/2*(a+I*a*tan(d*x+c))^(1/2)/a^2
/d/tan(d*x+c)^(3/2)+1/3/d/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.37, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3640, 3677, 3679, 12, 3625, 211}

$$-\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}-\frac{7\sqrt{a+ia\tan(c+dx)}}{2a^2d\tan^{\frac{3}{2}}(c+dx)}+\frac{13i\sqrt{a+ia\tan(c+dx)}}{2a^2d\sqrt{\tan(c+dx)}}+\frac{5}{2ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}+\frac{1}{3d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

```
[Out] ((-1/4 + I/4)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan
[c + d*x]]])/(a^(3/2)*d) + 1/(3*d*Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])
^(3/2)) + 5/(2*a*d*Tan[c + d*x]^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) - (7*Sqrt
[a + I*a*Tan[c + d*x]])/(2*a^2*d*Tan[c + d*x]^(3/2)) + (((13*I)/2)*Sqrt[a +
 I*a*Tan[c + d*x]])/(a^2*d*Sqrt[Tan[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
```

```
Q[c^2 + d^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{1}{3d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}} + \frac{\int \frac{\frac{9a}{2}-3ia\tan(}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia}}}{3a^2}$$

$$= \frac{1}{3d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}} + \frac{5}{2ad\tan^{\frac{3}{2}}(c+dx)\sqrt{}}$$

$$= \frac{1}{3d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}} + \frac{5}{2ad\tan^{\frac{3}{2}}(c+dx)\sqrt{}}$$

$$= \frac{1}{3d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}} + \frac{5}{2ad\tan^{\frac{3}{2}}(c+dx)\sqrt{}}$$

$$= \frac{1}{3d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}} + \frac{5}{2ad\tan^{\frac{3}{2}}(c+dx)\sqrt{}}$$

$$= \frac{1}{3d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}} + \frac{5}{2ad\tan^{\frac{3}{2}}(c+dx)\sqrt{}}$$

$$= -\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d} + \frac{}{3d\tan^{\frac{3}{2}}(c+d}$$

Mathematica [A]
time = 1.26, size = 186, normalized size = 0.93

$$\frac{ie^{-4i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(1+18e^{2i(c+dx)}-87e^{4i(c+dx)}+52e^{6i(c+dx)}+3e^{3i(c+dx)}\left(-1+e^{2i(c+dx)}\right)^{3/2}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)}{6\sqrt{2}\,a^2d\left(-1+e^{2i(c+dx)}\right)\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Tan[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

```
[Out] ((I/6)*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(1 + 18*E^((
2*I)*(c + d*x)) - 87*E^((4*I)*(c + d*x)) + 52*E^((6*I)*(c + d*x)) + 3*E^((3
*I)*(c + d*x))*(-1 + E^((2*I)*(c + d*x)))^(3/2)*ArcTanh[E^(I*(c + d*x))/Sqr
t[-1 + E^((2*I)*(c + d*x))]]))/(Sqrt[2]*a^2*d*E^((4*I)*(c + d*x))*(-1 + E^(
(2*I)*(c + d*x)))*Sqrt[Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(159) = 318$.
time = 0.18, size = 549, normalized size = 2.73

method	result
derivativedivides	$-\frac{\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\,\left(3i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1+i\tan\left(dx+c\right)\right)}}{\tan(dx+c)+i}\right.\right.}{}$
default	$-\frac{\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\,\left(3i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1+i\tan\left(dx+c\right)\right)}}{\tan(dx+c)+i}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24/d*(a*(1+I*tan(d*x+c)))^(1/2)/a^2/tan(d*x+c)^(3/2)*(3*I*2^(1/2)*ln(-(-
2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*
x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^5+384*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1
/2)*(-I*a)^(1/2)*tan(d*x+c)^3-9*I*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*t
an(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan
(d*x+c)^3+156*I*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+
c)^4+9*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))
^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4-32*tan(d*x+c)*(a*
tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)-276*I*(-I*a)^(1/2)*(a*tan(d
*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^2-3*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)
^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c
)+I))*a*tan(d*x+c)^2-16*I*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2
))/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(-tan(d*x+c)+I)^3/(-I*a)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(149) = 298$.
time = 0.44, size = 435, normalized size = 2.16

$$-\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(52\,e^{(8i\,dx+8i\,c)}-35\,e^{(6i\,dx+6i\,c)}-69\,e^{(4i\,dx+4i\,c)}+19\,e^{(2i\,dx+2i\,c)}+1\right)+3\left(a^{2}de^{(7i\,dx+7i\,c)}-2\,a^{2}de^{(5i\,dx+5i\,c)}+a^{2}de^{(3i\,dx+3i\,c)}\right)\sqrt{-\frac{i}{2\,a^{3}d^{2}}}\log\left(\frac{1}{2}\,a^{2}d\sqrt{-\frac{i}{2\,a^{3}d^{2}}}\,e^{(i\,dx+i\,c)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)\right)-3\left(a^{2}de^{(7i\,dx+7i\,c)}-2\,a^{2}de^{(5i\,dx+5i\,c)}+a^{2}de^{(3i\,dx+3i\,c)}\right)\sqrt{-\frac{i}{2\,a^{3}d^{2}}}\log\left(-\frac{1}{2}\,a^{2}d\sqrt{-\frac{i}{2\,a^{3}d^{2}}}\,e^{(i\,dx+i\,c)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)\right)}{12\left(a^{2}de^{(7i\,dx+7i\,c)}-2\,a^{2}de^{(5i\,dx+5i\,c)}+a^{2}de^{(3i\,dx+3i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas"
)

[Out] -1/12*(sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(52*e^(8*I*d*x + 8*I*c) - 35*e^(6*I*d*x + 6*I*c) - 69*e^(4*I*d*x + 4*I*c) + 19*e^(2*I*d*x + 2*I*c) + 1) + 3*(a^2*d*e^(7*I*d*x + 7*I*c) - 2*a^2*d*e^(5*I*d*x + 5*I*c) + a^2*d*e^(3*I*d*x + 3*I*c))*sqrt(-1/2*I/(a^3*d^2))*log(1/2*a^2*d*sqrt(-1/2*I/(a^3*d^2))*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)) - 3*(a^2*d*e^(7*I*d*x + 7*I*c) - 2*a^2*d*e^(5*I*d*x + 5*I*c) + a^2*d*e^(3*I*d*x + 3*I*c))*sqrt(-1/2*I/(a^3*d^2))*log(-1/2*a^2*d*sqrt(-1/2*I/(a^3*d^2))*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)))/(a^2*d*e^(7*I*d*x + 7*I*c) - 2*a^2*d*e^(5*I*d*x + 5*I*c) + a^2*d*e^(3*I*d*x + 3*I*c))

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

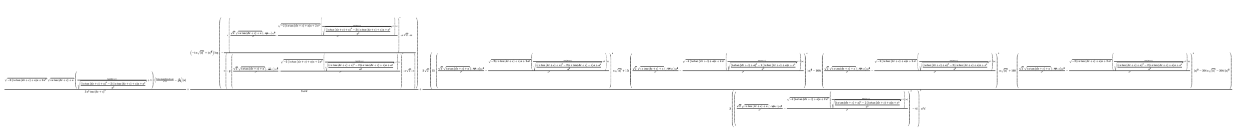
$$\int \frac{1}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}\tan^{\frac{5}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/tan(d*x+c)**(5/2)/(a+I*a*tan(d*x+c))**(3/2),x)

[Out] Integral(1/((I*a*(tan(c + d*x) - I))**(3/2)*tan(c + d*x)**(5/2)), x)

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1000 vs. 2(149) = 298.
time = 7.80, size = 1000, normalized size = 4.98



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")

[Out] 1/3*sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*sqrt(I*a*tan(d*x + c) + a)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*(5*(I*a*tan(d*x + c) + a)/(a^3*d) - 6/(a^2*d))*abs(a)/(a^2*tan(d*x + c)^2) + 1/8*(-I*a*sqrt(abs(a)) + abs(a)^(3/2))*log(-1/2*(I*(sqrt(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^

```
2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^2 + 8*sqrt(2) -
 12)/(-1/2*I*(sqrt(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(
3/2)/a^2 - sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I
*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)
/a^2)^2 + 4*sqrt(2) + 6))/(a^3*d) + 2/3*sqrt(2)*(15*(sqrt(2)*sqrt(I*a*tan(d
*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*tan(d*x + c)
 + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(
d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^4*a*sqrt(abs(a)) + 15*I*(sqrt(
2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*
(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a
)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^4*abs(a)^(3/2
) - 168*I*(sqrt(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2
)/a^2 - sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*
tan(d*x + c) + a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^
2)^2*a*sqrt(abs(a)) + 168*(sqrt(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a
+ 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x
+ c)/sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2
) + 1)*abs(a)/a^2)^2*abs(a)^(3/2) - 304*a*sqrt(abs(a)) - 304*I*abs(a)^(3/2)
)/(((sqrt(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2
- sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*
x + c) + a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^2 -
 4*I)^3*a^3*d)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\tan\left(c+d\,x\right)^{5/2}\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(3/2)),x)
[Out] int(1/(tan(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(3/2)), x)
```

3.225 $\int \frac{\tan^{\frac{9}{2}}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=257

$$\frac{5(-1)^{3/4}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d}-\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d}-\frac{\tan^{\frac{7}{2}}(c-}{5d(a+ia\tan(}$$

```
[Out] 5*(-1)^(3/4)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^
(1/2))/a^(5/2)/d-(1/8+1/8*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*
tan(d*x+c))^(1/2))/a^(5/2)/d+21/4*I*tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/
2)/a^3/d+41/12*tan(d*x+c)^(3/2)/a^2/d/(a+I*a*tan(d*x+c))^(1/2)-1/5*tan(d*x+
c)^(7/2)/d/(a+I*a*tan(d*x+c))^(5/2)+19/30*I*tan(d*x+c)^(5/2)/a/d/(a+I*a*tan
(d*x+c))^(3/2)
```

Rubi [A]
time = 0.56, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3639, 3676, 3678, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{5(-1)^{3/4}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d}-\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d}+\frac{21i\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{4a^3d}+\frac{41\tan^{\frac{3}{2}}(c+dx)}{12a^2d\sqrt{a+ia\tan(c+dx)}}-\frac{\tan^{\frac{7}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}+\frac{19i\tan^{\frac{5}{2}}(c+dx)}{30ad(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(9/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (5*(-1)^(3/4)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*T
an[c + d*x]]])/(a^(5/2)*d) - ((1/8 + I/8)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan
[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/(a^(5/2)*d) - Tan[c + d*x]^(7/2)/(
5*d*(a + I*a*Tan[c + d*x])^(5/2)) + (((19*I)/30)*Tan[c + d*x]^(5/2))/(a*d*(
a + I*a*Tan[c + d*x])^(3/2)) + (41*Tan[c + d*x]^(3/2))/(12*a^2*d*Sqrt[a + I
*a*Tan[c + d*x]]) + (((21*I)/4)*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x
]])/(a^3*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rule 3678

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Dist[
```

```
1/(a*(m + n)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Simp
[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{\tan^{\frac{9}{2}}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{\tan^{\frac{7}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} - \frac{\int \frac{\tan^{\frac{5}{2}}(c+dx)\left(-\frac{7a}{2}+6ia\tan(c+dx)\right)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{5a^2}$$

$$= -\frac{\tan^{\frac{7}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{19i\tan^{\frac{5}{2}}(c+dx)}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{\int \frac{\tan^{\frac{3}{2}}(c+dx)\left(-9\right.}{\sqrt{a+ia}}}{}$$

$$= -\frac{\tan^{\frac{7}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{19i\tan^{\frac{5}{2}}(c+dx)}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{41\tan^{\frac{3}{2}}(c}{12a^2d\sqrt{a+ia}}$$

$$= -\frac{\tan^{\frac{7}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{19i\tan^{\frac{5}{2}}(c+dx)}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{41\tan^{\frac{3}{2}}(c}{12a^2d\sqrt{a+ia}}$$

$$= -\frac{\tan^{\frac{7}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{19i\tan^{\frac{5}{2}}(c+dx)}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{41\tan^{\frac{3}{2}}(c}{12a^2d\sqrt{a+ia}}$$

$$= -\frac{\tan^{\frac{7}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{19i\tan^{\frac{5}{2}}(c+dx)}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{41\tan^{\frac{3}{2}}(c}{12a^2d\sqrt{a+ia}}$$

$$= -\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d} - \frac{\tan^{\frac{7}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} +$$

$$= -\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d} - \frac{\tan^{\frac{7}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} +$$

$$= \frac{5(-1)^{3/4}\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d} - \frac{\left(\frac{1}{8}+\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\mathrm{t}}}{\sqrt{a+ia\,\mathrm{t}}}\right.}{a^{5/2}d}$$

Mathematica [A]

time = 3.97, size = 270, normalized size = 1.05

$$-\frac{ie^{-6i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(-\sqrt{-1+e^{2i(c+dx)}}\left(3-28e^{2i(c+dx)}+252e^{4i(c+dx)}+403e^{6i(c+dx)}\right)+15e^{5i(c+dx)}\left(1+e^{2i(c+dx)}\right)\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)+300\sqrt{2}\,e^{5i(c+dx)}\left(1+e^{2i(c+dx)}\right)\tanh^{-1}\left(\frac{\sqrt{2}\,e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\tan(c+dx)}}{60\sqrt{2}\,a^3d\sqrt{-1+e^{2i(c+dx)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(9/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-1/60*I)*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(-(Sqrt[
-1 + E^((2*I)*(c + d*x))]*(3 - 28*E^((2*I)*(c + d*x)) + 252*E^((4*I)*(c + d
```

```
*x)) + 403*E^((6*I)*(c + d*x)))) + 15*E^((5*I)*(c + d*x))*(1 + E^((2*I)*(c
+ d*x)))*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]] + 300*Sqrt
[2]*E^((5*I)*(c + d*x))*(1 + E^((2*I)*(c + d*x)))*ArcTanh[(Sqrt[2]*E^(I*(c
+ d*x)))/Sqrt[-1 + E^((2*I)*(c + d*x))]])*Sqrt[Tan[c + d*x]])/(Sqrt[2]*a^3*
d*E^((6*I)*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1006 vs. $2(201) = 402$.
time = 0.19, size = 1007, normalized size = 3.92 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(9/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/240/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)/a^3*(2228*(a*tan(d*x+c)
*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^3+3600*I*ln(1/
2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/
(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^2+1260*I*(a*tan(d*x+c)*(1+I*tan(d*x+
c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)-60*I*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2
)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(t
an(d*x+c)+I))*a*tan(d*x+c)-15*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1
/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I
))*a*tan(d*x+c)^4+240*I*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(
-I*a)^(1/2)*tan(d*x+c)^4-4948*I*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)
^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^2-600*I*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*
x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan
(d*x+c)^4+90*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)
*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2
-2400*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a
)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^3+60*I*(I*a)^(1/2)*2^(1/2
)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*
a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3-600*I*ln(1/2*(2*I*a*tan(d*x+c)
+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)
^(1/2)*a-15*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*
(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a+2400*ln(1/2*(
2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*
a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)-4220*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1
/2)*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c))/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1
/2)/(-tan(d*x+c)+I)^4/(I*a)^(1/2)/(-I*a)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(9/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 623 vs. 2(189) = 378.
time = 0.52, size = 623, normalized size = 2.42

$$\frac{\left(30\,a^{3}d\sqrt{\frac{i}{8\,a^{5}d^{2}}}\,e^{\left(5i\,dx+5i\,c\right)}\log\left(i\,a^{3}d\sqrt{\frac{i}{8\,a^{5}d^{2}}}\,e^{\left(i\,dx+i\,c\right)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}\sqrt{\frac{-i\,e^{\left(2i\,dx+2i\,c\right)}+i}{e^{\left(2i\,dx+2i\,c\right)}+1}}\left(e^{\left(2i\,dx+2i\,c\right)}+1\right)\right)-30\,a^{3}d\sqrt{\frac{i}{8\,a^{5}d^{2}}}\,e^{\left(5i\,dx+5i\,c\right)}\log\left(-i\,a^{3}d\sqrt{\frac{i}{8\,a^{5}d^{2}}}\,e^{\left(i\,dx+i\,c\right)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}\sqrt{\frac{-i\,e^{\left(2i\,dx+2i\,c\right)}+i}{e^{\left(2i\,dx+2i\,c\right)}+1}}\left(e^{\left(2i\,dx+2i\,c\right)}+1\right)\right)-30\,a^{3}d\sqrt{\frac{25i}{a^{5}d^{2}}}\,e^{\left(5i\,dx+5i\,c\right)}\log\left(\frac{104\left(10\sqrt{2}\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}\sqrt{\frac{-i\,e^{\left(2i\,dx+2i\,c\right)}+i}{e^{\left(2i\,dx+2i\,c\right)}+1}}\left(e^{\left(3i\,dx+3i\,c\right)}+e^{\left(i\,dx+i\,c\right)}\right)-\left(3i\,a^{3}d\,e^{\left(2i\,dx+2i\,c\right)}-i\,a^{3}d\right)\sqrt{\frac{25i}{a^{5}d^{2}}}\right)}{3025\left(e^{\left(2i\,dx+2i\,c\right)}+1\right)}\right)+30\,a^{3}d\sqrt{\frac{25i}{a^{5}d^{2}}}\,e^{\left(5i\,dx+5i\,c\right)}\log\left(\frac{104\left(10\sqrt{2}\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}\sqrt{\frac{-i\,e^{\left(2i\,dx+2i\,c\right)}+i}{e^{\left(2i\,dx+2i\,c\right)}+1}}\left(e^{\left(3i\,dx+3i\,c\right)}+e^{\left(i\,dx+i\,c\right)}\right)-\left(-3i\,a^{3}d\,e^{\left(2i\,dx+2i\,c\right)}+i\,a^{3}d\right)\sqrt{\frac{25i}{a^{5}d^{2}}}\right)}{3025\left(e^{\left(2i\,dx+2i\,c\right)}+1\right)}\right)+\sqrt{2}\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}\sqrt{\frac{-i\,e^{\left(2i\,dx+2i\,c\right)}+i}{e^{\left(2i\,dx+2i\,c\right)}+1}}\left(403i\,e^{\left(6i\,dx+6i\,c\right)}+252i\,e^{\left(4i\,dx+4i\,c\right)}-28i\,e^{\left(2i\,dx+2i\,c\right)}+3i\right)\right)e^{\left(-5i\,dx-5i\,c\right)}}{120\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(9/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/120*(30*a^3*d*sqrt(1/8*I/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(I*a^3*d*sqrt(
1/8*I/(a^5*d^2))*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*
d*x + 2*I*c) + 1)) - 30*a^3*d*sqrt(1/8*I/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log
(-I*a^3*d*sqrt(1/8*I/(a^5*d^2))*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*
I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c)
 + 1))*(e^(2*I*d*x + 2*I*c) + 1)) - 30*a^3*d*sqrt(25*I/(a^5*d^2))*e^(5*I*d*
x + 5*I*c)*log(104/3025*(10*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(
(-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(3*I*d*x + 3*I*c
) + e^(I*d*x + I*c)) - (3*I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt(25*I/
(a^5*d^2)))/(e^(2*I*d*x + 2*I*c) + 1)) + 30*a^3*d*sqrt(25*I/(a^5*d^2))*e^(5
*I*d*x + 5*I*c)*log(104/3025*(10*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*
sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(3*I*d*x +
3*I*c) + e^(I*d*x + I*c)) - (-3*I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt
(25*I/(a^5*d^2)))/(e^(2*I*d*x + 2*I*c) + 1)) + sqrt(2)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*
(403*I*e^(6*I*d*x + 6*I*c) + 252*I*e^(4*I*d*x + 4*I*c) - 28*I*e^(2*I*d*x +
2*I*c) + 3*I))*e^(-5*I*d*x - 5*I*c)/(a^3*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(9/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7317 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(9/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(c+d\,x)^{9/2}}{{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(9/2)/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] int(tan(c + d*x)^(9/2)/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.226 $\int \frac{\tan^{\frac{7}{2}}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=218

$$\frac{2\sqrt[4]{-1}\,\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d}+\frac{\left(\frac{1}{8}-\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d}-\frac{\tan^{\frac{5}{2}}(c+d}{5d(a+ia\tan(c+}$$

```
[Out] 2*(-1)^(1/4)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^
(1/2))/a^(5/2)/d+(1/8-1/8*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*
tan(d*x+c))^(1/2))/a^(5/2)/d+7/4*tan(d*x+c)^(1/2)/a^2/d/(a+I*a*tan(d*x+c))^
(1/2)-1/5*tan(d*x+c)^(5/2)/d/(a+I*a*tan(d*x+c))^(5/2)+1/2*I*tan(d*x+c)^(3/2
)/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.46, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {3639, 3676, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{2\sqrt[4]{-1}\,\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d}+\frac{\left(\frac{1}{8}-\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d}+\frac{7\sqrt{\tan(c+dx)}}{4a^2d\sqrt{a+ia\tan(c+dx)}}-\frac{\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}+\frac{i\tan^{\frac{3}{2}}(c+dx)}{2ad(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(7/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (2*(-1)^(1/4)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*T
an[c + d*x]]])/(a^(5/2)*d) + ((1/8 - I/8)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan
[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/(a^(5/2)*d) - Tan[c + d*x]^(5/2)/(
5*d*(a + I*a*Tan[c + d*x])^(5/2)) + ((I/2)*Tan[c + d*x]^(3/2))/(a*d*(a + I*
a*Tan[c + d*x])^(3/2)) + (7*Sqrt[Tan[c + d*x]])/(4*a^2*d*Sqrt[a + I*a*Tan[c
 + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{\tan^{\frac{7}{2}}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} - \frac{\int \frac{\tan^{\frac{3}{2}}(c+dx)\left(-\frac{5a}{2}+5ia\tan(c+dx)\right)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{5a^2}$$

$$= -\frac{\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{i\tan^{\frac{3}{2}}(c+dx)}{2ad(a+ia\tan(c+dx))^{3/2}} + \frac{\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{a+}}}{}$$

$$= -\frac{\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{i\tan^{\frac{3}{2}}(c+dx)}{2ad(a+ia\tan(c+dx))^{3/2}} + \frac{7\sqrt{\tan(c+}}{4a^2d\sqrt{a+ia\tan}}$$

$$= -\frac{\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{i\tan^{\frac{3}{2}}(c+dx)}{2ad(a+ia\tan(c+dx))^{3/2}} + \frac{7\sqrt{\tan(c+}}{4a^2d\sqrt{a+ia\tan}}$$

$$= -\frac{\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{i\tan^{\frac{3}{2}}(c+dx)}{2ad(a+ia\tan(c+dx))^{3/2}} + \frac{7\sqrt{\tan(c+}}{4a^2d\sqrt{a+ia\tan}}$$

$$= \frac{\left(\frac{1}{8}-\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d} - \frac{\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{}{2a}$$

$$= \frac{\left(\frac{1}{8}-\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d} - \frac{\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{}{2a}$$

$$= \frac{2\sqrt[4]{-1}\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d} + \frac{\left(\frac{1}{8}-\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan}}{\sqrt{a+ia\tan}}\right)}{a^{5/2}d}$$

Mathematica [A]

time = 2.81, size = 241, normalized size = 1.11

$$\frac{ie^{-6i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(1-8e^{2i(c+dx)}+48e^{4i(c+dx)}-41e^{6i(c+dx)}-5e^{5i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)+40\sqrt{2}\,e^{5i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{\sqrt{2}\,e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)}{20\sqrt{2}\,a^3d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(7/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((I/20)*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(1 - 8*E^((
2*I)*(c + d*x)) + 48*E^((4*I)*(c + d*x)) - 41*E^((6*I)*(c + d*x)) - 5*E^((5
*I)*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]*ArcTanh[E^(I*(c + d*x))/Sqrt[
-1 + E^((2*I)*(c + d*x))]] + 40*Sqrt[2]*E^((5*I)*(c + d*x))*Sqrt[-1 + E^((2
*I)*(c + d*x))]*ArcTanh[(Sqrt[2]*E^(I*(c + d*x)))/Sqrt[-1 + E^((2*I)*(c + d
*x))]]))/(Sqrt[2]*a^3*d*E^((6*I)*(c + d*x))*Sqrt[Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 962 vs. 2(170) = 340.
time = 0.20, size = 963, normalized size = 4.42 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/80/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)/a^3*(-320*I*ln(1/2*(2*I*
a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(
1/2))*(-I*a)^(1/2)*a*tan(d*x+c)-516*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(
I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^2+460*I*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^
(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)-5*I*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2
^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c
))/(tan(d*x+c)+I))*a+30*I*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*
(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a
*tan(d*x+c)^2-80*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))
^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^4-20*(I*a)^(1/
2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/
2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3-5*I*(I*a)^(1/2)*2^(1/
2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3
*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4-196*I*(a*tan(d*x+c)*(1+I*tan(
d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^3+320*I*ln(1/2*(2*I*a*ta
n(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2)
)*(-I*a)^(1/2)*a*tan(d*x+c)^3+480*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*
(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+
c)^2+20*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+
I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*(I*a)^(1/2)*2^(1/2)*a*tan(d*x+c)-80*ln(
1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a
)/(I*a)^(1/2))*a*(-I*a)^(1/2)+140*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*
a)^(1/2)*(-I*a)^(1/2))/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(-tan(d*x+c)+I
)^4/(I*a)^(1/2)/(-I*a)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 621 vs. $2(160) = 320$.
time = 0.67, size = 621, normalized size = 2.85

$$\frac{\left(10\,a^{3}d\sqrt{-\frac{i}{8\,a^{5}d^{2}}}\,e^{(5\,i\,dx+5\,i\,c)}\log\left(a^{3}d\sqrt{-\frac{i}{8\,a^{5}d^{2}}}\,e^{(i\,dx+i\,c)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{(2\,i\,dx+2\,i\,c)}+1}}\sqrt{\frac{-i\,e^{(2\,i\,dx+2\,i\,c)}+i}{e^{(2\,i\,dx+2\,i\,c)}+1}}\left(e^{(2\,i\,dx+2\,i\,c)}+1\right)\right)-10\,a^{3}d\sqrt{-\frac{i}{8\,a^{5}d^{2}}}\,e^{(5\,i\,dx+5\,i\,c)}\log\left(-a^{3}d\sqrt{-\frac{i}{8\,a^{5}d^{2}}}\,e^{(i\,dx+i\,c)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{(2\,i\,dx+2\,i\,c)}+1}}\sqrt{\frac{-i\,e^{(2\,i\,dx+2\,i\,c)}+i}{e^{(2\,i\,dx+2\,i\,c)}+1}}\left(e^{(2\,i\,dx+2\,i\,c)}+1\right)\right)-10\,a^{3}d\sqrt{-\frac{4\,i}{a^{5}d^{2}}}\,e^{(5\,i\,dx+5\,i\,c)}\log\left(\frac{52\left(4\sqrt{2}\sqrt{\frac{a}{e^{(2\,i\,dx+2\,i\,c)}+1}}\sqrt{\frac{-i\,e^{(2\,i\,dx+2\,i\,c)}+i}{e^{(2\,i\,dx+2\,i\,c)}+1}}\left(e^{(3\,i\,dx+3\,i\,c)}+e^{(i\,dx+i\,c)}\right)+\left(3\,a^{3}d\,e^{(2\,i\,dx+2\,i\,c)}-a^{3}d\right)\sqrt{-\frac{4\,i}{a^{5}d^{2}}}\right)}{605\left(e^{(2\,i\,dx+2\,i\,c)}+1\right)}\right)+10\,a^{3}d\sqrt{-\frac{4\,i}{a^{5}d^{2}}}\,e^{(5\,i\,dx+5\,i\,c)}\log\left(\frac{52\left(4\sqrt{2}\sqrt{\frac{a}{e^{(2\,i\,dx+2\,i\,c)}+1}}\sqrt{\frac{-i\,e^{(2\,i\,dx+2\,i\,c)}+i}{e^{(2\,i\,dx+2\,i\,c)}+1}}\left(e^{(3\,i\,dx+3\,i\,c)}+e^{(i\,dx+i\,c)}\right)-\left(3\,a^{3}d\,e^{(2\,i\,dx+2\,i\,c)}-a^{3}d\right)\sqrt{-\frac{4\,i}{a^{5}d^{2}}}\right)}{605\left(e^{(2\,i\,dx+2\,i\,c)}+1\right)}\right)+\sqrt{2}\sqrt{\frac{a}{e^{(2\,i\,dx+2\,i\,c)}+1}}\sqrt{\frac{-i\,e^{(2\,i\,dx+2\,i\,c)}+i}{e^{(2\,i\,dx+2\,i\,c)}+1}}\left(41\,e^{(6\,i\,dx+6\,i\,c)}+34\,e^{(4\,i\,dx+4\,i\,c)}-6\,e^{(2\,i\,dx+2\,i\,c)}+1\right)\right)e^{(-5\,i\,dx-5\,i\,c)}}{40\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/40*(10*a^3*d*sqrt(-1/8*I/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(a^3*d*sqrt(-1
/8*I/(a^5*d^2))*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d
*x + 2*I*c) + 1)) - 10*a^3*d*sqrt(-1/8*I/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log
(-a^3*d*sqrt(-1/8*I/(a^5*d^2))*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I
*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c)
+ 1))*(e^(2*I*d*x + 2*I*c) + 1)) - 10*a^3*d*sqrt(-4*I/(a^5*d^2))*e^(5*I*d*x
 + 5*I*c)*log(52/605*(4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*
e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(3*I*d*x + 3*I*c) +
e^(I*d*x + I*c)) + (3*a^3*d*e^(2*I*d*x + 2*I*c) - a^3*d)*sqrt(-4*I/(a^5*d^2
)))/(e^(2*I*d*x + 2*I*c) + 1)) + 10*a^3*d*sqrt(-4*I/(a^5*d^2))*e^(5*I*d*x +
 5*I*c)*log(52/605*(4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^
(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(3*I*d*x + 3*I*c) + e^
(I*d*x + I*c)) - (3*a^3*d*e^(2*I*d*x + 2*I*c) - a^3*d)*sqrt(-4*I/(a^5*d^2))
)/(e^(2*I*d*x + 2*I*c) + 1)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sq
rt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(41*e^(6*I*d*x +
 6*I*c) + 34*e^(4*I*d*x + 4*I*c) - 6*e^(2*I*d*x + 2*I*c) + 1))*e^(-5*I*d*x
- 5*I*c)/(a^3*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(7/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(c+d\,x)^{7/2}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(7/2)/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] int(tan(c + d*x)^(7/2)/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.227 $\int \frac{\tan^{\frac{5}{2}}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=166

$$\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d}+\frac{i\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}+\frac{\tan^{\frac{3}{2}}(c+dx)}{6ad(a+ia\tan(c+dx))^{3/2}}-\frac{i\sqrt{}}{4a^2d\sqrt{}}$$

```
[Out] (1/8+1/8*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
)/a^(5/2)/d-1/4*I*tan(d*x+c)^(1/2)/a^2/d/(a+I*a*tan(d*x+c))^(1/2)+1/5*I*tan
(d*x+c)^(5/2)/d/(a+I*a*tan(d*x+c))^(5/2)+1/6*tan(d*x+c)^(3/2)/a/d/(a+I*a*ta
n(d*x+c))^(3/2)
```

Rubi [A]
time = 0.19, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3627, 3625, 211}

$$\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d}-\frac{i\sqrt{\tan(c+dx)}}{4a^2d\sqrt{a+ia\tan(c+dx)}}+\frac{i\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}+\frac{\tan^{\frac{3}{2}}(c+dx)}{6ad(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(5/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((1/8 + I/8)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[
c + d*x]]])/(a^(5/2)*d) + ((I/5)*Tan[c + d*x]^(5/2))/(d*(a + I*a*Tan[c + d*
x])^(5/2)) + Tan[c + d*x]^(3/2)/(6*a*d*(a + I*a*Tan[c + d*x])^(3/2)) - ((I/
4)*Sqrt[Tan[c + d*x]])/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3627

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^n/(2*b*f*m)), x] - Dist[(a*c - b*d)/(2*b^2), Int[(a + b*Tan[e + f*
x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && EqQ
[m + n, 0] && LeQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{i\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} - \frac{i\int \frac{\tan^{\frac{3}{2}}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{2a}$$

$$= \frac{i\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{\tan^{\frac{3}{2}}(c+dx)}{6ad(a+ia\tan(c+dx))^{3/2}} - \frac{\int \frac{\sqrt{\tan(c+}}{\sqrt{a+ia\tan(c}}}{4a^2}$$

$$= \frac{i\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{\tan^{\frac{3}{2}}(c+dx)}{6ad(a+ia\tan(c+dx))^{3/2}} - \frac{i\sqrt{\tan(c+}}{4a^2d\sqrt{a+ia\tan}}$$

$$= \frac{i\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{\tan^{\frac{3}{2}}(c+dx)}{6ad(a+ia\tan(c+dx))^{3/2}} - \frac{i\sqrt{\tan(c+}}{4a^2d\sqrt{a+ia\tan}}$$

$$= \frac{\left(\frac{1}{8}+\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d} + \frac{i\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} +$$

Mathematica [A]
time = 2.22, size = 177, normalized size = 1.07

$$\frac{ie^{-4i(c+dx)}\left(\sqrt{-1+e^{2i(c+dx)}}\left(-3+11e^{2i(c+dx)}-23e^{4i(c+dx)}\right)+15e^{5i(c+dx)}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\tan(c+dx)}}{60\sqrt{2}\,a^2d\sqrt{-1+e^{2i(c+dx)}}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(5/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((I/60)*(Sqrt[-1 + E^((2*I)*(c + d*x))]*(-3 + 11*E^((2*I)*(c + d*x)) - 23*E
^((4*I)*(c + d*x))) + 15*E^((5*I)*(c + d*x))*ArcTanh[E^(I*(c + d*x))/Sqrt[-
1 + E^((2*I)*(c + d*x))]])*Sqrt[Tan[c + d*x]])/(Sqrt[2]*a^2*d*E^((4*I)*(c +
 d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^(
(2*I)*(c + d*x)))])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 574 vs. $2(130) = 260$.
time = 0.17, size = 575, normalized size = 3.46

method	result
derivativedivides	$-\frac{\left(\sqrt{\tan}\,(dx+c)\right)\sqrt{a\,(1+i\tan\,(dx+c))}\,\left(148\sqrt{a\tan\,(dx+c)\,(1+i\tan\,(dx+c))}\,\sqrt{-ia}\right.}{}$
default	$-\frac{\left(\sqrt{\tan}\,(dx+c)\right)\sqrt{a\,(1+i\tan\,(dx+c))}\,\left(148\sqrt{a\tan\,(dx+c)\,(1+i\tan\,(dx+c))}\,\sqrt{-ia}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/240/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)/a^3*(148*(a*tan(d*x+c)
*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^3+60*I*2^(1/2)*ln(-(-2*2^(
1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))
/(tan(d*x+c)+I))*a*tan(d*x+c)^3-15*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*
tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*ta
n(d*x+c)^4-60*I*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan
(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)-308*I*(a*t
an(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^2+90*2^(1/2)*ln(-
(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(
d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2+60*I*(a*tan(d*x+c)*(1+I*tan(d*x+c)))
^(1/2)*(-I*a)^(1/2)-15*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(
1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a-220*tan(d*x+c)
*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2))/(a*tan(d*x+c)*(1+I*tan
(d*x+c)))^(1/2)/(-tan(d*x+c)+I)^4/(-I*a)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(120) = 240$.
time = 0.58, size = 332, normalized size = 2.00

$$-\frac{\left(30\,a^{3}d\sqrt{\frac{i}{8\,a^{5}d^{2}}}\,e^{(5i\,dx+5i\,c)}\log\left(i\,a^{3}d\sqrt{\frac{i}{8\,a^{5}d^{2}}}\,e^{(i\,dx+i\,c)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)\right)-30\,a^{3}d\sqrt{\frac{i}{8\,a^{5}d^{2}}}\,e^{(5i\,dx+5i\,c)}\log\left(-i\,a^{3}d\sqrt{\frac{i}{8\,a^{5}d^{2}}}\,e^{(i\,dx+i\,c)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)\right)-\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(-23i\,e^{(6i\,dx+6i\,c)}-12i\,e^{(4i\,dx+4i\,c)}+8i\,e^{(2i\,dx+2i\,c)}-3i\right)\right)e^{(-5i\,dx-5i\,c)}}{120\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")

[Out] -1/120*(30*a^3*d*sqrt(1/8*I/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(I*a^3*d*sqrt(1/8*I/(a^5*d^2))*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)) - 30*a^3*d*sqrt(1/8*I/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(-I*a^3*d*sqrt(1/8*I/(a^5*d^2))*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(-23*I*e^(6*I*d*x + 6*I*c) - 12*I*e^(4*I*d*x + 4*I*c) + 8*I*e^(2*I*d*x + 2*I*c) - 3*I))*e^(-5*I*d*x - 5*I*c)/(a^3*d)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)**(5/2)/(a+I*a*tan(d*x+c))**(5/2),x)

[Out] Integral(tan(c + d*x)**(5/2)/(I*a*(tan(c + d*x) - I))**(5/2), x)

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")

[Out] Timed out

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^{5/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d*x)^(5/2)/(a + a*tan(c + d*x)*1i)^(5/2),x)

[Out] int(tan(c + d*x)^(5/2)/(a + a*tan(c + d*x)*1i)^(5/2), x)

3.228 $\int \frac{\tan^{\frac{3}{2}}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=164

$$-\frac{\left(\frac{1}{8}-\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d}+\frac{\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}+\frac{i\tan^{\frac{3}{2}}(c+dx)}{6ad(a+ia\tan(c+dx))^{3/2}}+\frac{}{4a^2d}$$

```
[Out] (-1/8+1/8*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2
))/a^(5/2)/d+1/4*tan(d*x+c)^(1/2)/a^2/d/(a+I*a*tan(d*x+c))^(1/2)+1/5*tan(d*
x+c)^(5/2)/d/(a+I*a*tan(d*x+c))^(5/2)+1/6*I*tan(d*x+c)^(3/2)/a/d/(a+I*a*tan
(d*x+c))^(3/2)
```

Rubi [A]

time = 0.18, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3628, 3627, 3625, 211}

$$-\frac{\left(\frac{1}{8}-\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d}+\frac{\sqrt{\tan(c+dx)}}{4a^2d\sqrt{a+ia\tan(c+dx)}}+\frac{\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}+\frac{i\tan^{\frac{3}{2}}(c+dx)}{6ad(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(3/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-1/8 + I/8)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan
[c + d*x]]])/(a^(5/2)*d) + Tan[c + d*x]^(5/2)/(5*d*(a + I*a*Tan[c + d*x])^(
5/2)) + ((I/6)*Tan[c + d*x]^(3/2))/(a*d*(a + I*a*Tan[c + d*x])^(3/2)) + Sqr
t[Tan[c + d*x]]/(4*a^2*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3627

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^n/(2*b*f*m)), x] - Dist[(a*c - b*d)/(2*b^2), Int[(a + b*Tan[e + f*
x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && EqQ
[m + n, 0] && LeQ[m, -2^(-1)]
```

Rule 3628

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a), Int[(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && EqQ[m
 + n + 1, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{\int \frac{\tan^{\frac{3}{2}}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{2a}$$

$$= \frac{\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{i\tan^{\frac{3}{2}}(c+dx)}{6ad(a+ia\tan(c+dx))^{3/2}} - \frac{i\int \frac{\sqrt{\tan(c+}}{\sqrt{a+ia\tan(}}}{4a^2}$$

$$= \frac{\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{i\tan^{\frac{3}{2}}(c+dx)}{6ad(a+ia\tan(c+dx))^{3/2}} + \frac{\sqrt{\tan(c+}}{4a^2d\sqrt{a+ia\tan}}$$

$$= \frac{\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{i\tan^{\frac{3}{2}}(c+dx)}{6ad(a+ia\tan(c+dx))^{3/2}} + \frac{\sqrt{\tan(c+}}{4a^2d\sqrt{a+ia\tan}}$$

$$= -\frac{\left(\frac{1}{8}-\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d} + \frac{\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

Mathematica [A]
time = 2.15, size = 171, normalized size = 1.04

$$-\frac{ie^{-6i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3-4e^{2i(c+dx)}-16e^{4i(c+dx)}+17e^{6i(c+dx)}-15e^{5i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)}{60\sqrt{2}\,a^3d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(3/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-1/60*I)*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(3 - 4*E
^((2*I)*(c + d*x)) - 16*E^((4*I)*(c + d*x)) + 17*E^((6*I)*(c + d*x)) - 15*E
^((5*I)*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]*ArcTanh[E^(I*(c + d*x))/S
qrt[-1 + E^((2*I)*(c + d*x))]]))/(Sqrt[2]*a^3*d*E^((6*I)*(c + d*x))*Sqrt[Ta
n[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 575 vs. $2(129) = 258$.
time = 0.19, size = 576, normalized size = 3.51

method	result
derivativedivides	$\frac{\left(\sqrt{\tan}\left(dx+c\right)\right)\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\left(15i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1+i\right.}}{\tan(dx+c)+i}\right.\right.}{}$
default	$\frac{\left(\sqrt{\tan}\left(dx+c\right)\right)\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\left(15i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1+i\right.}}{\tan(dx+c)+i}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/240/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)/a^3*(15*I*2^(1/2)*ln(-(
-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d
*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4-212*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*t
an(d*x+c)))^(1/2)*tan(d*x+c)^2-90*I*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a
*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*t
an(d*x+c)^2-52*I*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x
+c)^3+60*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)
))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3+15*I*2^(1/2)*ln
(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*ta
n(d*x+c))/(tan(d*x+c)+I))*a+220*I*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I
*a)^(1/2)*tan(d*x+c)-60*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*
(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)+60
*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2))/(a*tan(d*x+c)*(1+I*tan
(d*x+c)))^(1/2)/(-tan(d*x+c)+I)^4/(-I*a)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(120) = 240$.
time = 0.52, size = 331, normalized size = 2.02

$$-\frac{\left(30\,a^{3}d\sqrt{-\frac{i}{8\,a^{5}d^{2}}}\,e^{(5i\,dx+5i\,c)}\log\left(a^{3}d\sqrt{-\frac{i}{8\,a^{5}d^{2}}}\,e^{(i\,dx+i\,c)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)\right)-30\,a^{3}d\sqrt{-\frac{i}{8\,a^{5}d^{2}}}\,e^{(5i\,dx+5i\,c)}\log\left(-a^{3}d\sqrt{-\frac{i}{8\,a^{5}d^{2}}}\,e^{(i\,dx+i\,c)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)\right)-\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(17\,e^{(6i\,dx+6i\,c)}+18\,e^{(4i\,dx+4i\,c)}-2\,e^{(2i\,dx+2i\,c)}-3\right)\right)e^{(-5i\,dx-5i\,c)}}{120\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/120*(30*a^3*d*sqrt(-1/8*I/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(a^3*d*sqrt(
-1/8*I/(a^5*d^2))*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c)
 + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I
*d*x + 2*I*c) + 1)) - 30*a^3*d*sqrt(-1/8*I/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*l
og(-a^3*d*sqrt(-1/8*I/(a^5*d^2))*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c
) + 1))*(e^(2*I*d*x + 2*I*c) + 1)) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(17*e^(6*I
*d*x + 6*I*c) + 18*e^(4*I*d*x + 4*I*c) - 2*e^(2*I*d*x + 2*I*c) - 3))*e^(-5*
I*d*x - 5*I*c)/(a^3*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{3}{2}}\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(3/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(tan(c + d*x)**(3/2)/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^{3/2}}{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(3/2)/(a + a*tan(c + d*x)*1i)^(5/2),x)
[Out] int(tan(c + d*x)^(3/2)/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.229 $\int \frac{\sqrt{\tan(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=168

$$-\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d}+\frac{i\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}+\frac{i\sqrt{\tan(c+dx)}}{10ad(a+ia\tan(c+dx))^{3/2}}-\frac{}{20}$$

```
[Out] (-1/8-1/8*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2
))/a^(5/2)/d-1/20*I*tan(d*x+c)^(1/2)/a^2/d/(a+I*a*tan(d*x+c))^(1/2)+1/5*I*t
an(d*x+c)^(1/2)/d/(a+I*a*tan(d*x+c))^(5/2)+1/10*I*tan(d*x+c)^(1/2)/a/d/(a+I
*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.28, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3638, 3677, 12, 3625, 211}

$$-\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d}-\frac{i\sqrt{\tan(c+dx)}}{20a^2d\sqrt{a+ia\tan(c+dx)}}+\frac{i\sqrt{\tan(c+dx)}}{10ad(a+ia\tan(c+dx))^{3/2}}+\frac{i\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[c + d*x]]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-1/8 - I/8)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan
[c + d*x]]])/(a^(5/2)*d) + ((I/5)*Sqrt[Tan[c + d*x]])/(d*(a + I*a*Tan[c + d
*x])^(5/2)) + ((I/10)*Sqrt[Tan[c + d*x]])/(a*d*(a + I*a*Tan[c + d*x])^(3/2)
) - ((I/20)*Sqrt[Tan[c + d*x]])/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
```

```
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3638

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*Sqrt[(c_.) + (d_.)*tan[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-b)*(a + b*Tan[e + f*x])^m*(Sqrt[c + d
*Tan[e + f*x]]/(2*a*f*m)), x] + Dist[1/(4*a^2*m), Int[(a + b*Tan[e + f*x])^
(m + 1)*(Simp[2*a*c*m + b*d + a*d*(2*m + 1)*Tan[e + f*x], x]/Sqrt[c + d*Tan
[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && IntegersQ[2*m]
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{\tan(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{i\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}} - \frac{\int \frac{ia-4a\tan(c+dx)}{\sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^{3/2}}\,dx}{10a^2}$$

$$= \frac{i\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{i\sqrt{\tan(c+dx)}}{10ad(a+ia\tan(c+dx))^{3/2}} - \frac{\int \frac{\frac{9ia}{2}}{\sqrt{\tan(c+dx}}}{}$$

$$= \frac{i\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{i\sqrt{\tan(c+dx)}}{10ad(a+ia\tan(c+dx))^{3/2}} - \frac{i\sqrt{\tan(c}}{20a^2d\sqrt{a+ia}}$$

$$= \frac{i\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{i\sqrt{\tan(c+dx)}}{10ad(a+ia\tan(c+dx))^{3/2}} - \frac{i\sqrt{\tan(c}}{20a^2d\sqrt{a+ia}}$$

$$= \frac{i\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{i\sqrt{\tan(c+dx)}}{10ad(a+ia\tan(c+dx))^{3/2}} - \frac{i\sqrt{\tan(c}}{20a^2d\sqrt{a+ia}}$$

$$= -\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d} + \frac{i\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}$$

Mathematica [A]
time = 1.91, size = 175, normalized size = 1.04

$$\frac{ie^{-4i(c+dx)}\left(\sqrt{-1+e^{2i(c+dx)}}\left(1+3e^{2i(c+dx)}+e^{4i(c+dx)}\right)-5e^{5i(c+dx)}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\tan(c+dx)}}{20\sqrt{2}\,a^2d\sqrt{-1+e^{2i(c+dx)}}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tan[c + d*x]]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((I/20)*(Sqrt[-1 + E^((2*I)*(c + d*x))]*(1 + 3*E^((2*I)*(c + d*x)) + E^((4*
I)*(c + d*x))) - 5*E^((5*I)*(c + d*x))*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^
((2*I)*(c + d*x))]])*Sqrt[Tan[c + d*x]])/(Sqrt[2]*a^2*d*E^((4*I)*(c + d*x))
*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*
(c + d*x)))])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 574 vs. $2(131) = 262$.
time = 0.19, size = 575, normalized size = 3.42

method	result
derivativedivides	$\frac{\left(\sqrt{\tan}\,(dx+c)\right)\sqrt{a\,(1+i\tan\,(dx+c))}\,\left(-4\sqrt{a\tan\,(dx+c)\,(1+i\tan\,(dx+c))}\,\sqrt{-ia}\,(\mathrm{t}\right.}{}$
default	$\frac{\left(\sqrt{\tan}\,(dx+c)\right)\sqrt{a\,(1+i\tan\,(dx+c))}\,\left(-4\sqrt{a\tan\,(dx+c)\,(1+i\tan\,(dx+c))}\,\sqrt{-ia}\,(\mathrm{t}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/80/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)/a^3*(-4*(a*tan(d*x+c)*(1
+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^3+20*I*2^(1/2)*ln(-(-2*2^(1/2
)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(t
an(d*x+c)+I))*a*tan(d*x+c)^3-5*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(
d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*
x+c)^4-20*I*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x
+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)+4*I*(-I*a)^(1/
2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^2+30*2^(1/2)*ln(-(-2*2^
(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c)
)/(tan(d*x+c)+I))*a*tan(d*x+c)^2+20*I*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d
*x+c)))^(1/2)-5*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan
(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a-20*tan(d*x+c)*(a*tan(
d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2))/(a*tan(d*x+c)*(1+I*tan(d*x+c))
)^(1/2)/(-I*a)^(1/2)/(-tan(d*x+c)+I)^4
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(120) = 240$.
time = 0.68, size = 331, normalized size = 1.97

$$\frac{\left(10\,a^3d\sqrt{\frac{i}{8\,a^5d^2}}\,e^{(5i\,dx+5i\,c)}\log\left(i\,a^3d\sqrt{\frac{i}{8\,a^5d^2}}\,e^{(i\,dx+i\,c)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)\right)-10\,a^3d\sqrt{\frac{i}{8\,a^5d^2}}\,e^{(5i\,dx+5i\,c)}\log\left(-i\,a^3d\sqrt{\frac{i}{8\,a^5d^2}}\,e^{(i\,dx+i\,c)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)\right)+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(i\,e^{(6i\,dx+6i\,c)}+4i\,e^{(4i\,dx+4i\,c)}+4i\,e^{(2i\,dx+2i\,c)}+i\right)\right)e^{(-5i\,dx-5i\,c)}}{40\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/40*(10*a^3*d*sqrt(1/8*I/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(I*a^3*d*sqrt(1
/8*I/(a^5*d^2))*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d
*x + 2*I*c) + 1)) - 10*a^3*d*sqrt(1/8*I/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(
-I*a^3*d*sqrt(1/8*I/(a^5*d^2))*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I
*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c)
+ 1))*(e^(2*I*d*x + 2*I*c) + 1)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^(6*I*d*
x + 6*I*c) + 4*I*e^(4*I*d*x + 4*I*c) + 4*I*e^(2*I*d*x + 2*I*c) + I))*e^(-5*
I*d*x - 5*I*c)/(a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\tan{(c+dx)}}}{(ia(\tan{(c+dx)}-i))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(sqrt(tan(c + d*x))/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\tan\left(c+d\,x\right)}}{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] int(tan(c + d*x)^(1/2)/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.230 $\int \frac{1}{\sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=162

$$\frac{\left(\frac{1}{8}-\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d}+\frac{\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}+\frac{13\sqrt{\tan(c+dx)}}{30ad(a+ia\tan(c+dx))^{3/2}}+\frac{6}{60a^2d}$$

```
[Out] (1/8-1/8*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
)/a^(5/2)/d+67/60*tan(d*x+c)^(1/2)/a^2/d/(a+I*a*tan(d*x+c))^(1/2)+1/5*tan(d
*x+c)^(1/2)/d/(a+I*a*tan(d*x+c))^(5/2)+13/30*tan(d*x+c)^(1/2)/a/d/(a+I*a*ta
n(d*x+c))^(3/2)
```

Rubi [A]
time = 0.29, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3640, 3677, 12, 3625, 211}

$$\frac{\left(\frac{1}{8}-\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d}+\frac{67\sqrt{\tan(c+dx)}}{60a^2d\sqrt{a+ia\tan(c+dx)}}+\frac{13\sqrt{\tan(c+dx)}}{30ad(a+ia\tan(c+dx))^{3/2}}+\frac{\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2)),x]
```

```
[Out] ((1/8 - I/8)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[
c + d*x]]])/(a^(5/2)*d) + Sqrt[Tan[c + d*x]]/(5*d*(a + I*a*Tan[c + d*x])^(5
/2)) + (13*Sqrt[Tan[c + d*x]])/(30*a*d*(a + I*a*Tan[c + d*x])^(3/2)) + (67*
Sqrt[Tan[c + d*x]])/(60*a^2*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
```

```
Q[c^2 + d^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{\int \frac{\frac{9a}{2}-2ia\tan(c+dx)}{\sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^{3/2}}}{5a^2}$$

$$= \frac{\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{13\sqrt{\tan(c+dx)}}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{\int}{}$$

$$= \frac{\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{13\sqrt{\tan(c+dx)}}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{}{6}$$

$$= \frac{\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{13\sqrt{\tan(c+dx)}}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{}{6}$$

$$= \frac{\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{13\sqrt{\tan(c+dx)}}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{}{6}$$

$$= \frac{\left(\frac{1}{8}-\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d} + \frac{\sqrt{\tan(c+}}{5d(a+ia\tan(}$$

Mathematica [A]
time = 2.27, size = 171, normalized size = 1.06

$$-\frac{ie^{-6i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(-3-16e^{2i(c+dx)}-64e^{4i(c+dx)}+83e^{6i(c+dx)}+15e^{5i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)}{60\sqrt{2}\,a^3d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2)),x]
```

```
[Out] ((-1/60*I)*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(-3 - 16
*E^((2*I)*(c + d*x)) - 64*E^((4*I)*(c + d*x)) + 83*E^((6*I)*(c + d*x)) + 15
*E^((5*I)*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]*ArcTanh[E^(I*(c + d*x))
/Sqrt[-1 + E^((2*I)*(c + d*x))]]))/(Sqrt[2]*a^3*d*E^((6*I)*(c + d*x))*Sqrt[
Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 575 vs. 2(128) = 256.
time = 0.19, size = 576, normalized size = 3.56

method	result
derivativedivides	$-\frac{\left(\sqrt{\tan}\,(dx+c)\right)\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\left(15i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1\right.}}{\tan(dx+c}\right.\right.}{}$
default	$-\frac{\left(\sqrt{\tan}\,(dx+c)\right)\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\left(15i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1\right.}}{\tan(dx+c}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/240/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)*(15*I*2^(1/2)*ln(-(-2*
2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+
c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4-90*I*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2
)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))
*a*tan(d*x+c)^2+268*I*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*ta
n(d*x+c)^3+60*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d
*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3+15*I*2^(1/
2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3
*a*tan(d*x+c))/(tan(d*x+c)+I))*a-1060*I*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/
2)*(-I*a)^(1/2)*tan(d*x+c)-60*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d
*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x
+c)+908*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^2-420
*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2))/a^3/(a*tan(d*x+c)*(1+I
*tan(d*x+c)))^(1/2)/(-tan(d*x+c)+I)^4/(-I*a)^(1/2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 330 vs. 2(120) = 240.
time = 0.61, size = 330, normalized size = 2.04

$$\frac{\left(30\,a^3d\sqrt{-\frac{i}{8\,a^5d^2}}\,e^{(5i\,dx+5i\,c)}\log\left(a^3d\sqrt{-\frac{i}{8\,a^5d^2}}\,e^{(i\,dx+i\,c)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)\right)-30\,a^3d\sqrt{-\frac{i}{8\,a^5d^2}}\,e^{(5i\,dx+5i\,c)}\log\left(-a^3d\sqrt{-\frac{i}{8\,a^5d^2}}\,e^{(i\,dx+i\,c)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)\right)+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(83\,e^{(6i\,dx+6i\,c)}+102\,e^{(4i\,dx+4i\,c)}+22\,e^{(2i\,dx+2i\,c)}+3\right)\right)e^{(-5i\,dx-5i\,c)}}{120\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/120*(30*a^3*d*sqrt(-1/8*I/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(a^3*d*sqrt(-
1/8*I/(a^5*d^2))*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*
d*x + 2*I*c) + 1)) - 30*a^3*d*sqrt(-1/8*I/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*lo
g(-a^3*d*sqrt(-1/8*I/(a^5*d^2))*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*
I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c)
 + 1))*(e^(2*I*d*x + 2*I*c) + 1)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(83*e^(6*I*
d*x + 6*I*c) + 102*e^(4*I*d*x + 4*I*c) + 22*e^(2*I*d*x + 2*I*c) + 3))*e^(-5
*I*d*x - 5*I*c)/(a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}\sqrt{\tan(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(1/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(1/((I*a*(tan(c + d*x) - I))**(5/2)*sqrt(tan(c + d*x))), x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1348 vs. 2(120) = 240.
time = 4.83, size = 1348, normalized size = 8.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*(I*a*sqrt(abs(a)) - abs(a)^(3/2))*log(-1/2*(I*(sqrt(2)*sqrt(I*a*tan(d*
x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*tan(d*x + c)
+ a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(d
*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^2 + 8*sqrt(2) - 12)/(-1/2*I*(sq
rt(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(
-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c)
+ a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^2 + 4*sqrt
(2) + 6))/(a^4*d) - 1/15*sqrt(2)*(15*(sqrt(2)*sqrt(I*a*tan(d*x + c) + a)*(-
I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2
)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(d*x + c) + a)*a
```

```
 + a^2)/a^2) + 1)*abs(a)/a^2)^8*a*sqrt(abs(a)) + 15*I*(sqrt(2)*sqrt(I*a*tan
(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*tan(d*x +
c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*a*ta
n(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^8*abs(a)^(3/2) - 480*I*(sqrt
(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2
*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) +
a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^6*a*sqrt(abs
(a)) + 480*(sqrt(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/
2)/a^2 - sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a
*tan(d*x + c) + a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a
^2)^6*abs(a)^(3/2) - 8800*(sqrt(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a
+ 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x
+ c)/sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2
) + 1)*abs(a)/a^2)^4*a*sqrt(abs(a)) - 8800*I*(sqrt(2)*sqrt(I*a*tan(d*x + c)
 + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*tan(d*x + c) + a)*a
 + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(d*x + c
) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^4*abs(a)^(3/2) + 20480*I*(sqrt(2)*sqr
t(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*t
an(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2 -
2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^2*a*sqrt(abs(a)) -
20480*(sqrt(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^
2 - sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(
d*x + c) + a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^2
*abs(a)^(3/2) + 17152*a*sqrt(abs(a)) + 17152*I*abs(a)^(3/2))/(((sqrt(2)*sqr
t(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*t
an(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2 -
2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^2 - 4*I)^5*a^4*d)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\tan\left(c+d\,x\right)}\,\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(5/2)),x)
[Out] int(1/(tan(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(5/2)), x)
```

3.231 $\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=199

$$\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d}+\frac{1}{5d\sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^{5/2}}+\frac{1}{30ad\sqrt{\tan(c+dx)}\,(\cdots}$$

```
[Out] (1/8+1/8*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
)/a^(5/2)/d+151/60/a^2/d/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-317/60*(
a+I*a*tan(d*x+c))^(1/2)/a^3/d/tan(d*x+c)^(1/2)+1/5/d/tan(d*x+c)^(1/2)/(a+I*
a*tan(d*x+c))^(5/2)+17/30/a/d/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.37, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3640, 3677, 3679, 12, 3625, 211}

$$\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d}-\frac{317\sqrt{a+ia\tan(c+dx)}}{60a^3d\sqrt{\tan(c+dx)}}+\frac{151}{60a^2d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{17}{30ad\sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^{3/2}}+\frac{1}{5d\sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(5/2)),x]
```

```
[Out] ((1/8 + I/8)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[
c + d*x]]])/(a^(5/2)*d) + 1/(5*d*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^
(5/2)) + 17/(30*a*d*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)) + 151/
(60*a^2*d*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (317*Sqrt[a + I*
a*Tan[c + d*x]])/(60*a^3*d*Sqrt[Tan[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
```

```
Q[c^2 + d^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{1}{5d\sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^{5/2}} + \frac{\int \frac{\frac{11a}{2}-3ia\tan(c+\cdots}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(\cdots}}{5a^2}$$

$$= \frac{1}{5d\sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^{5/2}} + \frac{\cdots}{30ad\sqrt{\tan(c+dx)}\cdots}$$

$$= \frac{1}{5d\sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^{5/2}} + \frac{\cdots}{30ad\sqrt{\tan(c+dx)}\cdots}$$

$$= \frac{1}{5d\sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^{5/2}} + \frac{\cdots}{30ad\sqrt{\tan(c+dx)}\cdots}$$

$$= \frac{1}{5d\sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^{5/2}} + \frac{\cdots}{30ad\sqrt{\tan(c+dx)}\cdots}$$

$$= \frac{1}{5d\sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^{5/2}} + \frac{\cdots}{30ad\sqrt{\tan(c+dx)}\cdots}$$

$$= \frac{\left(\frac{1}{8}+\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d} + \frac{\cdots}{5d\sqrt{\tan(c+dx)}\cdots}$$

Mathematica [A]
time = 1.35, size = 203, normalized size = 1.02

$$\frac{ie^{-4i(c+dx)}\left(\sqrt{-1+e^{2i(c+dx)}}\left(3+26e^{2i(c+dx)}+194e^{4i(c+dx)}-463e^{6i(c+dx)}\right)+15e^{5i(c+dx)}\left(-1+e^{2i(c+dx)}\right)\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\tan(c+dx)}}{60\sqrt{2}\,a^2d\left(-1+e^{2i(c+dx)}\right)^{3/2}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(5/2)),x]
```

```
[Out] ((I/60)*(Sqrt[-1 + E^((2*I)*(c + d*x))]*(3 + 26*E^((2*I)*(c + d*x)) + 194*E
^((4*I)*(c + d*x)) - 463*E^((6*I)*(c + d*x))) + 15*E^((5*I)*(c + d*x))*(-1
+ E^((2*I)*(c + d*x)))*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x)
)]])*Sqrt[Tan[c + d*x]])/(Sqrt[2]*a^2*d*E^((4*I)*(c + d*x))*(-1 + E^((2*I)*
(c + d*x)))^(3/2)*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(158) = 316$.
time = 0.17, size = 620, normalized size = 3.12

method	result
derivativedivides	$-\frac{\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\,\left(1268\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1+i\tan\left(dx+c\right)\right)}\,\left(\tan^{4}(dx+c)\right)+\right.}{}$
default	$-\frac{\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\,\left(1268\sqrt{-ia}\,\sqrt{a\tan\left(dx+c\right)\left(1+i\tan\left(dx+c\right)\right)}\,\left(\tan^{4}(dx+c)\right)+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/240/d*(a*(1+I*tan(d*x+c)))^(1/2)/a^3*(1268*(-I*a)^(1/2)*(a*tan(d*x+c)*(1
+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^4+60*I*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2
)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))
*a*tan(d*x+c)^4-15*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*
tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^5-5660*
(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^2-60*I*2^(1/2
)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*
a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2-4468*I*(-I*a)^(1/2)*(a*tan(d*x
+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^3+90*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^
(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)
+I))*a*tan(d*x+c)^3+2940*I*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/
2)*tan(d*x+c)-15*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*ta
n(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)+480*(a*ta
n(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2))/tan(d*x+c)^(1/2)/(a*tan(d*x+
c)*(1+I*tan(d*x+c)))^(1/2)/(-tan(d*x+c)+I)^4/(-I*a)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(149) = 298$.
time = 0.74, size = 393, normalized size = 1.97

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(-463i\,e^{(8i\,dx+8i\,c)}-269i\,e^{(6i\,dx+6i\,c)}+220i\,e^{(4i\,dx+4i\,c)}+29i\,e^{(2i\,dx+2i\,c)}+3i\right)-30\left(a^3de^{(7i\,dx+7i\,c)}-a^3de^{(5i\,dx+5i\,c)}\right)\sqrt{\frac{i}{8\,a^5d^2}}\log\left(i\,a^3d\sqrt{\frac{i}{8\,a^5d^2}}e^{(i\,dx+i\,c)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)\right)+30\left(a^3de^{(7i\,dx+7i\,c)}-a^3de^{(5i\,dx+5i\,c)}\right)\sqrt{\frac{i}{8\,a^5d^2}}\log\left(-i\,a^3d\sqrt{\frac{i}{8\,a^5d^2}}e^{(i\,dx+i\,c)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)\right)}{120\left(a^3de^{(7i\,dx+7i\,c)}-a^3de^{(5i\,dx+5i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/120*(sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*
c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(-463*I*e^(8*I*d*x + 8*I*c) - 269*I*e^(6
*I*d*x + 6*I*c) + 220*I*e^(4*I*d*x + 4*I*c) + 29*I*e^(2*I*d*x + 2*I*c) + 3*
I) - 30*(a^3*d*e^(7*I*d*x + 7*I*c) - a^3*d*e^(5*I*d*x + 5*I*c))*sqrt(1/8*I/
(a^5*d^2))*log(I*a^3*d*sqrt(1/8*I/(a^5*d^2))*e^(I*d*x + I*c) + 1/4*sqrt(2)*
sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I
*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)) + 30*(a^3*d*e^(7*I*d*x + 7*I
*c) - a^3*d*e^(5*I*d*x + 5*I*c))*sqrt(1/8*I/(a^5*d^2))*log(-I*a^3*d*sqrt(1/
8*I/(a^5*d^2))*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*
x + 2*I*c) + 1)))/(a^3*d*e^(7*I*d*x + 7*I*c) - a^3*d*e^(5*I*d*x + 5*I*c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(3/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1446 vs. $2(149) = 298$.
time = 18.88, size = 1446, normalized size = 7.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*(a*sqrt(abs(a)) + I*abs(a)^(3/2))*log(-1/2*(-I*(sqrt(2)*sqrt(I*a*tan(d
*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*tan(d*x + c)
 + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(
d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^2 + 8*sqrt(2) + 12)/(1/2*I*(sq
```

```
rt(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(
-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c)
+ a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^2 + 4*sqrt
(2) - 6))/(a^4*d) - sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*sqrt(I*a*tan(
d*x + c) + a)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(d*x
 + c) + a)*a + a^2)/a^2) + 1)*abs(a)/(a^5*d*tan(d*x + c)) + 1/15*sqrt(2)*(1
05*I*(sqrt(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2
 - sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d
*x + c) + a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^8*
a*sqrt(abs(a)) - 105*(sqrt(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*
abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/
sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1
)*abs(a)/a^2)^8*abs(a)^(3/2) + 2400*(sqrt(2)*sqrt(I*a*tan(d*x + c) + a)*(-I
*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)
*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(d*x + c) + a)*a
+ a^2)/a^2) + 1)*abs(a)/a^2)^6*a*sqrt(abs(a)) + 2400*I*(sqrt(2)*sqrt(I*a*ta
n(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*tan(d*x +
 c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*a*t
an(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^6*abs(a)^(3/2) - 21920*I*(s
qrt(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt
(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c)
 + a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^4*a*sqrt(
abs(a)) + 21920*(sqrt(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a
)^(3/2)/a^2 - sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(
((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs
(a)/a^2)^4*abs(a)^(3/2) - 56320*(sqrt(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs
(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(ta
n(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^
2)/a^2) + 1)*abs(a)/a^2)^2*a*sqrt(abs(a)) - 56320*I*(sqrt(2)*sqrt(I*a*tan(d
*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*tan(d*x + c)
 + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(
d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^2*abs(a)^(3/2) + 50432*I*a*sqr
t(abs(a)) - 50432*abs(a)^(3/2))/(((sqrt(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*a
bs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(
tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(d*x + c) + a)*a +
a^2)/a^2) + 1)*abs(a)/a^2)^2 - 4*I)^5*a^4*d)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\tan\left(c+d\,x\right)^{3/2}\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

```
[Out] int(1/(tan(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(5/2)), x)
```

3.232 $\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=238

$$-\frac{\left(\frac{1}{8}-\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d}+\frac{1}{5d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}+\frac{}{10ad\tan^{\frac{3}{2}}(c+dx)(}$$

```
[Out] (-1/8+1/8*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2
))/a^(5/2)/d+707/60*I*(a+I*a*tan(d*x+c))^(1/2)/a^3/d/tan(d*x+c)^(1/2)+89/20
/a^2/d/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(3/2)-361/60*(a+I*a*tan(d*x+c))^
(1/2)/a^3/d/tan(d*x+c)^(3/2)+1/5/d/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(5/2
)+7/10/a/d/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.48, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3640, 3677, 3679, 12, 3625, 211}

$$-\frac{\left(\frac{1}{8}-\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d}-\frac{361\sqrt{a+ia\tan(c+dx)}}{60a^3d\tan^{\frac{3}{2}}(c+dx)}+\frac{707i\sqrt{a+ia\tan(c+dx)}}{60a^3d\sqrt{\tan(c+dx)}}+\frac{89}{20a^2d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}+\frac{7}{10ad\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}+\frac{1}{5d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^(5/2)),x]
```

```
[Out] ((-1/8 + I/8)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan
[c + d*x]]])/(a^(5/2)*d) + 1/(5*d*Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])
^(5/2)) + 7/(10*a*d*Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)) + 89/(
20*a^2*d*Tan[c + d*x]^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) - (361*Sqrt[a + I*a
*Tan[c + d*x]])/(60*a^3*d*Tan[c + d*x]^(3/2)) + (((707*I)/60)*Sqrt[a + I*a*
Tan[c + d*x]])/(a^3*d*Sqrt[Tan[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
```

```
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{1}{5d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}} + \frac{\int \frac{\frac{13a}{2}-4ia\tan(c+a}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c}}{5a^2}$$

$$= \frac{1}{5d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}} + \frac{}{10ad\tan^{\frac{3}{2}}(c+dx)(a}$$

$$= \frac{1}{5d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}} + \frac{}{10ad\tan^{\frac{3}{2}}(c+dx)(a}$$

$$= \frac{1}{5d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}} + \frac{}{10ad\tan^{\frac{3}{2}}(c+dx)(a}$$

$$= \frac{1}{5d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}} + \frac{}{10ad\tan^{\frac{3}{2}}(c+dx)(a}$$

$$= \frac{1}{5d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}} + \frac{}{10ad\tan^{\frac{3}{2}}(c+dx)(a}$$

$$= \frac{1}{5d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}} + \frac{}{10ad\tan^{\frac{3}{2}}(c+dx)(a}$$

$$= -\frac{\left(\frac{1}{8}-\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d} + \frac{}{5d\tan^{\frac{3}{2}}(c+d}$$

Mathematica [A]
time = 1.57, size = 199, normalized size = 0.84

$$\frac{ie^{-6i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3+33e^{2i(c+dx)}+348e^{4i(c+dx)}-1527e^{6i(c+dx)}+983e^{8i(c+dx)}+15e^{5i(c+dx)}\left(-1+e^{2i(c+dx)}\right)^{3/2}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)}{60\sqrt{2}\,a^3d\left(-1+e^{2i(c+dx)}\right)\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Tan[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^(5/2)),x]
```

```
[Out] ((I/60)*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(3 + 33*E^(
(2*I)*(c + d*x)) + 348*E^((4*I)*(c + d*x)) - 1527*E^((6*I)*(c + d*x)) + 983
*E^((8*I)*(c + d*x)) + 15*E^((5*I)*(c + d*x))*(-1 + E^((2*I)*(c + d*x)))^(3
```

```
/2)*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]]))/(Sqrt[2]*a^3*
d*E^((6*I)*(c + d*x))*(-1 + E^((2*I)*(c + d*x)))*Sqrt[Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 660 vs. $2(189) = 378$.
time = 0.20, size = 661, normalized size = 2.78

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(dx+c))}\left(15i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}}{\tan(dx+c)+i}\right.\right.}{}$
default	$\frac{\sqrt{a(1+i\tan(dx+c))}\left(15i\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}}{\tan(dx+c)+i}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/240/d*(a*(1+I*tan(d*x+c)))^(1/2)*(15*I*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/
2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I)
)*a*tan(d*x+c)^6-90*I*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1
+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4+28
28*I*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^5+60*2^(
1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a
-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^5+15*I*2^(1/2)*ln(-(-2*2^(1/2
)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(t
an(d*x+c)+I))*a*tan(d*x+c)^2-12260*I*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*
x+c)))^(1/2)*tan(d*x+c)^3-60*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*
x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+
c)^3+9868*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^4+6
40*I*(-I*a)^(1/2)*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-6020*(-I
*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^2-160*(a*tan(d*x
+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2))/a^3/tan(d*x+c)^(3/2)/(a*tan(d*x+c
)*(1+I*tan(d*x+c)))^(1/2)/(-tan(d*x+c)+I)^4/(-I*a)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima"
)
```

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(178) = 356$.
time = 0.77, size = 445, normalized size = 1.87

$$-\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(983\,e^{(10i\,dx+10i\,c)}-544\,e^{(8i\,dx+8i\,c)}-1179\,e^{(6i\,dx+6i\,c)}+381\,e^{(4i\,dx+4i\,c)}+36\,e^{(2i\,dx+2i\,c)}+3\right)+30\left(a^3de^{(9i\,dx+9i\,c)}-2\,a^3de^{(7i\,dx+7i\,c)}+a^3de^{(5i\,dx+5i\,c)}\right)\sqrt{-\frac{i}{8\,a^5d^2}}\log\left(a^3d\sqrt{-\frac{i}{8\,a^5d^2}}e^{(i\,dx+i\,c)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)\right)-30\left(a^3de^{(9i\,dx+9i\,c)}-2\,a^3de^{(7i\,dx+7i\,c)}+a^3de^{(5i\,dx+5i\,c)}\right)\sqrt{-\frac{i}{8\,a^5d^2}}\log\left(-a^3d\sqrt{-\frac{i}{8\,a^5d^2}}e^{(i\,dx+i\,c)}+\frac{1}{4}\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)\right)}{120\left(a^3de^{(9i\,dx+9i\,c)}-2\,a^3de^{(7i\,dx+7i\,c)}+a^3de^{(5i\,dx+5i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas"
)
```

```
[Out] -1/120*(sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I
*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(983*e^(10*I*d*x + 10*I*c) - 544*e^(8*I
*d*x + 8*I*c) - 1179*e^(6*I*d*x + 6*I*c) + 381*e^(4*I*d*x + 4*I*c) + 36*e^(
2*I*d*x + 2*I*c) + 3) + 30*(a^3*d*e^(9*I*d*x + 9*I*c) - 2*a^3*d*e^(7*I*d*x
+ 7*I*c) + a^3*d*e^(5*I*d*x + 5*I*c))*sqrt(-1/8*I/(a^5*d^2))*log(a^3*d*sqrt
(-1/8*I/(a^5*d^2))*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*
I*d*x + 2*I*c) + 1)) - 30*(a^3*d*e^(9*I*d*x + 9*I*c) - 2*a^3*d*e^(7*I*d*x +
 7*I*c) + a^3*d*e^(5*I*d*x + 5*I*c))*sqrt(-1/8*I/(a^5*d^2))*log(-a^3*d*sqrt
(-1/8*I/(a^5*d^2))*e^(I*d*x + I*c) + 1/4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*
I*d*x + 2*I*c) + 1)))/(a^3*d*e^(9*I*d*x + 9*I*c) - 2*a^3*d*e^(7*I*d*x + 7*I
*c) + a^3*d*e^(5*I*d*x + 5*I*c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(5/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1472 vs. $2(178) = 356$.
time = 17.82, size = 1472, normalized size = 6.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*sqrt(I*a*tan(d*x + c) + a)*(t
an(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a
^2)/a^2) + 1)*(8*(I*a*tan(d*x + c) + a)/(a^4*d) - 9/(a^3*d))*abs(a)/(a^2*ta
n(d*x + c)^2) + 1/16*(I*a*sqrt(abs(a)) - abs(a)^(3/2))*log(-1/2*(-I*(sqrt(2
)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(
I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)
^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^2 + 8*sqrt(2)
+ 12)/(1/2*I*(sqrt(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(
3/2)/a^2 - sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I
*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)
/a^2)^2 + 4*sqrt(2) - 6))/(a^4*d) + 1/5*sqrt(2)*(85*(sqrt(2)*sqrt(I*a*tan(d
*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*tan(d*x + c)
 + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(
d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^8*a*sqrt(abs(a)) + 85*I*(sqrt(
2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*
(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a
)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^8*abs(a)^(3/2
) - 1760*I*(sqrt(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/
2)/a^2 - sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a
*tan(d*x + c) + a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a
^2)^6*a*sqrt(abs(a)) + 1760*(sqrt(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/
a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*
x + c)/sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a
^2) + 1)*abs(a)/a^2)^6*abs(a)^(3/2) - 13600*(sqrt(2)*sqrt(I*a*tan(d*x + c)
+ a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*tan(d*x + c) + a)*a
+ 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(d*x + c)
 + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^4*a*sqrt(abs(a)) - 13600*I*(sqrt(2)*sq
rt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*
tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan(d*x + c) + a)^2 -
 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^4*abs(a)^(3/2) + 3
5840*I*(sqrt(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a
^2 - sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan
(d*x + c) + a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^
2*a*sqrt(abs(a)) - 35840*(sqrt(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a +
 1)*abs(a)^(3/2)/a^2 - sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x +
 c)/sqrt(((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2)
 + 1)*abs(a)/a^2)^2*abs(a)^(3/2) + 33024*a*sqrt(abs(a)) + 33024*I*abs(a)^(3
/2))/(((sqrt(2)*sqrt(I*a*tan(d*x + c) + a)*(-I*abs(a)/a + 1)*abs(a)^(3/2)/a
^2 - sqrt(-2*(I*a*tan(d*x + c) + a)*a + 2*a^2)*(tan(d*x + c)/sqrt(((I*a*tan
(d*x + c) + a)^2 - 2*(I*a*tan(d*x + c) + a)*a + a^2)/a^2) + 1)*abs(a)/a^2)^
2 - 4*I)^5*a^4*d)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\tan\left(c+d\,x\right)^{5/2}\,{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(5/2)),x)
[Out] int(1/(tan(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(5/2)), x)
```

3.233 $\int \frac{\tan^{\frac{10}{3}}(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=343

$$\frac{7\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{12ad}-\frac{7\text{ArcTan}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{12ad}-\frac{5i\text{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)}{2\sqrt{3}\,ad}-\frac{7\text{Arc}}{}$$

```
[Out] -7/12*arctan(-3^(1/2)+2*tan(d*x+c)^(1/3))/a/d-7/12*arctan(3^(1/2)+2*tan(d*x
+c)^(1/3))/a/d-7/6*arctan(tan(d*x+c)^(1/3))/a/d-5/6*I*ln(1+tan(d*x+c)^(2/3)
)/a/d+5/12*I*ln(1-tan(d*x+c)^(2/3)+tan(d*x+c)^(4/3))/a/d-5/6*I*arctan(1/3*(
1-2*tan(d*x+c)^(2/3))*3^(1/2))/a/d*3^(1/2)+7/24*ln(1-3^(1/2)*tan(d*x+c)^(1/
3)+tan(d*x+c)^(2/3))/a/d*3^(1/2)-7/24*ln(1+3^(1/2)*tan(d*x+c)^(1/3)+tan(d*x
+c)^(2/3))/a/d*3^(1/2)+7/2*tan(d*x+c)^(1/3)/a/d-5/4*I*tan(d*x+c)^(4/3)/a/d-
1/2*tan(d*x+c)^(7/3)/d/(a+I*a*tan(d*x+c))
```

Rubi [A]
time = 0.32, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 14, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3631, 3609, 3619, 3557, 335, 215, 648, 632, 210, 642, 209, 281, 298, 31}

$$-\frac{5i\text{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)}{2\sqrt{3}\,ad}+\frac{7\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{12ad}-\frac{7\text{ArcTan}\left(2\sqrt[3]{\tan(c+dx)}+\sqrt{3}\right)}{12ad}-\frac{7\text{ArcTan}\left(\sqrt[3]{\tan(c+dx)}\right)}{6ad}-\frac{\tan^{\frac{7}{3}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{5i\tan^{\frac{4}{3}}(c+dx)}{4ad}+\frac{7\sqrt[3]{\tan(c+dx)}}{2ad}-\frac{5i\log\left(\tan^{\frac{2}{3}}(c+dx)+1\right)}{6ad}+\frac{7\log\left(\tan^{\frac{2}{3}}(c+dx)-\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{8\sqrt{3}\,ad}-\frac{7\log\left(\tan^{\frac{2}{3}}(c+dx)+\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{8\sqrt{3}\,ad}+\frac{5i\log\left(\tan^{\frac{4}{3}}(c+dx)-\tan^{\frac{2}{3}}(c+dx)+1\right)}{12ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(10/3)/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (7*ArcTan[Sqrt[3] - 2*Tan[c + d*x]^(1/3)])/(12*a*d) - (7*ArcTan[Sqrt[3] + 2
*Tan[c + d*x]^(1/3)])/(12*a*d) - (((5*I)/2)*ArcTan[(1 - 2*Tan[c + d*x]^(2/3
))/Sqrt[3]])/(Sqrt[3]*a*d) - (7*ArcTan[Tan[c + d*x]^(1/3)])/(6*a*d) - (((5*
I)/6)*Log[1 + Tan[c + d*x]^(2/3)])/(a*d) + (7*Log[1 - Sqrt[3]*Tan[c + d*x]^
(1/3) + Tan[c + d*x]^(2/3)])/(8*Sqrt[3]*a*d) - (7*Log[1 + Sqrt[3]*Tan[c + d
*x]^(1/3) + Tan[c + d*x]^(2/3)])/(8*Sqrt[3]*a*d) + (((5*I)/12)*Log[1 - Tan[
c + d*x]^(2/3) + Tan[c + d*x]^(4/3)])/(a*d) + (7*Tan[c + d*x]^(1/3))/(2*a*d
) - (((5*I)/4)*Tan[c + d*x]^(4/3))/(a*d) - Tan[c + d*x]^(7/3)/(2*d*(a + I*a
*Tan[c + d*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3631

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((c + d*Tan[e + f*x])^(n - 1)/(
2*a*f*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a^2), Int[(c + d*Tan[e + f*x])
^(n - 2)*Simp[a*c^2 + a*d^2*(n - 1) - b*c*d*n - d*(a*c*(n - 2) + b*d*n)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\tan^{\frac{10}{3}}(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{\tan^{\frac{7}{3}}(c+dx)}{2d(a+ia\tan(c+dx))} + \frac{\int \tan^{\frac{4}{3}}(c+dx)\left(\frac{7a}{3}-\frac{10}{3}ia\tan(c+dx)\right)\,dx}{2a^2}$$

$$= -\frac{5i\tan^{\frac{4}{3}}(c+dx)}{4ad} - \frac{\tan^{\frac{7}{3}}(c+dx)}{2d(a+ia\tan(c+dx))} + \frac{\int \sqrt[3]{\tan(c+dx)}\left(\frac{10ia}{3}+\frac{7}{3}a\tan(c\right.}{2a^2}$$

$$= \frac{7\sqrt[3]{\tan(c+dx)}}{2ad} - \frac{5i\tan^{\frac{4}{3}}(c+dx)}{4ad} - \frac{\tan^{\frac{7}{3}}(c+dx)}{2d(a+ia\tan(c+dx))} + \frac{\int \frac{-\frac{7a}{3}+\frac{10}{3}ia\tan(c}{\tan^{\frac{2}{3}}(c+dx)}}{2a^2}$$

$$= \frac{7\sqrt[3]{\tan(c+dx)}}{2ad} - \frac{5i\tan^{\frac{4}{3}}(c+dx)}{4ad} - \frac{\tan^{\frac{7}{3}}(c+dx)}{2d(a+ia\tan(c+dx))} + \frac{(5i)\int \sqrt[3]{\tan(c}}{3a}$$

$$= \frac{7\sqrt[3]{\tan(c+dx)}}{2ad} - \frac{5i\tan^{\frac{4}{3}}(c+dx)}{4ad} - \frac{\tan^{\frac{7}{3}}(c+dx)}{2d(a+ia\tan(c+dx))} + \frac{(5i)\mathrm{Subst}\left(\int \right.}{}$$

$$= \frac{7\sqrt[3]{\tan(c+dx)}}{2ad} - \frac{5i\tan^{\frac{4}{3}}(c+dx)}{4ad} - \frac{\tan^{\frac{7}{3}}(c+dx)}{2d(a+ia\tan(c+dx))} + \frac{(5i)\mathrm{Subst}\left(\int \right.}{}$$

$$= \frac{7\sqrt[3]{\tan(c+dx)}}{2ad} - \frac{5i\tan^{\frac{4}{3}}(c+dx)}{4ad} - \frac{\tan^{\frac{7}{3}}(c+dx)}{2d(a+ia\tan(c+dx))} + \frac{(5i)\mathrm{Subst}\left(\int \right.}{}$$

$$= -\frac{7\tan^{-1}\left(\sqrt[3]{\tan(c+dx)}\right)}{6ad} + \frac{7\sqrt[3]{\tan(c+dx)}}{2ad} - \frac{5i\tan^{\frac{4}{3}}(c+dx)}{4ad} - \frac{\tan^{\frac{7}{3}}(}{2d(a+ia}$$

$$= -\frac{7\tan^{-1}\left(\sqrt[3]{\tan(c+dx)}\right)}{6ad} - \frac{5i\log\left(1+\tan^{\frac{2}{3}}(c+dx)\right)}{6ad} + \frac{7\log\left(1-\sqrt{3}\sqrt[3]{\tan\right.}{}$$

$$= \frac{7\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{12ad} - \frac{7\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{12ad} - \frac{7\tan}{}$$

$$= \frac{7\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{12ad} - \frac{7\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{12ad} - \frac{5i\tan}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 1.83, size = 181, normalized size = 0.53

$$-\frac{i\sec^2(c+dx)\left(34+22\cos(2(c+dx))+3\,2^{2/3}\left(1+e^{2i(c+dx)}\right)^{4/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{1}{2}\left(1-e^{2i(c+dx)}\right)\right)-34\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};-\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)\left(1+\cos(2(c+dx))+i\sin(2(c+dx))\right)+18i\sin(2(c+dx))\right)\sqrt[3]{\tan(c+dx)}}{16ad(-i+\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(10/3)/(a + I*a*Tan[c + d*x]),x]
```

[Out] ((-1/16*I)*Sec[c + d*x]^2*(34 + 22*Cos[2*(c + d*x)] + 3*2^(2/3)*(1 + E^((2*I)*(c + d*x)))^(4/3)*Hypergeometric2F1[1/3, 1/3, 4/3, (1 - E^((2*I)*(c + d*x)))/2] - 34*Hypergeometric2F1[1/3, 1, 4/3, -((-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))]*(1 + Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) + (18*I)*Sin[2*(c + d*x)])*Tan[c + d*x]^(1/3))/(a*d*(-I + Tan[c + d*x]))

Maple [A]
time = 0.19, size = 211, normalized size = 0.62

method	result
derivativedivides	$\frac{3\left(\tan^{\frac{1}{3}}(dx+c)\right)-\frac{3i\left(\tan^{\frac{4}{3}}(dx+c)\right)}{4}-\frac{17i\ln\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}{12}+\frac{1}{6\left(\tan^{\frac{1}{3}}(dx+c)\right)+6i}+\frac{i\ln\left(i\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)-1\right)}{8}+\cdots}{}$
default	$\frac{3\left(\tan^{\frac{1}{3}}(dx+c)\right)-\frac{3i\left(\tan^{\frac{4}{3}}(dx+c)\right)}{4}-\frac{17i\ln\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}{12}+\frac{1}{6\left(\tan^{\frac{1}{3}}(dx+c)\right)+6i}+\frac{i\ln\left(i\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)-1\right)}{8}+\cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)^(10/3)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d/a*(3*tan(d*x+c)^(1/3)-3/4*I*tan(d*x+c)^(4/3)-17/12*I*ln(tan(d*x+c)^(1/3)+I)+1/6/(tan(d*x+c)^(1/3)+I)+1/8*I*ln(I*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3)-1)+1/4*3^(1/2)*arctanh(1/3*(I+2*tan(d*x+c)^(1/3))*3^(1/2))+1/12*(-2*tan(d*x+c)^(1/3)-2*I)/(-I*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3)-1)+17/24*I*ln(-I*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3)-1)-17/12*3^(1/2)*arctanh(1/3*(-I+2*tan(d*x+c)^(1/3))*3^(1/2))-1/4*I*ln(tan(d*x+c)^(1/3)-I))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^(10/3)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 645 vs. 2(270) = 540.
time = 0.80, size = 645, normalized size = 1.88

$$\frac{3\left(\sqrt{3}\left(ade^{(4idx+4ic)}+ade^{(2idx+2ic)}\right)\sqrt{\frac{1}{a^2d^2}}+ie^{(4idx+4ic)}+ie^{(2idx+2ic)}\right)\log\left(\frac{1}{2}\sqrt{3}ad\sqrt{\frac{1}{a^2d^2}}+\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}+\frac{1}{2}i\right)-3\left(\sqrt{3}\left(ade^{(4idx+4ic)}+ade^{(2idx+2ic)}\right)\sqrt{\frac{1}{a^2d^2}}-ie^{(4idx+4ic)}-ie^{(2idx+2ic)}\right)\log\left(-\frac{1}{2}\sqrt{3}ad\sqrt{\frac{1}{a^2d^2}}+\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}+\frac{1}{2}i\right)-17\left(3\sqrt{\frac{1}{3}}\left(ade^{(4idx+4ic)}+ade^{(2idx+2ic)}\right)\sqrt{\frac{1}{a^2d^2}}-ie^{(4idx+4ic)}-ie^{(2idx+2ic)}\right)\log\left(\frac{3}{2}\sqrt{\frac{1}{3}}ad\sqrt{\frac{1}{a^2d^2}}+\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}-\frac{1}{2}i\right)+17\left(3\sqrt{\frac{1}{3}}\left(ade^{(4idx+4ic)}+ade^{(2idx+2ic)}\right)\sqrt{\frac{1}{a^2d^2}}+ie^{(4idx+4ic)}+ie^{(2idx+2ic)}\right)\log\left(-\frac{3}{2}\sqrt{\frac{1}{3}}ad\sqrt{\frac{1}{a^2d^2}}+\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}-\frac{1}{2}i\right)-34\left(ie^{(4idx+4ic)}+ie^{(2idx+2ic)}\right)\log\left(\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}+i\right)-6\left(ie^{(4idx+4ic)}+ie^{(2idx+2ic)}\right)\log\left(\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}-i\right)+6\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}\left(10e^{(4idx+4ic)}+17e^{(2idx+2ic)}+1\right)}{24\left(ade^{(4idx+4ic)}+ade^{(2idx+2ic)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(10/3)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(3*(sqrt(3)*(a*d*e^(4*I*d*x + 4*I*c) + a*d*e^(2*I*d*x + 2*I*c))*sqrt(1
/(a^2*d^2)) + I*e^(4*I*d*x + 4*I*c) + I*e^(2*I*d*x + 2*I*c))*log(1/2*sqrt(3
)*a*d*sqrt(1/(a^2*d^2)) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c
) + 1))^(1/3) + 1/2*I) - 3*(sqrt(3)*(a*d*e^(4*I*d*x + 4*I*c) + a*d*e^(2*I*d
*x + 2*I*c))*sqrt(1/(a^2*d^2)) - I*e^(4*I*d*x + 4*I*c) - I*e^(2*I*d*x + 2*I
*c))*log(-1/2*sqrt(3)*a*d*sqrt(1/(a^2*d^2)) + ((-I*e^(2*I*d*x + 2*I*c) + I)
/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) + 1/2*I) - 17*(3*sqrt(1/3)*(a*d*e^(4*I*d*
x + 4*I*c) + a*d*e^(2*I*d*x + 2*I*c))*sqrt(1/(a^2*d^2)) - I*e^(4*I*d*x + 4*
I*c) - I*e^(2*I*d*x + 2*I*c))*log(3/2*sqrt(1/3)*a*d*sqrt(1/(a^2*d^2)) + ((-
I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) - 1/2*I) + 17*(
3*sqrt(1/3)*(a*d*e^(4*I*d*x + 4*I*c) + a*d*e^(2*I*d*x + 2*I*c))*sqrt(1/(a^2
*d^2)) + I*e^(4*I*d*x + 4*I*c) + I*e^(2*I*d*x + 2*I*c))*log(-3/2*sqrt(1/3)*
a*d*sqrt(1/(a^2*d^2)) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c)
+ 1))^(1/3) - 1/2*I) - 34*(I*e^(4*I*d*x + 4*I*c) + I*e^(2*I*d*x + 2*I*c))*l
og(((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) + I) - 6*
(I*e^(4*I*d*x + 4*I*c) + I*e^(2*I*d*x + 2*I*c))*log(((-I*e^(2*I*d*x + 2*I*c
) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) - I) + 6*((-I*e^(2*I*d*x + 2*I*c) +
 I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*(10*e^(4*I*d*x + 4*I*c) + 17*e^(2*I*d*
x + 2*I*c) + 1))/(a*d*e^(4*I*d*x + 4*I*c) + a*d*e^(2*I*d*x + 2*I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{\tan^{\frac{10}{3}}(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(10/3)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I*Integral(tan(c + d*x)**(10/3)/(tan(c + d*x) - I), x)/a
```

Giac [A]
time = 0.60, size = 256, normalized size = 0.75

$$\frac{17\sqrt{3}\log\left(-\frac{\sqrt{3}-2\tan(dx+c)^{\frac{1}{3}}+i}{\sqrt{3}+2\tan(dx+c)^{\frac{1}{3}}-i}\right)}{24\,ad}-\frac{\sqrt{3}\log\left(-\frac{\sqrt{3}-2\tan(dx+c)^{\frac{1}{3}}-i}{\sqrt{3}+2\tan(dx+c)^{\frac{1}{3}}+i}\right)}{8\,ad}+\frac{i\log\left(\tan(dx+c)^{\frac{2}{3}}+i\tan(dx+c)^{\frac{1}{3}}-1\right)}{8\,ad}+\frac{17i\log\left(\tan(dx+c)^{\frac{2}{3}}-i\tan(dx+c)^{\frac{1}{3}}-1\right)}{24\,ad}-\frac{17i\log\left(\tan(dx+c)^{\frac{1}{3}}+i\right)}{12\,ad}-\frac{i\log\left(\tan(dx+c)^{\frac{1}{3}}-i\right)}{4\,ad}-\frac{i\tan(dx+c)^{\frac{1}{3}}}{2\,ad(\tan(dx+c)-i)}-\frac{3\left(i\,a^3d^3\tan(dx+c)^{\frac{4}{3}}-4\,a^3d^3\tan(dx+c)^{\frac{1}{3}}\right)}{4\,a^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(10/3)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 17/24*sqrt(3)*log(-(sqrt(3) - 2*tan(d*x + c)^(1/3) + I)/(sqrt(3) + 2*tan(d*
x + c)^(1/3) - I))/(a*d) - 1/8*sqrt(3)*log(-(sqrt(3) - 2*tan(d*x + c)^(1/3)
 - I)/(sqrt(3) + 2*tan(d*x + c)^(1/3) + I))/(a*d) + 1/8*I*log(tan(d*x + c)^
```

```
(2/3) + I*tan(d*x + c)^(1/3) - 1)/(a*d) + 17/24*I*log(tan(d*x + c)^(2/3) -
I*tan(d*x + c)^(1/3) - 1)/(a*d) - 17/12*I*log(tan(d*x + c)^(1/3) + I)/(a*d)
 - 1/4*I*log(tan(d*x + c)^(1/3) - I)/(a*d) - 1/2*I*tan(d*x + c)^(1/3)/(a*d*
(tan(d*x + c) - I)) - 3/4*(I*a^3*d^3*tan(d*x + c)^(4/3) - 4*a^3*d^3*tan(d*x
 + c)^(1/3))/(a^4*d^4)
```

Mupad [B]
time = 5.72, size = 655, normalized size = 1.91

$$\ln\left(\left(a^3 d^3\,703584\mathrm{i}-414720\,a^5 d^5 \tan(c+d\,x)^{1/3}\left(\frac{1\mathrm{i}}{64\,a^3 d^3}\right)^{2/3}\right)\left(\frac{1\mathrm{i}}{64\,a^3 d^3}\right)^{1/3}+a^2 d^2 \tan(c+d\,x)^{1/3}\,182376\mathrm{i}\right)\left(\frac{1\mathrm{i}}{64\,a^3 d^3}\right)^{1/3}+\ln\left(\left(a^3 d^3\,703584\mathrm{i}-414720\,a^5 d^5 \tan(c+d\,x)^{1/3}\left(\frac{4913\mathrm{i}}{1728\,a^3 d^3}\right)^{2/3}\right)\left(\frac{4913\mathrm{i}}{1728\,a^3 d^3}\right)^{1/3}+a^2 d^2 \tan(c+d\,x)^{1/3}\,182376\mathrm{i}\right)\left(\frac{4913\mathrm{i}}{1728\,a^3 d^3}\right)^{1/3}+\frac{3\tan(c+d\,x)^{1/3}}{a\,d}-\frac{\tan(c+d\,x)^{4/3}\,3\mathrm{i}}{4\,a\,d}+\ldots$$ [illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(10/3)/(a + a*tan(c + d*x)*1i),x)
```

```
[Out] log((a^3*d^3*703584i - 414720*a^5*d^5*tan(c + d*x)^(1/3)*(1i/(64*a^3*d^3))^
(2/3))*(1i/(64*a^3*d^3))^(1/3) + a^2*d^2*tan(c + d*x)^(1/3)*182376i)*(1i/(6
4*a^3*d^3))^(1/3) + log((a^3*d^3*703584i - 414720*a^5*d^5*tan(c + d*x)^(1/3
)*(4913i/(1728*a^3*d^3))^(2/3))*(4913i/(1728*a^3*d^3))^(1/3) + a^2*d^2*tan(
c + d*x)^(1/3)*182376i)*(4913i/(1728*a^3*d^3))^(1/3) + (3*tan(c + d*x)^(1/3
))/(a*d) - (tan(c + d*x)^(4/3)*3i)/(4*a*d) + (log(((3^(1/2)*1i - 1)*(a^3*d^
3*703584i - 103680*a^5*d^5*tan(c + d*x)^(1/3)*(3^(1/2)*1i - 1)^2*(1i/(64*a^
3*d^3))^(2/3))*(1i/(64*a^3*d^3))^(1/3))/2 + a^2*d^2*tan(c + d*x)^(1/3)*1823
76i)*(3^(1/2)*1i - 1)*(1i/(64*a^3*d^3))^(1/3))/2 - (log(((3^(1/2)*1i + 1)*(
a^3*d^3*703584i - 103680*a^5*d^5*tan(c + d*x)^(1/3)*(3^(1/2)*1i + 1)^2*(1i/
(64*a^3*d^3))^(2/3))*(1i/(64*a^3*d^3))^(1/3))/2 - a^2*d^2*tan(c + d*x)^(1/3
)*182376i)*(3^(1/2)*1i + 1)*(1i/(64*a^3*d^3))^(1/3))/2 + (log(((3^(1/2)*1i
- 1)*(a^3*d^3*703584i - 103680*a^5*d^5*tan(c + d*x)^(1/3)*(3^(1/2)*1i - 1)^
2*(4913i/(1728*a^3*d^3))^(2/3))*(4913i/(1728*a^3*d^3))^(1/3))/2 + a^2*d^2*t
an(c + d*x)^(1/3)*182376i)*(3^(1/2)*1i - 1)*(4913i/(1728*a^3*d^3))^(1/3))/2
 - (log(((3^(1/2)*1i + 1)*(a^3*d^3*703584i - 103680*a^5*d^5*tan(c + d*x)^(1
/3)*(3^(1/2)*1i + 1)^2*(4913i/(1728*a^3*d^3))^(2/3))*(4913i/(1728*a^3*d^3))
^(1/3))/2 - a^2*d^2*tan(c + d*x)^(1/3)*182376i)*(3^(1/2)*1i + 1)*(4913i/(17
28*a^3*d^3))^(1/3))/2 + tan(c + d*x)^(1/3)/(2*a*d*(tan(c + d*x)*1i + 1))
```

3.234 $\int \frac{\tan^{\frac{8}{3}}(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=319

$$-\frac{5\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{12ad}+\frac{5\text{ArcTan}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{12ad}-\frac{2i\text{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)}{\sqrt{3}\,ad}+\frac{5\text{A}}{}$$

```
[Out] 5/12*arctan(-3^(1/2)+2*tan(d*x+c)^(1/3))/a/d+5/12*arctan(3^(1/2)+2*tan(d*x+
c)^(1/3))/a/d+5/6*arctan(tan(d*x+c)^(1/3))/a/d+2/3*I*ln(1+tan(d*x+c)^(2/3))
/a/d-1/3*I*ln(1-tan(d*x+c)^(2/3)+tan(d*x+c)^(4/3))/a/d-2/3*I*arctan(1/3*(1-
2*tan(d*x+c)^(2/3))*3^(1/2))/a/d*3^(1/2)+5/24*ln(1-3^(1/2)*tan(d*x+c)^(1/3)
+tan(d*x+c)^(2/3))/a/d*3^(1/2)-5/24*ln(1+3^(1/2)*tan(d*x+c)^(1/3)+tan(d*x+c
)^(2/3))/a/d*3^(1/2)-2*I*tan(d*x+c)^(2/3)/a/d-1/2*tan(d*x+c)^(5/3)/d/(a+I*a
*tan(d*x+c))
```

Rubi [A]
time = 0.33, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 14, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3631, 3609, 3619, 3557, 335, 281, 206, 31, 648, 632, 210, 642, 301, 209}

$$-\frac{2i\text{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)}{\sqrt{3}\,ad}-\frac{5\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{12ad}+\frac{5\text{ArcTan}\left(2\sqrt[3]{\tan(c+dx)}+\sqrt{3}\right)}{12ad}+\frac{5\text{ArcTan}\left(\sqrt[3]{\tan(c+dx)}\right)}{6ad}-\frac{\tan^{\frac{5}{3}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{2i\tan^{\frac{2}{3}}(c+dx)}{ad}+\frac{2i\log\left(\tan^{\frac{2}{3}}(c+dx)+1\right)}{3ad}+\frac{5\log\left(\tan^{\frac{2}{3}}(c+dx)-\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{8\sqrt{3}\,ad}-\frac{5\log\left(\tan^{\frac{2}{3}}(c+dx)+\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{8\sqrt{3}\,ad}-\frac{i\log\left(\tan^{\frac{4}{3}}(c+dx)-\tan^{\frac{2}{3}}(c+dx)+1\right)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(8/3)/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (-5*ArcTan[Sqrt[3] - 2*Tan[c + d*x]^(1/3)])/(12*a*d) + (5*ArcTan[Sqrt[3] +
2*Tan[c + d*x]^(1/3)])/(12*a*d) - ((2*I)*ArcTan[(1 - 2*Tan[c + d*x]^(2/3))/
Sqrt[3]])/(Sqrt[3]*a*d) + (5*ArcTan[Tan[c + d*x]^(1/3)])/(6*a*d) + (((2*I)/
3)*Log[1 + Tan[c + d*x]^(2/3)])/(a*d) + (5*Log[1 - Sqrt[3]*Tan[c + d*x]^(1/
3) + Tan[c + d*x]^(2/3)])/(8*Sqrt[3]*a*d) - (5*Log[1 + Sqrt[3]*Tan[c + d*x]
^(1/3) + Tan[c + d*x]^(2/3)])/(8*Sqrt[3]*a*d) - ((I/3)*Log[1 - Tan[c + d*x]
^(2/3) + Tan[c + d*x]^(4/3)])/(a*d) - ((2*I)*Tan[c + d*x]^(2/3))/(a*d) - Ta
n[c + d*x]^(5/3)/(2*d*(a + I*a*Tan[c + d*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3631

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((c + d*Tan[e + f*x])^(n - 1)/(
2*a*f*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a^2), Int[(c + d*Tan[e + f*x])
^(n - 2)*Simp[a*c^2 + a*d^2*(n - 1) - b*c*d*n - d*(a*c*(n - 2) + b*d*n)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\tan^{\frac{8}{3}}(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{\tan^{\frac{5}{3}}(c+dx)}{2d(a+ia\tan(c+dx))} + \frac{\int \tan^{\frac{2}{3}}(c+dx)\left(\frac{5a}{3}-\frac{8}{3}ia\tan(c+dx)\right)\,dx}{2a^2}$$

$$= -\frac{2i\tan^{\frac{2}{3}}(c+dx)}{ad} - \frac{\tan^{\frac{5}{3}}(c+dx)}{2d(a+ia\tan(c+dx))} + \frac{\int \frac{\frac{8ia}{3}+\frac{5}{3}a\tan(c+dx)}{\sqrt[3]{\tan(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{2i\tan^{\frac{2}{3}}(c+dx)}{ad} - \frac{\tan^{\frac{5}{3}}(c+dx)}{2d(a+ia\tan(c+dx))} + \frac{(4i)\int \frac{1}{\sqrt[3]{\tan(c+dx)}}\,dx}{3a} + \frac{5\int \tan$$

$$= -\frac{2i\tan^{\frac{2}{3}}(c+dx)}{ad} - \frac{\tan^{\frac{5}{3}}(c+dx)}{2d(a+ia\tan(c+dx))} + \frac{(4i)\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\,(1+x^2)}\,dx, x, \tan(\right.}{3ad}$$

$$= -\frac{2i\tan^{\frac{2}{3}}(c+dx)}{ad} - \frac{\tan^{\frac{5}{3}}(c+dx)}{2d(a+ia\tan(c+dx))} + \frac{(4i)\text{Subst}\left(\int \frac{x}{1+x^6}\,dx, x, \sqrt[3]{\tan(c+\right.}{ad}$$

$$= -\frac{2i\tan^{\frac{2}{3}}(c+dx)}{ad} - \frac{\tan^{\frac{5}{3}}(c+dx)}{2d(a+ia\tan(c+dx))} + \frac{(2i)\text{Subst}\left(\int \frac{1}{1+x^3}\,dx, x, \tan^{\frac{2}{3}}(c+\right.}{ad}$$

$$= \frac{5\tan^{-1}\left(\sqrt[3]{\tan(c+dx)}\right)}{6ad} - \frac{2i\tan^{\frac{2}{3}}(c+dx)}{ad} - \frac{\tan^{\frac{5}{3}}(c+dx)}{2d(a+ia\tan(c+dx))} + \frac{(2i)\text{Sul}}{}$$

$$= \frac{5\tan^{-1}\left(\sqrt[3]{\tan(c+dx)}\right)}{6ad} + \frac{2i\log\left(1+\tan^{\frac{2}{3}}(c+dx)\right)}{3ad} + \frac{5\log\left(1-\sqrt{3}\,\sqrt[3]{\tan(c\right.}{8\sqrt{}}$$

$$= -\frac{5\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{12ad} + \frac{5\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{12ad} + \frac{5\tan}{}$$

$$= -\frac{5\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{12ad} + \frac{5\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{12ad} - \frac{2i\tan}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.27, size = 163, normalized size = 0.51

$$\frac{ie^{-2i(c+dx)}\left(-4-28e^{2i(c+dx)}+3\sqrt[3]{2}\,e^{2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{1}{2}\left(1-e^{2i(c+dx)}\right)\right)+26e^{2i(c+dx)}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)\right)\tan^{\frac{2}{3}}(c+dx)}{16ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(8/3)/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((I/16)*(-4 - 28*E^((2*I)*(c + d*x)) + 3*2^(1/3)*E^((2*I)*(c + d*x))*(1 + E
^((2*I)*(c + d*x)))^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, (1 - E^((2*I)*(c
 + d*x)))/2] + 26*E^((2*I)*(c + d*x))*Hypergeometric2F1[2/3, 1, 5/3, -((-1
+ E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))])*Tan[c + d*x]^(2/3))/(a*
d*E^((2*I)*(c + d*x)))
```

Maple [A]
time = 0.16, size = 201, normalized size = 0.63

method	result
derivativedivides	$\frac{-\frac{3i\left(\tan^{\frac{2}{3}}(dx+c)\right)}{2}+\frac{13i\ln\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}{12}-\frac{1}{6\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}-\frac{i\ln\left(i\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)-1\right)}{8}+\frac{\sqrt{3}\ \operatorname{arctanh}\left(\left(\right.\right.}{}}{}$
default	$\frac{-\frac{3i\left(\tan^{\frac{2}{3}}(dx+c)\right)}{2}+\frac{13i\ln\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}{12}-\frac{1}{6\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}-\frac{i\ln\left(i\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)-1\right)}{8}+\frac{\sqrt{3}\ \operatorname{arctanh}\left(\left(\right.\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(8/3)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-3/2*I*tan(d*x+c)^(2/3)+13/12*I*ln(tan(d*x+c)^(1/3)+I)-1/6/(tan(d*x+
c)^(1/3)+I)-1/8*I*ln(I*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3)-1)+1/4*3^(1/2)*arc
tanh(1/3*(I+2*tan(d*x+c)^(1/3))*3^(1/2))-1/12*(4*tan(d*x+c)^(1/3)-2*I)/(-I*
tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3)-1)-13/24*I*ln(-I*tan(d*x+c)^(1/3)+tan(d*x
+c)^(2/3)-1)-13/12*3^(1/2)*arctanh(1/3*(-I+2*tan(d*x+c)^(1/3))*3^(1/2))+1/4
*I*ln(tan(d*x+c)^(1/3)-I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(8/3)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.69, size = 494, normalized size = 1.55

$$\frac{\left(3\left(\sqrt{3}\,ad\sqrt{\frac{1}{a^2d^2}}\,e^{(2i\,dx+2i\,c)}-i\,e^{(2i\,dx+2i\,c)}\right)\log\left(\frac{1}{2}\sqrt{3}\,ad\sqrt{\frac{1}{a^2d^2}}+\left(\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}+\frac{1}{2}i\right)-3\left(\sqrt{3}\,ad\sqrt{\frac{1}{a^2d^2}}\,e^{(2i\,dx+2i\,c)}+i\,e^{(2i\,dx+2i\,c)}\right)\log\left(-\frac{1}{2}\sqrt{3}\,ad\sqrt{\frac{1}{a^2d^2}}+\left(\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}+\frac{1}{2}i\right)-13\left(3\sqrt{\frac{1}{3}}\,ad\sqrt{\frac{1}{a^2d^2}}\,e^{(2i\,dx+2i\,c)}+i\,e^{(2i\,dx+2i\,c)}\right)\log\left(\frac{3}{2}\sqrt{\frac{1}{3}}\,ad\sqrt{\frac{1}{a^2d^2}}+\left(\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}-\frac{1}{2}i\right)+13\left(3\sqrt{\frac{1}{3}}\,ad\sqrt{\frac{1}{a^2d^2}}\,e^{(2i\,dx+2i\,c)}-i\,e^{(2i\,dx+2i\,c)}\right)\log\left(-\frac{3}{2}\sqrt{\frac{1}{3}}\,ad\sqrt{\frac{1}{a^2d^2}}+\left(\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}-\frac{1}{2}i\right)+26i\,e^{(2i\,dx+2i\,c)}\log\left(\left(\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}+i\right)+6i\,e^{(2i\,dx+2i\,c)}\log\left(\left(\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}-i\right)-6\left(\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{2}{3}}\left(7i\,e^{(2i\,dx+2i\,c)}+i\right)\right)e^{(-2i\,dx-2i\,c)}}{24\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(8/3)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(3*(sqrt(3)*a*d*sqrt(1/(a^2*d^2))*e^(2*I*d*x + 2*I*c) - I*e^(2*I*d*x +
 2*I*c))*log(1/2*sqrt(3)*a*d*sqrt(1/(a^2*d^2)) + ((-I*e^(2*I*d*x + 2*I*c) +
 I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) + 1/2*I) - 3*(sqrt(3)*a*d*sqrt(1/(a^2*
d^2))*e^(2*I*d*x + 2*I*c) + I*e^(2*I*d*x + 2*I*c))*log(-1/2*sqrt(3)*a*d*sqr
t(1/(a^2*d^2)) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(
1/3) + 1/2*I) - 13*(3*sqrt(1/3)*a*d*sqrt(1/(a^2*d^2))*e^(2*I*d*x + 2*I*c) +
 I*e^(2*I*d*x + 2*I*c))*log(3/2*sqrt(1/3)*a*d*sqrt(1/(a^2*d^2)) + ((-I*e^(2
*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) - 1/2*I) + 13*(3*sqrt
(1/3)*a*d*sqrt(1/(a^2*d^2))*e^(2*I*d*x + 2*I*c) - I*e^(2*I*d*x + 2*I*c))*lo
g(-3/2*sqrt(1/3)*a*d*sqrt(1/(a^2*d^2)) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(
2*I*d*x + 2*I*c) + 1))^(1/3) - 1/2*I) + 26*I*e^(2*I*d*x + 2*I*c)*log(((-I*e
^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) + I) + 6*I*e^(2*I*
d*x + 2*I*c)*log(((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(
1/3) - I) - 6*((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(2/3
)*(7*I*e^(2*I*d*x + 2*I*c) + I))*e^(-2*I*d*x - 2*I*c)/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int \frac{\tan^{\frac{8}{3}}(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(8/3)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I*Integral(tan(c + d*x)**(8/3)/(tan(c + d*x) - I), x)/a
```

Giac [A]
time = 0.71, size = 231, normalized size = 0.72

$$\frac{13\sqrt{3}\log\left(-\frac{\sqrt{3}-2\tan(dx+c)^{\frac{1}{3}}+i}{\sqrt{3}+2\tan(dx+c)^{\frac{1}{3}}-i}\right)}{24\,ad}-\frac{\sqrt{3}\log\left(-\frac{\sqrt{3}-2\tan(dx+c)^{\frac{1}{3}}-i}{\sqrt{3}+2\tan(dx+c)^{\frac{1}{3}}+i}\right)}{8\,ad}-\frac{i\log\left(\tan(dx+c)^{\frac{2}{3}}+i\tan(dx+c)^{\frac{1}{3}}-1\right)}{8\,ad}-\frac{13i\log\left(\tan(dx+c)^{\frac{2}{3}}-i\tan(dx+c)^{\frac{1}{3}}-1\right)}{24\,ad}+\frac{13i\log\left(\tan(dx+c)^{\frac{1}{3}}+i\right)}{12\,ad}+\frac{i\log\left(\tan(dx+c)^{\frac{1}{3}}-i\right)}{4\,ad}-\frac{3i\tan(dx+c)^{\frac{2}{3}}}{2\,ad}-\frac{\tan(dx+c)^{\frac{2}{3}}}{2\,ad(\tan(dx+c)-i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(8/3)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 13/24*sqrt(3)*log(-(sqrt(3) - 2*tan(d*x + c)^(1/3) + I)/(sqrt(3) + 2*tan(d*
x + c)^(1/3) - I))/(a*d) - 1/8*sqrt(3)*log(-(sqrt(3) - 2*tan(d*x + c)^(1/3)
 - I)/(sqrt(3) + 2*tan(d*x + c)^(1/3) + I))/(a*d) - 1/8*I*log(tan(d*x + c)^
(2/3) + I*tan(d*x + c)^(1/3) - 1)/(a*d) - 13/24*I*log(tan(d*x + c)^(2/3) -
I*tan(d*x + c)^(1/3) - 1)/(a*d) + 13/12*I*log(tan(d*x + c)^(1/3) + I)/(a*d)
 + 1/4*I*log(tan(d*x + c)^(1/3) - I)/(a*d) - 3/2*I*tan(d*x + c)^(2/3)/(a*d)
 - 1/2*tan(d*x + c)^(2/3)/(a*d*(tan(d*x + c) - I))
```

Mupad [B]
time = 5.44, size = 616, normalized size = 1.93

$$\ln\left(\left(a^3d^3\,312480\mathrm{i}-a^4d^4\tan(c+dx)^{1/3}\left(-\frac{1\mathrm{i}}{64a^3d^3}\right)^{1/3}307584\mathrm{i}\right)\left(-\frac{1\mathrm{i}}{64a^3d^3}\right)^{2/3}+24336\,a\,d\tan(c+dx)^{1/3}\right)\left(-\frac{1\mathrm{i}}{64a^3d^3}\right)^{1/3}+\ln\left(\left(a^3d^3\,312480\mathrm{i}-a^4d^4\tan(c+dx)^{1/3}\left(-\frac{2197\mathrm{i}}{1728a^3d^3}\right)^{1/3}307584\mathrm{i}\right)\left(-\frac{2197\mathrm{i}}{1728a^3d^3}\right)^{2/3}+24336\,a\,d\tan(c+dx)^{1/3}\right)\left(-\frac{2197\mathrm{i}}{1728a^3d^3}\right)^{1/3}-\frac{\tan(c+dx)^{2/3}\,3\mathrm{i}}{2ad}+\frac{\ln\left(\frac{\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2\left(a^3d^3\,312480\mathrm{i}-a^4d^4\tan(c+dx)^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)\left(-\frac{1\mathrm{i}}{64a^3d^3}\right)^{1/3}153792\mathrm{i}\right)\left(-\frac{1\mathrm{i}}{64a^3d^3}\right)^{2/3}}{4}+24336\,a\,d\tan(c+dx)^{1/3}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)\left(-\frac{1\mathrm{i}}{64a^3d^3}\right)^{1/3}}{2}-\frac{\ln\left(\frac{\left(1+\sqrt{3}\,1\mathrm{i}\right)^2\left(a^3d^3\,312480\mathrm{i}+a^4d^4\tan(c+dx)^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)\left(-\frac{1\mathrm{i}}{64a^3d^3}\right)^{1/3}153792\mathrm{i}\right)\left(-\frac{1\mathrm{i}}{64a^3d^3}\right)^{2/3}}{4}+24336\,a\,d\tan(c+dx)^{1/3}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)\left(-\frac{1\mathrm{i}}{64a^3d^3}\right)^{1/3}}{2}+\frac{\ln\left(\frac{\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2\left(a^3d^3\,312480\mathrm{i}-a^4d^4\tan(c+dx)^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)\left(-\frac{2197\mathrm{i}}{1728a^3d^3}\right)^{1/3}153792\mathrm{i}\right)\left(-\frac{2197\mathrm{i}}{1728a^3d^3}\right)^{2/3}}{4}+24336\,a\,d\tan(c+dx)^{1/3}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)\left(-\frac{2197\mathrm{i}}{1728a^3d^3}\right)^{1/3}}{2}-\frac{\ln\left(\frac{\left(1+\sqrt{3}\,1\mathrm{i}\right)^2\left(a^3d^3\,312480\mathrm{i}+a^4d^4\tan(c+dx)^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)\left(-\frac{2197\mathrm{i}}{1728a^3d^3}\right)^{1/3}153792\mathrm{i}\right)\left(-\frac{2197\mathrm{i}}{1728a^3d^3}\right)^{2/3}}{4}+24336\,a\,d\tan(c+dx)^{1/3}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)\left(-\frac{2197\mathrm{i}}{1728a^3d^3}\right)^{1/3}}{2}-\frac{\tan(c+dx)^{2/3}\,1\mathrm{i}}{2ad\left(\tan(c+dx)\,1\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(8/3)/(a + a*tan(c + d*x)*1i),x)
```

```
[Out] log((a^3*d^3*312480i - a^4*d^4*tan(c + d*x)^(1/3)*(-1i/(64*a^3*d^3))^(1/3)*
307584i)*(-1i/(64*a^3*d^3))^(2/3) + 24336*a*d*tan(c + d*x)^(1/3))*(-1i/(64*
a^3*d^3))^(1/3) + log((a^3*d^3*312480i - a^4*d^4*tan(c + d*x)^(1/3)*(-2197i
/(1728*a^3*d^3))^(1/3)*307584i)*(-2197i/(1728*a^3*d^3))^(2/3) + 24336*a*d*t
an(c + d*x)^(1/3))*(-2197i/(1728*a^3*d^3))^(1/3) - (tan(c + d*x)^(2/3)*3i)/
(2*a*d) + (log(((3^(1/2)*1i - 1)^2*(a^3*d^3*312480i - a^4*d^4*tan(c + d*x)^
(1/3)*(3^(1/2)*1i - 1)*(-1i/(64*a^3*d^3))^(1/3)*153792i)*(-1i/(64*a^3*d^3))
^(2/3))/4 + 24336*a*d*tan(c + d*x)^(1/3))*(3^(1/2)*1i - 1)*(-1i/(64*a^3*d^3
))^(1/3))/2 - (log(((3^(1/2)*1i + 1)^2*(a^3*d^3*312480i + a^4*d^4*tan(c + d
*x)^(1/3)*(3^(1/2)*1i + 1)*(-1i/(64*a^3*d^3))^(1/3)*153792i)*(-1i/(64*a^3*d
^3))^(2/3))/4 + 24336*a*d*tan(c + d*x)^(1/3))*(3^(1/2)*1i + 1)*(-1i/(64*a^3
*d^3))^(1/3))/2 + (log(((3^(1/2)*1i - 1)^2*(a^3*d^3*312480i - a^4*d^4*tan(c
 + d*x)^(1/3)*(3^(1/2)*1i - 1)*(-2197i/(1728*a^3*d^3))^(1/3)*153792i)*(-219
7i/(1728*a^3*d^3))^(2/3))/4 + 24336*a*d*tan(c + d*x)^(1/3))*(3^(1/2)*1i - 1
)*(-2197i/(1728*a^3*d^3))^(1/3))/2 - (log(((3^(1/2)*1i + 1)^2*(a^3*d^3*3124
80i + a^4*d^4*tan(c + d*x)^(1/3)*(3^(1/2)*1i + 1)*(-2197i/(1728*a^3*d^3))^(
1/3)*153792i)*(-2197i/(1728*a^3*d^3))^(2/3))/4 + 24336*a*d*tan(c + d*x)^(1/
3))*(3^(1/2)*1i + 1)*(-2197i/(1728*a^3*d^3))^(1/3))/2 - (tan(c + d*x)^(2/3)
*1i)/(2*a*d*(tan(c + d*x)*1i + 1))
```

3.235 $\int \frac{\tan^{\frac{4}{3}}(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=299

$$-\frac{\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{12ad}+\frac{\text{ArcTan}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{12ad}+\frac{i\text{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)}{\sqrt{3}\,ad}+\frac{\text{ArcTan}}{}$$

```
[Out] 1/12*arctan(-3^(1/2)+2*tan(d*x+c)^(1/3))/a/d+1/12*arctan(3^(1/2)+2*tan(d*x+
c)^(1/3))/a/d+1/6*arctan(tan(d*x+c)^(1/3))/a/d+1/3*I*ln(1+tan(d*x+c)^(2/3))
/a/d-1/6*I*ln(1-tan(d*x+c)^(2/3)+tan(d*x+c)^(4/3))/a/d+1/3*I*arctan(1/3*(1-
2*tan(d*x+c)^(2/3))*3^(1/2))/a/d*3^(1/2)-1/24*ln(1-3^(1/2)*tan(d*x+c)^(1/3)
+tan(d*x+c)^(2/3))/a/d*3^(1/2)+1/24*ln(1+3^(1/2)*tan(d*x+c)^(1/3)+tan(d*x+c
)^(2/3))/a/d*3^(1/2)-1/2*tan(d*x+c)^(1/3)/d/(a+I*a*tan(d*x+c))
```

Rubi [A]
time = 0.26, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 13, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3631, 3619, 3557, 335, 215, 648, 632, 210, 642, 209, 281, 298, 31}

$$\frac{i\text{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)}{\sqrt{3}\,ad}-\frac{\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{12ad}+\frac{\text{ArcTan}\left(2\sqrt[3]{\tan(c+dx)}+\sqrt{3}\right)}{12ad}+\frac{\text{ArcTan}\left(\sqrt[3]{\tan(c+dx)}\right)}{6ad}-\frac{\sqrt[3]{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}+\frac{i\log\left(\tan^{\frac{2}{3}}(c+dx)+1\right)}{3ad}-\frac{\log\left(\tan^{\frac{2}{3}}(c+dx)-\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{8\sqrt{3}\,ad}+\frac{\log\left(\tan^{\frac{2}{3}}(c+dx)+\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{8\sqrt{3}\,ad}-\frac{i\log\left(\tan^{\frac{4}{3}}(c+dx)-\tan^{\frac{2}{3}}(c+dx)+1\right)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(4/3)/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] -1/12*ArcTan[Sqrt[3] - 2*Tan[c + d*x]^(1/3)]/(a*d) + ArcTan[Sqrt[3] + 2*Tan
[c + d*x]^(1/3)]/(12*a*d) + (I*ArcTan[(1 - 2*Tan[c + d*x]^(2/3))/Sqrt[3]])/
(Sqrt[3]*a*d) + ArcTan[Tan[c + d*x]^(1/3)]/(6*a*d) + ((I/3)*Log[1 + Tan[c +
 d*x]^(2/3)])/(a*d) - Log[1 - Sqrt[3]*Tan[c + d*x]^(1/3) + Tan[c + d*x]^(2/
3)]/(8*Sqrt[3]*a*d) + Log[1 + Sqrt[3]*Tan[c + d*x]^(1/3) + Tan[c + d*x]^(2/
3)]/(8*Sqrt[3]*a*d) - ((I/6)*Log[1 - Tan[c + d*x]^(2/3) + Tan[c + d*x]^(4/3
)])/(a*d) - Tan[c + d*x]^(1/3)/(2*d*(a + I*a*Tan[c + d*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3631

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((c + d*Tan[e + f*x])^(n - 1)/(
2*a*f*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a^2), Int[(c + d*Tan[e + f*x])
^(n - 2)*Simp[a*c^2 + a*d^2*(n - 1) - b*c*d*n - d*(a*c*(n - 2) + b*d*n)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\tan^{\frac{4}{3}}(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{\sqrt[3]{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}+\frac{\int \frac{\frac{a}{3}-\frac{4}{3}ia\tan(c+dx)}{\tan^{\frac{2}{3}}(c+dx)}\,dx}{2a^2}$$

$$= -\frac{\sqrt[3]{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}-\frac{(2i)\int \sqrt[3]{\tan(c+dx)}\,dx}{3a}+\frac{\int \frac{1}{\tan^{\frac{2}{3}}(c+dx)}\,dx}{6a}$$

$$= -\frac{\sqrt[3]{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}-\frac{(2i)\text{Subst}\left(\int \frac{\sqrt[3]{x}}{1+x^2}\,dx,x,\tan(c+dx)\right)}{3ad}+\frac{\text{Subst}\Big(\int \frac{}{x}$$

$$= -\frac{\sqrt[3]{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}-\frac{(2i)\text{Subst}\left(\int \frac{x^3}{1+x^6}\,dx,x,\sqrt[3]{\tan(c+dx)}\right)}{ad}+\frac{\text{Subst}\Big(}{}$$

$$= -\frac{\sqrt[3]{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}-\frac{i\text{Subst}\left(\int \frac{x}{1+x^3}\,dx,x,\tan^{\frac{2}{3}}(c+dx)\right)}{ad}+\frac{\text{Subst}\Big(\int \frac{1}{1+x}}{}$$

$$= \frac{\tan^{-1}\left(\sqrt[3]{\tan(c+dx)}\right)}{6ad}-\frac{\sqrt[3]{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}+\frac{i\text{Subst}\Big(\int \frac{1}{1+x}\,dx,x,\tan^{\frac{2}{3}}(}{3ad}$$

$$= \frac{\tan^{-1}\left(\sqrt[3]{\tan(c+dx)}\right)}{6ad}+\frac{i\log\left(1+\tan^{\frac{2}{3}}(c+dx)\right)}{3ad}-\frac{\log\left(1-\sqrt{3}\sqrt[3]{\tan(c+}\right.}{8\sqrt{3}}$$

$$= -\frac{\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{12ad}+\frac{\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{12ad}+\frac{\tan^{-1}}{}$$

$$= -\frac{\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{12ad}+\frac{\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{12ad}+\frac{i\tan^{-}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.17, size = 162, normalized size = 0.54

$$-\frac{e^{-2i(c+dx)}\left(3\,2^{2/3}e^{2i(c+dx)}\sqrt[3]{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{1}{2}\left(1-e^{2i(c+dx)}\right)\right)+2\left(1+e^{2i(c+dx)}-5e^{2i(c+dx)}\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};-\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)\right)\right)\sqrt[3]{\tan(c+dx)}}{8ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(4/3)/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] -1/8*((3*2^(2/3)*E^((2*I)*(c + d*x))*(1 + E^((2*I)*(c + d*x)))^(1/3)*Hyperg
eometric2F1[1/3, 1/3, 4/3, (1 - E^((2*I)*(c + d*x)))/2] + 2*(1 + E^((2*I)*(
c + d*x)) - 5*E^((2*I)*(c + d*x))*Hypergeometric2F1[1/3, 1, 4/3, -((-1 + E^
```

((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]]))*Tan[c + d*x]^(1/3))/(a*d*E
^((2*I)*(c + d*x)))

Maple [A]
time = 0.18, size = 190, normalized size = 0.64

method	result
derivativedivides	$\frac{\frac{5 i \ln\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}{12}-\frac{1}{6\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}-\frac{i \ln\left(i\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)-1\right)}{8}-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(i+2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)\sqrt{3}}{3}\right.}{4}}{}$
default	$\frac{\frac{5 i \ln\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}{12}-\frac{1}{6\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}-\frac{i \ln\left(i\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)-1\right)}{8}-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(i+2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)\sqrt{3}}{3}\right.}{4}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(4/3)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(5/12*I*ln(tan(d*x+c)^(1/3)+I)-1/6/(tan(d*x+c)^(1/3)+I)-1/8*I*ln(I*ta
n(d*x+c)^(1/3)+tan(d*x+c)^(2/3)-1)-1/4*3^(1/2)*arctanh(1/3*(I+2*tan(d*x+c)^
(1/3))*3^(1/2))-1/12*(-2*tan(d*x+c)^(1/3)-2*I)/(-I*tan(d*x+c)^(1/3)+tan(d*x
+c)^(2/3)-1)-5/24*I*ln(-I*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3)-1)+5/12*3^(1/2)
*arctanh(1/3*(-I+2*tan(d*x+c)^(1/3))*3^(1/2))+1/4*I*ln(tan(d*x+c)^(1/3)-I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(4/3)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 492 vs. 2(238) = 476.
time = 0.77, size = 492, normalized size = 1.65

$$-\frac{\left(3\left(\sqrt{3}\,ad\sqrt{\frac{1}{a^2d^2}}\,e^{(2i\,dx+2i\,c)}+i\,e^{(2i\,dx+2i\,c)}\right)\log\left(\frac{1}{2}\sqrt{3}\,ad\sqrt{\frac{1}{a^2d^2}}+\left(\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}+\frac{1}{2}i\right)-3\left(\sqrt{3}\,ad\sqrt{\frac{1}{a^2d^2}}\,e^{(2i\,dx+2i\,c)}-i\,e^{(2i\,dx+2i\,c)}\right)\log\left(-\frac{1}{2}\sqrt{3}\,ad\sqrt{\frac{1}{a^2d^2}}+\left(\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}+\frac{1}{2}i\right)-5\left(3\sqrt{\frac{1}{3}}\,ad\sqrt{\frac{1}{a^2d^2}}\,e^{(2i\,dx+2i\,c)}-i\,e^{(2i\,dx+2i\,c)}\right)\log\left(\frac{3}{2}\sqrt{\frac{1}{3}}\,ad\sqrt{\frac{1}{a^2d^2}}+\left(\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}-\frac{1}{2}i\right)+5\left(3\sqrt{\frac{1}{3}}\,ad\sqrt{\frac{1}{a^2d^2}}\,e^{(2i\,dx+2i\,c)}+i\,e^{(2i\,dx+2i\,c)}\right)\log\left(-\frac{3}{2}\sqrt{\frac{1}{3}}\,ad\sqrt{\frac{1}{a^2d^2}}+\left(\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}-\frac{1}{2}i\right)-10i\,e^{(2i\,dx+2i\,c)}\log\left(\left(\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}+i\right)-6i\,e^{(2i\,dx+2i\,c)}\log\left(\left(\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}-i\right)+6\left(\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}\left(e^{(2i\,dx+2i\,c)}+1\right)\right)e^{(-2i\,dx-2i\,c)}}{24\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(4/3)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/24*(3*(sqrt(3)*a*d*sqrt(1/(a^2*d^2))*e^(2*I*d*x + 2*I*c) + I*e^(2*I*d*x
+ 2*I*c))*log(1/2*sqrt(3)*a*d*sqrt(1/(a^2*d^2)) + ((-I*e^(2*I*d*x + 2*I*c)
+ I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) + 1/2*I) - 3*(sqrt(3)*a*d*sqrt(1/(a^2
*d^2))*e^(2*I*d*x + 2*I*c) - I*e^(2*I*d*x + 2*I*c))*log(-1/2*sqrt(3)*a*d*sq
rt(1/(a^2*d^2)) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^
(1/3) + 1/2*I) - 5*(3*sqrt(1/3)*a*d*sqrt(1/(a^2*d^2))*e^(2*I*d*x + 2*I*c) -
 I*e^(2*I*d*x + 2*I*c))*log(3/2*sqrt(1/3)*a*d*sqrt(1/(a^2*d^2)) + ((-I*e^(2
*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) - 1/2*I) + 5*(3*sqrt(
1/3)*a*d*sqrt(1/(a^2*d^2))*e^(2*I*d*x + 2*I*c) + I*e^(2*I*d*x + 2*I*c))*log
(-3/2*sqrt(1/3)*a*d*sqrt(1/(a^2*d^2)) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2
*I*d*x + 2*I*c) + 1))^(1/3) - 1/2*I) - 10*I*e^(2*I*d*x + 2*I*c)*log(((-I*e^
(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) + I) - 6*I*e^(2*I*d
*x + 2*I*c)*log(((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1
/3) - I) + 6*((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)
*(e^(2*I*d*x + 2*I*c) + 1))*e^(-2*I*d*x - 2*I*c)/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int \frac{\tan^{\frac{4}{3}}(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(4/3)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I*Integral(tan(c + d*x)**(4/3)/(tan(c + d*x) - I), x)/a
```

Giac [A]
time = 0.69, size = 215, normalized size = 0.72

$$-\frac{5\sqrt{3}\log\left(-\frac{\sqrt{3}-2\tan(dx+c)^{\frac{1}{3}}+i}{\sqrt{3}+2\tan(dx+c)^{\frac{1}{3}}-i}\right)}{24\,ad}+\frac{\sqrt{3}\log\left(-\frac{\sqrt{3}-2\tan(dx+c)^{\frac{1}{3}}-i}{\sqrt{3}+2\tan(dx+c)^{\frac{1}{3}}+i}\right)}{8\,ad}-\frac{i\log\left(\tan(dx+c)^{\frac{2}{3}}+i\tan(dx+c)^{\frac{1}{3}}-1\right)}{8\,ad}-\frac{5i\log\left(\tan(dx+c)^{\frac{2}{3}}-i\tan(dx+c)^{\frac{1}{3}}-1\right)}{24\,ad}+\frac{5i\log\left(\tan(dx+c)^{\frac{1}{3}}+i\right)}{12\,ad}+\frac{i\log\left(\tan(dx+c)^{\frac{1}{3}}-i\right)}{4\,ad}+\frac{i\tan(dx+c)^{\frac{1}{3}}}{2\,ad(\tan(dx+c)-i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(4/3)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -5/24*sqrt(3)*log(-(sqrt(3) - 2*tan(d*x + c)^(1/3) + I)/(sqrt(3) + 2*tan(d*
x + c)^(1/3) - I))/(a*d) + 1/8*sqrt(3)*log(-(sqrt(3) - 2*tan(d*x + c)^(1/3)
 - I)/(sqrt(3) + 2*tan(d*x + c)^(1/3) + I))/(a*d) - 1/8*I*log(tan(d*x + c)^
(2/3) + I*tan(d*x + c)^(1/3) - 1)/(a*d) - 5/24*I*log(tan(d*x + c)^(2/3) - I
*tan(d*x + c)^(1/3) - 1)/(a*d) + 5/12*I*log(tan(d*x + c)^(1/3) + I)/(a*d) +
 1/4*I*log(tan(d*x + c)^(1/3) - I)/(a*d) + 1/2*I*tan(d*x + c)^(1/3)/(a*d*(t
an(d*x + c) - I))
```

Mupad [B]
time = 5.16, size = 622, normalized size = 2.08

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(4/3)/(a + a*tan(c + d*x)*1i),x)
```

```
[Out] log((a^3*d^3*14112i - 165888*a^5*d^5*tan(c + d*x)^(1/3)*(-1i/(64*a^3*d^3))^
(2/3))*(-1i/(64*a^3*d^3))^(1/3) - a^2*d^2*tan(c + d*x)^(1/3)*6120i)*(-1i/(6
4*a^3*d^3))^(1/3) + log((a^3*d^3*14112i - 165888*a^5*d^5*tan(c + d*x)^(1/3)
*(-125i/(1728*a^3*d^3))^(2/3))*(-125i/(1728*a^3*d^3))^(1/3) - a^2*d^2*tan(c
 + d*x)^(1/3)*6120i)*(-125i/(1728*a^3*d^3))^(1/3) + (log(((3^(1/2)*1i - 1)*
(a^3*d^3*14112i - 41472*a^5*d^5*tan(c + d*x)^(1/3)*(3^(1/2)*1i - 1)^2*(-1i/
(64*a^3*d^3))^(2/3))*(-1i/(64*a^3*d^3))^(1/3))/2 - a^2*d^2*tan(c + d*x)^(1/
3)*6120i)*(3^(1/2)*1i - 1)*(-1i/(64*a^3*d^3))^(1/3))/2 - (log(((3^(1/2)*1i
+ 1)*(a^3*d^3*14112i - 41472*a^5*d^5*tan(c + d*x)^(1/3)*(3^(1/2)*1i + 1)^2*
(-1i/(64*a^3*d^3))^(2/3))*(-1i/(64*a^3*d^3))^(1/3))/2 + a^2*d^2*tan(c + d*x
)^(1/3)*6120i)*(3^(1/2)*1i + 1)*(-1i/(64*a^3*d^3))^(1/3))/2 + (log(((3^(1/2
)*1i - 1)*(a^3*d^3*14112i - 41472*a^5*d^5*tan(c + d*x)^(1/3)*(3^(1/2)*1i -
1)^2*(-125i/(1728*a^3*d^3))^(2/3))*(-125i/(1728*a^3*d^3))^(1/3))/2 - a^2*d^
2*tan(c + d*x)^(1/3)*6120i)*(3^(1/2)*1i - 1)*(-125i/(1728*a^3*d^3))^(1/3))/
2 - (log(((3^(1/2)*1i + 1)*(a^3*d^3*14112i - 41472*a^5*d^5*tan(c + d*x)^(1/
3)*(3^(1/2)*1i + 1)^2*(-125i/(1728*a^3*d^3))^(2/3))*(-125i/(1728*a^3*d^3))^
(1/3))/2 + a^2*d^2*tan(c + d*x)^(1/3)*6120i)*(3^(1/2)*1i + 1)*(-125i/(1728*
a^3*d^3))^(1/3))/2 - tan(c + d*x)^(1/3)/(2*a*d*(tan(c + d*x)*1i + 1))
```

3.236 $\int \frac{\tan^{\frac{2}{3}}(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=303

$$-\frac{\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{12ad}+\frac{\text{ArcTan}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{12ad}+\frac{i\text{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)}{2\sqrt{3}\,ad}+\frac{\text{ArcT}}{}$$

```
[Out] 1/12*arctan(-3^(1/2)+2*tan(d*x+c)^(1/3))/a/d+1/12*arctan(3^(1/2)+2*tan(d*x+
c)^(1/3))/a/d+1/6*arctan(tan(d*x+c)^(1/3))/a/d-1/6*I*ln(1+tan(d*x+c)^(2/3))
/a/d+1/12*I*ln(1-tan(d*x+c)^(2/3)+tan(d*x+c)^(4/3))/a/d+1/6*I*arctan(1/3*(1
-2*tan(d*x+c)^(2/3))*3^(1/2))/a/d*3^(1/2)+1/24*ln(1-3^(1/2)*tan(d*x+c)^(1/3
)+tan(d*x+c)^(2/3))/a/d*3^(1/2)-1/24*ln(1+3^(1/2)*tan(d*x+c)^(1/3)+tan(d*x+
c)^(2/3))/a/d*3^(1/2)+1/2*I*tan(d*x+c)^(2/3)/d/(a+I*a*tan(d*x+c))
```

Rubi [A]
time = 0.32, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 13, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3630, 3619, 3557, 335, 281, 206, 31, 648, 632, 210, 642, 301, 209}

$$\frac{i\text{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)}{2\sqrt{3}\,ad}-\frac{\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{12ad}+\frac{\text{ArcTan}\left(2\sqrt[3]{\tan(c+dx)}+\sqrt{3}\right)}{12ad}+\frac{\text{ArcTan}\left(\sqrt[3]{\tan(c+dx)}\right)}{6ad}+\frac{i\tan^{\frac{2}{3}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{i\log\left(\tan^{\frac{2}{3}}(c+dx)+1\right)}{6ad}+\frac{\log\left(\tan^{\frac{2}{3}}(c+dx)-\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{8\sqrt{3}\,ad}-\frac{\log\left(\tan^{\frac{2}{3}}(c+dx)+\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{8\sqrt{3}\,ad}+\frac{i\log\left(\tan^{\frac{4}{3}}(c+dx)-\tan^{\frac{2}{3}}(c+dx)+1\right)}{12ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(2/3)/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] -1/12*ArcTan[Sqrt[3] - 2*Tan[c + d*x]^(1/3)]/(a*d) + ArcTan[Sqrt[3] + 2*Tan
[c + d*x]^(1/3)]/(12*a*d) + ((I/2)*ArcTan[(1 - 2*Tan[c + d*x]^(2/3))/Sqrt[3
]])/(Sqrt[3]*a*d) + ArcTan[Tan[c + d*x]^(1/3)]/(6*a*d) - ((I/6)*Log[1 + Tan
[c + d*x]^(2/3)])/(a*d) + Log[1 - Sqrt[3]*Tan[c + d*x]^(1/3) + Tan[c + d*x]
^(2/3)]/(8*Sqrt[3]*a*d) - Log[1 + Sqrt[3]*Tan[c + d*x]^(1/3) + Tan[c + d*x]
^(2/3)]/(8*Sqrt[3]*a*d) + ((I/12)*Log[1 - Tan[c + d*x]^(2/3) + Tan[c + d*x]
^(4/3)])/(a*d) + ((I/2)*Tan[c + d*x]^(2/3))/(d*(a + I*a*Tan[c + d*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3630

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(a*c + b*d))*((c + d*Tan[e + f*x])^n/(2*(
b*c - a*d)*f*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c
+ d*Tan[e + f*x])^(n - 1)*Simp[a*c*d*(n - 1) + b*c^2 + b*d^2*n - d*(b*c - a
*d)*(n - 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[0, n, 1]
```

Rubi steps

$$\int \frac{\tan^{\frac{2}{3}}(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{i\tan^{\frac{2}{3}}(c+dx)}{2d(a+ia\tan(c+dx))} - \frac{\int \frac{\frac{2ia}{3}-\frac{1}{3}a\tan(c+dx)}{\sqrt[3]{\tan(c+dx)}}\,dx}{2a^2}$$

$$= \frac{i\tan^{\frac{2}{3}}(c+dx)}{2d(a+ia\tan(c+dx))} - \frac{i\int \frac{1}{\sqrt[3]{\tan(c+dx)}}\,dx}{3a} + \frac{\int \tan^{\frac{2}{3}}(c+dx)\,dx}{6a}$$

$$= \frac{i\tan^{\frac{2}{3}}(c+dx)}{2d(a+ia\tan(c+dx))} - \frac{i\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\,(1+x^2)}\,dx, x, \tan(c+dx)\right)}{3ad} + \frac{\text{Subst}\left(\int \frac{x^2}{1+\right.}{}$$

$$= \frac{i\tan^{\frac{2}{3}}(c+dx)}{2d(a+ia\tan(c+dx))} - \frac{i\text{Subst}\left(\int \frac{x}{1+x^6}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{ad} + \frac{\text{Subst}\left(\int \frac{x^4}{1+x^6}\right.}{}$$

$$= \frac{i\tan^{\frac{2}{3}}(c+dx)}{2d(a+ia\tan(c+dx))} - \frac{i\text{Subst}\left(\int \frac{1}{1+x^3}\,dx, x, \tan^{\frac{2}{3}}(c+dx)\right)}{2ad} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx\right.}{}$$

$$= \frac{\tan^{-1}\left(\sqrt[3]{\tan(c+dx)}\right)}{6ad} + \frac{i\tan^{\frac{2}{3}}(c+dx)}{2d(a+ia\tan(c+dx))} - \frac{i\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \tan^{\frac{2}{3}}(c\right.}{6ad}$$

$$= \frac{\tan^{-1}\left(\sqrt[3]{\tan(c+dx)}\right)}{6ad} - \frac{i\log\left(1+\tan^{\frac{2}{3}}(c+dx)\right)}{6ad} + \frac{\log\left(1-\sqrt{3}\,\sqrt[3]{\tan(c+d\right.}{8\sqrt{3}\,a}$$

$$= -\frac{\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{12ad} + \frac{\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{12ad} + \frac{\tan^{-1}\left(\right.}{}$$

$$= -\frac{\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{12ad} + \frac{\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{12ad} + \frac{i\tan^{-1}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.08, size = 163, normalized size = 0.54

$$\frac{ie^{-2i(c+dx)}\left(4+4e^{2i(c+dx)}-3\sqrt[3]{2}\,e^{2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{1}{2}\left(1-e^{2i(c+dx)}\right)\right)-2e^{2i(c+dx)}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)\right)\tan^{\frac{2}{3}}(c+dx)}{16ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(2/3)/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((I/16)*(4 + 4*E^((2*I)*(c + d*x)) - 3*2^(1/3)*E^((2*I)*(c + d*x))*(1 + E^(
(2*I)*(c + d*x)))^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, (1 - E^((2*I)*(c +
 d*x)))/2] - 2*E^((2*I)*(c + d*x))*Hypergeometric2F1[2/3, 1, 5/3, -((-1 + E
```

```
^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))])*Tan[c + d*x]^(2/3))/(a*d*E
^((2*I)*(c + d*x)))
```

Maple [A]
time = 0.20, size = 190, normalized size = 0.63

method	result
derivativedivides	$\frac{-\frac{i \ln\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}{12}+\frac{1}{6\left(\tan^{\frac{1}{3}}(dx+c)\right)+6i}+\frac{i \ln\left(i\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)-1\right)}{8}-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(i+2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{3}\right.}{4}}{}$
default	$\frac{-\frac{i \ln\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}{12}+\frac{1}{6\left(\tan^{\frac{1}{3}}(dx+c)\right)+6i}+\frac{i \ln\left(i\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)-1\right)}{8}-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(i+2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{3}\right.}{4}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(2/3)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/12*I*ln(tan(d*x+c)^(1/3)+I)+1/6/(tan(d*x+c)^(1/3)+I)+1/8*I*ln(I*t
an(d*x+c)^(1/3)+tan(d*x+c)^(2/3)-1)-1/4*3^(1/2)*arctanh(1/3*(I+2*tan(d*x+c)
^(1/3))*3^(1/2))+1/12*(4*tan(d*x+c)^(1/3)-2*I)/(-I*tan(d*x+c)^(1/3)+tan(d*x
+c)^(2/3)-1)+1/24*I*ln(-I*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3)-1)+1/12*3^(1/2)
*arctanh(1/3*(-I+2*tan(d*x+c)^(1/3))*3^(1/2))-1/4*I*ln(tan(d*x+c)^(1/3)-I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(2/3)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 493 vs. 2(238) = 476.
time = 0.72, size = 493, normalized size = 1.63

$$-\frac{\left(3\left(\sqrt{3}\,ad\sqrt{\frac{1}{a^2d^2}}\,e^{(2i\,dx+2i\,c)}-i\,e^{(2i\,dx+2i\,c)}\right)\log\left(\frac{1}{2}\sqrt{3}\,ad\sqrt{\frac{1}{a^2d^2}}+\left(\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}+\frac{1}{2}i\right)-3\left(\sqrt{3}\,ad\sqrt{\frac{1}{a^2d^2}}\,e^{(2i\,dx+2i\,c)}+i\,e^{(2i\,dx+2i\,c)}\right)\log\left(-\frac{1}{2}\sqrt{3}\,ad\sqrt{\frac{1}{a^2d^2}}+\left(\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}+\frac{1}{2}i\right)-\left(3\sqrt{\frac{1}{3}}\,ad\sqrt{\frac{1}{a^2d^2}}\,e^{(2i\,dx+2i\,c)}+i\,e^{(2i\,dx+2i\,c)}\right)\log\left(\frac{3}{2}\sqrt{\frac{1}{3}}\,ad\sqrt{\frac{1}{a^2d^2}}+\left(\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}-\frac{1}{2}i\right)+\left(3\sqrt{\frac{1}{3}}\,ad\sqrt{\frac{1}{a^2d^2}}\,e^{(2i\,dx+2i\,c)}-i\,e^{(2i\,dx+2i\,c)}\right)\log\left(-\frac{3}{2}\sqrt{\frac{1}{3}}\,ad\sqrt{\frac{1}{a^2d^2}}+\left(\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}-\frac{1}{2}i\right)+2i\,e^{(2i\,dx+2i\,c)}\log\left(\left(\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}+i\right)+6i\,e^{(2i\,dx+2i\,c)}\log\left(\left(\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}-i\right)+6\left(\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{2}{3}}\left(-i\,e^{(2i\,dx+2i\,c)}-i\right)\right)e^{(-2i\,dx-2i\,c)}}{24\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(2/3)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/24*(3*(sqrt(3)*a*d*sqrt(1/(a^2*d^2))*e^(2*I*d*x + 2*I*c) - I*e^(2*I*d*x
+ 2*I*c))*log(1/2*sqrt(3)*a*d*sqrt(1/(a^2*d^2)) + ((-I*e^(2*I*d*x + 2*I*c)
+ I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) + 1/2*I) - 3*(sqrt(3)*a*d*sqrt(1/(a^2
*d^2))*e^(2*I*d*x + 2*I*c) + I*e^(2*I*d*x + 2*I*c))*log(-1/2*sqrt(3)*a*d*sq
rt(1/(a^2*d^2)) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^
(1/3) + 1/2*I) - (3*sqrt(1/3)*a*d*sqrt(1/(a^2*d^2))*e^(2*I*d*x + 2*I*c) + I
*e^(2*I*d*x + 2*I*c))*log(3/2*sqrt(1/3)*a*d*sqrt(1/(a^2*d^2)) + ((-I*e^(2*I
*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) - 1/2*I) + (3*sqrt(1/3)
*a*d*sqrt(1/(a^2*d^2))*e^(2*I*d*x + 2*I*c) - I*e^(2*I*d*x + 2*I*c))*log(-3/
2*sqrt(1/3)*a*d*sqrt(1/(a^2*d^2)) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d
*x + 2*I*c) + 1))^(1/3) - 1/2*I) + 2*I*e^(2*I*d*x + 2*I*c)*log(((-I*e^(2*I*
d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) + I) + 6*I*e^(2*I*d*x +
2*I*c)*log(((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) -
 I) + 6*((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(-I*
e^(2*I*d*x + 2*I*c) - I))*e^(-2*I*d*x - 2*I*c)/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{\tan^{\frac{2}{3}}(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(2/3)/(a+I*a*tan(d*x+c)),x)
[Out] -I*Integral(tan(c + d*x)**(2/3)/(tan(c + d*x) - I), x)/a
```

Giac [A]
time = 0.61, size = 215, normalized size = 0.71

$$-\frac{\sqrt{3}\log\left(-\frac{\sqrt{3}-2\tan(dx+c)^{\frac{1}{3}}+i}{\sqrt{3}+2\tan(dx+c)^{\frac{1}{3}}-i}\right)}{24\,ad}+\frac{\sqrt{3}\log\left(-\frac{\sqrt{3}-2\tan(dx+c)^{\frac{1}{3}}-i}{\sqrt{3}+2\tan(dx+c)^{\frac{1}{3}}+i}\right)}{8\,ad}+\frac{i\log\left(\tan(dx+c)^{\frac{2}{3}}+i\tan(dx+c)^{\frac{1}{3}}-1\right)}{8\,ad}+\frac{i\log\left(\tan(dx+c)^{\frac{2}{3}}-i\tan(dx+c)^{\frac{1}{3}}-1\right)}{24\,ad}-\frac{i\log\left(\tan(dx+c)^{\frac{1}{3}}+i\right)}{12\,ad}-\frac{i\log\left(\tan(dx+c)^{\frac{1}{3}}-i\right)}{4\,ad}+\frac{\tan(dx+c)^{\frac{2}{3}}}{2\,ad(\tan(dx+c)-i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(2/3)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] -1/24*sqrt(3)*log(-(sqrt(3) - 2*tan(d*x + c)^(1/3) + I)/(sqrt(3) + 2*tan(d*
x + c)^(1/3) - I))/(a*d) + 1/8*sqrt(3)*log(-(sqrt(3) - 2*tan(d*x + c)^(1/3)
 - I)/(sqrt(3) + 2*tan(d*x + c)^(1/3) + I))/(a*d) + 1/8*I*log(tan(d*x + c)^
(2/3) + I*tan(d*x + c)^(1/3) - 1)/(a*d) + 1/24*I*log(tan(d*x + c)^(2/3) - I
*tan(d*x + c)^(1/3) - 1)/(a*d) - 1/12*I*log(tan(d*x + c)^(1/3) + I)/(a*d) -
 1/4*I*log(tan(d*x + c)^(1/3) - I)/(a*d) + 1/2*tan(d*x + c)^(2/3)/(a*d*(tan
(d*x + c) - I))
```

Mupad [B]
time = 5.29, size = 599, normalized size = 1.98

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d*x)^(2/3)/(a + a*tan(c + d*x)*1i),x)

[Out] log((a^3*d^3*3744i - a^4*d^4*tan(c + d*x)^(1/3)*(1i/(64*a^3*d^3))^(1/3)*17280i)*(1i/(64*a^3*d^3))^(2/3) - 36*a*d*tan(c + d*x)^(1/3))*(1i/(64*a^3*d^3))^(1/3) + log((a^3*d^3*3744i - a^4*d^4*tan(c + d*x)^(1/3)*(1i/(1728*a^3*d^3))^(1/3)*17280i)*(1i/(1728*a^3*d^3))^(2/3) - 36*a*d*tan(c + d*x)^(1/3))*(1i/(1728*a^3*d^3))^(1/3) + (log(((3^(1/2)*1i - 1)^2*(a^3*d^3*3744i - a^4*d^4*tan(c + d*x)^(1/3)*(3^(1/2)*1i - 1)*(1i/(64*a^3*d^3))^(1/3)*8640i)*(1i/(64*a^3*d^3))^(2/3))/4 - 36*a*d*tan(c + d*x)^(1/3))*(3^(1/2)*1i - 1)*(1i/(64*a^3*d^3))^(1/3))/2 - (log(((3^(1/2)*1i + 1)^2*(a^3*d^3*3744i + a^4*d^4*tan(c + d*x)^(1/3)*(3^(1/2)*1i + 1)*(1i/(64*a^3*d^3))^(1/3)*8640i)*(1i/(64*a^3*d^3))^(2/3))/4 - 36*a*d*tan(c + d*x)^(1/3))*(3^(1/2)*1i + 1)*(1i/(64*a^3*d^3))^(1/3))/2 + (log(((3^(1/2)*1i - 1)^2*(a^3*d^3*3744i - a^4*d^4*tan(c + d*x)^(1/3)*(3^(1/2)*1i - 1)*(1i/(1728*a^3*d^3))^(1/3)*8640i)*(1i/(1728*a^3*d^3))^(2/3))/4 - 36*a*d*tan(c + d*x)^(1/3))*(3^(1/2)*1i - 1)*(1i/(1728*a^3*d^3))^(1/3))/2 - (log(((3^(1/2)*1i + 1)^2*(a^3*d^3*3744i + a^4*d^4*tan(c + d*x)^(1/3)*(3^(1/2)*1i + 1)*(1i/(1728*a^3*d^3))^(1/3)*8640i)*(1i/(1728*a^3*d^3))^(2/3))/4 - 36*a*d*tan(c + d*x)^(1/3))*(3^(1/2)*1i + 1)*(1i/(1728*a^3*d^3))^(1/3))/2 + (tan(c + d*x)^(2/3)*1i)/(2*a*d*(tan(c + d*x)*1i + 1))

3.237 $\int \frac{1}{\sqrt[3]{\tan(c+dx)}\,(a+ia\tan(c+dx))}\,dx$

Optimal. Leaf size=303

$$\frac{i\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{12ad}-\frac{i\text{ArcTan}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{12ad}-\frac{\text{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)}{\sqrt{3}\,ad}-\frac{i\text{ArcTan}}{}$$

```
[Out] -1/12*I*arctan(-3^(1/2)+2*tan(d*x+c)^(1/3))/a/d-1/12*I*arctan(3^(1/2)+2*tan
(d*x+c)^(1/3))/a/d-1/6*I*arctan(tan(d*x+c)^(1/3))/a/d+1/3*ln(1+tan(d*x+c)^(
2/3))/a/d-1/6*ln(1-tan(d*x+c)^(2/3)+tan(d*x+c)^(4/3))/a/d-1/3*arctan(1/3*(1
-2*tan(d*x+c)^(2/3))*3^(1/2))/a/d*3^(1/2)-1/24*I*ln(1-3^(1/2)*tan(d*x+c)^(1
/3)+tan(d*x+c)^(2/3))/a/d*3^(1/2)+1/24*I*ln(1+3^(1/2)*tan(d*x+c)^(1/3)+tan(
d*x+c)^(2/3))/a/d*3^(1/2)+1/2*tan(d*x+c)^(2/3)/d/(a+I*a*tan(d*x+c))
```

Rubi [A]
time = 0.32, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 13, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3633, 3619, 3557, 335, 281, 206, 31, 648, 632, 210, 642, 301, 209}

$$-\frac{\text{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)}{\sqrt{3}\,ad}+\frac{i\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{12ad}-\frac{i\text{ArcTan}\left(2\sqrt[3]{\tan(c+dx)}+\sqrt{3}\right)}{12ad}-\frac{i\text{ArcTan}\left(\sqrt[3]{\tan(c+dx)}\right)}{6ad}+\frac{\tan^{\frac{2}{3}}(c+dx)}{2d(a+ia\tan(c+dx))}+\frac{\log\left(\tan^{\frac{2}{3}}(c+dx)+1\right)}{3ad}-\frac{i\log\left(\tan^{\frac{2}{3}}(c+dx)-\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{8\sqrt{3}\,ad}+\frac{i\log\left(\tan^{\frac{2}{3}}(c+dx)+\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{8\sqrt{3}\,ad}-\frac{\log\left(\tan^{\frac{4}{3}}(c+dx)-\tan^{\frac{2}{3}}(c+dx)+1\right)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(1/3)*(a + I*a*Tan[c + d*x])),x]
```

```
[Out] ((I/12)*ArcTan[Sqrt[3] - 2*Tan[c + d*x]^(1/3)])/(a*d) - ((I/12)*ArcTan[Sqrt
[3] + 2*Tan[c + d*x]^(1/3)])/(a*d) - ArcTan[(1 - 2*Tan[c + d*x]^(2/3))/Sqrt
[3]]/(Sqrt[3]*a*d) - ((I/6)*ArcTan[Tan[c + d*x]^(1/3)])/(a*d) + Log[1 + Tan
[c + d*x]^(2/3)]/(3*a*d) - ((I/8)*Log[1 - Sqrt[3]*Tan[c + d*x]^(1/3) + Tan[
c + d*x]^(2/3)])/(Sqrt[3]*a*d) + ((I/8)*Log[1 + Sqrt[3]*Tan[c + d*x]^(1/3)
+ Tan[c + d*x]^(2/3)])/(Sqrt[3]*a*d) - Log[1 - Tan[c + d*x]^(2/3) + Tan[c +
 d*x]^(4/3)]/(6*a*d) + Tan[c + d*x]^(2/3)/(2*d*(a + I*a*Tan[c + d*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3633

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-a)*((c + d*Tan[e + f*x])^(n + 1)/(2*f*(b*
c - a*d)*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*
Tan[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &
& NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{\tan(c+dx)}\,(a+ia\tan(c+dx))}\,dx = \frac{\tan^{\frac{2}{3}}(c+dx)}{2d(a+ia\tan(c+dx))} - \frac{\int \frac{-\frac{4a}{3}+\frac{1}{3}ia\tan(c+dx)}{\sqrt[3]{\tan(c+dx)}}\,dx}{2a^2}$$

$$= \frac{\tan^{\frac{2}{3}}(c+dx)}{2d(a+ia\tan(c+dx))} - \frac{i\int \tan^{\frac{2}{3}}(c+dx)\,dx}{6a} + \frac{2\int \frac{1}{\sqrt[3]{\tan(c+}}}{3a}$$

$$= \frac{\tan^{\frac{2}{3}}(c+dx)}{2d(a+ia\tan(c+dx))} - \frac{i\text{Subst}\left(\int \frac{x^{2/3}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{6ad} +$$

$$= \frac{\tan^{\frac{2}{3}}(c+dx)}{2d(a+ia\tan(c+dx))} - \frac{i\text{Subst}\left(\int \frac{x^4}{1+x^6}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{2ad}$$

$$= \frac{\tan^{\frac{2}{3}}(c+dx)}{2d(a+ia\tan(c+dx))} - \frac{i\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{6ad}$$

$$= -\frac{i\tan^{-1}\left(\sqrt[3]{\tan(c+dx)}\right)}{6ad} + \frac{\tan^{\frac{2}{3}}(c+dx)}{2d(a+ia\tan(c+dx))} - \frac{i\text{Subs}}{}$$

$$= -\frac{i\tan^{-1}\left(\sqrt[3]{\tan(c+dx)}\right)}{6ad} + \frac{\log\left(1+\tan^{\frac{2}{3}}(c+dx)\right)}{3ad} - \frac{i\log}{}$$

$$= \frac{i\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{12ad} - \frac{i\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(c}\right.}{12ad}$$

$$= \frac{i\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{12ad} - \frac{i\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(c}\right.}{12ad}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.15, size = 161, normalized size = 0.53

$$\frac{e^{-2i(c+dx)}\left(4+4e^{2i(c+dx)}+3\sqrt[3]{2}\,e^{2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{1}{2}\left(1-e^{2i(c+dx)}\right)\right)+10e^{2i(c+dx)}{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)\right)\tan^{\frac{2}{3}}(c+dx)}{16ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Tan[c + d*x]^(1/3)*(a + I*a*Tan[c + d*x])),x]
```

```
[Out] ((4 + 4*E^((2*I)*(c + d*x)) + 3*2^(1/3)*E^((2*I)*(c + d*x))*(1 + E^((2*I)*(
c + d*x)))^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, (1 - E^((2*I)*(c + d*x)))
/2] + 10*E^((2*I)*(c + d*x))*Hypergeometric2F1[2/3, 1, 5/3, -((-1 + E^((2*I
```

```
)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))])*Tan[c + d*x]^(2/3))/(16*a*d*E^((
2*I)*(c + d*x)))
```

Maple [A]
time = 0.17, size = 189, normalized size = 0.62

method	result
derivativedivides	$\frac{-\frac{i}{6\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}+\frac{5\ln\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}{12}-\frac{\ln\left(i\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)-1\right)}{8}-\frac{i\sqrt{3}\ \mathrm{arctanh}\left(\frac{\left(i+2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)\sqrt{3}}{3}\right.}{4}}{}$
default	$\frac{-\frac{i}{6\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}+\frac{5\ln\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}{12}-\frac{\ln\left(i\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)-1\right)}{8}-\frac{i\sqrt{3}\ \mathrm{arctanh}\left(\frac{\left(i+2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)\sqrt{3}}{3}\right.}{4}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(1/3)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/6*I/(tan(d*x+c)^(1/3)+I)+5/12*ln(tan(d*x+c)^(1/3)+I)-1/8*ln(I*tan
(d*x+c)^(1/3)+tan(d*x+c)^(2/3)-1)-1/4*I*3^(1/2)*arctanh(1/3*(I+2*tan(d*x+c)
^(1/3))*3^(1/2))+1/12*(-4*I*tan(d*x+c)^(1/3)-2)/(-I*tan(d*x+c)^(1/3)+tan(d*
x+c)^(2/3)-1)-5/24*ln(-I*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3)-1)+5/12*I*3^(1/2
)*arctanh(1/3*(-I+2*tan(d*x+c)^(1/3))*3^(1/2))+1/4*ln(tan(d*x+c)^(1/3)-I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/3)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 486 vs. 2(238) = 476.
time = 0.70, size = 486, normalized size = 1.60

$$-\frac{\left(3\left(i\sqrt{3}\,ad\sqrt{\frac{1}{a^2d^2}}e^{(2idx+2ic)}+e^{(2idx+2ic)}\right)\log\left(\frac{1}{2}\sqrt{3}\,ad\sqrt{\frac{1}{a^2d^2}}+\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}+\frac{1}{2}i\right)+3\left(-i\sqrt{3}\,ad\sqrt{\frac{1}{a^2d^2}}e^{(2idx+2ic)}+e^{(2idx+2ic)}\right)\log\left(-\frac{1}{2}\sqrt{3}\,ad\sqrt{\frac{1}{a^2d^2}}+\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}+\frac{1}{2}i\right)+5\left(-3i\sqrt{\frac{1}{3}}\,ad\sqrt{\frac{1}{a^2d^2}}e^{(2idx+2ic)}+e^{(2idx+2ic)}\right)\log\left(\frac{3}{2}\sqrt{\frac{1}{3}}\,ad\sqrt{\frac{1}{a^2d^2}}+\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}-\frac{1}{2}i\right)+5\left(3i\sqrt{\frac{1}{3}}\,ad\sqrt{\frac{1}{a^2d^2}}e^{(2idx+2ic)}+e^{(2idx+2ic)}\right)\log\left(-\frac{3}{2}\sqrt{\frac{1}{3}}\,ad\sqrt{\frac{1}{a^2d^2}}+\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}-\frac{1}{2}i\right)-10e^{(2idx+2ic)}\log\left(\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}+i\right)-6e^{(2idx+2ic)}\log\left(\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}-i\right)-6\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{2}{3}}\left(e^{(2idx+2ic)}+1\right)\right)e^{(-2idx-2ic)}}{24\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/3)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/24*(3*(I*sqrt(3)*a*d*sqrt(1/(a^2*d^2))*e^(2*I*d*x + 2*I*c) + e^(2*I*d*x
+ 2*I*c))*log(1/2*sqrt(3)*a*d*sqrt(1/(a^2*d^2)) + ((-I*e^(2*I*d*x + 2*I*c)
+ I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) + 1/2*I) + 3*(-I*sqrt(3)*a*d*sqrt(1/(
a^2*d^2))*e^(2*I*d*x + 2*I*c) + e^(2*I*d*x + 2*I*c))*log(-1/2*sqrt(3)*a*d*s
qrt(1/(a^2*d^2)) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))
^(1/3) + 1/2*I) + 5*(-3*I*sqrt(1/3)*a*d*sqrt(1/(a^2*d^2))*e^(2*I*d*x + 2*I*
c) + e^(2*I*d*x + 2*I*c))*log(3/2*sqrt(1/3)*a*d*sqrt(1/(a^2*d^2)) + ((-I*e^
(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) - 1/2*I) + 5*(3*I*s
qrt(1/3)*a*d*sqrt(1/(a^2*d^2))*e^(2*I*d*x + 2*I*c) + e^(2*I*d*x + 2*I*c))*l
og(-3/2*sqrt(1/3)*a*d*sqrt(1/(a^2*d^2)) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^
(2*I*d*x + 2*I*c) + 1))^(1/3) - 1/2*I) - 10*e^(2*I*d*x + 2*I*c)*log(((-I*e^
(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) + I) - 6*e^(2*I*d*x
 + 2*I*c)*log(((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3
) - I) - 6*((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(
e^(2*I*d*x + 2*I*c) + 1))*e^(-2*I*d*x - 2*I*c)/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int \frac{1}{\tan^{\frac{4}{3}}(c+dx)-i\sqrt[3]{\tan(c+dx)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(1/3)/(a+I*a*tan(d*x+c)),x)
[Out] -I*Integral(1/(tan(c + d*x)**(4/3) - I*tan(c + d*x)**(1/3)), x)/a
```

Giac [A]
time = 0.64, size = 215, normalized size = 0.71

$$-\frac{5i\sqrt{3}\log\left(-\frac{\sqrt{3}-2\tan(dx+c)^{\frac{1}{3}}+i}{\sqrt{3}+2\tan(dx+c)^{\frac{1}{3}}-i}\right)}{24\,ad}+\frac{i\sqrt{3}\log\left(-\frac{\sqrt{3}-2\tan(dx+c)^{\frac{1}{3}}-i}{\sqrt{3}+2\tan(dx+c)^{\frac{1}{3}}+i}\right)}{8\,ad}-\frac{\log\left(\tan(dx+c)^{\frac{2}{3}}+i\tan(dx+c)^{\frac{1}{3}}-1\right)}{8\,ad}-\frac{5\log\left(\tan(dx+c)^{\frac{2}{3}}-i\tan(dx+c)^{\frac{1}{3}}-1\right)}{24\,ad}+\frac{5\log\left(\tan(dx+c)^{\frac{1}{3}}+i\right)}{12\,ad}+\frac{\log\left(\tan(dx+c)^{\frac{1}{3}}-i\right)}{4\,ad}-\frac{i\tan(dx+c)^{\frac{2}{3}}}{2\,ad(\tan(dx+c)-i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/3)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] -5/24*I*sqrt(3)*log(-(sqrt(3) - 2*tan(d*x + c)^(1/3) + I)/(sqrt(3) + 2*tan(
d*x + c)^(1/3) - I))/(a*d) + 1/8*I*sqrt(3)*log(-(sqrt(3) - 2*tan(d*x + c)^(
1/3) - I)/(sqrt(3) + 2*tan(d*x + c)^(1/3) + I))/(a*d) - 1/8*log(tan(d*x + c
)^(2/3) + I*tan(d*x + c)^(1/3) - 1)/(a*d) - 5/24*log(tan(d*x + c)^(2/3) - I
*tan(d*x + c)^(1/3) - 1)/(a*d) + 5/12*log(tan(d*x + c)^(1/3) + I)/(a*d) + 1
/4*log(tan(d*x + c)^(1/3) - I)/(a*d) - 1/2*I*tan(d*x + c)^(2/3)/(a*d*(tan(d
*x + c) - I))
```

Mupad [B]
time = 5.31, size = 587, normalized size = 1.94

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(1/3)*(a + a*tan(c + d*x)*1i)),x)
```

```
[Out] (5*log((25*(a^3*d^3*14112i + 24480*a^4*d^4*tan(c + d*x)^(1/3)*(1/(a^3*d^3))
^(1/3))*(1/(a^3*d^3))^(2/3))/144 - 1800*a*d*tan(c + d*x)^(1/3))*(1/(a^3*d^3
))^(1/3))/12 + log((a^3*d^3*14112i + 58752*a^4*d^4*tan(c + d*x)^(1/3)*(1/(6
4*a^3*d^3))^(1/3))*(1/(64*a^3*d^3))^(2/3) - 1800*a*d*tan(c + d*x)^(1/3))*(1
/(64*a^3*d^3))^(1/3) + (5*log((25*(3^(1/2)*1i - 1)^2*(a^3*d^3*14112i + 1224
0*a^4*d^4*tan(c + d*x)^(1/3)*(3^(1/2)*1i - 1)*(1/(a^3*d^3))^(1/3))*(1/(a^3*
d^3))^(2/3))/576 - 1800*a*d*tan(c + d*x)^(1/3))*(3^(1/2)*1i - 1)*(1/(a^3*d^
3))^(1/3))/24 - (5*log((25*(3^(1/2)*1i + 1)^2*(a^3*d^3*14112i - 12240*a^4*d
^4*tan(c + d*x)^(1/3)*(3^(1/2)*1i + 1)*(1/(a^3*d^3))^(1/3))*(1/(a^3*d^3))^(
2/3))/576 - 1800*a*d*tan(c + d*x)^(1/3))*(3^(1/2)*1i + 1)*(1/(a^3*d^3))^(1/
3))/24 + log(((3^(1/2)*1i)/2 - 1/2)^2*(a^3*d^3*14112i + 58752*a^4*d^4*tan(c
 + d*x)^(1/3)*((3^(1/2)*1i)/2 - 1/2)*(1/(64*a^3*d^3))^(1/3))*(1/(64*a^3*d^3
))^(2/3) - 1800*a*d*tan(c + d*x)^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(1/(64*a^3*d
^3))^(1/3) - log(((3^(1/2)*1i)/2 + 1/2)^2*(a^3*d^3*14112i - 58752*a^4*d^4*t
an(c + d*x)^(1/3)*((3^(1/2)*1i)/2 + 1/2)*(1/(64*a^3*d^3))^(1/3))*(1/(64*a^3
*d^3))^(2/3) - 1800*a*d*tan(c + d*x)^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(1/(64*a
^3*d^3))^(1/3) + tan(c + d*x)^(2/3)/(2*a*d*(tan(c + d*x)*1i + 1))
```

3.238 $\int \frac{1}{\tan^{\frac{5}{3}}(c+dx)(a+ia\tan(c+dx))}\,dx$

Optimal. Leaf size=321

$$\frac{5i\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{12ad}-\frac{5i\text{ArcTan}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{12ad}+\frac{2\text{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)}{\sqrt{3}\,ad}-\frac{5i\text{A}}{}$$

```
[Out] -5/12*I*arctan(-3^(1/2)+2*tan(d*x+c)^(1/3))/a/d-5/12*I*arctan(3^(1/2)+2*tan
(d*x+c)^(1/3))/a/d-5/6*I*arctan(tan(d*x+c)^(1/3))/a/d+2/3*ln(1+tan(d*x+c)^(
2/3))/a/d-1/3*ln(1-tan(d*x+c)^(2/3)+tan(d*x+c)^(4/3))/a/d+2/3*arctan(1/3*(1
-2*tan(d*x+c)^(2/3))*3^(1/2))/a/d*3^(1/2)+5/24*I*ln(1-3^(1/2)*tan(d*x+c)^(1
/3)+tan(d*x+c)^(2/3))/a/d*3^(1/2)-5/24*I*ln(1+3^(1/2)*tan(d*x+c)^(1/3)+tan(
d*x+c)^(2/3))/a/d*3^(1/2)-2/a/d/tan(d*x+c)^(2/3)+1/2/d/tan(d*x+c)^(2/3)/(a+
I*a*tan(d*x+c))
```

Rubi [A]
time = 0.28, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 14, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3633, 3610, 3619, 3557, 335, 215, 648, 632, 210, 642, 209, 281, 298, 31}

$$\frac{2\text{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)}{\sqrt{3}\,ad}+\frac{5i\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{12ad}-\frac{5i\text{ArcTan}\left(2\sqrt[3]{\tan(c+dx)}+\sqrt{3}\right)}{12ad}-\frac{5i\text{ArcTan}\left(\sqrt[3]{\tan(c+dx)}\right)}{6ad}-\frac{2}{ad\tan^{\frac{2}{3}}(c+dx)}+\frac{1}{2d\tan^{\frac{2}{3}}(c+dx)(a+ia\tan(c+dx))}+\frac{2\log\left(\tan^{\frac{2}{3}}(c+dx)+1\right)}{3ad}+\frac{5i\log\left(\tan^{\frac{2}{3}}(c+dx)-\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{8\sqrt{3}\,ad}-\frac{5i\log\left(\tan^{\frac{2}{3}}(c+dx)+\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{8\sqrt{3}\,ad}-\frac{\log\left(\tan^{\frac{4}{3}}(c+dx)-\tan^{\frac{2}{3}}(c+dx)+1\right)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(5/3)*(a + I*a*Tan[c + d*x])),x]
```

```
[Out] (((5*I)/12)*ArcTan[Sqrt[3] - 2*Tan[c + d*x]^(1/3)])/(a*d) - (((5*I)/12)*Arc
Tan[Sqrt[3] + 2*Tan[c + d*x]^(1/3)])/(a*d) + (2*ArcTan[(1 - 2*Tan[c + d*x]^
(2/3))/Sqrt[3]])/(Sqrt[3]*a*d) - (((5*I)/6)*ArcTan[Tan[c + d*x]^(1/3)])/(a*
d) + (2*Log[1 + Tan[c + d*x]^(2/3)])/(3*a*d) + (((5*I)/8)*Log[1 - Sqrt[3]*T
an[c + d*x]^(1/3) + Tan[c + d*x]^(2/3)])/(Sqrt[3]*a*d) - (((5*I)/8)*Log[1 +
 Sqrt[3]*Tan[c + d*x]^(1/3) + Tan[c + d*x]^(2/3)])/(Sqrt[3]*a*d) - Log[1 -
Tan[c + d*x]^(2/3) + Tan[c + d*x]^(4/3)]/(3*a*d) - 2/(a*d*Tan[c + d*x]^(2/3
)) + 1/(2*d*Tan[c + d*x]^(2/3)*(a + I*a*Tan[c + d*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3633

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-a)*((c + d*Tan[e + f*x])^(n + 1)/(2*f*(b*
c - a*d)*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*
Tan[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &
& NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\tan^{\frac{5}{3}}(c+dx)(a+ia\tan(c+dx))}\,dx = \frac{1}{2d\tan^{\frac{2}{3}}(c+dx)(a+ia\tan(c+dx))} - \frac{\int \frac{-\frac{8a}{3}+\frac{5}{3}ia\tan(c+dx)}{\tan^{\frac{5}{3}}(c+dx)}\,dx}{2a^2}$$

$$= -\frac{2}{ad\tan^{\frac{2}{3}}(c+dx)} + \frac{1}{2d\tan^{\frac{2}{3}}(c+dx)(a+ia\tan(c+dx))} - \frac{\int \frac{\frac{5ia}{3} \cdots}{\mathrm{t} \cdots}}{\cdots}$$

$$= -\frac{2}{ad\tan^{\frac{2}{3}}(c+dx)} + \frac{1}{2d\tan^{\frac{2}{3}}(c+dx)(a+ia\tan(c+dx))} - \frac{(5i)\int \cdots}{\cdots}$$

$$= -\frac{2}{ad\tan^{\frac{2}{3}}(c+dx)} + \frac{1}{2d\tan^{\frac{2}{3}}(c+dx)(a+ia\tan(c+dx))} - \frac{(5i)\mathrm{S} \cdots}{\cdots}$$

$$= -\frac{2}{ad\tan^{\frac{2}{3}}(c+dx)} + \frac{1}{2d\tan^{\frac{2}{3}}(c+dx)(a+ia\tan(c+dx))} - \frac{(5i)\mathrm{S} \cdots}{\cdots}$$

$$= -\frac{2}{ad\tan^{\frac{2}{3}}(c+dx)} + \frac{1}{2d\tan^{\frac{2}{3}}(c+dx)(a+ia\tan(c+dx))} - \frac{(5i)\mathrm{S} \cdots}{\cdots}$$

$$= -\frac{5i\tan^{-1}\left(\sqrt[3]{\tan(c+dx)}\right)}{6ad} - \frac{2}{ad\tan^{\frac{2}{3}}(c+dx)} + \frac{\cdots}{2d\tan^{\frac{2}{3}}(c+dx \cdots}$$

$$= -\frac{5i\tan^{-1}\left(\sqrt[3]{\tan(c+dx)}\right)}{6ad} + \frac{2\log\left(1+\tan^{\frac{2}{3}}(c+dx)\right)}{3ad} + \frac{5i\log \cdots}{\cdots}$$

$$= \frac{5i\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{12ad} - \frac{5i\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(c+ \cdots}\right.}{12ad}$$

$$= \frac{5i\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{12ad} - \frac{5i\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(c+ \cdots}\right.}{12ad}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.72, size = 201, normalized size = 0.63

$$\frac{i\csc(c+dx)\sec(c+dx)\left(-32^{2/3}\left(-1+e^{2i(c+dx)}\right)\sqrt[3]{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{1}{2}\left(1-e^{2i(c+dx)}\right)\right)+2\left(6+6\cos(2(c+dx))+13\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};-\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)(-1+\cos(2(c+dx))+i\sin(2(c+dx)))+8i\sin(2(c+dx))\right)\right)\sqrt[3]{\tan(c+dx)}}{16ad(-i+\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Tan[c + d*x]^(5/3)*(a + I*a*Tan[c + d*x])),x]
```

```
[Out] ((I/16)*Csc[c + d*x]*Sec[c + d*x]*(-3*2^(2/3)*(-1 + E^((2*I)*(c + d*x)))*(1
 + E^((2*I)*(c + d*x)))^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, (1 - E^((2*I
)*(c + d*x)))/2] + 2*(6 + 6*Cos[2*(c + d*x)] + 13*Hypergeometric2F1[1/3, 1,
 4/3, -((-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))]*(-1 + Cos[2*
(c + d*x)] + I*Sin[2*(c + d*x)]) + (8*I)*Sin[2*(c + d*x)]))*Tan[c + d*x]^(1
/3))/(a*d*(-I + Tan[c + d*x]))
```

Maple [A]
time = 0.15, size = 199, normalized size = 0.62

method	result
derivativedivides	$-\frac{i}{6\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}+\frac{13\ln\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}{12}-\frac{\ln\left(i\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)-1\right)}{8}+\frac{i\sqrt{3}\ \operatorname{arctanh}\left(\frac{\left(i+2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{3}\right.}{4}$
default	$-\frac{i}{6\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}+\frac{13\ln\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}{12}-\frac{\ln\left(i\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)-1\right)}{8}+\frac{i\sqrt{3}\ \operatorname{arctanh}\left(\frac{\left(i+2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{3}\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(5/3)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/6*I/(tan(d*x+c)^(1/3)+I)+13/12*ln(tan(d*x+c)^(1/3)+I)-1/8*ln(I*ta
n(d*x+c)^(1/3)+tan(d*x+c)^(2/3)-1)+1/4*I*3^(1/2)*arctanh(1/3*(I+2*tan(d*x+c
)^(1/3))*3^(1/2))+1/12*(2*I*tan(d*x+c)^(1/3)-2)/(-I*tan(d*x+c)^(1/3)+tan(d*
x+c)^(2/3)-1)-13/24*ln(-I*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3)-1)-13/12*I*3^(1
/2)*arctanh(1/3*(-I+2*tan(d*x+c)^(1/3))*3^(1/2))-3/2/tan(d*x+c)^(2/3)+1/4*l
n(tan(d*x+c)^(1/3)-I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(5/3)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 642 vs. $2(254) = 508$.
time = 0.76, size = 642, normalized size = 2.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(5/3)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/24*(3*(sqrt(3)*(-I*a*d*e^(4*I*d*x + 4*I*c) + I*a*d*e^(2*I*d*x + 2*I*c))*
sqrt(1/(a^2*d^2)) + e^(4*I*d*x + 4*I*c) - e^(2*I*d*x + 2*I*c))*log(1/2*sqrt
(3)*a*d*sqrt(1/(a^2*d^2)) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I
*c) + 1))^(1/3) + 1/2*I) + 3*(sqrt(3)*(I*a*d*e^(4*I*d*x + 4*I*c) - I*a*d*e^
(2*I*d*x + 2*I*c))*sqrt(1/(a^2*d^2)) + e^(4*I*d*x + 4*I*c) - e^(2*I*d*x + 2
*I*c))*log(-1/2*sqrt(3)*a*d*sqrt(1/(a^2*d^2)) + ((-I*e^(2*I*d*x + 2*I*c) +
I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) + 1/2*I) + 13*(3*sqrt(1/3)*(I*a*d*e^(4*
I*d*x + 4*I*c) - I*a*d*e^(2*I*d*x + 2*I*c))*sqrt(1/(a^2*d^2)) + e^(4*I*d*x
+ 4*I*c) - e^(2*I*d*x + 2*I*c))*log(3/2*sqrt(1/3)*a*d*sqrt(1/(a^2*d^2)) + (
(-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) - 1/2*I) + 13
*(3*sqrt(1/3)*(-I*a*d*e^(4*I*d*x + 4*I*c) + I*a*d*e^(2*I*d*x + 2*I*c))*sqrt
(1/(a^2*d^2)) + e^(4*I*d*x + 4*I*c) - e^(2*I*d*x + 2*I*c))*log(-3/2*sqrt(1/
3)*a*d*sqrt(1/(a^2*d^2)) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*
c) + 1))^(1/3) - 1/2*I) - 26*(e^(4*I*d*x + 4*I*c) - e^(2*I*d*x + 2*I*c))*lo
g(((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) + I) - 6*(
e^(4*I*d*x + 4*I*c) - e^(2*I*d*x + 2*I*c))*log(((-I*e^(2*I*d*x + 2*I*c) + I
)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) - I) + 6*((-I*e^(2*I*d*x + 2*I*c) + I)/(
e^(2*I*d*x + 2*I*c) + 1))^(1/3)*(7*I*e^(4*I*d*x + 4*I*c) + 6*I*e^(2*I*d*x +
 2*I*c) - I))/(a*d*e^(4*I*d*x + 4*I*c) - a*d*e^(2*I*d*x + 2*I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{1}{\tan^{\frac{8}{3}}(c+dx)-i\tan^{\frac{5}{3}}(c+dx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(5/3)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I*Integral(1/(tan(c + d*x)**(8/3) - I*tan(c + d*x)**(5/3)), x)/a
```

Giac [A]
time = 1.23, size = 231, normalized size = 0.72

$$\frac{13i\sqrt{3}\log\left(-\frac{\sqrt{3}-2\tan(dx+c)^{\frac{1}{3}}+i}{\sqrt{3}+2\tan(dx+c)^{\frac{1}{3}}-i}\right)}{24ad}-\frac{i\sqrt{3}\log\left(-\frac{\sqrt{3}-2\tan(dx+c)^{\frac{1}{3}}-i}{\sqrt{3}+2\tan(dx+c)^{\frac{1}{3}}+i}\right)}{8ad}-\frac{\log\left(\tan(dx+c)^{\frac{2}{3}}+i\tan(dx+c)^{\frac{1}{3}}-1\right)}{8ad}-\frac{13\log\left(\tan(dx+c)^{\frac{2}{3}}-i\tan(dx+c)^{\frac{1}{3}}-1\right)}{24ad}+\frac{13\log\left(\tan(dx+c)^{\frac{1}{3}}+i\right)}{12ad}+\frac{\log\left(\tan(dx+c)^{\frac{1}{3}}-i\right)}{4ad}-\frac{3}{2ad\tan(dx+c)^{\frac{2}{3}}}-\frac{\tan(dx+c)^{\frac{1}{3}}}{2ad(\tan(dx+c)-i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(5/3)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 13/24*I*sqrt(3)*log(-(sqrt(3) - 2*tan(d*x + c)^(1/3) + I)/(sqrt(3) + 2*tan(
d*x + c)^(1/3) - I))/(a*d) - 1/8*I*sqrt(3)*log(-(sqrt(3) - 2*tan(d*x + c)^(
1/3) - I)/(sqrt(3) + 2*tan(d*x + c)^(1/3) + I))/(a*d) - 1/8*log(tan(d*x + c
```

```
)^(2/3) + I*tan(d*x + c)^(1/3) - 1)/(a*d) - 13/24*log(tan(d*x + c)^(2/3) -
I*tan(d*x + c)^(1/3) - 1)/(a*d) + 13/12*log(tan(d*x + c)^(1/3) + I)/(a*d) +
 1/4*log(tan(d*x + c)^(1/3) - I)/(a*d) - 3/2/(a*d*tan(d*x + c)^(2/3)) - 1/2
*tan(d*x + c)^(1/3)/(a*d*(tan(d*x + c) - I))
```

Mupad [B]
time = 5.36, size = 630, normalized size = 1.96

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(5/3)*(a + a*tan(c + d*x)*1i)),x)
```

```
[Out] log((a^3*d^3*312480i + 331776*a^5*d^5*tan(c + d*x)^(1/3)*(1/(64*a^3*d^3))^(
2/3))*(1/(64*a^3*d^3))^(1/3) - 83304*a^2*d^2*tan(c + d*x)^(1/3))*(1/(64*a^3
*d^3))^(1/3) - (3/(2*a*d) + (tan(c + d*x)*2i)/(a*d))/(tan(c + d*x)^(2/3) +
tan(c + d*x)^(5/3)*1i) + (13*log((13*(a^3*d^3*312480i + 389376*a^5*d^5*tan(
c + d*x)^(1/3)*(1/(a^3*d^3))^(2/3))*(1/(a^3*d^3))^(1/3))/12 - 83304*a^2*d^2
*tan(c + d*x)^(1/3))*(1/(a^3*d^3))^(1/3))/12 + (13*log((13*(3^(1/2)*1i - 1)
*(a^3*d^3*312480i + 97344*a^5*d^5*tan(c + d*x)^(1/3)*(3^(1/2)*1i - 1)^2*(1/
(a^3*d^3))^(2/3))*(1/(a^3*d^3))^(1/3))/24 - 83304*a^2*d^2*tan(c + d*x)^(1/3
))*(3^(1/2)*1i - 1)*(1/(a^3*d^3))^(1/3))/24 - (13*log((13*(3^(1/2)*1i + 1)*
(a^3*d^3*312480i + 97344*a^5*d^5*tan(c + d*x)^(1/3)*(3^(1/2)*1i + 1)^2*(1/(
a^3*d^3))^(2/3))*(1/(a^3*d^3))^(1/3))/24 + 83304*a^2*d^2*tan(c + d*x)^(1/3)
)*(3^(1/2)*1i + 1)*(1/(a^3*d^3))^(1/3))/24 + log(((3^(1/2)*1i)/2 - 1/2)*(a^
3*d^3*312480i + 331776*a^5*d^5*tan(c + d*x)^(1/3)*((3^(1/2)*1i)/2 - 1/2)^2*
(1/(64*a^3*d^3))^(2/3))*(1/(64*a^3*d^3))^(1/3) - 83304*a^2*d^2*tan(c + d*x)
^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(1/(64*a^3*d^3))^(1/3) - log(((3^(1/2)*1i)/2
 + 1/2)*(a^3*d^3*312480i + 331776*a^5*d^5*tan(c + d*x)^(1/3)*((3^(1/2)*1i)/
2 + 1/2)^2*(1/(64*a^3*d^3))^(2/3))*(1/(64*a^3*d^3))^(1/3) + 83304*a^2*d^2*t
an(c + d*x)^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(1/(64*a^3*d^3))^(1/3)
```

3.239 $\int \frac{1}{\tan^{\frac{7}{3}}(c+dx)(a+ia\tan(c+dx))}\,dx$

Optimal. Leaf size=347

$$-\frac{7i\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{12ad}+\frac{7i\text{ArcTan}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{12ad}+\frac{5\text{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)}{2\sqrt{3}\,ad}+\frac{7i\ldots}{}$$

```
[Out] 7/12*I*arctan(-3^(1/2)+2*tan(d*x+c)^(1/3))/a/d+7/12*I*arctan(3^(1/2)+2*tan(
d*x+c)^(1/3))/a/d+7/6*I*arctan(tan(d*x+c)^(1/3))/a/d-5/6*ln(1+tan(d*x+c)^(2
/3))/a/d+5/12*ln(1-tan(d*x+c)^(2/3)+tan(d*x+c)^(4/3))/a/d+5/6*arctan(1/3*(1
-2*tan(d*x+c)^(2/3))*3^(1/2))/a/d*3^(1/2)+7/24*I*ln(1-3^(1/2)*tan(d*x+c)^(1
/3)+tan(d*x+c)^(2/3))/a/d*3^(1/2)-7/24*I*ln(1+3^(1/2)*tan(d*x+c)^(1/3)+tan(
d*x+c)^(2/3))/a/d*3^(1/2)-5/4/a/d/tan(d*x+c)^(4/3)+7/2*I/a/d/tan(d*x+c)^(1/
3)+1/2/d/tan(d*x+c)^(4/3)/(a+I*a*tan(d*x+c))
```

Rubi [A]

time = 0.36, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 14, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3633, 3610, 3619, 3557, 335, 281, 206, 31, 648, 632, 210, 642, 301, 209}

$$\frac{5\text{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)}{2\sqrt{3}\,ad}-\frac{7i\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{12ad}+\frac{7i\text{ArcTan}\left(2\sqrt[3]{\tan(c+dx)}+\sqrt{3}\right)}{12ad}+\frac{7i\text{ArcTan}\left(\sqrt[3]{\tan(c+dx)}\right)}{6ad}+\frac{1}{2d\tan^{\frac{4}{3}}(c+dx)(a+ia\tan(c+dx))}-\frac{5}{4ad\tan^{\frac{4}{3}}(c+dx)}+\frac{7i}{2ad\sqrt[3]{\tan(c+dx)}}-\frac{5\log\left(\tan^{\frac{2}{3}}(c+dx)+1\right)}{6ad}+\frac{7i\log\left(\tan^{\frac{2}{3}}(c+dx)-\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{8\sqrt{3}\,ad}-\frac{7i\log\left(\tan^{\frac{2}{3}}(c+dx)+\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{8\sqrt{3}\,ad}+\frac{5\log\left(\tan^{\frac{4}{3}}(c+dx)-\tan^{\frac{2}{3}}(c+dx)+1\right)}{12ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(7/3)*(a + I*a*Tan[c + d*x])),x]
```

```
[Out] (((-7*I)/12)*ArcTan[Sqrt[3] - 2*Tan[c + d*x]^(1/3)])/(a*d) + (((7*I)/12)*Ar
cTan[Sqrt[3] + 2*Tan[c + d*x]^(1/3)])/(a*d) + (5*ArcTan[(1 - 2*Tan[c + d*x]
^(2/3))/Sqrt[3]])/(2*Sqrt[3]*a*d) + (((7*I)/6)*ArcTan[Tan[c + d*x]^(1/3)])/
(a*d) - (5*Log[1 + Tan[c + d*x]^(2/3)])/(6*a*d) + (((7*I)/8)*Log[1 - Sqrt[3
]*Tan[c + d*x]^(1/3) + Tan[c + d*x]^(2/3)])/(Sqrt[3]*a*d) - (((7*I)/8)*Log[
1 + Sqrt[3]*Tan[c + d*x]^(1/3) + Tan[c + d*x]^(2/3)])/(Sqrt[3]*a*d) + (5*Lo
g[1 - Tan[c + d*x]^(2/3) + Tan[c + d*x]^(4/3)])/(12*a*d) - 5/(4*a*d*Tan[c +
 d*x]^(4/3)) + ((7*I)/2)/(a*d*Tan[c + d*x]^(1/3)) + 1/(2*d*Tan[c + d*x]^(4/
3)*(a + I*a*Tan[c + d*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3633

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-a)*((c + d*Tan[e + f*x])^(n + 1)/(2*f*(b*
c - a*d)*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*
Tan[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &
& NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\tan^{\frac{7}{3}}(c+dx)(a+ia\tan(c+dx))}\,dx = \frac{1}{2d\tan^{\frac{4}{3}}(c+dx)(a+ia\tan(c+dx))} - \frac{\int \frac{-\frac{10a}{3}+\frac{7}{3}ia\tan(c+dx)}{\tan^{\frac{7}{3}}(c+dx)}\,dx}{2a^2}$$

$$= -\frac{5}{4ad\tan^{\frac{4}{3}}(c+dx)} + \frac{1}{2d\tan^{\frac{4}{3}}(c+dx)(a+ia\tan(c+dx))} - \frac{\int}{}$$

$$= -\frac{5}{4ad\tan^{\frac{4}{3}}(c+dx)} + \frac{7i}{2ad\sqrt[3]{\tan(c+dx)}} + \frac{1}{2d\tan^{\frac{4}{3}}(c+dx)(a+}$$

$$= -\frac{5}{4ad\tan^{\frac{4}{3}}(c+dx)} + \frac{7i}{2ad\sqrt[3]{\tan(c+dx)}} + \frac{1}{2d\tan^{\frac{4}{3}}(c+dx)(a+}$$

$$= -\frac{5}{4ad\tan^{\frac{4}{3}}(c+dx)} + \frac{7i}{2ad\sqrt[3]{\tan(c+dx)}} + \frac{1}{2d\tan^{\frac{4}{3}}(c+dx)(a+}$$

$$= -\frac{5}{4ad\tan^{\frac{4}{3}}(c+dx)} + \frac{7i}{2ad\sqrt[3]{\tan(c+dx)}} + \frac{1}{2d\tan^{\frac{4}{3}}(c+dx)(a+}$$

$$= -\frac{5}{4ad\tan^{\frac{4}{3}}(c+dx)} + \frac{7i}{2ad\sqrt[3]{\tan(c+dx)}} + \frac{1}{2d\tan^{\frac{4}{3}}(c+dx)(a+}$$

$$= \frac{7i\tan^{-1}\left(\sqrt[3]{\tan(c+dx)}\right)}{6ad} - \frac{5}{4ad\tan^{\frac{4}{3}}(c+dx)} + \frac{7i}{2ad\sqrt[3]{\tan(c+}}$$

$$= \frac{7i\tan^{-1}\left(\sqrt[3]{\tan(c+dx)}\right)}{6ad} - \frac{5\log\left(1+\tan^{\frac{2}{3}}(c+dx)\right)}{6ad} + \frac{7i\log}{}$$

$$= -\frac{7i\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{12ad} + \frac{7i\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(}\right.}{12ad}$$

$$= -\frac{7i\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{12ad} + \frac{7i\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(}\right.}{12ad}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 2.03, size = 236, normalized size = 0.68

$$-\frac{i\csc^2(c+dx)\sec(c+dx)\left(3\sqrt[3]{2}\,e^{-i(c+dx)}\left(-1+e^{2i(c+dx)}\right)^2\left(1+e^{2i(c+dx)}\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{1}{2}\left(1-e^{2i(c+dx)}\right)\right)+4\left(-23\cos(c+dx)+11\cos(3(c+dx))+9i\sin(c+dx)-34\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)(\cos(c+dx)+i\sin(c+dx))\sin^2(c+dx)+9i\sin(3(c+dx))\right)\right)\tan^{\frac{2}{3}}(c+dx)}{64ad(-i+\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Tan[c + d*x]^(7/3)*(a + I*a*Tan[c + d*x])),x]
```

```
[Out] ((-1/64*I)*Csc[c + d*x]^2*Sec[c + d*x]*((3*2^(1/3)*(-1 + E^((2*I)*(c + d*x)
))^2*(1 + E^((2*I)*(c + d*x)))^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, (1 -
E^((2*I)*(c + d*x)))/2])/E^(I*(c + d*x)) + 4*(-23*Cos[c + d*x] + 11*Cos[3*(
c + d*x)] + (9*I)*Sin[c + d*x] - 34*Hypergeometric2F1[2/3, 1, 5/3, -((-1 +
E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))]*(Cos[c + d*x] + I*Sin[c +
d*x])*Sin[c + d*x]^2 + (9*I)*Sin[3*(c + d*x)]))*Tan[c + d*x]^(2/3))/(a*d*(-
I + Tan[c + d*x]))
```

Maple [A]
time = 0.14, size = 210, normalized size = 0.61

method	result
derivativedivides	$\frac{\frac{i}{6\left(\tan^{\frac{1}{3}}(dx+c)\right)+6i}-\frac{17\ln\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}{12}+\frac{\ln\left(i\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)-1\right)}{8}+\frac{i\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(i+2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)\sqrt{\cdots}}{3}\cdots\right.}{4}\cdots}{\cdots}$
default	$\frac{\frac{i}{6\left(\tan^{\frac{1}{3}}(dx+c)\right)+6i}-\frac{17\ln\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}{12}+\frac{\ln\left(i\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)-1\right)}{8}+\frac{i\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(i+2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)\sqrt{\cdots}}{3}\cdots\right.}{4}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(7/3)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(1/6*I/(tan(d*x+c)^(1/3)+I)-17/12*ln(tan(d*x+c)^(1/3)+I)+1/8*ln(I*tan
(d*x+c)^(1/3)+tan(d*x+c)^(2/3)-1)+1/4*I*3^(1/2)*arctanh(1/3*(I+2*tan(d*x+c)
^(1/3))*3^(1/2))-1/12*(-4*I*tan(d*x+c)^(1/3)-2)/(-I*tan(d*x+c)^(1/3)+tan(d*
x+c)^(2/3)-1)+17/24*ln(-I*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3)-1)-17/12*I*3^(1
/2)*arctanh(1/3*(-I+2*tan(d*x+c)^(1/3))*3^(1/2))-3/4/tan(d*x+c)^(4/3)+3*I/t
an(d*x+c)^(1/3)-1/4*ln(tan(d*x+c)^(1/3)-I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(7/3)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 787 vs. $2(270) = 540$.
time = 0.68, size = 787, normalized size = 2.27

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(7/3)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/24*(3*(sqrt(3)*(-I*a*d*e^(6*I*d*x + 6*I*c) + 2*I*a*d*e^(4*I*d*x + 4*I*c)
 - I*a*d*e^(2*I*d*x + 2*I*c))*sqrt(1/(a^2*d^2)) - e^(6*I*d*x + 6*I*c) + 2*e
^(4*I*d*x + 4*I*c) - e^(2*I*d*x + 2*I*c))*log(1/2*sqrt(3)*a*d*sqrt(1/(a^2*d
^2)) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) + 1/2
*I) + 3*(sqrt(3)*(I*a*d*e^(6*I*d*x + 6*I*c) - 2*I*a*d*e^(4*I*d*x + 4*I*c) +
 I*a*d*e^(2*I*d*x + 2*I*c))*sqrt(1/(a^2*d^2)) - e^(6*I*d*x + 6*I*c) + 2*e^(
4*I*d*x + 4*I*c) - e^(2*I*d*x + 2*I*c))*log(-1/2*sqrt(3)*a*d*sqrt(1/(a^2*d^
2)) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) + 1/2*
I) + 17*(3*sqrt(1/3)*(I*a*d*e^(6*I*d*x + 6*I*c) - 2*I*a*d*e^(4*I*d*x + 4*I*
c) + I*a*d*e^(2*I*d*x + 2*I*c))*sqrt(1/(a^2*d^2)) - e^(6*I*d*x + 6*I*c) + 2
*e^(4*I*d*x + 4*I*c) - e^(2*I*d*x + 2*I*c))*log(3/2*sqrt(1/3)*a*d*sqrt(1/(a
^2*d^2)) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) -
 1/2*I) + 17*(3*sqrt(1/3)*(-I*a*d*e^(6*I*d*x + 6*I*c) + 2*I*a*d*e^(4*I*d*x
+ 4*I*c) - I*a*d*e^(2*I*d*x + 2*I*c))*sqrt(1/(a^2*d^2)) - e^(6*I*d*x + 6*I*
c) + 2*e^(4*I*d*x + 4*I*c) - e^(2*I*d*x + 2*I*c))*log(-3/2*sqrt(1/3)*a*d*sq
rt(1/(a^2*d^2)) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^
(1/3) - 1/2*I) + 34*(e^(6*I*d*x + 6*I*c) - 2*e^(4*I*d*x + 4*I*c) + e^(2*I*d
*x + 2*I*c))*log(((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(
1/3) + I) + 6*(e^(6*I*d*x + 6*I*c) - 2*e^(4*I*d*x + 4*I*c) + e^(2*I*d*x + 2
*I*c))*log(((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) -
 I) + 6*((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(10*
e^(6*I*d*x + 6*I*c) - 7*e^(4*I*d*x + 4*I*c) - 16*e^(2*I*d*x + 2*I*c) + 1))/
(a*d*e^(6*I*d*x + 6*I*c) - 2*a*d*e^(4*I*d*x + 4*I*c) + a*d*e^(2*I*d*x + 2*I
*c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{1}{\tan^{\frac{10}{3}}(c+dx)-i\tan^{\frac{7}{3}}(c+dx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(7/3)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I*Integral(1/(tan(c + d*x)**(10/3) - I*tan(c + d*x)**(7/3)), x)/a
```

Giac [A]

time = 0.79, size = 241, normalized size = 0.69

$$\frac{17i\sqrt{3}\log\left(-\frac{\sqrt{3}-2\tan(dx+c)^{\frac{1}{3}}+i}{\sqrt{3}+2\tan(dx+c)^{\frac{1}{3}}-i}\right)}{24\,ad}-\frac{i\sqrt{3}\log\left(-\frac{\sqrt{3}-2\tan(dx+c)^{\frac{1}{3}}-i}{\sqrt{3}+2\tan(dx+c)^{\frac{1}{3}}+i}\right)}{8\,ad}+\frac{\log\left(\tan(dx+c)^{\frac{2}{3}}+i\tan(dx+c)^{\frac{1}{3}}-1\right)}{8\,ad}+\frac{17\log\left(\tan(dx+c)^{\frac{2}{3}}-i\tan(dx+c)^{\frac{1}{3}}-1\right)}{24\,ad}-\frac{17\log\left(\tan(dx+c)^{\frac{1}{3}}+i\right)}{12\,ad}-\frac{\log\left(\tan(dx+c)^{\frac{1}{3}}-i\right)}{4\,ad}-\frac{3(-4i\tan(dx+c)+1)}{4\,ad\tan(dx+c)^{\frac{4}{3}}}+\frac{i\tan(dx+c)^{\frac{2}{3}}}{2\,ad(\tan(dx+c)-i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(7/3)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 17/24*I*sqrt(3)*log(-(sqrt(3) - 2*tan(d*x + c)^(1/3) + I)/(sqrt(3) + 2*tan(
d*x + c)^(1/3) - I))/(a*d) - 1/8*I*sqrt(3)*log(-(sqrt(3) - 2*tan(d*x + c)^(
1/3) - I)/(sqrt(3) + 2*tan(d*x + c)^(1/3) + I))/(a*d) + 1/8*log(tan(d*x + c
)^(2/3) + I*tan(d*x + c)^(1/3) - 1)/(a*d) + 17/24*log(tan(d*x + c)^(2/3) -
I*tan(d*x + c)^(1/3) - 1)/(a*d) - 17/12*log(tan(d*x + c)^(1/3) + I)/(a*d) -
 1/4*log(tan(d*x + c)^(1/3) - I)/(a*d) - 3/4*(-4*I*tan(d*x + c) + 1)/(a*d*t
an(d*x + c)^(4/3)) + 1/2*I*tan(d*x + c)^(2/3)/(a*d*(tan(d*x + c) - I))
```

Mupad [B]

time = 5.75, size = 611, normalized size = 1.76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(7/3)*(a + a*tan(c + d*x)*1i)),x)
```

```
[Out] log(52020*a*d*tan(c + d*x)^(1/3) - (a^3*d^3*703584i - 514944*a^4*d^4*tan(c
+ d*x)^(1/3)*(-1/(64*a^3*d^3))^(1/3))*(-1/(64*a^3*d^3))^(2/3))*(-1/(64*a^3*
d^3))^(1/3) + (17*log(52020*a*d*tan(c + d*x)^(1/3) - (289*(a^3*d^3*703584i
- 729504*a^4*d^4*tan(c + d*x)^(1/3)*(-1/(a^3*d^3))^(1/3))*(-1/(a^3*d^3))^(2
/3))/144)*(-1/(a^3*d^3))^(1/3))/12 - (3/(4*a*d) - (tan(c + d*x)*9i)/(4*a*d)
 + (7*tan(c + d*x)^2)/(2*a*d))/(tan(c + d*x)^(4/3) + tan(c + d*x)^(7/3)*1i)
 + (17*log(52020*a*d*tan(c + d*x)^(1/3) - (289*(3^(1/2)*1i - 1)^2*(a^3*d^3*
703584i - 364752*a^4*d^4*tan(c + d*x)^(1/3)*(3^(1/2)*1i - 1)*(-1/(a^3*d^3))
^(1/3))*(-1/(a^3*d^3))^(2/3))/576)*(3^(1/2)*1i - 1)*(-1/(a^3*d^3))^(1/3))/2
4 - (17*log(52020*a*d*tan(c + d*x)^(1/3) - (289*(3^(1/2)*1i + 1)^2*(a^3*d^3
*703584i + 364752*a^4*d^4*tan(c + d*x)^(1/3)*(3^(1/2)*1i + 1)*(-1/(a^3*d^3)
)^(1/3))*(-1/(a^3*d^3))^(2/3))/576)*(3^(1/2)*1i + 1)*(-1/(a^3*d^3))^(1/3))/
24 + log(52020*a*d*tan(c + d*x)^(1/3) - ((3^(1/2)*1i)/2 - 1/2)^2*(a^3*d^3*7
03584i - 514944*a^4*d^4*tan(c + d*x)^(1/3)*((3^(1/2)*1i)/2 - 1/2)*(-1/(64*a
^3*d^3))^(1/3))*(-1/(64*a^3*d^3))^(2/3))*((3^(1/2)*1i)/2 - 1/2)*(-1/(64*a^3
*d^3))^(1/3) - log(52020*a*d*tan(c + d*x)^(1/3) - ((3^(1/2)*1i)/2 + 1/2)^2*
(a^3*d^3*703584i + 514944*a^4*d^4*tan(c + d*x)^(1/3)*((3^(1/2)*1i)/2 + 1/2)
*(-1/(64*a^3*d^3))^(1/3))*(-1/(64*a^3*d^3))^(2/3))*((3^(1/2)*1i)/2 + 1/2)*(
-1/(64*a^3*d^3))^(1/3)
```

3.240 $\int \frac{\tan^{\frac{14}{3}}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=379

$$-\frac{121\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d}+\frac{121\text{ArcTan}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d}-\frac{14i\text{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)}{3\sqrt{3}\,a^2d}$$

```
[Out] 121/72*arctan(-3^(1/2)+2*tan(d*x+c)^(1/3))/a^2/d+121/72*arctan(3^(1/2)+2*ta
n(d*x+c)^(1/3))/a^2/d+121/36*arctan(tan(d*x+c)^(1/3))/a^2/d+14/9*I*ln(1+tan
(d*x+c)^(2/3))/a^2/d-7/9*I*ln(1-tan(d*x+c)^(2/3)+tan(d*x+c)^(4/3))/a^2/d-14
/9*I*arctan(1/3*(1-2*tan(d*x+c)^(2/3))*3^(1/2))/a^2/d*3^(1/2)+121/144*ln(1-
3^(1/2)*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3))/a^2/d*3^(1/2)-121/144*ln(1+3^(1/
2)*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3))/a^2/d*3^(1/2)-14/3*I*tan(d*x+c)^(2/3)
/a^2/d-121/60*tan(d*x+c)^(5/3)/a^2/d+7/6*I*tan(d*x+c)^(8/3)/a^2/d/(1+I*tan(
d*x+c))-1/4*tan(d*x+c)^(11/3)/d/(a+I*a*tan(d*x+c))^2
```

Rubi [A]

time = 0.46, antiderivative size = 379, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 15, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {3639, 3676, 3609, 3619, 3557, 335, 281, 206, 31, 648, 632, 210, 642, 301, 209}

$$-\frac{14i\text{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)}{3\sqrt{3}\,a^2d}-\frac{121\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d}+\frac{121\text{ArcTan}\left(2\sqrt[3]{\tan(c+dx)}+\sqrt{3}\right)}{72a^2d}+\frac{121\text{ArcTan}\left(\sqrt[3]{\tan(c+dx)}\right)}{36a^2d}+\frac{7i\tan^{\frac{8}{3}}(c+dx)}{6a^2d(1+i\tan(c+dx))}-\frac{121\tan^{\frac{5}{3}}(c+dx)}{60a^2d}-\frac{14i\tan^{\frac{2}{3}}(c+dx)}{3a^2d}+\frac{14i\log\left(\tan^{\frac{2}{3}}(c+dx)+1\right)}{9a^2d}+\frac{121\log\left(\tan^{\frac{2}{3}}(c+dx)-\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{48\sqrt{3}\,a^2d}-\frac{121\log\left(\tan^{\frac{2}{3}}(c+dx)+\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{48\sqrt{3}\,a^2d}-\frac{7i\log\left(\tan^{\frac{4}{3}}(c+dx)-\tan^{\frac{2}{3}}(c+dx)+1\right)}{9a^2d}-\frac{\tan^{\frac{11}{3}}(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(14/3)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (-121*ArcTan[Sqrt[3] - 2*Tan[c + d*x]^(1/3)])/(72*a^2*d) + (121*ArcTan[Sqrt
[3] + 2*Tan[c + d*x]^(1/3)])/(72*a^2*d) - (((14*I)/3)*ArcTan[(1 - 2*Tan[c +
 d*x]^(2/3))/Sqrt[3]])/(Sqrt[3]*a^2*d) + (121*ArcTan[Tan[c + d*x]^(1/3)])/(
36*a^2*d) + (((14*I)/9)*Log[1 + Tan[c + d*x]^(2/3)])/(a^2*d) + (121*Log[1 -
 Sqrt[3]*Tan[c + d*x]^(1/3) + Tan[c + d*x]^(2/3)])/(48*Sqrt[3]*a^2*d) - (12
1*Log[1 + Sqrt[3]*Tan[c + d*x]^(1/3) + Tan[c + d*x]^(2/3)])/(48*Sqrt[3]*a^2
*d) - (((7*I)/9)*Log[1 - Tan[c + d*x]^(2/3) + Tan[c + d*x]^(4/3)])/(a^2*d)
- (((14*I)/3)*Tan[c + d*x]^(2/3))/(a^2*d) - (121*Tan[c + d*x]^(5/3))/(60*a^
2*d) + (((7*I)/6)*Tan[c + d*x]^(8/3))/(a^2*d*(1 + I*Tan[c + d*x])) - Tan[c
+ d*x]^(11/3)/(4*d*(a + I*a*Tan[c + d*x])^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\tan^{\frac{14}{3}}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\tan^{\frac{11}{3}}(c+dx)}{4d(a+ia\tan(c+dx))^2} - \frac{\int \frac{\tan^{\frac{8}{3}}(c+dx)\left(-\frac{11a}{3}+\frac{17}{3}ia\tan(c+dx)\right)}{a+ia\tan(c+dx)}\,dx}{4a^2}$$

$$= \frac{7i\tan^{\frac{8}{3}}(c+dx)}{6a^2d(1+i\tan(c+dx))} - \frac{\tan^{\frac{11}{3}}(c+dx)}{4d(a+ia\tan(c+dx))^2} + \frac{\int \tan^{\frac{5}{3}}(c+dx)\left(-\frac{224ia}{9}\right.}{}$$

$$= -\frac{121\tan^{\frac{5}{3}}(c+dx)}{60a^2d} + \frac{7i\tan^{\frac{8}{3}}(c+dx)}{6a^2d(1+i\tan(c+dx))} - \frac{\tan^{\frac{11}{3}}(c+dx)}{4d(a+ia\tan(c+dx))^2} +$$

$$= -\frac{14i\tan^{\frac{2}{3}}(c+dx)}{3a^2d} - \frac{121\tan^{\frac{5}{3}}(c+dx)}{60a^2d} + \frac{7i\tan^{\frac{8}{3}}(c+dx)}{6a^2d(1+i\tan(c+dx))} - \frac{\tan}{4d(a+}$$

$$= -\frac{14i\tan^{\frac{2}{3}}(c+dx)}{3a^2d} - \frac{121\tan^{\frac{5}{3}}(c+dx)}{60a^2d} + \frac{7i\tan^{\frac{8}{3}}(c+dx)}{6a^2d(1+i\tan(c+dx))} - \frac{\tan}{4d(a+}$$

$$= -\frac{14i\tan^{\frac{2}{3}}(c+dx)}{3a^2d} - \frac{121\tan^{\frac{5}{3}}(c+dx)}{60a^2d} + \frac{7i\tan^{\frac{8}{3}}(c+dx)}{6a^2d(1+i\tan(c+dx))} - \frac{\tan}{4d(a+}$$

$$= -\frac{14i\tan^{\frac{2}{3}}(c+dx)}{3a^2d} - \frac{121\tan^{\frac{5}{3}}(c+dx)}{60a^2d} + \frac{7i\tan^{\frac{8}{3}}(c+dx)}{6a^2d(1+i\tan(c+dx))} - \frac{\tan}{4d(a+}$$

$$= -\frac{14i\tan^{\frac{2}{3}}(c+dx)}{3a^2d} - \frac{121\tan^{\frac{5}{3}}(c+dx)}{60a^2d} + \frac{7i\tan^{\frac{8}{3}}(c+dx)}{6a^2d(1+i\tan(c+dx))} - \frac{\tan}{4d(a+}$$

$$= \frac{121\tan^{-1}\left(\sqrt[3]{\tan(c+dx)}\right)}{36a^2d} - \frac{14i\tan^{\frac{2}{3}}(c+dx)}{3a^2d} - \frac{121\tan^{\frac{5}{3}}(c+dx)}{60a^2d} + \frac{7}{6a^2d}$$

$$= \frac{121\tan^{-1}\left(\sqrt[3]{\tan(c+dx)}\right)}{36a^2d} + \frac{14i\log\left(1+\tan^{\frac{2}{3}}(c+dx)\right)}{9a^2d} + \frac{121\log\left(1-\sqrt{}\right.}{}$$

$$= -\frac{121\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d} + \frac{121\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right.}{72a^2d}$$

$$= -\frac{121\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d} + \frac{121\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right.}{72a^2d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 3.59, size = 210, normalized size = 0.55

$$\frac{\sec^2(c+dx)\left(90i\sqrt[3]{2}\,e^{2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{1}{2}\left(1-e^{2i(c+dx)}\right)\right)+4\left(344i+776i\cos(2(c+dx))+1165\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)(-i\cos(2(c+dx))+\sin(2(c+dx)))-403\sec(c+dx)\sin(3(c+dx))-547\tan(c+dx)\right)\right)\tan^{\frac{2}{3}}(c+dx)}{960a^2d(-i+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(14/3)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^2*((90*I)*2^(1/3)*E^((2*I)*(c + d*x))*(1 + E^((2*I)*(c + d*x)
))^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, (1 - E^((2*I)*(c + d*x)))/2] + 4*
(344*I + (776*I)*Cos[2*(c + d*x)] + 1165*Hypergeometric2F1[2/3, 1, 5/3, -((
-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))]*((-I)*Cos[2*(c + d*x)
] + Sin[2*(c + d*x)]) - 403*Sec[c + d*x]*Sin[3*(c + d*x)] - 547*Tan[c + d*x
]))*Tan[c + d*x]^(2/3))/(960*a^2*d*(-I + Tan[c + d*x])^2)
```

Maple [A]
time = 0.17, size = 246, normalized size = 0.65

method	result
derivativedivides	$-\frac{3\left(\tan^{\frac{5}{3}}(dx+c)\right)}{5}-3i\left(\tan^{\frac{2}{3}}(dx+c)\right)+\frac{i}{36\left(\tan^{\frac{1}{3}}(dx+c)+i\right)^{2}}+\frac{233i\ln\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}{72}-\frac{23}{36\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}+\frac{i\ln\left(i\left(\tan^{\frac{1}{3}}(dx\right.\right.}{}$
default	$-\frac{3\left(\tan^{\frac{5}{3}}(dx+c)\right)}{5}-3i\left(\tan^{\frac{2}{3}}(dx+c)\right)+\frac{i}{36\left(\tan^{\frac{1}{3}}(dx+c)+i\right)^{2}}+\frac{233i\ln\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}{72}-\frac{23}{36\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}+\frac{i\ln\left(i\left(\tan^{\frac{1}{3}}(dx\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(14/3)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(-3/5*tan(d*x+c)^(5/3)-3*I*tan(d*x+c)^(2/3)+1/36*I/(tan(d*x+c)^(1/3
)+I)^2+233/72*I*ln(tan(d*x+c)^(1/3)+I)-23/36/(tan(d*x+c)^(1/3)+I)+1/16*I*ln
(I*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3)-1)-1/8*3^(1/2)*arctanh(1/3*(I+2*tan(d*
x+c)^(1/3))*3^(1/2))-1/72*(92*tan(d*x+c)-136*I*tan(d*x+c)^(2/3)-130*tan(d*x
+c)^(1/3)+44*I)/(-I*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3)-1)^2-233/144*I*ln(-I*
tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3)-1)-233/72*3^(1/2)*arctanh(1/3*(-I+2*tan(d
*x+c)^(1/3))*3^(1/2))-1/8*I*ln(tan(d*x+c)^(1/3)-I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(14/3)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 684 vs. 2(298) = 596.
time = 0.63, size = 684, normalized size = 1.80

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(14/3)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/720*(45*(sqrt(3)*(a^2*d*e^(6*I*d*x + 6*I*c) + a^2*d*e^(4*I*d*x + 4*I*c))
*sqrt(1/(a^4*d^2)) - I*e^(6*I*d*x + 6*I*c) - I*e^(4*I*d*x + 4*I*c))*log(1/2
*sqrt(3)*a^2*d*sqrt(1/(a^4*d^2)) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*
x + 2*I*c) + 1))^(1/3) + 1/2*I) - 45*(sqrt(3)*(a^2*d*e^(6*I*d*x + 6*I*c) +
a^2*d*e^(4*I*d*x + 4*I*c))*sqrt(1/(a^4*d^2)) + I*e^(6*I*d*x + 6*I*c) + I*e^
(4*I*d*x + 4*I*c))*log(-1/2*sqrt(3)*a^2*d*sqrt(1/(a^4*d^2)) + ((-I*e^(2*I*d
*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) + 1/2*I) + 1165*(3*sqrt(1
/3)*(a^2*d*e^(6*I*d*x + 6*I*c) + a^2*d*e^(4*I*d*x + 4*I*c))*sqrt(1/(a^4*d^2
)) + I*e^(6*I*d*x + 6*I*c) + I*e^(4*I*d*x + 4*I*c))*log(3/2*sqrt(1/3)*a^2*d
*sqrt(1/(a^4*d^2)) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1
))^(1/3) - 1/2*I) - 1165*(3*sqrt(1/3)*(a^2*d*e^(6*I*d*x + 6*I*c) + a^2*d*e^
(4*I*d*x + 4*I*c))*sqrt(1/(a^4*d^2)) - I*e^(6*I*d*x + 6*I*c) - I*e^(4*I*d*x
 + 4*I*c))*log(-3/2*sqrt(1/3)*a^2*d*sqrt(1/(a^4*d^2)) + ((-I*e^(2*I*d*x + 2
*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) - 1/2*I) + 2330*(-I*e^(6*I*d*x
+ 6*I*c) - I*e^(4*I*d*x + 4*I*c))*log(((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I
*d*x + 2*I*c) + 1))^(1/3) + I) + 90*(I*e^(6*I*d*x + 6*I*c) + I*e^(4*I*d*x +
 4*I*c))*log(((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)
 - I) + 3*((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(7
91*I*e^(6*I*d*x + 6*I*c) + 1279*I*e^(4*I*d*x + 4*I*c) + 185*I*e^(2*I*d*x +
2*I*c) - 15*I))/(a^2*d*e^(6*I*d*x + 6*I*c) + a^2*d*e^(4*I*d*x + 4*I*c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(14/3)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4962 deep
```

Giac [A]
time = 0.80, size = 268, normalized size = 0.71

$$\frac{233\,\sqrt{3}\,\log\left(-\frac{\sqrt{3}-2\,\tan(dx+c)^{\frac{1}{3}}+i}{\sqrt{3}+2\,\tan(dx+c)^{\frac{1}{3}}-i}\right)}{144\,a^2d}+\frac{\sqrt{3}\,\log\left(-\frac{\sqrt{3}-2\,\tan(dx+c)^{\frac{1}{3}}-i}{\sqrt{3}+2\,\tan(dx+c)^{\frac{1}{3}}+i}\right)}{16\,a^2d}+\frac{i\,\log\left(\tan\left(dx+c\right)^{\frac{2}{3}}+i\,\tan\left(dx+c\right)^{\frac{1}{3}}-1\right)}{16\,a^2d}-\frac{233i\,\log\left(\tan\left(dx+c\right)^{\frac{2}{3}}-i\,\tan\left(dx+c\right)^{\frac{1}{3}}-1\right)}{144\,a^2d}+\frac{233i\,\log\left(\tan\left(dx+c\right)^{\frac{1}{3}}+i\right)}{72\,a^2d}-\frac{i\,\log\left(\tan\left(dx+c\right)^{\frac{1}{3}}-i\right)}{8\,a^2d}-\frac{23\,\tan\left(dx+c\right)^{\frac{5}{3}}-20i\,\tan\left(dx+c\right)^{\frac{2}{3}}}{12\,a^2d(\tan\left(dx+c\right)-i)^2}-\frac{3\left(a^8d^4\tan\left(dx+c\right)^{\frac{5}{3}}+5i\,a^8d^4\tan\left(dx+c\right)^{\frac{2}{3}}\right)}{5\,a^{10}d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(14/3)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 233/144*sqrt(3)*log(-(sqrt(3) - 2*tan(d*x + c)^(1/3) + I)/(sqrt(3) + 2*tan(
d*x + c)^(1/3) - I))/(a^2*d) + 1/16*sqrt(3)*log(-(sqrt(3) - 2*tan(d*x + c)^
(1/3) - I)/(sqrt(3) + 2*tan(d*x + c)^(1/3) + I))/(a^2*d) + 1/16*I*log(tan(d
*x + c)^(2/3) + I*tan(d*x + c)^(1/3) - 1)/(a^2*d) - 233/144*I*log(tan(d*x +
 c)^(2/3) - I*tan(d*x + c)^(1/3) - 1)/(a^2*d) + 233/72*I*log(tan(d*x + c)^(
1/3) + I)/(a^2*d) - 1/8*I*log(tan(d*x + c)^(1/3) - I)/(a^2*d) - 1/12*(23*ta
n(d*x + c)^(5/3) - 20*I*tan(d*x + c)^(2/3))/(a^2*d*(tan(d*x + c) - I)^2) -
3/5*(a^8*d^4*tan(d*x + c)^(5/3) + 5*I*a^8*d^4*tan(d*x + c)^(2/3))/(a^10*d^5
)
```

Mupad [B]
time = 5.46, size = 674, normalized size = 1.78

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(14/3)/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] log(((a^6*d^3*1619208448i)/3 - a^8*d^4*tan(c + d*x)^(1/3)*(1i/(512*a^6*d^3)
)^(1/3)*167024640i)*(1i/(512*a^6*d^3))^(2/3) + (24321472*a^2*d*tan(c + d*x)
^(1/3))/3)*(1i/(512*a^6*d^3))^(1/3) + log(((a^6*d^3*1619208448i)/3 - a^8*d^
4*tan(c + d*x)^(1/3)*(-12649337i/(373248*a^6*d^3))^(1/3)*167024640i)*(-1264
9337i/(373248*a^6*d^3))^(2/3) + (24321472*a^2*d*tan(c + d*x)^(1/3))/3)*(-12
649337i/(373248*a^6*d^3))^(1/3) - ((5*tan(c + d*x)^(2/3))/(3*a^2*d) + (tan(
c + d*x)^(5/3)*23i)/(12*a^2*d))/(2*tan(c + d*x) + tan(c + d*x)^2*1i - 1i) -
 (tan(c + d*x)^(2/3)*3i)/(a^2*d) - (3*tan(c + d*x)^(5/3))/(5*a^2*d) + (log(
(24321472*a^2*d*tan(c + d*x)^(1/3))/3 + ((3^(1/2)*1i - 1)^2*((a^6*d^3*16192
08448i)/3 - a^8*d^4*tan(c + d*x)^(1/3)*(3^(1/2)*1i - 1)*(1i/(512*a^6*d^3))^
(1/3)*83512320i)*(1i/(512*a^6*d^3))^(2/3))/4)*(3^(1/2)*1i - 1)*(1i/(512*a^6
*d^3))^(1/3))/2 - (log((24321472*a^2*d*tan(c + d*x)^(1/3))/3 + ((3^(1/2)*1i
 + 1)^2*((a^6*d^3*1619208448i)/3 + a^8*d^4*tan(c + d*x)^(1/3)*(3^(1/2)*1i +
 1)*(1i/(512*a^6*d^3))^(1/3)*83512320i)*(1i/(512*a^6*d^3))^(2/3))/4)*(3^(1/
2)*1i + 1)*(1i/(512*a^6*d^3))^(1/3))/2 + (log((24321472*a^2*d*tan(c + d*x)^
(1/3))/3 + ((3^(1/2)*1i - 1)^2*((a^6*d^3*1619208448i)/3 - a^8*d^4*tan(c + d
*x)^(1/3)*(3^(1/2)*1i - 1)*(-12649337i/(373248*a^6*d^3))^(1/3)*83512320i)*(
-12649337i/(373248*a^6*d^3))^(2/3))/4)*(3^(1/2)*1i - 1)*(-12649337i/(373248
*a^6*d^3))^(1/3))/2 - (log((24321472*a^2*d*tan(c + d*x)^(1/3))/3 + ((3^(1/2
)*1i + 1)^2*((a^6*d^3*1619208448i)/3 + a^8*d^4*tan(c + d*x)^(1/3)*(3^(1/2)*
1i + 1)*(-12649337i/(373248*a^6*d^3))^(1/3)*83512320i)*(-12649337i/(373248*
a^6*d^3))^(2/3))/4)*(3^(1/2)*1i + 1)*(-12649337i/(373248*a^6*d^3))^(1/3))/2
```

3.241 $\int \frac{\tan^{\frac{10}{3}}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=357

$$-\frac{49\mathrm{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d}+\frac{49\mathrm{ArcTan}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d}+\frac{5i\mathrm{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)}{3\sqrt{3}\,a^2d}+$$

```
[Out] 49/72*arctan(-3^(1/2)+2*tan(d*x+c)^(1/3))/a^2/d+49/72*arctan(3^(1/2)+2*tan(
d*x+c)^(1/3))/a^2/d+49/36*arctan(tan(d*x+c)^(1/3))/a^2/d+5/9*I*ln(1+tan(d*x
+c)^(2/3))/a^2/d-5/18*I*ln(1-tan(d*x+c)^(2/3)+tan(d*x+c)^(4/3))/a^2/d+5/9*I
*arctan(1/3*(1-2*tan(d*x+c)^(2/3))*3^(1/2))/a^2/d*3^(1/2)-49/144*ln(1-3^(1/
2)*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3))/a^2/d*3^(1/2)+49/144*ln(1+3^(1/2)*tan
(d*x+c)^(1/3)+tan(d*x+c)^(2/3))/a^2/d*3^(1/2)-49/12*tan(d*x+c)^(1/3)/a^2/d+
5/6*I*tan(d*x+c)^(4/3)/a^2/d/(1+I*tan(d*x+c))-1/4*tan(d*x+c)^(7/3)/d/(a+I*a
*tan(d*x+c))^2
```

Rubi [A]

time = 0.37, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 15, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {3639, 3676, 3609, 3619, 3557, 335, 215, 648, 632, 210, 642, 209, 281, 298, 31}

$$\frac{5i\mathrm{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)}{3\sqrt{3}\,a^2d}-\frac{49\mathrm{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d}+\frac{49\mathrm{ArcTan}\left(2\sqrt[3]{\tan(c+dx)}+\sqrt{3}\right)}{72a^2d}+\frac{49\mathrm{ArcTan}\left(\sqrt[3]{\tan(c+dx)}\right)}{36a^2d}+\frac{5i\tan^{\frac{4}{3}}(c+dx)}{6a^2d(1+i\tan(c+dx))}-\frac{49\sqrt[3]{\tan(c+dx)}}{12a^2d}+\frac{5i\log\left(\tan^{\frac{2}{3}}(c+dx)+1\right)}{9a^2d}-\frac{49\log\left(\tan^{\frac{2}{3}}(c+dx)-\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{48\sqrt{3}\,a^2d}+\frac{49\log\left(\tan^{\frac{2}{3}}(c+dx)+\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{48\sqrt{3}\,a^2d}-\frac{5i\log\left(\tan^{\frac{4}{3}}(c+dx)-\tan^{\frac{2}{3}}(c+dx)+1\right)}{18a^2d}-\frac{\tan^{\frac{7}{3}}(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(10/3)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (-49*ArcTan[Sqrt[3] - 2*Tan[c + d*x]^(1/3)])/(72*a^2*d) + (49*ArcTan[Sqrt[3
] + 2*Tan[c + d*x]^(1/3)])/(72*a^2*d) + (((5*I)/3)*ArcTan[(1 - 2*Tan[c + d*
x]^(2/3))/Sqrt[3]])/(Sqrt[3]*a^2*d) + (49*ArcTan[Tan[c + d*x]^(1/3)])/(36*a
^2*d) + (((5*I)/9)*Log[1 + Tan[c + d*x]^(2/3)])/(a^2*d) - (49*Log[1 - Sqrt[
3]*Tan[c + d*x]^(1/3) + Tan[c + d*x]^(2/3)])/(48*Sqrt[3]*a^2*d) + (49*Log[1
 + Sqrt[3]*Tan[c + d*x]^(1/3) + Tan[c + d*x]^(2/3)])/(48*Sqrt[3]*a^2*d) - (
((5*I)/18)*Log[1 - Tan[c + d*x]^(2/3) + Tan[c + d*x]^(4/3)])/(a^2*d) - (49*
Tan[c + d*x]^(1/3))/(12*a^2*d) + (((5*I)/6)*Tan[c + d*x]^(4/3))/(a^2*d*(1 +
 I*Tan[c + d*x])) - Tan[c + d*x]^(7/3)/(4*d*(a + I*a*Tan[c + d*x])^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\tan^{\frac{10}{3}}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\tan^{\frac{7}{3}}(c+dx)}{4d(a+ia\tan(c+dx))^2} - \frac{\int \frac{\tan^{\frac{4}{3}}(c+dx)\left(-\frac{7a}{3}+\frac{13}{3}ia\tan(c+dx)\right)}{a+ia\tan(c+dx)}\,dx}{4a^2}$$

$$= \frac{5i\tan^{\frac{4}{3}}(c+dx)}{6a^2d(1+i\tan(c+dx))} - \frac{\tan^{\frac{7}{3}}(c+dx)}{4d(a+ia\tan(c+dx))^2} + \frac{\int \sqrt[3]{\tan(c+dx)}\left(-\frac{80ia^2}{9}\right.}{8}$$

$$= -\frac{49\sqrt[3]{\tan(c+dx)}}{12a^2d} + \frac{5i\tan^{\frac{4}{3}}(c+dx)}{6a^2d(1+i\tan(c+dx))} - \frac{\tan^{\frac{7}{3}}(c+dx)}{4d(a+ia\tan(c+dx))^2} + \int$$

$$= -\frac{49\sqrt[3]{\tan(c+dx)}}{12a^2d} + \frac{5i\tan^{\frac{4}{3}}(c+dx)}{6a^2d(1+i\tan(c+dx))} - \frac{\tan^{\frac{7}{3}}(c+dx)}{4d(a+ia\tan(c+dx))^2} - (1$$

$$= -\frac{49\sqrt[3]{\tan(c+dx)}}{12a^2d} + \frac{5i\tan^{\frac{4}{3}}(c+dx)}{6a^2d(1+i\tan(c+dx))} - \frac{\tan^{\frac{7}{3}}(c+dx)}{4d(a+ia\tan(c+dx))^2} - (1$$

$$= -\frac{49\sqrt[3]{\tan(c+dx)}}{12a^2d} + \frac{5i\tan^{\frac{4}{3}}(c+dx)}{6a^2d(1+i\tan(c+dx))} - \frac{\tan^{\frac{7}{3}}(c+dx)}{4d(a+ia\tan(c+dx))^2} - (1$$

$$= -\frac{49\sqrt[3]{\tan(c+dx)}}{12a^2d} + \frac{5i\tan^{\frac{4}{3}}(c+dx)}{6a^2d(1+i\tan(c+dx))} - \frac{\tan^{\frac{7}{3}}(c+dx)}{4d(a+ia\tan(c+dx))^2} - (5$$

$$= \frac{49\tan^{-1}\left(\sqrt[3]{\tan(c+dx)}\right)}{36a^2d} - \frac{49\sqrt[3]{\tan(c+dx)}}{12a^2d} + \frac{5i\tan^{\frac{4}{3}}(c+dx)}{6a^2d(1+i\tan(c+dx))} -$$

$$= \frac{49\tan^{-1}\left(\sqrt[3]{\tan(c+dx)}\right)}{36a^2d} + \frac{5i\log\left(1+\tan^{\frac{2}{3}}(c+dx)\right)}{9a^2d} - \frac{49\log\left(1-\sqrt{3}\,\sqrt[3]{\right.}}{}$$

$$= -\frac{49\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d} + \frac{49\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d} +$$

$$= -\frac{49\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d} + \frac{49\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.84, size = 192, normalized size = 0.54

$$-\frac{\sec^2(c+dx)\left(9\,2^{2/3}e^{2i(c+dx)}\sqrt[3]{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{1}{2}\left(1-e^{2i(c+dx)}\right)\right)+2\left(-13-85\cos(2(c+dx))+89\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};-\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)(\cos(2(c+dx))+i\sin(2(c+dx)))-88i\sin(2(c+dx))\right)\right)\sqrt[3]{\tan(c+dx)}}{48a^2d(-i+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(10/3)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -1/48*(Sec[c + d*x]^2*(9*2^(2/3)*E^((2*I)*(c + d*x))*(1 + E^((2*I)*(c + d*x
)))^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, (1 - E^((2*I)*(c + d*x)))/2] + 2
*(-13 - 85*Cos[2*(c + d*x)] + 89*Hypergeometric2F1[1/3, 1, 4/3, -((-1 + E^(
(2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))]*(Cos[2*(c + d*x)] + I*Sin[2*(
c + d*x)]) - (88*I)*Sin[2*(c + d*x)]))*Tan[c + d*x]^(1/3))/(a^2*d*(-I + Tan
[c + d*x])^2)
```

Maple [A]
time = 0.18, size = 235, normalized size = 0.66

method	result
derivativedivides	$\frac{-3\left(\tan^{\frac{1}{3}}(dx+c)\right)+\frac{i}{36\left(\tan^{\frac{1}{3}}(dx+c)+i\right)^2}+\frac{89i\ln\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}{72}-\frac{5}{12\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}+\frac{i\ln\left(i\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)\right.}{16}}{}$
default	$\frac{-3\left(\tan^{\frac{1}{3}}(dx+c)\right)+\frac{i}{36\left(\tan^{\frac{1}{3}}(dx+c)+i\right)^2}+\frac{89i\ln\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}{72}-\frac{5}{12\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}+\frac{i\ln\left(i\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)\right.}{16}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(10/3)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(-3*tan(d*x+c)^(1/3)+1/36*I/(tan(d*x+c)^(1/3)+I)^2+89/72*I*ln(tan(d
*x+c)^(1/3)+I)-5/12/(tan(d*x+c)^(1/3)+I)+1/16*I*ln(I*tan(d*x+c)^(1/3)+tan(d
*x+c)^(2/3)-1)+1/8*3^(1/2)*arctanh(1/3*(I+2*tan(d*x+c)^(1/3))*3^(1/2))-1/72
*(-30*tan(d*x+c)-4*I*tan(d*x+c)^(2/3)-4*tan(d*x+c)^(1/3)+28*I)/(-I*tan(d*x+
c)^(1/3)+tan(d*x+c)^(2/3)-1)^2-89/144*I*ln(-I*tan(d*x+c)^(1/3)+tan(d*x+c)^(
2/3)-1)+89/72*3^(1/2)*arctanh(1/3*(-I+2*tan(d*x+c)^(1/3))*3^(1/2))-1/8*I*ln
(tan(d*x+c)^(1/3)-I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(10/3)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.70, size = 521, normalized size = 1.46

$$\frac{\left(9 \left(\sqrt{3} a^{2} d \sqrt{\frac{1}{a^{4} d^{2}}} e^{\left(4 i d x + 4 i c\right)} + i e^{\left(4 i d x + 4 i c\right)}\right) \log\left(\frac{1}{2} \sqrt{3} a^{2} d \sqrt{\frac{1}{a^{4} d^{2}}} + \left(\frac{-i e^{\left(2 i d x + 2 i c\right)} + i}{e^{\left(2 i d x + 2 i c\right)} + 1}\right)^{\frac{1}{3}} + \frac{1}{2} i\right) - 9 \left(\sqrt{3} a^{2} d \sqrt{\frac{1}{a^{4} d^{2}}} e^{\left(4 i d x + 4 i c\right)} - i e^{\left(4 i d x + 4 i c\right)}\right) \log\left(-\frac{1}{2} \sqrt{3} a^{2} d \sqrt{\frac{1}{a^{4} d^{2}}} + \left(\frac{-i e^{\left(2 i d x + 2 i c\right)} + i}{e^{\left(2 i d x + 2 i c\right)} + 1}\right)^{\frac{1}{3}} + \frac{1}{2} i\right) + 89 \left(3 \sqrt{\frac{1}{3}} a^{2} d \sqrt{\frac{1}{a^{4} d^{2}}} e^{\left(4 i d x + 4 i c\right)} - i e^{\left(4 i d x + 4 i c\right)}\right) \log\left(\frac{3}{2} \sqrt{\frac{1}{3}} a^{2} d \sqrt{\frac{1}{a^{4} d^{2}}} + \left(\frac{-i e^{\left(2 i d x + 2 i c\right)} + i}{e^{\left(2 i d x + 2 i c\right)} + 1}\right)^{\frac{1}{3}} - \frac{1}{2} i\right) - 89 \left(3 \sqrt{\frac{1}{3}} a^{2} d \sqrt{\frac{1}{a^{4} d^{2}}} e^{\left(4 i d x + 4 i c\right)} + i e^{\left(4 i d x + 4 i c\right)}\right) \log\left(-\frac{3}{2} \sqrt{\frac{1}{3}} a^{2} d \sqrt{\frac{1}{a^{4} d^{2}}} + \left(\frac{-i e^{\left(2 i d x + 2 i c\right)} + i}{e^{\left(2 i d x + 2 i c\right)} + 1}\right)^{\frac{1}{3}} - \frac{1}{2} i\right) + 178 i e^{\left(4 i d x + 4 i c\right)} \log\left(\left(\frac{-i e^{\left(2 i d x + 2 i c\right)} + i}{e^{\left(2 i d x + 2 i c\right)} + 1}\right)^{\frac{1}{3}} + i\right) - 18 i e^{\left(4 i d x + 4 i c\right)} \log\left(\left(\frac{-i e^{\left(2 i d x + 2 i c\right)} + i}{e^{\left(2 i d x + 2 i c\right)} + 1}\right)^{\frac{1}{3}} - i\right) - 3 \left(\frac{-i e^{\left(2 i d x + 2 i c\right)} + i}{e^{\left(2 i d x + 2 i c\right)} + 1}\right)^{\frac{1}{3}} \left(173 e^{\left(4 i d x + 4 i c\right)} + 26 e^{\left(2 i d x + 2 i c\right)} - 3\right)\right) e^{\left(-4 i d x - 4 i c\right)}}{144 a^{2} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(10/3)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/144*(9*(sqrt(3)*a^2*d*sqrt(1/(a^4*d^2))*e^(4*I*d*x + 4*I*c) + I*e^(4*I*d*
x + 4*I*c))*log(1/2*sqrt(3)*a^2*d*sqrt(1/(a^4*d^2)) + ((-I*e^(2*I*d*x + 2*I
*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) + 1/2*I) - 9*(sqrt(3)*a^2*d*sqrt(
1/(a^4*d^2))*e^(4*I*d*x + 4*I*c) - I*e^(4*I*d*x + 4*I*c))*log(-1/2*sqrt(3)*
a^2*d*sqrt(1/(a^4*d^2)) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c
) + 1))^(1/3) + 1/2*I) + 89*(3*sqrt(1/3)*a^2*d*sqrt(1/(a^4*d^2))*e^(4*I*d*x
 + 4*I*c) - I*e^(4*I*d*x + 4*I*c))*log(3/2*sqrt(1/3)*a^2*d*sqrt(1/(a^4*d^2)
) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) - 1/2*I)
 - 89*(3*sqrt(1/3)*a^2*d*sqrt(1/(a^4*d^2))*e^(4*I*d*x + 4*I*c) + I*e^(4*I*d
*x + 4*I*c))*log(-3/2*sqrt(1/3)*a^2*d*sqrt(1/(a^4*d^2)) + ((-I*e^(2*I*d*x +
 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) - 1/2*I) + 178*I*e^(4*I*d*x +
 4*I*c)*log(((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)
+ I) - 18*I*e^(4*I*d*x + 4*I*c)*log(((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d
*x + 2*I*c) + 1))^(1/3) - I) - 3*((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x
+ 2*I*c) + 1))^(1/3)*(173*e^(4*I*d*x + 4*I*c) + 26*e^(2*I*d*x + 2*I*c) - 3)
)*e^(-4*I*d*x - 4*I*c)/(a^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(10/3)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.70, size = 244, normalized size = 0.68

$$-\frac{89 \sqrt{3} \log\left(-\frac{\sqrt{3} - 2 \tan(dx+c)^{\frac{1}{3}} + i}{\sqrt{3} + 2 \tan(dx+c)^{\frac{1}{3}} - i}\right)}{144 a^{2} d} - \frac{\sqrt{3} \log\left(-\frac{\sqrt{3} - 2 \tan(dx+c)^{\frac{1}{3}} - i}{\sqrt{3} + 2 \tan(dx+c)^{\frac{1}{3}} + i}\right)}{16 a^{2} d} + \frac{i \log\left(\tan\left(dx+c\right)^{\frac{2}{3}} + i \tan\left(dx+c\right)^{\frac{1}{3}} - 1\right)}{16 a^{2} d} - \frac{89 i \log\left(\tan\left(dx+c\right)^{\frac{2}{3}} - i \tan\left(dx+c\right)^{\frac{1}{3}} - 1\right)}{144 a^{2} d} + \frac{89 i \log\left(\tan\left(dx+c\right)^{\frac{1}{3}} + i\right)}{72 a^{2} d} - \frac{i \log\left(\tan\left(dx+c\right)^{\frac{1}{3}} - i\right)}{8 a^{2} d} - \frac{3 \tan\left(dx+c\right)^{\frac{1}{3}}}{a^{2} d} - \frac{-16 i \tan\left(dx+c\right)^{\frac{4}{3}} - 13 \tan\left(dx+c\right)^{\frac{1}{3}}}{12 a^{2} d \left(\tan\left(dx+c\right) - i\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(10/3)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -89/144*sqrt(3)*log(-(sqrt(3) - 2*tan(d*x + c)^(1/3) + I)/(sqrt(3) + 2*tan(
d*x + c)^(1/3) - I))/(a^2*d) - 1/16*sqrt(3)*log(-(sqrt(3) - 2*tan(d*x + c)^
(1/3) - I)/(sqrt(3) + 2*tan(d*x + c)^(1/3) + I))/(a^2*d) + 1/16*I*log(tan(d
*x + c)^(2/3) + I*tan(d*x + c)^(1/3) - 1)/(a^2*d) - 89/144*I*log(tan(d*x +
c)^(2/3) - I*tan(d*x + c)^(1/3) - 1)/(a^2*d) + 89/72*I*log(tan(d*x + c)^(1/
3) + I)/(a^2*d) - 1/8*I*log(tan(d*x + c)^(1/3) - I)/(a^2*d) - 3*tan(d*x + c
)^(1/3)/(a^2*d) - 1/12*(-16*I*tan(d*x + c)^(4/3) - 13*tan(d*x + c)^(1/3))/(
a^2*d*(tan(d*x + c) - I)^2)
```

Mupad [B]
time = 5.24, size = 668, normalized size = 1.87

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(10/3)/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] log(((a^6*d^3*90329344i)/3 - 17694720*a^10*d^5*tan(c + d*x)^(1/3)*(1i/(512*
a^6*d^3))^(2/3))*(1i/(512*a^6*d^3))^(1/3) + (a^4*d^2*tan(c + d*x)^(1/3)*113
94848i)/3)*(1i/(512*a^6*d^3))^(1/3) + log(((a^6*d^3*90329344i)/3 - 17694720
*a^10*d^5*tan(c + d*x)^(1/3)*(-704969i/(373248*a^6*d^3))^(2/3))*(-704969i/(
373248*a^6*d^3))^(1/3) + (a^4*d^2*tan(c + d*x)^(1/3)*11394848i)/3)*(-704969
i/(373248*a^6*d^3))^(1/3) + ((tan(c + d*x)^(1/3)*13i)/(12*a^2*d) - (4*tan(c
 + d*x)^(4/3))/(3*a^2*d))/(2*tan(c + d*x) + tan(c + d*x)^2*1i - 1i) - (3*ta
n(c + d*x)^(1/3))/(a^2*d) + (log(((3^(1/2)*1i - 1)*((a^6*d^3*90329344i)/3 -
 4423680*a^10*d^5*tan(c + d*x)^(1/3)*(3^(1/2)*1i - 1)^2*(1i/(512*a^6*d^3))^
(2/3))*(1i/(512*a^6*d^3))^(1/3))/2 + (a^4*d^2*tan(c + d*x)^(1/3)*11394848i)
/3)*(3^(1/2)*1i - 1)*(1i/(512*a^6*d^3))^(1/3))/2 - (log(((3^(1/2)*1i + 1)*(
(a^6*d^3*90329344i)/3 - 4423680*a^10*d^5*tan(c + d*x)^(1/3)*(3^(1/2)*1i + 1
)^2*(1i/(512*a^6*d^3))^(2/3))*(1i/(512*a^6*d^3))^(1/3))/2 - (a^4*d^2*tan(c
+ d*x)^(1/3)*11394848i)/3)*(3^(1/2)*1i + 1)*(1i/(512*a^6*d^3))^(1/3))/2 + (
log(((3^(1/2)*1i - 1)*((a^6*d^3*90329344i)/3 - 4423680*a^10*d^5*tan(c + d*x
)^(1/3)*(3^(1/2)*1i - 1)^2*(-704969i/(373248*a^6*d^3))^(2/3))*(-704969i/(37
3248*a^6*d^3))^(1/3))/2 + (a^4*d^2*tan(c + d*x)^(1/3)*11394848i)/3)*(3^(1/2
)*1i - 1)*(-704969i/(373248*a^6*d^3))^(1/3))/2 - (log(((3^(1/2)*1i + 1)*((a
^6*d^3*90329344i)/3 - 4423680*a^10*d^5*tan(c + d*x)^(1/3)*(3^(1/2)*1i + 1)^
2*(-704969i/(373248*a^6*d^3))^(2/3))*(-704969i/(373248*a^6*d^3))^(1/3))/2 -
 (a^4*d^2*tan(c + d*x)^(1/3)*11394848i)/3)*(3^(1/2)*1i + 1)*(-704969i/(3732
48*a^6*d^3))^(1/3))/2
```

3.242 $\int \frac{\tan^{\frac{8}{3}}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=337

$$\frac{25\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d}-\frac{25\text{ArcTan}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d}+\frac{2i\text{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)}{3\sqrt{3}\,a^2d}-\frac{25}{}$$

```
[Out] -25/72*arctan(-3^(1/2)+2*tan(d*x+c)^(1/3))/a^2/d-25/72*arctan(3^(1/2)+2*tan
(d*x+c)^(1/3))/a^2/d-25/36*arctan(tan(d*x+c)^(1/3))/a^2/d-2/9*I*ln(1+tan(d*
x+c)^(2/3))/a^2/d+1/9*I*ln(1-tan(d*x+c)^(2/3)+tan(d*x+c)^(4/3))/a^2/d+2/9*I
*arctan(1/3*(1-2*tan(d*x+c)^(2/3))*3^(1/2))/a^2/d*3^(1/2)-25/144*ln(1-3^(1/
2)*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3))/a^2/d*3^(1/2)+25/144*ln(1+3^(1/2)*tan
(d*x+c)^(1/3)+tan(d*x+c)^(2/3))/a^2/d*3^(1/2)+2/3*I*tan(d*x+c)^(2/3)/a^2/d/
(1+I*tan(d*x+c))-1/4*tan(d*x+c)^(5/3)/d/(a+I*a*tan(d*x+c))^2
```

Rubi [A]
time = 0.40, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 14, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3639, 3676, 3619, 3557, 335, 281, 206, 31, 648, 632, 210, 642, 301, 209}

$$\frac{2i\text{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)}{3\sqrt{3}\,a^2d}+\frac{25\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d}-\frac{25\text{ArcTan}\left(2\sqrt[3]{\tan(c+dx)}+\sqrt{3}\right)}{72a^2d}-\frac{25\text{ArcTan}\left(\sqrt[3]{\tan(c+dx)}\right)}{36a^2d}+\frac{2i\tan^{\frac{2}{3}}(c+dx)}{3a^2d(1+i\tan(c+dx))}-\frac{2i\log\left(\tan^{\frac{2}{3}}(c+dx)+1\right)}{9a^2d}-\frac{25\log\left(\tan^{\frac{2}{3}}(c+dx)-\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{48\sqrt{3}\,a^2d}+\frac{25\log\left(\tan^{\frac{2}{3}}(c+dx)+\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{48\sqrt{3}\,a^2d}+\frac{i\log\left(\tan^{\frac{4}{3}}(c+dx)-\tan^{\frac{2}{3}}(c+dx)+1\right)}{9a^2d}-\frac{\tan^{\frac{5}{3}}(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(8/3)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (25*ArcTan[Sqrt[3] - 2*Tan[c + d*x]^(1/3)])/(72*a^2*d) - (25*ArcTan[Sqrt[3]
 + 2*Tan[c + d*x]^(1/3)])/(72*a^2*d) + (((2*I)/3)*ArcTan[(1 - 2*Tan[c + d*x
]^(2/3))/Sqrt[3]])/(Sqrt[3]*a^2*d) - (25*ArcTan[Tan[c + d*x]^(1/3)])/(36*a^
2*d) - (((2*I)/9)*Log[1 + Tan[c + d*x]^(2/3)])/(a^2*d) - (25*Log[1 - Sqrt[3
]*Tan[c + d*x]^(1/3) + Tan[c + d*x]^(2/3)])/(48*Sqrt[3]*a^2*d) + (25*Log[1
+ Sqrt[3]*Tan[c + d*x]^(1/3) + Tan[c + d*x]^(2/3)])/(48*Sqrt[3]*a^2*d) + ((
I/9)*Log[1 - Tan[c + d*x]^(2/3) + Tan[c + d*x]^(4/3)])/(a^2*d) + (((2*I)/3)
*Tan[c + d*x]^(2/3))/(a^2*d*(1 + I*Tan[c + d*x])) - Tan[c + d*x]^(5/3)/(4*d
*(a + I*a*Tan[c + d*x])^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\tan^{\frac{8}{3}}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\tan^{\frac{5}{3}}(c+dx)}{4d(a+ia\tan(c+dx))^2} - \frac{\int \frac{\tan^{\frac{2}{3}}(c+dx)\left(-\frac{5a}{3}+\frac{11}{3}ia\tan(c+dx)\right)}{a+ia\tan(c+dx)}\,dx}{4a^2}$$

$$= \frac{2i\tan^{\frac{2}{3}}(c+dx)}{3a^2d(1+i\tan(c+dx))} - \frac{\tan^{\frac{5}{3}}(c+dx)}{4d(a+ia\tan(c+dx))^2} + \frac{\int \frac{-\frac{32ia^2}{9}-\frac{50}{9}a^2\tan(c+dx)}{\sqrt[3]{\tan(c+dx)}}\,d}{8a^4}$$

$$= \frac{2i\tan^{\frac{2}{3}}(c+dx)}{3a^2d(1+i\tan(c+dx))} - \frac{\tan^{\frac{5}{3}}(c+dx)}{4d(a+ia\tan(c+dx))^2} - \frac{(4i)\int \frac{1}{\sqrt[3]{\tan(c+dx)}}\,d}{9a^2}$$

$$= \frac{2i\tan^{\frac{2}{3}}(c+dx)}{3a^2d(1+i\tan(c+dx))} - \frac{\tan^{\frac{5}{3}}(c+dx)}{4d(a+ia\tan(c+dx))^2} - \frac{(4i)\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\,(1+x^2)}\right.}{9a^2}$$

$$= \frac{2i\tan^{\frac{2}{3}}(c+dx)}{3a^2d(1+i\tan(c+dx))} - \frac{\tan^{\frac{5}{3}}(c+dx)}{4d(a+ia\tan(c+dx))^2} - \frac{(4i)\text{Subst}\left(\int \frac{x}{1+x^6}\,dx, x\right.}{3a^2d}$$

$$= \frac{2i\tan^{\frac{2}{3}}(c+dx)}{3a^2d(1+i\tan(c+dx))} - \frac{\tan^{\frac{5}{3}}(c+dx)}{4d(a+ia\tan(c+dx))^2} - \frac{(2i)\text{Subst}\left(\int \frac{1}{1+x^3}\,dx, x\right.}{3a^2d}$$

$$= -\frac{25\tan^{-1}\left(\sqrt[3]{\tan(c+dx)}\right)}{36a^2d} + \frac{2i\tan^{\frac{2}{3}}(c+dx)}{3a^2d(1+i\tan(c+dx))} - \frac{\tan^{\frac{5}{3}}(c+dx)}{4d(a+ia\tan(c+}$$

$$= -\frac{25\tan^{-1}\left(\sqrt[3]{\tan(c+dx)}\right)}{36a^2d} - \frac{2i\log\left(1+\tan^{\frac{2}{3}}(c+dx)\right)}{9a^2d} - \frac{25\log\left(1-\sqrt{3}\right.}{}$$

$$= \frac{25\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d} - \frac{25\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d} -$$

$$= \frac{25\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d} - \frac{25\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.76, size = 194, normalized size = 0.58

$$-\frac{i\sec^2(c+dx)\left(9\sqrt[3]{2}\,e^{2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{1}{2}\left(1-e^{2i(c+dx)}\right)\right)-82\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)(\cos(2(c+dx))+i\sin(2(c+dx)))+4(8+8\cos(2(c+dx))+11i\sin(2(c+dx)))\right)\tan^{\frac{2}{3}}(c+dx)}{96a^2d(-i+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(8/3)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((-1/96*I)*Sec[c + d*x]^2*(9*2^(1/3)*E^((2*I)*(c + d*x))*(1 + E^((2*I)*(c +
 d*x)))^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, (1 - E^((2*I)*(c + d*x)))/2]
 - 82*Hypergeometric2F1[2/3, 1, 5/3, -((-1 + E^((2*I)*(c + d*x)))/(1 + E^((
2*I)*(c + d*x))))]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) + 4*(8 + 8*Cos[2
*(c + d*x)] + (11*I)*Sin[2*(c + d*x)]))*Tan[c + d*x]^(2/3))/(a^2*d*(-I + Ta
n[c + d*x])^2)
```

Maple [A]
time = 0.20, size = 225, normalized size = 0.67

method	result
derivativedivides	$\frac{-\frac{41 i \ln\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}{72}-\frac{i}{36\left(\tan^{\frac{1}{3}}(dx+c)+i\right)^{2}}+\frac{11}{36\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}-\frac{i \ln\left(i\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)-1\right)}{16}+\frac{\sqrt{3}\ \mathrm{arctanh}}{}}{}$
default	$\frac{-\frac{41 i \ln\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}{72}-\frac{i}{36\left(\tan^{\frac{1}{3}}(dx+c)+i\right)^{2}}+\frac{11}{36\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}-\frac{i \ln\left(i\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)-1\right)}{16}+\frac{\sqrt{3}\ \mathrm{arctanh}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(8/3)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(-41/72*I*ln(tan(d*x+c)^(1/3)+I)-1/36*I/(tan(d*x+c)^(1/3)+I)^2+11/3
6/(tan(d*x+c)^(1/3)+I)-1/16*I*ln(I*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3)-1)+1/8
*3^(1/2)*arctanh(1/3*(I+2*tan(d*x+c)^(1/3))*3^(1/2))+1/72*(44*tan(d*x+c)-64
*I*tan(d*x+c)^(2/3)-58*tan(d*x+c)^(1/3)+20*I)/(-I*tan(d*x+c)^(1/3)+tan(d*x+
c)^(2/3)-1)^2+41/144*I*ln(-I*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3)-1)+41/72*3^(
1/2)*arctanh(1/3*(-I+2*tan(d*x+c)^(1/3))*3^(1/2))+1/8*I*ln(tan(d*x+c)^(1/3)
-I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(8/3)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.72, size = 521, normalized size = 1.55

$$\frac{\left(9\left(\sqrt{3}a^2d\sqrt{\frac{1}{a^4d^2}}e^{(4idx+4ic)}-ie^{(4idx+4ic)}\right)\log\left(\frac{1}{2}\sqrt{3}a^2d\sqrt{\frac{1}{a^4d^2}}+\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}+\frac{1}{2}i\right)-9\left(\sqrt{3}a^2d\sqrt{\frac{1}{a^4d^2}}e^{(4idx+4ic)}+ie^{(4idx+4ic)}\right)\log\left(-\frac{1}{2}\sqrt{3}a^2d\sqrt{\frac{1}{a^4d^2}}+\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}+\frac{1}{2}i\right)+41\left(3\sqrt{\frac{1}{3}}a^2d\sqrt{\frac{1}{a^4d^2}}e^{(4idx+4ic)}+ie^{(4idx+4ic)}\right)\log\left(\frac{3}{2}\sqrt{\frac{1}{3}}a^2d\sqrt{\frac{1}{a^4d^2}}+\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}-\frac{1}{2}i\right)-41\left(3\sqrt{\frac{1}{3}}a^2d\sqrt{\frac{1}{a^4d^2}}e^{(4idx+4ic)}-ie^{(4idx+4ic)}\right)\log\left(-\frac{3}{2}\sqrt{\frac{1}{3}}a^2d\sqrt{\frac{1}{a^4d^2}}+\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}-\frac{1}{2}i\right)-82ie^{(4idx+4ic)}\log\left(\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}+i\right)+18ie^{(4idx+4ic)}\log\left(\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}-i\right)-3\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{2}{3}}\left(-19ie^{(4idx+4ic)}-16ie^{(2idx+2ic)}+3i\right)\right)e^{(-4idx-4ic)}}{144a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(8/3)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/144*(9*(sqrt(3)*a^2*d*sqrt(1/(a^4*d^2))*e^(4*I*d*x + 4*I*c) - I*e^(4*I*d*
x + 4*I*c))*log(1/2*sqrt(3)*a^2*d*sqrt(1/(a^4*d^2)) + ((-I*e^(2*I*d*x + 2*I
*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) + 1/2*I) - 9*(sqrt(3)*a^2*d*sqrt(
1/(a^4*d^2))*e^(4*I*d*x + 4*I*c) + I*e^(4*I*d*x + 4*I*c))*log(-1/2*sqrt(3)*
a^2*d*sqrt(1/(a^4*d^2)) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c
) + 1))^(1/3) + 1/2*I) + 41*(3*sqrt(1/3)*a^2*d*sqrt(1/(a^4*d^2))*e^(4*I*d*x
 + 4*I*c) + I*e^(4*I*d*x + 4*I*c))*log(3/2*sqrt(1/3)*a^2*d*sqrt(1/(a^4*d^2)
) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) - 1/2*I)
 - 41*(3*sqrt(1/3)*a^2*d*sqrt(1/(a^4*d^2))*e^(4*I*d*x + 4*I*c) - I*e^(4*I*d
*x + 4*I*c))*log(-3/2*sqrt(1/3)*a^2*d*sqrt(1/(a^4*d^2)) + ((-I*e^(2*I*d*x +
 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) - 1/2*I) - 82*I*e^(4*I*d*x +
4*I*c)*log(((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) +
 I) + 18*I*e^(4*I*d*x + 4*I*c)*log(((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*
x + 2*I*c) + 1))^(1/3) - I) - 3*((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
 2*I*c) + 1))^(2/3)*(-19*I*e^(4*I*d*x + 4*I*c) - 16*I*e^(2*I*d*x + 2*I*c) +
 3*I))*e^(-4*I*d*x - 4*I*c)/(a^2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(8/3)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.94, size = 228, normalized size = 0.68

$$-\frac{41\sqrt{3}\log\left(-\frac{\sqrt{3}-2\tan(dx+c)^{\frac{1}{3}}+i}{\sqrt{3}+2\tan(dx+c)^{\frac{1}{3}}-i}\right)}{144\,a^2d}-\frac{\sqrt{3}\log\left(-\frac{\sqrt{3}-2\tan(dx+c)^{\frac{1}{3}}-i}{\sqrt{3}+2\tan(dx+c)^{\frac{1}{3}}+i}\right)}{16\,a^2d}-\frac{i\log\left(\tan\left(dx+c\right)^{\frac{2}{3}}+i\tan\left(dx+c\right)^{\frac{1}{3}}-1\right)}{16\,a^2d}+\frac{41i\log\left(\tan\left(dx+c\right)^{\frac{2}{3}}-i\tan\left(dx+c\right)^{\frac{1}{3}}-1\right)}{144\,a^2d}-\frac{41i\log\left(\tan\left(dx+c\right)^{\frac{1}{3}}+i\right)}{72\,a^2d}+\frac{i\log\left(\tan\left(dx+c\right)^{\frac{1}{3}}-i\right)}{8\,a^2d}+\frac{11\tan\left(dx+c\right)^{\frac{5}{3}}-8i\tan\left(dx+c\right)^{\frac{2}{3}}}{12\,a^2d(\tan\left(dx+c\right)-i)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(8/3)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -41/144*sqrt(3)*log(-(sqrt(3) - 2*tan(d*x + c)^(1/3) + I)/(sqrt(3) + 2*tan(
d*x + c)^(1/3) - I))/(a^2*d) - 1/16*sqrt(3)*log(-(sqrt(3) - 2*tan(d*x + c)^
(1/3) - I)/(sqrt(3) + 2*tan(d*x + c)^(1/3) + I))/(a^2*d) - 1/16*I*log(tan(d
*x + c)^(2/3) + I*tan(d*x + c)^(1/3) - 1)/(a^2*d) + 41/144*I*log(tan(d*x +
c)^(2/3) - I*tan(d*x + c)^(1/3) - 1)/(a^2*d) - 41/72*I*log(tan(d*x + c)^(1/
3) + I)/(a^2*d) + 1/8*I*log(tan(d*x + c)^(1/3) - I)/(a^2*d) + 1/12*(11*tan(
d*x + c)^(5/3) - 8*I*tan(d*x + c)^(2/3))/(a^2*d*(tan(d*x + c) - I)^2)
```

Mupad [B]
time = 5.26, size = 642, normalized size = 1.91

$$\ln\left(-\left(\frac{a^6\,d^3\,8915200\mathrm{i}}{3}+a^8\,d^4\,\tan\left(c+d\,x\right)^{1/3}\,{\left(-\frac{1\mathrm{i}}{512\,a^6\,d^3}\right)}^{1/3}\,5412864\mathrm{i}\right)\,{\left(-\frac{1\mathrm{i}}{512\,a^6\,d^3}\right)}^{2/3}-\frac{107584\,a^2\,d\,\tan\left(c+d\,x\right)^{1/3}}{3}\right)\,{\left(-\frac{1\mathrm{i}}{512\,a^6\,d^3}\right)}^{1/3}+\ln\left(-\left(\frac{a^6\,d^3\,8915200\mathrm{i}}{3}+a^8\,d^4\,\tan\left(c+d\,x\right)^{1/3}\,{\left(\frac{68921\mathrm{i}}{373248\,a^6\,d^3}\right)}^{1/3}\,5412864\mathrm{i}\right)\,{\left(\frac{68921\mathrm{i}}{373248\,a^6\,d^3}\right)}^{2/3}-\frac{107584\,a^2\,d\,\tan\left(c+d\,x\right)^{1/3}}{3}\right)\,{\left(\frac{68921\mathrm{i}}{373248\,a^6\,d^3}\right)}^{1/3}+\frac{\frac{2\,\tan\left(c+d\,x\right)^{2/3}}{3\,a^2\,d}+\frac{\tan\left(c+d\,x\right)^{5/3}\,11\mathrm{i}}{12\,a^2\,d}}{\tan\left(c+d\,x\right)^2\,1\mathrm{i}+2\,\tan\left(c+d\,x\right)-\mathrm{i}}+\frac{\ln\left(-\frac{107584\,a^2\,d\,\tan\left(c+d\,x\right)^{1/3}}{3}-\frac{{\left(-1+\sqrt{3}\,1\mathrm{i}\right)}^2\,\left(\frac{a^6\,d^3\,8915200\mathrm{i}}{3}+a^8\,d^4\,\tan\left(c+d\,x\right)^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,{\left(-\frac{1\mathrm{i}}{512\,a^6\,d^3}\right)}^{1/3}\,2706432\mathrm{i}\right)\,{\left(-\frac{1\mathrm{i}}{512\,a^6\,d^3}\right)}^{2/3}}{4}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,{\left(-\frac{1\mathrm{i}}{512\,a^6\,d^3}\right)}^{1/3}}{2}-\frac{\ln\left(-\frac{107584\,a^2\,d\,\tan\left(c+d\,x\right)^{1/3}}{3}-\frac{{\left(1+\sqrt{3}\,1\mathrm{i}\right)}^2\,\left(\frac{a^6\,d^3\,8915200\mathrm{i}}{3}-a^8\,d^4\,\tan\left(c+d\,x\right)^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,{\left(-\frac{1\mathrm{i}}{512\,a^6\,d^3}\right)}^{1/3}\,2706432\mathrm{i}\right)\,{\left(-\frac{1\mathrm{i}}{512\,a^6\,d^3}\right)}^{2/3}}{4}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,{\left(-\frac{1\mathrm{i}}{512\,a^6\,d^3}\right)}^{1/3}}{2}+\frac{\ln\left(-\frac{107584\,a^2\,d\,\tan\left(c+d\,x\right)^{1/3}}{3}-\frac{{\left(-1+\sqrt{3}\,1\mathrm{i}\right)}^2\,\left(\frac{a^6\,d^3\,8915200\mathrm{i}}{3}+a^8\,d^4\,\tan\left(c+d\,x\right)^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,{\left(\frac{68921\mathrm{i}}{373248\,a^6\,d^3}\right)}^{1/3}\,2706432\mathrm{i}\right)\,{\left(\frac{68921\mathrm{i}}{373248\,a^6\,d^3}\right)}^{2/3}}{4}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,{\left(\frac{68921\mathrm{i}}{373248\,a^6\,d^3}\right)}^{1/3}}{2}-\frac{\ln\left(-\frac{107584\,a^2\,d\,\tan\left(c+d\,x\right)^{1/3}}{3}-\frac{{\left(1+\sqrt{3}\,1\mathrm{i}\right)}^2\,\left(\frac{a^6\,d^3\,8915200\mathrm{i}}{3}-a^8\,d^4\,\tan\left(c+d\,x\right)^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,{\left(\frac{68921\mathrm{i}}{373248\,a^6\,d^3}\right)}^{1/3}\,2706432\mathrm{i}\right)\,{\left(\frac{68921\mathrm{i}}{373248\,a^6\,d^3}\right)}^{2/3}}{4}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,{\left(\frac{68921\mathrm{i}}{373248\,a^6\,d^3}\right)}^{1/3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(8/3)/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] log(- ((a^6*d^3*8915200i)/3 + a^8*d^4*tan(c + d*x)^(1/3)*(-1i/(512*a^6*d^3)
)^(1/3)*5412864i)*(-1i/(512*a^6*d^3))^(2/3) - (107584*a^2*d*tan(c + d*x)^(1
/3))/3)*(-1i/(512*a^6*d^3))^(1/3) + log(- ((a^6*d^3*8915200i)/3 + a^8*d^4*t
an(c + d*x)^(1/3)*(68921i/(373248*a^6*d^3))^(1/3)*5412864i)*(68921i/(373248
*a^6*d^3))^(2/3) - (107584*a^2*d*tan(c + d*x)^(1/3))/3)*(68921i/(373248*a^6
*d^3))^(1/3) + ((2*tan(c + d*x)^(2/3))/(3*a^2*d) + (tan(c + d*x)^(5/3)*11i)
/(12*a^2*d))/(2*tan(c + d*x) + tan(c + d*x)^2*1i - 1i) + (log(- (107584*a^2
*d*tan(c + d*x)^(1/3))/3 - ((3^(1/2)*1i - 1)^2*((a^6*d^3*8915200i)/3 + a^8*
d^4*tan(c + d*x)^(1/3)*(3^(1/2)*1i - 1)*(-1i/(512*a^6*d^3))^(1/3)*2706432i)
*(-1i/(512*a^6*d^3))^(2/3))/4)*(3^(1/2)*1i - 1)*(-1i/(512*a^6*d^3))^(1/3))/
2 - (log(- (107584*a^2*d*tan(c + d*x)^(1/3))/3 - ((3^(1/2)*1i + 1)^2*((a^6*
d^3*8915200i)/3 - a^8*d^4*tan(c + d*x)^(1/3)*(3^(1/2)*1i + 1)*(-1i/(512*a^6
*d^3))^(1/3)*2706432i)*(-1i/(512*a^6*d^3))^(2/3))/4)*(3^(1/2)*1i + 1)*(-1i/
(512*a^6*d^3))^(1/3))/2 + (log(- (107584*a^2*d*tan(c + d*x)^(1/3))/3 - ((3^
(1/2)*1i - 1)^2*((a^6*d^3*8915200i)/3 + a^8*d^4*tan(c + d*x)^(1/3)*(3^(1/2)
*1i - 1)*(68921i/(373248*a^6*d^3))^(1/3)*2706432i)*(68921i/(373248*a^6*d^3)
)^(2/3))/4)*(3^(1/2)*1i - 1)*(68921i/(373248*a^6*d^3))^(1/3))/2 - (log(- (1
07584*a^2*d*tan(c + d*x)^(1/3))/3 - ((3^(1/2)*1i + 1)^2*((a^6*d^3*8915200i)
/3 - a^8*d^4*tan(c + d*x)^(1/3)*(3^(1/2)*1i + 1)*(68921i/(373248*a^6*d^3))^
(1/3)*2706432i)*(68921i/(373248*a^6*d^3))^(2/3))/4)*(3^(1/2)*1i + 1)*(68921
i/(373248*a^6*d^3))^(1/3))/2
```

3.243 $\int \frac{\tan^{\frac{4}{3}}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=335

$$\frac{\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d}-\frac{\text{ArcTan}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d}+\frac{i\text{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)}{3\sqrt{3}\,a^2d}-\text{ArcTan}$$

```
[Out] -1/72*arctan(-3^(1/2)+2*tan(d*x+c)^(1/3))/a^2/d-1/72*arctan(3^(1/2)+2*tan(d
*x+c)^(1/3))/a^2/d-1/36*arctan(tan(d*x+c)^(1/3))/a^2/d+1/9*I*ln(1+tan(d*x+c
)^(2/3))/a^2/d-1/18*I*ln(1-tan(d*x+c)^(2/3)+tan(d*x+c)^(4/3))/a^2/d+1/9*I*a
rctan(1/3*(1-2*tan(d*x+c)^(2/3))*3^(1/2))/a^2/d*3^(1/2)+1/144*ln(1-3^(1/2)*
tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3))/a^2/d*3^(1/2)-1/144*ln(1+3^(1/2)*tan(d*x
+c)^(1/3)+tan(d*x+c)^(2/3))/a^2/d*3^(1/2)+1/3*tan(d*x+c)^(1/3)/a^2/d/(1+I*t
an(d*x+c))-1/4*tan(d*x+c)^(1/3)/d/(a+I*a*tan(d*x+c))^2
```

Rubi [A]
time = 0.35, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 14, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3639, 3677, 3619, 3557, 335, 215, 648, 632, 210, 642, 209, 281, 298, 31}

$$\frac{i\text{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)}{3\sqrt{3}\,a^2d}+\frac{\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d}-\frac{\text{ArcTan}\left(2\sqrt[3]{\tan(c+dx)}+\sqrt{3}\right)}{72a^2d}-\frac{\text{ArcTan}\left(\sqrt[3]{\tan(c+dx)}\right)}{36a^2d}+\frac{\sqrt[3]{\tan(c+dx)}}{3a^2d(1+i\tan(c+dx))}+\frac{i\log\left(\tan^{\frac{2}{3}}(c+dx)+1\right)}{9a^2d}+\frac{\log\left(\tan^{\frac{2}{3}}(c+dx)-\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{48\sqrt{3}\,a^2d}-\frac{\log\left(\tan^{\frac{2}{3}}(c+dx)+\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{48\sqrt{3}\,a^2d}-\frac{i\log\left(\tan^{\frac{4}{3}}(c+dx)-\tan^{\frac{2}{3}}(c+dx)+1\right)}{18a^2d}-\frac{\sqrt[3]{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(4/3)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ArcTan[Sqrt[3] - 2*Tan[c + d*x]^(1/3)]/(72*a^2*d) - ArcTan[Sqrt[3] + 2*Tan[
c + d*x]^(1/3)]/(72*a^2*d) + ((I/3)*ArcTan[(1 - 2*Tan[c + d*x]^(2/3))/Sqrt[
3]])/(Sqrt[3]*a^2*d) - ArcTan[Tan[c + d*x]^(1/3)]/(36*a^2*d) + ((I/9)*Log[1
 + Tan[c + d*x]^(2/3)])/(a^2*d) + Log[1 - Sqrt[3]*Tan[c + d*x]^(1/3) + Tan[
c + d*x]^(2/3)]/(48*Sqrt[3]*a^2*d) - Log[1 + Sqrt[3]*Tan[c + d*x]^(1/3) + T
an[c + d*x]^(2/3)]/(48*Sqrt[3]*a^2*d) - ((I/18)*Log[1 - Tan[c + d*x]^(2/3)
+ Tan[c + d*x]^(4/3)])/(a^2*d) + Tan[c + d*x]^(1/3)/(3*a^2*d*(1 + I*Tan[c +
 d*x])) - Tan[c + d*x]^(1/3)/(4*d*(a + I*a*Tan[c + d*x])^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\tan^{\frac{4}{3}}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\sqrt[3]{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2} - \frac{\int \frac{-\frac{a}{3}+\frac{7}{3}ia\tan(c+dx)}{\tan^{\frac{2}{3}}(c+dx)(a+ia\tan(c+dx))}\,dx}{4a^2}$$

$$= \frac{\sqrt[3]{\tan(c+dx)}}{3a^2d(1+i\tan(c+dx))} - \frac{\sqrt[3]{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2} - \frac{\int \frac{\frac{2a^2}{9}+\frac{16}{9}ia^2\tan(c+dx)}{\tan^{\frac{2}{3}}(c+dx)}\,dx}{8a^4}$$

$$= \frac{\sqrt[3]{\tan(c+dx)}}{3a^2d(1+i\tan(c+dx))} - \frac{\sqrt[3]{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2} - \frac{(2i)\int \sqrt[3]{\tan(c+dx)}\,dx}{9a^2}$$

$$= \frac{\sqrt[3]{\tan(c+dx)}}{3a^2d(1+i\tan(c+dx))} - \frac{\sqrt[3]{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2} - \frac{(2i)\text{Subst}\left(\int \frac{\sqrt[3]{x}}{1+x^2}\,dx, x,\right.}{9a^2d}$$

$$= \frac{\sqrt[3]{\tan(c+dx)}}{3a^2d(1+i\tan(c+dx))} - \frac{\sqrt[3]{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2} - \frac{(2i)\text{Subst}\left(\int \frac{x^3}{1+x^6}\,dx, x,\right.}{3a^2d}$$

$$= \frac{\sqrt[3]{\tan(c+dx)}}{3a^2d(1+i\tan(c+dx))} - \frac{\sqrt[3]{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2} - \frac{i\text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \tan\right.}{3a^2d}$$

$$= -\frac{\tan^{-1}\left(\sqrt[3]{\tan(c+dx)}\right)}{36a^2d} + \frac{\sqrt[3]{\tan(c+dx)}}{3a^2d(1+i\tan(c+dx))} - \frac{\sqrt[3]{\tan(c+dx)}}{4d(a+ia\tan(c+dx)}$$

$$= -\frac{\tan^{-1}\left(\sqrt[3]{\tan(c+dx)}\right)}{36a^2d} + \frac{i\log\left(1+\tan^{\frac{2}{3}}(c+dx)\right)}{9a^2d} + \frac{\log\left(1-\sqrt{3}\sqrt[3]{\tan(c\right.}{48}$$

$$= \frac{\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d} - \frac{\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d} - \frac{\tan^{-1}}{}$$

$$= \frac{\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d} - \frac{\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d} + \frac{i\tan^{-1}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.70, size = 190, normalized size = 0.57

$$\frac{\sec^2(c+dx)\left(9\,2^{2/3}e^{2i(c+dx)}\sqrt[3]{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{1}{2}\left(1-e^{2i(c+dx)}\right)\right)-2\left(1+\cos(2(c+dx))+7\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};-\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)(\cos(2(c+dx))+i\sin(2(c+dx)))+4i\sin(2(c+dx))\right)\right)\sqrt[3]{\tan(c+dx)}}{48a^2d(-i+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(4/3)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^2*(9*2^(2/3)*E^((2*I)*(c + d*x))*(1 + E^((2*I)*(c + d*x)))^(1
/3)*Hypergeometric2F1[1/3, 1/3, 4/3, (1 - E^((2*I)*(c + d*x)))/2] - 2*(1 +
Cos[2*(c + d*x)] + 7*Hypergeometric2F1[1/3, 1, 4/3, -((-1 + E^((2*I)*(c + d
*x)))/(1 + E^((2*I)*(c + d*x))))]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) +
 (4*I)*Sin[2*(c + d*x)]))*Tan[c + d*x]^(1/3))/(48*a^2*d*(-I + Tan[c + d*x])
^2)
```

Maple [A]
time = 0.18, size = 225, normalized size = 0.67

method	result
derivativedivides	$\frac{-\frac{i}{36\left(\tan^{\frac{1}{3}}(dx+c)+i\right)^{2}}+\frac{7i\ln\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}{72}+\frac{1}{12\left(\tan^{\frac{1}{3}}(dx+c)\right)+12i}-\frac{i\ln\left(i\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)-1\right)}{16}-\frac{\sqrt{3}\ \text{arcta}}{}}{}$
default	$\frac{-\frac{i}{36\left(\tan^{\frac{1}{3}}(dx+c)+i\right)^{2}}+\frac{7i\ln\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}{72}+\frac{1}{12\left(\tan^{\frac{1}{3}}(dx+c)\right)+12i}-\frac{i\ln\left(i\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)-1\right)}{16}-\frac{\sqrt{3}\ \text{arcta}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(4/3)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(-1/36*I/(tan(d*x+c)^(1/3)+I)^2+7/72*I*ln(tan(d*x+c)^(1/3)+I)+1/12/
(tan(d*x+c)^(1/3)+I)-1/16*I*ln(I*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3)-1)-1/8*3
^(1/2)*arctanh(1/3*(I+2*tan(d*x+c)^(1/3))*3^(1/2))-1/72*(6*tan(d*x+c)+4*I*t
an(d*x+c)^(2/3)+4*tan(d*x+c)^(1/3)-4*I)/(-I*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/
3)-1)^2-7/144*I*ln(-I*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3)-1)+7/72*3^(1/2)*arc
tanh(1/3*(-I+2*tan(d*x+c)^(1/3))*3^(1/2))+1/8*I*ln(tan(d*x+c)^(1/3)-I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(4/3)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.62, size = 521, normalized size = 1.56

$$-\frac{\left(9\left(\sqrt{3}\,a^{2}d\sqrt{\frac{1}{a^{4}d^{2}}}\,e^{(4idx+4ic)}+i\,e^{(4idx+4ic)}\right)\log\left(\frac{1}{2}\sqrt{3}\,a^{2}d\sqrt{\frac{1}{a^{4}d^{2}}}+\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}+\frac{1}{2}i\right)-9\left(\sqrt{3}\,a^{2}d\sqrt{\frac{1}{a^{4}d^{2}}}\,e^{(4idx+4ic)}-i\,e^{(4idx+4ic)}\right)\log\left(-\frac{1}{2}\sqrt{3}\,a^{2}d\sqrt{\frac{1}{a^{4}d^{2}}}+\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}+\frac{1}{2}i\right)-7\left(3\sqrt{\frac{1}{3}}\,a^{2}d\sqrt{\frac{1}{a^{4}d^{2}}}\,e^{(4idx+4ic)}-i\,e^{(4idx+4ic)}\right)\log\left(\frac{3}{2}\sqrt{\frac{1}{3}}\,a^{2}d\sqrt{\frac{1}{a^{4}d^{2}}}+\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}-\frac{1}{2}i\right)+7\left(3\sqrt{\frac{1}{3}}\,a^{2}d\sqrt{\frac{1}{a^{4}d^{2}}}\,e^{(4idx+4ic)}+i\,e^{(4idx+4ic)}\right)\log\left(-\frac{3}{2}\sqrt{\frac{1}{3}}\,a^{2}d\sqrt{\frac{1}{a^{4}d^{2}}}+\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}-\frac{1}{2}i\right)-14i\,e^{(4idx+4ic)}\log\left(\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}+i\right)-18i\,e^{(4idx+4ic)}\log\left(\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}-i\right)-3\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}\left(5e^{(4idx+4ic)}+2e^{(2idx+2ic)}-3\right)\right)e^{(-4idx-4ic)}}{144\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(4/3)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/144*(9*(sqrt(3)*a^2*d*sqrt(1/(a^4*d^2))*e^(4*I*d*x + 4*I*c) + I*e^(4*I*d
*x + 4*I*c))*log(1/2*sqrt(3)*a^2*d*sqrt(1/(a^4*d^2)) + ((-I*e^(2*I*d*x + 2*
I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) + 1/2*I) - 9*(sqrt(3)*a^2*d*sqrt
(1/(a^4*d^2))*e^(4*I*d*x + 4*I*c) - I*e^(4*I*d*x + 4*I*c))*log(-1/2*sqrt(3)
*a^2*d*sqrt(1/(a^4*d^2)) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*
c) + 1))^(1/3) + 1/2*I) - 7*(3*sqrt(1/3)*a^2*d*sqrt(1/(a^4*d^2))*e^(4*I*d*x
 + 4*I*c) - I*e^(4*I*d*x + 4*I*c))*log(3/2*sqrt(1/3)*a^2*d*sqrt(1/(a^4*d^2)
) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) - 1/2*I)
 + 7*(3*sqrt(1/3)*a^2*d*sqrt(1/(a^4*d^2))*e^(4*I*d*x + 4*I*c) + I*e^(4*I*d*
x + 4*I*c))*log(-3/2*sqrt(1/3)*a^2*d*sqrt(1/(a^4*d^2)) + ((-I*e^(2*I*d*x +
2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) - 1/2*I) - 14*I*e^(4*I*d*x + 4
*I*c)*log(((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) +
I) - 18*I*e^(4*I*d*x + 4*I*c)*log(((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x
 + 2*I*c) + 1))^(1/3) - I) - 3*((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
2*I*c) + 1))^(1/3)*(5*e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) - 3))*e^(
-4*I*d*x - 4*I*c)/(a^2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(4/3)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.76, size = 228, normalized size = 0.68

$$-\frac{7\sqrt{3}\log\left(-\frac{\sqrt{3}-2\tan(dx+c)^{\frac{1}{3}}+i}{\sqrt{3}+2\tan(dx+c)^{\frac{1}{3}}-i}\right)}{144a^2d}+\frac{\sqrt{3}\log\left(-\frac{\sqrt{3}-2\tan(dx+c)^{\frac{1}{3}}-i}{\sqrt{3}+2\tan(dx+c)^{\frac{1}{3}}+i}\right)}{16a^2d}-\frac{i\log\left(\tan(dx+c)^{\frac{2}{3}}+i\tan(dx+c)^{\frac{1}{3}}-1\right)}{16a^2d}-\frac{7i\log\left(\tan(dx+c)^{\frac{2}{3}}-i\tan(dx+c)^{\frac{1}{3}}-1\right)}{144a^2d}+\frac{7i\log\left(\tan(dx+c)^{\frac{1}{3}}+i\right)}{72a^2d}+\frac{i\log\left(\tan(dx+c)^{\frac{1}{3}}-i\right)}{8a^2d}+\frac{-4i\tan(dx+c)^{\frac{4}{3}}-\tan(dx+c)^{\frac{1}{3}}}{12a^2d(\tan(dx+c)-i)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(4/3)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -7/144*sqrt(3)*log(-(sqrt(3) - 2*tan(d*x + c)^(1/3) + I)/(sqrt(3) + 2*tan(d
*x + c)^(1/3) - I))/(a^2*d) + 1/16*sqrt(3)*log(-(sqrt(3) - 2*tan(d*x + c)^(
1/3) - I)/(sqrt(3) + 2*tan(d*x + c)^(1/3) + I))/(a^2*d) - 1/16*I*log(tan(d*
x + c)^(2/3) + I*tan(d*x + c)^(1/3) - 1)/(a^2*d) - 7/144*I*log(tan(d*x + c)
^(2/3) - I*tan(d*x + c)^(1/3) - 1)/(a^2*d) + 7/72*I*log(tan(d*x + c)^(1/3)
+ I)/(a^2*d) + 1/8*I*log(tan(d*x + c)^(1/3) - I)/(a^2*d) + 1/12*(-4*I*tan(d
*x + c)^(4/3) - tan(d*x + c)^(1/3))/(a^2*d*(tan(d*x + c) - I)^2)
```

Mupad [B]
time = 5.18, size = 653, normalized size = 1.95

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(4/3)/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] log(((a^6*d^3*49408i)/3 + 3538944*a^10*d^5*tan(c + d*x)^(1/3)*(-1i/(512*a^6
*d^3))^(2/3))*(-1i/(512*a^6*d^3))^(1/3) + (a^4*d^2*tan(c + d*x)^(1/3)*14560
i)/3)*(-1i/(512*a^6*d^3))^(1/3) - ((tan(c + d*x)^(1/3)*1i)/(12*a^2*d) - tan
(c + d*x)^(4/3)/(3*a^2*d))/(2*tan(c + d*x) + tan(c + d*x)^2*1i - 1i) + log(
((a^6*d^3*49408i)/3 + 3538944*a^10*d^5*tan(c + d*x)^(1/3)*(-343i/(373248*a^
6*d^3))^(2/3))*(-343i/(373248*a^6*d^3))^(1/3) + (a^4*d^2*tan(c + d*x)^(1/3)
*14560i)/3)*(-343i/(373248*a^6*d^3))^(1/3) + (log(((3^(1/2)*1i - 1)*((a^6*d
^3*49408i)/3 + 884736*a^10*d^5*tan(c + d*x)^(1/3)*(3^(1/2)*1i - 1)^2*(-1i/(
512*a^6*d^3))^(2/3))*(-1i/(512*a^6*d^3))^(1/3))/2 + (a^4*d^2*tan(c + d*x)^(
1/3)*14560i)/3)*(3^(1/2)*1i - 1)*(-1i/(512*a^6*d^3))^(1/3))/2 - (log(((3^(1
/2)*1i + 1)*((a^6*d^3*49408i)/3 + 884736*a^10*d^5*tan(c + d*x)^(1/3)*(3^(1/
2)*1i + 1)^2*(-1i/(512*a^6*d^3))^(2/3))*(-1i/(512*a^6*d^3))^(1/3))/2 - (a^4
*d^2*tan(c + d*x)^(1/3)*14560i)/3)*(3^(1/2)*1i + 1)*(-1i/(512*a^6*d^3))^(1/
3))/2 + (log(((3^(1/2)*1i - 1)*((a^6*d^3*49408i)/3 + 884736*a^10*d^5*tan(c
+ d*x)^(1/3)*(3^(1/2)*1i - 1)^2*(-343i/(373248*a^6*d^3))^(2/3))*(-343i/(373
248*a^6*d^3))^(1/3))/2 + (a^4*d^2*tan(c + d*x)^(1/3)*14560i)/3)*(3^(1/2)*1i
 - 1)*(-343i/(373248*a^6*d^3))^(1/3))/2 - (log(((3^(1/2)*1i + 1)*((a^6*d^3*
49408i)/3 + 884736*a^10*d^5*tan(c + d*x)^(1/3)*(3^(1/2)*1i + 1)^2*(-343i/(3
73248*a^6*d^3))^(2/3))*(-343i/(373248*a^6*d^3))^(1/3))/2 - (a^4*d^2*tan(c +
 d*x)^(1/3)*14560i)/3)*(3^(1/2)*1i + 1)*(-343i/(373248*a^6*d^3))^(1/3))/2
```

3.244 $\int \frac{\tan^{\frac{2}{3}}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=337

$$-\frac{\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d}+\frac{\text{ArcTan}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d}+\frac{i\text{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)}{3\sqrt{3}\,a^2d}+\frac{\text{ArcTa}}{}$$

```
[Out] 1/72*arctan(-3^(1/2)+2*tan(d*x+c)^(1/3))/a^2/d+1/72*arctan(3^(1/2)+2*tan(d*
x+c)^(1/3))/a^2/d+1/36*arctan(tan(d*x+c)^(1/3))/a^2/d-1/9*I*ln(1+tan(d*x+c)
^(2/3))/a^2/d+1/18*I*ln(1-tan(d*x+c)^(2/3)+tan(d*x+c)^(4/3))/a^2/d+1/9*I*ar
ctan(1/3*(1-2*tan(d*x+c)^(2/3))*3^(1/2))/a^2/d*3^(1/2)+1/144*ln(1-3^(1/2)*t
an(d*x+c)^(1/3)+tan(d*x+c)^(2/3))/a^2/d*3^(1/2)-1/144*ln(1+3^(1/2)*tan(d*x+
c)^(1/3)+tan(d*x+c)^(2/3))/a^2/d*3^(1/2)+1/3*I*tan(d*x+c)^(2/3)/a^2/d/(1+I*
tan(d*x+c))+1/4*tan(d*x+c)^(5/3)/d/(a+I*a*tan(d*x+c))^2
```

Rubi [A]

time = 0.39, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 14, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3640, 3676, 3619, 3557, 335, 281, 206, 31, 648, 632, 210, 642, 301, 209}

$$\frac{i\text{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)}{3\sqrt{3}\,a^2d}-\frac{\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d}+\frac{\text{ArcTan}\left(2\sqrt[3]{\tan(c+dx)}+\sqrt{3}\right)}{72a^2d}+\frac{\text{ArcTan}\left(\sqrt[3]{\tan(c+dx)}\right)}{36a^2d}+\frac{i\tan^{\frac{2}{3}}(c+dx)}{3a^2d(1+i\tan(c+dx))}-\frac{i\log\left(\tan^{\frac{2}{3}}(c+dx)+1\right)}{9a^2d}+\frac{\log\left(\tan^{\frac{2}{3}}(c+dx)-\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{48\sqrt{3}\,a^2d}-\frac{\log\left(\tan^{\frac{2}{3}}(c+dx)+\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{48\sqrt{3}\,a^2d}+\frac{i\log\left(\tan^{\frac{4}{3}}(c+dx)-\tan^{\frac{2}{3}}(c+dx)+1\right)}{18a^2d}+\frac{\tan^{\frac{5}{3}}(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(2/3)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -1/72*ArcTan[Sqrt[3] - 2*Tan[c + d*x]^(1/3)]/(a^2*d) + ArcTan[Sqrt[3] + 2*T
an[c + d*x]^(1/3)]/(72*a^2*d) + ((I/3)*ArcTan[(1 - 2*Tan[c + d*x]^(2/3))/Sq
rt[3]])/(Sqrt[3]*a^2*d) + ArcTan[Tan[c + d*x]^(1/3)]/(36*a^2*d) - ((I/9)*Lo
g[1 + Tan[c + d*x]^(2/3)])/(a^2*d) + Log[1 - Sqrt[3]*Tan[c + d*x]^(1/3) + T
an[c + d*x]^(2/3)]/(48*Sqrt[3]*a^2*d) - Log[1 + Sqrt[3]*Tan[c + d*x]^(1/3)
+ Tan[c + d*x]^(2/3)]/(48*Sqrt[3]*a^2*d) + ((I/18)*Log[1 - Tan[c + d*x]^(2/
3) + Tan[c + d*x]^(4/3)])/(a^2*d) + ((I/3)*Tan[c + d*x]^(2/3))/(a^2*d*(1 +
I*Tan[c + d*x])) + Tan[c + d*x]^(5/3)/(4*d*(a + I*a*Tan[c + d*x])^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\tan^{\frac{2}{3}}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\tan^{\frac{5}{3}}(c+dx)}{4d(a+ia\tan(c+dx))^2} + \frac{\int \frac{\tan^{\frac{2}{3}}(c+dx)\left(\frac{7a}{3}-\frac{1}{3}ia\tan(c+dx)\right)}{a+ia\tan(c+dx)}\,dx}{4a^2}$$

$$= \frac{i\tan^{\frac{2}{3}}(c+dx)}{3a^2d(1+i\tan(c+dx))} + \frac{\tan^{\frac{5}{3}}(c+dx)}{4d(a+ia\tan(c+dx))^2} - \frac{\int \frac{\frac{16ia^2}{9}-\frac{2}{9}a^2\tan(c+dx)}{\sqrt[3]{\tan(c+dx)}}\,dx}{8a^4}$$

$$= \frac{i\tan^{\frac{2}{3}}(c+dx)}{3a^2d(1+i\tan(c+dx))} + \frac{\tan^{\frac{5}{3}}(c+dx)}{4d(a+ia\tan(c+dx))^2} - \frac{(2i)\int \frac{1}{\sqrt[3]{\tan(c+dx)}}\,d}{9a^2}$$

$$= \frac{i\tan^{\frac{2}{3}}(c+dx)}{3a^2d(1+i\tan(c+dx))} + \frac{\tan^{\frac{5}{3}}(c+dx)}{4d(a+ia\tan(c+dx))^2} - \frac{(2i)\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\,(1+x^2)}\right.}{9a^2}$$

$$= \frac{i\tan^{\frac{2}{3}}(c+dx)}{3a^2d(1+i\tan(c+dx))} + \frac{\tan^{\frac{5}{3}}(c+dx)}{4d(a+ia\tan(c+dx))^2} - \frac{(2i)\text{Subst}\left(\int \frac{x}{1+x^6}\,dx, x\right.}{3a^2d}$$

$$= \frac{i\tan^{\frac{2}{3}}(c+dx)}{3a^2d(1+i\tan(c+dx))} + \frac{\tan^{\frac{5}{3}}(c+dx)}{4d(a+ia\tan(c+dx))^2} - \frac{i\text{Subst}\left(\int \frac{1}{1+x^3}\,dx, x, \text{ta}\right.}{3a^2d}$$

$$= \frac{\tan^{-1}\left(\sqrt[3]{\tan(c+dx)}\right)}{36a^2d} + \frac{i\tan^{\frac{2}{3}}(c+dx)}{3a^2d(1+i\tan(c+dx))} + \frac{\tan^{\frac{5}{3}}(c+dx)}{4d(a+ia\tan(c+dx))}$$

$$= \frac{\tan^{-1}\left(\sqrt[3]{\tan(c+dx)}\right)}{36a^2d} - \frac{i\log\left(1+\tan^{\frac{2}{3}}(c+dx)\right)}{9a^2d} + \frac{\log\left(1-\sqrt{3}\,\sqrt[3]{\tan(c\right.}{48}$$

$$= -\frac{\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d} + \frac{\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d} + \frac{\tan}{}$$

$$= -\frac{\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d} + \frac{\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d} + \frac{i\,\text{ta}}{}$$

<u>Mathematica</u> [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.59, size = 194, normalized size = 0.58

$$\frac{i\sec^2(c+dx)\left(9\sqrt[3]{2}\,e^{2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{1}{2}\left(1-e^{2i(c+dx)}\right)\right)+14\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)(\cos(2(c+dx))+i\sin(2(c+dx)))-4(4+4\cos(2(c+dx))+i\sin(2(c+dx)))\right)\tan^{\frac{2}{3}}(c+dx)}{96a^2d(-i+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(2/3)/(a + I*a*Tan[c + d*x])^2,x]
```

[Out] ((I/96)*Sec[c + d*x]^2*(9*2^(1/3)*E^((2*I)*(c + d*x))*(1 + E^((2*I)*(c + d*x)))^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, (1 - E^((2*I)*(c + d*x)))/2] + 14*Hypergeometric2F1[2/3, 1, 5/3, -((-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) - 4*(4 + 4*Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]))*Tan[c + d*x]^(2/3))/(a^2*d*(-I + Tan[c + d*x])^2)

Maple [A]
time = 0.17, size = 225, normalized size = 0.67

method	result
derivativedivides	$-\frac{7i\ln\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}{72}+\frac{i}{36\left(\tan^{\frac{1}{3}}(dx+c)+i\right)^{2}}+\frac{1}{36\left(\tan^{\frac{1}{3}}(dx+c)\right)+36i}+\frac{i\ln\left(i\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)-1\right)}{16}-\frac{\sqrt{3}\ \mathrm{arctanh}}{}$
default	$-\frac{7i\ln\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}{72}+\frac{i}{36\left(\tan^{\frac{1}{3}}(dx+c)+i\right)^{2}}+\frac{1}{36\left(\tan^{\frac{1}{3}}(dx+c)\right)+36i}+\frac{i\ln\left(i\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)-1\right)}{16}-\frac{\sqrt{3}\ \mathrm{arctanh}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)^(2/3)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d/a^2*(-7/72*I*ln(tan(d*x+c)^(1/3)+I)+1/36*I/(tan(d*x+c)^(1/3)+I)^2+1/36/(tan(d*x+c)^(1/3)+I)+1/16*I*ln(I*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3)-1)-1/8*3^(1/2)*arctanh(1/3*(I+2*tan(d*x+c)^(1/3))*3^(1/2))+1/72*(4*tan(d*x+c)-8*I*tan(d*x+c)^(2/3)-14*tan(d*x+c)^(1/3)+4*I)/(-I*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3)-1)^2+7/144*I*ln(-I*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3)-1)+7/72*3^(1/2)*arctanh(1/3*(-I+2*tan(d*x+c)^(1/3))*3^(1/2))-1/8*I*ln(tan(d*x+c)^(1/3)-I))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^(2/3)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

Fricas [A]
time = 0.70, size = 521, normalized size = 1.55

$$-\frac{\left(9\left(\sqrt{3}a^{2}d\sqrt{\frac{1}{a^{4}d^{2}}}e^{(4idx+4ic)}-ie^{(4idx+4ic)}\right)\log\left(\frac{1}{2}\sqrt{3}a^{2}d\sqrt{\frac{1}{a^{4}d^{2}}}+\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}+\frac{1}{2}i\right)-9\left(\sqrt{3}a^{2}d\sqrt{\frac{1}{a^{4}d^{2}}}e^{(4idx+4ic)}+ie^{(4idx+4ic)}\right)\log\left(-\frac{1}{2}\sqrt{3}a^{2}d\sqrt{\frac{1}{a^{4}d^{2}}}+\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}+\frac{1}{2}i\right)-7\left(3\sqrt{\frac{1}{3}}a^{2}d\sqrt{\frac{1}{a^{4}d^{2}}}e^{(4idx+4ic)}+ie^{(4idx+4ic)}\right)\log\left(\frac{3}{2}\sqrt{\frac{1}{3}}a^{2}d\sqrt{\frac{1}{a^{4}d^{2}}}+\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}-\frac{1}{2}i\right)+7\left(3\sqrt{\frac{1}{3}}a^{2}d\sqrt{\frac{1}{a^{4}d^{2}}}e^{(4idx+4ic)}-ie^{(4idx+4ic)}\right)\log\left(-\frac{3}{2}\sqrt{\frac{1}{3}}a^{2}d\sqrt{\frac{1}{a^{4}d^{2}}}+\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}-\frac{1}{2}i\right)+14ie^{(4idx+4ic)}\log\left(\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}+i\right)+18ie^{(4idx+4ic)}\log\left(\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}-i\right)+3\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{2}{3}}\left(-5ie^{(4idx+4ic)}-8ie^{(2idx+2ic)}-3i\right)\right)e^{(-4idx-4ic)}}{144a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(2/3)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/144*(9*(sqrt(3)*a^2*d*sqrt(1/(a^4*d^2))*e^(4*I*d*x + 4*I*c) - I*e^(4*I*d
*x + 4*I*c))*log(1/2*sqrt(3)*a^2*d*sqrt(1/(a^4*d^2)) + ((-I*e^(2*I*d*x + 2*
I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) + 1/2*I) - 9*(sqrt(3)*a^2*d*sqrt
(1/(a^4*d^2))*e^(4*I*d*x + 4*I*c) + I*e^(4*I*d*x + 4*I*c))*log(-1/2*sqrt(3)
*a^2*d*sqrt(1/(a^4*d^2)) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*
c) + 1))^(1/3) + 1/2*I) - 7*(3*sqrt(1/3)*a^2*d*sqrt(1/(a^4*d^2))*e^(4*I*d*x
 + 4*I*c) + I*e^(4*I*d*x + 4*I*c))*log(3/2*sqrt(1/3)*a^2*d*sqrt(1/(a^4*d^2)
) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) - 1/2*I)
 + 7*(3*sqrt(1/3)*a^2*d*sqrt(1/(a^4*d^2))*e^(4*I*d*x + 4*I*c) - I*e^(4*I*d*
x + 4*I*c))*log(-3/2*sqrt(1/3)*a^2*d*sqrt(1/(a^4*d^2)) + ((-I*e^(2*I*d*x +
2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) - 1/2*I) + 14*I*e^(4*I*d*x + 4
*I*c)*log(((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) +
I) + 18*I*e^(4*I*d*x + 4*I*c)*log(((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x
 + 2*I*c) + 1))^(1/3) - I) + 3*((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
2*I*c) + 1))^(2/3)*(-5*I*e^(4*I*d*x + 4*I*c) - 8*I*e^(2*I*d*x + 2*I*c) - 3*
I))*e^(-4*I*d*x - 4*I*c)/(a^2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(2/3)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.73, size = 226, normalized size = 0.67

$$-\frac{7\sqrt{3}\,\log\left(-\frac{\sqrt{3}-2\,\tan(dx+c)^{\frac{1}{3}}+i}{\sqrt{3}+2\,\tan(dx+c)^{\frac{1}{3}}-i}\right)}{144\,a^2d}+\frac{\sqrt{3}\,\log\left(-\frac{\sqrt{3}-2\,\tan(dx+c)^{\frac{1}{3}}-i}{\sqrt{3}+2\,\tan(dx+c)^{\frac{1}{3}}+i}\right)}{16\,a^2d}+\frac{i\,\log\left(\tan\left(dx+c\right)^{\frac{2}{3}}+i\,\tan\left(dx+c\right)^{\frac{1}{3}}-1\right)}{16\,a^2d}+\frac{7i\,\log\left(\tan\left(dx+c\right)^{\frac{2}{3}}-i\,\tan\left(dx+c\right)^{\frac{1}{3}}-1\right)}{144\,a^2d}-\frac{7i\,\log\left(\tan\left(dx+c\right)^{\frac{1}{3}}+i\right)}{72\,a^2d}-\frac{i\,\log\left(\tan\left(dx+c\right)^{\frac{1}{3}}-i\right)}{8\,a^2d}+\frac{\tan\left(dx+c\right)^{\frac{5}{3}}-4i\,\tan\left(dx+c\right)^{\frac{2}{3}}}{12\,a^2d(\tan\left(dx+c\right)-i)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(2/3)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -7/144*sqrt(3)*log(-(sqrt(3) - 2*tan(d*x + c)^(1/3) + I)/(sqrt(3) + 2*tan(d
*x + c)^(1/3) - I))/(a^2*d) + 1/16*sqrt(3)*log(-(sqrt(3) - 2*tan(d*x + c)^(
1/3) - I)/(sqrt(3) + 2*tan(d*x + c)^(1/3) + I))/(a^2*d) + 1/16*I*log(tan(d*
x + c)^(2/3) + I*tan(d*x + c)^(1/3) - 1)/(a^2*d) + 7/144*I*log(tan(d*x + c)
^(2/3) - I*tan(d*x + c)^(1/3) - 1)/(a^2*d) - 7/72*I*log(tan(d*x + c)^(1/3)
+ I)/(a^2*d) - 1/8*I*log(tan(d*x + c)^(1/3) - I)/(a^2*d) + 1/12*(tan(d*x +
c)^(5/3) - 4*I*tan(d*x + c)^(2/3))/(a^2*d*(tan(d*x + c) - I)^2)
```

Mupad [B]
time = 5.06, size = 640, normalized size = 1.90

$$\ln\left(\left(\frac{a^6\,d^3\,49408{}\mathrm{i}}{3}-a^8\,d^4\,{\tan\left(c+d\,x\right)}^{1/3}\,{\left(\frac{1{}\mathrm{i}}{512\,a^6\,d^3}\right)}^{1/3}\,399360{}\mathrm{i}\right)\,{\left(\frac{1{}\mathrm{i}}{512\,a^6\,d^3}\right)}^{2/3}-\frac{1568\,a^2\,d\,{\tan\left(c+d\,x\right)}^{1/3}}{3}\right)\,{\left(\frac{1{}\mathrm{i}}{512\,a^6\,d^3}\right)}^{1/3}+\ln\left(\left(\frac{a^6\,d^3\,49408{}\mathrm{i}}{3}-a^8\,d^4\,{\tan\left(c+d\,x\right)}^{1/3}\,{\left(\frac{343{}\mathrm{i}}{373248\,a^6\,d^3}\right)}^{1/3}\,399360{}\mathrm{i}\right)\,{\left(\frac{343{}\mathrm{i}}{373248\,a^6\,d^3}\right)}^{2/3}-\frac{1568\,a^2\,d\,{\tan\left(c+d\,x\right)}^{1/3}}{3}\right)\,{\left(\frac{343{}\mathrm{i}}{373248\,a^6\,d^3}\right)}^{1/3}+\frac{\frac{{\tan\left(c+d\,x\right)}^{2/3}}{3\,a^2\,d}+\frac{{\tan\left(c+d\,x\right)}^{5/3}\,1{}\mathrm{i}}{12\,a^2\,d}}{{\tan\left(c+d\,x\right)}^2\,1{}\mathrm{i}+2\,\tan\left(c+d\,x\right)-\mathrm{i}}+\frac{\ln\left(-\frac{1568\,a^2\,d\,{\tan\left(c+d\,x\right)}^{1/3}}{3}+\frac{{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,\left(\frac{a^6\,d^3\,49408{}\mathrm{i}}{3}-a^8\,d^4\,{\tan\left(c+d\,x\right)}^{1/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(\frac{1{}\mathrm{i}}{512\,a^6\,d^3}\right)}^{1/3}\,199680{}\mathrm{i}\right)\,{\left(\frac{1{}\mathrm{i}}{512\,a^6\,d^3}\right)}^{2/3}}{4}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(\frac{1{}\mathrm{i}}{512\,a^6\,d^3}\right)}^{1/3}}{2}-\frac{\ln\left(-\frac{1568\,a^2\,d\,{\tan\left(c+d\,x\right)}^{1/3}}{3}+\frac{{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,\left(\frac{a^6\,d^3\,49408{}\mathrm{i}}{3}+a^8\,d^4\,{\tan\left(c+d\,x\right)}^{1/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(\frac{1{}\mathrm{i}}{512\,a^6\,d^3}\right)}^{1/3}\,199680{}\mathrm{i}\right)\,{\left(\frac{1{}\mathrm{i}}{512\,a^6\,d^3}\right)}^{2/3}}{4}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(\frac{1{}\mathrm{i}}{512\,a^6\,d^3}\right)}^{1/3}}{2}+\frac{\ln\left(-\frac{1568\,a^2\,d\,{\tan\left(c+d\,x\right)}^{1/3}}{3}+\frac{{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,\left(\frac{a^6\,d^3\,49408{}\mathrm{i}}{3}-a^8\,d^4\,{\tan\left(c+d\,x\right)}^{1/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(\frac{343{}\mathrm{i}}{373248\,a^6\,d^3}\right)}^{1/3}\,199680{}\mathrm{i}\right)\,{\left(\frac{343{}\mathrm{i}}{373248\,a^6\,d^3}\right)}^{2/3}}{4}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(\frac{343{}\mathrm{i}}{373248\,a^6\,d^3}\right)}^{1/3}}{2}-\frac{\ln\left(-\frac{1568\,a^2\,d\,{\tan\left(c+d\,x\right)}^{1/3}}{3}+\frac{{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,\left(\frac{a^6\,d^3\,49408{}\mathrm{i}}{3}+a^8\,d^4\,{\tan\left(c+d\,x\right)}^{1/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(\frac{343{}\mathrm{i}}{373248\,a^6\,d^3}\right)}^{1/3}\,199680{}\mathrm{i}\right)\,{\left(\frac{343{}\mathrm{i}}{373248\,a^6\,d^3}\right)}^{2/3}}{4}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(\frac{343{}\mathrm{i}}{373248\,a^6\,d^3}\right)}^{1/3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(2/3)/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] log(((a^6*d^3*49408i)/3 - a^8*d^4*tan(c + d*x)^(1/3)*(1i/(512*a^6*d^3))^(1/
3)*399360i)*(1i/(512*a^6*d^3))^(2/3) - (1568*a^2*d*tan(c + d*x)^(1/3))/3)*(
1i/(512*a^6*d^3))^(1/3) + log(((a^6*d^3*49408i)/3 - a^8*d^4*tan(c + d*x)^(1
/3)*(343i/(373248*a^6*d^3))^(1/3)*399360i)*(343i/(373248*a^6*d^3))^(2/3) -
(1568*a^2*d*tan(c + d*x)^(1/3))/3)*(343i/(373248*a^6*d^3))^(1/3) + (tan(c +
 d*x)^(2/3)/(3*a^2*d) + (tan(c + d*x)^(5/3)*1i)/(12*a^2*d))/(2*tan(c + d*x)
 + tan(c + d*x)^2*1i - 1i) + (log(((3^(1/2)*1i - 1)^2*((a^6*d^3*49408i)/3 -
 a^8*d^4*tan(c + d*x)^(1/3)*(3^(1/2)*1i - 1)*(1i/(512*a^6*d^3))^(1/3)*19968
0i)*(1i/(512*a^6*d^3))^(2/3))/4 - (1568*a^2*d*tan(c + d*x)^(1/3))/3)*(3^(1/
2)*1i - 1)*(1i/(512*a^6*d^3))^(1/3))/2 - (log(((3^(1/2)*1i + 1)^2*((a^6*d^3
*49408i)/3 + a^8*d^4*tan(c + d*x)^(1/3)*(3^(1/2)*1i + 1)*(1i/(512*a^6*d^3))
^(1/3)*199680i)*(1i/(512*a^6*d^3))^(2/3))/4 - (1568*a^2*d*tan(c + d*x)^(1/3
))/3)*(3^(1/2)*1i + 1)*(1i/(512*a^6*d^3))^(1/3))/2 + (log(((3^(1/2)*1i - 1)
^2*((a^6*d^3*49408i)/3 - a^8*d^4*tan(c + d*x)^(1/3)*(3^(1/2)*1i - 1)*(343i/
(373248*a^6*d^3))^(1/3)*199680i)*(343i/(373248*a^6*d^3))^(2/3))/4 - (1568*a
^2*d*tan(c + d*x)^(1/3))/3)*(3^(1/2)*1i - 1)*(343i/(373248*a^6*d^3))^(1/3))
/2 - (log(((3^(1/2)*1i + 1)^2*((a^6*d^3*49408i)/3 + a^8*d^4*tan(c + d*x)^(1
/3)*(3^(1/2)*1i + 1)*(343i/(373248*a^6*d^3))^(1/3)*199680i)*(343i/(373248*a
^6*d^3))^(2/3))/4 - (1568*a^2*d*tan(c + d*x)^(1/3))/3)*(3^(1/2)*1i + 1)*(34
3i/(373248*a^6*d^3))^(1/3))/2
```

3.245 $\int \frac{1}{\sqrt[3]{\tan(c+dx)}\,(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=339

$$\frac{7i\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d}-\frac{7i\text{ArcTan}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d}-\frac{2\text{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)}{3\sqrt{3}\,a^2d}-\frac{7i\text{A}}{}$$

```
[Out] -7/72*I*arctan(-3^(1/2)+2*tan(d*x+c)^(1/3))/a^2/d-7/72*I*arctan(3^(1/2)+2*t
an(d*x+c)^(1/3))/a^2/d-7/36*I*arctan(tan(d*x+c)^(1/3))/a^2/d+2/9*ln(1+tan(d
*x+c)^(2/3))/a^2/d-1/9*ln(1-tan(d*x+c)^(2/3)+tan(d*x+c)^(4/3))/a^2/d-2/9*ar
ctan(1/3*(1-2*tan(d*x+c)^(2/3))*3^(1/2))/a^2/d*3^(1/2)-7/144*I*ln(1-3^(1/2)
*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3))/a^2/d*3^(1/2)+7/144*I*ln(1+3^(1/2)*tan(
d*x+c)^(1/3)+tan(d*x+c)^(2/3))/a^2/d*3^(1/2)+7/12*tan(d*x+c)^(2/3)/a^2/d/(1
+I*tan(d*x+c))+1/4*tan(d*x+c)^(2/3)/d/(a+I*a*tan(d*x+c))^2
```

Rubi [A]
time = 0.39, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 14, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3640, 3677, 3619, 3557, 335, 281, 206, 31, 648, 632, 210, 642, 301, 209}

$$-\frac{2\text{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)}{3\sqrt{3}\,a^2d}+\frac{7i\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d}-\frac{7i\text{ArcTan}\left(2\sqrt[3]{\tan(c+dx)}+\sqrt{3}\right)}{72a^2d}-\frac{7i\text{ArcTan}\left(\sqrt[3]{\tan(c+dx)}\right)}{36a^2d}+\frac{7\tan^{\frac{2}{3}}(c+dx)}{12a^2d(1+i\tan(c+dx))}+\frac{2\log\left(\tan^{\frac{2}{3}}(c+dx)+1\right)}{9a^2d}-\frac{7i\log\left(\tan^{\frac{2}{3}}(c+dx)-\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{48\sqrt{3}\,a^2d}+\frac{7i\log\left(\tan^{\frac{2}{3}}(c+dx)+\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{48\sqrt{3}\,a^2d}-\frac{\log\left(\tan^{\frac{4}{3}}(c+dx)-\tan^{\frac{2}{3}}(c+dx)+1\right)}{9a^2d}+\frac{\tan^{\frac{2}{3}}(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(1/3)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (((7*I)/72)*ArcTan[Sqrt[3] - 2*Tan[c + d*x]^(1/3)])/(a^2*d) - (((7*I)/72)*A
rcTan[Sqrt[3] + 2*Tan[c + d*x]^(1/3)])/(a^2*d) - (2*ArcTan[(1 - 2*Tan[c + d
*x]^(2/3))/Sqrt[3]])/(3*Sqrt[3]*a^2*d) - (((7*I)/36)*ArcTan[Tan[c + d*x]^(1
/3)])/(a^2*d) + (2*Log[1 + Tan[c + d*x]^(2/3)])/(9*a^2*d) - (((7*I)/48)*Log
[1 - Sqrt[3]*Tan[c + d*x]^(1/3) + Tan[c + d*x]^(2/3)])/(Sqrt[3]*a^2*d) + ((
(7*I)/48)*Log[1 + Sqrt[3]*Tan[c + d*x]^(1/3) + Tan[c + d*x]^(2/3)])/(Sqrt[3
]*a^2*d) - Log[1 - Tan[c + d*x]^(2/3) + Tan[c + d*x]^(4/3)]/(9*a^2*d) + (7*
Tan[c + d*x]^(2/3))/(12*a^2*d*(1 + I*Tan[c + d*x])) + Tan[c + d*x]^(2/3)/(4
*d*(a + I*a*Tan[c + d*x])^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{\tan(c+dx)}\,(a+ia\tan(c+dx))^2}\,dx = \frac{\tan^{\frac{2}{3}}(c+dx)}{4d(a+ia\tan(c+dx))^2} + \frac{\int \frac{\frac{10a}{3}-\frac{4}{3}ia\tan(c+dx)}{\sqrt[3]{\tan(c+dx)}\,(a+ia\tan(c+dx))}\,dx}{4a^2}$$

$$= \frac{7\tan^{\frac{2}{3}}(c+dx)}{12a^2d(1+i\tan(c+dx))} + \frac{\tan^{\frac{2}{3}}(c+dx)}{4d(a+ia\tan(c+dx))^2} + \frac{\int \frac{\frac{32a^2}{9}-\ldots}{\sqrt[3]{\tan\ldots}}}{\ldots}$$

$$= \frac{7\tan^{\frac{2}{3}}(c+dx)}{12a^2d(1+i\tan(c+dx))} + \frac{\tan^{\frac{2}{3}}(c+dx)}{4d(a+ia\tan(c+dx))^2} - \frac{(7i)\int \tan\ldots}{\ldots}$$

$$= \frac{7\tan^{\frac{2}{3}}(c+dx)}{12a^2d(1+i\tan(c+dx))} + \frac{\tan^{\frac{2}{3}}(c+dx)}{4d(a+ia\tan(c+dx))^2} - \frac{(7i)\text{Subs}\ldots}{\ldots}$$

$$= \frac{7\tan^{\frac{2}{3}}(c+dx)}{12a^2d(1+i\tan(c+dx))} + \frac{\tan^{\frac{2}{3}}(c+dx)}{4d(a+ia\tan(c+dx))^2} - \frac{(7i)\text{Subs}\ldots}{\ldots}$$

$$= \frac{7\tan^{\frac{2}{3}}(c+dx)}{12a^2d(1+i\tan(c+dx))} + \frac{\tan^{\frac{2}{3}}(c+dx)}{4d(a+ia\tan(c+dx))^2} - \frac{(7i)\text{Subs}\ldots}{\ldots}$$

$$= -\frac{7i\tan^{-1}\left(\sqrt[3]{\tan(c+dx)}\right)}{36a^2d} + \frac{7\tan^{\frac{2}{3}}(c+dx)}{12a^2d(1+i\tan(c+dx))} + \frac{\ldots}{4d(\ldots}$$

$$= -\frac{7i\tan^{-1}\left(\sqrt[3]{\tan(c+dx)}\right)}{36a^2d} + \frac{2\log\left(1+\tan^{\frac{2}{3}}(c+dx)\right)}{9a^2d} - \frac{7i\ldots}{\ldots}$$

$$= \frac{7i\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d} - \frac{7i\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(\ldots}\right.}{72a^2d}$$

$$= \frac{7i\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d} - \frac{7i\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(\ldots}\right.}{72a^2d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.70, size = 189, normalized size = 0.56

$$-\frac{\sec^2(c+dx)\left(40+40\cos(2(c+dx))+9\sqrt[3]{2}\,e^{2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{1}{2}\left(1-e^{2i(c+dx)}\right)\right)+46\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)(\cos(2(c+dx))+i\sin(2(c+dx)))+28i\sin(2(c+dx))\right)\tan^{\frac{2}{3}}(c+dx)}{96a^2d(-i+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Tan[c + d*x]^(1/3)*(a + I*a*Tan[c + d*x])^2),x]
```

[Out] -1/96*(Sec[c + d*x]^2*(40 + 40*Cos[2*(c + d*x)] + 9*2^(1/3)*E^((2*I)*(c + d*x))*(1 + E^((2*I)*(c + d*x)))^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, (1 - E^((2*I)*(c + d*x)))/2] + 46*Hypergeometric2F1[2/3, 1, 5/3, -((-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) + (28*I)*Sin[2*(c + d*x)])*Tan[c + d*x]^(2/3))/(a^2*d*(-I + Tan[c + d*x])^2)

Maple [A]
time = 0.20, size = 223, normalized size = 0.66

method	result
derivativedivides	$-\frac{7i}{36\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}+\frac{1}{36\left(\tan^{\frac{1}{3}}(dx+c)+i\right)^{2}}+\frac{23\ln\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}{72}-\frac{\ln\left(i\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)-1\right)}{16}-\frac{i\sqrt{3}\ \mathrm{arctanh}\ldots}{\ldots}$
default	$-\frac{7i}{36\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}+\frac{1}{36\left(\tan^{\frac{1}{3}}(dx+c)+i\right)^{2}}+\frac{23\ln\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}{72}-\frac{\ln\left(i\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)-1\right)}{16}-\frac{i\sqrt{3}\ \mathrm{arctanh}\ldots}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/tan(d*x+c)^(1/3)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d/a^2*(-7/36*I/(tan(d*x+c)^(1/3)+I)+1/36/(tan(d*x+c)^(1/3)+I)^2+23/72*ln(tan(d*x+c)^(1/3)+I)-1/16*ln(I*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3)-1)-1/8*I*3^(1/2)*arctanh(1/3*(I+2*tan(d*x+c)^(1/3))*3^(1/2))+1/72*(-28*I*tan(d*x+c)-44*tan(d*x+c)^(2/3)+50*I*tan(d*x+c)^(1/3)+16)/(-I*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3)-1)^2-23/144*ln(-I*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3)-1)+23/72*I*3^(1/2)*arctanh(1/3*(-I+2*tan(d*x+c)^(1/3))*3^(1/2))+1/8*ln(tan(d*x+c)^(1/3)-I))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/tan(d*x+c)^(1/3)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

Fricas [A]
time = 0.68, size = 515, normalized size = 1.52

$$-\frac{\left(9\left(i\sqrt{3}a^{2}d\sqrt{\frac{1}{a^{4}d^{2}}}e^{(4idx+4ic)}+e^{(4idx+4ic)}\right)\log\left(\frac{1}{2}\sqrt{3}a^{2}d\sqrt{\frac{1}{a^{4}d^{2}}}+\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}+\frac{1}{2}i\right)+9\left(-i\sqrt{3}a^{2}d\sqrt{\frac{1}{a^{4}d^{2}}}e^{(4idx+4ic)}+e^{(4idx+4ic)}\right)\log\left(-\frac{1}{2}\sqrt{3}a^{2}d\sqrt{\frac{1}{a^{4}d^{2}}}+\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}+\frac{1}{2}i\right)+23\left(-3i\sqrt{\frac{1}{3}}a^{2}d\sqrt{\frac{1}{a^{4}d^{2}}}e^{(4idx+4ic)}+e^{(4idx+4ic)}\right)\log\left(\frac{3}{2}\sqrt{\frac{1}{3}}a^{2}d\sqrt{\frac{1}{a^{4}d^{2}}}+\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}-\frac{1}{2}i\right)+23\left(3i\sqrt{\frac{1}{3}}a^{2}d\sqrt{\frac{1}{a^{4}d^{2}}}e^{(4idx+4ic)}+e^{(4idx+4ic)}\right)\log\left(-\frac{3}{2}\sqrt{\frac{1}{3}}a^{2}d\sqrt{\frac{1}{a^{4}d^{2}}}+\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}-\frac{1}{2}i\right)-46e^{(4idx+4ic)}\log\left(\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}+i\right)-18e^{(4idx+4ic)}\log\left(\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}-i\right)-3\left(\frac{-ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}+1}\right)^{\frac{2}{3}}\left(17e^{(4idx+4ic)}+20e^{(2idx+2ic)}+3\right)\right)e^{(-4idx-4ic)}}{144a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/tan(d*x+c)^(1/3)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")

[Out] -1/144*(9*(I*sqrt(3)*a^2*d*sqrt(1/(a^4*d^2))*e^(4*I*d*x + 4*I*c) + e^(4*I*d*x + 4*I*c))*log(1/2*sqrt(3)*a^2*d*sqrt(1/(a^4*d^2)) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) + 1/2*I) + 9*(-I*sqrt(3)*a^2*d*sqrt(1/(a^4*d^2))*e^(4*I*d*x + 4*I*c) + e^(4*I*d*x + 4*I*c))*log(-1/2*sqrt(3)*a^2*d*sqrt(1/(a^4*d^2)) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) + 1/2*I) + 23*(-3*I*sqrt(1/3)*a^2*d*sqrt(1/(a^4*d^2))*e^(4*I*d*x + 4*I*c) + e^(4*I*d*x + 4*I*c))*log(3/2*sqrt(1/3)*a^2*d*sqrt(1/(a^4*d^2)) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) - 1/2*I) + 23*(3*I*sqrt(1/3)*a^2*d*sqrt(1/(a^4*d^2))*e^(4*I*d*x + 4*I*c) + e^(4*I*d*x + 4*I*c))*log(-3/2*sqrt(1/3)*a^2*d*sqrt(1/(a^4*d^2)) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) - 1/2*I) - 46*e^(4*I*d*x + 4*I*c)*log(((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) + I) - 18*e^(4*I*d*x + 4*I*c)*log(((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) - I) - 3*((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(17*e^(4*I*d*x + 4*I*c) + 20*e^(2*I*d*x + 2*I*c) + 3))*e^(-4*I*d*x - 4*I*c)/(a^2*d)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{\tan^{\frac{7}{3}}(c+dx)-2i\tan^{\frac{4}{3}}(c+dx)-\sqrt[3]{\tan(c+dx)}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/tan(d*x+c)**(1/3)/(a+I*a*tan(d*x+c))**2,x)

[Out] -Integral(1/(tan(c + d*x)**(7/3) - 2*I*tan(c + d*x)**(4/3) - tan(c + d*x)**(1/3)), x)/a**2

Giac [A]
time = 0.85, size = 228, normalized size = 0.67

$$-\frac{23i\sqrt{3}\log\left(-\frac{\sqrt{3}-2\tan(dx+c)^{\frac{1}{3}}+i}{\sqrt{3}+2\tan(dx+c)^{\frac{1}{3}}-i}\right)}{144a^2d}+\frac{i\sqrt{3}\log\left(-\frac{\sqrt{3}-2\tan(dx+c)^{\frac{1}{3}}-i}{\sqrt{3}+2\tan(dx+c)^{\frac{1}{3}}+i}\right)}{16a^2d}-\frac{\log\left(\tan(dx+c)^{\frac{2}{3}}+i\tan(dx+c)^{\frac{1}{3}}-1\right)}{16a^2d}-\frac{23\log\left(\tan(dx+c)^{\frac{2}{3}}-i\tan(dx+c)^{\frac{1}{3}}-1\right)}{144a^2d}+\frac{23\log\left(\tan(dx+c)^{\frac{1}{3}}+i\right)}{72a^2d}+\frac{\log\left(\tan(dx+c)^{\frac{1}{3}}-i\right)}{8a^2d}+\frac{-7i\tan(dx+c)^{\frac{5}{3}}-10\tan(dx+c)^{\frac{2}{3}}}{12a^2d(\tan(dx+c)-i)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/tan(d*x+c)^(1/3)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")

[Out] -23/144*I*sqrt(3)*log(-(sqrt(3) - 2*tan(d*x + c)^(1/3) + I)/(sqrt(3) + 2*tan(d*x + c)^(1/3) - I))/(a^2*d) + 1/16*I*sqrt(3)*log(-(sqrt(3) - 2*tan(d*x + c)^(1/3) - I)/(sqrt(3) + 2*tan(d*x + c)^(1/3) + I))/(a^2*d) - 1/16*log(tan(d*x + c)^(2/3) + I*tan(d*x + c)^(1/3) - 1)/(a^2*d) - 23/144*log(tan(d*x + c)^(2/3) - I*tan(d*x + c)^(1/3) - 1)/(a^2*d) + 23/72*log(tan(d*x + c)^(1/3)

```
 + I)/(a^2*d) + 1/8*log(tan(d*x + c)^(1/3) - I)/(a^2*d) + 1/12*(-7*I*tan(d*
x + c)^(5/3) - 10*tan(d*x + c)^(2/3))/(a^2*d*(tan(d*x + c) - I)^2)
```

Mupad [B]
time = 5.19, size = 630, normalized size = 1.86

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(1/3)*(a + a*tan(c + d*x)*1i)^2),x)
```

```
[Out] (23*log((529*((a^6*d^3*1464064i)/3 + (1795840*a^8*d^4*tan(c + d*x)^(1/3)*(1
/(a^6*d^3))^(1/3))/3)*(1/(a^6*d^3))^(2/3))/5184 - (33856*a^2*d*tan(c + d*x)
^(1/3))/3)*(1/(a^6*d^3))^(1/3))/72 + log(((a^6*d^3*1464064i)/3 + 1873920*a^
8*d^4*tan(c + d*x)^(1/3)*(1/(512*a^6*d^3))^(1/3))*(1/(512*a^6*d^3))^(2/3) -
 (33856*a^2*d*tan(c + d*x)^(1/3))/3)*(1/(512*a^6*d^3))^(1/3) - ((tan(c + d*
x)^(2/3)*5i)/(6*a^2*d) - (7*tan(c + d*x)^(5/3))/(12*a^2*d))/(2*tan(c + d*x)
 + tan(c + d*x)^2*1i - 1i) + (23*log((529*(3^(1/2)*1i - 1)^2*((a^6*d^3*1464
064i)/3 + (897920*a^8*d^4*tan(c + d*x)^(1/3)*(3^(1/2)*1i - 1)*(1/(a^6*d^3))
^(1/3))/3)*(1/(a^6*d^3))^(2/3))/20736 - (33856*a^2*d*tan(c + d*x)^(1/3))/3)
*(3^(1/2)*1i - 1)*(1/(a^6*d^3))^(1/3))/144 - (23*log((529*(3^(1/2)*1i + 1)^
2*((a^6*d^3*1464064i)/3 - (897920*a^8*d^4*tan(c + d*x)^(1/3)*(3^(1/2)*1i +
1)*(1/(a^6*d^3))^(1/3))/3)*(1/(a^6*d^3))^(2/3))/20736 - (33856*a^2*d*tan(c
+ d*x)^(1/3))/3)*(3^(1/2)*1i + 1)*(1/(a^6*d^3))^(1/3))/144 + log(((3^(1/2)*
1i)/2 - 1/2)^2*((a^6*d^3*1464064i)/3 + 1873920*a^8*d^4*tan(c + d*x)^(1/3)*(
(3^(1/2)*1i)/2 - 1/2)*(1/(512*a^6*d^3))^(1/3))*(1/(512*a^6*d^3))^(2/3) - (3
3856*a^2*d*tan(c + d*x)^(1/3))/3)*((3^(1/2)*1i)/2 - 1/2)*(1/(512*a^6*d^3))^
(1/3) - log(((3^(1/2)*1i)/2 + 1/2)^2*((a^6*d^3*1464064i)/3 - 1873920*a^8*d^
4*tan(c + d*x)^(1/3)*((3^(1/2)*1i)/2 + 1/2)*(1/(512*a^6*d^3))^(1/3))*(1/(51
2*a^6*d^3))^(2/3) - (33856*a^2*d*tan(c + d*x)^(1/3))/3)*((3^(1/2)*1i)/2 + 1
/2)*(1/(512*a^6*d^3))^(1/3)
```

3.246 $\int \frac{1}{\tan^{\frac{5}{3}}(c+dx)(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=359

$$\frac{55i\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d}-\frac{55i\text{ArcTan}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d}+\frac{8\text{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)}{3\sqrt{3}\,a^2d}-\frac{55}{}$$

```
[Out] -55/72*I*arctan(-3^(1/2)+2*tan(d*x+c)^(1/3))/a^2/d-55/72*I*arctan(3^(1/2)+2
*tan(d*x+c)^(1/3))/a^2/d-55/36*I*arctan(tan(d*x+c)^(1/3))/a^2/d+8/9*ln(1+ta
n(d*x+c)^(2/3))/a^2/d-4/9*ln(1-tan(d*x+c)^(2/3)+tan(d*x+c)^(4/3))/a^2/d+8/9
*arctan(1/3*(1-2*tan(d*x+c)^(2/3))*3^(1/2))/a^2/d*3^(1/2)+55/144*I*ln(1-3^(
1/2)*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3))/a^2/d*3^(1/2)-55/144*I*ln(1+3^(1/2)
*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3))/a^2/d*3^(1/2)-8/3/a^2/d/tan(d*x+c)^(2/3
)+11/12/a^2/d/(1+I*tan(d*x+c))/tan(d*x+c)^(2/3)+1/4/d/tan(d*x+c)^(2/3)/(a+I
*a*tan(d*x+c))^2
```

Rubi [A]
time = 0.37, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 15, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {3640, 3677, 3610, 3619, 3557, 335, 215, 648, 632, 210, 642, 209, 281, 298, 31}

$$\frac{8\text{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)}{3\sqrt{3}\,a^2d}+\frac{55i\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d}-\frac{55i\text{ArcTan}\left(2\sqrt[3]{\tan(c+dx)}+\sqrt{3}\right)}{72a^2d}-\frac{55i\text{ArcTan}\left(\sqrt[3]{\tan(c+dx)}\right)}{36a^2d}-\frac{8}{3a^2d\tan^{\frac{2}{3}}(c+dx)}+\frac{11}{12a^2d(1+i\tan(c+dx))\tan^{\frac{2}{3}}(c+dx)}+\frac{8\log\left(\tan^{\frac{2}{3}}(c+dx)+1\right)}{9a^2d}+\frac{55i\log\left(\tan^{\frac{2}{3}}(c+dx)-\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{48\sqrt{3}\,a^2d}-\frac{55i\log\left(\tan^{\frac{2}{3}}(c+dx)+\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{48\sqrt{3}\,a^2d}-\frac{4\log\left(\tan^{\frac{4}{3}}(c+dx)-\tan^{\frac{2}{3}}(c+dx)+1\right)}{9a^2d}+\frac{1}{4d\tan^{\frac{2}{3}}(c+dx)(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(5/3)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (((55*I)/72)*ArcTan[Sqrt[3] - 2*Tan[c + d*x]^(1/3)])/(a^2*d) - (((55*I)/72)
*ArcTan[Sqrt[3] + 2*Tan[c + d*x]^(1/3)])/(a^2*d) + (8*ArcTan[(1 - 2*Tan[c +
 d*x]^(2/3))/Sqrt[3]])/(3*Sqrt[3]*a^2*d) - (((55*I)/36)*ArcTan[Tan[c + d*x]
^(1/3)])/(a^2*d) + (8*Log[1 + Tan[c + d*x]^(2/3)])/(9*a^2*d) + (((55*I)/48)
*Log[1 - Sqrt[3]*Tan[c + d*x]^(1/3) + Tan[c + d*x]^(2/3)])/(Sqrt[3]*a^2*d)
- (((55*I)/48)*Log[1 + Sqrt[3]*Tan[c + d*x]^(1/3) + Tan[c + d*x]^(2/3)])/(S
qrt[3]*a^2*d) - (4*Log[1 - Tan[c + d*x]^(2/3) + Tan[c + d*x]^(4/3)])/(9*a^2
*d) - 8/(3*a^2*d*Tan[c + d*x]^(2/3)) + 11/(12*a^2*d*(1 + I*Tan[c + d*x])*Ta
n[c + d*x]^(2/3)) + 1/(4*d*Tan[c + d*x]^(2/3)*(a + I*a*Tan[c + d*x])^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\tan^{\frac{5}{3}}(c+dx)(a+ia\tan(c+dx))^2}\,dx = \frac{1}{4d\tan^{\frac{2}{3}}(c+dx)(a+ia\tan(c+dx))^2} + \frac{\int \frac{\frac{14a}{3}-\frac{8}{3}ia\tan(c+dx)}{\tan^{\frac{5}{3}}(c+dx)(a+ia\tan(c+dx))}}{4a^2}$$

$$= \frac{11}{12a^2d(1+i\tan(c+dx))\tan^{\frac{2}{3}}(c+dx)} + \frac{1}{4d\tan^{\frac{2}{3}}(c+dx)(a+ia}$$

$$= -\frac{8}{3a^2d\tan^{\frac{2}{3}}(c+dx)} + \frac{11}{12a^2d(1+i\tan(c+dx))\tan^{\frac{2}{3}}(c+dx)} +$$

$$= -\frac{8}{3a^2d\tan^{\frac{2}{3}}(c+dx)} + \frac{11}{12a^2d(1+i\tan(c+dx))\tan^{\frac{2}{3}}(c+dx)} +$$

$$= -\frac{8}{3a^2d\tan^{\frac{2}{3}}(c+dx)} + \frac{11}{12a^2d(1+i\tan(c+dx))\tan^{\frac{2}{3}}(c+dx)} +$$

$$= -\frac{8}{3a^2d\tan^{\frac{2}{3}}(c+dx)} + \frac{11}{12a^2d(1+i\tan(c+dx))\tan^{\frac{2}{3}}(c+dx)} +$$

$$= -\frac{8}{3a^2d\tan^{\frac{2}{3}}(c+dx)} + \frac{11}{12a^2d(1+i\tan(c+dx))\tan^{\frac{2}{3}}(c+dx)} +$$

$$= -\frac{55i\tan^{-1}\left(\sqrt[3]{\tan(c+dx)}\right)}{36a^2d} - \frac{8}{3a^2d\tan^{\frac{2}{3}}(c+dx)} + \frac{}{12a^2d(1+}$$

$$= -\frac{55i\tan^{-1}\left(\sqrt[3]{\tan(c+dx)}\right)}{36a^2d} + \frac{8\log\left(1+\tan^{\frac{2}{3}}(c+dx)\right)}{9a^2d} + \frac{55i}{}$$

$$= \frac{55i\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d} - \frac{55i\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(}\right.}{72a^2d}$$

$$= \frac{55i\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d} - \frac{55i\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(}\right.}{72a^2d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 3.98, size = 205, normalized size = 0.57

$$\frac{\sec(c+dx)\left(-\frac{36i2^{2/3}e^{3i(c+dx)}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{1}{2}\left(1-e^{2i(c+dx)}\right)\right)}{\left(1+e^{2i(c+dx)}\right)^{2/3}}+476i\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};-\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)(\cos(2(c+dx))\sec(c+dx)+2i\sin(c+dx))+4\csc(c+dx)(-14+50\cos(2(c+dx))+53i\sin(2(c+dx)))\right)\sqrt[3]{\tan(c+dx)}}{96a^2d(-i+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Tan[c + d*x]^(5/3)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (Sec[c + d*x]*(((-36*I)*2^(2/3)*E^((3*I)*(c + d*x))*Hypergeometric2F1[1/3,
1/3, 4/3, (1 - E^((2*I)*(c + d*x)))/2])/(1 + E^((2*I)*(c + d*x)))^(2/3) + (
476*I)*Hypergeometric2F1[1/3, 1, 4/3, -((-1 + E^((2*I)*(c + d*x)))/(1 + E^(
(2*I)*(c + d*x))))]*(Cos[2*(c + d*x)]*Sec[c + d*x] + (2*I)*Sin[c + d*x]) +
4*Csc[c + d*x]*(-14 + 50*Cos[2*(c + d*x)] + (53*I)*Sin[2*(c + d*x)]))*Tan[c
 + d*x]^(1/3))/(96*a^2*d*(-I + Tan[c + d*x])^2)
```

Maple [A]
time = 0.21, size = 233, normalized size = 0.65

method	result
derivativedivides	$\frac{-\frac{5i}{12\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}+\frac{1}{36\left(\tan^{\frac{1}{3}}(dx+c)+i\right)^{2}}+\frac{119\ln\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}{72}-\frac{\ln\left(i\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)-1\right)}{16}+\frac{i\sqrt{3}\ \mathrm{arctan}}{}}{}$
default	$\frac{-\frac{5i}{12\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}+\frac{1}{36\left(\tan^{\frac{1}{3}}(dx+c)+i\right)^{2}}+\frac{119\ln\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}{72}-\frac{\ln\left(i\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)-1\right)}{16}+\frac{i\sqrt{3}\ \mathrm{arctan}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(5/3)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(-5/12*I/(tan(d*x+c)^(1/3)+I)+1/36/(tan(d*x+c)^(1/3)+I)^2+119/72*ln
(tan(d*x+c)^(1/3)+I)-1/16*ln(I*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3)-1)+1/8*I*3
^(1/2)*arctanh(1/3*(I+2*tan(d*x+c)^(1/3))*3^(1/2))+1/72*(30*I*tan(d*x+c)+4*
tan(d*x+c)^(2/3)-4*I*tan(d*x+c)^(1/3)+32)/(-I*tan(d*x+c)^(1/3)+tan(d*x+c)^(
2/3)-1)^2-119/144*ln(-I*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3)-1)-119/72*I*3^(1/
2)*arctanh(1/3*(-I+2*tan(d*x+c)^(1/3))*3^(1/2))-3/2/tan(d*x+c)^(2/3)+1/8*ln
(tan(d*x+c)^(1/3)-I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(5/3)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 681 vs. $2(282) = 564$.
time = 0.63, size = 681, normalized size = 1.90

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(5/3)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/144*(9*(sqrt(3)*(-I*a^2*d*e^(6*I*d*x + 6*I*c) + I*a^2*d*e^(4*I*d*x + 4*I
*c))*sqrt(1/(a^4*d^2)) + e^(6*I*d*x + 6*I*c) - e^(4*I*d*x + 4*I*c))*log(1/2
*sqrt(3)*a^2*d*sqrt(1/(a^4*d^2)) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*
x + 2*I*c) + 1))^(1/3) + 1/2*I) + 9*(sqrt(3)*(I*a^2*d*e^(6*I*d*x + 6*I*c) -
 I*a^2*d*e^(4*I*d*x + 4*I*c))*sqrt(1/(a^4*d^2)) + e^(6*I*d*x + 6*I*c) - e^(
4*I*d*x + 4*I*c))*log(-1/2*sqrt(3)*a^2*d*sqrt(1/(a^4*d^2)) + ((-I*e^(2*I*d*
x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) + 1/2*I) + 119*(3*sqrt(1/3
)*(I*a^2*d*e^(6*I*d*x + 6*I*c) - I*a^2*d*e^(4*I*d*x + 4*I*c))*sqrt(1/(a^4*d
^2)) + e^(6*I*d*x + 6*I*c) - e^(4*I*d*x + 4*I*c))*log(3/2*sqrt(1/3)*a^2*d*s
qrt(1/(a^4*d^2)) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))
^(1/3) - 1/2*I) + 119*(3*sqrt(1/3)*(-I*a^2*d*e^(6*I*d*x + 6*I*c) + I*a^2*d*
e^(4*I*d*x + 4*I*c))*sqrt(1/(a^4*d^2)) + e^(6*I*d*x + 6*I*c) - e^(4*I*d*x +
 4*I*c))*log(-3/2*sqrt(1/3)*a^2*d*sqrt(1/(a^4*d^2)) + ((-I*e^(2*I*d*x + 2*I
*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) - 1/2*I) - 238*(e^(6*I*d*x + 6*I*
c) - e^(4*I*d*x + 4*I*c))*log(((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2
*I*c) + 1))^(1/3) + I) - 18*(e^(6*I*d*x + 6*I*c) - e^(4*I*d*x + 4*I*c))*log
(((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) - I) + 3*((
-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*(103*I*e^(6*I*
d*x + 6*I*c) + 75*I*e^(4*I*d*x + 4*I*c) - 31*I*e^(2*I*d*x + 2*I*c) - 3*I))/
(a^2*d*e^(6*I*d*x + 6*I*c) - a^2*d*e^(4*I*d*x + 4*I*c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(5/3)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 2.05, size = 238, normalized size = 0.66

$$\frac{119i\sqrt{3}\log\left(-\frac{\sqrt{3}-2\tan(dx+c)^{\frac{1}{3}}+i}{\sqrt{3}+2\tan(dx+c)^{\frac{1}{3}}-i}\right)}{144a^2d}-\frac{i\sqrt{3}\log\left(-\frac{\sqrt{3}-2\tan(dx+c)^{\frac{1}{3}}-i}{\sqrt{3}+2\tan(dx+c)^{\frac{1}{3}}+i}\right)}{16a^2d}-\frac{\log\left(\tan(dx+c)^{\frac{2}{3}}+i\tan(dx+c)^{\frac{1}{3}}-1\right)}{16a^2d}-\frac{119\log\left(\tan(dx+c)^{\frac{2}{3}}-i\tan(dx+c)^{\frac{1}{3}}-1\right)}{144a^2d}+\frac{119\log\left(\tan(dx+c)^{\frac{1}{3}}+i\right)}{72a^2d}+\frac{\log\left(\tan(dx+c)^{\frac{1}{3}}-i\right)}{8a^2d}-\frac{32\tan(dx+c)^2-53i\tan(dx+c)-18}{12\left(\tan(dx+c)^{\frac{4}{3}}-i\tan(dx+c)^{\frac{1}{3}}\right)^2a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(5/3)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 119/144*I*sqrt(3)*log(-(sqrt(3) - 2*tan(d*x + c)^(1/3) + I)/(sqrt(3) + 2*ta
n(d*x + c)^(1/3) - I))/(a^2*d) - 1/16*I*sqrt(3)*log(-(sqrt(3) - 2*tan(d*x +
 c)^(1/3) - I)/(sqrt(3) + 2*tan(d*x + c)^(1/3) + I))/(a^2*d) - 1/16*log(tan
(d*x + c)^(2/3) + I*tan(d*x + c)^(1/3) - 1)/(a^2*d) - 119/144*log(tan(d*x +
 c)^(2/3) - I*tan(d*x + c)^(1/3) - 1)/(a^2*d) + 119/72*log(tan(d*x + c)^(1/
3) + I)/(a^2*d) + 1/8*log(tan(d*x + c)^(1/3) - I)/(a^2*d) - 1/12*(32*tan(d*
x + c)^2 - 53*I*tan(d*x + c) - 18)/((tan(d*x + c)^(4/3) - I*tan(d*x + c)^(1
/3))^2*a^2*d)
```

Mupad [B]
time = 5.26, size = 660, normalized size = 1.84

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(5/3)*(a + a*tan(c + d*x)*1i)^2),x)
```

```
[Out] log(((a^6*d^3*215607040i)/3 + 28311552*a^10*d^5*tan(c + d*x)^(1/3)*(1/(512*
a^6*d^3))^(2/3))*(1/(512*a^6*d^3))^(1/3) - (27116768*a^4*d^2*tan(c + d*x)^(
1/3))/3)*(1/(512*a^6*d^3))^(1/3) + (119*log((119*((a^6*d^3*215607040i)/3 +
(232013824*a^10*d^5*tan(c + d*x)^(1/3)*(1/(a^6*d^3))^(2/3))/3)*(1/(a^6*d^3)
)^(1/3))/72 - (27116768*a^4*d^2*tan(c + d*x)^(1/3))/3)*(1/(a^6*d^3))^(1/3))
/72 - ((53*tan(c + d*x))/(12*a^2*d) - 3i/(2*a^2*d) + (tan(c + d*x)^2*8i)/(3
*a^2*d))/(2*tan(c + d*x)^(5/3) - tan(c + d*x)^(2/3)*1i + tan(c + d*x)^(8/3)
*1i) + log(((3^(1/2)*1i)/2 - 1/2)*((a^6*d^3*215607040i)/3 + 28311552*a^10*d
^5*tan(c + d*x)^(1/3)*((3^(1/2)*1i)/2 - 1/2)^2*(1/(512*a^6*d^3))^(2/3))*(1/
(512*a^6*d^3))^(1/3) - (27116768*a^4*d^2*tan(c + d*x)^(1/3))/3)*((3^(1/2)*1
i)/2 - 1/2)*(1/(512*a^6*d^3))^(1/3) - log(((3^(1/2)*1i)/2 + 1/2)*((a^6*d^3*
215607040i)/3 + 28311552*a^10*d^5*tan(c + d*x)^(1/3)*((3^(1/2)*1i)/2 + 1/2)
^2*(1/(512*a^6*d^3))^(2/3))*(1/(512*a^6*d^3))^(1/3) + (27116768*a^4*d^2*tan
(c + d*x)^(1/3))/3)*((3^(1/2)*1i)/2 + 1/2)*(1/(512*a^6*d^3))^(1/3) + (119*l
og((119*(3^(1/2)*1i - 1)*((a^6*d^3*215607040i)/3 + (58003456*a^10*d^5*tan(c
 + d*x)^(1/3)*(3^(1/2)*1i - 1)^2*(1/(a^6*d^3))^(2/3))/3)*(1/(a^6*d^3))^(1/3
))/144 - (27116768*a^4*d^2*tan(c + d*x)^(1/3))/3)*(3^(1/2)*1i - 1)*(1/(a^6*
d^3))^(1/3))/144 - (119*log((119*(3^(1/2)*1i + 1)*((a^6*d^3*215607040i)/3 +
 (58003456*a^10*d^5*tan(c + d*x)^(1/3)*(3^(1/2)*1i + 1)^2*(1/(a^6*d^3))^(2/
3))/3)*(1/(a^6*d^3))^(1/3))/144 + (27116768*a^4*d^2*tan(c + d*x)^(1/3))/3)*
(3^(1/2)*1i + 1)*(1/(a^6*d^3))^(1/3))/144
```

3.247 $\int \frac{1}{\tan^{\frac{7}{3}}(c+dx)(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=381

$$-\frac{91i\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d}+\frac{91i\text{ArcTan}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d}+\frac{25\text{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)}{6\sqrt{3}\,a^2d}+$$

```
[Out] 91/72*I*arctan(-3^(1/2)+2*tan(d*x+c)^(1/3))/a^2/d+91/72*I*arctan(3^(1/2)+2*
tan(d*x+c)^(1/3))/a^2/d+91/36*I*arctan(tan(d*x+c)^(1/3))/a^2/d-25/18*ln(1+t
an(d*x+c)^(2/3))/a^2/d+25/36*ln(1-tan(d*x+c)^(2/3)+tan(d*x+c)^(4/3))/a^2/d+
25/18*arctan(1/3*(1-2*tan(d*x+c)^(2/3))*3^(1/2))/a^2/d*3^(1/2)+91/144*I*ln(
1-3^(1/2)*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3))/a^2/d*3^(1/2)-91/144*I*ln(1+3^
(1/2)*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3))/a^2/d*3^(1/2)-25/12/a^2/d/tan(d*x+
c)^(4/3)+13/12/a^2/d/(1+I*tan(d*x+c))/tan(d*x+c)^(4/3)+91/12*I/a^2/d/tan(d*
x+c)^(1/3)+1/4/d/tan(d*x+c)^(4/3)/(a+I*a*tan(d*x+c))^2
```

Rubi [A]
time = 0.45, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 15, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {3640, 3677, 3610, 3619, 3557, 335, 281, 206, 31, 648, 632, 210, 642, 301, 209}

$$\frac{25\text{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)}{6\sqrt{3}\,a^2d}-\frac{91i\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d}+\frac{91i\text{ArcTan}\left(2\sqrt[3]{\tan(c+dx)}+\sqrt{3}\right)}{72a^2d}+\frac{91i\text{ArcTan}\left(\sqrt[3]{\tan(c+dx)}\right)}{36a^2d}-\frac{25}{12a^2d\tan^{\frac{4}{3}}(c+dx)}+\frac{13}{12a^2d(1+i\tan(c+dx))\tan^{\frac{4}{3}}(c+dx)}+\frac{91i}{12a^2d\sqrt[3]{\tan(c+dx)}}-\frac{25\log\left(\tan^{\frac{2}{3}}(c+dx)+1\right)}{18a^2d}+\frac{91i\log\left(\tan^{\frac{2}{3}}(c+dx)-\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{48\sqrt{3}\,a^2d}-\frac{91i\log\left(\tan^{\frac{2}{3}}(c+dx)+\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{48\sqrt{3}\,a^2d}+\frac{25\log\left(\tan^{\frac{4}{3}}(c+dx)-\tan^{\frac{2}{3}}(c+dx)+1\right)}{36a^2d}+\frac{1}{4d\tan^{\frac{4}{3}}(c+dx)(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(7/3)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (((-91*I)/72)*ArcTan[Sqrt[3] - 2*Tan[c + d*x]^(1/3)])/(a^2*d) + (((91*I)/72
)*ArcTan[Sqrt[3] + 2*Tan[c + d*x]^(1/3)])/(a^2*d) + (25*ArcTan[(1 - 2*Tan[c
 + d*x]^(2/3))/Sqrt[3]])/(6*Sqrt[3]*a^2*d) + (((91*I)/36)*ArcTan[Tan[c + d*
x]^(1/3)])/(a^2*d) - (25*Log[1 + Tan[c + d*x]^(2/3)])/(18*a^2*d) + (((91*I)
/48)*Log[1 - Sqrt[3]*Tan[c + d*x]^(1/3) + Tan[c + d*x]^(2/3)])/(Sqrt[3]*a^2
*d) - (((91*I)/48)*Log[1 + Sqrt[3]*Tan[c + d*x]^(1/3) + Tan[c + d*x]^(2/3)]
)/(Sqrt[3]*a^2*d) + (25*Log[1 - Tan[c + d*x]^(2/3) + Tan[c + d*x]^(4/3)])/(
36*a^2*d) - 25/(12*a^2*d*Tan[c + d*x]^(4/3)) + 13/(12*a^2*d*(1 + I*Tan[c +
d*x])*Tan[c + d*x]^(4/3)) + ((91*I)/12)/(a^2*d*Tan[c + d*x]^(1/3)) + 1/(4*d
*Tan[c + d*x]^(4/3)*(a + I*a*Tan[c + d*x])^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\tan^{\frac{7}{3}}(c+dx)(a+ia\tan(c+dx))^2}\,dx = \frac{1}{4d\tan^{\frac{4}{3}}(c+dx)(a+ia\tan(c+dx))^2} + \frac{\int \frac{\frac{16a}{3}-\frac{10}{3}ia\tan(c+dx)}{\tan^{\frac{7}{3}}(c+dx)(a+ia\tan(c+dx))}}{4a^2}$$

$$= \frac{13}{12a^2d(1+i\tan(c+dx))\tan^{\frac{4}{3}}(c+dx)} + \frac{1}{4d\tan^{\frac{4}{3}}(c+dx)(a+ia$$

$$= -\frac{25}{12a^2d\tan^{\frac{4}{3}}(c+dx)} + \frac{13}{12a^2d(1+i\tan(c+dx))\tan^{\frac{4}{3}}(c+dx)} +$$

$$= -\frac{25}{12a^2d\tan^{\frac{4}{3}}(c+dx)} + \frac{13}{12a^2d(1+i\tan(c+dx))\tan^{\frac{4}{3}}(c+dx)} +$$

$$= -\frac{25}{12a^2d\tan^{\frac{4}{3}}(c+dx)} + \frac{13}{12a^2d(1+i\tan(c+dx))\tan^{\frac{4}{3}}(c+dx)} +$$

$$= -\frac{25}{12a^2d\tan^{\frac{4}{3}}(c+dx)} + \frac{13}{12a^2d(1+i\tan(c+dx))\tan^{\frac{4}{3}}(c+dx)} +$$

$$= -\frac{25}{12a^2d\tan^{\frac{4}{3}}(c+dx)} + \frac{13}{12a^2d(1+i\tan(c+dx))\tan^{\frac{4}{3}}(c+dx)} +$$

$$= -\frac{25}{12a^2d\tan^{\frac{4}{3}}(c+dx)} + \frac{13}{12a^2d(1+i\tan(c+dx))\tan^{\frac{4}{3}}(c+dx)} +$$

$$= \frac{91i\tan^{-1}\left(\sqrt[3]{\tan(c+dx)}\right)}{36a^2d} - \frac{25}{12a^2d\tan^{\frac{4}{3}}(c+dx)} + \frac{}{12a^2d(1+i}$$

$$= \frac{91i\tan^{-1}\left(\sqrt[3]{\tan(c+dx)}\right)}{36a^2d} - \frac{25\log\left(1+\tan^{\frac{2}{3}}(c+dx)\right)}{18a^2d} + \frac{91i\,l}{}$$

$$= -\frac{91i\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d} + \frac{91i\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{ta}\right.}{72a^2d}$$

$$= -\frac{91i\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{72a^2d} + \frac{91i\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{ta}\right.}{72a^2d}$$

<u>Mathematica</u> [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 2.11, size = 281, normalized size = 0.74

$$\frac{e^{-2i(c+2dx)}\left(6+76e^{2i(c+dx)}-736e^{4i(c+dx)}-220e^{6i(c+dx)}+586e^{8i(c+dx)}+9\sqrt[3]{2}\,e^{4i(c+dx)}\left(-1+e^{2i(c+dx)}\right)^{2}\left(1+e^{2i(c+dx)}\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{1}{2}\left(1-e^{2i(c+dx)}\right)\right)+382e^{4i(c+dx)}\left(-1+e^{2i(c+dx)}\right)^{2}{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)\right)\sec^2(c+dx)(\cos(dx)+i\sin(dx))^2\tan^{\frac{2}{3}}(c+dx)}{96a^2d\left(-1+e^{2i(c+dx)}\right)^2(-i+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Tan[c + d*x]^(7/3)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] ((6 + 76*E^((2*I)*(c + d*x)) - 736*E^((4*I)*(c + d*x)) - 220*E^((6*I)*(c +
d*x)) + 586*E^((8*I)*(c + d*x)) + 9*2^(1/3)*E^((4*I)*(c + d*x))*(-1 + E^((2
*I)*(c + d*x)))^2*(1 + E^((2*I)*(c + d*x)))^(2/3)*Hypergeometric2F1[2/3, 2/
3, 5/3, (1 - E^((2*I)*(c + d*x)))/2] + 382*E^((4*I)*(c + d*x))*(-1 + E^((2*
I)*(c + d*x)))^2*Hypergeometric2F1[2/3, 1, 5/3, -((-1 + E^((2*I)*(c + d*x))
)/(1 + E^((2*I)*(c + d*x))))])*Sec[c + d*x]^2*(Cos[d*x] + I*Sin[d*x])^2*Tan
[c + d*x]^(2/3))/(96*a^2*d*E^((2*I)*(c + 2*d*x))*(-1 + E^((2*I)*(c + d*x)))
^2*(-I + Tan[c + d*x])^2)
```

Maple [A]
time = 0.18, size = 244, normalized size = 0.64

method	result
derivativedivides	$\frac{\frac{19i}{36\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}-\frac{1}{36\left(\tan^{\frac{1}{3}}(dx+c)+i\right)^{2}}-\frac{191\ln\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}{72}+\frac{\ln\left(i\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)-1\right)}{16}+\frac{i\sqrt{3}\ \operatorname{arctanh}}{}}{}$
default	$\frac{\frac{19i}{36\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}-\frac{1}{36\left(\tan^{\frac{1}{3}}(dx+c)+i\right)^{2}}-\frac{191\ln\left(\tan^{\frac{1}{3}}(dx+c)+i\right)}{72}+\frac{\ln\left(i\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)-1\right)}{16}+\frac{i\sqrt{3}\ \operatorname{arctanh}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(7/3)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(19/36*I/(tan(d*x+c)^(1/3)+I)-1/36/(tan(d*x+c)^(1/3)+I)^2-191/72*ln
(tan(d*x+c)^(1/3)+I)+1/16*ln(I*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3)-1)+1/8*I*3
^(1/2)*arctanh(1/3*(I+2*tan(d*x+c)^(1/3))*3^(1/2))-1/72*(-76*I*tan(d*x+c)-1
16*tan(d*x+c)^(2/3)+122*I*tan(d*x+c)^(1/3)+40)/(-I*tan(d*x+c)^(1/3)+tan(d*x
+c)^(2/3)-1)^2+191/144*ln(-I*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3)-1)-191/72*I*
3^(1/2)*arctanh(1/3*(-I+2*tan(d*x+c)^(1/3))*3^(1/2))-3/4/tan(d*x+c)^(4/3)+6
*I/tan(d*x+c)^(1/3)-1/8*ln(tan(d*x+c)^(1/3)-I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(7/3)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 836 vs. $2(298) = 596$.
time = 0.81, size = 836, normalized size = 2.19

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(7/3)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/144*(9*(sqrt(3)*(-I*a^2*d*e^(8*I*d*x + 8*I*c) + 2*I*a^2*d*e^(6*I*d*x + 6
*I*c) - I*a^2*d*e^(4*I*d*x + 4*I*c))*sqrt(1/(a^4*d^2)) - e^(8*I*d*x + 8*I*c
) + 2*e^(6*I*d*x + 6*I*c) - e^(4*I*d*x + 4*I*c))*log(1/2*sqrt(3)*a^2*d*sqrt
(1/(a^4*d^2)) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1
/3) + 1/2*I) + 9*(sqrt(3)*(I*a^2*d*e^(8*I*d*x + 8*I*c) - 2*I*a^2*d*e^(6*I*d
*x + 6*I*c) + I*a^2*d*e^(4*I*d*x + 4*I*c))*sqrt(1/(a^4*d^2)) - e^(8*I*d*x +
 8*I*c) + 2*e^(6*I*d*x + 6*I*c) - e^(4*I*d*x + 4*I*c))*log(-1/2*sqrt(3)*a^2
*d*sqrt(1/(a^4*d^2)) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) +
 1))^(1/3) + 1/2*I) + 191*(3*sqrt(1/3)*(I*a^2*d*e^(8*I*d*x + 8*I*c) - 2*I*a
^2*d*e^(6*I*d*x + 6*I*c) + I*a^2*d*e^(4*I*d*x + 4*I*c))*sqrt(1/(a^4*d^2)) -
 e^(8*I*d*x + 8*I*c) + 2*e^(6*I*d*x + 6*I*c) - e^(4*I*d*x + 4*I*c))*log(3/2
*sqrt(1/3)*a^2*d*sqrt(1/(a^4*d^2)) + ((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*
d*x + 2*I*c) + 1))^(1/3) - 1/2*I) + 191*(3*sqrt(1/3)*(-I*a^2*d*e^(8*I*d*x +
 8*I*c) + 2*I*a^2*d*e^(6*I*d*x + 6*I*c) - I*a^2*d*e^(4*I*d*x + 4*I*c))*sqrt
(1/(a^4*d^2)) - e^(8*I*d*x + 8*I*c) + 2*e^(6*I*d*x + 6*I*c) - e^(4*I*d*x +
4*I*c))*log(-3/2*sqrt(1/3)*a^2*d*sqrt(1/(a^4*d^2)) + ((-I*e^(2*I*d*x + 2*I*
c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) - 1/2*I) + 382*(e^(8*I*d*x + 8*I*c
) - 2*e^(6*I*d*x + 6*I*c) + e^(4*I*d*x + 4*I*c))*log(((-I*e^(2*I*d*x + 2*I*
c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) + I) + 18*(e^(8*I*d*x + 8*I*c) - 2
*e^(6*I*d*x + 6*I*c) + e^(4*I*d*x + 4*I*c))*log(((-I*e^(2*I*d*x + 2*I*c) +
I)/(e^(2*I*d*x + 2*I*c) + 1))^(1/3) - I) + 3*((-I*e^(2*I*d*x + 2*I*c) + I)/
(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(293*e^(8*I*d*x + 8*I*c) - 110*e^(6*I*d*x
+ 6*I*c) - 368*e^(4*I*d*x + 4*I*c) + 38*e^(2*I*d*x + 2*I*c) + 3))/(a^2*d*e^
(8*I*d*x + 8*I*c) - 2*a^2*d*e^(6*I*d*x + 6*I*c) + a^2*d*e^(4*I*d*x + 4*I*c)
)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(7/3)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.18, size = 254, normalized size = 0.67

$$\frac{191i\sqrt{3}\log\left(-\frac{\sqrt{3}-2\tan(dx+c)^{\frac{1}{3}}+i}{\sqrt{3}+2\tan(dx+c)^{\frac{1}{3}}-i}\right)}{144a^2d}-\frac{i\sqrt{3}\log\left(-\frac{\sqrt{3}-2\tan(dx+c)^{\frac{1}{3}}-i}{\sqrt{3}+2\tan(dx+c)^{\frac{1}{3}}+i}\right)}{16a^2d}+\frac{\log\left(\tan(dx+c)^{\frac{2}{3}}+i\tan(dx+c)^{\frac{1}{3}}-1\right)}{16a^2d}+\frac{191\log\left(\tan(dx+c)^{\frac{2}{3}}-i\tan(dx+c)^{\frac{1}{3}}-1\right)}{144a^2d}-\frac{191\log\left(\tan(dx+c)^{\frac{1}{3}}+i\right)}{72a^2d}-\frac{\log\left(\tan(dx+c)^{\frac{1}{3}}-i\right)}{8a^2d}-\frac{3(-8i\tan(dx+c)+1)}{4a^2d\tan(dx+c)^{\frac{4}{3}}}+\frac{19i\tan(dx+c)^{\frac{5}{3}}+22\tan(dx+c)^{\frac{2}{3}}}{12a^2d(\tan(dx+c)-i)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(7/3)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 191/144*I*sqrt(3)*log(-(sqrt(3) - 2*tan(d*x + c)^(1/3) + I)/(sqrt(3) + 2*ta
n(d*x + c)^(1/3) - I))/(a^2*d) - 1/16*I*sqrt(3)*log(-(sqrt(3) - 2*tan(d*x +
 c)^(1/3) - I)/(sqrt(3) + 2*tan(d*x + c)^(1/3) + I))/(a^2*d) + 1/16*log(tan
(d*x + c)^(2/3) + I*tan(d*x + c)^(1/3) - 1)/(a^2*d) + 191/144*log(tan(d*x +
 c)^(2/3) - I*tan(d*x + c)^(1/3) - 1)/(a^2*d) - 191/72*log(tan(d*x + c)^(1/
3) + I)/(a^2*d) - 1/8*log(tan(d*x + c)^(1/3) - I)/(a^2*d) - 3/4*(-8*I*tan(d
*x + c) + 1)/(a^2*d*tan(d*x + c)^(4/3)) + 1/12*(19*I*tan(d*x + c)^(5/3) + 2
2*tan(d*x + c)^(2/3))/(a^2*d*(tan(d*x + c) - I)^2)
```

Mupad [B]
time = 5.46, size = 652, normalized size = 1.71

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(7/3)*(a + a*tan(c + d*x)*1i)^2),x)
```

```
[Out] (191*log((14592400*a^2*d*tan(c + d*x)^(1/3))/3 - (36481*((a^6*d^3*891794176
i)/3 - (893867776*a^8*d^4*tan(c + d*x)^(1/3)*(-1/(a^6*d^3))^(1/3))/3)*(-1/(
a^6*d^3))^(2/3))/5184)*(-1/(a^6*d^3))^(1/3))/72 + (3i/(4*a^2*d) + (9*tan(c
+ d*x))/(2*a^2*d) + (tan(c + d*x)^2*157i)/(12*a^2*d) - (91*tan(c + d*x)^3)/
(12*a^2*d))/(2*tan(c + d*x)^(7/3) - tan(c + d*x)^(4/3)*1i + tan(c + d*x)^(1
0/3)*1i) + log((14592400*a^2*d*tan(c + d*x)^(1/3))/3 - ((a^6*d^3*891794176i
)/3 - 112318464*a^8*d^4*tan(c + d*x)^(1/3)*(-1/(512*a^6*d^3))^(1/3))*(-1/(5
12*a^6*d^3))^(2/3))*(-1/(512*a^6*d^3))^(1/3) + log((14592400*a^2*d*tan(c +
d*x)^(1/3))/3 - ((3^(1/2)*1i)/2 - 1/2)^2*((a^6*d^3*891794176i)/3 - 11231846
4*a^8*d^4*tan(c + d*x)^(1/3)*((3^(1/2)*1i)/2 - 1/2)*(-1/(512*a^6*d^3))^(1/3
))*(-1/(512*a^6*d^3))^(2/3))*((3^(1/2)*1i)/2 - 1/2)*(-1/(512*a^6*d^3))^(1/3
) - log((14592400*a^2*d*tan(c + d*x)^(1/3))/3 - ((3^(1/2)*1i)/2 + 1/2)^2*((
a^6*d^3*891794176i)/3 + 112318464*a^8*d^4*tan(c + d*x)^(1/3)*((3^(1/2)*1i)/
2 + 1/2)*(-1/(512*a^6*d^3))^(1/3))*(-1/(512*a^6*d^3))^(2/3))*((3^(1/2)*1i)/
2 + 1/2)*(-1/(512*a^6*d^3))^(1/3) + (191*log((14592400*a^2*d*tan(c + d*x)^(
```

```
1/3))/3 - (36481*(3^(1/2)*1i - 1)^2*((a^6*d^3*891794176i)/3 - (446933888*a^
8*d^4*tan(c + d*x)^(1/3)*(3^(1/2)*1i - 1)*(-1/(a^6*d^3))^(1/3))/3)*(-1/(a^6
*d^3))^(2/3))/20736)*(3^(1/2)*1i - 1)*(-1/(a^6*d^3))^(1/3))/144 - (191*log(
(14592400*a^2*d*tan(c + d*x)^(1/3))/3 - (36481*(3^(1/2)*1i + 1)^2*((a^6*d^3
*891794176i)/3 + (446933888*a^8*d^4*tan(c + d*x)^(1/3)*(3^(1/2)*1i + 1)*(-1
/(a^6*d^3))^(1/3))/3)*(-1/(a^6*d^3))^(2/3))/20736)*(3^(1/2)*1i + 1)*(-1/(a^
6*d^3))^(1/3))/144
```

3.248 $\int \tan^{\frac{4}{3}}(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=82

$$\frac{3aF_1\left(\frac{7}{3};\frac{1}{2},1;\frac{10}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{1+i\tan(c+dx)}\,\tan^{\frac{7}{3}}(c+dx)}{7d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 3/7*a*AppellF1(7/3,1/2,1,10/3,-I*tan(d*x+c),I*tan(d*x+c))*(1+I*tan(d*x+c))^
(1/2)*tan(d*x+c)^(7/3)/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3645, 129, 525, 524}

$$\frac{3a\sqrt{1+i\tan(c+dx)}\,\tan^{\frac{7}{3}}(c+dx)F_1\left(\frac{7}{3};\frac{1}{2},1;\frac{10}{3};-i\tan(c+dx),i\tan(c+dx)\right)}{7d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(4/3)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (3*a*AppellF1[7/3, 1/2, 1, 10/3, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*Sqrt[1
+ I*Tan[c + d*x]]*Tan[c + d*x]^(7/3))/(7*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\int \tan^{\frac{4}{3}}(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{(ia^2)\,\text{Subst}\left(\int \frac{\left(-\frac{ix}{a}\right)^{4/3}}{\sqrt{a+x}\,(-a^2+ax)}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(3a^3)\,\text{Subst}\left(\int \frac{x^6}{\sqrt{a+iax^3}\,(-a^2+ia^2x^3)}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{d}$$

$$= -\frac{\left(3a^3\sqrt{1+i\tan(c+dx)}\right)\text{Subst}\left(\int \frac{x^6}{\sqrt{1+ix^3}\,(-a^2+ia^2x^3)}\,dx, x,$$

$$\frac{}{d\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{3aF_1\left(\frac{7}{3}; \frac{1}{2}, 1; \frac{10}{3}; -i\tan(c+dx), i\tan(c+dx)\right)\sqrt{1+i\tan(c+da}}{7d\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [F]
time = 5.55, size = 0, normalized size = 0.00

$$\int \tan^{\frac{4}{3}}(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^(4/3)*Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] Integrate[Tan[c + d*x]^(4/3)*Sqrt[a + I*a*Tan[c + d*x]], x]
```

Maple [F]
time = 1.09, size = 0, normalized size = 0.00

$$\int \left(\tan^{\frac{4}{3}}(dx+c)\right)\sqrt{a+ia\tan(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(4/3)*(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] int(tan(d*x+c)^(4/3)*(a+I*a*tan(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(4/3)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*tan(d*x + c)^(4/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(4/3)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\tan^{\frac{4}{3}}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(4/3)*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*tan(c + d*x)**(4/3), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(4/3)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 4.13
Done
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^{4/3}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(4/3)*(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int(tan(c + d*x)^(4/3)*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.249 $\int \tan^{\frac{2}{3}}(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=82

$$\frac{3aF_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{1+i\tan(c+dx)}\tan^{\frac{5}{3}}(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 3/5*a*AppellF1(5/3,1/2,1,8/3,-I*tan(d*x+c),I*tan(d*x+c))*(1+I*tan(d*x+c))^(
1/2)*tan(d*x+c)^(5/3)/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3645, 129, 525, 524}

$$\frac{3a\sqrt{1+i\tan(c+dx)}\tan^{\frac{5}{3}}(c+dx)F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-i\tan(c+dx),i\tan(c+dx)\right)}{5d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(2/3)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (3*a*AppellF1[5/3, 1/2, 1, 8/3, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*Sqrt[1 +
 I*Tan[c + d*x]]*Tan[c + d*x]^(5/3))/(5*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\int \tan^{\frac{2}{3}}(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{(ia^2)\,\text{Subst}\left(\int \frac{\left(-\frac{ix}{a}\right)^{2/3}}{\sqrt{a+x}\,(-a^2+ax)}\,dx, x, ia\tan(c+dx)\right)}{d}$$
$$= -\frac{(3a^3)\,\text{Subst}\left(\int \frac{x^4}{\sqrt{a+iax^3}\,(-a^2+ia^2x^3)}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{d}$$
$$= -\frac{\left(3a^3\sqrt{1+i\tan(c+dx)}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{1+ix^3}\,(-a^2+ia^2x^3)}\,dx, x, \right.}{d\sqrt{a+ia\tan(c+dx)}}$$
$$= \frac{3aF_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{1+i\tan(c+dx)}}{5d\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [**F**]
time = 16.66, size = 0, normalized size = 0.00

$$\int \tan^{\frac{2}{3}}(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^(2/3)*Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] Integrate[Tan[c + d*x]^(2/3)*Sqrt[a + I*a*Tan[c + d*x]], x]
```

Maple [**F**]
time = 1.07, size = 0, normalized size = 0.00

$$\int \left(\tan^{\frac{2}{3}}(dx+c)\right)\sqrt{a+ia\tan(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(2/3)*(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] int(tan(d*x+c)^(2/3)*(a+I*a*tan(d*x+c))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(2/3)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*tan(d*x + c)^(2/3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(2/3)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,\tan^{\frac{2}{3}}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(2/3)*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*tan(c + d*x)**(2/3), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(2/3)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 4.34
Done
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^{2/3}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(2/3)*(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int(tan(c + d*x)^(2/3)*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.250 $\int \sqrt[3]{\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=82

$$\frac{3aF_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{1+i\tan(c+dx)}\,\tan^{\frac{4}{3}}(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 3/4*a*AppellF1(4/3,1/2,1,7/3,-I*tan(d*x+c),I*tan(d*x+c))*(1+I*tan(d*x+c))^(
1/2)*tan(d*x+c)^(4/3)/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3645, 129, 525, 524}

$$\frac{3a\sqrt{1+i\tan(c+dx)}\,\tan^{\frac{4}{3}}(c+dx)F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-i\tan(c+dx),i\tan(c+dx)\right)}{4d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(1/3)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (3*a*AppellF1[4/3, 1/2, 1, 7/3, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*Sqrt[1 +
 I*Tan[c + d*x]]*Tan[c + d*x]^(4/3))/(4*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\int \sqrt[3]{\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}\,dx = \frac{(ia^2)\,\text{Subst}\left(\int \frac{\sqrt[3]{-\frac{ix}{a}}}{\sqrt{a+x}\,(-a^2+ax)}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(3a^3)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{a+iax^3}\,(-a^2+ia^2x^3)}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{d}$$

$$= -\frac{\left(3a^3\sqrt{1+i\tan(c+dx)}\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{1+ix^3}\,(-a^2+ia^2x^3)}\,dx,\right.}{d\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{3aF_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{1+i\tan(c+d}}{4d\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [F]
time = 1.13, size = 0, normalized size = 0.00

$$\int \sqrt[3]{\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^(1/3)*Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] Integrate[Tan[c + d*x]^(1/3)*Sqrt[a + I*a*Tan[c + d*x]], x]
```

Maple [F]
time = 1.06, size = 0, normalized size = 0.00

$$\int \left(\tan^{\frac{1}{3}}(dx+c)\right)\sqrt{a+ia\tan(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/3)*(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] int(tan(d*x+c)^(1/3)*(a+I*a*tan(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/3)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*tan(d*x + c)^(1/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/3)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,\sqrt[3]{\tan\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/3)*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*tan(c + d*x)**(1/3), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/3)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
```

```
dex_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 4.07
Done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+dx)^{1/3}\,\sqrt{a+a\tan(c+dx)\ 1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/3)*(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int(tan(c + d*x)^(1/3)*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.251 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt[3]{\tan(c+dx)}}\,dx$

Optimal. Leaf size=82

$$\frac{3aF_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{1+i\tan(c+dx)}\tan^{\frac{2}{3}}(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 3/2*a*AppellF1(2/3,1/2,1,5/3,-I*tan(d*x+c),I*tan(d*x+c))*(1+I*tan(d*x+c))^(
1/2)*tan(d*x+c)^(2/3)/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3645, 129, 525, 524}

$$\frac{3a\sqrt{1+i\tan(c+dx)}\tan^{\frac{2}{3}}(c+dx)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-i\tan(c+dx),i\tan(c+dx)\right)}{2d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[c + d*x]]/Tan[c + d*x]^(1/3),x]
```

```
[Out] (3*a*AppellF1[2/3, 1/2, 1, 5/3, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*Sqrt[1 +
 I*Tan[c + d*x]]*Tan[c + d*x]^(2/3))/(2*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
```

```
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt[3]{\tan(c+dx)}}\,dx = \frac{(ia^2)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{-\frac{ix}{a}}\,\sqrt{a+x}\,(-a^2+ax)}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(3a^3)\,\text{Subst}\left(\int \frac{x}{\sqrt{a+iax^3}\,(-a^2+ia^2x^3)}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{d}$$

$$= -\frac{\left(3a^3\sqrt{1+i\tan(c+dx)}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1+ix^3}\,(-a^2+ia^2x^3)}\,dx, x, \sqrt[3]{\tan(c+d$$

$$= \frac{3aF_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{1+i\tan(c+dx)}\,\tan^{\frac{2}{3}}(c+d}{2d\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [F]
time = 1.11, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt[3]{\tan(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/Tan[c + d*x]^(1/3),x]
[Out] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/Tan[c + d*x]^(1/3), x]
```

Maple [F]
time = 1.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+ia\tan(dx+c)}}{\tan(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/3),x)
```

```
[Out] int((a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)/tan(d*x + c)^(1/3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}{\sqrt[3]{\tan\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)/tan(d*x+c)**(1/3),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))/tan(c + d*x)**(1/3), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
```

```
dex_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 4.13
Done
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}{{\tan\left(c+d\,x\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/tan(c + d*x)^(1/3),x)
[Out] int((a + a*tan(c + d*x)*1i)^(1/2)/tan(c + d*x)^(1/3), x)
```

3.252 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{\tan^{\frac{2}{3}}(c+dx)}\,dx$

Optimal. Leaf size=80

$$\frac{3aF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{1+i\tan(c+dx)}\sqrt[3]{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 3*a*AppellF1(1/3,1/2,1,4/3,-I*tan(d*x+c),I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/
2)*tan(d*x+c)^(1/3)/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3645, 129, 441, 440}

$$\frac{3a\sqrt{1+i\tan(c+dx)}\sqrt[3]{\tan(c+dx)}\,F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-i\tan(c+dx),i\tan(c+dx)\right)}{d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[c + d*x]]/Tan[c + d*x]^(2/3),x]
```

```
[Out] (3*a*AppellF1[1/3, 1/2, 1, 4/3, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*Sqrt[1 +
 I*Tan[c + d*x]]*Tan[c + d*x]^(1/3))/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\tan^{\frac{2}{3}}(c+dx)}\,dx = \frac{(ia^2)\operatorname{Subst}\left(\int \frac{1}{\left(-\frac{ix}{a}\right)^{2/3}\sqrt{a+x}\,(-a^2+ax)}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(3a^3)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+iax^3}\,(-a^2+ia^2x^3)}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{d}$$

$$= -\frac{\left(3a^3\sqrt{1+i\tan(c+dx)}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+ix^3}\,(-a^2+ia^2x^3)}\,dx, x, \sqrt[3]{\tan(c+d}\right.}{d\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{3aF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{1+i\tan(c+dx)}\,\sqrt[3]{\tan(c+}}{d\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [**F**]
time = 1.08, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\tan^{\frac{2}{3}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/Tan[c + d*x]^(2/3),x]
[Out] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/Tan[c + d*x]^(2/3), x]
```

Maple [**F**]
time = 1.06, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+ia\tan(dx+c)}}{\tan(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(2/3),x)
```

```
[Out] int((a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(2/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(2/3),x, algorithm="maxima")
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)/tan(d*x + c)^(2/3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(2/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}{\tan^{\frac{2}{3}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)/tan(d*x+c)**(2/3),x)
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))/tan(c + d*x)**(2/3), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(2/3),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 4.4D
one
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}{\tan\left(c+d\,x\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/tan(c + d*x)^(2/3),x)
[Out] int((a + a*tan(c + d*x)*1i)^(1/2)/tan(c + d*x)^(2/3), x)
```

3.253 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{\tan^{\frac{4}{3}}(c+dx)}\,dx$

Optimal. Leaf size=80

$$-\frac{3aF_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{1+i\tan(c+dx)}}{d\sqrt[3]{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] -3*a*AppellF1(-1/3,1/2,1,2/3,-I*tan(d*x+c),I*tan(d*x+c))*(1+I*tan(d*x+c))^(
1/2)/d/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/3)
```

Rubi [A]
time = 0.10, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3645, 129, 525, 524}

$$-\frac{3a\sqrt{1+i\tan(c+dx)}\,F_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};-i\tan(c+dx),i\tan(c+dx)\right)}{d\sqrt[3]{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[c + d*x]]/Tan[c + d*x]^(4/3),x]
```

```
[Out] (-3*a*AppellF1[-1/3, 1/2, 1, 2/3, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*Sqrt[1
 + I*Tan[c + d*x]])/(d*Tan[c + d*x]^(1/3)*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
```

```
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\tan^{\frac{4}{3}}(c+dx)}\,dx = \frac{(ia^2)\,\text{Subst}\left(\int \frac{1}{\left(-\frac{ix}{a}\right)^{4/3}\sqrt{a+x}\,(-a^2+ax)}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(3a^3)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+iax^3}\,(-a^2+ia^2x^3)}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{d}$$

$$= -\frac{\left(3a^3\sqrt{1+i\tan(c+dx)}\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+ix^3}\,(-a^2+ia^2x^3)}\,dx, x, \sqrt[3]{\tan(c+}\right.}{d\sqrt{a+ia\tan(c+dx)}}$$

$$= -\frac{3aF_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{1+i\tan(c+dx)}}{d\sqrt[3]{\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [F]
time = 12.14, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\tan^{\frac{4}{3}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/Tan[c + d*x]^(4/3),x]
[Out] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/Tan[c + d*x]^(4/3), x]
```

Maple [F]
time = 1.05, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+ia\tan\left(dx+c\right)}}{\tan\left(dx+c\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(4/3),x)

[Out] int((a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(4/3),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(4/3),x, algorithm="maxima")

[Out] integrate(sqrt(I*a*tan(d*x + c) + a)/tan(d*x + c)^(4/3), x)

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(4/3),x, algorithm="fricas")

[Out] Timed out

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}{\tan^{\frac{4}{3}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I*a*tan(d*x+c))**(1/2)/tan(d*x+c)**(4/3),x)

[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))/tan(c + d*x)**(4/3), x)

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(4/3),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in

```
dex_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 4.26
Done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{{\tan\left(c+d\,x\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/tan(c + d*x)^(4/3),x)
[Out] int((a + a*tan(c + d*x)*1i)^(1/2)/tan(c + d*x)^(4/3), x)
```

3.254 $\int \tan^{\frac{4}{3}}(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=82

$$\frac{3aF_1\left(\frac{7}{3};-\frac{1}{2},1;\frac{10}{3};-i\tan(c+dx),i\tan(c+dx)\right)\tan^{\frac{7}{3}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d\sqrt{1+i\tan(c+dx)}}$$

```
[Out] 3/7*a*AppellF1(7/3,-1/2,1,10/3,-I*tan(d*x+c),I*tan(d*x+c))*(a+I*a*tan(d*x+c
))^(1/2)*tan(d*x+c)^(7/3)/d/(1+I*tan(d*x+c))^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3645, 129, 525, 524}

$$\frac{3a\tan^{\frac{7}{3}}(c+dx)\sqrt{a+ia\tan(c+dx)}\,F_1\left(\frac{7}{3};-\frac{1}{2},1;\frac{10}{3};-i\tan(c+dx),i\tan(c+dx)\right)}{7d\sqrt{1+i\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(4/3)*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (3*a*AppellF1[7/3, -1/2, 1, 10/3, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*Tan[c
+ d*x]^(7/3)*Sqrt[a + I*a*Tan[c + d*x]])/(7*d*Sqrt[1 + I*Tan[c + d*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\int \tan^{\frac{4}{3}}(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{(ia^2)\,\text{Subst}\left(\int \frac{\left(-\frac{ix}{a}\right)^{4/3}\sqrt{a+x}}{-a^2+ax}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(3a^3)\,\text{Subst}\left(\int \frac{x^6\sqrt{a+iax^3}}{-a^2+ia^2x^3}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{d}$$

$$= -\frac{\left(3a^3\sqrt{a+ia\tan(c+dx)}\right)\text{Subst}\left(\int \frac{x^6\sqrt{1+ix^3}}{-a^2+ia^2x^3}\,dx, x, \sqrt[3]{\tan\right.}{d\sqrt{1+i\tan(c+dx)}}$$

$$= \frac{3aF_1\left(\frac{7}{3}; -\frac{1}{2}, 1; \frac{10}{3}; -i\tan(c+dx), i\tan(c+dx)\right)\tan^{\frac{7}{3}}(c+dx)\sqrt{}}{7d\sqrt{1+i\tan(c+dx)}}$$

Mathematica [F]
time = 8.07, size = 0, normalized size = 0.00

$$\int \tan^{\frac{4}{3}}(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^(4/3)*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] Integrate[Tan[c + d*x]^(4/3)*(a + I*a*Tan[c + d*x])^(3/2), x]
```

Maple [F]
time = 1.01, size = 0, normalized size = 0.00

$$\int \left(\tan^{\frac{4}{3}}(dx+c)\right)(a+ia\tan(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(4/3)*(a+I*a*tan(d*x+c))^(3/2),x)
[Out] int(tan(d*x+c)^(4/3)*(a+I*a*tan(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(4/3)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*tan(d*x + c)^(4/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(4/3)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(4/3)*(a+I*a*tan(d*x+c))**(3/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(4/3)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*tan(d*x + c)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^{4/3}\,{\left(a+a\,\tan(c+d\,x)\,1{}\mathrm{i}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(4/3)*(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int(tan(c + d*x)^(4/3)*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.255 $\int \tan^{\frac{2}{3}}(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=82

$$\frac{3aF_1\left(\frac{5}{3};-\frac{1}{2},1;\frac{8}{3};-i\tan(c+dx),i\tan(c+dx)\right)\tan^{\frac{5}{3}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d\sqrt{1+i\tan(c+dx)}}$$

```
[Out] 3/5*a*AppellF1(5/3,-1/2,1,8/3,-I*tan(d*x+c),I*tan(d*x+c))*(a+I*a*tan(d*x+c)
)^(1/2)*tan(d*x+c)^(5/3)/d/(1+I*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3645, 129, 525, 524}

$$\frac{3a\tan^{\frac{5}{3}}(c+dx)\sqrt{a+ia\tan(c+dx)}\,F_1\left(\frac{5}{3};-\frac{1}{2},1;\frac{8}{3};-i\tan(c+dx),i\tan(c+dx)\right)}{5d\sqrt{1+i\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(2/3)*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (3*a*AppellF1[5/3, -1/2, 1, 8/3, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*Tan[c +
 d*x]^(5/3)*Sqrt[a + I*a*Tan[c + d*x]])/(5*d*Sqrt[1 + I*Tan[c + d*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\int \tan^{\frac{2}{3}}(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{(ia^2)\,\text{Subst}\left(\int \frac{\left(-\frac{ix}{a}\right)^{2/3}\sqrt{a+x}}{-a^2+ax}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(3a^3)\,\text{Subst}\left(\int \frac{x^4\sqrt{a+iax^3}}{-a^2+ia^2x^3}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{d}$$

$$= -\frac{\left(3a^3\sqrt{a+ia\tan(c+dx)}\right)\text{Subst}\left(\int \frac{x^4\sqrt{1+ix^3}}{-a^2+ia^2x^3}\,dx, x, \sqrt[3]{\tan}\right.}{d\sqrt{1+i\tan(c+dx)}}$$

$$= \frac{3aF_1\left(\frac{5}{3}; -\frac{1}{2}, 1; \frac{8}{3}; -i\tan(c+dx), i\tan(c+dx)\right)\tan^{\frac{5}{3}}(c+dx)\sqrt{}}{5d\sqrt{1+i\tan(c+dx)}}$$

Mathematica [F]
time = 13.99, size = 0, normalized size = 0.00

$$\int \tan^{\frac{2}{3}}(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^(2/3)*(a + I*a*Tan[c + d*x])^(3/2),x]
[Out] Integrate[Tan[c + d*x]^(2/3)*(a + I*a*Tan[c + d*x])^(3/2), x]
```

Maple [F]
time = 1.03, size = 0, normalized size = 0.00

$$\int \left(\tan^{\frac{2}{3}}(dx+c)\right)(a+ia\tan(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(2/3)*(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] int(tan(d*x+c)^(2/3)*(a+I*a*tan(d*x+c))^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(2/3)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*tan(d*x + c)^(2/3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(2/3)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan (c+dx)-i))^{\frac{3}{2}} \tan^{\frac{2}{3}} (c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(2/3)*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)*tan(c + d*x)**(2/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(2/3)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*tan(d*x + c)^(2/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^{2/3}\,(a+a\tan(c+d\,x)\;1\mathrm{i})^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(2/3)*(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int(tan(c + d*x)^(2/3)*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.256 $\int \sqrt[3]{\tan(c+dx)}\,(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=82

$$\frac{3aF_1\left(\frac{4}{3};-\frac{1}{2},1;\frac{7}{3};-i\tan(c+dx),i\tan(c+dx)\right)\tan^{\frac{4}{3}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d\sqrt{1+i\tan(c+dx)}}$$

```
[Out] 3/4*a*AppellF1(4/3,-1/2,1,7/3,-I*tan(d*x+c),I*tan(d*x+c))*(a+I*a*tan(d*x+c)
)^(1/2)*tan(d*x+c)^(4/3)/d/(1+I*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3645, 129, 525, 524}

$$\frac{3a\tan^{\frac{4}{3}}(c+dx)\sqrt{a+ia\tan(c+dx)}\,F_1\left(\frac{4}{3};-\frac{1}{2},1;\frac{7}{3};-i\tan(c+dx),i\tan(c+dx)\right)}{4d\sqrt{1+i\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(1/3)*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (3*a*AppellF1[4/3, -1/2, 1, 7/3, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*Tan[c +
 d*x]^(4/3)*Sqrt[a + I*a*Tan[c + d*x]])/(4*d*Sqrt[1 + I*Tan[c + d*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\int \sqrt[3]{\tan(c+dx)}\,(a+ia\tan(c+dx))^{3/2}\,dx = \frac{(ia^2)\,\text{Subst}\left(\int \frac{\sqrt[3]{-\frac{ix}{a}}\,\sqrt{a+x}}{-a^2+ax}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(3a^3)\,\text{Subst}\left(\int \frac{x^3\sqrt{a+iax^3}}{-a^2+ia^2x^3}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{d}$$

$$= -\frac{\left(3a^3\sqrt{a+ia\tan(c+dx)}\right)\text{Subst}\left(\int \frac{x^3\sqrt{1+ix^3}}{-a^2+ia^2x^3}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{d\sqrt{1+i\tan(c+dx)}}$$

$$= \frac{3aF_1\left(\frac{4}{3};-\frac{1}{2},1;\frac{7}{3};-i\tan(c+dx),i\tan(c+dx)\right)\tan^{\frac{4}{3}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d\sqrt{1+i\tan(c+dx)}}$$

Mathematica [F]
time = 7.41, size = 0, normalized size = 0.00

$$\int \sqrt[3]{\tan(c+dx)}\,(a+ia\tan(c+dx))^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^(1/3)*(a + I*a*Tan[c + d*x])^(3/2),x]
[Out] Integrate[Tan[c + d*x]^(1/3)*(a + I*a*Tan[c + d*x])^(3/2), x]
```

Maple [F]
time = 1.00, size = 0, normalized size = 0.00

$$\int \left(\tan^{\frac{1}{3}}(dx+c)\right)(a+ia\tan(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/3)*(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] int(tan(d*x+c)^(1/3)*(a+I*a*tan(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/3)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*tan(d*x + c)^(1/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/3)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c+dx)-i))^{\frac{3}{2}}\sqrt[3]{\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/3)*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)*tan(c + d*x)**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/3)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*tan(d*x + c)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^{1/3}\,{\left(a+a\,\tan(c+d\,x)\;1\mathrm{i}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/3)*(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int(tan(c + d*x)^(1/3)*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.257 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt[3]{\tan(c+dx)}}\,dx$

Optimal. Leaf size=82

$$\frac{3aF_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};-i\tan(c+dx),i\tan(c+dx)\right)\tan^{\frac{2}{3}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d\sqrt{1+i\tan(c+dx)}}$$

```
[Out] 3/2*a*AppellF1(2/3,-1/2,1,5/3,-I*tan(d*x+c),I*tan(d*x+c))*(a+I*a*tan(d*x+c)
)^(1/2)*tan(d*x+c)^(2/3)/d/(1+I*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3645, 129, 525, 524}

$$\frac{3a\tan^{\frac{2}{3}}(c+dx)\sqrt{a+ia\tan(c+dx)}\,F_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};-i\tan(c+dx),i\tan(c+dx)\right)}{2d\sqrt{1+i\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(3/2)/Tan[c + d*x]^(1/3),x]
```

```
[Out] (3*a*AppellF1[2/3, -1/2, 1, 5/3, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*Tan[c +
 d*x]^(2/3)*Sqrt[a + I*a*Tan[c + d*x]])/(2*d*Sqrt[1 + I*Tan[c + d*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
```

```
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt[3]{\tan(c+dx)}}\,dx &= \frac{(ia^2)\operatorname{Subst}\left(\int \frac{\sqrt{a+x}}{\sqrt[3]{-\frac{ix}{a}}\,(-a^2+ax)}\,dx, x, ia\tan(c+dx)\right)}{d} \\
&= -\frac{(3a^3)\operatorname{Subst}\left(\int \frac{x\sqrt{a+iax^3}}{-a^2+ia^2x^3}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{d} \\
&= -\frac{\left(3a^3\sqrt{a+ia\tan(c+dx)}\right)\operatorname{Subst}\left(\int \frac{x\sqrt{1+ix^3}}{-a^2+ia^2x^3}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{d\sqrt{1+i\tan(c+dx)}} \\
&= \frac{3aF_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};-i\tan(c+dx),i\tan(c+dx)\right)\tan^{\frac{2}{3}}(c+dx)\sqrt{a+ia\tan(c}{2d\sqrt{1+i\tan(c+dx)}}
\end{aligned}
$$

Mathematica [F]
time = 16.90, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt[3]{\tan(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(3/2)/Tan[c + d*x]^(1/3),x]
[Out] Integrate[(a + I*a*Tan[c + d*x])^(3/2)/Tan[c + d*x]^(1/3), x]
```

Maple [F]
time = 1.04, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(dx+c))^{\frac{3}{2}}}{\tan(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(1/3),x)
[Out] int((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(1/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(1/3),x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)/tan(d*x + c)^(1/3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(1/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}{\sqrt[3]{\tan(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(3/2)/tan(d*x+c)**(1/3),x)
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)/tan(c + d*x)**(1/3), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(1/3),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
```

```
dex_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 4.4D
one
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^{3/2}}{\tan\left(c+d\,x\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(3/2)/tan(c + d*x)^(1/3),x)
[Out] int((a + a*tan(c + d*x)*1i)^(3/2)/tan(c + d*x)^(1/3), x)
```

3.258 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{\tan^{\frac{2}{3}}(c+dx)}\,dx$

Optimal. Leaf size=80

$$\frac{3aF_1\left(\frac{1}{3};-\frac{1}{2},1;\frac{4}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt[3]{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d\sqrt{1+i\tan(c+dx)}}$$

```
[Out] 3*a*AppellF1(1/3,-1/2,1,4/3,-I*tan(d*x+c),I*tan(d*x+c))*(a+I*a*tan(d*x+c))^
(1/2)*tan(d*x+c)^(1/3)/d/(1+I*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3645, 129, 441, 440}

$$\frac{3a\sqrt[3]{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}F_1\left(\frac{1}{3};-\frac{1}{2},1;\frac{4}{3};-i\tan(c+dx),i\tan(c+dx)\right)}{d\sqrt{1+i\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(3/2)/Tan[c + d*x]^(2/3),x]
```

```
[Out] (3*a*AppellF1[1/3, -1/2, 1, 4/3, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*Tan[c +
 d*x]^(1/3)*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[1 + I*Tan[c + d*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\tan^{\frac{2}{3}}(c+dx)}\,dx = \frac{(ia^2)\,\text{Subst}\left(\int \frac{\sqrt{a+x}}{\left(-\frac{ix}{a}\right)^{2/3}(-a^2+ax)}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(3a^3)\,\text{Subst}\left(\int \frac{\sqrt{a+iax^3}}{-a^2+ia^2x^3}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{d}$$

$$= -\frac{\left(3a^3\sqrt{a+ia\tan(c+dx)}\right)\text{Subst}\left(\int \frac{\sqrt{1+ix^3}}{-a^2+ia^2x^3}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{d\sqrt{1+i\tan(c+dx)}}$$

$$= \frac{3aF_1\left(\frac{1}{3}; -\frac{1}{2}, 1; \frac{4}{3}; -i\tan(c+dx), i\tan(c+dx)\right)\sqrt[3]{\tan(c+dx)}\sqrt{a+ia\tan}}{d\sqrt{1+i\tan(c+dx)}}$$

Mathematica [F]
time = 3.80, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\tan^{\frac{2}{3}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(3/2)/Tan[c + d*x]^(2/3),x]
[Out] Integrate[(a + I*a*Tan[c + d*x])^(3/2)/Tan[c + d*x]^(2/3), x]
```

Maple [F]
time = 1.09, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(dx+c))^{\frac{3}{2}}}{\tan(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(2/3),x)
```

```
[Out] int((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(2/3),x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)/tan(d*x + c)^(2/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(2/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}{\tan^{\frac{2}{3}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(3/2)/tan(d*x+c)**(2/3),x)
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)/tan(c + d*x)**(2/3), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(2/3),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 4.64
Done
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\tan(c + d\,x)\ 1\mathrm{i})^{3/2}}{\tan{(c + d\,x)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(3/2)/tan(c + d*x)^(2/3),x)
[Out] int((a + a*tan(c + d*x)*1i)^(3/2)/tan(c + d*x)^(2/3), x)
```

3.259 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{\tan^{\frac{4}{3}}(c+dx)}\,dx$

Optimal. Leaf size=80

$$-\frac{3aF_1\left(-\frac{1}{3};-\frac{1}{2},1;\frac{2}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{1+i\tan(c+dx)}\,\sqrt[3]{\tan(c+dx)}}$$

```
[Out] -3*a*AppellF1(-1/3,-1/2,1,2/3,-I*tan(d*x+c),I*tan(d*x+c))*(a+I*a*tan(d*x+c)
)^(1/2)/d/(1+I*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/3)
```

Rubi [A]
time = 0.10, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3645, 129, 525, 524}

$$-\frac{3a\sqrt{a+ia\tan(c+dx)}\,F_1\left(-\frac{1}{3};-\frac{1}{2},1;\frac{2}{3};-i\tan(c+dx),i\tan(c+dx)\right)}{d\sqrt{1+i\tan(c+dx)}\,\sqrt[3]{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(3/2)/Tan[c + d*x]^(4/3),x]
```

```
[Out] (-3*a*AppellF1[-1/3, -1/2, 1, 2/3, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*Sqrt[
a + I*a*Tan[c + d*x]])/(d*Sqrt[1 + I*Tan[c + d*x]]*Tan[c + d*x]^(1/3))
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+ia\tan(c+dx))^{3/2}}{\tan^{\frac{4}{3}}(c+dx)}\,dx &= \frac{(ia^2)\,\text{Subst}\left(\int \frac{\sqrt{a+x}}{\left(-\frac{ix}{a}\right)^{4/3}(-a^2+ax)}\,dx, x, ia\tan(c+dx)\right)}{d} \\
&= -\frac{(3a^3)\,\text{Subst}\left(\int \frac{\sqrt{a+iax^3}}{x^2(-a^2+ia^2x^3)}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{d} \\
&= -\frac{\left(3a^3\sqrt{a+ia\tan(c+dx)}\right)\text{Subst}\left(\int \frac{\sqrt{1+ix^3}}{x^2(-a^2+ia^2x^3)}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{d\sqrt{1+i\tan(c+dx)}} \\
&= -\frac{3aF_1\left(-\frac{1}{3}; -\frac{1}{2}, 1; \frac{2}{3}; -i\tan(c+dx), i\tan(c+dx)\right)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{1+i\tan(c+dx)}\,\sqrt[3]{\tan(c+dx)}}
\end{aligned}
$$

Mathematica [F]
time = 14.55, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\tan^{\frac{4}{3}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(3/2)/Tan[c + d*x]^(4/3),x]
[Out] Integrate[(a + I*a*Tan[c + d*x])^(3/2)/Tan[c + d*x]^(4/3), x]
```

Maple [F]
time = 1.01, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(dx+c))^{\frac{3}{2}}}{\tan(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(4/3),x)
[Out] int((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(4/3),x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)/tan(d*x + c)^(4/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(4/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}{\tan^{\frac{4}{3}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(3/2)/tan(d*x+c)**(4/3),x)
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)/tan(c + d*x)**(4/3), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/tan(d*x+c)^(4/3),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 4.49
Done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^{3/2}}{\tan\left(c+d\,x\right)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(3/2)/tan(c + d*x)^(4/3),x)
[Out] int((a + a*tan(c + d*x)*1i)^(3/2)/tan(c + d*x)^(4/3), x)
```

3.260 $\int \frac{\tan^{\frac{4}{3}}(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=81

$$\frac{3F_1\left(\frac{7}{3};\frac{3}{2},1;\frac{10}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{1+i\tan(c+dx)}\,\tan^{\frac{7}{3}}(c+dx)}{7d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 3/7*AppellF1(7/3,3/2,1,10/3,-I*tan(d*x+c),I*tan(d*x+c))*(1+I*tan(d*x+c))^(1
/2)*tan(d*x+c)^(7/3)/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3645, 129, 525, 524}

$$\frac{3\sqrt{1+i\tan(c+dx)}\,\tan^{\frac{7}{3}}(c+dx)F_1\left(\frac{7}{3};\frac{3}{2},1;\frac{10}{3};-i\tan(c+dx),i\tan(c+dx)\right)}{7d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(4/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (3*AppellF1[7/3, 3/2, 1, 10/3, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*Sqrt[1 +
I*Tan[c + d*x]]*Tan[c + d*x]^(7/3))/(7*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
```

```
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{\frac{4}{3}}(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx &= \frac{(ia^2)\operatorname{Subst}\left(\int \frac{\left(-\frac{ix}{a}\right)^{4/3}}{(a+x)^{3/2}(-a^2+ax)}\,dx, x, ia\tan(c+dx)\right)}{d}\\ &= -\frac{(3a^3)\operatorname{Subst}\left(\int \frac{x^6}{(a+iax^3)^{3/2}(-a^2+ia^2x^3)}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{d}\\ &= -\frac{\left(3a^2\sqrt{1+i\tan(c+dx)}\right)\operatorname{Subst}\left(\int \frac{x^6}{(1+ix^3)^{3/2}(-a^2+ia^2x^3)}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{d\sqrt{a+ia\tan(c+dx)}}\\ &= \frac{3F_1\left(\frac{7}{3};\frac{3}{2},1;\frac{10}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{1+i\tan(c+dx)}\,\tan^{\frac{7}{3}}(c+dx)}{7d\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

Mathematica [F]
time = 6.04, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{4}{3}}(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^(4/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] Integrate[Tan[c + d*x]^(4/3)/Sqrt[a + I*a*Tan[c + d*x]], x]
```

Maple [F]
time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{4}{3}}(dx+c)}{\sqrt{a+ia\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(4/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] int(tan(d*x+c)^(4/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(4/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^(4/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(4/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{4}{3}}(c+dx)}{\sqrt{ia\,(\tan{(c+dx)}-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(4/3)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**(4/3)/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(4/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^(4/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\tan(c+d\,x)}^{4/3}}{\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(4/3)/(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int(tan(c + d*x)^(4/3)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.261 $\int \frac{\tan^{\frac{2}{3}}(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=81

$$\frac{3F_1\left(\frac{5}{3};\frac{3}{2},1;\frac{8}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{1+i\tan(c+dx)}\,\tan^{\frac{5}{3}}(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 3/5*AppellF1(5/3,3/2,1,8/3,-I*tan(d*x+c),I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/
2)*tan(d*x+c)^(5/3)/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3645, 129, 525, 524}

$$\frac{3\sqrt{1+i\tan(c+dx)}\,\tan^{\frac{5}{3}}(c+dx)F_1\left(\frac{5}{3};\frac{3}{2},1;\frac{8}{3};-i\tan(c+dx),i\tan(c+dx)\right)}{5d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(2/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (3*AppellF1[5/3, 3/2, 1, 8/3, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*Sqrt[1 + I
*Tan[c + d*x]]*Tan[c + d*x]^(5/3))/(5*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
```

```
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\int \frac{\tan^{\frac{2}{3}}(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{(ia^2)\,\text{Subst}\left(\int \frac{\left(-\frac{ix}{a}\right)^{2/3}}{(a+x)^{3/2}(-a^2+ax)}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(3a^3)\,\text{Subst}\left(\int \frac{x^4}{(a+iax^3)^{3/2}(-a^2+ia^2x^3)}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{d}$$

$$= -\frac{\left(3a^2\sqrt{1+i\tan(c+dx)}\right)\text{Subst}\left(\int \frac{x^4}{(1+ix^3)^{3/2}(-a^2+ia^2x^3)}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{d\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{3F_1\left(\frac{5}{3};\frac{3}{2},1;\frac{8}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{1+i\tan(c+dx)}\,\tan^{\frac{5}{3}}(c+d}{5d\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [F]
time = 2.86, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{2}{3}}(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^(2/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] Integrate[Tan[c + d*x]^(2/3)/Sqrt[a + I*a*Tan[c + d*x]], x]
```

Maple [F]
time = 1.03, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{2}{3}}(dx+c)}{\sqrt{a+ia\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(2/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] int(tan(d*x+c)^(2/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(2/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^(2/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(2/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*((-I*e^(2*I*d*x + 2*I*c) + I)
/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(-I*e^(4*I*d*x + 4*I*c) - 2*I*e^(2*I*d*x
+ 2*I*c) - I) - (a*d*e^(3*I*d*x + 3*I*c) - 4*a*d*e^(2*I*d*x + 2*I*c) + 4*a*
d*e^(I*d*x + I*c))*integral(1/6*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(
(-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(-3*I*e^(5*I*
d*x + 5*I*c) + 66*I*e^(4*I*d*x + 4*I*c) - 32*I*e^(3*I*d*x + 3*I*c) + 64*I*e
^(2*I*d*x + 2*I*c) - 29*I*e^(I*d*x + I*c) - 2*I)/(a*d*e^(5*I*d*x + 5*I*c) -
 6*a*d*e^(4*I*d*x + 4*I*c) + 11*a*d*e^(3*I*d*x + 3*I*c) - 2*a*d*e^(2*I*d*x
+ 2*I*c) - 12*a*d*e^(I*d*x + I*c) + 8*a*d), x))/(a*d*e^(3*I*d*x + 3*I*c) -
4*a*d*e^(2*I*d*x + 2*I*c) + 4*a*d*e^(I*d*x + I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{2}{3}}(c+dx)}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(2/3)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**(2/3)/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(2/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^(2/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\tan\left(c+d\,x\right)}^{2/3}}{\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(2/3)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] int(tan(c + d*x)^(2/3)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.262 $\int \frac{\sqrt[3]{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=81

$$\frac{3F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{1+i\tan(c+dx)}\,\tan^{\frac{4}{3}}(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 3/4*AppellF1(4/3,3/2,1,7/3,-I*tan(d*x+c),I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/
2)*tan(d*x+c)^(4/3)/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3645, 129, 525, 524}

$$\frac{3\sqrt{1+i\tan(c+dx)}\,\tan^{\frac{4}{3}}(c+dx)F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-i\tan(c+dx),i\tan(c+dx)\right)}{4d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(1/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (3*AppellF1[4/3, 3/2, 1, 7/3, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*Sqrt[1 + I
*Tan[c + d*x]]*Tan[c + d*x]^(4/3))/(4*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
```

```
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{(ia^2)\,\text{Subst}\left(\int \frac{\sqrt[3]{-\frac{ix}{a}}}{(a+x)^{3/2}(-a^2+ax)}\,dx, x, ia\tan(c+dx)\right)}{d}$$
$$= -\frac{(3a^3)\,\text{Subst}\left(\int \frac{x^3}{(a+iax^3)^{3/2}(-a^2+ia^2x^3)}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{d}$$
$$= -\frac{\left(3a^2\sqrt{1+i\tan(c+dx)}\right)\text{Subst}\left(\int \frac{x^3}{(1+ix^3)^{3/2}(-a^2+ia^2x^3)}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{d\sqrt{a+ia\tan(c+dx)}}$$
$$= \frac{3F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{1+i\tan(c+dx)}\,\tan^{\frac{4}{3}}(c+d}{4d\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [F]
time = 5.54, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^(1/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] Integrate[Tan[c + d*x]^(1/3)/Sqrt[a + I*a*Tan[c + d*x]], x]
```

Maple [F]
time = 1.08, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{1}{3}}(dx+c)}{\sqrt{a+ia\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x)
[Out] int(tan(d*x+c)^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(tan(d*x + c)^(1/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\tan{\left(c + dx \right)}}}{\sqrt{ia \left(\tan{\left(c + dx \right)} - i\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/3)/(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral(tan(c + d*x)**(1/3)/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(tan(d*x + c)^(1/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^{1/3}}{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/3)/(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int(tan(c + d*x)^(1/3)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.263 $\int \frac{1}{\sqrt[3]{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=81

$$\frac{3F_1\left(\frac{2}{3};\frac{3}{2},1;\frac{5}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{1+i\tan(c+dx)}\,\tan^{\frac{2}{3}}(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 3/2*AppellF1(2/3,3/2,1,5/3,-I*tan(d*x+c),I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/
2)*tan(d*x+c)^(2/3)/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3645, 129, 525, 524}

$$\frac{3\sqrt{1+i\tan(c+dx)}\,\tan^{\frac{2}{3}}(c+dx)F_1\left(\frac{2}{3};\frac{3}{2},1;\frac{5}{3};-i\tan(c+dx),i\tan(c+dx)\right)}{2d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(1/3)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] (3*AppellF1[2/3, 3/2, 1, 5/3, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*Sqrt[1 + I
*Tan[c + d*x]]*Tan[c + d*x]^(2/3))/(2*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{(ia^2)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{-\frac{ix}{a}}\,(a+x)^{3/2}(-a^2+ax)}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(3a^3)\,\text{Subst}\left(\int \frac{x}{(a+iax^3)^{3/2}(-a^2+ia^2x^3)}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{d}$$

$$= -\frac{\left(3a^2\sqrt{1+i\tan(c+dx)}\right)\text{Subst}\left(\int \frac{x}{(1+ix^3)^{3/2}(-a^2+ia^2x^3)}\,dx,\right.}{d\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{3F_1\left(\frac{2}{3};\frac{3}{2},1;\frac{5}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{1+i\tan(c+}}{2d\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [F]
time = 3.80, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Tan[c + d*x]^(1/3)*Sqrt[a + I*a*Tan[c + d*x]]),x]
[Out] Integrate[1/(Tan[c + d*x]^(1/3)*Sqrt[a + I*a*Tan[c + d*x]]), x]
```

Maple [F]
time = 1.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+ia\tan(dx+c)}\,\tan(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/3),x)
```

```
[Out] int(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/3),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(I*a*tan(d*x + c) + a)*tan(d*x + c)^(1/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/3),x, algorithm="fricas"
)
```

```
[Out] (2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*((-I*e^(2*I*d*x + 2*I*c) + I)/
(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*
c) + 1) + (a*d*e^(3*I*d*x + 3*I*c) - 4*a*d*e^(2*I*d*x + 2*I*c) + 4*a*d*e^(I
*d*x + I*c))*integral(1/6*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*((-I*e^
(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(3*e^(5*I*d*x + 5*I
*c) + 30*e^(4*I*d*x + 4*I*c) + 46*e^(3*I*d*x + 3*I*c) - 20*e^(2*I*d*x + 2*I
*c) + 43*e^(I*d*x + I*c) - 50)/(a*d*e^(5*I*d*x + 5*I*c) - 6*a*d*e^(4*I*d*x
+ 4*I*c) + 11*a*d*e^(3*I*d*x + 3*I*c) - 2*a*d*e^(2*I*d*x + 2*I*c) - 12*a*d*
e^(I*d*x + I*c) + 8*a*d), x))/(a*d*e^(3*I*d*x + 3*I*c) - 4*a*d*e^(2*I*d*x +
 2*I*c) + 4*a*d*e^(I*d*x + I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,\sqrt[3]{\tan\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))**(1/2)/tan(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/(sqrt(I*a*(tan(c + d*x) - I))*tan(c + d*x)**(1/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/3),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\tan\left(c+d\,x\right)^{1/3}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(1/3)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
[Out] int(1/(tan(c + d*x)^(1/3)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

3.264 $\int \frac{1}{\tan^{\frac{2}{3}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=79

$$\frac{3F_1\left(\frac{1}{3};\frac{3}{2},1;\frac{4}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{1+i\tan(c+dx)}\,\sqrt[3]{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 3*AppellF1(1/3,3/2,1,4/3,-I*tan(d*x+c),I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/2)
*tan(d*x+c)^(1/3)/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3645, 129, 441, 440}

$$\frac{3\sqrt{1+i\tan(c+dx)}\,\sqrt[3]{\tan(c+dx)}\,F_1\left(\frac{1}{3};\frac{3}{2},1;\frac{4}{3};-i\tan(c+dx),i\tan(c+dx)\right)}{d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(2/3)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] (3*AppellF1[1/3, 3/2, 1, 4/3, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*Sqrt[1 + I
*Tan[c + d*x]]*Tan[c + d*x]^(1/3))/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\int \frac{1}{\tan^{\frac{2}{3}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{(ia^2)\,\text{Subst}\left(\int \frac{1}{\left(-\frac{ix}{a}\right)^{2/3}(a+x)^{3/2}(-a^2+ax)}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(3a^3)\,\text{Subst}\left(\int \frac{1}{(a+iax^3)^{3/2}(-a^2+ia^2x^3)}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{d}$$

$$= -\frac{\left(3a^2\sqrt{1+i\tan(c+dx)}\right)\text{Subst}\left(\int \frac{1}{(1+ix^3)^{3/2}(-a^2+ia^2x^3)}\,dx, x,\right.}{d\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{3F_1\left(\frac{1}{3};\frac{3}{2},1;\frac{4}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{1+i\tan(c+dx}}{d\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [F]
time = 6.35, size = 0, normalized size = 0.00

$$\int \frac{1}{\tan^{\frac{2}{3}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Tan[c + d*x]^(2/3)*Sqrt[a + I*a*Tan[c + d*x]]),x]
[Out] Integrate[1/(Tan[c + d*x]^(2/3)*Sqrt[a + I*a*Tan[c + d*x]]), x]
```

Maple [F]
time = 1.18, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+ia\tan(dx+c)}\,\tan(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(2/3),x)
```

```
[Out] int(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(2/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(2/3),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(I*a*tan(d*x + c) + a)*tan(d*x + c)^(2/3)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(2/3),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,\tan^{\frac{2}{3}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))**(1/2)/tan(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/(sqrt(I*a*(tan(c + d*x) - I))*tan(c + d*x)**(2/3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\tan (c+d\,x)^{2/3}\,\sqrt{a+a\tan (c+d\,x)\ 1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(2/3)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
[Out] int(1/(tan(c + d*x)^(2/3)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

3.265 $\int \frac{1}{\tan^{\frac{4}{3}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=79

$$-\frac{3F_1\left(-\frac{1}{3};\frac{3}{2},1;\frac{2}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{1+i\tan(c+dx)}}{d\sqrt[3]{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] -3*AppellF1(-1/3,3/2,1,2/3,-I*tan(d*x+c),I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/
2)/d/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/3)
```

Rubi [A]
time = 0.10, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3645, 129, 525, 524}

$$-\frac{3\sqrt{1+i\tan(c+dx)}\,F_1\left(-\frac{1}{3};\frac{3}{2},1;\frac{2}{3};-i\tan(c+dx),i\tan(c+dx)\right)}{d\sqrt[3]{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(4/3)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] (-3*AppellF1[-1/3, 3/2, 1, 2/3, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*Sqrt[1 +
 I*Tan[c + d*x]])/(d*Tan[c + d*x]^(1/3)*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\int \frac{1}{\tan^{\frac{4}{3}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{(ia^2)\,\text{Subst}\left(\int \frac{1}{\left(-\frac{ix}{a}\right)^{4/3}(a+x)^{3/2}(-a^2+ax)}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(3a^3)\,\text{Subst}\left(\int \frac{1}{x^2(a+iax^3)^{3/2}(-a^2+ia^2x^3)}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{d}$$

$$= -\frac{\left(3a^2\sqrt{1+i\tan(c+dx)}\right)\text{Subst}\left(\int \frac{1}{x^2(1+ix^3)^{3/2}(-a^2+ia^2x^3)}\,dx, \right.}{d\sqrt{a+ia\tan(c+dx)}}$$

$$= -\frac{3F_1\left(-\frac{1}{3};\frac{3}{2},1;\frac{2}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{1+i\tan(c+}}{d\sqrt[3]{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [F]
time = 11.64, size = 0, normalized size = 0.00

$$\int \frac{1}{\tan^{\frac{4}{3}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Tan[c + d*x]^(4/3)*Sqrt[a + I*a*Tan[c + d*x]]),x]
[Out] Integrate[1/(Tan[c + d*x]^(4/3)*Sqrt[a + I*a*Tan[c + d*x]]), x]
```

Maple [F]
time = 1.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+ia\tan(dx+c)}\,\tan(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(4/3),x)
```

```
[Out] int(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(4/3),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(I*a*tan(d*x + c) + a)*tan(d*x + c)^(4/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(4/3),x, algorithm="fricas"
)
```

```
[Out] 1/2*(sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*((-I*e^(2*I*d*x + 2*I*c) + I
)/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(-19*I*e^(6*I*d*x + 6*I*c) + 12*I*e^(5*I
*d*x + 5*I*c) - 34*I*e^(4*I*d*x + 4*I*c) + 24*I*e^(3*I*d*x + 3*I*c) - 11*I*
e^(2*I*d*x + 2*I*c) + 12*I*e^(I*d*x + I*c) + 4*I) + 2*(a*d*e^(5*I*d*x + 5*I
*c) - 4*a*d*e^(4*I*d*x + 4*I*c) + 3*a*d*e^(3*I*d*x + 3*I*c) + 4*a*d*e^(2*I*
d*x + 2*I*c) - 4*a*d*e^(I*d*x + I*c))*integral(1/6*sqrt(2)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^
(2/3)*(3*I*e^(5*I*d*x + 5*I*c) - 210*I*e^(4*I*d*x + 4*I*c) + 11*I*e^(3*I*d*
x + 3*I*c) - 130*I*e^(2*I*d*x + 2*I*c) + 8*I*e^(I*d*x + I*c) + 80*I)/(a*d*e
^(5*I*d*x + 5*I*c) - 6*a*d*e^(4*I*d*x + 4*I*c) + 11*a*d*e^(3*I*d*x + 3*I*c)
 - 2*a*d*e^(2*I*d*x + 2*I*c) - 12*a*d*e^(I*d*x + I*c) + 8*a*d), x))/(a*d*e^
(5*I*d*x + 5*I*c) - 4*a*d*e^(4*I*d*x + 4*I*c) + 3*a*d*e^(3*I*d*x + 3*I*c) +
 4*a*d*e^(2*I*d*x + 2*I*c) - 4*a*d*e^(I*d*x + I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,\tan^{\frac{4}{3}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))**(1/2)/tan(d*x+c)**(4/3),x)
```

```
[Out] Integral(1/(sqrt(I*a*(tan(c + d*x) - I))*tan(c + d*x)**(4/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(4/3),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\tan\left(c+d\,x\right)^{4/3}\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(4/3)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
[Out] int(1/(tan(c + d*x)^(4/3)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

3.266 $\int \frac{\tan^{\frac{4}{3}}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=84

$$\frac{3F_1\left(\frac{7}{3};\frac{5}{2},1;\frac{10}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{1+i\tan(c+dx)}\,\tan^{\frac{7}{3}}(c+dx)}{7ad\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 3/7*AppellF1(7/3,5/2,1,10/3,-I*tan(d*x+c),I*tan(d*x+c))*(1+I*tan(d*x+c))^(1
/2)*tan(d*x+c)^(7/3)/a/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3645, 129, 525, 524}

$$\frac{3\sqrt{1+i\tan(c+dx)}\,\tan^{\frac{7}{3}}(c+dx)F_1\left(\frac{7}{3};\frac{5}{2},1;\frac{10}{3};-i\tan(c+dx),i\tan(c+dx)\right)}{7ad\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(4/3)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (3*AppellF1[7/3, 5/2, 1, 10/3, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*Sqrt[1 +
I*Tan[c + d*x]]*Tan[c + d*x]^(7/3))/(7*a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\int \frac{\tan^{\frac{4}{3}}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{(ia^2)\,\mathrm{Subst}\left(\int \frac{\left(-\frac{ix}{a}\right)^{4/3}}{(a+x)^{5/2}(-a^2+ax)}\,dx, x, ia\tan(c+dx)\right)}{d}$$
$$= -\frac{(3a^3)\,\mathrm{Subst}\left(\int \frac{x^6}{(a+iax^3)^{5/2}(-a^2+ia^2x^3)}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{d}$$
$$= -\frac{\left(3a\sqrt{1+i\tan(c+dx)}\right)\mathrm{Subst}\left(\int \frac{x^6}{(1+ix^3)^{5/2}(-a^2+ia^2x^3)}\,dx, x, \sqrt[3]{\tan(c+dx}\right.}{d\sqrt{a+ia\tan(c+dx)}}$$
$$= \frac{3F_1\left(\frac{7}{3};\frac{5}{2},1;\frac{10}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{1+i\tan(c+dx)}\,\tan^{\frac{7}{3}}(c+}{7ad\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [F]
time = 6.76, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{4}{3}}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^(4/3)/(a + I*a*Tan[c + d*x])^(3/2),x]
[Out] Integrate[Tan[c + d*x]^(4/3)/(a + I*a*Tan[c + d*x])^(3/2), x]
```

Maple [F]
time = 1.05, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{4}{3}}(dx+c)}{(a+ia\tan(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(4/3)/(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] int(tan(d*x+c)^(4/3)/(a+I*a*tan(d*x+c))^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(4/3)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(4/3)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{4}{3}}(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(4/3)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**(4/3)/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(4/3)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^(4/3)/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^{4/3}}{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(4/3)/(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int(tan(c + d*x)^(4/3)/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.267 $\int \frac{\tan^{\frac{2}{3}}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=84

$$\frac{3F_1\left(\frac{5}{3};\frac{5}{2},1;\frac{8}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{1+i\tan(c+dx)}\,\tan^{\frac{5}{3}}(c+dx)}{5ad\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 3/5*AppellF1(5/3,5/2,1,8/3,-I*tan(d*x+c),I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/
2)*tan(d*x+c)^(5/3)/a/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3645, 129, 525, 524}

$$\frac{3\sqrt{1+i\tan(c+dx)}\,\tan^{\frac{5}{3}}(c+dx)F_1\left(\frac{5}{3};\frac{5}{2},1;\frac{8}{3};-i\tan(c+dx),i\tan(c+dx)\right)}{5ad\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(2/3)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (3*AppellF1[5/3, 5/2, 1, 8/3, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*Sqrt[1 + I
*Tan[c + d*x]]*Tan[c + d*x]^(5/3))/(5*a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\int \frac{\tan^{\frac{2}{3}}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{(ia^2)\operatorname{Subst}\left(\int \frac{\left(-\frac{ix}{a}\right)^{2/3}}{(a+x)^{5/2}(-a^2+ax)}\,dx, x, ia\tan(c+dx)\right)}{d}$$
$$= -\frac{(3a^3)\operatorname{Subst}\left(\int \frac{x^4}{(a+iax^3)^{5/2}(-a^2+ia^2x^3)}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{d}$$
$$= -\frac{\left(3a\sqrt{1+i\tan(c+dx)}\right)\operatorname{Subst}\left(\int \frac{x^4}{(1+ix^3)^{5/2}(-a^2+ia^2x^3)}\,dx, x, \sqrt[3]{\tan(c+dx}\right.}{d\sqrt{a+ia\tan(c+dx)}}$$
$$= \frac{3F_1\left(\frac{5}{3};\frac{5}{2},1;\frac{8}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{1+i\tan(c+dx)}\,\tan^{\frac{5}{3}}(c+}{5ad\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [F]
time = 12.19, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{2}{3}}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^(2/3)/(a + I*a*Tan[c + d*x])^(3/2),x]
[Out] Integrate[Tan[c + d*x]^(2/3)/(a + I*a*Tan[c + d*x])^(3/2), x]
```

Maple [F]
time = 1.03, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{2}{3}}(dx+c)}{(a+ia\tan(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(2/3)/(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] int(tan(d*x+c)^(2/3)/(a+I*a*tan(d*x+c))^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(2/3)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(2/3)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/36*(sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*((-I*e^(2*I*d*x + 2*I*c) +
I)/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(7*I*e^(6*I*d*x + 6*I*c) - 12*I*e^(5*I*
d*x + 5*I*c) + 26*I*e^(4*I*d*x + 4*I*c) - 24*I*e^(3*I*d*x + 3*I*c) + 31*I*e
^(2*I*d*x + 2*I*c) - 12*I*e^(I*d*x + I*c) + 12*I) + 36*(a^2*d*e^(5*I*d*x +
5*I*c) - 4*a^2*d*e^(4*I*d*x + 4*I*c) + 4*a^2*d*e^(3*I*d*x + 3*I*c))*integra
l(1/108*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*((-I*e^(2*I*d*x + 2*I*c)
+ I)/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(-27*I*e^(5*I*d*x + 5*I*c) + 210*I*e^
(4*I*d*x + 4*I*c) - 344*I*e^(3*I*d*x + 3*I*c) + 400*I*e^(2*I*d*x + 2*I*c) -
 317*I*e^(I*d*x + I*c) + 190*I)/(a^2*d*e^(5*I*d*x + 5*I*c) - 6*a^2*d*e^(4*I
*d*x + 4*I*c) + 11*a^2*d*e^(3*I*d*x + 3*I*c) - 2*a^2*d*e^(2*I*d*x + 2*I*c)
- 12*a^2*d*e^(I*d*x + I*c) + 8*a^2*d), x))/(a^2*d*e^(5*I*d*x + 5*I*c) - 4*a
^2*d*e^(4*I*d*x + 4*I*c) + 4*a^2*d*e^(3*I*d*x + 3*I*c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{2}{3}}(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(2/3)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**(2/3)/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(2/3)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(tan(d*x + c)^(2/3)/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^{2/3}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(2/3)/(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int(tan(c + d*x)^(2/3)/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.268 $\int \frac{\sqrt[3]{\tan(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=84

$$\frac{3F_1\left(\frac{4}{3};\frac{5}{2},1;\frac{7}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{1+i\tan(c+dx)}\,\tan^{\frac{4}{3}}(c+dx)}{4ad\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 3/4*AppellF1(4/3,5/2,1,7/3,-I*tan(d*x+c),I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/
2)*tan(d*x+c)^(4/3)/a/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3645, 129, 525, 524}

$$\frac{3\sqrt{1+i\tan(c+dx)}\,\tan^{\frac{4}{3}}(c+dx)F_1\left(\frac{4}{3};\frac{5}{2},1;\frac{7}{3};-i\tan(c+dx),i\tan(c+dx)\right)}{4ad\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(1/3)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (3*AppellF1[4/3, 5/2, 1, 7/3, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*Sqrt[1 + I
*Tan[c + d*x]]*Tan[c + d*x]^(4/3))/(4*a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
```

```
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{\tan(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{(ia^2)\,\text{Subst}\left(\int \frac{\sqrt[3]{-\frac{ix}{a}}}{(a+x)^{5/2}(-a^2+ax)}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(3a^3)\,\text{Subst}\left(\int \frac{x^3}{(a+iax^3)^{5/2}(-a^2+ia^2x^3)}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{d}$$

$$= -\frac{\left(3a\sqrt{1+i\tan(c+dx)}\right)\text{Subst}\left(\int \frac{x^3}{(1+ix^3)^{5/2}(-a^2+ia^2x^3)}\,dx, x, \sqrt[3]{\tan(c+d}\right.}{d\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{3F_1\left(\frac{4}{3};\frac{5}{2},1;\frac{7}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{1+i\tan(c+dx)}\,\tan^{\frac{4}{3}}(c+}{4ad\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [F]
time = 6.41, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\tan(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^(1/3)/(a + I*a*Tan[c + d*x])^(3/2),x]
[Out] Integrate[Tan[c + d*x]^(1/3)/(a + I*a*Tan[c + d*x])^(3/2), x]
```

Maple [F]
time = 1.04, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{1}{3}}(dx+c)}{(a+ia\tan(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/3)/(a+I*a*tan(d*x+c))^(3/2),x)
[Out] int(tan(d*x+c)^(1/3)/(a+I*a*tan(d*x+c))^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/3)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/3)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\tan(c+dx)}}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/3)/(a+I*a*tan(d*x+c))**(3/2),x)
[Out] Integral(tan(c + d*x)**(1/3)/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/3)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(tan(d*x + c)^(1/3)/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^{1/3}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/3)/(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int(tan(c + d*x)^(1/3)/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.269 $\int \frac{1}{\sqrt[3]{\tan(c+dx)}\,(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=84

$$\frac{3F_1\left(\frac{2}{3};\frac{5}{2},1;\frac{5}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{1+i\tan(c+dx)}\,\tan^{\frac{2}{3}}(c+dx)}{2ad\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 3/2*AppellF1(2/3,5/2,1,5/3,-I*tan(d*x+c),I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/
2)*tan(d*x+c)^(2/3)/a/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3645, 129, 525, 524}

$$\frac{3\sqrt{1+i\tan(c+dx)}\,\tan^{\frac{2}{3}}(c+dx)F_1\left(\frac{2}{3};\frac{5}{2},1;\frac{5}{3};-i\tan(c+dx),i\tan(c+dx)\right)}{2ad\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(1/3)*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

```
[Out] (3*AppellF1[2/3, 5/2, 1, 5/3, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*Sqrt[1 + I
*Tan[c + d*x]]*Tan[c + d*x]^(2/3))/(2*a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{\tan(c+dx)}\,(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{(ia^2)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{-\frac{ix}{a}}\,(a+x)^{5/2}(-a^2+ax)}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(3a^3)\,\text{Subst}\left(\int \frac{x}{(a+iax^3)^{5/2}(-a^2+ia^2x^3)}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{d}$$

$$= -\frac{\left(3a\sqrt{1+i\tan(c+dx)}\right)\text{Subst}\left(\int \frac{x}{(1+ix^3)^{5/2}(-a^2+ia^2x^3)}\,dx,\right.}{d\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{3F_1\left(\frac{2}{3};\frac{5}{2},1;\frac{5}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{1+i\tan(c+}}{2ad\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [F]
time = 11.46, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{\tan(c+dx)}\,(a+ia\tan(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Tan[c + d*x]^(1/3)*(a + I*a*Tan[c + d*x])^(3/2)),x]
[Out] Integrate[1/(Tan[c + d*x]^(1/3)*(a + I*a*Tan[c + d*x])^(3/2)), x]
```

Maple [F]
time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{1}{\tan(dx+c)^{\frac{1}{3}}\,(a+ia\tan(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(1/3)/(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] int(1/tan(d*x+c)^(1/3)/(a+I*a*tan(d*x+c))^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/3)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/3)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/36*(sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*((-I*e^(2*I*d*x + 2*I*c) +
I)/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(79*e^(6*I*d*x + 6*I*c) - 12*e^(5*I*d*x
 + 5*I*c) + 170*e^(4*I*d*x + 4*I*c) - 24*e^(3*I*d*x + 3*I*c) + 103*e^(2*I*d
*x + 2*I*c) - 12*e^(I*d*x + I*c) + 12) + 36*(a^2*d*e^(5*I*d*x + 5*I*c) - 4*
a^2*d*e^(4*I*d*x + 4*I*c) + 4*a^2*d*e^(3*I*d*x + 3*I*c))*integral(1/108*sqr
t(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*
I*d*x + 2*I*c) + 1))^(2/3)*(27*e^(5*I*d*x + 5*I*c) + 750*e^(4*I*d*x + 4*I*c
) + 484*e^(3*I*d*x + 3*I*c) + 40*e^(2*I*d*x + 2*I*c) + 457*e^(I*d*x + I*c)
- 710)/(a^2*d*e^(5*I*d*x + 5*I*c) - 6*a^2*d*e^(4*I*d*x + 4*I*c) + 11*a^2*d*
e^(3*I*d*x + 3*I*c) - 2*a^2*d*e^(2*I*d*x + 2*I*c) - 12*a^2*d*e^(I*d*x + I*c
) + 8*a^2*d), x))/(a^2*d*e^(5*I*d*x + 5*I*c) - 4*a^2*d*e^(4*I*d*x + 4*I*c)
+ 4*a^2*d*e^(3*I*d*x + 3*I*c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{3}{2}}\sqrt[3]{\tan\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(1/3)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/((I*a*(tan(c + d*x) - I))**(3/2)*tan(c + d*x)**(1/3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/3)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\tan\left(c+d\,x\right)^{1/3}\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(1/3)*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

```
[Out] int(1/(tan(c + d*x)^(1/3)*(a + a*tan(c + d*x)*1i)^(3/2)), x)
```

3.270 $\int \frac{1}{\tan^{\frac{2}{3}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=82

$$\frac{3F_1\left(\frac{1}{3};\frac{5}{2},1;\frac{4}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{1+i\tan(c+dx)}\,\sqrt[3]{\tan(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 3*AppellF1(1/3,5/2,1,4/3,-I*tan(d*x+c),I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/2)
*tan(d*x+c)^(1/3)/a/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3645, 129, 441, 440}

$$\frac{3\sqrt{1+i\tan(c+dx)}\,\sqrt[3]{\tan(c+dx)}\,F_1\left(\frac{1}{3};\frac{5}{2},1;\frac{4}{3};-i\tan(c+dx),i\tan(c+dx)\right)}{ad\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(2/3)*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

```
[Out] (3*AppellF1[1/3, 5/2, 1, 4/3, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*Sqrt[1 + I
*Tan[c + d*x]]*Tan[c + d*x]^(1/3))/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\int \frac{1}{\tan^{\frac{2}{3}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{(ia^2)\,\text{Subst}\left(\int \frac{1}{\left(-\frac{ix}{a}\right)^{2/3}(a+x)^{5/2}(-a^2+ax)}\,dx, x, ia\tan(c+dx)\right)}{d}$$
$$= -\frac{(3a^3)\,\text{Subst}\left(\int \frac{1}{(a+iax^3)^{5/2}(-a^2+ia^2x^3)}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{d}$$
$$= -\frac{\left(3a\sqrt{1+i\tan(c+dx)}\right)\text{Subst}\left(\int \frac{1}{(1+ix^3)^{5/2}(-a^2+ia^2x^3)}\,dx, x,$$
$$= \frac{3F_1\left(\frac{1}{3}; \frac{5}{2}, 1; \frac{4}{3}; -i\tan(c+dx), i\tan(c+dx)\right)\sqrt{1+i\tan(c+d}{ad\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [F]
time = 7.30, size = 0, normalized size = 0.00

$$\int \frac{1}{\tan^{\frac{2}{3}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Tan[c + d*x]^(2/3)*(a + I*a*Tan[c + d*x])^(3/2)),x]
[Out] Integrate[1/(Tan[c + d*x]^(2/3)*(a + I*a*Tan[c + d*x])^(3/2)), x]
```

Maple [F]
time = 1.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\tan\left(dx+c\right)^{\frac{2}{3}}\left(a+ia\tan\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(2/3)/(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] int(1/tan(d*x+c)^(2/3)/(a+I*a*tan(d*x+c))^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(2/3)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(2/3)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(2/3)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(2/3)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\tan \left(c+d\,x\right)^{2/3}\,{\left(a+a\,\tan \left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(2/3)*(a + a*tan(c + d*x)*1i)^(3/2)),x)
[Out] int(1/(tan(c + d*x)^(2/3)*(a + a*tan(c + d*x)*1i)^(3/2)), x)
```

3.271 $\int \frac{1}{\tan^{\frac{4}{3}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=82

$$-\frac{3F_1\left(-\frac{1}{3};\frac{5}{2},1;\frac{2}{3};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{1+i\tan(c+dx)}}{ad\sqrt[3]{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] -3*AppellF1(-1/3,5/2,1,2/3,-I*tan(d*x+c),I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/
2)/a/d/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/3)
```

Rubi [A]
time = 0.11, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3645, 129, 525, 524}

$$-\frac{3\sqrt{1+i\tan(c+dx)}\,F_1\left(-\frac{1}{3};\frac{5}{2},1;\frac{2}{3};-i\tan(c+dx),i\tan(c+dx)\right)}{ad\sqrt[3]{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(4/3)*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

```
[Out] (-3*AppellF1[-1/3, 5/2, 1, 2/3, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*Sqrt[1 +
 I*Tan[c + d*x]])/(a*d*Tan[c + d*x]^(1/3)*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\int \frac{1}{\tan^{\frac{4}{3}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{(ia^2)\,\text{Subst}\left(\int \frac{1}{\left(-\frac{ix}{a}\right)^{4/3}(a+x)^{5/2}(-a^2+ax)}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(3a^3)\,\text{Subst}\left(\int \frac{1}{x^2(a+iax^3)^{5/2}(-a^2+ia^2x^3)}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{d}$$

$$= -\frac{\left(3a\sqrt{1+i\tan(c+dx)}\right)\text{Subst}\left(\int \frac{1}{x^2(1+ix^3)^{5/2}(-a^2+ia^2x^3)}\,dx, \ldots\right.}{d\sqrt{a+ia\tan(c+dx)}}$$

$$= -\frac{3F_1\left(-\frac{1}{3}; \frac{5}{2}, 1; \frac{2}{3}; -i\tan(c+dx), i\tan(c+dx)\right)\sqrt{1+i\tan(c\ldots}}{ad\sqrt[3]{\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [F]
time = 12.34, size = 0, normalized size = 0.00

$$\int \frac{1}{\tan^{\frac{4}{3}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Tan[c + d*x]^(4/3)*(a + I*a*Tan[c + d*x])^(3/2)),x]
[Out] Integrate[1/(Tan[c + d*x]^(4/3)*(a + I*a*Tan[c + d*x])^(3/2)), x]
```

Maple [F]
time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{1}{\tan(dx+c)^{\frac{4}{3}}\,(a+ia\tan(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(4/3)/(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] int(1/tan(d*x+c)^(4/3)/(a+I*a*tan(d*x+c))^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(4/3)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(4/3)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/36*(sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*((-I*e^(2*I*d*x + 2*I*c) +
I)/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(-421*I*e^(8*I*d*x + 8*I*c) + 228*I*e^(
7*I*d*x + 7*I*c) - 703*I*e^(6*I*d*x + 6*I*c) + 444*I*e^(5*I*d*x + 5*I*c) -
131*I*e^(4*I*d*x + 4*I*c) + 204*I*e^(3*I*d*x + 3*I*c) + 163*I*e^(2*I*d*x +
2*I*c) - 12*I*e^(I*d*x + I*c) + 12*I) + 36*(a^2*d*e^(7*I*d*x + 7*I*c) - 4*a
^2*d*e^(6*I*d*x + 6*I*c) + 3*a^2*d*e^(5*I*d*x + 5*I*c) + 4*a^2*d*e^(4*I*d*x
 + 4*I*c) - 4*a^2*d*e^(3*I*d*x + 3*I*c))*integral(1/108*sqrt(2)*sqrt(a/(e^(
2*I*d*x + 2*I*c) + 1))*((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) +
 1))^(2/3)*(27*I*e^(5*I*d*x + 5*I*c) - 4530*I*e^(4*I*d*x + 4*I*c) - 286*I*e
^(3*I*d*x + 3*I*c) - 2380*I*e^(2*I*d*x + 2*I*c) - 313*I*e^(I*d*x + I*c) + 2
150*I)/(a^2*d*e^(5*I*d*x + 5*I*c) - 6*a^2*d*e^(4*I*d*x + 4*I*c) + 11*a^2*d*
e^(3*I*d*x + 3*I*c) - 2*a^2*d*e^(2*I*d*x + 2*I*c) - 12*a^2*d*e^(I*d*x + I*c
) + 8*a^2*d), x))/(a^2*d*e^(7*I*d*x + 7*I*c) - 4*a^2*d*e^(6*I*d*x + 6*I*c)
+ 3*a^2*d*e^(5*I*d*x + 5*I*c) + 4*a^2*d*e^(4*I*d*x + 4*I*c) - 4*a^2*d*e^(3*
I*d*x + 3*I*c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(4/3)/(a+I*a*tan(d*x+c))**(3/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(4/3)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\tan\left(c+d\,x\right)^{4/3}\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(4/3)*(a + a*tan(c + d*x)*1i)^(3/2)),x)
[Out] int(1/(tan(c + d*x)^(4/3)*(a + a*tan(c + d*x)*1i)^(3/2)), x)
```

3.272 $\int \tan^3(c+dx)\sqrt[3]{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=234

$$-\frac{i\sqrt[3]{a}\,x}{2\,2^{2/3}}+\frac{\sqrt{3}\,\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2^{2/3}d}-\frac{\sqrt[3]{a}\,\log(\cos(c+dx))}{2\,2^{2/3}d}-\frac{3\sqrt[3]{a}\,\log\left(\sqrt[3]{2}\,\sqrt[3]{a}\,-\right.}{2\,2}$$

[Out] -1/4*I*a^(1/3)*x*2^(1/3)-1/4*a^(1/3)*ln(cos(d*x+c))*2^(1/3)/d-3/4*a^(1/3)*ln(2^(1/3)*a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))*2^(1/3)/d+1/2*a^(1/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)*2^(1/3)/d-18/7*(a+I*a*tan(d*x+c))^(1/3)/d+3/7*tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/3)/d-3/28*(a+I*a*tan(d*x+c))^(4/3)/a/d

Rubi [A]
time = 0.21, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3641, 3673, 3608, 3562, 59, 631, 210, 31}

$$\frac{\sqrt{3}\,\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2^{2/3}d}+\frac{3\tan^2(c+dx)\sqrt[3]{a+ia\tan(c+dx)}}{7d}-\frac{3(a+ia\tan(c+dx))^{4/3}}{28ad}-\frac{18\sqrt[3]{a+ia\tan(c+dx)}}{7d}-\frac{3\sqrt[3]{a}\,\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{2\,2^{2/3}d}-\frac{\sqrt[3]{a}\,\log(\cos(c+dx))}{2\,2^{2/3}d}-\frac{i\sqrt[3]{a}\,x}{2\,2^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d*x]^3*(a + I*a*Tan[c + d*x])^(1/3),x]

[Out] ((-1/2*I)*a^(1/3)*x)/2^(2/3) + (Sqrt[3]*a^(1/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + I*a*Tan[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/(2^(2/3)*d) - (a^(1/3)*Log[Cos[c + d*x]])/(2*2^(2/3)*d) - (3*a^(1/3)*Log[2^(1/3)*a^(1/3) - (a + I*a*Tan[c + d*x])^(1/3)])/(2*2^(2/3)*d) - (18*(a + I*a*Tan[c + d*x])^(1/3))/(7*d) + (3*Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^(1/3))/(7*d) - (3*(a + I*a*Tan[c + d*x])^(4/3))/(28*a*d)

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rule 3641

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[d*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n - 1)/(f*(m + n - 1))), x] - Dist[1/(a*(m + n - 1)), Int[(a + b*
Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 2)*Simp[d*(b*c*m + a*d*(-1 + n))
- a*c^2*(m + n - 1) + d*(b*d*m - a*c*(m + 2*n - 2))*Tan[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1] && NeQ[m + n - 1, 0] && (IntegerQ[n]
|| IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \tan^3(c+dx)\sqrt[3]{a+ia\tan(c+dx)}\,dx = \frac{3\tan^2(c+dx)\sqrt[3]{a+ia\tan(c+dx)}}{7d} - \frac{3\int \tan(c+dx)\left(2a+\frac{1}{3}ia\right.}{}$$

$$= \frac{3\tan^2(c+dx)\sqrt[3]{a+ia\tan(c+dx)}}{7d} - \frac{3(a+ia\tan(c+dx))^{4/3}}{28ad} -$$

$$= -\frac{18\sqrt[3]{a+ia\tan(c+dx)}}{7d} + \frac{3\tan^2(c+dx)\sqrt[3]{a+ia\tan(c+dx)}}{7d}$$

$$= -\frac{18\sqrt[3]{a+ia\tan(c+dx)}}{7d} + \frac{3\tan^2(c+dx)\sqrt[3]{a+ia\tan(c+dx)}}{7d}$$

$$= -\frac{i\sqrt[3]{a}\,x}{2\,2^{2/3}} - \frac{\sqrt[3]{a}\,\log(\cos(c+dx))}{2\,2^{2/3}d} - \frac{18\sqrt[3]{a+ia\tan(c+dx)}}{7d} + 3$$

$$= -\frac{i\sqrt[3]{a}\,x}{2\,2^{2/3}} - \frac{\sqrt[3]{a}\,\log(\cos(c+dx))}{2\,2^{2/3}d} - \frac{3\sqrt[3]{a}\,\log\left(\sqrt[3]{2}\sqrt[3]{a} - \sqrt[3]{a+i}\right.}{2\,2^{2/3}d}$$

$$= -\frac{i\sqrt[3]{a}\,x}{2\,2^{2/3}} + \frac{\sqrt{3}\sqrt[3]{a}\,\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2^{2/3}d} - \sqrt[3]{a}$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^3*(a + I*a*Tan[c + d*x])^(1/3),x]
```

```
[Out] $Aborted
```

Maple [A]
time = 0.17, size = 190, normalized size = 0.81

method	result

derivativedivides	$-\frac{3\left(\frac{(a+ia\tan(dx+c))^{\frac{7}{3}}}{7}-\frac{a(a+ia\tan(dx+c))^{\frac{4}{3}}}{4}+a^{2}(a+ia\tan(dx+c))^{\frac{1}{3}}+\left(\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}\ln\left((a+\cdots\right.}{\cdots}\right.\right.}{d\,a^{2}}$
default	$-\frac{3\left(\frac{(a+ia\tan(dx+c))^{\frac{7}{3}}}{7}-\frac{a(a+ia\tan(dx+c))^{\frac{4}{3}}}{4}+a^{2}(a+ia\tan(dx+c))^{\frac{1}{3}}+\left(\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}\ln\left((a+\cdots\right.}{\cdots}\right.\right.}{d\,a^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/d/a^2*(1/7*(a+I*a*tan(d*x+c))^(7/3)-1/4*a*(a+I*a*tan(d*x+c))^(4/3)+a^2*(
a+I*a*tan(d*x+c))^(1/3)+(1/6*2^(1/3)/a^(2/3)*ln((a+I*a*tan(d*x+c))^(1/3)-2^
(1/3)*a^(1/3))-1/12*2^(1/3)/a^(2/3)*ln((a+I*a*tan(d*x+c))^(2/3)+2^(1/3)*a^(
1/3)*(a+I*a*tan(d*x+c))^(1/3)+2^(2/3)*a^(2/3))-1/6*2^(1/3)/a^(2/3)*3^(1/2)*
arctan(1/3*3^(1/2)*(2^(2/3)/a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+1)))*a^3)
```

Maxima [A]
time = 0.53, size = 190, normalized size = 0.81

$$\frac{14\,\sqrt{3}\,2^{\frac{1}{3}}a^{\frac{13}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)+7\cdot 2^{\frac{1}{3}}a^{\frac{13}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(i\,a\tan(dx+c)+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{2}{3}}\right)-14\cdot 2^{\frac{1}{3}}a^{\frac{13}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)-12\,(i\,a\tan(dx+c)+a)^{\frac{7}{3}}a^{2}+21\,(i\,a\tan(dx+c)+a)^{\frac{4}{3}}a^{3}-84\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}a^{4}}{28\,a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] 1/28*(14*sqrt(3)*2^(1/3)*a^(13/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/
3) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) + 7*2^(1/3)*a^(13/3)*log(2^(2
/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x +
 c) + a)^(2/3)) - 14*2^(1/3)*a^(13/3)*log(-2^(1/3)*a^(1/3) + (I*a*tan(d*x +
 c) + a)^(1/3)) - 12*(I*a*tan(d*x + c) + a)^(7/3)*a^2 + 21*(I*a*tan(d*x + c
) + a)^(4/3)*a^3 - 84*(I*a*tan(d*x + c) + a)^(1/3)*a^4)/(a^4*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 393 vs. 2(172) = 344.
time = 0.63, size = 393, normalized size = 1.68

$$-\frac{3\cdot 2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}\left(15\,e^{(4i\,dx+4i\,c)}+21\,e^{(2i\,dx+2i\,c)}+14\right)e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}+7\left(\frac{1}{4}\right)^{\frac{1}{3}}\left(\left(-i\sqrt{3}\,d+d\right)e^{(4i\,dx+4i\,c)}+2\left(-i\sqrt{3}\,d+d\right)e^{(2i\,dx+2i\,c)}-i\sqrt{3}\,d+d\right)\left(-\frac{a}{d^{3}}\right)^{\frac{1}{3}}\log\left(\left(\frac{1}{4}\right)^{\frac{1}{3}}\left(i\sqrt{3}\,d-d\right)\left(-\frac{a}{d^{3}}\right)^{\frac{1}{3}}+2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)+7\left(\frac{1}{4}\right)^{\frac{1}{3}}\left(\left(i\sqrt{3}\,d+d\right)e^{(4i\,dx+4i\,c)}+2\left(i\sqrt{3}\,d+d\right)e^{(2i\,dx+2i\,c)}+i\sqrt{3}\,d+d\right)\left(-\frac{a}{d^{3}}\right)^{\frac{1}{3}}\log\left(\left(\frac{1}{4}\right)^{\frac{1}{3}}\left(-i\sqrt{3}\,d-d\right)\left(-\frac{a}{d^{3}}\right)^{\frac{1}{3}}+2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)-14\left(\frac{1}{4}\right)^{\frac{1}{3}}\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)\left(-\frac{a}{d^{3}}\right)^{\frac{1}{3}}\log\left(2\left(\frac{1}{4}\right)^{\frac{1}{3}}d\left(-\frac{a}{d^{3}}\right)^{\frac{1}{3}}+2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)}{14\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] -1/14*(3*2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*(15*e^(4*I*d*x + 4*I*c
) + 21*e^(2*I*d*x + 2*I*c) + 14)*e^(2/3*I*d*x + 2/3*I*c) + 7*(1/4)^(1/3)*((
-I*sqrt(3)*d + d)*e^(4*I*d*x + 4*I*c) + 2*(-I*sqrt(3)*d + d)*e^(2*I*d*x + 2
*I*c) - I*sqrt(3)*d + d)*(-a/d^3)^(1/3)*log((1/4)^(1/3)*(I*sqrt(3)*d - d)*(
-a/d^3)^(1/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x +
2/3*I*c)) + 7*(1/4)^(1/3)*((I*sqrt(3)*d + d)*e^(4*I*d*x + 4*I*c) + 2*(I*sqr
t(3)*d + d)*e^(2*I*d*x + 2*I*c) + I*sqrt(3)*d + d)*(-a/d^3)^(1/3)*log((1/4)
^(1/3)*(-I*sqrt(3)*d - d)*(-a/d^3)^(1/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c)
+ 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)) - 14*(1/4)^(1/3)*(d*e^(4*I*d*x + 4*I*c
) + 2*d*e^(2*I*d*x + 2*I*c) + d)*(-a/d^3)^(1/3)*log(2*(1/4)^(1/3)*d*(-a/d^3
)^(1/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*
c)))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{ia\left(\tan\left(c+dx\right)-i\right)}\,\tan^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3*(a+I*a*tan(d*x+c))**(1/3),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(1/3)*tan(c + d*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(1/3)*tan(d*x + c)^3, x)
```

Mupad [B]
time = 0.63, size = 235, normalized size = 1.00

$$-\frac{3\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{1/3}}{d}+\frac{3\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{4/3}}{4\,a\,d}-\frac{3\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/3}}{7\,a^2\,d}+\frac{2^{1/3}\,{\left(-a\right)}^{1/3}\,\ln\left(9\,a\,{\left(a\,\left(1+\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)\right)}^{1/3}-9\,2^{1/3}\,{\left(-a\right)}^{4/3}\right)}{2\,d}+\frac{4^{2/3}\,{\left(-a\right)}^{1/3}\,\ln\left(\frac{9\,a\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{1/3}}{d}-\frac{9\,2^{1/3}\,{\left(-a\right)}^{4/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,d}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{4\,d}-\frac{4^{2/3}\,{\left(-a\right)}^{1/3}\,\ln\left(\frac{9\,a\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{1/3}}{d}+\frac{9\,2^{1/3}\,{\left(-a\right)}^{4/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,d}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(1/3),x)
```

```
[Out] (3*(a + a*tan(c + d*x)*1i)^(4/3))/(4*a*d) - (3*(a + a*tan(c + d*x)*1i)^(1/3
))/d - (3*(a + a*tan(c + d*x)*1i)^(7/3))/(7*a^2*d) + (2^(1/3)*(-a)^(1/3)*lo
```

```
g(9*a*(a*(tan(c + d*x)*1i + 1))^(1/3) - 9*2^(1/3)*(-a)^(4/3)))/(2*d) + (4^(
2/3)*(-a)^(1/3)*log((9*a*(a + a*tan(c + d*x)*1i)^(1/3))/d - (9*2^(1/3)*(-a)
^(4/3)*(3^(1/2)*1i - 1))/(2*d))*((3^(1/2)*1i)/2 - 1/2))/(4*d) - (4^(2/3)*(-
a)^(1/3)*log((9*a*(a + a*tan(c + d*x)*1i)^(1/3))/d + (9*2^(1/3)*(-a)^(4/3)*
(3^(1/2)*1i + 1))/(2*d))*((3^(1/2)*1i)/2 + 1/2))/(4*d)
```

3.273 $\int \tan^2(c+dx)\sqrt[3]{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=185

$$\frac{\sqrt[3]{a}\,x}{2\,2^{2/3}}+\frac{i\sqrt{3}\,\sqrt[3]{a}\,\mathrm{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2^{2/3}d}-\frac{i\sqrt[3]{a}\,\log(\cos(c+dx))}{2\,2^{2/3}d}-\frac{3i\sqrt[3]{a}\,\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\right.}{2\,2^{\ldots}}$$

[Out] 1/4*a^(1/3)*x*2^(1/3)-1/4*I*a^(1/3)*ln(cos(d*x+c))*2^(1/3)/d-3/4*I*a^(1/3)*ln(2^(1/3)*a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))*2^(1/3)/d+1/2*I*a^(1/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)*2^(1/3)/d-3/4*I*(a+I*a*tan(d*x+c))^(4/3)/a/d

Rubi [A]
time = 0.09, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.231$, Rules used = {3624, 3562, 59, 631, 210, 31}

$$\frac{i\sqrt{3}\,\sqrt[3]{a}\,\mathrm{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2^{2/3}d}-\frac{3i(a+ia\tan(c+dx))^{4/3}}{4ad}-\frac{3i\sqrt[3]{a}\,\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{2\,2^{2/3}d}-\frac{i\sqrt[3]{a}\,\log(\cos(c+dx))}{2\,2^{2/3}d}+\frac{\sqrt[3]{a}\,x}{2\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^(1/3),x]
```

```
[Out] (a^(1/3)*x)/(2*2^(2/3)) + (I*Sqrt[3]*a^(1/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a +
 I*a*Tan[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/(2^(2/3)*d) - ((I/2)*a^(1/3)*
Log[Cos[c + d*x]])/(2^(2/3)*d) - (((3*I)/2)*a^(1/3)*Log[2^(1/3)*a^(1/3) - (
a + I*a*Tan[c + d*x])^(1/3)])/(2^(2/3)*d) - (((3*I)/4)*(a + I*a*Tan[c + d*x
])^(4/3))/(a*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \tan^2(c+dx)\sqrt[3]{a+ia\tan(c+dx)}\,dx = -\frac{3i(a+ia\tan(c+dx))^{4/3}}{4ad} - \int \sqrt[3]{a+ia\tan(c+dx)}\,dx$$

$$= -\frac{3i(a+ia\tan(c+dx))^{4/3}}{4ad} + \frac{(ia)\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{2/3}}\,dx, x, ia$$

$$= \frac{\sqrt[3]{a}\,x}{2\,2^{2/3}} - \frac{i\sqrt[3]{a}\,\log(\cos(c+dx))}{2\,2^{2/3}d} - \frac{3i(a+ia\tan(c+dx))^{4/3}}{4ad} + $$

$$= \frac{\sqrt[3]{a}\,x}{2\,2^{2/3}} - \frac{i\sqrt[3]{a}\,\log(\cos(c+dx))}{2\,2^{2/3}d} - \frac{3i\sqrt[3]{a}\,\log\left(\sqrt[3]{2}\,\sqrt[3]{a} - \sqrt[3]{a+}\right.}{2\,2^{2/3}d}$$

$$= \frac{\sqrt[3]{a}\,x}{2\,2^{2/3}} + \frac{i\sqrt{3}\,\sqrt[3]{a}\,\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2^{2/3}d} - i\sqrt[3]{a}$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$$\text{\$Aborted}$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^(1/3),x]
```

```
[Out] $Aborted
```

Maple [A]
time = 0.12, size = 156, normalized size = 0.84

method	result
derivativedivides	$-\frac{3i\left(\frac{(a+ia\tan(dx+c))^{\frac{4}{3}}}{4}+\left(\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right.}{12a^{\frac{2}{3}}}\right.}{da}$
default	$-\frac{3i\left(\frac{(a+ia\tan(dx+c))^{\frac{4}{3}}}{4}+\left(\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right.}{12a^{\frac{2}{3}}}\right.}{da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3*I/d/a*(1/4*(a+I*a*tan(d*x+c))^(4/3)+(1/6*2^(1/3)/a^(2/3)*ln((a+I*a*tan(d
*x+c))^(1/3)-2^(1/3)*a^(1/3))-1/12*2^(1/3)/a^(2/3)*ln((a+I*a*tan(d*x+c))^(2
/3)+2^(1/3)*a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+2^(2/3)*a^(2/3))-1/6*2^(1/3)/a
^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2^(2/3)/a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)
+1)))*a^2)
```

Maxima [A]
time = 0.51, size = 153, normalized size = 0.83

$$\frac{i\left(2\sqrt{3}\,2^{\frac{1}{3}}a^{\frac{10}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)+2^{\frac{1}{3}}a^{\frac{10}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(i\,a\tan(dx+c)+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{2}{3}}\right)-2\cdot 2^{\frac{1}{3}}a^{\frac{10}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)-3\,(i\,a\tan(dx+c)+a)^{\frac{4}{3}}a^{2}\right)}{4\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] 1/4*I*(2*sqrt(3)*2^(1/3)*a^(10/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/
3) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) + 2^(1/3)*a^(10/3)*log(2^(2/3
)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x + c
) + a)^(2/3)) - 2*2^(1/3)*a^(10/3)*log(-2^(1/3)*a^(1/3) + (I*a*tan(d*x + c)
 + a)^(1/3)) - 3*(I*a*tan(d*x + c) + a)^(4/3)*a^2)/(a^3*d)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(128) = 256$.
time = 0.67, size = 294, normalized size = 1.59

$$\frac{\left(\left(i\sqrt{3}\,d-d\right)e^{(2i\,dx+2i\,c)}+i\sqrt{3}\,d-d\right)\left(\frac{ia}{4\,d^3}\right)^{\frac{1}{3}}\log\left(2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}+\left(\sqrt{3}\,d+i\,d\right)\left(\frac{ia}{4\,d^3}\right)^{\frac{1}{3}}\right)+\left(\left(-i\sqrt{3}\,d-d\right)e^{(2i\,dx+2i\,c)}-i\sqrt{3}\,d-d\right)\left(\frac{ia}{4\,d^3}\right)^{\frac{1}{3}}\log\left(2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}-\left(\sqrt{3}\,d-i\,d\right)\left(\frac{ia}{4\,d^3}\right)^{\frac{1}{3}}\right)+2\left(de^{(2i\,dx+2i\,c)}+d\right)\left(\frac{ia}{4\,d^3}\right)^{\frac{1}{3}}\log\left(2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}-2i\,d\left(\frac{ia}{4\,d^3}\right)^{\frac{1}{3}}\right)-3i\cdot 2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{8}{3}i\,dx+\frac{8}{3}i\,c\right)}}{2\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*(((I*sqrt(3)*d - d)*e^(2*I*d*x + 2*I*c) + I*sqrt(3)*d - d)*(1/4*I*a/d^3
)^(1/3)*log(2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*
I*c) + (sqrt(3)*d + I*d)*(1/4*I*a/d^3)^(1/3)) + ((-I*sqrt(3)*d - d)*e^(2*I*
d*x + 2*I*c) - I*sqrt(3)*d - d)*(1/4*I*a/d^3)^(1/3)*log(2^(1/3)*(a/(e^(2*I*
d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) - (sqrt(3)*d - I*d)*(1/4*I
*a/d^3)^(1/3)) + 2*(d*e^(2*I*d*x + 2*I*c) + d)*(1/4*I*a/d^3)^(1/3)*log(2^(1
/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) - 2*I*d*(1/
4*I*a/d^3)^(1/3)) - 3*I*2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(8/3*
I*d*x + 8/3*I*c))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{ia\left(\tan\left(c+dx\right)-i\right)}\,\tan^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2*(a+I*a*tan(d*x+c))**(1/3),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(1/3)*tan(c + d*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(1/3)*tan(d*x + c)^2, x)
```

Mupad [B]
time = 4.07, size = 195, normalized size = 1.05

$$-\frac{(a+a\tan(c+d\,x)\,1\mathrm{i})^{4/3}\,3\mathrm{i}}{4\,a\,d}+\frac{\left(\frac{1}{4}\mathrm{i}\right)^{1/3}a^{1/3}\,\ln\left(18\,\left(\frac{1}{4}\mathrm{i}\right)^{1/3}a^{4/3}\,d^2+a\,d^2\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{1/3}\,9\mathrm{i}\right)}{d}+\frac{\left(\frac{1}{4}\mathrm{i}\right)^{1/3}a^{1/3}\,\ln\left(a\,d^2\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{1/3}\,9\mathrm{i}+18\,\left(\frac{1}{4}\mathrm{i}\right)^{1/3}a^{4/3}\,d^2\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{d}-\frac{\left(\frac{1}{4}\mathrm{i}\right)^{1/3}a^{1/3}\,\ln\left(a\,d^2\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{1/3}\,9\mathrm{i}-18\,\left(\frac{1}{4}\mathrm{i}\right)^{1/3}a^{4/3}\,d^2\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(1/3),x)
```

```
[Out] ((1i/4)^(1/3)*a^(1/3)*log(18*(1i/4)^(1/3)*a^(4/3)*d^2 + a*d^2*(a + a*tan(c
+ d*x)*1i)^(1/3)*9i))/d - ((a + a*tan(c + d*x)*1i)^(4/3)*3i)/(4*a*d) + ((1i
/4)^(1/3)*a^(1/3)*log(a*d^2*(a + a*tan(c + d*x)*1i)^(1/3)*9i + 18*(1i/4)^(1
/3)*a^(4/3)*d^2*((3^(1/2)*1i)/2 - 1/2))*((3^(1/2)*1i)/2 - 1/2))/d - ((1i/4)
^(1/3)*a^(1/3)*log(a*d^2*(a + a*tan(c + d*x)*1i)^(1/3)*9i - 18*(1i/4)^(1/3)
*a^(4/3)*d^2*((3^(1/2)*1i)/2 + 1/2))*((3^(1/2)*1i)/2 + 1/2))/d
```

3.274 $\int \tan(c+dx)\sqrt[3]{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=174

$$\frac{i\sqrt[3]{a}\,x}{2\ 2^{2/3}}-\frac{\sqrt{3}\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{2^{2/3}d}+\frac{\sqrt[3]{a}\,\log(\cos(c+dx))}{2\ 2^{2/3}d}+\frac{3\sqrt[3]{a}\,\log\left(\sqrt[3]{2}\sqrt[3]{a}-\ldots\right.}{2\ 2\ldots}$$

```
[Out] 1/4*I*a^(1/3)*x*2^(1/3)+1/4*a^(1/3)*ln(cos(d*x+c))*2^(1/3)/d+3/4*a^(1/3)*ln
(2^(1/3)*a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))*2^(1/3)/d-1/2*a^(1/3)*arctan(1/3
*(a^(1/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)*2^(1/3
)/d+3*(a+I*a*tan(d*x+c))^(1/3)/d
```

Rubi [A]
time = 0.09, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3608, 3562, 59, 631, 210, 31}

$$-\frac{\sqrt{3}\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{2^{2/3}d}+\frac{3\sqrt[3]{a+ia\tan(c+dx)}}{d}+\frac{3\sqrt[3]{a}\,\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{2\ 2^{2/3}d}+\frac{\sqrt[3]{a}\,\log(\cos(c+dx))}{2\ 2^{2/3}d}+\frac{i\sqrt[3]{a}\,x}{2\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]*(a + I*a*Tan[c + d*x])^(1/3),x]
```

```
[Out] ((I/2)*a^(1/3)*x)/2^(2/3) - (Sqrt[3]*a^(1/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a +
 I*a*Tan[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/(2^(2/3)*d) + (a^(1/3)*Log[Co
s[c + d*x]])/(2*2^(2/3)*d) + (3*a^(1/3)*Log[2^(1/3)*a^(1/3) - (a + I*a*Tan[
c + d*x])^(1/3)])/(2*2^(2/3)*d) + (3*(a + I*a*Tan[c + d*x])^(1/3))/d
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \tan(c+dx)\sqrt[3]{a+ia\tan(c+dx)}\,dx &= \frac{3\sqrt[3]{a+ia\tan(c+dx)}}{d} - i\int \sqrt[3]{a+ia\tan(c+dx)}\,dx \\
&= \frac{3\sqrt[3]{a+ia\tan(c+dx)}}{d} - \frac{a\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{2/3}}\,dx, x, ia\tan(c+
}{d} \\
&= \frac{i\sqrt[3]{a}\,x}{2\;2^{2/3}} + \frac{\sqrt[3]{a}\,\log(\cos(c+dx))}{2\;2^{2/3}d} + \frac{3\sqrt[3]{a+ia\tan(c+dx)}}{d} - \frac{(3\sqrt[3]{a}
}{} \\
&= \frac{i\sqrt[3]{a}\,x}{2\;2^{2/3}} + \frac{\sqrt[3]{a}\,\log(\cos(c+dx))}{2\;2^{2/3}d} + \frac{3\sqrt[3]{a}\,\log\left(\sqrt[3]{2}\,\sqrt[3]{a} - \sqrt[3]{a+ia\,\mathrm{t}
}{2\;2^{2/3}d} \\
&= \frac{i\sqrt[3]{a}\,x}{2\;2^{2/3}} - \frac{\sqrt{3}\,\sqrt[3]{a}\,\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2^{2/3}d} + \frac{\sqrt[3]{a}\,\mathrm{lo}
}{}
\end{aligned}
$$

Mathematica [F]

time = 180.00, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[c + d*x]*(a + I*a*Tan[c + d*x])^(1/3),x]
```

```
[Out] $Aborted
```

Maple [A]
time = 0.10, size = 150, normalized size = 0.86

method	result
derivativedivides	$\frac{3(a+ia\tan(dx+c))^{\frac{1}{3}}+3\left(\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{12a^{\frac{2}{3}}}\right.}{d}$
default	$\frac{3(a+ia\tan(dx+c))^{\frac{1}{3}}+3\left(\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{12a^{\frac{2}{3}}}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)*(a+I*a*tan(d*x+c))^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(3*(a+I*a*tan(d*x+c))^(1/3)+3*(1/6*2^(1/3)/a^(2/3)*ln((a+I*a*tan(d*x+c)
)^(1/3)-2^(1/3)*a^(1/3))-1/12*2^(1/3)/a^(2/3)*ln((a+I*a*tan(d*x+c))^(2/3)+2
^(1/3)*a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+2^(2/3)*a^(2/3))-1/6*2^(1/3)/a^(2/3
)*3^(1/2)*arctan(1/3*3^(1/2)*(2^(2/3)/a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+1)))
*a)
```

Maxima [A]
time = 0.52, size = 153, normalized size = 0.88

$$-\frac{2\sqrt{3}\,2^{\frac{1}{3}}a^{\frac{7}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)+2^{\frac{1}{3}}a^{\frac{7}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(i\,a\tan(dx+c)+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{2}{3}}\right)-2\cdot 2^{\frac{1}{3}}a^{\frac{7}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)-12\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}a^{2}}{4\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(3)*2^(1/3)*a^(7/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3)
 + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) + 2^(1/3)*a^(7/3)*log(2^(2/3)*a
^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x + c) +
 a)^(2/3)) - 2*2^(1/3)*a^(7/3)*log(-2^(1/3)*a^(1/3) + (I*a*tan(d*x + c) + a
)^(1/3)) - 12*(I*a*tan(d*x + c) + a)^(1/3)*a^2)/(a^2*d)
```

Fricas [A]
time = 0.55, size = 239, normalized size = 1.37

$$\frac{\left(\frac{1}{4}\right)^{\frac{1}{3}}\left(-i\sqrt{3}\,d-d\right)\left(\frac{a}{d^3}\right)^{\frac{1}{3}}\log\left(\left(\frac{1}{4}\right)^{\frac{1}{3}}\left(i\sqrt{3}\,d+d\right)\left(\frac{a}{d^3}\right)^{\frac{1}{3}}+2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)+\left(\frac{1}{4}\right)^{\frac{1}{3}}\left(i\sqrt{3}\,d-d\right)\left(\frac{a}{d^3}\right)^{\frac{1}{3}}\log\left(\left(\frac{1}{4}\right)^{\frac{1}{3}}\left(-i\sqrt{3}\,d+d\right)\left(\frac{a}{d^3}\right)^{\frac{1}{3}}+2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)+2\left(\frac{1}{4}\right)^{\frac{1}{3}}d\left(\frac{a}{d^3}\right)^{\frac{1}{3}}\log\left(-2\left(\frac{1}{4}\right)^{\frac{1}{3}}d\left(\frac{a}{d^3}\right)^{\frac{1}{3}}+2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)+6\cdot 2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*((1/4)^(1/3)*(-I*sqrt(3)*d - d)*(a/d^3)^(1/3)*log((1/4)^(1/3)*(I*sqrt(3
)*d + d)*(a/d^3)^(1/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3
*I*d*x + 2/3*I*c)) + (1/4)^(1/3)*(I*sqrt(3)*d - d)*(a/d^3)^(1/3)*log((1/4)^
(1/3)*(-I*sqrt(3)*d + d)*(a/d^3)^(1/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) +
1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)) + 2*(1/4)^(1/3)*d*(a/d^3)^(1/3)*log(-2*(
1/4)^(1/3)*d*(a/d^3)^(1/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^
(2/3*I*d*x + 2/3*I*c)) + 6*2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2
/3*I*d*x + 2/3*I*c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{ia\left(\tan\left(c+dx\right)-i\right)}\,\tan\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))**(1/3),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(1/3)*tan(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(1/3)*tan(d*x + c), x)
```

Mupad [B]
time = 4.28, size = 176, normalized size = 1.01

$$\frac{3\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{1/3}}{d}+\frac{2^{1/3}\,a^{1/3}\,\ln\left(\left(a\left(1+\tan(c+d\,x)\,1\mathrm{i}\right)\right)^{1/3}-2^{1/3}\,a^{1/3}\right)}{2\,d}+\frac{4^{2/3}\,a^{1/3}\,\ln\left(\frac{9\,a\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{1/3}}{d}-\frac{9\,2^{1/3}\,a^{4/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,d}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{4\,d}-\frac{4^{2/3}\,a^{1/3}\,\ln\left(\frac{9\,a\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{1/3}}{d}+\frac{9\,2^{1/3}\,a^{4/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,d}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + a*tan(c + d*x)*1i)^(1/3),x)
```

```
[Out] (3*(a + a*tan(c + d*x)*1i)^(1/3))/d + (2^(1/3)*a^(1/3)*log((a*(tan(c + d*x)
*1i + 1))^(1/3) - 2^(1/3)*a^(1/3)))/(2*d) + (4^(2/3)*a^(1/3)*log((9*a*(a +
a*tan(c + d*x)*1i)^(1/3))/d - (9*2^(1/3)*a^(4/3)*(3^(1/2)*1i - 1))/(2*d))*(
(3^(1/2)*1i)/2 - 1/2))/(4*d) - (4^(2/3)*a^(1/3)*log((9*a*(a + a*tan(c + d*x
)*1i)^(1/3))/d + (9*2^(1/3)*a^(4/3)*(3^(1/2)*1i + 1))/(2*d))*((3^(1/2)*1i)/
2 + 1/2))/(4*d)
```

3.275 $\int \sqrt[3]{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=156

$$-\frac{\sqrt[3]{a}\,x}{2\ 2^{2/3}}-\frac{i\sqrt{3}\,\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2^{2/3}d}+\frac{i\sqrt[3]{a}\,\log(\cos(c+dx))}{2\ 2^{2/3}d}+\frac{3i\sqrt[3]{a}\,\log\left(\sqrt[3]{2}\,\sqrt[3]{a}\right.}{2}$$

```
[Out] -1/4*a^(1/3)*x*2^(1/3)+1/4*I*a^(1/3)*ln(cos(d*x+c))*2^(1/3)/d+3/4*I*a^(1/3)
*ln(2^(1/3)*a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))*2^(1/3)/d-1/2*I*a^(1/3)*arcta
n(1/3*(a^(1/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)*2
^(1/3)/d
```

Rubi [A]
time = 0.06, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3562, 59, 631, 210, 31}

$$-\frac{i\sqrt{3}\,\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2^{2/3}d}+\frac{3i\sqrt[3]{a}\,\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{2\ 2^{2/3}d}+\frac{i\sqrt[3]{a}\,\log(\cos(c+dx))}{2\ 2^{2/3}d}-\frac{\sqrt[3]{a}\,x}{2\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(1/3),x]
```

```
[Out] -1/2*(a^(1/3)*x)/2^(2/3) - (I*Sqrt[3]*a^(1/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a
+ I*a*Tan[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/(2^(2/3)*d) + ((I/2)*a^(1/3)
*Log[Cos[c + d*x]])/(2^(2/3)*d) + (((3*I)/2)*a^(1/3)*Log[2^(1/3)*a^(1/3) -
(a + I*a*Tan[c + d*x])^(1/3)])/(2^(2/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \sqrt[3]{a+ia\tan(c+dx)}\,dx = -\frac{(ia)\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{2/3}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{\sqrt[3]{a}\,x}{2\,2^{2/3}} + \frac{i\sqrt[3]{a}\,\log(\cos(c+dx))}{2\,2^{2/3}d} - \frac{\left(3i\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}\,\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a \cdots}\right.}{2\,2^{2/3}d}$$

$$= -\frac{\sqrt[3]{a}\,x}{2\,2^{2/3}} + \frac{i\sqrt[3]{a}\,\log(\cos(c+dx))}{2\,2^{2/3}d} + \frac{3i\sqrt[3]{a}\,\log\left(\sqrt[3]{2}\,\sqrt[3]{a} - \sqrt[3]{a+ia\tan(c+\cdots}\right.}{2\,2^{2/3}d}$$

$$= -\frac{\sqrt[3]{a}\,x}{2\,2^{2/3}} - \frac{i\sqrt{3}\,\sqrt[3]{a}\,\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2^{2/3}d} + \frac{i\sqrt[3]{a}\,\log(\cos(c\cdots}{2\,2^{2/3}d\cdots}$$

Mathematica [A]
time = 1.02, size = 223, normalized size = 1.43

$$-\frac{ie^{-\frac{2}{3}i(c+dx)}\sqrt[3]{1+e^{2i(c+dx)}}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{1+\frac{2e^{\frac{2}{3}i(c+dx)}}{\sqrt[3]{1+e^{2i(c+dx)}}}}{\sqrt{3}}\right)-2\log\left(1-\frac{e^{\frac{2}{3}i(c+dx)}}{\sqrt[3]{1+e^{2i(c+dx)}}}\right)+\log\left(\frac{e^{\frac{4}{3}i(c+dx)}+e^{\frac{2}{3}i(c+dx)}\sqrt[3]{1+e^{2i(c+dx)}}+\left(1+e^{2i(c+dx)}\right)^{2/3}}{\left(1+e^{2i(c+dx)}\right)^{2/3}}\right)\right)\sqrt[3]{a+ia\tan(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(1/3),x]
```

[Out] ((-1/4*I)*(1 + E^((2*I)*(c + d*x)))^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*E^(((2*I)/3)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))^(1/3))/Sqrt[3]] - 2*Log[1 - E^(((2*I)/3)*(c + d*x))/(1 + E^((2*I)*(c + d*x)))^(1/3)] + Log[(E^(((4*I)/3)*(c + d*x)) + E^(((2*I)/3)*(c + d*x))*(1 + E^((2*I)*(c + d*x)))^(1/3) + (1 + E^((2*I)*(c + d*x)))^(2/3))/(1 + E^((2*I)*(c + d*x)))^(2/3)])*(a + I*a*Tan[c + d*x])^(1/3))/(d*E^(((2*I)/3)*(c + d*x)))

Maple [A]
time = 0.08, size = 133, normalized size = 0.85

method	result
derivativedivides	$\frac{3ia\left(\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{12a^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}\sqrt{3}\arctan\left(\sqrt{\cdots}\right.}{\cdots}\right.}{d}$
default	$\frac{3ia\left(\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{12a^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}\sqrt{3}\arctan\left(\sqrt{\cdots}\right.}{\cdots}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I*a*tan(d*x+c))^(1/3),x,method=_RETURNVERBOSE)

[Out] 3*I/d*a*(1/6*2^(1/3)/a^(2/3)*ln((a+I*a*tan(d*x+c))^(1/3)-2^(1/3)*a^(1/3))-1/12*2^(1/3)/a^(2/3)*ln((a+I*a*tan(d*x+c))^(2/3)+2^(1/3)*a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+2^(2/3)*a^(2/3))-1/6*2^(1/3)/a^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2^(2/3)/a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+1)))

Maxima [A]
time = 0.52, size = 135, normalized size = 0.87

$$-\frac{i\left(2\sqrt{3}\,2^{\frac{1}{3}}a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)+2^{\frac{1}{3}}a^{\frac{4}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(i\,a\tan(dx+c)+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{2}{3}}\right)-2\cdot2^{\frac{1}{3}}a^{\frac{4}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)\right)}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I*a*tan(d*x+c))^(1/3),x, algorithm="maxima")

[Out] -1/4*I*(2*sqrt(3)*2^(1/3)*a^(4/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) + 2^(1/3)*a^(4/3)*log(2^(2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x + c)

```
+ a)^(2/3)) - 2*2^(1/3)*a^(4/3)*log(-2^(1/3)*a^(1/3) + (I*a*tan(d*x + c) +
a)^(1/3)))/(a*d)
```

Fricas [A]
time = 0.51, size = 188, normalized size = 1.21

$$\frac{1}{2}\left(i\sqrt{3}-1\right)\left(-\frac{ia}{4d^3}\right)^{\frac{1}{3}}\log\left(2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}-\left(\sqrt{3}\,d+i\,d\right)\left(-\frac{ia}{4d^3}\right)^{\frac{1}{3}}\right)+\frac{1}{2}\left(-i\sqrt{3}-1\right)\left(-\frac{ia}{4d^3}\right)^{\frac{1}{3}}\log\left(2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}+\left(\sqrt{3}\,d-i\,d\right)\left(-\frac{ia}{4d^3}\right)^{\frac{1}{3}}\right)+\left(-\frac{ia}{4d^3}\right)^{\frac{1}{3}}\log\left(2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}+2i\,d\left(-\frac{ia}{4d^3}\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*(I*sqrt(3) - 1)*(-1/4*I*a/d^3)^(1/3)*log(2^(1/3)*(a/(e^(2*I*d*x + 2*I*c
) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) - (sqrt(3)*d + I*d)*(-1/4*I*a/d^3)^(1
/3)) + 1/2*(-I*sqrt(3) - 1)*(-1/4*I*a/d^3)^(1/3)*log(2^(1/3)*(a/(e^(2*I*d*x
 + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + (sqrt(3)*d - I*d)*(-1/4*I*a
/d^3)^(1/3)) + (-1/4*I*a/d^3)^(1/3)*log(2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1
))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + 2*I*d*(-1/4*I*a/d^3)^(1/3))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{ia\tan\left(c+dx\right)+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(1/3),x)
```

```
[Out] Integral((I*a*tan(c + d*x) + a)**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(1/3), x)
```

Mupad [B]
time = 3.97, size = 184, normalized size = 1.18

$$\frac{\left(\frac{1}{4}\mathrm{i}\right)^{1/3}(-a)^{1/3}\ln\left(18\left(\frac{1}{4}\mathrm{i}\right)^{1/3}(-a)^{4/3}d^2+a\,d^2\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{1/3}9\mathrm{i}\right)}{d}+\frac{\left(\frac{1}{4}\mathrm{i}\right)^{1/3}(-a)^{1/3}\ln\left(a\,d^2\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{1/3}9\mathrm{i}+18\left(\frac{1}{4}\mathrm{i}\right)^{1/3}(-a)^{4/3}d^2\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{d}-\frac{\left(\frac{1}{4}\mathrm{i}\right)^{1/3}(-a)^{1/3}\ln\left(a\,d^2\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{1/3}9\mathrm{i}-18\left(\frac{1}{4}\mathrm{i}\right)^{1/3}(-a)^{4/3}d^2\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(1/3),x)
```

```
[Out] ((1i/4)^(1/3)*(-a)^(1/3)*log(18*(1i/4)^(1/3)*(-a)^(4/3)*d^2 + a*d^2*(a + a*
tan(c + d*x)*1i)^(1/3)*9i))/d + ((1i/4)^(1/3)*(-a)^(1/3)*log(a*d^2*(a + a*t
an(c + d*x)*1i)^(1/3)*9i + 18*(1i/4)^(1/3)*(-a)^(4/3)*d^2*((3^(1/2)*1i)/2 -
 1/2))*((3^(1/2)*1i)/2 - 1/2))/d - ((1i/4)^(1/3)*(-a)^(1/3)*log(a*d^2*(a +
a*tan(c + d*x)*1i)^(1/3)*9i - 18*(1i/4)^(1/3)*(-a)^(4/3)*d^2*((3^(1/2)*1i)/
2 + 1/2))*((3^(1/2)*1i)/2 + 1/2))/d
```

3.276 $\int \cot(c+dx)\sqrt[3]{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=260

$$-\frac{i\sqrt[3]{a}\,x}{2\,2^{2/3}}-\frac{\sqrt{3}\,\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{d}+\frac{\sqrt{3}\,\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2^{2/3}d}$$

```
[Out] -1/4*I*a^(1/3)*x*2^(1/3)-1/4*a^(1/3)*ln(cos(d*x+c))*2^(1/3)/d-1/2*a^(1/3)*l
n(tan(d*x+c))/d+3/2*a^(1/3)*ln(a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))/d-3/4*a^(1
/3)*ln(2^(1/3)*a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))*2^(1/3)/d-a^(1/3)*arctan(1
/3*(a^(1/3)+2*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)/d+1/2*a^(1
/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))*
3^(1/2)*2^(1/3)/d
```

Rubi [A]
time = 0.21, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {3643, 3562, 59, 631, 210, 31, 3680}

$$-\frac{\sqrt{3}\,\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{d}+\frac{\sqrt{3}\,\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2^{2/3}d}-\frac{\sqrt[3]{a}\,\log(\tan(c+dx))}{2d}+\frac{3\sqrt[3]{a}\,\log\left(\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{2d}-\frac{3\sqrt[3]{a}\,\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{2\,2^{2/3}d}-\frac{\sqrt[3]{a}\,\log(\cos(c+dx))}{2\,2^{2/3}d}-\frac{i\sqrt[3]{a}\,x}{2\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^(1/3),x]
```

```
[Out] ((-1/2*I)*a^(1/3)*x)/2^(2/3) - (Sqrt[3]*a^(1/3)*ArcTan[(a^(1/3) + 2*(a + I*
a*Tan[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/d + (Sqrt[3]*a^(1/3)*ArcTan[(a^(
1/3) + 2^(2/3)*(a + I*a*Tan[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/(2^(2/3)*d
) - (a^(1/3)*Log[Cos[c + d*x]])/(2*2^(2/3)*d) - (a^(1/3)*Log[Tan[c + d*x]])
/(2*d) + (3*a^(1/3)*Log[a^(1/3) - (a + I*a*Tan[c + d*x])^(1/3)])/(2*d) - (3
*a^(1/3)*Log[2^(1/3)*a^(1/3) - (a + I*a*Tan[c + d*x])^(1/3)])/(2*2^(2/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3643

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)/((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[a/(a*c - b*d), Int[(a + b*Tan[e + f*x])^m,
x], x] - Dist[d/(a*c - b*d), Int[(a + b*Tan[e + f*x])^m*((b + a*Tan[e + f*x
])/(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rubi steps

$$\int \cot(c+dx)\sqrt[3]{a+ia\tan(c+dx)}\,dx = i\int \sqrt[3]{a+ia\tan(c+dx)}\,dx - \frac{i\int \cot(c+dx)\sqrt[3]{a+ia\tan(c+d}{a}$$

$$= \frac{a\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{2/3}}\,dx, x, ia\tan(c+dx)\right)}{d} + \frac{a\text{Subst}\left(\int \frac{1}{x(a+iax)}\right.}{}$$

$$= -\frac{i\sqrt[3]{a}\,x}{2\,2^{2/3}} - \frac{\sqrt[3]{a}\,\log(\cos(c+dx))}{2\,2^{2/3}d} - \frac{\sqrt[3]{a}\,\log(\tan(c+dx))}{2d} - \frac{(3\sqrt[3]{}}{}$$

$$= -\frac{i\sqrt[3]{a}\,x}{2\,2^{2/3}} - \frac{\sqrt[3]{a}\,\log(\cos(c+dx))}{2\,2^{2/3}d} - \frac{\sqrt[3]{a}\,\log(\tan(c+dx))}{2d} + \frac{3\sqrt[3]{a}}{}$$

$$= -\frac{i\sqrt[3]{a}\,x}{2\,2^{2/3}} - \frac{\sqrt{3}\,\sqrt[3]{a}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{d} + \frac{\sqrt{3}\,\sqrt[3]{}}{}$$

Mathematica [F]
time = 0.71, size = 0, normalized size = 0.00

$$\int \cot(c+dx)\sqrt[3]{a+ia\tan(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^(1/3),x]
[Out] Integrate[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^(1/3), x]
```

Maple [F]
time = 0.42, size = 0, normalized size = 0.00

$$\int \cot\left(dx+c\right)\left(a+ia\tan\left(dx+c\right)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+I*a*tan(d*x+c))^(1/3),x)
[Out] int(cot(d*x+c)*(a+I*a*tan(d*x+c))^(1/3),x)
```

Maxima [A]
time = 0.53, size = 233, normalized size = 0.90

$$\frac{2\sqrt{3}\,2^{\frac{1}{3}}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right) - 4\sqrt{3}\,a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right) + 2^{\frac{1}{3}}a^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(i\,a\tan(dx+c)+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{2}{3}}\right) - 2\cdot 2^{\frac{1}{3}}a^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right) - 2\,a^{\frac{1}{3}}\log\left((i\,a\tan(dx+c)+a)^{\frac{2}{3}}+(i\,a\tan(dx+c)+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right) + 4\,a^{\frac{1}{3}}\log\left((i\,a\tan(dx+c)+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(3)*2^(1/3)*a^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3)
+ 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) - 4*sqrt(3)*a^(1/3)*arctan(1/3*s
qrt(3)*(2*(I*a*tan(d*x + c) + a)^(1/3) + a^(1/3))/a^(1/3)) + 2^(1/3)*a^(1/3
)*log(2^(2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a
*tan(d*x + c) + a)^(2/3)) - 2*2^(1/3)*a^(1/3)*log(-2^(1/3)*a^(1/3) + (I*a*t
an(d*x + c) + a)^(1/3)) - 2*a^(1/3)*log((I*a*tan(d*x + c) + a)^(2/3) + (I*a
*tan(d*x + c) + a)^(1/3)*a^(1/3) + a^(2/3)) + 4*a^(1/3)*log((I*a*tan(d*x +
c) + a)^(1/3) - a^(1/3)))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 387 vs. 2(189) = 378.
time = 0.62, size = 387, normalized size = 1.49

$$\frac{1}{2} \left(\frac{1}{4}\right)^{\frac{1}{3}} \left(i \sqrt{3} - 1\right) \left(-\frac{a}{d^{3}}\right)^{\frac{1}{3}} \log\left(\left(\frac{1}{4}\right)^{\frac{1}{3}} \left(i \sqrt{3} d - d\right) \left(-\frac{a}{d^{3}}\right)^{\frac{1}{3}} + 2^{\frac{1}{3}} \left(\frac{a}{e^{\left(2 i d x + 2 i c\right)} + 1}\right)^{\frac{1}{3}} e^{\left(\frac{2}{3} i d x + \frac{2}{3} i c\right)}\right) + \frac{1}{2} \left(\frac{1}{4}\right)^{\frac{1}{3}} \left(-i \sqrt{3} - 1\right) \left(-\frac{a}{d^{3}}\right)^{\frac{1}{3}} \log\left(\left(\frac{1}{4}\right)^{\frac{1}{3}} \left(-i \sqrt{3} d - d\right) \left(-\frac{a}{d^{3}}\right)^{\frac{1}{3}} + 2^{\frac{1}{3}} \left(\frac{a}{e^{\left(2 i d x + 2 i c\right)} + 1}\right)^{\frac{1}{3}} e^{\left(\frac{2}{3} i d x + \frac{2}{3} i c\right)}\right) + \frac{1}{2} \left(-i \sqrt{3} - 1\right) \left(\frac{a}{d^{3}}\right)^{\frac{1}{3}} \log\left(2^{\frac{1}{3}} \left(\frac{a}{e^{\left(2 i d x + 2 i c\right)} + 1}\right)^{\frac{1}{3}} e^{\left(\frac{2}{3} i d x + \frac{2}{3} i c\right)} + \frac{1}{2} \left(i \sqrt{3} d + d\right) \left(\frac{a}{d^{3}}\right)^{\frac{1}{3}}\right) + \frac{1}{2} \left(i \sqrt{3} - 1\right) \left(\frac{a}{d^{3}}\right)^{\frac{1}{3}} \log\left(2^{\frac{1}{3}} \left(\frac{a}{e^{\left(2 i d x + 2 i c\right)} + 1}\right)^{\frac{1}{3}} e^{\left(\frac{2}{3} i d x + \frac{2}{3} i c\right)} + \frac{1}{2} \left(-i \sqrt{3} d + d\right) \left(\frac{a}{d^{3}}\right)^{\frac{1}{3}}\right) + \left(\frac{1}{4}\right)^{\frac{1}{3}} \left(-\frac{a}{d^{3}}\right)^{\frac{1}{3}} \log\left(2 \left(\frac{1}{4}\right)^{\frac{1}{3}} d \left(-\frac{a}{d^{3}}\right)^{\frac{1}{3}} + 2^{\frac{1}{3}} \left(\frac{a}{e^{\left(2 i d x + 2 i c\right)} + 1}\right)^{\frac{1}{3}} e^{\left(\frac{2}{3} i d x + \frac{2}{3} i c\right)}\right) + \left(\frac{a}{d^{3}}\right)^{\frac{1}{3}} \log\left(2^{\frac{1}{3}} \left(\frac{a}{e^{\left(2 i d x + 2 i c\right)} + 1}\right)^{\frac{1}{3}} e^{\left(\frac{2}{3} i d x + \frac{2}{3} i c\right)} - d \left(\frac{a}{d^{3}}\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*(1/4)^(1/3)*(I*sqrt(3) - 1)*(-a/d^3)^(1/3)*log((1/4)^(1/3)*(I*sqrt(3)*d
 - d)*(-a/d^3)^(1/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I
*d*x + 2/3*I*c)) + 1/2*(1/4)^(1/3)*(-I*sqrt(3) - 1)*(-a/d^3)^(1/3)*log((1/4
)^(1/3)*(-I*sqrt(3)*d - d)*(-a/d^3)^(1/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c)
 + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)) + 1/2*(-I*sqrt(3) - 1)*(a/d^3)^(1/3)*
log(2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + 1
/2*(I*sqrt(3)*d + d)*(a/d^3)^(1/3)) + 1/2*(I*sqrt(3) - 1)*(a/d^3)^(1/3)*log
(2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + 1/2*
(-I*sqrt(3)*d + d)*(a/d^3)^(1/3)) + (1/4)^(1/3)*(-a/d^3)^(1/3)*log(2*(1/4)^
(1/3)*d*(-a/d^3)^(1/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3
*I*d*x + 2/3*I*c)) + (a/d^3)^(1/3)*log(2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1)
)^(1/3)*e^(2/3*I*d*x + 2/3*I*c) - d*(a/d^3)^(1/3))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{ia\left(\tan\left(c + dx\right) - i\right)} \cot\left(c + dx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))**(1/3),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(1/3)*cot(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(1/3)*cot(d*x + c), x)
```

Mupad [B]
time = 4.31, size = 328, normalized size = 1.26

$$\ln\left(\left(a\,\left(1+\tan(c+d\,x)\,1\mathrm{i}\right)\right)^{1/3}-d\,\left(\frac{a}{d^3}\right)^{1/3}\right)\,\left(\frac{a}{d^3}\right)^{1/3}+\ln\left(\left(a\,\left(1+\tan(c+d\,x)\,1\mathrm{i}\right)\right)^{1/3}+2^{1/3}\,d\,\left(-\frac{a}{d^3}\right)^{1/3}\right)\,\left(-\frac{a}{4\,d^3}\right)^{1/3}-\ln\left(\frac{2^{1/3}\,d\,\left(-\frac{a}{d^3}\right)^{1/3}}{2}-\left(a\,\left(1+\tan(c+d\,x)\,1\mathrm{i}\right)\right)^{1/3}+\frac{2^{1/3}\,\sqrt{3}\,d\,\left(-\frac{a}{d^3}\right)^{1/3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-\frac{a}{4\,d^3}\right)^{1/3}+\frac{\ln\left(2\,\left(a\,\left(1+\tan(c+d\,x)\,1\mathrm{i}\right)\right)^{1/3}+d\,\left(\frac{a}{d^3}\right)^{1/3}-\sqrt{3}\,d\,\left(\frac{a}{d^3}\right)^{1/3}\,1\mathrm{i}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,\left(\frac{a}{d^3}\right)^{1/3}}{2}-\frac{\ln\left(2\,\left(a\,\left(1+\tan(c+d\,x)\,1\mathrm{i}\right)\right)^{1/3}+d\,\left(\frac{a}{d^3}\right)^{1/3}+\sqrt{3}\,d\,\left(\frac{a}{d^3}\right)^{1/3}\,1\mathrm{i}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,\left(\frac{a}{d^3}\right)^{1/3}}{2}+\ln\left(\left(a\,\left(1+\tan(c+d\,x)\,1\mathrm{i}\right)\right)^{1/3}-\frac{2^{1/3}\,d\,\left(-\frac{a}{d^3}\right)^{1/3}}{2}+\frac{2^{1/3}\,\sqrt{3}\,d\,\left(-\frac{a}{d^3}\right)^{1/3}\,1\mathrm{i}}{2}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-\frac{a}{4\,d^3}\right)^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + a*tan(c + d*x)*1i)^(1/3),x)
```

```
[Out] log((a*(tan(c + d*x)*1i + 1))^(1/3) - d*(a/d^3)^(1/3))*(a/d^3)^(1/3) + log(
(a*(tan(c + d*x)*1i + 1))^(1/3) + 2^(1/3)*d*(-a/d^3)^(1/3))*(-a/(4*d^3))^(1
/3) - log((2^(1/3)*d*(-a/d^3)^(1/3))/2 - (a*(tan(c + d*x)*1i + 1))^(1/3) +
(2^(1/3)*3^(1/2)*d*(-a/d^3)^(1/3)*1i)/2)*((3^(1/2)*1i)/2 + 1/2)*(-a/(4*d^3)
)^(1/3) + (log(2*(a*(tan(c + d*x)*1i + 1))^(1/3) + d*(a/d^3)^(1/3) - 3^(1/2
)*d*(a/d^3)^(1/3)*1i)*(3^(1/2)*1i - 1)*(a/d^3)^(1/3))/2 - (log(2*(a*(tan(c
+ d*x)*1i + 1))^(1/3) + d*(a/d^3)^(1/3) + 3^(1/2)*d*(a/d^3)^(1/3)*1i)*(3^(1
/2)*1i + 1)*(a/d^3)^(1/3))/2 + log((a*(tan(c + d*x)*1i + 1))^(1/3) - (2^(1/
3)*d*(-a/d^3)^(1/3))/2 + (2^(1/3)*3^(1/2)*d*(-a/d^3)^(1/3)*1i)/2)*((3^(1/2)
*1i)/2 - 1/2)*(-a/(4*d^3))^(1/3)
```

3.277 $\int \cot^2(c+dx)\sqrt[3]{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=299

$$\frac{\sqrt[3]{a}\,x}{2\ 2^{2/3}}-\frac{i\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,d}+\frac{i\sqrt{3}\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{2^{2/3}d}-$$

```
[Out] 1/4*a^(1/3)*x*2^(1/3)-1/4*I*a^(1/3)*ln(cos(d*x+c))*2^(1/3)/d-1/6*I*a^(1/3)*
ln(tan(d*x+c))/d+1/2*I*a^(1/3)*ln(a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))/d-3/4*I
*a^(1/3)*ln(2^(1/3)*a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))*2^(1/3)/d-1/3*I*a^(1/
3)*arctan(1/3*(a^(1/3)+2*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))/d*3^(1/
2)+1/2*I*a^(1/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))/a^(1
/3)*3^(1/2))*3^(1/2)*2^(1/3)/d-cot(d*x+c)*(a+I*a*tan(d*x+c))^(1/3)/d
```

Rubi [A]
time = 0.29, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3642, 3681, 3562, 59, 631, 210, 31, 3680}

$$-\frac{i\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,d}+\frac{i\sqrt{3}\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{2^{2/3}d}-\frac{i\sqrt[3]{a}\log(\tan(c+dx))}{6d}+\frac{i\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{2d}-\frac{3i\sqrt[3]{a}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{2\ 2^{2/3}d}-\frac{i\sqrt[3]{a}\log(\cos(c+dx))}{2\ 2^{2/3}d}-\frac{\cot(c+dx)\sqrt[3]{a+ia\tan(c+dx)}}{d}+\frac{\sqrt[3]{a}\,x}{2\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^(1/3),x]
```

```
[Out] (a^(1/3)*x)/(2*2^(2/3)) - (I*a^(1/3)*ArcTan[(a^(1/3) + 2*(a + I*a*Tan[c + d
*x])^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*d) + (I*Sqrt[3]*a^(1/3)*ArcTan[(a^
(1/3) + 2^(2/3)*(a + I*a*Tan[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/(2^(2/3)*
d) - ((I/2)*a^(1/3)*Log[Cos[c + d*x]])/(2^(2/3)*d) - ((I/6)*a^(1/3)*Log[Tan
[c + d*x]])/d + ((I/2)*a^(1/3)*Log[a^(1/3) - (a + I*a*Tan[c + d*x])^(1/3)])
/d - (((3*I)/2)*a^(1/3)*Log[2^(1/3)*a^(1/3) - (a + I*a*Tan[c + d*x])^(1/3)]
)/(2^(2/3)*d) - (Cot[c + d*x]*(a + I*a*Tan[c + d*x])^(1/3))/d
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3642

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[d*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(a*(c^2 + d^2)*(n +
1)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[b*d*m - a
*c*(n + 1) + a*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
 e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0
] && LtQ[n, -1] && (IntegerQ[n] || IntegersQ[2*m, 2*n])
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3681

```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(
A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \cot^2(c+dx)\sqrt[3]{a+ia\tan(c+dx)}\,dx = -\frac{\cot(c+dx)\sqrt[3]{a+ia\tan(c+dx)}}{d}+\frac{\int \cot(c+dx)\left(\frac{ia}{3}-\frac{2}{3}a\tan(\right.}{}$$

$$= -\frac{\cot(c+dx)\sqrt[3]{a+ia\tan(c+dx)}}{d}+\frac{i\int \cot(c+dx)(a-ia\tan(}{}$$

$$= -\frac{\cot(c+dx)\sqrt[3]{a+ia\tan(c+dx)}}{d}+\frac{(ia)\text{Subst}\left(\int \frac{1}{x(a+iax)^{2/3}}\,dx,\right.}{3d}$$

$$= \frac{\sqrt[3]{a}\,x}{2\ 2^{2/3}}-\frac{i\sqrt[3]{a}\,\log(\cos(c+dx))}{2\ 2^{2/3}d}-\frac{i\sqrt[3]{a}\,\log(\tan(c+dx))}{6d}-\frac{\cot(c}{}$$

$$= \frac{\sqrt[3]{a}\,x}{2\ 2^{2/3}}-\frac{i\sqrt[3]{a}\,\log(\cos(c+dx))}{2\ 2^{2/3}d}-\frac{i\sqrt[3]{a}\,\log(\tan(c+dx))}{6d}+\frac{i\sqrt[3]{a}\ \mathrm{l}}{}$$

$$= \frac{\sqrt[3]{a}\,x}{2\ 2^{2/3}}-\frac{i\sqrt[3]{a}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,d}+\frac{i\sqrt{3}\,\sqrt[3]{a}\ \mathrm{ta}}{}$$

Mathematica [F]
time = 180.01, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^(1/3),x]
[Out] $Aborted
```

Maple [F]
time = 0.35, size = 0, normalized size = 0.00

$$\int \left(\cot^2(dx+c)\right)(a+ia\tan(dx+c))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/3),x)
[Out] int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/3),x)
```

Maxima [**A**]
time = 0.53, size = 261, normalized size = 0.87

$$\frac{i\left(\frac{6\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}-\frac{4\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}+\frac{3\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(i\,a\tan(dx+c)+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{2}{3}}\right)}{a^{\frac{2}{3}}}-\frac{6\cdot 2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)}{a^{\frac{2}{3}}}-\frac{2\log\left((i\,a\tan(dx+c)+a)^{\frac{2}{3}}+(i\,a\tan(dx+c)+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{2}{3}}}+\frac{4\log\left((i\,a\tan(dx+c)+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{2}{3}}}+\frac{12i\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}}{a\tan(dx+c)}\right)a}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] 1/12*I*(6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(
I*a*tan(d*x + c) + a)^(1/3))/a^(1/3))/a^(2/3) - 4*sqrt(3)*arctan(1/3*sqrt(3
)*(2*(I*a*tan(d*x + c) + a)^(1/3) + a^(1/3))/a^(1/3))/a^(2/3) + 3*2^(1/3)*l
og(2^(2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*ta
n(d*x + c) + a)^(2/3))/a^(2/3) - 6*2^(1/3)*log(-2^(1/3)*a^(1/3) + (I*a*tan(
d*x + c) + a)^(1/3))/a^(2/3) - 2*log((I*a*tan(d*x + c) + a)^(2/3) + (I*a*ta
n(d*x + c) + a)^(1/3)*a^(1/3) + a^(2/3))/a^(2/3) + 4*log((I*a*tan(d*x + c)
+ a)^(1/3) - a^(1/3))/a^(2/3) + 12*I*(I*a*tan(d*x + c) + a)^(1/3)/(a*tan(d*
x + c)))*a/d
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 555 vs. 2(213) = 426.
time = 0.57, size = 555, normalized size = 1.86

$$-\frac{2\cdot 2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}\left(i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}-\left(\left(i\sqrt{3}\,d-d\right)e^{(2i\,dx+2i\,c)}-i\sqrt{3}\,d+d\right)\left(\frac{i\,a}{4\,d^{3}}\right)^{\frac{1}{3}}\log\left(2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}+\left(\sqrt{3}\,d+i\,d\right)\left(\frac{i\,a}{4\,d^{3}}\right)^{\frac{1}{3}}\right)-\left(\left(-i\sqrt{3}\,d-d\right)e^{(2i\,dx+2i\,c)}+i\sqrt{3}\,d+d\right)\left(\frac{i\,a}{4\,d^{3}}\right)^{\frac{1}{3}}\log\left(2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}-\left(\sqrt{3}\,d-i\,d\right)\left(\frac{i\,a}{4\,d^{3}}\right)^{\frac{1}{3}}\right)-2\left(d e^{(2i\,dx+2i\,c)}-d\right)\left(\frac{i\,a}{4\,d^{3}}\right)^{\frac{1}{3}}\log\left(2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}-2i\,d\left(\frac{i\,a}{4\,d^{3}}\right)^{\frac{1}{3}}\right)-\left(\left(i\sqrt{3}\,d-d\right)e^{(2i\,dx+2i\,c)}-i\sqrt{3}\,d+d\right)\left(-\frac{i\,a}{27\,d^{3}}\right)^{\frac{1}{3}}\log\left(2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}-\frac{3}{2}\left(\sqrt{3}\,d+i\,d\right)\left(-\frac{i\,a}{27\,d^{3}}\right)^{\frac{1}{3}}\right)-\left(\left(-i\sqrt{3}\,d-d\right)e^{(2i\,dx+2i\,c)}+i\sqrt{3}\,d+d\right)\left(-\frac{i\,a}{27\,d^{3}}\right)^{\frac{1}{3}}\log\left(2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}+\frac{3}{2}\left(\sqrt{3}\,d-i\,d\right)\left(-\frac{i\,a}{27\,d^{3}}\right)^{\frac{1}{3}}\right)-2\left(d e^{(2i\,dx+2i\,c)}-d\right)\left(-\frac{i\,a}{27\,d^{3}}\right)^{\frac{1}{3}}\log\left(2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}+3i\,d\left(-\frac{i\,a}{27\,d^{3}}\right)^{\frac{1}{3}}\right)}{2\left(d e^{(2i\,dx+2i\,c)}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*(I*e^(2*I*d*x + 2*I*c)
+ I)*e^(2/3*I*d*x + 2/3*I*c) - ((I*sqrt(3)*d - d)*e^(2*I*d*x + 2*I*c) - I*s
qrt(3)*d + d)*(1/4*I*a/d^3)^(1/3)*log(2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))
^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + (sqrt(3)*d + I*d)*(1/4*I*a/d^3)^(1/3)) - (
(-I*sqrt(3)*d - d)*e^(2*I*d*x + 2*I*c) + I*sqrt(3)*d + d)*(1/4*I*a/d^3)^(1/
3)*log(2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)
- (sqrt(3)*d - I*d)*(1/4*I*a/d^3)^(1/3)) - 2*(d*e^(2*I*d*x + 2*I*c) - d)*(1
/4*I*a/d^3)^(1/3)*log(2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*
d*x + 2/3*I*c) - 2*I*d*(1/4*I*a/d^3)^(1/3)) - ((I*sqrt(3)*d - d)*e^(2*I*d*x
 + 2*I*c) - I*sqrt(3)*d + d)*(-1/27*I*a/d^3)^(1/3)*log(2^(1/3)*(a/(e^(2*I*d
*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) - 3/2*(sqrt(3)*d + I*d)*(-1
/27*I*a/d^3)^(1/3)) - ((-I*sqrt(3)*d - d)*e^(2*I*d*x + 2*I*c) + I*sqrt(3)*d
 + d)*(-1/27*I*a/d^3)^(1/3)*log(2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)
*e^(2/3*I*d*x + 2/3*I*c) + 3/2*(sqrt(3)*d - I*d)*(-1/27*I*a/d^3)^(1/3)) - 2
*(d*e^(2*I*d*x + 2*I*c) - d)*(-1/27*I*a/d^3)^(1/3)*log(2^(1/3)*(a/(e^(2*I*d
*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + 3*I*d*(-1/27*I*a/d^3)^(1/
3)))/(d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{ia\left(\tan\left(c+dx\right)-i\right)}\,\cot^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**2*(a+I*a*tan(d*x+c))**(1/3),x)

[Out] Integral((I*a*(tan(c + d*x) - I))**(1/3)*cot(c + d*x)**2, x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/3),x, algorithm="giac")

[Out] integrate((I*a*tan(d*x + c) + a)^(1/3)*cot(d*x + c)^2, x)

Mupad [B]
time = 4.20, size = 806, normalized size = 2.70

$$\ln\left(\left(\left(a^7 d^5 \left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{1/3} 81\mathrm{i}-1458\,a^7 d^6 \left(\frac{a\,1\mathrm{i}}{4\,d^3}\right)^{1/3}\right)\left(\frac{a\,1\mathrm{i}}{4\,d^3}\right)^{2/3}+a^8 d^3\,225\mathrm{i}\right)\left(\frac{a\,1\mathrm{i}}{4\,d^3}\right)^{1/3}+90\,a^8 d^2 \left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{1/3}\right)\left(\frac{a\,1\mathrm{i}}{4\,d^3}\right)^{1/3}+\ln\left(\left(\left(a^7 d^5 \left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{1/3} 81\mathrm{i}-1458\,a^7 d^6 \left(-\frac{a\,1\mathrm{i}}{27\,d^3}\right)^{1/3}\right)\left(-\frac{a\,1\mathrm{i}}{27\,d^3}\right)^{2/3}+a^8 d^3\,225\mathrm{i}\right)\left(-\frac{a\,1\mathrm{i}}{27\,d^3}\right)^{1/3}+90\,a^8 d^2 \left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{1/3}\right)\left(-\frac{a\,1\mathrm{i}}{27\,d^3}\right)^{1/3}+\ldots$$

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(1/3),x)

[Out] log(((a^7*d^5*(a + a*tan(c + d*x)*1i)^(1/3)*81i - 1458*a^7*d^6*((a*1i)/(4*d^3))^(1/3))*((a*1i)/(4*d^3))^(2/3) + a^8*d^3*225i)*((a*1i)/(4*d^3))^(1/3) + 90*a^8*d^2*(a + a*tan(c + d*x)*1i)^(1/3))*((a*1i)/(4*d^3))^(1/3) + log(((a^7*d^5*(a + a*tan(c + d*x)*1i)^(1/3)*81i - 1458*a^7*d^6*(-(a*1i)/(27*d^3))^(1/3))*(-(a*1i)/(27*d^3))^(2/3) + a^8*d^3*225i)*(-(a*1i)/(27*d^3))^(1/3) + 90*a^8*d^2*(a + a*tan(c + d*x)*1i)^(1/3))*(-(a*1i)/(27*d^3))^(1/3) + (log(90*a^8*d^2*(a + a*tan(c + d*x)*1i)^(1/3) + ((3^(1/2)*1i - 1)*(a^8*d^3*225i + ((3^(1/2)*1i - 1)^2*(a^7*d^5*(a + a*tan(c + d*x)*1i)^(1/3)*81i - 729*a^7*d^6*(3^(1/2)*1i - 1)*((a*1i)/(4*d^3))^(1/3))*((a*1i)/(4*d^3))^(2/3))/4)*((a*1i)/(4*d^3))^(1/3))/2)*(3^(1/2)*1i - 1)*((a*1i)/(4*d^3))^(1/3))/2 - (log(90*a^8*d^2*(a + a*tan(c + d*x)*1i)^(1/3) - ((3^(1/2)*1i + 1)*(a^8*d^3*225i + ((3^(1/2)*1i + 1)^2*(a^7*d^5*(a + a*tan(c + d*x)*1i)^(1/3)*81i + 729*a^7*d^6*(3^(1/2)*1i + 1)*((a*1i)/(4*d^3))^(1/3))*((a*1i)/(4*d^3))^(2/3))/4)*((a*1i)/(4*d^3))^(1/3))/2)*(3^(1/2)*1i + 1)*((a*1i)/(4*d^3))^(1/3))/2 + (log(90*a^8*d^2*(a + a*tan(c + d*x)*1i)^(1/3) + ((3^(1/2)*1i - 1)*(a^8*d^3*225i + ((3^(1/2)*1i - 1)^2*(a^7*d^5*(a + a*tan(c + d*x)*1i)^(1/3)*81i - 729*a^7*d^6*(3^(1/2)*1i - 1)*(-(a*1i)/(27*d^3))^(1/3))*(-(a*1i)/(27*d^3))^(2/3))/4)*(-

```
(a*1i)/(27*d^3))^(1/3))/2)*(3^(1/2)*1i - 1)*(-(a*1i)/(27*d^3))^(1/3))/2 - (
log(90*a^8*d^2*(a + a*tan(c + d*x)*1i)^(1/3) - ((3^(1/2)*1i + 1)*(a^8*d^3*2
25i + ((3^(1/2)*1i + 1)^2*(a^7*d^5*(a + a*tan(c + d*x)*1i)^(1/3)*81i + 729*
a^7*d^6*(3^(1/2)*1i + 1)*(-(a*1i)/(27*d^3))^(1/3))*(-(a*1i)/(27*d^3))^(2/3)
)/4)*(-(a*1i)/(27*d^3))^(1/3))/2)*(3^(1/2)*1i + 1)*(-(a*1i)/(27*d^3))^(1/3)
)/2 - (a + a*tan(c + d*x)*1i)^(1/3)/(d*tan(c + d*x))
```

3.278 $\int \cot^3(c+dx)\sqrt[3]{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=327

$$\frac{i\sqrt[3]{a}\,x}{2\ 2^{2/3}}+\frac{8\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,d}-\frac{\sqrt{3}\,\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2^{2/3}d}+$$

```
[Out] 1/4*I*a^(1/3)*x*2^(1/3)+1/4*a^(1/3)*ln(cos(d*x+c))*2^(1/3)/d+4/9*a^(1/3)*ln
(tan(d*x+c))/d-4/3*a^(1/3)*ln(a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))/d+3/4*a^(1/
3)*ln(2^(1/3)*a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))*2^(1/3)/d+8/9*a^(1/3)*arcta
n(1/3*(a^(1/3)+2*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)/d-1/2*a
^(1/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2
))*3^(1/2)*2^(1/3)/d-1/6*I*cot(d*x+c)*(a+I*a*tan(d*x+c))^(1/3)/d-1/2*cot(d*
x+c)^2*(a+I*a*tan(d*x+c))^(1/3)/d
```

Rubi [A]
time = 0.39, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3642, 3679, 3681, 3562, 59, 631, 210, 31, 3680}

$$\frac{8\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,d}-\frac{\sqrt{3}\,\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2^{2/3}d}+\frac{4\sqrt[3]{a}\,\log(\tan(c+dx))}{9d}-\frac{4\sqrt[3]{a}\,\log\left(\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{3d}+\frac{3\sqrt[3]{a}\,\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{2\ 2^{2/3}d}+\frac{\sqrt[3]{a}\,\log(\cos(c+dx))}{2\ 2^{2/3}d}-\frac{\cot^2(c+dx)\sqrt[3]{a+ia\tan(c+dx)}}{2d}-\frac{i\cot(c+dx)\sqrt[3]{a+ia\tan(c+dx)}}{6d}+\frac{i\sqrt[3]{a}\,x}{2\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + I*a*Tan[c + d*x])^(1/3),x]
```

```
[Out] ((I/2)*a^(1/3)*x)/2^(2/3) + (8*a^(1/3)*ArcTan[(a^(1/3) + 2*(a + I*a*Tan[c +
 d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*d) - (Sqrt[3]*a^(1/3)*ArcTan[(
a^(1/3) + 2^(2/3)*(a + I*a*Tan[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/(2^(2/3
)*d) + (a^(1/3)*Log[Cos[c + d*x]])/(2*2^(2/3)*d) + (4*a^(1/3)*Log[Tan[c + d
*x]])/(9*d) - (4*a^(1/3)*Log[a^(1/3) - (a + I*a*Tan[c + d*x])^(1/3)])/(3*d)
 + (3*a^(1/3)*Log[2^(1/3)*a^(1/3) - (a + I*a*Tan[c + d*x])^(1/3)])/(2*2^(2/
3)*d) - ((I/6)*Cot[c + d*x]*(a + I*a*Tan[c + d*x])^(1/3))/d - (Cot[c + d*x]
^2*(a + I*a*Tan[c + d*x])^(1/3))/(2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
```

```
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3642

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[d*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(a*(c^2 + d^2)*(n +
1)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[b*d*m - a
*c*(n + 1) + a*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
 e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0
] && LtQ[n, -1] && (IntegerQ[n] || IntegersQ[2*m, 2*n])
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
```

```
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3681

```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(
A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \cot^3(c+dx)\sqrt[3]{a+ia\tan(c+dx)}\,dx = -\frac{\cot^2(c+dx)\sqrt[3]{a+ia\tan(c+dx)}}{2d} + \frac{\int \cot^2(c+dx)\left(\frac{ia}{3}-\frac{5}{3}a\tan\right.}{}$$

$$= -\frac{i\cot(c+dx)\sqrt[3]{a+ia\tan(c+dx)}}{6d} - \frac{\cot^2(c+dx)\sqrt[3]{a+ia\tan(}}{2d}$$

$$= -\frac{i\cot(c+dx)\sqrt[3]{a+ia\tan(c+dx)}}{6d} - \frac{\cot^2(c+dx)\sqrt[3]{a+ia\tan(}}{2d}$$

$$= -\frac{i\cot(c+dx)\sqrt[3]{a+ia\tan(c+dx)}}{6d} - \frac{\cot^2(c+dx)\sqrt[3]{a+ia\tan(}}{2d}$$

$$= \frac{i\sqrt[3]{a}\,x}{2\,2^{2/3}} + \frac{\sqrt[3]{a}\,\log(\cos(c+dx))}{2\,2^{2/3}d} + \frac{4\sqrt[3]{a}\,\log(\tan(c+dx))}{9d} - \frac{i\cot(}{}$$

$$= \frac{i\sqrt[3]{a}\,x}{2\,2^{2/3}} + \frac{\sqrt[3]{a}\,\log(\cos(c+dx))}{2\,2^{2/3}d} + \frac{4\sqrt[3]{a}\,\log(\tan(c+dx))}{9d} - \frac{4\sqrt[3]{a}}{}$$

$$= \frac{i\sqrt[3]{a}\,x}{2\,2^{2/3}} + \frac{8\sqrt[3]{a}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,d} - \frac{\sqrt{3}\sqrt[3]{a}\,\mathrm{ta}}{}$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[Cot[c + d*x]^3*(a + I*a*Tan[c + d*x])^(1/3),x]
```

```
[Out] $Aborted
```

Maple [F]
time = 0.40, size = 0, normalized size = 0.00

$$\int \left(\cot^{3}\left(dx+c\right)\right)\left(a+ia\tan\left(dx+c\right)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/3),x)
```

```
[Out] int(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/3),x)
```

Maxima [A]
time = 0.53, size = 306, normalized size = 0.94

$$-\frac{a^{2}\left(\frac{18\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(ia\tan\left(dx+c\right)+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{a^{\frac{5}{3}}}-\frac{6\left(\left(ia\tan\left(dx+c\right)+a\right)^{\frac{4}{3}}+2\left(ia\tan\left(dx+c\right)+a\right)^{\frac{1}{3}}a\right)}{\left(ia\tan\left(dx+c\right)+a\right)^{2}a-2\left(ia\tan\left(dx+c\right)+a\right)a^{2}+a^{3}}-\frac{32\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(ia\tan\left(dx+c\right)+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{5}{3}}}+\frac{9\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(ia\tan\left(dx+c\right)+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(ia\tan\left(dx+c\right)+a\right)^{\frac{2}{3}}\right)}{a^{\frac{5}{3}}}-\frac{18\cdot 2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(ia\tan\left(dx+c\right)+a\right)^{\frac{1}{3}}\right)}{a^{\frac{5}{3}}}-\frac{16\log\left(\left(ia\tan\left(dx+c\right)+a\right)^{\frac{2}{3}}+\left(ia\tan\left(dx+c\right)+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{5}{3}}}+\frac{32\log\left(\left(ia\tan\left(dx+c\right)+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{5}{3}}}\right)}{36d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] -1/36*a^2*(18*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) +
 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3))/a^(5/3) - 6*((I*a*tan(d*x + c) +
a)^(4/3) + 2*(I*a*tan(d*x + c) + a)^(1/3)*a)/((I*a*tan(d*x + c) + a)^2*a -
2*(I*a*tan(d*x + c) + a)*a^2 + a^3) - 32*sqrt(3)*arctan(1/3*sqrt(3)*(2*(I*a
*tan(d*x + c) + a)^(1/3) + a^(1/3))/a^(1/3))/a^(5/3) + 9*2^(1/3)*log(2^(2/3
)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x + c
) + a)^(2/3))/a^(5/3) - 18*2^(1/3)*log(-2^(1/3)*a^(1/3) + (I*a*tan(d*x + c)
 + a)^(1/3))/a^(5/3) - 16*log((I*a*tan(d*x + c) + a)^(2/3) + (I*a*tan(d*x +
 c) + a)^(1/3)*a^(1/3) + a^(2/3))/a^(5/3) + 32*log((I*a*tan(d*x + c) + a)^(
1/3) - a^(1/3))/a^(5/3))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 678 vs. 2(239) = 478.
time = 0.60, size = 678, normalized size = 2.07

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/18*(6*2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*(2*e^(4*I*d*x + 4*I*c)
+ 3*e^(2*I*d*x + 2*I*c) + 1)*e^(2/3*I*d*x + 2/3*I*c) - 9*(1/4)^(1/3)*((I*sq
```

```
rt(3)*d + d)*e^(4*I*d*x + 4*I*c) + 2*(-I*sqrt(3)*d - d)*e^(2*I*d*x + 2*I*c)
 + I*sqrt(3)*d + d)*(a/d^3)^(1/3)*log((1/4)^(1/3)*(I*sqrt(3)*d + d)*(a/d^3)
^(1/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c
)) - 9*(1/4)^(1/3)*((-I*sqrt(3)*d + d)*e^(4*I*d*x + 4*I*c) + 2*(I*sqrt(3)*d
 - d)*e^(2*I*d*x + 2*I*c) - I*sqrt(3)*d + d)*(a/d^3)^(1/3)*log((1/4)^(1/3)*
(-I*sqrt(3)*d + d)*(a/d^3)^(1/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1
/3)*e^(2/3*I*d*x + 2/3*I*c)) + 18*(1/4)^(1/3)*(d*e^(4*I*d*x + 4*I*c) - 2*d*
e^(2*I*d*x + 2*I*c) + d)*(a/d^3)^(1/3)*log(-2*(1/4)^(1/3)*d*(a/d^3)^(1/3) +
 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)) - 8*(
(I*sqrt(3)*d + d)*e^(4*I*d*x + 4*I*c) + 2*(-I*sqrt(3)*d - d)*e^(2*I*d*x + 2
*I*c) + I*sqrt(3)*d + d)*(-a/d^3)^(1/3)*log(2^(1/3)*(a/(e^(2*I*d*x + 2*I*c)
 + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) - 1/2*(I*sqrt(3)*d + d)*(-a/d^3)^(1/3)
) - 8*((-I*sqrt(3)*d + d)*e^(4*I*d*x + 4*I*c) + 2*(I*sqrt(3)*d - d)*e^(2*I*
d*x + 2*I*c) - I*sqrt(3)*d + d)*(-a/d^3)^(1/3)*log(2^(1/3)*(a/(e^(2*I*d*x +
 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) - 1/2*(-I*sqrt(3)*d + d)*(-a/d^
3)^(1/3)) + 16*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*(-a/d^
3)^(1/3)*log(2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3
*I*c) + d*(-a/d^3)^(1/3)))/(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c)
 + d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{ia\left(\tan\left(c+dx\right)-i\right)}\,\cot^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+I*a*tan(d*x+c))**(1/3),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(1/3)*cot(c + d*x)**3, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Not invertible Error: Bad Argument Va
lue
```

Mupad [**B**]
time = 4.41, size = 417, normalized size = 1.28

$$\frac{8\ln\left(\left(a\left(1+\tan(c+d\,x)\ 1\mathrm{i}\right)\right)^{1/3}+d\left(-\frac{a}{d^3}\right)^{1/3}\right)\left(-\frac{a}{d^3}\right)^{1/3}}{9}+\ln\left(\left(a\left(1+\tan(c+d\,x)\ 1\mathrm{i}\right)\right)^{1/3}-2^{1/3}\,d\left(\frac{a}{d^3}\right)^{1/3}\right)\left(\frac{a}{4\,d^3}\right)^{1/3}+\frac{\frac{a\,{\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)}^{4/3}}{6}+\frac{a^2\,{\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)}^{1/3}}{3}}{d\,{\left(a+a\tan\left(c+d\,x\right)\ 1\mathrm{i}\right)}^2+a^2\,d-2\,a\,d\,\left(a+a\tan\left(c+d\,x\right)\ 1\mathrm{i}\right)}-\frac{4\ln\left(d\left(-\frac{a}{d^3}\right)^{1/3}-2\left(a\left(1+\tan(c+d\,x)\ 1\mathrm{i}\right)\right)^{1/3}+\sqrt{3}\,d\left(-\frac{a}{d^3}\right)^{1/3}1\mathrm{i}\right)\left(1+\sqrt{3}\ 1\mathrm{i}\right)\left(-\frac{a}{d^3}\right)^{1/3}}{9}+\frac{4\ln\left(2\left(a\left(1+\tan(c+d\,x)\ 1\mathrm{i}\right)\right)^{1/3}-d\left(-\frac{a}{d^3}\right)^{1/3}+\sqrt{3}\,d\left(-\frac{a}{d^3}\right)^{1/3}1\mathrm{i}\right)\left(-1+\sqrt{3}\ 1\mathrm{i}\right)\left(-\frac{a}{d^3}\right)^{1/3}}{9}+\ln\left(\left(a\left(1+\tan(c+d\,x)\ 1\mathrm{i}\right)\right)^{1/3}+\frac{2^{1/3}\,d\left(\frac{a}{d^3}\right)^{1/3}}{2}-\frac{2^{1/3}\,\sqrt{3}\,d\left(\frac{a}{d^3}\right)^{1/3}1\mathrm{i}}{2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)\left(\frac{a}{4\,d^3}\right)^{1/3}-\ln\left(\left(a\left(1+\tan(c+d\,x)\ 1\mathrm{i}\right)\right)^{1/3}+\frac{2^{1/3}\,d\left(\frac{a}{d^3}\right)^{1/3}}{2}+\frac{2^{1/3}\,\sqrt{3}\,d\left(\frac{a}{d^3}\right)^{1/3}1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)\left(\frac{a}{4\,d^3}\right)^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(1/3),x)
```

```
[Out] (8*log((a*(tan(c + d*x)*1i + 1))^(1/3) + d*(-a/d^3)^(1/3))*(-a/d^3)^(1/3))/
9 + log((a*(tan(c + d*x)*1i + 1))^(1/3) - 2^(1/3)*d*(a/d^3)^(1/3))*(a/(4*d^
3))^(1/3) + ((a*(a + a*tan(c + d*x)*1i)^(4/3))/6 + (a^2*(a + a*tan(c + d*x)
*1i)^(1/3))/3)/(d*(a + a*tan(c + d*x)*1i)^2 + a^2*d - 2*a*d*(a + a*tan(c +
d*x)*1i)) - (4*log(d*(-a/d^3)^(1/3) - 2*(a*(tan(c + d*x)*1i + 1))^(1/3) + 3
^(1/2)*d*(-a/d^3)^(1/3)*1i)*(3^(1/2)*1i + 1)*(-a/d^3)^(1/3))/9 + (4*log(2*(
a*(tan(c + d*x)*1i + 1))^(1/3) - d*(-a/d^3)^(1/3) + 3^(1/2)*d*(-a/d^3)^(1/3
)*1i)*(3^(1/2)*1i - 1)*(-a/d^3)^(1/3))/9 + log((a*(tan(c + d*x)*1i + 1))^(1
/3) + (2^(1/3)*d*(a/d^3)^(1/3))/2 - (2^(1/3)*3^(1/2)*d*(a/d^3)^(1/3)*1i)/2)
*((3^(1/2)*1i)/2 - 1/2)*(a/(4*d^3))^(1/3) - log((a*(tan(c + d*x)*1i + 1))^(
1/3) + (2^(1/3)*d*(a/d^3)^(1/3))/2 + (2^(1/3)*3^(1/2)*d*(a/d^3)^(1/3)*1i)/2
)*((3^(1/2)*1i)/2 + 1/2)*(a/(4*d^3))^(1/3)
```

3.279 $\int (a+ia\tan(c+dx))^{2/3}\,dx$

Optimal. Leaf size=156

$$-\frac{a^{2/3}x}{2\sqrt[3]{2}}+\frac{i\sqrt{3}\,a^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt[3]{2}\,d}+\frac{ia^{2/3}\log(\cos(c+dx))}{2\sqrt[3]{2}\,d}+\frac{3ia^{2/3}\log\left(\sqrt[3]{2}\,\sqrt[3]{a}\,-\right.}{2\sqrt[3]{}}$$

```
[Out] -1/4*a^(2/3)*x*2^(2/3)+1/4*I*a^(2/3)*ln(cos(d*x+c))*2^(2/3)/d+3/4*I*a^(2/3)
*ln(2^(1/3)*a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))*2^(2/3)/d+1/2*I*a^(2/3)*arcta
n(1/3*(a^(1/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)*2
^(2/3)/d
```

Rubi [A]
time = 0.06, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3562, 57, 631, 210, 31}

$$\frac{i\sqrt{3}\,a^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt[3]{2}\,d}+\frac{3ia^{2/3}\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{2\sqrt[3]{2}\,d}+\frac{ia^{2/3}\log(\cos(c+dx))}{2\sqrt[3]{2}\,d}-\frac{a^{2/3}x}{2\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(2/3),x]
```

```
[Out] -1/2*(a^(2/3)*x)/2^(1/3) + (I*Sqrt[3]*a^(2/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a
+ I*a*Tan[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/(2^(1/3)*d) + ((I/2)*a^(2/3)
*Log[Cos[c + d*x]])/(2^(1/3)*d) + (((3*I)/2)*a^(2/3)*Log[2^(1/3)*a^(1/3) -
(a + I*a*Tan[c + d*x])^(1/3)])/(2^(1/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a+ia\tan(c+dx))^{2/3}\,dx = -\frac{(ia)\mathrm{Subst}\left(\int \frac{1}{(a-x)\sqrt[3]{a+x}}\,dx,x,ia\tan(c+dx)\right)}{d}$$

$$= -\frac{a^{2/3}x}{2\sqrt[3]{2}}+\frac{ia^{2/3}\log(\cos(c+dx))}{2\sqrt[3]{2}\,d}-\frac{\left(3ia^{2/3}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a-\cdots}\right.}{2\sqrt[3]{2}\,d}$$

$$= -\frac{a^{2/3}x}{2\sqrt[3]{2}}+\frac{ia^{2/3}\log(\cos(c+dx))}{2\sqrt[3]{2}\,d}+\frac{3ia^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+\cdots}\right.}{2\sqrt[3]{2}\,d}$$

$$= -\frac{a^{2/3}x}{2\sqrt[3]{2}}+\frac{i\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,d}+\frac{ia^{2/3}\log(\cos(c\cdots}{2\sqrt[3]{2}\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.41, size = 81, normalized size = 0.52

$$-\frac{3i\left(\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{2/3}{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)}{2\sqrt[3]{2}\,d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + I*a*Tan[c + d*x])^(2/3),x]

[Out] (((-3*I)/2)*((a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, E^((2*I)*(c + d*x))/(1 + E^((2*I)*(c + d*x)))])/(2^(1/3)*d)

Maple [A]
time = 0.08, size = 133, normalized size = 0.85

method	result
derivativedivides	$\frac{3ia\left(\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{12a^{\frac{1}{3}}}+\frac{\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\sqrt{\right.}{}\right.}{}\right.}{d}$
default	$\frac{3ia\left(\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{12a^{\frac{1}{3}}}+\frac{\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\sqrt{\right.}{}\right.}{}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I*a*tan(d*x+c))^(2/3),x,method=_RETURNVERBOSE)

[Out] 3*I/d*a*(1/6*2^(2/3)/a^(1/3)*ln((a+I*a*tan(d*x+c))^(1/3)-2^(1/3)*a^(1/3))-1/12*2^(2/3)/a^(1/3)*ln((a+I*a*tan(d*x+c))^(2/3)+2^(1/3)*a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+2^(2/3)*a^(2/3))+1/6*3^(1/2)*2^(2/3)/a^(1/3)*arctan(1/3*3^(1/2)*(2^(2/3)/a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+1)))

Maxima [A]
time = 0.54, size = 136, normalized size = 0.87

$$\frac{i\left(2\sqrt{3}\,2^{\frac{2}{3}}a^{\frac{5}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)-2^{\frac{2}{3}}a^{\frac{5}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(i\,a\tan(dx+c)+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{2}{3}}\right)+2\cdot 2^{\frac{2}{3}}a^{\frac{5}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)\right)}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I*a*tan(d*x+c))^(2/3),x, algorithm="maxima")

[Out] 1/4*I*(2*sqrt(3)*2^(2/3)*a^(5/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) - 2^(2/3)*a^(5/3)*log(2^(2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x + c) + a)^(2/3)) + 2*2^(2/3)*a^(5/3)*log(-2^(1/3)*a^(1/3) + (I*a*tan(d*x + c) + a)^(1/3)))/(a*d)

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(107) = 214.
time = 0.53, size = 226, normalized size = 1.45

$$\frac{1}{2}\left(-i\sqrt{3}-1\right)\left(-\frac{ia^2}{2d^3}\right)^{\frac{1}{3}}\log\left(\frac{2^{\frac{1}{3}}a\left(\frac{a}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}idx+\frac{2}{3}ic\right)}+\left(i\sqrt{3}d^2-d^2\right)\left(-\frac{ia^2}{2d^3}\right)^{\frac{2}{3}}}{a}\right)+\frac{1}{2}\left(i\sqrt{3}-1\right)\left(-\frac{ia^2}{2d^3}\right)^{\frac{1}{3}}\log\left(\frac{2^{\frac{1}{3}}a\left(\frac{a}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}idx+\frac{2}{3}ic\right)}+\left(-i\sqrt{3}d^2-d^2\right)\left(-\frac{ia^2}{2d^3}\right)^{\frac{2}{3}}}{a}\right)+\left(-\frac{ia^2}{2d^3}\right)^{\frac{1}{3}}\log\left(\frac{2d^2\left(-\frac{ia^2}{2d^3}\right)^{\frac{2}{3}}+2^{\frac{1}{3}}a\left(\frac{a}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}idx+\frac{2}{3}ic\right)}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*sqrt(3) - 1)*(-1/2*I*a^2/d^3)^(1/3)*log((2^(1/3)*a*(a/(e^(2*I*d*x +
 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + (I*sqrt(3)*d^2 - d^2)*(-1/2*I
*a^2/d^3)^(2/3))/a) + 1/2*(I*sqrt(3) - 1)*(-1/2*I*a^2/d^3)^(1/3)*log((2^(1/
3)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + (-I*sqrt
(3)*d^2 - d^2)*(-1/2*I*a^2/d^3)^(2/3))/a) + (-1/2*I*a^2/d^3)^(1/3)*log((2*d
^2*(-1/2*I*a^2/d^3)^(2/3) + 2^(1/3)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e
^(2/3*I*d*x + 2/3*I*c))/a)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia\tan(c+dx)+a)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(2/3),x)
```

```
[Out] Integral((I*a*tan(c + d*x) + a)**(2/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(2/3), x)
```

Mupad [**B**]
time = 4.56, size = 171, normalized size = 1.10

$$-\frac{\left(\frac{1}{2}\mathrm{i}\right)^{1/3}a^{2/3}\ln\left(\left(a\left(1+\tan(c+dx)\,1\mathrm{i}\right)\right)^{1/3}+(-1)^{1/3}2^{1/3}a^{1/3}\right)}{d}+\frac{\left(\frac{1}{2}\mathrm{i}\right)^{1/3}a^{2/3}\ln\left(-\frac{9a^2\left(a+a\tan(c+dx)\,1\mathrm{i}\right)^{1/3}}{d^2}-\frac{9(-1)^{1/3}2^{1/3}a^{7/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2d^2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{d}-\frac{\left(\frac{1}{2}\mathrm{i}\right)^{1/3}a^{2/3}\ln\left(-\frac{9a^2\left(a+a\tan(c+dx)\,1\mathrm{i}\right)^{1/3}}{d^2}+\frac{9(-1)^{1/3}2^{1/3}a^{7/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2d^2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(2/3),x)
```

```
[Out] ((1i/2)^(1/3)*a^(2/3)*log(- (9*a^2*(a + a*tan(c + d*x)*1i)^(1/3))/d^2 - (9*
(-1)^(1/3)*2^(1/3)*a^(7/3)*(3^(1/2)*1i - 1))/(2*d^2))*((3^(1/2)*1i)/2 + 1/2
))/d - ((1i/2)^(1/3)*a^(2/3)*log((a*(tan(c + d*x)*1i + 1))^(1/3) + (-1)^(1/
3)*2^(1/3)*a^(1/3)))/d - ((1i/2)^(1/3)*a^(2/3)*log((9*(-1)^(1/3)*2^(1/3)*a^
(7/3)*(3^(1/2)*1i + 1))/(2*d^2) - (9*a^2*(a + a*tan(c + d*x)*1i)^(1/3))/d^2
)*((3^(1/2)*1i)/2 - 1/2))/d
```

3.280 $\int \tan^3(c+dx)(a+ia\tan(c+dx))^{4/3}\,dx$

Optimal. Leaf size=251

$$-\frac{ia^{4/3}x}{2^{2/3}}+\frac{\sqrt[3]{2}\,\sqrt{3}\,a^{4/3}\mathrm{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{d}-\frac{a^{4/3}\log(\cos(c+dx))}{2^{2/3}d}-\frac{3a^{4/3}\log\left(\sqrt[3]{2}\,\sqrt[3]{a}\right.}{}$$

```
[Out] -1/2*I*a^(4/3)*x*2^(1/3)-1/2*a^(4/3)*ln(cos(d*x+c))*2^(1/3)/d-3/2*a^(4/3)*l
n(2^(1/3)*a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))*2^(1/3)/d+2^(1/3)*a^(4/3)*arcta
n(1/3*(a^(1/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)/d
-3*a*(a+I*a*tan(d*x+c))^(1/3)/d-9/20*(a+I*a*tan(d*x+c))^(4/3)/d+3/10*tan(d*
x+c)^2*(a+I*a*tan(d*x+c))^(4/3)/d-6/35*(a+I*a*tan(d*x+c))^(7/3)/a/d
```

Rubi [A]
time = 0.23, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3641, 3673, 3608, 3559, 3562, 59, 631, 210, 31}

$$\frac{\sqrt[3]{2}\,\sqrt{3}\,a^{4/3}\mathrm{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{d}-\frac{3a^{4/3}\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{2^{2/3}d}-\frac{a^{4/3}\log(\cos(c+dx))}{2^{2/3}d}-\frac{ia^{4/3}x}{2^{2/3}}+\frac{3\tan^2(c+dx)(a+ia\tan(c+dx))^{4/3}}{10d}-\frac{6(a+ia\tan(c+dx))^{7/3}}{35ad}-\frac{9(a+ia\tan(c+dx))^{4/3}}{20d}-\frac{3a\sqrt[3]{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3*(a + I*a*Tan[c + d*x])^(4/3),x]
```

```
[Out] ((-I)*a^(4/3)*x)/2^(2/3) + (2^(1/3)*Sqrt[3]*a^(4/3)*ArcTan[(a^(1/3) + 2^(2/
3)*(a + I*a*Tan[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/d - (a^(4/3)*Log[Cos[c
 + d*x]])/(2^(2/3)*d) - (3*a^(4/3)*Log[2^(1/3)*a^(1/3) - (a + I*a*Tan[c + d
*x])^(1/3)])/(2^(2/3)*d) - (3*a*(a + I*a*Tan[c + d*x])^(1/3))/d - (9*(a + I
*a*Tan[c + d*x])^(4/3))/(20*d) + (3*Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^(
4/3))/(10*d) - (6*(a + I*a*Tan[c + d*x])^(7/3))/(35*a*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rule 3641

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[d*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n - 1)/(f*(m + n - 1))), x] - Dist[1/(a*(m + n - 1)), Int[(a + b*
Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 2)*Simp[d*(b*c*m + a*d*(-1 + n))
- a*c^2*(m + n - 1) + d*(b*d*m - a*c*(m + 2*n - 2))*Tan[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1] && NeQ[m + n - 1, 0] && (IntegerQ[n]
|| IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
```

```
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \tan^3(c+dx)(a+ia\tan(c+dx))^{4/3}\,dx = \frac{3\tan^2(c+dx)(a+ia\tan(c+dx))^{4/3}}{10d} - \frac{3\int \tan(c+dx)(a+ia\ldots}{\ldots}$$

$$= \frac{3\tan^2(c+dx)(a+ia\tan(c+dx))^{4/3}}{10d} - \frac{6(a+ia\tan(c+dx))^{7\ldots}}{35ad}$$

$$= -\frac{9(a+ia\tan(c+dx))^{4/3}}{20d} + \frac{3\tan^2(c+dx)(a+ia\tan(c+dx)\ldots}{10d}$$

$$= -\frac{3a\sqrt[3]{a+ia\tan(c+dx)}}{d} - \frac{9(a+ia\tan(c+dx))^{4/3}}{20d} + \frac{3\tan^2\ldots}{\ldots}$$

$$= -\frac{3a\sqrt[3]{a+ia\tan(c+dx)}}{d} - \frac{9(a+ia\tan(c+dx))^{4/3}}{20d} + \frac{3\tan^2\ldots}{\ldots}$$

$$= -\frac{ia^{4/3}x}{2^{2/3}} - \frac{a^{4/3}\log(\cos(c+dx))}{2^{2/3}d} - \frac{3a\sqrt[3]{a+ia\tan(c+dx)}}{d} - \ldots$$

$$= -\frac{ia^{4/3}x}{2^{2/3}} - \frac{a^{4/3}\log(\cos(c+dx))}{2^{2/3}d} - \frac{3a^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{a} - \sqrt[3]{a+\ldots}\right.}{2^{2/3}d}$$

$$= -\frac{ia^{4/3}x}{2^{2/3}} + \frac{\sqrt[3]{2}\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{d}$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^3*(a + I*a*Tan[c + d*x])^(4/3),x]
```

```
[Out] $Aborted
```

Maple [A]
time = 0.10, size = 210, normalized size = 0.84

method	result
derivativedivides	$-\frac{3\left(\frac{(a+ia\tan(dx+c))^{\frac{10}{3}}}{10}-\frac{a(a+ia\tan(dx+c))^{\frac{7}{3}}}{7}+\frac{a^2(a+ia\tan(dx+c))^{\frac{4}{3}}}{4}+a^3(a+ia\tan(dx+c))^{\frac{1}{3}}+2\left(\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}\right.}{6a^{\frac{2}{3}}}\right.\right.}{}$
default	$-\frac{3\left(\frac{(a+ia\tan(dx+c))^{\frac{10}{3}}}{10}-\frac{a(a+ia\tan(dx+c))^{\frac{7}{3}}}{7}+\frac{a^2(a+ia\tan(dx+c))^{\frac{4}{3}}}{4}+a^3(a+ia\tan(dx+c))^{\frac{1}{3}}+2\left(\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}\right.}{6a^{\frac{2}{3}}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/d/a^2*(1/10*(a+I*a*tan(d*x+c))^(10/3)-1/7*a*(a+I*a*tan(d*x+c))^(7/3)+1/4
*a^2*(a+I*a*tan(d*x+c))^(4/3)+a^3*(a+I*a*tan(d*x+c))^(1/3)+2*(1/6*2^(1/3)/a
^(2/3)*ln((a+I*a*tan(d*x+c))^(1/3)-2^(1/3)*a^(1/3))-1/12*2^(1/3)/a^(2/3)*ln
((a+I*a*tan(d*x+c))^(2/3)+2^(1/3)*a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+2^(2/3)*
a^(2/3))-1/6*2^(1/3)/a^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2^(2/3)/a^(1/3)*(a
+I*a*tan(d*x+c))^(1/3)+1)))*a^4)
```

Maxima [A]
time = 0.53, size = 208, normalized size = 0.83

$$\frac{140\sqrt{3}\,2^{\frac{1}{3}}a^{\frac{16}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)+70\cdot2^{\frac{1}{3}}a^{\frac{16}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(i\,a\tan(dx+c)+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{2}{3}}\right)-140\cdot2^{\frac{1}{3}}a^{\frac{16}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)-42\,(i\,a\tan(dx+c)+a)^{\frac{10}{3}}a^2+60\,(i\,a\tan(dx+c)+a)^{\frac{7}{3}}a^3-105\,(i\,a\tan(dx+c)+a)^{\frac{4}{3}}a^4-420\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}a^5}{140\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] 1/140*(140*sqrt(3)*2^(1/3)*a^(16/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(
1/3) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) + 70*2^(1/3)*a^(16/3)*log(2
^(2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*
x + c) + a)^(2/3)) - 140*2^(1/3)*a^(16/3)*log(-2^(1/3)*a^(1/3) + (I*a*tan(d
*x + c) + a)^(1/3)) - 42*(I*a*tan(d*x + c) + a)^(10/3)*a^2 + 60*(I*a*tan(d*
x + c) + a)^(7/3)*a^3 - 105*(I*a*tan(d*x + c) + a)^(4/3)*a^4 - 420*(I*a*tan
(d*x + c) + a)^(1/3)*a^5)/(a^4*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 501 vs. 2(190) = 380.

time = 0.50, size = 501, normalized size = 2.00

$$-\frac{3\cdot 2^{\frac{1}{3}}\left(121\,ae^{(6i\,dx+6i\,c)}+240\,ae^{(4i\,dx+4i\,c)}+245\,ae^{(2i\,dx+2i\,c)}+70\,a\right)\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}+35\cdot 2^{\frac{1}{3}}\left(\left(-i\sqrt{3}\,d+d\right)e^{(6i\,dx+6i\,c)}+3\left(-i\sqrt{3}\,d+d\right)e^{(4i\,dx+4i\,c)}+3\left(-i\sqrt{3}\,d+d\right)e^{(2i\,dx+2i\,c)}-i\sqrt{3}\,d+d\right)\left(-\frac{a^4}{d^3}\right)^{\frac{1}{3}}\log\left(\frac{2\cdot 2^{\frac{1}{3}}a\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}+2^{\frac{1}{3}}\left(i\sqrt{3}\,d-d\right)\left(-\frac{a^4}{d^3}\right)^{\frac{1}{3}}}{2\,a}\right)+35\cdot 2^{\frac{1}{3}}\left(\left(i\sqrt{3}\,d+d\right)e^{(6i\,dx+6i\,c)}+3\left(i\sqrt{3}\,d+d\right)e^{(4i\,dx+4i\,c)}+3\left(i\sqrt{3}\,d+d\right)e^{(2i\,dx+2i\,c)}+i\sqrt{3}\,d+d\right)\left(-\frac{a^4}{d^3}\right)^{\frac{1}{3}}\log\left(\frac{2\cdot 2^{\frac{1}{3}}a\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}+2^{\frac{1}{3}}\left(-i\sqrt{3}\,d-d\right)\left(-\frac{a^4}{d^3}\right)^{\frac{1}{3}}}{2\,a}\right)-70\cdot 2^{\frac{1}{3}}\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)\left(-\frac{a^4}{d^3}\right)^{\frac{1}{3}}\log\left(\frac{2^{\frac{1}{3}}a\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}+2^{\frac{1}{3}}\left(-\frac{a^4}{d^3}\right)^{\frac{1}{3}}d}{a}\right)}{70\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] -1/70*(3*2^(1/3)*(121*a*e^(6*I*d*x + 6*I*c) + 240*a*e^(4*I*d*x + 4*I*c) + 2
45*a*e^(2*I*d*x + 2*I*c) + 70*a)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3
*I*d*x + 2/3*I*c) + 35*2^(1/3)*((-I*sqrt(3)*d + d)*e^(6*I*d*x + 6*I*c) + 3*
(-I*sqrt(3)*d + d)*e^(4*I*d*x + 4*I*c) + 3*(-I*sqrt(3)*d + d)*e^(2*I*d*x +
2*I*c) - I*sqrt(3)*d + d)*(-a^4/d^3)^(1/3)*log(1/2*(2*2^(1/3)*a*(a/(e^(2*I*
d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + 2^(1/3)*(I*sqrt(3)*d - d
)*(-a^4/d^3)^(1/3))/a) + 35*2^(1/3)*((I*sqrt(3)*d + d)*e^(6*I*d*x + 6*I*c)
+ 3*(I*sqrt(3)*d + d)*e^(4*I*d*x + 4*I*c) + 3*(I*sqrt(3)*d + d)*e^(2*I*d*x
+ 2*I*c) + I*sqrt(3)*d + d)*(-a^4/d^3)^(1/3)*log(1/2*(2*2^(1/3)*a*(a/(e^(2*
I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + 2^(1/3)*(-I*sqrt(3)*d
- d)*(-a^4/d^3)^(1/3))/a) - 70*2^(1/3)*(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*
d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)*(-a^4/d^3)^(1/3)*log((2^(1/3)*a
*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + 2^(1/3)*(-a^
4/d^3)^(1/3)*d)/a))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*
e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c+dx)-i))^{\frac{4}{3}}\tan^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3*(a+I*a*tan(d*x+c))**(4/3),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(4/3)*tan(c + d*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(4/3)*tan(d*x + c)^3, x)
```

Mupad [B]
time = 4.38, size = 243, normalized size = 0.97

$$-\frac{3\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{4/3}}{4\,d}+\frac{3\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/3}}{7\,a\,d}-\frac{3\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{10/3}}{10\,a^2\,d}-\frac{3\,a\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{1/3}}{d}-\frac{2^{1/3}\,a^{4/3}\,\ln\left({\left(a\,\left(1+\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)\right)}^{1/3}-2^{1/3}\,a^{1/3}\right)}{d}-\frac{2^{1/3}\,a^{4/3}\,\ln\left(\frac{18\,a^2\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{1/3}}{d}-\frac{9\,2^{1/3}\,a^{7/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{d}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{d}+\frac{2^{1/3}\,a^{4/3}\,\ln\left(\frac{18\,a^2\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{1/3}}{d}+\frac{9\,2^{1/3}\,a^{7/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{d}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(4/3),x)
```

```
[Out] (3*(a + a*tan(c + d*x)*1i)^(7/3))/(7*a*d) - (3*(a + a*tan(c + d*x)*1i)^(4/3
))/(4*d) - (3*(a + a*tan(c + d*x)*1i)^(10/3))/(10*a^2*d) - (3*a*(a + a*tan(
c + d*x)*1i)^(1/3))/d - (2^(1/3)*a^(4/3)*log((a*(tan(c + d*x)*1i + 1))^(1/3
) - 2^(1/3)*a^(1/3)))/d - (2^(1/3)*a^(4/3)*log((18*a^2*(a + a*tan(c + d*x)*
1i)^(1/3))/d - (9*2^(1/3)*a^(7/3)*(3^(1/2)*1i - 1))/d)*((3^(1/2)*1i)/2 - 1/
2))/d + (2^(1/3)*a^(4/3)*log((18*a^2*(a + a*tan(c + d*x)*1i)^(1/3))/d + (9*
2^(1/3)*a^(7/3)*(3^(1/2)*1i + 1))/d)*((3^(1/2)*1i)/2 + 1/2))/d
```

3.281 $\int \tan^2(c+dx)(a+ia\tan(c+dx))^{4/3}\,dx$

Optimal. Leaf size=203

$$\frac{a^{4/3}x}{2^{2/3}}+\frac{i\sqrt[3]{2}\,\sqrt{3}\,a^{4/3}\mathrm{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{d}-\frac{ia^{4/3}\log(\cos(c+dx))}{2^{2/3}d}-\frac{3ia^{4/3}\log\left(\sqrt[3]{2}\,\sqrt[3]{a}\right.}{}$$

```
[Out] 1/2*a^(4/3)*x*2^(1/3)-1/2*I*a^(4/3)*ln(cos(d*x+c))*2^(1/3)/d-3/2*I*a^(4/3)*
ln(2^(1/3)*a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))*2^(1/3)/d+I*2^(1/3)*a^(4/3)*ar
ctan(1/3*(a^(1/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))*3^(1/2
)/d-3*I*a*(a+I*a*tan(d*x+c))^(1/3)/d-3/7*I*(a+I*a*tan(d*x+c))^(7/3)/a/d
```

Rubi [A]
time = 0.12, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3624, 3559, 3562, 59, 631, 210, 31}

$$\frac{i\sqrt[3]{2}\,\sqrt{3}\,a^{4/3}\mathrm{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{d}-\frac{3ia^{4/3}\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{2^{2/3}d}-\frac{ia^{4/3}\log(\cos(c+dx))}{2^{2/3}d}+\frac{a^{4/3}x}{2^{2/3}}-\frac{3i(a+ia\tan(c+dx))^{7/3}}{7ad}-\frac{3ia\sqrt[3]{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^(4/3),x]
```

```
[Out] (a^(4/3)*x)/2^(2/3) + (I*2^(1/3)*Sqrt[3]*a^(4/3)*ArcTan[(a^(1/3) + 2^(2/3)*
(a + I*a*Tan[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/d - (I*a^(4/3)*Log[Cos[c
+ d*x]])/(2^(2/3)*d) - ((3*I)*a^(4/3)*Log[2^(1/3)*a^(1/3) - (a + I*a*Tan[c
+ d*x])^(1/3)])/(2^(2/3)*d) - ((3*I)*a*(a + I*a*Tan[c + d*x])^(1/3))/d - ((
(3*I)/7)*(a + I*a*Tan[c + d*x])^(7/3))/(a*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^{4/3}\,dx = -\frac{3i(a+ia\tan(c+dx))^{7/3}}{7ad} - \int (a+ia\tan(c+dx))^{4/3}\,dx$$

$$= -\frac{3ia\sqrt[3]{a+ia\tan(c+dx)}}{d} - \frac{3i(a+ia\tan(c+dx))^{7/3}}{7ad} - (2a)$$

$$= -\frac{3ia\sqrt[3]{a+ia\tan(c+dx)}}{d} - \frac{3i(a+ia\tan(c+dx))^{7/3}}{7ad} + \frac{(2ia}{}$$

$$= \frac{a^{4/3}x}{2^{2/3}} - \frac{ia^{4/3}\log(\cos(c+dx))}{2^{2/3}d} - \frac{3ia\sqrt[3]{a+ia\tan(c+dx)}}{d} - \frac{3}{}$$

$$= \frac{a^{4/3}x}{2^{2/3}} - \frac{ia^{4/3}\log(\cos(c+dx))}{2^{2/3}d} - \frac{3ia^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{a} - \sqrt[3]{a+}\right.}{2^{2/3}d}$$

$$= \frac{a^{4/3}x}{2^{2/3}} + \frac{i\sqrt[3]{2}\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{d} -$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^(4/3),x]
[Out] $Aborted
```

Maple [A]
time = 0.10, size = 175, normalized size = 0.86

method	result
derivativedivides	$-\frac{3i\left(\frac{(a+ia\tan(dx+c))^{\frac{7}{3}}}{7}+a^2(a+ia\tan(dx+c))^{\frac{1}{3}}+2\left(\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a\right.}{12a}\right.\right.}{da}$

default	$-\frac{3i\left(\frac{(a+ia\tan(dx+c))^{\frac{7}{3}}}{7}+a^{2}(a+ia\tan(dx+c))^{\frac{1}{3}}+2\left(\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}\right.}{12a^{\frac{2}{3}}}\right.\right.}{da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3*I/d/a*(1/7*(a+I*a*tan(d*x+c))^(7/3)+a^2*(a+I*a*tan(d*x+c))^(1/3)+2*(1/6*
2^(1/3)/a^(2/3)*ln((a+I*a*tan(d*x+c))^(1/3)-2^(1/3)*a^(1/3))-1/12*2^(1/3)/a
^(2/3)*ln((a+I*a*tan(d*x+c))^(2/3)+2^(1/3)*a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)
+2^(2/3)*a^(2/3))-1/6*2^(1/3)/a^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2^(2/3)/a
^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+1)))*a^3)
```

Maxima [A]
time = 0.53, size = 172, normalized size = 0.85

$$\frac{i\left(14\sqrt{3}\,2^{\frac{1}{3}}a^{\frac{13}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)+7\cdot 2^{\frac{1}{3}}a^{\frac{13}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(i\,a\tan(dx+c)+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{2}{3}}\right)-14\cdot 2^{\frac{1}{3}}a^{\frac{13}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)-6\,(i\,a\tan(dx+c)+a)^{\frac{7}{3}}a^{2}-42\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}a^{4}\right)}{14\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] 1/14*I*(14*sqrt(3)*2^(1/3)*a^(13/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(
1/3) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) + 7*2^(1/3)*a^(13/3)*log(2^
(2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x
 + c) + a)^(2/3)) - 14*2^(1/3)*a^(13/3)*log(-2^(1/3)*a^(1/3) + (I*a*tan(d*x
 + c) + a)^(1/3)) - 6*(I*a*tan(d*x + c) + a)^(7/3)*a^2 - 42*(I*a*tan(d*x +
c) + a)^(1/3)*a^4)/(a^3*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 409 vs. 2(147) = 294.
time = 0.57, size = 409, normalized size = 2.01

$$-\frac{6\cdot 2^{\frac{1}{3}}\left(11i\,ae^{(4i\,dx+4i\,c)}+14i\,ae^{(2i\,dx+2i\,c)}+7i\,a\right)\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}+7\left(\left(-i\sqrt{3}\,d+d\right)e^{(4i\,dx+4i\,c)}+2\left(-i\sqrt{3}\,d+d\right)e^{(2i\,dx+2i\,c)}-i\sqrt{3}\,d+d\right)\left(\frac{2i\,a^{4}}{d^{3}}\right)^{\frac{1}{3}}\log\left(\frac{2\cdot 2^{\frac{1}{3}}a\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}+\left(\sqrt{3}\,d+i\,d\right)\left(\frac{2i\,a^{4}}{d^{3}}\right)^{\frac{1}{3}}}{2\,a}\right)+7\left(\left(i\sqrt{3}\,d+d\right)e^{(4i\,dx+4i\,c)}+2\left(i\sqrt{3}\,d+d\right)e^{(2i\,dx+2i\,c)}+i\sqrt{3}\,d+d\right)\left(\frac{2i\,a^{4}}{d^{3}}\right)^{\frac{1}{3}}\log\left(\frac{2\cdot 2^{\frac{1}{3}}a\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}-\left(\sqrt{3}\,d-i\,d\right)\left(\frac{2i\,a^{4}}{d^{3}}\right)^{\frac{1}{3}}}{2\,a}\right)-14\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)\left(\frac{2i\,a^{4}}{d^{3}}\right)^{\frac{1}{3}}\log\left(\frac{2^{\frac{1}{3}}a\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}-i\left(\frac{2i\,a^{4}}{d^{3}}\right)^{\frac{1}{3}}d}{a}\right)}{14\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] -1/14*(6*2^(1/3)*(11*I*a*e^(4*I*d*x + 4*I*c) + 14*I*a*e^(2*I*d*x + 2*I*c) +
 7*I*a)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + 7*((-
I*sqrt(3)*d + d)*e^(4*I*d*x + 4*I*c) + 2*(-I*sqrt(3)*d + d)*e^(2*I*d*x + 2*
I*c) - I*sqrt(3)*d + d)*(2*I*a^4/d^3)^(1/3)*log(1/2*(2*2^(1/3)*a*(a/(e^(2*I
```

```
*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + (sqrt(3)*d + I*d)*(2*I*
a^4/d^3)^(1/3))/a) + 7*((I*sqrt(3)*d + d)*e^(4*I*d*x + 4*I*c) + 2*(I*sqrt(3
)*d + d)*e^(2*I*d*x + 2*I*c) + I*sqrt(3)*d + d)*(2*I*a^4/d^3)^(1/3)*log(1/2
*(2*2^(1/3)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) -
 (sqrt(3)*d - I*d)*(2*I*a^4/d^3)^(1/3))/a) - 14*(d*e^(4*I*d*x + 4*I*c) + 2*
d*e^(2*I*d*x + 2*I*c) + d)*(2*I*a^4/d^3)^(1/3)*log((2^(1/3)*a*(a/(e^(2*I*d*
x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) - I*(2*I*a^4/d^3)^(1/3)*d)/a
))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ia(\tan(c+dx)-i)\right)^{\frac{4}{3}} \tan^2(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2*(a+I*a*tan(d*x+c))**(4/3),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(4/3)*tan(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(4/3)*tan(d*x + c)^2, x)
```

Mupad [B]
time = 4.07, size = 218, normalized size = 1.07

$$-\frac{(a+a\tan(c+dx)\,1\mathrm{i})^{7/3}\,3\mathrm{i}}{7\,a\,d}-\frac{a\,(a+a\tan(c+dx)\,1\mathrm{i})^{1/3}\,3\mathrm{i}}{d}+\frac{(2\mathrm{i})^{1/3}\,a^{4/3}\,\ln\left(a^2\,d^2\,(a+a\tan(c+dx)\,1\mathrm{i})^{1/3}\,18\mathrm{i}+18\,(2\mathrm{i})^{1/3}\,a^{7/3}\,d^2\right)}{d}+\frac{(2\mathrm{i})^{1/3}\,a^{4/3}\,\ln\left(a^2\,d^2\,(a+a\tan(c+dx)\,1\mathrm{i})^{1/3}\,18\mathrm{i}+18\,(2\mathrm{i})^{1/3}\,a^{7/3}\,d^2\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{d}-\frac{(2\mathrm{i})^{1/3}\,a^{4/3}\,\ln\left(a^2\,d^2\,(a+a\tan(c+dx)\,1\mathrm{i})^{1/3}\,18\mathrm{i}-18\,(2\mathrm{i})^{1/3}\,a^{7/3}\,d^2\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(4/3),x)
```

```
[Out] (2i^(1/3)*a^(4/3)*log(a^2*d^2*(a + a*tan(c + d*x)*1i)^(1/3)*18i + 18*2i^(1/
3)*a^(7/3)*d^2))/d - (a*(a + a*tan(c + d*x)*1i)^(1/3)*3i)/d - ((a + a*tan(c
 + d*x)*1i)^(7/3)*3i)/(7*a*d) + (2i^(1/3)*a^(4/3)*log(a^2*d^2*(a + a*tan(c
+ d*x)*1i)^(1/3)*18i + 18*2i^(1/3)*a^(7/3)*d^2*((3^(1/2)*1i)/2 - 1/2))*((3^
(1/2)*1i)/2 - 1/2))/d - (2i^(1/3)*a^(4/3)*log(a^2*d^2*(a + a*tan(c + d*x)*1
i)^(1/3)*18i - 18*2i^(1/3)*a^(7/3)*d^2*((3^(1/2)*1i)/2 + 1/2))*((3^(1/2)*1i
)/2 + 1/2))/d
```

3.282 $\int \tan(c+dx)(a+ia\tan(c+dx))^{4/3}\,dx$

Optimal. Leaf size=192

$$\frac{ia^{4/3}x}{2^{2/3}}-\frac{\sqrt[3]{2}\,\sqrt{3}\,a^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{d}+\frac{a^{4/3}\log(\cos(c+dx))}{2^{2/3}d}+\frac{3a^{4/3}\log\left(\sqrt[3]{2}\,\sqrt[3]{a}\,-\right.}{2}$$

```
[Out] 1/2*I*a^(4/3)*x*2^(1/3)+1/2*a^(4/3)*ln(cos(d*x+c))*2^(1/3)/d+3/2*a^(4/3)*ln
(2^(1/3)*a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))*2^(1/3)/d-2^(1/3)*a^(4/3)*arctan
(1/3*(a^(1/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)/d+
3*a*(a+I*a*tan(d*x+c))^(1/3)/d+3/4*(a+I*a*tan(d*x+c))^(4/3)/d
```

Rubi [A]
time = 0.12, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {3608, 3559, 3562, 59, 631, 210, 31}

$$-\frac{\sqrt[3]{2}\,\sqrt{3}\,a^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{d}+\frac{3a^{4/3}\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{2^{2/3}d}+\frac{a^{4/3}\log(\cos(c+dx))}{2^{2/3}d}+\frac{ia^{4/3}x}{2^{2/3}}+\frac{3a\sqrt[3]{a+ia\tan(c+dx)}}{d}+\frac{3(a+ia\tan(c+dx))^{4/3}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]*(a + I*a*Tan[c + d*x])^(4/3),x]
```

```
[Out] (I*a^(4/3)*x)/2^(2/3) - (2^(1/3)*Sqrt[3]*a^(4/3)*ArcTan[(a^(1/3) + 2^(2/3)*
(a + I*a*Tan[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/d + (a^(4/3)*Log[Cos[c +
d*x]])/(2^(2/3)*d) + (3*a^(4/3)*Log[2^(1/3)*a^(1/3) - (a + I*a*Tan[c + d*x]
)^(1/3)])/(2^(2/3)*d) + (3*a*(a + I*a*Tan[c + d*x])^(1/3))/d + (3*(a + I*a*
Tan[c + d*x])^(4/3))/(4*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rubi steps

$$\int \tan(c+dx)(a+ia\tan(c+dx))^{4/3}\,dx = \frac{3(a+ia\tan(c+dx))^{4/3}}{4d} - i\int (a+ia\tan(c+dx))^{4/3}\,dx$$

$$= \frac{3a\sqrt[3]{a+ia\tan(c+dx)}}{d} + \frac{3(a+ia\tan(c+dx))^{4/3}}{4d} - (2ia)\int \sqrt[3]{\ldots}$$

$$= \frac{3a\sqrt[3]{a+ia\tan(c+dx)}}{d} + \frac{3(a+ia\tan(c+dx))^{4/3}}{4d} - \frac{(2a^2)\,\mathrm{Subs}\ldots}{\ldots}$$

$$= \frac{ia^{4/3}x}{2^{2/3}} + \frac{a^{4/3}\log(\cos(c+dx))}{2^{2/3}d} + \frac{3a\sqrt[3]{a+ia\tan(c+dx)}}{d} + \frac{3(a\ldots}{\ldots}$$

$$= \frac{ia^{4/3}x}{2^{2/3}} + \frac{a^{4/3}\log(\cos(c+dx))}{2^{2/3}d} + \frac{3a^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{a} - \sqrt[3]{a+ia\,\mathrm{t}\ldots}\right.}{2^{2/3}d}$$

$$= \frac{ia^{4/3}x}{2^{2/3}} - \frac{\sqrt[3]{2}\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{d} + \frac{a^{\ldots}}{\ldots}$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[c + d*x]*(a + I*a*Tan[c + d*x])^(4/3),x]
[Out] $Aborted
```

Maple [A]
time = 0.10, size = 169, normalized size = 0.88

method	result
derivativedivides	$\frac{\frac{3(a+ia\tan(dx+c))^{\frac{4}{3}}}{4}+3a(a+ia\tan(dx+c))^{\frac{1}{3}}+6\left(\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\ldots\right.}{12a^{\frac{2}{3}}}\ldots\right.}{d}$

default	$\frac{\frac{3(a+ia\tan(dx+c))^{\frac{4}{3}}}{4}+3a(a+ia\tan(dx+c))^{\frac{1}{3}}+6\left(\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+\right.}{12a^{\frac{2}{3}}}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)*(a+I*a*tan(d*x+c))^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(3/4*(a+I*a*tan(d*x+c))^(4/3)+3*a*(a+I*a*tan(d*x+c))^(1/3)+6*(1/6*2^(1/
3)/a^(2/3)*ln((a+I*a*tan(d*x+c))^(1/3)-2^(1/3)*a^(1/3))-1/12*2^(1/3)/a^(2/3
)*ln((a+I*a*tan(d*x+c))^(2/3)+2^(1/3)*a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+2^(2
/3)*a^(2/3))-1/6*2^(1/3)/a^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2^(2/3)/a^(1/3
)*(a+I*a*tan(d*x+c))^(1/3)+1)))*a^2)
```

Maxima [A]
time = 0.53, size = 172, normalized size = 0.90

$$-\frac{4\sqrt{3}\,2^{\frac{1}{3}}a^{\frac{10}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)+2\cdot 2^{\frac{1}{3}}a^{\frac{10}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(i\,a\tan(dx+c)+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{2}{3}}\right)-4\cdot 2^{\frac{1}{3}}a^{\frac{10}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)-3\,(i\,a\tan(dx+c)+a)^{\frac{4}{3}}a^{2}-12\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}a^{3}}{4\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] -1/4*(4*sqrt(3)*2^(1/3)*a^(10/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3
) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) + 2*2^(1/3)*a^(10/3)*log(2^(2/
3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x +
c) + a)^(2/3)) - 4*2^(1/3)*a^(10/3)*log(-2^(1/3)*a^(1/3) + (I*a*tan(d*x + c
) + a)^(1/3)) - 3*(I*a*tan(d*x + c) + a)^(4/3)*a^2 - 12*(I*a*tan(d*x + c) +
 a)^(1/3)*a^3)/(a^2*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 350 vs. 2(144) = 288.
time = 0.60, size = 350, normalized size = 1.82

$$\frac{3\cdot 2^{\frac{1}{3}}\left(3\,ae^{(2i\,dx+2i\,c)}+2\,a\right)\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}+2^{\frac{1}{3}}\left(\left(-i\sqrt{3}\,d-d\right)e^{(2i\,dx+2i\,c)}-i\sqrt{3}\,d-d\right)\left(\frac{a^4}{d^3}\right)^{\frac{1}{3}}\log\left(\frac{2\cdot 2^{\frac{1}{3}}a\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}+2^{\frac{1}{3}}\left(i\sqrt{3}\,d+d\right)\left(\frac{a^4}{d^3}\right)^{\frac{1}{3}}}{2\,a}\right)+2^{\frac{1}{3}}\left(\left(i\sqrt{3}\,d-d\right)e^{(2i\,dx+2i\,c)}+i\sqrt{3}\,d-d\right)\left(\frac{a^4}{d^3}\right)^{\frac{1}{3}}\log\left(\frac{2\cdot 2^{\frac{1}{3}}a\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}+2^{\frac{1}{3}}\left(-i\sqrt{3}\,d+d\right)\left(\frac{a^4}{d^3}\right)^{\frac{1}{3}}}{2\,a}\right)+2\cdot 2^{\frac{1}{3}}\left(de^{(2i\,dx+2i\,c)}+d\right)\left(\frac{a^4}{d^3}\right)^{\frac{1}{3}}\log\left(\frac{2^{\frac{1}{3}}a\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}-2^{\frac{1}{3}}\left(\frac{a^4}{d^3}\right)^{\frac{1}{3}}d}{a}\right)}{2\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] 1/2*(3*2^(1/3)*(3*a*e^(2*I*d*x + 2*I*c) + 2*a)*(a/(e^(2*I*d*x + 2*I*c) + 1)
)^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + 2^(1/3)*((-I*sqrt(3)*d - d)*e^(2*I*d*x +
2*I*c) - I*sqrt(3)*d - d)*(a^4/d^3)^(1/3)*log(1/2*(2*2^(1/3)*a*(a/(e^(2*I*d
*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + 2^(1/3)*(I*sqrt(3)*d + d)
```

```
*(a^4/d^3)^(1/3))/a) + 2^(1/3)*((I*sqrt(3)*d - d)*e^(2*I*d*x + 2*I*c) + I*s
qrt(3)*d - d)*(a^4/d^3)^(1/3)*log(1/2*(2*2^(1/3)*a*(a/(e^(2*I*d*x + 2*I*c)
+ 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + 2^(1/3)*(-I*sqrt(3)*d + d)*(a^4/d^3)^
(1/3))/a) + 2*2^(1/3)*(d*e^(2*I*d*x + 2*I*c) + d)*(a^4/d^3)^(1/3)*log((2^(1
/3)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) - 2^(1/3)
*(a^4/d^3)^(1/3)*d)/a))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan{(c+dx)}-i))^{\frac{4}{3}}\tan{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))**(4/3),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(4/3)*tan(c + d*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(4/3)*tan(d*x + c), x)
```

Mupad [**B**]
time = 4.32, size = 198, normalized size = 1.03

$$\frac{3\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{4/3}}{4\,d}+\frac{3\,a\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{1/3}}{d}+\frac{2^{1/3}\,a^{4/3}\,\ln\left({\left(a\,\left(1+\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)\right)}^{1/3}-2^{1/3}\,a^{1/3}\right)}{d}+\frac{2^{1/3}\,a^{4/3}\,\ln\left(\frac{18\,a^2\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{1/3}}{d}-\frac{9\,2^{1/3}\,a^{7/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{d}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{d}-\frac{2^{1/3}\,a^{4/3}\,\ln\left(\frac{18\,a^2\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{1/3}}{d}+\frac{9\,2^{1/3}\,a^{7/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{d}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + a*tan(c + d*x)*1i)^(4/3),x)
```

```
[Out] (3*(a + a*tan(c + d*x)*1i)^(4/3))/(4*d) + (3*a*(a + a*tan(c + d*x)*1i)^(1/3
))/d + (2^(1/3)*a^(4/3)*log((a*(tan(c + d*x)*1i + 1))^(1/3) - 2^(1/3)*a^(1/
3)))/d + (2^(1/3)*a^(4/3)*log((18*a^2*(a + a*tan(c + d*x)*1i)^(1/3))/d - (9
*2^(1/3)*a^(7/3)*(3^(1/2)*1i - 1))/d)*((3^(1/2)*1i)/2 - 1/2))/d - (2^(1/3)*
a^(4/3)*log((18*a^2*(a + a*tan(c + d*x)*1i)^(1/3))/d + (9*2^(1/3)*a^(7/3)*(
3^(1/2)*1i + 1))/d)*((3^(1/2)*1i)/2 + 1/2))/d
```

3.283 $\int (a+ia\tan(c+dx))^{4/3}\,dx$

Optimal. Leaf size=175

$$-\frac{a^{4/3}x}{2^{2/3}}-\frac{i\sqrt[3]{2}\,\sqrt{3}\,a^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{d}+\frac{ia^{4/3}\log(\cos(c+dx))}{2^{2/3}d}+\frac{3ia^{4/3}\log\left(\sqrt[3]{2}\,\sqrt[3]{}\right.}{}$$

```
[Out] -1/2*a^(4/3)*x*2^(1/3)+1/2*I*a^(4/3)*ln(cos(d*x+c))*2^(1/3)/d+3/2*I*a^(4/3)
*ln(2^(1/3)*a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))*2^(1/3)/d-I*2^(1/3)*a^(4/3)*a
rctan(1/3*(a^(1/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))*3^(1/
2)/d+3*I*a*(a+I*a*tan(d*x+c))^(1/3)/d
```

Rubi [A]
time = 0.08, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3559, 3562, 59, 631, 210, 31}

$$-\frac{i\sqrt[3]{2}\,\sqrt{3}\,a^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{d}+\frac{3ia^{4/3}\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{2^{2/3}d}+\frac{ia^{4/3}\log(\cos(c+dx))}{2^{2/3}d}-\frac{a^{4/3}x}{2^{2/3}}+\frac{3ia\sqrt[3]{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(4/3),x]
```

```
[Out] -((a^(4/3)*x)/2^(2/3)) - (I*2^(1/3)*Sqrt[3]*a^(4/3)*ArcTan[(a^(1/3) + 2^(2/
3)*(a + I*a*Tan[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/d + (I*a^(4/3)*Log[Cos
[c + d*x]])/(2^(2/3)*d) + ((3*I)*a^(4/3)*Log[2^(1/3)*a^(1/3) - (a + I*a*Tan
[c + d*x])^(1/3)])/(2^(2/3)*d) + ((3*I)*a*(a + I*a*Tan[c + d*x])^(1/3))/d
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a+ia\tan(c+dx))^{4/3}\,dx = \frac{3ia\sqrt[3]{a+ia\tan(c+dx)}}{d} + (2a)\int \sqrt[3]{a+ia\tan(c+dx)}\,dx$$

$$= \frac{3ia\sqrt[3]{a+ia\tan(c+dx)}}{d} - \frac{(2ia^2)\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{2/3}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{a^{4/3}x}{2^{2/3}} + \frac{ia^{4/3}\log(\cos(c+dx))}{2^{2/3}d} + \frac{3ia\sqrt[3]{a+ia\tan(c+dx)}}{d} - \frac{(3ia^{4/3})\,\text{Subs}}{}$$

$$= -\frac{a^{4/3}x}{2^{2/3}} + \frac{ia^{4/3}\log(\cos(c+dx))}{2^{2/3}d} + \frac{3ia^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{a} - \sqrt[3]{a+ia\tan(c+dx}\right.}{2^{2/3}d}$$

$$= -\frac{a^{4/3}x}{2^{2/3}} - \frac{i\sqrt[3]{2}\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{d} + \frac{ia^{4/3}\log(\cos}{2^{2/3}}$$

Mathematica [A]

time = 1.26, size = 294, normalized size = 1.68

$$\frac{iae^{\frac{1}{3}i(c+dx)}\cos(c+dx)\left(6e^{\frac{2}{3}i(c+dx)}-2\sqrt{3}\,\sqrt[3]{1+e^{2i(c+dx)}}\operatorname{ArcTan}\left(\frac{1+\frac{2e^{\frac{2}{3}i(c+dx)}}{\sqrt[3]{1+e^{2i(c+dx)}}}}{\sqrt{3}}\right)+2\sqrt[3]{1+e^{2i(c+dx)}}\log\left(1-\frac{e^{\frac{2}{3}i(c+dx)}}{\sqrt[3]{1+e^{2i(c+dx)}}}\right)-\sqrt[3]{1+e^{2i(c+dx)}}\log\left(\frac{e^{\frac{4}{3}i(c+dx)}+e^{\frac{2}{3}i(c+dx)}\sqrt[3]{1+e^{2i(c+dx)}}+(1+e^{2i(c+dx)})^{2/3}}{(1+e^{2i(c+dx)})^{2/3}}\right)\right)\sqrt[3]{a+ia\tan(c+dx)}}{d\left(1+e^{2i(c+dx)}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(4/3),x]
```

```
[Out] (I*a*E^((I/3)*(c + d*x))*Cos[c + d*x]*(6*E^(((2*I)/3)*(c + d*x)) - 2*Sqrt[3
]*(1 + E^((2*I)*(c + d*x)))^(1/3)*ArcTan[(1 + (2*E^(((2*I)/3)*(c + d*x)))/(
1 + E^((2*I)*(c + d*x)))^(1/3))/Sqrt[3]] + 2*(1 + E^((2*I)*(c + d*x)))^(1/3
)*Log[1 - E^(((2*I)/3)*(c + d*x))/(1 + E^((2*I)*(c + d*x)))^(1/3)] - (1 + E
^((2*I)*(c + d*x)))^(1/3)*Log[(E^(((4*I)/3)*(c + d*x)) + E^(((2*I)/3)*(c +
d*x))*(1 + E^((2*I)*(c + d*x)))^(1/3) + (1 + E^((2*I)*(c + d*x)))^(2/3))/(1
 + E^((2*I)*(c + d*x)))^(2/3)])*(a + I*a*Tan[c + d*x])^(1/3))/(d*(1 + E^((2
*I)*(c + d*x))))
```

Maple [A]

time = 0.10, size = 151, normalized size = 0.86

method	result
derivativedivides	$\frac{3ia\left((a+ia\tan(dx+c))^{\frac{1}{3}}+2\left(\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}\right.}{12a^{\frac{2}{3}}}\right.\right.}{d}$
default	$\frac{3ia\left((a+ia\tan(dx+c))^{\frac{1}{3}}+2\left(\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}\right.}{12a^{\frac{2}{3}}}\right.\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3*I/d*a*((a+I*a*tan(d*x+c))^(1/3)+2*(1/6*2^(1/3)/a^(2/3)*ln((a+I*a*tan(d*x+
c))^(1/3)-2^(1/3)*a^(1/3))-1/12*2^(1/3)/a^(2/3)*ln((a+I*a*tan(d*x+c))^(2/3)
+2^(1/3)*a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+2^(2/3)*a^(2/3))-1/6*2^(1/3)/a^(2
/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2^(2/3)/a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+1)
))*a)
```

Maxima [A]
time = 0.52, size = 153, normalized size = 0.87

$$-\frac{i\left(2\sqrt{3}\,2^{\frac{1}{3}}a^{\frac{7}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(ia\tan\left(dx+c\right)+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)+2^{\frac{1}{3}}a^{\frac{7}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(ia\tan\left(dx+c\right)+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(ia\tan\left(dx+c\right)+a\right)^{\frac{2}{3}}\right)-2\cdot 2^{\frac{1}{3}}a^{\frac{7}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(ia\tan\left(dx+c\right)+a\right)^{\frac{1}{3}}\right)-6\left(ia\tan\left(dx+c\right)+a\right)^{\frac{1}{3}}a^{2}\right)}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] -1/2*I*(2*sqrt(3)*2^(1/3)*a^(7/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/
3) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) + 2^(1/3)*a^(7/3)*log(2^(2/3)
*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x + c)
 + a)^(2/3)) - 2*2^(1/3)*a^(7/3)*log(-2^(1/3)*a^(1/3) + (I*a*tan(d*x + c) +
 a)^(1/3)) - 6*(I*a*tan(d*x + c) + a)^(1/3)*a^2)/(a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(126) = 252.
time = 0.80, size = 262, normalized size = 1.50

$$\frac{6i\cdot 2^{\frac{1}{3}}a\left(\frac{a}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}idx+\frac{2}{3}ic\right)}+\left(i\sqrt{3}\,d-d\right)\left(-\frac{2ia^4}{d^3}\right)^{\frac{1}{3}}\log\left(\frac{2\cdot 2^{\frac{1}{3}}a\left(\frac{a}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}idx+\frac{2}{3}ic\right)}-\left(\sqrt{3}\,d+id\right)\left(-\frac{2ia^4}{d^3}\right)^{\frac{1}{3}}}{2a}\right)+\left(-i\sqrt{3}\,d-d\right)\left(-\frac{2ia^4}{d^3}\right)^{\frac{1}{3}}\log\left(\frac{2\cdot 2^{\frac{1}{3}}a\left(\frac{a}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}idx+\frac{2}{3}ic\right)}+\left(\sqrt{3}\,d-id\right)\left(-\frac{2ia^4}{d^3}\right)^{\frac{1}{3}}}{2a}\right)+2\left(-\frac{2ia^4}{d^3}\right)^{\frac{1}{3}}d\log\left(\frac{2^{\frac{1}{3}}a\left(\frac{a}{e^{(2idx+2ic)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}idx+\frac{2}{3}ic\right)}+i\left(-\frac{2ia^4}{d^3}\right)^{\frac{1}{3}}d}{a}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] 1/2*(6*I*2^(1/3)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I
*c) + (I*sqrt(3)*d - d)*(-2*I*a^4/d^3)^(1/3)*log(1/2*(2*2^(1/3)*a*(a/(e^(2*
I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) - (sqrt(3)*d + I*d)*(-2*
I*a^4/d^3)^(1/3))/a) + (-I*sqrt(3)*d - d)*(-2*I*a^4/d^3)^(1/3)*log(1/2*(2*2
^(1/3)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + (sqr
t(3)*d - I*d)*(-2*I*a^4/d^3)^(1/3))/a) + 2*(-2*I*a^4/d^3)^(1/3)*d*log((2^(1
/3)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + I*(-2*I
*a^4/d^3)^(1/3)*d)/a))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ia\tan\left(c+dx\right)+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(4/3),x)
```

```
[Out] Integral((I*a*tan(c + d*x) + a)**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(4/3), x)
```

Mupad [B]
time = 3.99, size = 195, normalized size = 1.11

$$\frac{a\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{1/3}\,3{}\mathrm{i}}{d}-\frac{{\left(2{}\mathrm{i}\right)}^{1/3}\,a^{4/3}\,\ln\left(a^2\,d^2\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{1/3}\,18{}\mathrm{i}+18\,{\left(2{}\mathrm{i}\right)}^{1/3}\,a^{7/3}\,d^2\right)}{d}-\frac{{\left(2{}\mathrm{i}\right)}^{1/3}\,a^{4/3}\,\ln\left(a^2\,d^2\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{1/3}\,18{}\mathrm{i}+18\,{\left(2{}\mathrm{i}\right)}^{1/3}\,a^{7/3}\,d^2\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{d}+\frac{{\left(2{}\mathrm{i}\right)}^{1/3}\,a^{4/3}\,\ln\left(a^2\,d^2\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{1/3}\,18{}\mathrm{i}-18\,{\left(2{}\mathrm{i}\right)}^{1/3}\,a^{7/3}\,d^2\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(4/3),x)
```

```
[Out] (a*(a + a*tan(c + d*x)*1i)^(1/3)*3i)/d - (2i^(1/3)*a^(4/3)*log(a^2*d^2*(a +
 a*tan(c + d*x)*1i)^(1/3)*18i + 18*2i^(1/3)*a^(7/3)*d^2))/d - (2i^(1/3)*a^(
4/3)*log(a^2*d^2*(a + a*tan(c + d*x)*1i)^(1/3)*18i + 18*2i^(1/3)*a^(7/3)*d^
2*((3^(1/2)*1i)/2 - 1/2))*((3^(1/2)*1i)/2 - 1/2))/d + (2i^(1/3)*a^(4/3)*log
(a^2*d^2*(a + a*tan(c + d*x)*1i)^(1/3)*18i - 18*2i^(1/3)*a^(7/3)*d^2*((3^(1
/2)*1i)/2 + 1/2))*((3^(1/2)*1i)/2 + 1/2))/d
```

3.284 $\int \cot(c+dx)(a+ia\tan(c+dx))^{4/3}\,dx$

Optimal. Leaf size=254

$$-\frac{ia^{4/3}x}{2^{2/3}}-\frac{\sqrt{3}\,a^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{d}+\frac{\sqrt[3]{2}\,\sqrt{3}\,a^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{d}$$

```
[Out] -1/2*I*a^(4/3)*x*2^(1/3)-1/2*a^(4/3)*ln(cos(d*x+c))*2^(1/3)/d-1/2*a^(4/3)*l
n(tan(d*x+c))/d+3/2*a^(4/3)*ln(a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))/d-3/2*a^(4
/3)*ln(2^(1/3)*a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))*2^(1/3)/d-a^(4/3)*arctan(1
/3*(a^(1/3)+2*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)/d+2^(1/3)*
a^(4/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/
2))*3^(1/2)/d
```

Rubi [A]
time = 0.31, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3643, 3559, 3562, 59, 631, 210, 31, 3675, 3680}

$$-\frac{\sqrt{3}\,a^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{d}+\frac{\sqrt[3]{2}\,\sqrt{3}\,a^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{d}-\frac{a^{4/3}\log(\tan(c+dx))}{2d}+\frac{3a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{2d}-\frac{3a^{4/3}\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{2^{2/3}d}-\frac{a^{4/3}\log(\cos(c+dx))}{2^{2/3}d}-\frac{ia^{4/3}x}{2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^(4/3),x]
```

```
[Out] ((-I)*a^(4/3)*x)/2^(2/3) - (Sqrt[3]*a^(4/3)*ArcTan[(a^(1/3) + 2*(a + I*a*Ta
n[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/d + (2^(1/3)*Sqrt[3]*a^(4/3)*ArcTan[
(a^(1/3) + 2^(2/3)*(a + I*a*Tan[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/d - (a
^(4/3)*Log[Cos[c + d*x]])/(2^(2/3)*d) - (a^(4/3)*Log[Tan[c + d*x]])/(2*d) +
 (3*a^(4/3)*Log[a^(1/3) - (a + I*a*Tan[c + d*x])^(1/3)])/(2*d) - (3*a^(4/3)
*Log[2^(1/3)*a^(1/3) - (a + I*a*Tan[c + d*x])^(1/3)])/(2^(2/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3643

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)/((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[a/(a*c - b*d), Int[(a + b*Tan[e + f*x])^m,
x], x] - Dist[d/(a*c - b*d), Int[(a + b*Tan[e + f*x])^m*((b + a*Tan[e + f*x
])/(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3675

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[
e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && G
tQ[m, 1] &&  !LtQ[n, -1]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
```

```
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rubi steps

$$\int \cot(c+dx)(a+ia\tan(c+dx))^{4/3}\,dx = i\int (a+ia\tan(c+dx))^{4/3}\,dx - \frac{i\int \cot(c+dx)(a+ia\tan(c+dx)}{a}$$

$$= -\frac{(3i)\int \cot(c+dx)\sqrt[3]{a+ia\tan(c+dx)}\left(\frac{ia^2}{3}+\frac{1}{3}a^2\tan(c+dx)\right)}{a}$$

$$= \frac{a^2\text{Subst}\left(\int \frac{1}{x(a+iax)^{2/3}}\,dx, x, \tan(c+dx)\right)}{d} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{(a-x)(a}\right.}{}$$

$$= -\frac{ia^{4/3}x}{2^{2/3}} - \frac{a^{4/3}\log(\cos(c+dx))}{2^{2/3}d} - \frac{a^{4/3}\log(\tan(c+dx))}{2d} - \frac{(3a^{4/3}}{}$$

$$= -\frac{ia^{4/3}x}{2^{2/3}} - \frac{a^{4/3}\log(\cos(c+dx))}{2^{2/3}d} - \frac{a^{4/3}\log(\tan(c+dx))}{2d} + \frac{3a^{4/3}}{}$$

$$= -\frac{ia^{4/3}x}{2^{2/3}} - \frac{\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{d} + \frac{\sqrt[3]{2}\,\sqrt{}}{}$$

Mathematica [A]

time = 4.35, size = 411, normalized size = 1.62

$$\frac{ae^{-\frac{2}{3}i(c+dx)}\sqrt[3]{1+e^{2i(c+dx)}}\left(4\sqrt{3}\,\text{ArcTan}\left(\frac{1+\frac{2e^{\frac{2}{3}i(c+dx)}}{\sqrt[3]{1+e^{2i(c+dx)}}}}{\sqrt{3}}\right)-2\,2^{2/3}\sqrt{3}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{2}\,e^{\frac{2}{3}i(c+dx)}}{\sqrt[3]{1+e^{2i(c+dx)}}}}{\sqrt{3}}\right)-4\log\left(1-\frac{e^{\frac{2}{3}i(c+dx)}}{\sqrt[3]{1+e^{2i(c+dx)}}}\right)+2\,2^{2/3}\log\left(1-\frac{\sqrt[3]{2}\,e^{\frac{2}{3}i(c+dx)}}{\sqrt[3]{1+e^{2i(c+dx)}}}\right)-2^{2/3}\log\left(1+\frac{2^{2/3}e^{\frac{4}{3}i(c+dx)}}{(1+e^{2i(c+dx)})^{2/3}}+\frac{\sqrt[3]{2}\,e^{\frac{2}{3}i(c+dx)}}{\sqrt[3]{1+e^{2i(c+dx)}}}\right)+2\log\left(\frac{e^{\frac{4}{3}i(c+dx)}+e^{\frac{2}{3}i(c+dx)}\sqrt[3]{1+e^{2i(c+dx)}}+(1+e^{2i(c+dx)})^{2/3}}{(1+e^{2i(c+dx)})^{2/3}}\right)\right)\sqrt[3]{a+ia\tan(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^(4/3),x]
```

```
[Out] (a*(1 + E^((2*I)*(c + d*x)))^(1/3)*(4*Sqrt[3]*ArcTan[(1 + (2*E^(((2*I)/3)*(
c + d*x)))/(1 + E^((2*I)*(c + d*x)))^(1/3))/Sqrt[3]] - 2*2^(2/3)*Sqrt[3]*Ar
cTan[(1 + (2*2^(1/3)*E^(((2*I)/3)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))^(1/
3))/Sqrt[3]] - 4*Log[1 - E^(((2*I)/3)*(c + d*x))/(1 + E^((2*I)*(c + d*x)))^
(1/3)] + 2*2^(2/3)*Log[1 - (2^(1/3)*E^(((2*I)/3)*(c + d*x)))/(1 + E^((2*I)*
(c + d*x)))^(1/3)] - 2^(2/3)*Log[1 + (2^(2/3)*E^(((4*I)/3)*(c + d*x)))/(1 +
 E^((2*I)*(c + d*x)))^(2/3) + (2^(1/3)*E^(((2*I)/3)*(c + d*x)))/(1 + E^((2*
```

```
I)*(c + d*x)))^(1/3)] + 2*Log[(E^(((4*I)/3)*(c + d*x)) + E^(((2*I)/3)*(c +
d*x))*(1 + E^((2*I)*(c + d*x)))^(1/3) + (1 + E^((2*I)*(c + d*x)))^(2/3))/(1
 + E^((2*I)*(c + d*x)))^(2/3)])*(a + I*a*Tan[c + d*x])^(1/3))/(4*d*E^(((2*I
)/3)*(c + d*x)))
```

Maple [F]
time = 0.42, size = 0, normalized size = 0.00

$$\int \cot\left(dx+c\right)\left(a+ia\tan\left(dx+c\right)\right)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+I*a*tan(d*x+c))^(4/3),x)
```

```
[Out] int(cot(d*x+c)*(a+I*a*tan(d*x+c))^(4/3),x)
```

Maxima [A]
time = 0.55, size = 233, normalized size = 0.92

$$\frac{2\sqrt{3}\,2^{\frac{1}{3}}a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(ia\tan(dx+c)+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)-2\sqrt{3}\,a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(ia\tan(dx+c)+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)+2^{\frac{1}{3}}a^{\frac{4}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(ia\tan\left(dx+c\right)+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(ia\tan\left(dx+c\right)+a\right)^{\frac{2}{3}}\right)-2\cdot 2^{\frac{1}{3}}a^{\frac{4}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(ia\tan\left(dx+c\right)+a\right)^{\frac{1}{3}}\right)-a^{\frac{4}{3}}\log\left(\left(ia\tan\left(dx+c\right)+a\right)^{\frac{2}{3}}+\left(ia\tan\left(dx+c\right)+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2a^{\frac{4}{3}}\log\left(\left(ia\tan\left(dx+c\right)+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sqrt(3)*2^(1/3)*a^(4/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3)
+ 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) - 2*sqrt(3)*a^(4/3)*arctan(1/3*s
qrt(3)*(2*(I*a*tan(d*x + c) + a)^(1/3) + a^(1/3))/a^(1/3)) + 2^(1/3)*a^(4/3
)*log(2^(2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a
*tan(d*x + c) + a)^(2/3)) - 2*2^(1/3)*a^(4/3)*log(-2^(1/3)*a^(1/3) + (I*a*t
an(d*x + c) + a)^(1/3)) - a^(4/3)*log((I*a*tan(d*x + c) + a)^(2/3) + (I*a*t
an(d*x + c) + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*a^(4/3)*log((I*a*tan(d*x + c)
 + a)^(1/3) - a^(1/3)))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(188) = 376$.
time = 1.16, size = 446, normalized size = 1.76

$$\frac{1}{2}\cdot 2^{\frac{1}{3}}\left(-\frac{a^4}{d^3}\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)\log\left(\frac{2\cdot 2^{\frac{1}{3}}a\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}+2^{\frac{1}{3}}\left(i\sqrt{3}\,d-d\right)\left(-\frac{a^4}{d^3}\right)^{\frac{1}{3}}}{2a}\right)+\frac{1}{2}\cdot 2^{\frac{1}{3}}\left(-\frac{a^4}{d^3}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\log\left(\frac{2\cdot 2^{\frac{1}{3}}a\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}+2^{\frac{1}{3}}\left(-i\sqrt{3}\,d-d\right)\left(-\frac{a^4}{d^3}\right)^{\frac{1}{3}}}{2a}\right)+\frac{1}{2}\left(\frac{a^4}{d^3}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\log\left(\frac{2\cdot 2^{\frac{1}{3}}a\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}+\left(i\sqrt{3}\,d+d\right)\left(\frac{a^4}{d^3}\right)^{\frac{1}{3}}}{2a}\right)+\frac{1}{2}\left(\frac{a^4}{d^3}\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)\log\left(\frac{2\cdot 2^{\frac{1}{3}}a\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}+\left(-i\sqrt{3}\,d+d\right)\left(\frac{a^4}{d^3}\right)^{\frac{1}{3}}}{2a}\right)+2^{\frac{1}{3}}\left(-\frac{a^4}{d^3}\right)^{\frac{1}{3}}\log\left(\frac{2^{\frac{1}{3}}a\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}+2^{\frac{1}{3}}\left(-\frac{a^4}{d^3}\right)^{\frac{1}{3}}d}{a}\right)+\left(\frac{a^4}{d^3}\right)^{\frac{1}{3}}\log\left(\frac{2^{\frac{1}{3}}a\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}-\left(\frac{a^4}{d^3}\right)^{\frac{1}{3}}d}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] 1/2*2^(1/3)*(-a^4/d^3)^(1/3)*(I*sqrt(3) - 1)*log(1/2*(2*2^(1/3)*a*(a/(e^(2*
I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + 2^(1/3)*(I*sqrt(3)*d -
 d)*(-a^4/d^3)^(1/3))/a) + 1/2*2^(1/3)*(-a^4/d^3)^(1/3)*(-I*sqrt(3) - 1)*lo
g(1/2*(2*2^(1/3)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I
```

```
*c) + 2^(1/3)*(-I*sqrt(3)*d - d)*(-a^4/d^3)^(1/3))/a) + 1/2*(a^4/d^3)^(1/3)
*(-I*sqrt(3) - 1)*log(1/2*(2*2^(1/3)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*
e^(2/3*I*d*x + 2/3*I*c) + (I*sqrt(3)*d + d)*(a^4/d^3)^(1/3))/a) + 1/2*(a^4/
d^3)^(1/3)*(I*sqrt(3) - 1)*log(1/2*(2*2^(1/3)*a*(a/(e^(2*I*d*x + 2*I*c) + 1
))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + (-I*sqrt(3)*d + d)*(a^4/d^3)^(1/3))/a) +
 2^(1/3)*(-a^4/d^3)^(1/3)*log((2^(1/3)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3
)*e^(2/3*I*d*x + 2/3*I*c) + 2^(1/3)*(-a^4/d^3)^(1/3)*d)/a) + (a^4/d^3)^(1/3
)*log((2^(1/3)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c
) - (a^4/d^3)^(1/3)*d)/a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan{(c+dx)}-i))^{\frac{4}{3}}\cot{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))**(4/3),x)
[Out] Integral((I*a*(tan(c + d*x) - I))**(4/3)*cot(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(4/3)*cot(d*x + c), x)
```

Mupad [B]
time = 4.30, size = 369, normalized size = 1.45

$$\ln\left(-d\left(\frac{a^4}{d^3}\right)^{1/3}+a\,(a\,(1+\tan(c+d\,x)\,1\mathrm{i}))^{1/3}\right)\left(\frac{a^4}{d^3}\right)^{1/3}+\ln\left(a\,(a\,(1+\tan(c+d\,x)\,1\mathrm{i}))^{1/3}+2^{1/3}\,d\left(-\frac{a^4}{d^3}\right)^{1/3}\right)\left(-\frac{2\,a^4}{d^3}\right)^{1/3}+\frac{\ln\left(d\left(\frac{a^4}{d^3}\right)^{1/3}+2\,a\,(a\,(1+\tan(c+d\,x)\,1\mathrm{i}))^{1/3}-\sqrt{3}\,d\left(\frac{a^4}{d^3}\right)^{1/3}1\mathrm{i}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)\left(\frac{a^4}{d^3}\right)^{1/3}}{2}-\frac{\ln\left(d\left(\frac{a^4}{d^3}\right)^{1/3}+2\,a\,(a\,(1+\tan(c+d\,x)\,1\mathrm{i}))^{1/3}+\sqrt{3}\,d\left(\frac{a^4}{d^3}\right)^{1/3}1\mathrm{i}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)\left(\frac{a^4}{d^3}\right)^{1/3}}{2}-\ln\left(2^{1/3}\,d\left(-\frac{a^4}{d^3}\right)^{1/3}-2\,a\,(a\,(1+\tan(c+d\,x)\,1\mathrm{i}))^{1/3}+2^{1/3}\,\sqrt{3}\,d\left(-\frac{a^4}{d^3}\right)^{1/3}1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{2\,a^4}{d^3}\right)^{1/3}+\ln\left(2\,a\,(a\,(1+\tan(c+d\,x)\,1\mathrm{i}))^{1/3}-2^{1/3}\,d\left(-\frac{a^4}{d^3}\right)^{1/3}+2^{1/3}\,\sqrt{3}\,d\left(-\frac{a^4}{d^3}\right)^{1/3}1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{2\,a^4}{d^3}\right)^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + a*tan(c + d*x)*1i)^(4/3),x)
[Out] log(a*(a*(tan(c + d*x)*1i + 1))^(1/3) - d*(a^4/d^3)^(1/3))*(a^4/d^3)^(1/3)
+ log(a*(a*(tan(c + d*x)*1i + 1))^(1/3) + 2^(1/3)*d*(-a^4/d^3)^(1/3))*(-(2*
a^4)/d^3)^(1/3) + (log(d*(a^4/d^3)^(1/3) + 2*a*(a*(tan(c + d*x)*1i + 1))^(1
/3) - 3^(1/2)*d*(a^4/d^3)^(1/3)*1i)*(3^(1/2)*1i - 1)*(a^4/d^3)^(1/3))/2 - (
log(d*(a^4/d^3)^(1/3) + 2*a*(a*(tan(c + d*x)*1i + 1))^(1/3) + 3^(1/2)*d*(a^
4/d^3)^(1/3)*1i)*(3^(1/2)*1i + 1)*(a^4/d^3)^(1/3))/2 - log(2^(1/3)*d*(-a^4/
d^3)^(1/3) - 2*a*(a*(tan(c + d*x)*1i + 1))^(1/3) + 2^(1/3)*3^(1/2)*d*(-a^4/
d^3)^(1/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*(-(2*a^4)/d^3)^(1/3) + log(2*a*(a*(ta
n(c + d*x)*1i + 1))^(1/3) - 2^(1/3)*d*(-a^4/d^3)^(1/3) + 2^(1/3)*3^(1/2)*d*
(-a^4/d^3)^(1/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(-(2*a^4)/d^3)^(1/3)
```

3.285 $\int \cot^2(c+dx)(a+ia\tan(c+dx))^{4/3}\,dx$

Optimal. Leaf size=315

$$\frac{a^{4/3}x}{2^{2/3}}-\frac{4ia^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,d}+\frac{i\sqrt[3]{2}\sqrt{3}\,a^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{d}$$

```
[Out] 1/2*a^(4/3)*x*2^(1/3)-1/2*I*a^(4/3)*ln(cos(d*x+c))*2^(1/3)/d-2/3*I*a^(4/3)*
ln(tan(d*x+c))/d+2*I*a^(4/3)*ln(a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))/d-3/2*I*a
^(4/3)*ln(2^(1/3)*a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))*2^(1/3)/d-4/3*I*a^(4/3)
*arctan(1/3*(a^(1/3)+2*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))/d*3^(1/2)
+I*2^(1/3)*a^(4/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))/a^
(1/3)*3^(1/2))*3^(1/2)/d+I*a*(a+I*a*tan(d*x+c))^(1/3)/d-cot(d*x+c)*(a+I*a*t
an(d*x+c))^(4/3)/d
```

Rubi [A]
time = 0.39, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3642, 3675, 3681, 3562, 59, 631, 210, 31, 3680}

$$-\frac{4ia^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,d}+\frac{i\sqrt[3]{2}\sqrt{3}\,a^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{d}-\frac{2ia^{4/3}\log(\tan(c+dx))}{3d}+\frac{2ia^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{d}-\frac{3ia^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{2^{2/3}d}-\frac{ia^{4/3}\log(\cos(c+dx))}{2^{2/3}d}+\frac{a^{4/3}x}{2^{2/3}}+\frac{ia\sqrt[3]{a+ia\tan(c+dx)}}{d}-\frac{\cot(c+dx)(a+ia\tan(c+dx))^{4/3}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^(4/3),x]
```

```
[Out] (a^(4/3)*x)/2^(2/3) - ((4*I)*a^(4/3)*ArcTan[(a^(1/3) + 2*(a + I*a*Tan[c + d
*x])^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*d) + (I*2^(1/3)*Sqrt[3]*a^(4/3)*Ar
cTan[(a^(1/3) + 2^(2/3)*(a + I*a*Tan[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/d
 - (I*a^(4/3)*Log[Cos[c + d*x]])/(2^(2/3)*d) - (((2*I)/3)*a^(4/3)*Log[Tan[c
 + d*x]])/d + ((2*I)*a^(4/3)*Log[a^(1/3) - (a + I*a*Tan[c + d*x])^(1/3)])/d
 - ((3*I)*a^(4/3)*Log[2^(1/3)*a^(1/3) - (a + I*a*Tan[c + d*x])^(1/3)])/(2^(
2/3)*d) + (I*a*(a + I*a*Tan[c + d*x])^(1/3))/d - (Cot[c + d*x]*(a + I*a*Tan
[c + d*x])^(4/3))/d
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
```

```
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3642

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[d*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(a*(c^2 + d^2)*(n +
1)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[b*d*m - a
*c*(n + 1) + a*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
 e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0
] && LtQ[n, -1] && (IntegerQ[n] || IntegersQ[2*m, 2*n])
```

Rule 3675

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[
e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && G
tQ[m, 1] &&  !LtQ[n, -1]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
```

```
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3681

```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(
A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^{4/3}\,dx = -\frac{\cot(c+dx)(a+ia\tan(c+dx))^{4/3}}{d} + \frac{\int \cot(c+dx)(a+ia\tan$$

$$= \frac{ia\sqrt[3]{a+ia\tan(c+dx)}}{d} - \frac{\cot(c+dx)(a+ia\tan(c+dx))^{4/3}}{d}$$

$$= \frac{ia\sqrt[3]{a+ia\tan(c+dx)}}{d} - \frac{\cot(c+dx)(a+ia\tan(c+dx))^{4/3}}{d}$$

$$= \frac{ia\sqrt[3]{a+ia\tan(c+dx)}}{d} - \frac{\cot(c+dx)(a+ia\tan(c+dx))^{4/3}}{d}$$

$$= \frac{a^{4/3}x}{2^{2/3}} - \frac{ia^{4/3}\log(\cos(c+dx))}{2^{2/3}d} - \frac{2ia^{4/3}\log(\tan(c+dx))}{3d} + \frac{ia\sqrt[3]{}}{}$$

$$= \frac{a^{4/3}x}{2^{2/3}} - \frac{ia^{4/3}\log(\cos(c+dx))}{2^{2/3}d} - \frac{2ia^{4/3}\log(\tan(c+dx))}{3d} + \frac{2ia}{}$$

$$= \frac{a^{4/3}x}{2^{2/3}} - \frac{4ia^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,d} + \frac{i\sqrt[3]{2}\sqrt{3}}{}$$

Mathematica [A]
time = 7.76, size = 587, normalized size = 1.86

$$\frac{ie^{-\frac{1}{3}i(5c+2dx)}\sqrt[3]{e^{idx}}\sqrt[3]{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt[3]{1+e^{2i(c+dx)}}\left(6\sqrt{3}\,\text{ArcTan}\left(\frac{1+\frac{2e^{\frac{2}{3}i(c+dx)}}{\sqrt[3]{1+e^{2i(c+dx)}}}}{\sqrt{3}}\right)-4\,2^{2/3}\sqrt{3}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{2}\,e^{\frac{2}{3}i(c+dx)}}{\sqrt[3]{1+e^{2i(c+dx)}}}}{\sqrt{3}}\right)-6\log\left(1-\frac{e^{\frac{2}{3}i(c+dx)}}{\sqrt[3]{1+e^{2i(c+dx)}}}\right)+4\,2^{2/3}\log\left(1-\frac{\sqrt[3]{2}\,e^{\frac{2}{3}i(c+dx)}}{\sqrt[3]{1+e^{2i(c+dx)}}}\right)+3\log\left(\frac{e^{\frac{4}{3}i(c+dx)}+e^{\frac{2}{3}i(c+dx)}\sqrt[3]{1+e^{2i(c+dx)}}+(1+e^{2i(c+dx)})^{2/3}}{(1+e^{2i(c+dx)})^{2/3}}\right)-2\,2^{2/3}\log\left(\frac{2^{2/3}e^{\frac{4}{3}i(c+dx)}+\sqrt[3]{2}\,e^{\frac{2}{3}i(c+dx)}\sqrt[3]{1+e^{2i(c+dx)}}+(1+e^{2i(c+dx)})^{2/3}}{(1+e^{2i(c+dx)})^{2/3}}\right)\right)(a+ia\tan(c+dx))^{4/3}}{3\,2^{2/3}d\sec^{\frac{4}{3}}(c+dx)(\cos(dx)+i\sin(dx))^{4/3}}+\frac{\cos(c+dx)(\cot(c)(-\cos(c)+i\sin(c))+\csc(c)\csc(c+dx)(\cos(c)-i\sin(c))\sin(dx))(a+ia\tan(c+dx))^{4/3}}{d(\cos(dx)+i\sin(dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^(4/3),x]
```

```
[Out] ((I/3)*(E^(I*d*x))^(1/3)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(1/3)*
(1 + E^((2*I)*(c + d*x)))^(1/3)*(6*Sqrt[3]*ArcTan[(1 + (2*E^(((2*I)/3)*(c +
 d*x)))/(1 + E^((2*I)*(c + d*x)))^(1/3))/Sqrt[3]] - 4*2^(2/3)*Sqrt[3]*ArcTa
n[(1 + (2*2^(1/3)*E^(((2*I)/3)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))^(1/3))
/Sqrt[3]] - 6*Log[1 - E^(((2*I)/3)*(c + d*x))/(1 + E^((2*I)*(c + d*x)))^(1/
3)] + 4*2^(2/3)*Log[1 - (2^(1/3)*E^(((2*I)/3)*(c + d*x)))/(1 + E^((2*I)*(c
+ d*x)))^(1/3)] + 3*Log[(E^(((4*I)/3)*(c + d*x)) + E^(((2*I)/3)*(c + d*x))*
(1 + E^((2*I)*(c + d*x)))^(1/3) + (1 + E^((2*I)*(c + d*x)))^(2/3))/(1 + E^(
(2*I)*(c + d*x)))^(2/3)] - 2*2^(2/3)*Log[(2^(2/3)*E^(((4*I)/3)*(c + d*x)) +
 2^(1/3)*E^(((2*I)/3)*(c + d*x))*(1 + E^((2*I)*(c + d*x)))^(1/3) + (1 + E^(
(2*I)*(c + d*x)))^(2/3))/(1 + E^((2*I)*(c + d*x)))^(2/3)])*(a + I*a*Tan[c +
 d*x])^(4/3))/(2^(2/3)*d*E^((I/3)*(5*c + 2*d*x))*Sec[c + d*x]^(4/3)*(Cos[d*
x] + I*Sin[d*x])^(4/3)) + (Cos[c + d*x]*(Cot[c]*(-Cos[c] + I*Sin[c]) + Csc[
c]*Csc[c + d*x]*(Cos[c] - I*Sin[c])*Sin[d*x])*(a + I*a*Tan[c + d*x])^(4/3))
/(d*(Cos[d*x] + I*Sin[d*x]))
```

Maple [F]
time = 0.36, size = 0, normalized size = 0.00

$$\int \left(\cot^{2}\left(dx+c\right)\right) \left(a+ia\tan\left(dx+c\right)\right)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(4/3),x)
```

```
[Out] int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(4/3),x)
```

Maxima [A]
time = 0.55, size = 258, normalized size = 0.82

$$\frac{i\left(6\sqrt{3}\,2^{\frac{1}{3}}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(ia\tan(dx+c)+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)-8\sqrt{3}\,a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(ia\tan(dx+c)+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)+3\cdot 2^{\frac{1}{3}}a^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(ia\tan\left(dx+c\right)+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(ia\tan\left(dx+c\right)+a\right)^{\frac{2}{3}}\right)-6\cdot 2^{\frac{1}{3}}a^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(ia\tan\left(dx+c\right)+a\right)^{\frac{1}{3}}\right)-4a^{\frac{1}{3}}\log\left(\left(ia\tan\left(dx+c\right)+a\right)^{\frac{2}{3}}+\left(ia\tan\left(dx+c\right)+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+8a^{\frac{1}{3}}\log\left(\left(ia\tan\left(dx+c\right)+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+\frac{6i\left(ia\tan(dx+c)+a\right)^{\frac{1}{3}}}{\tan(dx+c)}\right)a}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] 1/6*I*(6*sqrt(3)*2^(1/3)*a^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3
) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) - 8*sqrt(3)*a^(1/3)*arctan(1/3
*sqrt(3)*(2*(I*a*tan(d*x + c) + a)^(1/3) + a^(1/3))/a^(1/3)) + 3*2^(1/3)*a^
(1/3)*log(2^(2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) +
(I*a*tan(d*x + c) + a)^(2/3)) - 6*2^(1/3)*a^(1/3)*log(-2^(1/3)*a^(1/3) + (I
*a*tan(d*x + c) + a)^(1/3)) - 4*a^(1/3)*log((I*a*tan(d*x + c) + a)^(2/3) +
(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + a^(2/3)) + 8*a^(1/3)*log((I*a*tan(d*
```

```
x + c) + a)^(1/3) - a^(1/3)) + 6*I*(I*a*tan(d*x + c) + a)^(1/3)/tan(d*x + c
))*a/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 622 vs. $2(232) = 464$.
time = 1.46, size = 622, normalized size = 1.97

$$-\frac{2\cdot 2^{\frac{1}{3}}\left(i a e^{(2 i d x+2 i c)}+i a\right)\left(\frac{a}{e^{(2 i d x+2 i c)}+1}\right)^{\frac{1}{3}} e^{\left(\frac{2}{3} i d x+\frac{2}{3} i c\right)}-\left(\left(i \sqrt{3} d-d\right) e^{(2 i d x+2 i c)}-i \sqrt{3} d+d\right)\left(-\frac{64 i a^{4}}{27 d^{3}}\right)^{\frac{1}{3}} \log\left(\frac{8\cdot 2^{\frac{1}{3}} a\left(\frac{a}{e^{(2 i d x+2 i c)}+1}\right)^{\frac{1}{3}} e^{\left(\frac{2}{3} i d x+\frac{2}{3} i c\right)}-3\left(\sqrt{3} d+i d\right)\left(-\frac{64 i a^{4}}{27 d^{3}}\right)^{\frac{1}{3}}}{8 a}\right)-\left(\left(-i \sqrt{3} d-d\right) e^{(2 i d x+2 i c)}+i \sqrt{3} d+d\right)\left(-\frac{64 i a^{4}}{27 d^{3}}\right)^{\frac{1}{3}} \log\left(\frac{8\cdot 2^{\frac{1}{3}} a\left(\frac{a}{e^{(2 i d x+2 i c)}+1}\right)^{\frac{1}{3}} e^{\left(\frac{2}{3} i d x+\frac{2}{3} i c\right)}+3\left(\sqrt{3} d-i d\right)\left(-\frac{64 i a^{4}}{27 d^{3}}\right)^{\frac{1}{3}}}{8 a}\right)-2\left(d e^{(2 i d x+2 i c)}-d\right)\left(-\frac{64 i a^{4}}{27 d^{3}}\right)^{\frac{1}{3}} \log\left(\frac{4\cdot 2^{\frac{1}{3}} a\left(\frac{a}{e^{(2 i d x+2 i c)}+1}\right)^{\frac{1}{3}} e^{\left(\frac{2}{3} i d x+\frac{2}{3} i c\right)}+3 i\left(-\frac{64 i a^{4}}{27 d^{3}}\right)^{\frac{1}{3}} d}{4 a}\right)-\left(\left(i \sqrt{3} d-d\right) e^{(2 i d x+2 i c)}-i \sqrt{3} d+d\right)\left(\frac{2 i a^{4}}{d^{3}}\right)^{\frac{1}{3}} \log\left(\frac{2\cdot 2^{\frac{1}{3}} a\left(\frac{a}{e^{(2 i d x+2 i c)}+1}\right)^{\frac{1}{3}} e^{\left(\frac{2}{3} i d x+\frac{2}{3} i c\right)}+\left(\sqrt{3} d+i d\right)\left(\frac{2 i a^{4}}{d^{3}}\right)^{\frac{1}{3}}}{2 a}\right)-\left(\left(-i \sqrt{3} d-d\right) e^{(2 i d x+2 i c)}+i \sqrt{3} d+d\right)\left(\frac{2 i a^{4}}{d^{3}}\right)^{\frac{1}{3}} \log\left(\frac{2\cdot 2^{\frac{1}{3}} a\left(\frac{a}{e^{(2 i d x+2 i c)}+1}\right)^{\frac{1}{3}} e^{\left(\frac{2}{3} i d x+\frac{2}{3} i c\right)}-\left(\sqrt{3} d-i d\right)\left(\frac{2 i a^{4}}{d^{3}}\right)^{\frac{1}{3}}}{2 a}\right)-2\left(d e^{(2 i d x+2 i c)}-d\right)\left(\frac{2 i a^{4}}{d^{3}}\right)^{\frac{1}{3}} \log\left(\frac{2^{\frac{1}{3}} a\left(\frac{a}{e^{(2 i d x+2 i c)}+1}\right)^{\frac{1}{3}} e^{\left(\frac{2}{3} i d x+\frac{2}{3} i c\right)}-i\left(\frac{2 i a^{4}}{d^{3}}\right)^{\frac{1}{3}} d}{a}\right)}{2\left(d e^{(2 i d x+2 i c)}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*2^(1/3)*(I*a*e^(2*I*d*x + 2*I*c) + I*a)*(a/(e^(2*I*d*x + 2*I*c) + 1
))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) - ((I*sqrt(3)*d - d)*e^(2*I*d*x + 2*I*c) -
 I*sqrt(3)*d + d)*(-64/27*I*a^4/d^3)^(1/3)*log(1/8*(8*2^(1/3)*a*(a/(e^(2*I*
d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) - 3*(sqrt(3)*d + I*d)*(-64
/27*I*a^4/d^3)^(1/3))/a) - ((-I*sqrt(3)*d - d)*e^(2*I*d*x + 2*I*c) + I*sqrt
(3)*d + d)*(-64/27*I*a^4/d^3)^(1/3)*log(1/8*(8*2^(1/3)*a*(a/(e^(2*I*d*x + 2
*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + 3*(sqrt(3)*d - I*d)*(-64/27*I*a
^4/d^3)^(1/3))/a) - 2*(d*e^(2*I*d*x + 2*I*c) - d)*(-64/27*I*a^4/d^3)^(1/3)*
log(1/4*(4*2^(1/3)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3
*I*c) + 3*I*(-64/27*I*a^4/d^3)^(1/3)*d)/a) - ((I*sqrt(3)*d - d)*e^(2*I*d*x
+ 2*I*c) - I*sqrt(3)*d + d)*(2*I*a^4/d^3)^(1/3)*log(1/2*(2*2^(1/3)*a*(a/(e^
(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + (sqrt(3)*d + I*d)*(
2*I*a^4/d^3)^(1/3))/a) - ((-I*sqrt(3)*d - d)*e^(2*I*d*x + 2*I*c) + I*sqrt(3
)*d + d)*(2*I*a^4/d^3)^(1/3)*log(1/2*(2*2^(1/3)*a*(a/(e^(2*I*d*x + 2*I*c) +
 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) - (sqrt(3)*d - I*d)*(2*I*a^4/d^3)^(1/3))
/a) - 2*(d*e^(2*I*d*x + 2*I*c) - d)*(2*I*a^4/d^3)^(1/3)*log((2^(1/3)*a*(a/(
e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) - I*(2*I*a^4/d^3)^(
1/3)*d)/a))/(d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan (c+dx)-i))^{\frac{4}{3}} \cot^2 (c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+I*a*tan(d*x+c))**(4/3),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(4/3)*cot(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(4/3)*cot(d*x + c)^2, x)
```

Mupad [B]
time = 0.53, size = 855, normalized size = 2.71

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(4/3),x)
```

```
[Out] log(((1458*a^7*d^6*((a^4*2i)/d^3)^(1/3) - a^8*d^5*(a + a*tan(c + d*x)*1i)^(
1/3)*810i)*((a^4*2i)/d^3)^(2/3) - a^11*d^3*792i)*((a^4*2i)/d^3)^(1/3) - 374
4*a^12*d^2*(a + a*tan(c + d*x)*1i)^(1/3))*((a^4*2i)/d^3)^(1/3) + log(((1458
*a^7*d^6*(-(a^4*64i)/(27*d^3))^(1/3) - a^8*d^5*(a + a*tan(c + d*x)*1i)^(1/3
)*810i)*(-(a^4*64i)/(27*d^3))^(2/3) - a^11*d^3*792i)*(-(a^4*64i)/(27*d^3))^
(1/3) - 3744*a^12*d^2*(a + a*tan(c + d*x)*1i)^(1/3))*(-(a^4*64i)/(27*d^3))^
(1/3) + (log(3744*a^12*d^2*(a + a*tan(c + d*x)*1i)^(1/3) + ((3^(1/2)*1i - 1
)*(a^11*d^3*792i + ((3^(1/2)*1i - 1)^2*(a^8*d^5*(a + a*tan(c + d*x)*1i)^(1/
3)*810i - 729*a^7*d^6*(3^(1/2)*1i - 1)*((a^4*2i)/d^3)^(1/3))*((a^4*2i)/d^3)
^(2/3))/4)*((a^4*2i)/d^3)^(1/3))/2)*(3^(1/2)*1i - 1)*((a^4*2i)/d^3)^(1/3))/
2 - (log(3744*a^12*d^2*(a + a*tan(c + d*x)*1i)^(1/3) - ((3^(1/2)*1i + 1)*(a
^11*d^3*792i + ((3^(1/2)*1i + 1)^2*(a^8*d^5*(a + a*tan(c + d*x)*1i)^(1/3)*8
10i + 729*a^7*d^6*(3^(1/2)*1i + 1)*((a^4*2i)/d^3)^(1/3))*((a^4*2i)/d^3)^(2/
3))/4)*((a^4*2i)/d^3)^(1/3))/2)*(3^(1/2)*1i + 1)*((a^4*2i)/d^3)^(1/3))/2 +
(log(3744*a^12*d^2*(a + a*tan(c + d*x)*1i)^(1/3) + ((3^(1/2)*1i - 1)*(a^11*
d^3*792i + ((3^(1/2)*1i - 1)^2*(a^8*d^5*(a + a*tan(c + d*x)*1i)^(1/3)*810i
- 729*a^7*d^6*(3^(1/2)*1i - 1)*(-(a^4*64i)/(27*d^3))^(1/3))*(-(a^4*64i)/(27
*d^3))^(2/3))/4)*(-(a^4*64i)/(27*d^3))^(1/3))/2)*(3^(1/2)*1i - 1)*(-(a^4*64
i)/(27*d^3))^(1/3))/2 - (log(3744*a^12*d^2*(a + a*tan(c + d*x)*1i)^(1/3) -
((3^(1/2)*1i + 1)*(a^11*d^3*792i + ((3^(1/2)*1i + 1)^2*(a^8*d^5*(a + a*tan(
c + d*x)*1i)^(1/3)*810i + 729*a^7*d^6*(3^(1/2)*1i + 1)*(-(a^4*64i)/(27*d^3)
)^(1/3))*(-(a^4*64i)/(27*d^3))^(2/3))/4)*(-(a^4*64i)/(27*d^3))^(1/3))/2)*(3
^(1/2)*1i + 1)*(-(a^4*64i)/(27*d^3))^(1/3))/2 - (a*(a + a*tan(c + d*x)*1i)^
(1/3))/(d*tan(c + d*x))
```

3.286 $\int \cot^3(c+dx)(a+ia\tan(c+dx))^{4/3}\,dx$

Optimal. Leaf size=321

$$\frac{ia^{4/3}x}{2^{2/3}}+\frac{11a^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,d}-\frac{\sqrt[3]{2}\sqrt{3}\,a^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx}}{\sqrt{3}\sqrt[3]{a}}\right.}{d}$$

```
[Out] 1/2*I*a^(4/3)*x*2^(1/3)+1/2*a^(4/3)*ln(cos(d*x+c))*2^(1/3)/d+11/18*a^(4/3)*
ln(tan(d*x+c))/d-11/6*a^(4/3)*ln(a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))/d+3/2*a^
(4/3)*ln(2^(1/3)*a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))*2^(1/3)/d+11/9*a^(4/3)*a
rctan(1/3*(a^(1/3)+2*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)/d-2
^(1/3)*a^(4/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3
)*3^(1/2))*3^(1/2)/d-2/3*I*a*cot(d*x+c)*(a+I*a*tan(d*x+c))^(1/3)/d-1/2*cot(
d*x+c)^2*(a+I*a*tan(d*x+c))^(4/3)/d
```

Rubi [A]
time = 0.39, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3642, 3674, 3681, 3562, 59, 631, 210, 31, 3680}

$$\frac{11a^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,d}-\frac{\sqrt[3]{2}\sqrt{3}\,a^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{d}+\frac{11a^{4/3}\log(\tan(c+dx))}{18d}-\frac{11a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{6d}+\frac{3a^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{2^{2/3}d}+\frac{a^{4/3}\log(\cos(c+dx))}{2^{2/3}d}+\frac{ia^{4/3}x}{2^{2/3}}-\frac{\cot^2(c+dx)(a+ia\tan(c+dx))^{4/3}}{2d}-\frac{2ia\cot(c+dx)\sqrt[3]{a+ia\tan(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + I*a*Tan[c + d*x])^(4/3),x]
```

```
[Out] (I*a^(4/3)*x)/2^(2/3) + (11*a^(4/3)*ArcTan[(a^(1/3) + 2*(a + I*a*Tan[c + d*
x])^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*d) - (2^(1/3)*Sqrt[3]*a^(4/3)*Arc
Tan[(a^(1/3) + 2^(2/3)*(a + I*a*Tan[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/d
+ (a^(4/3)*Log[Cos[c + d*x]])/(2^(2/3)*d) + (11*a^(4/3)*Log[Tan[c + d*x]])/
(18*d) - (11*a^(4/3)*Log[a^(1/3) - (a + I*a*Tan[c + d*x])^(1/3)])/(6*d) + (
3*a^(4/3)*Log[2^(1/3)*a^(1/3) - (a + I*a*Tan[c + d*x])^(1/3)])/(2^(2/3)*d)
- (((2*I)/3)*a*Cot[c + d*x]*(a + I*a*Tan[c + d*x])^(1/3))/d - (Cot[c + d*x]
^2*(a + I*a*Tan[c + d*x])^(4/3))/(2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
```

```
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3642

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[d*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(a*(c^2 + d^2)*(n +
1)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[b*d*m - a
*c*(n + 1) + a*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
 e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0
] && LtQ[n, -1] && (IntegerQ[n] || IntegersQ[2*m, 2*n])
```

Rule 3674

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(
a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
- 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*
d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
```

```
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3681

```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(
A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^{4/3}\,dx = -\frac{\cot^2(c+dx)(a+ia\tan(c+dx))^{4/3}}{2d} + \frac{\int \cot^2(c+dx)\left(\frac{4ia}{3} - \frac{2}{3}\right.}{}$$

$$= -\frac{2ia\cot(c+dx)\sqrt[3]{a+ia\tan(c+dx)}}{3d} - \frac{\cot^2(c+dx)(a+ia\,\mathrm{t}}{2d}$$

$$= -\frac{2ia\cot(c+dx)\sqrt[3]{a+ia\tan(c+dx)}}{3d} - \frac{\cot^2(c+dx)(a+ia\,\mathrm{t}}{2d}$$

$$= -\frac{2ia\cot(c+dx)\sqrt[3]{a+ia\tan(c+dx)}}{3d} - \frac{\cot^2(c+dx)(a+ia\,\mathrm{t}}{2d}$$

$$= \frac{ia^{4/3}x}{2^{2/3}} + \frac{a^{4/3}\log(\cos(c+dx))}{2^{2/3}d} + \frac{11a^{4/3}\log(\tan(c+dx))}{18d} - \frac{2ia}{}$$

$$= \frac{ia^{4/3}x}{2^{2/3}} + \frac{a^{4/3}\log(\cos(c+dx))}{2^{2/3}d} + \frac{11a^{4/3}\log(\tan(c+dx))}{18d} - \frac{11a}{}$$

$$= \frac{ia^{4/3}x}{2^{2/3}} + \frac{11a^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,d} - \frac{\sqrt[3]{2}\,\sqrt{3}}{}$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[Cot[c + d*x]^3*(a + I*a*Tan[c + d*x])^(4/3),x]
```

```
[Out] $Aborted
```

Maple [F]
time = 0.39, size = 0, normalized size = 0.00

$$\int \left(\cot^{3}\left(dx+c\right)\right)\left(a+ia\tan\left(dx+c\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(4/3),x)
```

```
[Out] int(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(4/3),x)
```

Maxima [A]
time = 0.55, size = 304, normalized size = 0.95

$$-\frac{\left(\frac{18\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(ia\tan\left(dx+c\right)+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}-\frac{22\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(ia\tan\left(dx+c\right)+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}+\frac{9\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(ia\tan\left(dx+c\right)+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(ia\tan\left(dx+c\right)+a\right)^{\frac{2}{3}}\right)}{a^{\frac{2}{3}}}-\frac{18\cdot 2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(ia\tan\left(dx+c\right)+a\right)^{\frac{1}{3}}\right)}{a^{\frac{2}{3}}}-\frac{11\log\left(\left(ia\tan\left(dx+c\right)+a\right)^{\frac{2}{3}}+\left(ia\tan\left(dx+c\right)+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{2}{3}}}+\frac{22\log\left(\left(ia\tan\left(dx+c\right)+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{2}{3}}}-\frac{3\left(7\left(ia\tan\left(dx+c\right)+a\right)^{\frac{4}{3}}-4\left(ia\tan\left(dx+c\right)+a\right)^{\frac{1}{3}}a\right)}{\left(ia\tan\left(dx+c\right)+a\right)^{2}-2\left(ia\tan\left(dx+c\right)+a\right)a+a^{2}}\right)a^{2}}{18d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] -1/18*(18*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(
I*a*tan(d*x + c) + a)^(1/3))/a^(1/3))/a^(2/3) - 22*sqrt(3)*arctan(1/3*sqrt(
3)*(2*(I*a*tan(d*x + c) + a)^(1/3) + a^(1/3))/a^(1/3))/a^(2/3) + 9*2^(1/3)*
log(2^(2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*t
an(d*x + c) + a)^(2/3))/a^(2/3) - 18*2^(1/3)*log(-2^(1/3)*a^(1/3) + (I*a*ta
n(d*x + c) + a)^(1/3))/a^(2/3) - 11*log((I*a*tan(d*x + c) + a)^(2/3) + (I*a
*tan(d*x + c) + a)^(1/3)*a^(1/3) + a^(2/3))/a^(2/3) + 22*log((I*a*tan(d*x +
 c) + a)^(1/3) - a^(1/3))/a^(2/3) - 3*(7*(I*a*tan(d*x + c) + a)^(4/3) - 4*(
I*a*tan(d*x + c) + a)^(1/3)*a)/((I*a*tan(d*x + c) + a)^2 - 2*(I*a*tan(d*x +
 c) + a)*a + a^2))*a^2/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 744 vs. $2(240) = 480$.
time = 1.20, size = 744, normalized size = 2.32

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] 1/18*(6*2^(1/3)*(5*a*e^(4*I*d*x + 4*I*c) + 3*a*e^(2*I*d*x + 2*I*c) - 2*a)*(
a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) - 9*2^(1/3)*((I*
```

```
sqrt(3)*d + d)*e^(4*I*d*x + 4*I*c) + 2*(-I*sqrt(3)*d - d)*e^(2*I*d*x + 2*I*
c) + I*sqrt(3)*d + d)*(a^4/d^3)^(1/3)*log(1/2*(2*2^(1/3)*a*(a/(e^(2*I*d*x +
 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + 2^(1/3)*(I*sqrt(3)*d + d)*(a^
4/d^3)^(1/3))/a) - 9*2^(1/3)*((-I*sqrt(3)*d + d)*e^(4*I*d*x + 4*I*c) + 2*(I
*sqrt(3)*d - d)*e^(2*I*d*x + 2*I*c) - I*sqrt(3)*d + d)*(a^4/d^3)^(1/3)*log(
1/2*(2*2^(1/3)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c
) + 2^(1/3)*(-I*sqrt(3)*d + d)*(a^4/d^3)^(1/3))/a) + 18*2^(1/3)*(d*e^(4*I*d
*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*(a^4/d^3)^(1/3)*log((2^(1/3)*a*(
a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) - 2^(1/3)*(a^4/d
^3)^(1/3)*d)/a) - 11*((I*sqrt(3)*d + d)*e^(4*I*d*x + 4*I*c) + 2*(-I*sqrt(3)
*d - d)*e^(2*I*d*x + 2*I*c) + I*sqrt(3)*d + d)*(-a^4/d^3)^(1/3)*log(1/2*(2*
2^(1/3)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) - (I*
sqrt(3)*d + d)*(-a^4/d^3)^(1/3))/a) - 11*((-I*sqrt(3)*d + d)*e^(4*I*d*x + 4
*I*c) + 2*(I*sqrt(3)*d - d)*e^(2*I*d*x + 2*I*c) - I*sqrt(3)*d + d)*(-a^4/d^
3)^(1/3)*log(1/2*(2*2^(1/3)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*
d*x + 2/3*I*c) - (-I*sqrt(3)*d + d)*(-a^4/d^3)^(1/3))/a) + 22*(d*e^(4*I*d*x
 + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*(-a^4/d^3)^(1/3)*log((2^(1/3)*a*(a
/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + (-a^4/d^3)^(1/3
)*d)/a))/(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+I*a*tan(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(4/3)*cot(d*x + c)^3, x)
```

Mupad [**B**]
time = 4.41, size = 460, normalized size = 1.43

$$-\frac{\frac{2a^3\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{1/3}}{3}-\frac{7a^2\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{4/3}}{6}}{d\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2+a^2\,d-2\,a\,d\,\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}+\frac{11\,\ln\left(d\,{\left(-\frac{a^4}{d^3}\right)}^{1/3}+a\,{\left(a\,\left(1+\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)\right)}^{1/3}\right)\,{\left(-\frac{a^4}{d^3}\right)}^{1/3}}{9}+\ln\left(a\,{\left(a\,\left(1+\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)\right)}^{1/3}-2^{1/3}\,d\,{\left(\frac{a^4}{d^3}\right)}^{1/3}\right)\,{\left(\frac{2\,a^4}{d^3}\right)}^{1/3}-\frac{11\,\ln\left(d\,{\left(-\frac{a^4}{d^3}\right)}^{1/3}-2\,a\,{\left(a\,\left(1+\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)\right)}^{1/3}+\sqrt{3}\,d\,{\left(-\frac{a^4}{d^3}\right)}^{1/3}\,1{}\mathrm{i}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(-\frac{a^4}{d^3}\right)}^{1/3}}{18}+\frac{11\,\ln\left(-d\,{\left(-\frac{a^4}{d^3}\right)}^{1/3}+2\,a\,{\left(a\,\left(1+\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)\right)}^{1/3}+\sqrt{3}\,d\,{\left(-\frac{a^4}{d^3}\right)}^{1/3}\,1{}\mathrm{i}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(-\frac{a^4}{d^3}\right)}^{1/3}}{18}+\ln\left(2\,a\,{\left(a\,\left(1+\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)\right)}^{1/3}+2^{1/3}\,d\,{\left(\frac{a^4}{d^3}\right)}^{1/3}-2^{1/3}\,\sqrt{3}\,d\,{\left(\frac{a^4}{d^3}\right)}^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(\frac{2\,a^4}{d^3}\right)}^{1/3}-\ln\left(2\,a\,{\left(a\,\left(1+\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)\right)}^{1/3}+2^{1/3}\,d\,{\left(\frac{a^4}{d^3}\right)}^{1/3}+2^{1/3}\,\sqrt{3}\,d\,{\left(\frac{a^4}{d^3}\right)}^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(\frac{2\,a^4}{d^3}\right)}^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(4/3),x)
```

```
[Out] (11*log(d*(-a^4/d^3)^(1/3) + a*(a*(tan(c + d*x)*1i + 1))^(1/3))*(-a^4/d^3)^
(1/3))/9 - ((2*a^3*(a + a*tan(c + d*x)*1i)^(1/3))/3 - (7*a^2*(a + a*tan(c +
 d*x)*1i)^(4/3))/6)/(d*(a + a*tan(c + d*x)*1i)^2 + a^2*d - 2*a*d*(a + a*tan
(c + d*x)*1i)) + log(a*(a*(tan(c + d*x)*1i + 1))^(1/3) - 2^(1/3)*d*(a^4/d^3
)^(1/3))*((2*a^4)/d^3)^(1/3) - (11*log(d*(-a^4/d^3)^(1/3) - 2*a*(a*(tan(c +
 d*x)*1i + 1))^(1/3) + 3^(1/2)*d*(-a^4/d^3)^(1/3)*1i)*(3^(1/2)*1i + 1)*(-a^
4/d^3)^(1/3))/18 + (11*log(2*a*(a*(tan(c + d*x)*1i + 1))^(1/3) - d*(-a^4/d^
3)^(1/3) + 3^(1/2)*d*(-a^4/d^3)^(1/3)*1i)*(3^(1/2)*1i - 1)*(-a^4/d^3)^(1/3)
)/18 + log(2*a*(a*(tan(c + d*x)*1i + 1))^(1/3) + 2^(1/3)*d*(a^4/d^3)^(1/3)
- 2^(1/3)*3^(1/2)*d*(a^4/d^3)^(1/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*((2*a^4)/d^3
)^(1/3) - log(2*a*(a*(tan(c + d*x)*1i + 1))^(1/3) + 2^(1/3)*d*(a^4/d^3)^(1/
3) + 2^(1/3)*3^(1/2)*d*(a^4/d^3)^(1/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*((2*a^4)/
d^3)^(1/3)
```

3.287 $\int (a+ia\tan(c+dx))^{5/3}\,dx$

Optimal. Leaf size=177

$$-\frac{a^{5/3}x}{\sqrt[3]{2}}+\frac{i2^{2/3}\sqrt{3}\,a^{5/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{d}+\frac{ia^{5/3}\log(\cos(c+dx))}{\sqrt[3]{2}\,d}+\frac{3ia^{5/3}\log\left(\sqrt[3]{2}\,\sqrt[3]{\right.}$$

```
[Out] -1/2*a^(5/3)*x*2^(2/3)+1/2*I*a^(5/3)*ln(cos(d*x+c))*2^(2/3)/d+3/2*I*a^(5/3)
*ln(2^(1/3)*a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))*2^(2/3)/d+I*2^(2/3)*a^(5/3)*a
rctan(1/3*(a^(1/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))*3^(1/
2)/d+3/2*I*a*(a+I*a*tan(d*x+c))^(2/3)/d
```

Rubi [A]

time = 0.09, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3559, 3562, 57, 631, 210, 31}

$$\frac{i2^{2/3}\sqrt{3}\,a^{5/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{d}+\frac{3ia^{5/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{\sqrt[3]{2}\,d}+\frac{ia^{5/3}\log(\cos(c+dx))}{\sqrt[3]{2}\,d}-\frac{a^{5/3}x}{\sqrt[3]{2}}+\frac{3ia(a+ia\tan(c+dx))^{2/3}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(5/3),x]
```

```
[Out] -((a^(5/3)*x)/2^(1/3)) + (I*2^(2/3)*Sqrt[3]*a^(5/3)*ArcTan[(a^(1/3) + 2^(2/
3)*(a + I*a*Tan[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/d + (I*a^(5/3)*Log[Cos
[c + d*x]])/(2^(1/3)*d) + ((3*I)*a^(5/3)*Log[2^(1/3)*a^(1/3) - (a + I*a*Tan
[c + d*x])^(1/3)])/(2^(1/3)*d) + (((3*I)/2)*a*(a + I*a*Tan[c + d*x])^(2/3))
/d
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a+ia\tan(c+dx))^{5/3}\,dx=\frac{3ia(a+ia\tan(c+dx))^{2/3}}{2d}+(2a)\int (a+ia\tan(c+dx))^{2/3}\,dx$$

$$=\frac{3ia(a+ia\tan(c+dx))^{2/3}}{2d}-\frac{(2ia^2)\,\text{Subst}\left(\int\frac{1}{(a-x)\sqrt[3]{a+x}}\,dx,x,ia\tan(c+d$$

$$=-\frac{a^{5/3}x}{\sqrt[3]{2}}+\frac{ia^{5/3}\log(\cos(c+dx))}{\sqrt[3]{2}\,d}+\frac{3ia(a+ia\tan(c+dx))^{2/3}}{2d}-\frac{(3ia^{5/3})\,\text{Sub}$$

$$=-\frac{a^{5/3}x}{\sqrt[3]{2}}+\frac{ia^{5/3}\log(\cos(c+dx))}{\sqrt[3]{2}\,d}+\frac{3ia^{5/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx}\right.}{\sqrt[3]{2}\,d}$$

$$=-\frac{a^{5/3}x}{\sqrt[3]{2}}+\frac{i2^{2/3}\sqrt{3}\,a^{5/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{d}+\frac{ia^{5/3}\log(\cos}{\sqrt[3]{2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.73, size = 82, normalized size = 0.46

$$-\frac{3ia\left(\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{2/3}\left(-1+{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)\right)}{\sqrt[3]{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(5/3),x]
```

```
[Out] ((-3*I)*a*((a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))^(2/3)*(-1 + H
ypergeometric2F1[2/3, 1, 5/3, E^((2*I)*(c + d*x))/(1 + E^((2*I)*(c + d*x)))
]))/(2^(1/3)*d)
```

Maple [A]
time = 0.10, size = 153, normalized size = 0.86

method	result
derivativedivides	$\frac{3ia\left(\frac{(a+ia\tan(dx+c))^{\frac{2}{3}}}{2}+2\left(\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a\right.}{12a^{\frac{1}{3}}}\right.\right.}{d}$
default	$\frac{3ia\left(\frac{(a+ia\tan(dx+c))^{\frac{2}{3}}}{2}+2\left(\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a\right.}{12a^{\frac{1}{3}}}\right.\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(5/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3*I/d*a*(1/2*(a+I*a*tan(d*x+c))^(2/3)+2*(1/6*2^(2/3)/a^(1/3)*ln((a+I*a*tan(
d*x+c))^(1/3)-2^(1/3)*a^(1/3))-1/12*2^(2/3)/a^(1/3)*ln((a+I*a*tan(d*x+c))^(
2/3)+2^(1/3)*a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+2^(2/3)*a^(2/3))+1/6*3^(1/2)*
2^(2/3)/a^(1/3)*arctan(1/3*3^(1/2)*(2^(2/3)/a^(1/3)*(a+I*a*tan(d*x+c))^(1/3
)+1)))*a)
```

Maxima [A]
time = 0.52, size = 154, normalized size = 0.87

$$\frac{i\left(2\sqrt{3}\,2^{\frac{2}{3}}a^{\frac{8}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)-2^{\frac{2}{3}}a^{\frac{8}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(i\,a\tan(dx+c)+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{2}{3}}\right)+2\cdot2^{\frac{2}{3}}a^{\frac{8}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)+3\,(i\,a\tan(dx+c)+a)^{\frac{2}{3}}a^{2}\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] 1/2*I*(2*sqrt(3)*2^(2/3)*a^(8/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3
) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) - 2^(2/3)*a^(8/3)*log(2^(2/3)*
a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x + c)
+ a)^(2/3)) + 2*2^(2/3)*a^(8/3)*log(-2^(1/3)*a^(1/3) + (I*a*tan(d*x + c) +
a)^(1/3)) + 3*(I*a*tan(d*x + c) + a)^(2/3)*a^2)/(a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(126) = 252$.
time = 0.97, size = 280, normalized size = 1.58

$$\frac{3i\cdot 2^{\frac{2}{3}}a\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{2}{3}}e^{\left(\frac{4}{3}i\,dx+\frac{4}{3}i\,c\right)}+\left(-\frac{4i\,a^5}{d^3}\right)^{\frac{1}{3}}\left(-i\sqrt{3}\,d-d\right)\log\left(\frac{4\cdot 2^{\frac{1}{3}}a^3\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}+\left(i\sqrt{3}\,d^2-d^2\right)\left(-\frac{4i\,a^5}{d^3}\right)^{\frac{2}{3}}}{4\,a^3}\right)+\left(-\frac{4i\,a^5}{d^3}\right)^{\frac{1}{3}}\left(i\sqrt{3}\,d-d\right)\log\left(\frac{4\cdot 2^{\frac{1}{3}}a^3\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}+\left(-i\sqrt{3}\,d^2-d^2\right)\left(-\frac{4i\,a^5}{d^3}\right)^{\frac{2}{3}}}{4\,a^3}\right)+2\left(-\frac{4i\,a^5}{d^3}\right)^{\frac{1}{3}}d\log\left(\frac{2\cdot 2^{\frac{1}{3}}a^3\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}+\left(-\frac{4i\,a^5}{d^3}\right)^{\frac{2}{3}}d^2}{2\,a^3}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] 1/2*(3*I*2^(2/3)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*e^(4/3*I*d*x + 4/3*I
*c) + (-4*I*a^5/d^3)^(1/3)*(-I*sqrt(3)*d - d)*log(1/4*(4*2^(1/3)*a^3*(a/(e^
(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + (I*sqrt(3)*d^2 - d^
2)*(-4*I*a^5/d^3)^(2/3))/a^3) + (-4*I*a^5/d^3)^(1/3)*(I*sqrt(3)*d - d)*log(
1/4*(4*2^(1/3)*a^3*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I
*c) + (-I*sqrt(3)*d^2 - d^2)*(-4*I*a^5/d^3)^(2/3))/a^3) + 2*(-4*I*a^5/d^3)^
(1/3)*d*log(1/2*(2*2^(1/3)*a^3*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I
*d*x + 2/3*I*c) + (-4*I*a^5/d^3)^(2/3)*d^2)/a^3))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia\tan(c+dx)+a)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(5/3),x)
```

```
[Out] Integral((I*a*tan(c + d*x) + a)**(5/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/3), x)
```

Mupad [B]
time = 4.54, size = 208, normalized size = 1.18

$$\frac{a\,(a+a\tan(c+d\,x)\ 1\mathrm{i})^{2/3}\,3\mathrm{i}}{2\,d}+\frac{(4\mathrm{i})^{1/3}\,(-a)^{5/3}\,\ln\left(36\,a^4\,(a\,(1+\tan(c+d\,x)\ 1\mathrm{i}))^{1/3}-36\,(-1)^{1/3}\,2^{1/3}\,(-a)^{13/3}\right)}{d}-\frac{(4\mathrm{i})^{1/3}\,(-a)^{5/3}\,\ln\left(-\frac{36\,a^4\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{1/3}}{d^2}+\frac{18\,(-1)^{1/3}\,2^{1/3}\,(-a)^{13/3}\,\left(-1+\sqrt{3}\ 1\mathrm{i}\right)}{d^2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)}{d}+\frac{(4\mathrm{i})^{1/3}\,(-a)^{5/3}\,\ln\left(-\frac{36\,a^4\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{1/3}}{d^2}-\frac{18\,(-1)^{1/3}\,2^{1/3}\,(-a)^{13/3}\,\left(1+\sqrt{3}\ 1\mathrm{i}\right)}{d^2}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(5/3),x)
```

```
[Out] (a*(a + a*tan(c + d*x)*1i)^(2/3)*3i)/(2*d) + (4i^(1/3)*(-a)^(5/3)*log(36*a^
4*(a*(tan(c + d*x)*1i + 1))^(1/3) - 36*(-1)^(1/3)*2^(1/3)*(-a)^(13/3)))/d -
 (4i^(1/3)*(-a)^(5/3)*log((18*(-1)^(1/3)*2^(1/3)*(-a)^(13/3)*(3^(1/2)*1i -
1))/d^2 - (36*a^4*(a + a*tan(c + d*x)*1i)^(1/3))/d^2)*((3^(1/2)*1i)/2 + 1/2
))/d + (4i^(1/3)*(-a)^(5/3)*log(- (36*a^4*(a + a*tan(c + d*x)*1i)^(1/3))/d^
2 - (18*(-1)^(1/3)*2^(1/3)*(-a)^(13/3)*(3^(1/2)*1i + 1))/d^2)*((3^(1/2)*1i)
/2 - 1/2))/d
```

3.288 $\int \frac{\tan^m(c+dx)}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=83

$$\frac{F_1\left(1+m;\frac{4}{3},1;2+m;-i\tan(c+dx),i\tan(c+dx)\right)\sqrt[3]{1+i\tan(c+dx)}\,\tan^{1+m}(c+dx)}{d(1+m)\sqrt[3]{a+ia\tan(c+dx)}}$$

```
[Out] AppellF1(1+m,4/3,1,2+m,-I*tan(d*x+c),I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/3)*t
an(d*x+c)^(1+m)/d/(1+m)/(a+I*a*tan(d*x+c))^(1/3)
```

Rubi [A]
time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3645, 140, 138}

$$\frac{\sqrt[3]{1+i\tan(c+dx)}\,\tan^{m+1}(c+dx)F_1\left(m+1;\frac{4}{3},1;m+2;-i\tan(c+dx),i\tan(c+dx)\right)}{d(m+1)\sqrt[3]{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^m/(a + I*a*Tan[c + d*x])^(1/3),x]
```

```
[Out] (AppellF1[1 + m, 4/3, 1, 2 + m, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*(1 + I*T
an[c + d*x])^(1/3)*Tan[c + d*x]^(1 + m))/(d*(1 + m)*(a + I*a*Tan[c + d*x])^
(1/3))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
```

2, 0]

Rubi steps

$$\int \frac{\tan^m(c+dx)}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx = \frac{(ia^2)\,\text{Subst}\left(\int \frac{\left(-\frac{ix}{a}\right)^m}{(a+x)^{4/3}(-a^2+ax)}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= \frac{\left(ia\sqrt[3]{1+i\tan(c+dx)}\right)\text{Subst}\left(\int \frac{\left(-\frac{ix}{a}\right)^m}{\left(1+\frac{x}{a}\right)^{4/3}(-a^2+ax)}\,dx, x, ia\tan(c+dx)\right)}{d\sqrt[3]{a+ia\tan(c+dx)}}$$

$$= \frac{F_1\left(1+m; \frac{4}{3}, 1; 2+m; -i\tan(c+dx), i\tan(c+dx)\right)\sqrt[3]{1+i\tan(c+dx)}\,\tan}{d(1+m)\sqrt[3]{a+ia\tan(c+dx)}}$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^m/(a + I*a*Tan[c + d*x])^(1/3),x]
[Out] $Aborted
```

Maple [F]
time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\tan^m(dx+c)}{(a+ia\tan(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^m/(a+I*a*tan(d*x+c))^(1/3),x)
[Out] int(tan(d*x+c)^m/(a+I*a*tan(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m/(a+I*a*tan(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^m/(I*a*tan(d*x + c) + a)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m/(a+I*a*tan(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(1/2*2^(2/3)*((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1
))^m*(a/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(e^(2*I*d*x + 2*I*c) + 1)*e^(-2/3*
I*d*x - 2/3*I*c)/a, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{m}\left(c+dx\right)}{\sqrt[3]{ia\left(\tan\left(c+dx\right)-i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**m/(a+I*a*tan(d*x+c))**(1/3),x)
```

```
[Out] Integral(tan(c + d*x)**m/(I*a*(tan(c + d*x) - I))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m/(a+I*a*tan(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^m/(I*a*tan(d*x + c) + a)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{tan}\left(c+d\,x\right)}^m}{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^m/(a + a*tan(c + d*x)*1i)^(1/3),x)
```

```
[Out] int(tan(c + d*x)^m/(a + a*tan(c + d*x)*1i)^(1/3), x)
```

3.289 $\int \frac{\sqrt{\tan(c+dx)}}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=81

$$\frac{2F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt[3]{1+i\tan(c+dx)}\,\tan^{\frac{3}{2}}(c+dx)}{3d\sqrt[3]{a+ia\tan(c+dx)}}$$

```
[Out] 2/3*AppellF1(3/2,4/3,1,5/2,-I*tan(d*x+c),I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/
3)*tan(d*x+c)^(3/2)/d/(a+I*a*tan(d*x+c))^(1/3)
```

Rubi [A]
time = 0.10, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3645, 129, 525, 524}

$$\frac{2\sqrt[3]{1+i\tan(c+dx)}\,\tan^{\frac{3}{2}}(c+dx)F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-i\tan(c+dx),i\tan(c+dx)\right)}{3d\sqrt[3]{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[c + d*x]]/(a + I*a*Tan[c + d*x])^(1/3),x]
```

```
[Out] (2*AppellF1[3/2, 4/3, 1, 5/2, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*(1 + I*Tan
[c + d*x])^(1/3)*Tan[c + d*x]^(3/2))/(3*d*(a + I*a*Tan[c + d*x])^(1/3))
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
```

```
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\tan(c+dx)}}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx &= \frac{(ia^2)\operatorname{Subst}\left(\int \frac{\sqrt{-\frac{ix}{a}}}{(a+x)^{4/3}(-a^2+ax)}\,dx, x, ia\tan(c+dx)\right)}{d} \\
&= -\frac{(2a^3)\operatorname{Subst}\left(\int \frac{x^2}{(a+iax^2)^{4/3}(-a^2+ia^2x^2)}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d} \\
&= -\frac{\left(2a^2\sqrt[3]{1+i\tan(c+dx)}\right)\operatorname{Subst}\left(\int \frac{x^2}{(1+ix^2)^{4/3}(-a^2+ia^2x^2)}\,dx, x, \sqrt{\tan(c+dx)}\right.}{d\sqrt[3]{a+ia\tan(c+dx)}} \\
&= \frac{2F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt[3]{1+i\tan(c+dx)}\tan^{\frac{3}{2}}(c+dx}{3d\sqrt[3]{a+ia\tan(c+dx)}}
\end{aligned}$$

Mathematica [F]
time = 5.28, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\tan(c+dx)}}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[Tan[c + d*x]]/(a + I*a*Tan[c + d*x])^(1/3),x]
[Out] Integrate[Sqrt[Tan[c + d*x]]/(a + I*a*Tan[c + d*x])^(1/3), x]
```

Maple [F]
time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\tan}\,(dx+c)}{(a+ia\tan(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/3),x)
```

```
[Out] int(tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(tan(d*x + c))/(I*a*tan(d*x + c) + a)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] -(3*2^(2/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*sqrt((-I*e^(2*I*d*x + 2*I*c
) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(-I*e^(4*I*d*x + 4*I*c) - 2*I*e^(2*I*d*x
+ 2*I*c) - I)*e^(4/3*I*d*x + 4/3*I*c) - (a*d*e^(4*I*d*x + 4*I*c) - 4*a*d*e^
(3*I*d*x + 3*I*c) + 4*a*d*e^(2*I*d*x + 2*I*c))*integral(1/2*2^(2/3)*(a/(e^(
2*I*d*x + 2*I*c) + 1))^(2/3)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x
+ 2*I*c) + 1))*(-I*e^(6*I*d*x + 6*I*c) + 30*I*e^(5*I*d*x + 5*I*c) - 8*I*e^(
4*I*d*x + 4*I*c) + 24*I*e^(3*I*d*x + 3*I*c) - 11*I*e^(2*I*d*x + 2*I*c) - 6*
I*e^(I*d*x + I*c) - 4*I)*e^(4/3*I*d*x + 4/3*I*c)/(a*d*e^(7*I*d*x + 7*I*c) -
 6*a*d*e^(6*I*d*x + 6*I*c) + 11*a*d*e^(5*I*d*x + 5*I*c) - 2*a*d*e^(4*I*d*x
+ 4*I*c) - 12*a*d*e^(3*I*d*x + 3*I*c) + 8*a*d*e^(2*I*d*x + 2*I*c)), x))/(a*
d*e^(4*I*d*x + 4*I*c) - 4*a*d*e^(3*I*d*x + 3*I*c) + 4*a*d*e^(2*I*d*x + 2*I*
c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\tan(c+dx)}}{\sqrt[3]{ia(\tan(c+dx)-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/2)/(a+I*a*tan(d*x+c))**(1/3),x)
```

```
[Out] Integral(sqrt(tan(c + d*x))/(I*a*(tan(c + d*x) - I))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(tan(d*x + c))/(I*a*tan(d*x + c) + a)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\tan\left(c+d\,x\right)}}{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)/(a + a*tan(c + d*x)*1i)^(1/3),x)
```

```
[Out] int(tan(c + d*x)^(1/2)/(a + a*tan(c + d*x)*1i)^(1/3), x)
```

3.290 $\int \frac{\tan^4(c+dx)}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=282

$$-\frac{x}{4\sqrt[3]{2}\sqrt[3]{a}}+\frac{i\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt[3]{2}\sqrt[3]{a}\,d}+\frac{i\log(\cos(c+dx))}{4\sqrt[3]{2}\sqrt[3]{a}\,d}+\frac{3i\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+\right.}}{4\sqrt[3]{2}\sqrt[3]{a}}$$

```
[Out] -1/8*x*2^(2/3)/a^(1/3)+1/8*I*ln(cos(d*x+c))*2^(2/3)/a^(1/3)/d+3/8*I*ln(2^(1
/3)*a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))*2^(2/3)/a^(1/3)/d+1/4*I*arctan(1/3*(a
^(1/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)*2^(2/3)/a
^(1/3)/d-15/8*I*tan(d*x+c)^2/d/(a+I*a*tan(d*x+c))^(1/3)+3/8*tan(d*x+c)^3/d/
(a+I*a*tan(d*x+c))^(1/3)+45/8*I*(a+I*a*tan(d*x+c))^(2/3)/a/d-39/20*I*(a+I*a
*tan(d*x+c))^(5/3)/a^2/d
```

Rubi [A]

time = 0.32, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3641, 3676, 3673, 3608, 3562, 57, 631, 210, 31}

$$-\frac{39i(a+ia\tan(c+dx))^{5/3}}{20a^2d}+\frac{i\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt[3]{2}\sqrt[3]{a}\,d}+\frac{3\tan^3(c+dx)}{8d\sqrt[3]{a+ia\tan(c+dx)}}-\frac{15i\tan^2(c+dx)}{8d\sqrt[3]{a+ia\tan(c+dx)}}+\frac{45i(a+ia\tan(c+dx))^{2/3}}{8ad}+\frac{3i\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{4\sqrt[3]{2}\sqrt[3]{a}\,d}+\frac{i\log(\cos(c+dx))}{4\sqrt[3]{2}\sqrt[3]{a}\,d}-\frac{x}{4\sqrt[3]{2}\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + I*a*Tan[c + d*x])^(1/3),x]
```

```
[Out] -1/4*x/(2^(1/3)*a^(1/3)) + ((I/2)*Sqrt[3]*ArcTan[(a^(1/3) + 2^(2/3)*(a + I*
a*Tan[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/(2^(1/3)*a^(1/3)*d) + ((I/4)*Log
[Cos[c + d*x]])/(2^(1/3)*a^(1/3)*d) + (((3*I)/4)*Log[2^(1/3)*a^(1/3) - (a +
 I*a*Tan[c + d*x])^(1/3)])/(2^(1/3)*a^(1/3)*d) - (((15*I)/8)*Tan[c + d*x]^2
)/(d*(a + I*a*Tan[c + d*x])^(1/3)) + (3*Tan[c + d*x]^3)/(8*d*(a + I*a*Tan[c
 + d*x])^(1/3)) + (((45*I)/8)*(a + I*a*Tan[c + d*x])^(2/3))/(a*d) - (((39*I
)/20)*(a + I*a*Tan[c + d*x])^(5/3))/(a^2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rule 3641

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[d*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n - 1)/(f*(m + n - 1))), x] - Dist[1/(a*(m + n - 1)), Int[(a + b*
Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 2)*Simp[d*(b*c*m + a*d*(-1 + n))
- a*c^2*(m + n - 1) + d*(b*d*m - a*c*(m + 2*n - 2))*Tan[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1] && NeQ[m + n - 1, 0] && (IntegerQ[n]
|| IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx = \frac{3\tan^3(c+dx)}{8d\sqrt[3]{a+ia\tan(c+dx)}} - \frac{3\int \frac{\tan^2(c+dx)\left(3a-\frac{1}{3}ia\tan(c+dx)\right)}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx}{8a}$$

$$= -\frac{15i\tan^2(c+dx)}{8d\sqrt[3]{a+ia\tan(c+dx)}} + \frac{3\tan^3(c+dx)}{8d\sqrt[3]{a+ia\tan(c+dx)}} + \frac{9\int \tan(c+dx)(a+$$

$$= -\frac{15i\tan^2(c+dx)}{8d\sqrt[3]{a+ia\tan(c+dx)}} + \frac{3\tan^3(c+dx)}{8d\sqrt[3]{a+ia\tan(c+dx)}} - \frac{39i(a+ia\tan(c+}{20a^2d}$$

$$= -\frac{15i\tan^2(c+dx)}{8d\sqrt[3]{a+ia\tan(c+dx)}} + \frac{3\tan^3(c+dx)}{8d\sqrt[3]{a+ia\tan(c+dx)}} + \frac{45i(a+ia\tan(c+}{8ad}$$

$$= -\frac{15i\tan^2(c+dx)}{8d\sqrt[3]{a+ia\tan(c+dx)}} + \frac{3\tan^3(c+dx)}{8d\sqrt[3]{a+ia\tan(c+dx)}} + \frac{45i(a+ia\tan(c+}{8ad}$$

$$= -\frac{x}{4\sqrt[3]{2}\sqrt[3]{a}} + \frac{i\log(\cos(c+dx))}{4\sqrt[3]{2}\sqrt[3]{a}\,d} - \frac{15i\tan^2(c+dx)}{8d\sqrt[3]{a+ia\tan(c+dx)}} + \frac{3\tan^3(c}{8d\sqrt[3]{a+ia\,\mathrm{ta}}}$$

$$= -\frac{x}{4\sqrt[3]{2}\sqrt[3]{a}} + \frac{i\log(\cos(c+dx))}{4\sqrt[3]{2}\sqrt[3]{a}\,d} + \frac{3i\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right.}{4\sqrt[3]{2}\sqrt[3]{a}\,d}$$

$$= -\frac{x}{4\sqrt[3]{2}\sqrt[3]{a}} + \frac{i\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt[3]{a}\,d} + \frac{i\log(\cos(c+d}{4\sqrt[3]{2}\sqrt[3]{a}\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 1.42, size = 125, normalized size = 0.44

$$\frac{3i\sec^3(c+dx)(37\cos(c+dx)+12\cos(3(c+dx))+2i\sin(c+dx)+7i\sin(3(c+dx)))+15\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)(-i+\tan(c+dx))}{40d\sqrt[3]{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d*x]^4/(a + I*a*Tan[c + d*x])^(1/3),x]

[Out] ((3*I)*Sec[c + d*x]^3*(37*Cos[c + d*x] + 12*Cos[3*(c + d*x)] + (2*I)*Sin[c + d*x] + (7*I)*Sin[3*(c + d*x)]) + 15*Hypergeometric2F1[2/3, 1, 5/3, E^((2*I)*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(-I + Tan[c + d*x]))/(40*d*(a + I*a*Tan[c + d*x])^(1/3))

Maple [A]
time = 0.12, size = 211, normalized size = 0.75

method	result
derivativedivides	$\frac{3i\left(\frac{(a+ia\tan(dx+c))^{\frac{8}{3}}}{8}-\frac{2a(a+ia\tan(dx+c))^{\frac{5}{3}}}{5}+a^2(a+ia\tan(dx+c))^{\frac{2}{3}}+\frac{\left(\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left((a+ia\,t\right.}{}\right.}{}\right.}{}$
default	$\frac{3i\left(\frac{(a+ia\tan(dx+c))^{\frac{8}{3}}}{8}-\frac{2a(a+ia\tan(dx+c))^{\frac{5}{3}}}{5}+a^2(a+ia\tan(dx+c))^{\frac{2}{3}}+\frac{\left(\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left((a+ia\,t\right.}{}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3*I/d/a^3*(1/8*(a+I*a*tan(d*x+c))^(8/3)-2/5*a*(a+I*a*tan(d*x+c))^(5/3)+a^2*
(a+I*a*tan(d*x+c))^(2/3)+1/2*(1/6*2^(2/3)/a^(1/3)*ln((a+I*a*tan(d*x+c))^(1/
3)-2^(1/3)*a^(1/3))-1/12*2^(2/3)/a^(1/3)*ln((a+I*a*tan(d*x+c))^(2/3)+2^(1/3
)*a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+2^(2/3)*a^(2/3))+1/6*3^(1/2)*2^(2/3)/a^(
1/3)*arctan(1/3*3^(1/2)*(2^(2/3)/a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+1)))*a^3+
1/2*a^3/(a+I*a*tan(d*x+c))^(1/3))
```

Maxima [A]
time = 0.54, size = 208, normalized size = 0.74

$$\frac{i\left(10\sqrt{3}\,2^{\frac{2}{3}}a^{\frac{14}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)-5\cdot 2^{\frac{2}{3}}a^{\frac{14}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(i\,a\tan(dx+c)+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{2}{3}}\right)+10\cdot 2^{\frac{2}{3}}a^{\frac{14}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)+15\,(i\,a\tan(dx+c)+a)^{\frac{8}{3}}a^{2}-48\,(i\,a\tan(dx+c)+a)^{\frac{5}{3}}a^{3}+120\,(i\,a\tan(dx+c)+a)^{\frac{2}{3}}a^{4}+\frac{60\,a^{5}}{(i\,a\tan(dx+c)+a)^{\frac{1}{3}}}\right)}{40\,a^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] 1/40*I*(10*sqrt(3)*2^(2/3)*a^(14/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(
1/3) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) - 5*2^(2/3)*a^(14/3)*log(2^
(2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x
 + c) + a)^(2/3)) + 10*2^(2/3)*a^(14/3)*log(-2^(1/3)*a^(1/3) + (I*a*tan(d*x
 + c) + a)^(1/3)) + 15*(I*a*tan(d*x + c) + a)^(8/3)*a^2 - 48*(I*a*tan(d*x +
 c) + a)^(5/3)*a^3 + 120*(I*a*tan(d*x + c) + a)^(2/3)*a^4 + 60*a^5/(I*a*tan
(d*x + c) + a)^(1/3))/(a^5*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 479 vs. 2(201) = 402.
time = 1.16, size = 479, normalized size = 1.70

$$-\frac{3\cdot 2^{\frac{2}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{2}{3}}\left(-19i\,e^{(6i\,dx+6i\,c)}-39i\,e^{(4i\,dx+4i\,c)}-35i\,e^{(2i\,dx+2i\,c)}-5i\right)e^{\left(\frac{4}{3}i\,dx+\frac{4}{3}i\,c\right)}-20\left(ade^{(6i\,dx+6i\,c)}+2\,ade^{(4i\,dx+4i\,c)}+ade^{(2i\,dx+2i\,c)}\right)\left(-\frac{i}{16\,ad^{3}}\right)^{\frac{1}{3}}\log\left(8\,ad^{2}\left(-\frac{i}{16\,ad^{3}}\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)+10\left(\left(-i\sqrt{3}\,ad+ad\right)e^{(6i\,dx+6i\,c)}+2\left(-i\sqrt{3}\,ad+ad\right)e^{(4i\,dx+4i\,c)}+\left(-i\sqrt{3}\,ad+ad\right)e^{(2i\,dx+2i\,c)}\right)\left(-\frac{i}{16\,ad^{3}}\right)^{\frac{1}{3}}\log\left(-4\left(i\sqrt{3}\,ad^{2}+ad^{2}\right)\left(-\frac{i}{16\,ad^{3}}\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)+10\left(\left(i\sqrt{3}\,ad+ad\right)e^{(6i\,dx+6i\,c)}+2\left(i\sqrt{3}\,ad+ad\right)e^{(4i\,dx+4i\,c)}+\left(i\sqrt{3}\,ad+ad\right)e^{(2i\,dx+2i\,c)}\right)\left(-\frac{i}{16\,ad^{3}}\right)^{\frac{1}{3}}\log\left(-4\left(-i\sqrt{3}\,ad^{2}+ad^{2}\right)\left(-\frac{i}{16\,ad^{3}}\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)}{20\left(ade^{(6i\,dx+6i\,c)}+2\,ade^{(4i\,dx+4i\,c)}+ade^{(2i\,dx+2i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] -1/20*(3*2^(2/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(-19*I*e^(6*I*d*x + 6*
I*c) - 39*I*e^(4*I*d*x + 4*I*c) - 35*I*e^(2*I*d*x + 2*I*c) - 5*I)*e^(4/3*I*
d*x + 4/3*I*c) - 20*(a*d*e^(6*I*d*x + 6*I*c) + 2*a*d*e^(4*I*d*x + 4*I*c) +
a*d*e^(2*I*d*x + 2*I*c))*(-1/16*I/(a*d^3))^(1/3)*log(8*a*d^2*(-1/16*I/(a*d^
3))^(2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*
I*c)) + 10*((-I*sqrt(3)*a*d + a*d)*e^(6*I*d*x + 6*I*c) + 2*(-I*sqrt(3)*a*d
+ a*d)*e^(4*I*d*x + 4*I*c) + (-I*sqrt(3)*a*d + a*d)*e^(2*I*d*x + 2*I*c))*(-
1/16*I/(a*d^3))^(1/3)*log(-4*(I*sqrt(3)*a*d^2 + a*d^2)*(-1/16*I/(a*d^3))^(2
/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c))
+ 10*((I*sqrt(3)*a*d + a*d)*e^(6*I*d*x + 6*I*c) + 2*(I*sqrt(3)*a*d + a*d)*e
^(4*I*d*x + 4*I*c) + (I*sqrt(3)*a*d + a*d)*e^(2*I*d*x + 2*I*c))*(-1/16*I/(a
*d^3))^(1/3)*log(-4*(-I*sqrt(3)*a*d^2 + a*d^2)*(-1/16*I/(a*d^3))^(2/3) + 2^
```

```
(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)))/(a*d*e^
(6*I*d*x + 6*I*c) + 2*a*d*e^(4*I*d*x + 4*I*c) + a*d*e^(2*I*d*x + 2*I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(c+dx)}{\sqrt[3]{ia(\tan(c+dx)-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+I*a*tan(d*x+c))**(1/3),x)
```

```
[Out] Integral(tan(c + d*x)**4/(I*a*(tan(c + d*x) - I))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^4/(I*a*tan(d*x + c) + a)^(1/3), x)
```

Mupad [B]
time = 4.63, size = 266, normalized size = 0.94

$$\frac{3\mathrm{i}}{2\,d\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{1/3}}+\frac{(a+a\tan(c+d\,x)\,1\mathrm{i})^{2/3}\,3\mathrm{i}}{a\,d}-\frac{(a+a\tan(c+d\,x)\,1\mathrm{i})^{5/3}\,6\mathrm{i}}{5\,a^2\,d}+\frac{(a+a\tan(c+d\,x)\,1\mathrm{i})^{8/3}\,3\mathrm{i}}{8\,a^3\,d}+\frac{\left(\frac{1}{16}\mathrm{i}\right)^{1/3}\ln\left((a\,(1+\tan(c+d\,x)\,1\mathrm{i}))^{1/3}-(-1)^{1/3}\,2^{1/3}\,(-a)^{1/3}\right)}{(-a)^{1/3}\,d}-\frac{\left(\frac{1}{16}\mathrm{i}\right)^{1/3}\ln\left(-\frac{9\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{1/3}}{4\,d^2}+\frac{9\,(-1)^{1/3}\,2^{1/3}\,(-a)^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{8\,d^2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{(-a)^{1/3}\,d}+\frac{\left(\frac{1}{16}\mathrm{i}\right)^{1/3}\ln\left(-\frac{9\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{1/3}}{4\,d^2}-\frac{9\,(-1)^{1/3}\,2^{1/3}\,(-a)^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{8\,d^2}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{(-a)^{1/3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(1/3),x)
```

```
[Out] 3i/(2*d*(a + a*tan(c + d*x)*1i)^(1/3)) + ((a + a*tan(c + d*x)*1i)^(2/3)*3i)
/(a*d) - ((a + a*tan(c + d*x)*1i)^(5/3)*6i)/(5*a^2*d) + ((a + a*tan(c + d*x
)*1i)^(8/3)*3i)/(8*a^3*d) + ((1i/16)^(1/3)*log((a*(tan(c + d*x)*1i + 1))^(1
/3) - (-1)^(1/3)*2^(1/3)*(-a)^(1/3)))/((-a)^(1/3)*d) - ((1i/16)^(1/3)*log((
9*(-1)^(1/3)*2^(1/3)*(-a)^(1/3)*(3^(1/2)*1i - 1))/(8*d^2) - (9*(a + a*tan(c
 + d*x)*1i)^(1/3))/(4*d^2))*((3^(1/2)*1i)/2 + 1/2))/((-a)^(1/3)*d) + ((1i/1
6)^(1/3)*log(- (9*(a + a*tan(c + d*x)*1i)^(1/3))/(4*d^2) - (9*(-1)^(1/3)*2^
(1/3)*(-a)^(1/3)*(3^(1/2)*1i + 1))/(8*d^2))*((3^(1/2)*1i)/2 - 1/2))/((-a)^(
1/3)*d)
```

3.291 $\int \frac{\tan^3(c+dx)}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=237

$$-\frac{ix}{4\sqrt[3]{2}\sqrt[3]{a}}-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt[3]{2}\sqrt[3]{a}\,d}-\frac{\log(\cos(c+dx))}{4\sqrt[3]{2}\sqrt[3]{a}\,d}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+ia}\right.}{4\sqrt[3]{2}\sqrt[3]{a}\,d}$$

```
[Out] -1/8*I*x*2^(2/3)/a^(1/3)-1/8*ln(cos(d*x+c))*2^(2/3)/a^(1/3)/d-3/8*ln(2^(1/3
)*a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))*2^(2/3)/a^(1/3)/d-1/4*arctan(1/3*(a^(1/
3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)*2^(2/3)/a^(1/
3)/d+21/10/d/(a+I*a*tan(d*x+c))^(1/3)+3/5*tan(d*x+c)^2/d/(a+I*a*tan(d*x+c))
^(1/3)+3/10*(a+I*a*tan(d*x+c))^(2/3)/a/d
```

Rubi [A]
time = 0.20, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3641, 3673, 3607, 3562, 57, 631, 210, 31}

$$-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt[3]{2}\sqrt[3]{a}\,d}+\frac{3\tan^2(c+dx)}{5d\sqrt[3]{a+ia\tan(c+dx)}}+\frac{3(a+ia\tan(c+dx))^{2/3}}{10ad}+\frac{21}{10d\sqrt[3]{a+ia\tan(c+dx)}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{4\sqrt[3]{2}\sqrt[3]{a}\,d}-\frac{\log(\cos(c+dx))}{4\sqrt[3]{2}\sqrt[3]{a}\,d}-\frac{ix}{4\sqrt[3]{2}\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + I*a*Tan[c + d*x])^(1/3),x]
```

```
[Out] ((-1/4*I)*x)/(2^(1/3)*a^(1/3)) - (Sqrt[3]*ArcTan[(a^(1/3) + 2^(2/3)*(a + I*
a*Tan[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/(2*2^(1/3)*a^(1/3)*d) - Log[Cos[
c + d*x]]/(4*2^(1/3)*a^(1/3)*d) - (3*Log[2^(1/3)*a^(1/3) - (a + I*a*Tan[c +
 d*x])^(1/3)])/(4*2^(1/3)*a^(1/3)*d) + 21/(10*d*(a + I*a*Tan[c + d*x])^(1/3
)) + (3*Tan[c + d*x]^2)/(5*d*(a + I*a*Tan[c + d*x])^(1/3)) + (3*(a + I*a*Ta
n[c + d*x])^(2/3))/(10*a*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rule 3641

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[d*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n - 1)/(f*(m + n - 1))), x] - Dist[1/(a*(m + n - 1)), Int[(a + b*
Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 2)*Simp[d*(b*c*m + a*d*(-1 + n))
- a*c^2*(m + n - 1) + d*(b*d*m - a*c*(m + 2*n - 2))*Tan[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1] && NeQ[m + n - 1, 0] && (IntegerQ[n]
|| IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx = \frac{3\tan^2(c+dx)}{5d\sqrt[3]{a+ia\tan(c+dx)}} - \frac{3\int \frac{\tan(c+dx)\left(2a-\frac{1}{3}ia\tan(c+dx)\right)}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx}{5a}$$

$$= \frac{3\tan^2(c+dx)}{5d\sqrt[3]{a+ia\tan(c+dx)}} + \frac{3(a+ia\tan(c+dx))^{2/3}}{10ad} - \frac{3\int \frac{\frac{ia}{3}+2a\tan(c+dx)}{\sqrt[3]{a+ia\tan(c+}}}{5a}$$

$$= \frac{21}{10d\sqrt[3]{a+ia\tan(c+dx)}} + \frac{3\tan^2(c+dx)}{5d\sqrt[3]{a+ia\tan(c+dx)}} + \frac{3(a+ia\tan(c+dx)}{10ad}$$

$$= \frac{21}{10d\sqrt[3]{a+ia\tan(c+dx)}} + \frac{3\tan^2(c+dx)}{5d\sqrt[3]{a+ia\tan(c+dx)}} + \frac{3(a+ia\tan(c+dx)}{10ad}$$

$$= -\frac{ix}{4\sqrt[3]{2}\sqrt[3]{a}} - \frac{\log(\cos(c+dx))}{4\sqrt[3]{2}\sqrt[3]{a}\,d} + \frac{21}{10d\sqrt[3]{a+ia\tan(c+dx)}} + \frac{3\tan^2(c}{5d\sqrt[3]{a+ia\,\mathrm{ta}}}$$

$$= -\frac{ix}{4\sqrt[3]{2}\sqrt[3]{a}} - \frac{\log(\cos(c+dx))}{4\sqrt[3]{2}\sqrt[3]{a}\,d} - \frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{4\sqrt[3]{2}\sqrt[3]{a}\,d}$$

$$= -\frac{ix}{4\sqrt[3]{2}\sqrt[3]{a}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt[3]{a}\,d} - \frac{\log(\cos(c+dx)}{4\sqrt[3]{2}\sqrt[3]{a}\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.90, size = 115, normalized size = 0.49

$$\frac{3\sec^2(c+dx)\left(40+24\cos(2(c+dx))+5\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)(1+\cos(2(c+dx))+i\sin(2(c+dx)))+4i\sin(2(c+dx))\right)}{80d\sqrt[3]{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + I*a*Tan[c + d*x])^(1/3),x]
```

```
[Out] (3*Sec[c + d*x]^2*(40 + 24*Cos[2*(c + d*x)] + 5*Hypergeometric2F1[2/3, 1, 5
/3, E^((2*I)*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(1 + Cos[2*(c + d*x)] +
I*Sin[2*(c + d*x)]) + (4*I)*Sin[2*(c + d*x)]))/(80*d*(a + I*a*Tan[c + d*x])
^(1/3))
```

Maple [A]

time = 0.10, size = 192, normalized size = 0.81

method	result
derivativedivides	$-\frac{3\left(\frac{(a+ia\tan(dx+c))^{\frac{5}{3}}}{5}-\frac{a(a+ia\tan(dx+c))^{\frac{2}{3}}}{2}+\frac{\left(\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia}{12a^{\frac{1}{3}}}\right.}{}\right.}{d\,a^{2}}$
default	$-\frac{3\left(\frac{(a+ia\tan(dx+c))^{\frac{5}{3}}}{5}-\frac{a(a+ia\tan(dx+c))^{\frac{2}{3}}}{2}+\frac{\left(\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia}{12a^{\frac{1}{3}}}\right.}{}\right.}{d\,a^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/d/a^2*(1/5*(a+I*a*tan(d*x+c))^(5/3)-1/2*a*(a+I*a*tan(d*x+c))^(2/3)+1/2*(
1/6*2^(2/3)/a^(1/3)*ln((a+I*a*tan(d*x+c))^(1/3)-2^(1/3)*a^(1/3))-1/12*2^(2/
3)/a^(1/3)*ln((a+I*a*tan(d*x+c))^(2/3)+2^(1/3)*a^(1/3)*(a+I*a*tan(d*x+c))^(
1/3)+2^(2/3)*a^(2/3))+1/6*3^(1/2)*2^(2/3)/a^(1/3)*arctan(1/3*3^(1/2)*(2^(2/
3)/a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+1)))*a^2-1/2*a^2/(a+I*a*tan(d*x+c))^(1/
3))
```

Maxima [A]

time = 0.54, size = 190, normalized size = 0.80

$$-\frac{10\,\sqrt{3}\,2^{\frac{2}{3}}a^{\frac{11}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)-5\cdot 2^{\frac{2}{3}}a^{\frac{11}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(i\,a\tan(dx+c)+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{2}{3}}\right)+10\cdot 2^{\frac{2}{3}}a^{\frac{11}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)+24\,(i\,a\tan(dx+c)+a)^{\frac{5}{3}}a^{2}-60\,(i\,a\tan(dx+c)+a)^{\frac{2}{3}}a^{3}-\frac{60\,a^{4}}{(i\,a\tan(dx+c)+a)^{\frac{1}{3}}}}{40\,a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] -1/40*(10*sqrt(3)*2^(2/3)*a^(11/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1
/3) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) - 5*2^(2/3)*a^(11/3)*log(2^(
2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x
+ c) + a)^(2/3)) + 10*2^(2/3)*a^(11/3)*log(-2^(1/3)*a^(1/3) + (I*a*tan(d*x
+ c) + a)^(1/3)) + 24*(I*a*tan(d*x + c) + a)^(5/3)*a^2 - 60*(I*a*tan(d*x +
c) + a)^(2/3)*a^3 - 60*a^4/(I*a*tan(d*x + c) + a)^(1/3))/(a^4*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(172) = 344$.
time = 1.49, size = 418, normalized size = 1.76

$$\frac{3 \cdot 2^{\frac{2}{3}} \left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{2}{3}} \left(7\,e^{(4i\,dx+4i\,c)} + 20\,e^{(2i\,dx+2i\,c)} + 5\right) e^{\left(\frac{4}{3}i\,dx+\frac{4}{3}i\,c\right)} + 10 \left(\frac{1}{2}\right)^{\frac{1}{3}} \left(ade^{(4i\,dx+4i\,c)} + ade^{(2i\,dx+2i\,c)}\right) \left(-\frac{1}{ad^3}\right)^{\frac{1}{3}} \log\left(-2\left(\frac{1}{2}\right)^{\frac{2}{3}} ad^2 \left(-\frac{1}{ad^3}\right)^{\frac{2}{3}} + 2^{\frac{1}{3}} \left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}} e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right) - 5\left(\frac{1}{2}\right)^{\frac{1}{3}} \left(\left(i\sqrt{3}\,ad + ad\right)e^{(4i\,dx+4i\,c)} + \left(i\sqrt{3}\,ad + ad\right)e^{(2i\,dx+2i\,c)}\right) \left(-\frac{1}{ad^3}\right)^{\frac{1}{3}} \log\left(-\left(\frac{1}{2}\right)^{\frac{2}{3}} \left(i\sqrt{3}\,ad^2 - ad^2\right) \left(-\frac{1}{ad^3}\right)^{\frac{2}{3}} + 2^{\frac{1}{3}} \left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}} e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right) - 5\left(\frac{1}{2}\right)^{\frac{1}{3}} \left(\left(-i\sqrt{3}\,ad + ad\right)e^{(4i\,dx+4i\,c)} + \left(-i\sqrt{3}\,ad + ad\right)e^{(2i\,dx+2i\,c)}\right) \left(-\frac{1}{ad^3}\right)^{\frac{1}{3}} \log\left(-\left(\frac{1}{2}\right)^{\frac{2}{3}} \left(-i\sqrt{3}\,ad^2 - ad^2\right) \left(-\frac{1}{ad^3}\right)^{\frac{2}{3}} + 2^{\frac{1}{3}} \left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}} e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)}{20\left(ade^{(4i\,dx+4i\,c)} + ade^{(2i\,dx+2i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/20*(3*2^(2/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(7*e^(4*I*d*x + 4*I*c)
+ 20*e^(2*I*d*x + 2*I*c) + 5)*e^(4/3*I*d*x + 4/3*I*c) + 10*(1/2)^(1/3)*(a*d
*e^(4*I*d*x + 4*I*c) + a*d*e^(2*I*d*x + 2*I*c))*(-1/(a*d^3))^(1/3)*log(-2*(
1/2)^(2/3)*a*d^2*(-1/(a*d^3))^(2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))
^(1/3)*e^(2/3*I*d*x + 2/3*I*c)) - 5*(1/2)^(1/3)*((I*sqrt(3)*a*d + a*d)*e^(4
*I*d*x + 4*I*c) + (I*sqrt(3)*a*d + a*d)*e^(2*I*d*x + 2*I*c))*(-1/(a*d^3))^(
1/3)*log(-(1/2)^(2/3)*(I*sqrt(3)*a*d^2 - a*d^2)*(-1/(a*d^3))^(2/3) + 2^(1/3
)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)) - 5*(1/2)^(1
/3)*((-I*sqrt(3)*a*d + a*d)*e^(4*I*d*x + 4*I*c) + (-I*sqrt(3)*a*d + a*d)*e^
(2*I*d*x + 2*I*c))*(-1/(a*d^3))^(1/3)*log(-(1/2)^(2/3)*(-I*sqrt(3)*a*d^2 -
a*d^2)*(-1/(a*d^3))^(2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(
2/3*I*d*x + 2/3*I*c)))/(a*d*e^(4*I*d*x + 4*I*c) + a*d*e^(2*I*d*x + 2*I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(c+dx)}{\sqrt[3]{ia(\tan(c+dx)-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+I*a*tan(d*x+c))**(1/3),x)
```

```
[Out] Integral(tan(c + d*x)**3/(I*a*(tan(c + d*x) - I))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^3/(I*a*tan(d*x + c) + a)^(1/3), x)
```

Mupad [B]
time = 0.36, size = 228, normalized size = 0.96

$$\frac{3}{2\,d\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{1/3}}+\frac{3\,{\left(a+a\,\tan(c+d\,x)\,1{}\mathrm{i}\right)}^{2/3}}{2\,a\,d}-\frac{3\,{\left(a+a\,\tan(c+d\,x)\,1{}\mathrm{i}\right)}^{5/3}}{5\,a^2\,d}+\frac{4^{1/3}\,\ln\left(18\,d\,{\left(a+a\,\tan(c+d\,x)\,1{}\mathrm{i}\right)}^{1/3}+9\,4^{2/3}\,{\left(-a\right)}^{1/3}\,d\right)}{4\,{\left(-a\right)}^{1/3}\,d}+\frac{4^{1/3}\,\ln\left(18\,d\,{\left(a+a\,\tan(c+d\,x)\,1{}\mathrm{i}\right)}^{1/3}+144\,4^{2/3}\,{\left(-a\right)}^{1/3}\,d\,{\left(-\frac{1}{8}+\frac{\sqrt{3}\,1{}\mathrm{i}}{8}\right)}^2\right)\,\left(-\frac{1}{8}+\frac{\sqrt{3}\,1{}\mathrm{i}}{8}\right)}{{\left(-a\right)}^{1/3}\,d}-\frac{4^{1/3}\,\ln\left(18\,d\,{\left(a+a\,\tan(c+d\,x)\,1{}\mathrm{i}\right)}^{1/3}+144\,4^{2/3}\,{\left(-a\right)}^{1/3}\,d\,{\left(\frac{1}{8}+\frac{\sqrt{3}\,1{}\mathrm{i}}{8}\right)}^2\right)\,\left(\frac{1}{8}+\frac{\sqrt{3}\,1{}\mathrm{i}}{8}\right)}{{\left(-a\right)}^{1/3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(1/3),x)
```

```
[Out] 3/(2*d*(a + a*tan(c + d*x)*1i)^(1/3)) + (3*(a + a*tan(c + d*x)*1i)^(2/3))/(
2*a*d) - (3*(a + a*tan(c + d*x)*1i)^(5/3))/(5*a^2*d) + (4^(1/3)*log(18*d*(a
 + a*tan(c + d*x)*1i)^(1/3) + 9*4^(2/3)*(-a)^(1/3)*d))/(4*(-a)^(1/3)*d) + (
4^(1/3)*log(18*d*(a + a*tan(c + d*x)*1i)^(1/3) + 144*4^(2/3)*(-a)^(1/3)*d*(
(3^(1/2)*1i)/8 - 1/8)^2)*((3^(1/2)*1i)/8 - 1/8))/((-a)^(1/3)*d) - (4^(1/3)*
log(18*d*(a + a*tan(c + d*x)*1i)^(1/3) + 144*4^(2/3)*(-a)^(1/3)*d*((3^(1/2)
*1i)/8 + 1/8)^2)*((3^(1/2)*1i)/8 + 1/8))/((-a)^(1/3)*d)
```

3.292 $\int \frac{\tan^2(c+dx)}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=213

$$\frac{x}{4\sqrt[3]{2}\sqrt[3]{a}}-\frac{i\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt[3]{2}\sqrt[3]{a}\,d}-\frac{i\log(\cos(c+dx))}{4\sqrt[3]{2}\sqrt[3]{a}\,d}-\frac{3i\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+i\right.}{4\sqrt[3]{2}\sqrt[3]{a}\,d}$$

```
[Out] 1/8*x*2^(2/3)/a^(1/3)-1/8*I*ln(cos(d*x+c))*2^(2/3)/a^(1/3)/d-3/8*I*ln(2^(1/
3)*a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))*2^(2/3)/a^(1/3)/d-1/4*I*arctan(1/3*(a^
(1/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)*2^(2/3)/a^
(1/3)/d-3/2*I/d/(a+I*a*tan(d*x+c))^(1/3)-3/2*I*(a+I*a*tan(d*x+c))^(2/3)/a/d
```

Rubi [A]
time = 0.12, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3624, 3560, 3562, 57, 631, 210, 31}

$$-\frac{i\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt[3]{2}\sqrt[3]{a}\,d}-\frac{3i(a+ia\tan(c+dx))^{2/3}}{2ad}-\frac{3i}{2d\sqrt[3]{a+ia\tan(c+dx)}}-\frac{3i\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{4\sqrt[3]{2}\sqrt[3]{a}\,d}-\frac{i\log(\cos(c+dx))}{4\sqrt[3]{2}\sqrt[3]{a}\,d}+\frac{x}{4\sqrt[3]{2}\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + I*a*Tan[c + d*x])^(1/3),x]
```

```
[Out] x/(4*2^(1/3)*a^(1/3)) - ((I/2)*Sqrt[3]*ArcTan[(a^(1/3) + 2^(2/3)*(a + I*a*T
an[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/(2^(1/3)*a^(1/3)*d) - ((I/4)*Log[Co
s[c + d*x]])/(2^(1/3)*a^(1/3)*d) - (((3*I)/4)*Log[2^(1/3)*a^(1/3) - (a + I*
a*Tan[c + d*x])^(1/3)])/(2^(1/3)*a^(1/3)*d) - ((3*I)/2)/(d*(a + I*a*Tan[c +
 d*x])^(1/3)) - (((3*I)/2)*(a + I*a*Tan[c + d*x])^(2/3))/(a*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx = -\frac{3i(a+ia\tan(c+dx))^{2/3}}{2ad} - \int \frac{1}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx$$

$$= -\frac{3i}{2d\sqrt[3]{a+ia\tan(c+dx)}} - \frac{3i(a+ia\tan(c+dx))^{2/3}}{2ad} - \frac{\int (a+ia\tan(c+d}{2a}$$

$$= -\frac{3i}{2d\sqrt[3]{a+ia\tan(c+dx)}} - \frac{3i(a+ia\tan(c+dx))^{2/3}}{2ad} + \frac{i\text{Subst}\left(\int \frac{1}{(a-x)\sqrt[3]{a}}\right.}{}$$

$$= \frac{x}{4\sqrt[3]{2}\sqrt[3]{a}} - \frac{i\log(\cos(c+dx))}{4\sqrt[3]{2}\sqrt[3]{a}\,d} - \frac{3i}{2d\sqrt[3]{a+ia\tan(c+dx)}} - \frac{3i(a+ia\tan(c}{2ad}$$

$$= \frac{x}{4\sqrt[3]{2}\sqrt[3]{a}} - \frac{i\log(\cos(c+dx))}{4\sqrt[3]{2}\sqrt[3]{a}\,d} - \frac{3i\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{4\sqrt[3]{2}\sqrt[3]{a}\,d}$$

$$= \frac{x}{4\sqrt[3]{2}\sqrt[3]{a}} - \frac{i\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt[3]{a}\,d} - \frac{i\log(\cos(c+dx)}{4\sqrt[3]{2}\sqrt[3]{a}\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.71, size = 82, normalized size = 0.38

$$-\frac{3\left(8i-4\tan(c+dx)+{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)(-i+\tan(c+dx))\right)}{8d\sqrt[3]{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + I*a*Tan[c + d*x])^(1/3),x]
```

```
[Out] (-3*(8*I - 4*Tan[c + d*x] + Hypergeometric2F1[2/3, 1, 5/3, E^((2*I)*(c + d*
x))/(1 + E^((2*I)*(c + d*x)))]*(-I + Tan[c + d*x])))/(8*d*(a + I*a*Tan[c +
d*x])^(1/3))
```

Maple [A]
time = 0.09, size = 172, normalized size = 0.81

method	result

derivativedivides	$-\frac{3i\left(\frac{(a+ia\tan(dx+c))^{\frac{2}{3}}}{2}+\frac{\left(\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{12a^{\frac{1}{3}}}\right.}{2}\right.}{da}$
default	$-\frac{3i\left(\frac{(a+ia\tan(dx+c))^{\frac{2}{3}}}{2}+\frac{\left(\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{12a^{\frac{1}{3}}}\right.}{2}\right.}{da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3*I/d/a*(1/2*(a+I*a*tan(d*x+c))^(2/3)+1/2*(1/6*2^(2/3)/a^(1/3)*ln((a+I*a*t
an(d*x+c))^(1/3)-2^(1/3)*a^(1/3))-1/12*2^(2/3)/a^(1/3)*ln((a+I*a*tan(d*x+c)
)^(2/3)+2^(1/3)*a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+2^(2/3)*a^(2/3))+1/6*3^(1/
2)*2^(2/3)/a^(1/3)*arctan(1/3*3^(1/2)*(2^(2/3)/a^(1/3)*(a+I*a*tan(d*x+c))^(
1/3)+1)))*a+1/2*a/(a+I*a*tan(d*x+c))^(1/3))
```

Maxima [A]
time = 0.52, size = 172, normalized size = 0.81

$$-\frac{i\left(2\sqrt{3}\,2^{\frac{2}{3}}a^{\frac{8}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)-2^{\frac{2}{3}}a^{\frac{8}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(i\,a\tan(dx+c)+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{2}{3}}\right)+2\cdot 2^{\frac{2}{3}}a^{\frac{8}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)+12\,(i\,a\tan(dx+c)+a)^{\frac{2}{3}}a^2+\frac{12\,a^3}{(i\,a\tan(dx+c)+a)^{\frac{1}{3}}}\right)}{8\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] -1/8*I*(2*sqrt(3)*2^(2/3)*a^(8/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/
3) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) - 2^(2/3)*a^(8/3)*log(2^(2/3)
*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x + c)
 + a)^(2/3)) + 2*2^(2/3)*a^(8/3)*log(-2^(1/3)*a^(1/3) + (I*a*tan(d*x + c) +
 a)^(1/3)) + 12*(I*a*tan(d*x + c) + a)^(2/3)*a^2 + 12*a^3/(I*a*tan(d*x + c)
 + a)^(1/3))/(a^3*d)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(146) = 292$.
time = 1.32, size = 315, normalized size = 1.48

$$\frac{\left(4\,ad\left(\frac{i}{16\,ad^3}\right)^{\frac{1}{3}}e^{(2i\,dx+2i\,c)}\log\left(8\,ad^2\left(\frac{i}{16\,ad^3}\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)-3\cdot 2^{\frac{2}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{2}{3}}\left(3i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(\frac{4}{3}i\,dx+\frac{4}{3}i\,c\right)}-2\left(-i\sqrt{3}\,ad+ad\right)\left(\frac{i}{16\,ad^3}\right)^{\frac{1}{3}}e^{(2i\,dx+2i\,c)}\log\left(-4\left(i\sqrt{3}\,ad^2+ad^2\right)\left(\frac{i}{16\,ad^3}\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)-2\left(i\sqrt{3}\,ad+ad\right)\left(\frac{i}{16\,ad^3}\right)^{\frac{1}{3}}e^{(2i\,dx+2i\,c)}\log\left(-4\left(-i\sqrt{3}\,ad^2+ad^2\right)\left(\frac{i}{16\,ad^3}\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)\right)e^{(-2i\,dx-2i\,c)}}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/4*(4*a*d*(1/16*I/(a*d^3))^(1/3)*e^(2*I*d*x + 2*I*c)*log(8*a*d^2*(1/16*I/(
a*d^3))^(2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x +
2/3*I*c)) - 3*2^(2/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(3*I*e^(2*I*d*x +
 2*I*c) + I)*e^(4/3*I*d*x + 4/3*I*c) - 2*(-I*sqrt(3)*a*d + a*d)*(1/16*I/(a*
d^3))^(1/3)*e^(2*I*d*x + 2*I*c)*log(-4*(I*sqrt(3)*a*d^2 + a*d^2)*(1/16*I/(a
*d^3))^(2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2
/3*I*c)) - 2*(I*sqrt(3)*a*d + a*d)*(1/16*I/(a*d^3))^(1/3)*e^(2*I*d*x + 2*I*
c)*log(-4*(-I*sqrt(3)*a*d^2 + a*d^2)*(1/16*I/(a*d^3))^(2/3) + 2^(1/3)*(a/(e
^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)))*e^(-2*I*d*x - 2*I*
c)/(a*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(c+dx)}{\sqrt[3]{ia(\tan(c+dx)-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+I*a*tan(d*x+c))**(1/3),x)
```

```
[Out] Integral(tan(c + d*x)**2/(I*a*(tan(c + d*x) - I))**(1/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^2/(I*a*tan(d*x + c) + a)^(1/3), x)
```

Mupad [B]
time = 4.48, size = 210, normalized size = 0.99

$$-\frac{3\mathrm{i}}{2\,d\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{1/3}}-\frac{\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{2/3}\,3\mathrm{i}}{2\,a\,d}+\frac{\left(\frac{1}{16}\mathrm{i}\right)^{1/3}\,\ln\left(9\,\left(a\,\left(1+\tan(c+d\,x)\,1\mathrm{i}\right)\right)^{1/3}+9\,\left(-1\right)^{1/3}\,2^{1/3}\,a^{1/3}\right)}{a^{1/3}\,d}-\frac{\left(\frac{1}{16}\mathrm{i}\right)^{1/3}\,\ln\left(-\frac{9\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{1/3}}{4\,d^2}-\frac{9\,\left(-1\right)^{1/3}\,2^{1/3}\,a^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{8\,d^2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{1/3}\,d}+\frac{\left(\frac{1}{16}\mathrm{i}\right)^{1/3}\,\ln\left(-\frac{9\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{1/3}}{4\,d^2}+\frac{9\,\left(-1\right)^{1/3}\,2^{1/3}\,a^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{8\,d^2}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{1/3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(1/3),x)
```

```
[Out] ((1i/16)^(1/3)*log(9*(a*(tan(c + d*x)*1i + 1))^(1/3) + 9*(-1)^(1/3)*2^(1/3)
*a^(1/3)))/(a^(1/3)*d) - ((a + a*tan(c + d*x)*1i)^(2/3)*3i)/(2*a*d) - 3i/(2
*d*(a + a*tan(c + d*x)*1i)^(1/3)) - ((1i/16)^(1/3)*log(- (9*(a + a*tan(c +
d*x)*1i)^(1/3))/(4*d^2) - (9*(-1)^(1/3)*2^(1/3)*a^(1/3)*(3^(1/2)*1i - 1))/(
8*d^2))*((3^(1/2)*1i)/2 + 1/2))/(a^(1/3)*d) + ((1i/16)^(1/3)*log((9*(-1)^(1
/3)*2^(1/3)*a^(1/3)*(3^(1/2)*1i + 1))/(8*d^2) - (9*(a + a*tan(c + d*x)*1i)^
(1/3))/(4*d^2))*((3^(1/2)*1i)/2 - 1/2))/(a^(1/3)*d)
```

3.293 $\int \frac{\tan(c+dx)}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=178

$$\frac{ix}{4\sqrt[3]{2}\sqrt[3]{a}}+\frac{\sqrt{3}\,\mathrm{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt[3]{2}\sqrt[3]{a}\,d}+\frac{\log(\cos(c+dx))}{4\sqrt[3]{2}\sqrt[3]{a}\,d}+\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{4\sqrt[3]{2}\sqrt[3]{a}\,d}$$

```
[Out] 1/8*I*x*2^(2/3)/a^(1/3)+1/8*ln(cos(d*x+c))*2^(2/3)/a^(1/3)/d+3/8*ln(2^(1/3)
*a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))*2^(2/3)/a^(1/3)/d+1/4*arctan(1/3*(a^(1/3
)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)*2^(2/3)/a^(1/3
)/d-3/2/d/(a+I*a*tan(d*x+c))^(1/3)
```

Rubi [A]
time = 0.09, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3607, 3562, 57, 631, 210, 31}

$$\frac{\sqrt{3}\,\mathrm{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt[3]{2}\sqrt[3]{a}\,d}-\frac{3}{2d\sqrt[3]{a+ia\tan(c+dx)}}+\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{4\sqrt[3]{2}\sqrt[3]{a}\,d}+\frac{\log(\cos(c+dx))}{4\sqrt[3]{2}\sqrt[3]{a}\,d}+\frac{ix}{4\sqrt[3]{2}\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + I*a*Tan[c + d*x])^(1/3),x]
```

```
[Out] ((I/4)*x)/(2^(1/3)*a^(1/3)) + (Sqrt[3]*ArcTan[(a^(1/3) + 2^(2/3)*(a + I*a*T
an[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/(2*2^(1/3)*a^(1/3)*d) + Log[Cos[c +
 d*x]]/(4*2^(1/3)*a^(1/3)*d) + (3*Log[2^(1/3)*a^(1/3) - (a + I*a*Tan[c + d*
x])^(1/3)])/(4*2^(1/3)*a^(1/3)*d) - 3/(2*d*(a + I*a*Tan[c + d*x])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan(c+dx)}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx &= -\frac{3}{2d\sqrt[3]{a+ia\tan(c+dx)}} - \frac{i\int (a+ia\tan(c+dx))^{2/3}\,dx}{2a} \\
&= -\frac{3}{2d\sqrt[3]{a+ia\tan(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{(a-x)\sqrt[3]{a+x}}\,dx, x, ia\tan(c+dx)\right)}{2d} \\
&= \frac{ix}{4\sqrt[3]{2}\sqrt[3]{a}} + \frac{\log(\cos(c+dx))}{4\sqrt[3]{2}\sqrt[3]{a}\,d} - \frac{3}{2d\sqrt[3]{a+ia\tan(c+dx)}} + \frac{3\text{Subst}\left(\int \frac{}{2^{2/3}a^{2/3}\cdots}\right.}{} \cdots \\
&= \frac{ix}{4\sqrt[3]{2}\sqrt[3]{a}} + \frac{\log(\cos(c+dx))}{4\sqrt[3]{2}\sqrt[3]{a}\,d} + \frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{4\sqrt[3]{2}\sqrt[3]{a}\,d} - \cdots \\
&= \frac{ix}{4\sqrt[3]{2}\sqrt[3]{a}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt[3]{a}\,d} + \frac{\log(\cos(c+dx))}{4\sqrt[3]{2}\sqrt[3]{a}\,d} + \cdots
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.64, size = 140, normalized size = 0.79

$$-\frac{3\left(2\left(1+e^{2idx}\right)\cos(c)+e^{2idx}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)(\cos(c)+i\sin(c))+2i\left(-1+e^{2idx}\right)\sin(c)\right)}{4d\left(\left(1+e^{2idx}\right)\cos(c)+i\left(-1+e^{2idx}\right)\sin(c)\right)\sqrt[3]{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + I*a*Tan[c + d*x])^(1/3),x]
```

```
[Out] (-3*(2*(1 + E^((2*I)*d*x))*Cos[c] + E^((2*I)*d*x)*Hypergeometric2F1[2/3, 1,
 5/3, E^((2*I)*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(Cos[c] + I*Sin[c]) +
(2*I)*(-1 + E^((2*I)*d*x))*Sin[c]))/(4*d*((1 + E^((2*I)*d*x))*Cos[c] + I*(-
1 + E^((2*I)*d*x))*Sin[c])*(a + I*a*Tan[c + d*x])^(1/3))
```

Maple [A]
time = 0.12, size = 146, normalized size = 0.82

method	result
derivativedivides	$\frac{\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{4a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{8a^{\frac{1}{3}}}+\frac{\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}}{\cdots}\right.}{\cdots}}{d}$
default	$\frac{\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{4a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{8a^{\frac{1}{3}}}+\frac{\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}}{\cdots}\right.}{\cdots}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+I*a*tan(d*x+c))^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*2^(2/3)/a^(1/3)*ln((a+I*a*tan(d*x+c))^(1/3)-2^(1/3)*a^(1/3))-1/8*2
^(2/3)/a^(1/3)*ln((a+I*a*tan(d*x+c))^(2/3)+2^(1/3)*a^(1/3)*(a+I*a*tan(d*x+c
))^(1/3)+2^(2/3)*a^(2/3))+1/4*3^(1/2)*2^(2/3)/a^(1/3)*arctan(1/3*3^(1/2)*(2
^(2/3)/a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+1))-3/2/(a+I*a*tan(d*x+c))^(1/3))
```

Maxima [A]
time = 0.53, size = 154, normalized size = 0.87

$$\frac{2\sqrt{3}\,2^{\frac{2}{3}}a^{\frac{5}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(ia\tan(dx+c)+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)-2^{\frac{2}{3}}a^{\frac{5}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(ia\tan(dx+c)+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(ia\tan(dx+c)+a)^{\frac{2}{3}}\right)+2\cdot 2^{\frac{2}{3}}a^{\frac{5}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(ia\tan(dx+c)+a)^{\frac{1}{3}}\right)-\frac{12a^2}{(ia\tan(dx+c)+a)^{\frac{1}{3}}}}{8a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] 1/8*(2*sqrt(3)*2^(2/3)*a^(5/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3)
+ 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) - 2^(2/3)*a^(5/3)*log(2^(2/3)*a^
(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x + c) +
a)^(2/3)) + 2*2^(2/3)*a^(5/3)*log(-2^(1/3)*a^(1/3) + (I*a*tan(d*x + c) + a)
^(1/3)) - 12*a^2/(I*a*tan(d*x + c) + a)^(1/3))/(a^2*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(125) = 250$.
time = 1.17, size = 327, normalized size = 1.84

$$\frac{\left(2\left(\frac{1}{2}\right)^{\frac{1}{3}}ad\left(\frac{1}{ad^3}\right)^{\frac{1}{3}}e^{(2i\,dx+2i\,c)}\log\left(-2\left(\frac{1}{2}\right)^{\frac{2}{3}}ad^2\left(\frac{1}{ad^3}\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)-\left(\frac{1}{2}\right)^{\frac{1}{3}}\left(i\sqrt{3}\,ad+ad\right)\left(\frac{1}{ad^3}\right)^{\frac{1}{3}}e^{(2i\,dx+2i\,c)}\log\left(-\left(\frac{1}{2}\right)^{\frac{2}{3}}\left(i\sqrt{3}\,ad^2-ad^2\right)\left(\frac{1}{ad^3}\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)-\left(\frac{1}{2}\right)^{\frac{1}{3}}\left(-i\sqrt{3}\,ad+ad\right)\left(\frac{1}{ad^3}\right)^{\frac{1}{3}}e^{(2i\,dx+2i\,c)}\log\left(-\left(\frac{1}{2}\right)^{\frac{2}{3}}\left(-i\sqrt{3}\,ad^2-ad^2\right)\left(\frac{1}{ad^3}\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)-3\cdot2^{\frac{2}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{2}{3}}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{\left(\frac{4}{3}i\,dx+\frac{4}{3}i\,c\right)}\right)e^{(-2i\,dx-2i\,c)}}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(1/2)^(1/3)*a*d*(1/(a*d^3))^(1/3)*e^(2*I*d*x + 2*I*c)*log(-2*(1/2)^(
2/3)*a*d^2*(1/(a*d^3))^(2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*
e^(2/3*I*d*x + 2/3*I*c)) - (1/2)^(1/3)*(I*sqrt(3)*a*d + a*d)*(1/(a*d^3))^(1
/3)*e^(2*I*d*x + 2*I*c)*log(-(1/2)^(2/3)*(I*sqrt(3)*a*d^2 - a*d^2)*(1/(a*d^
3))^(2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*
I*c)) - (1/2)^(1/3)*(-I*sqrt(3)*a*d + a*d)*(1/(a*d^3))^(1/3)*e^(2*I*d*x + 2
*I*c)*log(-(1/2)^(2/3)*(-I*sqrt(3)*a*d^2 - a*d^2)*(1/(a*d^3))^(2/3) + 2^(1/
3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)) - 3*2^(2/3)
*(a/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(e^(2*I*d*x + 2*I*c) + 1)*e^(4/3*I*d*x
 + 4/3*I*c))*e^(-2*I*d*x - 2*I*c)/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(c+dx\right)}{\sqrt[3]{ia\left(\tan\left(c+dx\right)-i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c))**(1/3),x)
```

```
[Out] Integral(tan(c + d*x)/(I*a*(tan(c + d*x) - I))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)/(I*a*tan(d*x + c) + a)^(1/3), x)
```

Mupad [B]
time = 0.24, size = 172, normalized size = 0.97

$$-\frac{3}{2\,d\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{1/3}}+\frac{4^{1/3}\,\ln\left(18\,d\,{\left(a+a\,\tan(c+d\,x)\,1{}\mathrm{i}\right)}^{1/3}-9\,4^{2/3}\,a^{1/3}\,d\right)}{4\,a^{1/3}\,d}+\frac{4^{1/3}\,\ln\left(18\,d\,{\left(a+a\,\tan(c+d\,x)\,1{}\mathrm{i}\right)}^{1/3}-144\,4^{2/3}\,a^{1/3}\,d\,{\left(-\frac{1}{8}+\frac{\sqrt{3}\,1{}\mathrm{i}}{8}\right)}^2\right)\,\left(-\frac{1}{8}+\frac{\sqrt{3}\,1{}\mathrm{i}}{8}\right)}{a^{1/3}\,d}-\frac{4^{1/3}\,\ln\left(18\,d\,{\left(a+a\,\tan(c+d\,x)\,1{}\mathrm{i}\right)}^{1/3}-144\,4^{2/3}\,a^{1/3}\,d\,{\left(\frac{1}{8}+\frac{\sqrt{3}\,1{}\mathrm{i}}{8}\right)}^2\right)\,\left(\frac{1}{8}+\frac{\sqrt{3}\,1{}\mathrm{i}}{8}\right)}{a^{1/3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + a*tan(c + d*x)*1i)^(1/3),x)
```

```
[Out] (4^(1/3)*log(18*d*(a + a*tan(c + d*x)*1i)^(1/3) - 9*4^(2/3)*a^(1/3)*d))/(4*
a^(1/3)*d) - 3/(2*d*(a + a*tan(c + d*x)*1i)^(1/3)) + (4^(1/3)*log(18*d*(a +
 a*tan(c + d*x)*1i)^(1/3) - 144*4^(2/3)*a^(1/3)*d*((3^(1/2)*1i)/8 - 1/8)^2)
*((3^(1/2)*1i)/8 - 1/8))/(a^(1/3)*d) - (4^(1/3)*log(18*d*(a + a*tan(c + d*x
)*1i)^(1/3) - 144*4^(2/3)*a^(1/3)*d*((3^(1/2)*1i)/8 + 1/8)^2)*((3^(1/2)*1i)
/8 + 1/8))/(a^(1/3)*d)
```

3.294 $\int \frac{1}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=184

$$-\frac{x}{4\sqrt[3]{2}\sqrt[3]{a}}+\frac{i\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt[3]{2}\sqrt[3]{a}\,d}+\frac{i\log(\cos(c+dx))}{4\sqrt[3]{2}\sqrt[3]{a}\,d}+\frac{3i\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+i\right.}{4\sqrt[3]{2}\sqrt[3]{a}\,d}$$

```
[Out] -1/8*x*2^(2/3)/a^(1/3)+1/8*I*ln(cos(d*x+c))*2^(2/3)/a^(1/3)/d+3/8*I*ln(2^(1
/3)*a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))*2^(2/3)/a^(1/3)/d+1/4*I*arctan(1/3*(a
^(1/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)*2^(2/3)/a
^(1/3)/d+3/2*I/d/(a+I*a*tan(d*x+c))^(1/3)
```

Rubi [A]
time = 0.08, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3560, 3562, 57, 631, 210, 31}

$$\frac{i\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt[3]{2}\sqrt[3]{a}\,d}+\frac{3i}{2d\sqrt[3]{a+ia\tan(c+dx)}}+\frac{3i\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{4\sqrt[3]{2}\sqrt[3]{a}\,d}+\frac{i\log(\cos(c+dx))}{4\sqrt[3]{2}\sqrt[3]{a}\,d}-\frac{x}{4\sqrt[3]{2}\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(-1/3),x]
```

```
[Out] -1/4*x/(2^(1/3)*a^(1/3)) + ((I/2)*Sqrt[3]*ArcTan[(a^(1/3) + 2^(2/3)*(a + I*
a*Tan[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/(2^(1/3)*a^(1/3)*d) + ((I/4)*Log
[Cos[c + d*x]])/(2^(1/3)*a^(1/3)*d) + (((3*I)/4)*Log[2^(1/3)*a^(1/3) - (a +
 I*a*Tan[c + d*x])^(1/3)])/(2^(1/3)*a^(1/3)*d) + ((3*I)/2)/(d*(a + I*a*Tan[
c + d*x])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx = \frac{3i}{2d\sqrt[3]{a+ia\tan(c+dx)}} + \frac{\int (a+ia\tan(c+dx))^{2/3}\,dx}{2a}$$

$$= \frac{3i}{2d\sqrt[3]{a+ia\tan(c+dx)}} - \frac{i\text{Subst}\left(\int \frac{1}{(a-x)\sqrt[3]{a+x}}\,dx, x, ia\tan(c+dx)\right)}{2d}$$

$$= -\frac{x}{4\sqrt[3]{2}\sqrt[3]{a}} + \frac{i\log(\cos(c+dx))}{4\sqrt[3]{2}\sqrt[3]{a}\,d} + \frac{3i}{2d\sqrt[3]{a+ia\tan(c+dx)}} + \frac{(3i)\text{Subst}\left(\int \right.}{}$$

$$= -\frac{x}{4\sqrt[3]{2}\sqrt[3]{a}} + \frac{i\log(\cos(c+dx))}{4\sqrt[3]{2}\sqrt[3]{a}\,d} + \frac{3i\log\left(\sqrt[3]{2}\sqrt[3]{a} - \sqrt[3]{a+ia\tan(c+dx)}\right)}{4\sqrt[3]{2}\sqrt[3]{a}\,d}$$

$$= -\frac{x}{4\sqrt[3]{2}\sqrt[3]{a}} + \frac{i\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt[3]{a}\,d} + \frac{i\log(\cos(c+d}{4\sqrt[3]{2}\sqrt[3]{a}\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.45, size = 141, normalized size = 0.77

$$\frac{3\left(-2\left(1+e^{2idx}\right)\cos(c)+e^{2idx}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)(\cos(c)+i\sin(c))-2i\left(-1+e^{2idx}\right)\sin(c)\right)}{4d\left(i\left(1+e^{2idx}\right)\cos(c)-\left(-1+e^{2idx}\right)\sin(c)\right)\sqrt[3]{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(-1/3),x]
```

```
[Out] (3*(-2*(1 + E^((2*I)*d*x))*Cos[c] + E^((2*I)*d*x)*Hypergeometric2F1[2/3, 1,
 5/3, E^((2*I)*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(Cos[c] + I*Sin[c]) -
(2*I)*(-1 + E^((2*I)*d*x))*Sin[c]))/(4*d*(I*(1 + E^((2*I)*d*x))*Cos[c] - (-
1 + E^((2*I)*d*x))*Sin[c])*(a + I*a*Tan[c + d*x])^(1/3))
```

Maple [A]
time = 0.08, size = 158, normalized size = 0.86

method	result
derivativedivides	$\frac{3ia\left(\frac{\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{12a^{\frac{1}{3}}}+\frac{\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\sqrt{\ldots}\right.}{\ldots}}{2a}\ldots\right.}{d}$
default	$\frac{3ia\left(\frac{\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{12a^{\frac{1}{3}}}+\frac{\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\sqrt{\ldots}\right.}{\ldots}}{2a}\ldots\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(d*x+c))^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3*I/d*a*(1/2*(1/6*2^(2/3)/a^(1/3)*ln((a+I*a*tan(d*x+c))^(1/3)-2^(1/3)*a^(1/
3))-1/12*2^(2/3)/a^(1/3)*ln((a+I*a*tan(d*x+c))^(2/3)+2^(1/3)*a^(1/3)*(a+I*a
*tan(d*x+c))^(1/3)+2^(2/3)*a^(2/3))+1/6*3^(1/2)*2^(2/3)/a^(1/3)*arctan(1/3*
3^(1/2)*(2^(2/3)/a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+1)))/a+1/2/a/(a+I*a*tan(d
*x+c))^(1/3))
```

Maxima [A]
time = 0.54, size = 152, normalized size = 0.83

$$\frac{i\left(2\sqrt{3}\,2^{\frac{2}{3}}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(i\,a\tan\left(dx+c\right)+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)-2^{\frac{2}{3}}a^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(i\,a\tan\left(dx+c\right)+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan\left(dx+c\right)+a)^{\frac{2}{3}}\right)+2\cdot 2^{\frac{2}{3}}a^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan\left(dx+c\right)+a)^{\frac{1}{3}}\right)+\frac{12\,a}{(i\,a\tan(dx+c)+a)^{\frac{1}{3}}}\right)}{8\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] 1/8*I*(2*sqrt(3)*2^(2/3)*a^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3
) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) - 2^(2/3)*a^(2/3)*log(2^(2/3)*
a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x + c)
+ a)^(2/3)) + 2*2^(2/3)*a^(2/3)*log(-2^(1/3)*a^(1/3) + (I*a*tan(d*x + c) +
a)^(1/3)) + 12*a/(I*a*tan(d*x + c) + a)^(1/3))/(a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 315 vs. 2(125) = 250.
time = 0.94, size = 315, normalized size = 1.71

$$\frac{\left(4\,ad\left(-\frac{i}{16\,ad^3}\right)^{\frac{1}{3}}e^{(2i\,dx+2i\,c)}\log\left(8\,ad^2\left(-\frac{i}{16\,ad^3}\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)-3\cdot 2^{\frac{2}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{2}{3}}\left(-i\,e^{(2i\,dx+2i\,c)}-i\right)e^{\left(\frac{4}{3}i\,dx+\frac{4}{3}i\,c\right)}-2\left(-i\sqrt{3}\,ad+ad\right)\left(-\frac{i}{16\,ad^3}\right)^{\frac{1}{3}}e^{(2i\,dx+2i\,c)}\log\left(-4\left(i\sqrt{3}\,ad^2+ad^2\right)\left(-\frac{i}{16\,ad^3}\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)-2\left(i\sqrt{3}\,ad+ad\right)\left(-\frac{i}{16\,ad^3}\right)^{\frac{1}{3}}e^{(2i\,dx+2i\,c)}\log\left(-4\left(-i\sqrt{3}\,ad^2+ad^2\right)\left(-\frac{i}{16\,ad^3}\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)\right)e^{(-2i\,dx-2i\,c)}}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/4*(4*a*d*(-1/16*I/(a*d^3))^(1/3)*e^(2*I*d*x + 2*I*c)*log(8*a*d^2*(-1/16*I
/(a*d^3))^(2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x
+ 2/3*I*c)) - 3*2^(2/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(-I*e^(2*I*d*x
+ 2*I*c) - I)*e^(4/3*I*d*x + 4/3*I*c) - 2*(-I*sqrt(3)*a*d + a*d)*(-1/16*I/(
a*d^3))^(1/3)*e^(2*I*d*x + 2*I*c)*log(-4*(I*sqrt(3)*a*d^2 + a*d^2)*(-1/16*I
/(a*d^3))^(2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x
+ 2/3*I*c)) - 2*(I*sqrt(3)*a*d + a*d)*(-1/16*I/(a*d^3))^(1/3)*e^(2*I*d*x +
2*I*c)*log(-4*(-I*sqrt(3)*a*d^2 + a*d^2)*(-1/16*I/(a*d^3))^(2/3) + 2^(1/3)*
(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)))*e^(-2*I*d*x -
 2*I*c)/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{ia\tan\left(c+dx\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))**(1/3),x)
```

```
[Out] Integral((I*a*tan(c + d*x) + a)**(-1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(-1/3), x)
```

Mupad [B]
time = 4.39, size = 197, normalized size = 1.07

$$\frac{3\mathrm{i}}{2\,d\,{\left(a+a\,\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}^{1/3}}+\frac{{\left(\frac{1}{16}\mathrm{i}\right)}^{1/3}\,\ln\left({\left(a\,\left(1+\tan(c+d\,x)\,1\mathrm{i}\right)\right)}^{1/3}-{\left(-1\right)}^{1/3}\,2^{1/3}\,{\left(-a\right)}^{1/3}\right)}{{\left(-a\right)}^{1/3}\,d}-\frac{{\left(\frac{1}{16}\mathrm{i}\right)}^{1/3}\,\ln\left(-\frac{9\,{\left(a+a\,\tan(c+d\,x)\,1\mathrm{i}\right)}^{1/3}}{4\,d^2}+\frac{9\,{\left(-1\right)}^{1/3}\,2^{1/3}\,{\left(-a\right)}^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{8\,d^2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{{\left(-a\right)}^{1/3}\,d}+\frac{{\left(\frac{1}{16}\mathrm{i}\right)}^{1/3}\,\ln\left(-\frac{9\,{\left(a+a\,\tan(c+d\,x)\,1\mathrm{i}\right)}^{1/3}}{4\,d^2}-\frac{9\,{\left(-1\right)}^{1/3}\,2^{1/3}\,{\left(-a\right)}^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{8\,d^2}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{{\left(-a\right)}^{1/3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*tan(c + d*x)*1i)^(1/3),x)
```

```
[Out] 3i/(2*d*(a + a*tan(c + d*x)*1i)^(1/3)) + ((1i/16)^(1/3)*log((a*(tan(c + d*x
)*1i + 1))^(1/3) - (-1)^(1/3)*2^(1/3)*(-a)^(1/3)))/((-a)^(1/3)*d) - ((1i/16
)^(1/3)*log((9*(-1)^(1/3)*2^(1/3)*(-a)^(1/3)*(3^(1/2)*1i - 1))/(8*d^2) - (9
*(a + a*tan(c + d*x)*1i)^(1/3))/(4*d^2))*((3^(1/2)*1i)/2 + 1/2))/((-a)^(1/3
)*d) + ((1i/16)^(1/3)*log(- (9*(a + a*tan(c + d*x)*1i)^(1/3))/(4*d^2) - (9*
(-1)^(1/3)*2^(1/3)*(-a)^(1/3)*(3^(1/2)*1i + 1))/(8*d^2))*((3^(1/2)*1i)/2 -
1/2))/((-a)^(1/3)*d)
```

3.295 $\int \frac{\cot(c+dx)}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=286

$$-\frac{ix}{4\sqrt[3]{2}\sqrt[3]{a}}+\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}\,d}-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt[3]{2}\sqrt[3]{a}\,d}-$$

```
[Out] -1/8*I*x*2^(2/3)/a^(1/3)-1/8*ln(cos(d*x+c))*2^(2/3)/a^(1/3)/d-1/2*ln(tan(d*
x+c))/a^(1/3)/d+3/2*ln(a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)/d-3/8*ln(2
^(1/3)*a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))*2^(2/3)/a^(1/3)/d+arctan(1/3*(a^(1
/3)+2*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)/a^(1/3)/d-1/4*arct
an(1/3*(a^(1/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)*
2^(2/3)/a^(1/3)/d+3/2/d/(a+I*a*tan(d*x+c))^(1/3)
```

Rubi [A]
time = 0.30, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3643, 3560, 3562, 57, 631, 210, 31, 3677, 3680}

$$\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}\,d}-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt[3]{2}\sqrt[3]{a}\,d}+\frac{3}{2d\sqrt[3]{a+ia\tan(c+dx)}}-\frac{\log(\tan(c+dx))}{2\sqrt[3]{a}\,d}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{2\sqrt[3]{a}\,d}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{4\sqrt[3]{2}\sqrt[3]{a}\,d}-\frac{\log(\cos(c+dx))}{4\sqrt[3]{2}\sqrt[3]{a}\,d}-\frac{ix}{4\sqrt[3]{2}\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + I*a*Tan[c + d*x])^(1/3),x]
```

```
[Out] ((-1/4*I)*x)/(2^(1/3)*a^(1/3)) + (Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + I*a*Tan[
c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/(a^(1/3)*d) - (Sqrt[3]*ArcTan[(a^(1/3)
 + 2^(2/3)*(a + I*a*Tan[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/(2*2^(1/3)*a^(
1/3)*d) - Log[Cos[c + d*x]]/(4*2^(1/3)*a^(1/3)*d) - Log[Tan[c + d*x]]/(2*a^
(1/3)*d) + (3*Log[a^(1/3) - (a + I*a*Tan[c + d*x])^(1/3)])/(2*a^(1/3)*d) -
(3*Log[2^(1/3)*a^(1/3) - (a + I*a*Tan[c + d*x])^(1/3)])/(4*2^(1/3)*a^(1/3)*
d) + 3/(2*d*(a + I*a*Tan[c + d*x])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3643

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)/((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[a/(a*c - b*d), Int[(a + b*Tan[e + f*x])^m,
x], x] - Dist[d/(a*c - b*d), Int[(a + b*Tan[e + f*x])^m*((b + a*Tan[e + f*x
])/(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx = i\int \frac{1}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx - \frac{i\int \frac{\cot(c+dx)(ia+a\tan(c+dx))}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx}{a}$$

$$= \frac{3}{2d\sqrt[3]{a+ia\tan(c+dx)}} - \frac{(3i)\int \cot(c+dx)(a+ia\tan(c+dx))^{2/3}\left(\frac{2ia^2}{3}+\right.}{2a^3}$$

$$= \frac{3}{2d\sqrt[3]{a+ia\tan(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{(a-x)\sqrt[3]{a+x}}\,dx, x, ia\tan(c+dx)\right)}{2d} +$$

$$= -\frac{ix}{4\sqrt[3]{2}\sqrt[3]{a}} - \frac{\log(\cos(c+dx))}{4\sqrt[3]{2}\sqrt[3]{a}\,d} - \frac{\log(\tan(c+dx))}{2\sqrt[3]{a}\,d} + \frac{3}{2d\sqrt[3]{a+ia\tan(c+d}}$$

$$= -\frac{ix}{4\sqrt[3]{2}\sqrt[3]{a}} - \frac{\log(\cos(c+dx))}{4\sqrt[3]{2}\sqrt[3]{a}\,d} - \frac{\log(\tan(c+dx))}{2\sqrt[3]{a}\,d} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+i}\right.}{2\sqrt[3]{a}\,d}$$

$$= -\frac{ix}{4\sqrt[3]{2}\sqrt[3]{a}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}\,d} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2^{2/}}{}}{}\right.}{2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.29, size = 195, normalized size = 0.68

$$\frac{3\left(2\left(1+e^{2idx}\right)\cos(c)+e^{2idx}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)(\cos(c)+i\sin(c))-4e^{2idx}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{2e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)(\cos(c)+i\sin(c))+2i\left(-1+e^{2idx}\right)\sin(c)\right)}{4d\left(\left(1+e^{2idx}\right)\cos(c)+i\left(-1+e^{2idx}\right)\sin(c)\right)\sqrt[3]{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + I*a*Tan[c + d*x])^(1/3),x]
```

[Out] (3*(2*(1 + E^((2*I)*d*x))*Cos[c] + E^((2*I)*d*x)*Hypergeometric2F1[2/3, 1, 5/3, E^((2*I)*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(Cos[c] + I*Sin[c]) - 4 *E^((2*I)*d*x)*Hypergeometric2F1[2/3, 1, 5/3, (2*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(Cos[c] + I*Sin[c]) + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c]))/(4*d*((1 + E^((2*I)*d*x))*Cos[c] + I*(-1 + E^((2*I)*d*x))*Sin[c])*(a + I*a*Tan[c + d*x])^(1/3))

Maple [F]
time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\cot (dx+c)}{(a+ia\tan (dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)/(a+I*a*tan(d*x+c))^(1/3),x)

[Out] int(cot(d*x+c)/(a+I*a*tan(d*x+c))^(1/3),x)

Maxima [A]
time = 0.54, size = 249, normalized size = 0.87

$$-\frac{\frac{2\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(i a\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(i a\tan(dx+c)+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(i a\tan(dx+c)+a)^{\frac{2}{3}}\right)}{a^{\frac{1}{3}}}+\frac{2\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(i a\tan(dx+c)+a)^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}-\frac{8\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(i a\tan(dx+c)+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}+\frac{4\log\left((i a\tan(dx+c)+a)^{\frac{2}{3}}+(i a\tan(dx+c)+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{1}{3}}}-\frac{8\log\left((i a\tan(dx+c)+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}-\frac{12}{(i a\tan(dx+c)+a)^{\frac{1}{3}}}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c))^(1/3),x, algorithm="maxima")

[Out] -1/8*(2*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3))/a^(1/3) - 2^(2/3)*log(2^(2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x + c) + a)^(2/3))/a^(1/3) + 2*2^(2/3)*log(-2^(1/3)*a^(1/3) + (I*a*tan(d*x + c) + a)^(1/3)) /a^(1/3) - 8*sqrt(3)*arctan(1/3*sqrt(3)*(2*(I*a*tan(d*x + c) + a)^(1/3) + a ^(1/3))/a^(1/3))/a^(1/3) + 4*log((I*a*tan(d*x + c) + a)^(2/3) + (I*a*tan(d* x + c) + a)^(1/3)*a^(1/3) + a^(2/3))/a^(1/3) - 8*log((I*a*tan(d*x + c) + a) ^(1/3) - a^(1/3))/a^(1/3) - 12/(I*a*tan(d*x + c) + a)^(1/3))/d

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(206) = 412.
time = 0.96, size = 580, normalized size = 2.03

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c))^(1/3),x, algorithm="fricas")

[Out] 1/4*(2*(1/2)^(1/3)*a*d*(-1/(a*d^3))^(1/3)*e^(2*I*d*x + 2*I*c)*log(-2*(1/2)^ (2/3)*a*d^2*(-1/(a*d^3))^(2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3

```
)*e^(2/3*I*d*x + 2/3*I*c)) + 4*a*d*(1/(a*d^3))^(1/3)*e^(2*I*d*x + 2*I*c)*lo
g(-a*d^2*(1/(a*d^3))^(2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^
(2/3*I*d*x + 2/3*I*c)) - (1/2)^(1/3)*(I*sqrt(3)*a*d + a*d)*(-1/(a*d^3))^(1/
3)*e^(2*I*d*x + 2*I*c)*log(-(1/2)^(2/3)*(I*sqrt(3)*a*d^2 - a*d^2)*(-1/(a*d^
3))^(2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*
I*c)) - (1/2)^(1/3)*(-I*sqrt(3)*a*d + a*d)*(-1/(a*d^3))^(1/3)*e^(2*I*d*x +
2*I*c)*log(-(1/2)^(2/3)*(-I*sqrt(3)*a*d^2 - a*d^2)*(-1/(a*d^3))^(2/3) + 2^(
1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)) + 3*2^(2/
3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(e^(2*I*d*x + 2*I*c) + 1)*e^(4/3*I*d
*x + 4/3*I*c) - 2*(-I*sqrt(3)*a*d + a*d)*(1/(a*d^3))^(1/3)*e^(2*I*d*x + 2*I
*c)*log(1/2*(I*sqrt(3)*a*d^2 + a*d^2)*(1/(a*d^3))^(2/3) + 2^(1/3)*(a/(e^(2*
I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)) - 2*(I*sqrt(3)*a*d + a*
d)*(1/(a*d^3))^(1/3)*e^(2*I*d*x + 2*I*c)*log(1/2*(-I*sqrt(3)*a*d^2 + a*d^2)
*(1/(a*d^3))^(2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d
*x + 2/3*I*c)))*e^(-2*I*d*x - 2*I*c)/(a*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(c+dx)}{\sqrt[3]{ia(\tan(c+dx)-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c))**(1/3),x)
[Out] Integral(cot(c + d*x)/(I*a*(tan(c + d*x) - I))**(1/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate(cot(d*x + c)/(I*a*tan(d*x + c) + a)^(1/3), x)
```

Mupad [**B**]
time = 0.25, size = 559, normalized size = 1.95

$$\frac{3}{2\,d\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{1/3}}+\ln\left(\left(746496\,a^7\,d^9\,{\left(\frac{1}{a\,d^3}\right)}^{2/3}-528768\,a^6\,d^7\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{1/3}\right)\,{\left(\frac{1}{a\,d^3}\right)}^{1/3}-217728\,a^6\,d^6\right)\,{\left(\frac{1}{a\,d^3}\right)}^{1/3}+\ln\left(\left(746496\,a^7\,d^9\,{\left(-\frac{1}{16\,a\,d^3}\right)}^{2/3}-528768\,a^6\,d^7\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{1/3}\right)\,{\left(-\frac{1}{16\,a\,d^3}\right)}^{1/3}-217728\,a^6\,d^6\right)\,{\left(-\frac{1}{16\,a\,d^3}\right)}^{1/3}+\frac{\ln\left(217728\,a^6\,d^6+\frac{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)\,\left(528768\,a^6\,d^7\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{1/3}-186624\,a^7\,d^9\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,{\left(\frac{1}{a\,d^3}\right)}^{2/3}\right)\,{\left(\frac{1}{a\,d^3}\right)}^{1/3}}{2}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(\frac{1}{a\,d^3}\right)}^{1/3}}{2}-\frac{\ln\left(217728\,a^6\,d^6-\frac{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,\left(528768\,a^6\,d^7\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{1/3}-186624\,a^7\,d^9\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,{\left(\frac{1}{a\,d^3}\right)}^{2/3}\right)\,{\left(\frac{1}{a\,d^3}\right)}^{1/3}}{2}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(\frac{1}{a\,d^3}\right)}^{1/3}}{2}+\ln\left(217728\,a^6\,d^6+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(528768\,a^6\,d^7\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{1/3}-746496\,a^7\,d^9\,{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2\,{\left(-\frac{1}{16\,a\,d^3}\right)}^{2/3}\right)\,{\left(-\frac{1}{16\,a\,d^3}\right)}^{1/3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(-\frac{1}{16\,a\,d^3}\right)}^{1/3}-\ln\left(217728\,a^6\,d^6-\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(528768\,a^6\,d^7\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{1/3}-746496\,a^7\,d^9\,{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2\,{\left(-\frac{1}{16\,a\,d^3}\right)}^{2/3}\right)\,{\left(-\frac{1}{16\,a\,d^3}\right)}^{1/3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(-\frac{1}{16\,a\,d^3}\right)}^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + a*tan(c + d*x)*1i)^(1/3),x)
[Out] 3/(2*d*(a + a*tan(c + d*x)*1i)^(1/3)) + log((746496*a^7*d^9*(1/(a*d^3))^(2/
3) - 528768*a^6*d^7*(a + a*tan(c + d*x)*1i)^(1/3))*(1/(a*d^3))^(1/3) - 2177
```

```
28*a^6*d^6)*(1/(a*d^3))^(1/3) + log((746496*a^7*d^9*(-1/(16*a*d^3))^(2/3) -
 528768*a^6*d^7*(a + a*tan(c + d*x)*1i)^(1/3))*(-1/(16*a*d^3))^(1/3) - 2177
28*a^6*d^6)*(-1/(16*a*d^3))^(1/3) + (log(217728*a^6*d^6 + ((3^(1/2)*1i - 1)
*(528768*a^6*d^7*(a + a*tan(c + d*x)*1i)^(1/3) - 186624*a^7*d^9*(3^(1/2)*1i
 - 1)^2*(1/(a*d^3))^(2/3))*(1/(a*d^3))^(1/3))/2)*(3^(1/2)*1i - 1)*(1/(a*d^3
))^(1/3))/2 - (log(217728*a^6*d^6 - ((3^(1/2)*1i + 1)*(528768*a^6*d^7*(a +
a*tan(c + d*x)*1i)^(1/3) - 186624*a^7*d^9*(3^(1/2)*1i + 1)^2*(1/(a*d^3))^(2
/3))*(1/(a*d^3))^(1/3))/2)*(3^(1/2)*1i + 1)*(1/(a*d^3))^(1/3))/2 + log(2177
28*a^6*d^6 + ((3^(1/2)*1i)/2 - 1/2)*(528768*a^6*d^7*(a + a*tan(c + d*x)*1i)
^(1/3) - 746496*a^7*d^9*((3^(1/2)*1i)/2 - 1/2)^2*(-1/(16*a*d^3))^(2/3))*(-1
/(16*a*d^3))^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(-1/(16*a*d^3))^(1/3) - log(2177
28*a^6*d^6 - ((3^(1/2)*1i)/2 + 1/2)*(528768*a^6*d^7*(a + a*tan(c + d*x)*1i)
^(1/3) - 746496*a^7*d^9*((3^(1/2)*1i)/2 + 1/2)^2*(-1/(16*a*d^3))^(2/3))*(-1
/(16*a*d^3))^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(-1/(16*a*d^3))^(1/3)
```

3.296 $\int \frac{\cot^2(c+dx)}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=327

$$\frac{x}{4\sqrt[3]{2}\sqrt[3]{a}}-\frac{i\text{ArcTan}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,d}-\frac{i\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt[3]{2}\sqrt[3]{a}\,d}-\frac{i\log}{}$$

```
[Out] 1/8*x*2^(2/3)/a^(1/3)-1/8*I*ln(cos(d*x+c))*2^(2/3)/a^(1/3)/d+1/6*I*ln(tan(d
*x+c))/a^(1/3)/d-1/2*I*ln(a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)/d-3/8*I
*ln(2^(1/3)*a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))*2^(2/3)/a^(1/3)/d-1/3*I*arcta
n(1/3*(a^(1/3)+2*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))/a^(1/3)/d*3^(1/
2)-1/4*I*arctan(1/3*(a^(1/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1
/2))*3^(1/2)*2^(2/3)/a^(1/3)/d-5/2*I/d/(a+I*a*tan(d*x+c))^(1/3)-cot(d*x+c)/
d/(a+I*a*tan(d*x+c))^(1/3)
```

Rubi [A]
time = 0.39, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3642, 3677, 3681, 3562, 57, 631, 210, 31, 3680}

$$-\frac{i\text{ArcTan}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,d}-\frac{i\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt[3]{2}\sqrt[3]{a}\,d}-\frac{5i}{2d\sqrt[3]{a+ia\tan(c+dx)}}+\frac{i\log(\tan(c+dx))}{6\sqrt[3]{a}\,d}-\frac{i\log\left(\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{2\sqrt[3]{a}\,d}-\frac{3i\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{4\sqrt[3]{2}\sqrt[3]{a}\,d}-\frac{i\log(\cos(c+dx))}{4\sqrt[3]{2}\sqrt[3]{a}\,d}-\frac{\cot(c+dx)}{d\sqrt[3]{a+ia\tan(c+dx)}}+\frac{x}{4\sqrt[3]{2}\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + I*a*Tan[c + d*x])^(1/3),x]
```

```
[Out] x/(4*2^(1/3)*a^(1/3)) - (I*ArcTan[(a^(1/3) + 2*(a + I*a*Tan[c + d*x])^(1/3)
)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*d) - ((I/2)*Sqrt[3]*ArcTan[(a^(1/3)
+ 2^(2/3)*(a + I*a*Tan[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/(2^(1/3)*a^(1/3
)*d) - ((I/4)*Log[Cos[c + d*x]])/(2^(1/3)*a^(1/3)*d) + ((I/6)*Log[Tan[c + d
*x]])/(a^(1/3)*d) - ((I/2)*Log[a^(1/3) - (a + I*a*Tan[c + d*x])^(1/3)])/(a^
(1/3)*d) - (((3*I)/4)*Log[2^(1/3)*a^(1/3) - (a + I*a*Tan[c + d*x])^(1/3)])/
(2^(1/3)*a^(1/3)*d) - ((5*I)/2)/(d*(a + I*a*Tan[c + d*x])^(1/3)) - Cot[c +
d*x]/(d*(a + I*a*Tan[c + d*x])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
```

```
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3642

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[d*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(a*(c^2 + d^2)*(n +
1)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[b*d*m - a
*c*(n + 1) + a*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
 e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0
] && LtQ[n, -1] && (IntegerQ[n] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
```

```
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3681

```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(
A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx = -\frac{\cot(c+dx)}{d\sqrt[3]{a+ia\tan(c+dx)}} + \frac{\int \frac{\cot(c+dx)\left(-\frac{ia}{3}-\frac{4}{3}a\tan(c+dx)\right)}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx}{a}$$

$$= -\frac{5i}{2d\sqrt[3]{a+ia\tan(c+dx)}} - \frac{\cot(c+dx)}{d\sqrt[3]{a+ia\tan(c+dx)}} + \frac{3\int \cot(c+dx)(a+\cdots}{\cdots}$$

$$= -\frac{5i}{2d\sqrt[3]{a+ia\tan(c+dx)}} - \frac{\cot(c+dx)}{d\sqrt[3]{a+ia\tan(c+dx)}} - \frac{i\int \cot(c+dx)(a-\cdots}{\cdots}$$

$$= -\frac{5i}{2d\sqrt[3]{a+ia\tan(c+dx)}} - \frac{\cot(c+dx)}{d\sqrt[3]{a+ia\tan(c+dx)}} - \frac{i\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+iax\cdots}}\cdots\right)}{3\cdots}$$

$$= \frac{x}{4\sqrt[3]{2}\sqrt[3]{a}} - \frac{i\log(\cos(c+dx))}{4\sqrt[3]{2}\sqrt[3]{a}\,d} + \frac{i\log(\tan(c+dx))}{6\sqrt[3]{a}\,d} - \frac{5i}{2d\sqrt[3]{a+ia\tan(c+d\cdots}}$$

$$= \frac{x}{4\sqrt[3]{2}\sqrt[3]{a}} - \frac{i\log(\cos(c+dx))}{4\sqrt[3]{2}\sqrt[3]{a}\,d} + \frac{i\log(\tan(c+dx))}{6\sqrt[3]{a}\,d} - \frac{i\log\left(\sqrt[3]{a}-\sqrt[3]{a+i\cdots}\right.}{2\sqrt[3]{a}\,\cdots}$$

$$= \frac{x}{4\sqrt[3]{2}\sqrt[3]{a}} - \frac{i\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}\,d} - \frac{i\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a\cdots}}{\cdots}}{\cdots}\right)}{2\sqrt[3]{2}\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 1.51, size = 179, normalized size = 0.55

$$\frac{\csc(c+dx)\sec(c+dx)\left(-8-8\cos(2(c+dx))+3\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)(-1+\cos(2(c+dx))+i\sin(2(c+dx)))+4\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{2e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)(-1+\cos(2(c+dx))+i\sin(2(c+dx)))-20i\sin(2(c+dx))\right)}{16d\sqrt[3]{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + I*a*Tan[c + d*x])^(1/3),x]
```

```
[Out] (Csc[c + d*x]*Sec[c + d*x]*(-8 - 8*Cos[2*(c + d*x)] + 3*Hypergeometric2F1[2
/3, 1, 5/3, E^((2*I)*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(-1 + Cos[2*(c +
 d*x)] + I*Sin[2*(c + d*x)]) + 4*Hypergeometric2F1[2/3, 1, 5/3, (2*E^((2*I)
*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(-1 + Cos[2*(c + d*x)] + I*Sin[2*(c
 + d*x)]) - (20*I)*Sin[2*(c + d*x)]))/(16*d*(a + I*a*Tan[c + d*x])^(1/3))
```

Maple [F]

time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(dx+c)}{(a+ia\tan(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/3),x)
```

```
[Out] int(cot(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/3),x)
```

Maxima [A]

time = 0.53, size = 286, normalized size = 0.87

$$-\frac{ia\left(\frac{6\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2(ia\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}}-\frac{3\cdot2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(ia\tan(dx+c)+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(ia\tan(dx+c)+a)^{\frac{2}{3}}\right)}{a^{\frac{4}{3}}}+\frac{6\cdot2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(ia\tan(dx+c)+a)^{\frac{1}{3}}\right)}{a^{\frac{4}{3}}}+\frac{8\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2(ia\tan(dx+c)+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}}-\frac{12(-5ia\tan(dx+c)-2a)}{(ia\tan(dx+c)+a)^{\frac{4}{3}}a-(ia\tan(dx+c)+a)^{\frac{1}{3}}a^2}-\frac{4\log\left((ia\tan(dx+c)+a)^{\frac{2}{3}}+(ia\tan(dx+c)+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{4}{3}}}+\frac{8\log\left((ia\tan(dx+c)+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{4}{3}}}\right)}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] -1/24*I*a*(6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) +
2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3))/a^(4/3) - 3*2^(2/3)*log(2^(2/3)*a^
(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x + c) +
a)^(2/3))/a^(4/3) + 6*2^(2/3)*log(-2^(1/3)*a^(1/3) + (I*a*tan(d*x + c) + a)
^(1/3))/a^(4/3) + 8*sqrt(3)*arctan(1/3*sqrt(3)*(2*(I*a*tan(d*x + c) + a)^(1
/3) + a^(1/3))/a^(1/3))/a^(4/3) - 12*(-5*I*a*tan(d*x + c) - 2*a)/((I*a*tan(
d*x + c) + a)^(4/3)*a - (I*a*tan(d*x + c) + a)^(1/3)*a^2) - 4*log((I*a*tan(
d*x + c) + a)^(2/3) + (I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + a^(2/3))/a^(4/
3) + 8*log((I*a*tan(d*x + c) + a)^(1/3) - a^(1/3))/a^(4/3))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 717 vs. 2(231) = 462.

time = 0.96, size = 717, normalized size = 2.19

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/4*(2^(2/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(-7*I*e^(4*I*d*x + 4*I*c)
- 4*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(4/3*I*d*x + 4/3*I*c) + 4*(a*d*e^(4*I*d*
x + 4*I*c) - a*d*e^(2*I*d*x + 2*I*c))*(1/16*I/(a*d^3))^(1/3)*log(8*a*d^2*(1
/16*I/(a*d^3))^(2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I
*d*x + 2/3*I*c)) + 4*(a*d*e^(4*I*d*x + 4*I*c) - a*d*e^(2*I*d*x + 2*I*c))*(1
/27*I/(a*d^3))^(1/3)*log(9*a*d^2*(1/27*I/(a*d^3))^(2/3) + 2^(1/3)*(a/(e^(2*
I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)) - 2*((-I*sqrt(3)*a*d +
a*d)*e^(4*I*d*x + 4*I*c) + (I*sqrt(3)*a*d - a*d)*e^(2*I*d*x + 2*I*c))*(1/16
*I/(a*d^3))^(1/3)*log(-4*(I*sqrt(3)*a*d^2 + a*d^2)*(1/16*I/(a*d^3))^(2/3) +
 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)) - 2*(
(I*sqrt(3)*a*d + a*d)*e^(4*I*d*x + 4*I*c) + (-I*sqrt(3)*a*d - a*d)*e^(2*I*d
*x + 2*I*c))*(1/16*I/(a*d^3))^(1/3)*log(-4*(-I*sqrt(3)*a*d^2 + a*d^2)*(1/16
*I/(a*d^3))^(2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*
x + 2/3*I*c)) - 2*((-I*sqrt(3)*a*d + a*d)*e^(4*I*d*x + 4*I*c) + (I*sqrt(3)*
a*d - a*d)*e^(2*I*d*x + 2*I*c))*(1/27*I/(a*d^3))^(1/3)*log(-9/2*(I*sqrt(3)*
a*d^2 + a*d^2)*(1/27*I/(a*d^3))^(2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1
))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)) - 2*((I*sqrt(3)*a*d + a*d)*e^(4*I*d*x + 4
*I*c) + (-I*sqrt(3)*a*d - a*d)*e^(2*I*d*x + 2*I*c))*(1/27*I/(a*d^3))^(1/3)*
log(-9/2*(-I*sqrt(3)*a*d^2 + a*d^2)*(1/27*I/(a*d^3))^(2/3) + 2^(1/3)*(a/(e^
(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)))/(a*d*e^(4*I*d*x + 4
*I*c) - a*d*e^(2*I*d*x + 2*I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(c+dx)}{\sqrt[3]{ia(\tan(c+dx)-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+I*a*tan(d*x+c))**(1/3),x)
```

```
[Out] Integral(cot(c + d*x)**2/(I*a*(tan(c + d*x) - I))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)^2/(I*a*tan(d*x + c) + a)^(1/3), x)
```

Mupad [B]
time = 4.57, size = 887, normalized size = 2.71

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(1/3),x)
```

```
[Out] log(a^5*d*(a + a*tan(c + d*x)*1i)^(1/3)*36i - ((5832*a^7*d^6*(1i/(16*a*d^3)
)^(2/3) + 675*a^6*d^4*(a + a*tan(c + d*x)*1i)^(1/3))*(1i/(16*a*d^3))^(1/3)
- a^6*d^3*315i)*(1i/(16*a*d^3))^(2/3))*(1i/(16*a*d^3))^(1/3) + log(a^5*d*(a
 + a*tan(c + d*x)*1i)^(1/3)*36i - ((5832*a^7*d^6*(1i/(27*a*d^3))^(2/3) + 67
5*a^6*d^4*(a + a*tan(c + d*x)*1i)^(1/3))*(1i/(27*a*d^3))^(1/3) - a^6*d^3*31
5i)*(1i/(27*a*d^3))^(2/3))*(1i/(27*a*d^3))^(1/3) + (((a + a*tan(c + d*x)*1i
)*5i)/(2*d) - (a*3i)/(2*d))/(a*(a + a*tan(c + d*x)*1i)^(1/3) - (a + a*tan(c
 + d*x)*1i)^(4/3)) + (log(((3^(1/2)*1i - 1)^2*(a^6*d^3*315i - ((3^(1/2)*1i
- 1)*(675*a^6*d^4*(a + a*tan(c + d*x)*1i)^(1/3) + 1458*a^7*d^6*(3^(1/2)*1i
- 1)^2*(1i/(16*a*d^3))^(2/3))*(1i/(16*a*d^3))^(1/3))/2)*(1i/(16*a*d^3))^(2/
3))/4 + a^5*d*(a + a*tan(c + d*x)*1i)^(1/3)*36i)*(3^(1/2)*1i - 1)*(1i/(16*a
*d^3))^(1/3))/2 - (log(((3^(1/2)*1i + 1)^2*(a^6*d^3*315i + ((3^(1/2)*1i + 1
)*(675*a^6*d^4*(a + a*tan(c + d*x)*1i)^(1/3) + 1458*a^7*d^6*(3^(1/2)*1i + 1
)^2*(1i/(16*a*d^3))^(2/3))*(1i/(16*a*d^3))^(1/3))/2)*(1i/(16*a*d^3))^(2/3))
/4 + a^5*d*(a + a*tan(c + d*x)*1i)^(1/3)*36i)*(3^(1/2)*1i + 1)*(1i/(16*a*d^
3))^(1/3))/2 + (log(((3^(1/2)*1i - 1)^2*(a^6*d^3*315i - ((3^(1/2)*1i - 1)*(
675*a^6*d^4*(a + a*tan(c + d*x)*1i)^(1/3) + 1458*a^7*d^6*(3^(1/2)*1i - 1)^2
*(1i/(27*a*d^3))^(2/3))*(1i/(27*a*d^3))^(1/3))/2)*(1i/(27*a*d^3))^(2/3))/4
+ a^5*d*(a + a*tan(c + d*x)*1i)^(1/3)*36i)*(3^(1/2)*1i - 1)*(1i/(27*a*d^3))
^(1/3))/2 - (log(((3^(1/2)*1i + 1)^2*(a^6*d^3*315i + ((3^(1/2)*1i + 1)*(675
*a^6*d^4*(a + a*tan(c + d*x)*1i)^(1/3) + 1458*a^7*d^6*(3^(1/2)*1i + 1)^2*(1
i/(27*a*d^3))^(2/3))*(1i/(27*a*d^3))^(1/3))/2)*(1i/(27*a*d^3))^(2/3))/4 + a
^5*d*(a + a*tan(c + d*x)*1i)^(1/3)*36i)*(3^(1/2)*1i + 1)*(1i/(27*a*d^3))^(1
/3))/2
```

3.297 $\int \frac{1}{(a+ia\tan(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=184

$$-\frac{x}{4\ 2^{2/3}a^{2/3}}-\frac{i\sqrt{3}\ \text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\ 2^{2/3}a^{2/3}d}+\frac{i\log(\cos(c+dx))}{4\ 2^{2/3}a^{2/3}d}+\frac{3i\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+\dots}\right)}{4\ 2^{2/3}a^{2/\dots}}$$

```
[Out] -1/8*x*2^(1/3)/a^(2/3)+1/8*I*ln(cos(d*x+c))*2^(1/3)/a^(2/3)/d+3/8*I*ln(2^(1
/3)*a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))*2^(1/3)/a^(2/3)/d-1/4*I*arctan(1/3*(a
^(1/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)*2^(1/3)/a
^(2/3)/d+3/4*I/d/(a+I*a*tan(d*x+c))^(2/3)
```

Rubi [A]

time = 0.08, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3560, 3562, 59, 631, 210, 31}

$$-\frac{i\sqrt{3}\ \text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\ 2^{2/3}a^{2/3}d}+\frac{3i\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{4\ 2^{2/3}a^{2/3}d}+\frac{i\log(\cos(c+dx))}{4\ 2^{2/3}a^{2/3}d}-\frac{x}{4\ 2^{2/3}a^{2/3}}+\frac{3i}{4d(a+ia\tan(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(-2/3),x]
```

```
[Out] -1/4*x/(2^(2/3)*a^(2/3)) - ((I/2)*Sqrt[3]*ArcTan[(a^(1/3) + 2^(2/3)*(a + I*
a*Tan[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/(2^(2/3)*a^(2/3)*d) + ((I/4)*Log
[Cos[c + d*x]])/(2^(2/3)*a^(2/3)*d) + (((3*I)/4)*Log[2^(1/3)*a^(1/3) - (a +
 I*a*Tan[c + d*x])^(1/3)])/(2^(2/3)*a^(2/3)*d) + ((3*I)/4)/(d*(a + I*a*Tan[
c + d*x])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(c+dx))^{2/3}}\,dx = \frac{3i}{4d(a+ia\tan(c+dx))^{2/3}} + \frac{\int \sqrt[3]{a+ia\tan(c+dx)}\,dx}{2a}$$

$$= \frac{3i}{4d(a+ia\tan(c+dx))^{2/3}} - \frac{i\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{2/3}}\,dx, x, ia\tan(c+dx)\right)}{2d}$$

$$= -\frac{x}{4\,2^{2/3}a^{2/3}} + \frac{i\log(\cos(c+dx))}{4\,2^{2/3}a^{2/3}d} + \frac{3i}{4d(a+ia\tan(c+dx))^{2/3}} - \frac{(3i)\text{Subst}\left(\right.}{}$$

$$= -\frac{x}{4\,2^{2/3}a^{2/3}} + \frac{i\log(\cos(c+dx))}{4\,2^{2/3}a^{2/3}d} + \frac{3i\log\left(\sqrt[3]{2}\sqrt[3]{a} - \sqrt[3]{a+ia\tan(c+dx)}\right.}{4\,2^{2/3}a^{2/3}d}$$

$$= -\frac{x}{4\,2^{2/3}a^{2/3}} - \frac{i\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2\,2^{2/3}a^{2/3}d} + \frac{i\log(\cos(c+d}{4\,2^{2/3}a^{2/3}d}$$

Mathematica [A]

time = 1.10, size = 290, normalized size = 1.58

$$\frac{i\left(3\left(1+e^{2i(c+dx)}\right)^{2/3}-2\sqrt{3}\,e^{\frac{4}{3}i(c+dx)}\text{ArcTan}\left(\frac{1+\frac{2e^{\frac{2}{3}i(c+dx)}}{\sqrt[3]{1+e^{2i(c+dx)}}}}{\sqrt{3}}\right)+2e^{\frac{4}{3}i(c+dx)}\log\left(1-\frac{e^{\frac{2}{3}i(c+dx)}}{\sqrt[3]{1+e^{2i(c+dx)}}}\right)-e^{\frac{4}{3}i(c+dx)}\log\left(\frac{e^{\frac{4}{3}i(c+dx)}+e^{\frac{2}{3}i(c+dx)}\sqrt[3]{1+e^{2i(c+dx)}}+\left(1+e^{2i(c+dx)}\right)^{2/3}}{\left(1+e^{2i(c+dx)}\right)^{2/3}}\right)\right)}{4\,2^{2/3}d\left(\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{2/3}\left(1+e^{2i(c+dx)}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(-2/3),x]
```

```
[Out] ((I/4)*(3*(1 + E^((2*I)*(c + d*x)))^(2/3) - 2*Sqrt[3]*E^(((4*I)/3)*(c + d*x
))*ArcTan[(1 + (2*E^(((2*I)/3)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))^(1/3))
/Sqrt[3]] + 2*E^(((4*I)/3)*(c + d*x))*Log[1 - E^(((2*I)/3)*(c + d*x))/(1 +
E^((2*I)*(c + d*x)))^(1/3)] - E^(((4*I)/3)*(c + d*x))*Log[(E^(((4*I)/3)*(c
+ d*x)) + E^(((2*I)/3)*(c + d*x))*(1 + E^((2*I)*(c + d*x)))^(1/3) + (1 + E^
((2*I)*(c + d*x)))^(2/3))/(1 + E^((2*I)*(c + d*x)))^(2/3)]))/(2^(2/3)*d*((a
*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))^(2/3)*(1 + E^((2*I)*(c + d
*x)))^(2/3))
```

Maple [A]
time = 0.09, size = 158, normalized size = 0.86

method	result
derivativedivides	$\frac{3ia\left(\frac{\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{12a^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\ldots\right.}{\ldots}}{2a}\ldots\right)}{d}$
default	$\frac{3ia\left(\frac{\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{12a^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\ldots\right.}{\ldots}}{2a}\ldots\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(d*x+c))^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3*I/d*a*(1/2*(1/6*2^(1/3)/a^(2/3)*ln((a+I*a*tan(d*x+c))^(1/3)-2^(1/3)*a^(1/
3))-1/12*2^(1/3)/a^(2/3)*ln((a+I*a*tan(d*x+c))^(2/3)+2^(1/3)*a^(1/3)*(a+I*a
```

```
*tan(d*x+c))^(1/3)+2^(2/3)*a^(2/3))-1/6*2^(1/3)/a^(2/3)*3^(1/2)*arctan(1/3*
3^(1/2)*(2^(2/3)/a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+1)))/a+1/4/a/(a+I*a*tan(d
*x+c))^(2/3))
```

Maxima [A]
time = 0.56, size = 151, normalized size = 0.82

$$-\frac{i\left(2\sqrt{3}\,2^{\frac{1}{3}}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)+2^{\frac{1}{3}}a^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(i\,a\tan(dx+c)+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{2}{3}}\right)-2\cdot 2^{\frac{1}{3}}a^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)-\frac{6\,a}{(i\,a\tan(dx+c)+a)^{\frac{2}{3}}}\right)}{8\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] -1/8*I*(2*sqrt(3)*2^(1/3)*a^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/
3) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) + 2^(1/3)*a^(1/3)*log(2^(2/3)
*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x + c)
 + a)^(2/3)) - 2*2^(1/3)*a^(1/3)*log(-2^(1/3)*a^(1/3) + (I*a*tan(d*x + c) +
 a)^(1/3)) - 6*a/(I*a*tan(d*x + c) + a)^(2/3))/(a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(125) = 250.
time = 1.33, size = 305, normalized size = 1.66

$$\frac{\left(8\,ad\left(-\frac{i}{32\,a^2d^3}\right)^{\frac{1}{3}}e^{(2i\,dx+2i\,c)}\log\left(4i\,ad\left(-\frac{i}{32\,a^2d^3}\right)^{\frac{1}{3}}+2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)-4\left(-i\sqrt{3}\,ad+ad\right)\left(-\frac{i}{32\,a^2d^3}\right)^{\frac{1}{3}}e^{(2i\,dx+2i\,c)}\log\left(2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}-2\left(\sqrt{3}\,ad+i\,ad\right)\left(-\frac{i}{32\,a^2d^3}\right)^{\frac{1}{3}}\right)-4\left(i\sqrt{3}\,ad+ad\right)\left(-\frac{i}{32\,a^2d^3}\right)^{\frac{1}{3}}e^{(2i\,dx+2i\,c)}\log\left(2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}+2\left(\sqrt{3}\,ad-i\,ad\right)\left(-\frac{i}{32\,a^2d^3}\right)^{\frac{1}{3}}\right)-3\cdot 2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}\left(-i\,e^{(2i\,dx+2i\,c)}-i\right)e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)e^{(-2i\,dx-2i\,c)}}{8\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] 1/8*(8*a*d*(-1/32*I/(a^2*d^3))^(1/3)*e^(2*I*d*x + 2*I*c)*log(4*I*a*d*(-1/32
*I/(a^2*d^3))^(1/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*
d*x + 2/3*I*c)) - 4*(-I*sqrt(3)*a*d + a*d)*(-1/32*I/(a^2*d^3))^(1/3)*e^(2*I
*d*x + 2*I*c)*log(2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x
+ 2/3*I*c) - 2*(sqrt(3)*a*d + I*a*d)*(-1/32*I/(a^2*d^3))^(1/3)) - 4*(I*sqrt
(3)*a*d + a*d)*(-1/32*I/(a^2*d^3))^(1/3)*e^(2*I*d*x + 2*I*c)*log(2^(1/3)*(a
/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + 2*(sqrt(3)*a*d
- I*a*d)*(-1/32*I/(a^2*d^3))^(1/3)) - 3*2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1
))^(1/3)*(-I*e^(2*I*d*x + 2*I*c) - I)*e^(2/3*I*d*x + 2/3*I*c))*e^(-2*I*d*x
- 2*I*c)/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia\tan(c+dx)+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))**(2/3),x)
```

```
[Out] Integral((I*a*tan(c + d*x) + a)**(-2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(-2/3), x)
```

Mupad [B]
time = 0.22, size = 190, normalized size = 1.03

$$\frac{3\mathrm{i}}{4\,d\,{\left(a+a\,\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}^{2/3}}-\frac{{\left(\frac{1}{32}\mathrm{i}\right)}^{1/3}\,\ln\left(d^2\,{\left(a+a\,\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}^{1/3}\,36\mathrm{i}+144\,{\left(\frac{1}{32}\mathrm{i}\right)}^{1/3}\,a^{1/3}\,d^2\right)}{a^{2/3}\,d}-\frac{{\left(\frac{1}{32}\mathrm{i}\right)}^{1/3}\,\ln\left(d^2\,{\left(a+a\,\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}^{1/3}\,36\mathrm{i}+144\,{\left(\frac{1}{32}\mathrm{i}\right)}^{1/3}\,a^{1/3}\,d^2\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{2/3}\,d}+\frac{{\left(\frac{1}{32}\mathrm{i}\right)}^{1/3}\,\ln\left(d^2\,{\left(a+a\,\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}^{1/3}\,36\mathrm{i}-144\,{\left(\frac{1}{32}\mathrm{i}\right)}^{1/3}\,a^{1/3}\,d^2\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{2/3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*tan(c + d*x)*1i)^(2/3),x)
```

```
[Out] 3i/(4*d*(a + a*tan(c + d*x)*1i)^(2/3)) - ((1i/32)^(1/3)*log(d^2*(a + a*tan(
c + d*x)*1i)^(1/3)*36i + 144*(1i/32)^(1/3)*a^(1/3)*d^2))/(a^(2/3)*d) - ((1i
/32)^(1/3)*log(d^2*(a + a*tan(c + d*x)*1i)^(1/3)*36i + 144*(1i/32)^(1/3)*a^
(1/3)*d^2*((3^(1/2)*1i)/2 - 1/2))*((3^(1/2)*1i)/2 - 1/2))/(a^(2/3)*d) + ((1
i/32)^(1/3)*log(d^2*(a + a*tan(c + d*x)*1i)^(1/3)*36i - 144*(1i/32)^(1/3)*a
^(1/3)*d^2*((3^(1/2)*1i)/2 + 1/2))*((3^(1/2)*1i)/2 + 1/2))/(a^(2/3)*d)
```

3.298 $\int \frac{\tan^m(c+dx)}{(a+ia\tan(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=86

$$\frac{F_1\left(1+m;\frac{7}{3},1;2+m;-i\tan(c+dx),i\tan(c+dx)\right)\sqrt[3]{1+i\tan(c+dx)}\,\tan^{1+m}(c+dx)}{ad(1+m)\sqrt[3]{a+ia\tan(c+dx)}}$$

```
[Out] AppellF1(1+m,7/3,1,2+m,-I*tan(d*x+c),I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/3)*t
an(d*x+c)^(1+m)/a/d/(1+m)/(a+I*a*tan(d*x+c))^(1/3)
```

Rubi [A]
time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3645, 140, 138}

$$\frac{\sqrt[3]{1+i\tan(c+dx)}\,\tan^{m+1}(c+dx)F_1\left(m+1;\frac{7}{3},1;m+2;-i\tan(c+dx),i\tan(c+dx)\right)}{ad(m+1)\sqrt[3]{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^m/(a + I*a*Tan[c + d*x])^(4/3),x]
```

```
[Out] (AppellF1[1 + m, 7/3, 1, 2 + m, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*(1 + I*T
an[c + d*x])^(1/3)*Tan[c + d*x]^(1 + m))/(a*d*(1 + m)*(a + I*a*Tan[c + d*x]
)^(1/3))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
```

2, 0]

Rubi steps

$$\int \frac{\tan^m(c+dx)}{(a+ia\tan(c+dx))^{4/3}}\,dx = \frac{(ia^2)\,\text{Subst}\left(\int \frac{\left(-\frac{ix}{a}\right)^m}{(a+x)^{7/3}(-a^2+ax)}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= \frac{\left(i\sqrt[3]{1+i\tan(c+dx)}\right)\text{Subst}\left(\int \frac{\left(-\frac{ix}{a}\right)^m}{\left(1+\frac{x}{a}\right)^{7/3}(-a^2+ax)}\,dx, x, ia\tan(c+dx)\right)}{d\sqrt[3]{a+ia\tan(c+dx)}}$$

$$= \frac{F_1\left(1+m;\frac{7}{3},1;2+m;-i\tan(c+dx), i\tan(c+dx)\right)\sqrt[3]{1+i\tan(c+dx)}\ \text{ta}}{ad(1+m)\sqrt[3]{a+ia\tan(c+dx)}}$$

Mathematica [F]
time = 34.66, size = 0, normalized size = 0.00

$$\int \frac{\tan^m(c+dx)}{(a+ia\tan(c+dx))^{4/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^m/(a + I*a*Tan[c + d*x])^(4/3),x]
[Out] Integrate[Tan[c + d*x]^m/(a + I*a*Tan[c + d*x])^(4/3), x]
```

Maple [F]
time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\tan^m(dx+c)}{(a+ia\tan(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^m/(a+I*a*tan(d*x+c))^(4/3),x)
[Out] int(tan(d*x+c)^m/(a+I*a*tan(d*x+c))^(4/3),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m/(a+I*a*tan(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m/(a+I*a*tan(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(1/4*2^(2/3)*((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1
))^m*(a/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*
x + 2*I*c) + 1)*e^(-8/3*I*d*x - 8/3*I*c)/a^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{m}\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**m/(a+I*a*tan(d*x+c))**(4/3),x)
```

```
[Out] Integral(tan(c + d*x)**m/(I*a*(tan(c + d*x) - I))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m/(a+I*a*tan(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^m/(I*a*tan(d*x + c) + a)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\tan\left(c+d\,x\right)}^m}{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^m/(a + a*tan(c + d*x)*1i)^(4/3),x)
```

```
[Out] int(tan(c + d*x)^m/(a + a*tan(c + d*x)*1i)^(4/3), x)
```

3.299 $\int \frac{\sqrt{\tan(c+dx)}}{(a+ia\tan(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=84

$$\frac{2F_1\left(\frac{3}{2};\frac{7}{3},1;\frac{5}{2};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt[3]{1+i\tan(c+dx)}\tan^{\frac{3}{2}}(c+dx)}{3ad\sqrt[3]{a+ia\tan(c+dx)}}$$

```
[Out] 2/3*AppellF1(3/2,7/3,1,5/2,-I*tan(d*x+c),I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/
3)*tan(d*x+c)^(3/2)/a/d/(a+I*a*tan(d*x+c))^(1/3)
```

Rubi [A]
time = 0.11, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3645, 129, 525, 524}

$$\frac{2\sqrt[3]{1+i\tan(c+dx)}\tan^{\frac{3}{2}}(c+dx)F_1\left(\frac{3}{2};\frac{7}{3},1;\frac{5}{2};-i\tan(c+dx),i\tan(c+dx)\right)}{3ad\sqrt[3]{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[c + d*x]]/(a + I*a*Tan[c + d*x])^(4/3),x]
```

```
[Out] (2*AppellF1[3/2, 7/3, 1, 5/2, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*(1 + I*Tan
[c + d*x])^(1/3)*Tan[c + d*x]^(3/2))/(3*a*d*(a + I*a*Tan[c + d*x])^(1/3))
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
```

```
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\tan(c+dx)}}{(a+ia\tan(c+dx))^{4/3}}\,dx = \frac{(ia^2)\,\text{Subst}\left(\int \frac{\sqrt{-\frac{ix}{a}}}{(a+x)^{7/3}(-a^2+ax)}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(2a^3)\,\text{Subst}\left(\int \frac{x^2}{(a+iax^2)^{7/3}(-a^2+ia^2x^2)}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d}$$

$$= -\frac{\left(2a\sqrt[3]{1+i\tan(c+dx)}\right)\text{Subst}\left(\int \frac{x^2}{(1+ix^2)^{7/3}(-a^2+ia^2x^2)}\,dx, x, \sqrt{\tan(c+dx)}\right.}{d\sqrt[3]{a+ia\tan(c+dx)}}$$

$$= \frac{2F_1\left(\frac{3}{2}; \frac{7}{3}, 1; \frac{5}{2}; -i\tan(c+dx), i\tan(c+dx)\right)\sqrt[3]{1+i\tan(c+dx)}\,\tan^{\frac{3}{2}}(c+d}{3ad\sqrt[3]{a+ia\tan(c+dx)}}$$

Mathematica [F]
time = 14.65, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\tan(c+dx)}}{(a+ia\tan(c+dx))^{4/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[Tan[c + d*x]]/(a + I*a*Tan[c + d*x])^(4/3),x]
[Out] Integrate[Sqrt[Tan[c + d*x]]/(a + I*a*Tan[c + d*x])^(4/3), x]
```

Maple [F]
time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\tan}\,(dx+c)}{(a+ia\tan(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(4/3),x)
```

```
[Out] int(tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(4/3),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] -1/32*(3*2^(2/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*sqrt((-I*e^(2*I*d*x +
2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(-5*I*e^(6*I*d*x + 6*I*c) + 4*I*e^(5
*I*d*x + 5*I*c) - 14*I*e^(4*I*d*x + 4*I*c) + 8*I*e^(3*I*d*x + 3*I*c) - 13*I
*e^(2*I*d*x + 2*I*c) + 4*I*e^(I*d*x + I*c) - 4*I)*e^(4/3*I*d*x + 4/3*I*c) -
 32*(a^2*d*e^(6*I*d*x + 6*I*c) - 4*a^2*d*e^(5*I*d*x + 5*I*c) + 4*a^2*d*e^(4
*I*d*x + 4*I*c))*integral(1/16*2^(2/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*
sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(-4*I*e^(6*I*d
*x + 6*I*c) + 48*I*e^(5*I*d*x + 5*I*c) - 47*I*e^(4*I*d*x + 4*I*c) + 66*I*e^
(3*I*d*x + 3*I*c) - 47*I*e^(2*I*d*x + 2*I*c) + 18*I*e^(I*d*x + I*c) - 4*I)*
e^(4/3*I*d*x + 4/3*I*c)/(a^2*d*e^(7*I*d*x + 7*I*c) - 6*a^2*d*e^(6*I*d*x + 6
*I*c) + 11*a^2*d*e^(5*I*d*x + 5*I*c) - 2*a^2*d*e^(4*I*d*x + 4*I*c) - 12*a^2
*d*e^(3*I*d*x + 3*I*c) + 8*a^2*d*e^(2*I*d*x + 2*I*c)), x))/(a^2*d*e^(6*I*d*
x + 6*I*c) - 4*a^2*d*e^(5*I*d*x + 5*I*c) + 4*a^2*d*e^(4*I*d*x + 4*I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\tan{\left(c + dx \right)}}}{\left(ia \left(\tan{\left(c + dx \right)} - i\right)\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/2)/(a+I*a*tan(d*x+c))**(4/3),x)
[Out] Integral(sqrt(tan(c + d*x))/(I*a*(tan(c + d*x) - I))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate(sqrt(tan(d*x + c))/(I*a*tan(d*x + c) + a)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\tan\left(c+d\,x\right)}}{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)/(a + a*tan(c + d*x)*1i)^(4/3),x)
[Out] int(tan(c + d*x)^(1/2)/(a + a*tan(c + d*x)*1i)^(4/3), x)
```

3.300 $\int \frac{\tan^4(c+dx)}{(a+ia\tan(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=282

$$-\frac{x}{8\sqrt[3]{2}\,a^{4/3}}+\frac{i\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{4\sqrt[3]{2}\,a^{4/3}d}+\frac{i\log(\cos(c+dx))}{8\sqrt[3]{2}\,a^{4/3}d}+\frac{3i\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+\right.}{8\sqrt[3]{2}\,a^{4/3}}$$

```
[Out] -1/16*x*2^(2/3)/a^(4/3)+1/16*I*ln(cos(d*x+c))*2^(2/3)/a^(4/3)/d+3/16*I*ln(2
^(1/3)*a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))*2^(2/3)/a^(4/3)/d+1/8*I*arctan(1/3
*(a^(1/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)*2^(2/3
)/a^(4/3)/d-39/40*I*tan(d*x+c)^2/d/(a+I*a*tan(d*x+c))^(4/3)+3/5*tan(d*x+c)^
3/d/(a+I*a*tan(d*x+c))^(4/3)-51/10*I/a/d/(a+I*a*tan(d*x+c))^(1/3)-87/40*I*(
a+I*a*tan(d*x+c))^(2/3)/a^2/d
```

Rubi [A]

time = 0.32, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3641, 3676, 3673, 3607, 3562, 57, 631, 210, 31}

$$\frac{i\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{4\sqrt[3]{2}\,a^{4/3}d}+\frac{3i\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{8\sqrt[3]{2}\,a^{4/3}d}+\frac{i\log(\cos(c+dx))}{8\sqrt[3]{2}\,a^{4/3}d}-\frac{x}{8\sqrt[3]{2}\,a^{4/3}}-\frac{87i(a+ia\tan(c+dx))^{2/3}}{40a^2d}+\frac{3\tan^3(c+dx)}{5d(a+ia\tan(c+dx))^{4/3}}-\frac{39i\tan^2(c+dx)}{40d(a+ia\tan(c+dx))^{4/3}}-\frac{51i}{10ad\sqrt[3]{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + I*a*Tan[c + d*x])^(4/3),x]
```

```
[Out] -1/8*x/(2^(1/3)*a^(4/3)) + ((I/4)*Sqrt[3]*ArcTan[(a^(1/3) + 2^(2/3)*(a + I*
a*Tan[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/(2^(1/3)*a^(4/3)*d) + ((I/8)*Log
[Cos[c + d*x]])/(2^(1/3)*a^(4/3)*d) + (((3*I)/8)*Log[2^(1/3)*a^(1/3) - (a +
 I*a*Tan[c + d*x])^(1/3)])/(2^(1/3)*a^(4/3)*d) - (((39*I)/40)*Tan[c + d*x]^
2)/(d*(a + I*a*Tan[c + d*x])^(4/3)) + (3*Tan[c + d*x]^3)/(5*d*(a + I*a*Tan[
c + d*x])^(4/3)) - ((51*I)/10)/(a*d*(a + I*a*Tan[c + d*x])^(1/3)) - (((87*I
)/40)*(a + I*a*Tan[c + d*x])^(2/3))/(a^2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rule 3641

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[d*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n - 1)/(f*(m + n - 1))), x] - Dist[1/(a*(m + n - 1)), Int[(a + b*
Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 2)*Simp[d*(b*c*m + a*d*(-1 + n))
- a*c^2*(m + n - 1) + d*(b*d*m - a*c*(m + 2*n - 2))*Tan[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1] && NeQ[m + n - 1, 0] && (IntegerQ[n]
|| IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+ia\tan(c+dx))^{4/3}}\,dx = \frac{3\tan^3(c+dx)}{5d(a+ia\tan(c+dx))^{4/3}} - \frac{3\int \frac{\tan^2(c+dx)\left(3a-\frac{4}{3}ia\tan(c+dx)\right)}{(a+ia\tan(c+dx))^{4/3}}\,dx}{5a}$$

$$= -\frac{39i\tan^2(c+dx)}{40d(a+ia\tan(c+dx))^{4/3}} + \frac{3\tan^3(c+dx)}{5d(a+ia\tan(c+dx))^{4/3}} + \frac{9\int \frac{\tan(c+dx)\left(\frac{26i}{3}\right.}{\sqrt[3]{a+ic}}}{}$$

$$= -\frac{39i\tan^2(c+dx)}{40d(a+ia\tan(c+dx))^{4/3}} + \frac{3\tan^3(c+dx)}{5d(a+ia\tan(c+dx))^{4/3}} - \frac{87i(a+ia\tan(}{40a^2}$$

$$= -\frac{39i\tan^2(c+dx)}{40d(a+ia\tan(c+dx))^{4/3}} + \frac{3\tan^3(c+dx)}{5d(a+ia\tan(c+dx))^{4/3}} - \frac{51i}{10ad\sqrt[3]{a+ia\,\mathrm{t}}}$$

$$= -\frac{39i\tan^2(c+dx)}{40d(a+ia\tan(c+dx))^{4/3}} + \frac{3\tan^3(c+dx)}{5d(a+ia\tan(c+dx))^{4/3}} - \frac{51i}{10ad\sqrt[3]{a+ia\,\mathrm{t}}}$$

$$= -\frac{x}{8\sqrt[3]{2}\,a^{4/3}} + \frac{i\log(\cos(c+dx))}{8\sqrt[3]{2}\,a^{4/3}d} - \frac{39i\tan^2(c+dx)}{40d(a+ia\tan(c+dx))^{4/3}} + \frac{3\tan}{5d(a+ia}$$

$$= -\frac{x}{8\sqrt[3]{2}\,a^{4/3}} + \frac{i\log(\cos(c+dx))}{8\sqrt[3]{2}\,a^{4/3}d} + \frac{3i\log\left(\sqrt[3]{2}\sqrt[3]{a} - \sqrt[3]{a+ia\tan(c+dx)}\right)}{8\sqrt[3]{2}\,a^{4/3}d}$$

$$= -\frac{x}{8\sqrt[3]{2}\,a^{4/3}} + \frac{i\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\,a^{4/3}d} + \frac{i\log(\cos(c+}{8\sqrt[3]{2}\,a^{4/3}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 1.64, size = 145, normalized size = 0.51

$$-\frac{3\sec^2(c+dx)\left(81+113\cos(2(c+dx))+5\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)(\cos(2(c+dx))+i\sin(2(c+dx)))+59i\sec(c+dx)\sin(3(c+dx))+75i\tan(c+dx)\right)}{80ad(-i+\tan(c+dx))\sqrt[3]{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + I*a*Tan[c + d*x])^(4/3),x]
```

```
[Out] (-3*Sec[c + d*x]^2*(81 + 113*Cos[2*(c + d*x)] + 5*Hypergeometric2F1[2/3, 1,
 5/3, E^((2*I)*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(Cos[2*(c + d*x)] + I*
Sin[2*(c + d*x)]) + (59*I)*Sec[c + d*x]*Sin[3*(c + d*x)] + (75*I)*Tan[c + d
*x]))/(80*a*d*(-I + Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(1/3))
```

Maple [A]
time = 0.13, size = 212, normalized size = 0.75

method	result
derivativedivides	$3i\left(\frac{(a+ia\tan(dx+c))^{\frac{5}{3}}}{5}-a(a+ia\tan(dx+c))^{\frac{2}{3}}+\frac{\left(\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\right.}{12a^{\frac{1}{3}}}\right.}{}\right.$
default	$3i\left(\frac{(a+ia\tan(dx+c))^{\frac{5}{3}}}{5}-a(a+ia\tan(dx+c))^{\frac{2}{3}}+\frac{\left(\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\right.}{12a^{\frac{1}{3}}}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+I*a*tan(d*x+c))^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3*I/d/a^3*(1/5*(a+I*a*tan(d*x+c))^(5/3)-a*(a+I*a*tan(d*x+c))^(2/3)+1/4*(1/6
*2^(2/3)/a^(1/3)*ln((a+I*a*tan(d*x+c))^(1/3)-2^(1/3)*a^(1/3))-1/12*2^(2/3)/
a^(1/3)*ln((a+I*a*tan(d*x+c))^(2/3)+2^(1/3)*a^(1/3)*(a+I*a*tan(d*x+c))^(1/3
)+2^(2/3)*a^(2/3))+1/6*3^(1/2)*2^(2/3)/a^(1/3)*arctan(1/3*3^(1/2)*(2^(2/3)/
a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+1)))*a^2-7/4*a^2/(a+I*a*tan(d*x+c))^(1/3)+
1/8*a^3/(a+I*a*tan(d*x+c))^(4/3))
```

Maxima [**A**]
time = 0.53, size = 209, normalized size = 0.74

$$\frac{i\left(10\sqrt{3}\,2^{\frac{2}{3}}a^{\frac{11}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)-5\cdot2^{\frac{2}{3}}a^{\frac{11}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{2}{3}}\right)+10\cdot2^{\frac{2}{3}}a^{\frac{11}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{1}{3}}\right)+48\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{3}}a^{2}-240\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{2}{3}}a^{3}-\frac{30\left(14\left(i\,a\tan\left(dx+c\right)+a\right)a^{4}-a^{5}\right)}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{4}{3}}}\right)}{80\,a^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+I*a*tan(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] 1/80*I*(10*sqrt(3)*2^(2/3)*a^(11/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(
1/3) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) - 5*2^(2/3)*a^(11/3)*log(2^
(2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x
 + c) + a)^(2/3)) + 10*2^(2/3)*a^(11/3)*log(-2^(1/3)*a^(1/3) + (I*a*tan(d*x
 + c) + a)^(1/3)) + 48*(I*a*tan(d*x + c) + a)^(5/3)*a^2 - 240*(I*a*tan(d*x
+ c) + a)^(2/3)*a^3 - 30*(14*(I*a*tan(d*x + c) + a)*a^4 - a^5)/(I*a*tan(d*x
 + c) + a)^(4/3))/(a^5*d)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 443 vs. 2(201) = 402.
time = 1.34, size = 443, normalized size = 1.57

$$-\frac{3\cdot2^{\frac{2}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{2}{3}}\left(231i\,e^{(6i\,dx+6i\,c)}+425i\,e^{(4i\,dx+4i\,c)}+125i\,e^{(2i\,dx+2i\,c)}-5i\right)e^{\left(\frac{4}{3}i\,dx+\frac{4}{3}i\,c\right)}-160\left(a^{2}de^{(6i\,dx+6i\,c)}+a^{2}de^{(4i\,dx+4i\,c)}\right)\left(-\frac{i}{128\,a^{4}d^{3}}\right)^{\frac{1}{3}}\log\left(32\,a^{3}d^{2}\left(-\frac{i}{128\,a^{4}d^{3}}\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)+80\left(\left(-i\sqrt{3}\,a^{2}d+a^{2}d\right)e^{(6i\,dx+6i\,c)}+\left(-i\sqrt{3}\,a^{2}d+a^{2}d\right)e^{(4i\,dx+4i\,c)}\right)\left(-\frac{i}{128\,a^{4}d^{3}}\right)^{\frac{1}{3}}\log\left(-16\left(i\sqrt{3}\,a^{3}d^{2}+a^{3}d^{2}\right)\left(-\frac{i}{128\,a^{4}d^{3}}\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)+80\left(\left(i\sqrt{3}\,a^{2}d+a^{2}d\right)e^{(6i\,dx+6i\,c)}+\left(i\sqrt{3}\,a^{2}d+a^{2}d\right)e^{(4i\,dx+4i\,c)}\right)\left(-\frac{i}{128\,a^{4}d^{3}}\right)^{\frac{1}{3}}\log\left(-16\left(-i\sqrt{3}\,a^{3}d^{2}+a^{3}d^{2}\right)\left(-\frac{i}{128\,a^{4}d^{3}}\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)}{160\left(a^{2}de^{(6i\,dx+6i\,c)}+a^{2}de^{(4i\,dx+4i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+I*a*tan(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] -1/160*(3*2^(2/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(231*I*e^(6*I*d*x + 6
*I*c) + 425*I*e^(4*I*d*x + 4*I*c) + 125*I*e^(2*I*d*x + 2*I*c) - 5*I)*e^(4/3
*I*d*x + 4/3*I*c) - 160*(a^2*d*e^(6*I*d*x + 6*I*c) + a^2*d*e^(4*I*d*x + 4*I
*c))*(-1/128*I/(a^4*d^3))^(1/3)*log(32*a^3*d^2*(-1/128*I/(a^4*d^3))^(2/3) +
 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)) + 80*
((-I*sqrt(3)*a^2*d + a^2*d)*e^(6*I*d*x + 6*I*c) + (-I*sqrt(3)*a^2*d + a^2*d
)*e^(4*I*d*x + 4*I*c))*(-1/128*I/(a^4*d^3))^(1/3)*log(-16*(I*sqrt(3)*a^3*d^
2 + a^3*d^2)*(-1/128*I/(a^4*d^3))^(2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) +
 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)) + 80*((I*sqrt(3)*a^2*d + a^2*d)*e^(6*I*
d*x + 6*I*c) + (I*sqrt(3)*a^2*d + a^2*d)*e^(4*I*d*x + 4*I*c))*(-1/128*I/(a^
4*d^3))^(1/3)*log(-16*(-I*sqrt(3)*a^3*d^2 + a^3*d^2)*(-1/128*I/(a^4*d^3))^(
2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c))
)/(a^2*d*e^(6*I*d*x + 6*I*c) + a^2*d*e^(4*I*d*x + 4*I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+I*a*tan(d*x+c))**(4/3),x)
```

```
[Out] Integral(tan(c + d*x)**4/(I*a*(tan(c + d*x) - I))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+I*a*tan(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^4/(I*a*tan(d*x + c) + a)^(4/3), x)
```

Mupad [B]
time = 4.54, size = 263, normalized size = 0.93

$$\frac{\frac{3\mathrm{i}}{8\,d}-\frac{(a+a\tan(c+d\,x)\,1\mathrm{i})\,21\mathrm{i}}{4\,a\,d}}{(a+a\tan(c+d\,x)\,1\mathrm{i})^{4/3}}-\frac{(a+a\tan(c+d\,x)\,1\mathrm{i})^{2/3}\,3\mathrm{i}}{a^2\,d}+\frac{(a+a\tan(c+d\,x)\,1\mathrm{i})^{5/3}\,3\mathrm{i}}{5\,a^3\,d}-\frac{\left(\frac{1}{128}\mathrm{i}\right)^{1/3}\ln\left(9\,(a\,(1+\tan(c+d\,x)\,1\mathrm{i}))^{1/3}+9\,(-1)^{1/3}\,2^{1/3}\,a^{1/3}\right)}{a^{4/3}\,d}+\frac{\left(\frac{1}{128}\mathrm{i}\right)^{1/3}\ln\left(-\frac{9\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{1/3}}{16\,a^2\,d^2}-\frac{9\,(-1)^{1/3}\,2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{32\,a^{5/3}\,d^2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{4/3}\,d}-\frac{\left(\frac{1}{128}\mathrm{i}\right)^{1/3}\ln\left(-\frac{9\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{1/3}}{16\,a^2\,d^2}+\frac{9\,(-1)^{1/3}\,2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{32\,a^{5/3}\,d^2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{4/3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(4/3),x)
```

```
[Out] (3i/(8*d) - ((a + a*tan(c + d*x)*1i)*21i)/(4*a*d))/(a + a*tan(c + d*x)*1i)^
(4/3) - ((a + a*tan(c + d*x)*1i)^(2/3)*3i)/(a^2*d) + ((a + a*tan(c + d*x)*1
i)^(5/3)*3i)/(5*a^3*d) - ((1i/128)^(1/3)*log(9*(a*(tan(c + d*x)*1i + 1))^(1
/3) + 9*(-1)^(1/3)*2^(1/3)*a^(1/3)))/(a^(4/3)*d) + ((1i/128)^(1/3)*log(- (9
*(a + a*tan(c + d*x)*1i)^(1/3))/(16*a^2*d^2) - (9*(-1)^(1/3)*2^(1/3)*(3^(1/
2)*1i - 1))/(32*a^(5/3)*d^2))*((3^(1/2)*1i)/2 + 1/2))/(a^(4/3)*d) - ((1i/12
8)^(1/3)*log((9*(-1)^(1/3)*2^(1/3)*(3^(1/2)*1i + 1))/(32*a^(5/3)*d^2) - (9*
(a + a*tan(c + d*x)*1i)^(1/3))/(16*a^2*d^2))*((3^(1/2)*1i)/2 - 1/2))/(a^(4/
3)*d)
```

3.301 $\int \frac{\tan^3(c+dx)}{(a+ia\tan(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=237

$$-\frac{ix}{8\sqrt[3]{2}\,a^{4/3}}-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{4\sqrt[3]{2}\,a^{4/3}d}-\frac{\log(\cos(c+dx))}{8\sqrt[3]{2}\,a^{4/3}d}-\frac{3\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+ia}}{8\sqrt[3]{2}\,a^{4/3}d}$$

```
[Out] -1/16*I*x*2^(2/3)/a^(4/3)-1/16*ln(cos(d*x+c))*2^(2/3)/a^(4/3)/d-3/16*ln(2^(
1/3)*a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))*2^(2/3)/a^(4/3)/d-1/8*arctan(1/3*(a^
(1/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)*2^(2/3)/a^
(4/3)/d+15/8/d/(a+I*a*tan(d*x+c))^(4/3)+3/2*tan(d*x+c)^2/d/(a+I*a*tan(d*x+c
))^(4/3)-27/4/a/d/(a+I*a*tan(d*x+c))^(1/3)
```

Rubi [A]
time = 0.23, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3641, 3671, 3607, 3562, 57, 631, 210, 31}

$$-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{4\sqrt[3]{2}\,a^{4/3}d}-\frac{3\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{8\sqrt[3]{2}\,a^{4/3}d}-\frac{\log(\cos(c+dx))}{8\sqrt[3]{2}\,a^{4/3}d}-\frac{ix}{8\sqrt[3]{2}\,a^{4/3}}+\frac{3\tan^2(c+dx)}{2d(a+ia\tan(c+dx))^{4/3}}-\frac{27}{4ad\sqrt[3]{a+ia\tan(c+dx)}}+\frac{15}{8d(a+ia\tan(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + I*a*Tan[c + d*x])^(4/3),x]
```

```
[Out] ((-1/8*I)*x)/(2^(1/3)*a^(4/3)) - (Sqrt[3]*ArcTan[(a^(1/3) + 2^(2/3)*(a + I*
a*Tan[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/(4*2^(1/3)*a^(4/3)*d) - Log[Cos[
c + d*x]]/(8*2^(1/3)*a^(4/3)*d) - (3*Log[2^(1/3)*a^(1/3) - (a + I*a*Tan[c +
 d*x])^(1/3)])/(8*2^(1/3)*a^(4/3)*d) + 15/(8*d*(a + I*a*Tan[c + d*x])^(4/3)
) + (3*Tan[c + d*x]^2)/(2*d*(a + I*a*Tan[c + d*x])^(4/3)) - 27/(4*a*d*(a +
I*a*Tan[c + d*x])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rule 3641

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[d*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n - 1)/(f*(m + n - 1))), x] - Dist[1/(a*(m + n - 1)), Int[(a + b*
Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 2)*Simp[d*(b*c*m + a*d*(-1 + n))
- a*c^2*(m + n - 1) + d*(b*d*m - a*c*(m + 2*n - 2))*Tan[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1] && NeQ[m + n - 1, 0] && (IntegerQ[n]
|| IntegersQ[2*m, 2*n])
```

Rule 3671

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(
A*b - a*B))*(a*c + b*d)*((a + b*Tan[e + f*x])^m/(2*a^2*f*m)), x] + Dist[1/(
2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[A*b*c + a*B*c + a*A*d + b*B*d
 + 2*a*B*d*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{(a+ia\tan(c+dx))^{4/3}}\,dx = \frac{3\tan^2(c+dx)}{2d(a+ia\tan(c+dx))^{4/3}} - \frac{3\int \frac{\tan(c+dx)\left(2a-\frac{4}{3}ia\tan(c+dx)\right)}{(a+ia\tan(c+dx))^{4/3}}\,dx}{2a}$$

$$= \frac{15}{8d(a+ia\tan(c+dx))^{4/3}} + \frac{3\tan^2(c+dx)}{2d(a+ia\tan(c+dx))^{4/3}} + \frac{(3i)\int \frac{\frac{10a^2}{3}-\frac{8}{3}ia^2}{\sqrt[3]{a+ia\tan}}}{4a^3}$$

$$= \frac{15}{8d(a+ia\tan(c+dx))^{4/3}} + \frac{3\tan^2(c+dx)}{2d(a+ia\tan(c+dx))^{4/3}} - \frac{27}{4ad\sqrt[3]{a+ia\tan(}}$$

$$= \frac{15}{8d(a+ia\tan(c+dx))^{4/3}} + \frac{3\tan^2(c+dx)}{2d(a+ia\tan(c+dx))^{4/3}} - \frac{27}{4ad\sqrt[3]{a+ia\tan(}}$$

$$= -\frac{ix}{8\sqrt[3]{2}\,a^{4/3}} - \frac{\log(\cos(c+dx))}{8\sqrt[3]{2}\,a^{4/3}d} + \frac{15}{8d(a+ia\tan(c+dx))^{4/3}} + \frac{3\tan^2(}{2d(a+ia\tan}$$

$$= -\frac{ix}{8\sqrt[3]{2}\,a^{4/3}} - \frac{\log(\cos(c+dx))}{8\sqrt[3]{2}\,a^{4/3}d} - \frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{8\sqrt[3]{2}\,a^{4/3}d}$$

$$= -\frac{ix}{8\sqrt[3]{2}\,a^{4/3}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\,a^{4/3}d} - \frac{\log(\cos(c+dx}{8\sqrt[3]{2}\,a^{4/3}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.02, size = 130, normalized size = 0.55

$$\frac{3\sec^2(c+dx)\left(9i+17i\cos(2(c+dx))-18\sin(2(c+dx))+{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)(-i\cos(2(c+dx))+\sin(2(c+dx)))\right)}{16ad(-i+\tan(c+dx))\sqrt[3]{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + I*a*Tan[c + d*x])^(4/3),x]
```

```
[Out] (3*Sec[c + d*x]^2*(9*I + (17*I)*Cos[2*(c + d*x)] - 18*Sin[2*(c + d*x)] + Hy
pergeometric2F1[2/3, 1, 5/3, E^((2*I)*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]
*((-I)*Cos[2*(c + d*x)] + Sin[2*(c + d*x)])))/(16*a*d*(-I + Tan[c + d*x])*(
a + I*a*Tan[c + d*x])^(1/3))
```

Maple [A]
time = 0.12, size = 190, normalized size = 0.80

method	result
derivativedivides	$-\frac{3\left(\frac{(a+ia\tan(dx+c))^{\frac{2}{3}}}{2}+\frac{\left(\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{12a^{\frac{1}{3}}}}{4}}{d\,a^2}$
default	$-\frac{3\left(\frac{(a+ia\tan(dx+c))^{\frac{2}{3}}}{2}+\frac{\left(\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{12a^{\frac{1}{3}}}}{4}}{d\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+I*a*tan(d*x+c))^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/d/a^2*(1/2*(a+I*a*tan(d*x+c))^(2/3)+1/4*(1/6*2^(2/3)/a^(1/3)*ln((a+I*a*t
an(d*x+c))^(1/3)-2^(1/3)*a^(1/3))-1/12*2^(2/3)/a^(1/3)*ln((a+I*a*tan(d*x+c)
)^(2/3)+2^(1/3)*a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+2^(2/3)*a^(2/3))+1/6*3^(1/
2)*2^(2/3)/a^(1/3)*arctan(1/3*3^(1/2)*(2^(2/3)/a^(1/3)*(a+I*a*tan(d*x+c))^(
1/3)+1)))*a+5/4*a/(a+I*a*tan(d*x+c))^(1/3)-1/8*a^2/(a+I*a*tan(d*x+c))^(4/3)
)
```

Maxima [A]

time = 0.52, size = 191, normalized size = 0.81

$$-\frac{2\sqrt{3}\,2^{\frac{2}{3}}a^{\frac{8}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)-2^{\frac{2}{3}}a^{\frac{8}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(i\,a\tan(dx+c)+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{2}{3}}\right)+2\cdot 2^{\frac{2}{3}}a^{\frac{8}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)+24\,(i\,a\tan(dx+c)+a)^{\frac{2}{3}}a^2+\frac{6\,(10\,(i\,a\tan(dx+c)+a)a^3-a^4)}{(i\,a\tan(dx+c)+a)^{\frac{4}{3}}}}{16\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+I*a*tan(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] -1/16*(2*sqrt(3)*2^(2/3)*a^(8/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3
) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) - 2^(2/3)*a^(8/3)*log(2^(2/3)*
a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x + c)
+ a)^(2/3)) + 2*2^(2/3)*a^(8/3)*log(-2^(1/3)*a^(1/3) + (I*a*tan(d*x + c) +
a)^(1/3)) + 24*(I*a*tan(d*x + c) + a)^(2/3)*a^2 + 6*(10*(I*a*tan(d*x + c) +
 a)*a^3 - a^4)/(I*a*tan(d*x + c) + a)^(4/3))/(a^4*d)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(172) = 344$.
time = 1.22, size = 366, normalized size = 1.54

$$\frac{\left(8\left(\frac{1}{2}\right)^{\frac{1}{3}}a^{2}d\left(-\frac{1}{a^{4}d^{3}}\right)^{\frac{1}{3}}e^{(4i\,dx+4i\,c)}\log\left(-2\left(\frac{1}{2}\right)^{\frac{2}{3}}a^{3}d^{2}\left(-\frac{1}{a^{4}d^{3}}\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)-4\left(\frac{1}{2}\right)^{\frac{1}{3}}\left(i\sqrt{3}\,a^{2}d+a^{2}d\right)\left(-\frac{1}{a^{4}d^{3}}\right)^{\frac{1}{3}}e^{(4i\,dx+4i\,c)}\log\left(-\left(\frac{1}{2}\right)^{\frac{2}{3}}\left(i\sqrt{3}\,a^{3}d^{2}-a^{3}d^{2}\right)\left(-\frac{1}{a^{4}d^{3}}\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)-4\left(\frac{1}{2}\right)^{\frac{1}{3}}\left(-i\sqrt{3}\,a^{2}d+a^{2}d\right)\left(-\frac{1}{a^{4}d^{3}}\right)^{\frac{1}{3}}e^{(4i\,dx+4i\,c)}\log\left(-\left(\frac{1}{2}\right)^{\frac{2}{3}}\left(-i\sqrt{3}\,a^{3}d^{2}-a^{3}d^{2}\right)\left(-\frac{1}{a^{4}d^{3}}\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)-3\cdot 2^{\frac{2}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{2}{3}}\left(35\,e^{(4i\,dx+4i\,c)}+18\,e^{(2i\,dx+2i\,c)}-1\right)e^{\left(\frac{4}{3}i\,dx+\frac{4}{3}i\,c\right)}\right)e^{(-4i\,dx-4i\,c)}}{32\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+I*a*tan(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] 1/32*(8*(1/2)^(1/3)*a^2*d*(-1/(a^4*d^3))^(1/3)*e^(4*I*d*x + 4*I*c)*log(-2*(
1/2)^(2/3)*a^3*d^2*(-1/(a^4*d^3))^(2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) +
 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)) - 4*(1/2)^(1/3)*(I*sqrt(3)*a^2*d + a^2*
d)*(-1/(a^4*d^3))^(1/3)*e^(4*I*d*x + 4*I*c)*log(-(1/2)^(2/3)*(I*sqrt(3)*a^3
*d^2 - a^3*d^2)*(-1/(a^4*d^3))^(2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1)
)^(1/3)*e^(2/3*I*d*x + 2/3*I*c)) - 4*(1/2)^(1/3)*(-I*sqrt(3)*a^2*d + a^2*d)
*(-1/(a^4*d^3))^(1/3)*e^(4*I*d*x + 4*I*c)*log(-(1/2)^(2/3)*(-I*sqrt(3)*a^3*
d^2 - a^3*d^2)*(-1/(a^4*d^3))^(2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))
^(1/3)*e^(2/3*I*d*x + 2/3*I*c)) - 3*2^(2/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(
2/3)*(35*e^(4*I*d*x + 4*I*c) + 18*e^(2*I*d*x + 2*I*c) - 1)*e^(4/3*I*d*x + 4
/3*I*c))*e^(-4*I*d*x - 4*I*c)/(a^2*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{3}\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+I*a*tan(d*x+c))**(4/3),x)
```

```
[Out] Integral(tan(c + d*x)**3/(I*a*(tan(c + d*x) - I))**(4/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+I*a*tan(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^3/(I*a*tan(d*x + c) + a)^(4/3), x)
```

Mupad [B]
time = 0.19, size = 214, normalized size = 0.90

$$-\frac{3\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{2/3}}{2\,a^2\,d}-\frac{4^{1/3}\,\ln\left(36\,a\,d\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{1/3}-18\,4^{2/3}\,a^{4/3}\,d\right)}{8\,a^{4/3}\,d}-\frac{\frac{27\,a}{8}+\frac{a\,\mathrm{tan}\left(c+d\,x\right)\,15{}\mathrm{i}}{4}}{a\,d\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{4/3}}-\frac{4^{1/3}\,\ln\left(36\,a\,d\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{1/3}-1152\,4^{2/3}\,a^{4/3}\,d\,{\left(-\frac{1}{16}+\frac{\sqrt{3}\,1{}\mathrm{i}}{16}\right)}^2\right)\,\left(-\frac{1}{16}+\frac{\sqrt{3}\,1{}\mathrm{i}}{16}\right)}{a^{4/3}\,d}+\frac{4^{1/3}\,\ln\left(36\,a\,d\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{1/3}-1152\,4^{2/3}\,a^{4/3}\,d\,{\left(\frac{1}{16}+\frac{\sqrt{3}\,1{}\mathrm{i}}{16}\right)}^2\right)\,\left(\frac{1}{16}+\frac{\sqrt{3}\,1{}\mathrm{i}}{16}\right)}{a^{4/3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(4/3),x)
```

```
[Out] (4^(1/3)*log(36*a*d*(a + a*tan(c + d*x)*1i)^(1/3) - 1152*4^(2/3)*a^(4/3)*d*
((3^(1/2)*1i)/16 + 1/16)^2)*((3^(1/2)*1i)/16 + 1/16))/(a^(4/3)*d) - (4^(1/3
)*log(36*a*d*(a + a*tan(c + d*x)*1i)^(1/3) - 18*4^(2/3)*a^(4/3)*d))/(8*a^(4
/3)*d) - ((27*a)/8 + (a*tan(c + d*x)*15i)/4)/(a*d*(a + a*tan(c + d*x)*1i)^(
4/3)) - (4^(1/3)*log(36*a*d*(a + a*tan(c + d*x)*1i)^(1/3) - 1152*4^(2/3)*a^
(4/3)*d*((3^(1/2)*1i)/16 - 1/16)^2)*((3^(1/2)*1i)/16 - 1/16))/(a^(4/3)*d) -
 (3*(a + a*tan(c + d*x)*1i)^(2/3))/(2*a^2*d)
```

3.302 $\int \frac{\tan^2(c+dx)}{(a+ia\tan(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=213

$$\frac{x}{8\sqrt[3]{2}\,a^{4/3}}-\frac{i\sqrt{3}\,\mathrm{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{4\sqrt[3]{2}\,a^{4/3}d}-\frac{i\log(\cos(c+dx))}{8\sqrt[3]{2}\,a^{4/3}d}-\frac{3i\log\left(\sqrt[3]{2}\,\sqrt[3]{a}\;-\sqrt[3]{a+i\right.}{8\sqrt[3]{2}\,a^{4/3}d}$$

[Out] 1/16*x*2^(2/3)/a^(4/3)-1/16*I*ln(cos(d*x+c))*2^(2/3)/a^(4/3)/d-3/16*I*ln(2^(1/3)*a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))*2^(2/3)/a^(4/3)/d-1/8*I*arctan(1/3*(a^(1/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)*2^(2/3)/a^(4/3)/d-3/8*I/d/(a+I*a*tan(d*x+c))^(4/3)+9/4*I/a/d/(a+I*a*tan(d*x+c))^(1/3)

Rubi [A]
time = 0.15, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3621, 3607, 3562, 57, 631, 210, 31}

$$-\frac{i\sqrt{3}\,\mathrm{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{4\sqrt[3]{2}\,a^{4/3}d}-\frac{3i\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{8\sqrt[3]{2}\,a^{4/3}d}-\frac{i\log(\cos(c+dx))}{8\sqrt[3]{2}\,a^{4/3}d}+\frac{x}{8\sqrt[3]{2}\,a^{4/3}}+\frac{9i}{4ad\sqrt[3]{a+ia\tan(c+dx)}}-\frac{3i}{8d(a+ia\tan(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d*x]^2/(a + I*a*Tan[c + d*x])^(4/3),x]

[Out] x/(8*2^(1/3)*a^(4/3)) - ((I/4)*Sqrt[3]*ArcTan[(a^(1/3) + 2^(2/3)*(a + I*a*Tan[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/(2^(1/3)*a^(4/3)*d) - ((I/8)*Log[Cos[c + d*x]])/(2^(1/3)*a^(4/3)*d) - (((3*I)/8)*Log[2^(1/3)*a^(1/3) - (a + I*a*Tan[c + d*x])^(1/3)])/(2^(1/3)*a^(4/3)*d) - ((3*I)/8)/(d*(a + I*a*Tan[c + d*x])^(4/3)) + ((9*I)/4)/(a*d*(a + I*a*Tan[c + d*x])^(1/3))

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rule 3621

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(a*c + b*d)^2*((a + b*Tan[e + f*x])^
m/(2*a^3*f*m)), x] + Dist[1/(2*a^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[
a*c^2 - 2*b*c*d + a*d^2 - 2*b*d^2*Tan[e + f*x], x], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LeQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{(a+ia\tan(c+dx))^{4/3}}\,dx = -\frac{3i}{8d(a+ia\tan(c+dx))^{4/3}} + \frac{\int \frac{a-2ia\tan(c+dx)}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{3i}{8d(a+ia\tan(c+dx))^{4/3}} + \frac{9i}{4ad\sqrt[3]{a+ia\tan(c+dx)}} - \frac{\int (a+ia\tan(c-}{4a^2}$$

$$= -\frac{3i}{8d(a+ia\tan(c+dx))^{4/3}} + \frac{9i}{4ad\sqrt[3]{a+ia\tan(c+dx)}} + \frac{i\text{Subst}\left(\int \frac{}{(a-x)}\right.}{}$$

$$= \frac{x}{8\sqrt[3]{2}\,a^{4/3}} - \frac{i\log(\cos(c+dx))}{8\sqrt[3]{2}\,a^{4/3}d} - \frac{3i}{8d(a+ia\tan(c+dx))^{4/3}} + \frac{9}{4ad\sqrt[3]{a+ia}}$$

$$= \frac{x}{8\sqrt[3]{2}\,a^{4/3}} - \frac{i\log(\cos(c+dx))}{8\sqrt[3]{2}\,a^{4/3}d} - \frac{3i\log\left(\sqrt[3]{2}\sqrt[3]{a} - \sqrt[3]{a+ia\tan(c+dx)}\right)}{8\sqrt[3]{2}\,a^{4/3}d}$$

$$= \frac{x}{8\sqrt[3]{2}\,a^{4/3}} - \frac{i\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\,a^{4/3}d} - \frac{i\log(\cos(c+dx}{8\sqrt[3]{2}\,a^{4/3}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.78, size = 128, normalized size = 0.60

$$\frac{3\sec^2(c+dx)\left(5+5\cos(2(c+dx))+{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)(\cos(2(c+dx))+i\sin(2(c+dx)))+6i\sin(2(c+dx))\right)}{16ad(-i+\tan(c+dx))\sqrt[3]{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + I*a*Tan[c + d*x])^(4/3),x]
```

```
[Out] (3*Sec[c + d*x]^2*(5 + 5*Cos[2*(c + d*x)] + Hypergeometric2F1[2/3, 1, 5/3,
E^((2*I)*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(Cos[2*(c + d*x)] + I*Sin[2*
(c + d*x)]) + (6*I)*Sin[2*(c + d*x)]))/(16*a*d*(-I + Tan[c + d*x])*(a + I*a
*Tan[c + d*x])^(1/3))
```

Maple [A]
time = 0.11, size = 168, normalized size = 0.79

method	result

derivativedivides	$-\frac{3i\left(\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{24a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{48a^{\frac{1}{3}}}+\frac{\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\sqrt{\ldots}\right.}{\ldots}\right.}{da}$
default	$-\frac{3i\left(\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{24a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{48a^{\frac{1}{3}}}+\frac{\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\sqrt{\ldots}\right.}{\ldots}\right.}{da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+I*a*tan(d*x+c))^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3*I/d/a*(1/24*2^(2/3)/a^(1/3)*ln((a+I*a*tan(d*x+c))^(1/3)-2^(1/3)*a^(1/3))
-1/48*2^(2/3)/a^(1/3)*ln((a+I*a*tan(d*x+c))^(2/3)+2^(1/3)*a^(1/3)*(a+I*a*ta
n(d*x+c))^(1/3)+2^(2/3)*a^(2/3))+1/24*3^(1/2)*2^(2/3)/a^(1/3)*arctan(1/3*3^
(1/2)*(2^(2/3)/a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+1))-3/4/(a+I*a*tan(d*x+c))^
(1/3)+1/8*a/(a+I*a*tan(d*x+c))^(4/3))
```

Maxima [A]
time = 0.60, size = 173, normalized size = 0.81

$$-\frac{i\left(2\sqrt{3}\,2^{\frac{2}{3}}a^{\frac{5}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)-2^{\frac{2}{3}}a^{\frac{5}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(i\,a\tan(dx+c)+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{2}{3}}\right)+2\cdot 2^{\frac{2}{3}}a^{\frac{5}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)-\frac{6\,(6\,(i\,a\tan(dx+c)+a)a^2-a^3)}{(i\,a\tan(dx+c)+a)^{\frac{4}{3}}}\right)}{16\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+I*a*tan(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] -1/16*I*(2*sqrt(3)*2^(2/3)*a^(5/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1
/3) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) - 2^(2/3)*a^(5/3)*log(2^(2/3
)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x + c
) + a)^(2/3)) + 2*2^(2/3)*a^(5/3)*log(-2^(1/3)*a^(1/3) + (I*a*tan(d*x + c)
+ a)^(1/3)) - 6*(6*(I*a*tan(d*x + c) + a)*a^2 - a^3)/(I*a*tan(d*x + c) + a)
^(4/3))/(a^3*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 346 vs. 2(146) = 292.
time = 1.01, size = 346, normalized size = 1.62

$$\frac{\left(32\,a^2d\left(\frac{i}{128\,a^4d^3}\right)^{\frac{1}{3}}e^{(4i\,dx+4i\,c)}\log\left(32\,a^3d^2\left(\frac{i}{128\,a^4d^3}\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)-3\cdot 2^{\frac{2}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{2}{3}}\left(-11i\,e^{(4i\,dx+4i\,c)}-10i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(\frac{4}{3}i\,dx+\frac{4}{3}i\,c\right)}-16\left(-i\sqrt{3}\,a^2d+a^2d\right)\left(\frac{i}{128\,a^4d^3}\right)^{\frac{1}{3}}e^{(4i\,dx+4i\,c)}\log\left(-16\left(i\sqrt{3}\,a^3d^2+a^3d^2\right)\left(\frac{i}{128\,a^4d^3}\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)-16\left(i\sqrt{3}\,a^2d+a^2d\right)\left(\frac{i}{128\,a^4d^3}\right)^{\frac{1}{3}}e^{(4i\,dx+4i\,c)}\log\left(-16\left(-i\sqrt{3}\,a^3d^2+a^3d^2\right)\left(\frac{i}{128\,a^4d^3}\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)\right)e^{(-4i\,dx-4i\,c)}}{32\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+I*a*tan(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] 1/32*(32*a^2*d*(1/128*I/(a^4*d^3))^(1/3)*e^(4*I*d*x + 4*I*c)*log(32*a^3*d^2
*(1/128*I/(a^4*d^3))^(2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^
(2/3*I*d*x + 2/3*I*c)) - 3*2^(2/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(-11
*I*e^(4*I*d*x + 4*I*c) - 10*I*e^(2*I*d*x + 2*I*c) + I)*e^(4/3*I*d*x + 4/3*I
*c) - 16*(-I*sqrt(3)*a^2*d + a^2*d)*(1/128*I/(a^4*d^3))^(1/3)*e^(4*I*d*x +
4*I*c)*log(-16*(I*sqrt(3)*a^3*d^2 + a^3*d^2)*(1/128*I/(a^4*d^3))^(2/3) + 2^
(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)) - 16*(I*
sqrt(3)*a^2*d + a^2*d)*(1/128*I/(a^4*d^3))^(1/3)*e^(4*I*d*x + 4*I*c)*log(-1
6*(-I*sqrt(3)*a^3*d^2 + a^3*d^2)*(1/128*I/(a^4*d^3))^(2/3) + 2^(1/3)*(a/(e^
(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)))*e^(-4*I*d*x - 4*I*c
)/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+I*a*tan(d*x+c))**(4/3),x)
```

```
[Out] Integral(tan(c + d*x)**2/(I*a*(tan(c + d*x) - I))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+I*a*tan(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^2/(I*a*tan(d*x + c) + a)^(4/3), x)
```

Mupad [B]
time = 0.68, size = 217, normalized size = 1.02

$$-\frac{\frac{3\mathrm{i}}{8\,d}-\frac{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1\mathrm{i}\right)\,9\mathrm{i}}{4\,a\,d}}{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1\mathrm{i}\right)}^{4/3}}+\frac{{\left(\frac{1}{128}\mathrm{i}\right)}^{1/3}\,\ln\left(9\,{\left(a\,\left(1+\mathrm{tan}\left(c+d\,x\right)\,1\mathrm{i}\right)\right)}^{1/3}+9\,{\left(-1\right)}^{1/3}\,2^{1/3}\,a^{1/3}\right)}{a^{4/3}\,d}-\frac{{\left(\frac{1}{128}\mathrm{i}\right)}^{1/3}\,\ln\left(-\frac{9\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1\mathrm{i}\right)}^{1/3}}{16\,a^2\,d^2}-\frac{9\,{\left(-1\right)}^{1/3}\,2^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{32\,a^{5/3}\,d^2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{4/3}\,d}+\frac{{\left(\frac{1}{128}\mathrm{i}\right)}^{1/3}\,\ln\left(-\frac{9\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1\mathrm{i}\right)}^{1/3}}{16\,a^2\,d^2}+\frac{9\,{\left(-1\right)}^{1/3}\,2^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{32\,a^{5/3}\,d^2}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{4/3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(4/3),x)
```

```
[Out] ((1i/128)^(1/3)*log(9*(a*(tan(c + d*x)*1i + 1))^(1/3) + 9*(-1)^(1/3)*2^(1/3
)*a^(1/3)))/(a^(4/3)*d) - (3i/(8*d) - ((a + a*tan(c + d*x)*1i)*9i)/(4*a*d))
```

```
/(a + a*tan(c + d*x)*1i)^(4/3) - ((1i/128)^(1/3)*log(- (9*(a + a*tan(c + d*
x)*1i)^(1/3))/(16*a^2*d^2) - (9*(-1)^(1/3)*2^(1/3)*(3^(1/2)*1i - 1))/(32*a^
(5/3)*d^2))*((3^(1/2)*1i)/2 + 1/2))/(a^(4/3)*d) + ((1i/128)^(1/3)*log((9*(-
1)^(1/3)*2^(1/3)*(3^(1/2)*1i + 1))/(32*a^(5/3)*d^2) - (9*(a + a*tan(c + d*x
)*1i)^(1/3))/(16*a^2*d^2))*((3^(1/2)*1i)/2 - 1/2))/(a^(4/3)*d)
```

3.303 $\int \frac{\tan(c+dx)}{(a+ia\tan(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=205

$$\frac{ix}{8\sqrt[3]{2}\,a^{4/3}}+\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{4\sqrt[3]{2}\,a^{4/3}d}+\frac{\log(\cos(c+dx))}{8\sqrt[3]{2}\,a^{4/3}d}+\frac{3\log\left(\sqrt[3]{2}\,\sqrt[3]{a}\,-\,\sqrt[3]{a+ia\tan(c+dx)}\right)}{8\sqrt[3]{2}\,a^{4/3}d}$$

```
[Out] 1/16*I*x*2^(2/3)/a^(4/3)+1/16*ln(cos(d*x+c))*2^(2/3)/a^(4/3)/d+3/16*ln(2^(1
/3)*a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))*2^(2/3)/a^(4/3)/d+1/8*arctan(1/3*(a^(
1/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)*2^(2/3)/a^(
4/3)/d-3/8/d/(a+I*a*tan(d*x+c))^(4/3)+3/4/a/d/(a+I*a*tan(d*x+c))^(1/3)
```

Rubi [A]
time = 0.11, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {3607, 3560, 3562, 57, 631, 210, 31}

$$\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{4\sqrt[3]{2}\,a^{4/3}d}+\frac{3\log\left(\sqrt[3]{2}\,\sqrt[3]{a}\,-\,\sqrt[3]{a+ia\tan(c+dx)}\right)}{8\sqrt[3]{2}\,a^{4/3}d}+\frac{\log(\cos(c+dx))}{8\sqrt[3]{2}\,a^{4/3}d}+\frac{ix}{8\sqrt[3]{2}\,a^{4/3}}+\frac{3}{4ad\sqrt[3]{a+ia\tan(c+dx)}}-\frac{3}{8d(a+ia\tan(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + I*a*Tan[c + d*x])^(4/3),x]
```

```
[Out] ((I/8)*x)/(2^(1/3)*a^(4/3)) + (Sqrt[3]*ArcTan[(a^(1/3) + 2^(2/3)*(a + I*a*T
an[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/(4*2^(1/3)*a^(4/3)*d) + Log[Cos[c +
 d*x]]/(8*2^(1/3)*a^(4/3)*d) + (3*Log[2^(1/3)*a^(1/3) - (a + I*a*Tan[c + d*
x])^(1/3)])/(8*2^(1/3)*a^(4/3)*d) - 3/(8*d*(a + I*a*Tan[c + d*x])^(4/3)) +
3/(4*a*d*(a + I*a*Tan[c + d*x])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{(a+ia\tan(c+dx))^{4/3}}\,dx = -\frac{3}{8d(a+ia\tan(c+dx))^{4/3}} - \frac{i\int \frac{1}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx}{2a}$$

$$= -\frac{3}{8d(a+ia\tan(c+dx))^{4/3}} + \frac{3}{4ad\sqrt[3]{a+ia\tan(c+dx)}} - \frac{i\int(a+ia\tan(c}{4a^2}$$

$$= -\frac{3}{8d(a+ia\tan(c+dx))^{4/3}} + \frac{3}{4ad\sqrt[3]{a+ia\tan(c+dx)}} - \frac{\text{Subst}\Big(\int \frac{}{(a-x)\sqrt[3]{}}}{}$$

$$= \frac{ix}{8\sqrt[3]{2}\,a^{4/3}} + \frac{\log(\cos(c+dx))}{8\sqrt[3]{2}\,a^{4/3}d} - \frac{3}{8d(a+ia\tan(c+dx))^{4/3}} + \frac{3}{4ad\sqrt[3]{a+ia\,t}}$$

$$= \frac{ix}{8\sqrt[3]{2}\,a^{4/3}} + \frac{\log(\cos(c+dx))}{8\sqrt[3]{2}\,a^{4/3}d} + \frac{3\log\left(\sqrt[3]{2}\,\sqrt[3]{a} - \sqrt[3]{a+ia\tan(c+dx)}\right)}{8\sqrt[3]{2}\,a^{4/3}d} -$$

$$= \frac{ix}{8\sqrt[3]{2}\,a^{4/3}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\,a^{4/3}d} + \frac{\log(\cos(c+dx))}{8\sqrt[3]{2}\,a^{4/3}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.71, size = 130, normalized size = 0.63

$$\frac{3i\sec^2(c+dx)\left(-1-\cos(2(c+dx))+\,_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)(\cos(2(c+dx))+i\sin(2(c+dx)))-2i\sin(2(c+dx))\right)}{16ad(-i+\tan(c+dx))\sqrt[3]{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + I*a*Tan[c + d*x])^(4/3),x]
```

```
[Out] (((3*I)/16)*Sec[c + d*x]^2*(-1 - Cos[2*(c + d*x)] + Hypergeometric2F1[2/3,
1, 5/3, E^((2*I)*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(Cos[2*(c + d*x)] +
I*Sin[2*(c + d*x)]) - (2*I)*Sin[2*(c + d*x)]))/(a*d*(-I + Tan[c + d*x])*(a
+ I*a*Tan[c + d*x])^(1/3))
```

Maple [A]
time = 0.10, size = 171, normalized size = 0.83

method	result

derivativedivides	$\frac{\frac{\frac{2^{\frac{2}{3}} \ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}} a^{\frac{1}{3}}\right)}{8a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}} \ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}} a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}} a^{\frac{2}{3}}\right)}{16a^{\frac{1}{3}}}+\frac{\sqrt{3}\, 2^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\,(\cdots}{\cdots}\right.}{8\cdots}}{a}}{d}$
default	$\frac{\frac{\frac{2^{\frac{2}{3}} \ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}} a^{\frac{1}{3}}\right)}{8a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}} \ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}} a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}} a^{\frac{2}{3}}\right)}{16a^{\frac{1}{3}}}+\frac{\sqrt{3}\, 2^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\,(\cdots}{\cdots}\right.}{8\cdots}}{a}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+I*a*tan(d*x+c))^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(3/4*(1/6*2^(2/3)/a^(1/3)*ln((a+I*a*tan(d*x+c))^(1/3)-2^(1/3)*a^(1/3))-
1/12*2^(2/3)/a^(1/3)*ln((a+I*a*tan(d*x+c))^(2/3)+2^(1/3)*a^(1/3)*(a+I*a*tan
(d*x+c))^(1/3)+2^(2/3)*a^(2/3))+1/6*3^(1/2)*2^(2/3)/a^(1/3)*arctan(1/3*3^(1
/2)*(2^(2/3)/a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+1)))/a-3/8/(a+I*a*tan(d*x+c))
^(4/3)+3/4/a/(a+I*a*tan(d*x+c))^(1/3))
```

Maxima [A]
time = 0.51, size = 171, normalized size = 0.83

$$\frac{2\sqrt{3}\,2^{\frac{2}{3}}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)-2^{\frac{2}{3}}a^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(i\,a\tan(dx+c)+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{2}{3}}\right)+2\cdot 2^{\frac{2}{3}}a^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)+\frac{6\,(2\,(i\,a\tan(dx+c)+a)a-a^2)}{(i\,a\tan(dx+c)+a)^{\frac{4}{3}}}}{16\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] 1/16*(2*sqrt(3)*2^(2/3)*a^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3)
 + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) - 2^(2/3)*a^(2/3)*log(2^(2/3)*a
^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x + c) +
 a)^(2/3)) + 2*2^(2/3)*a^(2/3)*log(-2^(1/3)*a^(1/3) + (I*a*tan(d*x + c) + a
)^(1/3)) + 6*(2*(I*a*tan(d*x + c) + a)*a - a^2)/(I*a*tan(d*x + c) + a)^(4/3
))/(a^2*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(146) = 292.
time = 1.40, size = 360, normalized size = 1.76

$$\frac{\left(8\left(\frac{1}{2}\right)^{\frac{1}{3}}a^2d\left(\frac{1}{a^4d^3}\right)^{\frac{1}{3}}e^{(4i\,dx+4i\,c)}\log\left(-2\left(\frac{1}{2}\right)^{\frac{2}{3}}a^3d^2\left(\frac{1}{a^4d^3}\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)-4\left(\frac{1}{2}\right)^{\frac{1}{3}}\left(i\sqrt{3}\,a^2d+a^2d\right)\left(\frac{1}{a^4d^3}\right)^{\frac{1}{3}}e^{(4i\,dx+4i\,c)}\log\left(-\left(\frac{1}{2}\right)^{\frac{2}{3}}\left(i\sqrt{3}\,a^3d^2-a^3d^2\right)\left(\frac{1}{a^4d^3}\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)-4\left(\frac{1}{2}\right)^{\frac{1}{3}}\left(-i\sqrt{3}\,a^2d+a^2d\right)\left(\frac{1}{a^4d^3}\right)^{\frac{1}{3}}e^{(4i\,dx+4i\,c)}\log\left(-\left(\frac{1}{2}\right)^{\frac{2}{3}}\left(-i\sqrt{3}\,a^3d^2-a^3d^2\right)\left(\frac{1}{a^4d^3}\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)+3\cdot 2^{\frac{2}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{2}{3}}\left(3\,e^{(4i\,dx+4i\,c)}+2\,e^{(2i\,dx+2i\,c)}-1\right)e^{\left(\frac{4}{3}i\,dx+\frac{4}{3}i\,c\right)}\right)e^{(-4i\,dx-4i\,c)}}{32\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] 1/32*(8*(1/2)^(1/3)*a^2*d*(1/(a^4*d^3))^(1/3)*e^(4*I*d*x + 4*I*c)*log(-2*(1
/2)^(2/3)*a^3*d^2*(1/(a^4*d^3))^(2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1
))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)) - 4*(1/2)^(1/3)*(I*sqrt(3)*a^2*d + a^2*d)
*(1/(a^4*d^3))^(1/3)*e^(4*I*d*x + 4*I*c)*log(-(1/2)^(2/3)*(I*sqrt(3)*a^3*d^
2 - a^3*d^2)*(1/(a^4*d^3))^(2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1
/3)*e^(2/3*I*d*x + 2/3*I*c)) - 4*(1/2)^(1/3)*(-I*sqrt(3)*a^2*d + a^2*d)*(1/
(a^4*d^3))^(1/3)*e^(4*I*d*x + 4*I*c)*log(-(1/2)^(2/3)*(-I*sqrt(3)*a^3*d^2 -
 a^3*d^2)*(1/(a^4*d^3))^(2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)
*e^(2/3*I*d*x + 2/3*I*c)) + 3*2^(2/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(
3*e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) - 1)*e^(4/3*I*d*x + 4/3*I*c))
*e^(-4*I*d*x - 4*I*c)/(a^2*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan (c+dx)}{(ia(\tan (c+dx)-i))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c))**(4/3),x)
```

```
[Out] Integral(tan(c + d*x)/(I*a*(tan(c + d*x) - I))**(4/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+I*a*tan(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)/(I*a*tan(d*x + c) + a)^(4/3), x)
```

Mupad [**B**]
time = 3.97, size = 193, normalized size = 0.94

$$\frac{\frac{3\,(a+a\tan(c+d\,x)\,1\mathrm{i})}{4\,a}-\frac{3}{8}}{d\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{4/3}}+\frac{4^{1/3}\,\ln\left(36\,a\,d\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{1/3}-18\,4^{2/3}\,a^{4/3}\,d\right)}{8\,a^{4/3}\,d}+\frac{4^{1/3}\,\ln\left(36\,a\,d\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{1/3}-1152\,4^{2/3}\,a^{4/3}\,d\left(-\frac{1}{16}+\frac{\sqrt{3}\,1\mathrm{i}}{16}\right)^2\right)\left(-\frac{1}{16}+\frac{\sqrt{3}\,1\mathrm{i}}{16}\right)}{a^{4/3}\,d}-\frac{4^{1/3}\,\ln\left(36\,a\,d\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{1/3}-1152\,4^{2/3}\,a^{4/3}\,d\left(\frac{1}{16}+\frac{\sqrt{3}\,1\mathrm{i}}{16}\right)^2\right)\left(\frac{1}{16}+\frac{\sqrt{3}\,1\mathrm{i}}{16}\right)}{a^{4/3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + a*tan(c + d*x)*1i)^(4/3),x)
```

```
[Out] ((3*(a + a*tan(c + d*x)*1i))/(4*a) - 3/8)/(d*(a + a*tan(c + d*x)*1i)^(4/3))
 + (4^(1/3)*log(36*a*d*(a + a*tan(c + d*x)*1i)^(1/3) - 18*4^(2/3)*a^(4/3)*d
))/(8*a^(4/3)*d) + (4^(1/3)*log(36*a*d*(a + a*tan(c + d*x)*1i)^(1/3) - 1152
*4^(2/3)*a^(4/3)*d*((3^(1/2)*1i)/16 - 1/16)^2)*((3^(1/2)*1i)/16 - 1/16))/(a
^(4/3)*d) - (4^(1/3)*log(36*a*d*(a + a*tan(c + d*x)*1i)^(1/3) - 1152*4^(2/3
)*a^(4/3)*d*((3^(1/2)*1i)/16 + 1/16)^2)*((3^(1/2)*1i)/16 + 1/16))/(a^(4/3)*
d)
```

3.304 $\int \frac{1}{(a+ia\tan(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=213

$$-\frac{x}{8\sqrt[3]{2}\,a^{4/3}}+\frac{i\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{4\sqrt[3]{2}\,a^{4/3}d}+\frac{i\log(\cos(c+dx))}{8\sqrt[3]{2}\,a^{4/3}d}+\frac{3i\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+i\right.}}{8\sqrt[3]{2}\,a^{4/3}d}$$

```
[Out] -1/16*x*2^(2/3)/a^(4/3)+1/16*I*ln(cos(d*x+c))*2^(2/3)/a^(4/3)/d+3/16*I*ln(2
^(1/3)*a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))*2^(2/3)/a^(4/3)/d+1/8*I*arctan(1/3
*(a^(1/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)*2^(2/3
)/a^(4/3)/d+3/8*I/d/(a+I*a*tan(d*x+c))^(4/3)+3/4*I/a/d/(a+I*a*tan(d*x+c))^(
1/3)
```

Rubi [A]
time = 0.10, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3560, 3562, 57, 631, 210, 31}

$$\frac{i\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{4\sqrt[3]{2}\,a^{4/3}d}+\frac{3i\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{8\sqrt[3]{2}\,a^{4/3}d}+\frac{i\log(\cos(c+dx))}{8\sqrt[3]{2}\,a^{4/3}d}-\frac{x}{8\sqrt[3]{2}\,a^{4/3}}+\frac{3i}{4ad\sqrt[3]{a+ia\tan(c+dx)}}+\frac{3i}{8d(a+ia\tan(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(-4/3),x]
```

```
[Out] -1/8*x/(2^(1/3)*a^(4/3)) + ((I/4)*Sqrt[3]*ArcTan[(a^(1/3) + 2^(2/3)*(a + I*
a*Tan[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/(2^(1/3)*a^(4/3)*d) + ((I/8)*Log
[Cos[c + d*x]])/(2^(1/3)*a^(4/3)*d) + (((3*I)/8)*Log[2^(1/3)*a^(1/3) - (a +
 I*a*Tan[c + d*x])^(1/3)])/(2^(1/3)*a^(4/3)*d) + ((3*I)/8)/(d*(a + I*a*Tan[
c + d*x])^(4/3)) + ((3*I)/4)/(a*d*(a + I*a*Tan[c + d*x])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(c+dx))^{4/3}}\,dx = \frac{3i}{8d(a+ia\tan(c+dx))^{4/3}} + \frac{\int \frac{1}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx}{2a}$$

$$= \frac{3i}{8d(a+ia\tan(c+dx))^{4/3}} + \frac{3i}{4ad\sqrt[3]{a+ia\tan(c+dx)}} + \frac{\int(a+ia\tan(c+d}{4a^2}$$

$$= \frac{3i}{8d(a+ia\tan(c+dx))^{4/3}} + \frac{3i}{4ad\sqrt[3]{a+ia\tan(c+dx)}} - \frac{i\text{Subst}\left(\int \frac{1}{(a-x)\sqrt[3]{a}}\right.}{}$$

$$= -\frac{x}{8\sqrt[3]{2}\,a^{4/3}} + \frac{i\log(\cos(c+dx))}{8\sqrt[3]{2}\,a^{4/3}d} + \frac{3i}{8d(a+ia\tan(c+dx))^{4/3}} + \frac{3}{4ad\sqrt[3]{a+ia}}$$

$$= -\frac{x}{8\sqrt[3]{2}\,a^{4/3}} + \frac{i\log(\cos(c+dx))}{8\sqrt[3]{2}\,a^{4/3}d} + \frac{3i\log\left(\sqrt[3]{2}\sqrt[3]{a} - \sqrt[3]{a+ia\tan(c+dx)}\right)}{8\sqrt[3]{2}\,a^{4/3}d}$$

$$= -\frac{x}{8\sqrt[3]{2}\,a^{4/3}} + \frac{i\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\,a^{4/3}d} + \frac{i\log(\cos(c+d}{8\sqrt[3]{2}\,a^{4/3}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.46, size = 128, normalized size = 0.60

$$-\frac{3\sec^2(c+dx)\left(-3-3\cos(2(c+dx))+{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)(\cos(2(c+dx))+i\sin(2(c+dx)))-2i\sin(2(c+dx))\right)}{16ad(-i+\tan(c+dx))\sqrt[3]{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(-4/3),x]
```

```
[Out] (-3*Sec[c + d*x]^2*(-3 - 3*Cos[2*(c + d*x)] + Hypergeometric2F1[2/3, 1, 5/3
, E^((2*I)*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(Cos[2*(c + d*x)] + I*Sin[
2*(c + d*x)]) - (2*I)*Sin[2*(c + d*x)]))/(16*a*d*(-I + Tan[c + d*x])*(a + I
*a*Tan[c + d*x])^(1/3))
```

Maple [A]
time = 0.10, size = 177, normalized size = 0.83

method	result

derivativedivides	$\frac{3ia\left(\frac{\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{12a^{\frac{1}{3}}}+\frac{\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\ldots\right.}{}}{4a^{2}}\right.}{d}$
default	$\frac{3ia\left(\frac{\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{12a^{\frac{1}{3}}}+\frac{\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\ldots\right.}{}}{4a^{2}}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+I*a*tan(d*x+c))^(4/3),x,method=_RETURNVERBOSE)

[Out] 3*I/d*a*(1/4*(1/6*2^(2/3)/a^(1/3)*ln((a+I*a*tan(d*x+c))^(1/3)-2^(1/3)*a^(1/3))-1/12*2^(2/3)/a^(1/3)*ln((a+I*a*tan(d*x+c))^(2/3)+2^(1/3)*a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+2^(2/3)*a^(2/3))+1/6*3^(1/2)*2^(2/3)/a^(1/3)*arctan(1/3*3^(1/2)*(2^(2/3)/a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+1)))/a^2+1/4/a^2/(a+I*a*tan(d*x+c))^(1/3)+1/8/a/(a+I*a*tan(d*x+c))^(4/3))

Maxima [A]

time = 0.52, size = 164, normalized size = 0.77

$$\frac{i\left(\frac{2\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(ia\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(ia\tan(dx+c)+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(ia\tan(dx+c)+a)^{\frac{2}{3}}\right)}{a^{\frac{1}{3}}}+\frac{2\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(ia\tan(dx+c)+a)^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}+\frac{6\,(2ia\tan(dx+c)+3a)}{(ia\tan(dx+c)+a)^{\frac{4}{3}}}\right)}{16\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+I*a*tan(d*x+c))^(4/3),x, algorithm="maxima")

[Out] 1/16*I*(2*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3))/a^(1/3) - 2^(2/3)*log(2^(2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x + c) + a)^(2/3))/a^(1/3) + 2*2^(2/3)*log(-2^(1/3)*a^(1/3) + (I*a*tan(d*x + c) + a)^(1/3))/a^(1/3) + 6*(2*I*a*tan(d*x + c) + 3*a)/(I*a*tan(d*x + c) + a)^(4/3))/(a*d)

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 346 vs. 2(146) = 292.
time = 1.57, size = 346, normalized size = 1.62

$$\frac{\left(32\,a^2d\left(-\frac{i}{128\,a^4d^3}\right)^{\frac{1}{3}}e^{\left(4i\,dx+4i\,c\right)}\log\left(32\,a^3d^2\left(-\frac{i}{128\,a^4d^3}\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)-3\cdot2^{\frac{2}{3}}\left(\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}\right)^{\frac{2}{3}}\left(-5i\,e^{\left(4i\,dx+4i\,c\right)}-6i\,e^{\left(2i\,dx+2i\,c\right)}-i\right)e^{\left(\frac{4}{3}i\,dx+\frac{4}{3}i\,c\right)}-16\left(-i\sqrt{3}\,a^2d+a^2d\right)\left(-\frac{i}{128\,a^4d^3}\right)^{\frac{1}{3}}e^{\left(4i\,dx+4i\,c\right)}\log\left(-16\left(i\sqrt{3}\,a^3d^2+a^3d^2\right)\left(-\frac{i}{128\,a^4d^3}\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)-16\left(i\sqrt{3}\,a^2d+a^2d\right)\left(-\frac{i}{128\,a^4d^3}\right)^{\frac{1}{3}}e^{\left(4i\,dx+4i\,c\right)}\log\left(-16\left(-i\sqrt{3}\,a^3d^2+a^3d^2\right)\left(-\frac{i}{128\,a^4d^3}\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)\right)e^{\left(-4i\,dx-4i\,c\right)}}{32\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] 1/32*(32*a^2*d*(-1/128*I/(a^4*d^3))^(1/3)*e^(4*I*d*x + 4*I*c)*log(32*a^3*d^
2*(-1/128*I/(a^4*d^3))^(2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*
e^(2/3*I*d*x + 2/3*I*c)) - 3*2^(2/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(-
5*I*e^(4*I*d*x + 4*I*c) - 6*I*e^(2*I*d*x + 2*I*c) - I)*e^(4/3*I*d*x + 4/3*I
*c) - 16*(-I*sqrt(3)*a^2*d + a^2*d)*(-1/128*I/(a^4*d^3))^(1/3)*e^(4*I*d*x +
 4*I*c)*log(-16*(I*sqrt(3)*a^3*d^2 + a^3*d^2)*(-1/128*I/(a^4*d^3))^(2/3) +
2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)) - 16*(
I*sqrt(3)*a^2*d + a^2*d)*(-1/128*I/(a^4*d^3))^(1/3)*e^(4*I*d*x + 4*I*c)*log
(-16*(-I*sqrt(3)*a^3*d^2 + a^3*d^2)*(-1/128*I/(a^4*d^3))^(2/3) + 2^(1/3)*(a
/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)))*e^(-4*I*d*x - 4
*I*c)/(a^2*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\tan\left(c+dx\right)+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))**(4/3),x)
```

```
[Out] Integral((I*a*tan(c + d*x) + a)**(-4/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(-4/3), x)
```

Mupad [**B**]
time = 4.42, size = 217, normalized size = 1.02

$$\frac{\frac{3\mathrm{i}}{8\,d}+\frac{\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)3\mathrm{i}}{4\,a\,d}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{4/3}}-\frac{\left(\frac{1}{128}\mathrm{i}\right)^{1/3}\,\ln\left(9\left(a\left(1+\tan(c+d\,x)\,1\mathrm{i}\right)\right)^{1/3}+9\left(-1\right)^{1/3}2^{1/3}a^{1/3}\right)}{a^{4/3}\,d}+\frac{\left(\frac{1}{128}\mathrm{i}\right)^{1/3}\,\ln\left(-\frac{9\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{1/3}}{16\,a^2\,d^2}-\frac{9\left(-1\right)^{1/3}2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{32\,a^{5/3}\,d^2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{4/3}\,d}-\frac{\left(\frac{1}{128}\mathrm{i}\right)^{1/3}\,\ln\left(-\frac{9\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{1/3}}{16\,a^2\,d^2}+\frac{9\left(-1\right)^{1/3}2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{32\,a^{5/3}\,d^2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{4/3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*tan(c + d*x)*1i)^(4/3),x)
```

```
[Out] (3i/(8*d) + ((a + a*tan(c + d*x)*1i)*3i)/(4*a*d))/(a + a*tan(c + d*x)*1i)^(
4/3) - ((1i/128)^(1/3)*log(9*(a*(tan(c + d*x)*1i + 1))^(1/3) + 9*(-1)^(1/3)
*2^(1/3)*a^(1/3)))/(a^(4/3)*d) + ((1i/128)^(1/3)*log(- (9*(a + a*tan(c + d*
x)*1i)^(1/3))/(16*a^2*d^2) - (9*(-1)^(1/3)*2^(1/3)*(3^(1/2)*1i - 1))/(32*a^
(5/3)*d^2))*((3^(1/2)*1i)/2 + 1/2))/(a^(4/3)*d) - ((1i/128)^(1/3)*log((9*(-
1)^(1/3)*2^(1/3)*(3^(1/2)*1i + 1))/(32*a^(5/3)*d^2) - (9*(a + a*tan(c + d*x
)*1i)^(1/3))/(16*a^2*d^2))*((3^(1/2)*1i)/2 - 1/2))/(a^(4/3)*d)
```

3.305 $\int \frac{\cot(c+dx)}{(a+ia\tan(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=313

$$-\frac{ix}{8\sqrt[3]{2}\,a^{4/3}}+\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{a^{4/3}d}-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{4\sqrt[3]{2}\,a^{4/3}d}-\text{l}$$

```
[Out] -1/16*I*x*2^(2/3)/a^(4/3)-1/16*ln(cos(d*x+c))*2^(2/3)/a^(4/3)/d-1/2*ln(tan(
d*x+c))/a^(4/3)/d+3/2*ln(a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))/a^(4/3)/d-3/16*l
n(2^(1/3)*a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))*2^(2/3)/a^(4/3)/d+arctan(1/3*(a
^(1/3)+2*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)/a^(4/3)/d-1/8*a
rctan(1/3*(a^(1/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))*3^(1/
2)*2^(2/3)/a^(4/3)/d+3/8/d/(a+I*a*tan(d*x+c))^(4/3)+9/4/a/d/(a+I*a*tan(d*x+
c))^(1/3)
```

Rubi [A]
time = 0.42, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3643, 3560, 3562, 57, 631, 210, 31, 3677, 3680}

$$\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{a^{4/3}d}-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{4\sqrt[3]{2}\,a^{4/3}d}-\frac{\log(\tan(c+dx))}{2a^{4/3}d}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{2a^{4/3}d}-\frac{3\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{8\sqrt[3]{2}\,a^{4/3}d}-\frac{\log(\cos(c+dx))}{8\sqrt[3]{2}\,a^{4/3}d}-\frac{ix}{8\sqrt[3]{2}\,a^{4/3}}+\frac{9}{4ad\sqrt[3]{a+ia\tan(c+dx)}}+\frac{3}{8d(a+ia\tan(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + I*a*Tan[c + d*x])^(4/3),x]
```

```
[Out] ((-1/8*I)*x)/(2^(1/3)*a^(4/3)) + (Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + I*a*Tan[
c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/(a^(4/3)*d) - (Sqrt[3]*ArcTan[(a^(1/3)
 + 2^(2/3)*(a + I*a*Tan[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/(4*2^(1/3)*a^(
4/3)*d) - Log[Cos[c + d*x]]/(8*2^(1/3)*a^(4/3)*d) - Log[Tan[c + d*x]]/(2*a^
(4/3)*d) + (3*Log[a^(1/3) - (a + I*a*Tan[c + d*x])^(1/3)])/(2*a^(4/3)*d) -
(3*Log[2^(1/3)*a^(1/3) - (a + I*a*Tan[c + d*x])^(1/3)])/(8*2^(1/3)*a^(4/3)*
d) + 3/(8*d*(a + I*a*Tan[c + d*x])^(4/3)) + 9/(4*a*d*(a + I*a*Tan[c + d*x])
^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
```

```
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3643

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)/((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[a/(a*c - b*d), Int[(a + b*Tan[e + f*x])^m,
x], x] - Dist[d/(a*c - b*d), Int[(a + b*Tan[e + f*x])^m*((b + a*Tan[e + f*x
])/(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+ia\tan(c+dx))^{4/3}}\,dx = i\int \frac{1}{(a+ia\tan(c+dx))^{4/3}}\,dx - \frac{i\int \frac{\cot(c+dx)(ia+a\tan(c+dx))}{(a+ia\tan(c+dx))^{4/3}}\,dx}{a}$$

$$= \frac{3}{8d(a+ia\tan(c+dx))^{4/3}} - \frac{(3i)\int \frac{\cot(c+dx)\left(\frac{8ia^2}{3}+\frac{8}{3}a^2\tan(c+dx)\right)}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx}{8a^3} + \frac{i\int \frac{\cdots}{\sqrt[3]{a+\cdots}}}{\cdots}$$

$$= \frac{3}{8d(a+ia\tan(c+dx))^{4/3}} + \frac{9}{4ad\sqrt[3]{a+ia\tan(c+dx)}} - \frac{(9i)\int \cot(c+dx)(\cdots}{\cdots}$$

$$= \frac{3}{8d(a+ia\tan(c+dx))^{4/3}} + \frac{9}{4ad\sqrt[3]{a+ia\tan(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{(a-x)\sqrt[3]{a-\cdots}}\cdots\right.}{\cdots}$$

$$= -\frac{ix}{8\sqrt[3]{2}\,a^{4/3}} - \frac{\log(\cos(c+dx))}{8\sqrt[3]{2}\,a^{4/3}d} - \frac{\log(\tan(c+dx))}{2a^{4/3}d} + \frac{3}{8d(a+ia\tan(c+dx)\cdots}$$

$$= -\frac{ix}{8\sqrt[3]{2}\,a^{4/3}} - \frac{\log(\cos(c+dx))}{8\sqrt[3]{2}\,a^{4/3}d} - \frac{\log(\tan(c+dx))}{2a^{4/3}d} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+ia\cdots}\right.}{2a^{4/3}d}$$

$$= -\frac{ix}{8\sqrt[3]{2}\,a^{4/3}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}d} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\cdots}{\cdots}}{\cdots}\right)}{4\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.69, size = 189, normalized size = 0.60

$$-\frac{3i\sec^2(c+dx)\left(7+7\cos(2(c+dx))+{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)(\cos(2(c+dx))+i\sin(2(c+dx)))-8\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{2e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)(\cos(2(c+dx))+i\sin(2(c+dx)))+6i\sin(2(c+dx))\right)}{16ad(-i+\tan(c+dx))\sqrt[3]{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + I*a*Tan[c + d*x])^(4/3),x]
```

```
[Out] (((-3*I)/16)*Sec[c + d*x]^2*(7 + 7*Cos[2*(c + d*x)] + Hypergeometric2F1[2/3
, 1, 5/3, E^((2*I)*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(Cos[2*(c + d*x)]
+ I*Sin[2*(c + d*x)]) - 8*Hypergeometric2F1[2/3, 1, 5/3, (2*E^((2*I)*(c + d
*x)))/(1 + E^((2*I)*(c + d*x)))]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) +
(6*I)*Sin[2*(c + d*x)]))/(a*d*(-I + Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(1
/3))
```

Maple [**F**]
time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{\cot (dx + c)}{(a + ia\tan (dx + c))^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+I*a*tan(d*x+c))^(4/3),x)
```

```
[Out] int(cot(d*x+c)/(a+I*a*tan(d*x+c))^(4/3),x)
```

Maxima [**A**]
time = 0.60, size = 265, normalized size = 0.85

$$-\frac{\frac{2\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}}-\frac{2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(i\,a\tan(dx+c)+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{2}{3}}\right)}{a^{\frac{4}{3}}}+\frac{2\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)}{a^{\frac{4}{3}}}-\frac{16\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}}+\frac{8\log\left((i\,a\tan(dx+c)+a)^{\frac{2}{3}}+(i\,a\tan(dx+c)+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{4}{3}}}-\frac{16\log\left((i\,a\tan(dx+c)+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{4}{3}}}-\frac{6\,(6i\,a\tan(dx+c)+7\,a)}{(i\,a\tan(dx+c)+a)^{\frac{4}{3}}a}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] -1/16*(2*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(I
*a*tan(d*x + c) + a)^(1/3))/a^(1/3))/a^(4/3) - 2^(2/3)*log(2^(2/3)*a^(2/3)
+ 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x + c) + a)^(2/
3))/a^(4/3) + 2*2^(2/3)*log(-2^(1/3)*a^(1/3) + (I*a*tan(d*x + c) + a)^(1/3)
)/a^(4/3) - 16*sqrt(3)*arctan(1/3*sqrt(3)*(2*(I*a*tan(d*x + c) + a)^(1/3) +
 a^(1/3))/a^(1/3))/a^(4/3) + 8*log((I*a*tan(d*x + c) + a)^(2/3) + (I*a*tan(
d*x + c) + a)^(1/3)*a^(1/3) + a^(2/3))/a^(4/3) - 16*log((I*a*tan(d*x + c) +
 a)^(1/3) - a^(1/3))/a^(4/3) - 6*(6*I*a*tan(d*x + c) + 7*a)/((I*a*tan(d*x +
 c) + a)^(4/3)*a))/d
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 633 vs. 2(227) = 454.
time = 1.47, size = 633, normalized size = 2.02

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] 1/32*(8*(1/2)^(1/3)*a^2*d*(-1/(a^4*d^3))^(1/3)*e^(4*I*d*x + 4*I*c)*log(-2*(
1/2)^(2/3)*a^3*d^2*(-1/(a^4*d^3))^(2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) +
 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)) + 32*a^2*d*(1/(a^4*d^3))^(1/3)*e^(4*I*d
*x + 4*I*c)*log(-a^3*d^2*(1/(a^4*d^3))^(2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I
*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)) - 4*(1/2)^(1/3)*(I*sqrt(3)*a^2*d +
 a^2*d)*(-1/(a^4*d^3))^(1/3)*e^(4*I*d*x + 4*I*c)*log(-(1/2)^(2/3)*(I*sqrt(3
)*a^3*d^2 - a^3*d^2)*(-1/(a^4*d^3))^(2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c)
 + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)) - 4*(1/2)^(1/3)*(-I*sqrt(3)*a^2*d + a
^2*d)*(-1/(a^4*d^3))^(1/3)*e^(4*I*d*x + 4*I*c)*log(-(1/2)^(2/3)*(-I*sqrt(3)
*a^3*d^2 - a^3*d^2)*(-1/(a^4*d^3))^(2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c)
+ 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)) + 3*2^(2/3)*(a/(e^(2*I*d*x + 2*I*c) +
1))^(2/3)*(13*e^(4*I*d*x + 4*I*c) + 14*e^(2*I*d*x + 2*I*c) + 1)*e^(4/3*I*d*
x + 4/3*I*c) - 16*(-I*sqrt(3)*a^2*d + a^2*d)*(1/(a^4*d^3))^(1/3)*e^(4*I*d*x
 + 4*I*c)*log(1/2*(I*sqrt(3)*a^3*d^2 + a^3*d^2)*(1/(a^4*d^3))^(2/3) + 2^(1/
3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)) - 16*(I*sqr
t(3)*a^2*d + a^2*d)*(1/(a^4*d^3))^(1/3)*e^(4*I*d*x + 4*I*c)*log(1/2*(-I*sqr
t(3)*a^3*d^2 + a^3*d^2)*(1/(a^4*d^3))^(2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*
c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)))*e^(-4*I*d*x - 4*I*c)/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot (c + dx)}{(ia (\tan (c + dx) - i))^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c))**(4/3),x)
```

```
[Out] Integral(cot(c + d*x)/(I*a*(tan(c + d*x) - I))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+I*a*tan(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)/(I*a*tan(d*x + c) + a)^(4/3), x)
```

Mupad [B]
time = 3.98, size = 822, normalized size = 2.63

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + a*tan(c + d*x)*1i)^(4/3),x)
```

```
[Out] log(((382205952*a^16*d^9*(1/(a^4*d^3))^(2/3) - 258785280*a^13*d^7*(a + a*ta
n(c + d*x)*1i)^(1/3))*(1/(a^4*d^3))^(1/3) - 125411328*a^12*d^6)*(1/(a^4*d^3
))^(2/3) + 1990656*a^9*d^4*(a + a*tan(c + d*x)*1i)^(1/3))*(1/(a^4*d^3))^(1/
3) + log(((382205952*a^16*d^9*(-1/(128*a^4*d^3))^(2/3) - 258785280*a^13*d^7
*(a + a*tan(c + d*x)*1i)^(1/3))*(-1/(128*a^4*d^3))^(1/3) - 125411328*a^12*d
^6)*(-1/(128*a^4*d^3))^(2/3) + 1990656*a^9*d^4*(a + a*tan(c + d*x)*1i)^(1/3
))*(-1/(128*a^4*d^3))^(1/3) + (log(1990656*a^9*d^4*(a + a*tan(c + d*x)*1i)^
(1/3) - ((3^(1/2)*1i - 1)^2*(125411328*a^12*d^6 + ((3^(1/2)*1i - 1)*(258785
280*a^13*d^7*(a + a*tan(c + d*x)*1i)^(1/3) - 95551488*a^16*d^9*(3^(1/2)*1i
- 1)^2*(1/(a^4*d^3))^(2/3))*(1/(a^4*d^3))^(1/3))/2)*(1/(a^4*d^3))^(2/3))/4)
*(3^(1/2)*1i - 1)*(1/(a^4*d^3))^(1/3))/2 - (log(1990656*a^9*d^4*(a + a*tan(
c + d*x)*1i)^(1/3) - ((3^(1/2)*1i + 1)^2*(125411328*a^12*d^6 - ((3^(1/2)*1i
 + 1)*(258785280*a^13*d^7*(a + a*tan(c + d*x)*1i)^(1/3) - 95551488*a^16*d^9
*(3^(1/2)*1i + 1)^2*(1/(a^4*d^3))^(2/3))*(1/(a^4*d^3))^(1/3))/2)*(1/(a^4*d^
3))^(2/3))/4)*(3^(1/2)*1i + 1)*(1/(a^4*d^3))^(1/3))/2 + log(1990656*a^9*d^4
*(a + a*tan(c + d*x)*1i)^(1/3) - ((3^(1/2)*1i)/2 - 1/2)^2*(125411328*a^12*d
^6 + ((3^(1/2)*1i)/2 - 1/2)*(258785280*a^13*d^7*(a + a*tan(c + d*x)*1i)^(1/
3) - 382205952*a^16*d^9*((3^(1/2)*1i)/2 - 1/2)^2*(-1/(128*a^4*d^3))^(2/3))*
(-1/(128*a^4*d^3))^(1/3))*(-1/(128*a^4*d^3))^(2/3))*((3^(1/2)*1i)/2 - 1/2)*
(-1/(128*a^4*d^3))^(1/3) - log(1990656*a^9*d^4*(a + a*tan(c + d*x)*1i)^(1/3
) - ((3^(1/2)*1i)/2 + 1/2)^2*(125411328*a^12*d^6 - ((3^(1/2)*1i)/2 + 1/2)*(
258785280*a^13*d^7*(a + a*tan(c + d*x)*1i)^(1/3) - 382205952*a^16*d^9*((3^(
1/2)*1i)/2 + 1/2)^2*(-1/(128*a^4*d^3))^(2/3))*(-1/(128*a^4*d^3))^(1/3))*(-1
/(128*a^4*d^3))^(2/3))*((3^(1/2)*1i)/2 + 1/2)*(-1/(128*a^4*d^3))^(1/3) + ((
9*(a + a*tan(c + d*x)*1i))/(4*a) + 3/8)/(d*(a + a*tan(c + d*x)*1i)^(4/3))
```

3.306 $\int \frac{\cot^2(c+dx)}{(a+ia\tan(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=354

$$\frac{x}{8\sqrt[3]{2}\,a^{4/3}}-\frac{4i\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}d}-\frac{i\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{4\sqrt[3]{2}\,a^{4/3}d}-\frac{i\log}{8}$$

```
[Out] 1/16*x*2^(2/3)/a^(4/3)-1/16*I*ln(cos(d*x+c))*2^(2/3)/a^(4/3)/d+2/3*I*ln(tan
(d*x+c))/a^(4/3)/d-2*I*ln(a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))/a^(4/3)/d-3/16*
I*ln(2^(1/3)*a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))*2^(2/3)/a^(4/3)/d-4/3*I*arct
an(1/3*(a^(1/3)+2*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))/a^(4/3)/d*3^(1
/2)-1/8*I*arctan(1/3*(a^(1/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(
1/2))*3^(1/2)*2^(2/3)/a^(4/3)/d-11/8*I/d/(a+I*a*tan(d*x+c))^(4/3)-cot(d*x+c
)/d/(a+I*a*tan(d*x+c))^(4/3)-19/4*I/a/d/(a+I*a*tan(d*x+c))^(1/3)
```

Rubi [A]
time = 0.51, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3642, 3677, 3681, 3562, 57, 631, 210, 31, 3680}

$$-\frac{4i\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}d}-\frac{i\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{4\sqrt[3]{2}\,a^{4/3}d}+\frac{2i\log(\tan(c+dx))}{3a^{4/3}d}-\frac{2i\log\left(\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{a^{4/3}d}-\frac{3i\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{8\sqrt[3]{2}\,a^{4/3}d}-\frac{i\log(\cos(c+dx))}{8\sqrt[3]{2}\,a^{4/3}d}+\frac{x}{8\sqrt[3]{2}\,a^{4/3}}-\frac{19i}{4ad\sqrt[3]{a+ia\tan(c+dx)}}-\frac{11i}{8d(a+ia\tan(c+dx))^{4/3}}-\frac{\cot(c+dx)}{d(a+ia\tan(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + I*a*Tan[c + d*x])^(4/3),x]
```

```
[Out] x/(8*2^(1/3)*a^(4/3)) - ((4*I)*ArcTan[(a^(1/3) + 2*(a + I*a*Tan[c + d*x])^(
1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(4/3)*d) - ((I/4)*Sqrt[3]*ArcTan[(a^(1
/3) + 2^(2/3)*(a + I*a*Tan[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/(2^(1/3)*a^
(4/3)*d) - ((I/8)*Log[Cos[c + d*x]])/(2^(1/3)*a^(4/3)*d) + (((2*I)/3)*Log[T
an[c + d*x]])/(a^(4/3)*d) - ((2*I)*Log[a^(1/3) - (a + I*a*Tan[c + d*x])^(1/
3)])/(a^(4/3)*d) - (((3*I)/8)*Log[2^(1/3)*a^(1/3) - (a + I*a*Tan[c + d*x])^
(1/3)])/(2^(1/3)*a^(4/3)*d) - ((11*I)/8)/(d*(a + I*a*Tan[c + d*x])^(4/3)) -
 Cot[c + d*x]/(d*(a + I*a*Tan[c + d*x])^(4/3)) - ((19*I)/4)/(a*d*(a + I*a*T
an[c + d*x])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
```

```
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3642

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[d*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(a*(c^2 + d^2)*(n +
1)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[b*d*m - a
*c*(n + 1) + a*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
 e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0
] && LtQ[n, -1] && (IntegerQ[n] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3681

```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(
A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+ia\tan(c+dx))^{4/3}}\,dx = -\frac{\cot(c+dx)}{d(a+ia\tan(c+dx))^{4/3}} + \frac{\int \frac{\cot(c+dx)\left(-\frac{4ia}{3}-\frac{7}{3}a\tan(c+dx)\right)}{(a+ia\tan(c+dx))^{4/3}}\,dx}{a}$$

$$= -\frac{11i}{8d(a+ia\tan(c+dx))^{4/3}} - \frac{\cot(c+dx)}{d(a+ia\tan(c+dx))^{4/3}} + \frac{3\int \frac{\cot(c+dx)\left(-\frac{32ia^{}}{9}\right.}{\sqrt[3]{a+ia\,\mathrm{t}}}}{8}$$

$$= -\frac{11i}{8d(a+ia\tan(c+dx))^{4/3}} - \frac{\cot(c+dx)}{d(a+ia\tan(c+dx))^{4/3}} - \frac{19i}{4ad\sqrt[3]{a+ia\tan(}}$$

$$= -\frac{11i}{8d(a+ia\tan(c+dx))^{4/3}} - \frac{\cot(c+dx)}{d(a+ia\tan(c+dx))^{4/3}} - \frac{19i}{4ad\sqrt[3]{a+ia\tan(}}$$

$$= -\frac{11i}{8d(a+ia\tan(c+dx))^{4/3}} - \frac{\cot(c+dx)}{d(a+ia\tan(c+dx))^{4/3}} - \frac{19i}{4ad\sqrt[3]{a+ia\tan(}}$$

$$= \frac{x}{8\sqrt[3]{2}\,a^{4/3}} - \frac{i\log(\cos(c+dx))}{8\sqrt[3]{2}\,a^{4/3}d} + \frac{2i\log(\tan(c+dx))}{3a^{4/3}d} - \frac{11i}{8d(a+ia\tan(c+d}$$

$$= \frac{x}{8\sqrt[3]{2}\,a^{4/3}} - \frac{i\log(\cos(c+dx))}{8\sqrt[3]{2}\,a^{4/3}d} + \frac{2i\log(\tan(c+dx))}{3a^{4/3}d} - \frac{2i\log\left(\sqrt[3]{a}-\sqrt[3]{a}\right.}{a^{4/}}$$

$$= \frac{x}{8\sqrt[3]{2}\,a^{4/3}} - \frac{4i\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{4/3}d} - \frac{i\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{}}{}}{}\right.}{4\sqrt[3]{2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 2.07, size = 233, normalized size = 0.66

$$\frac{3+63e^{2i(c+dx)}-35e^{4i(c+dx)}-95e^{6i(c+dx)}+6e^{4i(c+dx)}\left(-1+e^{2i(c+dx)}\right)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)+64e^{4i(c+dx)}\left(-1+e^{2i(c+dx)}\right)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{2e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)}{8ad\left(-1+e^{2i(c+dx)}\right)\left(1+e^{2i(c+dx)}\right)^2\left(-i+\tan(c+dx)\right)\sqrt[3]{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + I*a*Tan[c + d*x])^(4/3),x]
```

```
[Out] (3 + 63*E^((2*I)*(c + d*x)) - 35*E^((4*I)*(c + d*x)) - 95*E^((6*I)*(c + d*x
)) + 6*E^((4*I)*(c + d*x))*(-1 + E^((2*I)*(c + d*x)))*Hypergeometric2F1[2/3
, 1, 5/3, E^((2*I)*(c + d*x))/(1 + E^((2*I)*(c + d*x)))] + 64*E^((4*I)*(c +
```

```
d*x))*(-1 + E^((2*I)*(c + d*x)))*Hypergeometric2F1[2/3, 1, 5/3, (2*E^((2*I
)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))])/(8*a*d*(-1 + E^((2*I)*(c + d*x)))
*(1 + E^((2*I)*(c + d*x)))^2*(-I + Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(1/
3))
```

Maple [F]
time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(dx+c)}{(a+ia\tan(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+I*a*tan(d*x+c))^(4/3),x)
```

```
[Out] int(cot(d*x+c)^2/(a+I*a*tan(d*x+c))^(4/3),x)
```

Maxima [A]
time = 0.53, size = 310, normalized size = 0.88

$$-\frac{ia\left(\frac{6\left(38\,(ia\tan(dx+c)+a)^2-27\,(ia\tan(dx+c)+a)a-3\,a^2\right)}{(ia\tan(dx+c)+a)^{\frac{7}{3}}a^2-(ia\tan(dx+c)+a)^{\frac{4}{3}}a^3}+\frac{6\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(ia\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{a^{\frac{7}{3}}}-\frac{3\cdot2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(ia\tan(dx+c)+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(ia\tan(dx+c)+a)^{\frac{2}{3}}\right)}{a^{\frac{7}{3}}}+\frac{6\cdot2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(ia\tan(dx+c)+a)^{\frac{1}{3}}\right)}{a^{\frac{7}{3}}}+\frac{64\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(ia\tan(dx+c)+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{7}{3}}}-\frac{32\log\left((ia\tan(dx+c)+a)^{\frac{2}{3}}+(ia\tan(dx+c)+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{7}{3}}}+\frac{64\log\left((ia\tan(dx+c)+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{7}{3}}}\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+I*a*tan(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] -1/48*I*a*(6*(38*(I*a*tan(d*x + c) + a)^2 - 27*(I*a*tan(d*x + c) + a)*a - 3
*a^2)/((I*a*tan(d*x + c) + a)^(7/3)*a^2 - (I*a*tan(d*x + c) + a)^(4/3)*a^3)
 + 6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(I*a*t
an(d*x + c) + a)^(1/3))/a^(1/3))/a^(7/3) - 3*2^(2/3)*log(2^(2/3)*a^(2/3) +
2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x + c) + a)^(2/3)
)/a^(7/3) + 6*2^(2/3)*log(-2^(1/3)*a^(1/3) + (I*a*tan(d*x + c) + a)^(1/3))/
a^(7/3) + 64*sqrt(3)*arctan(1/3*sqrt(3)*(2*(I*a*tan(d*x + c) + a)^(1/3) + a
^(1/3))/a^(1/3))/a^(7/3) - 32*log((I*a*tan(d*x + c) + a)^(2/3) + (I*a*tan(d
*x + c) + a)^(1/3)*a^(1/3) + a^(2/3))/a^(7/3) + 64*log((I*a*tan(d*x + c) +
a)^(1/3) - a^(1/3))/a^(7/3))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 792 vs. 2(252) = 504.
time = 1.59, size = 792, normalized size = 2.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+I*a*tan(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] 1/32*(2^(2/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(-95*I*e^(6*I*d*x + 6*I*c
) - 35*I*e^(4*I*d*x + 4*I*c) + 63*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(4/3*I*d*x
```

```
 + 4/3*I*c) + 32*(a^2*d*e^(6*I*d*x + 6*I*c) - a^2*d*e^(4*I*d*x + 4*I*c))*(6
4/27*I/(a^4*d^3))^(1/3)*log(9/16*a^3*d^2*(64/27*I/(a^4*d^3))^(2/3) + 2^(1/3
)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)) + 32*(a^2*d*
e^(6*I*d*x + 6*I*c) - a^2*d*e^(4*I*d*x + 4*I*c))*(1/128*I/(a^4*d^3))^(1/3)*
log(32*a^3*d^2*(1/128*I/(a^4*d^3))^(2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c)
+ 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)) - 16*((-I*sqrt(3)*a^2*d + a^2*d)*e^(6*
I*d*x + 6*I*c) + (I*sqrt(3)*a^2*d - a^2*d)*e^(4*I*d*x + 4*I*c))*(64/27*I/(a
^4*d^3))^(1/3)*log(-9/32*(I*sqrt(3)*a^3*d^2 + a^3*d^2)*(64/27*I/(a^4*d^3))^
(2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)
) - 16*((I*sqrt(3)*a^2*d + a^2*d)*e^(6*I*d*x + 6*I*c) + (-I*sqrt(3)*a^2*d -
 a^2*d)*e^(4*I*d*x + 4*I*c))*(64/27*I/(a^4*d^3))^(1/3)*log(-9/32*(-I*sqrt(3
)*a^3*d^2 + a^3*d^2)*(64/27*I/(a^4*d^3))^(2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2
*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)) - 16*((-I*sqrt(3)*a^2*d + a^2*d)
*e^(6*I*d*x + 6*I*c) + (I*sqrt(3)*a^2*d - a^2*d)*e^(4*I*d*x + 4*I*c))*(1/12
8*I/(a^4*d^3))^(1/3)*log(-16*(I*sqrt(3)*a^3*d^2 + a^3*d^2)*(1/128*I/(a^4*d^
3))^(2/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*
I*c)) - 16*((I*sqrt(3)*a^2*d + a^2*d)*e^(6*I*d*x + 6*I*c) + (-I*sqrt(3)*a^2
*d - a^2*d)*e^(4*I*d*x + 4*I*c))*(1/128*I/(a^4*d^3))^(1/3)*log(-16*(-I*sqrt
(3)*a^3*d^2 + a^3*d^2)*(1/128*I/(a^4*d^3))^(2/3) + 2^(1/3)*(a/(e^(2*I*d*x +
 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)))/(a^2*d*e^(6*I*d*x + 6*I*c) -
a^2*d*e^(4*I*d*x + 4*I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+I*a*tan(d*x+c))**(4/3),x)
[Out] Integral(cot(c + d*x)**2/(I*a*(tan(c + d*x) - I))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+I*a*tan(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate(cot(d*x + c)^2/(I*a*tan(d*x + c) + a)^(4/3), x)
```

Mupad [B]
time = 5.99, size = 893, normalized size = 2.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(4/3),x)
```

```
[Out] log(d*(a + a*tan(c + d*x)*1i)^(1/3)*1584i - ((46656*a^7*d^6*(64i/(27*a^4*d^
3))^(2/3) + 55782*a^4*d^4*(a + a*tan(c + d*x)*1i)^(1/3))*(64i/(27*a^4*d^3))
^(1/3) - a^3*d^3*37107i)*(64i/(27*a^4*d^3))^(2/3))*(64i/(27*a^4*d^3))^(1/3)
 - (((a + a*tan(c + d*x)*1i)*27i)/(8*d) + (a*3i)/(8*d) - ((a + a*tan(c + d*
x)*1i)^2*19i)/(4*a*d))/(a*(a + a*tan(c + d*x)*1i)^(4/3) - (a + a*tan(c + d*
x)*1i)^(7/3)) + log(d*(a + a*tan(c + d*x)*1i)^(1/3)*1584i - ((46656*a^7*d^6
*(1i/(128*a^4*d^3))^(2/3) + 55782*a^4*d^4*(a + a*tan(c + d*x)*1i)^(1/3))*(1
i/(128*a^4*d^3))^(1/3) - a^3*d^3*37107i)*(1i/(128*a^4*d^3))^(2/3))*(1i/(128
*a^4*d^3))^(1/3) + (log(d*(a + a*tan(c + d*x)*1i)^(1/3)*1584i + ((3^(1/2)*1
i - 1)^2*(a^3*d^3*37107i - ((3^(1/2)*1i - 1)*(55782*a^4*d^4*(a + a*tan(c +
d*x)*1i)^(1/3) + 11664*a^7*d^6*(3^(1/2)*1i - 1)^2*(64i/(27*a^4*d^3))^(2/3))
*(64i/(27*a^4*d^3))^(1/3))/2)*(64i/(27*a^4*d^3))^(2/3))/4)*(3^(1/2)*1i - 1)
*(64i/(27*a^4*d^3))^(1/3))/2 - (log(d*(a + a*tan(c + d*x)*1i)^(1/3)*1584i +
 ((3^(1/2)*1i + 1)^2*(a^3*d^3*37107i + ((3^(1/2)*1i + 1)*(55782*a^4*d^4*(a
+ a*tan(c + d*x)*1i)^(1/3) + 11664*a^7*d^6*(3^(1/2)*1i + 1)^2*(64i/(27*a^4*
d^3))^(2/3))*(64i/(27*a^4*d^3))^(1/3))/2)*(64i/(27*a^4*d^3))^(2/3))/4)*(3^(
1/2)*1i + 1)*(64i/(27*a^4*d^3))^(1/3))/2 + (log(d*(a + a*tan(c + d*x)*1i)^(
1/3)*1584i + ((3^(1/2)*1i - 1)^2*(a^3*d^3*37107i - ((3^(1/2)*1i - 1)*(55782
*a^4*d^4*(a + a*tan(c + d*x)*1i)^(1/3) + 11664*a^7*d^6*(3^(1/2)*1i - 1)^2*(
1i/(128*a^4*d^3))^(2/3))*(1i/(128*a^4*d^3))^(1/3))/2)*(1i/(128*a^4*d^3))^(2
/3))/4)*(3^(1/2)*1i - 1)*(1i/(128*a^4*d^3))^(1/3))/2 - (log(d*(a + a*tan(c
+ d*x)*1i)^(1/3)*1584i + ((3^(1/2)*1i + 1)^2*(a^3*d^3*37107i + ((3^(1/2)*1i
 + 1)*(55782*a^4*d^4*(a + a*tan(c + d*x)*1i)^(1/3) + 11664*a^7*d^6*(3^(1/2)
*1i + 1)^2*(1i/(128*a^4*d^3))^(2/3))*(1i/(128*a^4*d^3))^(1/3))/2)*(1i/(128*
a^4*d^3))^(2/3))/4)*(3^(1/2)*1i + 1)*(1i/(128*a^4*d^3))^(1/3))/2
```

3.307 $\int \frac{1}{(a+ia\tan(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=213

$$-\frac{x}{8\ 2^{2/3}a^{5/3}}-\frac{i\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{4\ 2^{2/3}a^{5/3}d}+\frac{i\log(\cos(c+dx))}{8\ 2^{2/3}a^{5/3}d}+\frac{3i\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+\cdots}\right.}{8\ 2^{2/3}a^{5/\cdots}}$$

```
[Out] -1/16*x*2^(1/3)/a^(5/3)+1/16*I*ln(cos(d*x+c))*2^(1/3)/a^(5/3)/d+3/16*I*ln(2
^(1/3)*a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))*2^(1/3)/a^(5/3)/d-1/8*I*arctan(1/3
*(a^(1/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)*2^(1/3
)/a^(5/3)/d+3/10*I/d/(a+I*a*tan(d*x+c))^(5/3)+3/8*I/a/d/(a+I*a*tan(d*x+c))^
(2/3)
```

Rubi [A]

time = 0.11, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3560, 3562, 59, 631, 210, 31}

$$-\frac{i\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{4\ 2^{2/3}a^{5/3}d}+\frac{3i\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{8\ 2^{2/3}a^{5/3}d}+\frac{i\log(\cos(c+dx))}{8\ 2^{2/3}a^{5/3}d}-\frac{x}{8\ 2^{2/3}a^{5/3}}+\frac{3i}{8ad(a+ia\tan(c+dx))^{2/3}}+\frac{3i}{10d(a+ia\tan(c+dx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(-5/3),x]
```

```
[Out] -1/8*x/(2^(2/3)*a^(5/3)) - ((I/4)*Sqrt[3]*ArcTan[(a^(1/3) + 2^(2/3)*(a + I*
a*Tan[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))])/(2^(2/3)*a^(5/3)*d) + ((I/8)*Log
[Cos[c + d*x]])/(2^(2/3)*a^(5/3)*d) + (((3*I)/8)*Log[2^(1/3)*a^(1/3) - (a +
 I*a*Tan[c + d*x])^(1/3)])/(2^(2/3)*a^(5/3)*d) + ((3*I)/10)/(d*(a + I*a*Tan
[c + d*x])^(5/3)) + ((3*I)/8)/(a*d*(a + I*a*Tan[c + d*x])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(c+dx))^{5/3}}\,dx = \frac{3i}{10d(a+ia\tan(c+dx))^{5/3}} + \frac{\int \frac{1}{(a+ia\tan(c+dx))^{2/3}}\,dx}{2a}$$

$$= \frac{3i}{10d(a+ia\tan(c+dx))^{5/3}} + \frac{3i}{8ad(a+ia\tan(c+dx))^{2/3}} + \frac{\int \sqrt[3]{a+ia\tan(}{4a^2}$$

$$= \frac{3i}{10d(a+ia\tan(c+dx))^{5/3}} + \frac{3i}{8ad(a+ia\tan(c+dx))^{2/3}} - \frac{i\text{Subst}\Big(\int \frac{}{(a-x)(}}{}$$

$$= -\frac{x}{8\ 2^{2/3}a^{5/3}} + \frac{i\log(\cos(c+dx))}{8\ 2^{2/3}a^{5/3}d} + \frac{3i}{10d(a+ia\tan(c+dx))^{5/3}} + \frac{}{8ad(a+i}$$

$$= -\frac{x}{8\ 2^{2/3}a^{5/3}} + \frac{i\log(\cos(c+dx))}{8\ 2^{2/3}a^{5/3}d} + \frac{3i\log\left(\sqrt[3]{2}\sqrt[3]{a} - \sqrt[3]{a+ia\tan(c+dx)}\right.}{8\ 2^{2/3}a^{5/3}d}$$

$$= -\frac{x}{8\ 2^{2/3}a^{5/3}} - \frac{i\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{4\ 2^{2/3}a^{5/3}d} + \frac{i\log(\cos(c+}{8\ 2^{2/3}a^{5/3}}$$

Mathematica [A]

time = 1.54, size = 331, normalized size = 1.55

$$\frac{ie^{-2i(c+dx)}\left(6+27e^{2i(c+dx)}+21e^{4i(c+dx)}-10\sqrt{3}e^{\frac{10}{3}i(c+dx)}\sqrt[3]{1+e^{2i(c+dx)}}\text{ArcTan}\left(\frac{1+\frac{2e^{\frac{2}{3}i(c+dx)}}{\sqrt[3]{1+e^{2i(c+dx)}}}}{\sqrt{3}}\right)+10e^{\frac{10}{3}i(c+dx)}\sqrt[3]{1+e^{2i(c+dx)}}\log\left(1-\frac{e^{\frac{2}{3}i(c+dx)}}{\sqrt[3]{1+e^{2i(c+dx)}}}\right)-5e^{\frac{10}{3}i(c+dx)}\sqrt[3]{1+e^{2i(c+dx)}}\log\left(\frac{e^{\frac{4}{3}i(c+dx)}+e^{\frac{2}{3}i(c+dx)}\sqrt[3]{1+e^{2i(c+dx)}}+\left(1+e^{2i(c+dx)}\right)^{2/3}}{\left(1+e^{2i(c+dx)}\right)^{2/3}}\right)\right)\sec^2(c+dx)}{80d(a+ia\tan(c+dx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(-5/3),x]
```

```
[Out] ((I/80)*(6 + 27*E^((2*I)*(c + d*x)) + 21*E^((4*I)*(c + d*x)) - 10*Sqrt[3]*E
^(((10*I)/3)*(c + d*x))*(1 + E^((2*I)*(c + d*x)))^(1/3)*ArcTan[(1 + (2*E^((
(2*I)/3)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))^(1/3))/Sqrt[3]] + 10*E^(((10
*I)/3)*(c + d*x))*(1 + E^((2*I)*(c + d*x)))^(1/3)*Log[1 - E^(((2*I)/3)*(c +
 d*x))/(1 + E^((2*I)*(c + d*x)))^(1/3)] - 5*E^(((10*I)/3)*(c + d*x))*(1 + E
^((2*I)*(c + d*x)))^(1/3)*Log[(E^(((4*I)/3)*(c + d*x)) + E^(((2*I)/3)*(c +
d*x))*(1 + E^((2*I)*(c + d*x)))^(1/3) + (1 + E^((2*I)*(c + d*x)))^(2/3))/(1
 + E^((2*I)*(c + d*x)))^(2/3)])*Sec[c + d*x]^2)/(d*E^((2*I)*(c + d*x))*(a +
 I*a*Tan[c + d*x])^(5/3))
```

Maple [A]

time = 0.10, size = 177, normalized size = 0.83

method	result
derivativedivides	$\frac{3ia\left(\frac{\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{12a^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}\sqrt{3}\arctan\left(\sqrt{\right.}}{}}{4a^2}\right.}{d}$
default	$\frac{3ia\left(\frac{\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{12a^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}\sqrt{3}\arctan\left(\sqrt{\right.}}{}}{4a^2}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(d*x+c))^(5/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3*I/d*a*(1/4*(1/6*2^(1/3)/a^(2/3)*ln((a+I*a*tan(d*x+c))^(1/3)-2^(1/3)*a^(1/
3))-1/12*2^(1/3)/a^(2/3)*ln((a+I*a*tan(d*x+c))^(2/3)+2^(1/3)*a^(1/3)*(a+I*a
*tan(d*x+c))^(1/3)+2^(2/3)*a^(2/3))-1/6*2^(1/3)/a^(2/3)*3^(1/2)*arctan(1/3*
3^(1/2)*(2^(2/3)/a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+1)))/a^2+1/8/a^2/(a+I*a*t
an(d*x+c))^(2/3)+1/10/a/(a+I*a*tan(d*x+c))^(5/3))
```

Maxima [A]

time = 0.51, size = 164, normalized size = 0.77

$$-\frac{i\left(\frac{10\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(ia\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}+\frac{5\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(ia\tan(dx+c)+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(ia\tan(dx+c)+a)^{\frac{2}{3}}\right)}{a^{\frac{2}{3}}}-\frac{10\cdot 2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(ia\tan(dx+c)+a)^{\frac{1}{3}}\right)}{a^{\frac{2}{3}}}-\frac{6\,(5ia\tan(dx+c)+9a)}{(ia\tan(dx+c)+a)^{\frac{5}{3}}}\right)}{80\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] -1/80*I*(10*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2
*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3))/a^(2/3) + 5*2^(1/3)*log(2^(2/3)*a^(
2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x + c) + a
)^(2/3))/a^(2/3) - 10*2^(1/3)*log(-2^(1/3)*a^(1/3) + (I*a*tan(d*x + c) + a)
^(1/3))/a^(2/3) - 6*(5*I*a*tan(d*x + c) + 9*a)/(I*a*tan(d*x + c) + a)^(5/3)
)/(a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 336 vs. 2(146) = 292.
time = 0.97, size = 336, normalized size = 1.58

$$\frac{\left(80\,a^2 d\left(-\frac{i}{256\,a^5 d^3}\right)^{\frac{1}{3}} e^{(4i\,dx+4i\,c)} \log\left(8i\,a^2 d\left(-\frac{i}{256\,a^5 d^3}\right)^{\frac{1}{3}} + 2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}} e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right) - 40\left(-i\sqrt{3}\,a^2 d + a^2 d\right)\left(-\frac{i}{256\,a^5 d^3}\right)^{\frac{1}{3}} e^{(4i\,dx+4i\,c)} \log\left(2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}} e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)} - 4\left(\sqrt{3}\,a^2 d + i\,a^2 d\right)\left(-\frac{i}{256\,a^5 d^3}\right)^{\frac{1}{3}}\right) - 40\left(i\sqrt{3}\,a^2 d + a^2 d\right)\left(-\frac{i}{256\,a^5 d^3}\right)^{\frac{1}{3}} e^{(4i\,dx+4i\,c)} \log\left(2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}} e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)} + 4\left(\sqrt{3}\,a^2 d - i\,a^2 d\right)\left(-\frac{i}{256\,a^5 d^3}\right)^{\frac{1}{3}}\right) - 3\cdot 2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}\left(-7i\,e^{(4i\,dx+4i\,c)} - 9i\,e^{(2i\,dx+2i\,c)} - 2i\right)e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)}\right)e^{(-4i\,dx-4i\,c)}}{80\,a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] 1/80*(80*a^2*d*(-1/256*I/(a^5*d^3))^(1/3)*e^(4*I*d*x + 4*I*c)*log(8*I*a^2*d
*(-1/256*I/(a^5*d^3))^(1/3) + 2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e
^(2/3*I*d*x + 2/3*I*c)) - 40*(-I*sqrt(3)*a^2*d + a^2*d)*(-1/256*I/(a^5*d^3)
)^(1/3)*e^(4*I*d*x + 4*I*c)*log(2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)
*e^(2/3*I*d*x + 2/3*I*c) - 4*(sqrt(3)*a^2*d + I*a^2*d)*(-1/256*I/(a^5*d^3))
^(1/3)) - 40*(I*sqrt(3)*a^2*d + a^2*d)*(-1/256*I/(a^5*d^3))^(1/3)*e^(4*I*d*
x + 4*I*c)*log(2^(1/3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2
/3*I*c) + 4*(sqrt(3)*a^2*d - I*a^2*d)*(-1/256*I/(a^5*d^3))^(1/3)) - 3*2^(1/
3)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*(-7*I*e^(4*I*d*x + 4*I*c) - 9*I*e^(2
*I*d*x + 2*I*c) - 2*I)*e^(2/3*I*d*x + 2/3*I*c))*e^(-4*I*d*x - 4*I*c)/(a^2*d
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\tan\left(c+dx\right)+a\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))**(5/3),x)
```

```
[Out] Integral((I*a*tan(c + d*x) + a)**(-5/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(-5/3), x)
```

Mupad [B]
time = 3.99, size = 233, normalized size = 1.09

$$\frac{\frac{3\mathrm{i}}{10\,d}+\frac{(a+a\tan(c+d\,x)\,1\mathrm{i})\,3\mathrm{i}}{8\,a\,d}}{(a+a\tan(c+d\,x)\,1\mathrm{i})^{5/3}}+\frac{\left(\frac{1}{256}\mathrm{i}\right)^{1/3}\ln\left(a^2\,d^2\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{1/3}\,144\mathrm{i}-1152\left(\frac{1}{256}\mathrm{i}\right)^{1/3}(-a)^{7/3}\,d^2\right)}{(-a)^{5/3}\,d}+\frac{\left(\frac{1}{256}\mathrm{i}\right)^{1/3}\ln\left(a^2\,d^2\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{1/3}\,144\mathrm{i}-1152\left(\frac{1}{256}\mathrm{i}\right)^{1/3}(-a)^{7/3}\,d^2\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{(-a)^{5/3}\,d}-\frac{\left(\frac{1}{256}\mathrm{i}\right)^{1/3}\ln\left(a^2\,d^2\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{1/3}\,144\mathrm{i}+1152\left(\frac{1}{256}\mathrm{i}\right)^{1/3}(-a)^{7/3}\,d^2\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{(-a)^{5/3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + a*tan(c + d*x)*1i)^(5/3),x)

[Out] (3i/(10*d) + ((a + a*tan(c + d*x)*1i)*3i)/(8*a*d))/(a + a*tan(c + d*x)*1i)^(5/3) + ((1i/256)^(1/3)*log(a^2*d^2*(a + a*tan(c + d*x)*1i)^(1/3)*144i - 1152*(1i/256)^(1/3)*(-a)^(7/3)*d^2))/((-a)^(5/3)*d) + ((1i/256)^(1/3)*log(a^2*d^2*(a + a*tan(c + d*x)*1i)^(1/3)*144i - 1152*(1i/256)^(1/3)*(-a)^(7/3)*d^2*((3^(1/2)*1i)/2 - 1/2))*((3^(1/2)*1i)/2 - 1/2))/((-a)^(5/3)*d) - ((1i/256)^(1/3)*log(a^2*d^2*(a + a*tan(c + d*x)*1i)^(1/3)*144i + 1152*(1i/256)^(1/3)*(-a)^(7/3)*d^2*((3^(1/2)*1i)/2 + 1/2))*((3^(1/2)*1i)/2 + 1/2))/((-a)^(5/3)*d)

3.308 $\int (e\tan(c+dx))^m(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=43

$$\frac{a\,{}_2F_1(1,1+m;2+m;i\tan(c+dx))(e\tan(c+dx))^{1+m}}{de(1+m)}$$

```
[Out] a*hypergeom([1, 1+m],[2+m],I*tan(d*x+c))*(e*tan(d*x+c))^(1+m)/d/e/(1+m)
```

Rubi [A]
time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3618, 66}

$$\frac{a(e\tan(c+dx))^{m+1}\,{}_2F_1(1,m+1;m+2;i\tan(c+dx))}{de(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^m*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (a*Hypergeometric2F1[1, 1 + m, 2 + m, I*Tan[c + d*x]]*(e*Tan[c + d*x])^(1 +
 m))/(d*e*(1 + m))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rubi steps

$$\begin{aligned}\int (e\tan(c+dx))^m(a+ia\tan(c+dx))\,dx &= \frac{(ia^2)\,\text{Subst}\left(\int \frac{\left(-\frac{iex}{a}\right)^m}{-a^2+ax}\,dx,x,ia\tan(c+dx)\right)}{d}\\ &= \frac{a\,{}_2F_1(1,1+m;2+m;i\tan(c+dx))(e\tan(c+dx))^{1+m}}{de(1+m)}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 159 vs. $2(43) = 86$.
time = 0.87, size = 159, normalized size = 3.70

$$\frac{2^{-1-m} a e^{-ic} \left(-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}\right)^{1+m} \left(1+e^{2i(c+dx)}\right)^{1+m} \cos(c+dx)\, {}_2F_1\left(1+m,1+m;2+m;\frac{1}{2}\left(1-e^{2i(c+dx)}\right)\right) (1+i\tan(c+dx)) \tan^{-m}(c+dx)(e\tan(c+dx))^m}{d(1+m)(\cos(dx)+i\sin(dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Tan[c + d*x])^m*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (2^(-1 - m)*a*(((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x))))
^(1 + m)*(1 + E^((2*I)*(c + d*x)))^(1 + m)*Cos[c + d*x]*Hypergeometric2F1[1
 + m, 1 + m, 2 + m, (1 - E^((2*I)*(c + d*x)))/2]*(1 + I*Tan[c + d*x])*(e*Ta
n[c + d*x])^m)/(d*E^(I*c)*(1 + m)*(Cos[d*x] + I*Sin[d*x])*Tan[c + d*x]^m)
```

Maple [F]
time = 0.40, size = 0, normalized size = 0.00

$$\int \left(e \tan \left(dx+c\right)\right)^{m} \left(a+i a \tan \left(dx+c\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^m*(a+I*a*tan(d*x+c)),x)
```

```
[Out] int((e*tan(d*x+c))^m*(a+I*a*tan(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(2*a*((I*e - I*e^(2*I*d*x + 2*I*c + 1))/(e^(2*I*d*x + 2*I*c) + 1))^
m*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-i(e\tan\left(c+dx\right))^{m}\right)\,dx+\int\left(e\tan\left(c+dx\right)\right)^{m}\tan\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**m*(a+I*a*tan(d*x+c)),x)
```

```
[Out] I*a*(Integral(-I*(e*tan(c + d*x))**m, x) + Integral((e*tan(c + d*x))**m*tan
(c + d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int{\left(e\,\mathrm{tan}\left(c+d\,x\right)\right)}^m\,\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^m*(a + a*tan(c + d*x)*1i),x)
```

```
[Out] int((e*tan(c + d*x))^m*(a + a*tan(c + d*x)*1i), x)
```

3.309 $\int (e\tan(c+dx))^m(a-ia\tan(c+dx))\,dx$

Optimal. Leaf size=43

$$\frac{a\,{}_2F_1(1,1+m;2+m;-i\tan(c+dx))(e\tan(c+dx))^{1+m}}{de(1+m)}$$

```
[Out] a*hypergeom([1, 1+m],[2+m],-I*tan(d*x+c))*(e*tan(d*x+c))^(1+m)/d/e/(1+m)
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3618, 66}

$$\frac{a(e\tan(c+dx))^{m+1}\,{}_2F_1(1,m+1;m+2;-i\tan(c+dx))}{de(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^m*(a - I*a*Tan[c + d*x]),x]
```

```
[Out] (a*Hypergeometric2F1[1, 1 + m, 2 + m, (-I)*Tan[c + d*x]]*(e*Tan[c + d*x])^(
1 + m))/(d*e*(1 + m))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rubi steps

$$\begin{aligned}\int (e\tan(c+dx))^m(a-ia\tan(c+dx))\,dx &= -\frac{(ia^2)\,\mathrm{Subst}\left(\int \frac{\left(\frac{iex}{a}\right)^m}{-a^2+ax}\,dx, x, -ia\tan(c+dx)\right)}{d}\\ &= \frac{a\,{}_2F_1(1,1+m;2+m;-i\tan(c+dx))(e\tan(c+dx))^{1+m}}{de(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 44, normalized size = 1.02

$$\frac{a\,{}_2F_1(1,1+m;2+m;-i\tan(c+dx))\tan(c+dx)(e\tan(c+dx))^m}{d(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Tan[c + d*x])^m*(a - I*a*Tan[c + d*x]),x]
```

```
[Out] (a*Hypergeometric2F1[1, 1 + m, 2 + m, (-I)*Tan[c + d*x]]*Tan[c + d*x]*(e*Ta
n[c + d*x])^m)/(d*(1 + m))
```

Maple [F]
time = 0.40, size = 0, normalized size = 0.00

$$\int \left(e\tan\left(dx+c\right)\right)^{m}\left(a-ia\tan\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^m*(a-I*a*tan(d*x+c)),x)
```

```
[Out] int((e*tan(d*x+c))^m*(a-I*a*tan(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m*(a-I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*tan(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m*(a-I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(2*a*((I*e - I*e^(2*I*d*x + 2*I*c + 1))/(e^(2*I*d*x + 2*I*c) + 1))^
m/(e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia\left(\int i(e\tan\left(c+dx\right))^{m}\,dx+\int\left(e\tan\left(c+dx\right)\right)^{m}\tan\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**m*(a-I*a*tan(d*x+c)),x)
```

```
[Out] -I*a*(Integral(I*(e*tan(c + d*x))**m, x) + Integral((e*tan(c + d*x))**m*tan
(c + d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m*(a-I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((-I*a*tan(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (e\tan(c+d\,x))^m\,(a-a\tan(c+d\,x)\ 1\mathrm{i})\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^m*(a - a*tan(c + d*x)*1i),x)
```

```
[Out] int((e*tan(c + d*x))^m*(a - a*tan(c + d*x)*1i), x)
```

3.310 $\int (d\tan(e+fx))^n (a+ia\tan(e+fx))^4\,dx$

Optimal. Leaf size=189

$$-\frac{2a^4(16+11n+2n^2)\,(d\tan(e+fx))^{1+n}}{df(1+n)(2+n)(3+n)}+\frac{8a^4\,{}_2F_1(1,1+n;2+n;i\tan(e+fx))(d\tan(e+fx))^{1+n}}{df(1+n)}-\frac{(d\tan$$

```
[Out] -2*a^4*(2*n^2+11*n+16)*(d*tan(f*x+e))^(1+n)/d/f/(3+n)/(n^2+3*n+2)+8*a^4*hyp
ergeom([1, 1+n],[2+n],I*tan(f*x+e))*(d*tan(f*x+e))^(1+n)/d/f/(1+n)-(d*tan(f
*x+e))^(1+n)*(a^2+I*a^2*tan(f*x+e))^2/d/f/(3+n)-2*(4+n)*(d*tan(f*x+e))^(1+n
)*(a^4+I*a^4*tan(f*x+e))/d/f/(2+n)/(3+n)
```

Rubi [A]
time = 0.37, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3637, 3675, 3673, 3618, 12, 66}

$$\frac{8a^4(d\tan(e+fx))^{n+1}\,{}_2F_1(1,n+1;n+2;i\tan(e+fx))}{df(n+1)}-\frac{2a^4(2n^2+11n+16)\,(d\tan(e+fx))^{n+1}}{df(n+1)(n+2)(n+3)}-\frac{2(n+4)\,(a^4+ia^4\tan(e+fx))\,(d\tan(e+fx))^{n+1}}{df(n+2)(n+3)}-\frac{(a^2+ia^2\tan(e+fx))^2\,(d\tan(e+fx))^{n+1}}{df(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^n*(a + I*a*Tan[e + f*x])^4,x]
```

```
[Out] (-2*a^4*(16 + 11*n + 2*n^2)*(d*Tan[e + f*x])^(1 + n))/(d*f*(1 + n)*(2 + n)*
(3 + n)) + (8*a^4*Hypergeometric2F1[1, 1 + n, 2 + n, I*Tan[e + f*x]]*(d*Tan
[e + f*x])^(1 + n))/(d*f*(1 + n)) - ((d*Tan[e + f*x])^(1 + n)*(a^2 + I*a^2*
Tan[e + f*x])^2)/(d*f*(3 + n)) - (2*(4 + n)*(d*Tan[e + f*x])^(1 + n)*(a^4 +
 I*a^4*Tan[e + f*x]))/(d*f*(2 + n)*(3 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rule 3675

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[
e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && G
tQ[m, 1] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (d\tan(e+fx))^n(a+ia\tan(e+fx))^4\,dx = -\frac{(d\tan(e+fx))^{1+n}\left(a^2+ia^2\tan(e+fx)\right)^2}{df(3+n)} + \frac{a\int(d\tan(e+}{}$$

$$= -\frac{(d\tan(e+fx))^{1+n}\left(a^2+ia^2\tan(e+fx)\right)^2}{df(3+n)} - \frac{2(4+n)(d\tan}{}$$

$$= -\frac{2a^4(16+11n+2n^2)(d\tan(e+fx))^{1+n}}{df(1+n)(2+n)(3+n)} - \frac{(d\tan(e+fx))}{}$$

$$= -\frac{2a^4(16+11n+2n^2)(d\tan(e+fx))^{1+n}}{df(1+n)(2+n)(3+n)} - \frac{(d\tan(e+fx))}{}$$

$$= -\frac{2a^4(16+11n+2n^2)(d\tan(e+fx))^{1+n}}{df(1+n)(2+n)(3+n)} - \frac{(d\tan(e+fx))}{}$$

$$= -\frac{2a^4(16+11n+2n^2)(d\tan(e+fx))^{1+n}}{df(1+n)(2+n)(3+n)} + \frac{8a^4\,{}_2F_1(1,1+n}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1065 vs. 2(189) = 378.
time = 8.95, size = 1065, normalized size = 5.63

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Tan[e + f*x])^n*(a + I*a*Tan[e + f*x])^4,x]
```

```
[Out] (Cos[e + f*x]^4*((Sec[e + f*x]^2*((-4*I)*Cos[4*e] - 4*Sin[4*e]))/(2 + n) +
((-3 - 2*n + Cos[2*e])*Sec[e]^2*((-2*I)*Cos[4*e] - 2*Sin[4*e]))/((1 + n)*(2
 + n)) + ((-Cos[e - f*x] + Cos[e + f*x])*Sec[e]^2*Sec[e + f*x]*((-2*I)*Cos[
4*e] - 2*Sin[4*e]))/(1 + n))*(d*Tan[e + f*x])^n*(a + I*a*Tan[e + f*x])^4)/(
f*(Cos[f*x] + I*Sin[f*x])^4) + (Cos[e + f*x]^4*((Sec[e]^2*(-1 + Cos[2*e] +
(2*I)*Sin[2*e])*((2*I)*Cos[4*e] + 2*Sin[4*e]))/(1 + n) + (Sec[e]^2*Sec[e +
f*x]*((2*I)*Cos[4*e] + 2*Sin[4*e])*(-Cos[e - f*x] + Cos[e + f*x] - (2*I)*Si
n[e - f*x] + (2*I)*Sin[e + f*x]))/(1 + n))*(d*Tan[e + f*x])^n*(a + I*a*Tan[
e + f*x])^4)/(f*(Cos[f*x] + I*Sin[f*x])^4) + (Cos[e + f*x]^4*((Sec[e]*Sec[e
 + f*x]^3*(Cos[4*e] - I*Sin[4*e])*Sin[f*x])/(3 + n) + (Sec[e]*Sec[e + f*x]*
(2*Cos[4*e] - (2*I)*Sin[4*e])*Sin[f*x])/((1 + n)*(3 + n)) + (Sec[e + f*x]^2
*(Cos[4*e] - I*Sin[4*e])*Tan[e])/(3 + n) + ((2*Cos[4*e] - (2*I)*Sin[4*e])*T
an[e])/((1 + n)*(3 + n)))*(d*Tan[e + f*x])^n*(a + I*a*Tan[e + f*x])^4)/(f*(
Cos[f*x] + I*Sin[f*x])^4) + (I*2^(3 - n)*(((-I)*(-1 + E^((2*I)*(e + f*x))))
/(1 + E^((2*I)*(e + f*x))))^n*Cos[e + f*x]^4*(2^n*Hypergeometric2F1[1, n, 1
```

```
 + n, -((-1 + E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f*x))))] - (1 + E^((2
*I)*(e + f*x)))^n*Hypergeometric2F1[n, n, 1 + n, (1 - E^((2*I)*(e + f*x)))/
2])*(d*Tan[e + f*x])^n*(a + I*a*Tan[e + f*x])^4)/((E^((2*I)*e) + E^((4*I)*e
))*f*n*(Cos[f*x] + I*Sin[f*x])^4*Tan[e + f*x]^n) - ((8*I)*(-1 + E^((2*I)*(e
 + f*x)))^n*(((-I)*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x))))^n
*Cos[e + f*x]^4*(-(Hypergeometric2F1[1, n, 1 + n, (1 - E^((2*I)*(e + f*x)))
/(1 + E^((2*I)*(e + f*x)))]/((1 + E^((2*I)*(e + f*x)))^n*n)) - ((1 + E^((2*
I)*e))*(-1 + E^((2*I)*(e + f*x)))*(1 + E^((2*I)*(e + f*x)))^(-1 - n)*Hyperg
eometric2F1[1, 1 + n, 2 + n, (1 - E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f
*x)))])/(1 + n) + Hypergeometric2F1[n, n, 1 + n, (1 - E^((2*I)*(e + f*x)))/
2]/(2^n*n))*(d*Tan[e + f*x])^n*(a + I*a*Tan[e + f*x])^4)/(E^((4*I)*e)*(1 +
E^((2*I)*e))*((-1 + E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f*x))))^n*f*(Co
s[f*x] + I*Sin[f*x])^4*Tan[e + f*x]^n)
```

Maple [**F**]
time = 1.12, size = 0, normalized size = 0.00

$$\int \left(d \tan \left(f x +e \right)\right)^{n} \left(a +i a \tan \left(f x +e \right)\right)^{4}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e))^4,x)
```

```
[Out] int((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e))^4,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^4*(d*tan(f*x + e))^n, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] integral(16*a^4*((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)
)^n*e^(8*I*f*x + 8*I*e)/(e^(8*I*f*x + 8*I*e) + 4*e^(6*I*f*x + 6*I*e) + 6*e^
(4*I*f*x + 4*I*e) + 4*e^(2*I*f*x + 2*I*e) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int (d\tan(e+fx))^n\,dx+\int\left(-6(d\tan(e+fx))^n\tan^2(e+fx)\right)dx+\int (d\tan(e+fx))^n\tan^4(e+fx)\,dx+\int 4i(d\tan(e+fx))^n\tan(e+fx)\,dx+\int\left(-4i(d\tan(e+fx))^n\tan^3(e+fx)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**n*(a+I*a*tan(f*x+e))**4,x)
```

```
[Out] a**4*(Integral((d*tan(e + f*x))**n, x) + Integral(-6*(d*tan(e + f*x))**n*ta
n(e + f*x)**2, x) + Integral((d*tan(e + f*x))**n*tan(e + f*x)**4, x) + Inte
gral(4*I*(d*tan(e + f*x))**n*tan(e + f*x), x) + Integral(-4*I*(d*tan(e + f*
x))**n*tan(e + f*x)**3, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e))^4,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^4*(d*tan(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\tan(e+f\,x))^n\,\left(a+a\tan(e+f\,x)\;1\mathrm{i}\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^n*(a + a*tan(e + f*x)*1i)^4,x)
```

```
[Out] int((d*tan(e + f*x))^n*(a + a*tan(e + f*x)*1i)^4, x)
```

3.311 $\int (d\tan(e+fx))^n(a+ia\tan(e+fx))^3\,dx$

Optimal. Leaf size=127

$$-\frac{a^3(5+2n)(d\tan(e+fx))^{1+n}}{df(1+n)(2+n)}+\frac{4a^3\,{}_2F_1(1,1+n;2+n;i\tan(e+fx))(d\tan(e+fx))^{1+n}}{df(1+n)}-\frac{(d\tan(e+fx))^1}{}$$

```
[Out] -a^3*(5+2*n)*(d*tan(f*x+e))^(1+n)/d/f/(1+n)/(2+n)+4*a^3*hypergeom([1, 1+n],
[2+n],I*tan(f*x+e))*(d*tan(f*x+e))^(1+n)/d/f/(1+n)-(d*tan(f*x+e))^(1+n)*(a^
3+I*a^3*tan(f*x+e))/d/f/(2+n)
```

Rubi [A]
time = 0.18, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3637, 3673, 3618, 12, 66}

$$\frac{4a^3(d\tan(e+fx))^{n+1}\,{}_2F_1(1,n+1;n+2;i\tan(e+fx))}{df(n+1)}-\frac{a^3(2n+5)(d\tan(e+fx))^{n+1}}{df(n+1)(n+2)}-\frac{\left(a^3+ia^3\tan(e+fx)\right)(d\tan(e+fx))^{n+1}}{df(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^n*(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] -((a^3*(5 + 2*n)*(d*Tan[e + f*x])^(1 + n))/(d*f*(1 + n)*(2 + n))) + (4*a^3*
Hypergeometric2F1[1, 1 + n, 2 + n, I*Tan[e + f*x]]*(d*Tan[e + f*x])^(1 + n)
)/(d*f*(1 + n)) - ((d*Tan[e + f*x])^(1 + n)*(a^3 + I*a^3*Tan[e + f*x]))/(d*
f*(2 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (d\tan(e+fx))^n(a+ia\tan(e+fx))^3\,dx = -\frac{(d\tan(e+fx))^{1+n}\left(a^3+ia^3\tan(e+fx)\right)}{df(2+n)} + \frac{a\int(d\tan(e+\cdots}{\cdots}$$

$$= -\frac{a^3(5+2n)(d\tan(e+fx))^{1+n}}{df(1+n)(2+n)} - \frac{(d\tan(e+fx))^{1+n}\left(a^3+i\cdots\right.}{df(2+n)}$$

$$= -\frac{a^3(5+2n)(d\tan(e+fx))^{1+n}}{df(1+n)(2+n)} - \frac{(d\tan(e+fx))^{1+n}\left(a^3+i\cdots\right.}{df(2+n)}$$

$$= -\frac{a^3(5+2n)(d\tan(e+fx))^{1+n}}{df(1+n)(2+n)} - \frac{(d\tan(e+fx))^{1+n}\left(a^3+i\cdots\right.}{df(2+n)}$$

$$= -\frac{a^3(5+2n)(d\tan(e+fx))^{1+n}}{df(1+n)(2+n)} + \frac{4a^3\,{}_2F_1(1,1+n;2+n;i\,\mathrm{t}\cdots}{df\cdots}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 900 vs. 2(127) = 254.
time = 8.29, size = 900, normalized size = 7.09

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Tan[e + f*x])^n*(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] (Cos[e + f*x]^3*((Sec[e + f*x]^2*((-I)*Cos[3*e] - Sin[3*e]))/(2 + n) + ((-3
 - 2*n + Cos[2*e])*Sec[e]^2*((-1/2*I)*Cos[3*e] - Sin[3*e]/2))/((1 + n)*(2 +
 n)) + ((-Cos[e - f*x] + Cos[e + f*x])*Sec[e]^2*Sec[e + f*x]*((-1/2*I)*Cos[
3*e] - Sin[3*e]/2))/(1 + n))*(d*Tan[e + f*x])^n*(a + I*a*Tan[e + f*x])^3)/(
f*(Cos[f*x] + I*Sin[f*x])^3) + (Cos[e + f*x]^3*((Sec[e]^2*(-1 + Cos[2*e] +
(3*I)*Sin[2*e])*((I/2)*Cos[3*e] + Sin[3*e]/2))/(1 + n) + (Sec[e]^2*Sec[e +
f*x]*((I/2)*Cos[3*e] + Sin[3*e]/2)*(-Cos[e - f*x] + Cos[e + f*x] - (3*I)*Si
n[e - f*x] + (3*I)*Sin[e + f*x]))/(1 + n))*(d*Tan[e + f*x])^n*(a + I*a*Tan[
e + f*x])^3)/(f*(Cos[f*x] + I*Sin[f*x])^3) + (I*2^(2 - n)*(((-I)*(-1 + E^((
2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x))))^n*Cos[e + f*x]^3*(2^n*Hypergeo
metric2F1[1, n, 1 + n, -((-1 + E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f*x)
)))] - (1 + E^((2*I)*(e + f*x)))^n*Hypergeometric2F1[n, n, 1 + n, (1 - E^((
2*I)*(e + f*x)))/2])*(d*Tan[e + f*x])^n*(a + I*a*Tan[e + f*x])^3)/((E^(I*e)
 + E^((3*I)*e))*f*n*(Cos[f*x] + I*Sin[f*x])^3*Tan[e + f*x]^n) - ((4*I)*(-1
+ E^((2*I)*(e + f*x)))^n*(((-I)*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(
e + f*x))))^n*Cos[e + f*x]^3*(-(Hypergeometric2F1[1, n, 1 + n, (1 - E^((2*I
)*(e + f*x)))/(1 + E^((2*I)*(e + f*x)))]/((1 + E^((2*I)*(e + f*x)))^n*n)) -
 ((1 + E^((2*I)*e))*(-1 + E^((2*I)*(e + f*x)))*(1 + E^((2*I)*(e + f*x)))^(-
1 - n)*Hypergeometric2F1[1, 1 + n, 2 + n, (1 - E^((2*I)*(e + f*x)))/(1 + E^
((2*I)*(e + f*x)))])/(1 + n) + Hypergeometric2F1[n, n, 1 + n, (1 - E^((2*I)
*(e + f*x)))/2]/(2^n*n))*(d*Tan[e + f*x])^n*(a + I*a*Tan[e + f*x])^3)/(E^((
3*I)*e)*(1 + E^((2*I)*e))*((-1 + E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f*
x))))^n*f*(Cos[f*x] + I*Sin[f*x])^3*Tan[e + f*x]^n)
```

Maple [F]
time = 1.03, size = 0, normalized size = 0.00

$$\int \left(d \tan \left(f x +e \right)\right)^{n} \left(a +i a \tan \left(f x +e \right)\right)^{3}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e))^3,x)
[Out] int((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e))^3,x, algorithm="maxima")
[Out] integrate((I*a*tan(f*x + e) + a)^3*(d*tan(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(8*a^3*((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))
^n*e^(6*I*f*x + 6*I*e)/(e^(6*I*f*x + 6*I*e) + 3*e^(4*I*f*x + 4*I*e) + 3*e^(
2*I*f*x + 2*I*e) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia^3\left(\int i(d\tan(e+fx))^n\,dx+\int\left(-3(d\tan(e+fx))^n\tan(e+fx)\right)dx+\int(d\tan(e+fx))^n\tan^3(e+fx)\,dx+\int\left(-3i(d\tan(e+fx))^n\tan^2(e+fx)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**n*(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] -I*a**3*(Integral(I*(d*tan(e + f*x))**n, x) + Integral(-3*(d*tan(e + f*x))*
*n*tan(e + f*x), x) + Integral((d*tan(e + f*x))**n*tan(e + f*x)**3, x) + In
tegral(-3*I*(d*tan(e + f*x))**n*tan(e + f*x)**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^3*(d*tan(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\tan(e+f\,x))^n\,(a+a\tan(e+f\,x)\ 1\mathrm{i})^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^n*(a + a*tan(e + f*x)*1i)^3,x)
```

```
[Out] int((d*tan(e + f*x))^n*(a + a*tan(e + f*x)*1i)^3, x)
```

3.312 $\int (d\tan(e+fx))^n(a+ia\tan(e+fx))^2\,dx$

Optimal. Leaf size=75

$$-\frac{a^2(d\tan(e+fx))^{1+n}}{df(1+n)}+\frac{2a^2\,{}_2F_1(1,1+n;2+n;i\tan(e+fx))(d\tan(e+fx))^{1+n}}{df(1+n)}$$

```
[Out] -a^2*(d*tan(f*x+e))^(1+n)/d/f/(1+n)+2*a^2*hypergeom([1, 1+n],[2+n],I*tan(f*
x+e))*(d*tan(f*x+e))^(1+n)/d/f/(1+n)
```

Rubi [A]
time = 0.08, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3624, 3618, 12, 66}

$$-\frac{a^2(d\tan(e+fx))^{n+1}}{df(n+1)}+\frac{2a^2(d\tan(e+fx))^{n+1}\,{}_2F_1(1,n+1;n+2;i\tan(e+fx))}{df(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^n*(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] -((a^2*(d*Tan[e + f*x])^(1 + n))/(d*f*(1 + n))) + (2*a^2*Hypergeometric2F1[
1, 1 + n, 2 + n, I*Tan[e + f*x]]*(d*Tan[e + f*x])^(1 + n))/(d*f*(1 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
```

```
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int (d\tan(e+fx))^n(a+ia\tan(e+fx))^2\,dx = -\frac{a^2(d\tan(e+fx))^{1+n}}{df(1+n)} + \int (d\tan(e+fx))^n\left(2a^2+2ia^2\tan\right.$$

$$= -\frac{a^2(d\tan(e+fx))^{1+n}}{df(1+n)} + \frac{(4ia^4)\,\text{Subst}\left(\int \frac{2^{-n}\left(-\frac{idx}{a^2}\right)^n}{-4a^4+2a^2x}\,dx, x, 2\right.}{f}$$

$$= -\frac{a^2(d\tan(e+fx))^{1+n}}{df(1+n)} + \frac{(i2^{2-n}a^4)\,\text{Subst}\left(\int \frac{\left(-\frac{idx}{a^2}\right)^n}{-4a^4+2a^2x}\,dx, x,\right.}{f}$$

$$= -\frac{a^2(d\tan(e+fx))^{1+n}}{df(1+n)} + \frac{2a^2\,{}_2F_1(1,1+n;2+n;i\tan(e+f}{df(1+n)}$$

Mathematica [F]
time = 2.60, size = 0, normalized size = 0.00

$$\int (d\tan(e+fx))^n(a+ia\tan(e+fx))^2\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Tan[e + f*x])^n*(a + I*a*Tan[e + f*x])^2,x]
[Out] Integrate[(d*Tan[e + f*x])^n*(a + I*a*Tan[e + f*x])^2, x]
```

Maple [F]
time = 0.50, size = 0, normalized size = 0.00

$$\int \left(d\tan\left(fx+e\right)\right)^n\left(a+ia\tan\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e))^2,x)
[Out] int((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^2*(d*tan(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(4*a^2*((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))
^n*e^(4*I*f*x + 4*I*e)/(e^(4*I*f*x + 4*I*e) + 2*e^(2*I*f*x + 2*I*e) + 1), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int\left(-(d\tan(e+fx))^n\right)dx+\int(d\tan(e+fx))^n\tan^2(e+fx)\,dx+\int\left(-2i(d\tan(e+fx))^n\tan(e+fx)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**n*(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -a**2*(Integral(-(d*tan(e + f*x))**n, x) + Integral((d*tan(e + f*x))**n*tan
(e + f*x)**2, x) + Integral(-2*I*(d*tan(e + f*x))**n*tan(e + f*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^2*(d*tan(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int(d\tan(e+f\,x))^n\,(a+a\tan(e+f\,x)\,1\mathrm{i})^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^n*(a + a*tan(e + f*x)*1i)^2,x)
```

```
[Out] int((d*tan(e + f*x))^n*(a + a*tan(e + f*x)*1i)^2, x)
```

3.313 $\int (d\tan(e+fx))^n(a+ia\tan(e+fx))\,dx$

Optimal. Leaf size=43

$$\frac{a\,{}_2F_1(1,1+n;2+n;i\tan(e+fx))(d\tan(e+fx))^{1+n}}{df(1+n)}$$

```
[Out] a*hypergeom([1, 1+n],[2+n],I*tan(f*x+e))*(d*tan(f*x+e))^(1+n)/d/f/(1+n)
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3618, 66}

$$\frac{a(d\tan(e+fx))^{n+1}\,{}_2F_1(1,n+1;n+2;i\tan(e+fx))}{df(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^n*(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (a*Hypergeometric2F1[1, 1 + n, 2 + n, I*Tan[e + f*x]]*(d*Tan[e + f*x])^(1 +
 n))/(d*f*(1 + n))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rubi steps

$$\begin{aligned}\int (d\tan(e+fx))^n(a+ia\tan(e+fx))\,dx &= \frac{(ia^2)\,\text{Subst}\left(\int \frac{\left(-\frac{idx}{a}\right)^n}{-a^2+ax}\,dx, x, ia\tan(e+fx)\right)}{f}\\ &= \frac{a\,{}_2F_1(1,1+n;2+n;i\tan(e+fx))(d\tan(e+fx))^{1+n}}{df(1+n)}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 159 vs. 2(43) = 86.
time = 0.81, size = 159, normalized size = 3.70

$$\frac{2^{-1-n}ae^{-ie}\left(-\frac{i(-1+e^{2i(e+fx)})}{1+e^{2i(e+fx)}}\right)^{1+n}\left(1+e^{2i(e+fx)}\right)^{1+n}\cos(e+fx)\,{}_2F_1\left(1+n,1+n;2+n;\frac{1}{2}\left(1-e^{2i(e+fx)}\right)\right)(1+i\tan(e+fx))\tan^{-n}(e+fx)(d\tan(e+fx))^n}{f(1+n)(\cos(fx)+i\sin(fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^n*(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (2^(-1 - n)*a*(((-I)*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x))))
^(1 + n)*(1 + E^((2*I)*(e + f*x)))^(1 + n)*Cos[e + f*x]*Hypergeometric2F1[1
 + n, 1 + n, 2 + n, (1 - E^((2*I)*(e + f*x)))/2]*(1 + I*Tan[e + f*x])*(d*Ta
n[e + f*x])^n)/(E^(I*e)*f*(1 + n)*(Cos[f*x] + I*Sin[f*x])*Tan[e + f*x]^n)
```

Maple [F]
time = 0.38, size = 0, normalized size = 0.00

$$\int (d\tan(fx+e))^n (a+ia\tan(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e)),x)
```

```
[Out] int((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)*(d*tan(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(2*a*((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))^n
*e^(2*I*f*x + 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-i(d\tan\left(e+fx\right))^{n}\right)\,dx+\int(d\tan\left(e+fx\right))^{n}\tan\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**n*(a+I*a*tan(f*x+e)),x)
```

```
[Out] I*a*(Integral(-I*(d*tan(e + f*x))**n, x) + Integral((d*tan(e + f*x))**n*tan
(e + f*x), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)*(d*tan(f*x + e))^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int(d\tan(e+f\,x))^{n}\,\left(a+a\tan(e+f\,x)\,1\mathrm{i}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^n*(a + a*tan(e + f*x)*1i),x)
```

```
[Out] int((d*tan(e + f*x))^n*(a + a*tan(e + f*x)*1i), x)
```

3.314 $\int \frac{(d\tan(e+fx))^n}{a+ia\tan(e+fx)}\,dx$

Optimal. Leaf size=158

$$\frac{(1-n)\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)(d\tan(e+fx))^{1+n}}{2adf(1+n)}+\frac{in\,{}_2F_1\left(1,\frac{2+n}{2};\frac{4+n}{2};-\tan^2(e+fx)\right)(d\tan(e+}{2ad^2f(2+n)}$$

```
[Out] 1/2*(1-n)*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],-tan(f*x+e)^2)*(d*tan(f*x+e)
)^(1+n)/a/d/f/(1+n)+1/2*I*n*hypergeom([1, 1+1/2*n],[2+1/2*n],-tan(f*x+e)^2)
*(d*tan(f*x+e))^(2+n)/a/d^2/f/(2+n)+1/2*(d*tan(f*x+e))^(1+n)/d/f/(a+I*a*tan
(f*x+e))
```

Rubi [A]
time = 0.13, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3633, 3619, 3557, 371}

$$\frac{in(d\tan(e+fx))^{n+2}\,{}_2F_1\left(1,\frac{n+2}{2};\frac{n+4}{2};-\tan^2(e+fx)\right)}{2ad^2f(n+2)}+\frac{(1-n)(d\tan(e+fx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(e+fx)\right)}{2adf(n+1)}+\frac{(d\tan(e+fx))^{n+1}}{2df(a+ia\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^n/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] ((1 - n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[e + f*x]^2]*(d*Tan
[e + f*x])^(1 + n))/(2*a*d*f*(1 + n)) + ((I/2)*n*Hypergeometric2F1[1, (2 +
n)/2, (4 + n)/2, -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(2 + n))/(a*d^2*f*(2 + n
)) + (d*Tan[e + f*x])^(1 + n)/(2*d*f*(a + I*a*Tan[e + f*x]))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
```

```
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3633

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-a)*((c + d*Tan[e + f*x])^(n + 1)/(2*f*(b*
c - a*d)*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*
Tan[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &
& NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^n}{a+ia\tan(e+fx)}\,dx = \frac{(d\tan(e+fx))^{1+n}}{2df(a+ia\tan(e+fx))} - \frac{\int (d\tan(e+fx))^n(-ad(1-n)-iadn\tan(e+fx))}{2a^2d}$$

$$= \frac{(d\tan(e+fx))^{1+n}}{2df(a+ia\tan(e+fx))} + \frac{(1-n)\int (d\tan(e+fx))^n\,dx}{2a} + \frac{(in)\int (d\tan(e+fx}{2ad}$$

$$= \frac{(d\tan(e+fx))^{1+n}}{2df(a+ia\tan(e+fx))} + \frac{(d(1-n))\text{Subst}\left(\int \frac{x^n}{d^2+x^2}\,dx, x, d\tan(e+fx)\right)}{2af} + \frac{(in}{}$$

$$= \frac{(1-n)\,{}_2F_1\left(1, \frac{1+n}{2}; \frac{3+n}{2}; -\tan^2(e+fx)\right)(d\tan(e+fx))^{1+n}}{2adf(1+n)} + \frac{in\,{}_2F_1\left(1, \frac{2+n}{2}; \frac{4+}{2}\right.}{}$$

Mathematica [F]
time = 15.87, size = 0, normalized size = 0.00

$$\int \frac{(d\tan(e+fx))^n}{a+ia\tan(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Tan[e + f*x])^n/(a + I*a*Tan[e + f*x]),x]
[Out] Integrate[(d*Tan[e + f*x])^n/(a + I*a*Tan[e + f*x]), x]
```

Maple [F]
time = 1.14, size = 0, normalized size = 0.00

$$\int \frac{(d\tan(fx+e))^n}{a+ia\tan(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^n/(a+I*a*tan(f*x+e)),x)
```

[Out] int((d*tan(f*x+e))^n/(a+I*a*tan(f*x+e)),x)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*tan(f*x+e))^n/(a+I*a*tan(f*x+e)),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*tan(f*x+e))^n/(a+I*a*tan(f*x+e)),x, algorithm="fricas")

[Out] integral(1/2*((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))^n*(e^(2*I*f*x + 2*I*e) + 1)*e^(-2*I*f*x - 2*I*e)/a, x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{(d\tan{(e+fx)})^n}{\tan{(e+fx)}-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*tan(f*x+e))**n/(a+I*a*tan(f*x+e)),x)

[Out] -I*Integral((d*tan(e + f*x))**n/(tan(e + f*x) - I), x)/a

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*tan(f*x+e))^n/(a+I*a*tan(f*x+e)),x, algorithm="giac")

[Out] integrate((d*tan(f*x + e))^n/(I*a*tan(f*x + e) + a), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\tan(e+fx))^n}{a+a\tan\left(e+fx\right)\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^n/(a + a*tan(e + f*x)*1i),x)
[Out] int((d*tan(e + f*x))^n/(a + a*tan(e + f*x)*1i), x)
```

3.315 $\int \frac{(d\tan(e+fx))^n}{(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=209

$$\frac{(1-n)^2\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)(d\tan(e+fx))^{1+n}}{4a^2df(1+n)}+\frac{(2-n)(d\tan(e+fx))^{1+n}}{4a^2df(1+i\tan(e+fx))}+\frac{i(2-n)n\,{}_2F_1\left(1,\right.}{}$$

```
[Out] 1/4*(1-n)^2*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],-tan(f*x+e)^2)*(d*tan(f*x+
e))^(1+n)/a^2/d/f/(1+n)+1/4*(2-n)*(d*tan(f*x+e))^(1+n)/a^2/d/f/(1+I*tan(f*x
+e))+1/4*I*(2-n)*n*hypergeom([1, 1+1/2*n],[2+1/2*n],-tan(f*x+e)^2)*(d*tan(f
*x+e))^(2+n)/a^2/d^2/f/(2+n)+1/4*(d*tan(f*x+e))^(1+n)/d/f/(a+I*a*tan(f*x+e)
)^2
```

Rubi [A]

time = 0.26, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3640, 3677, 3619, 3557, 371}

$$\frac{i(2-n)n(d\tan(e+fx))^{n+2}\,{}_2F_1\left(1,\frac{n+2}{2};\frac{n+4}{2};-\tan^2(e+fx)\right)}{4a^2d^2f(n+2)}+\frac{(1-n)^2(d\tan(e+fx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(e+fx)\right)}{4a^2df(n+1)}+\frac{(2-n)(d\tan(e+fx))^{n+1}}{4a^2df(1+i\tan(e+fx))}+\frac{(d\tan(e+fx))^{n+1}}{4df(a+ia\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^n/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] ((1 - n)^2*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[e + f*x]^2]*(d*T
an[e + f*x])^(1 + n))/(4*a^2*d*f*(1 + n)) + ((2 - n)*(d*Tan[e + f*x])^(1 +
n))/(4*a^2*d*f*(1 + I*Tan[e + f*x])) + ((I/4)*(2 - n)*n*Hypergeometric2F1[1
, (2 + n)/2, (4 + n)/2, -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(2 + n))/(a^2*d^2
*f*(2 + n)) + (d*Tan[e + f*x])^(1 + n)/(4*d*f*(a + I*a*Tan[e + f*x])^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d\tan(e+fx))^n}{(a+ia\tan(e+fx))^2}\,dx &= \frac{(d\tan(e+fx))^{1+n}}{4df(a+ia\tan(e+fx))^2} + \frac{\int \frac{(d\tan(e+fx))^n(ad(3-n)-iad(1-n)\tan(e+fx))}{a+ia\tan(e+fx)}\,dx}{4a^2d} \\
&= \frac{(2-n)(d\tan(e+fx))^{1+n}}{4a^2df(1+i\tan(e+fx))} + \frac{(d\tan(e+fx))^{1+n}}{4df(a+ia\tan(e+fx))^2} + \frac{\int (d\tan(e+fx))^n
\\
&= \frac{(2-n)(d\tan(e+fx))^{1+n}}{4a^2df(1+i\tan(e+fx))} + \frac{(d\tan(e+fx))^{1+n}}{4df(a+ia\tan(e+fx))^2} + \frac{(1-n)^2\int (d\tan(e}{4a^2}
\\
&= \frac{(2-n)(d\tan(e+fx))^{1+n}}{4a^2df(1+i\tan(e+fx))} + \frac{(d\tan(e+fx))^{1+n}}{4df(a+ia\tan(e+fx))^2} + \frac{(d(1-n)^2)\,\mathrm{Subst}(}{}
\\
&= \frac{(1-n)^2\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)(d\tan(e+fx))^{1+n}}{4a^2df(1+n)} + \frac{(2-n)(d\tan}{4a^2df(1+i}
\end{aligned}
$$

Mathematica [F]
time = 5.88, size = 0, normalized size = 0.00

$$\int \frac{(d\tan(e+fx))^n}{(a+ia\tan(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Tan[e + f*x])^n/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] Integrate[(d*Tan[e + f*x])^n/(a + I*a*Tan[e + f*x])^2, x]
```

Maple [F]
time = 1.23, size = 0, normalized size = 0.00

$$\int \frac{(d\tan (fx+e))^n}{(a+ia\tan (fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^2,x)
```

```
[Out] int((d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/4*((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))^n
*(e^(4*I*f*x + 4*I*e) + 2*e^(2*I*f*x + 2*I*e) + 1)*e^(-4*I*f*x - 4*I*e)/a^2
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{(d\tan (e+fx))^n}{\tan^2 (e+fx)-2i\tan (e+fx)-1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**n/(a+I*a*tan(f*x+e))**2,x)
[Out] -Integral((d*tan(e + f*x))**n/(tan(e + f*x)**2 - 2*I*tan(e + f*x) - 1), x)/
a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*tan(f*x + e))^n/(I*a*tan(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\tan(e+f\,x))^n}{(a+a\tan\left(e+f\,x\right)\,1\mathrm{i})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^n/(a + a*tan(e + f*x)*1i)^2,x)
[Out] int((d*tan(e + f*x))^n/(a + a*tan(e + f*x)*1i)^2, x)
```

3.316 $\int \frac{(d\tan(e+fx))^n}{(a+ia\tan(e+fx))^3}\,dx$

Optimal. Leaf size=274

$$\frac{(1-2n)(1-n)(3-n)\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)(d\tan(e+fx))^{1+n}}{24a^3df(1+n)}+\frac{i(5-2n)(2-n)n\,{}_2F_1\left(1,\frac{2+n}{2};\right.}{24a}$$

```
[Out] 1/24*(1-2*n)*(1-n)*(3-n)*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],-tan(f*x+e)^2
)*(d*tan(f*x+e))^(1+n)/a^3/d/f/(1+n)+1/24*I*(5-2*n)*(2-n)*n*hypergeom([1, 1
+1/2*n],[2+1/2*n],-tan(f*x+e)^2)*(d*tan(f*x+e))^(2+n)/a^3/d^2/f/(2+n)+1/6*(
d*tan(f*x+e))^(1+n)/d/f/(a+I*a*tan(f*x+e))^3+1/24*(7-2*n)*(d*tan(f*x+e))^(1
+n)/a/d/f/(a+I*a*tan(f*x+e))^2+1/24*(5-2*n)*(2-n)*(d*tan(f*x+e))^(1+n)/d/f/
(a^3+I*a^3*tan(f*x+e))
```

Rubi [A]
time = 0.46, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3640, 3677, 3619, 3557, 371}

$$\frac{i(5-2n)(2-n)n(d\tan(e+fx))^{n+2}\,{}_2F_1\left(1,\frac{n+2}{2};\frac{n+4}{2};-\tan^2(e+fx)\right)}{24a^3d^2f(n+2)}+\frac{(1-2n)(1-n)(3-n)(d\tan(e+fx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(e+fx)\right)}{24a^3df(n+1)}+\frac{(5-2n)(2-n)(d\tan(e+fx))^{n+1}}{24df\left(a^3+ia^3\tan(e+fx)\right)}+\frac{(7-2n)(d\tan(e+fx))^{n+1}}{24adf(a+ia\tan(e+fx))^2}+\frac{(d\tan(e+fx))^{n+1}}{6df(a+ia\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^n/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] ((1 - 2*n)*(1 - n)*(3 - n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[
e + f*x]^2]*(d*Tan[e + f*x])^(1 + n))/(24*a^3*d*f*(1 + n)) + ((I/24)*(5 - 2
*n)*(2 - n)*n*Hypergeometric2F1[1, (2 + n)/2, (4 + n)/2, -Tan[e + f*x]^2]*(
d*Tan[e + f*x])^(2 + n))/(a^3*d^2*f*(2 + n)) + (d*Tan[e + f*x])^(1 + n)/(6*
d*f*(a + I*a*Tan[e + f*x])^3) + ((7 - 2*n)*(d*Tan[e + f*x])^(1 + n))/(24*a*
d*f*(a + I*a*Tan[e + f*x])^2) + ((5 - 2*n)*(2 - n)*(d*Tan[e + f*x])^(1 + n)
)/(24*d*f*(a^3 + I*a^3*Tan[e + f*x]))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d\tan(e+fx))^n}{(a+ia\tan(e+fx))^3}\,dx &= \frac{(d\tan(e+fx))^{1+n}}{6df(a+ia\tan(e+fx))^3} + \frac{\int \frac{(d\tan(e+fx))^n(ad(5-n)-iad(2-n)\tan(e+fx))}{(a+ia\tan(e+fx))^2}\,dx}{6a^2d} \\
&= \frac{(d\tan(e+fx))^{1+n}}{6df(a+ia\tan(e+fx))^3} + \frac{(7-2n)(d\tan(e+fx))^{1+n}}{24adf(a+ia\tan(e+fx))^2} + \frac{\int \frac{(d\tan(e+fx))^n(a^2d^{\cdots}}{\cdots}}{\cdots} \\
&= \frac{(d\tan(e+fx))^{1+n}}{6df(a+ia\tan(e+fx))^3} + \frac{(7-2n)(d\tan(e+fx))^{1+n}}{24adf(a+ia\tan(e+fx))^2} + \frac{(5-2n)(2-n)(\cdots}{24df\,(a^3+ia^{3}\cdots} \\
&= \frac{(d\tan(e+fx))^{1+n}}{6df(a+ia\tan(e+fx))^3} + \frac{(7-2n)(d\tan(e+fx))^{1+n}}{24adf(a+ia\tan(e+fx))^2} + \frac{(5-2n)(2-n)(\cdots}{24df\,(a^3+ia^{3}\cdots} \\
&= \frac{(d\tan(e+fx))^{1+n}}{6df(a+ia\tan(e+fx))^3} + \frac{(7-2n)(d\tan(e+fx))^{1+n}}{24adf(a+ia\tan(e+fx))^2} + \frac{(5-2n)(2-n)(\cdots}{24df\,(a^3+ia^{3}\cdots} \\
&= \frac{(1-2n)(1-n)(3-n)\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)(d\tan(e+fx))^{1+n}}{24a^3df(1+n)}
\end{aligned}
$$

Mathematica [F]
time = 16.62, size = 0, normalized size = 0.00

$$\int \frac{(d\tan(e+fx))^n}{(a+ia\tan(e+fx))^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Tan[e + f*x])^n/(a + I*a*Tan[e + f*x])^3,x]
[Out] Integrate[(d*Tan[e + f*x])^n/(a + I*a*Tan[e + f*x])^3, x]
```

Maple [F]
time = 1.22, size = 0, normalized size = 0.00

$$\int \frac{(d\tan(fx+e))^n}{(a+ia\tan(fx+e))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^3,x)
[Out] int((d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^3,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^3,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^3,x, algorithm="fricas")
[Out] integral(1/8*((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))^n
*(e^(6*I*f*x + 6*I*e) + 3*e^(4*I*f*x + 4*I*e) + 3*e^(2*I*f*x + 2*I*e) + 1)*
e^(-6*I*f*x - 6*I*e)/a^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int \frac{(d\tan{(e+fx)})^{n}}{\tan^{3}{(e+fx)}-3i\tan^{2}{(e+fx)}-3\tan{(e+fx)}+i}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**n/(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] I*Integral((d*tan(e + f*x))**n/(tan(e + f*x)**3 - 3*I*tan(e + f*x)**2 - 3*t
an(e + f*x) + I), x)/a**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*tan(f*x + e))^n/(I*a*tan(f*x + e) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\tan(e+f\,x))^{n}}{(a+a\tan{(e+f\,x)}\ 1\mathrm{i})^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^n/(a + a*tan(e + f*x)*1i)^3,x)
```

```
[Out] int((d*tan(e + f*x))^n/(a + a*tan(e + f*x)*1i)^3, x)
```

3.317 $\int \frac{(d\tan(e+fx))^n}{(a+ia\tan(e+fx))^4}\,dx$

Optimal. Leaf size=326

$$\frac{(1-n)(3-n)\left(1-4n+n^2\right)\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)(d\tan(e+fx))^{1+n}}{48a^4df(1+n)}+\frac{\left(13-7n+n^2\right)(d\tan(e-}{48a^4df(1+i\tan(e+}$$

```
[Out] 1/48*(1-n)*(3-n)*(n^2-4*n+1)*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],-tan(f*x+
e)^2)*(d*tan(f*x+e))^(1+n)/a^4/d/f/(1+n)+1/48*(n^2-7*n+13)*(d*tan(f*x+e))^(
1+n)/a^4/d/f/(1+I*tan(f*x+e))^2+1/48*(2-n)^2*(4-n)*(d*tan(f*x+e))^(1+n)/a^4
/d/f/(1+I*tan(f*x+e))+1/48*I*(2-n)^2*(4-n)*n*hypergeom([1, 1+1/2*n],[2+1/2*
n],-tan(f*x+e)^2)*(d*tan(f*x+e))^(2+n)/a^4/d^2/f/(2+n)+1/8*(d*tan(f*x+e))^(
1+n)/d/f/(a+I*a*tan(f*x+e))^4+1/24*(5-n)*(d*tan(f*x+e))^(1+n)/a/d/f/(a+I*a*
tan(f*x+e))^3
```

Rubi [A]

time = 0.66, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3640, 3677, 3619, 3557, 371}

$$\frac{i(2-n)^2(4-n)n(d\tan(e+fx))^{n+2}\,{}_2F_1\left(1,\frac{n+2}{2};\frac{n+4}{2};-\tan^2(e+fx)\right)}{48a^4d^2f(n+2)}+\frac{(1-n)(3-n)\left(n^2-4n+1\right)(d\tan(e+fx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(e+fx)\right)}{48a^4df(n+1)}+\frac{\left(n^2-7n+13\right)(d\tan(e+fx))^{n+1}}{48a^4df(1+i\tan(e+fx))^2}+\frac{(2-n)^2(4-n)(d\tan(e+fx))^{n+1}}{48a^4df(1+i\tan(e+fx))}+\frac{(5-n)(d\tan(e+fx))^{n+1}}{24adf(a+ia\tan(e+fx))^3}+\frac{(d\tan(e+fx))^{n+1}}{8df(a+ia\tan(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^n/(a + I*a*Tan[e + f*x])^4,x]
```

```
[Out] ((1 - n)*(3 - n)*(1 - 4*n + n^2)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2,
 -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(1 + n))/(48*a^4*d*f*(1 + n)) + ((13 - 7
*n + n^2)*(d*Tan[e + f*x])^(1 + n))/(48*a^4*d*f*(1 + I*Tan[e + f*x])^2) + (
(2 - n)^2*(4 - n)*(d*Tan[e + f*x])^(1 + n))/(48*a^4*d*f*(1 + I*Tan[e + f*x]
)) + ((I/48)*(2 - n)^2*(4 - n)*n*Hypergeometric2F1[1, (2 + n)/2, (4 + n)/2,
 -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(2 + n))/(a^4*d^2*f*(2 + n)) + (d*Tan[e
+ f*x])^(1 + n)/(8*d*f*(a + I*a*Tan[e + f*x])^4) + ((5 - n)*(d*Tan[e + f*x]
)^(1 + n))/(24*a*d*f*(a + I*a*Tan[e + f*x])^3)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^n}{(a+ia\tan(e+fx))^4}\,dx = \frac{(d\tan(e+fx))^{1+n}}{8df(a+ia\tan(e+fx))^4} + \frac{\int \frac{(d\tan(e+fx))^n(ad(7-n)-iad(3-n)\tan(e+fx))}{(a+ia\tan(e+fx))^3}\,dx}{8a^2d}$$

$$= \frac{(d\tan(e+fx))^{1+n}}{8df(a+ia\tan(e+fx))^4} + \frac{(5-n)(d\tan(e+fx))^{1+n}}{24adf(a+ia\tan(e+fx))^3} + \frac{\int \frac{(d\tan(e+fx))^n(2a^2d^2}{}}{}$$

$$= \frac{(13-7n+n^2)(d\tan(e+fx))^{1+n}}{48a^4df(1+i\tan(e+fx))^2} + \frac{(d\tan(e+fx))^{1+n}}{8df(a+ia\tan(e+fx))^4} + \frac{(5-n)(d\,\mathrm{ta}}{24adf(a+i}$$

$$= \frac{(13-7n+n^2)(d\tan(e+fx))^{1+n}}{48a^4df(1+i\tan(e+fx))^2} + \frac{(d\tan(e+fx))^{1+n}}{8df(a+ia\tan(e+fx))^4} + \frac{(5-n)(d\,\mathrm{ta}}{24adf(a+i}$$

$$= \frac{(13-7n+n^2)(d\tan(e+fx))^{1+n}}{48a^4df(1+i\tan(e+fx))^2} + \frac{(d\tan(e+fx))^{1+n}}{8df(a+ia\tan(e+fx))^4} + \frac{(5-n)(d\,\mathrm{ta}}{24adf(a+i}$$

$$= \frac{(13-7n+n^2)(d\tan(e+fx))^{1+n}}{48a^4df(1+i\tan(e+fx))^2} + \frac{(d\tan(e+fx))^{1+n}}{8df(a+ia\tan(e+fx))^4} + \frac{(5-n)(d\,\mathrm{ta}}{24adf(a+i}$$

$$= \frac{(1-n)(3-n)(1-4n+n^2)\ {}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)(d\tan(e+fx)}{48a^4df(1+n)}$$

Mathematica [F]
time = 22.75, size = 0, normalized size = 0.00

$$\int \frac{(d\tan(e+fx))^n}{(a+ia\tan(e+fx))^4}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Tan[e + f*x])^n/(a + I*a*Tan[e + f*x])^4,x]
[Out] Integrate[(d*Tan[e + f*x])^n/(a + I*a*Tan[e + f*x])^4, x]
```

Maple [F]
time = 1.43, size = 0, normalized size = 0.00

$$\int \frac{(d\tan(fx+e))^n}{(a+ia\tan(fx+e))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^4,x)
[Out] int((d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^4,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] integral(1/16*((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))^
n*(e^(8*I*f*x + 8*I*e) + 4*e^(6*I*f*x + 6*I*e) + 6*e^(4*I*f*x + 4*I*e) + 4*
e^(2*I*f*x + 2*I*e) + 1)*e^(-8*I*f*x - 8*I*e)/a^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(d\tan{(e+fx)})^n}{\tan^4{(e+fx)}-4i\tan^3{(e+fx)}-6\tan^2{(e+fx)}+4i\tan{(e+fx)}+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**n/(a+I*a*tan(f*x+e))**4,x)
```

```
[Out] Integral((d*tan(e + f*x))**n/(tan(e + f*x)**4 - 4*I*tan(e + f*x)**3 - 6*tan
(e + f*x)**2 + 4*I*tan(e + f*x) + 1), x)/a**4
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^4,x, algorithm="giac")
```

```
[Out] integrate((d*tan(f*x + e))^n/(I*a*tan(f*x + e) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\tan(e+f\,x))^n}{(a+a\tan(e+f\,x)\;1\mathrm{i})^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^n/(a + a*tan(e + f*x)*1i)^4,x)
```

```
[Out] int((d*tan(e + f*x))^n/(a + a*tan(e + f*x)*1i)^4, x)
```

3.318 $\int (d\tan(e+fx))^n(a-ia\tan(e+fx))\,dx$

Optimal. Leaf size=43

$$\frac{a\,{}_2F_1(1,1+n;2+n;-i\tan(e+fx))(d\tan(e+fx))^{1+n}}{df(1+n)}$$

```
[Out] a*hypergeom([1, 1+n],[2+n],-I*tan(f*x+e))*(d*tan(f*x+e))^(1+n)/d/f/(1+n)
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3618, 66}

$$\frac{a(d\tan(e+fx))^{n+1}\,{}_2F_1(1,n+1;n+2;-i\tan(e+fx))}{df(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^n*(a - I*a*Tan[e + f*x]),x]
```

```
[Out] (a*Hypergeometric2F1[1, 1 + n, 2 + n, (-I)*Tan[e + f*x]]*(d*Tan[e + f*x])^(
1 + n))/(d*f*(1 + n))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rubi steps

$$\begin{aligned}\int (d\tan(e+fx))^n(a-ia\tan(e+fx))\,dx &= -\frac{(ia^2)\,\mathrm{Subst}\left(\int \frac{\left(\frac{idx}{a}\right)^n}{-a^2+ax}\,dx, x, -ia\tan(e+fx)\right)}{f}\\ &= \frac{a\,{}_2F_1(1,1+n;2+n;-i\tan(e+fx))(d\tan(e+fx))^{1+n}}{df(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 44, normalized size = 1.02

$$\frac{a\,{}_2F_1(1,1+n;2+n;-i\tan(e+fx))\tan(e+fx)(d\tan(e+fx))^n}{f(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^n*(a - I*a*Tan[e + f*x]),x]
```

```
[Out] (a*Hypergeometric2F1[1, 1 + n, 2 + n, (-I)*Tan[e + f*x]]*Tan[e + f*x]*(d*Ta
n[e + f*x])^n)/(f*(1 + n))
```

Maple [F]
time = 0.37, size = 0, normalized size = 0.00

$$\int (d\tan(fx+e))^n (a-ia\tan(fx+e))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^n*(a-I*a*tan(f*x+e)),x)
```

```
[Out] int((d*tan(f*x+e))^n*(a-I*a*tan(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a-I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*tan(f*x + e) + a)*(d*tan(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a-I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(2*a*((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))^n
/(e^(2*I*f*x + 2*I*e) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia\left(\int i(d\tan(e+fx))^n\, dx + \int (d\tan(e+fx))^n \tan(e+fx)\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**n*(a-I*a*tan(f*x+e)),x)
```

```
[Out] -I*a*(Integral(I*(d*tan(e + f*x))**n, x) + Integral((d*tan(e + f*x))**n*tan
(e + f*x), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a-I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((-I*a*tan(f*x + e) + a)*(d*tan(f*x + e))^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int (d\tan(e+f\,x))^n\,(a-a\tan(e+f\,x)\;1\mathrm{i})\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^n*(a - a*tan(e + f*x)*1i),x)
```

```
[Out] int((d*tan(e + f*x))^n*(a - a*tan(e + f*x)*1i), x)
```

3.319 $\int \frac{(d\tan(e+fx))^n}{a-ia\tan(e+fx)}\,dx$

Optimal. Leaf size=158

$$\frac{(1-n)\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)(d\tan(e+fx))^{1+n}}{2adf(1+n)}-\frac{in\,{}_2F_1\left(1,\frac{2+n}{2};\frac{4+n}{2};-\tan^2(e+fx)\right)(d\tan(e+$$

```
[Out] 1/2*(1-n)*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],-tan(f*x+e)^2)*(d*tan(f*x+e)
)^(1+n)/a/d/f/(1+n)-1/2*I*n*hypergeom([1, 1+1/2*n],[2+1/2*n],-tan(f*x+e)^2)
*(d*tan(f*x+e))^(2+n)/a/d^2/f/(2+n)+1/2*(d*tan(f*x+e))^(1+n)/d/f/(a-I*a*tan
(f*x+e))
```

Rubi [A]

time = 0.12, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3633, 3619, 3557, 371}

$$-\frac{in(d\tan(e+fx))^{n+2}\,{}_2F_1\left(1,\frac{n+2}{2};\frac{n+4}{2};-\tan^2(e+fx)\right)}{2ad^2f(n+2)}+\frac{(1-n)(d\tan(e+fx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(e+fx)\right)}{2adf(n+1)}+\frac{(d\tan(e+fx))^{n+1}}{2df(a-ia\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^n/(a - I*a*Tan[e + f*x]),x]
```

```
[Out] ((1 - n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[e + f*x]^2]*(d*Tan
[e + f*x])^(1 + n))/(2*a*d*f*(1 + n)) - ((I/2)*n*Hypergeometric2F1[1, (2 +
n)/2, (4 + n)/2, -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(2 + n))/(a*d^2*f*(2 + n
)) + (d*Tan[e + f*x])^(1 + n)/(2*d*f*(a - I*a*Tan[e + f*x]))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
```

```
+ d^2, 0] && !IntegerQ[2*m]
```

Rule 3633

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-a)*((c + d*Tan[e + f*x])^(n + 1)/(2*f*(b*
c - a*d)*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*
Tan[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &
& NeQ[c^2 + d^2, 0] && !GtQ[n, 0]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^n}{a-ia\tan(e+fx)}\,dx = \frac{(d\tan(e+fx))^{1+n}}{2df(a-ia\tan(e+fx))} - \frac{\int (d\tan(e+fx))^n(-ad(1-n)+iadn\tan(e+fx))\,d$$

$$= \frac{(d\tan(e+fx))^{1+n}}{2df(a-ia\tan(e+fx))} + \frac{(1-n)\int (d\tan(e+fx))^n\,dx}{2a} - \frac{(in)\int (d\tan(e+fx)}{2ad}$$

$$= \frac{(d\tan(e+fx))^{1+n}}{2df(a-ia\tan(e+fx))} + \frac{(d(1-n))\text{Subst}\left(\int \frac{x^n}{d^2+x^2}\,dx, x, d\tan(e+fx)\right)}{2af} - \frac{(in)}{}$$

$$= \frac{(1-n)\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)(d\tan(e+fx))^{1+n}}{2adf(1+n)} - \frac{in\,{}_2F_1\left(1,\frac{2+n}{2};\frac{4+n}{2}\right.}{}$$

Mathematica [A]
time = 0.77, size = 123, normalized size = 0.78

$$\frac{\tan(e+fx)(d\tan(e+fx))^n\left(-\frac{(-1+n)\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)}{a(1+n)} - \frac{in\,{}_2F_1\left(1,\frac{2+n}{2};\frac{4+n}{2};-\tan^2(e+fx)\right)\tan(e+fx)}{a(2+n)} + \frac{1}{a-ia\tan(e+fx)}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^n/(a - I*a*Tan[e + f*x]),x]
```

```
[Out] (Tan[e + f*x]*(d*Tan[e + f*x])^n*(-(((-1 + n)*Hypergeometric2F1[1, (1 + n)/
2, (3 + n)/2, -Tan[e + f*x]^2])/(a*(1 + n))) - (I*n*Hypergeometric2F1[1, (2
 + n)/2, (4 + n)/2, -Tan[e + f*x]^2]*Tan[e + f*x])/(a*(2 + n)) + (a - I*a*T
an[e + f*x])^(-1)))/(2*f)
```

Maple [F]
time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{(d\tan(fx+e))^n}{a-ia\tan(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^n/(a-I*a*tan(f*x+e)),x)
```

```
[Out] int((d*tan(f*x+e))^n/(a-I*a*tan(f*x+e)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n/(a-I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n/(a-I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/2*((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))^n
*(e^(2*I*f*x + 2*I*e) + 1)/a, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i \int \frac{(d \tan{(e+fx)})^n}{\tan{(e+fx)}+i}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**n/(a-I*a*tan(f*x+e)),x)
```

```
[Out] I*Integral((d*tan(e + f*x))**n/(tan(e + f*x) + I), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n/(a-I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*tan(f*x + e))^n/(-I*a*tan(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\tan(e+f\,x))^n}{a-a\tan\left(e+f\,x\right)\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^n/(a - a*tan(e + f*x)*1i),x)
[Out] int((d*tan(e + f*x))^n/(a - a*tan(e + f*x)*1i), x)
```

3.320 $\int (d\tan(e+fx))^n(a+ia\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=89

$$\frac{aF_1\left(1+n;-\frac{1}{2},1;2+n;-i\tan(e+fx),i\tan(e+fx)\right)(d\tan(e+fx))^{1+n}\sqrt{a+ia\tan(e+fx)}}{df(1+n)\sqrt{1+i\tan(e+fx)}}$$

```
[Out] a*AppellF1(1+n,-1/2,1,2+n,-I*tan(f*x+e),I*tan(f*x+e))*(a+I*a*tan(f*x+e))^(1
/2)*(d*tan(f*x+e))^(1+n)/d/f/(1+n)/(1+I*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3645, 140, 138}

$$\frac{a\sqrt{a+ia\tan(e+fx)}\,F_1\left(n+1;-\frac{1}{2},1;n+2;-i\tan(e+fx),i\tan(e+fx)\right)(d\tan(e+fx))^{n+1}}{df(n+1)\sqrt{1+i\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^n*(a + I*a*Tan[e + f*x])^(3/2),x]
```

```
[Out] (a*AppellF1[1 + n, -1/2, 1, 2 + n, (-I)*Tan[e + f*x], I*Tan[e + f*x]]*(d*Ta
n[e + f*x])^(1 + n)*Sqrt[a + I*a*Tan[e + f*x]])/(d*f*(1 + n)*Sqrt[1 + I*Tan
[e + f*x]])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
```

2, 0]

Rubi steps

$$\int (d\tan(e+fx))^n(a+ia\tan(e+fx))^{3/2}\,dx = \frac{(ia^2)\,\text{Subst}\left(\int \frac{\left(-\frac{idx}{a}\right)^n\sqrt{a+x}}{-a^2+ax}\,dx, x, ia\tan(e+fx)\right)}{f}$$

$$= \frac{\left(ia^2\sqrt{a+ia\tan(e+fx)}\right)\text{Subst}\left(\int \frac{\left(-\frac{idx}{a}\right)^n\sqrt{1+\frac{x}{a}}}{-a^2+ax}\,dx, x$$

$$= \frac{aF_1\left(1+n;-\frac{1}{2},1;2+n;-i\tan(e+fx),i\tan(e+fx)\right)(d\tan$$

$$df(1+n)\sqrt{1+i\tan(e+f}$$

Mathematica [F]
time = 2.24, size = 0, normalized size = 0.00

$$\int (d\tan(e+fx))^n(a+ia\tan(e+fx))^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Tan[e + f*x])^n*(a + I*a*Tan[e + f*x])^(3/2),x]
[Out] Integrate[(d*Tan[e + f*x])^n*(a + I*a*Tan[e + f*x])^(3/2), x]
```

Maple [F]
time = 0.95, size = 0, normalized size = 0.00

$$\int (d\tan(fx+e))^n(a+ia\tan(fx+e))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e))^(3/2),x)
[Out] int((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^(3/2)*(d*tan(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(2*sqrt(2)*a*((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e)
 + 1))^n*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*e^(3*I*f*x + 3*I*e)/(e^(2*I*f*x
+ 2*I*e) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\tan{(e+fx)})^n\,(ia(\tan{(e+fx)}-i))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**n*(a+I*a*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*tan(e + f*x))**n*(I*a*(tan(e + f*x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^(3/2)*(d*tan(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\tan(e+f\,x))^n\,\left(a+a\tan(e+f\,x)\,1\mathrm{i}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^n*(a + a*tan(e + f*x)*1i)^(3/2),x)
```

```
[Out] int((d*tan(e + f*x))^n*(a + a*tan(e + f*x)*1i)^(3/2), x)
```

3.321 $\int (d\tan(e+fx))^n \sqrt{a+ia\tan(e+fx)}\, dx$

Optimal. Leaf size=89

$$\frac{aF_1\left(1+n;\frac{1}{2},1;2+n;-i\tan(e+fx),i\tan(e+fx)\right)\sqrt{1+i\tan(e+fx)}\,(d\tan(e+fx))^{1+n}}{df(1+n)\sqrt{a+ia\tan(e+fx)}}$$

```
[Out] a*AppellF1(1+n,1/2,1,2+n,-I*tan(f*x+e),I*tan(f*x+e))*(1+I*tan(f*x+e))^(1/2)
*(d*tan(f*x+e))^(1+n)/d/f/(1+n)/(a+I*a*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3645, 140, 138}

$$\frac{a\sqrt{1+i\tan(e+fx)}\,F_1\left(n+1;\frac{1}{2},1;n+2;-i\tan(e+fx),i\tan(e+fx)\right)(d\tan(e+fx))^{n+1}}{df(n+1)\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^n*Sqrt[a + I*a*Tan[e + f*x]],x]
```

```
[Out] (a*AppellF1[1 + n, 1/2, 1, 2 + n, (-I)*Tan[e + f*x], I*Tan[e + f*x]]*Sqrt[1
 + I*Tan[e + f*x]]*(d*Tan[e + f*x])^(1 + n))/(d*f*(1 + n)*Sqrt[a + I*a*Tan[
e + f*x]])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
```

```
2, 0]
```

Rubi steps

$$\int (d\tan(e+fx))^n\sqrt{a+ia\tan(e+fx)}\,dx = \frac{(ia^2)\,\text{Subst}\left(\int \frac{\left(-\frac{idx}{a}\right)^n}{\sqrt{a+x}\,(-a^2+ax)}\,dx, x, ia\tan(e+fx)\right)}{f}$$

$$= \frac{\left(ia^2\sqrt{1+i\tan(e+fx)}\right)\text{Subst}\left(\int \frac{\left(-\frac{idx}{a}\right)^n}{\sqrt{1+\frac{x}{a}}\,(-a^2+ax)}\,dx, x,$$

$$= \frac{aF_1\left(1+n;\frac{1}{2},1;2+n;-i\tan(e+fx),i\tan(e+fx)\right)\sqrt{1+}}{df(1+n)\sqrt{a+ia\tan(e+fx}}$$

Mathematica [F]
time = 1.16, size = 0, normalized size = 0.00

$$\int (d\tan(e+fx))^n\sqrt{a+ia\tan(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Tan[e + f*x])^n*Sqrt[a + I*a*Tan[e + f*x]],x]
[Out] Integrate[(d*Tan[e + f*x])^n*Sqrt[a + I*a*Tan[e + f*x]], x]
```

Maple [F]
time = 1.31, size = 0, normalized size = 0.00

$$\int \left(d\tan\left(fx+e\right)\right)^n\sqrt{a+ia\tan\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e))^(1/2),x)
[Out] int((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*tan(f*x + e) + a)*(d*tan(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2)*((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1
))^n*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*e^(I*f*x + I*e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\tan(e+fx))^n\sqrt{ia(\tan(e+fx)-i)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**n*(a+I*a*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral((d*tan(e + f*x))**n*sqrt(I*a*(tan(e + f*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(f*x + e) + a)*(d*tan(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\tan(e+f\,x))^n\sqrt{a+a\tan(e+f\,x)\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^n*(a + a*tan(e + f*x)*1i)^(1/2),x)
```

```
[Out] int((d*tan(e + f*x))^n*(a + a*tan(e + f*x)*1i)^(1/2), x)
```

3.322 $\int \frac{(d\tan(e+fx))^n}{\sqrt{a+ia\tan(e+fx)}}\,dx$

Optimal. Leaf size=88

$$\frac{F_1\left(1+n;\frac{3}{2},1;2+n;-i\tan(e+fx),i\tan(e+fx)\right)\sqrt{1+i\tan(e+fx)}\,(d\tan(e+fx))^{1+n}}{df(1+n)\sqrt{a+ia\tan(e+fx)}}$$

```
[Out] AppellF1(1+n,3/2,1,2+n,-I*tan(f*x+e),I*tan(f*x+e))*(1+I*tan(f*x+e))^(1/2)*(
d*tan(f*x+e))^(1+n)/d/f/(1+n)/(a+I*a*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3645, 140, 138}

$$\frac{\sqrt{1+i\tan(e+fx)}\,F_1\left(n+1;\frac{3}{2},1;n+2;-i\tan(e+fx),i\tan(e+fx)\right)(d\tan(e+fx))^{n+1}}{df(n+1)\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^n/Sqrt[a + I*a*Tan[e + f*x]],x]
```

```
[Out] (AppellF1[1 + n, 3/2, 1, 2 + n, (-I)*Tan[e + f*x], I*Tan[e + f*x]]*Sqrt[1 +
 I*Tan[e + f*x]]*(d*Tan[e + f*x])^(1 + n))/(d*f*(1 + n)*Sqrt[a + I*a*Tan[e
+ f*x]])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
```

```
2, 0]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^n}{\sqrt{a+ia\tan(e+fx)}}\,dx = \frac{(ia^2)\,\mathrm{Subst}\left(\int \frac{\left(-\frac{idx}{a}\right)^n}{(a+x)^{3/2}(-a^2+ax)}\,dx, x, ia\tan(e+fx)\right)}{f}$$

$$= \frac{\left(ia\sqrt{1+i\tan(e+fx)}\right)\mathrm{Subst}\left(\int \frac{\left(-\frac{idx}{a}\right)^n}{\left(1+\frac{x}{a}\right)^{3/2}(-a^2+ax)}\,dx, x, ia\tan(e+fx)\right)}{f\sqrt{a+ia\tan(e+fx)}}$$

$$= \frac{F_1\left(1+n;\frac{3}{2},1;2+n;-i\tan(e+fx),i\tan(e+fx)\right)\sqrt{1+i\tan(e+fx)}\,(d\,\mathrm{ta}}{df(1+n)\sqrt{a+ia\tan(e+fx)}}$$

Mathematica [**F**]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(d*Tan[e + f*x])^n/Sqrt[a + I*a*Tan[e + f*x]],x]
[Out] $Aborted
```

Maple [**F**]
time = 1.32, size = 0, normalized size = 0.00

$$\int \frac{(d\tan(fx+e))^n}{\sqrt{a+ia\tan(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^(1/2),x)
[Out] int((d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(f*x + e))^n/sqrt(I*a*tan(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/2*sqrt(2)*((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e)
 + 1))^n*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*(e^(2*I*f*x + 2*I*e) + 1)*e^(-I*
f*x - I*e)/a, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\tan(e+fx))^n}{\sqrt{ia(\tan(e+fx)-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**n/(a+I*a*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral((d*tan(e + f*x))**n/sqrt(I*a*(tan(e + f*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*tan(f*x + e))^n/sqrt(I*a*tan(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\tan(e+f\,x))^n}{\sqrt{a+a\tan(e+f\,x)\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^n/(a + a*tan(e + f*x)*1i)^(1/2),x)
```

```
[Out] int((d*tan(e + f*x))^n/(a + a*tan(e + f*x)*1i)^(1/2), x)
```

3.323 $\int \frac{(d\tan(e+fx))^n}{(a+ia\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=91

$$\frac{F_1\left(1+n;\frac{5}{2},1;2+n;-i\tan(e+fx),i\tan(e+fx)\right)\sqrt{1+i\tan(e+fx)}\,(d\tan(e+fx))^{1+n}}{adf(1+n)\sqrt{a+ia\tan(e+fx)}}$$

```
[Out] AppellF1(1+n,5/2,1,2+n,-I*tan(f*x+e),I*tan(f*x+e))*(1+I*tan(f*x+e))^(1/2)*(
d*tan(f*x+e))^(1+n)/a/d/f/(1+n)/(a+I*a*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3645, 140, 138}

$$\frac{\sqrt{1+i\tan(e+fx)}\,F_1\left(n+1;\frac{5}{2},1;n+2;-i\tan(e+fx),i\tan(e+fx)\right)(d\tan(e+fx))^{n+1}}{adf(n+1)\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^n/(a + I*a*Tan[e + f*x])^(3/2),x]
```

```
[Out] (AppellF1[1 + n, 5/2, 1, 2 + n, (-I)*Tan[e + f*x], I*Tan[e + f*x]]*Sqrt[1 +
 I*Tan[e + f*x]]*(d*Tan[e + f*x])^(1 + n))/(a*d*f*(1 + n)*Sqrt[a + I*a*Tan[
e + f*x]])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
```

```
2, 0]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^n}{(a+ia\tan(e+fx))^{3/2}}\,dx = \frac{(ia^2)\,\text{Subst}\left(\int \frac{\left(-\frac{idx}{a}\right)^n}{(a+x)^{5/2}(-a^2+ax)}\,dx, x, ia\tan(e+fx)\right)}{f}$$

$$= \frac{\left(i\sqrt{1+i\tan(e+fx)}\right)\text{Subst}\left(\int \frac{\left(-\frac{idx}{a}\right)^n}{\left(1+\frac{x}{a}\right)^{5/2}(-a^2+ax)}\,dx, x, ia\tan(e+fx)\right)}{f\sqrt{a+ia\tan(e+fx)}}$$

$$= \frac{F_1\left(1+n;\frac{5}{2},1;2+n;-i\tan(e+fx),i\tan(e+fx)\right)\sqrt{1+i\tan(e+fx)}\,(c}{adf(1+n)\sqrt{a+ia\tan(e+fx)}}$$

Mathematica [F]
time = 35.58, size = 0, normalized size = 0.00

$$\int \frac{(d\tan(e+fx))^n}{(a+ia\tan(e+fx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Tan[e + f*x])^n/(a + I*a*Tan[e + f*x])^(3/2),x]
[Out] Integrate[(d*Tan[e + f*x])^n/(a + I*a*Tan[e + f*x])^(3/2), x]
```

Maple [F]
time = 1.24, size = 0, normalized size = 0.00

$$\int \frac{(d\tan(fx+e))^n}{(a+ia\tan(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^(3/2),x)
[Out] int((d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(1/4*sqrt(2)*((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e)
 + 1))^n*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*(e^(4*I*f*x + 4*I*e) + 2*e^(2*I*
f*x + 2*I*e) + 1)*e^(-3*I*f*x - 3*I*e)/a^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\tan{(e+fx)})^n}{(ia(\tan{(e+fx)}-i))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**n/(a+I*a*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*tan(e + f*x))**n/(I*a*(tan(e + f*x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*tan(f*x + e))^n/(I*a*tan(f*x + e) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\tan(e+f\,x))^n}{(a+a\tan{(e+f\,x)}\,1\mathrm{i})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^n/(a + a*tan(e + f*x)*1i)^(3/2),x)
```

```
[Out] int((d*tan(e + f*x))^n/(a + a*tan(e + f*x)*1i)^(3/2), x)
```

3.324 $\int (d\tan(e+fx))^n(a+ia\tan(e+fx))^m\,dx$

Optimal. Leaf size=88

$$\frac{F_1(1+n;1-m,1;2+n;-i\tan(e+fx),i\tan(e+fx))(1+i\tan(e+fx))^{-m}(d\tan(e+fx))^{1+n}(a+ia\tan(e+fx))^m}{df(1+n)}$$

```
[Out] AppellF1(1+n,1-m,1,2+n,-I*tan(f*x+e),I*tan(f*x+e))*(d*tan(f*x+e))^(1+n)*(a+
I*a*tan(f*x+e))^m/d/f/(1+n)/((1+I*tan(f*x+e))^m)
```

Rubi [A]
time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3645, 140, 138}

$$\frac{(1+i\tan(e+fx))^{-m}(a+ia\tan(e+fx))^m(d\tan(e+fx))^{n+1}F_1(n+1;1-m,1;n+2;-i\tan(e+fx),i\tan(e+fx))}{df(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^n*(a + I*a*Tan[e + f*x])^m,x]
```

```
[Out] (AppellF1[1 + n, 1 - m, 1, 2 + n, (-I)*Tan[e + f*x], I*Tan[e + f*x]]*(d*Tan
[e + f*x])^(1 + n)*(a + I*a*Tan[e + f*x])^m)/(d*f*(1 + n)*(1 + I*Tan[e + f*
x])^m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\int (d\tan(e+fx))^n(a+ia\tan(e+fx))^m\,dx = \frac{(ia^2)\,\mathrm{Subst}\left(\int \frac{\left(-\frac{idx}{a}\right)^n (a+x)^{-1+m}}{-a^2+ax}\,dx, x, ia\tan(e+fx)\right)}{f}$$

$$= \frac{(ia(1+i\tan(e+fx))^{-m}(a+ia\tan(e+fx))^m)\,\mathrm{Subst}\left(\int \frac{\left(-\frac{i}{}\right.}{}\right.}{f}$$

$$= \frac{F_1(1+n;1-m,1;2+n;-i\tan(e+fx),i\tan(e+fx))(1+}{df(1+}$$

Mathematica [F]
time = 5.48, size = 0, normalized size = 0.00

$$\int (d\tan(e+fx))^n(a+ia\tan(e+fx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Tan[e + f*x])^n*(a + I*a*Tan[e + f*x])^m,x]
[Out] Integrate[(d*Tan[e + f*x])^n*(a + I*a*Tan[e + f*x])^m, x]
```

Maple [F]
time = 0.78, size = 0, normalized size = 0.00

$$\int \left(d\tan\left(fx+e\right)\right)^{n}\left(a+ia\tan\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e))^m,x)
[Out] int((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((I*a*tan(f*x + e) + a)^m*(d*tan(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e))^m,x, algorithm="fricas")
[Out] integral((2*a*e^(2*I*f*x + 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1))^m*((-I*d*e^(2*
I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\tan{(e+fx)})^n\,(ia(\tan{(e+fx)}-i))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**n*(a+I*a*tan(f*x+e))**m,x)
[Out] Integral((d*tan(e + f*x))**n*(I*a*(tan(e + f*x) - I))**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+I*a*tan(f*x+e))^m,x, algorithm="giac")
[Out] integrate((I*a*tan(f*x + e) + a)^m*(d*tan(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\tan(e+f\,x))^n\,(a+a\tan(e+f\,x)\;1\mathrm{i})^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^n*(a + a*tan(e + f*x)*1i)^m,x)
[Out] int((d*tan(e + f*x))^n*(a + a*tan(e + f*x)*1i)^m, x)
```

3.325 $\int \tan^4(c+dx)(a+ia\tan(c+dx))^m\,dx$

Optimal. Leaf size=205

$$\frac{2i(a+ia\tan(c+dx))^m}{d\,(6+5m+m^2)}-\frac{i\,{}_2F_1\left(1,m;1+m;\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^m}{2dm}-\frac{im\tan^2(c+dx)(a}{d\,(6+5}$$

```
[Out] 2*I*(a+I*a*tan(d*x+c))^m/d/(m^2+5*m+6)-1/2*I*hypergeom([1, m],[1+m],1/2+1/2
*I*tan(d*x+c))*(a+I*a*tan(d*x+c))^m/d/m-I*m*tan(d*x+c)^2*(a+I*a*tan(d*x+c))
^m/d/(m^2+5*m+6)+tan(d*x+c)^3*(a+I*a*tan(d*x+c))^m/d/(3+m)+I*(m^2+3*m+6)*(a
+I*a*tan(d*x+c))^(1+m)/a/d/(3+m)/(m^2+3*m+2)
```

Rubi [A]

time = 0.25, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3641, 3678, 3673, 3608, 3562, 70}

$$-\frac{i(a+ia\tan(c+dx))^m\,{}_2F_1\left(1,m;m+1;\frac{1}{2}(i\tan(c+dx)+1)\right)}{2dm}-\frac{im\tan^2(c+dx)(a+ia\tan(c+dx))^m}{d\,(m^2+5m+6)}+\frac{2i(a+ia\tan(c+dx))^m}{d\,(m^2+5m+6)}+\frac{i(m^2+3m+6)\,(a+ia\tan(c+dx))^{m+1}}{ad(m+1)(m+2)(m+3)}+\frac{\tan^3(c+dx)(a+ia\tan(c+dx))^m}{d(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4*(a + I*a*Tan[c + d*x])^m,x]
```

```
[Out] ((2*I)*(a + I*a*Tan[c + d*x])^m)/(d*(6 + 5*m + m^2)) - ((I/2)*Hypergeometri
c2F1[1, m, 1 + m, (1 + I*Tan[c + d*x])/2]*(a + I*a*Tan[c + d*x])^m)/(d*m) -
 (I*m*Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^m)/(d*(6 + 5*m + m^2)) + (Tan[c
 + d*x]^3*(a + I*a*Tan[c + d*x])^m)/(d*(3 + m)) + (I*(6 + 3*m + m^2)*(a + I
*a*Tan[c + d*x])^(1 + m))/(a*d*(1 + m)*(2 + m)*(3 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rule 3641

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[d*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n - 1)/(f*(m + n - 1))), x] - Dist[1/(a*(m + n - 1)), Int[(a + b*
Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 2)*Simp[d*(b*c*m + a*d*(-1 + n))
- a*c^2*(m + n - 1) + d*(b*d*m - a*c*(m + 2*n - 2))*Tan[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1] && NeQ[m + n - 1, 0] && (IntegerQ[n]
|| IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rule 3678

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Dist[
1/(a*(m + n)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Simp
[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \tan^4(c+dx)(a+ia\tan(c+dx))^m\,dx = \frac{\tan^3(c+dx)(a+ia\tan(c+dx))^m}{d(3+m)} - \frac{\int \tan^2(c+dx)(a+ia\tan(c

```
[In] integrate(tan(d*x+c)^4*(a+I*a*tan(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^m*tan(d*x + c)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4*(a+I*a*tan(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*(e^(8*I*d*x
+ 8*I*c) - 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I*c) - 4*e^(2*I*d*x + 2
*I*c) + 1)/(e^(8*I*d*x + 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*
I*c) + 4*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c+dx)-i))^m \tan^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4*(a+I*a*tan(d*x+c))**m,x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**m*tan(c + d*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4*(a+I*a*tan(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^m*tan(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c+d\,x)^4\,\left(a+a\,\tan(c+d\,x)\;1\mathrm{i}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + a*tan(c + d*x)*1i)^m,x)
```

```
[Out] int(tan(c + d*x)^4*(a + a*tan(c + d*x)*1i)^m, x)
```

3.326 $\int \tan^3(c+dx)(a+ia\tan(c+dx))^m\,dx$

Optimal. Leaf size=144

$$-\frac{2(a+ia\tan(c+dx))^m}{dm(2+m)}+\frac{{}_2F_1\left(1,m;1+m;\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^m}{2dm}+\frac{\tan^2(c+dx)(a+ia\tan(c+dx))^m}{d(2+m)}$$

```
[Out] -2*(a+I*a*tan(d*x+c))^m/d/m/(2+m)+1/2*hypergeom([1, m],[1+m],1/2+1/2*I*tan(
d*x+c))*(a+I*a*tan(d*x+c))^m/d/m+tan(d*x+c)^2*(a+I*a*tan(d*x+c))^m/d/(2+m)-
m*(a+I*a*tan(d*x+c))^(1+m)/a/d/(m^2+3*m+2)
```

Rubi [A]
time = 0.14, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3641, 3673, 3608, 3562, 70}

$$\frac{(a+ia\tan(c+dx))^m\,{}_2F_1\left(1,m;m+1;\frac{1}{2}(i\tan(c+dx)+1)\right)}{2dm}-\frac{m(a+ia\tan(c+dx))^{m+1}}{ad(m^2+3m+2)}+\frac{\tan^2(c+dx)(a+ia\tan(c+dx))^m}{d(m+2)}-\frac{2(a+ia\tan(c+dx))^m}{dm(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3*(a + I*a*Tan[c + d*x])^m,x]
```

```
[Out] (-2*(a + I*a*Tan[c + d*x])^m)/(d*m*(2 + m)) + (Hypergeometric2F1[1, m, 1 +
m, (1 + I*Tan[c + d*x])/2]*(a + I*a*Tan[c + d*x])^m)/(2*d*m) + (Tan[c + d*x
]^2*(a + I*a*Tan[c + d*x])^m)/(d*(2 + m)) - (m*(a + I*a*Tan[c + d*x])^(1 +
m))/(a*d*(2 + 3*m + m^2))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rule 3641

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[d*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n - 1)/(f*(m + n - 1))), x] - Dist[1/(a*(m + n - 1)), Int[(a + b*
Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 2)*Simp[d*(b*c*m + a*d*(-1 + n))
- a*c^2*(m + n - 1) + d*(b*d*m - a*c*(m + 2*n - 2))*Tan[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1] && NeQ[m + n - 1, 0] && (IntegerQ[n]
|| IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \tan^3(c+dx)(a+ia\tan(c+dx))^m\,dx = \frac{\tan^2(c+dx)(a+ia\tan(c+dx))^m}{d(2+m)} - \frac{\int \tan(c+dx)(a+ia\tan(\ldots}{\ldots}$$

$$= \frac{\tan^2(c+dx)(a+ia\tan(c+dx))^m}{d(2+m)} - \frac{m(a+ia\tan(c+dx))^{1+m\ldots}}{ad\left(2+3m+m^2\right)}$$

$$= -\frac{2(a+ia\tan(c+dx))^m}{dm(2+m)} + \frac{\tan^2(c+dx)(a+ia\tan(c+dx))^m}{d(2+m)} \ldots$$

$$= -\frac{2(a+ia\tan(c+dx))^m}{dm(2+m)} + \frac{\tan^2(c+dx)(a+ia\tan(c+dx))^m}{d(2+m)} \ldots$$

$$= -\frac{2(a+ia\tan(c+dx))^m}{dm(2+m)} + \frac{{}_2F_1\left(1,m;1+m;\frac{1}{2}(1+i\tan(c+dx\ldots\right.}{2dm}$$

Mathematica [F]
time = 33.17, size = 0, normalized size = 0.00

$$\int \tan^3(c+dx)(a+ia\tan(c+dx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^3*(a + I*a*Tan[c + d*x])^m,x]
```

```
[Out] Integrate[Tan[c + d*x]^3*(a + I*a*Tan[c + d*x])^m, x]
```

Maple [F]
time = 1.09, size = 0, normalized size = 0.00

$$\int \left(\tan^{3}\left(dx+c\right)\right)\left(a+ia\tan\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^m,x)
```

```
[Out] int(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^m*tan(d*x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*(I*e^(6*I*d*
x + 6*I*c) - 3*I*e^(4*I*d*x + 4*I*c) + 3*I*e^(2*I*d*x + 2*I*c) - I)/(e^(6*I
*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ia(\tan\left(c+dx\right)-i)\right)^{m}\tan^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3*(a+I*a*tan(d*x+c))**m,x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**m*tan(c + d*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^m,x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^m*tan(d*x + c)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^3\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + a*tan(c + d*x)*1i)^m,x)
[Out] int(tan(c + d*x)^3*(a + a*tan(c + d*x)*1i)^m, x)
```

3.327 $\int \tan^2(c+dx)(a+ia\tan(c+dx))^m\,dx$

Optimal. Leaf size=82

$$\frac{i\,{}_2F_1\left(1,m;1+m;\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^m}{2dm}-\frac{i(a+ia\tan(c+dx))^{1+m}}{ad(1+m)}$$

```
[Out] 1/2*I*hypergeom([1, m],[1+m],1/2+1/2*I*tan(d*x+c))*(a+I*a*tan(d*x+c))^m/d/m
-I*(a+I*a*tan(d*x+c))^(1+m)/a/d/(1+m)
```

Rubi [A]
time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3624, 3562, 70}

$$\frac{i(a+ia\tan(c+dx))^m\,{}_2F_1\left(1,m;m+1;\frac{1}{2}(i\tan(c+dx)+1)\right)}{2dm}-\frac{i(a+ia\tan(c+dx))^{m+1}}{ad(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^m,x]
```

```
[Out] ((I/2)*Hypergeometric2F1[1, m, 1 + m, (1 + I*Tan[c + d*x])/2]*(a + I*a*Tan[
c + d*x])^m)/(d*m) - (I*(a + I*a*Tan[c + d*x])^(1 + m))/(a*d*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^m\,dx = -\frac{i(a+ia\tan(c+dx))^{1+m}}{ad(1+m)} - \int (a+ia\tan(c+dx))^m\,dx$$
$$= -\frac{i(a+ia\tan(c+dx))^{1+m}}{ad(1+m)} + \frac{(ia)\text{Subst}\left(\int \frac{(a+x)^{-1+m}}{a-x}\,dx, x, ia\tan\right.}{d}$$
$$= \frac{i\,{}_2F_1\left(1,m;1+m;\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^m}{2dm} -$$

Mathematica [F]
time = 16.30, size = 0, normalized size = 0.00

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^m,x]
[Out] Integrate[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^m, x]
```

Maple [F]
time = 0.99, size = 0, normalized size = 0.00

$$\int \left(\tan^2(dx+c)\right)(a+ia\tan(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^m,x)
[Out] int(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^m*tan(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral(-(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*(e^(4*I*d*x
 + 4*I*c) - 2*e^(2*I*d*x + 2*I*c) + 1)/(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x
+ 2*I*c) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan{(c+dx)}-i))^m \tan^2{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2*(a+I*a*tan(d*x+c))**m,x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**m*tan(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^m*tan(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^2\,\left(a+a\,\tan(c+d\,x)\,1\mathrm{i}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + a*tan(c + d*x)*1i)^m,x)
```

```
[Out] int(tan(c + d*x)^2*(a + a*tan(c + d*x)*1i)^m, x)
```

3.328 $\int \tan(c+dx)(a+ia\tan(c+dx))^m\,dx$

Optimal. Leaf size=70

$$\frac{(a+ia\tan(c+dx))^m}{dm}-\frac{{}_2F_1\left(1,m;1+m;\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^m}{2dm}$$

```
[Out] (a+I*a*tan(d*x+c))^m/d/m-1/2*hypergeom([1, m],[1+m],1/2+1/2*I*tan(d*x+c))*(
a+I*a*tan(d*x+c))^m/d/m
```

Rubi [A]
time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.136$, Rules used = {3608, 3562, 70}

$$\frac{(a+ia\tan(c+dx))^m}{dm}-\frac{(a+ia\tan(c+dx))^m\,{}_2F_1\left(1,m;m+1;\frac{1}{2}(i\tan(c+dx)+1)\right)}{2dm}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]*(a + I*a*Tan[c + d*x])^m,x]
```

```
[Out] (a + I*a*Tan[c + d*x])^m/(d*m) - (Hypergeometric2F1[1, m, 1 + m, (1 + I*Tan
[c + d*x])/2]*(a + I*a*Tan[c + d*x])^m)/(2*d*m)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \tan(c+dx)(a+ia\tan(c+dx))^m\,dx &= \frac{(a+ia\tan(c+dx))^m}{dm} - i\int (a+ia\tan(c+dx))^m\,dx \\
&= \frac{(a+ia\tan(c+dx))^m}{dm} - \frac{a\text{Subst}\left(\int \frac{(a+x)^{-1+m}}{a-x}\,dx, x, ia\tan(c+dx)\right)}{d} \\
&= \frac{(a+ia\tan(c+dx))^m}{dm} - \frac{{}_2F_1\left(1,m;1+m;\frac{1}{2}(1+i\tan(c+dx))\right)(a}{2dm}
\end{aligned}
$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 153 vs. 2(70) = 140.
time = 4.97, size = 153, normalized size = 2.19

$$\frac{2^{-1+m}\left(e^{idx}\right)^m\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^m\left(1+m-e^{2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^m m\,{}_2F_1\left(1+m,1+m;2+m;-e^{2i(c+dx)}\right)\right)\sec^{-m}(c+dx)(\cos(dx)+i\sin(dx))^{-m}(a+ia\tan(c+dx))^m}{dm(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]*(a + I*a*Tan[c + d*x])^m,x]
```

```
[Out] (2^(-1 + m)*(E^(I*d*x))^m*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^m*(1
+ m - E^((2*I)*(c + d*x))*(1 + E^((2*I)*(c + d*x)))^m*m*Hypergeometric2F1[1
 + m, 1 + m, 2 + m, -E^((2*I)*(c + d*x))])*(a + I*a*Tan[c + d*x])^m)/(d*m*(
1 + m)*Sec[c + d*x]^m*(Cos[d*x] + I*Sin[d*x])^m)
```

Maple [F]
time = 0.78, size = 0, normalized size = 0.00

$$\int \tan(dx+c)\left(a+ia\tan(dx+c)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)*(a+I*a*tan(d*x+c))^m,x)
```

```
[Out] int(tan(d*x+c)*(a+I*a*tan(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^m*tan(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^m,x, algorithm="fricas")
[Out] integral((2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*(-I*e^(2*I*d
*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c+dx)-i))^m \tan(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))**m,x)
[Out] Integral((I*a*(tan(c + d*x) - I))**m*tan(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^m,x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^m*tan(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + a*tan(c + d*x)*1i)^m,x)
[Out] int(tan(c + d*x)*(a + a*tan(c + d*x)*1i)^m, x)
```

3.329 $\int (a+ia\tan(c+dx))^m\,dx$

Optimal. Leaf size=49

$$-\frac{i\,{}_2F_1\left(1,m;1+m;\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^m}{2dm}$$

```
[Out] -1/2*I*hypergeom([1, m],[1+m],1/2+1/2*I*tan(d*x+c))*(a+I*a*tan(d*x+c))^m/d/
m
```

Rubi [A]
time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3562, 70}

$$-\frac{i(a+ia\tan(c+dx))^m\,{}_2F_1\left(1,m;m+1;\frac{1}{2}(i\tan(c+dx)+1)\right)}{2dm}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^m,x]
```

```
[Out] ((-1/2*I)*Hypergeometric2F1[1, m, 1 + m, (1 + I*Tan[c + d*x])/2]*(a + I*a*T
an[c + d*x])^m)/(d*m)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+ia\tan(c+dx))^m\,dx &= -\frac{(ia)\text{Subst}\left(\int \frac{(a+x)^{-1+m}}{a-x}\,dx, x, ia\tan(c+dx)\right)}{d}\\ &= -\frac{i\,{}_2F_1\left(1,m;1+m;\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^m}{2dm}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 128 vs. 2(49) = 98.
time = 0.50, size = 128, normalized size = 2.61

$$-\frac{i2^{-1+m}\left(e^{idx}\right)^m\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^m\left(1+e^{2i(c+dx)}\right)\,{}_2F_1\left(1,1;1+m;-e^{2i(c+dx)}\right)\sec^{-m}(c+dx)(\cos(dx)+i\sin(dx))^{-m}(a+ia\tan(c+dx))^m}{dm}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^m,x]
```

```
[Out] ((-I)*2^(-1 + m)*(E^(I*d*x))^m*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^
m*(1 + E^((2*I)*(c + d*x)))*Hypergeometric2F1[1, 1, 1 + m, -E^((2*I)*(c + d
*x))]*(a + I*a*Tan[c + d*x])^m)/(d*m*Sec[c + d*x]^m*(Cos[d*x] + I*Sin[d*x])
^m)
```

Maple [F]
time = 0.80, size = 0, normalized size = 0.00

$$\int (a+ia\tan(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^m,x)
```

```
[Out] int((a+I*a*tan(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia\tan(c+dx)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**m,x)
```

```
[Out] Integral((I*a*tan(c + d*x) + a)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (a+a\tan(c+d\,x)\;1\mathrm{i})^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^m,x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^m, x)
```

3.330 $\int \cot(c+dx)(a+ia\tan(c+dx))^m\,dx$

Optimal. Leaf size=89

$$\frac{{}_2F_1\left(1,m;1+m;\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^m}{2dm}-\frac{{}_2F_1(1,m;1+m;1+i\tan(c+dx))(a+ia\tan(c+dx))^m}{dm}$$

```
[Out] 1/2*hypergeom([1, m],[1+m],1/2+1/2*I*tan(d*x+c))*(a+I*a*tan(d*x+c))^m/d/m-h
ypergeom([1, m],[1+m],1+I*tan(d*x+c))*(a+I*a*tan(d*x+c))^m/d/m
```

Rubi [A]
time = 0.10, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.227$, Rules used = {3643, 3562, 70, 3680, 67}

$$\frac{(a+ia\tan(c+dx))^m\,{}_2F_1\left(1,m;m+1;\frac{1}{2}(i\tan(c+dx)+1)\right)}{2dm}-\frac{(a+ia\tan(c+dx))^m\,{}_2F_1(1,m;m+1;i\tan(c+dx)+1)}{dm}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^m,x]
```

```
[Out] (Hypergeometric2F1[1, m, 1 + m, (1 + I*Tan[c + d*x])/2]*(a + I*a*Tan[c + d*
x])^m)/(2*d*m) - (Hypergeometric2F1[1, m, 1 + m, 1 + I*Tan[c + d*x]]*(a + I
*a*Tan[c + d*x])^m)/(d*m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3643

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)/((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[a/(a*c - b*d), Int[(a + b*Tan[e + f*x])^m,
x], x] - Dist[d/(a*c - b*d), Int[(a + b*Tan[e + f*x])^m*((b + a*Tan[e + f*x
])/(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rubi steps

$$\int \cot(c+dx)(a+ia\tan(c+dx))^m\,dx = i\int (a+ia\tan(c+dx))^m\,dx - \frac{i\int \cot(c+dx)(a+ia\tan(c+dx))^{}}{a}$$

$$= \frac{a\text{Subst}\left(\int \frac{(a+x)^{-1+m}}{a-x}\,dx, x, ia\tan(c+dx)\right)}{d} + \frac{a\text{Subst}\left(\int \frac{(a+iax)^{-1+m}}{x}\right.}{d}$$

$$= \frac{{}_2F_1\left(1,m;1+m;\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^m}{2dm} - {}_2$$

Mathematica [F]
time = 10.67, size = 0, normalized size = 0.00

$$\int \cot(c+dx)(a+ia\tan(c+dx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^m,x]
[Out] Integrate[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^m, x]
```

Maple [F]
time = 1.58, size = 0, normalized size = 0.00

$$\int \cot(dx+c)(a+ia\tan(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+I*a*tan(d*x+c))^m,x)
```

```
[Out] int(cot(d*x+c)*(a+I*a*tan(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^m*cot(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^m,x, algorithm="fricas")
[Out] integral((2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*(I*e^(2*I*d*
x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan{(c+dx)}-i))^m \cot{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))**m,x)
[Out] Integral((I*a*(tan(c + d*x) - I))**m*cot(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^m,x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^m*cot(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + a*tan(c + d*x)*1i)^m,x)
[Out] int(cot(c + d*x)*(a + a*tan(c + d*x)*1i)^m, x)
```

3.331 $\int \cot^2(c+dx)(a+ia\tan(c+dx))^m\,dx$

Optimal. Leaf size=116

$$-\frac{\cot(c+dx)(a+ia\tan(c+dx))^m}{d}+\frac{i\,{}_2F_1\left(1,m;1+m;\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^m}{2dm}-\frac{i\,{}_2F_1(1,$$

```
[Out] -cot(d*x+c)*(a+I*a*tan(d*x+c))^m/d+1/2*I*hypergeom([1, m],[1+m],1/2+1/2*I*t
an(d*x+c))*(a+I*a*tan(d*x+c))^m/d/m-I*hypergeom([1, m],[1+m],1+I*tan(d*x+c)
)*(a+I*a*tan(d*x+c))^m/d
```

Rubi [A]
time = 0.17, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3642, 3681, 3562, 70, 3680, 67}

$$\frac{i(a+ia\tan(c+dx))^m\,{}_2F_1\left(1,m;m+1;\frac{1}{2}(i\tan(c+dx)+1)\right)}{2dm}-\frac{i(a+ia\tan(c+dx))^m\,{}_2F_1(1,m;m+1;i\tan(c+dx)+1)}{d}-\frac{\cot(c+dx)(a+ia\tan(c+dx))^m}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^m,x]
```

```
[Out] -((Cot[c + d*x]*(a + I*a*Tan[c + d*x])^m)/d) + ((I/2)*Hypergeometric2F1[1,
m, 1 + m, (1 + I*Tan[c + d*x])/2]*(a + I*a*Tan[c + d*x])^m)/(d*m) - (I*Hype
rgeometric2F1[1, m, 1 + m, 1 + I*Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^m)/d
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3642

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[d*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(a*(c^2 + d^2)*(n +
1)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[b*d*m - a
*c*(n + 1) + a*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
 e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0
] && LtQ[n, -1] && (IntegerQ[n] || IntegersQ[2*m, 2*n])
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3681

```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(
A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^m\,dx = -\frac{\cot(c+dx)(a+ia\tan(c+dx))^m}{d} + \frac{\int \cot(c+dx)(a+ia\tan(}{}$$
$$= -\frac{\cot(c+dx)(a+ia\tan(c+dx))^m}{d} + \frac{(im)\int \cot(c+dx)(a-ia}{}$$
$$= -\frac{\cot(c+dx)(a+ia\tan(c+dx))^m}{d} + \frac{(ia)\text{Subst}\left(\int \frac{(a+x)^{-1+m}}{a-x}\,dx\right.}{d}$$
$$= -\frac{\cot(c+dx)(a+ia\tan(c+dx))^m}{d} + \frac{i\,{}_2F_1\left(1,m;1+m;\frac{1}{2}(1+i\right.}{}$$

Mathematica [F]
time = 18.73, size = 0, normalized size = 0.00

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^m,x]
```

```
[Out] Integrate[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^m, x]
```

Maple [F]
time = 1.74, size = 0, normalized size = 0.00

$$\int \left(\cot^2\left(dx+c\right)\right)\left(a+ia\tan\left(dx+c\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^m,x)
```

```
[Out] int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^m*cot(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral(-(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*(e^(4*I*d*x
 + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)/(e^(4*I*d*x + 4*I*c) - 2*e^(2*I*d*x
+ 2*I*c) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ia\left(\tan\left(c+dx\right)-i\right)\right)^m\cot^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+I*a*tan(d*x+c))**m,x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**m*cot(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^m,x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^m*cot(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^2\,\left(a+a\tan(c+d\,x)\;1\mathrm{i}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + a*tan(c + d*x)*1i)^m,x)
[Out] int(cot(c + d*x)^2*(a + a*tan(c + d*x)*1i)^m, x)
```

3.332 $\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^m\,dx$

Optimal. Leaf size=81

$$\frac{2F_1\left(\frac{5}{2};1-m,1;\frac{7}{2};-i\tan(c+dx),i\tan(c+dx)\right)(1+i\tan(c+dx))^{-m}\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^m}{5d}$$

```
[Out] 2/5*AppellF1(5/2,1-m,1,7/2,-I*tan(d*x+c),I*tan(d*x+c))*tan(d*x+c)^(5/2)*(a+
I*a*tan(d*x+c))^m/d/((1+I*tan(d*x+c))^m)
```

Rubi [A]
time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3645, 129, 525, 524}

$$\frac{2\tan^{\frac{5}{2}}(c+dx)(1+i\tan(c+dx))^{-m}(a+ia\tan(c+dx))^m F_1\left(\frac{5}{2};1-m,1;\frac{7}{2};-i\tan(c+dx),i\tan(c+dx)\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^m,x]
```

```
[Out] (2*AppellF1[5/2, 1 - m, 1, 7/2, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*Tan[c +
d*x]^(5/2)*(a + I*a*Tan[c + d*x])^m)/(5*d*(1 + I*Tan[c + d*x])^m)
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^m\,dx = \frac{(ia^2)\,\text{Subst}\left(\int \frac{\left(-\frac{ix}{a}\right)^{3/2}(a+x)^{-1+m}}{-a^2+ax}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(2a^3)\,\text{Subst}\left(\int \frac{x^4(a+iax^2)^{-1+m}}{-a^2+ia^2x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d}$$

$$= -\frac{(2a^2(1+i\tan(c+dx))^{-m}(a+ia\tan(c+dx))^m)\,\text{Subst}\left(\int \frac{x^4(\cdots}{-\cdots}\right.}{d}$$

$$= \frac{2F_1\left(\frac{5}{2};1-m,1;\frac{7}{2};-i\tan(c+dx),i\tan(c+dx)\right)(1+i\tan(c+\cdots}{5d}$$

Mathematica [F]
time = 11.36, size = 0, normalized size = 0.00

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^m,x]
[Out] Integrate[Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^m, x]
```

Maple [F]
time = 1.65, size = 0, normalized size = 0.00

$$\int \left(\tan^{\frac{3}{2}}(dx+c)\right)(a+ia\tan(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^m,x)
[Out] int(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^m*tan(d*x + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^m,x, algorithm="fricas")
[Out] integral((2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*sqrt((-I*e^(
2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(-I*e^(2*I*d*x + 2*I*c) +
I)/(e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c+dx)-i))^m \tan^{\frac{3}{2}}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(3/2)*(a+I*a*tan(d*x+c))**m,x)
[Out] Integral((I*a*(tan(c + d*x) - I))**m*tan(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^m,x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^m*tan(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^{3/2}\,\left(a+a\,\tan(c+d\,x)\;1\mathrm{i}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^m,x)
[Out] int(tan(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^m, x)
```

3.333 $\int \sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^m\,dx$

Optimal. Leaf size=81

$$\frac{2F_1\left(\frac{3}{2};1-m,1;\frac{5}{2};-i\tan(c+dx),i\tan(c+dx)\right)(1+i\tan(c+dx))^{-m}\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^m}{3d}$$

```
[Out] 2/3*AppellF1(3/2,1-m,1,5/2,-I*tan(d*x+c),I*tan(d*x+c))*tan(d*x+c)^(3/2)*(a+
I*a*tan(d*x+c))^m/d/((1+I*tan(d*x+c))^m)
```

Rubi [A]

time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3645, 129, 525, 524}

$$\frac{2\tan^{\frac{3}{2}}(c+dx)(1+i\tan(c+dx))^{-m}(a+ia\tan(c+dx))^m F_1\left(\frac{3}{2};1-m,1;\frac{5}{2};-i\tan(c+dx),i\tan(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^m,x]
```

```
[Out] (2*AppellF1[3/2, 1 - m, 1, 5/2, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*Tan[c +
d*x]^(3/2)*(a + I*a*Tan[c + d*x])^m)/(3*d*(1 + I*Tan[c + d*x])^m)
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\int \sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^m\,dx = \frac{(ia^2)\,\text{Subst}\left(\int \frac{\sqrt{-\frac{ix}{a}}\,(a+x)^{-1+m}}{-a^2+ax}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(2a^3)\,\text{Subst}\left(\int \frac{x^2(a+iax^2)^{-1+m}}{-a^2+ia^2x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d}$$

$$= -\frac{(2a^2(1+i\tan(c+dx))^{-m}(a+ia\tan(c+dx))^m)\,\text{Subst}\left(\int \frac{x^2(\right.}{d}$$

$$= \frac{2F_1\left(\frac{3}{2}; 1-m, 1; \frac{5}{2}; -i\tan(c+dx), i\tan(c+dx)\right)(1+i\tan(c+}{3d}$$

Mathematica [F]
time = 5.21, size = 0, normalized size = 0.00

$$\int \sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^m,x]
[Out] Integrate[Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^m, x]
```

Maple [F]
time = 1.73, size = 0, normalized size = 0.00

$$\int \left(\sqrt{\tan}\,(dx+c)\right)(a+ia\tan(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^m,x)
```

```
[Out] int(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^m*sqrt(tan(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^m,x, algorithm="fricas")
[Out] integral((2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*sqrt((-I*e^(
2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c+dx)-i))^m \sqrt{\tan(c+dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/2)*(a+I*a*tan(d*x+c))**m,x)
[Out] Integral((I*a*(tan(c + d*x) - I))**m*sqrt(tan(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^m,x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^m*sqrt(tan(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\tan(c+d\,x)}\,\left(a+a\,\tan(c+d\,x)\,1{}\mathrm{i}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^m,x)
[Out] int(tan(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^m, x)
```

3.334 $\int \frac{(a+ia\tan(c+dx))^m}{\sqrt{\tan(c+dx)}}\,dx$

Optimal. Leaf size=79

$$\frac{2F_1\left(\frac{1}{2};1-m,1;\frac{3}{2};-i\tan(c+dx),i\tan(c+dx)\right)(1+i\tan(c+dx))^{-m}\sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^m}{d}$$

```
[Out] 2*AppellF1(1/2,1-m,1,3/2,-I*tan(d*x+c),I*tan(d*x+c))*tan(d*x+c)^(1/2)*(a+I*
a*tan(d*x+c))^m/d/((1+I*tan(d*x+c))^m)
```

Rubi [A]
time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3645, 129, 441, 440}

$$\frac{2\sqrt{\tan(c+dx)}\,(1+i\tan(c+dx))^{-m}(a+ia\tan(c+dx))^m F_1\left(\frac{1}{2};1-m,1;\frac{3}{2};-i\tan(c+dx),i\tan(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^m/Sqrt[Tan[c + d*x]],x]
```

```
[Out] (2*AppellF1[1/2, 1 - m, 1, 3/2, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*Sqrt[Tan
[c + d*x]]*(a + I*a*Tan[c + d*x])^m)/(d*(1 + I*Tan[c + d*x])^m)
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^m}{\sqrt{\tan(c+dx)}}\,dx = \frac{(ia^2)\,\text{Subst}\left(\int \frac{(a+x)^{-1+m}}{\sqrt{-\frac{ix}{a}}\,(-a^2+ax)}\,dx, x, ia\tan(c+dx)\right)}{d}$$
$$= -\frac{(2a^3)\,\text{Subst}\left(\int \frac{(a+iax^2)^{-1+m}}{-a^2+ia^2x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d}$$
$$= -\frac{(2a^2(1+i\tan(c+dx))^{-m}(a+ia\tan(c+dx))^m)\,\text{Subst}\left(\int \frac{(1+ix^2)^{-1+m}}{-a^2+ia^2x^2}\,dx, x\right.}{d}$$
$$= \frac{2F_1\left(\frac{1}{2}; 1-m, 1; \frac{3}{2}; -i\tan(c+dx), i\tan(c+dx)\right)(1+i\tan(c+dx))^{-m}\sqrt{\tan}}{d}$$

Mathematica [F]
time = 5.26, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(c+dx))^m}{\sqrt{\tan(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^m/Sqrt[Tan[c + d*x]],x]
[Out] Integrate[(a + I*a*Tan[c + d*x])^m/Sqrt[Tan[c + d*x]], x]
```

Maple [F]
time = 1.70, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(dx+c))^m}{\sqrt{\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^m/tan(d*x+c)^(1/2),x)
```

```
[Out] int((a+I*a*tan(d*x+c))^m/tan(d*x+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^m/tan(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^m/sqrt(tan(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^m/tan(d*x+c)^(1/2),x, algorithm="fricas")
[Out] integral((2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*sqrt((-I*e^(
2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^(2*I*d*x + 2*I*c) + I
)/(e^(2*I*d*x + 2*I*c) - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(c+dx)-i))^m}{\sqrt{\tan(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**m/tan(d*x+c)**(1/2),x)
[Out] Integral((I*a*(tan(c + d*x) - I))**m/sqrt(tan(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^m/tan(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^m/sqrt(tan(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^m}{\sqrt{\tan\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^m/tan(c + d*x)^(1/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^m/tan(c + d*x)^(1/2), x)
```

3.335 $\int \frac{(a+ia\tan(c+dx))^m}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=79

$$-\frac{2F_1\left(-\frac{1}{2};1-m,1;\frac{1}{2};-i\tan(c+dx),i\tan(c+dx)\right)(1+i\tan(c+dx))^{-m}(a+ia\tan(c+dx))^m}{d\sqrt{\tan(c+dx)}}$$

```
[Out] -2*AppellF1(-1/2,1-m,1,1/2,-I*tan(d*x+c),I*tan(d*x+c))*(a+I*a*tan(d*x+c))^m
/d/tan(d*x+c)^(1/2)/((1+I*tan(d*x+c))^m)
```

Rubi [A]
time = 0.09, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3645, 129, 525, 524}

$$-\frac{2(1+i\tan(c+dx))^{-m}(a+ia\tan(c+dx))^m F_1\left(-\frac{1}{2};1-m,1;\frac{1}{2};-i\tan(c+dx),i\tan(c+dx)\right)}{d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^m/Tan[c + d*x]^(3/2),x]
```

```
[Out] (-2*AppellF1[-1/2, 1 - m, 1, 1/2, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*(a + I
*a*Tan[c + d*x])^m)/(d*(1 + I*Tan[c + d*x])^m*Sqrt[Tan[c + d*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^m}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \frac{(ia^2)\,\mathrm{Subst}\left(\int \frac{(a+x)^{-1+m}}{\left(-\frac{ix}{a}\right)^{3/2}(-a^2+ax)}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(2a^3)\,\mathrm{Subst}\left(\int \frac{(a+iax^2)^{-1+m}}{x^2(-a^2+ia^2x^2)}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d}$$

$$= -\frac{(2a^2(1+i\tan(c+dx))^{-m}(a+ia\tan(c+dx))^m)\,\mathrm{Subst}\left(\int \frac{(1+ix^2)^{-1+m}}{x^2(-a^2+ia^2x^2)}\,dx,\right.}{d}$$

$$= -\frac{2F_1\left(-\frac{1}{2};1-m,1;\frac{1}{2};-i\tan(c+dx),i\tan(c+dx)\right)(1+i\tan(c+dx))^{-m}(}{d\sqrt{\tan(c+dx)}}$$

Mathematica [F]
time = 3.21, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(c+dx))^m}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^m/Tan[c + d*x]^(3/2),x]
[Out] Integrate[(a + I*a*Tan[c + d*x])^m/Tan[c + d*x]^(3/2), x]
```

Maple [F]
time = 1.64, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(dx+c))^m}{\tan(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^m/tan(d*x+c)^(3/2),x)
```

```
[Out] int((a+I*a*tan(d*x+c))^m/tan(d*x+c)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^m/tan(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^m/tan(d*x + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^m/tan(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*sqrt((-I*e^
(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*(e^(4*I*d*x + 4*I*c) + 2*
e^(2*I*d*x + 2*I*c) + 1)/(e^(4*I*d*x + 4*I*c) - 2*e^(2*I*d*x + 2*I*c) + 1),
 x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(c+dx)-i))^m}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**m/tan(d*x+c)**(3/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**m/tan(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^m/tan(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^m/tan(d*x + c)^(3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^m}{\tan\left(c+d\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^m/tan(c + d*x)^(3/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^m/tan(c + d*x)^(3/2), x)
```

3.336 $\int (d\tan(e+fx))^{5/2}(a+a\tan(e+fx))\,dx$

Optimal. Leaf size=115

$$\frac{\sqrt{2}\,ad^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{f}-\frac{2ad^2\sqrt{d\tan(e+fx)}}{f}+\frac{2ad(d\tan(e+fx))^{3/2}}{3f}+\frac{2a(d\tan(e+fx))^{5/2}}{5f}$$

```
[Out] a*d^(5/2)*arctanh(1/2*(d^(1/2)+d^(1/2)*tan(f*x+e))*2^(1/2)/(d*tan(f*x+e))^(
1/2))*2^(1/2)/f-2*a*d^2*(d*tan(f*x+e))^(1/2)/f+2/3*a*d*(d*tan(f*x+e))^(3/2)
/f+2/5*a*(d*tan(f*x+e))^(5/2)/f
```

Rubi [A]
time = 0.11, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3609, 3613, 214}

$$\frac{\sqrt{2}\,ad^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\tan(e+fx)+\sqrt{d}}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{f}-\frac{2ad^2\sqrt{d\tan(e+fx)}}{f}+\frac{2ad(d\tan(e+fx))^{3/2}}{3f}+\frac{2a(d\tan(e+fx))^{5/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(5/2)*(a + a*Tan[e + f*x]),x]
```

```
[Out] (Sqrt[2]*a*d^(5/2)*ArcTanh[(Sqrt[d] + Sqrt[d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[d
*Tan[e + f*x]])])/f - (2*a*d^2*Sqrt[d*Tan[e + f*x]])/f + (2*a*d*(d*Tan[e +
f*x])^(3/2))/(3*f) + (2*a*(d*Tan[e + f*x])^(5/2))/(5*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
```

```
EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (d\tan(e+fx))^{5/2}(a+a\tan(e+fx))\,dx = \frac{2a(d\tan(e+fx))^{5/2}}{5f} + \int (d\tan(e+fx))^{3/2}(-ad+ad\tan(e$$
$$= \frac{2ad(d\tan(e+fx))^{3/2}}{3f} + \frac{2a(d\tan(e+fx))^{5/2}}{5f} + \int \sqrt{d\tan(e$$
$$= -\frac{2ad^2\sqrt{d\tan(e+fx)}}{f} + \frac{2ad(d\tan(e+fx))^{3/2}}{3f} + \frac{2a(d\tan($$
$$= -\frac{2ad^2\sqrt{d\tan(e+fx)}}{f} + \frac{2ad(d\tan(e+fx))^{3/2}}{3f} + \frac{2a(d\tan($$
$$= \frac{\sqrt{2}\,ad^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{f} - \frac{2ad^2\sqrt{d\tan(e-}}{f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.85, size = 117, normalized size = 1.02

$$\frac{\left(\frac{1}{15}+\frac{i}{15}\right)a(d\tan(e+fx))^{5/2}\left(-15\sqrt[4]{-1}\,\mathrm{ArcTan}\left((-1)^{3/4}\sqrt{\tan(e+fx)}\right)+15(-1)^{3/4}\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(e+fx)}\right)+(1-i)\sqrt{\tan(e+fx)}\left(-15+5\tan(e+fx)+3\tan^2(e+fx)\right)\right)}{f\tan^{\frac{5}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(5/2)*(a + a*Tan[e + f*x]),x]
```

```
[Out] ((1/15 + I/15)*a*(d*Tan[e + f*x])^(5/2)*(-15*(-1)^(1/4)*ArcTan[(-1)^(3/4)*S
qrt[Tan[e + f*x]]] + 15*(-1)^(3/4)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[e + f*x]]] +
 (1 - I)*Sqrt[Tan[e + f*x]]*(-15 + 5*Tan[e + f*x] + 3*Tan[e + f*x]^2)))/(f*
Tan[e + f*x]^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(94) = 188$.
time = 0.49, size = 318, normalized size = 2.77

method	result
derivativedivides	$a\left(\frac{2(d\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{2d(d\tan(fx+e))^{\frac{3}{2}}}{3}-2d^2\sqrt{d\tan(fx+e)}+2d^3\left(\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\,\mathrm{t}}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\,\mathrm{t}}}\right.\right.}{}\right.\right.$

default	$a\left(\frac{2(d\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{2d(d\tan(fx+e))^{\frac{3}{2}}}{3}-2d^2\sqrt{d\tan(fx+e)}+2d^3\left(\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan}}\right.\right.}{}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(5/2)*(a+a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a*(2/5*(d*tan(f*x+e))^(5/2)+2/3*d*(d*tan(f*x+e))^(3/2)-2*d^2*(d*tan(f*x
+e))^(1/2)+2*d^3*(1/8/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(
d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f
*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+
e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))-1/8/(d^
2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)
+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^
(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1
/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))))
```

Maxima [A]
time = 0.50, size = 141, normalized size = 1.23

$$\frac{15\,ad^4\left(\frac{\sqrt{2}\,\log\left(d\tan(fx+e)+\sqrt{2}\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{d}+d\right)}{\sqrt{d}}-\frac{\sqrt{2}\,\log\left(d\tan(fx+e)-\sqrt{2}\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{d}+d\right)}{\sqrt{d}}\right)+12\left(d\tan\left(fx+e\right)\right)^{\frac{5}{2}}ad+20\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}ad^2-60\,\sqrt{d\tan\left(fx+e\right)}\,ad^3}{30\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)*(a+a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/30*(15*a*d^4*(sqrt(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*s
qrt(d) + d)/sqrt(d) - sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x +
 e))*sqrt(d) + d)/sqrt(d)) + 12*(d*tan(f*x + e))^(5/2)*a*d + 20*(d*tan(f*x
+ e))^(3/2)*a*d^2 - 60*sqrt(d*tan(f*x + e))*a*d^3)/(d*f)
```

Fricas [A]
time = 1.49, size = 241, normalized size = 2.10

$$\left[\frac{15\,\sqrt{2}\,ad^{\frac{5}{2}}\log\left(\frac{d\tan(fx+e)^2+2\,\sqrt{2}\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{d}\,(\tan(fx+e)+1)+4\,d\tan(fx+e)+d}{\tan(fx+e)^2+1}\right)+4\left(3\,ad^2\tan\left(fx+e\right)^2+5\,ad^2\tan\left(fx+e\right)-15\,ad^2\right)\sqrt{d\tan\left(fx+e\right)}}{30\,f},-\frac{15\,\sqrt{2}\,a\sqrt{-d}\,d^2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{-d}\,(\tan(fx+e)+1)}{2\,d\tan(fx+e)}\right)-2\left(3\,ad^2\tan\left(fx+e\right)^2+5\,ad^2\tan\left(fx+e\right)-15\,ad^2\right)\sqrt{d\tan\left(fx+e\right)}}{15\,f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)*(a+a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/30*(15*sqrt(2)*a*d^(5/2)*log((d*tan(f*x + e)^2 + 2*sqrt(2)*sqrt(d*tan(f*
x + e))*sqrt(d)*(tan(f*x + e) + 1) + 4*d*tan(f*x + e) + d)/(tan(f*x + e)^2
+ 1)) + 4*(3*a*d^2*tan(f*x + e)^2 + 5*a*d^2*tan(f*x + e) - 15*a*d^2)*sqrt(d
```

```
*tan(f*x + e)))/f, -1/15*(15*sqrt(2)*a*sqrt(-d)*d^2*arctan(1/2*sqrt(2)*sqrt
(d*tan(f*x + e))*sqrt(-d)*(tan(f*x + e) + 1)/(d*tan(f*x + e))) - 2*(3*a*d^2
*tan(f*x + e)^2 + 5*a*d^2*tan(f*x + e) - 15*a*d^2)*sqrt(d*tan(f*x + e)))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int (d\tan{(e+fx)})^{\frac{5}{2}}\,dx + \int (d\tan{(e+fx)})^{\frac{5}{2}}\tan{(e+fx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(5/2)*(a+a*tan(f*x+e)),x)
```

```
[Out] a*(Integral((d*tan(e + f*x))**(5/2), x) + Integral((d*tan(e + f*x))**(5/2)*
tan(e + f*x), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(99) = 198.
time = 0.73, size = 314, normalized size = 2.73

$$\frac{\sqrt{2}\left(ad^2\sqrt{|d|}-ad|d|^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan{(fx+e)}}\right)}{2\sqrt{|d|}}\right)}{2f}+\frac{\sqrt{2}\left(ad^2\sqrt{|d|}-ad|d|^{\frac{3}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan{(fx+e)}}\right)}{2\sqrt{|d|}}\right)}{2f}+\frac{\sqrt{2}\left(ad^2\sqrt{|d|}+ad|d|^{\frac{3}{2}}\right)\log\left(d\tan{(fx+e)}+\sqrt{2}\sqrt{d\tan{(fx+e)}}\sqrt{|d|}+|d|\right)}{4f}-\frac{\sqrt{2}\left(ad^2\sqrt{|d|}+ad|d|^{\frac{3}{2}}\right)\log\left(d\tan{(fx+e)}-\sqrt{2}\sqrt{d\tan{(fx+e)}}\sqrt{|d|}+|d|\right)}{4f}+\frac{2\left(3\sqrt{d\tan{(fx+e)}}ad^2f^4\tan{(fx+e)}^2+5\sqrt{d\tan{(fx+e)}}ad^2f^4\tan{(fx+e)}-15\sqrt{d\tan{(fx+e)}}ad^2f^4\right)}{15f^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)*(a+a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(a*d^2*sqrt(abs(d)) - a*d*abs(d)^(3/2))*arctan(1/2*sqrt(2)*(sqr
t(2)*sqrt(abs(d)) + 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/f + 1/2*sqrt(2)*(
a*d^2*sqrt(abs(d)) - a*d*abs(d)^(3/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(ab
s(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/f + 1/4*sqrt(2)*(a*d^2*sqrt(a
bs(d)) + a*d*abs(d)^(3/2))*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e)
)*sqrt(abs(d)) + abs(d))/f - 1/4*sqrt(2)*(a*d^2*sqrt(abs(d)) + a*d*abs(d)^(
3/2))*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(
d))/f + 2/15*(3*sqrt(d*tan(f*x + e))*a*d^2*f^4*tan(f*x + e)^2 + 5*sqrt(d*ta
n(f*x + e))*a*d^2*f^4*tan(f*x + e) - 15*sqrt(d*tan(f*x + e))*a*d^2*f^4)/f^5
```

Mupad [B]
time = 5.47, size = 144, normalized size = 1.25

$$\frac{2\,a\,{\left(d\,\mathrm{tan}\left(e+f\,x\right)\right)}^{5/2}}{5\,f}+\frac{2\,a\,d\,{\left(d\,\mathrm{tan}\left(e+f\,x\right)\right)}^{3/2}}{3\,f}-\frac{2\,a\,d^2\,\sqrt{d\,\mathrm{tan}\left(e+f\,x\right)}}{f}-\frac{{\left(-1\right)}^{1/4}\,a\,d^{5/2}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{d\,\mathrm{tan}\left(e+f\,x\right)}\,1{}\mathrm{i}}{\sqrt{d}}\right)}{f}+\frac{{\left(-1\right)}^{1/4}\,a\,d^{5/2}\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{d\,\mathrm{tan}\left(e+f\,x\right)}}{\sqrt{d}}\right)}{f}+\frac{{\left(-1\right)}^{1/4}\,a\,d^{5/2}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{d\,\mathrm{tan}\left(e+f\,x\right)}}{\sqrt{d}}\right)\,\left(-1-\mathrm{i}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(5/2)*(a + a*tan(e + f*x)),x)
```

```
[Out] (2*a*(d*tan(e + f*x))^(5/2))/(5*f) + (2*a*d*(d*tan(e + f*x))^(3/2))/(3*f) -
 (2*a*d^2*(d*tan(e + f*x))^(1/2))/f - ((-1)^(1/4)*a*d^(5/2)*atan(((-1)^(1/4
)*(d*tan(e + f*x))^(1/2))/d^(1/2))*(1 + 1i))/f - ((-1)^(1/4)*a*d^(5/2)*atan
(((-1)^(1/4)*(d*tan(e + f*x))^(1/2)*1i)/d^(1/2)))/f + ((-1)^(1/4)*a*d^(5/2)
*atanh(((-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2)))/f
```

3.337 $\int (d\tan(e+fx))^{3/2}(a+a\tan(e+fx))\,dx$

Optimal. Leaf size=93

$$\frac{\sqrt{2}\,ad^{3/2}\text{ArcTan}\left(\frac{\sqrt{d}-\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{f}+\frac{2ad\sqrt{d\tan(e+fx)}}{f}+\frac{2a(d\tan(e+fx))^{3/2}}{3f}$$

```
[Out] a*d^(3/2)*arctan(1/2*(d^(1/2)-d^(1/2)*tan(f*x+e))*2^(1/2)/(d*tan(f*x+e))^(1
/2))*2^(1/2)/f+2*a*d*(d*tan(f*x+e))^(1/2)/f+2/3*a*(d*tan(f*x+e))^(3/2)/f
```

Rubi [A]
time = 0.08, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3609, 3613, 211}

$$\frac{\sqrt{2}\,ad^{3/2}\text{ArcTan}\left(\frac{\sqrt{d}-\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{f}+\frac{2ad\sqrt{d\tan(e+fx)}}{f}+\frac{2a(d\tan(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(3/2)*(a + a*Tan[e + f*x]),x]
```

```
[Out] (Sqrt[2]*a*d^(3/2)*ArcTan[(Sqrt[d] - Sqrt[d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[d*
Tan[e + f*x]])])/f + (2*a*d*Sqrt[d*Tan[e + f*x]])/f + (2*a*(d*Tan[e + f*x])
^(3/2))/(3*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (d\tan(e+fx))^{3/2}(a+a\tan(e+fx))\,dx = \frac{2a(d\tan(e+fx))^{3/2}}{3f} + \int \sqrt{d\tan(e+fx)}\,(-ad+ad\tan(e$$

$$= \frac{2ad\sqrt{d\tan(e+fx)}}{f} + \frac{2a(d\tan(e+fx))^{3/2}}{3f} + \int \frac{-ad^2-a}{\sqrt{d\,\mathrm{ta}}}$$

$$= \frac{2ad\sqrt{d\tan(e+fx)}}{f} + \frac{2a(d\tan(e+fx))^{3/2}}{3f} - \frac{(2a^2d^4)\,\mathrm{Subs}}{}$$

$$= \frac{\sqrt{2}\,ad^{3/2}\tan^{-1}\left(\frac{\sqrt{d}-\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{f} + \frac{2ad\sqrt{d\tan(e+}}{f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.47, size = 105, normalized size = 1.13

$$\frac{\left(\frac{1}{3}+\frac{i}{3}\right)a(d\tan(e+fx))^{3/2}\left(-3(-1)^{3/4}\mathrm{ArcTan}\left((-1)^{3/4}\sqrt{\tan(e+fx)}\right)+3\sqrt[4]{-1}\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(e+fx)}\right)+(1-i)\sqrt{\tan(e+fx)}\,(3+\tan(e+fx))\right)}{f\tan^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(3/2)*(a + a*Tan[e + f*x]),x]
```

```
[Out] ((1/3 + I/3)*a*(d*Tan[e + f*x])^(3/2)*(-3*(-1)^(3/4)*ArcTan[(-1)^(3/4)*Sqrt
[Tan[e + f*x]]] + 3*(-1)^(1/4)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[e + f*x]]] + (1
- I)*Sqrt[Tan[e + f*x]]*(3 + Tan[e + f*x])))/(f*Tan[e + f*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(76) = 152.
time = 0.28, size = 303, normalized size = 3.26

method	result
derivativedivides	$\frac{a\left(\frac{2(d\tan(fx+e))^{\frac{3}{2}}}{3}+2d\sqrt{d\tan(fx+e)}-2d^2\left(\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{}}\right.\right.}{}\right.\right.}{}$

default	$\frac{a\left(\frac{2(d\tan(fx+e))^{\frac{3}{2}}}{3}+2d\sqrt{d\tan(fx+e)}-2d^2\left(\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}}\right.\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(3/2)*(a+a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a*(2/3*(d*tan(f*x+e))^(3/2)+2*d*(d*tan(f*x+e))^(1/2)-2*d^2*(1/8/d*(d^2)
^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(
d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1
/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2
)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))+1/8/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f
*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(
d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2
)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e)
)^(1/2)+1))))
```

Maxima [A]
time = 0.52, size = 122, normalized size = 1.31

$$-\frac{3\,ad^3\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}\right)-2\,(d\tan(fx+e))^{\frac{3}{2}}\,ad-6\,\sqrt{d\tan(fx+e)}\,ad^2}{3\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)*(a+a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/3*(3*a*d^3*(sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(f
*x + e)))/sqrt(d))/sqrt(d) + sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) -
 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d)) - 2*(d*tan(f*x + e))^(3/2)*a*d -
 6*sqrt(d*tan(f*x + e))*a*d^2)/(d*f)
```

Fricas [A]
time = 1.47, size = 196, normalized size = 2.11

$$\left[\frac{3\sqrt{2}\,a\sqrt{-d}\,d\log\left(\frac{d\tan(fx+e)^2-2\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{-d}\,(\tan(fx+e)-1)-4\,d\tan(fx+e)+d}{\tan(fx+e)^2+1}\right)+4\,(ad\tan(fx+e)+3\,ad)\sqrt{d\tan(fx+e)}}{6\,f},-\frac{3\sqrt{2}\,ad^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+e)}\,(\tan(fx+e)-1)}{2\sqrt{d}\,\tan(fx+e)}\right)-2\,(ad\tan(fx+e)+3\,ad)\sqrt{d\tan(fx+e)}}{3\,f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)*(a+a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(2)*a*sqrt(-d)*d*log((d*tan(f*x + e)^2 - 2*sqrt(2)*sqrt(d*tan(f
*x + e))*sqrt(-d)*(tan(f*x + e) - 1) - 4*d*tan(f*x + e) + d)/(tan(f*x + e)^
```

```
2 + 1)) + 4*(a*d*tan(f*x + e) + 3*a*d)*sqrt(d*tan(f*x + e)))/f, -1/3*(3*sqr
t(2)*a*d^(3/2)*arctan(1/2*sqrt(2)*sqrt(d*tan(f*x + e))*(tan(f*x + e) - 1)/(
sqrt(d)*tan(f*x + e))) - 2*(a*d*tan(f*x + e) + 3*a*d)*sqrt(d*tan(f*x + e)))
/f]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int (d\tan(e+fx))^{\frac{3}{2}}\,dx+\int (d\tan(e+fx))^{\frac{3}{2}}\tan(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(3/2)*(a+a*tan(f*x+e)),x)
```

```
[Out] a*(Integral((d*tan(e + f*x))**(3/2), x) + Integral((d*tan(e + f*x))**(3/2)*
tan(e + f*x), x))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 290 vs. 2(81) = 162.
time = 0.64, size = 290, normalized size = 3.12

$$-\frac{1}{12}\,d\left(\frac{6\sqrt{2}\left(ad\sqrt{|d|}+a|d|^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{df}+\frac{6\sqrt{2}\left(ad\sqrt{|d|}+a|d|^{\frac{3}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{df}+\frac{3\sqrt{2}\left(ad\sqrt{|d|}-a|d|^{\frac{3}{2}}\right)\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{|d|}+|d|\right)}{df}-\frac{3\sqrt{2}\left(ad\sqrt{|d|}-a|d|^{\frac{3}{2}}\right)\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{|d|}+|d|\right)}{df}-\frac{8\left(\sqrt{d\tan(fx+e)}\,ad^3f^2\tan(fx+e)+3\sqrt{d\tan(fx+e)}\,ad^3f^2\right)}{d^3f^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)*(a+a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/12*d*(6*sqrt(2)*(a*d*sqrt(abs(d)) + a*abs(d)^(3/2))*arctan(1/2*sqrt(2)*(
sqrt(2)*sqrt(abs(d)) + 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(d*f) + 6*sqrt
(2)*(a*d*sqrt(abs(d)) + a*abs(d)^(3/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(a
bs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(d*f) + 3*sqrt(2)*(a*d*sqrt(
abs(d)) - a*abs(d)^(3/2))*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))
*sqrt(abs(d)) + abs(d))/(d*f) - 3*sqrt(2)*(a*d*sqrt(abs(d)) - a*abs(d)^(3/2
))*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))
/(d*f) - 8*(sqrt(d*tan(f*x + e))*a*d^3*f^2*tan(f*x + e) + 3*sqrt(d*tan(f*x
+ e))*a*d^3*f^2)/(d^3*f^3))
```

Mupad [**B**]
time = 4.84, size = 98, normalized size = 1.05

$$\frac{2\,a\,{\left(d\tan(e+f\,x)\right)}^{3/2}}{3\,f}+\frac{2\,a\,d\,\sqrt{d\,\tan\left(e+f\,x\right)}}{f}+\frac{{\left(-1\right)}^{1/4}\,a\,d^{3/2}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{d\,\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)\,\left(-1+1{}\mathrm{i}\right)}{f}+\frac{{\left(-1\right)}^{1/4}\,a\,d^{3/2}\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{d\,\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)\,\left(1+1{}\mathrm{i}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(3/2)*(a + a*tan(e + f*x)),x)
```

```
[Out] (2*a*(d*tan(e + f*x))^(3/2))/(3*f) + (2*a*d*(d*tan(e + f*x))^(1/2))/f - ((-
1)^(1/4)*a*d^(3/2)*atan(((-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2))*(1 - 1
i))/f + ((-1)^(1/4)*a*d^(3/2)*atanh(((-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(
1/2))*(1 + 1i))/f
```

3.338 $\int \sqrt{d\tan(e+fx)}\,(a+a\tan(e+fx))\,dx$

Optimal. Leaf size=72

$$-\frac{\sqrt{2}\,a\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{f}+\frac{2a\sqrt{d\tan(e+fx)}}{f}$$

```
[Out] -a*arctanh(1/2*(d^(1/2)+d^(1/2)*tan(f*x+e))*2^(1/2)/(d*tan(f*x+e))^(1/2))*2
^(1/2)*d^(1/2)/f+2*a*(d*tan(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3609, 3613, 214}

$$\frac{2a\sqrt{d\tan(e+fx)}}{f}-\frac{\sqrt{2}\,a\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\tan(e+fx)+\sqrt{d}}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Tan[e + f*x]]*(a + a*Tan[e + f*x]),x]
```

```
[Out] -((Sqrt[2]*a*Sqrt[d]*ArcTanh[(Sqrt[d] + Sqrt[d]*Tan[e + f*x])/(Sqrt[2]*Sqrt
[d*Tan[e + f*x]])])/f) + (2*a*Sqrt[d*Tan[e + f*x]])/f
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{d\tan(e+fx)}\,(a+a\tan(e+fx))\,dx = \frac{2a\sqrt{d\tan(e+fx)}}{f} + \int \frac{-ad+ad\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$$

$$= \frac{2a\sqrt{d\tan(e+fx)}}{f} - \frac{(2a^2d^2)\,\mathrm{Subst}\left(\int \frac{1}{-2a^2d^2+dx^2}\,dx, x, \frac{-ad}{\sqrt{d\tan(e+fx)}}\right)}{f}$$

$$= -\frac{\sqrt{2}\,a\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{f} + \frac{2a\sqrt{d\tan(e+fx)}}{f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.11, size = 92, normalized size = 1.28

$$\frac{(1+i)a\left(\sqrt[4]{-1}\,\mathrm{ArcTan}\left((-1)^{3/4}\sqrt{\tan(e+fx)}\right)-(-1)^{3/4}\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(e+fx)}\right)+(1-i)\sqrt{\tan(e+fx)}\right)\sqrt{d\tan(e+fx)}}{f\sqrt{\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Tan[e + f*x]]*(a + a*Tan[e + f*x]),x]
```

```
[Out] ((1 + I)*a*((-1)^(1/4)*ArcTan[(-1)^(3/4)*Sqrt[Tan[e + f*x]]] - (-1)^(3/4)*A
rcTanh[(-1)^(3/4)*Sqrt[Tan[e + f*x]]] + (1 - I)*Sqrt[Tan[e + f*x]])*Sqrt[d*
Tan[e + f*x]])/(f*Sqrt[Tan[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(59) = 118.
time = 0.23, size = 288, normalized size = 4.00

method	result
derivativedivides	$a\left(2\sqrt{d\tan(fx+e)}-2d\left(\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\,\mathrm{arcta}\right.}{}\right.\right.$
default	$a\left(2\sqrt{d\tan(fx+e)}-2d\left(\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\,\mathrm{arcta}\right.}{}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(1/2)*(a+a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a*(2*(d*tan(f*x+e))^(1/2)-2*d*(1/8/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x
+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^
2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^
(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^
(1/2)+1))-1/8/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+
e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/
2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+
1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))))
```

Maxima [A]
time = 0.50, size = 106, normalized size = 1.47

$$-\frac{ad^2\left(\frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{d}+d\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{d}+d\right)}{\sqrt{d}}\right)-4\sqrt{d\tan\left(fx+e\right)}\,ad}{2\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)*(a+a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/2*(a*d^2*(sqrt(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt
(d) + d)/sqrt(d) - sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e)
)*sqrt(d) + d)/sqrt(d)) - 4*sqrt(d*tan(f*x + e))*a*d)/(d*f)
```

Fricas [A]
time = 1.54, size = 167, normalized size = 2.32

$$\left[\frac{\sqrt{2}\,a\sqrt{d}\,\log\left(\frac{d\tan(fx+e)^2-2\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{d}\,(\tan(fx+e)+1)+4\,d\tan(fx+e)+d}{\tan(fx+e)^2+1}\right)+4\sqrt{d\tan\left(fx+e\right)}\,a}{2\,f},\ \frac{\sqrt{2}\,a\sqrt{-d}\,\arctan\left(\frac{\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{-d}\,(\tan(fx+e)+1)}{2\,d\tan(fx+e)}\right)+2\sqrt{d\tan\left(fx+e\right)}\,a}{f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)*(a+a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*a*sqrt(d)*log((d*tan(f*x + e)^2 - 2*sqrt(2)*sqrt(d*tan(f*x +
e))*sqrt(d)*(tan(f*x + e) + 1) + 4*d*tan(f*x + e) + d)/(tan(f*x + e)^2 + 1)
) + 4*sqrt(d*tan(f*x + e))*a)/f, (sqrt(2)*a*sqrt(-d)*arctan(1/2*sqrt(2)*sqr
t(d*tan(f*x + e))*sqrt(-d)*(tan(f*x + e) + 1)/(d*tan(f*x + e))) + 2*sqrt(d*
tan(f*x + e))*a)/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sqrt{d\tan\left(e+fx\right)}\,dx+\int\sqrt{d\tan\left(e+fx\right)}\,\tan\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(1/2)*(a+a*tan(f*x+e)),x)
```

```
[Out] a*(Integral(sqrt(d*tan(e + f*x)), x) + Integral(sqrt(d*tan(e + f*x))*tan(e
+ f*x), x))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(62) = 124$.
time = 0.60, size = 249, normalized size = 3.46

$$\frac{2\sqrt{d\tan(fx+e)}\,a}{f} - \frac{\sqrt{2}\left(ad\sqrt{|d|} - a|d|^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{2\,df} - \frac{\sqrt{2}\left(ad\sqrt{|d|} - a|d|^{\frac{3}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{2\,df} - \frac{\sqrt{2}\left(ad\sqrt{|d|} + a|d|^{\frac{3}{2}}\right)\log\left(d\tan(fx+e) + \sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{|d|} + |d|\right)}{4\,df} + \frac{\sqrt{2}\left(ad\sqrt{|d|} + a|d|^{\frac{3}{2}}\right)\log\left(d\tan(fx+e) - \sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{|d|} + |d|\right)}{4\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)*(a+a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] 2*sqrt(d*tan(f*x + e))*a/f - 1/2*sqrt(2)*(a*d*sqrt(abs(d)) - a*abs(d)^(3/2)
)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqrt(d*tan(f*x + e)))/sqrt(a
bs(d)))/(d*f) - 1/2*sqrt(2)*(a*d*sqrt(abs(d)) - a*abs(d)^(3/2))*arctan(-1/2
*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(d*f
) - 1/4*sqrt(2)*(a*d*sqrt(abs(d)) + a*abs(d)^(3/2))*log(d*tan(f*x + e) + sq
rt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(d*f) + 1/4*sqrt(2)*(a*d*
sqrt(abs(d)) + a*abs(d)^(3/2))*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x
+ e))*sqrt(abs(d)) + abs(d))/(d*f)
```

Mupad [**B**]
time = 4.46, size = 125, normalized size = 1.74

$$\frac{2\,a\,\sqrt{d\tan(e+f\,x)}}{f} + \frac{(-1)^{1/4}\,a\,\sqrt{d}\,\left(\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan(e+f\,x)}}{\sqrt{d}}\right) - \mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan(e+f\,x)}}{\sqrt{d}}\right)\right)}{f} + \frac{(-1)^{1/4}\,a\,\sqrt{d}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{f} + \frac{(-1)^{1/4}\,a\,\sqrt{d}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(1/2)*(a + a*tan(e + f*x)),x)
```

```
[Out] (2*a*(d*tan(e + f*x))^(1/2))/f + ((-1)^(1/4)*a*d^(1/2)*atan(((-1)^(1/4)*(d*
tan(e + f*x))^(1/2))/d^(1/2))*1i)/f + ((-1)^(1/4)*a*d^(1/2)*atanh(((-1)^(1/
4)*(d*tan(e + f*x))^(1/2))/d^(1/2))*1i)/f + ((-1)^(1/4)*a*d^(1/2)*(atan(((-
1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2)) - atanh(((-1)^(1/4)*(d*tan(e + f*
x))^(1/2))/d^(1/2))))/f
```

3.339 $\int \frac{a+a\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$

Optimal. Leaf size=50

$$-\frac{\sqrt{2}\,a\text{ArcTan}\left(\frac{\sqrt{d}\,(1-\tan(e+fx))}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{\sqrt{d}\,f}$$

```
[Out] -a*arctan(1/2*d^(1/2)*(1-tan(f*x+e))*2^(1/2)/(d*tan(f*x+e))^(1/2))*2^(1/2)/
f/d^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3613, 211}

$$-\frac{\sqrt{2}\,a\text{ArcTan}\left(\frac{\sqrt{d}\,(1-\tan(e+fx))}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{\sqrt{d}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[e + f*x])/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] -((Sqrt[2]*a*ArcTan[(Sqrt[d]*(1 - Tan[e + f*x]))/(Sqrt[2]*Sqrt[d*Tan[e + f*
x]])])/(Sqrt[d]*f))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{a+a\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = -\frac{(2a^2)\,\mathrm{Subst}\left(\int \frac{1}{2a^2+dx^2}\,dx, x, \frac{a-a\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\right)}{f}$$
$$= -\frac{\sqrt{2}\,a\tan^{-1}\left(\frac{\sqrt{d}\,(1-\tan(e+fx))}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{\sqrt{d}\,f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.09, size = 74, normalized size = 1.48

$$-\frac{(1-i)\sqrt[4]{-1}\,a\left(\mathrm{ArcTan}\left((-1)^{3/4}\sqrt{\tan(e+fx)}\right)+i\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(e+fx)}\right)\right)\sqrt{\tan(e+fx)}}{f\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[e + f*x])/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] ((-1 + I)*(-1)^(1/4)*a*(ArcTan[(-1)^(3/4)*Sqrt[Tan[e + f*x]]] + I*ArcTanh[(
-1)^(3/4)*Sqrt[Tan[e + f*x]]])*Sqrt[Tan[e + f*x]])/(f*Sqrt[d*Tan[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(41) = 82.
time = 0.25, size = 272, normalized size = 5.44

method	result
derivativedivides	$a\left(\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\,\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\tan(fx+e}}{(d^2)^{\frac{1}{4}}}\right.\right.}{4d}\right.$
default	$a\left(\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\,\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\tan(fx+e}}{(d^2)^{\frac{1}{4}}}\right.\right.}{4d}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(f*x+e))/(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a*(1/4/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e
))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2
```

```
)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1
)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))+1/4/(d^2)^(1/4)*2^
(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2
))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*a
rctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(
1/4)*(d*tan(f*x+e))^(1/2)+1)))
```

Maxima [A]
time = 0.51, size = 80, normalized size = 1.60

$$\frac{a\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] a*(sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(f*x + e)))/sq
rt(d))/sqrt(d) + sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*sqrt(d*ta
n(f*x + e)))/sqrt(d))/sqrt(d))/f
```

Fricas [A]
time = 1.13, size = 133, normalized size = 2.66

$$\left[\frac{\sqrt{2}\,a\sqrt{-\frac{1}{d}}\log\left(\frac{2\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{-\frac{1}{d}}(\tan(fx+e)-1)+\tan(fx+e)^2-4\tan(fx+e)+1}{\tan(fx+e)^2+1}\right)}{2f},\ \frac{\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+e)}(\tan(fx+e)-1)}{2\sqrt{d}\tan(fx+e)}\right)}{\sqrt{d}\,f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*a*sqrt(-1/d)*log((2*sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(-1/d)*(t
an(f*x + e) - 1) + tan(f*x + e)^2 - 4*tan(f*x + e) + 1)/(tan(f*x + e)^2 + 1
))/f, sqrt(2)*a*arctan(1/2*sqrt(2)*sqrt(d*tan(f*x + e))*(tan(f*x + e) - 1)/
(sqrt(d)*tan(f*x + e)))/(sqrt(d)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\frac{1}{\sqrt{d\tan(e+fx)}}\,dx+\int\frac{\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))/(d*tan(f*x+e))**(1/2),x)
```

```
[Out] a*(Integral(1/sqrt(d*tan(e + f*x)), x) + Integral(tan(e + f*x)/sqrt(d*tan(e
 + f*x)), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(41) = 82.

time = 0.62, size = 232, normalized size = 4.64

$$\frac{\sqrt{2}\left(ad\sqrt{|d|}+a|d|^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{2\,d^{2}f}+\frac{\sqrt{2}\left(ad\sqrt{|d|}+a|d|^{\frac{3}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{2\,d^{2}f}+\frac{\sqrt{2}\left(ad\sqrt{|d|}-a|d|^{\frac{3}{2}}\right)\log\left(d\tan\left(fx+e\right)+\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{|d|}+|d|\right)}{4\,d^{2}f}-\frac{\sqrt{2}\left(ad\sqrt{|d|}-a|d|^{\frac{3}{2}}\right)\log\left(d\tan\left(fx+e\right)-\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{|d|}+|d|\right)}{4\,d^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(a*d*sqrt(abs(d)) + a*abs(d)^(3/2))*arctan(1/2*sqrt(2)*(sqrt(2)
*sqrt(abs(d)) + 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(d^2*f) + 1/2*sqrt(2)
*(a*d*sqrt(abs(d)) + a*abs(d)^(3/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(
d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(d^2*f) + 1/4*sqrt(2)*(a*d*sqrt
(abs(d)) - a*abs(d)^(3/2))*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e)
)*sqrt(abs(d)) + abs(d))/(d^2*f) - 1/4*sqrt(2)*(a*d*sqrt(abs(d)) - a*abs(d)
^(3/2))*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + ab
s(d))/(d^2*f)
```

Mupad [B]

time = 4.30, size = 65, normalized size = 1.30

$$\frac{(-1)^{1/4}\,a\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)\,\left(1-\mathrm{i}\right)}{\sqrt{d}\,f}+\frac{(-1)^{1/4}\,a\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)\,\left(-1-\mathrm{i}\right)}{\sqrt{d}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x))/(d*tan(e + f*x))^(1/2),x)
```

```
[Out] ((-1)^(1/4)*a*atan(((-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2))*(1 - 1i))/(
d^(1/2)*f) - ((-1)^(1/4)*a*atanh(((-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2
))*(1 + 1i))/(d^(1/2)*f)
```

3.340 $\int \frac{a+a\tan(e+fx)}{(d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=74

$$\frac{\sqrt{2}\,a\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{d^{3/2}f}-\frac{2a}{df\,\sqrt{d\tan(e+fx)}}$$

```
[Out] a*arctanh(1/2*(d^(1/2)+d^(1/2)*tan(f*x+e))*2^(1/2)/(d*tan(f*x+e))^(1/2))*2^
(1/2)/d^(3/2)/f-2*a/d/f/(d*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3610, 3613, 214}

$$\frac{\sqrt{2}\,a\tanh^{-1}\left(\frac{\sqrt{d}\,\tan(e+fx)+\sqrt{d}}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{d^{3/2}f}-\frac{2a}{df\,\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[e + f*x])/(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (Sqrt[2]*a*ArcTanh[(Sqrt[d] + Sqrt[d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[d*Tan[e +
 f*x]])])/(d^(3/2)*f) - (2*a)/(d*f*Sqrt[d*Tan[e + f*x]])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\tan(e+fx)}{(d\tan(e+fx))^{3/2}}\,dx &= -\frac{2a}{df\sqrt{d\tan(e+fx)}}+\frac{\int \frac{ad-ad\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx}{d^2}\\ &= -\frac{2a}{df\sqrt{d\tan(e+fx)}}-\frac{(2a^2)\,\mathrm{Subst}\left(\int \frac{1}{-2a^2d^2+dx^2}\,dx,x,\frac{ad+ad\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\right)}{f}\\ &= \frac{\sqrt{2}\,a\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{d^{3/2}f}-\frac{2a}{df\sqrt{d\tan(e+fx)}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.14, size = 64, normalized size = 0.86

$$-\frac{(1+i)a\left({}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-i\tan(e+fx)\right)-i\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};i\tan(e+fx)\right)\right)}{df\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[e + f*x])/(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-1 - I)*a*(Hypergeometric2F1[-1/2, 1, 1/2, (-I)*Tan[e + f*x]] - I*Hyperge
ometric2F1[-1/2, 1, 1/2, I*Tan[e + f*x]]))/(d*f*Sqrt[d*Tan[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 292 vs. 2(61) = 122.
time = 0.20, size = 293, normalized size = 3.96

method	result
derivativedivides	$a\left(-\frac{2}{d\sqrt{d\tan(fx+e)}}+\frac{\frac{(d^2)^{\frac14}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac14}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac14}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan \ldots\right.}{4d \ldots}}{\ldots}\right.$

default	$a\left(-\frac{2}{d\sqrt{d\tan(fx+e)}}+\frac{\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\right.\right.}{4d}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(f*x+e))/(d*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a*(-2/d/(d*tan(f*x+e))^(1/2)+2/d*(1/8/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(
f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-
(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^
2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e
))^(1/2)+1))-1/8/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f
*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^
(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/
2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))))
```

Maxima [A]
time = 0.50, size = 102, normalized size = 1.38

$$\frac{a\left(\frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)-\frac{4a}{\sqrt{d\tan(fx+e)}}}{2df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))/(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(a*(sqrt(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) +
 d)/sqrt(d) - sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqr
t(d) + d)/sqrt(d)) - 4*a/sqrt(d*tan(f*x + e)))/(d*f)
```

Fricas [A]
time = 1.36, size = 205, normalized size = 2.77

$$\left[\frac{\sqrt{2}\,a\sqrt{d}\,\log\left(\frac{\tan(fx+e)^2+\frac{2\sqrt{2}\sqrt{d\tan(fx+e)}(\tan(fx+e)+1)}{\sqrt{d}}+4\tan(fx+e)+1}{\tan(fx+e)^2+1}\right)\tan(fx+e)-4\sqrt{d\tan(fx+e)}\,a}{2d^2f\tan(fx+e)},-\frac{\sqrt{2}\,ad\sqrt{-\frac{1}{d}}\,\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{-\frac{1}{d}}(\tan(fx+e)+1)}{2\tan(fx+e)}\right)\tan(fx+e)+2\sqrt{d\tan(fx+e)}\,a}{d^2f\tan(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))/(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*a*sqrt(d)*log((tan(f*x + e)^2 + 2*sqrt(2)*sqrt(d*tan(f*x + e)
)*(tan(f*x + e) + 1)/sqrt(d) + 4*tan(f*x + e) + 1)/(tan(f*x + e)^2 + 1))*ta
n(f*x + e) - 4*sqrt(d*tan(f*x + e))*a)/(d^2*f*tan(f*x + e)), -(sqrt(2)*a*d*
sqrt(-1/d)*arctan(1/2*sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(-1/d)*(tan(f*x + e)
 + 1)/tan(f*x + e))*tan(f*x + e) + 2*sqrt(d*tan(f*x + e))*a)/(d^2*f*tan(f*x
 + e))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{1}{\left(d\tan\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx+\int \frac{\tan\left(e+fx\right)}{\left(d\tan\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))/(d*tan(f*x+e))**(3/2),x)
```

```
[Out] a*(Integral((d*tan(e + f*x))**(-3/2), x) + Integral(tan(e + f*x)/(d*tan(e +
 f*x))**(3/2), x))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(64) = 128.
time = 0.66, size = 253, normalized size = 3.42

$$-\frac{\frac{8a}{\sqrt{d\tan\left(fx+e\right)}f}-\frac{2\sqrt{2}\left(ad\sqrt{|d|}-a|d|^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{d^{2}f}-\frac{2\sqrt{2}\left(ad\sqrt{|d|}-a|d|^{\frac{3}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{d^{2}f}-\frac{\sqrt{2}\left(ad\sqrt{|d|}+a|d|^{\frac{3}{2}}\right)\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{|d|}+|d|\right)}{d^{2}f}+\frac{\sqrt{2}\left(ad\sqrt{|d|}+a|d|^{\frac{3}{2}}\right)\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{|d|}+|d|\right)}{d^{2}f}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))/(d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*(8*a/(sqrt(d*tan(f*x + e))*f) - 2*sqrt(2)*(a*d*sqrt(abs(d)) - a*abs(d)
^(3/2))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqrt(d*tan(f*x + e)))/
sqrt(abs(d)))/(d^2*f) - 2*sqrt(2)*(a*d*sqrt(abs(d)) - a*abs(d)^(3/2))*arcta
n(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d))
)/(d^2*f) - sqrt(2)*(a*d*sqrt(abs(d)) + a*abs(d)^(3/2))*log(d*tan(f*x + e)
+ sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(d^2*f) + sqrt(2)*(a*
d*sqrt(abs(d)) + a*abs(d)^(3/2))*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*
x + e))*sqrt(abs(d)) + abs(d))/(d^2*f))/d
```

Mupad [**B**]
time = 4.58, size = 84, normalized size = 1.14

$$-\frac{2\,a}{d\,f\,\sqrt{d\,\tan\left(e+f\,x\right)}}+\frac{{\left(-1\right)}^{1/4}\,a\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{d\,\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)\,\left(-1-\mathrm{i}\right)}{d^{3/2}\,f}+\frac{{\left(-1\right)}^{1/4}\,a\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{d\,\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)\,\left(1-\mathrm{i}\right)}{d^{3/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x))/(d*tan(e + f*x))^(3/2),x)
```

```
[Out] ((-1)^(1/4)*a*atanh(((-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2))*(1 - 1i))/
(d^(3/2)*f) - ((-1)^(1/4)*a*atan(((-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2
))*(1 + 1i))/(d^(3/2)*f) - (2*a)/(d*f*(d*tan(e + f*x))^(1/2))
```

3.341 $\int \frac{a+a\tan(e+fx)}{(d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=98

$$\frac{\sqrt{2}\,a\text{ArcTan}\left(\frac{\sqrt{d}-\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{d^{5/2}f}-\frac{2a}{3df(d\tan(e+fx))^{3/2}}-\frac{2a}{d^2f\sqrt{d\tan(e+fx)}}$$

```
[Out] a*arctan(1/2*(d^(1/2)-d^(1/2)*tan(f*x+e))*2^(1/2)/(d*tan(f*x+e))^(1/2))*2^(
1/2)/d^(5/2)/f-2*a/d^2/f/(d*tan(f*x+e))^(1/2)-2/3*a/d/f/(d*tan(f*x+e))^(3/2
)
```

Rubi [A]
time = 0.09, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3610, 3613, 211}

$$\frac{\sqrt{2}\,a\text{ArcTan}\left(\frac{\sqrt{d}-\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{d^{5/2}f}-\frac{2a}{d^2f\sqrt{d\tan(e+fx)}}-\frac{2a}{3df(d\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[e + f*x])/(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[2]*a*ArcTan[(Sqrt[d] - Sqrt[d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[d*Tan[e +
f*x]])])/(d^(5/2)*f) - (2*a)/(3*d*f*(d*Tan[e + f*x])^(3/2)) - (2*a)/(d^2*f*
Sqrt[d*Tan[e + f*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
```

```
d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{a+a\tan(e+fx)}{(d\tan(e+fx))^{5/2}}\,dx = -\frac{2a}{3df(d\tan(e+fx))^{3/2}} + \frac{\int \frac{ad-ad\tan(e+fx)}{(d\tan(e+fx))^{3/2}}\,dx}{d^2}$$

$$= -\frac{2a}{3df(d\tan(e+fx))^{3/2}} - \frac{2a}{d^2f\sqrt{d\tan(e+fx)}} + \frac{\int \frac{-ad^2-ad^2\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx}{d^4}$$

$$= -\frac{2a}{3df(d\tan(e+fx))^{3/2}} - \frac{2a}{d^2f\sqrt{d\tan(e+fx)}} - \frac{(2a^2)\,\mathrm{Subst}\left(\int \frac{1}{2a^2d^4+dx^2}\,dx, x\right.}{f}$$

$$= \frac{\sqrt{2}\,a\tan^{-1}\left(\frac{\sqrt{d}-\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{d^{5/2}f} - \frac{2a}{3df(d\tan(e+fx))^{3/2}} - \frac{2a}{d^2f\sqrt{d\tan(e-}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.16, size = 68, normalized size = 0.69

$$-\frac{\left(\frac{1}{3}+\frac{i}{3}\right)a\left({}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-i\tan(e+fx)\right)-i\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};i\tan(e+fx)\right)\right)}{df(d\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[e + f*x])/(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-1/3 - I/3)*a*(Hypergeometric2F1[-3/2, 1, -1/2, (-I)*Tan[e + f*x]] - I*Hy
pergeometric2F1[-3/2, 1, -1/2, I*Tan[e + f*x]]))/(d*f*(d*Tan[e + f*x])^(3/2
))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 307 vs. 2(81) = 162.
time = 0.20, size = 308, normalized size = 3.14

method	result

derivativedivides	$\frac{a\left(-\frac{2}{d^2\sqrt{d\tan(fx+e)}}-\frac{2}{3d(d\tan(fx+e))^{\frac{3}{2}}}+\frac{-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}}\right.\right.}{}}{}\right)}{}$
default	$\frac{a\left(-\frac{2}{d^2\sqrt{d\tan(fx+e)}}-\frac{2}{3d(d\tan(fx+e))^{\frac{3}{2}}}+\frac{-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}}\right.\right.}{}}{}\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(f*x+e))/(d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a*(-2/d^2/(d*tan(f*x+e))^(1/2)-2/3/d/(d*tan(f*x+e))^(3/2)+2/d^2*(-1/8/d
*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(
1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d
^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-
2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))-1/8/(d^2)^(1/4)*2^(1/2)*(ln((d
*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*
x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2
)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(
f*x+e))^(1/2)+1))))
```

Maxima [A]
time = 0.52, size = 121, normalized size = 1.23

$$-\frac{\frac{3a\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}\right)}{d}+\frac{2(3ad\tan(fx+e)+ad)}{(d\tan(fx+e))^{\frac{3}{2}}d}}{3df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))/(d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*a*(sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(f*x +
 e)))/sqrt(d))/sqrt(d) + sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*s
```

```
qrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d))/d + 2*(3*a*d*tan(f*x + e) + a*d)/((d
*tan(f*x + e))^(3/2)*d))/(d*f)
```

Fricas [A]
time = 0.97, size = 236, normalized size = 2.41

$$\left[\frac{3\sqrt{2}\,ad\sqrt{-\frac{1}{d}}\log\left(-\frac{2\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{-\frac{1}{d}}\,(\tan(fx+e)-1)-\tan(fx+e)^2+4\tan(fx+e)-1}{\tan(fx+e)^2+1}\right)\tan\left(fx+e\right)^2-4\left(3\,a\tan\left(fx+e\right)+a\right)\sqrt{d\tan\left(fx+e\right)}}{6\,d^3f\tan\left(fx+e\right)^2},-\frac{3\sqrt{2}\,a\sqrt{d}\,\arctan\left(\frac{\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\,(\tan(fx+e)-1)}{2\sqrt{d}\,\tan(fx+e)}\right)\tan\left(fx+e\right)^2+2\left(3\,a\tan\left(fx+e\right)+a\right)\sqrt{d\tan\left(fx+e\right)}}{3\,d^3f\tan\left(fx+e\right)^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))/(d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(2)*a*d*sqrt(-1/d)*log(-(2*sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(-1
/d)*(tan(f*x + e) - 1) - tan(f*x + e)^2 + 4*tan(f*x + e) - 1)/(tan(f*x + e)
^2 + 1))*tan(f*x + e)^2 - 4*(3*a*tan(f*x + e) + a)*sqrt(d*tan(f*x + e)))/(d
^3*f*tan(f*x + e)^2), -1/3*(3*sqrt(2)*a*sqrt(d)*arctan(1/2*sqrt(2)*sqrt(d*t
an(f*x + e))*(tan(f*x + e) - 1)/(sqrt(d)*tan(f*x + e)))*tan(f*x + e)^2 + 2*
(3*a*tan(f*x + e) + a)*sqrt(d*tan(f*x + e)))/(d^3*f*tan(f*x + e)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{1}{\left(d\tan\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx+\int \frac{\tan\left(e+fx\right)}{\left(d\tan\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))/(d*tan(f*x+e))**(5/2),x)
```

```
[Out] a*(Integral((d*tan(e + f*x))**(-5/2), x) + Integral(tan(e + f*x)/(d*tan(e +
 f*x))**(5/2), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(86) = 172.
time = 0.73, size = 275, normalized size = 2.81

$$-\frac{\sqrt{2}\left(ad\sqrt{|d|}+a|d|^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{2\,d^4f}-\frac{\sqrt{2}\left(ad\sqrt{|d|}+a|d|^{\frac{3}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{2\,d^4f}-\frac{\sqrt{2}\left(ad\sqrt{|d|}-a|d|^{\frac{3}{2}}\right)\log\left(d\tan\left(fx+e\right)+\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{|d|}+|d|\right)}{4\,d^4f}+\frac{\sqrt{2}\left(ad\sqrt{|d|}-a|d|^{\frac{3}{2}}\right)\log\left(d\tan\left(fx+e\right)-\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{|d|}+|d|\right)}{4\,d^4f}-\frac{2\left(3\,ad\tan\left(fx+e\right)+ad\right)}{3\sqrt{d\tan\left(fx+e\right)}\,d^3f\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))/(d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(a*d*sqrt(abs(d)) + a*abs(d)^(3/2))*arctan(1/2*sqrt(2)*(sqrt(2
)*sqrt(abs(d)) + 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(d^4*f) - 1/2*sqrt(2
)*(a*d*sqrt(abs(d)) + a*abs(d)^(3/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs
(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(d^4*f) - 1/4*sqrt(2)*(a*d*sqr
t(abs(d)) - a*abs(d)^(3/2))*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e
```

```
))*sqrt(abs(d)) + abs(d))/(d^4*f) + 1/4*sqrt(2)*(a*d*sqrt(abs(d)) - a*abs(d
)^(3/2))*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + a
bs(d))/(d^4*f) - 2/3*(3*a*d*tan(f*x + e) + a*d)/(sqrt(d*tan(f*x + e))*d^3*f
*tan(f*x + e))
```

Mupad [B]
time = 5.19, size = 103, normalized size = 1.05

$$-\frac{2a}{d^2 f \sqrt{d \tan(e+fx)}}-\frac{2a}{3 d f (d \tan(e+fx))^{3/2}}+\frac{(-1)^{1/4} a \operatorname{atan}\left(\frac{(-1)^{1/4} \sqrt{d \tan(e+fx)}}{\sqrt{d}}\right) (-1+1i)}{d^{5/2} f}+\frac{(-1)^{1/4} a \operatorname{atanh}\left(\frac{(-1)^{1/4} \sqrt{d \tan(e+fx)}}{\sqrt{d}}\right) (1+1i)}{d^{5/2} f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x))/(d*tan(e + f*x))^(5/2),x)
```

```
[Out] ((-1)^(1/4)*a*atanh(((-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2))*(1 + 1i))/
(d^(5/2)*f) - (2*a)/(3*d*f*(d*tan(e + f*x))^(3/2)) - ((-1)^(1/4)*a*atan(((-
1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2))*(1 - 1i))/(d^(5/2)*f) - (2*a)/(d^
2*f*(d*tan(e + f*x))^(1/2))
```

3.342 $\int \frac{a+a\tan(e+fx)}{(d\tan(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=121

$$-\frac{\sqrt{2}\,a\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{d^{7/2}f}-\frac{2a}{5df(d\tan(e+fx))^{5/2}}-\frac{2a}{3d^2f(d\tan(e+fx))^{3/2}}+\frac{2a}{d^3f\sqrt{d\tan(e+}}$$

```
[Out] -a*arctanh(1/2*(d^(1/2)+d^(1/2)*tan(f*x+e))*2^(1/2)/(d*tan(f*x+e))^(1/2))*2
^(1/2)/d^(7/2)/f+2*a/d^3/f/(d*tan(f*x+e))^(1/2)-2/5*a/d/f/(d*tan(f*x+e))^(5
/2)-2/3*a/d^2/f/(d*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.12, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3610, 3613, 214}

$$-\frac{\sqrt{2}\,a\tanh^{-1}\left(\frac{\sqrt{d}\,\tan(e+fx)+\sqrt{d}}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{d^{7/2}f}+\frac{2a}{d^3f\sqrt{d\tan(e+fx)}}-\frac{2a}{3d^2f(d\tan(e+fx))^{3/2}}-\frac{2a}{5df(d\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[e + f*x])/(d*Tan[e + f*x])^(7/2),x]
```

```
[Out] -((Sqrt[2]*a*ArcTanh[(Sqrt[d] + Sqrt[d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[d*Tan[e
 + f*x]])])/(d^(7/2)*f)) - (2*a)/(5*d*f*(d*Tan[e + f*x])^(5/2)) - (2*a)/(3*
d^2*f*(d*Tan[e + f*x])^(3/2)) + (2*a)/(d^3*f*Sqrt[d*Tan[e + f*x]])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
```

```
EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{a+a\tan(e+fx)}{(d\tan(e+fx))^{7/2}}\,dx = -\frac{2a}{5df(d\tan(e+fx))^{5/2}} + \frac{\int \frac{ad-ad\tan(e+fx)}{(d\tan(e+fx))^{5/2}}\,dx}{d^2}$$

$$= -\frac{2a}{5df(d\tan(e+fx))^{5/2}} - \frac{2a}{3d^2f(d\tan(e+fx))^{3/2}} + \frac{\int \frac{-ad^2-ad^2\tan(e+fx)}{(d\tan(e+fx))^{3/2}}\,dx}{d^4}$$

$$= -\frac{2a}{5df(d\tan(e+fx))^{5/2}} - \frac{2a}{3d^2f(d\tan(e+fx))^{3/2}} + \frac{2a}{d^3f\sqrt{d\tan(e+fx)}} + \frac{\int \frac{-}{\sqrt{}}}{}$$

$$= -\frac{2a}{5df(d\tan(e+fx))^{5/2}} - \frac{2a}{3d^2f(d\tan(e+fx))^{3/2}} + \frac{2a}{d^3f\sqrt{d\tan(e+fx)}} - \frac{(2a^2}{}$$

$$= -\frac{\sqrt{2}\,a\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{d^{7/2}f} - \frac{2a}{5df(d\tan(e+fx))^{5/2}} - \frac{2a}{3d^2f(d\tan(e}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.20, size = 68, normalized size = 0.56

$$-\frac{\left(\frac{1}{5}+\frac{i}{5}\right)a\left({}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-i\tan(e+fx)\right)-i\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};i\tan(e+fx)\right)\right)}{df(d\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[e + f*x])/(d*Tan[e + f*x])^(7/2),x]
```

```
[Out] ((-1/5 - I/5)*a*(Hypergeometric2F1[-5/2, 1, -3/2, (-I)*Tan[e + f*x]] - I*Hy
pergeometric2F1[-5/2, 1, -3/2, I*Tan[e + f*x]]))/(d*f*(d*Tan[e + f*x])^(5/2
))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(100) = 200.
time = 0.20, size = 323, normalized size = 2.67

method	result

derivativedivides	$\frac{a\left(-\frac{2}{3d^2(d\tan(fx+e))^{\frac{3}{2}}}-\frac{2}{5d(d\tan(fx+e))^{\frac{5}{2}}}+\frac{2}{d^3\sqrt{d\tan(fx+e)}}+\frac{-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan \cdots}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan \cdots}}\right)\cdots}{\cdots}}{\cdots}\right)}{\cdots}$
default	$\frac{a\left(-\frac{2}{3d^2(d\tan(fx+e))^{\frac{3}{2}}}-\frac{2}{5d(d\tan(fx+e))^{\frac{5}{2}}}+\frac{2}{d^3\sqrt{d\tan(fx+e)}}+\frac{-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan \cdots}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan \cdots}}\right)\cdots}{\cdots}}{\cdots}\right)}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(f*x+e))/(d*tan(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a*(-2/3/d^2/(d*tan(f*x+e))^(3/2)-2/5/d/(d*tan(f*x+e))^(5/2)+2/d^3/(d*ta
n(f*x+e))^(1/2)+2/d^3*(-1/8/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(
1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d
*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*ta
n(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))+1
/8/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2
^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+
(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan
(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))))
```

Maxima [A]
time = 0.53, size = 143, normalized size = 1.18

$$-\frac{\frac{15\,a\left(\frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)}{d^2}-\frac{4\left(15\,ad^2\tan(fx+e)^2-5\,ad^2\tan(fx+e)-3\,ad^2\right)}{(d\tan(fx+e))^{\frac{5}{2}}d^2}}{30\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))/(d*tan(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] -1/30*(15*a*(sqrt(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt
(d) + d)/sqrt(d) - sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e)
)*sqrt(d) + d)/sqrt(d))/d^2 - 4*(15*a*d^2*tan(f*x + e)^2 - 5*a*d^2*tan(f*x
+ e) - 3*a*d^2)/((d*tan(f*x + e))^(5/2)*d^2))/(d*f)
```

Fricas [A]
time = 0.88, size = 261, normalized size = 2.16

$$\left[\frac{15\,\sqrt{2}\,a\sqrt{d}\,\log\left(\frac{\tan\left(fx+e\right)^{2}-\frac{2\,\sqrt{2}\,\sqrt{d\tan\left(fx+e\right)}\,\left(\tan\left(fx+e\right)+1\right)}{\sqrt{d}}+4\,\tan\left(fx+e\right)+1}{\tan\left(fx+e\right)^{2}+1}\right)\tan\left(fx+e\right)^{3}+4\,\left(15\,a\tan\left(fx+e\right)^{2}-5\,a\tan\left(fx+e\right)-3\,a\right)\sqrt{d\tan\left(fx+e\right)}}{30\,d^{4}f\tan\left(fx+e\right)^{3}},\ \frac{15\,\sqrt{2}\,ad\sqrt{-\frac{1}{d}}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{-\frac{1}{d}}\,\left(\tan\left(fx+e\right)+1\right)}{2\,\tan\left(fx+e\right)}\right)\tan\left(fx+e\right)^{3}+2\,\left(15\,a\tan\left(fx+e\right)^{2}-5\,a\tan\left(fx+e\right)-3\,a\right)\sqrt{d\tan\left(fx+e\right)}}{15\,d^{4}f\tan\left(fx+e\right)^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))/(d*tan(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*sqrt(2)*a*sqrt(d)*log((tan(f*x + e)^2 - 2*sqrt(2)*sqrt(d*tan(f*x
+ e))*(tan(f*x + e) + 1)/sqrt(d) + 4*tan(f*x + e) + 1)/(tan(f*x + e)^2 + 1)
)*tan(f*x + e)^3 + 4*(15*a*tan(f*x + e)^2 - 5*a*tan(f*x + e) - 3*a)*sqrt(d*
tan(f*x + e)))/(d^4*f*tan(f*x + e)^3), 1/15*(15*sqrt(2)*a*d*sqrt(-1/d)*arct
an(1/2*sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(-1/d)*(tan(f*x + e) + 1)/tan(f*x +
 e))*tan(f*x + e)^3 + 2*(15*a*tan(f*x + e)^2 - 5*a*tan(f*x + e) - 3*a)*sqrt
(d*tan(f*x + e)))/(d^4*f*tan(f*x + e)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{1}{\left(d\tan\left(e+fx\right)\right)^{\frac{7}{2}}}\,dx+\int \frac{\tan\left(e+fx\right)}{\left(d\tan\left(e+fx\right)\right)^{\frac{7}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))/(d*tan(f*x+e))**(7/2),x)
```

```
[Out] a*(Integral((d*tan(e + f*x))**(-7/2), x) + Integral(tan(e + f*x)/(d*tan(e +
 f*x))**(7/2), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(105) = 210$.
time = 0.77, size = 295, normalized size = 2.44

$$-\frac{\sqrt{2}\,\left(ad\sqrt{|d|}-a|d|^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{2}\,\left(\sqrt{2}\,\sqrt{|d|}+2\,\sqrt{d\tan\left(fx+e\right)}\right)}{2\,\sqrt{|d|}}\right)}{2\,d^{5}f}-\frac{\sqrt{2}\,\left(ad\sqrt{|d|}-a|d|^{\frac{3}{2}}\right)\arctan\left(-\frac{\sqrt{2}\,\left(\sqrt{2}\,\sqrt{|d|}-2\,\sqrt{d\tan\left(fx+e\right)}\right)}{2\,\sqrt{|d|}}\right)}{2\,d^{5}f}-\frac{\sqrt{2}\,\left(ad\sqrt{|d|}+a|d|^{\frac{3}{2}}\right)\log\left(d\tan\left(fx+e\right)+\sqrt{2}\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{|d|}+|d|\right)}{4\,d^{5}f}+\frac{\sqrt{2}\,\left(ad\sqrt{|d|}+a|d|^{\frac{3}{2}}\right)\log\left(d\tan\left(fx+e\right)-\sqrt{2}\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{|d|}+|d|\right)}{4\,d^{5}f}+\frac{2\,\left(15\,ad^{2}\tan\left(fx+e\right)^{2}-5\,ad^{2}\tan\left(fx+e\right)-3\,ad^{2}\right)}{15\,\sqrt{d\tan\left(fx+e\right)}\,d^{5}f\tan\left(fx+e\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))/(d*tan(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(a*d*sqrt(abs(d)) - a*abs(d)^(3/2))*arctan(1/2*sqrt(2)*(sqrt(2
)*sqrt(abs(d)) + 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(d^5*f) - 1/2*sqrt(2
)*(a*d*sqrt(abs(d)) - a*abs(d)^(3/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs
(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(d^5*f) - 1/4*sqrt(2)*(a*d*sqr
t(abs(d)) + a*abs(d)^(3/2))*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e
))*sqrt(abs(d)) + abs(d))/(d^5*f) + 1/4*sqrt(2)*(a*d*sqrt(abs(d)) + a*abs(d
)^(3/2))*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + a
```

```
bs(d))/(d^5*f) + 2/15*(15*a*d^2*tan(f*x + e)^2 - 5*a*d^2*tan(f*x + e) - 3*a
*d^2)/(sqrt(d*tan(f*x + e))*d^5*f*tan(f*x + e)^2)
```

Mupad [B]
time = 5.80, size = 120, normalized size = 0.99

$$-\frac{\frac{2a}{5d}-\frac{2a\tan(e+fx)^2}{d}}{f\,(d\tan(e+fx))^{5/2}}-\frac{2a}{3d^2f\,(d\tan(e+fx))^{3/2}}+\frac{(-1)^{1/4}a\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)(1+1\mathrm{i})}{d^{7/2}f}+\frac{(-1)^{1/4}a\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)(-1+1\mathrm{i})}{d^{7/2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x))/(d*tan(e + f*x))^(7/2),x)
```

```
[Out] ((-1)^(1/4)*a*atan(((-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2))*(1 + 1i))/(
d^(7/2)*f) - (2*a)/(3*d^2*f*(d*tan(e + f*x))^(3/2)) - ((2*a)/(5*d) - (2*a*t
an(e + f*x)^2)/d)/(f*(d*tan(e + f*x))^(5/2)) - ((-1)^(1/4)*a*atanh(((-1)^(1
/4)*(d*tan(e + f*x))^(1/2))/d^(1/2))*(1 - 1i))/(d^(7/2)*f)
```

3.343 $\int (d\tan(e+fx))^{5/2}(a+a\tan(e+fx))^2\,dx$

Optimal. Leaf size=269

$$-\frac{\sqrt{2}\,a^2d^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}+\frac{\sqrt{2}\,a^2d^{5/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}-\frac{a^2d^{5/2}\,\mathrm{lo}}{}$$

```
[Out] -1/2*a^2*d^(5/2)*ln(d^(1/2)-2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e)
)/f*2^(1/2)+1/2*a^2*d^(5/2)*ln(d^(1/2)+2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)
*tan(f*x+e))/f*2^(1/2)-a^2*d^(5/2)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^
(1/2))*2^(1/2)/f+a^2*d^(5/2)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))
*2^(1/2)/f-4*a^2*d^2*(d*tan(f*x+e))^(1/2)/f+4/5*a^2*(d*tan(f*x+e))^(5/2)/f+
2/7*a^2*(d*tan(f*x+e))^(7/2)/d/f
```

Rubi [A]

time = 0.20, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3624, 12, 16, 3554, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\sqrt{2}\,a^2d^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}+\frac{\sqrt{2}\,a^2d^{5/2}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{f}-\frac{a^2d^{5/2}\log\left(\sqrt{d}\,\tan(e+fx)-\sqrt{2}\,\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,f}+\frac{a^2d^{5/2}\log\left(\sqrt{d}\,\tan(e+fx)+\sqrt{2}\,\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,f}-\frac{4a^2d^2\sqrt{d\tan(e+fx)}}{f}+\frac{2a^2(d\tan(e+fx))^{7/2}}{7df}+\frac{4a^2(d\tan(e+fx))^{5/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(5/2)*(a + a*Tan[e + f*x])^2,x]
```

```
[Out] -((Sqrt[2]*a^2*d^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/
f) + (Sqrt[2]*a^2*d^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]
])/f - (a^2*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan
[e + f*x]]])/(Sqrt[2]*f) + (a^2*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x]
+ Sqrt[2]*Sqrt[d*Tan[e + f*x]]])/(Sqrt[2]*f) - (4*a^2*d^2*Sqrt[d*Tan[e + f*
x]])/f + (4*a^2*(d*Tan[e + f*x])^(5/2))/(5*f) + (2*a^2*(d*Tan[e + f*x])^(7/
2))/(7*d*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\int (d\tan(e+fx))^{5/2}(a+a\tan(e+fx))^2\,dx &= \frac{2a^2(d\tan(e+fx))^{7/2}}{7df} + \int 2a^2\tan(e+fx)(d\tan(e+fx))^{5/} \\
&= \frac{2a^2(d\tan(e+fx))^{7/2}}{7df} + \left(2a^2\right)\int \tan(e+fx)(d\tan(e+fx) \\
&= \frac{2a^2(d\tan(e+fx))^{7/2}}{7df} + \frac{\left(2a^2\right)\int (d\tan(e+fx))^{7/2}\,dx}{d} \\
&= \frac{4a^2(d\tan(e+fx))^{5/2}}{5f} + \frac{2a^2(d\tan(e+fx))^{7/2}}{7df} - \left(2a^2d\right)\int (\\
&= -\frac{4a^2d^2\sqrt{d\tan(e+fx)}}{f} + \frac{4a^2(d\tan(e+fx))^{5/2}}{5f} + \frac{2a^2(d\,\mathrm{t}}{} \\
&= -\frac{4a^2d^2\sqrt{d\tan(e+fx)}}{f} + \frac{4a^2(d\tan(e+fx))^{5/2}}{5f} + \frac{2a^2(d\,\mathrm{t}}{} \\
&= -\frac{4a^2d^2\sqrt{d\tan(e+fx)}}{f} + \frac{4a^2(d\tan(e+fx))^{5/2}}{5f} + \frac{2a^2(d\,\mathrm{t}}{} \\
&= -\frac{4a^2d^2\sqrt{d\tan(e+fx)}}{f} + \frac{4a^2(d\tan(e+fx))^{5/2}}{5f} + \frac{2a^2(d\,\mathrm{t}}{} \\
&= -\frac{4a^2d^2\sqrt{d\tan(e+fx)}}{f} + \frac{4a^2(d\tan(e+fx))^{5/2}}{5f} + \frac{2a^2(d\,\mathrm{t}}{} \\
&= -\frac{a^2d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right.}{\sqrt{2}f} \\
&= -\frac{\sqrt{2}a^2d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f} + \frac{\sqrt{2}a^2d^{5}}{}
\end{aligned}
$$

Mathematica [A]
time = 1.29, size = 187, normalized size = 0.70

$$
\frac{a^2(d\tan(e+fx))^{5/2}\left(-70\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)+70\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(e+fx)}\right)-35\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)+35\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)-280\sqrt{\tan(e+fx)}+56\tan^{\frac{5}{2}}(e+fx)+20\tan^{\frac{7}{2}}(e+fx)\right)}{70f\tan^{\frac{5}{2}}(e+fx)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(5/2)*(a + a*Tan[e + f*x])^2,x]
```

```
[Out] (a^2*(d*Tan[e + f*x])^(5/2)*(-70*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*
x]]] + 70*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] - 35*Sqrt[2]*Log[1
```

```
 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] + 35*Sqrt[2]*Log[1 + Sqrt[2]*
Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 280*Sqrt[Tan[e + f*x]] + 56*Tan[e + f*
x]^(5/2) + 20*Tan[e + f*x]^(7/2)))/(70*f*Tan[e + f*x]^(5/2))
```

Maple [A]
time = 0.25, size = 187, normalized size = 0.70

method	result
derivativedivides	$\frac{2a^2\left(\frac{(d\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{2d(d\tan(fx+e))^{\frac{5}{2}}}{5}-2d^3\sqrt{d\tan(fx+e)}+\frac{d^3(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(}\right.\right.}{}\right.}{}$
default	$\frac{2a^2\left(\frac{(d\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{2d(d\tan(fx+e))^{\frac{5}{2}}}{5}-2d^3\sqrt{d\tan(fx+e)}+\frac{d^3(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(}\right.\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(5/2)*(a+a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^2/d*(1/7*(d*tan(f*x+e))^(7/2)+2/5*d*(d*tan(f*x+e))^(5/2)-2*d^3*(d*tan
(f*x+e))^(1/2)+1/4*d^3*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d
*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*
x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e
))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)))
```

Maxima [A]
time = 0.51, size = 219, normalized size = 0.81

$$\frac{20\,(d\tan(fx+e))^{\frac{7}{2}}a^2+56\,(d\tan(fx+e))^{\frac{5}{2}}a^2d-280\sqrt{d\tan(fx+e)}\,a^2d^3+35\left(2\sqrt{2}\,d^{\frac{7}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)+2\sqrt{2}\,d^{\frac{7}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)+\sqrt{2}\,d^{\frac{7}{2}}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)-\sqrt{2}\,d^{\frac{7}{2}}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)\right)a^2}{70\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)*(a+a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/70*(20*(d*tan(f*x + e))^(7/2)*a^2 + 56*(d*tan(f*x + e))^(5/2)*a^2*d - 280
*sqrt(d*tan(f*x + e))*a^2*d^3 + 35*(2*sqrt(2)*d^(7/2)*arctan(1/2*sqrt(2)*(s
qrt(2)*sqrt(d) + 2*sqrt(d*tan(f*x + e)))/sqrt(d)) + 2*sqrt(2)*d^(7/2)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d)) + sqrt(2
)*d^(7/2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d) -
sqrt(2)*d^(7/2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) +
 d))*a^2)/(d*f)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 775 vs. $2(225) = 450$.
time = 1.49, size = 775, normalized size = 2.88

$$-\frac{140\sqrt{2}\left(\frac{a^8d^{10}}{f^4}\right)^{\frac14} f \arctan\left(-\frac{a^8d^{10} + \sqrt{2}\left(\frac{a^8d^{10}}{f^4}\right)^{\frac34} a^2d^2f^3\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}} - \sqrt{2}\left(\frac{a^8d^{10}}{f^4}\right)^{\frac34} f^3\sqrt{\frac{a^4d^5\sin(fx+e) + \sqrt{2}\left(\frac{a^8d^{10}}{f^4}\right)^{\frac14} a^2d^2f\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e) + \sqrt{\frac{a^8d^{10}}{f^4}} f^2\cos(fx+e)}{\cos(fx+e)}}}{a^8d^{10}}\right)\cos(fx+e)^3 + 140\sqrt{2}\left(\frac{a^8d^{10}}{f^4}\right)^{\frac14} f \arctan\left(\frac{a^8d^{10} - \sqrt{2}\left(\frac{a^8d^{10}}{f^4}\right)^{\frac34} a^2d^2f^3\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}} + \sqrt{2}\left(\frac{a^8d^{10}}{f^4}\right)^{\frac34} f^3\sqrt{\frac{a^4d^5\sin(fx+e) - \sqrt{2}\left(\frac{a^8d^{10}}{f^4}\right)^{\frac14} a^2d^2f\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e) + \sqrt{\frac{a^8d^{10}}{f^4}} f^2\cos(fx+e)}{\cos(fx+e)}}}{a^8d^{10}}\right)\cos(fx+e)^3 - 35\sqrt{2}\left(\frac{a^8d^{10}}{f^4}\right)^{\frac14} f\cos(fx+e)^3\log\left(\frac{a^4d^5\sin(fx+e) + \sqrt{2}\left(\frac{a^8d^{10}}{f^4}\right)^{\frac14} a^2d^2f\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e) + \sqrt{\frac{a^8d^{10}}{f^4}} f^2\cos(fx+e)}{\cos(fx+e)}\right) + 35\sqrt{2}\left(\frac{a^8d^{10}}{f^4}\right)^{\frac14} f\cos(fx+e)^3\log\left(\frac{a^4d^5\sin(fx+e) - \sqrt{2}\left(\frac{a^8d^{10}}{f^4}\right)^{\frac14} a^2d^2f\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e) + \sqrt{\frac{a^8d^{10}}{f^4}} f^2\cos(fx+e)}{\cos(fx+e)}\right) + 4\left(84a^2d^2\cos(fx+e)^3 - 14a^2d^2\cos(fx+e) + 5\left(a^2d^2\cos(fx+e)^2 - a^2d^2\right)\sin(fx+e)\right)\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}}{70f\cos(fx+e)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)*(a+a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/70*(140*sqrt(2)*(a^8*d^10/f^4)^(1/4)*f*arctan(-(a^8*d^10 + sqrt(2)*(a^8*
d^10/f^4)^(3/4)*a^2*d^2*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e)) - sqrt(2)*(a^
8*d^10/f^4)^(3/4)*f^3*sqrt((a^4*d^5*sin(f*x + e) + sqrt(2)*(a^8*d^10/f^4)^(
1/4)*a^2*d^2*f*sqrt(d*sin(f*x + e)/cos(f*x + e))*cos(f*x + e) + sqrt(a^8*d^
10/f^4)*f^2*cos(f*x + e))/cos(f*x + e)))/(a^8*d^10))*cos(f*x + e)^3 + 140*s
qrt(2)*(a^8*d^10/f^4)^(1/4)*f*arctan((a^8*d^10 - sqrt(2)*(a^8*d^10/f^4)^(3/
4)*a^2*d^2*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e)) + sqrt(2)*(a^8*d^10/f^4)^(
3/4)*f^3*sqrt((a^4*d^5*sin(f*x + e) - sqrt(2)*(a^8*d^10/f^4)^(1/4)*a^2*d^2*
f*sqrt(d*sin(f*x + e)/cos(f*x + e))*cos(f*x + e) + sqrt(a^8*d^10/f^4)*f^2*c
os(f*x + e))/cos(f*x + e)))/(a^8*d^10))*cos(f*x + e)^3 - 35*sqrt(2)*(a^8*d^
10/f^4)^(1/4)*f*cos(f*x + e)^3*log((a^4*d^5*sin(f*x + e) + sqrt(2)*(a^8*d^1
0/f^4)^(1/4)*a^2*d^2*f*sqrt(d*sin(f*x + e)/cos(f*x + e))*cos(f*x + e) + sqr
t(a^8*d^10/f^4)*f^2*cos(f*x + e))/cos(f*x + e)) + 35*sqrt(2)*(a^8*d^10/f^4)
^(1/4)*f*cos(f*x + e)^3*log((a^4*d^5*sin(f*x + e) - sqrt(2)*(a^8*d^10/f^4)^
(1/4)*a^2*d^2*f*sqrt(d*sin(f*x + e)/cos(f*x + e))*cos(f*x + e) + sqrt(a^8*d
^10/f^4)*f^2*cos(f*x + e))/cos(f*x + e)) + 4*(84*a^2*d^2*cos(f*x + e)^3 - 1
4*a^2*d^2*cos(f*x + e) + 5*(a^2*d^2*cos(f*x + e)^2 - a^2*d^2)*sin(f*x + e))
*sqrt(d*sin(f*x + e)/cos(f*x + e)))/(f*cos(f*x + e)^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int (d\tan(e+fx))^{\frac52}\,dx + \int 2(d\tan(e+fx))^{\frac52}\tan(e+fx)\,dx + \int (d\tan(e+fx))^{\frac52}\tan^2(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(5/2)*(a+a*tan(f*x+e))**2,x)
```

```
[Out] a**2*(Integral((d*tan(e + f*x))**(5/2), x) + Integral(2*(d*tan(e + f*x))**(
5/2)*tan(e + f*x), x) + Integral((d*tan(e + f*x))**(5/2)*tan(e + f*x)**2, x
))
```

Giac [**A**]
time = 0.79, size = 293, normalized size = 1.09

$$\frac{\sqrt{2}a^2d^2\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{f} + \frac{\sqrt{2}a^2d^2\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{f} + \frac{\sqrt{2}a^2d^2\sqrt{|d|}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{|d|}+|d|\right)}{2f} - \frac{\sqrt{2}a^2d^2\sqrt{|d|}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{|d|}+|d|\right)}{2f} + \frac{2\left(5\sqrt{d\tan(fx+e)}a^2d^9f^6\tan(fx+e)^3 + 14\sqrt{d\tan(fx+e)}a^2d^9f^6\tan(fx+e)^2 - 70\sqrt{d\tan(fx+e)}a^2d^9f^6\right)}{35d^7f^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)*(a+a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] sqrt(2)*a^2*d^2*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*s
qrt(d*tan(f*x + e)))/sqrt(abs(d)))/f + sqrt(2)*a^2*d^2*sqrt(abs(d))*arctan(
-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/
f + 1/2*sqrt(2)*a^2*d^2*sqrt(abs(d))*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*ta
n(f*x + e))*sqrt(abs(d)) + abs(d))/f - 1/2*sqrt(2)*a^2*d^2*sqrt(abs(d))*log
(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/f + 2
/35*(5*sqrt(d*tan(f*x + e))*a^2*d^9*f^6*tan(f*x + e)^3 + 14*sqrt(d*tan(f*x
+ e))*a^2*d^9*f^6*tan(f*x + e)^2 - 70*sqrt(d*tan(f*x + e))*a^2*d^9*f^6)/(d^
7*f^7)
```

Mupad [B]
time = 5.30, size = 125, normalized size = 0.46

$$\frac{4\,a^2\,{\left(d\,\tan(e+f\,x)\right)}^{5/2}}{5\,f}-\frac{4\,a^2\,d^2\,\sqrt{d\,\tan\left(e+f\,x\right)}}{f}+\frac{2\,a^2\,{\left(d\,\tan(e+f\,x)\right)}^{7/2}}{7\,d\,f}-\frac{{\left(-1\right)}^{1/4}\,a^2\,d^{5/2}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{d\,\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)\,2{}\mathrm{i}}{f}-\frac{2\,{\left(-1\right)}^{1/4}\,a^2\,d^{5/2}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{d\,\tan\left(e+f\,x\right)}\,1{}\mathrm{i}}{\sqrt{d}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(5/2)*(a + a*tan(e + f*x))^2,x)
```

```
[Out] (4*a^2*(d*tan(e + f*x))^(5/2))/(5*f) - (4*a^2*d^2*(d*tan(e + f*x))^(1/2))/f
 + (2*a^2*(d*tan(e + f*x))^(7/2))/(7*d*f) - ((-1)^(1/4)*a^2*d^(5/2)*atan(((
-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2))*2i)/f - (2*(-1)^(1/4)*a^2*d^(5/2
)*atan(((-1)^(1/4)*(d*tan(e + f*x))^(1/2)*1i)/d^(1/2)))/f
```

3.344 $\int (d\tan(e+fx))^{3/2}(a+a\tan(e+fx))^2\,dx$

Optimal. Leaf size=246

$$\frac{\sqrt{2}\,a^2d^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}-\frac{\sqrt{2}\,a^2d^{3/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}-\frac{a^2d^{3/2}\,\text{lo}}{}$$

```
[Out] -1/2*a^2*d^(3/2)*ln(d^(1/2)-2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e)
)/f*2^(1/2)+1/2*a^2*d^(3/2)*ln(d^(1/2)+2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)
*tan(f*x+e))/f*2^(1/2)+a^2*d^(3/2)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^
(1/2))*2^(1/2)/f-a^2*d^(3/2)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))
*2^(1/2)/f+4/3*a^2*(d*tan(f*x+e))^(3/2)/f+2/5*a^2*(d*tan(f*x+e))^(5/2)/d/f
```

Rubi [A]
time = 0.16, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3624, 12, 16, 3554, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{\sqrt{2}\,a^2d^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}-\frac{\sqrt{2}\,a^2d^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{f}-\frac{a^2d^{3/2}\log\left(\sqrt{d}\,\tan(e+fx)-\sqrt{2}\,\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,f}+\frac{a^2d^{3/2}\log\left(\sqrt{d}\,\tan(e+fx)+\sqrt{2}\,\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,f}+\frac{2a^2(d\tan(e+fx))^{5/2}}{5df}+\frac{4a^2(d\tan(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(3/2)*(a + a*Tan[e + f*x])^2,x]
```

```
[Out] (Sqrt[2]*a^2*d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/f
- (Sqrt[2]*a^2*d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/
f - (a^2*d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e
+ f*x]]])/(Sqrt[2]*f) + (a^2*d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + S
qrt[2]*Sqrt[d*Tan[e + f*x]]])/(Sqrt[2]*f) + (4*a^2*(d*Tan[e + f*x])^(3/2))/
(3*f) + (2*a^2*(d*Tan[e + f*x])^(5/2))/(5*d*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int (d\tan(e+fx))^{3/2}(a+a\tan(e+fx))^2\,dx = \frac{2a^2(d\tan(e+fx))^{5/2}}{5df} + \int 2a^2\tan(e+fx)(d\tan(e+fx))^{3/2}$$

$$= \frac{2a^2(d\tan(e+fx))^{5/2}}{5df} + (2a^2)\int \tan(e+fx)(d\tan(e+fx))^{3}$$

$$= \frac{2a^2(d\tan(e+fx))^{5/2}}{5df} + \frac{(2a^2)\int (d\tan(e+fx))^{5/2}\,dx}{d}$$

$$= \frac{4a^2(d\tan(e+fx))^{3/2}}{3f} + \frac{2a^2(d\tan(e+fx))^{5/2}}{5df} - (2a^2d)\int \sqrt{}$$

$$= \frac{4a^2(d\tan(e+fx))^{3/2}}{3f} + \frac{2a^2(d\tan(e+fx))^{5/2}}{5df} - \frac{(2a^2d^2)\,\mathrm{Sub}}{}$$

$$= \frac{4a^2(d\tan(e+fx))^{3/2}}{3f} + \frac{2a^2(d\tan(e+fx))^{5/2}}{5df} - \frac{(4a^2d^2)\,\mathrm{Sub}}{}$$

$$= \frac{4a^2(d\tan(e+fx))^{3/2}}{3f} + \frac{2a^2(d\tan(e+fx))^{5/2}}{5df} + \frac{(2a^2d^2)\,\mathrm{Sub}}{}$$

$$= \frac{4a^2(d\tan(e+fx))^{3/2}}{3f} + \frac{2a^2(d\tan(e+fx))^{5/2}}{5df} - \frac{(a^2d^{3/2})\,\mathrm{Su}}{}$$

$$= -\frac{a^2d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right.}{\sqrt{2}\,f}$$

$$= \frac{\sqrt{2}\,a^2d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f} - \frac{\sqrt{2}\,a^2d^{3/2}\,\mathrm{ta}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.40, size = 52, normalized size = 0.21

$$\frac{2a^2(d\tan(e+fx))^{3/2}\left(10-10\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(e+fx)\right)+3\tan(e+fx)\right)}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(3/2)*(a + a*Tan[e + f*x])^2,x]
```

```
[Out] (2*a^2*(d*Tan[e + f*x])^(3/2)*(10 - 10*Hypergeometric2F1[3/4, 1, 7/4, -Tan[
e + f*x]^2] + 3*Tan[e + f*x]))/(15*f)
```

Maple [A]
time = 0.26, size = 172, normalized size = 0.70

method	result
derivativedivides	$\frac{2a^2\left(\frac{(d\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{2d(d\tan(fx+e))^{\frac{3}{2}}}{3}-\frac{d^3\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\,\mathrm{ar}\right.}{}\right.}{fd}$
default	$\frac{2a^2\left(\frac{(d\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{2d(d\tan(fx+e))^{\frac{3}{2}}}{3}-\frac{d^3\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\,\mathrm{ar}\right.}{}\right.}{fd}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(3/2)*(a+a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^2/d*(1/5*(d*tan(f*x+e))^(5/2)+2/3*d*(d*tan(f*x+e))^(3/2)-1/4*d^3/(d^2
)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+
(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(
1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/
2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)))
```

Maxima [A]
time = 0.52, size = 203, normalized size = 0.83

$$-\frac{15\,a^2d^3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)-12\,(d\tan(fx+e))^{\frac{5}{2}}a^2-40\,(d\tan(fx+e))^{\frac{3}{2}}a^2d}{30\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)*(a+a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/30*(15*a^2*d^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d
*tan(f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*s
qrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(f*x +
 e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan
(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d)) - 12*(d*tan(
f*x + e))^(5/2)*a^2 - 40*(d*tan(f*x + e))^(3/2)*a^2*d)/(d*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(203) = 406$.
time = 1.18, size = 774, normalized size = 3.15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)*(a+a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/30*(60*sqrt(2)*(a^8*d^6/f^4)^(1/4)*f*arctan(-(a^8*d^6 + sqrt(2)*(a^8*d^6/
f^4)^(1/4)*a^6*d^4*f*sqrt(d*sin(f*x + e)/cos(f*x + e)) - sqrt(2)*(a^8*d^6/f
^4)^(1/4)*f*sqrt((a^12*d^9*sin(f*x + e) + sqrt(a^8*d^6/f^4)*a^8*d^6*f^2*cos
(f*x + e) + sqrt(2)*(a^8*d^6/f^4)^(3/4)*a^6*d^4*f^3*sqrt(d*sin(f*x + e)/cos
(f*x + e))*cos(f*x + e))/cos(f*x + e)))/(a^8*d^6))*cos(f*x + e)^2 + 60*sqrt
(2)*(a^8*d^6/f^4)^(1/4)*f*arctan((a^8*d^6 - sqrt(2)*(a^8*d^6/f^4)^(1/4)*a^6
*d^4*f*sqrt(d*sin(f*x + e)/cos(f*x + e)) + sqrt(2)*(a^8*d^6/f^4)^(1/4)*f*sq
rt((a^12*d^9*sin(f*x + e) + sqrt(a^8*d^6/f^4)*a^8*d^6*f^2*cos(f*x + e) - sq
rt(2)*(a^8*d^6/f^4)^(3/4)*a^6*d^4*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e))*cos
(f*x + e))/cos(f*x + e)))/(a^8*d^6))*cos(f*x + e)^2 + 15*sqrt(2)*(a^8*d^6/f
^4)^(1/4)*f*cos(f*x + e)^2*log((a^12*d^9*sin(f*x + e) + sqrt(a^8*d^6/f^4)*a
^8*d^6*f^2*cos(f*x + e) + sqrt(2)*(a^8*d^6/f^4)^(3/4)*a^6*d^4*f^3*sqrt(d*si
n(f*x + e)/cos(f*x + e))*cos(f*x + e))/cos(f*x + e)) - 15*sqrt(2)*(a^8*d^6/
f^4)^(1/4)*f*cos(f*x + e)^2*log((a^12*d^9*sin(f*x + e) + sqrt(a^8*d^6/f^4)*
a^8*d^6*f^2*cos(f*x + e) - sqrt(2)*(a^8*d^6/f^4)^(3/4)*a^6*d^4*f^3*sqrt(d*s
in(f*x + e)/cos(f*x + e))*cos(f*x + e))/cos(f*x + e)) - 4*(3*a^2*d*cos(f*x
+ e)^2 - 10*a^2*d*cos(f*x + e)*sin(f*x + e) - 3*a^2*d)*sqrt(d*sin(f*x + e)/
cos(f*x + e)))/(f*cos(f*x + e)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int (d\tan(e+fx))^{\frac{3}{2}}\,dx + \int 2(d\tan(e+fx))^{\frac{3}{2}}\tan(e+fx)\,dx + \int (d\tan(e+fx))^{\frac{3}{2}}\tan^2(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(3/2)*(a+a*tan(f*x+e))**2,x)
```

```
[Out] a**2*(Integral((d*tan(e + f*x))**(3/2), x) + Integral(2*(d*tan(e + f*x))**(
3/2)*tan(e + f*x), x) + Integral((d*tan(e + f*x))**(3/2)*tan(e + f*x)**2, x
))
```

Giac [A]
time = 0.68, size = 274, normalized size = 1.11

$$-\frac{1}{30}\left(\frac{30\sqrt{2}\,a^2|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{df}+\frac{30\sqrt{2}\,a^2|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{df}-\frac{15\sqrt{2}\,a^2|d|^{\frac{3}{2}}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{|d|}+|d|\right)}{df}+\frac{15\sqrt{2}\,a^2|d|^{\frac{3}{2}}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{|d|}+|d|\right)}{df}-\frac{4\left(3\sqrt{d\tan(fx+e)}\,a^2d^{10}f^4\tan(fx+e)^2+10\sqrt{d\tan(fx+e)}\,a^2d^{10}f^4\tan(fx+e)\right)}{d^{10}f^5}\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)*(a+a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/30*(30*sqrt(2)*a^2*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d))
 + 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(d*f) + 30*sqrt(2)*a^2*abs(d)^(3/2
```

```
)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(
abs(d)))/(d*f) - 15*sqrt(2)*a^2*abs(d)^(3/2)*log(d*tan(f*x + e) + sqrt(2)*s
qrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(d*f) + 15*sqrt(2)*a^2*abs(d)^(3
/2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d)
)/(d*f) - 4*(3*sqrt(d*tan(f*x + e))*a^2*d^10*f^4*tan(f*x + e)^2 + 10*sqrt(d
*tan(f*x + e))*a^2*d^10*f^4*tan(f*x + e))/(d^10*f^5))*d
```

Mupad [B]
time = 4.64, size = 104, normalized size = 0.42

$$\frac{4\,a^2\,{\left(d\,\tan(e+f\,x)\right)}^{3/2}}{3\,f}+\frac{2\,a^2\,{\left(d\,\tan(e+f\,x)\right)}^{5/2}}{5\,d\,f}-\frac{2\,{\left(-1\right)}^{1/4}\,a^2\,d^{3/2}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{d\,\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)}{f}-\frac{{\left(-1\right)}^{1/4}\,a^2\,d^{3/2}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{d\,\tan\left(e+f\,x\right)}\,1{}\mathrm{i}}{\sqrt{d}}\right)\,2{}\mathrm{i}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(3/2)*(a + a*tan(e + f*x))^2,x)
```

```
[Out] (4*a^2*(d*tan(e + f*x))^(3/2))/(3*f) + (2*a^2*(d*tan(e + f*x))^(5/2))/(5*d*
f) - (2*(-1)^(1/4)*a^2*d^(3/2)*atan(((-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(
1/2)))/f - ((-1)^(1/4)*a^2*d^(3/2)*atan(((-1)^(1/4)*(d*tan(e + f*x))^(1/2)*
1i)/d^(1/2))*2i)/f
```

3.345 $\int \sqrt{d\tan(e+fx)}\,(a+a\tan(e+fx))^2\,dx$

Optimal. Leaf size=244

$$\frac{\sqrt{2}\,a^2\sqrt{d}\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}-\frac{\sqrt{2}\,a^2\sqrt{d}\,\mathrm{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}+\frac{a^2\sqrt{d}\,\log}{}$$

```
[Out] 1/2*a^2*ln(d^(1/2)-2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))*d^(1/2)
/f*2^(1/2)-1/2*a^2*ln(d^(1/2)+2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+
e))*d^(1/2)/f*2^(1/2)+a^2*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))*2^
(1/2)*d^(1/2)/f-a^2*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))*2^(1/2)*
d^(1/2)/f+4*a^2*(d*tan(f*x+e))^(1/2)/f+2/3*a^2*(d*tan(f*x+e))^(3/2)/d/f
```

Rubi [A]
time = 0.16, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$,
Rules used = {3624, 12, 16, 3554, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{\sqrt{2}\,a^2\sqrt{d}\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f}-\frac{\sqrt{2}\,a^2\sqrt{d}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{f}+\frac{2a^2(d\tan(e+fx))^{3/2}}{3df}+\frac{4a^2\sqrt{d\tan(e+fx)}}{f}+\frac{a^2\sqrt{d}\,\log\left(\sqrt{d}\,\tan(e+fx)-\sqrt{2}\,\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,f}-\frac{a^2\sqrt{d}\,\log\left(\sqrt{d}\,\tan(e+fx)+\sqrt{2}\,\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Tan[e + f*x]]*(a + a*Tan[e + f*x])^2,x]
```

```
[Out] (Sqrt[2]*a^2*Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/f
- (Sqrt[2]*a^2*Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/
f + (a^2*Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e
+ f*x]]])/(Sqrt[2]*f) - (a^2*Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + S
qrt[2]*Sqrt[d*Tan[e + f*x]]])/(Sqrt[2]*f) + (4*a^2*Sqrt[d*Tan[e + f*x]])/f
+ (2*a^2*(d*Tan[e + f*x])^(3/2))/(3*d*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && !LeQ
[m, -1] && !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \sqrt{d\tan(e+fx)}\,(a+a\tan(e+fx))^2\,dx = \frac{2a^2(d\tan(e+fx))^{3/2}}{3df} + \int 2a^2\tan(e+fx)\sqrt{d\tan(e+fx)}$$

$$= \frac{2a^2(d\tan(e+fx))^{3/2}}{3df} + (2a^2)\int \tan(e+fx)\sqrt{d\tan(e+fx}$$

$$= \frac{2a^2(d\tan(e+fx))^{3/2}}{3df} + \frac{(2a^2)\int(d\tan(e+fx))^{3/2}\,dx}{d}$$

$$= \frac{4a^2\sqrt{d\tan(e+fx)}}{f} + \frac{2a^2(d\tan(e+fx))^{3/2}}{3df} - (2a^2d)\int -$$

$$= \frac{4a^2\sqrt{d\tan(e+fx)}}{f} + \frac{2a^2(d\tan(e+fx))^{3/2}}{3df} - \frac{(2a^2d^2)\,\mathrm{Sub}}{}$$

$$= \frac{4a^2\sqrt{d\tan(e+fx)}}{f} + \frac{2a^2(d\tan(e+fx))^{3/2}}{3df} - \frac{(4a^2d^2)\,\mathrm{Sub}}{}$$

$$= \frac{4a^2\sqrt{d\tan(e+fx)}}{f} + \frac{2a^2(d\tan(e+fx))^{3/2}}{3df} - \frac{(2a^2d)\,\mathrm{Subs}}{}$$

$$= \frac{4a^2\sqrt{d\tan(e+fx)}}{f} + \frac{2a^2(d\tan(e+fx))^{3/2}}{3df} + \frac{\left(a^2\sqrt{d}\right)\mathrm{Su}}{}$$

$$= \frac{a^2\sqrt{d}\,\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{\sqrt{2}f}$$

$$= \frac{\sqrt{2}a^2\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{f} - \frac{\sqrt{2}a^2\sqrt{d}}{}$$

Mathematica [A]
time = 0.46, size = 175, normalized size = 0.72

$$\frac{a^2\sqrt{d\tan(e+fx)}\left(6\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)-6\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(e+fx)}\right)+3\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)-3\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)+24\sqrt{\tan(e+fx)}+4\tan^{\frac{3}{2}}(e+fx)\right)}{6f\sqrt{\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Tan[e + f*x]]*(a + a*Tan[e + f*x])^2,x]
[Out] (a^2*Sqrt[d*Tan[e + f*x]]*(6*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]]
 - 6*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] + 3*Sqrt[2]*Log[1 - Sqr
t[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 3*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Ta
n[e + f*x]] + Tan[e + f*x]] + 24*Sqrt[Tan[e + f*x]] + 4*Tan[e + f*x]^(3/2))
)/(6*f*Sqrt[Tan[e + f*x]])
```

Maple [A]
time = 0.25, size = 170, normalized size = 0.70

method	result
derivativedivides	$\frac{2a^2\left(\frac{(d\tan(fx+e))^{\frac{3}{2}}}{3}+2d\sqrt{d\tan(fx+e)}-\frac{d(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{\cdots}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{\cdots}}\right)\cdots\right.}{\cdots}\right)}{fd}$
default	$\frac{2a^2\left(\frac{(d\tan(fx+e))^{\frac{3}{2}}}{3}+2d\sqrt{d\tan(fx+e)}-\frac{d(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{\cdots}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{\cdots}}\right)\cdots\right.}{\cdots}\right)}{fd}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(1/2)*(a+a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^2/d*(1/3*(d*tan(f*x+e))^(3/2)+2*d*(d*tan(f*x+e))^(1/2)-1/4*d*(d^2)^(1
/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2
)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)
))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(
d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)))
```

Maxima [A]
time = 0.52, size = 200, normalized size = 0.82

$$\frac{4\,(d\tan(fx+e))^{\frac{3}{2}}a^2+24\sqrt{d\tan(fx+e)}\,a^2d-3\left(2\sqrt{2}\,d^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)+2\sqrt{2}\,d^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)+\sqrt{2}\,d^{\frac{3}{2}}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)-\sqrt{2}\,d^{\frac{3}{2}}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)\right)a^2}{6\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)*(a+a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(4*(d*tan(f*x + e))^(3/2)*a^2 + 24*sqrt(d*tan(f*x + e))*a^2*d - 3*(2*sq
rt(2)*d^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(f*x + e)))
/sqrt(d)) + 2*sqrt(2)*d^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*sqrt
(d*tan(f*x + e)))/sqrt(d)) + sqrt(2)*d^(3/2)*log(d*tan(f*x + e) + sqrt(2)*s
qrt(d*tan(f*x + e))*sqrt(d) + d) - sqrt(2)*d^(3/2)*log(d*tan(f*x + e) - sqr
t(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d))*a^2)/(d*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 698 vs. $2(203) = 406$.
time = 1.04, size = 698, normalized size = 2.86

$$\frac{12\sqrt{2}\left(\frac{a^8d^2}{f^4}\right)^{\frac{1}{4}}f\arctan\left(-\frac{a^6d^2+\sqrt{2}\left(\frac{a^8d^2}{f^4}\right)^{\frac{3}{4}}a^2f^3\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}-\sqrt{2}\left(\frac{a^8d^2}{f^4}\right)^{\frac{3}{4}}f^3\sqrt{\frac{a^4d\sin(fx+e)+\sqrt{2}\left(\frac{a^8d^2}{f^4}\right)^{\frac{1}{4}}a^2f\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)+\sqrt{\frac{a^8d^2}{f^4}}f^2\cos(fx+e)}{\cos(fx+e)}}}{a^6d^2}\right)\cos(fx+e)+12\sqrt{2}\left(\frac{a^8d^2}{f^4}\right)^{\frac{1}{4}}f\arctan\left(\frac{a^6d^2-\sqrt{2}\left(\frac{a^8d^2}{f^4}\right)^{\frac{3}{4}}a^2f^3\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}+\sqrt{2}\left(\frac{a^8d^2}{f^4}\right)^{\frac{3}{4}}f^3\sqrt{\frac{a^4d\sin(fx+e)-\sqrt{2}\left(\frac{a^8d^2}{f^4}\right)^{\frac{1}{4}}a^2f\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)+\sqrt{\frac{a^8d^2}{f^4}}f^2\cos(fx+e)}{\cos(fx+e)}}}{a^6d^2}\right)\cos(fx+e)-3\sqrt{2}\left(\frac{a^8d^2}{f^4}\right)^{\frac{1}{4}}f\cos(fx+e)\log\left(\frac{a^4d\sin(fx+e)+\sqrt{2}\left(\frac{a^8d^2}{f^4}\right)^{\frac{1}{4}}a^2f\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)+\sqrt{\frac{a^8d^2}{f^4}}f^2\cos(fx+e)}{\cos(fx+e)}\right)+3\sqrt{2}\left(\frac{a^8d^2}{f^4}\right)^{\frac{1}{4}}f\cos(fx+e)\log\left(\frac{a^4d\sin(fx+e)-\sqrt{2}\left(\frac{a^8d^2}{f^4}\right)^{\frac{1}{4}}a^2f\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)+\sqrt{\frac{a^8d^2}{f^4}}f^2\cos(fx+e)}{\cos(fx+e)}\right)+4\left(6a^2\cos(fx+e)+a^2\sin(fx+e)\right)\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}}{6f\cos(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)*(a+a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(12*sqrt(2)*(a^8*d^2/f^4)^(1/4)*f*arctan(-(a^8*d^2 + sqrt(2)*(a^8*d^2/f
^4)^(3/4)*a^2*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e)) - sqrt(2)*(a^8*d^2/f^4)
^(3/4)*f^3*sqrt((a^4*d*sin(f*x + e) + sqrt(2)*(a^8*d^2/f^4)^(1/4)*a^2*f*sqr
t(d*sin(f*x + e)/cos(f*x + e))*cos(f*x + e) + sqrt(a^8*d^2/f^4)*f^2*cos(f*x
 + e))/cos(f*x + e)))/(a^8*d^2))*cos(f*x + e) + 12*sqrt(2)*(a^8*d^2/f^4)^(1
/4)*f*arctan((a^8*d^2 - sqrt(2)*(a^8*d^2/f^4)^(3/4)*a^2*f^3*sqrt(d*sin(f*x
+ e)/cos(f*x + e)) + sqrt(2)*(a^8*d^2/f^4)^(3/4)*f^3*sqrt((a^4*d*sin(f*x +
e) - sqrt(2)*(a^8*d^2/f^4)^(1/4)*a^2*f*sqrt(d*sin(f*x + e)/cos(f*x + e))*co
s(f*x + e) + sqrt(a^8*d^2/f^4)*f^2*cos(f*x + e))/cos(f*x + e)))/(a^8*d^2))*
cos(f*x + e) - 3*sqrt(2)*(a^8*d^2/f^4)^(1/4)*f*cos(f*x + e)*log((a^4*d*sin(
f*x + e) + sqrt(2)*(a^8*d^2/f^4)^(1/4)*a^2*f*sqrt(d*sin(f*x + e)/cos(f*x +
e))*cos(f*x + e) + sqrt(a^8*d^2/f^4)*f^2*cos(f*x + e))/cos(f*x + e)) + 3*sq
rt(2)*(a^8*d^2/f^4)^(1/4)*f*cos(f*x + e)*log((a^4*d*sin(f*x + e) - sqrt(2)*
(a^8*d^2/f^4)^(1/4)*a^2*f*sqrt(d*sin(f*x + e)/cos(f*x + e))*cos(f*x + e) +
sqrt(a^8*d^2/f^4)*f^2*cos(f*x + e))/cos(f*x + e)) + 4*(6*a^2*cos(f*x + e) +
 a^2*sin(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e)))/(f*cos(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\sqrt{d\tan\left(e+fx\right)}\,dx+\int 2\sqrt{d\tan\left(e+fx\right)}\tan\left(e+fx\right)dx+\int\sqrt{d\tan\left(e+fx\right)}\tan^2\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(1/2)*(a+a*tan(f*x+e))**2,x)
```

```
[Out] a**2*(Integral(sqrt(d*tan(e + f*x)), x) + Integral(2*sqrt(d*tan(e + f*x))*t
an(e + f*x), x) + Integral(sqrt(d*tan(e + f*x))*tan(e + f*x)**2, x))
```

Giac [**A**]
time = 0.76, size = 249, normalized size = 1.02

$$-\frac{\sqrt{2}a^2\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{f}-\frac{\sqrt{2}a^2\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{f}-\frac{\sqrt{2}a^2\sqrt{|d|}\log\left(d\tan\left(fx+e\right)+\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{|d|}+|d|\right)}{2f}+\frac{\sqrt{2}a^2\sqrt{|d|}\log\left(d\tan\left(fx+e\right)-\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{|d|}+|d|\right)}{2f}+\frac{2\left(\sqrt{d\tan\left(fx+e\right)}a^2d^3f^2\tan\left(fx+e\right)+6\sqrt{d\tan\left(fx+e\right)}a^2d^3f^2\right)}{3d^3f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)*(a+a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -sqrt(2)*a^2*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqrt
(d*tan(f*x + e)))/sqrt(abs(d)))/f - sqrt(2)*a^2*sqrt(abs(d))*arctan(-1/2*sq
rt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/f - 1/2
*sqrt(2)*a^2*sqrt(abs(d))*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))
*sqrt(abs(d)) + abs(d))/f + 1/2*sqrt(2)*a^2*sqrt(abs(d))*log(d*tan(f*x + e)
 - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/f + 2/3*(sqrt(d*tan(
```

```
f*x + e))*a^2*d^3*f^2*tan(f*x + e) + 6*sqrt(d*tan(f*x + e))*a^2*d^3*f^2)/(d
^3*f^3)
```

Mupad [B]
time = 4.38, size = 104, normalized size = 0.43

$$\frac{4\,a^2\,\sqrt{d\tan\left(e+f\,x\right)}}{f}+\frac{2\,a^2\,{\left(d\tan(e+f\,x)\right)}^{3/2}}{3\,d\,f}+\frac{{\left(-1\right)}^{1/4}\,a^2\,\sqrt{d}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{d\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)\,2{}\mathrm{i}}{f}+\frac{2\,{\left(-1\right)}^{1/4}\,a^2\,\sqrt{d}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{d\tan\left(e+f\,x\right)}\,1{}\mathrm{i}}{\sqrt{d}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(1/2)*(a + a*tan(e + f*x))^2,x)
```

```
[Out] (4*a^2*(d*tan(e + f*x))^(1/2))/f + (2*a^2*(d*tan(e + f*x))^(3/2))/(3*d*f) +
 ((-1)^(1/4)*a^2*d^(1/2)*atan(((-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2))*
2i)/f + (2*(-1)^(1/4)*a^2*d^(1/2)*atan(((-1)^(1/4)*(d*tan(e + f*x))^(1/2)*1
i)/d^(1/2)))/f
```

3.346 $\int \frac{(a+a\tan(e+fx))^2}{\sqrt{d\tan(e+fx)}}\,dx$

Optimal. Leaf size=222

$$-\frac{\sqrt{2}\,a^2\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{d}\,f}+\frac{\sqrt{2}\,a^2\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{d}\,f}+\frac{a^2\log\left(\sqrt{d}\,+\right.}{}$$

```
[Out] 1/2*a^2*ln(d^(1/2)-2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/f*2^(1/
2)/d^(1/2)-1/2*a^2*ln(d^(1/2)+2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+
e))/f*2^(1/2)/d^(1/2)-a^2*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))*2^
(1/2)/f/d^(1/2)+a^2*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/
f/d^(1/2)+2*a^2*(d*tan(f*x+e))^(1/2)/d/f
```

Rubi [A]
time = 0.14, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3624, 12, 16, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\sqrt{2}\,a^2\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{d}\,f}+\frac{\sqrt{2}\,a^2\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{d}\,f}+\frac{2a^2\sqrt{d\tan(e+fx)}}{df}+\frac{a^2\log\left(\sqrt{d}\,\tan(e+fx)-\sqrt{2}\,\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,\sqrt{d}\,f}-\frac{a^2\log\left(\sqrt{d}\,\tan(e+fx)+\sqrt{2}\,\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,\sqrt{d}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[e + f*x])^2/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] -((Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(Sqrt[d]
*f)) + (Sqrt[2]*a^2*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(Sq
rt[d]*f) + (a^2*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e +
 f*x]]])/(Sqrt[2]*Sqrt[d]*f) - (a^2*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sq
rt[2]*Sqrt[d*Tan[e + f*x]]])/(Sqrt[2]*Sqrt[d]*f) + (2*a^2*Sqrt[d*Tan[e + f*
x]])/(d*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\tan(e+fx))^2}{\sqrt{d\tan(e+fx)}}\,dx &= \frac{2a^2\sqrt{d\tan(e+fx)}}{df}+\int \frac{2a^2\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx \\
&= \frac{2a^2\sqrt{d\tan(e+fx)}}{df}+\left(2a^2\right)\int \frac{\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx \\
&= \frac{2a^2\sqrt{d\tan(e+fx)}}{df}+\frac{\left(2a^2\right)\int \sqrt{d\tan(e+fx)}\,dx}{d} \\
&= \frac{2a^2\sqrt{d\tan(e+fx)}}{df}+\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx,x,d\tan(e+fx)\right)}{f} \\
&= \frac{2a^2\sqrt{d\tan(e+fx)}}{df}+\frac{\left(4a^2\right)\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx,x,\sqrt{d\tan(e+fx)}\right)}{f} \\
&= \frac{2a^2\sqrt{d\tan(e+fx)}}{df}-\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx,x,\sqrt{d\tan(e+fx)}\right)}{f}+\frac{\left(2a^2\right)}{} \\
&= \frac{2a^2\sqrt{d\tan(e+fx)}}{df}+\frac{a^2\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}\,x+x^2}\,dx,x,\sqrt{d\tan(e+fx)}\right)}{f} \\
&= \frac{a^2\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{\sqrt{2}\sqrt{d}\,f}-\frac{a^2\log\left(\sqrt{d}+\sqrt{}\right.}{} \\
&= -\frac{\sqrt{2}\,a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{d}\,f}+\frac{\sqrt{2}\,a^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan}}{\sqrt{}}\right.}{\sqrt{d}\,f}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.26, size = 53, normalized size = 0.24

$$\frac{2a^2\sqrt{d\tan(e+fx)}\,\left(3+2\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(e+fx)\right)\tan(e+fx)\right)}{3df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[e + f*x])^2/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*a^2*Sqrt[d*Tan[e + f*x]]*(3 + 2*Hypergeometric2F1[3/4, 1, 7/4, -Tan[e +
f*x]^2]*Tan[e + f*x]))/(3*d*f)
```

Maple [A]

time = 0.22, size = 155, normalized size = 0.70

method	result
derivativedivides	$\frac{2a^2\left(\sqrt{d\tan(fx+e)}+\frac{d\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\ \cdots}{\cdots}\right.\cdots}{4(d^2)^{\frac{1}{4}}}\cdots\right)}{fd}$
default	$\frac{2a^2\left(\sqrt{d\tan(fx+e)}+\frac{d\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\ \cdots}{\cdots}\right.\cdots}{4(d^2)^{\frac{1}{4}}}\cdots\right)}{fd}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(f*x+e))^2/(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^2/d*((d*tan(f*x+e))^(1/2)+1/4*d/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)
-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^
(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/
4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/
2)+1)))
```

Maxima [A]

time = 0.51, size = 183, normalized size = 0.82

$$\frac{a^2d\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)+4\sqrt{d\tan(fx+e)}\,a^2}{2\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))^2/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(a^2*d*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(f*
x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
- 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(f*x + e) + s
qrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(f*x +
e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d)) + 4*sqrt(d*tan(f*x
+ e))*a^2)/(d*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(184) = 368$.
time = 1.15, size = 671, normalized size = 3.02

$$-\frac{4\sqrt{2}\left(\frac{a^8}{d^2f^4}\right)^{\frac{1}{4}}df\arctan\left(-\frac{\sqrt{2}\left(\frac{a^8}{d^2f^4}\right)^{\frac{1}{4}}a^6f\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}+a^8-\sqrt{2}\left(\frac{a^8}{d^2f^4}\right)^{\frac{1}{4}}f\sqrt{\frac{a^{12}d\sin(fx+e)+\sqrt{2}\left(\frac{a^8}{d^2f^4}\right)^{\frac{3}{4}}a^6d^2f^3\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)+\sqrt{\frac{a^8}{d^2f^4}}a^8d^2f^2\cos(fx+e)}{\cos(fx+e)}}}{a^8}\right)+4\sqrt{2}\left(\frac{a^8}{d^2f^4}\right)^{\frac{1}{4}}df\arctan\left(-\frac{\sqrt{2}\left(\frac{a^8}{d^2f^4}\right)^{\frac{1}{4}}a^6f\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}-a^8-\sqrt{2}\left(\frac{a^8}{d^2f^4}\right)^{\frac{1}{4}}f\sqrt{\frac{a^{12}d\sin(fx+e)-\sqrt{2}\left(\frac{a^8}{d^2f^4}\right)^{\frac{3}{4}}a^6d^2f^3\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)+\sqrt{\frac{a^8}{d^2f^4}}a^8d^2f^2\cos(fx+e)}{\cos(fx+e)}}}{a^8}\right)+\sqrt{2}\left(\frac{a^8}{d^2f^4}\right)^{\frac{1}{4}}df\log\left(\frac{a^{12}d\sin(fx+e)+\sqrt{2}\left(\frac{a^8}{d^2f^4}\right)^{\frac{3}{4}}a^6d^2f^3\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)+\sqrt{\frac{a^8}{d^2f^4}}a^8d^2f^2\cos(fx+e)}{\cos(fx+e)}\right)-\sqrt{2}\left(\frac{a^8}{d^2f^4}\right)^{\frac{1}{4}}df\log\left(\frac{a^{12}d\sin(fx+e)-\sqrt{2}\left(\frac{a^8}{d^2f^4}\right)^{\frac{3}{4}}a^6d^2f^3\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)+\sqrt{\frac{a^8}{d^2f^4}}a^8d^2f^2\cos(fx+e)}{\cos(fx+e)}\right)-4a^2\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}}{2df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))^2/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(4*sqrt(2)*(a^8/(d^2*f^4))^(1/4)*d*f*arctan(-(sqrt(2)*(a^8/(d^2*f^4))^
(1/4)*a^6*f*sqrt(d*sin(f*x + e)/cos(f*x + e)) + a^8 - sqrt(2)*(a^8/(d^2*f^4
))^(1/4)*f*sqrt((a^12*d*sin(f*x + e) + sqrt(2)*(a^8/(d^2*f^4))^(3/4)*a^6*d^
2*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e))*cos(f*x + e) + sqrt(a^8/(d^2*f^4))*
a^8*d^2*f^2*cos(f*x + e))/cos(f*x + e)))/a^8) + 4*sqrt(2)*(a^8/(d^2*f^4))^(
1/4)*d*f*arctan(-(sqrt(2)*(a^8/(d^2*f^4))^(1/4)*a^6*f*sqrt(d*sin(f*x + e)/c
os(f*x + e)) - a^8 - sqrt(2)*(a^8/(d^2*f^4))^(1/4)*f*sqrt((a^12*d*sin(f*x +
 e) - sqrt(2)*(a^8/(d^2*f^4))^(3/4)*a^6*d^2*f^3*sqrt(d*sin(f*x + e)/cos(f*x
 + e))*cos(f*x + e) + sqrt(a^8/(d^2*f^4))*a^8*d^2*f^2*cos(f*x + e))/cos(f*x
 + e)))/a^8) + sqrt(2)*(a^8/(d^2*f^4))^(1/4)*d*f*log((a^12*d*sin(f*x + e) +
 sqrt(2)*(a^8/(d^2*f^4))^(3/4)*a^6*d^2*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e)
)*cos(f*x + e) + sqrt(a^8/(d^2*f^4))*a^8*d^2*f^2*cos(f*x + e))/cos(f*x + e)
) - sqrt(2)*(a^8/(d^2*f^4))^(1/4)*d*f*log((a^12*d*sin(f*x + e) - sqrt(2)*(a
^8/(d^2*f^4))^(3/4)*a^6*d^2*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e))*cos(f*x +
 e) + sqrt(a^8/(d^2*f^4))*a^8*d^2*f^2*cos(f*x + e))/cos(f*x + e)) - 4*a^2*s
qrt(d*sin(f*x + e)/cos(f*x + e)))/(d*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\frac{1}{\sqrt{d\tan(e+fx)}}\,dx+\int\frac{2\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx+\int\frac{\tan^2(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))**2/(d*tan(f*x+e))**(1/2),x)
```

```
[Out] a**2*(Integral(1/sqrt(d*tan(e + f*x)), x) + Integral(2*tan(e + f*x)/sqrt(d*
tan(e + f*x)), x) + Integral(tan(e + f*x)**2/sqrt(d*tan(e + f*x)), x))
```

Giac [A]
time = 0.66, size = 222, normalized size = 1.00

$$\frac{\sqrt{2}\,a^2|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{|d|}+2\,\sqrt{d\tan\left(fx+e\right)}\right)}{2\,\sqrt{|d|}}\right)}{d^2f}+\frac{\sqrt{2}\,a^2|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{|d|}-2\,\sqrt{d\tan\left(fx+e\right)}\right)}{2\,\sqrt{|d|}}\right)}{d^2f}-\frac{\sqrt{2}\,a^2|d|^{\frac{3}{2}}\log\left(d\tan\left(fx+e\right)+\sqrt{2}\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{|d|}+|d|\right)}{2\,d^2f}+\frac{\sqrt{2}\,a^2|d|^{\frac{3}{2}}\log\left(d\tan\left(fx+e\right)-\sqrt{2}\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{|d|}+|d|\right)}{2\,d^2f}+\frac{2\,\sqrt{d\tan\left(fx+e\right)}\,a^2}{df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))^2/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*a^2*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqrt(
d*tan(f*x + e)))/sqrt(abs(d)))/(d^2*f) + sqrt(2)*a^2*abs(d)^(3/2)*arctan(-1
/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(d
^2*f) - 1/2*sqrt(2)*a^2*abs(d)^(3/2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*ta
n(f*x + e))*sqrt(abs(d)) + abs(d))/(d^2*f) + 1/2*sqrt(2)*a^2*abs(d)^(3/2)*l
og(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(d^
2*f) + 2*sqrt(d*tan(f*x + e))*a^2/(d*f)
```

Mupad [B]
time = 4.06, size = 86, normalized size = 0.39

$$\frac{2\,a^2\,\sqrt{d\tan\left(e+f\,x\right)}}{d\,f}+\frac{2\,{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{d\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)}{\sqrt{d}\,f}-\frac{2\,{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{d\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)}{\sqrt{d}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x))^2/(d*tan(e + f*x))^(1/2),x)
```

```
[Out] (2*a^2*(d*tan(e + f*x))^(1/2))/(d*f) + (2*(-1)^(1/4)*a^2*atan(((-1)^(1/4)*(
d*tan(e + f*x))^(1/2))/d^(1/2)))/(d^(1/2)*f) - (2*(-1)^(1/4)*a^2*atanh(((-1
)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2)))/(d^(1/2)*f)
```

3.347 $\int \frac{(a+a\tan(e+fx))^2}{(d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=222

$$-\frac{\sqrt{2}\,a^2\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{3/2}f}+\frac{\sqrt{2}\,a^2\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{3/2}f}-\frac{a^2\log\left(\sqrt{d}+\right.}{}$$

```
[Out] -1/2*a^2*ln(d^(1/2)-2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/d^(3/2
)/f*2^(1/2)+1/2*a^2*ln(d^(1/2)+2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x
+e))/d^(3/2)/f*2^(1/2)-a^2*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))*2
^(1/2)/d^(3/2)/f+a^2*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))*2^(1/2)
/d^(3/2)/f-2*a^2/d/f/(d*tan(f*x+e))^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3623, 12, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\sqrt{2}\,a^2\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{3/2}f}+\frac{\sqrt{2}\,a^2\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{d^{3/2}f}-\frac{a^2\log\left(\sqrt{d}\,\tan(e+fx)-\sqrt{2}\,\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,d^{3/2}f}+\frac{a^2\log\left(\sqrt{d}\,\tan(e+fx)+\sqrt{2}\,\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,d^{3/2}f}-\frac{2a^2}{df\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[e + f*x])^2/(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] -((Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(d^(3/2)
*f)) + (Sqrt[2]*a^2*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(d^
(3/2)*f) - (a^2*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e +
 f*x]]])/(Sqrt[2]*d^(3/2)*f) + (a^2*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sq
rt[2]*Sqrt[d*Tan[e + f*x]]])/(Sqrt[2]*d^(3/2)*f) - (2*a^2)/(d*f*Sqrt[d*Tan[
e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\tan(e+fx))^2}{(d\tan(e+fx))^{3/2}}\,dx &= -\frac{2a^2}{df\sqrt{d\tan(e+fx)}}+\frac{\int \frac{2a^2d}{\sqrt{d\tan(e+fx)}}\,dx}{d^2}\\
&= -\frac{2a^2}{df\sqrt{d\tan(e+fx)}}+\frac{(2a^2)\int \frac{1}{\sqrt{d\tan(e+fx)}}\,dx}{d}\\
&= -\frac{2a^2}{df\sqrt{d\tan(e+fx)}}+\frac{(2a^2)\operatorname{Subst}\left(\int \frac{1}{\sqrt{x}\,(d^2+x^2)}\,dx,x,d\tan(e+fx)\right)}{f}\\
&= -\frac{2a^2}{df\sqrt{d\tan(e+fx)}}+\frac{(4a^2)\operatorname{Subst}\left(\int \frac{1}{d^2+x^4}\,dx,x,\sqrt{d\tan(e+fx)}\right)}{f}\\
&= -\frac{2a^2}{df\sqrt{d\tan(e+fx)}}+\frac{(2a^2)\operatorname{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx,x,\sqrt{d\tan(e+fx)}\right)}{df}+\frac{(2a^2}{}\\
&= -\frac{2a^2}{df\sqrt{d\tan(e+fx)}}-\frac{a^2\operatorname{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx,x,\sqrt{d\tan(e+fx)}\right)}{\sqrt{2}\,d^{3/2}f}\\
&= -\frac{a^2\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{\sqrt{2}\,d^{3/2}f}+\frac{a^2\log\left(\sqrt{d}+\right.}{}\\
&= -\frac{\sqrt{2}\,a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{3/2}f}+\frac{\sqrt{2}\,a^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan}}{\sqrt{}}\right.}{d^{3/2}f}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.87, size = 232, normalized size = 1.05

$$-\frac{a^2(1+\tan(e+fx))^2\left(6\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\tan^2(e+fx)\right)\sin(2(e+fx))-4\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(e+fx)\right)\sin^2(e+fx)\tan(e+fx)+3\sqrt{2}\cos^2(e+fx)\left(2\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)-2\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(e+fx)}\right)+\log\left(1-\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)-\log\left(1+\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)\right)\tan^{\frac{3}{2}}(e+fx)\right)}{6f(\cos(e+fx)+\sin(e+fx))^2(d\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Tan[e + f*x])^2/(d*Tan[e + f*x])^(3/2),x]

[Out] -1/6*(a^2*(1 + Tan[e + f*x])^2*(6*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[e + f*x]^2]*Sin[2*(e + f*x)] - 4*Hypergeometric2F1[3/4, 1, 7/4, -Tan[e + f*x]^2]*Sin[e + f*x]^2*Tan[e + f*x] + 3*Sqrt[2]*Cos[e + f*x]^2*(2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] + Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]])*Tan[e + f*x]^(3/2)))/(f*(Cos[e + f*x] + Sin[e + f*x])^2*(d*Tan[e + f*x])^(3/2))

Maple [A]
time = 0.19, size = 159, normalized size = 0.72

method	result
derivativedivides	$\frac{2a^2\left(-\frac{1}{\sqrt{d\tan(fx+e)}}+\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\right.\right.}{4d}\right)}{fd}$
default	$\frac{2a^2\left(-\frac{1}{\sqrt{d\tan(fx+e)}}+\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\right.\right.}{4d}\right)}{fd}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*tan(f*x+e))^2/(d*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/f*a^2/d*(-1/(d*tan(f*x+e))^(1/2)+1/4/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)))

Maxima [A]
time = 0.52, size = 182, normalized size = 0.82

$$\frac{a^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)-\frac{4a^2}{\sqrt{d\tan(fx+e)}}}{2df}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*tan(f*x+e))^2/(d*tan(f*x+e))^(3/2),x, algorithm="maxima")

[Out] 1/2*(a^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) -

```
2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) + sqrt(2)*log(d*tan(f*x + e) + sqr
t(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) - sqrt(2)*log(d*tan(f*x + e)
 - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d)) - 4*a^2/sqrt(d*tan(f*
x + e)))/(d*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 790 vs. $2(184) = 368$.
time = 1.14, size = 790, normalized size = 3.56

$$\frac{4a^2\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e) - 4\left(\sqrt{2}d^2f\cos(fx+e)^2 - \sqrt{2}d^2f\right)\left(\frac{a^8}{d^6f^4}\right)^{\frac14}\arctan\left(-\frac{\sqrt{2}a^2d^4f^3\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\left(\frac{a^8}{d^6f^4}\right)^{\frac34} - \sqrt{2}d^4f^3\sqrt{\frac{d^4f^2\sqrt{\frac{a^8}{d^6f^4}}\cos(fx+e) + \sqrt{2}a^2d^2f\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\left(\frac{a^8}{d^6f^4}\right)^{\frac14}\cos(fx+e) + a^4d\sin(fx+e)}{\cos(fx+e)}}\left(\frac{a^8}{d^6f^4}\right)^{\frac34} + a^8}{a^8}\right) - 4\left(\sqrt{2}d^2f\cos(fx+e)^2 - \sqrt{2}d^2f\right)\left(\frac{a^8}{d^6f^4}\right)^{\frac14}\arctan\left(-\frac{\sqrt{2}a^2d^4f^3\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\left(\frac{a^8}{d^6f^4}\right)^{\frac34} - \sqrt{2}d^4f^3\sqrt{\frac{d^4f^2\sqrt{\frac{a^8}{d^6f^4}}\cos(fx+e) - \sqrt{2}a^2d^2f\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\left(\frac{a^8}{d^6f^4}\right)^{\frac14}\cos(fx+e) + a^4d\sin(fx+e)}{\cos(fx+e)}}\left(\frac{a^8}{d^6f^4}\right)^{\frac34} - a^8}{a^8}\right) + \left(\sqrt{2}d^2f\cos(fx+e)^2 - \sqrt{2}d^2f\right)\left(\frac{a^8}{d^6f^4}\right)^{\frac14}\log\left(\frac{d^4f^2\sqrt{\frac{a^8}{d^6f^4}}\cos(fx+e) + \sqrt{2}a^2d^2f\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\left(\frac{a^8}{d^6f^4}\right)^{\frac14}\cos(fx+e) + a^4d\sin(fx+e)}{\cos(fx+e)}\right) - \left(\sqrt{2}d^2f\cos(fx+e)^2 - \sqrt{2}d^2f\right)\left(\frac{a^8}{d^6f^4}\right)^{\frac14}\log\left(\frac{d^4f^2\sqrt{\frac{a^8}{d^6f^4}}\cos(fx+e) - \sqrt{2}a^2d^2f\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\left(\frac{a^8}{d^6f^4}\right)^{\frac14}\cos(fx+e) + a^4d\sin(fx+e)}{\cos(fx+e)}\right)}{2\left(d^2f\cos(fx+e)^2 - d^2f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))^2/(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^2*sqrt(d*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 4*
(sqrt(2)*d^2*f*cos(f*x + e)^2 - sqrt(2)*d^2*f)*(a^8/(d^6*f^4))^(1/4)*arctan
(-(sqrt(2)*a^2*d^4*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e))*(a^8/(d^6*f^4))^(3
/4) - sqrt(2)*d^4*f^3*sqrt((d^4*f^2*sqrt(a^8/(d^6*f^4))*cos(f*x + e) + sqrt
(2)*a^2*d^2*f*sqrt(d*sin(f*x + e)/cos(f*x + e))*(a^8/(d^6*f^4))^(1/4)*cos(f
*x + e) + a^4*d*sin(f*x + e))/cos(f*x + e))*(a^8/(d^6*f^4))^(3/4) + a^8)/a^
8) - 4*(sqrt(2)*d^2*f*cos(f*x + e)^2 - sqrt(2)*d^2*f)*(a^8/(d^6*f^4))^(1/4)
*arctan(-(sqrt(2)*a^2*d^4*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e))*(a^8/(d^6*f
^4))^(3/4) - sqrt(2)*d^4*f^3*sqrt((d^4*f^2*sqrt(a^8/(d^6*f^4))*cos(f*x + e)
 - sqrt(2)*a^2*d^2*f*sqrt(d*sin(f*x + e)/cos(f*x + e))*(a^8/(d^6*f^4))^(1/4
)*cos(f*x + e) + a^4*d*sin(f*x + e))/cos(f*x + e))*(a^8/(d^6*f^4))^(3/4) -
a^8)/a^8) + (sqrt(2)*d^2*f*cos(f*x + e)^2 - sqrt(2)*d^2*f)*(a^8/(d^6*f^4))^
(1/4)*log((d^4*f^2*sqrt(a^8/(d^6*f^4))*cos(f*x + e) + sqrt(2)*a^2*d^2*f*sqr
t(d*sin(f*x + e)/cos(f*x + e))*(a^8/(d^6*f^4))^(1/4)*cos(f*x + e) + a^4*d*s
in(f*x + e))/cos(f*x + e)) - (sqrt(2)*d^2*f*cos(f*x + e)^2 - sqrt(2)*d^2*f)
*(a^8/(d^6*f^4))^(1/4)*log((d^4*f^2*sqrt(a^8/(d^6*f^4))*cos(f*x + e) - sqrt
(2)*a^2*d^2*f*sqrt(d*sin(f*x + e)/cos(f*x + e))*(a^8/(d^6*f^4))^(1/4)*cos(f
*x + e) + a^4*d*sin(f*x + e))/cos(f*x + e)))/(d^2*f*cos(f*x + e)^2 - d^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{1}{(d\tan(e+fx))^{\frac32}}\,dx + \int \frac{2\tan(e+fx)}{(d\tan(e+fx))^{\frac32}}\,dx + \int \frac{\tan^2(e+fx)}{(d\tan(e+fx))^{\frac32}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))**2/(d*tan(f*x+e))**(3/2),x)
```

```
[Out] a**2*(Integral((d*tan(e + f*x))**(-3/2), x) + Integral(2*tan(e + f*x)/(d*ta
n(e + f*x))**(3/2), x) + Integral(tan(e + f*x)**2/(d*tan(e + f*x))**(3/2),
x))
```

Giac [A]
time = 0.73, size = 225, normalized size = 1.01

$$\frac{\frac{2\sqrt{2}\,a^{2}\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{df}+\frac{2\sqrt{2}\,a^{2}\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{df}+\frac{\sqrt{2}\,a^{2}\sqrt{|d|}\log\left(d\tan\left(fx+e\right)+\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{|d|}+|d|\right)}{df}-\frac{\sqrt{2}\,a^{2}\sqrt{|d|}\log\left(d\tan\left(fx+e\right)-\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{|d|}+|d|\right)}{df}-\frac{4a^{2}}{\sqrt{d\tan\left(fx+e\right)}\,f}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))^2/(d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(2)*a^2*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) +
2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(d*f) + 2*sqrt(2)*a^2*sqrt(abs(d))*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(
d)))/(d*f) + sqrt(2)*a^2*sqrt(abs(d))*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*t
an(f*x + e))*sqrt(abs(d)) + abs(d))/(d*f) - sqrt(2)*a^2*sqrt(abs(d))*log(d*
tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(d*f) -
4*a^2/(sqrt(d*tan(f*x + e))*f))/d
```

Mupad [B]
time = 4.25, size = 86, normalized size = 0.39

$$-\frac{2\,a^2}{d\,f\,\sqrt{d\,\mathrm{tan}\left(e+f\,x\right)}}-\frac{{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{d\,\mathrm{tan}\left(e+f\,x\right)}}{\sqrt{d}}\right)\,2{}\mathrm{i}}{d^{3/2}\,f}-\frac{{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{d\,\mathrm{tan}\left(e+f\,x\right)}}{\sqrt{d}}\right)\,2{}\mathrm{i}}{d^{3/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x))^2/(d*tan(e + f*x))^(3/2),x)
```

```
[Out] - (2*a^2)/(d*f*(d*tan(e + f*x))^(1/2)) - ((-1)^(1/4)*a^2*atan(((-1)^(1/4)*(
d*tan(e + f*x))^(1/2))/d^(1/2))*2i)/(d^(3/2)*f) - ((-1)^(1/4)*a^2*atanh(((-
1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2))*2i)/(d^(3/2)*f)
```

3.348 $\int \frac{(a+a\tan(e+fx))^2}{(d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=247

$$\frac{\sqrt{2}\,a^2\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{5/2}f}-\frac{\sqrt{2}\,a^2\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{5/2}f}-\frac{a^2\log\left(\sqrt{d}+\right.}{}$$

```
[Out] -1/2*a^2*ln(d^(1/2)-2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/d^(5/2
)/f*2^(1/2)+1/2*a^2*ln(d^(1/2)+2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x
+e))/d^(5/2)/f*2^(1/2)+a^2*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))*2
^(1/2)/d^(5/2)/f-a^2*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))*2^(1/2)
/d^(5/2)/f-4*a^2/d^2/f/(d*tan(f*x+e))^(1/2)-2/3*a^2/d/f/(d*tan(f*x+e))^(3/2
)
```

Rubi [A]

time = 0.17, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3623, 12, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{\sqrt{2}\,a^2\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{5/2}f}-\frac{\sqrt{2}\,a^2\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{d^{5/2}f}-\frac{a^2\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\,\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,d^{5/2}f}+\frac{a^2\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\,\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{\sqrt{2}\,d^{5/2}f}-\frac{4a^2}{d^2f\sqrt{d\tan(e+fx)}}-\frac{2a^2}{3df(d\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[e + f*x])^2/(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(d^(5/2)*f
) - (Sqrt[2]*a^2*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(d^(5/
2)*f) - (a^2*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e + f*
x]]])/(Sqrt[2]*d^(5/2)*f) + (a^2*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqrt[
2]*Sqrt[d*Tan[e + f*x]]])/(Sqrt[2]*d^(5/2)*f) - (2*a^2)/(3*d*f*(d*Tan[e + f
*x])^(3/2)) - (4*a^2)/(d^2*f*Sqrt[d*Tan[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\tan(e+fx))^2}{(d\tan(e+fx))^{5/2}}\,dx = -\frac{2a^2}{3df(d\tan(e+fx))^{3/2}} + \frac{\int \frac{2a^2d}{(d\tan(e+fx))^{3/2}}\,dx}{d^2}$$

$$= -\frac{2a^2}{3df(d\tan(e+fx))^{3/2}} + \frac{(2a^2)\int \frac{1}{(d\tan(e+fx))^{3/2}}\,dx}{d}$$

$$= -\frac{2a^2}{3df(d\tan(e+fx))^{3/2}} - \frac{4a^2}{d^2f\sqrt{d\tan(e+fx)}} - \frac{(2a^2)\int \sqrt{d\tan(e+fx)}\,dx}{d^3}$$

$$= -\frac{2a^2}{3df(d\tan(e+fx))^{3/2}} - \frac{4a^2}{d^2f\sqrt{d\tan(e+fx)}} - \frac{(2a^2)\,\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\right.}{d^2f}$$

$$= -\frac{2a^2}{3df(d\tan(e+fx))^{3/2}} - \frac{4a^2}{d^2f\sqrt{d\tan(e+fx)}} - \frac{(4a^2)\,\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{\right.}}{d^2f}$$

$$= -\frac{2a^2}{3df(d\tan(e+fx))^{3/2}} - \frac{4a^2}{d^2f\sqrt{d\tan(e+fx)}} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{\right.}}{d^2f}$$

$$= -\frac{2a^2}{3df(d\tan(e+fx))^{3/2}} - \frac{4a^2}{d^2f\sqrt{d\tan(e+fx)}} - \frac{a^2\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}\,x-x^2}\right.}{\sqrt{2}\,d^{5}}$$

$$= -\frac{a^2\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{\sqrt{2}\,d^{5/2}f} + \frac{a^2\log\left(\sqrt{d}+\sqrt{\right.}}{}$$

$$= \frac{\sqrt{2}\,a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{d^{5/2}f} - \frac{\sqrt{2}\,a^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan(e\right.}}{\sqrt{d}}}{d^{5/2}f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.39, size = 229, normalized size = 0.93

$$-\frac{a^2(1+\cot(e+fx))^2\left(48\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\tan^2(e+fx)\right)\sin^2(e+fx)+4\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\tan^2(e+fx)\right)\sin(2(e+fx))+3\sqrt{2}\cos^2(e+fx)\left(2\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)-2\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(e+fx)}\right)+\log\left(1-\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)-\log\left(1+\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)\right)\tan^{\frac{5}{2}}(e+fx)\right)}{12d^2f(\cos(e+fx)+\sin(e+fx))^2\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[e + f*x])^2/(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] -1/12*(a^2*(1 + Cot[e + f*x])^2*(48*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[e
+ f*x]^2]*Sin[e + f*x]^2 + 4*Hypergeometric2F1[-3/4, 1, 1/4, -Tan[e + f*x]^
2]*Sin[2*(e + f*x)] + 3*Sqrt[2]*Cos[e + f*x]^2*(2*ArcTan[1 - Sqrt[2]*Sqrt[T
an[e + f*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] + Log[1 - Sqrt[2]*
```

```
Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + T
an[e + f*x]])*Tan[e + f*x]^(5/2)))/(d^2*f*(Cos[e + f*x] + Sin[e + f*x])^2*S
qrt[d*Tan[e + f*x]])
```

Maple [A]

time = 0.19, size = 174, normalized size = 0.70

method	result
derivativedivides	$\frac{2a^2\left(-\frac{1}{3(d\tan(fx+e))^{\frac{3}{2}}}-\frac{2}{d\sqrt{d\tan(fx+e)}}-\frac{\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d}}{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d}}\right.\right.}{}\right.}{fd}$
default	$\frac{2a^2\left(-\frac{1}{3(d\tan(fx+e))^{\frac{3}{2}}}-\frac{2}{d\sqrt{d\tan(fx+e)}}-\frac{\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d}}{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d}}\right.\right.}{}\right.}{fd}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(f*x+e))^2/(d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^2/d*(-1/3/(d*tan(f*x+e))^(3/2)-2/d/(d*tan(f*x+e))^(1/2)-1/4/d/(d^2)^(
1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^
2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2
)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/
(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)))
```

Maxima [A]

time = 0.51, size = 205, normalized size = 0.83

$$-\frac{\frac{3a^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)}{d}+\frac{4(6a^2d\tan(fx+e)+a^2d)}{(d\tan(fx+e))^{\frac{3}{2}}d}}{6df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))^2/(d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/6*(3*a^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(f
*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
 - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(f*x + e) +
sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(f*x +
 e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d))/d + 4*(6*a^2*d*tan
(f*x + e) + a^2*d)/((d*tan(f*x + e))^(3/2)*d))/(d*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 823 vs. $2(206) = 412$.

time = 0.94, size = 823, normalized size = 3.33

$$\frac{12\left(\sqrt{2}\,d^3f\cos\left(fx+e\right)^2-\sqrt{2}\,d^3f\right)\left(\frac{a^8}{d^{10}f^4}\right)^{\frac14}\arctan\left(-\frac{\sqrt{2}\,a^6d^2f\sqrt{\frac{d\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\left(\frac{a^8}{d^{10}f^4}\right)^{\frac14}+a^8-\sqrt{2}\,d^2f\sqrt{\frac{\sqrt{2}\,a^6d^8f^3\sqrt{\frac{d\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\left(\frac{a^8}{d^{10}f^4}\right)^{\frac34}\cos\left(fx+e\right)+a^8d^6f^2\sqrt{\frac{a^8}{d^{10}f^4}}\cos\left(fx+e\right)+a^{12}d\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\left(\frac{a^8}{d^{10}f^4}\right)^{\frac14}}{a^8}\right)+12\left(\sqrt{2}\,d^3f\cos\left(fx+e\right)^2-\sqrt{2}\,d^3f\right)\left(\frac{a^8}{d^{10}f^4}\right)^{\frac14}\arctan\left(-\frac{\sqrt{2}\,a^6d^2f\sqrt{\frac{d\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\left(\frac{a^8}{d^{10}f^4}\right)^{\frac14}-a^8-\sqrt{2}\,d^2f\sqrt{-\frac{\sqrt{2}\,a^6d^8f^3\sqrt{\frac{d\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\left(\frac{a^8}{d^{10}f^4}\right)^{\frac34}\cos\left(fx+e\right)-a^8d^6f^2\sqrt{\frac{a^8}{d^{10}f^4}}\cos\left(fx+e\right)-a^{12}d\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\left(\frac{a^8}{d^{10}f^4}\right)^{\frac14}}{a^8}\right)+3\left(\sqrt{2}\,d^3f\cos\left(fx+e\right)^2-\sqrt{2}\,d^3f\right)\left(\frac{a^8}{d^{10}f^4}\right)^{\frac14}\log\left(\frac{\sqrt{2}\,a^6d^8f^3\sqrt{\frac{d\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\left(\frac{a^8}{d^{10}f^4}\right)^{\frac34}\cos\left(fx+e\right)+a^8d^6f^2\sqrt{\frac{a^8}{d^{10}f^4}}\cos\left(fx+e\right)+a^{12}d\sin\left(fx+e\right)}{\cos\left(fx+e\right)}\right)-3\left(\sqrt{2}\,d^3f\cos\left(fx+e\right)^2-\sqrt{2}\,d^3f\right)\left(\frac{a^8}{d^{10}f^4}\right)^{\frac14}\log\left(-\frac{\sqrt{2}\,a^6d^8f^3\sqrt{\frac{d\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\left(\frac{a^8}{d^{10}f^4}\right)^{\frac34}\cos\left(fx+e\right)-a^8d^6f^2\sqrt{\frac{a^8}{d^{10}f^4}}\cos\left(fx+e\right)-a^{12}d\sin\left(fx+e\right)}{\cos\left(fx+e\right)}\right)+4\left(a^2\cos\left(fx+e\right)^2+6a^2\cos\left(fx+e\right)\sin\left(fx+e\right)\right)\sqrt{\frac{d\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}}{6\left(d^3f\cos\left(fx+e\right)^2-d^3f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))^2/(d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(12*(sqrt(2)*d^3*f*cos(f*x + e)^2 - sqrt(2)*d^3*f)*(a^8/(d^10*f^4))^(1/
4)*arctan(-(sqrt(2)*a^6*d^2*f*sqrt(d*sin(f*x + e)/cos(f*x + e))*(a^8/(d^10*
f^4))^(1/4) + a^8 - sqrt(2)*d^2*f*sqrt((sqrt(2)*a^6*d^8*f^3*sqrt(d*sin(f*x
+ e)/cos(f*x + e))*(a^8/(d^10*f^4))^(3/4)*cos(f*x + e) + a^8*d^6*f^2*sqrt(a
^8/(d^10*f^4))*cos(f*x + e) + a^12*d*sin(f*x + e))/cos(f*x + e))*(a^8/(d^10
*f^4))^(1/4))/a^8) + 12*(sqrt(2)*d^3*f*cos(f*x + e)^2 - sqrt(2)*d^3*f)*(a^8
/(d^10*f^4))^(1/4)*arctan(-(sqrt(2)*a^6*d^2*f*sqrt(d*sin(f*x + e)/cos(f*x +
 e))*(a^8/(d^10*f^4))^(1/4) - a^8 - sqrt(2)*d^2*f*sqrt(-(sqrt(2)*a^6*d^8*f^
3*sqrt(d*sin(f*x + e)/cos(f*x + e))*(a^8/(d^10*f^4))^(3/4)*cos(f*x + e) - a
^8*d^6*f^2*sqrt(a^8/(d^10*f^4))*cos(f*x + e) - a^12*d*sin(f*x + e))/cos(f*x
 + e))*(a^8/(d^10*f^4))^(1/4))/a^8) + 3*(sqrt(2)*d^3*f*cos(f*x + e)^2 - sqr
t(2)*d^3*f)*(a^8/(d^10*f^4))^(1/4)*log((sqrt(2)*a^6*d^8*f^3*sqrt(d*sin(f*x
+ e)/cos(f*x + e))*(a^8/(d^10*f^4))^(3/4)*cos(f*x + e) + a^8*d^6*f^2*sqrt(a
^8/(d^10*f^4))*cos(f*x + e) + a^12*d*sin(f*x + e))/cos(f*x + e)) - 3*(sqrt(
2)*d^3*f*cos(f*x + e)^2 - sqrt(2)*d^3*f)*(a^8/(d^10*f^4))^(1/4)*log(-(sqrt(
2)*a^6*d^8*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e))*(a^8/(d^10*f^4))^(3/4)*cos
(f*x + e) - a^8*d^6*f^2*sqrt(a^8/(d^10*f^4))*cos(f*x + e) - a^12*d*sin(f*x
+ e))/cos(f*x + e)) + 4*(a^2*cos(f*x + e)^2 + 6*a^2*cos(f*x + e)*sin(f*x +
e))*sqrt(d*sin(f*x + e)/cos(f*x + e)))/(d^3*f*cos(f*x + e)^2 - d^3*f)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int\frac{1}{\left(d\tan\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx+\int\frac{2\tan\left(e+fx\right)}{\left(d\tan\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx+\int\frac{\tan^{2}\left(e+fx\right)}{\left(d\tan\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))**2/(d*tan(f*x+e))**(5/2),x)
```

```
[Out] a**2*(Integral((d*tan(e + f*x))**(-5/2), x) + Integral(2*tan(e + f*x)/(d*ta
n(e + f*x))**(5/2), x) + Integral(tan(e + f*x)**2/(d*tan(e + f*x))**(5/2),
x))
```

Giac [A]

time = 0.72, size = 249, normalized size = 1.01

$$-\frac{\sqrt{2}\,a^2|d|^{\frac32}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{|d|}+2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{d^4f}-\frac{\sqrt{2}\,a^2|d|^{\frac32}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{|d|}-2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{d^4f}+\frac{\sqrt{2}\,a^2|d|^{\frac32}\log\left(d\tan\left(fx+e\right)+\sqrt{2}\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{|d|}+|d|\right)}{2\,d^4f}-\frac{\sqrt{2}\,a^2|d|^{\frac32}\log\left(d\tan\left(fx+e\right)-\sqrt{2}\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{|d|}+|d|\right)}{2\,d^4f}-\frac{2\left(6\,a^2d\tan\left(fx+e\right)+a^2d\right)}{3\sqrt{d\tan\left(fx+e\right)}\,d^3f\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))^2/(d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*a^2*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqrt
(d*tan(f*x + e)))/sqrt(abs(d)))/(d^4*f) - sqrt(2)*a^2*abs(d)^(3/2)*arctan(-
1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(
d^4*f) + 1/2*sqrt(2)*a^2*abs(d)^(3/2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*t
an(f*x + e))*sqrt(abs(d)) + abs(d))/(d^4*f) - 1/2*sqrt(2)*a^2*abs(d)^(3/2)*
log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(d
^4*f) - 2/3*(6*a^2*d*tan(f*x + e) + a^2*d)/(sqrt(d*tan(f*x + e))*d^3*f*tan(
f*x + e))
```

Mupad [B]
time = 4.35, size = 100, normalized size = 0.40

$$\frac{2\,{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{d\,\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)}{d^{5/2}\,f}-\frac{2\,{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{d\,\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)}{d^{5/2}\,f}-\frac{4\,a^2\,\tan(e+f\,x)+\frac{2\,a^2}{3}}{d\,f\,{\left(d\,\tan\left(e+f\,x\right)\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x))^2/(d*tan(e + f*x))^(5/2),x)
```

```
[Out] (2*(-1)^(1/4)*a^2*atanh(((-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2)))/(d^(5
/2)*f) - (2*(-1)^(1/4)*a^2*atan(((-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2)
))/(d^(5/2)*f) - (4*a^2*tan(e + f*x) + (2*a^2)/3)/(d*f*(d*tan(e + f*x))^(3/
2))
```

3.349 $\int (d\tan(e+fx))^{7/2}(a+a\tan(e+fx))^3\,dx$

Optimal. Leaf size=210

$$-\frac{2\sqrt{2}\,a^3d^{7/2}\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{f}+\frac{4a^3d^3\sqrt{d\tan(e+fx)}}{f}-\frac{4a^3d^2(d\tan(e+fx))^{3/2}}{3f}-\frac{4a^3d(d\tan$$

```
[Out] -2*a^3*d^(7/2)*arctanh(1/2*(d^(1/2)+d^(1/2)*tan(f*x+e))*2^(1/2)/(d*tan(f*x+
e))^(1/2))*2^(1/2)/f+4*a^3*d^3*(d*tan(f*x+e))^(1/2)/f-4/3*a^3*d^2*(d*tan(f*
x+e))^(3/2)/f-4/5*a^3*d*(d*tan(f*x+e))^(5/2)/f+4/7*a^3*(d*tan(f*x+e))^(7/2)
/f+16/33*a^3*(d*tan(f*x+e))^(9/2)/d/f+2/11*(d*tan(f*x+e))^(9/2)*(a^3+a^3*ta
n(f*x+e))/d/f
```

Rubi [A]
time = 0.23, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3647, 3711, 3609, 3613, 214}

$$-\frac{2\sqrt{2}\,a^3d^{7/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\tan(e+fx)+\sqrt{d}}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{f}+\frac{4a^3d^3\sqrt{d\tan(e+fx)}}{f}-\frac{4a^3d^2(d\tan(e+fx))^{3/2}}{3f}+\frac{2(a^3\tan(e+fx)+a^3)\,(d\tan(e+fx))^{9/2}}{11df}+\frac{16a^3(d\tan(e+fx))^{9/2}}{33df}+\frac{4a^3(d\tan(e+fx))^{7/2}}{7f}-\frac{4a^3d(d\tan(e+fx))^{5/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(7/2)*(a + a*Tan[e + f*x])^3,x]
```

```
[Out] (-2*Sqrt[2]*a^3*d^(7/2)*ArcTanh[(Sqrt[d] + Sqrt[d]*Tan[e + f*x])/(Sqrt[2]*S
qrt[d*Tan[e + f*x]])])/f + (4*a^3*d^3*Sqrt[d*Tan[e + f*x]])/f - (4*a^3*d^2*
(d*Tan[e + f*x])^(3/2))/(3*f) - (4*a^3*d*(d*Tan[e + f*x])^(5/2))/(5*f) + (4
*a^3*(d*Tan[e + f*x])^(7/2))/(7*f) + (16*a^3*(d*Tan[e + f*x])^(9/2))/(33*d*
f) + (2*(d*Tan[e + f*x])^(9/2)*(a^3 + a^3*Tan[e + f*x]))/(11*d*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
```

```
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (d\tan(e+fx))^{7/2}(a+a\tan(e+fx))^3\,dx &= \frac{2(d\tan(e+fx))^{9/2}\left(a^3+a^3\tan(e+fx)\right)}{11df}+\frac{2\int(d\tan(e+fx\\
&= \frac{16a^3(d\tan(e+fx))^{9/2}}{33df}+\frac{2(d\tan(e+fx))^{9/2}\left(a^3+a^3\tan(e+\right.}{11df}\\
&= \frac{4a^3(d\tan(e+fx))^{7/2}}{7f}+\frac{16a^3(d\tan(e+fx))^{9/2}}{33df}+\frac{2(d\tan(e-}{}\\
&= -\frac{4a^3d(d\tan(e+fx))^{5/2}}{5f}+\frac{4a^3(d\tan(e+fx))^{7/2}}{7f}+\frac{16a^3(d\tan}{}\\
&= -\frac{4a^3d^2(d\tan(e+fx))^{3/2}}{3f}-\frac{4a^3d(d\tan(e+fx))^{5/2}}{5f}+\frac{4a^3(d}{}\\
&= \frac{4a^3d^3\sqrt{d\tan(e+fx)}}{f}-\frac{4a^3d^2(d\tan(e+fx))^{3/2}}{3f}-\frac{4a^3d(d\tan}{}\\
&= \frac{4a^3d^3\sqrt{d\tan(e+fx)}}{f}-\frac{4a^3d^2(d\tan(e+fx))^{3/2}}{3f}-\frac{4a^3d(d\tan}{}\\
&= -\frac{2\sqrt{2}\,a^3d^{7/2}\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{f}+\frac{4a^3d^3\sqrt{d\tan}}{f}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 4.54, size = 375, normalized size = 1.79

$$\frac{a^3d^3\cos(e+fx)\sqrt{d\tan(e+fx)}(1+\tan(e+fx))^3\left(2310\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\cos^2(e+fx)-2310\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(e+fx)}\right)\cos^2(e+fx)+1155\sqrt{2}\cos^2(e+fx)\log\left(1-\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)-1155\sqrt{2}\cos^2(e+fx)\log\left(1+\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)+9240\cos^2(e+fx)\sqrt{\tan(e+fx)}-3080\cos^2(e+fx)\tan^{\frac{3}{2}}(e+fx)+3080\cos^2(e+fx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(e+fx)\right)\tan^{\frac{3}{2}}(e+fx)-1848\cos^2(e+fx)\tan^{\frac{5}{2}}(e+fx)+1320\cos^2(e+fx)\tan^{\frac{7}{2}}(e+fx)+420\sin^2(e+fx)\tan^{\frac{7}{2}}(e+fx)+770\sin(2(e+fx))\tan^{\frac{7}{2}}(e+fx)\right)}{2310f(\cos(e+fx)+\sin(e+fx))^3\sqrt{\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(7/2)*(a + a*Tan[e + f*x])^3,x]
```

```
[Out] (a^3*d^3*Cos[e + f*x]*Sqrt[d*Tan[e + f*x]]*(1 + Tan[e + f*x])^3*(2310*Sqrt[
2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]]*Cos[e + f*x]^2 - 2310*Sqrt[2]*Arc
Tan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]]*Cos[e + f*x]^2 + 1155*Sqrt[2]*Cos[e + f
*x]^2*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 1155*Sqrt[2]*Cos
[e + f*x]^2*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] + 9240*Cos[e
 + f*x]^2*Sqrt[Tan[e + f*x]] - 3080*Cos[e + f*x]^2*Tan[e + f*x]^(3/2) + 308
0*Cos[e + f*x]^2*Hypergeometric2F1[3/4, 1, 7/4, -Tan[e + f*x]^2]*Tan[e + f*
x]^(3/2) - 1848*Cos[e + f*x]^2*Tan[e + f*x]^(5/2) + 1320*Cos[e + f*x]^2*Tan
[e + f*x]^(7/2) + 420*Sin[e + f*x]^2*Tan[e + f*x]^(7/2) + 770*Sin[2*(e + f*
x)]*Tan[e + f*x]^(7/2)))/(2310*f*(Cos[e + f*x] + Sin[e + f*x])^3*Sqrt[Tan[e
 + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(177) = 354$.
time = 0.44, size = 369, normalized size = 1.76

method	result
derivativedivides	$\frac{2a^3\left(\frac{(d\tan(fx+e))^{\frac{11}{2}}}{11}+\frac{d(d\tan(fx+e))^{\frac{9}{2}}}{3}+\frac{2d^2(d\tan(fx+e))^{\frac{7}{2}}}{7}-\frac{2d^3(d\tan(fx+e))^{\frac{5}{2}}}{5}-\frac{2d^4(d\tan(fx+e))^{\frac{3}{2}}}{3}+2d^5\sqrt{d\tan(f}\right.}{}$
default	$\frac{2a^3\left(\frac{(d\tan(fx+e))^{\frac{11}{2}}}{11}+\frac{d(d\tan(fx+e))^{\frac{9}{2}}}{3}+\frac{2d^2(d\tan(fx+e))^{\frac{7}{2}}}{7}-\frac{2d^3(d\tan(fx+e))^{\frac{5}{2}}}{5}-\frac{2d^4(d\tan(fx+e))^{\frac{3}{2}}}{3}+2d^5\sqrt{d\tan(f}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(7/2)*(a+a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3/d^2*(1/11*(d*tan(f*x+e))^(11/2)+1/3*d*(d*tan(f*x+e))^(9/2)+2/7*d^2*
(d*tan(f*x+e))^(7/2)-2/5*d^3*(d*tan(f*x+e))^(5/2)-2/3*d^4*(d*tan(f*x+e))^(3
/2)+2*d^5*(d*tan(f*x+e))^(1/2)-2*d^6*(1/8/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(
f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-
(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^
2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e
))^(1/2)+1))-1/8/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f
*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^
(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/
2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))))
```

Maxima [A]
time = 0.52, size = 209, normalized size = 1.00

$$-\frac{1155\,a^3d^5\left(\frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\,\sqrt{d\tan(fx+e)}\,\sqrt{d}+d\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\,\sqrt{d\tan(fx+e)}\,\sqrt{d}+d\right)}{\sqrt{d}}\right)-\frac{2\left(105\,(d\tan(fx+e))^{\frac{11}{2}}a^3+385\,(d\tan(fx+e))^{\frac{9}{2}}a^3d+330\,(d\tan(fx+e))^{\frac{7}{2}}a^3d^2-462\,(d\tan(fx+e))^{\frac{5}{2}}a^3d^3-770\,(d\tan(fx+e))^{\frac{3}{2}}a^3d^4+2310\,\sqrt{d\tan(fx+e)}\,a^3d^5\right)}{d}}{1155\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(7/2)*(a+a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/1155*(1155*a^3*d^5*(sqrt(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x
+ e))*sqrt(d) + d)/sqrt(d) - sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*ta
n(f*x + e))*sqrt(d) + d)/sqrt(d)) - 2*(105*(d*tan(f*x + e))^(11/2)*a^3 + 38
5*(d*tan(f*x + e))^(9/2)*a^3*d + 330*(d*tan(f*x + e))^(7/2)*a^3*d^2 - 462*(
d*tan(f*x + e))^(5/2)*a^3*d^3 - 770*(d*tan(f*x + e))^(3/2)*a^3*d^4 + 2310*s
qrt(d*tan(f*x + e))*a^3*d^5)/d)/(d*f)
```

Fricas [A]
time = 1.02, size = 358, normalized size = 1.70

$$\left[\frac{1155\,\sqrt{2}\,a^{3}d^{\frac{7}{2}}\log\left(\frac{d\tan(fx+e)^{2}-2\,\sqrt{2}\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{d}\,(\tan(fx+e)+1)+4\,d\tan(fx+e)+d}{\tan(fx+e)^{2}+1}\right)+2\,\left(105\,a^{3}d^{3}\tan\left(fx+e\right)^{5}+385\,a^{3}d^{3}\tan\left(fx+e\right)^{4}+330\,a^{3}d^{3}\tan\left(fx+e\right)^{3}-462\,a^{3}d^{3}\tan\left(fx+e\right)^{2}-770\,a^{3}d^{3}\tan\left(fx+e\right)+2310\,a^{3}d^{3}\right)\sqrt{d\tan\left(fx+e\right)}}{1155\,f}, \frac{2\,\left(1155\,\sqrt{2}\,a^{3}\sqrt{-d}\,d^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{-d}\,(\tan(fx+e)+1)}{2\,d\tan(fx+e)}\right)+\left(105\,a^{3}d^{3}\tan\left(fx+e\right)^{5}+385\,a^{3}d^{3}\tan\left(fx+e\right)^{4}+330\,a^{3}d^{3}\tan\left(fx+e\right)^{3}-462\,a^{3}d^{3}\tan\left(fx+e\right)^{2}-770\,a^{3}d^{3}\tan\left(fx+e\right)+2310\,a^{3}d^{3}\right)\sqrt{d\tan\left(fx+e\right)}\right)}{1155\,f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(7/2)*(a+a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/1155*(1155*sqrt(2)*a^3*d^(7/2)*log((d*tan(f*x + e)^2 - 2*sqrt(2)*sqrt(d*
tan(f*x + e))*sqrt(d)*(tan(f*x + e) + 1) + 4*d*tan(f*x + e) + d)/(tan(f*x +
 e)^2 + 1)) + 2*(105*a^3*d^3*tan(f*x + e)^5 + 385*a^3*d^3*tan(f*x + e)^4 +
330*a^3*d^3*tan(f*x + e)^3 - 462*a^3*d^3*tan(f*x + e)^2 - 770*a^3*d^3*tan(f
*x + e) + 2310*a^3*d^3)*sqrt(d*tan(f*x + e)))/f, 2/1155*(1155*sqrt(2)*a^3*s
qrt(-d)*d^3*arctan(1/2*sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(-d)*(tan(f*x + e)
+ 1)/(d*tan(f*x + e))) + (105*a^3*d^3*tan(f*x + e)^5 + 385*a^3*d^3*tan(f*x
+ e)^4 + 330*a^3*d^3*tan(f*x + e)^3 - 462*a^3*d^3*tan(f*x + e)^2 - 770*a^3*
d^3*tan(f*x + e) + 2310*a^3*d^3)*sqrt(d*tan(f*x + e)))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int\left(d\tan\left(e+fx\right)\right)^{\frac{7}{2}}\,dx+\int 3\left(d\tan\left(e+fx\right)\right)^{\frac{7}{2}}\tan\left(e+fx\right)dx+\int 3\left(d\tan\left(e+fx\right)\right)^{\frac{7}{2}}\tan^{2}\left(e+fx\right)dx+\int\left(d\tan\left(e+fx\right)\right)^{\frac{7}{2}}\tan^{3}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(7/2)*(a+a*tan(f*x+e))**3,x)
```

```
[Out] a**3*(Integral((d*tan(e + f*x))**(7/2), x) + Integral(3*(d*tan(e + f*x))**(
7/2)*tan(e + f*x), x) + Integral(3*(d*tan(e + f*x))**(7/2)*tan(e + f*x)**2,
 x) + Integral((d*tan(e + f*x))**(7/2)*tan(e + f*x)**3, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 446 vs. 2(186) = 372.
time = 1.02, size = 446, normalized size = 2.12

$$-\frac{\sqrt{2}\left(a^{3}d^{3}\sqrt{|d|}+a^{3}d^{2}|d|^{\frac{3}{2}}\right)\log\left(d\tan\left(fx+e\right)+\sqrt{2}\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{|d|}+|d|\right)}{2\,f}+\frac{\sqrt{2}\left(a^{3}d^{3}\sqrt{|d|}+a^{3}d^{2}|d|^{\frac{3}{2}}\right)\log\left(d\tan\left(fx+e\right)-\sqrt{2}\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{|d|}+|d|\right)}{2\,f}-\frac{\left(\sqrt{2}\,a^{3}d^{3}\sqrt{|d|}-\sqrt{2}\,a^{3}d^{2}|d|^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{|d|}+2\,\sqrt{d\tan\left(fx+e\right)}\right)}{2\,\sqrt{|d|}}\right)}{f}-\frac{\left(\sqrt{2}\,a^{3}d^{3}\sqrt{|d|}-\sqrt{2}\,a^{3}d^{2}|d|^{\frac{3}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{|d|}-2\,\sqrt{d\tan\left(fx+e\right)}\right)}{2\,\sqrt{|d|}}\right)}{f}+\frac{2\left(105\,\sqrt{d\tan\left(fx+e\right)}\,a^{3}d^{25}f^{10}\tan\left(fx+e\right)^{5}+385\,\sqrt{d\tan\left(fx+e\right)}\,a^{3}d^{25}f^{10}\tan\left(fx+e\right)^{4}+330\,\sqrt{d\tan\left(fx+e\right)}\,a^{3}d^{25}f^{10}\tan\left(fx+e\right)^{3}-462\,\sqrt{d\tan\left(fx+e\right)}\,a^{3}d^{25}f^{10}\tan\left(fx+e\right)^{2}-770\,\sqrt{d\tan\left(fx+e\right)}\,a^{3}d^{25}f^{10}\tan\left(fx+e\right)+2310\,\sqrt{d\tan\left(fx+e\right)}\,a^{3}d^{25}f^{10}\right)}{1155\,d^{22}f^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(7/2)*(a+a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(a^3*d^3*sqrt(abs(d)) + a^3*d^2*abs(d)^(3/2))*log(d*tan(f*x +
e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/f + 1/2*sqrt(2)*(a
^3*d^3*sqrt(abs(d)) + a^3*d^2*abs(d)^(3/2))*log(d*tan(f*x + e) - sqrt(2)*sq
rt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/f - (sqrt(2)*a^3*d^3*sqrt(abs(d))
 - sqrt(2)*a^3*d^2*abs(d)^(3/2))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) +
 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/f - (sqrt(2)*a^3*d^3*sqrt(abs(d)) -
```

```
sqrt(2)*a^3*d^2*abs(d)^(3/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2
*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/f + 2/1155*(105*sqrt(d*tan(f*x + e))*a
^3*d^25*f^10*tan(f*x + e)^5 + 385*sqrt(d*tan(f*x + e))*a^3*d^25*f^10*tan(f*
x + e)^4 + 330*sqrt(d*tan(f*x + e))*a^3*d^25*f^10*tan(f*x + e)^3 - 462*sqrt
(d*tan(f*x + e))*a^3*d^25*f^10*tan(f*x + e)^2 - 770*sqrt(d*tan(f*x + e))*a^
3*d^25*f^10*tan(f*x + e) + 2310*sqrt(d*tan(f*x + e))*a^3*d^25*f^10)/(d^22*f
^11)
```

Mupad [B]
time = 7.04, size = 185, normalized size = 0.88

$$\frac{4\,a^3\,{\left(d\,\tan(e+f\,x)\right)}^{7/2}}{7\,f}+\frac{4\,a^3\,d^3\,\sqrt{d\,\tan\left(e+f\,x\right)}}{f}-\frac{4\,a^3\,d^2\,{\left(d\,\tan(e+f\,x)\right)}^{3/2}}{3\,f}+\frac{2\,a^3\,{\left(d\,\tan(e+f\,x)\right)}^{9/2}}{3\,d\,f}+\frac{2\,a^3\,{\left(d\,\tan(e+f\,x)\right)}^{11/2}}{11\,d^2\,f}-\frac{4\,a^3\,d\,{\left(d\,\tan(e+f\,x)\right)}^{5/2}}{5\,f}+\frac{\sqrt{2}\,a^3\,d^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,a^6\,d^{17/2}\,\sqrt{d\,\tan\left(e+f\,x\right)}\,32{}\mathrm{i}}{32\,a^6\,d^9+32\,a^6\,d^9\,\tan(e+f\,x)}\right)\,2{}\mathrm{i}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(7/2)*(a + a*tan(e + f*x))^3,x)
```

```
[Out] (4*a^3*(d*tan(e + f*x))^(7/2))/(7*f) + (4*a^3*d^3*(d*tan(e + f*x))^(1/2))/f
 - (4*a^3*d^2*(d*tan(e + f*x))^(3/2))/(3*f) + (2*a^3*(d*tan(e + f*x))^(9/2)
)/(3*d*f) + (2*a^3*(d*tan(e + f*x))^(11/2))/(11*d^2*f) - (4*a^3*d*(d*tan(e
+ f*x))^(5/2))/(5*f) + (2^(1/2)*a^3*d^(7/2)*atan((2^(1/2)*a^6*d^(17/2)*(d*t
an(e + f*x))^(1/2)*32i)/(32*a^6*d^9 + 32*a^6*d^9*tan(e + f*x)))*2i)/f
```

3.350 $\int (d\tan(e+fx))^{5/2}(a+a\tan(e+fx))^3\,dx$

Optimal. Leaf size=186

$$-\frac{2\sqrt{2}\,a^3d^{5/2}\text{ArcTan}\left(\frac{\sqrt{d}-\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{f}-\frac{4a^3d^2\sqrt{d\tan(e+fx)}}{f}-\frac{4a^3d(d\tan(e+fx))^{3/2}}{3f}+\frac{4a^3(d\tan(\ldots}{5\ldots}$$

```
[Out] -2*a^3*d^(5/2)*arctan(1/2*(d^(1/2)-d^(1/2)*tan(f*x+e))*2^(1/2)/(d*tan(f*x+e
))^(1/2))*2^(1/2)/f-4*a^3*d^2*(d*tan(f*x+e))^(1/2)/f-4/3*a^3*d*(d*tan(f*x+e
))^(3/2)/f+4/5*a^3*(d*tan(f*x+e))^(5/2)/f+40/63*a^3*(d*tan(f*x+e))^(7/2)/d/
f+2/9*(d*tan(f*x+e))^(7/2)*(a^3+a^3*tan(f*x+e))/d/f
```

Rubi [A]
time = 0.19, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3647, 3711, 3609, 3613, 211}

$$-\frac{2\sqrt{2}\,a^3d^{5/2}\text{ArcTan}\left(\frac{\sqrt{d}-\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{f}-\frac{4a^3d^2\sqrt{d\tan(e+fx)}}{f}+\frac{2\left(a^3\tan(e+fx)+a^3\right)(d\tan(e+fx))^{7/2}}{9df}+\frac{40a^3(d\tan(e+fx))^{7/2}}{63df}+\frac{4a^3(d\tan(e+fx))^{5/2}}{5f}-\frac{4a^3d(d\tan(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(5/2)*(a + a*Tan[e + f*x])^3,x]
```

```
[Out] (-2*Sqrt[2]*a^3*d^(5/2)*ArcTan[(Sqrt[d] - Sqrt[d]*Tan[e + f*x])/(Sqrt[2]*Sq
rt[d*Tan[e + f*x]])])/f - (4*a^3*d^2*Sqrt[d*Tan[e + f*x]])/f - (4*a^3*d*(d*
Tan[e + f*x])^(3/2))/(3*f) + (4*a^3*(d*Tan[e + f*x])^(5/2))/(5*f) + (40*a^3
*(d*Tan[e + f*x])^(7/2))/(63*d*f) + (2*(d*Tan[e + f*x])^(7/2)*(a^3 + a^3*Ta
n[e + f*x]))/(9*d*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
```

```
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (d\tan(e+fx))^{5/2}(a+a\tan(e+fx))^3\,dx = \frac{2(d\tan(e+fx))^{7/2}\left(a^3+a^3\tan(e+fx)\right)}{9df} + \frac{2\int (d\tan(e+f$$

$$= \frac{40a^3(d\tan(e+fx))^{7/2}}{63df} + \frac{2(d\tan(e+fx))^{7/2}\left(a^3+a^3\tan(e\right.}{9df}$$

$$= \frac{4a^3(d\tan(e+fx))^{5/2}}{5f} + \frac{40a^3(d\tan(e+fx))^{7/2}}{63df} + \frac{2(d\tan(e}{}$$

$$= -\frac{4a^3d(d\tan(e+fx))^{3/2}}{3f} + \frac{4a^3(d\tan(e+fx))^{5/2}}{5f} + \frac{40a^3(d}{}$$

$$= -\frac{4a^3d^2\sqrt{d\tan(e+fx)}}{f} - \frac{4a^3d(d\tan(e+fx))^{3/2}}{3f} + \frac{4a^3(d}{}$$

$$= -\frac{4a^3d^2\sqrt{d\tan(e+fx)}}{f} - \frac{4a^3d(d\tan(e+fx))^{3/2}}{3f} + \frac{4a^3(d}{}$$

$$= -\frac{2\sqrt{2}\,a^3d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}-\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{f} - \frac{4a^3d^2\sqrt{d\,\mathrm{ta}}}{f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.10, size = 729, normalized size = 3.92

$$\frac{4\cos^3(e+fx)(d\tan(e+fx))^{5/2}(a+a\tan(e+fx))^3}{5f(\cos(e+fx)+\sin(e+fx))^3}-\frac{4\cos^3(e+fx)\cot(e+fx)(d\tan(e+fx))^{5/2}(a+a\tan(e+fx))^3}{3f(\cos(e+fx)+\sin(e+fx))^3}-\frac{4\cos^3(e+fx)\cot^2(e+fx)(d\tan(e+fx))^{5/2}(a+a\tan(e+fx))^3}{f(\cos(e+fx)+\sin(e+fx))^3}+\frac{4\cos^3(e+fx)\cot(e+fx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(e+fx)\right)(d\tan(e+fx))^{5/2}(a+a\tan(e+fx))^3}{3f(\cos(e+fx)+\sin(e+fx))^3}+\frac{6\cos^2(e+fx)\sin(e+fx)(d\tan(e+fx))^{5/2}(a+a\tan(e+fx))^3}{7f(\cos(e+fx)+\sin(e+fx))^3}+\frac{2\cos(e+fx)\sin^2(e+fx)(d\tan(e+fx))^{5/2}(a+a\tan(e+fx))^3}{9f(\cos(e+fx)+\sin(e+fx))^3}-\frac{\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\cos^3(e+fx)(d\tan(e+fx))^{5/2}(a+a\tan(e+fx))^3}{f(\cos(e+fx)+\sin(e+fx))^3\tan^{\frac{5}{2}}(e+fx)}+\frac{\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(e+fx)}\right)\cos^3(e+fx)(d\tan(e+fx))^{5/2}(a+a\tan(e+fx))^3}{f(\cos(e+fx)+\sin(e+fx))^3\tan^{\frac{5}{2}}(e+fx)}-\frac{\cos^3(e+fx)\log\left(1-\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)(d\tan(e+fx))^{5/2}(a+a\tan(e+fx))^3}{\sqrt{2}f(\cos(e+fx)+\sin(e+fx))^3\tan^{\frac{5}{2}}(e+fx)}+\frac{\cos^3(e+fx)\log\left(1+\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)(d\tan(e+fx))^{5/2}(a+a\tan(e+fx))^3}{\sqrt{2}f(\cos(e+fx)+\sin(e+fx))^3\tan^{\frac{5}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(5/2)*(a + a*Tan[e + f*x])^3,x]
```

```
[Out] (4*Cos[e + f*x]^3*(d*Tan[e + f*x])^(5/2)*(a + a*Tan[e + f*x])^3)/(5*f*(Cos[
e + f*x] + Sin[e + f*x])^3) - (4*Cos[e + f*x]^3*Cot[e + f*x]*(d*Tan[e + f*x
])^(5/2)*(a + a*Tan[e + f*x])^3)/(3*f*(Cos[e + f*x] + Sin[e + f*x])^3) - (4
*Cos[e + f*x]^3*Cot[e + f*x]^2*(d*Tan[e + f*x])^(5/2)*(a + a*Tan[e + f*x])^
3)/(f*(Cos[e + f*x] + Sin[e + f*x])^3) + (4*Cos[e + f*x]^3*Cot[e + f*x]*Hyp
ergeometric2F1[3/4, 1, 7/4, -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(5/2)*(a + a*
Tan[e + f*x])^3)/(3*f*(Cos[e + f*x] + Sin[e + f*x])^3) + (6*Cos[e + f*x]^2*
Sin[e + f*x]*(d*Tan[e + f*x])^(5/2)*(a + a*Tan[e + f*x])^3)/(7*f*(Cos[e + f
*x] + Sin[e + f*x])^3) + (2*Cos[e + f*x]*Sin[e + f*x]^2*(d*Tan[e + f*x])^(5
/2)*(a + a*Tan[e + f*x])^3)/(9*f*(Cos[e + f*x] + Sin[e + f*x])^3) - (Sqrt[2
]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]]*Cos[e + f*x]^3*(d*Tan[e + f*x])^(5
/2)*(a + a*Tan[e + f*x])^3)/(f*(Cos[e + f*x] + Sin[e + f*x])^3*Tan[e + f*x]
^(5/2)) + (Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]]*Cos[e + f*x]^3*(d
*Tan[e + f*x])^(5/2)*(a + a*Tan[e + f*x])^3)/(f*(Cos[e + f*x] + Sin[e + f*x
])^3*Tan[e + f*x]^(5/2)) - (Cos[e + f*x]^3*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x
]] + Tan[e + f*x]]*(d*Tan[e + f*x])^(5/2)*(a + a*Tan[e + f*x])^3)/(Sqrt[2]*
f*(Cos[e + f*x] + Sin[e + f*x])^3*Tan[e + f*x]^(5/2)) + (Cos[e + f*x]^3*Log
[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]*(d*Tan[e + f*x])^(5/2)*(a +
 a*Tan[e + f*x])^3)/(Sqrt[2]*f*(Cos[e + f*x] + Sin[e + f*x])^3*Tan[e + f*x]
^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(157) = 314$.
time = 0.32, size = 354, normalized size = 1.90

method	result
derivativedivides	$2a^3\left(\frac{(d\tan(fx+e))^{\frac{9}{2}}}{9}+\frac{3d(d\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{2d^2(d\tan(fx+e))^{\frac{5}{2}}}{5}-\frac{2d^3(d\tan(fx+e))^{\frac{3}{2}}}{3}-2d^4\sqrt{d\tan(fx+e)}+2d^5\left(\frac{(d^2)}{}\right.\right.$

default	$\frac{2a^3\left(\frac{(d\tan(fx+e))^{\frac{9}{2}}}{9}+\frac{3d(d\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{2d^2(d\tan(fx+e))^{\frac{5}{2}}}{5}-\frac{2d^3(d\tan(fx+e))^{\frac{3}{2}}}{3}-2d^4\sqrt{d\tan(fx+e)}+2d^5\left(\frac{(d}{\cdots}\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(5/2)*(a+a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3/d^2*(1/9*(d*tan(f*x+e))^(9/2)+3/7*d*(d*tan(f*x+e))^(7/2)+2/5*d^2*(d
*tan(f*x+e))^(5/2)-2/3*d^3*(d*tan(f*x+e))^(3/2)-2*d^4*(d*tan(f*x+e))^(1/2)+
2*d^5*(1/8/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e
))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2
)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1
)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))+1/8/(d^2)^(1/4)*2^
(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2
))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*a
rctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(
1/4)*(d*tan(f*x+e))^(1/2)+1))))
```

Maxima [A]
time = 0.52, size = 187, normalized size = 1.01

$$\frac{2\left(315\,a^3d^4\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}\right)+\frac{35\,(d\tan(fx+e))^{\frac{9}{2}}a^3+135\,(d\tan(fx+e))^{\frac{7}{2}}a^3d+126\,(d\tan(fx+e))^{\frac{5}{2}}a^3d^2-210\,(d\tan(fx+e))^{\frac{3}{2}}a^3d^3-630\,\sqrt{d\tan(fx+e)}\,a^3d^4}{d}\right)}{315\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)*(a+a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 2/315*(315*a^3*d^4*(sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*
tan(f*x + e)))/sqrt(d))/sqrt(d) + sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt
(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d)) + (35*(d*tan(f*x + e))^(9/2
)*a^3 + 135*(d*tan(f*x + e))^(7/2)*a^3*d + 126*(d*tan(f*x + e))^(5/2)*a^3*d
^2 - 210*(d*tan(f*x + e))^(3/2)*a^3*d^3 - 630*sqrt(d*tan(f*x + e))*a^3*d^4)
/d)/(d*f)
```

Fricas [A]
time = 1.12, size = 321, normalized size = 1.73

$$\left[\frac{315\,\sqrt{2}\,a^3\sqrt{-d}\,d^2\log\left(\frac{d\tan(fx+e)^2+2\sqrt{2}\,\sqrt{d\tan(fx+e)}\,\sqrt{-d}\,(\tan(fx+e)-1)-4\,d\tan(fx+e)+d}{\tan(fx+e)^2+1}\right)+2\left(35\,a^3d^2\tan(fx+e)^4+135\,a^3d^2\tan(fx+e)^3+126\,a^3d^2\tan(fx+e)^2-210\,a^3d^2\tan(fx+e)-630\,a^3d^2\right)\sqrt{d\tan(fx+e)}}{315\,f},\ \frac{2\left(315\,\sqrt{2}\,a^3d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\,\sqrt{d\tan(fx+e)}\,(\tan(fx+e)-1)}{2\sqrt{d}\,\tan(fx+e)}\right)+\left(35\,a^3d^2\tan(fx+e)^4+135\,a^3d^2\tan(fx+e)^3+126\,a^3d^2\tan(fx+e)^2-210\,a^3d^2\tan(fx+e)-630\,a^3d^2\right)\sqrt{d\tan(fx+e)}\right)}{315\,f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)*(a+a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/315*(315*sqrt(2)*a^3*sqrt(-d)*d^2*log((d*tan(f*x + e)^2 + 2*sqrt(2)*sqrt
(d*tan(f*x + e))*sqrt(-d)*(tan(f*x + e) - 1) - 4*d*tan(f*x + e) + d)/(tan(f
*x + e)^2 + 1)) + 2*(35*a^3*d^2*tan(f*x + e)^4 + 135*a^3*d^2*tan(f*x + e)^3
 + 126*a^3*d^2*tan(f*x + e)^2 - 210*a^3*d^2*tan(f*x + e) - 630*a^3*d^2)*sqr
t(d*tan(f*x + e)))/f, 2/315*(315*sqrt(2)*a^3*d^(5/2)*arctan(1/2*sqrt(2)*sqr
t(d*tan(f*x + e))*(tan(f*x + e) - 1)/(sqrt(d)*tan(f*x + e))) + (35*a^3*d^2*
tan(f*x + e)^4 + 135*a^3*d^2*tan(f*x + e)^3 + 126*a^3*d^2*tan(f*x + e)^2 -
210*a^3*d^2*tan(f*x + e) - 630*a^3*d^2)*sqrt(d*tan(f*x + e)))/f]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int (d\tan(e+fx))^{\frac{5}{2}}\,dx+\int 3(d\tan(e+fx))^{\frac{5}{2}}\tan(e+fx)\,dx+\int 3(d\tan(e+fx))^{\frac{5}{2}}\tan^2(e+fx)\,dx+\int (d\tan(e+fx))^{\frac{5}{2}}\tan^3(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(5/2)*(a+a*tan(f*x+e))**3,x)
```

```
[Out] a**3*(Integral((d*tan(e + f*x))**(5/2), x) + Integral(3*(d*tan(e + f*x))**(
5/2)*tan(e + f*x), x) + Integral(3*(d*tan(e + f*x))**(5/2)*tan(e + f*x)**2,
 x) + Integral((d*tan(e + f*x))**(5/2)*tan(e + f*x)**3, x))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(166) = 332.
time = 0.99, size = 405, normalized size = 2.18

$$\frac{\sqrt{2}\left(a^3d^2\sqrt{|d|}-a^3d|d|^{\frac{3}{2}}\right)\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{|d|}+|d|\right)}{2f}-\frac{\sqrt{2}\left(a^3d^2\sqrt{|d|}-a^3d|d|^{\frac{3}{2}}\right)\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{|d|}+|d|\right)}{2f}+\frac{\left(\sqrt{2}a^3d^2\sqrt{|d|}+\sqrt{2}a^3d|d|^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{f}+\frac{\left(\sqrt{2}a^3d^2\sqrt{|d|}+\sqrt{2}a^3d|d|^{\frac{3}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{f}+\frac{2\left(35\sqrt{d\tan(fx+e)}a^3d^{20}f^8\tan(fx+e)^4+135\sqrt{d\tan(fx+e)}a^3d^{20}f^8\tan(fx+e)^3+126\sqrt{d\tan(fx+e)}a^3d^{20}f^8\tan(fx+e)^2-210\sqrt{d\tan(fx+e)}a^3d^{20}f^8\tan(fx+e)-630\sqrt{d\tan(fx+e)}a^3d^{20}f^8\right)}{315d^{18}f^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)*(a+a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(a^3*d^2*sqrt(abs(d)) - a^3*d*abs(d)^(3/2))*log(d*tan(f*x + e)
+ sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/f - 1/2*sqrt(2)*(a^3*
d^2*sqrt(abs(d)) - a^3*d*abs(d)^(3/2))*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*
tan(f*x + e))*sqrt(abs(d)) + abs(d))/f + (sqrt(2)*a^3*d^2*sqrt(abs(d)) + sq
rt(2)*a^3*d*abs(d)^(3/2))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqrt
(d*tan(f*x + e)))/sqrt(abs(d)))/f + (sqrt(2)*a^3*d^2*sqrt(abs(d)) + sqrt(2)
*a^3*d*abs(d)^(3/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*t
an(f*x + e)))/sqrt(abs(d)))/f + 2/315*(35*sqrt(d*tan(f*x + e))*a^3*d^20*f^8
*tan(f*x + e)^4 + 135*sqrt(d*tan(f*x + e))*a^3*d^20*f^8*tan(f*x + e)^3 + 12
6*sqrt(d*tan(f*x + e))*a^3*d^20*f^8*tan(f*x + e)^2 - 210*sqrt(d*tan(f*x + e
))*a^3*d^20*f^8*tan(f*x + e) - 630*sqrt(d*tan(f*x + e))*a^3*d^20*f^8)/(d^18
*f^9)
```

Mupad [**B**]
time = 5.96, size = 176, normalized size = 0.95

$$\frac{4a^3(d\tan(e+fx))^{5/2}}{5f}-\frac{4a^3d^2\sqrt{d\tan(e+fx)}}{f}+\frac{6a^3(d\tan(e+fx))^{7/2}}{7df}+\frac{2a^3(d\tan(e+fx))^{9/2}}{9d^2f}-\frac{4a^3d(d\tan(e+fx))^{3/2}}{3f}+\frac{\sqrt{2}a^3d^{5/2}\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{2\sqrt{d}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{2\sqrt{d}}+\frac{\sqrt{2}(d\tan(e+fx))^{3/2}}{2d^{3/2}}\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(5/2)*(a + a*tan(e + f*x))^3,x)
```

```
[Out] (4*a^3*(d*tan(e + f*x))^(5/2))/(5*f) - (4*a^3*d^2*(d*tan(e + f*x))^(1/2))/f
 + (6*a^3*(d*tan(e + f*x))^(7/2))/(7*d*f) + (2*a^3*(d*tan(e + f*x))^(9/2))/
(9*d^2*f) - (4*a^3*d*(d*tan(e + f*x))^(3/2))/(3*f) + (2^(1/2)*a^3*d^(5/2)*(
2*atan((2^(1/2)*(d*tan(e + f*x))^(1/2))/(2*d^(1/2))) + 2*atan((2^(1/2)*(d*t
an(e + f*x))^(1/2))/(2*d^(1/2)) + (2^(1/2)*(d*tan(e + f*x))^(3/2))/(2*d^(3/
2)))))/f
```

3.351 $\int (d\tan(e+fx))^{3/2}(a+a\tan(e+fx))^3\,dx$

Optimal. Leaf size=160

$$\frac{2\sqrt{2}\,a^3d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{f}-\frac{4a^3d\sqrt{d\tan(e+fx)}}{f}+\frac{4a^3(d\tan(e+fx))^{3/2}}{3f}+\frac{32a^3(d\tan(e+\ldots}{35df}$$

```
[Out] 2*a^3*d^(3/2)*arctanh(1/2*(d^(1/2)+d^(1/2)*tan(f*x+e))*2^(1/2)/(d*tan(f*x+e
))^(1/2))*2^(1/2)/f-4*a^3*d*(d*tan(f*x+e))^(1/2)/f+4/3*a^3*(d*tan(f*x+e))^(
3/2)/f+32/35*a^3*(d*tan(f*x+e))^(5/2)/d/f+2/7*(d*tan(f*x+e))^(5/2)*(a^3+a^3
*tan(f*x+e))/d/f
```

Rubi [A]
time = 0.17, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3647, 3711, 3609, 3613, 214}

$$\frac{2\sqrt{2}\,a^3d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\tan(e+fx)+\sqrt{d}}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{f}+\frac{32a^3(d\tan(e+fx))^{5/2}}{35df}+\frac{4a^3(d\tan(e+fx))^{3/2}}{3f}-\frac{4a^3d\sqrt{d\tan(e+fx)}}{f}+\frac{2(a^3\tan(e+fx)+a^3)(d\tan(e+fx))^{5/2}}{7df}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(3/2)*(a + a*Tan[e + f*x])^3,x]
```

```
[Out] (2*Sqrt[2]*a^3*d^(3/2)*ArcTanh[(Sqrt[d] + Sqrt[d]*Tan[e + f*x])/(Sqrt[2]*Sq
rt[d*Tan[e + f*x]])])/f - (4*a^3*d*Sqrt[d*Tan[e + f*x]])/f + (4*a^3*(d*Tan[
e + f*x])^(3/2))/(3*f) + (32*a^3*(d*Tan[e + f*x])^(5/2))/(35*d*f) + (2*(d*T
an[e + f*x])^(5/2)*(a^3 + a^3*Tan[e + f*x]))/(7*d*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
```

```
EqQ[c^2 - d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (d\tan(e+fx))^{3/2}(a+a\tan(e+fx))^3\,dx = \frac{2(d\tan(e+fx))^{5/2}\left(a^3+a^3\tan(e+fx)\right)}{7df} + \frac{2\int(d\tan(e+f}{}$$

$$= \frac{32a^3(d\tan(e+fx))^{5/2}}{35df} + \frac{2(d\tan(e+fx))^{5/2}\left(a^3+a^3\tan(e\right.}{7df}$$

$$= \frac{4a^3(d\tan(e+fx))^{3/2}}{3f} + \frac{32a^3(d\tan(e+fx))^{5/2}}{35df} + \frac{2(d\tan(e}{}$$

$$= -\frac{4a^3d\sqrt{d\tan(e+fx)}}{f} + \frac{4a^3(d\tan(e+fx))^{3/2}}{3f} + \frac{32a^3(d\,\mathrm{t}}{}$$

$$= -\frac{4a^3d\sqrt{d\tan(e+fx)}}{f} + \frac{4a^3(d\tan(e+fx))^{3/2}}{3f} + \frac{32a^3(d\,\mathrm{t}}{}$$

$$= \frac{2\sqrt{2}\,a^3d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{f} - \frac{4a^3d\sqrt{d\tan}}{f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 2.78, size = 332, normalized size = 2.08

$$-\frac{a^3\cos(e+fx)(d\tan(e+fx))^{3/2}(1+\tan(e+fx))^3\left(210\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(e+fx)}\right)\cos^2(e+fx)-210\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(e+fx)}\right)\cos^2(e+fx)+105\sqrt{2}\,\cos^2(e+fx)\log\left(1-\sqrt{2}\,\sqrt{\tan(e+fx)}+\tan(e+fx)\right)-105\sqrt{2}\,\cos^2(e+fx)\log\left(1+\sqrt{2}\,\sqrt{\tan(e+fx)}+\tan(e+fx)\right)+840\cos^2(e+fx)\sqrt{\tan(e+fx)}-280\cos^2(e+fx)\tan^{\frac{3}{2}}(e+fx)+280\cos^2(e+fx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(e+fx)\right)\tan^{\frac{3}{2}}(e+fx)-60\sin^2(e+fx)\tan^{\frac{3}{2}}(e+fx)-126\sin(2(e+fx))\tan^{\frac{3}{2}}(e+fx)\right)}{210f(\cos(e+fx)+\sin(e+fx))^3\tan^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(3/2)*(a + a*Tan[e + f*x])^3,x]
```

```
[Out] -1/210*(a^3*Cos[e + f*x]*(d*Tan[e + f*x])^(3/2)*(1 + Tan[e + f*x])^3*(210*S
qrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]]*Cos[e + f*x]^2 - 210*Sqrt[2]*
ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]]*Cos[e + f*x]^2 + 105*Sqrt[2]*Cos[e +
 f*x]^2*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 105*Sqrt[2]*Co
s[e + f*x]^2*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] + 840*Cos[e
 + f*x]^2*Sqrt[Tan[e + f*x]] - 280*Cos[e + f*x]^2*Tan[e + f*x]^(3/2) + 280*
Cos[e + f*x]^2*Hypergeometric2F1[3/4, 1, 7/4, -Tan[e + f*x]^2]*Tan[e + f*x]
^(3/2) - 60*Sin[e + f*x]^2*Tan[e + f*x]^(3/2) - 126*Sin[2*(e + f*x)]*Tan[e
+ f*x]^(3/2)))/(f*(Cos[e + f*x] + Sin[e + f*x])^3*Tan[e + f*x]^(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(135) = 270$.

time = 0.36, size = 339, normalized size = 2.12

method	result
derivativedivides	$\frac{2a^3\left(\frac{(d\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{3d(d\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{2d^2(d\tan(fx+e))^{\frac{3}{2}}}{3}-2d^3\sqrt{d\tan(fx+e)}+2d^4\left(\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(f\ldots}{d\tan(f\ldots}\right.\right.}{\ldots}\right.\right.}{\ldots}$
default	$\frac{2a^3\left(\frac{(d\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{3d(d\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{2d^2(d\tan(fx+e))^{\frac{3}{2}}}{3}-2d^3\sqrt{d\tan(fx+e)}+2d^4\left(\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(f\ldots}{d\tan(f\ldots}\right.\right.}{\ldots}\right.\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(3/2)*(a+a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3/d^2*(1/7*(d*tan(f*x+e))^(7/2)+3/5*d*(d*tan(f*x+e))^(5/2)+2/3*d^2*(d
*tan(f*x+e))^(3/2)-2*d^3*(d*tan(f*x+e))^(1/2)+2*d^4*(1/8/d*(d^2)^(1/4)*2^(1
/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))
/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arc
tan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/
4)*(d*tan(f*x+e))^(1/2)+1))-1/8/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)
^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*
(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*
```

```
tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))
))
```

Maxima [A]
time = 0.52, size = 171, normalized size = 1.07

$$\frac{105\,a^3d^3\left(\frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{d}+d\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{d}+d\right)}{\sqrt{d}}\right)+\frac{2\left(15\left(d\tan(fx+e)\right)^{\frac{7}{2}}a^3+63\left(d\tan(fx+e)\right)^{\frac{5}{2}}a^3d+70\left(d\tan(fx+e)\right)^{\frac{3}{2}}a^3d^2-210\sqrt{d\tan\left(fx+e\right)}a^3d^3\right)}{d}}{105\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)*(a+a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/105*(105*a^3*d^3*(sqrt(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e
))*sqrt(d) + d)/sqrt(d) - sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f
*x + e))*sqrt(d) + d)/sqrt(d)) + 2*(15*(d*tan(f*x + e))^(7/2)*a^3 + 63*(d*t
an(f*x + e))^(5/2)*a^3*d + 70*(d*tan(f*x + e))^(3/2)*a^3*d^2 - 210*sqrt(d*t
an(f*x + e))*a^3*d^3)/d)/(d*f)
```

Fricas [A]
time = 1.35, size = 273, normalized size = 1.71

$$\left[\frac{105\sqrt{2}\,a^3d^{\frac{3}{2}}\log\left(\frac{d\tan(fx+e)^2+2\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{d}\left(\tan(fx+e)+1\right)+4\,d\tan(fx+e)+d}{\tan(fx+e)^2+1}\right)+2\left(15\,a^3d\tan\left(fx+e\right)^3+63\,a^3d\tan\left(fx+e\right)^2+70\,a^3d\tan\left(fx+e\right)-210\,a^3d\right)\sqrt{d\tan\left(fx+e\right)}}{105\,f},\ -\frac{2\left(105\sqrt{2}\,a^3\sqrt{-d}\,d\arctan\left(\frac{\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{-d}\left(\tan(fx+e)+1\right)}{2\,d\tan(fx+e)}\right)-\left(15\,a^3d\tan\left(fx+e\right)^3+63\,a^3d\tan\left(fx+e\right)^2+70\,a^3d\tan\left(fx+e\right)-210\,a^3d\right)\sqrt{d\tan\left(fx+e\right)}\right)}{105\,f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)*(a+a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/105*(105*sqrt(2)*a^3*d^(3/2)*log((d*tan(f*x + e)^2 + 2*sqrt(2)*sqrt(d*ta
n(f*x + e))*sqrt(d)*(tan(f*x + e) + 1) + 4*d*tan(f*x + e) + d)/(tan(f*x + e
)^2 + 1)) + 2*(15*a^3*d*tan(f*x + e)^3 + 63*a^3*d*tan(f*x + e)^2 + 70*a^3*d
*tan(f*x + e) - 210*a^3*d)*sqrt(d*tan(f*x + e)))/f, -2/105*(105*sqrt(2)*a^3
*sqrt(-d)*d*arctan(1/2*sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(-d)*(tan(f*x + e)
+ 1)/(d*tan(f*x + e))) - (15*a^3*d*tan(f*x + e)^3 + 63*a^3*d*tan(f*x + e)^2
 + 70*a^3*d*tan(f*x + e) - 210*a^3*d)*sqrt(d*tan(f*x + e)))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int\left(d\tan\left(e+fx\right)\right)^{\frac{3}{2}}\,dx+\int 3\left(d\tan\left(e+fx\right)\right)^{\frac{3}{2}}\tan\left(e+fx\right)dx+\int 3\left(d\tan\left(e+fx\right)\right)^{\frac{3}{2}}\tan^2\left(e+fx\right)dx+\int\left(d\tan\left(e+fx\right)\right)^{\frac{3}{2}}\tan^3\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(3/2)*(a+a*tan(f*x+e))**3,x)
```

```
[Out] a**3*(Integral((d*tan(e + f*x))**(3/2), x) + Integral(3*(d*tan(e + f*x))**(
3/2)*tan(e + f*x), x) + Integral(3*(d*tan(e + f*x))**(3/2)*tan(e + f*x)**2,
 x) + Integral((d*tan(e + f*x))**(3/2)*tan(e + f*x)**3, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(142) = 284.
time = 0.86, size = 379, normalized size = 2.37

$$\frac{1}{210}\,d\left(\frac{105\,\sqrt{2}\left(a^3d\sqrt{|d|}+a^3|d|^{\frac{3}{2}}\right)\log\left(d\tan\left(fx+e\right)+\sqrt{2}\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{|d|}+|d|\right)}{df}-\frac{105\,\sqrt{2}\left(a^3d\sqrt{|d|}+a^3|d|^{\frac{3}{2}}\right)\log\left(d\tan\left(fx+e\right)-\sqrt{2}\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{|d|}+|d|\right)}{df}+\frac{210\left(\sqrt{2}\,a^3d\sqrt{|d|}-\sqrt{2}\,a^3|d|^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{|d|}+2\,\sqrt{d\tan\left(fx+e\right)}\right)}{2\,\sqrt{|d|}}\right)}{df}+\frac{210\left(\sqrt{2}\,a^3d\sqrt{|d|}-\sqrt{2}\,a^3|d|^{\frac{3}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{|d|}-2\,\sqrt{d\tan\left(fx+e\right)}\right)}{2\,\sqrt{|d|}}\right)}{df}+\frac{4\left(15\,\sqrt{d\tan\left(fx+e\right)}\,a^3d^{21}f^6\tan\left(fx+e\right)^3+63\,\sqrt{d\tan\left(fx+e\right)}\,a^3d^{21}f^6\tan\left(fx+e\right)^2+70\,\sqrt{d\tan\left(fx+e\right)}\,a^3d^{21}f^6\tan\left(fx+e\right)-210\,\sqrt{d\tan\left(fx+e\right)}\,a^3d^{21}f^6\right)}{d^{21}f^7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)*(a+a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/210*d*(105*sqrt(2)*(a^3*d*sqrt(abs(d)) + a^3*abs(d)^(3/2))*log(d*tan(f*x
+ e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(d*f) - 105*sqrt
(2)*(a^3*d*sqrt(abs(d)) + a^3*abs(d)^(3/2))*log(d*tan(f*x + e) - sqrt(2)*sq
rt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(d*f) + 210*(sqrt(2)*a^3*d*sqrt(a
bs(d)) - sqrt(2)*a^3*abs(d)^(3/2))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d))
 + 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(d*f) + 210*(sqrt(2)*a^3*d*sqrt(ab
s(d)) - sqrt(2)*a^3*abs(d)^(3/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d))
 - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(d*f) + 4*(15*sqrt(d*tan(f*x + e))
*a^3*d^21*f^6*tan(f*x + e)^3 + 63*sqrt(d*tan(f*x + e))*a^3*d^21*f^6*tan(f*x
 + e)^2 + 70*sqrt(d*tan(f*x + e))*a^3*d^21*f^6*tan(f*x + e) - 210*sqrt(d*ta
n(f*x + e))*a^3*d^21*f^6)/(d^21*f^7))
```

Mupad [B]
time = 5.15, size = 143, normalized size = 0.89

$$\frac{4\,a^3\,{\left(d\tan(e+f\,x)\right)}^{3/2}}{3\,f}+\frac{6\,a^3\,{\left(d\tan(e+f\,x)\right)}^{5/2}}{5\,d\,f}+\frac{2\,a^3\,{\left(d\tan(e+f\,x)\right)}^{7/2}}{7\,d^2\,f}-\frac{4\,a^3\,d\,\sqrt{d\tan\left(e+f\,x\right)}}{f}-\frac{\sqrt{2}\,a^3\,d^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,a^6\,d^{9/2}\,\sqrt{d\tan\left(e+f\,x\right)}\,32{}\mathrm{i}}{32\,a^6\,d^5+32\,a^6\,d^5\,\tan(e+f\,x)}\right)\,2{}\mathrm{i}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(3/2)*(a + a*tan(e + f*x))^3,x)
```

```
[Out] (4*a^3*(d*tan(e + f*x))^(3/2))/(3*f) + (6*a^3*(d*tan(e + f*x))^(5/2))/(5*d*
f) + (2*a^3*(d*tan(e + f*x))^(7/2))/(7*d^2*f) - (4*a^3*d*(d*tan(e + f*x))^(
1/2))/f - (2^(1/2)*a^3*d^(3/2)*atan((2^(1/2)*a^6*d^(9/2)*(d*tan(e + f*x))^(
1/2)*32i)/(32*a^6*d^5 + 32*a^6*d^5*tan(e + f*x)))*2i)/f
```

3.352 $\int \sqrt{d\tan(e+fx)}\,(a+a\tan(e+fx))^3\,dx$

Optimal. Leaf size=138

$$\frac{2\sqrt{2}\,a^3\sqrt{d}\,\mathrm{ArcTan}\left(\frac{\sqrt{d}-\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{f}+\frac{4a^3\sqrt{d\tan(e+fx)}}{f}+\frac{8a^3(d\tan(e+fx))^{3/2}}{5df}+\frac{2(d\tan(e+f$$

```
[Out] 2*a^3*arctan(1/2*(d^(1/2)-d^(1/2)*tan(f*x+e))*2^(1/2)/(d*tan(f*x+e))^(1/2))
*2^(1/2)*d^(1/2)/f+4*a^3*(d*tan(f*x+e))^(1/2)/f+8/5*a^3*(d*tan(f*x+e))^(3/2
)/d/f+2/5*(d*tan(f*x+e))^(3/2)*(a^3+a^3*tan(f*x+e))/d/f
```

Rubi [A]
time = 0.13, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3647, 3711, 3609, 3613, 211}

$$\frac{2\sqrt{2}\,a^3\sqrt{d}\,\mathrm{ArcTan}\left(\frac{\sqrt{d}-\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{f}+\frac{8a^3(d\tan(e+fx))^{3/2}}{5df}+\frac{4a^3\sqrt{d\tan(e+fx)}}{f}+\frac{2(a^3\tan(e+fx)+a^3)\,(d\tan(e+fx))^{3/2}}{5df}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Tan[e + f*x]]*(a + a*Tan[e + f*x])^3,x]
```

```
[Out] (2*Sqrt[2]*a^3*Sqrt[d]*ArcTan[(Sqrt[d] - Sqrt[d]*Tan[e + f*x])/(Sqrt[2]*Sqr
t[d*Tan[e + f*x]])])/f + (4*a^3*Sqrt[d*Tan[e + f*x]])/f + (8*a^3*(d*Tan[e +
 f*x])^(3/2))/(5*d*f) + (2*(d*Tan[e + f*x])^(3/2)*(a^3 + a^3*Tan[e + f*x]))
/(5*d*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
```

```
EqQ[c^2 - d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \sqrt{d\tan(e+fx)}\,(a+a\tan(e+fx))^3\,dx = \frac{2(d\tan(e+fx))^{3/2}\left(a^3+a^3\tan(e+fx)\right)}{5df} + \frac{2\int \sqrt{d\tan(e+f}}{}$$

$$= \frac{8a^3(d\tan(e+fx))^{3/2}}{5df} + \frac{2(d\tan(e+fx))^{3/2}\left(a^3+a^3\tan(e+\right.}{5df}$$

$$= \frac{4a^3\sqrt{d\tan(e+fx)}}{f} + \frac{8a^3(d\tan(e+fx))^{3/2}}{5df} + \frac{2(d\tan(e+f}{}$$

$$= \frac{4a^3\sqrt{d\tan(e+fx)}}{f} + \frac{8a^3(d\tan(e+fx))^{3/2}}{5df} + \frac{2(d\tan(e+f}{}$$

$$= \frac{2\sqrt{2}\,a^3\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{d}-\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{f} + \frac{4a^3\sqrt{d\tan(e+}}{f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.84, size = 315, normalized size = 2.28

$$\frac{a^3\cos(e+fx)\left(3\left(10\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(e+fx)}\right)\cos^2(e+fx)-10\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(e+fx)}\right)\cos^2(e+fx)+5\sqrt{2}\,\cos^2(e+fx)\log\left(1-\sqrt{2}\,\sqrt{\tan(e+fx)}+\tan(e+fx)\right)-5\sqrt{2}\,\cos^2(e+fx)\log\left(1+\sqrt{2}\,\sqrt{\tan(e+fx)}+\tan(e+fx)\right)+40\cos^2(e+fx)\sqrt{\tan(e+fx)}+4\sin^2(e+fx)\sqrt{\tan(e+fx)}+10\sin(2(e+fx))\sqrt{\tan(e+fx)}\right)-20\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(e+fx)\right)\sin(2(e+fx))\sqrt{\tan(e+fx)}\right)\sqrt{d\tan(e+fx)}\,(1+\tan(e+fx))^3}{30f(\cos(e+fx)+\sin(e+fx))^3\sqrt{\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Tan[e + f*x]]*(a + a*Tan[e + f*x])^3,x]
```

```
[Out] (a^3*Cos[e + f*x]*(3*(10*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]]*Cos
[e + f*x]^2 - 10*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]]*Cos[e + f*x
]^2 + 5*Sqrt[2]*Cos[e + f*x]^2*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e +
 f*x]] - 5*Sqrt[2]*Cos[e + f*x]^2*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[
e + f*x]] + 40*Cos[e + f*x]^2*Sqrt[Tan[e + f*x]] + 4*Sin[e + f*x]^2*Sqrt[Ta
n[e + f*x]] + 10*Sin[2*(e + f*x)]*Sqrt[Tan[e + f*x]]) - 20*Hypergeometric2F
1[3/4, 1, 7/4, -Tan[e + f*x]^2]*Sin[2*(e + f*x)]*Sqrt[Tan[e + f*x]])*Sqrt[d
*Tan[e + f*x]]*(1 + Tan[e + f*x])^3)/(30*f*(Cos[e + f*x] + Sin[e + f*x])^3*
Sqrt[Tan[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(117) = 234$.

time = 0.33, size = 323, normalized size = 2.34

method	result
derivativedivides	$\frac{2a^3\left(\frac{(d\tan(fx+e))^{\frac{5}{2}}}{5}+d(d\tan(fx+e))^{\frac{3}{2}}+2d^2\sqrt{d\tan(fx+e)}-2d^3\left(\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d}}\right.\right.}{}\right.\right.}{}$
default	$\frac{2a^3\left(\frac{(d\tan(fx+e))^{\frac{5}{2}}}{5}+d(d\tan(fx+e))^{\frac{3}{2}}+2d^2\sqrt{d\tan(fx+e)}-2d^3\left(\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d}}\right.\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(1/2)*(a+a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3/d^2*(1/5*(d*tan(f*x+e))^(5/2)+d*(d*tan(f*x+e))^(3/2)+2*d^2*(d*tan(f
*x+e))^(1/2)-2*d^3*(1/8/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)
*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan
(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*
x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))+1/8/(
d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/
2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2
)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^
(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)))
```

Maxima [A]

time = 0.51, size = 149, normalized size = 1.08

$$-\frac{2\left(5\,a^{3}d^{2}\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}\right)-\frac{\left(d\tan\left(fx+e\right)\right)^{\frac{5}{2}}a^{3}+5\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}a^{3}d+10\sqrt{d\tan\left(fx+e\right)}\,a^{3}d^{2}}{d}\right)}{5\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)*(a+a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/5*(5*a^3*d^2*(sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan
(f*x + e)))/sqrt(d))/sqrt(d) + sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
 - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d)) - ((d*tan(f*x + e))^(5/2)*a^3
+ 5*(d*tan(f*x + e))^(3/2)*a^3*d + 10*sqrt(d*tan(f*x + e))*a^3*d^2)/d)/(d*f
)
```

Fricas [A]
time = 1.09, size = 231, normalized size = 1.67

$$\left[\frac{5\sqrt{2}\,a^{3}\sqrt{-d}\,\log\left(\frac{d\tan\left(fx+e\right)^{2}-2\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{-d}\left(\tan\left(fx+e\right)-1\right)-4\,d\tan\left(fx+e\right)+d}{\tan\left(fx+e\right)^{2}+1}\right)+2\left(a^{3}\tan\left(fx+e\right)^{2}+5\,a^{3}\tan\left(fx+e\right)+10\,a^{3}\right)\sqrt{d\tan\left(fx+e\right)}}{5\,f},\ -\frac{2\left(5\sqrt{2}\,a^{3}\sqrt{d}\,\arctan\left(\frac{\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\left(\tan\left(fx+e\right)-1\right)}{2\sqrt{d}\tan\left(fx+e\right)}\right)-\left(a^{3}\tan\left(fx+e\right)^{2}+5\,a^{3}\tan\left(fx+e\right)+10\,a^{3}\right)\sqrt{d\tan\left(fx+e\right)}\right)}{5\,f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)*(a+a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/5*(5*sqrt(2)*a^3*sqrt(-d)*log((d*tan(f*x + e)^2 - 2*sqrt(2)*sqrt(d*tan(f
*x + e))*sqrt(-d)*(tan(f*x + e) - 1) - 4*d*tan(f*x + e) + d)/(tan(f*x + e)^
2 + 1)) + 2*(a^3*tan(f*x + e)^2 + 5*a^3*tan(f*x + e) + 10*a^3)*sqrt(d*tan(f
*x + e)))/f, -2/5*(5*sqrt(2)*a^3*sqrt(d)*arctan(1/2*sqrt(2)*sqrt(d*tan(f*x
+ e))*(tan(f*x + e) - 1)/(sqrt(d)*tan(f*x + e))) - (a^3*tan(f*x + e)^2 + 5*
a^3*tan(f*x + e) + 10*a^3)*sqrt(d*tan(f*x + e)))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int\sqrt{d\tan\left(e+fx\right)}\,dx+\int 3\sqrt{d\tan\left(e+fx\right)}\tan\left(e+fx\right)dx+\int 3\sqrt{d\tan\left(e+fx\right)}\tan^{2}\left(e+fx\right)dx+\int\sqrt{d\tan\left(e+fx\right)}\tan^{3}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(1/2)*(a+a*tan(f*x+e))**3,x)
```

```
[Out] a**3*(Integral(sqrt(d*tan(e + f*x)), x) + Integral(3*sqrt(d*tan(e + f*x))*t
an(e + f*x), x) + Integral(3*sqrt(d*tan(e + f*x))*tan(e + f*x)**2, x) + Int
egral(sqrt(d*tan(e + f*x))*tan(e + f*x)**3, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(124) = 248.
time = 0.73, size = 344, normalized size = 2.49

$$-\frac{\sqrt{2}\left(a^{3}d\sqrt{|d|}-a^{3}|d|^{\frac{3}{2}}\right)\log\left(d\tan\left(fx+e\right)+\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{|d|}+|d|\right)}{2\,df}+\frac{\sqrt{2}\left(a^{3}d\sqrt{|d|}-a^{3}|d|^{\frac{3}{2}}\right)\log\left(d\tan\left(fx+e\right)-\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{|d|}+|d|\right)}{2\,df}-\frac{\left(\sqrt{2}\,a^{3}d\sqrt{|d|}+\sqrt{2}\,a^{3}|d|^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{df}-\frac{\left(\sqrt{2}\,a^{3}d\sqrt{|d|}+\sqrt{2}\,a^{3}|d|^{\frac{3}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{df}+\frac{2\left(\sqrt{d\tan\left(fx+e\right)}\,a^{3}d^{10}f^{4}\tan\left(fx+e\right)^{2}+5\sqrt{d\tan\left(fx+e\right)}\,a^{3}d^{10}f^{4}\tan\left(fx+e\right)+10\sqrt{d\tan\left(fx+e\right)}\,a^{3}d^{10}f^{4}\right)}{5\,d^{10}f^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)*(a+a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(a^3*d*sqrt(abs(d)) - a^3*abs(d)^(3/2))*log(d*tan(f*x + e) + s
qrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(d*f) + 1/2*sqrt(2)*(a^3
*d*sqrt(abs(d)) - a^3*abs(d)^(3/2))*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan
(f*x + e))*sqrt(abs(d)) + abs(d))/(d*f) - (sqrt(2)*a^3*d*sqrt(abs(d)) + sqr
t(2)*a^3*abs(d)^(3/2))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqrt(d*
tan(f*x + e)))/sqrt(abs(d)))/(d*f) - (sqrt(2)*a^3*d*sqrt(abs(d)) + sqrt(2)*
a^3*abs(d)^(3/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(
f*x + e)))/sqrt(abs(d)))/(d*f) + 2/5*(sqrt(d*tan(f*x + e))*a^3*d^10*f^4*tan
(f*x + e)^2 + 5*sqrt(d*tan(f*x + e))*a^3*d^10*f^4*tan(f*x + e) + 10*sqrt(d*
tan(f*x + e))*a^3*d^10*f^4)/(d^10*f^5)
```

Mupad [B]
time = 4.62, size = 137, normalized size = 0.99

$$\frac{4\,a^3\,\sqrt{d\tan\left(e+f\,x\right)}}{f}+\frac{2\,a^3\,{\left(d\tan\left(e+f\,x\right)\right)}^{3/2}}{d\,f}+\frac{2\,a^3\,{\left(d\tan\left(e+f\,x\right)\right)}^{5/2}}{5\,d^2\,f}-\frac{\sqrt{2}\,a^3\,\sqrt{d}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{d\tan\left(e+f\,x\right)}}{2\,\sqrt{d}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{d\tan\left(e+f\,x\right)}}{2\,\sqrt{d}}+\frac{\sqrt{2}\,{\left(d\tan\left(e+f\,x\right)\right)}^{3/2}}{2\,d^{3/2}}\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(1/2)*(a + a*tan(e + f*x))^3,x)
```

```
[Out] (4*a^3*(d*tan(e + f*x))^(1/2))/f + (2*a^3*(d*tan(e + f*x))^(3/2))/(d*f) + (
2*a^3*(d*tan(e + f*x))^(5/2))/(5*d^2*f) - (2^(1/2)*a^3*d^(1/2)*(2*atan((2^(
1/2)*(d*tan(e + f*x))^(1/2))/(2*d^(1/2))) + 2*atan((2^(1/2)*(d*tan(e + f*x)
)^(1/2))/(2*d^(1/2)) + (2^(1/2)*(d*tan(e + f*x))^(3/2))/(2*d^(3/2)))))/f
```

3.353 $\int \frac{(a+a\tan(e+fx))^3}{\sqrt{d\tan(e+fx)}}\,dx$

Optimal. Leaf size=117

$$-\frac{2\sqrt{2}\,a^3\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{\sqrt{d}\,f}+\frac{16a^3\sqrt{d\tan(e+fx)}}{3df}+\frac{2\sqrt{d\tan(e+fx)}\,(a^3+a^3\tan(e+fx))}{3df}$$

```
[Out] -2*a^3*arctanh(1/2*(d^(1/2)+d^(1/2)*tan(f*x+e))*2^(1/2)/(d*tan(f*x+e))^(1/2
))*2^(1/2)/f/d^(1/2)+16/3*a^3*(d*tan(f*x+e))^(1/2)/d/f+2/3*(d*tan(f*x+e))^(
1/2)*(a^3+a^3*tan(f*x+e))/d/f
```

Rubi [A]
time = 0.11, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3647, 3711, 3613, 214}

$$\frac{16a^3\sqrt{d\tan(e+fx)}}{3df}+\frac{2(a^3\tan(e+fx)+a^3)\sqrt{d\tan(e+fx)}}{3df}-\frac{2\sqrt{2}\,a^3\tanh^{-1}\left(\frac{\sqrt{d}\,\tan(e+fx)+\sqrt{d}}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{\sqrt{d}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[e + f*x])^3/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (-2*Sqrt[2]*a^3*ArcTanh[(Sqrt[d] + Sqrt[d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[d*Ta
n[e + f*x]])])/(Sqrt[d]*f) + (16*a^3*Sqrt[d*Tan[e + f*x]])/(3*d*f) + (2*Sqr
t[d*Tan[e + f*x]]*(a^3 + a^3*Tan[e + f*x]))/(3*d*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
```

```
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\tan(e+fx))^3}{\sqrt{d\tan(e+fx)}}\,dx = \frac{2\sqrt{d\tan(e+fx)}\,(a^3+a^3\tan(e+fx))}{3df} + \frac{2\int \frac{a^3d+3a^3d\tan(e+fx)+4a^3d\tan^2(e+fx)}{\sqrt{d\tan(e+fx)}}}{3d}$$

$$= \frac{16a^3\sqrt{d\tan(e+fx)}}{3df} + \frac{2\sqrt{d\tan(e+fx)}\,(a^3+a^3\tan(e+fx))}{3df} + \frac{2\int \frac{-3a^3d}{\sqrt{d\mathrm{t}}}}{}$$

$$= \frac{16a^3\sqrt{d\tan(e+fx)}}{3df} + \frac{2\sqrt{d\tan(e+fx)}\,(a^3+a^3\tan(e+fx))}{3df} - \frac{(12a^6d)\,\mathrm{S}}{}$$

$$= -\frac{2\sqrt{2}\,a^3\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{\sqrt{d}\,f} + \frac{16a^3\sqrt{d\tan(e+fx)}}{3df} + \frac{2\sqrt{d\mathrm{t}}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.09, size = 553, normalized size = 4.73

$$\frac{6\cos^2(e+fx)\sin(e+fx)(a+a\tan(e+fx))^3}{f(\cos(e+fx)+\sin(e+fx))^3\sqrt{d\tan(e+fx)}} + \frac{2\cos(e+fx)\sin^2(e+fx)(a+a\tan(e+fx))^3}{3f(\cos(e+fx)+\sin(e+fx))^3\sqrt{d\tan(e+fx)}} + \frac{4\cos(e+fx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(e+fx)\right)\sin^2(e+fx)(a+a\tan(e+fx))^3}{3f(\cos(e+fx)+\sin(e+fx))^3\sqrt{d\tan(e+fx)}} + \frac{\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\cos^3(e+fx)\sqrt{\tan(e+fx)}\,(a+a\tan(e+fx))^3}{f(\cos(e+fx)+\sin(e+fx))^3\sqrt{d\tan(e+fx)}} - \frac{\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(e+fx)}\right)\cos^3(e+fx)\sqrt{\tan(e+fx)}\,(a+a\tan(e+fx))^3}{f(\cos(e+fx)+\sin(e+fx))^3\sqrt{d\tan(e+fx)}} + \frac{\cos^3(e+fx)\log\left(1-\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)\sqrt{\tan(e+fx)}\,(a+a\tan(e+fx))^3}{\sqrt{2}\,f(\cos(e+fx)+\sin(e+fx))^3\sqrt{d\tan(e+fx)}} - \frac{\cos^3(e+fx)\log\left(1+\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)\sqrt{\tan(e+fx)}\,(a+a\tan(e+fx))^3}{\sqrt{2}\,f(\cos(e+fx)+\sin(e+fx))^3\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[e + f*x])^3/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (6*Cos[e + f*x]^2*Sin[e + f*x]*(a + a*Tan[e + f*x])^3)/(f*(Cos[e + f*x] + S
in[e + f*x])^3*Sqrt[d*Tan[e + f*x]]) + (2*Cos[e + f*x]*Sin[e + f*x]^2*(a +
```

```
a*Tan[e + f*x])^3)/(3*f*(Cos[e + f*x] + Sin[e + f*x])^3*Sqrt[d*Tan[e + f*x]
]) + (4*Cos[e + f*x]*Hypergeometric2F1[3/4, 1, 7/4, -Tan[e + f*x]^2]*Sin[e
+ f*x]^2*(a + a*Tan[e + f*x])^3)/(3*f*(Cos[e + f*x] + Sin[e + f*x])^3*Sqrt[
d*Tan[e + f*x]]) + (Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]]*Cos[e +
f*x]^3*Sqrt[Tan[e + f*x]]*(a + a*Tan[e + f*x])^3)/(f*(Cos[e + f*x] + Sin[e
+ f*x])^3*Sqrt[d*Tan[e + f*x]]) - (Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e +
f*x]]]*Cos[e + f*x]^3*Sqrt[Tan[e + f*x]]*(a + a*Tan[e + f*x])^3)/(f*(Cos[e
+ f*x] + Sin[e + f*x])^3*Sqrt[d*Tan[e + f*x]]) + (Cos[e + f*x]^3*Log[1 - Sq
rt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]*Sqrt[Tan[e + f*x]]*(a + a*Tan[e +
f*x])^3)/(Sqrt[2]*f*(Cos[e + f*x] + Sin[e + f*x])^3*Sqrt[d*Tan[e + f*x]]) -
 (Cos[e + f*x]^3*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]*Sqrt[Ta
n[e + f*x]]*(a + a*Tan[e + f*x])^3)/(Sqrt[2]*f*(Cos[e + f*x] + Sin[e + f*x]
)^3*Sqrt[d*Tan[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(98) = 196$.

time = 0.27, size = 309, normalized size = 2.64

method	result
derivativedivides	$\frac{2a^3\left(\frac{(d\tan(fx+e))^{\frac{3}{2}}}{3}+3d\sqrt{d\tan(fx+e)}-2d^2\left(\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}}\right.\right.}{}\right.\right.}{}$
default	$\frac{2a^3\left(\frac{(d\tan(fx+e))^{\frac{3}{2}}}{3}+3d\sqrt{d\tan(fx+e)}-2d^2\left(\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}}\right.\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(f*x+e))^3/(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3/d^2*(1/3*(d*tan(f*x+e))^(3/2)+3*d*(d*tan(f*x+e))^(1/2)-2*d^2*(1/8/d
*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(
1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d
^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-
2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))-1/8/(d^2)^(1/4)*2^(1/2)*(ln((d
*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*
x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2
)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(
f*x+e))^(1/2)+1))))
```

Maxima [A]

time = 0.50, size = 130, normalized size = 1.11

$$-\frac{3\,a^3 d\left(\frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)-\frac{2\left((d\tan(fx+e))^{\frac{3}{2}}a^3+9\sqrt{d\tan(fx+e)}\,a^3 d\right)}{d}}{3\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))^3/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*a^3*d*(sqrt(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sq
rt(d) + d)/sqrt(d) - sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x +
e))*sqrt(d) + d)/sqrt(d)) - 2*((d*tan(f*x + e))^(3/2)*a^3 + 9*sqrt(d*tan(f*
x + e))*a^3*d)/d)/(d*f)
```

Fricas [A]
time = 0.87, size = 210, normalized size = 1.79

$$\left[\frac{3\sqrt{2}\,a^3\sqrt{d}\,\log\left(\frac{\tan(fx+e)^2-\frac{2\sqrt{2}\sqrt{d\tan(fx+e)}\,(\tan(fx+e)+1)}{\sqrt{d}}+4\tan(fx+e)+1}{\tan(fx+e)^2+1}\right)+2\left(a^3\tan(fx+e)+9a^3\right)\sqrt{d\tan(fx+e)}}{3\,df},\ \frac{2\left(3\sqrt{2}\,a^3 d\sqrt{-\frac{1}{d}}\,\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{-\frac{1}{d}}\,(\tan(fx+e)+1)}{2\tan(fx+e)}\right)+\left(a^3\tan(fx+e)+9a^3\right)\sqrt{d\tan(fx+e)}\right)}{3\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))^3/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(2)*a^3*sqrt(d)*log((tan(f*x + e)^2 - 2*sqrt(2)*sqrt(d*tan(f*x
+ e))*(tan(f*x + e) + 1)/sqrt(d) + 4*tan(f*x + e) + 1)/(tan(f*x + e)^2 + 1)
) + 2*(a^3*tan(f*x + e) + 9*a^3)*sqrt(d*tan(f*x + e)))/(d*f), 2/3*(3*sqrt(2
)*a^3*d*sqrt(-1/d)*arctan(1/2*sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(-1/d)*(tan(
f*x + e) + 1)/tan(f*x + e)) + (a^3*tan(f*x + e) + 9*a^3)*sqrt(d*tan(f*x + e
)))/(d*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int\frac{1}{\sqrt{d\tan(e+fx)}}\,dx+\int\frac{3\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx+\int\frac{3\tan^2(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx+\int\frac{\tan^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))**3/(d*tan(f*x+e))**(1/2),x)
```

```
[Out] a**3*(Integral(1/sqrt(d*tan(e + f*x)), x) + Integral(3*tan(e + f*x)/sqrt(d*
tan(e + f*x)), x) + Integral(3*tan(e + f*x)**2/sqrt(d*tan(e + f*x)), x) + I
ntegral(tan(e + f*x)**3/sqrt(d*tan(e + f*x)), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(103) = 206.
time = 0.71, size = 313, normalized size = 2.68

$$-\frac{\sqrt{2}\left(a^3 d\sqrt{|d|}+a^3|d|^{\frac{3}{2}}\right)\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{|d|}+|d|\right)}{2\,d^2 f}+\frac{\sqrt{2}\left(a^3 d\sqrt{|d|}+a^3|d|^{\frac{3}{2}}\right)\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{|d|}+|d|\right)}{2\,d^2 f}-\frac{\left(\sqrt{2}\,a^3 d\sqrt{|d|}-\sqrt{2}\,a^3|d|^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{d^2 f}-\frac{\left(\sqrt{2}\,a^3 d\sqrt{|d|}-\sqrt{2}\,a^3|d|^{\frac{3}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{d^2 f}+\frac{2\left(\sqrt{d\tan(fx+e)}\,a^3 d^5 f^2\tan(fx+e)+9\sqrt{d\tan(fx+e)}\,a^3 d^5 f^2\right)}{3\,d^6 f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))^3/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(a^3*d*sqrt(abs(d)) + a^3*abs(d)^(3/2))*log(d*tan(f*x + e) + s
qrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(d^2*f) + 1/2*sqrt(2)*(a
^3*d*sqrt(abs(d)) + a^3*abs(d)^(3/2))*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*t
an(f*x + e))*sqrt(abs(d)) + abs(d))/(d^2*f) - (sqrt(2)*a^3*d*sqrt(abs(d)) -
 sqrt(2)*a^3*abs(d)^(3/2))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqr
t(d*tan(f*x + e)))/sqrt(abs(d)))/(d^2*f) - (sqrt(2)*a^3*d*sqrt(abs(d)) - sq
rt(2)*a^3*abs(d)^(3/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(
d*tan(f*x + e)))/sqrt(abs(d)))/(d^2*f) + 2/3*(sqrt(d*tan(f*x + e))*a^3*d^5*
f^2*tan(f*x + e) + 9*sqrt(d*tan(f*x + e))*a^3*d^5*f^2)/(d^6*f^3)
```

Mupad [B]
time = 4.26, size = 100, normalized size = 0.85

$$\frac{6\,a^3\,\sqrt{d\tan\left(e+f\,x\right)}}{d\,f}+\frac{2\,a^3\,{\left(d\tan(e+f\,x)\right)}^{3/2}}{3\,d^2\,f}-\frac{2\,\sqrt{2}\,a^3\,\mathrm{atanh}\left(\frac{32\,\sqrt{2}\,a^6\,\sqrt{d}\,\sqrt{d\tan\left(e+f\,x\right)}}{32\,a^6\,d+32\,a^6\,d\tan(e+f\,x)}\right)}{\sqrt{d}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x))^3/(d*tan(e + f*x))^(1/2),x)
```

```
[Out] (6*a^3*(d*tan(e + f*x))^(1/2))/(d*f) + (2*a^3*(d*tan(e + f*x))^(3/2))/(3*d^
2*f) - (2*2^(1/2)*a^3*atanh((32*2^(1/2)*a^6*d^(1/2)*(d*tan(e + f*x))^(1/2))
/(32*a^6*d + 32*a^6*d*tan(e + f*x))))/(d^(1/2)*f)
```

3.354 $\int \frac{(a+a\tan(e+fx))^3}{(d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=114

$$-\frac{2\sqrt{2}\,a^3\operatorname{ArcTan}\left(\frac{\sqrt{d}-\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{d^{3/2}f}+\frac{4a^3\sqrt{d\tan(e+fx)}}{d^2f}-\frac{2(a^3+a^3\tan(e+fx))}{df\sqrt{d\tan(e+fx)}}$$

```
[Out] -2*a^3*arctan(1/2*(d^(1/2)-d^(1/2)*tan(f*x+e))*2^(1/2)/(d*tan(f*x+e))^(1/2)
)*2^(1/2)/d^(3/2)/f+4*a^3*(d*tan(f*x+e))^(1/2)/d^2/f-2*(a^3+a^3*tan(f*x+e))
/d/f/(d*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3646, 3711, 3613, 211}

$$-\frac{2\sqrt{2}\,a^3\operatorname{ArcTan}\left(\frac{\sqrt{d}-\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{d^{3/2}f}+\frac{4a^3\sqrt{d\tan(e+fx)}}{d^2f}-\frac{2(a^3\tan(e+fx)+a^3)}{df\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[e + f*x])^3/(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[2]*a^3*ArcTan[(Sqrt[d] - Sqrt[d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[d*Tan
[e + f*x]])])/(d^(3/2)*f) + (4*a^3*Sqrt[d*Tan[e + f*x]])/(d^2*f) - (2*(a^3
+ a^3*Tan[e + f*x]))/(d*f*Sqrt[d*Tan[e + f*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
```

```
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\tan(e+fx))^3}{(d\tan(e+fx))^{3/2}}\,dx = -\frac{2(a^3+a^3\tan(e+fx))}{df\sqrt{d\tan(e+fx)}}+\frac{2\int\frac{2a^3d^2+a^3d^2\tan(e+fx)+a^3d^2\tan^2(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx}{d^3}$$

$$=\frac{4a^3\sqrt{d\tan(e+fx)}}{d^2f}-\frac{2(a^3+a^3\tan(e+fx))}{df\sqrt{d\tan(e+fx)}}+\frac{2\int\frac{a^3d^2+a^3d^2\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx}{d^3}$$

$$=\frac{4a^3\sqrt{d\tan(e+fx)}}{d^2f}-\frac{2(a^3+a^3\tan(e+fx))}{df\sqrt{d\tan(e+fx)}}-\frac{(4a^6d)\operatorname{Subst}\left(\int\frac{1}{2a^6d^4+dx^2}\,dx,\right.}{f}$$

$$=-\frac{2\sqrt{2}\,a^3\tan^{-1}\left(\frac{\sqrt{d}-\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{d^{3/2}f}+\frac{4a^3\sqrt{d\tan(e+fx)}}{d^2f}-\frac{2(a^3+a^3\text{t}}{df\sqrt{d\,\text{ta}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.76, size = 314, normalized size = 2.75

$$\frac{(a+a\tan(e+fx))^3\left(-4\cos^2(e+fx)\,{}_2F_1\left(-\tfrac{1}{4},1;\tfrac{3}{4};-\tan^2(e+fx)\right)\sin(e+fx)+4\cos(e+fx)\sin^2(e+fx)+4\,{}_2F_1\left(\tfrac{3}{4},1;\tfrac{7}{4};-\tan^2(e+fx)\right)\sin^3(e+fx)-2\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(e+fx)}\right)\cos^3(e+fx)\tan^{\frac{3}{2}}(e+fx)+2\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(e+fx)}\right)\cos^3(e+fx)\tan^{\frac{3}{2}}(e+fx)-\sqrt{2}\,\cos^3(e+fx)\log\left(1-\sqrt{2}\,\sqrt{\tan(e+fx)}+\tan(e+fx)\right)\tan^{\frac{3}{2}}(e+fx)+\sqrt{2}\,\cos^3(e+fx)\log\left(1+\sqrt{2}\,\sqrt{\tan(e+fx)}+\tan(e+fx)\right)\tan^{\frac{3}{2}}(e+fx)\right)}{2f(\cos(e+fx)+\sin(e+fx))^3(d\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[e + f*x])^3/(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((a + a*Tan[e + f*x])^3*(-4*Cos[e + f*x]^2*Hypergeometric2F1[-1/4, 1, 3/4,
-Tan[e + f*x]^2]*Sin[e + f*x] + 4*Cos[e + f*x]*Sin[e + f*x]^2 + 4*Hypergeom
etric2F1[3/4, 1, 7/4, -Tan[e + f*x]^2]*Sin[e + f*x]^3 - 2*Sqrt[2]*ArcTan[1
- Sqrt[2]*Sqrt[Tan[e + f*x]]]*Cos[e + f*x]^3*Tan[e + f*x]^(3/2) + 2*Sqrt[2]
*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]]*Cos[e + f*x]^3*Tan[e + f*x]^(3/2) -
 Sqrt[2]*Cos[e + f*x]^3*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]*
Tan[e + f*x]^(3/2) + Sqrt[2]*Cos[e + f*x]^3*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*
x]] + Tan[e + f*x]]*Tan[e + f*x]^(3/2)))/(2*f*(Cos[e + f*x] + Sin[e + f*x])
^3*(d*Tan[e + f*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(99) = 198$.

time = 0.21, size = 305, normalized size = 2.68

method	result
derivativedivides	$\frac{2a^3\left(\sqrt{d\tan(fx+e)}-\frac{d}{\sqrt{d\tan(fx+e)}}+2d\left(\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+}}\right.\right.}{}\right.\right.}{}$
default	$\frac{2a^3\left(\sqrt{d\tan(fx+e)}-\frac{d}{\sqrt{d\tan(fx+e)}}+2d\left(\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+}}\right.\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(f*x+e))^3/(d*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3/d^2*((d*tan(f*x+e))^(1/2)-d/(d*tan(f*x+e))^(1/2)+2*d*(1/8/d*(d^2)^(
1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^
2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2
)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/
(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))+1/8/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x
+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^
2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^
(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^
(1/2)+1))))
```

Maxima [A]

time = 0.50, size = 122, normalized size = 1.07

$$\frac{2\left(a^3\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}\right)-\frac{a^3}{\sqrt{d\tan(fx+e)}}+\frac{\sqrt{d\tan(fx+e)}a^3}{d}\right)}{df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))^3/(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] 2*(a^3*(sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(f*x + e)
))/sqrt(d))/sqrt(d) + sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*sqrt
(d*tan(f*x + e)))/sqrt(d))/sqrt(d)) - a^3/sqrt(d*tan(f*x + e)) + sqrt(d*tan
(f*x + e))*a^3/d)/(d*f)
```

Fricas [A]
time = 0.89, size = 239, normalized size = 2.10

$$\left[\frac{\sqrt{2}\, a^{3} d \sqrt{-\frac{1}{d}} \log\left(\frac{2 \sqrt{2}\, \sqrt{d \tan\left(f x + e\right)} \sqrt{-\frac{1}{d}} \left(\tan\left(f x + e\right) - 1\right) + \tan\left(f x + e\right)^{2} - 4 \tan\left(f x + e\right) + 1}{\tan\left(f x + e\right)^{2} + 1}\right) \tan\left(f x + e\right) + 2 \left(a^{3} \tan\left(f x + e\right) - a^{3}\right) \sqrt{d \tan\left(f x + e\right)}}{d^{2} f \tan\left(f x + e\right)}, \frac{2 \left(\sqrt{2}\, a^{3} \sqrt{d} \arctan\left(\frac{\sqrt{2}\, \sqrt{d \tan\left(f x + e\right)} \left(\tan\left(f x + e\right) - 1\right)}{2 \sqrt{d} \tan\left(f x + e\right)}\right) \tan\left(f x + e\right) + \left(a^{3} \tan\left(f x + e\right) - a^{3}\right) \sqrt{d \tan\left(f x + e\right)}\right)}{d^{2} f \tan\left(f x + e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))^3/(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(2)*a^3*d*sqrt(-1/d)*log((2*sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(-1/d)*(
tan(f*x + e) - 1) + tan(f*x + e)^2 - 4*tan(f*x + e) + 1)/(tan(f*x + e)^2 +
1))*tan(f*x + e) + 2*(a^3*tan(f*x + e) - a^3)*sqrt(d*tan(f*x + e)))/(d^2*f*
tan(f*x + e)), 2*(sqrt(2)*a^3*sqrt(d)*arctan(1/2*sqrt(2)*sqrt(d*tan(f*x + e
))*(tan(f*x + e) - 1)/(sqrt(d)*tan(f*x + e)))*tan(f*x + e) + (a^3*tan(f*x +
 e) - a^3)*sqrt(d*tan(f*x + e)))/(d^2*f*tan(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3} \left(\int \frac{1}{\left(d \tan{\left(e + f x \right)}\right)^{\frac{3}{2}}}\, dx + \int \frac{3 \tan{\left(e + f x \right)}}{\left(d \tan{\left(e + f x \right)}\right)^{\frac{3}{2}}}\, dx + \int \frac{3 \tan^{2}{\left(e + f x \right)}}{\left(d \tan{\left(e + f x \right)}\right)^{\frac{3}{2}}}\, dx + \int \frac{\tan^{3}{\left(e + f x \right)}}{\left(d \tan{\left(e + f x \right)}\right)^{\frac{3}{2}}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))**3/(d*tan(f*x+e))**(3/2),x)
```

```
[Out] a**3*(Integral((d*tan(e + f*x))**(-3/2), x) + Integral(3*tan(e + f*x)/(d*ta
n(e + f*x))**(3/2), x) + Integral(3*tan(e + f*x)**2/(d*tan(e + f*x))**(3/2)
, x) + Integral(tan(e + f*x)**3/(d*tan(e + f*x))**(3/2), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(105) = 210.
time = 0.73, size = 299, normalized size = 2.62

$$-\frac{\frac{4 a^{3}}{\sqrt{d \tan\left(f x + e\right)} f} - \frac{4 \sqrt{d \tan\left(f x + e\right)} a^{3}}{d f} - \frac{\sqrt{2} \left(a^{3} d \sqrt{|d|} - a^{3} |d|^{\frac{3}{2}}\right) \log\left(d \tan\left(f x + e\right) + \sqrt{2} \sqrt{d \tan\left(f x + e\right)} \sqrt{|d|} + |d|\right)}{d^{2} f} + \frac{\sqrt{2} \left(a^{3} d \sqrt{|d|} - a^{3} |d|^{\frac{3}{2}}\right) \log\left(d \tan\left(f x + e\right) - \sqrt{2} \sqrt{d \tan\left(f x + e\right)} \sqrt{|d|} + |d|\right)}{d^{2} f} - \frac{2 \left(\sqrt{2} a^{3} d \sqrt{|d|} + \sqrt{2} a^{3} |d|^{\frac{3}{2}}\right) \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \sqrt{|d|} + 2 \sqrt{d \tan\left(f x + e\right)}\right)}{2 \sqrt{|d|}}\right)}{d^{2} f} - \frac{2 \left(\sqrt{2} a^{3} d \sqrt{|d|} + \sqrt{2} a^{3} |d|^{\frac{3}{2}}\right) \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} \sqrt{|d|} - 2 \sqrt{d \tan\left(f x + e\right)}\right)}{2 \sqrt{|d|}}\right)}{d^{2} f}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))^3/(d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(4*a^3/(sqrt(d*tan(f*x + e))*f) - 4*sqrt(d*tan(f*x + e))*a^3/(d*f) - s
qrt(2)*(a^3*d*sqrt(abs(d)) - a^3*abs(d)^(3/2))*log(d*tan(f*x + e) + sqrt(2)
*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(d^2*f) + sqrt(2)*(a^3*d*sqrt(
abs(d)) - a^3*abs(d)^(3/2))*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e
))*sqrt(abs(d)) + abs(d))/(d^2*f) - 2*(sqrt(2)*a^3*d*sqrt(abs(d)) + sqrt(2)
*a^3*abs(d)^(3/2))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqrt(d*tan(
f*x + e)))/sqrt(abs(d)))/(d^2*f) - 2*(sqrt(2)*a^3*d*sqrt(abs(d)) + sqrt(2)*
a^3*abs(d)^(3/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(
f*x + e)))/sqrt(abs(d)))/(d^2*f))/d
```

Mupad [B]
time = 4.20, size = 118, normalized size = 1.04

$$\frac{2\,a^3\,\sqrt{d\tan\left(e+f\,x\right)}}{d^2\,f}-\frac{2\,a^3}{d\,f\,\sqrt{d\tan\left(e+f\,x\right)}}+\frac{\sqrt{2}\,a^3\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{d\tan\left(e+f\,x\right)}}{2\,\sqrt{d}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{d\tan\left(e+f\,x\right)}}{2\,\sqrt{d}}+\frac{\sqrt{2}\,{\left(d\tan\left(e+f\,x\right)\right)}^{3/2}}{2\,d^{3/2}}\right)\right)}{d^{3/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x))^3/(d*tan(e + f*x))^(3/2),x)
```

```
[Out] (2*a^3*(d*tan(e + f*x))^(1/2))/(d^2*f) - (2*a^3)/(d*f*(d*tan(e + f*x))^(1/2
)) + (2^(1/2)*a^3*(2*atan((2^(1/2)*(d*tan(e + f*x))^(1/2))/(2*d^(1/2))) + 2
*atan((2^(1/2)*(d*tan(e + f*x))^(1/2))/(2*d^(1/2)) + (2^(1/2)*(d*tan(e + f*
x))^(3/2))/(2*d^(3/2)))))/(d^(3/2)*f)
```

3.355 $\int \frac{(a+a\tan(e+fx))^3}{(d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=117

$$\frac{2\sqrt{2}\,a^3\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{d^{5/2}f}-\frac{16a^3}{3d^2f\sqrt{d\tan(e+fx)}}-\frac{2(a^3+a^3\tan(e+fx))}{3df(d\tan(e+fx))^{3/2}}$$

```
[Out] 2*a^3*arctanh(1/2*(d^(1/2)+d^(1/2)*tan(f*x+e))*2^(1/2)/(d*tan(f*x+e))^(1/2)
)*2^(1/2)/d^(5/2)/f-16/3*a^3/d^2/f/(d*tan(f*x+e))^(1/2)-2/3*(a^3+a^3*tan(f*
x+e))/d/f/(d*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.12, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3646, 3709, 3613, 214}

$$\frac{2\sqrt{2}\,a^3\tanh^{-1}\left(\frac{\sqrt{d}\,\tan(e+fx)+\sqrt{d}}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{d^{5/2}f}-\frac{16a^3}{3d^2f\sqrt{d\tan(e+fx)}}-\frac{2(a^3\tan(e+fx)+a^3)}{3df(d\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[e + f*x])^3/(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (2*Sqrt[2]*a^3*ArcTanh[(Sqrt[d] + Sqrt[d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[d*Tan
[e + f*x]])])/(d^(5/2)*f) - (16*a^3)/(3*d^2*f*Sqrt[d*Tan[e + f*x]]) - (2*(a
^3 + a^3*Tan[e + f*x]))/(3*d*f*(d*Tan[e + f*x])^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
```

```
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\tan(e+fx))^3}{(d\tan(e+fx))^{5/2}}\,dx = -\frac{2(a^3+a^3\tan(e+fx))}{3df(d\tan(e+fx))^{3/2}} + \frac{2\int \frac{4a^3d^2+3a^3d^2\tan(e+fx)+a^3d^2\tan^2(e+fx)}{(d\tan(e+fx))^{3/2}}\,dx}{3d^3}$$

$$= -\frac{16a^3}{3d^2f\sqrt{d\tan(e+fx)}} - \frac{2(a^3+a^3\tan(e+fx))}{3df(d\tan(e+fx))^{3/2}} + \frac{2\int \frac{3a^3d^3-3a^3d^3\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,d}{3d^5}$$

$$= -\frac{16a^3}{3d^2f\sqrt{d\tan(e+fx)}} - \frac{2(a^3+a^3\tan(e+fx))}{3df(d\tan(e+fx))^{3/2}} - \frac{(12a^6d)\,\text{Subst}\left(\int \frac{1}{-18a^6d^6}\right.}{}$$

$$= \frac{2\sqrt{2}\,a^3\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{d^{5/2}f} - \frac{16a^3}{3d^2f\sqrt{d\tan(e+fx)}} - \frac{2(a^3+a}{3df(d\,\text{ta}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 4.17, size = 272, normalized size = 2.32

$$\frac{a^3(1+\tan(e+fx))^3\left(-8\cos^2(e+fx)\,{}_2F_1\left(-\tfrac{3}{4},1;\tfrac{1}{4};-\tan^2(e+fx)\right)\sin(e+fx)-72\cos(e+fx)\,{}_2F_1\left(-\tfrac{1}{4},1;\tfrac{3}{4};-\tan^2(e+fx)\right)\sin^2(e+fx)+8\,{}_2F_1\left(\tfrac{3}{4},1;\tfrac{7}{4};-\tan^2(e+fx)\right)\sin^3(e+fx)\tan(e+fx)-9\sqrt{2}\cos^3(e+fx)\left(2\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)-2\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(e+fx)}\right)+\log\left(1-\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)-\log\left(1+\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)\right)\tan^{\frac{5}{2}}(e+fx)\right)}{12f(\cos(e+fx)+\sin(e+fx))^3(d\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[e + f*x])^3/(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (a^3*(1 + Tan[e + f*x])^3*(-8*Cos[e + f*x]^2*Hypergeometric2F1[-3/4, 1, 1/4
, -Tan[e + f*x]^2]*Sin[e + f*x] - 72*Cos[e + f*x]*Hypergeometric2F1[-1/4, 1
, 3/4, -Tan[e + f*x]^2]*Sin[e + f*x]^2 + 8*Hypergeometric2F1[3/4, 1, 7/4, -
Tan[e + f*x]^2]*Sin[e + f*x]^3*Tan[e + f*x] - 9*Sqrt[2]*Cos[e + f*x]^3*(2*A
rcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f
*x]]] + Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - Log[1 + Sqrt[2
]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]])*Tan[e + f*x]^(5/2)))/(12*f*(Cos[e + f
*x] + Sin[e + f*x])^3*(d*Tan[e + f*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(98) = 196$.
time = 0.21, size = 303, normalized size = 2.59

method	result
derivativedivides	$\frac{2a^3\left(-\frac{d}{3(d\tan(fx+e))^{\frac{3}{2}}}-\frac{3}{\sqrt{d\tan(fx+e)}}+\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\cdots}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\cdots}\right.\right.}{\cdots}\cdots\right.}{\cdots}$
default	$\frac{2a^3\left(-\frac{d}{3(d\tan(fx+e))^{\frac{3}{2}}}-\frac{3}{\sqrt{d\tan(fx+e)}}+\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\cdots}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\cdots}\right.\right.}{\cdots}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(f*x+e))^3/(d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3/d^2*(-1/3*d/(d*tan(f*x+e))^(3/2)-3/(d*tan(f*x+e))^(1/2)+1/4/d*(d^2)
^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(
d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1
/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2
)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))-1/4/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f
*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(
d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2
)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e)
)^(1/2)+1)))
```

Maxima [A]
time = 0.51, size = 129, normalized size = 1.10

$$\frac{\frac{3a^3\left(\frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)}{d}-\frac{2\left(9a^3d\tan(fx+e)+a^3d\right)}{(d\tan(fx+e))^{\frac{3}{2}}d}}{3df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))^3/(d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*a^3*(sqrt(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(
d) + d)/sqrt(d) - sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))
*sqrt(d) + d)/sqrt(d))/d - 2*(9*a^3*d*tan(f*x + e) + a^3*d)/((d*tan(f*x + e
))^(3/2)*d))/(d*f)
```

Fricas [A]
time = 1.03, size = 244, normalized size = 2.09

$$\left[\frac{3\sqrt{2}\,a^3\sqrt{d}\,\log\left(\frac{\tan(fx+e)^2+\frac{2\sqrt{2}\,\sqrt{d\tan(fx+e)}\,(\tan(fx+e)+1)}{\sqrt{d}}+4\,\tan(fx+e)+1}{\tan(fx+e)^2+1}\right)\tan(fx+e)^2-2\left(9\,a^3\tan(fx+e)+a^3\right)\sqrt{d\tan(fx+e)}}{3\,d^3f\tan(fx+e)^2},\ -\frac{2\left(3\sqrt{2}\,a^3d\sqrt{-\frac{1}{d}}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{d\tan(fx+e)}\,\sqrt{-\frac{1}{d}}\,(\tan(fx+e)+1)}{2\,\tan(fx+e)}\right)\tan(fx+e)^2+\left(9\,a^3\tan(fx+e)+a^3\right)\sqrt{d\tan(fx+e)}\right)}{3\,d^3f\tan(fx+e)^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))^3/(d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(2)*a^3*sqrt(d)*log((tan(f*x + e)^2 + 2*sqrt(2)*sqrt(d*tan(f*x
+ e))*(tan(f*x + e) + 1)/sqrt(d) + 4*tan(f*x + e) + 1)/(tan(f*x + e)^2 + 1)
)*tan(f*x + e)^2 - 2*(9*a^3*tan(f*x + e) + a^3)*sqrt(d*tan(f*x + e)))/(d^3*
f*tan(f*x + e)^2), -2/3*(3*sqrt(2)*a^3*d*sqrt(-1/d)*arctan(1/2*sqrt(2)*sqrt
(d*tan(f*x + e))*sqrt(-1/d)*(tan(f*x + e) + 1)/tan(f*x + e))*tan(f*x + e)^2
 + (9*a^3*tan(f*x + e) + a^3)*sqrt(d*tan(f*x + e)))/(d^3*f*tan(f*x + e)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int\frac{1}{(d\tan(e+fx))^{\frac{5}{2}}}\,dx+\int\frac{3\tan(e+fx)}{(d\tan(e+fx))^{\frac{5}{2}}}\,dx+\int\frac{3\tan^2(e+fx)}{(d\tan(e+fx))^{\frac{5}{2}}}\,dx+\int\frac{\tan^3(e+fx)}{(d\tan(e+fx))^{\frac{5}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))**3/(d*tan(f*x+e))**(5/2),x)
```

```
[Out] a**3*(Integral((d*tan(e + f*x))**(-5/2), x) + Integral(3*tan(e + f*x)/(d*ta
n(e + f*x))**(5/2), x) + Integral(3*tan(e + f*x)**2/(d*tan(e + f*x))**(5/2)
, x) + Integral(tan(e + f*x)**3/(d*tan(e + f*x))**(5/2), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(103) = 206.
time = 0.87, size = 299, normalized size = 2.56

$$\frac{\sqrt{2}\left(a^3d\sqrt{|d|}+a^3|d|^{\frac{3}{2}}\right)\log\left(d\tan(fx+e)+\sqrt{2}\,\sqrt{d\tan(fx+e)}\,\sqrt{|d|}+|d|\right)}{2\,d^4f}-\frac{\sqrt{2}\left(a^3d\sqrt{|d|}+a^3|d|^{\frac{3}{2}}\right)\log\left(d\tan(fx+e)-\sqrt{2}\,\sqrt{d\tan(fx+e)}\,\sqrt{|d|}+|d|\right)}{2\,d^4f}+\frac{\left(\sqrt{2}\,a^3d\sqrt{|d|}-\sqrt{2}\,a^3|d|^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{|d|}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{d^4f}+\frac{\left(\sqrt{2}\,a^3d\sqrt{|d|}-\sqrt{2}\,a^3|d|^{\frac{3}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{|d|}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{d^4f}-\frac{2\left(9\,a^3d\tan(fx+e)+a^3d\right)}{3\sqrt{d\tan(fx+e)}\,d^3f\tan(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))^3/(d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(a^3*d*sqrt(abs(d)) + a^3*abs(d)^(3/2))*log(d*tan(f*x + e) + sq
rt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(d^4*f) - 1/2*sqrt(2)*(a^
3*d*sqrt(abs(d)) + a^3*abs(d)^(3/2))*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*ta
n(f*x + e))*sqrt(abs(d)) + abs(d))/(d^4*f) + (sqrt(2)*a^3*d*sqrt(abs(d)) -
sqrt(2)*a^3*abs(d)^(3/2))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqrt
(d*tan(f*x + e)))/sqrt(abs(d)))/(d^4*f) + (sqrt(2)*a^3*d*sqrt(abs(d)) - sqr
t(2)*a^3*abs(d)^(3/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d
*tan(f*x + e)))/sqrt(abs(d)))/(d^4*f) - 2/3*(9*a^3*d*tan(f*x + e) + a^3*d)/
(sqrt(d*tan(f*x + e))*d^3*f*tan(f*x + e))
```

Mupad [B]
time = 4.32, size = 102, normalized size = 0.87

$$\frac{2\sqrt{2}\,a^3\,\mathrm{atanh}\left(\frac{32\sqrt{2}\,a^6\,d^{5/2}\,f\,\sqrt{d\tan\left(e+f\,x\right)}}{32\,a^6\,d^3\,f+32\,a^6\,d^3\,f\tan(e+f\,x)}\right)}{d^{5/2}\,f}-\frac{\frac{2\,a^3\,d}{3}+6\,a^3\,d\tan(e+f\,x)}{d^2\,f\,\left(d\tan\left(e+f\,x\right)\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x))^3/(d*tan(e + f*x))^(5/2),x)
```

```
[Out] (2*2^(1/2)*a^3*atanh((32*2^(1/2)*a^6*d^(5/2)*f*(d*tan(e + f*x))^(1/2))/(32*
a^6*d^3*f + 32*a^6*d^3*f*tan(e + f*x))))/(d^(5/2)*f) - ((2*a^3*d)/3 + 6*a^3
*d*tan(e + f*x))/(d^2*f*(d*tan(e + f*x))^(3/2))
```

3.356 $\int \frac{(a+a\tan(e+fx))^3}{(d\tan(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=141

$$\frac{2\sqrt{2}\,a^3\mathrm{ArcTan}\left(\frac{\sqrt{d}-\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{d^{7/2}f}-\frac{8a^3}{5d^2f(d\tan(e+fx))^{3/2}}-\frac{4a^3}{d^3f\sqrt{d\tan(e+fx)}}-\frac{2(a^3+a^3\tan(e+ \ldots}{5df(d\tan(e+f \ldots}$$

```
[Out] 2*a^3*arctan(1/2*(d^(1/2)-d^(1/2)*tan(f*x+e))*2^(1/2)/(d*tan(f*x+e))^(1/2))
*2^(1/2)/d^(7/2)/f-4*a^3/d^3/f/(d*tan(f*x+e))^(1/2)-8/5*a^3/d^2/f/(d*tan(f*
x+e))^(3/2)-2/5*(a^3+a^3*tan(f*x+e))/d/f/(d*tan(f*x+e))^(5/2)
```

Rubi [A]
time = 0.15, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3646, 3709, 3610, 3613, 211}

$$\frac{2\sqrt{2}\,a^3\mathrm{ArcTan}\left(\frac{\sqrt{d}-\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{d^{7/2}f}-\frac{4a^3}{d^3f\sqrt{d\tan(e+fx)}}-\frac{8a^3}{5d^2f(d\tan(e+fx))^{3/2}}-\frac{2(a^3\tan(e+fx)+a^3)}{5df(d\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[e + f*x])^3/(d*Tan[e + f*x])^(7/2),x]
```

```
[Out] (2*Sqrt[2]*a^3*ArcTan[(Sqrt[d] - Sqrt[d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[d*Tan[
e + f*x]])])/(d^(7/2)*f) - (8*a^3)/(5*d^2*f*(d*Tan[e + f*x])^(3/2)) - (4*a^
3)/(d^3*f*Sqrt[d*Tan[e + f*x]]) - (2*(a^3 + a^3*Tan[e + f*x]))/(5*d*f*(d*Ta
n[e + f*x])^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
```

```
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\tan(e+fx))^3}{(d\tan(e+fx))^{7/2}}\,dx = -\frac{2(a^3+a^3\tan(e+fx))}{5df(d\tan(e+fx))^{5/2}}+\frac{2\int \frac{6a^3d^2+5a^3d^2\tan(e+fx)+a^3d^2\tan^2(e+fx)}{(d\tan(e+fx))^{5/2}}\,dx}{5d^3}$$

$$= -\frac{8a^3}{5d^2f(d\tan(e+fx))^{3/2}}-\frac{2(a^3+a^3\tan(e+fx))}{5df(d\tan(e+fx))^{5/2}}+\frac{2\int \frac{5a^3d^3-5a^3d^3\tan(e+fx)}{(d\tan(e+fx))^{3/2}}\,dx}{5d^5}$$

$$= -\frac{8a^3}{5d^2f(d\tan(e+fx))^{3/2}}-\frac{4a^3}{d^3f\sqrt{d\tan(e+fx)}}-\frac{2(a^3+a^3\tan(e+fx))}{5df(d\tan(e+fx))^{5/2}}+$$

$$= -\frac{8a^3}{5d^2f(d\tan(e+fx))^{3/2}}-\frac{4a^3}{d^3f\sqrt{d\tan(e+fx)}}-\frac{2(a^3+a^3\tan(e+fx))}{5df(d\tan(e+fx))^{5/2}}-$$

$$= \frac{2\sqrt{2}\,a^3\tan^{-1}\left(\frac{\sqrt{d}-\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{d^{7/2}f}-\frac{8a^3}{5d^2f(d\tan(e+fx))^{3/2}}-\frac{4}{d^3f\sqrt{d\tan}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 2.26, size = 271, normalized size = 1.92

$$-\frac{a^3(1+\cot(e+fx))^3\left(8\cos^2(e+fx)\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\tan^2(e+fx)\right)\sin(e+fx)+5\left(8\cos(e+fx)\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\tan^2(e+fx)\right)\sin^2(e+fx)+24\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\tan^2(e+fx)\right)\sin^3(e+fx)+\sqrt{2}\cos^3(e+fx)\left(2\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)-2\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(e+fx)}\right)+\log\left(1-\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)-\log\left(1+\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)\right)\tan^{\frac{7}{2}}(e+fx)\right)\right)}{20d^3f(\cos(e+fx)+\sin(e+fx))^3\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[e + f*x])^3/(d*Tan[e + f*x])^(7/2),x]
```

```
[Out] -1/20*(a^3*(1 + Cot[e + f*x])^3*(8*Cos[e + f*x]^2*Hypergeometric2F1[-5/4, 1
, -1/4, -Tan[e + f*x]^2]*Sin[e + f*x] + 5*(8*Cos[e + f*x]*Hypergeometric2F1
[-3/4, 1, 1/4, -Tan[e + f*x]^2]*Sin[e + f*x]^2 + 24*Hypergeometric2F1[-1/4,
 1, 3/4, -Tan[e + f*x]^2]*Sin[e + f*x]^3 + Sqrt[2]*Cos[e + f*x]^3*(2*ArcTan
[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]]
 + Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - Log[1 + Sqrt[2]*Sqr
t[Tan[e + f*x]] + Tan[e + f*x]])*Tan[e + f*x]^(7/2))))/(d^3*f*(Cos[e + f*x]
 + Sin[e + f*x])^3*Sqrt[d*Tan[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(120) = 240$.
time = 0.21, size = 323, normalized size = 2.29

method	result
derivativedivides	$2a^3\left(-\frac{d}{5(d\tan(fx+e))^{\frac{5}{2}}}-\frac{1}{(d\tan(fx+e))^{\frac{3}{2}}}-\frac{2}{d\sqrt{d\tan(fx+e)}}+\frac{-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan}}\right.\right.}{}}{}\right.$
default	$2a^3\left(-\frac{d}{5(d\tan(fx+e))^{\frac{5}{2}}}-\frac{1}{(d\tan(fx+e))^{\frac{3}{2}}}-\frac{2}{d\sqrt{d\tan(fx+e)}}+\frac{-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan}}\right.\right.}{}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(f*x+e))^3/(d*tan(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3/d^2*(-1/5*d/(d*tan(f*x+e))^(5/2)-1/(d*tan(f*x+e))^(3/2)-2/d/(d*tan(
f*x+e))^(1/2)+1/d*(-1/4/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)
*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan
```

```
(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*
x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))-1/4/(
d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/
2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2
)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^
(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))))
```

Maxima [A]
time = 0.53, size = 147, normalized size = 1.04

$$-\frac{2\left(\frac{5\,a^3\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}\right)}{d^2}+\frac{10\,a^3d^2\tan\left(fx+e\right)^2+5\,a^3d^2\tan\left(fx+e\right)+a^3d^2}{\left(d\tan\left(fx+e\right)\right)^{\frac{5}{2}}d^2}\right)}{5\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))^3/(d*tan(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] -2/5*(5*a^3*(sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(f*x
 + e)))/sqrt(d))/sqrt(d) + sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2
*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d))/d^2 + (10*a^3*d^2*tan(f*x + e)^2 +
 5*a^3*d^2*tan(f*x + e) + a^3*d^2)/((d*tan(f*x + e))^(5/2)*d^2))/(d*f)
```

Fricas [A]
time = 0.91, size = 275, normalized size = 1.95

$$\left[\frac{5\sqrt{2}\,a^3d\sqrt{-\frac{1}{d}}\log\left(-\frac{2\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{-\frac{1}{d}}\left(\tan\left(fx+e\right)-1\right)-\tan\left(fx+e\right)^2+4\tan\left(fx+e\right)-1}{\tan\left(fx+e\right)^2+1}\right)\tan\left(fx+e\right)^3-2\left(10\,a^3\tan\left(fx+e\right)^2+5\,a^3\tan\left(fx+e\right)+a^3\right)\sqrt{d\tan\left(fx+e\right)}}{5\,d^4f\tan\left(fx+e\right)^3},\ -\frac{2\left(5\sqrt{2}\,a^3\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\left(\tan\left(fx+e\right)-1\right)}{2\sqrt{d}\tan\left(fx+e\right)}\right)\tan\left(fx+e\right)^3+\left(10\,a^3\tan\left(fx+e\right)^2+5\,a^3\tan\left(fx+e\right)+a^3\right)\sqrt{d\tan\left(fx+e\right)}\right)}{5\,d^4f\tan\left(fx+e\right)^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))^3/(d*tan(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] [1/5*(5*sqrt(2)*a^3*d*sqrt(-1/d)*log(-(2*sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(
-1/d)*(tan(f*x + e) - 1) - tan(f*x + e)^2 + 4*tan(f*x + e) - 1)/(tan(f*x +
e)^2 + 1))*tan(f*x + e)^3 - 2*(10*a^3*tan(f*x + e)^2 + 5*a^3*tan(f*x + e) +
 a^3)*sqrt(d*tan(f*x + e)))/(d^4*f*tan(f*x + e)^3), -2/5*(5*sqrt(2)*a^3*sqr
t(d)*arctan(1/2*sqrt(2)*sqrt(d*tan(f*x + e))*(tan(f*x + e) - 1)/(sqrt(d)*ta
n(f*x + e)))*tan(f*x + e)^3 + (10*a^3*tan(f*x + e)^2 + 5*a^3*tan(f*x + e) +
 a^3)*sqrt(d*tan(f*x + e)))/(d^4*f*tan(f*x + e)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int \frac{1}{\left(d\tan\left(e+fx\right)\right)^{\frac{7}{2}}}\,dx+\int \frac{3\tan\left(e+fx\right)}{\left(d\tan\left(e+fx\right)\right)^{\frac{7}{2}}}\,dx+\int \frac{3\tan^2\left(e+fx\right)}{\left(d\tan\left(e+fx\right)\right)^{\frac{7}{2}}}\,dx+\int \frac{\tan^3\left(e+fx\right)}{\left(d\tan\left(e+fx\right)\right)^{\frac{7}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))**3/(d*tan(f*x+e))**(7/2),x)
```

```
[Out] a**3*(Integral((d*tan(e + f*x))**(-7/2), x) + Integral(3*tan(e + f*x)/(d*ta
n(e + f*x))**(7/2), x) + Integral(3*tan(e + f*x)**2/(d*tan(e + f*x))**(7/2)
, x) + Integral(tan(e + f*x)**3/(d*tan(e + f*x))**(7/2), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(127) = 254.
time = 0.85, size = 322, normalized size = 2.28

$$-\frac{\sqrt{2}\left(a^3d\sqrt{|d|}-a^3|d|^{\frac{3}{2}}\right)\log\left(d\tan\left(fx+e\right)+\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{|d|}+|d|\right)}{2\,d^5f}+\frac{\sqrt{2}\left(a^3d\sqrt{|d|}-a^3|d|^{\frac{3}{2}}\right)\log\left(d\tan\left(fx+e\right)-\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{|d|}+|d|\right)}{2\,d^5f}-\frac{\left(\sqrt{2}a^3d\sqrt{|d|}+\sqrt{2}a^3|d|^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{d^5f}-\frac{\left(\sqrt{2}a^3d\sqrt{|d|}+\sqrt{2}a^3|d|^{\frac{3}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{d^5f}-\frac{2\left(10\,a^3d^2\tan\left(fx+e\right)^2+5\,a^3d^2\tan\left(fx+e\right)+a^3d^2\right)}{5\sqrt{d\tan\left(fx+e\right)}\,d^5f\tan\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))^3/(d*tan(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(a^3*d*sqrt(abs(d)) - a^3*abs(d)^(3/2))*log(d*tan(f*x + e) + s
qrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(d^5*f) + 1/2*sqrt(2)*(a
^3*d*sqrt(abs(d)) - a^3*abs(d)^(3/2))*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*t
an(f*x + e))*sqrt(abs(d)) + abs(d))/(d^5*f) - (sqrt(2)*a^3*d*sqrt(abs(d)) +
 sqrt(2)*a^3*abs(d)^(3/2))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqr
t(d*tan(f*x + e)))/sqrt(abs(d)))/(d^5*f) - (sqrt(2)*a^3*d*sqrt(abs(d)) + sq
rt(2)*a^3*abs(d)^(3/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(
d*tan(f*x + e)))/sqrt(abs(d)))/(d^5*f) - 2/5*(10*a^3*d^2*tan(f*x + e)^2 + 5
*a^3*d^2*tan(f*x + e) + a^3*d^2)/(sqrt(d*tan(f*x + e))*d^5*f*tan(f*x + e)^2
)
```

Mupad [B]
time = 5.08, size = 128, normalized size = 0.91

$$-\frac{4\,d\,a^3\,{\mathrm{tan}\left(e+f\,x\right)}^2+2\,d\,a^3\,\mathrm{tan}\left(e+f\,x\right)+\frac{2\,d\,a^3}{5}}{d^2\,f\,{\left(d\,\mathrm{tan}\left(e+f\,x\right)\right)}^{5/2}}-\frac{\sqrt{2}\,a^3\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{d\,\mathrm{tan}\left(e+f\,x\right)}}{2\,\sqrt{d}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{d\,\mathrm{tan}\left(e+f\,x\right)}}{2\,\sqrt{d}}+\frac{\sqrt{2}\,{\left(d\,\mathrm{tan}\left(e+f\,x\right)\right)}^{3/2}}{2\,d^{3/2}}\right)\right)}{d^{7/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x))^3/(d*tan(e + f*x))^(7/2),x)
```

```
[Out] - ((2*a^3*d)/5 + 4*a^3*d*tan(e + f*x)^2 + 2*a^3*d*tan(e + f*x))/(d^2*f*(d*t
an(e + f*x))^(5/2)) - (2^(1/2)*a^3*(2*atan((2^(1/2)*(d*tan(e + f*x))^(1/2))
/(2*d^(1/2))) + 2*atan((2^(1/2)*(d*tan(e + f*x))^(1/2))/(2*d^(1/2)) + (2^(1
/2)*(d*tan(e + f*x))^(3/2))/(2*d^(3/2)))))/(d^(7/2)*f)
```

3.357 $\int \frac{(a+a\tan(e+fx))^3}{(d\tan(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=165

$$-\frac{2\sqrt{2}\,a^3\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{d^{9/2}f}-\frac{32a^3}{35d^2f(d\tan(e+fx))^{5/2}}-\frac{4a^3}{3d^3f(d\tan(e+fx))^{3/2}}+\frac{4a^3}{d^4f\sqrt{d\tan(e+fx)}}$$

```
[Out] -2*a^3*arctanh(1/2*(d^(1/2)+d^(1/2)*tan(f*x+e))*2^(1/2)/(d*tan(f*x+e))^(1/2
))*2^(1/2)/d^(9/2)/f+4*a^3/d^4/f/(d*tan(f*x+e))^(1/2)-32/35*a^3/d^2/f/(d*ta
n(f*x+e))^(5/2)-4/3*a^3/d^3/f/(d*tan(f*x+e))^(3/2)-2/7*(a^3+a^3*tan(f*x+e))
/d/f/(d*tan(f*x+e))^(7/2)
```

Rubi [A]
time = 0.19, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3646, 3709, 3610, 3613, 214}

$$-\frac{2\sqrt{2}\,a^3\tanh^{-1}\left(\frac{\sqrt{d}\,\tan(e+fx)+\sqrt{d}}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{d^{9/2}f}+\frac{4a^3}{d^4f\sqrt{d\tan(e+fx)}}-\frac{4a^3}{3d^3f(d\tan(e+fx))^{3/2}}-\frac{32a^3}{35d^2f(d\tan(e+fx))^{5/2}}-\frac{2(a^3\tan(e+fx)+a^3)}{7df(d\tan(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[e + f*x])^3/(d*Tan[e + f*x])^(9/2),x]
```

```
[Out] (-2*Sqrt[2]*a^3*ArcTanh[(Sqrt[d] + Sqrt[d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[d*Ta
n[e + f*x]])])/(d^(9/2)*f) - (32*a^3)/(35*d^2*f*(d*Tan[e + f*x])^(5/2)) - (
4*a^3)/(3*d^3*f*(d*Tan[e + f*x])^(3/2)) + (4*a^3)/(d^4*f*Sqrt[d*Tan[e + f*x
]]) - (2*(a^3 + a^3*Tan[e + f*x]))/(7*d*f*(d*Tan[e + f*x])^(7/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
```

```
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\tan(e+fx))^3}{(d\tan(e+fx))^{9/2}}\,dx = -\frac{2(a^3+a^3\tan(e+fx))}{7df(d\tan(e+fx))^{7/2}} + \frac{2\int \frac{8a^3d^2+7a^3d^2\tan(e+fx)+a^3d^2\tan^2(e+fx)}{(d\tan(e+fx))^{7/2}}dx}{7d^3}$$

$$= -\frac{32a^3}{35d^2f(d\tan(e+fx))^{5/2}} - \frac{2(a^3+a^3\tan(e+fx))}{7df(d\tan(e+fx))^{7/2}} + \frac{2\int \frac{7a^3d^3-7a^3d^3\tan(e+fx)}{(d\tan(e+fx))^{5/2}}}{7d^5}$$

$$= -\frac{32a^3}{35d^2f(d\tan(e+fx))^{5/2}} - \frac{4a^3}{3d^3f(d\tan(e+fx))^{3/2}} - \frac{2(a^3+a^3\tan(e+fx))}{7df(d\tan(e+fx))^{7/2}}$$

$$= -\frac{32a^3}{35d^2f(d\tan(e+fx))^{5/2}} - \frac{4a^3}{3d^3f(d\tan(e+fx))^{3/2}} + \frac{4a^3}{d^4f\sqrt{d\tan(e+fx)}} -$$

$$= -\frac{32a^3}{35d^2f(d\tan(e+fx))^{5/2}} - \frac{4a^3}{3d^3f(d\tan(e+fx))^{3/2}} + \frac{4a^3}{d^4f\sqrt{d\tan(e+fx)}} -$$

$$= -\frac{2\sqrt{2}\,a^3\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{d^{9/2}f} - \frac{32a^3}{35d^2f(d\tan(e+fx))^{5/2}} - \frac{}{3d^3f(}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.90, size = 175, normalized size = 1.06

$$-\frac{a^3\cos(e+fx)(1+\cot(e+fx))^3\left(10\cos^2(e+fx)\cot(e+fx)\,{}_2F_1\left(-\frac{7}{4},1;-\frac{3}{4};-\tan^2(e+fx)\right)+42\cos^2(e+fx)\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\tan^2(e+fx)\right)+70\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\tan^2(e+fx)\right)\sin^2(e+fx)+35\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\tan^2(e+fx)\right)\sin(2(e+fx))\right)\sqrt{d\tan(e+fx)}}{35d^5f(\cos(e+fx)+\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[e + f*x])^3/(d*Tan[e + f*x])^(9/2),x]
```

```
[Out] -1/35*(a^3*Cos[e + f*x]*(1 + Cot[e + f*x])^3*(10*Cos[e + f*x]^2*Cot[e + f*x
]*Hypergeometric2F1[-7/4, 1, -3/4, -Tan[e + f*x]^2] + 42*Cos[e + f*x]^2*Hyp
ergeometric2F1[-5/4, 1, -1/4, -Tan[e + f*x]^2] + 70*Hypergeometric2F1[-1/4,
 1, 3/4, -Tan[e + f*x]^2]*Sin[e + f*x]^2 + 35*Hypergeometric2F1[-3/4, 1, 1/
4, -Tan[e + f*x]^2]*Sin[2*(e + f*x)])*Sqrt[d*Tan[e + f*x]])/(d^5*f*(Cos[e +
 f*x] + Sin[e + f*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(140) = 280.
time = 0.20, size = 338, normalized size = 2.05

method	result

derivativedivides	$\frac{2a^3\left(-\frac{d}{7(d\tan(fx+e))^{\frac{7}{2}}}-\frac{3}{5(d\tan(fx+e))^{\frac{5}{2}}}+\frac{2}{d^2\sqrt{d\tan(fx+e)}}-\frac{2}{3d(d\tan(fx+e))^{\frac{3}{2}}}+\frac{-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+}{d\tan(fx+}\right.\right.}{}}{}\right.}{}$
default	$\frac{2a^3\left(-\frac{d}{7(d\tan(fx+e))^{\frac{7}{2}}}-\frac{3}{5(d\tan(fx+e))^{\frac{5}{2}}}+\frac{2}{d^2\sqrt{d\tan(fx+e)}}-\frac{2}{3d(d\tan(fx+e))^{\frac{3}{2}}}+\frac{-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+}{d\tan(fx+}\right.\right.}{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(f*x+e))^3/(d*tan(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3/d^2*(-1/7*d/(d*tan(f*x+e))^(7/2)-3/5/(d*tan(f*x+e))^(5/2)+2/d^2/(d*
tan(f*x+e))^(1/2)-2/3/d/(d*tan(f*x+e))^(3/2)+1/d^2*(-1/4/d*(d^2)^(1/4)*2^(1
/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))
/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arc
tan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/
4)*(d*tan(f*x+e))^(1/2)+1))+1/4/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)
^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*
(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*
tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))
))
```

Maxima [A]
time = 0.53, size = 168, normalized size = 1.02

$$-\frac{\frac{105\,a^3\left(\frac{\sqrt{2}\,\log\left(d\tan(fx+e)+\sqrt{2}\,\sqrt{d\tan(fx+e)}\,\sqrt{d}+d\right)}{\sqrt{d}}-\frac{\sqrt{2}\,\log\left(d\tan(fx+e)-\sqrt{2}\,\sqrt{d\tan(fx+e)}\,\sqrt{d}+d\right)}{\sqrt{d}}\right)}{d^3}-\frac{2\left(210\,a^3d^3\tan(fx+e)^3-70\,a^3d^3\tan(fx+e)^2-63\,a^3d^3\tan(fx+e)-15\,a^3d^3\right)}{(d\tan(fx+e))^{\frac{7}{2}}d^3}}{105\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))^3/(d*tan(f*x+e))^(9/2),x, algorithm="maxima")
```

```
[Out] -1/105*(105*a^3*(sqrt(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*
sqrt(d) + d)/sqrt(d) - sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x
+ e))*sqrt(d) + d)/sqrt(d))/d^3 - 2*(210*a^3*d^3*tan(f*x + e)^3 - 70*a^3*d^
3*tan(f*x + e)^2 - 63*a^3*d^3*tan(f*x + e) - 15*a^3*d^3)/((d*tan(f*x + e))^
(7/2)*d^3))/(d*f)
```

Fricas [A]
time = 1.01, size = 304, normalized size = 1.84

$$\left[\frac{105\,\sqrt{2}\,a^{3}\sqrt{d}\,\log\left(\frac{\tan\left(fx+e\right)^{2}-\frac{2\,\sqrt{2}\,\sqrt{d\tan\left(fx+e\right)}\,\left(\tan\left(fx+e\right)+1\right)}{\sqrt{d}}+4\,\tan\left(fx+e\right)+1}{\tan\left(fx+e\right)^{2}+1}\right)\tan\left(fx+e\right)^{4}+2\,\left(210\,a^{3}\tan\left(fx+e\right)^{3}-70\,a^{3}\tan\left(fx+e\right)^{2}-63\,a^{3}\tan\left(fx+e\right)-15\,a^{3}\right)\sqrt{d\tan\left(fx+e\right)}}{105\,d^{5}f\tan\left(fx+e\right)^{4}},\ \frac{2\,\left(105\,\sqrt{2}\,a^{3}d\sqrt{-\frac{1}{d}}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{-\frac{1}{d}}\,\left(\tan\left(fx+e\right)+1\right)}{2\,\tan\left(fx+e\right)}\right)\tan\left(fx+e\right)^{4}+\left(210\,a^{3}\tan\left(fx+e\right)^{3}-70\,a^{3}\tan\left(fx+e\right)^{2}-63\,a^{3}\tan\left(fx+e\right)-15\,a^{3}\right)\sqrt{d\tan\left(fx+e\right)}\right)}{105\,d^{5}f\tan\left(fx+e\right)^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))^3/(d*tan(f*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] [1/105*(105*sqrt(2)*a^3*sqrt(d)*log((tan(f*x + e)^2 - 2*sqrt(2)*sqrt(d*tan(
f*x + e))*(tan(f*x + e) + 1)/sqrt(d) + 4*tan(f*x + e) + 1)/(tan(f*x + e)^2
+ 1))*tan(f*x + e)^4 + 2*(210*a^3*tan(f*x + e)^3 - 70*a^3*tan(f*x + e)^2 -
63*a^3*tan(f*x + e) - 15*a^3)*sqrt(d*tan(f*x + e)))/(d^5*f*tan(f*x + e)^4),
 2/105*(105*sqrt(2)*a^3*d*sqrt(-1/d)*arctan(1/2*sqrt(2)*sqrt(d*tan(f*x + e)
)*sqrt(-1/d)*(tan(f*x + e) + 1)/tan(f*x + e))*tan(f*x + e)^4 + (210*a^3*tan
(f*x + e)^3 - 70*a^3*tan(f*x + e)^2 - 63*a^3*tan(f*x + e) - 15*a^3)*sqrt(d*
tan(f*x + e)))/(d^5*f*tan(f*x + e)^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int\frac{1}{\left(d\tan\left(e+fx\right)\right)^{\frac{9}{2}}}\,dx+\int\frac{3\tan\left(e+fx\right)}{\left(d\tan\left(e+fx\right)\right)^{\frac{9}{2}}}\,dx+\int\frac{3\tan^{2}\left(e+fx\right)}{\left(d\tan\left(e+fx\right)\right)^{\frac{9}{2}}}\,dx+\int\frac{\tan^{3}\left(e+fx\right)}{\left(d\tan\left(e+fx\right)\right)^{\frac{9}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))**3/(d*tan(f*x+e))**(9/2),x)
```

```
[Out] a**3*(Integral((d*tan(e + f*x))**(-9/2), x) + Integral(3*tan(e + f*x)/(d*ta
n(e + f*x))**(9/2), x) + Integral(3*tan(e + f*x)**2/(d*tan(e + f*x))**(9/2)
, x) + Integral(tan(e + f*x)**3/(d*tan(e + f*x))**(9/2), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 340 vs. 2(147) = 294.
time = 0.97, size = 340, normalized size = 2.06

$$-\frac{\sqrt{2}\,\left(a^{3}d\sqrt{|d|}+a^{3}|d|^{\frac{3}{2}}\right)\log\left(d\tan\left(fx+e\right)+\sqrt{2}\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{|d|}+|d|\right)}{2\,d^{6}f}+\frac{\sqrt{2}\,\left(a^{3}d\sqrt{|d|}+a^{3}|d|^{\frac{3}{2}}\right)\log\left(d\tan\left(fx+e\right)-\sqrt{2}\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{|d|}+|d|\right)}{2\,d^{6}f}-\frac{\left(\sqrt{2}\,a^{3}d\sqrt{|d|}-\sqrt{2}\,a^{3}|d|^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{2}\,\left(\sqrt{2}\,\sqrt{|d|}+2\,\sqrt{d\tan\left(fx+e\right)}\right)}{2\,\sqrt{|d|}}\right)}{d^{6}f}-\frac{\left(\sqrt{2}\,a^{3}d\sqrt{|d|}-\sqrt{2}\,a^{3}|d|^{\frac{3}{2}}\right)\arctan\left(-\frac{\sqrt{2}\,\left(\sqrt{2}\,\sqrt{|d|}-2\,\sqrt{d\tan\left(fx+e\right)}\right)}{2\,\sqrt{|d|}}\right)}{d^{6}f}+\frac{2\,\left(210\,a^{3}d^{3}\tan\left(fx+e\right)^{3}-70\,a^{3}d^{3}\tan\left(fx+e\right)^{2}-63\,a^{3}d^{3}\tan\left(fx+e\right)-15\,a^{3}d^{3}\right)}{105\,\sqrt{d\tan\left(fx+e\right)}\,d^{8}f\tan\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(f*x+e))^3/(d*tan(f*x+e))^(9/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(a^3*d*sqrt(abs(d)) + a^3*abs(d)^(3/2))*log(d*tan(f*x + e) + s
qrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(d^6*f) + 1/2*sqrt(2)*(a
^3*d*sqrt(abs(d)) + a^3*abs(d)^(3/2))*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*t
an(f*x + e))*sqrt(abs(d)) + abs(d))/(d^6*f) - (sqrt(2)*a^3*d*sqrt(abs(d)) -
 sqrt(2)*a^3*abs(d)^(3/2))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqr
```

```
t(d*tan(f*x + e)))/sqrt(abs(d)))/(d^6*f) - (sqrt(2)*a^3*d*sqrt(abs(d)) - sq
rt(2)*a^3*abs(d)^(3/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(
d*tan(f*x + e)))/sqrt(abs(d)))/(d^6*f) + 2/105*(210*a^3*d^3*tan(f*x + e)^3
- 70*a^3*d^3*tan(f*x + e)^2 - 63*a^3*d^3*tan(f*x + e) - 15*a^3*d^3)/(sqrt(d
*tan(f*x + e))*d^7*f*tan(f*x + e)^3)
```

Mupad [B]
time = 5.75, size = 130, normalized size = 0.79

$$-\frac{-4\,d\,a^3\,\tan(e+f\,x)^3+\frac{4\,d\,a^3\,\tan(e+f\,x)^2}{3}+\frac{6\,d\,a^3\,\tan(e+f\,x)}{5}+\frac{2\,d\,a^3}{7}}{d^2\,f\,\left(d\tan\left(e+f\,x\right)\right)^{7/2}}-\frac{2\,\sqrt{2}\,a^3\,\mathrm{atanh}\left(\frac{32\,\sqrt{2}\,a^6\,d^{9/2}\,f\,\sqrt{d\tan\left(e+f\,x\right)}}{32\,a^6\,d^5\,f+32\,a^6\,d^5\,f\tan(e+f\,x)}\right)}{d^{9/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x))^3/(d*tan(e + f*x))^(9/2),x)
```

```
[Out] - ((2*a^3*d)/7 + (4*a^3*d*tan(e + f*x)^2)/3 - 4*a^3*d*tan(e + f*x)^3 + (6*a
^3*d*tan(e + f*x))/5)/(d^2*f*(d*tan(e + f*x))^(7/2)) - (2*2^(1/2)*a^3*atanh
((32*2^(1/2)*a^6*d^(9/2)*f*(d*tan(e + f*x))^(1/2))/(32*a^6*d^5*f + 32*a^6*d
^5*f*tan(e + f*x))))/(d^(9/2)*f)
```

3.358 $\int \frac{(d\tan(e+fx))^{5/2}}{a+a\tan(e+fx)}\,dx$

Optimal. Leaf size=111

$$-\frac{d^{5/2}\text{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{af}+\frac{d^{5/2}\text{ArcTan}\left(\frac{\sqrt{d}-\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{\sqrt{2}\,af}+\frac{2d^2\sqrt{d\tan(e+fx)}}{af}$$

```
[Out] -d^(5/2)*arctan((d*tan(f*x+e))^(1/2)/d^(1/2))/a/f+1/2*d^(5/2)*arctan(1/2*(d
^(1/2)-d^(1/2)*tan(f*x+e))*2^(1/2)/(d*tan(f*x+e))^(1/2))/a/f*2^(1/2)+2*d^2*
(d*tan(f*x+e))^(1/2)/a/f
```

Rubi [A]
time = 0.27, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3647, 3734, 3613, 211, 3715, 65}

$$-\frac{d^{5/2}\text{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{af}+\frac{d^{5/2}\text{ArcTan}\left(\frac{\sqrt{d}-\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{\sqrt{2}\,af}+\frac{2d^2\sqrt{d\tan(e+fx)}}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(5/2)/(a + a*Tan[e + f*x]),x]
```

```
[Out] -((d^(5/2)*ArcTan[Sqrt[d*Tan[e + f*x]]/Sqrt[d]])/(a*f)) + (d^(5/2)*ArcTan[(
Sqrt[d] - Sqrt[d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[d*Tan[e + f*x]])])/(Sqrt[2]*a
*f) + (2*d^2*Sqrt[d*Tan[e + f*x]])/(a*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
```

```
EqQ[c^2 - d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^{5/2}}{a+a\tan(e+fx)}\,dx = \frac{2d^2\sqrt{d\tan(e+fx)}}{af} + \frac{2\int \frac{-\frac{ad^3}{2}-\frac{1}{2}ad^3\tan(e+fx)-\frac{1}{2}ad^3\tan^2(e+fx)}{\sqrt{d\tan(e+fx)}(a+a\tan(e+fx))}dx}{a}$$

$$= \frac{2d^2\sqrt{d\tan(e+fx)}}{af} + \frac{\int \frac{-\frac{1}{2}a^2d^3-\frac{1}{2}a^2d^3\tan(e+fx)}{\sqrt{d\tan(e+fx)}}dx}{a^3} - \frac{1}{2}d^3\int \frac{1+\tan^2(e}{\sqrt{d\tan(e+fx)}(a+}$$

$$= \frac{2d^2\sqrt{d\tan(e+fx)}}{af} - \frac{d^3\text{Subst}\left(\int \frac{1}{\sqrt{dx}(a+ax)}dx, x, \tan(e+fx)\right)}{2f} - \frac{(ad^6)\,\text{Subs}}{}$$

$$= \frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}-\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{\sqrt{2}af} + \frac{2d^2\sqrt{d\tan(e+fx)}}{af} - \frac{d^2\text{Subst}\left(\int \frac{1}{a+\frac{ax^2}{d}}\right.}{}$$

$$= -\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{af} + \frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}-\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{\sqrt{2}af} + \frac{2d^2\sqrt{}}{}$$

Mathematica [A]
time = 0.96, size = 110, normalized size = 0.99

$$\frac{\left(\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)-\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(e+fx)}\right)-2\text{ArcTan}\left(\sqrt{\tan(e+fx)}\right)+4\sqrt{\tan(e+fx)}\right)(d\tan(e+fx))^{5/2}}{2af\tan^{\frac{5}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(5/2)/(a + a*Tan[e + f*x]),x]
```

```
[Out] ((Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] - Sqrt[2]*ArcTan[1 + Sqrt[
2]*Sqrt[Tan[e + f*x]]] - 2*ArcTan[Sqrt[Tan[e + f*x]]] + 4*Sqrt[Tan[e + f*x]
])*(d*Tan[e + f*x])^(5/2))/(2*a*f*Tan[e + f*x]^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(93) = 186.
time = 0.31, size = 312, normalized size = 2.81

method	result

derivativedivides	$\frac{2d^2\left(\sqrt{d\tan(fx+e)}-\frac{\sqrt{d}\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{2}-\frac{d\left(\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cdots}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cdots}}\right)\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$
default	$\frac{2d^2\left(\sqrt{d\tan(fx+e)}-\frac{\sqrt{d}\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{2}-\frac{d\left(\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cdots}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cdots}}\right)\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(5/2)/(a+a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a*d^2*((d*tan(f*x+e))^(1/2)-1/2*d^(1/2)*arctan((d*tan(f*x+e))^(1/2)/d^(
1/2))-1/2*d*(1/8/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*tan
(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e)
)^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(
1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))+1/8/(d^2)^(1
/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2
)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)
))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(
d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))))
```

Maxima [A]

time = 0.52, size = 134, normalized size = 1.21

$$-\frac{\frac{d^4\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}\right)}{a}+\frac{2d^{\frac{7}{2}}\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{a}-\frac{4\sqrt{d\tan(fx+e)}\,d^3}{a}}{2df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)/(a+a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/2*(d^4*(sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(f*x +
 e)))/sqrt(d))/sqrt(d) + sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*s
qrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d))/a + 2*d^(7/2)*arctan(sqrt(d*tan(f*x
+ e))/sqrt(d))/a - 4*sqrt(d*tan(f*x + e))*d^3/a)/(d*f)
```

Fricas [A]
time = 0.83, size = 261, normalized size = 2.35

$$\left[\frac{\sqrt{2}\sqrt{-d}\,d^2\log\left(\frac{d\tan(fx+e)^2-2\sqrt{d\tan(fx+e)}\left(\sqrt{2}\tan(fx+e)-\sqrt{2}\right)\sqrt{-d}-4d\tan(fx+e)+d}{\tan(fx+e)^2+1}\right)+2\sqrt{-d}\,d^2\log\left(\frac{d\tan(fx+e)-2\sqrt{d\tan(fx+e)}\sqrt{-d}-d}{\tan(fx+e)+1}\right)+8\sqrt{d\tan(fx+e)}\,d^2}{4af},-\frac{\sqrt{2}\,d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{d\tan(fx+e)}\left(\sqrt{2}\tan(fx+e)-\sqrt{2}\right)}{2\sqrt{d}\tan(fx+e)}\right)+2d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)-4\sqrt{d\tan(fx+e)}\,d^2}{2af}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)/(a+a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*sqrt(-d)*d^2*log((d*tan(f*x + e)^2 - 2*sqrt(d*tan(f*x + e))*(
sqrt(2)*tan(f*x + e) - sqrt(2))*sqrt(-d) - 4*d*tan(f*x + e) + d)/(tan(f*x +
 e)^2 + 1)) + 2*sqrt(-d)*d^2*log((d*tan(f*x + e) - 2*sqrt(d*tan(f*x + e))*s
qrt(-d) - d)/(tan(f*x + e) + 1)) + 8*sqrt(d*tan(f*x + e))*d^2)/(a*f), -1/2*
(sqrt(2)*d^(5/2)*arctan(1/2*sqrt(d*tan(f*x + e))*(sqrt(2)*tan(f*x + e) - sq
rt(2))/(sqrt(d)*tan(f*x + e))) + 2*d^(5/2)*arctan(sqrt(d*tan(f*x + e))/sqrt
(d)) - 4*sqrt(d*tan(f*x + e))*d^2)/(a*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(d\tan(e+fx))^{\frac{5}{2}}}{\tan(e+fx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(5/2)/(a+a*tan(f*x+e)),x)
```

```
[Out] Integral((d*tan(e + f*x))**(5/2)/(tan(e + f*x) + 1), x)/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(98) = 196.
time = 0.61, size = 284, normalized size = 2.56

$$-\frac{1}{8}d^2\left(\frac{2\sqrt{2}\left(d\sqrt{|d|}+|d|^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{adf}+\frac{2\sqrt{2}\left(d\sqrt{|d|}+|d|^{\frac{3}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{adf}+\frac{8\sqrt{d}\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{af}+\frac{\sqrt{2}\left(d\sqrt{|d|}-|d|^{\frac{3}{2}}\right)\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{|d|}+|d|\right)}{adf}-\frac{\sqrt{2}\left(d\sqrt{|d|}-|d|^{\frac{3}{2}}\right)\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{|d|}+|d|\right)}{adf}-\frac{16\sqrt{d\tan(fx+e)}}{af}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)/(a+a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/8*d^2*(2*sqrt(2)*(d*sqrt(abs(d)) + abs(d)^(3/2))*arctan(1/2*sqrt(2)*(sqr
t(2)*sqrt(abs(d)) + 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(a*d*f) + 2*sqrt(
2)*(d*sqrt(abs(d)) + abs(d)^(3/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)
) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(a*d*f) + 8*sqrt(d)*arctan(sqrt(d
```

```
*tan(f*x + e))/sqrt(d))/(a*f) + sqrt(2)*(d*sqrt(abs(d)) - abs(d)^(3/2))*log
(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(a*d*
f) - sqrt(2)*(d*sqrt(abs(d)) - abs(d)^(3/2))*log(d*tan(f*x + e) - sqrt(2)*s
qrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(a*d*f) - 16*sqrt(d*tan(f*x + e)
)/(a*f))
```

Mupad [B]
time = 4.45, size = 124, normalized size = 1.12

$$\frac{2\,d^2\,\sqrt{d\tan\left(e+f\,x\right)}}{a\,f}-\frac{d^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{d\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)}{a\,f}-\frac{\sqrt{2}\,d^{5/2}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{d\tan\left(e+f\,x\right)}}{2\,\sqrt{d}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{d\tan\left(e+f\,x\right)}}{2\,\sqrt{d}}+\frac{\sqrt{2}\,{\left(d\tan\left(e+f\,x\right)\right)}^{3/2}}{2\,d^{3/2}}\right)\right)}{4\,a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(5/2)/(a + a*tan(e + f*x)),x)
```

```
[Out] (2*d^2*(d*tan(e + f*x))^(1/2))/(a*f) - (d^(5/2)*atan((d*tan(e + f*x))^(1/2)
/d^(1/2)))/(a*f) - (2^(1/2)*d^(5/2)*(2*atan((2^(1/2)*(d*tan(e + f*x))^(1/2)
)/(2*d^(1/2))) + 2*atan((2^(1/2)*(d*tan(e + f*x))^(1/2))/(2*d^(1/2)) + (2^(
1/2)*(d*tan(e + f*x))^(3/2))/(2*d^(3/2)))))/(4*a*f)
```

3.359 $\int \frac{(d\tan(e+fx))^{3/2}}{a+a\tan(e+fx)}\,dx$

Optimal. Leaf size=87

$$\frac{d^{3/2}\text{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{af}-\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{\sqrt{2}\,af}$$

```
[Out] d^(3/2)*arctan((d*tan(f*x+e))^(1/2)/d^(1/2))/a/f-1/2*d^(3/2)*arctanh(1/2*(d
^(1/2)+d^(1/2)*tan(f*x+e))*2^(1/2)/(d*tan(f*x+e))^(1/2))/a/f*2^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3654, 3613, 214, 3715, 65, 211}

$$\frac{d^{3/2}\text{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{af}-\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\tan(e+fx)+\sqrt{d}}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{\sqrt{2}\,af}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(3/2)/(a + a*Tan[e + f*x]),x]
```

```
[Out] (d^(3/2)*ArcTan[Sqrt[d*Tan[e + f*x]]/Sqrt[d]])/(a*f) - (d^(3/2)*ArcTanh[(Sq
rt[d] + Sqrt[d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[d*Tan[e + f*x]])])/(Sqrt[2]*a*f
)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3654

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[Simp[a^2*c - b^2*c + 2
*a*b*d + (2*a*b*c - a^2*d + b^2*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x]
], x], x] + Dist[(b*c - a*d)^2/(c^2 + d^2), Int[(1 + Tan[e + f*x]^2)/(Sqrt[
a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^{3/2}}{a+a\tan(e+fx)}\,dx = \frac{\int \frac{-ad^2+ad^2\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx}{2a^2}+\frac{1}{2}d^2\int \frac{1+\tan^2(e+fx)}{\sqrt{d\tan(e+fx)}\,(a+a\tan(e+fx))}\,dx$$

$$= \frac{d^2\text{Subst}\left(\int \frac{1}{\sqrt{dx}\,(a+ax)}\,dx,x,\tan(e+fx)\right)}{2f}-\frac{d^4\text{Subst}\left(\int \frac{1}{-2a^2d^4+dx^2}\,dx,x,\frac{-ad^2\cdots}{\sqrt{d\cdots}}\right.}{f}$$

$$= -\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{\sqrt{2}\,af}+\frac{d\text{Subst}\left(\int \frac{1}{a+\frac{ax^2}{d}}\,dx,x,\sqrt{d\tan(e+f\cdots}\right.}{f}$$

$$= \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{af}-\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{\sqrt{2}\,af}$$

Mathematica [A]

time = 0.35, size = 107, normalized size = 1.23

$$\frac{\left(4\text{ArcTan}\left(\sqrt{\tan(e+fx)}\right)+\sqrt{2}\left(\log\left(-1+\sqrt{2}\sqrt{\tan(e+fx)}-\tan(e+fx)\right)-\log\left(1+\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)\right)\right)(d\tan(e+fx))^{3/2}}{4af\tan^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(3/2)/(a + a*Tan[e + f*x]),x]
```

```
[Out] ((4*ArcTan[Sqrt[Tan[e + f*x]]] + Sqrt[2]*(Log[-1 + Sqrt[2]*Sqrt[Tan[e + f*x
]] - Tan[e + f*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]))*(
d*Tan[e + f*x])^(3/2))/(4*a*f*Tan[e + f*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(70) = 140$.
time = 0.29, size = 298, normalized size = 3.43

method	result
derivativedivides	$\frac{2d^2\left(\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{2\sqrt{d}}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\cdots\right.}{\cdots}\right.}{\cdots}$
default	$\frac{2d^2\left(\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{2\sqrt{d}}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\cdots\right.}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(3/2)/(a+a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a*d^2*(1/2/d^(1/2)*arctan((d*tan(f*x+e))^(1/2)/d^(1/2))-1/16/d*(d^2)^(1
/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2
)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)
))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(
d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))+1/16/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x
+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^
2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^
(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^
(1/2)+1)))
```

Maxima [A]
time = 0.53, size = 117, normalized size = 1.34

$$-\frac{\frac{d^3\left(\frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)}{a}-\frac{4d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{a}}{4df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(a+a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/4*(d^3*(sqrt(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d
) + d)/sqrt(d) - sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*
sqrt(d) + d)/sqrt(d))/a - 4*d^(5/2)*arctan(sqrt(d*tan(f*x + e))/sqrt(d))/a)
/(d*f)
```

Fricas [A]
time = 1.05, size = 223, normalized size = 2.56

$$\left[\frac{\sqrt{2}\,\sqrt{-d}\,d\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}\left(\sqrt{2}\,\tan(fx+e)+\sqrt{2}\right)\sqrt{-d}}{2\,d\tan(fx+e)}\right)+\sqrt{-d}\,d\log\left(\frac{d\tan(fx+e)+2\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{-d}-d}{\tan(fx+e)+1}\right)}{2\,af},\ \frac{\sqrt{2}\,d^{\frac{3}{2}}\log\left(\frac{d\tan(fx+e)^{2}-2\,\sqrt{d\tan\left(fx+e\right)}\left(\sqrt{2}\,\tan(fx+e)+\sqrt{2}\right)\sqrt{d}+4\,d\tan(fx+e)+d}{\tan(fx+e)^{2}+1}\right)+4\,d^{\frac{3}{2}}\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}}{\sqrt{d}}\right)}{4\,af}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(a+a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*sqrt(-d)*d*arctan(1/2*sqrt(d*tan(f*x + e))*(sqrt(2)*tan(f*x +
 e) + sqrt(2))*sqrt(-d)/(d*tan(f*x + e))) + sqrt(-d)*d*log((d*tan(f*x + e)
+ 2*sqrt(d*tan(f*x + e))*sqrt(-d) - d)/(tan(f*x + e) + 1)))/(a*f), 1/4*(sqr
t(2)*d^(3/2)*log((d*tan(f*x + e)^2 - 2*sqrt(d*tan(f*x + e))*(sqrt(2)*tan(f*
x + e) + sqrt(2))*sqrt(d) + 4*d*tan(f*x + e) + d)/(tan(f*x + e)^2 + 1)) + 4
*d^(3/2)*arctan(sqrt(d*tan(f*x + e))/sqrt(d)))/(a*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\left(d\tan\left(e+fx\right)\right)^{\frac{3}{2}}}{\tan\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(3/2)/(a+a*tan(f*x+e)),x)
```

```
[Out] Integral((d*tan(e + f*x))**(3/2)/(tan(e + f*x) + 1), x)/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(73) = 146.
time = 0.57, size = 263, normalized size = 3.02

$$-\frac{1}{8}\,d\left(\frac{2\,\sqrt{2}\left(d\sqrt{|d|}-|d|^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{|d|}+2\,\sqrt{d\tan\left(fx+e\right)}\right)}{2\,\sqrt{|d|}}\right)}{adf}+\frac{2\,\sqrt{2}\left(d\sqrt{|d|}-|d|^{\frac{3}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{|d|}-2\,\sqrt{d\tan\left(fx+e\right)}\right)}{2\,\sqrt{|d|}}\right)}{adf}-\frac{8\,\sqrt{d}\,\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}}{\sqrt{d}}\right)}{af}+\frac{\sqrt{2}\left(d\sqrt{|d|}+|d|^{\frac{3}{2}}\right)\log\left(d\tan\left(fx+e\right)+\sqrt{2}\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{|d|}+|d|\right)}{adf}-\frac{\sqrt{2}\left(d\sqrt{|d|}+|d|^{\frac{3}{2}}\right)\log\left(d\tan\left(fx+e\right)-\sqrt{2}\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{|d|}+|d|\right)}{adf}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(a+a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/8*d*(2*sqrt(2)*(d*sqrt(abs(d)) - abs(d)^(3/2))*arctan(1/2*sqrt(2)*(sqrt(
2)*sqrt(abs(d)) + 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(a*d*f) + 2*sqrt(2)
*(d*sqrt(abs(d)) - abs(d)^(3/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d))
```

```
- 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(a*d*f) - 8*sqrt(d)*arctan(sqrt(d*t
an(f*x + e))/sqrt(d))/(a*f) + sqrt(2)*(d*sqrt(abs(d)) + abs(d)^(3/2))*log(d
*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(a*d*f)
 - sqrt(2)*(d*sqrt(abs(d)) + abs(d)^(3/2))*log(d*tan(f*x + e) - sqrt(2)*sqr
t(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(a*d*f))
```

Mupad [B]
time = 4.23, size = 78, normalized size = 0.90

$$\frac{d^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{d\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)}{a\,f}-\frac{\sqrt{2}\,d^{3/2}\,\mathrm{atanh}\left(\frac{12\,\sqrt{2}\,d^{25/2}\,\sqrt{d\tan\left(e+f\,x\right)}}{12\,d^{13}\,\tan(e+f\,x)+12\,d^{13}}\right)}{2\,a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(3/2)/(a + a*tan(e + f*x)),x)
```

```
[Out] (d^(3/2)*atan((d*tan(e + f*x))^(1/2)/d^(1/2)))/(a*f) - (2^(1/2)*d^(3/2)*ata
nh((12*2^(1/2)*d^(25/2)*(d*tan(e + f*x))^(1/2))/(12*d^13*tan(e + f*x) + 12*
d^13)))/(2*a*f)
```

3.360 $\int \frac{\sqrt{d\tan(e+fx)}}{a+a\tan(e+fx)}\,dx$

Optimal. Leaf size=89

$$-\frac{\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{af}-\frac{\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{d}-\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{\sqrt{2}\,af}$$

```
[Out] -arctan((d*tan(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/a/f-1/2*arctan(1/2*(d^(1/2)-d
^(1/2)*tan(f*x+e))*2^(1/2)/(d*tan(f*x+e))^(1/2))*d^(1/2)/a/f*2^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3653, 3613, 211, 3715, 65}

$$-\frac{\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{af}-\frac{\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{d}-\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{\sqrt{2}\,af}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Tan[e + f*x]]/(a + a*Tan[e + f*x]),x]
```

```
[Out] -((Sqrt[d]*ArcTan[Sqrt[d*Tan[e + f*x]]/Sqrt[d]])/(a*f)) - (Sqrt[d]*ArcTan[(
Sqrt[d] - Sqrt[d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[d*Tan[e + f*x]])])/(Sqrt[2]*a
*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
```

```
EqQ[c^2 - d^2, 0]
```

Rule 3653

```
Int[Sqrt[(a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[Simp[a*c + b*d + (b*c -
a*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x]], x], x] - Dist[d*((b*c - a*d
)/(c^2 + d^2)), Int[(1 + Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]]*(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{d\tan(e+fx)}}{a+a\tan(e+fx)}\,dx &= \frac{\int \frac{ad+ad\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx}{2a^2} - \frac{1}{2}d\int \frac{1+\tan^2(e+fx)}{\sqrt{d\tan(e+fx)}\,(a+a\tan(e+fx))}\,dx\\ &= -\frac{d\text{Subst}\left(\int \frac{1}{\sqrt{dx}\,(a+ax)}\,dx, x, \tan(e+fx)\right)}{2f} - \frac{d^2\text{Subst}\left(\int \frac{1}{2a^2d^2+dx^2}\,dx, x, \frac{ad-ac}{\sqrt{d\tan}}\right.}{f}\\ &= -\frac{\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{d}-\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{\sqrt{2}\,af} - \frac{\text{Subst}\left(\int \frac{1}{a+\frac{ax^2}{d}}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{f}\\ &= -\frac{\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{af} - \frac{\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{d}-\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{\sqrt{2}\,af}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 98, normalized size = 1.10

$$-\frac{\left(\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)-\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(e+fx)}\right)+2\text{ArcTan}\left(\sqrt{\tan(e+fx)}\right)\right)\sqrt{d\tan(e+fx)}}{2af\sqrt{\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Tan[e + f*x]]/(a + a*Tan[e + f*x]),x]
```

```
[Out] -1/2*((Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] - Sqrt[2]*ArcTan[1 +
Sqrt[2]*Sqrt[Tan[e + f*x]]] + 2*ArcTan[Sqrt[Tan[e + f*x]]])*Sqrt[d*Tan[e +
f*x]])/(a*f*Sqrt[Tan[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(72) = 144$.
time = 0.17, size = 304, normalized size = 3.42

method	result
derivativedivides	$2d^2\left(-\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{2d^{\frac{3}{2}}}+\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right.\right.}{}\right.$
default	$2d^2\left(-\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{2d^{\frac{3}{2}}}+\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right.\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(1/2)/(a+a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a*d^2*(-1/2/d^(3/2)*arctan((d*tan(f*x+e))^(1/2)/d^(1/2))+1/2/d*(1/8/d*(
d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/
2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2
)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^
(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))+1/8/(d^2)^(1/4)*2^(1/2)*(ln((d*t
an(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+
e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/
(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*
x+e))^(1/2)+1))))
```

Maxima [A]
time = 0.51, size = 115, normalized size = 1.29

$$\frac{\frac{d^2\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}\right)}{a}-\frac{2d^{\frac{3}{2}}\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{a}}{2df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(a+a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*(d^2*(sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(f*x +
e)))/sqrt(d))/sqrt(d) + sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*sq
rt(d*tan(f*x + e)))/sqrt(d))/sqrt(d))/a - 2*d^(3/2)*arctan(sqrt(d*tan(f*x +
 e))/sqrt(d))/a)/(d*f)
```

Fricas [**A**]
time = 1.08, size = 223, normalized size = 2.51

$$\left[\frac{\sqrt{2}\,\sqrt{-d}\,\log\left(\frac{d\tan(fx+e)^2+2\,\sqrt{d\tan\left(fx+e\right)}\,\left(\sqrt{2}\,\tan(fx+e)-\sqrt{2}\right)\sqrt{-d}\,-4\,d\tan(fx+e)+d}{\tan(fx+e)^2+1}\right)+2\,\sqrt{-d}\,\log\left(\frac{d\tan(fx+e)-2\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{-d}\,-d}{\tan(fx+e)+1}\right)}{4\,af},\ \frac{\sqrt{2}\,\sqrt{d}\,\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}\,\left(\sqrt{2}\,\tan(fx+e)-\sqrt{2}\right)}{2\,\sqrt{d}\,\tan(fx+e)}\right)-2\,\sqrt{d}\,\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}}{\sqrt{d}}\right)}{2\,af}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(a+a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*sqrt(-d)*log((d*tan(f*x + e)^2 + 2*sqrt(d*tan(f*x + e))*(sqrt
(2)*tan(f*x + e) - sqrt(2))*sqrt(-d) - 4*d*tan(f*x + e) + d)/(tan(f*x + e)^
2 + 1)) + 2*sqrt(-d)*log((d*tan(f*x + e) - 2*sqrt(d*tan(f*x + e))*sqrt(-d)
- d)/(tan(f*x + e) + 1)))/(a*f), 1/2*(sqrt(2)*sqrt(d)*arctan(1/2*sqrt(d*tan
(f*x + e))*(sqrt(2)*tan(f*x + e) - sqrt(2))/(sqrt(d)*tan(f*x + e))) - 2*sqr
t(d)*arctan(sqrt(d*tan(f*x + e))/sqrt(d)))/(a*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{d\tan\left(e+fx\right)}}{\tan\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(1/2)/(a+a*tan(f*x+e)),x)
```

```
[Out] Integral(sqrt(d*tan(e + f*x))/(tan(e + f*x) + 1), x)/a
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(76) = 152.
time = 0.58, size = 253, normalized size = 2.84

$$\frac{\frac{2\,\sqrt{2}\,\left(d\sqrt{|d|}+|d|^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{2}\,\left(\sqrt{2}\,\sqrt{|d|}+2\,\sqrt{d\tan\left(fx+e\right)}\right)}{2\,\sqrt{|d|}}\right)}{af}+\frac{2\,\sqrt{2}\,\left(d\sqrt{|d|}+|d|^{\frac{3}{2}}\right)\arctan\left(-\frac{\sqrt{2}\,\left(\sqrt{2}\,\sqrt{|d|}-2\,\sqrt{d\tan\left(fx+e\right)}\right)}{2\,\sqrt{|d|}}\right)}{af}-\frac{8\,d^{\frac{3}{2}}\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}}{\sqrt{d}}\right)}{af}+\frac{\sqrt{2}\,\left(d\sqrt{|d|}-|d|^{\frac{3}{2}}\right)\log\left(d\tan(fx+e)+\sqrt{2}\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{|d|}+|d|\right)}{af}-\frac{\sqrt{2}\,\left(d\sqrt{|d|}-|d|^{\frac{3}{2}}\right)\log\left(d\tan(fx+e)-\sqrt{2}\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{|d|}+|d|\right)}{af}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(a+a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/8*(2*sqrt(2)*(d*sqrt(abs(d)) + abs(d)^(3/2))*arctan(1/2*sqrt(2)*(sqrt(2)*
sqrt(abs(d)) + 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(a*f) + 2*sqrt(2)*(d*s
qrt(abs(d)) + abs(d)^(3/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*s
qrt(d*tan(f*x + e)))/sqrt(abs(d)))/(a*f) - 8*d^(3/2)*arctan(sqrt(d*tan(f*x
+ e))/sqrt(d))/(a*f) + sqrt(2)*(d*sqrt(abs(d)) - abs(d)^(3/2))*log(d*tan(f*
x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(a*f) - sqrt(2
)*(d*sqrt(abs(d)) - abs(d)^(3/2))*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f
*x + e))*sqrt(abs(d)) + abs(d))/(a*f))/d
```

Mupad [B]
time = 4.25, size = 103, normalized size = 1.16

$$\frac{\sqrt{2}\,\sqrt{d}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{d\tan\left(e+f\,x\right)}}{2\,\sqrt{d}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{d\tan\left(e+f\,x\right)}}{2\,\sqrt{d}}+\frac{\sqrt{2}\,{\left(d\tan\left(e+f\,x\right)\right)}^{3/2}}{2\,d^{3/2}}\right)\right)}{4\,a\,f}-\frac{\sqrt{d}\,\mathrm{atan}\left(\frac{\sqrt{d\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(1/2)/(a + a*tan(e + f*x)),x)
```

```
[Out] (2^(1/2)*d^(1/2)*(2*atan((2^(1/2)*(d*tan(e + f*x))^(1/2))/(2*d^(1/2))) + 2*
atan((2^(1/2)*(d*tan(e + f*x))^(1/2))/(2*d^(1/2)) + (2^(1/2)*(d*tan(e + f*x
))^(3/2))/(2*d^(3/2)))))/(4*a*f) - (d^(1/2)*atan((d*tan(e + f*x))^(1/2)/d^(
1/2)))/(a*f)
```

3.361 $\int \frac{1}{\sqrt{d\tan(e+fx)}\,(a+a\tan(e+fx))}\,dx$

Optimal. Leaf size=81

$$\frac{\text{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{a\sqrt{d}\,f}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,(1+\tan(e+fx))}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{\sqrt{2}\,a\sqrt{d}\,f}$$

```
[Out] arctan((d*tan(f*x+e))^(1/2)/d^(1/2))/a/f/d^(1/2)+1/2*arctanh(1/2*d^(1/2)*(1
+tan(f*x+e))*2^(1/2)/(d*tan(f*x+e))^(1/2))/a/f*2^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3655, 3613, 214, 3715, 65, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{a\sqrt{d}\,f}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,(\tan(e+fx)+1)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{\sqrt{2}\,a\sqrt{d}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*Tan[e + f*x]]*(a + a*Tan[e + f*x])),x]
```

```
[Out] ArcTan[Sqrt[d*Tan[e + f*x]]/Sqrt[d]]/(a*Sqrt[d]*f) + ArcTanh[(Sqrt[d]*(1 +
Tan[e + f*x]))/(Sqrt[2]*Sqrt[d*Tan[e + f*x]])]/(Sqrt[2]*a*Sqrt[d]*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)/((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[(a + b*Tan[e + f*x])^m*
(c - d*Tan[e + f*x]), x], x] + Dist[d^2/(c^2 + d^2), Int[(a + b*Tan[e + f*x
])^m*((1 + Tan[e + f*x]^2)/(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{d\tan(e+fx)}\,(a+a\tan(e+fx))}\,dx &= \frac{1}{2}\int \frac{1+\tan^2(e+fx)}{\sqrt{d\tan(e+fx)}\,(a+a\tan(e+fx))}\,dx + \frac{\int \frac{a-a\tan(e+}{\sqrt{d\tan(e-}}}{2a^2}\\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{dx}\,(a+ax)}\,dx, x, \tan(e+fx)\right)}{2f} - \frac{\text{Subst}\left(\int \frac{1}{-2a^2+dx}\right.}{}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{d}\,(1+\tan(e+fx))}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{\sqrt{2}\,a\sqrt{d}\,f} + \frac{\text{Subst}\left(\int \frac{1}{a+\frac{ax^2}{d}}\,dx, x, \sqrt{d}\right.}{df}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{a\sqrt{d}\,f} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}\,(1+\tan(e+fx))}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{\sqrt{2}\,a\sqrt{d}\,f}
\end{aligned}$$

Mathematica [A]
time = 0.49, size = 107, normalized size = 1.32

$$\frac{\left(4\text{ArcTan}\left(\sqrt{\tan(e+fx)}\right)+\sqrt{2}\left(-\log\left(-1+\sqrt{2}\,\sqrt{\tan(e+fx)}-\tan(e+fx)\right)+\log\left(1+\sqrt{2}\,\sqrt{\tan(e+fx)}+\tan(e+fx)\right)\right)\right)\sqrt{\tan(e+fx)}}{4af\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*Tan[e + f*x]]*(a + a*Tan[e + f*x])),x]
```

```
[Out] ((4*ArcTan[Sqrt[Tan[e + f*x]]] + Sqrt[2]*(-Log[-1 + Sqrt[2]*Sqrt[Tan[e + f*
x]] - Tan[e + f*x]] + Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]))*
Sqrt[Tan[e + f*x]])/(4*a*f*Sqrt[d*Tan[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(67) = 134$.

time = 0.19, size = 304, normalized size = 3.75

method	result
derivativedivides	$2d^2\left(\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{2d^{\frac{5}{2}}}+\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+\right.}{}\right.$
default	$2d^2\left(\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{2d^{\frac{5}{2}}}+\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*tan(f*x+e))^(1/2)/(a+a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a*d^2*(1/2/d^(5/2)*arctan((d*tan(f*x+e))^(1/2)/d^(1/2))+1/2/d^2*(1/8/d*
(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1
/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^
2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2
^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))-1/8/(d^2)^(1/4)*2^(1/2)*(ln((d*
tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x
+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)
/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f
*x+e))^(1/2)+1))))
```

Maxima [A]

time = 0.51, size = 115, normalized size = 1.42

$$\frac{\frac{d\left(\frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)}{a}+\frac{4\sqrt{d}\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{a}}{4\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(1/2)/(a+a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*(d*(sqrt(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) +
 d)/sqrt(d) - sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqr
t(d) + d)/sqrt(d))/a + 4*sqrt(d)*arctan(sqrt(d*tan(f*x + e))/sqrt(d))/a)/(d
*f)
```

Fricas [**A**]
time = 1.10, size = 221, normalized size = 2.73

$$\left[-\frac{\sqrt{2}\sqrt{-d}\arctan\left(\frac{\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{-d}\,(\tan(fx+e)+1)}{2\,d\tan(fx+e)}\right)+\sqrt{-d}\,\log\left(\frac{d\tan(fx+e)-2\sqrt{d\tan\left(fx+e\right)}\sqrt{-d}-d}{\tan(fx+e)+1}\right)}{2\,adf},\ \frac{\sqrt{2}\sqrt{d}\,\log\left(\frac{d\tan(fx+e)^2+2\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{d}\,(\tan(fx+e)+1)+4\,d\tan(fx+e)+d}{\tan(fx+e)^2+1}\right)+4\sqrt{d}\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}}{\sqrt{d}}\right)}{4\,adf}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(1/2)/(a+a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(2)*sqrt(-d)*arctan(1/2*sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(-d)*(t
an(f*x + e) + 1)/(d*tan(f*x + e))) + sqrt(-d)*log((d*tan(f*x + e) - 2*sqrt(
d*tan(f*x + e))*sqrt(-d) - d)/(tan(f*x + e) + 1)))/(a*d*f), 1/4*(sqrt(2)*sq
rt(d)*log((d*tan(f*x + e)^2 + 2*sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d)*(tan(f
*x + e) + 1) + 4*d*tan(f*x + e) + d)/(tan(f*x + e)^2 + 1)) + 4*sqrt(d)*arct
an(sqrt(d*tan(f*x + e))/sqrt(d)))/(a*d*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{d\tan\left(e+fx\right)}\tan\left(e+fx\right)+\sqrt{d\tan\left(e+fx\right)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))**(1/2)/(a+a*tan(f*x+e)),x)
```

```
[Out] Integral(1/(sqrt(d*tan(e + f*x))*tan(e + f*x) + sqrt(d*tan(e + f*x))), x)/a
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. 2(70) = 140.
time = 0.58, size = 260, normalized size = 3.21

$$\frac{\sqrt{2}\left(d\sqrt{|d|}-|d|^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{4\,ad^2f}+\frac{\sqrt{2}\left(d\sqrt{|d|}-|d|^{\frac{3}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{4\,ad^2f}+\frac{\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}}{\sqrt{d}}\right)}{a\sqrt{d}\,f}+\frac{\sqrt{2}\left(d\sqrt{|d|}+|d|^{\frac{3}{2}}\right)\log\left(d\tan\left(fx+e\right)+\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{|d|}+|d|\right)}{8\,ad^2f}-\frac{\sqrt{2}\left(d\sqrt{|d|}+|d|^{\frac{3}{2}}\right)\log\left(d\tan\left(fx+e\right)-\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{|d|}+|d|\right)}{8\,ad^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(1/2)/(a+a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(d*sqrt(abs(d)) - abs(d)^(3/2))*arctan(1/2*sqrt(2)*(sqrt(2)*sqr
t(abs(d)) + 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(a*d^2*f) + 1/4*sqrt(2)*(
```

```
d*sqrt(abs(d)) - abs(d)^(3/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) -
2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(a*d^2*f) + arctan(sqrt(d*tan(f*x + e
))/sqrt(d))/(a*sqrt(d)*f) + 1/8*sqrt(2)*(d*sqrt(abs(d)) + abs(d)^(3/2))*log
(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(a*d^
2*f) - 1/8*sqrt(2)*(d*sqrt(abs(d)) + abs(d)^(3/2))*log(d*tan(f*x + e) - sqr
t(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(a*d^2*f)
```

Mupad [B]
time = 4.34, size = 78, normalized size = 0.96

$$\frac{\operatorname{atan}\left(\frac{\sqrt{d\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)}{a\,\sqrt{d}\,f}+\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{12\,\sqrt{2}\,d^{9/2}\,\sqrt{d\tan\left(e+f\,x\right)}}{12\,d^5\,\tan(e+f\,x)+12\,d^5}\right)}{2\,a\,\sqrt{d}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*tan(e + f*x))^(1/2)*(a + a*tan(e + f*x))),x)
```

```
[Out] atan((d*tan(e + f*x))^(1/2)/d^(1/2))/(a*d^(1/2)*f) + (2^(1/2)*atanh((12*2^(
1/2)*d^(9/2)*(d*tan(e + f*x))^(1/2))/(12*d^5*tan(e + f*x) + 12*d^5)))/(2*a*
d^(1/2)*f)
```

3.362 $\int \frac{1}{(d\tan(e+fx))^{3/2}(a+a\tan(e+fx))}\,dx$

Optimal. Leaf size=111

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{ad^{3/2}f}+\frac{\text{ArcTan}\left(\frac{\sqrt{d}-\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{\sqrt{2}\,ad^{3/2}f}-\frac{2}{adf\sqrt{d\tan(e+fx)}}$$

```
[Out] -arctan((d*tan(f*x+e))^(1/2)/d^(1/2))/a/d^(3/2)/f+1/2*arctan(1/2*(d^(1/2)-d
^(1/2)*tan(f*x+e))*2^(1/2)/(d*tan(f*x+e))^(1/2))/a/d^(3/2)/f*2^(1/2)-2/a/d/
f/(d*tan(f*x+e))^(1/2)
```

Rubi [A]

time = 0.27, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3650, 3734, 3613, 211, 3715, 65}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{ad^{3/2}f}+\frac{\text{ArcTan}\left(\frac{\sqrt{d}-\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{\sqrt{2}\,ad^{3/2}f}-\frac{2}{adf\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Tan[e + f*x])^(3/2)*(a + a*Tan[e + f*x])),x]
```

```
[Out] -(ArcTan[Sqrt[d*Tan[e + f*x]]/Sqrt[d]]/(a*d^(3/2)*f)) + ArcTan[(Sqrt[d] - S
qrt[d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[d*Tan[e + f*x]])]/(Sqrt[2]*a*d^(3/2)*f)
- 2/(a*d*f*Sqrt[d*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
```

```
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{(d\tan(e+fx))^{3/2}(a+a\tan(e+fx))}\,dx = -\frac{2}{adf\sqrt{d\tan(e+fx)}} - \frac{2\int \frac{\frac{ad^2}{2}+\frac{1}{2}ad^2\tan(e+fx)+\frac{1}{2}ad^2\tan^2(e+fx)}{\sqrt{d\tan(e+fx)}\,(a+a\tan(e+fx))}}{ad^3}$$

$$= -\frac{2}{adf\sqrt{d\tan(e+fx)}} - \frac{\int \frac{\frac{a^2d^2}{2}+\frac{1}{2}a^2d^2\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx}{a^3d^3} - \frac{\int \frac{}{\sqrt{d\,t}}}{}$$

$$= -\frac{2}{adf\sqrt{d\tan(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{dx}\,(a+ax)}\,dx, x, \tan(e-\right.}{2df}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{d}-\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{\sqrt{2}\,ad^{3/2}f} - \frac{2}{adf\sqrt{d\tan(e+fx)}} - \frac{\text{S}}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{ad^{3/2}f} + \frac{\tan^{-1}\left(\frac{\sqrt{d}-\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx}}\right.}{\sqrt{2}\,ad^{3/2}f}$$

Mathematica [**A**]
time = 1.78, size = 122, normalized size = 1.10

$$-\frac{4-\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\sqrt{\tan(e+fx)}+\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(e+fx)}\right)\sqrt{\tan(e+fx)}+2\text{ArcTan}\left(\sqrt{\tan(e+fx)}\right)\sqrt{\tan(e+fx)}}{2adf\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Tan[e + f*x])^(3/2)*(a + a*Tan[e + f*x])),x]
```

```
[Out] -1/2*(4 - Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]]*Sqrt[Tan[e + f*x]]
 + Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]]*Sqrt[Tan[e + f*x]] + 2*Ar
cTan[Sqrt[Tan[e + f*x]]]*Sqrt[Tan[e + f*x]])/(a*d*f*Sqrt[d*Tan[e + f*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(93) = 186.
time = 0.17, size = 319, normalized size = 2.87

method	result

derivativedivides	$\frac{2d^2\left(-\frac{1}{d^3\sqrt{d\tan(fx+e)}}-\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{2d^{\frac{7}{2}}}+\frac{-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan}}\right.\right.}{}}{}\right.}{}$
default	$\frac{2d^2\left(-\frac{1}{d^3\sqrt{d\tan(fx+e)}}-\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{2d^{\frac{7}{2}}}+\frac{-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan}}\right.\right.}{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*tan(f*x+e))^(3/2)/(a+a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a*d^2*(-1/d^3/(d*tan(f*x+e))^(1/2)-1/2/d^(7/2)*arctan((d*tan(f*x+e))^(1
/2)/d^(1/2))+1/2/d^3*(-1/8/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1
/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*
tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan
(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))-1/
8/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^
(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(
d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(
-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))))
```

Maxima [A]
time = 0.53, size = 128, normalized size = 1.15

$$-\frac{\frac{\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}}{a}+\frac{2\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{a\sqrt{d}}+\frac{4}{\sqrt{d\tan(fx+e)}\,a}}{2\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(3/2)/(a+a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/2*((sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(f*x + e))
)/sqrt(d))/sqrt(d) + sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*sqrt(
d*tan(f*x + e)))/sqrt(d))/sqrt(d))/a + 2*arctan(sqrt(d*tan(f*x + e))/sqrt(d
))/(a*sqrt(d)) + 4/(sqrt(d*tan(f*x + e))*a))/(d*f)
```

Fricas [A]
time = 0.91, size = 291, normalized size = 2.62

$$\left[-\frac{\sqrt{2}\sqrt{-d}\log\left(\frac{d\tan(fx+e)^2+2\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{-d}(\tan(fx+e)-1)-4d\tan(fx+e)+d}{\tan(fx+e)^2+1}\right)\tan\left(fx+e\right)+2\sqrt{-d}\log\left(\frac{d\tan(fx+e)+2\sqrt{d\tan\left(fx+e\right)}\sqrt{-d}-d}{\tan(fx+e)+1}\right)\tan\left(fx+e\right)+8\sqrt{d\tan\left(fx+e\right)}}{4\,ad^2f\tan\left(fx+e\right)},-\frac{\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{d\tan\left(fx+e\right)}(\tan(fx+e)-1)}{2\sqrt{d}\tan(fx+e)}\right)\tan\left(fx+e\right)+2\sqrt{d}\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}}{\sqrt{d}}\right)\tan\left(fx+e\right)+4\sqrt{d\tan\left(fx+e\right)}}{2\,ad^2f\tan\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(3/2)/(a+a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(2)*sqrt(-d)*log((d*tan(f*x + e)^2 + 2*sqrt(2)*sqrt(d*tan(f*x +
e))*sqrt(-d)*(tan(f*x + e) - 1) - 4*d*tan(f*x + e) + d)/(tan(f*x + e)^2 + 1
))*tan(f*x + e) + 2*sqrt(-d)*log((d*tan(f*x + e) + 2*sqrt(d*tan(f*x + e))*s
qrt(-d) - d)/(tan(f*x + e) + 1))*tan(f*x + e) + 8*sqrt(d*tan(f*x + e)))/(a*
d^2*f*tan(f*x + e)), -1/2*(sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*sqrt(d*tan(f*
x + e))*(tan(f*x + e) - 1)/(sqrt(d)*tan(f*x + e)))*tan(f*x + e) + 2*sqrt(d)
*arctan(sqrt(d*tan(f*x + e))/sqrt(d))*tan(f*x + e) + 4*sqrt(d*tan(f*x + e))
)/(a*d^2*f*tan(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\left(d\tan\left(e+fx\right)\right)^{\frac{3}{2}}\tan\left(e+fx\right)+\left(d\tan\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))**(3/2)/(a+a*tan(f*x+e)),x)
```

```
[Out] Integral(1/((d*tan(e + f*x))**(3/2)*tan(e + f*x) + (d*tan(e + f*x))**(3/2))
, x)/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(98) = 196.
time = 0.65, size = 284, normalized size = 2.56

$$-\frac{\frac{2\sqrt{2}\left(d\sqrt{|d|}+|d|^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{ad^2f}+\frac{2\sqrt{2}\left(d\sqrt{|d|}+|d|^{\frac{3}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{ad^2f}+\frac{8\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}}{\sqrt{d}}\right)}{a\sqrt{d}f}+\frac{\sqrt{2}\left(d\sqrt{|d|}-|d|^{\frac{3}{2}}\right)\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{|d|}+|d|\right)}{ad^2f}-\frac{\sqrt{2}\left(d\sqrt{|d|}-|d|^{\frac{3}{2}}\right)\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{|d|}+|d|\right)}{ad^2f}+\frac{16}{\sqrt{d\tan\left(fx+e\right)}af}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(3/2)/(a+a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/8*(2*sqrt(2)*(d*sqrt(abs(d)) + abs(d)^(3/2))*arctan(1/2*sqrt(2)*(sqrt(2)
*sqrt(abs(d)) + 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(a*d^2*f) + 2*sqrt(2)
*(d*sqrt(abs(d)) + abs(d)^(3/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d))
- 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(a*d^2*f) + 8*arctan(sqrt(d*tan(f*x
 + e))/sqrt(d))/(a*sqrt(d)*f) + sqrt(2)*(d*sqrt(abs(d)) - abs(d)^(3/2))*log
(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(a*d^
```

```
2*f) - sqrt(2)*(d*sqrt(abs(d)) - abs(d)^(3/2))*log(d*tan(f*x + e) - sqrt(2)
*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(a*d^2*f) + 16/(sqrt(d*tan(f*x
 + e))*a*f))/d
```

Mupad [B]

time = 4.44, size = 124, normalized size = 1.12

$$-\frac{2}{a\,d\,f\,\sqrt{d\tan\left(e+f\,x\right)}}-\frac{\mathrm{atan}\left(\frac{\sqrt{d\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)}{a\,d^{3/2}\,f}-\frac{\sqrt{2}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{d\tan\left(e+f\,x\right)}}{2\,\sqrt{d}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{d\tan\left(e+f\,x\right)}}{2\,\sqrt{d}}+\frac{\sqrt{2}\,{\left(d\tan\left(e+f\,x\right)\right)}^{3/2}}{2\,d^{3/2}}\right)\right)}{4\,a\,d^{3/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*tan(e + f*x))^(3/2)*(a + a*tan(e + f*x))),x)
```

```
[Out] - 2/(a*d*f*(d*tan(e + f*x))^(1/2)) - atan((d*tan(e + f*x))^(1/2)/d^(1/2))/(
a*d^(3/2)*f) - (2^(1/2)*(2*atan((2^(1/2)*(d*tan(e + f*x))^(1/2))/(2*d^(1/2)
)) + 2*atan((2^(1/2)*(d*tan(e + f*x))^(1/2))/(2*d^(1/2)) + (2^(1/2)*(d*tan(
e + f*x))^(3/2))/(2*d^(3/2)))))/(4*a*d^(3/2)*f)
```

3.363 $\int \frac{1}{(d\tan(e+fx))^{5/2}(a+a\tan(e+fx))}\,dx$

Optimal. Leaf size=135

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{ad^{5/2}f}-\frac{\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{\sqrt{2}ad^{5/2}f}-\frac{2}{3adf(d\tan(e+fx))^{3/2}}+\frac{2}{ad^2f\sqrt{d\tan(e+fx)}}$$

```
[Out] arctan((d*tan(f*x+e))^(1/2)/d^(1/2))/a/d^(5/2)/f-1/2*arctanh(1/2*(d^(1/2)+d
^(1/2)*tan(f*x+e))*2^(1/2)/(d*tan(f*x+e))^(1/2))/a/d^(5/2)/f*2^(1/2)+2/a/d^
2/f/(d*tan(f*x+e))^(1/2)-2/3/a/d/f/(d*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.33, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3650, 3730, 12, 16, 3654, 3613, 214, 3715, 65, 211}

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{ad^{5/2}f}-\frac{\tanh^{-1}\left(\frac{\sqrt{d}\tan(e+fx)+\sqrt{d}}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{\sqrt{2}ad^{5/2}f}+\frac{2}{ad^2f\sqrt{d\tan(e+fx)}}-\frac{2}{3adf(d\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Tan[e + f*x])^(5/2)*(a + a*Tan[e + f*x])),x]
```

```
[Out] ArcTan[Sqrt[d*Tan[e + f*x]]/Sqrt[d]]/(a*d^(5/2)*f) - ArcTanh[(Sqrt[d] + Sqr
t[d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[d*Tan[e + f*x]])]/(Sqrt[2]*a*d^(5/2)*f) -
2/(3*a*d*f*(d*Tan[e + f*x])^(3/2)) + 2/(a*d^2*f*Sqrt[d*Tan[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3654

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[Simp[a^2*c - b^2*c + 2
*a*b*d + (2*a*b*c - a^2*d + b^2*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x]
], x], x] + Dist[(b*c - a*d)^2/(c^2 + d^2), Int[(1 + Tan[e + f*x]^2)/(Sqrt[
a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{(d\tan(e+fx))^{5/2}(a+a\tan(e+fx))}\,dx = -\frac{2}{3adf(d\tan(e+fx))^{3/2}} - \frac{2\int \frac{\frac{3ad^2}{2}+\frac{3}{2}ad^2\tan(e+fx)+\frac{3}{2}ad^2\tan^2(e+}{(d\tan(e+fx))^{3/2}(a+a\tan(e+fx))}}{3ad^3}$$

$$= -\frac{2}{3adf(d\tan(e+fx))^{3/2}} + \frac{2}{ad^2f\sqrt{d\tan(e+fx)}} + \frac{4\int \frac{}{4\sqrt{}}}{}$$

$$= -\frac{2}{3adf(d\tan(e+fx))^{3/2}} + \frac{2}{ad^2f\sqrt{d\tan(e+fx)}} + \frac{\int \frac{}{\sqrt{d}}}{}$$

$$= -\frac{2}{3adf(d\tan(e+fx))^{3/2}} + \frac{2}{ad^2f\sqrt{d\tan(e+fx)}} + \frac{\int \frac{(d\tan}{a+a}}{}$$

$$= -\frac{2}{3adf(d\tan(e+fx))^{3/2}} + \frac{2}{ad^2f\sqrt{d\tan(e+fx)}} + \frac{\int \frac{-ad^2}{\sqrt{d}}}{}$$

$$= -\frac{2}{3adf(d\tan(e+fx))^{3/2}} + \frac{2}{ad^2f\sqrt{d\tan(e+fx)}} - \frac{\text{Subst}\Big(}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{\sqrt{2}\,ad^{5/2}f} - \frac{2}{3adf(d\tan(e+fx))^{3/2}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{ad^{5/2}f} - \frac{\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{\sqrt{2}\,ad^{5/2}f}$$

Mathematica [A]

time = 1.47, size = 130, normalized size = 0.96

$$\frac{-8+24\tan(e+fx)+12\mathrm{ArcTan}\left(\sqrt{\tan(e+fx)}\right)\tan^{\frac{3}{2}}(e+fx)+3\sqrt{2}\left(\log\left(-1+\sqrt{2}\sqrt{\tan(e+fx)}-\tan(e+fx)\right)-\log\left(1+\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)\right)\tan^{\frac{3}{2}}(e+fx)}{12adf(d\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Tan[e + f*x])^(5/2)*(a + a*Tan[e + f*x])),x]
```

```
[Out] (-8 + 24*Tan[e + f*x] + 12*ArcTan[Sqrt[Tan[e + f*x]]]*Tan[e + f*x]^(3/2) +
3*Sqrt[2]*(Log[-1 + Sqrt[2]*Sqrt[Tan[e + f*x]] - Tan[e + f*x]] - Log[1 + Sq
rt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]])*Tan[e + f*x]^(3/2))/(12*a*d*f*(d*
Tan[e + f*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(112) = 224$.
time = 0.16, size = 333, normalized size = 2.47

method	result
derivativedivides	$2d^2\left(-\frac{1}{3d^3(d\tan(fx+e))^{\frac{3}{2}}}+\frac{1}{d^4\sqrt{d\tan(fx+e)}}+\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{2d^{\frac{9}{2}}}+\frac{-(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+\cdots}{d\tan(fx+\cdots}\right)\cdots\right.}{\cdots}\right.$
default	$2d^2\left(-\frac{1}{3d^3(d\tan(fx+e))^{\frac{3}{2}}}+\frac{1}{d^4\sqrt{d\tan(fx+e)}}+\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{2d^{\frac{9}{2}}}+\frac{-(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+\cdots}{d\tan(fx+\cdots}\right)\cdots\right.}{\cdots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*tan(f*x+e))^(5/2)/(a+a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a*d^2*(-1/3/d^3/(d*tan(f*x+e))^(3/2)+1/d^4/(d*tan(f*x+e))^(1/2)+1/2/d^(
9/2)*arctan((d*tan(f*x+e))^(1/2)/d^(1/2))+1/2/d^4*(-1/8/d*(d^2)^(1/4)*2^(1/
2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/
(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arct
an(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4
)*(d*tan(f*x+e))^(1/2)+1))+1/8/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^
(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(
d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*t
```

```
an(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)))
)
```

Maxima [A]
time = 0.51, size = 151, normalized size = 1.12

$$-\frac{\frac{3\left(\frac{\sqrt{2}\log\left(d\tan\left(fx+e\right)+\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{d}+d\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan\left(fx+e\right)-\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{d}+d\right)}{\sqrt{d}}\right)}{ad}-\frac{12\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}}{\sqrt{d}}\right)}{ad^{\frac{3}{2}}}-\frac{8\left(3\,d\tan\left(fx+e\right)-d\right)}{\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}ad}}{12\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(5/2)/(a+a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/12*(3*(sqrt(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d)
 + d)/sqrt(d) - sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*s
qrt(d) + d)/sqrt(d))/(a*d) - 12*arctan(sqrt(d*tan(f*x + e))/sqrt(d))/(a*d^(
3/2)) - 8*(3*d*tan(f*x + e) - d)/((d*tan(f*x + e))^(3/2)*a*d))/(d*f)
```

Fricas [A]
time = 0.98, size = 326, normalized size = 2.41

$$\left[\frac{3\sqrt{2}\sqrt{-d}\arctan\left(\frac{\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{-d}\left(\tan\left(fx+e\right)+1\right)}{2\,d\tan\left(fx+e\right)}\right)\tan\left(fx+e\right)^{2}-3\sqrt{-d}\log\left(\frac{d\tan\left(fx+e\right)-2\sqrt{d\tan\left(fx+e\right)}\sqrt{-d}-d}{\tan\left(fx+e\right)+1}\right)\tan\left(fx+e\right)^{2}+4\sqrt{d\tan\left(fx+e\right)}\left(3\tan\left(fx+e\right)-1\right)}{6\,ad^{3}f\tan\left(fx+e\right)^{2}},\ \frac{3\sqrt{2}\sqrt{d}\log\left(\frac{d\tan\left(fx+e\right)^{2}-2\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{d}\left(\tan\left(fx+e\right)+1\right)+4\,d\tan\left(fx+e\right)+d}{\tan\left(fx+e\right)^{2}+1}\right)\tan\left(fx+e\right)^{2}+12\sqrt{d}\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}}{\sqrt{d}}\right)\tan\left(fx+e\right)^{2}+8\sqrt{d\tan\left(fx+e\right)}\left(3\tan\left(fx+e\right)-1\right)}{12\,ad^{3}f\tan\left(fx+e\right)^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(5/2)/(a+a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(2)*sqrt(-d)*arctan(1/2*sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(-d)*(
tan(f*x + e) + 1)/(d*tan(f*x + e)))*tan(f*x + e)^2 - 3*sqrt(-d)*log((d*tan(
f*x + e) - 2*sqrt(d*tan(f*x + e))*sqrt(-d) - d)/(tan(f*x + e) + 1))*tan(f*x
 + e)^2 + 4*sqrt(d*tan(f*x + e))*(3*tan(f*x + e) - 1))/(a*d^3*f*tan(f*x + e
)^2), 1/12*(3*sqrt(2)*sqrt(d)*log((d*tan(f*x + e)^2 - 2*sqrt(2)*sqrt(d*tan(
f*x + e))*sqrt(d)*(tan(f*x + e) + 1) + 4*d*tan(f*x + e) + d)/(tan(f*x + e)^
2 + 1))*tan(f*x + e)^2 + 12*sqrt(d)*arctan(sqrt(d*tan(f*x + e))/sqrt(d))*ta
n(f*x + e)^2 + 8*sqrt(d*tan(f*x + e))*(3*tan(f*x + e) - 1))/(a*d^3*f*tan(f*
x + e)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\left(d\tan\left(e+fx\right)\right)^{\frac{5}{2}}\tan\left(e+fx\right)+\left(d\tan\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))**(5/2)/(a+a*tan(f*x+e)),x)
```

```
[Out] Integral(1/((d*tan(e + f*x))**(5/2)*tan(e + f*x) + (d*tan(e + f*x))**(5/2))
, x)/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(117) = 234.
time = 0.75, size = 305, normalized size = 2.26

$$-\frac{\sqrt{2}\left(d\sqrt{|d|}-|d|^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{4\,ad^4f}-\frac{\sqrt{2}\left(d\sqrt{|d|}-|d|^{\frac{3}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{4\,ad^4f}+\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{ad^{\frac{5}{2}}f}-\frac{\sqrt{2}\left(d\sqrt{|d|}+|d|^{\frac{3}{2}}\right)\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{|d|}+|d|\right)}{8\,ad^4f}+\frac{\sqrt{2}\left(d\sqrt{|d|}+|d|^{\frac{3}{2}}\right)\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{|d|}+|d|\right)}{8\,ad^4f}+\frac{2\,(3\,d\tan(fx+e)-d)}{3\sqrt{d\tan(fx+e)}\,ad^3f\tan(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(5/2)/(a+a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(d*sqrt(abs(d)) - abs(d)^(3/2))*arctan(1/2*sqrt(2)*(sqrt(2)*sq
rt(abs(d)) + 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(a*d^4*f) - 1/4*sqrt(2)*
(d*sqrt(abs(d)) - abs(d)^(3/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) -
 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(a*d^4*f) + arctan(sqrt(d*tan(f*x +
e))/sqrt(d))/(a*d^(5/2)*f) - 1/8*sqrt(2)*(d*sqrt(abs(d)) + abs(d)^(3/2))*lo
g(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(a*d
^4*f) + 1/8*sqrt(2)*(d*sqrt(abs(d)) + abs(d)^(3/2))*log(d*tan(f*x + e) - sq
rt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(a*d^4*f) + 2/3*(3*d*tan(
f*x + e) - d)/(sqrt(d*tan(f*x + e))*a*d^3*f*tan(f*x + e))
```

Mupad [B]
time = 4.82, size = 130, normalized size = 0.96

$$\frac{\frac{2\tan(e+f\,x)}{d}-\frac{2}{3\,d}}{a\,f\,(d\tan(e+f\,x))^{3/2}}+\frac{\mathrm{atan}\left(\frac{\sqrt{d\tan(e+f\,x)}}{\sqrt{d}}\right)}{a\,d^{5/2}\,f}-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{12\sqrt{2}\,a^3\,d^{21/2}\,f^3\,\sqrt{d\tan(e+f\,x)}}{12\,a^3\,d^{11}\,f^3+12\,a^3\,d^{11}\,f^3\tan(e+f\,x)}\right)}{2\,a\,d^{5/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*tan(e + f*x))^(5/2)*(a + a*tan(e + f*x))),x)
```

```
[Out] ((2*tan(e + f*x))/d - 2/(3*d))/(a*f*(d*tan(e + f*x))^(3/2)) + atan((d*tan(e
 + f*x))^(1/2)/d^(1/2))/(a*d^(5/2)*f) - (2^(1/2)*atanh((12*2^(1/2)*a^3*d^(2
1/2)*f^3*(d*tan(e + f*x))^(1/2))/(12*a^3*d^11*f^3 + 12*a^3*d^11*f^3*tan(e +
 f*x))))/(2*a*d^(5/2)*f)
```

3.364 $\int \frac{(d\tan(e+fx))^{5/2}}{(a+a\tan(e+fx))^2}\,dx$

Optimal. Leaf size=281

$$\frac{3d^{5/2}\text{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2f}+\frac{d^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2\sqrt{2}\,a^2f}-\frac{d^{5/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2\sqrt{2}\,a^2f}$$

```
[Out] 3/2*d^(5/2)*arctan((d*tan(f*x+e))^(1/2)/d^(1/2))/a^2/f+1/4*d^(5/2)*arctan(1
-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a^2/f*2^(1/2)-1/4*d^(5/2)*arctan(1+2
^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a^2/f*2^(1/2)+1/8*d^(5/2)*ln(d^(1/2)-2
^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/a^2/f*2^(1/2)-1/8*d^(5/2)*l
n(d^(1/2)+2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/a^2/f*2^(1/2)-1/
2*d^2*(d*tan(f*x+e))^(1/2)/f/(a^2+a^2*tan(f*x+e))
```

Rubi [A]
time = 0.34, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3646, 3734, 12, 3557, 335, 217, 1179, 642, 1176, 631, 210, 3715, 65, 211}

$$\frac{3d^{5/2}\text{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2f}+\frac{d^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2\sqrt{2}\,a^2f}-\frac{d^{5/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{2\sqrt{2}\,a^2f}+\frac{d^{5/2}\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{4\sqrt{2}\,a^2f}-\frac{d^{5/2}\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{4\sqrt{2}\,a^2f}-\frac{d^2\sqrt{d\tan(e+fx)}}{2f\left(a^2\tan(e+fx)+a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(5/2)/(a + a*Tan[e + f*x])^2,x]
```

```
[Out] (3*d^(5/2)*ArcTan[Sqrt[d*Tan[e + f*x]]/Sqrt[d]])/(2*a^2*f) + (d^(5/2)*ArcTa
n[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(2*Sqrt[2]*a^2*f) - (d^(5/2)
*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(2*Sqrt[2]*a^2*f) + (d
^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e + f*x]]])/
(4*Sqrt[2]*a^2*f) - (d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqrt[2]*S
qrt[d*Tan[e + f*x]]])/(4*Sqrt[2]*a^2*f) - (d^2*Sqrt[d*Tan[e + f*x]])/(2*f*(
a^2 + a^2*Tan[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^{5/2}}{(a+a\tan(e+fx))^2}\,dx = -\frac{d^2\sqrt{d\tan(e+fx)}}{2f\left(a^2+a^2\tan(e+fx)\right)} + \frac{\int \frac{\frac{a^2d^3}{2}-a^2d^3\tan(e+fx)+\frac{3}{2}a^2d^3\tan^2(e+fx)}{\sqrt{d\tan(e+fx)}\,(a+a\tan(e+fx))}\,dx}{2a^3}$$

$$= -\frac{d^2\sqrt{d\tan(e+fx)}}{2f\left(a^2+a^2\tan(e+fx)\right)} + \frac{\int -\frac{2a^3d^3}{\sqrt{d\tan(e+fx)}}\,dx}{4a^5} + \frac{(3d^3)\int \frac{1+\tan}{\sqrt{d\tan(e+f}}}{4a}$$

$$= -\frac{d^2\sqrt{d\tan(e+fx)}}{2f\left(a^2+a^2\tan(e+fx)\right)} - \frac{d^3\int \frac{1}{\sqrt{d\tan(e+fx)}}\,dx}{2a^2} + \frac{(3d^3)\,\text{Subst}\left(\int \frac{}{\sqrt{dx}}\right.}{}$$

$$= -\frac{d^2\sqrt{d\tan(e+fx)}}{2f\left(a^2+a^2\tan(e+fx)\right)} + \frac{(3d^2)\,\text{Subst}\left(\int \frac{1}{a+\frac{ax^2}{d}}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{2af} -$$

$$= \frac{3d^{5/2}\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2f} - \frac{d^2\sqrt{d\tan(e+fx)}}{2f\left(a^2+a^2\tan(e+fx)\right)} - \frac{d^4\,\text{Subst}\left(\int \frac{1}{d^2+}\right.}{}$$

$$= \frac{3d^{5/2}\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2f} - \frac{d^2\sqrt{d\tan(e+fx)}}{2f\left(a^2+a^2\tan(e+fx)\right)} - \frac{d^3\,\text{Subst}\left(\int \frac{d-}{d^2+}\right.}{}$$

$$= \frac{3d^{5/2}\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2f} - \frac{d^2\sqrt{d\tan(e+fx)}}{2f\left(a^2+a^2\tan(e+fx)\right)} + \frac{d^{5/2}\,\text{Subst}\left(\int -\right.}{}$$

$$= \frac{3d^{5/2}\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2f} + \frac{d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{}\right.}{4\sqrt{2}\,a^2f}$$

$$= \frac{3d^{5/2}\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2f} + \frac{d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2\sqrt{2}\,a^2f} -$$

Mathematica [A]

time = 3.55, size = 226, normalized size = 0.80

$$\frac{\csc(e+fx)(\cos(e+fx)+\sin(e+fx))^2\left(-\frac{4\cot(e+fx)}{\cos(e+fx)+\sin(e+fx)}+\frac{\left(2\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)-2\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(e+fx)}\right)+12\text{ArcTan}\left(\sqrt{\tan(e+fx)}\right)+\sqrt{2}\log\left(-1+\sqrt{2}\sqrt{\tan(e+fx)}-\tan(e+fx)\right)-\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)\right)\sec(e+fx)}{\tan^{\frac{3}{2}}(e+fx)}\right)(d\tan(e+fx))^{5/2}}{8a^2f(1+\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(5/2)/(a + a*Tan[e + f*x])^2,x]
```

```
[Out] (Csc[e + f*x]*(Cos[e + f*x] + Sin[e + f*x])^2*((-4*Cot[e + f*x])/(Cos[e + f
*x] + Sin[e + f*x]) + ((2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] -
```

```
2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] + 12*ArcTan[Sqrt[Tan[e + f
*x]]] + Sqrt[2]*Log[-1 + Sqrt[2]*Sqrt[Tan[e + f*x]] - Tan[e + f*x]] - Sqrt[
2]*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]])*Sec[e + f*x])/Tan[e
+ f*x]^(3/2))*(d*Tan[e + f*x])^(5/2))/(8*a^2*f*(1 + Tan[e + f*x])^2)
```

Maple [A]
time = 0.16, size = 191, normalized size = 0.68

method	result
derivativedivides	$\frac{2d^{3}\left(-\frac{\sqrt{d\tan\left(fx+e\right)}}{4(d\tan(fx+e)+d)}+\frac{3\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}}{\sqrt{d}}\right)}{4\sqrt{d}}-\frac{\left(d^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+\left(d^{2}\right)^{\frac{1}{4}}\sqrt{d\tan}}{d\tan(fx+e)-\left(d^{2}\right)^{\frac{1}{4}}\sqrt{d\tan}}\right.\right.}{}\right.}{}$
default	$\frac{2d^{3}\left(-\frac{\sqrt{d\tan\left(fx+e\right)}}{4(d\tan(fx+e)+d)}+\frac{3\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}}{\sqrt{d}}\right)}{4\sqrt{d}}-\frac{\left(d^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+\left(d^{2}\right)^{\frac{1}{4}}\sqrt{d\tan}}{d\tan(fx+e)-\left(d^{2}\right)^{\frac{1}{4}}\sqrt{d\tan}}\right.\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(5/2)/(a+a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^2*d^3*(-1/4*(d*tan(f*x+e))^(1/2)/(d*tan(f*x+e)+d)+3/4/d^(1/2)*arctan(
(d*tan(f*x+e))^(1/2)/d^(1/2))-1/16/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+
(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(
1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4
)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2
)+1)))
```

Maxima [A]
time = 0.53, size = 226, normalized size = 0.80

$$-\frac{\frac{4\sqrt{d\tan\left(fx+e\right)}\,d^{4}}{a^{2}d\tan(fx+e)+a^{2}d}-\frac{12\,d^{\frac{7}{2}}\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}}{\sqrt{d}}\right)}{a^{2}}+\frac{2\sqrt{2}\,d^{\frac{7}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{d}}\right)+2\sqrt{2}\,d^{\frac{7}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{d}}\right)+\sqrt{2}\,d^{\frac{7}{2}}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{d}+d\right)-\sqrt{2}\,d^{\frac{7}{2}}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{d}+d\right)}{a^{2}}}{8\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)/(a+a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(4*sqrt(d*tan(f*x + e))*d^4/(a^2*d*tan(f*x + e) + a^2*d) - 12*d^(7/2)*
arctan(sqrt(d*tan(f*x + e))/sqrt(d))/a^2 + (2*sqrt(2)*d^(7/2)*arctan(1/2*sq
rt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(f*x + e)))/sqrt(d)) + 2*sqrt(2)*d^(7/
2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))
+ sqrt(2)*d^(7/2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d)
```

```
 + d) - sqrt(2)*d^(7/2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*s
qrt(d) + d))/a^2)/(d*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 879 vs. $2(226) = 452$.
time = 1.50, size = 1840, normalized size = 6.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)/(a+a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/8*(3*(2*d^2*cos(f*x + e)*sin(f*x + e) + d^2)*sqrt(-d)*log(-(6*d^2*cos(f*
x + e)*sin(f*x + e) - d^2 + 4*(d*cos(f*x + e)^2 - d*cos(f*x + e)*sin(f*x +
e))*sqrt(-d)*sqrt(d*sin(f*x + e)/cos(f*x + e)))/(2*cos(f*x + e)*sin(f*x + e
) + 1)) + 4*(2*sqrt(2)*a^2*f*cos(f*x + e)*sin(f*x + e) + sqrt(2)*a^2*f)*(d^
10/(a^8*f^4))^(1/4)*arctan(-(sqrt(2)*a^6*d^2*f^3*sqrt(d*sin(f*x + e)/cos(f*
x + e))*(d^10/(a^8*f^4))^(3/4) - sqrt(2)*a^6*f^3*sqrt((a^4*f^2*sqrt(d^10/(a
^8*f^4))*cos(f*x + e) + sqrt(2)*a^2*d^2*f*sqrt(d*sin(f*x + e)/cos(f*x + e))
*(d^10/(a^8*f^4))^(1/4)*cos(f*x + e) + d^5*sin(f*x + e))/cos(f*x + e))*(d^1
0/(a^8*f^4))^(3/4) + d^10)/d^10) + 4*(2*sqrt(2)*a^2*f*cos(f*x + e)*sin(f*x
+ e) + sqrt(2)*a^2*f)*(d^10/(a^8*f^4))^(1/4)*arctan(-(sqrt(2)*a^6*d^2*f^3*s
qrt(d*sin(f*x + e)/cos(f*x + e))*(d^10/(a^8*f^4))^(3/4) - sqrt(2)*a^6*f^3*s
qrt((a^4*f^2*sqrt(d^10/(a^8*f^4))*cos(f*x + e) - sqrt(2)*a^2*d^2*f*sqrt(d*s
in(f*x + e)/cos(f*x + e))*(d^10/(a^8*f^4))^(1/4)*cos(f*x + e) + d^5*sin(f*x
 + e))/cos(f*x + e))*(d^10/(a^8*f^4))^(3/4) - d^10)/d^10) - (2*sqrt(2)*a^2*
f*cos(f*x + e)*sin(f*x + e) + sqrt(2)*a^2*f)*(d^10/(a^8*f^4))^(1/4)*log((a^
4*f^2*sqrt(d^10/(a^8*f^4))*cos(f*x + e) + sqrt(2)*a^2*d^2*f*sqrt(d*sin(f*x
+ e)/cos(f*x + e))*(d^10/(a^8*f^4))^(1/4)*cos(f*x + e) + d^5*sin(f*x + e))/
cos(f*x + e)) + (2*sqrt(2)*a^2*f*cos(f*x + e)*sin(f*x + e) + sqrt(2)*a^2*f)
*(d^10/(a^8*f^4))^(1/4)*log((a^4*f^2*sqrt(d^10/(a^8*f^4))*cos(f*x + e) - sq
rt(2)*a^2*d^2*f*sqrt(d*sin(f*x + e)/cos(f*x + e))*(d^10/(a^8*f^4))^(1/4)*co
s(f*x + e) + d^5*sin(f*x + e))/cos(f*x + e)) - 4*(d^2*cos(f*x + e)^2 + d^2*
cos(f*x + e)*sin(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e)))/(2*a^2*f*cos(
f*x + e)*sin(f*x + e) + a^2*f), 1/8*(12*(2*d^2*cos(f*x + e)*sin(f*x + e) +
d^2)*sqrt(d)*arctan(sqrt(d*sin(f*x + e)/cos(f*x + e))/sqrt(d)) + 4*(2*sqrt(
2)*a^2*f*cos(f*x + e)*sin(f*x + e) + sqrt(2)*a^2*f)*(d^10/(a^8*f^4))^(1/4)*
arctan(-(sqrt(2)*a^6*d^2*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e))*(d^10/(a^8*f
^4))^(3/4) - sqrt(2)*a^6*f^3*sqrt((a^4*f^2*sqrt(d^10/(a^8*f^4))*cos(f*x + e
) + sqrt(2)*a^2*d^2*f*sqrt(d*sin(f*x + e)/cos(f*x + e))*(d^10/(a^8*f^4))^(1
/4)*cos(f*x + e) + d^5*sin(f*x + e))/cos(f*x + e))*(d^10/(a^8*f^4))^(3/4) +
 d^10)/d^10) + 4*(2*sqrt(2)*a^2*f*cos(f*x + e)*sin(f*x + e) + sqrt(2)*a^2*f
)*(d^10/(a^8*f^4))^(1/4)*arctan(-(sqrt(2)*a^6*d^2*f^3*sqrt(d*sin(f*x + e)/c
os(f*x + e))*(d^10/(a^8*f^4))^(3/4) - sqrt(2)*a^6*f^3*sqrt((a^4*f^2*sqrt(d^
10/(a^8*f^4))*cos(f*x + e) - sqrt(2)*a^2*d^2*f*sqrt(d*sin(f*x + e)/cos(f*x
```

```
+ e))*(d^10/(a^8*f^4))^(1/4)*cos(f*x + e) + d^5*sin(f*x + e))/cos(f*x + e))
*(d^10/(a^8*f^4))^(3/4) - d^10)/d^10) - (2*sqrt(2)*a^2*f*cos(f*x + e)*sin(f
*x + e) + sqrt(2)*a^2*f)*(d^10/(a^8*f^4))^(1/4)*log((a^4*f^2*sqrt(d^10/(a^8
*f^4))*cos(f*x + e) + sqrt(2)*a^2*d^2*f*sqrt(d*sin(f*x + e)/cos(f*x + e))*(
d^10/(a^8*f^4))^(1/4)*cos(f*x + e) + d^5*sin(f*x + e))/cos(f*x + e)) + (2*s
qrt(2)*a^2*f*cos(f*x + e)*sin(f*x + e) + sqrt(2)*a^2*f)*(d^10/(a^8*f^4))^(1
/4)*log((a^4*f^2*sqrt(d^10/(a^8*f^4))*cos(f*x + e) - sqrt(2)*a^2*d^2*f*sqrt
(d*sin(f*x + e)/cos(f*x + e))*(d^10/(a^8*f^4))^(1/4)*cos(f*x + e) + d^5*sin
(f*x + e))/cos(f*x + e)) - 4*(d^2*cos(f*x + e)^2 + d^2*cos(f*x + e)*sin(f*x
 + e))*sqrt(d*sin(f*x + e)/cos(f*x + e)))/(2*a^2*f*cos(f*x + e)*sin(f*x + e
) + a^2*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(d\tan(e+fx))^{\frac{5}{2}}}{\tan^2(e+fx)+2\tan(e+fx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(5/2)/(a+a*tan(f*x+e))**2,x)
```

```
[Out] Integral((d*tan(e + f*x))**(5/2)/(tan(e + f*x)**2 + 2*tan(e + f*x) + 1), x)
/a**2
```

Giac [A]
time = 0.78, size = 254, normalized size = 0.90

$$-\frac{1}{8}d^2\left(\frac{2\sqrt{2}\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{a^2f}+\frac{2\sqrt{2}\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{a^2f}+\frac{\sqrt{2}\sqrt{|d|}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{|d|}+|d|\right)}{a^2f}-\frac{\sqrt{2}\sqrt{|d|}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{|d|}+|d|\right)}{a^2f}-\frac{12\sqrt{d}\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{a^2f}+\frac{4\sqrt{d\tan(fx+e)}\,d}{(d\tan(fx+e)+d)a^2f}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)/(a+a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/8*d^2*(2*sqrt(2)*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) +
 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(a^2*f) + 2*sqrt(2)*sqrt(abs(d))*arc
tan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d
)))/(a^2*f) + sqrt(2)*sqrt(abs(d))*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(
f*x + e))*sqrt(abs(d)) + abs(d))/(a^2*f) - sqrt(2)*sqrt(abs(d))*log(d*tan(f
*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(a^2*f) - 12*
sqrt(d)*arctan(sqrt(d*tan(f*x + e))/sqrt(d))/(a^2*f) + 4*sqrt(d*tan(f*x + e
))*d/((d*tan(f*x + e) + d)*a^2*f))
```

Mupad [B]
time = 4.69, size = 376, normalized size = 1.34

$$\frac{\mathrm{atan}\left(\frac{4\,d^{20}\,\sqrt{d\tan(e+f\,x)}\,\left(-\frac{d^{10}}{a^8\,f^4}\right)^{1/4}}{\frac{36\,d^{23}}{a^2\,f}-4\,a^2\,d^{18}\,f\,\sqrt{-\frac{d^{10}}{a^8\,f^4}}}+\frac{36\,d^{15}\,\sqrt{d\tan(e+f\,x)}\,\left(-\frac{d^{10}}{a^8\,f^4}\right)^{3/4}}{\frac{36\,d^{23}}{a^6\,f^3}-\frac{4\,d^{18}\,\sqrt{-\frac{d^{10}}{a^8\,f^4}}}{a^2\,f}}\right)\left(-\frac{d^{10}}{a^8\,f^4}\right)^{1/4}}{2}+\mathrm{atan}\left(\frac{d^{20}\,\sqrt{d\tan(e+f\,x)}\,\left(-\frac{d^{10}}{256\,a^8\,f^4}\right)^{1/4}16\mathrm{i}}{\frac{36\,d^{23}}{a^2\,f}+64\,a^2\,d^{18}\,f\,\sqrt{-\frac{d^{10}}{256\,a^8\,f^4}}}-\frac{d^{15}\,\sqrt{d\tan(e+f\,x)}\,\left(-\frac{d^{10}}{256\,a^8\,f^4}\right)^{3/4}2304\mathrm{i}}{\frac{36\,d^{23}}{a^6\,f^3}+\frac{64\,d^{18}\,\sqrt{-\frac{d^{10}}{256\,a^8\,f^4}}}{a^2\,f}}\right)\left(-\frac{d^{10}}{256\,a^8\,f^4}\right)^{1/4}2\mathrm{i}-\frac{d^3\,\sqrt{d\tan(e+f\,x)}}{2\,\left(a^2\,d\,f+a^2\,d\,f\tan(e+f\,x)\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{d\tan(e+f\,x)}\,\sqrt{-d^5}\,1\mathrm{i}}{d^3}\right)\sqrt{-d^5}\,3\mathrm{i}}{2\,a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(5/2)/(a + a*tan(e + f*x))^2,x)
```

```
[Out] (atan((4*d^20*(d*tan(e + f*x))^(1/2)*(-d^10/(a^8*f^4))^(1/4))/((36*d^23)/(a
^2*f) - 4*a^2*d^18*f*(-d^10/(a^8*f^4))^(1/2)) + (36*d^15*(d*tan(e + f*x))^(
1/2)*(-d^10/(a^8*f^4))^(3/4))/((36*d^23)/(a^6*f^3) - (4*d^18*(-d^10/(a^8*f^
4))^(1/2))/(a^2*f)))*(-d^10/(a^8*f^4))^(1/4))/2 + atan((d^20*(d*tan(e + f*x
))^(1/2)*(-d^10/(256*a^8*f^4))^(1/4)*16i)/((36*d^23)/(a^2*f) + 64*a^2*d^18*
f*(-d^10/(256*a^8*f^4))^(1/2)) - (d^15*(d*tan(e + f*x))^(1/2)*(-d^10/(256*a
^8*f^4))^(3/4)*2304i)/((36*d^23)/(a^6*f^3) + (64*d^18*(-d^10/(256*a^8*f^4))
^(1/2))/(a^2*f)))*(-d^10/(256*a^8*f^4))^(1/4)*2i - (d^3*(d*tan(e + f*x))^(1
/2))/(2*(a^2*d*f + a^2*d*f*tan(e + f*x))) + (atan(((d*tan(e + f*x))^(1/2)*(
-d^5)^(1/2)*1i)/d^3)*(-d^5)^(1/2)*3i)/(2*a^2*f)
```

3.365 $\int \frac{(d\tan(e+fx))^{3/2}}{(a+a\tan(e+fx))^2}\,dx$

Optimal. Leaf size=279

$$-\frac{d^{3/2}\text{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2f}-\frac{d^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2\sqrt{2}a^2f}+\frac{d^{3/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2\sqrt{2}a^2f}$$

```
[Out] -1/2*d^(3/2)*arctan((d*tan(f*x+e))^(1/2)/d^(1/2))/a^2/f-1/4*d^(3/2)*arctan(
1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a^2/f*2^(1/2)+1/4*d^(3/2)*arctan(1+
2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a^2/f*2^(1/2)+1/8*d^(3/2)*ln(d^(1/2)-
2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/a^2/f*2^(1/2)-1/8*d^(3/2)*
ln(d^(1/2)+2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/a^2/f*2^(1/2)+1
/2*d*(d*tan(f*x+e))^(1/2)/f/(a^2+a^2*tan(f*x+e))
```

Rubi [A]
time = 0.35, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3648, 3734, 12, 16, 3557, 335, 303, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{d^{3/2}\text{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2f}-\frac{d^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2\sqrt{2}a^2f}+\frac{d^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{2\sqrt{2}a^2f}+\frac{d^{3/2}\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{4\sqrt{2}a^2f}-\frac{d^{3/2}\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{4\sqrt{2}a^2f}+\frac{d\sqrt{d\tan(e+fx)}}{2f\left(a^2\tan(e+fx)+a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(3/2)/(a + a*Tan[e + f*x])^2,x]
```

```
[Out] -1/2*(d^(3/2)*ArcTan[Sqrt[d*Tan[e + f*x]]/Sqrt[d]])/(a^2*f) - (d^(3/2)*ArcT
an[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(2*Sqrt[2]*a^2*f) + (d^(3/2
)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(2*Sqrt[2]*a^2*f) + (
d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e + f*x]]])
/(4*Sqrt[2]*a^2*f) - (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqrt[2]*
Sqrt[d*Tan[e + f*x]]])/(4*Sqrt[2]*a^2*f) + (d*Sqrt[d*Tan[e + f*x]])/(2*f*(a
^2 + a^2*Tan[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
```

```
!GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^{3/2}}{(a+a\tan(e+fx))^2}\,dx = \frac{d\sqrt{d\tan(e+fx)}}{2f\left(a^2+a^2\tan(e+fx)\right)} - \frac{\int \frac{\frac{ad^2}{2}-ad^2\tan(e+fx)-\frac{1}{2}ad^2\tan^2(e+fx)}{\sqrt{d\tan(e+fx)}\,(a+a\tan(e+fx))}\,dx}{2a^2}$$

$$= \frac{d\sqrt{d\tan(e+fx)}}{2f\left(a^2+a^2\tan(e+fx)\right)} - \frac{\int -\frac{2a^2d^2\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx}{4a^4} - \frac{d^2\int \frac{1+\tan^2(e+}{\sqrt{d\tan(e+fx)}\,(a}}{4a}$$

$$= \frac{d\sqrt{d\tan(e+fx)}}{2f\left(a^2+a^2\tan(e+fx)\right)} + \frac{d^2\int \frac{\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx}{2a^2} - \frac{d^2\text{Subst}\Big(\int \frac{1}{\sqrt{dx}\,(a+ax}}{4a}$$

$$= \frac{d\sqrt{d\tan(e+fx)}}{2f\left(a^2+a^2\tan(e+fx)\right)} + \frac{d\int \sqrt{d\tan(e+fx)}\,dx}{2a^2} - \frac{d\text{Subst}\Big(\int \frac{1}{a+\frac{ax^2}{d}}\,dx, x,}{2af}$$

$$= -\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2f} + \frac{d\sqrt{d\tan(e+fx)}}{2f\left(a^2+a^2\tan(e+fx)\right)} + \frac{d^2\text{Subst}\Big(\int \frac{\sqrt{}}{d^2}}{}$$

$$= -\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2f} + \frac{d\sqrt{d\tan(e+fx)}}{2f\left(a^2+a^2\tan(e+fx)\right)} + \frac{d^2\text{Subst}\Big(\int \frac{x}{d^2+}}{}$$

$$= -\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2f} + \frac{d\sqrt{d\tan(e+fx)}}{2f\left(a^2+a^2\tan(e+fx)\right)} - \frac{d^2\text{Subst}\Big(\int \frac{d-}{d^2+}}{}$$

$$= -\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2f} + \frac{d\sqrt{d\tan(e+fx)}}{2f\left(a^2+a^2\tan(e+fx)\right)} + \frac{d^{3/2}\text{Subst}\Big(\int}{}$$

$$= -\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2f} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\right.}{4\sqrt{2}\,a^2f}$$

$$= -\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2f} - \frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2\sqrt{2}\,a^2f} +$$

Mathematica [A]

time = 2.17, size = 229, normalized size = 0.82

$$\frac{\sec(e+fx)(\cos(e+fx)+\sin(e+fx))^2\left(\frac{2\cot(e+fx)}{\cos(e+fx)+\sin(e+fx)}-\frac{\csc(e+fx)\left(2\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)-2\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(e+fx)}\right)+4\text{ArcTan}\left(\sqrt{\tan(e+fx)}\right)-\sqrt{2}\log\left(-1+\sqrt{2}\sqrt{\tan(e+fx)}-\tan(e+fx)\right)+\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)\right)}{2\sqrt{\tan(e+fx)}}\right)(d\tan(e+fx))^{3/2}}{4a^2f(1+\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(3/2)/(a + a*Tan[e + f*x])^2,x]
```

```
[Out] (Sec[e + f*x]*(Cos[e + f*x] + Sin[e + f*x])^2*((2*Cot[e + f*x])/(Cos[e + f*
x] + Sin[e + f*x]) - (Csc[e + f*x]*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e
 + f*x]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] + 4*ArcTan[Sqr
t[Tan[e + f*x]]] - Sqrt[2]*Log[-1 + Sqrt[2]*Sqrt[Tan[e + f*x]] - Tan[e + f*
x]] + Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]))/(2*Sqrt[
Tan[e + f*x]]))*(d*Tan[e + f*x])^(3/2))/(4*a^2*f*(1 + Tan[e + f*x])^2)
```

Maple [A]

time = 0.14, size = 197, normalized size = 0.71

method	result
derivativedivides	$\frac{2d^3\left(-\frac{-\frac{\sqrt{d\tan(fx+e)}}{2(d\tan(fx+e)+d)}+\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{2\sqrt{d}}}{2d}+\frac{\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx \ldots}{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx \ldots}\right.\ldots}{\ldots}\right.}{f \ldots}$
default	$\frac{2d^3\left(-\frac{-\frac{\sqrt{d\tan(fx+e)}}{2(d\tan(fx+e)+d)}+\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{2\sqrt{d}}}{2d}+\frac{\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx \ldots}{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx \ldots}\right.\ldots}{\ldots}\right.}{f \ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(3/2)/(a+a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^2*d^3*(-1/2/d*(-1/2*(d*tan(f*x+e))^(1/2)/(d*tan(f*x+e)+d)+1/2/d^(1/2)
*arctan((d*tan(f*x+e))^(1/2)/d^(1/2)))+1/16/d/(d^2)^(1/4)*2^(1/2)*(ln((d*ta
n(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e
)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(
d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x
+e))^(1/2)+1)))
```

Maxima [A]

time = 0.51, size = 229, normalized size = 0.82

$$\frac{\frac{4\sqrt{d\tan(fx+e)}\,d^3}{a^2d\tan(fx+e)+a^2d}+\frac{d^3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)}{a^2}-\frac{4d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{a^2}}{8\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(a+a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(4*sqrt(d*tan(f*x + e))*d^3/(a^2*d*tan(f*x + e) + a^2*d) + d^3*(2*sqrt(
2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(f*x + e)))/sqrt(d))/s
qrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*sqrt(d*tan(f*x
+ e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f
*x + e))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d
*tan(f*x + e))*sqrt(d) + d)/sqrt(d))/a^2 - 4*d^(5/2)*arctan(sqrt(d*tan(f*x
+ e))/sqrt(d))/a^2)/(d*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 887 vs. $2(224) = 448$.
time = 1.64, size = 1855, normalized size = 6.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(a+a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/8*((2*d*cos(f*x + e)*sin(f*x + e) + d)*sqrt(-d)*log(-(6*d^2*cos(f*x + e)
*sin(f*x + e) - d^2 - 4*(d*cos(f*x + e)^2 - d*cos(f*x + e)*sin(f*x + e))*sq
rt(-d)*sqrt(d*sin(f*x + e)/cos(f*x + e)))/(2*cos(f*x + e)*sin(f*x + e) + 1)
) - 4*(2*sqrt(2)*a^2*f*cos(f*x + e)*sin(f*x + e) + sqrt(2)*a^2*f)*(d^6/(a^8
*f^4))^(1/4)*arctan(-(sqrt(2)*a^2*d^4*f*sqrt(d*sin(f*x + e)/cos(f*x + e))*(
d^6/(a^8*f^4))^(1/4) + d^6 - sqrt(2)*a^2*f*sqrt((sqrt(2)*a^6*d^4*f^3*sqrt(d
*sin(f*x + e)/cos(f*x + e))*(d^6/(a^8*f^4))^(3/4)*cos(f*x + e) + a^4*d^6*f^
2*sqrt(d^6/(a^8*f^4))*cos(f*x + e) + d^9*sin(f*x + e))/cos(f*x + e))*(d^6/(
a^8*f^4))^(1/4))/d^6) - 4*(2*sqrt(2)*a^2*f*cos(f*x + e)*sin(f*x + e) + sqrt
(2)*a^2*f)*(d^6/(a^8*f^4))^(1/4)*arctan(-(sqrt(2)*a^2*d^4*f*sqrt(d*sin(f*x
+ e)/cos(f*x + e))*(d^6/(a^8*f^4))^(1/4) - d^6 - sqrt(2)*a^2*f*sqrt(-(sqrt(
2)*a^6*d^4*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e))*(d^6/(a^8*f^4))^(3/4)*cos(
f*x + e) - a^4*d^6*f^2*sqrt(d^6/(a^8*f^4))*cos(f*x + e) - d^9*sin(f*x + e))
/cos(f*x + e))*(d^6/(a^8*f^4))^(1/4))/d^6) - (2*sqrt(2)*a^2*f*cos(f*x + e)*
sin(f*x + e) + sqrt(2)*a^2*f)*(d^6/(a^8*f^4))^(1/4)*log((sqrt(2)*a^6*d^4*f^
3*sqrt(d*sin(f*x + e)/cos(f*x + e))*(d^6/(a^8*f^4))^(3/4)*cos(f*x + e) + a^
4*d^6*f^2*sqrt(d^6/(a^8*f^4))*cos(f*x + e) + d^9*sin(f*x + e))/cos(f*x + e)
) + (2*sqrt(2)*a^2*f*cos(f*x + e)*sin(f*x + e) + sqrt(2)*a^2*f)*(d^6/(a^8*f
^4))^(1/4)*log(-(sqrt(2)*a^6*d^4*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e))*(d^6
/(a^8*f^4))^(3/4)*cos(f*x + e) - a^4*d^6*f^2*sqrt(d^6/(a^8*f^4))*cos(f*x +
e) - d^9*sin(f*x + e))/cos(f*x + e)) + 4*(d*cos(f*x + e)^2 + d*cos(f*x + e)
*sin(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e)))/(2*a^2*f*cos(f*x + e)*sin
(f*x + e) + a^2*f), -1/8*(4*(2*d*cos(f*x + e)*sin(f*x + e) + d)*sqrt(d)*arc
tan(sqrt(d*sin(f*x + e)/cos(f*x + e))/sqrt(d)) + 4*(2*sqrt(2)*a^2*f*cos(f*x
 + e)*sin(f*x + e) + sqrt(2)*a^2*f)*(d^6/(a^8*f^4))^(1/4)*arctan(-(sqrt(2)*
```

```
a^2*d^4*f*sqrt(d*sin(f*x + e)/cos(f*x + e))*(d^6/(a^8*f^4))^(1/4) + d^6 - s
qrt(2)*a^2*f*sqrt((sqrt(2)*a^6*d^4*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e))*(d
^6/(a^8*f^4))^(3/4)*cos(f*x + e) + a^4*d^6*f^2*sqrt(d^6/(a^8*f^4))*cos(f*x
+ e) + d^9*sin(f*x + e))/cos(f*x + e))*(d^6/(a^8*f^4))^(1/4))/d^6) + 4*(2*s
qrt(2)*a^2*f*cos(f*x + e)*sin(f*x + e) + sqrt(2)*a^2*f)*(d^6/(a^8*f^4))^(1/
4)*arctan(-(sqrt(2)*a^2*d^4*f*sqrt(d*sin(f*x + e)/cos(f*x + e))*(d^6/(a^8*f
^4))^(1/4) - d^6 - sqrt(2)*a^2*f*sqrt(-(sqrt(2)*a^6*d^4*f^3*sqrt(d*sin(f*x
+ e)/cos(f*x + e))*(d^6/(a^8*f^4))^(3/4)*cos(f*x + e) - a^4*d^6*f^2*sqrt(d^
6/(a^8*f^4))*cos(f*x + e) - d^9*sin(f*x + e))/cos(f*x + e))*(d^6/(a^8*f^4))
^(1/4))/d^6) + (2*sqrt(2)*a^2*f*cos(f*x + e)*sin(f*x + e) + sqrt(2)*a^2*f)*
(d^6/(a^8*f^4))^(1/4)*log((sqrt(2)*a^6*d^4*f^3*sqrt(d*sin(f*x + e)/cos(f*x
+ e))*(d^6/(a^8*f^4))^(3/4)*cos(f*x + e) + a^4*d^6*f^2*sqrt(d^6/(a^8*f^4))*
cos(f*x + e) + d^9*sin(f*x + e))/cos(f*x + e)) - (2*sqrt(2)*a^2*f*cos(f*x +
 e)*sin(f*x + e) + sqrt(2)*a^2*f)*(d^6/(a^8*f^4))^(1/4)*log(-(sqrt(2)*a^6*d
^4*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e))*(d^6/(a^8*f^4))^(3/4)*cos(f*x + e)
 - a^4*d^6*f^2*sqrt(d^6/(a^8*f^4))*cos(f*x + e) - d^9*sin(f*x + e))/cos(f*x
 + e)) - 4*(d*cos(f*x + e)^2 + d*cos(f*x + e)*sin(f*x + e))*sqrt(d*sin(f*x
+ e)/cos(f*x + e)))/(2*a^2*f*cos(f*x + e)*sin(f*x + e) + a^2*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(d\tan(e+fx))^{\frac{3}{2}}}{\tan^2(e+fx)+2\tan(e+fx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(3/2)/(a+a*tan(f*x+e))**2,x)
```

```
[Out] Integral((d*tan(e + f*x))**(3/2)/(tan(e + f*x)**2 + 2*tan(e + f*x) + 1), x)
/a**2
```

Giac [**A**]
time = 0.72, size = 264, normalized size = 0.95

$$\frac{1}{8}d\left(\frac{2\sqrt{2}\,|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{a^2df}+\frac{2\sqrt{2}\,|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{a^2df}-\frac{\sqrt{2}\,|d|^{\frac{3}{2}}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{|d|}+|d|\right)}{a^2df}+\frac{\sqrt{2}\,|d|^{\frac{3}{2}}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{|d|}+|d|\right)}{a^2df}-\frac{4\sqrt{d}\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{a^2f}+\frac{4\sqrt{d\tan(fx+e)}\,d}{(d\tan(fx+e)+d)a^2f}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(a+a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/8*d*(2*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*
sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(a^2*d*f) + 2*sqrt(2)*abs(d)^(3/2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)
))/(a^2*d*f) - sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan
(f*x + e))*sqrt(abs(d)) + abs(d))/(a^2*d*f) + sqrt(2)*abs(d)^(3/2)*log(d*ta
n(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(a^2*d*f)
```

```
- 4*sqrt(d)*arctan(sqrt(d*tan(f*x + e))/sqrt(d))/(a^2*f) + 4*sqrt(d*tan(f*x
 + e))*d/((d*tan(f*x + e) + d)*a^2*f))
```

Mupad [B]

time = 4.61, size = 375, normalized size = 1.34

$$\frac{\mathrm{atan}\left(\frac{4\,d^{16}\,\sqrt{d\,\tan\left(e+f\,x\right)}\,{\left(-\frac{d^6}{a^8\,f^4}\right)}^{1/4}}{\frac{4\,d^{18}}{a^2\,f}+4\,a^2\,d^{15}\,f\,\sqrt{-\frac{d^6}{a^8\,f^4}}}+\frac{4\,d^{13}\,\sqrt{d\,\tan\left(e+f\,x\right)}\,{\left(-\frac{d^6}{a^8\,f^4}\right)}^{3/4}}{\frac{4\,d^{18}}{a^6\,f^3}+\frac{4\,d^{15}\,\sqrt{-\frac{d^6}{a^8\,f^4}}}{a^2\,f}}\right)\,{\left(-\frac{d^6}{a^8\,f^4}\right)}^{1/4}}{2}+\mathrm{atan}\left(\frac{d^{16}\,\sqrt{d\,\tan\left(e+f\,x\right)}\,{\left(-\frac{d^6}{256\,a^8\,f^4}\right)}^{1/4}\,16{}\mathrm{i}}{\frac{4\,d^{18}}{a^2\,f}-64\,a^2\,d^{15}\,f\,\sqrt{-\frac{d^6}{256\,a^8\,f^4}}}-\frac{d^{13}\,\sqrt{d\,\tan\left(e+f\,x\right)}\,{\left(-\frac{d^6}{256\,a^8\,f^4}\right)}^{3/4}\,256{}\mathrm{i}}{\frac{4\,d^{18}}{a^6\,f^3}-\frac{64\,d^{15}\,\sqrt{-\frac{d^6}{256\,a^8\,f^4}}}{a^2\,f}}\right)\,{\left(-\frac{d^6}{256\,a^8\,f^4}\right)}^{1/4}\,2{}\mathrm{i}+\frac{d^2\,\sqrt{d\,\tan\left(e+f\,x\right)}}{2\,\left(a^2\,d\,f+a^2\,d\,f\,\tan\left(e+f\,x\right)\right)}-\frac{\mathrm{atan}\left(\frac{\sqrt{d\,\tan\left(e+f\,x\right)}\,\sqrt{-d^3}\,1{}\mathrm{i}}{d^2}\right)\,\sqrt{-d^3}\,1{}\mathrm{i}}{2\,a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(3/2)/(a + a*tan(e + f*x))^2,x)
```

```
[Out] (atan((4*d^16*(d*tan(e + f*x))^(1/2)*(-d^6/(a^8*f^4))^(1/4))/((4*d^18)/(a^2
*f) + 4*a^2*d^15*f*(-d^6/(a^8*f^4))^(1/2)) + (4*d^13*(d*tan(e + f*x))^(1/2)
*(-d^6/(a^8*f^4))^(3/4))/((4*d^18)/(a^6*f^3) + (4*d^15*(-d^6/(a^8*f^4))^(1/
2))/(a^2*f)))*(-d^6/(a^8*f^4))^(1/4))/2 + atan((d^16*(d*tan(e + f*x))^(1/2)
*(-d^6/(256*a^8*f^4))^(1/4)*16i)/((4*d^18)/(a^2*f) - 64*a^2*d^15*f*(-d^6/(2
56*a^8*f^4))^(1/2)) - (d^13*(d*tan(e + f*x))^(1/2)*(-d^6/(256*a^8*f^4))^(3/
4)*256i)/((4*d^18)/(a^6*f^3) - (64*d^15*(-d^6/(256*a^8*f^4))^(1/2))/(a^2*f)
))*(-d^6/(256*a^8*f^4))^(1/4)*2i + (d^2*(d*tan(e + f*x))^(1/2))/(2*(a^2*d*f
 + a^2*d*f*tan(e + f*x))) - (atan(((d*tan(e + f*x))^(1/2)*(-d^3)^(1/2)*1i)/
d^2)*(-d^3)^(1/2)*1i)/(2*a^2*f)
```

3.366 $\int \frac{\sqrt{d\tan(e+fx)}}{(a+a\tan(e+fx))^2}\,dx$

Optimal. Leaf size=278

$$-\frac{\sqrt{d}\,\mathrm{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2f}-\frac{\sqrt{d}\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2\sqrt{2}\,a^2f}+\frac{\sqrt{d}\,\mathrm{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\mathrm{t}}}{}\right)}{2\sqrt{2}\,a^2f}$$

```
[Out] -1/2*arctan((d*tan(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/a^2/f-1/4*arctan(1-2^(1/2
)*(d*tan(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/a^2/f*2^(1/2)+1/4*arctan(1+2^(1/2)*
(d*tan(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/a^2/f*2^(1/2)-1/8*ln(d^(1/2)-2^(1/2)*
(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))*d^(1/2)/a^2/f*2^(1/2)+1/8*ln(d^(1/
2)+2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))*d^(1/2)/a^2/f*2^(1/2)-1
/2*(d*tan(f*x+e))^(1/2)/f/(a^2+a^2*tan(f*x+e))
```

Rubi [A]
time = 0.32, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3649, 3734, 12, 3557, 335, 217, 1179, 642, 1176, 631, 210, 3715, 65, 211}

$$-\frac{\sqrt{d}\,\mathrm{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2f}-\frac{\sqrt{d}\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2\sqrt{2}\,a^2f}+\frac{\sqrt{d}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{2\sqrt{2}\,a^2f}-\frac{\sqrt{d\tan(e+fx)}}{2f\left(a^2\tan(e+fx)+a^2\right)}-\frac{\sqrt{d}\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{4\sqrt{2}\,a^2f}+\frac{\sqrt{d}\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{4\sqrt{2}\,a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Tan[e + f*x]]/(a + a*Tan[e + f*x])^2,x]
```

```
[Out] -1/2*(Sqrt[d]*ArcTan[Sqrt[d*Tan[e + f*x]]/Sqrt[d]])/(a^2*f) - (Sqrt[d]*ArcT
an[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(2*Sqrt[2]*a^2*f) + (Sqrt[d
]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(2*Sqrt[2]*a^2*f) - (
Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e + f*x]]])
/(4*Sqrt[2]*a^2*f) + (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqrt[2]*
Sqrt[d*Tan[e + f*x]]])/(4*Sqrt[2]*a^2*f) - Sqrt[d*Tan[e + f*x]]/(2*f*(a^2 +
 a^2*Tan[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{d\tan(e+fx)}}{(a+a\tan(e+fx))^2}\,dx = -\frac{\sqrt{d\tan(e+fx)}}{2f\left(a^2+a^2\tan(e+fx)\right)} - \frac{\int \frac{-\frac{ad}{2}-ad\tan(e+fx)+\frac{1}{2}ad\tan^2(e+fx)}{\sqrt{d\tan(e+fx)}\,(a+a\tan(e+fx))}\,dx}{2a^2}$$

$$= -\frac{\sqrt{d\tan(e+fx)}}{2f\left(a^2+a^2\tan(e+fx)\right)} - \frac{\int -\frac{2a^2d}{\sqrt{d\tan(e+fx)}}\,dx}{4a^4} - \frac{d\int \frac{1+\tan^2(e+\cdots}{\sqrt{d\tan(e+fx)}\,\cdots}}{4a}\cdots$$

$$= -\frac{\sqrt{d\tan(e+fx)}}{2f\left(a^2+a^2\tan(e+fx)\right)} + \frac{d\int \frac{1}{\sqrt{d\tan(e+fx)}}\,dx}{2a^2} - \frac{d\text{Subst}\left(\int \frac{1}{\sqrt{dx}\,(a+ax\cdots}\cdots\right)}{4a\cdots}\cdots$$

$$= -\frac{\sqrt{d\tan(e+fx)}}{2f\left(a^2+a^2\tan(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{a+\frac{ax^2}{d}}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{2af} + \frac{d^2\text{Su}\cdots}{\cdots}$$

$$= -\frac{\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2f} - \frac{\sqrt{d\tan(e+fx)}}{2f\left(a^2+a^2\tan(e+fx)\right)} + \frac{d^2\text{Subst}\left(\int \frac{\cdots}{d^2\cdots}\cdots\right)}{\cdots}$$

$$= -\frac{\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2f} - \frac{\sqrt{d\tan(e+fx)}}{2f\left(a^2+a^2\tan(e+fx)\right)} + \frac{d\text{Subst}\left(\int \frac{d-\cdots}{d^2+\cdots}\cdots\right)}{\cdots}$$

$$= -\frac{\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2f} - \frac{\sqrt{d\tan(e+fx)}}{2f\left(a^2+a^2\tan(e+fx)\right)} - \frac{\sqrt{d}\,\text{Subst}\left(\int\cdots\right)}{\cdots}$$

$$= -\frac{\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2f} - \frac{\sqrt{d}\,\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\,\cdots\right)}{4\sqrt{2}\,a^2f}\cdots$$

$$= -\frac{\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2f} - \frac{\sqrt{d}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2\sqrt{2}\,a^2f} + \cdots$$

Mathematica [A]
time = 1.42, size = 192, normalized size = 0.69

$$\frac{\left(-\frac{2\cos(e+fx)}{\cos(e+fx)+\sin(e+fx)} - \frac{2\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(e+fx)}\right)-2\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(e+fx)}\right)+4\text{ArcTan}\left(\sqrt{\tan(e+fx)}\right)+\sqrt{2}\,\log\left(-1+\sqrt{2}\,\sqrt{\tan(e+fx)}-\tan(e+fx)\right)-\sqrt{2}\,\log\left(1+\sqrt{2}\,\sqrt{\tan(e+fx)}+\tan(e+fx)\right)}{2\sqrt{\tan(e+fx)}}\right)\sqrt{d\tan(e+fx)}}{4a^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Tan[e + f*x]]/(a + a*Tan[e + f*x])^2,x]
```

```
[Out] (((-2*Cos[e + f*x])/(Cos[e + f*x] + Sin[e + f*x]) - (2*Sqrt[2]*ArcTan[1 - S
qrt[2]*Sqrt[Tan[e + f*x]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]
```

```
]] + 4*ArcTan[Sqrt[Tan[e + f*x]]] + Sqrt[2]*Log[-1 + Sqrt[2]*Sqrt[Tan[e + f
*x]] - Tan[e + f*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e +
 f*x]])/(2*Sqrt[Tan[e + f*x]]))*Sqrt[d*Tan[e + f*x]])/(4*a^2*f)
```

Maple [A]

time = 0.18, size = 200, normalized size = 0.72

method	result
derivativedivides	$\frac{2d^3\left(-\frac{\frac{\sqrt{d\tan(fx+e)}}{2d(d\tan(fx+e)+d)}+\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{2d^{\frac{3}{2}}}}{2d}+\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(}}\right.\right.}{}\right.}{}$
default	$\frac{2d^3\left(-\frac{\frac{\sqrt{d\tan(fx+e)}}{2d(d\tan(fx+e)+d)}+\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{2d^{\frac{3}{2}}}}{2d}+\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(}}{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(}}\right.\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(1/2)/(a+a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^2*d^3*(-1/2/d*(1/2*(d*tan(f*x+e))^(1/2)/d/(d*tan(f*x+e)+d)+1/2/d^(3/2
)*arctan((d*tan(f*x+e))^(1/2)/d^(1/2)))+1/16/d^3*(d^2)^(1/4)*2^(1/2)*(ln((d
*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*
x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2
)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(
f*x+e))^(1/2)+1)))
```

Maxima [A]

time = 0.50, size = 227, normalized size = 0.82

$$-\frac{\frac{4\sqrt{d\tan(fx+e)}\,d^2}{a^2d\tan(fx+e)+a^2d}+\frac{4d^{\frac{3}{2}}\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{a^2}-\frac{2\sqrt{2}\,d^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)+2\sqrt{2}\,d^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)+\sqrt{2}\,d^{\frac{3}{2}}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)-\sqrt{2}\,d^{\frac{3}{2}}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{a^2}}{8\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(a+a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(4*sqrt(d*tan(f*x + e))*d^2/(a^2*d*tan(f*x + e) + a^2*d) + 4*d^(3/2)*a
rctan(sqrt(d*tan(f*x + e))/sqrt(d))/a^2 - (2*sqrt(2)*d^(3/2)*arctan(1/2*sqr
t(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(f*x + e)))/sqrt(d)) + 2*sqrt(2)*d^(3/2
)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d)) +
 sqrt(2)*d^(3/2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d)
```

```
+ d) - sqrt(2)*d^(3/2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sq
rt(d) + d))/a^2)/(d*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 841 vs. $2(223) = 446$.
time = 1.36, size = 1763, normalized size = 6.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(a+a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/8*((2*cos(f*x + e)*sin(f*x + e) + 1)*sqrt(-d)*log(-(6*d^2*cos(f*x + e)*s
in(f*x + e) - d^2 - 4*(d*cos(f*x + e)^2 - d*cos(f*x + e)*sin(f*x + e))*sqrt
(-d)*sqrt(d*sin(f*x + e)/cos(f*x + e)))/(2*cos(f*x + e)*sin(f*x + e) + 1))
- 4*(2*sqrt(2)*a^2*f*cos(f*x + e)*sin(f*x + e) + sqrt(2)*a^2*f)*(d^2/(a^8*f
^4))^(1/4)*arctan(-(sqrt(2)*a^6*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e))*(d^2/
(a^8*f^4))^(3/4) - sqrt(2)*a^6*f^3*sqrt((a^4*f^2*sqrt(d^2/(a^8*f^4))*cos(f*
x + e) + sqrt(2)*a^2*f*sqrt(d*sin(f*x + e)/cos(f*x + e))*(d^2/(a^8*f^4))^(1
/4)*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))*(d^2/(a^8*f^4))^(3/4) + d^
2)/d^2) - 4*(2*sqrt(2)*a^2*f*cos(f*x + e)*sin(f*x + e) + sqrt(2)*a^2*f)*(d^
2/(a^8*f^4))^(1/4)*arctan(-(sqrt(2)*a^6*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e
))*(d^2/(a^8*f^4))^(3/4) - sqrt(2)*a^6*f^3*sqrt((a^4*f^2*sqrt(d^2/(a^8*f^4)
)*cos(f*x + e) - sqrt(2)*a^2*f*sqrt(d*sin(f*x + e)/cos(f*x + e))*(d^2/(a^8*
f^4))^(1/4)*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))*(d^2/(a^8*f^4))^(3
/4) - d^2)/d^2) + (2*sqrt(2)*a^2*f*cos(f*x + e)*sin(f*x + e) + sqrt(2)*a^2*
f)*(d^2/(a^8*f^4))^(1/4)*log((a^4*f^2*sqrt(d^2/(a^8*f^4))*cos(f*x + e) + sq
rt(2)*a^2*f*sqrt(d*sin(f*x + e)/cos(f*x + e))*(d^2/(a^8*f^4))^(1/4)*cos(f*x
 + e) + d*sin(f*x + e))/cos(f*x + e)) - (2*sqrt(2)*a^2*f*cos(f*x + e)*sin(f
*x + e) + sqrt(2)*a^2*f)*(d^2/(a^8*f^4))^(1/4)*log((a^4*f^2*sqrt(d^2/(a^8*f
^4))*cos(f*x + e) - sqrt(2)*a^2*f*sqrt(d*sin(f*x + e)/cos(f*x + e))*(d^2/(a
^8*f^4))^(1/4)*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e)) - 4*(cos(f*x +
e)^2 + cos(f*x + e)*sin(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e)))/(2*a^2
*f*cos(f*x + e)*sin(f*x + e) + a^2*f), -1/8*(4*(2*cos(f*x + e)*sin(f*x + e)
 + 1)*sqrt(d)*arctan(sqrt(d*sin(f*x + e)/cos(f*x + e))/sqrt(d)) + 4*(2*sqrt
(2)*a^2*f*cos(f*x + e)*sin(f*x + e) + sqrt(2)*a^2*f)*(d^2/(a^8*f^4))^(1/4)*
arctan(-(sqrt(2)*a^6*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e))*(d^2/(a^8*f^4))^
(3/4) - sqrt(2)*a^6*f^3*sqrt((a^4*f^2*sqrt(d^2/(a^8*f^4))*cos(f*x + e) + sq
rt(2)*a^2*f*sqrt(d*sin(f*x + e)/cos(f*x + e))*(d^2/(a^8*f^4))^(1/4)*cos(f*x
 + e) + d*sin(f*x + e))/cos(f*x + e))*(d^2/(a^8*f^4))^(3/4) + d^2)/d^2) + 4
*(2*sqrt(2)*a^2*f*cos(f*x + e)*sin(f*x + e) + sqrt(2)*a^2*f)*(d^2/(a^8*f^4)
)^(1/4)*arctan(-(sqrt(2)*a^6*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e))*(d^2/(a^
8*f^4))^(3/4) - sqrt(2)*a^6*f^3*sqrt((a^4*f^2*sqrt(d^2/(a^8*f^4))*cos(f*x +
 e) - sqrt(2)*a^2*f*sqrt(d*sin(f*x + e)/cos(f*x + e))*(d^2/(a^8*f^4))^(1/4)
*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))*(d^2/(a^8*f^4))^(3/4) - d^2)/
```

```
d^2) - (2*sqrt(2)*a^2*f*cos(f*x + e)*sin(f*x + e) + sqrt(2)*a^2*f)*(d^2/(a^
8*f^4))^(1/4)*log((a^4*f^2*sqrt(d^2/(a^8*f^4))*cos(f*x + e) + sqrt(2)*a^2*f
*sqrt(d*sin(f*x + e)/cos(f*x + e))*(d^2/(a^8*f^4))^(1/4)*cos(f*x + e) + d*s
in(f*x + e))/cos(f*x + e)) + (2*sqrt(2)*a^2*f*cos(f*x + e)*sin(f*x + e) + s
qrt(2)*a^2*f)*(d^2/(a^8*f^4))^(1/4)*log((a^4*f^2*sqrt(d^2/(a^8*f^4))*cos(f*
x + e) - sqrt(2)*a^2*f*sqrt(d*sin(f*x + e)/cos(f*x + e))*(d^2/(a^8*f^4))^(1
/4)*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e)) + 4*(cos(f*x + e)^2 + cos(
f*x + e)*sin(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e)))/(2*a^2*f*cos(f*x
+ e)*sin(f*x + e) + a^2*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{d\tan(e+fx)}}{\tan^2(e+fx)+2\tan(e+fx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(1/2)/(a+a*tan(f*x+e))**2,x)
```

```
[Out] Integral(sqrt(d*tan(e + f*x))/(tan(e + f*x)**2 + 2*tan(e + f*x) + 1), x)/a*
*2
```

Giac [A]
time = 0.65, size = 260, normalized size = 0.94

$$\frac{\frac{2\sqrt{2}\,d\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{a^2f}+\frac{2\sqrt{2}\,d\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{a^2f}+\frac{\sqrt{2}\,d\sqrt{|d|}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{|d|}+|d|\right)}{a^2f}-\frac{\sqrt{2}\,d\sqrt{|d|}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{|d|}+|d|\right)}{a^2f}-\frac{4d^{\frac{3}{2}}\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{a^2f}-\frac{4\sqrt{d\tan(fx+e)}\,d^2}{(d\tan(fx+e)+d)a^2f}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(a+a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/8*(2*sqrt(2)*d*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*
sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(a^2*f) + 2*sqrt(2)*d*sqrt(abs(d))*arct
an(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)
))/(a^2*f) + sqrt(2)*d*sqrt(abs(d))*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan
(f*x + e))*sqrt(abs(d)) + abs(d))/(a^2*f) - sqrt(2)*d*sqrt(abs(d))*log(d*ta
n(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(a^2*f) -
4*d^(3/2)*arctan(sqrt(d*tan(f*x + e))/sqrt(d))/(a^2*f) - 4*sqrt(d*tan(f*x +
 e))*d^2/((d*tan(f*x + e) + d)*a^2*f))/d
```

Mupad [B]
time = 4.58, size = 367, normalized size = 1.32

$$-\frac{\mathrm{atan}\left(\frac{4\,d^{12}\,\sqrt{d\tan(e+f\,x)}\,{\left(-\frac{d^2}{a^8\,f^4}\right)}^{1/4}}{\frac{4\,d^{13}}{a^2\,f}-4\,a^2\,d^{12}\,f\,\sqrt{-\frac{d^2}{a^8\,f^4}}}+\frac{4\,d^{11}\,\sqrt{d\tan(e+f\,x)}\,{\left(-\frac{d^2}{a^8\,f^4}\right)}^{3/4}}{\frac{4\,d^{13}}{a^6\,f^3}-\frac{4\,d^{12}\,\sqrt{-\frac{d^2}{a^8\,f^4}}}{a^2\,f}}\right)\,{\left(-\frac{d^2}{a^8\,f^4}\right)}^{1/4}}{2}-\mathrm{atan}\left(\frac{d^{12}\,\sqrt{d\tan(e+f\,x)}\,{\left(-\frac{d^2}{256\,a^8\,f^4}\right)}^{1/4}\,16{}\mathrm{i}}{\frac{4\,d^{13}}{a^2\,f}+64\,a^2\,d^{12}\,f\,\sqrt{-\frac{d^2}{256\,a^8\,f^4}}}-\frac{d^{11}\,\sqrt{d\tan(e+f\,x)}\,{\left(-\frac{d^2}{256\,a^8\,f^4}\right)}^{3/4}\,256{}\mathrm{i}}{\frac{4\,d^{13}}{a^6\,f^3}+\frac{64\,d^{12}\,\sqrt{-\frac{d^2}{256\,a^8\,f^4}}}{a^2\,f}}\right)\,{\left(-\frac{d^2}{256\,a^8\,f^4}\right)}^{1/4}\,2{}\mathrm{i}-\frac{d\,\sqrt{d\tan(e+f\,x)}}{2\,\left(a^2\,d\,f+a^2\,d\,f\tan(e+f\,x)\right)}+\frac{\sqrt{-d}\,\mathrm{atan}\left(\frac{\sqrt{d\tan(e+f\,x)}\,1{}\mathrm{i}}{\sqrt{-d}}\right)\,1{}\mathrm{i}}{2\,a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(1/2)/(a + a*tan(e + f*x))^2,x)
```

```
[Out] ((-d)^(1/2)*atan(((d*tan(e + f*x))^(1/2)*1i)/(-d)^(1/2))*1i)/(2*a^2*f) - at
an((d^12*(d*tan(e + f*x))^(1/2)*(-d^2/(256*a^8*f^4))^(1/4)*16i)/((4*d^13)/(
a^2*f) + 64*a^2*d^12*f*(-d^2/(256*a^8*f^4))^(1/2)) - (d^11*(d*tan(e + f*x))
^(1/2)*(-d^2/(256*a^8*f^4))^(3/4)*256i)/((4*d^13)/(a^6*f^3) + (64*d^12*(-d^
2/(256*a^8*f^4))^(1/2))/(a^2*f)))*(-d^2/(256*a^8*f^4))^(1/4)*2i - (d*(d*tan
(e + f*x))^(1/2))/(2*(a^2*d*f + a^2*d*f*tan(e + f*x))) - (atan((4*d^12*(d*t
an(e + f*x))^(1/2)*(-d^2/(a^8*f^4))^(1/4))/((4*d^13)/(a^2*f) - 4*a^2*d^12*f
*(-d^2/(a^8*f^4))^(1/2)) + (4*d^11*(d*tan(e + f*x))^(1/2)*(-d^2/(a^8*f^4))^
(3/4))/((4*d^13)/(a^6*f^3) - (4*d^12*(-d^2/(a^8*f^4))^(1/2))/(a^2*f)))*(-d^
2/(a^8*f^4))^(1/4))/2
```

3.367 $\int \frac{1}{\sqrt{d\tan(e+fx)}\,(a+a\tan(e+fx))^2}\,dx$

Optimal. Leaf size=281

$$\frac{3\text{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2\sqrt{d}\,f}+\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2\sqrt{2}\,a^2\sqrt{d}\,f}-\frac{\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2\sqrt{2}\,a^2\sqrt{d}\,f}$$

```
[Out] 3/2*arctan((d*tan(f*x+e))^(1/2)/d^(1/2))/a^2/f/d^(1/2)+1/4*arctan(1-2^(1/2)
*(d*tan(f*x+e))^(1/2)/d^(1/2))/a^2/f*2^(1/2)/d^(1/2)-1/4*arctan(1+2^(1/2)*(
d*tan(f*x+e))^(1/2)/d^(1/2))/a^2/f*2^(1/2)/d^(1/2)-1/8*ln(d^(1/2)-2^(1/2)*(
d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/a^2/f*2^(1/2)/d^(1/2)+1/8*ln(d^(1/2
)+2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/a^2/f*2^(1/2)/d^(1/2)+1/
2*(d*tan(f*x+e))^(1/2)/d/f/(a^2+a^2*tan(f*x+e))
```

Rubi [A]

time = 0.35, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3650, 3734, 12, 16, 3557, 335, 303, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{3\text{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2\sqrt{d}\,f}+\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2\sqrt{2}\,a^2\sqrt{d}\,f}-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{2\sqrt{2}\,a^2\sqrt{d}\,f}+\frac{\sqrt{d\tan(e+fx)}}{2df\left(a^2\tan(e+fx)+a^2\right)}-\frac{\log\left(\sqrt{d}\,\tan(e+fx)-\sqrt{2}\,\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{4\sqrt{2}\,a^2\sqrt{d}\,f}+\frac{\log\left(\sqrt{d}\,\tan(e+fx)+\sqrt{2}\,\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{4\sqrt{2}\,a^2\sqrt{d}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*Tan[e + f*x]]*(a + a*Tan[e + f*x])^2),x]
```

```
[Out] (3*ArcTan[Sqrt[d*Tan[e + f*x]]/Sqrt[d]])/(2*a^2*Sqrt[d]*f) + ArcTan[1 - (Sq
rt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]]/(2*Sqrt[2]*a^2*Sqrt[d]*f) - ArcTan[1 +
 (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]]/(2*Sqrt[2]*a^2*Sqrt[d]*f) - Log[Sq
rt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e + f*x]]]/(4*Sqrt[2]*a^2
*Sqrt[d]*f) + Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqrt[2]*Sqrt[d*Tan[e + f
*x]]]/(4*Sqrt[2]*a^2*Sqrt[d]*f) + Sqrt[d*Tan[e + f*x]]/(2*d*f*(a^2 + a^2*Ta
n[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
```

```
!GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{d\tan(e+fx)}\,(a+a\tan(e+fx))^2}\,dx &= \frac{\sqrt{d\tan(e+fx)}}{2df\left(a^2+a^2\tan(e+fx)\right)}+\frac{\int \frac{\frac{3a^2d}{2}-a^2d\tan(e+fx)+\frac{1}{2}a^2d\tan^2(e+fx)}{\sqrt{d\tan(e+fx)}\,(a+a\tan(e+fx))}}{2a^3d}\\
&= \frac{\sqrt{d\tan(e+fx)}}{2df\left(a^2+a^2\tan(e+fx)\right)}+\frac{3\int \frac{1+\tan^2(e+fx)}{\sqrt{d\tan(e+fx)}\,(a+a\tan(e+fx)}}{4a}\\
&= \frac{\sqrt{d\tan(e+fx)}}{2df\left(a^2+a^2\tan(e+fx)\right)}-\frac{\int \frac{\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx}{2a^2}+\frac{3\text{Subst}}{}\\
&= \frac{\sqrt{d\tan(e+fx)}}{2df\left(a^2+a^2\tan(e+fx)\right)}-\frac{\int \sqrt{d\tan(e+fx)}\,dx}{2a^2d}+\frac{3\text{Subst}}{}\\
&= \frac{3\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2\sqrt{d}\,f}+\frac{\sqrt{d\tan(e+fx)}}{2df\left(a^2+a^2\tan(e+fx)\right)}-\\
&= \frac{3\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2\sqrt{d}\,f}+\frac{\sqrt{d\tan(e+fx)}}{2df\left(a^2+a^2\tan(e+fx)\right)}-\\
&= \frac{3\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2\sqrt{d}\,f}+\frac{\sqrt{d\tan(e+fx)}}{2df\left(a^2+a^2\tan(e+fx)\right)}+\\
&= \frac{3\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2\sqrt{d}\,f}+\frac{\sqrt{d\tan(e+fx)}}{2df\left(a^2+a^2\tan(e+fx)\right)}-\\
&= \frac{3\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2\sqrt{d}\,f}-\frac{\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)\right.}{4\sqrt{2}\,a^2}\\
&= \frac{3\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2\sqrt{d}\,f}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+}}{\sqrt{d}}\right.}{2\sqrt{2}\,a^2\sqrt{d}\,f}
\end{aligned}
$$

Mathematica [A]
time = 0.74, size = 337, normalized size = 1.20

$$
\frac{\left(12\text{ArcTan}\left(\sqrt{\tan(e+fx)}\right)\cos(e+fx)-\sqrt{2}\cos(e+fx)\log\left(-1+\sqrt{2}\sqrt{\tan(e+fx)}-\tan(e+fx)\right)+\sqrt{2}\cos(e+fx)\log\left(1+\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)+12\text{ArcTan}\left(\sqrt{\tan(e+fx)}\right)\sin(e+fx)-\sqrt{2}\log\left(-1+\sqrt{2}\sqrt{\tan(e+fx)}-\tan(e+fx)\right)\sin(e+fx)+\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)\sin(e+fx)+2\sqrt{2}\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)(\cos(e+fx)+\sin(e+fx))-2\sqrt{2}\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(e+fx)}\right)(\cos(e+fx)+\sin(e+fx))+4\cos(e+fx)\sqrt{\tan(e+fx)}\right)\sqrt{\tan(e+fx)}}{8a^2f(\cos(e+fx)+\sin(e+fx))\sqrt{d\tan(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*Tan[e + f*x]]*(a + a*Tan[e + f*x])^2),x]
```

```
[Out] ((12*ArcTan[Sqrt[Tan[e + f*x]]]*Cos[e + f*x] - Sqrt[2]*Cos[e + f*x]*Log[-1
+ Sqrt[2]*Sqrt[Tan[e + f*x]] - Tan[e + f*x]] + Sqrt[2]*Cos[e + f*x]*Log[1 +
 Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] + 12*ArcTan[Sqrt[Tan[e + f*x]]]
*Sin[e + f*x] - Sqrt[2]*Log[-1 + Sqrt[2]*Sqrt[Tan[e + f*x]] - Tan[e + f*x]]
*Sin[e + f*x] + Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]*
Sin[e + f*x] + 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]]*(Cos[e + f*
x] + Sin[e + f*x]) - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]]*(Cos[
e + f*x] + Sin[e + f*x]) + 4*Cos[e + f*x]*Sqrt[Tan[e + f*x]])*Sqrt[Tan[e +
f*x]])/(8*a^2*f*(Cos[e + f*x] + Sin[e + f*x])*Sqrt[d*Tan[e + f*x]])
```

Maple [A]
time = 0.20, size = 197, normalized size = 0.70

method	result
derivativedivides	$\frac{2d^3\left(\frac{\frac{\sqrt{d\tan(fx+e)}}{2d\tan(fx+e)+2d}+\frac{3\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{2\sqrt{d}}}{2d^3}-\frac{\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+}}{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+}}\right.\right.}{}\right)}{f\,a}$
default	$\frac{2d^3\left(\frac{\frac{\sqrt{d\tan(fx+e)}}{2d\tan(fx+e)+2d}+\frac{3\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{2\sqrt{d}}}{2d^3}-\frac{\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+}}{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+}}\right.\right.}{}\right)}{f\,a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*tan(f*x+e))^(1/2)/(a+a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^2*d^3*(1/2/d^3*(1/2*(d*tan(f*x+e))^(1/2)/(d*tan(f*x+e)+d)+3/2/d^(1/2)
*arctan((d*tan(f*x+e))^(1/2)/d^(1/2)))-1/16/d^3/(d^2)^(1/4)*2^(1/2)*(ln((d*
tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x
+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)
/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f
*x+e))^(1/2)+1)))
```

Maxima [A]

time = 0.53, size = 226, normalized size = 0.80

$$\frac{\frac{4\sqrt{d\tan(fx+e)}\,d}{a^2d\tan(fx+e)+a^2d}-\frac{d\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)}{a^2}+\frac{12\sqrt{d}\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{a^2}}{8\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(1/2)/(a+a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(4*sqrt(d*tan(f*x + e))*d/(a^2*d*tan(f*x + e) + a^2*d) - d*(2*sqrt(2)*a
rctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(
d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)
))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x +
 e))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan
(f*x + e))*sqrt(d) + d)/sqrt(d))/a^2 + 12*sqrt(d)*arctan(sqrt(d*tan(f*x + e
))/sqrt(d))/a^2)/(d*f)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 863 vs. $2(226) = 452$.
time = 2.63, size = 1808, normalized size = 6.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(1/2)/(a+a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(3*(2*cos(f*x + e)*sin(f*x + e) + 1)*sqrt(-d)*log(-(6*d^2*cos(f*x + e
)*sin(f*x + e) - d^2 - 4*(d*cos(f*x + e)^2 - d*cos(f*x + e)*sin(f*x + e))*s
qrt(-d)*sqrt(d*sin(f*x + e)/cos(f*x + e)))/(2*cos(f*x + e)*sin(f*x + e) + 1
)) - 4*(2*sqrt(2)*a^2*d*f*cos(f*x + e)*sin(f*x + e) + sqrt(2)*a^2*d*f)*(1/(
a^8*d^2*f^4))^(1/4)*arctan(-sqrt(2)*a^2*f*sqrt(d*sin(f*x + e)/cos(f*x + e))
*(1/(a^8*d^2*f^4))^(1/4) + sqrt(2)*a^2*f*sqrt((sqrt(2)*a^6*d^2*f^3*sqrt(d*s
in(f*x + e)/cos(f*x + e))*(1/(a^8*d^2*f^4))^(3/4)*cos(f*x + e) + a^4*d^2*f^
2*sqrt(1/(a^8*d^2*f^4))*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))*(1/(a^
8*d^2*f^4))^(1/4) - 1) - 4*(2*sqrt(2)*a^2*d*f*cos(f*x + e)*sin(f*x + e) + s
qrt(2)*a^2*d*f)*(1/(a^8*d^2*f^4))^(1/4)*arctan(-sqrt(2)*a^2*f*sqrt(d*sin(f*
x + e)/cos(f*x + e))*(1/(a^8*d^2*f^4))^(1/4) + sqrt(2)*a^2*f*sqrt(-(sqrt(2)
*a^6*d^2*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e))*(1/(a^8*d^2*f^4))^(3/4)*cos(
f*x + e) - a^4*d^2*f^2*sqrt(1/(a^8*d^2*f^4))*cos(f*x + e) - d*sin(f*x + e))
/cos(f*x + e))*(1/(a^8*d^2*f^4))^(1/4) + 1) - (2*sqrt(2)*a^2*d*f*cos(f*x +
e)*sin(f*x + e) + sqrt(2)*a^2*d*f)*(1/(a^8*d^2*f^4))^(1/4)*log((sqrt(2)*a^6
*d^2*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e))*(1/(a^8*d^2*f^4))^(3/4)*cos(f*x
+ e) + a^4*d^2*f^2*sqrt(1/(a^8*d^2*f^4))*cos(f*x + e) + d*sin(f*x + e))/cos
(f*x + e)) + (2*sqrt(2)*a^2*d*f*cos(f*x + e)*sin(f*x + e) + sqrt(2)*a^2*d*f
)*(1/(a^8*d^2*f^4))^(1/4)*log(-(sqrt(2)*a^6*d^2*f^3*sqrt(d*sin(f*x + e)/cos
```

```
(f*x + e))*(1/(a^8*d^2*f^4))^(3/4)*cos(f*x + e) - a^4*d^2*f^2*sqrt(1/(a^8*d
^2*f^4))*cos(f*x + e) - d*sin(f*x + e))/cos(f*x + e)) - 4*(cos(f*x + e)^2 +
 cos(f*x + e)*sin(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e)))/(2*a^2*d*f*c
os(f*x + e)*sin(f*x + e) + a^2*d*f), 1/8*(12*(2*cos(f*x + e)*sin(f*x + e) +
 1)*sqrt(d)*arctan(sqrt(d*sin(f*x + e)/cos(f*x + e))/sqrt(d)) + 4*(2*sqrt(2
)*a^2*d*f*cos(f*x + e)*sin(f*x + e) + sqrt(2)*a^2*d*f)*(1/(a^8*d^2*f^4))^(1
/4)*arctan(-sqrt(2)*a^2*f*sqrt(d*sin(f*x + e)/cos(f*x + e))*(1/(a^8*d^2*f^4
))^(1/4) + sqrt(2)*a^2*f*sqrt((sqrt(2)*a^6*d^2*f^3*sqrt(d*sin(f*x + e)/cos(
f*x + e))*(1/(a^8*d^2*f^4))^(3/4)*cos(f*x + e) + a^4*d^2*f^2*sqrt(1/(a^8*d^
2*f^4))*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))*(1/(a^8*d^2*f^4))^(1/4
) - 1) + 4*(2*sqrt(2)*a^2*d*f*cos(f*x + e)*sin(f*x + e) + sqrt(2)*a^2*d*f)*
(1/(a^8*d^2*f^4))^(1/4)*arctan(-sqrt(2)*a^2*f*sqrt(d*sin(f*x + e)/cos(f*x +
 e))*(1/(a^8*d^2*f^4))^(1/4) + sqrt(2)*a^2*f*sqrt(-(sqrt(2)*a^6*d^2*f^3*sqr
t(d*sin(f*x + e)/cos(f*x + e))*(1/(a^8*d^2*f^4))^(3/4)*cos(f*x + e) - a^4*d
^2*f^2*sqrt(1/(a^8*d^2*f^4))*cos(f*x + e) - d*sin(f*x + e))/cos(f*x + e))*(
1/(a^8*d^2*f^4))^(1/4) + 1) + (2*sqrt(2)*a^2*d*f*cos(f*x + e)*sin(f*x + e)
+ sqrt(2)*a^2*d*f)*(1/(a^8*d^2*f^4))^(1/4)*log((sqrt(2)*a^6*d^2*f^3*sqrt(d*
sin(f*x + e)/cos(f*x + e))*(1/(a^8*d^2*f^4))^(3/4)*cos(f*x + e) + a^4*d^2*f
^2*sqrt(1/(a^8*d^2*f^4))*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e)) - (2*
sqrt(2)*a^2*d*f*cos(f*x + e)*sin(f*x + e) + sqrt(2)*a^2*d*f)*(1/(a^8*d^2*f^
4))^(1/4)*log(-(sqrt(2)*a^6*d^2*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e))*(1/(a
^8*d^2*f^4))^(3/4)*cos(f*x + e) - a^4*d^2*f^2*sqrt(1/(a^8*d^2*f^4))*cos(f*x
 + e) - d*sin(f*x + e))/cos(f*x + e)) + 4*(cos(f*x + e)^2 + cos(f*x + e)*si
n(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e)))/(2*a^2*d*f*cos(f*x + e)*sin(
f*x + e) + a^2*d*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{d\tan(e+fx)}\tan^2(e+fx)+2\sqrt{d\tan(e+fx)}\tan(e+fx)+\sqrt{d\tan(e+fx)}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))**(1/2)/(a+a*tan(f*x+e))**2,x)
```

```
[Out] Integral(1/(sqrt(d*tan(e + f*x))*tan(e + f*x)**2 + 2*sqrt(d*tan(e + f*x))*t
an(e + f*x) + sqrt(d*tan(e + f*x))), x)/a**2
```

Giac [A]
time = 0.68, size = 261, normalized size = 0.93

$$-\frac{\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{4a^2d^2f}-\frac{\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{4a^2d^2f}+\frac{\sqrt{2}|d|^{\frac{3}{2}}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{|d|}+|d|\right)}{8a^2d^2f}-\frac{\sqrt{2}|d|^{\frac{3}{2}}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{|d|}+|d|\right)}{8a^2d^2f}+\frac{3\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{2a^2\sqrt{d}f}+\frac{\sqrt{d\tan(fx+e)}}{2(d\tan(fx+e)+d)a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(1/2)/(a+a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqrt
(d*tan(f*x + e)))/sqrt(abs(d)))/(a^2*d^2*f) - 1/4*sqrt(2)*abs(d)^(3/2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)
))/(a^2*d^2*f) + 1/8*sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x + e) + sqrt(2)*sqrt
(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(a^2*d^2*f) - 1/8*sqrt(2)*abs(d)^(3
/2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d)
)/(a^2*d^2*f) + 3/2*arctan(sqrt(d*tan(f*x + e))/sqrt(d))/(a^2*sqrt(d)*f) +
1/2*sqrt(d*tan(f*x + e))/((d*tan(f*x + e) + d)*a^2*f)
```

Mupad [B]
time = 4.65, size = 365, normalized size = 1.30

$$\frac{\sqrt{d\tan(e+fx)}}{2\left(a^2\,d\,f+a^2\,d\,f\tan(e+fx)\right)}-\frac{\mathrm{atan}\left(\frac{4\,d^8\sqrt{d\tan(e+fx)}\left(-\frac{1}{a^8\,d^2\,f^4}\right)^{1/4}}{\frac{4\,d^8}{a^2\,f}+36\,a^2\,d^9\,f\sqrt{-\frac{1}{a^8\,d^2\,f^4}}}+\frac{36\,d^9\sqrt{d\tan(e+fx)}\left(-\frac{1}{a^8\,d^2\,f^4}\right)^{3/4}}{\frac{4\,d^8}{a^6\,f^3}+\frac{36\,d^9\sqrt{-\frac{1}{a^8\,d^2\,f^4}}}{a^2\,f}}\right)\left(-\frac{1}{a^8\,d^2\,f^4}\right)^{1/4}}{2}-\mathrm{atan}\left(\frac{d^8\sqrt{d\tan(e+fx)}\left(-\frac{1}{256\,a^8\,d^2\,f^4}\right)^{1/4}16\mathrm{i}}{\frac{4\,d^8}{a^2\,f}-576\,a^2\,d^9\,f\sqrt{-\frac{1}{256\,a^8\,d^2\,f^4}}}-\frac{d^9\sqrt{d\tan(e+fx)}\left(-\frac{1}{256\,a^8\,d^2\,f^4}\right)^{3/4}2304\mathrm{i}}{\frac{4\,d^8}{a^6\,f^3}-\frac{576\,d^9\sqrt{-\frac{1}{256\,a^8\,d^2\,f^4}}}{a^2\,f}}\right)\left(-\frac{1}{256\,a^8\,d^2\,f^4}\right)^{1/4}2\mathrm{i}+\frac{\mathrm{atan}\left(\frac{\sqrt{d\tan(e+fx)}\,1\mathrm{i}}{\sqrt{-d}}\right)3\mathrm{i}}{2\,a^2\sqrt{-d}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*tan(e + f*x))^(1/2)*(a + a*tan(e + f*x))^2),x)
```

```
[Out] (d*tan(e + f*x))^(1/2)/(2*(a^2*d*f + a^2*d*f*tan(e + f*x))) - (atan((4*d^8*
(d*tan(e + f*x))^(1/2)*(-1/(a^8*d^2*f^4))^(1/4))/((4*d^8)/(a^2*f) + 36*a^2*
d^9*f*(-1/(a^8*d^2*f^4))^(1/2)) + (36*d^9*(d*tan(e + f*x))^(1/2)*(-1/(a^8*d
^2*f^4))^(3/4))/((4*d^8)/(a^6*f^3) + (36*d^9*(-1/(a^8*d^2*f^4))^(1/2))/(a^2
*f)))*(-1/(a^8*d^2*f^4))^(1/4))/2 - atan((d^8*(d*tan(e + f*x))^(1/2)*(-1/(2
56*a^8*d^2*f^4))^(1/4)*16i)/((4*d^8)/(a^2*f) - 576*a^2*d^9*f*(-1/(256*a^8*d
^2*f^4))^(1/2)) - (d^9*(d*tan(e + f*x))^(1/2)*(-1/(256*a^8*d^2*f^4))^(3/4)*
2304i)/((4*d^8)/(a^6*f^3) - (576*d^9*(-1/(256*a^8*d^2*f^4))^(1/2))/(a^2*f))
)*(-1/(256*a^8*d^2*f^4))^(1/4)*2i + (atan(((d*tan(e + f*x))^(1/2)*1i)/(-d)^
(1/2))*3i)/(2*a^2*(-d)^(1/2)*f)
```

3.368 $\int \frac{1}{(d\tan(e+fx))^{3/2}(a+a\tan(e+fx))^2}\,dx$

Optimal. Leaf size=306

$$-\frac{5\text{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2d^{3/2}f}+\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2\sqrt{2}\,a^2d^{3/2}f}-\frac{\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right.}{2\sqrt{2}\,a^2d^{3/2}f}$$

```
[Out] -5/2*arctan((d*tan(f*x+e))^(1/2)/d^(1/2))/a^2/d^(3/2)/f+1/4*arctan(1-2^(1/2
)*(d*tan(f*x+e))^(1/2)/d^(1/2))/a^2/d^(3/2)/f*2^(1/2)-1/4*arctan(1+2^(1/2)*
(d*tan(f*x+e))^(1/2)/d^(1/2))/a^2/d^(3/2)/f*2^(1/2)+1/8*ln(d^(1/2)-2^(1/2)*
(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/a^2/d^(3/2)/f*2^(1/2)-1/8*ln(d^(1/
2)+2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/a^2/d^(3/2)/f*2^(1/2)-5
/2/a^2/d/f/(d*tan(f*x+e))^(1/2)+1/2/d/f/(d*tan(f*x+e))^(1/2)/(a^2+a^2*tan(f
*x+e))
```

Rubi [A]
time = 0.48, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3650, 3730, 3734, 12, 3557, 335, 217, 1179, 642, 1176, 631, 210, 3715, 65, 211}

$$-\frac{5\text{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2d^{3/2}f}+\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2\sqrt{2}\,a^2d^{3/2}f}-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{2\sqrt{2}\,a^2d^{3/2}f}+\frac{\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{4\sqrt{2}\,a^2d^{3/2}f}-\frac{\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{4\sqrt{2}\,a^2d^{3/2}f}-\frac{5}{2a^2df\sqrt{d\tan(e+fx)}}+\frac{1}{2df\left(a^2\tan(e+fx)+a^2\right)\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Tan[e + f*x])^(3/2)*(a + a*Tan[e + f*x])^2),x]
```

```
[Out] (-5*ArcTan[Sqrt[d*Tan[e + f*x]]/Sqrt[d]])/(2*a^2*d^(3/2)*f) + ArcTan[1 - (S
qrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]]/(2*Sqrt[2]*a^2*d^(3/2)*f) - ArcTan[1
+ (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]]/(2*Sqrt[2]*a^2*d^(3/2)*f) + Log[S
qrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e + f*x]]]/(4*Sqrt[2]*a^
2*d^(3/2)*f) - Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqrt[2]*Sqrt[d*Tan[e +
f*x]]]/(4*Sqrt[2]*a^2*d^(3/2)*f) - 5/(2*a^2*d*f*Sqrt[d*Tan[e + f*x]]) + 1/(
2*d*f*Sqrt[d*Tan[e + f*x]]*(a^2 + a^2*Tan[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
```

```
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{(d\tan(e+fx))^{3/2}(a+a\tan(e+fx))^2}\,dx = \frac{1}{2df\sqrt{d\tan(e+fx)}\,(a^2+a^2\tan(e+fx))} + \frac{\int \frac{\frac{5a^2d}{2}-a^2d\tan(}{(d\tan(e+fx}}{}$$

$$= -\frac{5}{2a^2df\sqrt{d\tan(e+fx)}} + \frac{1}{2df\sqrt{d\tan(e+fx)}\,(a^2+a^2\,\mathrm{ta}}$$

$$= -\frac{5}{2a^2df\sqrt{d\tan(e+fx)}} + \frac{1}{2df\sqrt{d\tan(e+fx)}\,(a^2+a^2\,\mathrm{ta}}$$

$$= -\frac{5}{2a^2df\sqrt{d\tan(e+fx)}} + \frac{1}{2df\sqrt{d\tan(e+fx)}\,(a^2+a^2\,\mathrm{ta}}$$

$$= -\frac{5}{2a^2df\sqrt{d\tan(e+fx)}} + \frac{1}{2df\sqrt{d\tan(e+fx)}\,(a^2+a^2\,\mathrm{ta}}$$

$$= -\frac{5\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2d^{3/2}f} - \frac{5}{2a^2df\sqrt{d\tan(e+fx)}} +$$

$$= -\frac{5\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2d^{3/2}f} - \frac{5}{2a^2df\sqrt{d\tan(e+fx)}} +$$

$$= -\frac{5\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2d^{3/2}f} - \frac{5}{2a^2df\sqrt{d\tan(e+fx)}} +$$

$$= -\frac{5\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2d^{3/2}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\tan(e+\right.}{4\sqrt{2}}$$

$$= -\frac{5\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2d^{3/2}f} + \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(}}{\sqrt{d}}\right.}{2\sqrt{2}\,a^2d^{3/2}f}$$

Mathematica [A]
time = 1.38, size = 203, normalized size = 0.66

$$\frac{\left(\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)-\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(e+fx)}\right)-10\mathrm{ArcTan}\left(\sqrt{\tan(e+fx)}\right)+\frac{\log\left(-1+\sqrt{2}\sqrt{\tan(e+fx)}-\tan(e+fx)\right)-\log\left(1+\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)}{\sqrt{2}}-\frac{2(4\cos(e+fx)+5\sin(e+fx))}{(\cos(e+fx)+\sin(e+fx))\sqrt{\tan(e+fx)}}\right)\tan^{\frac{3}{2}}(e+fx)}{4a^2f(d\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Tan[e + f*x])^(3/2)*(a + a*Tan[e + f*x])^2),x]
```

```
[Out] ((Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] - Sqrt[2]*ArcTan[1 + Sqrt[
2]*Sqrt[Tan[e + f*x]]] - 10*ArcTan[Sqrt[Tan[e + f*x]]] + (Log[-1 + Sqrt[2]*
Sqrt[Tan[e + f*x]] - Tan[e + f*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + T
an[e + f*x]])/Sqrt[2] - (2*(4*Cos[e + f*x] + 5*Sin[e + f*x]))/((Cos[e + f*x
] + Sin[e + f*x])*Sqrt[Tan[e + f*x]]))*Tan[e + f*x]^(3/2))/(4*a^2*f*(d*Tan[
e + f*x])^(3/2))
```

Maple [A]
time = 0.17, size = 212, normalized size = 0.69

method	result
derivativedivides	$\frac{2d^3\left(-\frac{1}{d^4\sqrt{d\tan(fx+e)}}-\frac{\frac{\sqrt{d\tan(fx+e)}}{2d\tan(fx+e)+2d}+\frac{5\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{2\sqrt{d}}}{2d^4}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\,\text{ta}}{d\,\text{ta}}\right.\right.}{}\right.}{}$
default	$\frac{2d^3\left(-\frac{1}{d^4\sqrt{d\tan(fx+e)}}-\frac{\frac{\sqrt{d\tan(fx+e)}}{2d\tan(fx+e)+2d}+\frac{5\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{2\sqrt{d}}}{2d^4}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\,\text{ta}}{d\,\text{ta}}\right.\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*tan(f*x+e))^(3/2)/(a+a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^2*d^3*(-1/d^4/(d*tan(f*x+e))^(1/2)-1/2/d^4*(1/2*(d*tan(f*x+e))^(1/2)/
(d*tan(f*x+e)+d)+5/2/d^(1/2)*arctan((d*tan(f*x+e))^(1/2)/d^(1/2)))-1/16/d^5
*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(
1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d
^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-
2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)))
```

Maxima [A]
time = 0.52, size = 240, normalized size = 0.78

$$-\frac{\frac{4\,(5\,d\tan(fx+e)+4\,d)}{(d\tan(fx+e))^{\frac{3}{2}}a^2+\sqrt{d\tan(fx+e)}\,a^2d}+\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}}{a^2}+\frac{20\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{a^2\sqrt{d}}}{8\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(3/2)/(a+a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(4*(5*d*tan(f*x + e) + 4*d)/((d*tan(f*x + e))^(3/2)*a^2 + sqrt(d*tan(f
*x + e))*a^2*d) + (2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d
*tan(f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*s
qrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) + sqrt(2)*log(d*tan(f*x +
 e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) - sqrt(2)*log(d*tan
(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d))/a^2 + 20*arc
tan(sqrt(d*tan(f*x + e))/sqrt(d))/(a^2*sqrt(d)))/(d*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1151 vs. $2(248) = 496$.
time = 2.18, size = 2384, normalized size = 7.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(3/2)/(a+a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(5*(cos(f*x + e)^2 + 2*(cos(f*x + e)^3 - cos(f*x + e))*sin(f*x + e) -
 1)*sqrt(-d)*log(-(6*d^2*cos(f*x + e)*sin(f*x + e) - d^2 + 4*(d*cos(f*x + e
)^2 - d*cos(f*x + e)*sin(f*x + e))*sqrt(-d)*sqrt(d*sin(f*x + e)/cos(f*x + e
)))/(2*cos(f*x + e)*sin(f*x + e) + 1)) - 4*(sqrt(2)*a^2*d^2*f*cos(f*x + e)^
2 - sqrt(2)*a^2*d^2*f + 2*(sqrt(2)*a^2*d^2*f*cos(f*x + e)^3 - sqrt(2)*a^2*d
^2*f*cos(f*x + e))*sin(f*x + e))*(1/(a^8*d^6*f^4))^(1/4)*arctan(-sqrt(2)*a^
6*d^4*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e))*(1/(a^8*d^6*f^4))^(3/4) + sqrt(
2)*a^6*d^4*f^3*sqrt((a^4*d^4*f^2*sqrt(1/(a^8*d^6*f^4))*cos(f*x + e) + sqrt(
2)*a^2*d^2*f*sqrt(d*sin(f*x + e)/cos(f*x + e))*(1/(a^8*d^6*f^4))^(1/4)*cos(
f*x + e) + d*sin(f*x + e))/cos(f*x + e))*(1/(a^8*d^6*f^4))^(3/4) - 1) - 4*(
sqrt(2)*a^2*d^2*f*cos(f*x + e)^2 - sqrt(2)*a^2*d^2*f + 2*(sqrt(2)*a^2*d^2*f
*cos(f*x + e)^3 - sqrt(2)*a^2*d^2*f*cos(f*x + e))*sin(f*x + e))*(1/(a^8*d^6
*f^4))^(1/4)*arctan(-sqrt(2)*a^6*d^4*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e))*
(1/(a^8*d^6*f^4))^(3/4) + sqrt(2)*a^6*d^4*f^3*sqrt((a^4*d^4*f^2*sqrt(1/(a^8
*d^6*f^4))*cos(f*x + e) - sqrt(2)*a^2*d^2*f*sqrt(d*sin(f*x + e)/cos(f*x + e
))*(1/(a^8*d^6*f^4))^(1/4)*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))*(1/
(a^8*d^6*f^4))^(3/4) + 1) + (sqrt(2)*a^2*d^2*f*cos(f*x + e)^2 - sqrt(2)*a^2
*d^2*f + 2*(sqrt(2)*a^2*d^2*f*cos(f*x + e)^3 - sqrt(2)*a^2*d^2*f*cos(f*x +
e))*sin(f*x + e))*(1/(a^8*d^6*f^4))^(1/4)*log((a^4*d^4*f^2*sqrt(1/(a^8*d^6*
f^4))*cos(f*x + e) + sqrt(2)*a^2*d^2*f*sqrt(d*sin(f*x + e)/cos(f*x + e))*(1
/(a^8*d^6*f^4))^(1/4)*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e)) - (sqrt(
2)*a^2*d^2*f*cos(f*x + e)^2 - sqrt(2)*a^2*d^2*f + 2*(sqrt(2)*a^2*d^2*f*cos(
f*x + e)^3 - sqrt(2)*a^2*d^2*f*cos(f*x + e))*sin(f*x + e))*(1/(a^8*d^6*f^4)
)^(1/4)*log((a^4*d^4*f^2*sqrt(1/(a^8*d^6*f^4))*cos(f*x + e) - sqrt(2)*a^2*d
^2*f*sqrt(d*sin(f*x + e)/cos(f*x + e))*(1/(a^8*d^6*f^4))^(1/4)*cos(f*x + e)
 + d*sin(f*x + e))/cos(f*x + e)) + 4*(9*cos(f*x + e)^4 - 9*cos(f*x + e)^2 +
 (cos(f*x + e)^3 - 5*cos(f*x + e))*sin(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*
x + e)))/(a^2*d^2*f*cos(f*x + e)^2 - a^2*d^2*f + 2*(a^2*d^2*f*cos(f*x + e)^
```

```
3 - a^2*d^2*f*cos(f*x + e))*sin(f*x + e)), -1/8*(20*(cos(f*x + e)^2 + 2*(co
s(f*x + e)^3 - cos(f*x + e))*sin(f*x + e) - 1)*sqrt(d)*arctan(sqrt(d*sin(f*
x + e)/cos(f*x + e))/sqrt(d)) - 4*(sqrt(2)*a^2*d^2*f*cos(f*x + e)^2 - sqrt(
2)*a^2*d^2*f + 2*(sqrt(2)*a^2*d^2*f*cos(f*x + e)^3 - sqrt(2)*a^2*d^2*f*cos(
f*x + e))*sin(f*x + e))*(1/(a^8*d^6*f^4))^(1/4)*arctan(-sqrt(2)*a^6*d^4*f^3
*sqrt(d*sin(f*x + e)/cos(f*x + e))*(1/(a^8*d^6*f^4))^(3/4) + sqrt(2)*a^6*d^
4*f^3*sqrt((a^4*d^4*f^2*sqrt(1/(a^8*d^6*f^4))*cos(f*x + e) + sqrt(2)*a^2*d^
2*f*sqrt(d*sin(f*x + e)/cos(f*x + e))*(1/(a^8*d^6*f^4))^(1/4)*cos(f*x + e)
+ d*sin(f*x + e))/cos(f*x + e))*(1/(a^8*d^6*f^4))^(3/4) - 1) - 4*(sqrt(2)*a
^2*d^2*f*cos(f*x + e)^2 - sqrt(2)*a^2*d^2*f + 2*(sqrt(2)*a^2*d^2*f*cos(f*x
+ e)^3 - sqrt(2)*a^2*d^2*f*cos(f*x + e))*sin(f*x + e))*(1/(a^8*d^6*f^4))^(1
/4)*arctan(-sqrt(2)*a^6*d^4*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e))*(1/(a^8*d
^6*f^4))^(3/4) + sqrt(2)*a^6*d^4*f^3*sqrt((a^4*d^4*f^2*sqrt(1/(a^8*d^6*f^4)
)*cos(f*x + e) - sqrt(2)*a^2*d^2*f*sqrt(d*sin(f*x + e)/cos(f*x + e))*(1/(a^
8*d^6*f^4))^(1/4)*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))*(1/(a^8*d^6*
f^4))^(3/4) + 1) + (sqrt(2)*a^2*d^2*f*cos(f*x + e)^2 - sqrt(2)*a^2*d^2*f +
2*(sqrt(2)*a^2*d^2*f*cos(f*x + e)^3 - sqrt(2)*a^2*d^2*f*cos(f*x + e))*sin(f
*x + e))*(1/(a^8*d^6*f^4))^(1/4)*log((a^4*d^4*f^2*sqrt(1/(a^8*d^6*f^4))*cos
(f*x + e) + sqrt(2)*a^2*d^2*f*sqrt(d*sin(f*x + e)/cos(f*x + e))*(1/(a^8*d^6
*f^4))^(1/4)*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e)) - (sqrt(2)*a^2*d^
2*f*cos(f*x + e)^2 - sqrt(2)*a^2*d^2*f + 2*(sqrt(2)*a^2*d^2*f*cos(f*x + e)^
3 - sqrt(2)*a^2*d^2*f*cos(f*x + e))*sin(f*x + e))*(1/(a^8*d^6*f^4))^(1/4)*l
og((a^4*d^4*f^2*sqrt(1/(a^8*d^6*f^4))*cos(f*x + e) - sqrt(2)*a^2*d^2*f*sqrt
(d*sin(f*x + e)/cos(f*x + e))*(1/(a^8*d^6*f^4))^(1/4)*cos(f*x + e) + d*sin(
f*x + e))/cos(f*x + e)) + 4*(9*cos(f*x + e)^4 - 9*cos(f*x + e)^2 + (cos(f*x
 + e)^3 - 5*cos(f*x + e))*sin(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e)))/
(a^2*d^2*f*cos(f*x + e)^2 - a^2*d^2*f + 2*(a^2*d^2*f*cos(f*x + e)^3 - a^2*d
^2*f*cos(f*x + e))*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{(d\tan(e+fx))^{\frac{3}{2}}\tan^2(e+fx)+2(d\tan(e+fx))^{\frac{3}{2}}\tan(e+fx)+(d\tan(e+fx))^{\frac{3}{2}}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))**(3/2)/(a+a*tan(f*x+e))**2,x)
```

```
[Out] Integral(1/((d*tan(e + f*x))**(3/2)*tan(e + f*x)**2 + 2*(d*tan(e + f*x))**(
3/2)*tan(e + f*x) + (d*tan(e + f*x))**(3/2)), x)/a**2
```

Giac [A]
time = 0.75, size = 291, normalized size = 0.95

$$-\frac{\frac{2\sqrt{2}\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{a^2df}+\frac{2\sqrt{2}\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{a^2df}+\frac{\sqrt{2}\sqrt{|d|}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{|d|}+|d|\right)}{a^2df}-\frac{\sqrt{2}\sqrt{|d|}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{|d|}+|d|\right)}{a^2df}+\frac{20\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{a^2\sqrt{d}f}+\frac{4(5d\tan(fx+e)+4d)}{\left(\sqrt{d\tan(fx+e)}\,d\tan(fx+e)+\sqrt{d\tan(fx+e)}\,d\right)a^2f}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(3/2)/(a+a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/8*(2*sqrt(2)*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*s
qrt(d*tan(f*x + e)))/sqrt(abs(d)))/(a^2*d*f) + 2*sqrt(2)*sqrt(abs(d))*arcta
n(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d))
)/(a^2*d*f) + sqrt(2)*sqrt(abs(d))*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(
f*x + e))*sqrt(abs(d)) + abs(d))/(a^2*d*f) - sqrt(2)*sqrt(abs(d))*log(d*tan
(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(a^2*d*f) +
 20*arctan(sqrt(d*tan(f*x + e))/sqrt(d))/(a^2*sqrt(d)*f) + 4*(5*d*tan(f*x +
 e) + 4*d)/((sqrt(d*tan(f*x + e))*d*tan(f*x + e) + sqrt(d*tan(f*x + e))*d)*
a^2*f))/d
```

Mupad [B]
time = 4.72, size = 415, normalized size = 1.36

$$\frac{\mathrm{atan}\left(\frac{2048\,a^{10}\,d^{13}\,f^5\,\sqrt{d\,\tan\left(e+f\,x\right)}\,{\left(-\frac{1}{a^8\,d^6\,f^4}\right)}^{1/4}}{51200\,a^8\,d^{12}\,f^4-2048\,a^{12}\,d^{15}\,f^6\,\sqrt{-\frac{1}{a^8\,d^6\,f^4}}}+\frac{51200\,a^{14}\,d^{16}\,f^7\,\sqrt{d\,\tan\left(e+f\,x\right)}\,{\left(-\frac{1}{a^8\,d^6\,f^4}\right)}^{3/4}}{51200\,a^8\,d^{12}\,f^4-2048\,a^{12}\,d^{15}\,f^6\,\sqrt{-\frac{1}{a^8\,d^6\,f^4}}}\right)\,{\left(-\frac{1}{a^8\,d^6\,f^4}\right)}^{1/4}}{2}+\mathrm{atan}\left(\frac{a^{10}\,d^{13}\,f^5\,\sqrt{d\,\tan\left(e+f\,x\right)}\,{\left(-\frac{1}{256\,a^8\,d^6\,f^4}\right)}^{1/4}\,8192{}\mathrm{i}}{51200\,a^8\,d^{12}\,f^4+32768\,a^{12}\,d^{15}\,f^6\,\sqrt{-\frac{1}{256\,a^8\,d^6\,f^4}}}-\frac{a^{14}\,d^{16}\,f^7\,\sqrt{d\,\tan\left(e+f\,x\right)}\,{\left(-\frac{1}{256\,a^8\,d^6\,f^4}\right)}^{3/4}\,3276800{}\mathrm{i}}{51200\,a^8\,d^{12}\,f^4+32768\,a^{12}\,d^{15}\,f^6\,\sqrt{-\frac{1}{256\,a^8\,d^6\,f^4}}}\right)\,{\left(-\frac{1}{256\,a^8\,d^6\,f^4}\right)}^{1/4}\,2{}\mathrm{i}-\frac{\frac{5\,\tan\left(e+f\,x\right)}{2}+2}{a^2\,f\,{\left(d\,\tan\left(e+f\,x\right)\right)}^{3/2}+a^2\,d\,f\,\sqrt{d\,\tan\left(e+f\,x\right)}}-\frac{\mathrm{atan}\left(\frac{\sqrt{d\,\tan\left(e+f\,x\right)}\,\sqrt{-d^3}\,1{}\mathrm{i}}{d^2}\right)\,\sqrt{-d^3}\,5{}\mathrm{i}}{2\,a^2\,d^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*tan(e + f*x))^(3/2)*(a + a*tan(e + f*x))^2),x)
```

```
[Out] (atan((2048*a^10*d^13*f^5*(d*tan(e + f*x))^(1/2)*(-1/(a^8*d^6*f^4))^(1/4))/
(51200*a^8*d^12*f^4 - 2048*a^12*d^15*f^6*(-1/(a^8*d^6*f^4))^(1/2)) + (51200
*a^14*d^16*f^7*(d*tan(e + f*x))^(1/2)*(-1/(a^8*d^6*f^4))^(3/4))/(51200*a^8*
d^12*f^4 - 2048*a^12*d^15*f^6*(-1/(a^8*d^6*f^4))^(1/2)))*(-1/(a^8*d^6*f^4))
^(1/4))/2 + atan((a^10*d^13*f^5*(d*tan(e + f*x))^(1/2)*(-1/(256*a^8*d^6*f^4
))^(1/4)*8192i)/(51200*a^8*d^12*f^4 + 32768*a^12*d^15*f^6*(-1/(256*a^8*d^6*
f^4))^(1/2)) - (a^14*d^16*f^7*(d*tan(e + f*x))^(1/2)*(-1/(256*a^8*d^6*f^4))
^(3/4)*3276800i)/(51200*a^8*d^12*f^4 + 32768*a^12*d^15*f^6*(-1/(256*a^8*d^6
*f^4))^(1/2)))*(-1/(256*a^8*d^6*f^4))^(1/4)*2i - ((5*tan(e + f*x))/2 + 2)/(
a^2*f*(d*tan(e + f*x))^(3/2) + a^2*d*f*(d*tan(e + f*x))^(1/2)) - (atan(((d*
tan(e + f*x))^(1/2)*(-d^3)^(1/2)*1i)/d^2)*(-d^3)^(1/2)*5i)/(2*a^2*d^3*f)
```

3.369 $\int \frac{1}{(d\tan(e+fx))^{5/2}(a+a\tan(e+fx))^2}\,dx$

Optimal. Leaf size=331

$$\frac{7\text{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2d^{5/2}f}-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2\sqrt{2}\,a^2d^{5/2}f}+\frac{\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2\sqrt{2}\,a^2d^{5/2}f}$$

```
[Out] 7/2*arctan((d*tan(f*x+e))^(1/2)/d^(1/2))/a^2/d^(5/2)/f-1/4*arctan(1-2^(1/2)
*(d*tan(f*x+e))^(1/2)/d^(1/2))/a^2/d^(5/2)/f*2^(1/2)+1/4*arctan(1+2^(1/2)*(
d*tan(f*x+e))^(1/2)/d^(1/2))/a^2/d^(5/2)/f*2^(1/2)+1/8*ln(d^(1/2)-2^(1/2)*(
d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/a^2/d^(5/2)/f*2^(1/2)-1/8*ln(d^(1/2
)+2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/a^2/d^(5/2)/f*2^(1/2)+9/
2/a^2/d^2/f/(d*tan(f*x+e))^(1/2)-7/6/a^2/d/f/(d*tan(f*x+e))^(3/2)+1/2/d/f/(
d*tan(f*x+e))^(3/2)/(a^2+a^2*tan(f*x+e))
```

Rubi [A]
time = 0.66, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3650, 3730, 3734, 12, 16, 3557, 335, 303, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{7\text{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2d^{5/2}f}-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2\sqrt{2}\,a^2d^{5/2}f}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{2\sqrt{2}\,a^2d^{5/2}f}+\frac{\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{4\sqrt{2}\,a^2d^{5/2}f}-\frac{\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{4\sqrt{2}\,a^2d^{5/2}f}+\frac{9}{2a^2d^2f\sqrt{d\tan(e+fx)}}+\frac{1}{2df\left(a^2\tan(e+fx)+a^2\right)(d\tan(e+fx))^{3/2}}-\frac{7}{6a^2df(d\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Tan[e + f*x])^(5/2)*(a + a*Tan[e + f*x])^2),x]
```

```
[Out] (7*ArcTan[Sqrt[d*Tan[e + f*x]]/Sqrt[d]])/(2*a^2*d^(5/2)*f) - ArcTan[1 - (Sq
rt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]]/(2*Sqrt[2]*a^2*d^(5/2)*f) + ArcTan[1 +
 (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]]/(2*Sqrt[2]*a^2*d^(5/2)*f) + Log[Sq
rt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e + f*x]]]/(4*Sqrt[2]*a^2
*d^(5/2)*f) - Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqrt[2]*Sqrt[d*Tan[e + f
*x]]]/(4*Sqrt[2]*a^2*d^(5/2)*f) - 7/(6*a^2*d*f*(d*Tan[e + f*x])^(3/2)) + 9/
(2*a^2*d^2*f*Sqrt[d*Tan[e + f*x]]) + 1/(2*d*f*(d*Tan[e + f*x])^(3/2)*(a^2 +
 a^2*Tan[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
```

```
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{(d\tan(e+fx))^{5/2}(a+a\tan(e+fx))^2}\,dx = \frac{1}{2df(d\tan(e+fx))^{3/2}\left(a^2+a^2\tan(e+fx)\right)} + \frac{\int \frac{\frac{7a^2d}{2}-a^2d\tan(\cdots}{(d\tan(e+fx\cdots}}{\cdots}$$

$$= -\frac{7}{6a^2df(d\tan(e+fx))^{3/2}} + \frac{1}{2df(d\tan(e+fx))^{3/2}\left(a^2+a^2\tan\cdots\right.}$$

$$= -\frac{7}{6a^2df(d\tan(e+fx))^{3/2}} + \frac{9}{2a^2d^2f\sqrt{d\tan(e+fx)}} + \frac{\cdots}{2df(\cdots}$$

$$= -\frac{7}{6a^2df(d\tan(e+fx))^{3/2}} + \frac{9}{2a^2d^2f\sqrt{d\tan(e+fx)}} + \frac{\cdots}{2df(\cdots}$$

$$= -\frac{7}{6a^2df(d\tan(e+fx))^{3/2}} + \frac{9}{2a^2d^2f\sqrt{d\tan(e+fx)}} + \frac{\cdots}{2df(\cdots}$$

$$= -\frac{7}{6a^2df(d\tan(e+fx))^{3/2}} + \frac{9}{2a^2d^2f\sqrt{d\tan(e+fx)}} + \frac{\cdots}{2df(\cdots}$$

$$= \frac{7\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2d^{5/2}f} - \frac{7}{6a^2df(d\tan(e+fx))^{3/2}} + \frac{\cdots}{2\cdots}$$

$$= \frac{7\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2d^{5/2}f} - \frac{7}{6a^2df(d\tan(e+fx))^{3/2}} + \frac{\cdots}{2\cdots}$$

$$= \frac{7\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2d^{5/2}f} - \frac{7}{6a^2df(d\tan(e+fx))^{3/2}} + \frac{\cdots}{2\cdots}$$

$$= \frac{7\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2d^{5/2}f} - \frac{7}{6a^2df(d\tan(e+fx))^{3/2}} + \frac{\cdots}{2\cdots}$$

$$= \frac{7\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2d^{5/2}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx\cdots\right.}{4\sqrt{2}\,a^2\cdots}$$

$$= \frac{7\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{2a^2d^{5/2}f} - \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+\cdots}}{\sqrt{d}}\right.}{2\sqrt{2}\,a^2d^{5/2}f}$$

Mathematica [A]

time = 4.17, size = 203, normalized size = 0.61

$$\frac{\left(-\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(e+fx)}\right)+\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(e+fx)}\right)+14\mathrm{ArcTan}\left(\sqrt{\tan(e+fx)}\right)+\frac{\log\left(-1+\sqrt{2}\,\sqrt{\tan(e+fx)}-\tan(e+fx)\right)-\log\left(1+\sqrt{2}\,\sqrt{\tan(e+fx)}+\tan(e+fx)\right)}{\sqrt{2}}+\frac{2(31+20\cot(e+fx)-4\csc^2(e+fx))}{3(1+\cot(e+fx))\sqrt{\tan(e+fx)}}\right)\tan^{\frac{5}{2}}(e+fx)}{4a^2f(d\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Tan[e + f*x])^(5/2)*(a + a*Tan[e + f*x])^2),x]
```

```
[Out] ((-(Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]]) + Sqrt[2]*ArcTan[1 + Sq
rt[2]*Sqrt[Tan[e + f*x]]] + 14*ArcTan[Sqrt[Tan[e + f*x]]] + (Log[-1 + Sqrt[
2]*Sqrt[Tan[e + f*x]] - Tan[e + f*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]
+ Tan[e + f*x]])/Sqrt[2] + (2*(31 + 20*Cot[e + f*x] - 4*Csc[e + f*x]^2))/(3
*(1 + Cot[e + f*x])*Sqrt[Tan[e + f*x]]))*Tan[e + f*x]^(5/2))/(4*a^2*f*(d*Ta
n[e + f*x])^(5/2))
```

Maple [A]

time = 0.17, size = 227, normalized size = 0.69

method	result
derivativedivides	$2d^3\left(-\frac{1}{3d^4(d\tan(fx+e))^{\frac{3}{2}}}+\frac{2}{d^5\sqrt{d\tan(fx+e)}}+\frac{\frac{\sqrt{d\tan(fx+e)}}{2d\tan(fx+e)+2d}+\frac{7\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{2\sqrt{d}}}{2d^5}\right.$
default	$2d^3\left(-\frac{1}{3d^4(d\tan(fx+e))^{\frac{3}{2}}}+\frac{2}{d^5\sqrt{d\tan(fx+e)}}+\frac{\frac{\sqrt{d\tan(fx+e)}}{2d\tan(fx+e)+2d}+\frac{7\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{2\sqrt{d}}}{2d^5}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*tan(f*x+e))^(5/2)/(a+a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^2*d^3*(-1/3/d^4/(d*tan(f*x+e))^(3/2)+2/d^5/(d*tan(f*x+e))^(1/2)+1/2/d
^5*(1/2*(d*tan(f*x+e))^(1/2)/(d*tan(f*x+e)+d)+7/2/d^(1/2)*arctan((d*tan(f*x
+e))^(1/2)/d^(1/2)))+1/16/d^5/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(
1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d
*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*ta
n(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)))
```

Maxima [A]
time = 0.52, size = 265, normalized size = 0.80

$$\frac{\frac{4\left(27\,d^{2}\tan\left(fx+e\right)^{2}+20\,d^{2}\tan\left(fx+e\right)-4\,d^{2}\right)}{\left(d\tan\left(fx+e\right)\right)^{\frac{5}{2}}a^{2}d+\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}a^{2}d^{2}}+\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan\left(fx+e\right)+\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(d\tan\left(fx+e\right)-\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{d}+d\right)}{\sqrt{d}}\right)}{a^{2}d}+\frac{84\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}}{\sqrt{d}}\right)}{a^{2}d^{\frac{3}{2}}}}{24\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(5/2)/(a+a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/24*(4*(27*d^2*tan(f*x + e)^2 + 20*d^2*tan(f*x + e) - 4*d^2)/((d*tan(f*x +
 e))^(5/2)*a^2*d + (d*tan(f*x + e))^(3/2)*a^2*d^2) + 3*(2*sqrt(2)*arctan(1/
2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) + 2*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(
d))/sqrt(d) - sqrt(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqr
t(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e
))*sqrt(d) + d)/sqrt(d))/(a^2*d) + 84*arctan(sqrt(d*tan(f*x + e))/sqrt(d))/
(a^2*d^(3/2)))/(d*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1158 vs. $2(270) = 540$.
time = 2.01, size = 2398, normalized size = 7.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(5/2)/(a+a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [-1/24*(21*(cos(f*x + e)^2 + 2*(cos(f*x + e)^3 - cos(f*x + e))*sin(f*x + e)
 - 1)*sqrt(-d)*log(-(6*d^2*cos(f*x + e)*sin(f*x + e) - d^2 - 4*(d*cos(f*x +
 e)^2 - d*cos(f*x + e)*sin(f*x + e))*sqrt(-d)*sqrt(d*sin(f*x + e)/cos(f*x +
 e)))/(2*cos(f*x + e)*sin(f*x + e) + 1)) + 12*(sqrt(2)*a^2*d^3*f*cos(f*x +
e)^2 - sqrt(2)*a^2*d^3*f + 2*(sqrt(2)*a^2*d^3*f*cos(f*x + e)^3 - sqrt(2)*a^
2*d^3*f*cos(f*x + e))*sin(f*x + e))*(1/(a^8*d^10*f^4))^(1/4)*arctan(-sqrt(2
)*a^2*d^2*f*sqrt(d*sin(f*x + e)/cos(f*x + e))*(1/(a^8*d^10*f^4))^(1/4) + sq
rt(2)*a^2*d^2*f*sqrt((sqrt(2)*a^6*d^8*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e))
*(1/(a^8*d^10*f^4))^(3/4)*cos(f*x + e) + a^4*d^6*f^2*sqrt(1/(a^8*d^10*f^4))
*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))*(1/(a^8*d^10*f^4))^(1/4) - 1)
 + 12*(sqrt(2)*a^2*d^3*f*cos(f*x + e)^2 - sqrt(2)*a^2*d^3*f + 2*(sqrt(2)*a^
2*d^3*f*cos(f*x + e)^3 - sqrt(2)*a^2*d^3*f*cos(f*x + e))*sin(f*x + e))*(1/(
a^8*d^10*f^4))^(1/4)*arctan(-sqrt(2)*a^2*d^2*f*sqrt(d*sin(f*x + e)/cos(f*x
+ e))*(1/(a^8*d^10*f^4))^(1/4) + sqrt(2)*a^2*d^2*f*sqrt(-(sqrt(2)*a^6*d^8*f
^3*sqrt(d*sin(f*x + e)/cos(f*x + e))*(1/(a^8*d^10*f^4))^(3/4)*cos(f*x + e)
- a^4*d^6*f^2*sqrt(1/(a^8*d^10*f^4))*cos(f*x + e) - d*sin(f*x + e))/cos(f*x
 + e))*(1/(a^8*d^10*f^4))^(1/4) + 1) + 3*(sqrt(2)*a^2*d^3*f*cos(f*x + e)^2
```

```
- sqrt(2)*a^2*d^3*f + 2*(sqrt(2)*a^2*d^3*f*cos(f*x + e)^3 - sqrt(2)*a^2*d^3
*f*cos(f*x + e))*sin(f*x + e))*(1/(a^8*d^10*f^4))^(1/4)*log((sqrt(2)*a^6*d^
8*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e))*(1/(a^8*d^10*f^4))^(3/4)*cos(f*x +
e) + a^4*d^6*f^2*sqrt(1/(a^8*d^10*f^4))*cos(f*x + e) + d*sin(f*x + e))/cos(
f*x + e)) - 3*(sqrt(2)*a^2*d^3*f*cos(f*x + e)^2 - sqrt(2)*a^2*d^3*f + 2*(sq
rt(2)*a^2*d^3*f*cos(f*x + e)^3 - sqrt(2)*a^2*d^3*f*cos(f*x + e))*sin(f*x +
e))*(1/(a^8*d^10*f^4))^(1/4)*log(-(sqrt(2)*a^6*d^8*f^3*sqrt(d*sin(f*x + e)/
cos(f*x + e))*(1/(a^8*d^10*f^4))^(3/4)*cos(f*x + e) - a^4*d^6*f^2*sqrt(1/(a
^8*d^10*f^4))*cos(f*x + e) - d*sin(f*x + e))/cos(f*x + e)) - 4*(51*cos(f*x
+ e)^4 - 47*cos(f*x + e)^2 + (11*cos(f*x + e)^3 - 27*cos(f*x + e))*sin(f*x
+ e))*sqrt(d*sin(f*x + e)/cos(f*x + e)))/(a^2*d^3*f*cos(f*x + e)^2 - a^2*d^
3*f + 2*(a^2*d^3*f*cos(f*x + e)^3 - a^2*d^3*f*cos(f*x + e))*sin(f*x + e)),
1/24*(84*(cos(f*x + e)^2 + 2*(cos(f*x + e)^3 - cos(f*x + e))*sin(f*x + e) -
 1)*sqrt(d)*arctan(sqrt(d*sin(f*x + e)/cos(f*x + e))/sqrt(d)) - 12*(sqrt(2)
*a^2*d^3*f*cos(f*x + e)^2 - sqrt(2)*a^2*d^3*f + 2*(sqrt(2)*a^2*d^3*f*cos(f*
x + e)^3 - sqrt(2)*a^2*d^3*f*cos(f*x + e))*sin(f*x + e))*(1/(a^8*d^10*f^4))
^(1/4)*arctan(-sqrt(2)*a^2*d^2*f*sqrt(d*sin(f*x + e)/cos(f*x + e))*(1/(a^8*
d^10*f^4))^(1/4) + sqrt(2)*a^2*d^2*f*sqrt((sqrt(2)*a^6*d^8*f^3*sqrt(d*sin(f
*x + e)/cos(f*x + e))*(1/(a^8*d^10*f^4))^(3/4)*cos(f*x + e) + a^4*d^6*f^2*s
qrt(1/(a^8*d^10*f^4))*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))*(1/(a^8*
d^10*f^4))^(1/4) - 1) - 12*(sqrt(2)*a^2*d^3*f*cos(f*x + e)^2 - sqrt(2)*a^2*
d^3*f + 2*(sqrt(2)*a^2*d^3*f*cos(f*x + e)^3 - sqrt(2)*a^2*d^3*f*cos(f*x + e
))*sin(f*x + e))*(1/(a^8*d^10*f^4))^(1/4)*arctan(-sqrt(2)*a^2*d^2*f*sqrt(d*
sin(f*x + e)/cos(f*x + e))*(1/(a^8*d^10*f^4))^(1/4) + sqrt(2)*a^2*d^2*f*sqr
t(-(sqrt(2)*a^6*d^8*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e))*(1/(a^8*d^10*f^4)
)^(3/4)*cos(f*x + e) - a^4*d^6*f^2*sqrt(1/(a^8*d^10*f^4))*cos(f*x + e) - d*
sin(f*x + e))/cos(f*x + e))*(1/(a^8*d^10*f^4))^(1/4) + 1) - 3*(sqrt(2)*a^2*
d^3*f*cos(f*x + e)^2 - sqrt(2)*a^2*d^3*f + 2*(sqrt(2)*a^2*d^3*f*cos(f*x + e
)^3 - sqrt(2)*a^2*d^3*f*cos(f*x + e))*sin(f*x + e))*(1/(a^8*d^10*f^4))^(1/4
)*log((sqrt(2)*a^6*d^8*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e))*(1/(a^8*d^10*f
^4))^(3/4)*cos(f*x + e) + a^4*d^6*f^2*sqrt(1/(a^8*d^10*f^4))*cos(f*x + e) +
 d*sin(f*x + e))/cos(f*x + e)) + 3*(sqrt(2)*a^2*d^3*f*cos(f*x + e)^2 - sqrt
(2)*a^2*d^3*f + 2*(sqrt(2)*a^2*d^3*f*cos(f*x + e)^3 - sqrt(2)*a^2*d^3*f*cos
(f*x + e))*sin(f*x + e))*(1/(a^8*d^10*f^4))^(1/4)*log(-(sqrt(2)*a^6*d^8*f^3
*sqrt(d*sin(f*x + e)/cos(f*x + e))*(1/(a^8*d^10*f^4))^(3/4)*cos(f*x + e) -
a^4*d^6*f^2*sqrt(1/(a^8*d^10*f^4))*cos(f*x + e) - d*sin(f*x + e))/cos(f*x +
 e)) + 4*(51*cos(f*x + e)^4 - 47*cos(f*x + e)^2 + (11*cos(f*x + e)^3 - 27*c
os(f*x + e))*sin(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e)))/(a^2*d^3*f*co
s(f*x + e)^2 - a^2*d^3*f + 2*(a^2*d^3*f*cos(f*x + e)^3 - a^2*d^3*f*cos(f*x
+ e))*sin(f*x + e))]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{(d\tan(e+fx))^{\frac{5}{2}}\tan^2(e+fx)+2(d\tan(e+fx))^{\frac{5}{2}}\tan(e+fx)+(d\tan(e+fx))^{\frac{5}{2}}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))**(5/2)/(a+a*tan(f*x+e))**2,x)
```

```
[Out] Integral(1/((d*tan(e + f*x))**(5/2)*tan(e + f*x)**2 + 2*(d*tan(e + f*x))**(
5/2)*tan(e + f*x) + (d*tan(e + f*x))**(5/2)), x)/a**2
```

Giac [A]
time = 0.80, size = 309, normalized size = 0.93

$$\frac{\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{4a^2d^4f}+\frac{\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{4a^2d^4f}-\frac{\sqrt{2}|d|^{\frac{3}{2}}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{|d|}+|d|\right)}{8a^2d^4f}+\frac{\sqrt{2}|d|^{\frac{3}{2}}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{|d|}+|d|\right)}{8a^2d^4f}+\frac{7\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{2a^2d^{\frac{5}{2}}f}+\frac{\sqrt{d\tan(fx+e)}}{2(d\tan(fx+e)+d)a^2d^2f}+\frac{2(6d\tan(fx+e)-d)}{3\sqrt{d\tan(fx+e)}\,a^2d^3f\tan(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(5/2)/(a+a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqrt(
d*tan(f*x + e)))/sqrt(abs(d)))/(a^2*d^4*f) + 1/4*sqrt(2)*abs(d)^(3/2)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d))
)/(a^2*d^4*f) - 1/8*sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(
d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(a^2*d^4*f) + 1/8*sqrt(2)*abs(d)^(3/
2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))
/(a^2*d^4*f) + 7/2*arctan(sqrt(d*tan(f*x + e))/sqrt(d))/(a^2*d^(5/2)*f) + 1
/2*sqrt(d*tan(f*x + e))/((d*tan(f*x + e) + d)*a^2*d^2*f) + 2/3*(6*d*tan(f*x
 + e) - d)/(sqrt(d*tan(f*x + e))*a^2*d^3*f*tan(f*x + e))
```

Mupad [B]
time = 4.98, size = 424, normalized size = 1.28

$$\frac{\frac{9\tan(e+fx)^2}{2}+\frac{10\tan(e+fx)}{3}-\frac{2}{3}}{a^2f(d\tan(e+fx))^{5/2}+a^2df(d\tan(e+fx))^{3/2}}+\frac{\mathrm{atan}\left(\frac{2048a^{10}d^{18}f^5\sqrt{d\tan(e+fx)}\left(-\frac{1}{a^8d^{10}f^4}\right)^{1/4}}{2048a^8d^{16}f^4+100352a^{12}d^{21}f^6\sqrt{-\frac{1}{a^8d^{10}f^4}}}+\frac{100352a^{14}d^{23}f^7\sqrt{d\tan(e+fx)}\left(-\frac{1}{a^8d^{10}f^4}\right)^{3/4}}{2048a^8d^{16}f^4+100352a^{12}d^{21}f^6\sqrt{-\frac{1}{a^8d^{10}f^4}}}\right)\left(-\frac{1}{a^8d^{10}f^4}\right)^{1/4}}{2}+\mathrm{atan}\left(\frac{a^{10}d^{18}f^5\sqrt{d\tan(e+fx)}\left(-\frac{1}{256a^8d^{10}f^4}\right)^{1/4}8192\mathrm{i}}{2048a^8d^{16}f^4-1605632a^{12}d^{21}f^6\sqrt{-\frac{1}{256a^8d^{10}f^4}}}-\frac{a^{14}d^{23}f^7\sqrt{d\tan(e+fx)}\left(-\frac{1}{256a^8d^{10}f^4}\right)^{3/4}6422528\mathrm{i}}{2048a^8d^{16}f^4-1605632a^{12}d^{21}f^6\sqrt{-\frac{1}{256a^8d^{10}f^4}}}\right)\left(-\frac{1}{256a^8d^{10}f^4}\right)^{1/4}2\mathrm{i}+\frac{\mathrm{atan}\left(\frac{\sqrt{d\tan(e+fx)}\sqrt{-d^5}\,1\mathrm{i}}{d^3}\right)\sqrt{-d^5}\,7\mathrm{i}}{2a^2d^5f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*tan(e + f*x))^(5/2)*(a + a*tan(e + f*x))^2),x)
```

```
[Out] ((10*tan(e + f*x))/3 + (9*tan(e + f*x)^2)/2 - 2/3)/(a^2*f*(d*tan(e + f*x))^
(5/2) + a^2*d*f*(d*tan(e + f*x))^(3/2)) + (atan((2048*a^10*d^18*f^5*(d*tan(
e + f*x))^(1/2)*(-1/(a^8*d^10*f^4))^(1/4))/(2048*a^8*d^16*f^4 + 100352*a^12
*d^21*f^6*(-1/(a^8*d^10*f^4))^(1/2)) + (100352*a^14*d^23*f^7*(d*tan(e + f*x
))^(1/2)*(-1/(a^8*d^10*f^4))^(3/4))/(2048*a^8*d^16*f^4 + 100352*a^12*d^21*f
^6*(-1/(a^8*d^10*f^4))^(1/2)))*(-1/(a^8*d^10*f^4))^(1/4))/2 + atan((a^10*d^
18*f^5*(d*tan(e + f*x))^(1/2)*(-1/(256*a^8*d^10*f^4))^(1/4)*8192i)/(2048*a^
8*d^16*f^4 - 1605632*a^12*d^21*f^6*(-1/(256*a^8*d^10*f^4))^(1/2)) - (a^14*d
```

```
^23*f^7*(d*tan(e + f*x))^(1/2)*(-1/(256*a^8*d^10*f^4))^(3/4)*6422528i)/(204
8*a^8*d^16*f^4 - 1605632*a^12*d^21*f^6*(-1/(256*a^8*d^10*f^4))^(1/2)))*(-1/
(256*a^8*d^10*f^4))^(1/4)*2i + (atan(((d*tan(e + f*x))^(1/2)*(-d^5)^(1/2)*1
i)/d^3)*(-d^5)^(1/2)*7i)/(2*a^2*d^5*f)
```

3.370 $\int \frac{(d\tan(e+fx))^{9/2}}{(a+a\tan(e+fx))^3}\,dx$

Optimal. Leaf size=189

$$-\frac{31d^{9/2}\text{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8a^3f}+\frac{d^{9/2}\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{2\sqrt{2}a^3f}+\frac{27d^4\sqrt{d\tan(e+fx)}}{8a^3f}-\frac{9d^3(d}{8a^3f(1}$$

```
[Out] -31/8*d^(9/2)*arctan((d*tan(f*x+e))^(1/2)/d^(1/2))/a^3/f+1/4*d^(9/2)*arctan
h(1/2*(d^(1/2)+d^(1/2)*tan(f*x+e))*2^(1/2)/(d*tan(f*x+e))^(1/2))/a^3/f*2^(1
/2)+27/8*d^4*(d*tan(f*x+e))^(1/2)/a^3/f-9/8*d^3*(d*tan(f*x+e))^(3/2)/a^3/f/
(1+tan(f*x+e))-1/4*d^2*(d*tan(f*x+e))^(5/2)/a/f/(a+a*tan(f*x+e))^2
```

Rubi [A]
time = 0.49, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3646, 3726, 3728, 3735, 3613, 214, 3715, 65, 211}

$$-\frac{31d^{9/2}\text{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8a^3f}+\frac{d^{9/2}\tanh^{-1}\left(\frac{\sqrt{d}\tan(e+fx)+\sqrt{d}}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{2\sqrt{2}a^3f}+\frac{27d^4\sqrt{d\tan(e+fx)}}{8a^3f}-\frac{9d^3(d\tan(e+fx))^{3/2}}{8a^3f(\tan(e+fx)+1)}-\frac{d^2(d\tan(e+fx))^{5/2}}{4af(a\tan(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(9/2)/(a + a*Tan[e + f*x])^3,x]
```

```
[Out] (-31*d^(9/2)*ArcTan[Sqrt[d*Tan[e + f*x]]/Sqrt[d]])/(8*a^3*f) + (d^(9/2)*Arc
Tanh[(Sqrt[d] + Sqrt[d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[d*Tan[e + f*x]])])/(2*S
qrt[2]*a^3*f) + (27*d^4*Sqrt[d*Tan[e + f*x]])/(8*a^3*f) - (9*d^3*(d*Tan[e +
 f*x])^(3/2))/(8*a^3*f*(1 + Tan[e + f*x])) - (d^2*(d*Tan[e + f*x])^(5/2))/(
4*a*f*(a + a*Tan[e + f*x])^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
```

```
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3735

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dis
t[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*Simp[a*(A - C) - (A*b - b*C)*Ta
n[e + f*x], x], x], x] + Dist[(A*b^2 + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e
 + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a,
 b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0] &&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^{9/2}}{(a+a\tan(e+fx))^3}\,dx = -\frac{d^2(d\tan(e+fx))^{5/2}}{4af(a+a\tan(e+fx))^2} + \frac{\int \frac{(d\tan(e+fx))^{3/2}\left(\frac{5a^2d^3}{2}-2a^2d^3\tan(e+fx)+\frac{9}{2}a^2d^3\tan^2(e+f\cdots\right.}{(a+a\tan(e+fx))^2}}{4a^3}$$

$$= -\frac{9d^3(d\tan(e+fx))^{3/2}}{8a^3f(1+\tan(e+fx))} - \frac{d^2(d\tan(e+fx))^{5/2}}{4af(a+a\tan(e+fx))^2} + \frac{\int \sqrt{d\tan(e+fx)}\,\left(\cdots\right.}{\cdots}$$

$$= \frac{27d^4\sqrt{d\tan(e+fx)}}{8a^3f} - \frac{9d^3(d\tan(e+fx))^{3/2}}{8a^3f(1+\tan(e+fx))} - \frac{d^2(d\tan(e+fx))^{5/2}}{4af(a+a\tan(e+fx))^2} \cdots$$

$$= \frac{27d^4\sqrt{d\tan(e+fx)}}{8a^3f} - \frac{9d^3(d\tan(e+fx))^{3/2}}{8a^3f(1+\tan(e+fx))} - \frac{d^2(d\tan(e+fx))^{5/2}}{4af(a+a\tan(e+fx))^2} \cdots$$

$$= \frac{27d^4\sqrt{d\tan(e+fx)}}{8a^3f} - \frac{9d^3(d\tan(e+fx))^{3/2}}{8a^3f(1+\tan(e+fx))} - \frac{d^2(d\tan(e+fx))^{5/2}}{4af(a+a\tan(e+fx))^2} \cdots$$

$$= \frac{d^{9/2}\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{2\sqrt{2}\,a^3f} + \frac{27d^4\sqrt{d\tan(e+fx)}}{8a^3f} - \frac{9d^3(d\tan(e\cdots}{8a^3f(1+\tan\cdots}$$

$$= -\frac{31d^{9/2}\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8a^3f} + \frac{d^{9/2}\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{2\sqrt{2}\,a^3f} \cdots$$

Mathematica [A]
time = 6.26, size = 346, normalized size = 1.83

$$\frac{\cot(e+fx)\csc^3(e+fx)(\cos(e+fx)+\sin(e+fx))^3\left(\frac{7}{2}-\frac{1}{8(\cos(e+fx)+\sin(e+fx))^2}-\frac{11\sin(e+fx)}{8(\cos(e+fx)+\sin(e+fx))}\right)(d\tan(e+fx))^{9/2}}{f(a+a\tan(e+fx))^3}+\frac{\sec^3(e+fx)(\cos(e+fx)+\sin(e+fx))^3(d\tan(e+fx))^{9/2}\left(-\frac{62\text{ArcTan}\left(\sqrt{\tan(e+fx)}\right)\csc(e+fx)\sec^3(e+fx)(1+\tan(e+fx))}{(1+\cot(e+fx))(1+\tan^2(e+fx))^2}+\frac{2\sqrt{2}\cos(2(e+fx))\csc(e+fx)\left(-\log\left(-1+\sqrt{2}\sqrt{\tan(e+fx)}-\tan(e+fx)\right)+\log\left(1+\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)\right)\sec^3(e+fx)}{(1+\cot(e+fx))(1-\tan(e+fx))(1+\tan^2(e+fx))}\right)}{16f\tan^{\frac{9}{2}}(e+fx)(a+a\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(9/2)/(a + a*Tan[e + f*x])^3,x]
```

```
[Out] (Cot[e + f*x]*Csc[e + f*x]^3*(Cos[e + f*x] + Sin[e + f*x])^3*(7/2 - 1/(8*(C
os[e + f*x] + Sin[e + f*x])^2) - (11*Sin[e + f*x])/(8*(Cos[e + f*x] + Sin[e
 + f*x])))*(d*Tan[e + f*x])^(9/2))/(f*(a + a*Tan[e + f*x])^3) + (Sec[e + f*
x]^3*(Cos[e + f*x] + Sin[e + f*x])^3*(d*Tan[e + f*x])^(9/2)*((-62*ArcTan[Sq
rt[Tan[e + f*x]]]*Csc[e + f*x]*Sec[e + f*x]^3*(1 + Tan[e + f*x]))/((1 + Cot
[e + f*x])*(1 + Tan[e + f*x]^2)^2) + (2*Sqrt[2]*Cos[2*(e + f*x)]*Csc[e + f*
x]*(-Log[-1 + Sqrt[2]*Sqrt[Tan[e + f*x]] - Tan[e + f*x]] + Log[1 + Sqrt[2]*
Sqrt[Tan[e + f*x]] + Tan[e + f*x]])*Sec[e + f*x]^3)/((1 + Cot[e + f*x])*(1
- Tan[e + f*x])*(1 + Tan[e + f*x]^2))))/(16*f*Tan[e + f*x]^(9/2)*(a + a*Tan
[e + f*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(156) = 312$.
time = 0.19, size = 355, normalized size = 1.88

method	result
derivativedivides	$\frac{2d^4\left(\sqrt{d\tan(fx+e)}-\frac{d\left(\frac{-\frac{13(d\tan(fx+e))^{\frac{3}{2}}}{4}-\frac{11d\sqrt{d\tan(fx+e)}}{4}}{(d\tan(fx+e)+d)^2}+\frac{31\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{4\sqrt{d}}\right.}{4}\right.}{}$
default	$\frac{2d^4\left(\sqrt{d\tan(fx+e)}-\frac{d\left(\frac{-\frac{13(d\tan(fx+e))^{\frac{3}{2}}}{4}-\frac{11d\sqrt{d\tan(fx+e)}}{4}}{(d\tan(fx+e)+d)^2}+\frac{31\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{4\sqrt{d}}\right.}{4}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(9/2)/(a+a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3*d^4*((d*tan(f*x+e))^(1/2)-1/4*d*((-13/4*(d*tan(f*x+e))^(3/2)-11/4*d
*(d*tan(f*x+e))^(1/2))/(d*tan(f*x+e)+d)^2+31/4/d^(1/2)*arctan((d*tan(f*x+e)
)^(1/2)/d^(1/2)))+1/4*d*(1/8/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^
(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(
d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*t
an(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))-
1/8/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*
2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)
+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arcta
n(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))))
```

Maxima [A]
time = 0.53, size = 211, normalized size = 1.12

$$\frac{\frac{d^6\left(\frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)}{a^3}-\frac{31d^{\frac{11}{2}}\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{a^3}+\frac{16\sqrt{d\tan(fx+e)}\,d^5}{a^3}+\frac{13(d\tan(fx+e))^{\frac{3}{2}}d^6+11\sqrt{d\tan(fx+e)}\,d^7}{a^3d^2\tan(fx+e)^2+2a^3d^2\tan(fx+e)+a^3d^2}}{8df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(9/2)/(a+a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(d^6*(sqrt(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d)
 + d)/sqrt(d) - sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*s
qrt(d) + d)/sqrt(d))/a^3 - 31*d^(11/2)*arctan(sqrt(d*tan(f*x + e))/sqrt(d))
/a^3 + 16*sqrt(d*tan(f*x + e))*d^5/a^3 + (13*(d*tan(f*x + e))^(3/2)*d^6 + 1
1*sqrt(d*tan(f*x + e))*d^7)/(a^3*d^2*tan(f*x + e)^2 + 2*a^3*d^2*tan(f*x + e
) + a^3*d^2))/(d*f)
```

Fricas [A]
time = 1.22, size = 502, normalized size = 2.66

$$\left[-\frac{4\left(\sqrt{2}\,d^{4}\tan\left(fx+e\right)^{2}+2\sqrt{2}\,d^{4}\tan\left(fx+e\right)+\sqrt{2}\,d^{4}\right)\sqrt{-d}\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}\left(\sqrt{2}\tan\left(fx+e\right)+\sqrt{2}\right)\sqrt{-d}}{2\,d\tan\left(fx+e\right)}\right)-31\left(d^{4}\tan\left(fx+e\right)^{2}+2\,d^{4}\tan\left(fx+e\right)+d^{4}\right)\sqrt{-d}\log\left(\frac{d\tan\left(fx+e\right)-2\sqrt{d\tan\left(fx+e\right)}\sqrt{-d}-d}{\tan\left(fx+e\right)+1}\right)-2\left(16\,d^{4}\tan\left(fx+e\right)^{2}+45\,d^{4}\tan\left(fx+e\right)+27\,d^{4}\right)\sqrt{d\tan\left(fx+e\right)}}{16\left(a^{3}f\tan\left(fx+e\right)^{2}+2\,a^{3}f\tan\left(fx+e\right)+a^{3}f\right)},-\frac{31\left(d^{4}\tan\left(fx+e\right)^{2}+2\,d^{4}\tan\left(fx+e\right)+d^{4}\right)\sqrt{d}\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}}{\sqrt{d}}\right)-\left(\sqrt{2}\,d^{4}\tan\left(fx+e\right)^{2}+2\sqrt{2}\,d^{4}\tan\left(fx+e\right)+\sqrt{2}\,d^{4}\right)\sqrt{d}\log\left(\frac{d\tan\left(fx+e\right)^{2}+2\sqrt{d\tan\left(fx+e\right)}\left(\sqrt{2}\tan\left(fx+e\right)+\sqrt{2}\right)\sqrt{d}+4\,d\tan\left(fx+e\right)+d}{\tan\left(fx+e\right)^{2}+1}\right)-\left(16\,d^{4}\tan\left(fx+e\right)^{2}+45\,d^{4}\tan\left(fx+e\right)+27\,d^{4}\right)\sqrt{d\tan\left(fx+e\right)}}{8\left(a^{3}f\tan\left(fx+e\right)^{2}+2\,a^{3}f\tan\left(fx+e\right)+a^{3}f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(9/2)/(a+a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(sqrt(2)*d^4*tan(f*x + e)^2 + 2*sqrt(2)*d^4*tan(f*x + e) + sqrt(2
)*d^4)*sqrt(-d)*arctan(1/2*sqrt(d*tan(f*x + e))*(sqrt(2)*tan(f*x + e) + sqr
t(2))*sqrt(-d)/(d*tan(f*x + e))) - 31*(d^4*tan(f*x + e)^2 + 2*d^4*tan(f*x +
 e) + d^4)*sqrt(-d)*log((d*tan(f*x + e) - 2*sqrt(d*tan(f*x + e))*sqrt(-d) -
 d)/(tan(f*x + e) + 1)) - 2*(16*d^4*tan(f*x + e)^2 + 45*d^4*tan(f*x + e) +
27*d^4)*sqrt(d*tan(f*x + e)))/(a^3*f*tan(f*x + e)^2 + 2*a^3*f*tan(f*x + e)
+ a^3*f), -1/8*(31*(d^4*tan(f*x + e)^2 + 2*d^4*tan(f*x + e) + d^4)*sqrt(d)*
arctan(sqrt(d*tan(f*x + e))/sqrt(d)) - (sqrt(2)*d^4*tan(f*x + e)^2 + 2*sqrt
(2)*d^4*tan(f*x + e) + sqrt(2)*d^4)*sqrt(d)*log((d*tan(f*x + e)^2 + 2*sqrt(
d*tan(f*x + e))*(sqrt(2)*tan(f*x + e) + sqrt(2))*sqrt(d) + 4*d*tan(f*x + e)
 + d)/(tan(f*x + e)^2 + 1)) - (16*d^4*tan(f*x + e)^2 + 45*d^4*tan(f*x + e)
+ 27*d^4)*sqrt(d*tan(f*x + e)))/(a^3*f*tan(f*x + e)^2 + 2*a^3*f*tan(f*x + e
) + a^3*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\left(d\tan\left(e+fx\right)\right)^{\frac{9}{2}}}{\tan^{3}\left(e+fx\right)+3\tan^{2}\left(e+fx\right)+3\tan\left(e+fx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(9/2)/(a+a*tan(f*x+e))**3,x)
```

```
[Out] Integral((d*tan(e + f*x))**(9/2)/(tan(e + f*x)**3 + 3*tan(e + f*x)**2 + 3*t
an(e + f*x) + 1), x)/a**3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(164) = 328.

time = 0.88, size = 345, normalized size = 1.83

$$\frac{1}{16}d^4\left(\frac{2\sqrt{2}\left(d\sqrt{|d|}-|d|^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{a^3df}+\frac{2\sqrt{2}\left(d\sqrt{|d|}-|d|^{\frac{3}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{a^3df}-\frac{62\sqrt{d}\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{a^3f}+\frac{\sqrt{2}\left(d\sqrt{|d|}+|d|^{\frac{3}{2}}\right)\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{|d|}+|d|\right)}{a^3df}-\frac{\sqrt{2}\left(d\sqrt{|d|}+|d|^{\frac{3}{2}}\right)\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{|d|}+|d|\right)}{a^3df}+\frac{32\sqrt{d\tan(fx+e)}}{a^3f}+\frac{2\left(13\sqrt{d\tan(fx+e)}d^2\tan(fx+e)+11\sqrt{d\tan(fx+e)}d^2\right)}{(d\tan(fx+e)+d)^2a^3f}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(9/2)/(a+a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/16*d^4*(2*sqrt(2)*(d*sqrt(abs(d)) - abs(d)^(3/2))*arctan(1/2*sqrt(2)*(sqr
t(2)*sqrt(abs(d)) + 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(a^3*d*f) + 2*sqr
t(2)*(d*sqrt(abs(d)) - abs(d)^(3/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(
d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(a^3*d*f) - 62*sqrt(d)*arctan(s
qrt(d*tan(f*x + e))/sqrt(d))/(a^3*f) + sqrt(2)*(d*sqrt(abs(d)) + abs(d)^(3/
2))*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d)
)/(a^3*d*f) - sqrt(2)*(d*sqrt(abs(d)) + abs(d)^(3/2))*log(d*tan(f*x + e) -
sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(a^3*d*f) + 32*sqrt(d*t
an(f*x + e))/(a^3*f) + 2*(13*sqrt(d*tan(f*x + e))*d^2*tan(f*x + e) + 11*sqr
t(d*tan(f*x + e))*d^2)/((d*tan(f*x + e) + d)^2*a^3*f))
```

Mupad [B]

time = 4.95, size = 176, normalized size = 0.93

$$\frac{\frac{11\,d^6\,\sqrt{d\tan(e+f\,x)}}{8}+\frac{13\,d^5\,(d\tan(e+f\,x))^{3/2}}{8}}{f\,a^3\,d^2\tan(e+f\,x)^2+2\,f\,a^3\,d^2\tan(e+f\,x)+f\,a^3\,d^2}+\frac{2\,d^4\,\sqrt{d\tan(e+f\,x)}}{a^3\,f}-\frac{31\,d^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{d\tan(e+f\,x)}}{\sqrt{d}}\right)}{8\,a^3\,f}-\frac{\sqrt{2}\,d^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,d^{49/2}\,\sqrt{d\tan(e+f\,x)}\,969\mathrm{i}}{32\left(\frac{969\,d^{25}\tan(e+f\,x)}{32}+\frac{969\,d^{25}}{32}\right)}\right)\,1\mathrm{i}}{4\,a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(9/2)/(a + a*tan(e + f*x))^3,x)
```

```
[Out] ((11*d^6*(d*tan(e + f*x))^(1/2))/8 + (13*d^5*(d*tan(e + f*x))^(3/2))/8)/(a^
3*d^2*f + a^3*d^2*f*tan(e + f*x)^2 + 2*a^3*d^2*f*tan(e + f*x)) + (2*d^4*(d*
tan(e + f*x))^(1/2))/(a^3*f) - (31*d^(9/2)*atan((d*tan(e + f*x))^(1/2)/d^(1
/2)))/(8*a^3*f) - (2^(1/2)*d^(9/2)*atan((2^(1/2)*d^(49/2)*(d*tan(e + f*x))^
(1/2)*969i)/(32*((969*d^25*tan(e + f*x))/32 + (969*d^25)/32)))*1i)/(4*a^3*f
)
```

3.371 $\int \frac{(d\tan(e+fx))^{7/2}}{(a+a\tan(e+fx))^3}\,dx$

Optimal. Leaf size=165

$$\frac{11d^{7/2}\text{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8a^3f}+\frac{d^{7/2}\text{ArcTan}\left(\frac{\sqrt{d}-\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{2\sqrt{2}a^3f}-\frac{7d^3\sqrt{d\tan(e+fx)}}{8a^3f(1+\tan(e+fx))}-\frac{d^2(d}{4af(a}$$

[Out] 11/8*d^(7/2)*arctan((d*tan(f*x+e))^(1/2)/d^(1/2))/a^3/f+1/4*d^(7/2)*arctan(1/2*(d^(1/2)-d^(1/2)*tan(f*x+e))*2^(1/2)/(d*tan(f*x+e))^(1/2))/a^3/f*2^(1/2)-7/8*d^3*(d*tan(f*x+e))^(1/2)/a^3/f/(1+tan(f*x+e))-1/4*d^2*(d*tan(f*x+e))^(3/2)/a/f/(a+a*tan(f*x+e))^2

Rubi [A]
time = 0.39, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3646, 3726, 3734, 3613, 211, 3715, 65}

$$\frac{11d^{7/2}\text{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8a^3f}+\frac{d^{7/2}\text{ArcTan}\left(\frac{\sqrt{d}-\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{2\sqrt{2}a^3f}-\frac{7d^3\sqrt{d\tan(e+fx)}}{8a^3f(\tan(e+fx)+1)}-\frac{d^2(d\tan(e+fx))^{3/2}}{4af(a\tan(e+fx)+a)^2}$$

Antiderivative was successfully verified.

[In] Int[(d*Tan[e + f*x])^(7/2)/(a + a*Tan[e + f*x])^3,x]

[Out] (11*d^(7/2)*ArcTan[Sqrt[d*Tan[e + f*x]]/Sqrt[d]])/(8*a^3*f) + (d^(7/2)*ArcTan[(Sqrt[d] - Sqrt[d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[d*Tan[e + f*x]])])/(2*Sqrt[2]*a^3*f) - (7*d^3*Sqrt[d*Tan[e + f*x]])/(8*a^3*f*(1 + Tan[e + f*x])) - (d^2*(d*Tan[e + f*x])^(3/2))/(4*a*f*(a + a*Tan[e + f*x])^2)

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
```

```
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
```

```
!GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^{7/2}}{(a+a\tan(e+fx))^3}\,dx = -\frac{d^2(d\tan(e+fx))^{3/2}}{4af(a+a\tan(e+fx))^2} + \frac{\int \frac{\sqrt{d\tan(e+fx)}\left(\frac{3a^2d^3}{2}-2a^2d^3\tan(e+fx)+\frac{7}{2}a^2d^3\tan^{\cdots}\right.}{(a+a\tan(e+fx))^2}}{4a^3}$$

$$= -\frac{7d^3\sqrt{d\tan(e+fx)}}{8a^3f(1+\tan(e+fx))} - \frac{d^2(d\tan(e+fx))^{3/2}}{4af(a+a\tan(e+fx))^2} + \frac{\int \frac{\frac{7a^4d^4}{2}-4a^4d^4\tan(e+fx)+\cdots}{\sqrt{d\tan(e+fx)}\cdots}}{8a^6}$$

$$= -\frac{7d^3\sqrt{d\tan(e+fx)}}{8a^3f(1+\tan(e+fx))} - \frac{d^2(d\tan(e+fx))^{3/2}}{4af(a+a\tan(e+fx))^2} + \frac{\int \frac{-4a^5d^4-4a^5d^4\tan(e+fx)\cdots}{\sqrt{d\tan(e+fx)}\cdots}}{16a^8}$$

$$= -\frac{7d^3\sqrt{d\tan(e+fx)}}{8a^3f(1+\tan(e+fx))} - \frac{d^2(d\tan(e+fx))^{3/2}}{4af(a+a\tan(e+fx))^2} + \frac{(11d^4)\,\mathrm{Subst}\left(\int \frac{\cdots}{\sqrt{dx}\cdots}\right.}{\cdots}$$

$$= \frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}-\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{2\sqrt{2}\,a^3f} - \frac{7d^3\sqrt{d\tan(e+fx)}}{8a^3f(1+\tan(e+fx))} - \frac{d^2(d\tan(e\cdots}{4af(a+a\,\mathrm{ta}\cdots}$$

$$= \frac{11d^{7/2}\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8a^3f} + \frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}-\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{2\sqrt{2}\,a^3f} - \frac{\cdots}{8\cdots}$$

Mathematica [A]
time = 3.06, size = 183, normalized size = 1.11

$$\frac{(\cos(e+fx)+\sin(e+fx))^3\left(-\frac{\csc^5(e+fx)(7+7\cos(2(e+fx))+9\sin(2(e+fx)))}{(1+\cot(e+fx))^2}+\frac{2\left(2\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)-2\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(e+fx)}\right)+11\,\mathrm{ArcTan}\left(\sqrt{\tan(e+fx)}\right)\right)\csc(e+fx)\sec^2(e+fx)}{\tan^{\frac{5}{2}}(e+fx)}\right)(d\tan(e+fx))^{7/2}}{16a^3f(1+\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(7/2)/(a + a*Tan[e + f*x])^3,x]
```

```
[Out] ((Cos[e + f*x] + Sin[e + f*x])^3*(-((Csc[e + f*x]^5*(7 + 7*Cos[2*(e + f*x)]
 + 9*Sin[2*(e + f*x)]))/(1 + Cot[e + f*x])^2) + (2*(2*Sqrt[2]*ArcTan[1 - Sq
rt[2]*Sqrt[Tan[e + f*x]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]
] + 11*ArcTan[Sqrt[Tan[e + f*x]]])*Csc[e + f*x]*Sec[e + f*x]^2)/Tan[e + f*x
]^(5/2))*(d*Tan[e + f*x])^(7/2))/(16*a^3*f*(1 + Tan[e + f*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(136) = 272$.
time = 0.18, size = 338, normalized size = 2.05

method	result
derivativedivides	$2d^4\left(\frac{-\frac{9(d\tan(fx+e))^{\frac{3}{2}}}{4}-\frac{7d\sqrt{d\tan(fx+e)}}{4}}{4(d\tan(fx+e)+d)^2}+\frac{11\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{16\sqrt{d}}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)}{d\tan(fx+e)}\right.\right.}{}\right.$
default	$2d^4\left(\frac{-\frac{9(d\tan(fx+e))^{\frac{3}{2}}}{4}-\frac{7d\sqrt{d\tan(fx+e)}}{4}}{4(d\tan(fx+e)+d)^2}+\frac{11\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{16\sqrt{d}}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)}{d\tan(fx+e)}\right.\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(7/2)/(a+a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3*d^4*(1/4*(-9/4*(d*tan(f*x+e))^(3/2)-7/4*d*(d*tan(f*x+e))^(1/2))/(d*
tan(f*x+e)+d)^2+11/16/d^(1/2)*arctan((d*tan(f*x+e))^(1/2)/d^(1/2))-1/32/d*(
d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/
2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2
)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^
(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))-1/32/(d^2)^(1/4)*2^(1/2)*(ln((d*
tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x
+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)
/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f
*x+e))^(1/2)+1)))
```

Maxima [A]

time = 0.55, size = 191, normalized size = 1.16

$$-\frac{\frac{2d^5\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}\right)}{a^3}-\frac{11d^{\frac{9}{2}}\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{a^3}+\frac{9(d\tan(fx+e))^{\frac{3}{2}}d^5+7\sqrt{d\tan(fx+e)}\,d^6}{a^3d^2\tan(fx+e)^2+2a^3d^2\tan(fx+e)+a^3d^2}}{8df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(7/2)/(a+a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/8*(2*d^5*(sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(f*x
 + e)))/sqrt(d))/sqrt(d) + sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2
*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d))/a^3 - 11*d^(9/2)*arctan(sqrt(d*tan
(f*x + e))/sqrt(d))/a^3 + (9*(d*tan(f*x + e))^(3/2)*d^5 + 7*sqrt(d*tan(f*x
+ e))*d^6)/(a^3*d^2*tan(f*x + e)^2 + 2*a^3*d^2*tan(f*x + e) + a^3*d^2))/(d*
f)
```

Fricas [A]
time = 1.14, size = 475, normalized size = 2.88

$$\left[\frac{2\left(\sqrt{2}\,d^{3}\tan\left(fx+e\right)^{2}+2\,\sqrt{2}\,d^{3}\tan\left(fx+e\right)+\sqrt{2}\,d^{3}\right)\sqrt{-d}\,\log\left(\frac{d\tan(fx+e)^{2}-2\,\sqrt{d\tan\left(fx+e\right)}\left(\sqrt{2}\tan(fx+e)-\sqrt{2}\right)\sqrt{-d}-4\,d\tan(fx+e)+d}{\tan(fx+e)^{2}+1}\right)+11\left(d^{3}\tan\left(fx+e\right)^{2}+2\,d^{3}\tan\left(fx+e\right)+d^{3}\right)\sqrt{-d}\,\log\left(\frac{d\tan(fx+e)+2\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{-d}-d}{\tan(fx+e)+1}\right)-2\left(9\,d^{3}\tan\left(fx+e\right)+7\,d^{3}\right)\sqrt{d\tan\left(fx+e\right)}}{16\left(a^{3}f\tan\left(fx+e\right)^{2}+2\,a^{3}f\tan\left(fx+e\right)+a^{3}f\right)},\ \frac{11\left(d^{3}\tan\left(fx+e\right)^{2}+2\,d^{3}\tan\left(fx+e\right)+d^{3}\right)\sqrt{d}\,\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}}{\sqrt{d}}\right)-2\left(\sqrt{2}\,d^{3}\tan\left(fx+e\right)^{2}+2\,\sqrt{2}\,d^{3}\tan\left(fx+e\right)+\sqrt{2}\,d^{3}\right)\sqrt{d}\,\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}\left(\sqrt{2}\tan(fx+e)-\sqrt{2}\right)}{2\,\sqrt{d}\tan(fx+e)}\right)-\left(9\,d^{3}\tan\left(fx+e\right)+7\,d^{3}\right)\sqrt{d\tan\left(fx+e\right)}}{8\left(a^{3}f\tan\left(fx+e\right)^{2}+2\,a^{3}f\tan\left(fx+e\right)+a^{3}f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(7/2)/(a+a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(sqrt(2)*d^3*tan(f*x + e)^2 + 2*sqrt(2)*d^3*tan(f*x + e) + sqrt(2)
*d^3)*sqrt(-d)*log((d*tan(f*x + e)^2 - 2*sqrt(d*tan(f*x + e))*(sqrt(2)*tan(
f*x + e) - sqrt(2))*sqrt(-d) - 4*d*tan(f*x + e) + d)/(tan(f*x + e)^2 + 1))
+ 11*(d^3*tan(f*x + e)^2 + 2*d^3*tan(f*x + e) + d^3)*sqrt(-d)*log((d*tan(f*
x + e) + 2*sqrt(d*tan(f*x + e))*sqrt(-d) - d)/(tan(f*x + e) + 1)) - 2*(9*d^
3*tan(f*x + e) + 7*d^3)*sqrt(d*tan(f*x + e)))/(a^3*f*tan(f*x + e)^2 + 2*a^3
*f*tan(f*x + e) + a^3*f), 1/8*(11*(d^3*tan(f*x + e)^2 + 2*d^3*tan(f*x + e)
+ d^3)*sqrt(d)*arctan(sqrt(d*tan(f*x + e))/sqrt(d)) - 2*(sqrt(2)*d^3*tan(f*
x + e)^2 + 2*sqrt(2)*d^3*tan(f*x + e) + sqrt(2)*d^3)*sqrt(d)*arctan(1/2*sqr
t(d*tan(f*x + e))*(sqrt(2)*tan(f*x + e) - sqrt(2))/(sqrt(d)*tan(f*x + e)))
- (9*d^3*tan(f*x + e) + 7*d^3)*sqrt(d*tan(f*x + e)))/(a^3*f*tan(f*x + e)^2
+ 2*a^3*f*tan(f*x + e) + a^3*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(d\tan(e+fx))^{\frac{7}{2}}}{\tan^{3}(e+fx)+3\tan^{2}(e+fx)+3\tan(e+fx)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(7/2)/(a+a*tan(f*x+e))**3,x)
```

```
[Out] Integral((d*tan(e + f*x))**(7/2)/(tan(e + f*x)**3 + 3*tan(e + f*x)**2 + 3*t
an(e + f*x) + 1), x)/a**3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(144) = 288.
time = 0.85, size = 326, normalized size = 1.98

$$-\frac{1}{16}\,d^{3}\left(\frac{2\,\sqrt{2}\left(d\sqrt{|d|}+|d|^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\,\sqrt{d\tan\left(fx+e\right)}\right)}{2\,\sqrt{|d|}}\right)}{a^{3}df}+\frac{2\,\sqrt{2}\left(d\sqrt{|d|}+|d|^{\frac{3}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\,\sqrt{d\tan\left(fx+e\right)}\right)}{2\,\sqrt{|d|}}\right)}{a^{3}df}-\frac{22\,\sqrt{d}\,\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}}{\sqrt{d}}\right)}{a^{3}f}+\frac{\sqrt{2}\left(d\sqrt{|d|}-|d|^{\frac{3}{2}}\right)\log\left(d\tan\left(fx+e\right)+\sqrt{2}\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{|d|}+|d|\right)}{a^{3}df}-\frac{\sqrt{2}\left(d\sqrt{|d|}-|d|^{\frac{3}{2}}\right)\log\left(d\tan\left(fx+e\right)-\sqrt{2}\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{|d|}+|d|\right)}{a^{3}df}+\frac{2\left(9\,\sqrt{d\tan\left(fx+e\right)}\,d^{2}\tan\left(fx+e\right)+7\,\sqrt{d\tan\left(fx+e\right)}\,d^{2}\right)}{\left(d\tan\left(fx+e\right)+d\right)^{2}a^{3}f}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(7/2)/(a+a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/16*d^3*(2*sqrt(2)*(d*sqrt(abs(d)) + abs(d)^(3/2))*arctan(1/2*sqrt(2)*(sq
rt(2)*sqrt(abs(d)) + 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(a^3*d*f) + 2*sq
rt(2)*(d*sqrt(abs(d)) + abs(d)^(3/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs
```

```
(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(a^3*d*f) - 22*sqrt(d)*arctan(
sqrt(d*tan(f*x + e))/sqrt(d))/(a^3*f) + sqrt(2)*(d*sqrt(abs(d)) - abs(d)^(3
/2))*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d
))/(a^3*d*f) - sqrt(2)*(d*sqrt(abs(d)) - abs(d)^(3/2))*log(d*tan(f*x + e) -
 sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(a^3*d*f) + 2*(9*sqrt(
d*tan(f*x + e))*d^2*tan(f*x + e) + 7*sqrt(d*tan(f*x + e))*d^2)/((d*tan(f*x
+ e) + d)^2*a^3*f))
```

Mupad [B]
time = 4.86, size = 178, normalized size = 1.08

$$\frac{11\,d^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{d\,\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)}{8\,a^3\,f}-\frac{\frac{7\,d^5\,\sqrt{d\,\tan\left(e+f\,x\right)}}{8}+\frac{9\,d^4\,{\left(d\,\tan\left(e+f\,x\right)\right)}^{3/2}}{8}}{f\,a^3\,d^2\,{\tan\left(e+f\,x\right)}^2+2\,f\,a^3\,d^2\,\tan\left(e+f\,x\right)+f\,a^3\,d^2}-\frac{\sqrt{2}\,d^{7/2}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{d\,\tan\left(e+f\,x\right)}}{2\,\sqrt{d}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{d\,\tan\left(e+f\,x\right)}}{2\,\sqrt{d}}+\frac{\sqrt{2}\,{\left(d\,\tan\left(e+f\,x\right)\right)}^{3/2}}{2\,d^{3/2}}\right)\right)}{8\,a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(7/2)/(a + a*tan(e + f*x))^3,x)
```

```
[Out] (11*d^(7/2)*atan((d*tan(e + f*x))^(1/2)/d^(1/2)))/(8*a^3*f) - ((7*d^5*(d*ta
n(e + f*x))^(1/2))/8 + (9*d^4*(d*tan(e + f*x))^(3/2))/8)/(a^3*d^2*f + a^3*d
^2*f*tan(e + f*x)^2 + 2*a^3*d^2*f*tan(e + f*x)) - (2^(1/2)*d^(7/2)*(2*atan(
(2^(1/2)*(d*tan(e + f*x))^(1/2))/(2*d^(1/2))) + 2*atan((2^(1/2)*(d*tan(e +
f*x))^(1/2))/(2*d^(1/2)) + (2^(1/2)*(d*tan(e + f*x))^(3/2))/(2*d^(3/2)))))/
(8*a^3*f)
```

3.372 $\int \frac{(d\tan(e+fx))^{5/2}}{(a+a\tan(e+fx))^3}\,dx$

Optimal. Leaf size=164

$$\frac{d^{5/2}\text{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8a^3f}-\frac{d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{2\sqrt{2}a^3f}+\frac{5d^2\sqrt{d\tan(e+fx)}}{8a^3f(1+\tan(e+fx))}-\frac{d^2\sqrt{d}}{4af(a+}$$

```
[Out] 1/8*d^(5/2)*arctan((d*tan(f*x+e))^(1/2)/d^(1/2))/a^3/f-1/4*d^(5/2)*arctanh(
1/2*(d^(1/2)+d^(1/2)*tan(f*x+e))*2^(1/2)/(d*tan(f*x+e))^(1/2))/a^3/f*2^(1/2
)+5/8*d^2*(d*tan(f*x+e))^(1/2)/a^3/f/(1+tan(f*x+e))-1/4*d^2*(d*tan(f*x+e))^
(1/2)/a/f/(a+a*tan(f*x+e))^2
```

Rubi [A]
time = 0.36, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3646, 3730, 3735, 3613, 214, 3715, 65, 211}

$$\frac{d^{5/2}\text{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8a^3f}-\frac{d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\tan(e+fx)+\sqrt{d}}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{2\sqrt{2}a^3f}+\frac{5d^2\sqrt{d\tan(e+fx)}}{8a^3f(\tan(e+fx)+1)}-\frac{d^2\sqrt{d\tan(e+fx)}}{4af(a\tan(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(5/2)/(a + a*Tan[e + f*x])^3,x]
```

```
[Out] (d^(5/2)*ArcTan[Sqrt[d*Tan[e + f*x]]/Sqrt[d]])/(8*a^3*f) - (d^(5/2)*ArcTanh
[(Sqrt[d] + Sqrt[d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[d*Tan[e + f*x]])])/(2*Sqrt[
2]*a^3*f) + (5*d^2*Sqrt[d*Tan[e + f*x]])/(8*a^3*f*(1 + Tan[e + f*x])) - (d^
2*Sqrt[d*Tan[e + f*x]])/(4*a*f*(a + a*Tan[e + f*x])^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3735

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dis
t[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*Simp[a*(A - C) - (A*b - b*C)*Ta
```

```
n[e + f*x], x], x], x] + Dist[(A*b^2 + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e
 + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a,
 b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0] && !GtQ[n, 0] && !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^{5/2}}{(a+a\tan(e+fx))^3}\,dx = -\frac{d^2\sqrt{d\tan(e+fx)}}{4af(a+a\tan(e+fx))^2} + \frac{\int \frac{\frac{a^2d^3}{2}-2a^2d^3\tan(e+fx)+\frac{5}{2}a^2d^3\tan^2(e+fx)}{\sqrt{d\tan(e+fx)}\,(a+a\tan(e+fx))^2}\,dx}{4a^3}$$

$$= \frac{5d^2\sqrt{d\tan(e+fx)}}{8a^3f(1+\tan(e+fx))} - \frac{d^2\sqrt{d\tan(e+fx)}}{4af(a+a\tan(e+fx))^2} + \frac{\int \frac{-\frac{3}{2}a^4d^4+\frac{5}{2}a^4d^4\tan^2(e}{\sqrt{d\tan(e+fx)}\,(a+a}}{8a^6d}$$

$$= \frac{5d^2\sqrt{d\tan(e+fx)}}{8a^3f(1+\tan(e+fx))} - \frac{d^2\sqrt{d\tan(e+fx)}}{4af(a+a\tan(e+fx))^2} + \frac{\int \frac{-4a^5d^4+4a^5d^4\tan(e+fx)}{\sqrt{d\tan(e+fx)}}\,a}{16a^8d}$$

$$= \frac{5d^2\sqrt{d\tan(e+fx)}}{8a^3f(1+\tan(e+fx))} - \frac{d^2\sqrt{d\tan(e+fx)}}{4af(a+a\tan(e+fx))^2} + \frac{d^3\text{Subst}\left(\int \frac{1}{\sqrt{dx}\,(a+ax)}\right.}{16a^2}$$

$$= -\frac{d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{2\sqrt{2}\,a^3f} + \frac{5d^2\sqrt{d\tan(e+fx)}}{8a^3f(1+\tan(e+fx))} - \frac{d^2\sqrt{d}}{4af(a+}$$

$$= \frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8a^3f} - \frac{d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{2\sqrt{2}\,a^3f} + \frac{5}{8a}$$

Mathematica [A]
time = 3.25, size = 192, normalized size = 1.17

$$\frac{\sec(e+fx)(\cos(e+fx)+\sin(e+fx))^3\left(\frac{\csc^4(e+fx)(3+3\cos(2(e+fx))+5\sin(2(e+fx)))}{(1+\cot(e+fx))^2}+\frac{2\csc(e+fx)\left(\text{ArcTan}\left(\sqrt{\tan(e+fx)}\right)+\sqrt{2}\left(\log\left(-1+\sqrt{2}\sqrt{\tan(e+fx)}-\tan(e+fx)\right)-\log\left(1+\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)\right)\right)\sec(e+fx)}{\tan^{\frac{3}{2}}(e+fx)}\right)(d\tan(e+fx))^{5/2}}{16a^3f(1+\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(5/2)/(a + a*Tan[e + f*x])^3,x]
```

```
[Out] (Sec[e + f*x]*(Cos[e + f*x] + Sin[e + f*x])^3*((Csc[e + f*x]^4*(3 + 3*Cos[2
*(e + f*x)] + 5*Sin[2*(e + f*x)]))/(1 + Cot[e + f*x])^2 + (2*Csc[e + f*x]*(
ArcTan[Sqrt[Tan[e + f*x]]] + Sqrt[2]*(Log[-1 + Sqrt[2]*Sqrt[Tan[e + f*x]] -
 Tan[e + f*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]))*Sec[e
 + f*x])/Tan[e + f*x]^(3/2))*(d*Tan[e + f*x])^(5/2))/(16*a^3*f*(1 + Tan[e +
 f*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(135) = 270$.

time = 0.18, size = 349, normalized size = 2.13

method	result
derivativedivides	$\frac{2d^4\left(\frac{\frac{\frac{5(d\tan(fx+e))^{\frac{3}{2}}}{4}+\frac{3d\sqrt{d\tan(fx+e)}}{4}}{(d\tan(fx+e)+d)^2}+\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{4\sqrt{d}}}{4d}+\frac{-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+\cdots}{d\tan(fx+e)-\cdots}\right)\cdots\right)}{\cdots}}{\cdots}\cdots\right.}{\cdots}$
default	$\frac{2d^4\left(\frac{\frac{\frac{5(d\tan(fx+e))^{\frac{3}{2}}}{4}+\frac{3d\sqrt{d\tan(fx+e)}}{4}}{(d\tan(fx+e)+d)^2}+\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{4\sqrt{d}}}{4d}+\frac{-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+\cdots}{d\tan(fx+e)-\cdots}\right)\cdots\right)}{\cdots}}{\cdots}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(5/2)/(a+a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3*d^4*(1/4/d*((5/4*(d*tan(f*x+e))^(3/2)+3/4*d*(d*tan(f*x+e))^(1/2))/(
d*tan(f*x+e)+d)^2+1/4/d^(1/2)*arctan((d*tan(f*x+e))^(1/2)/d^(1/2)))+1/4/d*(
-1/8/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/
2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1
/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*ar
ctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))+1/8/(d^2)^(1/4)*2^(1/2)*
(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*
tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(
2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(
d*tan(f*x+e))^(1/2)+1))))
```

Maxima [A]

time = 0.54, size = 193, normalized size = 1.18

$$-\frac{\frac{d^4\left(\frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)}{a^3}-\frac{d^{\frac{7}{2}}\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{a^3}-\frac{5(d\tan(fx+e))^{\frac{3}{2}}d^4+3\sqrt{d\tan(fx+e)}\,d^5}{a^3d^2\tan(fx+e)^2+2a^3d^2\tan(fx+e)+a^3d^2}}{8df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)/(a+a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/8*(d^4*(sqrt(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d
) + d)/sqrt(d) - sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*
sqrt(d) + d)/sqrt(d))/a^3 - d^(7/2)*arctan(sqrt(d*tan(f*x + e))/sqrt(d))/a^
3 - (5*(d*tan(f*x + e))^(3/2)*d^4 + 3*sqrt(d*tan(f*x + e))*d^5)/(a^3*d^2*ta
n(f*x + e)^2 + 2*a^3*d^2*tan(f*x + e) + a^3*d^2))/(d*f)
```

Fricas [A]
time = 1.09, size = 470, normalized size = 2.87

$$\left[\frac{4\left(\sqrt{2}d^{2}\tan\left(fx+e\right)^{2}+2\sqrt{2}d^{2}\tan\left(fx+e\right)+\sqrt{2}d^{2}\right)\sqrt{-d}\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}\left(\sqrt{2}\tan\left(fx+e\right)+\sqrt{2}\right)\sqrt{-d}}{2d\tan\left(fx+e\right)}\right)+\left(d^{2}\tan\left(fx+e\right)^{2}+2d^{2}\tan\left(fx+e\right)+d^{2}\right)\sqrt{-d}\log\left(\frac{d\tan\left(fx+e\right)+2\sqrt{d\tan\left(fx+e\right)}\sqrt{-d}-d}{\tan\left(fx+e\right)+1}\right)+2\left(5d^{2}\tan\left(fx+e\right)+3d^{2}\right)\sqrt{d\tan\left(fx+e\right)}}{16\left(a^{3}f\tan\left(fx+e\right)^{2}+2a^{3}f\tan\left(fx+e\right)+a^{3}f\right)}, \frac{\left(d^{2}\tan\left(fx+e\right)^{2}+2d^{2}\tan\left(fx+e\right)+d^{2}\right)\sqrt{d}\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}}{\sqrt{d}}\right)+\left(\sqrt{2}d^{2}\tan\left(fx+e\right)^{2}+2\sqrt{2}d^{2}\tan\left(fx+e\right)+\sqrt{2}d^{2}\right)\sqrt{d}\log\left(\frac{d\tan\left(fx+e\right)^{2}-2\sqrt{d\tan\left(fx+e\right)}\left(\sqrt{2}\tan\left(fx+e\right)+\sqrt{2}\right)\sqrt{d}+4d\tan\left(fx+e\right)+d}{\tan\left(fx+e\right)^{2}+1}\right)+\left(5d^{2}\tan\left(fx+e\right)+3d^{2}\right)\sqrt{d\tan\left(fx+e\right)}}{8\left(a^{3}f\tan\left(fx+e\right)^{2}+2a^{3}f\tan\left(fx+e\right)+a^{3}f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)/(a+a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(sqrt(2)*d^2*tan(f*x + e)^2 + 2*sqrt(2)*d^2*tan(f*x + e) + sqrt(2)
*d^2)*sqrt(-d)*arctan(1/2*sqrt(d*tan(f*x + e))*(sqrt(2)*tan(f*x + e) + sqrt
(2))*sqrt(-d)/(d*tan(f*x + e))) + (d^2*tan(f*x + e)^2 + 2*d^2*tan(f*x + e)
+ d^2)*sqrt(-d)*log((d*tan(f*x + e) + 2*sqrt(d*tan(f*x + e))*sqrt(-d) - d)/
(tan(f*x + e) + 1)) + 2*(5*d^2*tan(f*x + e) + 3*d^2)*sqrt(d*tan(f*x + e)))/
(a^3*f*tan(f*x + e)^2 + 2*a^3*f*tan(f*x + e) + a^3*f), 1/8*((d^2*tan(f*x +
e)^2 + 2*d^2*tan(f*x + e) + d^2)*sqrt(d)*arctan(sqrt(d*tan(f*x + e))/sqrt(d
)) + (sqrt(2)*d^2*tan(f*x + e)^2 + 2*sqrt(2)*d^2*tan(f*x + e) + sqrt(2)*d^2
)*sqrt(d)*log((d*tan(f*x + e)^2 - 2*sqrt(d*tan(f*x + e))*(sqrt(2)*tan(f*x +
 e) + sqrt(2))*sqrt(d) + 4*d*tan(f*x + e) + d)/(tan(f*x + e)^2 + 1)) + (5*d
^2*tan(f*x + e) + 3*d^2)*sqrt(d*tan(f*x + e)))/(a^3*f*tan(f*x + e)^2 + 2*a^
3*f*tan(f*x + e) + a^3*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\left(d\tan\left(e+fx\right)\right)^{\frac{5}{2}}}{\tan^{3}\left(e+fx\right)+3\tan^{2}\left(e+fx\right)+3\tan\left(e+fx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(5/2)/(a+a*tan(f*x+e))**3,x)
```

```
[Out] Integral((d*tan(e + f*x))**(5/2)/(tan(e + f*x)**3 + 3*tan(e + f*x)**2 + 3*t
an(e + f*x) + 1), x)/a**3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(142) = 284.
time = 0.79, size = 326, normalized size = 1.99

$$-\frac{1}{16}d^{2}\left(\frac{2\sqrt{2}\left(d\sqrt{|d|}-|d|^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{a^{3}df}+\frac{2\sqrt{2}\left(d\sqrt{|d|}-|d|^{\frac{3}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{a^{3}df}-\frac{2\sqrt{d}\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}}{\sqrt{d}}\right)}{a^{3}f}+\frac{\sqrt{2}\left(d\sqrt{|d|}+|d|^{\frac{3}{2}}\right)\log\left(d\tan\left(fx+e\right)+\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{|d|}+|d|\right)}{a^{3}df}-\frac{\sqrt{2}\left(d\sqrt{|d|}+|d|^{\frac{3}{2}}\right)\log\left(d\tan\left(fx+e\right)-\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{|d|}+|d|\right)}{a^{3}df}-\frac{2\left(5\sqrt{d\tan\left(fx+e\right)}d^{2}\tan\left(fx+e\right)+3\sqrt{d\tan\left(fx+e\right)}d^{2}\right)}{\left(d\tan\left(fx+e\right)+d\right)^{2}a^{3}f}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2)/(a+a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/16*d^2*(2*sqrt(2)*(d*sqrt(abs(d)) - abs(d)^(3/2))*arctan(1/2*sqrt(2)*(sq
rt(2)*sqrt(abs(d)) + 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(a^3*d*f) + 2*sq
rt(2)*(d*sqrt(abs(d)) - abs(d)^(3/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs
(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(a^3*d*f) - 2*sqrt(d)*arctan(s
qrt(d*tan(f*x + e))/sqrt(d))/(a^3*f) + sqrt(2)*(d*sqrt(abs(d)) + abs(d)^(3/
2))*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d)
)/(a^3*d*f) - sqrt(2)*(d*sqrt(abs(d)) + abs(d)^(3/2))*log(d*tan(f*x + e) -
sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(a^3*d*f) - 2*(5*sqrt(d
*tan(f*x + e))*d^2*tan(f*x + e) + 3*sqrt(d*tan(f*x + e))*d^2)/((d*tan(f*x +
 e) + d)^2*a^3*f))
```

Mupad [B]
time = 4.80, size = 153, normalized size = 0.93

$$\frac{\frac{3\,d^4\,\sqrt{d\tan\left(e+f\,x\right)}}{8}+\frac{5\,d^3\,{\left(d\tan\left(e+f\,x\right)\right)}^{3/2}}{8}}{f\,a^3\,d^2\,{\tan\left(e+f\,x\right)}^2+2\,f\,a^3\,d^2\tan\left(e+f\,x\right)+f\,a^3\,d^2}+\frac{d^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{d\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)}{8\,a^3\,f}-\frac{\sqrt{2}\,d^{5/2}\,\mathrm{atanh}\left(\frac{9\,\sqrt{2}\,d^{33/2}\,\sqrt{d\tan\left(e+f\,x\right)}}{32\,\left(\frac{9\,d^{17}\,\tan\left(e+f\,x\right)}{32}+\frac{9\,d^{17}}{32}\right)}\right)}{4\,a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(5/2)/(a + a*tan(e + f*x))^3,x)
```

```
[Out] ((3*d^4*(d*tan(e + f*x))^(1/2))/8 + (5*d^3*(d*tan(e + f*x))^(3/2))/8)/(a^3*
d^2*f + a^3*d^2*f*tan(e + f*x)^2 + 2*a^3*d^2*f*tan(e + f*x)) + (d^(5/2)*ata
n((d*tan(e + f*x))^(1/2)/d^(1/2)))/(8*a^3*f) - (2^(1/2)*d^(5/2)*atanh((9*2^
(1/2)*d^(33/2)*(d*tan(e + f*x))^(1/2))/(32*((9*d^17*tan(e + f*x))/32 + (9*d
^17)/32))))/(4*a^3*f)
```

3.373 $\int \frac{(d\tan(e+fx))^{3/2}}{(a+a\tan(e+fx))^3}\,dx$

Optimal. Leaf size=164

$$-\frac{5d^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8a^3f}-\frac{d^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{d}-\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{2\sqrt{2}\,a^3f}+\frac{d\sqrt{d\tan(e+fx)}}{4af(a+a\tan(e+fx))^2}-\frac{d\sqrt{d\tan(e+fx)}}{8f(a^3+a^3\tan(e+fx))}$$

```
[Out] -5/8*d^(3/2)*arctan((d*tan(f*x+e))^(1/2)/d^(1/2))/a^3/f-1/4*d^(3/2)*arctan(
1/2*(d^(1/2)-d^(1/2)*tan(f*x+e))*2^(1/2)/(d*tan(f*x+e))^(1/2))/a^3/f*2^(1/2
)+1/4*d*(d*tan(f*x+e))^(1/2)/a/f/(a+a*tan(f*x+e))^2-1/8*d*(d*tan(f*x+e))^(1
/2)/f/(a^3+a^3*tan(f*x+e))
```

Rubi [A]

time = 0.39, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3648, 3730, 3734, 3613, 211, 3715, 65}

$$-\frac{5d^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8a^3f}-\frac{d^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{d}-\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{2\sqrt{2}\,a^3f}-\frac{d\sqrt{d\tan(e+fx)}}{8f\,(a^3\tan(e+fx)+a^3)}+\frac{d\sqrt{d\tan(e+fx)}}{4af(a\tan(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(3/2)/(a + a*Tan[e + f*x])^3,x]
```

```
[Out] (-5*d^(3/2)*ArcTan[Sqrt[d*Tan[e + f*x]]/Sqrt[d]])/(8*a^3*f) - (d^(3/2)*ArcT
an[(Sqrt[d] - Sqrt[d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[d*Tan[e + f*x]])])/(2*Sqr
t[2]*a^3*f) + (d*Sqrt[d*Tan[e + f*x]])/(4*a*f*(a + a*Tan[e + f*x])^2) - (d*
Sqrt[d*Tan[e + f*x]])/(8*f*(a^3 + a^3*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
```

```
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
```

```
!GtQ[n, 0] && !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^{3/2}}{(a+a\tan(e+fx))^3}\,dx = \frac{d\sqrt{d\tan(e+fx)}}{4af(a+a\tan(e+fx))^2} - \frac{\int \frac{\frac{ad^2}{2}-2ad^2\tan(e+fx)-\frac{3}{2}ad^2\tan^2(e+fx)}{\sqrt{d\tan(e+fx)}\,(a+a\tan(e+fx))^2}\,dx}{4a^2}$$

$$= \frac{d\sqrt{d\tan(e+fx)}}{4af(a+a\tan(e+fx))^2} - \frac{d\sqrt{d\tan(e+fx)}}{8f\,(a^3+a^3\tan(e+fx))} - \frac{\int \frac{\frac{a^3d^3}{2}-4a^3d^3\tan(e+fx)+}{\sqrt{d\tan(e+fx)}\,(}}{8a^5d}$$

$$= \frac{d\sqrt{d\tan(e+fx)}}{4af(a+a\tan(e+fx))^2} - \frac{d\sqrt{d\tan(e+fx)}}{8f\,(a^3+a^3\tan(e+fx))} - \frac{\int \frac{-4a^4d^3-4a^4d^3\tan(e+fx)}{\sqrt{d\tan(e+fx)}}}{16a^7d}$$

$$= \frac{d\sqrt{d\tan(e+fx)}}{4af(a+a\tan(e+fx))^2} - \frac{d\sqrt{d\tan(e+fx)}}{8f\,(a^3+a^3\tan(e+fx))} - \frac{(5d^2)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{dx}\,(}\right.}{1}$$

$$= -\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}-\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{2\sqrt{2}\,a^3f} + \frac{d\sqrt{d\tan(e+fx)}}{4af(a+a\tan(e+fx))^2} - \frac{d\sqrt{d}}{8f\,(a^3+}$$

$$= -\frac{5d^{3/2}\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8a^3f} - \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}-\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{2\sqrt{2}\,a^3f} + $$

Mathematica [A]
time = 2.50, size = 127, normalized size = 0.77

$$\frac{d\left(\frac{(-1+\cot(e+fx))\cot(e+fx)}{(1+\cot(e+fx))^2}+\frac{-2\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)+2\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(e+fx)}\right)-5\mathrm{ArcTan}\left(\sqrt{\tan(e+fx)}\right)}{\sqrt{\tan(e+fx)}}\right)\sqrt{d\tan(e+fx)}}{8a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(3/2)/(a + a*Tan[e + f*x])^3,x]
```

```
[Out] (d*(((-1 + Cot[e + f*x])*Cot[e + f*x])/(1 + Cot[e + f*x])^2 + (-2*Sqrt[2]*A
rcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] + 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[T
an[e + f*x]]] - 5*ArcTan[Sqrt[Tan[e + f*x]]])/Sqrt[Tan[e + f*x]])*Sqrt[d*Ta
n[e + f*x]])/(8*a^3*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(135) = 270$.
time = 0.18, size = 349, normalized size = 2.13

method	result
derivativedivides	$\dfrac{2d^4\left(-\dfrac{\frac{\frac{(d\tan(fx+e))^{\frac{3}{2}}}{4}-\frac{d\sqrt{d\tan(fx+e)}}{4}}{(d\tan(fx+e)+d)^2}+\frac{5\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{4\sqrt{d}}}{4d^2}+\dfrac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+}{d\tan(fx+e)-}\right.\right.}{}\right.}{}$
default	$\dfrac{2d^4\left(-\dfrac{\frac{\frac{(d\tan(fx+e))^{\frac{3}{2}}}{4}-\frac{d\sqrt{d\tan(fx+e)}}{4}}{(d\tan(fx+e)+d)^2}+\frac{5\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{4\sqrt{d}}}{4d^2}+\dfrac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+}{d\tan(fx+e)-}\right.\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(3/2)/(a+a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3*d^4*(-1/4/d^2*((1/4*(d*tan(f*x+e))^(3/2)-1/4*d*(d*tan(f*x+e))^(1/2)
)/(d*tan(f*x+e)+d)^2+5/4/d^(1/2)*arctan((d*tan(f*x+e))^(1/2)/d^(1/2)))+1/4/
d^2*(1/8/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))
^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*
2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-
2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))+1/8/(d^2)^(1/4)*2^(1
/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))
/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arc
tan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/
4)*(d*tan(f*x+e))^(1/2)+1))))
```

Maxima [A]

time = 0.54, size = 191, normalized size = 1.16

$$\frac{\frac{2d^3\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}\right)}{a^3}-\frac{5d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{a^3}-\frac{(d\tan(fx+e))^{\frac{3}{2}}d^3-\sqrt{d\tan(fx+e)}\,d^4}{a^3d^2\tan(fx+e)^2+2a^3d^2\tan(fx+e)+a^3d^2}}{8df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(a+a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(2*d^3*(sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(f*x
+ e)))/sqrt(d))/sqrt(d) + sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*
```

```
sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d))/a^3 - 5*d^(5/2)*arctan(sqrt(d*tan(f
*x + e))/sqrt(d))/a^3 - ((d*tan(f*x + e))^(3/2)*d^3 - sqrt(d*tan(f*x + e))*
d^4)/(a^3*d^2*tan(f*x + e)^2 + 2*a^3*d^2*tan(f*x + e) + a^3*d^2))/(d*f)
```

Fricas [A]
time = 0.88, size = 440, normalized size = 2.68

$$\left[\frac{2\left(\sqrt{2}\,d\tan\left(fx+e\right)^{2}+2\,\sqrt{2}\,d\tan\left(fx+e\right)+\sqrt{2}\,d\right)\sqrt{-d}\,\log\left(\frac{d\tan(fx+e)^{2}+2\,\sqrt{d\tan\left(fx+e\right)}\left(\sqrt{2}\,\tan(fx+e)-\sqrt{2}\right)\sqrt{-d}-4\,d\tan(fx+e)+d}{\tan(fx+e)^{2}+1}\right)+5\left(d\tan\left(fx+e\right)^{2}+2\,d\tan\left(fx+e\right)+d\right)\sqrt{-d}\,\log\left(\frac{d\tan(fx+e)-2\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{-d}-d}{\tan(fx+e)+1}\right)-2\left(d\tan\left(fx+e\right)-d\right)\sqrt{d\tan\left(fx+e\right)}}{16\left(a^{3}f\tan\left(fx+e\right)^{2}+2\,a^{3}f\tan\left(fx+e\right)+a^{3}f\right)},-\frac{5\left(d\tan\left(fx+e\right)^{2}+2\,d\tan\left(fx+e\right)+d\right)\sqrt{d}\,\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}}{\sqrt{d}}\right)-2\left(\sqrt{2}\,d\tan\left(fx+e\right)^{2}+2\,\sqrt{2}\,d\tan\left(fx+e\right)+\sqrt{2}\,d\right)\sqrt{d}\,\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}\left(\sqrt{2}\,\tan(fx+e)-\sqrt{2}\right)}{2\,\sqrt{d}\,\tan(fx+e)}\right)+\left(d\tan\left(fx+e\right)-d\right)\sqrt{d\tan\left(fx+e\right)}}{8\left(a^{3}f\tan\left(fx+e\right)^{2}+2\,a^{3}f\tan\left(fx+e\right)+a^{3}f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(a+a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(sqrt(2)*d*tan(f*x + e)^2 + 2*sqrt(2)*d*tan(f*x + e) + sqrt(2)*d)*
sqrt(-d)*log((d*tan(f*x + e)^2 + 2*sqrt(d*tan(f*x + e))*(sqrt(2)*tan(f*x +
e) - sqrt(2))*sqrt(-d) - 4*d*tan(f*x + e) + d)/(tan(f*x + e)^2 + 1)) + 5*(d
*tan(f*x + e)^2 + 2*d*tan(f*x + e) + d)*sqrt(-d)*log((d*tan(f*x + e) - 2*sq
rt(d*tan(f*x + e))*sqrt(-d) - d)/(tan(f*x + e) + 1)) - 2*(d*tan(f*x + e) -
d)*sqrt(d*tan(f*x + e)))/(a^3*f*tan(f*x + e)^2 + 2*a^3*f*tan(f*x + e) + a^3
*f), -1/8*(5*(d*tan(f*x + e)^2 + 2*d*tan(f*x + e) + d)*sqrt(d)*arctan(sqrt(
d*tan(f*x + e))/sqrt(d)) - 2*(sqrt(2)*d*tan(f*x + e)^2 + 2*sqrt(2)*d*tan(f*
x + e) + sqrt(2)*d)*sqrt(d)*arctan(1/2*sqrt(d*tan(f*x + e))*(sqrt(2)*tan(f*
x + e) - sqrt(2))/(sqrt(d)*tan(f*x + e))) + (d*tan(f*x + e) - d)*sqrt(d*tan
(f*x + e)))/(a^3*f*tan(f*x + e)^2 + 2*a^3*f*tan(f*x + e) + a^3*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\left(d\tan\left(e+fx\right)\right)^{\frac{3}{2}}}{\tan^{3}\left(e+fx\right)+3\tan^{2}\left(e+fx\right)+3\tan\left(e+fx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(3/2)/(a+a*tan(f*x+e))**3,x)
```

```
[Out] Integral((d*tan(e + f*x))**(3/2)/(tan(e + f*x)**3 + 3*tan(e + f*x)**2 + 3*t
an(e + f*x) + 1), x)/a**3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(143) = 286.
time = 0.77, size = 323, normalized size = 1.97

$$\frac{1}{16}\,d\left(\frac{2\,\sqrt{2}\left(d\sqrt{|d|}+|d|^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{|d|}+2\,\sqrt{d\tan\left(fx+e\right)}\right)}{2\,\sqrt{|d|}}\right)}{a^{3}df}+\frac{2\,\sqrt{2}\left(d\sqrt{|d|}+|d|^{\frac{3}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{|d|}-2\,\sqrt{d\tan\left(fx+e\right)}\right)}{2\,\sqrt{|d|}}\right)}{a^{3}df}-\frac{10\,\sqrt{d}\,\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}}{\sqrt{d}}\right)}{a^{3}f}+\frac{\sqrt{2}\left(d\sqrt{|d|}-|d|^{\frac{3}{2}}\right)\log\left(d\tan\left(fx+e\right)+\sqrt{2}\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{|d|}+|d|\right)}{a^{3}df}-\frac{\sqrt{2}\left(d\sqrt{|d|}-|d|^{\frac{3}{2}}\right)\log\left(d\tan\left(fx+e\right)-\sqrt{2}\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{|d|}+|d|\right)}{a^{3}df}-\frac{2\left(\sqrt{d\tan\left(fx+e\right)}\,d^{2}\tan\left(fx+e\right)-\sqrt{d\tan\left(fx+e\right)}\,d^{2}\right)}{\left(d\tan\left(fx+e\right)+d\right)^{2}a^{3}f}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(a+a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/16*d*(2*sqrt(2)*(d*sqrt(abs(d)) + abs(d)^(3/2))*arctan(1/2*sqrt(2)*(sqrt(
2)*sqrt(abs(d)) + 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(a^3*d*f) + 2*sqrt(
2)*(d*sqrt(abs(d)) + abs(d)^(3/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)
) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(a^3*d*f) - 10*sqrt(d)*arctan(sqr
t(d*tan(f*x + e))/sqrt(d))/(a^3*f) + sqrt(2)*(d*sqrt(abs(d)) - abs(d)^(3/2)
)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/
(a^3*d*f) - sqrt(2)*(d*sqrt(abs(d)) - abs(d)^(3/2))*log(d*tan(f*x + e) - sq
rt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(a^3*d*f) - 2*(sqrt(d*tan
(f*x + e))*d^2*tan(f*x + e) - sqrt(d*tan(f*x + e))*d^2)/((d*tan(f*x + e) +
d)^2*a^3*f))
```

Mupad [B]
time = 4.77, size = 177, normalized size = 1.08

$$\frac{\frac{d^3\,\sqrt{d\tan\left(e+f\,x\right)}}{8}-\frac{d^2\,{\left(d\tan\left(e+f\,x\right)\right)}^{3/2}}{8}}{f\,a^3\,d^2\,{\tan\left(e+f\,x\right)}^2+2\,f\,a^3\,d^2\tan\left(e+f\,x\right)+f\,a^3\,d^2}-\frac{5\,d^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{d\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)}{8\,a^3\,f}+\frac{\sqrt{2}\,d^{3/2}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{d\tan\left(e+f\,x\right)}}{2\,\sqrt{d}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{d\tan\left(e+f\,x\right)}}{2\,\sqrt{d}}+\frac{\sqrt{2}\,{\left(d\tan\left(e+f\,x\right)\right)}^{3/2}}{2\,d^{3/2}}\right)\right)}{8\,a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(3/2)/(a + a*tan(e + f*x))^3,x)
```

```
[Out] ((d^3*(d*tan(e + f*x))^(1/2))/8 - (d^2*(d*tan(e + f*x))^(3/2))/8)/(a^3*d^2*
f + a^3*d^2*f*tan(e + f*x)^2 + 2*a^3*d^2*f*tan(e + f*x)) - (5*d^(3/2)*atan(
(d*tan(e + f*x))^(1/2)/d^(1/2)))/(8*a^3*f) + (2^(1/2)*d^(3/2)*(2*atan((2^(1
/2)*(d*tan(e + f*x))^(1/2))/(2*d^(1/2))) + 2*atan((2^(1/2)*(d*tan(e + f*x))
^(1/2))/(2*d^(1/2)) + (2^(1/2)*(d*tan(e + f*x))^(3/2))/(2*d^(3/2)))))/(8*a^
3*f)
```

3.374 $\int \frac{\sqrt{d\tan(e+fx)}}{(a+a\tan(e+fx))^3}\,dx$

Optimal. Leaf size=161

$$\frac{\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8a^3f}+\frac{\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{2\sqrt{2}\,a^3f}-\frac{\sqrt{d\tan(e+fx)}}{4af(a+a\tan(e+fx))^2}-\frac{3\sqrt{}}{8f\,(a^3}$$

```
[Out] 1/8*arctan((d*tan(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/a^3/f+1/4*arctanh(1/2*(d^(
1/2)+d^(1/2)*tan(f*x+e))*2^(1/2)/(d*tan(f*x+e))^(1/2))*d^(1/2)/a^3/f*2^(1/2
)-1/4*(d*tan(f*x+e))^(1/2)/a/f/(a+a*tan(f*x+e))^2-3/8*(d*tan(f*x+e))^(1/2)/
f/(a^3+a^3*tan(f*x+e))
```

Rubi [A]
time = 0.35, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3649, 3730, 3735, 3613, 214, 3715, 65, 211}

$$\frac{\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8a^3f}-\frac{3\sqrt{d\tan(e+fx)}}{8f\left(a^3\tan(e+fx)+a^3\right)}+\frac{\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\tan(e+fx)+\sqrt{d}}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{2\sqrt{2}\,a^3f}-\frac{\sqrt{d\tan(e+fx)}}{4af(a\tan(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Tan[e + f*x]]/(a + a*Tan[e + f*x])^3,x]
```

```
[Out] (Sqrt[d]*ArcTan[Sqrt[d*Tan[e + f*x]]/Sqrt[d]])/(8*a^3*f) + (Sqrt[d]*ArcTanh
[(Sqrt[d] + Sqrt[d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[d*Tan[e + f*x]])])/(2*Sqrt[
2]*a^3*f) - Sqrt[d*Tan[e + f*x]]/(4*a*f*(a + a*Tan[e + f*x])^2) - (3*Sqrt[d
*Tan[e + f*x]])/(8*f*(a^3 + a^3*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3735

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dis
```

```
t[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*Simp[a*(A - C) - (A*b - b*C)*Ta
n[e + f*x], x], x], x] + Dist[(A*b^2 + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e
 + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a,
 b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0] &&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{d\tan(e+fx)}}{(a+a\tan(e+fx))^3}\,dx = -\frac{\sqrt{d\tan(e+fx)}}{4af(a+a\tan(e+fx))^2} - \frac{\int \frac{-\frac{ad}{2}-2ad\tan(e+fx)+\frac{3}{2}ad\tan^2(e+fx)}{\sqrt{d\tan(e+fx)}\,(a+a\tan(e+fx))^2}\,dx}{4a^2}$$

$$= -\frac{\sqrt{d\tan(e+fx)}}{4af(a+a\tan(e+fx))^2} - \frac{3\sqrt{d\tan(e+fx)}}{8f\,(a^3+a^3\tan(e+fx))} - \frac{\int \frac{-\frac{5}{2}a^3d^2+\frac{3}{2}a^3d^2\,\mathrm{t}}{\sqrt{d\tan(e+fx)}}}{8a^5d}$$

$$= -\frac{\sqrt{d\tan(e+fx)}}{4af(a+a\tan(e+fx))^2} - \frac{3\sqrt{d\tan(e+fx)}}{8f\,(a^3+a^3\tan(e+fx))} - \frac{\int \frac{-4a^4d^2+4a^4d^2\tan(e+}{\sqrt{d\tan(e+fx)}}}{16a^7d}$$

$$= -\frac{\sqrt{d\tan(e+fx)}}{4af(a+a\tan(e+fx))^2} - \frac{3\sqrt{d\tan(e+fx)}}{8f\,(a^3+a^3\tan(e+fx))} + \frac{d\mathrm{Subst}\left(\int \frac{1}{\sqrt{dx}\,(a+}\right.}{1}$$

$$= \frac{\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{2\sqrt{2}\,a^3f} - \frac{\sqrt{d\tan(e+fx)}}{4af(a+a\tan(e+fx))^2} - \frac{3\sqrt{d}}{8f\,(a^3+}$$

$$= \frac{\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8a^3f} + \frac{\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{2\sqrt{2}\,a^3f} - \frac{}{4}$$

Mathematica [A]
time = 0.87, size = 253, normalized size = 1.57

$$-\frac{\left(2\sqrt{2}\log\left(-1+\sqrt{2}\sqrt{\tan(e+fx)}-\tan(e+fx)\right)-2\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)-2\mathrm{ArcTan}\left(\sqrt{\tan(e+fx)}\right)(1+\sin(2(e+fx)))+\sin(2(e+fx))\left(2\sqrt{2}\left(\log\left(-1+\sqrt{2}\sqrt{\tan(e+fx)}-\tan(e+fx)\right)-\log\left(1+\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)\right)+3\sqrt{\tan(e+fx)}\right)+5\sqrt{\tan(e+fx)}+5\cos(2(e+fx))\sqrt{\tan(e+fx)}\right)\sqrt{d\tan(e+fx)}}{16a^3f(\cos(e+fx)+\sin(e+fx))^2\sqrt{\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Tan[e + f*x]]/(a + a*Tan[e + f*x])^3,x]
```

```
[Out] -1/16*((2*Sqrt[2]*Log[-1 + Sqrt[2]*Sqrt[Tan[e + f*x]] - Tan[e + f*x]] - 2*S
qrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2*ArcTan[Sqrt[T
an[e + f*x]]]*(1 + Sin[2*(e + f*x)]) + Sin[2*(e + f*x)]*(2*Sqrt[2]*(Log[-1
+ Sqrt[2]*Sqrt[Tan[e + f*x]] - Tan[e + f*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[e +
 f*x]] + Tan[e + f*x]]) + 3*Sqrt[Tan[e + f*x]]) + 5*Sqrt[Tan[e + f*x]] + 5*
```

```
Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]])*Sqrt[d*Tan[e + f*x]])/(a^3*f*(Cos[e +
f*x] + Sin[e + f*x])^2*Sqrt[Tan[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(132) = 264$.

time = 0.21, size = 349, normalized size = 2.17

method	result
derivativedivides	$\frac{2d^4\left(-\frac{\frac{\frac{3(d\tan(fx+e))^{\frac{3}{2}}}{4}+\frac{5d\sqrt{d\tan(fx+e)}}{4}}{(d\tan(fx+e)+d)^2}-\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{4\sqrt{d}}}{4d^3}+\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+}{d\tan(fx+e)-}\right.\right.}{}\right.}{}$
default	$\frac{2d^4\left(-\frac{\frac{\frac{3(d\tan(fx+e))^{\frac{3}{2}}}{4}+\frac{5d\sqrt{d\tan(fx+e)}}{4}}{(d\tan(fx+e)+d)^2}-\frac{\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{4\sqrt{d}}}{4d^3}+\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)+}{d\tan(fx+e)-}\right.\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(1/2)/(a+a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3*d^4*(-1/4/d^3*((3/4*(d*tan(f*x+e))^(3/2)+5/4*d*(d*tan(f*x+e))^(1/2)
)/(d*tan(f*x+e)+d)^2-1/4/d^(1/2)*arctan((d*tan(f*x+e))^(1/2)/d^(1/2)))+1/4/
d^3*(1/8/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))
^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*
2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-
2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))-1/8/(d^2)^(1/4)*2^(1
/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))
/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arc
tan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/
4)*(d*tan(f*x+e))^(1/2)+1))))
```

Maxima [A]

time = 0.51, size = 193, normalized size = 1.20

$$-\frac{\frac{3\,(d\tan(fx+e))^{\frac{3}{2}}d^2+5\,\sqrt{d\tan(fx+e)}\,d^3}{a^3d^2\tan(fx+e)^2+2\,a^3d^2\tan(fx+e)+a^3d^2}-\frac{d^2\left(\frac{\sqrt{2}\,\log\left(d\tan(fx+e)+\sqrt{2}\,\sqrt{d\tan(fx+e)}\,\sqrt{d}+d\right)}{\sqrt{d}}-\frac{\sqrt{2}\,\log\left(d\tan(fx+e)-\sqrt{2}\,\sqrt{d\tan(fx+e)}\,\sqrt{d}+d\right)}{\sqrt{d}}\right)}{a^3}-\frac{d^{\frac{3}{2}}\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{a^3}}{8\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(a+a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/8*((3*(d*tan(f*x + e))^(3/2)*d^2 + 5*sqrt(d*tan(f*x + e))*d^3)/(a^3*d^2*
tan(f*x + e)^2 + 2*a^3*d^2*tan(f*x + e) + a^3*d^2) - d^2*(sqrt(2)*log(d*tan
(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) - sqrt(2)*log
(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d))/a^3 -
d^(3/2)*arctan(sqrt(d*tan(f*x + e))/sqrt(d))/a^3)/(d*f)
```

Fricas [**A**]
time = 0.85, size = 420, normalized size = 2.61

$$\left[-\frac{4\left(\sqrt{2}\tan\left(fx+e\right)^{2}+2\sqrt{2}\tan\left(fx+e\right)+\sqrt{2}\right)\sqrt{-d}\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}\left(\sqrt{2}\tan(fx+e)+\sqrt{2}\right)\sqrt{-d}}{2\,d\tan(fx+e)}\right)-\left(\tan\left(fx+e\right)^{2}+2\tan\left(fx+e\right)+1\right)\sqrt{-d}\log\left(\frac{d\tan(fx+e)+2\sqrt{d\tan\left(fx+e\right)}\sqrt{-d}-d}{\tan(fx+e)+1}\right)+2\sqrt{d\tan\left(fx+e\right)}\left(3\tan\left(fx+e\right)+5\right)}{16\left(a^{3}f\tan\left(fx+e\right)^{2}+2\,a^{3}f\tan\left(fx+e\right)+a^{3}f\right)},\ \frac{\left(\tan\left(fx+e\right)^{2}+2\tan\left(fx+e\right)+1\right)\sqrt{d}\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}}{\sqrt{d}}\right)+\left(\sqrt{2}\tan\left(fx+e\right)^{2}+2\sqrt{2}\tan\left(fx+e\right)+\sqrt{2}\right)\sqrt{d}\log\left(\frac{d\tan(fx+e)^{2}+2\sqrt{d\tan\left(fx+e\right)}\left(\sqrt{2}\tan(fx+e)+\sqrt{2}\right)\sqrt{d}+4\,d\tan(fx+e)+d}{\tan(fx+e)^{2}+1}\right)-\sqrt{d\tan\left(fx+e\right)}\left(3\tan\left(fx+e\right)+5\right)}{8\left(a^{3}f\tan\left(fx+e\right)^{2}+2\,a^{3}f\tan\left(fx+e\right)+a^{3}f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(a+a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(sqrt(2)*tan(f*x + e)^2 + 2*sqrt(2)*tan(f*x + e) + sqrt(2))*sqrt(
-d)*arctan(1/2*sqrt(d*tan(f*x + e))*(sqrt(2)*tan(f*x + e) + sqrt(2))*sqrt(-
d)/(d*tan(f*x + e))) - (tan(f*x + e)^2 + 2*tan(f*x + e) + 1)*sqrt(-d)*log((
d*tan(f*x + e) + 2*sqrt(d*tan(f*x + e))*sqrt(-d) - d)/(tan(f*x + e) + 1)) +
 2*sqrt(d*tan(f*x + e))*(3*tan(f*x + e) + 5))/(a^3*f*tan(f*x + e)^2 + 2*a^3
*f*tan(f*x + e) + a^3*f), 1/8*((tan(f*x + e)^2 + 2*tan(f*x + e) + 1)*sqrt(d
)*arctan(sqrt(d*tan(f*x + e))/sqrt(d)) + (sqrt(2)*tan(f*x + e)^2 + 2*sqrt(2
)*tan(f*x + e) + sqrt(2))*sqrt(d)*log((d*tan(f*x + e)^2 + 2*sqrt(d*tan(f*x
+ e))*(sqrt(2)*tan(f*x + e) + sqrt(2))*sqrt(d) + 4*d*tan(f*x + e) + d)/(tan
(f*x + e)^2 + 1)) - sqrt(d*tan(f*x + e))*(3*tan(f*x + e) + 5))/(a^3*f*tan(f
*x + e)^2 + 2*a^3*f*tan(f*x + e) + a^3*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{d\tan\left(e+fx\right)}}{\tan^{3}\left(e+fx\right)+3\tan^{2}\left(e+fx\right)+3\tan\left(e+fx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(1/2)/(a+a*tan(f*x+e))**3,x)
```

```
[Out] Integral(sqrt(d*tan(e + f*x))/(tan(e + f*x)**3 + 3*tan(e + f*x)**2 + 3*tan(
e + f*x) + 1), x)/a**3
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(139) = 278.
time = 0.73, size = 314, normalized size = 1.95

$$\frac{\frac{2\sqrt{2}\left(d\sqrt{|d|}-|d|^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{a^{3}f}+\frac{2\sqrt{2}\left(d\sqrt{|d|}-|d|^{\frac{3}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{a^{3}f}+\frac{2\,d^{\frac{3}{2}}\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}}{\sqrt{d}}\right)}{a^{3}f}+\frac{\sqrt{2}\left(d\sqrt{|d|}+|d|^{\frac{3}{2}}\right)\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{|d|}+|d|\right)}{a^{3}f}-\frac{\sqrt{2}\left(d\sqrt{|d|}+|d|^{\frac{3}{2}}\right)\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{|d|}+|d|\right)}{a^{3}f}-\frac{2\left(3\sqrt{d\tan\left(fx+e\right)}\,d^{3}\tan(fx+e)+5\sqrt{d\tan\left(fx+e\right)}\,d^{3}\right)}{\left(d\tan(fx+e)+d\right)^{2}a^{3}f}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(a+a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(2)*(d*sqrt(abs(d)) - abs(d)^(3/2))*arctan(1/2*sqrt(2)*(sqrt(2)
*sqrt(abs(d)) + 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(a^3*f) + 2*sqrt(2)*(
d*sqrt(abs(d)) - abs(d)^(3/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) -
2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(a^3*f) + 2*d^(3/2)*arctan(sqrt(d*tan
(f*x + e))/sqrt(d))/(a^3*f) + sqrt(2)*(d*sqrt(abs(d)) + abs(d)^(3/2))*log(d
*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(a^3*f)
 - sqrt(2)*(d*sqrt(abs(d)) + abs(d)^(3/2))*log(d*tan(f*x + e) - sqrt(2)*sqr
t(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(a^3*f) - 2*(3*sqrt(d*tan(f*x + e)
)*d^3*tan(f*x + e) + 5*sqrt(d*tan(f*x + e))*d^3)/((d*tan(f*x + e) + d)^2*a^
3*f))/d
```

Mupad [B]
time = 4.68, size = 152, normalized size = 0.94

$$\frac{\sqrt{d}\,\mathrm{atan}\left(\frac{\sqrt{d\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)}{8\,a^3\,f}-\frac{\frac{3\,d\,{\left(d\tan\left(e+f\,x\right)\right)}^{3/2}}{8}+\frac{5\,d^2\,\sqrt{d\tan\left(e+f\,x\right)}}{8}}{f\,a^3\,d^2\,{\tan\left(e+f\,x\right)}^2+2\,f\,a^3\,d^2\tan\left(e+f\,x\right)+f\,a^3\,d^2}+\frac{\sqrt{2}\,\sqrt{d}\,\mathrm{atanh}\left(\frac{9\,\sqrt{2}\,d^{17/2}\,\sqrt{d\tan\left(e+f\,x\right)}}{32\,\left(\frac{9\,d^9\tan\left(e+f\,x\right)}{32}+\frac{9\,d^9}{32}\right)}\right)}{4\,a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(1/2)/(a + a*tan(e + f*x))^3,x)
```

```
[Out] (d^(1/2)*atan((d*tan(e + f*x))^(1/2)/d^(1/2)))/(8*a^3*f) - ((3*d*(d*tan(e +
 f*x))^(3/2))/8 + (5*d^2*(d*tan(e + f*x))^(1/2))/8)/(a^3*d^2*f + a^3*d^2*f*
tan(e + f*x)^2 + 2*a^3*d^2*f*tan(e + f*x)) + (2^(1/2)*d^(1/2)*atanh((9*2^(1
/2)*d^(17/2)*(d*tan(e + f*x))^(1/2))/(32*((9*d^9*tan(e + f*x))/32 + (9*d^9)
/32))))/(4*a^3*f)
```

3.375 $\int \frac{1}{\sqrt{d\tan(e+fx)}\,(a+a\tan(e+fx))^3}\,dx$

Optimal. Leaf size=165

$$\frac{11\text{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8a^3\sqrt{d}\,f}+\frac{\text{ArcTan}\left(\frac{\sqrt{d}-\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{2\sqrt{2}\,a^3\sqrt{d}\,f}+\frac{7\sqrt{d\tan(e+fx)}}{8a^3df(1+\tan(e+fx))}+\frac{\sqrt{d\tan(e+fx)}}{4adf(a+a\tan(e+fx))^2}$$

```
[Out] 11/8*arctan((d*tan(f*x+e))^(1/2)/d^(1/2))/a^3/f/d^(1/2)+1/4*arctan(1/2*(d^(
1/2)-d^(1/2)*tan(f*x+e))*2^(1/2)/(d*tan(f*x+e))^(1/2))/a^3/f*2^(1/2)/d^(1/2
)+7/8*(d*tan(f*x+e))^(1/2)/a^3/d/f/(1+tan(f*x+e))+1/4*(d*tan(f*x+e))^(1/2)/
a/d/f/(a+a*tan(f*x+e))^2
```

Rubi [A]
time = 0.37, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3650, 3730, 3734, 3613, 211, 3715, 65}

$$\frac{11\text{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8a^3\sqrt{d}\,f}+\frac{\text{ArcTan}\left(\frac{\sqrt{d}-\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{2\sqrt{2}\,a^3\sqrt{d}\,f}+\frac{7\sqrt{d\tan(e+fx)}}{8a^3df(\tan(e+fx)+1)}+\frac{\sqrt{d\tan(e+fx)}}{4adf(a\tan(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*Tan[e + f*x]]*(a + a*Tan[e + f*x])^3),x]
```

```
[Out] (11*ArcTan[Sqrt[d*Tan[e + f*x]]/Sqrt[d]])/(8*a^3*Sqrt[d]*f) + ArcTan[(Sqrt[
d] - Sqrt[d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[d*Tan[e + f*x]])]/(2*Sqrt[2]*a^3*S
qrt[d]*f) + (7*Sqrt[d*Tan[e + f*x]])/(8*a^3*d*f*(1 + Tan[e + f*x])) + Sqrt[
d*Tan[e + f*x]]/(4*a*d*f*(a + a*Tan[e + f*x])^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
```

```
d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
```

```
!GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{d\tan(e+fx)}\,(a+a\tan(e+fx))^3}\,dx = \frac{\sqrt{d\tan(e+fx)}}{4adf(a+a\tan(e+fx))^2} + \frac{\int \frac{\frac{7a^2d}{2}-2a^2d\tan(e+fx)+\frac{3}{2}a^2d\tan^2(e+f\ldots}{\sqrt{d\tan(e+fx)}\,(a+a\tan(e+fx)\ldots}}{4a^3d}$$

$$= \frac{7\sqrt{d\tan(e+fx)}}{8a^3df(1+\tan(e+fx))} + \frac{\sqrt{d\tan(e+fx)}}{4adf(a+a\tan(e+fx))^2} + \frac{\int \frac{7a^4}{2}\ldots}{\ldots}$$

$$= \frac{7\sqrt{d\tan(e+fx)}}{8a^3df(1+\tan(e+fx))} + \frac{\sqrt{d\tan(e+fx)}}{4adf(a+a\tan(e+fx))^2} + \frac{11\int \ldots}{\ldots}$$

$$= \frac{7\sqrt{d\tan(e+fx)}}{8a^3df(1+\tan(e+fx))} + \frac{\sqrt{d\tan(e+fx)}}{4adf(a+a\tan(e+fx))^2} + \frac{11\mathrm{Su}\ldots}{\ldots}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{d}-\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{2\sqrt{2}\,a^3\sqrt{d}\,f} + \frac{7\sqrt{d\tan(e+fx)}}{8a^3df(1+\tan(e+fx))} + \ldots$$

$$= \frac{11\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8a^3\sqrt{d}\,f} + \frac{\tan^{-1}\left(\frac{\sqrt{d}-\sqrt{d}\tan(e+fx\ldots}{\sqrt{2}\sqrt{d\tan(e+fx\ldots}}\right)}{2\sqrt{2}\,a^3\sqrt{d}\,f}\ldots$$

Mathematica [A]
time = 1.08, size = 217, normalized size = 1.32

$$\frac{\left(22\mathrm{ArcTan}\left(\sqrt{\tan(e+fx)}\right)+4\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)(\cos(e+fx)+\sin(e+fx))^2-4\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(e+fx)}\right)(\cos(e+fx)+\sin(e+fx))^2+22\mathrm{ArcTan}\left(\sqrt{\tan(e+fx)}\right)\sin(2(e+fx))+9\sqrt{\tan(e+fx)}+9\cos(2(e+fx))\sqrt{\tan(e+fx)}+7\sin(2(e+fx))\sqrt{\tan(e+fx)}\right)\sqrt{\tan(e+fx)}}{16a^3f(\cos(e+fx)+\sin(e+fx))^2\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*Tan[e + f*x]]*(a + a*Tan[e + f*x])^3),x]
```

```
[Out] ((22*ArcTan[Sqrt[Tan[e + f*x]]] + 4*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e +
 f*x]]]*(Cos[e + f*x] + Sin[e + f*x])^2 - 4*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt
[Tan[e + f*x]]]*(Cos[e + f*x] + Sin[e + f*x])^2 + 22*ArcTan[Sqrt[Tan[e + f*
x]]]*Sin[2*(e + f*x)] + 9*Sqrt[Tan[e + f*x]] + 9*Cos[2*(e + f*x)]*Sqrt[Tan[
e + f*x]] + 7*Sin[2*(e + f*x)]*Sqrt[Tan[e + f*x]])*Sqrt[Tan[e + f*x]])/(16*
a^3*f*(Cos[e + f*x] + Sin[e + f*x])^2*Sqrt[d*Tan[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(136) = 272$.
time = 0.24, size = 349, normalized size = 2.12

method	result
derivativedivides	$\frac{2d^4\left(\frac{\frac{\frac{7(d\tan(fx+e))^{\frac{3}{2}}}{4}+\frac{9d\sqrt{d\tan(fx+e)}}{4}}{(d\tan(fx+e)+d)^2}+\frac{11\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{4\sqrt{d}}}{4d^4}+\frac{-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e}{d\tan(fx+e}\right.\right.}{}}{}\right.}{}$
default	$\frac{2d^4\left(\frac{\frac{\frac{7(d\tan(fx+e))^{\frac{3}{2}}}{4}+\frac{9d\sqrt{d\tan(fx+e)}}{4}}{(d\tan(fx+e)+d)^2}+\frac{11\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{4\sqrt{d}}}{4d^4}+\frac{-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e}{d\tan(fx+e}\right.\right.}{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*tan(f*x+e))^(1/2)/(a+a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3*d^4*(1/4/d^4*((7/4*(d*tan(f*x+e))^(3/2)+9/4*d*(d*tan(f*x+e))^(1/2))
/(d*tan(f*x+e)+d)^2+11/4/d^(1/2)*arctan((d*tan(f*x+e))^(1/2)/d^(1/2)))+1/4/
d^4*(-1/8/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e)
)^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)
*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)
-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))-1/8/(d^2)^(1/4)*2^(
1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)
)/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*ar
ctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1
/4)*(d*tan(f*x+e))^(1/2)+1))))
```

Maxima [A]

time = 0.53, size = 187, normalized size = 1.13

$$\frac{\frac{7(d\tan(fx+e))^{\frac{3}{2}}d+9\sqrt{d\tan(fx+e)}\,d^2}{a^3d^2\tan(fx+e)^2+2a^3d^2\tan(fx+e)+a^3d^2}-\frac{2d\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}\right)}{a^3}+\frac{11\sqrt{d}\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{a^3}}{8\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(1/2)/(a+a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/8*((7*(d*tan(f*x + e))^(3/2)*d + 9*sqrt(d*tan(f*x + e))*d^2)/(a^3*d^2*tan
(f*x + e)^2 + 2*a^3*d^2*tan(f*x + e) + a^3*d^2) - 2*d*(sqrt(2)*arctan(1/2*s
```

```
qrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) + sqrt(2
)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/s
qrt(d))/a^3 + 11*sqrt(d)*arctan(sqrt(d*tan(f*x + e))/sqrt(d))/a^3)/(d*f)
```

Fricas [A]
time = 0.93, size = 407, normalized size = 2.47

$$\left[-\frac{2\sqrt{2}\left(\tan\left(fx+e\right)^{2}+2\tan\left(fx+e\right)+1\right)\sqrt{-d}\log\left(\frac{d\tan(fx+e)^{2}+2\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{-d}\left(\tan(fx+e)-1\right)-4d\tan(fx+e)+d}{\tan(fx+e)^{2}+1}\right)+11\left(\tan\left(fx+e\right)^{2}+2\tan\left(fx+e\right)+1\right)\sqrt{-d}\log\left(\frac{d\tan(fx+e)-2\sqrt{d\tan\left(fx+e\right)}\sqrt{-d}-d}{\tan(fx+e)+1}\right)-2\sqrt{d\tan\left(fx+e\right)}\left(7\tan\left(fx+e\right)+9\right)}{16\left(a^{3}df\tan\left(fx+e\right)^{2}+2a^{3}df\tan\left(fx+e\right)+a^{3}df\right)}, -\frac{2\sqrt{2}\left(\tan\left(fx+e\right)^{2}+2\tan\left(fx+e\right)+1\right)\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\left(\tan(fx+e)-1\right)}{2\sqrt{d}\tan(fx+e)}\right)-11\left(\tan\left(fx+e\right)^{2}+2\tan\left(fx+e\right)+1\right)\sqrt{d}\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}}{\sqrt{d}}\right)-\sqrt{d\tan\left(fx+e\right)}\left(7\tan\left(fx+e\right)+9\right)}{8\left(a^{3}df\tan\left(fx+e\right)^{2}+2a^{3}df\tan\left(fx+e\right)+a^{3}df\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(1/2)/(a+a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*sqrt(2)*(tan(f*x + e)^2 + 2*tan(f*x + e) + 1)*sqrt(-d)*log((d*tan
(f*x + e)^2 + 2*sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(-d)*(tan(f*x + e) - 1) -
4*d*tan(f*x + e) + d)/(tan(f*x + e)^2 + 1)) + 11*(tan(f*x + e)^2 + 2*tan(f*
x + e) + 1)*sqrt(-d)*log((d*tan(f*x + e) - 2*sqrt(d*tan(f*x + e))*sqrt(-d)
- d)/(tan(f*x + e) + 1)) - 2*sqrt(d*tan(f*x + e))*(7*tan(f*x + e) + 9))/(a^
3*d*f*tan(f*x + e)^2 + 2*a^3*d*f*tan(f*x + e) + a^3*d*f), -1/8*(2*sqrt(2)*(
tan(f*x + e)^2 + 2*tan(f*x + e) + 1)*sqrt(d)*arctan(1/2*sqrt(2)*sqrt(d*tan(
f*x + e))*(tan(f*x + e) - 1)/(sqrt(d)*tan(f*x + e))) - 11*(tan(f*x + e)^2 +
 2*tan(f*x + e) + 1)*sqrt(d)*arctan(sqrt(d*tan(f*x + e))/sqrt(d)) - sqrt(d*
tan(f*x + e))*(7*tan(f*x + e) + 9))/(a^3*d*f*tan(f*x + e)^2 + 2*a^3*d*f*tan
(f*x + e) + a^3*d*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{d\tan\left(e+fx\right)}\tan^{3}\left(e+fx\right)+3\sqrt{d\tan\left(e+fx\right)}\tan^{2}\left(e+fx\right)+3\sqrt{d\tan\left(e+fx\right)}\tan\left(e+fx\right)+\sqrt{d\tan\left(e+fx\right)}}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))**(1/2)/(a+a*tan(f*x+e))**3,x)
```

```
[Out] Integral(1/(sqrt(d*tan(e + f*x))*tan(e + f*x)**3 + 3*sqrt(d*tan(e + f*x))*t
an(e + f*x)**2 + 3*sqrt(d*tan(e + f*x))*tan(e + f*x) + sqrt(d*tan(e + f*x))
), x)/a**3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(144) = 288.
time = 0.75, size = 318, normalized size = 1.93

$$-\frac{\sqrt{2}\left(d\sqrt{|d|}+|d|^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{8a^{3}d^{2}f}-\frac{\sqrt{2}\left(d\sqrt{|d|}+|d|^{\frac{3}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{8a^{3}d^{2}f}+\frac{11\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}}{\sqrt{d}}\right)}{8a^{3}\sqrt{d}f}-\frac{\sqrt{2}\left(d\sqrt{|d|}-|d|^{\frac{3}{2}}\right)\log\left(d\tan\left(fx+e\right)+\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{|d|}+|d|\right)}{16a^{3}d^{2}f}+\frac{\sqrt{2}\left(d\sqrt{|d|}-|d|^{\frac{3}{2}}\right)\log\left(d\tan\left(fx+e\right)-\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{|d|}+|d|\right)}{16a^{3}d^{2}f}+\frac{7\sqrt{d\tan\left(fx+e\right)}d\tan\left(fx+e\right)+9\sqrt{d\tan\left(fx+e\right)}d}{8\left(d\tan\left(fx+e\right)+d\right)^{2}a^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(1/2)/(a+a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*(d*sqrt(abs(d)) + abs(d)^(3/2))*arctan(1/2*sqrt(2)*(sqrt(2)*sq
rt(abs(d)) + 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(a^3*d^2*f) - 1/8*sqrt(2
)*(d*sqrt(abs(d)) + abs(d)^(3/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d))
 - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(a^3*d^2*f) + 11/8*arctan(sqrt(d*t
an(f*x + e))/sqrt(d))/(a^3*sqrt(d)*f) - 1/16*sqrt(2)*(d*sqrt(abs(d)) - abs(
d)^(3/2))*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) +
abs(d))/(a^3*d^2*f) + 1/16*sqrt(2)*(d*sqrt(abs(d)) - abs(d)^(3/2))*log(d*ta
n(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(a^3*d^2*f
) + 1/8*(7*sqrt(d*tan(f*x + e))*d*tan(f*x + e) + 9*sqrt(d*tan(f*x + e))*d)/
((d*tan(f*x + e) + d)^2*a^3*f)
```

Mupad [B]
time = 4.78, size = 172, normalized size = 1.04

$$\frac{\frac{9\,d\,\sqrt{d\tan\left(e+f\,x\right)}}{8}+\frac{7\,{\left(d\tan\left(e+f\,x\right)\right)}^{3/2}}{8}}{f\,a^3\,d^2\,{\tan\left(e+f\,x\right)}^2+2\,f\,a^3\,d^2\tan\left(e+f\,x\right)+f\,a^3\,d^2}+\frac{11\,\mathrm{atan}\left(\frac{\sqrt{d\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)}{8\,a^3\,\sqrt{d}\,f}-\frac{\sqrt{2}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{d\tan\left(e+f\,x\right)}}{2\,\sqrt{d}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{d\tan\left(e+f\,x\right)}}{2\,\sqrt{d}}+\frac{\sqrt{2}\,{\left(d\tan\left(e+f\,x\right)\right)}^{3/2}}{2\,d^{3/2}}\right)\right)}{8\,a^3\,\sqrt{d}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*tan(e + f*x))^(1/2)*(a + a*tan(e + f*x))^3),x)
```

```
[Out] ((9*d*(d*tan(e + f*x))^(1/2))/8 + (7*(d*tan(e + f*x))^(3/2))/8)/(a^3*d^2*f
+ a^3*d^2*f*tan(e + f*x)^2 + 2*a^3*d^2*f*tan(e + f*x)) + (11*atan((d*tan(e
+ f*x))^(1/2)/d^(1/2)))/(8*a^3*d^(1/2)*f) - (2^(1/2)*(2*atan((2^(1/2)*(d*ta
n(e + f*x))^(1/2))/(2*d^(1/2))) + 2*atan((2^(1/2)*(d*tan(e + f*x))^(1/2))/(
2*d^(1/2)) + (2^(1/2)*(d*tan(e + f*x))^(3/2))/(2*d^(3/2)))))/(8*a^3*d^(1/2)
*f)
```

3.376 $\int \frac{1}{(d\tan(e+fx))^{3/2}(a+a\tan(e+fx))^3}\,dx$

Optimal. Leaf size=189

$$-\frac{31\text{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8a^3d^{3/2}f}-\frac{\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{2\sqrt{2}\,a^3d^{3/2}f}-\frac{27}{8a^3df\sqrt{d\tan(e+fx)}}+\frac{}{8a^3df\sqrt{d\tan}}$$

[Out] -31/8*arctan((d*tan(f*x+e))^(1/2)/d^(1/2))/a^3/d^(3/2)/f-1/4*arctanh(1/2*(d^(1/2)+d^(1/2)*tan(f*x+e))*2^(1/2)/(d*tan(f*x+e))^(1/2))/a^3/d^(3/2)/f*2^(1/2)-27/8/a^3/d/f/(d*tan(f*x+e))^(1/2)+9/8/a^3/d/f/(d*tan(f*x+e))^(1/2)/(1+tan(f*x+e))+1/4/a/d/f/(d*tan(f*x+e))^(1/2)/(a+a*tan(f*x+e))^2

Rubi [A]
time = 0.50, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3650, 3730, 3735, 3613, 214, 3715, 65, 211}

$$-\frac{31\text{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8a^3d^{3/2}f}-\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,\tan(e+fx)+\sqrt{d}}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{2\sqrt{2}\,a^3d^{3/2}f}-\frac{27}{8a^3df\sqrt{d\tan(e+fx)}}+\frac{9}{8a^3df(\tan(e+fx)+1)\sqrt{d\tan(e+fx)}}+\frac{1}{4adf(a\tan(e+fx)+a)^2\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[1/((d*Tan[e + f*x])^(3/2)*(a + a*Tan[e + f*x])^3),x]

[Out] (-31*ArcTan[Sqrt[d*Tan[e + f*x]]/Sqrt[d]])/(8*a^3*d^(3/2)*f) - ArcTanh[(Sqrt[d] + Sqrt[d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[d*Tan[e + f*x]])]/(2*Sqrt[2]*a^3*d^(3/2)*f) - 27/(8*a^3*d*f*Sqrt[d*Tan[e + f*x]]) + 9/(8*a^3*d*f*Sqrt[d*Tan[e + f*x]]*(1 + Tan[e + f*x])) + 1/(4*a*d*f*Sqrt[d*Tan[e + f*x]]*(a + a*Tan[e + f*x])^2)

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3735

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dis
t[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*Simp[a*(A - C) - (A*b - b*C)*Ta
n[e + f*x], x], x], x] + Dist[(A*b^2 + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e
```

```
 + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a,
 b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0] &&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{(d\tan(e+fx))^{3/2}(a+a\tan(e+fx))^3}\,dx = \frac{1}{4adf\sqrt{d\tan(e+fx)}\,(a+a\tan(e+fx))^2} + \frac{\int \frac{\frac{9a^2d}{2}-2a^2d\tan}{(d\tan(e+fx}}{}$$

$$= \frac{9}{8a^3df\sqrt{d\tan(e+fx)}\,(1+\tan(e+fx))} + \frac{}{4adf\sqrt{d\tan(e}}$$

$$= -\frac{27}{8a^3df\sqrt{d\tan(e+fx)}} + \frac{9}{8a^3df\sqrt{d\tan(e+fx)}\,(1+\tan}$$

$$= -\frac{27}{8a^3df\sqrt{d\tan(e+fx)}} + \frac{9}{8a^3df\sqrt{d\tan(e+fx)}\,(1+\tan}$$

$$= -\frac{27}{8a^3df\sqrt{d\tan(e+fx)}} + \frac{9}{8a^3df\sqrt{d\tan(e+fx)}\,(1+\tan}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{2\sqrt{2}\,a^3d^{3/2}f} - \frac{27}{8a^3df\sqrt{d\tan(e+fx)}}$$

$$= -\frac{31\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8a^3d^{3/2}f} - \frac{\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{d}\tan(}{\sqrt{2}\sqrt{d\tan(e}}\right)}{2\sqrt{2}\,a^3d^{3/2}f}$$

Mathematica [A]
time = 1.48, size = 173, normalized size = 0.92

$$\frac{\left(-62\text{ArcTan}\left(\sqrt{\tan(e+fx)}\right)+2\sqrt{2}\left(\log\left(-1+\sqrt{2}\sqrt{\tan(e+fx)}-\tan(e+fx)\right)-\log\left(1+\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)\right)-\frac{(90+90\cos(2(e+fx))+75\cot(e+fx)-11\cos(3(e+fx))\csc(e+fx))\sqrt{\tan(e+fx)}}{2(\cos(e+fx)+\sin(e+fx))^2}\right)\tan^{\frac{3}{2}}(e+fx)}{16a^3f(d\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Tan[e + f*x])^(3/2)*(a + a*Tan[e + f*x])^3),x]
```

```
[Out] ((-62*ArcTan[Sqrt[Tan[e + f*x]]] + 2*Sqrt[2]*(Log[-1 + Sqrt[2]*Sqrt[Tan[e +
 f*x]] - Tan[e + f*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]
) - ((90 + 90*Cos[2*(e + f*x)] + 75*Cot[e + f*x] - 11*Cos[3*(e + f*x)]*Csc[
```

```
e + f*x])*Sqrt[Tan[e + f*x]])/(2*(Cos[e + f*x] + Sin[e + f*x])^2))*Tan[e +
f*x]^(3/2))/(16*a^3*f*(d*Tan[e + f*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(156) = 312$.
time = 0.19, size = 364, normalized size = 1.93

method	result
derivativedivides	$\frac{2d^4\left(-\frac{1}{d^5\sqrt{d\tan(fx+e)}}-\frac{\frac{\frac{11(d\tan(fx+e))^{\frac{3}{2}}}{4}+\frac{13d\sqrt{d\tan(fx+e)}}{4}}{(d\tan(fx+e)+d)^2}+\frac{31\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{4\sqrt{d}}}{4d^5}\right.}{}$
default	$\frac{2d^4\left(-\frac{1}{d^5\sqrt{d\tan(fx+e)}}-\frac{\frac{\frac{11(d\tan(fx+e))^{\frac{3}{2}}}{4}+\frac{13d\sqrt{d\tan(fx+e)}}{4}}{(d\tan(fx+e)+d)^2}+\frac{31\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{4\sqrt{d}}}{4d^5}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*tan(f*x+e))^(3/2)/(a+a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3*d^4*(-1/d^5/(d*tan(f*x+e))^(1/2)-1/4/d^5*((11/4*(d*tan(f*x+e))^(3/2
)+13/4*d*(d*tan(f*x+e))^(1/2))/(d*tan(f*x+e)+d)^2+31/4/d^(1/2)*arctan((d*ta
n(f*x+e))^(1/2)/d^(1/2)))+1/4/d^5*(-1/8/d*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*
x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)-(d
^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)
^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))
^(1/2)+1))+1/8/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x
+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1
/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)
+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))))
```

Maxima [A]
time = 0.53, size = 200, normalized size = 1.06

$$-\frac{\frac{27\,d^2\tan(fx+e)^2+45\,d^2\tan(fx+e)+16\,d^2}{(d\tan(fx+e))^{\frac{5}{2}}a^3+2\,(d\tan(fx+e))^{\frac{3}{2}}a^3d+\sqrt{d\tan(fx+e)}\,a^3d^2}+\frac{\frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}}{a^3}+\frac{31\arctan\left(\frac{\sqrt{d\tan(fx+e)}}{\sqrt{d}}\right)}{a^3\sqrt{d}}}{8\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(3/2)/(a+a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/8*((27*d^2*tan(f*x + e)^2 + 45*d^2*tan(f*x + e) + 16*d^2)/((d*tan(f*x +
e))^(5/2)*a^3 + 2*(d*tan(f*x + e))^(3/2)*a^3*d + sqrt(d*tan(f*x + e))*a^3*d
^2) + (sqrt(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) +
d)/sqrt(d) - sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt
(d) + d)/sqrt(d))/a^3 + 31*arctan(sqrt(d*tan(f*x + e))/sqrt(d))/(a^3*sqrt(d
)))/(d*f)
```

Fricas [A]
time = 0.96, size = 493, normalized size = 2.61

$$\left[\frac{4\sqrt{2}\left(\tan\left(fx+e\right)^{3}+2\tan\left(fx+e\right)^{2}+\tan\left(fx+e\right)\right)\sqrt{-d}\arctan\left(\frac{\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{-d}\left(\tan\left(fx+e\right)+1\right)}{2\,d\tan\left(fx+e\right)}\right)-31\left(\tan\left(fx+e\right)^{3}+2\tan\left(fx+e\right)^{2}+\tan\left(fx+e\right)\right)\sqrt{-d}\log\left(\frac{d\tan\left(fx+e\right)+2\sqrt{d\tan\left(fx+e\right)}\sqrt{-d}-d}{\tan\left(fx+e\right)+1}\right)-2\sqrt{d\tan\left(fx+e\right)}\left(27\tan\left(fx+e\right)^{2}+45\tan\left(fx+e\right)+16\right)}{16\left(a^{3}d^{2}f\tan\left(fx+e\right)^{3}+2\,a^{3}d^{2}f\tan\left(fx+e\right)^{2}+a^{3}d^{2}f\tan\left(fx+e\right)\right)},\ \frac{\sqrt{2}\left(\tan\left(fx+e\right)^{3}+2\tan\left(fx+e\right)^{2}+\tan\left(fx+e\right)\right)\sqrt{d}\log\left(\frac{d\tan\left(fx+e\right)^{2}-2\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{d}\left(\tan\left(fx+e\right)+1\right)+4\,d\tan\left(fx+e\right)+d}{\tan\left(fx+e\right)^{2}+1}\right)-31\left(\tan\left(fx+e\right)^{3}+2\tan\left(fx+e\right)^{2}+\tan\left(fx+e\right)\right)\sqrt{d}\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}}{\sqrt{d}}\right)-\sqrt{d\tan\left(fx+e\right)}\left(27\tan\left(fx+e\right)^{2}+45\tan\left(fx+e\right)+16\right)}{8\left(a^{3}d^{2}f\tan\left(fx+e\right)^{3}+2\,a^{3}d^{2}f\tan\left(fx+e\right)^{2}+a^{3}d^{2}f\tan\left(fx+e\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(3/2)/(a+a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/16*(4*sqrt(2)*(tan(f*x + e)^3 + 2*tan(f*x + e)^2 + tan(f*x + e))*sqrt(-d
)*arctan(1/2*sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(-d)*(tan(f*x + e) + 1)/(d*ta
n(f*x + e))) - 31*(tan(f*x + e)^3 + 2*tan(f*x + e)^2 + tan(f*x + e))*sqrt(-
d)*log((d*tan(f*x + e) + 2*sqrt(d*tan(f*x + e))*sqrt(-d) - d)/(tan(f*x + e)
 + 1)) - 2*sqrt(d*tan(f*x + e))*(27*tan(f*x + e)^2 + 45*tan(f*x + e) + 16))
/(a^3*d^2*f*tan(f*x + e)^3 + 2*a^3*d^2*f*tan(f*x + e)^2 + a^3*d^2*f*tan(f*x
 + e)), 1/8*(sqrt(2)*(tan(f*x + e)^3 + 2*tan(f*x + e)^2 + tan(f*x + e))*sqr
t(d)*log((d*tan(f*x + e)^2 - 2*sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d)*(tan(f*
x + e) + 1) + 4*d*tan(f*x + e) + d)/(tan(f*x + e)^2 + 1)) - 31*(tan(f*x + e
)^3 + 2*tan(f*x + e)^2 + tan(f*x + e))*sqrt(d)*arctan(sqrt(d*tan(f*x + e))/
sqrt(d)) - sqrt(d*tan(f*x + e))*(27*tan(f*x + e)^2 + 45*tan(f*x + e) + 16))
/(a^3*d^2*f*tan(f*x + e)^3 + 2*a^3*d^2*f*tan(f*x + e)^2 + a^3*d^2*f*tan(f*x
 + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\left(d\tan\left(e+fx\right)\right)^{\frac{3}{2}}\tan^{3}\left(e+fx\right)+3\left(d\tan\left(e+fx\right)\right)^{\frac{3}{2}}\tan^{2}\left(e+fx\right)+3\left(d\tan\left(e+fx\right)\right)^{\frac{3}{2}}\tan\left(e+fx\right)+\left(d\tan\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))**(3/2)/(a+a*tan(f*x+e))**3,x)
```

```
[Out] Integral(1/((d*tan(e + f*x))**(3/2)*tan(e + f*x)**3 + 3*(d*tan(e + f*x))**(
3/2)*tan(e + f*x)**2 + 3*(d*tan(e + f*x))**(3/2)*tan(e + f*x) + (d*tan(e +
f*x))**(3/2)), x)/a**3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(164) = 328.
time = 0.86, size = 341, normalized size = 1.80

$$-\frac{\frac{2\sqrt{2}\left(d\sqrt{|d|}-|d|^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{a^{3}d^{2}f}+\frac{2\sqrt{2}\left(d\sqrt{|d|}-|d|^{\frac{3}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{a^{3}d^{2}f}+\frac{62\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}}{\sqrt{d}}\right)}{a^{3}\sqrt{d}f}+\frac{\sqrt{2}\left(d\sqrt{|d|}+|d|^{\frac{3}{2}}\right)\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{|d|}+|d|\right)}{a^{3}d^{2}f}-\frac{\sqrt{2}\left(d\sqrt{|d|}+|d|^{\frac{3}{2}}\right)\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{|d|}+|d|\right)}{a^{3}d^{2}f}+\frac{32}{\sqrt{d\tan\left(fx+e\right)}a^{3}f}+\frac{2\left(11\sqrt{d\tan\left(fx+e\right)}d\tan(fx+e)+13\sqrt{d\tan\left(fx+e\right)}d\right)}{(d\tan(fx+e)+d)^{2}a^{3}f}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(3/2)/(a+a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/16*(2*sqrt(2)*(d*sqrt(abs(d)) - abs(d)^(3/2))*arctan(1/2*sqrt(2)*(sqrt(2
)*sqrt(abs(d)) + 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(a^3*d^2*f) + 2*sqrt
(2)*(d*sqrt(abs(d)) - abs(d)^(3/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d
)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(a^3*d^2*f) + 62*arctan(sqrt(d*t
an(f*x + e))/sqrt(d))/(a^3*sqrt(d)*f) + sqrt(2)*(d*sqrt(abs(d)) + abs(d)^(3
/2))*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d
))/(a^3*d^2*f) - sqrt(2)*(d*sqrt(abs(d)) + abs(d)^(3/2))*log(d*tan(f*x + e)
 - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(a^3*d^2*f) + 32/(sq
rt(d*tan(f*x + e))*a^3*f) + 2*(11*sqrt(d*tan(f*x + e))*d*tan(f*x + e) + 13*
sqrt(d*tan(f*x + e))*d)/((d*tan(f*x + e) + d)^2*a^3*f))/d
```

Mupad [B]
time = 4.82, size = 176, normalized size = 0.93

$$-\frac{\frac{27\,d\,\tan(e+f\,x)^2}{8}+\frac{45\,d\,\tan(e+f\,x)}{8}+2\,d}{a^3\,f\,{\left(d\,\tan\left(e+f\,x\right)\right)}^{5/2}+2\,a^3\,d\,f\,{\left(d\,\tan\left(e+f\,x\right)\right)}^{3/2}+a^3\,d^2\,f\,\sqrt{d\,\tan\left(e+f\,x\right)}}-\frac{31\,\mathrm{atan}\left(\frac{\sqrt{d\,\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)}{8\,a^3\,d^{3/2}\,f}-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{63504384\,\sqrt{2}\,a^9\,d^{15/2}\,f^3\,\sqrt{d\,\tan\left(e+f\,x\right)}}{63504384\,a^9\,d^8\,f^3+63504384\,a^9\,d^8\,f^3\,\tan(e+f\,x)}\right)}{4\,a^3\,d^{3/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*tan(e + f*x))^(3/2)*(a + a*tan(e + f*x))^3),x)
```

```
[Out] - (2*d + (45*d*tan(e + f*x))/8 + (27*d*tan(e + f*x)^2)/8)/(a^3*f*(d*tan(e +
 f*x))^(5/2) + 2*a^3*d*f*(d*tan(e + f*x))^(3/2) + a^3*d^2*f*(d*tan(e + f*x)
)^(1/2)) - (31*atan((d*tan(e + f*x))^(1/2)/d^(1/2)))/(8*a^3*d^(3/2)*f) - (2
^(1/2)*atanh((63504384*2^(1/2)*a^9*d^(15/2)*f^3*(d*tan(e + f*x))^(1/2))/(63
504384*a^9*d^8*f^3 + 63504384*a^9*d^8*f^3*tan(e + f*x))))/(4*a^3*d^(3/2)*f)
```

3.377 $\int \frac{1}{(d\tan(e+fx))^{5/2}(a+a\tan(e+fx))^3}\,dx$

Optimal. Leaf size=215

$$\frac{59\text{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8a^3d^{5/2}f}-\frac{\text{ArcTan}\left(\frac{\sqrt{d}-\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{2\sqrt{2}\,a^3d^{5/2}f}-\frac{55}{24a^3df(d\tan(e+fx))^{3/2}}+\frac{6\cdots}{8a^3d^2f\sqrt{d\cdots}}$$

```
[Out] 59/8*arctan((d*tan(f*x+e))^(1/2)/d^(1/2))/a^3/d^(5/2)/f-1/4*arctan(1/2*(d^(
1/2)-d^(1/2)*tan(f*x+e))*2^(1/2)/(d*tan(f*x+e))^(1/2))/a^3/d^(5/2)/f*2^(1/2
)+63/8/a^3/d^2/f/(d*tan(f*x+e))^(1/2)-55/24/a^3/d/f/(d*tan(f*x+e))^(3/2)+11
/8/a^3/d/f/(d*tan(f*x+e))^(3/2)/(1+tan(f*x+e))+1/4/a/d/f/(d*tan(f*x+e))^(3/
2)/(a+a*tan(f*x+e))^2
```

Rubi [A]
time = 0.65, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3650, 3730, 3731, 3734, 3613, 211, 3715, 65}

$$\frac{59\text{ArcTan}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8a^3d^{5/2}f}-\frac{\text{ArcTan}\left(\frac{\sqrt{d}-\sqrt{d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{d\tan(e+fx)}}\right)}{2\sqrt{2}\,a^3d^{5/2}f}+\frac{63}{8a^3d^2f\sqrt{d\tan(e+fx)}}+\frac{11}{8a^3df(\tan(e+fx)+1)(d\tan(e+fx))^{3/2}}-\frac{55}{24a^3df(d\tan(e+fx))^{3/2}}+\frac{1}{4adf(a\tan(e+fx)+a)^2(d\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Tan[e + f*x])^(5/2)*(a + a*Tan[e + f*x])^3),x]
```

```
[Out] (59*ArcTan[Sqrt[d*Tan[e + f*x]]/Sqrt[d]])/(8*a^3*d^(5/2)*f) - ArcTan[(Sqrt[
d] - Sqrt[d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[d*Tan[e + f*x]])]/(2*Sqrt[2]*a^3*d
^(5/2)*f) - 55/(24*a^3*d*f*(d*Tan[e + f*x])^(3/2)) + 63/(8*a^3*d^2*f*Sqrt[d
*Tan[e + f*x]]) + 11/(8*a^3*d*f*(d*Tan[e + f*x])^(3/2)*(1 + Tan[e + f*x]))
+ 1/(4*a*d*f*(d*Tan[e + f*x])^(3/2)*(a + a*Tan[e + f*x])^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3731

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)
*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp
[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - a*C*(b*c*(m + 1) + a*d*(n
```

```
+ 1)) - (m + 1)*(b*c - a*d)*(A*b - b*C)*Tan[e + f*x] - d*(A*b^2 + a^2*C)*(m
 + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m,
 -1] &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{(d\tan(e+fx))^{5/2}(a+a\tan(e+fx))^3}\,dx = \frac{1}{4adf(d\tan(e+fx))^{3/2}(a+a\tan(e+fx))^2} + \frac{\int \frac{\frac{11a^2d}{2}-2a^2d\tan}{(d\tan(e+fx}}{}$$

$$= \frac{11}{8a^3df(d\tan(e+fx))^{3/2}(1+\tan(e+fx))} + \frac{}{4adf(d\tan(e+}$$

$$= -\frac{55}{24a^3df(d\tan(e+fx))^{3/2}} + \frac{11}{8a^3df(d\tan(e+fx))^{3/2}(1+\text{ta}}$$

$$= -\frac{55}{24a^3df(d\tan(e+fx))^{3/2}} + \frac{63}{8a^3d^2f\sqrt{d\tan(e+fx)}} + \frac{}{8a^3}$$

$$= -\frac{55}{24a^3df(d\tan(e+fx))^{3/2}} + \frac{63}{8a^3d^2f\sqrt{d\tan(e+fx)}} + \frac{}{8a^3}$$

$$= -\frac{55}{24a^3df(d\tan(e+fx))^{3/2}} + \frac{63}{8a^3d^2f\sqrt{d\tan(e+fx)}} + \frac{}{8a^3}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{d}-\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx)}}\right)}{2\sqrt{2}a^3d^{5/2}f} - \frac{55}{24a^3df(d\tan(e+fx))^{3/}}$$

$$= \frac{59\tan^{-1}\left(\frac{\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{8a^3d^{5/2}f} - \frac{\tan^{-1}\left(\frac{\sqrt{d}-\sqrt{d}\tan(e+fx)}{\sqrt{2}\sqrt{d\tan(e+fx}}\right.}{2\sqrt{2}a^3d^{5/2}f}$$

Mathematica [A]
time = 6.30, size = 368, normalized size = 1.71

$$\frac{\sec^3(e+fx)(\cos(e+fx)+\sin(e+fx))^3\left(\frac{8}{3}+6\cot(e+fx)-\frac{2}{3}\csc^2(e+fx)+\frac{1}{8(\cos(e+fx)+\sin(e+fx))^2}-\frac{17\sin(e+fx)}{8(\cos(e+fx)+\sin(e+fx))}\right)\tan^3(e+fx)}{f(d\tan(e+fx))^{5/2}(a+a\tan(e+fx))^3}+\frac{\sec^3(e+fx)(\cos(e+fx)+\sin(e+fx))^3\tan^{\frac{5}{2}}(e+fx)\left(\frac{126\text{ArcTan}\left(\sqrt{\tan(e+fx)}\right)\csc(e+fx)\sec^3(e+fx)(1+\tan(e+fx))}{(1+\cot(e+fx))(1+\tan^2(e+fx))^2}+\frac{2\left(\sqrt{2}\left(-\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)+\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(e+fx)}\right)\right)-2\text{ArcTan}\left(\sqrt{\tan(e+fx)}\right)\right)\csc^2(e+fx)\sec^2(e+fx)\sin(2(e+fx))(1+\tan(e+fx))}{(1+\cot(e+fx))(1+\tan^2(e+fx))}\right)}{16f(d\tan(e+fx))^{5/2}(a+a\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Tan[e + f*x])^(5/2)*(a + a*Tan[e + f*x])^3),x]
```

```
[Out] (Sec[e + f*x]^3*(Cos[e + f*x] + Sin[e + f*x])^3*(8/3 + 6*Cot[e + f*x] - (2*
Csc[e + f*x]^2)/3 + 1/(8*(Cos[e + f*x] + Sin[e + f*x])^2) - (17*Sin[e + f*x
])/(8*(Cos[e + f*x] + Sin[e + f*x])))*Tan[e + f*x]^3)/(f*(d*Tan[e + f*x])^(
5/2)*(a + a*Tan[e + f*x])^3) + (Sec[e + f*x]^3*(Cos[e + f*x] + Sin[e + f*x]
)^3*Tan[e + f*x]^(5/2)*((126*ArcTan[Sqrt[Tan[e + f*x]]]*Csc[e + f*x]*Sec[e
+ f*x]^3*(1 + Tan[e + f*x]))/((1 + Cot[e + f*x])*(1 + Tan[e + f*x]^2)^2) +
```

```
(2*(Sqrt[2]*(-ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] + ArcTan[1 + Sqrt[2]*S
qrt[Tan[e + f*x]]]) - 2*ArcTan[Sqrt[Tan[e + f*x]]])*Csc[e + f*x]^2*Sec[e +
f*x]^2*Sin[2*(e + f*x)]*(1 + Tan[e + f*x]))/((1 + Cot[e + f*x])*(1 + Tan[e
+ f*x]^2))))/(16*f*(d*Tan[e + f*x])^(5/2)*(a + a*Tan[e + f*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(178) = 356$.

time = 0.20, size = 379, normalized size = 1.76

method	result
derivativedivides	$2d^4\left(-\frac{1}{3d^5(d\tan(fx+e))^{\frac{3}{2}}}+\frac{3}{d^6\sqrt{d\tan(fx+e)}}+\frac{\frac{\frac{15(d\tan(fx+e))^{\frac{3}{2}}}{4}+\frac{17d\sqrt{d\tan(fx+e)}}{4}}{(d\tan(fx+e)+d)^2}+\frac{59\arctan\left(\frac{\sqrt{\cdot}}{\cdot}\right.}{\cdot}}{4d^6}\right.$
default	$2d^4\left(-\frac{1}{3d^5(d\tan(fx+e))^{\frac{3}{2}}}+\frac{3}{d^6\sqrt{d\tan(fx+e)}}+\frac{\frac{\frac{15(d\tan(fx+e))^{\frac{3}{2}}}{4}+\frac{17d\sqrt{d\tan(fx+e)}}{4}}{(d\tan(fx+e)+d)^2}+\frac{59\arctan\left(\frac{\sqrt{\cdot}}{\cdot}\right.}{\cdot}}{4d^6}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*tan(f*x+e))^(5/2)/(a+a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3*d^4*(-1/3/d^5/(d*tan(f*x+e))^(3/2)+3/d^6/(d*tan(f*x+e))^(1/2)+1/4/d
^6*((15/4*(d*tan(f*x+e))^(3/2)+17/4*d*(d*tan(f*x+e))^(1/2))/(d*tan(f*x+e)+d
)^2+59/4/d^(1/2)*arctan((d*tan(f*x+e))^(1/2)/d^(1/2)))+1/4/d^6*(1/8/d*(d^2)
^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(
d^2)^(1/2))/(d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1
/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2
)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))+1/8/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f
*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(
d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2
)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e)
)^(1/2)+1))))
```

Maxima [A]

time = 0.51, size = 219, normalized size = 1.02

$$\frac{\frac{189\,d^3\tan(fx+e)^3+323\,d^3\tan(fx+e)^2+112\,d^3\tan(fx+e)-16\,d^3}{(d\tan(fx+e))^{\frac{7}{2}}a^3d+2\,(d\tan(fx+e))^{\frac{5}{2}}a^3d^2+(d\tan(fx+e))^{\frac{3}{2}}a^3d^3}+\frac{6\left(\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left(\sqrt{2}\,\sqrt{d}+2\,\sqrt{d\tan\left(fx+e\right)}\right)}{2\,\sqrt{d}}\right)}{\sqrt{d}}+\frac{\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\,\left(\sqrt{2}\,\sqrt{d}-2\,\sqrt{d\tan\left(fx+e\right)}\right)}{2\,\sqrt{d}}\right)}{\sqrt{d}}\right)}{a^3d}+\frac{177\,\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}}{\sqrt{d}}\right)}{a^3d^{\frac{3}{2}}}}{24\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(5/2)/(a+a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/24*((189*d^3*tan(f*x + e)^3 + 323*d^3*tan(f*x + e)^2 + 112*d^3*tan(f*x +
e) - 16*d^3)/((d*tan(f*x + e))^(7/2)*a^3*d + 2*(d*tan(f*x + e))^(5/2)*a^3*d
^2 + (d*tan(f*x + e))^(3/2)*a^3*d^3) + 6*(sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(
2)*sqrt(d) + 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) + sqrt(2)*arctan(-1/2
*sqrt(2)*(sqrt(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d))/(a^3*
d) + 177*arctan(sqrt(d*tan(f*x + e))/sqrt(d))/(a^3*d^(3/2)))/(d*f)
```

Fricas [A]
time = 0.85, size = 524, normalized size = 2.44

$$\left[-\frac{6\,\sqrt{2}\,\left(\tan\left(fx+e\right)^4+2\,\tan\left(fx+e\right)^3+\tan\left(fx+e\right)^2\right)\sqrt{-d}\,\log\left(\frac{d\tan(fx+e)^2-2\,\sqrt{2}\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{-d}\,(\tan(fx+e)-1)-4\,d\tan(fx+e)+d}{\tan(fx+e)^2+1}\right)+177\,\left(\tan\left(fx+e\right)^4+2\,\tan\left(fx+e\right)^3+\tan\left(fx+e\right)^2\right)\sqrt{-d}\,\log\left(\frac{d\tan(fx+e)-2\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{-d}-d}{\tan(fx+e)+1}\right)-2\,\left(189\,\tan\left(fx+e\right)^3+323\,\tan\left(fx+e\right)^2+112\,\tan\left(fx+e\right)-16\right)\sqrt{d\tan\left(fx+e\right)}}{48\,\left(a^3d^3f\tan\left(fx+e\right)^4+2\,a^3d^3f\tan\left(fx+e\right)^3+a^3d^3f\tan\left(fx+e\right)^2\right)},\ \frac{6\,\sqrt{2}\,\left(\tan\left(fx+e\right)^4+2\,\tan\left(fx+e\right)^3+\tan\left(fx+e\right)^2\right)\sqrt{d}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{d\tan\left(fx+e\right)}\,(\tan(fx+e)-1)}{2\,\sqrt{d}\,\tan(fx+e)}\right)+177\,\left(\tan\left(fx+e\right)^4+2\,\tan\left(fx+e\right)^3+\tan\left(fx+e\right)^2\right)\sqrt{d}\,\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}}{\sqrt{d}}\right)+\left(189\,\tan\left(fx+e\right)^3+323\,\tan\left(fx+e\right)^2+112\,\tan\left(fx+e\right)-16\right)\sqrt{d\tan\left(fx+e\right)}}{24\,\left(a^3d^3f\tan\left(fx+e\right)^4+2\,a^3d^3f\tan\left(fx+e\right)^3+a^3d^3f\tan\left(fx+e\right)^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(f*x+e))^(5/2)/(a+a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [-1/48*(6*sqrt(2)*(tan(f*x + e)^4 + 2*tan(f*x + e)^3 + tan(f*x + e)^2)*sqrt
(-d)*log((d*tan(f*x + e)^2 - 2*sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(-d)*(tan(f
*x + e) - 1) - 4*d*tan(f*x + e) + d)/(tan(f*x + e)^2 + 1)) + 177*(tan(f*x +
 e)^4 + 2*tan(f*x + e)^3 + tan(f*x + e)^2)*sqrt(-d)*log((d*tan(f*x + e) - 2
*sqrt(d*tan(f*x + e))*sqrt(-d) - d)/(tan(f*x + e) + 1)) - 2*(189*tan(f*x +
e)^3 + 323*tan(f*x + e)^2 + 112*tan(f*x + e) - 16)*sqrt(d*tan(f*x + e)))/(a
^3*d^3*f*tan(f*x + e)^4 + 2*a^3*d^3*f*tan(f*x + e)^3 + a^3*d^3*f*tan(f*x +
e)^2), 1/24*(6*sqrt(2)*(tan(f*x + e)^4 + 2*tan(f*x + e)^3 + tan(f*x + e)^2)
*sqrt(d)*arctan(1/2*sqrt(2)*sqrt(d*tan(f*x + e))*(tan(f*x + e) - 1)/(sqrt(d
)*tan(f*x + e))) + 177*(tan(f*x + e)^4 + 2*tan(f*x + e)^3 + tan(f*x + e)^2)
*sqrt(d)*arctan(sqrt(d*tan(f*x + e))/sqrt(d)) + (189*tan(f*x + e)^3 + 323*t
an(f*x + e)^2 + 112*tan(f*x + e) - 16)*sqrt(d*tan(f*x + e)))/(a^3*d^3*f*tan
(f*x + e)^4 + 2*a^3*d^3*f*tan(f*x + e)^3 + a^3*d^3*f*tan(f*x + e)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{1}{(d\tan(e+fx))^{\frac{5}{2}}\tan^3(e+fx)+3(d\tan(e+fx))^{\frac{5}{2}}\tan^2(e+fx)+3(d\tan(e+fx))^{\frac{5}{2}}\tan(e+fx)+(d\tan(e+fx))^{\frac{5}{2}}}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*tan(f*x+e))**(5/2)/(a+a*tan(f*x+e))**3,x)

[Out] Integral(1/((d*tan(e + f*x))**(5/2)*tan(e + f*x)**3 + 3*(d*tan(e + f*x))**(5/2)*tan(e + f*x)**2 + 3*(d*tan(e + f*x))**(5/2)*tan(e + f*x) + (d*tan(e + f*x))**(5/2)), x)/a**3

Giac [A]
time = 1.00, size = 366, normalized size = 1.70

$$\frac{\sqrt{2}\left(d\sqrt{|d|}+|d|^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{8\,a^{3}d^{4}f}+\frac{\sqrt{2}\left(d\sqrt{|d|}+|d|^{\frac{3}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{8\,a^{3}d^{4}f}+\frac{59\arctan\left(\frac{\sqrt{d\tan\left(fx+e\right)}}{\sqrt{d}}\right)}{8\,a^{3}d^{\frac{5}{2}}f}+\frac{\sqrt{2}\left(d\sqrt{|d|}-|d|^{\frac{3}{2}}\right)\log\left(d\tan\left(fx+e\right)+\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{|d|}+|d|\right)}{16\,a^{3}d^{4}f}-\frac{\sqrt{2}\left(d\sqrt{|d|}-|d|^{\frac{3}{2}}\right)\log\left(d\tan\left(fx+e\right)-\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{|d|}+|d|\right)}{16\,a^{3}d^{4}f}+\frac{15\sqrt{d\tan\left(fx+e\right)}\,d\tan\left(fx+e\right)+17\sqrt{d\tan\left(fx+e\right)}\,d}{8\left(d\tan\left(fx+e\right)+d\right)^{2}a^{3}d^{2}f}+\frac{2\left(9\,d\tan\left(fx+e\right)-d\right)}{3\sqrt{d\tan\left(fx+e\right)}\,a^{3}d^{3}f\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*tan(f*x+e))^(5/2)/(a+a*tan(f*x+e))^3,x, algorithm="giac")

[Out] 1/8*sqrt(2)*(d*sqrt(abs(d)) + abs(d)^(3/2))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(a^3*d^4*f) + 1/8*sqrt(2)*(d*sqrt(abs(d)) + abs(d)^(3/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(a^3*d^4*f) + 59/8*arctan(sqrt(d*tan(f*x + e))/sqrt(d))/(a^3*d^(5/2)*f) + 1/16*sqrt(2)*(d*sqrt(abs(d)) - abs(d)^(3/2))*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(a^3*d^4*f) - 1/16*sqrt(2)*(d*sqrt(abs(d)) - abs(d)^(3/2))*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(a^3*d^4*f) + 1/8*(15*sqrt(d*tan(f*x + e))*d*tan(f*x + e) + 17*sqrt(d*tan(f*x + e))*d)/((d*tan(f*x + e) + d)^2*a^3*d^2*f) + 2/3*(9*d*tan(f*x + e) - d)/(sqrt(d*tan(f*x + e))*a^3*d^3*f*tan(f*x + e))

Mupad [B]
time = 5.02, size = 192, normalized size = 0.89

$$\frac{\frac{63\,d\,{\mathrm{tan}\left(e+f\,x\right)}^3}{8}+\frac{323\,d\,{\mathrm{tan}\left(e+f\,x\right)}^2}{24}+\frac{14\,d\,\mathrm{tan}\left(e+f\,x\right)}{3}-\frac{2\,d}{3}}{a^3\,f\,{\left(d\,\mathrm{tan}\left(e+f\,x\right)\right)}^{7/2}+2\,a^3\,d\,f\,{\left(d\,\mathrm{tan}\left(e+f\,x\right)\right)}^{5/2}+a^3\,d^2\,f\,{\left(d\,\mathrm{tan}\left(e+f\,x\right)\right)}^{3/2}}+\frac{59\,\mathrm{atan}\left(\frac{\sqrt{d\,\mathrm{tan}\left(e+f\,x\right)}}{\sqrt{d}}\right)}{8\,a^3\,d^{5/2}\,f}+\frac{\sqrt{2}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{d\,\mathrm{tan}\left(e+f\,x\right)}}{2\,\sqrt{d}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{d\,\mathrm{tan}\left(e+f\,x\right)}}{2\,\sqrt{d}}+\frac{\sqrt{2}\,{\left(d\,\mathrm{tan}\left(e+f\,x\right)\right)}^{3/2}}{2\,d^{3/2}}\right)\right)}{8\,a^3\,d^{5/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((d*tan(e + f*x))^(5/2)*(a + a*tan(e + f*x))^3),x)

[Out] ((14*d*tan(e + f*x))/3 - (2*d)/3 + (323*d*tan(e + f*x)^2)/24 + (63*d*tan(e + f*x)^3)/8)/(a^3*f*(d*tan(e + f*x))^(7/2) + 2*a^3*d*f*(d*tan(e + f*x))^(5/2) + a^3*d^2*f*(d*tan(e + f*x))^(3/2)) + (59*atan((d*tan(e + f*x))^(1/2)/d^(1/2)))/(8*a^3*d^(5/2)*f) + (2^(1/2)*(2*atan((2^(1/2)*(d*tan(e + f*x))^(1/2))/(2*d^(1/2))) + 2*atan((2^(1/2)*(d*tan(e + f*x))^(1/2))/(2*d^(1/2)) + (2^(1/2)*(d*tan(e + f*x))^(3/2))/(2*d^(3/2)))))/(8*a^3*d^(5/2)*f)

3.378 $\int \tan^5(e+fx)\sqrt{1+\tan(e+fx)}\,dx$

Optimal. Leaf size=264

$$-\frac{\sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{4-3\sqrt{2}+\left(2-\sqrt{2}\right)\tan(e+fx)}{2\sqrt{-7+5\sqrt{2}}\,\sqrt{1+\tan(e+fx)}}\right)}{f}-\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\tanh^{-1}\left(\frac{4+3\sqrt{2}+}{2\sqrt{7+5\sqrt{2}}}\right.}{f}$$

```
[Out] -1/2*arctan(1/2*(4-3*2^(1/2)+(2-2^(1/2))*tan(f*x+e))/(-7+5*2^(1/2))^(1/2)/(
1+tan(f*x+e))^(1/2))*(-2+2*2^(1/2))^(1/2)/f-1/2*arctanh(1/2*(4+3*2^(1/2)+(2
+2^(1/2))*tan(f*x+e))/(7+5*2^(1/2))^(1/2)/(1+tan(f*x+e))^(1/2))*(2+2*2^(1/2
))^(1/2)/f+2*(1+tan(f*x+e))^(1/2)/f+52/315*(1+tan(f*x+e))^(3/2)/f-26/105*ta
n(f*x+e)*(1+tan(f*x+e))^(3/2)/f-4/21*tan(f*x+e)^2*(1+tan(f*x+e))^(3/2)/f+2/
9*tan(f*x+e)^3*(1+tan(f*x+e))^(3/2)/f
```

Rubi [A]
time = 0.38, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3647, 3728, 3729, 3711, 12, 3609, 3617, 3616, 209, 213}

$$-\frac{\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\,\text{ArcTan}\left(\frac{\left(2-\sqrt{2}\right)\tan(e+fx)-3\sqrt{2}+4}{2\sqrt{5\sqrt{2}-7}\,\sqrt{\tan(e+fx)+1}}\right)}{f}+\frac{2(\tan(e+fx)+1)^{3/2}\tan^3(e+fx)}{9f}-\frac{4(\tan(e+fx)+1)^{3/2}\tan^2(e+fx)}{21f}-\frac{26(\tan(e+fx)+1)^{3/2}\tan(e+fx)}{105f}+\frac{52(\tan(e+fx)+1)^{3/2}}{315f}+\frac{2\sqrt{\tan(e+fx)+1}}{f}-\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\tanh^{-1}\left(\frac{\left(2+\sqrt{2}\right)\tan(e+fx)+3\sqrt{2}+4}{2\sqrt{7+5\sqrt{2}}\,\sqrt{\tan(e+fx)+1}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5*Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] -((Sqrt[(-1 + Sqrt[2])/2]*ArcTan[(4 - 3*Sqrt[2] + (2 - Sqrt[2])*Tan[e + f*x
])/(2*Sqrt[-7 + 5*Sqrt[2]]*Sqrt[1 + Tan[e + f*x]])]])/f) - (Sqrt[(1 + Sqrt[2
])/2]*ArcTanh[(4 + 3*Sqrt[2] + (2 + Sqrt[2])*Tan[e + f*x])/(2*Sqrt[7 + 5*Sq
rt[2]]*Sqrt[1 + Tan[e + f*x]])]])/f + (2*Sqrt[1 + Tan[e + f*x]])/f + (52*(1
+ Tan[e + f*x])^(3/2))/(315*f) - (26*Tan[e + f*x]*(1 + Tan[e + f*x])^(3/2))
/(105*f) - (4*Tan[e + f*x]^2*(1 + Tan[e + f*x])^(3/2))/(21*f) + (2*Tan[e +
f*x]^3*(1 + Tan[e + f*x])^(3/2))/(9*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3616

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*b*c*d - 4*a*d^2 +
 x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

Rule 3617

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2*q), Int[(a
*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
x], x] - Dist[1/(2*q), Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f*x
])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d - b
*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
```

```
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3729

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m + n +
1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*
Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*(b*c*m + a*d*(n + 1)) + d*(A*b -
 b*C)*(m + n + 1)*Tan[e + f*x] - C*m*(b*c - a*d)*Tan[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 +
 b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[
m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \tan^5(e+fx)\sqrt{1+\tan(e+fx)}\,dx = \frac{2\tan^3(e+fx)(1+\tan(e+fx))^{3/2}}{9f} + \frac{2}{9}\int \tan^2(e+fx)\sqrt{1+\tan$$

$$= -\frac{4\tan^2(e+fx)(1+\tan(e+fx))^{3/2}}{21f} + \frac{2\tan^3(e+fx)(1+\tan(e}{9f}$$

$$= -\frac{26\tan(e+fx)(1+\tan(e+fx))^{3/2}}{105f} - \frac{4\tan^2(e+fx)(1+\tan(e}{21f}$$

$$= \frac{52(1+\tan(e+fx))^{3/2}}{315f} - \frac{26\tan(e+fx)(1+\tan(e+fx))^{3/2}}{105f} -$$

$$= \frac{52(1+\tan(e+fx))^{3/2}}{315f} - \frac{26\tan(e+fx)(1+\tan(e+fx))^{3/2}}{105f} -$$

$$= \frac{2\sqrt{1+\tan(e+fx)}}{f} + \frac{52(1+\tan(e+fx))^{3/2}}{315f} - 26\tan(e+fx)($$

$$= \frac{2\sqrt{1+\tan(e+fx)}}{f} + \frac{52(1+\tan(e+fx))^{3/2}}{315f} - 26\tan(e+fx)($$

$$= \frac{2\sqrt{1+\tan(e+fx)}}{f} + \frac{52(1+\tan(e+fx))^{3/2}}{315f} - 26\tan(e+fx)($$

$$= -\frac{\sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\tan^{-1}\left(\frac{4-3\sqrt{2}+\left(2-\sqrt{2}\right)\tan(e+fx)}{2\sqrt{-7+5\sqrt{2}}\sqrt{1+\tan(e+fx)}}\right)}{f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.86, size = 150, normalized size = 0.57

$$\frac{\cos(e+fx)(1+\tan(e+fx))\left(-315\sqrt{1-i}\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1-i}}\right)-315\sqrt{1+i}\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1+i}}\right)+2\sqrt{1+\tan(e+fx)}\left(445+35\sec^4(e+fx)-18\tan(e+fx)+\sec^2(e+fx)(-139+5\tan(e+fx))\right)\right)}{315f(\cos(e+fx)+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5*Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*(1 + Tan[e + f*x])*(-315*Sqrt[1 - I]*ArcTanh[Sqrt[1 + Tan[e +
 f*x]]/Sqrt[1 - I]] - 315*Sqrt[1 + I]*ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1
 + I]] + 2*Sqrt[1 + Tan[e + f*x]]*(445 + 35*Sec[e + f*x]^4 - 18*Tan[e + f*x
] + Sec[e + f*x]^2*(-139 + 5*Tan[e + f*x]))))/(315*f*(Cos[e + f*x] + Sin[e
+ f*x]))
```

Maple [A]
time = 0.42, size = 242, normalized size = 0.92

method	result
derivativedivides	$\frac{\frac{2(1+\tan(fx+e))^{\frac{9}{2}}}{9}-\frac{6(1+\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{4(1+\tan(fx+e))^{\frac{5}{2}}}{5}+2\sqrt{1+\tan(fx+e)}+\frac{\sqrt{2\sqrt{2}+2}\,\ln\left(1+\sqrt{2}-\sqrt{\right.}}{}}{}$
default	$\frac{\frac{2(1+\tan(fx+e))^{\frac{9}{2}}}{9}-\frac{6(1+\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{4(1+\tan(fx+e))^{\frac{5}{2}}}{5}+2\sqrt{1+\tan(fx+e)}+\frac{\sqrt{2\sqrt{2}+2}\,\ln\left(1+\sqrt{2}-\sqrt{\right.}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+tan(f*x+e))^(1/2)*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2/9*(1+tan(f*x+e))^(9/2)-6/7*(1+tan(f*x+e))^(7/2)+4/5*(1+tan(f*x+e))^(
5/2)+2*(1+tan(f*x+e))^(1/2)+1/4*(2*2^(1/2)+2)^(1/2)*ln(1+2^(1/2)-(2*2^(1/2)
+2)^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))-(2^(1/2)-1)/(-2+2*2^(1/2))^(1/2)
*arctan((2*(1+tan(f*x+e))^(1/2)-(2*2^(1/2)+2)^(1/2))/(-2+2*2^(1/2))^(1/2))-
1/4*(2*2^(1/2)+2)^(1/2)*ln(1+2^(1/2)+(2*2^(1/2)+2)^(1/2)*(1+tan(f*x+e))^(1/
2)+tan(f*x+e))+(1-2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan(((2*2^(1/2)+2)^(1/2)
+2*(1+tan(f*x+e))^(1/2))/(-2+2*2^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(f*x+e))^(1/2)*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(tan(f*x + e) + 1)*tan(f*x + e)^5, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1099 vs. $2(220) = 440$.
time = 0.99, size = 1099, normalized size = 4.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(f*x+e))^(1/2)*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] 1/2520*(1260*2^(3/4)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f*(f^(-4))^(1/4)
*arctan(1/2*2^(3/4)*sqrt(1/2)*(f^5*sqrt(f^(-4)) + sqrt(2)*f^3)*sqrt(-2*sqrt
(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) + 2
```

```
^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4))*cos(f*x + e) + 2*f*cos(f*x + e))*sqrt(-2*s
qrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e
))*(f^(-4))^(1/4) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e))*(f^(-4))
^(3/4) - 1/2*2^(3/4)*(f^5*sqrt(f^(-4)) + sqrt(2)*f^3)*sqrt(-2*sqrt(2)*f^2*s
qrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^
(3/4) - f^2*sqrt(f^(-4)) - sqrt(2))*cos(f*x + e)^4 + 1260*2^(3/4)*sqrt(-2*s
qrt(2)*f^2*sqrt(f^(-4)) + 4)*f*(f^(-4))^(1/4)*arctan(1/2*2^(3/4)*sqrt(1/2)*
(f^5*sqrt(f^(-4)) + sqrt(2)*f^3)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt
((2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) - 2^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4
))*cos(f*x + e) + 2*f*cos(f*x + e))*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*s
qrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4) + 2*cos(f*x
+ e) + 2*sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) - 1/2*2^(3/4)*(f^5*sqrt
(f^(-4)) + sqrt(2)*f^3)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x
 + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) + f^2*sqrt(f^(-4)) + sqr
t(2))*cos(f*x + e)^4 - 315*2^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4))*cos(f*x + e)^4
 + 2*f*cos(f*x + e)^4)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)
*log(1/2*(2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) + 2^(1/4)*(sqrt(2)*f^3*sq
rt(f^(-4))*cos(f*x + e) + 2*f*cos(f*x + e))*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)
) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4) + 2*
cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e)) + 315*2^(1/4)*(sqrt(2)*f^3*sqr
t(f^(-4))*cos(f*x + e)^4 + 2*f*cos(f*x + e)^4)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(
-4)) + 4)*(f^(-4))^(1/4)*log(1/2*(2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) -
 2^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4))*cos(f*x + e) + 2*f*cos(f*x + e))*sqrt(-2
*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x +
 e))*(f^(-4))^(1/4) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e)) + 16*(
445*cos(f*x + e)^4 - 139*cos(f*x + e)^2 - (18*cos(f*x + e)^3 - 5*cos(f*x +
e))*sin(f*x + e) + 35)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e)))/(f
*cos(f*x + e)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\tan(e + fx) + 1} \tan^5(e + fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(f*x+e))**(1/2)*tan(f*x+e)**5,x)
```

```
[Out] Integral(sqrt(tan(e + f*x) + 1)*tan(e + f*x)**5, x)
```

Giac [A]
time = 0.68, size = 278, normalized size = 1.05

$$-\frac{\sqrt{2\sqrt{2}-2}\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{2f}-\frac{\sqrt{2\sqrt{2}-2}\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{2f}-\frac{\sqrt{2\sqrt{2}+2}\log\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{4f}+\frac{\sqrt{2\sqrt{2}+2}\log\left(-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{4f}+\frac{2\left(35f^8(\tan(fx+e)+1)^{\frac{9}{2}}-135f^8(\tan(fx+e)+1)^{\frac{7}{2}}+126f^8(\tan(fx+e)+1)^{\frac{5}{2}}+315f^8\sqrt{\tan(fx+e)+1}\right)}{315f^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(f*x+e))^(1/2)*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2*sqrt(2) - 2)*arctan(1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*
sqrt(tan(f*x + e) + 1))/sqrt(-sqrt(2) + 2))/f - 1/2*sqrt(2*sqrt(2) - 2)*arc
tan(-1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(tan(f*x + e) + 1))/sqr
t(-sqrt(2) + 2))/f - 1/4*sqrt(2*sqrt(2) + 2)*log(2^(1/4)*sqrt(sqrt(2) + 2)*
sqrt(tan(f*x + e) + 1) + sqrt(2) + tan(f*x + e) + 1)/f + 1/4*sqrt(2*sqrt(2)
 + 2)*log(-2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x + e) + 1) + sqrt(2) + tan
(f*x + e) + 1)/f + 2/315*(35*f^8*(tan(f*x + e) + 1)^(9/2) - 135*f^8*(tan(f*
x + e) + 1)^(7/2) + 126*f^8*(tan(f*x + e) + 1)^(5/2) + 315*f^8*sqrt(tan(f*x
 + e) + 1))/f^9
```

Mupad [B]
time = 6.03, size = 135, normalized size = 0.51

$$\frac{2\sqrt{\tan(e+fx)+1}}{f}+\frac{4(\tan(e+fx)+1)^{5/2}}{5f}-\frac{6(\tan(e+fx)+1)^{7/2}}{7f}+\frac{2(\tan(e+fx)+1)^{9/2}}{9f}-\mathrm{atan}\left(f\sqrt{\frac{\frac{1}{4}-\frac{1}{4}\mathrm{i}}{f^2}}\sqrt{\tan(e+fx)+1}\,(1-\mathrm{i})\right)\sqrt{\frac{\frac{1}{4}-\frac{1}{4}\mathrm{i}}{f^2}}\,2\mathrm{i}+\mathrm{atan}\left(f\sqrt{\frac{\frac{1}{4}+\frac{1}{4}\mathrm{i}}{f^2}}\sqrt{\tan(e+fx)+1}\,(1+1\mathrm{i})\right)\sqrt{\frac{\frac{1}{4}+\frac{1}{4}\mathrm{i}}{f^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5*(tan(e + f*x) + 1)^(1/2),x)
```

```
[Out] (2*(tan(e + f*x) + 1)^(1/2))/f + (4*(tan(e + f*x) + 1)^(5/2))/(5*f) - (6*(t
an(e + f*x) + 1)^(7/2))/(7*f) + (2*(tan(e + f*x) + 1)^(9/2))/(9*f) - atan(f
*((1/4 - 1i/4)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1/2)*(1 - 1i))*((1/4 - 1i/4)/
f^2)^(1/2)*2i + atan(f*((1/4 + 1i/4)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1/2)*(1
 + 1i))*((1/4 + 1i/4)/f^2)^(1/2)*2i
```

3.379 $\int \tan^3(e+fx)\sqrt{1+\tan(e+fx)}\,dx$

Optimal. Leaf size=208

$$\frac{\sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{4-3\sqrt{2}+\left(2-\sqrt{2}\right)\tan(e+fx)}{2\sqrt{-7+5\sqrt{2}}\,\sqrt{1+\tan(e+fx)}}\right)}{f}+\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\tanh^{-1}\left(\frac{4+3\sqrt{2}+}{2\sqrt{7+5\sqrt{2}}}\right.}{f}$$

```
[Out] 1/2*arctan(1/2*(4-3*2^(1/2)+(2-2^(1/2))*tan(f*x+e))/(-7+5*2^(1/2))^(1/2)/(1
+tan(f*x+e))^(1/2))*(-2+2*2^(1/2))^(1/2)/f+1/2*arctanh(1/2*(4+3*2^(1/2)+(2+
2^(1/2))*tan(f*x+e))/(7+5*2^(1/2))^(1/2)/(1+tan(f*x+e))^(1/2))*(2+2*2^(1/2)
)^(1/2)/f-2*(1+tan(f*x+e))^(1/2)/f-4/15*(1+tan(f*x+e))^(3/2)/f+2/5*tan(f*x+
e)*(1+tan(f*x+e))^(3/2)/f
```

Rubi [A]
time = 0.18, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3647, 3711, 12, 3609, 3617, 3616, 209, 213}

$$\frac{\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\,\text{ArcTan}\left(\frac{\left(2-\sqrt{2}\right)\tan(e+fx)-3\sqrt{2}+4}{2\sqrt{5\sqrt{2}-7}\,\sqrt{\tan(e+fx)+1}}\right)}{f}+\frac{2\tan(e+fx)(\tan(e+fx)+1)^{3/2}}{5f}-\frac{4(\tan(e+fx)+1)^{3/2}}{15f}-\frac{2\sqrt{\tan(e+fx)+1}}{f}+\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\tanh^{-1}\left(\frac{\left(2+\sqrt{2}\right)\tan(e+fx)+3\sqrt{2}+4}{2\sqrt{7+5\sqrt{2}}\,\sqrt{\tan(e+fx)+1}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3*Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] (Sqrt[(-1 + Sqrt[2])/2]*ArcTan[(4 - 3*Sqrt[2] + (2 - Sqrt[2])*Tan[e + f*x])
/(2*Sqrt[-7 + 5*Sqrt[2]]*Sqrt[1 + Tan[e + f*x]])])/f + (Sqrt[(1 + Sqrt[2])/
2]*ArcTanh[(4 + 3*Sqrt[2] + (2 + Sqrt[2])*Tan[e + f*x])/(2*Sqrt[7 + 5*Sqrt[
2]]*Sqrt[1 + Tan[e + f*x]])])/f - (2*Sqrt[1 + Tan[e + f*x]])/f - (4*(1 + Ta
n[e + f*x])^(3/2))/(15*f) + (2*Tan[e + f*x]*(1 + Tan[e + f*x])^(3/2))/(5*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3616

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*b*c*d - 4*a*d^2 +
 x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

Rule 3617

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2*q), Int[(a
*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
x], x] - Dist[1/(2*q), Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f*x
])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d - b
*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \tan^3(e+fx)\sqrt{1+\tan(e+fx)}\,dx = \frac{2\tan(e+fx)(1+\tan(e+fx))^{3/2}}{5f} + \frac{2}{5}\int \sqrt{1+\tan(e+fx)}\,\Big(-$$

$$= -\frac{4(1+\tan(e+fx))^{3/2}}{15f} + \frac{2\tan(e+fx)(1+\tan(e+fx))^{3/2}}{5f} + \frac{2}{5}$$

$$= -\frac{4(1+\tan(e+fx))^{3/2}}{15f} + \frac{2\tan(e+fx)(1+\tan(e+fx))^{3/2}}{5f} - \int$$

$$= -\frac{2\sqrt{1+\tan(e+fx)}}{f} - \frac{4(1+\tan(e+fx))^{3/2}}{15f} + \frac{2\tan(e+fx)(}{}$$

$$= -\frac{2\sqrt{1+\tan(e+fx)}}{f} - \frac{4(1+\tan(e+fx))^{3/2}}{15f} + \frac{2\tan(e+fx)(}{}$$

$$= -\frac{2\sqrt{1+\tan(e+fx)}}{f} - \frac{4(1+\tan(e+fx))^{3/2}}{15f} + \frac{2\tan(e+fx)(}{}$$

$$= \frac{\sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\,\tan^{-1}\left(\frac{4-3\sqrt{2}+\left(2-\sqrt{2}\right)\tan(e+fx)}{2\sqrt{-7+5\sqrt{2}}\,\sqrt{1+\tan(e+fx)}}\right)}{f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.36, size = 100, normalized size = 0.48

$$\frac{15\sqrt{1-i}\,\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1-i}}\right) + 15\sqrt{1+i}\,\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1+i}}\right) + 2\sqrt{1+\tan(e+fx)}\,(-17+\tan(e+fx)+3\tan^2(e+fx))}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3*Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] (15*Sqrt[1 - I]*ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1 - I]] + 15*Sqrt[1 + I
]*ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1 + I]] + 2*Sqrt[1 + Tan[e + f*x]]*(-
17 + Tan[e + f*x] + 3*Tan[e + f*x]^2))/(15*f)
```

Maple [A]
time = 0.16, size = 230, normalized size = 1.11

method	result
derivativedivides	$\frac{\frac{2(1+\tan(fx+e))^{\frac{5}{2}}}{5}-\frac{2(1+\tan(fx+e))^{\frac{3}{2}}}{3}-2\sqrt{1+\tan(fx+e)}-\frac{\sqrt{2\sqrt{2}+2}\,\ln\left(1+\sqrt{2}-\sqrt{2\sqrt{2}+2}\,\sqrt{\right.}}{4}}{}$
default	$\frac{\frac{2(1+\tan(fx+e))^{\frac{5}{2}}}{5}-\frac{2(1+\tan(fx+e))^{\frac{3}{2}}}{3}-2\sqrt{1+\tan(fx+e)}-\frac{\sqrt{2\sqrt{2}+2}\,\ln\left(1+\sqrt{2}-\sqrt{2\sqrt{2}+2}\,\sqrt{\right.}}{4}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+tan(f*x+e))^(1/2)*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2/5*(1+tan(f*x+e))^(5/2)-2/3*(1+tan(f*x+e))^(3/2)-2*(1+tan(f*x+e))^(1/
2)-1/4*(2*2^(1/2)+2)^(1/2)*ln(1+2^(1/2)-(2*2^(1/2)+2)^(1/2)*(1+tan(f*x+e))^
(1/2)+tan(f*x+e))-(1-2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+tan(f*x+e))
^(1/2)-(2*2^(1/2)+2)^(1/2))/(-2+2*2^(1/2))^(1/2))+1/4*(2*2^(1/2)+2)^(1/2)*l
n(1+2^(1/2)+(2*2^(1/2)+2)^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))+(2^(1/2)-1
)/(-2+2*2^(1/2))^(1/2)*arctan(((2*2^(1/2)+2)^(1/2)+2*(1+tan(f*x+e))^(1/2))/
(-2+2*2^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(f*x+e))^(1/2)*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(tan(f*x + e) + 1)*tan(f*x + e)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1074 vs. $2(170) = 340$.
time = 1.24, size = 1074, normalized size = 5.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(f*x+e))^(1/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] -1/120*(60*2^(3/4)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f*(f^(-4))^(1/4)*a
rctan(1/2*2^(3/4)*sqrt(1/2)*(f^5*sqrt(f^(-4)) + sqrt(2)*f^3)*sqrt(-2*sqrt(2
)*f^2*sqrt(f^(-4)) + 4)*sqrt((2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) + 2^(
```

```
1/4)*(sqrt(2)*f^3*sqrt(f^(-4))*cos(f*x + e) + 2*f*cos(f*x + e))*sqrt(-2*sqr
t(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))
*(f^(-4))^(1/4) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e))*(f^(-4))^(
3/4) - 1/2*2^(3/4)*(f^5*sqrt(f^(-4)) + sqrt(2)*f^3)*sqrt(-2*sqrt(2)*f^2*sqr
t(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3
/4) - f^2*sqrt(f^(-4)) - sqrt(2))*cos(f*x + e)^2 + 60*2^(3/4)*sqrt(-2*sqrt(
2)*f^2*sqrt(f^(-4)) + 4)*f*(f^(-4))^(1/4)*arctan(1/2*2^(3/4)*sqrt(1/2)*(f^5
*sqrt(f^(-4)) + sqrt(2)*f^3)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((2*
sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) - 2^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4))*c
os(f*x + e) + 2*f*cos(f*x + e))*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt(
(cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4) + 2*cos(f*x + e)
 + 2*sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) - 1/2*2^(3/4)*(f^5*sqrt(f^(
-4)) + sqrt(2)*f^3)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e
) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) + f^2*sqrt(f^(-4)) + sqrt(2)
)*cos(f*x + e)^2 - 15*2^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4))*cos(f*x + e)^2 + 2*
f*cos(f*x + e)^2)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*log(
1/2*(2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) + 2^(1/4)*(sqrt(2)*f^3*sqrt(f^
(-4))*cos(f*x + e) + 2*f*cos(f*x + e))*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4
)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4) + 2*cos(f
*x + e) + 2*sin(f*x + e))/cos(f*x + e)) + 15*2^(1/4)*(sqrt(2)*f^3*sqrt(f^(-
4))*cos(f*x + e)^2 + 2*f*cos(f*x + e)^2)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) +
 4)*(f^(-4))^(1/4)*log(1/2*(2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) - 2^(1/
4)*(sqrt(2)*f^3*sqrt(f^(-4))*cos(f*x + e) + 2*f*cos(f*x + e))*sqrt(-2*sqrt(
2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(
f^(-4))^(1/4) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e)) + 16*(20*cos
(f*x + e)^2 - cos(f*x + e)*sin(f*x + e) - 3)*sqrt((cos(f*x + e) + sin(f*x +
 e))/cos(f*x + e)))/(f*cos(f*x + e)^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\tan(e+fx)+1}\,\tan^3(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(f*x+e))**(1/2)*tan(f*x+e)**3,x)
```

```
[Out] Integral(sqrt(tan(e + f*x) + 1)*tan(e + f*x)**3, x)
```

Giac [**A**]
time = 0.67, size = 262, normalized size = 1.26

$$\frac{\sqrt{2\sqrt{2}-2}\,\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{2f}+\frac{\sqrt{2\sqrt{2}-2}\,\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{2f}+\frac{\sqrt{2\sqrt{2}+2}\,\log\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{4f}-\frac{\sqrt{2\sqrt{2}+2}\,\log\left(-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{4f}+\frac{2\left(3f^4(\tan(fx+e)+1)^{\frac{5}{2}}-5f^4(\tan(fx+e)+1)^{\frac{3}{2}}-15f^4\sqrt{\tan(fx+e)+1}\right)}{15f^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(f*x+e))^(1/2)*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2*sqrt(2) - 2)*arctan(1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*s
qrt(tan(f*x + e) + 1))/sqrt(-sqrt(2) + 2))/f + 1/2*sqrt(2*sqrt(2) - 2)*arct
an(-1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(tan(f*x + e) + 1))/sqrt
(-sqrt(2) + 2))/f + 1/4*sqrt(2*sqrt(2) + 2)*log(2^(1/4)*sqrt(sqrt(2) + 2)*s
qrt(tan(f*x + e) + 1) + sqrt(2) + tan(f*x + e) + 1)/f - 1/4*sqrt(2*sqrt(2)
+ 2)*log(-2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x + e) + 1) + sqrt(2) + tan(
f*x + e) + 1)/f + 2/15*(3*f^4*(tan(f*x + e) + 1)^(5/2) - 5*f^4*(tan(f*x + e
) + 1)^(3/2) - 15*f^4*sqrt(tan(f*x + e) + 1))/f^5
```

Mupad [B]
time = 4.57, size = 120, normalized size = 0.58

$$\frac{2\,{\left(\tan\left(e+f\,x\right)+1\right)}^{5/2}}{5\,f}-\frac{2\,{\left(\tan\left(e+f\,x\right)+1\right)}^{3/2}}{3\,f}-\frac{2\,\sqrt{\tan\left(e+f\,x\right)+1}}{f}+\mathrm{atan}\left(f\,\sqrt{\frac{\frac{1}{4}-\frac{1}{4}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\,\left(1-\mathrm{i}\right)\right)\,\sqrt{\frac{\frac{1}{4}-\frac{1}{4}\mathrm{i}}{f^2}}\,2\mathrm{i}-\mathrm{atan}\left(f\,\sqrt{\frac{\frac{1}{4}+\frac{1}{4}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\,\left(1+1\mathrm{i}\right)\right)\,\sqrt{\frac{\frac{1}{4}+\frac{1}{4}\mathrm{i}}{f^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(tan(e + f*x) + 1)^(1/2),x)
```

```
[Out] (2*(tan(e + f*x) + 1)^(5/2))/(5*f) - (2*(tan(e + f*x) + 1)^(3/2))/(3*f) - (
2*(tan(e + f*x) + 1)^(1/2))/f + atan(f*((1/4 - 1i/4)/f^2)^(1/2)*(tan(e + f*
x) + 1)^(1/2)*(1 - 1i))*((1/4 - 1i/4)/f^2)^(1/2)*2i - atan(f*((1/4 + 1i/4)/
f^2)^(1/2)*(tan(e + f*x) + 1)^(1/2)*(1 + 1i))*((1/4 + 1i/4)/f^2)^(1/2)*2i
```

3.380 $\int \tan(e+fx)\sqrt{1+\tan(e+fx)}\,dx$

Optimal. Leaf size=166

$$-\frac{\sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{4-3\sqrt{2}+\left(2-\sqrt{2}\right)\tan(e+fx)}{2\sqrt{-7+5\sqrt{2}}\,\sqrt{1+\tan(e+fx)}}\right)}{f}-\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\tanh^{-1}\left(\frac{4+3\sqrt{2}}{2\sqrt{7+5\sqrt{}}}\right.}{f}$$

```
[Out] -1/2*arctan(1/2*(4-3*2^(1/2)+(2-2^(1/2))*tan(f*x+e))/(-7+5*2^(1/2))^(1/2)/(
1+tan(f*x+e))^(1/2))*(-2+2*2^(1/2))^(1/2)/f-1/2*arctanh(1/2*(4+3*2^(1/2)+(2
+2^(1/2))*tan(f*x+e))/(7+5*2^(1/2))^(1/2)/(1+tan(f*x+e))^(1/2))*(2+2*2^(1/2
))^(1/2)/f+2*(1+tan(f*x+e))^(1/2)/f
```

Rubi [A]

time = 0.11, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3609, 3617, 3616, 209, 213}

$$-\frac{\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\,\text{ArcTan}\left(\frac{\left(2-\sqrt{2}\right)\tan(e+fx)-3\sqrt{2}+4}{2\sqrt{5\sqrt{2}-7}\,\sqrt{\tan(e+fx)+1}}\right)}{f}+\frac{2\sqrt{\tan(e+fx)+1}}{f}-\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\tanh^{-1}\left(\frac{\left(2+\sqrt{2}\right)\tan(e+fx)+3\sqrt{2}+4}{2\sqrt{7+5\sqrt{2}}\,\sqrt{\tan(e+fx)+1}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]*Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] -((Sqrt[(-1 + Sqrt[2])/2]*ArcTan[(4 - 3*Sqrt[2] + (2 - Sqrt[2])*Tan[e + f*x
])/(2*Sqrt[-7 + 5*Sqrt[2]]*Sqrt[1 + Tan[e + f*x]])]])/f) - (Sqrt[(1 + Sqrt[2
])/2]*ArcTanh[(4 + 3*Sqrt[2] + (2 + Sqrt[2])*Tan[e + f*x])/(2*Sqrt[7 + 5*Sq
rt[2]]*Sqrt[1 + Tan[e + f*x]])]])/f + (2*Sqrt[1 + Tan[e + f*x]])/f
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
```

```
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3616

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*b*c*d - 4*a*d^2 +
 x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

Rule 3617

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2*q), Int[(a
*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
x], x] - Dist[1/(2*q), Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f*x
])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d - b
*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])
```

Rubi steps

$$\int \tan(e+fx)\sqrt{1+\tan(e+fx)}\,dx = \frac{2\sqrt{1+\tan(e+fx)}}{f} + \int \frac{-1+\tan(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx$$

$$= \frac{2\sqrt{1+\tan(e+fx)}}{f} - \frac{\int \frac{\sqrt{2}+\left(-2-\sqrt{2}\right)\tan(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx}{2\sqrt{2}} + \frac{\int \frac{-\sqrt{2}+\left(-\right.}{\sqrt{1+}}$$

$$= \frac{2\sqrt{1+\tan(e+fx)}}{f} + \frac{\left(4-3\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{-2\sqrt{2}\left(-2+\sqrt{2}\right)-4\left(-\right.}\right.}{}$$

$$= -\frac{\sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\tan^{-1}\left(\frac{4-3\sqrt{2}+\left(2-\sqrt{2}\right)\tan(e+fx)}{2\sqrt{-7+5\sqrt{2}}\sqrt{1+\tan(e+fx)}}\right)}{f} -$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.07, size = 78, normalized size = 0.47

$$-\frac{\sqrt{1-i}\,\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1-i}}\right)+\sqrt{1+i}\,\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1+i}}\right)-2\sqrt{1+\tan(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]*Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] -((Sqrt[1 - I]*ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1 - I]] + Sqrt[1 + I]*Ar
cTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1 + I]] - 2*Sqrt[1 + Tan[e + f*x]])/f)
```

Maple [A]
time = 0.10, size = 206, normalized size = 1.24

method	result
derivativedivides	$\frac{2\sqrt{1+\tan(fx+e)}+\frac{\sqrt{2\sqrt{2}+2}\,\ln\left(1+\sqrt{2}-\sqrt{2\sqrt{2}+2}\,\sqrt{1+\tan(fx+e)}+\tan(fx+e)\right)}{4}}{}$
default	$\frac{2\sqrt{1+\tan(fx+e)}+\frac{\sqrt{2\sqrt{2}+2}\,\ln\left(1+\sqrt{2}-\sqrt{2\sqrt{2}+2}\,\sqrt{1+\tan(fx+e)}+\tan(fx+e)\right)}{4}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+tan(f*x+e))^(1/2)*tan(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*(1+tan(f*x+e))^(1/2)+1/4*(2*2^(1/2)+2)^(1/2)*ln(1+2^(1/2)-(2*2^(1/2)
+2)^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))-(2^(1/2)-1)/(-2+2*2^(1/2))^(1/2)
*arctan((2*(1+tan(f*x+e))^(1/2)-(2*2^(1/2)+2)^(1/2))/(-2+2*2^(1/2))^(1/2))-
1/4*(2*2^(1/2)+2)^(1/2)*ln(1+2^(1/2)+(2*2^(1/2)+2)^(1/2)*(1+tan(f*x+e))^(1/
2)+tan(f*x+e))+(1-2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan(((2*2^(1/2)+2)^(1/2)
+2*(1+tan(f*x+e))^(1/2))/(-2+2*2^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(f*x+e))^(1/2)*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(tan(f*x + e) + 1)*tan(f*x + e), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 981 vs. $2(131) = 262$.

time = 1.01, size = 981, normalized size = 5.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(f*x+e))^(1/2)*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] 1/8*(4*2^(3/4)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f*(f^(-4))^(1/4)*arcta
n(1/2*2^(3/4)*sqrt(1/2)*(f^5*sqrt(f^(-4)) + sqrt(2)*f^3)*sqrt(-2*sqrt(2)*f^
2*sqrt(f^(-4)) + 4)*sqrt((2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) + 2^(1/4)
*(sqrt(2)*f^3*sqrt(f^(-4))*cos(f*x + e) + 2*f*cos(f*x + e))*sqrt(-2*sqrt(2)
*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^
(-4))^(1/4) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4)
 - 1/2*2^(3/4)*(f^5*sqrt(f^(-4)) + sqrt(2)*f^3)*sqrt(-2*sqrt(2)*f^2*sqrt(f^
(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4)
- f^2*sqrt(f^(-4)) - sqrt(2)) + 4*2^(3/4)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4))
+ 4)*f*(f^(-4))^(1/4)*arctan(1/2*2^(3/4)*sqrt(1/2)*(f^5*sqrt(f^(-4)) + sqrt
(2)*f^3)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((2*sqrt(2)*f^2*sqrt(f^(
-4))*cos(f*x + e) - 2^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4))*cos(f*x + e) + 2*f*co
s(f*x + e))*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(
f*x + e))/cos(f*x + e))*(f^(-4))^(1/4) + 2*cos(f*x + e) + 2*sin(f*x + e))/c
os(f*x + e))*(f^(-4))^(3/4) - 1/2*2^(3/4)*(f^5*sqrt(f^(-4)) + sqrt(2)*f^3)*
sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/co
s(f*x + e))*(f^(-4))^(3/4) + f^2*sqrt(f^(-4)) + sqrt(2)) - 2^(1/4)*(sqrt(2)
*f^3*sqrt(f^(-4)) + 2*f)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/
4)*log(1/2*(2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) + 2^(1/4)*(sqrt(2)*f^3*
sqrt(f^(-4))*cos(f*x + e) + 2*f*cos(f*x + e))*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-
4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4) +
2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e)) + 2^(1/4)*(sqrt(2)*f^3*sqrt(
f^(-4)) + 2*f)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*log(1/2
*(2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) - 2^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4
))*cos(f*x + e) + 2*f*cos(f*x + e))*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*s
qrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4) + 2*cos(f*x
+ e) + 2*sin(f*x + e))/cos(f*x + e)) + 16*sqrt((cos(f*x + e) + sin(f*x + e)
)/cos(f*x + e)))/f
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\tan{\left(e + fx \right)} + 1} \tan{\left(e + fx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(f*x+e))**(1/2)*tan(f*x+e),x)
```

```
[Out] Integral(sqrt(tan(e + f*x) + 1)*tan(e + f*x), x)
```

Giac [A]
time = 0.64, size = 224, normalized size = 1.35

$$-\frac{\sqrt{2\sqrt{2}-2}\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{\tan\left(fx+e\right)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{2f}-\frac{\sqrt{2\sqrt{2}-2}\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{\tan\left(fx+e\right)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{2f}-\frac{\sqrt{2\sqrt{2}+2}\log\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan\left(fx+e\right)+1}+\sqrt{2}+\tan\left(fx+e\right)+1\right)}{4f}+\frac{\sqrt{2\sqrt{2}+2}\log\left(-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan\left(fx+e\right)+1}+\sqrt{2}+\tan\left(fx+e\right)+1\right)}{4f}+\frac{2\sqrt{\tan\left(fx+e\right)+1}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(f*x+e))^(1/2)*tan(f*x+e),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2*sqrt(2) - 2)*arctan(1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*
sqrt(tan(f*x + e) + 1))/sqrt(-sqrt(2) + 2))/f - 1/2*sqrt(2*sqrt(2) - 2)*arc
tan(-1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(tan(f*x + e) + 1))/sqr
t(-sqrt(2) + 2))/f - 1/4*sqrt(2*sqrt(2) + 2)*log(2^(1/4)*sqrt(sqrt(2) + 2)*
sqrt(tan(f*x + e) + 1) + sqrt(2) + tan(f*x + e) + 1)/f + 1/4*sqrt(2*sqrt(2)
 + 2)*log(-2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x + e) + 1) + sqrt(2) + tan
(f*x + e) + 1)/f + 2*sqrt(tan(f*x + e) + 1)/f
```

Mupad [B]
time = 4.10, size = 90, normalized size = 0.54

$$\frac{2\sqrt{\tan\left(e+f\,x\right)+1}}{f}-\mathrm{atan}\left(f\,\sqrt{\frac{\frac{1}{4}-\frac{1}{4}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\,\left(1-\mathrm{i}\right)\right)\,\sqrt{\frac{\frac{1}{4}-\frac{1}{4}\mathrm{i}}{f^2}}\,2\mathrm{i}+\mathrm{atan}\left(f\,\sqrt{\frac{\frac{1}{4}+\frac{1}{4}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\,\left(1+1\mathrm{i}\right)\right)\,\sqrt{\frac{\frac{1}{4}+\frac{1}{4}\mathrm{i}}{f^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(tan(e + f*x) + 1)^(1/2),x)
```

```
[Out] (2*(tan(e + f*x) + 1)^(1/2))/f - atan(f*((1/4 - 1i/4)/f^2)^(1/2)*(tan(e + f
*x) + 1)^(1/2)*(1 - 1i))*((1/4 - 1i/4)/f^2)^(1/2)*2i + atan(f*((1/4 + 1i/4)
/f^2)^(1/2)*(tan(e + f*x) + 1)^(1/2)*(1 + 1i))*((1/4 + 1i/4)/f^2)^(1/2)*2i
```

3.381 $\int \cot(e+fx)\sqrt{1+\tan(e+fx)}\,dx$

Optimal. Leaf size=165

$$\frac{\sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\,\mathrm{ArcTan}\left(\frac{4-3\sqrt{2}+\left(2-\sqrt{2}\right)\tan(e+fx)}{2\sqrt{-7+5\sqrt{2}}\,\sqrt{1+\tan(e+fx)}}\right)}{f}-\frac{2\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{f}+\frac{\sqrt{\frac{1}{2}\left(1\right.}}{}$$

```
[Out] -2*arctanh((1+tan(f*x+e))^(1/2))/f+1/2*arctan(1/2*(4-3*2^(1/2)+(2-2^(1/2))*
tan(f*x+e))/(-7+5*2^(1/2))^(1/2)/(1+tan(f*x+e))^(1/2))*(-2+2*2^(1/2))^(1/2)
/f+1/2*arctanh(1/2*(4+3*2^(1/2)+(2+2^(1/2))*tan(f*x+e))/(7+5*2^(1/2))^(1/2)
/(1+tan(f*x+e))^(1/2))*(2+2*2^(1/2))^(1/2)/f
```

Rubi [A]
time = 0.18, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3653, 3617, 3616, 209, 213, 3715, 65}

$$\frac{\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\,\mathrm{ArcTan}\left(\frac{\left(2-\sqrt{2}\right)\tan(e+fx)-3\sqrt{2}+4}{2\sqrt{5\sqrt{2}-7}\,\sqrt{\tan(e+fx)+1}}\right)}{f}-\frac{2\tanh^{-1}\left(\sqrt{\tan(e+fx)+1}\right)}{f}+\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\tanh^{-1}\left(\frac{\left(2+\sqrt{2}\right)\tan(e+fx)+3\sqrt{2}+4}{2\sqrt{7+5\sqrt{2}}\,\sqrt{\tan(e+fx)+1}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] (Sqrt[(-1 + Sqrt[2])/2]*ArcTan[(4 - 3*Sqrt[2] + (2 - Sqrt[2])*Tan[e + f*x])
/(2*Sqrt[-7 + 5*Sqrt[2]]*Sqrt[1 + Tan[e + f*x]])])/f - (2*ArcTanh[Sqrt[1 +
Tan[e + f*x]]])/f + (Sqrt[(1 + Sqrt[2])/2]*ArcTanh[(4 + 3*Sqrt[2] + (2 + Sq
rt[2])*Tan[e + f*x])/(2*Sqrt[7 + 5*Sqrt[2]]*Sqrt[1 + Tan[e + f*x]])])/f
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3616

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*b*c*d - 4*a*d^2 +
 x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

Rule 3617

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2*q), Int[(a
*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
x], x] - Dist[1/(2*q), Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f*x
])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d - b
*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])
```

Rule 3653

```
Int[Sqrt[(a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[Simp[a*c + b*d + (b*c -
a*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x]], x], x] - Dist[d*((b*c - a*d
)/(c^2 + d^2)), Int[(1 + Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]]*(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \cot(e+fx)\sqrt{1+\tan(e+fx)}\,dx = \int \frac{1-\tan(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx + \int \frac{\cot(e+fx)\left(1+\tan^2(e+fx)\right)}{\sqrt{1+\tan(e+fx)}}\,dx$$

$$= \frac{\int \frac{\sqrt{2}+\left(2-\sqrt{2}\right)\tan(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx}{2\sqrt{2}} - \frac{\int \frac{-\sqrt{2}+\left(2+\sqrt{2}\right)\tan(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx}{2\sqrt{2}} + \frac{\text{Subs}}{}$$

$$= \frac{2\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\tan(e+fx)}\right)}{f} + \frac{\left(4-3\sqrt{2}\right)\text{Subst}\Big(\int}{}$$

$$= \frac{\sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\tan^{-1}\left(\frac{4-3\sqrt{2}+\left(2-\sqrt{2}\right)\tan(e+fx)}{2\sqrt{-7+5\sqrt{2}}\sqrt{1+\tan(e+fx)}}\right)}{f} - \frac{2}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.09, size = 78, normalized size = 0.47

$$\frac{-2\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)+\sqrt{1-i}\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1-i}}\right)+\sqrt{1+i}\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1+i}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[1 + Tan[e + f*x]]] + Sqrt[1 - I]*ArcTanh[Sqrt[1 + Tan[e +
f*x]]/Sqrt[1 - I]] + Sqrt[1 + I]*ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1 + I]
])/f
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 18.54, size = 2750, normalized size = 16.67

method	result	size
default	Expression too large to display	2750

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(1+tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f*((cos(f*x+e)+sin(f*x+e))/cos(f*x+e))^(1/2)*(cos(f*x+e)+1)^2*(cos(f*x+
e)-1)^2*(1+sin(f*x+e))*(8*I*2^(1/2)*((sin(f*x+e)-1)*(1+2^(1/2))/cos(f*x+e))
^(1/2)*((2^(1/2)*sin(f*x+e)-2^(1/2)+cos(f*x+e)-sin(f*x+e)+1)/cos(f*x+e))^(1
/2)*(-(2^(1/2)*sin(f*x+e)-2^(1/2)-cos(f*x+e)+sin(f*x+e)-1)/cos(f*x+e))^(1/2
```

```
)*EllipticPi(1/2*((sin(f*x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*2^(1
/2),I*2^(1/2)/(2+2^(1/2)),I*((2-2^(1/2))/(2+2^(1/2)))^(1/2))-8*I*2^(1/2)*((
sin(f*x+e)-1)*(1+2^(1/2))/cos(f*x+e))^(1/2)*((2^(1/2)*sin(f*x+e)-2^(1/2)+co
s(f*x+e)-sin(f*x+e)+1)/cos(f*x+e))^(1/2)*(-(2^(1/2)*sin(f*x+e)-2^(1/2)-cos(
f*x+e)+sin(f*x+e)-1)/cos(f*x+e))^(1/2)*EllipticPi(1/2*((sin(f*x+e)-1)/cos(f
*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*2^(1/2),-I*2^(1/2)/(2+2^(1/2)),I*((2-2^(1/
2))/(2+2^(1/2)))^(1/2))-8*2^(1/2)*((sin(f*x+e)-1)*(1+2^(1/2))/cos(f*x+e))^(
1/2)*((2^(1/2)*sin(f*x+e)-2^(1/2)+cos(f*x+e)-sin(f*x+e)+1)/cos(f*x+e))^(1/2
)*(-(2^(1/2)*sin(f*x+e)-2^(1/2)-cos(f*x+e)+sin(f*x+e)-1)/cos(f*x+e))^(1/2)*
EllipticPi(1/2*((sin(f*x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*2^(1/2
),I*2^(1/2)/(2+2^(1/2)),I*((2-2^(1/2))/(2+2^(1/2)))^(1/2))-((sin(f*x+e)-1)/
cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*2^(1/2)*((2^(1/2)*cos(f*x+e)-2^(1/2)*
sin(f*x+e)+2^(1/2)+2*sin(f*x+e)-2)/cos(f*x+e)*2^(1/2))^(1/2)*((2^(1/2)*cos(
f*x+e)-2^(1/2)*sin(f*x+e)+2^(1/2)-2*sin(f*x+e)+2)/cos(f*x+e)*2^(1/2))^(1/2)
*EllipticE(1/2*((sin(f*x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*2^(1/2
),I*((2-2^(1/2))/(2+2^(1/2)))^(1/2))+((sin(f*x+e)-1)/cos(f*x+e)*(2+2^(1/2))
*2^(1/2))^(1/2)*2^(1/2)*((2^(1/2)*cos(f*x+e)-2^(1/2)*sin(f*x+e)+2^(1/2)+2*s
in(f*x+e)-2)/cos(f*x+e)*2^(1/2))^(1/2)*((2^(1/2)*cos(f*x+e)-2^(1/2)*sin(f*x
+e)+2^(1/2)-2*sin(f*x+e)+2)/cos(f*x+e)*2^(1/2))^(1/2)*EllipticF(1/2*((sin(f
*x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*2^(1/2),I*((2-2^(1/2))/(2+2^
(1/2)))^(1/2))+2*2^(1/2)*EllipticE(1/2*((sin(f*x+e)-1)/cos(f*x+e)*(2+2^(1/2
))*2^(1/2))^(1/2)*2^(1/2),I*((2-2^(1/2))/(2+2^(1/2)))^(1/2))*((sin(f*x+e)-1
)*(1+2^(1/2))/cos(f*x+e))^(1/2)*((2^(1/2)*sin(f*x+e)-2^(1/2)+cos(f*x+e)-sin
(f*x+e)+1)/cos(f*x+e))^(1/2)*(-(2^(1/2)*sin(f*x+e)-2^(1/2)-cos(f*x+e)+sin(f
*x+e)-1)/cos(f*x+e))^(1/2)+20*2^(1/2)*EllipticPi(1/2*((sin(f*x+e)-1)/cos(f*
x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*2^(1/2),2^(1/2)/(2+2^(1/2)),I*((2-2^(1/2))/
(2+2^(1/2)))^(1/2))*((sin(f*x+e)-1)*(1+2^(1/2))/cos(f*x+e))^(1/2)*((2^(1/2)
*sin(f*x+e)-2^(1/2)+cos(f*x+e)-sin(f*x+e)+1)/cos(f*x+e))^(1/2)*(-(2^(1/2)*s
in(f*x+e)-2^(1/2)-cos(f*x+e)+sin(f*x+e)-1)/cos(f*x+e))^(1/2)-8*2^(1/2)*((si
n(f*x+e)-1)*(1+2^(1/2))/cos(f*x+e))^(1/2)*((2^(1/2)*sin(f*x+e)-2^(1/2)+cos(
f*x+e)-sin(f*x+e)+1)/cos(f*x+e))^(1/2)*(-(2^(1/2)*sin(f*x+e)-2^(1/2)-cos(f*
x+e)+sin(f*x+e)-1)/cos(f*x+e))^(1/2)*EllipticF(1/2*((sin(f*x+e)-1)/cos(f*x+
e)*(2+2^(1/2))*2^(1/2))^(1/2)*2^(1/2),I*((2-2^(1/2))/(2+2^(1/2)))^(1/2))-8*
2^(1/2)*((sin(f*x+e)-1)*(1+2^(1/2))/cos(f*x+e))^(1/2)*((2^(1/2)*sin(f*x+e)-
2^(1/2)+cos(f*x+e)-sin(f*x+e)+1)/cos(f*x+e))^(1/2)*(-(2^(1/2)*sin(f*x+e)-2^
(1/2)-cos(f*x+e)+sin(f*x+e)-1)/cos(f*x+e))^(1/2)*EllipticPi(1/2*((sin(f*x+e
)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*2^(1/2),-I*2^(1/2)/(2+2^(1/2)),I
*((2-2^(1/2))/(2+2^(1/2)))^(1/2))-((sin(f*x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^
(1/2))^(1/2)*((2^(1/2)*cos(f*x+e)-2^(1/2)*sin(f*x+e)+2^(1/2)+2*sin(f*x+e)-2
)/cos(f*x+e)*2^(1/2))^(1/2)*((2^(1/2)*cos(f*x+e)-2^(1/2)*sin(f*x+e)+2^(1/2)
-2*sin(f*x+e)+2)/cos(f*x+e)*2^(1/2))^(1/2)*EllipticE(1/2*((sin(f*x+e)-1)/co
s(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*2^(1/2),I*((2-2^(1/2))/(2+2^(1/2)))^(1/
2))-6*((sin(f*x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*((2^(1/2)*cos(f
*x+e)-2^(1/2)*sin(f*x+e)+2^(1/2)+2*sin(f*x+e)-2)/cos(f*x+e)*2^(1/2))^(1/2)*
((2^(1/2)*cos(f*x+e)-2^(1/2)*sin(f*x+e)+2^(1/2)-2*sin(f*x+e)+2)/cos(f*x+e)*
```

```
2^(1/2))^(1/2)*EllipticPi(1/2*((sin(f*x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2
))^(1/2)*2^(1/2),2^(1/2)/(2+2^(1/2)),I*((2-2^(1/2))/(2+2^(1/2)))^(1/2))+4*(
(sin(f*x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*((2^(1/2)*cos(f*x+e)-2
^(1/2)*sin(f*x+e)+2^(1/2)+2*sin(f*x+e)-2)/cos(f*x+e)*2^(1/2))^(1/2)*((2^(1/
2)*cos(f*x+e)-2^(1/2)*sin(f*x+e)+2^(1/2)-2*sin(f*x+e)+2)/cos(f*x+e)*2^(1/2)
)^(1/2)*EllipticPi(1/2*((sin(f*x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2
)*2^(1/2),-2^(1/2)/(2+2^(1/2)),I*((2-2^(1/2))/(2+2^(1/2)))^(1/2))+4*((sin(f
*x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*((2^(1/2)*cos(f*x+e)-2^(1/2)
*sin(f*x+e)+2^(1/2)+2*sin(f*x+e)-2)/cos(f*x+e)*2^(1/2))^(1/2)*((2^(1/2)*cos
(f*x+e)-2^(1/2)*sin(f*x+e)+2^(1/2)-2*sin(f*x+e)+2)/cos(f*x+e)*2^(1/2))^(1/2
)*EllipticF(1/2*((sin(f*x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*2^(1/
2),I*((2-2^(1/2))/(2+2^(1/2)))^(1/2))+4*EllipticE(1/2*((sin(f*x+e)-1)/cos(f
*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*2^(1/2),I*((2-2^(1/2))/(2+2^(1/2)))^(1/2))
*((sin(f*x+e)-1)*(1+2^(1/2))/cos(f*x+e))^(1/2)*((2^(1/2)*sin(f*x+e)-2^(1/2)
+cos(f*x+e)-sin(f*x+e)+1)/cos(f*x+e))^(1/2)*(-(2^(1/2)*sin(f*x+e)-2^(1/2)-c
os(f*x+e)+sin(f*x+e)-1)/cos(f*x+e))^(1/2)-4*((sin(f*x+e)-1)*(1+2^(1/2))/cos
(f*x+e))^(1/2)*((2^(1/2)*sin(f*x+e)-2^(1/2)+cos(f*x+e)-sin(f*x+e)+1)/cos(f*
x+e))^(1/2)*(-(2^(1/2)*sin(f*x+e)-2^(1/2)-cos(f...
```

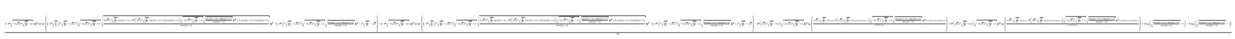
Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(1+tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(tan(f*x + e) + 1)*cot(f*x + e), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1016 vs. 2(132) = 264.
time = 0.97, size = 1016, normalized size = 6.16



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(1+tan(f*x+e))^(1/2),x, algorithm="fricas")
[Out] -1/8*(4*2^(3/4)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f*(f^(-4))^(1/4)*arct
an(1/2*2^(3/4)*sqrt(1/2)*(f^5*sqrt(f^(-4)) + sqrt(2)*f^3)*sqrt(-2*sqrt(2)*f
^2*sqrt(f^(-4)) + 4)*sqrt((2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) + 2^(1/4
)*(sqrt(2)*f^3*sqrt(f^(-4))*cos(f*x + e) + 2*f*cos(f*x + e))*sqrt(-2*sqrt(2
)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f
^(-4))^(1/4) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4
) - 1/2*2^(3/4)*(f^5*sqrt(f^(-4)) + sqrt(2)*f^3)*sqrt(-2*sqrt(2)*f^2*sqrt(f
^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4)
```

```
 - f^2*sqrt(f^(-4)) - sqrt(2)) + 4*2^(3/4)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4))
 + 4)*f*(f^(-4))^(1/4)*arctan(1/2*2^(3/4)*sqrt(1/2)*(f^5*sqrt(f^(-4)) + sqr
t(2)*f^3)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((2*sqrt(2)*f^2*sqrt(f^
(-4))*cos(f*x + e) - 2^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4))*cos(f*x + e) + 2*f*c
os(f*x + e))*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin
(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4) + 2*cos(f*x + e) + 2*sin(f*x + e))/
cos(f*x + e))*(f^(-4))^(3/4) - 1/2*2^(3/4)*(f^5*sqrt(f^(-4)) + sqrt(2)*f^3)
*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/c
os(f*x + e))*(f^(-4))^(3/4) + f^2*sqrt(f^(-4)) + sqrt(2)) - 2^(1/4)*(sqrt(2
)*f^3*sqrt(f^(-4)) + 2*f)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1
/4)*log(1/2*(2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) + 2^(1/4)*(sqrt(2)*f^3
*sqrt(f^(-4))*cos(f*x + e) + 2*f*cos(f*x + e))*sqrt(-2*sqrt(2)*f^2*sqrt(f^(
-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4) +
 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e)) + 2^(1/4)*(sqrt(2)*f^3*sqrt
(f^(-4)) + 2*f)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*log(1/
2*(2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) - 2^(1/4)*(sqrt(2)*f^3*sqrt(f^(-
4))*cos(f*x + e) + 2*f*cos(f*x + e))*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*
sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4) + 2*cos(f*x
 + e) + 2*sin(f*x + e))/cos(f*x + e)) + 8*log(sqrt((cos(f*x + e) + sin(f*x
+ e))/cos(f*x + e)) + 1) - 8*log(sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x
 + e)) - 1))/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\tan{\left(e + fx \right)} + 1} \cot{\left(e + fx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(1+tan(f*x+e))**(1/2),x)
[Out] Integral(sqrt(tan(e + f*x) + 1)*cot(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(1+tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(tan(f*x + e) + 1)*cot(f*x + e), x)
```

Mupad [B]
time = 0.16, size = 90, normalized size = 0.55

$$-\frac{2\,\mathrm{atanh}\left(\sqrt{\mathrm{tan}\left(e+f\,x\right)+1}\right)}{f}-\mathrm{atan}\left(f\,\sqrt{\frac{\frac{1}{4}+\frac{1}{4}{}\mathrm{i}}{f^2}}\,\sqrt{\mathrm{tan}\left(e+f\,x\right)+1}\,\left(1+1{}\mathrm{i}\right)\right)\,\sqrt{\frac{\frac{1}{4}+\frac{1}{4}{}\mathrm{i}}{f^2}}\,2{}\mathrm{i}+2\,\mathrm{atanh}\left(f\,\sqrt{\frac{\frac{1}{4}-\frac{1}{4}{}\mathrm{i}}{f^2}}\,\sqrt{\mathrm{tan}\left(e+f\,x\right)+1}\,\left(1+1{}\mathrm{i}\right)\right)\,\sqrt{\frac{\frac{1}{4}-\frac{1}{4}{}\mathrm{i}}{f^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(tan(e + f*x) + 1)^(1/2),x)
```

```
[Out] 2*atanh(f*((1/4 - 1i/4)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1/2)*(1 + 1i))*((1/4
 - 1i/4)/f^2)^(1/2) - atan(f*((1/4 + 1i/4)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1
/2)*(1 + 1i))*((1/4 + 1i/4)/f^2)^(1/2)*2i - (2*atanh((tan(e + f*x) + 1)^(1/
2)))/f
```

3.382 $\int \cot^3(e+fx)\sqrt{1+\tan(e+fx)}\,dx$

Optimal. Leaf size=221

$$-\frac{\sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{4-3\sqrt{2}+\left(2-\sqrt{2}\right)\tan(e+fx)}{2\sqrt{-7+5\sqrt{2}}\,\sqrt{1+\tan(e+fx)}}\right)}{f}+\frac{9\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{4f}-\frac{\sqrt{\frac{1}{2}}}{}$$

```
[Out] 9/4*arctanh((1+tan(f*x+e))^(1/2))/f-1/2*arctan(1/2*(4-3*2^(1/2)+(2-2^(1/2))
*tan(f*x+e))/(-7+5*2^(1/2))^(1/2)/(1+tan(f*x+e))^(1/2))*(-2+2*2^(1/2))^(1/2
)/f-1/2*arctanh(1/2*(4+3*2^(1/2)+(2+2^(1/2))*tan(f*x+e))/(7+5*2^(1/2))^(1/2
)/(1+tan(f*x+e))^(1/2))*(2+2*2^(1/2))^(1/2)/f-1/4*cot(f*x+e)*(1+tan(f*x+e))
^(1/2)/f-1/2*cot(f*x+e)^2*(1+tan(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.34, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3649, 3730, 3734, 3617, 3616, 209, 213, 3715, 65}

$$-\frac{\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\,\text{ArcTan}\left(\frac{\left(2-\sqrt{2}\right)\tan(e+fx)-3\sqrt{2}+4}{2\sqrt{5\sqrt{2}-7}\,\sqrt{\tan(e+fx)+1}}\right)}{f}+\frac{9\tanh^{-1}\left(\sqrt{\tan(e+fx)+1}\right)}{4f}-\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\tanh^{-1}\left(\frac{\left(2+\sqrt{2}\right)\tan(e+fx)+3\sqrt{2}+4}{2\sqrt{7+5\sqrt{2}}\,\sqrt{\tan(e+fx)+1}}\right)}{f}-\frac{\sqrt{\tan(e+fx)+1}\,\cot^2(e+fx)}{2f}-\frac{\sqrt{\tan(e+fx)+1}\,\cot(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] -((Sqrt[(-1 + Sqrt[2])/2]*ArcTan[(4 - 3*Sqrt[2] + (2 - Sqrt[2])*Tan[e + f*x
])/(2*Sqrt[-7 + 5*Sqrt[2]]*Sqrt[1 + Tan[e + f*x]])]])/f) + (9*ArcTanh[Sqrt[1
 + Tan[e + f*x]]])/(4*f) - (Sqrt[(1 + Sqrt[2])/2]*ArcTanh[(4 + 3*Sqrt[2] +
(2 + Sqrt[2])*Tan[e + f*x])/(2*Sqrt[7 + 5*Sqrt[2]]*Sqrt[1 + Tan[e + f*x]])]
)/f - (Cot[e + f*x]*Sqrt[1 + Tan[e + f*x]])/(4*f) - (Cot[e + f*x]^2*Sqrt[1
+ Tan[e + f*x]])/(2*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3616

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*b*c*d - 4*a*d^2 +
 x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

Rule 3617

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2*q), Int[(a
*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
x], x] - Dist[1/(2*q), Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f*x
])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d - b
*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \cot^3(e+fx)\sqrt{1+\tan(e+fx)}\,dx = -\frac{\cot^2(e+fx)\sqrt{1+\tan(e+fx)}}{2f} - \frac{1}{2}\int \frac{\cot^2(e+fx)\left(-\frac{1}{2}+2\tan\ldots\right)}{\sqrt{1+\ldots}}$$

$$= -\frac{\cot(e+fx)\sqrt{1+\tan(e+fx)}}{4f} - \frac{\cot^2(e+fx)\sqrt{1+\tan(e+f\ldots)}}{2f}$$

$$= -\frac{\cot(e+fx)\sqrt{1+\tan(e+fx)}}{4f} - \frac{\cot^2(e+fx)\sqrt{1+\tan(e+f\ldots)}}{2f}$$

$$= -\frac{\cot(e+fx)\sqrt{1+\tan(e+fx)}}{4f} - \frac{\cot^2(e+fx)\sqrt{1+\tan(e+f\ldots)}}{2f}$$

$$= -\frac{\cot(e+fx)\sqrt{1+\tan(e+fx)}}{4f} - \frac{\cot^2(e+fx)\sqrt{1+\tan(e+f\ldots)}}{2f}$$

$$= -\frac{\sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\tan^{-1}\left(\frac{4-3\sqrt{2}+\left(2-\sqrt{2}\right)\tan(e+fx)}{2\sqrt{-7+5\sqrt{2}}\sqrt{1+\tan(e+fx)}}\right)}{f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.49, size = 124, normalized size = 0.56

$$-\frac{-9\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)+4\sqrt{1-i}\,\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1-i}}\right)+4\sqrt{1+i}\,\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1+i}}\right)+\cot(e+fx)\sqrt{1+\tan(e+fx)}+2\cot^2(e+fx)\sqrt{1+\tan(e+fx)}}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] -1/4*(-9*ArcTanh[Sqrt[1 + Tan[e + f*x]]] + 4*Sqrt[1 - I]*ArcTanh[Sqrt[1 + T
an[e + f*x]]/Sqrt[1 - I]] + 4*Sqrt[1 + I]*ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sq
rt[1 + I]] + Cot[e + f*x]*Sqrt[1 + Tan[e + f*x]] + 2*Cot[e + f*x]^2*Sqrt[1
+ Tan[e + f*x]])/f
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.90, size = 11217, normalized size = 50.76

method	result	size
default	Expression too large to display	11217

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(1+tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(1+tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(tan(f*x + e) + 1)*cot(f*x + e)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1206 vs. 2(180) = 360.
time = 0.78, size = 1206, normalized size = 5.46

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(1+tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*(2^(1/4)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*(2*f*cos(f*x + e)^2 + s
qrt(2)*(f^3*cos(f*x + e)^2 - f^3)*sqrt(f^(-4)) - 2*f)*(f^(-4))^(1/4)*log(1/
```

```
2*(2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) + 2^(1/4)*(sqrt(2)*f^3*sqrt(f^(-
4))*cos(f*x + e) + 2*f*cos(f*x + e))*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*
sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4) + 2*cos(f*x
 + e) + 2*sin(f*x + e))/cos(f*x + e)) - 2^(1/4)*sqrt(-2*sqrt(2)*f^2*sqrt(f^
(-4)) + 4)*(2*f*cos(f*x + e)^2 + sqrt(2)*(f^3*cos(f*x + e)^2 - f^3)*sqrt(f^
(-4)) - 2*f)*(f^(-4))^(1/4)*log(1/2*(2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e
) - 2^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4))*cos(f*x + e) + 2*f*cos(f*x + e))*sqrt
(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*
x + e))*(f^(-4))^(1/4) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e)) - 9
*(cos(f*x + e)^2 - 1)*log(sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))
+ 1) + 9*(cos(f*x + e)^2 - 1)*log(sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*
x + e)) - 1) - 2*(2*cos(f*x + e)^2 + cos(f*x + e)*sin(f*x + e))*sqrt((cos(f
*x + e) + sin(f*x + e))/cos(f*x + e)) - 4*2^(3/4)*(f^5*cos(f*x + e)^2 - f^5
)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*arctan(1/2*2^(3/4)*s
qrt(1/2)*(f^5*sqrt(f^(-4)) + sqrt(2)*f^3)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4))
+ 4)*sqrt((2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) + 2^(1/4)*(sqrt(2)*f^3*s
qrt(f^(-4))*cos(f*x + e) + 2*f*cos(f*x + e))*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4
)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4) + 2
*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) - 1/2*2^(3/4)*
(f^5*sqrt(f^(-4)) + sqrt(2)*f^3)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt
((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) - f^2*sqrt(f^(-
4)) - sqrt(2))/f^4 - 4*2^(3/4)*(f^5*cos(f*x + e)^2 - f^5)*sqrt(-2*sqrt(2)*f
^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*arctan(1/2*2^(3/4)*sqrt(1/2)*(f^5*sqrt(
f^(-4)) + sqrt(2)*f^3)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((2*sqrt(2
)*f^2*sqrt(f^(-4))*cos(f*x + e) - 2^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4))*cos(f*x
 + e) + 2*f*cos(f*x + e))*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f
*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4) + 2*cos(f*x + e) + 2*s
in(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) - 1/2*2^(3/4)*(f^5*sqrt(f^(-4)) +
 sqrt(2)*f^3)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + si
n(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) + f^2*sqrt(f^(-4)) + sqrt(2))/f^4)
/(f*cos(f*x + e)^2 - f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\tan(e + fx) + 1} \cot^3(e + fx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(1+tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(tan(e + f*x) + 1)*cot(e + f*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(1+tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(tan(f*x + e) + 1)*cot(f*x + e)^3, x)
```

Mupad [B]
time = 0.19, size = 149, normalized size = 0.67

$$-\frac{\mathrm{atan}\left(\sqrt{\tan\left(e+f\,x\right)+1}\,1\mathrm{i}\right)\,9\mathrm{i}}{4\,f}-\frac{\frac{\sqrt{\tan\left(e+f\,x\right)+1}}{4}+\frac{{\left(\tan\left(e+f\,x\right)+1\right)}^{3/2}}{4}}{f-2\,f\,\left(\tan\left(e+f\,x\right)+1\right)+f\,{\left(\tan\left(e+f\,x\right)+1\right)}^2}-\mathrm{atan}\left(f\,\sqrt{\frac{\frac{1}{4}-\frac{1}{4}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\,\left(1-\mathrm{i}\right)\right)\,\sqrt{\frac{\frac{1}{4}-\frac{1}{4}\mathrm{i}}{f^2}}\,2\mathrm{i}+\mathrm{atan}\left(f\,\sqrt{\frac{\frac{1}{4}+\frac{1}{4}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\,\left(1+1\mathrm{i}\right)\right)\,\sqrt{\frac{\frac{1}{4}+\frac{1}{4}\mathrm{i}}{f^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(tan(e + f*x) + 1)^(1/2),x)
```

```
[Out] atan(f*((1/4 + 1i/4)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1/2)*(1 + 1i))*((1/4 +
1i/4)/f^2)^(1/2)*2i - ((tan(e + f*x) + 1)^(1/2)/4 + (tan(e + f*x) + 1)^(3/2
)/4)/(f - 2*f*(tan(e + f*x) + 1) + f*(tan(e + f*x) + 1)^2) - atan(f*((1/4 -
 1i/4)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1/2)*(1 - 1i))*((1/4 - 1i/4)/f^2)^(1/
2)*2i - (atan((tan(e + f*x) + 1)^(1/2)*1i)*9i)/(4*f)
```

3.383 $\int \cot^5(e+fx)\sqrt{1+\tan(e+fx)}\,dx$

Optimal. Leaf size=273

$$\frac{\sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\,\mathrm{ArcTan}\left(\frac{4-3\sqrt{2}+\left(2-\sqrt{2}\right)\tan(e+fx)}{2\sqrt{-7+5\sqrt{2}}\,\sqrt{1+\tan(e+fx)}}\right)}{f}-\frac{139\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{64f}+$$

[Out] -139/64*arctanh((1+tan(f*x+e))^(1/2))/f+1/2*arctan(1/2*(4-3*2^(1/2)+(2-2^(1/2))*tan(f*x+e))/(-7+5*2^(1/2))^(1/2)/(1+tan(f*x+e))^(1/2))*(-2+2*2^(1/2))^(1/2)/f+1/2*arctanh(1/2*(4+3*2^(1/2)+(2+2^(1/2))*tan(f*x+e))/(7+5*2^(1/2))^(1/2)/(1+tan(f*x+e))^(1/2))*(2+2*2^(1/2))^(1/2)/f+11/64*cot(f*x+e)*(1+tan(f*x+e))^(1/2)/f+53/96*cot(f*x+e)^2*(1+tan(f*x+e))^(1/2)/f-1/24*cot(f*x+e)^3*(1+tan(f*x+e))^(1/2)/f-1/4*cot(f*x+e)^4*(1+tan(f*x+e))^(1/2)/f

Rubi [A]
time = 0.47, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3649, 3730, 3734, 3617, 3616, 209, 213, 3715, 65}

$$\frac{\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\,\mathrm{ArcTan}\left(\frac{\left(2-\sqrt{2}\right)\tan(e+fx)-3\sqrt{2}+4}{2\sqrt{5\sqrt{2}-7}\,\sqrt{\tan(e+fx)+1}}\right)}{f}-\frac{139\tanh^{-1}\left(\sqrt{\tan(e+fx)+1}\right)}{64f}+\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\tanh^{-1}\left(\frac{\left(2+\sqrt{2}\right)\tan(e+fx)+3\sqrt{2}+4}{2\sqrt{7+5\sqrt{2}}\,\sqrt{\tan(e+fx)+1}}\right)}{f}-\frac{\sqrt{\tan(e+fx)+1}\,\cot^4(e+fx)}{4f}-\frac{\sqrt{\tan(e+fx)+1}\,\cot^3(e+fx)}{24f}+\frac{53\sqrt{\tan(e+fx)+1}\,\cot^2(e+fx)}{96f}+\frac{11\sqrt{\tan(e+fx)+1}\,\cot(e+fx)}{64f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] (Sqrt[(-1 + Sqrt[2])/2]*ArcTan[(4 - 3*Sqrt[2] + (2 - Sqrt[2])*Tan[e + f*x])/(2*Sqrt[-7 + 5*Sqrt[2]]*Sqrt[1 + Tan[e + f*x]])])/f - (139*ArcTanh[Sqrt[1 + Tan[e + f*x]]])/(64*f) + (Sqrt[(1 + Sqrt[2])/2]*ArcTanh[(4 + 3*Sqrt[2] + (2 + Sqrt[2])*Tan[e + f*x])/(2*Sqrt[7 + 5*Sqrt[2]]*Sqrt[1 + Tan[e + f*x]])])/f + (11*Cot[e + f*x]*Sqrt[1 + Tan[e + f*x]])/(64*f) + (53*Cot[e + f*x]^2*Sqrt[1 + Tan[e + f*x]])/(96*f) - (Cot[e + f*x]^3*Sqrt[1 + Tan[e + f*x]])/(24*f) - (Cot[e + f*x]^4*Sqrt[1 + Tan[e + f*x]])/(4*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3616

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*b*c*d - 4*a*d^2 +
 x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

Rule 3617

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2*q), Int[(a
*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
x], x] - Dist[1/(2*q), Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f*x
])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d - b
*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \cot^5(e+fx)\sqrt{1+\tan(e+fx)}\,dx = -\frac{\cot^4(e+fx)\sqrt{1+\tan(e+fx)}}{4f} - \frac{1}{4}\int \frac{\cot^4(e+fx)\left(-\frac{1}{2}+4\tan\right.}{\sqrt{1+\tan}}$$

$$= -\frac{\cot^3(e+fx)\sqrt{1+\tan(e+fx)}}{24f} - \frac{\cot^4(e+fx)\sqrt{1+\tan(e+fx}}{4f}$$

$$= \frac{53\cot^2(e+fx)\sqrt{1+\tan(e+fx)}}{96f} - \frac{\cot^3(e+fx)\sqrt{1+\tan(e+f}}{24f}$$

$$= \frac{11\cot(e+fx)\sqrt{1+\tan(e+fx)}}{64f} + \frac{53\cot^2(e+fx)\sqrt{1+\tan(e+}}{96f}$$

$$= \frac{11\cot(e+fx)\sqrt{1+\tan(e+fx)}}{64f} + \frac{53\cot^2(e+fx)\sqrt{1+\tan(e+}}{96f}$$

$$= \frac{11\cot(e+fx)\sqrt{1+\tan(e+fx)}}{64f} + \frac{53\cot^2(e+fx)\sqrt{1+\tan(e+}}{96f}$$

$$= \frac{11\cot(e+fx)\sqrt{1+\tan(e+fx)}}{64f} + \frac{53\cot^2(e+fx)\sqrt{1+\tan(e+}}{96f}$$

$$= \frac{\sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\tan^{-1}\left(\frac{4-3\sqrt{2}+\left(2-\sqrt{2}\right)\tan(e+fx)}{2\sqrt{-7+5\sqrt{2}}\sqrt{1+\tan(e+fx)}}\right)}{f} -$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.89, size = 169, normalized size = 0.62

$$\frac{-417\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)+192\sqrt{1-i}\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1-i}}\right)+192\sqrt{1+i}\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1+i}}\right)+33\cot(e+fx)\sqrt{1+\tan(e+fx)}+106\cot^2(e+fx)\sqrt{1+\tan(e+fx)}-8\cot^3(e+fx)\sqrt{1+\tan(e+fx)}-48\cot^4(e+fx)\sqrt{1+\tan(e+fx)}}{192f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] (-417*ArcTanh[Sqrt[1 + Tan[e + f*x]]] + 192*Sqrt[1 - I]*ArcTanh[Sqrt[1 + Ta
n[e + f*x]]/Sqrt[1 - I]] + 192*Sqrt[1 + I]*ArcTanh[Sqrt[1 + Tan[e + f*x]]/S
qrt[1 + I]] + 33*Cot[e + f*x]*Sqrt[1 + Tan[e + f*x]] + 106*Cot[e + f*x]^2*S
qrt[1 + Tan[e + f*x]] - 8*Cot[e + f*x]^3*Sqrt[1 + Tan[e + f*x]] - 48*Cot[e
+ f*x]^4*Sqrt[1 + Tan[e + f*x]])/(192*f)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.89, size = 16923, normalized size = 61.99

method	result	size
default	Expression too large to display	16923

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(1+tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(1+tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(tan(f*x + e) + 1)*cot(f*x + e)^5, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1336 vs. 2(230) = 460.
time = 1.31, size = 1336, normalized size = 4.89

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(1+tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/384*(48*2^(1/4)*(2*f*cos(f*x + e)^4 - 4*f*cos(f*x + e)^2 + sqrt(2)*(f^3*c
os(f*x + e)^4 - 2*f^3*cos(f*x + e)^2 + f^3)*sqrt(f^(-4)) + 2*f)*sqrt(-2*sqr
t(2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*log(1/2*(2*sqrt(2)*f^2*sqrt(f^(-4
))*cos(f*x + e) + 2^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4))*cos(f*x + e) + 2*f*cos(
f*x + e))*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*
x + e))/cos(f*x + e))*(f^(-4))^(1/4) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos
(f*x + e)) - 48*2^(1/4)*(2*f*cos(f*x + e)^4 - 4*f*cos(f*x + e)^2 + sqrt(2)*
(f^3*cos(f*x + e)^4 - 2*f^3*cos(f*x + e)^2 + f^3)*sqrt(f^(-4)) + 2*f)*sqrt(
-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*log(1/2*(2*sqrt(2)*f^2*sqrt
(f^(-4))*cos(f*x + e) - 2^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4))*cos(f*x + e) + 2*
f*cos(f*x + e))*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) +
sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4) + 2*cos(f*x + e) + 2*sin(f*x + e
))/cos(f*x + e)) - 417*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*log(sqrt((co
s(f*x + e) + sin(f*x + e))/cos(f*x + e)) + 1) + 417*(cos(f*x + e)^4 - 2*cos
(f*x + e)^2 + 1)*log(sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e)) - 1)
- 2*(154*cos(f*x + e)^4 - 106*cos(f*x + e)^2 + (41*cos(f*x + e)^3 - 33*cos(
f*x + e))*sin(f*x + e))*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e)) -
```

```
192*2^(3/4)*(f^5*cos(f*x + e)^4 - 2*f^5*cos(f*x + e)^2 + f^5)*sqrt(-2*sqrt(
2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*arctan(1/2*2^(3/4)*sqrt(1/2)*(f^5*s
qrt(f^(-4)) + sqrt(2)*f^3)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((2*sq
rt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) + 2^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4))*cos
(f*x + e) + 2*f*cos(f*x + e))*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((c
os(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4) + 2*cos(f*x + e) +
 2*sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) - 1/2*2^(3/4)*(f^5*sqrt(f^(-4
)) + sqrt(2)*f^3)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e)
+ sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) - f^2*sqrt(f^(-4)) - sqrt(2))/
f^4 - 192*2^(3/4)*(f^5*cos(f*x + e)^4 - 2*f^5*cos(f*x + e)^2 + f^5)*sqrt(-2
*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*arctan(1/2*2^(3/4)*sqrt(1/2)*
(f^5*sqrt(f^(-4)) + sqrt(2)*f^3)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt
((2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) - 2^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4
))*cos(f*x + e) + 2*f*cos(f*x + e))*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*s
qrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4) + 2*cos(f*x
+ e) + 2*sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) - 1/2*2^(3/4)*(f^5*sqrt
(f^(-4)) + sqrt(2)*f^3)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x
 + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) + f^2*sqrt(f^(-4)) + sqr
t(2))/f^4)/(f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^2 + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\tan\left(e+fx\right)+1}\,\cot^{5}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(1+tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(tan(e + f*x) + 1)*cot(e + f*x)**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(1+tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(tan(f*x + e) + 1)*cot(f*x + e)^5, x)
```

Mupad [B]
time = 0.20, size = 198, normalized size = 0.73

$$\frac{\mathrm{atan}\left(\sqrt{\tan\left(e+f\,x\right)+1}\,1{}\mathrm{i}\right)\,139{}\mathrm{i}}{64\,f}+\frac{\frac{11\,\sqrt{\tan\left(e+f\,x\right)+1}}{64}-\frac{121\,{\left(\tan\left(e+f\,x\right)+1\right)}^{3/2}}{192}+\frac{7\,{\left(\tan\left(e+f\,x\right)+1\right)}^{5/2}}{192}+\frac{11\,{\left(\tan\left(e+f\,x\right)+1\right)}^{7/2}}{64}}{f-4\,f\,\left(\tan\left(e+f\,x\right)+1\right)+6\,f\,{\left(\tan\left(e+f\,x\right)+1\right)}^2-4\,f\,{\left(\tan\left(e+f\,x\right)+1\right)}^3+f\,{\left(\tan\left(e+f\,x\right)+1\right)}^4}+\mathrm{atan}\left(f\,\sqrt{\frac{\frac{1}{4}-\frac{1}{4}{}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\,\left(1-\mathrm{i}\right)\right)\,\sqrt{\frac{\frac{1}{4}-\frac{1}{4}{}\mathrm{i}}{f^2}}\,2{}\mathrm{i}-\mathrm{atan}\left(f\,\sqrt{\frac{\frac{1}{4}+\frac{1}{4}{}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\,\left(1+1{}\mathrm{i}\right)\right)\,\sqrt{\frac{\frac{1}{4}+\frac{1}{4}{}\mathrm{i}}{f^2}}\,2{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5*(tan(e + f*x) + 1)^(1/2),x)
```

```
[Out] (atan((tan(e + f*x) + 1)^(1/2)*1i)*139i)/(64*f) + ((11*(tan(e + f*x) + 1)^(
1/2))/64 - (121*(tan(e + f*x) + 1)^(3/2))/192 + (7*(tan(e + f*x) + 1)^(5/2)
)/192 + (11*(tan(e + f*x) + 1)^(7/2))/64)/(f - 4*f*(tan(e + f*x) + 1) + 6*f
*(tan(e + f*x) + 1)^2 - 4*f*(tan(e + f*x) + 1)^3 + f*(tan(e + f*x) + 1)^4)
+ atan(f*((1/4 - 1i/4)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1/2)*(1 - 1i))*((1/4
- 1i/4)/f^2)^(1/2)*2i - atan(f*((1/4 + 1i/4)/f^2)^(1/2)*(tan(e + f*x) + 1)^
(1/2)*(1 + 1i))*((1/4 + 1i/4)/f^2)^(1/2)*2i
```

3.384 $\int \tan^4(e+fx)\sqrt{1+\tan(e+fx)}\,dx$

Optimal. Leaf size=318

$$-\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\tan(e+fx)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{f}+\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{\right.}}}{}\right.}{f}$$

```
[Out] -1/2*arctan(((2+2*2^(1/2))^(1/2)-2*(1+tan(f*x+e))^(1/2))/(-2+2*2^(1/2))^(1/
2))*(2+2*2^(1/2))^(1/2)/f+1/2*arctan(((2+2*2^(1/2))^(1/2)+2*(1+tan(f*x+e))^
(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)/f+1/2*ln(1+2^(1/2)-(2+2*2^
(1/2))^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))/f/(2+2*2^(1/2))^(1/2)-1/2*ln(
1+2^(1/2)+(2+2*2^(1/2))^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))/f/(2+2*2^(1/
2))^(1/2)-18/35*(1+tan(f*x+e))^(3/2)/f-8/35*tan(f*x+e)*(1+tan(f*x+e))^(3/2)
/f+2/7*tan(f*x+e)^2*(1+tan(f*x+e))^(3/2)/f
```

Rubi [A]
time = 0.31, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3647, 3728, 3712, 3566, 714, 1141, 1175, 632, 210, 1178, 642}

$$-\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\tan(e+fx)+1}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{f}+\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{2\sqrt{\tan(e+fx)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{f}+\frac{2(\tan(e+fx)+1)^{3/2}\tan^2(e+fx)}{7f}-\frac{8(\tan(e+fx)+1)^{3/2}\tan(e+fx)}{35f}-\frac{18(\tan(e+fx)+1)^{3/2}}{35f}+\frac{\log\left(\tan(e+fx)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\tan(e+fx)+1}+\sqrt{2}+1\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}f}-\frac{\log\left(\tan(e+fx)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\tan(e+fx)+1}+\sqrt{2}+1\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4*Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] -((Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + Tan[e +
 f*x]])/Sqrt[2*(-1 + Sqrt[2])]])/f) + (Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2
*(1 + Sqrt[2])] + 2*Sqrt[1 + Tan[e + f*x]])/Sqrt[2*(-1 + Sqrt[2])]])/f + Lo
g[1 + Sqrt[2] + Tan[e + f*x] - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + Tan[e + f*x]]
]/(2*Sqrt[2*(1 + Sqrt[2])]*f) - Log[1 + Sqrt[2] + Tan[e + f*x] + Sqrt[2*(1
+ Sqrt[2])]*Sqrt[1 + Tan[e + f*x]]]/(2*Sqrt[2*(1 + Sqrt[2])]*f) - (18*(1 +
Tan[e + f*x])^(3/2))/(35*f) - (8*Tan[e + f*x]*(1 + Tan[e + f*x])^(3/2))/(35
*f) + (2*Tan[e + f*x]^2*(1 + Tan[e + f*x])^(3/2))/(7*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 714

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, S
ubst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x]
 /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3712

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \tan^4(e+fx)\sqrt{1+\tan(e+fx)}\,dx = \frac{2\tan^2(e+fx)(1+\tan(e+fx))^{3/2}}{7f} + \frac{2}{7}\int \tan(e+fx)\sqrt{1+\tan($$

$$= -\frac{8\tan(e+fx)(1+\tan(e+fx))^{3/2}}{35f} + \frac{2\tan^2(e+fx)(1+\tan(e}{7f}$$

$$= -\frac{18(1+\tan(e+fx))^{3/2}}{35f} - \frac{8\tan(e+fx)(1+\tan(e+fx))^{3/2}}{35f} +$$

$$= -\frac{18(1+\tan(e+fx))^{3/2}}{35f} - \frac{8\tan(e+fx)(1+\tan(e+fx))^{3/2}}{35f} +$$

$$= -\frac{18(1+\tan(e+fx))^{3/2}}{35f} - \frac{8\tan(e+fx)(1+\tan(e+fx))^{3/2}}{35f} +$$

$$= -\frac{18(1+\tan(e+fx))^{3/2}}{35f} - \frac{8\tan(e+fx)(1+\tan(e+fx))^{3/2}}{35f} +$$

$$= -\frac{18(1+\tan(e+fx))^{3/2}}{35f} - \frac{8\tan(e+fx)(1+\tan(e+fx))^{3/2}}{35f} +$$

$$= \frac{\log\left(1+\sqrt{2}+\tan(e+fx)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\tan(e+fx}\right.}{2\sqrt{2\left(1+\sqrt{2}\right)}\,f}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\tan(e+fx)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{\sqrt{2\left(-1+\sqrt{2}\right)}\,f} + \frac{\tan^{-1}\left(\frac{\sqrt{2}}{}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.75, size = 118, normalized size = 0.37

$$\frac{-35i\sqrt{1-i}\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1-i}}\right)+35i\sqrt{1+i}\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1+i}}\right)+2\sqrt{1+\tan(e+fx)}\left(\sec^2(e+fx)(1+5\tan(e+fx))-2(5+9\tan(e+fx))\right)}{35f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4*Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] ((-35*I)*Sqrt[1 - I]*ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1 - I]] + (35*I)*S
qrt[1 + I]*ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1 + I]] + 2*Sqrt[1 + Tan[e +
 f*x]]*(Sec[e + f*x]^2*(1 + 5*Tan[e + f*x]) - 2*(5 + 9*Tan[e + f*x])))/(35*
f)
```

Maple [A]
time = 0.16, size = 241, normalized size = 0.76

method	result
derivativedivides	$\frac{\frac{2(1+\tan(fx+e))^{\frac{7}{2}}}{7}-\frac{4(1+\tan(fx+e))^{\frac{5}{2}}}{5}-\frac{\sqrt{2\sqrt{2}+2}\,(\sqrt{2}-1)\left(-\frac{\ln\left(1+\sqrt{2}-\sqrt{2\sqrt{2}+2}\,\sqrt{1+\tan(fx}\right.}{2}\right.}{}}{}$
default	$\frac{\frac{2(1+\tan(fx+e))^{\frac{7}{2}}}{7}-\frac{4(1+\tan(fx+e))^{\frac{5}{2}}}{5}-\frac{\sqrt{2\sqrt{2}+2}\,(\sqrt{2}-1)\left(-\frac{\ln\left(1+\sqrt{2}-\sqrt{2\sqrt{2}+2}\,\sqrt{1+\tan(fx}\right.}{2}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+tan(f*x+e))^(1/2)*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2/7*(1+tan(f*x+e))^(7/2)-4/5*(1+tan(f*x+e))^(5/2)-1/2*(2*2^(1/2)+2)^(1
/2)*(2^(1/2)-1)*(-1/2*ln(1+2^(1/2)-(2*2^(1/2)+2)^(1/2)*(1+tan(f*x+e))^(1/2)
+tan(f*x+e))-(2*2^(1/2)+2)^(1/2)/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+tan(f*x+
e))^(1/2)-(2*2^(1/2)+2)^(1/2))/(-2+2*2^(1/2))^(1/2)))-1/2*(2*2^(1/2)+2)^(1/
2)*(2^(1/2)-1)*(1/2*ln(1+2^(1/2)+(2*2^(1/2)+2)^(1/2)*(1+tan(f*x+e))^(1/2)+t
an(f*x+e))-(2*2^(1/2)+2)^(1/2)/(-2+2*2^(1/2))^(1/2)*arctan(((2*2^(1/2)+2)^(
1/2)+2*(1+tan(f*x+e))^(1/2))/(-2+2*2^(1/2))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

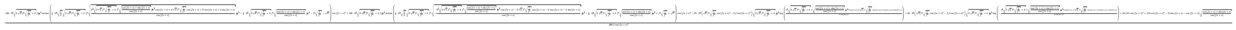
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(f*x+e))^(1/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(tan(f*x + e) + 1)*tan(f*x + e)^4, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 950 vs. $2(258) = 516$.

time = 1.17, size = 950, normalized size = 2.99



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(f*x+e))^(1/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] -1/280*(140*2^(3/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f*(f^(-4))^(1/4)*a
rctan(1/2*2^(3/4)*sqrt(1/2)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^5*sqrt((
2^(3/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^3*sqrt((cos(f*x + e) + sin(f
*x + e))/cos(f*x + e))*(f^(-4))^(3/4)*cos(f*x + e) + 2*sqrt(2)*f^2*sqrt(f^(
-4))*cos(f*x + e) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e))*(f^(-4))
^(5/4) - 1/2*2^(3/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^5*sqrt((cos(f*x
 + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(5/4) - f^2*sqrt(f^(-4)) - sqr
t(2))*cos(f*x + e)^3 + 140*2^(3/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f*(
f^(-4))^(1/4)*arctan(1/2*2^(3/4)*sqrt(1/2)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4))
+ 4)*f^5*sqrt(-(2^(3/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^3*sqrt((cos(
f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4)*cos(f*x + e) - 2*sqrt
(2)*f^2*sqrt(f^(-4))*cos(f*x + e) - 2*cos(f*x + e) - 2*sin(f*x + e))/cos(f*
x + e))*(f^(-4))^(5/4) - 1/2*2^(3/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f
^5*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(5/4) + f^2*sq
rt(f^(-4)) + sqrt(2))*cos(f*x + e)^3 - 35*2^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4))
*cos(f*x + e)^3 - 2*f*cos(f*x + e)^3)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*
(f^(-4))^(1/4)*log(1/2*(2^(3/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^3*sq
rt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4)*cos(f*x + e)
+ 2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) + 2*cos(f*x + e) + 2*sin(f*x + e)
)/cos(f*x + e)) + 35*2^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4))*cos(f*x + e)^3 - 2*f
*cos(f*x + e)^3)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*log(-1
/2*(2^(3/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^3*sqrt((cos(f*x + e) + s
in(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4)*cos(f*x + e) - 2*sqrt(2)*f^2*sqrt
(f^(-4))*cos(f*x + e) - 2*cos(f*x + e) - 2*sin(f*x + e))/cos(f*x + e)) + 16
*(10*cos(f*x + e)^3 + (18*cos(f*x + e)^2 - 5)*sin(f*x + e) - cos(f*x + e))*
sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e)))/(f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\tan (e + fx) + 1} \tan^4 (e + fx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(f*x+e))**(1/2)*tan(f*x+e)**4,x)
```

```
[Out] Integral(sqrt(tan(e + f*x) + 1)*tan(e + f*x)**4, x)
```

Giac [A]
time = 0.72, size = 246, normalized size = 0.77

$$\frac{\sqrt{2\sqrt{2}+2}\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{2f}+\frac{\sqrt{2\sqrt{2}+2}\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{2f}-\frac{\sqrt{2\sqrt{2}-2}\log\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{4f}+\frac{\sqrt{2\sqrt{2}-2}\log\left(-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{4f}+\frac{2\left(5f^6(\tan(fx+e)+1)^{\frac{7}{2}}-14f^6(\tan(fx+e)+1)^{\frac{5}{2}}\right)}{35f^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(f*x+e))^(1/2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2*sqrt(2) + 2)*arctan(1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*s
qrt(tan(f*x + e) + 1))/sqrt(-sqrt(2) + 2))/f + 1/2*sqrt(2*sqrt(2) + 2)*arct
an(-1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(tan(f*x + e) + 1))/sqrt
(-sqrt(2) + 2))/f - 1/4*sqrt(2*sqrt(2) - 2)*log(2^(1/4)*sqrt(sqrt(2) + 2)*s
qrt(tan(f*x + e) + 1) + sqrt(2) + tan(f*x + e) + 1)/f + 1/4*sqrt(2*sqrt(2)
- 2)*log(-2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x + e) + 1) + sqrt(2) + tan(
f*x + e) + 1)/f + 2/35*(5*f^6*(tan(f*x + e) + 1)^(7/2) - 14*f^6*(tan(f*x +
e) + 1)^(5/2))/f^7
```

Mupad [B]
time = 5.11, size = 107, normalized size = 0.34

$$\frac{2\left(\tan(e+fx)+1\right)^{7/2}}{7f}-\frac{4\left(\tan(e+fx)+1\right)^{5/2}}{5f}-\mathrm{atan}\left(f^3\left(\frac{-\frac{1}{4}-\frac{1}{4}\mathrm{i}}{f^2}\right)^{3/2}\sqrt{\tan(e+fx)+1}\,4\mathrm{i}\right)\sqrt{\frac{-\frac{1}{4}-\frac{1}{4}\mathrm{i}}{f^2}}\,2\mathrm{i}-\mathrm{atan}\left(f^3\left(\frac{-\frac{1}{4}+\frac{1}{4}\mathrm{i}}{f^2}\right)^{3/2}\sqrt{\tan(e+fx)+1}\,4\mathrm{i}\right)\sqrt{\frac{-\frac{1}{4}+\frac{1}{4}\mathrm{i}}{f^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(tan(e + f*x) + 1)^(1/2),x)
```

```
[Out] (2*(tan(e + f*x) + 1)^(7/2))/(7*f) - atan(f^3*((- 1/4 + 1i/4)/f^2)^(3/2)*(t
an(e + f*x) + 1)^(1/2)*4i)*((- 1/4 + 1i/4)/f^2)^(1/2)*2i - (4*(tan(e + f*x)
 + 1)^(5/2))/(5*f) - atan(f^3*((- 1/4 - 1i/4)/f^2)^(3/2)*(tan(e + f*x) + 1)
^(1/2)*4i)*((- 1/4 - 1i/4)/f^2)^(1/2)*2i
```

3.385 $\int \tan^2(e+fx)\sqrt{1+\tan(e+fx)}\,dx$

Optimal. Leaf size=266

$$\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\tan(e+fx)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{f}-\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{}\right.}}{\sqrt{}}\right)}{f}$$

```
[Out] 1/2*arctan(((2+2*2^(1/2))^(1/2)-2*(1+tan(f*x+e))^(1/2))/(-2+2*2^(1/2))^(1/2
))*(2+2*2^(1/2))^(1/2)/f-1/2*arctan(((2+2*2^(1/2))^(1/2)+2*(1+tan(f*x+e))^(
1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)/f-1/2*ln(1+2^(1/2)-(2+2*2^(
1/2))^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))/f/(2+2*2^(1/2))^(1/2)+1/2*ln(1
+2^(1/2)+(2+2*2^(1/2))^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))/f/(2+2*2^(1/2
))^(1/2)+2/3*(1+tan(f*x+e))^(3/2)/f
```

Rubi [A]

time = 0.16, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3624, 3566, 714, 1141, 1175, 632, 210, 1178, 642}

$$\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\tan(e+fx)+1}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{f}-\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{2\sqrt{\tan(e+fx)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{f}+\frac{2(\tan(e+fx)+1)^{3/2}}{3f}-\frac{\log\left(\tan(e+fx)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\tan(e+fx)+1}+\sqrt{2}+1\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}f}+\frac{\log\left(\tan(e+fx)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\tan(e+fx)+1}+\sqrt{2}+1\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2*Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] (Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + Tan[e + f
*x]])/Sqrt[2*(-1 + Sqrt[2])]])/f - (Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1
 + Sqrt[2])] + 2*Sqrt[1 + Tan[e + f*x]])/Sqrt[2*(-1 + Sqrt[2])]])/f - Log[1
 + Sqrt[2] + Tan[e + f*x] - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + Tan[e + f*x]]]/(
2*Sqrt[2*(1 + Sqrt[2])]*f) + Log[1 + Sqrt[2] + Tan[e + f*x] + Sqrt[2*(1 + S
qrt[2])]*Sqrt[1 + Tan[e + f*x]]]/(2*Sqrt[2*(1 + Sqrt[2])]*f) + (2*(1 + Tan[
e + f*x])^(3/2))/(3*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 714

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, S
ubst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x]
 /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \tan^2(e+fx)\sqrt{1+\tan(e+fx)}\,dx = \frac{2(1+\tan(e+fx))^{3/2}}{3f} - \int \sqrt{1+\tan(e+fx)}\,dx$$

$$= \frac{2(1+\tan(e+fx))^{3/2}}{3f} - \frac{\text{Subst}\left(\int \frac{\sqrt{1+x}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{2(1+\tan(e+fx))^{3/2}}{3f} - \frac{2\text{Subst}\left(\int \frac{x^2}{2-2x^2+x^4}\,dx, x, \sqrt{1+\tan(e+\ldots}\right.}{f}$$

$$= \frac{2(1+\tan(e+fx))^{3/2}}{3f} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-x^2}{2-2x^2+x^4}\,dx, x, \sqrt{1+\tan(e+f\ldots}\right.}{f}$$

$$= \frac{2(1+\tan(e+fx))^{3/2}}{3f} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2}\,dx, x\ldots\right.}{2f}$$

$$= -\frac{\log\left(1+\sqrt{2}+\tan(e+fx)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\tan(e+f\ldots}\right.}{2\sqrt{2\left(1+\sqrt{2}\right)}\,f}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\tan(e+fx)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{\sqrt{2\left(-1+\sqrt{2}\right)}\,f} - \frac{\tan^{-1}\left(\frac{\sqrt{2\left(\ldots\right.}}{\ldots}\right.}{\ldots}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.18, size = 86, normalized size = 0.32

$$\frac{3i\sqrt{1-i}\,\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1-i}}\right)-3i\sqrt{1+i}\,\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1+i}}\right)+2(1+\tan(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2*Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] ((3*I)*Sqrt[1 - I]*ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1 - I]] - (3*I)*Sqrt
[1 + I]*ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1 + I]] + 2*(1 + Tan[e + f*x])^
(3/2))/(3*f)
```

Maple [A]
time = 0.12, size = 229, normalized size = 0.86

method	result
derivativedivides	$\frac{\frac{2(1+\tan(fx+e))^{\frac{3}{2}}}{3}+\frac{\sqrt{2\sqrt{2}+2}\left(\sqrt{2}-1\right)\left(-\frac{\ln\left(1+\sqrt{2}-\sqrt{2\sqrt{2}+2}\,\sqrt{1+\tan(fx+e)}+\tan(fx+e\right)}{2}\right.}{2}}{}$
default	$\frac{\frac{2(1+\tan(fx+e))^{\frac{3}{2}}}{3}+\frac{\sqrt{2\sqrt{2}+2}\left(\sqrt{2}-1\right)\left(-\frac{\ln\left(1+\sqrt{2}-\sqrt{2\sqrt{2}+2}\,\sqrt{1+\tan(fx+e)}+\tan(fx+e\right)}{2}\right.}{2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+tan(f*x+e))^(1/2)*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2/3*(1+tan(f*x+e))^(3/2)+1/2*(2*2^(1/2)+2)^(1/2)*(2^(1/2)-1)*(-1/2*ln(
1+2^(1/2)-(2*2^(1/2)+2)^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))-(2*2^(1/2)+2
)^(1/2)/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+tan(f*x+e))^(1/2)-(2*2^(1/2)+2)^(
1/2))/(-2+2*2^(1/2))^(1/2)))+1/2*(2*2^(1/2)+2)^(1/2)*(2^(1/2)-1)*(1/2*ln(1+
2^(1/2)+(2*2^(1/2)+2)^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))-(2*2^(1/2)+2)^
(1/2)/(-2+2*2^(1/2))^(1/2)*arctan(((2*2^(1/2)+2)^(1/2)+2*(1+tan(f*x+e))^(1/
2))/(-2+2*2^(1/2))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(f*x+e))^(1/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(tan(f*x + e) + 1)*tan(f*x + e)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 911 vs. $2(210) = 420$.
time = 1.09, size = 911, normalized size = 3.42

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(f*x+e))^(1/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(12*2^(3/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f*(f^(-4))^(1/4)*arct
an(1/2*2^(3/4)*sqrt(1/2)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^5*sqrt((2^(
3/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^3*sqrt((cos(f*x + e) + sin(f*x
+ e))/cos(f*x + e))*(f^(-4))^(3/4)*cos(f*x + e) + 2*sqrt(2)*f^2*sqrt(f^(-4)
)*cos(f*x + e) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e))*(f^(-4))^(5
/4) - 1/2*2^(3/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^5*sqrt((cos(f*x +
e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(5/4) - f^2*sqrt(f^(-4)) - sqrt(2
))*cos(f*x + e) + 12*2^(3/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f*(f^(-4)
)^(1/4)*arctan(1/2*2^(3/4)*sqrt(1/2)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f
^5*sqrt(-(2^(3/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^3*sqrt((cos(f*x +
e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4)*cos(f*x + e) - 2*sqrt(2)*f^
2*sqrt(f^(-4))*cos(f*x + e) - 2*cos(f*x + e) - 2*sin(f*x + e))/cos(f*x + e)
)*(f^(-4))^(5/4) - 1/2*2^(3/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^5*sqr
t((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(5/4) + f^2*sqrt(f^(
-4)) + sqrt(2))*cos(f*x + e) - 3*2^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4))*cos(f*x
+ e) - 2*f*cos(f*x + e))*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4
)*log(1/2*(2^(3/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^3*sqrt((cos(f*x +
 e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4)*cos(f*x + e) + 2*sqrt(2)*f
^2*sqrt(f^(-4))*cos(f*x + e) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e
)) + 3*2^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4))*cos(f*x + e) - 2*f*cos(f*x + e))*s
qrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*log(-1/2*(2^(3/4)*sqrt(2
*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^3*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f
*x + e))*(f^(-4))^(3/4)*cos(f*x + e) - 2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x +
 e) - 2*cos(f*x + e) - 2*sin(f*x + e))/cos(f*x + e)) + 16*sqrt((cos(f*x + e
) + sin(f*x + e))/cos(f*x + e))*(cos(f*x + e) + sin(f*x + e)))/(f*cos(f*x +
 e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\tan(e + fx) + 1}\, \tan^2(e + fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(f*x+e))**(1/2)*tan(f*x+e)**2,x)
```

```
[Out] Integral(sqrt(tan(e + f*x) + 1)*tan(e + f*x)**2, x)
```

Giac [A]
time = 0.71, size = 224, normalized size = 0.84

$$\frac{2\,(\tan(fx+e)+1)^{\frac{3}{2}}}{3\,f}-\frac{\sqrt{2\sqrt{2}+2}\,\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{2\,f}-\frac{\sqrt{2\sqrt{2}+2}\,\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{2\,f}+\frac{\sqrt{2\sqrt{2}-2}\,\log\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\,\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{4\,f}-\frac{\sqrt{2\sqrt{2}-2}\,\log\left(-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\,\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(f*x+e))^(1/2)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] 2/3*(tan(f*x + e) + 1)^(3/2)/f - 1/2*sqrt(2*sqrt(2) + 2)*arctan(1/2*2^(3/4)
*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*sqrt(tan(f*x + e) + 1))/sqrt(-sqrt(2) + 2))
/f - 1/2*sqrt(2*sqrt(2) + 2)*arctan(-1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2)
 - 2*sqrt(tan(f*x + e) + 1))/sqrt(-sqrt(2) + 2))/f + 1/4*sqrt(2*sqrt(2) - 2
)*log(2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x + e) + 1) + sqrt(2) + tan(f*x
+ e) + 1)/f - 1/4*sqrt(2*sqrt(2) - 2)*log(-2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(t
an(f*x + e) + 1) + sqrt(2) + tan(f*x + e) + 1)/f
```

Mupad [B]
time = 4.14, size = 88, normalized size = 0.33

$$\frac{2\,{\left(\tan\left(e+f\,x\right)+1\right)}^{3/2}}{3\,f}-2\,\mathrm{atanh}\left(4\,f^3\,{\left(\frac{-\frac{1}{4}+\frac{1}{4}\mathrm{i}}{f^2}\right)}^{3/2}\,\sqrt{\tan\left(e+f\,x\right)+1}\right)\,\sqrt{\frac{-\frac{1}{4}+\frac{1}{4}\mathrm{i}}{f^2}}-2\,\mathrm{atanh}\left(4\,f^3\,{\left(\frac{-\frac{1}{4}-\frac{1}{4}\mathrm{i}}{f^2}\right)}^{3/2}\,\sqrt{\tan\left(e+f\,x\right)+1}\right)\,\sqrt{\frac{-\frac{1}{4}-\frac{1}{4}\mathrm{i}}{f^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(tan(e + f*x) + 1)^(1/2),x)
```

```
[Out] (2*(tan(e + f*x) + 1)^(3/2))/(3*f) - 2*atanh(4*f^3*((- 1/4 + 1i/4)/f^2)^(3/
2)*(tan(e + f*x) + 1)^(1/2))*((- 1/4 + 1i/4)/f^2)^(1/2) - 2*atanh(4*f^3*((-
 1/4 - 1i/4)/f^2)^(3/2)*(tan(e + f*x) + 1)^(1/2))*((- 1/4 - 1i/4)/f^2)^(1/2
)
```

3.386 $\int \sqrt{1+\tan(e+fx)}\,dx$

Optimal. Leaf size=247

$$-\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\mathrm{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\tan(e+fx)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{f}+\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\mathrm{ArcTan}\left(\frac{\sqrt{2\left(1+\right.}}{}\right.}{f}$$

```
[Out] -1/2*arctan(((2+2*2^(1/2))^(1/2)-2*(1+tan(f*x+e))^(1/2))/(-2+2*2^(1/2))^(1/
2))*(2+2*2^(1/2))^(1/2)/f+1/2*arctan(((2+2*2^(1/2))^(1/2)+2*(1+tan(f*x+e))^
(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)/f+1/2*ln(1+2^(1/2)-(2+2*2^
(1/2))^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))/f/(2+2*2^(1/2))^(1/2)-1/2*ln(
1+2^(1/2)+(2+2*2^(1/2))^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))/f/(2+2*2^(1/
2))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3566, 714, 1141, 1175, 632, 210, 1178, 642}

$$-\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\mathrm{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\tan(e+fx)+1}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{f}+\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\mathrm{ArcTan}\left(\frac{2\sqrt{\tan(e+fx)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{f}+\frac{\log\left(\tan(e+fx)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\tan(e+fx)+1}+\sqrt{2}+1\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}f}-\frac{\log\left(\tan(e+fx)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\tan(e+fx)+1}+\sqrt{2}+1\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] -((Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + Tan[e +
 f*x]])/Sqrt[2*(-1 + Sqrt[2])]])/f) + (Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2
*(1 + Sqrt[2])] + 2*Sqrt[1 + Tan[e + f*x]])/Sqrt[2*(-1 + Sqrt[2])]])/f + Lo
g[1 + Sqrt[2] + Tan[e + f*x] - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + Tan[e + f*x]]
]/(2*Sqrt[2*(1 + Sqrt[2])]*f) - Log[1 + Sqrt[2] + Tan[e + f*x] + Sqrt[2*(1
+ Sqrt[2])]*Sqrt[1 + Tan[e + f*x]]]/(2*Sqrt[2*(1 + Sqrt[2])]*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 714

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, S
ubst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x]
 /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \sqrt{1+\tan(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{1+x}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{2\text{Subst}\left(\int \frac{x^2}{2-2x^2+x^4}\,dx, x, \sqrt{1+\tan(e+fx)}\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{2}-x^2}{2-2x^2+x^4}\,dx, x, \sqrt{1+\tan(e+fx)}\right)}{f} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+x^2}{2-2x^2+x^4}\,dx, x, \sqrt{1\right.}}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2}\,dx, x, \sqrt{1+\tan(e+fx)}\right)}{2f} + \frac{\text{Subst}\left(\int -\right.}{}$$

$$= \frac{\log\left(1+\sqrt{2}+\tan(e+fx)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\tan(e+fx)}\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}\,f} - \frac{\log\left(1\right.}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\tan(e+fx)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{\sqrt{2\left(-1+\sqrt{2}\right)}\,f} + \frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(-\right.}}\right.}{\sqrt{2\left(-1+\right.}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.03, size = 67, normalized size = 0.27

$$-\frac{i\left(\sqrt{1-i}\,\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1-i}}\right)-\sqrt{1+i}\,\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1+i}}\right)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Tan[e + f*x]],x]
[Out] ((-I)*(Sqrt[1 - I]*ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1 - I]] - Sqrt[1 + I
]*ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1 + I]]))/f
```

Maple [A]
time = 0.23, size = 217, normalized size = 0.88

method	result
derivativedivides	$-\frac{\sqrt{2\sqrt{2}+2}\,\left(\sqrt{2}-1\right)\left(-\frac{\ln\left(1+\sqrt{2}-\sqrt{2\sqrt{2}+2}\,\sqrt{1+\tan(fx+e)}+\tan(fx+e)\right)}{2}-\frac{\sqrt{2\sqrt{2}+\cdots}}{\cdots}\right.}{2}$
default	$-\frac{\sqrt{2\sqrt{2}+2}\,\left(\sqrt{2}-1\right)\left(-\frac{\ln\left(1+\sqrt{2}-\sqrt{2\sqrt{2}+2}\,\sqrt{1+\tan(fx+e)}+\tan(fx+e)\right)}{2}-\frac{\sqrt{2\sqrt{2}+\cdots}}{\cdots}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/2*(2*2^(1/2)+2)^(1/2)*(2^(1/2)-1)*(-1/2*ln(1+2^(1/2)-(2*2^(1/2)+2)^
(1/2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))-(2*2^(1/2)+2)^(1/2)/(-2+2*2^(1/2))^(
1/2)*arctan((2*(1+tan(f*x+e))^(1/2)-(2*2^(1/2)+2)^(1/2))/(-2+2*2^(1/2))^(1/
2)))-1/2*(2*2^(1/2)+2)^(1/2)*(2^(1/2)-1)*(1/2*ln(1+2^(1/2)+(2*2^(1/2)+2)^(1
/2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))-(2*2^(1/2)+2)^(1/2)/(-2+2*2^(1/2))^(1/
2)*arctan(((2*2^(1/2)+2)^(1/2)+2*(1+tan(f*x+e))^(1/2))/(-2+2*2^(1/2))^(1/2)
)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  1which is not of the expected type LIST
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 805 vs. $2(194) = 388$.
time = 1.52, size = 805, normalized size = 3.26

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*2^(1/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*(sqrt(2)*f^2*sqrt(f^(-4))
- 2)*(f^(-4))^(1/4)*log(1/2*(2^(3/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f
^3*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4)*cos(f*x
+ e) + 2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) + 2*cos(f*x + e) + 2*sin(f*x
 + e))/cos(f*x + e)) - 1/8*2^(1/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*(sq
rt(2)*f^2*sqrt(f^(-4)) - 2)*(f^(-4))^(1/4)*log(-1/2*(2^(3/4)*sqrt(2*sqrt(2)
*f^2*sqrt(f^(-4)) + 4)*f^3*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))
*(f^(-4))^(3/4)*cos(f*x + e) - 2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) - 2*
cos(f*x + e) - 2*sin(f*x + e))/cos(f*x + e)) - 1/2*2^(3/4)*sqrt(2*sqrt(2)*f
^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*arctan(1/2*2^(3/4)*sqrt(1/2)*sqrt(2*sqr
t(2)*f^2*sqrt(f^(-4)) + 4)*f^5*sqrt((2^(3/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)
) + 4)*f^3*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4)*
cos(f*x + e) + 2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) + 2*cos(f*x + e) + 2
*sin(f*x + e))/cos(f*x + e))*(f^(-4))^(5/4) - 1/2*2^(3/4)*sqrt(2*sqrt(2)*f^
2*sqrt(f^(-4)) + 4)*f^5*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f
^(-4))^(5/4) - f^2*sqrt(f^(-4)) - sqrt(2)) - 1/2*2^(3/4)*sqrt(2*sqrt(2)*f^2
*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*arctan(1/2*2^(3/4)*sqrt(1/2)*sqrt(2*sqrt(
2)*f^2*sqrt(f^(-4)) + 4)*f^5*sqrt(-(2^(3/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4))
 + 4)*f^3*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4)*c
os(f*x + e) - 2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) - 2*cos(f*x + e) - 2*
sin(f*x + e))/cos(f*x + e))*(f^(-4))^(5/4) - 1/2*2^(3/4)*sqrt(2*sqrt(2)*f^2
*sqrt(f^(-4)) + 4)*f^5*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^
(-4))^(5/4) + f^2*sqrt(f^(-4)) + sqrt(2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\tan(e+fx)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(tan(e + f*x) + 1), x)
```

Giac [A]
time = 0.79, size = 266, normalized size = 1.08

$$\frac{\left(f^2\sqrt{\sqrt{2}+1}+f\sqrt{\sqrt{2}-1}\,|f|\right)\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{2f^3}+\frac{\left(f^2\sqrt{\sqrt{2}+1}+f\sqrt{\sqrt{2}-1}\,|f|\right)\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{2f^3}+\frac{\left(f^2\sqrt{\sqrt{2}-1}-f\sqrt{\sqrt{2}+1}\,|f|\right)\log\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{4f^3}-\frac{\left(f^2\sqrt{\sqrt{2}-1}-f\sqrt{\sqrt{2}+1}\,|f|\right)\log\left(-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{4f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(f^2*sqrt(sqrt(2) + 1) + f*sqrt(sqrt(2) - 1)*abs(f))*arctan(1/2*2^(3/4)
*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*sqrt(tan(f*x + e) + 1))/sqrt(-sqrt(2) + 2))
/f^3 + 1/2*(f^2*sqrt(sqrt(2) + 1) + f*sqrt(sqrt(2) - 1)*abs(f))*arctan(-1/2
```

```
*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(tan(f*x + e) + 1))/sqrt(-sqrt(
2) + 2))/f^3 + 1/4*(f^2*sqrt(sqrt(2) - 1) - f*sqrt(sqrt(2) + 1)*abs(f))*log
(2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x + e) + 1) + sqrt(2) + tan(f*x + e)
+ 1)/f^3 - 1/4*(f^2*sqrt(sqrt(2) - 1) - f*sqrt(sqrt(2) + 1)*abs(f))*log(-2^
(1/4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x + e) + 1) + sqrt(2) + tan(f*x + e) + 1
)/f^3
```

Mupad [B]
time = 0.18, size = 73, normalized size = 0.30

$$2\,\mathrm{atanh}\left(4\,f^3\,{\left(\frac{-\frac{1}{4}-\frac{1}{4}\mathrm{i}}{f^2}\right)}^{3/2}\,\sqrt{\tan\left(e+f\,x\right)+1}\right)\,\sqrt{\frac{-\frac{1}{4}-\frac{1}{4}\mathrm{i}}{f^2}}+2\,\mathrm{atanh}\left(4\,f^3\,{\left(\frac{-\frac{1}{4}+\frac{1}{4}\mathrm{i}}{f^2}\right)}^{3/2}\,\sqrt{\tan\left(e+f\,x\right)+1}\right)\,\sqrt{\frac{-\frac{1}{4}+\frac{1}{4}\mathrm{i}}{f^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(e + f*x) + 1)^(1/2),x)
```

```
[Out] 2*atanh(4*f^3*((- 1/4 - 1i/4)/f^2)^(3/2)*(tan(e + f*x) + 1)^(1/2))*((- 1/4
- 1i/4)/f^2)^(1/2) + 2*atanh(4*f^3*((- 1/4 + 1i/4)/f^2)^(3/2)*(tan(e + f*x)
 + 1)^(1/2))*((- 1/4 + 1i/4)/f^2)^(1/2)
```

3.387 $\int \cot^2(e+fx)\sqrt{1+\tan(e+fx)}\,dx$

Optimal. Leaf size=288

$$\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\tan(e+fx)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{f}-\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{\right.}}}{\sqrt{}}\right)}{f}$$

```
[Out] -arctanh((1+tan(f*x+e))^(1/2))/f+1/2*arctan(((2+2*2^(1/2))^(1/2)-2*(1+tan(f
*x+e))^(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)/f-1/2*arctan(((2+2*
2^(1/2))^(1/2)+2*(1+tan(f*x+e))^(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^
(1/2)/f-1/2*ln(1+2^(1/2)-(2+2*2^(1/2))^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e
))/f/(2+2*2^(1/2))^(1/2)+1/2*ln(1+2^(1/2)+(2+2*2^(1/2))^(1/2)*(1+tan(f*x+e)
)^(1/2)+tan(f*x+e))/f/(2+2*2^(1/2))^(1/2)-cot(f*x+e)*(1+tan(f*x+e))^(1/2)/f
```

Rubi [A]

time = 0.25, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3649, 3734, 3566, 714, 1141, 1175, 632, 210, 1178, 642, 3715, 65, 213}

$$\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\tan(e+fx)+1}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{f}-\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{2\sqrt{\tan(e+fx)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{f}-\frac{\log\left(\tan(e+fx)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\tan(e+fx)+1}+\sqrt{2}+1\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}f}+\frac{\log\left(\tan(e+fx)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\tan(e+fx)+1}+\sqrt{2}+1\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}f}-\frac{\tanh^{-1}\left(\sqrt{\tan(e+fx)+1}\right)}{f}-\frac{\sqrt{\tan(e+fx)+1}\cot(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] (Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + Tan[e + f
*x]])/Sqrt[2*(-1 + Sqrt[2])]])/f - (Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1
 + Sqrt[2])] + 2*Sqrt[1 + Tan[e + f*x]])/Sqrt[2*(-1 + Sqrt[2])]])/f - ArcTa
nh[Sqrt[1 + Tan[e + f*x]]]/f - Log[1 + Sqrt[2] + Tan[e + f*x] - Sqrt[2*(1 +
 Sqrt[2])]*Sqrt[1 + Tan[e + f*x]]]/(2*Sqrt[2*(1 + Sqrt[2])]*f) + Log[1 + Sq
rt[2] + Tan[e + f*x] + Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + Tan[e + f*x]]]/(2*Sqr
t[2*(1 + Sqrt[2])]*f) - (Cot[e + f*x]*Sqrt[1 + Tan[e + f*x]])/f
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 714

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, S
ubst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x]
 /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \cot^2(e+fx)\sqrt{1+\tan(e+fx)}\,dx = -\frac{\cot(e+fx)\sqrt{1+\tan(e+fx)}}{f} - \int \frac{\cot(e+fx)\left(-\frac{1}{2}+\tan(e+\right.}{\sqrt{1+\tan(e}}$$

$$= -\frac{\cot(e+fx)\sqrt{1+\tan(e+fx)}}{f} + \frac{1}{2}\int \frac{\cot(e+fx)\left(1+\tan^2(e+\right.}{\sqrt{1+\tan(e+fx)}}$$

$$= -\frac{\cot(e+fx)\sqrt{1+\tan(e+fx)}}{f} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \tan(e\right.}{2f}$$

$$= -\frac{\cot(e+fx)\sqrt{1+\tan(e+fx)}}{f} + \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\tan\right.}{f}$$

$$= -\frac{\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{f} - \frac{\cot(e+fx)\sqrt{1+\tan(e+fx)}}{f}$$

$$= -\frac{\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{f} - \frac{\cot(e+fx)\sqrt{1+\tan(e+fx)}}{f}$$

$$= -\frac{\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{f} - \frac{\log\left(1+\sqrt{2}+\tan(e+fx)-\right.}{2\sqrt{2\left(\right.}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\tan(e+fx)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{\sqrt{2\left(-1+\sqrt{2}\right)}\,f} - \frac{\tan^{-1}\left(\frac{\sqrt{2\left(1\right.}}{}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.26, size = 102, normalized size = 0.35

$$-\frac{\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)-i\sqrt{1-i}\,\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1-i}}\right)+i\sqrt{1+i}\,\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1+i}}\right)+\cot(e+fx)\sqrt{1+\tan(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] -((ArcTanh[Sqrt[1 + Tan[e + f*x]]] - I*Sqrt[1 - I]*ArcTanh[Sqrt[1 + Tan[e +
 f*x]]/Sqrt[1 - I]] + I*Sqrt[1 + I]*ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1 +
 I]] + Cot[e + f*x]*Sqrt[1 + Tan[e + f*x]])/f)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.80, size = 8262, normalized size = 28.69

method	result	size
default	Expression too large to display	8262

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(1+tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(1+tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(tan(f*x + e) + 1)*cot(f*x + e)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1058 vs. 2(234) = 468.
time = 1.15, size = 1058, normalized size = 3.67

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(1+tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(2^(1/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*(2*f*cos(f*x + e)^2 - sqr
t(2)*(f^3*cos(f*x + e)^2 - f^3)*sqrt(f^(-4)) - 2*f)*(f^(-4))^(1/4)*log(1/2*
(2^(3/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^3*sqrt((cos(f*x + e) + sin(
f*x + e))/cos(f*x + e))*(f^(-4))^(3/4)*cos(f*x + e) + 2*sqrt(2)*f^2*sqrt(f^
(-4))*cos(f*x + e) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e)) - 2^(1/
4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*(2*f*cos(f*x + e)^2 - sqrt(2)*(f^3*
cos(f*x + e)^2 - f^3)*sqrt(f^(-4)) - 2*f)*(f^(-4))^(1/4)*log(-1/2*(2^(3/4)*
sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^3*sqrt((cos(f*x + e) + sin(f*x + e))
/cos(f*x + e))*(f^(-4))^(3/4)*cos(f*x + e) - 2*sqrt(2)*f^2*sqrt(f^(-4))*cos
(f*x + e) - 2*cos(f*x + e) - 2*sin(f*x + e))/cos(f*x + e)) + 8*sqrt((cos(f*
x + e) + sin(f*x + e))/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 4*(cos(f*x
 + e)^2 - 1)*log(sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e)) + 1) + 4*
```

```
(cos(f*x + e)^2 - 1)*log(sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e)) -
 1) + 4*2^(3/4)*(f^5*cos(f*x + e)^2 - f^5)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4))
+ 4)*(f^(-4))^(1/4)*arctan(1/2*2^(3/4)*sqrt(1/2)*sqrt(2*sqrt(2)*f^2*sqrt(f^
(-4)) + 4)*f^5*sqrt((2^(3/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^3*sqrt(
(cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4)*cos(f*x + e) + 2
*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) + 2*cos(f*x + e) + 2*sin(f*x + e))/c
os(f*x + e))*(f^(-4))^(5/4) - 1/2*2^(3/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) +
 4)*f^5*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(5/4) - f
^2*sqrt(f^(-4)) - sqrt(2))/f^4 + 4*2^(3/4)*(f^5*cos(f*x + e)^2 - f^5)*sqrt(
2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*arctan(1/2*2^(3/4)*sqrt(1/2)
*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^5*sqrt(-(2^(3/4)*sqrt(2*sqrt(2)*f^2
*sqrt(f^(-4)) + 4)*f^3*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^
(-4))^(3/4)*cos(f*x + e) - 2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) - 2*cos(
f*x + e) - 2*sin(f*x + e))/cos(f*x + e))*(f^(-4))^(5/4) - 1/2*2^(3/4)*sqrt(
2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^5*sqrt((cos(f*x + e) + sin(f*x + e))/cos(
f*x + e))*(f^(-4))^(5/4) + f^2*sqrt(f^(-4)) + sqrt(2))/f^4)/(f*cos(f*x + e)
^2 - f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\tan(e+fx)+1}\,\cot^2(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(1+tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(tan(e + f*x) + 1)*cot(e + f*x)**2, x)
```

Giac [A]
time = 1.10, size = 330, normalized size = 1.15

$$-\frac{\log\left(\sqrt{\tan(fx+e)+1}+1\right)}{2f}+\frac{\log\left(\left|\sqrt{\tan(fx+e)+1}-1\right|\right)}{2f}-\frac{\left(f^2\sqrt{\sqrt{2}+1}+f\sqrt{\sqrt{2}-1}\,|f|\right)\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{2f^3}-\frac{\left(f^2\sqrt{\sqrt{2}+1}+f\sqrt{\sqrt{2}-1}\,|f|\right)\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{2f^3}-\frac{\left(f^2\sqrt{\sqrt{2}-1}-f\sqrt{\sqrt{2}+1}\,|f|\right)\log\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{4f^3}+\frac{\left(f^2\sqrt{\sqrt{2}-1}-f\sqrt{\sqrt{2}+1}\,|f|\right)\log\left(-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{4f^3}-\frac{\sqrt{\tan(fx+e)+1}}{f\tan(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(1+tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(sqrt(tan(f*x + e) + 1) + 1)/f + 1/2*log(abs(sqrt(tan(f*x + e) + 1)
 - 1))/f - 1/2*(f^2*sqrt(sqrt(2) + 1) + f*sqrt(sqrt(2) - 1)*abs(f))*arctan(
1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*sqrt(tan(f*x + e) + 1))/sqrt(-sq
rt(2) + 2))/f^3 - 1/2*(f^2*sqrt(sqrt(2) + 1) + f*sqrt(sqrt(2) - 1)*abs(f))*
arctan(-1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(tan(f*x + e) + 1))/
sqrt(-sqrt(2) + 2))/f^3 - 1/4*(f^2*sqrt(sqrt(2) - 1) - f*sqrt(sqrt(2) + 1)*
abs(f))*log(2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x + e) + 1) + sqrt(2) + ta
n(f*x + e) + 1)/f^3 + 1/4*(f^2*sqrt(sqrt(2) - 1) - f*sqrt(sqrt(2) + 1)*abs(
```

```
f))*log(-2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x + e) + 1) + sqrt(2) + tan(f
*x + e) + 1)/f^3 - sqrt(tan(f*x + e) + 1)/(f*tan(f*x + e))
```

Mupad [B]
time = 3.89, size = 121, normalized size = 0.42

$$\frac{\mathrm{atan}\left(\sqrt{\tan\left(e+f\,x\right)+1}\,1\mathrm{i}\right)\,1\mathrm{i}}{f}+\frac{\sqrt{\tan\left(e+f\,x\right)+1}}{f-f\,\left(\tan\left(e+f\,x\right)+1\right)}+\mathrm{atan}\left(f\,\sqrt{\frac{-\frac{1}{4}-\frac{1}{4}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\,\left(1-\mathrm{i}\right)\right)\,\sqrt{\frac{-\frac{1}{4}-\frac{1}{4}\mathrm{i}}{f^2}}\,2\mathrm{i}-\mathrm{atan}\left(f\,\sqrt{\frac{-\frac{1}{4}+\frac{1}{4}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\,\left(1+1\mathrm{i}\right)\right)\,\sqrt{\frac{-\frac{1}{4}+\frac{1}{4}\mathrm{i}}{f^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(tan(e + f*x) + 1)^(1/2),x)
```

```
[Out] (atan((tan(e + f*x) + 1)^(1/2)*1i)*1i)/f + (tan(e + f*x) + 1)^(1/2)/(f - f*
(tan(e + f*x) + 1)) + atan(f*((- 1/4 - 1i/4)/f^2)^(1/2)*(tan(e + f*x) + 1)^
(1/2)*(1 - 1i))*((- 1/4 - 1i/4)/f^2)^(1/2)*2i - atan(f*((- 1/4 + 1i/4)/f^2)
^(1/2)*(tan(e + f*x) + 1)^(1/2)*(1 + 1i))*((- 1/4 + 1i/4)/f^2)^(1/2)*2i
```

3.388 $\int \cot^4(e+fx)\sqrt{1+\tan(e+fx)}\,dx$

Optimal. Leaf size=346

$$-\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\mathrm{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\tan(e+fx)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{f}+\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\mathrm{ArcTan}\left(\frac{\sqrt{2\left(1+\cdots\right.}}{\cdots}\right.}{f}$$

```
[Out] 7/8*arctanh((1+tan(f*x+e))^(1/2))/f-1/2*arctan(((2+2*2^(1/2))^(1/2)-2*(1+ta
n(f*x+e))^(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)/f+1/2*arctan(((2
+2*2^(1/2))^(1/2)+2*(1+tan(f*x+e))^(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2
))^(1/2)/f+1/2*ln(1+2^(1/2)-(2+2*2^(1/2))^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f*
x+e))/f/(2+2*2^(1/2))^(1/2)-1/2*ln(1+2^(1/2)+(2+2*2^(1/2))^(1/2)*(1+tan(f*x
+e))^(1/2)+tan(f*x+e))/f/(2+2*2^(1/2))^(1/2)+9/8*cot(f*x+e)*(1+tan(f*x+e))^
(1/2)/f-1/12*cot(f*x+e)^2*(1+tan(f*x+e))^(1/2)/f-1/3*cot(f*x+e)^3*(1+tan(f*
x+e))^(1/2)/f
```

Rubi [A]
time = 0.43, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3649, 3730, 3734, 21, 3566, 714, 1141, 1175, 632, 210, 1178, 642, 3715, 65, 213}

$$-\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\mathrm{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\tan(e+fx)+1}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{f}+\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\mathrm{ArcTan}\left(\frac{2\sqrt{\tan(e+fx)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{f}+\frac{\log\left(\tan(e+fx)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\tan(e+fx)+1}+\sqrt{2}+1\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}f}-\frac{\log\left(\tan(e+fx)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\tan(e+fx)+1}+\sqrt{2}+1\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}f}+\frac{7\tanh^{-1}\left(\sqrt{\tan(e+fx)+1}\right)}{8f}-\frac{\sqrt{\tan(e+fx)+1}\cot^3(e+fx)}{3f}-\frac{\sqrt{\tan(e+fx)+1}\cot^2(e+fx)}{12f}+\frac{9\sqrt{\tan(e+fx)+1}\cot(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] -((Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + Tan[e +
 f*x]])/Sqrt[2*(-1 + Sqrt[2])]])/f) + (Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2
*(1 + Sqrt[2])] + 2*Sqrt[1 + Tan[e + f*x]])/Sqrt[2*(-1 + Sqrt[2])]])/f + (7
*ArcTanh[Sqrt[1 + Tan[e + f*x]]])/(8*f) + Log[1 + Sqrt[2] + Tan[e + f*x] -
Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + Tan[e + f*x]]]/(2*Sqrt[2*(1 + Sqrt[2])]*f) -
 Log[1 + Sqrt[2] + Tan[e + f*x] + Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + Tan[e + f*
x]]]/(2*Sqrt[2*(1 + Sqrt[2])]*f) + (9*Cot[e + f*x]*Sqrt[1 + Tan[e + f*x]])/
(8*f) - (Cot[e + f*x]^2*Sqrt[1 + Tan[e + f*x]])/(12*f) - (Cot[e + f*x]^3*Sq
rt[1 + Tan[e + f*x]])/(3*f)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 714

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, S
ubst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x]
 /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
```

```
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \cot^4(e+fx)\sqrt{1+\tan(e+fx)}\,dx = -\frac{\cot^3(e+fx)\sqrt{1+\tan(e+fx)}}{3f} - \frac{1}{3}\int \frac{\cot^3(e+fx)\left(-\frac{1}{2}+3\tan\right.}{\sqrt{1+\tan}}$$

$$= -\frac{\cot^2(e+fx)\sqrt{1+\tan(e+fx)}}{12f} - \frac{\cot^3(e+fx)\sqrt{1+\tan(e+fx)}}{3f}$$

$$= \frac{9\cot(e+fx)\sqrt{1+\tan(e+fx)}}{8f} - \frac{\cot^2(e+fx)\sqrt{1+\tan(e+fx)}}{12f}$$

$$= \frac{9\cot(e+fx)\sqrt{1+\tan(e+fx)}}{8f} - \frac{\cot^2(e+fx)\sqrt{1+\tan(e+fx)}}{12f}$$

$$= \frac{9\cot(e+fx)\sqrt{1+\tan(e+fx)}}{8f} - \frac{\cot^2(e+fx)\sqrt{1+\tan(e+fx)}}{12f}$$

$$= \frac{9\cot(e+fx)\sqrt{1+\tan(e+fx)}}{8f} - \frac{\cot^2(e+fx)\sqrt{1+\tan(e+fx)}}{12f}$$

$$= \frac{7\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{8f} + \frac{9\cot(e+fx)\sqrt{1+\tan(e+fx)}}{8f}$$

$$= \frac{7\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{8f} + \frac{9\cot(e+fx)\sqrt{1+\tan(e+fx)}}{8f}$$

$$= \frac{7\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{8f} + \frac{9\cot(e+fx)\sqrt{1+\tan(e+fx)}}{8f}$$

$$= \frac{7\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{8f} + \frac{\log\left(1+\sqrt{2}+\tan(e+fx)-\right.}{2\sqrt{2\left(1\right.}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\tan(e+fx)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{\sqrt{2\left(-1+\sqrt{2}\right)}\,f} + \frac{\tan^{-1}\left(\frac{\sqrt{2\left(\right.}}{}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.44, size = 151, normalized size = 0.44

$$\frac{21\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)-24i\sqrt{1-i}\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1-i}}\right)+24i\sqrt{1+i}\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1+i}}\right)+27\cot(e+fx)\sqrt{1+\tan(e+fx)}-2\cot^2(e+fx)\sqrt{1+\tan(e+fx)}-8\cot^3(e+fx)\sqrt{1+\tan(e+fx)}}{24f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] (21*ArcTanh[Sqrt[1 + Tan[e + f*x]]] - (24*I)*Sqrt[1 - I]*ArcTanh[Sqrt[1 + T
an[e + f*x]]/Sqrt[1 - I]] + (24*I)*Sqrt[1 + I]*ArcTanh[Sqrt[1 + Tan[e + f*x
]]/Sqrt[1 + I]] + 27*Cot[e + f*x]*Sqrt[1 + Tan[e + f*x]] - 2*Cot[e + f*x]^2
*Sqrt[1 + Tan[e + f*x]] - 8*Cot[e + f*x]^3*Sqrt[1 + Tan[e + f*x]])/(24*f)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.87, size = 13941, normalized size = 40.29

method	result	size
default	Expression too large to display	13941

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(1+tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(1+tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(tan(f*x + e) + 1)*cot(f*x + e)^4, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1202 vs. 2(284) = 568.
time = 1.18, size = 1202, normalized size = 3.47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(1+tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/48*(6*2^(1/4)*(2*f*cos(f*x + e)^4 - 4*f*cos(f*x + e)^2 - sqrt(2)*(f^3*co
s(f*x + e)^4 - 2*f^3*cos(f*x + e)^2 + f^3)*sqrt(f^(-4)) + 2*f)*sqrt(2*sqrt(
```

```
2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*log(1/2*(2^(3/4)*sqrt(2*sqrt(2)*f^2
*sqrt(f^(-4)) + 4)*f^3*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^
(-4))^(3/4)*cos(f*x + e) + 2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) + 2*cos(
f*x + e) + 2*sin(f*x + e))/cos(f*x + e)) - 6*2^(1/4)*(2*f*cos(f*x + e)^4 -
4*f*cos(f*x + e)^2 - sqrt(2)*(f^3*cos(f*x + e)^4 - 2*f^3*cos(f*x + e)^2 + f
^3)*sqrt(f^(-4)) + 2*f)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)
*log(-1/2*(2^(3/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^3*sqrt((cos(f*x +
 e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4)*cos(f*x + e) - 2*sqrt(2)*f
^2*sqrt(f^(-4))*cos(f*x + e) - 2*cos(f*x + e) - 2*sin(f*x + e))/cos(f*x + e
)) - 21*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*log(sqrt((cos(f*x + e) + si
n(f*x + e))/cos(f*x + e)) + 1) + 21*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)
*log(sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e)) - 1) - 2*(2*cos(f*x +
 e)^4 - 2*cos(f*x + e)^2 - (35*cos(f*x + e)^3 - 27*cos(f*x + e))*sin(f*x +
e))*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e)) + 24*2^(3/4)*(f^5*cos(
f*x + e)^4 - 2*f^5*cos(f*x + e)^2 + f^5)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) +
4)*(f^(-4))^(1/4)*arctan(1/2*2^(3/4)*sqrt(1/2)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-
4)) + 4)*f^5*sqrt((2^(3/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^3*sqrt((c
os(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4)*cos(f*x + e) + 2*s
qrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos
(f*x + e))*(f^(-4))^(5/4) - 1/2*2^(3/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4
)*f^5*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(5/4) - f^2
*sqrt(f^(-4)) - sqrt(2))/f^4 + 24*2^(3/4)*(f^5*cos(f*x + e)^4 - 2*f^5*cos(f
*x + e)^2 + f^5)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*arctan
(1/2*2^(3/4)*sqrt(1/2)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^5*sqrt(-(2^(3
/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^3*sqrt((cos(f*x + e) + sin(f*x +
 e))/cos(f*x + e))*(f^(-4))^(3/4)*cos(f*x + e) - 2*sqrt(2)*f^2*sqrt(f^(-4))
*cos(f*x + e) - 2*cos(f*x + e) - 2*sin(f*x + e))/cos(f*x + e))*(f^(-4))^(5/
4) - 1/2*2^(3/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^5*sqrt((cos(f*x + e
) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(5/4) + f^2*sqrt(f^(-4)) + sqrt(2)
)/f^4)/(f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^2 + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\tan \left(e + f x\right) + 1} \cot^{4}{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(1+tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(tan(e + f*x) + 1)*cot(e + f*x)**4, x)
```

Giac [A]
time = 1.20, size = 359, normalized size = 1.04

$$\frac{7 \log\left(\sqrt{\tan\left(fx+e\right)+1}+1\right)}{16\,f} - \frac{7 \log\left(\left|\sqrt{\tan\left(fx+e\right)+1}-1\right|\right)}{16\,f} + \frac{\left(f^{2}\sqrt{\sqrt{2}+1}+f\sqrt{\sqrt{2}-1}\,|f|\right)\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{\tan\left(fx+e\right)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{2\,f^{3}} + \frac{\left(f^{2}\sqrt{\sqrt{2}+1}+f\sqrt{\sqrt{2}-1}\,|f|\right)\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{\tan\left(fx+e\right)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{2\,f^{3}} + \frac{\left(f^{2}\sqrt{\sqrt{2}-1}-f\sqrt{\sqrt{2}+1}\,|f|\right)\log\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan\left(fx+e\right)+1}+\sqrt{2}+\tan\left(fx+e\right)+1\right)}{4\,f^{3}} - \frac{\left(f^{2}\sqrt{\sqrt{2}-1}-f\sqrt{\sqrt{2}+1}\,|f|\right)\log\left(-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan\left(fx+e\right)+1}+\sqrt{2}+\tan\left(fx+e\right)+1\right)}{4\,f^{3}} + \frac{27\left(\tan\left(fx+e\right)+1\right)^{\frac{5}{2}}-56\left(\tan\left(fx+e\right)+1\right)^{\frac{3}{2}}+21\sqrt{\tan\left(fx+e\right)+1}}{24\,f\tan\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(1+tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 7/16*log(sqrt(tan(f*x + e) + 1) + 1)/f - 7/16*log(abs(sqrt(tan(f*x + e) + 1
) - 1))/f + 1/2*(f^2*sqrt(sqrt(2) + 1) + f*sqrt(sqrt(2) - 1)*abs(f))*arctan
(1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*sqrt(tan(f*x + e) + 1))/sqrt(-s
qrt(2) + 2))/f^3 + 1/2*(f^2*sqrt(sqrt(2) + 1) + f*sqrt(sqrt(2) - 1)*abs(f))
*arctan(-1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(tan(f*x + e) + 1))
/sqrt(-sqrt(2) + 2))/f^3 + 1/4*(f^2*sqrt(sqrt(2) - 1) - f*sqrt(sqrt(2) + 1)
*abs(f))*log(2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x + e) + 1) + sqrt(2) + t
an(f*x + e) + 1)/f^3 - 1/4*(f^2*sqrt(sqrt(2) - 1) - f*sqrt(sqrt(2) + 1)*abs
(f))*log(-2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x + e) + 1) + sqrt(2) + tan(
f*x + e) + 1)/f^3 + 1/24*(27*(tan(f*x + e) + 1)^(5/2) - 56*(tan(f*x + e) +
1)^(3/2) + 21*sqrt(tan(f*x + e) + 1))/(f*tan(f*x + e)^3)
```

Mupad [B]
time = 0.19, size = 175, normalized size = 0.51

$$-\frac{\mathrm{atan}\left(\sqrt{\tan\left(e+f\,x\right)+1}\,1{}\mathrm{i}\right)\,7{}\mathrm{i}}{8\,f}-\frac{\frac{7\,\sqrt{\tan\left(e+f\,x\right)+1}}{8}-\frac{7\,{\left(\tan\left(e+f\,x\right)+1\right)}^{3/2}}{3}+\frac{9\,{\left(\tan\left(e+f\,x\right)+1\right)}^{5/2}}{8}}{f-3\,f\,\left(\tan\left(e+f\,x\right)+1\right)+3\,f\,{\left(\tan\left(e+f\,x\right)+1\right)}^2-f\,{\left(\tan\left(e+f\,x\right)+1\right)}^3}-\mathrm{atan}\left(f\,\sqrt{\frac{-\frac{1}{4}-\frac{1}{4}{}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\,\left(1-{}\mathrm{i}\right)\right)\,\sqrt{\frac{-\frac{1}{4}-\frac{1}{4}{}\mathrm{i}}{f^2}}\,2{}\mathrm{i}+\mathrm{atan}\left(f\,\sqrt{\frac{-\frac{1}{4}+\frac{1}{4}{}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\,\left(1+1{}\mathrm{i}\right)\right)\,\sqrt{\frac{-\frac{1}{4}+\frac{1}{4}{}\mathrm{i}}{f^2}}\,2{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(tan(e + f*x) + 1)^(1/2),x)
```

```
[Out] atan(f*((- 1/4 + 1i/4)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1/2)*(1 + 1i))*((- 1/
4 + 1i/4)/f^2)^(1/2)*2i - ((7*(tan(e + f*x) + 1)^(1/2))/8 - (7*(tan(e + f*x
) + 1)^(3/2))/3 + (9*(tan(e + f*x) + 1)^(5/2))/8)/(f - 3*f*(tan(e + f*x) +
1) + 3*f*(tan(e + f*x) + 1)^2 - f*(tan(e + f*x) + 1)^3) - atan(f*((- 1/4 -
1i/4)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1/2)*(1 - 1i))*((- 1/4 - 1i/4)/f^2)^(1
/2)*2i - (atan((tan(e + f*x) + 1)^(1/2)*1i)*7i)/(8*f)
```

3.389 $\int \tan^5(e+fx)(1+\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=369

$$\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\tan(e+fx)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{f}-\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+\cdots}}{\sqrt{2\left(-1+\cdots\right.}}\right)}{f}$$

```
[Out] 1/2*ln(1+2^(1/2)-(2+2*2^(1/2))^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))/f/(1+
2^(1/2))^(1/2)-1/2*ln(1+2^(1/2)+(2+2*2^(1/2))^(1/2)*(1+tan(f*x+e))^(1/2)+ta
n(f*x+e))/f/(1+2^(1/2))^(1/2)+arctan(((2+2*2^(1/2))^(1/2)-2*(1+tan(f*x+e))^
(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^(1/2))^(1/2)/f-arctan(((2+2*2^(1/2))^(1/2
)+2*(1+tan(f*x+e))^(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^(1/2))^(1/2)/f+2*(1+ta
n(f*x+e))^(1/2)/f+2/3*(1+tan(f*x+e))^(3/2)/f+20/231*(1+tan(f*x+e))^(5/2)/f-
50/231*tan(f*x+e)*(1+tan(f*x+e))^(5/2)/f-4/33*tan(f*x+e)^2*(1+tan(f*x+e))^(
5/2)/f+2/11*tan(f*x+e)^3*(1+tan(f*x+e))^(5/2)/f
```

Rubi [A]

time = 0.38, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3647, 3728, 3729, 3711, 12, 3609, 3566, 722, 1108, 648, 632, 210, 642}

$$\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\tan(e+fx)+1}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{f}-\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{2\sqrt{\tan(e+fx)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{f}+\frac{2(\tan(e+fx)+1)^{5/2}\tan^3(e+fx)}{11f}-\frac{4(\tan(e+fx)+1)^{5/2}\tan^2(e+fx)}{33f}-\frac{50(\tan(e+fx)+1)^{5/2}\tan(e+fx)}{231f}+\frac{20(\tan(e+fx)+1)^{5/2}}{231f}+\frac{2(\tan(e+fx)+1)^{3/2}}{3f}+\frac{2\sqrt{\tan(e+fx)+1}}{f}+\frac{\log\left(\tan(e+fx)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\tan(e+fx)+1}+\sqrt{2}+1\right)}{2\sqrt{1+\sqrt{2}}\,f}-\frac{\log\left(\tan(e+fx)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\tan(e+fx)+1}+\sqrt{2}+1\right)}{2\sqrt{1+\sqrt{2}}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5*(1 + Tan[e + f*x])^(3/2),x]
```

```
[Out] (Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + Tan[e + f*x]]
)/Sqrt[2*(-1 + Sqrt[2])]]])/f - (Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[
2])] + 2*Sqrt[1 + Tan[e + f*x]])/Sqrt[2*(-1 + Sqrt[2])]]])/f + Log[1 + Sqrt[
2] + Tan[e + f*x] - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + Tan[e + f*x]]]/(2*Sqrt[1
 + Sqrt[2]]*f) - Log[1 + Sqrt[2] + Tan[e + f*x] + Sqrt[2*(1 + Sqrt[2])]*Sqr
t[1 + Tan[e + f*x]]]/(2*Sqrt[1 + Sqrt[2]]*f) + (2*Sqrt[1 + Tan[e + f*x]])/f
 + (2*(1 + Tan[e + f*x])^(3/2))/(3*f) + (20*(1 + Tan[e + f*x])^(5/2))/(231*
f) - (50*Tan[e + f*x]*(1 + Tan[e + f*x])^(5/2))/(231*f) - (4*Tan[e + f*x]^2
*(1 + Tan[e + f*x])^(5/2))/(33*f) + (2*Tan[e + f*x]^3*(1 + Tan[e + f*x])^(5
/2))/(11*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 722

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
```

```
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3729

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m + n +
1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*
Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*(b*c*m + a*d*(n + 1)) + d*(A*b -
 b*C)*(m + n + 1)*Tan[e + f*x] - C*m*(b*c - a*d)*Tan[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 +
```

```
 b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[
m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \tan^5(e+fx)(1+\tan(e+fx))^{3/2}\,dx = \frac{2\tan^3(e+fx)(1+\tan(e+fx))^{5/2}}{11f} + \frac{2}{11}\int \tan^2(e+fx)(1+\tan($$

$$= -\frac{4\tan^2(e+fx)(1+\tan(e+fx))^{5/2}}{33f} + \frac{2\tan^3(e+fx)(1+\tan(e}{11f}$$

$$= -\frac{50\tan(e+fx)(1+\tan(e+fx))^{5/2}}{231f} - \frac{4\tan^2(e+fx)(1+\tan(e}{33f}$$

$$= \frac{20(1+\tan(e+fx))^{5/2}}{231f} - \frac{50\tan(e+fx)(1+\tan(e+fx))^{5/2}}{231f} - 4$$

$$= \frac{20(1+\tan(e+fx))^{5/2}}{231f} - \frac{50\tan(e+fx)(1+\tan(e+fx))^{5/2}}{231f} - 4$$

$$= \frac{2(1+\tan(e+fx))^{3/2}}{3f} + \frac{20(1+\tan(e+fx))^{5/2}}{231f} - \frac{50\tan(e+fx)}{}$$

$$= \frac{2\sqrt{1+\tan(e+fx)}}{f} + \frac{2(1+\tan(e+fx))^{3/2}}{3f} + \frac{20(1+\tan(e+f}{231f}$$

$$= \frac{2\sqrt{1+\tan(e+fx)}}{f} + \frac{2(1+\tan(e+fx))^{3/2}}{3f} + \frac{20(1+\tan(e+f}{231f}$$

$$= \frac{2\sqrt{1+\tan(e+fx)}}{f} + \frac{2(1+\tan(e+fx))^{3/2}}{3f} + \frac{20(1+\tan(e+f}{231f}$$

$$= \frac{2\sqrt{1+\tan(e+fx)}}{f} + \frac{2(1+\tan(e+fx))^{3/2}}{3f} + \frac{20(1+\tan(e+f}{231f}$$

$$= \frac{2\sqrt{1+\tan(e+fx)}}{f} + \frac{2(1+\tan(e+fx))^{3/2}}{3f} + \frac{20(1+\tan(e+f}{231f}$$

$$= \frac{2\sqrt{1+\tan(e+fx)}}{f} + \frac{2(1+\tan(e+fx))^{3/2}}{3f} + \frac{20(1+\tan(e+f}{231f}$$

$$= \frac{\log\left(1+\sqrt{2}+\tan(e+fx)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\tan(e+fx)}\right.}{2\sqrt{1+\sqrt{2}}\,f}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\tan(e+fx)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{\sqrt{-1+\sqrt{2}}\,f} - \frac{\tan^{-1}\left(\frac{\sqrt{2\left(\right.}}{}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.90, size = 188, normalized size = 0.51

$$\frac{\cos(e+fx)\left((-1+i)\left(\sqrt{1-i}\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1-i}}\right)+i\sqrt{1+i}\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1+i}}\right)\right)\cos(e+fx)(1+\tan(e+fx))^2+\frac{2}{231}\sec^3(e+fx)(1+\tan(e+fx))^{5/2}\left(28-110\cos^2(e+fx)+400\cos^4(e+fx)+125\cos^3(e+fx)\sin(e+fx)-37\sin(2(e+fx))+21\tan(e+fx)\right)\right)}{f(\cos(e+fx)+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5*(1 + Tan[e + f*x])^(3/2),x]
```

```
[Out] (Cos[e + f*x]*((-1 + I)*(Sqrt[1 - I]*ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1
- I]] + I*Sqrt[1 + I]*ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1 + I]])*Cos[e +
f*x]*(1 + Tan[e + f*x])^2 + (2*Sec[e + f*x]^3*(1 + Tan[e + f*x])^(5/2)*(28
- 110*Cos[e + f*x]^2 + 400*Cos[e + f*x]^4 + 125*Cos[e + f*x]^3*Sin[e + f*x]
 - 37*Sin[2*(e + f*x)] + 21*Tan[e + f*x]))/231))/(f*(Cos[e + f*x] + Sin[e +
 f*x])^2)
```

Maple [A]
time = 0.13, size = 362, normalized size = 0.98

method	result
derivativedivides	$\frac{\frac{2(1+\tan(fx+e))^{\frac{11}{2}}}{11}-\frac{2(1+\tan(fx+e))^{\frac{9}{2}}}{3}+\frac{4(1+\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{2(1+\tan(fx+e))^{\frac{3}{2}}}{3}+2\sqrt{1+\tan(fx+e)}+\frac{\left(-\sqrt{2\sqrt{}}\right.}{}}{}$
default	$\frac{\frac{2(1+\tan(fx+e))^{\frac{11}{2}}}{11}-\frac{2(1+\tan(fx+e))^{\frac{9}{2}}}{3}+\frac{4(1+\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{2(1+\tan(fx+e))^{\frac{3}{2}}}{3}+2\sqrt{1+\tan(fx+e)}+\frac{\left(-\sqrt{2\sqrt{}}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5*(1+tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2/11*(1+tan(f*x+e))^(11/2)-2/3*(1+tan(f*x+e))^(9/2)+4/7*(1+tan(f*x+e))
^(7/2)+2/3*(1+tan(f*x+e))^(3/2)+2*(1+tan(f*x+e))^(1/2)+1/4*(-(2*2^(1/2)+2)^
(1/2)*2^(1/2)+2*(2*2^(1/2)+2)^(1/2))*ln(1+2^(1/2)-(2*2^(1/2)+2)^(1/2)*(1+ta
n(f*x+e))^(1/2)+tan(f*x+e))+(-2*2^(1/2)+1/2*(-(2*2^(1/2)+2)^(1/2)*2^(1/2)+2
*(2*2^(1/2)+2)^(1/2))*(2*2^(1/2)+2)^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(
1+tan(f*x+e))^(1/2)-(2*2^(1/2)+2)^(1/2))/(-2+2*2^(1/2))^(1/2))-1/4*(-(2*2^(
1/2)+2)^(1/2)*2^(1/2)+2*(2*2^(1/2)+2)^(1/2))*ln(1+2^(1/2)+(2*2^(1/2)+2)^(1/
2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))-(2*2^(1/2)-1/2*(-(2*2^(1/2)+2)^(1/2)*2^
(1/2)+2*(2*2^(1/2)+2)^(1/2))*(2*2^(1/2)+2)^(1/2))/(-2+2*2^(1/2))^(1/2)*arct
an(((2*2^(1/2)+2)^(1/2)+2*(1+tan(f*x+e))^(1/2))/(-2+2*2^(1/2))^(1/2)))
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(1+tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 971 vs. $2(306) = 612$.
time = 1.22, size = 971, normalized size = 2.63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(1+tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1848*(924*8^(1/4)*sqrt(2)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f*(f^(-4))
^(1/4)*arctan(-1/8*8^(3/4)*sqrt(2)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^3
*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) + 1/8*8^(3
/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^3*sqrt((2*sqrt(2)*f^2*sqrt(f^(-4
))*cos(f*x + e) + 8^(1/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f*sqrt((cos(
f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4)*cos(f*x + e) + 2*cos(
f*x + e) + 2*sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) - f^2*sqrt(f^(-4))
- sqrt(2))*cos(f*x + e)^5 + 924*8^(1/4)*sqrt(2)*sqrt(2*sqrt(2)*f^2*sqrt(f^(
-4)) + 4)*f*(f^(-4))^(1/4)*arctan(-1/8*8^(3/4)*sqrt(2)*sqrt(2*sqrt(2)*f^2*s
qrt(f^(-4)) + 4)*f^3*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-
4))^(3/4) + 1/8*8^(3/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^3*sqrt((2*sq
rt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) - 8^(1/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4
)) + 4)*f*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4)*c
os(f*x + e) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4)
 + f^2*sqrt(f^(-4)) + sqrt(2))*cos(f*x + e)^5 + 231*8^(1/4)*(sqrt(2)*f^3*sq
rt(f^(-4))*cos(f*x + e)^5 - 2*f*cos(f*x + e)^5)*sqrt(2*sqrt(2)*f^2*sqrt(f^(
-4)) + 4)*(f^(-4))^(1/4)*log(2*(2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) + 8
^(1/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f*sqrt((cos(f*x + e) + sin(f*x
+ e))/cos(f*x + e))*(f^(-4))^(1/4)*cos(f*x + e) + 2*cos(f*x + e) + 2*sin(f*
x + e))/cos(f*x + e)) - 231*8^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4))*cos(f*x + e)^
5 - 2*f*cos(f*x + e)^5)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)
*log(2*(2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) - 8^(1/4)*sqrt(2*sqrt(2)*f^
2*sqrt(f^(-4)) + 4)*f*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(
-4))^(1/4)*cos(f*x + e) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e)) +
16*(400*cos(f*x + e)^5 - 110*cos(f*x + e)^3 + (125*cos(f*x + e)^4 - 74*cos(
f*x + e)^2 + 21)*sin(f*x + e) + 28*cos(f*x + e))*sqrt((cos(f*x + e) + sin(f
*x + e))/cos(f*x + e)))/(f*cos(f*x + e)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (\tan(e + fx) + 1)^{\frac{3}{2}} \tan^5(e + fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5*(1+tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((tan(e + f*x) + 1)**(3/2)*tan(e + f*x)**5, x)
```

Giac [A]
time = 0.84, size = 286, normalized size = 0.78

$$-\frac{\sqrt{\sqrt{2}+1}\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{f}-\frac{\sqrt{\sqrt{2}+1}\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{f}-\frac{\sqrt{\sqrt{2}-1}\log\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{2f}+\frac{\sqrt{\sqrt{2}-1}\log\left(-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{2f}+\frac{2\left(21f^{10}(\tan(fx+e)+1)^{\frac{11}{2}}-77f^{10}(\tan(fx+e)+1)^{\frac{9}{2}}+66f^{10}(\tan(fx+e)+1)^{\frac{7}{2}}+77f^{10}(\tan(fx+e)+1)^{\frac{3}{2}}+231f^{10}\sqrt{\tan(fx+e)+1}\right)}{231f^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(1+tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt(sqrt(2) + 1)*arctan(1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*sqrt(t
an(f*x + e) + 1))/sqrt(-sqrt(2) + 2))/f - sqrt(sqrt(2) + 1)*arctan(-1/2*2^(
3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(tan(f*x + e) + 1))/sqrt(-sqrt(2) +
 2))/f - 1/2*sqrt(sqrt(2) - 1)*log(2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x +
 e) + 1) + sqrt(2) + tan(f*x + e) + 1)/f + 1/2*sqrt(sqrt(2) - 1)*log(-2^(1/
4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x + e) + 1) + sqrt(2) + tan(f*x + e) + 1)/f
 + 2/231*(21*f^10*(tan(f*x + e) + 1)^(11/2) - 77*f^10*(tan(f*x + e) + 1)^(9
/2) + 66*f^10*(tan(f*x + e) + 1)^(7/2) + 77*f^10*(tan(f*x + e) + 1)^(3/2) +
 231*f^10*sqrt(tan(f*x + e) + 1))/f^11
```

Mupad [B]
time = 6.90, size = 144, normalized size = 0.39

$$\frac{2\sqrt{\tan(e+fx)+1}}{f}+\frac{2(\tan(e+fx)+1)^{3/2}}{3f}+\frac{4(\tan(e+fx)+1)^{7/2}}{7f}-\frac{2(\tan(e+fx)+1)^{9/2}}{3f}+\frac{2(\tan(e+fx)+1)^{11/2}}{11f}-\mathrm{atan}\left(f\sqrt{\frac{-\frac{1}{2}-\frac{1}{2}\mathrm{i}}{f^2}}\sqrt{\tan(e+fx)+1}\right)\sqrt{\frac{-\frac{1}{2}-\frac{1}{2}\mathrm{i}}{f^2}}\,2\mathrm{i}+\mathrm{atan}\left(f\sqrt{\frac{-\frac{1}{2}+\frac{1}{2}\mathrm{i}}{f^2}}\sqrt{\tan(e+fx)+1}\right)\sqrt{\frac{-\frac{1}{2}+\frac{1}{2}\mathrm{i}}{f^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5*(tan(e + f*x) + 1)^(3/2),x)
```

```
[Out] (2*(tan(e + f*x) + 1)^(1/2))/f + (2*(tan(e + f*x) + 1)^(3/2))/(3*f) + (4*(t
an(e + f*x) + 1)^(7/2))/(7*f) - (2*(tan(e + f*x) + 1)^(9/2))/(3*f) + (2*(ta
n(e + f*x) + 1)^(11/2))/(11*f) - atan(f*((- 1/2 - 1i/2)/f^2)^(1/2)*(tan(e +
 f*x) + 1)^(1/2))*((- 1/2 - 1i/2)/f^2)^(1/2)*2i + atan(f*((- 1/2 + 1i/2)/f^
2)^(1/2)*(tan(e + f*x) + 1)^(1/2))*((- 1/2 + 1i/2)/f^2)^(1/2)*2i
```

3.390 $\int \tan^3(e+fx)(1+\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=315

$$-\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\tan(e+fx)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{f}+\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{\ldots}}{\sqrt{2\left(-1+\ldots\right.}}\right.}{f}$$

```
[Out] -1/2*ln(1+2^(1/2)-(2+2*2^(1/2))^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))/f/(1
+2^(1/2))^(1/2)+1/2*ln(1+2^(1/2)+(2+2*2^(1/2))^(1/2)*(1+tan(f*x+e))^(1/2)+t
an(f*x+e))/f/(1+2^(1/2))^(1/2)-arctan(((2+2*2^(1/2))^(1/2)-2*(1+tan(f*x+e))
^(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^(1/2))^(1/2)/f+arctan(((2+2*2^(1/2))^(1/
2)+2*(1+tan(f*x+e))^(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^(1/2))^(1/2)/f-2*(1+t
an(f*x+e))^(1/2)/f-2/3*(1+tan(f*x+e))^(3/2)/f-4/35*(1+tan(f*x+e))^(5/2)/f+2
/7*tan(f*x+e)*(1+tan(f*x+e))^(5/2)/f
```

Rubi [A]

time = 0.24, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3647, 3711, 12, 3609, 3566, 722, 1108, 648, 632, 210, 642}

$$-\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\tan(e+fx)+1}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{f}+\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{2\sqrt{\tan(e+fx)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{f}+\frac{2\tan(e+fx)(\tan(e+fx)+1)^{5/2}}{7f}-\frac{4(\tan(e+fx)+1)^{5/2}}{35f}-\frac{2(\tan(e+fx)+1)^{3/2}}{3f}-\frac{2\sqrt{\tan(e+fx)+1}}{f}-\frac{\log\left(\tan(e+fx)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\tan(e+fx)+1}+\sqrt{2}+1\right)}{2\sqrt{1+\sqrt{2}}\,f}+\frac{\log\left(\tan(e+fx)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\tan(e+fx)+1}+\sqrt{2}+1\right)}{2\sqrt{1+\sqrt{2}}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3*(1 + Tan[e + f*x])^(3/2),x]
```

```
[Out] -((Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + Tan[e + f*x
]])/Sqrt[2*(-1 + Sqrt[2])]])/f) + (Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sq
rt[2])] + 2*Sqrt[1 + Tan[e + f*x]])/Sqrt[2*(-1 + Sqrt[2])]])/f - Log[1 + Sq
rt[2] + Tan[e + f*x] - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + Tan[e + f*x]]]/(2*Sqr
t[1 + Sqrt[2]]*f) + Log[1 + Sqrt[2] + Tan[e + f*x] + Sqrt[2*(1 + Sqrt[2])]*
Sqrt[1 + Tan[e + f*x]]]/(2*Sqrt[1 + Sqrt[2]]*f) - (2*Sqrt[1 + Tan[e + f*x]]
)/f - (2*(1 + Tan[e + f*x])^(3/2))/(3*f) - (4*(1 + Tan[e + f*x])^(5/2))/(35
*f) + (2*Tan[e + f*x]*(1 + Tan[e + f*x])^(5/2))/(7*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 722

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
```

```
 0] && GtQ[m, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \tan^3(e+fx)(1+\tan(e+fx))^{3/2}\,dx = \frac{2\tan(e+fx)(1+\tan(e+fx))^{5/2}}{7f} + \frac{2}{7}\int (1+\tan(e+fx))^{3/2}\Big($$

$$= -\frac{4(1+\tan(e+fx))^{5/2}}{35f} + \frac{2\tan(e+fx)(1+\tan(e+fx))^{5/2}}{7f} +$$

$$= -\frac{4(1+\tan(e+fx))^{5/2}}{35f} + \frac{2\tan(e+fx)(1+\tan(e+fx))^{5/2}}{7f} -$$

$$= -\frac{2(1+\tan(e+fx))^{3/2}}{3f} - \frac{4(1+\tan(e+fx))^{5/2}}{35f} + \frac{2\tan(e+fx}{}$$

$$= -\frac{2\sqrt{1+\tan(e+fx)}}{f} - \frac{2(1+\tan(e+fx))^{3/2}}{3f} - \frac{4(1+\tan(e+}{35f}$$

$$= -\frac{2\sqrt{1+\tan(e+fx)}}{f} - \frac{2(1+\tan(e+fx))^{3/2}}{3f} - \frac{4(1+\tan(e+}{35f}$$

$$= -\frac{2\sqrt{1+\tan(e+fx)}}{f} - \frac{2(1+\tan(e+fx))^{3/2}}{3f} - \frac{4(1+\tan(e+}{35f}$$

$$= -\frac{2\sqrt{1+\tan(e+fx)}}{f} - \frac{2(1+\tan(e+fx))^{3/2}}{3f} - \frac{4(1+\tan(e+}{35f}$$

$$= -\frac{2\sqrt{1+\tan(e+fx)}}{f} - \frac{2(1+\tan(e+fx))^{3/2}}{3f} - \frac{4(1+\tan(e+}{35f}$$

$$= -\frac{2\sqrt{1+\tan(e+fx)}}{f} - \frac{2(1+\tan(e+fx))^{3/2}}{3f} - \frac{4(1+\tan(e+}{35f}$$

$$= -\frac{\log\left(1+\sqrt{2}+\tan(e+fx)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\tan(e+}\right.}{2\sqrt{1+\sqrt{2}}\,f}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\tan(e+fx)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{\sqrt{-1+\sqrt{2}}\,f} + \frac{\tan^{-1}\Big(\sqrt{}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.64, size = 112, normalized size = 0.36

$$\frac{105(1-i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1-i}}\right)+105(1+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1+i}}\right)+2\sqrt{1+\tan(e+fx)}\,(-146-32\tan(e+fx)+24\tan^2(e+fx)+15\tan^3(e+fx))}{105f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3*(1 + Tan[e + f*x])^(3/2),x]
```

```
[Out] (105*(1 - I)^(3/2)*ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1 - I]] + 105*(1 + I
)^(3/2)*ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1 + I]] + 2*Sqrt[1 + Tan[e + f*
x]]*(-146 - 32*Tan[e + f*x] + 24*Tan[e + f*x]^2 + 15*Tan[e + f*x]^3))/(105*
f)
```

Maple [A]
time = 0.15, size = 350, normalized size = 1.11

method	result
derivativedivides	$\frac{\frac{2(1+\tan(fx+e))^{\frac{7}{2}}}{7}-\frac{2(1+\tan(fx+e))^{\frac{5}{2}}}{5}-\frac{2(1+\tan(fx+e))^{\frac{3}{2}}}{3}-2\sqrt{1+\tan(fx+e)}-\frac{\left(-\sqrt{2\sqrt{2}+2}\,\sqrt{2}+2\sqrt{2}\right.}{}}{}$
default	$\frac{\frac{2(1+\tan(fx+e))^{\frac{7}{2}}}{7}-\frac{2(1+\tan(fx+e))^{\frac{5}{2}}}{5}-\frac{2(1+\tan(fx+e))^{\frac{3}{2}}}{3}-2\sqrt{1+\tan(fx+e)}-\frac{\left(-\sqrt{2\sqrt{2}+2}\,\sqrt{2}+2\sqrt{2}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3*(1+tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2/7*(1+tan(f*x+e))^(7/2)-2/5*(1+tan(f*x+e))^(5/2)-2/3*(1+tan(f*x+e))^(
3/2)-2*(1+tan(f*x+e))^(1/2)-1/4*(-(2*2^(1/2)+2)^(1/2)*2^(1/2)+2*(2*2^(1/2)+
2)^(1/2))*ln(1+2^(1/2)-(2*2^(1/2)+2)^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))
-(-2*2^(1/2)+1/2*(-(2*2^(1/2)+2)^(1/2)*2^(1/2)+2*(2*2^(1/2)+2)^(1/2))*(2*2^
(1/2)+2)^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+tan(f*x+e))^(1/2)-(2*2^(1
/2)+2)^(1/2))/(-2+2*2^(1/2))^(1/2))+1/4*(-(2*2^(1/2)+2)^(1/2)*2^(1/2)+2*(2*
2^(1/2)+2)^(1/2))*ln(1+2^(1/2)+(2*2^(1/2)+2)^(1/2)*(1+tan(f*x+e))^(1/2)+tan
(f*x+e))+(2*2^(1/2)-1/2*(-(2*2^(1/2)+2)^(1/2)*2^(1/2)+2*(2*2^(1/2)+2)^(1/2)
)*(2*2^(1/2)+2)^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan(((2*2^(1/2)+2)^(1/2)+2*(
1+tan(f*x+e))^(1/2))/(-2+2*2^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(1+tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((tan(f*x + e) + 1)^(3/2)*tan(f*x + e)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 949 vs. $2(256) = 512$.
time = 1.45, size = 949, normalized size = 3.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(1+tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/840*(420*8^(1/4)*sqrt(2)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f*(f^(-4))
^(1/4)*arctan(-1/8*8^(3/4)*sqrt(2)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^3
*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) + 1/8*8^(3
/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^3*sqrt((2*sqrt(2)*f^2*sqrt(f^(-4
))*cos(f*x + e) + 8^(1/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f*sqrt((cos(
f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4)*cos(f*x + e) + 2*cos(
f*x + e) + 2*sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) - f^2*sqrt(f^(-4))
- sqrt(2))*cos(f*x + e)^3 + 420*8^(1/4)*sqrt(2)*sqrt(2*sqrt(2)*f^2*sqrt(f^(
-4)) + 4)*f*(f^(-4))^(1/4)*arctan(-1/8*8^(3/4)*sqrt(2)*sqrt(2*sqrt(2)*f^2*s
qrt(f^(-4)) + 4)*f^3*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-
4))^(3/4) + 1/8*8^(3/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^3*sqrt((2*sq
rt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) - 8^(1/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4
)) + 4)*f*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4)*c
os(f*x + e) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4)
 + f^2*sqrt(f^(-4)) + sqrt(2))*cos(f*x + e)^3 + 105*8^(1/4)*(sqrt(2)*f^3*sq
rt(f^(-4))*cos(f*x + e)^3 - 2*f*cos(f*x + e)^3)*sqrt(2*sqrt(2)*f^2*sqrt(f^(
-4)) + 4)*(f^(-4))^(1/4)*log(2*(2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) + 8
^(1/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f*sqrt((cos(f*x + e) + sin(f*x
+ e))/cos(f*x + e))*(f^(-4))^(1/4)*cos(f*x + e) + 2*cos(f*x + e) + 2*sin(f*
x + e))/cos(f*x + e)) - 105*8^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4))*cos(f*x + e)^
3 - 2*f*cos(f*x + e)^3)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)
*log(2*(2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) - 8^(1/4)*sqrt(2*sqrt(2)*f^
2*sqrt(f^(-4)) + 4)*f*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(
-4))^(1/4)*cos(f*x + e) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e)) +
16*(170*cos(f*x + e)^3 + (47*cos(f*x + e)^2 - 15)*sin(f*x + e) - 24*cos(f*x
 + e))*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e)))/(f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (\tan(e + fx) + 1)^{\frac{3}{2}} \tan^3(e + fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3*(1+tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((tan(e + f*x) + 1)**(3/2)*tan(e + f*x)**3, x)
```

Giac [A]
time = 0.80, size = 268, normalized size = 0.85

$$\frac{\sqrt{\sqrt{2}+1}\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{\tan\left(fx+e\right)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{f}+\frac{\sqrt{\sqrt{2}+1}\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{\tan\left(fx+e\right)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{f}+\frac{\sqrt{\sqrt{2}-1}\log\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan\left(fx+e\right)+1}+\sqrt{2}+\tan\left(fx+e\right)+1\right)}{2f}-\frac{\sqrt{\sqrt{2}-1}\log\left(-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan\left(fx+e\right)+1}+\sqrt{2}+\tan\left(fx+e\right)+1\right)}{2f}+\frac{2\left(15f^{6}(\tan\left(fx+e\right)+1)^{\frac{7}{2}}-21f^{6}(\tan\left(fx+e\right)+1)^{\frac{5}{2}}-35f^{6}(\tan\left(fx+e\right)+1)^{\frac{3}{2}}-105f^{6}\sqrt{\tan\left(fx+e\right)+1}\right)}{105f^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(1+tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(sqrt(2) + 1)*arctan(1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*sqrt(ta
n(f*x + e) + 1))/sqrt(-sqrt(2) + 2))/f + sqrt(sqrt(2) + 1)*arctan(-1/2*2^(3
/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(tan(f*x + e) + 1))/sqrt(-sqrt(2) +
2))/f + 1/2*sqrt(sqrt(2) - 1)*log(2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x +
e) + 1) + sqrt(2) + tan(f*x + e) + 1)/f - 1/2*sqrt(sqrt(2) - 1)*log(-2^(1/4
)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x + e) + 1) + sqrt(2) + tan(f*x + e) + 1)/f
+ 2/105*(15*f^6*(tan(f*x + e) + 1)^(7/2) - 21*f^6*(tan(f*x + e) + 1)^(5/2)
- 35*f^6*(tan(f*x + e) + 1)^(3/2) - 105*f^6*sqrt(tan(f*x + e) + 1))/f^7
```

Mupad [B]
time = 5.24, size = 129, normalized size = 0.41

$$\frac{2\,{\left(\tan\left(e+f\,x\right)+1\right)}^{7/2}}{7\,f}-\frac{2\,{\left(\tan\left(e+f\,x\right)+1\right)}^{3/2}}{3\,f}-\frac{2\,{\left(\tan\left(e+f\,x\right)+1\right)}^{5/2}}{5\,f}-\frac{2\,\sqrt{\tan\left(e+f\,x\right)+1}}{f}+\mathrm{atan}\left(f\,\sqrt{\frac{-\frac{1}{2}-\frac{1}{2}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\right)\,\sqrt{\frac{-\frac{1}{2}-\frac{1}{2}\mathrm{i}}{f^2}}\,2\mathrm{i}-\mathrm{atan}\left(f\,\sqrt{\frac{-\frac{1}{2}+\frac{1}{2}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\right)\,\sqrt{\frac{-\frac{1}{2}+\frac{1}{2}\mathrm{i}}{f^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(tan(e + f*x) + 1)^(3/2),x)
```

```
[Out] (2*(tan(e + f*x) + 1)^(7/2))/(7*f) - (2*(tan(e + f*x) + 1)^(3/2))/(3*f) - (
2*(tan(e + f*x) + 1)^(5/2))/(5*f) - (2*(tan(e + f*x) + 1)^(1/2))/f + atan(f
*((- 1/2 - 1i/2)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1/2))*((- 1/2 - 1i/2)/f^2)^
(1/2)*2i - atan(f*((- 1/2 + 1i/2)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1/2))*((-
1/2 + 1i/2)/f^2)^(1/2)*2i
```

3.391 $\int \tan(e+fx)(1+\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=271

$$\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\tan(e+fx)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{f}-\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1}}{\sqrt{2\left(-1+\right.}}\right.}{f}$$

```
[Out] 1/2*ln(1+2^(1/2)-(2+2*2^(1/2))^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))/f/(1+
2^(1/2))^(1/2)-1/2*ln(1+2^(1/2)+(2+2*2^(1/2))^(1/2)*(1+tan(f*x+e))^(1/2)+ta
n(f*x+e))/f/(1+2^(1/2))^(1/2)+arctan(((2+2*2^(1/2))^(1/2)-2*(1+tan(f*x+e))^
(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^(1/2))^(1/2)/f-arctan(((2+2*2^(1/2))^(1/2
)+2*(1+tan(f*x+e))^(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^(1/2))^(1/2)/f+2*(1+ta
n(f*x+e))^(1/2)/f+2/3*(1+tan(f*x+e))^(3/2)/f
```

Rubi [A]

time = 0.16, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3609, 12, 3566, 722, 1108, 648, 632, 210, 642}

$$\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\tan(e+fx)+1}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{f}-\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{2\sqrt{\tan(e+fx)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{f}+\frac{2(\tan(e+fx)+1)^{3/2}}{3f}+\frac{2\sqrt{\tan(e+fx)+1}}{f}+\frac{\log\left(\tan(e+fx)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\tan(e+fx)+1}+\sqrt{2}+1\right)}{2\sqrt{1+\sqrt{2}}\,f}-\frac{\log\left(\tan(e+fx)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\tan(e+fx)+1}+\sqrt{2}+1\right)}{2\sqrt{1+\sqrt{2}}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]*(1 + Tan[e + f*x])^(3/2),x]
```

```
[Out] (Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + Tan[e + f*x]]
)/Sqrt[2*(-1 + Sqrt[2])]])/f - (Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[
2])] + 2*Sqrt[1 + Tan[e + f*x]])/Sqrt[2*(-1 + Sqrt[2])]])/f + Log[1 + Sqrt[
2] + Tan[e + f*x] - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + Tan[e + f*x]]]/(2*Sqrt[1
 + Sqrt[2]]*f) - Log[1 + Sqrt[2] + Tan[e + f*x] + Sqrt[2*(1 + Sqrt[2])]*Sqr
t[1 + Tan[e + f*x]]]/(2*Sqrt[1 + Sqrt[2]]*f) + (2*Sqrt[1 + Tan[e + f*x]])/f
 + (2*(1 + Tan[e + f*x])^(3/2))/(3*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 722

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
```

```
0] && GtQ[m, 0]
```

Rubi steps

$$\int \tan(e+fx)(1+\tan(e+fx))^{3/2}\,dx = \frac{2(1+\tan(e+fx))^{3/2}}{3f} + \int (-1+\tan(e+fx))\sqrt{1+\tan(e+fx)}$$

$$= \frac{2\sqrt{1+\tan(e+fx)}}{f} + \frac{2(1+\tan(e+fx))^{3/2}}{3f} + \int -\frac{2}{\sqrt{1+\tan(e}}$$

$$= \frac{2\sqrt{1+\tan(e+fx)}}{f} + \frac{2(1+\tan(e+fx))^{3/2}}{3f} - 2\int \frac{1}{\sqrt{1+\tan(e}}$$

$$= \frac{2\sqrt{1+\tan(e+fx)}}{f} + \frac{2(1+\tan(e+fx))^{3/2}}{3f} - \frac{2\text{Subst}\left(\int \frac{}{\sqrt{1+}}\right.}{}$$

$$= \frac{2\sqrt{1+\tan(e+fx)}}{f} + \frac{2(1+\tan(e+fx))^{3/2}}{3f} - \frac{4\text{Subst}\left(\int \frac{1}{2-2x^2+}\right.}{}$$

$$= \frac{2\sqrt{1+\tan(e+fx)}}{f} + \frac{2(1+\tan(e+fx))^{3/2}}{3f} - \frac{\text{Subst}\left(\int \frac{\sqrt{}}{\sqrt{2}-}\right.}{}$$

$$= \frac{2\sqrt{1+\tan(e+fx)}}{f} + \frac{2(1+\tan(e+fx))^{3/2}}{3f} - \frac{\text{Subst}\left(\int \frac{}{\sqrt{2}-}\right.}{}$$

$$= \frac{\log\left(1+\sqrt{2}+\tan(e+fx)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\tan(e+fx}\right.}{2\sqrt{1+\sqrt{2}}\,f}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\tan(e+fx)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{\sqrt{-1+\sqrt{2}}\,f} - \frac{\tan^{-1}\left(\frac{\sqrt{2\left(\right.}}{}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.23, size = 90, normalized size = 0.33

$$\frac{-3(1-i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1-i}}\right)-3(1+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1+i}}\right)+2\sqrt{1+\tan(e+fx)}\,(4+\tan(e+fx))}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]*(1 + Tan[e + f*x])^(3/2),x]
```

```
[Out] (-3*(1 - I)^(3/2)*ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1 - I]] - 3*(1 + I)^(
3/2)*ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1 + I]] + 2*Sqrt[1 + Tan[e + f*x]]
*(4 + Tan[e + f*x]))/(3*f)
```

Maple [A]
time = 0.08, size = 326, normalized size = 1.20

method	result
derivativedivides	$\frac{2(1+\tan(fx+e))^{\frac{3}{2}}}{3}+2\sqrt{1+\tan(fx+e)}+\frac{\left(-\sqrt{2\sqrt{2}+2}\,\sqrt{2}+2\sqrt{2\sqrt{2}+2}\right)\ln\left(1+\sqrt{2}-\sqrt{2\sqrt{2}\right.}{4}$
default	$\frac{2(1+\tan(fx+e))^{\frac{3}{2}}}{3}+2\sqrt{1+\tan(fx+e)}+\frac{\left(-\sqrt{2\sqrt{2}+2}\,\sqrt{2}+2\sqrt{2\sqrt{2}+2}\right)\ln\left(1+\sqrt{2}-\sqrt{2\sqrt{2}\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)*(1+tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2/3*(1+tan(f*x+e))^(3/2)+2*(1+tan(f*x+e))^(1/2)+1/4*(-(2*2^(1/2)+2)^(1
/2)*2^(1/2)+2*(2*2^(1/2)+2)^(1/2))*ln(1+2^(1/2)-(2*2^(1/2)+2)^(1/2)*(1+tan(
f*x+e))^(1/2)+tan(f*x+e))+(-2*2^(1/2)+1/2*(-(2*2^(1/2)+2)^(1/2)*2^(1/2)+2*(
2*2^(1/2)+2)^(1/2))*(2*2^(1/2)+2)^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+
tan(f*x+e))^(1/2)-(2*2^(1/2)+2)^(1/2))/(-2+2*2^(1/2))^(1/2))-1/4*(-(2*2^(1/
2)+2)^(1/2)*2^(1/2)+2*(2*2^(1/2)+2)^(1/2))*ln(1+2^(1/2)+(2*2^(1/2)+2)^(1/2)
*(1+tan(f*x+e))^(1/2)+tan(f*x+e))-(2*2^(1/2)-1/2*(-(2*2^(1/2)+2)^(1/2)*2^(1
/2)+2*(2*2^(1/2)+2)^(1/2))*(2*2^(1/2)+2)^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan
(((2*2^(1/2)+2)^(1/2)+2*(1+tan(f*x+e))^(1/2))/(-2+2*2^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(1+tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((tan(f*x + e) + 1)^(3/2)*tan(f*x + e), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 912 vs. $2(217) = 434$.
time = 1.35, size = 912, normalized size = 3.37

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(1+tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(12*8^(1/4)*sqrt(2)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f*(f^(-4))^(1
/4)*arctan(-1/8*8^(3/4)*sqrt(2)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^3*sq
rt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) + 1/8*8^(3/4)
*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^3*sqrt((2*sqrt(2)*f^2*sqrt(f^(-4))*
cos(f*x + e) + 8^(1/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f*sqrt((cos(f*x
 + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4)*cos(f*x + e) + 2*cos(f*x
 + e) + 2*sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) - f^2*sqrt(f^(-4)) - s
qrt(2))*cos(f*x + e) + 12*8^(1/4)*sqrt(2)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) +
 4)*f*(f^(-4))^(1/4)*arctan(-1/8*8^(3/4)*sqrt(2)*sqrt(2*sqrt(2)*f^2*sqrt(f^
(-4)) + 4)*f^3*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3
/4) + 1/8*8^(3/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^3*sqrt((2*sqrt(2)*
f^2*sqrt(f^(-4))*cos(f*x + e) - 8^(1/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4
)*f*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4)*cos(f*x
 + e) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) + f^2
*sqrt(f^(-4)) + sqrt(2))*cos(f*x + e) + 3*8^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4))
*cos(f*x + e) - 2*f*cos(f*x + e))*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*(f^(
-4))^(1/4)*log(2*(2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) + 8^(1/4)*sqrt(2*
sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x
+ e))*(f^(-4))^(1/4)*cos(f*x + e) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*
x + e)) - 3*8^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4))*cos(f*x + e) - 2*f*cos(f*x +
e))*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*log(2*(2*sqrt(2)*f^
2*sqrt(f^(-4))*cos(f*x + e) - 8^(1/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*
f*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4)*cos(f*x +
 e) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e)) + 16*sqrt((cos(f*x + e
) + sin(f*x + e))/cos(f*x + e))*(4*cos(f*x + e) + sin(f*x + e)))/(f*cos(f*x
 + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (\tan(e+fx)+1)^{\frac{3}{2}}\tan(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(1+tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((tan(e + f*x) + 1)**(3/2)*tan(e + f*x), x)
```

Giac [A]
time = 0.79, size = 237, normalized size = 0.87

$$-\frac{\sqrt{\sqrt{2}+1}\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{\tan\left(fx+e\right)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{f}-\frac{\sqrt{\sqrt{2}+1}\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{\tan\left(fx+e\right)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{f}-\frac{\sqrt{\sqrt{2}-1}\log\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan\left(fx+e\right)+1}+\sqrt{2}+\tan\left(fx+e\right)+1\right)}{2f}+\frac{\sqrt{\sqrt{2}-1}\log\left(-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan\left(fx+e\right)+1}+\sqrt{2}+\tan\left(fx+e\right)+1\right)}{2f}+\frac{2\left(f^{2}(\tan\left(fx+e\right)+1)^{\frac{3}{2}}+3f^{2}\sqrt{\tan\left(fx+e\right)+1}\right)}{3f^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(1+tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt(sqrt(2) + 1)*arctan(1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*sqrt(t
an(f*x + e) + 1))/sqrt(-sqrt(2) + 2))/f - sqrt(sqrt(2) + 1)*arctan(-1/2*2^(
3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(tan(f*x + e) + 1))/sqrt(-sqrt(2) +
 2))/f - 1/2*sqrt(sqrt(2) - 1)*log(2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x +
 e) + 1) + sqrt(2) + tan(f*x + e) + 1)/f + 1/2*sqrt(sqrt(2) - 1)*log(-2^(1/
4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x + e) + 1) + sqrt(2) + tan(f*x + e) + 1)/f
 + 2/3*(f^2*(tan(f*x + e) + 1)^(3/2) + 3*f^2*sqrt(tan(f*x + e) + 1))/f^3
```

Mupad [B]
time = 4.37, size = 99, normalized size = 0.37

$$\frac{2\sqrt{\tan\left(e+f\,x\right)+1}}{f}+\frac{2\,{\left(\tan\left(e+f\,x\right)+1\right)}^{3/2}}{3\,f}-\mathrm{atan}\left(f\,\sqrt{\frac{-\frac{1}{2}-\frac{1}{2}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\right)\,\sqrt{\frac{-\frac{1}{2}-\frac{1}{2}\mathrm{i}}{f^2}}\,2\mathrm{i}+\mathrm{atan}\left(f\,\sqrt{\frac{-\frac{1}{2}+\frac{1}{2}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\right)\,\sqrt{\frac{-\frac{1}{2}+\frac{1}{2}\mathrm{i}}{f^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(tan(e + f*x) + 1)^(3/2),x)
```

```
[Out] (2*(tan(e + f*x) + 1)^(1/2))/f + (2*(tan(e + f*x) + 1)^(3/2))/(3*f) - atan(
f*((- 1/2 - 1i/2)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1/2))*((- 1/2 - 1i/2)/f^2)
^(1/2)*2i + atan(f*((- 1/2 + 1i/2)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1/2))*((-
 1/2 + 1i/2)/f^2)^(1/2)*2i
```

3.392 $\int \cot(e+fx)(1+\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=253

$$-\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\tan(e+fx)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{f}+\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{\cdots}}{\sqrt{2\left(-1\cdots\right.}}\right.}{f}$$

```
[Out] -2*arctanh((1+tan(f*x+e))^(1/2))/f-1/2*ln(1+2^(1/2)-(2+2*2^(1/2))^(1/2)*(1+
tan(f*x+e))^(1/2)+tan(f*x+e))/f/(1+2^(1/2))^(1/2)+1/2*ln(1+2^(1/2)+(2+2*2^(
1/2))^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))/f/(1+2^(1/2))^(1/2)-arctan(((2
+2*2^(1/2))^(1/2)-2*(1+tan(f*x+e))^(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^(1/2))
^(1/2)/f+arctan(((2+2*2^(1/2))^(1/2)+2*(1+tan(f*x+e))^(1/2))/(-2+2*2^(1/2))
^(1/2))*(1+2^(1/2))^(1/2)/f
```

Rubi [A]

time = 0.19, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {3654, 12, 3566, 722, 1108, 648, 632, 210, 642, 3715, 65, 213}

$$-\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\tan(e+fx)+1}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{f}+\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{2\sqrt{\tan(e+fx)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{f}-\frac{\log\left(\tan(e+fx)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\tan(e+fx)+1}+\sqrt{2}+1\right)}{2\sqrt{1+\sqrt{2}}\,f}+\frac{\log\left(\tan(e+fx)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\tan(e+fx)+1}+\sqrt{2}+1\right)}{2\sqrt{1+\sqrt{2}}\,f}-\frac{2\tanh^{-1}\left(\sqrt{\tan(e+fx)+1}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(1 + Tan[e + f*x])^(3/2),x]
```

```
[Out] -((Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + Tan[e + f*x
]])/Sqrt[2*(-1 + Sqrt[2])]])/f) + (Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sq
rt[2])] + 2*Sqrt[1 + Tan[e + f*x]])/Sqrt[2*(-1 + Sqrt[2])]])/f - (2*ArcTanh
[Sqrt[1 + Tan[e + f*x]]])/f - Log[1 + Sqrt[2] + Tan[e + f*x] - Sqrt[2*(1 +
Sqrt[2])]*Sqrt[1 + Tan[e + f*x]]]/(2*Sqrt[1 + Sqrt[2]]*f) + Log[1 + Sqrt[2]
 + Tan[e + f*x] + Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + Tan[e + f*x]]]/(2*Sqrt[1 +
 Sqrt[2]]*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 722

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3654

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[Simp[a^2*c - b^2*c + 2
*a*b*d + (2*a*b*c - a^2*d + b^2*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x]
], x], x] + Dist[(b*c - a*d)^2/(c^2 + d^2), Int[(1 + Tan[e + f*x]^2)/(Sqrt[
a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \cot(e+fx)(1+\tan(e+fx))^{3/2}\,dx = \int \frac{2}{\sqrt{1+\tan(e+fx)}}\,dx + \int \frac{\cot(e+fx)\left(1+\tan^2(e+fx)\right)}{\sqrt{1+\tan(e+fx)}}\,dx$$

$$= 2\int \frac{1}{\sqrt{1+\tan(e+fx)}}\,dx + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \tan(e+fx)\right.}{f}$$

$$= \frac{2\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\tan(e+fx)}\right)}{f} + \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1+x}\,(\ldots}\right.}{}$$

$$= -\frac{2\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{f} + \frac{4\text{Subst}\left(\int \frac{1}{2-2x^2+x^4}\,dx, x, \sqrt{1+\ldots}\right.}{f}$$

$$= -\frac{2\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{f} + \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-\ldots}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\ldots}\right.}{\sqrt{1+\ldots}}$$

$$= -\frac{2\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{f} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\ldots}\right.}{\sqrt{\ldots}}$$

$$= -\frac{2\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{f} - \frac{\log\left(1+\sqrt{2}+\tan(e+fx)-\ldots\right.}{2\sqrt{\ldots}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\tan(e+fx)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{\sqrt{-1+\sqrt{2}}\,f} + \frac{\tan^{-1}\left(\frac{\sqrt{2\left(\ldots\right.}}{}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.19, size = 78, normalized size = 0.31

$$\frac{-2\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)+(1-i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1-i}}\right)+(1+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1+i}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(1 + Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[1 + Tan[e + f*x]]] + (1 - I)^(3/2)*ArcTanh[Sqrt[1 + Tan[e
+ f*x]]/Sqrt[1 - I]] + (1 + I)^(3/2)*ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1
+ I]])/f
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.72, size = 2377, normalized size = 9.40

method	result	size
default	Expression too large to display	2377

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(1+tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/f*((cos(f*x+e)+sin(f*x+e))/cos(f*x+e))^(1/2)*(cos(f*x+e)+1)^2*(cos(f*x
+e)-1)^2*(1+sin(f*x+e))*(16*I*2^(1/2)*((sin(f*x+e)-1)*(1+2^(1/2))/cos(f*x+e
))^(1/2)*((2^(1/2)*sin(f*x+e)-2^(1/2)+cos(f*x+e)-sin(f*x+e)+1)/cos(f*x+e))^
(1/2)*((-2^(1/2)*sin(f*x+e)+2^(1/2)+cos(f*x+e)-sin(f*x+e)+1)/cos(f*x+e))^(1
/2)*EllipticPi(1/2*((sin(f*x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*2^
(1/2),-I*2^(1/2)/(2+2^(1/2)),I*((2-2^(1/2))/(2+2^(1/2)))^(1/2))-16*I*2^(1/2
)*((sin(f*x+e)-1)*(1+2^(1/2))/cos(f*x+e))^(1/2)*((2^(1/2)*sin(f*x+e)-2^(1/2
)+cos(f*x+e)-sin(f*x+e)+1)/cos(f*x+e))^(1/2)*((-2^(1/2)*sin(f*x+e)+2^(1/2)+
cos(f*x+e)-sin(f*x+e)+1)/cos(f*x+e))^(1/2)*EllipticPi(1/2*((sin(f*x+e)-1)/c
os(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*2^(1/2),I*2^(1/2)/(2+2^(1/2)),I*((2-2^
(1/2))/(2+2^(1/2)))^(1/2))+3*((sin(f*x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2)
)^(1/2)*2^(1/2)*((2^(1/2)*cos(f*x+e)-2^(1/2)*sin(f*x+e)+2^(1/2)+2*sin(f*x+e
)-2)/cos(f*x+e)*2^(1/2))^(1/2)*((2^(1/2)*cos(f*x+e)-2^(1/2)*sin(f*x+e)+2^(1
/2)-2*sin(f*x+e)+2)/cos(f*x+e)*2^(1/2))^(1/2)*EllipticE(1/2*((sin(f*x+e)-1)
/cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*2^(1/2),I*((2-2^(1/2))/(2+2^(1/2)))^
(1/2))-3*((sin(f*x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*2^(1/2)*((2^
(1/2)*cos(f*x+e)-2^(1/2)*sin(f*x+e)+2^(1/2)+2*sin(f*x+e)-2)/cos(f*x+e)*2^(1
/2))^(1/2)*((2^(1/2)*cos(f*x+e)-2^(1/2)*sin(f*x+e)+2^(1/2)-2*sin(f*x+e)+2)/
cos(f*x+e)*2^(1/2))^(1/2)*EllipticF(1/2*((sin(f*x+e)-1)/cos(f*x+e)*(2+2^(1/
2))*2^(1/2))^(1/2)*2^(1/2),I*((2-2^(1/2))/(2+2^(1/2)))^(1/2))-6*EllipticE(1
/2*((sin(f*x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*2^(1/2),I*((2-2^(1
/2))/(2+2^(1/2)))^(1/2))*2^(1/2)*((-2^(1/2)*sin(f*x+e)+2^(1/2)+cos(f*x+e)-s
in(f*x+e)+1)/cos(f*x+e))^(1/2)*((2^(1/2)*sin(f*x+e)-2^(1/2)+cos(f*x+e)-sin(
f*x+e)+1)/cos(f*x+e))^(1/2)*((sin(f*x+e)-1)*(1+2^(1/2))/cos(f*x+e))^(1/2)+1
6*EllipticF(1/2*((sin(f*x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*2^(1/
2),I*((2-2^(1/2))/(2+2^(1/2)))^(1/2))*2^(1/2)*((-2^(1/2)*sin(f*x+e)+2^(1/2)
+cos(f*x+e)-sin(f*x+e)+1)/cos(f*x+e))^(1/2)*((2^(1/2)*sin(f*x+e)-2^(1/2)+co
s(f*x+e)-sin(f*x+e)+1)/cos(f*x+e))^(1/2)*((sin(f*x+e)-1)*(1+2^(1/2))/cos(f*
x+e))^(1/2)-12*EllipticPi(1/2*((sin(f*x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2
))^(1/2)*2^(1/2),2^(1/2)/(2+2^(1/2)),I*((2-2^(1/2))/(2+2^(1/2)))^(1/2))*2^(
1/2)*((-2^(1/2)*sin(f*x+e)+2^(1/2)+cos(f*x+e)-sin(f*x+e)+1)/cos(f*x+e))^(1/
```

```
2)*((2^(1/2)*sin(f*x+e)-2^(1/2)+cos(f*x+e)-sin(f*x+e)+1)/cos(f*x+e))^(1/2)*
((sin(f*x+e)-1)*(1+2^(1/2))/cos(f*x+e))^(1/2)+3*((sin(f*x+e)-1)/cos(f*x+e)*
(2+2^(1/2))*2^(1/2))^(1/2)*((2^(1/2)*cos(f*x+e)-2^(1/2)*sin(f*x+e)+2^(1/2)+
2*sin(f*x+e)-2)/cos(f*x+e)*2^(1/2))^(1/2)*((2^(1/2)*cos(f*x+e)-2^(1/2)*sin(
f*x+e)+2^(1/2)-2*sin(f*x+e)+2)/cos(f*x+e)*2^(1/2))^(1/2)*EllipticE(1/2*((si
n(f*x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*2^(1/2),I*((2-2^(1/2))/(2
+2^(1/2)))^(1/2))-4*((sin(f*x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*(
(2^(1/2)*cos(f*x+e)-2^(1/2)*sin(f*x+e)+2^(1/2)+2*sin(f*x+e)-2)/cos(f*x+e)*2
^(1/2))^(1/2)*((2^(1/2)*cos(f*x+e)-2^(1/2)*sin(f*x+e)+2^(1/2)-2*sin(f*x+e)+
2)/cos(f*x+e)*2^(1/2))^(1/2)*EllipticF(1/2*((sin(f*x+e)-1)/cos(f*x+e)*(2+2^
(1/2))*2^(1/2))^(1/2)*2^(1/2),I*((2-2^(1/2))/(2+2^(1/2)))^(1/2))+2*((sin(f*
x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*((2^(1/2)*cos(f*x+e)-2^(1/2)*
sin(f*x+e)+2^(1/2)+2*sin(f*x+e)-2)/cos(f*x+e)*2^(1/2))^(1/2)*((2^(1/2)*cos(
f*x+e)-2^(1/2)*sin(f*x+e)+2^(1/2)-2*sin(f*x+e)+2)/cos(f*x+e)*2^(1/2))^(1/2)
*EllipticPi(1/2*((sin(f*x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*2^(1/
2),2^(1/2)/(2+2^(1/2)),I*((2-2^(1/2))/(2+2^(1/2)))^(1/2))-4*((sin(f*x+e)-1)
/cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*((2^(1/2)*cos(f*x+e)-2^(1/2)*sin(f*x
+e)+2^(1/2)+2*sin(f*x+e)-2)/cos(f*x+e)*2^(1/2))^(1/2)*((2^(1/2)*cos(f*x+e)-
2^(1/2)*sin(f*x+e)+2^(1/2)-2*sin(f*x+e)+2)/cos(f*x+e)*2^(1/2))^(1/2)*Ellipt
icPi(1/2*((sin(f*x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*2^(1/2),-2^(
1/2)/(2+2^(1/2)),I*((2-2^(1/2))/(2+2^(1/2)))^(1/2))-12*EllipticE(1/2*((sin(
f*x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*2^(1/2),I*((2-2^(1/2))/(2+2
^(1/2)))^(1/2))*((-2^(1/2)*sin(f*x+e)+2^(1/2)+cos(f*x+e)-sin(f*x+e)+1)/cos(
f*x+e))^(1/2)*((2^(1/2)*sin(f*x+e)-2^(1/2)+cos(f*x+e)-sin(f*x+e)+1)/cos(f*x
+e))^(1/2)*((sin(f*x+e)-1)*(1+2^(1/2))/cos(f*x+e))^(1/2)+12*EllipticF(1/2*(
(sin(f*x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*2^(1/2),I*((2-2^(1/2))
/(2+2^(1/2)))^(1/2))*((-2^(1/2)*sin(f*x+e)+2^(1/2)+cos(f*x+e)-sin(f*x+e)+1)
/cos(f*x+e))^(1/2)*((2^(1/2)*sin(f*x+e)-2^(1/2)+cos(f*x+e)-sin(f*x+e)+1)/co
s(f*x+e))^(1/2)*((sin(f*x+e)-1)*(1+2^(1/2))/cos(f*x+e))^(1/2))*4^(1/2)/(cos
(f*x+e)+sin(f*x+e))/sin(f*x+e)^4/(2+2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(1+tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((tan(f*x + e) + 1)^(3/2)*cot(f*x + e), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 878 vs. 2(202) = 404.
time = 1.15, size = 878, normalized size = 3.47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(1+tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*(4*8^(1/4)*sqrt(2)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f*(f^(-4))^(1/
4)*arctan(-1/8*8^(3/4)*sqrt(2)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^3*sqr
t((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) + 1/8*8^(3/4)*
sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^3*sqrt((2*sqrt(2)*f^2*sqrt(f^(-4))*c
os(f*x + e) + 8^(1/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f*sqrt((cos(f*x
+ e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4)*cos(f*x + e) + 2*cos(f*x
+ e) + 2*sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) - f^2*sqrt(f^(-4)) - sq
rt(2)) + 4*8^(1/4)*sqrt(2)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f*(f^(-4))^
(1/4)*arctan(-1/8*8^(3/4)*sqrt(2)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^3*
sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) + 1/8*8^(3/
4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^3*sqrt((2*sqrt(2)*f^2*sqrt(f^(-4)
)*cos(f*x + e) - 8^(1/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f*sqrt((cos(f
*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4)*cos(f*x + e) + 2*cos(f
*x + e) + 2*sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) + f^2*sqrt(f^(-4)) +
 sqrt(2)) + 8^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4)) - 2*f)*sqrt(2*sqrt(2)*f^2*sqr
t(f^(-4)) + 4)*(f^(-4))^(1/4)*log(2*(2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e
) + 8^(1/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f*sqrt((cos(f*x + e) + sin
(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4)*cos(f*x + e) + 2*cos(f*x + e) + 2*s
in(f*x + e))/cos(f*x + e)) - 8^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4)) - 2*f)*sqrt(
2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*log(2*(2*sqrt(2)*f^2*sqrt(f^
(-4))*cos(f*x + e) - 8^(1/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f*sqrt((c
os(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4)*cos(f*x + e) + 2*c
os(f*x + e) + 2*sin(f*x + e))/cos(f*x + e)) + 8*log(sqrt((cos(f*x + e) + si
n(f*x + e))/cos(f*x + e)) + 1) - 8*log(sqrt((cos(f*x + e) + sin(f*x + e))/c
os(f*x + e)) - 1))/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (\tan (e + fx) + 1)^{\frac{3}{2}} \cot (e + fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(1+tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((tan(e + f*x) + 1)**(3/2)*cot(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(1+tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((tan(f*x + e) + 1)^(3/2)*cot(f*x + e), x)
```

Mupad [B]
time = 0.15, size = 85, normalized size = 0.34

$$-\frac{2\,\mathrm{atanh}\left(\sqrt{\tan\left(e+f\,x\right)+1}\right)}{f}+\mathrm{atan}\left(f\,\sqrt{\frac{-\frac{1}{2}-\frac{1}{2}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\right)\,\sqrt{\frac{-\frac{1}{2}-\frac{1}{2}\mathrm{i}}{f^2}}\,2\mathrm{i}-\mathrm{atan}\left(f\,\sqrt{\frac{-\frac{1}{2}+\frac{1}{2}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\right)\,\sqrt{\frac{-\frac{1}{2}+\frac{1}{2}\mathrm{i}}{f^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(tan(e + f*x) + 1)^(3/2),x)
```

```
[Out] atan(f*((- 1/2 - 1i/2)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1/2))*((- 1/2 - 1i/2)
/f^2)^(1/2)*2i - (2*atanh((tan(e + f*x) + 1)^(1/2)))/f - atan(f*((- 1/2 + 1
i/2)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1/2))*((- 1/2 + 1i/2)/f^2)^(1/2)*2i
```

3.393 $\int \cot^3(e+fx)(1+\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=307

$$\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\tan(e+fx)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{f}-\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1\ldots}}{\sqrt{2\left(-1+\ldots\right)}}\right)}{f}$$

```
[Out] 5/4*arctanh((1+tan(f*x+e))^(1/2))/f+1/2*ln(1+2^(1/2)-(2+2*2^(1/2))^(1/2)*(1
+tan(f*x+e))^(1/2)+tan(f*x+e))/f/(1+2^(1/2))^(1/2)-1/2*ln(1+2^(1/2)+(2+2*2^
(1/2))^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))/f/(1+2^(1/2))^(1/2)+arctan(((
2+2*2^(1/2))^(1/2)-2*(1+tan(f*x+e))^(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^(1/2)
)^(1/2)/f-arctan(((2+2*2^(1/2))^(1/2)+2*(1+tan(f*x+e))^(1/2))/(-2+2*2^(1/2)
)^(1/2))*(1+2^(1/2))^(1/2)/f-5/4*cot(f*x+e)*(1+tan(f*x+e))^(1/2)/f-1/2*cot(
f*x+e)^2*(1+tan(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.31, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3648, 3731, 3734, 12, 3566, 722, 1108, 648, 632, 210, 642, 3715, 65, 213}

$$\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\tan(e+fx)+1}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{f}-\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{2\sqrt{\tan(e+fx)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{f}+\frac{\log\left(\tan(e+fx)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\tan(e+fx)+1}+\sqrt{2}+1\right)}{2\sqrt{1+\sqrt{2}}\,f}-\frac{\log\left(\tan(e+fx)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\tan(e+fx)+1}+\sqrt{2}+1\right)}{2\sqrt{1+\sqrt{2}}\,f}+\frac{5\tanh^{-1}\left(\sqrt{\tan(e+fx)+1}\right)}{4f}-\frac{\sqrt{\tan(e+fx)+1}\cot^2(e+fx)}{2f}-\frac{5\sqrt{\tan(e+fx)+1}\cot(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(1 + Tan[e + f*x])^(3/2),x]
```

```
[Out] (Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + Tan[e + f*x]]
)/Sqrt[2*(-1 + Sqrt[2])]])/f - (Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[
2])] + 2*Sqrt[1 + Tan[e + f*x]])/Sqrt[2*(-1 + Sqrt[2])]])/f + (5*ArcTanh[Sq
rt[1 + Tan[e + f*x]]])/(4*f) + Log[1 + Sqrt[2] + Tan[e + f*x] - Sqrt[2*(1 +
 Sqrt[2])]*Sqrt[1 + Tan[e + f*x]]]/(2*Sqrt[1 + Sqrt[2]]*f) - Log[1 + Sqrt[2
] + Tan[e + f*x] + Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + Tan[e + f*x]]]/(2*Sqrt[1
+ Sqrt[2]]*f) - (5*Cot[e + f*x]*Sqrt[1 + Tan[e + f*x]])/(4*f) - (Cot[e + f*
x]^2*Sqrt[1 + Tan[e + f*x]])/(2*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 722

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
```

```
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3731

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)
*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp
[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - a*C*(b*c*(m + 1) + a*d*(n
+ 1)) - (m + 1)*(b*c - a*d)*(A*b - b*C)*Tan[e + f*x] - d*(A*b^2 + a^2*C)*(m
 + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m,
 -1] &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
```

```
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \cot^3(e+fx)(1+\tan(e+fx))^{3/2}\,dx = -\frac{\cot^2(e+fx)\sqrt{1+\tan(e+fx)}}{2f} - \frac{1}{2}\int \frac{\cot^2(e+fx)\left(-\frac{5}{2}+\frac{3}{2}\text{t}\ldots\right.}{\sqrt{1+\tan(e+\ldots}}$$

$$= -\frac{5\cot(e+fx)\sqrt{1+\tan(e+fx)}}{4f} - \frac{\cot^2(e+fx)\sqrt{1+\tan(e+\ldots}}{2f}$$

$$= -\frac{5\cot(e+fx)\sqrt{1+\tan(e+fx)}}{4f} - \frac{\cot^2(e+fx)\sqrt{1+\tan(e+\ldots}}{2f}$$

$$= -\frac{5\cot(e+fx)\sqrt{1+\tan(e+fx)}}{4f} - \frac{\cot^2(e+fx)\sqrt{1+\tan(e+\ldots}}{2f}$$

$$= -\frac{5\cot(e+fx)\sqrt{1+\tan(e+fx)}}{4f} - \frac{\cot^2(e+fx)\sqrt{1+\tan(e+\ldots}}{2f}$$

$$= \frac{5\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{4f} - \frac{5\cot(e+fx)\sqrt{1+\tan(e+f\ldots}}{4f}$$

$$= \frac{5\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{4f} - \frac{5\cot(e+fx)\sqrt{1+\tan(e+f\ldots}}{4f}$$

$$= \frac{5\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{4f} - \frac{5\cot(e+fx)\sqrt{1+\tan(e+f\ldots}}{4f}$$

$$= \frac{5\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{4f} + \frac{\log\left(1+\sqrt{2}+\tan(e+fx)-\ldots\right.}{2\sqrt{\ldots}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\tan(e+fx)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{\sqrt{-1+\sqrt{2}}\,f} - \frac{\tan^{-1}\left(\frac{\sqrt{2\,(\ldots}}{\ldots}\right.}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order

3 in optimal.
time = 27.71, size = 4055, normalized size = 13.21

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^3*(1 + Tan[e + f*x])^(3/2),x]
```

```
[Out] (Cos[e + f*x]*(1/2 - (5*Cot[e + f*x])/4 - Csc[e + f*x]^2/2)*(1 + Tan[e + f*
x])^(3/2))/(f*(Cos[e + f*x] + Sin[e + f*x])) + (Cos[e + f*x]*(-5*EllipticF[
ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[
2 + Sqrt[2]]], -3 - 2*Sqrt[2]] + 5*EllipticPi[-1 - Sqrt[2], ArcSin[(2^(1/4)
*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]],
-3 - 2*Sqrt[2]] + (16*I)*EllipticPi[(-I)*(1 + Sqrt[2]), ArcSin[(2^(1/4)*Sqr
t[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 -
 2*Sqrt[2]] - (16*I)*EllipticPi[I*(1 + Sqrt[2]), ArcSin[(2^(1/4)*Sqrt[(1 +
Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqrt
[2]] + 5*EllipticPi[1 + Sqrt[2], ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2]
)/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqrt[2]])*((-5*Csc[e
 + f*x]*Sqrt[Sec[e + f*x]])/(8*Sqrt[Cos[e + f*x] + Sin[e + f*x]]) - (Csc[e
+ f*x]*Sqrt[Sec[e + f*x]]*Sin[2*(e + f*x)])/Sqrt[Cos[e + f*x] + Sin[e + f*x
]])*Sqrt[-((1 + Tan[(e + f*x)/2])/((-2 + Sqrt[2])*(-1 + Tan[(e + f*x)/2])))
]*(1 + Tan[e + f*x])^(3/2))/(2*2^(1/4)*f*(Cos[e + f*x] + Sin[e + f*x])*Sqrt
[(Cos[e + f*x] + Sin[e + f*x])/(-1 + Sin[e + f*x])]*(((-5*EllipticF[ArcSin[
(2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqr
t[2]]], -3 - 2*Sqrt[2]] + 5*EllipticPi[-1 - Sqrt[2], ArcSin[(2^(1/4)*Sqrt[(
1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*
Sqrt[2]] + (16*I)*EllipticPi[(-I)*(1 + Sqrt[2]), ArcSin[(2^(1/4)*Sqrt[(1 +
Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqrt
[2]] - (16*I)*EllipticPi[I*(1 + Sqrt[2]), ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(e
+ f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqrt[2]] +
5*EllipticPi[1 + Sqrt[2], ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 +
 Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqrt[2]])*Sqrt[Sec[e + f*x]
]*(Cos[e + f*x] - Sin[e + f*x])*Sqrt[-((1 + Tan[(e + f*x)/2])/((-2 + Sqrt[2
])*(-1 + Tan[(e + f*x)/2])))])/(4*2^(1/4)*Sqrt[Cos[e + f*x] + Sin[e + f*x]]
*Sqrt[(Cos[e + f*x] + Sin[e + f*x])/(-1 + Sin[e + f*x])]) + ((-5*EllipticF[
ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[
2 + Sqrt[2]]], -3 - 2*Sqrt[2]] + 5*EllipticPi[-1 - Sqrt[2], ArcSin[(2^(1/4)
*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]],
-3 - 2*Sqrt[2]] + (16*I)*EllipticPi[(-I)*(1 + Sqrt[2]), ArcSin[(2^(1/4)*Sqr
t[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 -
 2*Sqrt[2]] - (16*I)*EllipticPi[I*(1 + Sqrt[2]), ArcSin[(2^(1/4)*Sqrt[(1 +
Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqrt
[2]] + 5*EllipticPi[1 + Sqrt[2], ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2]
)/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqrt[2]])*Sec[e + f*
```

```
x]^(3/2)*Sin[e + f*x]*Sqrt[Cos[e + f*x] + Sin[e + f*x]]*Sqrt[-((1 + Tan[(e
+ f*x)/2])/((-2 + Sqrt[2])*(-1 + Tan[(e + f*x)/2])))])/(4*2^(1/4)*Sqrt[(Cos
[e + f*x] + Sin[e + f*x])/(-1 + Sin[e + f*x])]) - ((-5*EllipticF[ArcSin[(2^
(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2
]]], -3 - 2*Sqrt[2]] + 5*EllipticPi[-1 - Sqrt[2], ArcSin[(2^(1/4)*Sqrt[(1 +
 Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqr
t[2]] + (16*I)*EllipticPi[(-I)*(1 + Sqrt[2]), ArcSin[(2^(1/4)*Sqrt[(1 + Tan
[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqrt[2]
] - (16*I)*EllipticPi[I*(1 + Sqrt[2]), ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(e + f
*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqrt[2]] + 5*E
llipticPi[1 + Sqrt[2], ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Ta
n[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqrt[2]])*Sqrt[Sec[e + f*x]]*S
qrt[Cos[e + f*x] + Sin[e + f*x]]*((Cos[e + f*x] - Sin[e + f*x])/(-1 + Sin[e
 + f*x]) - (Cos[e + f*x]*(Cos[e + f*x] + Sin[e + f*x]))/(-1 + Sin[e + f*x])
^2)*Sqrt[-((1 + Tan[(e + f*x)/2])/((-2 + Sqrt[2])*(-1 + Tan[(e + f*x)/2])))
])/(4*2^(1/4)*((Cos[e + f*x] + Sin[e + f*x])/(-1 + Sin[e + f*x]))^(3/2)) +
((-5*EllipticF[ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f
*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqrt[2]] + 5*EllipticPi[-1 - Sqrt[2],
ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[
2 + Sqrt[2]]], -3 - 2*Sqrt[2]] + (16*I)*EllipticPi[(-I)*(1 + Sqrt[2]), ArcS
in[(2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 +
Sqrt[2]]], -3 - 2*Sqrt[2]] - (16*I)*EllipticPi[I*(1 + Sqrt[2]), ArcSin[(2^(
1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]
]], -3 - 2*Sqrt[2]] + 5*EllipticPi[1 + Sqrt[2], ArcSin[(2^(1/4)*Sqrt[(1 + T
an[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqrt[
2]])*Sqrt[Sec[e + f*x]]*Sqrt[Cos[e + f*x] + Sin[e + f*x]]*(-1/2*Sec[(e + f*
x)/2]^2/((-2 + Sqrt[2])*(-1 + Tan[(e + f*x)/2])) + (Sec[(e + f*x)/2]^2*(1 +
 Tan[(e + f*x)/2]))/(2*(-2 + Sqrt[2])*(-1 + Tan[(e + f*x)/2])^2)))/(4*2^(1/
4)*Sqrt[(Cos[e + f*x] + Sin[e + f*x])/(-1 + Sin[e + f*x])]*Sqrt[-((1 + Tan[
(e + f*x)/2])/((-2 + Sqrt[2])*(-1 + Tan[(e + f*...
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.72, size = 9665, normalized size = 31.48

method	result	size
default	Expression too large to display	9665

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(1+tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^3*(1+tan(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate((tan(f*x + e) + 1)^(3/2)*cot(f*x + e)^3, x)

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1071 vs. $2(250) = 500$.
time = 1.13, size = 1071, normalized size = 3.49

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^3*(1+tan(f*x+e))^(3/2),x, algorithm="fricas")

[Out] -1/8*(8^(1/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*(2*f*cos(f*x + e)^2 - sq
rt(2)*(f^3*cos(f*x + e)^2 - f^3)*sqrt(f^(-4)) - 2*f)*(f^(-4))^(1/4)*log(2*(
2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) + 8^(1/4)*sqrt(2*sqrt(2)*f^2*sqrt(f
^(-4)) + 4)*f*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/
4)*cos(f*x + e) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e)) - 8^(1/4)*
sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*(2*f*cos(f*x + e)^2 - sqrt(2)*(f^3*cos
(f*x + e)^2 - f^3)*sqrt(f^(-4)) - 2*f)*(f^(-4))^(1/4)*log(2*(2*sqrt(2)*f^2*
sqrt(f^(-4))*cos(f*x + e) - 8^(1/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f*
sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4)*cos(f*x + e
) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e)) - 5*(cos(f*x + e)^2 - 1)
*log(sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e)) + 1) + 5*(cos(f*x + e
)^2 - 1)*log(sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e)) - 1) - 2*(2*c
os(f*x + e)^2 + 5*cos(f*x + e)*sin(f*x + e))*sqrt((cos(f*x + e) + sin(f*x +
 e))/cos(f*x + e)) - 4*8^(1/4)*sqrt(2)*(f^5*cos(f*x + e)^2 - f^5)*sqrt(2*sq
rt(2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*arctan(-1/8*8^(3/4)*sqrt(2)*sqrt
(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^3*sqrt((cos(f*x + e) + sin(f*x + e))/cos
(f*x + e))*(f^(-4))^(3/4) + 1/8*8^(3/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4
)*f^3*sqrt((2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) + 8^(1/4)*sqrt(2*sqrt(2
)*f^2*sqrt(f^(-4)) + 4)*f*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*
(f^(-4))^(1/4)*cos(f*x + e) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e)
)*(f^(-4))^(3/4) - f^2*sqrt(f^(-4)) - sqrt(2))/f^4 - 4*8^(1/4)*sqrt(2)*(f^5
*cos(f*x + e)^2 - f^5)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*
arctan(-1/8*8^(3/4)*sqrt(2)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^3*sqrt((
cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) + 1/8*8^(3/4)*sqr
t(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^3*sqrt((2*sqrt(2)*f^2*sqrt(f^(-4))*cos(
f*x + e) - 8^(1/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f*sqrt((cos(f*x + e
) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4)*cos(f*x + e) + 2*cos(f*x + e
) + 2*sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) + f^2*sqrt(f^(-4)) + sqrt(
2))/f^4)/(f*cos(f*x + e)^2 - f)

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int (\tan (e + fx) + 1)^{\frac{3}{2}} \cot^3 (e + fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(1+tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((tan(e + f*x) + 1)**(3/2)*cot(e + f*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(1+tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((tan(f*x + e) + 1)^(3/2)*cot(f*x + e)^3, x)
```

Mupad [B]
time = 3.97, size = 142, normalized size = 0.46

$$\frac{\frac{3\,\sqrt{\tan (e+f\,x)+1}}{4}-\frac{5\,{(\tan (e+f\,x)+1)}^{3/2}}{4}}{f-2\,f\,\left(\tan (e+f\,x)+1\right)+f\,{\left(\tan (e+f\,x)+1\right)}^2}-\frac{\mathrm{atan}\left(\sqrt{\tan (e+f\,x)+1}\,1{}\mathrm{i}\right)\,5{}\mathrm{i}}{4\,f}-\mathrm{atan}\left(f\,\sqrt{\frac{-\frac{1}{2}-\frac{1}{2}{}\mathrm{i}}{f^2}}\,\sqrt{\tan (e+f\,x)+1}\right)\,\sqrt{\frac{-\frac{1}{2}-\frac{1}{2}{}\mathrm{i}}{f^2}}\,2{}\mathrm{i}+\mathrm{atan}\left(f\,\sqrt{\frac{-\frac{1}{2}+\frac{1}{2}{}\mathrm{i}}{f^2}}\,\sqrt{\tan (e+f\,x)+1}\right)\,\sqrt{\frac{-\frac{1}{2}+\frac{1}{2}{}\mathrm{i}}{f^2}}\,2{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(tan(e + f*x) + 1)^(3/2),x)
```

```
[Out] ((3*(tan(e + f*x) + 1)^(1/2))/4 - (5*(tan(e + f*x) + 1)^(3/2))/4)/(f - 2*f*
(tan(e + f*x) + 1) + f*(tan(e + f*x) + 1)^2) - (atan((tan(e + f*x) + 1)^(1/
2)*1i)*5i)/(4*f) - atan(f*((- 1/2 - 1i/2)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1/
2))*((- 1/2 - 1i/2)/f^2)^(1/2)*2i + atan(f*((- 1/2 + 1i/2)/f^2)^(1/2)*(tan(
e + f*x) + 1)^(1/2))*((- 1/2 + 1i/2)/f^2)^(1/2)*2i
```

3.394 $\int \cot^5(e+fx)(1+\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=361

$$-\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\tan(e+fx)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{f}+\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{\ldots}}{\sqrt{2\left(-1+\ldots\right.}}\right.}{f}$$

```
[Out] -83/64*arctanh((1+tan(f*x+e))^(1/2))/f-1/2*ln(1+2^(1/2)-(2+2*2^(1/2))^(1/2)
*(1+tan(f*x+e))^(1/2)+tan(f*x+e))/f/(1+2^(1/2))^(1/2)+1/2*ln(1+2^(1/2)+(2+2
*2^(1/2))^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))/f/(1+2^(1/2))^(1/2)-arctan
(((2+2*2^(1/2))^(1/2)-2*(1+tan(f*x+e))^(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^(1
/2))^(1/2)/f+arctan(((2+2*2^(1/2))^(1/2)+2*(1+tan(f*x+e))^(1/2))/(-2+2*2^(1
/2))^(1/2))*(1+2^(1/2))^(1/2)/f+83/64*cot(f*x+e)*(1+tan(f*x+e))^(1/2)/f+15/
32*cot(f*x+e)^2*(1+tan(f*x+e))^(1/2)/f-3/8*cot(f*x+e)^3*(1+tan(f*x+e))^(1/2
)/f-1/4*cot(f*x+e)^4*(1+tan(f*x+e))^(1/2)/f
```

Rubi [A]

time = 0.46, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3648, 3731, 3730, 3734, 12, 3566, 722, 1108, 648, 632, 210, 642, 3715, 65, 213}

$$-\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\tan(e+fx)+1}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{f}+\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{2\sqrt{\tan(e+fx)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{f}-\frac{\log\left(\tan(e+fx)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\tan(e+fx)+1}+\sqrt{2}+1\right)}{2\sqrt{1+\sqrt{2}}\,f}+\frac{\log\left(\tan(e+fx)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\tan(e+fx)+1}+\sqrt{2}+1\right)}{2\sqrt{1+\sqrt{2}}\,f}-\frac{83\tanh^{-1}\left(\sqrt{\tan(e+fx)+1}\right)}{64f}-\frac{\sqrt{\tan(e+fx)+1}\,\cot^4(e+fx)}{4f}-\frac{3\sqrt{\tan(e+fx)+1}\,\cot^3(e+fx)}{8f}+\frac{15\sqrt{\tan(e+fx)+1}\,\cot^2(e+fx)}{32f}+\frac{83\sqrt{\tan(e+fx)+1}\,\cot(e+fx)}{64f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*(1 + Tan[e + f*x])^(3/2),x]
```

```
[Out] -((Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + Tan[e + f*x
]])/Sqrt[2*(-1 + Sqrt[2])]])/f) + (Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sq
rt[2])] + 2*Sqrt[1 + Tan[e + f*x]])/Sqrt[2*(-1 + Sqrt[2])]])/f - (83*ArcTan
h[Sqrt[1 + Tan[e + f*x]]])/(64*f) - Log[1 + Sqrt[2] + Tan[e + f*x] - Sqrt[2
*(1 + Sqrt[2])]*Sqrt[1 + Tan[e + f*x]]]/(2*Sqrt[1 + Sqrt[2]]*f) + Log[1 + S
qrt[2] + Tan[e + f*x] + Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + Tan[e + f*x]]]/(2*Sq
rt[1 + Sqrt[2]]*f) + (83*Cot[e + f*x]*Sqrt[1 + Tan[e + f*x]])/(64*f) + (15*
Cot[e + f*x]^2*Sqrt[1 + Tan[e + f*x]])/(32*f) - (3*Cot[e + f*x]^3*Sqrt[1 +
Tan[e + f*x]])/(8*f) - (Cot[e + f*x]^4*Sqrt[1 + Tan[e + f*x]])/(4*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 722

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3731

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)
*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp
[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - a*C*(b*c*(m + 1) + a*d*(n
+ 1)) - (m + 1)*(b*c - a*d)*(A*b - b*C)*Tan[e + f*x] - d*(A*b^2 + a^2*C)*(m
 + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m,
 -1] &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \cot^5(e+fx)(1+\tan(e+fx))^{3/2}\,dx = -\frac{\cot^4(e+fx)\sqrt{1+\tan(e+fx)}}{4f} - \frac{1}{4}\int \frac{\cot^4(e+fx)\left(-\frac{9}{2}+\frac{7}{2}\text{ta}\right.}{\sqrt{1+\tan(e+}}$$

$$= -\frac{3\cot^3(e+fx)\sqrt{1+\tan(e+fx)}}{8f} - \frac{\cot^4(e+fx)\sqrt{1+\tan(e+}}{4f}$$

$$= \frac{15\cot^2(e+fx)\sqrt{1+\tan(e+fx)}}{32f} - \frac{3\cot^3(e+fx)\sqrt{1+\tan(e+}}{8f}$$

$$= \frac{83\cot(e+fx)\sqrt{1+\tan(e+fx)}}{64f} + \frac{15\cot^2(e+fx)\sqrt{1+\tan(e+}}{32f}$$

$$= \frac{83\cot(e+fx)\sqrt{1+\tan(e+fx)}}{64f} + \frac{15\cot^2(e+fx)\sqrt{1+\tan(e+}}{32f}$$

$$= \frac{83\cot(e+fx)\sqrt{1+\tan(e+fx)}}{64f} + \frac{15\cot^2(e+fx)\sqrt{1+\tan(e+}}{32f}$$

$$= \frac{83\cot(e+fx)\sqrt{1+\tan(e+fx)}}{64f} + \frac{15\cot^2(e+fx)\sqrt{1+\tan(e+}}{32f}$$

$$= -\frac{83\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{64f} + \frac{83\cot(e+fx)\sqrt{1+\tan(e+}}{64f}$$

$$= -\frac{83\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{64f} + \frac{83\cot(e+fx)\sqrt{1+\tan(e+}}{64f}$$

$$= -\frac{83\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{64f} + \frac{83\cot(e+fx)\sqrt{1+\tan(e+}}{64f}$$

$$= -\frac{83\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{64f} - \frac{\log\left(1+\sqrt{2}+\tan(e+fx)\right.}{2}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\tan(e+fx)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{\sqrt{-1+\sqrt{2}}\,f} + \frac{\tan^{-1}\left(\frac{\sqrt{2}\right.}{}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 21.98, size = 4084, normalized size = 11.31

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^5*(1 + Tan[e + f*x])^(3/2),x]
```

```
[Out] (Cos[e + f*x]*(-23/32 + (107*Cot[e + f*x])/64 + (31*Csc[e + f*x]^2)/32 - (3
*Cot[e + f*x]*Csc[e + f*x]^2)/8 - Csc[e + f*x]^4/4)*(1 + Tan[e + f*x])^(3/2
))/(f*(Cos[e + f*x] + Sin[e + f*x])) + (Cos[e + f*x]*(83*EllipticF[ArcSin[(
2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt
[2]]], -3 - 2*Sqrt[2]] - 83*EllipticPi[-1 - Sqrt[2], ArcSin[(2^(1/4)*Sqrt[(
1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*
Sqrt[2]] - (256*I)*EllipticPi[(-I)*(1 + Sqrt[2]), ArcSin[(2^(1/4)*Sqrt[(1 +
 Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqr
t[2]] + (256*I)*EllipticPi[I*(1 + Sqrt[2]), ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(
e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqrt[2]]
- 83*EllipticPi[1 + Sqrt[2], ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-
1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqrt[2]])*((83*Csc[e + f
*x]*Sqrt[Sec[e + f*x]])/(128*Sqrt[Cos[e + f*x] + Sin[e + f*x]]) + (Csc[e +
f*x]*Sqrt[Sec[e + f*x]]*Sin[2*(e + f*x)])/Sqrt[Cos[e + f*x] + Sin[e + f*x]]
)*Sqrt[-((1 + Tan[(e + f*x)/2])/((-2 + Sqrt[2])*(-1 + Tan[(e + f*x)/2])))]*
(1 + Tan[e + f*x])^(3/2))/(32*2^(1/4)*f*(Cos[e + f*x] + Sin[e + f*x])*Sqrt[
(Cos[e + f*x] + Sin[e + f*x])/(-1 + Sin[e + f*x])]*(((83*EllipticF[ArcSin[(
2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt
[2]]], -3 - 2*Sqrt[2]] - 83*EllipticPi[-1 - Sqrt[2], ArcSin[(2^(1/4)*Sqrt[(
1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*
Sqrt[2]] - (256*I)*EllipticPi[(-I)*(1 + Sqrt[2]), ArcSin[(2^(1/4)*Sqrt[(1 +
 Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqr
t[2]] + (256*I)*EllipticPi[I*(1 + Sqrt[2]), ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(
e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqrt[2]]
- 83*EllipticPi[1 + Sqrt[2], ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-
1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqrt[2]])*Sqrt[Sec[e + f
*x]]*(Cos[e + f*x] - Sin[e + f*x])*Sqrt[-((1 + Tan[(e + f*x)/2])/((-2 + Sqr
t[2])*(-1 + Tan[(e + f*x)/2])))])/(64*2^(1/4)*Sqrt[Cos[e + f*x] + Sin[e + f
*x]]*Sqrt[(Cos[e + f*x] + Sin[e + f*x])/(-1 + Sin[e + f*x])]) + ((83*Ellipt
icF[ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/S
qrt[2 + Sqrt[2]]], -3 - 2*Sqrt[2]] - 83*EllipticPi[-1 - Sqrt[2], ArcSin[(2^
(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2
]]], -3 - 2*Sqrt[2]] - (256*I)*EllipticPi[(-I)*(1 + Sqrt[2]), ArcSin[(2^(1/
4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]]
, -3 - 2*Sqrt[2]] + (256*I)*EllipticPi[I*(1 + Sqrt[2]), ArcSin[(2^(1/4)*Sqr
t[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 -
```

```
 2*Sqrt[2]] - 83*EllipticPi[1 + Sqrt[2], ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(e +
 f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqrt[2]])*Se
c[e + f*x]^(3/2)*Sin[e + f*x]*Sqrt[Cos[e + f*x] + Sin[e + f*x]]*Sqrt[-((1 +
 Tan[(e + f*x)/2])/((-2 + Sqrt[2])*(-1 + Tan[(e + f*x)/2])))])/(64*2^(1/4)*
Sqrt[(Cos[e + f*x] + Sin[e + f*x])/(-1 + Sin[e + f*x])]) - ((83*EllipticF[A
rcSin[(2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2
 + Sqrt[2]]], -3 - 2*Sqrt[2]] - 83*EllipticPi[-1 - Sqrt[2], ArcSin[(2^(1/4)
*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]],
-3 - 2*Sqrt[2]] - (256*I)*EllipticPi[(-I)*(1 + Sqrt[2]), ArcSin[(2^(1/4)*Sq
rt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3
- 2*Sqrt[2]] + (256*I)*EllipticPi[I*(1 + Sqrt[2]), ArcSin[(2^(1/4)*Sqrt[(1
+ Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sq
rt[2]] - 83*EllipticPi[1 + Sqrt[2], ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(e + f*x)
/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqrt[2]])*Sqrt[Se
c[e + f*x]]*Sqrt[Cos[e + f*x] + Sin[e + f*x]]*((Cos[e + f*x] - Sin[e + f*x]
)/(-1 + Sin[e + f*x]) - (Cos[e + f*x]*(Cos[e + f*x] + Sin[e + f*x]))/(-1 +
Sin[e + f*x])^2)*Sqrt[-((1 + Tan[(e + f*x)/2])/((-2 + Sqrt[2])*(-1 + Tan[(e
 + f*x)/2])))])/(64*2^(1/4)*((Cos[e + f*x] + Sin[e + f*x])/(-1 + Sin[e + f*
x]))^(3/2)) + ((83*EllipticF[ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-
1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqrt[2]] - 83*EllipticPi
[-1 - Sqrt[2], ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f
*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqrt[2]] - (256*I)*EllipticPi[(-I)*(1
+ Sqrt[2]), ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)
/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqrt[2]] + (256*I)*EllipticPi[I*(1 + Sqrt
[2]), ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])
/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqrt[2]] - 83*EllipticPi[1 + Sqrt[2], ArcSin[(2
^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[
2]]], -3 - 2*Sqrt[2]])*Sqrt[Sec[e + f*x]]*Sqrt[Cos[e + f*x] + Sin[e + f*x]]
*(-1/2*Sec[(e + f*x)/2]^2/((-2 + Sqrt[2])*(-1 + Tan[(e + f*x)/2])) + (Sec[(
e + f*x)/2]^2*(1 + Tan[(e + f*x)/2]))/(2*(-2 + Sqrt[2])*(-1 + Tan[(e + f*x)
/2])^2)))/(64*2^(1/4)*Sqrt[(Cos[e + f*x] + Sin[...
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.77, size = 14663, normalized size = 40.62

method	result	size
default	Expression too large to display	14663

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(1+tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(1+tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1200 vs. $2(300) = 600$.
time = 1.26, size = 1200, normalized size = 3.32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(1+tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/128*(16*8^(1/4)*(2*f*cos(f*x + e)^4 - 4*f*cos(f*x + e)^2 - sqrt(2)*(f^3*c
os(f*x + e)^4 - 2*f^3*cos(f*x + e)^2 + f^3)*sqrt(f^(-4)) + 2*f)*sqrt(2*sqrt
(2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*log(2*(2*sqrt(2)*f^2*sqrt(f^(-4))*
cos(f*x + e) + 8^(1/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f*sqrt((cos(f*x
 + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4)*cos(f*x + e) + 2*cos(f*x
 + e) + 2*sin(f*x + e))/cos(f*x + e)) - 16*8^(1/4)*(2*f*cos(f*x + e)^4 - 4*
f*cos(f*x + e)^2 - sqrt(2)*(f^3*cos(f*x + e)^4 - 2*f^3*cos(f*x + e)^2 + f^3
)*sqrt(f^(-4)) + 2*f)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*l
og(2*(2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) - 8^(1/4)*sqrt(2*sqrt(2)*f^2*
sqrt(f^(-4)) + 4)*f*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4
))^(1/4)*cos(f*x + e) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e)) - 83
*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*log(sqrt((cos(f*x + e) + sin(f*x +
 e))/cos(f*x + e)) + 1) + 83*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*log(sq
rt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e)) - 1) - 2*(46*cos(f*x + e)^4
- 30*cos(f*x + e)^2 + (107*cos(f*x + e)^3 - 83*cos(f*x + e))*sin(f*x + e))*
sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e)) - 64*8^(1/4)*sqrt(2)*(f^5*
cos(f*x + e)^4 - 2*f^5*cos(f*x + e)^2 + f^5)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)
) + 4)*(f^(-4))^(1/4)*arctan(-1/8*8^(3/4)*sqrt(2)*sqrt(2*sqrt(2)*f^2*sqrt(f
^(-4)) + 4)*f^3*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(
3/4) + 1/8*8^(3/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^3*sqrt((2*sqrt(2)
*f^2*sqrt(f^(-4))*cos(f*x + e) + 8^(1/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) +
4)*f*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4)*cos(f*
x + e) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) - f^
2*sqrt(f^(-4)) - sqrt(2))/f^4 - 64*8^(1/4)*sqrt(2)*(f^5*cos(f*x + e)^4 - 2*
f^5*cos(f*x + e)^2 + f^5)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/
4)*arctan(-1/8*8^(3/4)*sqrt(2)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^3*sqr
t((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) + 1/8*8^(3/4)*
sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f^3*sqrt((2*sqrt(2)*f^2*sqrt(f^(-4))*c
os(f*x + e) - 8^(1/4)*sqrt(2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f*sqrt((cos(f*x
+ e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4)*cos(f*x + e) + 2*cos(f*x
+ e) + 2*sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) + f^2*sqrt(f^(-4)) + sq
rt(2))/f^4)/(f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^2 + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (\tan(e+fx)+1)^{\frac{3}{2}}\cot^5(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(1+tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((tan(e + f*x) + 1)**(3/2)*cot(e + f*x)**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(1+tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((tan(f*x + e) + 1)^(3/2)*cot(f*x + e)^5, x)
```

Mupad [B]
time = 3.98, size = 193, normalized size = 0.53

$$\frac{\mathrm{atan}\left(\sqrt{\tan(e+fx)+1}\,1\mathrm{i}\right)83\mathrm{i}}{64f}-\frac{\frac{45\sqrt{\tan(e+fx)+1}}{64}-\frac{165(\tan(e+fx)+1)^{3/2}}{64}+\frac{219(\tan(e+fx)+1)^{5/2}}{64}-\frac{83(\tan(e+fx)+1)^{7/2}}{64}}{f-4f(\tan(e+fx)+1)+6f(\tan(e+fx)+1)^2-4f(\tan(e+fx)+1)^3+f(\tan(e+fx)+1)^4}+\mathrm{atan}\left(f\sqrt{\frac{-\frac{1}{2}-\frac{1}{2}\mathrm{i}}{f^2}}\sqrt{\tan(e+fx)+1}\right)\sqrt{\frac{-\frac{1}{2}-\frac{1}{2}\mathrm{i}}{f^2}}\,2\mathrm{i}-\mathrm{atan}\left(f\sqrt{\frac{-\frac{1}{2}+\frac{1}{2}\mathrm{i}}{f^2}}\sqrt{\tan(e+fx)+1}\right)\sqrt{\frac{-\frac{1}{2}+\frac{1}{2}\mathrm{i}}{f^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5*(tan(e + f*x) + 1)^(3/2),x)
```

```
[Out] (atan((tan(e + f*x) + 1)^(1/2)*1i)*83i)/(64*f) - ((45*(tan(e + f*x) + 1)^(1
/2))/64 - (165*(tan(e + f*x) + 1)^(3/2))/64 + (219*(tan(e + f*x) + 1)^(5/2)
)/64 - (83*(tan(e + f*x) + 1)^(7/2))/64)/(f - 4*f*(tan(e + f*x) + 1) + 6*f*
(tan(e + f*x) + 1)^2 - 4*f*(tan(e + f*x) + 1)^3 + f*(tan(e + f*x) + 1)^4) +
 atan(f*((- 1/2 - 1i/2)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1/2))*((- 1/2 - 1i/2
)/f^2)^(1/2)*2i - atan(f*((- 1/2 + 1i/2)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1/2
))*((- 1/2 + 1i/2)/f^2)^(1/2)*2i
```

3.395 $\int \tan^4(e+fx)(1+\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=227

$$-\frac{\sqrt{-1+\sqrt{2}}\ \text{ArcTan}\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\tan(e+fx)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\ \sqrt{1+\tan(e+fx)}}\right)}{f}-\frac{\sqrt{1+\sqrt{2}}\ \tanh^{-1}\left(\frac{3+2\sqrt{2}+\left(1-\right.}{\sqrt{2\left(7+5\sqrt{2}\right)}}\right.}{f}$$

```
[Out] -arctan((3-2*2^(1/2)+(1-2^(1/2))*tan(f*x+e))/(-14+10*2^(1/2))^(1/2)/(1+tan(
f*x+e))^(1/2))*(2^(1/2)-1)^(1/2)/f-arctanh((3+2*2^(1/2)+(1+2^(1/2))*tan(f*x
+e))/(14+10*2^(1/2))^(1/2)/(1+tan(f*x+e))^(1/2))*(1+2^(1/2))^(1/2)/f+2*(1+t
an(f*x+e))^(1/2)/f-22/63*(1+tan(f*x+e))^(5/2)/f-8/63*tan(f*x+e)*(1+tan(f*x+
e))^(5/2)/f+2/9*tan(f*x+e)^2*(1+tan(f*x+e))^(5/2)/f
```

Rubi [A]
time = 0.29, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3647, 3728, 3712, 3563, 12, 3617, 3616, 209, 213}

$$-\frac{\sqrt{\sqrt{2}-1}\ \text{ArcTan}\left(\frac{\left(1-\sqrt{2}\right)\tan(e+fx)-2\sqrt{2}+3}{\sqrt{2\left(5\sqrt{2}-7\right)}\ \sqrt{\tan(e+fx)+1}}\right)}{f}+\frac{2\tan^2(e+fx)(\tan(e+fx)+1)^{5/2}}{9f}-\frac{8\tan(e+fx)(\tan(e+fx)+1)^{5/2}}{63f}-\frac{22(\tan(e+fx)+1)^{5/2}}{63f}+\frac{2\sqrt{\tan(e+fx)+1}}{f}-\frac{\sqrt{1+\sqrt{2}}\ \tanh^{-1}\left(\frac{\left(1+\sqrt{2}\right)\tan(e+fx)+2\sqrt{2}+3}{\sqrt{2\left(7+5\sqrt{2}\right)}\ \sqrt{\tan(e+fx)+1}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4*(1 + Tan[e + f*x])^(3/2),x]
```

```
[Out] -((Sqrt[-1 + Sqrt[2]]*ArcTan[(3 - 2*Sqrt[2] + (1 - Sqrt[2])*Tan[e + f*x])/(
Sqrt[2*(-7 + 5*Sqrt[2])]*Sqrt[1 + Tan[e + f*x]])])/f) - (Sqrt[1 + Sqrt[2]]*
ArcTanh[(3 + 2*Sqrt[2] + (1 + Sqrt[2])*Tan[e + f*x])/(Sqrt[2*(7 + 5*Sqrt[2]
)]*Sqrt[1 + Tan[e + f*x]])])/f + (2*Sqrt[1 + Tan[e + f*x]])/f - (22*(1 + Ta
n[e + f*x])^(5/2))/(63*f) - (8*Tan[e + f*x]*(1 + Tan[e + f*x])^(5/2))/(63*f
) + (2*Tan[e + f*x]^2*(1 + Tan[e + f*x])^(5/2))/(9*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3563

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3616

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*b*c*d - 4*a*d^2 +
 x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

Rule 3617

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2*q), Int[(a
*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
x], x] - Dist[1/(2*q), Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f*x
])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d - b
*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3712

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
```

```
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \tan^4(e+fx)(1+\tan(e+fx))^{3/2}\,dx = \frac{2\tan^2(e+fx)(1+\tan(e+fx))^{5/2}}{9f}+\frac{2}{9}\int \tan(e+fx)(1+\tan($$

$$= -\frac{8\tan(e+fx)(1+\tan(e+fx))^{5/2}}{63f}+\frac{2\tan^2(e+fx)(1+\tan(e}{9f}$$

$$= -\frac{22(1+\tan(e+fx))^{5/2}}{63f}-\frac{8\tan(e+fx)(1+\tan(e+fx))^{5/2}}{63f}+$$

$$= \frac{2\sqrt{1+\tan(e+fx)}}{f}-\frac{22(1+\tan(e+fx))^{5/2}}{63f}-\frac{8\tan(e+fx)(}{}$$

$$= \frac{2\sqrt{1+\tan(e+fx)}}{f}-\frac{22(1+\tan(e+fx))^{5/2}}{63f}-\frac{8\tan(e+fx)(}{}$$

$$= \frac{2\sqrt{1+\tan(e+fx)}}{f}-\frac{22(1+\tan(e+fx))^{5/2}}{63f}-\frac{8\tan(e+fx)(}{}$$

$$= \frac{2\sqrt{1+\tan(e+fx)}}{f}-\frac{22(1+\tan(e+fx))^{5/2}}{63f}-\frac{8\tan(e+fx)(}{}$$

$$= -\frac{\sqrt{-1+\sqrt{2}}\,\tan^{-1}\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\tan(e+fx)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\,\sqrt{1+\tan(e+fx)}}\right)}{f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.51, size = 155, normalized size = 0.68

$$\frac{2\cos^2(e+fx)\left(-63\left(\frac{\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1-i}}\right)}{\sqrt{1-i}}+\frac{\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1+i}}\right)}{\sqrt{1+i}}\right)(1+\tan(e+fx))^2+(1+\tan(e+fx))^{5/2}(71+7\sec^4(e+fx)-36\tan(e+fx)+2\sec^2(e+fx)(-13+5\tan(e+fx)))\right)}{63f(\cos(e+fx)+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4*(1 + Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*Cos[e + f*x]^2*(-63*(ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1 - I]]/Sqrt[1
- I] + ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1 + I]]/Sqrt[1 + I])*(1 + Tan[e
+ f*x])^2 + (1 + Tan[e + f*x])^(5/2)*(71 + 7*Sec[e + f*x]^4 - 36*Tan[e + f*
x] + 2*Sec[e + f*x]^2*(-13 + 5*Tan[e + f*x]))))/(63*f*(Cos[e + f*x] + Sin[e
 + f*x])^2)
```

Maple [A]
time = 0.16, size = 245, normalized size = 1.08

method	result
derivativedivides	$\frac{\frac{2(1+\tan(fx+e))^{\frac{9}{2}}}{9}-\frac{4(1+\tan(fx+e))^{\frac{7}{2}}}{7}+2\sqrt{1+\tan(fx+e)}-\frac{\sqrt{2}\left(-\frac{\sqrt{2\sqrt{2}+2}\ln\left(1+\sqrt{2}-\sqrt{2\sqrt{2}}\right.}{}\right.}{}}{}$
default	$\frac{\frac{2(1+\tan(fx+e))^{\frac{9}{2}}}{9}-\frac{4(1+\tan(fx+e))^{\frac{7}{2}}}{7}+2\sqrt{1+\tan(fx+e)}-\frac{\sqrt{2}\left(-\frac{\sqrt{2\sqrt{2}+2}\ln\left(1+\sqrt{2}-\sqrt{2\sqrt{2}}\right.}{}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4*(1+tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2/9*(1+tan(f*x+e))^(9/2)-4/7*(1+tan(f*x+e))^(7/2)+2*(1+tan(f*x+e))^(1/
2)-1/2*2^(1/2)*(-1/2*(2*2^(1/2)+2)^(1/2)*ln(1+2^(1/2)-(2*2^(1/2)+2)^(1/2)*(
1+tan(f*x+e))^(1/2)+tan(f*x+e))+2*(1-2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((
2*(1+tan(f*x+e))^(1/2)-(2*2^(1/2)+2)^(1/2))/(-2+2*2^(1/2))^(1/2)))-1/2*2^(1
/2)*(1/2*(2*2^(1/2)+2)^(1/2)*ln(1+2^(1/2)+(2*2^(1/2)+2)^(1/2)*(1+tan(f*x+e)
)^(1/2)+tan(f*x+e))+2*(1-2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan(((2*2^(1/2)+2
)^(1/2)+2*(1+tan(f*x+e))^(1/2))/(-2+2*2^(1/2))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(1+tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((tan(f*x + e) + 1)^(3/2)*tan(f*x + e)^4, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1105 vs. $2(190) = 380$.
time = 1.04, size = 1105, normalized size = 4.87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(1+tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/504*(252*8^(1/4)*sqrt(2)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f*(f^(-4)
)^(1/4)*arctan(1/16*8^(3/4)*sqrt(2)*(2*f^5*sqrt(f^(-4)) + sqrt(2)*f^3)*sqrt
(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x
+ e) + 8^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4))*cos(f*x + e) + f*cos(f*x + e))*sqr
t(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f
*x + e))*(f^(-4))^(1/4) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e))*(f
^(-4))^(3/4) - 1/8*8^(3/4)*(2*f^5*sqrt(f^(-4)) + sqrt(2)*f^3)*sqrt(-2*sqrt(
2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(
f^(-4))^(3/4) - f^2*sqrt(f^(-4)) - sqrt(2))*cos(f*x + e)^4 + 252*8^(1/4)*sq
rt(2)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f*(f^(-4))^(1/4)*arctan(1/16*8^
(3/4)*sqrt(2)*(2*f^5*sqrt(f^(-4)) + sqrt(2)*f^3)*sqrt(-2*sqrt(2)*f^2*sqrt(f
^(-4)) + 4)*sqrt((2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) - 8^(1/4)*(sqrt(2
)*f^3*sqrt(f^(-4))*cos(f*x + e) + f*cos(f*x + e))*sqrt(-2*sqrt(2)*f^2*sqrt(
f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4
) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) - 1/8*8^(
3/4)*(2*f^5*sqrt(f^(-4)) + sqrt(2)*f^3)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) +
4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) + f^2*sq
rt(f^(-4)) + sqrt(2))*cos(f*x + e)^4 + 63*8^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4))
*cos(f*x + e)^4 + 2*f*cos(f*x + e)^4)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)
*(f^(-4))^(1/4)*log(2*(2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) + 8^(1/4)*(s
qrt(2)*f^3*sqrt(f^(-4))*cos(f*x + e) + f*cos(f*x + e))*sqrt(-2*sqrt(2)*f^2*
sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))
^(1/4) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e)) - 63*8^(1/4)*(sqrt(
2)*f^3*sqrt(f^(-4))*cos(f*x + e)^4 + 2*f*cos(f*x + e)^4)*sqrt(-2*sqrt(2)*f^
2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*log(2*(2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*
x + e) - 8^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4))*cos(f*x + e) + f*cos(f*x + e))*s
qrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos
(f*x + e))*(f^(-4))^(1/4) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e))
- 16*(71*cos(f*x + e)^4 - 26*cos(f*x + e)^2 - 2*(18*cos(f*x + e)^3 - 5*cos(
f*x + e))*sin(f*x + e) + 7)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e)
))/(f*cos(f*x + e)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (\tan(e+fx)+1)^{\frac{3}{2}}\tan^4(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4*(1+tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((tan(e + f*x) + 1)**(3/2)*tan(e + f*x)**4, x)
```

Giac [A]
time = 0.84, size = 252, normalized size = 1.11

$$\frac{\sqrt{\sqrt{2}-1}\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{f}+\frac{\sqrt{\sqrt{2}-1}\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{f}-\frac{\sqrt{\sqrt{2}+1}\log\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{2f}+\frac{\sqrt{\sqrt{2}+1}\log\left(-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{2f}+\frac{2\left(7f^8(\tan(fx+e)+1)^{\frac{9}{2}}-18f^8(\tan(fx+e)+1)^{\frac{7}{2}}+63f^8\sqrt{\tan(fx+e)+1}\right)}{63f^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(1+tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(sqrt(2) - 1)*arctan(1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*sqrt(ta
n(f*x + e) + 1))/sqrt(-sqrt(2) + 2))/f + sqrt(sqrt(2) - 1)*arctan(-1/2*2^(3
/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(tan(f*x + e) + 1))/sqrt(-sqrt(2) +
2))/f - 1/2*sqrt(sqrt(2) + 1)*log(2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x +
e) + 1) + sqrt(2) + tan(f*x + e) + 1)/f + 1/2*sqrt(sqrt(2) + 1)*log(-2^(1/4
)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x + e) + 1) + sqrt(2) + tan(f*x + e) + 1)/f
+ 2/63*(7*f^8*(tan(f*x + e) + 1)^(9/2) - 18*f^8*(tan(f*x + e) + 1)^(7/2) +
63*f^8*sqrt(tan(f*x + e) + 1))/f^9
```

Mupad [B]
time = 6.00, size = 118, normalized size = 0.52

$$\frac{2\sqrt{\tan(e+fx)+1}}{f}-\frac{4(\tan(e+fx)+1)^{7/2}}{7f}+\frac{2(\tan(e+fx)+1)^{9/2}}{9f}+\mathrm{atan}\left(f\sqrt{\frac{\frac{1}{2}-\frac{1}{2}\mathrm{i}}{f^2}}\sqrt{\tan(e+fx)+1}\,1\mathrm{i}\right)\sqrt{\frac{\frac{1}{2}-\frac{1}{2}\mathrm{i}}{f^2}}\,2\mathrm{i}+\mathrm{atan}\left(f\sqrt{\frac{\frac{1}{2}+\frac{1}{2}\mathrm{i}}{f^2}}\sqrt{\tan(e+fx)+1}\,1\mathrm{i}\right)\sqrt{\frac{\frac{1}{2}+\frac{1}{2}\mathrm{i}}{f^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(tan(e + f*x) + 1)^(3/2),x)
```

```
[Out] (2*(tan(e + f*x) + 1)^(1/2))/f - (4*(tan(e + f*x) + 1)^(7/2))/(7*f) + (2*(t
an(e + f*x) + 1)^(9/2))/(9*f) + atan(f*((1/2 - 1i/2)/f^2)^(1/2)*(tan(e + f*
x) + 1)^(1/2)*1i)*((1/2 - 1i/2)/f^2)^(1/2)*2i + atan(f*((1/2 + 1i/2)/f^2)^(
1/2)*(tan(e + f*x) + 1)^(1/2)*1i)*((1/2 + 1i/2)/f^2)^(1/2)*2i
```

3.396 $\int \tan^2(e+fx)(1+\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=173

$$\frac{\sqrt{-1+\sqrt{2}}\,\text{ArcTan}\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\tan(e+fx)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\,\sqrt{1+\tan(e+fx)}}\right)}{f}+\frac{\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{3+2\sqrt{2}+\left(1+\sqrt{2}\right)\tan(e+fx)}{\sqrt{2\left(7+5\sqrt{2}\right)}\,\sqrt{1+\tan(e+fx)}}\right)}{f}$$

```
[Out] arctan((3-2*2^(1/2)+(1-2^(1/2))*tan(f*x+e))/(-14+10*2^(1/2))^(1/2)/(1+tan(f
*x+e))^(1/2))*(2^(1/2)-1)^(1/2)/f+arctanh((3+2*2^(1/2)+(1+2^(1/2))*tan(f*x+
e))/(14+10*2^(1/2))^(1/2)/(1+tan(f*x+e))^(1/2))*(1+2^(1/2))^(1/2)/f-2*(1+ta
n(f*x+e))^(1/2)/f+2/5*(1+tan(f*x+e))^(5/2)/f
```

Rubi [A]
time = 0.13, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3624, 3563, 12, 3617, 3616, 209, 213}

$$\frac{\sqrt{\sqrt{2}-1}\,\text{ArcTan}\left(\frac{\left(1-\sqrt{2}\right)\tan(e+fx)-2\sqrt{2}+3}{\sqrt{2\left(5\sqrt{2}-7\right)}\,\sqrt{\tan(e+fx)+1}}\right)}{f}+\frac{2(\tan(e+fx)+1)^{5/2}}{5f}-\frac{2\sqrt{\tan(e+fx)+1}}{f}+\frac{\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\left(1+\sqrt{2}\right)\tan(e+fx)+2\sqrt{2}+3}{\sqrt{2\left(7+5\sqrt{2}\right)}\,\sqrt{\tan(e+fx)+1}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2*(1 + Tan[e + f*x])^(3/2),x]
```

```
[Out] (Sqrt[-1 + Sqrt[2]]*ArcTan[(3 - 2*Sqrt[2] + (1 - Sqrt[2])*Tan[e + f*x])/(Sq
rt[2*(-7 + 5*Sqrt[2])]*Sqrt[1 + Tan[e + f*x]])])/f + (Sqrt[1 + Sqrt[2]]*Arc
Tanh[(3 + 2*Sqrt[2] + (1 + Sqrt[2])*Tan[e + f*x])/(Sqrt[2*(7 + 5*Sqrt[2])]*
Sqrt[1 + Tan[e + f*x]])])/f - (2*Sqrt[1 + Tan[e + f*x]])/f + (2*(1 + Tan[e
+ f*x])^(5/2))/(5*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 3563

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3616

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*b*c*d - 4*a*d^2 +
 x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

Rule 3617

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2*q), Int[(a
*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
x], x] - Dist[1/(2*q), Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f*x
])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d - b
*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \tan^2(e+fx)(1+\tan(e+fx))^{3/2}\,dx = \frac{2(1+\tan(e+fx))^{5/2}}{5f} - \int (1+\tan(e+fx))^{3/2}\,dx$$
$$= -\frac{2\sqrt{1+\tan(e+fx)}}{f} + \frac{2(1+\tan(e+fx))^{5/2}}{5f} - \int \frac{2\tan(e}{\sqrt{1+\tan(}}$$
$$= -\frac{2\sqrt{1+\tan(e+fx)}}{f} + \frac{2(1+\tan(e+fx))^{5/2}}{5f} - 2\int \frac{\tan(e}{\sqrt{1+\tan}}$$
$$= -\frac{2\sqrt{1+\tan(e+fx)}}{f} + \frac{2(1+\tan(e+fx))^{5/2}}{5f} + \frac{\int \frac{1+\left(-1-\sqrt{2}\right)}{\sqrt{1+\tan(}}}{\sqrt{2}}$$
$$= -\frac{2\sqrt{1+\tan(e+fx)}}{f} + \frac{2(1+\tan(e+fx))^{5/2}}{5f} - \frac{\left(4-3\sqrt{2}\right)\,S}{}$$
$$= \frac{\sqrt{-1+\sqrt{2}}\,\tan^{-1}\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\tan(e+fx)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\sqrt{1+\tan(e+fx)}}\right)}{f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.40, size = 100, normalized size = 0.58

$$\frac{\frac{10\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1-i}}\right)}{\sqrt{1-i}} + \frac{10\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1+i}}\right)}{\sqrt{1+i}} + 2\sqrt{1+\tan(e+fx)}\,(-4+2\tan(e+fx)+\tan^2(e+fx))}{5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2*(1 + Tan[e + f*x])^(3/2),x]
```

```
[Out] ((10*ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1 - I]])/Sqrt[1 - I] + (10*ArcTanh
[Sqrt[1 + Tan[e + f*x]]/Sqrt[1 + I]])/Sqrt[1 + I] + 2*Sqrt[1 + Tan[e + f*x]
]*(-4 + 2*Tan[e + f*x] + Tan[e + f*x]^2))/(5*f)
```

Maple [A]
time = 0.11, size = 233, normalized size = 1.35

method	result

derivativedivides	$\frac{\frac{2(1+\tan(fx+e))^{\frac{5}{2}}}{5}-2\sqrt{1+\tan(fx+e)}+\frac{\sqrt{2}\left(-\frac{\sqrt{2\sqrt{2}+2}\,\ln\left(1+\sqrt{2}-\sqrt{2\sqrt{2}+2}\,\sqrt{1+\tan}\right.}{2}\right.}{}}{}$
default	$\frac{\frac{2(1+\tan(fx+e))^{\frac{5}{2}}}{5}-2\sqrt{1+\tan(fx+e)}+\frac{\sqrt{2}\left(-\frac{\sqrt{2\sqrt{2}+2}\,\ln\left(1+\sqrt{2}-\sqrt{2\sqrt{2}+2}\,\sqrt{1+\tan}\right.}{2}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2*(1+tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2/5*(1+tan(f*x+e))^(5/2)-2*(1+tan(f*x+e))^(1/2)+1/2*2^(1/2)*(-1/2*(2*2
^(1/2)+2)^(1/2)*ln(1+2^(1/2)-(2*2^(1/2)+2)^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f
*x+e))+2*(1-2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+tan(f*x+e))^(1/2)-(2
*2^(1/2)+2)^(1/2))/(-2+2*2^(1/2))^(1/2)))+1/2*2^(1/2)*(1/2*(2*2^(1/2)+2)^(1
/2)*ln(1+2^(1/2)+(2*2^(1/2)+2)^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))+2*(1-
2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan(((2*2^(1/2)+2)^(1/2)+2*(1+tan(f*x+e))^
(1/2))/(-2+2*2^(1/2))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(1+tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((tan(f*x + e) + 1)^(3/2)*tan(f*x + e)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1080 vs. $2(140) = 280$.
time = 1.28, size = 1080, normalized size = 6.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(1+tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/40*(20*8^(1/4)*sqrt(2)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f*(f^(-4))^(
1/4)*arctan(1/16*8^(3/4)*sqrt(2)*(2*f^5*sqrt(f^(-4)) + sqrt(2)*f^3)*sqrt(-2
```

```
*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e
) + 8^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4))*cos(f*x + e) + f*cos(f*x + e))*sqrt(-
2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x
+ e))*(f^(-4))^(1/4) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e))*(f^(-
4))^(3/4) - 1/8*8^(3/4)*(2*f^5*sqrt(f^(-4)) + sqrt(2)*f^3)*sqrt(-2*sqrt(2)*
f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(
-4))^(3/4) - f^2*sqrt(f^(-4)) - sqrt(2))*cos(f*x + e)^2 + 20*8^(1/4)*sqrt(2
)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f*(f^(-4))^(1/4)*arctan(1/16*8^(3/4
)*sqrt(2)*(2*f^5*sqrt(f^(-4)) + sqrt(2)*f^3)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4
)) + 4)*sqrt((2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) - 8^(1/4)*(sqrt(2)*f^
3*sqrt(f^(-4))*cos(f*x + e) + f*cos(f*x + e))*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-
4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4) +
2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) - 1/8*8^(3/4)
*(2*f^5*sqrt(f^(-4)) + sqrt(2)*f^3)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*s
qrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) + f^2*sqrt(f
^(-4)) + sqrt(2))*cos(f*x + e)^2 + 5*8^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4))*cos(
f*x + e)^2 + 2*f*cos(f*x + e)^2)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*(f^(
-4))^(1/4)*log(2*(2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) + 8^(1/4)*(sqrt(2
)*f^3*sqrt(f^(-4))*cos(f*x + e) + f*cos(f*x + e))*sqrt(-2*sqrt(2)*f^2*sqrt(
f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4
) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e)) - 5*8^(1/4)*(sqrt(2)*f^3
*sqrt(f^(-4))*cos(f*x + e)^2 + 2*f*cos(f*x + e)^2)*sqrt(-2*sqrt(2)*f^2*sqrt
(f^(-4)) + 4)*(f^(-4))^(1/4)*log(2*(2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e)
 - 8^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4))*cos(f*x + e) + f*cos(f*x + e))*sqrt(-2
*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x +
 e))*(f^(-4))^(1/4) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e)) - 16*(
5*cos(f*x + e)^2 - 2*cos(f*x + e)*sin(f*x + e) - 1)*sqrt((cos(f*x + e) + si
n(f*x + e))/cos(f*x + e)))/(f*cos(f*x + e)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (\tan(e+fx)+1)^{\frac{3}{2}} \tan^2(e+fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2*(1+tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((tan(e + f*x) + 1)**(3/2)*tan(e + f*x)**2, x)
```

Giac [A]
time = 0.77, size = 237, normalized size = 1.37

$$-\frac{\sqrt{\sqrt{2}-1}\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{f}-\frac{\sqrt{\sqrt{2}-1}\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{f}+\frac{\sqrt{\sqrt{2}+1}\log\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{2f}-\frac{\sqrt{\sqrt{2}+1}\log\left(-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{2f}+\frac{2\left(f^4(\tan(fx+e)+1)^{\frac{5}{2}}-5f^4\sqrt{\tan(fx+e)+1}\right)}{5f^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(1+tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt(sqrt(2) - 1)*arctan(1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*sqrt(t
an(f*x + e) + 1))/sqrt(-sqrt(2) + 2))/f - sqrt(sqrt(2) - 1)*arctan(-1/2*2^(
3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(tan(f*x + e) + 1))/sqrt(-sqrt(2) +
 2))/f + 1/2*sqrt(sqrt(2) + 1)*log(2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x +
 e) + 1) + sqrt(2) + tan(f*x + e) + 1)/f - 1/2*sqrt(sqrt(2) + 1)*log(-2^(1/
4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x + e) + 1) + sqrt(2) + tan(f*x + e) + 1)/f
 + 2/5*(f^4*(tan(f*x + e) + 1)^(5/2) - 5*f^4*sqrt(tan(f*x + e) + 1))/f^5
```

Mupad [B]
time = 0.88, size = 103, normalized size = 0.60

$$\frac{2\,{\left(\mathrm{tan}\left(e+f\,x\right)+1\right)}^{5/2}}{5\,f}-\frac{2\,\sqrt{\mathrm{tan}\left(e+f\,x\right)+1}}{f}-\mathrm{atan}\left(f\,\sqrt{\frac{\frac{1}{2}-\frac{1}{2}\mathrm{i}}{f^2}}\,\sqrt{\mathrm{tan}\left(e+f\,x\right)+1}\,1\mathrm{i}\right)\,\sqrt{\frac{\frac{1}{2}-\frac{1}{2}\mathrm{i}}{f^2}}\,2\mathrm{i}-\mathrm{atan}\left(f\,\sqrt{\frac{\frac{1}{2}+\frac{1}{2}\mathrm{i}}{f^2}}\,\sqrt{\mathrm{tan}\left(e+f\,x\right)+1}\,1\mathrm{i}\right)\,\sqrt{\frac{\frac{1}{2}+\frac{1}{2}\mathrm{i}}{f^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(tan(e + f*x) + 1)^(3/2),x)
```

```
[Out] (2*(tan(e + f*x) + 1)^(5/2))/(5*f) - (2*(tan(e + f*x) + 1)^(1/2))/f - atan(
f*((1/2 - 1i/2)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1/2)*1i)*((1/2 - 1i/2)/f^2)^
(1/2)*2i - atan(f*((1/2 + 1i/2)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1/2)*1i)*((1
/2 + 1i/2)/f^2)^(1/2)*2i
```

3.397 $\int (1+\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=156

$$-\frac{\sqrt{-1+\sqrt{2}}\,\text{ArcTan}\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\tan(e+fx)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\,\sqrt{1+\tan(e+fx)}}\right)}{f}-\frac{\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{3+2\sqrt{2}+\left(1-\right.}{\sqrt{2\left(7+5\sqrt{2}\right)}}\right.}{f}$$

```
[Out] -arctan((3-2*2^(1/2)+(1-2^(1/2))*tan(f*x+e))/(-14+10*2^(1/2))^(1/2)/(1+tan(
f*x+e))^(1/2))*(2^(1/2)-1)^(1/2)/f-arctanh((3+2*2^(1/2)+(1+2^(1/2))*tan(f*x
+e))/(14+10*2^(1/2))^(1/2)/(1+tan(f*x+e))^(1/2))*(1+2^(1/2))^(1/2)/f+2*(1+t
an(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.10, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3563, 12, 3617, 3616, 209, 213}

$$-\frac{\sqrt{\sqrt{2}-1}\,\text{ArcTan}\left(\frac{\left(1-\sqrt{2}\right)\tan(e+fx)-2\sqrt{2}+3}{\sqrt{2\left(5\sqrt{2}-7\right)}\,\sqrt{\tan(e+fx)+1}}\right)}{f}+\frac{2\sqrt{\tan(e+fx)+1}}{f}-\frac{\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\left(1+\sqrt{2}\right)\tan(e+fx)+2\sqrt{2}+3}{\sqrt{2\left(7+5\sqrt{2}\right)}\,\sqrt{\tan(e+fx)+1}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Tan[e + f*x])^(3/2),x]
```

```
[Out] -((Sqrt[-1 + Sqrt[2]]*ArcTan[(3 - 2*Sqrt[2] + (1 - Sqrt[2])*Tan[e + f*x])/(
Sqrt[2*(-7 + 5*Sqrt[2])]*Sqrt[1 + Tan[e + f*x]])])/f) - (Sqrt[1 + Sqrt[2]]*
ArcTanh[(3 + 2*Sqrt[2] + (1 + Sqrt[2])*Tan[e + f*x])/(Sqrt[2*(7 + 5*Sqrt[2]
)]*Sqrt[1 + Tan[e + f*x]])])/f + (2*Sqrt[1 + Tan[e + f*x]])/f
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 3563

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3616

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*b*c*d - 4*a*d^2 +
 x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

Rule 3617

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2*q), Int[(a
*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
x], x] - Dist[1/(2*q), Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f*x
])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d - b
*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])
```

Rubi steps

$$\int (1+\tan(e+fx))^{3/2}\,dx = \frac{2\sqrt{1+\tan(e+fx)}}{f} + \int \frac{2\tan(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx$$

$$= \frac{2\sqrt{1+\tan(e+fx)}}{f} + 2\int \frac{\tan(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx$$

$$= \frac{2\sqrt{1+\tan(e+fx)}}{f} - \frac{\int \frac{1+\left(-1-\sqrt{2}\right)\tan(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx}{\sqrt{2}} + \frac{\int \frac{1+\left(-1+\sqrt{2}\right)\tan(e+fx)}{\sqrt{1+\tan(e+fx)}}\,d}{\sqrt{2}}$$

$$= \frac{2\sqrt{1+\tan(e+fx)}}{f} + \frac{\left(4-3\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{2\left(-1+\sqrt{2}\right)-4\left(-1+\sqrt{2}\right)^2+x^2}\,dx,\right.}{f}$$

$$= -\frac{\sqrt{-1+\sqrt{2}}\,\tan^{-1}\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\tan(e+fx)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\,\sqrt{1+\tan(e+fx)}}\right)}{f} - \frac{\sqrt{1+\sqrt{2}}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.07, size = 79, normalized size = 0.51

$$\frac{-\frac{2\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1-i}}\right)}{\sqrt{1-i}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1+i}}\right)}{\sqrt{1+i}} + 2\sqrt{1+\tan(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-2*ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1 - I]])/Sqrt[1 - I] - (2*ArcTanh[
Sqrt[1 + Tan[e + f*x]]/Sqrt[1 + I]])/Sqrt[1 + I] + 2*Sqrt[1 + Tan[e + f*x]]
)/f
```

Maple [A]
time = 0.09, size = 221, normalized size = 1.42

method	result

derivativedivides	$2\sqrt{1+\tan(fx+e)}-\frac{\sqrt{2}\left(-\frac{\sqrt{2\sqrt{2}+2}\,\ln\left(1+\sqrt{2}-\sqrt{2\sqrt{2}+2}\,\sqrt{1+\tan(fx+e)}+\tan\right.}{2}\right.}{2}$
default	$2\sqrt{1+\tan(fx+e)}-\frac{\sqrt{2}\left(-\frac{\sqrt{2\sqrt{2}+2}\,\ln\left(1+\sqrt{2}-\sqrt{2\sqrt{2}+2}\,\sqrt{1+\tan(fx+e)}+\tan\right.}{2}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*(1+tan(f*x+e))^(1/2)-1/2*2^(1/2)*(-1/2*(2*2^(1/2)+2)^(1/2)*ln(1+2^(1
/2)-(2*2^(1/2)+2)^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))+2*(1-2^(1/2))/(-2+
2*2^(1/2))^(1/2)*arctan((2*(1+tan(f*x+e))^(1/2)-(2*2^(1/2)+2)^(1/2))/(-2+2*
2^(1/2))^(1/2)))-1/2*2^(1/2)*(1/2*(2*2^(1/2)+2)^(1/2)*ln(1+2^(1/2)+(2*2^(1/
2)+2)^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))+2*(1-2^(1/2))/(-2+2*2^(1/2))^(
1/2)*arctan(((2*2^(1/2)+2)^(1/2)+2*(1+tan(f*x+e))^(1/2))/(-2+2*2^(1/2))^(1/
2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  1which is not of the expected type LIST
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 987 vs. $2(126) = 252$.
time = 1.36, size = 987, normalized size = 6.33

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*(4*8^(1/4)*sqrt(2)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*f*(f^(-4))^(1
/4)*arctan(1/16*8^(3/4)*sqrt(2)*(2*f^5*sqrt(f^(-4)) + sqrt(2)*f^3)*sqrt(-2*
sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e)
 + 8^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4))*cos(f*x + e) + f*cos(f*x + e))*sqrt(-2
*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x +
 e))*(f^(-4))^(1/4) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e))*(f^(-4
))^(3/4) - 1/8*8^(3/4)*(2*f^5*sqrt(f^(-4)) + sqrt(2)*f^3)*sqrt(-2*sqrt(2)*f
^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-
4))^(3/4) - f^2*sqrt(f^(-4)) - sqrt(2)) + 4*8^(1/4)*sqrt(2)*sqrt(-2*sqrt(2)
*f^2*sqrt(f^(-4)) + 4)*f*(f^(-4))^(1/4)*arctan(1/16*8^(3/4)*sqrt(2)*(2*f^5*
sqrt(f^(-4)) + sqrt(2)*f^3)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((2*s
qrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) - 8^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4))*co
s(f*x + e) + f*cos(f*x + e))*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((co
s(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4) + 2*cos(f*x + e) +
2*sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) - 1/8*8^(3/4)*(2*f^5*sqrt(f^(-
4)) + sqrt(2)*f^3)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e)
 + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) + f^2*sqrt(f^(-4)) + sqrt(2))
 + 8^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4)) + 2*f)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)
) + 4)*(f^(-4))^(1/4)*log(2*(2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) + 8^(1
/4)*(sqrt(2)*f^3*sqrt(f^(-4))*cos(f*x + e) + f*cos(f*x + e))*sqrt(-2*sqrt(2
)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f
^(-4))^(1/4) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e)) - 8^(1/4)*(sq
rt(2)*f^3*sqrt(f^(-4)) + 2*f)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4)
)^(1/4)*log(2*(2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) - 8^(1/4)*(sqrt(2)*f
^3*sqrt(f^(-4))*cos(f*x + e) + f*cos(f*x + e))*sqrt(-2*sqrt(2)*f^2*sqrt(f^(
-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4) +
 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e)) - 16*sqrt((cos(f*x + e) + s
in(f*x + e))/cos(f*x + e)))/f
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (\tan(e + fx) + 1)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((tan(e + f*x) + 1)**(3/2), x)
```

Giac [**A**]
time = 0.75, size = 214, normalized size = 1.37

$$\frac{\sqrt{\sqrt{2}-1}\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{f}+\frac{\sqrt{\sqrt{2}-1}\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{f}-\frac{\sqrt{\sqrt{2}+1}\log\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{2f}+\frac{\sqrt{\sqrt{2}+1}\log\left(-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{2f}+\frac{2\sqrt{\tan(fx+e)+1}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(sqrt(2) - 1)*arctan(1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*sqrt(ta
n(f*x + e) + 1))/sqrt(-sqrt(2) + 2))/f + sqrt(sqrt(2) - 1)*arctan(-1/2*2^(3
/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(tan(f*x + e) + 1))/sqrt(-sqrt(2) +
2))/f - 1/2*sqrt(sqrt(2) + 1)*log(2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x +
e) + 1) + sqrt(2) + tan(f*x + e) + 1)/f + 1/2*sqrt(sqrt(2) + 1)*log(-2^(1/4
)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x + e) + 1) + sqrt(2) + tan(f*x + e) + 1)/f
+ 2*sqrt(tan(f*x + e) + 1)/f
```

Mupad [B]
time = 4.15, size = 82, normalized size = 0.53

$$\frac{2\sqrt{\tan(e+fx)+1}}{f}-2\,\mathrm{atanh}\left(f\sqrt{\frac{\frac{1}{2}-\frac{1}{2}\mathrm{i}}{f^2}}\,\sqrt{\tan(e+fx)+1}\right)\sqrt{\frac{\frac{1}{2}-\frac{1}{2}\mathrm{i}}{f^2}}-2\,\mathrm{atanh}\left(f\sqrt{\frac{\frac{1}{2}+\frac{1}{2}\mathrm{i}}{f^2}}\,\sqrt{\tan(e+fx)+1}\right)\sqrt{\frac{\frac{1}{2}+\frac{1}{2}\mathrm{i}}{f^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(e + f*x) + 1)^(3/2),x)
```

```
[Out] (2*(tan(e + f*x) + 1)^(1/2))/f - 2*atanh(f*((1/2 - 1i/2)/f^2)^(1/2)*(tan(e
+ f*x) + 1)^(1/2))*((1/2 - 1i/2)/f^2)^(1/2) - 2*atanh(f*((1/2 + 1i/2)/f^2)^
(1/2)*(tan(e + f*x) + 1)^(1/2))*((1/2 + 1i/2)/f^2)^(1/2)
```

3.398 $\int \cot^2(e+fx)(1+\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=178

$$\frac{\sqrt{-1+\sqrt{2}}\,\mathrm{ArcTan}\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\tan(e+fx)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\,\sqrt{1+\tan(e+fx)}}\right)}{f}-\frac{3\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{f}+\frac{\sqrt{1+}}{}$$

[Out] -3*arctanh((1+tan(f*x+e))^(1/2))/f+arctan((3-2*2^(1/2)+(1-2^(1/2))*tan(f*x+e))/(-14+10*2^(1/2))^(1/2)/(1+tan(f*x+e))^(1/2))*(2^(1/2)-1)^(1/2)/f+arctanh((3+2*2^(1/2)+(1+2^(1/2))*tan(f*x+e))/(14+10*2^(1/2))^(1/2)/(1+tan(f*x+e))^(1/2))*(1+2^(1/2))^(1/2)/f-cot(f*x+e)*(1+tan(f*x+e))^(1/2)/f

Rubi [A]

time = 0.21, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3648, 3735, 12, 3617, 3616, 209, 213, 3715, 65}

$$\frac{\sqrt{\sqrt{2}-1}\,\mathrm{ArcTan}\left(\frac{\left(1-\sqrt{2}\right)\tan(e+fx)-2\sqrt{2}+3}{\sqrt{2\left(5\sqrt{2}-7\right)}\,\sqrt{\tan(e+fx)+1}}\right)}{f}-\frac{3\tanh^{-1}\left(\sqrt{\tan(e+fx)+1}\right)}{f}+\frac{\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\left(1+\sqrt{2}\right)\tan(e+fx)+2\sqrt{2}+3}{\sqrt{2\left(7+5\sqrt{2}\right)}\,\sqrt{\tan(e+fx)+1}}\right)}{f}-\frac{\sqrt{\tan(e+fx)+1}\,\cot(e+fx)}{f}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^2*(1 + Tan[e + f*x])^(3/2),x]

[Out] (Sqrt[-1 + Sqrt[2]]*ArcTan[(3 - 2*Sqrt[2] + (1 - Sqrt[2])*Tan[e + f*x])/(Sqrt[2*(-7 + 5*Sqrt[2])]*Sqrt[1 + Tan[e + f*x]])])/f - (3*ArcTanh[Sqrt[1 + Tan[e + f*x]]])/f + (Sqrt[1 + Sqrt[2]]*ArcTanh[(3 + 2*Sqrt[2] + (1 + Sqrt[2])*Tan[e + f*x])/(Sqrt[2*(7 + 5*Sqrt[2])]*Sqrt[1 + Tan[e + f*x]])])/f - (Cot[e + f*x]*Sqrt[1 + Tan[e + f*x]])/f

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3616

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*b*c*d - 4*a*d^2 +
 x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

Rule 3617

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2*q), Int[(a
*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
x], x] - Dist[1/(2*q), Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f*x
])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d - b
*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
```

```
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3735

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dis
t[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*Simp[a*(A - C) - (A*b - b*C)*Ta
n[e + f*x], x], x], x] + Dist[(A*b^2 + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e
 + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a,
 b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0] &&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \cot^2(e+fx)(1+\tan(e+fx))^{3/2}\,dx = -\frac{\cot(e+fx)\sqrt{1+\tan(e+fx)}}{f} - \int \frac{\cot(e+fx)\left(-\frac{3}{2}+\frac{1}{2}\tan^2}{\sqrt{1+\tan(e+fx}}$$

$$= -\frac{\cot(e+fx)\sqrt{1+\tan(e+fx)}}{f} + \frac{3}{2}\int \frac{\cot(e+fx)\left(1+\tan^2(e\right.}{\sqrt{1+\tan(e+fx}}$$

$$= -\frac{\cot(e+fx)\sqrt{1+\tan(e+fx)}}{f} - 2\int \frac{\tan(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx$$

$$= -\frac{\cot(e+fx)\sqrt{1+\tan(e+fx)}}{f} + \frac{\int \frac{1+\left(-1-\sqrt{2}\right)\tan(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx}{\sqrt{2}} -$$

$$= -\frac{3\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{f} - \frac{\cot(e+fx)\sqrt{1+\tan(e+f}}{f}$$

$$= \frac{\sqrt{-1+\sqrt{2}}\,\tan^{-1}\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\tan(e+fx)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\sqrt{1+\tan(e+fx)}}\right)}{f} -$$

Mathematica [C] Result contains complex when optimal does not.
time = 17.77, size = 245, normalized size = 1.38

$$\frac{(-1+\cot(e+fx))\left(1+\cot(e+fx)+\sqrt{-2-2i}\,\mathrm{ArcTan}\left(\sqrt{-\frac{1}{2}-\frac{i}{2}}\sqrt{1+\sec(e+fx)\sqrt{\sin^2(e+fx)}}\right)\sqrt{1+\sec(e+fx)\sqrt{\sin^2(e+fx)}}+\sqrt{-2+2i}\,\mathrm{ArcTan}\left(\sqrt{-\frac{1}{2}+\frac{i}{2}}\sqrt{1+\sec(e+fx)\sqrt{\sin^2(e+fx)}}\right)\sqrt{1+\sec(e+fx)\sqrt{\sin^2(e+fx)}}+3\tanh^{-1}\left(\sqrt{1+\sec(e+fx)\sqrt{\sin^2(e+fx)}}\right)\sqrt{1+\sec(e+fx)\sqrt{\sin^2(e+fx)}}\right)\tan(e+fx)\sqrt{1+\tan(e+fx)}}{f\left(-2+\sec^2(e+fx)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^2*(1 + Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-1 + Cot[e + f*x])*(1 + Cot[e + f*x] + Sqrt[-2 - 2*I]*ArcTan[Sqrt[-1/2 -
I/2]*Sqrt[1 + Sec[e + f*x]*Sqrt[Sin[e + f*x]^2]]]*Sqrt[1 + Sec[e + f*x]*Sqr
t[Sin[e + f*x]^2]] + Sqrt[-2 + 2*I]*ArcTan[Sqrt[-1/2 + I/2]*Sqrt[1 + Sec[e
+ f*x]*Sqrt[Sin[e + f*x]^2]]]*Sqrt[1 + Sec[e + f*x]*Sqrt[Sin[e + f*x]^2]] +
 3*ArcTanh[Sqrt[1 + Sec[e + f*x]*Sqrt[Sin[e + f*x]^2]]]*Sqrt[1 + Sec[e + f*
x]*Sqrt[Sin[e + f*x]^2]])*Tan[e + f*x]*Sqrt[1 + Tan[e + f*x]])/(f*(-2 + Sec
[e + f*x]^2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.68, size = 6656, normalized size = 37.39

method	result	size
default	Expression too large to display	6656

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(1+tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

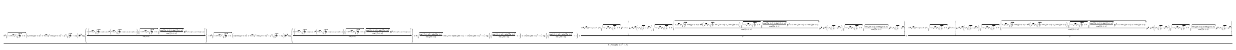
Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(1+tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((tan(f*x + e) + 1)^(3/2)*cot(f*x + e)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1199 vs. 2(148) = 296.
time = 1.04, size = 1199, normalized size = 6.74



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(1+tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(8^(1/4)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*(2*f*cos(f*x + e)^2 + sq
rt(2)*(f^3*cos(f*x + e)^2 - f^3)*sqrt(f^(-4)) - 2*f)*(f^(-4))^(1/4)*log(2*(
2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) + 8^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4))
*cos(f*x + e) + f*cos(f*x + e))*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt(
(cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4) + 2*cos(f*x + e)
 + 2*sin(f*x + e))/cos(f*x + e)) - 8^(1/4)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4))
```

```
 + 4)*(2*f*cos(f*x + e)^2 + sqrt(2)*(f^3*cos(f*x + e)^2 - f^3)*sqrt(f^(-4))
 - 2*f)*(f^(-4))^(1/4)*log(2*(2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) - 8^(
1/4)*(sqrt(2)*f^3*sqrt(f^(-4))*cos(f*x + e) + f*cos(f*x + e))*sqrt(-2*sqrt(
2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(
f^(-4))^(1/4) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x + e)) + 8*sqrt((co
s(f*x + e) + sin(f*x + e))/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 12*(co
s(f*x + e)^2 - 1)*log(sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e)) + 1)
 + 12*(cos(f*x + e)^2 - 1)*log(sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x +
 e)) - 1) + 4*8^(1/4)*sqrt(2)*(f^5*cos(f*x + e)^2 - f^5)*sqrt(-2*sqrt(2)*f^
2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*arctan(1/16*8^(3/4)*sqrt(2)*(2*f^5*sqrt(
f^(-4)) + sqrt(2)*f^3)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((2*sqrt(2
)*f^2*sqrt(f^(-4))*cos(f*x + e) + 8^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4))*cos(f*x
 + e) + f*cos(f*x + e))*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x
 + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4) + 2*cos(f*x + e) + 2*sin
(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) - 1/8*8^(3/4)*(2*f^5*sqrt(f^(-4)) +
 sqrt(2)*f^3)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + si
n(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) - f^2*sqrt(f^(-4)) - sqrt(2))/f^4
+ 4*8^(1/4)*sqrt(2)*(f^5*cos(f*x + e)^2 - f^5)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(
-4)) + 4)*(f^(-4))^(1/4)*arctan(1/16*8^(3/4)*sqrt(2)*(2*f^5*sqrt(f^(-4)) +
sqrt(2)*f^3)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((2*sqrt(2)*f^2*sqrt
(f^(-4))*cos(f*x + e) - 8^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4))*cos(f*x + e) + f*
cos(f*x + e))*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + si
n(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4) + 2*cos(f*x + e) + 2*sin(f*x + e))
/cos(f*x + e))*(f^(-4))^(3/4) - 1/8*8^(3/4)*(2*f^5*sqrt(f^(-4)) + sqrt(2)*f
^3)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e)
)/cos(f*x + e))*(f^(-4))^(3/4) + f^2*sqrt(f^(-4)) + sqrt(2))/f^4)/(f*cos(f*
x + e)^2 - f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (\tan (e + fx) + 1)^{\frac{3}{2}} \cot^2 (e + fx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(1+tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((tan(e + f*x) + 1)**(3/2)*cot(e + f*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 346 vs. 2(148) = 296.
time = 1.42, size = 346, normalized size = 1.94

$$-\frac{3 \log\left(\sqrt{\tan(fx+e)+1}+1\right)}{2f} + \frac{3 \log\left(\left|\sqrt{\tan(fx+e)+1}-1\right|\right)}{2f} - \frac{\left(f^2\sqrt{2\sqrt{2}+2} - f\sqrt{2\sqrt{2}-2}\,|f|\right)\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{2f^3} - \frac{\left(f^2\sqrt{2\sqrt{2}+2} - f\sqrt{2\sqrt{2}-2}\,|f|\right)\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{2f^3} + \frac{\left(f^2\sqrt{2\sqrt{2}-2} + f\sqrt{2\sqrt{2}+2}\,|f|\right)\log\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{4f^3} - \frac{\left(f^2\sqrt{2\sqrt{2}-2} + f\sqrt{2\sqrt{2}+2}\,|f|\right)\log\left(-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{4f^3} - \frac{\sqrt{\tan(fx+e)+1}}{f\tan(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(1+tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] -3/2*log(sqrt(tan(f*x + e) + 1) + 1)/f + 3/2*log(abs(sqrt(tan(f*x + e) + 1)
 - 1))/f - 1/2*(f^2*sqrt(2*sqrt(2) + 2) - f*sqrt(2*sqrt(2) - 2)*abs(f))*arc
tan(1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*sqrt(tan(f*x + e) + 1))/sqrt
(-sqrt(2) + 2))/f^3 - 1/2*(f^2*sqrt(2*sqrt(2) + 2) - f*sqrt(2*sqrt(2) - 2)*
abs(f))*arctan(-1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(tan(f*x + e
) + 1))/sqrt(-sqrt(2) + 2))/f^3 + 1/4*(f^2*sqrt(2*sqrt(2) - 2) + f*sqrt(2*s
qrt(2) + 2)*abs(f))*log(2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x + e) + 1) +
sqrt(2) + tan(f*x + e) + 1)/f^3 - 1/4*(f^2*sqrt(2*sqrt(2) - 2) + f*sqrt(2*s
qrt(2) + 2)*abs(f))*log(-2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x + e) + 1) +
 sqrt(2) + tan(f*x + e) + 1)/f^3 - sqrt(tan(f*x + e) + 1)/(f*tan(f*x + e))
```

Mupad [B]
time = 0.18, size = 119, normalized size = 0.67

$$\frac{\mathrm{atan}\left(\sqrt{\tan\left(e+f\,x\right)+1}\,1{}\mathrm{i}\right)\,3{}\mathrm{i}}{f}+\frac{\sqrt{\tan\left(e+f\,x\right)+1}}{f-f\,\left(\tan\left(e+f\,x\right)+1\right)}-\mathrm{atan}\left(f\,\sqrt{\frac{\frac{1}{2}-\frac{1}{2}{}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\,1{}\mathrm{i}\right)\,\sqrt{\frac{\frac{1}{2}-\frac{1}{2}{}\mathrm{i}}{f^2}}\,2{}\mathrm{i}-\mathrm{atan}\left(f\,\sqrt{\frac{\frac{1}{2}+\frac{1}{2}{}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\,1{}\mathrm{i}\right)\,\sqrt{\frac{\frac{1}{2}+\frac{1}{2}{}\mathrm{i}}{f^2}}\,2{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(tan(e + f*x) + 1)^(3/2),x)
```

```
[Out] (atan((tan(e + f*x) + 1)^(1/2)*1i)*3i)/f + (tan(e + f*x) + 1)^(1/2)/(f - f*
(tan(e + f*x) + 1)) - atan(f*((1/2 - 1i/2)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1
/2)*1i)*((1/2 - 1i/2)/f^2)^(1/2)*2i - atan(f*((1/2 + 1i/2)/f^2)^(1/2)*(tan(
e + f*x) + 1)^(1/2)*1i)*((1/2 + 1i/2)/f^2)^(1/2)*2i
```

3.399 $\int \cot^4(e+fx)(1+\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=238

$$-\frac{\sqrt{-1+\sqrt{2}}\,\text{ArcTan}\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\tan(e+fx)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\,\sqrt{1+\tan(e+fx)}}\right)}{f}+\frac{25\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{8f}-\frac{\sqrt{1}}{}$$

```
[Out] 25/8*arctanh((1+tan(f*x+e))^(1/2))/f-arctan((3-2*2^(1/2)+(1-2^(1/2))*tan(f*
x+e))/(-14+10*2^(1/2))^(1/2)/(1+tan(f*x+e))^(1/2))*(2^(1/2)-1)^(1/2)/f-arct
anh((3+2*2^(1/2)+(1+2^(1/2))*tan(f*x+e))/(14+10*2^(1/2))^(1/2)/(1+tan(f*x+e
))^(1/2))*(1+2^(1/2))^(1/2)/f+7/8*cot(f*x+e)*(1+tan(f*x+e))^(1/2)/f-7/12*co
t(f*x+e)^2*(1+tan(f*x+e))^(1/2)/f-1/3*cot(f*x+e)^3*(1+tan(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.36, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3648, 3731, 3730, 3735, 12, 3617, 3616, 209, 213, 3715, 65}

$$-\frac{\sqrt{\sqrt{2}-1}\,\text{ArcTan}\left(\frac{\left(1-\sqrt{2}\right)\tan(e+fx)-2\sqrt{2}+3}{\sqrt{2\left(5\sqrt{2}-7\right)}\,\sqrt{\tan(e+fx)+1}}\right)}{f}+\frac{25\tanh^{-1}\left(\sqrt{\tan(e+fx)+1}\right)}{8f}-\frac{\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\left(1+\sqrt{2}\right)\tan(e+fx)+2\sqrt{2}+3}{\sqrt{2\left(7+5\sqrt{2}\right)}\,\sqrt{\tan(e+fx)+1}}\right)}{f}-\frac{\sqrt{\tan(e+fx)+1}\,\cot^3(e+fx)}{3f}-\frac{7\sqrt{\tan(e+fx)+1}\,\cot^2(e+fx)}{12f}+\frac{7\sqrt{\tan(e+fx)+1}\,\cot(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(1 + Tan[e + f*x])^(3/2),x]
```

```
[Out] -((Sqrt[-1 + Sqrt[2]]*ArcTan[(3 - 2*Sqrt[2] + (1 - Sqrt[2])*Tan[e + f*x])/(
Sqrt[2*(-7 + 5*Sqrt[2])]*Sqrt[1 + Tan[e + f*x]])])/f) + (25*ArcTanh[Sqrt[1
+ Tan[e + f*x]]])/(8*f) - (Sqrt[1 + Sqrt[2]]*ArcTanh[(3 + 2*Sqrt[2] + (1 +
Sqrt[2])*Tan[e + f*x])/(Sqrt[2*(7 + 5*Sqrt[2])]*Sqrt[1 + Tan[e + f*x]])])/f
 + (7*Cot[e + f*x]*Sqrt[1 + Tan[e + f*x]])/(8*f) - (7*Cot[e + f*x]^2*Sqrt[1
 + Tan[e + f*x]])/(12*f) - (Cot[e + f*x]^3*Sqrt[1 + Tan[e + f*x]])/(3*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3616

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*b*c*d - 4*a*d^2 +
 x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

Rule 3617

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2*q), Int[(a
*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
x], x] - Dist[1/(2*q), Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f*x
])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d - b
*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3731

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)
*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp
[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - a*C*(b*c*(m + 1) + a*d*(n
+ 1)) - (m + 1)*(b*c - a*d)*(A*b - b*C)*Tan[e + f*x] - d*(A*b^2 + a^2*C)*(m
 + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m,
 -1] &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3735

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dis
t[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*Simp[a*(A - C) - (A*b - b*C)*Ta
n[e + f*x], x], x], x] + Dist[(A*b^2 + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e
 + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a,
 b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0] &&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \cot^4(e+fx)(1+\tan(e+fx))^{3/2}\,dx = -\frac{\cot^3(e+fx)\sqrt{1+\tan(e+fx)}}{3f} - \frac{1}{3}\int \frac{\cot^3(e+fx)\left(-\frac{7}{2}+\frac{5}{2}\text{ta}\right.}{\sqrt{1+\tan(e+}}$$

$$= -\frac{7\cot^2(e+fx)\sqrt{1+\tan(e+fx)}}{12f} - \frac{\cot^3(e+fx)\sqrt{1+\tan(e+}}{3f}$$

$$= \frac{7\cot(e+fx)\sqrt{1+\tan(e+fx)}}{8f} - \frac{7\cot^2(e+fx)\sqrt{1+\tan(e+}}{12f}$$

$$= \frac{7\cot(e+fx)\sqrt{1+\tan(e+fx)}}{8f} - \frac{7\cot^2(e+fx)\sqrt{1+\tan(e+}}{12f}$$

$$= \frac{7\cot(e+fx)\sqrt{1+\tan(e+fx)}}{8f} - \frac{7\cot^2(e+fx)\sqrt{1+\tan(e+}}{12f}$$

$$= \frac{7\cot(e+fx)\sqrt{1+\tan(e+fx)}}{8f} - \frac{7\cot^2(e+fx)\sqrt{1+\tan(e+}}{12f}$$

$$= \frac{25\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{8f} + \frac{7\cot(e+fx)\sqrt{1+\tan(e+fx}}{8f}$$

$$= -\frac{\sqrt{-1+\sqrt{2}}\,\tan^{-1}\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\tan(e+fx)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\sqrt{1+\tan(e+fx)}}\right)}{f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 19.14, size = 321, normalized size = 1.35

$$\frac{\cos(2(e+fx))\csc^4(e+fx)\sec(e+fx)\left(34\cos(e+fx)+30\cos(3(e+fx))-82\sin(e+fx)-192\sqrt{-2-2i}\,\text{ArcTan}\left(\sqrt{-\frac{1}{2}-\frac{i}{2}}\sqrt{1+\sec(e+fx)\sqrt{\sin^2(e+fx)}}\right)\sin^3(e+fx)\sqrt{1+\sec(e+fx)\sqrt{\sin^2(e+fx)}}-192\sqrt{-2+2i}\,\text{ArcTan}\left(\sqrt{-\frac{1}{2}+\frac{i}{2}}\sqrt{1+\sec(e+fx)\sqrt{\sin^2(e+fx)}}\right)\sin^3(e+fx)\sqrt{1+\sec(e+fx)\sqrt{\sin^2(e+fx)}}-600\tanh^{-1}\left(\sqrt{1+\sec(e+fx)\sqrt{\sin^2(e+fx)}}\right)\sin^3(e+fx)\sqrt{1+\sec(e+fx)\sqrt{\sin^2(e+fx)}}+86\sin(3(e+fx))\right)\sqrt{1+\tan(e+fx)}}{192f(1+\cot(e+fx))\left(-2+\sec^2(e+fx)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^4*(1 + Tan[e + f*x])^(3/2),x]
```

```
[Out] (Cos[2*(e + f*x)]*Csc[e + f*x]^4*Sec[e + f*x]*(34*Cos[e + f*x] + 30*Cos[3*(
e + f*x)] - 82*Sin[e + f*x] - 192*Sqrt[-2 - 2*I]*ArcTan[Sqrt[-1/2 - I/2]*Sq
rt[1 + Sec[e + f*x]*Sqrt[Sin[e + f*x]^2]]]*Sin[e + f*x]^3*Sqrt[1 + Sec[e +
f*x]*Sqrt[Sin[e + f*x]^2]] - 192*Sqrt[-2 + 2*I]*ArcTan[Sqrt[-1/2 + I/2]*Sqr
t[1 + Sec[e + f*x]*Sqrt[Sin[e + f*x]^2]]]*Sin[e + f*x]^3*Sqrt[1 + Sec[e + f
*x]*Sqrt[Sin[e + f*x]^2]] - 600*ArcTanh[Sqrt[1 + Sec[e + f*x]*Sqrt[Sin[e +
```

```
f*x]^2]]]*Sin[e + f*x]^3*Sqrt[1 + Sec[e + f*x]*Sqrt[Sin[e + f*x]^2]] + 86*S
in[3*(e + f*x)])*Sqrt[1 + Tan[e + f*x]])/(192*f*(1 + Cot[e + f*x])*(-2 + Se
c[e + f*x]^2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.74, size = 11278, normalized size = 47.39

method	result	size
default	Expression too large to display	11278

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(1+tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(1+tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((tan(f*x + e) + 1)^(3/2)*cot(f*x + e)^4, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1343 vs. 2(200) = 400.
time = 1.33, size = 1343, normalized size = 5.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(1+tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/48*(6*8^(1/4)*(2*f*cos(f*x + e)^4 - 4*f*cos(f*x + e)^2 + sqrt(2)*(f^3*co
s(f*x + e)^4 - 2*f^3*cos(f*x + e)^2 + f^3)*sqrt(f^(-4)) + 2*f)*sqrt(-2*sqrt
(2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*log(2*(2*sqrt(2)*f^2*sqrt(f^(-4))*
cos(f*x + e) + 8^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4))*cos(f*x + e) + f*cos(f*x +
 e))*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e
))/cos(f*x + e))*(f^(-4))^(1/4) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x
+ e)) - 6*8^(1/4)*(2*f*cos(f*x + e)^4 - 4*f*cos(f*x + e)^2 + sqrt(2)*(f^3*c
os(f*x + e)^4 - 2*f^3*cos(f*x + e)^2 + f^3)*sqrt(f^(-4)) + 2*f)*sqrt(-2*sqr
t(2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*log(2*(2*sqrt(2)*f^2*sqrt(f^(-4))
*cos(f*x + e) - 8^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4))*cos(f*x + e) + f*cos(f*x
+ e))*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x +
e))/cos(f*x + e))*(f^(-4))^(1/4) + 2*cos(f*x + e) + 2*sin(f*x + e))/cos(f*x
```

```
 + e)) - 75*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*log(sqrt((cos(f*x + e)
+ sin(f*x + e))/cos(f*x + e)) + 1) + 75*(cos(f*x + e)^4 - 2*cos(f*x + e)^2
+ 1)*log(sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e)) - 1) - 2*(14*cos(
f*x + e)^4 - 14*cos(f*x + e)^2 - (29*cos(f*x + e)^3 - 21*cos(f*x + e))*sin(
f*x + e))*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e)) + 24*8^(1/4)*sqr
t(2)*(f^5*cos(f*x + e)^4 - 2*f^5*cos(f*x + e)^2 + f^5)*sqrt(-2*sqrt(2)*f^2*
sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*arctan(1/16*8^(3/4)*sqrt(2)*(2*f^5*sqrt(f^
(-4)) + sqrt(2)*f^3)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((2*sqrt(2)*
f^2*sqrt(f^(-4))*cos(f*x + e) + 8^(1/4)*(sqrt(2)*f^3*sqrt(f^(-4))*cos(f*x +
 e) + f*cos(f*x + e))*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x +
 e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4) + 2*cos(f*x + e) + 2*sin(f
*x + e))/cos(f*x + e))*(f^(-4))^(3/4) - 1/8*8^(3/4)*(2*f^5*sqrt(f^(-4)) + s
qrt(2)*f^3)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(
f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) - f^2*sqrt(f^(-4)) - sqrt(2))/f^4 +
24*8^(1/4)*sqrt(2)*(f^5*cos(f*x + e)^4 - 2*f^5*cos(f*x + e)^2 + f^5)*sqrt(-
2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*arctan(1/16*8^(3/4)*sqrt(2)*
(2*f^5*sqrt(f^(-4)) + sqrt(2)*f^3)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sq
rt((2*sqrt(2)*f^2*sqrt(f^(-4))*cos(f*x + e) - 8^(1/4)*(sqrt(2)*f^3*sqrt(f^(
-4))*cos(f*x + e) + f*cos(f*x + e))*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*s
qrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4) + 2*cos(f*x
+ e) + 2*sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) - 1/8*8^(3/4)*(2*f^5*sq
rt(f^(-4)) + sqrt(2)*f^3)*sqrt(-2*sqrt(2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f
*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) + f^2*sqrt(f^(-4)) + s
qrt(2))/f^4)/(f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^2 + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (\tan(e + fx) + 1)^{\frac{3}{2}} \cot^4(e + fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(1+tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((tan(e + f*x) + 1)**(3/2)*cot(e + f*x)**4, x)
```

Giac [A]
time = 1.56, size = 375, normalized size = 1.58

$$\frac{25 \log\left(\sqrt{\tan(fx+e)+1}+1\right)}{16 f} - \frac{25 \log\left(\left|\sqrt{\tan(fx+e)+1}-1\right|\right)}{16 f} + \frac{\left(f^2\sqrt{2\sqrt{2}+2} - f\sqrt{2\sqrt{2}-2}\,|f|\right)\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{2 f^3} + \frac{\left(f^2\sqrt{2\sqrt{2}+2} - f\sqrt{2\sqrt{2}-2}\,|f|\right)\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{2 f^3} - \frac{\left(f^2\sqrt{2\sqrt{2}-2} + f\sqrt{2\sqrt{2}+2}\,|f|\right)\log\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{4 f^3} + \frac{\left(f^2\sqrt{2\sqrt{2}-2} + f\sqrt{2\sqrt{2}+2}\,|f|\right)\log\left(-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{4 f^3} + \frac{21(\tan(fx+e)+1)^{\frac{5}{2}} - 56(\tan(fx+e)+1)^{\frac{3}{2}} + 27\sqrt{\tan(fx+e)+1}}{24 f \tan(fx+e)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(1+tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] 25/16*log(sqrt(tan(f*x + e) + 1) + 1)/f - 25/16*log(abs(sqrt(tan(f*x + e) +
 1) - 1))/f + 1/2*(f^2*sqrt(2*sqrt(2) + 2) - f*sqrt(2*sqrt(2) - 2)*abs(f))*
```

```
arctan(1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*sqrt(tan(f*x + e) + 1))/s
qrt(-sqrt(2) + 2))/f^3 + 1/2*(f^2*sqrt(2*sqrt(2) + 2) - f*sqrt(2*sqrt(2) -
2)*abs(f))*arctan(-1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(tan(f*x
+ e) + 1))/sqrt(-sqrt(2) + 2))/f^3 - 1/4*(f^2*sqrt(2*sqrt(2) - 2) + f*sqrt(
2*sqrt(2) + 2)*abs(f))*log(2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x + e) + 1)
 + sqrt(2) + tan(f*x + e) + 1)/f^3 + 1/4*(f^2*sqrt(2*sqrt(2) - 2) + f*sqrt(
2*sqrt(2) + 2)*abs(f))*log(-2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x + e) + 1
) + sqrt(2) + tan(f*x + e) + 1)/f^3 + 1/24*(21*(tan(f*x + e) + 1)^(5/2) - 5
6*(tan(f*x + e) + 1)^(3/2) + 27*sqrt(tan(f*x + e) + 1))/(f*tan(f*x + e)^3)
```

Mupad [B]

time = 0.19, size = 173, normalized size = 0.73

$$-\frac{\mathrm{atan}\left(\sqrt{\tan\left(e+f\,x\right)+1}\,1\mathrm{i}\right)\,25\mathrm{i}}{8\,f}-\frac{\frac{9\,\sqrt{\tan\left(e+f\,x\right)+1}}{8}-\frac{7\,{\left(\tan\left(e+f\,x\right)+1\right)}^{3/2}}{3}+\frac{7\,{\left(\tan\left(e+f\,x\right)+1\right)}^{5/2}}{8}}{f-3\,f\,\left(\tan\left(e+f\,x\right)+1\right)+3\,f\,{\left(\tan\left(e+f\,x\right)+1\right)}^2-f\,{\left(\tan\left(e+f\,x\right)+1\right)}^3}+\mathrm{atan}\left(f\,\sqrt{\frac{\frac{1}{2}-\frac{1}{2}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\,1\mathrm{i}\right)\,\sqrt{\frac{\frac{1}{2}-\frac{1}{2}\mathrm{i}}{f^2}}\,2\mathrm{i}+\mathrm{atan}\left(f\,\sqrt{\frac{\frac{1}{2}+\frac{1}{2}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\,1\mathrm{i}\right)\,\sqrt{\frac{\frac{1}{2}+\frac{1}{2}\mathrm{i}}{f^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(tan(e + f*x) + 1)^(3/2),x)
```

```
[Out] atan(f*((1/2 - 1i/2)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1/2)*1i)*((1/2 - 1i/2)/
f^2)^(1/2)*2i - ((9*(tan(e + f*x) + 1)^(1/2))/8 - (7*(tan(e + f*x) + 1)^(3/
2))/3 + (7*(tan(e + f*x) + 1)^(5/2))/8)/(f - 3*f*(tan(e + f*x) + 1) + 3*f*(
tan(e + f*x) + 1)^2 - f*(tan(e + f*x) + 1)^3) - (atan((tan(e + f*x) + 1)^(1
/2)*1i)*25i)/(8*f) + atan(f*((1/2 + 1i/2)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1/
2)*1i)*((1/2 + 1i/2)/f^2)^(1/2)*2i
```

3.400 $\int \frac{\tan^5(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx$

Optimal. Leaf size=241

$$-\frac{\sqrt{-1+\sqrt{2}}\,\text{ArcTan}\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\tan(e+fx)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\sqrt{1+\tan(e+fx)}}\right)}{2f}-\frac{\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{3+2\sqrt{2}+\left(1+\cdots\right.}{\sqrt{2\left(7+5\sqrt{2}\right)}\cdots}\right.}{2f}$$

```
[Out] -1/2*arctan((3-2*2^(1/2)+(1-2^(1/2))*tan(f*x+e))/(-14+10*2^(1/2))^(1/2)/(1+
tan(f*x+e))^(1/2))*(2^(1/2)-1)^(1/2)/f-1/2*arctanh((3+2*2^(1/2)+(1+2^(1/2))
*tan(f*x+e))/(14+10*2^(1/2))^(1/2)/(1+tan(f*x+e))^(1/2))*(1+2^(1/2))^(1/2)/
f+44/105*(1+tan(f*x+e))^(1/2)/f-22/105*(1+tan(f*x+e))^(1/2)*tan(f*x+e)/f-12
/35*(1+tan(f*x+e))^(1/2)*tan(f*x+e)^2/f+2/7*(1+tan(f*x+e))^(1/2)*tan(f*x+e)
^3/f
```

Rubi [A]

time = 0.27, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3647, 3728, 3729, 3711, 12, 3617, 3616, 209, 213}

$$-\frac{\sqrt{\sqrt{2}-1}\,\text{ArcTan}\left(\frac{\left(1-\sqrt{2}\right)\tan(e+fx)-2\sqrt{2}+3}{\sqrt{2\left(5\sqrt{2}-7\right)}\sqrt{\tan(e+fx)+1}}\right)}{2f}+\frac{2\sqrt{\tan(e+fx)+1}\tan^3(e+fx)}{7f}-\frac{12\sqrt{\tan(e+fx)+1}\tan^2(e+fx)}{35f}-\frac{22\sqrt{\tan(e+fx)+1}\tan(e+fx)}{105f}+\frac{44\sqrt{\tan(e+fx)+1}}{105f}-\frac{\sqrt{1+\sqrt{2}}\tanh^{-1}\left(\frac{\left(1+\sqrt{2}\right)\tan(e+fx)+2\sqrt{2}+3}{\sqrt{2\left(7+5\sqrt{2}\right)}\sqrt{\tan(e+fx)+1}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] -1/2*(Sqrt[-1 + Sqrt[2]]*ArcTan[(3 - 2*Sqrt[2] + (1 - Sqrt[2])*Tan[e + f*x]
)/(Sqrt[2*(-7 + 5*Sqrt[2])]*Sqrt[1 + Tan[e + f*x]])])/f - (Sqrt[1 + Sqrt[2]
]*ArcTanh[(3 + 2*Sqrt[2] + (1 + Sqrt[2])*Tan[e + f*x])/(Sqrt[2*(7 + 5*Sqrt[
2])]*Sqrt[1 + Tan[e + f*x]])])/(2*f) + (44*Sqrt[1 + Tan[e + f*x]])/(105*f)
- (22*Tan[e + f*x]*Sqrt[1 + Tan[e + f*x]])/(105*f) - (12*Tan[e + f*x]^2*Sqr
t[1 + Tan[e + f*x]])/(35*f) + (2*Tan[e + f*x]^3*Sqrt[1 + Tan[e + f*x]])/(7*
f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3616

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*b*c*d - 4*a*d^2 +
 x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

Rule 3617

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2*q), Int[(a
*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
x], x] - Dist[1/(2*q), Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f*x
])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d - b
*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3729

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m + n +
1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*
Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*(b*c*m + a*d*(n + 1)) + d*(A*b -
 b*C)*(m + n + 1)*Tan[e + f*x] - C*m*(b*c - a*d)*Tan[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 +
 b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[
m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx = \frac{2\tan^3(e+fx)\sqrt{1+\tan(e+fx)}}{7f} + \frac{2}{7}\int \frac{\tan^2(e+fx)\left(-3-\frac{7}{2}\tan(e+fx)-\right.}{\sqrt{1+\tan(e+fx)}}$$

$$= -\frac{12\tan^2(e+fx)\sqrt{1+\tan(e+fx)}}{35f} + \frac{2\tan^3(e+fx)\sqrt{1+\tan(e+fx)}}{7f} + $$

$$= -\frac{22\tan(e+fx)\sqrt{1+\tan(e+fx)}}{105f} - \frac{12\tan^2(e+fx)\sqrt{1+\tan(e+fx)}}{35f} + $$

$$= \frac{44\sqrt{1+\tan(e+fx)}}{105f} - \frac{22\tan(e+fx)\sqrt{1+\tan(e+fx)}}{105f} - \frac{12\tan^2(e+fx)}{}$$

$$= \frac{44\sqrt{1+\tan(e+fx)}}{105f} - \frac{22\tan(e+fx)\sqrt{1+\tan(e+fx)}}{105f} - \frac{12\tan^2(e+fx)}{}$$

$$= \frac{44\sqrt{1+\tan(e+fx)}}{105f} - \frac{22\tan(e+fx)\sqrt{1+\tan(e+fx)}}{105f} - \frac{12\tan^2(e+fx)}{}$$

$$= \frac{44\sqrt{1+\tan(e+fx)}}{105f} - \frac{22\tan(e+fx)\sqrt{1+\tan(e+fx)}}{105f} - \frac{12\tan^2(e+fx)}{}$$

$$= -\frac{\sqrt{-1+\sqrt{2}}\tan^{-1}\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\tan(e+fx)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\sqrt{1+\tan(e+fx)}}\right)}{2f} - \frac{\sqrt{1+\sqrt{2}}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.61, size = 110, normalized size = 0.46

$$-\frac{\frac{\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1-i}}\right)}{\sqrt{1-i}}+\frac{\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1+i}}\right)}{\sqrt{1+i}}-\frac{2}{105}\sqrt{1+\tan(e+fx)}\left(40-26\tan(e+fx)+3\sec^2(e+fx)(-6+5\tan(e+fx))\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] -((ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1 - I]]/Sqrt[1 - I] + ArcTanh[Sqrt[1
 + Tan[e + f*x]]/Sqrt[1 + I]]/Sqrt[1 + I] - (2*Sqrt[1 + Tan[e + f*x]]*(40 -
 26*Tan[e + f*x] + 3*Sec[e + f*x]^2*(-6 + 5*Tan[e + f*x])))/105)/f)
```

Maple [A]
time = 0.13, size = 245, normalized size = 1.02

method	result
derivativedivides	$\frac{2(1+\tan(fx+e))^{\frac{7}{2}}}{7}-\frac{6(1+\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{4(1+\tan(fx+e))^{\frac{3}{2}}}{3}-\frac{\sqrt{2}\left(-\frac{\sqrt{2\sqrt{2}+2}\,\ln\left(1+\sqrt{2}-\sqrt{2\sqrt{2}+2}\,\sqrt{1\ldots}\right.}{2}\ldots\right.}{\ldots}$
default	$\frac{2(1+\tan(fx+e))^{\frac{7}{2}}}{7}-\frac{6(1+\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{4(1+\tan(fx+e))^{\frac{3}{2}}}{3}-\frac{\sqrt{2}\left(-\frac{\sqrt{2\sqrt{2}+2}\,\ln\left(1+\sqrt{2}-\sqrt{2\sqrt{2}+2}\,\sqrt{1\ldots}\right.}{2}\ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(1+tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2/7*(1+tan(f*x+e))^(7/2)-6/5*(1+tan(f*x+e))^(5/2)+4/3*(1+tan(f*x+e))^(
3/2)-1/4*2^(1/2)*(-1/2*(2*2^(1/2)+2)^(1/2)*ln(1+2^(1/2)-(2*2^(1/2)+2)^(1/2)
*(1+tan(f*x+e))^(1/2)+tan(f*x+e))+2*(1-2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan
((2*(1+tan(f*x+e))^(1/2)-(2*2^(1/2)+2)^(1/2))/(-2+2*2^(1/2))^(1/2)))-1/4*2^
(1/2)*(1/2*(2*2^(1/2)+2)^(1/2)*ln(1+2^(1/2)+(2*2^(1/2)+2)^(1/2)*(1+tan(f*x+
e))^(1/2)+tan(f*x+e))+2*(1-2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan(((2*2^(1/2)
+2)^(1/2)+2*(1+tan(f*x+e))^(1/2))/(-2+2*2^(1/2))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(1+tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^5/sqrt(tan(f*x + e) + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1064 vs. $2(199) = 398$.
time = 1.33, size = 1064, normalized size = 4.41

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(1+tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/420*(420*(1/2)^(3/4)*sqrt(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*f*(f^(-4))^
(1/4)*arctan(2*(1/2)^(3/4)*(f^5*sqrt(f^(-4)) + sqrt(1/2)*f^3)*sqrt(-4*sqrt(
1/2)*f^2*sqrt(f^(-4)) + 4)*sqrt((2*sqrt(1/2)*f^2*sqrt(f^(-4))*cos(f*x + e)
+ (1/2)^(1/4)*(2*sqrt(1/2)*f^3*sqrt(f^(-4))*cos(f*x + e) + f*cos(f*x + e))*
sqrt(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/
cos(f*x + e))*(f^(-4))^(1/4) + cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(
f^(-4))^(3/4) - 2*(1/2)^(3/4)*(f^5*sqrt(f^(-4)) + sqrt(1/2)*f^3)*sqrt(-4*sq
rt(1/2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x +
e))*(f^(-4))^(3/4) - f^2*sqrt(f^(-4)) - 2*sqrt(1/2))*cos(f*x + e)^3 + 420*(
1/2)^(3/4)*sqrt(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*f*(f^(-4))^(1/4)*arctan(
2*(1/2)^(3/4)*(f^5*sqrt(f^(-4)) + sqrt(1/2)*f^3)*sqrt(-4*sqrt(1/2)*f^2*sqrt
(f^(-4)) + 4)*sqrt((2*sqrt(1/2)*f^2*sqrt(f^(-4))*cos(f*x + e) - (1/2)^(1/4)
*(2*sqrt(1/2)*f^3*sqrt(f^(-4))*cos(f*x + e) + f*cos(f*x + e))*sqrt(-4*sqrt(
1/2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))
*(f^(-4))^(1/4) + cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4)
 - 2*(1/2)^(3/4)*(f^5*sqrt(f^(-4)) + sqrt(1/2)*f^3)*sqrt(-4*sqrt(1/2)*f^2*s
qrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^
(3/4) + f^2*sqrt(f^(-4)) + 2*sqrt(1/2))*cos(f*x + e)^3 + 105*(1/2)^(1/4)*(s
qrt(1/2)*f^3*sqrt(f^(-4))*cos(f*x + e)^3 + f*cos(f*x + e)^3)*sqrt(-4*sqrt(1
/2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*log((2*sqrt(1/2)*f^2*sqrt(f^(-4))*
cos(f*x + e) + (1/2)^(1/4)*(2*sqrt(1/2)*f^3*sqrt(f^(-4))*cos(f*x + e) + f*c
os(f*x + e))*sqrt(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + s
in(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4) + cos(f*x + e) + sin(f*x + e))/co
s(f*x + e)) - 105*(1/2)^(1/4)*(sqrt(1/2)*f^3*sqrt(f^(-4))*cos(f*x + e)^3 +
f*cos(f*x + e)^3)*sqrt(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*lo
g((2*sqrt(1/2)*f^2*sqrt(f^(-4))*cos(f*x + e) - (1/2)^(1/4)*(2*sqrt(1/2)*f^3
*sqrt(f^(-4))*cos(f*x + e) + f*cos(f*x + e))*sqrt(-4*sqrt(1/2)*f^2*sqrt(f^(
-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4) +
 cos(f*x + e) + sin(f*x + e))/cos(f*x + e)) - 8*(40*cos(f*x + e)^3 - (26*co
s(f*x + e)^2 - 15)*sin(f*x + e) - 18*cos(f*x + e))*sqrt((cos(f*x + e) + sin
(f*x + e))/cos(f*x + e)))/(f*cos(f*x + e)^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5{(e+fx)}}{\sqrt{\tan{(e+fx)}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(1+tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**5/sqrt(tan(e + f*x) + 1), x)
```

Giac [**A**]
time = 0.87, size = 254, normalized size = 1.05

$$\frac{\sqrt{\sqrt{2}-1}\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{2f}+\frac{\sqrt{\sqrt{2}-1}\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{2f}-\frac{\sqrt{\sqrt{2}+1}\log\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{4f}+\frac{\sqrt{\sqrt{2}+1}\log\left(-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{4f}+\frac{2\left(15f^6(\tan(fx+e)+1)^{\frac{7}{2}}-63f^6(\tan(fx+e)+1)^{\frac{5}{2}}+70f^6(\tan(fx+e)+1)^{\frac{3}{2}}\right)}{105f^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(1+tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(sqrt(2) - 1)*arctan(1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*sqr
t(tan(f*x + e) + 1))/sqrt(-sqrt(2) + 2))/f + 1/2*sqrt(sqrt(2) - 1)*arctan(-
1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(tan(f*x + e) + 1))/sqrt(-sq
rt(2) + 2))/f - 1/4*sqrt(sqrt(2) + 1)*log(2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(ta
n(f*x + e) + 1) + sqrt(2) + tan(f*x + e) + 1)/f + 1/4*sqrt(sqrt(2) + 1)*log
(-2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x + e) + 1) + sqrt(2) + tan(f*x + e)
 + 1)/f + 2/105*(15*f^6*(tan(f*x + e) + 1)^(7/2) - 63*f^6*(tan(f*x + e) + 1
)^(5/2) + 70*f^6*(tan(f*x + e) + 1)^(3/2))/f^7
```

Mupad [B]
time = 4.42, size = 118, normalized size = 0.49

$$\frac{4\,{\left(\tan(e+f\,x)+1\right)}^{3/2}}{3\,f}-\frac{6\,{\left(\tan(e+f\,x)+1\right)}^{5/2}}{5\,f}+\frac{2\,{\left(\tan(e+f\,x)+1\right)}^{7/2}}{7\,f}+\mathrm{atan}\left(f\,\sqrt{\frac{\frac{1}{8}-\frac{1}{8}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\,2\mathrm{i}\right)\,\sqrt{\frac{\frac{1}{8}-\frac{1}{8}\mathrm{i}}{f^2}}\,2\mathrm{i}+\mathrm{atan}\left(f\,\sqrt{\frac{\frac{1}{8}+\frac{1}{8}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\,2\mathrm{i}\right)\,\sqrt{\frac{\frac{1}{8}+\frac{1}{8}\mathrm{i}}{f^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(tan(e + f*x) + 1)^(1/2),x)
```

```
[Out] (4*(tan(e + f*x) + 1)^(3/2))/(3*f) - (6*(tan(e + f*x) + 1)^(5/2))/(5*f) + (
2*(tan(e + f*x) + 1)^(7/2))/(7*f) + atan(f*((1/8 - 1i/8)/f^2)^(1/2)*(tan(e
+ f*x) + 1)^(1/2)*2i)*((1/8 - 1i/8)/f^2)^(1/2)*2i + atan(f*((1/8 + 1i/8)/f^
2)^(1/2)*(tan(e + f*x) + 1)^(1/2)*2i)*((1/8 + 1i/8)/f^2)^(1/2)*2i
```

3.401 $\int \frac{\tan^3(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx$

Optimal. Leaf size=187

$$\frac{\sqrt{-1+\sqrt{2}}\,\text{ArcTan}\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\tan(e+fx)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\,\sqrt{1+\tan(e+fx)}}\right)}{2f}+\frac{\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{3+2\sqrt{2}+\left(1+\right.}{\sqrt{2\left(7+5\sqrt{2}\right)}\,\right.}\right.}{2f}$$

[Out] 1/2*arctan((3-2*2^(1/2)+(1-2^(1/2))*tan(f*x+e))/(-14+10*2^(1/2))^(1/2)/(1+tan(f*x+e))^(1/2))*(2^(1/2)-1)^(1/2)/f+1/2*arctanh((3+2*2^(1/2)+(1+2^(1/2))*tan(f*x+e))/(14+10*2^(1/2))^(1/2)/(1+tan(f*x+e))^(1/2))*(1+2^(1/2))^(1/2)/f-4/3*(1+tan(f*x+e))^(1/2)/f+2/3*(1+tan(f*x+e))^(1/2)*tan(f*x+e)/f

Rubi [A]
time = 0.16, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3647, 3711, 12, 3617, 3616, 209, 213}

$$\frac{\sqrt{\sqrt{2}-1}\,\text{ArcTan}\left(\frac{\left(1-\sqrt{2}\right)\tan(e+fx)-2\sqrt{2}+3}{\sqrt{2\left(5\sqrt{2}-7\right)}\,\sqrt{\tan(e+fx)+1}}\right)}{2f}+\frac{2\tan(e+fx)\sqrt{\tan(e+fx)+1}}{3f}-\frac{4\sqrt{\tan(e+fx)+1}}{3f}+\frac{\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\left(1+\sqrt{2}\right)\tan(e+fx)+2\sqrt{2}+3}{\sqrt{2\left(7+5\sqrt{2}\right)}\,\sqrt{\tan(e+fx)+1}}\right)}{2f}$$

Antiderivative was successfully verified.

[In] Int[Tan[e + f*x]^3/Sqrt[1 + Tan[e + f*x]],x]

[Out] (Sqrt[-1 + Sqrt[2]]*ArcTan[(3 - 2*Sqrt[2] + (1 - Sqrt[2])*Tan[e + f*x])/(Sqrt[2*(-7 + 5*Sqrt[2])]*Sqrt[1 + Tan[e + f*x]])])/(2*f) + (Sqrt[1 + Sqrt[2]]*ArcTanh[(3 + 2*Sqrt[2] + (1 + Sqrt[2])*Tan[e + f*x])/(Sqrt[2*(7 + 5*Sqrt[2])]*Sqrt[1 + Tan[e + f*x]])])/(2*f) - (4*Sqrt[1 + Tan[e + f*x]])/(3*f) + (2*Tan[e + f*x]*Sqrt[1 + Tan[e + f*x]])/(3*f)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3616

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*b*c*d - 4*a*d^2 +
 x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

Rule 3617

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2*q), Int[(a
*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
x], x] - Dist[1/(2*q), Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f*x
])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d - b
*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx = \frac{2\tan(e+fx)\sqrt{1+\tan(e+fx)}}{3f} + \frac{2}{3}\int \frac{-1-\frac{3}{2}\tan(e+fx)-\tan^2(e+fx)}{\sqrt{1+\tan(e+fx)}}\,d$$

$$= -\frac{4\sqrt{1+\tan(e+fx)}}{3f} + \frac{2\tan(e+fx)\sqrt{1+\tan(e+fx)}}{3f} + \frac{2}{3}\int -\frac{3\tan(}{2\sqrt{1+\text{ta}}}$$

$$= -\frac{4\sqrt{1+\tan(e+fx)}}{3f} + \frac{2\tan(e+fx)\sqrt{1+\tan(e+fx)}}{3f} - \int \frac{\tan(e+f}{\sqrt{1+\tan(e}}$$

$$= -\frac{4\sqrt{1+\tan(e+fx)}}{3f} + \frac{2\tan(e+fx)\sqrt{1+\tan(e+fx)}}{3f} + \frac{\int \frac{1+\left(-1-\sqrt{2}\right)\text{ta}}{\sqrt{1+\tan(e}}}{2\sqrt{2}}$$

$$= -\frac{4\sqrt{1+\tan(e+fx)}}{3f} + \frac{2\tan(e+fx)\sqrt{1+\tan(e+fx)}}{3f} - \frac{\left(4-3\sqrt{2}\right)\text{Su}}{}$$

$$= \frac{\sqrt{-1+\sqrt{2}}\,\tan^{-1}\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\tan(e+fx)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\sqrt{1+\tan(e+fx)}}\right)}{2f} + \frac{\sqrt{1+\sqrt{2}}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.22, size = 90, normalized size = 0.48

$$\frac{\frac{6\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1-i}}\right)}{\sqrt{1-i}} + \frac{6\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1+i}}\right)}{\sqrt{1+i}} + 4(-2+\tan(e+fx))\sqrt{1+\tan(e+fx)}}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] ((6*ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1 - I]])/Sqrt[1 - I] + (6*ArcTanh[S
qrt[1 + Tan[e + f*x]]/Sqrt[1 + I]])/Sqrt[1 + I] + 4*(-2 + Tan[e + f*x])*Sqr
t[1 + Tan[e + f*x]])/(6*f)
```

Maple [A]
time = 0.15, size = 233, normalized size = 1.25

method	result

derivativedivides	$\frac{\frac{2(1+\tan(fx+e))^{\frac{3}{2}}}{3}-2\sqrt{1+\tan(fx+e)}+\frac{\sqrt{2}\left(-\frac{\sqrt{2\sqrt{2}+2}\,\ln\left(1+\sqrt{2}-\sqrt{2\sqrt{2}+2}\,\sqrt{1+\tan}\right.}{2}\right.}{}}{}$
default	$\frac{\frac{2(1+\tan(fx+e))^{\frac{3}{2}}}{3}-2\sqrt{1+\tan(fx+e)}+\frac{\sqrt{2}\left(-\frac{\sqrt{2\sqrt{2}+2}\,\ln\left(1+\sqrt{2}-\sqrt{2\sqrt{2}+2}\,\sqrt{1+\tan}\right.}{2}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(1+tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2/3*(1+tan(f*x+e))^(3/2)-2*(1+tan(f*x+e))^(1/2)+1/4*2^(1/2)*(-1/2*(2*2
^(1/2)+2)^(1/2)*ln(1+2^(1/2)-(2*2^(1/2)+2)^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f
*x+e))+2*(1-2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+tan(f*x+e))^(1/2)-(2
*2^(1/2)+2)^(1/2))/(-2+2*2^(1/2))^(1/2)))+1/4*2^(1/2)*(1/2*(2*2^(1/2)+2)^(1
/2)*ln(1+2^(1/2)+(2*2^(1/2)+2)^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))+2*(1-
2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan(((2*2^(1/2)+2)^(1/2)+2*(1+tan(f*x+e))^
(1/2))/(-2+2*2^(1/2))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(1+tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^3/sqrt(tan(f*x + e) + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1028 vs. $2(149) = 298$.
time = 1.26, size = 1028, normalized size = 5.50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(1+tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(12*(1/2)^(3/4)*sqrt(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*f*(f^(-4))^(1/
4)*arctan(2*(1/2)^(3/4)*(f^5*sqrt(f^(-4)) + sqrt(1/2)*f^3)*sqrt(-4*sqrt(1/2
```

```
)*f^2*sqrt(f^(-4)) + 4)*sqrt((2*sqrt(1/2)*f^2*sqrt(f^(-4))*cos(f*x + e) + (
1/2)^(1/4)*(2*sqrt(1/2)*f^3*sqrt(f^(-4))*cos(f*x + e) + f*cos(f*x + e))*sqr
t(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos
(f*x + e))*(f^(-4))^(1/4) + cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(
-4))^(3/4) - 2*(1/2)^(3/4)*(f^5*sqrt(f^(-4)) + sqrt(1/2)*f^3)*sqrt(-4*sqrt(
1/2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))
*(f^(-4))^(3/4) - f^2*sqrt(f^(-4)) - 2*sqrt(1/2))*cos(f*x + e) + 12*(1/2)^(
3/4)*sqrt(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*f*(f^(-4))^(1/4)*arctan(2*(1/2
)^(3/4)*(f^5*sqrt(f^(-4)) + sqrt(1/2)*f^3)*sqrt(-4*sqrt(1/2)*f^2*sqrt(f^(-4
)) + 4)*sqrt((2*sqrt(1/2)*f^2*sqrt(f^(-4))*cos(f*x + e) - (1/2)^(1/4)*(2*sq
rt(1/2)*f^3*sqrt(f^(-4))*cos(f*x + e) + f*cos(f*x + e))*sqrt(-4*sqrt(1/2)*f
^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-
4))^(1/4) + cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) - 2*(
1/2)^(3/4)*(f^5*sqrt(f^(-4)) + sqrt(1/2)*f^3)*sqrt(-4*sqrt(1/2)*f^2*sqrt(f^
(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4)
+ f^2*sqrt(f^(-4)) + 2*sqrt(1/2))*cos(f*x + e) + 3*(1/2)^(1/4)*(sqrt(1/2)*f
^3*sqrt(f^(-4))*cos(f*x + e) + f*cos(f*x + e))*sqrt(-4*sqrt(1/2)*f^2*sqrt(f
^(-4)) + 4)*(f^(-4))^(1/4)*log((2*sqrt(1/2)*f^2*sqrt(f^(-4))*cos(f*x + e) +
 (1/2)^(1/4)*(2*sqrt(1/2)*f^3*sqrt(f^(-4))*cos(f*x + e) + f*cos(f*x + e))*s
qrt(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/c
os(f*x + e))*(f^(-4))^(1/4) + cos(f*x + e) + sin(f*x + e))/cos(f*x + e)) -
3*(1/2)^(1/4)*(sqrt(1/2)*f^3*sqrt(f^(-4))*cos(f*x + e) + f*cos(f*x + e))*sq
rt(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*log((2*sqrt(1/2)*f^2*s
qrt(f^(-4))*cos(f*x + e) - (1/2)^(1/4)*(2*sqrt(1/2)*f^3*sqrt(f^(-4))*cos(f*
x + e) + f*cos(f*x + e))*sqrt(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(
f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4) + cos(f*x + e) + sin(
f*x + e))/cos(f*x + e)) - 8*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e)
)*(2*cos(f*x + e) - sin(f*x + e)))/(f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(e+fx)}{\sqrt{\tan(e+fx)+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(1+tan(f*x+e))**(1/2),x)
[Out] Integral(tan(e + f*x)**3/sqrt(tan(e + f*x) + 1), x)
```

Giac [A]
time = 0.68, size = 237, normalized size = 1.27

$$-\frac{\sqrt{\sqrt{2}-1}\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{2f}-\frac{\sqrt{\sqrt{2}-1}\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{2f}+\frac{\sqrt{\sqrt{2}+1}\log\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{4f}-\frac{\sqrt{\sqrt{2}+1}\log\left(-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{4f}+\frac{2\left(f^2(\tan(fx+e)+1)^{\frac{3}{2}}-3f^2\sqrt{\tan(fx+e)+1}\right)}{3f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(1+tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(sqrt(2) - 1)*arctan(1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*sq
rt(tan(f*x + e) + 1))/sqrt(-sqrt(2) + 2))/f - 1/2*sqrt(sqrt(2) - 1)*arctan(
-1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(tan(f*x + e) + 1))/sqrt(-s
qrt(2) + 2))/f + 1/4*sqrt(sqrt(2) + 1)*log(2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(t
an(f*x + e) + 1) + sqrt(2) + tan(f*x + e) + 1)/f - 1/4*sqrt(sqrt(2) + 1)*lo
g(-2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x + e) + 1) + sqrt(2) + tan(f*x + e
) + 1)/f + 2/3*(f^2*(tan(f*x + e) + 1)^(3/2) - 3*f^2*sqrt(tan(f*x + e) + 1)
)/f^3
```

Mupad [B]
time = 4.13, size = 103, normalized size = 0.55

$$\frac{2\,{\left(\tan\left(e+f\,x\right)+1\right)}^{3/2}}{3\,f}-\frac{2\,\sqrt{\tan\left(e+f\,x\right)+1}}{f}-\mathrm{atan}\left(f\,\sqrt{\frac{\frac{1}{8}-\frac{1}{8}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\,2\mathrm{i}\right)\,\sqrt{\frac{\frac{1}{8}-\frac{1}{8}\mathrm{i}}{f^2}}\,2\mathrm{i}-\mathrm{atan}\left(f\,\sqrt{\frac{\frac{1}{8}+\frac{1}{8}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\,2\mathrm{i}\right)\,\sqrt{\frac{\frac{1}{8}+\frac{1}{8}\mathrm{i}}{f^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(tan(e + f*x) + 1)^(1/2),x)
```

```
[Out] (2*(tan(e + f*x) + 1)^(3/2))/(3*f) - (2*(tan(e + f*x) + 1)^(1/2))/f - atan(
f*((1/8 - 1i/8)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1/2)*2i)*((1/8 - 1i/8)/f^2)^
(1/2)*2i - atan(f*((1/8 + 1i/8)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1/2)*2i)*((1
/8 + 1i/8)/f^2)^(1/2)*2i
```

3.402 $\int \frac{\tan(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx$

Optimal. Leaf size=143

$$-\frac{\sqrt{-1+\sqrt{2}}\,\text{ArcTan}\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\tan(e+fx)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\,\sqrt{1+\tan(e+fx)}}\right)}{2f}-\frac{\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{3+2\sqrt{2}+\left(1-\ldots\right.}{\sqrt{2\left(7+5\sqrt{2}\right)}\ldots}\right.}{2f}$$

```
[Out] -1/2*arctan((3-2*2^(1/2)+(1-2^(1/2))*tan(f*x+e))/(-14+10*2^(1/2))^(1/2)/(1+
tan(f*x+e))^(1/2))*(2^(1/2)-1)^(1/2)/f-1/2*arctanh((3+2*2^(1/2)+(1+2^(1/2))
*tan(f*x+e))/(14+10*2^(1/2))^(1/2)/(1+tan(f*x+e))^(1/2))*(1+2^(1/2))^(1/2)/
f
```

Rubi [A]
time = 0.09, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3617, 3616, 209, 213}

$$-\frac{\sqrt{\sqrt{2}-1}\,\text{ArcTan}\left(\frac{\left(1-\sqrt{2}\right)\tan(e+fx)-2\sqrt{2}+3}{\sqrt{2\left(5\sqrt{2}-7\right)}\,\sqrt{\tan(e+fx)+1}}\right)}{2f}-\frac{\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\left(1+\sqrt{2}\right)\tan(e+fx)+2\sqrt{2}+3}{\sqrt{2\left(7+5\sqrt{2}\right)}\,\sqrt{\tan(e+fx)+1}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] -1/2*(Sqrt[-1 + Sqrt[2]]*ArcTan[(3 - 2*Sqrt[2] + (1 - Sqrt[2])*Tan[e + f*x]
)/(Sqrt[2*(-7 + 5*Sqrt[2])]*Sqrt[1 + Tan[e + f*x]])])/f - (Sqrt[1 + Sqrt[2]
]*ArcTanh[(3 + 2*Sqrt[2] + (1 + Sqrt[2])*Tan[e + f*x])/(Sqrt[2*(7 + 5*Sqrt[
2])]*Sqrt[1 + Tan[e + f*x]])])/(2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3616

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*b*c*d - 4*a*d^2 +
 x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

Rule 3617

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2*q), Int[(a
*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
x], x] - Dist[1/(2*q), Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f*x
])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d - b
*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])
```

Rubi steps

$$\int \frac{\tan(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx = -\frac{\int \frac{1+\left(-1-\sqrt{2}\right)\tan(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx}{2\sqrt{2}} + \frac{\int \frac{1+\left(-1+\sqrt{2}\right)\tan(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx}{2\sqrt{2}}$$

$$= \frac{\left(4-3\sqrt{2}\right)\operatorname{Subst}\left(\int \frac{1}{2\left(-1+\sqrt{2}\right)-4\left(-1+\sqrt{2}\right)^2+x^2}\,dx, x, \frac{1-2\left(-1+\sqrt{2}\right)-\left(-1+\sqrt{2}\right.}{\sqrt{1+\tan(e+f}}\right.}{2f}$$

$$= -\frac{\sqrt{-1+\sqrt{2}}\,\tan^{-1}\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\tan(e+fx)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\sqrt{1+\tan(e+fx)}}\right)}{2f} - \frac{\sqrt{1+\sqrt{2}}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.04, size = 67, normalized size = 0.47

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1-i}}\right)}{\sqrt{1-i}\,f} - \frac{\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1+i}}\right)}{\sqrt{1+i}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] -(ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1 - I]]/(Sqrt[1 - I]*f)) - ArcTanh[Sq
rt[1 + Tan[e + f*x]]/Sqrt[1 + I]]/(Sqrt[1 + I]*f)
```

Maple [A]
time = 0.13, size = 209, normalized size = 1.46

method	result
derivativedivides	$-\frac{\sqrt{2}\left(-\frac{\sqrt{2\sqrt{2}+2}\,\ln\left(1+\sqrt{2}-\sqrt{2\sqrt{2}+2}\,\sqrt{1+\tan(fx+e)}+\tan(fx+e)\right)}{2}+\frac{2\left(1-\sqrt{2}\right)\mathrm{arct}\cdots}{\cdots}\right.}{4}\cdots$
default	$-\frac{\sqrt{2}\left(-\frac{\sqrt{2\sqrt{2}+2}\,\ln\left(1+\sqrt{2}-\sqrt{2\sqrt{2}+2}\,\sqrt{1+\tan(fx+e)}+\tan(fx+e)\right)}{2}+\frac{2\left(1-\sqrt{2}\right)\mathrm{arct}\cdots}{\cdots}\right.}{4}\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(1+tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/4*2^(1/2)*(-1/2*(2*2^(1/2)+2)^(1/2)*ln(1+2^(1/2)-(2*2^(1/2)+2)^(1/2
)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))+2*(1-2^(1/2))/(-2+2*2^(1/2))^(1/2)*arcta
n((2*(1+tan(f*x+e))^(1/2)-(2*2^(1/2)+2)^(1/2))/(-2+2*2^(1/2))^(1/2)))-1/4*2
^(1/2)*(1/2*(2*2^(1/2)+2)^(1/2)*ln(1+2^(1/2)+(2*2^(1/2)+2)^(1/2)*(1+tan(f*x
+e))^(1/2)+tan(f*x+e))+2*(1-2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan(((2*2^(1/2
)+2)^(1/2)+2*(1+tan(f*x+e))^(1/2))/(-2+2*2^(1/2))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(1+tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)/sqrt(tan(f*x + e) + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 920 vs. $2(110) = 220$.
time = 1.15, size = 920, normalized size = 6.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(1+tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(1/2)^(1/4)*(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)*sqrt(-4*sqrt(1/2)*f^2*sqr
t(f^(-4)) + 4)*(f^(-4))^(1/4)*log((2*sqrt(1/2)*f^2*sqrt(f^(-4))*cos(f*x + e
) + (1/2)^(1/4)*(2*sqrt(1/2)*f^3*sqrt(f^(-4))*cos(f*x + e) + f*cos(f*x + e)
)*sqrt(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e)
)/cos(f*x + e))*(f^(-4))^(1/4) + cos(f*x + e) + sin(f*x + e))/cos(f*x + e))
 + 1/4*(1/2)^(1/4)*(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)*sqrt(-4*sqrt(1/2)*f^2*s
qrt(f^(-4)) + 4)*(f^(-4))^(1/4)*log((2*sqrt(1/2)*f^2*sqrt(f^(-4))*cos(f*x +
 e) - (1/2)^(1/4)*(2*sqrt(1/2)*f^3*sqrt(f^(-4))*cos(f*x + e) + f*cos(f*x +
e))*sqrt(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x +
e))/cos(f*x + e))*(f^(-4))^(1/4) + cos(f*x + e) + sin(f*x + e))/cos(f*x + e
)) - (1/2)^(3/4)*sqrt(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*arc
tan(2*(1/2)^(3/4)*(f^5*sqrt(f^(-4)) + sqrt(1/2)*f^3)*sqrt(-4*sqrt(1/2)*f^2*
sqrt(f^(-4)) + 4)*sqrt((2*sqrt(1/2)*f^2*sqrt(f^(-4))*cos(f*x + e) + (1/2)^(
1/4)*(2*sqrt(1/2)*f^3*sqrt(f^(-4))*cos(f*x + e) + f*cos(f*x + e))*sqrt(-4*s
qrt(1/2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x +
 e))*(f^(-4))^(1/4) + cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(
3/4) - 2*(1/2)^(3/4)*(f^5*sqrt(f^(-4)) + sqrt(1/2)*f^3)*sqrt(-4*sqrt(1/2)*f
^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-
4))^(3/4) - f^2*sqrt(f^(-4)) - 2*sqrt(1/2)) - (1/2)^(3/4)*sqrt(-4*sqrt(1/2)
*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*arctan(2*(1/2)^(3/4)*(f^5*sqrt(f^(-4)
) + sqrt(1/2)*f^3)*sqrt(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*sqrt((2*sqrt(1/2
)*f^2*sqrt(f^(-4))*cos(f*x + e) - (1/2)^(1/4)*(2*sqrt(1/2)*f^3*sqrt(f^(-4))
*cos(f*x + e) + f*cos(f*x + e))*sqrt(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*sqr
t((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4) + cos(f*x + e)
 + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) - 2*(1/2)^(3/4)*(f^5*sqrt(f^(
-4)) + sqrt(1/2)*f^3)*sqrt(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x
 + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) + f^2*sqrt(f^(-4)) + 2*s
qrt(1/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(e + fx)}{\sqrt{\tan(e + fx) + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(1+tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)/sqrt(tan(e + f*x) + 1), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(110) = 220.

time = 0.78, size = 282, normalized size = 1.97

$$\frac{\left(f^{2} \sqrt{2 \sqrt{2}+2}-f \sqrt{2 \sqrt{2}-2}\, |f|\right) \arctan \left(\frac{2^{\frac{3}{4}} \left(2^{\frac{1}{4}} \sqrt{\sqrt{2}+2}+2 \sqrt{\tan \left(f x+e\right)+1}\right)}{2 \sqrt{-\sqrt{2}+2}}\right)}{4 f^{3}}+\frac{\left(f^{2} \sqrt{2 \sqrt{2}+2}-f \sqrt{2 \sqrt{2}-2}\, |f|\right) \arctan \left(-\frac{2^{\frac{3}{4}} \left(2^{\frac{1}{4}} \sqrt{\sqrt{2}+2}-2 \sqrt{\tan \left(f x+e\right)+1}\right)}{2 \sqrt{-\sqrt{2}+2}}\right)}{4 f^{3}}-\frac{\left(f^{2} \sqrt{2 \sqrt{2}-2}+f \sqrt{2 \sqrt{2}+2}\, |f|\right) \log \left(2^{\frac{1}{4}} \sqrt{\sqrt{2}+2} \sqrt{\tan \left(f x+e\right)+1}+\sqrt{2}+\tan \left(f x+e\right)+1\right)}{8 f^{3}}+\frac{\left(f^{2} \sqrt{2 \sqrt{2}-2}+f \sqrt{2 \sqrt{2}+2}\, |f|\right) \log \left(-2^{\frac{1}{4}} \sqrt{\sqrt{2}+2} \sqrt{\tan \left(f x+e\right)+1}+\sqrt{2}+\tan \left(f x+e\right)+1\right)}{8 f^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(1+tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(f^2*sqrt(2*sqrt(2) + 2) - f*sqrt(2*sqrt(2) - 2)*abs(f))*arctan(1/2*2^(
3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*sqrt(tan(f*x + e) + 1))/sqrt(-sqrt(2) +
 2))/f^3 + 1/4*(f^2*sqrt(2*sqrt(2) + 2) - f*sqrt(2*sqrt(2) - 2)*abs(f))*arc
tan(-1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(tan(f*x + e) + 1))/sqr
t(-sqrt(2) + 2))/f^3 - 1/8*(f^2*sqrt(2*sqrt(2) - 2) + f*sqrt(2*sqrt(2) + 2)
*abs(f))*log(2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x + e) + 1) + sqrt(2) + t
an(f*x + e) + 1)/f^3 + 1/8*(f^2*sqrt(2*sqrt(2) - 2) + f*sqrt(2*sqrt(2) + 2)
*abs(f))*log(-2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x + e) + 1) + sqrt(2) +
tan(f*x + e) + 1)/f^3
```

Mupad [B]
time = 0.26, size = 69, normalized size = 0.48

$$-2\,\mathrm{atanh}\left(2\,f\,\sqrt{\frac{\frac{1}{8}-\frac{1}{8}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\right)\,\sqrt{\frac{\frac{1}{8}-\frac{1}{8}\mathrm{i}}{f^2}}-2\,\mathrm{atanh}\left(2\,f\,\sqrt{\frac{\frac{1}{8}+\frac{1}{8}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\right)\,\sqrt{\frac{\frac{1}{8}+\frac{1}{8}\mathrm{i}}{f^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(tan(e + f*x) + 1)^(1/2),x)
```

```
[Out] - 2*atanh(2*f*((1/8 - 1i/8)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1/2))*((1/8 - 1i
/8)/f^2)^(1/2) - 2*atanh(2*f*((1/8 + 1i/8)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1
/2))*((1/8 + 1i/8)/f^2)^(1/2)
```

3.403 $\int \frac{\cot(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx$

Optimal. Leaf size=161

$$\frac{\sqrt{-1+\sqrt{2}}\,\text{ArcTan}\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\tan(e+fx)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\,\sqrt{1+\tan(e+fx)}}\right)}{2f}-\frac{2\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{f}+\frac{\sqrt{1+\sqrt{}}}{}$$

```
[Out] -2*arctanh((1+tan(f*x+e))^(1/2))/f+1/2*arctan((3-2*2^(1/2)+(1-2^(1/2))*tan(
f*x+e))/(-14+10*2^(1/2))^(1/2)/(1+tan(f*x+e))^(1/2))*(2^(1/2)-1)^(1/2)/f+1/
2*arctanh((3+2*2^(1/2)+(1+2^(1/2))*tan(f*x+e))/(14+10*2^(1/2))^(1/2)/(1+tan
(f*x+e))^(1/2))*(1+2^(1/2))^(1/2)/f
```

Rubi [A]

time = 0.15, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3655, 3617, 3616, 209, 213, 3715, 65}

$$\frac{\sqrt{\sqrt{2}-1}\,\text{ArcTan}\left(\frac{\left(1-\sqrt{2}\right)\tan(e+fx)-2\sqrt{2}+3}{\sqrt{2\left(5\sqrt{2}-7\right)}\,\sqrt{\tan(e+fx)+1}}\right)}{2f}-\frac{2\tanh^{-1}\left(\sqrt{\tan(e+fx)+1}\right)}{f}+\frac{\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\left(1+\sqrt{2}\right)\tan(e+fx)+2\sqrt{2}+3}{\sqrt{2\left(7+5\sqrt{2}\right)}\,\sqrt{\tan(e+fx)+1}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] (Sqrt[-1 + Sqrt[2]]*ArcTan[(3 - 2*Sqrt[2] + (1 - Sqrt[2])*Tan[e + f*x])/(Sq
rt[2*(-7 + 5*Sqrt[2])]*Sqrt[1 + Tan[e + f*x]])])/(2*f) - (2*ArcTanh[Sqrt[1
+ Tan[e + f*x]]])/f + (Sqrt[1 + Sqrt[2]]*ArcTanh[(3 + 2*Sqrt[2] + (1 + Sqrt
[2])*Tan[e + f*x])/(Sqrt[2*(7 + 5*Sqrt[2])]*Sqrt[1 + Tan[e + f*x]])])/(2*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3616

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*b*c*d - 4*a*d^2 +
 x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

Rule 3617

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2*q), Int[(a
*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
x], x] - Dist[1/(2*q), Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f*x
])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d - b
*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)/((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[(a + b*Tan[e + f*x])^m*
(c - d*Tan[e + f*x]), x], x] + Dist[d^2/(c^2 + d^2), Int[(a + b*Tan[e + f*x
])^m*((1 + Tan[e + f*x]^2)/(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{\cot(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx = -\int \frac{\tan(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx + \int \frac{\cot(e+fx)\left(1+\tan^2(e+fx)\right)}{\sqrt{1+\tan(e+fx)}}\,dx$$

$$= \frac{\int \frac{1+\left(-1-\sqrt{2}\right)\tan(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx}{2\sqrt{2}} - \frac{\int \frac{1+\left(-1+\sqrt{2}\right)\tan(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx}{2\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,d\ldots\right.}{f}$$

$$= \frac{2\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\tan(e+fx)}\right)}{f} - \frac{\left(4-3\sqrt{2}\right)\text{Subst}\left(\int \frac{\ldots}{2\left(-1+\sqrt{2}\ldots\right.}\right.}{\ldots}$$

$$= \frac{\sqrt{-1+\sqrt{2}}\,\tan^{-1}\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\tan(e+fx)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\sqrt{1+\tan(e+fx)}}\right)}{2f} - \frac{2\tanh^{-1}\left(\sqrt{\ldots}\right.}{\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.10, size = 83, normalized size = 0.52

$$-\frac{2\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{f} + \frac{\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1-i}}\right)}{\sqrt{1-i}\,f} + \frac{\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1+i}}\right)}{\sqrt{1+i}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[1 + Tan[e + f*x]]])/f + ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqr
t[1 - I]]/(Sqrt[1 - I]*f) + ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1 + I]]/(Sq
rt[1 + I]*f)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 1.02, size = 2043, normalized size = 12.69

method	result	size
default	Expression too large to display	2043

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(1+tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/f*((cos(f*x+e)+sin(f*x+e))/cos(f*x+e))^(1/2)*(cos(f*x+e)+1)^2*(cos(f*
x+e)-1)^2*(1+sin(f*x+e))*(((sin(f*x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(
```

```
1/2)*2^(1/2)*((2^(1/2)*cos(f*x+e)-2^(1/2)*sin(f*x+e)+2^(1/2)+2*sin(f*x+e)-2
)/cos(f*x+e)*2^(1/2))^(1/2)*((2^(1/2)*cos(f*x+e)-2^(1/2)*sin(f*x+e)+2^(1/2)
-2*sin(f*x+e)+2)/cos(f*x+e)*2^(1/2))^(1/2)*EllipticF(1/2*((sin(f*x+e)-1)/co
s(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*2^(1/2),I*((2-2^(1/2))/(2+2^(1/2)))^(1/
2))-((sin(f*x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*2^(1/2)*((2^(1/2)
*cos(f*x+e)-2^(1/2)*sin(f*x+e)+2^(1/2)+2*sin(f*x+e)-2)/cos(f*x+e)*2^(1/2))^
(1/2)*((2^(1/2)*cos(f*x+e)-2^(1/2)*sin(f*x+e)+2^(1/2)-2*sin(f*x+e)+2)/cos(f
*x+e)*2^(1/2))^(1/2)*EllipticE(1/2*((sin(f*x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2
^(1/2))^(1/2)*2^(1/2),I*((2-2^(1/2))/(2+2^(1/2)))^(1/2))+2*2^(1/2)*Elliptic
E(1/2*((sin(f*x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*2^(1/2),I*((2-2
^(1/2))/(2+2^(1/2)))^(1/2))*((sin(f*x+e)-1)*(1+2^(1/2))/cos(f*x+e))^(1/2)*(
(2^(1/2)*sin(f*x+e)-2^(1/2)+cos(f*x+e)-sin(f*x+e)+1)/cos(f*x+e))^(1/2)*(-(2
^(1/2)*sin(f*x+e)-2^(1/2)-cos(f*x+e)+sin(f*x+e)-1)/cos(f*x+e))^(1/2)-28*2^(
1/2)*EllipticPi(1/2*((sin(f*x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*2
^(1/2),2^(1/2)/(2+2^(1/2)),I*((2-2^(1/2))/(2+2^(1/2)))^(1/2))*((sin(f*x+e)-
1)*(1+2^(1/2))/cos(f*x+e))^(1/2)*((2^(1/2)*sin(f*x+e)-2^(1/2)+cos(f*x+e)-si
n(f*x+e)+1)/cos(f*x+e))^(1/2)*(-(2^(1/2)*sin(f*x+e)-2^(1/2)-cos(f*x+e)+sin(
f*x+e)-1)/cos(f*x+e))^(1/2)+16*2^(1/2)*((sin(f*x+e)-1)*(1+2^(1/2))/cos(f*x+
e))^(1/2)*((2^(1/2)*sin(f*x+e)-2^(1/2)+cos(f*x+e)-sin(f*x+e)+1)/cos(f*x+e))
^(1/2)*(-(2^(1/2)*sin(f*x+e)-2^(1/2)-cos(f*x+e)+sin(f*x+e)-1)/cos(f*x+e))^(
1/2)*EllipticPi(1/2*((sin(f*x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*2
^(1/2),-I*2^(1/2)/(2+2^(1/2)),I*((2-2^(1/2))/(2+2^(1/2)))^(1/2))+16*2^(1/2)
*((sin(f*x+e)-1)*(1+2^(1/2))/cos(f*x+e))^(1/2)*((2^(1/2)*sin(f*x+e)-2^(1/2)
+cos(f*x+e)-sin(f*x+e)+1)/cos(f*x+e))^(1/2)*(-(2^(1/2)*sin(f*x+e)-2^(1/2)-c
os(f*x+e)+sin(f*x+e)-1)/cos(f*x+e))^(1/2)*EllipticPi(1/2*((sin(f*x+e)-1)/co
s(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*2^(1/2),I*2^(1/2)/(2+2^(1/2)),I*((2-2^(
1/2))/(2+2^(1/2)))^(1/2))-((sin(f*x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(
1/2)*((2^(1/2)*cos(f*x+e)-2^(1/2)*sin(f*x+e)+2^(1/2)+2*sin(f*x+e)-2)/cos(f*
x+e)*2^(1/2))^(1/2)*((2^(1/2)*cos(f*x+e)-2^(1/2)*sin(f*x+e)+2^(1/2)-2*sin(f
*x+e)+2)/cos(f*x+e)*2^(1/2))^(1/2)*EllipticE(1/2*((sin(f*x+e)-1)/cos(f*x+e)
*(2+2^(1/2))*2^(1/2))^(1/2)*2^(1/2),I*((2-2^(1/2))/(2+2^(1/2)))^(1/2))+6*((
sin(f*x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*((2^(1/2)*cos(f*x+e)-2^
(1/2)*sin(f*x+e)+2^(1/2)+2*sin(f*x+e)-2)/cos(f*x+e)*2^(1/2))^(1/2)*((2^(1/2
)*cos(f*x+e)-2^(1/2)*sin(f*x+e)+2^(1/2)-2*sin(f*x+e)+2)/cos(f*x+e)*2^(1/2))
^(1/2)*EllipticPi(1/2*((sin(f*x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)
*2^(1/2),2^(1/2)/(2+2^(1/2)),I*((2-2^(1/2))/(2+2^(1/2)))^(1/2))-8*((sin(f*x
+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*((2^(1/2)*cos(f*x+e)-2^(1/2)*s
in(f*x+e)+2^(1/2)+2*sin(f*x+e)-2)/cos(f*x+e)*2^(1/2))^(1/2)*((2^(1/2)*cos(f
*x+e)-2^(1/2)*sin(f*x+e)+2^(1/2)-2*sin(f*x+e)+2)/cos(f*x+e)*2^(1/2))^(1/2)*
EllipticPi(1/2*((sin(f*x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*2^(1/2
),-2^(1/2)/(2+2^(1/2)),I*((2-2^(1/2))/(2+2^(1/2)))^(1/2))-4*((sin(f*x+e)-1)
*(1+2^(1/2))/cos(f*x+e))^(1/2)*((2^(1/2)*sin(f*x+e)-2^(1/2)+cos(f*x+e)-sin(
f*x+e)+1)/cos(f*x+e))^(1/2)*(-(2^(1/2)*sin(f*x+e)-2^(1/2)-cos(f*x+e)+sin(f*
x+e)-1)/cos(f*x+e))^(1/2)*EllipticF(1/2*((sin(f*x+e)-1)/cos(f*x+e)*(2+2^(1/
2))*2^(1/2))^(1/2)*2^(1/2),I*((2-2^(1/2))/(2+2^(1/2)))^(1/2))+4*EllipticE(1
```

```
/2*((sin(f*x+e)-1)/cos(f*x+e)*(2+2^(1/2))*2^(1/2))^(1/2)*2^(1/2),I*((2-2^(1
/2))/(2+2^(1/2)))^(1/2))*((sin(f*x+e)-1)*(1+2^(1/2))/cos(f*x+e))^(1/2)*((2^
(1/2)*sin(f*x+e)-2^(1/2)+cos(f*x+e)-sin(f*x+e)+1)/cos(f*x+e))^(1/2)*(-(2^(1
/2)*sin(f*x+e)-2^(1/2)-cos(f*x+e)+sin(f*x+e)-1)/cos(f*x+e))^(1/2))*4^(1/2)/
sin(f*x+e)^4/(cos(f*x+e)+sin(f*x+e))/(2+2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

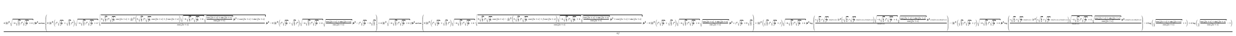
Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(1+tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)/sqrt(tan(f*x + e) + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 990 vs. 2(127) = 254.
time = 1.42, size = 990, normalized size = 6.15



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(1+tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*(1/2)^(3/4)*sqrt(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*f*(f^(-4))^(1/4)
*arctan(2*(1/2)^(3/4)*(f^5*sqrt(f^(-4)) + sqrt(1/2)*f^3)*sqrt(-4*sqrt(1/2)*
f^2*sqrt(f^(-4)) + 4)*sqrt((2*sqrt(1/2)*f^2*sqrt(f^(-4))*cos(f*x + e) + (1/
2)^(1/4)*(2*sqrt(1/2)*f^3*sqrt(f^(-4))*cos(f*x + e) + f*cos(f*x + e))*sqrt(
-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f
*x + e))*(f^(-4))^(1/4) + cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4
))^(3/4) - 2*(1/2)^(3/4)*(f^5*sqrt(f^(-4)) + sqrt(1/2)*f^3)*sqrt(-4*sqrt(1/
2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(
f^(-4))^(3/4) - f^2*sqrt(f^(-4)) - 2*sqrt(1/2)) + 4*(1/2)^(3/4)*sqrt(-4*sqr
t(1/2)*f^2*sqrt(f^(-4)) + 4)*f*(f^(-4))^(1/4)*arctan(2*(1/2)^(3/4)*(f^5*sqr
t(f^(-4)) + sqrt(1/2)*f^3)*sqrt(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*sqrt((2*
sqrt(1/2)*f^2*sqrt(f^(-4))*cos(f*x + e) - (1/2)^(1/4)*(2*sqrt(1/2)*f^3*sqrt
(f^(-4))*cos(f*x + e) + f*cos(f*x + e))*sqrt(-4*sqrt(1/2)*f^2*sqrt(f^(-4))
+ 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4) + cos(
f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) - 2*(1/2)^(3/4)*(f^5*
sqrt(f^(-4)) + sqrt(1/2)*f^3)*sqrt(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*sqrt(
(cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) + f^2*sqrt(f^(-4
)) + 2*sqrt(1/2)) + (1/2)^(1/4)*(sqrt(1/2)*f^3*sqrt(f^(-4)) + f)*sqrt(-4*sq
rt(1/2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*log((2*sqrt(1/2)*f^2*sqrt(f^(-
4))*cos(f*x + e) + (1/2)^(1/4)*(2*sqrt(1/2)*f^3*sqrt(f^(-4))*cos(f*x + e) +
 f*cos(f*x + e))*sqrt(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e)
```

```
 + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4) + cos(f*x + e) + sin(f*x + e)
)/cos(f*x + e)) - (1/2)^(1/4)*(sqrt(1/2)*f^3*sqrt(f^(-4)) + f)*sqrt(-4*sqrt
(1/2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*log((2*sqrt(1/2)*f^2*sqrt(f^(-4)
)*cos(f*x + e) - (1/2)^(1/4)*(2*sqrt(1/2)*f^3*sqrt(f^(-4))*cos(f*x + e) + f
*cos(f*x + e))*sqrt(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) +
 sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4) + cos(f*x + e) + sin(f*x + e))/
cos(f*x + e)) - 4*log(sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e)) + 1)
 + 4*log(sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e)) - 1))/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(e+fx)}{\sqrt{\tan(e+fx)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(1+tan(f*x+e))**(1/2),x)
[Out] Integral(cot(e + f*x)/sqrt(tan(e + f*x) + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(1+tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(cot(f*x + e)/sqrt(tan(f*x + e) + 1), x)
```

Mupad [B]
time = 0.16, size = 85, normalized size = 0.53

$$-\frac{2\,\mathrm{atanh}\left(\sqrt{\tan(e+f\,x)+1}\right)}{f}+2\,\mathrm{atanh}\left(2\,f\,\sqrt{\frac{\frac{1}{8}-\frac{1}{8}\mathrm{i}}{f^2}}\,\sqrt{\tan(e+f\,x)+1}\right)\sqrt{\frac{\frac{1}{8}-\frac{1}{8}\mathrm{i}}{f^2}}+2\,\mathrm{atanh}\left(2\,f\,\sqrt{\frac{\frac{1}{8}+\frac{1}{8}\mathrm{i}}{f^2}}\,\sqrt{\tan(e+f\,x)+1}\right)\sqrt{\frac{\frac{1}{8}+\frac{1}{8}\mathrm{i}}{f^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(tan(e + f*x) + 1)^(1/2),x)
[Out] 2*atanh(2*f*((1/8 - 1i/8)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1/2))*((1/8 - 1i/8
)/f^2)^(1/2) - (2*atanh((tan(e + f*x) + 1)^(1/2)))/f + 2*atanh(2*f*((1/8 +
1i/8)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1/2))*((1/8 + 1i/8)/f^2)^(1/2)
```

3.404 $\int \frac{\cot^3(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx$

Optimal. Leaf size=215

$$-\frac{\sqrt{-1+\sqrt{2}}\,\text{ArcTan}\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\tan(e+fx)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\,\sqrt{1+\tan(e+fx)}}\right)}{2f}+\frac{5\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{4f}-\frac{\sqrt{1+}}{}$$

[Out] 5/4*arctanh((1+tan(f*x+e))^(1/2))/f-1/2*arctan((3-2*2^(1/2)+(1-2^(1/2))*tan(f*x+e))/(-14+10*2^(1/2))^(1/2)/(1+tan(f*x+e))^(1/2))*(2^(1/2)-1)^(1/2)/f-1/2*arctanh((3+2*2^(1/2)+(1+2^(1/2))*tan(f*x+e))/(14+10*2^(1/2))^(1/2)/(1+tan(f*x+e))^(1/2))*(1+2^(1/2))^(1/2)/f+3/4*cot(f*x+e)*(1+tan(f*x+e))^(1/2)/f-1/2*cot(f*x+e)^2*(1+tan(f*x+e))^(1/2)/f

Rubi [A]
time = 0.28, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3650, 3730, 3735, 12, 3617, 3616, 209, 213, 3715, 65}

$$-\frac{\sqrt{\sqrt{2}-1}\,\text{ArcTan}\left(\frac{\left(1-\sqrt{2}\right)\tan(e+fx)-2\sqrt{2}+3}{\sqrt{2\left(5\sqrt{2}-7\right)}\,\sqrt{\tan(e+fx)+1}}\right)}{2f}+\frac{5\tanh^{-1}\left(\sqrt{\tan(e+fx)+1}\right)}{4f}-\frac{\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\left(1+\sqrt{2}\right)\tan(e+fx)+2\sqrt{2}+3}{\sqrt{2\left(7+5\sqrt{2}\right)}\,\sqrt{\tan(e+fx)+1}}\right)}{2f}-\frac{\sqrt{\tan(e+fx)+1}\cot^2(e+fx)}{2f}+\frac{3\sqrt{\tan(e+fx)+1}\cot(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] -1/2*(Sqrt[-1 + Sqrt[2]]*ArcTan[(3 - 2*Sqrt[2] + (1 - Sqrt[2])*Tan[e + f*x]
)/(Sqrt[2*(-7 + 5*Sqrt[2])]*Sqrt[1 + Tan[e + f*x]])])/f + (5*ArcTanh[Sqrt[1
 + Tan[e + f*x]]])/(4*f) - (Sqrt[1 + Sqrt[2]]*ArcTanh[(3 + 2*Sqrt[2] + (1 +
 Sqrt[2])*Tan[e + f*x])/(Sqrt[2*(7 + 5*Sqrt[2])]*Sqrt[1 + Tan[e + f*x]])])/
(2*f) + (3*Cot[e + f*x]*Sqrt[1 + Tan[e + f*x]])/(4*f) - (Cot[e + f*x]^2*Sqr
t[1 + Tan[e + f*x]])/(2*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3616

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*b*c*d - 4*a*d^2 +
 x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

Rule 3617

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2*q), Int[(a
*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
x], x] - Dist[1/(2*q), Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f*x
])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d - b
*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3735

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dis
t[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*Simp[a*(A - C) - (A*b - b*C)*Ta
n[e + f*x], x], x], x] + Dist[(A*b^2 + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e
 + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a,
 b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0] &&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx = -\frac{\cot^2(e+fx)\sqrt{1+\tan(e+fx)}}{2f} - \frac{1}{2}\int \frac{\cot^2(e+fx)\left(\frac{3}{2}+2\tan(e+fx)+\frac{3}{2}\right.}{\sqrt{1+\tan(e+fx)}}$$

$$= \frac{3\cot(e+fx)\sqrt{1+\tan(e+fx)}}{4f} - \frac{\cot^2(e+fx)\sqrt{1+\tan(e+fx)}}{2f} + \frac{1}{2}\int$$

$$= \frac{3\cot(e+fx)\sqrt{1+\tan(e+fx)}}{4f} - \frac{\cot^2(e+fx)\sqrt{1+\tan(e+fx)}}{2f} + \frac{1}{2}\int$$

$$= \frac{3\cot(e+fx)\sqrt{1+\tan(e+fx)}}{4f} - \frac{\cot^2(e+fx)\sqrt{1+\tan(e+fx)}}{2f} - \frac{5\mathrm{Subst}}{}$$

$$= \frac{3\cot(e+fx)\sqrt{1+\tan(e+fx)}}{4f} - \frac{\cot^2(e+fx)\sqrt{1+\tan(e+fx)}}{2f} - \frac{\int \frac{1+(}{\sqrt{1}}}{}$$

$$= \frac{5\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{4f} + \frac{3\cot(e+fx)\sqrt{1+\tan(e+fx)}}{4f} - \cot^2(e$$

$$= -\frac{\sqrt{-1+\sqrt{2}}\,\tan^{-1}\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\tan(e+fx)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\sqrt{1+\tan(e+fx)}}\right)}{2f} + \frac{5\tanh^{-1}(}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.34, size = 125, normalized size = 0.58

$$\frac{5\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right) - \frac{4\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1-i}}\right)}{\sqrt{1-i}} - \frac{4\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1+i}}\right)}{\sqrt{1+i}} + 3\cot(e+fx)\sqrt{1+\tan(e+fx)} - 2\cot^2(e+fx)\sqrt{1+\tan(e+fx)}}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] (5*ArcTanh[Sqrt[1 + Tan[e + f*x]]] - (4*ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt
[1 - I]])/Sqrt[1 - I] - (4*ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1 + I]])/Sqr
t[1 + I] + 3*Cot[e + f*x]*Sqrt[1 + Tan[e + f*x]] - 2*Cot[e + f*x]^2*Sqrt[1
+ Tan[e + f*x]])/(4*f)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.78, size = 8963, normalized size = 41.69

method	result	size
default	Expression too large to display	8963

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(1+tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(1+tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)^3/sqrt(tan(f*x + e) + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1184 vs. $2(175) = 350$.
time = 1.17, size = 1184, normalized size = 5.51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(1+tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*(2*(1/2)^(1/4)*sqrt(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*(f*cos(f*x + e)
^2 + sqrt(1/2)*(f^3*cos(f*x + e)^2 - f^3)*sqrt(f^(-4)) - f)*(f^(-4))^(1/4)*
log((2*sqrt(1/2)*f^2*sqrt(f^(-4))*cos(f*x + e) + (1/2)^(1/4)*(2*sqrt(1/2)*f
^3*sqrt(f^(-4))*cos(f*x + e) + f*cos(f*x + e))*sqrt(-4*sqrt(1/2)*f^2*sqrt(f
^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4)
 + cos(f*x + e) + sin(f*x + e))/cos(f*x + e)) - 2*(1/2)^(1/4)*sqrt(-4*sqrt(
1/2)*f^2*sqrt(f^(-4)) + 4)*(f*cos(f*x + e)^2 + sqrt(1/2)*(f^3*cos(f*x + e)^
2 - f^3)*sqrt(f^(-4)) - f)*(f^(-4))^(1/4)*log((2*sqrt(1/2)*f^2*sqrt(f^(-4))
*cos(f*x + e) - (1/2)^(1/4)*(2*sqrt(1/2)*f^3*sqrt(f^(-4))*cos(f*x + e) + f*
cos(f*x + e))*sqrt(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) +
sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4) + cos(f*x + e) + sin(f*x + e))/c
os(f*x + e)) - 5*(cos(f*x + e)^2 - 1)*log(sqrt((cos(f*x + e) + sin(f*x + e)
)/cos(f*x + e)) + 1) + 5*(cos(f*x + e)^2 - 1)*log(sqrt((cos(f*x + e) + sin(
f*x + e))/cos(f*x + e)) - 1) - 2*(2*cos(f*x + e)^2 - 3*cos(f*x + e)*sin(f*x
 + e))*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e)) + 8*(1/2)^(3/4)*(f^
5*cos(f*x + e)^2 - f^5)*sqrt(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1
/4)*arctan(2*(1/2)^(3/4)*(f^5*sqrt(f^(-4)) + sqrt(1/2)*f^3)*sqrt(-4*sqrt(1/
```

```
2)*f^2*sqrt(f^(-4)) + 4)*sqrt((2*sqrt(1/2)*f^2*sqrt(f^(-4))*cos(f*x + e) +
(1/2)^(1/4)*(2*sqrt(1/2)*f^3*sqrt(f^(-4))*cos(f*x + e) + f*cos(f*x + e))*sq
rt(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/co
s(f*x + e))*(f^(-4))^(1/4) + cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^
(-4))^(3/4) - 2*(1/2)^(3/4)*(f^5*sqrt(f^(-4)) + sqrt(1/2)*f^3)*sqrt(-4*sqrt
(1/2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e)
)*(f^(-4))^(3/4) - f^2*sqrt(f^(-4)) - 2*sqrt(1/2))/f^4 + 8*(1/2)^(3/4)*(f^5
*cos(f*x + e)^2 - f^5)*sqrt(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/
4)*arctan(2*(1/2)^(3/4)*(f^5*sqrt(f^(-4)) + sqrt(1/2)*f^3)*sqrt(-4*sqrt(1/2
)*f^2*sqrt(f^(-4)) + 4)*sqrt((2*sqrt(1/2)*f^2*sqrt(f^(-4))*cos(f*x + e) - (
1/2)^(1/4)*(2*sqrt(1/2)*f^3*sqrt(f^(-4))*cos(f*x + e) + f*cos(f*x + e))*sqr
t(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos
(f*x + e))*(f^(-4))^(1/4) + cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(
-4))^(3/4) - 2*(1/2)^(3/4)*(f^5*sqrt(f^(-4)) + sqrt(1/2)*f^3)*sqrt(-4*sqrt(
1/2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))
*(f^(-4))^(3/4) + f^2*sqrt(f^(-4)) + 2*sqrt(1/2))/f^4)/(f*cos(f*x + e)^2 -
f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(e+fx)}{\sqrt{\tan(e+fx)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(1+tan(f*x+e))**(1/2),x)
[Out] Integral(cot(e + f*x)**3/sqrt(tan(e + f*x) + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(1+tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(cot(f*x + e)^3/sqrt(tan(f*x + e) + 1), x)
```

Mupad [B]
time = 4.17, size = 147, normalized size = 0.68

$$-\frac{\mathrm{atan}\left(\sqrt{\tan(e+f\,x)+1}\,1\mathrm{i}\right)\,5\mathrm{i}}{4\,f}-\frac{\frac{5\,\sqrt{\tan(e+f\,x)+1}}{4}-\frac{3\,(\tan(e+f\,x)+1)^{3/2}}{4}}{f-2\,f\,(\tan(e+f\,x)+1)+f\,(\tan(e+f\,x)+1)^2}+\mathrm{atan}\left(f\,\sqrt{\frac{\frac{1}{8}-\frac{1}{8}\mathrm{i}}{f^2}}\,\sqrt{\tan(e+f\,x)+1}\,2\mathrm{i}\right)\,\sqrt{\frac{\frac{1}{8}-\frac{1}{8}\mathrm{i}}{f^2}}\,2\mathrm{i}+\mathrm{atan}\left(f\,\sqrt{\frac{\frac{1}{8}+\frac{1}{8}\mathrm{i}}{f^2}}\,\sqrt{\tan(e+f\,x)+1}\,2\mathrm{i}\right)\,\sqrt{\frac{\frac{1}{8}+\frac{1}{8}\mathrm{i}}{f^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(tan(e + f*x) + 1)^(1/2),x)
```

```
[Out] atan(f*((1/8 - 1i/8)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1/2)*2i)*((1/8 - 1i/8)/
f^2)^(1/2)*2i - ((5*(tan(e + f*x) + 1)^(1/2))/4 - (3*(tan(e + f*x) + 1)^(3/
2))/4)/(f - 2*f*(tan(e + f*x) + 1) + f*(tan(e + f*x) + 1)^2) - (atan((tan(e
 + f*x) + 1)^(1/2)*1i)*5i)/(4*f) + atan(f*((1/8 + 1i/8)/f^2)^(1/2)*(tan(e +
 f*x) + 1)^(1/2)*2i)*((1/8 + 1i/8)/f^2)^(1/2)*2i
```

3.405 $\int \frac{\cot^5(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx$

Optimal. Leaf size=269

$$\frac{\sqrt{-1+\sqrt{2}}\,\mathrm{ArcTan}\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\tan(e+fx)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\sqrt{1+\tan(e+fx)}}\right)}{2f}-\frac{115\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{64f}+\frac{\sqrt{1}}{}$$

```
[Out] -115/64*arctanh((1+tan(f*x+e))^(1/2))/f+1/2*arctan((3-2*2^(1/2)+(1-2^(1/2))
*tan(f*x+e))/(-14+10*2^(1/2))^(1/2)/(1+tan(f*x+e))^(1/2))*(2^(1/2)-1)^(1/2)
/f+1/2*arctanh((3+2*2^(1/2)+(1+2^(1/2))*tan(f*x+e))/(14+10*2^(1/2))^(1/2)/(
1+tan(f*x+e))^(1/2))*(1+2^(1/2))^(1/2)/f-13/64*cot(f*x+e)*(1+tan(f*x+e))^(1
/2)/f+13/96*cot(f*x+e)^2*(1+tan(f*x+e))^(1/2)/f+7/24*cot(f*x+e)^3*(1+tan(f*
x+e))^(1/2)/f-1/4*cot(f*x+e)^4*(1+tan(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.43, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3650, 3730, 3731, 3735, 12, 3617, 3616, 209, 213, 3715, 65}

$$\frac{\sqrt{\sqrt{2}-1}\,\mathrm{ArcTan}\left(\frac{\left(1-\sqrt{2}\right)\tan(e+fx)-2\sqrt{2}+3}{\sqrt{2\left(5\sqrt{2}-7\right)}\sqrt{\tan(e+fx)+1}}\right)}{2f}-\frac{115\tanh^{-1}\left(\sqrt{\tan(e+fx)+1}\right)}{64f}+\frac{\sqrt{1+\sqrt{2}}\tanh^{-1}\left(\frac{\left(1+\sqrt{2}\right)\tan(e+fx)+2\sqrt{2}+3}{\sqrt{2\left(7+5\sqrt{2}\right)}\sqrt{\tan(e+fx)+1}}\right)}{2f}-\frac{\sqrt{\tan(e+fx)+1}\cot^4(e+fx)}{4f}+\frac{7\sqrt{\tan(e+fx)+1}\cot^3(e+fx)}{24f}+\frac{13\sqrt{\tan(e+fx)+1}\cot^2(e+fx)}{96f}-\frac{13\sqrt{\tan(e+fx)+1}\cot(e+fx)}{64f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] (Sqrt[-1 + Sqrt[2]]*ArcTan[(3 - 2*Sqrt[2] + (1 - Sqrt[2])*Tan[e + f*x])/(Sq
rt[2*(-7 + 5*Sqrt[2])]*Sqrt[1 + Tan[e + f*x]])])/(2*f) - (115*ArcTanh[Sqrt[
1 + Tan[e + f*x]]])/(64*f) + (Sqrt[1 + Sqrt[2]]*ArcTanh[(3 + 2*Sqrt[2] + (1
 + Sqrt[2])*Tan[e + f*x])/(Sqrt[2*(7 + 5*Sqrt[2])]*Sqrt[1 + Tan[e + f*x]])]
)/(2*f) - (13*Cot[e + f*x]*Sqrt[1 + Tan[e + f*x]])/(64*f) + (13*Cot[e + f*x
]^2*Sqrt[1 + Tan[e + f*x]])/(96*f) + (7*Cot[e + f*x]^3*Sqrt[1 + Tan[e + f*x
]])/(24*f) - (Cot[e + f*x]^4*Sqrt[1 + Tan[e + f*x]])/(4*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3616

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*b*c*d - 4*a*d^2 +
 x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

Rule 3617

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2*q), Int[(a
*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
x], x] - Dist[1/(2*q), Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f*x
])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d - b
*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3731

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)
*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp
[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - a*C*(b*c*(m + 1) + a*d*(n
+ 1)) - (m + 1)*(b*c - a*d)*(A*b - b*C)*Tan[e + f*x] - d*(A*b^2 + a^2*C)*(m
 + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m,
 -1] &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3735

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dis
t[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*Simp[a*(A - C) - (A*b - b*C)*Ta
n[e + f*x], x], x], x] + Dist[(A*b^2 + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e
 + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a,
 b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0] &&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx = -\frac{\cot^4(e+fx)\sqrt{1+\tan(e+fx)}}{4f} - \frac{1}{4}\int \frac{\cot^4(e+fx)\left(\frac{7}{2}+4\tan(e+fx)+\frac{7}{2}\mathrm{t}\right.}{\sqrt{1+\tan(e+fx)}}$$

$$= \frac{7\cot^3(e+fx)\sqrt{1+\tan(e+fx)}}{24f} - \frac{\cot^4(e+fx)\sqrt{1+\tan(e+fx)}}{4f} + \frac{1}{12}\int$$

$$= \frac{13\cot^2(e+fx)\sqrt{1+\tan(e+fx)}}{96f} + \frac{7\cot^3(e+fx)\sqrt{1+\tan(e+fx)}}{24f} - \cot^4$$

$$= -\frac{13\cot(e+fx)\sqrt{1+\tan(e+fx)}}{64f} + \frac{13\cot^2(e+fx)\sqrt{1+\tan(e+fx)}}{96f} + 7$$

$$= -\frac{13\cot(e+fx)\sqrt{1+\tan(e+fx)}}{64f} + \frac{13\cot^2(e+fx)\sqrt{1+\tan(e+fx)}}{96f} + 7$$

$$= -\frac{13\cot(e+fx)\sqrt{1+\tan(e+fx)}}{64f} + \frac{13\cot^2(e+fx)\sqrt{1+\tan(e+fx)}}{96f} + 7$$

$$= -\frac{13\cot(e+fx)\sqrt{1+\tan(e+fx)}}{64f} + \frac{13\cot^2(e+fx)\sqrt{1+\tan(e+fx)}}{96f} + 7$$

$$= -\frac{115\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{64f} - \frac{13\cot(e+fx)\sqrt{1+\tan(e+fx)}}{64f} + 13$$

$$= \frac{\sqrt{-1+\sqrt{2}}\,\tan^{-1}\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\tan(e+fx)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\sqrt{1+\tan(e+fx)}}\right)}{2f} - \frac{115\tanh^{-1}\left(\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 18.04, size = 440, normalized size = 1.64

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^5/Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] (338 + 190*Cot[e + f*x] - 356*Csc[e + f*x]^2 - 112*Cot[e + f*x]*Csc[e + f*x
]^2 + 96*Csc[e + f*x]^4 - 678*Csc[2*(e + f*x)] + 712*Csc[2*(e + f*x)]^2 - 3
84*Csc[2*(e + f*x)]^3 + 152*Csc[e + f*x]^3*Sec[e + f*x] - 169*Sec[e + f*x]^
```

```
2 - 48*Csc[e + f*x]^4*Sec[e + f*x]^2 + 122*Csc[e + f*x]*Sec[e + f*x]^3 + 96
*Sqrt[-2 - 2*I]*ArcTan[Sqrt[-1/2 - I/2]*Sqrt[1 + Sec[e + f*x]*Sqrt[Sin[e +
f*x]^2]]]*Cos[2*(e + f*x)]*Sec[e + f*x]^2*Sqrt[1 + Sec[e + f*x]*Sqrt[Sin[e
+ f*x]^2]] + 96*Sqrt[-2 + 2*I]*ArcTan[Sqrt[-1/2 + I/2]*Sqrt[1 + Sec[e + f*x
]*Sqrt[Sin[e + f*x]^2]]]*Cos[2*(e + f*x)]*Sec[e + f*x]^2*Sqrt[1 + Sec[e + f
*x]*Sqrt[Sin[e + f*x]^2]] + 345*ArcTanh[Sqrt[1 + Sec[e + f*x]*Sqrt[Sin[e +
f*x]^2]]]*Cos[2*(e + f*x)]*Sec[e + f*x]^2*Sqrt[1 + Sec[e + f*x]*Sqrt[Sin[e
+ f*x]^2]] + 148*Tan[e + f*x] - 74*Sec[e + f*x]^2*Tan[e + f*x])/(192*f*(-2
+ Sec[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.86, size = 13527, normalized size = 50.29

method	result	size
default	Expression too large to display	13527

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(1+tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(1+tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)^5/sqrt(tan(f*x + e) + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1309 vs. $2(225) = 450$.
time = 1.19, size = 1309, normalized size = 4.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(1+tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/384*(96*(1/2)^(1/4)*(f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^2 + sqrt(1/2)*(f
^3*cos(f*x + e)^4 - 2*f^3*cos(f*x + e)^2 + f^3)*sqrt(f^(-4)) + f)*sqrt(-4*s
qrt(1/2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*log((2*sqrt(1/2)*f^2*sqrt(f^(
-4))*cos(f*x + e) + (1/2)^(1/4)*(2*sqrt(1/2)*f^3*sqrt(f^(-4))*cos(f*x + e)
+ f*cos(f*x + e))*sqrt(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e
) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4) + cos(f*x + e) + sin(f*x + e
```

```
))/cos(f*x + e)) - 96*(1/2)^(1/4)*(f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^2 +
sqrt(1/2)*(f^3*cos(f*x + e)^4 - 2*f^3*cos(f*x + e)^2 + f^3)*sqrt(f^(-4)) +
f)*sqrt(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*log((2*sqrt(1/2)*
f^2*sqrt(f^(-4))*cos(f*x + e) - (1/2)^(1/4)*(2*sqrt(1/2)*f^3*sqrt(f^(-4))*c
os(f*x + e) + f*cos(f*x + e))*sqrt(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*sqrt(
(cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4) + cos(f*x + e) +
 sin(f*x + e))/cos(f*x + e)) - 345*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*
log(sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e)) + 1) + 345*(cos(f*x +
e)^4 - 2*cos(f*x + e)^2 + 1)*log(sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x
 + e)) - 1) - 2*(74*cos(f*x + e)^4 - 26*cos(f*x + e)^2 - (95*cos(f*x + e)^3
 - 39*cos(f*x + e))*sin(f*x + e))*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*
x + e)) + 384*(1/2)^(3/4)*(f^5*cos(f*x + e)^4 - 2*f^5*cos(f*x + e)^2 + f^5)
*sqrt(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(1/4)*arctan(2*(1/2)^(3/4
)*(f^5*sqrt(f^(-4)) + sqrt(1/2)*f^3)*sqrt(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4
)*sqrt((2*sqrt(1/2)*f^2*sqrt(f^(-4))*cos(f*x + e) + (1/2)^(1/4)*(2*sqrt(1/2
)*f^3*sqrt(f^(-4))*cos(f*x + e) + f*cos(f*x + e))*sqrt(-4*sqrt(1/2)*f^2*sqr
t(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1
/4) + cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) - 2*(1/2)^(
3/4)*(f^5*sqrt(f^(-4)) + sqrt(1/2)*f^3)*sqrt(-4*sqrt(1/2)*f^2*sqrt(f^(-4))
+ 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) - f^2*
sqrt(f^(-4)) - 2*sqrt(1/2))/f^4 + 384*(1/2)^(3/4)*(f^5*cos(f*x + e)^4 - 2*f
^5*cos(f*x + e)^2 + f^5)*sqrt(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*(f^(-4))^(
1/4)*arctan(2*(1/2)^(3/4)*(f^5*sqrt(f^(-4)) + sqrt(1/2)*f^3)*sqrt(-4*sqrt(1
/2)*f^2*sqrt(f^(-4)) + 4)*sqrt((2*sqrt(1/2)*f^2*sqrt(f^(-4))*cos(f*x + e) -
 (1/2)^(1/4)*(2*sqrt(1/2)*f^3*sqrt(f^(-4))*cos(f*x + e) + f*cos(f*x + e))*s
qrt(-4*sqrt(1/2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/c
os(f*x + e))*(f^(-4))^(1/4) + cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f
^(-4))^(3/4) - 2*(1/2)^(3/4)*(f^5*sqrt(f^(-4)) + sqrt(1/2)*f^3)*sqrt(-4*sqr
t(1/2)*f^2*sqrt(f^(-4)) + 4)*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e
))*(f^(-4))^(3/4) + f^2*sqrt(f^(-4)) + 2*sqrt(1/2))/f^4)/(f*cos(f*x + e)^4
- 2*f*cos(f*x + e)^2 + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5(e+fx)}{\sqrt{\tan(e+fx)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(1+tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**5/sqrt(tan(e + f*x) + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(1+tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^5/sqrt(tan(f*x + e) + 1), x)
```

Mupad [B]
time = 4.22, size = 197, normalized size = 0.73

$$\frac{\mathrm{atan}\left(\sqrt{\tan\left(e+f\,x\right)+1}\,1{}\mathrm{i}\right)\,115{}\mathrm{i}}{64\,f}-\frac{\frac{13\,\sqrt{\tan\left(e+f\,x\right)+1}}{64}+\frac{113\,{\left(\tan\left(e+f\,x\right)+1\right)}^{3/2}}{192}-\frac{143\,{\left(\tan\left(e+f\,x\right)+1\right)}^{5/2}}{192}+\frac{13\,{\left(\tan\left(e+f\,x\right)+1\right)}^{7/2}}{64}}{f-4\,f\,\left(\tan\left(e+f\,x\right)+1\right)+6\,f\,{\left(\tan\left(e+f\,x\right)+1\right)}^2-4\,f\,{\left(\tan\left(e+f\,x\right)+1\right)}^3+f\,{\left(\tan\left(e+f\,x\right)+1\right)}^4}-\mathrm{atan}\left(f\,\sqrt{\frac{\frac{1}{8}-\frac{1}{8}{}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\,2{}\mathrm{i}\right)\,\sqrt{\frac{\frac{1}{8}-\frac{1}{8}{}\mathrm{i}}{f^2}}\,2{}\mathrm{i}-\mathrm{atan}\left(f\,\sqrt{\frac{\frac{1}{8}+\frac{1}{8}{}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\,2{}\mathrm{i}\right)\,\sqrt{\frac{\frac{1}{8}+\frac{1}{8}{}\mathrm{i}}{f^2}}\,2{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5/(tan(e + f*x) + 1)^(1/2),x)
```

```
[Out] (atan((tan(e + f*x) + 1)^(1/2)*1i)*115i)/(64*f) - ((13*(tan(e + f*x) + 1)^(
1/2))/64 + (113*(tan(e + f*x) + 1)^(3/2))/192 - (143*(tan(e + f*x) + 1)^(5/
2))/192 + (13*(tan(e + f*x) + 1)^(7/2))/64)/(f - 4*f*(tan(e + f*x) + 1) + 6
*f*(tan(e + f*x) + 1)^2 - 4*f*(tan(e + f*x) + 1)^3 + f*(tan(e + f*x) + 1)^4
) - atan(f*((1/8 - 1i/8)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1/2)*2i)*((1/8 - 1i
/8)/f^2)^(1/2)*2i - atan(f*((1/8 + 1i/8)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1/2
)*2i)*((1/8 + 1i/8)/f^2)^(1/2)*2i
```

3.406 $\int \frac{\tan^4(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx$

Optimal. Leaf size=311

$$-\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\tan(e+fx)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2f}+\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{}}{\sqrt{2\left(-1+\right.}}\right.}{2f}$$

```
[Out] -1/4*ln(1+2^(1/2)-(2+2*2^(1/2))^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))/f/(1
+2^(1/2))^(1/2)+1/4*ln(1+2^(1/2)+(2+2*2^(1/2))^(1/2)*(1+tan(f*x+e))^(1/2)+t
an(f*x+e))/f/(1+2^(1/2))^(1/2)-1/2*arctan(((2+2*2^(1/2))^(1/2)-2*(1+tan(f*x
+e))^(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^(1/2))^(1/2)/f+1/2*arctan(((2+2*2^(1
/2))^(1/2)+2*(1+tan(f*x+e))^(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^(1/2))^(1/2)/
f-14/15*(1+tan(f*x+e))^(1/2)/f-8/15*(1+tan(f*x+e))^(1/2)*tan(f*x+e)/f+2/5*(
1+tan(f*x+e))^(1/2)*tan(f*x+e)^2/f
```

Rubi [A]
time = 0.24, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3647, 3728, 3712, 3566, 722, 1108, 648, 632, 210, 642}

$$-\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\tan(e+fx)+1}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{2f}+\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{2\sqrt{\tan(e+fx)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{2f}+\frac{2\sqrt{\tan(e+fx)+1}\tan^2(e+fx)}{5f}-\frac{8\sqrt{\tan(e+fx)+1}\tan(e+fx)}{15f}-\frac{14\sqrt{\tan(e+fx)+1}}{15f}-\frac{\log\left(\tan(e+fx)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\tan(e+fx)+1}+\sqrt{2}+1\right)}{4\sqrt{1+\sqrt{2}}\,f}+\frac{\log\left(\tan(e+fx)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\tan(e+fx)+1}+\sqrt{2}+1\right)}{4\sqrt{1+\sqrt{2}}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] -1/2*(Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + Tan[e +
f*x]])/Sqrt[2*(-1 + Sqrt[2])]])/f + (Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 +
Sqrt[2])] + 2*Sqrt[1 + Tan[e + f*x]])/Sqrt[2*(-1 + Sqrt[2])]])/(2*f) - Log[
1 + Sqrt[2] + Tan[e + f*x] - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + Tan[e + f*x]]]/
(4*Sqrt[1 + Sqrt[2]]*f) + Log[1 + Sqrt[2] + Tan[e + f*x] + Sqrt[2*(1 + Sqrt
[2])]*Sqrt[1 + Tan[e + f*x]]]/(4*Sqrt[1 + Sqrt[2]]*f) - (14*Sqrt[1 + Tan[e
+ f*x]])/(15*f) - (8*Tan[e + f*x]*Sqrt[1 + Tan[e + f*x]])/(15*f) + (2*Tan[e
 + f*x]^2*Sqrt[1 + Tan[e + f*x]])/(5*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 722

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
```

```
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3712

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx = \frac{2\tan^2(e+fx)\sqrt{1+\tan(e+fx)}}{5f} + \frac{2}{5}\int \frac{\tan(e+fx)\left(-2-\frac{5}{2}\tan(e+fx)-\right.}{\sqrt{1+\tan(e+fx)}}$$

$$= -\frac{8\tan(e+fx)\sqrt{1+\tan(e+fx)}}{15f} + \frac{2\tan^2(e+fx)\sqrt{1+\tan(e+fx)}}{5f} + \frac{4}{15}$$

$$= -\frac{14\sqrt{1+\tan(e+fx)}}{15f} - \frac{8\tan(e+fx)\sqrt{1+\tan(e+fx)}}{15f} + \frac{2\tan^2(e+fx)}{}$$

$$= -\frac{14\sqrt{1+\tan(e+fx)}}{15f} - \frac{8\tan(e+fx)\sqrt{1+\tan(e+fx)}}{15f} + \frac{2\tan^2(e+fx)}{}$$

$$= -\frac{14\sqrt{1+\tan(e+fx)}}{15f} - \frac{8\tan(e+fx)\sqrt{1+\tan(e+fx)}}{15f} + \frac{2\tan^2(e+fx)}{}$$

$$= -\frac{14\sqrt{1+\tan(e+fx)}}{15f} - \frac{8\tan(e+fx)\sqrt{1+\tan(e+fx)}}{15f} + \frac{2\tan^2(e+fx)}{}$$

$$= -\frac{14\sqrt{1+\tan(e+fx)}}{15f} - \frac{8\tan(e+fx)\sqrt{1+\tan(e+fx)}}{15f} + \frac{2\tan^2(e+fx)}{}$$

$$= -\frac{\log\left(1+\sqrt{2}+\tan(e+fx)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\tan(e+fx)}\right)}{4\sqrt{1+\sqrt{2}}\,f} + \frac{\log}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\tan(e+fx)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2\sqrt{-1+\sqrt{2}}\,f} + \frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(-\right.}}\right.}{2\sqrt{-1+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.73, size = 100, normalized size = 0.32

$$\frac{(1-i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1-i}}\right)}{2f} + \frac{(1+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1+i}}\right)}{2f} + \frac{2(1+\tan(e+fx))^{3/2}(-7+3\tan(e+fx))}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] ((1 - I)^(3/2)*ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1 - I]])/(2*f) + ((1 + I
)^(3/2)*ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1 + I]])/(2*f) + (2*(1 + Tan[e
+ f*x])^(3/2)*(-7 + 3*Tan[e + f*x]))/(15*f)
```

Maple [A]
time = 0.13, size = 327, normalized size = 1.05

method	result
derivativedivides	$\frac{\frac{2(1+\tan(fx+e))^{\frac{5}{2}}}{5}-\frac{4(1+\tan(fx+e))^{\frac{3}{2}}}{3}-\frac{\left(-\sqrt{2\sqrt{2}+2}\,\sqrt{2}+2\sqrt{2\sqrt{2}+2}\right)\ln\left(1+\sqrt{2}-\sqrt{2\sqrt{2}+2}\,\sqrt{\right.}}{8}}{}$
default	$\frac{\frac{2(1+\tan(fx+e))^{\frac{5}{2}}}{5}-\frac{4(1+\tan(fx+e))^{\frac{3}{2}}}{3}-\frac{\left(-\sqrt{2\sqrt{2}+2}\,\sqrt{2}+2\sqrt{2\sqrt{2}+2}\right)\ln\left(1+\sqrt{2}-\sqrt{2\sqrt{2}+2}\,\sqrt{\right.}}{8}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(1+tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2/5*(1+tan(f*x+e))^(5/2)-4/3*(1+tan(f*x+e))^(3/2)-1/8*(-(2*2^(1/2)+2)^
(1/2)*2^(1/2)+2*(2*2^(1/2)+2)^(1/2))*ln(1+2^(1/2)-(2*2^(1/2)+2)^(1/2)*(1+ta
n(f*x+e))^(1/2)+tan(f*x+e))-1/2*(-2*2^(1/2)+1/2*(-(2*2^(1/2)+2)^(1/2)*2^(1/
2)+2*(2*2^(1/2)+2)^(1/2))*(2*2^(1/2)+2)^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan(
(2*(1+tan(f*x+e))^(1/2)-(2*2^(1/2)+2)^(1/2))/(-2+2*2^(1/2))^(1/2))+1/8*(-(2
*2^(1/2)+2)^(1/2)*2^(1/2)+2*(2*2^(1/2)+2)^(1/2))*ln(1+2^(1/2)+(2*2^(1/2)+2)
^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))+1/2*(2*2^(1/2)-1/2*(-(2*2^(1/2)+2)^
(1/2)*2^(1/2)+2*(2*2^(1/2)+2)^(1/2))*(2*2^(1/2)+2)^(1/2))/(-2+2*2^(1/2))^(1
/2)*arctan(((2*2^(1/2)+2)^(1/2)+2*(1+tan(f*x+e))^(1/2))/(-2+2*2^(1/2))^(1/2
)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(1+tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^4/sqrt(tan(f*x + e) + 1), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(250) = 500$.
time = 1.26, size = 898, normalized size = 2.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(1+tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/30*(60*(1/2)^(3/4)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)*f*(f^(-4))^(1/4)
*arctan(2*(1/2)^(1/4)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)*f^3*sqrt((2*sqrt
(1/2)*f^2*sqrt(f^(-4))*cos(f*x + e) + 2*(1/2)^(1/4)*sqrt(sqrt(1/2)*f^2*sqrt
(f^(-4)) + 1)*f*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(
1/4)*cos(f*x + e) + cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/
4) - 2*(1/2)^(1/4)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)*f^3*sqrt((cos(f*x +
 e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) - f^2*sqrt(f^(-4)) - 2*sqr
t(1/2))*cos(f*x + e)^2 + 60*(1/2)^(3/4)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1
)*f*(f^(-4))^(1/4)*arctan(2*(1/2)^(1/4)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1
)*f^3*sqrt((2*sqrt(1/2)*f^2*sqrt(f^(-4))*cos(f*x + e) - 2*(1/2)^(1/4)*sqrt(
sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)*f*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*
x + e))*(f^(-4))^(1/4)*cos(f*x + e) + cos(f*x + e) + sin(f*x + e))/cos(f*x
+ e))*(f^(-4))^(3/4) - 2*(1/2)^(1/4)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)*f
^3*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) + f^2*sq
rt(f^(-4)) + 2*sqrt(1/2))*cos(f*x + e)^2 + 15*(1/2)^(1/4)*(sqrt(1/2)*f^3*sq
rt(f^(-4))*cos(f*x + e)^2 - f*cos(f*x + e)^2)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4
)) + 1)*(f^(-4))^(1/4)*log((2*sqrt(1/2)*f^2*sqrt(f^(-4))*cos(f*x + e) + 2*(
1/2)^(1/4)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)*f*sqrt((cos(f*x + e) + sin(
f*x + e))/cos(f*x + e))*(f^(-4))^(1/4)*cos(f*x + e) + cos(f*x + e) + sin(f*
x + e))/cos(f*x + e)) - 15*(1/2)^(1/4)*(sqrt(1/2)*f^3*sqrt(f^(-4))*cos(f*x
+ e)^2 - f*cos(f*x + e)^2)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)*(f^(-4))^(1
/4)*log((2*sqrt(1/2)*f^2*sqrt(f^(-4))*cos(f*x + e) - 2*(1/2)^(1/4)*sqrt(sqr
t(1/2)*f^2*sqrt(f^(-4)) + 1)*f*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x +
 e))*(f^(-4))^(1/4)*cos(f*x + e) + cos(f*x + e) + sin(f*x + e))/cos(f*x + e
)) + 4*(10*cos(f*x + e)^2 + 4*cos(f*x + e)*sin(f*x + e) - 3)*sqrt((cos(f*x
+ e) + sin(f*x + e))/cos(f*x + e)))/(f*cos(f*x + e)^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(e + fx)}{\sqrt{\tan(e + fx) + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(1+tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**4/sqrt(tan(e + f*x) + 1), x)
```

Giac [A]
time = 0.72, size = 238, normalized size = 0.77

$$\frac{\sqrt{\sqrt{2}+1}\,\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{\tan\left(fx+e\right)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{2f}+\frac{\sqrt{\sqrt{2}+1}\,\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{\tan\left(fx+e\right)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{2f}+\frac{\sqrt{\sqrt{2}-1}\,\log\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\,\sqrt{\tan\left(fx+e\right)+1}+\sqrt{2}+\tan\left(fx+e\right)+1\right)}{4f}-\frac{\sqrt{\sqrt{2}-1}\,\log\left(-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\,\sqrt{\tan\left(fx+e\right)+1}+\sqrt{2}+\tan\left(fx+e\right)+1\right)}{4f}+\frac{2\left(3f^{4}(\tan\left(fx+e\right)+1)^{\frac{5}{2}}-10f^{4}(\tan\left(fx+e\right)+1)^{\frac{3}{2}}\right)}{15f^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(1+tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(sqrt(2) + 1)*arctan(1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*sqr
t(tan(f*x + e) + 1))/sqrt(-sqrt(2) + 2))/f + 1/2*sqrt(sqrt(2) + 1)*arctan(-
1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(tan(f*x + e) + 1))/sqrt(-sq
rt(2) + 2))/f + 1/4*sqrt(sqrt(2) - 1)*log(2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(ta
n(f*x + e) + 1) + sqrt(2) + tan(f*x + e) + 1)/f - 1/4*sqrt(sqrt(2) - 1)*log
(-2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x + e) + 1) + sqrt(2) + tan(f*x + e)
 + 1)/f + 2/15*(3*f^4*(tan(f*x + e) + 1)^(5/2) - 10*f^4*(tan(f*x + e) + 1)^
(3/2))/f^5
```

Mupad [B]
time = 0.55, size = 101, normalized size = 0.32

$$\frac{2\left(\tan(e+f\,x)+1\right)^{5/2}}{5\,f}-\frac{4\left(\tan(e+f\,x)+1\right)^{3/2}}{3\,f}+\mathrm{atan}\left(2\,f\,\sqrt{\frac{-\frac{1}{8}-\frac{1}{8}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\right)\,\sqrt{\frac{-\frac{1}{8}-\frac{1}{8}\mathrm{i}}{f^2}}\,2\mathrm{i}-\mathrm{atan}\left(2\,f\,\sqrt{\frac{-\frac{1}{8}+\frac{1}{8}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\right)\,\sqrt{\frac{-\frac{1}{8}+\frac{1}{8}\mathrm{i}}{f^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(tan(e + f*x) + 1)^(1/2),x)
```

```
[Out] (2*(tan(e + f*x) + 1)^(5/2))/(5*f) - (4*(tan(e + f*x) + 1)^(3/2))/(3*f) + a
tan(2*f*((- 1/8 - 1i/8)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1/2))*((- 1/8 - 1i/8
)/f^2)^(1/2)*2i - atan(2*f*((- 1/8 + 1i/8)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1
/2))*((- 1/8 + 1i/8)/f^2)^(1/2)*2i
```

3.407 $\int \frac{\tan^2(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx$

Optimal. Leaf size=257

$$\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\tan(e+fx)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2f}-\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+\tan(e+fx)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2f}$$

```
[Out] 1/4*ln(1+2^(1/2)-(2+2*2^(1/2))^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))/f/(1+
2^(1/2))^(1/2)-1/4*ln(1+2^(1/2)+(2+2*2^(1/2))^(1/2)*(1+tan(f*x+e))^(1/2)+ta
n(f*x+e))/f/(1+2^(1/2))^(1/2)+1/2*arctan(((2+2*2^(1/2))^(1/2)-2*(1+tan(f*x+
e))^(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^(1/2))^(1/2)/f-1/2*arctan(((2+2*2^(1/
2))^(1/2)+2*(1+tan(f*x+e))^(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^(1/2))^(1/2)/f
+2*(1+tan(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.14, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3624, 3566, 722, 1108, 648, 632, 210, 642}

$$\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\tan(e+fx)+1}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{2f}-\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{2\sqrt{\tan(e+fx)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{2f}+\frac{2\sqrt{\tan(e+fx)+1}}{f}+\frac{\log\left(\tan(e+fx)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\tan(e+fx)+1}+\sqrt{2}+1\right)}{4\sqrt{1+\sqrt{2}}\,f}-\frac{\log\left(\tan(e+fx)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\tan(e+fx)+1}+\sqrt{2}+1\right)}{4\sqrt{1+\sqrt{2}}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] (Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + Tan[e + f*x]]
)/Sqrt[2*(-1 + Sqrt[2])]])/(2*f) - (Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + S
qrt[2])] + 2*Sqrt[1 + Tan[e + f*x]])/Sqrt[2*(-1 + Sqrt[2])]])/(2*f) + Log[1
 + Sqrt[2] + Tan[e + f*x] - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + Tan[e + f*x]]]/(
4*Sqrt[1 + Sqrt[2]]*f) - Log[1 + Sqrt[2] + Tan[e + f*x] + Sqrt[2*(1 + Sqrt[
2])]*Sqrt[1 + Tan[e + f*x]]]/(4*Sqrt[1 + Sqrt[2]]*f) + (2*Sqrt[1 + Tan[e +
f*x]])/f
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 722

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx = \frac{2\sqrt{1+\tan(e+fx)}}{f} - \int \frac{1}{\sqrt{1+\tan(e+fx)}}\,dx$$

$$= \frac{2\sqrt{1+\tan(e+fx)}}{f} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x}\,(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{2\sqrt{1+\tan(e+fx)}}{f} - \frac{2\text{Subst}\left(\int \frac{1}{2-2x^2+x^4}\,dx, x, \sqrt{1+\tan(e+fx)}\right)}{f}$$

$$= \frac{2\sqrt{1+\tan(e+fx)}}{f} - \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-x}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2}\,dx, x, \sqrt{1+\tan(e\ldots}\right.}{2\sqrt{1+\sqrt{2}}\,f}$$

$$= \frac{2\sqrt{1+\tan(e+fx)}}{f} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2}\,dx, x, \sqrt{1+\tan(e\ldots}\right.}{2\sqrt{2}\,f}$$

$$= \frac{\log\left(1+\sqrt{2}+\tan(e+fx)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\tan(e+fx)}\right)}{4\sqrt{1+\sqrt{2}}\,f} - \frac{\log\left(1\ldots\right.}{\ldots}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\tan(e+fx)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2\sqrt{-1+\sqrt{2}}\,f} - \frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\ldots}{\sqrt{2\left(-1\ldots\right.}}\right.}{2\sqrt{-1+\ldots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.11, size = 80, normalized size = 0.31

$$-\frac{(1-i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1-i}}\right)+(1+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1+i}}\right)-4\sqrt{1+\tan(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/Sqrt[1 + Tan[e + f*x]],x]
```

[Out] -1/2*((1 - I)^(3/2)*ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1 - I]] + (1 + I)^(
3/2)*ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1 + I]] - 4*Sqrt[1 + Tan[e + f*x]]
)/f

Maple [A]
time = 0.12, size = 315, normalized size = 1.23

method	result
derivativedivides	$2\sqrt{1+\tan(fx+e)}+\frac{\left(-\sqrt{2\sqrt{2}+2}\,\sqrt{2}+2\sqrt{2\sqrt{2}+2}\right)\ln\left(1+\sqrt{2}-\sqrt{2\sqrt{2}+2}\,\sqrt{1+t\right.}{8}$
default	$2\sqrt{1+\tan(fx+e)}+\frac{\left(-\sqrt{2\sqrt{2}+2}\,\sqrt{2}+2\sqrt{2\sqrt{2}+2}\right)\ln\left(1+\sqrt{2}-\sqrt{2\sqrt{2}+2}\,\sqrt{1+t\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(f*x+e)^2/(1+tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/f*(2*(1+tan(f*x+e))^(1/2)+1/8*(-(2*2^(1/2)+2)^(1/2)*2^(1/2)+2*(2*2^(1/2)+
2)^(1/2))*ln(1+2^(1/2)-(2*2^(1/2)+2)^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))
+1/2*(-2*2^(1/2)+1/2*(-(2*2^(1/2)+2)^(1/2)*2^(1/2)+2*(2*2^(1/2)+2)^(1/2))*(
2*2^(1/2)+2)^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+tan(f*x+e))^(1/2)-(2*
2^(1/2)+2)^(1/2))/(-2+2*2^(1/2))^(1/2))-1/8*(-(2*2^(1/2)+2)^(1/2)*2^(1/2)+2
*(2*2^(1/2)+2)^(1/2))*ln(1+2^(1/2)+(2*2^(1/2)+2)^(1/2)*(1+tan(f*x+e))^(1/2)
+tan(f*x+e))-1/2*(2*2^(1/2)-1/2*(-(2*2^(1/2)+2)^(1/2)*2^(1/2)+2*(2*2^(1/2)+
2)^(1/2))*(2*2^(1/2)+2)^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan(((2*2^(1/2)+2)^(
1/2)+2*(1+tan(f*x+e))^(1/2))/(-2+2*2^(1/2))^(1/2)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^2/(1+tan(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(tan(f*x + e)^2/sqrt(tan(f*x + e) + 1), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 805 vs. 2(202) = 404.
time = 1.04, size = 805, normalized size = 3.13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(1+tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*(1/2)^(3/4)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)*f*(f^(-4))^(1/4)*ar
ctan(2*(1/2)^(1/4)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)*f^3*sqrt((2*sqrt(1/
2)*f^2*sqrt(f^(-4))*cos(f*x + e) + 2*(1/2)^(1/4)*sqrt(sqrt(1/2)*f^2*sqrt(f^
(-4)) + 1)*f*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4
)*cos(f*x + e) + cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4)
- 2*(1/2)^(1/4)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)*f^3*sqrt((cos(f*x + e)
 + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) - f^2*sqrt(f^(-4)) - 2*sqrt(1
/2)) + 4*(1/2)^(3/4)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)*f*(f^(-4))^(1/4)*
arctan(2*(1/2)^(1/4)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)*f^3*sqrt((2*sqrt(
1/2)*f^2*sqrt(f^(-4))*cos(f*x + e) - 2*(1/2)^(1/4)*sqrt(sqrt(1/2)*f^2*sqrt(
f^(-4)) + 1)*f*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1
/4)*cos(f*x + e) + cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4
) - 2*(1/2)^(1/4)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)*f^3*sqrt((cos(f*x +
e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) + f^2*sqrt(f^(-4)) + 2*sqrt
(1/2)) + (1/2)^(1/4)*(sqrt(1/2)*f^3*sqrt(f^(-4)) - f)*sqrt(sqrt(1/2)*f^2*sq
rt(f^(-4)) + 1)*(f^(-4))^(1/4)*log((2*sqrt(1/2)*f^2*sqrt(f^(-4))*cos(f*x +
e) + 2*(1/2)^(1/4)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)*f*sqrt((cos(f*x + e
) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4)*cos(f*x + e) + cos(f*x + e)
+ sin(f*x + e))/cos(f*x + e)) - (1/2)^(1/4)*(sqrt(1/2)*f^3*sqrt(f^(-4)) - f
)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)*(f^(-4))^(1/4)*log((2*sqrt(1/2)*f^2*
sqrt(f^(-4))*cos(f*x + e) - 2*(1/2)^(1/4)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4)) +
 1)*f*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4)*cos(f
*x + e) + cos(f*x + e) + sin(f*x + e))/cos(f*x + e)) + 4*sqrt((cos(f*x + e)
 + sin(f*x + e))/cos(f*x + e)))/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(e+fx)}{\sqrt{\tan(e+fx)+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(1+tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/sqrt(tan(e + f*x) + 1), x)
```

Giac [A]
time = 0.68, size = 216, normalized size = 0.84

$$-\frac{\sqrt{\sqrt{2}+1}\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{2f}-\frac{\sqrt{\sqrt{2}+1}\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{2f}-\frac{\sqrt{\sqrt{2}-1}\log\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{4f}+\frac{\sqrt{\sqrt{2}-1}\log\left(-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{4f}+\frac{2\sqrt{\tan(fx+e)+1}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(1+tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(sqrt(2) + 1)*arctan(1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*sq
rt(tan(f*x + e) + 1))/sqrt(-sqrt(2) + 2))/f - 1/2*sqrt(sqrt(2) + 1)*arctan(
-1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(tan(f*x + e) + 1))/sqrt(-s
qrt(2) + 2))/f - 1/4*sqrt(sqrt(2) - 1)*log(2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(t
an(f*x + e) + 1) + sqrt(2) + tan(f*x + e) + 1)/f + 1/4*sqrt(sqrt(2) - 1)*lo
g(-2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x + e) + 1) + sqrt(2) + tan(f*x + e
) + 1)/f + 2*sqrt(tan(f*x + e) + 1)/f
```

Mupad [B]
time = 0.28, size = 86, normalized size = 0.33

$$\frac{2\,\sqrt{\tan\left(e+f\,x\right)+1}}{f}-\mathrm{atan}\left(2\,f\,\sqrt{\frac{-\frac{1}{8}-\frac{1}{8}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\right)\,\sqrt{\frac{-\frac{1}{8}-\frac{1}{8}\mathrm{i}}{f^2}}\,2\mathrm{i}+\mathrm{atan}\left(2\,f\,\sqrt{\frac{-\frac{1}{8}+\frac{1}{8}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\right)\,\sqrt{\frac{-\frac{1}{8}+\frac{1}{8}\mathrm{i}}{f^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(tan(e + f*x) + 1)^(1/2),x)
```

```
[Out] (2*(tan(e + f*x) + 1)^(1/2))/f - atan(2*f*((- 1/8 - 1i/8)/f^2)^(1/2)*(tan(e
 + f*x) + 1)^(1/2))*((- 1/8 - 1i/8)/f^2)^(1/2)*2i + atan(2*f*((- 1/8 + 1i/8
)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1/2))*((- 1/8 + 1i/8)/f^2)^(1/2)*2i
```

3.408 $\int \frac{1}{\sqrt{1+\tan(e+fx)}}\,dx$

Optimal. Leaf size=240

$$-\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\tan(e+fx)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2f}+\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{\cdots}}{\sqrt{2\left(-1+\cdots\right.}}\right.}{2f}$$

```
[Out] -1/4*ln(1+2^(1/2)-(2+2*2^(1/2))^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))/f/(1
+2^(1/2))^(1/2)+1/4*ln(1+2^(1/2)+(2+2*2^(1/2))^(1/2)*(1+tan(f*x+e))^(1/2)+t
an(f*x+e))/f/(1+2^(1/2))^(1/2)-1/2*arctan(((2+2*2^(1/2))^(1/2)-2*(1+tan(f*x
+e))^(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^(1/2))^(1/2)/f+1/2*arctan(((2+2*2^(1
/2))^(1/2)+2*(1+tan(f*x+e))^(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^(1/2))^(1/2)/
f
```

Rubi [A]
time = 0.12, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3566, 722, 1108, 648, 632, 210, 642}

$$-\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\tan(e+fx)+1}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{2f}+\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{2\sqrt{\tan(e+fx)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{2f}-\frac{\log\left(\tan(e+fx)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\tan(e+fx)+1}+\sqrt{2}+1\right)}{4\sqrt{1+\sqrt{2}}\,f}+\frac{\log\left(\tan(e+fx)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\tan(e+fx)+1}+\sqrt{2}+1\right)}{4\sqrt{1+\sqrt{2}}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] -1/2*(Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + Tan[e +
f*x]])/Sqrt[2*(-1 + Sqrt[2])]])/f + (Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 +
Sqrt[2])] + 2*Sqrt[1 + Tan[e + f*x]])/Sqrt[2*(-1 + Sqrt[2])]])/(2*f) - Log[
1 + Sqrt[2] + Tan[e + f*x] - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + Tan[e + f*x]]]/
(4*Sqrt[1 + Sqrt[2]]*f) + Log[1 + Sqrt[2] + Tan[e + f*x] + Sqrt[2*(1 + Sqrt
[2])]*Sqrt[1 + Tan[e + f*x]]]/(4*Sqrt[1 + Sqrt[2]]*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 722

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+\tan(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x}\,(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{2\text{Subst}\left(\int \frac{1}{2-2x^2+x^4}\,dx, x, \sqrt{1+\tan(e+fx)}\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-x}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2}\,dx, x, \sqrt{1+\tan(e+fx)}\right)}{2\sqrt{1+\sqrt{2}}\,f} + \frac{\text{Subst}\left(\int -\right.}{}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2}\,dx, x, \sqrt{1+\tan(e+fx)}\right)}{2\sqrt{2}\,f} + \frac{\text{Subst}\left(\int -\right.}{}$$

$$= -\frac{\log\left(1+\sqrt{2}+\tan(e+fx)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\tan(e+fx)}\right)}{4\sqrt{1+\sqrt{2}}\,f} + \frac{\log}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\tan(e+fx)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2\sqrt{-1+\sqrt{2}}\,f} + \frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(-\right.}}\right.}{2\sqrt{-1+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.03, size = 66, normalized size = 0.28

$$\frac{(1-i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1-i}}\right)+(1+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1+i}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] ((1 - I)^(3/2)*ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1 - I]] + (1 + I)^(3/2)*
ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sqrt[1 + I]])/(2*f)
```

Maple [A]
time = 0.14, size = 303, normalized size = 1.26

method	result
derivativedivides	$-\frac{\frac{\left(-\sqrt{2\sqrt{2}+2}\,\sqrt{2}+2\sqrt{2\sqrt{2}+2}\right)\ln\left(1+\sqrt{2}-\sqrt{2\sqrt{2}+2}\,\sqrt{1+\tan\left(fx+e\right)}+\tan(fx+e)\right.}{8}}{}$
default	$-\frac{\frac{\left(-\sqrt{2\sqrt{2}+2}\,\sqrt{2}+2\sqrt{2\sqrt{2}+2}\right)\ln\left(1+\sqrt{2}-\sqrt{2\sqrt{2}+2}\,\sqrt{1+\tan\left(fx+e\right)}+\tan(fx+e)\right.}{8}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/8*(-(2*2^(1/2)+2)^(1/2)*2^(1/2)+2*(2*2^(1/2)+2)^(1/2))*ln(1+2^(1/2)
-(2*2^(1/2)+2)^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))-1/2*(-2*2^(1/2)+1/2*(
-(2*2^(1/2)+2)^(1/2)*2^(1/2)+2*(2*2^(1/2)+2)^(1/2))*(2*2^(1/2)+2)^(1/2))/(-
2+2*2^(1/2))^(1/2)*arctan((2*(1+tan(f*x+e))^(1/2)-(2*2^(1/2)+2)^(1/2))/(-2+
2*2^(1/2))^(1/2))+1/8*(-(2*2^(1/2)+2)^(1/2)*2^(1/2)+2*(2*2^(1/2)+2)^(1/2))*
ln(1+2^(1/2)+(2*2^(1/2)+2)^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))+1/2*(2*2^
(1/2)-1/2*(-(2*2^(1/2)+2)^(1/2)*2^(1/2)+2*(2*2^(1/2)+2)^(1/2))*(2*2^(1/2)+2
)^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan(((2*2^(1/2)+2)^(1/2)+2*(1+tan(f*x+e))^
(1/2))/(-2+2*2^(1/2))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  1which is not of the expected type LIST
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 766 vs. 2(186) = 372.
time = 0.92, size = 766, normalized size = 3.19

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(1/2)^(1/4)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)*(sqrt(1/2)*f^2*sqrt(f
^(-4)) - 1)*(f^(-4))^(1/4)*log((2*sqrt(1/2)*f^2*sqrt(f^(-4))*cos(f*x + e) +
 2*(1/2)^(1/4)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)*f*sqrt((cos(f*x + e) +
sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4)*cos(f*x + e) + cos(f*x + e) + si
n(f*x + e))/cos(f*x + e)) + 1/2*(1/2)^(1/4)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4))
 + 1)*(sqrt(1/2)*f^2*sqrt(f^(-4)) - 1)*(f^(-4))^(1/4)*log((2*sqrt(1/2)*f^2*
sqrt(f^(-4))*cos(f*x + e) - 2*(1/2)^(1/4)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4)) +
 1)*f*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4)*cos(f
*x + e) + cos(f*x + e) + sin(f*x + e))/cos(f*x + e)) - 2*(1/2)^(3/4)*sqrt(s
qrt(1/2)*f^2*sqrt(f^(-4)) + 1)*(f^(-4))^(1/4)*arctan(2*(1/2)^(1/4)*sqrt(sqr
t(1/2)*f^2*sqrt(f^(-4)) + 1)*f^3*sqrt((2*sqrt(1/2)*f^2*sqrt(f^(-4))*cos(f*x
 + e) + 2*(1/2)^(1/4)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)*f*sqrt((cos(f*x
+ e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4)*cos(f*x + e) + cos(f*x +
e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) - 2*(1/2)^(1/4)*sqrt(sqrt(1
/2)*f^2*sqrt(f^(-4)) + 1)*f^3*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x +
e))*(f^(-4))^(3/4) - f^2*sqrt(f^(-4)) - 2*sqrt(1/2)) - 2*(1/2)^(3/4)*sqrt(s
qrt(1/2)*f^2*sqrt(f^(-4)) + 1)*(f^(-4))^(1/4)*arctan(2*(1/2)^(1/4)*sqrt(sqr
t(1/2)*f^2*sqrt(f^(-4)) + 1)*f^3*sqrt((2*sqrt(1/2)*f^2*sqrt(f^(-4))*cos(f*x
 + e) - 2*(1/2)^(1/4)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)*f*sqrt((cos(f*x
+ e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4)*cos(f*x + e) + cos(f*x +
e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) - 2*(1/2)^(1/4)*sqrt(sqrt(1
/2)*f^2*sqrt(f^(-4)) + 1)*f^3*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x +
e))*(f^(-4))^(3/4) + f^2*sqrt(f^(-4)) + 2*sqrt(1/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\tan\left(e+fx\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/sqrt(tan(e + f*x) + 1), x)
```

Giac [A]
time = 0.75, size = 282, normalized size = 1.18

$$\frac{\left(f^2\sqrt{2\sqrt{2}-2}+f\sqrt{2\sqrt{2}+2}\,|f|\right)\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{\tan\left(fx+e\right)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{4f^3}+\frac{\left(f^2\sqrt{2\sqrt{2}-2}+f\sqrt{2\sqrt{2}+2}\,|f|\right)\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{\tan\left(fx+e\right)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{4f^3}+\frac{\left(f^2\sqrt{2\sqrt{2}+2}-f\sqrt{2\sqrt{2}-2}\,|f|\right)\log\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\,\sqrt{\tan\left(fx+e\right)+1}+\sqrt{2}+\tan\left(fx+e\right)+1\right)}{8f^3}-\frac{\left(f^2\sqrt{2\sqrt{2}+2}-f\sqrt{2\sqrt{2}-2}\,|f|\right)\log\left(-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\,\sqrt{\tan\left(fx+e\right)+1}+\sqrt{2}+\tan\left(fx+e\right)+1\right)}{8f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(f^2*sqrt(2*sqrt(2) - 2) + f*sqrt(2*sqrt(2) + 2)*abs(f))*arctan(1/2*2^(
3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*sqrt(tan(f*x + e) + 1))/sqrt(-sqrt(2) +
```

```
 2))/f^3 + 1/4*(f^2*sqrt(2*sqrt(2) - 2) + f*sqrt(2*sqrt(2) + 2)*abs(f))*arc
tan(-1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(tan(f*x + e) + 1))/sqr
t(-sqrt(2) + 2))/f^3 + 1/8*(f^2*sqrt(2*sqrt(2) + 2) - f*sqrt(2*sqrt(2) - 2)
*abs(f))*log(2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x + e) + 1) + sqrt(2) + t
an(f*x + e) + 1)/f^3 - 1/8*(f^2*sqrt(2*sqrt(2) + 2) - f*sqrt(2*sqrt(2) - 2)
*abs(f))*log(-2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x + e) + 1) + sqrt(2) +
tan(f*x + e) + 1)/f^3
```

Mupad [B]
time = 4.21, size = 71, normalized size = 0.30

$$\mathrm{atan}\left(2\,f\,\sqrt{\frac{-\frac{1}{8}-\frac{1}{8}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\right)\,\sqrt{\frac{-\frac{1}{8}-\frac{1}{8}\mathrm{i}}{f^2}}\,2\mathrm{i}-\mathrm{atan}\left(2\,f\,\sqrt{\frac{-\frac{1}{8}+\frac{1}{8}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\right)\,\sqrt{\frac{-\frac{1}{8}+\frac{1}{8}\mathrm{i}}{f^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(e + f*x) + 1)^(1/2),x)
```

```
[Out] atan(2*f*((- 1/8 - 1i/8)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1/2))*((- 1/8 - 1i/
8)/f^2)^(1/2)*2i - atan(2*f*((- 1/8 + 1i/8)/f^2)^(1/2)*(tan(e + f*x) + 1)^(
1/2))*((- 1/8 + 1i/8)/f^2)^(1/2)*2i
```

3.409 $\int \frac{\cot^2(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx$

Optimal. Leaf size=280

$$\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\tan(e+fx)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2f}-\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+\tan(e+fx)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2f}$$

```
[Out] arctanh((1+tan(f*x+e))^(1/2))/f+1/4*ln(1+2^(1/2)-(2+2*2^(1/2))^(1/2)*(1+tan
(f*x+e))^(1/2)+tan(f*x+e))/f/(1+2^(1/2))^(1/2)-1/4*ln(1+2^(1/2)+(2+2*2^(1/2
))^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))/f/(1+2^(1/2))^(1/2)+1/2*arctan(((
2+2*2^(1/2))^(1/2)-2*(1+tan(f*x+e))^(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^(1/2)
)^(1/2)/f-1/2*arctan(((2+2*2^(1/2))^(1/2)+2*(1+tan(f*x+e))^(1/2))/(-2+2*2^(
1/2))^(1/2))*(1+2^(1/2))^(1/2)/f-cot(f*x+e)*(1+tan(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.27, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3650, 3713, 21, 3654, 12, 3566, 722, 1108, 648, 632, 210, 642, 3715, 65, 213}

$$\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\tan(e+fx)+1}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{2f}-\frac{\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{2\sqrt{\tan(e+fx)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{2f}+\frac{\log\left(\tan(e+fx)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\tan(e+fx)+1}+\sqrt{2}+1\right)}{4\sqrt{1+\sqrt{2}}\,f}-\frac{\log\left(\tan(e+fx)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\tan(e+fx)+1}+\sqrt{2}+1\right)}{4\sqrt{1+\sqrt{2}}\,f}+\frac{\tanh^{-1}\left(\sqrt{\tan(e+fx)+1}\right)}{f}-\frac{\sqrt{\tan(e+fx)+1}\cot(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] (Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + Tan[e + f*x]]
)/Sqrt[2*(-1 + Sqrt[2])]])/(2*f) - (Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + S
qrt[2])] + 2*Sqrt[1 + Tan[e + f*x]])/Sqrt[2*(-1 + Sqrt[2])]])/(2*f) + ArcTa
nh[Sqrt[1 + Tan[e + f*x]]]/f + Log[1 + Sqrt[2] + Tan[e + f*x] - Sqrt[2*(1 +
 Sqrt[2])]*Sqrt[1 + Tan[e + f*x]]]/(4*Sqrt[1 + Sqrt[2]]*f) - Log[1 + Sqrt[2
] + Tan[e + f*x] + Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + Tan[e + f*x]]]/(4*Sqrt[1
+ Sqrt[2]]*f) - (Cot[e + f*x]*Sqrt[1 + Tan[e + f*x]])/f
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 722

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
```

```
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3654

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[Simp[a^2*c - b^2*c + 2
*a*b*d + (2*a*b*c - a^2*d + b^2*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x]
], x], x] + Dist[(b*c - a*d)^2/(c^2 + d^2), Int[(1 + Tan[e + f*x]^2)/(Sqrt[
a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx = -\frac{\cot(e+fx)\sqrt{1+\tan(e+fx)}}{f} - \int \frac{\cot(e+fx)\left(\frac{1}{2}+\tan(e+fx)+\frac{1}{2}\tan^2(\right.}{\sqrt{1+\tan(e+fx)}}$$

$$= -\frac{\cot(e+fx)\sqrt{1+\tan(e+fx)}}{f} - \int \cot(e+fx)\left(\frac{1}{2}+\frac{1}{2}\tan(e+fx)\right)\sqrt{1}$$

$$= -\frac{\cot(e+fx)\sqrt{1+\tan(e+fx)}}{f} - \frac{1}{2}\int \cot(e+fx)(1+\tan(e+fx))^{3/2}\,dx$$

$$= -\frac{\cot(e+fx)\sqrt{1+\tan(e+fx)}}{f} - \frac{1}{2}\int \frac{2}{\sqrt{1+\tan(e+fx)}}\,dx - \frac{1}{2}\int \frac{\cot(e}{}$$

$$= -\frac{\cot(e+fx)\sqrt{1+\tan(e+fx)}}{f} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \tan(e+fx)\right)}{2f} -$$

$$= -\frac{\cot(e+fx)\sqrt{1+\tan(e+fx)}}{f} - \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\tan(e+fx)}\right)}{f}$$

$$= \frac{\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{f} - \frac{\cot(e+fx)\sqrt{1+\tan(e+fx)}}{f} - \frac{2\text{Subst}\left(\int\right.}{}$$

$$= \frac{\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{f} - \frac{\cot(e+fx)\sqrt{1+\tan(e+fx)}}{f} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{f} - \frac{\cot(e+fx)\sqrt{1+\tan(e+fx)}}{f} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{f} + \frac{\log\left(1+\sqrt{2}+\tan(e+fx)-\sqrt{2\left(1+\sqrt{2}\right.}\right.}{4\sqrt{1+\sqrt{2}}\,f}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\tan(e+fx)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2\sqrt{-1+\sqrt{2}}\,f} - \frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2}{\sqrt{2\left(-1\right.}}\right.}{2\sqrt{-1+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.26, size = 101, normalized size = 0.36

$$-\frac{-2\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)+(1-i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1-i}}\right)+(1+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+\tan(e+fx)}}{\sqrt{1+i}}\right)+2\cot(e+fx)\sqrt{1+\tan(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] -1/2*(-2*ArcTanh[Sqrt[1 + Tan[e + f*x]]] + (1 - I)^(3/2)*ArcTanh[Sqrt[1 + T
an[e + f*x]]/Sqrt[1 - I]] + (1 + I)^(3/2)*ArcTanh[Sqrt[1 + Tan[e + f*x]]/Sq
rt[1 + I]] + 2*Cot[e + f*x]*Sqrt[1 + Tan[e + f*x]])/f
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.65, size = 7175, normalized size = 25.62

method	result	size
default	Expression too large to display	7175

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(1+tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(1+tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)^2/sqrt(tan(f*x + e) + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1016 vs. 2(225) = 450.
time = 1.00, size = 1016, normalized size = 3.63

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(1+tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((1/2)^(1/4)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)*(f*cos(f*x + e)^2 -
sqrt(1/2)*(f^3*cos(f*x + e)^2 - f^3)*sqrt(f^(-4)) - f)*(f^(-4))^(1/4)*log((
2*sqrt(1/2)*f^2*sqrt(f^(-4))*cos(f*x + e) + 2*(1/2)^(1/4)*sqrt(sqrt(1/2)*f^
```

```
2*sqrt(f^(-4)) + 1)*f*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(
-4))^(1/4)*cos(f*x + e) + cos(f*x + e) + sin(f*x + e))/cos(f*x + e)) - (1/2
)^(1/4)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)*(f*cos(f*x + e)^2 - sqrt(1/2)*
(f^3*cos(f*x + e)^2 - f^3)*sqrt(f^(-4)) - f)*(f^(-4))^(1/4)*log((2*sqrt(1/2
)*f^2*sqrt(f^(-4))*cos(f*x + e) - 2*(1/2)^(1/4)*sqrt(sqrt(1/2)*f^2*sqrt(f^(
-4)) + 1)*f*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4)
*cos(f*x + e) + cos(f*x + e) + sin(f*x + e))/cos(f*x + e)) - 2*sqrt((cos(f*
x + e) + sin(f*x + e))/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - (cos(f*x +
 e)^2 - 1)*log(sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e)) + 1) + (cos
(f*x + e)^2 - 1)*log(sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e)) - 1)
- 4*(1/2)^(3/4)*(f^5*cos(f*x + e)^2 - f^5)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4))
+ 1)*(f^(-4))^(1/4)*arctan(2*(1/2)^(1/4)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4)) +
1)*f^3*sqrt((2*sqrt(1/2)*f^2*sqrt(f^(-4))*cos(f*x + e) + 2*(1/2)^(1/4)*sqrt
(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)*f*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f
*x + e))*(f^(-4))^(1/4)*cos(f*x + e) + cos(f*x + e) + sin(f*x + e))/cos(f*x
 + e))*(f^(-4))^(3/4) - 2*(1/2)^(1/4)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)*
f^3*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) - f^2*s
qrt(f^(-4)) - 2*sqrt(1/2))/f^4 - 4*(1/2)^(3/4)*(f^5*cos(f*x + e)^2 - f^5)*s
qrt(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)*(f^(-4))^(1/4)*arctan(2*(1/2)^(1/4)*sqr
t(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)*f^3*sqrt((2*sqrt(1/2)*f^2*sqrt(f^(-4))*co
s(f*x + e) - 2*(1/2)^(1/4)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)*f*sqrt((cos
(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4)*cos(f*x + e) + cos(f
*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) - 2*(1/2)^(1/4)*sqrt(s
qrt(1/2)*f^2*sqrt(f^(-4)) + 1)*f^3*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f
*x + e))*(f^(-4))^(3/4) + f^2*sqrt(f^(-4)) + 2*sqrt(1/2))/f^4)/(f*cos(f*x +
 e)^2 - f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(e+fx)}{\sqrt{\tan(e+fx)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(1+tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/sqrt(tan(e + f*x) + 1), x)
```

Giac [**A**]
time = 0.86, size = 346, normalized size = 1.24

$$\frac{\log\left(\sqrt{\tan(fx+e)+1}+1\right)}{2f} - \frac{\log\left(\left|\sqrt{\tan(fx+e)+1}-1\right|\right)}{2f} - \frac{\left(f^2\sqrt{2\sqrt{2}-2}+f\sqrt{2\sqrt{2}+2}\,|f|\right)\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{4f^3} - \frac{\left(f^2\sqrt{2\sqrt{2}-2}+f\sqrt{2\sqrt{2}+2}\,|f|\right)\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{4f^3} - \frac{\left(f^2\sqrt{2\sqrt{2}+2}-f\sqrt{2\sqrt{2}-2}\,|f|\right)\log\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{8f^3} + \frac{\left(f^2\sqrt{2\sqrt{2}+2}-f\sqrt{2\sqrt{2}-2}\,|f|\right)\log\left(-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{8f^3} - \frac{\sqrt{\tan(fx+e)+1}}{f\tan(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(1+tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*log(sqrt(tan(f*x + e) + 1) + 1)/f - 1/2*log(abs(sqrt(tan(f*x + e) + 1)
- 1))/f - 1/4*(f^2*sqrt(2*sqrt(2) - 2) + f*sqrt(2*sqrt(2) + 2)*abs(f))*arct
an(1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*sqrt(tan(f*x + e) + 1))/sqrt(
-sqrt(2) + 2))/f^3 - 1/4*(f^2*sqrt(2*sqrt(2) - 2) + f*sqrt(2*sqrt(2) + 2)*a
bs(f))*arctan(-1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(tan(f*x + e)
 + 1))/sqrt(-sqrt(2) + 2))/f^3 - 1/8*(f^2*sqrt(2*sqrt(2) + 2) - f*sqrt(2*sq
rt(2) - 2)*abs(f))*log(2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x + e) + 1) + s
qrt(2) + tan(f*x + e) + 1)/f^3 + 1/8*(f^2*sqrt(2*sqrt(2) + 2) - f*sqrt(2*sq
rt(2) - 2)*abs(f))*log(-2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x + e) + 1) +
sqrt(2) + tan(f*x + e) + 1)/f^3 - sqrt(tan(f*x + e) + 1)/(f*tan(f*x + e))
```

Mupad [B]
time = 0.17, size = 117, normalized size = 0.42

$$\frac{\sqrt{\tan\left(e+f\,x\right)+1}}{f-f\,\left(\tan\left(e+f\,x\right)+1\right)}-\frac{\mathrm{atan}\left(\sqrt{\tan\left(e+f\,x\right)+1}\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{f}-\mathrm{atan}\left(2\,f\,\sqrt{\frac{-\frac{1}{8}-\frac{1}{8}{}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\right)\,\sqrt{\frac{-\frac{1}{8}-\frac{1}{8}{}\mathrm{i}}{f^2}}\,2{}\mathrm{i}+\mathrm{atan}\left(2\,f\,\sqrt{\frac{-\frac{1}{8}+\frac{1}{8}{}\mathrm{i}}{f^2}}\,\sqrt{\tan\left(e+f\,x\right)+1}\right)\,\sqrt{\frac{-\frac{1}{8}+\frac{1}{8}{}\mathrm{i}}{f^2}}\,2{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(tan(e + f*x) + 1)^(1/2),x)
```

```
[Out] (tan(e + f*x) + 1)^(1/2)/(f - f*(tan(e + f*x) + 1)) - (atan((tan(e + f*x) +
 1)^(1/2)*1i)*1i)/f - atan(2*f*((- 1/8 - 1i/8)/f^2)^(1/2)*(tan(e + f*x) + 1
)^(1/2))*((- 1/8 - 1i/8)/f^2)^(1/2)*2i + atan(2*f*((- 1/8 + 1i/8)/f^2)^(1/2
)*(tan(e + f*x) + 1)^(1/2))*((- 1/8 + 1i/8)/f^2)^(1/2)*2i
```

3.410 $\int \frac{\cot^4(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx$

Optimal. Leaf size=339

$$-\frac{\sqrt{1+\sqrt{2}}\,\mathrm{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\tan(e+fx)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2f}+\frac{\sqrt{1+\sqrt{2}}\,\mathrm{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{\ldots}}{\sqrt{2\left(-1\ldots\right.}}\right.}{2f}$$

```
[Out] -3/8*arctanh((1+tan(f*x+e))^(1/2))/f-1/4*ln(1+2^(1/2)-(2+2*2^(1/2))^(1/2)*(
1+tan(f*x+e))^(1/2)+tan(f*x+e))/f/(1+2^(1/2))^(1/2)+1/4*ln(1+2^(1/2)+(2+2*2
^(1/2))^(1/2)*(1+tan(f*x+e))^(1/2)+tan(f*x+e))/f/(1+2^(1/2))^(1/2)-1/2*arct
an(((2+2*2^(1/2))^(1/2)-2*(1+tan(f*x+e))^(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^
(1/2))^(1/2)/f+1/2*arctan(((2+2*2^(1/2))^(1/2)+2*(1+tan(f*x+e))^(1/2))/(-2+
2*2^(1/2))^(1/2))*(1+2^(1/2))^(1/2)/f+3/8*cot(f*x+e)*(1+tan(f*x+e))^(1/2)/f
+5/12*cot(f*x+e)^2*(1+tan(f*x+e))^(1/2)/f-1/3*cot(f*x+e)^3*(1+tan(f*x+e))^(
1/2)/f
```

Rubi [A]
time = 0.39, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3650, 3730, 3731, 3734, 12, 3566, 722, 1108, 648, 632, 210, 642, 3715, 65, 213}

$$-\frac{\sqrt{1+\sqrt{2}}\,\mathrm{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\tan(e+fx)+1}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{2f}+\frac{\sqrt{1+\sqrt{2}}\,\mathrm{ArcTan}\left(\frac{2\sqrt{\tan(e+fx)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{2f}-\frac{\log\left(\tan(e+fx)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\tan(e+fx)+1}+\sqrt{2}+1\right)}{4\sqrt{1+\sqrt{2}}\,f}+\frac{\log\left(\tan(e+fx)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\tan(e+fx)+1}+\sqrt{2}+1\right)}{4\sqrt{1+\sqrt{2}}\,f}-\frac{3\tanh^{-1}\left(\sqrt{\tan(e+fx)+1}\right)}{8f}-\frac{\sqrt{\tan(e+fx)+1}\cot^3(e+fx)}{3f}+\frac{5\sqrt{\tan(e+fx)+1}\cot^2(e+fx)}{12f}+\frac{3\sqrt{\tan(e+fx)+1}\cot(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] -1/2*(Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + Tan[e +
f*x]])/Sqrt[2*(-1 + Sqrt[2])]])/f + (Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 +
Sqrt[2])] + 2*Sqrt[1 + Tan[e + f*x]])/Sqrt[2*(-1 + Sqrt[2])]])/(2*f) - (3*A
rcTanh[Sqrt[1 + Tan[e + f*x]]])/(8*f) - Log[1 + Sqrt[2] + Tan[e + f*x] - Sq
rt[2*(1 + Sqrt[2])]*Sqrt[1 + Tan[e + f*x]]]/(4*Sqrt[1 + Sqrt[2]]*f) + Log[1
 + Sqrt[2] + Tan[e + f*x] + Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + Tan[e + f*x]]]/(
4*Sqrt[1 + Sqrt[2]]*f) + (3*Cot[e + f*x]*Sqrt[1 + Tan[e + f*x]])/(8*f) + (5
*Cot[e + f*x]^2*Sqrt[1 + Tan[e + f*x]])/(12*f) - (Cot[e + f*x]^3*Sqrt[1 + T
an[e + f*x]])/(3*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 722

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3731

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)
*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp
[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - a*C*(b*c*(m + 1) + a*d*(n
+ 1)) - (m + 1)*(b*c - a*d)*(A*b - b*C)*Tan[e + f*x] - d*(A*b^2 + a^2*C)*(m
 + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m,
 -1] &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\sqrt{1+\tan(e+fx)}}\,dx = -\frac{\cot^3(e+fx)\sqrt{1+\tan(e+fx)}}{3f} - \frac{1}{3}\int \frac{\cot^3(e+fx)\left(\frac{5}{2}+3\tan(e+fx)+\frac{5}{2}\right.}{\sqrt{1+\tan(e+fx)}}$$

$$= \frac{5\cot^2(e+fx)\sqrt{1+\tan(e+fx)}}{12f} - \frac{\cot^3(e+fx)\sqrt{1+\tan(e+fx)}}{3f} + \frac{1}{6}\int$$

$$= \frac{3\cot(e+fx)\sqrt{1+\tan(e+fx)}}{8f} + \frac{5\cot^2(e+fx)\sqrt{1+\tan(e+fx)}}{12f} - \frac{\cot^3(}{}$$

$$= \frac{3\cot(e+fx)\sqrt{1+\tan(e+fx)}}{8f} + \frac{5\cot^2(e+fx)\sqrt{1+\tan(e+fx)}}{12f} - \frac{\cot^3(}{}$$

$$= \frac{3\cot(e+fx)\sqrt{1+\tan(e+fx)}}{8f} + \frac{5\cot^2(e+fx)\sqrt{1+\tan(e+fx)}}{12f} - \frac{\cot^3(}{}$$

$$= \frac{3\cot(e+fx)\sqrt{1+\tan(e+fx)}}{8f} + \frac{5\cot^2(e+fx)\sqrt{1+\tan(e+fx)}}{12f} - \frac{\cot^3(}{}$$

$$= -\frac{3\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{8f} + \frac{3\cot(e+fx)\sqrt{1+\tan(e+fx)}}{8f} + \frac{5\cot^2}{}$$

$$= -\frac{3\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{8f} + \frac{3\cot(e+fx)\sqrt{1+\tan(e+fx)}}{8f} + \frac{5\cot^2}{}$$

$$= -\frac{3\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{8f} + \frac{3\cot(e+fx)\sqrt{1+\tan(e+fx)}}{8f} + \frac{5\cot^2}{}$$

$$= -\frac{3\tanh^{-1}\left(\sqrt{1+\tan(e+fx)}\right)}{8f} - \frac{\log\left(1+\sqrt{2}+\tan(e+fx)-\sqrt{2\left(1+\right.}\right.}{4\sqrt{1+\sqrt{2}}\,f}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\tan(e+fx)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2\sqrt{-1+\sqrt{2}}\,f} + \frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(-\right.}}\right.}{2\sqrt{-1+}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 21.95, size = 4071, normalized size = 12.01

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^4/Sqrt[1 + Tan[e + f*x]],x]
```

```
[Out] ((-5/12 + (17*Cot[e + f*x])/24 + (5*Csc[e + f*x]^2)/12 - (Cot[e + f*x]*Csc[
e + f*x]^2)/3)*Sec[e + f*x]*(Cos[e + f*x] + Sin[e + f*x]))/(f*Sqrt[1 + Tan[
e + f*x]]) + ((3*EllipticF[ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1
+ Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqrt[2]] - 3*EllipticPi[-1
 - Sqrt[2], ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)
/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqrt[2]] - (16*I)*EllipticPi[(-I)*(1 + Sq
rt[2]), ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])
])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqrt[2]] + (16*I)*EllipticPi[I*(1 + Sqrt[2]),
 ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt
[2 + Sqrt[2]]], -3 - 2*Sqrt[2]] - 3*EllipticPi[1 + Sqrt[2], ArcSin[(2^(1/4)
*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]],
-3 - 2*Sqrt[2]])*Sec[e + f*x]*(Cos[e + f*x] + Sin[e + f*x])*((3*Csc[e + f*x
]*Sqrt[Sec[e + f*x]])/(16*Sqrt[Cos[e + f*x] + Sin[e + f*x]]) + (Csc[e + f*x
]*Sqrt[Sec[e + f*x]]*Sin[2*(e + f*x)])/(2*Sqrt[Cos[e + f*x] + Sin[e + f*x]]
))*Sqrt[-((1 + Tan[(e + f*x)/2])/((-2 + Sqrt[2])*(-1 + Tan[(e + f*x)/2])))]
)/(4*2^(1/4)*f*Sqrt[(Cos[e + f*x] + Sin[e + f*x])/(-1 + Sin[e + f*x])]*(((3
*EllipticF[ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/
2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqrt[2]] - 3*EllipticPi[-1 - Sqrt[2], ArcS
in[(2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 +
Sqrt[2]]], -3 - 2*Sqrt[2]] - (16*I)*EllipticPi[(-I)*(1 + Sqrt[2]), ArcSin[(
2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt
[2]]], -3 - 2*Sqrt[2]] + (16*I)*EllipticPi[I*(1 + Sqrt[2]), ArcSin[(2^(1/4)
*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]],
-3 - 2*Sqrt[2]] - 3*EllipticPi[1 + Sqrt[2], ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(
e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqrt[2]])
*Sqrt[Sec[e + f*x]]*(Cos[e + f*x] - Sin[e + f*x])*Sqrt[-((1 + Tan[(e + f*x)
/2])/((-2 + Sqrt[2])*(-1 + Tan[(e + f*x)/2])))])/(8*2^(1/4)*Sqrt[Cos[e + f*
x] + Sin[e + f*x]]*Sqrt[(Cos[e + f*x] + Sin[e + f*x])/(-1 + Sin[e + f*x])])
 + ((3*EllipticF[ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e +
 f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqrt[2]] - 3*EllipticPi[-1 - Sqrt[2]
, ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqr
t[2 + Sqrt[2]]], -3 - 2*Sqrt[2]] - (16*I)*EllipticPi[(-I)*(1 + Sqrt[2]), Ar
cSin[(2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2
+ Sqrt[2]]], -3 - 2*Sqrt[2]] + (16*I)*EllipticPi[I*(1 + Sqrt[2]), ArcSin[(2
^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[
2]]], -3 - 2*Sqrt[2]] - 3*EllipticPi[1 + Sqrt[2], ArcSin[(2^(1/4)*Sqrt[(1 +
```

```
 Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqr
t[2]])*Sec[e + f*x]^(3/2)*Sin[e + f*x]*Sqrt[Cos[e + f*x] + Sin[e + f*x]]*Sq
rt[-((1 + Tan[(e + f*x)/2])/((-2 + Sqrt[2])*(-1 + Tan[(e + f*x)/2])))])/(8*
2^(1/4)*Sqrt[(Cos[e + f*x] + Sin[e + f*x])/(-1 + Sin[e + f*x])]) - ((3*Elli
pticF[ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])
/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqrt[2]] - 3*EllipticPi[-1 - Sqrt[2], ArcSin[(2
^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[
2]]], -3 - 2*Sqrt[2]] - (16*I)*EllipticPi[(-I)*(1 + Sqrt[2]), ArcSin[(2^(1/
4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]]
, -3 - 2*Sqrt[2]] + (16*I)*EllipticPi[I*(1 + Sqrt[2]), ArcSin[(2^(1/4)*Sqrt
[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 -
2*Sqrt[2]] - 3*EllipticPi[1 + Sqrt[2], ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(e + f
*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqrt[2]])*Sqrt
[Sec[e + f*x]]*Sqrt[Cos[e + f*x] + Sin[e + f*x]]*((Cos[e + f*x] - Sin[e + f
*x])/(-1 + Sin[e + f*x]) - (Cos[e + f*x]*(Cos[e + f*x] + Sin[e + f*x]))/(-1
 + Sin[e + f*x])^2)*Sqrt[-((1 + Tan[(e + f*x)/2])/((-2 + Sqrt[2])*(-1 + Tan
[(e + f*x)/2])))])/(8*2^(1/4)*((Cos[e + f*x] + Sin[e + f*x])/(-1 + Sin[e +
f*x]))^(3/2)) + ((3*EllipticF[ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(
-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqrt[2]] - 3*EllipticPi
[-1 - Sqrt[2], ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f
*x)/2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqrt[2]] - (16*I)*EllipticPi[(-I)*(1 +
 Sqrt[2]), ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/
2])])/Sqrt[2 + Sqrt[2]]], -3 - 2*Sqrt[2]] + (16*I)*EllipticPi[I*(1 + Sqrt[2
]), ArcSin[(2^(1/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/S
qrt[2 + Sqrt[2]]], -3 - 2*Sqrt[2]] - 3*EllipticPi[1 + Sqrt[2], ArcSin[(2^(1
/4)*Sqrt[(1 + Tan[(e + f*x)/2])/(-1 + Tan[(e + f*x)/2])])/Sqrt[2 + Sqrt[2]]
], -3 - 2*Sqrt[2]])*Sqrt[Sec[e + f*x]]*Sqrt[Cos[e + f*x] + Sin[e + f*x]]*(-
1/2*Sec[(e + f*x)/2]^2/((-2 + Sqrt[2])*(-1 + Tan[(e + f*x)/2])) + (Sec[(e +
 f*x)/2]^2*(1 + Tan[(e + f*x)/2]))/(2*(-2 + Sqrt[2])*(-1 + Tan[(e + f*x)/2]
)^2)))/(8*2^(1/4)*Sqrt[(Cos[e + f*x] + Sin[e + f*x])/(-1 + Sin[e + f*x])]*S
qrt[-((1 + Tan[(e + f*x)/2])/((-2 + Sqrt[2])*(-...
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 1.22, size = 12089, normalized size = 35.66

method	result	size
default	Expression too large to display	12089

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(1+tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(1+tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)^4/sqrt(tan(f*x + e) + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1156 vs. $2(276) = 552$.
time = 1.06, size = 1156, normalized size = 3.41

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(1+tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/48*(24*(1/2)^(1/4)*(f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^2 - sqrt(1/2)*(f^
3*cos(f*x + e)^4 - 2*f^3*cos(f*x + e)^2 + f^3)*sqrt(f^(-4)) + f)*sqrt(sqrt(
1/2)*f^2*sqrt(f^(-4)) + 1)*(f^(-4))^(1/4)*log((2*sqrt(1/2)*f^2*sqrt(f^(-4))
*cos(f*x + e) + 2*(1/2)^(1/4)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)*f*sqrt((
cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(1/4)*cos(f*x + e) + co
s(f*x + e) + sin(f*x + e))/cos(f*x + e)) - 24*(1/2)^(1/4)*(f*cos(f*x + e)^4
 - 2*f*cos(f*x + e)^2 - sqrt(1/2)*(f^3*cos(f*x + e)^4 - 2*f^3*cos(f*x + e)^
2 + f^3)*sqrt(f^(-4)) + f)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)*(f^(-4))^(1
/4)*log((2*sqrt(1/2)*f^2*sqrt(f^(-4))*cos(f*x + e) - 2*(1/2)^(1/4)*sqrt(sqr
t(1/2)*f^2*sqrt(f^(-4)) + 1)*f*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x +
 e))*(f^(-4))^(1/4)*cos(f*x + e) + cos(f*x + e) + sin(f*x + e))/cos(f*x + e
)) - 9*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*log(sqrt((cos(f*x + e) + sin
(f*x + e))/cos(f*x + e)) + 1) + 9*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*l
og(sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e)) - 1) - 2*(10*cos(f*x +
e)^4 - 10*cos(f*x + e)^2 + (17*cos(f*x + e)^3 - 9*cos(f*x + e))*sin(f*x + e
))*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e)) - 96*(1/2)^(3/4)*(f^5*c
os(f*x + e)^4 - 2*f^5*cos(f*x + e)^2 + f^5)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4))
 + 1)*(f^(-4))^(1/4)*arctan(2*(1/2)^(1/4)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4)) +
 1)*f^3*sqrt((2*sqrt(1/2)*f^2*sqrt(f^(-4))*cos(f*x + e) + 2*(1/2)^(1/4)*sqr
t(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)*f*sqrt((cos(f*x + e) + sin(f*x + e))/cos(
f*x + e))*(f^(-4))^(1/4)*cos(f*x + e) + cos(f*x + e) + sin(f*x + e))/cos(f*
x + e))*(f^(-4))^(3/4) - 2*(1/2)^(1/4)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)
*f^3*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) - f^2*
sqrt(f^(-4)) - 2*sqrt(1/2))/f^4 - 96*(1/2)^(3/4)*(f^5*cos(f*x + e)^4 - 2*f^
5*cos(f*x + e)^2 + f^5)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)*(f^(-4))^(1/4)
*arctan(2*(1/2)^(1/4)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)*f^3*sqrt((2*sqrt
(1/2)*f^2*sqrt(f^(-4))*cos(f*x + e) - 2*(1/2)^(1/4)*sqrt(sqrt(1/2)*f^2*sqrt
(f^(-4)) + 1)*f*sqrt((cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(
1/4)*cos(f*x + e) + cos(f*x + e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/
4) - 2*(1/2)^(1/4)*sqrt(sqrt(1/2)*f^2*sqrt(f^(-4)) + 1)*f^3*sqrt((cos(f*x +
 e) + sin(f*x + e))/cos(f*x + e))*(f^(-4))^(3/4) + f^2*sqrt(f^(-4)) + 2*sqr
t(1/2))/f^4)/(f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^2 + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(e+fx)}{\sqrt{\tan(e+fx)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(1+tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**4/sqrt(tan(e + f*x) + 1), x)
```

Giac [A]
time = 0.91, size = 375, normalized size = 1.11

$$-\frac{3\log\left(\sqrt{\tan(fx+e)+1}+1\right)}{16f}+\frac{3\log\left(\left|\sqrt{\tan(fx+e)+1}-1\right|\right)}{16f}+\frac{\left(f^2\sqrt{2\sqrt{2}-2}+f\sqrt{2\sqrt{2}+2}\,|f|\right)\arctan\left(\frac{2^{3/4}\left(2^{1/4}\sqrt{\sqrt{2}+2}+2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{4f^3}+\frac{\left(f^2\sqrt{2\sqrt{2}-2}+f\sqrt{2\sqrt{2}+2}\,|f|\right)\arctan\left(-\frac{2^{3/4}\left(2^{1/4}\sqrt{\sqrt{2}+2}-2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{4f^3}+\frac{\left(f^2\sqrt{2\sqrt{2}+2}-f\sqrt{2\sqrt{2}-2}\,|f|\right)\log\left(2^{1/4}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{8f^3}-\frac{\left(f^2\sqrt{2\sqrt{2}+2}-f\sqrt{2\sqrt{2}-2}\,|f|\right)\log\left(-2^{1/4}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{8f^3}+\frac{9(\tan(fx+e)+1)^{5/2}-8(\tan(fx+e)+1)^{3/2}-9\sqrt{\tan(fx+e)+1}}{24f\tan(fx+e)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(1+tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] -3/16*log(sqrt(tan(f*x + e) + 1) + 1)/f + 3/16*log(abs(sqrt(tan(f*x + e) +
1) - 1))/f + 1/4*(f^2*sqrt(2*sqrt(2) - 2) + f*sqrt(2*sqrt(2) + 2)*abs(f))*a
rctan(1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*sqrt(tan(f*x + e) + 1))/sq
rt(-sqrt(2) + 2))/f^3 + 1/4*(f^2*sqrt(2*sqrt(2) - 2) + f*sqrt(2*sqrt(2) + 2
)*abs(f))*arctan(-1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(tan(f*x +
 e) + 1))/sqrt(-sqrt(2) + 2))/f^3 + 1/8*(f^2*sqrt(2*sqrt(2) + 2) - f*sqrt(2
*sqrt(2) - 2)*abs(f))*log(2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x + e) + 1)
+ sqrt(2) + tan(f*x + e) + 1)/f^3 - 1/8*(f^2*sqrt(2*sqrt(2) + 2) - f*sqrt(2
*sqrt(2) - 2)*abs(f))*log(-2^(1/4)*sqrt(sqrt(2) + 2)*sqrt(tan(f*x + e) + 1)
 + sqrt(2) + tan(f*x + e) + 1)/f^3 + 1/24*(9*(tan(f*x + e) + 1)^(5/2) - 8*(
tan(f*x + e) + 1)^(3/2) - 9*sqrt(tan(f*x + e) + 1))/(f*tan(f*x + e)^3)
```

Mupad [B]
time = 4.09, size = 170, normalized size = 0.50

$$\frac{\mathrm{atan}\left(\sqrt{\tan(e+fx)+1}\,1\mathrm{i}\right)3\mathrm{i}}{8f}+\frac{\frac{3\sqrt{\tan(e+fx)+1}}{8}+\frac{(\tan(e+fx)+1)^{3/2}}{3}-\frac{3(\tan(e+fx)+1)^{5/2}}{8}}{f-3f(\tan(e+fx)+1)+3f(\tan(e+fx)+1)^2-f(\tan(e+fx)+1)^3}+\mathrm{atan}\left(2f\sqrt{\frac{-\frac{1}{8}-\frac{1}{8}\mathrm{i}}{f^2}}\sqrt{\tan(e+fx)+1}\right)\sqrt{\frac{-\frac{1}{8}-\frac{1}{8}\mathrm{i}}{f^2}}\,2\mathrm{i}-\mathrm{atan}\left(2f\sqrt{\frac{-\frac{1}{8}+\frac{1}{8}\mathrm{i}}{f^2}}\sqrt{\tan(e+fx)+1}\right)\sqrt{\frac{-\frac{1}{8}+\frac{1}{8}\mathrm{i}}{f^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(tan(e + f*x) + 1)^(1/2),x)
```

```
[Out] (atan((tan(e + f*x) + 1)^(1/2)*1i)*3i)/(8*f) + ((3*(tan(e + f*x) + 1)^(1/2)
)/8 + (tan(e + f*x) + 1)^(3/2)/3 - (3*(tan(e + f*x) + 1)^(5/2))/8)/(f - 3*f
*(tan(e + f*x) + 1) + 3*f*(tan(e + f*x) + 1)^2 - f*(tan(e + f*x) + 1)^3) +
atan(2*f*((- 1/8 - 1i/8)/f^2)^(1/2)*(tan(e + f*x) + 1)^(1/2))*((- 1/8 - 1i/
8)/f^2)^(1/2)*2i - atan(2*f*((- 1/8 + 1i/8)/f^2)^(1/2)*(tan(e + f*x) + 1)^(
1/2))*((- 1/8 + 1i/8)/f^2)^(1/2)*2i
```

3.411 $\int (d\tan(e+fx))^n(a+a\tan(e+fx))^m\,dx$

Optimal. Leaf size=161

$$\frac{F_1(1+n;-m,1;2+n;-\tan(e+fx),-i\tan(e+fx))(d\tan(e+fx))^{1+n}(1+\tan(e+fx))^{-m}(a+a\tan(e+$$
$$2df(1+n)$$

```
[Out] 1/2*AppellF1(1+n,1,-m,2+n,-I*tan(f*x+e),-tan(f*x+e))*(d*tan(f*x+e))^(1+n)*(
a+a*tan(f*x+e))^m/d/f/(1+n)/((1+tan(f*x+e))^m)+1/2*AppellF1(1+n,1,-m,2+n,I*
tan(f*x+e),-tan(f*x+e))*(d*tan(f*x+e))^(1+n)*(a+a*tan(f*x+e))^m/d/f/(1+n)/(
(1+tan(f*x+e))^m)
```

Rubi [A]
time = 0.12, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3656, 926, 140, 138}

$$\frac{(\tan(e+fx)+1)^{-m}(a\tan(e+fx)+a)^m(d\tan(e+fx))^{n+1}F_1(n+1;-m,1;n+2;-\tan(e+fx),-i\tan(e+fx))}{2df(n+1)}+\frac{(\tan(e+fx)+1)^{-m}(a\tan(e+fx)+a)^m(d\tan(e+fx))^{n+1}F_1(n+1;-m,1;n+2;-\tan(e+fx),i\tan(e+fx))}{2df(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^n*(a + a*Tan[e + f*x])^m,x]
```

```
[Out] (AppellF1[1 + n, -m, 1, 2 + n, -Tan[e + f*x], (-I)*Tan[e + f*x]]*(d*Tan[e +
 f*x])^(1 + n)*(a + a*Tan[e + f*x])^m)/(2*d*f*(1 + n)*(1 + Tan[e + f*x])^m)
 + (AppellF1[1 + n, -m, 1, 2 + n, -Tan[e + f*x], I*Tan[e + f*x]]*(d*Tan[e +
 f*x])^(1 + n)*(a + a*Tan[e + f*x])^m)/(2*d*f*(1 + n)*(1 + Tan[e + f*x])^m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
```

```
& !IntegerQ[m] && !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int (d\tan(e+fx))^n(a+a\tan(e+fx))^m\,dx = \frac{\text{Subst}\left(\int \frac{(dx)^n(a+ax)^m}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{i(dx)^n(a+ax)^m}{2(i-x)}+\frac{i(dx)^n(a+ax)^m}{2(i+x)}\right)\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{i\text{Subst}\left(\int \frac{(dx)^n(a+ax)^m}{i-x}\,dx, x, \tan(e+fx)\right)}{2f} + \frac{i\text{Subst}\left(\int \frac{(dx)^n(a}{i+}\right.}{}$$
$$= \frac{(i(1+\tan(e+fx))^{-m}(a+a\tan(e+fx))^m)\,\text{Subst}\left(\int \frac{(dx)^n(1-}{i-x}\right.}{2f}$$
$$= \frac{F_1(1+n;-m,1;2+n;-\tan(e+fx),-i\tan(e+fx))(d\tan}{2df(1+}$$

Mathematica [F]
time = 0.82, size = 0, normalized size = 0.00

$$\int (d\tan(e+fx))^n(a+a\tan(e+fx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Tan[e + f*x])^n*(a + a*Tan[e + f*x])^m,x]
[Out] Integrate[(d*Tan[e + f*x])^n*(a + a*Tan[e + f*x])^m, x]
```

Maple [F]
time = 0.40, size = 0, normalized size = 0.00

$$\int \left(d\tan\left(fx+e\right)\right)^{n}\left(a+a\tan\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^n*(a+a*tan(f*x+e))^m,x)
[Out] int((d*tan(f*x+e))^n*(a+a*tan(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+a*tan(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*tan(f*x + e) + a)^m*(d*tan(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+a*tan(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*tan(f*x + e) + a)^m*(d*tan(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\tan(e + fx) + 1))^m (d\tan(e + fx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**n*(a+a*tan(f*x+e))**m,x)
[Out] Integral((a*(tan(e + f*x) + 1))**m*(d*tan(e + f*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+a*tan(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*tan(f*x + e) + a)^m*(d*tan(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\tan(e + f\,x))^n\,(a + a\tan(e + f\,x))^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^n*(a + a*tan(e + f*x))^m,x)
[Out] int((d*tan(e + f*x))^n*(a + a*tan(e + f*x))^m, x)
```

3.412 $\int \tan^5(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=93

$$-bx-\frac{a\log(\cos(c+dx))}{d}+\frac{b\tan(c+dx)}{d}-\frac{a\tan^2(c+dx)}{2d}-\frac{b\tan^3(c+dx)}{3d}+\frac{a\tan^4(c+dx)}{4d}+\frac{b\tan^5(c+dx)}{5d}$$

```
[Out] -b*x-a*ln(cos(d*x+c))/d+b*tan(d*x+c)/d-1/2*a*tan(d*x+c)^2/d-1/3*b*tan(d*x+c
)^3/d+1/4*a*tan(d*x+c)^4/d+1/5*b*tan(d*x+c)^5/d
```

Rubi [A]
time = 0.08, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3609, 3606, 3556}

$$\frac{a\tan^4(c+dx)}{4d}-\frac{a\tan^2(c+dx)}{2d}-\frac{a\log(\cos(c+dx))}{d}+\frac{b\tan^5(c+dx)}{5d}-\frac{b\tan^3(c+dx)}{3d}+\frac{b\tan(c+dx)}{d}-bx$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```

```
[Out] -(b*x) - (a*Log[Cos[c + d*x]])/d + (b*Tan[c + d*x])/d - (a*Tan[c + d*x]^2)/
(2*d) - (b*Tan[c + d*x]^3)/(3*d) + (a*Tan[c + d*x]^4)/(4*d) + (b*Tan[c + d*
x]^5)/(5*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\int \tan^5(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\tan^5(c+dx)}{5d} + \int \tan^4(c+dx)(-b+a\tan(c+dx))\,dx$$
$$= \frac{a\tan^4(c+dx)}{4d} + \frac{b\tan^5(c+dx)}{5d} + \int \tan^3(c+dx)(-a-b\tan(c+$$
$$= -\frac{b\tan^3(c+dx)}{3d} + \frac{a\tan^4(c+dx)}{4d} + \frac{b\tan^5(c+dx)}{5d} + \int \tan^2(c+$$
$$= -\frac{a\tan^2(c+dx)}{2d} - \frac{b\tan^3(c+dx)}{3d} + \frac{a\tan^4(c+dx)}{4d} + \frac{b\tan^5(c+d}{5d}$$
$$= -bx + \frac{b\tan(c+dx)}{d} - \frac{a\tan^2(c+dx)}{2d} - \frac{b\tan^3(c+dx)}{3d} + \frac{a\tan^4(c}{4d}$$
$$= -bx - \frac{a\log(\cos(c+dx))}{d} + \frac{b\tan(c+dx)}{d} - \frac{a\tan^2(c+dx)}{2d} - \frac{b\tan}{}$$

Mathematica [A]
time = 0.37, size = 95, normalized size = 1.02

$$-\frac{b\text{ArcTan}(\tan(c+dx))}{d} + \frac{b\tan(c+dx)}{d} - \frac{b\tan^3(c+dx)}{3d} + \frac{b\tan^5(c+dx)}{5d} - \frac{a(4\log(\cos(c+dx)) + 2\tan^2(c+dx) - \tan^4(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```

```
[Out] -((b*ArcTan[Tan[c + d*x]])/d) + (b*Tan[c + d*x])/d - (b*Tan[c + d*x]^3)/(3*
d) + (b*Tan[c + d*x]^5)/(5*d) - (a*(4*Log[Cos[c + d*x]] + 2*Tan[c + d*x]^2
- Tan[c + d*x]^4))/(4*d)
```

Maple [A]
time = 0.07, size = 82, normalized size = 0.88

method	result
derivativedivides	$\frac{\frac{b\left(\tan^5(dx+c)\right)}{5}+\frac{a\left(\tan^4(dx+c)\right)}{4}-\frac{b\left(\tan^3(dx+c)\right)}{3}-\frac{a\left(\tan^2(dx+c)\right)}{2}+b\tan(dx+c)+\frac{a\ln\left(1+\tan^2(dx+c)\right)}{2}-b\arctan(\tan(dx+c)}{d}$
default	$\frac{\frac{b\left(\tan^5(dx+c)\right)}{5}+\frac{a\left(\tan^4(dx+c)\right)}{4}-\frac{b\left(\tan^3(dx+c)\right)}{3}-\frac{a\left(\tan^2(dx+c)\right)}{2}+b\tan(dx+c)+\frac{a\ln\left(1+\tan^2(dx+c)\right)}{2}-b\arctan(\tan(dx+c)}{d}$
norman	$\frac{b\tan(dx+c)}{d}-bx-\frac{a\left(\tan^2(dx+c)\right)}{2d}+\frac{a\left(\tan^4(dx+c)\right)}{4d}-\frac{b\left(\tan^3(dx+c)\right)}{3d}+\frac{b\left(\tan^5(dx+c)\right)}{5d}+\frac{a\ln\left(1+\tan^2(dx\right.}{2d}$
risch	$-bx+iax+\frac{2iac}{d}+\frac{2i\left(30ia\,e^{8i(dx+c)}+45b\,e^{8i(dx+c)}+60ia\,e^{6i(dx+c)}+90b\,e^{6i(dx+c)}+60ia\,e^{4i(dx+c)}+140b\,e^{4i(dx\right.}{15d\left(e^{2i(dx+c)}+1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*b*tan(d*x+c)^5+1/4*a*tan(d*x+c)^4-1/3*b*tan(d*x+c)^3-1/2*a*tan(d*x
+c)^2+b*tan(d*x+c)+1/2*a*ln(1+tan(d*x+c)^2)-b*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.54, size = 81, normalized size = 0.87

$$\frac{12\,b\tan\left(dx+c\right)^{5}+15\,a\tan\left(dx+c\right)^{4}-20\,b\tan\left(dx+c\right)^{3}-30\,a\tan\left(dx+c\right)^{2}-60\,(dx+c)b+30\,a\log\left(\tan\left(dx+c\right)^{2}+1\right)+60\,b\tan\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(12*b*tan(d*x + c)^5 + 15*a*tan(d*x + c)^4 - 20*b*tan(d*x + c)^3 - 30*
a*tan(d*x + c)^2 - 60*(d*x + c)*b + 30*a*log(tan(d*x + c)^2 + 1) + 60*b*tan
(d*x + c))/d
```

Fricas [A]
time = 1.64, size = 80, normalized size = 0.86

$$\frac{12\,b\tan\left(dx+c\right)^{5}+15\,a\tan\left(dx+c\right)^{4}-20\,b\tan\left(dx+c\right)^{3}-60\,bdx-30\,a\tan\left(dx+c\right)^{2}-30\,a\log\left(\frac{1}{\tan(dx+c)^{2}+1}\right)+60\,b\tan\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/60*(12*b*tan(d*x + c)^5 + 15*a*tan(d*x + c)^4 - 20*b*tan(d*x + c)^3 - 60*
b*d*x - 30*a*tan(d*x + c)^2 - 30*a*log(1/(tan(d*x + c)^2 + 1)) + 60*b*tan(d
*x + c))/d
```

Sympy [A]
time = 0.15, size = 97, normalized size = 1.04

$$\begin{cases} \frac{a\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d}+\frac{a\tan^{4}\left(c+dx\right)}{4d}-\frac{a\tan^{2}\left(c+dx\right)}{2d}-bx+\frac{b\tan^{5}\left(c+dx\right)}{5d}-\frac{b\tan^{3}\left(c+dx\right)}{3d}+\frac{b\tan\left(c+dx\right)}{d} & \text{for}\ d\neq 0\\ x(a+b\tan\left(c\right))\tan^{5}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5*(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise((a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**4/(4*d) - a*t
an(c + d*x)**2/(2*d) - b*x + b*tan(c + d*x)**5/(5*d) - b*tan(c + d*x)**3/(3
*d) + b*tan(c + d*x)/d, Ne(d, 0)), (x*(a + b*tan(c))*tan(c)**5, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 947 vs. 2(85) = 170.
time = 2.64, size = 947, normalized size = 10.18

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="giac")

[Out] -1/60*(60*b*d*x*tan(d*x)^5*tan(c)^5 + 30*a*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^5*tan(c)^5 - 300*b*d*x*tan(d*x)^4*tan(c)^4 + 45*a*tan(d*x)^5*tan(c)^5 - 150*a*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 + 60*b*tan(d*x)^5*tan(c)^4 + 60*b*tan(d*x)^4*tan(c)^5 + 600*b*d*x*tan(d*x)^3*tan(c)^3 + 30*a*tan(d*x)^5*tan(c)^3 - 165*a*tan(d*x)^4*tan(c)^4 + 30*a*tan(d*x)^3*tan(c)^5 - 20*b*tan(d*x)^5*tan(c)^2 + 300*a*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 - 300*b*tan(d*x)^4*tan(c)^3 - 300*b*tan(d*x)^3*tan(c)^4 - 20*b*tan(d*x)^2*tan(c)^5 - 15*a*tan(d*x)^5*tan(c) - 600*b*d*x*tan(d*x)^2*tan(c)^2 - 150*a*tan(d*x)^4*tan(c)^2 + 180*a*tan(d*x)^3*tan(c)^3 - 150*a*tan(d*x)^2*tan(c)^4 - 15*a*tan(d*x)*tan(c)^5 + 12*b*tan(d*x)^5 + 100*b*tan(d*x)^4*tan(c) - 300*a*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + 600*b*tan(d*x)^3*tan(c)^2 + 600*b*tan(d*x)^2*tan(c)^3 + 100*b*tan(d*x)*tan(c)^4 + 12*b*tan(c)^5 + 15*a*tan(d*x)^4 + 300*b*d*x*tan(d*x)*tan(c) + 150*a*tan(d*x)^3*tan(c) - 180*a*tan(d*x)^2*tan(c)^2 + 150*a*tan(d*x)*tan(c)^3 + 15*a*tan(c)^4 - 20*b*tan(d*x)^3 + 150*a*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)*tan(c) - 300*b*tan(d*x)^2*tan(c) - 300*b*tan(d*x)*tan(c)^2 - 20*b*tan(c)^3 - 60*b*d*x - 30*a*tan(d*x)^2 + 165*a*tan(d*x)*tan(c) - 30*a*tan(c)^2 - 30*a*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) + 60*b*tan(d*x) + 60*b*tan(c) - 45*a)/(d*tan(d*x)^5*tan(c)^5 - 5*d*tan(d*x)^4*tan(c)^4 + 10*d*tan(d*x)^3*tan(c)^3 - 10*d*tan(d*x)^2*tan(c)^2 + 5*d*tan(d*x)*tan(c) - d)

Mupad [B]
time = 4.10, size = 76, normalized size = 0.82

$$\frac{b\tan(c+d\,x)+\frac{a\,\ln\left(\tan(c+d\,x)^2+1\right)}{2}-\frac{a\tan(c+d\,x)^2}{2}+\frac{a\tan(c+d\,x)^4}{4}-\frac{b\tan(c+d\,x)^3}{3}+\frac{b\tan(c+d\,x)^5}{5}-b\,d\,x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d*x)^5*(a + b*tan(c + d*x)),x)

[Out] (b*tan(c + d*x) + (a*log(tan(c + d*x)^2 + 1))/2 - (a*tan(c + d*x)^2)/2 + (a*tan(c + d*x)^4)/4 - (b*tan(c + d*x)^3)/3 + (b*tan(c + d*x)^5)/5 - b*d*x)/d

3.413 $\int \tan^4(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=77

$$ax-\frac{b\log(\cos(c+dx))}{d}-\frac{a\tan(c+dx)}{d}-\frac{b\tan^2(c+dx)}{2d}+\frac{a\tan^3(c+dx)}{3d}+\frac{b\tan^4(c+dx)}{4d}$$

```
[Out] a*x-b*ln(cos(d*x+c))/d-a*tan(d*x+c)/d-1/2*b*tan(d*x+c)^2/d+1/3*a*tan(d*x+c)
^3/d+1/4*b*tan(d*x+c)^4/d
```

Rubi [A]
time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3609, 3606, 3556}

$$\frac{a\tan^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+ax+\frac{b\tan^4(c+dx)}{4d}-\frac{b\tan^2(c+dx)}{2d}-\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

```
[Out] a*x - (b*Log[Cos[c + d*x]])/d - (a*Tan[c + d*x])/d - (b*Tan[c + d*x]^2)/(2*
d) + (a*Tan[c + d*x]^3)/(3*d) + (b*Tan[c + d*x]^4)/(4*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\int \tan^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\tan^4(c+dx)}{4d} + \int \tan^3(c+dx)(-b+a\tan(c+dx))\,dx$$
$$= \frac{a\tan^3(c+dx)}{3d} + \frac{b\tan^4(c+dx)}{4d} + \int \tan^2(c+dx)(-a-b\tan(c+$$
$$= -\frac{b\tan^2(c+dx)}{2d} + \frac{a\tan^3(c+dx)}{3d} + \frac{b\tan^4(c+dx)}{4d} + \int \tan(c+d$$
$$= ax - \frac{a\tan(c+dx)}{d} - \frac{b\tan^2(c+dx)}{2d} + \frac{a\tan^3(c+dx)}{3d} + \frac{b\tan^4(c-}{4d}$$
$$= ax - \frac{b\log(\cos(c+dx))}{d} - \frac{a\tan(c+dx)}{d} - \frac{b\tan^2(c+dx)}{2d} + \frac{a\tan^3}{}$$

Mathematica [A]
time = 0.22, size = 79, normalized size = 1.03

$$\frac{a\mathrm{ArcTan}(\tan(c+dx))}{d} - \frac{a\tan(c+dx)}{d} + \frac{a\tan^3(c+dx)}{3d} - \frac{b(4\log(\cos(c+dx)) + 2\tan^2(c+dx) - \tan^4(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*ArcTan[Tan[c + d*x]])/d - (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
- (b*(4*Log[Cos[c + d*x]] + 2*Tan[c + d*x]^2 - Tan[c + d*x]^4))/(4*d)
```

Maple [A]
time = 0.03, size = 71, normalized size = 0.92

method	result
derivativedivides	$\frac{\frac{b\left(\tan^4(dx+c)\right)}{4}+\frac{a\left(\tan^3(dx+c)\right)}{3}-\frac{b\left(\tan^2(dx+c)\right)}{2}-a\tan(dx+c)+\frac{b\ln\left(1+\tan^2(dx+c)\right)}{2}+a\arctan(\tan(dx+c))}{d}$
default	$\frac{\frac{b\left(\tan^4(dx+c)\right)}{4}+\frac{a\left(\tan^3(dx+c)\right)}{3}-\frac{b\left(\tan^2(dx+c)\right)}{2}-a\tan(dx+c)+\frac{b\ln\left(1+\tan^2(dx+c)\right)}{2}+a\arctan(\tan(dx+c))}{d}$
norman	$ax-\frac{a\tan(dx+c)}{d}+\frac{a(\tan^3(dx+c))}{3d}-\frac{b(\tan^2(dx+c))}{2d}+\frac{b(\tan^4(dx+c))}{4d}+\frac{b\ln(1+\tan^2(dx+c))}{2d}$
risch	$ibx+ax+\frac{2ibc}{d}-\frac{4\left(3ia\,e^{6i(dx+c)}+3b\,e^{6i(dx+c)}+6ia\,e^{4i(dx+c)}+3b\,e^{4i(dx+c)}+5ia\,e^{2i(dx+c)}+3b\,e^{2i(dx+c)}+2ia\right)}{3d\left(e^{2i(dx+c)}+1\right)^4}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*b*tan(d*x+c)^4+1/3*a*tan(d*x+c)^3-1/2*b*tan(d*x+c)^2-a*tan(d*x+c)+
1/2*b*ln(1+tan(d*x+c)^2)+a*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.51, size = 70, normalized size = 0.91

$$\frac{3\,b\tan\left(dx+c\right)^{4}+4\,a\tan\left(dx+c\right)^{3}-6\,b\tan\left(dx+c\right)^{2}+12\,(dx+c)a+6\,b\log\left(\tan\left(dx+c\right)^{2}+1\right)-12\,a\tan\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*b*tan(d*x + c)^4 + 4*a*tan(d*x + c)^3 - 6*b*tan(d*x + c)^2 + 12*(d*
x + c)*a + 6*b*log(tan(d*x + c)^2 + 1) - 12*a*tan(d*x + c))/d
```

Fricas [A]
time = 1.24, size = 69, normalized size = 0.90

$$\frac{3\,b\tan\left(dx+c\right)^{4}+4\,a\tan\left(dx+c\right)^{3}+12\,adx-6\,b\tan\left(dx+c\right)^{2}-6\,b\log\left(\frac{1}{\tan(dx+c)^{2}+1}\right)-12\,a\tan\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*tan(d*x + c)^4 + 4*a*tan(d*x + c)^3 + 12*a*d*x - 6*b*tan(d*x + c)
^2 - 6*b*log(1/(tan(d*x + c)^2 + 1)) - 12*a*tan(d*x + c))/d
```

Sympy [A]
time = 0.11, size = 83, normalized size = 1.08

$$\begin{cases} ax+\frac{a\tan^{3}\left(c+dx\right)}{3d}-\frac{a\tan\left(c+dx\right)}{d}+\frac{b\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d}+\frac{b\tan^{4}\left(c+dx\right)}{4d}-\frac{b\tan^{2}\left(c+dx\right)}{2d} & \text{for } d\neq 0\\ x(a+b\tan\left(c\right))\tan^{4}\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4*(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise((a*x + a*tan(c + d*x)**3/(3*d) - a*tan(c + d*x)/d + b*log(tan(c +
 d*x)**2 + 1)/(2*d) + b*tan(c + d*x)**4/(4*d) - b*tan(c + d*x)**2/(2*d), Ne
(d, 0)), (x*(a + b*tan(c))*tan(c)**4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 716 vs. 2(71) = 142.
time = 1.48, size = 716, normalized size = 9.30

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*a*d*x*tan(d*x)^4*tan(c)^4 - 6*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan
(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/
(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 - 48*a*d*x*tan(d*x)^3*tan(c)^3 - 9*b*ta
n(d*x)^4*tan(c)^4 + 24*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) +
 tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*
tan(d*x)^3*tan(c)^3 + 12*a*tan(d*x)^4*tan(c)^3 + 12*a*tan(d*x)^3*tan(c)^4 +
 72*a*d*x*tan(d*x)^2*tan(c)^2 - 6*b*tan(d*x)^4*tan(c)^2 + 24*b*tan(d*x)^3*t
an(c)^3 - 6*b*tan(d*x)^2*tan(c)^4 - 4*a*tan(d*x)^4*tan(c) - 36*b*log(4*(tan
(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 -
 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 - 48*a*tan(d*x)
^3*tan(c)^2 - 48*a*tan(d*x)^2*tan(c)^3 - 4*a*tan(d*x)*tan(c)^4 + 3*b*tan(d*
x)^4 - 48*a*d*x*tan(d*x)*tan(c) + 24*b*tan(d*x)^3*tan(c) - 12*b*tan(d*x)^2*
tan(c)^2 + 24*b*tan(d*x)*tan(c)^3 + 3*b*tan(c)^4 + 4*a*tan(d*x)^3 + 24*b*lo
g(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(
d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)*tan(c) + 48*a*tan(
d*x)^2*tan(c) + 48*a*tan(d*x)*tan(c)^2 + 4*a*tan(c)^3 + 12*a*d*x - 6*b*tan(
d*x)^2 + 24*b*tan(d*x)*tan(c) - 6*b*tan(c)^2 - 6*b*log(4*(tan(d*x)^4*tan(c)
^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*ta
n(c) + 1)/(tan(c)^2 + 1)) - 12*a*tan(d*x) - 12*a*tan(c) - 9*b)/(d*tan(d*x)^
4*tan(c)^4 - 4*d*tan(d*x)^3*tan(c)^3 + 6*d*tan(d*x)^2*tan(c)^2 - 4*d*tan(d*
x)*tan(c) + d)
```

Mupad [B]
time = 4.01, size = 65, normalized size = 0.84

$$\frac{\frac{b\,\ln\left(\tan(c+d\,x)^2+1\right)}{2}-a\tan(c+d\,x)+\frac{a\tan(c+d\,x)^3}{3}-\frac{b\tan(c+d\,x)^2}{2}+\frac{b\tan(c+d\,x)^4}{4}+a\,d\,x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + b*tan(c + d*x)),x)
```

```
[Out] ((b*log(tan(c + d*x)^2 + 1))/2 - a*tan(c + d*x) + (a*tan(c + d*x)^3)/3 - (b
*tan(c + d*x)^2)/2 + (b*tan(c + d*x)^4)/4 + a*d*x)/d
```

3.414 $\int \tan^3(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=60

$$bx+\frac{a\log(\cos(c+dx))}{d}-\frac{b\tan(c+dx)}{d}+\frac{a\tan^2(c+dx)}{2d}+\frac{b\tan^3(c+dx)}{3d}$$

```
[Out] b*x+a*ln(cos(d*x+c))/d-b*tan(d*x+c)/d+1/2*a*tan(d*x+c)^2/d+1/3*b*tan(d*x+c)
^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.158$, Rules used = {3609, 3606, 3556}

$$\frac{a\tan^2(c+dx)}{2d}+\frac{a\log(\cos(c+dx))}{d}+\frac{b\tan^3(c+dx)}{3d}-\frac{b\tan(c+dx)}{d}+bx$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

```
[Out] b*x + (a*Log[Cos[c + d*x]])/d - (b*Tan[c + d*x])/d + (a*Tan[c + d*x]^2)/(2*
d) + (b*Tan[c + d*x]^3)/(3*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\int \tan^3(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\tan^3(c+dx)}{3d} + \int \tan^2(c+dx)(-b+a\tan(c+dx))\,dx$$
$$= \frac{a\tan^2(c+dx)}{2d} + \frac{b\tan^3(c+dx)}{3d} + \int \tan(c+dx)(-a-b\tan(c+$$
$$= bx - \frac{b\tan(c+dx)}{d} + \frac{a\tan^2(c+dx)}{2d} + \frac{b\tan^3(c+dx)}{3d} - a\int \tan($$
$$= bx + \frac{a\log(\cos(c+dx))}{d} - \frac{b\tan(c+dx)}{d} + \frac{a\tan^2(c+dx)}{2d} + \frac{b\tan^3}{}$$

Mathematica [A]
time = 0.15, size = 67, normalized size = 1.12

$$\frac{b\text{ArcTan}(\tan(c+dx))}{d} - \frac{b\tan(c+dx)}{d} + \frac{b\tan^3(c+dx)}{3d} + \frac{a(2\log(\cos(c+dx))+\tan^2(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3*(a + b*Tan[c + d*x]),x]
[Out] (b*ArcTan[Tan[c + d*x]])/d - (b*Tan[c + d*x])/d + (b*Tan[c + d*x]^3)/(3*d)
+ (a*(2*Log[Cos[c + d*x]] + Tan[c + d*x]^2))/(2*d)
```

Maple [A]
time = 0.03, size = 60, normalized size = 1.00

method	result	size
derivativedivides	$\frac{\frac{b\left(\tan^3(dx+c)\right)}{3}+\frac{a\left(\tan^2(dx+c)\right)}{2}-b\tan(dx+c)-\frac{a\ln\left(1+\tan^2(dx+c)\right)}{2}+b\arctan(\tan(dx+c))}{d}$	60
default	$\frac{\frac{b\left(\tan^3(dx+c)\right)}{3}+\frac{a\left(\tan^2(dx+c)\right)}{2}-b\tan(dx+c)-\frac{a\ln\left(1+\tan^2(dx+c)\right)}{2}+b\arctan(\tan(dx+c))}{d}$	60
norman	$bx+\frac{a(\tan^2(dx+c))}{2d}-\frac{b\tan(dx+c)}{d}+\frac{b(\tan^3(dx+c))}{3d}-\frac{a\ln(1+\tan^2(dx+c))}{2d}$	62
risch	$bx-iax-\frac{2iac}{d}-\frac{2i(3ia\,e^{4i(dx+c)}+6b\,e^{4i(dx+c)}+3ia\,e^{2i(dx+c)}+6b\,e^{2i(dx+c)}+4b)}{3d(e^{2i(dx+c)}+1)^3}+\frac{a\ln(e^{2i(dx+c)}+1)}{d}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/3*b*tan(d*x+c)^3+1/2*a*tan(d*x+c)^2-b*tan(d*x+c)-1/2*a*ln(1+tan(d*x+
c)^2)+b*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.51, size = 59, normalized size = 0.98

$$\frac{2\,b\tan{(dx+c)}^3+3\,a\tan{(dx+c)}^2+6\,(dx+c)b-3\,a\log\left(\tan{(dx+c)}^2+1\right)-6\,b\tan{(dx+c)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*b*tan(d*x + c)^3 + 3*a*tan(d*x + c)^2 + 6*(d*x + c)*b - 3*a*log(tan(
d*x + c)^2 + 1) - 6*b*tan(d*x + c))/d
```

Fricas [A]
time = 1.90, size = 58, normalized size = 0.97

$$\frac{2\,b\tan\left(dx+c\right)^{3}+6\,bdx+3\,a\tan\left(dx+c\right)^{2}+3\,a\log\left(\frac{1}{\tan\left(dx+c\right)^{2}+1}\right)-6\,b\tan\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*tan(d*x + c)^3 + 6*b*d*x + 3*a*tan(d*x + c)^2 + 3*a*log(1/(tan(d*x
 + c)^2 + 1)) - 6*b*tan(d*x + c))/d
```

Sympy [A]
time = 0.10, size = 70, normalized size = 1.17

$$\begin{cases} -\frac{a\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d}+\frac{a\tan^{2}\left(c+dx\right)}{2d}+bx+\frac{b\tan^{3}\left(c+dx\right)}{3d}-\frac{b\tan\left(c+dx\right)}{d} & \text{for}\ d\neq 0\\ x(a+b\tan\left(c\right))\tan^{3}\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3*(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise((-a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**2/(2*d) + b*
x + b*tan(c + d*x)**3/(3*d) - b*tan(c + d*x)/d, Ne(d, 0)), (x*(a + b*tan(c)
)*tan(c)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 515 vs. 2(56) = 112.
time = 1.09, size = 515, normalized size = 8.58

$$\frac{6\,bdx\tan\left(dx\right)^{3}\tan\left(c\right)^{3}+3\,a\log\left(\frac{4\left(\tan(dx)^{4}\tan(c)^{2}-2\tan(dx)^{3}\tan(c)+\tan(dx)^{2}\tan(c)^{2}+\tan(dx)^{2}-2\tan(dx)\tan(c)+1\right)}{\tan(c)^{2}+1}\right)\tan\left(dx\right)^{3}\tan\left(c\right)^{3}-18\,bdx\tan\left(dx\right)^{2}\tan\left(c\right)^{2}+3\,a\tan\left(dx\right)^{3}\tan\left(c\right)^{3}-9\,a\log\left(\frac{4\left(\tan(dx)^{4}\tan(c)^{2}-2\tan(dx)^{3}\tan(c)+\tan(dx)^{2}\tan(c)^{2}+\tan(dx)^{2}-2\tan(dx)\tan(c)+1\right)}{\tan(c)^{2}+1}\right)\tan\left(dx\right)^{2}\tan\left(c\right)^{2}+6\,b\tan\left(dx\right)^{3}\tan\left(c\right)^{2}+6\,b\tan\left(dx\right)^{2}\tan\left(c\right)^{3}+18\,bdx\tan\left(dx\right)\tan\left(c\right)+3\,a\tan\left(dx\right)^{3}\tan\left(c\right)-3\,a\tan\left(dx\right)^{2}\tan\left(c\right)^{2}+3\,a\tan\left(dx\right)\tan\left(c\right)^{3}-2\,b\tan\left(dx\right)^{3}+9\,a\log\left(\frac{4\left(\tan(dx)^{4}\tan(c)^{2}-2\tan(dx)^{3}\tan(c)+\tan(dx)^{2}\tan(c)^{2}+\tan(dx)^{2}-2\tan(dx)\tan(c)+1\right)}{\tan(c)^{2}+1}\right)\tan\left(dx\right)\tan\left(c\right)-18\,b\tan\left(dx\right)^{2}\tan\left(c\right)-18\,b\tan\left(dx\right)\tan\left(c\right)^{2}-2\,b\tan\left(c\right)^{3}-6\,bdx-3\,a\tan\left(dx\right)^{2}+3\,a\tan\left(dx\right)\tan\left(c\right)-3\,a\tan\left(c\right)^{2}-3\,a\log\left(\frac{4\left(\tan(dx)^{4}\tan(c)^{2}-2\tan(dx)^{3}\tan(c)+\tan(dx)^{2}\tan(c)^{2}+\tan(dx)^{2}-2\tan(dx)\tan(c)+1\right)}{\tan(c)^{2}+1}\right)+6\,b\tan\left(dx\right)+6\,b\tan\left(c\right)-3\,a}{6\left(d\tan\left(dx\right)^{3}\tan\left(c\right)^{3}-3\,d\tan\left(dx\right)^{2}\tan\left(c\right)^{2}+3\,d\tan\left(dx\right)\tan\left(c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*b*d*x*tan(d*x)^3*tan(c)^3 + 3*a*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d
*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(t
an(c)^2 + 1))*tan(d*x)^3*tan(c)^3 - 18*b*d*x*tan(d*x)^2*tan(c)^2 + 3*a*tan(
d*x)^3*tan(c)^3 - 9*a*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + ta
n(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan
```

```
(d*x)^2*tan(c)^2 + 6*b*tan(d*x)^3*tan(c)^2 + 6*b*tan(d*x)^2*tan(c)^3 + 18*b
*d*x*tan(d*x)*tan(c) + 3*a*tan(d*x)^3*tan(c) - 3*a*tan(d*x)^2*tan(c)^2 + 3*
a*tan(d*x)*tan(c)^3 - 2*b*tan(d*x)^3 + 9*a*log(4*(tan(d*x)^4*tan(c)^2 - 2*t
an(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1
)/(tan(c)^2 + 1))*tan(d*x)*tan(c) - 18*b*tan(d*x)^2*tan(c) - 18*b*tan(d*x)*
tan(c)^2 - 2*b*tan(c)^3 - 6*b*d*x - 3*a*tan(d*x)^2 + 3*a*tan(d*x)*tan(c) -
3*a*tan(c)^2 - 3*a*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d
*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) + 6*b*
tan(d*x) + 6*b*tan(c) - 3*a)/(d*tan(d*x)^3*tan(c)^3 - 3*d*tan(d*x)^2*tan(c)
^2 + 3*d*tan(d*x)*tan(c) - d)
```

Mupad [B]
time = 4.04, size = 54, normalized size = 0.90

$$\frac{\frac{a\tan(c+d\,x)^2}{2}-\frac{a\,\ln\left(\tan(c+d\,x)^2+1\right)}{2}-b\tan(c+d\,x)+\frac{b\tan(c+d\,x)^3}{3}+b\,d\,x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + b*tan(c + d*x)),x)
```

```
[Out] ((a*tan(c + d*x)^2)/2 - (a*log(tan(c + d*x)^2 + 1))/2 - b*tan(c + d*x) + (b
*tan(c + d*x)^3)/3 + b*d*x)/d
```

3.415 $\int \tan^2(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=44

$$-ax+\frac{b\log(\cos(c+dx))}{d}+\frac{a\tan(c+dx)}{d}+\frac{b\tan^2(c+dx)}{2d}$$

```
[Out] -a*x+b*ln(cos(d*x+c))/d+a*tan(d*x+c)/d+1/2*b*tan(d*x+c)^2/d
```

Rubi [A]
time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3609, 3606, 3556}

$$\frac{a\tan(c+dx)}{d}-ax+\frac{b\tan^2(c+dx)}{2d}+\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2*(a + b*Tan[c + d*x]),x]
```

```
[Out] -(a*x) + (b*Log[Cos[c + d*x]])/d + (a*Tan[c + d*x])/d + (b*Tan[c + d*x]^2)/
(2*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\int \tan^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\tan^2(c+dx)}{2d} + \int \tan(c+dx)(-b+a\tan(c+dx))\,dx$$
$$= -ax + \frac{a\tan(c+dx)}{d} + \frac{b\tan^2(c+dx)}{2d} - b\int \tan(c+dx)\,dx$$
$$= -ax + \frac{b\log(\cos(c+dx))}{d} + \frac{a\tan(c+dx)}{d} + \frac{b\tan^2(c+dx)}{2d}$$

Mathematica [A]
time = 0.11, size = 51, normalized size = 1.16

$$-\frac{a\text{ArcTan}(\tan(c+dx))}{d} + \frac{a\tan(c+dx)}{d} + \frac{b(2\log(\cos(c+dx)) + \tan^2(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2*(a + b*Tan[c + d*x]),x]
[Out] -((a*ArcTan[Tan[c + d*x]])/d) + (a*Tan[c + d*x])/d + (b*(2*Log[Cos[c + d*x]
] + Tan[c + d*x]^2))/(2*d)
```

Maple [A]
time = 0.02, size = 49, normalized size = 1.11

method	result	size
norman	$\frac{a\tan(dx+c)}{d} - ax + \frac{b(\tan^2(dx+c))}{2d} - \frac{b\ln(1+\tan^2(dx+c))}{2d}$	48
derivativedivides	$\frac{\frac{b\left(\tan^2(dx+c)\right)}{2} + a\tan(dx+c) - \frac{b\ln\left(1+\tan^2(dx+c)\right)}{2} - a\arctan(\tan(dx+c))}{d}$	49
default	$\frac{\frac{b\left(\tan^2(dx+c)\right)}{2} + a\tan(dx+c) - \frac{b\ln\left(1+\tan^2(dx+c)\right)}{2} - a\arctan(\tan(dx+c))}{d}$	49
risch	$-ibx - ax - \frac{2ibc}{d} + \frac{2ia\,e^{2i(dx+c)} + 2b\,e^{2i(dx+c)} + 2ia}{d\left(e^{2i(dx+c)}+1\right)^2} + \frac{b\ln\left(e^{2i(dx+c)}+1\right)}{d}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/2*b*tan(d*x+c)^2+a*tan(d*x+c)-1/2*b*ln(1+tan(d*x+c)^2)-a*arctan(tan(
d*x+c)))
```

Maxima [A]
time = 0.51, size = 47, normalized size = 1.07

$$\frac{b\tan(dx+c)^2 - 2(dx+c)a - b\log\left(\tan(dx+c)^2+1\right) + 2a\tan(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(b*tan(d*x + c)^2 - 2*(d*x + c)*a - b*log(tan(d*x + c)^2 + 1) + 2*a*tan
(d*x + c))/d
```

Fricas [A]
time = 1.34, size = 47, normalized size = 1.07

$$-\frac{2\,adx-b\tan\left(dx+c\right)^2-b\log\left(\frac{1}{\tan(dx+c)^2+1}\right)-2\,a\tan\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d*x - b*tan(d*x + c)^2 - b*log(1/(tan(d*x + c)^2 + 1)) - 2*a*tan(
d*x + c))/d
```

Sympy [A]
time = 0.08, size = 56, normalized size = 1.27

$$\begin{cases} -ax+\frac{a\tan\left(c+dx\right)}{d}-\frac{b\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{b\tan^2\left(c+dx\right)}{2d} & \text{for } d\neq 0\\ x(a+b\tan\left(c\right))\tan^2\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2*(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise((-a*x + a*tan(c + d*x)/d - b*log(tan(c + d*x)**2 + 1)/(2*d) + b*t
an(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*tan(c))*tan(c)**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 327 vs. 2(42) = 84.
time = 0.72, size = 327, normalized size = 7.43

$$-\frac{2\,adx\tan\left(dx\right)^2\tan\left(c\right)^2-b\log\left(\frac{4\left(\tan(dx)^4\tan(c)^2-2\,\tan(dx)^3\tan(c)+\tan(dx)^2\tan(c)^2+\tan(dx)^2-2\,\tan(dx)\tan(c)+1\right)}{\tan(c)^2+1}\right)\tan\left(dx\right)^2\tan\left(c\right)^2-4\,adx\tan\left(dx\right)\tan\left(c\right)-b\tan\left(dx\right)^2\tan\left(c\right)^2+2\,b\log\left(\frac{4\left(\tan(dx)^4\tan(c)^2-2\,\tan(dx)^3\tan(c)+\tan(dx)^2\tan(c)^2+\tan(dx)^2-2\,\tan(dx)\tan(c)+1\right)}{\tan(c)^2+1}\right)\tan\left(dx\right)\tan\left(c\right)+2\,a\tan\left(dx\right)^2\tan\left(c\right)+2\,a\tan\left(dx\right)\tan\left(c\right)^2+2\,adx-b\tan\left(dx\right)^2-b\tan\left(c\right)^2-b\log\left(\frac{4\left(\tan(dx)^4\tan(c)^2-2\,\tan(dx)^3\tan(c)+\tan(dx)^2\tan(c)^2+\tan(dx)^2-2\,\tan(dx)\tan(c)+1\right)}{\tan(c)^2+1}\right)-2\,a\tan\left(dx\right)-2\,a\tan\left(c\right)-b}{2\left(d\tan\left(dx\right)^2\tan\left(c\right)^2-2\,d\tan\left(dx\right)\tan\left(c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*a*d*x*tan(d*x)^2*tan(c)^2 - b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*
x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(ta
n(c)^2 + 1))*tan(d*x)^2*tan(c)^2 - 4*a*d*x*tan(d*x)*tan(c) - b*tan(d*x)^2*t
an(c)^2 + 2*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2
*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)*ta
n(c) + 2*a*tan(d*x)^2*tan(c) + 2*a*tan(d*x)*tan(c)^2 + 2*a*d*x - b*tan(d*x)
```

```
^2 - b*tan(c)^2 - b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(
d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) - 2*a
*tan(d*x) - 2*a*tan(c) - b)/(d*tan(d*x)^2*tan(c)^2 - 2*d*tan(d*x)*tan(c) +
d)
```

Mupad [B]
time = 4.05, size = 43, normalized size = 0.98

$$\frac{a\tan(c+d\,x)-\frac{b\,\ln\left(\tan(c+d\,x)^2+1\right)}{2}+\frac{b\tan(c+d\,x)^2}{2}-a\,d\,x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + b*tan(c + d*x)),x)
```

```
[Out] (a*tan(c + d*x) - (b*log(tan(c + d*x)^2 + 1))/2 + (b*tan(c + d*x)^2)/2 - a*
d*x)/d
```

3.416 $\int \tan(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=29

$$-bx-\frac{a\log(\cos(c+dx))}{d}+\frac{b\tan(c+dx)}{d}$$

```
[Out] -b*x-a*ln(cos(d*x+c))/d+b*tan(d*x+c)/d
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3606, 3556}

$$-\frac{a\log(\cos(c+dx))}{d}+\frac{b\tan(c+dx)}{d}-bx$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]*(a + b*Tan[c + d*x]),x]
[Out] -(b*x) - (a*Log[Cos[c + d*x]])/d + (b*Tan[c + d*x])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \tan(c+dx)(a+b\tan(c+dx))\,dx &= -bx+\frac{b\tan(c+dx)}{d}+a\int \tan(c+dx)\,dx\\ &= -bx-\frac{a\log(\cos(c+dx))}{d}+\frac{b\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 38, normalized size = 1.31

$$-\frac{b\text{ArcTan}(\tan(c+dx))}{d}-\frac{a\log(\cos(c+dx))}{d}+\frac{b\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]*(a + b*Tan[c + d*x]),x]
```

```
[Out] -((b*ArcTan[Tan[c + d*x]])/d) - (a*Log[Cos[c + d*x]])/d + (b*Tan[c + d*x])/
d
```

Maple [A]
time = 0.02, size = 38, normalized size = 1.31

method	result	size
norman	$\frac{b\tan(dx+c)}{d}-bx+\frac{a\ln(1+\tan^2(dx+c))}{2d}$	34
derivativedivides	$\frac{b\tan(dx+c)+\frac{a\ln\left(1+\tan^2(dx+c)\right)}{2}-b\arctan(\tan(dx+c))}{d}$	38
default	$\frac{b\tan(dx+c)+\frac{a\ln\left(1+\tan^2(dx+c)\right)}{2}-b\arctan(\tan(dx+c))}{d}$	38
risch	$-bx+iax+\frac{2iac}{d}+\frac{2ib}{d\left(e^{2i(dx+c)}+1\right)}-\frac{a\ln\left(e^{2i(dx+c)}+1\right)}{d}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*tan(d*x+c)+1/2*a*ln(1+tan(d*x+c)^2)-b*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.52, size = 37, normalized size = 1.28

$$-\frac{2\,(dx+c)b-a\log\left(\tan\left(dx+c\right)^2+1\right)-2\,b\tan\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*(d*x + c)*b - a*log(tan(d*x + c)^2 + 1) - 2*b*tan(d*x + c))/d
```

Fricas [A]
time = 1.51, size = 35, normalized size = 1.21

$$-\frac{2\,bdx+a\log\left(\frac{1}{\tan(dx+c)^2+1}\right)-2\,b\tan\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*x + a*log(1/(tan(d*x + c)^2 + 1)) - 2*b*tan(d*x + c))/d
```

Sympy [A]
time = 0.07, size = 41, normalized size = 1.41

$$\begin{cases} \frac{a\log\left(\tan^2\left(c+dx\right)+1\right)}{2d} - bx + \frac{b\tan\left(c+dx\right)}{d} & \text{for } d \neq 0 \\ x(a+b\tan\left(c\right))\tan\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise((a*log(tan(c + d*x)**2 + 1)/(2*d) - b*x + b*tan(c + d*x)/d, Ne(d,
 0)), (x*(a + b*tan(c))*tan(c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(29) = 58.
time = 0.72, size = 174, normalized size = 6.00

$$-\frac{2\,bdx\tan\left(dx\right)\tan\left(c\right)+a\log\left(\frac{4\left(\tan(dx)^4\tan(c)^2-2\tan(dx)^3\tan(c)+\tan(dx)^2\tan(c)^2+\tan(dx)^2-2\tan(dx)\tan(c)+1\right)}{\tan(c)^2+1}\right)\tan\left(dx\right)\tan\left(c\right)-2\,bdx-a\log\left(\frac{4\left(\tan(dx)^4\tan(c)^2-2\tan(dx)^3\tan(c)+\tan(dx)^2\tan(c)^2+\tan(dx)^2-2\tan(dx)\tan(c)+1\right)}{\tan(c)^2+1}\right)+2\,b\tan\left(dx\right)+2\,b\tan\left(c\right)}{2\left(d\tan\left(dx\right)\tan\left(c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*d*x*tan(d*x)*tan(c) + a*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3
*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)
^2 + 1))*tan(d*x)*tan(c) - 2*b*d*x - a*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d
*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(t
an(c)^2 + 1)) + 2*b*tan(d*x) + 2*b*tan(c))/(d*tan(d*x)*tan(c) - d)
```

Mupad [B]
time = 4.06, size = 32, normalized size = 1.10

$$\frac{b\tan(c+d\,x)+\frac{a\,\ln\left(\tan(c+d\,x)^2+1\right)}{2}-b\,d\,x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + b*tan(c + d*x)),x)
```

```
[Out] (b*tan(c + d*x) + (a*log(tan(c + d*x)^2 + 1))/2 - b*d*x)/d
```

3.417 $\int (a+b\tan(c+dx))\,dx$

Optimal. Leaf size=17

$$ax-\frac{b\log(\cos(c+dx))}{d}$$

```
[Out] a*x-b*ln(cos(d*x+c))/d
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3556}

$$ax-\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Tan[c + d*x],x]
[Out] a*x - (b*Log[Cos[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\tan(c+dx))\,dx &= ax+b\int \tan(c+dx)\,dx\\ &= ax-\frac{b\log(\cos(c+dx))}{d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 1.00

$$ax-\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Tan[c + d*x],x]
[Out] a*x - (b*Log[Cos[c + d*x]])/d
```

Maple [A]
time = 0.02, size = 22, normalized size = 1.29

method	result	size
default	$ax+\frac{b\ln\left(1+\tan^{2}\left(dx+c\right)\right)}{2d}$	22
norman	$ax+\frac{b\ln\left(1+\tan^{2}\left(dx+c\right)\right)}{2d}$	22
derivativedivides	$\frac{\frac{b\ln\left(1+\tan^{2}\left(dx+c\right)\right)}{2}+a\arctan\left(\tan\left(dx+c\right)\right)}{d}$	29
risch	$ibx+\frac{2ibc}{d}+ax-\frac{b\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{d}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*tan(d*x+c),x,method=_RETURNVERBOSE)
[Out] a*x+1/2*b/d*ln(1+tan(d*x+c)^2)
```

Maxima [A]
time = 0.28, size = 16, normalized size = 0.94

$$ax+\frac{b\log\left(\sec\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(d*x+c),x, algorithm="maxima")
[Out] a*x + b*log(sec(d*x + c))/d
```

Fricas [A]
time = 1.13, size = 27, normalized size = 1.59

$$\frac{2\,adx-b\log\left(\frac{1}{\tan\left(dx+c\right)^{2}+1}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(d*x+c),x, algorithm="fricas")
[Out] 1/2*(2*a*d*x - b*log(1/(tan(d*x + c)^2 + 1)))/d
```

Sympy [A]
time = 0.04, size = 24, normalized size = 1.41

$$ax+b\left(\begin{cases}\frac{\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d} & \text{for}\ d\neq 0\\ x\tan\left(c\right) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(d*x+c),x)
```

```
[Out] a*x + b*Piecewise((log(tan(c + d*x)**2 + 1)/(2*d), Ne(d, 0)), (x*tan(c), Tr
ue))
```

Giac [A]
time = 0.46, size = 18, normalized size = 1.06

$$ax - \frac{b \log(|\cos(dx + c)|)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(d*x+c),x, algorithm="giac")
[Out] a*x - b*log(abs(cos(d*x + c)))/d
```

Mupad [B]
time = 3.98, size = 21, normalized size = 1.24

$$a\,x + \frac{b\,\ln\left(\tan(c + d\,x)^2 + 1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*tan(c + d*x),x)
[Out] a*x + (b*log(tan(c + d*x)^2 + 1))/(2*d)
```

3.418 $\int \cot(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=16

$$bx+\frac{a\log(\sin(c+dx))}{d}$$

[Out] b*x+a*ln(sin(d*x+c))/d

Rubi [A]
time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3612, 3556}

$$\frac{a\log(\sin(c+dx))}{d}+bx$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]*(a + b*Tan[c + d*x]),x]

[Out] b*x + (a*Log[Sin[c + d*x]])/d

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)(a+b\tan(c+dx))\,dx &= bx+a\int \cot(c+dx)\,dx\\ &= bx+\frac{a\log(\sin(c+dx))}{d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 24, normalized size = 1.50

$$bx+\frac{a(\log(\cos(c+dx))+\log(\tan(c+dx)))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Tan[c + d*x]),x]
```

```
[Out] b*x + (a*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]]))/d
```

Maple [A]
time = 0.16, size = 22, normalized size = 1.38

method	result	size
derivativedivides	$\frac{a\ln(\sin(dx+c))+b(dx+c)}{d}$	22
default	$\frac{a\ln(\sin(dx+c))+b(dx+c)}{d}$	22
norman	$bx+\frac{a\ln(\tan(dx+c))}{d}-\frac{a\ln(1+\tan^2(dx+c))}{2d}$	34
risch	$bx-iax-\frac{2iac}{d}+\frac{a\ln(e^{2i(dx+c)}-1)}{d}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*ln(sin(d*x+c))+b*(d*x+c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(16) = 32.
time = 0.55, size = 38, normalized size = 2.38

$$\frac{2\,(dx+c)b-a\log\left(\tan\left(dx+c\right)^2+1\right)+2\,a\log\left(\tan\left(dx+c\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(d*x + c)*b - a*log(tan(d*x + c)^2 + 1) + 2*a*log(tan(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(16) = 32.
time = 1.01, size = 35, normalized size = 2.19

$$\frac{2\,bdx+a\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*x + a*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1)))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(14) = 28$.
time = 0.14, size = 42, normalized size = 2.62

$$\begin{cases} -\frac{a\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d} + \frac{a\log\left(\tan\left(c+dx\right)\right)}{d} + bx & \text{for}\ d \neq 0 \\ x(a + b\tan\left(c\right))\cot\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise((-a*log(tan(c + d*x)**2 + 1)/(2*d) + a*log(tan(c + d*x))/d + b*x,
 Ne(d, 0)), (x*(a + b*tan(c))*cot(c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(16) = 32$.
time = 0.52, size = 42, normalized size = 2.62

$$\frac{(dx+c)b - a\log\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{2} + 1\right) + a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)*b - a*log(tan(1/2*d*x + 1/2*c)^2 + 1) + a*log(abs(tan(1/2*d*x +
1/2*c))))/d
```

Mupad [B]
time = 4.15, size = 79, normalized size = 4.94

$$\frac{a\,\ln\left(\mathrm{tan}(c+d\,x)\right)}{d}-\frac{a\,\ln\left(\mathrm{tan}(c+d\,x)-\mathrm{i}\right)}{2\,d}-\frac{a\,\ln\left(\mathrm{tan}(c+d\,x)+1\mathrm{i}\right)}{2\,d}-\frac{b\,\ln\left(\mathrm{tan}(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{2\,d}+\frac{b\,\ln\left(\mathrm{tan}(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + b*tan(c + d*x)),x)
```

```
[Out] (a*log(tan(c + d*x)))/d - (a*log(tan(c + d*x) - 1i))/(2*d) - (a*log(tan(c +
 d*x) + 1i))/(2*d) - (b*log(tan(c + d*x) - 1i)*1i)/(2*d) + (b*log(tan(c + d
*x) + 1i)*1i)/(2*d)
```

3.419 $\int \cot^2(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=29

$$-ax-\frac{a\cot(c+dx)}{d}+\frac{b\log(\sin(c+dx))}{d}$$

```
[Out] -a*x-a*cot(d*x+c)/d+b*ln(sin(d*x+c))/d
```

Rubi [A]
time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.158$, Rules used = {3610, 3612, 3556}

$$-\frac{a\cot(c+dx)}{d}-ax+\frac{b\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Tan[c + d*x]),x]
```

```
[Out] -(a*x) - (a*Cot[c + d*x])/d + (b*Log[Sin[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \cot^2(c+dx)(a+b\tan(c+dx))\,dx = -\frac{a\cot(c+dx)}{d} + \int \cot(c+dx)(b-a\tan(c+dx))\,dx$$
$$= -ax - \frac{a\cot(c+dx)}{d} + b\int \cot(c+dx)\,dx$$
$$= -ax - \frac{a\cot(c+dx)}{d} + \frac{b\log(\sin(c+dx))}{d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.13, size = 51, normalized size = 1.76

$$-\frac{a\cot(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)}{d} + \frac{b(\log(\cos(c+dx))+\log(\tan(c+dx)))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Tan[c + d*x]),x]
```

```
[Out] -((a*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2])/d) + (b
*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]]))/d
```

Maple [A]
time = 0.12, size = 33, normalized size = 1.14

method	result	size
derivativedivides	$\frac{a(-\cot(dx+c)-dx-c)+b\ln(\sin(dx+c))}{d}$	33
default	$\frac{a(-\cot(dx+c)-dx-c)+b\ln(\sin(dx+c))}{d}$	33
risch	$-ibx-ax-\frac{2ibc}{d}-\frac{2ia}{d\left(e^{2i(dx+c)}-1\right)}+\frac{b\ln\left(e^{2i(dx+c)}-1\right)}{d}$	56
norman	$\frac{-\frac{a}{d}-ax\tan(dx+c)}{\tan(dx+c)}+\frac{b\ln(\tan(dx+c))}{d}-\frac{b\ln\left(1+\tan^2(dx+c)\right)}{2d}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-cot(d*x+c)-d*x-c)+b*ln(sin(d*x+c)))
```

Maxima [A]
time = 0.52, size = 48, normalized size = 1.66

$$-\frac{2\,(dx+c)a + b\log\left(\tan\left(dx+c\right)^2+1\right) - 2\,b\log\left(\tan\left(dx+c\right)\right) + \frac{2\,a}{\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*(d*x + c)*a + b*log(tan(d*x + c)^2 + 1) - 2*b*log(tan(d*x + c)) + 2
*a/tan(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(29) = 58$.
time = 1.21, size = 59, normalized size = 2.03

$$-\frac{2\,adx\tan\left(dx+c\right)-b\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)+2\,a}{2\,d\tan\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d*x*tan(d*x + c) - b*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan
(d*x + c) + 2*a)/(d*tan(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(24) = 48$.
time = 0.35, size = 70, normalized size = 2.41

$$\begin{cases} \tilde{\infty}ax & \text{for}\ (c=0\vee c=-dx)\wedge(c=-dx\vee d=0) \\ x(a+b\tan\left(c\right))\cot^2\left(c\right) & \text{for}\ d=0 \\ -ax-\frac{a}{d\tan\left(c+dx\right)}-\frac{b\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{b\log\left(\tan\left(c+dx\right)\right)}{d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise((zoo*a*x, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (
x*(a + b*tan(c))*cot(c)**2, Eq(d, 0)), (-a*x - a/(d*tan(c + d*x)) - b*log(t
an(c + d*x)**2 + 1)/(2*d) + b*log(tan(c + d*x))/d, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(29) = 58$.
time = 0.58, size = 83, normalized size = 2.86

$$-\frac{2\left(dx+c\right)a+2\,b\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)-2\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*(d*x + c)*a + 2*b*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 2*b*log(abs(tan
(1/2*d*x + 1/2*c))) - a*tan(1/2*d*x + 1/2*c) + (2*b*tan(1/2*d*x + 1/2*c) +
a)/tan(1/2*d*x + 1/2*c))/d
```

Mupad [B]
time = 4.10, size = 70, normalized size = 2.41

$$\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-\frac{b}{2}+\frac{a\,1\mathrm{i}}{2}\right)}{d}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(\frac{b}{2}+\frac{a\,1\mathrm{i}}{2}\right)}{d}+\frac{b\,\ln\left(\tan(c+d\,x)\right)}{d}-\frac{a\cot(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + b*tan(c + d*x)),x)
```

```
[Out] (log(tan(c + d*x) - 1i)*((a*1i)/2 - b/2))/d - (log(tan(c + d*x) + 1i)*((a*1
i)/2 + b/2))/d + (b*log(tan(c + d*x)))/d - (a*cot(c + d*x))/d
```

3.420 $\int \cot^3(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=46

$$-bx-\frac{b\cot(c+dx)}{d}-\frac{a\cot^2(c+dx)}{2d}-\frac{a\log(\sin(c+dx))}{d}$$

```
[Out] -b*x-b*cot(d*x+c)/d-1/2*a*cot(d*x+c)^2/d-a*ln(sin(d*x+c))/d
```

Rubi [A]
time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3610, 3612, 3556}

$$-\frac{a\cot^2(c+dx)}{2d}-\frac{a\log(\sin(c+dx))}{d}-\frac{b\cot(c+dx)}{d}-bx$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

```
[Out] -(b*x) - (b*Cot[c + d*x])/d - (a*Cot[c + d*x]^2)/(2*d) - (a*Log[Sin[c + d*x
]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \cot^3(c+dx)(a+b\tan(c+dx))\,dx = -\frac{a\cot^2(c+dx)}{2d} + \int \cot^2(c+dx)(b-a\tan(c+dx))\,dx$$
$$= -\frac{b\cot(c+dx)}{d} - \frac{a\cot^2(c+dx)}{2d} + \int \cot(c+dx)(-a-b\tan(c+dx$$
$$= -bx - \frac{b\cot(c+dx)}{d} - \frac{a\cot^2(c+dx)}{2d} - a\int \cot(c+dx)\,dx$$
$$= -bx - \frac{b\cot(c+dx)}{d} - \frac{a\cot^2(c+dx)}{2d} - \frac{a\log(\sin(c+dx))}{d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.29, size = 66, normalized size = 1.43

$$-\frac{b\cot(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)}{d} - \frac{a(\cot^2(c+dx)+2\log(\cos(c+dx))+2\log(\tan(c+dx)))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Tan[c + d*x]),x]
[Out] -((b*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2])/d) - (a
*(Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[Tan[c + d*x]]))/(2*d)
```

Maple [A]
time = 0.16, size = 46, normalized size = 1.00

method	result	size
derivativedivides	$\frac{a\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+b(-\cot(dx+c)-dx-c)}{d}$	46
default	$\frac{a\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+b(-\cot(dx+c)-dx-c)}{d}$	46
norman	$\frac{-\frac{a}{2d}-bx\left(\tan^2(dx+c)\right)-\frac{b\tan(dx+c)}{d}}{\tan(dx+c)^2}-\frac{a\ln(\tan(dx+c))}{d}+\frac{a\ln\left(1+\tan^2(dx+c)\right)}{2d}$	72
risch	$-bx+iax+\frac{2iac}{d}-\frac{2i\left(ia\,e^{2i(dx+c)}+b\,e^{2i(dx+c)}-b\right)}{d\left(e^{2i(dx+c)}-1\right)^2}-\frac{a\ln\left(e^{2i(dx+c)}-1\right)}{d}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+b*(-cot(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.52, size = 58, normalized size = 1.26

$$-\frac{2\,(dx+c)b - a\log\left(\tan\left(dx+c\right)^{2}+1\right) + 2\,a\log\left(\tan\left(dx+c\right)\right) + \frac{2\,b\tan(dx+c)+a}{\tan(dx+c)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*(d*x + c)*b - a*log(tan(d*x + c)^2 + 1) + 2*a*log(tan(d*x + c)) + (
2*b*tan(d*x + c) + a)/tan(d*x + c)^2)/d
```

Fricas [A]
time = 0.98, size = 72, normalized size = 1.57

$$-\frac{a\log\left(\frac{\tan(dx+c)^{2}}{\tan(dx+c)^{2}+1}\right)\tan\left(dx+c\right)^{2} + (2\,bdx+a)\tan\left(dx+c\right)^{2} + 2\,b\tan\left(dx+c\right) + a}{2\,d\tan\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^2 + (2*b*d*x
+ a)*tan(d*x + c)^2 + 2*b*tan(d*x + c) + a)/(d*tan(d*x + c)^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(39) = 78$.
time = 0.50, size = 83, normalized size = 1.80

$$\begin{cases} \tilde{\infty} ax & \text{for}\: (c=0 \vee c=-dx) \wedge (c=-dx \vee d=0) \\ x(a+b\tan\left(c\right))\cot^{3}\left(c\right) & \text{for}\: d=0 \\ \frac{a\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d} - \frac{a\log\left(\tan\left(c+dx\right)\right)}{d} - \frac{a}{2d\tan^{2}\left(c+dx\right)} - bx - \frac{b}{d\tan\left(c+dx\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise((zoo*a*x, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (
x*(a + b*tan(c))*cot(c)**3, Eq(d, 0)), (a*log(tan(c + d*x)**2 + 1)/(2*d) -
a*log(tan(c + d*x))/d - a/(2*d*tan(c + d*x)**2) - b*x - b/(d*tan(c + d*x)),
 True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(44) = 88$.
time = 0.63, size = 113, normalized size = 2.46

$$-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2} + 8\,(dx+c)b - 8\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right) + 8\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right) - 4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - \frac{12\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/8*(a*tan(1/2*d*x + 1/2*c)^2 + 8*(d*x + c)*b - 8*a*log(tan(1/2*d*x + 1/2*
c)^2 + 1) + 8*a*log(abs(tan(1/2*d*x + 1/2*c))) - 4*b*tan(1/2*d*x + 1/2*c) -
 (12*a*tan(1/2*d*x + 1/2*c)^2 - 4*b*tan(1/2*d*x + 1/2*c) - a)/tan(1/2*d*x +
 1/2*c)^2)/d
```

Mupad [B]
time = 4.07, size = 83, normalized size = 1.80

$$-\frac{\cot(c+d\,x)^2\,\left(\frac{a}{2}+b\tan(c+d\,x)\right)}{d}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(\frac{a}{2}+\frac{b\,1\mathrm{i}}{2}\right)}{d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(\frac{a}{2}-\frac{b\,1\mathrm{i}}{2}\right)}{d}-\frac{a\,\ln\left(\tan(c+d\,x)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + b*tan(c + d*x)),x)
```

```
[Out] (log(tan(c + d*x) - 1i)*(a/2 + (b*1i)/2))/d - (cot(c + d*x)^2*(a/2 + b*tan(
c + d*x)))/d + (log(tan(c + d*x) + 1i)*(a/2 - (b*1i)/2))/d - (a*log(tan(c +
 d*x)))/d
```

3.421 $\int \cot^4(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=60

$$ax+\frac{a\cot(c+dx)}{d}-\frac{b\cot^2(c+dx)}{2d}-\frac{a\cot^3(c+dx)}{3d}-\frac{b\log(\sin(c+dx))}{d}$$

```
[Out] a*x+a*cot(d*x+c)/d-1/2*b*cot(d*x+c)^2/d-1/3*a*cot(d*x+c)^3/d-b*ln(sin(d*x+c
))/d
```

Rubi [A]
time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.158$, Rules used = {3610, 3612, 3556}

$$-\frac{a\cot^3(c+dx)}{3d}+\frac{a\cot(c+dx)}{d}+ax-\frac{b\cot^2(c+dx)}{2d}-\frac{b\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

```
[Out] a*x + (a*Cot[c + d*x])/d - (b*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]^3)/(3
*d) - (b*Log[Sin[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \cot^4(c+dx)(a+b\tan(c+dx))\,dx = -\frac{a\cot^3(c+dx)}{3d} + \int \cot^3(c+dx)(b-a\tan(c+dx))\,dx$$
$$= -\frac{b\cot^2(c+dx)}{2d} - \frac{a\cot^3(c+dx)}{3d} + \int \cot^2(c+dx)(-a-b\tan(c+$$
$$= \frac{a\cot(c+dx)}{d} - \frac{b\cot^2(c+dx)}{2d} - \frac{a\cot^3(c+dx)}{3d} + \int \cot(c+dx)(-$$
$$= ax + \frac{a\cot(c+dx)}{d} - \frac{b\cot^2(c+dx)}{2d} - \frac{a\cot^3(c+dx)}{3d} - b\int \cot(c+$$
$$= ax + \frac{a\cot(c+dx)}{d} - \frac{b\cot^2(c+dx)}{2d} - \frac{a\cot^3(c+dx)}{3d} - \frac{b\log(\sin(c-}{d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.30, size = 70, normalized size = 1.17

$$-\frac{a\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(c+dx)\right)}{3d} - \frac{b(\cot^2(c+dx)+2\log(\cos(c+dx))+2\log(\tan(c+dx)))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Tan[c + d*x]),x]
[Out] -1/3*(a*Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/d
 - (b*(Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[Tan[c + d*x]]))/(2*d)
```

Maple [A]
time = 0.16, size = 51, normalized size = 0.85

method	result	size
derivativedivides	$\frac{a\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+b\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)}{d}$	51
default	$\frac{a\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+b\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)}{d}$	51
norman	$\frac{ax\left(\tan^3(dx+c)\right)+\frac{a\left(\tan^2(dx+c)\right)}{d}-\frac{a}{3d}-\frac{b\tan(dx+c)}{2d}}{\tan(dx+c)^3}-\frac{b\ln(\tan(dx+c))}{d}+\frac{b\ln\left(1+\tan^2(dx+c)\right)}{2d}$	84
risch	$ibx+ax+\frac{2ibc}{d}+\frac{4ia\,e^{4i(dx+c)}+2b\,e^{4i(dx+c)}-4ia\,e^{2i(dx+c)}-2b\,e^{2i(dx+c)}+\frac{8ia}{3}}{d\left(e^{2i(dx+c)}-1\right)^3}-\frac{b\ln\left(e^{2i(dx+c)}-1\right)}{d}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+b*(-1/2*cot(d*x+c)^2-ln(sin(d*x
+c))))
```

Maxima [A]
time = 0.51, size = 71, normalized size = 1.18

$$\frac{6(dx+c)a+3b\log\left(\tan(dx+c)^2+1\right)-6b\log(\tan(dx+c))+\frac{6a\tan(dx+c)^2-3b\tan(dx+c)-2a}{\tan(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(6*(d*x + c)*a + 3*b*log(tan(d*x + c)^2 + 1) - 6*b*log(tan(d*x + c)) +
(6*a*tan(d*x + c)^2 - 3*b*tan(d*x + c) - 2*a)/tan(d*x + c)^3)/d
```

Fricas [A]
time = 0.77, size = 89, normalized size = 1.48

$$-\frac{3b\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan(dx+c)^3-3(2adx-b)\tan(dx+c)^3-6a\tan(dx+c)^2+3b\tan(dx+c)+2a}{6d\tan(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(3*b*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^3 - 3*(2*a*
d*x - b)*tan(d*x + c)^3 - 6*a*tan(d*x + c)^2 + 3*b*tan(d*x + c) + 2*a)/(d*t
an(d*x + c)^3)
```

Sympy [A]
time = 0.80, size = 97, normalized size = 1.62

$$\begin{cases} \tilde{\infty}ax & \text{for}\: (c=0 \vee c=-dx) \wedge (c=-dx \vee d=0) \\ x(a+b\tan(c))\cot^4(c) & \text{for}\: d=0 \\ ax+\frac{a}{d\tan(c+dx)}-\frac{a}{3d\tan^3(c+dx)}+\frac{b\log(\tan^2(c+dx)+1)}{2d}-\frac{b\log(\tan(c+dx))}{d}-\frac{b}{2d\tan^2(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise((zoo*a*x, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (
x*(a + b*tan(c))*cot(c)**4, Eq(d, 0)), (a*x + a/(d*tan(c + d*x)) - a/(3*d*t
an(c + d*x)**3) + b*log(tan(c + d*x)**2 + 1)/(2*d) - b*log(tan(c + d*x))/d
- b/(2*d*tan(c + d*x)**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(56) = 112.
time = 0.62, size = 140, normalized size = 2.33

$$\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+24(dx+c)a+24b\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)-24b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)-15a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{44b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+15a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*d*x + 1/2*c)^3 - 3*b*tan(1/2*d*x + 1/2*c)^2 + 24*(d*x + c)*
a + 24*b*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 24*b*log(abs(tan(1/2*d*x + 1/2*c
))) - 15*a*tan(1/2*d*x + 1/2*c) + (44*b*tan(1/2*d*x + 1/2*c)^3 + 15*a*tan(1
/2*d*x + 1/2*c)^2 - 3*b*tan(1/2*d*x + 1/2*c) - a)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B]
time = 3.98, size = 96, normalized size = 1.60

$$-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-\frac{b}{2}+\frac{a\,1\mathrm{i}}{2}\right)}{d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(\frac{b}{2}+\frac{a\,1\mathrm{i}}{2}\right)}{d}-\frac{b\,\ln\left(\tan(c+d\,x)\right)}{d}-\frac{\cot(c+d\,x)^3\,\left(-a\tan(c+d\,x)^2+\frac{b\tan(c+d\,x)}{2}+\frac{a}{3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + b*tan(c + d*x)),x)
```

```
[Out] (log(tan(c + d*x) + 1i)*((a*1i)/2 + b/2))/d - (log(tan(c + d*x) - 1i)*((a*1
i)/2 - b/2))/d - (b*log(tan(c + d*x)))/d - (cot(c + d*x)^3*(a/3 + (b*tan(c
+ d*x))/2 - a*tan(c + d*x)^2))/d
```

3.422 $\int \cot^5(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=75

$$bx+\frac{b\cot(c+dx)}{d}+\frac{a\cot^2(c+dx)}{2d}-\frac{b\cot^3(c+dx)}{3d}-\frac{a\cot^4(c+dx)}{4d}+\frac{a\log(\sin(c+dx))}{d}$$

```
[Out] b*x+b*cot(d*x+c)/d+1/2*a*cot(d*x+c)^2/d-1/3*b*cot(d*x+c)^3/d-1/4*a*cot(d*x+
c)^4/d+a*ln(sin(d*x+c))/d
```

Rubi [A]
time = 0.08, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3610, 3612, 3556}

$$-\frac{a\cot^4(c+dx)}{4d}+\frac{a\cot^2(c+dx)}{2d}+\frac{a\log(\sin(c+dx))}{d}-\frac{b\cot^3(c+dx)}{3d}+\frac{b\cot(c+dx)}{d}+bx$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```

```
[Out] b*x + (b*Cot[c + d*x])/d + (a*Cot[c + d*x]^2)/(2*d) - (b*Cot[c + d*x]^3)/(3
*d) - (a*Cot[c + d*x]^4)/(4*d) + (a*Log[Sin[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \cot^5(c+dx)(a+b\tan(c+dx))\,dx = -\frac{a\cot^4(c+dx)}{4d} + \int \cot^4(c+dx)(b-a\tan(c+dx))\,dx$$
$$= -\frac{b\cot^3(c+dx)}{3d} - \frac{a\cot^4(c+dx)}{4d} + \int \cot^3(c+dx)(-a-b\tan(c+$$
$$= \frac{a\cot^2(c+dx)}{2d} - \frac{b\cot^3(c+dx)}{3d} - \frac{a\cot^4(c+dx)}{4d} + \int \cot^2(c+dx)($$
$$= \frac{b\cot(c+dx)}{d} + \frac{a\cot^2(c+dx)}{2d} - \frac{b\cot^3(c+dx)}{3d} - \frac{a\cot^4(c+dx)}{4d} +$$
$$= bx + \frac{b\cot(c+dx)}{d} + \frac{a\cot^2(c+dx)}{2d} - \frac{b\cot^3(c+dx)}{3d} - \frac{a\cot^4(c+d}{4d}$$
$$= bx + \frac{b\cot(c+dx)}{d} + \frac{a\cot^2(c+dx)}{2d} - \frac{b\cot^3(c+dx)}{3d} - \frac{a\cot^4(c+d}{4d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.41, size = 82, normalized size = 1.09

$$-\frac{b\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(c+dx)\right)}{3d} + \frac{a(2\cot^2(c+dx)-\cot^4(c+dx)+4\log(\cos(c+dx))+4\log(\tan(c+dx)))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Tan[c + d*x]),x]
[Out] -1/3*(b*Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/d
 + (a*(2*Cot[c + d*x]^2 - Cot[c + d*x]^4 + 4*Log[Cos[c + d*x]] + 4*Log[Tan[
c + d*x]]))/(4*d)
```

Maple [A]
time = 0.15, size = 59, normalized size = 0.79

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+b\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)}{d}$
default	$\frac{a\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+b\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)}{d}$
norman	$\frac{bx\left(\tan^4(dx+c)\right)+\frac{b\left(\tan^3(dx+c)\right)}{d}-\frac{a}{4d}+\frac{a\left(\tan^2(dx+c)\right)}{2d}-\frac{b\tan(dx+c)}{3d}}{\tan(dx+c)^4}+\frac{a\ln(\tan(dx+c))}{d}-\frac{a\ln(1+\tan^2(dx+c))}{2d}$
risch	$bx-iax-\frac{2iac}{d}+\frac{4i\left(3ia\,e^{6i(dx+c)}+3b\,e^{6i(dx+c)}-3ia\,e^{4i(dx+c)}-6b\,e^{4i(dx+c)}+3ia\,e^{2i(dx+c)}+5b\,e^{2i(dx+c)}-2b\right)}{3d\left(e^{2i(dx+c)}-1\right)^4}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+b*(-1/3*cot(d*x+
c)^3+cot(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.52, size = 82, normalized size = 1.09

$$\frac{12\,(dx+c)b-6\,a\log\left(\tan\left(dx+c\right)^{2}+1\right)+12\,a\log\left(\tan\left(dx+c\right)\right)+\frac{12\,b\tan(dx+c)^{3}+6\,a\tan(dx+c)^{2}-4\,b\tan(dx+c)-3\,a}{\tan(dx+c)^{4}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(12*(d*x + c)*b - 6*a*log(tan(d*x + c)^2 + 1) + 12*a*log(tan(d*x + c))
 + (12*b*tan(d*x + c)^3 + 6*a*tan(d*x + c)^2 - 4*b*tan(d*x + c) - 3*a)/tan(
d*x + c)^4)/d
```

Fricas [A]
time = 0.65, size = 100, normalized size = 1.33

$$\frac{6\,a\log\left(\frac{\tan(dx+c)^{2}}{\tan(dx+c)^{2}+1}\right)\tan\left(dx+c\right)^{4}+3\left(4\,bdx+3\,a\right)\tan\left(dx+c\right)^{4}+12\,b\tan\left(dx+c\right)^{3}+6\,a\tan\left(dx+c\right)^{2}-4\,b\tan\left(dx+c\right)-3\,a}{12\,d\tan\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(6*a*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^4 + 3*(4*b*
d*x + 3*a)*tan(d*x + c)^4 + 12*b*tan(d*x + c)^3 + 6*a*tan(d*x + c)^2 - 4*b*
tan(d*x + c) - 3*a)/(d*tan(d*x + c)^4)
```

Sympy [A]
time = 1.03, size = 110, normalized size = 1.47

$$\begin{cases} \tilde{\infty}ax & \text{for}\ (c=0\vee c=-dx)\wedge(c=-dx\vee d=0) \\ x(a+b\tan(c))\cot^{5}(c) & \text{for}\ d=0 \\ -\frac{a\log(\tan^{2}(c+dx)+1)}{2d}+\frac{a\log(\tan(c+dx))}{d}+\frac{a}{2d\tan^{2}(c+dx)}-\frac{a}{4d\tan^{4}(c+dx)}+bx+\frac{b}{d\tan(c+dx)}-\frac{b}{3d\tan^{3}(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise((zoo*a*x, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (
x*(a + b*tan(c))*cot(c)**5, Eq(d, 0)), (-a*log(tan(c + d*x)**2 + 1)/(2*d) +
 a*log(tan(c + d*x))/d + a/(2*d*tan(c + d*x)**2) - a/(4*d*tan(c + d*x)**4)
+ b*x + b/(d*tan(c + d*x)) - b/(3*d*tan(c + d*x)**3), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(69) = 138.
time = 0.77, size = 169, normalized size = 2.25

$$-\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-8\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-36\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-192\,(dx+c)b+192\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)-192\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+120\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{400\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-120\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-36\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+8\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/192*(3*a*tan(1/2*d*x + 1/2*c)^4 - 8*b*tan(1/2*d*x + 1/2*c)^3 - 36*a*tan(
1/2*d*x + 1/2*c)^2 - 192*(d*x + c)*b + 192*a*log(tan(1/2*d*x + 1/2*c)^2 + 1
) - 192*a*log(abs(tan(1/2*d*x + 1/2*c))) + 120*b*tan(1/2*d*x + 1/2*c) + (40
0*a*tan(1/2*d*x + 1/2*c)^4 - 120*b*tan(1/2*d*x + 1/2*c)^3 - 36*a*tan(1/2*d*
x + 1/2*c)^2 + 8*b*tan(1/2*d*x + 1/2*c) + 3*a)/tan(1/2*d*x + 1/2*c)^4)/d
```

Mupad [B]
time = 4.11, size = 107, normalized size = 1.43

$$\frac{a\ln\left(\tan(c+d\,x)\right)}{d}-\frac{\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)\,\left(\frac{a}{2}-\frac{b\,1{}\mathrm{i}}{2}\right)}{d}-\frac{\cot(c+d\,x)^4\,\left(-b\tan(c+d\,x)^3-\frac{a\tan(c+d\,x)^2}{2}+\frac{b\tan(c+d\,x)}{3}+\frac{a}{4}\right)}{d}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(\frac{a}{2}+\frac{b\,1{}\mathrm{i}}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(a + b*tan(c + d*x)),x)
```

```
[Out] (a*log(tan(c + d*x)))/d - (log(tan(c + d*x) + 1i)*(a/2 - (b*1i)/2))/d - (co
t(c + d*x)^4*(a/4 + (b*tan(c + d*x))/3 - (a*tan(c + d*x)^2)/2 - b*tan(c + d
*x)^3))/d - (log(tan(c + d*x) - 1i)*(a/2 + (b*1i)/2))/d
```

3.423 $\int \cot^6(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=93

$$-ax-\frac{a\cot(c+dx)}{d}+\frac{b\cot^2(c+dx)}{2d}+\frac{a\cot^3(c+dx)}{3d}-\frac{b\cot^4(c+dx)}{4d}-\frac{a\cot^5(c+dx)}{5d}+\frac{b\log(\sin(c+dx))}{d}$$

```
[Out] -a*x-a*cot(d*x+c)/d+1/2*b*cot(d*x+c)^2/d+1/3*a*cot(d*x+c)^3/d-1/4*b*cot(d*x
+c)^4/d-1/5*a*cot(d*x+c)^5/d+b*ln(sin(d*x+c))/d
```

Rubi [A]
time = 0.10, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3610, 3612, 3556}

$$-\frac{a\cot^5(c+dx)}{5d}+\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-ax-\frac{b\cot^4(c+dx)}{4d}+\frac{b\cot^2(c+dx)}{2d}+\frac{b\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + b*Tan[c + d*x]),x]
```

```
[Out] -(a*x) - (a*Cot[c + d*x])/d + (b*Cot[c + d*x]^2)/(2*d) + (a*Cot[c + d*x]^3)
/(3*d) - (b*Cot[c + d*x]^4)/(4*d) - (a*Cot[c + d*x]^5)/(5*d) + (b*Log[Sin[c
 + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \cot^6(c+dx)(a+b\tan(c+dx))\,dx = -\frac{a\cot^5(c+dx)}{5d} + \int \cot^5(c+dx)(b-a\tan(c+dx))\,dx$$
$$= -\frac{b\cot^4(c+dx)}{4d} - \frac{a\cot^5(c+dx)}{5d} + \int \cot^4(c+dx)(-a-b\tan(c+$$
$$= \frac{a\cot^3(c+dx)}{3d} - \frac{b\cot^4(c+dx)}{4d} - \frac{a\cot^5(c+dx)}{5d} + \int \cot^3(c+dx)($$
$$= \frac{b\cot^2(c+dx)}{2d} + \frac{a\cot^3(c+dx)}{3d} - \frac{b\cot^4(c+dx)}{4d} - \frac{a\cot^5(c+dx)}{5d} +$$
$$= -\frac{a\cot(c+dx)}{d} + \frac{b\cot^2(c+dx)}{2d} + \frac{a\cot^3(c+dx)}{3d} - \frac{b\cot^4(c+dx)}{4d}$$
$$= -ax - \frac{a\cot(c+dx)}{d} + \frac{b\cot^2(c+dx)}{2d} + \frac{a\cot^3(c+dx)}{3d} - \frac{b\cot^4(c+}{4d}$$
$$= -ax - \frac{a\cot(c+dx)}{d} + \frac{b\cot^2(c+dx)}{2d} + \frac{a\cot^3(c+dx)}{3d} - \frac{b\cot^4(c+}{4d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.38, size = 82, normalized size = 0.88

$$-\frac{a\cot^5(c+dx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\tan^2(c+dx)\right)}{5d} + \frac{b(2\cot^2(c+dx)-\cot^4(c+dx)+4\log(\cos(c+dx))+4\log(\tan(c+dx)))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + b*Tan[c + d*x]),x]
```

```
[Out] -1/5*(a*Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x]^2])/d
 + (b*(2*Cot[c + d*x]^2 - Cot[c + d*x]^4 + 4*Log[Cos[c + d*x]] + 4*Log[Tan[
c + d*x]]))/(4*d)
```

Maple [A]
time = 0.15, size = 74, normalized size = 0.80

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-dx-c\right)+b\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)}{d}$
default	$\frac{a\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-dx-c\right)+b\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)}{d}$
norman	$\frac{-\frac{a}{5d}-ax\left(\tan^5(dx+c)\right)+\frac{a\left(\tan^2(dx+c)\right)}{3d}-\frac{a\left(\tan^4(dx+c)\right)}{d}-\frac{b\tan(dx+c)}{4d}+\frac{b\left(\tan^3(dx+c)\right)}{2d}}{\tan(dx+c)^5}+\frac{b\ln(\tan(dx+c))}{d}-\frac{b\ln(1+\tan}{2d}$

risch	$-ibx-ax-\frac{2ibc}{d}-\frac{2\left(45ia\,e^{8i(dx+c)}+30b\,e^{8i(dx+c)}-90ia\,e^{6i(dx+c)}-60b\,e^{6i(dx+c)}+140ia\,e^{4i(dx+c)}+60b\,e^{4i(dx+}\right.}{15d\left(e^{2i(dx+c)}-1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c)+b*(-1/4*cot(d*
x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c))))
```

Maxima [A]
time = 0.53, size = 93, normalized size = 1.00

$$-\frac{60\,(dx+c)a+30\,b\log\left(\tan\left(dx+c\right)^2+1\right)-60\,b\log\left(\tan\left(dx+c\right)\right)+\frac{60\,a\tan(dx+c)^4-30\,b\tan(dx+c)^3-20\,a\tan(dx+c)^2+15\,b\tan(dx+c)+12\,a}{\tan(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="maxima")
[Out] -1/60*(60*(d*x + c)*a + 30*b*log(tan(d*x + c)^2 + 1) - 60*b*log(tan(d*x + c
)) + (60*a*tan(d*x + c)^4 - 30*b*tan(d*x + c)^3 - 20*a*tan(d*x + c)^2 + 15*
b*tan(d*x + c) + 12*a)/tan(d*x + c)^5)/d
```

Fricas [A]
time = 0.92, size = 111, normalized size = 1.19

$$\frac{30\,b\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^5-15\left(4\,adx-3\,b\right)\tan\left(dx+c\right)^5-60\,a\tan\left(dx+c\right)^4+30\,b\tan\left(dx+c\right)^3+20\,a\tan\left(dx+c\right)^2-15\,b\tan\left(dx+c\right)-12\,a}{60\,d\tan\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="fricas")
[Out] 1/60*(30*b*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^5 - 15*(4*
a*d*x - 3*b)*tan(d*x + c)^5 - 60*a*tan(d*x + c)^4 + 30*b*tan(d*x + c)^3 + 2
0*a*tan(d*x + c)^2 - 15*b*tan(d*x + c) - 12*a)/(d*tan(d*x + c)^5)
```

Sympy [A]
time = 1.73, size = 124, normalized size = 1.33

$$\begin{cases} \tilde{\infty}ax & \text{for}\: \left(c = 0 \vee c = -dx\right) \wedge \left(c = -dx \vee d = 0\right) \\ x\left(a+b\tan\left(c\right)\right)\cot^6\left(c\right) & \text{for}\: d = 0 \\ -ax-\frac{a}{d\tan\left(c+dx\right)}+\frac{a}{3d\tan^3\left(c+dx\right)}-\frac{a}{5d\tan^5\left(c+dx\right)}-\frac{b\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{b\log\left(\tan\left(c+dx\right)\right)}{d}+\frac{b}{2d\tan^2\left(c+dx\right)}-\frac{b}{4d\tan^4\left(c+dx\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+b*tan(d*x+c)),x)
[Out] Piecewise((zoo*a*x, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (
x*(a + b*tan(c))*cot(c)**6, Eq(d, 0)), (-a*x - a/(d*tan(c + d*x)) + a/(3*d*
```

```
tan(c + d*x)**3) - a/(5*d*tan(c + d*x)**5) - b*log(tan(c + d*x)**2 + 1)/(2*
d) + b*log(tan(c + d*x))/d + b/(2*d*tan(c + d*x)**2) - b/(4*d*tan(c + d*x)*
*4), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 198 vs. 2(85) = 170.
time = 0.79, size = 198, normalized size = 2.13

$$\frac{6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-15\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-70\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+180\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-960\,(dx+c)a-960\,b\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)+960\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+660\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{2192\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+660\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-180\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-70\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+15\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*(6*a*tan(1/2*d*x + 1/2*c)^5 - 15*b*tan(1/2*d*x + 1/2*c)^4 - 70*a*tan(
1/2*d*x + 1/2*c)^3 + 180*b*tan(1/2*d*x + 1/2*c)^2 - 960*(d*x + c)*a - 960*b
*log(tan(1/2*d*x + 1/2*c)^2 + 1) + 960*b*log(abs(tan(1/2*d*x + 1/2*c))) + 6
60*a*tan(1/2*d*x + 1/2*c) - (2192*b*tan(1/2*d*x + 1/2*c)^5 + 660*a*tan(1/2*
d*x + 1/2*c)^4 - 180*b*tan(1/2*d*x + 1/2*c)^3 - 70*a*tan(1/2*d*x + 1/2*c)^2
 + 15*b*tan(1/2*d*x + 1/2*c) + 6*a)/tan(1/2*d*x + 1/2*c)^5)/d
```

Mupad [B]
time = 4.14, size = 116, normalized size = 1.25

$$\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-\frac{b}{2}+\frac{a\,1\mathrm{i}}{2}\right)}{d}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(\frac{b}{2}+\frac{a\,1\mathrm{i}}{2}\right)}{d}+\frac{b\,\ln\left(\tan(c+d\,x)\right)}{d}-\frac{\cot(c+d\,x)^{5}\,\left(a\tan(c+d\,x)^{4}-\frac{b\tan(c+d\,x)^{3}}{2}-\frac{a\tan(c+d\,x)^{2}}{3}+\frac{b\tan(c+d\,x)}{4}+\frac{a}{5}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(a + b*tan(c + d*x)),x)
```

```
[Out] (log(tan(c + d*x) - 1i)*((a*1i)/2 - b/2))/d - (log(tan(c + d*x) + 1i)*((a*1
i)/2 + b/2))/d + (b*log(tan(c + d*x)))/d - (cot(c + d*x)^5*(a/5 + (b*tan(c
+ d*x))/4 - (a*tan(c + d*x)^2)/3 + a*tan(c + d*x)^4 - (b*tan(c + d*x)^3)/2)
)/d
```

3.424 $\int \tan^4(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=120

$$\left(a^2-b^2\right)x-\frac{2ab\log(\cos(c+dx))}{d}-\frac{\left(a^2-b^2\right)\tan(c+dx)}{d}-\frac{ab\tan^2(c+dx)}{d}+\frac{\left(a^2-b^2\right)\tan^3(c+dx)}{3d}+\frac{ab\tan^4(c+dx)}{2d}+\cdots$$

```
[Out] (a^2-b^2)*x-2*a*b*ln(cos(d*x+c))/d-(a^2-b^2)*tan(d*x+c)/d-a*b*tan(d*x+c)^2/
d+1/3*(a^2-b^2)*tan(d*x+c)^3/d+1/2*a*b*tan(d*x+c)^4/d+1/5*b^2*tan(d*x+c)^5/
d
```

Rubi [A]
time = 0.12, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3624, 3609, 3606, 3556}

$$\frac{\left(a^2-b^2\right)\tan^3(c+dx)}{3d}-\frac{\left(a^2-b^2\right)\tan(c+dx)}{d}+x\left(a^2-b^2\right)+\frac{ab\tan^4(c+dx)}{2d}-\frac{ab\tan^2(c+dx)}{d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\tan^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (a^2 - b^2)*x - (2*a*b*Log[Cos[c + d*x]])/d - ((a^2 - b^2)*Tan[c + d*x])/d
- (a*b*Tan[c + d*x]^2)/d + ((a^2 - b^2)*Tan[c + d*x]^3)/(3*d) + (a*b*Tan[c
+ d*x]^4)/(2*d) + (b^2*Tan[c + d*x]^5)/(5*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \tan^4(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{b^2\tan^5(c+dx)}{5d} + \int \tan^4(c+dx)\left(a^2-b^2+2ab\tan(c+dx)\right)\,dx$$
$$= \frac{ab\tan^4(c+dx)}{2d} + \frac{b^2\tan^5(c+dx)}{5d} + \int \tan^3(c+dx)\left(-2ab+(a^2\right.$$
$$= \frac{(a^2-b^2)\tan^3(c+dx)}{3d} + \frac{ab\tan^4(c+dx)}{2d} + \frac{b^2\tan^5(c+dx)}{5d} + \int \text{ta}$$
$$= -\frac{ab\tan^2(c+dx)}{d} + \frac{(a^2-b^2)\tan^3(c+dx)}{3d} + \frac{ab\tan^4(c+dx)}{2d} + \frac{b^2}{}$$
$$= \left(a^2-b^2\right)x - \frac{(a^2-b^2)\tan(c+dx)}{d} - \frac{ab\tan^2(c+dx)}{d} + \frac{(a^2-b^2)\text{t}}{}$$
$$= \left(a^2-b^2\right)x - \frac{2ab\log(\cos(c+dx))}{d} - \frac{(a^2-b^2)\tan(c+dx)}{d} - \frac{ab\,\text{ta}}{}$$

Mathematica [A]
time = 0.64, size = 110, normalized size = 0.92

$$\frac{30(a^2-b^2)\operatorname{ArcTan}(\tan(c+dx)) - 60ab\log(\cos(c+dx)) - 30(a^2-b^2)\tan(c+dx) - 30ab\tan^2(c+dx) + 10(a^2-b^2)\tan^3(c+dx) + 15ab\tan^4(c+dx) + 6b^2\tan^5(c+dx)}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (30*(a^2 - b^2)*ArcTan[Tan[c + d*x]] - 60*a*b*Log[Cos[c + d*x]] - 30*(a^2 -
 b^2)*Tan[c + d*x] - 30*a*b*Tan[c + d*x]^2 + 10*(a^2 - b^2)*Tan[c + d*x]^3
+ 15*a*b*Tan[c + d*x]^4 + 6*b^2*Tan[c + d*x]^5)/(30*d)
```

Maple [A]
time = 0.04, size = 121, normalized size = 1.01

method	result
norman	$\left(a^2-b^2\right)x + \frac{b^2\left(\tan^5(dx+c)\right)}{5d} - \frac{(a^2-b^2)\tan(dx+c)}{d} + \frac{(a^2-b^2)\left(\tan^3(dx+c)\right)}{3d} - \frac{ab\left(\tan^2(dx+c)\right)}{d} + \frac{ab\left(\tan^4(\right.}{2d}$
derivativedivides	$\frac{\frac{b^2\left(\tan^5(dx+c)\right)}{5} + \frac{ab\left(\tan^4(dx+c)\right)}{2} + \frac{a^2\left(\tan^3(dx+c)\right)}{3} - \frac{b^2\left(\tan^3(dx+c)\right)}{3} - ab\left(\tan^2(dx+c)\right) - a^2\tan(dx+c) + b^2\tan(dx+c) + ab\,\text{l}}{d}$

default	$\frac{\frac{b^2\left(\tan^5(dx+c)\right)}{5}+\frac{ab\left(\tan^4(dx+c)\right)}{2}+\frac{a^2\left(\tan^3(dx+c)\right)}{3}-\frac{b^2\left(\tan^3(dx+c)\right)}{3}-ab\left(\tan^2(dx+c)\right)-a^2\tan(dx+c)+b^2\tan(dx+c)+a}{d}$
risch	$2iabx+a^2x-b^2x+\frac{4iabc}{d}-\frac{2i\left(-60iab\,e^{8i(dx+c)}+30a^2e^{8i(dx+c)}-45b^2e^{8i(dx+c)}-120iab\,e^{6i(dx+c)}+90a^2e^{6i(}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*b^2*tan(d*x+c)^5+1/2*a*b*tan(d*x+c)^4+1/3*a^2*tan(d*x+c)^3-1/3*b^2
*tan(d*x+c)^3-a*b*tan(d*x+c)^2-a^2*tan(d*x+c)+b^2*tan(d*x+c)+a*b*ln(1+tan(d
*x+c)^2)+(a^2-b^2)*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.51, size = 110, normalized size = 0.92

$$\frac{6\,b^2\tan\left(dx+c\right)^5+15\,ab\tan\left(dx+c\right)^4-30\,ab\tan\left(dx+c\right)^2+10\,(a^2-b^2)\tan\left(dx+c\right)^3+30\,ab\log\left(\tan\left(dx+c\right)^2+1\right)+30\,(a^2-b^2)(dx+c)-30\,(a^2-b^2)\tan\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/30*(6*b^2*tan(d*x + c)^5 + 15*a*b*tan(d*x + c)^4 - 30*a*b*tan(d*x + c)^2
+ 10*(a^2 - b^2)*tan(d*x + c)^3 + 30*a*b*log(tan(d*x + c)^2 + 1) + 30*(a^2
- b^2)*(d*x + c) - 30*(a^2 - b^2)*tan(d*x + c))/d
```

Fricas [A]
time = 1.00, size = 109, normalized size = 0.91

$$\frac{6\,b^2\tan\left(dx+c\right)^5+15\,ab\tan\left(dx+c\right)^4-30\,ab\tan\left(dx+c\right)^2+10\,(a^2-b^2)\tan\left(dx+c\right)^3+30\,(a^2-b^2)dx-30\,ab\log\left(\frac{1}{\tan\left(dx+c\right)^2+1}\right)-30\,(a^2-b^2)\tan\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/30*(6*b^2*tan(d*x + c)^5 + 15*a*b*tan(d*x + c)^4 - 30*a*b*tan(d*x + c)^2
+ 10*(a^2 - b^2)*tan(d*x + c)^3 + 30*(a^2 - b^2)*d*x - 30*a*b*log(1/(tan(d*
x + c)^2 + 1)) - 30*(a^2 - b^2)*tan(d*x + c))/d
```

Sympy [A]
time = 0.18, size = 139, normalized size = 1.16

$$\begin{cases} a^2x+\frac{a^2\tan^3\left(c+dx\right)}{3d}-\frac{a^2\tan\left(c+dx\right)}{d}+\frac{ab\log\left(\tan^2\left(c+dx\right)+1\right)}{d}+\frac{ab\tan^4\left(c+dx\right)}{2d}-\frac{ab\tan^2\left(c+dx\right)}{d}-b^2x+\frac{b^2\tan^5\left(c+dx\right)}{5d}-\frac{b^2\tan^3\left(c+dx\right)}{3d}+\frac{b^2\tan\left(c+dx\right)}{d} & \text{for}\ d\neq 0\\ x\left(a+b\tan\left(c\right)\right)^2\tan^4\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x + a**2*tan(c + d*x)**3/(3*d) - a**2*tan(c + d*x)/d + a*b*
log(tan(c + d*x)**2 + 1)/d + a*b*tan(c + d*x)**4/(2*d) - a*b*tan(c + d*x)**
2/d - b**2*x + b**2*tan(c + d*x)**5/(5*d) - b**2*tan(c + d*x)**3/(3*d) + b*
*2*tan(c + d*x)/d, Ne(d, 0)), (x*(a + b*tan(c))**2*tan(c)**4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1315 vs. $2(114) = 228$.
time = 2.30, size = 1315, normalized size = 10.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/30*(30*a^2*d*x*tan(d*x)^5*tan(c)^5 - 30*b^2*d*x*tan(d*x)^5*tan(c)^5 - 30*
a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2
+ tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^5*tan(c)^5 -
 150*a^2*d*x*tan(d*x)^4*tan(c)^4 + 150*b^2*d*x*tan(d*x)^4*tan(c)^4 - 45*a*b
*tan(d*x)^5*tan(c)^5 + 150*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*ta
n(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2
+ 1))*tan(d*x)^4*tan(c)^4 + 30*a^2*tan(d*x)^5*tan(c)^4 - 30*b^2*tan(d*x)^5*
tan(c)^4 + 30*a^2*tan(d*x)^4*tan(c)^5 - 30*b^2*tan(d*x)^4*tan(c)^5 + 300*a^
2*d*x*tan(d*x)^3*tan(c)^3 - 300*b^2*d*x*tan(d*x)^3*tan(c)^3 - 30*a*b*tan(d*
x)^5*tan(c)^3 + 165*a*b*tan(d*x)^4*tan(c)^4 - 30*a*b*tan(d*x)^3*tan(c)^5 -
10*a^2*tan(d*x)^5*tan(c)^2 + 10*b^2*tan(d*x)^5*tan(c)^2 - 300*a*b*log(4*(ta
n(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2
- 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 - 150*a^2*tan(
d*x)^4*tan(c)^3 + 150*b^2*tan(d*x)^4*tan(c)^3 - 150*a^2*tan(d*x)^3*tan(c)^4
 + 150*b^2*tan(d*x)^3*tan(c)^4 - 10*a^2*tan(d*x)^2*tan(c)^5 + 10*b^2*tan(d*
x)^2*tan(c)^5 + 15*a*b*tan(d*x)^5*tan(c) - 300*a^2*d*x*tan(d*x)^2*tan(c)^2
+ 300*b^2*d*x*tan(d*x)^2*tan(c)^2 + 150*a*b*tan(d*x)^4*tan(c)^2 - 180*a*b*t
an(d*x)^3*tan(c)^3 + 150*a*b*tan(d*x)^2*tan(c)^4 + 15*a*b*tan(d*x)*tan(c)^5
 - 6*b^2*tan(d*x)^5 + 20*a^2*tan(d*x)^4*tan(c) - 50*b^2*tan(d*x)^4*tan(c) +
 300*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(
c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c
)^2 + 240*a^2*tan(d*x)^3*tan(c)^2 - 300*b^2*tan(d*x)^3*tan(c)^2 + 240*a^2*t
an(d*x)^2*tan(c)^3 - 300*b^2*tan(d*x)^2*tan(c)^3 + 20*a^2*tan(d*x)*tan(c)^4
 - 50*b^2*tan(d*x)*tan(c)^4 - 6*b^2*tan(c)^5 - 15*a*b*tan(d*x)^4 + 150*a^2*
d*x*tan(d*x)*tan(c) - 150*b^2*d*x*tan(d*x)*tan(c) - 150*a*b*tan(d*x)^3*tan(
c) + 180*a*b*tan(d*x)^2*tan(c)^2 - 150*a*b*tan(d*x)*tan(c)^3 - 15*a*b*tan(c
)^4 - 10*a^2*tan(d*x)^3 + 10*b^2*tan(d*x)^3 - 150*a*b*log(4*(tan(d*x)^4*tan
(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)
*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)*tan(c) - 150*a^2*tan(d*x)^2*tan(c) +
150*b^2*tan(d*x)^2*tan(c) - 150*a^2*tan(d*x)*tan(c)^2 + 150*b^2*tan(d*x)*ta
n(c)^2 - 10*a^2*tan(c)^3 + 10*b^2*tan(c)^3 - 30*a^2*d*x + 30*b^2*d*x + 30*a
```

```
*b*tan(d*x)^2 - 165*a*b*tan(d*x)*tan(c) + 30*a*b*tan(c)^2 + 30*a*b*log(4*(t
an(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2
 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) + 30*a^2*tan(d*x) - 30*b^2*tan(d*
x) + 30*a^2*tan(c) - 30*b^2*tan(c) + 45*a*b)/(d*tan(d*x)^5*tan(c)^5 - 5*d*t
an(d*x)^4*tan(c)^4 + 10*d*tan(d*x)^3*tan(c)^3 - 10*d*tan(d*x)^2*tan(c)^2 +
5*d*tan(d*x)*tan(c) - d)
```

Mupad [B]
time = 4.09, size = 146, normalized size = 1.22

$$\frac{\tan(c+d\,x)^3\,\left(\frac{a^2}{3}-\frac{b^2}{3}\right)}{d}+\frac{b^2\tan(c+d\,x)^5}{5\,d}-\frac{\tan(c+d\,x)\,\left(a^2-b^2\right)}{d}+\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(a+b)\,(a-b)}{a^2-b^2}\right)\,(a+b)\,(a-b)}{d}+\frac{a\,b\,\ln\left(\tan(c+d\,x)^2+1\right)}{d}-\frac{a\,b\tan(c+d\,x)^2}{d}+\frac{a\,b\tan(c+d\,x)^4}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + b*tan(c + d*x))^2,x)
```

```
[Out] (tan(c + d*x)^3*(a^2/3 - b^2/3))/d + (b^2*tan(c + d*x)^5)/(5*d) - (tan(c +
d*x)*(a^2 - b^2))/d + (atan((tan(c + d*x)*(a + b)*(a - b))/(a^2 - b^2))*(a
+ b)*(a - b))/d + (a*b*log(tan(c + d*x)^2 + 1))/d - (a*b*tan(c + d*x)^2)/d
+ (a*b*tan(c + d*x)^4)/(2*d)
```

3.425 $\int \tan^3(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=98

$$2abx+\frac{(a^2-b^2)\log(\cos(c+dx))}{d}-\frac{2ab\tan(c+dx)}{d}+\frac{(a^2-b^2)\tan^2(c+dx)}{2d}+\frac{2ab\tan^3(c+dx)}{3d}+\frac{b^2\tan^4(c+d}{4d}$$

```
[Out] 2*a*b*x+(a^2-b^2)*ln(cos(d*x+c))/d-2*a*b*tan(d*x+c)/d+1/2*(a^2-b^2)*tan(d*x
+c)^2/d+2/3*a*b*tan(d*x+c)^3/d+1/4*b^2*tan(d*x+c)^4/d
```

Rubi [A]
time = 0.10, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3624, 3609, 3606, 3556}

$$\frac{(a^2-b^2)\tan^2(c+dx)}{2d}+\frac{(a^2-b^2)\log(\cos(c+dx))}{d}+\frac{2ab\tan^3(c+dx)}{3d}-\frac{2ab\tan(c+dx)}{d}+2abx+\frac{b^2\tan^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] 2*a*b*x + ((a^2 - b^2)*Log[Cos[c + d*x]])/d - (2*a*b*Tan[c + d*x])/d + ((a^
2 - b^2)*Tan[c + d*x]^2)/(2*d) + (2*a*b*Tan[c + d*x]^3)/(3*d) + (b^2*Tan[c
+ d*x]^4)/(4*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \tan^3(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{b^2\tan^4(c+dx)}{4d} + \int \tan^3(c+dx)\left(a^2-b^2+2ab\tan(c+dx)\right)\,dx$$
$$= \frac{2ab\tan^3(c+dx)}{3d} + \frac{b^2\tan^4(c+dx)}{4d} + \int \tan^2(c+dx)\left(-2ab+(a$$
$$= \frac{(a^2-b^2)\tan^2(c+dx)}{2d} + \frac{2ab\tan^3(c+dx)}{3d} + \frac{b^2\tan^4(c+dx)}{4d} + \int$$
$$= 2abx - \frac{2ab\tan(c+dx)}{d} + \frac{(a^2-b^2)\tan^2(c+dx)}{2d} + \frac{2ab\tan^3(c+}{3d}$$
$$= 2abx + \frac{(a^2-b^2)\log(\cos(c+dx))}{d} - \frac{2ab\tan(c+dx)}{d} + \frac{(a^2-b^2)\,t}{}$$

Mathematica [A]

time = 0.46, size = 113, normalized size = 1.15

$$\frac{2ab\text{ArcTan}(\tan(c+dx))}{d} - \frac{2ab\tan(c+dx)}{d} + \frac{2ab\tan^3(c+dx)}{3d} + \frac{a^2(2\log(\cos(c+dx))+\tan^2(c+dx))}{2d} - \frac{b^2(4\log(\cos(c+dx))+2\tan^2(c+dx)-\tan^4(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
[Out] (2*a*b*ArcTan[Tan[c + d*x]])/d - (2*a*b*Tan[c + d*x])/d + (2*a*b*Tan[c + d*
x]^3)/(3*d) + (a^2*(2*Log[Cos[c + d*x]] + Tan[c + d*x]^2))/(2*d) - (b^2*(4*
Log[Cos[c + d*x]] + 2*Tan[c + d*x]^2 - Tan[c + d*x]^4))/(4*d)
```

Maple [A]

time = 0.04, size = 100, normalized size = 1.02

method	result
norman	$2abx + \frac{b^2(\tan^4(dx+c))}{4d} + \frac{(a^2-b^2)(\tan^2(dx+c))}{2d} - \frac{2ab\tan(dx+c)}{d} + \frac{2ab(\tan^3(dx+c))}{3d} - \frac{(a^2-b^2)\ln(1+\tan^2}{2d}$
derivativedivides	$\frac{\frac{b^2(\tan^4(dx+c))}{4} + \frac{2ab(\tan^3(dx+c))}{3} + \frac{a^2(\tan^2(dx+c))}{2} - \frac{b^2(\tan^2(dx+c))}{2} - 2ab\tan(dx+c) + \frac{(-a^2+b^2)\ln(1+\tan^2(dx+c))}{2} + 2a}{d}$
default	$\frac{\frac{b^2(\tan^4(dx+c))}{4} + \frac{2ab(\tan^3(dx+c))}{3} + \frac{a^2(\tan^2(dx+c))}{2} - \frac{b^2(\tan^2(dx+c))}{2} - 2ab\tan(dx+c) + \frac{(-a^2+b^2)\ln(1+\tan^2(dx+c))}{2} + 2a}{d}$

risch	$2abx - ia^2x + ib^2x - \frac{2ia^2c}{d} + \frac{2ib^2c}{d} + \frac{-8iab\,e^{6i(dx+c)}+2a^2e^{6i(dx+c)}-4b^2e^{6i(dx+c)}-16iab\,e^{4i(dx+c)}+4a^2e^{4i(}}{d(e^{2i(d}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*b^2*tan(d*x+c)^4+2/3*a*b*tan(d*x+c)^3+1/2*a^2*tan(d*x+c)^2-1/2*b^2
*tan(d*x+c)^2-2*a*b*tan(d*x+c)+1/2*(-a^2+b^2)*ln(1+tan(d*x+c)^2)+2*a*b*arct
an(tan(d*x+c)))
```

Maxima [A]
time = 0.51, size = 91, normalized size = 0.93

$$\frac{3\,b^2\tan\left(dx+c\right)^4+8\,ab\tan\left(dx+c\right)^3+24\,(dx+c)ab-24\,ab\tan\left(dx+c\right)+6\,(a^2-b^2)\tan\left(dx+c\right)^2-6\,(a^2-b^2)\log\left(\tan\left(dx+c\right)^2+1\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/12*(3*b^2*tan(d*x + c)^4 + 8*a*b*tan(d*x + c)^3 + 24*(d*x + c)*a*b - 24*a
*b*tan(d*x + c) + 6*(a^2 - b^2)*tan(d*x + c)^2 - 6*(a^2 - b^2)*log(tan(d*x
+ c)^2 + 1))/d
```

Fricas [A]
time = 0.79, size = 90, normalized size = 0.92

$$\frac{3\,b^2\tan\left(dx+c\right)^4+8\,ab\tan\left(dx+c\right)^3+24\,abdx-24\,ab\tan\left(dx+c\right)+6\,(a^2-b^2)\tan\left(dx+c\right)^2+6\,(a^2-b^2)\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*tan(d*x + c)^4 + 8*a*b*tan(d*x + c)^3 + 24*a*b*d*x - 24*a*b*tan
(d*x + c) + 6*(a^2 - b^2)*tan(d*x + c)^2 + 6*(a^2 - b^2)*log(1/(tan(d*x + c
)^2 + 1)))/d
```

Sympy [A]
time = 0.15, size = 134, normalized size = 1.37

$$\begin{cases} -\frac{a^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a^2\tan^2\left(c+dx\right)}{2d}+2abx+\frac{2ab\tan^3\left(c+dx\right)}{3d}-\frac{2ab\tan\left(c+dx\right)}{d}+\frac{b^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{b^2\tan^4\left(c+dx\right)}{4d}-\frac{b^2\tan^2\left(c+dx\right)}{2d} & \text{for } d\neq 0 \\ x\left(a+b\tan\left(c\right)\right)^2\tan^3\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Piecewise((-a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**2/(2*d
) + 2*a*b*x + 2*a*b*tan(c + d*x)**3/(3*d) - 2*a*b*tan(c + d*x)/d + b**2*log
```

```
(tan(c + d*x)**2 + 1)/(2*d) + b**2*tan(c + d*x)**4/(4*d) - b**2*tan(c + d*x
)**2/(2*d), Ne(d, 0)), (x*(a + b*tan(c))**2*tan(c)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1262 vs. $2(92) = 184$.
time = 1.72, size = 1262, normalized size = 12.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(24*a*b*d*x*tan(d*x)^4*tan(c)^4 + 6*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2
*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) +
 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 - 6*b^2*log(4*(tan(d*x)^4*tan(c)^2
- 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c
) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 - 96*a*b*d*x*tan(d*x)^3*tan(c)^3
 + 6*a^2*tan(d*x)^4*tan(c)^4 - 9*b^2*tan(d*x)^4*tan(c)^4 - 24*a^2*log(4*(ta
n(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2
- 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 + 24*b^2*log(4
*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x
)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 + 24*a*b*t
an(d*x)^4*tan(c)^3 + 24*a*b*tan(d*x)^3*tan(c)^4 + 144*a*b*d*x*tan(d*x)^2*ta
n(c)^2 + 6*a^2*tan(d*x)^4*tan(c)^2 - 6*b^2*tan(d*x)^4*tan(c)^2 - 12*a^2*tan
(d*x)^3*tan(c)^3 + 24*b^2*tan(d*x)^3*tan(c)^3 + 6*a^2*tan(d*x)^2*tan(c)^4 -
 6*b^2*tan(d*x)^2*tan(c)^4 - 8*a*b*tan(d*x)^4*tan(c) + 36*a^2*log(4*(tan(d*
x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*
tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 - 36*b^2*log(4*(ta
n(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2
- 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 - 96*a*b*tan(d
*x)^3*tan(c)^2 - 96*a*b*tan(d*x)^2*tan(c)^3 - 8*a*b*tan(d*x)*tan(c)^4 + 3*b
^2*tan(d*x)^4 - 96*a*b*d*x*tan(d*x)*tan(c) - 12*a^2*tan(d*x)^3*tan(c) + 24*
b^2*tan(d*x)^3*tan(c) + 12*a^2*tan(d*x)^2*tan(c)^2 - 12*b^2*tan(d*x)^2*tan(
c)^2 - 12*a^2*tan(d*x)*tan(c)^3 + 24*b^2*tan(d*x)*tan(c)^3 + 3*b^2*tan(c)^4
 + 8*a*b*tan(d*x)^3 - 24*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(
c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 +
1))*tan(d*x)*tan(c) + 24*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(
c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 +
1))*tan(d*x)*tan(c) + 96*a*b*tan(d*x)^2*tan(c) + 96*a*b*tan(d*x)*tan(c)^2 +
 8*a*b*tan(c)^3 + 24*a*b*d*x + 6*a^2*tan(d*x)^2 - 6*b^2*tan(d*x)^2 - 12*a^2
*tan(d*x)*tan(c) + 24*b^2*tan(d*x)*tan(c) + 6*a^2*tan(c)^2 - 6*b^2*tan(c)^2
 + 6*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(
c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) - 6*b^2*log(4*(t
an(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2
 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) - 24*a*b*tan(d*x) - 24*a*b*tan(c)
```

```
 + 6*a^2 - 9*b^2)/(d*tan(d*x)^4*tan(c)^4 - 4*d*tan(d*x)^3*tan(c)^3 + 6*d*ta
n(d*x)^2*tan(c)^2 - 4*d*tan(d*x)*tan(c) + d)
```

Mupad [B]
time = 4.07, size = 90, normalized size = 0.92

$$\frac{\tan(c+d\,x)^2\,\left(\frac{a^2}{2}-\frac{b^2}{2}\right)-\ln\left(\tan(c+d\,x)^2+1\right)\,\left(\frac{a^2}{2}-\frac{b^2}{2}\right)+\frac{b^2\tan(c+d\,x)^4}{4}-2\,a\,b\tan(c+d\,x)+\frac{2\,a\,b\tan(c+d\,x)^3}{3}+2\,a\,b\,d\,x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + b*tan(c + d*x))^2,x)
[Out] (tan(c + d*x)^2*(a^2/2 - b^2/2) - log(tan(c + d*x)^2 + 1)*(a^2/2 - b^2/2) +
 (b^2*tan(c + d*x)^4)/4 - 2*a*b*tan(c + d*x) + (2*a*b*tan(c + d*x)^3)/3 + 2
*a*b*d*x)/d
```

3.426 $\int \tan^2(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=63

$$-\left(\left(a^2-b^2\right)x\right)+\frac{2ab\log(\cos(c+dx))}{d}-\frac{b^2\tan(c+dx)}{d}+\frac{(a+b\tan(c+dx))^3}{3bd}$$

```
[Out] -(a^2-b^2)*x+2*a*b*ln(cos(d*x+c))/d-b^2*tan(d*x+c)/d+1/3*(a+b*tan(d*x+c))^3
/b/d
```

Rubi [A]
time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3624, 3558, 3556}

$$-x\left(a^2-b^2\right)+\frac{(a+b\tan(c+dx))^3}{3bd}+\frac{2ab\log(\cos(c+dx))}{d}-\frac{b^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -((a^2 - b^2)*x) + (2*a*b*Log[Cos[c + d*x]])/d - (b^2*Tan[c + d*x])/d + (a
+ b*Tan[c + d*x])^3/(3*b*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{(a+b\tan(c+dx))^3}{3bd} - \int (a+b\tan(c+dx))^2\,dx$$
$$= -\left(a^2-b^2\right)x - \frac{b^2\tan(c+dx)}{d} + \frac{(a+b\tan(c+dx))^3}{3bd} - (2ab)\int \text{ta}$$
$$= -\left(a^2-b^2\right)x + \frac{2ab\log(\cos(c+dx))}{d} - \frac{b^2\tan(c+dx)}{d} + \frac{(a+b\tan}{3l}$$

Mathematica [A]
time = 0.32, size = 99, normalized size = 1.57

$$-\frac{a^2\text{ArcTan}(\tan(c+dx))}{d} + \frac{b^2\text{ArcTan}(\tan(c+dx))}{d} + \frac{a^2\tan(c+dx)}{d} - \frac{b^2\tan(c+dx)}{d} + \frac{b^2\tan^3(c+dx)}{3d} + \frac{ab(2\log(\cos(c+dx))+\tan^2(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -((a^2*ArcTan[Tan[c + d*x]])/d) + (b^2*ArcTan[Tan[c + d*x]])/d + (a^2*Tan[c
 + d*x])/d - (b^2*Tan[c + d*x])/d + (b^2*Tan[c + d*x]^3)/(3*d) + (a*b*(2*Lo
g[Cos[c + d*x]] + Tan[c + d*x]^2))/d
```

Maple [A]
time = 0.03, size = 83, normalized size = 1.32

method	result
norman	$\left(-a^2+b^2\right)x + \frac{\left(a^2-b^2\right)\tan(dx+c)}{d} + \frac{ab\left(\tan^2(dx+c)\right)}{d} + \frac{b^2\left(\tan^3(dx+c)\right)}{3d} - \frac{ab\ln\left(1+\tan^2(dx+c)\right)}{d}$
derivativedivides	$\frac{\frac{b^2\left(\tan^3(dx+c)\right)}{3}+ab\left(\tan^2(dx+c)\right)+a^2\tan(dx+c)-b^2\tan(dx+c)-ab\ln\left(1+\tan^2(dx+c)\right)+\left(-a^2+b^2\right)\arctan(\tan(dx+c))}{d}$
default	$\frac{\frac{b^2\left(\tan^3(dx+c)\right)}{3}+ab\left(\tan^2(dx+c)\right)+a^2\tan(dx+c)-b^2\tan(dx+c)-ab\ln\left(1+\tan^2(dx+c)\right)+\left(-a^2+b^2\right)\arctan(\tan(dx+c))}{d}$
risch	$-2iabx-a^2x+b^2x-\frac{4iabc}{d}+\frac{2i\left(-6iab\,e^{4i(dx+c)}+3a^2e^{4i(dx+c)}-6b^2e^{4i(dx+c)}-6iab\,e^{2i(dx+c)}+6a^2e^{2i(dx+c)}-\right.}{3d\left(e^{2i(dx+c)}+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*b^2*tan(d*x+c)^3+a*b*tan(d*x+c)^2+a^2*tan(d*x+c)-b^2*tan(d*x+c)-a*
b*ln(1+tan(d*x+c)^2)+(-a^2+b^2)*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.51, size = 78, normalized size = 1.24

$$\frac{b^2\tan(dx+c)^3 + 3\,ab\tan(dx+c)^2 - 3\,ab\log\left(\tan(dx+c)^2+1\right) - 3\left(a^2-b^2\right)(dx+c) + 3\left(a^2-b^2\right)\tan(dx+c)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*tan(d*x + c)^3 + 3*a*b*tan(d*x + c)^2 - 3*a*b*log(tan(d*x + c)^2 +
 1) - 3*(a^2 - b^2)*(d*x + c) + 3*(a^2 - b^2)*tan(d*x + c))/d
```

Fricas [A]
time = 0.88, size = 77, normalized size = 1.22

$$\frac{b^2\tan\left(dx+c\right)^3+3\,ab\tan\left(dx+c\right)^2-3\,(a^2-b^2)dx+3\,ab\log\left(\frac{1}{\tan(dx+c)^2+1}\right)+3\,(a^2-b^2)\tan\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*tan(d*x + c)^3 + 3*a*b*tan(d*x + c)^2 - 3*(a^2 - b^2)*d*x + 3*a*b*
log(1/(tan(d*x + c)^2 + 1)) + 3*(a^2 - b^2)*tan(d*x + c))/d
```

Sympy [A]
time = 0.11, size = 94, normalized size = 1.49

$$\begin{cases} -a^2x+\frac{a^2\tan\left(c+dx\right)}{d}-\frac{ab\log\left(\tan^2\left(c+dx\right)+1\right)}{d}+\frac{ab\tan^2\left(c+dx\right)}{d}+b^2x+\frac{b^2\tan^3\left(c+dx\right)}{3d}-\frac{b^2\tan\left(c+dx\right)}{d} & \text{for } d\neq 0\\ x(a+b\tan\left(c\right))^2\tan^2\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Piecewise((-a**2*x + a**2*tan(c + d*x)/d - a*b*log(tan(c + d*x)**2 + 1)/d +
 a*b*tan(c + d*x)**2/d + b**2*x + b**2*tan(c + d*x)**3/(3*d) - b**2*tan(c +
 d*x)/d, Ne(d, 0)), (x*(a + b*tan(c))**2*tan(c)**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 675 vs. 2(61) = 122.
time = 0.97, size = 675, normalized size = 10.71

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*a^2*d*x*tan(d*x)^3*tan(c)^3 - 3*b^2*d*x*tan(d*x)^3*tan(c)^3 - 3*a*b
*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + t
an(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 - 9*
a^2*d*x*tan(d*x)^2*tan(c)^2 + 9*b^2*d*x*tan(d*x)^2*tan(c)^2 - 3*a*b*tan(d*x
)^3*tan(c)^3 + 9*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan
```

```
(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(
d*x)^2*tan(c)^2 + 3*a^2*tan(d*x)^3*tan(c)^2 - 3*b^2*tan(d*x)^3*tan(c)^2 + 3
*a^2*tan(d*x)^2*tan(c)^3 - 3*b^2*tan(d*x)^2*tan(c)^3 + 9*a^2*d*x*tan(d*x)*t
an(c) - 9*b^2*d*x*tan(d*x)*tan(c) - 3*a*b*tan(d*x)^3*tan(c) + 3*a*b*tan(d*x
)^2*tan(c)^2 - 3*a*b*tan(d*x)*tan(c)^3 + b^2*tan(d*x)^3 - 9*a*b*log(4*(tan(
d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 -
2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)*tan(c) - 6*a^2*tan(d*x)^2*t
an(c) + 9*b^2*tan(d*x)^2*tan(c) - 6*a^2*tan(d*x)*tan(c)^2 + 9*b^2*tan(d*x)*
tan(c)^2 + b^2*tan(c)^3 - 3*a^2*d*x + 3*b^2*d*x + 3*a*b*tan(d*x)^2 - 3*a*b*
tan(d*x)*tan(c) + 3*a*b*tan(c)^2 + 3*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan
(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/
(tan(c)^2 + 1)) + 3*a^2*tan(d*x) - 3*b^2*tan(d*x) + 3*a^2*tan(c) - 3*b^2*ta
n(c) + 3*a*b)/(d*tan(d*x)^3*tan(c)^3 - 3*d*tan(d*x)^2*tan(c)^2 + 3*d*tan(d*
x)*tan(c) - d)
```

Mupad [B]
time = 4.07, size = 108, normalized size = 1.71

$$\frac{b^2\tan(c+d\,x)^3}{3\,d}+\frac{\tan(c+d\,x)\,\left(a^2-b^2\right)}{d}-\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(a+b)\,(a-b)}{a^2-b^2}\right)\,(a+b)\,(a-b)}{d}-\frac{a\,b\,\ln\left(\tan(c+d\,x)^2+1\right)}{d}+\frac{a\,b\tan(c+d\,x)^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + b*tan(c + d*x))^2,x)
```

```
[Out] (b^2*tan(c + d*x)^3)/(3*d) + (tan(c + d*x)*(a^2 - b^2))/d - (atan((tan(c +
d*x)*(a + b)*(a - b))/(a^2 - b^2))*(a + b)*(a - b))/d - (a*b*log(tan(c + d*
x)^2 + 1))/d + (a*b*tan(c + d*x)^2)/d
```

3.427 $\int \tan(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=58

$$-2abx-\frac{(a^2-b^2)\log(\cos(c+dx))}{d}+\frac{ab\tan(c+dx)}{d}+\frac{(a+b\tan(c+dx))^2}{2d}$$

```
[Out] -2*a*b*x-(a^2-b^2)*ln(cos(d*x+c))/d+a*b*tan(d*x+c)/d+1/2*(a+b*tan(d*x+c))^2
/d
```

Rubi [A]
time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3609, 3606, 3556}

$$-\frac{(a^2-b^2)\log(\cos(c+dx))}{d}+\frac{(a+b\tan(c+dx))^2}{2d}+\frac{ab\tan(c+dx)}{d}-2abx$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -2*a*b*x - ((a^2 - b^2)*Log[Cos[c + d*x]])/d + (a*b*Tan[c + d*x])/d + (a +
b*Tan[c + d*x])^2/(2*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\int \tan(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{(a+b\tan(c+dx))^2}{2d} + \int (-b+a\tan(c+dx))(a+b\tan(c+dx))\,dx$$
$$= -2abx + \frac{ab\tan(c+dx)}{d} + \frac{(a+b\tan(c+dx))^2}{2d} + (a^2-b^2)\int \tan(c$$
$$= -2abx - \frac{(a^2-b^2)\log(\cos(c+dx))}{d} + \frac{ab\tan(c+dx)}{d} + \frac{(a+b\tan(c}{2d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.29, size = 74, normalized size = 1.28

$$\frac{(a+ib)^2\log(i-\tan(c+dx)) + (a-ib)^2\log(i+\tan(c+dx)) + 4ab\tan(c+dx) + b^2\tan^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((a + I*b)^2*Log[I - Tan[c + d*x]] + (a - I*b)^2*Log[I + Tan[c + d*x]] + 4*
a*b*Tan[c + d*x] + b^2*Tan[c + d*x]^2)/(2*d)
```

Maple [A]
time = 0.03, size = 62, normalized size = 1.07

method	result
norman	$-2abx+\frac{b^2(\tan^2(dx+c))}{2d}+\frac{2ab\tan(dx+c)}{d}+\frac{(a^2-b^2)\ln(1+\tan^2(dx+c))}{2d}$
derivativedivides	$\frac{\frac{b^2\left(\tan^2(dx+c)\right)}{2}+2ab\tan(dx+c)+\frac{\left(a^2-b^2\right)\ln\left(1+\tan^2(dx+c)\right)}{2}-2ab\arctan(\tan(dx+c))}{d}$
default	$\frac{\frac{b^2\left(\tan^2(dx+c)\right)}{2}+2ab\tan(dx+c)+\frac{\left(a^2-b^2\right)\ln\left(1+\tan^2(dx+c)\right)}{2}-2ab\arctan(\tan(dx+c))}{d}$
risch	$-2abx+ia^2x-ib^2x+\frac{2ia^2c}{d}-\frac{2ib^2c}{d}+\frac{2ib\left(-ib\,e^{2i(dx+c)}+2a\,e^{2i(dx+c)}+2a\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{a^2\ln\left(e^{2i(dx+c)}+1\right)}{d}+\frac{\ln(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*b^2*tan(d*x+c)^2+2*a*b*tan(d*x+c)+1/2*(a^2-b^2)*ln(1+tan(d*x+c)^2)
-2*a*b*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.52, size = 58, normalized size = 1.00

$$\frac{b^2\tan(dx+c)^2 - 4(dx+c)ab + 4ab\tan(dx+c) + (a^2-b^2)\log\left(\tan(dx+c)^2+1\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*tan(d*x + c)^2 - 4*(d*x + c)*a*b + 4*a*b*tan(d*x + c) + (a^2 - b^2
)*log(tan(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.75, size = 58, normalized size = 1.00

$$-\frac{4\,abdx-b^{2}\tan\left(dx+c\right)^{2}-4\,ab\tan\left(dx+c\right)+\left(a^{2}-b^{2}\right)\log\left(\frac{1}{\tan\left(dx+c\right)^{2}+1}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a*b*d*x - b^2*tan(d*x + c)^2 - 4*a*b*tan(d*x + c) + (a^2 - b^2)*log
(1/(tan(d*x + c)^2 + 1)))/d
```

Sympy [A]
time = 0.10, size = 85, normalized size = 1.47

$$\begin{cases} \frac{a^{2}\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d}-2abx+\frac{2ab\tan\left(c+dx\right)}{d}-\frac{b^{2}\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d}+\frac{b^{2}\tan^{2}\left(c+dx\right)}{2d} & \text{for}\ d\neq 0\\ x\left(a+b\tan\left(c\right)\right)^{2}\tan\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*log(tan(c + d*x)**2 + 1)/(2*d) - 2*a*b*x + 2*a*b*tan(c + d*
x)/d - b**2*log(tan(c + d*x)**2 + 1)/(2*d) + b**2*tan(c + d*x)**2/(2*d), Ne
(d, 0)), (x*(a + b*tan(c))**2*tan(c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(56) =$
112.
time = 0.87, size = 554, normalized size = 9.55

$$-\frac{4\,abdx\tan\left(dx\right)^{2}\tan\left(c\right)^{2}+a^{2}\log\left(\frac{4\left(\tan(dx)^{4}\tan(c)^{2}-2\tan(dx)^{3}\tan(c)+\tan(dx)^{2}\tan(c)^{2}+\tan(dx)^{2}-2\tan(dx)\tan(c)+1\right)}{\tan(c)^{2}+1}\right)\tan\left(dx\right)^{2}\tan\left(c\right)^{2}-b^{2}\log\left(\frac{4\left(\tan(dx)^{4}\tan(c)^{2}-2\tan(dx)^{3}\tan(c)+\tan(dx)^{2}\tan(c)^{2}+\tan(dx)^{2}-2\tan(dx)\tan(c)+1\right)}{\tan(c)^{2}+1}\right)\tan\left(dx\right)^{2}\tan\left(c\right)^{2}-8\,abdx\tan\left(dx\right)\tan\left(c\right)-b^{2}\tan\left(dx\right)^{2}\tan\left(c\right)^{2}-2\,a^{2}\log\left(\frac{4\left(\tan(dx)^{4}\tan(c)^{2}-2\tan(dx)^{3}\tan(c)+\tan(dx)^{2}\tan(c)^{2}+\tan(dx)^{2}-2\tan(dx)\tan(c)+1\right)}{\tan(c)^{2}+1}\right)\tan\left(dx\right)\tan\left(c\right)+2\,b^{2}\log\left(\frac{4\left(\tan(dx)^{4}\tan(c)^{2}-2\tan(dx)^{3}\tan(c)+\tan(dx)^{2}\tan(c)^{2}+\tan(dx)^{2}-2\tan(dx)\tan(c)+1\right)}{\tan(c)^{2}+1}\right)\tan\left(dx\right)\tan\left(c\right)+4\,ab\tan\left(dx\right)^{2}\tan\left(c\right)+4\,ab\tan\left(dx\right)\tan\left(c\right)^{2}+4\,abdx-b^{2}\tan\left(dx\right)^{2}-b^{2}\tan\left(c\right)^{2}+a^{2}\log\left(\frac{4\left(\tan(dx)^{4}\tan(c)^{2}-2\tan(dx)^{3}\tan(c)+\tan(dx)^{2}\tan(c)^{2}+\tan(dx)^{2}-2\tan(dx)\tan(c)+1\right)}{\tan(c)^{2}+1}\right)-b^{2}\log\left(\frac{4\left(\tan(dx)^{4}\tan(c)^{2}-2\tan(dx)^{3}\tan(c)+\tan(dx)^{2}\tan(c)^{2}+\tan(dx)^{2}-2\tan(dx)\tan(c)+1\right)}{\tan(c)^{2}+1}\right)-4\,ab\tan\left(dx\right)-4\,ab\tan\left(c\right)-b^{2}}{2\left(d\tan\left(dx\right)^{2}\tan\left(c\right)^{2}-2\,d\tan\left(dx\right)\tan\left(c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*a*b*d*x*tan(d*x)^2*tan(c)^2 + a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*ta
n(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)
/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 - b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*t
an(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1
)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 - 8*a*b*d*x*tan(d*x)*tan(c) - b^2*tan
```

```
(d*x)^2*tan(c)^2 - 2*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) +
 tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*
tan(d*x)*tan(c) + 2*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) +
tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*t
an(d*x)*tan(c) + 4*a*b*tan(d*x)^2*tan(c) + 4*a*b*tan(d*x)*tan(c)^2 + 4*a*b*
d*x - b^2*tan(d*x)^2 - b^2*tan(c)^2 + a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*ta
n(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)
/(tan(c)^2 + 1)) - b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + t
an(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) -
4*a*b*tan(d*x) - 4*a*b*tan(c) - b^2)/(d*tan(d*x)^2*tan(c)^2 - 2*d*tan(d*x)*
tan(c) + d)
```

Mupad [B]
time = 4.02, size = 57, normalized size = 0.98

$$\frac{\ln\left(\tan(c+d\,x)^2+1\right)\,\left(\frac{a^2}{2}-\frac{b^2}{2}\right)+\frac{b^2\tan(c+d\,x)^2}{2}+2\,a\,b\tan(c+d\,x)-2\,a\,b\,d\,x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + b*tan(c + d*x))^2,x)
```

```
[Out] (log(tan(c + d*x)^2 + 1)*(a^2/2 - b^2/2) + (b^2*tan(c + d*x)^2)/2 + 2*a*b*t
an(c + d*x) - 2*a*b*d*x)/d
```

3.428 $\int (a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=39

$$\left(a^2-b^2\right)x-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

```
[Out] (a^2-b^2)*x-2*a*b*ln(cos(d*x+c))/d+b^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3558, 3556}

$$x\left(a^2-b^2\right)-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (a^2 - b^2)*x - (2*a*b*Log[Cos[c + d*x]])/d + (b^2*Tan[c + d*x])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\tan(c+dx))^2\,dx&=\left(a^2-b^2\right)x+\frac{b^2\tan(c+dx)}{d}+(2ab)\int \tan(c+dx)\,dx\\&=\left(a^2-b^2\right)x-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.10, size = 69, normalized size = 1.77

$$\frac{-i((a+ib)^2\log(i-\tan(c+dx))-(a-ib)^2\log(i+\tan(c+dx)))+2b^2\tan(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((-I)*((a + I*b)^2*Log[I - Tan[c + d*x]] - (a - I*b)^2*Log[I + Tan[c + d*x]
]) + 2*b^2*Tan[c + d*x])/(2*d)
```

Maple [A]
time = 0.02, size = 47, normalized size = 1.21

method	result	size
norman	$\left(a^2-b^2\right)x+\frac{b^2\tan(dx+c)}{d}+\frac{ab\ln\left(1+\tan^2(dx+c)\right)}{d}$	43
derivativedivides	$\frac{b^2\tan(dx+c)+ab\ln\left(1+\tan^2(dx+c)\right)+\left(a^2-b^2\right)\arctan(\tan(dx+c))}{d}$	47
default	$\frac{b^2\tan(dx+c)+ab\ln\left(1+\tan^2(dx+c)\right)+\left(a^2-b^2\right)\arctan(\tan(dx+c))}{d}$	47
risch	$2iabx+a^2x-b^2x+\frac{4iabc}{d}+\frac{2ib^2}{d\left(e^{2i(dx+c)}+1\right)}-\frac{2ab\ln\left(e^{2i(dx+c)}+1\right)}{d}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*tan(d*x+c)+a*b*ln(1+tan(d*x+c)^2)+(a^2-b^2)*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.55, size = 41, normalized size = 1.05

$$a^2x-\frac{(dx+c-\tan(dx+c))b^2}{d}+\frac{2\,ab\log\left(\sec\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] a^2*x - (d*x + c - tan(d*x + c))*b^2/d + 2*a*b*log(sec(d*x + c))/d
```

Fricas [A]
time = 1.05, size = 44, normalized size = 1.13

$$\frac{(a^2-b^2)dx-ab\log\left(\frac{1}{\tan(dx+c)^2+1}\right)+b^2\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((a^2 - b^2)*d*x - a*b*log(1/(tan(d*x + c)^2 + 1)) + b^2*tan(d*x + c))/d
```

Sympy [A]
time = 0.07, size = 48, normalized size = 1.23

$$\begin{cases} a^2x + \frac{ab\log\left(\tan^2(c+dx)+1\right)}{d} - b^2x + \frac{b^2\tan(c+dx)}{d} & \text{for } d \neq 0 \\ x(a+b\tan(c))^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x + a*b*log(tan(c + d*x)**2 + 1)/d - b**2*x + b**2*tan(c +
d*x)/d, Ne(d, 0)), (x*(a + b*tan(c))**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(39) = 78.
time = 0.56, size = 201, normalized size = 5.15

$$\frac{a^2dx\tan(dx)\tan(c) - b^2dx\tan(dx)\tan(c) - ab\log\left(\frac{4\left(\tan(dx)^4\tan(c)^2 - 2\tan(dx)^3\tan(c) + \tan(dx)^2\tan(c)^2 + \tan(dx)^2 - 2\tan(dx)\tan(c) + 1\right)}{\tan(c)^2+1}\right)\tan(dx)\tan(c) - a^2dx + b^2dx + ab\log\left(\frac{4\left(\tan(dx)^4\tan(c)^2 - 2\tan(dx)^3\tan(c) + \tan(dx)^2\tan(c)^2 + \tan(dx)^2 - 2\tan(dx)\tan(c) + 1\right)}{\tan(c)^2+1}\right) - b^2\tan(dx) - b^2\tan(c)}{d\tan(dx)\tan(c) - d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (a^2*d*x*tan(d*x)*tan(c) - b^2*d*x*tan(d*x)*tan(c) - a*b*log(4*(tan(d*x)^4*
tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d
*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)*tan(c) - a^2*d*x + b^2*d*x + a*b*l
og(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan
(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) - b^2*tan(d*x) - b^2*tan(c
))/(d*tan(d*x)*tan(c) - d)
```

Mupad [B]
time = 4.09, size = 136, normalized size = 3.49

$$\frac{a^2\,\mathrm{atan}\left(\frac{a^2\tan(c+d\,x)}{a^2-b^2}-\frac{b^2\tan(c+d\,x)}{a^2-b^2}\right)}{d}-\frac{b^2\,\mathrm{atan}\left(\frac{a^2\tan(c+d\,x)}{a^2-b^2}-\frac{b^2\tan(c+d\,x)}{a^2-b^2}\right)}{d}+\frac{b^2\tan(c+d\,x)}{d}+\frac{a\,b\,\ln\left(\tan(c+d\,x)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^2,x)
```

```
[Out] (a^2*atan((a^2*tan(c + d*x))/(a^2 - b^2) - (b^2*tan(c + d*x))/(a^2 - b^2)))
/d - (b^2*atan((a^2*tan(c + d*x))/(a^2 - b^2) - (b^2*tan(c + d*x))/(a^2 - b
^2)))/d + (b^2*tan(c + d*x))/d + (a*b*log(tan(c + d*x)^2 + 1))/d
```

3.429 $\int \cot(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=35

$$2abx-\frac{b^2\log(\cos(c+dx))}{d}+\frac{a^2\log(\sin(c+dx))}{d}$$

```
[Out] 2*a*b*x-b^2*ln(cos(d*x+c))/d+a^2*ln(sin(d*x+c))/d
```

Rubi [A]
time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3622, 3556}

$$\frac{a^2\log(\sin(c+dx))}{d}+2abx-\frac{b^2\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] 2*a*b*x - (b^2*Log[Cos[c + d*x]])/d + (a^2*Log[Sin[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3622

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^2/((a_.) + (b_.)*tan[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[d*(2*b*c - a*d)*(x/b^2), x] + (Dist[d^2/b, In
t[Tan[e + f*x], x], x] + Dist[(b*c - a*d)^2/b^2, Int[1/(a + b*Tan[e + f*x])
, x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 +
 b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)(a+b\tan(c+dx))^2\,dx &= 2abx+a^2\int \cot(c+dx)\,dx+b^2\int \tan(c+dx)\,dx\\ &= 2abx-\frac{b^2\log(\cos(c+dx))}{d}+\frac{a^2\log(\sin(c+dx))}{d}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 43, normalized size = 1.23

$$2abx-\frac{b^2\log(\cos(c+dx))}{d}+\frac{a^2(\log(\cos(c+dx))+\log(\tan(c+dx)))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] 2*a*b*x - (b^2*Log[Cos[c + d*x]])/d + (a^2*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]]))/d
```

Maple [A]
time = 0.18, size = 38, normalized size = 1.09

method	result	size
derivativedivides	$\frac{a^2 \ln(\sin(dx+c))+2ab(dx+c)-b^2 \ln(\cos(dx+c))}{d}$	38
default	$\frac{a^2 \ln(\sin(dx+c))+2ab(dx+c)-b^2 \ln(\cos(dx+c))}{d}$	38
norman	$2abx+\frac{a^2 \ln(\tan(dx+c))}{d}-\frac{(a^2-b^2) \ln(1+\tan^2(dx+c))}{2d}$	46
risch	$2abx-ia^2x+ib^2x-\frac{2ia^2c}{d}+\frac{2ib^2c}{d}+\frac{a^2 \ln(e^{2i(dx+c)}-1)}{d}-\frac{\ln(e^{2i(dx+c)}+1)b^2}{d}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*ln(sin(d*x+c))+2*a*b*(d*x+c)-b^2*ln(cos(d*x+c)))
```

Maxima [A]
time = 0.51, size = 49, normalized size = 1.40

$$\frac{4\,(dx+c)ab+2\,a^2\log\left(\tan\left(dx+c\right)\right)-\left(a^2-b^2\right)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(4*(d*x + c)*a*b + 2*a^2*log(tan(d*x + c)) - (a^2 - b^2)*log(tan(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.93, size = 56, normalized size = 1.60

$$\frac{4\,abdx+a^2\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)-b^2\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*b*d*x + a^2*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1)) - b^2*log(1/(tan(d*x + c)^2 + 1)))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(32) = 64$.
time = 0.21, size = 70, normalized size = 2.00

$$\begin{cases} -\frac{a^2 \log\left(\tan^2\left(c+dx\right)+1\right)}{2d} + \frac{a^2 \log\left(\tan\left(c+dx\right)\right)}{d} + 2abx + \frac{b^2 \log\left(\tan^2\left(c+dx\right)+1\right)}{2d} & \text{for } d \neq 0 \\ x(a + b\tan(c))^2 \cot(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Piecewise((-a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*log(tan(c + d*x))/d
+ 2*a*b*x + b**2*log(tan(c + d*x)**2 + 1)/(2*d), Ne(d, 0)), (x*(a + b*tan(c
))**2*cot(c), True))
```

Giac [A]
time = 0.67, size = 50, normalized size = 1.43

$$\frac{4(dx+c)ab + 2a^2 \log\left(|\tan\left(dx+c\right)|\right) - (a^2-b^2)\log\left(\tan\left(dx+c\right)^2+1\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*(d*x + c)*a*b + 2*a^2*log(abs(tan(d*x + c))) - (a^2 - b^2)*log(tan(d
*x + c)^2 + 1))/d
```

Mupad [B]
time = 4.19, size = 61, normalized size = 1.74

$$\frac{a^2 \ln\left(\tan(c+d\,x)\right)}{d} + \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(b+a\,1\mathrm{i}\right)^2}{2\,d} - \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(a+b\,1\mathrm{i}\right)^2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + b*tan(c + d*x))^2,x)
```

```
[Out] (log(tan(c + d*x) + 1i)*(a*1i + b)^2)/(2*d) - (log(tan(c + d*x) - 1i)*(a +
b*1i)^2)/(2*d) + (a^2*log(tan(c + d*x)))/d
```

3.430 $\int \cot^2(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=41

$$-\left(\left(a^2-b^2\right)x\right)-\frac{a^2\cot(c+dx)}{d}+\frac{2ab\log(\sin(c+dx))}{d}$$

```
[Out] -(a^2-b^2)*x-a^2*cot(d*x+c)/d+2*a*b*ln(sin(d*x+c))/d
```

Rubi [A]
time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3623, 3612, 3556}

$$-x\left(a^2-b^2\right)-\frac{a^2\cot(c+dx)}{d}+\frac{2ab\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -((a^2 - b^2)*x) - (a^2*Cot[c + d*x])/d + (2*a*b*Log[Sin[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \cot^2(c+dx)(a+b\tan(c+dx))^2\,dx &= -\frac{a^2\cot(c+dx)}{d} + \int \cot(c+dx)\left(2ab-\left(a^2-b^2\right)\tan(c+dx)\right)\,dx \\
&= -\left(a^2-b^2\right)x - \frac{a^2\cot(c+dx)}{d} + (2ab)\int \cot(c+dx)\,dx \\
&= -\left(a^2-b^2\right)x - \frac{a^2\cot(c+dx)}{d} + \frac{2ab\log(\sin(c+dx))}{d}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.16, size = 82, normalized size = 2.00

$$
\frac{-2a^2\cot(c+dx)+i\left((a+ib)^2\log(i-\tan(c+dx))-4iab\log(\tan(c+dx))-(a-ib)^2\log(i+\tan(c+dx))\right)}{2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
[Out] (-2*a^2*Cot[c + d*x] + I*((a + I*b)^2*Log[I - Tan[c + d*x]] - (4*I)*a*b*Log
[Tan[c + d*x]] - (a - I*b)^2*Log[I + Tan[c + d*x]]))/(2*d)
```

Maple [A]
time = 0.14, size = 46, normalized size = 1.12

method	result	size
derivativedivides	$\frac{a^2(-\cot(dx+c)-dx-c)+2ab\ln(\sin(dx+c))+b^2(dx+c)}{d}$	46
default	$\frac{a^2(-\cot(dx+c)-dx-c)+2ab\ln(\sin(dx+c))+b^2(dx+c)}{d}$	46
norman	$\frac{(-a^2+b^2)x\tan(dx+c)-\frac{a^2}{d}}{\tan(dx+c)}+\frac{2ab\ln(\tan(dx+c))}{d}-\frac{ab\ln(1+\tan^2(dx+c))}{d}$	69
risch	$-2iabx-a^2x+b^2x-\frac{4iabc}{d}-\frac{2ia^2}{d(e^{2i(dx+c)}-1)}+\frac{2ab\ln(e^{2i(dx+c)}-1)}{d}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(a^2*(-cot(d*x+c)-d*x-c)+2*a*b*ln(sin(d*x+c))+b^2*(d*x+c))
```

Maxima [A]
time = 0.52, size = 58, normalized size = 1.41

$$
-\frac{ab\log\left(\tan(dx+c)^2+1\right)-2\,ab\log\left(\tan(dx+c)\right)+(a^2-b^2)(dx+c)+\frac{a^2}{\tan(dx+c)}}{d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(a*b*log(tan(d*x + c)^2 + 1) - 2*a*b*log(tan(d*x + c)) + (a^2 - b^2)*(d*x
+ c) + a^2/tan(d*x + c))/d
```

Fricas [A]
time = 1.17, size = 67, normalized size = 1.63

$$-\frac{(a^2-b^2)dx\tan(dx+c)-ab\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan(dx+c)+a^2}{d\tan(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -((a^2 - b^2)*d*x*tan(d*x + c) - a*b*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1
))*tan(d*x + c) + a^2)/(d*tan(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(34) = 68.
time = 0.46, size = 85, normalized size = 2.07

$$\begin{cases} \tilde{\infty}a^2x & \text{for } (c=0 \vee c=-dx) \wedge (c=-dx \vee d=0) \\ x(a+b\tan(c))^2\cot^2(c) & \text{for } d=0 \\ -a^2x-\frac{a^2}{d\tan(c+dx)}-\frac{ab\log(\tan^2(c+dx)+1)}{d}+\frac{2ab\log(\tan(c+dx))}{d}+b^2x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Piecewise((zoo*a**2*x, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x)))
, (x*(a + b*tan(c))**2*cot(c)**2, Eq(d, 0)), (-a**2*x - a**2/(d*tan(c + d*x
)) - a*b*log(tan(c + d*x)**2 + 1)/d + 2*a*b*log(tan(c + d*x))/d + b**2*x, T
rue))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(41) = 82.
time = 0.75, size = 98, normalized size = 2.39

$$-\frac{4\,ab\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)-4\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,(a^2-b^2)(dx+c)+\frac{4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*a*b*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 4*a*b*log(abs(tan(1/2*d*x + 1
/2*c))) - a^2*tan(1/2*d*x + 1/2*c) + 2*(a^2 - b^2)*(d*x + c) + (4*a*b*tan(1
/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*c))/d
```

Mupad [B]
time = 4.07, size = 79, normalized size = 1.93

$$\frac{2\,a\,b\,\ln\left(\tan(c+d\,x)\right)}{d}-\frac{a^2\cot(c+d\,x)}{d}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(a-b\,1\mathrm{i}\right)^2\,1\mathrm{i}}{2\,d}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-b+a\,1\mathrm{i}\right)^2\,1\mathrm{i}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + b*tan(c + d*x))^2,x)
```

```
[Out] (2*a*b*log(tan(c + d*x)))/d - (a^2*cot(c + d*x))/d - (log(tan(c + d*x) - 1i
)*(a*1i - b)^2*1i)/(2*d) - (log(tan(c + d*x) + 1i)*(a - b*1i)^2*1i)/(2*d)
```

3.431 $\int \cot^3(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=58

$$-2abx-\frac{2ab\cot(c+dx)}{d}-\frac{a^2\cot^2(c+dx)}{2d}-\frac{(a^2-b^2)\log(\sin(c+dx))}{d}$$

```
[Out] -2*a*b*x-2*a*b*cot(d*x+c)/d-1/2*a^2*cot(d*x+c)^2/d-(a^2-b^2)*ln(sin(d*x+c))
/d
```

Rubi [A]
time = 0.07, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3623, 3610, 3612, 3556}

$$-\frac{(a^2-b^2)\log(\sin(c+dx))}{d}-\frac{a^2\cot^2(c+dx)}{2d}-\frac{2ab\cot(c+dx)}{d}-2abx$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -2*a*b*x - (2*a*b*Cot[c + d*x])/d - (a^2*Cot[c + d*x]^2)/(2*d) - ((a^2 - b^
2)*Log[Sin[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cot^3(c+dx)(a+b\tan(c+dx))^2\,dx &= -\frac{a^2\cot^2(c+dx)}{2d}+\int \cot^2(c+dx)\left(2ab-\left(a^2-b^2\right)\tan(c+dx)\right)\,d\\ &= -\frac{2ab\cot(c+dx)}{d}-\frac{a^2\cot^2(c+dx)}{2d}+\int \cot(c+dx)\left(-a^2+b^2-2\right.\\ &= -2abx-\frac{2ab\cot(c+dx)}{d}-\frac{a^2\cot^2(c+dx)}{2d}+\left(-a^2+b^2\right)\int \cot(c-\\ &= -2abx-\frac{2ab\cot(c+dx)}{d}-\frac{a^2\cot^2(c+dx)}{2d}-\frac{(a^2-b^2)\log(\sin(c+}{d}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.30, size = 92, normalized size = 1.59

$$\frac{-4ab\cot(c+dx)-a^2\cot^2(c+dx)+(a+ib)^2\log(i-\tan(c+dx))-2(a-b)(a+b)\log(\tan(c+dx))+(a-ib)^2\log(i+\tan(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-4*a*b*Cot[c + d*x] - a^2*Cot[c + d*x]^2 + (a + I*b)^2*Log[I - Tan[c + d*x
]] - 2*(a - b)*(a + b)*Log[Tan[c + d*x]] + (a - I*b)^2*Log[I + Tan[c + d*x]
])/(2*d)
```

Maple [A]
time = 0.20, size = 61, normalized size = 1.05

method	result
derivativedivides	$\frac{a^2\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+2ab(-\cot(dx+c)-dx-c)+b^2\ln(\sin(dx+c))}{d}$
default	$\frac{a^2\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+2ab(-\cot(dx+c)-dx-c)+b^2\ln(\sin(dx+c))}{d}$
norman	$\frac{-\frac{a^2}{2d}-2abx\left(\tan^2(dx+c)\right)-\frac{2ab\tan(dx+c)}{d}}{\tan(dx+c)^2}-\frac{(a^2-b^2)\ln(\tan(dx+c))}{d}+\frac{(a^2-b^2)\ln(1+\tan^2(dx+c))}{2d}$

risch	$-2abx+ia^2x-ib^2x+\frac{2ia^2c}{d}-\frac{2ib^2c}{d}+\frac{2a\left(a\,e^{2i(dx+c)}-2ib\,e^{2i(dx+c)}+2ib\right)}{d\left(e^{2i(dx+c)}-1\right)^2}-\frac{a^2\ln\left(e^{2i(dx+c)}-1\right)}{d}+\ln($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+2*a*b*(-cot(d*x+c)-d*x-c)+b^2*l
n(sin(d*x+c)))
```

Maxima [A]
time = 0.51, size = 78, normalized size = 1.34

$$-\frac{4\,(dx+c)ab-(a^2-b^2)\log\left(\tan\left(dx+c\right)^2+1\right)+2\,(a^2-b^2)\log\left(\tan\left(dx+c\right)\right)+\frac{4\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(4*(d*x + c)*a*b - (a^2 - b^2)*log(tan(d*x + c)^2 + 1) + 2*(a^2 - b^2)
*log(tan(d*x + c)) + (4*a*b*tan(d*x + c) + a^2)/tan(d*x + c)^2)/d
```

Fricas [A]
time = 1.19, size = 86, normalized size = 1.48

$$-\frac{(a^2-b^2)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^2+4\,ab\tan\left(dx+c\right)+(4\,abdx+a^2)\tan\left(dx+c\right)^2+a^2}{2\,d\tan\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*((a^2 - b^2)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^2 +
 4*a*b*tan(d*x + c) + (4*a*b*d*x + a^2)*tan(d*x + c)^2 + a^2)/(d*tan(d*x +
c)^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(53) = 106.
time = 0.68, size = 131, normalized size = 2.26

$$\begin{cases} \tilde{\infty}a^2x & \text{for}\ (c=0\vee c=-dx)\wedge(c=-dx\vee d=0) \\ x(a+b\tan\left(c\right))^2\cot^3\left(c\right) & \text{for}\ d=0 \\ \frac{a^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}-\frac{a^2\log\left(\tan\left(c+dx\right)\right)}{d}-\frac{a^2}{2d\tan^2\left(c+dx\right)}-2abx-\frac{2ab}{d\tan\left(c+dx\right)}-\frac{b^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{b^2\log\left(\tan\left(c+dx\right)\right)}{d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Piecewise((zoo*a**2*x, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x)))
, (x*(a + b*tan(c))**2*cot(c)**3, Eq(d, 0)), (a**2*log(tan(c + d*x)**2 + 1)
/(2*d) - a**2*log(tan(c + d*x))/d - a**2/(2*d*tan(c + d*x)**2) - 2*a*b*x -
2*a*b/(d*tan(c + d*x)) - b**2*log(tan(c + d*x)**2 + 1)/(2*d) + b**2*log(tan
(c + d*x))/d, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(56) = 112.
time = 0.84, size = 154, normalized size = 2.66

$$-\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+16\,(dx+c)ab-8\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-8\,(a^2-b^2)\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)+8\,(a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-8\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/8*(a^2*tan(1/2*d*x + 1/2*c)^2 + 16*(d*x + c)*a*b - 8*a*b*tan(1/2*d*x + 1
/2*c) - 8*(a^2 - b^2)*log(tan(1/2*d*x + 1/2*c)^2 + 1) + 8*(a^2 - b^2)*log(a
bs(tan(1/2*d*x + 1/2*c))) - (12*a^2*tan(1/2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2
*d*x + 1/2*c)^2 - 8*a*b*tan(1/2*d*x + 1/2*c) - a^2)/tan(1/2*d*x + 1/2*c)^2)
/d
```

Mupad [B]
time = 4.01, size = 97, normalized size = 1.67

$$\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,{\left(a+b\,1\mathrm{i}\right)}^2}{2\,d}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,{\left(b+a\,1\mathrm{i}\right)}^2}{2\,d}-\frac{\ln\left(\tan(c+d\,x)\right)\,\left(a^2-b^2\right)}{d}-\frac{{\mathrm{cot}(c+d\,x)}^2\,\left(\frac{a^2}{2}+2\,b\,\tan(c+d\,x)\,a\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + b*tan(c + d*x))^2,x)
```

```
[Out] (log(tan(c + d*x) - 1i)*(a + b*1i)^2)/(2*d) - (log(tan(c + d*x) + 1i)*(a*1i
 + b)^2)/(2*d) - (log(tan(c + d*x))*(a^2 - b^2))/d - (cot(c + d*x)^2*(a^2/2
 + 2*a*b*tan(c + d*x)))/d
```

3.432 $\int \cot^4(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=78

$$\left(a^2-b^2\right)x+\frac{\left(a^2-b^2\right)\cot(c+dx)}{d}-\frac{ab\cot^2(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-\frac{2ab\log(\sin(c+dx))}{d}$$

```
[Out] (a^2-b^2)*x+(a^2-b^2)*cot(d*x+c)/d-a*b*cot(d*x+c)^2/d-1/3*a^2*cot(d*x+c)^3/
d-2*a*b*ln(sin(d*x+c))/d
```

Rubi [A]
time = 0.10, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3623, 3610, 3612, 3556}

$$\frac{\left(a^2-b^2\right)\cot(c+dx)}{d}+x\left(a^2-b^2\right)-\frac{a^2\cot^3(c+dx)}{3d}-\frac{ab\cot^2(c+dx)}{d}-\frac{2ab\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (a^2 - b^2)*x + ((a^2 - b^2)*Cot[c + d*x])/d - (a*b*Cot[c + d*x]^2)/d - (a^
2*Cot[c + d*x]^3)/(3*d) - (2*a*b*Log[Sin[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cot^4(c+dx)(a+b\tan(c+dx))^2\,dx &= -\frac{a^2\cot^3(c+dx)}{3d}+\int \cot^3(c+dx)\left(2ab-\left(a^2-b^2\right)\tan(c+dx)\right)\,d\\ &= -\frac{ab\cot^2(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}+\int \cot^2(c+dx)\left(-a^2+b^2-\right.\\ &= \frac{(a^2-b^2)\cot(c+dx)}{d}-\frac{ab\cot^2(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}+\int \cot(\\ &= \left(a^2-b^2\right)x+\frac{(a^2-b^2)\cot(c+dx)}{d}-\frac{ab\cot^2(c+dx)}{d}-\frac{a^2\cot^3(c+}{3d}\\ &= \left(a^2-b^2\right)x+\frac{(a^2-b^2)\cot(c+dx)}{d}-\frac{ab\cot^2(c+dx)}{d}-\frac{a^2\cot^3(c+}{3d}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.74, size = 103, normalized size = 1.32

$$-\frac{a^2\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(c+dx)\right)}{3d}-\frac{b^2\cot(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)}{d}-\frac{ab\left(\cot^2(c+dx)+2\log(\cos(c+dx))+2\log(\tan(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -1/3*(a^2*Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])
/d - (b^2*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2])/d
- (a*b*(Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[Tan[c + d*x]]))/d
```

Maple [A]
time = 0.16, size = 75, normalized size = 0.96

method	result
derivativedivides	$\frac{a^2\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+2ab\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+b^2(-\cot(dx+c)-dx-c)}{d}$

default	$\frac{a^2\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+2ab\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+b^2(-\cot(dx+c)-dx-c)}{d}$
norman	$\frac{(a^2-b^2)x(\tan^3(dx+c))+\frac{\left(a^2-b^2\right)\left(\tan^2(dx+c)\right)}{d}-\frac{a^2}{3d}-\frac{ab\tan(dx+c)}{d}}{\tan(dx+c)^3}+\frac{ab\ln(1+\tan^2(dx+c))}{d}-\frac{2ab\ln(\tan(dx+c))}{d}$
risch	$2iabx+a^2x-b^2x+\frac{4iabc}{d}-\frac{2i(6iab\,e^{4i(dx+c)}-6a^2e^{4i(dx+c)}+3b^2e^{4i(dx+c)}-6iab\,e^{2i(dx+c)}+6a^2e^{2i(dx+c)}-6}{3d(e^{2i(dx+c)}-1)^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^4*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^2*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+2*a*b*(-1/2*cot(d*x+c)^2-ln(s in(d*x+c)))+b^2*(-cot(d*x+c)-d*x-c))

Maxima [A]
time = 0.52, size = 91, normalized size = 1.17

$$\frac{3\,ab\log\left(\tan\left(dx+c\right)^2+1\right)-6\,ab\log\left(\tan\left(dx+c\right)\right)+3\left(a^2-b^2\right)(dx+c)-\frac{3\,ab\tan(dx+c)-3\left(a^2-b^2\right)\tan(dx+c)^2+a^2}{\tan(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="maxima")

[Out] 1/3*(3*a*b*log(tan(d*x + c)^2 + 1) - 6*a*b*log(tan(d*x + c)) + 3*(a^2 - b^2)*(d*x + c) - (3*a*b*tan(d*x + c) - 3*(a^2 - b^2)*tan(d*x + c)^2 + a^2)/tan (d*x + c)^3)/d

Fricas [A]
time = 1.01, size = 107, normalized size = 1.37

$$-\frac{3\,ab\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^3-3\left(\left(a^2-b^2\right)dx-ab\right)\tan\left(dx+c\right)^3+3\,ab\tan\left(dx+c\right)-3\left(a^2-b^2\right)\tan\left(dx+c\right)^2+a^2}{3\,d\tan\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="fricas")

[Out] -1/3*(3*a*b*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^3 - 3*((a ^2 - b^2)*d*x - a*b)*tan(d*x + c)^3 + 3*a*b*tan(d*x + c) - 3*(a^2 - b^2)*ta n(d*x + c)^2 + a^2)/(d*tan(d*x + c)^3)

Sympy [A]
time = 1.01, size = 126, normalized size = 1.62

$$\begin{cases} \tilde{\infty}a^2x & \text{for } (c=0\vee c=-dx)\wedge(c=-dx\vee d=0) \\ x(a+b\tan(c))^2\cot^4(c) & \text{for } d=0 \\ a^2x+\frac{a^2}{d\tan(c+dx)}-\frac{a^2}{3d\tan^3(c+dx)}+\frac{ab\log\left(\tan^2(c+dx)+1\right)}{d}-\frac{2ab\log\left(\tan(c+dx)\right)}{d}-\frac{ab}{d\tan^2(c+dx)}-b^2x-\frac{b^2}{d\tan(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Piecewise((zoo*a**2*x, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x)))
, (x*(a + b*tan(c))**2*cot(c)**4, Eq(d, 0)), (a**2*x + a**2/(d*tan(c + d*x)
) - a**2/(3*d*tan(c + d*x)**3) + a*b*log(tan(c + d*x)**2 + 1)/d - 2*a*b*log
(tan(c + d*x))/d - a*b/(d*tan(c + d*x)**2) - b**2*x - b**2/(d*tan(c + d*x))
, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(76) = 152.
time = 0.90, size = 191, normalized size = 2.45

$$\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+48\,ab\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)-48\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+24\,(a^2-b^2)(dx+c)+\frac{88\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-6\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 - 6*a*b*tan(1/2*d*x + 1/2*c)^2 + 48*a*b*lo
g(tan(1/2*d*x + 1/2*c)^2 + 1) - 48*a*b*log(abs(tan(1/2*d*x + 1/2*c))) - 15*
a^2*tan(1/2*d*x + 1/2*c) + 12*b^2*tan(1/2*d*x + 1/2*c) + 24*(a^2 - b^2)*(d*
x + c) + (88*a*b*tan(1/2*d*x + 1/2*c)^3 + 15*a^2*tan(1/2*d*x + 1/2*c)^2 - 1
2*b^2*tan(1/2*d*x + 1/2*c)^2 - 6*a*b*tan(1/2*d*x + 1/2*c) - a^2)/tan(1/2*d*
x + 1/2*c)^3)/d
```

Mupad [B]
time = 4.02, size = 112, normalized size = 1.44

$$-\frac{\cot(c+d\,x)^3\,\left(\frac{a^2}{3}-\tan(c+d\,x)^2\,(a^2-b^2)+a\,b\tan(c+d\,x)\right)}{d}-\frac{2\,a\,b\,\ln\left(\tan(c+d\,x)\right)}{d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(a-b\,1\mathrm{i}\right)^2\,1\mathrm{i}}{2\,d}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-b+a\,1\mathrm{i}\right)^2\,1\mathrm{i}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + b*tan(c + d*x))^2,x)
```

```
[Out] (log(tan(c + d*x) + 1i)*(a - b*1i)^2*1i)/(2*d) - (cot(c + d*x)^3*(a^2/3 - t
an(c + d*x)^2*(a^2 - b^2) + a*b*tan(c + d*x)))/d + (log(tan(c + d*x) - 1i)*
(a*1i - b)^2*1i)/(2*d) - (2*a*b*log(tan(c + d*x)))/d
```

3.433 $\int \cot^5(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=98

$$2abx+\frac{2ab\cot(c+dx)}{d}+\frac{(a^2-b^2)\cot^2(c+dx)}{2d}-\frac{2ab\cot^3(c+dx)}{3d}-\frac{a^2\cot^4(c+dx)}{4d}+\frac{(a^2-b^2)\log(\sin(c+dx))}{d}$$

```
[Out] 2*a*b*x+2*a*b*cot(d*x+c)/d+1/2*(a^2-b^2)*cot(d*x+c)^2/d-2/3*a*b*cot(d*x+c)^
3/d-1/4*a^2*cot(d*x+c)^4/d+(a^2-b^2)*ln(sin(d*x+c))/d
```

Rubi [A]
time = 0.13, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3623, 3610, 3612, 3556}

$$\frac{(a^2-b^2)\cot^2(c+dx)}{2d}+\frac{(a^2-b^2)\log(\sin(c+dx))}{d}-\frac{a^2\cot^4(c+dx)}{4d}-\frac{2ab\cot^3(c+dx)}{3d}+\frac{2ab\cot(c+dx)}{d}+2abx$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] 2*a*b*x + (2*a*b*Cot[c + d*x])/d + ((a^2 - b^2)*Cot[c + d*x]^2)/(2*d) - (2*
a*b*Cot[c + d*x]^3)/(3*d) - (a^2*Cot[c + d*x]^4)/(4*d) + ((a^2 - b^2)*Log[S
in[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{a^2\cot^4(c+dx)}{4d} + \int \cot^4(c+dx)\left(2ab-\left(a^2-b^2\right)\tan(c+dx)\right)$$

$$= -\frac{2ab\cot^3(c+dx)}{3d} - \frac{a^2\cot^4(c+dx)}{4d} + \int \cot^3(c+dx)\left(-a^2+b^2-\right.$$

$$= \frac{\left(a^2-b^2\right)\cot^2(c+dx)}{2d} - \frac{2ab\cot^3(c+dx)}{3d} - \frac{a^2\cot^4(c+dx)}{4d} + \int$$

$$= \frac{2ab\cot(c+dx)}{d} + \frac{\left(a^2-b^2\right)\cot^2(c+dx)}{2d} - \frac{2ab\cot^3(c+dx)}{3d} -$$

$$= 2abx + \frac{2ab\cot(c+dx)}{d} + \frac{\left(a^2-b^2\right)\cot^2(c+dx)}{2d} - \frac{2ab\cot^3(c+dx)}{3d}$$

$$= 2abx + \frac{2ab\cot(c+dx)}{d} + \frac{\left(a^2-b^2\right)\cot^2(c+dx)}{2d} - \frac{2ab\cot^3(c+dx)}{3d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.57, size = 122, normalized size = 1.24

$$-\frac{2ab\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(c+dx)\right)}{3d} - \frac{b^2\left(\cot^2(c+dx)+2\log(\cos(c+dx))+2\log(\tan(c+dx))\right)}{2d} + \frac{a^2\left(2\cot^2(c+dx)-\cot^4(c+dx)+4\log(\cos(c+dx))+4\log(\tan(c+dx))\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-2*a*b*Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(
3*d) - (b^2*(Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[Tan[c + d*x]]))/(
2*d) + (a^2*(2*Cot[c + d*x]^2 - Cot[c + d*x]^4 + 4*Log[Cos[c + d*x]] + 4*Lo
g[Tan[c + d*x]]))/(4*d)
```

Maple [A]
time = 0.17, size = 87, normalized size = 0.89

method	result

derivativedivides	$\frac{a^2\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+2ab\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+b^2\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\text{l}\right.}{d}$
default	$\frac{a^2\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+2ab\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+b^2\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\text{l}\right.}{d}$
norman	$\frac{-\frac{a^2}{4d}+\frac{(a^2-b^2)\left(\tan^2(dx+c)\right)}{2d}+2abx\left(\tan^4(dx+c)\right)-\frac{2ab\tan(dx+c)}{3d}+\frac{2ab\left(\tan^3(dx+c)\right)}{d}}{\tan(dx+c)^4}+\frac{(a^2-b^2)\ln(\tan(dx+c))}{d}-\frac{(a^2-}{}$
risch	$2abx-ia^2x+ib^2x-\frac{2ia^2c}{d}+\frac{2ib^2c}{d}-\frac{2(-12iab\,e^{6i(dx+c)}+6a^2e^{6i(dx+c)}-3b^2e^{6i(dx+c)}+24iab\,e^{4i(dx+c)}-6c}{3d(e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+2*a*b*(-1/3*co
t(d*x+c)^3+cot(d*x+c)+d*x+c)+b^2*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c))))
```

Maxima [A]
time = 0.53, size = 111, normalized size = 1.13

$$\frac{24\,(dx+c)ab-6\,(a^2-b^2)\log\left(\tan\left(dx+c\right)^2+1\right)+12\,(a^2-b^2)\log\left(\tan\left(dx+c\right)\right)+\frac{24\,ab\tan(dx+c)^3-8\,ab\tan(dx+c)+6\,(a^2-b^2)\tan(dx+c)^2-3\,a^2}{\tan(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/12*(24*(d*x + c)*a*b - 6*(a^2 - b^2)*log(tan(d*x + c)^2 + 1) + 12*(a^2 -
b^2)*log(tan(d*x + c)) + (24*a*b*tan(d*x + c)^3 - 8*a*b*tan(d*x + c) + 6*(a
^2 - b^2)*tan(d*x + c)^2 - 3*a^2)/tan(d*x + c)^4)/d
```

Fricas [A]
time = 1.14, size = 128, normalized size = 1.31

$$\frac{6\,(a^2-b^2)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^4+24\,ab\tan\left(dx+c\right)^3+3\,(8\,abdx+3\,a^2-2\,b^2)\tan\left(dx+c\right)^4-8\,ab\tan\left(dx+c\right)+6\,(a^2-b^2)\tan\left(dx+c\right)^2-3\,a^2}{12\,d\tan\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*(a^2 - b^2)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^4
 + 24*a*b*tan(d*x + c)^3 + 3*(8*a*b*d*x + 3*a^2 - 2*b^2)*tan(d*x + c)^4 - 8
*a*b*tan(d*x + c) + 6*(a^2 - b^2)*tan(d*x + c)^2 - 3*a^2)/(d*tan(d*x + c)^4
)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(88) = 176$.

time = 1.43, size = 178, normalized size = 1.82

$$\begin{cases} \tilde{\infty}a^2x & \text{for } (c = 0 \vee c = -dx) \wedge (c = -dx \vee d = 0) \\ x(a + b\tan(c))^2 \cot^5(c) & \text{for } d = 0 \\ -\frac{a^2 \log(\tan^2(c+dx)+1)}{2d} + \frac{a^2 \log(\tan(c+dx))}{d} + \frac{a^2}{2d\tan^2(c+dx)} - \frac{a^2}{4d\tan^4(c+dx)} + 2abx + \frac{2ab}{d\tan(c+dx)} - \frac{2ab}{3d\tan^3(c+dx)} + \frac{b^2 \log(\tan^2(c+dx)+1)}{2d} - \frac{b^2 \log(\tan(c+dx))}{d} - \frac{b^2}{2d\tan^2(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Piecewise((zoo*a**2*x, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x)))
, (x*(a + b*tan(c))**2*cot(c)**5, Eq(d, 0)), (-a**2*log(tan(c + d*x)**2 + 1
)/(2*d) + a**2*log(tan(c + d*x))/d + a**2/(2*d*tan(c + d*x)**2) - a**2/(4*d
*tan(c + d*x)**4) + 2*a*b*x + 2*a*b/(d*tan(c + d*x)) - 2*a*b/(3*d*tan(c + d
*x)**3) + b**2*log(tan(c + d*x)**2 + 1)/(2*d) - b**2*log(tan(c + d*x))/d -
b**2/(2*d*tan(c + d*x)**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(92) = 184.

time = 1.06, size = 248, normalized size = 2.53

$$-\frac{3 a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 - 16 ab \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 36 a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 24 b^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 384 (dx + c)ab + 240 ab \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 192 (a^2 - b^2) \log\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 1\right) - 192 (a^2 - b^2) \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right) + \frac{400 a^2 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4 - 400 b^2 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4 - 240 ab \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3 - 36 a^2 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2 + 24 b^2 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2 + 16 ab \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right) + 3 a^2}{\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4}}{192 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/192*(3*a^2*tan(1/2*d*x + 1/2*c)^4 - 16*a*b*tan(1/2*d*x + 1/2*c)^3 - 36*a
^2*tan(1/2*d*x + 1/2*c)^2 + 24*b^2*tan(1/2*d*x + 1/2*c)^2 - 384*(d*x + c)*a
*b + 240*a*b*tan(1/2*d*x + 1/2*c) + 192*(a^2 - b^2)*log(tan(1/2*d*x + 1/2*c
)^2 + 1) - 192*(a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + (400*a^2*tan(1/
2*d*x + 1/2*c)^4 - 400*b^2*tan(1/2*d*x + 1/2*c)^4 - 240*a*b*tan(1/2*d*x + 1
/2*c)^3 - 36*a^2*tan(1/2*d*x + 1/2*c)^2 + 24*b^2*tan(1/2*d*x + 1/2*c)^2 + 1
6*a*b*tan(1/2*d*x + 1/2*c) + 3*a^2)/tan(1/2*d*x + 1/2*c)^4)/d
```

Mupad [B]
time = 3.99, size = 129, normalized size = 1.32

$$\frac{\ln(\tan(c + d\,x))\,(a^2 - b^2)}{d} + \frac{\ln(\tan(c + d\,x) + 1\mathrm{i})\,(b + a\,1\mathrm{i})^2}{2\,d} - \frac{\ln(\tan(c + d\,x) - \mathrm{i})\,(a + b\,1\mathrm{i})^2}{2\,d} - \frac{\cot(c + d\,x)^4\left(\frac{a^2}{4} - \tan(c + d\,x)^2\left(\frac{a^2}{2} - \frac{b^2}{2}\right) + \frac{2\,a\,b\tan(c+d\,x)}{3} - 2\,a\,b\tan(c + d\,x)^3\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(a + b*tan(c + d*x))^2,x)
```

```
[Out] (log(tan(c + d*x) + 1i)*(a*1i + b)^2)/(2*d) - (log(tan(c + d*x) - 1i)*(a +
b*1i)^2)/(2*d) + (log(tan(c + d*x))*(a^2 - b^2))/d - (cot(c + d*x)^4*(a^2/4
 - tan(c + d*x)^2*(a^2/2 - b^2/2) + (2*a*b*tan(c + d*x))/3 - 2*a*b*tan(c +
d*x)^3))/d
```

3.434 $\int \cot^6(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=120

$$-\left(\left(a^2-b^2\right)x\right)-\frac{\left(a^2-b^2\right)\cot(c+dx)}{d}+\frac{ab\cot^2(c+dx)}{d}+\frac{\left(a^2-b^2\right)\cot^3(c+dx)}{3d}-\frac{ab\cot^4(c+dx)}{2d}-\frac{a^2\cot^5(c}{5c}$$

```
[Out] -(a^2-b^2)*x-(a^2-b^2)*cot(d*x+c)/d+a*b*cot(d*x+c)^2/d+1/3*(a^2-b^2)*cot(d*
x+c)^3/d-1/2*a*b*cot(d*x+c)^4/d-1/5*a^2*cot(d*x+c)^5/d+2*a*b*ln(sin(d*x+c))
/d
```

Rubi [A]
time = 0.14, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3623, 3610, 3612, 3556}

$$\frac{\left(a^2-b^2\right)\cot^3(c+dx)}{3d}-\frac{\left(a^2-b^2\right)\cot(c+dx)}{d}-x\left(a^2-b^2\right)-\frac{a^2\cot^5(c+dx)}{5d}-\frac{ab\cot^4(c+dx)}{2d}+\frac{ab\cot^2(c+dx)}{d}+\frac{2ab\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -((a^2 - b^2)*x) - ((a^2 - b^2)*Cot[c + d*x])/d + (a*b*Cot[c + d*x]^2)/d +
((a^2 - b^2)*Cot[c + d*x]^3)/(3*d) - (a*b*Cot[c + d*x]^4)/(2*d) - (a^2*Cot[
c + d*x]^5)/(5*d) + (2*a*b*Log[Sin[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cot^6(c+dx)(a+b\tan(c+dx))^2\,dx &= -\frac{a^2\cot^5(c+dx)}{5d}+\int \cot^5(c+dx)\left(2ab-\left(a^2-b^2\right)\tan(c+dx)\right)\ d\\ &= -\frac{ab\cot^4(c+dx)}{2d}-\frac{a^2\cot^5(c+dx)}{5d}+\int \cot^4(c+dx)\left(-a^2+b^2-\right.\\ &= \frac{(a^2-b^2)\cot^3(c+dx)}{3d}-\frac{ab\cot^4(c+dx)}{2d}-\frac{a^2\cot^5(c+dx)}{5d}+\int \mathrm{co}\\ &= \frac{ab\cot^2(c+dx)}{d}+\frac{(a^2-b^2)\cot^3(c+dx)}{3d}-\frac{ab\cot^4(c+dx)}{2d}-\frac{a^2\,\mathrm{co}}{}\\ &= -\frac{(a^2-b^2)\cot(c+dx)}{d}+\frac{ab\cot^2(c+dx)}{d}+\frac{(a^2-b^2)\cot^3(c+dx)}{3d}\\ &= -\left(a^2-b^2\right)x-\frac{(a^2-b^2)\cot(c+dx)}{d}+\frac{ab\cot^2(c+dx)}{d}+\frac{(a^2-b^2)}{}\\ &= -\left(a^2-b^2\right)x-\frac{(a^2-b^2)\cot(c+dx)}{d}+\frac{ab\cot^2(c+dx)}{d}+\frac{(a^2-b^2)}{}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.01, size = 121, normalized size = 1.01

$$-\frac{a^2\cot^5(c+dx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\tan^2(c+dx)\right)}{5d}-\frac{b^2\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(c+dx)\right)}{3d}+\frac{ab\left(2\cot^2(c+dx)-\cot^4(c+dx)+4\log(\cos(c+dx))+4\log(\tan(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -1/5*(a^2*Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x]^2])
/d - (b^2*Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])
/(3*d) + (a*b*(2*Cot[c + d*x]^2 - Cot[c + d*x]^4 + 4*Log[Cos[c + d*x]] + 4*
Log[Tan[c + d*x]]))/(2*d)
```

Maple [A]
time = 0.19, size = 103, normalized size = 0.86

method	result
derivativedivides	$\frac{a^2\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-dx-c\right)+2ab\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+b^2\left(-\frac{(}{}\right.}{d}$
default	$\frac{a^2\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-dx-c\right)+2ab\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+b^2\left(-\frac{(}{}\right.}{d}$
norman	$\frac{(-a^2+b^2)x(\tan^5(dx+c))+\frac{ab\left(\tan^3(dx+c)\right)}{d}-\frac{a^2}{5d}-\frac{\left(a^2-b^2\right)\left(\tan^4(dx+c)\right)}{d}+\frac{\left(a^2-b^2\right)\left(\tan^2(dx+c)\right)}{3d}-\frac{ab\tan(dx+c)}{2d}}{\tan(dx+c)^5}+\frac{2ab\ln}{}$
risch	$-2iabx-a^2x+b^2x-\frac{4iabc}{d}-\frac{2i\left(-60iab\,e^{8i(dx+c)}+45a^2e^{8i(dx+c)}-30b^2e^{8i(dx+c)}+120iab\,e^{6i(dx+c)}-90a^2e\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c)+2*a*b*(-1/4*
cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+b^2*(-1/3*cot(d*x+c)^3+cot(d*
x+c)+d*x+c))
```

Maxima [A]
time = 0.54, size = 124, normalized size = 1.03

$$-\frac{30\,ab\log\left(\tan\left(dx+c\right)^2+1\right)-60\,ab\log\left(\tan\left(dx+c\right)\right)+30\,(a^2-b^2)(dx+c)-\frac{30\,ab\tan(dx+c)^3-30\,(a^2-b^2)\tan(dx+c)^4-15\,ab\tan(dx+c)+10\,(a^2-b^2)\tan(dx+c)^2-6\,a^2}{\tan(dx+c)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/30*(30*a*b*log(tan(d*x + c)^2 + 1) - 60*a*b*log(tan(d*x + c)) + 30*(a^2
- b^2)*(d*x + c) - (30*a*b*tan(d*x + c)^3 - 30*(a^2 - b^2)*tan(d*x + c)^4 -
 15*a*b*tan(d*x + c) + 10*(a^2 - b^2)*tan(d*x + c)^2 - 6*a^2)/tan(d*x + c)^
5)/d
```

Fricas [A]
time = 1.09, size = 141, normalized size = 1.18

$$\frac{30\,ab\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^5-15\left(2\left(a^2-b^2\right)dx-3\,ab\right)\tan\left(dx+c\right)^5+30\,ab\tan\left(dx+c\right)^3-30\left(a^2-b^2\right)\tan\left(dx+c\right)^4-15\,ab\tan\left(dx+c\right)+10\left(a^2-b^2\right)\tan\left(dx+c\right)^2-6\,a^2}{30\,d\tan\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/30*(30*a*b*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^5 - 15*(
2*(a^2 - b^2)*d*x - 3*a*b)*tan(d*x + c)^5 + 30*a*b*tan(d*x + c)^3 - 30*(a^2
 - b^2)*tan(d*x + c)^4 - 15*a*b*tan(d*x + c) + 10*(a^2 - b^2)*tan(d*x + c)^
2 - 6*a^2)/(d*tan(d*x + c)^5)
```

Sympy [A]
time = 2.31, size = 172, normalized size = 1.43

$$\begin{cases} \tilde{\infty} a^2 x & \text{for } (c = 0 \vee c = -dx) \wedge (c = -dx \vee d = 0) \\ x(a + b\tan(c))^2 \cot^6(c) & \text{for } d = 0 \\ -a^2x - \frac{a^2}{d\tan(c+dx)} + \frac{a^2}{3d\tan^3(c+dx)} - \frac{a^2}{5d\tan^5(c+dx)} - \frac{ab\log(\tan^2(c+dx)+1)}{d} + \frac{2ab\log(\tan(c+dx))}{d} + \frac{ab}{d\tan^2(c+dx)} - \frac{ab}{2d\tan^4(c+dx)} + b^2x + \frac{b^2}{d\tan(c+dx)} - \frac{b^2}{3d\tan^3(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Piecewise((zoo*a**2*x, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x)))
, (x*(a + b*tan(c))**2*cot(c)**6, Eq(d, 0)), (-a**2*x - a**2/(d*tan(c + d*x
)) + a**2/(3*d*tan(c + d*x)**3) - a**2/(5*d*tan(c + d*x)**5) - a*b*log(tan(
c + d*x)**2 + 1)/d + 2*a*b*log(tan(c + d*x))/d + a*b/(d*tan(c + d*x)**2) -
a*b/(2*d*tan(c + d*x)**4) + b**2*x + b**2/(d*tan(c + d*x)) - b**2/(3*d*tan(
c + d*x)**3), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(114) = 228.
time = 1.15, size = 287, normalized size = 2.39

$$\frac{3a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 15ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 35a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 20b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 180ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 960ab\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right) + 960ab\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right) + 330a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 300b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 480(a^2-b^2)(dx+c) - \frac{2192ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 330a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 300b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 180ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 35a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 20b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 15ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + 3a^2}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5}}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 15*a*b*tan(1/2*d*x + 1/2*c)^4 - 35*a^
2*tan(1/2*d*x + 1/2*c)^3 + 20*b^2*tan(1/2*d*x + 1/2*c)^3 + 180*a*b*tan(1/2*
d*x + 1/2*c)^2 - 960*a*b*log(tan(1/2*d*x + 1/2*c)^2 + 1) + 960*a*b*log(abs(
tan(1/2*d*x + 1/2*c))) + 330*a^2*tan(1/2*d*x + 1/2*c) - 300*b^2*tan(1/2*d*x
 + 1/2*c) - 480*(a^2 - b^2)*(d*x + c) - (2192*a*b*tan(1/2*d*x + 1/2*c)^5 +
330*a^2*tan(1/2*d*x + 1/2*c)^4 - 300*b^2*tan(1/2*d*x + 1/2*c)^4 - 180*a*b*t
an(1/2*d*x + 1/2*c)^3 - 35*a^2*tan(1/2*d*x + 1/2*c)^2 + 20*b^2*tan(1/2*d*x
+ 1/2*c)^2 + 15*a*b*tan(1/2*d*x + 1/2*c) + 3*a^2)/tan(1/2*d*x + 1/2*c)^5)/d
```

Mupad [B]
time = 3.97, size = 145, normalized size = 1.21

$$\frac{2\,a\,b\,\ln\left(\tan(c+d\,x)\right)}{d} - \frac{\cot(c+d\,x)^5\left(\tan(c+d\,x)^4\left(a^2-b^2\right) + \frac{a^2}{5} - \tan(c+d\,x)^2\left(\frac{a^2}{3}-\frac{b^2}{3}\right) + \frac{a\,b\tan(c+d\,x)}{2} - a\,b\tan(c+d\,x)^3\right)}{d} - \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\left(a-b\,1\mathrm{i}\right)^2\,1\mathrm{i}}{2\,d} - \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\left(-b+a\,1\mathrm{i}\right)^2\,1\mathrm{i}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(a + b*tan(c + d*x))^2,x)
```

```
[Out] (2*a*b*log(tan(c + d*x)))/d - (log(tan(c + d*x) + 1i)*(a - b*1i)^2*1i)/(2*d
) - (log(tan(c + d*x) - 1i)*(a*1i - b)^2*1i)/(2*d) - (cot(c + d*x)^5*(tan(c
 + d*x)^4*(a^2 - b^2) + a^2/5 - tan(c + d*x)^2*(a^2/3 - b^2/3) + (a*b*tan(c
 + d*x))/2 - a*b*tan(c + d*x)^3))/d
```

3.435 $\int \tan^3(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=147

$$b(3a^2-b^2)\,x+\frac{a(a^2-3b^2)\log(\cos(c+dx))}{d}-\frac{b(a^2-b^2)\tan(c+dx)}{d}-\frac{a(a+b\tan(c+dx))^2}{2d}-\frac{(a+b\tan}{3d}$$

```
[Out] b*(3*a^2-b^2)*x+a*(a^2-3*b^2)*ln(cos(d*x+c))/d-b*(a^2-b^2)*tan(d*x+c)/d-1/2
*a*(a+b*tan(d*x+c))^2/d-1/3*(a+b*tan(d*x+c))^3/d-1/20*a*(a+b*tan(d*x+c))^4/
b^2/d+1/5*tan(d*x+c)*(a+b*tan(d*x+c))^4/b/d
```

Rubi [A]
time = 0.14, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3647, 3711, 12, 3609, 3606, 3556}

$$-\frac{b(a^2-b^2)\tan(c+dx)}{d}+\frac{a(a^2-3b^2)\log(\cos(c+dx))}{d}+bx(3a^2-b^2)-\frac{a(a+b\tan(c+dx))^4}{20b^2d}+\frac{\tan(c+dx)(a+b\tan(c+dx))^4}{5bd}-\frac{(a+b\tan(c+dx))^3}{3d}-\frac{a(a+b\tan(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] b*(3*a^2 - b^2)*x + (a*(a^2 - 3*b^2)*Log[Cos[c + d*x]])/d - (b*(a^2 - b^2)*
Tan[c + d*x])/d - (a*(a + b*Tan[c + d*x])^2)/(2*d) - (a + b*Tan[c + d*x])^3
/(3*d) - (a*(a + b*Tan[c + d*x])^4)/(20*b^2*d) + (Tan[c + d*x]*(a + b*Tan[c
 + d*x])^4)/(5*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
```

```
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \tan^3(c+dx)(a+b\tan(c+dx))^3\,dx &= \frac{\tan(c+dx)(a+b\tan(c+dx))^4}{5bd}+\frac{\int (a+b\tan(c+dx))^3\,(-a-5b\\
&= -\frac{a(a+b\tan(c+dx))^4}{20b^2d}+\frac{\tan(c+dx)(a+b\tan(c+dx))^4}{5bd}+\frac{\int -5\\
&= -\frac{a(a+b\tan(c+dx))^4}{20b^2d}+\frac{\tan(c+dx)(a+b\tan(c+dx))^4}{5bd}-\int \tan\\
&= -\frac{(a+b\tan(c+dx))^3}{3d}-\frac{a(a+b\tan(c+dx))^4}{20b^2d}+\frac{\tan(c+dx)(a+\\
&= -\frac{a(a+b\tan(c+dx))^2}{2d}-\frac{(a+b\tan(c+dx))^3}{3d}-\frac{a(a+b\tan(c+d\\
&= b(3a^2-b^2)\,x-\frac{b(a^2-b^2)\tan(c+dx)}{d}-\frac{a(a+b\tan(c+dx))^2}{2d}-\\
&= b(3a^2-b^2)\,x+\frac{a(a^2-3b^2)\log(\cos(c+dx))}{d}-\frac{b(a^2-b^2)\tan(c+d
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.81, size = 161, normalized size = 1.10

$$\frac{-3(a^5+10(a+ib)^3b^2\log(i-\tan(c+dx))+10(a-ib)^3b^2\log(i+\tan(c+dx)))+60b^3(-3a^2+b^2)\tan(c+dx)+30ab^2(a^2-3b^2)\tan^2(c+dx)-20b^3(-3a^2+b^2)\tan^3(c+dx)+45ab^4\tan^4(c+dx)+12b^5\tan^5(c+dx)}{60b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-3*(a^5 + 10*(a + I*b)^3*b^2*Log[I - Tan[c + d*x]] + 10*(a - I*b)^3*b^2*Lo
g[I + Tan[c + d*x]]) + 60*b^3*(-3*a^2 + b^2)*Tan[c + d*x] + 30*a*b^2*(a^2 -
 3*b^2)*Tan[c + d*x]^2 - 20*b^3*(-3*a^2 + b^2)*Tan[c + d*x]^3 + 45*a*b^4*Ta
n[c + d*x]^4 + 12*b^5*Tan[c + d*x]^5)/(60*b^2*d)
```

Maple [A]
time = 0.05, size = 153, normalized size = 1.04

method	result
norman	$b(3a^2-b^2)\,x+\frac{b^3(\tan^5(dx+c))}{5d}+\frac{a(a^2-3b^2)(\tan^2(dx+c))}{2d}-\frac{b(3a^2-b^2)\tan(dx+c)}{d}+\frac{b(3a^2-b^2)(\tan^3(dx\text{-}}{3d}$
derivativedivides	$\frac{\frac{b^3(\tan^5(dx+c))}{5}+\frac{3b^2a(\tan^4(dx+c))}{4}+a^2b(\tan^3(dx+c))-\frac{b^3(\tan^3(dx+c))}{3}+\frac{a^3(\tan^2(dx+c))}{2}-\frac{3b^2a(\tan^2(dx+c))}{2}-3a^2b\tan(}{d}$
default	$\frac{\frac{b^3(\tan^5(dx+c))}{5}+\frac{3b^2a(\tan^4(dx+c))}{4}+a^2b(\tan^3(dx+c))-\frac{b^3(\tan^3(dx+c))}{3}+\frac{a^3(\tan^2(dx+c))}{2}-\frac{3b^2a(\tan^2(dx+c))}{2}-3a^2b\tan(}{d}$
risch	$3a^2bx-b^3x-ia^3x+3ia\,b^2x-\frac{2ia^3c}{d}+\frac{6ia\,b^2c}{d}+\frac{2i(-15ia^3e^{8i(dx+c)}-15ia^3e^{2i(dx+c)}-90a^2b\,e^{8i(dx+c)}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*b^3*tan(d*x+c)^5+3/4*b^2*a*tan(d*x+c)^4+a^2*b*tan(d*x+c)^3-1/3*b^3
*tan(d*x+c)^3+1/2*a^3*tan(d*x+c)^2-3/2*b^2*a*tan(d*x+c)^2-3*a^2*b*tan(d*x+c
)+b^3*tan(d*x+c)+1/2*(-a^3+3*a*b^2)*ln(1+tan(d*x+c)^2)+(3*a^2*b-b^3)*arctan
(tan(d*x+c)))
```

Maxima [A]
time = 0.52, size = 137, normalized size = 0.93

$$\frac{12\,b^3\tan\left(dx+c\right)^5+45\,ab^2\tan\left(dx+c\right)^4+20\,(3\,a^2b-b^3)\tan\left(dx+c\right)^3+30\,(a^3-3\,ab^2)\tan\left(dx+c\right)^2+60\,(3\,a^2b-b^3)(dx+c)-30\,(a^3-3\,ab^2)\log\left(\tan\left(dx+c\right)^2+1\right)-60\,(3\,a^2b-b^3)\tan\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(12*b^3*tan(d*x + c)^5 + 45*a*b^2*tan(d*x + c)^4 + 20*(3*a^2*b - b^3)*
tan(d*x + c)^3 + 30*(a^3 - 3*a*b^2)*tan(d*x + c)^2 + 60*(3*a^2*b - b^3)*(d*
x + c) - 30*(a^3 - 3*a*b^2)*log(tan(d*x + c)^2 + 1) - 60*(3*a^2*b - b^3)*ta
n(d*x + c))/d
```

Fricas [A]
time = 1.08, size = 136, normalized size = 0.93

$$\frac{12\,b^3\tan\left(dx+c\right)^5+45\,ab^2\tan\left(dx+c\right)^4+20\left(3\,a^2b-b^3\right)\tan\left(dx+c\right)^3+60\left(3\,a^2b-b^3\right)dx+30\left(a^3-3\,ab^2\right)\tan\left(dx+c\right)^2+30\left(a^3-3\,ab^2\right)\log\left(\frac{1}{\tan(dx+c)^2+1}\right)-60\left(3\,a^2b-b^3\right)\tan\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(12*b^3*tan(d*x + c)^5 + 45*a*b^2*tan(d*x + c)^4 + 20*(3*a^2*b - b^3)*
tan(d*x + c)^3 + 60*(3*a^2*b - b^3)*d*x + 30*(a^3 - 3*a*b^2)*tan(d*x + c)^2
 + 30*(a^3 - 3*a*b^2)*log(1/(tan(d*x + c)^2 + 1)) - 60*(3*a^2*b - b^3)*tan(
d*x + c))/d
```

Sympy [A]
time = 0.20, size = 194, normalized size = 1.32

$$\begin{cases} -\frac{a^3\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a^3\tan^2\left(c+dx\right)}{2d}+3a^2bx+\frac{a^2b\tan^3\left(c+dx\right)}{d}-\frac{3a^2b\tan\left(c+dx\right)}{d}+\frac{3ab^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{3ab^2\tan^4\left(c+dx\right)}{4d}-\frac{3ab^2\tan^2\left(c+dx\right)}{2d}-b^3x+\frac{b^3\tan^5\left(c+dx\right)}{5d}-\frac{b^3\tan^3\left(c+dx\right)}{3d}+\frac{b^3\tan\left(c+dx\right)}{d} & \text{for}\ d\neq 0 \\ x\left(a+b\tan\left(c\right)\right)^3\tan^3\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Piecewise((-a**3*log(tan(c + d*x)**2 + 1)/(2*d) + a**3*tan(c + d*x)**2/(2*d
) + 3*a**2*b*x + a**2*b*tan(c + d*x)**3/d - 3*a**2*b*tan(c + d*x)/d + 3*a*b
**2*log(tan(c + d*x)**2 + 1)/(2*d) + 3*a*b**2*tan(c + d*x)**4/(4*d) - 3*a*b
**2*tan(c + d*x)**2/(2*d) - b**3*x + b**3*tan(c + d*x)**5/(5*d) - b**3*tan(
c + d*x)**3/(3*d) + b**3*tan(c + d*x)/d, Ne(d, 0)), (x*(a + b*tan(c))**3*ta
n(c)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1997 vs. $2(139) = 278$.
time = 3.00, size = 1997, normalized size = 13.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(180*a^2*b*d*x*tan(d*x)^5*tan(c)^5 - 60*b^3*d*x*tan(d*x)^5*tan(c)^5 +
30*a^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)
^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^5*tan(c)^
5 - 90*a*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*
tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^5*t
an(c)^5 - 900*a^2*b*d*x*tan(d*x)^4*tan(c)^4 + 300*b^3*d*x*tan(d*x)^4*tan(c)
^4 + 30*a^3*tan(d*x)^5*tan(c)^5 - 135*a*b^2*tan(d*x)^5*tan(c)^5 - 150*a^3*l
og(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan
```

(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 + 450*
a*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^
2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4
 + 180*a^2*b*tan(d*x)^5*tan(c)^4 - 60*b^3*tan(d*x)^5*tan(c)^4 + 180*a^2*b*t
an(d*x)^4*tan(c)^5 - 60*b^3*tan(d*x)^4*tan(c)^5 + 1800*a^2*b*d*x*tan(d*x)^3
*tan(c)^3 - 600*b^3*d*x*tan(d*x)^3*tan(c)^3 + 30*a^3*tan(d*x)^5*tan(c)^3 -
90*a*b^2*tan(d*x)^5*tan(c)^3 - 90*a^3*tan(d*x)^4*tan(c)^4 + 495*a*b^2*tan(d
*x)^4*tan(c)^4 + 30*a^3*tan(d*x)^3*tan(c)^5 - 90*a*b^2*tan(d*x)^3*tan(c)^5
- 60*a^2*b*tan(d*x)^5*tan(c)^2 + 20*b^3*tan(d*x)^5*tan(c)^2 + 300*a^3*log(4
*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x
)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 - 900*a*b^
2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 +
tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 - 9
00*a^2*b*tan(d*x)^4*tan(c)^3 + 300*b^3*tan(d*x)^4*tan(c)^3 - 900*a^2*b*tan(
d*x)^3*tan(c)^4 + 300*b^3*tan(d*x)^3*tan(c)^4 - 60*a^2*b*tan(d*x)^2*tan(c)^
5 + 20*b^3*tan(d*x)^2*tan(c)^5 + 45*a*b^2*tan(d*x)^5*tan(c) - 1800*a^2*b*d*
x*tan(d*x)^2*tan(c)^2 + 600*b^3*d*x*tan(d*x)^2*tan(c)^2 - 90*a^3*tan(d*x)^4
*tan(c)^2 + 450*a*b^2*tan(d*x)^4*tan(c)^2 + 120*a^3*tan(d*x)^3*tan(c)^3 - 5
40*a*b^2*tan(d*x)^3*tan(c)^3 - 90*a^3*tan(d*x)^2*tan(c)^4 + 450*a*b^2*tan(d
*x)^2*tan(c)^4 + 45*a*b^2*tan(d*x)*tan(c)^5 - 12*b^3*tan(d*x)^5 + 120*a^2*b
*tan(d*x)^4*tan(c) - 100*b^3*tan(d*x)^4*tan(c) - 300*a^3*log(4*(tan(d*x)^4*
tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d
*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + 900*a*b^2*log(4*(tan(
d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 -
2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + 1440*a^2*b*tan
(d*x)^3*tan(c)^2 - 600*b^3*tan(d*x)^3*tan(c)^2 + 1440*a^2*b*tan(d*x)^2*tan(
c)^3 - 600*b^3*tan(d*x)^2*tan(c)^3 + 120*a^2*b*tan(d*x)*tan(c)^4 - 100*b^3*
tan(d*x)*tan(c)^4 - 12*b^3*tan(c)^5 - 45*a*b^2*tan(d*x)^4 + 900*a^2*b*d*x*t
an(d*x)*tan(c) - 300*b^3*d*x*tan(d*x)*tan(c) + 90*a^3*tan(d*x)^3*tan(c) - 4
50*a*b^2*tan(d*x)^3*tan(c) - 120*a^3*tan(d*x)^2*tan(c)^2 + 540*a*b^2*tan(d*
x)^2*tan(c)^2 + 90*a^3*tan(d*x)*tan(c)^3 - 450*a*b^2*tan(d*x)*tan(c)^3 - 45
*a*b^2*tan(c)^4 - 60*a^2*b*tan(d*x)^3 + 20*b^3*tan(d*x)^3 + 150*a^3*log(4*(
tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^
2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)*tan(c) - 450*a*b^2*log(
4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*
x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)*tan(c) - 900*a^2*b*t
an(d*x)^2*tan(c) + 300*b^3*tan(d*x)^2*tan(c) - 900*a^2*b*tan(d*x)*tan(c)^2
+ 300*b^3*tan(d*x)*tan(c)^2 - 60*a^2*b*tan(c)^3 + 20*b^3*tan(c)^3 - 180*a^2
*b*d*x + 60*b^3*d*x - 30*a^3*tan(d*x)^2 + 90*a*b^2*tan(d*x)^2 + 90*a^3*tan(
d*x)*tan(c) - 495*a*b^2*tan(d*x)*tan(c) - 30*a^3*tan(c)^2 + 90*a*b^2*tan(c)
^2 - 30*a^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*t
an(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) + 90*a*b^2*lo
g(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(
d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) + 180*a^2*b*tan(d*x) - 60*b
^3*tan(d*x) + 180*a^2*b*tan(c) - 60*b^3*tan(c) - 30*a^3 + 135*a*b^2)/(d*tan

```
(d*x)^5*tan(c)^5 - 5*d*tan(d*x)^4*tan(c)^4 + 10*d*tan(d*x)^3*tan(c)^3 - 10*
d*tan(d*x)^2*tan(c)^2 + 5*d*tan(d*x)*tan(c) - d)
```

Mupad [B]
time = 3.88, size = 182, normalized size = 1.24

$$\frac{b^3\tan\left(c+d\,x\right)^5}{5\,d}-\frac{\tan\left(c+d\,x\right)\,\left(3\,a^2\,b-b^3\right)}{d}+\frac{\ln\left(\tan\left(c+d\,x\right)^2+1\right)\,\left(\frac{3\,a\,b^2}{2}-\frac{a^3}{2}\right)}{d}-\frac{\tan\left(c+d\,x\right)^2\,\left(\frac{3\,a\,b^2}{2}-\frac{a^3}{2}\right)}{d}+\frac{\tan\left(c+d\,x\right)^3\,\left(a^2\,b-\frac{b^3}{3}\right)}{d}+\frac{3\,a\,b^2\tan\left(c+d\,x\right)^4}{4\,d}+\frac{b\,\mathrm{atan}\left(\frac{b\tan\left(c+d\,x\right)\,\left(3\,a^2-b^2\right)}{3\,a^2\,b-b^3}\right)\,\left(3\,a^2-b^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + b*tan(c + d*x))^3,x)
```

```
[Out] (b^3*tan(c + d*x)^5)/(5*d) - (tan(c + d*x)*(3*a^2*b - b^3))/d + (log(tan(c
+ d*x)^2 + 1)*((3*a*b^2)/2 - a^3/2))/d - (tan(c + d*x)^2*((3*a*b^2)/2 - a^3
/2))/d + (tan(c + d*x)^3*(a^2*b - b^3/3))/d + (3*a*b^2*tan(c + d*x)^4)/(4*d
) + (b*atan((b*tan(c + d*x)*(3*a^2 - b^2))/(3*a^2*b - b^3))*(3*a^2 - b^2))/
d
```

3.436 $\int \tan^2(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=94

$$-a\left(a^2-3b^2\right)x+\frac{b(3a^2-b^2)\log(\cos(c+dx))}{d}-\frac{2ab^2\tan(c+dx)}{d}-\frac{b(a+b\tan(c+dx))^2}{2d}+\frac{(a+b\tan(c+dx))^4}{4bd}$$

```
[Out] -a*(a^2-3*b^2)*x+b*(3*a^2-b^2)*ln(cos(d*x+c))/d-2*a*b^2*tan(d*x+c)/d-1/2*b*
(a+b*tan(d*x+c))^2/d+1/4*(a+b*tan(d*x+c))^4/b/d
```

Rubi [A]
time = 0.07, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3624, 3563, 3606, 3556}

$$\frac{b(3a^2-b^2)\log(\cos(c+dx))}{d}-ax\left(a^2-3b^2\right)-\frac{2ab^2\tan(c+dx)}{d}+\frac{(a+b\tan(c+dx))^4}{4bd}-\frac{b(a+b\tan(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -(a*(a^2 - 3*b^2)*x) + (b*(3*a^2 - b^2)*Log[Cos[c + d*x]])/d - (2*a*b^2*Tan
[c + d*x])/d - (b*(a + b*Tan[c + d*x])^2)/(2*d) + (a + b*Tan[c + d*x])^4/(4
*b*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3563

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
```

```
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{(a+b\tan(c+dx))^4}{4bd} - \int (a+b\tan(c+dx))^3\,dx$$
$$= -\frac{b(a+b\tan(c+dx))^2}{2d} + \frac{(a+b\tan(c+dx))^4}{4bd} - \int (a+b\tan(c+$$
$$= -a\left(a^2-3b^2\right)x - \frac{2ab^2\tan(c+dx)}{d} - \frac{b(a+b\tan(c+dx))^2}{2d} + \frac{(a+$$
$$= -a\left(a^2-3b^2\right)x + \frac{b(3a^2-b^2)\log(\cos(c+dx))}{d} - \frac{2ab^2\tan(c+dx)}{d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.05, size = 97, normalized size = 1.03

$$\frac{2i(a+ib)^3\log(i-\tan(c+dx)) + 2(ia+b)^3\log(i+\tan(c+dx)) - 12ab^2\tan(c+dx) - 2b^3\tan^2(c+dx) + \frac{(a+b\tan(c+dx))^4}{b}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((2*I)*(a + I*b)^3*Log[I - Tan[c + d*x]] + 2*(I*a + b)^3*Log[I + Tan[c + d*
x]] - 12*a*b^2*Tan[c + d*x] - 2*b^3*Tan[c + d*x]^2 + (a + b*Tan[c + d*x])^4
/b)/(4*d)
```

Maple [A]
time = 0.04, size = 124, normalized size = 1.32

method	result
norman	$\left(-a^3+3b^2a\right)x + \frac{a(a^2-3b^2)\tan(dx+c)}{d} + \frac{b^2a(\tan^3(dx+c))}{d} + \frac{b^3(\tan^4(dx+c))}{4d} + \frac{b(3a^2-b^2)(\tan^2(dx+c))}{2d}$
derivativedivides	$\frac{\frac{b^3(\tan^4(dx+c))}{4} + b^2a(\tan^3(dx+c)) + \frac{3a^2b(\tan^2(dx+c))}{2} - \frac{b^3(\tan^2(dx+c))}{2} + a^3\tan(dx+c) - 3b^2a\tan(dx+c) + \frac{(-3a^2b+b^3)\ln(}{2}}{d}$
default	$\frac{\frac{b^3(\tan^4(dx+c))}{4} + b^2a(\tan^3(dx+c)) + \frac{3a^2b(\tan^2(dx+c))}{2} - \frac{b^3(\tan^2(dx+c))}{2} + a^3\tan(dx+c) - 3b^2a\tan(dx+c) + \frac{(-3a^2b+b^3)\ln(}{2}}{d}$
risch	$-3ixb\,a^2 + ib^3x - a^3x + 3a\,b^2x - \frac{6ib\,a^2c}{d} + \frac{2ib^3c}{d} - \frac{2(-ia^3e^{6i(dx+c)} + 6ia\,b^2e^{6i(dx+c)} - 3a^2b\,e^{6i(dx+c)} + 2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*b^3*tan(d*x+c)^4+b^2*a*tan(d*x+c)^3+3/2*a^2*b*tan(d*x+c)^2-1/2*b^3
*tan(d*x+c)^2+a^3*tan(d*x+c)-3*b^2*a*tan(d*x+c)+1/2*(-3*a^2*b+b^3)*ln(1+tan
(d*x+c)^2)+(-a^3+3*a*b^2)*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.52, size = 114, normalized size = 1.21

$$\frac{b^3 \tan\left(dx+c\right)^4 + 4\,ab^2 \tan\left(dx+c\right)^3 + 2\left(3\,a^2b - b^3\right)\tan\left(dx+c\right)^2 - 4\left(a^3 - 3\,ab^2\right)(dx+c) - 2\left(3\,a^2b - b^3\right)\log\left(\tan\left(dx+c\right)^2 + 1\right) + 4\left(a^3 - 3\,ab^2\right)\tan\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(b^3*tan(d*x + c)^4 + 4*a*b^2*tan(d*x + c)^3 + 2*(3*a^2*b - b^3)*tan(d*
x + c)^2 - 4*(a^3 - 3*a*b^2)*(d*x + c) - 2*(3*a^2*b - b^3)*log(tan(d*x + c)
^2 + 1) + 4*(a^3 - 3*a*b^2)*tan(d*x + c))/d
```

Fricas [A]
time = 1.24, size = 113, normalized size = 1.20

$$\frac{b^3 \tan\left(dx+c\right)^4 + 4\,ab^2 \tan\left(dx+c\right)^3 - 4\left(a^3 - 3\,ab^2\right)dx + 2\left(3\,a^2b - b^3\right)\tan\left(dx+c\right)^2 + 2\left(3\,a^2b - b^3\right)\log\left(\frac{1}{\tan(dx+c)^2+1}\right) + 4\left(a^3 - 3\,ab^2\right)\tan\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*tan(d*x + c)^4 + 4*a*b^2*tan(d*x + c)^3 - 4*(a^3 - 3*a*b^2)*d*x +
2*(3*a^2*b - b^3)*tan(d*x + c)^2 + 2*(3*a^2*b - b^3)*log(1/(tan(d*x + c)^2
+ 1)) + 4*(a^3 - 3*a*b^2)*tan(d*x + c))/d
```

Sympy [A]
time = 0.15, size = 160, normalized size = 1.70

$$\begin{cases} -a^3x + \frac{a^3\tan(c+dx)}{d} - \frac{3a^2b\log(\tan^2(c+dx)+1)}{2d} + \frac{3a^2b\tan^2(c+dx)}{2d} + 3ab^2x + \frac{ab^2\tan^3(c+dx)}{d} - \frac{3ab^2\tan(c+dx)}{d} + \frac{b^3\log(\tan^2(c+dx)+1)}{2d} + \frac{b^3\tan^4(c+dx)}{4d} - \frac{b^3\tan^2(c+dx)}{2d} & \text{for}\ d \neq 0 \\ x(a+b\tan(c))^3\tan^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Piecewise((-a**3*x + a**3*tan(c + d*x)/d - 3*a**2*b*log(tan(c + d*x)**2 + 1
)/(2*d) + 3*a**2*b*tan(c + d*x)**2/(2*d) + 3*a*b**2*x + a*b**2*tan(c + d*x)
**3/d - 3*a*b**2*tan(c + d*x)/d + b**3*log(tan(c + d*x)**2 + 1)/(2*d) + b**
3*tan(c + d*x)**4/(4*d) - b**3*tan(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*
tan(c))**3*tan(c)**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1485 vs. $2(90) = 180$.
time = 1.70, size = 1485, normalized size = 15.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/4*(4*a^3*d*x*tan(d*x)^4*tan(c)^4 - 12*a*b^2*d*x*tan(d*x)^4*tan(c)^4 - 6*
a^2*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^
2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4
 + 2*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(
c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c
)^4 - 16*a^3*d*x*tan(d*x)^3*tan(c)^3 + 48*a*b^2*d*x*tan(d*x)^3*tan(c)^3 - 6
*a^2*b*tan(d*x)^4*tan(c)^4 + 3*b^3*tan(d*x)^4*tan(c)^4 + 24*a^2*b*log(4*(ta
n(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2
- 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 - 8*b^3*log(4*
(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)
^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 + 4*a^3*tan
(d*x)^4*tan(c)^3 - 12*a*b^2*tan(d*x)^4*tan(c)^3 + 4*a^3*tan(d*x)^3*tan(c)^4
 - 12*a*b^2*tan(d*x)^3*tan(c)^4 + 24*a^3*d*x*tan(d*x)^2*tan(c)^2 - 72*a*b^2
*d*x*tan(d*x)^2*tan(c)^2 - 6*a^2*b*tan(d*x)^4*tan(c)^2 + 2*b^3*tan(d*x)^4*t
an(c)^2 + 12*a^2*b*tan(d*x)^3*tan(c)^3 - 8*b^3*tan(d*x)^3*tan(c)^3 - 6*a^2*
b*tan(d*x)^2*tan(c)^4 + 2*b^3*tan(d*x)^2*tan(c)^4 + 4*a*b^2*tan(d*x)^4*tan(
c) - 36*a^2*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2
*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*
tan(c)^2 + 12*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*
x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x
)^2*tan(c)^2 - 12*a^3*tan(d*x)^3*tan(c)^2 + 48*a*b^2*tan(d*x)^3*tan(c)^2 -
12*a^3*tan(d*x)^2*tan(c)^3 + 48*a*b^2*tan(d*x)^2*tan(c)^3 + 4*a*b^2*tan(d*x
)*tan(c)^4 - b^3*tan(d*x)^4 - 16*a^3*d*x*tan(d*x)*tan(c) + 48*a*b^2*d*x*tan
(d*x)*tan(c) + 12*a^2*b*tan(d*x)^3*tan(c) - 8*b^3*tan(d*x)^3*tan(c) - 12*a^
2*b*tan(d*x)^2*tan(c)^2 + 4*b^3*tan(d*x)^2*tan(c)^2 + 12*a^2*b*tan(d*x)*tan
(c)^3 - 8*b^3*tan(d*x)*tan(c)^3 - b^3*tan(c)^4 - 4*a*b^2*tan(d*x)^3 + 24*a^
2*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2
+ tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)*tan(c) - 8*b
^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 +
 tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)*tan(c) + 12*a
^3*tan(d*x)^2*tan(c) - 48*a*b^2*tan(d*x)^2*tan(c) + 12*a^3*tan(d*x)*tan(c)^
2 - 48*a*b^2*tan(d*x)*tan(c)^2 - 4*a*b^2*tan(c)^3 + 4*a^3*d*x - 12*a*b^2*d*
x - 6*a^2*b*tan(d*x)^2 + 2*b^3*tan(d*x)^2 + 12*a^2*b*tan(d*x)*tan(c) - 8*b^
3*tan(d*x)*tan(c) - 6*a^2*b*tan(c)^2 + 2*b^3*tan(c)^2 - 6*a^2*b*log(4*(tan(
d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 -
2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) + 2*b^3*log(4*(tan(d*x)^4*tan(c)^2 -
 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c)
 + 1)/(tan(c)^2 + 1)) - 4*a^3*tan(d*x) + 12*a*b^2*tan(d*x) - 4*a^3*tan(c) +
 12*a*b^2*tan(c) - 6*a^2*b + 3*b^3)/(d*tan(d*x)^4*tan(c)^4 - 4*d*tan(d*x)^3
*tan(c)^3 + 6*d*tan(d*x)^2*tan(c)^2 - 4*d*tan(d*x)*tan(c) + d)
```

Mupad [B]

time = 3.84, size = 154, normalized size = 1.64

$$\frac{b^3 \tan(c+dx)^4}{4d} - \frac{\tan(c+dx)\,(3ab^2-a^3)}{d} - \frac{\ln\left(\tan(c+dx)^2+1\right)\left(\frac{3a^2b}{2}-\frac{b^3}{2}\right)}{d} + \frac{\tan(c+dx)^2\left(\frac{3a^2b}{2}-\frac{b^3}{2}\right)}{d} + \frac{ab^2\tan(c+dx)^3}{d} + \frac{a\,\mathrm{atan}\left(\frac{a\tan(c+dx)\,(a^2-3b^2)}{3ab^2-a^3}\right)(a^2-3b^2)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + b*tan(c + d*x))^3,x)
```

```
[Out] (b^3*tan(c + d*x)^4)/(4*d) - (tan(c + d*x)*(3*a*b^2 - a^3))/d - (log(tan(c
+ d*x)^2 + 1)*((3*a^2*b)/2 - b^3/2))/d + (tan(c + d*x)^2*((3*a^2*b)/2 - b^3
/2))/d + (a*b^2*tan(c + d*x)^3)/d + (a*atan((a*tan(c + d*x)*(a^2 - 3*b^2))/
(3*a*b^2 - a^3))*(a^2 - 3*b^2))/d
```

3.437 $\int \tan(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=97

$$-b(3a^2-b^2)\,x-\frac{a(a^2-3b^2)\log(\cos(c+dx))}{d}+\frac{b(a^2-b^2)\tan(c+dx)}{d}+\frac{a(a+b\tan(c+dx))^2}{2d}+\frac{(a+b\tan(c+dx))^3}{3d}$$

```
[Out] -b*(3*a^2-b^2)*x-a*(a^2-3*b^2)*ln(cos(d*x+c))/d+b*(a^2-b^2)*tan(d*x+c)/d+1/
2*a*(a+b*tan(d*x+c))^2/d+1/3*(a+b*tan(d*x+c))^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3609, 3606, 3556}

$$\frac{b(a^2-b^2)\tan(c+dx)}{d}-\frac{a(a^2-3b^2)\log(\cos(c+dx))}{d}-bx(3a^2-b^2)+\frac{(a+b\tan(c+dx))^3}{3d}+\frac{a(a+b\tan(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -(b*(3*a^2 - b^2)*x) - (a*(a^2 - 3*b^2)*Log[Cos[c + d*x]])/d + (b*(a^2 - b^
2)*Tan[c + d*x])/d + (a*(a + b*Tan[c + d*x])^2)/(2*d) + (a + b*Tan[c + d*x]
)^3/(3*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\int \tan(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{(a+b\tan(c+dx))^3}{3d} + \int (-b+a\tan(c+dx))(a+b\tan(c+dx))^2$$
$$= \frac{a(a+b\tan(c+dx))^2}{2d} + \frac{(a+b\tan(c+dx))^3}{3d} + \int (a+b\tan(c+d$$
$$= -b(3a^2-b^2)\,x + \frac{b(a^2-b^2)\tan(c+dx)}{d} + \frac{a(a+b\tan(c+dx))^2}{2d}$$
$$= -b(3a^2-b^2)\,x - \frac{a(a^2-3b^2)\log(\cos(c+dx))}{d} + \frac{b(a^2-b^2)\tan(c}{d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.58, size = 100, normalized size = 1.03

$$\frac{3((a+ib)^3\log(i-\tan(c+dx))+(a-ib)^3\log(i+\tan(c+dx)))-6b(-3a^2+b^2)\tan(c+dx)+9ab^2\tan^2(c+dx)+2b^3\tan^3(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*((a + I*b)^3*Log[I - Tan[c + d*x]] + (a - I*b)^3*Log[I + Tan[c + d*x]])
- 6*b*(-3*a^2 + b^2)*Tan[c + d*x] + 9*a*b^2*Tan[c + d*x]^2 + 2*b^3*Tan[c +
d*x]^3)/(6*d)
```

Maple [A]
time = 0.04, size = 97, normalized size = 1.00

method	result
norman	$(-3a^2b+b^3)\,x+\frac{b(3a^2-b^2)\tan(dx+c)}{d}+\frac{b^3(\tan^3(dx+c))}{3d}+\frac{3b^2a(\tan^2(dx+c))}{2d}+\frac{a(a^2-3b^2)\ln(1+\tan^2(d}{2d}$
derivativedivides	$\frac{\frac{b^3(\tan^3(dx+c))}{3}+\frac{3b^2a(\tan^2(dx+c))}{2}+3a^2b\tan(dx+c)-b^3\tan(dx+c)+\frac{(a^3-3b^2a)\ln(1+\tan^2(dx+c))}{2}+(-3a^2b+b^3)\arctan}{d}$
default	$\frac{\frac{b^3(\tan^3(dx+c))}{3}+\frac{3b^2a(\tan^2(dx+c))}{2}+3a^2b\tan(dx+c)-b^3\tan(dx+c)+\frac{(a^3-3b^2a)\ln(1+\tan^2(dx+c))}{2}+(-3a^2b+b^3)\arctan}{d}$
risch	$-3a^2bx+b^3x+ia^3x-3ia\,b^2x+\frac{2ia^3c}{d}-\frac{6ia\,b^2c}{d}-\frac{2ib(9iab\,e^{4i(dx+c)}-9a^2e^{4i(dx+c)}+6b^2e^{4i(dx+c)}+9i}{3d(e^{2i(}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*b^3*tan(d*x+c)^3+3/2*b^2*a*tan(d*x+c)^2+3*a^2*b*tan(d*x+c)-b^3*tan
(d*x+c)+1/2*(a^3-3*a*b^2)*ln(1+tan(d*x+c)^2)+(-3*a^2*b+b^3)*arctan(tan(d*x+
c)))
```

Maxima [A]
time = 0.52, size = 95, normalized size = 0.98

$$\frac{2\,b^{3}\tan\left(dx+c\right)^{3}+9\,ab^{2}\tan\left(dx+c\right)^{2}-6\,(3\,a^{2}b-b^{3})(dx+c)+3\,(a^{3}-3\,ab^{2})\log\left(\tan\left(dx+c\right)^{2}+1\right)+6\,(3\,a^{2}b-b^{3})\tan\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^3*tan(d*x + c)^3 + 9*a*b^2*tan(d*x + c)^2 - 6*(3*a^2*b - b^3)*(d*x
 + c) + 3*(a^3 - 3*a*b^2)*log(tan(d*x + c)^2 + 1) + 6*(3*a^2*b - b^3)*tan(d
*x + c))/d
```

Fricas [A]
time = 1.41, size = 94, normalized size = 0.97

$$\frac{2\,b^{3}\tan\left(dx+c\right)^{3}+9\,ab^{2}\tan\left(dx+c\right)^{2}-6\,(3\,a^{2}b-b^{3})dx-3\,(a^{3}-3\,ab^{2})\log\left(\frac{1}{\tan(dx+c)^{2}+1}\right)+6\,(3\,a^{2}b-b^{3})\tan\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*tan(d*x + c)^3 + 9*a*b^2*tan(d*x + c)^2 - 6*(3*a^2*b - b^3)*d*x
- 3*(a^3 - 3*a*b^2)*log(1/(tan(d*x + c)^2 + 1)) + 6*(3*a^2*b - b^3)*tan(d*x
 + c))/d
```

Sympy [A]
time = 0.11, size = 128, normalized size = 1.32

$$\begin{cases} \frac{a^{3}\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d}-3a^{2}bx+\frac{3a^{2}b\tan\left(c+dx\right)}{d}-\frac{3ab^{2}\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d}+\frac{3ab^{2}\tan^{2}\left(c+dx\right)}{2d}+b^{3}x+\frac{b^{3}\tan^{3}\left(c+dx\right)}{3d}-\frac{b^{3}\tan\left(c+dx\right)}{d} & \text{for}\ d\neq 0 \\ x\left(a+b\tan\left(c\right)\right)^{3}\tan\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*log(tan(c + d*x)**2 + 1)/(2*d) - 3*a**2*b*x + 3*a**2*b*tan(
c + d*x)/d - 3*a*b**2*log(tan(c + d*x)**2 + 1)/(2*d) + 3*a*b**2*tan(c + d*x
)**2/(2*d) + b**3*x + b**3*tan(c + d*x)**3/(3*d) - b**3*tan(c + d*x)/d, Ne(
d, 0)), (x*(a + b*tan(c))**3*tan(c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 993 vs. 2(93) = 186.
time = 1.32, size = 993, normalized size = 10.24

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="giac")

[Out] -1/6*(18*a^2*b*d*x*tan(d*x)^3*tan(c)^3 - 6*b^3*d*x*tan(d*x)^3*tan(c)^3 + 3*a^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 - 9*a*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 - 54*a^2*b*d*x*tan(d*x)^2*tan(c)^2 + 18*b^3*d*x*tan(d*x)^2*tan(c)^2 - 9*a*b^2*tan(d*x)^3*tan(c)^3 - 9*a^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + 27*a*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + 18*a^2*b*tan(d*x)^3*tan(c)^2 - 6*b^3*tan(d*x)^3*tan(c)^2 + 18*a^2*b*tan(d*x)^2*tan(c)^3 - 6*b^3*tan(d*x)^2*tan(c)^3 + 54*a^2*b*d*x*tan(d*x)*tan(c) - 18*b^3*d*x*tan(d*x)*tan(c) - 9*a*b^2*tan(d*x)^3*tan(c) + 9*a*b^2*tan(d*x)^2*tan(c)^2 - 9*a*b^2*tan(d*x)*tan(c)^3 + 2*b^3*tan(d*x)^3 + 9*a^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)*tan(c) - 27*a*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)*tan(c) - 36*a^2*b*tan(d*x)^2*tan(c) + 18*b^3*tan(d*x)^2*tan(c) - 36*a^2*b*tan(d*x)*tan(c)^2 + 18*b^3*tan(d*x)*tan(c)^2 + 2*b^3*tan(c)^3 - 18*a^2*b*d*x + 6*b^3*d*x + 9*a*b^2*tan(d*x)^2 - 9*a*b^2*tan(d*x)*tan(c) + 9*a*b^2*tan(c)^2 - 3*a^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) + 9*a*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) + 18*a^2*b*tan(d*x) - 6*b^3*tan(d*x) + 18*a^2*b*tan(c) - 6*b^3*tan(c) + 9*a*b^2)/(d*tan(d*x)^3*tan(c)^3 - 3*d*tan(d*x)^2*tan(c)^2 + 3*d*tan(d*x)*tan(c) - d)

Mupad [B]
time = 3.83, size = 135, normalized size = 1.39

$$\frac{\tan(c+d\,x)\,(3\,a^2\,b-b^3)}{d}+\frac{b^3\tan(c+d\,x)^3}{3\,d}-\frac{\ln\left(\tan(c+d\,x)^2+1\right)\,\left(\frac{3\,a\,b^2}{2}-\frac{a^3}{2}\right)}{d}+\frac{3\,a\,b^2\tan(c+d\,x)^2}{2\,d}-\frac{b\,\mathrm{atan}\left(\frac{b\tan(c+d\,x)\,(3\,a^2-b^2)}{3\,a^2\,b-b^3}\right)\,(3\,a^2-b^2)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d*x)*(a + b*tan(c + d*x))^3,x)

[Out] (tan(c + d*x)*(3*a^2*b - b^3))/d + (b^3*tan(c + d*x)^3)/(3*d) - (log(tan(c + d*x)^2 + 1)*((3*a*b^2)/2 - a^3/2))/d + (3*a*b^2*tan(c + d*x)^2)/(2*d) - (b*atan((b*tan(c + d*x)*(3*a^2 - b^2))/(3*a^2*b - b^3))*(3*a^2 - b^2))/d

3.438 $\int (a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=72

$$a\left(a^2-3b^2\right)x-\frac{b(3a^2-b^2)\log(\cos(c+dx))}{d}+\frac{2ab^2\tan(c+dx)}{d}+\frac{b(a+b\tan(c+dx))^2}{2d}$$

```
[Out] a*(a^2-3*b^2)*x-b*(3*a^2-b^2)*ln(cos(d*x+c))/d+2*a*b^2*tan(d*x+c)/d+1/2*b*(
a+b*tan(d*x+c))^2/d
```

Rubi [A]
time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {3563, 3606, 3556}

$$-\frac{b(3a^2-b^2)\log(\cos(c+dx))}{d}+ax\left(a^2-3b^2\right)+\frac{2ab^2\tan(c+dx)}{d}+\frac{b(a+b\tan(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^3,x]
```

```
[Out] a*(a^2 - 3*b^2)*x - (b*(3*a^2 - b^2)*Log[Cos[c + d*x]])/d + (2*a*b^2*Tan[c
+ d*x])/d + (b*(a + b*Tan[c + d*x])^2)/(2*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3563

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rubi steps

$$\int (a+b\tan(c+dx))^3\,dx = \frac{b(a+b\tan(c+dx))^2}{2d} + \int (a+b\tan(c+dx))\left(a^2-b^2+2ab\tan(c+dx)\right)\,dx$$
$$= a\left(a^2-3b^2\right)x + \frac{2ab^2\tan(c+dx)}{d} + \frac{b(a+b\tan(c+dx))^2}{2d} + \left(b\left(3a^2-b^2\right)\right)\int \mathrm{t}$$
$$= a\left(a^2-3b^2\right)x - \frac{b(3a^2-b^2)\log(\cos(c+dx))}{d} + \frac{2ab^2\tan(c+dx)}{d} + \frac{b(a+b\tan}{2c}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.25, size = 79, normalized size = 1.10

$$\frac{(ia-b)^3\log(i-\tan(c+dx)) - (ia+b)^3\log(i+\tan(c+dx)) + 6ab^2\tan(c+dx) + b^3\tan^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^3,x]
[Out] ((I*a - b)^3*Log[I - Tan[c + d*x]] - (I*a + b)^3*Log[I + Tan[c + d*x]] + 6*
a*b^2*Tan[c + d*x] + b^3*Tan[c + d*x]^2)/(2*d)
```

Maple [A]
time = 0.03, size = 74, normalized size = 1.03

method	result
norman	$(a^3-3b^2a)\,x+\frac{b^3(\tan^2(dx+c))}{2d}+\frac{3a\,b^2\tan(dx+c)}{d}+\frac{b(3a^2-b^2)\ln(1+\tan^2(dx+c))}{2d}$
derivativedivides	$\frac{\frac{b^3\left(\tan^2(dx+c)\right)}{2}+3b^2a\tan(dx+c)+\frac{\left(3a^2b-b^3\right)\ln\left(1+\tan^2(dx+c)\right)}{2}+(a^3-3b^2a)\arctan(\tan(dx+c))}{d}$
default	$\frac{\frac{b^3\left(\tan^2(dx+c)\right)}{2}+3b^2a\tan(dx+c)+\frac{\left(3a^2b-b^3\right)\ln\left(1+\tan^2(dx+c)\right)}{2}+(a^3-3b^2a)\arctan(\tan(dx+c))}{d}$
risch	$3ixb\,a^2-ib^3x+a^3x-3a\,b^2x+\frac{6ib\,a^2c}{d}-\frac{2ib^3c}{d}+\frac{2b^2\left(3ia\,e^{2i(dx+c)}+b\,e^{2i(dx+c)}+3ia\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{3b\ln\left(e^{2i(dx+\right.}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/d*(1/2*b^3*tan(d*x+c)^2+3*b^2*a*tan(d*x+c)+1/2*(3*a^2*b-b^3)*ln(1+tan(d*x
+c)^2)+(a^3-3*a*b^2)*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.52, size = 78, normalized size = 1.08

$$a^3x - \frac{3\,(dx+c-\tan(dx+c))ab^2}{d} - \frac{b^3\left(\frac{1}{\sin(dx+c)^2-1} - \log\left(\sin(dx+c)^2-1\right)\right)}{2\,d} + \frac{3\,a^2b\log(\sec(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x - 3*(d*x + c - tan(d*x + c))*a*b^2/d - 1/2*b^3*(1/(sin(d*x + c)^2 - 1
) - log(sin(d*x + c)^2 - 1))/d + 3*a^2*b*log(sec(d*x + c))/d
```

Fricas [A]
time = 1.66, size = 71, normalized size = 0.99

$$\frac{b^3 \tan\left(dx+c\right)^2 + 6\,ab^2 \tan\left(dx+c\right) + 2\left(a^3 - 3\,ab^2\right)dx - \left(3\,a^2b - b^3\right)\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*tan(d*x + c)^2 + 6*a*b^2*tan(d*x + c) + 2*(a^3 - 3*a*b^2)*d*x - (3
*a^2*b - b^3)*log(1/(tan(d*x + c)^2 + 1)))/d
```

Sympy [A]
time = 0.09, size = 94, normalized size = 1.31

$$\begin{cases} a^3x + \frac{3a^2b\log\left(\tan^2\left(c+dx\right)+1\right)}{2d} - 3ab^2x + \frac{3ab^2\tan\left(c+dx\right)}{d} - \frac{b^3\log\left(\tan^2\left(c+dx\right)+1\right)}{2d} + \frac{b^3\tan^2\left(c+dx\right)}{2d} & \text{for } d \neq 0 \\ x\left(a+b\tan\left(c\right)\right)^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*log(tan(c + d*x)**2 + 1)/(2*d) - 3*a*b**2*x +
3*a*b**2*tan(c + d*x)/d - b**3*log(tan(c + d*x)**2 + 1)/(2*d) + b**3*tan(c
+ d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*tan(c))**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 603 vs. 2(70) = 140.
time = 0.80, size = 603, normalized size = 8.38

$$\frac{2\,a^3dx\tan\left(dx\right)^2\tan\left(c\right)^2 - 6\,ab^2dx\tan\left(dx\right)^2\tan\left(c\right)^2 - 3\,a^2b\log\left(\frac{4\left(\tan(dx)^4\tan(c)^2 - 2\tan(dx)^3\tan(c) + \tan(dx)^2\tan(c)^2 + \tan(dx)^2 - 2\tan(dx)\tan(c) + 1\right)}{\tan(c)^2+1}\right)\tan\left(dx\right)^2\tan\left(c\right)^2 + b^3\log\left(\frac{4\left(\tan(dx)^4\tan(c)^2 - 2\tan(dx)^3\tan(c) + \tan(dx)^2\tan(c)^2 + \tan(dx)^2 - 2\tan(dx)\tan(c) + 1\right)}{\tan(c)^2+1}\right)\tan\left(dx\right)^2\tan\left(c\right)^2 - 4\,a^3dx\tan\left(dx\right)\tan\left(c\right) + 12\,ab^2dx\tan\left(dx\right)\tan\left(c\right) + b^3\tan\left(dx\right)^2\tan\left(c\right)^2 + 6\,a^2b\log\left(\frac{4\left(\tan(dx)^4\tan(c)^2 - 2\tan(dx)^3\tan(c) + \tan(dx)^2\tan(c)^2 + \tan(dx)^2 - 2\tan(dx)\tan(c) + 1\right)}{\tan(c)^2+1}\right)\tan\left(dx\right)\tan\left(c\right) - 2\,b^3\log\left(\frac{4\left(\tan(dx)^4\tan(c)^2 - 2\tan(dx)^3\tan(c) + \tan(dx)^2\tan(c)^2 + \tan(dx)^2 - 2\tan(dx)\tan(c) + 1\right)}{\tan(c)^2+1}\right)\tan\left(dx\right)\tan\left(c\right) - 6\,ab^2\tan\left(dx\right)^2\tan\left(c\right) - 6\,ab^2\tan\left(dx\right)\tan\left(c\right)^2 + 2\,a^3dx - 6\,ab^2dx + b^3\tan\left(dx\right)^2 + b^3\tan\left(c\right)^2 - 3\,a^2b\log\left(\frac{4\left(\tan(dx)^4\tan(c)^2 - 2\tan(dx)^3\tan(c) + \tan(dx)^2\tan(c)^2 + \tan(dx)^2 - 2\tan(dx)\tan(c) + 1\right)}{\tan(c)^2+1}\right) + b^3\log\left(\frac{4\left(\tan(dx)^4\tan(c)^2 - 2\tan(dx)^3\tan(c) + \tan(dx)^2\tan(c)^2 + \tan(dx)^2 - 2\tan(dx)\tan(c) + 1\right)}{\tan(c)^2+1}\right) + 6\,ab^2\tan\left(dx\right) + 6\,ab^2\tan\left(c\right) + b^3}{2\left(d\tan\left(dx\right)^2\tan\left(c\right)^2 - 2\,d\tan\left(dx\right)\tan\left(c\right) + d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*a^3*d*x*tan(d*x)^2*tan(c)^2 - 6*a*b^2*d*x*tan(d*x)^2*tan(c)^2 - 3*a^
2*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2
+ tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 +
 b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2
 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2
```

```
- 4*a^3*d*x*tan(d*x)*tan(c) + 12*a*b^2*d*x*tan(d*x)*tan(c) + b^3*tan(d*x)^2
*tan(c)^2 + 6*a^2*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(
d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d
*x)*tan(c) - 2*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d
*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*
x)*tan(c) - 6*a*b^2*tan(d*x)^2*tan(c) - 6*a*b^2*tan(d*x)*tan(c)^2 + 2*a^3*d
*x - 6*a*b^2*d*x + b^3*tan(d*x)^2 + b^3*tan(c)^2 - 3*a^2*b*log(4*(tan(d*x)^
4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan
(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) + b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(
d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(
tan(c)^2 + 1)) + 6*a*b^2*tan(d*x) + 6*a*b^2*tan(c) + b^3)/(d*tan(d*x)^2*tan
(c)^2 - 2*d*tan(d*x)*tan(c) + d)
```

Mupad [B]
time = 3.82, size = 106, normalized size = 1.47

$$\frac{b^3\tan(c+d\,x)^2}{2\,d}+\frac{\ln\left(\tan(c+d\,x)^2+1\right)\,\left(\frac{3\,a^2\,b}{2}-\frac{b^3}{2}\right)}{d}+\frac{3\,a\,b^2\tan(c+d\,x)}{d}-\frac{a\,\mathrm{atan}\left(\frac{a\tan(c+d\,x)\,\left(a^2-3\,b^2\right)}{3\,a\,b^2-a^3}\right)\,\left(a^2-3\,b^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^3,x)
```

```
[Out] (b^3*tan(c + d*x)^2)/(2*d) + (log(tan(c + d*x)^2 + 1)*((3*a^2*b)/2 - b^3/2)
)/d + (3*a*b^2*tan(c + d*x))/d - (a*atan((a*tan(c + d*x)*(a^2 - 3*b^2))/(3*
a*b^2 - a^3))*(a^2 - 3*b^2))/d
```

3.439 $\int \cot(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=62

$$b(3a^2-b^2)\,x-\frac{3ab^2\log(\cos(c+dx))}{d}+\frac{a^3\log(\sin(c+dx))}{d}+\frac{b^2(a+b\tan(c+dx))}{d}$$

```
[Out] b*(3*a^2-b^2)*x-3*a*b^2*ln(cos(d*x+c))/d+a^3*ln(sin(d*x+c))/d+b^2*(a+b*tan(
d*x+c))/d
```

Rubi [A]
time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.158$, Rules used = {3647, 3705, 3556}

$$\frac{a^3\log(\sin(c+dx))}{d}+bx(3a^2-b^2)+\frac{b^2(a+b\tan(c+dx))}{d}-\frac{3ab^2\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] b*(3*a^2 - b^2)*x - (3*a*b^2*Log[Cos[c + d*x]])/d + (a^3*Log[Sin[c + d*x]])
/d + (b^2*(a + b*Tan[c + d*x]))/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3705

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[B*x, x] + (Dist[A, Int[1/Tan[e
 + f*x], x], x] + Dist[C, Int[Tan[e + f*x], x], x]) /; FreeQ[{e, f, A, B, C
}, x] && NeQ[A, C]
```

Rubi steps

$$\int \cot(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{b^2(a+b\tan(c+dx))}{d} + \int \cot(c+dx)\left(a^3+b\left(3a^2-b^2\right)\tan(c+d$$
$$= b\left(3a^2-b^2\right)x + \frac{b^2(a+b\tan(c+dx))}{d} + a^3\int \cot(c+dx)\,dx + (3a$$
$$= b\left(3a^2-b^2\right)x - \frac{3ab^2\log(\cos(c+dx))}{d} + \frac{a^3\log(\sin(c+dx))}{d} + \frac{b^2(a$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.21, size = 79, normalized size = 1.27

$$-\frac{(a+ib)^3\log(i-\tan(c+dx))-2a^3\log(\tan(c+dx))+(a-ib)^3\log(i+\tan(c+dx))-2b^2(a+b\tan(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -1/2*((a + I*b)^3*Log[I - Tan[c + d*x]] - 2*a^3*Log[Tan[c + d*x]] + (a - I*
b)^3*Log[I + Tan[c + d*x]] - 2*b^2*(a + b*Tan[c + d*x]))/d
```

Maple [A]
time = 0.18, size = 59, normalized size = 0.95

method	result
derivativedivides	$\frac{a^3\ln(\sin(dx+c))+3a^2b(dx+c)-3b^2a\ln(\cos(dx+c))+b^3(\tan(dx+c)-dx-c)}{d}$
default	$\frac{a^3\ln(\sin(dx+c))+3a^2b(dx+c)-3b^2a\ln(\cos(dx+c))+b^3(\tan(dx+c)-dx-c)}{d}$
norman	$\left(3a^2b-b^3\right)x+\frac{b^3\tan(dx+c)}{d}+\frac{a^3\ln(\tan(dx+c))}{d}-\frac{a\left(a^2-3b^2\right)\ln\left(1+\tan^2(dx+c)\right)}{2d}$
risch	$3a^2bx-b^3x-ia^3x+3iab^2x+\frac{6iab^2c}{d}-\frac{2ia^3c}{d}+\frac{2ib^3}{d\left(e^{2i(dx+c)}+1\right)}-\frac{3a\ln\left(e^{2i(dx+c)}+1\right)b^2}{d}+\frac{a^3\ln(e$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*ln(sin(d*x+c))+3*a^2*b*(d*x+c)-3*b^2*a*ln(cos(d*x+c))+b^3*(tan(d*x
+c)-d*x-c))
```

Maxima [A]
time = 0.53, size = 71, normalized size = 1.15

$$\frac{2\,a^3\log\left(\tan\left(dx+c\right)\right)+2\,b^3\tan\left(dx+c\right)+2\,\left(3\,a^2b-b^3\right)(dx+c)-\left(a^3-3\,ab^2\right)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^3*log(tan(d*x + c)) + 2*b^3*tan(d*x + c) + 2*(3*a^2*b - b^3)*(d*x
+ c) - (a^3 - 3*a*b^2)*log(tan(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 2.29, size = 78, normalized size = 1.26

$$\frac{a^3 \log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right) - 3\,ab^2 \log\left(\frac{1}{\tan(dx+c)^2+1}\right) + 2\,b^3 \tan(dx+c) + 2\,(3\,a^2b - b^3)dx}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a^3*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1)) - 3*a*b^2*log(1/(tan(d*x
+ c)^2 + 1)) + 2*b^3*tan(d*x + c) + 2*(3*a^2*b - b^3)*d*x)/d
```

Sympy [A]
time = 0.32, size = 92, normalized size = 1.48

$$\begin{cases} -\frac{a^3 \log(\tan^2(c+dx)+1)}{2d} + \frac{a^3 \log(\tan(c+dx))}{d} + 3a^2bx + \frac{3ab^2 \log(\tan^2(c+dx)+1)}{2d} - b^3x + \frac{b^3 \tan(c+dx)}{d} & \text{for}\ d \neq 0 \\ x(a + b\tan(c))^3 \cot(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Piecewise((-a**3*log(tan(c + d*x)**2 + 1)/(2*d) + a**3*log(tan(c + d*x))/d
+ 3*a**2*b*x + 3*a*b**2*log(tan(c + d*x)**2 + 1)/(2*d) - b**3*x + b**3*tan(
c + d*x)/d, Ne(d, 0)), (x*(a + b*tan(c))**3*cot(c), True))
```

Giac [A]
time = 1.01, size = 72, normalized size = 1.16

$$\frac{2\,a^3 \log(|\tan(dx+c)|) + 2\,b^3 \tan(dx+c) + 2\,(3\,a^2b - b^3)(dx+c) - (a^3 - 3\,ab^2)\log\left(\tan(dx+c)^2 + 1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*a^3*log(abs(tan(d*x + c))) + 2*b^3*tan(d*x + c) + 2*(3*a^2*b - b^3)*
(d*x + c) - (a^3 - 3*a*b^2)*log(tan(d*x + c)^2 + 1))/d
```

Mupad [B]
time = 3.86, size = 75, normalized size = 1.21

$$\frac{b^3 \tan(c + d\,x)}{d} - \frac{\ln(\tan(c + d\,x) - \mathrm{i})\ (a + b\,1\mathrm{i})^3}{2\,d} + \frac{a^3\ \ln(\tan(c + d\,x))}{d} - \frac{\ln(\tan(c + d\,x) + 1\mathrm{i})\ (b + a\,1\mathrm{i})^3\,1\mathrm{i}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + b*tan(c + d*x))^3,x)
```

```
[Out] (b^3*tan(c + d*x))/d - (log(tan(c + d*x) + 1i)*(a*1i + b)^3*1i)/(2*d) - (lo
g(tan(c + d*x) - 1i)*(a + b*1i)^3)/(2*d) + (a^3*log(tan(c + d*x)))/d
```

3.440 $\int \cot^2(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=69

$$-a\left(a^2-3b^2\right)x-\frac{b^3\log(\cos(c+dx))}{d}+\frac{3a^2b\log(\sin(c+dx))}{d}-\frac{a^2\cot(c+dx)(a+b\tan(c+dx))}{d}$$

```
[Out] -a*(a^2-3*b^2)*x-b^3*ln(cos(d*x+c))/d+3*a^2*b*ln(sin(d*x+c))/d-a^2*cot(d*x+
c)*(a+b*tan(d*x+c))/d
```

Rubi [A]
time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {3646, 3705, 3556}

$$-ax\left(a^2-3b^2\right)+\frac{3a^2b\log(\sin(c+dx))}{d}-\frac{a^2\cot(c+dx)(a+b\tan(c+dx))}{d}-\frac{b^3\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -(a*(a^2 - 3*b^2)*x) - (b^3*Log[Cos[c + d*x]])/d + (3*a^2*b*Log[Sin[c + d*x
]])/d - (a^2*Cot[c + d*x]*(a + b*Tan[c + d*x]))/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3705

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[B*x, x] + (Dist[A, Int[1/Tan[e
 + f*x], x], x] + Dist[C, Int[Tan[e + f*x], x], x]) /; FreeQ[{e, f, A, B, C
```

```
}, x] && NeQ[A, C]
```

Rubi steps

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{a^2\cot(c+dx)(a+b\tan(c+dx))}{d} + \int \cot(c+dx)\left(3a^2b - a(a^2\right.$$

$$= -a\left(a^2-3b^2\right)x - \frac{a^2\cot(c+dx)(a+b\tan(c+dx))}{d} + \left(3a^2b\right)\int c$$

$$= -a\left(a^2-3b^2\right)x - \frac{b^3\log(\cos(c+dx))}{d} + \frac{3a^2b\log(\sin(c+dx))}{d} - \frac{a}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.15, size = 80, normalized size = 1.16

$$-\frac{a^3\cot(c+dx) - \frac{1}{2}(ia+b)^3\log(i-\cot(c+dx)) + \frac{1}{2}(ia-b)^3\log(i+\cot(c+dx)) - b^3\log(\tan(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
[Out] -((a^3*Cot[c + d*x] - ((I*a + b)^3*Log[I - Cot[c + d*x]])/2 + ((I*a - b)^3*
Log[I + Cot[c + d*x]])/2 - b^3*Log[Tan[c + d*x]])/d)
```

Maple [A]
time = 0.18, size = 62, normalized size = 0.90

method	result
derivativedivides	$\frac{a^3(-\cot(dx+c)-dx-c)+3a^2b\ln(\sin(dx+c))+3b^2a(dx+c)-b^3\ln(\cos(dx+c))}{d}$
default	$\frac{a^3(-\cot(dx+c)-dx-c)+3a^2b\ln(\sin(dx+c))+3b^2a(dx+c)-b^3\ln(\cos(dx+c))}{d}$
norman	$\frac{(-a^3+3b^2a)x\tan(dx+c)-\frac{a^3}{d}}{\tan(dx+c)} + \frac{3a^2b\ln(\tan(dx+c))}{d} - \frac{b(3a^2-b^2)\ln(1+\tan^2(dx+c))}{2d}$
risch	$-3ixb\,a^2 + ib^3x - a^3x + 3a\,b^2x - \frac{6ib\,a^2c}{d} + \frac{2ib^3c}{d} - \frac{2ia^3}{d(e^{2i(dx+c)}-1)} + \frac{3a^2b\ln(e^{2i(dx+c)}-1)}{d} - \frac{b^3\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/d*(a^3*(-cot(d*x+c)-d*x-c)+3*a^2*b*ln(sin(d*x+c))+3*b^2*a*(d*x+c)-b^3*ln(
cos(d*x+c)))
```

Maxima [A]
time = 0.51, size = 74, normalized size = 1.07

$$\frac{6\,a^2b\log(\tan(dx+c)) - 2\,(a^3-3\,ab^2)(dx+c) - (3\,a^2b-b^3)\log\left(\tan(dx+c)^2+1\right) - \frac{2\,a^3}{\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(6*a^2*b*log(tan(d*x + c)) - 2*(a^3 - 3*a*b^2)*(d*x + c) - (3*a^2*b - b
^3)*log(tan(d*x + c)^2 + 1) - 2*a^3/tan(d*x + c))/d
```

Fricas [A]
time = 1.04, size = 97, normalized size = 1.41

$$\frac{3\,a^2b\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)-b^3\log\left(\frac{1}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)-2\left(a^3-3\,ab^2\right)dx\tan\left(dx+c\right)-2\,a^3}{2\,d\tan\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(3*a^2*b*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c) - b^3*lo
g(1/(tan(d*x + c)^2 + 1))*tan(d*x + c) - 2*(a^3 - 3*a*b^2)*d*x*tan(d*x + c)
 - 2*a^3)/(d*tan(d*x + c))
```

Sympy [A]
time = 0.66, size = 114, normalized size = 1.65

$$\begin{cases} \tilde{\infty}a^3x & \text{for}\ (c=0\vee c=-dx)\wedge(c=-dx\vee d=0) \\ x(a+b\tan(c))^3\cot^2(c) & \text{for}\ d=0 \\ -a^3x-\frac{a^3}{d\tan(c+dx)}-\frac{3a^2b\log(\tan^2(c+dx)+1)}{2d}+\frac{3a^2b\log(\tan(c+dx))}{d}+3ab^2x+\frac{b^3\log(\tan^2(c+dx)+1)}{2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Piecewise((zoo*a**3*x, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x)))
, (x*(a + b*tan(c))**3*cot(c)**2, Eq(d, 0)), (-a**3*x - a**3/(d*tan(c + d*x
)) - 3*a**2*b*log(tan(c + d*x)**2 + 1)/(2*d) + 3*a**2*b*log(tan(c + d*x))/d
 + 3*a*b**2*x + b**3*log(tan(c + d*x)**2 + 1)/(2*d), True))
```

Giac [A]
time = 1.14, size = 88, normalized size = 1.28

$$\frac{6\,a^2b\log\left(\left|\tan\left(dx+c\right)\right|\right)-2\left(a^3-3\,ab^2\right)\left(dx+c\right)-\left(3\,a^2b-b^3\right)\log\left(\tan\left(dx+c\right)^2+1\right)-\frac{2\left(3\,a^2b\tan(dx+c)+a^3\right)}{\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(6*a^2*b*log(abs(tan(d*x + c))) - 2*(a^3 - 3*a*b^2)*(d*x + c) - (3*a^2*
b - b^3)*log(tan(d*x + c)^2 + 1) - 2*(3*a^2*b*tan(d*x + c) + a^3)/tan(d*x +
 c))/d
```

Mupad [B]
time = 3.91, size = 78, normalized size = 1.13

$$\frac{\ln(\tan(c+dx)+1\mathrm{i})\ (b+a\,1\mathrm{i})^3}{2\,d}-\frac{a^3\cot(c+dx)}{d}+\frac{3\,a^2\,b\,\ln(\tan(c+dx))}{d}+\frac{\ln(\tan(c+dx)-\mathrm{i})\ (a+b\,1\mathrm{i})^3\,1\mathrm{i}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + b*tan(c + d*x))^3,x)
```

```
[Out] (log(tan(c + d*x) - 1i)*(a + b*1i)^3*1i)/(2*d) + (log(tan(c + d*x) + 1i)*(a
*1i + b)^3)/(2*d) - (a^3*cot(c + d*x))/d + (3*a^2*b*log(tan(c + d*x)))/d
```

3.441 $\int \cot^3(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=83

$$-b(3a^2-b^2)\,x-\frac{5a^2b\cot(c+dx)}{2d}-\frac{a(a^2-3b^2)\log(\sin(c+dx))}{d}-\frac{a^2\cot^2(c+dx)(a+b\tan(c+dx))}{2d}$$

```
[Out] -b*(3*a^2-b^2)*x-5/2*a^2*b*cot(d*x+c)/d-a*(a^2-3*b^2)*ln(sin(d*x+c))/d-1/2*
a^2*cot(d*x+c)^2*(a+b*tan(d*x+c))/d
```

Rubi [A]

time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3646, 3709, 3612, 3556}

$$-\frac{a(a^2-3b^2)\log(\sin(c+dx))}{d}-bx(3a^2-b^2)-\frac{5a^2b\cot(c+dx)}{2d}-\frac{a^2\cot^2(c+dx)(a+b\tan(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -(b*(3*a^2 - b^2)*x) - (5*a^2*b*Cot[c + d*x])/(2*d) - (a*(a^2 - 3*b^2)*Log[
Sin[c + d*x]])/d - (a^2*Cot[c + d*x]^2*(a + b*Tan[c + d*x]))/(2*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
```

```
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{a^2\cot^2(c+dx)(a+b\tan(c+dx))}{2d}+\frac{1}{2}\int \cot^2(c+dx)\left(5a^2b-\right.$$

$$= -\frac{5a^2b\cot(c+dx)}{2d}-\frac{a^2\cot^2(c+dx)(a+b\tan(c+dx))}{2d}+\frac{1}{2}\int \mathrm{co}$$

$$= -b\left(3a^2-b^2\right)x-\frac{5a^2b\cot(c+dx)}{2d}-\frac{a^2\cot^2(c+dx)(a+b\tan(c}{2d}$$

$$= -b\left(3a^2-b^2\right)x-\frac{5a^2b\cot(c+dx)}{2d}-\frac{a(a^2-3b^2)\log(\sin(c+dx))}{d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.33, size = 96, normalized size = 1.16

$$\frac{-6a^2b\cot(c+dx)-a^3\cot^2(c+dx)+(a+ib)^3\log(i-\tan(c+dx))-2a(a^2-3b^2)\log(\tan(c+dx))+(a-ib)^3\log(i+\tan(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
[Out] (-6*a^2*b*Cot[c + d*x] - a^3*Cot[c + d*x]^2 + (a + I*b)^3*Log[I - Tan[c + d
*x]] - 2*a*(a^2 - 3*b^2)*Log[Tan[c + d*x]] + (a - I*b)^3*Log[I + Tan[c + d*
x]])/(2*d)
```

Maple [A]
time = 0.21, size = 74, normalized size = 0.89

method	result
derivativedivides	$\frac{a^3\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+3a^2b(-\cot(dx+c)-dx-c)+3b^2a\ln(\sin(dx+c))+b^3(dx+c)}{d}$

default	$\frac{a^3\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+3a^2b(-\cot(dx+c)-dx-c)+3b^2a\ln(\sin(dx+c))+b^3(dx+c)}{d}$
norman	$\frac{(-3a^2b+b^3)x(\tan^2(dx+c))-\frac{a^3}{2d}-\frac{3a^2b\tan(dx+c)}{d}}{\tan(dx+c)^2}-\frac{a(a^2-3b^2)\ln(\tan(dx+c))}{d}+\frac{a(a^2-3b^2)\ln(1+\tan^2(dx+c))}{2d}$
risch	$-3a^2bx+b^3x+ia^3x-3ia\,b^2x+\frac{2ia^3c}{d}-\frac{6ia\,b^2c}{d}+\frac{2a^2\left(a\,e^{2i(dx+c)}-3ib\,e^{2i(dx+c)}+3ib\right)}{d\left(e^{2i(dx+c)}-1\right)^2}-\frac{a^3\ln(e^{2i(dx}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^3*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^3*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+3*a^2*b*(-cot(d*x+c)-d*x-c)+3*b^2*a*ln(sin(d*x+c))+b^3*(d*x+c))

Maxima [A]
time = 0.53, size = 92, normalized size = 1.11

$$-\frac{2\,(3\,a^2b-b^3)(dx+c)-(a^3-3\,ab^2)\log\left(\tan\left(dx+c\right)^2+1\right)+2\,(a^3-3\,ab^2)\log\left(\tan\left(dx+c\right)\right)+\frac{6\,a^2b\tan(dx+c)+a^3}{\tan(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="maxima")

[Out] -1/2*(2*(3*a^2*b - b^3)*(d*x + c) - (a^3 - 3*a*b^2)*log(tan(d*x + c)^2 + 1) + 2*(a^3 - 3*a*b^2)*log(tan(d*x + c)) + (6*a^2*b*tan(d*x + c) + a^3)/tan(d*x + c)^2)/d

Fricas [A]
time = 1.35, size = 99, normalized size = 1.19

$$-\frac{6\,a^2b\tan\left(dx+c\right)+(a^3-3\,ab^2)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^2+a^3+(a^3+2\,(3\,a^2b-b^3)dx)\tan\left(dx+c\right)^2}{2\,d\tan\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="fricas")

[Out] -1/2*(6*a^2*b*tan(d*x + c) + (a^3 - 3*a*b^2)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^2 + a^3 + (a^3 + 2*(3*a^2*b - b^3)*d*x)*tan(d*x + c)^2)/(d*tan(d*x + c)^2)

Sympy [A]
time = 0.95, size = 146, normalized size = 1.76

$$\begin{cases} \tilde{\infty}a^3x & \text{for}\: (c=0 \vee c=-dx) \wedge (c=-dx \vee d=0) \\ x(a+b\tan(c))^3\cot^3(c) & \text{for}\: d=0 \\ \frac{a^3\log(\tan^2(c+dx)+1)}{2d}-\frac{a^3\log(\tan(c+dx))}{d}-\frac{a^3}{2d\tan^2(c+dx)}-3a^2bx-\frac{3a^2b}{d\tan(c+dx)}-\frac{3ab^2\log(\tan^2(c+dx)+1)}{2d}+\frac{3ab^2\log(\tan(c+dx))}{d}+b^3x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Piecewise((zoo*a**3*x, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x)))
, (x*(a + b*tan(c))**3*cot(c)**3, Eq(d, 0)), (a**3*log(tan(c + d*x)**2 + 1)
/(2*d) - a**3*log(tan(c + d*x))/d - a**3/(2*d*tan(c + d*x)**2) - 3*a**2*b*x
 - 3*a**2*b/(d*tan(c + d*x)) - 3*a*b**2*log(tan(c + d*x)**2 + 1)/(2*d) + 3*
a*b**2*log(tan(c + d*x))/d + b**3*x, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(79) = 158$.
time = 1.25, size = 171, normalized size = 2.06

$$-\frac{a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 12 a^2 b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 8\left(3 a^2 b - b^3\right)(dx + c) - 8\left(a^3 - 3 ab^2\right) \log\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 1\right) + 8\left(a^3 - 3 ab^2\right) \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right) - \frac{12 a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 36 ab^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 12 a^2 b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - a^3}{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2}}{8 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(a^3*tan(1/2*d*x + 1/2*c)^2 - 12*a^2*b*tan(1/2*d*x + 1/2*c) + 8*(3*a^2
*b - b^3)*(d*x + c) - 8*(a^3 - 3*a*b^2)*log(tan(1/2*d*x + 1/2*c)^2 + 1) + 8
*(a^3 - 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) - (12*a^3*tan(1/2*d*x + 1/2
*c)^2 - 36*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 12*a^2*b*tan(1/2*d*x + 1/2*c) - a
^3)/tan(1/2*d*x + 1/2*c)^2)/d
```

Mupad [B]
time = 4.07, size = 102, normalized size = 1.23

$$\frac{\ln\left(\tan(c + d\,x)\right)\,\left(3\,a\,b^2 - a^3\right)}{d} + \frac{\ln\left(\tan(c + d\,x) - \mathrm{i}\right)\,\left(a + b\,1\mathrm{i}\right)^3}{2\,d} - \frac{\cot(c + d\,x)^2\,\left(\frac{a^3}{2} + 3\,b\tan(c + d\,x)\,a^2\right)}{d} + \frac{\ln\left(\tan(c + d\,x) + 1\mathrm{i}\right)\,\left(b + a\,1\mathrm{i}\right)^3\,1\mathrm{i}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + b*tan(c + d*x))^3,x)
```

```
[Out] (log(tan(c + d*x))*(3*a*b^2 - a^3))/d + (log(tan(c + d*x) - 1i)*(a + b*1i)^
3)/(2*d) + (log(tan(c + d*x) + 1i)*(a*1i + b)^3*1i)/(2*d) - (cot(c + d*x)^2
*(a^3/2 + 3*a^2*b*tan(c + d*x)))/d
```

3.442 $\int \cot^4(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=104

$$a\left(a^2-3b^2\right)x+\frac{a(a^2-3b^2)\cot(c+dx)}{d}-\frac{7a^2b\cot^2(c+dx)}{6d}-\frac{b(3a^2-b^2)\log(\sin(c+dx))}{d}-\frac{a^2\cot^3(c+dx)(a+\ldots}{3d}$$

```
[Out] a*(a^2-3*b^2)*x+a*(a^2-3*b^2)*cot(d*x+c)/d-7/6*a^2*b*cot(d*x+c)^2/d-b*(3*a^
2-b^2)*ln(sin(d*x+c))/d-1/3*a^2*cot(d*x+c)^3*(a+b*tan(d*x+c))/d
```

Rubi [A]
time = 0.13, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3646, 3709, 3610, 3612, 3556}

$$\frac{a(a^2-3b^2)\cot(c+dx)}{d}-\frac{b(3a^2-b^2)\log(\sin(c+dx))}{d}+ax\left(a^2-3b^2\right)-\frac{7a^2b\cot^2(c+dx)}{6d}-\frac{a^2\cot^3(c+dx)(a+b\tan(c+dx))}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] a*(a^2 - 3*b^2)*x + (a*(a^2 - 3*b^2)*Cot[c + d*x])/d - (7*a^2*b*Cot[c + d*x
]^2)/(6*d) - (b*(3*a^2 - b^2)*Log[Sin[c + d*x]])/d - (a^2*Cot[c + d*x]^3*(a
 + b*Tan[c + d*x]))/(3*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \cot^4(c+dx)(a+b\tan(c+dx))^3\,dx &= -\frac{a^2\cot^3(c+dx)(a+b\tan(c+dx))}{3d}+\frac{1}{3}\int \cot^3(c+dx)\left(7a^2b-\right. \\
&= -\frac{7a^2b\cot^2(c+dx)}{6d}-\frac{a^2\cot^3(c+dx)(a+b\tan(c+dx))}{3d}+\frac{1}{3}\int \mathrm{c} \\
&= \frac{a(a^2-3b^2)\cot(c+dx)}{d}-\frac{7a^2b\cot^2(c+dx)}{6d}-\frac{a^2\cot^3(c+dx)(a-}{3d} \\
&= a\left(a^2-3b^2\right)x+\frac{a(a^2-3b^2)\cot(c+dx)}{d}-\frac{7a^2b\cot^2(c+dx)}{6d}-\frac{a^2}{} \\
&= a\left(a^2-3b^2\right)x+\frac{a(a^2-3b^2)\cot(c+dx)}{d}-\frac{7a^2b\cot^2(c+dx)}{6d}-\frac{b(}{}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.15, size = 120, normalized size = 1.15

$$
\frac{6a(a^2-3b^2)\cot(c+dx)-9a^2b\cot^2(c+dx)-2a^3\cot^3(c+dx)+3(ia-b)^3\log(i-\tan(c+dx))+6b(-3a^2+b^2)\log(\tan(c+dx))-3(ia+b)^3\log(i+\tan(c+dx))}{6d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Tan[c + d*x])^3,x]
```

[Out] (6*a*(a^2 - 3*b^2)*Cot[c + d*x] - 9*a^2*b*Cot[c + d*x]^2 - 2*a^3*Cot[c + d*x]^3 + 3*(I*a - b)^3*Log[I - Tan[c + d*x]] + 6*b*(-3*a^2 + b^2)*Log[Tan[c + d*x]] - 3*(I*a + b)^3*Log[I + Tan[c + d*x]])/(6*d)

Maple [A]
time = 0.17, size = 90, normalized size = 0.87

method	result
derivativedivides	$\frac{a^3\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+3a^2b\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+3b^2a(-\cot(dx+c)-dx-c)+b^3\ln(\sin(dx+c))}{d}$
default	$\frac{a^3\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+3a^2b\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+3b^2a(-\cot(dx+c)-dx-c)+b^3\ln(\sin(dx+c))}{d}$
norman	$\frac{\frac{a(a^2-3b^2)\left(\tan^2(dx+c)\right)}{d}+a(a^2-3b^2)x\left(\tan^3(dx+c)\right)-\frac{a^3}{3d}-\frac{3a^2b\tan(dx+c)}{2d}}{\tan(dx+c)^3}-\frac{b(3a^2-b^2)\ln(\tan(dx+c))}{d}+\frac{b(3a^2-b^2)\ln}{}$
risch	$3ixba^2-ib^3x+a^3x-3ab^2x+\frac{6iba^2c}{d}-\frac{2ib^3c}{d}+\frac{2ia\left(6a^2e^{4i(dx+c)}-9b^2e^{4i(dx+c)}-9iabe^{4i(dx+c)}-6a^2e^{2}\right.}{3d\left(e^{2i(dx+c}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^4*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^3*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+3*a^2*b*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+3*b^2*a*(-cot(d*x+c)-d*x-c)+b^3*ln(sin(d*x+c)))

Maxima [A]
time = 0.52, size = 117, normalized size = 1.12

$$\frac{6\left(a^3-3ab^2\right)(dx+c)+3\left(3a^2b-b^3\right)\log\left(\tan\left(dx+c\right)^2+1\right)-6\left(3a^2b-b^3\right)\log\left(\tan\left(dx+c\right)\right)-\frac{9a^2b\tan(dx+c)+2a^3-6\left(a^3-3ab^2\right)\tan(dx+c)^2}{\tan(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="maxima")

[Out] 1/6*(6*(a^3 - 3*a*b^2)*(d*x + c) + 3*(3*a^2*b - b^3)*log(tan(d*x + c)^2 + 1) - 6*(3*a^2*b - b^3)*log(tan(d*x + c)) - (9*a^2*b*tan(d*x + c) + 2*a^3 - 6*(a^3 - 3*a*b^2)*tan(d*x + c)^2)/tan(d*x + c)^3)/d

Fricas [A]
time = 1.06, size = 126, normalized size = 1.21

$$-\frac{3\left(3a^2b-b^3\right)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^3+9a^2b\tan\left(dx+c\right)+3\left(3a^2b-2\left(a^3-3ab^2\right)dx\right)\tan\left(dx+c\right)^3+2a^3-6\left(a^3-3ab^2\right)\tan\left(dx+c\right)^2}{6d\tan\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="fricas")

```
[Out] -1/6*(3*(3*a^2*b - b^3)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x +
c)^3 + 9*a^2*b*tan(d*x + c) + 3*(3*a^2*b - 2*(a^3 - 3*a*b^2)*d*x)*tan(d*x +
 c)^3 + 2*a^3 - 6*(a^3 - 3*a*b^2)*tan(d*x + c)^2)/(d*tan(d*x + c)^3)
```

Sympy [A]
time = 1.38, size = 177, normalized size = 1.70

$$\begin{cases} \tilde{\infty} a^3 x & \text{for}\ (c = 0 \vee c = -dx) \wedge (c = -dx \vee d = 0) \\ x(a + b\tan(c))^3 \cot^4(c) & \text{for}\ d = 0 \\ a^3 x + \frac{a^3}{d\tan(c+dx)} - \frac{a^3}{3d\tan^3(c+dx)} + \frac{3a^2 b\log(\tan^2(c+dx)+1)}{2d} - \frac{3a^2 b\log(\tan(c+dx))}{d} - \frac{3a^2 b}{2d\tan^2(c+dx)} - 3ab^2 x - \frac{3ab^2}{d\tan(c+dx)} - \frac{b^3\log(\tan^2(c+dx)+1)}{2d} + \frac{b^3\log(\tan(c+dx))}{d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Piecewise((zoo*a**3*x, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x)))
, (x*(a + b*tan(c))**3*cot(c)**4, Eq(d, 0)), (a**3*x + a**3/(d*tan(c + d*x)
) - a**3/(3*d*tan(c + d*x)**3) + 3*a**2*b*log(tan(c + d*x)**2 + 1)/(2*d) -
3*a**2*b*log(tan(c + d*x))/d - 3*a**2*b/(2*d*tan(c + d*x)**2) - 3*a*b**2*x
- 3*a*b**2/(d*tan(c + d*x)) - b**3*log(tan(c + d*x)**2 + 1)/(2*d) + b**3*lo
g(tan(c + d*x))/d, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(100) = 200.
time = 1.50, size = 236, normalized size = 2.27

$$\frac{a^3\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 9a^2 b\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 15a^3\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 36ab^2\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 24(a^3 - 3ab^2)(dx + c) + 24(3a^2 b - b^3)\log\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 1\right) - 24(3a^2 b - b^3)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right|\right) + \frac{132a^2 b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 44b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 15a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 36ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 9a^2 b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - a^3}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(a^3*tan(1/2*d*x + 1/2*c)^3 - 9*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 15*a^3*
tan(1/2*d*x + 1/2*c) + 36*a*b^2*tan(1/2*d*x + 1/2*c) + 24*(a^3 - 3*a*b^2)*(
d*x + c) + 24*(3*a^2*b - b^3)*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 24*(3*a^2*b
 - b^3)*log(abs(tan(1/2*d*x + 1/2*c))) + (132*a^2*b*tan(1/2*d*x + 1/2*c)^3
- 44*b^3*tan(1/2*d*x + 1/2*c)^3 + 15*a^3*tan(1/2*d*x + 1/2*c)^2 - 36*a*b^2*
tan(1/2*d*x + 1/2*c)^2 - 9*a^2*b*tan(1/2*d*x + 1/2*c) - a^3)/tan(1/2*d*x +
1/2*c)^3)/d
```

Mupad [B]
time = 3.96, size = 124, normalized size = 1.19

$$-\frac{\ln(\tan(c + d\,x))\,(3\,a^2\,b - b^3)}{d} - \frac{\ln(\tan(c + d\,x) + 1\mathrm{i})\,(b + a\,1\mathrm{i})^3}{2\,d} - \frac{\cot(c + d\,x)^3\,\left(\tan(c + d\,x)^2\,(3\,a\,b^2 - a^3) + \frac{a^3}{3} + \frac{3\,a^2\,b\tan(c+d\,x)}{2}\right)}{d} - \frac{\ln(\tan(c + d\,x) - \mathrm{i})\,(a + b\,1\mathrm{i})^3\,1\mathrm{i}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + b*tan(c + d*x))^3,x)
```

```
[Out] - (log(tan(c + d*x))*(3*a^2*b - b^3))/d - (log(tan(c + d*x) - 1i)*(a + b*1i
)^3*1i)/(2*d) - (log(tan(c + d*x) + 1i)*(a*1i + b)^3)/(2*d) - (cot(c + d*x)
^3*(tan(c + d*x)^2*(3*a*b^2 - a^3) + a^3/3 + (3*a^2*b*tan(c + d*x))/2))/d
```

3.443 $\int \cot^5(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=130

$$b(3a^2-b^2)\,x+\frac{b(3a^2-b^2)\cot(c+dx)}{d}+\frac{a(a^2-3b^2)\cot^2(c+dx)}{2d}-\frac{3a^2b\cot^3(c+dx)}{4d}+\frac{a(a^2-3b^2)\log(\sin(c+$$

```
[Out] b*(3*a^2-b^2)*x+b*(3*a^2-b^2)*cot(d*x+c)/d+1/2*a*(a^2-3*b^2)*cot(d*x+c)^2/d
-3/4*a^2*b*cot(d*x+c)^3/d+a*(a^2-3*b^2)*ln(sin(d*x+c))/d-1/4*a^2*cot(d*x+c)
^4*(a+b*tan(d*x+c))/d
```

Rubi [A]
time = 0.16, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3646, 3709, 3610, 3612, 3556}

$$\frac{a(a^2-3b^2)\cot^2(c+dx)}{2d}+\frac{b(3a^2-b^2)\cot(c+dx)}{d}+\frac{a(a^2-3b^2)\log(\sin(c+dx))}{d}+bx(3a^2-b^2)-\frac{3a^2b\cot^3(c+dx)}{4d}-\frac{a^2\cot^4(c+dx)(a+b\tan(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] b*(3*a^2 - b^2)*x + (b*(3*a^2 - b^2)*Cot[c + d*x])/d + (a*(a^2 - 3*b^2)*Cot
[c + d*x]^2)/(2*d) - (3*a^2*b*Cot[c + d*x]^3)/(4*d) + (a*(a^2 - 3*b^2)*Log[
Sin[c + d*x]])/d - (a^2*Cot[c + d*x]^4*(a + b*Tan[c + d*x]))/(4*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cot^5(c+dx)(a+b\tan(c+dx))^3\,dx &= -\frac{a^2\cot^4(c+dx)(a+b\tan(c+dx))}{4d}+\frac{1}{4}\int \cot^4(c+dx)\left(9a^2b-\right.\\ &= -\frac{3a^2b\cot^3(c+dx)}{4d}-\frac{a^2\cot^4(c+dx)(a+b\tan(c+dx))}{4d}+\frac{1}{4}\int \mathrm{c}\\ &= \frac{a(a^2-3b^2)\cot^2(c+dx)}{2d}-\frac{3a^2b\cot^3(c+dx)}{4d}-\frac{a^2\cot^4(c+dx)(a}{4d}\\ &= \frac{b(3a^2-b^2)\cot(c+dx)}{d}+\frac{a(a^2-3b^2)\cot^2(c+dx)}{2d}-\frac{3a^2b\cot^3(c}{4d}\\ &= b\left(3a^2-b^2\right)x+\frac{b(3a^2-b^2)\cot(c+dx)}{d}+\frac{a(a^2-3b^2)\cot^2(c+dx}{2d}\\ &= b\left(3a^2-b^2\right)x+\frac{b(3a^2-b^2)\cot(c+dx)}{d}+\frac{a(a^2-3b^2)\cot^2(c+dx}{2d}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.65, size = 118, normalized size = 0.91

$$-\frac{4b(-3a^2+b^2)\cot(c+dx)-2a(a^2-3b^2)\cot^2(c+dx)+4a^2b\cot^3(c+dx)+a^3\cot^4(c+dx)+2(a-ib)^3\log(i-\cot(c+dx))+2(a+ib)^3\log(i+\cot(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -1/4*(4*b*(-3*a^2 + b^2)*Cot[c + d*x] - 2*a*(a^2 - 3*b^2)*Cot[c + d*x]^2 +
4*a^2*b*Cot[c + d*x]^3 + a^3*Cot[c + d*x]^4 + 2*(a - I*b)^3*Log[I - Cot[c +
 d*x]] + 2*(a + I*b)^3*Log[I + Cot[c + d*x]])/d
```

Maple [A]
time = 0.19, size = 111, normalized size = 0.85

method	result
derivativedivides	$\frac{a^3\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+3a^2b\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+3b^2a\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\right.}{d}$
default	$\frac{a^3\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+3a^2b\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+3b^2a\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\right.}{d}$
norman	$\frac{\frac{b\left(3a^2-b^2\right)\left(\tan^3(dx+c)\right)}{d}+b(3a^2-b^2)x(\tan^4(dx+c))-\frac{a^3}{4d}+\frac{a\left(a^2-3b^2\right)\left(\tan^2(dx+c)\right)}{2d}-\frac{a^2b\tan(dx+c)}{d}}{\tan(dx+c)^4}+\frac{a(a^2-3b^2)\ln(\tan(c}{d}$
risch	$3a^2bx-b^3x-ia^3x+3ia\,b^2x-\frac{2ia^3c}{d}+\frac{6ia\,b^2c}{d}-\frac{2i\left(-2ia^3e^{6i(dx+c)}+3ia\,b^2e^{6i(dx+c)}-6a^2b\,e^{6i(dx+c)}+b^3\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+3*a^2*b*(-1/3*
cot(d*x+c)^3+cot(d*x+c)+d*x+c)+3*b^2*a*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+b
^3*(-cot(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.52, size = 135, normalized size = 1.04

$$\frac{4\,(3\,a^2b-b^3)(dx+c)-2\,(a^3-3\,ab^2)\log\left(\tan\left(dx+c\right)^2+1\right)+4\,(a^3-3\,ab^2)\log\left(\tan\left(dx+c\right)\right)-\frac{4\,a^2b\tan(dx+c)-4\,(3\,a^2b-b^3)\tan(dx+c)^3+a^3-2\,(a^3-3\,ab^2)\tan(dx+c)^2}{\tan(dx+c)^4}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(4*(3*a^2*b - b^3)*(d*x + c) - 2*(a^3 - 3*a*b^2)*log(tan(d*x + c)^2 + 1
) + 4*(a^3 - 3*a*b^2)*log(tan(d*x + c)) - (4*a^2*b*tan(d*x + c) - 4*(3*a^2*
b - b^3)*tan(d*x + c)^3 + a^3 - 2*(a^3 - 3*a*b^2)*tan(d*x + c)^2)/tan(d*x +
 c)^4)/d
```

Fricas [A]
time = 0.96, size = 152, normalized size = 1.17

$$\frac{2\,(a^3-3\,ab^2)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^4+(3\,a^3-6\,ab^2+4\,(3\,a^2b-b^3)dx)\tan\left(dx+c\right)^4-4\,a^2b\tan\left(dx+c\right)+4\,(3\,a^2b-b^3)\tan\left(dx+c\right)^3-a^3+2\,(a^3-3\,ab^2)\tan\left(dx+c\right)^2}{4\,d\tan\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="fricas")

[Out] 1/4*(2*(a^3 - 3*a*b^2)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^4 + (3*a^3 - 6*a*b^2 + 4*(3*a^2*b - b^3)*d*x)*tan(d*x + c)^4 - 4*a^2*b*tan(d*x + c) + 4*(3*a^2*b - b^3)*tan(d*x + c)^3 - a^3 + 2*(a^3 - 3*a*b^2)*tan(d*x + c)^2)/(d*tan(d*x + c)^4)

Sympy [A]
time = 2.33, size = 207, normalized size = 1.59

$$\begin{cases} \tilde{\infty} a^3 x & \text{for } (c = 0 \vee c = -dx) \wedge (c = -dx \vee d = 0) \\ x(a + b\tan(c))^3 \cot^5(c) & \text{for } d = 0 \\ -\frac{a^3\log(\tan^2(c+dx)+1)}{2d} + \frac{a^3\log(\tan(c+dx))}{d} + \frac{a^3}{2d\tan^2(c+dx)} - \frac{a^3}{4d\tan^4(c+dx)} + 3a^2bx + \frac{3a^2b}{d\tan(c+dx)} - \frac{a^2b}{d\tan^3(c+dx)} + \frac{3ab^2\log(\tan^2(c+dx)+1)}{2d} - \frac{3ab^2\log(\tan(c+dx))}{d} - \frac{3ab^2}{2d\tan^2(c+dx)} - b^3x - \frac{b^3}{d\tan(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**5*(a+b*tan(d*x+c))**3,x)

[Out] Piecewise((zoo*a**3*x, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (x*(a + b*tan(c))**3*cot(c)**5, Eq(d, 0)), (-a**3*log(tan(c + d*x)**2 + 1)/(2*d) + a**3*log(tan(c + d*x))/d + a**3/(2*d*tan(c + d*x)**2) - a**3/(4*d*tan(c + d*x)**4) + 3*a**2*b*x + 3*a**2*b/(d*tan(c + d*x)) - a**2*b/(d*tan(c + d*x)**3) + 3*a*b**2*log(tan(c + d*x)**2 + 1)/(2*d) - 3*a*b**2*log(tan(c + d*x))/d - 3*a*b**2/(2*d*tan(c + d*x)**2) - b**3*x - b**3/(d*tan(c + d*x)), True))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(124) = 248.
time = 1.85, size = 301, normalized size = 2.32

$$-\frac{3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 24a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 36a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 72ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 360a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 96b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 192(3a^2b-b^3)(dx+c) + 192(a^3-3ab^2)\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right) - 192(a^3-3ab^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right) + \frac{400a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 1200ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 360a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 96b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 36a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 72ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 24a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + 3a^3}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4}}{192d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="giac")

[Out] -1/192*(3*a^3*tan(1/2*d*x + 1/2*c)^4 - 24*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 36*a^3*tan(1/2*d*x + 1/2*c)^2 + 72*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 360*a^2*b*tan(1/2*d*x + 1/2*c) - 96*b^3*tan(1/2*d*x + 1/2*c) - 192*(3*a^2*b - b^3)*(d*x + c) + 192*(a^3 - 3*a*b^2)*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 192*(a^3 - 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + (400*a^3*tan(1/2*d*x + 1/2*c)^4 - 1200*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 360*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 96*b^3*tan(1/2*d*x + 1/2*c)^3 - 36*a^3*tan(1/2*d*x + 1/2*c)^2 + 72*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 24*a^2*b*tan(1/2*d*x + 1/2*c) + 3*a^3)/tan(1/2*d*x + 1/2*c)^4)/d

Mupad [B]
time = 3.93, size = 145, normalized size = 1.12

$$-\frac{\ln\left(\tan(c+d\,x)\right)\,\left(3\,a\,b^2-a^3\right)}{d}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,{\left(a+b\,1\mathrm{i}\right)}^3}{2\,d}-\frac{\cot(c+d\,x)^4\,\left(\tan(c+d\,x)^2\,\left(\frac{3\,a\,b^2}{2}-\frac{a^3}{2}\right)-\tan(c+d\,x)^3\,\left(3\,a^2\,b-b^3\right)+\frac{a^3}{4}+a^2\,b\,\tan(c+d\,x)\right)}{d}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,{\left(b+a\,1\mathrm{i}\right)}^3\,1\mathrm{i}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(a + b*tan(c + d*x))^3,x)
```

```
[Out] - (log(tan(c + d*x))*(3*a*b^2 - a^3))/d - (log(tan(c + d*x) - 1i)*(a + b*1i
)^3)/(2*d) - (log(tan(c + d*x) + 1i)*(a*1i + b)^3*1i)/(2*d) - (cot(c + d*x)
^4*(tan(c + d*x)^2*((3*a*b^2)/2 - a^3/2) - tan(c + d*x)^3*(3*a^2*b - b^3) +
 a^3/4 + a^2*b*tan(c + d*x)))/d
```

3.444 $\int \cot^6(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=157

$$-a\left(a^2-3b^2\right)x-\frac{a(a^2-3b^2)\cot(c+dx)}{d}+\frac{b(3a^2-b^2)\cot^2(c+dx)}{2d}+\frac{a(a^2-3b^2)\cot^3(c+dx)}{3d}-\frac{11a^2b\cot^4(c}{20d}$$

```
[Out] -a*(a^2-3*b^2)*x-a*(a^2-3*b^2)*cot(d*x+c)/d+1/2*b*(3*a^2-b^2)*cot(d*x+c)^2/
d+1/3*a*(a^2-3*b^2)*cot(d*x+c)^3/d-11/20*a^2*b*cot(d*x+c)^4/d+b*(3*a^2-b^2)
*ln(sin(d*x+c))/d-1/5*a^2*cot(d*x+c)^5*(a+b*tan(d*x+c))/d
```

Rubi [A]
time = 0.20, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3646, 3709, 3610, 3612, 3556}

$$\frac{a(a^2-3b^2)\cot^3(c+dx)}{3d}+\frac{b(3a^2-b^2)\cot^2(c+dx)}{2d}-\frac{a(a^2-3b^2)\cot(c+dx)}{d}+\frac{b(3a^2-b^2)\log(\sin(c+dx))}{d}-ax\left(a^2-3b^2\right)-\frac{11a^2b\cot^4(c+dx)}{20d}-\frac{a^2\cot^5(c+dx)(a+b\tan(c+dx))}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -(a*(a^2 - 3*b^2)*x) - (a*(a^2 - 3*b^2)*Cot[c + d*x])/d + (b*(3*a^2 - b^2)*
Cot[c + d*x]^2)/(2*d) + (a*(a^2 - 3*b^2)*Cot[c + d*x]^3)/(3*d) - (11*a^2*b*
Cot[c + d*x]^4)/(20*d) + (b*(3*a^2 - b^2)*Log[Sin[c + d*x]])/d - (a^2*Cot[c
 + d*x]^5*(a + b*Tan[c + d*x]))/(5*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \cot^6(c+dx)(a+b\tan(c+dx))^3\,dx &= -\frac{a^2\cot^5(c+dx)(a+b\tan(c+dx))}{5d}+\frac{1}{5}\int \cot^5(c+dx)\left(11a^2b-5\right.\\
&= -\frac{11a^2b\cot^4(c+dx)}{20d}-\frac{a^2\cot^5(c+dx)(a+b\tan(c+dx))}{5d}+\frac{1}{5}\int \mathrm{co}\\
&= \frac{a(a^2-3b^2)\cot^3(c+dx)}{3d}-\frac{11a^2b\cot^4(c+dx)}{20d}-\frac{a^2\cot^5(c+dx)(a}{5a}\\
&= \frac{b(3a^2-b^2)\cot^2(c+dx)}{2d}+\frac{a(a^2-3b^2)\cot^3(c+dx)}{3d}-\frac{11a^2b\cot^4(c}{20d}\\
&= -\frac{a(a^2-3b^2)\cot(c+dx)}{d}+\frac{b(3a^2-b^2)\cot^2(c+dx)}{2d}+\frac{a(a^2-3b^2)}{3}\\
&= -a\left(a^2-3b^2\right)x-\frac{a(a^2-3b^2)\cot(c+dx)}{d}+\frac{b(3a^2-b^2)\cot^2(c+dx}{2d}\\
&= -a\left(a^2-3b^2\right)x-\frac{a(a^2-3b^2)\cot(c+dx)}{d}+\frac{b(3a^2-b^2)\cot^2(c+dx}{2d}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.00, size = 152, normalized size = 0.97

$$
-\frac{a(a^2-3b^2)\cot(c+dx)-\frac{1}{2}b(3a^2-b^2)\cot^2(c+dx)-\frac{1}{3}a(a^2-3b^2)\cot^3(c+dx)+\frac{3}{4}a^2b\cot^4(c+dx)+\frac{1}{5}a^3\cot^5(c+dx)-\frac{1}{2}(ia+b)^3\log(i-\cot(c+dx))+\frac{1}{2}(ia-b)^3\log(i+\cot(c+dx))}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -((a*(a^2 - 3*b^2)*Cot[c + d*x] - (b*(3*a^2 - b^2)*Cot[c + d*x]^2)/2 - (a*(
a^2 - 3*b^2)*Cot[c + d*x]^3)/3 + (3*a^2*b*Cot[c + d*x]^4)/4 + (a^3*Cot[c +
d*x]^5)/5 - ((I*a + b)^3*Log[I - Cot[c + d*x]])/2 + ((I*a - b)^3*Log[I + Co
t[c + d*x]])/2)/d)
```

Maple [A]
time = 0.20, size = 131, normalized size = 0.83

method	result
derivativedivides	$\frac{a^3\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-dx-c\right)+3a^2b\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+3b^2a\left(\right.}{d}$
default	$\frac{a^3\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-dx-c\right)+3a^2b\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+3b^2a\left(\right.}{d}$
norman	$\frac{-\frac{a^3}{5d}+\frac{a\left(a^2-3b^2\right)\left(\tan^2(dx+c)\right)}{3d}-\frac{a\left(a^2-3b^2\right)\left(\tan^4(dx+c)\right)}{d}-a\left(a^2-3b^2\right)x\left(\tan^5(dx+c)\right)-\frac{3a^2b\tan(dx+c)}{4d}+\frac{b\left(3a^2-b^2\right)\left(\tan^3(\right.}{2d}}{\tan(dx+c)^5}$
risch	$-3ixb\,a^2+ib^3x-a^3x+3a\,b^2x-\frac{6ib\,a^2c}{d}+\frac{2ib^3c}{d}+\frac{44ia\,b^2\mathrm{e}^{4i(dx+c)}-\frac{46ia^3}{15}-12a^2b\,\mathrm{e}^{8i(dx+c)}+2b^3\mathrm{e}^{8i(}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c)+3*a^2*b*(-1/
4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+3*b^2*a*(-1/3*cot(d*x+c)^3+
cot(d*x+c)+d*x+c)+b^3*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c))))
```

Maxima [A]
time = 0.52, size = 158, normalized size = 1.01

$$-\frac{60\,(a^3-3\,ab^2)(dx+c)+30\,(3\,a^2b-b^3)\log\left(\tan\left(dx+c\right)^2+1\right)-60\,(3\,a^2b-b^3)\log\left(\tan\left(dx+c\right)\right)+\frac{60\,(a^3-3\,ab^2)\tan(dx+c)^4+45\,a^2b\tan(dx+c)-30\,(3\,a^2b-b^3)\tan(dx+c)^3+12\,a^3-20\,(a^3-3\,ab^2)\tan(dx+c)^2}{\tan(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(60*(a^3 - 3*a*b^2)*(d*x + c) + 30*(3*a^2*b - b^3)*log(tan(d*x + c)^2
 + 1) - 60*(3*a^2*b - b^3)*log(tan(d*x + c)) + (60*(a^3 - 3*a*b^2)*tan(d*x
+ c)^4 + 45*a^2*b*tan(d*x + c) - 30*(3*a^2*b - b^3)*tan(d*x + c)^3 + 12*a^3
 - 20*(a^3 - 3*a*b^2)*tan(d*x + c)^2)/tan(d*x + c)^5)/d
```

Fricas [A]
time = 1.09, size = 173, normalized size = 1.10

$$\frac{30\,(3\,a^2b-b^3)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^5+15\,(9\,a^2b-2\,b^3-4\,(a^3-3\,ab^2)dx)\tan\left(dx+c\right)^5-60\,(a^3-3\,ab^2)\tan\left(dx+c\right)^4-45\,a^2b\tan\left(dx+c\right)+30\,(3\,a^2b-b^3)\tan\left(dx+c\right)^3-12\,a^3+20\,(a^3-3\,ab^2)\tan\left(dx+c\right)^2}{60\,d\tan\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="fricas")

[Out] 1/60*(30*(3*a^2*b - b^3)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^5 + 15*(9*a^2*b - 2*b^3 - 4*(a^3 - 3*a*b^2)*d*x)*tan(d*x + c)^5 - 60*(a^3 - 3*a*b^2)*tan(d*x + c)^4 - 45*a^2*b*tan(d*x + c) + 30*(3*a^2*b - b^3)*tan(d*x + c)^3 - 12*a^3 + 20*(a^3 - 3*a*b^2)*tan(d*x + c)^2)/(d*tan(d*x + c)^5)

Sympy [A]
time = 3.05, size = 241, normalized size = 1.54

$$\begin{cases} \tilde{\infty} a^3 x & \text{for}\ (c = 0 \vee c = -dx) \wedge (c = -dx \vee d = 0) \\ x(a + b\tan(c))^3 \cot^6(c) & \text{for}\ d = 0 \\ -a^3 x - \frac{a^3}{d\tan(c+dx)} + \frac{a^3}{3d\tan^3(c+dx)} - \frac{a^3}{5d\tan^5(c+dx)} - \frac{3a^2 b\log(\tan^2(c+dx)+1)}{2d} + \frac{3a^2 b\log(\tan(c+dx))}{d} + \frac{3a^2 b}{2d\tan^2(c+dx)} - \frac{3a^2 b}{4d\tan^4(c+dx)} + 3ab^2 x + \frac{3ab^2}{d\tan(c+dx)} - \frac{ab^2}{d\tan^3(c+dx)} + \frac{b^3\log(\tan^2(c+dx)+1)}{2d} - \frac{b^3\log(\tan(c+dx))}{d} - \frac{b^3}{2d\tan^2(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**6*(a+b*tan(d*x+c))**3,x)

[Out] Piecewise((zoo*a**3*x, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (x*(a + b*tan(c))**3*cot(c)**6, Eq(d, 0)), (-a**3*x - a**3/(d*tan(c + d*x)) + a**3/(3*d*tan(c + d*x)**3) - a**3/(5*d*tan(c + d*x)**5) - 3*a**2*b*log(tan(c + d*x)**2 + 1)/(2*d) + 3*a**2*b*log(tan(c + d*x))/d + 3*a**2*b/(2*d*tan(c + d*x)**2) - 3*a**2*b/(4*d*tan(c + d*x)**4) + 3*a*b**2*x + 3*a*b**2/(d*tan(c + d*x)) - a*b**2/(d*tan(c + d*x)**3) + b**3*log(tan(c + d*x)**2 + 1)/(2*d) - b**3*log(tan(c + d*x))/d - b**3/(2*d*tan(c + d*x)**2), True))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(149) = 298.
time = 2.81, size = 370, normalized size = 2.36

$$\frac{6a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 45a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 70a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 120ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 540a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 120b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 660a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 1800ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 960(a^3-3ab^2)(dx+c) - 960(3a^2b-b^3)\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right) + 960(3a^2b-b^3)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right) - \frac{6576a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 2192b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 660a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 1800ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 540a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 120b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 70a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 120ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 45a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + 6a^3}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5}}{960d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="giac")

[Out] 1/960*(6*a^3*tan(1/2*d*x + 1/2*c)^5 - 45*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 70*a^3*tan(1/2*d*x + 1/2*c)^3 + 120*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 540*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 120*b^3*tan(1/2*d*x + 1/2*c)^2 + 660*a^3*tan(1/2*d*x + 1/2*c) - 1800*a*b^2*tan(1/2*d*x + 1/2*c) - 960*(a^3 - 3*a*b^2)*(d*x + c) - 960*(3*a^2*b - b^3)*log(tan(1/2*d*x + 1/2*c)^2 + 1) + 960*(3*a^2*b - b^3)*log(abs(tan(1/2*d*x + 1/2*c))) - (6576*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 2192*b^3*tan(1/2*d*x + 1/2*c)^5 + 660*a^3*tan(1/2*d*x + 1/2*c)^4 - 1800*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 540*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 120*b^3*tan(1/2*d*x + 1/2*c)^3 - 70*a^3*tan(1/2*d*x + 1/2*c)^2 + 120*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 45*a^2*b*tan(1/2*d*x + 1/2*c) + 6*a^3)/tan(1/2*d*x + 1/2*c)^5)/d

Mupad [B]
time = 4.10, size = 166, normalized size = 1.06

$$\frac{\ln\left(\tan(c+d\,x)\right)\,\left(3\,a^2\,b-b^3\right)}{d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,{\left(b+a\,1\mathrm{i}\right)}^3}{2\,d}-\frac{{\cot(c+d\,x)}^5\,\left({\tan(c+d\,x)}^2\,\left(a\,b^2-\frac{a^3}{3}\right)-{\tan(c+d\,x)}^4\,\left(3\,a\,b^2-a^3\right)-{\tan(c+d\,x)}^3\,\left(\frac{3\,a^2\,b}{2}-\frac{b^3}{2}\right)+\frac{a^3}{5}+\frac{3\,a^2\,b\,\tan(c+d\,x)}{4}\right)}{d}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,{\left(a+b\,1\mathrm{i}\right)}^3\,1\mathrm{i}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(a + b*tan(c + d*x))^3,x)
```

```
[Out] (log(tan(c + d*x))*(3*a^2*b - b^3))/d + (log(tan(c + d*x) - 1i)*(a + b*1i)^
3*1i)/(2*d) + (log(tan(c + d*x) + 1i)*(a*1i + b)^3)/(2*d) - (cot(c + d*x)^5
*(tan(c + d*x)^2*(a*b^2 - a^3/3) - tan(c + d*x)^4*(3*a*b^2 - a^3) - tan(c +
 d*x)^3*((3*a^2*b)/2 - b^3/2) + a^3/5 + (3*a^2*b*tan(c + d*x))/4))/d
```

3.445 $\int \tan^3(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=181

$$4ab(a^2-b^2)\,x+\frac{(a^4-6a^2b^2+b^4)\log(\cos(c+dx))}{d}-\frac{ab(a^2-3b^2)\tan(c+dx)}{d}-\frac{(a^2-b^2)\,(a+b\tan(c+dx))^2}{2d}$$

```
[Out] 4*a*b*(a^2-b^2)*x+(a^4-6*a^2*b^2+b^4)*ln(cos(d*x+c))/d-a*b*(a^2-3*b^2)*tan(
d*x+c)/d-1/2*(a^2-b^2)*(a+b*tan(d*x+c))^2/d-1/3*a*(a+b*tan(d*x+c))^3/d-1/4*
(a+b*tan(d*x+c))^4/d-1/30*a*(a+b*tan(d*x+c))^5/b^2/d+1/6*tan(d*x+c)*(a+b*ta
n(d*x+c))^5/b/d
```

Rubi [A]
time = 0.18, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3647, 3711, 12, 3609, 3606, 3556}

$$-\frac{(a^2-b^2)\,(a+b\tan(c+dx))^2}{2d}-\frac{ab(a^2-3b^2)\tan(c+dx)}{d}+4abx\,(a^2-b^2)+\frac{(a^4-6a^2b^2+b^4)\log(\cos(c+dx))}{d}-\frac{a(a+b\tan(c+dx))^5}{30b^2d}+\frac{\tan(c+dx)(a+b\tan(c+dx))^5}{6bd}-\frac{(a+b\tan(c+dx))^4}{4d}-\frac{a(a+b\tan(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] 4*a*b*(a^2 - b^2)*x + ((a^4 - 6*a^2*b^2 + b^4)*Log[Cos[c + d*x]])/d - (a*b*
(a^2 - 3*b^2)*Tan[c + d*x])/d - ((a^2 - b^2)*(a + b*Tan[c + d*x])^2)/(2*d)
- (a*(a + b*Tan[c + d*x])^3)/(3*d) - (a + b*Tan[c + d*x])^4/(4*d) - (a*(a +
 b*Tan[c + d*x])^5)/(30*b^2*d) + (Tan[c + d*x]*(a + b*Tan[c + d*x])^5)/(6*b
*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \tan^3(c+dx)(a+b\tan(c+dx))^4\,dx &= \frac{\tan(c+dx)(a+b\tan(c+dx))^5}{6bd} + \frac{\int (a+b\tan(c+dx))^4\,(-a-6b \\
&= -\frac{a(a+b\tan(c+dx))^5}{30b^2d} + \frac{\tan(c+dx)(a+b\tan(c+dx))^5}{6bd} + \frac{\int -6 \\
&= -\frac{a(a+b\tan(c+dx))^5}{30b^2d} + \frac{\tan(c+dx)(a+b\tan(c+dx))^5}{6bd} - \int \tan \\
&= -\frac{(a+b\tan(c+dx))^4}{4d} - \frac{a(a+b\tan(c+dx))^5}{30b^2d} + \frac{\tan(c+dx)(a+ \\
&= -\frac{a(a+b\tan(c+dx))^3}{3d} - \frac{(a+b\tan(c+dx))^4}{4d} - \frac{a(a+b\tan(c+d \\
&= -\frac{(a^2-b^2)\,(a+b\tan(c+dx))^2}{2d} - \frac{a(a+b\tan(c+dx))^3}{3d} - \frac{(a+b\,\mathrm{t} \\
&= 4ab(a^2-b^2)\,x - \frac{ab(a^2-3b^2)\tan(c+dx)}{d} - \frac{(a^2-b^2)\,(a+b\tan(c \\
&= 4ab(a^2-b^2)\,x + \frac{(a^4-6a^2b^2+b^4)\log(\cos(c+dx))}{d} - \frac{ab(a^2-3b^2)}{d}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.22, size = 190, normalized size = 1.05

$$
\frac{-2(a^6+15(a+ib)^4b^2\log(i-\tan(c+dx))+15(a-ib)^4b^2\log(i+\tan(c+dx)))-240ab^3(a^2-b^2)\tan(c+dx)+30b^2(a^4-6a^2b^2+b^4)\tan^2(c+dx)+80ab^3(a^2-b^2)\tan^3(c+dx)-15b^4(-6a^2+b^2)\tan^4(c+dx)+48ab^5\tan^5(c+dx)+10b^6\tan^6(c+dx)}{60b^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (-2*(a^6 + 15*(a + I*b)^4*b^2*Log[I - Tan[c + d*x]] + 15*(a - I*b)^4*b^2*Lo
g[I + Tan[c + d*x]]) - 240*a*b^3*(a^2 - b^2)*Tan[c + d*x] + 30*b^2*(a^4 - 6
*a^2*b^2 + b^4)*Tan[c + d*x]^2 + 80*a*b^3*(a^2 - b^2)*Tan[c + d*x]^3 - 15*b
^4*(-6*a^2 + b^2)*Tan[c + d*x]^4 + 48*a*b^5*Tan[c + d*x]^5 + 10*b^6*Tan[c +
 d*x]^6)/(60*b^2*d)
```

Maple [A]
time = 0.06, size = 209, normalized size = 1.15

method	result
norman	$\frac{b^4(\tan^6(dx+c))}{6d}+\frac{(a^4-6a^2b^2+b^4)(\tan^2(dx+c))}{2d}+4ab(a^2-b^2)\,x+\frac{4a\,b^3(\tan^5(dx+c))}{5d}+\frac{b^2(6a^2-b^2)(\tan^4(}{4d}$
derivativedivides	$\frac{\frac{b^4(\tan^6(dx+c))}{6}+\frac{4a\,b^3(\tan^5(dx+c))}{5}+\frac{3a^2b^2(\tan^4(dx+c))}{2}-\frac{b^4(\tan^4(dx+c))}{4}+\frac{4a^3b(\tan^3(dx+c))}{3}-\frac{4a\,b^3(\tan^3(dx+c))}{3}+\frac{a^4(\tan}{}}{}$
default	$\frac{\frac{b^4(\tan^6(dx+c))}{6}+\frac{4a\,b^3(\tan^5(dx+c))}{5}+\frac{3a^2b^2(\tan^4(dx+c))}{2}-\frac{b^4(\tan^4(dx+c))}{4}+\frac{4a^3b(\tan^3(dx+c))}{3}-\frac{4a\,b^3(\tan^3(dx+c))}{3}+\frac{a^4(\tan}{}}{}$

risch	$4a^3bx - 4a\,b^3x - ia^4x + 6ia^2b^2x - ib^4x - \frac{2ia^4c}{d} + \frac{12ia^2b^2c}{d} - \frac{2ib^4c}{d} + \frac{-72a^2b^2e^{4i(dx+c)}-96ia^3b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/6*b^4*tan(d*x+c)^6+4/5*a*b^3*tan(d*x+c)^5+3/2*a^2*b^2*tan(d*x+c)^4-1
/4*b^4*tan(d*x+c)^4+4/3*a^3*b*tan(d*x+c)^3-4/3*a*b^3*tan(d*x+c)^3+1/2*a^4*t
an(d*x+c)^2-3*a^2*b^2*tan(d*x+c)^2+1/2*b^4*tan(d*x+c)^2-4*a^3*b*tan(d*x+c)+
4*a*b^3*tan(d*x+c)+1/2*(-a^4+6*a^2*b^2-b^4)*ln(1+tan(d*x+c)^2)+(4*a^3*b-4*a
*b^3)*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.52, size = 171, normalized size = 0.94

$$\frac{10\,b^4\tan(dx+c)^6 + 48\,ab^3\tan(dx+c)^5 + 15\,(6\,a^2b^2 - b^4)\tan(dx+c)^4 + 80\,(a^3b - ab^3)\tan(dx+c)^3 + 30\,(a^4 - 6\,a^2b^2 + b^4)\tan(dx+c)^2 + 240\,(a^3b - ab^3)(dx+c) - 30\,(a^4 - 6\,a^2b^2 + b^4)\log\left(\tan(dx+c)^2 + 1\right) - 240\,(a^3b - ab^3)\tan(dx+c)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/60*(10*b^4*tan(d*x + c)^6 + 48*a*b^3*tan(d*x + c)^5 + 15*(6*a^2*b^2 - b^4
)*tan(d*x + c)^4 + 80*(a^3*b - a*b^3)*tan(d*x + c)^3 + 30*(a^4 - 6*a^2*b^2
+ b^4)*tan(d*x + c)^2 + 240*(a^3*b - a*b^3)*(d*x + c) - 30*(a^4 - 6*a^2*b^2
 + b^4)*log(tan(d*x + c)^2 + 1) - 240*(a^3*b - a*b^3)*tan(d*x + c))/d
```

Fricas [A]
time = 0.87, size = 170, normalized size = 0.94

$$\frac{10\,b^4\tan(dx+c)^6 + 48\,ab^3\tan(dx+c)^5 + 15\,(6\,a^2b^2 - b^4)\tan(dx+c)^4 + 80\,(a^3b - ab^3)\tan(dx+c)^3 + 240\,(a^3b - ab^3)dx + 30\,(a^4 - 6\,a^2b^2 + b^4)\tan(dx+c)^2 + 30\,(a^4 - 6\,a^2b^2 + b^4)\log\left(\frac{1}{\tan(dx+c)^2+1}\right) - 240\,(a^3b - ab^3)\tan(dx+c)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^4*tan(d*x + c)^6 + 48*a*b^3*tan(d*x + c)^5 + 15*(6*a^2*b^2 - b^4
)*tan(d*x + c)^4 + 80*(a^3*b - a*b^3)*tan(d*x + c)^3 + 240*(a^3*b - a*b^3)*
d*x + 30*(a^4 - 6*a^2*b^2 + b^4)*tan(d*x + c)^2 + 30*(a^4 - 6*a^2*b^2 + b^4
)*log(1/(tan(d*x + c)^2 + 1)) - 240*(a^3*b - a*b^3)*tan(d*x + c))/d
```

Sympy [A]
time = 0.22, size = 277, normalized size = 1.53

$$\begin{cases} -\frac{a^4\log\left(\tan^2(c+dx)+1\right)}{2d} + \frac{a^4\tan^2(c+dx)}{2d} + 4a^3bx + \frac{4a^3b\tan^3(c+dx)}{3d} - \frac{4a^3b\tan(c+dx)}{d} + \frac{3a^2b^2\log\left(\tan^2(c+dx)+1\right)}{d} + \frac{3a^2b^2\tan^4(c+dx)}{2d} - \frac{3a^2b^2\tan^2(c+dx)}{d} - 4ab^3x + \frac{4ab^3\tan^5(c+dx)}{5d} - \frac{4ab^3\tan^3(c+dx)}{3d} + \frac{4ab^3\tan(c+dx)}{d} - \frac{b^4\log\left(\tan^2(c+dx)+1\right)}{2d} + \frac{b^4\tan^6(c+dx)}{6d} - \frac{b^4\tan^4(c+dx)}{4d} + \frac{b^4\tan^2(c+dx)}{2d} & \text{for}\ d \neq 0 \\ x(a+b\tan(c))^4\tan^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Piecewise((-a**4*log(tan(c + d*x)**2 + 1)/(2*d) + a**4*tan(c + d*x)**2/(2*d
) + 4*a**3*b*x + 4*a**3*b*tan(c + d*x)**3/(3*d) - 4*a**3*b*tan(c + d*x)/d +
 3*a**2*b**2*log(tan(c + d*x)**2 + 1)/d + 3*a**2*b**2*tan(c + d*x)**4/(2*d)
 - 3*a**2*b**2*tan(c + d*x)**2/d - 4*a*b**3*x + 4*a*b**3*tan(c + d*x)**5/(5
*d) - 4*a*b**3*tan(c + d*x)**3/(3*d) + 4*a*b**3*tan(c + d*x)/d - b**4*log(t
an(c + d*x)**2 + 1)/(2*d) + b**4*tan(c + d*x)**6/(6*d) - b**4*tan(c + d*x)*
*4/(4*d) + b**4*tan(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*tan(c))**4*tan(
c)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 3385 vs. $2(171) = 342$.
time = 5.19, size = 3385, normalized size = 18.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/60*(240*a^3*b*d*x*tan(d*x)^6*tan(c)^6 - 240*a*b^3*d*x*tan(d*x)^6*tan(c)^6
 + 30*a^4*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan
(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^6*tan(
c)^6 - 180*a^2*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d
*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*
x)^6*tan(c)^6 + 30*b^4*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + t
an(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*ta
n(d*x)^6*tan(c)^6 - 1440*a^3*b*d*x*tan(d*x)^5*tan(c)^5 + 1440*a*b^3*d*x*tan
(d*x)^5*tan(c)^5 + 30*a^4*tan(d*x)^6*tan(c)^6 - 270*a^2*b^2*tan(d*x)^6*tan(
c)^6 + 55*b^4*tan(d*x)^6*tan(c)^6 - 180*a^4*log(4*(tan(d*x)^4*tan(c)^2 - 2*
tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) +
1)/(tan(c)^2 + 1))*tan(d*x)^5*tan(c)^5 + 1080*a^2*b^2*log(4*(tan(d*x)^4*tan
(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)
*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^5*tan(c)^5 - 180*b^4*log(4*(tan(d*x)^
4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan
(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^5*tan(c)^5 + 240*a^3*b*tan(d*x)^
6*tan(c)^5 - 240*a*b^3*tan(d*x)^6*tan(c)^5 + 240*a^3*b*tan(d*x)^5*tan(c)^6
 - 240*a*b^3*tan(d*x)^5*tan(c)^6 + 3600*a^3*b*d*x*tan(d*x)^4*tan(c)^4 - 3600
*a*b^3*d*x*tan(d*x)^4*tan(c)^4 + 30*a^4*tan(d*x)^6*tan(c)^4 - 180*a^2*b^2*t
an(d*x)^6*tan(c)^4 + 30*b^4*tan(d*x)^6*tan(c)^4 - 120*a^4*tan(d*x)^5*tan(c)
^5 + 1260*a^2*b^2*tan(d*x)^5*tan(c)^5 - 270*b^4*tan(d*x)^5*tan(c)^5 + 30*a^
4*tan(d*x)^4*tan(c)^6 - 180*a^2*b^2*tan(d*x)^4*tan(c)^6 + 30*b^4*tan(d*x)^4
*tan(c)^6 - 80*a^3*b*tan(d*x)^6*tan(c)^3 + 80*a*b^3*tan(d*x)^6*tan(c)^3 + 4
50*a^4*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)
^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^
4 - 2700*a^2*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x
)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)
```

```
^4*tan(c)^4 + 450*b^4*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + ta
n(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan
(d*x)^4*tan(c)^4 - 1440*a^3*b*tan(d*x)^5*tan(c)^4 + 1440*a*b^3*tan(d*x)^5*t
an(c)^4 - 1440*a^3*b*tan(d*x)^4*tan(c)^5 + 1440*a*b^3*tan(d*x)^4*tan(c)^5 -
 80*a^3*b*tan(d*x)^3*tan(c)^6 + 80*a*b^3*tan(d*x)^3*tan(c)^6 + 90*a^2*b^2*t
an(d*x)^6*tan(c)^2 - 15*b^4*tan(d*x)^6*tan(c)^2 - 4800*a^3*b*d*x*tan(d*x)^3
*tan(c)^3 + 4800*a*b^3*d*x*tan(d*x)^3*tan(c)^3 - 120*a^4*tan(d*x)^5*tan(c)^
3 + 1080*a^2*b^2*tan(d*x)^5*tan(c)^3 - 180*b^4*tan(d*x)^5*tan(c)^3 + 210*a^
4*tan(d*x)^4*tan(c)^4 - 2070*a^2*b^2*tan(d*x)^4*tan(c)^4 + 495*b^4*tan(d*x)
^4*tan(c)^4 - 120*a^4*tan(d*x)^3*tan(c)^5 + 1080*a^2*b^2*tan(d*x)^3*tan(c)^
5 - 180*b^4*tan(d*x)^3*tan(c)^5 + 90*a^2*b^2*tan(d*x)^2*tan(c)^6 - 15*b^4*t
an(d*x)^2*tan(c)^6 - 48*a*b^3*tan(d*x)^6*tan(c) + 240*a^3*b*tan(d*x)^5*tan(
c)^2 - 480*a*b^3*tan(d*x)^5*tan(c)^2 - 600*a^4*log(4*(tan(d*x)^4*tan(c)^2 -
 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c)
 + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 + 3600*a^2*b^2*log(4*(tan(d*x)^4*
tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d
*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 - 600*b^4*log(4*(tan(d*
x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*
tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 + 3120*a^3*b*tan(d
*x)^4*tan(c)^3 - 3600*a*b^3*tan(d*x)^4*tan(c)^3 + 3120*a^3*b*tan(d*x)^3*tan
(c)^4 - 3600*a*b^3*tan(d*x)^3*tan(c)^4 + 240*a^3*b*tan(d*x)^2*tan(c)^5 - 48
0*a*b^3*tan(d*x)^2*tan(c)^5 - 48*a*b^3*tan(d*x)*tan(c)^6 + 10*b^4*tan(d*x)^
6 - 180*a^2*b^2*tan(d*x)^5*tan(c) + 90*b^4*tan(d*x)^5*tan(c) + 3600*a^3*b*d
*x*tan(d*x)^2*tan(c)^2 - 3600*a*b^3*d*x*tan(d*x)^2*tan(c)^2 + 180*a^4*tan(d
*x)^4*tan(c)^2 - 1800*a^2*b^2*tan(d*x)^4*tan(c)^2 + 450*b^4*tan(d*x)^4*tan(
c)^2 - 240*a^4*tan(d*x)^3*tan(c)^3 + 2160*a^2*b^2*tan(d*x)^3*tan(c)^3 - 360
*b^4*tan(d*x)^3*tan(c)^3 + 180*a^4*tan(d*x)^2*tan(c)^4 - 1800*a^2*b^2*tan(d
*x)^2*tan(c)^4 + 450*b^4*tan(d*x)^2*tan(c)^4 - 180*a^2*b^2*tan(d*x)*tan(c)^
5 + 90*b^4*tan(d*x)*tan(c)^5 + 10*b^4*tan(c)^6 + 48*a*b^3*tan(d*x)^5 - 240*
a^3*b*tan(d*x)^4*tan(c) + 480*a*b^3*tan(d*x)^4*tan(c) + 450*a^4*log(4*(tan(
d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 -
2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 - 2700*a^2*b^2*l
og(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan
(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + 450*
b^4*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2
+ tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 -
 3120*a^3*b*tan(d*x)^3*tan(c)^2 + 3600*a*b^3*tan(d*x)^3*tan(c)^2 - 3120*a^3
*b*tan(d*x)^2*tan(c)^3 + 3600*a*b^3*tan(d*x)^2*tan(c)^3 - 240*a^3*b*tan(d*x
)*tan(c)^4 + 480*a*b^3*tan(d*x)*tan(c)^4 + 48*a*b^3*tan(c)^5 + 90*a^2*b^2*t
an(d*x)^4 - 15*b^4*tan(d*x)^4 - 1440*a^3*b*d*x*...
```

Mupad [B]

time = 3.97, size = 225, normalized size = 1.24

$$\frac{\tan(c+d\,x)^2\,\left(\frac{a^4}{2}-3\,a^2\,b^2+\frac{b^4}{2}\right)}{d}-\frac{\tan(c+d\,x)^4\,\left(\frac{b^4}{4}-\frac{3\,a^2\,b^2}{2}\right)}{d}-\frac{\tan(c+d\,x)^3\,\left(\frac{4\,a\,b^3}{3}-\frac{4\,a^3\,b}{3}\right)}{d}-\frac{\ln\left(\tan(c+d\,x)^2+1\right)\,\left(\frac{a^4}{2}-3\,a^2\,b^2+\frac{b^4}{2}\right)}{d}+\frac{b^4\,\tan(c+d\,x)^6}{6\,d}+\frac{\tan(c+d\,x)\,\left(4\,a\,b^3-4\,a^3\,b\right)}{d}+\frac{4\,a\,b^3\,\tan(c+d\,x)^5}{5\,d}-\frac{4\,a\,b\,\mathrm{atan}\left(\frac{4\,a\,b\,\tan(c+d\,x)\,(a+b)\,(a-b)}{4\,a\,b^3-4\,a^3\,b}\right)\,(a+b)\,(a-b)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + b*tan(c + d*x))^4,x)
```

```
[Out] (tan(c + d*x)^2*(a^4/2 + b^4/2 - 3*a^2*b^2))/d - (tan(c + d*x)^4*(b^4/4 - (
3*a^2*b^2)/2))/d - (tan(c + d*x)^3*((4*a*b^3)/3 - (4*a^3*b)/3))/d - (log(ta
n(c + d*x)^2 + 1)*(a^4/2 + b^4/2 - 3*a^2*b^2))/d + (b^4*tan(c + d*x)^6)/(6*
d) + (tan(c + d*x)*(4*a*b^3 - 4*a^3*b))/d + (4*a*b^3*tan(c + d*x)^5)/(5*d)
- (4*a*b*atan((4*a*b*tan(c + d*x)*(a + b)*(a - b))/(4*a*b^3 - 4*a^3*b))*(a
+ b)*(a - b))/d
```

3.446 $\int \tan^2(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=128

$$-\left(\left(a^4-6a^2b^2+b^4\right)x\right)+\frac{4ab(a^2-b^2)\log(\cos(c+dx))}{d}-\frac{b^2(3a^2-b^2)\tan(c+dx)}{d}-\frac{ab(a+b\tan(c+dx))^2}{d}-$$

```
[Out] -(a^4-6*a^2*b^2+b^4)*x+4*a*b*(a^2-b^2)*ln(cos(d*x+c))/d-b^2*(3*a^2-b^2)*tan
(d*x+c)/d-a*b*(a+b*tan(d*x+c))^2/d-1/3*b*(a+b*tan(d*x+c))^3/d+1/5*(a+b*tan(
d*x+c))^5/b/d
```

Rubi [A]
time = 0.11, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3624, 3563, 3609, 3606, 3556}

$$-\frac{b^2(3a^2-b^2)\tan(c+dx)}{d}+\frac{4ab(a^2-b^2)\log(\cos(c+dx))}{d}-x\left(a^4-6a^2b^2+b^4\right)+\frac{(a+b\tan(c+dx))^5}{5bd}-\frac{b(a+b\tan(c+dx))^3}{3d}-\frac{ab(a+b\tan(c+dx))^2}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -((a^4 - 6*a^2*b^2 + b^4)*x) + (4*a*b*(a^2 - b^2)*Log[Cos[c + d*x]])/d - (b
^2*(3*a^2 - b^2)*Tan[c + d*x])/d - (a*b*(a + b*Tan[c + d*x])^2)/d - (b*(a +
 b*Tan[c + d*x])^3)/(3*d) + (a + b*Tan[c + d*x])^5/(5*b*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3563

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
```

```
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^4\,dx = \frac{(a+b\tan(c+dx))^5}{5bd} - \int (a+b\tan(c+dx))^4\,dx$$
$$= -\frac{b(a+b\tan(c+dx))^3}{3d} + \frac{(a+b\tan(c+dx))^5}{5bd} - \int (a+b\tan(c+$$
$$= -\frac{ab(a+b\tan(c+dx))^2}{d} - \frac{b(a+b\tan(c+dx))^3}{3d} + \frac{(a+b\tan(c+}{5bd}$$
$$= -\left(a^4-6a^2b^2+b^4\right)x - \frac{b^2(3a^2-b^2)\tan(c+dx)}{d} - \frac{ab(a+b\tan(c}{d}$$
$$= -\left(a^4-6a^2b^2+b^4\right)x + \frac{4ab(a^2-b^2)\log(\cos(c+dx))}{d} - \frac{b^2(3a^2-b}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.77, size = 122, normalized size = 0.95

$$\frac{15i(a+ib)^4\log(i-\tan(c+dx)) - 15i(a-ib)^4\log(i+\tan(c+dx)) + 30b^2(-6a^2+b^2)\tan(c+dx) - 60ab^3\tan^2(c+dx) - 10b^4\tan^3(c+dx) + \frac{6(a+b\tan(c+dx))^5}{b}}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] ((15*I)*(a + I*b)^4*Log[I - Tan[c + d*x]] - (15*I)*(a - I*b)^4*Log[I + Tan[
c + d*x]] + 30*b^2*(-6*a^2 + b^2)*Tan[c + d*x] - 60*a*b^3*Tan[c + d*x]^2 -
10*b^4*Tan[c + d*x]^3 + (6*(a + b*Tan[c + d*x])^5)/b)/(30*d)
```

Maple [A]
time = 0.04, size = 176, normalized size = 1.38

method	result

norman	$\left(-a^4+6a^2b^2-b^4\right)x+\frac{\left(a^4-6a^2b^2+b^4\right)\tan(dx+c)}{d}+\frac{a\,b^3\left(\tan^4(dx+c)\right)}{d}+\frac{b^4\left(\tan^5(dx+c)\right)}{5d}+\frac{b^2\left(6a^2-b^2\right)}{}$
derivativedivides	$\frac{\frac{b^4\left(\tan^5(dx+c)\right)}{5}+a\,b^3\left(\tan^4(dx+c)\right)+2a^2b^2\left(\tan^3(dx+c)\right)-\frac{b^4\left(\tan^3(dx+c)\right)}{3}+2a^3b\left(\tan^2(dx+c)\right)-2a\,b^3\left(\tan^2(dx+c)\right)+a^4}{d}$
default	$\frac{\frac{b^4\left(\tan^5(dx+c)\right)}{5}+a\,b^3\left(\tan^4(dx+c)\right)+2a^2b^2\left(\tan^3(dx+c)\right)-\frac{b^4\left(\tan^3(dx+c)\right)}{3}+2a^3b\left(\tan^2(dx+c)\right)-2a\,b^3\left(\tan^2(dx+c)\right)+a^4}{d}$
risch	$-4ia^3bx+4ia\,b^3x-a^4x+6a^2b^2x-b^4x-\frac{8ia^3bc}{d}+\frac{8ia\,b^3c}{d}+\frac{2i\left(-60ia^3b\,e^{8i(dx+c)}-60ia^3b\,e^{2i(dx-}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*tan(d*x+c)^5*b^4+a*b^3*tan(d*x+c)^4+2*a^2*b^2*tan(d*x+c)^3-1/3*b^4
*tan(d*x+c)^3+2*a^3*b*tan(d*x+c)^2-2*a*b^3*tan(d*x+c)^2+a^4*tan(d*x+c)-6*a^
2*b^2*tan(d*x+c)+b^4*tan(d*x+c)+1/2*(-4*a^3*b+4*a*b^3)*ln(1+tan(d*x+c)^2)+(
-a^4+6*a^2*b^2-b^4)*arctan(tan(d*x+c)))
```

Maxima [A]

time = 0.52, size = 149, normalized size = 1.16

$$\frac{3\,b^4\tan\left(dx+c\right)^5+15\,ab^3\tan\left(dx+c\right)^4+5\left(6\,a^2b^2-b^4\right)\tan\left(dx+c\right)^3+30\left(a^3b-ab^3\right)\tan\left(dx+c\right)^2-15\left(a^4-6\,a^2b^2+b^4\right)\left(dx+c\right)-30\left(a^3b-ab^3\right)\log\left(\tan\left(dx+c\right)^2+1\right)+15\left(a^4-6\,a^2b^2+b^4\right)\tan\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/15*(3*b^4*tan(d*x + c)^5 + 15*a*b^3*tan(d*x + c)^4 + 5*(6*a^2*b^2 - b^4)*
tan(d*x + c)^3 + 30*(a^3*b - a*b^3)*tan(d*x + c)^2 - 15*(a^4 - 6*a^2*b^2 +
b^4)*(d*x + c) - 30*(a^3*b - a*b^3)*log(tan(d*x + c)^2 + 1) + 15*(a^4 - 6*a
^2*b^2 + b^4)*tan(d*x + c))/d
```

Fricas [A]

time = 0.99, size = 148, normalized size = 1.16

$$\frac{3\,b^4\tan\left(dx+c\right)^5+15\,ab^3\tan\left(dx+c\right)^4+5\left(6\,a^2b^2-b^4\right)\tan\left(dx+c\right)^3-15\left(a^4-6\,a^2b^2+b^4\right)dx+30\left(a^3b-ab^3\right)\tan\left(dx+c\right)^2+30\left(a^3b-ab^3\right)\log\left(\frac{1}{\tan\left(dx+c\right)^2+1}\right)+15\left(a^4-6\,a^2b^2+b^4\right)\tan\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^4*tan(d*x + c)^5 + 15*a*b^3*tan(d*x + c)^4 + 5*(6*a^2*b^2 - b^4)*
tan(d*x + c)^3 - 15*(a^4 - 6*a^2*b^2 + b^4)*d*x + 30*(a^3*b - a*b^3)*tan(d*
x + c)^2 + 30*(a^3*b - a*b^3)*log(1/(tan(d*x + c)^2 + 1)) + 15*(a^4 - 6*a^2
*b^2 + b^4)*tan(d*x + c))/d
```

Sympy [A]

time = 0.17, size = 214, normalized size = 1.67

$$\begin{cases} -a^4x+\frac{a^4\tan\left(c+dx\right)}{d}-\frac{2a^3b\log\left(\tan^2\left(c+dx\right)+1\right)}{d}+\frac{2a^3b\tan^2\left(c+dx\right)}{d}+6a^2b^2x+\frac{2a^2b^2\tan^3\left(c+dx\right)}{d}-\frac{6a^2b^2\tan\left(c+dx\right)}{d}+\frac{2ab^3\log\left(\tan^2\left(c+dx\right)+1\right)}{d}+\frac{ab^3\tan^4\left(c+dx\right)}{d}-\frac{2ab^3\tan^2\left(c+dx\right)}{d}-b^4x+\frac{b^4\tan^5\left(c+dx\right)}{5d}-\frac{b^4\tan^3\left(c+dx\right)}{3d}+\frac{b^4\tan\left(c+dx\right)}{d} & \text{for}\: d \neq 0 \\ x\left(a+b\tan\left(c\right)\right)^4\tan^2\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Piecewise((-a**4*x + a**4*tan(c + d*x)/d - 2*a**3*b*log(tan(c + d*x)**2 + 1
)/d + 2*a**3*b*tan(c + d*x)**2/d + 6*a**2*b**2*x + 2*a**2*b**2*tan(c + d*x)
**3/d - 6*a**2*b**2*tan(c + d*x)/d + 2*a*b**3*log(tan(c + d*x)**2 + 1)/d +
a*b**3*tan(c + d*x)**4/d - 2*a*b**3*tan(c + d*x)**2/d - b**4*x + b**4*tan(c
 + d*x)**5/(5*d) - b**4*tan(c + d*x)**3/(3*d) + b**4*tan(c + d*x)/d, Ne(d,
0)), (x*(a + b*tan(c))**4*tan(c)**2, True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2281 vs. $2(124) = 248$.
time = 2.83, size = 2281, normalized size = 17.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/15*(15*a^4*d*x*tan(d*x)^5*tan(c)^5 - 90*a^2*b^2*d*x*tan(d*x)^5*tan(c)^5
+ 15*b^4*d*x*tan(d*x)^5*tan(c)^5 - 30*a^3*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*
tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) +
1)/(tan(c)^2 + 1))*tan(d*x)^5*tan(c)^5 + 30*a*b^3*log(4*(tan(d*x)^4*tan(c)^
2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan
(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^5*tan(c)^5 - 75*a^4*d*x*tan(d*x)^4*tan(c)
^4 + 450*a^2*b^2*d*x*tan(d*x)^4*tan(c)^4 - 75*b^4*d*x*tan(d*x)^4*tan(c)^4 -
 30*a^3*b*tan(d*x)^5*tan(c)^5 + 45*a*b^3*tan(d*x)^5*tan(c)^5 + 150*a^3*b*lo
g(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(
d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 - 150*a
*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2
 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4
+ 15*a^4*tan(d*x)^5*tan(c)^4 - 90*a^2*b^2*tan(d*x)^5*tan(c)^4 + 15*b^4*tan(
d*x)^5*tan(c)^4 + 15*a^4*tan(d*x)^4*tan(c)^5 - 90*a^2*b^2*tan(d*x)^4*tan(c)
^5 + 15*b^4*tan(d*x)^4*tan(c)^5 + 150*a^4*d*x*tan(d*x)^3*tan(c)^3 - 900*a^2
*b^2*d*x*tan(d*x)^3*tan(c)^3 + 150*b^4*d*x*tan(d*x)^3*tan(c)^3 - 30*a^3*b*t
an(d*x)^5*tan(c)^3 + 30*a*b^3*tan(d*x)^5*tan(c)^3 + 90*a^3*b*tan(d*x)^4*tan
(c)^4 - 165*a*b^3*tan(d*x)^4*tan(c)^4 - 30*a^3*b*tan(d*x)^3*tan(c)^5 + 30*a
*b^3*tan(d*x)^3*tan(c)^5 + 30*a^2*b^2*tan(d*x)^5*tan(c)^2 - 5*b^4*tan(d*x)^
5*tan(c)^2 - 300*a^3*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + t
an(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*ta
n(d*x)^3*tan(c)^3 + 300*a*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan
(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 +
 1))*tan(d*x)^3*tan(c)^3 - 60*a^4*tan(d*x)^4*tan(c)^3 + 450*a^2*b^2*tan(d*x
)^4*tan(c)^3 - 75*b^4*tan(d*x)^4*tan(c)^3 - 60*a^4*tan(d*x)^3*tan(c)^4 + 45
0*a^2*b^2*tan(d*x)^3*tan(c)^4 - 75*b^4*tan(d*x)^3*tan(c)^4 + 30*a^2*b^2*tan
```

```
(d*x)^2*tan(c)^5 - 5*b^4*tan(d*x)^2*tan(c)^5 - 15*a*b^3*tan(d*x)^5*tan(c) -
 150*a^4*d*x*tan(d*x)^2*tan(c)^2 + 900*a^2*b^2*d*x*tan(d*x)^2*tan(c)^2 - 15
0*b^4*d*x*tan(d*x)^2*tan(c)^2 + 90*a^3*b*tan(d*x)^4*tan(c)^2 - 150*a*b^3*ta
n(d*x)^4*tan(c)^2 - 120*a^3*b*tan(d*x)^3*tan(c)^3 + 180*a*b^3*tan(d*x)^3*ta
n(c)^3 + 90*a^3*b*tan(d*x)^2*tan(c)^4 - 150*a*b^3*tan(d*x)^2*tan(c)^4 - 15*
a*b^3*tan(d*x)*tan(c)^5 + 3*b^4*tan(d*x)^5 - 60*a^2*b^2*tan(d*x)^4*tan(c) +
 25*b^4*tan(d*x)^4*tan(c) + 300*a^3*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*
x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(ta
n(c)^2 + 1))*tan(d*x)^2*tan(c)^2 - 300*a*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2
*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) +
 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + 90*a^4*tan(d*x)^3*tan(c)^2 - 720*
a^2*b^2*tan(d*x)^3*tan(c)^2 + 150*b^4*tan(d*x)^3*tan(c)^2 + 90*a^4*tan(d*x)
^2*tan(c)^3 - 720*a^2*b^2*tan(d*x)^2*tan(c)^3 + 150*b^4*tan(d*x)^2*tan(c)^3
 - 60*a^2*b^2*tan(d*x)*tan(c)^4 + 25*b^4*tan(d*x)*tan(c)^4 + 3*b^4*tan(c)^5
 + 15*a*b^3*tan(d*x)^4 + 75*a^4*d*x*tan(d*x)*tan(c) - 450*a^2*b^2*d*x*tan(d
*x)*tan(c) + 75*b^4*d*x*tan(d*x)*tan(c) - 90*a^3*b*tan(d*x)^3*tan(c) + 150*
a*b^3*tan(d*x)^3*tan(c) + 120*a^3*b*tan(d*x)^2*tan(c)^2 - 180*a*b^3*tan(d*x
)^2*tan(c)^2 - 90*a^3*b*tan(d*x)*tan(c)^3 + 150*a*b^3*tan(d*x)*tan(c)^3 + 1
5*a*b^3*tan(c)^4 + 30*a^2*b^2*tan(d*x)^3 - 5*b^4*tan(d*x)^3 - 150*a^3*b*log
(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d
*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)*tan(c) + 150*a*b^3*
log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + ta
n(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)*tan(c) - 60*a^4*
tan(d*x)^2*tan(c) + 450*a^2*b^2*tan(d*x)^2*tan(c) - 75*b^4*tan(d*x)^2*tan(c
) - 60*a^4*tan(d*x)*tan(c)^2 + 450*a^2*b^2*tan(d*x)*tan(c)^2 - 75*b^4*tan(d
*x)*tan(c)^2 + 30*a^2*b^2*tan(c)^3 - 5*b^4*tan(c)^3 - 15*a^4*d*x + 90*a^2*b
^2*d*x - 15*b^4*d*x + 30*a^3*b*tan(d*x)^2 - 30*a*b^3*tan(d*x)^2 - 90*a^3*b*
tan(d*x)*tan(c) + 165*a*b^3*tan(d*x)*tan(c) + 30*a^3*b*tan(c)^2 - 30*a*b^3*
tan(c)^2 + 30*a^3*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(
d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) - 30*
a*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^
2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) + 15*a^4*tan(d*x) -
 90*a^2*b^2*tan(d*x) + 15*b^4*tan(d*x) + 15*a^4*tan(c) - 90*a^2*b^2*tan(c)
+ 15*b^4*tan(c) + 30*a^3*b - 45*a*b^3)/(d*tan(d*x)^5*tan(c)^5 - 5*d*tan(d*x
)^4*tan(c)^4 + 10*d*tan(d*x)^3*tan(c)^3 - 10*d*tan(d*x)^2*tan(c)^2 + 5*d*ta
n(d*x)*tan(c) - d)
```

Mupad [B]
time = 3.98, size = 221, normalized size = 1.73

$$\frac{\ln\left(\tan(c+d\,x)^2+1\right)\,\left(2\,a\,b^3-2\,a^3\,b\right)}{d}-\frac{\tan(c+d\,x)^3\,\left(\frac{b^4}{3}-2\,a^2\,b^2\right)}{d}-\frac{\tan(c+d\,x)^2\,\left(2\,a\,b^3-2\,a^3\,b\right)}{d}+\frac{\tan(c+d\,x)\,\left(a^4-6\,a^2\,b^2+b^4\right)}{d}+\frac{b^4\,\tan(c+d\,x)^5}{5\,d}-\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,\left(-a^2+2\,a\,b+b^2\right)\,\left(a^2+2\,a\,b-b^2\right)}{a^4-6\,a^2\,b^2+b^4}\right)\,\left(-a^2+2\,a\,b+b^2\right)\,\left(a^2+2\,a\,b-b^2\right)}{d}+\frac{a\,b^3\,\tan(c+d\,x)^4}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + b*tan(c + d*x))^4,x)
```

```
[Out] (log(tan(c + d*x)^2 + 1)*(2*a*b^3 - 2*a^3*b))/d - (tan(c + d*x)^3*(b^4/3 -
2*a^2*b^2))/d - (tan(c + d*x)^2*(2*a*b^3 - 2*a^3*b))/d + (tan(c + d*x)*(a^4
 + b^4 - 6*a^2*b^2))/d + (b^4*tan(c + d*x)^5)/(5*d) - (atan((tan(c + d*x)*(
2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2))/(a^4 + b^4 - 6*a^2*b^2))*(2*a*b - a
^2 + b^2)*(2*a*b + a^2 - b^2))/d + (a*b^3*tan(c + d*x)^4)/d
```

3.447 $\int \tan(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=130

$$-4ab\left(a^2-b^2\right)x-\frac{\left(a^4-6a^2b^2+b^4\right)\log(\cos(c+dx))}{d}+\frac{ab\left(a^2-3b^2\right)\tan(c+dx)}{d}+\frac{\left(a^2-b^2\right)(a+b\tan(c+dx$$

```
[Out] -4*a*b*(a^2-b^2)*x-(a^4-6*a^2*b^2+b^4)*ln(cos(d*x+c))/d+a*b*(a^2-3*b^2)*tan
(d*x+c)/d+1/2*(a^2-b^2)*(a+b*tan(d*x+c))^2/d+1/3*a*(a+b*tan(d*x+c))^3/d+1/4
*(a+b*tan(d*x+c))^4/d
```

Rubi [A]
time = 0.10, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3609, 3606, 3556}

$$\frac{\left(a^2-b^2\right)(a+b\tan(c+dx))^2}{2d}+\frac{ab\left(a^2-3b^2\right)\tan(c+dx)}{d}-4abx\left(a^2-b^2\right)-\frac{\left(a^4-6a^2b^2+b^4\right)\log(\cos(c+dx))}{d}+\frac{(a+b\tan(c+dx))^4}{4d}+\frac{a(a+b\tan(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -4*a*b*(a^2 - b^2)*x - ((a^4 - 6*a^2*b^2 + b^4)*Log[Cos[c + d*x]])/d + (a*b
*(a^2 - 3*b^2)*Tan[c + d*x])/d + ((a^2 - b^2)*(a + b*Tan[c + d*x])^2)/(2*d)
 + (a*(a + b*Tan[c + d*x])^3)/(3*d) + (a + b*Tan[c + d*x])^4/(4*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\int \tan(c+dx)(a+b\tan(c+dx))^4\,dx = \frac{(a+b\tan(c+dx))^4}{4d} + \int (-b+a\tan(c+dx))(a+b\tan(c+dx))^3\,d$$
$$= \frac{a(a+b\tan(c+dx))^3}{3d} + \frac{(a+b\tan(c+dx))^4}{4d} + \int (a+b\tan(c+dx$$
$$= \frac{(a^2-b^2)(a+b\tan(c+dx))^2}{2d} + \frac{a(a+b\tan(c+dx))^3}{3d} + \frac{(a+b\tan(}{4}$$
$$= -4ab(a^2-b^2)\,x + \frac{ab(a^2-3b^2)\tan(c+dx)}{d} + \frac{(a^2-b^2)(a+b\tan(}{2d}$$
$$= -4ab(a^2-b^2)\,x - \frac{(a^4-6a^2b^2+b^4)\log(\cos(c+dx))}{d} + \frac{ab(a^2-3b^2}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.70, size = 123, normalized size = 0.95

$$\frac{6((a+ib)^4\log(i-\tan(c+dx))+(a-ib)^4\log(i+\tan(c+dx)))+48ab(a^2-b^2)\tan(c+dx)-6b^2(-6a^2+b^2)\tan^2(c+dx)+16ab^3\tan^3(c+dx)+3b^4\tan^4(c+dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (6*((a + I*b)^4*Log[I - Tan[c + d*x]] + (a - I*b)^4*Log[I + Tan[c + d*x]])
+ 48*a*b*(a^2 - b^2)*Tan[c + d*x] - 6*b^2*(-6*a^2 + b^2)*Tan[c + d*x]^2 + 1
6*a*b^3*Tan[c + d*x]^3 + 3*b^4*Tan[c + d*x]^4)/(12*d)
```

Maple [A]
time = 0.04, size = 135, normalized size = 1.04

method	result
norman	$\left(-4a^3b+4a\,b^3\right)x+\frac{b^4\left(\tan^4(dx+c)\right)}{4d}+\frac{4a\,b^3\left(\tan^3(dx+c)\right)}{3d}+\frac{b^2\left(6a^2-b^2\right)\left(\tan^2(dx+c)\right)}{2d}+\frac{4ab\left(a^2-b^2\right)\tan(d}{d}$
derivativedivides	$\frac{\frac{b^4\left(\tan^4(dx+c)\right)}{4}+\frac{4a\,b^3\left(\tan^3(dx+c)\right)}{3}+3a^2b^2\left(\tan^2(dx+c)\right)-\frac{b^4\left(\tan^2(dx+c)\right)}{2}+4a^3b\tan(dx+c)-4a\,b^3\tan(dx+c)+\frac{(a^4-6a^2b^2}{}}{d}$
default	$\frac{\frac{b^4\left(\tan^4(dx+c)\right)}{4}+\frac{4a\,b^3\left(\tan^3(dx+c)\right)}{3}+3a^2b^2\left(\tan^2(dx+c)\right)-\frac{b^4\left(\tan^2(dx+c)\right)}{2}+4a^3b\tan(dx+c)-4a\,b^3\tan(dx+c)+\frac{(a^4-6a^2b^2}{}}{d}$
risch	$-4a^3bx+4a\,b^3x+ia^4x-6ia^2b^2x+ib^4x+\frac{2ia^4c}{d}-\frac{12ia^2b^2c}{d}+\frac{2ib^4c}{d}+\frac{4ib\left(-9ia^2b\,e^{6i(dx+c)}+3ib^3\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*b^4*tan(d*x+c)^4+4/3*a*b^3*tan(d*x+c)^3+3*a^2*b^2*tan(d*x+c)^2-1/2
*b^4*tan(d*x+c)^2+4*a^3*b*tan(d*x+c)-4*a*b^3*tan(d*x+c)+1/2*(a^4-6*a^2*b^2+
b^4)*ln(1+tan(d*x+c)^2)+(-4*a^3*b+4*a*b^3)*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.52, size = 124, normalized size = 0.95

$$\frac{3\,b^{4}\tan\left(dx+c\right)^{4}+16\,ab^{3}\tan\left(dx+c\right)^{3}+6\,\left(6\,a^{2}b^{2}-b^{4}\right)\tan\left(dx+c\right)^{2}-48\,\left(a^{3}b-ab^{3}\right)\left(dx+c\right)+6\,\left(a^{4}-6\,a^{2}b^{2}+b^{4}\right)\log\left(\tan\left(dx+c\right)^{2}+1\right)+48\,\left(a^{3}b-ab^{3}\right)\tan\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/12*(3*b^4*tan(d*x + c)^4 + 16*a*b^3*tan(d*x + c)^3 + 6*(6*a^2*b^2 - b^4)*
tan(d*x + c)^2 - 48*(a^3*b - a*b^3)*(d*x + c) + 6*(a^4 - 6*a^2*b^2 + b^4)*l
og(tan(d*x + c)^2 + 1) + 48*(a^3*b - a*b^3)*tan(d*x + c))/d
```

Fricas [A]
time = 1.31, size = 123, normalized size = 0.95

$$\frac{3\,b^{4}\tan\left(dx+c\right)^{4}+16\,ab^{3}\tan\left(dx+c\right)^{3}-48\,\left(a^{3}b-ab^{3}\right)dx+6\,\left(6\,a^{2}b^{2}-b^{4}\right)\tan\left(dx+c\right)^{2}-6\,\left(a^{4}-6\,a^{2}b^{2}+b^{4}\right)\log\left(\frac{1}{\tan\left(dx+c\right)^{2}+1}\right)+48\,\left(a^{3}b-ab^{3}\right)\tan\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*tan(d*x + c)^4 + 16*a*b^3*tan(d*x + c)^3 - 48*(a^3*b - a*b^3)*d
*x + 6*(6*a^2*b^2 - b^4)*tan(d*x + c)^2 - 6*(a^4 - 6*a^2*b^2 + b^4)*log(1/(
tan(d*x + c)^2 + 1)) + 48*(a^3*b - a*b^3)*tan(d*x + c))/d
```

Sympy [A]
time = 0.16, size = 187, normalized size = 1.44

$$\begin{cases} \frac{a^{4}\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d}-4a^{3}bx+\frac{4a^{3}b\tan\left(c+dx\right)}{d}-\frac{3a^{2}b^{2}\log\left(\tan^{2}\left(c+dx\right)+1\right)}{d}+\frac{3a^{2}b^{2}\tan^{2}\left(c+dx\right)}{d}+4ab^{3}x+\frac{4ab^{3}\tan^{3}\left(c+dx\right)}{3d}-\frac{4ab^{3}\tan\left(c+dx\right)}{d}+\frac{b^{4}\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d}+\frac{b^{4}\tan^{4}\left(c+dx\right)}{4d}-\frac{b^{4}\tan^{2}\left(c+dx\right)}{2d} & \text{for}\: d\neq 0 \\ x\left(a+b\tan\left(c\right)\right)^{4}\tan\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*log(tan(c + d*x)**2 + 1)/(2*d) - 4*a**3*b*x + 4*a**3*b*tan(
c + d*x)/d - 3*a**2*b**2*log(tan(c + d*x)**2 + 1)/d + 3*a**2*b**2*tan(c + d
*x)**2/d + 4*a*b**3*x + 4*a*b**3*tan(c + d*x)**3/(3*d) - 4*a*b**3*tan(c + d
*x)/d + b**4*log(tan(c + d*x)**2 + 1)/(2*d) + b**4*tan(c + d*x)**4/(4*d) -
b**4*tan(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*tan(c))**4*tan(c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1886 vs. $2(124) = 248$.
time = 2.12, size = 1886, normalized size = 14.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="giac")

[Out] -1/12*(48*a^3*b*d*x*tan(d*x)^4*tan(c)^4 - 48*a*b^3*d*x*tan(d*x)^4*tan(c)^4 + 6*a^4*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 - 36*a^2*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 + 6*b^4*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 - 192*a^3*b*d*x*tan(d*x)^3*tan(c)^3 + 192*a*b^3*d*x*tan(d*x)^3*tan(c)^3 - 36*a^2*b^2*tan(d*x)^4*tan(c)^4 + 9*b^4*tan(d*x)^4*tan(c)^4 - 24*a^4*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 + 144*a^2*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 - 24*b^4*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 + 48*a^3*b*tan(d*x)^4*tan(c)^3 - 48*a*b^3*tan(d*x)^4*tan(c)^3 + 48*a^3*b*tan(d*x)^3*tan(c)^4 - 48*a*b^3*tan(d*x)^3*tan(c)^4 + 288*a^3*b*d*x*tan(d*x)^2*tan(c)^2 - 288*a*b^3*d*x*tan(d*x)^2*tan(c)^2 - 36*a^2*b^2*tan(d*x)^4*tan(c)^2 + 6*b^4*tan(d*x)^4*tan(c)^2 + 72*a^2*b^2*tan(d*x)^3*tan(c)^3 - 24*b^4*tan(d*x)^3*tan(c)^3 - 36*a^2*b^2*tan(d*x)^2*tan(c)^4 + 6*b^4*tan(d*x)^2*tan(c)^4 + 16*a*b^3*tan(d*x)^4*tan(c) + 36*a^4*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 - 216*a^2*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + 36*b^4*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 - 144*a^3*b*tan(d*x)^3*tan(c)^2 + 192*a*b^3*tan(d*x)^3*tan(c)^2 - 144*a^3*b*tan(d*x)^2*tan(c)^3 + 192*a*b^3*tan(d*x)^2*tan(c)^3 + 16*a*b^3*tan(d*x)*tan(c)^4 - 3*b^4*tan(d*x)^4 - 192*a^3*b*d*x*tan(d*x)*tan(c) + 192*a*b^3*d*x*tan(d*x)*tan(c) + 72*a^2*b^2*tan(d*x)^3*tan(c) - 24*b^4*tan(d*x)^3*tan(c) - 72*a^2*b^2*tan(d*x)^2*tan(c)^2 + 12*b^4*tan(d*x)^2*tan(c)^2 + 72*a^2*b^2*tan(d*x)*tan(c)^3 - 24*b^4*tan(d*x)*tan(c)^3 - 3*b^4*tan(c)^4 - 16*a*b^3*tan(d*x)^3 - 24*a^4*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)*tan(c) + 144*a^2*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)*tan(c) - 24*b^4*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)*tan(c) + 144*a^3*b*tan(d*x)^2*tan(c) - 192*a*b^3*tan(d*x)^2*tan(c) + 144*a^3*b*tan(d*x)*tan(c)^2 - 192*a*b^3*tan(d*x)*tan(c)^2 - 16*a*b^3*tan(c)^3 + 48*a^3*b*d*x - 48*a*b^3*d*x - 36*a^2*b^2*tan(d*x)^2 + 6*b^4*tan(d*x)^2 + 72*a^2*b^2*tan(d*x)*tan(c) - 24*b^4*tan(d*x)*tan(c) - 36*a^2*b^2*tan(c)^2 + 6*b^4*t

```
an(c)^2 + 6*a^4*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)
^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) - 36*a^2*
b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2
+ tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) + 6*b^4*log(4*(tan(d*
x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*
tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) - 48*a^3*b*tan(d*x) + 48*a*b^3*tan(d*x
) - 48*a^3*b*tan(c) + 48*a*b^3*tan(c) - 36*a^2*b^2 + 9*b^4)/(d*tan(d*x)^4*t
an(c)^4 - 4*d*tan(d*x)^3*tan(c)^3 + 6*d*tan(d*x)^2*tan(c)^2 - 4*d*tan(d*x)*
tan(c) + d)
```

Mupad [B]
time = 3.92, size = 168, normalized size = 1.29

$$\frac{\ln\left(\tan(c+d\,x)^2+1\right)\,\left(\frac{a^4}{2}-3\,a^2\,b^2+\frac{b^4}{2}\right)}{d}-\frac{\tan(c+d\,x)^2\,\left(\frac{b^4}{2}-3\,a^2\,b^2\right)}{d}+\frac{b^4\tan(c+d\,x)^4}{4\,d}-\frac{\tan(c+d\,x)\,\left(4\,a\,b^3-4\,a^3\,b\right)}{d}+\frac{4\,a\,b^3\tan(c+d\,x)^3}{3\,d}+\frac{4\,a\,b\,\mathrm{atan}\left(\frac{4\,a\,b\tan(c+d\,x)\,(a+b)\,(a-b)}{4\,a\,b^3-4\,a^3\,b}\right)\,(a+b)\,(a-b)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + b*tan(c + d*x))^4,x)
```

```
[Out] (log(tan(c + d*x)^2 + 1)*(a^4/2 + b^4/2 - 3*a^2*b^2))/d - (tan(c + d*x)^2*(
b^4/2 - 3*a^2*b^2))/d + (b^4*tan(c + d*x)^4)/(4*d) - (tan(c + d*x)*(4*a*b^3
 - 4*a^3*b))/d + (4*a*b^3*tan(c + d*x)^3)/(3*d) + (4*a*b*atan((4*a*b*tan(c
+ d*x)*(a + b)*(a - b))/(4*a*b^3 - 4*a^3*b))*(a + b)*(a - b))/d
```

3.448 $\int (a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=103

$$(a^4-6a^2b^2+b^4)\,x-\frac{4ab(a^2-b^2)\log(\cos(c+dx))}{d}+\frac{b^2(3a^2-b^2)\tan(c+dx)}{d}+\frac{ab(a+b\tan(c+dx))^2}{d}+\frac{b(a+$$

```
[Out] (a^4-6*a^2*b^2+b^4)*x-4*a*b*(a^2-b^2)*ln(cos(d*x+c))/d+b^2*(3*a^2-b^2)*tan(
d*x+c)/d+a*b*(a+b*tan(d*x+c))^2/d+1/3*b*(a+b*tan(d*x+c))^3/d
```

Rubi [A]
time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3563, 3609, 3606, 3556}

$$\frac{b^2(3a^2-b^2)\tan(c+dx)}{d}-\frac{4ab(a^2-b^2)\log(\cos(c+dx))}{d}+x(a^4-6a^2b^2+b^4)+\frac{b(a+b\tan(c+dx))^3}{3d}+\frac{ab(a+b\tan(c+dx))^2}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (a^4 - 6*a^2*b^2 + b^4)*x - (4*a*b*(a^2 - b^2)*Log[Cos[c + d*x]])/d + (b^2*
(3*a^2 - b^2)*Tan[c + d*x])/d + (a*b*(a + b*Tan[c + d*x])^2)/d + (b*(a + b*
Tan[c + d*x])^3)/(3*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3563

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
```

```
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (a+b\tan(c+dx))^4\,dx &= \frac{b(a+b\tan(c+dx))^3}{3d}+\int (a+b\tan(c+dx))^2\left(a^2-b^2+2ab\tan(c+dx)\right)\,dx\\ &= \frac{ab(a+b\tan(c+dx))^2}{d}+\frac{b(a+b\tan(c+dx))^3}{3d}+\int (a+b\tan(c+dx))\left(a(a^2\right.\\ &= \left(a^4-6a^2b^2+b^4\right)x+\frac{b^2(3a^2-b^2)\tan(c+dx)}{d}+\frac{ab(a+b\tan(c+dx))^2}{d}+\frac{b(a}{}\\ &= \left(a^4-6a^2b^2+b^4\right)x-\frac{4ab(a^2-b^2)\log(\cos(c+dx))}{d}+\frac{b^2(3a^2-b^2)\tan(c+dx)}{d}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.38, size = 105, normalized size = 1.02

$$\frac{-3i(a+ib)^4\log(i-\tan(c+dx))+3i(a-ib)^4\log(i+\tan(c+dx))-6b^2(-6a^2+b^2)\tan(c+dx)+12ab^3\tan^2(c+dx)+2b^4\tan^3(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^4,x]
[Out] ((-3*I)*(a + I*b)^4*Log[I - Tan[c + d*x]] + (3*I)*(a - I*b)^4*Log[I + Tan[c
 + d*x]] - 6*b^2*(-6*a^2 + b^2)*Tan[c + d*x] + 12*a*b^3*Tan[c + d*x]^2 + 2*
b^4*Tan[c + d*x]^3)/(6*d)
```

Maple [A]
time = 0.03, size = 107, normalized size = 1.04

method	result
norman	$\left(a^4-6a^2b^2+b^4\right)x+\frac{b^2(6a^2-b^2)\tan(dx+c)}{d}+\frac{b^4(\tan^3(dx+c))}{3d}+\frac{2a\,b^3(\tan^2(dx+c))}{d}+\frac{2ab(a^2-b^2)\ln(1}{d}$
derivativedivides	$\frac{\frac{b^4(\tan^3(dx+c))}{3}+2a\,b^3(\tan^2(dx+c))+6a^2b^2\tan(dx+c)-b^4\tan(dx+c)+\frac{(4a^3b-4a\,b^3)\ln(1+\tan^2(dx+c))}{2}+(a^4-6a^2b^2+b^4}{d}$
default	$\frac{\frac{b^4(\tan^3(dx+c))}{3}+2a\,b^3(\tan^2(dx+c))+6a^2b^2\tan(dx+c)-b^4\tan(dx+c)+\frac{(4a^3b-4a\,b^3)\ln(1+\tan^2(dx+c))}{2}+(a^4-6a^2b^2+b^4}{d}$
risch	$4ia^3bx-4ia\,b^3x+a^4x-6a^2b^2x+b^4x+\frac{8ia^3bc}{d}-\frac{8ia\,b^3c}{d}-\frac{4ib^2(-9a^2e^{4i(dx+c)}+3b^2e^{4i(dx+c)}+6ia}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

[Out] 1/d*(1/3*b^4*tan(d*x+c)^3+2*a*b^3*tan(d*x+c)^2+6*a^2*b^2*tan(d*x+c)-b^4*tan(d*x+c)+1/2*(4*a^3*b-4*a*b^3)*ln(1+tan(d*x+c)^2)+(a^4-6*a^2*b^2+b^4)*arctan(tan(d*x+c)))

Maxima [A]
time = 0.52, size = 113, normalized size = 1.10

$$a^4x - \frac{6\left(dx+c-\tan\left(dx+c\right)\right)a^2b^2}{d} + \frac{\left(\tan\left(dx+c\right)^3+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)b^4}{3\,d} - \frac{2\,ab^3\left(\frac{1}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)^2-1\right)\right)}{d} + \frac{4\,a^3b\log\left(\sec\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] a^4*x - 6*(d*x + c - tan(d*x + c))*a^2*b^2/d + 1/3*(tan(d*x + c)^3 + 3*d*x
+ 3*c - 3*tan(d*x + c))*b^4/d - 2*a*b^3*(1/(sin(d*x + c)^2 - 1) - log(sin(d
*x + c)^2 - 1))/d + 4*a^3*b*log(sec(d*x + c))/d
```

Fricas [A]
time = 0.73, size = 100, normalized size = 0.97

$$\frac{b^4\tan\left(dx+c\right)^3+6\,ab^3\tan\left(dx+c\right)^2+3\left(a^4-6\,a^2b^2+b^4\right)dx-6\left(a^3b-ab^3\right)\log\left(\frac{1}{\tan(dx+c)^2+1}\right)+3\left(6\,a^2b^2-b^4\right)\tan\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*tan(d*x + c)^3 + 6*a*b^3*tan(d*x + c)^2 + 3*(a^4 - 6*a^2*b^2 + b^4
)*d*x - 6*(a^3*b - a*b^3)*log(1/(tan(d*x + c)^2 + 1)) + 3*(6*a^2*b^2 - b^4)
*tan(d*x + c))/d
```

Sympy [A]
time = 0.12, size = 131, normalized size = 1.27

$$\begin{cases} a^4x + \frac{2a^3b\log\left(\tan^2\left(c+dx\right)+1\right)}{d} - 6a^2b^2x + \frac{6a^2b^2\tan\left(c+dx\right)}{d} - \frac{2ab^3\log\left(\tan^2\left(c+dx\right)+1\right)}{d} + \frac{2ab^3\tan^2\left(c+dx\right)}{d} + b^4x + \frac{b^4\tan^3\left(c+dx\right)}{3d} - \frac{b^4\tan\left(c+dx\right)}{d} & \text{for } d \neq 0 \\ x\left(a+b\tan\left(c\right)\right)^4 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*x + 2*a**3*b*log(tan(c + d*x)**2 + 1)/d - 6*a**2*b**2*x + 6
*a**2*b**2*tan(c + d*x)/d - 2*a*b**3*log(tan(c + d*x)**2 + 1)/d + 2*a*b**3*
tan(c + d*x)**2/d + b**4*x + b**4*tan(c + d*x)**3/(3*d) - b**4*tan(c + d*x)
/d, Ne(d, 0)), (x*(a + b*tan(c))**4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1071 vs. 2(101) = 202.
time = 0.94, size = 1071, normalized size = 10.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^4*d*x*tan(d*x)^3*tan(c)^3 - 18*a^2*b^2*d*x*tan(d*x)^3*tan(c)^3 + 3
*b^4*d*x*tan(d*x)^3*tan(c)^3 - 6*a^3*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d
*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(t
an(c)^2 + 1))*tan(d*x)^3*tan(c)^3 + 6*a*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*
tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) +
1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 - 9*a^4*d*x*tan(d*x)^2*tan(c)^2 + 54
*a^2*b^2*d*x*tan(d*x)^2*tan(c)^2 - 9*b^4*d*x*tan(d*x)^2*tan(c)^2 + 6*a*b^3*
tan(d*x)^3*tan(c)^3 + 18*a^3*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*ta
n(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2
+ 1))*tan(d*x)^2*tan(c)^2 - 18*a*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x
)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan
(c)^2 + 1))*tan(d*x)^2*tan(c)^2 - 18*a^2*b^2*tan(d*x)^3*tan(c)^2 + 3*b^4*ta
n(d*x)^3*tan(c)^2 - 18*a^2*b^2*tan(d*x)^2*tan(c)^3 + 3*b^4*tan(d*x)^2*tan(c
)^3 + 9*a^4*d*x*tan(d*x)*tan(c) - 54*a^2*b^2*d*x*tan(d*x)*tan(c) + 9*b^4*d*
x*tan(d*x)*tan(c) + 6*a*b^3*tan(d*x)^3*tan(c) - 6*a*b^3*tan(d*x)^2*tan(c)^2
 + 6*a*b^3*tan(d*x)*tan(c)^3 - b^4*tan(d*x)^3 - 18*a^3*b*log(4*(tan(d*x)^4*
tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d
*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)*tan(c) + 18*a*b^3*log(4*(tan(d*x)^
4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan
(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)*tan(c) + 36*a^2*b^2*tan(d*x)^2*t
an(c) - 9*b^4*tan(d*x)^2*tan(c) + 36*a^2*b^2*tan(d*x)*tan(c)^2 - 9*b^4*tan(
d*x)*tan(c)^2 - b^4*tan(c)^3 - 3*a^4*d*x + 18*a^2*b^2*d*x - 3*b^4*d*x - 6*a
*b^3*tan(d*x)^2 + 6*a*b^3*tan(d*x)*tan(c) - 6*a*b^3*tan(c)^2 + 6*a^3*b*log(
4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*
x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) - 6*a*b^3*log(4*(tan(d*x)^4*t
an(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*
x)*tan(c) + 1)/(tan(c)^2 + 1)) - 18*a^2*b^2*tan(d*x) + 3*b^4*tan(d*x) - 18*
a^2*b^2*tan(c) + 3*b^4*tan(c) - 6*a*b^3)/(d*tan(d*x)^3*tan(c)^3 - 3*d*tan(d
*x)^2*tan(c)^2 + 3*d*tan(d*x)*tan(c) - d)
```

Mupad [B]
time = 3.89, size = 167, normalized size = 1.62

$$\frac{b^4\tan(c+d\,x)^3}{3\,d}-\frac{\tan(c+d\,x)\,\left(b^4-6\,a^2\,b^2\right)}{d}-\frac{\ln\left(\tan(c+d\,x)^2+1\right)\,\left(2\,a\,b^3-2\,a^3\,b\right)}{d}+\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,\left(-a^2+2\,a\,b+b^2\right)\,\left(a^2+2\,a\,b-b^2\right)}{a^4-6\,a^2\,b^2+b^4}\right)\,\left(-a^2+2\,a\,b+b^2\right)\,\left(a^2+2\,a\,b-b^2\right)}{d}+\frac{2\,a\,b^3\tan(c+d\,x)^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^4,x)
```

```
[Out] (b^4*tan(c + d*x)^3)/(3*d) - (tan(c + d*x)*(b^4 - 6*a^2*b^2))/d - (log(tan(
c + d*x)^2 + 1)*(2*a*b^3 - 2*a^3*b))/d + (atan((tan(c + d*x)*(2*a*b - a^2 +
 b^2)*(2*a*b + a^2 - b^2))/(a^4 + b^4 - 6*a^2*b^2))*(2*a*b - a^2 + b^2)*(2*
a*b + a^2 - b^2))/d + (2*a*b^3*tan(c + d*x)^2)/d
```

3.449 $\int \cot(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=92

$$4ab\left(a^2-b^2\right)x-\frac{b^2\left(6a^2-b^2\right)\log(\cos(c+dx))}{d}+\frac{a^4\log(\sin(c+dx))}{d}+\frac{3ab^3\tan(c+dx)}{d}+\frac{b^2(a+b\tan(c+dx))^2}{2d}$$

```
[Out] 4*a*b*(a^2-b^2)*x-b^2*(6*a^2-b^2)*ln(cos(d*x+c))/d+a^4*ln(sin(d*x+c))/d+3*a
*b^3*tan(d*x+c)/d+1/2*b^2*(a+b*tan(d*x+c))^2/d
```

Rubi [A]
time = 0.13, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3647, 3718, 3705, 3556}

$$\frac{a^4\log(\sin(c+dx))}{d}-\frac{b^2\left(6a^2-b^2\right)\log(\cos(c+dx))}{d}+4abx\left(a^2-b^2\right)+\frac{3ab^3\tan(c+dx)}{d}+\frac{b^2(a+b\tan(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] 4*a*b*(a^2 - b^2)*x - (b^2*(6*a^2 - b^2)*Log[Cos[c + d*x]])/d + (a^4*Log[Si
n[c + d*x]])/d + (3*a*b^3*Tan[c + d*x])/d + (b^2*(a + b*Tan[c + d*x])^2)/(2
*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3705

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[B*x, x] + (Dist[A, Int[1/Tan[e
 + f*x], x], x] + Dist[C, Int[Tan[e + f*x], x], x]) /; FreeQ[{e, f, A, B, C
```

```
}, x] && NeQ[A, C]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rubi steps

$$\int \cot(c+dx)(a+b\tan(c+dx))^4\,dx = \frac{b^2(a+b\tan(c+dx))^2}{2d} + \frac{1}{2}\int \cot(c+dx)(a+b\tan(c+dx))\left(2a^3\right.$$

$$= \frac{3ab^3\tan(c+dx)}{d} + \frac{b^2(a+b\tan(c+dx))^2}{2d} - \frac{1}{2}\int \cot(c+dx)\left(-2a\right.$$

$$= 4ab\left(a^2-b^2\right)x + \frac{3ab^3\tan(c+dx)}{d} + \frac{b^2(a+b\tan(c+dx))^2}{2d} + a^4\int$$

$$= 4ab\left(a^2-b^2\right)x - \frac{b^2(6a^2-b^2)\log(\cos(c+dx))}{d} + \frac{a^4\log(\sin(c+dx}{d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.42, size = 94, normalized size = 1.02

$$\frac{-(a+ib)^4\log(i-\tan(c+dx)) + 2a^4\log(\tan(c+dx)) - (a-ib)^4\log(i+\tan(c+dx)) + 6ab^3\tan(c+dx) + b^2(a+b\tan(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (-((a + I*b)^4*Log[I - Tan[c + d*x]]) + 2*a^4*Log[Tan[c + d*x]] - (a - I*b)
^4*Log[I + Tan[c + d*x]] + 6*a*b^3*Tan[c + d*x] + b^2*(a + b*Tan[c + d*x])^
2)/(2*d)
```

Maple [A]
time = 0.20, size = 85, normalized size = 0.92

method	result

derivativedivides	$\frac{a^4\ln(\sin(dx+c))+4a^3b(dx+c)-6a^2b^2\ln(\cos(dx+c))+4a\,b^3(\tan(dx+c)-dx-c)+b^4\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)}{d}$
default	$\frac{a^4\ln(\sin(dx+c))+4a^3b(dx+c)-6a^2b^2\ln(\cos(dx+c))+4a\,b^3(\tan(dx+c)-dx-c)+b^4\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)}{d}$
norman	$(4a^3b-4a\,b^3)\,x+\frac{b^4\left(\tan^2(dx+c)\right)}{2d}+\frac{4a\,b^3\tan(dx+c)}{d}+\frac{a^4\ln(\tan(dx+c))}{d}-\frac{(a^4-6a^2b^2+b^4)\ln(1+\tan^2(dx+$
risch	$4a^3bx-4a\,b^3x-ia^4x+6ia^2b^2x-ib^4x+\frac{12ia^2b^2c}{d}-\frac{2ib^4c}{d}-\frac{2ia^4c}{d}+\frac{2ib^3(-ib\,e^{2i(dx+c)}+4a\,e^{2i(dx}}{d(e^{2i(dx+c)}+1)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*ln(sin(d*x+c))+4*a^3*b*(d*x+c)-6*a^2*b^2*ln(cos(d*x+c))+4*a*b^3*(t
an(d*x+c)-d*x-c)+b^4*(1/2*tan(d*x+c)^2+ln(cos(d*x+c))))
```

Maxima [A]

time = 0.52, size = 89, normalized size = 0.97

$$\frac{b^4\tan(dx+c)^2+2\,a^4\log(\tan(dx+c))+8\,ab^3\tan(dx+c)+8\,(a^3b-ab^3)(dx+c)-(a^4-6\,a^2b^2+b^4)\log\left(\tan(dx+c)^2+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/2*(b^4*tan(d*x + c)^2 + 2*a^4*log(tan(d*x + c)) + 8*a*b^3*tan(d*x + c) +
8*(a^3*b - a*b^3)*(d*x + c) - (a^4 - 6*a^2*b^2 + b^4)*log(tan(d*x + c)^2 +
1))/d
```

Fricas [A]

time = 1.01, size = 101, normalized size = 1.10

$$\frac{b^4\tan(dx+c)^2+a^4\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)+8\,ab^3\tan(dx+c)+8\,(a^3b-ab^3)dx-(6\,a^2b^2-b^4)\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*tan(d*x + c)^2 + a^4*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1)) + 8*
a*b^3*tan(d*x + c) + 8*(a^3*b - a*b^3)*d*x - (6*a^2*b^2 - b^4)*log(1/(tan(d
*x + c)^2 + 1)))/d
```

Sympy [A]

time = 0.53, size = 133, normalized size = 1.45

$$\begin{cases} -\frac{a^4\log(\tan^2(c+dx)+1)}{2d}+\frac{a^4\log(\tan(c+dx))}{d}+4a^3bx+\frac{3a^2b^2\log(\tan^2(c+dx)+1)}{d}-4ab^3x+\frac{4ab^3\tan(c+dx)}{d}-\frac{b^4\log(\tan^2(c+dx)+1)}{2d}+\frac{b^4\tan^2(c+dx)}{2d} & \text{for } d\neq 0 \\ x(a+b\tan(c))^4\cot(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Piecewise((-a**4*log(tan(c + d*x)**2 + 1)/(2*d) + a**4*log(tan(c + d*x))/d
+ 4*a**3*b*x + 3*a**2*b**2*log(tan(c + d*x)**2 + 1)/d - 4*a*b**3*x + 4*a*b*
*3*tan(c + d*x)/d - b**4*log(tan(c + d*x)**2 + 1)/(2*d) + b**4*tan(c + d*x)
**2/(2*d), Ne(d, 0)), (x*(a + b*tan(c))**4*cot(c), True))
```

Giac [A]
time = 1.43, size = 90, normalized size = 0.98

$$\frac{b^4\tan\left(dx+c\right)^2+2\,a^4\log\left(\left|\tan\left(dx+c\right)\right|\right)+8\,ab^3\tan\left(dx+c\right)+8\left(a^3b-ab^3\right)\left(dx+c\right)-\left(a^4-6\,a^2b^2+b^4\right)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/2*(b^4*tan(d*x + c)^2 + 2*a^4*log(abs(tan(d*x + c))) + 8*a*b^3*tan(d*x +
c) + 8*(a^3*b - a*b^3)*(d*x + c) - (a^4 - 6*a^2*b^2 + b^4)*log(tan(d*x + c)
^2 + 1))/d
```

Mupad [B]
time = 3.97, size = 92, normalized size = 1.00

$$\frac{b^4\tan(c+d\,x)^2}{2\,d}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(b+a\,1\mathrm{i}\right)^4}{2\,d}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(a+b\,1\mathrm{i}\right)^4}{2\,d}+\frac{a^4\,\ln\left(\tan(c+d\,x)\right)}{d}+\frac{4\,a\,b^3\tan(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + b*tan(c + d*x))^4,x)
```

```
[Out] (b^4*tan(c + d*x)^2)/(2*d) - (log(tan(c + d*x) + 1i)*(a*1i + b)^4)/(2*d) -
(log(tan(c + d*x) - 1i)*(a + b*1i)^4)/(2*d) + (a^4*log(tan(c + d*x)))/d + (
4*a*b^3*tan(c + d*x))/d
```

3.450 $\int \cot^2(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=97

$$-\left(\left(a^4-6a^2b^2+b^4\right)x\right)-\frac{4ab^3\log(\cos(c+dx))}{d}+\frac{4a^3b\log(\sin(c+dx))}{d}+\frac{b^2(a^2+b^2)\tan(c+dx)}{d}-\frac{a^2\cot(c+a}{}$$

```
[Out] -(a^4-6*a^2*b^2+b^4)*x-4*a*b^3*ln(cos(d*x+c))/d+4*a^3*b*ln(sin(d*x+c))/d+b^
2*(a^2+b^2)*tan(d*x+c)/d-a^2*cot(d*x+c)*(a+b*tan(d*x+c))^2/d
```

Rubi [A]
time = 0.13, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3646, 3718, 3705, 3556}

$$\frac{4a^3b\log(\sin(c+dx))}{d}+\frac{b^2(a^2+b^2)\tan(c+dx)}{d}-\frac{a^2\cot(c+dx)(a+b\tan(c+dx))^2}{d}-x\left(a^4-6a^2b^2+b^4\right)-\frac{4ab^3\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -((a^4 - 6*a^2*b^2 + b^4)*x) - (4*a*b^3*Log[Cos[c + d*x]])/d + (4*a^3*b*Log
[Sin[c + d*x]])/d + (b^2*(a^2 + b^2)*Tan[c + d*x])/d - (a^2*Cot[c + d*x]*(a
 + b*Tan[c + d*x])^2)/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3705

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[B*x, x] + (Dist[A, Int[1/Tan[e
 + f*x], x], x] + Dist[C, Int[Tan[e + f*x], x], x]) /; FreeQ[{e, f, A, B, C
```

```
}, x] && NeQ[A, C]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rubi steps

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^4\,dx = -\frac{a^2\cot(c+dx)(a+b\tan(c+dx))^2}{d} + \int \cot(c+dx)(a+b\tan(c$$
$$= \frac{b^2(a^2+b^2)\tan(c+dx)}{d} - \frac{a^2\cot(c+dx)(a+b\tan(c+dx))^2}{d} - \int$$
$$= -\left(a^4-6a^2b^2+b^4\right)x + \frac{b^2(a^2+b^2)\tan(c+dx)}{d} - \frac{a^2\cot(c+dx)($$
$$= -\left(a^4-6a^2b^2+b^4\right)x - \frac{4ab^3\log(\cos(c+dx))}{d} + \frac{4a^3b\log(\sin(c+$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.28, size = 94, normalized size = 0.97

$$-\frac{a^4\cot(c+dx)+\frac{1}{2}i(a-ib)^4\log(i-\cot(c+dx))-\frac{1}{2}i(a+ib)^4\log(i+\cot(c+dx))-4ab^3\log(\tan(c+dx))-b^4\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -((a^4*Cot[c + d*x] + (I/2)*(a - I*b)^4*Log[I - Cot[c + d*x]] - (I/2)*(a +
I*b)^4*Log[I + Cot[c + d*x]] - 4*a*b^3*Log[Tan[c + d*x]] - b^4*Tan[c + d*x]
)/d)
```

Maple [A]
time = 0.17, size = 83, normalized size = 0.86

method	result

derivativedivides	$\frac{a^4(-\cot(dx+c)-dx-c)+4a^3b\ln(\sin(dx+c))+6a^2b^2(dx+c)-4a\,b^3\ln(\cos(dx+c))+b^4(\tan(dx+c)-dx-c)}{d}$
default	$\frac{a^4(-\cot(dx+c)-dx-c)+4a^3b\ln(\sin(dx+c))+6a^2b^2(dx+c)-4a\,b^3\ln(\cos(dx+c))+b^4(\tan(dx+c)-dx-c)}{d}$
norman	$\frac{\frac{b^4\left(\tan^2(dx+c)\right)}{d}+(-a^4+6a^2b^2-b^4)x\tan(dx+c)-\frac{a^4}{d}}{\tan(dx+c)}+\frac{4a^3b\ln(\tan(dx+c))}{d}-\frac{2ab(a^2-b^2)\ln(1+\tan^2(dx+c))}{d}$
risch	$-4ia^3bx+4ia\,b^3x-a^4x+6a^2b^2x-b^4x+\frac{8ia\,b^3c}{d}-\frac{8ia^3bc}{d}-\frac{2i(a^4e^{2i(dx+c)}-b^4e^{2i(dx+c)}+a^4+b^4)}{d(e^{2i(dx+c)}-1)(e^{2i(dx+c)}+1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(-cot(d*x+c)-d*x-c)+4*a^3*b*ln(sin(d*x+c))+6*a^2*b^2*(d*x+c)-4*a*b
^3*ln(cos(d*x+c))+b^4*(tan(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.53, size = 88, normalized size = 0.91

$$\frac{4\,a^3b\log\left(\tan\left(dx+c\right)\right)+b^4\tan\left(dx+c\right)-\frac{a^4}{\tan(dx+c)}-\left(a^4-6\,a^2b^2+b^4\right)(dx+c)-2\left(a^3b-ab^3\right)\log\left(\tan\left(dx+c\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] (4*a^3*b*log(tan(d*x + c)) + b^4*tan(d*x + c) - a^4/tan(d*x + c) - (a^4 - 6
*a^2*b^2 + b^4)*(d*x + c) - 2*(a^3*b - a*b^3)*log(tan(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 1.41, size = 114, normalized size = 1.18

$$\frac{2\,a^3b\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)-2\,ab^3\log\left(\frac{1}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)+b^4\tan\left(dx+c\right)^2-a^4-\left(a^4-6\,a^2b^2+b^4\right)dx\tan\left(dx+c\right)}{d\tan\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] (2*a^3*b*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c) - 2*a*b^3*lo
g(1/(tan(d*x + c)^2 + 1))*tan(d*x + c) + b^4*tan(d*x + c)^2 - a^4 - (a^4 -
6*a^2*b^2 + b^4)*d*x*tan(d*x + c))/(d*tan(d*x + c))
```

Sympy [A]
time = 0.97, size = 133, normalized size = 1.37

$$\begin{cases} \tilde{\infty}a^4x & \text{for}\: (c=0 \vee c=-dx) \wedge (c=-dx \vee d=0) \\ x(a+b\tan(c))^4\cot^2(c) & \text{for}\: d=0 \\ -a^4x-\frac{a^4}{d\tan(c+dx)}-\frac{2a^3b\log(\tan^2(c+dx)+1)}{d}+\frac{4a^3b\log(\tan(c+dx))}{d}+6a^2b^2x+\frac{2ab^3\log(\tan^2(c+dx)+1)}{d}-b^4x+\frac{b^4\tan(c+dx)}{d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Piecewise((zoo*a**4*x, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x)))
, (x*(a + b*tan(c))**4*cot(c)**2, Eq(d, 0)), (-a**4*x - a**4/(d*tan(c + d*x
)) - 2*a**3*b*log(tan(c + d*x)**2 + 1)/d + 4*a**3*b*log(tan(c + d*x))/d + 6
*a**2*b**2*x + 2*a*b**3*log(tan(c + d*x)**2 + 1)/d - b**4*x + b**4*tan(c +
d*x)/d, True))
```

Giac [A]
time = 1.77, size = 102, normalized size = 1.05

$$\frac{4\,a^{3}b\log\left(\left|\tan\left(dx+c\right)\right|\right)+b^{4}\tan\left(dx+c\right)-\left(a^{4}-6\,a^{2}b^{2}+b^{4}\right)\left(dx+c\right)-2\left(a^{3}b-ab^{3}\right)\log\left(\tan\left(dx+c\right)^{2}+1\right)-\frac{4\,a^{3}b\tan\left(dx+c\right)+a^{4}}{\tan\left(dx+c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] (4*a^3*b*log(abs(tan(d*x + c))) + b^4*tan(d*x + c) - (a^4 - 6*a^2*b^2 + b^4
)*(d*x + c) - 2*(a^3*b - a*b^3)*log(tan(d*x + c)^2 + 1) - (4*a^3*b*tan(d*x
+ c) + a^4)/tan(d*x + c))/d
```

Mupad [B]
time = 3.96, size = 94, normalized size = 0.97

$$\frac{b^{4}\tan(c+d\,x)}{d}-\frac{a^{4}\cot(c+d\,x)}{d}+\frac{4\,a^{3}\,b\,\ln\left(\tan(c+d\,x)\right)}{d}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(a-b\,1\mathrm{i}\right)^{4}\,1\mathrm{i}}{2\,d}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-b+a\,1\mathrm{i}\right)^{4}\,1\mathrm{i}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + b*tan(c + d*x))^4,x)
```

```
[Out] (b^4*tan(c + d*x))/d - (a^4*cot(c + d*x))/d - (log(tan(c + d*x) + 1i)*(a -
b*1i)^4*1i)/(2*d) + (log(tan(c + d*x) - 1i)*(a*1i - b)^4*1i)/(2*d) + (4*a^3
*b*log(tan(c + d*x)))/d
```

3.451 $\int \cot^3(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=99

$$-4ab\left(a^2-b^2\right)x-\frac{3a^3b\cot(c+dx)}{d}-\frac{b^4\log(\cos(c+dx))}{d}-\frac{a^2(a^2-6b^2)\log(\sin(c+dx))}{d}-\frac{a^2\cot^2(c+dx)(a+}{2d}$$

```
[Out] -4*a*b*(a^2-b^2)*x-3*a^3*b*cot(d*x+c)/d-b^4*ln(cos(d*x+c))/d-a^2*(a^2-6*b^2
)*ln(sin(d*x+c))/d-1/2*a^2*cot(d*x+c)^2*(a+b*tan(d*x+c))^2/d
```

Rubi [A]
time = 0.14, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3646, 3716, 3705, 3556}

$$-\frac{3a^3b\cot(c+dx)}{d}-\frac{a^2(a^2-6b^2)\log(\sin(c+dx))}{d}-4abx\left(a^2-b^2\right)-\frac{a^2\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}-\frac{b^4\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -4*a*b*(a^2 - b^2)*x - (3*a^3*b*Cot[c + d*x])/d - (b^4*Log[Cos[c + d*x]])/d
 - (a^2*(a^2 - 6*b^2)*Log[Sin[c + d*x]])/d - (a^2*Cot[c + d*x]^2*(a + b*Tan
[c + d*x])^2)/(2*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3705

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[B*x, x] + (Dist[A, Int[1/Tan[e
 + f*x], x], x] + Dist[C, Int[Tan[e + f*x], x], x]) /; FreeQ[{e, f, A, B, C
```

```
}, x] && NeQ[A, C]
```

Rule 3716

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
 -1]
```

Rubi steps

$$\begin{aligned}\int \cot^3(c+dx)(a+b\tan(c+dx))^4\,dx &= -\frac{a^2\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}+\frac{1}{2}\int \cot^2(c+dx)(a+b\tan\\ &= -\frac{3a^3b\cot(c+dx)}{d}-\frac{a^2\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}+\frac{1}{2}\int \mathrm{c}\\ &= -4ab\left(a^2-b^2\right)x-\frac{3a^3b\cot(c+dx)}{d}-\frac{a^2\cot^2(c+dx)(a+b\tan(c\\ &= -4ab\left(a^2-b^2\right)x-\frac{3a^3b\cot(c+dx)}{d}-\frac{b^4\log(\cos(c+dx))}{d}-\frac{a^2(a\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.32, size = 90, normalized size = 0.91

$$-\frac{8a^3b\cot(c+dx)+a^4\cot^2(c+dx)-(a-ib)^4\log(i-\cot(c+dx))-(a+ib)^4\log(i+\cot(c+dx))-2b^4\log(\tan(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -1/2*(8*a^3*b*Cot[c + d*x] + a^4*Cot[c + d*x]^2 - (a - I*b)^4*Log[I - Cot[c
 + d*x]] - (a + I*b)^4*Log[I + Cot[c + d*x]] - 2*b^4*Log[Tan[c + d*x]])/d
```

Maple [A]
time = 0.21, size = 90, normalized size = 0.91

method	result

derivativedivides	$\frac{a^4\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+4a^3b(-\cot(dx+c)-dx-c)+6a^2b^2\ln(\sin(dx+c))+4a\,b^3(dx+c)-b^4\ln(\cos(dx+c))}{d}$
default	$\frac{a^4\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+4a^3b(-\cot(dx+c)-dx-c)+6a^2b^2\ln(\sin(dx+c))+4a\,b^3(dx+c)-b^4\ln(\cos(dx+c))}{d}$
norman	$\frac{(-4a^3b+4a\,b^3)x\left(\tan^2(dx+c)\right)-\frac{a^4}{2d}-\frac{4a^3b\tan(dx+c)}{d}}{\tan(dx+c)^2}+\frac{(a^4-6a^2b^2+b^4)\ln\left(1+\tan^2(dx+c)\right)}{2d}-\frac{a^2(a^2-6b^2)\ln(\tan(dx+}{d}$
risch	$-4a^3bx+4a\,b^3x+ia^4x-6ia^2b^2x+ib^4x+\frac{2ia^4c}{d}-\frac{12ia^2b^2c}{d}+\frac{2ib^4c}{d}+\frac{2a^3\left(a\,e^{2i(dx+c)}-4ib\,e^{2i(dx}\right.}{d\left(e^{2i(dx+c)}-1\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+4*a^3*b*(-cot(d*x+c)-d*x-c)+6*a
^2*b^2*ln(sin(d*x+c))+4*a*b^3*(d*x+c)-b^4*ln(cos(d*x+c)))
```

Maxima [A]

time = 0.51, size = 99, normalized size = 1.00

$$-\frac{8\,(a^3b-ab^3)(dx+c)-(a^4-6\,a^2b^2+b^4)\log\left(\tan\left(dx+c\right)^2+1\right)+2\,(a^4-6\,a^2b^2)\log\left(\tan\left(dx+c\right)\right)+\frac{8\,a^3b\tan(dx+c)+a^4}{\tan(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/2*(8*(a^3*b - a*b^3)*(d*x + c) - (a^4 - 6*a^2*b^2 + b^4)*log(tan(d*x + c
)^2 + 1) + 2*(a^4 - 6*a^2*b^2)*log(tan(d*x + c)) + (8*a^3*b*tan(d*x + c) +
a^4)/tan(d*x + c)^2)/d
```

Fricas [A]

time = 1.60, size = 126, normalized size = 1.27

$$-\frac{b^4\log\left(\frac{1}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^2+8\,a^3b\tan\left(dx+c\right)+a^4+(a^4-6\,a^2b^2)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^2+(a^4+8\,(a^3b-ab^3)dx)\tan\left(dx+c\right)^2}{2\,d\tan\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/2*(b^4*log(1/(tan(d*x + c)^2 + 1))*tan(d*x + c)^2 + 8*a^3*b*tan(d*x + c)
 + a^4 + (a^4 - 6*a^2*b^2)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x
 + c)^2 + (a^4 + 8*(a^3*b - a*b^3)*d*x)*tan(d*x + c)^2)/(d*tan(d*x + c)^2)
```

Sympy [A]

time = 1.36, size = 170, normalized size = 1.72

$$\begin{cases} \tilde{\infty}a^4x & \text{for}\: (c=0\vee c=-dx)\wedge(c=-dx\vee d=0) \\ x(a+b\tan(c))^4\cot^3(c) & \text{for}\: d=0 \\ \frac{a^4\log(\tan^2(c+dx)+1)}{2d}-\frac{a^4\log(\tan(c+dx))}{d}-\frac{a^4}{2d\tan^2(c+dx)}-4a^3bx-\frac{4a^3b}{d\tan(c+dx)}-\frac{3a^2b^2\log(\tan^2(c+dx)+1)}{d}+\frac{6a^2b^2\log(\tan(c+dx))}{d}+4ab^3x+\frac{b^4\log(\tan^2(c+dx)+1)}{2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Piecewise((zoo*a**4*x, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x)))
, (x*(a + b*tan(c))**4*cot(c)**3, Eq(d, 0)), (a**4*log(tan(c + d*x)**2 + 1)
/(2*d) - a**4*log(tan(c + d*x))/d - a**4/(2*d*tan(c + d*x)**2) - 4*a**3*b*x
 - 4*a**3*b/(d*tan(c + d*x)) - 3*a**2*b**2*log(tan(c + d*x)**2 + 1)/d + 6*a
**2*b**2*log(tan(c + d*x))/d + 4*a*b**3*x + b**4*log(tan(c + d*x)**2 + 1)/(
2*d), True))
```

Giac [A]
time = 1.85, size = 132, normalized size = 1.33

$$-\frac{8\left(a^{3}b-ab^{3}\right)(dx+c)-\left(a^{4}-6\,a^{2}b^{2}+b^{4}\right)\log\left(\tan\left(dx+c\right)^{2}+1\right)+2\left(a^{4}-6\,a^{2}b^{2}\right)\log\left(\left|\tan\left(dx+c\right)\right|\right)-\frac{3\,a^{4}\tan(dx+c)^{2}-18\,a^{2}b^{2}\tan(dx+c)^{2}-8\,a^{3}b\tan(dx+c)-a^{4}}{\tan(dx+c)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/2*(8*(a^3*b - a*b^3)*(d*x + c) - (a^4 - 6*a^2*b^2 + b^4)*log(tan(d*x + c
)^2 + 1) + 2*(a^4 - 6*a^2*b^2)*log(abs(tan(d*x + c))) - (3*a^4*tan(d*x + c)
^2 - 18*a^2*b^2*tan(d*x + c)^2 - 8*a^3*b*tan(d*x + c) - a^4)/tan(d*x + c)^2
)/d
```

Mupad [B]
time = 4.01, size = 102, normalized size = 1.03

$$-\frac{\ln\left(\tan(c+d\,x)\right)\,\left(a^4-6\,a^2\,b^2\right)}{d}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,{\left(a+b\,1\mathrm{i}\right)}^4}{2\,d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,{\left(b+a\,1\mathrm{i}\right)}^4}{2\,d}-\frac{{\mathrm{cot}(c+d\,x)}^2\,\left(\frac{a^4}{2}+4\,b\tan(c+d\,x)\,a^3\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + b*tan(c + d*x))^4,x)
```

```
[Out] (log(tan(c + d*x) - 1i)*(a + b*1i)^4)/(2*d) - (log(tan(c + d*x))*(a^4 - 6*a
^2*b^2))/d + (log(tan(c + d*x) + 1i)*(a*1i + b)^4)/(2*d) - (cot(c + d*x)^2*
(a^4/2 + 4*a^3*b*tan(c + d*x)))/d
```

3.452 $\int \cot^4(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=117

$$\left(a^4-6a^2b^2+b^4\right)x+\frac{a^2(3a^2-17b^2)\cot(c+dx)}{3d}-\frac{4a^3b\cot^2(c+dx)}{3d}-\frac{4ab(a^2-b^2)\log(\sin(c+dx))}{d}-\frac{a^2\cot^3(c+dx)(a+b\tan(c+dx))^2}{3d}$$

```
[Out] (a^4-6*a^2*b^2+b^4)*x+1/3*a^2*(3*a^2-17*b^2)*cot(d*x+c)/d-4/3*a^3*b*cot(d*x
+c)^2/d-4*a*b*(a^2-b^2)*ln(sin(d*x+c))/d-1/3*a^2*cot(d*x+c)^3*(a+b*tan(d*x+
c))^2/d
```

Rubi [A]
time = 0.18, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3646, 3716, 3709, 3612, 3556}

$$-\frac{4a^3b\cot^2(c+dx)}{3d}+\frac{a^2(3a^2-17b^2)\cot(c+dx)}{3d}-\frac{4ab(a^2-b^2)\log(\sin(c+dx))}{d}-\frac{a^2\cot^3(c+dx)(a+b\tan(c+dx))^2}{3d}+x\left(a^4-6a^2b^2+b^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (a^4 - 6*a^2*b^2 + b^4)*x + (a^2*(3*a^2 - 17*b^2)*Cot[c + d*x])/(3*d) - (4*
a^3*b*Cot[c + d*x]^2)/(3*d) - (4*a*b*(a^2 - b^2)*Log[Sin[c + d*x]])/d - (a^
2*Cot[c + d*x]^3*(a + b*Tan[c + d*x])^2)/(3*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
```

```
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3716

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
 -1]
```

Rubi steps

$$\begin{aligned}
\int \cot^4(c+dx)(a+b\tan(c+dx))^4\,dx &= -\frac{a^2\cot^3(c+dx)(a+b\tan(c+dx))^2}{3d}+\frac{1}{3}\int \cot^3(c+dx)(a+b\,\text{ta}\\
&= -\frac{4a^3b\cot^2(c+dx)}{3d}-\frac{a^2\cot^3(c+dx)(a+b\tan(c+dx))^2}{3d}+\frac{1}{3}\int\\
&= \frac{a^2(3a^2-17b^2)\cot(c+dx)}{3d}-\frac{4a^3b\cot^2(c+dx)}{3d}-\frac{a^2\cot^3(c+dx)}{}\\
&= \left(a^4-6a^2b^2+b^4\right)x+\frac{a^2(3a^2-17b^2)\cot(c+dx)}{3d}-\frac{4a^3b\cot^2(c+}{3d}\\
&= \left(a^4-6a^2b^2+b^4\right)x+\frac{a^2(3a^2-17b^2)\cot(c+dx)}{3d}-\frac{4a^3b\cot^2(c+}{3d}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.28, size = 125, normalized size = 1.07

$$-\frac{-6a^2(a^2-6b^2)\cot(c+dx)+12a^3b\cot^2(c+dx)+2a^4\cot^3(c+dx)+3i(a+ib)^4\log(i-\tan(c+dx))+24ab(a^2-b^2)\log(\tan(c+dx))-3i(a-ib)^4\log(i+\tan(c+dx))}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -1/6*(-6*a^2*(a^2 - 6*b^2)*Cot[c + d*x] + 12*a^3*b*Cot[c + d*x]^2 + 2*a^4*C
ot[c + d*x]^3 + (3*I)*(a + I*b)^4*Log[I - Tan[c + d*x]] + 24*a*b*(a^2 - b^2
)*Log[Tan[c + d*x]] - (3*I)*(a - I*b)^4*Log[I + Tan[c + d*x]])/d
```

Maple [A]
time = 0.19, size = 103, normalized size = 0.88

method	result
derivativedivides	$\frac{a^4\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+4a^3b\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+6a^2b^2(-\cot(dx+c)-dx-c)+4a\,b^3\ln(s}{d}$
default	$\frac{a^4\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+4a^3b\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+6a^2b^2(-\cot(dx+c)-dx-c)+4a\,b^3\ln(s}{d}$
norman	$\frac{(a^4-6a^2b^2+b^4)x(\tan^3(dx+c))+\frac{a^2\left(a^2-6b^2\right)\left(\tan^2(dx+c)\right)}{d}-\frac{a^4}{3d}-\frac{2a^3b\tan(dx+c)}{d}}{\tan(dx+c)^3}-\frac{4ab(a^2-b^2)\ln(\tan(dx+c))}{d}+\frac{2ab(a}{}$
risch	$4ia^3bx-4ia\,b^3x+a^4x-6a^2b^2x+b^4x+\frac{8ia^3bc}{d}-\frac{8ia\,b^3c}{d}+\frac{4ia^2\left(3a^2\mathrm{e}^{4i(dx+c)}-9b^2\mathrm{e}^{4i(dx+c)}-6iab\,\mathrm{e}^{4}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+4*a^3*b*(-1/2*cot(d*x+c)^2-ln
(sin(d*x+c)))+6*a^2*b^2*(-cot(d*x+c)-d*x-c)+4*a*b^3*ln(sin(d*x+c))+b^4*(d*x
+c))
```

Maxima [A]
time = 0.53, size = 122, normalized size = 1.04

$$\frac{3\,(a^4-6\,a^2b^2+b^4)(dx+c)+6\,(a^3b-ab^3)\log\left(\tan\left(dx+c\right)^2+1\right)-12\,(a^3b-ab^3)\log\left(\tan\left(dx+c\right)\right)-\frac{6\,a^3b\tan(dx+c)+a^4-3\,(a^4-6\,a^2b^2)\tan(dx+c)^2}{\tan(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(a^4 - 6*a^2*b^2 + b^4)*(d*x + c) + 6*(a^3*b - a*b^3)*log(tan(d*x +
c)^2 + 1) - 12*(a^3*b - a*b^3)*log(tan(d*x + c)) - (6*a^3*b*tan(d*x + c) +
a^4 - 3*(a^4 - 6*a^2*b^2)*tan(d*x + c)^2)/tan(d*x + c)^3)/d
```

Fricas [A]
time = 1.25, size = 131, normalized size = 1.12

$$-\frac{6\,a^3b\tan\left(dx+c\right)+6\,(a^3b-ab^3)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^3+a^4+3\,(2\,a^3b-(a^4-6\,a^2b^2+b^4)dx)\tan\left(dx+c\right)^3-3\,(a^4-6\,a^2b^2)\tan\left(dx+c\right)^2}{3\,d\tan\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(6*a^3*b*tan(d*x + c) + 6*(a^3*b - a*b^3)*log(tan(d*x + c)^2/(tan(d*x
+ c)^2 + 1))*tan(d*x + c)^3 + a^4 + 3*(2*a^3*b - (a^4 - 6*a^2*b^2 + b^4)*d*
x)*tan(d*x + c)^3 - 3*(a^4 - 6*a^2*b^2)*tan(d*x + c)^2)/(d*tan(d*x + c)^3)
```

Sympy [A]
time = 2.19, size = 187, normalized size = 1.60

$$\begin{cases} \tilde{\infty} a^4 x & \text{for}\ (c = 0 \vee c = -dx) \wedge (c = -dx \vee d = 0) \\ x(a + b\tan(c))^4 \cot^4(c) & \text{for}\ d = 0 \\ a^4 x + \frac{a^4}{d\tan(c+dx)} - \frac{a^4}{3d\tan^3(c+dx)} + \frac{2a^3b\log(\tan^2(c+dx)+1)}{d} - \frac{4a^3b\log(\tan(c+dx))}{d} - \frac{2a^3b}{d\tan^2(c+dx)} - 6a^2b^2x - \frac{6a^2b^2}{d\tan(c+dx)} - \frac{2ab^3\log(\tan^2(c+dx)+1)}{d} + \frac{4ab^3\log(\tan(c+dx))}{d} + b^4 x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Piecewise((zoo*a**4*x, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x)))
, (x*(a + b*tan(c))**4*cot(c)**4, Eq(d, 0)), (a**4*x + a**4/(d*tan(c + d*x)
) - a**4/(3*d*tan(c + d*x)**3) + 2*a**3*b*log(tan(c + d*x)**2 + 1)/d - 4*a*
*3*b*log(tan(c + d*x))/d - 2*a**3*b/(d*tan(c + d*x)**2) - 6*a**2*b**2*x - 6
*a**2*b**2/(d*tan(c + d*x)) - 2*a*b**3*log(tan(c + d*x)**2 + 1)/d + 4*a*b**
3*log(tan(c + d*x))/d + b**4*x, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(111) = 222.
time = 1.90, size = 246, normalized size = 2.10

$$\frac{a^4\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 12a^3b\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 15a^4\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 72a^2b^2\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 24(a^4 - 6a^2b^2 + b^4)(dx + c) + 96(a^3b - ab^3)\log\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 1\right) - 96(a^3b - ab^3)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right|\right) + \frac{176a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 176ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 15a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 72a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 12a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - a^4}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(a^4*tan(1/2*d*x + 1/2*c)^3 - 12*a^3*b*tan(1/2*d*x + 1/2*c)^2 - 15*a^4
*tan(1/2*d*x + 1/2*c) + 72*a^2*b^2*tan(1/2*d*x + 1/2*c) + 24*(a^4 - 6*a^2*b
^2 + b^4)*(d*x + c) + 96*(a^3*b - a*b^3)*log(tan(1/2*d*x + 1/2*c)^2 + 1) -
96*(a^3*b - a*b^3)*log(abs(tan(1/2*d*x + 1/2*c))) + (176*a^3*b*tan(1/2*d*x
+ 1/2*c)^3 - 176*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 15*a^4*tan(1/2*d*x + 1/2*c)
^2 - 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 12*a^3*b*tan(1/2*d*x + 1/2*c) - a^
4)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B]
time = 3.98, size = 127, normalized size = 1.09

$$-\frac{\cot(c+d\,x)^3\,\left(\frac{a^4}{3} - \tan(c+d\,x)^2\,(a^4 - 6\,a^2\,b^2) + 2\,a^3\,b\tan(c+d\,x)\right)}{d} - \frac{4\,a\,b\,\ln(\tan(c+d\,x))\,(a^2-b^2)}{d} + \frac{\ln(\tan(c+d\,x)+1\mathrm{i})\,(a-b\,1\mathrm{i})^4\,1\mathrm{i}}{2\,d} - \frac{\ln(\tan(c+d\,x)-\mathrm{i})\,(-b+a\,1\mathrm{i})^4\,1\mathrm{i}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + b*tan(c + d*x))^4,x)
```

```
[Out] (log(tan(c + d*x) + 1i)*(a - b*1i)^4*1i)/(2*d) - (cot(c + d*x)^3*(a^4/3 - t
an(c + d*x)^2*(a^4 - 6*a^2*b^2) + 2*a^3*b*tan(c + d*x)))/d - (log(tan(c + d
*x) - 1i)*(a*1i - b)^4*1i)/(2*d) - (4*a*b*log(tan(c + d*x))*(a^2 - b^2))/d
```

3.453 $\int \cot^5(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=141

$$4ab(a^2-b^2)\,x+\frac{4ab(a^2-b^2)\cot(c+dx)}{d}+\frac{a^2(2a^2-11b^2)\cot^2(c+dx)}{4d}-\frac{5a^3b\cot^3(c+dx)}{6d}+\frac{(a^4-6a^2b^2+b}{}$$

```
[Out] 4*a*b*(a^2-b^2)*x+4*a*b*(a^2-b^2)*cot(d*x+c)/d+1/4*a^2*(2*a^2-11*b^2)*cot(d
*x+c)^2/d-5/6*a^3*b*cot(d*x+c)^3/d+(a^4-6*a^2*b^2+b^4)*ln(sin(d*x+c))/d-1/4
*a^2*cot(d*x+c)^4*(a+b*tan(d*x+c))^2/d
```

Rubi [A]
time = 0.23, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3646, 3716, 3709, 3610, 3612, 3556}

$$-\frac{5a^3b\cot^3(c+dx)}{6d}+\frac{a^2(2a^2-11b^2)\cot^2(c+dx)}{4d}+\frac{4ab(a^2-b^2)\cot(c+dx)}{d}+4abx(a^2-b^2)-\frac{a^2\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}+\frac{(a^4-6a^2b^2+b^4)\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] 4*a*b*(a^2 - b^2)*x + (4*a*b*(a^2 - b^2)*Cot[c + d*x])/d + (a^2*(2*a^2 - 11
*b^2)*Cot[c + d*x]^2)/(4*d) - (5*a^3*b*Cot[c + d*x]^3)/(6*d) + ((a^4 - 6*a^
2*b^2 + b^4)*Log[Sin[c + d*x]])/d - (a^2*Cot[c + d*x]^4*(a + b*Tan[c + d*x]
)^2)/(4*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3716

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
 -1]
```

Rubi steps

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^4\,dx = -\frac{a^2\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d} + \frac{1}{4}\int \cot^4(c+dx)(a+b\,\text{ta}$$

$$= -\frac{5a^3b\cot^3(c+dx)}{6d} - \frac{a^2\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d} + \frac{1}{4}\int$$

$$= \frac{a^2(2a^2-11b^2)\cot^2(c+dx)}{4d} - \frac{5a^3b\cot^3(c+dx)}{6d} - \frac{a^2\cot^4(c+dx}{}$$

$$= \frac{4ab(a^2-b^2)\cot(c+dx)}{d} + \frac{a^2(2a^2-11b^2)\cot^2(c+dx)}{4d} - \frac{5a^3b\,\text{co}}{}$$

$$= 4ab(a^2-b^2)\,x + \frac{4ab(a^2-b^2)\cot(c+dx)}{d} + \frac{a^2(2a^2-11b^2)\cot^2(c}{4d}$$

$$= 4ab(a^2-b^2)\,x + \frac{4ab(a^2-b^2)\cot(c+dx)}{d} + \frac{a^2(2a^2-11b^2)\cot^2(c}{4d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.16, size = 147, normalized size = 1.04

$$\frac{48ab(a^2-b^2)\cot(c+dx)+6a^2(a^2-6b^2)\cot^2(c+dx)-16a^3b\cot^3(c+dx)-3a^4\cot^4(c+dx)-6((a+ib)^4\log(i-\tan(c+dx))-2(a^4-6a^2b^2+b^4)\log(\tan(c+dx))+(a-ib)^4\log(i+\tan(c+dx)))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (48*a*b*(a^2 - b^2)*Cot[c + d*x] + 6*a^2*(a^2 - 6*b^2)*Cot[c + d*x]^2 - 16*
a^3*b*Cot[c + d*x]^3 - 3*a^4*Cot[c + d*x]^4 - 6*((a + I*b)^4*Log[I - Tan[c
+ d*x]] - 2*(a^4 - 6*a^2*b^2 + b^4)*Log[Tan[c + d*x]] + (a - I*b)^4*Log[I +
 Tan[c + d*x]]))/(12*d)
```

Maple [A]
time = 0.19, size = 126, normalized size = 0.89

method	result
derivativedivides	$\frac{a^4\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+4a^3b\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+6a^2b^2\left(-\frac{\left(\cot^2(dx+c\right.}{2}\right.}{d}$
default	$\frac{a^4\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+4a^3b\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+6a^2b^2\left(-\frac{\left(\cot^2(dx+c\right.}{2}\right.}{d}$
norman	$\frac{-\frac{a^4}{4d}+\frac{a^2\left(a^2-6b^2\right)\left(\tan^2(dx+c)\right)}{2d}-\frac{4a^3b\tan(dx+c)}{3d}+\frac{4ab\left(a^2-b^2\right)\left(\tan^3(dx+c)\right)}{d}+4ab\left(a^2-b^2\right)x\left(\tan^4(dx+c)\right)}{\tan(dx+c)^4}+\frac{\left(a^4-6a^2b^2\right.}{}$
risch	$4a^3bx-4a\,b^3x-ia^4x+6ia^2b^2x-ib^4x-\frac{2ia^4c}{d}+\frac{12ia^2b^2c}{d}-\frac{2ib^4c}{d}-\frac{4ia\left(-3ia^3e^{6i(dx+c)}+9ia\,b^2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+4*a^3*b*(-1/3*
cot(d*x+c)^3+cot(d*x+c)+d*x+c)+6*a^2*b^2*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))
+4*a*b^3*(-cot(d*x+c)-d*x-c)+b^4*ln(sin(d*x+c)))
```

Maxima [A]
time = 0.52, size = 149, normalized size = 1.06

$$\frac{48\,(a^3b-ab^3)(dx+c)-6\,(a^4-6\,a^2b^2+b^4)\log\left(\tan\left(dx+c\right)^2+1\right)+12\,(a^4-6\,a^2b^2+b^4)\log\left(\tan\left(dx+c\right)\right)-\frac{16\,a^3b\tan(dx+c)+3\,a^4-48\,(a^3b-ab^3)\tan(dx+c)^3-6\,(a^4-6\,a^2b^2)\tan(dx+c)^2}{\tan(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/12*(48*(a^3*b - a*b^3)*(d*x + c) - 6*(a^4 - 6*a^2*b^2 + b^4)*log(tan(d*x
+ c)^2 + 1) + 12*(a^4 - 6*a^2*b^2 + b^4)*log(tan(d*x + c)) - (16*a^3*b*tan(
d*x + c) + 3*a^4 - 48*(a^3*b - a*b^3)*tan(d*x + c)^3 - 6*(a^4 - 6*a^2*b^2)*
tan(d*x + c)^2)/tan(d*x + c)^4)/d
```

Fricas [A]
time = 1.57, size = 162, normalized size = 1.15

$$\frac{6\,(a^4-6\,a^2b^2+b^4)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^4-16\,a^3b\tan\left(dx+c\right)+3\,(3\,a^4-12\,a^2b^2+16\,(a^3b-ab^3)dx)\tan\left(dx+c\right)^4-3\,a^4+48\,(a^3b-ab^3)\tan\left(dx+c\right)^3+6\,(a^4-6\,a^2b^2)\tan\left(dx+c\right)^2}{12\,d\tan\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/12*(6*(a^4 - 6*a^2*b^2 + b^4)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*ta
n(d*x + c)^4 - 16*a^3*b*tan(d*x + c) + 3*(3*a^4 - 12*a^2*b^2 + 16*(a^3*b -
a*b^3)*d*x)*tan(d*x + c)^4 - 3*a^4 + 48*(a^3*b - a*b^3)*tan(d*x + c)^3 + 6*
(a^4 - 6*a^2*b^2)*tan(d*x + c)^2)/(d*tan(d*x + c)^4)
```

Sympy [A]
time = 3.01, size = 252, normalized size = 1.79

$$\begin{cases} \tilde{\infty}a^4x & \text{for}\ (c=0\vee c=-dx)\wedge(c=-dx\vee d=0) \\ x(a+b\tan(c))^4\cot^5(c) & \text{for}\ d=0 \\ -\frac{a^4\log(\tan^2(c+dx)+1)}{2d}+\frac{a^4\log(\tan(c+dx))}{d}+\frac{a^4}{2d\tan^2(c+dx)}-\frac{a^4}{4d\tan^4(c+dx)}+4a^3bx+\frac{4a^3b}{d\tan(c+dx)}-\frac{4a^3b}{3d\tan^3(c+dx)}+\frac{3a^2b^2\log(\tan^2(c+dx)+1)}{d}-\frac{6a^2b^2\log(\tan(c+dx))}{d}-\frac{3a^2b^2}{d\tan^2(c+dx)}-4ab^3x-\frac{4ab^3}{d\tan(c+dx)}-\frac{b^4\log(\tan^2(c+dx)+1)}{2d}+\frac{b^4\log(\tan(c+dx))}{d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Piecewise((zoo*a**4*x, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x)))
, (x*(a + b*tan(c))**4*cot(c)**5, Eq(d, 0)), (-a**4*log(tan(c + d*x)**2 + 1
)/(2*d) + a**4*log(tan(c + d*x))/d + a**4/(2*d*tan(c + d*x)**2) - a**4/(4*d
*tan(c + d*x)**4) + 4*a**3*b*x + 4*a**3*b/(d*tan(c + d*x)) - 4*a**3*b/(3*d*
tan(c + d*x)**3) + 3*a**2*b**2*log(tan(c + d*x)**2 + 1)/d - 6*a**2*b**2*log
(tan(c + d*x))/d - 3*a**2*b**2/(d*tan(c + d*x)**2) - 4*a*b**3*x - 4*a*b**3/
```

```
(d*tan(c + d*x)) - b**4*log(tan(c + d*x)**2 + 1)/(2*d) + b**4*log(tan(c + d
*x))/d, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(135) = 270.
time = 1.85, size = 335, normalized size = 2.38

$$-\frac{3\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-32\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-36\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+144\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+480\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-384\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-768\,(a^3b-ab^3)(dx+c)+192\,(a^4-6\,a^2b^2+b^4)\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)-192\,(a^4-6\,a^2b^2+b^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+\frac{400\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2400\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+400\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-480\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+384\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-36\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+144\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+32\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a^4}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/192*(3*a^4*tan(1/2*d*x + 1/2*c)^4 - 32*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 36
*a^4*tan(1/2*d*x + 1/2*c)^2 + 144*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 480*a^3*
b*tan(1/2*d*x + 1/2*c) - 384*a*b^3*tan(1/2*d*x + 1/2*c) - 768*(a^3*b - a*b^
3)*(d*x + c) + 192*(a^4 - 6*a^2*b^2 + b^4)*log(tan(1/2*d*x + 1/2*c)^2 + 1)
- 192*(a^4 - 6*a^2*b^2 + b^4)*log(abs(tan(1/2*d*x + 1/2*c))) + (400*a^4*tan
(1/2*d*x + 1/2*c)^4 - 2400*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 400*b^4*tan(1/2
*d*x + 1/2*c)^4 - 480*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 384*a*b^3*tan(1/2*d*x
+ 1/2*c)^3 - 36*a^4*tan(1/2*d*x + 1/2*c)^2 + 144*a^2*b^2*tan(1/2*d*x + 1/2*
c)^2 + 32*a^3*b*tan(1/2*d*x + 1/2*c) + 3*a^4)/tan(1/2*d*x + 1/2*c)^4)/d
```

Mupad [B]
time = 3.97, size = 150, normalized size = 1.06

$$\frac{\ln\left(\tan(c+d\,x)\right)\,\left(a^4-6\,a^2\,b^2+b^4\right)}{d}-\frac{\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)\,{\left(b+a\,1{}\mathrm{i}\right)}^4}{2\,d}-\frac{\mathrm{cot}(c+d\,x)^4\,\left(\tan(c+d\,x)^3\,\left(4\,a\,b^3-4\,a^3\,b\right)-\tan(c+d\,x)^2\,\left(\frac{a^4}{2}-3\,a^2\,b^2\right)+\frac{a^4}{4}+\frac{4\,a^3\,b\,\tan(c+d\,x)}{3}\right)}{d}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,{\left(a+b\,1{}\mathrm{i}\right)}^4}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(a + b*tan(c + d*x))^4,x)
```

```
[Out] (log(tan(c + d*x))*(a^4 + b^4 - 6*a^2*b^2))/d - (log(tan(c + d*x) + 1i)*(a*
1i + b)^4)/(2*d) - (cot(c + d*x)^4*(tan(c + d*x)^3*(4*a*b^3 - 4*a^3*b) - ta
n(c + d*x)^2*(a^4/2 - 3*a^2*b^2) + a^4/4 + (4*a^3*b*tan(c + d*x))/3))/d - (
log(tan(c + d*x) - 1i)*(a + b*1i)^4)/(2*d)
```

3.454 $\int \cot^6(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=170

$$-\left(\left(a^4-6a^2b^2+b^4\right)x\right)-\frac{\left(a^4-6a^2b^2+b^4\right)\cot(c+dx)}{d}+\frac{2ab\left(a^2-b^2\right)\cot^2(c+dx)}{d}+\frac{a^2\left(5a^2-27b^2\right)\cot^3(c+dx)}{15d}$$

```
[Out] -(a^4-6*a^2*b^2+b^4)*x-(a^4-6*a^2*b^2+b^4)*cot(d*x+c)/d+2*a*b*(a^2-b^2)*cot
(d*x+c)^2/d+1/15*a^2*(5*a^2-27*b^2)*cot(d*x+c)^3/d-3/5*a^3*b*cot(d*x+c)^4/d
+4*a*b*(a^2-b^2)*ln(sin(d*x+c))/d-1/5*a^2*cot(d*x+c)^5*(a+b*tan(d*x+c))^2/d
```

Rubi [A]
time = 0.26, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3646, 3716, 3709, 3610, 3612, 3556}

$$-\frac{3a^3b\cot^4(c+dx)}{5d}+\frac{a^2\left(5a^2-27b^2\right)\cot^3(c+dx)}{15d}+\frac{2ab\left(a^2-b^2\right)\cot^2(c+dx)}{d}+\frac{4ab\left(a^2-b^2\right)\log(\sin(c+dx))}{d}-\frac{a^2\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}-\frac{\left(a^4-6a^2b^2+b^4\right)\cot(c+dx)}{d}-x\left(a^4-6a^2b^2+b^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -((a^4 - 6*a^2*b^2 + b^4)*x) - ((a^4 - 6*a^2*b^2 + b^4)*Cot[c + d*x])/d + (
2*a*b*(a^2 - b^2)*Cot[c + d*x]^2)/d + (a^2*(5*a^2 - 27*b^2)*Cot[c + d*x]^3)
/(15*d) - (3*a^3*b*Cot[c + d*x]^4)/(5*d) + (4*a*b*(a^2 - b^2)*Log[Sin[c + d
*x]])/d - (a^2*Cot[c + d*x]^5*(a + b*Tan[c + d*x])^2)/(5*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3716

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
 -1]
```

Rubi steps

$$
\begin{aligned}
\int \cot^6(c+dx)(a+b\tan(c+dx))^4\,dx &= -\frac{a^2\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d} + \frac{1}{5}\int \cot^5(c+dx)(a+b\tan \\
&= -\frac{3a^3b\cot^4(c+dx)}{5d} - \frac{a^2\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d} + \frac{1}{5}\int \mathrm{co} \\
&= \frac{a^2(5a^2-27b^2)\cot^3(c+dx)}{15d} - \frac{3a^3b\cot^4(c+dx)}{5d} - \frac{a^2\cot^5(c+dx)(}{} \\
&= \frac{2ab(a^2-b^2)\cot^2(c+dx)}{d} + \frac{a^2(5a^2-27b^2)\cot^3(c+dx)}{15d} - \frac{3a^3b\cot}{5} \\
&= -\frac{(a^4-6a^2b^2+b^4)\cot(c+dx)}{d} + \frac{2ab(a^2-b^2)\cot^2(c+dx)}{d} + \frac{a^2(5}{} \\
&= -\left(a^4-6a^2b^2+b^4\right)x - \frac{(a^4-6a^2b^2+b^4)\cot(c+dx)}{d} + \frac{2ab(a^2-b}{} \\
&= -\left(a^4-6a^2b^2+b^4\right)x - \frac{(a^4-6a^2b^2+b^4)\cot(c+dx)}{d} + \frac{2ab(a^2-b}{}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.45, size = 154, normalized size = 0.91

$$
-\frac{(a^4-6a^2b^2+b^4)\cot(c+dx) - 2a(a-b)b(a+b)\cot^2(c+dx) - \frac{1}{3}a^2(a^2-6b^2)\cot^3(c+dx) + a^3b\cot^4(c+dx) + \frac{1}{5}a^4\cot^5(c+dx) + \frac{1}{2}i(a-ib)^4\log(i-\cot(c+dx)) - \frac{1}{2}i(a+ib)^4\log(i+\cot(c+dx))}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -(((a^4 - 6*a^2*b^2 + b^4)*Cot[c + d*x] - 2*a*(a - b)*b*(a + b)*Cot[c + d*x
]^2 - (a^2*(a^2 - 6*b^2)*Cot[c + d*x]^3)/3 + a^3*b*Cot[c + d*x]^4 + (a^4*Co
t[c + d*x]^5)/5 + (I/2)*(a - I*b)^4*Log[I - Cot[c + d*x]] - (I/2)*(a + I*b)
^4*Log[I + Cot[c + d*x]])/d)
```

Maple [A]
time = 0.20, size = 155, normalized size = 0.91

method	result
derivativedivides	$\frac{a^4\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-dx-c\right)+4a^3b\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+6a^2b^2\left(\right.}{d}$
default	$\frac{a^4\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-dx-c\right)+4a^3b\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+6a^2b^2\left(\right.}{d}$
norman	$\frac{(-a^4+6a^2b^2-b^4)x\left(\tan^5(dx+c)\right)-\frac{a^4}{5d}-\frac{\left(a^4-6a^2b^2+b^4\right)\left(\tan^4(dx+c)\right)}{d}+\frac{a^2\left(a^2-6b^2\right)\left(\tan^2(dx+c)\right)}{3d}-\frac{a^3b\tan(dx+c)}{d}+\frac{2ab\left(a^2-\right.}{}}{\tan(dx+c)^5}$

risch	$-4ia^3bx + 4ia\,b^3x - a^4x + 6a^2b^2x - b^4x - \frac{8ia^3bc}{d} + \frac{8ia\,b^3c}{d} - \frac{2i(-120ia^3b\,e^{8i(dx+c)} + 120ia^3b\,e^{2i(c}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c)+4*a^3*b*(-1/
4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+6*a^2*b^2*(-1/3*cot(d*x+c)^
3+cot(d*x+c)+d*x+c)+4*a*b^3*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+b^4*(-cot(d*
x+c)-d*x-c))
```

Maxima [A]
time = 0.52, size = 170, normalized size = 1.00

$$-\frac{15\,(a^4-6\,a^2b^2+b^4)(dx+c)+30\,(a^3b-ab^3)\log\left(\tan\left(dx+c\right)^2+1\right)-60\,(a^3b-ab^3)\log\left(\tan\left(dx+c\right)\right)+\frac{15\,a^3b\tan(dx+c)+15\,(a^4-6\,a^2b^2+b^4)\tan(dx+c)^4+3\,a^4-30\,(a^3b-ab^3)\tan(dx+c)^3-5\,(a^4-6\,a^2b^2)\tan(dx+c)^2}{\tan(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/15*(15*(a^4 - 6*a^2*b^2 + b^4)*(d*x + c) + 30*(a^3*b - a*b^3)*log(tan(d*
x + c)^2 + 1) - 60*(a^3*b - a*b^3)*log(tan(d*x + c)) + (15*a^3*b*tan(d*x +
c) + 15*(a^4 - 6*a^2*b^2 + b^4)*tan(d*x + c)^4 + 3*a^4 - 30*(a^3*b - a*b^3)
*tan(d*x + c)^3 - 5*(a^4 - 6*a^2*b^2)*tan(d*x + c)^2)/tan(d*x + c)^5)/d
```

Fricas [A]
time = 1.02, size = 186, normalized size = 1.09

$$\frac{30\,(a^3b-ab^3)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^5+15\,(3\,a^3b-2\,ab^3-(a^4-6\,a^2b^2+b^4)dx)\tan\left(dx+c\right)^5-15\,a^3b\tan\left(dx+c\right)-15\,(a^4-6\,a^2b^2+b^4)\tan\left(dx+c\right)^4-3\,a^4+30\,(a^3b-ab^3)\tan\left(dx+c\right)^3+5\,(a^4-6\,a^2b^2)\tan\left(dx+c\right)^2}{15\,d\tan\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/15*(30*(a^3*b - a*b^3)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x +
 c)^5 + 15*(3*a^3*b - 2*a*b^3 - (a^4 - 6*a^2*b^2 + b^4)*d*x)*tan(d*x + c)^5
 - 15*a^3*b*tan(d*x + c) - 15*(a^4 - 6*a^2*b^2 + b^4)*tan(d*x + c)^4 - 3*a^
4 + 30*(a^3*b - a*b^3)*tan(d*x + c)^3 + 5*(a^4 - 6*a^2*b^2)*tan(d*x + c)^2)
/(d*tan(d*x + c)^5)
```

Sympy [A]
time = 5.07, size = 265, normalized size = 1.56

$$\begin{cases} \tilde{\infty}a^4x & \text{for}\ (c=0 \vee c=-dx) \wedge (c=-dx \vee d=0) \\ x(a+b\tan(c))^4\cot^6(c) & \text{for}\ d=0 \\ -a^4x - \frac{a^4}{d\tan(c+dx)} + \frac{a^4}{3d\tan^3(c+dx)} - \frac{a^4}{5d\tan^5(c+dx)} - \frac{2a^3b\log(\tan^2(c+dx)+1)}{d} + \frac{4a^3b\log(\tan(c+dx))}{d} + \frac{2a^3b}{d\tan^2(c+dx)} - \frac{a^3b}{d\tan^4(c+dx)} + 6a^2b^2x + \frac{6a^2b^2}{d\tan(c+dx)} - \frac{2a^2b^2}{d\tan^3(c+dx)} + \frac{2ab^3\log(\tan^2(c+dx)+1)}{d} - \frac{4ab^3\log(\tan(c+dx))}{d} - \frac{2ab^3}{d\tan^2(c+dx)} - b^4x - \frac{b^4}{d\tan(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Piecewise((zoo*a**4*x, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x)))
, (x*(a + b*tan(c))**4*cot(c)**6, Eq(d, 0)), (-a**4*x - a**4/(d*tan(c + d*x
)) + a**4/(3*d*tan(c + d*x)**3) - a**4/(5*d*tan(c + d*x)**5) - 2*a**3*b*log
(tan(c + d*x)**2 + 1)/d + 4*a**3*b*log(tan(c + d*x))/d + 2*a**3*b/(d*tan(c
+ d*x)**2) - a**3*b/(d*tan(c + d*x)**4) + 6*a**2*b**2*x + 6*a**2*b**2/(d*ta
n(c + d*x)) - 2*a**2*b**2/(d*tan(c + d*x)**3) + 2*a*b**3*log(tan(c + d*x)**
2 + 1)/d - 4*a*b**3*log(tan(c + d*x))/d - 2*a*b**3/(d*tan(c + d*x)**2) - b*
*4*x - b**4/(d*tan(c + d*x)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(164) = 328$.
time = 1.32, size = 416, normalized size = 2.45

$$\frac{3\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-30\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-35\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+120\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+360\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-240\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+330\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1800\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+240\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-480\,(a^4-6\,a^2b^2+b^4)(dx+c)-1920\,(a^3b-ab^3)\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)+1920\,(a^3b-ab^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{4384\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-4384\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+330\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1800\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+240\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-360\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+240\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-35\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+120\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+30\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a^4}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^4*tan(1/2*d*x + 1/2*c)^5 - 30*a^3*b*tan(1/2*d*x + 1/2*c)^4 - 35*
a^4*tan(1/2*d*x + 1/2*c)^3 + 120*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 360*a^3*b
*tan(1/2*d*x + 1/2*c)^2 - 240*a*b^3*tan(1/2*d*x + 1/2*c)^2 + 330*a^4*tan(1/
2*d*x + 1/2*c) - 1800*a^2*b^2*tan(1/2*d*x + 1/2*c) + 240*b^4*tan(1/2*d*x +
1/2*c) - 480*(a^4 - 6*a^2*b^2 + b^4)*(d*x + c) - 1920*(a^3*b - a*b^3)*log(t
an(1/2*d*x + 1/2*c)^2 + 1) + 1920*(a^3*b - a*b^3)*log(abs(tan(1/2*d*x + 1/2
*c))) - (4384*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 4384*a*b^3*tan(1/2*d*x + 1/2*c
)^5 + 330*a^4*tan(1/2*d*x + 1/2*c)^4 - 1800*a^2*b^2*tan(1/2*d*x + 1/2*c)^4
+ 240*b^4*tan(1/2*d*x + 1/2*c)^4 - 360*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 240*a
*b^3*tan(1/2*d*x + 1/2*c)^3 - 35*a^4*tan(1/2*d*x + 1/2*c)^2 + 120*a^2*b^2*t
an(1/2*d*x + 1/2*c)^2 + 30*a^3*b*tan(1/2*d*x + 1/2*c) + 3*a^4)/tan(1/2*d*x
+ 1/2*c)^5)/d
```

Mupad [B]
time = 4.06, size = 174, normalized size = 1.02

$$\frac{4\,a\,b\,\ln\left(\tan(c+d\,x)\right)\,\left(a^2-b^2\right)}{d}-\frac{\cot(c+d\,x)^5\,\left(\tan(c+d\,x)^3\,\left(2\,a\,b^3-2\,a^3\,b\right)-\tan(c+d\,x)^2\,\left(\frac{a^4}{3}-2\,a^2\,b^2\right)+\frac{a^4}{5}+\tan(c+d\,x)^4\,\left(a^4-6\,a^2\,b^2+b^4\right)+a^3\,b\,\tan(c+d\,x)\right)}{d}-\frac{\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)\,{\left(a-b\,1{}\mathrm{i}\right)}^4\,1{}\mathrm{i}}{2\,d}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,{\left(-b+a\,1{}\mathrm{i}\right)}^4\,1{}\mathrm{i}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(a + b*tan(c + d*x))^4,x)
```

```
[Out] (log(tan(c + d*x) - 1i)*(a*1i - b)^4*1i)/(2*d) - (cot(c + d*x)^5*(tan(c + d
*x)^3*(2*a*b^3 - 2*a^3*b) - tan(c + d*x)^2*(a^4/3 - 2*a^2*b^2) + a^4/5 + ta
n(c + d*x)^4*(a^4 + b^4 - 6*a^2*b^2) + a^3*b*tan(c + d*x)))/d - (log(tan(c
+ d*x) + 1i)*(a - b*1i)^4*1i)/(2*d) + (4*a*b*log(tan(c + d*x))*(a^2 - b^2))
/d
```

3.455 $\int \cot^7(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=198

$$-4ab\left(a^2-b^2\right)x-\frac{4ab(a^2-b^2)\cot(c+dx)}{d}-\frac{(a^4-6a^2b^2+b^4)\cot^2(c+dx)}{2d}+\frac{4ab(a^2-b^2)\cot^3(c+dx)}{3d}+\frac{a^2(\ldots$$

```
[Out] -4*a*b*(a^2-b^2)*x-4*a*b*(a^2-b^2)*cot(d*x+c)/d-1/2*(a^4-6*a^2*b^2+b^4)*cot
(d*x+c)^2/d+4/3*a*b*(a^2-b^2)*cot(d*x+c)^3/d+1/12*a^2*(3*a^2-16*b^2)*cot(d*
x+c)^4/d-7/15*a^3*b*cot(d*x+c)^5/d-(a^4-6*a^2*b^2+b^4)*ln(sin(d*x+c))/d-1/6
*a^2*cot(d*x+c)^6*(a+b*tan(d*x+c))^2/d
```

Rubi [A]
time = 0.30, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3646, 3716, 3709, 3610, 3612, 3556}

$$-\frac{7a^3b\cot^5(c+dx)}{15d}+\frac{a^2(3a^2-16b^2)\cot^4(c+dx)}{12d}+\frac{4ab(a^2-b^2)\cot^3(c+dx)}{3d}-\frac{4ab(a^2-b^2)\cot(c+dx)}{d}-4abx\left(a^2-b^2\right)-\frac{a^2\cot^6(c+dx)(a+b\tan(c+dx))^2}{6d}-\frac{(a^4-6a^2b^2+b^4)\cot^2(c+dx)}{2d}-\frac{(a^4-6a^2b^2+b^4)\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -4*a*b*(a^2 - b^2)*x - (4*a*b*(a^2 - b^2)*Cot[c + d*x])/d - ((a^4 - 6*a^2*b
^2 + b^4)*Cot[c + d*x]^2)/(2*d) + (4*a*b*(a^2 - b^2)*Cot[c + d*x]^3)/(3*d)
+ (a^2*(3*a^2 - 16*b^2)*Cot[c + d*x]^4)/(12*d) - (7*a^3*b*Cot[c + d*x]^5)/(
15*d) - ((a^4 - 6*a^2*b^2 + b^4)*Log[Sin[c + d*x]])/d - (a^2*Cot[c + d*x]^6
*(a + b*Tan[c + d*x])^2)/(6*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
```

```
Q[a*c + b*d, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3716

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
 -1]
```

Rubi steps

$$\int \cot^7(c+dx)(a+b\tan(c+dx))^4\,dx = -\frac{a^2\cot^6(c+dx)(a+b\tan(c+dx))^2}{6d} + \frac{1}{6}\int \cot^6(c+dx)(a+b\,\mathrm{ta}$$

$$= -\frac{7a^3b\cot^5(c+dx)}{15d} - \frac{a^2\cot^6(c+dx)(a+b\tan(c+dx))^2}{6d} + \frac{1}{6}\int$$

$$= \frac{a^2(3a^2-16b^2)\cot^4(c+dx)}{12d} - \frac{7a^3b\cot^5(c+dx)}{15d} - \frac{a^2\cot^6(c+dx}{}$$

$$= \frac{4ab(a^2-b^2)\cot^3(c+dx)}{3d} + \frac{a^2(3a^2-16b^2)\cot^4(c+dx)}{12d} - \frac{7a^3b\,\mathrm{co}}{}$$

$$= -\frac{(a^4-6a^2b^2+b^4)\cot^2(c+dx)}{2d} + \frac{4ab(a^2-b^2)\cot^3(c+dx)}{3d} + \frac{a^2}{}$$

$$= -\frac{4ab(a^2-b^2)\cot(c+dx)}{d} - \frac{(a^4-6a^2b^2+b^4)\cot^2(c+dx)}{2d} + \frac{4ab}{}$$

$$= -4ab\left(a^2-b^2\right)x - \frac{4ab(a^2-b^2)\cot(c+dx)}{d} - \frac{(a^4-6a^2b^2+b^4)\,\mathrm{c}}{2d}$$

$$= -4ab\left(a^2-b^2\right)x - \frac{4ab(a^2-b^2)\cot(c+dx)}{d} - \frac{(a^4-6a^2b^2+b^4)\,\mathrm{c}}{2d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.51, size = 178, normalized size = 0.90

$$-\frac{4a(a-b)b(a+b)\cot(c+dx)+\frac{1}{2}(a^4-6a^2b^2+b^4)\cot^2(c+dx)-\frac{4}{3}a(a-b)b(a+b)\cot^3(c+dx)-\frac{1}{4}a^2(a^2-6b^2)\cot^4(c+dx)+\frac{4}{5}a^3b\cot^5(c+dx)+\frac{1}{6}a^4\cot^6(c+dx)-\frac{1}{2}(a-ib)^4\log(i-\cot(c+dx))-\frac{1}{2}(a+ib)^4\log(i+\cot(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -((4*a*(a - b)*b*(a + b)*Cot[c + d*x] + ((a^4 - 6*a^2*b^2 + b^4)*Cot[c + d*
x]^2)/2 - (4*a*(a - b)*b*(a + b)*Cot[c + d*x]^3)/3 - (a^2*(a^2 - 6*b^2)*Cot
[c + d*x]^4)/4 + (4*a^3*b*Cot[c + d*x]^5)/5 + (a^4*Cot[c + d*x]^6)/6 - ((a
- I*b)^4*Log[I - Cot[c + d*x]])/2 - ((a + I*b)^4*Log[I + Cot[c + d*x]])/2)/
d)
```

Maple [A]
time = 0.24, size = 179, normalized size = 0.90

method	result
derivativedivides	$\frac{a^4\left(-\frac{\left(\cot^6(dx+c)\right)}{6}+\frac{\left(\cot^4(dx+c)\right)}{4}-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+4a^3b\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-\right.}{}$
default	$\frac{a^4\left(-\frac{\left(\cot^6(dx+c)\right)}{6}+\frac{\left(\cot^4(dx+c)\right)}{4}-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+4a^3b\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-\right.}{}$

norman	$\frac{-\frac{a^4}{6d}-\frac{\left(a^4-6a^2b^2+b^4\right)\left(\tan^4(dx+c)\right)}{2d}+\frac{a^2\left(a^2-6b^2\right)\left(\tan^2(dx+c)\right)}{4d}-\frac{4a^3b\tan(dx+c)}{5d}+\frac{4ab\left(a^2-b^2\right)\left(\tan^3(dx+c)\right)}{3d}-\frac{4ab\left(a^2-b^2\right)\left(\mathrm{ta}\right.}{d}}{\tan(dx+c)^6}$
risch	$-4a^3bx+4a\,b^3x+ia^4x-6ia^2b^2x+ib^4x+\frac{2ia^4c}{d}-\frac{12ia^2b^2c}{d}+\frac{2ib^4c}{d}+\frac{72a^2b^2\mathrm{e}^{4i(dx+c)}+112ia^3b\,\mathrm{e}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(-1/6*cot(d*x+c)^6+1/4*cot(d*x+c)^4-1/2*cot(d*x+c)^2-ln(sin(d*x+c)
))+4*a^3*b*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c)+6*a^2*b^2*
(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+4*a*b^3*(-1/3*cot(d*x+c
)^3+cot(d*x+c)+d*x+c)+b^4*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c))))
```

Maxima [A]
time = 0.53, size = 195, normalized size = 0.98

$$-\frac{240\,(a^3b-ab^3)(dx+c)-30\,(a^4-6\,a^2b^2+b^4)\log\left(\tan\left(dx+c\right)^2+1\right)+60\,(a^4-6\,a^2b^2+b^4)\log\left(\tan\left(dx+c\right)\right)+\frac{240\,(a^3b-ab^3)\tan(dx+c)^5+48\,a^3b\tan(dx+c)+30\,(a^4-6\,a^2b^2+b^4)\tan(dx+c)^4+10\,a^4-80\,(a^3b-ab^3)\tan(dx+c)^3-15\,(a^4-6\,a^2b^2)\tan(dx+c)^2}{\tan(dx+c)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/60*(240*(a^3*b - a*b^3)*(d*x + c) - 30*(a^4 - 6*a^2*b^2 + b^4)*log(tan(d
*x + c)^2 + 1) + 60*(a^4 - 6*a^2*b^2 + b^4)*log(tan(d*x + c)) + (240*(a^3*b
 - a*b^3)*tan(d*x + c)^5 + 48*a^3*b*tan(d*x + c) + 30*(a^4 - 6*a^2*b^2 + b^
4)*tan(d*x + c)^4 + 10*a^4 - 80*(a^3*b - a*b^3)*tan(d*x + c)^3 - 15*(a^4 -
6*a^2*b^2)*tan(d*x + c)^2)/tan(d*x + c)^6)/d
```

Fricas [A]
time = 1.41, size = 214, normalized size = 1.08

$$-\frac{30\,(a^4-6\,a^2b^2+b^4)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^6+5\,(11\,a^4-54\,a^2b^2+6\,b^4+48\,(a^3b-ab^3)dx)\tan\left(dx+c\right)^6+240\,(a^3b-ab^3)\tan\left(dx+c\right)^5+48\,a^3b\tan\left(dx+c\right)+30\,(a^4-6\,a^2b^2+b^4)\tan\left(dx+c\right)^4+10\,a^4-80\,(a^3b-ab^3)\tan\left(dx+c\right)^3-15\,(a^4-6\,a^2b^2)\tan\left(dx+c\right)^2}{60\,d\tan\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/60*(30*(a^4 - 6*a^2*b^2 + b^4)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*
tan(d*x + c)^6 + 5*(11*a^4 - 54*a^2*b^2 + 6*b^4 + 48*(a^3*b - a*b^3)*d*x)*t
an(d*x + c)^6 + 240*(a^3*b - a*b^3)*tan(d*x + c)^5 + 48*a^3*b*tan(d*x + c)
+ 30*(a^4 - 6*a^2*b^2 + b^4)*tan(d*x + c)^4 + 10*a^4 - 80*(a^3*b - a*b^3)*t
an(d*x + c)^3 - 15*(a^4 - 6*a^2*b^2)*tan(d*x + c)^2)/(d*tan(d*x + c)^6)
```

Sympy [A]
time = 6.67, size = 340, normalized size = 1.72

$$\begin{cases} \tilde{\infty}a^4x & \text{for } (c=0 \vee c=-dx) \wedge (c=-dx \vee d=0) \\ x(a+b\tan(c))^4\cot^7(c) & \text{for } d=0 \\ \frac{a^4\log(\tan^2(c+dx)+1)}{2d}-\frac{a^4\log(\tan(c+dx))}{d}-\frac{a^4}{2d\tan^2(c+dx)}+\frac{a^4}{4d\tan^4(c+dx)}-\frac{a^4}{6d\tan^6(c+dx)}-4a^3bx-\frac{4a^3b}{d\tan(c+dx)}+\frac{4a^3b}{3d\tan^3(c+dx)}-\frac{4a^3b}{5d\tan^5(c+dx)}-\frac{3a^2b^2\log(\tan^2(c+dx)+1)}{d}+\frac{6a^2b^2\log(\tan(c+dx))}{d}+\frac{3a^2b^2}{d\tan^2(c+dx)}-\frac{3a^2b^2}{2d\tan^4(c+dx)}+4ab^3x+\frac{4ab^3}{d\tan(c+dx)}-\frac{4ab^3}{3d\tan^3(c+dx)}+\frac{b^4\log(\tan^2(c+dx)+1)}{2d}-\frac{b^4\log(\tan(c+dx))}{d}-\frac{b^4}{2d\tan^2(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Piecewise((zoo*a**4*x, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x)))
, (x*(a + b*tan(c))**4*cot(c)**7, Eq(d, 0)), (a**4*log(tan(c + d*x)**2 + 1)
/(2*d) - a**4*log(tan(c + d*x))/d - a**4/(2*d*tan(c + d*x)**2) + a**4/(4*d*
tan(c + d*x)**4) - a**4/(6*d*tan(c + d*x)**6) - 4*a**3*b*x - 4*a**3*b/(d*ta
n(c + d*x)) + 4*a**3*b/(3*d*tan(c + d*x)**3) - 4*a**3*b/(5*d*tan(c + d*x)**
5) - 3*a**2*b**2*log(tan(c + d*x)**2 + 1)/d + 6*a**2*b**2*log(tan(c + d*x))
/d + 3*a**2*b**2/(d*tan(c + d*x)**2) - 3*a**2*b**2/(2*d*tan(c + d*x)**4) +
4*a*b**3*x + 4*a*b**3/(d*tan(c + d*x)) - 4*a*b**3/(3*d*tan(c + d*x)**3) + b
**4*log(tan(c + d*x)**2 + 1)/(2*d) - b**4*log(tan(c + d*x))/d - b**4/(2*d*t
an(c + d*x)**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 506 vs. 2(188) = 376.
time = 1.37, size = 506, normalized size = 2.56

$$-\frac{5 a^4 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^6 - 48 a^3 b \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^5 - 60 a^4 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^4 + 180 a^2 b^2 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^4 + 560 a^3 b \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^3 - 320 a b^3 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^3 + 435 a^4 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^2 - 2160 a^2 b^2 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^2 + 240 b^4 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^2 - 5280 a^3 b \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right) + 4800 a b^3 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right) + 7680 (a^3 b - a b^3)(d x + c) - 1920 (a^4 - 6 a^2 b^2 + b^4) \log\left(\tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^2 + 1\right) + 1920 (a^4 - 6 a^2 b^2 + b^4) \log\left(\left|\tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)\right|\right) - \frac{4704 a^4 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^6 - 28224 a^2 b^2 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^6 + 4704 b^4 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^6 - 5280 a^3 b \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^5 + 4800 a b^3 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^5 - 435 a^4 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^4 + 2160 a^2 b^2 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^4 - 240 b^4 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^4 + 560 a^3 b \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^3 - 320 a b^3 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^3 + 60 a^4 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^2 - 180 a^2 b^2 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^2 - 48 a^3 b \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right) - 5 a^4}{\tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^6}}{1920 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/1920*(5*a^4*tan(1/2*d*x + 1/2*c)^6 - 48*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 6
0*a^4*tan(1/2*d*x + 1/2*c)^4 + 180*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 560*a^3
*b*tan(1/2*d*x + 1/2*c)^3 - 320*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 435*a^4*tan(
1/2*d*x + 1/2*c)^2 - 2160*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 240*b^4*tan(1/2*
d*x + 1/2*c)^2 - 5280*a^3*b*tan(1/2*d*x + 1/2*c) + 4800*a*b^3*tan(1/2*d*x +
 1/2*c) + 7680*(a^3*b - a*b^3)*(d*x + c) - 1920*(a^4 - 6*a^2*b^2 + b^4)*log
(tan(1/2*d*x + 1/2*c)^2 + 1) + 1920*(a^4 - 6*a^2*b^2 + b^4)*log(abs(tan(1/2
*d*x + 1/2*c))) - (4704*a^4*tan(1/2*d*x + 1/2*c)^6 - 28224*a^2*b^2*tan(1/2*
d*x + 1/2*c)^6 + 4704*b^4*tan(1/2*d*x + 1/2*c)^6 - 5280*a^3*b*tan(1/2*d*x +
 1/2*c)^5 + 4800*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 435*a^4*tan(1/2*d*x + 1/2*c
)^4 + 2160*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 - 240*b^4*tan(1/2*d*x + 1/2*c)^4
+ 560*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 320*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 60*
a^4*tan(1/2*d*x + 1/2*c)^2 - 180*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 48*a^3*b*
tan(1/2*d*x + 1/2*c) - 5*a^4)/tan(1/2*d*x + 1/2*c)^6)/d
```

Mupad [B]
time = 4.25, size = 202, normalized size = 1.02

$$\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,{\left(a+b\,1\mathrm{i}\right)}^4}{2\,d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,{\left(b+a\,1\mathrm{i}\right)}^4}{2\,d}-\frac{{\mathrm{cot}(c+d\,x)}^6\,\left({\tan(c+d\,x)}^3\,\left(\frac{4\,a\,b^3}{3}-\frac{4\,a^3\,b}{3}\right)-{\tan(c+d\,x)}^5\,\left(4\,a\,b^3-4\,a^3\,b\right)-{\tan(c+d\,x)}^2\,\left(\frac{a^4}{4}-\frac{3\,a^2\,b^2}{2}\right)+{\tan(c+d\,x)}^4\,\left(\frac{a^4}{2}-3\,a^2\,b^2+\frac{b^4}{2}\right)+\frac{a^4}{6}+\frac{4\,a^3\,b\,\tan(c+d\,x)}{5}\right)}{d}-\frac{\ln\left(\tan(c+d\,x)\right)\,\left(a^4-6\,a^2\,b^2+b^4\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^7*(a + b*tan(c + d*x))^4,x)
```

```
[Out] (log(tan(c + d*x) - 1i)*(a + b*1i)^4)/(2*d) + (log(tan(c + d*x) + 1i)*(a*1i
 + b)^4)/(2*d) - (cot(c + d*x)^6*(tan(c + d*x)^3*((4*a*b^3)/3 - (4*a^3*b)/3
) - tan(c + d*x)^5*(4*a*b^3 - 4*a^3*b) - tan(c + d*x)^2*(a^4/4 - (3*a^2*b^2
)/2) + tan(c + d*x)^4*(a^4/2 + b^4/2 - 3*a^2*b^2) + a^4/6 + (4*a^3*b*tan(c
+ d*x))/5))/d - (log(tan(c + d*x))*(a^4 + b^4 - 6*a^2*b^2))/d
```

3.456 $\int \frac{\tan^6(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=154

$$-\frac{ax}{a^2+b^2}-\frac{b\log(\cos(c+dx))}{(a^2+b^2)\,d}+\frac{a^6\log(a+b\tan(c+dx))}{b^5\,(a^2+b^2)\,d}-\frac{a(a^2-b^2)\tan(c+dx)}{b^4d}+\frac{(a^2-b^2)\tan^2(c+dx)}{2b^3d}-$$

```
[Out] -a*x/(a^2+b^2)-b*ln(cos(d*x+c))/(a^2+b^2)/d+a^6*ln(a+b*tan(d*x+c))/b^5/(a^2
+b^2)/d-a*(a^2-b^2)*tan(d*x+c)/b^4/d+1/2*(a^2-b^2)*tan(d*x+c)^2/b^3/d-1/3*a
*tan(d*x+c)^3/b^2/d+1/4*tan(d*x+c)^4/b/d
```

Rubi [A]
time = 0.39, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3647, 3728, 3729, 3708, 3698, 31, 3556}

$$-\frac{b\log(\cos(c+dx))}{d\,(a^2+b^2)}-\frac{ax}{a^2+b^2}-\frac{a(a^2-b^2)\tan(c+dx)}{b^4d}+\frac{(a^2-b^2)\tan^2(c+dx)}{2b^3d}+\frac{a^6\log(a+b\tan(c+dx))}{b^5d\,(a^2+b^2)}-\frac{a\tan^3(c+dx)}{3b^2d}+\frac{\tan^4(c+dx)}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + b*Tan[c + d*x]),x]
```

```
[Out] -((a*x)/(a^2 + b^2)) - (b*Log[Cos[c + d*x]])/((a^2 + b^2)*d) + (a^6*Log[a +
 b*Tan[c + d*x]])/(b^5*(a^2 + b^2)*d) - (a*(a^2 - b^2)*Tan[c + d*x])/(b^4*d
) + ((a^2 - b^2)*Tan[c + d*x]^2)/(2*b^3*d) - (a*Tan[c + d*x]^3)/(3*b^2*d) +
 Tan[c + d*x]^4/(4*b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
```

```
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3708

```
Int[((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[a*(A - C)*(x/(a^2 + b^2)), x] + (Dist[(a^2*C +
A*b^2)/(a^2 + b^2), Int[(1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] -
 Dist[b*((A - C)/(a^2 + b^2)), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e,
 f, A, C}, x] && NeQ[a^2*C + A*b^2, 0] && NeQ[a^2 + b^2, 0] && NeQ[A, C]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3729

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m + n +
1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*
Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*(b*c*m + a*d*(n + 1)) + d*(A*b -
 b*C)*(m + n + 1)*Tan[e + f*x] - C*m*(b*c - a*d)*Tan[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 +
 b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[
m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{\tan^6(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\tan^4(c+dx)}{4bd} + \frac{\int \frac{\tan^3(c+dx)(-4a-4b\tan(c+dx)-4a\tan^2(c+dx))}{a+b\tan(c+dx)}\,dx}{4b}$$

$$= -\frac{a\tan^3(c+dx)}{3b^2d} + \frac{\tan^4(c+dx)}{4bd} + \frac{\int \frac{\tan^2(c+dx)(12a^2+12(a^2-b^2)\tan^2(c+dx))}{a+b\tan(c+dx)}\,dx}{12b^2}$$

$$= \frac{(a^2-b^2)\tan^2(c+dx)}{2b^3d} - \frac{a\tan^3(c+dx)}{3b^2d} + \frac{\tan^4(c+dx)}{4bd} + \frac{\int \frac{\tan(c+dx)(-24a(a^2-b^2)\cdot}{}}{}$$

$$= -\frac{a(a^2-b^2)\tan(c+dx)}{b^4d} + \frac{(a^2-b^2)\tan^2(c+dx)}{2b^3d} - \frac{a\tan^3(c+dx)}{3b^2d} + \frac{\tan^4(c+}{4bd}$$

$$= -\frac{ax}{a^2+b^2} - \frac{a(a^2-b^2)\tan(c+dx)}{b^4d} + \frac{(a^2-b^2)\tan^2(c+dx)}{2b^3d} - \frac{a\tan^3(c+dx)}{3b^2d} +$$

$$= -\frac{ax}{a^2+b^2} - \frac{b\log(\cos(c+dx))}{(a^2+b^2)\,d} - \frac{a(a^2-b^2)\tan(c+dx)}{b^4d} + \frac{(a^2-b^2)\tan^2(c+dx}{2b^3d}$$

$$= -\frac{ax}{a^2+b^2} - \frac{b\log(\cos(c+dx))}{(a^2+b^2)\,d} + \frac{a^6\log(a+b\tan(c+dx))}{b^5\,(a^2+b^2)\,d} - \frac{a(a^2-b^2)\tan(c+}{b^4d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.76, size = 167, normalized size = 1.08

$$\frac{6(b^5(ia+b)\log(i-\tan(c+dx))+b^5(-ia+b)\log(i+\tan(c+dx))+2a^6\log(a+b\tan(c+dx)))-12ab(a^4-b^4)\tan(c+dx)+6b^2(a^4-b^4)\tan^2(c+dx)-4ab^3(a^2+b^2)\tan^3(c+dx)+3b^4(a^2+b^2)\tan^4(c+dx)}{12b^5\,(a^2+b^2)\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^6/(a + b*Tan[c + d*x]),x]
```

```
[Out] (6*(b^5*(I*a + b)*Log[I - Tan[c + d*x]] + b^5*((-I)*a + b)*Log[I + Tan[c +
d*x]] + 2*a^6*Log[a + b*Tan[c + d*x]]) - 12*a*b*(a^4 - b^4)*Tan[c + d*x] +
6*b^2*(a^4 - b^4)*Tan[c + d*x]^2 - 4*a*b^3*(a^2 + b^2)*Tan[c + d*x]^3 + 3*b
^4*(a^2 + b^2)*Tan[c + d*x]^4)/(12*b^5*(a^2 + b^2)*d)
```

Maple [A]
time = 0.19, size = 138, normalized size = 0.90

method	result
derivativedivides	$-\frac{-\frac{b^3\left(\tan^4(dx+c)\right)}{4}+\frac{b^2a\left(\tan^3(dx+c)\right)}{3}-\frac{\left(a^2-b^2\right)\left(\tan^2(dx+c)\right)b}{2}+a\left(a^2-b^2\right)\tan(dx+c)}{b^4}+\frac{\frac{b\ln\left(1+\tan^2(dx+c)\right)}{2}-a\arctan(\tan(dx}{a^2+b^2}}{d}$
default	$-\frac{-\frac{b^3\left(\tan^4(dx+c)\right)}{4}+\frac{b^2a\left(\tan^3(dx+c)\right)}{3}-\frac{\left(a^2-b^2\right)\left(\tan^2(dx+c)\right)b}{2}+a\left(a^2-b^2\right)\tan(dx+c)}{b^4}+\frac{\frac{b\ln\left(1+\tan^2(dx+c)\right)}{2}-a\arctan(\tan(dx}{a^2+b^2}}{d}$

norman	$-\frac{ax}{a^2+b^2}+\frac{\tan^4(dx+c)}{4bd}-\frac{a(\tan^3(dx+c))}{3b^2d}+\frac{(a^2-b^2)(\tan^2(dx+c))}{2b^3d}-\frac{a(a^2-b^2)\tan(dx+c)}{b^4d}+\frac{a^6\ln(a+b\tan(dx+}{b^5(a^2+b^2)d}$
risch	$\frac{x}{ib-a}+\frac{2ia^4x}{b^5}+\frac{2ia^4c}{b^5d}-\frac{2ix\,a^2}{b^3}-\frac{2ia^2c}{b^3d}+\frac{2ix}{b}+\frac{2ic}{bd}-\frac{2ia^6x}{(a^2+b^2)b^5}-\frac{2ia^6c}{(a^2+b^2)b^5d}-\frac{2(3ia^3e^{6i(dx+c)}-6ia\,b^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^4*(-1/4*b^3*tan(d*x+c)^4+1/3*b^2*a*tan(d*x+c)^3-1/2*(a^2-b^2)*tan
(d*x+c)^2*b+a*(a^2-b^2)*tan(d*x+c))+1/(a^2+b^2)*(1/2*b*ln(1+tan(d*x+c)^2)-a
*arctan(tan(d*x+c)))+1/b^5*a^6/(a^2+b^2)*ln(a+b*tan(d*x+c)))
```

Maxima [A]
time = 0.52, size = 146, normalized size = 0.95

$$\frac{\frac{12\,a^6\log(b\tan(dx+c)+a)}{a^2b^5+b^7}-\frac{12\,(dx+c)a}{a^2+b^2}+\frac{6\,b\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}+\frac{3\,b^3\tan(dx+c)^4-4\,ab^2\tan(dx+c)^3+6\,(a^2b-b^3)\tan(dx+c)^2-12\,(a^3-ab^2)\tan(dx+c)}{b^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(12*a^6*log(b*tan(d*x + c) + a)/(a^2*b^5 + b^7) - 12*(d*x + c)*a/(a^2
+ b^2) + 6*b*log(tan(d*x + c)^2 + 1)/(a^2 + b^2) + (3*b^3*tan(d*x + c)^4 -
4*a*b^2*tan(d*x + c)^3 + 6*(a^2*b - b^3)*tan(d*x + c)^2 - 12*(a^3 - a*b^2)*
tan(d*x + c))/b^4)/d
```

Fricas [A]
time = 1.16, size = 181, normalized size = 1.18

$$-\frac{12\,ab^5dx-6\,a^6\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)-3\,(a^2b^4+b^6)\tan(dx+c)^4+4\,(a^3b^3+ab^5)\tan(dx+c)^3-6\,(a^4b^2-b^6)\tan(dx+c)^2+6\,(a^6+b^6)\log\left(\frac{1}{\tan(dx+c)^2+1}\right)+12\,(a^5b-ab^5)\tan(dx+c)}{12\,(a^2b^5+b^7)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(12*a*b^5*d*x - 6*a^6*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) +
a^2)/(tan(d*x + c)^2 + 1)) - 3*(a^2*b^4 + b^6)*tan(d*x + c)^4 + 4*(a^3*b^3
+ a*b^5)*tan(d*x + c)^3 - 6*(a^4*b^2 - b^6)*tan(d*x + c)^2 + 6*(a^6 + b^6)*
log(1/(tan(d*x + c)^2 + 1)) + 12*(a^5*b - a*b^5)*tan(d*x + c))/((a^2*b^5 +
b^7)*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.22, size = 947, normalized size = 6.15

$$\begin{cases} \tilde{\infty}x\tan^5(c) & \text{for}\ a=0\wedge b=0\wedge d=0 \\ \frac{-x+\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}+\frac{\tan(c+dx)}{d}}{a} & \text{for}\ b=0 \\ \frac{30idx\tan(c+dx)}{12bd\tan(c+dx)-12ibd}+\frac{30dx}{12bd\tan(c+dx)-12ibd}+\frac{18\log(\tan^2(c+dx)+1)\tan(c+dx)}{12bd\tan(c+dx)-12ibd}-\frac{18i\log(\tan^2(c+dx)+1)}{12bd\tan(c+dx)-12ibd}+\frac{3\tan^5(c+dx)}{12bd\tan(c+dx)-12ibd}+\frac{i\tan^4(c+dx)}{12bd\tan(c+dx)-12ibd}-\frac{8\tan^3(c+dx)}{12bd\tan(c+dx)-12ibd}-\frac{12i\tan^2(c+dx)}{12bd\tan(c+dx)-12ibd}-\frac{30i}{12bd\tan(c+dx)-12ibd} & \text{for}\ a=-ib \\ -\frac{30idx\tan(c+dx)}{12bd\tan(c+dx)+12ibd}+\frac{30dx}{12bd\tan(c+dx)+12ibd}+\frac{18\log(\tan^2(c+dx)+1)\tan(c+dx)}{12bd\tan(c+dx)+12ibd}+\frac{18i\log(\tan^2(c+dx)+1)}{12bd\tan(c+dx)+12ibd}+\frac{3\tan^5(c+dx)}{12bd\tan(c+dx)+12ibd}-\frac{i\tan^4(c+dx)}{12bd\tan(c+dx)+12ibd}-\frac{8\tan^3(c+dx)}{12bd\tan(c+dx)+12ibd}+\frac{12i\tan^2(c+dx)}{12bd\tan(c+dx)+12ibd}+\frac{30i}{12bd\tan(c+dx)+12ibd} & \text{for}\ a=ib \\ \frac{x\tan^6(c)}{a+b\tan(c)} & \text{for}\ d=0 \\ \frac{12a^6\log(\frac{a}{b}+\tan(c+dx))}{12a^2b^5d+12b^7d}-\frac{12a^5b\tan(c+dx)}{12a^2b^5d+12b^7d}+\frac{6a^4b^2\tan^2(c+dx)}{12a^2b^5d+12b^7d}-\frac{4a^3b^3\tan^3(c+dx)}{12a^2b^5d+12b^7d}+\frac{3a^2b^4\tan^4(c+dx)}{12a^2b^5d+12b^7d}-\frac{12ab^5dx}{12a^2b^5d+12b^7d}-\frac{4ab^5\tan^3(c+dx)}{12a^2b^5d+12b^7d}+\frac{12ab^5\tan(c+dx)}{12a^2b^5d+12b^7d}+\frac{6b^6\log(\tan^2(c+dx)+1)}{12a^2b^5d+12b^7d}+\frac{3b^6\tan^4(c+dx)}{12a^2b^5d+12b^7d}-\frac{6b^6\tan^2(c+dx)}{12a^2b^5d+12b^7d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*tan(c)**5, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((-x + tan(c +
 d*x)**5/(5*d) - tan(c + d*x)**3/(3*d) + tan(c + d*x)/d)/a, Eq(b, 0)), (30*
I*d*x*tan(c + d*x)/(12*b*d*tan(c + d*x) - 12*I*b*d) + 30*d*x/(12*b*d*tan(c
+ d*x) - 12*I*b*d) + 18*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(12*b*d*tan(c
 + d*x) - 12*I*b*d) - 18*I*log(tan(c + d*x)**2 + 1)/(12*b*d*tan(c + d*x) -
12*I*b*d) + 3*tan(c + d*x)**5/(12*b*d*tan(c + d*x) - 12*I*b*d) + I*tan(c +
d*x)**4/(12*b*d*tan(c + d*x) - 12*I*b*d) - 8*tan(c + d*x)**3/(12*b*d*tan(c
+ d*x) - 12*I*b*d) - 12*I*tan(c + d*x)**2/(12*b*d*tan(c + d*x) - 12*I*b*d)
- 30*I/(12*b*d*tan(c + d*x) - 12*I*b*d), Eq(a, -I*b)), (-30*I*d*x*tan(c + d
*x)/(12*b*d*tan(c + d*x) + 12*I*b*d) + 30*d*x/(12*b*d*tan(c + d*x) + 12*I*b
*d) + 18*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(12*b*d*tan(c + d*x) + 12*I*
b*d) + 18*I*log(tan(c + d*x)**2 + 1)/(12*b*d*tan(c + d*x) + 12*I*b*d) + 3*t
an(c + d*x)**5/(12*b*d*tan(c + d*x) + 12*I*b*d) - I*tan(c + d*x)**4/(12*b*d
*tan(c + d*x) + 12*I*b*d) - 8*tan(c + d*x)**3/(12*b*d*tan(c + d*x) + 12*I*b
*d) + 12*I*tan(c + d*x)**2/(12*b*d*tan(c + d*x) + 12*I*b*d) + 30*I/(12*b*d*
tan(c + d*x) + 12*I*b*d), Eq(a, I*b)), (x*tan(c)**6/(a + b*tan(c)), Eq(d, 0
)), (12*a**6*log(a/b + tan(c + d*x))/(12*a**2*b**5*d + 12*b**7*d) - 12*a**5
*b*tan(c + d*x)/(12*a**2*b**5*d + 12*b**7*d) + 6*a**4*b**2*tan(c + d*x)**2/
(12*a**2*b**5*d + 12*b**7*d) - 4*a**3*b**3*tan(c + d*x)**3/(12*a**2*b**5*d
+ 12*b**7*d) + 3*a**2*b**4*tan(c + d*x)**4/(12*a**2*b**5*d + 12*b**7*d) - 1
2*a*b**5*d*x/(12*a**2*b**5*d + 12*b**7*d) - 4*a*b**5*tan(c + d*x)**3/(12*a*
*2*b**5*d + 12*b**7*d) + 12*a*b**5*tan(c + d*x)/(12*a**2*b**5*d + 12*b**7*d
) + 6*b**6*log(tan(c + d*x)**2 + 1)/(12*a**2*b**5*d + 12*b**7*d) + 3*b**6*t
an(c + d*x)**4/(12*a**2*b**5*d + 12*b**7*d) - 6*b**6*tan(c + d*x)**2/(12*a*
*2*b**5*d + 12*b**7*d), True))
```

Giac [A]
time = 2.45, size = 158, normalized size = 1.03

$$\frac{\frac{12\,a^6\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^2b^5+b^7}-\frac{12\,\left(dx+c\right)a}{a^2+b^2}+\frac{6\,b\log\left(\tan\left(dx+c\right)^2+1\right)}{a^2+b^2}+\frac{3\,b^3\tan\left(dx+c\right)^4-4\,ab^2\tan\left(dx+c\right)^3+6\,a^2b\tan\left(dx+c\right)^2-6\,b^3\tan\left(dx+c\right)^2-12\,a^3\tan\left(dx+c\right)+12\,ab^2\tan\left(dx+c\right)}{b^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*a^6*log(abs(b*tan(d*x + c) + a))/(a^2*b^5 + b^7) - 12*(d*x + c)*a/
(a^2 + b^2) + 6*b*log(tan(d*x + c)^2 + 1)/(a^2 + b^2) + (3*b^3*tan(d*x + c)
^4 - 4*a*b^2*tan(d*x + c)^3 + 6*a^2*b*tan(d*x + c)^2 - 6*b^3*tan(d*x + c)^2
 - 12*a^3*tan(d*x + c) + 12*a*b^2*tan(d*x + c))/b^4)/d
```

Mupad [B]
time = 4.26, size = 165, normalized size = 1.07

$$\frac{\ln\left(\mathrm{tan}\left(c+d\,x\right)+1{}\mathrm{i}\right)}{2\,d\,\left(b+a\,1{}\mathrm{i}\right)}-\frac{{\mathrm{tan}\left(c+d\,x\right)}^2\,\left(\frac{1}{2\,b}-\frac{a^2}{2\,b^3}\right)}{d}+\frac{{\mathrm{tan}\left(c+d\,x\right)}^4}{4\,b\,d}-\frac{a\,{\mathrm{tan}\left(c+d\,x\right)}^3}{3\,b^2\,d}+\frac{a^6\,\ln\left(a+b\,\mathrm{tan}\left(c+d\,x\right)\right)}{d\,\left(a^2\,b^5+b^7\right)}+\frac{a\,\mathrm{tan}\left(c+d\,x\right)\,\left(\frac{1}{b}-\frac{a^2}{b^3}\right)}{b\,d}+\frac{\ln\left(\mathrm{tan}\left(c+d\,x\right)-\mathrm{i}\right)\,1{}\mathrm{i}}{2\,d\,\left(a+b\,1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d*x)^6/(a + b*tan(c + d*x)),x)

[Out] (log(tan(c + d*x) - 1i)*1i)/(2*d*(a + b*1i)) + log(tan(c + d*x) + 1i)/(2*d*(a*1i + b)) - (tan(c + d*x)^2*(1/(2*b) - a^2/(2*b^3)))/d + tan(c + d*x)^4/(4*b*d) - (a*tan(c + d*x)^3)/(3*b^2*d) + (a^6*log(a + b*tan(c + d*x)))/(d*(b^7 + a^2*b^5)) + (a*tan(c + d*x)*(1/b - a^2/b^3))/(b*d)

3.457 $\int \frac{\tan^5(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=125

$$\frac{bx}{a^2+b^2}-\frac{a\log(\cos(c+dx))}{(a^2+b^2)\,d}-\frac{a^5\log(a+b\tan(c+dx))}{b^4\,(a^2+b^2)\,d}+\frac{(a^2-b^2)\tan(c+dx)}{b^3d}-\frac{a\tan^2(c+dx)}{2b^2d}+\frac{\tan^3(c+d}{3bd}$$

```
[Out] b*x/(a^2+b^2)-a*ln(cos(d*x+c))/(a^2+b^2)/d-a^5*ln(a+b*tan(d*x+c))/b^4/(a^2+
b^2)/d+(a^2-b^2)*tan(d*x+c)/b^3/d-1/2*a*tan(d*x+c)^2/b^2/d+1/3*tan(d*x+c)^3
/b/d
```

Rubi [A]
time = 0.26, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3647, 3728, 3729, 3707, 3698, 31, 3556}

$$-\frac{a\log(\cos(c+dx))}{d\,(a^2+b^2)}+\frac{bx}{a^2+b^2}+\frac{(a^2-b^2)\tan(c+dx)}{b^3d}-\frac{a^5\log(a+b\tan(c+dx))}{b^4d\,(a^2+b^2)}-\frac{a\tan^2(c+dx)}{2b^2d}+\frac{\tan^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*x)/(a^2 + b^2) - (a*Log[Cos[c + d*x]])/((a^2 + b^2)*d) - (a^5*Log[a + b*
Tan[c + d*x]])/(b^4*(a^2 + b^2)*d) + ((a^2 - b^2)*Tan[c + d*x])/(b^3*d) - (
a*Tan[c + d*x]^2)/(2*b^2*d) + Tan[c + d*x]^3/(3*b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
```

```
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3729

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m + n +
1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*
Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*(b*c*m + a*d*(n + 1)) + d*(A*b -
 b*C)*(m + n + 1)*Tan[e + f*x] - C*m*(b*c - a*d)*Tan[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 +
 b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[
m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\tan^3(c+dx)}{3bd} + \frac{\int \frac{\tan^2(c+dx)(-3a-3b\tan(c+dx)-3a\tan^2(c+dx))}{a+b\tan(c+dx)}\,dx}{3b}$$
$$= -\frac{a\tan^2(c+dx)}{2b^2d} + \frac{\tan^3(c+dx)}{3bd} + \frac{\int \frac{\tan(c+dx)(6a^2+6(a^2-b^2)\tan^2(c+dx))}{a+b\tan(c+dx)}\,dx}{6b^2}$$
$$= \frac{(a^2-b^2)\tan(c+dx)}{b^3d} - \frac{a\tan^2(c+dx)}{2b^2d} + \frac{\tan^3(c+dx)}{3bd} + \frac{\int \frac{-6a(a^2-b^2)+6b^3\tan(c+dx}{a+b\tan}}{6}$$
$$= \frac{bx}{a^2+b^2} + \frac{(a^2-b^2)\tan(c+dx)}{b^3d} - \frac{a\tan^2(c+dx)}{2b^2d} + \frac{\tan^3(c+dx)}{3bd} + \frac{a\int \tan(c+}{a^2+b}$$
$$= \frac{bx}{a^2+b^2} - \frac{a\log(\cos(c+dx))}{(a^2+b^2)\,d} + \frac{(a^2-b^2)\tan(c+dx)}{b^3d} - \frac{a\tan^2(c+dx)}{2b^2d} + \frac{\tan^3(c}{3b}$$
$$= \frac{bx}{a^2+b^2} - \frac{a\log(\cos(c+dx))}{(a^2+b^2)\,d} - \frac{a^5\log(a+b\tan(c+dx))}{b^4(a^2+b^2)\,d} + \frac{(a^2-b^2)\tan(c+dx}{b^3d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.62, size = 155, normalized size = 1.24

$$\frac{\log(i-\tan(c+dx))}{2(a+ib)d} + \frac{\log(i+\tan(c+dx))}{2(a-ib)d} - \frac{a^5\log(a+b\tan(c+dx))}{b^4(a^2+b^2)\,d} + \frac{a^2\tan(c+dx)}{b^3d} - \frac{\tan(c+dx)}{bd} - \frac{a\tan^2(c+dx)}{2b^2d} + \frac{\tan^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + b*Tan[c + d*x]),x]
```

```
[Out] Log[I - Tan[c + d*x]]/(2*(a + I*b)*d) + Log[I + Tan[c + d*x]]/(2*(a - I*b)*
d) - (a^5*Log[a + b*Tan[c + d*x]])/(b^4*(a^2 + b^2)*d) + (a^2*Tan[c + d*x])
/(b^3*d) - Tan[c + d*x]/(b*d) - (a*Tan[c + d*x]^2)/(2*b^2*d) + Tan[c + d*x]
^3/(3*b*d)
```

Maple [A]
time = 0.18, size = 119, normalized size = 0.95

method	result
derivativedivides	$\frac{\frac{b^2\left(\tan^3(dx+c)\right)}{3}-\frac{ab\left(\tan^2(dx+c)\right)}{2}+a^2\tan(dx+c)-b^2\tan(dx+c)}{b^3}+\frac{\frac{a\ln\left(1+\tan^2(dx+c)\right)}{2}+b\arctan(\tan(dx+c))}{a^2+b^2}-\frac{a^5\ln(a+b\tan(dx+}{b^4\left(a^2+b^2\right)}}{d}$
default	$\frac{\frac{b^2\left(\tan^3(dx+c)\right)}{3}-\frac{ab\left(\tan^2(dx+c)\right)}{2}+a^2\tan(dx+c)-b^2\tan(dx+c)}{b^3}+\frac{\frac{a\ln\left(1+\tan^2(dx+c)\right)}{2}+b\arctan(\tan(dx+c))}{a^2+b^2}-\frac{a^5\ln(a+b\tan(dx+}{b^4\left(a^2+b^2\right)}}{d}$
norman	$\frac{bx}{a^2+b^2}+\frac{(a^2-b^2)\tan(dx+c)}{b^3d}+\frac{\tan^3(dx+c)}{3bd}-\frac{a\left(\tan^2(dx+c)\right)}{2b^2d}+\frac{a\ln\left(1+\tan^2(dx+c)\right)}{2d(a^2+b^2)}-\frac{a^5\ln(a+b\tan(dx+c))}{b^4(a^2+b^2)d}$

risch	$\frac{ix}{ib-a} - \frac{2ia^3x}{b^4} - \frac{2ia^3c}{b^4d} + \frac{2iax}{b^2} + \frac{2iac}{b^2d} + \frac{2ia^5x}{b^4(a^2+b^2)} + \frac{2ia^5c}{b^4d(a^2+b^2)} + \frac{2i(3iab\,e^{4i(dx+c)}+3a^2e^{4i(dx+c)}-6b^2e^{4i(d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^3*(1/3*b^2*tan(d*x+c)^3-1/2*a*b*tan(d*x+c)^2+a^2*tan(d*x+c)-b^2*ta
n(d*x+c))+1/(a^2+b^2)*(1/2*a*ln(1+tan(d*x+c)^2)+b*arctan(tan(d*x+c)))-1/b^4
*a^5/(a^2+b^2)*ln(a+b*tan(d*x+c)))
```

Maxima [A]
time = 0.53, size = 123, normalized size = 0.98

$$-\frac{\frac{6\,a^5\log\left(b\tan\left(dx+c\right)+a\right)}{a^2b^4+b^6} - \frac{6\,(dx+c)b}{a^2+b^2} - \frac{3\,a\log\left(\tan\left(dx+c\right)^2+1\right)}{a^2+b^2} - \frac{2\,b^2\tan\left(dx+c\right)^3-3\,ab\tan\left(dx+c\right)^2+6\,(a^2-b^2)\tan\left(dx+c\right)}{b^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(6*a^5*log(b*tan(d*x + c) + a)/(a^2*b^4 + b^6) - 6*(d*x + c)*b/(a^2 +
b^2) - 3*a*log(tan(d*x + c)^2 + 1)/(a^2 + b^2) - (2*b^2*tan(d*x + c)^3 - 3*
a*b*tan(d*x + c)^2 + 6*(a^2 - b^2)*tan(d*x + c))/b^3)/d
```

Fricas [A]
time = 1.72, size = 159, normalized size = 1.27

$$\frac{6\,b^5dx - 3\,a^5\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right) + 2\,(a^2b^3+b^5)\tan\left(dx+c\right)^3 - 3\,(a^3b^2+ab^4)\tan\left(dx+c\right)^2 + 3\,(a^5-ab^4)\log\left(\frac{1}{\tan(dx+c)^2+1}\right) + 6\,(a^4b-b^5)\tan\left(dx+c\right)}{6\,(a^2b^4+b^6)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(6*b^5*d*x - 3*a^5*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/
(tan(d*x + c)^2 + 1)) + 2*(a^2*b^3 + b^5)*tan(d*x + c)^3 - 3*(a^3*b^2 + a*b
^4)*tan(d*x + c)^2 + 3*(a^5 - a*b^4)*log(1/(tan(d*x + c)^2 + 1)) + 6*(a^4*b
 - b^5)*tan(d*x + c))/((a^2*b^4 + b^6)*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.79, size = 821, normalized size = 6.57

$$\begin{cases} \tilde{\infty}x\tan^4(c) & \text{for}\: a=0\wedge b=0\wedge d=0 \\ \frac{\frac{\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{\tan^4(c+dx)}{4d}-\frac{\tan^2(c+dx)}{2d}}{a} & \text{for}\: b=0 \\ \frac{15dx\tan(c+dx)}{6bd\tan(c+dx)-6ibd} - \frac{15idx}{6bd\tan(c+dx)-6ibd} - \frac{6i\log\left(\tan^2(c+dx)+1\right)\tan(c+dx)}{6bd\tan(c+dx)-6ibd} - \frac{6\log\left(\tan^2(c+dx)+1\right)}{6bd\tan(c+dx)-6ibd} + \frac{2\tan^4(c+dx)}{6bd\tan(c+dx)-6ibd} + \frac{i\tan^3(c+dx)}{6bd\tan(c+dx)-6ibd} - \frac{9\tan^2(c+dx)}{6bd\tan(c+dx)-6ibd} - \frac{15}{6bd\tan(c+dx)-6ibd} & \text{for}\: a=-ib \\ \frac{15dx\tan(c+dx)}{6bd\tan(c+dx)+6ibd} + \frac{15idx}{6bd\tan(c+dx)+6ibd} + \frac{6i\log\left(\tan^2(c+dx)+1\right)\tan(c+dx)}{6bd\tan(c+dx)+6ibd} - \frac{6\log\left(\tan^2(c+dx)+1\right)}{6bd\tan(c+dx)+6ibd} + \frac{2\tan^4(c+dx)}{6bd\tan(c+dx)+6ibd} - \frac{i\tan^3(c+dx)}{6bd\tan(c+dx)+6ibd} - \frac{9\tan^2(c+dx)}{6bd\tan(c+dx)+6ibd} - \frac{15}{6bd\tan(c+dx)+6ibd} & \text{for}\: a=ib \\ \frac{x\tan^5(c)}{a+b\tan(c)} & \text{for}\: d=0 \\ -\frac{6a^5\log\left(\frac{a}{b}+\tan(c+dx)\right)}{6a^2b^4d+6b^6d} + \frac{6a^4b\tan(c+dx)}{6a^2b^4d+6b^6d} - \frac{3a^3b^2\tan^2(c+dx)}{6a^2b^4d+6b^6d} + \frac{2a^2b^3\tan^3(c+dx)}{6a^2b^4d+6b^6d} + \frac{3ab^4\log\left(\tan^2(c+dx)+1\right)}{6a^2b^4d+6b^6d} - \frac{3ab^4\tan^2(c+dx)}{6a^2b^4d+6b^6d} + \frac{6b^5dx}{6a^2b^4d+6b^6d} + \frac{2b^5\tan^3(c+dx)}{6a^2b^4d+6b^6d} - \frac{6b^5\tan(c+dx)}{6a^2b^4d+6b^6d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*tan(c)**4, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((log(tan(c +
d*x)**2 + 1)/(2*d) + tan(c + d*x)**4/(4*d) - tan(c + d*x)**2/(2*d))/a, Eq(b
, 0)), (15*d*x*tan(c + d*x)/(6*b*d*tan(c + d*x) - 6*I*b*d) - 15*I*d*x/(6*b*
d*tan(c + d*x) - 6*I*b*d) - 6*I*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(6*b*
d*tan(c + d*x) - 6*I*b*d) - 6*log(tan(c + d*x)**2 + 1)/(6*b*d*tan(c + d*x)
- 6*I*b*d) + 2*tan(c + d*x)**4/(6*b*d*tan(c + d*x) - 6*I*b*d) + I*tan(c + d
*x)**3/(6*b*d*tan(c + d*x) - 6*I*b*d) - 9*tan(c + d*x)**2/(6*b*d*tan(c + d*
x) - 6*I*b*d) - 15/(6*b*d*tan(c + d*x) - 6*I*b*d), Eq(a, -I*b)), (15*d*x*ta
n(c + d*x)/(6*b*d*tan(c + d*x) + 6*I*b*d) + 15*I*d*x/(6*b*d*tan(c + d*x) +
6*I*b*d) + 6*I*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(6*b*d*tan(c + d*x) +
6*I*b*d) - 6*log(tan(c + d*x)**2 + 1)/(6*b*d*tan(c + d*x) + 6*I*b*d) + 2*ta
n(c + d*x)**4/(6*b*d*tan(c + d*x) + 6*I*b*d) - I*tan(c + d*x)**3/(6*b*d*tan
(c + d*x) + 6*I*b*d) - 9*tan(c + d*x)**2/(6*b*d*tan(c + d*x) + 6*I*b*d) - 1
5/(6*b*d*tan(c + d*x) + 6*I*b*d), Eq(a, I*b)), (x*tan(c)**5/(a + b*tan(c)),
 Eq(d, 0)), (-6*a**5*log(a/b + tan(c + d*x))/(6*a**2*b**4*d + 6*b**6*d) + 6
*a**4*b*tan(c + d*x)/(6*a**2*b**4*d + 6*b**6*d) - 3*a**3*b**2*tan(c + d*x)*
*2/(6*a**2*b**4*d + 6*b**6*d) + 2*a**2*b**3*tan(c + d*x)**3/(6*a**2*b**4*d
+ 6*b**6*d) + 3*a*b**4*log(tan(c + d*x)**2 + 1)/(6*a**2*b**4*d + 6*b**6*d)
- 3*a*b**4*tan(c + d*x)**2/(6*a**2*b**4*d + 6*b**6*d) + 6*b**5*d*x/(6*a**2*
b**4*d + 6*b**6*d) + 2*b**5*tan(c + d*x)**3/(6*a**2*b**4*d + 6*b**6*d) - 6*
b**5*tan(c + d*x)/(6*a**2*b**4*d + 6*b**6*d), True))
```

Giac [A]
time = 1.96, size = 129, normalized size = 1.03

$$-\frac{\frac{6\,a^5\log(|b\tan(dx+c)+a|)}{a^2b^4+b^6}-\frac{6\,(dx+c)b}{a^2+b^2}-\frac{3\,a\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}-\frac{2\,b^2\tan(dx+c)^3-3\,ab\tan(dx+c)^2+6\,a^2\tan(dx+c)-6\,b^2\tan(dx+c)}{b^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*a^5*log(abs(b*tan(d*x + c) + a))/(a^2*b^4 + b^6) - 6*(d*x + c)*b/(a
^2 + b^2) - 3*a*log(tan(d*x + c)^2 + 1)/(a^2 + b^2) - (2*b^2*tan(d*x + c)^3
 - 3*a*b*tan(d*x + c)^2 + 6*a^2*tan(d*x + c) - 6*b^2*tan(d*x + c))/b^3)/d
```

Mupad [B]
time = 4.08, size = 138, normalized size = 1.10

$$\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)}{2\,d\,\left(a-b\,1\mathrm{i}\right)}-\frac{\tan(c+d\,x)\,\left(\frac{1}{b}-\frac{a^2}{b^3}\right)}{d}+\frac{\tan(c+d\,x)^3}{3\,b\,d}-\frac{a\tan(c+d\,x)^2}{2\,b^2\,d}-\frac{a^5\,\ln\left(a+b\tan(c+d\,x)\right)}{d\,\left(a^2\,b^4+b^6\right)}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(-b+a\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + b*tan(c + d*x)),x)
```

```
[Out] log(tan(c + d*x) + 1i)/(2*d*(a - b*1i)) - (tan(c + d*x)*(1/b - a^2/b^3))/d
+ (log(tan(c + d*x) - 1i)*1i)/(2*d*(a*1i - b)) + tan(c + d*x)^3/(3*b*d) - (
a*tan(c + d*x)^2)/(2*b^2*d) - (a^5*log(a + b*tan(c + d*x)))/(d*(b^6 + a^2*b
^4))
```

3.458 $\int \frac{\tan^4(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=97

$$\frac{ax}{a^2+b^2}+\frac{b\log(\cos(c+dx))}{(a^2+b^2)\,d}+\frac{a^4\log(a+b\tan(c+dx))}{b^3\,(a^2+b^2)\,d}-\frac{a\tan(c+dx)}{b^2d}+\frac{\tan^2(c+dx)}{2bd}$$

```
[Out] a*x/(a^2+b^2)+b*ln(cos(d*x+c))/(a^2+b^2)/d+a^4*ln(a+b*tan(d*x+c))/b^3/(a^2+
b^2)/d-a*tan(d*x+c)/b^2/d+1/2*tan(d*x+c)^2/b/d
```

Rubi [A]
time = 0.15, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3647, 3728, 3708, 3698, 31, 3556}

$$\frac{b\log(\cos(c+dx))}{d\,(a^2+b^2)}+\frac{ax}{a^2+b^2}+\frac{a^4\log(a+b\tan(c+dx))}{b^3d\,(a^2+b^2)}-\frac{a\tan(c+dx)}{b^2d}+\frac{\tan^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*x)/(a^2 + b^2) + (b*Log[Cos[c + d*x]])/((a^2 + b^2)*d) + (a^4*Log[a + b*
Tan[c + d*x]])/(b^3*(a^2 + b^2)*d) - (a*Tan[c + d*x])/(b^2*d) + Tan[c + d*x
]^2/(2*b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3708

```
Int[((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[a*(A - C)*(x/(a^2 + b^2)), x] + (Dist[(a^2*C +
A*b^2)/(a^2 + b^2), Int[(1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] -
 Dist[b*((A - C)/(a^2 + b^2)), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e,
 f, A, C}, x] && NeQ[a^2*C + A*b^2, 0] && NeQ[a^2 + b^2, 0] && NeQ[A, C]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^4(c+dx)}{a+b\tan(c+dx)}\,dx &= \frac{\tan^2(c+dx)}{2bd}+\frac{\int \frac{\tan(c+dx)\left(-2a-2b\tan(c+dx)-2a\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx}{2b}\\
&= -\frac{a\tan(c+dx)}{b^2d}+\frac{\tan^2(c+dx)}{2bd}+\frac{\int \frac{2a^2+2\left(a^2-b^2\right)\tan^2(c+dx)}{a+b\tan(c+dx)}\,dx}{2b^2}\\
&= \frac{ax}{a^2+b^2}-\frac{a\tan(c+dx)}{b^2d}+\frac{\tan^2(c+dx)}{2bd}+\frac{a^4\int \frac{1+\tan^2(c+dx)}{a+b\tan(c+dx)}\,dx}{b^2\left(a^2+b^2\right)}-\frac{b\int \tan(c+dx)\,d}{a^2+b^2}\\
&= \frac{ax}{a^2+b^2}+\frac{b\log(\cos(c+dx))}{\left(a^2+b^2\right)d}-\frac{a\tan(c+dx)}{b^2d}+\frac{\tan^2(c+dx)}{2bd}+\frac{a^4\mathrm{Subst}\left(\int \frac{1}{a+x}\,dx\right.}{b^3\left(a^2\right.}\\
&= \frac{ax}{a^2+b^2}+\frac{b\log(\cos(c+dx))}{\left(a^2+b^2\right)d}+\frac{a^4\log(a+b\tan(c+dx))}{b^3\left(a^2+b^2\right)d}-\frac{a\tan(c+dx)}{b^2d}+\frac{\tan^2(}{2}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.45, size = 107, normalized size = 1.10

$$\frac{\frac{\log(i-\tan(c+dx))}{ia-b}-\frac{\log(i+\tan(c+dx))}{ia+b}+\frac{2a^4\log(a+b\tan(c+dx))}{b^3(a^2+b^2)}-\frac{2a\tan(c+dx)}{b^2}+\frac{\tan^2(c+dx)}{b}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

```
[Out] (Log[I - Tan[c + d*x]]/(I*a - b) - Log[I + Tan[c + d*x]]/(I*a + b) + (2*a^4
*Log[a + b*Tan[c + d*x]])/(b^3*(a^2 + b^2)) - (2*a*Tan[c + d*x])/b^2 + Tan[
c + d*x]^2/b)/(2*d)
```

Maple [A]
time = 0.12, size = 92, normalized size = 0.95

method	result
derivativedivides	$\frac{-\frac{-\frac{b\left(\tan^2(dx+c)\right)}{2}+a\tan(dx+c)}{b^2}+\frac{-\frac{b\ln\left(1+\tan^2(dx+c)\right)}{2}+a\arctan(\tan(dx+c))}{a^2+b^2}+\frac{a^4\ln(a+b\tan(dx+c))}{b^3\left(a^2+b^2\right)}}{d}$
default	$\frac{-\frac{-\frac{b\left(\tan^2(dx+c)\right)}{2}+a\tan(dx+c)}{b^2}+\frac{-\frac{b\ln\left(1+\tan^2(dx+c)\right)}{2}+a\arctan(\tan(dx+c))}{a^2+b^2}+\frac{a^4\ln(a+b\tan(dx+c))}{b^3\left(a^2+b^2\right)}}{d}$
norman	$\frac{ax}{a^2+b^2}+\frac{\tan^2(dx+c)}{2bd}-\frac{a\tan(dx+c)}{b^2d}+\frac{a^4\ln(a+b\tan(dx+c))}{b^3(a^2+b^2)d}-\frac{b\ln\left(1+\tan^2(dx+c)\right)}{2d(a^2+b^2)}$
risch	$-\frac{x}{ib-a}+\frac{2ixa^2}{b^3}+\frac{2ia^2c}{b^3d}-\frac{2ix}{b}-\frac{2ic}{bd}-\frac{2ia^4x}{(a^2+b^2)b^3}-\frac{2ia^4c}{(a^2+b^2)b^3d}+\frac{-2ia\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}-2ia}{b^2d\left(e^{2i(dx+c)}+1\right)^2}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^2*(-1/2*b*tan(d*x+c)^2+a*tan(d*x+c))+1/(a^2+b^2)*(-1/2*b*ln(1+tan
(d*x+c)^2)+a*arctan(tan(d*x+c)))+1/b^3*a^4/(a^2+b^2)*ln(a+b*tan(d*x+c)))
```

Maxima [A]
time = 0.52, size = 99, normalized size = 1.02

$$\frac{\frac{2\,a^4\log(b\tan(dx+c)+a)}{a^2b^3+b^5}+\frac{2\,(dx+c)a}{a^2+b^2}-\frac{b\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}+\frac{b\tan(dx+c)^2-2\,a\tan(dx+c)}{b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^4*log(b*tan(d*x + c) + a)/(a^2*b^3 + b^5) + 2*(d*x + c)*a/(a^2 + b
^2) - b*log(tan(d*x + c)^2 + 1)/(a^2 + b^2) + (b*tan(d*x + c)^2 - 2*a*tan(d
*x + c))/b^2)/d
```

Fricas [A]
time = 1.63, size = 134, normalized size = 1.38

$$\frac{2\,ab^3dx+a^4\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)+(a^2b^2+b^4)\tan\left(dx+c\right)^2-(a^4-b^4)\log\left(\frac{1}{\tan(dx+c)^2+1}\right)-2\left(a^3b+ab^3\right)\tan\left(dx+c\right)}{2\left(a^2b^3+b^5\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b^3*d*x + a^4*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/
(tan(d*x + c)^2 + 1)) + (a^2*b^2 + b^4)*tan(d*x + c)^2 - (a^4 - b^4)*log(1/
(tan(d*x + c)^2 + 1)) - 2*(a^3*b + a*b^3)*tan(d*x + c))/((a^2*b^3 + b^5)*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.70, size = 677, normalized size = 6.98

$$\begin{cases} \tilde{\infty}x\tan^3(c) & \text{for } a=0\wedge b=0\wedge d=0 \\ \frac{x+\frac{\tan^3(c+dx)}{3d}-\frac{\tan(c+dx)}{d}}{a} & \text{for } b=0 \\ -\frac{3idx\tan(c+dx)}{2bd\tan(c+dx)-2ibd}-\frac{3dx}{2bd\tan(c+dx)-2ibd}-\frac{2\log(\tan^2(c+dx)+1)\tan(c+dx)}{2bd\tan(c+dx)-2ibd}+\frac{2i\log(\tan^2(c+dx)+1)}{2bd\tan(c+dx)-2ibd}+\frac{\tan^3(c+dx)}{2bd\tan(c+dx)-2ibd}+\frac{i\tan^2(c+dx)}{2bd\tan(c+dx)-2ibd}+\frac{3i}{2bd\tan(c+dx)-2ibd} & \text{for } a=-ib \\ \frac{3idx\tan(c+dx)}{2bd\tan(c+dx)+2ibd}-\frac{3dx}{2bd\tan(c+dx)+2ibd}-\frac{2\log(\tan^2(c+dx)+1)\tan(c+dx)}{2bd\tan(c+dx)+2ibd}-\frac{2i\log(\tan^2(c+dx)+1)}{2bd\tan(c+dx)+2ibd}+\frac{\tan^3(c+dx)}{2bd\tan(c+dx)+2ibd}-\frac{i\tan^2(c+dx)}{2bd\tan(c+dx)+2ibd}-\frac{3i}{2bd\tan(c+dx)+2ibd} & \text{for } a=ib \\ \frac{x\tan^4(c)}{a+b\tan(c)} & \text{for } d=0 \\ \frac{2a^4\log\left(\frac{a}{b}+\tan(c+dx)\right)}{2a^2b^3d+2b^5d}-\frac{2a^3b\tan(c+dx)}{2a^2b^3d+2b^5d}+\frac{a^2b^2\tan^2(c+dx)}{2a^2b^3d+2b^5d}+\frac{2ab^3dx}{2a^2b^3d+2b^5d}-\frac{2ab^3\tan(c+dx)}{2a^2b^3d+2b^5d}-\frac{b^4\log(\tan^2(c+dx)+1)}{2a^2b^3d+2b^5d}+\frac{b^4\tan^2(c+dx)}{2a^2b^3d+2b^5d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*tan(c)**3, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x + tan(c +
d*x)**3/(3*d) - tan(c + d*x)/d)/a, Eq(b, 0)), (-3*I*d*x*tan(c + d*x)/(2*b*d
*tan(c + d*x) - 2*I*b*d) - 3*d*x/(2*b*d*tan(c + d*x) - 2*I*b*d) - 2*log(tan
(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) + 2*I*log(tan
(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x) - 2*I*b*d) + tan(c + d*x)**3/(2*b*d*t
an(c + d*x) - 2*I*b*d) + I*tan(c + d*x)**2/(2*b*d*tan(c + d*x) - 2*I*b*d) +
 3*I/(2*b*d*tan(c + d*x) - 2*I*b*d), Eq(a, -I*b)), (3*I*d*x*tan(c + d*x)/(2
*b*d*tan(c + d*x) + 2*I*b*d) - 3*d*x/(2*b*d*tan(c + d*x) + 2*I*b*d) - 2*log
(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) - 2*I*log
(tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x) + 2*I*b*d) + tan(c + d*x)**3/(2*b
*d*tan(c + d*x) + 2*I*b*d) - I*tan(c + d*x)**2/(2*b*d*tan(c + d*x) + 2*I*b*
d) - 3*I/(2*b*d*tan(c + d*x) + 2*I*b*d), Eq(a, I*b)), (x*tan(c)**4/(a + b*t
an(c)), Eq(d, 0)), (2*a**4*log(a/b + tan(c + d*x))/(2*a**2*b**3*d + 2*b**5*
d) - 2*a**3*b*tan(c + d*x)/(2*a**2*b**3*d + 2*b**5*d) + a**2*b**2*tan(c + d
*x)**2/(2*a**2*b**3*d + 2*b**5*d) + 2*a*b**3*d*x/(2*a**2*b**3*d + 2*b**5*d)
 - 2*a*b**3*tan(c + d*x)/(2*a**2*b**3*d + 2*b**5*d) - b**4*log(tan(c + d*x)
**2 + 1)/(2*a**2*b**3*d + 2*b**5*d) + b**4*tan(c + d*x)**2/(2*a**2*b**3*d +
 2*b**5*d), True))
```

Giac [A]
time = 1.25, size = 100, normalized size = 1.03

$$\frac{\frac{2\,a^4\log(|b\tan(dx+c)+a|)}{a^2b^3+b^5}+\frac{2\,(dx+c)a}{a^2+b^2}-\frac{b\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}+\frac{b\tan(dx+c)^2-2\,a\tan(dx+c)}{b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*a^4*log(abs(b*tan(d*x + c) + a))/(a^2*b^3 + b^5) + 2*(d*x + c)*a/(a^
2 + b^2) - b*log(tan(d*x + c)^2 + 1)/(a^2 + b^2) + (b*tan(d*x + c)^2 - 2*a*
tan(d*x + c))/b^2)/d
```

Mupad [B]
time = 4.08, size = 109, normalized size = 1.12

$$\frac{\tan(c+d\,x)^2}{2\,b\,d}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)}{2\,d\,\left(b+a\,1\mathrm{i}\right)}-\frac{a\tan(c+d\,x)}{b^2\,d}+\frac{a^4\,\ln\left(a+b\tan(c+d\,x)\right)}{b^3\,d\,\left(a^2+b^2\right)}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(a+b\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + b*tan(c + d*x)),x)
```

```
[Out] tan(c + d*x)^2/(2*b*d) - log(tan(c + d*x) + 1i)/(2*d*(a*1i + b)) - (log(tan
(c + d*x) - 1i)*1i)/(2*d*(a + b*1i)) - (a*tan(c + d*x))/(b^2*d) + (a^4*log(
a + b*tan(c + d*x)))/(b^3*d*(a^2 + b^2))
```

3.459 $\int \frac{\tan^3(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=79

$$-\frac{bx}{a^2+b^2}+\frac{a\log(\cos(c+dx))}{(a^2+b^2)\,d}-\frac{a^3\log(a+b\tan(c+dx))}{b^2\,(a^2+b^2)\,d}+\frac{\tan(c+dx)}{bd}$$

```
[Out] -b*x/(a^2+b^2)+a*ln(cos(d*x+c))/(a^2+b^2)/d-a^3*ln(a+b*tan(d*x+c))/b^2/(a^2
+b^2)/d+tan(d*x+c)/b/d
```

Rubi [A]
time = 0.09, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3647, 3707, 3698, 31, 3556}

$$\frac{a\log(\cos(c+dx))}{d\,(a^2+b^2)}-\frac{bx}{a^2+b^2}-\frac{a^3\log(a+b\tan(c+dx))}{b^2d\,(a^2+b^2)}+\frac{\tan(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

```
[Out] -((b*x)/(a^2 + b^2)) + (a*Log[Cos[c + d*x]])/((a^2 + b^2)*d) - (a^3*Log[a +
 b*Tan[c + d*x]])/(b^2*(a^2 + b^2)*d) + Tan[c + d*x]/(b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0]))
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^3(c+dx)}{a+b\tan(c+dx)}\,dx &= \frac{\tan(c+dx)}{bd}+\frac{\int \frac{-a-b\tan(c+dx)-a\tan^2(c+dx)}{a+b\tan(c+dx)}\,dx}{b}\\ &= -\frac{bx}{a^2+b^2}+\frac{\tan(c+dx)}{bd}-\frac{a\int \tan(c+dx)\,dx}{a^2+b^2}-\frac{a^3\int \frac{1+\tan^2(c+dx)}{a+b\tan(c+dx)}\,dx}{b\left(a^2+b^2\right)}\\ &= -\frac{bx}{a^2+b^2}+\frac{a\log(\cos(c+dx))}{\left(a^2+b^2\right)d}+\frac{\tan(c+dx)}{bd}-\frac{a^3\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\tan(c+d\right.}{b^2\left(a^2+b^2\right)d}\\ &= -\frac{bx}{a^2+b^2}+\frac{a\log(\cos(c+dx))}{\left(a^2+b^2\right)d}-\frac{a^3\log(a+b\tan(c+dx))}{b^2\left(a^2+b^2\right)d}+\frac{\tan(c+dx)}{bd}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.42, size = 91, normalized size = 1.15

$$-\frac{\frac{\log(i-\tan(c+dx))}{a+ib}+\frac{\log(i+\tan(c+dx))}{a-ib}+\frac{2a^3\log(a+b\tan(c+dx))}{b^2(a^2+b^2)}-\frac{2\tan(c+dx)}{b}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

```
[Out] -1/2*(Log[I - Tan[c + d*x]]/(a + I*b) + Log[I + Tan[c + d*x]]/(a - I*b) + (
2*a^3*Log[a + b*Tan[c + d*x]])/(b^2*(a^2 + b^2)) - (2*Tan[c + d*x])/b)/d
```

Maple [A]
time = 0.14, size = 79, normalized size = 1.00

method	result
derivativedivides	$\frac{\frac{\tan(dx+c)}{b}+\frac{-\frac{a\ln\left(1+\tan^2(dx+c)\right)}{2}-b\arctan(\tan(dx+c))}{a^2+b^2}-\frac{a^3\ln(a+b\tan(dx+c))}{b^2\left(a^2+b^2\right)}}{d}$
default	$\frac{\frac{\tan(dx+c)}{b}+\frac{-\frac{a\ln\left(1+\tan^2(dx+c)\right)}{2}-b\arctan(\tan(dx+c))}{a^2+b^2}-\frac{a^3\ln(a+b\tan(dx+c))}{b^2\left(a^2+b^2\right)}}{d}$
norman	$\frac{\tan(dx+c)}{bd}-\frac{bx}{a^2+b^2}-\frac{a\ln(1+\tan^2(dx+c))}{2d(a^2+b^2)}-\frac{a^3\ln(a+b\tan(dx+c))}{b^2(a^2+b^2)d}$
risch	$-\frac{ix}{ib-a}+\frac{2ia^3x}{b^2(a^2+b^2)}+\frac{2ia^3c}{b^2d(a^2+b^2)}-\frac{2iax}{b^2}-\frac{2iac}{b^2d}+\frac{2i}{bd(e^{2i(dx+c)}+1)}-\frac{a^3\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{b^2d(a^2+b^2)}+\frac{a\ln(e^{2i(d}}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b*tan(d*x+c)+1/(a^2+b^2)*(-1/2*a*ln(1+tan(d*x+c)^2)-b*arctan(tan(d*x
+c)))-1/b^2*a^3/(a^2+b^2)*ln(a+b*tan(d*x+c)))
```

Maxima [A]
time = 0.52, size = 85, normalized size = 1.08

$$-\frac{\frac{2\,a^3\log(b\tan(dx+c)+a)}{a^2b^2+b^4}+\frac{2\,(dx+c)b}{a^2+b^2}+\frac{a\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}-\frac{2\,\tan(dx+c)}{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a^3*log(b*tan(d*x + c) + a)/(a^2*b^2 + b^4) + 2*(d*x + c)*b/(a^2 +
b^2) + a*log(tan(d*x + c)^2 + 1)/(a^2 + b^2) - 2*tan(d*x + c)/b)/d
```

Fricas [A]
time = 1.30, size = 111, normalized size = 1.41

$$-\frac{2\,b^3dx+a^3\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)-(a^3+ab^2)\log\left(\frac{1}{\tan(dx+c)^2+1}\right)-2\,(a^2b+b^3)\tan\left(dx+c\right)}{2\,(a^2b^2+b^4)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*d*x + a^3*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(
tan(d*x + c)^2 + 1)) - (a^3 + a*b^2)*log(1/(tan(d*x + c)^2 + 1)) - 2*(a^2*b
 + b^3)*tan(d*x + c))/((a^2*b^2 + b^4)*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.57, size = 554, normalized size = 7.01

$$\begin{cases} \tilde{\infty}x\tan^2(c) & \text{for } a=0 \wedge b=0 \wedge d=0 \\ -\frac{3dx\tan(c+dx)}{2bd\tan(c+dx)-2ibd} + \frac{3idx}{2bd\tan(c+dx)-2ibd} + \frac{i\log(\tan^2(c+dx)+1)\tan(c+dx)}{2bd\tan(c+dx)-2ibd} + \frac{\log(\tan^2(c+dx)+1)}{2bd\tan(c+dx)-2ibd} + \frac{2\tan^2(c+dx)}{2bd\tan(c+dx)-2ibd} + \frac{3}{2bd\tan(c+dx)-2ibd} & \text{for } a=-ib \\ -\frac{3dx\tan(c+dx)}{2bd\tan(c+dx)+2ibd} - \frac{3idx}{2bd\tan(c+dx)+2ibd} - \frac{i\log(\tan^2(c+dx)+1)\tan(c+dx)}{2bd\tan(c+dx)+2ibd} + \frac{\log(\tan^2(c+dx)+1)}{2bd\tan(c+dx)+2ibd} + \frac{2\tan^2(c+dx)}{2bd\tan(c+dx)+2ibd} + \frac{3}{2bd\tan(c+dx)+2ibd} & \text{for } a=ib \\ \frac{-\frac{\log(\tan^2(c+dx)+1)}{2d}+\frac{\tan^2(c+dx)}{2d}}{a} & \text{for } b=0 \\ \frac{x\tan^3(c)}{a+b\tan(c)} & \text{for } d=0 \\ -\frac{2a^3\log\left(\frac{a}{b}+\tan(c+dx)\right)}{2a^2b^2d+2b^4d} + \frac{2a^2b\tan(c+dx)}{2a^2b^2d+2b^4d} - \frac{ab^2\log(\tan^2(c+dx)+1)}{2a^2b^2d+2b^4d} - \frac{2b^3dx}{2a^2b^2d+2b^4d} + \frac{2b^3\tan(c+dx)}{2a^2b^2d+2b^4d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*tan(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (-3*d*x*tan(c
+ d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) + 3*I*d*x/(2*b*d*tan(c + d*x) - 2*I*b
*d) + I*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d
) + log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x) - 2*I*b*d) + 2*tan(c + d*x
)**2/(2*b*d*tan(c + d*x) - 2*I*b*d) + 3/(2*b*d*tan(c + d*x) - 2*I*b*d), Eq(
a, -I*b)), (-3*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) - 3*I*d*x/(2
*b*d*tan(c + d*x) + 2*I*b*d) - I*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b
*d*tan(c + d*x) + 2*I*b*d) + log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x) +
 2*I*b*d) + 2*tan(c + d*x)**2/(2*b*d*tan(c + d*x) + 2*I*b*d) + 3/(2*b*d*tan
(c + d*x) + 2*I*b*d), Eq(a, I*b)), ((-log(tan(c + d*x)**2 + 1)/(2*d) + tan(
c + d*x)**2/(2*d))/a, Eq(b, 0)), (x*tan(c)**3/(a + b*tan(c)), Eq(d, 0)), (-
2*a**3*log(a/b + tan(c + d*x))/(2*a**2*b**2*d + 2*b**4*d) + 2*a**2*b*tan(c
+ d*x)/(2*a**2*b**2*d + 2*b**4*d) - a*b**2*log(tan(c + d*x)**2 + 1)/(2*a**2
*b**2*d + 2*b**4*d) - 2*b**3*d*x/(2*a**2*b**2*d + 2*b**4*d) + 2*b**3*tan(c
+ d*x)/(2*a**2*b**2*d + 2*b**4*d), True))
```

Giac [A]
time = 0.90, size = 86, normalized size = 1.09

$$-\frac{\frac{2\,a^3\log(|b\tan(dx+c)+a|)}{a^2b^2+b^4}+\frac{2\,(dx+c)b}{a^2+b^2}+\frac{a\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}-\frac{2\,\tan(dx+c)}{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*a^3*log(abs(b*tan(d*x + c) + a))/(a^2*b^2 + b^4) + 2*(d*x + c)*b/(a
^2 + b^2) + a*log(tan(d*x + c)^2 + 1)/(a^2 + b^2) - 2*tan(d*x + c)/b)/d
```

Mupad [B]
time = 4.11, size = 94, normalized size = 1.19

$$\frac{\tan(c+d\,x)}{b\,d}-\frac{\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)}{2\,d\,\left(a-b\,1{}\mathrm{i}\right)}-\frac{a^3\,\ln\left(a+b\tan(c+d\,x)\right)}{b^2\,d\,\left(a^2+b^2\right)}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,1{}\mathrm{i}}{2\,d\,\left(-b+a\,1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + b*tan(c + d*x)),x)
```

```
[Out] tan(c + d*x)/(b*d) - log(tan(c + d*x) + 1i)/(2*d*(a - b*1i)) - (log(tan(c +
 d*x) - 1i)*1i)/(2*d*(a*1i - b)) - (a^3*log(a + b*tan(c + d*x)))/(b^2*d*(a^
2 + b^2))
```

3.460 $\int \frac{\tan^2(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=66

$$-\frac{ax}{a^2+b^2}-\frac{\log(\cos(c+dx))}{bd}+\frac{a^2\log(a\cos(c+dx)+b\sin(c+dx))}{b\,(a^2+b^2)\,d}$$

```
[Out] -a*x/(a^2+b^2)-ln(cos(d*x+c))/b/d+a^2*ln(a*cos(d*x+c)+b*sin(d*x+c))/b/(a^2+
b^2)/d
```

Rubi [A]
time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3622, 3556, 3565, 3611}

$$\frac{a^2\log(a\cos(c+dx)+b\sin(c+dx))}{bd\,(a^2+b^2)}+\frac{a^3x}{b^2\,(a^2+b^2)}-\frac{ax}{b^2}-\frac{\log(\cos(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

```
[Out] -((a*x)/b^2) + (a^3*x)/(b^2*(a^2 + b^2)) - Log[Cos[c + d*x]]/(b*d) + (a^2*L
og[a*Cos[c + d*x] + b*Sin[c + d*x]])/(b*(a^2 + b^2)*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3565

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3622

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^2/((a_.) + (b_.)*tan[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[d*(2*b*c - a*d)*(x/b^2), x] + (Dist[d^2/b, In
```

```
t[Tan[e + f*x], x], x] + Dist[(b*c - a*d)^2/b^2, Int[1/(a + b*Tan[e + f*x])
, x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 +
 b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^2(c+dx)}{a+b\tan(c+dx)}\,dx &= -\frac{ax}{b^2}+\frac{a^2\int \frac{1}{a+b\tan(c+dx)}\,dx}{b^2}+\frac{\int \tan(c+dx)\,dx}{b}\\ &= -\frac{ax}{b^2}+\frac{a^3x}{b^2(a^2+b^2)}-\frac{\log(\cos(c+dx))}{bd}+\frac{a^2\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}\,dx}{b(a^2+b^2)}\\ &= -\frac{ax}{b^2}+\frac{a^3x}{b^2(a^2+b^2)}-\frac{\log(\cos(c+dx))}{bd}+\frac{a^2\log(a\cos(c+dx)+b\sin(c+dx))}{b(a^2+b^2)\,d}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.10, size = 78, normalized size = 1.18

$$\frac{b(ia+b)\log(i-\tan(c+dx))+b(-ia+b)\log(i+\tan(c+dx))+2a^2\log(a+b\tan(c+dx))}{2b(a^2+b^2)\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Tan[c + d*x]),x]
[Out] (b*(I*a + b)*Log[I - Tan[c + d*x]] + b*((-I)*a + b)*Log[I + Tan[c + d*x]] +
 2*a^2*Log[a + b*Tan[c + d*x]])/(2*b*(a^2 + b^2)*d)
```

Maple [A]
time = 0.11, size = 68, normalized size = 1.03

method	result	size
derivativedivides	$\frac{\frac{\frac{b\ln\left(1+\tan^2(dx+c)\right)}{2}-a\arctan(\tan(dx+c))}{a^2+b^2}+\frac{a^2\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)b}}{d}$	68
default	$\frac{\frac{\frac{b\ln\left(1+\tan^2(dx+c)\right)}{2}-a\arctan(\tan(dx+c))}{a^2+b^2}+\frac{a^2\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)b}}{d}$	68
norman	$-\frac{ax}{a^2+b^2}+\frac{a^2\ln(a+b\tan(dx+c))}{bd(a^2+b^2)}+\frac{b\ln(1+\tan^2(dx+c))}{2d(a^2+b^2)}$	71
risch	$\frac{x}{ib-a}+\frac{2ix}{b}+\frac{2ic}{bd}-\frac{2ia^2x}{b(a^2+b^2)}-\frac{2ia^2c}{bd(a^2+b^2)}-\frac{\ln\left(e^{2i(dx+c)}+1\right)}{bd}+\frac{a^2\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{bd(a^2+b^2)}$	140

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a^2+b^2)*(1/2*b*ln(1+tan(d*x+c)^2)-a*arctan(tan(d*x+c)))+a^2/(a^2+b
^2)/b*ln(a+b*tan(d*x+c)))
```

Maxima [A]
time = 0.53, size = 72, normalized size = 1.09

$$\frac{\frac{2\,a^2\log(b\tan(dx+c)+a)}{a^2b+b^3}-\frac{2\,(dx+c)a}{a^2+b^2}+\frac{b\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^2*log(b*tan(d*x + c) + a)/(a^2*b + b^3) - 2*(d*x + c)*a/(a^2 + b^2
) + b*log(tan(d*x + c)^2 + 1)/(a^2 + b^2))/d
```

Fricas [A]
time = 1.53, size = 89, normalized size = 1.35

$$-\frac{2\,abdx-a^2\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)+(a^2+b^2)\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,(a^2b+b^3)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*d*x - a^2*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(
tan(d*x + c)^2 + 1)) + (a^2 + b^2)*log(1/(tan(d*x + c)^2 + 1)))/((a^2*b + b
^3)*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.51, size = 405, normalized size = 6.14

$$\begin{cases} \tilde{\infty}x\tan(c) & \text{for}\: a=0\wedge b=0\wedge d=0 \\ \frac{idx\tan(c+dx)}{2bd\tan(c+dx)-2ibd}+\frac{dx}{2bd\tan(c+dx)-2ibd}+\frac{\log(\tan^2(c+dx)+1)\tan(c+dx)}{2bd\tan(c+dx)-2ibd}-\frac{i\log(\tan^2(c+dx)+1)}{2bd\tan(c+dx)-2ibd}-\frac{i}{2bd\tan(c+dx)-2ibd} & \text{for}\: a=-ib \\ -\frac{idx\tan(c+dx)}{2bd\tan(c+dx)+2ibd}+\frac{dx}{2bd\tan(c+dx)+2ibd}+\frac{\log(\tan^2(c+dx)+1)\tan(c+dx)}{2bd\tan(c+dx)+2ibd}+\frac{i\log(\tan^2(c+dx)+1)}{2bd\tan(c+dx)+2ibd}+\frac{i}{2bd\tan(c+dx)+2ibd} & \text{for}\: a=ib \\ \frac{-x+\frac{\tan(c+dx)}{d}}{a} & \text{for}\: b=0 \\ \frac{x\tan^2(c)}{a+b\tan(c)} & \text{for}\: d=0 \\ \frac{2a^2\log\left(\frac{a}{b}+\tan(c+dx)\right)}{2a^2bd+2b^3d}-\frac{2abdx}{2a^2bd+2b^3d}+\frac{b^2\log(\tan^2(c+dx)+1)}{2a^2bd+2b^3d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*tan(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (I*d*x*tan(c + d*
x)/(2*b*d*tan(c + d*x) - 2*I*b*d) + d*x/(2*b*d*tan(c + d*x) - 2*I*b*d) + lo
```

```
g(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) - I*log(
tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x) - 2*I*b*d) - I/(2*b*d*tan(c + d*x)
 - 2*I*b*d), Eq(a, -I*b)), (-I*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b
*d) + d*x/(2*b*d*tan(c + d*x) + 2*I*b*d) + log(tan(c + d*x)**2 + 1)*tan(c +
 d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) + I*log(tan(c + d*x)**2 + 1)/(2*b*d*ta
n(c + d*x) + 2*I*b*d) + I/(2*b*d*tan(c + d*x) + 2*I*b*d), Eq(a, I*b)), ((-x
 + tan(c + d*x)/d)/a, Eq(b, 0)), (x*tan(c)**2/(a + b*tan(c)), Eq(d, 0)), (2
*a**2*log(a/b + tan(c + d*x))/(2*a**2*b*d + 2*b**3*d) - 2*a*b*d*x/(2*a**2*b
*d + 2*b**3*d) + b**2*log(tan(c + d*x)**2 + 1)/(2*a**2*b*d + 2*b**3*d), Tru
e))
```

Giac [A]
time = 0.64, size = 73, normalized size = 1.11

$$\frac{\frac{2\,a^2\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^2b+b^3}-\frac{2\,(dx+c)a}{a^2+b^2}+\frac{b\log\left(\tan\left(dx+c\right)^2+1\right)}{a^2+b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2*log(abs(b*tan(d*x + c) + a))/(a^2*b + b^3) - 2*(d*x + c)*a/(a^2
+ b^2) + b*log(tan(d*x + c)^2 + 1)/(a^2 + b^2))/d
```

Mupad [B]
time = 4.05, size = 78, normalized size = 1.18

$$\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)}{2\,d\,\left(b+a\,1\mathrm{i}\right)}+\frac{a^2\,\ln\left(a+b\tan(c+d\,x)\right)}{b\,d\,\left(a^2+b^2\right)}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(a+b\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + b*tan(c + d*x)),x)
```

```
[Out] (log(tan(c + d*x) - 1i)*1i)/(2*d*(a + b*1i)) + log(tan(c + d*x) + 1i)/(2*d*
(a*1i + b)) + (a^2*log(a + b*tan(c + d*x)))/(b*d*(a^2 + b^2))
```

3.461 $\int \frac{\tan(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=46

$$\frac{bx}{a^2+b^2}-\frac{a\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)\,d}$$

```
[Out] b*x/(a^2+b^2)-a*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {3612, 3611}

$$\frac{bx}{a^2+b^2}-\frac{a\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*x)/(a^2 + b^2) - (a*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)*d
)
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(c+dx)}{a+b\tan(c+dx)}\,dx &= \frac{bx}{a^2+b^2}-\frac{a\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}\,dx}{a^2+b^2}\\ &= \frac{bx}{a^2+b^2}-\frac{a\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)\,d}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.13, size = 66, normalized size = 1.43

$$\frac{2(-ia+b)(c+dx)+2ia\text{ArcTan}(\tan(c+dx))-a\log\left((a\cos(c+dx)+b\sin(c+dx))^2\right)}{2\left(a^2+b^2\right)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + b*Tan[c + d*x]),x]
```

```
[Out] (2*((-I)*a + b)*(c + d*x) + (2*I)*a*ArcTan[Tan[c + d*x]] - a*Log[(a*Cos[c +
 d*x] + b*Sin[c + d*x])^2])/(2*(a^2 + b^2)*d)
```

Maple [A]
time = 0.09, size = 63, normalized size = 1.37

method	result	size
derivativedivides	$\frac{\frac{\frac{a\ln\left(1+\tan^2(dx+c)\right)}{2}+b\arctan(\tan(dx+c))}{a^2+b^2}-\frac{a\ln(a+b\tan(dx+c))}{a^2+b^2}}{d}$	63
default	$\frac{\frac{\frac{a\ln\left(1+\tan^2(dx+c)\right)}{2}+b\arctan(\tan(dx+c))}{a^2+b^2}-\frac{a\ln(a+b\tan(dx+c))}{a^2+b^2}}{d}$	63
norman	$\frac{bx}{a^2+b^2}+\frac{a\ln(1+\tan^2(dx+c))}{2d(a^2+b^2)}-\frac{a\ln(a+b\tan(dx+c))}{d(a^2+b^2)}$	66
risch	$\frac{ix}{ib-a}+\frac{2iax}{a^2+b^2}+\frac{2iac}{d(a^2+b^2)}-\frac{a\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{d(a^2+b^2)}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a^2+b^2)*(1/2*a*ln(1+tan(d*x+c)^2)+b*arctan(tan(d*x+c)))-a/(a^2+b^2
)*ln(a+b*tan(d*x+c)))
```

Maxima [A]
time = 0.55, size = 68, normalized size = 1.48

$$\frac{\frac{2\,(dx+c)b}{a^2+b^2}-\frac{2\,a\log(b\tan(dx+c)+a)}{a^2+b^2}+\frac{a\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(d*x + c)*b/(a^2 + b^2) - 2*a*log(b*tan(d*x + c) + a)/(a^2 + b^2) +
a*log(tan(d*x + c)^2 + 1)/(a^2 + b^2))/d
```

Fricas [A]
time = 1.41, size = 63, normalized size = 1.37

$$\frac{2\,bdx - a\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)}{2\,(a^2+b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*x - a*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d
*x + c)^2 + 1)))/((a^2 + b^2)*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.46, size = 255, normalized size = 5.54

$$\begin{cases} \tilde{\infty}x & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{dx\tan(c+dx)}{2bd\tan(c+dx)-2ibd} - \frac{idx}{2bd\tan(c+dx)-2ibd} - \frac{1}{2bd\tan(c+dx)-2ibd} & \text{for } a = -ib \\ \frac{dx\tan(c+dx)}{2bd\tan(c+dx)+2ibd} + \frac{idx}{2bd\tan(c+dx)+2ibd} - \frac{1}{2bd\tan(c+dx)+2ibd} & \text{for } a = ib \\ \frac{x\tan(c)}{a+b\tan(c)} & \text{for } d = 0 \\ \frac{\log(\tan^2(c+dx)+1)}{2ad} & \text{for } b = 0 \\ -\frac{2a\log\left(\frac{a}{b}+\tan(c+dx)\right)}{2a^2d+2b^2d} + \frac{a\log(\tan^2(c+dx)+1)}{2a^2d+2b^2d} + \frac{2bdx}{2a^2d+2b^2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (d*x*tan(c + d*x)/(2*b*d
*tan(c + d*x) - 2*I*b*d) - I*d*x/(2*b*d*tan(c + d*x) - 2*I*b*d) - 1/(2*b*d*
tan(c + d*x) - 2*I*b*d), Eq(a, -I*b)), (d*x*tan(c + d*x)/(2*b*d*tan(c + d*x
) + 2*I*b*d) + I*d*x/(2*b*d*tan(c + d*x) + 2*I*b*d) - 1/(2*b*d*tan(c + d*x)
 + 2*I*b*d), Eq(a, I*b)), (x*tan(c)/(a + b*tan(c)), Eq(d, 0)), (log(tan(c +
 d*x)**2 + 1)/(2*a*d), Eq(b, 0)), (-2*a*log(a/b + tan(c + d*x))/(2*a**2*d +
 2*b**2*d) + a*log(tan(c + d*x)**2 + 1)/(2*a**2*d + 2*b**2*d) + 2*b*d*x/(2*
a**2*d + 2*b**2*d), True))
```

Giac [A]
time = 0.52, size = 73, normalized size = 1.59

$$-\frac{\frac{2\,ab\log(|b\tan(dx+c)+a|)}{a^2b+b^3} - \frac{2\,(dx+c)b}{a^2+b^2} - \frac{a\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*a*b*log(abs(b*tan(d*x + c) + a))/(a^2*b + b^3) - 2*(d*x + c)*b/(a^2
 + b^2) - a*log(tan(d*x + c)^2 + 1)/(a^2 + b^2))/d
```

Mupad [B]
time = 4.03, size = 76, normalized size = 1.65

$$\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)}{2\,d\,\left(a-b\,1\mathrm{i}\right)}-\frac{a\,\ln\left(a+b\tan(c+d\,x)\right)}{d\,\left(a^2+b^2\right)}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(-b+a\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + b*tan(c + d*x)),x)
```

```
[Out] log(tan(c + d*x) + 1i)/(2*d*(a - b*1i)) + (log(tan(c + d*x) - 1i)*1i)/(2*d*
(a*1i - b)) - (a*log(a + b*tan(c + d*x)))/(d*(a^2 + b^2))
```

3.462 $\int \frac{1}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=45

$$\frac{ax}{a^2+b^2}+\frac{b\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)\,d}$$

```
[Out] a*x/(a^2+b^2)+b*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)/d
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3565, 3611}

$$\frac{b\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)}+\frac{ax}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^(-1),x]
```

```
[Out] (a*x)/(a^2 + b^2) + (b*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)*d
)
```

Rule 3565

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\tan(c+dx)}\,dx &= \frac{ax}{a^2+b^2}+\frac{b\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}\,dx}{a^2+b^2}\\ &= \frac{ax}{a^2+b^2}+\frac{b\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)\,d}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.06, size = 76, normalized size = 1.69

$$\frac{(-ia-b)\log(i-\tan(c+dx))+i(a+ib)\log(i+\tan(c+dx))+2b\log(a+b\tan(c+dx))}{2(a^2+b^2)\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^(-1),x]
```

```
[Out] (((-I)*a - b)*Log[I - Tan[c + d*x]] + I*(a + I*b)*Log[I + Tan[c + d*x]] + 2
*b*Log[a + b*Tan[c + d*x]])/(2*(a^2 + b^2)*d)
```

Maple [A]
time = 0.09, size = 62, normalized size = 1.38

method	result	size
derivativedivides	$\frac{\frac{-\frac{b\ln\left(1+\tan^2(dx+c)\right)}{2}+a\arctan(\tan(dx+c))}{a^2+b^2}+\frac{b\ln(a+b\tan(dx+c))}{a^2+b^2}}{d}$	62
default	$\frac{\frac{-\frac{b\ln\left(1+\tan^2(dx+c)\right)}{2}+a\arctan(\tan(dx+c))}{a^2+b^2}+\frac{b\ln(a+b\tan(dx+c))}{a^2+b^2}}{d}$	62
norman	$\frac{ax}{a^2+b^2}+\frac{b\ln(a+b\tan(dx+c))}{d(a^2+b^2)}-\frac{b\ln(1+\tan^2(dx+c))}{2d(a^2+b^2)}$	65
risch	$-\frac{x}{ib-a}-\frac{2ibx}{a^2+b^2}-\frac{2ibc}{d(a^2+b^2)}+\frac{b\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{d(a^2+b^2)}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a^2+b^2)*(-1/2*b*ln(1+tan(d*x+c)^2)+a*arctan(tan(d*x+c)))+b/(a^2+b^
2)*ln(a+b*tan(d*x+c)))
```

Maxima [A]
time = 0.52, size = 69, normalized size = 1.53

$$\frac{\frac{2\,(dx+c)a}{a^2+b^2}+\frac{2\,b\log(b\tan(dx+c)+a)}{a^2+b^2}-\frac{b\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(d*x + c)*a/(a^2 + b^2) + 2*b*log(b*tan(d*x + c) + a)/(a^2 + b^2) -
b*log(tan(d*x + c)^2 + 1)/(a^2 + b^2))/d
```

Fricas [A]
time = 1.27, size = 62, normalized size = 1.38

$$\frac{2\,adx + b\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)}{2\,(a^2+b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*d*x + b*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d
*x + c)^2 + 1)))/((a^2 + b^2)*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.41, size = 241, normalized size = 5.36

$$\begin{cases} \frac{\tilde{\infty}x}{\tan(c)} & \text{for } a=0 \wedge b=0 \wedge d=0 \\ \frac{idx\tan(c+dx)}{2bd\tan(c+dx)-2ibd} + \frac{dx}{2bd\tan(c+dx)-2ibd} + \frac{i}{2bd\tan(c+dx)-2ibd} & \text{for } a=-ib \\ -\frac{idx\tan(c+dx)}{2bd\tan(c+dx)+2ibd} + \frac{dx}{2bd\tan(c+dx)+2ibd} - \frac{i}{2bd\tan(c+dx)+2ibd} & \text{for } a=ib \\ \frac{x}{a+b\tan(c)} & \text{for } d=0 \\ \frac{x}{a} & \text{for } b=0 \\ \frac{2adx}{2a^2d+2b^2d} + \frac{2b\log\left(\frac{a}{b}+\tan(c+dx)\right)}{2a^2d+2b^2d} - \frac{b\log\left(\tan^2(c+dx)+1\right)}{2a^2d+2b^2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x/tan(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (I*d*x*tan(c + d*
x)/(2*b*d*tan(c + d*x) - 2*I*b*d) + d*x/(2*b*d*tan(c + d*x) - 2*I*b*d) + I/
(2*b*d*tan(c + d*x) - 2*I*b*d), Eq(a, -I*b)), (-I*d*x*tan(c + d*x)/(2*b*d*t
an(c + d*x) + 2*I*b*d) + d*x/(2*b*d*tan(c + d*x) + 2*I*b*d) - I/(2*b*d*tan(
c + d*x) + 2*I*b*d), Eq(a, I*b)), (x/(a + b*tan(c)), Eq(d, 0)), (x/a, Eq(b,
 0)), (2*a*d*x/(2*a**2*d + 2*b**2*d) + 2*b*log(a/b + tan(c + d*x))/(2*a**2*
d + 2*b**2*d) - b*log(tan(c + d*x)**2 + 1)/(2*a**2*d + 2*b**2*d), True))
```

Giac [A]
time = 0.59, size = 74, normalized size = 1.64

$$\frac{\frac{2\,b^2\log(|b\tan(dx+c)+a|)}{a^2b+b^3} + \frac{2\,(dx+c)a}{a^2+b^2} - \frac{b\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^2*log(abs(b*tan(d*x + c) + a))/(a^2*b + b^3) + 2*(d*x + c)*a/(a^2
+ b^2) - b*log(tan(d*x + c)^2 + 1)/(a^2 + b^2))/d
```

Mupad [B]
time = 4.18, size = 73, normalized size = 1.62

$$\frac{b\ln\left(a+b\tan(c+d\,x)\right)}{d\,\left(a^2+b^2\right)}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)}{2\,d\,\left(b+a\,1\mathrm{i}\right)}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(a+b\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(c + d*x)),x)
```

```
[Out] (b*log(a + b*tan(c + d*x)))/(d*(a^2 + b^2)) - log(tan(c + d*x) + 1i)/(2*d*(
a*1i + b)) - (log(tan(c + d*x) - 1i)*1i)/(2*d*(a + b*1i))
```

3.463 $\int \frac{\cot(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=66

$$-\frac{bx}{a^2+b^2}+\frac{\log(\sin(c+dx))}{ad}-\frac{b^2\log(a\cos(c+dx)+b\sin(c+dx))}{a\left(a^2+b^2\right)d}$$

```
[Out] -b*x/(a^2+b^2)+ln(sin(d*x+c))/a/d-b^2*ln(a*cos(d*x+c)+b*sin(d*x+c))/a/(a^2+
b^2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3652, 3611, 3556}

$$-\frac{b^2\log(a\cos(c+dx)+b\sin(c+dx))}{ad\left(a^2+b^2\right)}-\frac{bx}{a^2+b^2}+\frac{\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + b*Tan[c + d*x]),x]
```

```
[Out] -((b*x)/(a^2 + b^2)) + Log[Sin[c + d*x]]/(a*d) - (b^2*Log[a*Cos[c + d*x] +
b*Sin[c + d*x]])/(a*(a^2 + b^2)*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3652

```
Int[1/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f
_.)*(x_)])), x_Symbol] :> Simp[(a*c - b*d)*(x/((a^2 + b^2)*(c^2 + d^2))), x
] + (Dist[b^2/((b*c - a*d)*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Ta
n[e + f*x]), x], x] - Dist[d^2/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e
+ f*x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{bx}{a^2+b^2} + \frac{\int \cot(c+dx)\,dx}{a} - \frac{b^2 \int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}\,dx}{a\,(a^2+b^2)}$$
$$= -\frac{bx}{a^2+b^2} + \frac{\log(\sin(c+dx))}{ad} - \frac{b^2\log(a\cos(c+dx)+b\sin(c+dx))}{a\,(a^2+b^2)\,d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.21, size = 91, normalized size = 1.38

$$-\frac{\frac{\log(i-\tan(c+dx))}{a+ib} - \frac{2\log(\tan(c+dx))}{a} + \frac{\log(i+\tan(c+dx))}{a-ib} + \frac{2b^2\log(a+b\tan(c+dx))}{a^3+ab^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + b*Tan[c + d*x]),x]
[Out] -1/2*(Log[I - Tan[c + d*x]]/(a + I*b) - (2*Log[Tan[c + d*x]])/a + Log[I + T
an[c + d*x]]/(a - I*b) + (2*b^2*Log[a + b*Tan[c + d*x]])/(a^3 + a*b^2))/d
```

Maple [A]
time = 0.27, size = 80, normalized size = 1.21

method	result	size
derivativedivides	$\frac{\frac{\ln(\tan(dx+c))}{a} - \frac{b^2\ln(a+b\tan(dx+c))}{(a^2+b^2)a} + \frac{-\frac{a\ln\left(1+\tan^2(dx+c)\right)}{2} - b\arctan(\tan(dx+c))}{a^2+b^2}}{d}$	80
default	$\frac{\frac{\ln(\tan(dx+c))}{a} - \frac{b^2\ln(a+b\tan(dx+c))}{(a^2+b^2)a} + \frac{-\frac{a\ln\left(1+\tan^2(dx+c)\right)}{2} - b\arctan(\tan(dx+c))}{a^2+b^2}}{d}$	80
norman	$-\frac{bx}{a^2+b^2} + \frac{\ln(\tan(dx+c))}{da} - \frac{a\ln(1+\tan^2(dx+c))}{2d(a^2+b^2)} - \frac{b^2\ln(a+b\tan(dx+c))}{(a^2+b^2)da}$	86
risch	$-\frac{ix}{ib-a} + \frac{2ib^2x}{(a^2+b^2)a} + \frac{2ib^2c}{(a^2+b^2)da} - \frac{2ix}{a} - \frac{2ic}{da} - \frac{b^2\ln\left(e^{2i(dx+c)} - \frac{ib+a}{ib-a}\right)}{(a^2+b^2)da} + \frac{\ln(e^{2i(dx+c)}-1)}{ad}$	142

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/a*ln(tan(d*x+c))-b^2/(a^2+b^2)/a*ln(a+b*tan(d*x+c))+1/(a^2+b^2)*(-1/
2*a*ln(1+tan(d*x+c)^2)-b*arctan(tan(d*x+c))))
```

Maxima [A]
time = 0.53, size = 84, normalized size = 1.27

$$-\frac{\frac{2\,b^2\log(b\tan(dx+c)+a)}{a^3+ab^2} + \frac{2\,(dx+c)b}{a^2+b^2} + \frac{a\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2} - \frac{2\,\log(\tan(dx+c))}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^2*log(b*tan(d*x + c) + a)/(a^3 + a*b^2) + 2*(d*x + c)*b/(a^2 + b^
2) + a*log(tan(d*x + c)^2 + 1)/(a^2 + b^2) - 2*log(tan(d*x + c))/a)/d
```

Fricas [A]
time = 1.02, size = 98, normalized size = 1.48

$$-\frac{2\,abdx+b^2\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)-(a^2+b^2)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)}{2\,(a^3+ab^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*d*x + b^2*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(
tan(d*x + c)^2 + 1)) - (a^2 + b^2)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))
)/((a^3 + a*b^2)*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.89, size = 626, normalized size = 9.48

$$\begin{cases} \frac{\tilde{\infty}x\cot(c)}{\tan(c)} & \text{for}\: a=0\wedge b=0\wedge d=0 \\ \frac{-\frac{\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{\log(\tan(c+dx))}{d}}{a} & \text{for}\: b=0 \\ \frac{-x-\frac{1}{d\tan(c+dx)}}{b} & \text{for}\: a=0 \\ \frac{dx\tan(c+dx)}{2bd\tan(c+dx)-2ibd}-\frac{idx}{2bd\tan(c+dx)-2ibd}-\frac{i\log(\tan^2(c+dx)+1)\tan(c+dx)}{2bd\tan(c+dx)-2ibd}-\frac{\log(\tan^2(c+dx)+1)}{2bd\tan(c+dx)-2ibd}+\frac{2i\log(\tan(c+dx))\tan(c+dx)}{2bd\tan(c+dx)-2ibd}+\frac{2\log(\tan(c+dx))}{2bd\tan(c+dx)-2ibd}+\frac{1}{2bd\tan(c+dx)-2ibd} & \text{for}\: a=-ib \\ \frac{dx\tan(c+dx)}{2bd\tan(c+dx)+2ibd}+\frac{idx}{2bd\tan(c+dx)+2ibd}+\frac{i\log(\tan^2(c+dx)+1)\tan(c+dx)}{2bd\tan(c+dx)+2ibd}-\frac{\log(\tan^2(c+dx)+1)}{2bd\tan(c+dx)+2ibd}-\frac{2i\log(\tan(c+dx))\tan(c+dx)}{2bd\tan(c+dx)+2ibd}+\frac{2\log(\tan(c+dx))}{2bd\tan(c+dx)+2ibd}+\frac{1}{2bd\tan(c+dx)+2ibd} & \text{for}\: a=ib \\ \frac{x\cot(c)}{a+b\tan(c)} & \text{for}\: d=0 \\ -\frac{a^2\log(\tan^2(c+dx)+1)}{2a^3d+2ab^2d}+\frac{2a^2\log(\tan(c+dx))}{2a^3d+2ab^2d}-\frac{2abdx}{2a^3d+2ab^2d}-\frac{2b^2\log\left(\frac{a}{b}+\tan(c+dx)\right)}{2a^3d+2ab^2d}+\frac{2b^2\log(\tan(c+dx))}{2a^3d+2ab^2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*cot(c)/tan(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((-log(tan
(c + d*x)**2 + 1)/(2*d) + log(tan(c + d*x))/d)/a, Eq(b, 0)), ((-x - 1/(d*ta
n(c + d*x)))/b, Eq(a, 0)), (d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d)
 - I*d*x/(2*b*d*tan(c + d*x) - 2*I*b*d) - I*log(tan(c + d*x)**2 + 1)*tan(c
+ d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) - log(tan(c + d*x)**2 + 1)/(2*b*d*tan
(c + d*x) - 2*I*b*d) + 2*I*log(tan(c + d*x))*tan(c + d*x)/(2*b*d*tan(c + d*
x) - 2*I*b*d) + 2*log(tan(c + d*x))/(2*b*d*tan(c + d*x) - 2*I*b*d) + 1/(2*b
*d*tan(c + d*x) - 2*I*b*d), Eq(a, -I*b)), (d*x*tan(c + d*x)/(2*b*d*tan(c +
d*x) + 2*I*b*d) + I*d*x/(2*b*d*tan(c + d*x) + 2*I*b*d) + I*log(tan(c + d*x)
**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) - log(tan(c + d*x)**2
+ 1)/(2*b*d*tan(c + d*x) + 2*I*b*d) - 2*I*log(tan(c + d*x))*tan(c + d*x)/(2
*b*d*tan(c + d*x) + 2*I*b*d) + 2*log(tan(c + d*x))/(2*b*d*tan(c + d*x) + 2*
```

```
I*b*d) + 1/(2*b*d*tan(c + d*x) + 2*I*b*d), Eq(a, I*b)), (x*cot(c)/(a + b*ta
n(c)), Eq(d, 0)), (-a**2*log(tan(c + d*x)**2 + 1)/(2*a**3*d + 2*a*b**2*d) +
 2*a**2*log(tan(c + d*x))/(2*a**3*d + 2*a*b**2*d) - 2*a*b*d*x/(2*a**3*d + 2
*a*b**2*d) - 2*b**2*log(a/b + tan(c + d*x))/(2*a**3*d + 2*a*b**2*d) + 2*b**
2*log(tan(c + d*x))/(2*a**3*d + 2*a*b**2*d), True))
```

Giac [A]
time = 0.58, size = 88, normalized size = 1.33

$$-\frac{\frac{2\,b^3\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^3b+ab^3}+\frac{2\,(dx+c)b}{a^2+b^2}+\frac{a\log\left(\tan\left(dx+c\right)^2+1\right)}{a^2+b^2}-\frac{2\,\log\left(|\tan\left(dx+c\right)|\right)}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b^3*log(abs(b*tan(d*x + c) + a))/(a^3*b + a*b^3) + 2*(d*x + c)*b/(a
^2 + b^2) + a*log(tan(d*x + c)^2 + 1)/(a^2 + b^2) - 2*log(abs(tan(d*x + c))
)/a)/d
```

Mupad [B]
time = 4.13, size = 95, normalized size = 1.44

$$\frac{\ln\left(\tan(c+d\,x)\right)}{a\,d}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)}{2\,d\,\left(a-b\,1\mathrm{i}\right)}-\frac{b^2\,\ln\left(a+b\tan(c+d\,x)\right)}{a\,d\,\left(a^2+b^2\right)}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(-b+a\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + b*tan(c + d*x)),x)
```

```
[Out] log(tan(c + d*x))/(a*d) - (log(tan(c + d*x) - 1i)*1i)/(2*d*(a*1i - b)) - lo
g(tan(c + d*x) + 1i)/(2*d*(a - b*1i)) - (b^2*log(a + b*tan(c + d*x)))/(a*d*
(a^2 + b^2))
```

3.464 $\int \frac{\cot^2(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=81

$$-\frac{ax}{a^2+b^2}-\frac{\cot(c+dx)}{ad}-\frac{b\log(\sin(c+dx))}{a^2d}+\frac{b^3\log(a\cos(c+dx)+b\sin(c+dx))}{a^2\left(a^2+b^2\right)d}$$

```
[Out] -a*x/(a^2+b^2)-cot(d*x+c)/a/d-b*ln(sin(d*x+c))/a^2/d+b^3*ln(a*cos(d*x+c)+b*
sin(d*x+c))/a^2/(a^2+b^2)/d
```

Rubi [A]
time = 0.12, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3650, 3732, 3611, 3556}

$$-\frac{ax}{a^2+b^2}+\frac{b^3\log(a\cos(c+dx)+b\sin(c+dx))}{a^2d\left(a^2+b^2\right)}-\frac{b\log(\sin(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

```
[Out] -((a*x)/(a^2 + b^2)) - Cot[c + d*x]/(a*d) - (b*Log[Sin[c + d*x]])/(a^2*d) +
 (b^3*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/(a^2*(a^2 + b^2)*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
```

```
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^2(c+dx)}{a+b\tan(c+dx)}\,dx &= -\frac{\cot(c+dx)}{ad}-\frac{\int \frac{\cot(c+dx)\left(b+a\tan(c+dx)+b\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx}{a}\\ &= -\frac{ax}{a^2+b^2}-\frac{\cot(c+dx)}{ad}-\frac{b\int\cot(c+dx)\,dx}{a^2}+\frac{b^3\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}\,dx}{a^2\left(a^2+b^2\right)}\\ &= -\frac{ax}{a^2+b^2}-\frac{\cot(c+dx)}{ad}-\frac{b\log(\sin(c+dx))}{a^2d}+\frac{b^3\log(a\cos(c+dx)+b\sin(c+dx))}{a^2\left(a^2+b^2\right)d}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.50, size = 96, normalized size = 1.19

$$-\frac{\frac{\cot(c+dx)}{a}-\frac{\log(i-\cot(c+dx))}{2(ia+b)}+\frac{\log(i+\cot(c+dx))}{2(ia-b)}-\frac{b^3\log(b+a\cot(c+dx))}{a^2(a^2+b^2)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

```
[Out] -((Cot[c + d*x]/a - Log[I - Cot[c + d*x]]/(2*(I*a + b)) + Log[I + Cot[c + d
*x]]/(2*(I*a - b)) - (b^3*Log[b + a*Cot[c + d*x]])/(a^2*(a^2 + b^2)))/d)
```

Maple [A]
time = 0.26, size = 94, normalized size = 1.16

method	result
derivativedivides	$\frac{-\frac{1}{a\tan(dx+c)}-\frac{b\ln(\tan(dx+c))}{a^2}+\frac{b^3\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)a^2}+\frac{\frac{b\ln\left(1+\tan^2(dx+c)\right)}{2}-a\arctan(\tan(dx+c))}{a^2+b^2}}{d}$

default	$\frac{-\frac{1}{a\tan(dx+c)}-\frac{b\ln(\tan(dx+c))}{a^2}+\frac{b^3\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)a^2}+\frac{\frac{b\ln\left(1+\tan^2(dx+c)\right)}{2}-a\arctan(\tan(dx+c))}{a^2+b^2}}{d}$
norman	$\frac{-\frac{1}{da}-\frac{ax\tan(dx+c)}{a^2+b^2}}{\tan(dx+c)}+\frac{b^3\ln(a+b\tan(dx+c))}{a^2d(a^2+b^2)}-\frac{b\ln(\tan(dx+c))}{a^2d}+\frac{b\ln(1+\tan^2(dx+c))}{2d(a^2+b^2)}$
risch	$\frac{x}{ib-a}+\frac{2ibx}{a^2}+\frac{2ibc}{a^2d}-\frac{2ib^3x}{a^2(a^2+b^2)}-\frac{2ib^3c}{a^2d(a^2+b^2)}-\frac{2i}{da(e^{2i(dx+c)}-1)}-\frac{b\ln(e^{2i(dx+c)}-1)}{a^2d}+\frac{b^3\ln\left(e^{2i(dx+c)}\ldots\right.}{a^2d(a^2+b\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a/tan(d*x+c)-b/a^2*ln(tan(d*x+c))+b^3/(a^2+b^2)/a^2*ln(a+b*tan(d*x+
c))+1/(a^2+b^2)*(1/2*b*ln(1+tan(d*x+c)^2)-a*arctan(tan(d*x+c))))
```

Maxima [A]
time = 0.59, size = 100, normalized size = 1.23

$$\frac{\frac{2\,b^3\log(b\tan(dx+c)+a)}{a^4+a^2b^2}-\frac{2\,(dx+c)a}{a^2+b^2}+\frac{b\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}-\frac{2\,b\log(\tan(dx+c))}{a^2}-\frac{2}{a\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^3*log(b*tan(d*x + c) + a)/(a^4 + a^2*b^2) - 2*(d*x + c)*a/(a^2 + b
^2) + b*log(tan(d*x + c)^2 + 1)/(a^2 + b^2) - 2*b*log(tan(d*x + c))/a^2 - 2
/(a*tan(d*x + c)))/d
```

Fricas [A]
time = 0.92, size = 140, normalized size = 1.73

$$-\frac{2\,a^3dx\tan(dx+c)-b^3\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)\tan(dx+c)+2\,a^3+2\,ab^2+(a^2b+b^3)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan(dx+c)}{2\,(a^4+a^2b^2)d\tan(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3*d*x*tan(d*x + c) - b^3*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x
+ c) + a^2)/(tan(d*x + c)^2 + 1))*tan(d*x + c) + 2*a^3 + 2*a*b^2 + (a^2*b +
 b^3)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c))/((a^4 + a^2*b^
2)*d*tan(d*x + c))
```

Sympy [C] Result contains complex when optimal does not.
time = 1.51, size = 1080, normalized size = 13.33

$$\begin{cases} \tilde{\infty}x & \text{for } a=0\wedge b=0\wedge c=0\wedge d=0 \\ \frac{-x-\frac{\cot(c+dx)}{d}}{a} & \text{for } b=0 \\ \frac{\frac{\log\left(\tan^2(c+dx)+1\right)}{2d}-\frac{\log(\tan(c+dx))}{d}-\frac{1}{2d\tan^2(c+dx)}}{b} & \text{for } a=0 \\ -\frac{3idx\tan^2(c+dx)}{2bd\tan^2(c+dx)-2ibd\tan(c+dx)}-\frac{3dx\tan(c+dx)}{2bd\tan^2(c+dx)-2ibd\tan(c+dx)}-\frac{\log\left(\tan^2(c+dx)+1\right)\tan^2(c+dx)}{2bd\tan^2(c+dx)-2ibd\tan(c+dx)}+\frac{i\log\left(\tan^2(c+dx)+1\right)\tan(c+dx)}{2bd\tan^2(c+dx)-2ibd\tan(c+dx)}+\frac{2\log(\tan(c+dx))\tan^2(c+dx)}{2bd\tan^2(c+dx)-2ibd\tan(c+dx)}-\frac{2i\log(\tan(c+dx))\tan(c+dx)}{2bd\tan^2(c+dx)-2ibd\tan(c+dx)}-\frac{3i\tan(c+dx)}{2bd\tan^2(c+dx)-2ibd\tan(c+dx)}-\frac{2}{2bd\tan^2(c+dx)-2ibd\tan(c+dx)} & \text{for } a=-ib \\ \frac{3idx\tan^2(c+dx)}{2bd\tan^2(c+dx)+2ibd\tan(c+dx)}-\frac{3dx\tan(c+dx)}{2bd\tan^2(c+dx)+2ibd\tan(c+dx)}-\frac{\log\left(\tan^2(c+dx)+1\right)\tan^2(c+dx)}{2bd\tan^2(c+dx)+2ibd\tan(c+dx)}-\frac{i\log\left(\tan^2(c+dx)+1\right)\tan(c+dx)}{2bd\tan^2(c+dx)+2ibd\tan(c+dx)}+\frac{2\log(\tan(c+dx))\tan^2(c+dx)}{2bd\tan^2(c+dx)+2ibd\tan(c+dx)}+\frac{2i\log(\tan(c+dx))\tan(c+dx)}{2bd\tan^2(c+dx)+2ibd\tan(c+dx)}+\frac{3i\tan(c+dx)}{2bd\tan^2(c+dx)+2ibd\tan(c+dx)}-\frac{2}{2bd\tan^2(c+dx)+2ibd\tan(c+dx)} & \text{for } a=ib \\ \frac{\tilde{\infty}x}{a} & \text{for } c=-dx \\ \frac{x\cot^2(c)}{a+b\tan(c)} & \text{for } d=0 \\ -\frac{2a^3dx\tan(c+dx)}{2a^4d\tan(c+dx)+2a^2b^2d\tan(c+dx)}-\frac{2a^3}{2a^4d\tan(c+dx)+2a^2b^2d\tan(c+dx)}+\frac{a^2b\log\left(\tan^2(c+dx)+1\right)\tan(c+dx)}{2a^4d\tan(c+dx)+2a^2b^2d\tan(c+dx)}-\frac{2a^2b\log(\tan(c+dx))\tan(c+dx)}{2a^4d\tan(c+dx)+2a^2b^2d\tan(c+dx)}-\frac{2ab^2}{2a^4d\tan(c+dx)+2a^2b^2d\tan(c+dx)}+\frac{2b^3\log\left(\frac{a}{b}+\tan(c+dx)\right)\tan(c+dx)}{2a^4d\tan(c+dx)+2a^2b^2d\tan(c+dx)}-\frac{2b^3\log(\tan(c+dx))\tan(c+dx)}{2a^4d\tan(c+dx)+2a^2b^2d\tan(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**2/(a+b*tan(d*x+c)),x)

[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(c, 0) & Eq(d, 0)), ((-x - cot(c + d*x)/d)/a, Eq(b, 0)), ((log(tan(c + d*x)**2 + 1)/(2*d) - log(tan(c + d*x))/d - 1/(2*d*tan(c + d*x)**2))/b, Eq(a, 0)), (-3*I*d*x*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**2 - 2*I*b*d*tan(c + d*x)) - 3*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x)**2 - 2*I*b*d*tan(c + d*x)) - log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**2 - 2*I*b*d*tan(c + d*x)) + I*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x)**2 - 2*I*b*d*tan(c + d*x)) + 2*log(tan(c + d*x))*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**2 - 2*I*b*d*tan(c + d*x)) - 2*I*log(tan(c + d*x))*tan(c + d*x)/(2*b*d*tan(c + d*x)**2 - 2*I*b*d*tan(c + d*x)) - 3*I*tan(c + d*x)/(2*b*d*tan(c + d*x)**2 - 2*I*b*d*tan(c + d*x)) - 2/(2*b*d*tan(c + d*x)**2 - 2*I*b*d*tan(c + d*x)), Eq(a, -I*b)), (3*I*d*x*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**2 + 2*I*b*d*tan(c + d*x)) - 3*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x)**2 + 2*I*b*d*tan(c + d*x)) - log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**2 + 2*I*b*d*tan(c + d*x)) - I*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x)**2 + 2*I*b*d*tan(c + d*x)) + 2*log(tan(c + d*x))*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**2 + 2*I*b*d*tan(c + d*x)) + 2*I*log(tan(c + d*x))*tan(c + d*x)/(2*b*d*tan(c + d*x)**2 + 2*I*b*d*tan(c + d*x)) + 3*I*tan(c + d*x)/(2*b*d*tan(c + d*x)**2 + 2*I*b*d*tan(c + d*x)) - 2/(2*b*d*tan(c + d*x)**2 + 2*I*b*d*tan(c + d*x)), Eq(a, I*b)), (zoo*x/a, Eq(c, -d*x)), (x*cot(c)**2/(a + b*tan(c)), Eq(d, 0)), (-2*a**3*d*x*tan(c + d*x)/(2*a**4*d*tan(c + d*x) + 2*a**2*b**2*d*tan(c + d*x)) - 2*a**3/(2*a**4*d*tan(c + d*x) + 2*a**2*b**2*d*tan(c + d*x)) + a**2*b*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*a**4*d*tan(c + d*x) + 2*a**2*b**2*d*tan(c + d*x)) - 2*a**2*b*log(tan(c + d*x))*tan(c + d*x)/(2*a**4*d*tan(c + d*x) + 2*a**2*b**2*d*tan(c + d*x)) - 2*a*b**2/(2*a**4*d*tan(c + d*x) + 2*a**2*b**2*d*tan(c + d*x)) + 2*b**3*log(a/b + tan(c + d*x))*tan(c + d*x)/(2*a**4*d*tan(c + d*x) + 2*a**2*b**2*d*tan(c + d*x)) - 2*b**3*log(tan(c + d*x))*tan(c + d*x)/(2*a**4*d*tan(c + d*x) + 2*a**2*b**2*d*tan(c + d*x)), True))

Giac [A]
time = 0.72, size = 116, normalized size = 1.43

$$\frac{\frac{2\,b^4\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^4b+a^2b^3}-\frac{2\,(dx+c)a}{a^2+b^2}+\frac{b\log\left(\tan\left(dx+c\right)^2+1\right)}{a^2+b^2}-\frac{2\,b\log\left(|\tan\left(dx+c\right)|\right)}{a^2}+\frac{2\,(b\tan\left(dx+c\right)-a)}{a^2\tan\left(dx+c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="giac")

[Out] 1/2*(2*b^4*log(abs(b*tan(d*x + c) + a))/(a^4*b + a^2*b^3) - 2*(d*x + c)*a/(a^2 + b^2) + b*log(tan(d*x + c)^2 + 1)/(a^2 + b^2) - 2*b*log(abs(tan(d*x + c)))/a^2 + 2*(b*tan(d*x + c) - a)/(a^2*tan(d*x + c)))/d

Mupad [B]
time = 4.16, size = 108, normalized size = 1.33

$$\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)}{2\,d\,\left(b+a\,1\mathrm{i}\right)}-\frac{\cot(c+d\,x)}{a\,d}-\frac{b\,\ln\left(\tan(c+d\,x)\right)}{a^2\,d}+\frac{b^3\,\ln\left(a+b\tan(c+d\,x)\right)}{a^2\,d\,\left(a^2+b^2\right)}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(a+b\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + b*tan(c + d*x)),x)
```

```
[Out] (log(tan(c + d*x) - 1i)*1i)/(2*d*(a + b*1i)) + log(tan(c + d*x) + 1i)/(2*d*
(a*1i + b)) - cot(c + d*x)/(a*d) - (b*log(tan(c + d*x)))/(a^2*d) + (b^3*log
(a + b*tan(c + d*x)))/(a^2*d*(a^2 + b^2))
```

3.465 $\int \frac{\cot^3(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=107

$$\frac{bx}{a^2+b^2}+\frac{b\cot(c+dx)}{a^2d}-\frac{\cot^2(c+dx)}{2ad}-\frac{(a^2-b^2)\log(\sin(c+dx))}{a^3d}-\frac{b^4\log(a\cos(c+dx)+b\sin(c+dx))}{a^3(a^2+b^2)\,d}$$

```
[Out] b*x/(a^2+b^2)+b*cot(d*x+c)/a^2/d-1/2*cot(d*x+c)^2/a/d-(a^2-b^2)*ln(sin(d*x+
c))/a^3/d-b^4*ln(a*cos(d*x+c)+b*sin(d*x+c))/a^3/(a^2+b^2)/d
```

Rubi [A]
time = 0.21, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3650, 3730, 3733, 3611, 3556}

$$\frac{bx}{a^2+b^2}+\frac{b\cot(c+dx)}{a^2d}-\frac{(a^2-b^2)\log(\sin(c+dx))}{a^3d}-\frac{b^4\log(a\cos(c+dx)+b\sin(c+dx))}{a^3d\,(a^2+b^2)}-\frac{\cot^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*x)/(a^2 + b^2) + (b*Cot[c + d*x])/(a^2*d) - Cot[c + d*x]^2/(2*a*d) - ((a
^2 - b^2)*Log[Sin[c + d*x]])/(a^3*d) - (b^4*Log[a*Cos[c + d*x] + b*Sin[c +
d*x]])/(a^3*(a^2 + b^2)*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
```

```
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3733

```
Int[((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2)/(((a_) + (b_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(a*(
A*c - c*C) - b*(A*d - C*d))*(x/((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^
2 + a^2*C)/((b*c - a*d)*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e
 + f*x]), x], x] - Dist[(c^2*C + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d -
 c*Tan[e + f*x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f,
A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^3(c+dx)}{a+b\tan(c+dx)}\,dx &= -\frac{\cot^2(c+dx)}{2ad} - \frac{\int \frac{\cot^2(c+dx)\left(2b+2a\tan(c+dx)+2b\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx}{2a} \\
&= \frac{b\cot(c+dx)}{a^2d} - \frac{\cot^2(c+dx)}{2ad} + \frac{\int \frac{\cot(c+dx)\left(-2\left(a^2-b^2\right)+2b^2\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx}{2a^2} \\
&= \frac{bx}{a^2+b^2} + \frac{b\cot(c+dx)}{a^2d} - \frac{\cot^2(c+dx)}{2ad} - \frac{\left(a^2-b^2\right)\int\cot(c+dx)\,dx}{a^3} - \frac{b^4\int \frac{b-a\,\mathrm{t}}{a+b\,\mathrm{t}}}{a^3\,(a} \\
&= \frac{bx}{a^2+b^2} + \frac{b\cot(c+dx)}{a^2d} - \frac{\cot^2(c+dx)}{2ad} - \frac{\left(a^2-b^2\right)\log(\sin(c+dx))}{a^3d} - \frac{b^4\log(a\,\mathrm{c}}{}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.68, size = 106, normalized size = 0.99

$$-\frac{-\frac{2b\cot(c+dx)}{a^2}+\frac{\cot^2(c+dx)}{a}-\frac{\log(i-\cot(c+dx))}{a-ib}-\frac{\log(i+\cot(c+dx))}{a+ib}+\frac{2b^4\log(b+a\cot(c+dx))}{a^3(a^2+b^2)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

```
[Out] -1/2*((-2*b*Cot[c + d*x])/a^2 + Cot[c + d*x]^2/a - Log[I - Cot[c + d*x]]/(a
 - I*b) - Log[I + Cot[c + d*x]]/(a + I*b) + (2*b^4*Log[b + a*Cot[c + d*x]])
/(a^3*(a^2 + b^2)))/d
```

Maple [A]

time = 0.27, size = 114, normalized size = 1.07

method	result
derivativedivides	$\frac{-\frac{1}{2a\tan(dx+c)^2}+\frac{\left(-a^2+b^2\right)\ln(\tan(dx+c))}{a^3}+\frac{b}{a^2\tan(dx+c)}-\frac{b^4\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)a^3}+\frac{\frac{a\ln\left(1+\tan^2(dx+c)\right)}{2}+b\arctan(\tan(dx+c))}{a^2+b^2}}{d}$
default	$\frac{-\frac{1}{2a\tan(dx+c)^2}+\frac{\left(-a^2+b^2\right)\ln(\tan(dx+c))}{a^3}+\frac{b}{a^2\tan(dx+c)}-\frac{b^4\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)a^3}+\frac{\frac{a\ln\left(1+\tan^2(dx+c)\right)}{2}+b\arctan(\tan(dx+c))}{a^2+b^2}}{d}$
norman	$\frac{\frac{bx\left(\tan^2(dx+c)\right)}{a^2+b^2}+\frac{b\tan(dx+c)}{a^2d}-\frac{1}{2da}}{\tan(dx+c)^2}+\frac{a\ln(1+\tan^2(dx+c))}{2d(a^2+b^2)}-\frac{(a^2-b^2)\ln(\tan(dx+c))}{a^3d}-\frac{b^4\ln(a+b\tan(dx+c))}{(a^2+b^2)a^3d}$
risch	$\frac{ix}{ib-a}+\frac{2ix}{a}+\frac{2ic}{da}-\frac{2ib^2x}{a^3}-\frac{2ib^2c}{a^3d}+\frac{2ib^4x}{(a^2+b^2)a^3}+\frac{2ib^4c}{(a^2+b^2)a^3d}+\frac{2i\left(-ia\,e^{2i(dx+c)}+b\,e^{2i(dx+c)}-b\right)}{d\,a^2\left(e^{2i(dx+c)}-1\right)^2}-\ln(e^{2i}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/a/tan(d*x+c)^2+1/a^3*(-a^2+b^2)*ln(tan(d*x+c))+b/a^2/tan(d*x+c)-b
^4/(a^2+b^2)/a^3*ln(a+b*tan(d*x+c))+1/(a^2+b^2)*(1/2*a*ln(1+tan(d*x+c)^2)+b
*arctan(tan(d*x+c))))
```

Maxima [A]

time = 0.67, size = 122, normalized size = 1.14

$$-\frac{\frac{2\,b^4\log(b\tan(dx+c)+a)}{a^5+a^3b^2}-\frac{2\,(dx+c)b}{a^2+b^2}-\frac{a\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}+\frac{2\,(a^2-b^2)\log(\tan(dx+c))}{a^3}-\frac{2\,b\tan(dx+c)-a}{a^2\tan(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^4*log(b*tan(d*x + c) + a)/(a^5 + a^3*b^2) - 2*(d*x + c)*b/(a^2 +
b^2) - a*log(tan(d*x + c)^2 + 1)/(a^2 + b^2) + 2*(a^2 - b^2)*log(tan(d*x +
c))/a^3 - (2*b*tan(d*x + c) - a)/(a^2*tan(d*x + c)^2))/d
```

Fricas [A]
time = 1.10, size = 180, normalized size = 1.68

$$-\frac{b^{4} \log \left(\frac{b^{2} \tan (d x+c)^{2}+2 a b \tan (d x+c)+a^{2}}{\tan (d x+c)^{2}+1}\right) \tan (d x+c)^{2}+a^{4}+a^{2} b^{2}+\left(a^{4}-b^{4}\right) \log \left(\frac{\tan (d x+c)^{2}}{\tan (d x+c)^{2}+1}\right) \tan (d x+c)^{2}-\left(2 a^{3} b d x-a^{4}-a^{2} b^{2}\right) \tan (d x+c)^{2}-2\left(a^{3} b+a b^{3}\right) \tan (d x+c)}{2\left(a^{5}+a^{3} b^{2}\right) d \tan (d x+c)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(b^4*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)
^2 + 1))*tan(d*x + c)^2 + a^4 + a^2*b^2 + (a^4 - b^4)*log(tan(d*x + c)^2/(t
an(d*x + c)^2 + 1))*tan(d*x + c)^2 - (2*a^3*b*d*x - a^4 - a^2*b^2)*tan(d*x
+ c)^2 - 2*(a^3*b + a*b^3)*tan(d*x + c))/((a^5 + a^3*b^2)*d*tan(d*x + c)^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.38, size = 1336, normalized size = 12.49

$$\begin{cases} \tilde{\infty} x & \text{for}\: a = 0 \wedge b = 0 \wedge c = 0 \wedge d = 0 \\ \frac{\frac{\log\left(\tan^{2}\left(c + d x\right) + 1\right)}{2 d} - \frac{\log\left(\tan\left(c + d x\right)\right)}{d} - \frac{1}{2 d \tan^{2}\left(c + d x\right)}}{a} & \text{for}\: b = 0 \\ \frac{x + \frac{1}{d \tan\left(c + d x\right)} - \frac{1}{3 d \tan^{3}\left(c + d x\right)}}{b} & \text{for}\: a = 0 \\ - \frac{3 d x \tan^{3}\left(c + d x\right)}{2 b d \tan^{3}\left(c + d x\right) - 2 i b d \tan^{2}\left(c + d x\right)} + \frac{3 i d x \tan^{2}\left(c + d x\right)}{2 b d \tan^{3}\left(c + d x\right) - 2 i b d \tan^{2}\left(c + d x\right)} + \frac{2 i \log\left(\tan^{2}\left(c + d x\right) + 1\right) \tan^{3}\left(c + d x\right)}{2 b d \tan^{3}\left(c + d x\right) - 2 i b d \tan^{2}\left(c + d x\right)} + \frac{2 \log\left(\tan^{2}\left(c + d x\right) + 1\right) \tan^{2}\left(c + d x\right)}{2 b d \tan^{3}\left(c + d x\right) - 2 i b d \tan^{2}\left(c + d x\right)} - \frac{4 i \log\left(\tan\left(c + d x\right)\right) \tan^{3}\left(c + d x\right)}{2 b d \tan^{3}\left(c + d x\right) - 2 i b d \tan^{2}\left(c + d x\right)} - \frac{4 \log\left(\tan\left(c + d x\right)\right) \tan^{2}\left(c + d x\right)}{2 b d \tan^{3}\left(c + d x\right) - 2 i b d \tan^{2}\left(c + d x\right)} - \frac{3 \tan^{2}\left(c + d x\right)}{2 b d \tan^{3}\left(c + d x\right) - 2 i b d \tan^{2}\left(c + d x\right)} + \frac{i \tan\left(c + d x\right)}{2 b d \tan^{3}\left(c + d x\right) - 2 i b d \tan^{2}\left(c + d x\right)} - \frac{1}{2 b d \tan^{3}\left(c + d x\right) - 2 i b d \tan^{2}\left(c + d x\right)} & \text{for}\: a = - i b \\ - \frac{3 d x \tan^{3}\left(c + d x\right)}{2 b d \tan^{3}\left(c + d x\right) + 2 i b d \tan^{2}\left(c + d x\right)} - \frac{3 i d x \tan^{2}\left(c + d x\right)}{2 b d \tan^{3}\left(c + d x\right) + 2 i b d \tan^{2}\left(c + d x\right)} - \frac{2 i \log\left(\tan^{2}\left(c + d x\right) + 1\right) \tan^{3}\left(c + d x\right)}{2 b d \tan^{3}\left(c + d x\right) + 2 i b d \tan^{2}\left(c + d x\right)} + \frac{2 \log\left(\tan^{2}\left(c + d x\right) + 1\right) \tan^{2}\left(c + d x\right)}{2 b d \tan^{3}\left(c + d x\right) + 2 i b d \tan^{2}\left(c + d x\right)} + \frac{4 i \log\left(\tan\left(c + d x\right)\right) \tan^{3}\left(c + d x\right)}{2 b d \tan^{3}\left(c + d x\right) + 2 i b d \tan^{2}\left(c + d x\right)} - \frac{4 \log\left(\tan\left(c + d x\right)\right) \tan^{2}\left(c + d x\right)}{2 b d \tan^{3}\left(c + d x\right) + 2 i b d \tan^{2}\left(c + d x\right)} - \frac{3 \tan^{2}\left(c + d x\right)}{2 b d \tan^{3}\left(c + d x\right) + 2 i b d \tan^{2}\left(c + d x\right)} - \frac{i \tan\left(c + d x\right)}{2 b d \tan^{3}\left(c + d x\right) + 2 i b d \tan^{2}\left(c + d x\right)} - \frac{1}{2 b d \tan^{3}\left(c + d x\right) + 2 i b d \tan^{2}\left(c + d x\right)} & \text{for}\: a = i b \\ \frac{\tilde{\infty} x}{a} & \text{for}\: c = - d x \\ \frac{x \cot^{3}\left(c\right)}{a + b \tan\left(c\right)} & \text{for}\: d = 0 \\ \frac{a^{4} \log\left(\tan^{2}\left(c + d x\right) + 1\right) \tan^{2}\left(c + d x\right)}{2 a^{5} d \tan^{2}\left(c + d x\right) + 2 a^{3} b^{2} d \tan^{2}\left(c + d x\right)} - \frac{2 a^{4} \log\left(\tan\left(c + d x\right)\right) \tan^{2}\left(c + d x\right)}{2 a^{5} d \tan^{2}\left(c + d x\right) + 2 a^{3} b^{2} d \tan^{2}\left(c + d x\right)} - \frac{a^{4}}{2 a^{5} d \tan^{2}\left(c + d x\right) + 2 a^{3} b^{2} d \tan^{2}\left(c + d x\right)} + \frac{2 a^{3} b d x \tan^{2}\left(c + d x\right)}{2 a^{5} d \tan^{2}\left(c + d x\right) + 2 a^{3} b^{2} d \tan^{2}\left(c + d x\right)} + \frac{2 a^{3} b \tan\left(c + d x\right)}{2 a^{5} d \tan^{2}\left(c + d x\right) + 2 a^{3} b^{2} d \tan^{2}\left(c + d x\right)} - \frac{a^{2} b^{2}}{2 a^{5} d \tan^{2}\left(c + d x\right) + 2 a^{3} b^{2} d \tan^{2}\left(c + d x\right)} + \frac{2 a b^{3} \tan\left(c + d x\right)}{2 a^{5} d \tan^{2}\left(c + d x\right) + 2 a^{3} b^{2} d \tan^{2}\left(c + d x\right)} - \frac{2 b^{4} \log\left(\frac{a}{b} + \tan\left(c + d x\right)\right) \tan^{2}\left(c + d x\right)}{2 a^{5} d \tan^{2}\left(c + d x\right) + 2 a^{3} b^{2} d \tan^{2}\left(c + d x\right)} + \frac{2 b^{4} \log\left(\tan\left(c + d x\right)\right) \tan^{2}\left(c + d x\right)}{2 a^{5} d \tan^{2}\left(c + d x\right) + 2 a^{3} b^{2} d \tan^{2}\left(c + d x\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(c, 0) & Eq(d, 0)), ((log(tan(c +
 d*x)**2 + 1)/(2*d) - log(tan(c + d*x))/d - 1/(2*d*tan(c + d*x)**2))/a, Eq(
b, 0)), ((x + 1/(d*tan(c + d*x)) - 1/(3*d*tan(c + d*x)**3))/b, Eq(a, 0)), (
-3*d*x*tan(c + d*x)**3/(2*b*d*tan(c + d*x)**3 - 2*I*b*d*tan(c + d*x)**2) +
3*I*d*x*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**3 - 2*I*b*d*tan(c + d*x)**2) +
 2*I*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**3/(2*b*d*tan(c + d*x)**3 - 2*I*
b*d*tan(c + d*x)**2) + 2*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(2*b*d*ta
n(c + d*x)**3 - 2*I*b*d*tan(c + d*x)**2) - 4*I*log(tan(c + d*x))*tan(c + d*
x)**3/(2*b*d*tan(c + d*x)**3 - 2*I*b*d*tan(c + d*x)**2) - 4*log(tan(c + d*x
))*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**3 - 2*I*b*d*tan(c + d*x)**2) - 3*ta
n(c + d*x)**2/(2*b*d*tan(c + d*x)**3 - 2*I*b*d*tan(c + d*x)**2) + I*tan(c +
 d*x)/(2*b*d*tan(c + d*x)**3 - 2*I*b*d*tan(c + d*x)**2) - 1/(2*b*d*tan(c +
d*x)**3 - 2*I*b*d*tan(c + d*x)**2), Eq(a, -I*b)), (-3*d*x*tan(c + d*x)**3/(
2*b*d*tan(c + d*x)**3 + 2*I*b*d*tan(c + d*x)**2) - 3*I*d*x*tan(c + d*x)**2/
(2*b*d*tan(c + d*x)**3 + 2*I*b*d*tan(c + d*x)**2) - 2*I*log(tan(c + d*x)**2
 + 1)*tan(c + d*x)**3/(2*b*d*tan(c + d*x)**3 + 2*I*b*d*tan(c + d*x)**2) + 2
*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**3 + 2*I*b*d*
tan(c + d*x)**2) + 4*I*log(tan(c + d*x))*tan(c + d*x)**3/(2*b*d*tan(c + d*x
)**3 + 2*I*b*d*tan(c + d*x)**2) - 4*log(tan(c + d*x))*tan(c + d*x)**2/(2*b*
d*tan(c + d*x)**3 + 2*I*b*d*tan(c + d*x)**2) - 3*tan(c + d*x)**2/(2*b*d*tan
(c + d*x)**3 + 2*I*b*d*tan(c + d*x)**2) - I*tan(c + d*x)/(2*b*d*tan(c + d*x
```

```
)**3 + 2*I*b*d*tan(c + d*x)**2) - 1/(2*b*d*tan(c + d*x)**3 + 2*I*b*d*tan(c
+ d*x)**2), Eq(a, I*b)), (zoo*x/a, Eq(c, -d*x)), (x*cot(c)**3/(a + b*tan(c)
), Eq(d, 0)), (a**4*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(2*a**5*d*tan(
c + d*x)**2 + 2*a**3*b**2*d*tan(c + d*x)**2) - 2*a**4*log(tan(c + d*x))*tan
(c + d*x)**2/(2*a**5*d*tan(c + d*x)**2 + 2*a**3*b**2*d*tan(c + d*x)**2) - a
**4/(2*a**5*d*tan(c + d*x)**2 + 2*a**3*b**2*d*tan(c + d*x)**2) + 2*a**3*b*d
*x*tan(c + d*x)**2/(2*a**5*d*tan(c + d*x)**2 + 2*a**3*b**2*d*tan(c + d*x)**
2) + 2*a**3*b*tan(c + d*x)/(2*a**5*d*tan(c + d*x)**2 + 2*a**3*b**2*d*tan(c
+ d*x)**2) - a**2*b**2/(2*a**5*d*tan(c + d*x)**2 + 2*a**3*b**2*d*tan(c + d*
x)**2) + 2*a*b**3*tan(c + d*x)/(2*a**5*d*tan(c + d*x)**2 + 2*a**3*b**2*d*ta
n(c + d*x)**2) - 2*b**4*log(a/b + tan(c + d*x))*tan(c + d*x)**2/(2*a**5*d*t
an(c + d*x)**2 + 2*a**3*b**2*d*tan(c + d*x)**2) + 2*b**4*log(tan(c + d*x))*
tan(c + d*x)**2/(2*a**5*d*tan(c + d*x)**2 + 2*a**3*b**2*d*tan(c + d*x)**2),
 True))
```

Giac [A]
time = 0.75, size = 155, normalized size = 1.45

$$-\frac{\frac{2\,b^5\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^5b+a^3b^3}-\frac{2\,(dx+c)b}{a^2+b^2}-\frac{a\log\left(\tan\left(dx+c\right)^2+1\right)}{a^2+b^2}+\frac{2\,(a^2-b^2)\log\left(|\tan\left(dx+c\right)|\right)}{a^3}-\frac{3\,a^2\tan\left(dx+c\right)^2-3\,b^2\tan\left(dx+c\right)^2+2\,ab\tan\left(dx+c\right)-a^2}{a^3\tan\left(dx+c\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b^5*log(abs(b*tan(d*x + c) + a))/(a^5*b + a^3*b^3) - 2*(d*x + c)*b/
(a^2 + b^2) - a*log(tan(d*x + c)^2 + 1)/(a^2 + b^2) + 2*(a^2 - b^2)*log(abs
(tan(d*x + c)))/a^3 - (3*a^2*tan(d*x + c)^2 - 3*b^2*tan(d*x + c)^2 + 2*a*b*
tan(d*x + c) - a^2)/(a^3*tan(d*x + c)^2))/d
```

Mupad [B]
time = 4.20, size = 137, normalized size = 1.28

$$\frac{\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)}{2\,d\,\left(a-b\,1{}\mathrm{i}\right)}-\frac{\cot(c+d\,x)^2\,\left(\frac{1}{2\,a}-\frac{b\tan(c+d\,x)}{a^2}\right)}{d}-\frac{\ln\left(\tan(c+d\,x)\right)\,\left(a^2-b^2\right)}{a^3\,d}-\frac{b^4\,\ln\left(a+b\tan(c+d\,x)\right)}{d\,\left(a^5+a^3\,b^2\right)}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,1{}\mathrm{i}}{2\,d\,\left(-b+a\,1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + b*tan(c + d*x)),x)
```

```
[Out] log(tan(c + d*x) + 1i)/(2*d*(a - b*1i)) - (cot(c + d*x)^2*(1/(2*a) - (b*tan
(c + d*x))/a^2))/d + (log(tan(c + d*x) - 1i)*1i)/(2*d*(a*1i - b)) - (log(ta
n(c + d*x))*(a^2 - b^2))/(a^3*d) - (b^4*log(a + b*tan(c + d*x)))/(d*(a^5 +
a^3*b^2))
```

3.466 $\int \frac{\cot^4(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=133

$$\frac{ax}{a^2+b^2}+\frac{(a^2-b^2)\cot(c+dx)}{a^3d}+\frac{b\cot^2(c+dx)}{2a^2d}-\frac{\cot^3(c+dx)}{3ad}+\frac{b(a^2-b^2)\log(\sin(c+dx))}{a^4d}+\frac{b^5\log(a\cos(c+\cdots}{a^4(\cdots}$$

```
[Out] a*x/(a^2+b^2)+(a^2-b^2)*cot(d*x+c)/a^3/d+1/2*b*cot(d*x+c)^2/a^2/d-1/3*cot(d
*x+c)^3/a/d+b*(a^2-b^2)*ln(sin(d*x+c))/a^4/d+b^5*ln(a*cos(d*x+c)+b*sin(d*x+
c))/a^4/(a^2+b^2)/d
```

Rubi [A]

time = 0.34, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3650, 3730, 3731, 3732, 3611, 3556}

$$\frac{ax}{a^2+b^2}+\frac{b\cot^2(c+dx)}{2a^2d}+\frac{b(a^2-b^2)\log(\sin(c+dx))}{a^4d}+\frac{b^5\log(a\cos(c+dx)+b\sin(c+dx))}{a^4d\,(a^2+b^2)}+\frac{(a^2-b^2)\cot(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*x)/(a^2 + b^2) + ((a^2 - b^2)*Cot[c + d*x])/(a^3*d) + (b*Cot[c + d*x]^2)
/(2*a^2*d) - Cot[c + d*x]^3/(3*a*d) + (b*(a^2 - b^2)*Log[Sin[c + d*x]])/(a^
4*d) + (b^5*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/(a^4*(a^2 + b^2)*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
```

```
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3731

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)
*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp
[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - a*C*(b*c*(m + 1) + a*d*(n
+ 1)) - (m + 1)*(b*c - a*d)*(A*b - b*C)*Tan[e + f*x] - d*(A*b^2 + a^2*C)*(m
 + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m,
 -1] &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\cot^3(c+dx)}{3ad} - \frac{\int \frac{\cot^3(c+dx)(3b+3a\tan(c+dx)+3b\tan^2(c+dx))}{a+b\tan(c+dx)}\,dx}{3a}$$

$$= \frac{b\cot^2(c+dx)}{2a^2d} - \frac{\cot^3(c+dx)}{3ad} + \frac{\int \frac{\cot^2(c+dx)(-6(a^2-b^2)+6b^2\tan^2(c+dx))}{a+b\tan(c+dx)}\,dx}{6a^2}$$

$$= \frac{(a^2-b^2)\cot(c+dx)}{a^3d} + \frac{b\cot^2(c+dx)}{2a^2d} - \frac{\cot^3(c+dx)}{3ad} - \frac{\int \frac{\cot(c+dx)(-6b(a^2-b^2)-6a^3}{a+}}{}$$

$$= \frac{ax}{a^2+b^2} + \frac{(a^2-b^2)\cot(c+dx)}{a^3d} + \frac{b\cot^2(c+dx)}{2a^2d} - \frac{\cot^3(c+dx)}{3ad} + \frac{(b(a^2-b^2))}{}$$

$$= \frac{ax}{a^2+b^2} + \frac{(a^2-b^2)\cot(c+dx)}{a^3d} + \frac{b\cot^2(c+dx)}{2a^2d} - \frac{\cot^3(c+dx)}{3ad} + \frac{b(a^2-b^2)\log}{a}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.79, size = 131, normalized size = 0.98

$$-\frac{-\frac{6(a^2-b^2)\cot(c+dx)}{a^3} - \frac{3b\cot^2(c+dx)}{a^2} + \frac{2\cot^3(c+dx)}{a} + \frac{3\log(i-\cot(c+dx))}{ia+b} + \frac{3i\log(i+\cot(c+dx))}{a+ib} - \frac{6b^5\log(b+a\cot(c+dx))}{a^4(a^2+b^2)}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

```
[Out] -1/6*((-6*(a^2 - b^2)*Cot[c + d*x])/a^3 - (3*b*Cot[c + d*x]^2)/a^2 + (2*Cot
[c + d*x]^3)/a + (3*Log[I - Cot[c + d*x]])/(I*a + b) + ((3*I)*Log[I + Cot[c
 + d*x]])/(a + I*b) - (6*b^5*Log[b + a*Cot[c + d*x]])/(a^4*(a^2 + b^2)))/d
```

Maple [A]
time = 0.29, size = 137, normalized size = 1.03

method	result
derivativedivides	$\frac{-\frac{1}{3a\tan(dx+c)^3} - \frac{-a^2+b^2}{a^3\tan(dx+c)} + \frac{(a^2-b^2)b\ln(\tan(dx+c))}{a^4} + \frac{b}{2a^2\tan(dx+c)^2} + \frac{b^5\ln(a+b\tan(dx+c))}{(a^2+b^2)a^4} + \frac{-\frac{b\ln(1+\tan^2(dx+c))}{2}+a\,a}{a^2+b^2}}{d}$
default	$\frac{-\frac{1}{3a\tan(dx+c)^3} - \frac{-a^2+b^2}{a^3\tan(dx+c)} + \frac{(a^2-b^2)b\ln(\tan(dx+c))}{a^4} + \frac{b}{2a^2\tan(dx+c)^2} + \frac{b^5\ln(a+b\tan(dx+c))}{(a^2+b^2)a^4} + \frac{-\frac{b\ln(1+\tan^2(dx+c))}{2}+a\,a}{a^2+b^2}}{d}$
norman	$\frac{\frac{ax(\tan^3(dx+c))}{a^2+b^2} + \frac{(a^2-b^2)(\tan^2(dx+c))}{a^3d} - \frac{1}{3da} + \frac{b\tan(dx+c)}{2a^2d}}{\tan(dx+c)^3} + \frac{b^5\ln(a+b\tan(dx+c))}{(a^2+b^2)a^4d} + \frac{(a^2-b^2)b\ln(\tan(dx+c))}{a^4d} - \frac{b}{}$
risch	$-\frac{x}{ib-a} - \frac{2ibx}{a^2} - \frac{2ibc}{a^2d} + \frac{2ib^3x}{a^4} + \frac{2ib^3c}{a^4d} - \frac{2ib^5x}{(a^2+b^2)a^4} - \frac{2ib^5c}{(a^2+b^2)a^4d} - \frac{2i(-3iab\,e^{4i(dx+c)}-6a^2e^{4i(dx+c)}+3b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3/a/tan(d*x+c)^3-(-a^2+b^2)/a^3/tan(d*x+c)+(a^2-b^2)/a^4*b*ln(tan(d
*x+c))+1/2*b/a^2/tan(d*x+c)^2+b^5/(a^2+b^2)/a^4*ln(a+b*tan(d*x+c))+1/(a^2+b
^2)*(-1/2*b*ln(1+tan(d*x+c)^2)+a*arctan(tan(d*x+c))))
```

Maxima [A]
time = 1.02, size = 145, normalized size = 1.09

$$\frac{\frac{6\,b^5\log\left(b\tan\left(dx+c\right)+a\right)}{a^6+a^4b^2}+\frac{6\,(dx+c)a}{a^2+b^2}-\frac{3\,b\log\left(\tan\left(dx+c\right)^2+1\right)}{a^2+b^2}+\frac{6\,(a^2b-b^3)\log\left(\tan\left(dx+c\right)\right)}{a^4}+\frac{3\,ab\tan\left(dx+c\right)+6\,(a^2-b^2)\tan\left(dx+c\right)^2-2\,a^2}{a^3\tan\left(dx+c\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(6*b^5*log(b*tan(d*x + c) + a)/(a^6 + a^4*b^2) + 6*(d*x + c)*a/(a^2 + b
^2) - 3*b*log(tan(d*x + c)^2 + 1)/(a^2 + b^2) + 6*(a^2*b - b^3)*log(tan(d*x
 + c))/a^4 + (3*a*b*tan(d*x + c) + 6*(a^2 - b^2)*tan(d*x + c)^2 - 2*a^2)/(a
^3*tan(d*x + c)^3))/d
```

Fricas [A]
time = 1.83, size = 207, normalized size = 1.56

$$\frac{3\,b^5\log\left(\frac{b^2\tan\left(dx+c\right)^2+2\,ab\tan\left(dx+c\right)+a^2}{\tan\left(dx+c\right)^2+1}\right)\tan\left(dx+c\right)^3-2\,a^5-2\,a^3b^2+3\,(a^4b-b^5)\log\left(\frac{\tan\left(dx+c\right)^2}{\tan\left(dx+c\right)^2+1}\right)\tan\left(dx+c\right)^3+3\,(2\,a^5dx+a^4b+a^2b^3)\tan\left(dx+c\right)^3+6\,(a^5-ab^4)\tan\left(dx+c\right)^2+3\,(a^4b+a^2b^3)\tan\left(dx+c\right)}{6\,(a^6+a^4b^2)d\tan\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^5*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c
)^2 + 1))*tan(d*x + c)^3 - 2*a^5 - 2*a^3*b^2 + 3*(a^4*b - b^5)*log(tan(d*x
+ c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^3 + 3*(2*a^5*d*x + a^4*b + a^2*b^
3)*tan(d*x + c)^3 + 6*(a^5 - a*b^4)*tan(d*x + c)^2 + 3*(a^4*b + a^2*b^3)*ta
n(d*x + c))/((a^6 + a^4*b^2)*d*tan(d*x + c)^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.24, size = 1533, normalized size = 11.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(c, 0) & Eq(d, 0)), ((x - cot(c +
 d*x)**3/(3*d) + cot(c + d*x)/d)/a, Eq(b, 0)), ((-log(tan(c + d*x)**2 + 1)/
(2*d) + log(tan(c + d*x))/d + 1/(2*d*tan(c + d*x)**2) - 1/(4*d*tan(c + d*x)
**4))/b, Eq(a, 0)), (15*I*d*x*tan(c + d*x)**4/(6*b*d*tan(c + d*x)**4 - 6*I*
b*d*tan(c + d*x)**3) + 15*d*x*tan(c + d*x)**3/(6*b*d*tan(c + d*x)**4 - 6*I*
b*d*tan(c + d*x)**3) + 6*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**4/(6*b*d*ta
n(c + d*x)**4 - 6*I*b*d*tan(c + d*x)**3) - 6*I*log(tan(c + d*x)**2 + 1)*tan
(c + d*x)**3/(6*b*d*tan(c + d*x)**4 - 6*I*b*d*tan(c + d*x)**3) - 12*log(tan
(c + d*x))*tan(c + d*x)**4/(6*b*d*tan(c + d*x)**4 - 6*I*b*d*tan(c + d*x)**3
) + 12*I*log(tan(c + d*x))*tan(c + d*x)**3/(6*b*d*tan(c + d*x)**4 - 6*I*b*d
*tan(c + d*x)**3) + 15*I*tan(c + d*x)**3/(6*b*d*tan(c + d*x)**4 - 6*I*b*d*t
an(c + d*x)**3) + 9*tan(c + d*x)**2/(6*b*d*tan(c + d*x)**4 - 6*I*b*d*tan(c
+ d*x)**3) + I*tan(c + d*x)/(6*b*d*tan(c + d*x)**4 - 6*I*b*d*tan(c + d*x)**
3) - 2/(6*b*d*tan(c + d*x)**4 - 6*I*b*d*tan(c + d*x)**3), Eq(a, -I*b)), (-1
5*I*d*x*tan(c + d*x)**4/(6*b*d*tan(c + d*x)**4 + 6*I*b*d*tan(c + d*x)**3) +
 15*d*x*tan(c + d*x)**3/(6*b*d*tan(c + d*x)**4 + 6*I*b*d*tan(c + d*x)**3) +
 6*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**4/(6*b*d*tan(c + d*x)**4 + 6*I*b*
d*tan(c + d*x)**3) + 6*I*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**3/(6*b*d*ta
n(c + d*x)**4 + 6*I*b*d*tan(c + d*x)**3) - 12*log(tan(c + d*x))*tan(c + d*x
)**4/(6*b*d*tan(c + d*x)**4 + 6*I*b*d*tan(c + d*x)**3) - 12*I*log(tan(c + d
*x))*tan(c + d*x)**3/(6*b*d*tan(c + d*x)**4 + 6*I*b*d*tan(c + d*x)**3) - 15
*I*tan(c + d*x)**3/(6*b*d*tan(c + d*x)**4 + 6*I*b*d*tan(c + d*x)**3) + 9*ta
n(c + d*x)**2/(6*b*d*tan(c + d*x)**4 + 6*I*b*d*tan(c + d*x)**3) - I*tan(c +
 d*x)/(6*b*d*tan(c + d*x)**4 + 6*I*b*d*tan(c + d*x)**3) - 2/(6*b*d*tan(c +
d*x)**4 + 6*I*b*d*tan(c + d*x)**3), Eq(a, I*b)), (zoo*x/a, Eq(c, -d*x)), (x
*cot(c)**4/(a + b*tan(c)), Eq(d, 0)), (6*a**5*d*x*tan(c + d*x)**3/(6*a**6*d
*tan(c + d*x)**3 + 6*a**4*b**2*d*tan(c + d*x)**3) + 6*a**5*tan(c + d*x)**2/
(6*a**6*d*tan(c + d*x)**3 + 6*a**4*b**2*d*tan(c + d*x)**3) - 2*a**5/(6*a**6
*d*tan(c + d*x)**3 + 6*a**4*b**2*d*tan(c + d*x)**3) - 3*a**4*b*log(tan(c +
d*x)**2 + 1)*tan(c + d*x)**3/(6*a**6*d*tan(c + d*x)**3 + 6*a**4*b**2*d*tan(
c + d*x)**3) + 6*a**4*b*log(tan(c + d*x))*tan(c + d*x)**3/(6*a**6*d*tan(c +
 d*x)**3 + 6*a**4*b**2*d*tan(c + d*x)**3) + 3*a**4*b*tan(c + d*x)/(6*a**6*d
*tan(c + d*x)**3 + 6*a**4*b**2*d*tan(c + d*x)**3) - 2*a**3*b**2/(6*a**6*d*t
an(c + d*x)**3 + 6*a**4*b**2*d*tan(c + d*x)**3) + 3*a**2*b**3*tan(c + d*x)/
(6*a**6*d*tan(c + d*x)**3 + 6*a**4*b**2*d*tan(c + d*x)**3) - 6*a*b**4*tan(c
 + d*x)**2/(6*a**6*d*tan(c + d*x)**3 + 6*a**4*b**2*d*tan(c + d*x)**3) + 6*b
**5*log(a/b + tan(c + d*x))*tan(c + d*x)**3/(6*a**6*d*tan(c + d*x)**3 + 6*a
**4*b**2*d*tan(c + d*x)**3) - 6*b**5*log(tan(c + d*x))*tan(c + d*x)**3/(6*a
**6*d*tan(c + d*x)**3 + 6*a**4*b**2*d*tan(c + d*x)**3), True))
```

Giac [A]

time = 0.79, size = 187, normalized size = 1.41

$$\frac{\frac{6\,b^{6}\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^{6}b+a^{4}b^{3}}+\frac{6\,\left(dx+c\right)a}{a^{2}+b^{2}}-\frac{3\,b\log\left(\tan\left(dx+c\right)^{2}+1\right)}{a^{2}+b^{2}}+\frac{6\,\left(a^{2}b-b^{3}\right)\log\left(|\tan\left(dx+c\right)|\right)}{a^{4}}-\frac{11\,a^{2}b\tan\left(dx+c\right)^{3}-11\,b^{3}\tan\left(dx+c\right)^{3}-6\,a^{3}\tan\left(dx+c\right)^{2}+6\,ab^{2}\tan\left(dx+c\right)^{2}-3\,a^{2}b\tan\left(dx+c\right)+2\,a^{3}}{a^{4}\tan\left(dx+c\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*b^6*log(abs(b*tan(d*x + c) + a))/(a^6*b + a^4*b^3) + 6*(d*x + c)*a/(
a^2 + b^2) - 3*b*log(tan(d*x + c)^2 + 1)/(a^2 + b^2) + 6*(a^2*b - b^3)*log(
abs(tan(d*x + c)))/a^4 - (11*a^2*b*tan(d*x + c)^3 - 11*b^3*tan(d*x + c)^3 -
 6*a^3*tan(d*x + c)^2 + 6*a*b^2*tan(d*x + c)^2 - 3*a^2*b*tan(d*x + c) + 2*a
^3)/(a^4*tan(d*x + c)^3))/d
```

Mupad [B]
time = 4.26, size = 153, normalized size = 1.15

$$\frac{\cot(c+d\,x)^3\,\left(\frac{\tan(c+d\,x)^2\,(a^2-b^2)}{a^3}-\frac{1}{3\,a}+\frac{b\tan(c+d\,x)}{2\,a^2}\right)}{d}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)}{2\,d\,\left(b+a\,1\mathrm{i}\right)}+\frac{b^5\,\ln\left(a+b\tan(c+d\,x)\right)}{a^4\,d\,\left(a^2+b^2\right)}+\frac{b\,\ln\left(\tan(c+d\,x)\right)\,\left(a^2-b^2\right)}{a^4\,d}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(a+b\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + b*tan(c + d*x)),x)
```

```
[Out] (cot(c + d*x)^3*((tan(c + d*x)^2*(a^2 - b^2))/a^3 - 1/(3*a) + (b*tan(c + d*
x))/(2*a^2)))/d - log(tan(c + d*x) + 1i)/(2*d*(a*1i + b)) - (log(tan(c + d*
x) - 1i)*1i)/(2*d*(a + b*1i)) + (b^5*log(a + b*tan(c + d*x)))/(a^4*d*(a^2 +
 b^2)) + (b*log(tan(c + d*x))*(a^2 - b^2))/(a^4*d)
```

3.467 $\int \frac{\tan^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=239

$$-\frac{(a^2-b^2)\,x}{(a^2+b^2)^2}-\frac{2ab\log(\cos(c+dx))}{(a^2+b^2)^2\,d}-\frac{2a^5(2a^2+3b^2)\log(a+b\tan(c+dx))}{b^5\,(a^2+b^2)^2\,d}+\frac{(4a^4+2a^2b^2-b^4)\tan(c+dx)}{b^4\,(a^2+b^2)\,d}$$

```
[Out] -(a^2-b^2)*x/(a^2+b^2)^2-2*a*b*ln(cos(d*x+c))/(a^2+b^2)^2/d-2*a^5*(2*a^2+3*
b^2)*ln(a+b*tan(d*x+c))/b^5/(a^2+b^2)^2/d+(4*a^4+2*a^2*b^2-b^4)*tan(d*x+c)/
b^4/(a^2+b^2)/d-a*(2*a^2+b^2)*tan(d*x+c)^2/b^3/(a^2+b^2)/d+1/3*(4*a^2+b^2)*
tan(d*x+c)^3/b^2/(a^2+b^2)/d-a^2*tan(d*x+c)^4/b/(a^2+b^2)/d/(a+b*tan(d*x+c)
)
```

Rubi [A]
time = 0.51, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3646, 3728, 3707, 3698, 31, 3556}

$$-\frac{a^2\tan^4(c+dx)}{bd\,(a^2+b^2)\,(a+b\tan(c+dx))}+\frac{(4a^2+b^2)\tan^3(c+dx)}{3b^2d\,(a^2+b^2)}-\frac{2ab\log(\cos(c+dx))}{d\,(a^2+b^2)^2}-\frac{x(a^2-b^2)}{(a^2+b^2)^2}-\frac{a(2a^2+b^2)\tan^2(c+dx)}{b^3d\,(a^2+b^2)}-\frac{2a^5(2a^2+3b^2)\log(a+b\tan(c+dx))}{b^5d\,(a^2+b^2)^2}+\frac{(4a^4+2a^2b^2-b^4)\tan(c+dx)}{b^4d\,(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((a^2 - b^2)*x)/(a^2 + b^2)^2) - (2*a*b*Log[Cos[c + d*x]])/((a^2 + b^2)^2
*d) - (2*a^5*(2*a^2 + 3*b^2)*Log[a + b*Tan[c + d*x]])/(b^5*(a^2 + b^2)^2*d)
 + ((4*a^4 + 2*a^2*b^2 - b^4)*Tan[c + d*x])/(b^4*(a^2 + b^2)*d) - (a*(2*a^2
 + b^2)*Tan[c + d*x]^2)/(b^3*(a^2 + b^2)*d) + ((4*a^2 + b^2)*Tan[c + d*x]^3
)/(3*b^2*(a^2 + b^2)*d) - (a^2*Tan[c + d*x]^4)/(b*(a^2 + b^2)*d*(a + b*Tan[
c + d*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
```

```
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{\tan^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{a^2\tan^4(c+dx)}{b\,(a^2+b^2)\,d(a+b\tan(c+dx))} + \frac{\int \frac{\tan^3(c+dx)\left(4a^2-ab\tan(c+dx)+(4a^2+b^2)\tan^2(c+dx\right.}{a+b\tan(c+dx)}}{b\,(a^2+b^2)}$$

$$= \frac{(4a^2+b^2)\tan^3(c+dx)}{3b^2\,(a^2+b^2)\,d} - \frac{a^2\tan^4(c+dx)}{b\,(a^2+b^2)\,d(a+b\tan(c+dx))} + \frac{\int \frac{\tan^2(c+dx)(-3a(4\ldots}{\ldots}}{\ldots}$$

$$= -\frac{a(2a^2+b^2)\tan^2(c+dx)}{b^3\,(a^2+b^2)\,d} + \frac{(4a^2+b^2)\tan^3(c+dx)}{3b^2\,(a^2+b^2)\,d} - \frac{a^2\tan^4(c+d\ldots}{b\,(a^2+b^2)\,d(a+b\tan\ldots}$$

$$= \frac{(4a^4+2a^2b^2-b^4)\tan(c+dx)}{b^4\,(a^2+b^2)\,d} - \frac{a(2a^2+b^2)\tan^2(c+dx)}{b^3\,(a^2+b^2)\,d} + \frac{(4a^2+b^2)\tan^3(\ldots}{3b^2\,(a^2+b^2\ldots}$$

$$= -\frac{(a^2-b^2)\,x}{(a^2+b^2)^2} + \frac{(4a^4+2a^2b^2-b^4)\tan(c+dx)}{b^4\,(a^2+b^2)\,d} - \frac{a(2a^2+b^2)\tan^2(c+dx)}{b^3\,(a^2+b^2)\,d} + \ldots$$

$$= -\frac{(a^2-b^2)\,x}{(a^2+b^2)^2} - \frac{2ab\log(\cos(c+dx))}{(a^2+b^2)^2\,d} + \frac{(4a^4+2a^2b^2-b^4)\tan(c+dx)}{b^4\,(a^2+b^2)\,d} - \frac{a(2a\ldots}{\ldots}$$

$$= -\frac{(a^2-b^2)\,x}{(a^2+b^2)^2} - \frac{2ab\log(\cos(c+dx))}{(a^2+b^2)^2\,d} - \frac{2a^5(2a^2+3b^2)\log(a+b\tan(c+dx))}{b^5\,(a^2+b^2)^2\,d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.24, size = 242, normalized size = 1.01

$$\frac{\tan^4(c+dx)}{3bd(a+b\tan(c+dx))} + \frac{-\frac{2a\tan^3(c+dx)}{bd(a+b\tan(c+dx))} + \frac{\frac{3ib^2\log(i-\tan(c+dx))}{(a+ib)^2d} - \frac{3ib^2\log(i+\tan(c+dx))}{(a-ib)^2d} - \frac{12a^5(2a^2+3b^2)\log(a+b\tan(c+dx))}{b^3(a^2+b^2)^2d} - \frac{6\left(1-\frac{2a^2}{b^2}\right)\tan(c+dx)}{d} - \frac{6a^4(2a^2+b^2)}{b^3(a^2+b^2)d(a+b\tan(c+dx))}}{2b}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] Tan[c + d*x]^4/(3*b*d*(a + b*Tan[c + d*x])) + ((-2*a*Tan[c + d*x]^3)/(b*d*(
a + b*Tan[c + d*x])) + (((3*I)*b^2*Log[I - Tan[c + d*x]])/((a + I*b)^2*d) -
 ((3*I)*b^2*Log[I + Tan[c + d*x]])/((a - I*b)^2*d) - (12*a^5*(2*a^2 + 3*b^2
)*Log[a + b*Tan[c + d*x]])/(b^3*(a^2 + b^2)^2*d) - (6*(1 - (2*a^2)/b^2)*Tan
[c + d*x])/d - (6*a^4*(2*a^2 + b^2))/(b^3*(a^2 + b^2)*d*(a + b*Tan[c + d*x]
)))/(2*b))/(3*b)
```

Maple [A]
time = 0.16, size = 168, normalized size = 0.70

method	result

derivativedivides	$\frac{\frac{\frac{b^2\left(\tan^3(dx+c)\right)}{3}-ab\left(\tan^2(dx+c)\right)+3a^2\tan(dx+c)-b^2\tan(dx+c)}{b^4}+\frac{ab\ln\left(1+\tan^2(dx+c)\right)+\left(-a^2+b^2\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^2}-\frac{}{b^5\left(a^2+b^2\right.}}{d}$
default	$\frac{\frac{\frac{b^2\left(\tan^3(dx+c)\right)}{3}-ab\left(\tan^2(dx+c)\right)+3a^2\tan(dx+c)-b^2\tan(dx+c)}{b^4}+\frac{ab\ln\left(1+\tan^2(dx+c)\right)+\left(-a^2+b^2\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^2}-\frac{}{b^5\left(a^2+b^2\right.}}{d}$
norman	$\frac{\frac{\left(2a^2-b^2\right)\left(\tan^2(dx+c)\right)}{b^3d}+\frac{\tan^4(dx+c)}{3bd}-\frac{2a\left(\tan^3(dx+c)\right)}{3b^2d}-\frac{\left(a^2-b^2\right)ax}{a^4+2a^2b^2+b^4}-\frac{b\left(a^2-b^2\right)x\tan(dx+c)}{a^4+2a^2b^2+b^4}-\frac{\left(4a^5+2a^3b^2-a\,b^4\right)a}{d\,b^5\left(a^2+b^2\right)}}{a+b\tan(dx+c)}+\frac{ab\ln}{d(}$
risch	$\frac{x}{2iab-a^2+b^2}+\frac{8ia^7x}{(a^4+2a^2b^2+b^4)b^5}+\frac{8ia^7c}{(a^4+2a^2b^2+b^4)b^5d}+\frac{12ia^5x}{(a^4+2a^2b^2+b^4)b^3}+\frac{12ia^5c}{(a^4+2a^2b^2+b^4)b^3d}-\frac{8ia^3x}{b^5}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^4*(1/3*b^2*tan(d*x+c)^3-a*b*tan(d*x+c)^2+3*a^2*tan(d*x+c)-b^2*tan(
d*x+c))+1/(a^2+b^2)^2*(a*b*ln(1+tan(d*x+c)^2)+(-a^2+b^2)*arctan(tan(d*x+c))
)-1/b^5*a^6/(a^2+b^2)/(a+b*tan(d*x+c))-2/b^5*a^5*(2*a^2+3*b^2)/(a^2+b^2)^2*
ln(a+b*tan(d*x+c)))
```

Maxima [A]

time = 0.66, size = 206, normalized size = 0.86

$$-\frac{\frac{3\,a^6}{a^3b^5+ab^7+(a^2b^6+b^8)\tan(dx+c)}-\frac{3\,ab\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{3\,(a^2-b^2)(dx+c)}{a^4+2\,a^2b^2+b^4}+\frac{6\,(2\,a^7+3\,a^5b^2)\log(b\tan(dx+c)+a)}{a^4b^5+2\,a^2b^7+b^9}-\frac{b^2\tan(dx+c)^3-3\,ab\tan(dx+c)^2+3\,(3\,a^2-b^2)\tan(dx+c)}{b^4}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(3*a^6/(a^3*b^5 + a*b^7 + (a^2*b^6 + b^8)*tan(d*x + c)) - 3*a*b*log(ta
n(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 3*(a^2 - b^2)*(d*x + c)/(a^4 +
2*a^2*b^2 + b^4) + 6*(2*a^7 + 3*a^5*b^2)*log(b*tan(d*x + c) + a)/(a^4*b^5 +
 2*a^2*b^7 + b^9) - (b^2*tan(d*x + c)^3 - 3*a*b*tan(d*x + c)^2 + 3*(3*a^2 -
 b^2)*tan(d*x + c))/b^4)/d
```

Fricas [A]

time = 1.37, size = 387, normalized size = 1.62

$$-\frac{6\,a^6b^2+6\,a^4b^4+3\,a^2b^6-(a^4b^4+2\,a^2b^6+b^8)\tan(dx+c)^4+2\,(a^5b^3+2\,a^3b^5+ab^7)\tan(dx+c)^3+3\,(a^3b^5-ab^7)dx-3\,(2\,a^6b^2+3\,a^4b^4-b^8)\tan(dx+c)^2+3\,(2\,a^8+3\,a^6b^2+(2\,a^7b+3\,a^5b^3)\tan(dx+c))\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)-3\,(2\,a^8+3\,a^6b^2-a^2b^6+(2\,a^7b+3\,a^5b^3-ab^7)\tan(dx+c))\log\left(\frac{1}{\tan(dx+c)^2+1}\right)-3\,(4\,a^7b+4\,a^5b^3-a^3b^5-2\,ab^7-(a^2b^6-b^8)dx)\tan(dx+c)}{3\,((a^4b^6+2\,a^2b^8+b^{10})d\tan(dx+c)+(a^5b^5+2\,a^3b^7+ab^9)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(6*a^6*b^2 + 6*a^4*b^4 + 3*a^2*b^6 - (a^4*b^4 + 2*a^2*b^6 + b^8)*tan(d
*x + c)^4 + 2*(a^5*b^3 + 2*a^3*b^5 + a*b^7)*tan(d*x + c)^3 + 3*(a^3*b^5 - a
```

```
*b^7)*d*x - 3*(2*a^6*b^2 + 3*a^4*b^4 - b^8)*tan(d*x + c)^2 + 3*(2*a^8 + 3*a
^6*b^2 + (2*a^7*b + 3*a^5*b^3)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*
b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) - 3*(2*a^8 + 3*a^6*b^2 - a^2*b^
6 + (2*a^7*b + 3*a^5*b^3 - a*b^7)*tan(d*x + c))*log(1/(tan(d*x + c)^2 + 1))
 - 3*(4*a^7*b + 4*a^5*b^3 - a^3*b^5 - 2*a*b^7 - (a^2*b^6 - b^8)*d*x)*tan(d*
x + c))/((a^4*b^6 + 2*a^2*b^8 + b^10)*d*tan(d*x + c) + (a^5*b^5 + 2*a^3*b^7
 + a*b^9)*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.51, size = 3279, normalized size = 13.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Piecewise((zoo*x*tan(c)**4, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (75*d*x*tan(c
+ d*x)**2/(12*b**2*d*tan(c + d*x)**2 - 24*I*b**2*d*tan(c + d*x) - 12*b**2*d
) - 150*I*d*x*tan(c + d*x)/(12*b**2*d*tan(c + d*x)**2 - 24*I*b**2*d*tan(c +
 d*x) - 12*b**2*d) - 75*d*x/(12*b**2*d*tan(c + d*x)**2 - 24*I*b**2*d*tan(c
+ d*x) - 12*b**2*d) - 36*I*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(12*b**
2*d*tan(c + d*x)**2 - 24*I*b**2*d*tan(c + d*x) - 12*b**2*d) - 72*log(tan(c
+ d*x)**2 + 1)*tan(c + d*x)/(12*b**2*d*tan(c + d*x)**2 - 24*I*b**2*d*tan(c
+ d*x) - 12*b**2*d) + 36*I*log(tan(c + d*x)**2 + 1)/(12*b**2*d*tan(c + d*x)
**2 - 24*I*b**2*d*tan(c + d*x) - 12*b**2*d) + 4*tan(c + d*x)**5/(12*b**2*d*
tan(c + d*x)**2 - 24*I*b**2*d*tan(c + d*x) - 12*b**2*d) + 4*I*tan(c + d*x)*
*4/(12*b**2*d*tan(c + d*x)**2 - 24*I*b**2*d*tan(c + d*x) - 12*b**2*d) - 28*
tan(c + d*x)**3/(12*b**2*d*tan(c + d*x)**2 - 24*I*b**2*d*tan(c + d*x) - 12*
b**2*d) - 153*tan(c + d*x)/(12*b**2*d*tan(c + d*x)**2 - 24*I*b**2*d*tan(c +
 d*x) - 12*b**2*d) + 114*I/(12*b**2*d*tan(c + d*x)**2 - 24*I*b**2*d*tan(c +
 d*x) - 12*b**2*d), Eq(a, -I*b)), (75*d*x*tan(c + d*x)**2/(12*b**2*d*tan(c
+ d*x)**2 + 24*I*b**2*d*tan(c + d*x) - 12*b**2*d) + 150*I*d*x*tan(c + d*x)/
(12*b**2*d*tan(c + d*x)**2 + 24*I*b**2*d*tan(c + d*x) - 12*b**2*d) - 75*d*x
/(12*b**2*d*tan(c + d*x)**2 + 24*I*b**2*d*tan(c + d*x) - 12*b**2*d) + 36*I*
log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(12*b**2*d*tan(c + d*x)**2 + 24*I*
b**2*d*tan(c + d*x) - 12*b**2*d) - 72*log(tan(c + d*x)**2 + 1)*tan(c + d*x)
/(12*b**2*d*tan(c + d*x)**2 + 24*I*b**2*d*tan(c + d*x) - 12*b**2*d) - 36*I*
log(tan(c + d*x)**2 + 1)/(12*b**2*d*tan(c + d*x)**2 + 24*I*b**2*d*tan(c + d
*x) - 12*b**2*d) + 4*tan(c + d*x)**5/(12*b**2*d*tan(c + d*x)**2 + 24*I*b**2
*d*tan(c + d*x) - 12*b**2*d) - 4*I*tan(c + d*x)**4/(12*b**2*d*tan(c + d*x)*
*2 + 24*I*b**2*d*tan(c + d*x) - 12*b**2*d) - 28*tan(c + d*x)**3/(12*b**2*d*
tan(c + d*x)**2 + 24*I*b**2*d*tan(c + d*x) - 12*b**2*d) - 153*tan(c + d*x)/
(12*b**2*d*tan(c + d*x)**2 + 24*I*b**2*d*tan(c + d*x) - 12*b**2*d) - 114*I/
(12*b**2*d*tan(c + d*x)**2 + 24*I*b**2*d*tan(c + d*x) - 12*b**2*d), Eq(a, I
*b)), ((-x + tan(c + d*x)**5/(5*d) - tan(c + d*x)**3/(3*d) + tan(c + d*x)/d
```

```
)/a**2, Eq(b, 0)), (x*tan(c)**6/(a + b*tan(c))**2, Eq(d, 0)), (-12*a**8*log
(a/b + tan(c + d*x))/(3*a**5*b**5*d + 3*a**4*b**6*d*tan(c + d*x) + 6*a**3*b
**7*d + 6*a**2*b**8*d*tan(c + d*x) + 3*a*b**9*d + 3*b**10*d*tan(c + d*x)) -
 12*a**8/(3*a**5*b**5*d + 3*a**4*b**6*d*tan(c + d*x) + 6*a**3*b**7*d + 6*a*
*2*b**8*d*tan(c + d*x) + 3*a*b**9*d + 3*b**10*d*tan(c + d*x)) - 12*a**7*b*l
og(a/b + tan(c + d*x))*tan(c + d*x)/(3*a**5*b**5*d + 3*a**4*b**6*d*tan(c +
d*x) + 6*a**3*b**7*d + 6*a**2*b**8*d*tan(c + d*x) + 3*a*b**9*d + 3*b**10*d*
tan(c + d*x)) - 18*a**6*b**2*log(a/b + tan(c + d*x))/(3*a**5*b**5*d + 3*a**
4*b**6*d*tan(c + d*x) + 6*a**3*b**7*d + 6*a**2*b**8*d*tan(c + d*x) + 3*a*b*
*9*d + 3*b**10*d*tan(c + d*x)) + 6*a**6*b**2*tan(c + d*x)**2/(3*a**5*b**5*d
 + 3*a**4*b**6*d*tan(c + d*x) + 6*a**3*b**7*d + 6*a**2*b**8*d*tan(c + d*x)
+ 3*a*b**9*d + 3*b**10*d*tan(c + d*x)) - 18*a**6*b**2/(3*a**5*b**5*d + 3*a*
*4*b**6*d*tan(c + d*x) + 6*a**3*b**7*d + 6*a**2*b**8*d*tan(c + d*x) + 3*a*b
**9*d + 3*b**10*d*tan(c + d*x)) - 18*a**5*b**3*log(a/b + tan(c + d*x))*tan(
c + d*x)/(3*a**5*b**5*d + 3*a**4*b**6*d*tan(c + d*x) + 6*a**3*b**7*d + 6*a*
*2*b**8*d*tan(c + d*x) + 3*a*b**9*d + 3*b**10*d*tan(c + d*x)) - 2*a**5*b**3
*tan(c + d*x)**3/(3*a**5*b**5*d + 3*a**4*b**6*d*tan(c + d*x) + 6*a**3*b**7*
d + 6*a**2*b**8*d*tan(c + d*x) + 3*a*b**9*d + 3*b**10*d*tan(c + d*x)) + a**
4*b**4*tan(c + d*x)**4/(3*a**5*b**5*d + 3*a**4*b**6*d*tan(c + d*x) + 6*a**3
*b**7*d + 6*a**2*b**8*d*tan(c + d*x) + 3*a*b**9*d + 3*b**10*d*tan(c + d*x))
 + 9*a**4*b**4*tan(c + d*x)**2/(3*a**5*b**5*d + 3*a**4*b**6*d*tan(c + d*x)
+ 6*a**3*b**7*d + 6*a**2*b**8*d*tan(c + d*x) + 3*a*b**9*d + 3*b**10*d*tan(c
 + d*x)) - 3*a**4*b**4/(3*a**5*b**5*d + 3*a**4*b**6*d*tan(c + d*x) + 6*a**3
*b**7*d + 6*a**2*b**8*d*tan(c + d*x) + 3*a*b**9*d + 3*b**10*d*tan(c + d*x))
 - 3*a**3*b**5*d*x/(3*a**5*b**5*d + 3*a**4*b**6*d*tan(c + d*x) + 6*a**3*b**
7*d + 6*a**2*b**8*d*tan(c + d*x) + 3*a*b**9*d + 3*b**10*d*tan(c + d*x)) - 4
*a**3*b**5*tan(c + d*x)**3/(3*a**5*b**5*d + 3*a**4*b**6*d*tan(c + d*x) + 6*
a**3*b**7*d + 6*a**2*b**8*d*tan(c + d*x) + 3*a*b**9*d + 3*b**10*d*tan(c + d
*x)) - 3*a**2*b**6*d*x*tan(c + d*x)/(3*a**5*b**5*d + 3*a**4*b**6*d*tan(c +
d*x) + 6*a**3*b**7*d + 6*a**2*b**8*d*tan(c + d*x) + 3*a*b**9*d + 3*b**10*d*
tan(c + d*x)) + 3*a**2*b**6*log(tan(c + d*x)**2 + 1)/(3*a**5*b**5*d + 3*a**
4*b**6*d*tan(c + d*x) + 6*a**3*b**7*d + 6*a**2*b**8*d*tan(c + d*x) + 3*a*b*
*9*d + 3*b**10*d*tan(c + d*x)) + 2*a**2*b**6*tan(c + d*x)**4/(3*a**5*b**5*d
 + 3*a**4*b**6*d*tan(c + d*x) + 6*a**3*b**7*d + 6*a**2*b**8*d*tan(c + d*x)
+ 3*a*b**9*d + 3*b**10*d*tan(c + d*x)) + 3*a**2*b**6/(3*a**5*b**5*d + 3*a**
4*b**6*d*tan(c + d*x) + 6*a**3*b**7*d + 6*a**2*b**8*d*tan(c + d*x) + 3*a*b*
*9*d + 3*b**10*d*tan(c + d*x)) + 3*a*b**7*d*x/(3*a**5*b**5*d + 3*a**4*b**6*
d*tan(c + d*x) + 6*a**3*b**7*d + 6*a**2*b**8*d*...
```

Giac [A]

time = 2.44, size = 251, normalized size = 1.05

$$\frac{\frac{3\,ab\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4} - \frac{3\,(a^2-b^2)(dx+c)}{a^4+2\,a^2b^2+b^4} - \frac{6\,(2\,a^7+3\,a^5b^2)\log(|b\tan(dx+c)+a|)}{a^4b^5+2\,a^2b^7+b^9} + \frac{3\,(4\,a^7b\tan(dx+c)+6\,a^5b^3\tan(dx+c)+3\,a^8+5\,a^6b^2)}{(a^4b^5+2\,a^2b^7+b^9)(b\tan(dx+c)+a)} + \frac{b^4\tan(dx+c)^3-3\,ab^3\tan(dx+c)^2+9\,a^2b^2\tan(dx+c)-3\,b^4\tan(dx+c)}{b^6}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*a*b*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 3*(a^2 - b^2)*
(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - 6*(2*a^7 + 3*a^5*b^2)*log(abs(b*tan(d*x
 + c) + a))/(a^4*b^5 + 2*a^2*b^7 + b^9) + 3*(4*a^7*b*tan(d*x + c) + 6*a^5*b
^3*tan(d*x + c) + 3*a^8 + 5*a^6*b^2)/((a^4*b^5 + 2*a^2*b^7 + b^9)*(b*tan(d*
x + c) + a)) + (b^4*tan(d*x + c)^3 - 3*a*b^3*tan(d*x + c)^2 + 9*a^2*b^2*tan
(d*x + c) - 3*b^4*tan(d*x + c))/b^6)/d
```

Mupad [B]
time = 4.38, size = 213, normalized size = 0.89

$$\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)}{2\,d\,\left(-a^2\,1\mathrm{i}+2\,a\,b+b^2\,1\mathrm{i}\right)}-\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{4\,a^3}{b^5}-\frac{2\,a}{b^3}+\frac{2\,a\,b}{{\left(a^2+b^2\right)}^2}\right)}{d}+\frac{{\tan(c+d\,x)}^3}{3\,b^2\,d}-\frac{\tan(c+d\,x)\,\left(\frac{a^2+b^2}{b^4}-\frac{4\,a^2}{b^4}\right)}{d}-\frac{a\,{\tan(c+d\,x)}^2}{b^3\,d}-\frac{a^6}{b\,d\,\left(\tan\left(c+d\,x\right)\,b^5+a\,b^4\right)\,\left(a^2+b^2\right)}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(-a^2+a\,b\,2\mathrm{i}+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6/(a + b*tan(c + d*x))^2,x)
```

```
[Out] (log(tan(c + d*x) + 1i)*1i)/(2*d*(a*b*2i - a^2 + b^2)) + log(tan(c + d*x) -
 1i)/(2*d*(2*a*b - a^2*1i + b^2*1i)) - (log(a + b*tan(c + d*x))*((4*a^3)/b^
5 - (2*a)/b^3 + (2*a*b)/(a^2 + b^2)^2))/d + tan(c + d*x)^3/(3*b^2*d) - (tan
(c + d*x)*((a^2 + b^2)/b^4 - (4*a^2)/b^4))/d - (a*tan(c + d*x)^2)/(b^3*d) -
 a^6/(b*d*(a*b^4 + b^5*tan(c + d*x))*(a^2 + b^2))
```

3.468 $\int \frac{\tan^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=197

$$\frac{2abx}{(a^2+b^2)^2}-\frac{(a^2-b^2)\log(\cos(c+dx))}{(a^2+b^2)^2 d}+\frac{a^4(3a^2+5b^2)\log(a+b\tan(c+dx))}{b^4(a^2+b^2)^2 d}-\frac{a(3a^2+2b^2)\tan(c+dx)}{b^3(a^2+b^2)d}+\frac{(3a$$

```
[Out] 2*a*b*x/(a^2+b^2)^2-(a^2-b^2)*ln(cos(d*x+c))/(a^2+b^2)^2/d+a^4*(3*a^2+5*b^2
)*ln(a+b*tan(d*x+c))/b^4/(a^2+b^2)^2/d-a*(3*a^2+2*b^2)*tan(d*x+c)/b^3/(a^2+
b^2)/d+1/2*(3*a^2+b^2)*tan(d*x+c)^2/b^2/(a^2+b^2)/d-a^2*tan(d*x+c)^3/b/(a^2
+b^2)/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.35, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3646, 3728, 3707, 3698, 31, 3556}

$$-\frac{a^2\tan^3(c+dx)}{bd(a^2+b^2)(a+b\tan(c+dx))}+\frac{(3a^2+b^2)\tan^2(c+dx)}{2b^2d(a^2+b^2)}-\frac{(a^2-b^2)\log(\cos(c+dx))}{d(a^2+b^2)^2}+\frac{2abx}{(a^2+b^2)^2}-\frac{a(3a^2+2b^2)\tan(c+dx)}{b^3d(a^2+b^2)}+\frac{a^4(3a^2+5b^2)\log(a+b\tan(c+dx))}{b^4d(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (2*a*b*x)/(a^2 + b^2)^2 - ((a^2 - b^2)*Log[Cos[c + d*x]])/((a^2 + b^2)^2*d)
 + (a^4*(3*a^2 + 5*b^2)*Log[a + b*Tan[c + d*x]])/(b^4*(a^2 + b^2)^2*d) - (a
*(3*a^2 + 2*b^2)*Tan[c + d*x])/(b^3*(a^2 + b^2)*d) + ((3*a^2 + b^2)*Tan[c +
 d*x]^2)/(2*b^2*(a^2 + b^2)*d) - (a^2*Tan[c + d*x]^3)/(b*(a^2 + b^2)*d*(a +
 b*Tan[c + d*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
```

```
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{a^2\tan^3(c+dx)}{b(a^2+b^2)\,d(a+b\tan(c+dx))} + \frac{\int \frac{\tan^2(c+dx)(3a^2-ab\tan(c+dx)+(3a^2+b^2)\tan^2(c+dx))}{a+b\tan(c+dx)}}{b(a^2+b^2)}$$

$$= \frac{(3a^2+b^2)\tan^2(c+dx)}{2b^2(a^2+b^2)\,d} - \frac{a^2\tan^3(c+dx)}{b(a^2+b^2)\,d(a+b\tan(c+dx))} + \int \frac{\tan(c+dx)(-2a(3a^2+}{}$$

$$= -\frac{a(3a^2+2b^2)\tan(c+dx)}{b^3(a^2+b^2)\,d} + \frac{(3a^2+b^2)\tan^2(c+dx)}{2b^2(a^2+b^2)\,d} - \frac{a^2\tan^3(c+dx}{b(a^2+b^2)\,d(a+b\tan(}$$

$$= \frac{2abx}{(a^2+b^2)^2} - \frac{a(3a^2+2b^2)\tan(c+dx)}{b^3(a^2+b^2)\,d} + \frac{(3a^2+b^2)\tan^2(c+dx)}{2b^2(a^2+b^2)\,d} - \frac{a^2\,\mathrm{t}}{b(a^2+b^2)\,d}$$

$$= \frac{2abx}{(a^2+b^2)^2} - \frac{(a^2-b^2)\log(\cos(c+dx))}{(a^2+b^2)^2\,d} - \frac{a(3a^2+2b^2)\tan(c+dx)}{b^3(a^2+b^2)\,d} + \frac{(3a^2+b^2}{2b^2(}$$

$$= \frac{2abx}{(a^2+b^2)^2} - \frac{(a^2-b^2)\log(\cos(c+dx))}{(a^2+b^2)^2\,d} + \frac{a^4(3a^2+5b^2)\log(a+b\tan(c+dx))}{b^4(a^2+b^2)^2\,d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.02, size = 182, normalized size = 0.92

$$\frac{\frac{b\log(i-\tan(c+dx))}{(a+ib)^2} + \frac{b\log(i+\tan(c+dx))}{(a-ib)^2} + \frac{2a^4(3a^2+5b^2)\log(a+b\tan(c+dx))}{b^3(a^2+b^2)^2} + \frac{6a^5+4a^3b^2}{b^3(a^2+b^2)(a+b\tan(c+dx))} - \frac{3a\tan^2(c+dx)}{b(a+b\tan(c+dx))} + \frac{\tan^3(c+dx)}{a+b\tan(c+dx)}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((b*Log[I - Tan[c + d*x]])/(a + I*b)^2 + (b*Log[I + Tan[c + d*x]])/(a - I*b
)^2 + (2*a^4*(3*a^2 + 5*b^2)*Log[a + b*Tan[c + d*x]])/(b^3*(a^2 + b^2)^2) +
 (6*a^5 + 4*a^3*b^2)/(b^3*(a^2 + b^2)*(a + b*Tan[c + d*x])) - (3*a*Tan[c +
d*x]^2)/(b*(a + b*Tan[c + d*x])) + Tan[c + d*x]^3/(a + b*Tan[c + d*x]))/(2*
b*d)
```

Maple [A]
time = 0.17, size = 142, normalized size = 0.72

method	result
derivativedivides	$\frac{-\frac{-\frac{b\left(\tan^2(dx+c)\right)}{2}+2a\tan(dx+c)}{b^3}+\frac{\frac{\left(a^2-b^2\right)\ln\left(1+\tan^2(dx+c)\right)}{2}+2ab\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^2}+\frac{a^4\left(3a^2+5b^2\right)\ln(a+b\tan(dx+c))}{b^4\left(a^2+b^2\right)^2}+\frac{}{b^4\left(a^2\right.}}{d}$
default	$\frac{-\frac{-\frac{b\left(\tan^2(dx+c)\right)}{2}+2a\tan(dx+c)}{b^3}+\frac{\frac{\left(a^2-b^2\right)\ln\left(1+\tan^2(dx+c)\right)}{2}+2ab\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^2}+\frac{a^4\left(3a^2+5b^2\right)\ln(a+b\tan(dx+c))}{b^4\left(a^2+b^2\right)^2}+\frac{}{b^4\left(a^2\right.}}{d}$

norman	$\frac{\frac{(3a^4+2a^2b^2)a}{d\,b^4(a^2+b^2)}+\frac{\tan^3(dx+c)}{2bd}-\frac{3a(\tan^2(dx+c))}{2b^2d}+\frac{2a^2bx}{a^4+2a^2b^2+b^4}+\frac{2b^2ax\tan(dx+c)}{a^4+2a^2b^2+b^4}}{a+b\tan(dx+c)}+\frac{a^4(3a^2+5b^2)\ln(a+b\tan(dx+c))}{(a^4+2a^2b^2+b^4)b^4d}+$
risch	$\frac{ix}{2iab-a^2+b^2}+\frac{6ia^2x}{b^4}+\frac{6ia^2c}{b^4d}-\frac{2ix}{b^2}-\frac{2ic}{b^2d}-\frac{6ia^6x}{b^4(a^4+2a^2b^2+b^4)}-\frac{6ia^6c}{b^4d(a^4+2a^2b^2+b^4)}-\frac{10ia^4x}{b^2(a^4+2a^2b^2+b^4)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^3*(-1/2*b*tan(d*x+c)^2+2*a*tan(d*x+c))+1/(a^2+b^2)^2*(1/2*(a^2-b^
2)*ln(1+tan(d*x+c)^2)+2*a*b*arctan(tan(d*x+c)))+1/b^4*a^4*(3*a^2+5*b^2)/(a^
2+b^2)^2*ln(a+b*tan(d*x+c))+1/b^4*a^5/(a^2+b^2)/(a+b*tan(d*x+c)))
```

Maxima [A]
time = 1.08, size = 180, normalized size = 0.91

$$\frac{\frac{2\,a^5}{a^3b^4+ab^6+(a^2b^5+b^7)\tan(dx+c)}+\frac{4\,(dx+c)ab}{a^4+2\,a^2b^2+b^4}+\frac{2\,(3\,a^6+5\,a^4b^2)\log(b\tan(dx+c)+a)}{a^4b^4+2\,a^2b^6+b^8}+\frac{(a^2-b^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{b\tan(dx+c)^2-4\,a\tan(dx+c)}{b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^5/(a^3*b^4 + a*b^6 + (a^2*b^5 + b^7)*tan(d*x + c)) + 4*(d*x + c)*a
*b/(a^4 + 2*a^2*b^2 + b^4) + 2*(3*a^6 + 5*a^4*b^2)*log(b*tan(d*x + c) + a)/
(a^4*b^4 + 2*a^2*b^6 + b^8) + (a^2 - b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*
a^2*b^2 + b^4) + (b*tan(d*x + c)^2 - 4*a*tan(d*x + c))/b^3)/d
```

Fricas [A]
time = 1.07, size = 340, normalized size = 1.73

$$\frac{4\,a^2b^5dx+3\,a^5b^2+2\,a^3b^4+ab^6+(a^4b^3+2\,a^2b^5+b^7)\tan(dx+c)^3-3\,(a^5b^2+2\,a^3b^4+ab^6)\tan(dx+c)^2+(3\,a^7+5\,a^5b^2+(3\,a^6b+5\,a^4b^3)\tan(dx+c))\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)-(3\,a^7+5\,a^5b^2+a^3b^4-ab^6+(3\,a^6b+5\,a^4b^3+a^2b^5-b^7)\tan(dx+c))\log\left(\frac{1}{\tan(dx+c)^2+1}\right)+(4\,ab^6dx-6\,a^6b-7\,a^4b^3-2\,a^2b^5+b^7)\tan(dx+c)}{2\,((a^4b^5+2\,a^2b^7+b^9)d\tan(dx+c)+(a^5b^4+2\,a^3b^6+ab^8)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^2*b^5*d*x + 3*a^5*b^2 + 2*a^3*b^4 + a*b^6 + (a^4*b^3 + 2*a^2*b^5 +
 b^7)*tan(d*x + c)^3 - 3*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*tan(d*x + c)^2 + (3*
a^7 + 5*a^5*b^2 + (3*a^6*b + 5*a^4*b^3)*tan(d*x + c))*log((b^2*tan(d*x + c)
^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) - (3*a^7 + 5*a^5*b^2 +
 a^3*b^4 - a*b^6 + (3*a^6*b + 5*a^4*b^3 + a^2*b^5 - b^7)*tan(d*x + c))*log(
1/(tan(d*x + c)^2 + 1)) + (4*a*b^6*d*x - 6*a^6*b - 7*a^4*b^3 - 2*a^2*b^5 +
b^7)*tan(d*x + c))/((a^4*b^5 + 2*a^2*b^7 + b^9)*d*tan(d*x + c) + (a^5*b^4 +
 2*a^3*b^6 + a*b^8)*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.23, size = 2837, normalized size = 14.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)**5/(a+b*tan(d*x+c))**2,x)

[Out] Piecewise((zoo*x*tan(c)**3, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((log(tan(c + d*x)**2 + 1)/(2*d) + tan(c + d*x)**4/(4*d) - tan(c + d*x)**2/(2*d))/a**2, Eq(b, 0)), (-15*I*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 30*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 15*I*d*x/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 8*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 16*I*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 8*log(tan(c + d*x)**2 + 1)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 2*tan(c + d*x)**4/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 4*I*tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 29*I*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 22/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d), Eq(a, -I*b)), (15*I*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 30*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 15*I*d*x/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 8*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 16*I*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 8*log(tan(c + d*x)**2 + 1)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 2*tan(c + d*x)**4/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 4*I*tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 29*I*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 22/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d), Eq(a, I*b)), (x*tan(c)**5/(a + b*tan(c))**2, Eq(d, 0)), (6*a**7*log(a/b + tan(c + d*x))/(2*a**5*b**4*d + 2*a**4*b**5*d*tan(c + d*x) + 4*a**3*b**6*d + 4*a**2*b**7*d*tan(c + d*x) + 2*a*b**8*d + 2*b**9*d*tan(c + d*x)) + 6*a**7/(2*a**5*b**4*d + 2*a**4*b**5*d*tan(c + d*x) + 4*a**3*b**6*d + 4*a**2*b**7*d*tan(c + d*x) + 2*a*b**8*d + 2*b**9*d*tan(c + d*x)) + 6*a**6*b*log(a/b + tan(c + d*x))*tan(c + d*x)/(2*a**5*b**4*d + 2*a**4*b**5*d*tan(c + d*x) + 4*a**3*b**6*d + 4*a**2*b**7*d*tan(c + d*x) + 2*a*b**8*d + 2*b**9*d*tan(c + d*x)) + 10*a**5*b**2*log(a/b + tan(c + d*x))/(2*a**5*b**4*d + 2*a**4*b**5*d*tan(c + d*x) + 4*a**3*b**6*d + 4*a**2*b**7*d*tan(c + d*x) + 2*a*b**8*d + 2*b**9*d*tan(c + d*x)) - 3*a**5*b**2*tan(c + d*x)**2/(2*a**5*b**4*d + 2*a**4*b**5*d*tan(c + d*x) + 4*a**3*b**6*d + 4*a**2*b**7*d*tan(c + d*x) + 2*a*b**8*d + 2*b**9*d*tan(c + d*x)) + 10*a**5*b**2/(2*a**5*b**4*d + 2*a**4*b**5*d*tan(c + d*x) + 4*a**3*b**6*d + 4*a**2*b**7*d*tan(c + d*x) + 2*a*b**8*d + 2*b**9*d*tan(c + d*x)) + 10*a**4*b**3*log(a/b + tan(c + d*x))*tan(c + d*x)/(2*a**5*b**4*d + 2*a**4*b**5*d*tan(c + d*x) + 4*a**3*b**6*d + 4*a**2*b**7*d*tan(c + d

```
*x) + 2*a*b**8*d + 2*b**9*d*tan(c + d*x)) + a**4*b**3*tan(c + d*x)**3/(2*a*
*5*b**4*d + 2*a**4*b**5*d*tan(c + d*x) + 4*a**3*b**6*d + 4*a**2*b**7*d*tan(
c + d*x) + 2*a*b**8*d + 2*b**9*d*tan(c + d*x)) + a**3*b**4*log(tan(c + d*x)
**2 + 1)/(2*a**5*b**4*d + 2*a**4*b**5*d*tan(c + d*x) + 4*a**3*b**6*d + 4*a*
*2*b**7*d*tan(c + d*x) + 2*a*b**8*d + 2*b**9*d*tan(c + d*x)) - 6*a**3*b**4*
tan(c + d*x)**2/(2*a**5*b**4*d + 2*a**4*b**5*d*tan(c + d*x) + 4*a**3*b**6*d
 + 4*a**2*b**7*d*tan(c + d*x) + 2*a*b**8*d + 2*b**9*d*tan(c + d*x)) + 4*a**
3*b**4/(2*a**5*b**4*d + 2*a**4*b**5*d*tan(c + d*x) + 4*a**3*b**6*d + 4*a**2
*b**7*d*tan(c + d*x) + 2*a*b**8*d + 2*b**9*d*tan(c + d*x)) + 4*a**2*b**5*d*
x/(2*a**5*b**4*d + 2*a**4*b**5*d*tan(c + d*x) + 4*a**3*b**6*d + 4*a**2*b**7
*d*tan(c + d*x) + 2*a*b**8*d + 2*b**9*d*tan(c + d*x)) + a**2*b**5*log(tan(c
 + d*x)**2 + 1)*tan(c + d*x)/(2*a**5*b**4*d + 2*a**4*b**5*d*tan(c + d*x) +
4*a**3*b**6*d + 4*a**2*b**7*d*tan(c + d*x) + 2*a*b**8*d + 2*b**9*d*tan(c +
d*x)) + 2*a**2*b**5*tan(c + d*x)**3/(2*a**5*b**4*d + 2*a**4*b**5*d*tan(c +
d*x) + 4*a**3*b**6*d + 4*a**2*b**7*d*tan(c + d*x) + 2*a*b**8*d + 2*b**9*d*t
an(c + d*x)) + 4*a*b**6*d*x*tan(c + d*x)/(2*a**5*b**4*d + 2*a**4*b**5*d*tan
(c + d*x) + 4*a**3*b**6*d + 4*a**2*b**7*d*tan(c + d*x) + 2*a*b**8*d + 2*b**
9*d*tan(c + d*x)) - a*b**6*log(tan(c + d*x)**2 + 1)/(2*a**5*b**4*d + 2*a**4
*b**5*d*tan(c + d*x) + 4*a**3*b**6*d + 4*a**2*b**7*d*tan(c + d*x) + 2*a*b**
8*d + 2*b**9*d*tan(c + d*x)) - 3*a*b**6*tan(c + d*x)**2/(2*a**5*b**4*d + 2*
a**4*b**5*d*tan(c + d*x) + 4*a**3*b**6*d + 4*a**2*b**7*d*tan(c + d*x) + 2*a
*b**8*d + 2*b**9*d*tan(c + d*x)) - b**7*log(tan(c + d*x)**2 + 1)*tan(c + d*
x)/(2*a**5*b**4*d + 2*a**4*b**5*d*tan(c + d*x) + 4*a**3*b**6*d + 4*a**2*b**
7*d*tan(c + d*x) + 2*a*b**8*d + 2*b**9*d*tan(c + d*x)) + b**7*tan(c + d*x)*
*3/(2*a**5*b**4*d + 2*a**4*b**5*d*tan(c + d*x) + 4*a**3*b**6*d + 4*a**2*b**
7*d*tan(c + d*x) + 2*a*b**8*d + 2*b**9*d*tan(c ...
```

Giac [A]
time = 1.90, size = 221, normalized size = 1.12

$$\frac{\frac{4\,(dx+c)ab}{a^4+2\,a^2b^2+b^4}+\frac{(a^2-b^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\,(3\,a^6+5\,a^4b^2)\log(|b\tan(dx+c)+a|)}{a^4b^4+2\,a^2b^6+b^8}-\frac{2\,(3\,a^6b\tan(dx+c)+5\,a^4b^3\tan(dx+c)+2\,a^7+4\,a^5b^2)}{(a^4b^4+2\,a^2b^6+b^8)(b\tan(dx+c)+a)}+\frac{b^2\tan(dx+c)^2-4\,ab\tan(dx+c)}{b^4}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*(d*x + c)*a*b/(a^4 + 2*a^2*b^2 + b^4) + (a^2 - b^2)*log(tan(d*x + c)
^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 2*(3*a^6 + 5*a^4*b^2)*log(abs(b*tan(d*x +
 c) + a))/(a^4*b^4 + 2*a^2*b^6 + b^8) - 2*(3*a^6*b*tan(d*x + c) + 5*a^4*b^3
*tan(d*x + c) + 2*a^7 + 4*a^5*b^2)/((a^4*b^4 + 2*a^2*b^6 + b^8)*(b*tan(d*x
+ c) + a)) + (b^2*tan(d*x + c)^2 - 4*a*b*tan(d*x + c))/b^4)/d
```

Mupad [B]
time = 4.33, size = 185, normalized size = 0.94

$$\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)}{2\,d\,\left(a^2+a\,b\,2\mathrm{i}-b^2\right)}+\frac{{\tan(c+d\,x)}^2}{2\,b^2\,d}+\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(3\,a^6+5\,a^4\,b^2\right)}{d\,\left(a^4\,b^4+2\,a^2\,b^6+b^8\right)}-\frac{2\,a\tan(c+d\,x)}{b^3\,d}+\frac{a^5}{b\,d\,\left(\tan\left(c+d\,x\right)\,b^4+a\,b^3\right)\,\left(a^2+b^2\right)}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(a^2\,1\mathrm{i}+2\,a\,b-b^2\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + b*tan(c + d*x))^2,x)
```

```
[Out] log(tan(c + d*x) - 1i)/(2*d*(a*b*2i + a^2 - b^2)) + (log(tan(c + d*x) + 1i)
*1i)/(2*d*(2*a*b + a^2*1i - b^2*1i)) + tan(c + d*x)^2/(2*b^2*d) + (log(a +
b*tan(c + d*x))*(3*a^6 + 5*a^4*b^2))/(d*(b^8 + 2*a^2*b^6 + a^4*b^4)) - (2*a
*tan(c + d*x))/(b^3*d) + a^5/(b*d*(a*b^3 + b^4*tan(c + d*x))*(a^2 + b^2))
```

3.469 $\int \frac{\tan^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=155

$$\frac{(a^2-b^2)\,x}{(a^2+b^2)^2}+\frac{2ab\log(\cos(c+dx))}{(a^2+b^2)^2\,d}-\frac{2a^3(a^2+2b^2)\log(a+b\tan(c+dx))}{b^3\,(a^2+b^2)^2\,d}+\frac{(2a^2+b^2)\tan(c+dx)}{b^2\,(a^2+b^2)\,d}-\frac{a}{b\,(a^2+b^2}$$

```
[Out] (a^2-b^2)*x/(a^2+b^2)^2+2*a*b*ln(cos(d*x+c))/(a^2+b^2)^2/d-2*a^3*(a^2+2*b^2
)*ln(a+b*tan(d*x+c))/b^3/(a^2+b^2)^2/d+(2*a^2+b^2)*tan(d*x+c)/b^2/(a^2+b^2)
/d-a^2*tan(d*x+c)^2/b/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.21, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3646, 3728, 3707, 3698, 31, 3556}

$$-\frac{a^2\tan^2(c+dx)}{bd\,(a^2+b^2)\,(a+b\tan(c+dx))}+\frac{(2a^2+b^2)\tan(c+dx)}{b^2d\,(a^2+b^2)}+\frac{2ab\log(\cos(c+dx))}{d\,(a^2+b^2)^2}+\frac{x(a^2-b^2)}{(a^2+b^2)^2}-\frac{2a^3(a^2+2b^2)\log(a+b\tan(c+dx))}{b^3d\,(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((a^2 - b^2)*x)/(a^2 + b^2)^2 + (2*a*b*Log[Cos[c + d*x]])/((a^2 + b^2)^2*d)
 - (2*a^3*(a^2 + 2*b^2)*Log[a + b*Tan[c + d*x]])/(b^3*(a^2 + b^2)^2*d) + ((
2*a^2 + b^2)*Tan[c + d*x])/(b^2*(a^2 + b^2)*d) - (a^2*Tan[c + d*x]^2)/(b*(a
^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
```

```
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{a^2\tan^2(c+dx)}{b\,(a^2+b^2)\,d(a+b\tan(c+dx))} + \frac{\int \frac{\tan(c+dx)\left(2a^2-ab\tan(c+dx)+(2a^2+b^2)\tan^2(c+dx)\right)}{a+b\tan(c+dx)}}{b\,(a^2+b^2)}$$

$$= \frac{(2a^2+b^2)\tan(c+dx)}{b^2\,(a^2+b^2)\,d} - \frac{a^2\tan^2(c+dx)}{b\,(a^2+b^2)\,d(a+b\tan(c+dx))} + \frac{\int \frac{-a(2a^2+b^2)-b^3\tan(}{a+}}{b^2}$$

$$= \frac{(a^2-b^2)\,x}{(a^2+b^2)^2} + \frac{(2a^2+b^2)\tan(c+dx)}{b^2\,(a^2+b^2)\,d} - \frac{a^2\tan^2(c+dx)}{b\,(a^2+b^2)\,d(a+b\tan(c+dx))} - \frac{(2a}{}$$

$$= \frac{(a^2-b^2)\,x}{(a^2+b^2)^2} + \frac{2ab\log(\cos(c+dx))}{(a^2+b^2)^2\,d} + \frac{(2a^2+b^2)\tan(c+dx)}{b^2\,(a^2+b^2)\,d} - \frac{a^2\tan^2}{b\,(a^2+b^2)\,d(a}$$

$$= \frac{(a^2-b^2)\,x}{(a^2+b^2)^2} + \frac{2ab\log(\cos(c+dx))}{(a^2+b^2)^2\,d} - \frac{2a^3(a^2+2b^2)\log(a+b\tan(c+dx))}{b^3\,(a^2+b^2)^2\,d} + \frac{(}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.78, size = 329, normalized size = 2.12

$$\frac{a\left((a+ib)^2\left(-2ia^3-4a^2b+2iab^2+b^3\right)(c+dx)+2a(a^2+b^2)^2\log(\cos(c+dx))-a^3(a^2+2b^2)\log\left((a\cos(c+dx)+b\sin(c+dx))^2\right)\right)+b\left(2a^5+3a^3b^2+ab^4-2ia^5c-4ia^3b^2c+a^2b^3c-b^5c-2ia^5dx-4ia^3b^2dx+a^2b^3dx-b^5dx+2a(a^2+b^2)^2\log(\cos(c+dx))-a^3(a^2+2b^2)\log\left((a\cos(c+dx)+b\sin(c+dx))^2\right)\right)\tan(c+dx)+b^2(a^2+b^2)^2\tan^2(c+dx)+2ia^3(a^2+2b^2)\,\mathrm{ArcTan}(\tan(c+dx))(a+b\tan(c+dx))}{b^3\,(a^2+b^2)^2\,d(a+b\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (a*((a + I*b)^2*((-2*I)*a^3 - 4*a^2*b + (2*I)*a*b^2 + b^3)*(c + d*x) + 2*a*
(a^2 + b^2)^2*Log[Cos[c + d*x]] - a^3*(a^2 + 2*b^2)*Log[(a*Cos[c + d*x] + b
*Sin[c + d*x])^2]) + b*(2*a^5 + 3*a^3*b^2 + a*b^4 - (2*I)*a^5*c - (4*I)*a^3
*b^2*c + a^2*b^3*c - b^5*c - (2*I)*a^5*d*x - (4*I)*a^3*b^2*d*x + a^2*b^3*d*
x - b^5*d*x + 2*a*(a^2 + b^2)^2*Log[Cos[c + d*x]] - a^3*(a^2 + 2*b^2)*Log[(
a*Cos[c + d*x] + b*Sin[c + d*x])^2])*Tan[c + d*x] + b^2*(a^2 + b^2)^2*Tan[c
 + d*x]^2 + (2*I)*a^3*(a^2 + 2*b^2)*ArcTan[Tan[c + d*x]]*(a + b*Tan[c + d*x
]))/(b^3*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```

Maple [A]
time = 0.16, size = 125, normalized size = 0.81

method	result
derivativedivides	$\frac{\frac{\tan(dx+c)}{b^2}+\frac{-ab\ln\left(1+\tan^2(dx+c)\right)+\left(a^2-b^2\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^2}-\frac{a^4}{b^3\left(a^2+b^2\right)(a+b\tan(dx+c))}-\frac{2a^3\left(a^2+2b^2\right)\ln(a+b\tan(dx+c))}{b^3\left(a^2+b^2\right)^2}}{d}$
default	$\frac{\frac{\tan(dx+c)}{b^2}+\frac{-ab\ln\left(1+\tan^2(dx+c)\right)+\left(a^2-b^2\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^2}-\frac{a^4}{b^3\left(a^2+b^2\right)(a+b\tan(dx+c))}-\frac{2a^3\left(a^2+2b^2\right)\ln(a+b\tan(dx+c))}{b^3\left(a^2+b^2\right)^2}}{d}$

norman	$\frac{\frac{\tan^2(dx+c)}{bd}+\frac{(a^2-b^2)ax}{a^4+2a^2b^2+b^4}+\frac{b(a^2-b^2)x\tan(dx+c)}{a^4+2a^2b^2+b^4}-\frac{(2a^3+b^2a)a}{d\,b^3(a^2+b^2)}}{a+b\tan(dx+c)}-\frac{ab\ln(1+\tan^2(dx+c))}{d(a^4+2a^2b^2+b^4)}-\frac{2a^3(a^2+2b^2)\ln(a+b\tan(d}{(a^4+2a^2b^2+b^4)b^3d}$
risch	$-\frac{x}{2iab-a^2+b^2}+\frac{4ia^5x}{(a^4+2a^2b^2+b^4)b^3}+\frac{4ia^5c}{(a^4+2a^2b^2+b^4)b^3d}+\frac{8ia^3x}{(a^4+2a^2b^2+b^4)b}+\frac{8ia^3c}{(a^4+2a^2b^2+b^4)bd}-\frac{4iax}{b^3}-\frac{4}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^2*tan(d*x+c)+1/(a^2+b^2)^2*(-a*b*ln(1+tan(d*x+c)^2)+(a^2-b^2)*arct
an(tan(d*x+c)))-1/b^3*a^4/(a^2+b^2)/(a+b*tan(d*x+c))-2/b^3*a^3*(a^2+2*b^2)/
(a^2+b^2)^2*ln(a+b*tan(d*x+c)))
```

Maxima [A]
time = 0.63, size = 164, normalized size = 1.06

$$-\frac{\frac{a^4}{a^3b^3+ab^5+(a^2b^4+b^6)\tan(dx+c)}+\frac{ab\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{(a^2-b^2)(dx+c)}{a^4+2\,a^2b^2+b^4}+\frac{2\,(a^5+2\,a^3b^2)\log(b\tan(dx+c)+a)}{a^4b^3+2\,a^2b^5+b^7}-\frac{\tan(dx+c)}{b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(a^4/(a^3*b^3 + a*b^5 + (a^2*b^4 + b^6)*tan(d*x + c)) + a*b*log(tan(d*x +
c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - (a^2 - b^2)*(d*x + c)/(a^4 + 2*a^2*b^2
+ b^4) + 2*(a^5 + 2*a^3*b^2)*log(b*tan(d*x + c) + a)/(a^4*b^3 + 2*a^2*b^5 +
 b^7) - tan(d*x + c)/b^2)/d
```

Fricas [A]
time = 0.88, size = 288, normalized size = 1.86

$$-\frac{a^4b^2-(a^3b^3-ab^5)dx-(a^4b^2+2\,a^2b^4+b^6)\tan(dx+c)^2+(a^6+2\,a^4b^2+(a^5b+2\,a^3b^3)\tan(dx+c))\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)-(a^6+2\,a^4b^2+a^2b^4+(a^5b+2\,a^3b^3+ab^5)\tan(dx+c))\log\left(\frac{1}{\tan(dx+c)^2+1}\right)-(2\,a^5b+2\,a^3b^3+ab^5+(a^2b^4-b^6)dx)\tan(dx+c)}{(a^4b^4+2\,a^2b^6+b^8)d\tan(dx+c)+(a^5b^3+2\,a^3b^5+ab^7)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(a^4*b^2 - (a^3*b^3 - a*b^5)*d*x - (a^4*b^2 + 2*a^2*b^4 + b^6)*tan(d*x + c
)^2 + (a^6 + 2*a^4*b^2 + (a^5*b + 2*a^3*b^3)*tan(d*x + c))*log((b^2*tan(d*x
 + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) - (a^6 + 2*a^4*b^
2 + a^2*b^4 + (a^5*b + 2*a^3*b^3 + a*b^5)*tan(d*x + c))*log(1/(tan(d*x + c)
^2 + 1)) - (2*a^5*b + 2*a^3*b^3 + a*b^5 + (a^2*b^4 - b^6)*d*x)*tan(d*x + c)
)/((a^4*b^4 + 2*a^2*b^6 + b^8)*d*tan(d*x + c) + (a^5*b^3 + 2*a^3*b^5 + a*b^
7)*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.02, size = 2312, normalized size = 14.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)**4/(a+b*tan(d*x+c))**2,x)

[Out] Piecewise((zoo*x*tan(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x + tan(c + d*x)**3/(3*d) - tan(c + d*x)/d)/a**2, Eq(b, 0)), (-9*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 18*I*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 9*d*x/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 4*I*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 8*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 4*I*log(tan(c + d*x)**2 + 1)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 4*tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 19*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 14*I/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d), Eq(a, -I*b)), (-9*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 18*I*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 9*d*x/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 4*I*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 8*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 4*I*log(tan(c + d*x)**2 + 1)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 4*tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 19*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 14*I/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d), Eq(a, I*b)), (x*tan(c)**4/(a + b*tan(c))**2, Eq(d, 0)), (-2*a**6*log(a/b + tan(c + d*x))/(a**5*b**3*d + a**4*b**4*d*tan(c + d*x) + 2*a**3*b**5*d + 2*a**2*b**6*d*tan(c + d*x) + a*b**7*d + b**8*d*tan(c + d*x)) - 2*a**6/(a**5*b**3*d + a**4*b**4*d*tan(c + d*x) + 2*a**3*b**5*d + 2*a**2*b**6*d*tan(c + d*x) + a*b**7*d + b**8*d*tan(c + d*x)) - 2*a**5*b*log(a/b + tan(c + d*x))*tan(c + d*x)/(a**5*b**3*d + a**4*b**4*d*tan(c + d*x) + 2*a**3*b**5*d + 2*a**2*b**6*d*tan(c + d*x) + a*b**7*d + b**8*d*tan(c + d*x)) - 4*a**4*b**2*log(a/b + tan(c + d*x))/(a**5*b**3*d + a**4*b**4*d*tan(c + d*x) + 2*a**3*b**5*d + 2*a**2*b**6*d*tan(c + d*x) + a*b**7*d + b**8*d*tan(c + d*x)) + a**4*b**2*tan(c + d*x)**2/(a**5*b**3*d + a**4*b**4*d*tan(c + d*x) + 2*a**3*b**5*d + 2*a**2*b**6*d*tan(c + d*x) + a*b**7*d + b**8*d*tan(c + d*x)) - 3*a**4*b**2/(a**5*b**3*d + a**4*b**4*d*tan(c + d*x) + 2*a**3*b**5*d + 2*a**2*b**6*d*tan(c + d*x) + a*b**7*d + b**8*d*tan(c + d*x)) + a**3*b**3*d*x/(a**5*b**3*d + a**4*b**4*d*tan(c + d*x) + 2*a**3*b**5*d + 2*a**2*b**6*d*tan(c + d*x) + a*b**7*d + b**8*d*tan(c + d*x)) - 4*a**3*b**3*log(a/b + tan(c + d*x))*tan(c + d*x)/(a**5*b**3*d + a**4*b**4*d*tan(c + d*x) + 2*a**3*b**5*d + 2*a**2*b**6*d*tan(c + d*x) + a*b**7*d + b**8*d*tan(c + d*x)) + a**2*b**4*d*x*tan(c + d*x)/(a**5*b**3*d + a**4*b**4*d*tan(c + d*x) + 2*a**3*b**5*d + 2*a**2*b*

```
*6*d*tan(c + d*x) + a*b**7*d + b**8*d*tan(c + d*x)) - a**2*b**4*log(tan(c +
 d*x)**2 + 1)/(a**5*b**3*d + a**4*b**4*d*tan(c + d*x) + 2*a**3*b**5*d + 2*a
**2*b**6*d*tan(c + d*x) + a*b**7*d + b**8*d*tan(c + d*x)) + 2*a**2*b**4*tan
(c + d*x)**2/(a**5*b**3*d + a**4*b**4*d*tan(c + d*x) + 2*a**3*b**5*d + 2*a*
*2*b**6*d*tan(c + d*x) + a*b**7*d + b**8*d*tan(c + d*x)) - a**2*b**4/(a**5*
b**3*d + a**4*b**4*d*tan(c + d*x) + 2*a**3*b**5*d + 2*a**2*b**6*d*tan(c + d
*x) + a*b**7*d + b**8*d*tan(c + d*x)) - a*b**5*d*x/(a**5*b**3*d + a**4*b**4
*d*tan(c + d*x) + 2*a**3*b**5*d + 2*a**2*b**6*d*tan(c + d*x) + a*b**7*d + b
**8*d*tan(c + d*x)) - a*b**5*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(a**5*b*
*3*d + a**4*b**4*d*tan(c + d*x) + 2*a**3*b**5*d + 2*a**2*b**6*d*tan(c + d*x
) + a*b**7*d + b**8*d*tan(c + d*x)) - b**6*d*x*tan(c + d*x)/(a**5*b**3*d +
a**4*b**4*d*tan(c + d*x) + 2*a**3*b**5*d + 2*a**2*b**6*d*tan(c + d*x) + a*b
**7*d + b**8*d*tan(c + d*x)) + b**6*tan(c + d*x)**2/(a**5*b**3*d + a**4*b**
4*d*tan(c + d*x) + 2*a**3*b**5*d + 2*a**2*b**6*d*tan(c + d*x) + a*b**7*d +
b**8*d*tan(c + d*x)), True))
```

Giac [A]
time = 1.17, size = 201, normalized size = 1.30

$$-\frac{\frac{ab\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{(a^2-b^2)\,(dx+c)}{a^4+2\,a^2b^2+b^4}+\frac{2\,(a^5+2\,a^3b^2)\log(|b\tan(dx+c)+a|)}{a^4b^3+2\,a^2b^5+b^7}-\frac{2\,a^5b\tan(dx+c)+4\,a^3b^3\tan(dx+c)+a^6+3\,a^4b^2}{(a^4b^3+2\,a^2b^5+b^7)(b\tan(dx+c)+a)}-\frac{\tan(dx+c)}{b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(a*b*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - (a^2 - b^2)*(d*x +
c)/(a^4 + 2*a^2*b^2 + b^4) + 2*(a^5 + 2*a^3*b^2)*log(abs(b*tan(d*x + c) + a
))/(a^4*b^3 + 2*a^2*b^5 + b^7) - (2*a^5*b*tan(d*x + c) + 4*a^3*b^3*tan(d*x
+ c) + a^6 + 3*a^4*b^2)/((a^4*b^3 + 2*a^2*b^5 + b^7)*(b*tan(d*x + c) + a))
- tan(d*x + c)/b^2)/d
```

Mupad [B]
time = 4.27, size = 158, normalized size = 1.02

$$\frac{\tan(c+d\,x)}{b^2\,d}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)}{2\,d\,\left(-a^2\,1\mathrm{i}+2\,a\,b+b^2\,1\mathrm{i}\right)}-\frac{a^4}{b\,d\,\left(\tan\left(c+d\,x\right)\,b^3+a\,b^2\right)\,\left(a^2+b^2\right)}-\frac{2\,a^3\,\ln\left(a+b\tan(c+d\,x)\right)\,\left(a^2+2\,b^2\right)}{b^3\,d\,{\left(a^2+b^2\right)}^2}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(-a^2+a\,b\,2\mathrm{i}+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + b*tan(c + d*x))^2,x)
```

```
[Out] tan(c + d*x)/(b^2*d) - (log(tan(c + d*x) + 1i)*1i)/(2*d*(a*b*2i - a^2 + b^2
)) - log(tan(c + d*x) - 1i)/(2*d*(2*a*b - a^2*1i + b^2*1i)) - a^4/(b*d*(a*b
^2 + b^3*tan(c + d*x))*(a^2 + b^2)) - (2*a^3*log(a + b*tan(c + d*x))*(a^2 +
 2*b^2))/(b^3*d*(a^2 + b^2)^2)
```

3.470 $\int \frac{\tan^3(c+dx)}{(a+b\tan(c+dx))^2} \, dx$

Optimal. Leaf size=114

$$-\frac{2abx}{(a^2+b^2)^2}+\frac{(a^2-b^2)\log(\cos(c+dx))}{(a^2+b^2)^2 d}+\frac{a^2(a^2+3b^2)\log(a+b\tan(c+dx))}{b^2(a^2+b^2)^2 d}+\frac{a^3}{b^2(a^2+b^2)d(a+b\tan(c+dx))}$$

```
[Out] -2*a*b*x/(a^2+b^2)^2+(a^2-b^2)*ln(cos(d*x+c))/(a^2+b^2)^2/d+a^2*(a^2+3*b^2)
*ln(a+b*tan(d*x+c))/b^2/(a^2+b^2)^2/d+a^3/b^2/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.12, antiderivative size = 121, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3646, 3707, 3698, 31, 3556}

$$-\frac{a^2\tan(c+dx)}{bd(a^2+b^2)(a+b\tan(c+dx))}+\frac{a^2(a^2+3b^2)\log(a+b\tan(c+dx))}{b^2d(a^2+b^2)^2}+\frac{(a^2-b^2)\log(\cos(c+dx))}{d(a^2+b^2)^2}-\frac{2abx}{(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-2*a*b*x)/(a^2 + b^2)^2 + ((a^2 - b^2)*Log[Cos[c + d*x]])/((a^2 + b^2)^2*d
) + (a^2*(a^2 + 3*b^2)*Log[a + b*Tan[c + d*x]])/(b^2*(a^2 + b^2)^2*d) - (a^
2*Tan[c + d*x])/(b*(a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
```

```
n, -1] && IntegerQ[2*m]
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx &= -\frac{a^2\tan(c+dx)}{b(a^2+b^2)\,d(a+b\tan(c+dx))}+\frac{\int \frac{a^2-ab\tan(c+dx)+(a^2+b^2)\tan^2(c+dx)}{a+b\tan(c+dx)}\,dx}{b(a^2+b^2)} \\
&= -\frac{2abx}{(a^2+b^2)^2}-\frac{a^2\tan(c+dx)}{b(a^2+b^2)\,d(a+b\tan(c+dx))}-\frac{(a^2-b^2)\int\tan(c+dx)\,dx}{(a^2+b^2)^2}+ \\
&= -\frac{2abx}{(a^2+b^2)^2}+\frac{(a^2-b^2)\log(\cos(c+dx))}{(a^2+b^2)^2\,d}-\frac{a^2\tan(c+dx)}{b(a^2+b^2)\,d(a+b\tan(c+dx))}+ \\
&= -\frac{2abx}{(a^2+b^2)^2}+\frac{(a^2-b^2)\log(\cos(c+dx))}{(a^2+b^2)^2\,d}+\frac{a^2(a^2+3b^2)\log(a+b\tan(c+dx))}{b^2(a^2+b^2)^2\,d}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.11, size = 251, normalized size = 2.20

$$\frac{a\left(-2(a^2+b^2)^2\log(\cos(c+dx))+a(2(a+ib)^2(ia+2b)(c+dx)+a(a^2+3b^2)\log\left((a\cos(c+dx)+b\sin(c+dx))^2\right))\right)+b\left(-2(a^2+b^2)^2\log(\cos(c+dx))+a(2i(2ib^3(c+dx)+a^3(i+c+dx)+ab^2(i+3c+3dx))+a(a^2+3b^2)\log\left((a\cos(c+dx)+b\sin(c+dx))^2\right))\right)\tan(c+dx)-2ia^2(a^2+3b^2)\,\mathrm{ArcTan}(\tan(c+dx))(a+b\tan(c+dx))}{2b^2(a^2+b^2)^2\,d(a+b\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (a*(-2*(a^2 + b^2)^2*Log[Cos[c + d*x]] + a*(2*(a + I*b)^2*(I*a + 2*b)*(c +
d*x) + a*(a^2 + 3*b^2)*Log[(a*Cos[c + d*x] + b*Sin[c + d*x])^2])) + b*(-2*(
a^2 + b^2)^2*Log[Cos[c + d*x]] + a*((2*I)*((2*I)*b^3*(c + d*x) + a^3*(I + c
```

```
 + d*x) + a*b^2*(I + 3*c + 3*d*x)) + a*(a^2 + 3*b^2)*Log[(a*Cos[c + d*x] +
b*Sin[c + d*x])^2]))*Tan[c + d*x] - (2*I)*a^2*(a^2 + 3*b^2)*ArcTan[Tan[c +
d*x]]*(a + b*Tan[c + d*x]))/(2*b^2*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```

Maple [A]

time = 0.16, size = 114, normalized size = 1.00

method	result
derivativedivides	$\frac{\frac{\frac{(-a^2+b^2)\ln(1+\tan^2(dx+c))}{2}-2ab\arctan(\tan(dx+c))}{(a^2+b^2)^2}+\frac{a^2(a^2+3b^2)\ln(a+b\tan(dx+c))}{(a^2+b^2)^2 b^2}+\frac{a^3}{b^2(a^2+b^2)(a+b\tan(dx+c))}}{d}$
default	$\frac{\frac{\frac{(-a^2+b^2)\ln(1+\tan^2(dx+c))}{2}-2ab\arctan(\tan(dx+c))}{(a^2+b^2)^2}+\frac{a^2(a^2+3b^2)\ln(a+b\tan(dx+c))}{(a^2+b^2)^2 b^2}+\frac{a^3}{b^2(a^2+b^2)(a+b\tan(dx+c))}}{d}$
norman	$\frac{\frac{a^3}{d(a^2+b^2)b^2}-\frac{2a^2bx}{a^4+2a^2b^2+b^4}-\frac{2b^2ax\tan(dx+c)}{a^4+2a^2b^2+b^4}}{a+b\tan(dx+c)}+\frac{a^2(a^2+3b^2)\ln(a+b\tan(dx+c))}{(a^4+2a^2b^2+b^4)b^2d}-\frac{(a^2-b^2)\ln(1+\tan^2(dx+c))}{2d(a^4+2a^2b^2+b^4)}$
risch	$-\frac{ix}{2iab-a^2+b^2}-\frac{2ia^4x}{b^2(a^4+2a^2b^2+b^4)}-\frac{2ia^4c}{b^2d(a^4+2a^2b^2+b^4)}-\frac{6ia^2x}{a^4+2a^2b^2+b^4}-\frac{6ia^2c}{(a^4+2a^2b^2+b^4)d}+\frac{2ix}{b^2}+\frac{2ic}{b^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a^2+b^2)^2*(1/2*(-a^2+b^2)*ln(1+tan(d*x+c)^2)-2*a*b*arctan(tan(d*x+
c)))+a^2*(a^2+3*b^2)/(a^2+b^2)^2/b^2*ln(a+b*tan(d*x+c))+a^3/b^2/(a^2+b^2)/(
a+b*tan(d*x+c)))
```

Maxima [A]

time = 1.03, size = 155, normalized size = 1.36

$$\frac{\frac{2a^3}{a^3b^2+ab^4+(a^2b^3+b^5)\tan(dx+c)}-\frac{4(dx+c)ab}{a^4+2a^2b^2+b^4}+\frac{2(a^4+3a^2b^2)\log(b\tan(dx+c)+a)}{a^4b^2+2a^2b^4+b^6}-\frac{(a^2-b^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2a^2b^2+b^4}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^3/(a^3*b^2 + a*b^4 + (a^2*b^3 + b^5)*tan(d*x + c)) - 4*(d*x + c)*a
*b/(a^4 + 2*a^2*b^2 + b^4) + 2*(a^4 + 3*a^2*b^2)*log(b*tan(d*x + c) + a)/(a
^4*b^2 + 2*a^2*b^4 + b^6) - (a^2 - b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^
2*b^2 + b^4))/d
```

Fricas [A]

time = 1.01, size = 226, normalized size = 1.98

$$-\frac{4a^2b^3dx-2a^3b^2-(a^5+3a^3b^2+(a^4b+3a^2b^3)\tan(dx+c))\log\left(\frac{b^2\tan(dx+c)^2+2ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)+(a^5+2a^3b^2+ab^4+(a^4b+2a^2b^3+b^5)\tan(dx+c))\log\left(\frac{1}{\tan(dx+c)^2+1}\right)+2(2ab^4dx+a^4b)\tan(dx+c)}{2((a^4b^3+2a^2b^5+b^7)d\tan(dx+c)+(a^5b^2+2a^3b^4+ab^6)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="fricas")

[Out] -1/2*(4*a^2*b^3*d*x - 2*a^3*b^2 - (a^5 + 3*a^3*b^2 + (a^4*b + 3*a^2*b^3)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) + (a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*tan(d*x + c))*log(1/(tan(d*x + c)^2 + 1)) + 2*(2*a*b^4*d*x + a^4*b)*tan(d*x + c))/((a^4*b^3 + 2*a^2*b^5 + b^7)*d*tan(d*x + c) + (a^5*b^2 + 2*a^3*b^4 + a*b^6)*d)

Sympy [C] Result contains complex when optimal does not.
time = 0.89, size = 1992, normalized size = 17.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)**3/(a+b*tan(d*x+c))**2,x)

[Out] Piecewise((zoo*x*tan(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((-log(tan(c + d*x)**2 + 1)/(2*d) + tan(c + d*x)**2/(2*d))/a**2, Eq(b, 0)), (3*I*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 6*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 3*I*d*x/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 2*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 4*I*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 2*log(tan(c + d*x)**2 + 1)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 5*I*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 4/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d), Eq(a, -I*b)), (-3*I*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 6*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 3*I*d*x/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 2*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 4*I*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 2*log(tan(c + d*x)**2 + 1)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 5*I*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 4/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d), Eq(a, I*b)), (x*tan(c)**3/(a + b*tan(c))**2, Eq(d, 0)), (2*a**5*log(a/b + tan(c + d*x))/(2*a**5*b**2*d + 2*a**4*b**3*d*tan(c + d*x) + 4*a**3*b**4*d + 4*a**2*b**5*d*tan(c + d*x) + 2*a*b**6*d + 2*b**7*d*tan(c + d*x)) + 2*a**5/(2*a**5*b**2*d + 2*a**4*b**3*d*tan(c + d*x) + 4*a**3*b**4*d + 4*a**2*b**5*d*tan(c + d*x) + 2*a*b**6*d + 2*b**7*d*tan(c + d*x)) + 2*a**4*b*log(a/b + tan(c + d*x))*tan(c + d*x)/(2*a**5*b**2*d + 2*a**4*b**3*d*tan(c + d*x) + 4*a**3*b**4

```
*d + 4*a**2*b**5*d*tan(c + d*x) + 2*a*b**6*d + 2*b**7*d*tan(c + d*x)) + 6*a
**3*b**2*log(a/b + tan(c + d*x))/(2*a**5*b**2*d + 2*a**4*b**3*d*tan(c + d*x
) + 4*a**3*b**4*d + 4*a**2*b**5*d*tan(c + d*x) + 2*a*b**6*d + 2*b**7*d*tan(
c + d*x)) - a**3*b**2*log(tan(c + d*x)**2 + 1)/(2*a**5*b**2*d + 2*a**4*b**3
*d*tan(c + d*x) + 4*a**3*b**4*d + 4*a**2*b**5*d*tan(c + d*x) + 2*a*b**6*d +
 2*b**7*d*tan(c + d*x)) + 2*a**3*b**2/(2*a**5*b**2*d + 2*a**4*b**3*d*tan(c
+ d*x) + 4*a**3*b**4*d + 4*a**2*b**5*d*tan(c + d*x) + 2*a*b**6*d + 2*b**7*d
*tan(c + d*x)) - 4*a**2*b**3*d*x/(2*a**5*b**2*d + 2*a**4*b**3*d*tan(c + d*x
) + 4*a**3*b**4*d + 4*a**2*b**5*d*tan(c + d*x) + 2*a*b**6*d + 2*b**7*d*tan(
c + d*x)) + 6*a**2*b**3*log(a/b + tan(c + d*x))*tan(c + d*x)/(2*a**5*b**2*d
 + 2*a**4*b**3*d*tan(c + d*x) + 4*a**3*b**4*d + 4*a**2*b**5*d*tan(c + d*x)
+ 2*a*b**6*d + 2*b**7*d*tan(c + d*x)) - a**2*b**3*log(tan(c + d*x)**2 + 1)*
tan(c + d*x)/(2*a**5*b**2*d + 2*a**4*b**3*d*tan(c + d*x) + 4*a**3*b**4*d +
4*a**2*b**5*d*tan(c + d*x) + 2*a*b**6*d + 2*b**7*d*tan(c + d*x)) - 4*a*b**4
*d*x*tan(c + d*x)/(2*a**5*b**2*d + 2*a**4*b**3*d*tan(c + d*x) + 4*a**3*b**4
*d + 4*a**2*b**5*d*tan(c + d*x) + 2*a*b**6*d + 2*b**7*d*tan(c + d*x)) + a*b
**4*log(tan(c + d*x)**2 + 1)/(2*a**5*b**2*d + 2*a**4*b**3*d*tan(c + d*x) +
4*a**3*b**4*d + 4*a**2*b**5*d*tan(c + d*x) + 2*a*b**6*d + 2*b**7*d*tan(c +
d*x)) + b**5*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*a**5*b**2*d + 2*a**4*
b**3*d*tan(c + d*x) + 4*a**3*b**4*d + 4*a**2*b**5*d*tan(c + d*x) + 2*a*b**6
*d + 2*b**7*d*tan(c + d*x)), True))
```

Giac [A]

time = 0.91, size = 181, normalized size = 1.59

$$-\frac{\frac{4\,(dx+c)ab}{a^4+2\,a^2b^2+b^4}+\frac{(a^2-b^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\,(a^4+3\,a^2b^2)\log(|b\tan(dx+c)+a|)}{a^4b^2+2\,a^2b^4+b^6}+\frac{2\,(a^4\tan(dx+c)+3\,a^2b^2\tan(dx+c)+2\,a^3b)}{(a^4b+2\,a^2b^3+b^5)(b\tan(dx+c)+a)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*(d*x + c)*a*b/(a^4 + 2*a^2*b^2 + b^4) + (a^2 - b^2)*log(tan(d*x + c
)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*(a^4 + 3*a^2*b^2)*log(abs(b*tan(d*x +
c) + a))/(a^4*b^2 + 2*a^2*b^4 + b^6) + 2*(a^4*tan(d*x + c) + 3*a^2*b^2*tan(
d*x + c) + 2*a^3*b)/((a^4*b + 2*a^2*b^3 + b^5)*(b*tan(d*x + c) + a)))/d
```

Mupad [B]

time = 4.25, size = 137, normalized size = 1.20

$$\frac{a^3}{b^2\,d\,\left(a^2+b^2\right)\,\left(a+b\tan\left(c+d\,x\right)\right)}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)}{2\,d\,\left(a^2+a\,b\,2\mathrm{i}-b^2\right)}+\frac{a^2\,\ln\left(a+b\tan(c+d\,x)\right)\,\left(a^2+3\,b^2\right)}{b^2\,d\,{\left(a^2+b^2\right)}^2}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(a^2\,1\mathrm{i}+2\,a\,b-b^2\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + b*tan(c + d*x))^2,x)
```

```
[Out] a^3/(b^2*d*(a^2 + b^2)*(a + b*tan(c + d*x))) - (log(tan(c + d*x) + 1i)*1i)/
(2*d*(2*a*b + a^2*1i - b^2*1i)) - log(tan(c + d*x) - 1i)/(2*d*(a*b*2i + a^2
```

```
 - b^2)) + (a^2*log(a + b*tan(c + d*x))*(a^2 + 3*b^2))/(b^2*d*(a^2 + b^2)^2
)
```

3.471 $\int \frac{\tan^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=88

$$-\frac{(a^2-b^2)\,x}{(a^2+b^2)^2}-\frac{2ab\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^2\,d}-\frac{a^2}{b\,(a^2+b^2)\,d(a+b\tan(c+dx))}$$

```
[Out] -(a^2-b^2)*x/(a^2+b^2)^2-2*a*b*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^2/d-
a^2/b/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3623, 3612, 3611}

$$-\frac{a^2}{bd\,(a^2+b^2)\,(a+b\tan(c+dx))}-\frac{2ab\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^2}-\frac{x(a^2-b^2)}{(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((a^2 - b^2)*x)/(a^2 + b^2)^2) - (2*a*b*Log[a*Cos[c + d*x] + b*Sin[c + d*
x]])/((a^2 + b^2)^2*d) - a^2/(b*(a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

```
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{a^2}{b\,(a^2+b^2)\,d(a+b\tan(c+dx))} + \frac{\int \frac{-a+b\tan(c+dx)}{a+b\tan(c+dx)}\,dx}{a^2+b^2}$$

$$= -\frac{(a^2-b^2)\,x}{(a^2+b^2)^2} - \frac{a^2}{b\,(a^2+b^2)\,d(a+b\tan(c+dx))} - \frac{(2ab)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}\,dx}{(a^2+b^2)^2}$$

$$= -\frac{(a^2-b^2)\,x}{(a^2+b^2)^2} - \frac{2ab\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^2\,d} - \frac{a^2}{b\,(a^2+b^2)\,d(a+b\tan(}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.73, size = 161, normalized size = 1.83

$$\frac{-a((a+ib)^2(c+dx)+ab\log((a\cos(c+dx)+b\sin(c+dx))^2))+((a+ib)(a^2-ib^2(c+dx)-ab(i+c+dx))-ab^2\log((a\cos(c+dx)+b\sin(c+dx))^2))\tan(c+dx)+2iab\text{ArcTan}(\tan(c+dx))(a+b\tan(c+dx))}{(a^2+b^2)^2\,d(a+b\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-(a*((a + I*b)^2*(c + d*x) + a*b*Log[(a*Cos[c + d*x] + b*Sin[c + d*x])^2])
) + ((a + I*b)*(a^2 - I*b^2*(c + d*x) - a*b*(I + c + d*x)) - a*b^2*Log[(a*C
os[c + d*x] + b*Sin[c + d*x])^2])*Tan[c + d*x] + (2*I)*a*b*ArcTan[Tan[c + d
*x]]*(a + b*Tan[c + d*x]))/((a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```

Maple [A]
time = 0.12, size = 101, normalized size = 1.15

method	result
derivativedivides	$\frac{\frac{ab\ln\left(1+\tan^2(dx+c)\right)+\left(-a^2+b^2\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^2}-\frac{a^2}{\left(a^2+b^2\right)b(a+b\tan(dx+c))}-\frac{2ab\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^2}}{d}$
default	$\frac{\frac{ab\ln\left(1+\tan^2(dx+c)\right)+\left(-a^2+b^2\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^2}-\frac{a^2}{\left(a^2+b^2\right)b(a+b\tan(dx+c))}-\frac{2ab\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^2}}{d}$
norman	$\frac{-\frac{a^2}{bd\left(a^2+b^2\right)}-\frac{\left(a^2-b^2\right)ax}{a^4+2a^2b^2+b^4}-\frac{b\left(a^2-b^2\right)x\tan(dx+c)}{a^4+2a^2b^2+b^4}}{a+b\tan(dx+c)}+\frac{ab\ln(1+\tan^2(dx+c))}{d(a^4+2a^2b^2+b^4)}-\frac{2ab\ln(a+b\tan(dx+c))}{d(a^4+2a^2b^2+b^4)}$
risch	$\frac{x}{2iab-a^2+b^2}+\frac{4iabx}{a^4+2a^2b^2+b^4}+\frac{4iabc}{d(a^4+2a^2b^2+b^4)}+\frac{2ia^2}{(ib+a)d(-ib+a)^2\left(-ib\,e^{2i(dx+c)}+a\,e^{2i(dx+c)}+ib+a\right)}-\frac{2ab\ln}{d(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a^2+b^2)^2*(a*b*ln(1+tan(d*x+c)^2)+(-a^2+b^2)*arctan(tan(d*x+c)))-a
^2/(a^2+b^2)/b/(a+b*tan(d*x+c))-2*a*b/(a^2+b^2)^2*ln(a+b*tan(d*x+c)))
```

Maxima [**A**]
time = 1.01, size = 137, normalized size = 1.56

$$-\frac{\frac{2\,ab\log\left(b\tan\left(dx+c\right)+a\right)}{a^4+2\,a^2b^2+b^4}-\frac{ab\log\left(\tan\left(dx+c\right)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{\left(a^2-b^2\right)\left(dx+c\right)}{a^4+2\,a^2b^2+b^4}+\frac{a^2}{a^3b+ab^3+\left(a^2b^2+b^4\right)\tan\left(dx+c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(2*a*b*log(b*tan(d*x + c) + a)/(a^4 + 2*a^2*b^2 + b^4) - a*b*log(tan(d*x +
 c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^2 - b^2)*(d*x + c)/(a^4 + 2*a^2*b^2
 + b^4) + a^2/(a^3*b + a*b^3 + (a^2*b^2 + b^4)*tan(d*x + c)))/d
```

Fricas [**A**]
time = 0.87, size = 153, normalized size = 1.74

$$-\frac{a^2b+\left(a^3-ab^2\right)dx+\left(ab^2\tan\left(dx+c\right)+a^2b\right)\log\left(\frac{b^2\tan\left(dx+c\right)^2+2\,ab\tan\left(dx+c\right)+a^2}{\tan\left(dx+c\right)^2+1}\right)-\left(a^3-\left(a^2b-b^3\right)dx\right)\tan\left(dx+c\right)}{\left(a^4b+2\,a^2b^3+b^5\right)d\tan\left(dx+c\right)+\left(a^5+2\,a^3b^2+ab^4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(a^2*b + (a^3 - a*b^2)*d*x + (a*b^2*tan(d*x + c) + a^2*b)*log((b^2*tan(d*x
 + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) - (a^3 - (a^2*b -
 b^3)*d*x)*tan(d*x + c))/((a^4*b + 2*a^2*b^3 + b^5)*d*tan(d*x + c) + (a^5 +
 2*a^3*b^2 + a*b^4)*d)
```

Sympy [**C**] Result contains complex when optimal does not.
time = 0.79, size = 1314, normalized size = 14.93

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((-x + tan(c + d*x)/d)/a
**2, Eq(b, 0)), (d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d
*tan(c + d*x) - 4*b**2*d) - 2*I*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2
 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - d*x/(4*b**2*d*tan(c + d*x)**2 - 8*I
*b**2*d*tan(c + d*x) - 4*b**2*d) - 3*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2
 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 2*I/(4*b**2*d*tan(c + d*x)**2 - 8*
```

```
I*b**2*d*tan(c + d*x) - 4*b**2*d), Eq(a, -I*b)), (d*x*tan(c + d*x)**2/(4*b*
*2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 2*I*d*x*tan(c
+ d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - d*
x/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 3*tan(c
 + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 2
*I/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d), Eq(a, I
*b)), (x*tan(c)**2/(a + b*tan(c))**2, Eq(d, 0)), (-a**4/(a**5*b*d + a**4*b*
*2*d*tan(c + d*x) + 2*a**3*b**3*d + 2*a**2*b**4*d*tan(c + d*x) + a*b**5*d +
 b**6*d*tan(c + d*x)) - a**3*b*d*x/(a**5*b*d + a**4*b**2*d*tan(c + d*x) + 2
*a**3*b**3*d + 2*a**2*b**4*d*tan(c + d*x) + a*b**5*d + b**6*d*tan(c + d*x))
 - a**2*b**2*d*x*tan(c + d*x)/(a**5*b*d + a**4*b**2*d*tan(c + d*x) + 2*a**3
*b**3*d + 2*a**2*b**4*d*tan(c + d*x) + a*b**5*d + b**6*d*tan(c + d*x)) - 2*
a**2*b**2*log(a/b + tan(c + d*x))/(a**5*b*d + a**4*b**2*d*tan(c + d*x) + 2*
a**3*b**3*d + 2*a**2*b**4*d*tan(c + d*x) + a*b**5*d + b**6*d*tan(c + d*x))
+ a**2*b**2*log(tan(c + d*x)**2 + 1)/(a**5*b*d + a**4*b**2*d*tan(c + d*x) +
 2*a**3*b**3*d + 2*a**2*b**4*d*tan(c + d*x) + a*b**5*d + b**6*d*tan(c + d*x
)) - a**2*b**2/(a**5*b*d + a**4*b**2*d*tan(c + d*x) + 2*a**3*b**3*d + 2*a**
2*b**4*d*tan(c + d*x) + a*b**5*d + b**6*d*tan(c + d*x)) + a*b**3*d*x/(a**5*
b*d + a**4*b**2*d*tan(c + d*x) + 2*a**3*b**3*d + 2*a**2*b**4*d*tan(c + d*x)
 + a*b**5*d + b**6*d*tan(c + d*x)) - 2*a*b**3*log(a/b + tan(c + d*x))*tan(c
 + d*x)/(a**5*b*d + a**4*b**2*d*tan(c + d*x) + 2*a**3*b**3*d + 2*a**2*b**4*
d*tan(c + d*x) + a*b**5*d + b**6*d*tan(c + d*x)) + a*b**3*log(tan(c + d*x)*
*2 + 1)*tan(c + d*x)/(a**5*b*d + a**4*b**2*d*tan(c + d*x) + 2*a**3*b**3*d +
 2*a**2*b**4*d*tan(c + d*x) + a*b**5*d + b**6*d*tan(c + d*x)) + b**4*d*x*ta
n(c + d*x)/(a**5*b*d + a**4*b**2*d*tan(c + d*x) + 2*a**3*b**3*d + 2*a**2*b*
*4*d*tan(c + d*x) + a*b**5*d + b**6*d*tan(c + d*x)), True))
```

Giac [A]
time = 0.73, size = 165, normalized size = 1.88

$$-\frac{\frac{2\,ab^2\log(|b\tan(dx+c)+a|)}{a^4b+2\,a^2b^3+b^5}-\frac{ab\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{(a^2-b^2)(dx+c)}{a^4+2\,a^2b^2+b^4}-\frac{2\,ab^3\tan(dx+c)-a^4+a^2b^2}{(a^4b+2\,a^2b^3+b^5)(b\tan(dx+c)+a)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*a*b^2*log(abs(b*tan(d*x + c) + a))/(a^4*b + 2*a^2*b^3 + b^5) - a*b*log(
tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^2 - b^2)*(d*x + c)/(a^4 +
2*a^2*b^2 + b^4) - (2*a*b^3*tan(d*x + c) - a^4 + a^2*b^2)/((a^4*b + 2*a^2*b
^3 + b^5)*(b*tan(d*x + c) + a)))/d
```

Mupad [B]
time = 4.07, size = 126, normalized size = 1.43

$$\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)}{2\,d\,\left(-a^2\,1\mathrm{i}+2\,a\,b+b^2\,1\mathrm{i}\right)}-\frac{a^2}{b\,d\,\left(a^2+b^2\right)\,\left(a+b\tan\left(c+d\,x\right)\right)}-\frac{2\,a\,b\,\ln\left(a+b\tan(c+d\,x)\right)}{d\,{\left(a^2+b^2\right)}^2}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(-a^2+a\,b\,2\mathrm{i}+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + b*tan(c + d*x))^2,x)
```

```
[Out] (log(tan(c + d*x) + 1i)*1i)/(2*d*(a*b*2i - a^2 + b^2)) + log(tan(c + d*x) -
 1i)/(2*d*(2*a*b - a^2*1i + b^2*1i)) - a^2/(b*d*(a^2 + b^2)*(a + b*tan(c +
d*x))) - (2*a*b*log(a + b*tan(c + d*x)))/(d*(a^2 + b^2)^2)
```

3.472 $\int \frac{\tan(c+dx)}{(a+b\tan(c+dx))^2} dx$

Optimal. Leaf size=82

$$\frac{2abx}{(a^2+b^2)^2}-\frac{(a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^2 d}+\frac{a}{(a^2+b^2)\,d(a+b\tan(c+dx))}$$

```
[Out] 2*a*b*x/(a^2+b^2)^2-(a^2-b^2)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^2/d+a
/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3610, 3612, 3611}

$$\frac{a}{d\,(a^2+b^2)\,(a+b\tan(c+dx))}-\frac{(a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^2}+\frac{2abx}{(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (2*a*b*x)/(a^2 + b^2)^2 - ((a^2 - b^2)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]]
)/((a^2 + b^2)^2*d) + a/((a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan(c+dx)}{(a+b\tan(c+dx))^2}\,dx &= \frac{a}{(a^2+b^2)\,d(a+b\tan(c+dx))}+\frac{\int \frac{b+a\tan(c+dx)}{a+b\tan(c+dx)}\,dx}{a^2+b^2}\\
&= \frac{2abx}{(a^2+b^2)^2}+\frac{a}{(a^2+b^2)\,d(a+b\tan(c+dx))}-\frac{(a^2-b^2)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}\,dx}{(a^2+b^2)^2}\\
&= \frac{2abx}{(a^2+b^2)^2}-\frac{(a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^2\,d}+\frac{a}{(a^2+b^2)\,d(a+b\,\mathrm{t}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.49, size = 181, normalized size = 2.21

$$
\frac{a((a-ib)^2\log(i-\tan(c+dx))+(a+ib)^2\log(i+\tan(c+dx))+2(a^2+b^2+(-a^2+b^2)\log(a+b\tan(c+dx))))+b((a-ib)^2\log(i-\tan(c+dx))+(a+ib)^2\log(i+\tan(c+dx))+2(-a^2+b^2)\log(a+b\tan(c+dx)))\tan(c+dx)}{2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (a*((a - I*b)^2*Log[I - Tan[c + d*x]] + (a + I*b)^2*Log[I + Tan[c + d*x]] +
 2*(a^2 + b^2 + (-a^2 + b^2)*Log[a + b*Tan[c + d*x]])) + b*((a - I*b)^2*Log
[I - Tan[c + d*x]] + (a + I*b)^2*Log[I + Tan[c + d*x]] + 2*(-a^2 + b^2)*Log
[a + b*Tan[c + d*x]])*Tan[c + d*x])/(2*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])
)
```

Maple [A]
time = 0.13, size = 104, normalized size = 1.27

method	result
derivativedivides	$\frac{\frac{\frac{(a^2-b^2)\ln(1+\tan^2(dx+c))}{2}+2ab\arctan(\tan(dx+c))}{(a^2+b^2)^2}+\frac{a}{(a^2+b^2)(a+b\tan(dx+c))}-\frac{(a^2-b^2)\ln(a+b\tan(dx+c))}{(a^2+b^2)^2}}{d}$
default	$\frac{\frac{\frac{(a^2-b^2)\ln(1+\tan^2(dx+c))}{2}+2ab\arctan(\tan(dx+c))}{(a^2+b^2)^2}+\frac{a}{(a^2+b^2)(a+b\tan(dx+c))}-\frac{(a^2-b^2)\ln(a+b\tan(dx+c))}{(a^2+b^2)^2}}{d}$
norman	$\frac{-\frac{b\tan(dx+c)}{d(a^2+b^2)}+\frac{2a^2bx}{a^4+2a^2b^2+b^4}+\frac{2b^2ax\tan(dx+c)}{a^4+2a^2b^2+b^4}}{a+b\tan(dx+c)}+\frac{(a^2-b^2)\ln(1+\tan^2(dx+c))}{2d(a^4+2a^2b^2+b^4)}-\frac{(a^2-b^2)\ln(a+b\tan(dx+c))}{d(a^4+2a^2b^2+b^4)}$
risch	$\frac{ix}{2iab-a^2+b^2}+\frac{2ia^2x}{a^4+2a^2b^2+b^4}-\frac{2ib^2x}{a^4+2a^2b^2+b^4}+\frac{2ia^2c}{(a^4+2a^2b^2+b^4)d}-\frac{2ib^2c}{d(a^4+2a^2b^2+b^4)}-\frac{}{(ib+a)d(-ib+a)^2(-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

[Out] 1/d*(1/(a^2+b^2)^2*(1/2*(a^2-b^2)*ln(1+tan(d*x+c)^2)+2*a*b*arctan(tan(d*x+c)))+a/(a^2+b^2)/(a+b*tan(d*x+c))-(a^2-b^2)/(a^2+b^2)^2*ln(a+b*tan(d*x+c)))

Maxima [A]
time = 0.96, size = 139, normalized size = 1.70

$$\frac{\frac{4\,(dx+c)ab}{a^4+2\,a^2b^2+b^4}-\frac{2\,(a^2-b^2)\log(b\tan(dx+c)+a)}{a^4+2\,a^2b^2+b^4}+\frac{(a^2-b^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\,a}{a^3+ab^2+(a^2b+b^3)\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(4*(d*x + c)*a*b/(a^4 + 2*a^2*b^2 + b^4) - 2*(a^2 - b^2)*log(b*tan(d*x
+ c) + a)/(a^4 + 2*a^2*b^2 + b^4) + (a^2 - b^2)*log(tan(d*x + c)^2 + 1)/(a^
4 + 2*a^2*b^2 + b^4) + 2*a/(a^3 + a*b^2 + (a^2*b + b^3)*tan(d*x + c)))/d
```

Fricas [A]
time = 0.85, size = 157, normalized size = 1.91

$$\frac{4\,a^2bdx+2\,ab^2-(a^3-ab^2+(a^2b-b^3)\tan(dx+c))\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)+2\,(2\,ab^2dx-a^2b)\tan(dx+c)}{2\,((a^4b+2\,a^2b^3+b^5)d\tan(dx+c)+(a^5+2\,a^3b^2+ab^4)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^2*b*d*x + 2*a*b^2 - (a^3 - a*b^2 + (a^2*b - b^3)*tan(d*x + c))*log
((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) + 2*
(2*a*b^2*d*x - a^2*b)*tan(d*x + c))/((a^4*b + 2*a^2*b^3 + b^5)*d*tan(d*x +
c) + (a^5 + 2*a^3*b^2 + a*b^4)*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.76, size = 1476, normalized size = 18.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Piecewise((zoo*x/tan(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (log(tan(c + d*x)
**2 + 1)/(2*a**2*d), Eq(b, 0)), (I*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*
x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 2*d*x*tan(c + d*x)/(4*b**2*d*
tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - I*d*x/(4*b**2*d*tan
(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + I*tan(c + d*x)/(4*b**2
*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d), Eq(a, -I*b)), (-I
*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) -
```

```
4*b**2*d) + 2*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c
 + d*x) - 4*b**2*d) + I*d*x/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c +
d*x) - 4*b**2*d) - I*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*ta
n(c + d*x) - 4*b**2*d), Eq(a, I*b)), (x*tan(c)/(a + b*tan(c))**2, Eq(d, 0))
, (-2*a**3*log(a/b + tan(c + d*x))/(2*a**5*d + 2*a**4*b*d*tan(c + d*x) + 4*
a**3*b**2*d + 4*a**2*b**3*d*tan(c + d*x) + 2*a*b**4*d + 2*b**5*d*tan(c + d*
x)) + a**3*log(tan(c + d*x)**2 + 1)/(2*a**5*d + 2*a**4*b*d*tan(c + d*x) + 4
*a**3*b**2*d + 4*a**2*b**3*d*tan(c + d*x) + 2*a*b**4*d + 2*b**5*d*tan(c + d
*x)) + 2*a**3/(2*a**5*d + 2*a**4*b*d*tan(c + d*x) + 4*a**3*b**2*d + 4*a**2*
b**3*d*tan(c + d*x) + 2*a*b**4*d + 2*b**5*d*tan(c + d*x)) + 4*a**2*b*d*x/(2
*a**5*d + 2*a**4*b*d*tan(c + d*x) + 4*a**3*b**2*d + 4*a**2*b**3*d*tan(c + d
*x) + 2*a*b**4*d + 2*b**5*d*tan(c + d*x)) - 2*a**2*b*log(a/b + tan(c + d*x)
)*tan(c + d*x)/(2*a**5*d + 2*a**4*b*d*tan(c + d*x) + 4*a**3*b**2*d + 4*a**2
*b**3*d*tan(c + d*x) + 2*a*b**4*d + 2*b**5*d*tan(c + d*x)) + a**2*b*log(tan
(c + d*x)**2 + 1)*tan(c + d*x)/(2*a**5*d + 2*a**4*b*d*tan(c + d*x) + 4*a**3
*b**2*d + 4*a**2*b**3*d*tan(c + d*x) + 2*a*b**4*d + 2*b**5*d*tan(c + d*x))
 + 4*a*b**2*d*x*tan(c + d*x)/(2*a**5*d + 2*a**4*b*d*tan(c + d*x) + 4*a**3*b*
*2*d + 4*a**2*b**3*d*tan(c + d*x) + 2*a*b**4*d + 2*b**5*d*tan(c + d*x)) + 2
*a*b**2*log(a/b + tan(c + d*x))/(2*a**5*d + 2*a**4*b*d*tan(c + d*x) + 4*a**
3*b**2*d + 4*a**2*b**3*d*tan(c + d*x) + 2*a*b**4*d + 2*b**5*d*tan(c + d*x))
 - a*b**2*log(tan(c + d*x)**2 + 1)/(2*a**5*d + 2*a**4*b*d*tan(c + d*x) + 4*
a**3*b**2*d + 4*a**2*b**3*d*tan(c + d*x) + 2*a*b**4*d + 2*b**5*d*tan(c + d*
x)) + 2*a*b**2/(2*a**5*d + 2*a**4*b*d*tan(c + d*x) + 4*a**3*b**2*d + 4*a**2
*b**3*d*tan(c + d*x) + 2*a*b**4*d + 2*b**5*d*tan(c + d*x)) + 2*b**3*log(a/b
 + tan(c + d*x))*tan(c + d*x)/(2*a**5*d + 2*a**4*b*d*tan(c + d*x) + 4*a**3*
b**2*d + 4*a**2*b**3*d*tan(c + d*x) + 2*a*b**4*d + 2*b**5*d*tan(c + d*x)) -
 b**3*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*a**5*d + 2*a**4*b*d*tan(c +
d*x) + 4*a**3*b**2*d + 4*a**2*b**3*d*tan(c + d*x) + 2*a*b**4*d + 2*b**5*d*t
an(c + d*x)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(82) = 164$.

time = 0.59, size = 173, normalized size = 2.11

$$\frac{\frac{4\,(dx+c)ab}{a^4+2\,a^2b^2+b^4}+\frac{(a^2-b^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\,(a^2b-b^3)\log(|b\tan(dx+c)+a|)}{a^4b+2\,a^2b^3+b^5}+\frac{2\,(a^2b\tan(dx+c)-b^3\tan(dx+c)+2\,a^3)}{(a^4+2\,a^2b^2+b^4)(b\tan(dx+c)+a)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*(d*x + c)*a*b/(a^4 + 2*a^2*b^2 + b^4) + (a^2 - b^2)*log(tan(d*x + c)
^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*(a^2*b - b^3)*log(abs(b*tan(d*x + c) +
a))/(a^4*b + 2*a^2*b^3 + b^5) + 2*(a^2*b*tan(d*x + c) - b^3*tan(d*x + c) +
2*a^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*tan(d*x + c) + a)))/d
```

Mupad [B]
time = 4.17, size = 133, normalized size = 1.62

$$\frac{\ln(\tan(c+d\,x)-\mathrm{i})}{2\,d\,\left(a^2+a\,b\,2\mathrm{i}-b^2\right)}-\frac{\ln(a+b\tan(c+d\,x))\,\left(\frac{1}{a^2+b^2}-\frac{2\,b^2}{(a^2+b^2)^2}\right)}{d}+\frac{a}{d\,\left(a^2+b^2\right)\,\left(a+b\tan(c+d\,x)\right)}+\frac{\ln(\tan(c+d\,x)+1\mathrm{i})\;1\mathrm{i}}{2\,d\,\left(a^2\,1\mathrm{i}+2\,a\,b-b^2\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + b*tan(c + d*x))^2,x)
```

```
[Out] log(tan(c + d*x) - 1i)/(2*d*(a*b*2i + a^2 - b^2)) + (log(tan(c + d*x) + 1i)
*1i)/(2*d*(2*a*b + a^2*1i - b^2*1i)) - (log(a + b*tan(c + d*x))*(1/(a^2 + b
^2) - (2*b^2)/(a^2 + b^2)^2))/d + a/(d*(a^2 + b^2)*(a + b*tan(c + d*x)))
```

3.473 $\int \frac{1}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=82

$$\frac{(a^2-b^2)\,x}{(a^2+b^2)^2}+\frac{2ab\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^2\,d}-\frac{b}{(a^2+b^2)\,d(a+b\tan(c+dx))}$$

```
[Out] (a^2-b^2)*x/(a^2+b^2)^2+2*a*b*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^2/d-b
/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3564, 3612, 3611}

$$-\frac{b}{d\,(a^2+b^2)\,(a+b\tan(c+dx))}+\frac{2ab\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^2}+\frac{x(a^2-b^2)}{(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^(-2),x]
```

```
[Out] ((a^2 - b^2)*x)/(a^2 + b^2)^2 + (2*a*b*Log[a*Cos[c + d*x] + b*Sin[c + d*x]]
)/((a^2 + b^2)^2*d) - b/((a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\tan(c+dx))^2}\,dx = -\frac{b}{(a^2+b^2)\,d(a+b\tan(c+dx))} + \frac{\int \frac{a-b\tan(c+dx)}{a+b\tan(c+dx)}\,dx}{a^2+b^2}$$

$$= \frac{(a^2-b^2)\,x}{(a^2+b^2)^2} - \frac{b}{(a^2+b^2)\,d(a+b\tan(c+dx))} + \frac{(2ab)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}\,dx}{(a^2+b^2)^2}$$

$$= \frac{(a^2-b^2)\,x}{(a^2+b^2)^2} + \frac{2ab\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^2\,d} - \frac{b}{(a^2+b^2)\,d(a+b\tan(c+$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.36, size = 106, normalized size = 1.29

$$\frac{-\frac{i\log(i-\tan(c+dx))}{(a+ib)^2} + \frac{i\log(i+\tan(c+dx))}{(a-ib)^2} + \frac{2b\left(2a\log(a+b\tan(c+dx)) - \frac{a^2+b^2}{a+b\tan(c+dx)}\right)}{(a^2+b^2)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^(-2),x]
```

```
[Out] (((-I)*Log[I - Tan[c + d*x]])/(a + I*b)^2 + (I*Log[I + Tan[c + d*x]])/(a -
I*b)^2 + (2*b*(2*a*Log[a + b*Tan[c + d*x]] - (a^2 + b^2)/(a + b*Tan[c + d*x
]]))/(a^2 + b^2)^2)/(2*d)
```

Maple [A]
time = 0.10, size = 97, normalized size = 1.18

method	result
derivativedivides	$\frac{\frac{-ab\ln\left(1+\tan^2(dx+c)\right)+\left(a^2-b^2\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^2} - \frac{b}{\left(a^2+b^2\right)(a+b\tan(dx+c))} + \frac{2ab\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^2}}{d}$
default	$\frac{\frac{-ab\ln\left(1+\tan^2(dx+c)\right)+\left(a^2-b^2\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^2} - \frac{b}{\left(a^2+b^2\right)(a+b\tan(dx+c))} + \frac{2ab\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^2}}{d}$
norman	$\frac{\frac{\left(a^2-b^2\right)ax}{a^4+2a^2b^2+b^4} + \frac{b\left(a^2-b^2\right)x\tan(dx+c)}{a^4+2a^2b^2+b^4} + \frac{b^2\tan(dx+c)}{a\left(a^2+b^2\right)d}}{a+b\tan(dx+c)} - \frac{ab\ln\left(1+\tan^2(dx+c)\right)}{d(a^4+2a^2b^2+b^4)} + \frac{2ab\ln(a+b\tan(dx+c))}{d(a^4+2a^2b^2+b^4)}$
risch	$-\frac{x}{2iab-a^2+b^2} - \frac{4iabx}{a^4+2a^2b^2+b^4} - \frac{4iabc}{d(a^4+2a^2b^2+b^4)} - \frac{2ib^2}{(-ia+b)d(ia+b)^2\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)} + \frac{2ab\ln}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

[Out] 1/d*(1/(a^2+b^2)^2*(-a*b*ln(1+tan(d*x+c)^2)+(a^2-b^2)*arctan(tan(d*x+c)))-b/(a^2+b^2)/(a+b*tan(d*x+c))+2*a*b/(a^2+b^2)^2*ln(a+b*tan(d*x+c)))

Maxima [A]
time = 0.51, size = 131, normalized size = 1.60

$$\frac{\frac{2\,ab\log\left(b\tan\left(dx+c\right)+a\right)}{a^4+2\,a^2b^2+b^4}-\frac{ab\log\left(\tan\left(dx+c\right)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{\left(a^2-b^2\right)\left(dx+c\right)}{a^4+2\,a^2b^2+b^4}-\frac{b}{a^3+ab^2+\left(a^2b+b^3\right)\tan\left(dx+c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tan(d*x+c))^2,x, algorithm="maxima")

[Out] (2*a*b*log(b*tan(d*x + c) + a)/(a^4 + 2*a^2*b^2 + b^4) - a*b*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^2 - b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - b/(a^3 + a*b^2 + (a^2*b + b^3)*tan(d*x + c)))/d

Fricas [A]
time = 0.86, size = 154, normalized size = 1.88

$$-\frac{b^3-(a^3-ab^2)dx-(ab^2\tan\left(dx+c\right)+a^2b)\log\left(\frac{b^2\tan\left(dx+c\right)^2+2\,ab\tan\left(dx+c\right)+a^2}{\tan\left(dx+c\right)^2+1}\right)-(ab^2+(a^2b-b^3)dx)\tan\left(dx+c\right)}{(a^4b+2\,a^2b^3+b^5)d\tan\left(dx+c\right)+(a^5+2\,a^3b^2+ab^4)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tan(d*x+c))^2,x, algorithm="fricas")

[Out] -(b^3 - (a^3 - a*b^2)*d*x - (a*b^2*tan(d*x + c) + a^2*b)*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) - (a*b^2 + (a^2*b - b^3)*d*x)*tan(d*x + c))/((a^4*b + 2*a^2*b^3 + b^5)*d*tan(d*x + c) + (a^5 + 2*a^3*b^2 + a*b^4)*d)

Sympy [C] Result contains complex when optimal does not.
time = 0.72, size = 1260, normalized size = 15.37

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tan(d*x+c))**2,x)

[Out] Piecewise((zoo*x/tan(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/a**2, Eq(b, 0)), (-d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 2*I*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + d*x/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 2*I/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d), Eq(a, -I*b)), (-d*x*tan(c + d*x)**2/(4*b**2*d*tan(c

```
 + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 2*I*d*x*tan(c + d*x)/(4*
b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + d*x/(4*b**2*
d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - tan(c + d*x)/(4*b
**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 2*I/(4*b**2*d
*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d), Eq(a, I*b)), (x/(a
+ b*tan(c))**2, Eq(d, 0)), (a**3*d*x/(a**5*d + a**4*b*d*tan(c + d*x) + 2*a*
*3*b**2*d + 2*a**2*b**3*d*tan(c + d*x) + a*b**4*d + b**5*d*tan(c + d*x)) +
a**2*b*d*x*tan(c + d*x)/(a**5*d + a**4*b*d*tan(c + d*x) + 2*a**3*b**2*d + 2
*a**2*b**3*d*tan(c + d*x) + a*b**4*d + b**5*d*tan(c + d*x)) + 2*a**2*b*log(
a/b + tan(c + d*x))/(a**5*d + a**4*b*d*tan(c + d*x) + 2*a**3*b**2*d + 2*a**
2*b**3*d*tan(c + d*x) + a*b**4*d + b**5*d*tan(c + d*x)) - a**2*b*log(tan(c
+ d*x)**2 + 1)/(a**5*d + a**4*b*d*tan(c + d*x) + 2*a**3*b**2*d + 2*a**2*b**
3*d*tan(c + d*x) + a*b**4*d + b**5*d*tan(c + d*x)) - a**2*b/(a**5*d + a**4*
b*d*tan(c + d*x) + 2*a**3*b**2*d + 2*a**2*b**3*d*tan(c + d*x) + a*b**4*d +
b**5*d*tan(c + d*x)) - a*b**2*d*x/(a**5*d + a**4*b*d*tan(c + d*x) + 2*a**3*
b**2*d + 2*a**2*b**3*d*tan(c + d*x) + a*b**4*d + b**5*d*tan(c + d*x)) + 2*a
*b**2*log(a/b + tan(c + d*x))*tan(c + d*x)/(a**5*d + a**4*b*d*tan(c + d*x)
+ 2*a**3*b**2*d + 2*a**2*b**3*d*tan(c + d*x) + a*b**4*d + b**5*d*tan(c + d*
x)) - a*b**2*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(a**5*d + a**4*b*d*tan(c
 + d*x) + 2*a**3*b**2*d + 2*a**2*b**3*d*tan(c + d*x) + a*b**4*d + b**5*d*ta
n(c + d*x)) - b**3*d*x*tan(c + d*x)/(a**5*d + a**4*b*d*tan(c + d*x) + 2*a**
3*b**2*d + 2*a**2*b**3*d*tan(c + d*x) + a*b**4*d + b**5*d*tan(c + d*x)) - b
**3/(a**5*d + a**4*b*d*tan(c + d*x) + 2*a**3*b**2*d + 2*a**2*b**3*d*tan(c +
 d*x) + a*b**4*d + b**5*d*tan(c + d*x)), True))
```

Giac [A]
time = 0.47, size = 159, normalized size = 1.94

$$\frac{\frac{2\,ab^2\log(|b\tan(dx+c)+a|)}{a^4b+2\,a^2b^3+b^5}-\frac{ab\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{(a^2-b^2)(dx+c)}{a^4+2\,a^2b^2+b^4}-\frac{2\,ab^2\tan(dx+c)+3\,a^2b+b^3}{(a^4+2\,a^2b^2+b^4)(b\tan(dx+c)+a)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*a*b^2*log(abs(b*tan(d*x + c) + a))/(a^4*b + 2*a^2*b^3 + b^5) - a*b*log(t
an(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^2 - b^2)*(d*x + c)/(a^4 + 2
*a^2*b^2 + b^4) - (2*a*b^2*tan(d*x + c) + 3*a^2*b + b^3)/((a^4 + 2*a^2*b^2
+ b^4)*(b*tan(d*x + c) + a)))/d
```

Mupad [B]
time = 4.08, size = 121, normalized size = 1.48

$$\frac{2\,a\,b\,\ln\left(a+b\tan(c+d\,x)\right)}{d\,{\left(a^2+b^2\right)}^2}-\frac{b}{d\,\left(a^2+b^2\right)\,\left(a+b\tan\left(c+d\,x\right)\right)}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)}{2\,d\,\left(-a^2\,1\mathrm{i}+2\,a\,b+b^2\,1\mathrm{i}\right)}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(-a^2+a\,b\,2\mathrm{i}+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(c + d*x))^2,x)
```

```
[Out] (2*a*b*log(a + b*tan(c + d*x)))/(d*(a^2 + b^2)^2) - log(tan(c + d*x) - 1i)/
(2*d*(2*a*b - a^2*1i + b^2*1i)) - b/(d*(a^2 + b^2)*(a + b*tan(c + d*x))) -
(log(tan(c + d*x) + 1i)*1i)/(2*d*(a*b*2i - a^2 + b^2))
```

3.474 $\int \frac{\cot(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=107

$$-\frac{2abx}{\left(a^2+b^2\right)^2}+\frac{\log(\sin(c+dx))}{a^2d}-\frac{b^2\left(3a^2+b^2\right)\log(a\cos(c+dx)+b\sin(c+dx))}{a^2\left(a^2+b^2\right)^2d}+\frac{b^2}{a\left(a^2+b^2\right)d(a+b\tan(c+dx))}$$

```
[Out] -2*a*b*x/(a^2+b^2)^2+ln(sin(d*x+c))/a^2/d-b^2*(3*a^2+b^2)*ln(a*cos(d*x+c)+b
*sin(d*x+c))/a^2/(a^2+b^2)^2/d+b^2/a/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

Rubi [A]

time = 0.15, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3650, 3732, 3611, 3556}

$$\frac{b^2}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{b^2\left(3a^2+b^2\right)\log(a\cos(c+dx)+b\sin(c+dx))}{a^2d\left(a^2+b^2\right)^2}-\frac{2abx}{\left(a^2+b^2\right)^2}+\frac{\log(\sin(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-2*a*b*x)/(a^2 + b^2)^2 + Log[Sin[c + d*x]]/(a^2*d) - (b^2*(3*a^2 + b^2)*L
og[a*Cos[c + d*x] + b*Sin[c + d*x]])/(a^2*(a^2 + b^2)^2*d) + b^2/(a*(a^2 +
b^2)*d*(a + b*Tan[c + d*x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
```

```
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(c+dx)}{(a+b\tan(c+dx))^2}\,dx &= \frac{b^2}{a\left(a^2+b^2\right)d(a+b\tan(c+dx))}+\frac{\int \frac{\cot(c+dx)\left(a^2+b^2-ab\tan(c+dx)+b^2\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx}{a\left(a^2+b^2\right)} \\ &= -\frac{2abx}{\left(a^2+b^2\right)^2}+\frac{b^2}{a\left(a^2+b^2\right)d(a+b\tan(c+dx))}+\frac{\int \cot(c+dx)\,dx}{a^2}-\frac{(b^2(3a^2 \\ &= -\frac{2abx}{\left(a^2+b^2\right)^2}+\frac{\log(\sin(c+dx))}{a^2d}-\frac{b^2\left(3a^2+b^2\right)\log(a\cos(c+dx)+b\sin(c+d}{a^2\left(a^2+b^2\right)^2d}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.72, size = 154, normalized size = 1.44

$$\frac{-\frac{a(a-ib)\log(i-\tan(c+dx))}{2(a+ib)}+\frac{\left(a^2+b^2\right)\log(\tan(c+dx))}{a}-\frac{a(a+ib)\log(i+\tan(c+dx))}{2(a-ib)}-\frac{b^2\left(3a^2+b^2\right)\log(a+b\tan(c+dx))}{a\left(a^2+b^2\right)}+\frac{b^2}{a+b\tan(c+dx)}}{a\left(a^2+b^2\right)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-1/2*(a*(a - I*b)*Log[I - Tan[c + d*x]])/(a + I*b) + ((a^2 + b^2)*Log[Tan[
c + d*x]])/a - (a*(a + I*b)*Log[I + Tan[c + d*x]])/(2*(a - I*b)) - (b^2*(3*
a^2 + b^2)*Log[a + b*Tan[c + d*x]])/(a*(a^2 + b^2)) + b^2/(a + b*Tan[c + d*
x]))/(a*(a^2 + b^2)*d)
```

Maple [A]
time = 0.30, size = 126, normalized size = 1.18

method	result
derivativedivides	$\frac{\frac{\ln(\tan(dx+c))}{a^2}+\frac{b^2}{\left(a^2+b^2\right)a(a+b\tan(dx+c))}-\frac{b^2\left(3a^2+b^2\right)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^2a^2}+\frac{\frac{\left(-a^2+b^2\right)\ln\left(1+\tan^2(dx+c)\right)}{2}-2ab\arctan(\tan(dx+c\cdots}{\left(a^2+b^2\right)^2}}{d}$
default	$\frac{\frac{\ln(\tan(dx+c))}{a^2}+\frac{b^2}{\left(a^2+b^2\right)a(a+b\tan(dx+c))}-\frac{b^2\left(3a^2+b^2\right)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^2a^2}+\frac{\frac{\left(-a^2+b^2\right)\ln\left(1+\tan^2(dx+c)\right)}{2}-2ab\arctan(\tan(dx+c\cdots}{\left(a^2+b^2\right)^2}}{d}$
norman	$\frac{-\frac{b^3\tan(dx+c)}{d\,a^2\left(a^2+b^2\right)}-\frac{2a^2bx}{a^4+2a^2b^2+b^4}-\frac{2b^2ax\tan(dx+c)}{a^4+2a^2b^2+b^4}}{a+b\tan(dx+c)}+\frac{\ln(\tan(dx+c))}{a^2d}-\frac{(a^2-b^2)\ln(1+\tan^2(dx+c))}{2d(a^4+2a^2b^2+b^4)}-\frac{b^2(3a^2+b^2)\ln(a+\cdots}{(a^4+2a^2b^2\cdots}$
risch	$-\frac{ix}{2iab-a^2+b^2}+\frac{6ib^2x}{a^4+2a^2b^2+b^4}+\frac{6ib^2c}{d(a^4+2a^2b^2+b^4)}+\frac{2ib^4x}{(a^4+2a^2b^2+b^4)a^2}+\frac{2ib^4c}{(a^4+2a^2b^2+b^4)a^2d}-\frac{2ix}{a^2}-\frac{2ic}{a^2d}-\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a^2*ln(tan(d*x+c))+b^2/(a^2+b^2)/a/(a+b*tan(d*x+c))-b^2*(3*a^2+b^2)/
(a^2+b^2)^2/a^2*ln(a+b*tan(d*x+c))+1/(a^2+b^2)^2*(1/2*(-a^2+b^2)*ln(1+tan(d
*x+c)^2)-2*a*b*arctan(tan(d*x+c))))
```

Maxima [A]
time = 0.51, size = 164, normalized size = 1.53

$$-\frac{\frac{4\,(dx+c)ab}{a^4+2\,a^2b^2+b^4}-\frac{2\,b^2}{a^4+a^2b^2+(a^3b+ab^3)\tan(dx+c)}+\frac{2\,(3\,a^2b^2+b^4)\log(b\tan(dx+c)+a)}{a^6+2\,a^4b^2+a^2b^4}+\frac{(a^2-b^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\,\log(\tan(dx+c))}{a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(4*(d*x + c)*a*b/(a^4 + 2*a^2*b^2 + b^4) - 2*b^2/(a^4 + a^2*b^2 + (a^3
*b + a*b^3)*tan(d*x + c)) + 2*(3*a^2*b^2 + b^4)*log(b*tan(d*x + c) + a)/(a^
6 + 2*a^4*b^2 + a^2*b^4) + (a^2 - b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2
*b^2 + b^4) - 2*log(tan(d*x + c))/a^2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(107) = 214.
time = 0.87, size = 235, normalized size = 2.20

$$-\frac{4\,a^4bdx-2\,ab^4-\left(a^5+2\,a^3b^2+ab^4+\left(a^4b+2\,a^2b^3+b^5\right)\tan\left(dx+c\right)\right)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)+\left(3\,a^3b^2+ab^4+\left(3\,a^2b^3+b^5\right)\tan\left(dx+c\right)\right)\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)+2\,\left(2\,a^3b^2dx+a^2b^3\right)\tan\left(dx+c\right)}{2\,\left(\left(a^6b+2\,a^4b^3+a^2b^5\right)d\tan\left(dx+c\right)+\left(a^7+2\,a^5b^2+a^3b^4\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^4*b*d*x - 2*a*b^4 - (a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3
 + b^5)*tan(d*x + c))*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1)) + (3*a^3*b^2
```

```
 + a*b^4 + (3*a^2*b^3 + b^5)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*
tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) + 2*(2*a^3*b^2*d*x + a^2*b^3)*tan
(d*x + c))/((a^6*b + 2*a^4*b^3 + a^2*b^5)*d*tan(d*x + c) + (a^7 + 2*a^5*b^2
 + a^3*b^4)*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.67, size = 2927, normalized size = 27.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Piecewise((zoo*x*cot(c)/tan(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((-log(
tan(c + d*x)**2 + 1)/(2*d) + log(tan(c + d*x))/d)/a**2, Eq(b, 0)), ((log(ta
n(c + d*x)**2 + 1)/(2*d) - log(tan(c + d*x))/d - 1/(2*d*tan(c + d*x)**2))/b
**2, Eq(a, 0)), (3*I*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 - 8*I*b*
*2*d*tan(c + d*x) - 4*b**2*d) + 6*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**
2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 3*I*d*x/(4*b**2*d*tan(c + d*x)**2
 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 2*log(tan(c + d*x)**2 + 1)*tan(c +
 d*x)**2/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) -
4*I*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b
**2*d*tan(c + d*x) - 4*b**2*d) - 2*log(tan(c + d*x)**2 + 1)/(4*b**2*d*tan(c
 + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 4*log(tan(c + d*x))*tan(
c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d)
 + 8*I*log(tan(c + d*x))*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*
d*tan(c + d*x) - 4*b**2*d) + 4*log(tan(c + d*x))/(4*b**2*d*tan(c + d*x)**2
 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 3*I*tan(c + d*x)/(4*b**2*d*tan(c +
d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 4/(4*b**2*d*tan(c + d*x)**2
 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d), Eq(a, -I*b)), (-3*I*d*x*tan(c + d*x
)**2/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 6*d*
x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2
*d) + 3*I*d*x/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*
d) + 2*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 +
 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 4*I*log(tan(c + d*x)**2 + 1)*tan(c +
 d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 2*l
og(tan(c + d*x)**2 + 1)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x)
 - 4*b**2*d) - 4*log(tan(c + d*x))*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**
2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 8*I*log(tan(c + d*x))*tan(c + d*x
)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 4*log(t
an(c + d*x))/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d
) - 3*I*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) -
4*b**2*d) + 4/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*
d), Eq(a, I*b)), (x*cot(c)/(a + b*tan(c))**2, Eq(d, 0)), (-a**5*log(tan(c +
 d*x)**2 + 1)/(2*a**7*d + 2*a**6*b*d*tan(c + d*x) + 4*a**5*b**2*d + 4*a**4*
```

b**3*d*tan(c + d*x) + 2*a**3*b**4*d + 2*a**2*b**5*d*tan(c + d*x)) + 2*a**5*
log(tan(c + d*x))/(2*a**7*d + 2*a**6*b*d*tan(c + d*x) + 4*a**5*b**2*d + 4*a
4*b3*d*tan(c + d*x) + 2*a**3*b**4*d + 2*a**2*b**5*d*tan(c + d*x)) - 4*a
4*b*d*x/(2*a7*d + 2*a**6*b*d*tan(c + d*x) + 4*a**5*b**2*d + 4*a**4*b**3
*d*tan(c + d*x) + 2*a**3*b**4*d + 2*a**2*b**5*d*tan(c + d*x)) - a**4*b*log(
tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*a**7*d + 2*a**6*b*d*tan(c + d*x) + 4*a
5*b2*d + 4*a**4*b**3*d*tan(c + d*x) + 2*a**3*b**4*d + 2*a**2*b**5*d*tan
(c + d*x)) + 2*a**4*b*log(tan(c + d*x))*tan(c + d*x)/(2*a**7*d + 2*a**6*b*d
*tan(c + d*x) + 4*a**5*b**2*d + 4*a**4*b**3*d*tan(c + d*x) + 2*a**3*b**4*d
+ 2*a**2*b**5*d*tan(c + d*x)) - 4*a**3*b**2*d*x*tan(c + d*x)/(2*a**7*d + 2*
a**6*b*d*tan(c + d*x) + 4*a**5*b**2*d + 4*a**4*b**3*d*tan(c + d*x) + 2*a**3
*b**4*d + 2*a**2*b**5*d*tan(c + d*x)) - 6*a**3*b**2*log(a/b + tan(c + d*x))
/(2*a**7*d + 2*a**6*b*d*tan(c + d*x) + 4*a**5*b**2*d + 4*a**4*b**3*d*tan(c
+ d*x) + 2*a**3*b**4*d + 2*a**2*b**5*d*tan(c + d*x)) + a**3*b**2*log(tan(c
+ d*x)**2 + 1)/(2*a**7*d + 2*a**6*b*d*tan(c + d*x) + 4*a**5*b**2*d + 4*a**4
*b**3*d*tan(c + d*x) + 2*a**3*b**4*d + 2*a**2*b**5*d*tan(c + d*x)) + 4*a**3
*b**2*log(tan(c + d*x))/(2*a**7*d + 2*a**6*b*d*tan(c + d*x) + 4*a**5*b**2*d
 + 4*a**4*b**3*d*tan(c + d*x) + 2*a**3*b**4*d + 2*a**2*b**5*d*tan(c + d*x))
 + 2*a**3*b**2/(2*a**7*d + 2*a**6*b*d*tan(c + d*x) + 4*a**5*b**2*d + 4*a**4
*b**3*d*tan(c + d*x) + 2*a**3*b**4*d + 2*a**2*b**5*d*tan(c + d*x)) - 6*a**2
*b**3*log(a/b + tan(c + d*x))*tan(c + d*x)/(2*a**7*d + 2*a**6*b*d*tan(c + d
*x) + 4*a**5*b**2*d + 4*a**4*b**3*d*tan(c + d*x) + 2*a**3*b**4*d + 2*a**2*b
5*d*tan(c + d*x)) + a2*b**3*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*a*
*7*d + 2*a**6*b*d*tan(c + d*x) + 4*a**5*b**2*d + 4*a**4*b**3*d*tan(c + d*x)
 + 2*a**3*b**4*d + 2*a**2*b**5*d*tan(c + d*x)) + 4*a**2*b**3*log(tan(c + d*
x))*tan(c + d*x)/(2*a**7*d + 2*a**6*b*d*tan(c + d*x) + 4*a**5*b**2*d + 4*a*
*4*b**3*d*tan(c + d*x) + 2*a**3*b**4*d + 2*a**2*b**5*d*tan(c + d*x)) - 2*a*
b**4*log(a/b + tan(c + d*x))/(2*a**7*d + 2*a**6*b*d*tan(c + d*x) + 4*a**5*b
2*d + 4*a4*b**3*d*tan(c + d*x) + 2*a**3*b**4*d + 2*a**2*b**5*d*tan(c +
d*x)) + 2*a*b**4*log(tan(c + d*x))/(2*a**7*d + 2*a**6*b*d*tan(c + d*x) + 4*
a**5*b**2*d + 4*a**4*b**3*d*tan(c + d*x) + 2*a**3*b**4*d + 2*a**2*b**5*d*ta
n(c + d*x)) + 2*a*b**4/(2*a**7*d + 2*a**6*b*d*tan(c + d*x) + 4*a**5*b**2*d
+ 4*a**4*b**3*d*tan(c + d*x) + 2*a**3*b**4*d + 2*a**2*b**5*d*tan(c + d*x))
- 2*b**5*log(a/b + tan(c + d*x))*tan(c + d*x)/(2*a**7*d + 2*a**6*b*d*tan(c
+ d*x) + 4*a**5*b**2*d + 4*a**4*b**3*d*tan(c + ...

Giac [A]

time = 0.92, size = 206, normalized size = 1.93

$$-\frac{\frac{4\,(dx+c)ab}{a^4+2\,a^2b^2+b^4}+\frac{(a^2-b^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\,(3\,a^2b^3+b^5)\log(|b\tan(dx+c)+a|)}{a^6b+2\,a^4b^3+a^2b^5}-\frac{2\,(3\,a^2b^3\tan(dx+c)+b^5\tan(dx+c)+4\,a^3b^2+2\,ab^4)}{(a^6+2\,a^4b^2+a^2b^4)(b\tan(dx+c)+a)}-\frac{2\log(|\tan(dx+c)|)}{a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="giac")

[Out] -1/2*(4*(d*x + c)*a*b/(a^4 + 2*a^2*b^2 + b^4) + (a^2 - b^2)*log(tan(d*x + c
)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 2*(3*a^2*b^3 + b^5)*log(abs(b*tan(d*x +

```
c) + a))/(a^6*b + 2*a^4*b^3 + a^2*b^5) - 2*(3*a^2*b^3*tan(d*x + c) + b^5*ta
n(d*x + c) + 4*a^3*b^2 + 2*a*b^4)/((a^6 + 2*a^4*b^2 + a^2*b^4)*(b*tan(d*x +
 c) + a)) - 2*log(abs(tan(d*x + c)))/a^2)/d
```

Mupad [B]
time = 4.29, size = 152, normalized size = 1.42

$$\frac{\ln\left(\tan(c+d\,x)\right)}{a^2\,d}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)}{2\,d\,\left(a^2+a\,b\,2\mathrm{i}-b^2\right)}+\frac{b^2}{a\,d\,\left(a^2+b^2\right)\,\left(a+b\tan\left(c+d\,x\right)\right)}-\frac{b^2\,\ln\left(a+b\tan(c+d\,x)\right)\,\left(3\,a^2+b^2\right)}{a^2\,d\,{\left(a^2+b^2\right)}^2}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(a^2\,1\mathrm{i}+2\,a\,b-b^2\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + b*tan(c + d*x))^2,x)
```

```
[Out] log(tan(c + d*x))/(a^2*d) - (log(tan(c + d*x) + 1i)*1i)/(2*d*(2*a*b + a^2*1
i - b^2*1i)) - log(tan(c + d*x) - 1i)/(2*d*(a*b*2i + a^2 - b^2)) + b^2/(a*d
*(a^2 + b^2)*(a + b*tan(c + d*x))) - (b^2*log(a + b*tan(c + d*x))*(3*a^2 +
b^2))/(a^2*d*(a^2 + b^2)^2)
```

3.475 $\int \frac{\cot^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=150

$$-\frac{(a^2-b^2)\,x}{(a^2+b^2)^2}-\frac{2b\log(\sin(c+dx))}{a^3d}+\frac{2b^3(2a^2+b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{a^3\,(a^2+b^2)^2\,d}-\frac{b(a^2+2b^2)}{a^2\,(a^2+b^2)\,d(a+b\tan$$

```
[Out] -(a^2-b^2)*x/(a^2+b^2)^2-2*b*ln(sin(d*x+c))/a^3/d+2*b^3*(2*a^2+b^2)*ln(a*co
s(d*x+c)+b*sin(d*x+c))/a^3/(a^2+b^2)^2/d-b*(a^2+2*b^2)/a^2/(a^2+b^2)/d/(a+b
*tan(d*x+c))-cot(d*x+c)/a/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.26, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3650, 3730, 3732, 3611, 3556}

$$-\frac{2b\log(\sin(c+dx))}{a^3d}-\frac{b(a^2+2b^2)}{a^2d\,(a^2+b^2)\,(a+b\tan(c+dx))}-\frac{x(a^2-b^2)}{(a^2+b^2)^2}+\frac{2b^3(2a^2+b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{a^3d\,(a^2+b^2)^2}-\frac{\cot(c+dx)}{ad(a+b\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((a^2 - b^2)*x)/(a^2 + b^2)^2) - (2*b*Log[Sin[c + d*x]])/(a^3*d) + (2*b^3
*(2*a^2 + b^2)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/(a^3*(a^2 + b^2)^2*d)
- (b*(a^2 + 2*b^2))/(a^2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])) - Cot[c + d*x]
/(a*d*(a + b*Tan[c + d*x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
```

```
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\cot(c+dx)}{ad(a+b\tan(c+dx))} - \frac{\int \frac{\cot(c+dx)\left(2b+a\tan(c+dx)+2b\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx}{a}$$

$$= -\frac{b(a^2+2b^2)}{a^2\left(a^2+b^2\right)d(a+b\tan(c+dx))} - \frac{\cot(c+dx)}{ad(a+b\tan(c+dx))} - \frac{\int \frac{\cot(c+dx)\left(2b(a^2\cdots\right.}{\cdots}}{\cdots}$$

$$= -\frac{\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^2} - \frac{b(a^2+2b^2)}{a^2\left(a^2+b^2\right)d(a+b\tan(c+dx))} - \frac{\cot(c+dx)}{ad(a+b\tan(c+dx))} - \frac{(\cdots}{\cdots}$$

$$= -\frac{\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^2} - \frac{2b\log(\sin(c+dx))}{a^3d} + \frac{2b^3(2a^2+b^2)\log(a\cos(c+dx)+b\sin(\cdots}{a^3\left(a^2+b^2\right)^2d}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.78, size = 136, normalized size = 0.91

$$-\frac{\frac{\cot(c+dx)}{a^2}-\frac{b^4}{a^3(a^2+b^2)(b+a\cot(c+dx))}+\frac{i\log(i-\cot(c+dx))}{2(a-ib)^2}-\frac{i\log(i+\cot(c+dx))}{2(a+ib)^2}-\frac{2b^3(2a^2+b^2)\log(b+a\cot(c+dx))}{a^3(a^2+b^2)^2}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -((Cot[c + d*x]/a^2 - b^4/(a^3*(a^2 + b^2)*(b + a*Cot[c + d*x])) + ((I/2)*L
og[I - Cot[c + d*x]])/(a - I*b)^2 - ((I/2)*Log[I + Cot[c + d*x]])/(a + I*b)
^2 - (2*b^3*(2*a^2 + b^2)*Log[b + a*Cot[c + d*x]])/(a^3*(a^2 + b^2)^2))/d)
```

Maple [A]

time = 0.30, size = 140, normalized size = 0.93

method	result
derivativedivides	$\frac{-\frac{1}{a^2\tan(dx+c)}-\frac{2b\ln(\tan(dx+c))}{a^3}-\frac{b^3}{(a^2+b^2)a^2(a+b\tan(dx+c))}+\frac{2b^3(2a^2+b^2)\ln(a+b\tan(dx+c))}{(a^2+b^2)^2a^3}+\frac{ab\ln(1+\tan^2(dx+c))+(-a^2+}{(a^2+b^2)}}{d}$
default	$\frac{-\frac{1}{a^2\tan(dx+c)}-\frac{2b\ln(\tan(dx+c))}{a^3}-\frac{b^3}{(a^2+b^2)a^2(a+b\tan(dx+c))}+\frac{2b^3(2a^2+b^2)\ln(a+b\tan(dx+c))}{(a^2+b^2)^2a^3}+\frac{ab\ln(1+\tan^2(dx+c))+(-a^2+}{(a^2+b^2)}}{d}$
norman	$\frac{\frac{(-a^2b^2-2b^4)\tan(dx+c)}{a^2bd(a^2+b^2)}-\frac{1}{da}-\frac{b(a^2-b^2)x(\tan^2(dx+c))}{a^4+2a^2b^2+b^4}-\frac{(a^2-b^2)ax\tan(dx+c)}{a^4+2a^2b^2+b^4}}{\tan(dx+c)(a+b\tan(dx+c))}+\frac{ab\ln(1+\tan^2(dx+c))}{d(a^4+2a^2b^2+b^4)}-\frac{2b\ln(\tan(dx+}{a^3d}$
risch	$\frac{x}{2iab-a^2+b^2}+\frac{4ibx}{a^3}+\frac{4ibc}{a^3d}-\frac{8ib^3x}{a(a^4+2a^2b^2+b^4)}-\frac{8ib^3c}{ad(a^4+2a^2b^2+b^4)}-\frac{4ib^5x}{a^3(a^4+2a^2b^2+b^4)}-\frac{4ib^5c}{a^3d(a^4+2a^2b^2+b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a^2/tan(d*x+c)-2/a^3*b*ln(tan(d*x+c))-b^3/(a^2+b^2)/a^2/(a+b*tan(d*
x+c))+2*b^3*(2*a^2+b^2)/(a^2+b^2)^2/a^3*ln(a+b*tan(d*x+c))+1/(a^2+b^2)^2*(a
*b*ln(1+tan(d*x+c)^2)+(-a^2+b^2)*arctan(tan(d*x+c))))
```

Maxima [A]

time = 0.53, size = 200, normalized size = 1.33

$$\frac{\frac{ab\log\left(\tan(dx+c)^2+1\right)}{a^4+2a^2b^2+b^4}-\frac{(a^2-b^2)(dx+c)}{a^4+2a^2b^2+b^4}+\frac{2(2a^2b^3+b^5)\log(b\tan(dx+c)+a)}{a^7+2a^5b^2+a^3b^4}-\frac{a^3+ab^2+(a^2b+2b^3)\tan(dx+c)}{(a^4b+a^2b^3)\tan(dx+c)^2+(a^5+a^3b^2)\tan(dx+c)}-\frac{2b\log(\tan(dx+c))}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (a*b*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - (a^2 - b^2)*(d*x + c
)/(a^4 + 2*a^2*b^2 + b^4) + 2*(2*a^2*b^3 + b^5)*log(b*tan(d*x + c) + a)/(a^
```

```
7 + 2*a^5*b^2 + a^3*b^4) - (a^3 + a*b^2 + (a^2*b + 2*b^3)*tan(d*x + c))/((a
^4*b + a^2*b^3)*tan(d*x + c)^2 + (a^5 + a^3*b^2)*tan(d*x + c)) - 2*b*log(ta
n(d*x + c))/a^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(150) = 300$.
time = 1.25, size = 323, normalized size = 2.15

$$-\frac{a^{6} + 2\,a^{4}b^{2} + a^{2}b^{4} - (a^{2}b^{4} - (a^{5}b - a^{3}b^{3})dx)\tan\left(dx + c\right)^{2} + \left((a^{4}b^{2} + 2\,a^{2}b^{4} + b^{6})\tan\left(dx + c\right)^{2} + (a^{5}b + 2\,a^{3}b^{3} + ab^{5})\tan\left(dx + c\right)\right)\log\left(\frac{\tan(dx+c)^{2}}{\tan(dx+c)^{2}+1}\right) - \left((2\,a^{2}b^{4} + b^{6})\tan\left(dx + c\right)^{2} + (2\,a^{3}b^{3} + ab^{5})\tan\left(dx + c\right)\right)\log\left(\frac{b^{2}\tan(dx+c)^{2}+2\,ab\tan(dx+c)+a^{2}}{\tan(dx+c)^{2}+1}\right) + (a^{5}b + 2\,a^{3}b^{3} + 2\,ab^{5} + (a^{6} - a^{4}b^{2})dx)\tan\left(dx + c\right)}{(a^{7}b + 2\,a^{5}b^{3} + a^{3}b^{5})d\tan\left(dx + c\right)^{2} + (a^{8} + 2\,a^{6}b^{2} + a^{4}b^{4})d\tan\left(dx + c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(a^6 + 2*a^4*b^2 + a^2*b^4 - (a^2*b^4 - (a^5*b - a^3*b^3)*d*x)*tan(d*x + c
)^2 + ((a^4*b^2 + 2*a^2*b^4 + b^6)*tan(d*x + c)^2 + (a^5*b + 2*a^3*b^3 + a*
b^5)*tan(d*x + c))*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1)) - ((2*a^2*b^4 +
 b^6)*tan(d*x + c)^2 + (2*a^3*b^3 + a*b^5)*tan(d*x + c))*log((b^2*tan(d*x +
 c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) + (a^5*b + 2*a^3*b^
3 + 2*a*b^5 + (a^6 - a^4*b^2)*d*x)*tan(d*x + c))/((a^7*b + 2*a^5*b^3 + a^3*
b^5)*d*tan(d*x + c)^2 + (a^8 + 2*a^6*b^2 + a^4*b^4)*d*tan(d*x + c))
```

Sympy [C] Result contains complex when optimal does not.
time = 1.97, size = 4070, normalized size = 27.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(c, 0) & Eq(d, 0)), ((x + 1/(d*ta
n(c + d*x)) - 1/(3*d*tan(c + d*x)**3))/b**2, Eq(a, 0)), (9*d*x*tan(c + d*x)
**3/(4*b**2*d*tan(c + d*x)**3 - 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c
 + d*x)) - 18*I*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**3 - 8*I*b**2*d*
tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) - 9*d*x*tan(c + d*x)/(4*b**2*d*tan
(c + d*x)**3 - 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) - 4*I*lo
g(tan(c + d*x)**2 + 1)*tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**3 - 8*I*b**2
*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) - 8*log(tan(c + d*x)**2 + 1)*ta
n(c + d*x)**2/(4*b**2*d*tan(c + d*x)**3 - 8*I*b**2*d*tan(c + d*x)**2 - 4*b*
*2*d*tan(c + d*x)) + 4*I*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(4*b**2*d*ta
n(c + d*x)**3 - 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) + 8*I*l
og(tan(c + d*x))*tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**3 - 8*I*b**2*d*tan
(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) + 16*log(tan(c + d*x))*tan(c + d*x)**
2/(4*b**2*d*tan(c + d*x)**3 - 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c +
 d*x)) - 8*I*log(tan(c + d*x))*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**3 - 8*I
*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) + 9*tan(c + d*x)**2/(4*b**
2*d*tan(c + d*x)**3 - 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) -
```

14*I*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**3 - 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) - 4/(4*b**2*d*tan(c + d*x)**3 - 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)), Eq(a, -I*b)), (9*d*x*tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**3 + 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) + 18*I*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**3 + 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) - 9*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**3 + 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) + 4*I*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**3 + 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) - 8*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**3 + 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) - 4*I*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**3 + 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) - 8*I*log(tan(c + d*x))*tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**3 + 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) + 16*log(tan(c + d*x))*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**3 + 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) + 8*I*log(tan(c + d*x))*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**3 + 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) + 9*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**3 + 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) + 14*I*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**3 + 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) - 4/(4*b**2*d*tan(c + d*x)**3 + 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)), Eq(a, I*b)), (zoo*x/a**2, Eq(c, -d*x)), (x*cot(c)**2/(a + b*tan(c))**2, Eq(d, 0)), ((-x - cot(c + d*x)/d)/a**2, Eq(b, 0)), (-a**6*d*x*tan(c + d*x)/(a**8*d*tan(c + d*x) + a**7*b*d*tan(c + d*x)**2 + 2*a**6*b**2*d*tan(c + d*x) + 2*a**5*b**3*d*tan(c + d*x)**2 + a**4*b**4*d*tan(c + d*x) + a**3*b**5*d*tan(c + d*x)**2) - a**6/(a**8*d*tan(c + d*x) + a**7*b*d*tan(c + d*x)**2 + 2*a**6*b**2*d*tan(c + d*x) + 2*a**5*b**3*d*tan(c + d*x)**2 + a**4*b**4*d*tan(c + d*x) + a**3*b**5*d*tan(c + d*x)**2) - a**5*b*d*x*tan(c + d*x)**2/(a**8*d*tan(c + d*x) + a**7*b*d*tan(c + d*x)**2 + 2*a**6*b**2*d*tan(c + d*x) + 2*a**5*b**3*d*tan(c + d*x)**2 + a**4*b**4*d*tan(c + d*x) + a**3*b**5*d*tan(c + d*x)**2) + a**5*b*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(a**8*d*tan(c + d*x) + a**7*b*d*tan(c + d*x)**2 + 2*a**6*b**2*d*tan(c + d*x) + 2*a**5*b**3*d*tan(c + d*x)**2 + a**4*b**4*d*tan(c + d*x) + a**3*b**5*d*tan(c + d*x)**2) - 2*a**5*b*log(tan(c + d*x))*tan(c + d*x)/(a**8*d*tan(c + d*x) + a**7*b*d*tan(c + d*x)**2 + 2*a**6*b**2*d*tan(c + d*x) + 2*a**5*b**3*d*tan(c + d*x)**2 + a**4*b**4*d*tan(c + d*x) + a**3*b**5*d*tan(c + d*x)**2) - a**5*b*tan(c + d*x)/(a**8*d*tan(c + d*x) + a**7*b*d*tan(c + d*x)**2 + 2*a**6*b**2*d*tan(c + d*x) + 2*a**5*b**3*d*tan(c + d*x)**2 + a**4*b**4*d*tan(c + d*x) + a**3*b**5*d*tan(c + d*x)**2) + a**4*b**2*d*x*tan(c + d*x)/(a**8*d*tan(c + d*x) + a**7*b*d*tan(c + d*x)**2 + 2*a**6*b**2*d*tan(c + d*x) + 2*a**5*b**3*d*tan(c + d*x)**2 + a**4*b**4*d*tan(c + d*x) + a**3*b**5*d*tan(c + d*x)**2) + a**4*b**2*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(a**8*d*tan(c + d*x) + a**7*b*d*tan(c + d*x)**2 + 2*a**6*b**2*d*tan(c + d*x) + 2*a**5*b**3*d*tan(c + d*x)**2 + a**4*b**4*d*tan(c + d*x) + a**3*b**5*d*tan(c + d*x)**2) - 2*a**4*b**2*log(tan(c + d*x))*tan(c + d*x)**2/(a**8*d*tan(c + d*x) + a**7*b*d*tan(c + d*x)**2 + 2*a**6*b**2*d*tan(c + d*x) + 2*a**5*b**

```
3*d*tan(c + d*x)**2 + a**4*b**4*d*tan(c + d*x) + a**3*b**5*d*tan(c + d*x)**
2) - 2*a**4*b**2/(a**8*d*tan(c + d*x) + a**7*b*d*tan(c + d*x)**2 + 2*a**6*b
**2*d*tan(c + d*x) + 2*a**5*b**3*d*tan(c + d*x)**2 + a**4*b**4*d*tan(c + d*
x) + a**3*b**5*d*tan(c + d*x)**2) + a**3*b**3*d*x*tan(c + d*x)**2/(a**8*d*t
an(c + d*x) + a**7*b*d*tan(c + d*x)**2 + 2*a**6...
```

Giac [A]
time = 0.88, size = 235, normalized size = 1.57

$$\frac{\frac{ab\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{(a^2-b^2)(dx+c)}{a^4+2\,a^2b^2+b^4}+\frac{2\,(2\,a^2b^4+b^6)\log(|b\tan(dx+c)+a|)}{a^7b+2\,a^5b^3+a^3b^5}+\frac{a^3b^2\tan(dx+c)^2-3\,a^2b^3\tan(dx+c)-2\,b^5\tan(dx+c)-a^5-2\,a^3b^2-ab^4}{(a^6+2\,a^4b^2+a^2b^4)\left(b\tan(dx+c)^2+a\tan(dx+c)\right)}-\frac{2\,b\log(|\tan(dx+c)|)}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (a*b*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - (a^2 - b^2)*(d*x + c
)/(a^4 + 2*a^2*b^2 + b^4) + 2*(2*a^2*b^4 + b^6)*log(abs(b*tan(d*x + c) + a)
)/(a^7*b + 2*a^5*b^3 + a^3*b^5) + (a^3*b^2*tan(d*x + c)^2 - 3*a^2*b^3*tan(d
*x + c) - 2*b^5*tan(d*x + c) - a^5 - 2*a^3*b^2 - a*b^4)/((a^6 + 2*a^4*b^2 +
 a^2*b^4)*(b*tan(d*x + c)^2 + a*tan(d*x + c))) - 2*b*log(abs(tan(d*x + c)))
/a^3)/d
```

Mupad [B]
time = 4.35, size = 183, normalized size = 1.22

$$\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)}{2\,d\,\left(-a^2\,1\mathrm{i}+2\,a\,b+b^2\,1\mathrm{i}\right)}-\frac{\frac{1}{a}+\frac{\tan(c+d\,x)\,\left(a^2\,b+2\,b^3\right)}{a^2\,\left(a^2+b^2\right)}}{d\,\left(b\tan\left(c+d\,x\right)^2+a\tan\left(c+d\,x\right)\right)}-\frac{2\,b\,\ln\left(\tan(c+d\,x)\right)}{a^3\,d}+\frac{2\,b^3\,\ln\left(a+b\tan(c+d\,x)\right)\,\left(2\,a^2+b^2\right)}{a^3\,d\,{\left(a^2+b^2\right)}^2}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(-a^2+a\,b\,2\mathrm{i}+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + b*tan(c + d*x))^2,x)
```

```
[Out] (log(tan(c + d*x) + 1i)*1i)/(2*d*(a*b*2i - a^2 + b^2)) + log(tan(c + d*x) -
 1i)/(2*d*(2*a*b - a^2*1i + b^2*1i)) - (1/a + (tan(c + d*x)*(a^2*b + 2*b^3)
)/(a^2*(a^2 + b^2)))/(d*(a*tan(c + d*x) + b*tan(c + d*x)^2)) - (2*b*log(tan
(c + d*x)))/(a^3*d) + (2*b^3*log(a + b*tan(c + d*x))*(2*a^2 + b^2))/(a^3*d*
(a^2 + b^2)^2)
```

3.476 $\int \frac{\cot^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=189

$$\frac{2abx}{(a^2+b^2)^2}-\frac{(a^2-3b^2)\log(\sin(c+dx))}{a^4d}-\frac{b^4(5a^2+3b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{a^4(a^2+b^2)^2d}+\frac{b^2(2a^2+}{a^3(a^2+b^2)d(a+}$$

```
[Out] 2*a*b*x/(a^2+b^2)^2-(a^2-3*b^2)*ln(sin(d*x+c))/a^4/d-b^4*(5*a^2+3*b^2)*ln(a
*cos(d*x+c)+b*sin(d*x+c))/a^4/(a^2+b^2)^2/d+b^2*(2*a^2+3*b^2)/a^3/(a^2+b^2)
/d/(a+b*tan(d*x+c))+3/2*b*cot(d*x+c)/a^2/d/(a+b*tan(d*x+c))-1/2*cot(d*x+c)^
2/a/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.39, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3650, 3730, 3731, 3732, 3611, 3556}

$$\frac{2abx}{(a^2+b^2)^2}+\frac{3b\cot(c+dx)}{2a^2d(a+b\tan(c+dx))}-\frac{(a^2-3b^2)\log(\sin(c+dx))}{a^4d}-\frac{b^4(5a^2+3b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{a^4d(a^2+b^2)^2}+\frac{b^2(2a^2+3b^2)}{a^3d(a^2+b^2)(a+b\tan(c+dx))}-\frac{\cot^2(c+dx)}{2ad(a+b\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (2*a*b*x)/(a^2 + b^2)^2 - ((a^2 - 3*b^2)*Log[Sin[c + d*x]])/(a^4*d) - (b^4*
(5*a^2 + 3*b^2)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/(a^4*(a^2 + b^2)^2*d)
 + (b^2*(2*a^2 + 3*b^2))/(a^3*(a^2 + b^2)*d*(a + b*Tan[c + d*x])) + (3*b*Co
t[c + d*x])/(2*a^2*d*(a + b*Tan[c + d*x])) - Cot[c + d*x]^2/(2*a*d*(a + b*T
an[c + d*x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
```

```
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3731

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)
*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp
[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - a*C*(b*c*(m + 1) + a*d*(n
+ 1)) - (m + 1)*(b*c - a*d)*(A*b - b*C)*Tan[e + f*x] - d*(A*b^2 + a^2*C)*(m
 + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m,
 -1] &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\cot^2(c+dx)}{2ad(a+b\tan(c+dx))} - \frac{\int \frac{\cot^2(c+dx)(3b+2a\tan(c+dx)+3b\tan^2(c+dx))}{(a+b\tan(c+dx))^2}\,dx}{2a}$$

$$= \frac{3b\cot(c+dx)}{2a^2d(a+b\tan(c+dx))} - \frac{\cot^2(c+dx)}{2ad(a+b\tan(c+dx))} + \frac{\int \frac{\cot(c+dx)(-2(a^2-3b^2)+6b^2\tan}{(a+b\tan(c+dx))^2}}{2a^2}$$

$$= \frac{b^2(2a^2+3b^2)}{a^3(a^2+b^2)\,d(a+b\tan(c+dx))} + \frac{3b\cot(c+dx)}{2a^2d(a+b\tan(c+dx))} - \frac{\cot^2(c+dx}{2ad(a+b\tan(c}$$

$$= \frac{2abx}{(a^2+b^2)^2} + \frac{b^2(2a^2+3b^2)}{a^3(a^2+b^2)\,d(a+b\tan(c+dx))} + \frac{3b\cot(c+dx)}{2a^2d(a+b\tan(c+dx))} - \frac{}{2ad}$$

$$= \frac{2abx}{(a^2+b^2)^2} - \frac{(a^2-3b^2)\log(\sin(c+dx))}{a^4d} - \frac{b^4(5a^2+3b^2)\log(a\cos(c+dx)+b\,s}{a^4(a^2+b^2)^2\,d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.28, size = 146, normalized size = 0.77

$$-\frac{-\frac{4b\cot(c+dx)}{a^3}+\frac{\cot^2(c+dx)}{a^2}+\frac{2b^5}{a^4(a^2+b^2)(b+a\cot(c+dx))}-\frac{\log(i-\cot(c+dx))}{(a-ib)^2}-\frac{\log(i+\cot(c+dx))}{(a+ib)^2}+\frac{2b^4(5a^2+3b^2)\log(b+a\cot(c+dx))}{a^4(a^2+b^2)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -1/2*((-4*b*Cot[c + d*x])/a^3 + Cot[c + d*x]^2/a^2 + (2*b^5)/(a^4*(a^2 + b^
2)*(b + a*Cot[c + d*x])) - Log[I - Cot[c + d*x]]/(a - I*b)^2 - Log[I + Cot[
c + d*x]]/(a + I*b)^2 + (2*b^4*(5*a^2 + 3*b^2)*Log[b + a*Cot[c + d*x]])/(a^
4*(a^2 + b^2)^2))/d
```

Maple [A]
time = 0.33, size = 166, normalized size = 0.88

method	result
derivativedivides	$\frac{-\frac{1}{2a^2\tan(dx+c)^2}+\frac{(-a^2+3b^2)\ln(\tan(dx+c))}{a^4}+\frac{2b}{a^3\tan(dx+c)}+\frac{b^4}{(a^2+b^2)a^3(a+b\tan(dx+c))}-\frac{b^4(5a^2+3b^2)\ln(a+b\tan(dx+c))}{(a^2+b^2)^2a^4}+\frac{(a}{}}{d}$
default	$\frac{-\frac{1}{2a^2\tan(dx+c)^2}+\frac{(-a^2+3b^2)\ln(\tan(dx+c))}{a^4}+\frac{2b}{a^3\tan(dx+c)}+\frac{b^4}{(a^2+b^2)a^3(a+b\tan(dx+c))}-\frac{b^4(5a^2+3b^2)\ln(a+b\tan(dx+c))}{(a^2+b^2)^2a^4}+\frac{(a}{}}{d}$
norman	$\frac{\frac{(-2a^2b^2-3b^4)b(\tan^3(dx+c))}{d\,a^4(a^2+b^2)}-\frac{1}{2da}+\frac{3b\tan(dx+c)}{2a^2d}+\frac{2a^2bx(\tan^2(dx+c))}{a^4+2a^2b^2+b^4}+\frac{2b^2ax(\tan^3(dx+c))}{a^4+2a^2b^2+b^4}}{\tan(dx+c)^2(a+b\tan(dx+c))}-\frac{(a^2-3b^2)\ln(\tan(dx+c))}{a^4d}+$

risch	$\frac{ix}{2iab-a^2+b^2}+\frac{10ib^4x}{(a^4+2a^2b^2+b^4)a^2}+\frac{10ib^4c}{(a^4+2a^2b^2+b^4)a^2d}+\frac{6ib^6x}{(a^4+2a^2b^2+b^4)a^4}+\frac{6ib^6c}{(a^4+2a^2b^2+b^4)a^4d}+\frac{2ix}{a^2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/a^2/tan(d*x+c)^2+(-a^2+3*b^2)/a^4*ln(tan(d*x+c))+2/a^3*b/tan(d*x+
c)+b^4/(a^2+b^2)/a^3/(a+b*tan(d*x+c))-b^4*(5*a^2+3*b^2)/(a^2+b^2)^2/a^4*ln(
a+b*tan(d*x+c))+1/(a^2+b^2)^2*(1/2*(a^2-b^2)*ln(1+tan(d*x+c)^2)+2*a*b*arcta
n(tan(d*x+c))))
```

Maxima [A]
time = 0.50, size = 240, normalized size = 1.27

$$\frac{\frac{4(dx+c)ab}{a^4+2a^2b^2+b^4}-\frac{2(5a^2b^4+3b^6)\log(b\tan(dx+c)+a)}{a^8+2a^6b^2+a^4b^4}+\frac{(a^2-b^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2a^2b^2+b^4}-\frac{a^4+a^2b^2-2(2a^2b^2+3b^4)\tan(dx+c)^2-3(a^3b+ab^3)\tan(dx+c)}{(a^5b+a^3b^3)\tan(dx+c)^3+(a^6+a^4b^2)\tan(dx+c)^2}-\frac{2(a^2-3b^2)\log(\tan(dx+c))}{a^4}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(4*(d*x + c)*a*b/(a^4 + 2*a^2*b^2 + b^4) - 2*(5*a^2*b^4 + 3*b^6)*log(b*
tan(d*x + c) + a)/(a^8 + 2*a^6*b^2 + a^4*b^4) + (a^2 - b^2)*log(tan(d*x + c
)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - (a^4 + a^2*b^2 - 2*(2*a^2*b^2 + 3*b^4)*t
an(d*x + c)^2 - 3*(a^3*b + a*b^3)*tan(d*x + c))/((a^5*b + a^3*b^3)*tan(d*x
+ c)^3 + (a^6 + a^4*b^2)*tan(d*x + c)^2) - 2*(a^2 - 3*b^2)*log(tan(d*x + c)
)/a^4)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(185) = 370$.
time = 1.18, size = 386, normalized size = 2.04

$$-\frac{a^7+2a^5b^2+a^3b^4-(4a^5b^2dx-a^6b-2a^4b^3-3a^2b^5)\tan(dx+c)^3-(4a^6bdx-a^7+2a^5b^2+7a^3b^4+6ab^6)\tan(dx+c)^2+((a^6b-a^4b^3-5a^2b^5-3b^7)\tan(dx+c)^3+(a^7-a^5b^2-5a^3b^4-3ab^6)\tan(dx+c)^2)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)+((5a^2b^5+3b^7)\tan(dx+c)^3+(5a^3b^4+3ab^6)\tan(dx+c)^2)\log\left(\frac{b^2\tan(dx+c)^2+2ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)-3(a^6b+2a^4b^3+a^2b^5)\tan(dx+c)}{2((a^8b+2a^6b^3+a^4b^5)d\tan(dx+c)^3+(a^9+2a^7b^2+a^5b^4)d\tan(dx+c)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^7 + 2*a^5*b^2 + a^3*b^4 - (4*a^5*b^2*d*x - a^6*b - 2*a^4*b^3 - 3*a^
2*b^5)*tan(d*x + c)^3 - (4*a^6*b*d*x - a^7 + 2*a^5*b^2 + 7*a^3*b^4 + 6*a*b^
6)*tan(d*x + c)^2 + ((a^6*b - a^4*b^3 - 5*a^2*b^5 - 3*b^7)*tan(d*x + c)^3 +
 (a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^6)*tan(d*x + c)^2)*log(tan(d*x + c)^2/(
tan(d*x + c)^2 + 1)) + ((5*a^2*b^5 + 3*b^7)*tan(d*x + c)^3 + (5*a^3*b^4 + 3
*a*b^6)*tan(d*x + c)^2)*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)
/(tan(d*x + c)^2 + 1)) - 3*(a^6*b + 2*a^4*b^3 + a^2*b^5)*tan(d*x + c))/((a^
8*b + 2*a^6*b^3 + a^4*b^5)*d*tan(d*x + c)^3 + (a^9 + 2*a^7*b^2 + a^5*b^4)*d
*tan(d*x + c)^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.90, size = 5222, normalized size = 27.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(c, 0) & Eq(d, 0)), ((log(tan(c +
 d*x)**2 + 1)/(2*d) - log(tan(c + d*x))/d - 1/(2*d*tan(c + d*x)**2))/a**2,
Eq(b, 0)), ((-log(tan(c + d*x)**2 + 1)/(2*d) + log(tan(c + d*x))/d + 1/(2*d
*tan(c + d*x)**2) - 1/(4*d*tan(c + d*x)**4))/b**2, Eq(a, 0)), (-15*I*d*x*ta
n(c + d*x)**4/(4*b**2*d*tan(c + d*x)**4 - 8*I*b**2*d*tan(c + d*x)**3 - 4*b*
*2*d*tan(c + d*x)**2) - 30*d*x*tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**4 -
8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) + 15*I*d*x*tan(c + d
*x)**2/(4*b**2*d*tan(c + d*x)**4 - 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*ta
n(c + d*x)**2) - 8*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**4/(4*b**2*d*tan(c
 + d*x)**4 - 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) + 16*I*
log(tan(c + d*x)**2 + 1)*tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**4 - 8*I*b*
*2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) + 8*log(tan(c + d*x)**2 +
1)*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**4 - 8*I*b**2*d*tan(c + d*x)**3 -
 4*b**2*d*tan(c + d*x)**2) + 16*log(tan(c + d*x))*tan(c + d*x)**4/(4*b**2*d
*tan(c + d*x)**4 - 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) -
 32*I*log(tan(c + d*x))*tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**4 - 8*I*b**
2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) - 16*log(tan(c + d*x))*tan(
c + d*x)**2/(4*b**2*d*tan(c + d*x)**4 - 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2
*d*tan(c + d*x)**2) - 15*I*tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**4 - 8*I*
b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) - 22*tan(c + d*x)**2/(4*
b**2*d*tan(c + d*x)**4 - 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)
**2) + 4*I*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**4 - 8*I*b**2*d*tan(c + d*x)
**3 - 4*b**2*d*tan(c + d*x)**2) - 2/(4*b**2*d*tan(c + d*x)**4 - 8*I*b**2*d*
tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2), Eq(a, -I*b)), (15*I*d*x*tan(c
+ d*x)**4/(4*b**2*d*tan(c + d*x)**4 + 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d
*tan(c + d*x)**2) - 30*d*x*tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**4 + 8*I*
b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) - 15*I*d*x*tan(c + d*x)*
*2/(4*b**2*d*tan(c + d*x)**4 + 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c
+ d*x)**2) - 8*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**4/(4*b**2*d*tan(c + d
*x)**4 + 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) - 16*I*log(
tan(c + d*x)**2 + 1)*tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**4 + 8*I*b**2*d
*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) + 8*log(tan(c + d*x)**2 + 1)*t
an(c + d*x)**2/(4*b**2*d*tan(c + d*x)**4 + 8*I*b**2*d*tan(c + d*x)**3 - 4*b
**2*d*tan(c + d*x)**2) + 16*log(tan(c + d*x))*tan(c + d*x)**4/(4*b**2*d*tan
(c + d*x)**4 + 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) + 32*
I*log(tan(c + d*x))*tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**4 + 8*I*b**2*d*
tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) - 16*log(tan(c + d*x))*tan(c +
```

```
d*x)**2/(4*b**2*d*tan(c + d*x)**4 + 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*t
an(c + d*x)**2) + 15*I*tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**4 + 8*I*b**2
*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) - 22*tan(c + d*x)**2/(4*b**2
*d*tan(c + d*x)**4 + 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2)
 - 4*I*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**4 + 8*I*b**2*d*tan(c + d*x)**3
- 4*b**2*d*tan(c + d*x)**2) - 2/(4*b**2*d*tan(c + d*x)**4 + 8*I*b**2*d*tan(
c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2), Eq(a, I*b)), (zoo*x/a**2, Eq(c, -d
*x)), (x*cot(c)**3/(a + b*tan(c))**2, Eq(d, 0)), (a**7*log(tan(c + d*x)**2
+ 1)*tan(c + d*x)**2/(2*a**9*d*tan(c + d*x)**2 + 2*a**8*b*d*tan(c + d*x)**3
 + 4*a**7*b**2*d*tan(c + d*x)**2 + 4*a**6*b**3*d*tan(c + d*x)**3 + 2*a**5*b
**4*d*tan(c + d*x)**2 + 2*a**4*b**5*d*tan(c + d*x)**3) - 2*a**7*log(tan(c +
 d*x))*tan(c + d*x)**2/(2*a**9*d*tan(c + d*x)**2 + 2*a**8*b*d*tan(c + d*x)*
*3 + 4*a**7*b**2*d*tan(c + d*x)**2 + 4*a**6*b**3*d*tan(c + d*x)**3 + 2*a**5
*b**4*d*tan(c + d*x)**2 + 2*a**4*b**5*d*tan(c + d*x)**3) - a**7/(2*a**9*d*t
an(c + d*x)**2 + 2*a**8*b*d*tan(c + d*x)**3 + 4*a**7*b**2*d*tan(c + d*x)**2
 + 4*a**6*b**3*d*tan(c + d*x)**3 + 2*a**5*b**4*d*tan(c + d*x)**2 + 2*a**4*b
**5*d*tan(c + d*x)**3) + 4*a**6*b*d*x*tan(c + d*x)**2/(2*a**9*d*tan(c + d*x
)**2 + 2*a**8*b*d*tan(c + d*x)**3 + 4*a**7*b**2*d*tan(c + d*x)**2 + 4*a**6*
b**3*d*tan(c + d*x)**3 + 2*a**5*b**4*d*tan(c + d*x)**2 + 2*a**4*b**5*d*tan(
c + d*x)**3) + a**6*b*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**3/(2*a**9*d*ta
n(c + d*x)**2 + 2*a**8*b*d*tan(c + d*x)**3 + 4*a**7*b**2*d*tan(c + d*x)**2
+ 4*a**6*b**3*d*tan(c + d*x)**3 + 2*a**5*b**4*d*tan(c + d*x)**2 + 2*a**4*b*
*5*d*tan(c + d*x)**3) - 2*a**6*b*log(tan(c + d*x))*tan(c + d*x)**3/(2*a**9*
d*tan(c + d*x)**2 + 2*a**8*b*d*tan(c + d*x)**3 + 4*a**7*b**2*d*tan(c + d*x)
**2 + 4*a**6*b**3*d*tan(c + d*x)**3 + 2*a**5*b**4*d*tan(c + d*x)**2 + 2*a**
4*b**5*d*tan(c + d*x)**3) + 3*a**6*b*tan(c + d*x)/(2*a**9*d*tan(c + d*x)**2
 + 2*a**8*b*d*tan(c + d*x)**3 + 4*a**7*b**2*d*tan(c + d*x)**2 + 4*a**6*b**3
*d*tan(c + d*x)**3 + 2*a**5*b**4*d*tan(c + d*x)**2 + 2*a**4*b**5*d*tan(c +
d*x)**3) + 4*a**5*b**2*d*x*tan(c + d*x)**3/(2*a**9*d*tan(c + d*x)**2 + 2*a*
*8*b*d*tan(c + d*x)**3 + 4*a**7*b**2*d*tan(c + ...
```

Giac [A]

time = 0.94, size = 272, normalized size = 1.44

$$\frac{\frac{4\,(dx+c)ab}{a^4+2\,a^2b^2+b^4}+\frac{(a^2-b^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\,(5\,a^2b^5+3\,b^7)\log(|b\tan(dx+c)+a|)}{a^8b+2\,a^6b^3+a^4b^5}+\frac{2\,(5\,a^2b^5\tan(dx+c)+3\,b^7\tan(dx+c)+6\,a^3b^4+4\,ab^6)}{(a^8+2\,a^6b^2+a^4b^4)(b\tan(dx+c)+a)}-\frac{2\,(a^2-3\,b^2)\log(|\tan(dx+c)|)}{a^4}+\frac{3\,a^2\tan(dx+c)^2-9\,b^2\tan(dx+c)^2+4\,ab\tan(dx+c)-a^2}{a^4\tan(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*(d*x + c)*a*b/(a^4 + 2*a^2*b^2 + b^4) + (a^2 - b^2)*log(tan(d*x + c)
^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*(5*a^2*b^5 + 3*b^7)*log(abs(b*tan(d*x +
 c) + a))/(a^8*b + 2*a^6*b^3 + a^4*b^5) + 2*(5*a^2*b^5*tan(d*x + c) + 3*b^7
*tan(d*x + c) + 6*a^3*b^4 + 4*a*b^6)/((a^8 + 2*a^6*b^2 + a^4*b^4)*(b*tan(d*
x + c) + a)) - 2*(a^2 - 3*b^2)*log(abs(tan(d*x + c)))/a^4 + (3*a^2*tan(d*x
+ c)^2 - 9*b^2*tan(d*x + c)^2 + 4*a*b*tan(d*x + c) - a^2)/(a^4*tan(d*x + c)
^2))/d
```

Mupad [B]
time = 4.46, size = 222, normalized size = 1.17

$$\frac{\frac{3\,b\tan(c+d\,x)}{2\,a^2}-\frac{1}{2\,a}+\frac{\tan(c+d\,x)^2\,\left(2\,a^2\,b^2+3\,b^4\right)}{a^3\,\left(a^2+b^2\right)}}{d\,\left(b\tan\left(c+d\,x\right)^3+a\tan\left(c+d\,x\right)^2\right)}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)}{2\,d\,\left(a^2+a\,b\,2\mathrm{i}-b^2\right)}-\frac{\ln\left(\tan(c+d\,x)\right)\,\left(a^2-3\,b^2\right)}{a^4\,d}-\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(5\,a^2\,b^4+3\,b^6\right)}{d\,\left(a^8+2\,a^6\,b^2+a^4\,b^4\right)}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(a^2\,1\mathrm{i}+2\,a\,b-b^2\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + b*tan(c + d*x))^2,x)
```

```
[Out] ((3*b*tan(c + d*x))/(2*a^2) - 1/(2*a) + (tan(c + d*x)^2*(3*b^4 + 2*a^2*b^2)
)/(a^3*(a^2 + b^2)))/(d*(a*tan(c + d*x)^2 + b*tan(c + d*x)^3)) + log(tan(c
+ d*x) - 1i)/(2*d*(a*b*2i + a^2 - b^2)) + (log(tan(c + d*x) + 1i)*1i)/(2*d*
(2*a*b + a^2*1i - b^2*1i)) - (log(tan(c + d*x))*(a^2 - 3*b^2))/(a^4*d) - (l
og(a + b*tan(c + d*x))*(3*b^6 + 5*a^2*b^4))/(d*(a^8 + a^4*b^4 + 2*a^6*b^2))
```

3.477 $\int \frac{\tan^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=283

$$-\frac{a(a^2-3b^2)\,x}{(a^2+b^2)^3}-\frac{b(3a^2-b^2)\log(\cos(c+dx))}{(a^2+b^2)^3\,d}+\frac{a^4(6a^4+17a^2b^2+15b^4)\log(a+b\tan(c+dx))}{b^5\,(a^2+b^2)^3\,d}-\frac{a(6a^4+11$$

```
[Out] -a*(a^2-3*b^2)*x/(a^2+b^2)^3-b*(3*a^2-b^2)*ln(cos(d*x+c))/(a^2+b^2)^3/d+a^4
*(6*a^4+17*a^2*b^2+15*b^4)*ln(a+b*tan(d*x+c))/b^5/(a^2+b^2)^3/d-a*(6*a^4+11
*a^2*b^2+3*b^4)*tan(d*x+c)/b^4/(a^2+b^2)^2/d+1/2*(6*a^4+11*a^2*b^2+b^4)*tan
(d*x+c)^2/b^3/(a^2+b^2)^2/d-1/2*a^2*tan(d*x+c)^4/b/(a^2+b^2)/d/(a+b*tan(d*x
+c))^2-2*a^2*(a^2+2*b^2)*tan(d*x+c)^3/b^2/(a^2+b^2)^2/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.56, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3646, 3726, 3728, 3707, 3698, 31, 3556}

$$-\frac{a^2\tan^4(c+dx)}{2bd\,(a^2+b^2)\,(a+b\tan(c+dx))^2}-\frac{2a^2(a^2+2b^2)\tan^3(c+dx)}{b^2d\,(a^2+b^2)^2\,(a+b\tan(c+dx))}-\frac{b(3a^2-b^2)\log(\cos(c+dx))}{d\,(a^2+b^2)^3}-\frac{ax(a^2-3b^2)}{(a^2+b^2)^3}-\frac{a(6a^4+11a^2b^2+3b^4)\tan(c+dx)}{b^4d\,(a^2+b^2)^2}+\frac{a^4(6a^4+17a^2b^2+15b^4)\log(a+b\tan(c+dx))}{b^5d\,(a^2+b^2)^3}+\frac{(6a^4+11a^2b^2+b^4)\tan^2(c+dx)}{2b^3d\,(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -((a*(a^2 - 3*b^2)*x)/(a^2 + b^2)^3) - (b*(3*a^2 - b^2)*Log[Cos[c + d*x]])/
((a^2 + b^2)^3*d) + (a^4*(6*a^4 + 17*a^2*b^2 + 15*b^4)*Log[a + b*Tan[c + d*
x]])/(b^5*(a^2 + b^2)^3*d) - (a*(6*a^4 + 11*a^2*b^2 + 3*b^4)*Tan[c + d*x])/
(b^4*(a^2 + b^2)^2*d) + ((6*a^4 + 11*a^2*b^2 + b^4)*Tan[c + d*x]^2)/(2*b^3*
(a^2 + b^2)^2*d) - (a^2*Tan[c + d*x]^4)/(2*b*(a^2 + b^2)*d*(a + b*Tan[c + d
*x])^2) - (2*a^2*(a^2 + 2*b^2)*Tan[c + d*x]^3)/(b^2*(a^2 + b^2)^2*d*(a + b*
Tan[c + d*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
```

```
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
```

```
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx &= -\frac{a^2\tan^4(c+dx)}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} + \frac{\int \frac{\tan^3(c+dx)\left(4a^2-2ab\tan(c+dx)+2\left(2a^2+b^2\right)\tan^2(\right.}{(a+b\tan(c+dx))^2}}{2b\,(a^2+b^2)} \\
&= -\frac{a^2\tan^4(c+dx)}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} - \frac{2a^2(a^2+2b^2)\tan^3(c+dx)}{b^2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))} + \frac{\int \frac{\mathrm{t}}{}}{} \\
&= \frac{(6a^4+11a^2b^2+b^4)\tan^2(c+dx)}{2b^3\,(a^2+b^2)^2\,d} - \frac{a^2\tan^4(c+dx)}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} - \frac{2a}{b^2\,(a^2} \\
&= -\frac{a(6a^4+11a^2b^2+3b^4)\tan(c+dx)}{b^4\,(a^2+b^2)^2\,d} + \frac{(6a^4+11a^2b^2+b^4)\tan^2(c+dx)}{2b^3\,(a^2+b^2)^2\,d} - \frac{}{2b} \\
&= -\frac{a(a^2-3b^2)\,x}{(a^2+b^2)^3} - \frac{a(6a^4+11a^2b^2+3b^4)\tan(c+dx)}{b^4\,(a^2+b^2)^2\,d} + \frac{(6a^4+11a^2b^2+b^4)\,\mathrm{ta}}{2b^3\,(a^2+b^2)^2} \\
&= -\frac{a(a^2-3b^2)\,x}{(a^2+b^2)^3} - \frac{b(3a^2-b^2)\log(\cos(c+dx))}{(a^2+b^2)^3\,d} - \frac{a(6a^4+11a^2b^2+3b^4)\tan(c}{b^4\,(a^2+b^2)^2\,d} \\
&= -\frac{a(a^2-3b^2)\,x}{(a^2+b^2)^3} - \frac{b(3a^2-b^2)\log(\cos(c+dx))}{(a^2+b^2)^3\,d} + \frac{a^4(6a^4+17a^2b^2+15b^4)\log(}{b^5\,(a^2+b^2)^3}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.81, size = 243, normalized size = 0.86

$$\frac{\frac{ib\log(i-\tan(c+dx))}{(a+ib)^3}-\frac{b\log(i+\tan(c+dx))}{(ia+b)^3}+\frac{2a^4(6a^4+17a^2b^2+15b^4)\log(a+b\tan(c+dx))}{b^4(a^2+b^2)^3}-\frac{a^4(6a^2+5b^2)}{b^4(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{4a\tan^3(c+dx)}{b(a+b\tan(c+dx))^2}+\frac{\tan^4(c+dx)}{(a+b\tan(c+dx))^2}+\frac{4a^3(6a^4+11a^2b^2+4b^4)}{b^4(a^2+b^2)^2(a+b\tan(c+dx))}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^6/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((I*b*Log[I - Tan[c + d*x]])/(a + I*b)^3 - (b*Log[I + Tan[c + d*x]])/(I*a +
 b)^3 + (2*a^4*(6*a^4 + 17*a^2*b^2 + 15*b^4)*Log[a + b*Tan[c + d*x]])/(b^4*
(a^2 + b^2)^3) - (a^4*(6*a^2 + 5*b^2))/(b^4*(a^2 + b^2)*(a + b*Tan[c + d*x]
)^2) - (4*a*Tan[c + d*x]^3)/(b*(a + b*Tan[c + d*x])^2) + Tan[c + d*x]^4/(a
+ b*Tan[c + d*x])^2 + (4*a^3*(6*a^4 + 11*a^2*b^2 + 4*b^4))/(b^4*(a^2 + b^2)
^2*(a + b*Tan[c + d*x])))/(2*b*d)
```

Maple [A]
time = 0.21, size = 203, normalized size = 0.72 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^4*(-1/2*b*tan(d*x+c)^2+3*a*tan(d*x+c))+1/(a^2+b^2)^3*(1/2*(3*a^2*
b-b^3)*ln(1+tan(d*x+c)^2)+(-a^3+3*a*b^2)*arctan(tan(d*x+c)))-1/2/b^5*a^6/(a
^2+b^2)/(a+b*tan(d*x+c))^2+1/b^5*a^4*(6*a^4+17*a^2*b^2+15*b^4)/(a^2+b^2)^3*
ln(a+b*tan(d*x+c))+2/b^5*a^5*(2*a^2+3*b^2)/(a^2+b^2)^2/(a+b*tan(d*x+c)))
```

Maxima [A]
time = 0.51, size = 308, normalized size = 1.09

$$-\frac{\frac{2\,(a^3-3\,ab^2)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{2\,(6\,a^8+17\,a^6b^2+15\,a^4b^4)\log\left(b\tan\left(dx+c\right)+a\right)}{a^6b^5+3\,a^4b^7+3\,a^2b^9+b^{11}}-\frac{(3\,a^2b-b^3)\log\left(\tan\left(dx+c\right)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{7\,a^8+11\,a^6b^2+4\,(2\,a^7b+3\,a^5b^3)\tan\left(dx+c\right)}{a^6b^5+2\,a^4b^7+a^2b^9+(a^4b^7+2\,a^2b^9+b^{11})\tan\left(dx+c\right)^2+2\,(a^5b^6+2\,a^3b^8+ab^{10})\tan\left(dx+c\right)}-\frac{b\tan\left(dx+c\right)^2-6\,a\tan\left(dx+c\right)}{b^4}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(a^3 - 3*a*b^2)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 2*(
6*a^8 + 17*a^6*b^2 + 15*a^4*b^4)*log(b*tan(d*x + c) + a)/(a^6*b^5 + 3*a^4*b
^7 + 3*a^2*b^9 + b^11) - (3*a^2*b - b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a
^4*b^2 + 3*a^2*b^4 + b^6) - (7*a^8 + 11*a^6*b^2 + 4*(2*a^7*b + 3*a^5*b^3)*t
an(d*x + c))/(a^6*b^5 + 2*a^4*b^7 + a^2*b^9 + (a^4*b^7 + 2*a^2*b^9 + b^11)*
tan(d*x + c)^2 + 2*(a^5*b^6 + 2*a^3*b^8 + a*b^10)*tan(d*x + c)) - (b*tan(d*
x + c)^2 - 6*a*tan(d*x + c))/b^4)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 628 vs. 2(279) = 558.
time = 1.04, size = 628, normalized size = 2.22

$$\frac{6\,a^8b^2+14\,a^6b^4+3\,a^4b^6+a^2b^8+(a^6b^4+3\,a^4b^6+3\,a^2b^8+b^{10})\tan\left(dx+c\right)^4-4\,(a^7b^3+3\,a^5b^5+3\,a^3b^7+ab^9)\tan\left(dx+c\right)^3-2\,(a^5b^5-3\,a^3b^7)dx-(18\,a^8b^2+45\,a^6b^4+30\,a^4b^6+8\,a^2b^8-b^{10}+2\,(a^3b^7-3\,ab^9)dx)\tan\left(dx+c\right)^2+(6\,a^{10}+17\,a^8b^2+15\,a^6b^4+(6\,a^8b^2+17\,a^6b^4+15\,a^4b^6)\tan\left(dx+c\right)^2+2\,(6\,a^9b+17\,a^7b^3+15\,a^5b^5)\tan\left(dx+c\right))\log\left(\frac{b^2\tan\left(dx+c\right)^2+2\,ab\tan\left(dx+c\right)+a^2}{\tan\left(dx+c\right)^2+1}\right)-(6\,a^{10}+17\,a^8b^2+15\,a^6b^4+3\,a^4b^6-a^2b^8+(6\,a^8b^2+17\,a^6b^4+15\,a^4b^6+3\,a^2b^8-b^{10})\tan\left(dx+c\right)^2+2\,(6\,a^9b+17\,a^7b^3+15\,a^5b^5+3\,a^3b^7-ab^9)\tan\left(dx+c\right))\log\left(\frac{1}{\tan\left(dx+c\right)^2+1}\right)-2\,(6\,a^9b+11\,a^7b^3-ab^9+2\,(a^4b^6-3\,a^2b^8)dx)\tan\left(dx+c\right)}{2\,((a^6b^7+3\,a^4b^9+3\,a^2b^{11}+b^{13})d\tan\left(dx+c\right)^2+2\,(a^7b^6+3\,a^5b^8+3\,a^3b^{10}+ab^{12})d\tan\left(dx+c\right)+(a^8b^5+3\,a^6b^7+3\,a^4b^9+a^2b^{11})d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a^8*b^2 + 14*a^6*b^4 + 3*a^4*b^6 + a^2*b^8 + (a^6*b^4 + 3*a^4*b^6 +
3*a^2*b^8 + b^10)*tan(d*x + c)^4 - 4*(a^7*b^3 + 3*a^5*b^5 + 3*a^3*b^7 + a*b
^9)*tan(d*x + c)^3 - 2*(a^5*b^5 - 3*a^3*b^7)*d*x - (18*a^8*b^2 + 45*a^6*b^4
 + 30*a^4*b^6 + 8*a^2*b^8 - b^10 + 2*(a^3*b^7 - 3*a*b^9)*d*x)*tan(d*x + c)^
2 + (6*a^10 + 17*a^8*b^2 + 15*a^6*b^4 + (6*a^8*b^2 + 17*a^6*b^4 + 15*a^4*b^
6)*tan(d*x + c)^2 + 2*(6*a^9*b + 17*a^7*b^3 + 15*a^5*b^5)*tan(d*x + c))*log
((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) - (6
*a^10 + 17*a^8*b^2 + 15*a^6*b^4 + 3*a^4*b^6 - a^2*b^8 + (6*a^8*b^2 + 17*a^6
*b^4 + 15*a^4*b^6 + 3*a^2*b^8 - b^10)*tan(d*x + c)^2 + 2*(6*a^9*b + 17*a^7*
b^3 + 15*a^5*b^5 + 3*a^3*b^7 - a*b^9)*tan(d*x + c))*log(1/(tan(d*x + c)^2 +
 1)) - 2*(6*a^9*b + 11*a^7*b^3 - a*b^9 + 2*(a^4*b^6 - 3*a^2*b^8)*d*x)*tan(d
*x + c))/((a^6*b^7 + 3*a^4*b^9 + 3*a^2*b^11 + b^13)*d*tan(d*x + c)^2 + 2*(a
```

```
^7*b^6 + 3*a^5*b^8 + 3*a^3*b^10 + a*b^12)*d*tan(d*x + c) + (a^8*b^5 + 3*a^6
*b^7 + 3*a^4*b^9 + a^2*b^11)*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [A]
time = 2.51, size = 345, normalized size = 1.22

$$-\frac{\frac{2\left(a^{3}-3\,ab^{2}\right)(dx+c)}{a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}}-\frac{\left(3\,a^{2}b-b^{3}\right)\log\left(\tan(dx+c)^{2}+1\right)}{a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}}-\frac{2\left(6\,a^{8}+17\,a^{6}b^{2}+15\,a^{4}b^{4}\right)\log(|b\tan(dx+c)+a|)}{a^{6}b^{5}+3\,a^{4}b^{7}+3\,a^{2}b^{9}+b^{11}}+\frac{18\,a^{8}b^{2}\tan(dx+c)^{2}+51\,a^{6}b^{4}\tan(dx+c)^{2}+45\,a^{4}b^{6}\tan(dx+c)^{2}+28\,a^{9}b\tan(dx+c)+82\,a^{7}b^{3}\tan(dx+c)+78\,a^{5}b^{5}\tan(dx+c)+11\,a^{10}+33\,a^{8}b^{2}+34\,a^{6}b^{4}}{\left(a^{6}b^{5}+3\,a^{4}b^{7}+3\,a^{2}b^{9}+b^{11}\right)(b\tan(dx+c)+a)^{2}}-\frac{b^{3}\tan(dx+c)^{2}-6\,ab^{2}\tan(dx+c)}{b^{6}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*(a^3 - 3*a*b^2)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (3*
a^2*b - b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) -
2*(6*a^8 + 17*a^6*b^2 + 15*a^4*b^4)*log(abs(b*tan(d*x + c) + a))/(a^6*b^5 +
 3*a^4*b^7 + 3*a^2*b^9 + b^11) + (18*a^8*b^2*tan(d*x + c)^2 + 51*a^6*b^4*ta
n(d*x + c)^2 + 45*a^4*b^6*tan(d*x + c)^2 + 28*a^9*b*tan(d*x + c) + 82*a^7*b
^3*tan(d*x + c) + 78*a^5*b^5*tan(d*x + c) + 11*a^10 + 33*a^8*b^2 + 34*a^6*b
^4)/((a^6*b^5 + 3*a^4*b^7 + 3*a^2*b^9 + b^11)*(b*tan(d*x + c) + a)^2) - (b^
3*tan(d*x + c)^2 - 6*a*b^2*tan(d*x + c))/b^6)/d
```

Mupad [B]
time = 4.35, size = 284, normalized size = 1.00

$$\frac{\frac{2\tan(c+d\,x)\,\left(2\,a^{7}+3\,a^{5}\,b^{2}\right)}{a^{4}+2\,a^{2}\,b^{2}+b^{4}}+\frac{7\,a^{8}+11\,a^{6}\,b^{2}}{2\,b\,\left(a^{4}+2\,a^{2}\,b^{2}+b^{4}\right)}}{d\,\left(a^{2}\,b^{4}+2\,a\,b^{5}\tan\left(c+d\,x\right)+b^{6}\tan\left(c+d\,x\right)^{2}\right)}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)}{2\,d\,\left(-a^{3}\,1\mathrm{i}-3\,a^{2}\,b+a\,b^{2}\,3\mathrm{i}+b^{3}\right)}+\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{b}{(a^{2}+b^{2})^{2}}-\frac{1}{b^{3}}+\frac{6\,a^{2}}{b^{5}}-\frac{4\,a^{2}\,b}{(a^{2}+b^{2})^{3}}\right)}{d}+\frac{\tan(c+d\,x)^{2}}{2\,b^{3}\,d}-\frac{3\,a\tan(c+d\,x)}{b^{4}\,d}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(-a^{3}-a^{2}\,b\,3\mathrm{i}+3\,a\,b^{2}+b^{3}\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6/(a + b*tan(c + d*x))^3,x)
```

```
[Out] ((2*tan(c + d*x)*(2*a^7 + 3*a^5*b^2))/(a^4 + b^4 + 2*a^2*b^2) + (7*a^8 + 11
*a^6*b^2)/(2*b*(a^4 + b^4 + 2*a^2*b^2)))/(d*(a^2*b^4 + b^6*tan(c + d*x)^2 +
 2*a*b^5*tan(c + d*x))) - log(tan(c + d*x) + 1i)/(2*d*(a*b^2*3i - 3*a^2*b -
 a^3*1i + b^3)) - (log(tan(c + d*x) - 1i)*1i)/(2*d*(3*a*b^2 - a^2*b*3i - a^
3 + b^3*1i)) + (log(a + b*tan(c + d*x))*(b/(a^2 + b^2)^2 - 1/b^3 + (6*a^2)/
b^5 - (4*a^2*b)/(a^2 + b^2)^3))/d + tan(c + d*x)^2/(2*b^3*d) - (3*a*tan(c +
 d*x))/(b^4*d)
```

3.478 $\int \frac{\tan^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=239

$$\frac{b(3a^2-b^2)\,x}{(a^2+b^2)^3}-\frac{a(a^2-3b^2)\log(\cos(c+dx))}{(a^2+b^2)^3\,d}-\frac{a^3(3a^4+9a^2b^2+10b^4)\log(a+b\tan(c+dx))}{b^4\,(a^2+b^2)^3\,d}+\frac{(3a^4+6a^2b^2+\cdots}{b^3\,(a^2-\cdots}$$

```
[Out] b*(3*a^2-b^2)*x/(a^2+b^2)^3-a*(a^2-3*b^2)*ln(cos(d*x+c))/(a^2+b^2)^3/d-a^3*
(3*a^4+9*a^2*b^2+10*b^4)*ln(a+b*tan(d*x+c))/b^4/(a^2+b^2)^3/d+(3*a^4+6*a^2*
b^2+b^4)*tan(d*x+c)/b^3/(a^2+b^2)^2/d-1/2*a^2*tan(d*x+c)^3/b/(a^2+b^2)/d/(a
+b*tan(d*x+c))^2-1/2*a^2*(3*a^2+7*b^2)*tan(d*x+c)^2/b^2/(a^2+b^2)^2/d/(a+b*
tan(d*x+c))
```

Rubi [A]

time = 0.39, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3646, 3726, 3728, 3707, 3698, 31, 3556}

$$-\frac{a^2\tan^3(c+dx)}{2bd\,(a^2+b^2)\,(a+b\tan(c+dx))^2}-\frac{a^2(3a^2+7b^2)\tan^2(c+dx)}{2b^2d\,(a^2+b^2)^2\,(a+b\tan(c+dx))}-\frac{a(a^2-3b^2)\log(\cos(c+dx))}{d\,(a^2+b^2)^3}+\frac{bx(3a^2-b^2)}{(a^2+b^2)^3}+\frac{(3a^4+6a^2b^2+b^4)\tan(c+dx)}{b^3d\,(a^2+b^2)^2}-\frac{a^3(3a^4+9a^2b^2+10b^4)\log(a+b\tan(c+dx))}{b^4d\,(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (b*(3*a^2 - b^2)*x)/(a^2 + b^2)^3 - (a*(a^2 - 3*b^2)*Log[Cos[c + d*x]])/((a
^2 + b^2)^3*d) - (a^3*(3*a^4 + 9*a^2*b^2 + 10*b^4)*Log[a + b*Tan[c + d*x]])
/(b^4*(a^2 + b^2)^3*d) + ((3*a^4 + 6*a^2*b^2 + b^4)*Tan[c + d*x])/(b^3*(a^2
 + b^2)^2*d) - (a^2*Tan[c + d*x]^3)/(2*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])
^2) - (a^2*(3*a^2 + 7*b^2)*Tan[c + d*x]^2)/(2*b^2*(a^2 + b^2)^2*d*(a + b*Ta
n[c + d*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
```

```
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
```

```
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{a^2\tan^3(c+dx)}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} + \frac{\int \frac{\tan^2(c+dx)(3a^2-2ab\tan(c+dx)+(3a^2+2b^2)\tan^2(c\cdots}{(a+b\tan(c+dx))^2}}{2b\,(a^2+b^2)}$$

$$= -\frac{a^2\tan^3(c+dx)}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} - \frac{a^2(3a^2+7b^2)\tan^2(c+dx)}{2b^2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))} + \cdots$$

$$= \frac{(3a^4+6a^2b^2+b^4)\tan(c+dx)}{b^3\,(a^2+b^2)^2\,d} - \frac{a^2\tan^3(c+dx)}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} - \frac{a^2(3a\cdots}{2b^2\,(a^2+\cdots}$$

$$= \frac{b(3a^2-b^2)\,x}{(a^2+b^2)^3} + \frac{(3a^4+6a^2b^2+b^4)\tan(c+dx)}{b^3\,(a^2+b^2)^2\,d} - \frac{a^2\tan^3(c+dx)}{2b\,(a^2+b^2)\,d(a+b\tan(c+d\cdots}$$

$$= \frac{b(3a^2-b^2)\,x}{(a^2+b^2)^3} - \frac{a(a^2-3b^2)\log(\cos(c+dx))}{(a^2+b^2)^3\,d} + \frac{(3a^4+6a^2b^2+b^4)\tan(c+dx)}{b^3\,(a^2+b^2)^2\,d}$$

$$= \frac{b(3a^2-b^2)\,x}{(a^2+b^2)^3} - \frac{a(a^2-3b^2)\log(\cos(c+dx))}{(a^2+b^2)^3\,d} - \frac{a^3(3a^4+9a^2b^2+10b^4)\log(a+\cdots}{b^4\,(a^2+b^2)^3\,d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.33, size = 694, normalized size = 2.90

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (a^5*Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x]))/(2*(a - I*b)^2*(a +
I*b)^2*b^2*d*(a + b*Tan[c + d*x])^3) + (b*(3*a^2 - b^2)*(c + d*x)*Sec[c + d
*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)/((a - I*b)^3*(a + I*b)^3*d*(a +
b*Tan[c + d*x])^3) - (I*(3*a^12*b^3 - (3*I)*a^11*b^4 + 15*a^10*b^5 - (15*I)
*a^9*b^6 + 31*a^8*b^7 - (31*I)*a^7*b^8 + 29*a^6*b^9 - (29*I)*a^5*b^10 + 10*
a^4*b^11 - (10*I)*a^3*b^12)*(c + d*x)*Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Si
n[c + d*x])^3)/((a - I*b)^6*(a + I*b)^5*b^7*d*(a + b*Tan[c + d*x])^3) - (I*
(-3*a^7 - 9*a^5*b^2 - 10*a^3*b^4)*ArcTan[Tan[c + d*x]]*Sec[c + d*x]^3*(a*Co
s[c + d*x] + b*Sin[c + d*x])^3)/(b^4*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x])^3
) + (3*a*Log[Cos[c + d*x]]*Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])
^3)/(b^4*d*(a + b*Tan[c + d*x])^3) + ((-3*a^7 - 9*a^5*b^2 - 10*a^3*b^4)*Log
[(a*Cos[c + d*x] + b*Sin[c + d*x])^2]*Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Si
```

```
n[c + d*x])^3)/(2*b^4*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x])^3) + (Sec[c + d*
x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^2*(2*a^5*Sin[c + d*x] + 5*a^3*b^2*Si
n[c + d*x]))/((a - I*b)^2*(a + I*b)^2*b^3*d*(a + b*Tan[c + d*x])^3) + (Sec[
c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^3*Tan[c + d*x])/(b^3*d*(a + b*
Tan[c + d*x])^3)
```

Maple [A]

time = 0.24, size = 186, normalized size = 0.78

method	result
derivativedivides	$\frac{\frac{\tan(dx+c)}{b^3}+\frac{\frac{\left(a^3-3b^2a\right)\ln\left(1+\tan^2(dx+c)\right)}{2}+\left(3a^2b-b^3\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^3}-\frac{a^4\left(3a^2+5b^2\right)}{b^4\left(a^2+b^2\right)^2(a+b\tan(dx+c))}+\frac{a^5}{2b^4\left(a^2+b^2\right)(a+b\tan \cdots}}{d}$
default	$\frac{\frac{\tan(dx+c)}{b^3}+\frac{\frac{\left(a^3-3b^2a\right)\ln\left(1+\tan^2(dx+c)\right)}{2}+\left(3a^2b-b^3\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^3}-\frac{a^4\left(3a^2+5b^2\right)}{b^4\left(a^2+b^2\right)^2(a+b\tan(dx+c))}+\frac{a^5}{2b^4\left(a^2+b^2\right)(a+b\tan \cdots}}{d}$
norman	$\frac{\frac{\tan^3(dx+c)}{bd}+\frac{b^3\left(3a^2-b^2\right)x\left(\tan^2(dx+c)\right)}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}+\frac{\left(3a^2-b^2\right)a^2bx}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}-\frac{a^2\left(9a^5+17a^3b^2+4a\,b^4\right)}{2d\,b^4\left(a^4+2a^2b^2+b^4\right)}-\frac{a\left(6a^5+11a^3b^2+3a\,b^4\right)\tan(dx \cdots}{d\,b^3\left(a^4+2a^2b^2+b^4\right)}}{(a+b\tan(dx+c))^2}$
risch	$\frac{ix}{3ib\,a^2-ib^3-a^3+3b^2a}+\frac{6ia^7x}{b^4(a^6+3a^4b^2+3a^2b^4+b^6)}+\frac{6ia^7c}{b^4d(a^6+3a^4b^2+3a^2b^4+b^6)}+\frac{18ia^5x}{b^2(a^6+3a^4b^2+3a^2b^4+b^6)}+ \cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^3*tan(d*x+c)+1/(a^2+b^2)^3*(1/2*(a^3-3*a*b^2)*ln(1+tan(d*x+c)^2)+(
3*a^2*b-b^3)*arctan(tan(d*x+c)))-1/b^4*a^4*(3*a^2+5*b^2)/(a^2+b^2)^2/(a+b*t
an(d*x+c))+1/2/b^4*a^5/(a^2+b^2)/(a+b*tan(d*x+c))^2-1/b^4*a^3*(3*a^4+9*a^2*
b^2+10*b^4)/(a^2+b^2)^3*ln(a+b*tan(d*x+c)))
```

Maxima [A]

time = 0.52, size = 293, normalized size = 1.23

$$\frac{\frac{2\,(3\,a^2b-b^3)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{2\,(3\,a^7+9\,a^5b^2+10\,a^3b^4)\log(b\tan(dx+c)+a)}{a^6b^4+3\,a^4b^6+3\,a^2b^8+b^{10}}+\frac{(a^3-3\,ab^2)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{5\,a^7+9\,a^5b^2+2\,(3\,a^6b+5\,a^4b^3)\tan(dx+c)}{a^6b^4+2\,a^4b^6+a^2b^8+(a^4b^6+2\,a^2b^8+b^{10})\tan(dx+c)^2+2\,(a^5b^5+2\,a^3b^7+ab^9)\tan(dx+c)}+\frac{2\,\tan(dx+c)}{b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(3*a^2*b - b^3)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 2*(3
*a^7 + 9*a^5*b^2 + 10*a^3*b^4)*log(b*tan(d*x + c) + a)/(a^6*b^4 + 3*a^4*b^6
 + 3*a^2*b^8 + b^10) + (a^3 - 3*a*b^2)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4
*b^2 + 3*a^2*b^4 + b^6) - (5*a^7 + 9*a^5*b^2 + 2*(3*a^6*b + 5*a^4*b^3)*tan(
d*x + c))/(a^6*b^4 + 2*a^4*b^6 + a^2*b^8 + (a^4*b^6 + 2*a^2*b^8 + b^10)*tan
(d*x + c)^2 + 2*(a^5*b^5 + 2*a^3*b^7 + a*b^9)*tan(d*x + c)) + 2*tan(d*x + c
)/b^3)/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(235) = 470$.
time = 1.11, size = 549, normalized size = 2.30

$$-\frac{3 a^{7} b^{2} + 9 a^{5} b^{4} - 2 (a^{6} b^{3} + 3 a^{4} b^{5} + 3 a^{2} b^{7} + b^{9}) \tan (dx + c)^{3} - 2 (3 a^{4} b^{5} - a^{2} b^{7}) dx - (9 a^{7} b^{2} + 23 a^{5} b^{4} + 12 a^{3} b^{6} + 4 a b^{8} + 2 (3 a^{2} b^{7} - b^{9}) dx) \tan (dx + c)^{2} + (3 a^{9} + 9 a^{7} b^{2} + 10 a^{5} b^{4} + (3 a^{7} b^{2} + 9 a^{5} b^{4} + 10 a^{3} b^{6}) \tan (dx + c)^{2} + 2 (3 a^{8} b + 9 a^{6} b^{3} + 10 a^{4} b^{5}) \tan (dx + c)) \log \left(\frac{b^{2} \tan(dx+c)^{2} + 2 a b \tan(dx+c) + a^{2}}{\tan(dx+c)^{2} + 1}\right) - 3 (a^{9} + 3 a^{7} b^{2} + 3 a^{5} b^{4} + a^{3} b^{6} + (a^{7} b^{2} + 3 a^{5} b^{4} + 3 a^{3} b^{6} + a b^{8}) \tan (dx + c)^{2} + 2 (a^{8} b + 3 a^{6} b^{3} + 3 a^{4} b^{5} + a^{2} b^{7}) \tan (dx + c)) \log \left(\frac{1}{\tan(dx+c)^{2} + 1}\right) - 2 (3 a^{8} b + 6 a^{6} b^{3} - 2 a^{4} b^{5} + a^{2} b^{7} + 2 (3 a^{3} b^{6} - a b^{8}) dx) \tan (dx + c)}{2 ((a^{6} b^{6} + 3 a^{4} b^{8} + 3 a^{2} b^{10} + b^{12}) d \tan (dx + c)^{2} + 2 (a^{7} b^{5} + 3 a^{5} b^{7} + 3 a^{3} b^{9} + a b^{11}) d \tan (dx + c) + (a^{8} b^{4} + 3 a^{6} b^{6} + 3 a^{4} b^{8} + a^{2} b^{10}) d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(3*a^7*b^2 + 9*a^5*b^4 - 2*(a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9)*tan
(d*x + c)^3 - 2*(3*a^4*b^5 - a^2*b^7)*d*x - (9*a^7*b^2 + 23*a^5*b^4 + 12*a^
3*b^6 + 4*a*b^8 + 2*(3*a^2*b^7 - b^9)*d*x)*tan(d*x + c)^2 + (3*a^9 + 9*a^7*
b^2 + 10*a^5*b^4 + (3*a^7*b^2 + 9*a^5*b^4 + 10*a^3*b^6)*tan(d*x + c)^2 + 2*
(3*a^8*b + 9*a^6*b^3 + 10*a^4*b^5)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 +
2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) - 3*(a^9 + 3*a^7*b^2 + 3*a^
5*b^4 + a^3*b^6 + (a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*tan(d*x + c)^2
+ 2*(a^8*b + 3*a^6*b^3 + 3*a^4*b^5 + a^2*b^7)*tan(d*x + c))*log(1/(tan(d*x
+ c)^2 + 1)) - 2*(3*a^8*b + 6*a^6*b^3 - 2*a^4*b^5 + a^2*b^7 + 2*(3*a^3*b^6
- a*b^8)*d*x)*tan(d*x + c))/((a^6*b^6 + 3*a^4*b^8 + 3*a^2*b^10 + b^12)*d*ta
n(d*x + c)^2 + 2*(a^7*b^5 + 3*a^5*b^7 + 3*a^3*b^9 + a*b^11)*d*tan(d*x + c)
+ (a^8*b^4 + 3*a^6*b^6 + 3*a^4*b^8 + a^2*b^10)*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [**A**]
time = 2.26, size = 325, normalized size = 1.36

$$\frac{\frac{2 (3 a^{2} b - b^{3})(dx + c)}{a^{6} + 3 a^{4} b^{2} + 3 a^{2} b^{4} + b^{6}} + \frac{(a^{3} - 3 a b^{2}) \log \left(\tan(dx+c)^{2} + 1\right)}{a^{6} + 3 a^{4} b^{2} + 3 a^{2} b^{4} + b^{6}} - \frac{2 (3 a^{7} + 9 a^{5} b^{2} + 10 a^{3} b^{4}) \log(|b \tan(dx+c) + a|)}{a^{6} b^{4} + 3 a^{4} b^{6} + 3 a^{2} b^{8} + b^{10}} + \frac{9 a^{7} b^{2} \tan(dx+c)^{2} + 27 a^{5} b^{4} \tan(dx+c)^{2} + 30 a^{3} b^{6} \tan(dx+c)^{2} + 12 a^{8} b \tan(dx+c) + 38 a^{6} b^{3} \tan(dx+c) + 50 a^{4} b^{5} \tan(dx+c) + 4 a^{9} + 13 a^{7} b^{2} + 21 a^{5} b^{4}}{(a^{6} b^{4} + 3 a^{4} b^{6} + 3 a^{2} b^{8} + b^{10})(b \tan(dx+c) + a)^{2}} + \frac{2 \tan(dx+c)}{b^{3}}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(3*a^2*b - b^3)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (a^3
 - 3*a*b^2)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 2
*(3*a^7 + 9*a^5*b^2 + 10*a^3*b^4)*log(abs(b*tan(d*x + c) + a))/(a^6*b^4 + 3
*a^4*b^6 + 3*a^2*b^8 + b^10) + (9*a^7*b^2*tan(d*x + c)^2 + 27*a^5*b^4*tan(d
```

```
*x + c)^2 + 30*a^3*b^6*tan(d*x + c)^2 + 12*a^8*b*tan(d*x + c) + 38*a^6*b^3*
tan(d*x + c) + 50*a^4*b^5*tan(d*x + c) + 4*a^9 + 13*a^7*b^2 + 21*a^5*b^4)/(
(a^6*b^4 + 3*a^4*b^6 + 3*a^2*b^8 + b^10)*(b*tan(d*x + c) + a)^2) + 2*tan(d*
x + c)/b^3)/d
```

Mupad [B]
time = 4.25, size = 263, normalized size = 1.10

$$\frac{\tan(c+dx)}{b^3 d}-\frac{\frac{\tan(c+dx)\,\left(3a^6+5a^4b^2\right)}{a^4+2a^2b^2+b^4}+\frac{5a^7+9a^5b^2}{2b\,\left(a^4+2a^2b^2+b^4\right)}}{d\,\left(a^2b^3+2ab^4\tan\left(c+dx\right)+b^5\tan\left(c+dx\right)^2\right)}-\frac{\ln\left(\tan(c+dx)+1\mathrm{i}\right)}{2d\,\left(-a^3+a^2b\,3\mathrm{i}+3ab^2-b^3\,1\mathrm{i}\right)}-\frac{a^3\,\ln\left(a+b\tan(c+dx)\right)\,\left(3a^4+9a^2b^2+10b^4\right)}{b^4d\,\left(a^2+b^2\right)^3}-\frac{\ln\left(\tan(c+dx)-\mathrm{i}\right)\,1\mathrm{i}}{2d\,\left(-a^3\,1\mathrm{i}+3a^2b+ab^2\,3\mathrm{i}-b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + b*tan(c + d*x))^3,x)
```

```
[Out] tan(c + d*x)/(b^3*d) - ((tan(c + d*x)*(3*a^6 + 5*a^4*b^2))/(a^4 + b^4 + 2*a
^2*b^2) + (5*a^7 + 9*a^5*b^2)/(2*b*(a^4 + b^4 + 2*a^2*b^2)))/(d*(a^2*b^3 +
b^5*tan(c + d*x)^2 + 2*a*b^4*tan(c + d*x))) - (log(tan(c + d*x) - 1i)*1i)/(
2*d*(a*b^2*3i + 3*a^2*b - a^3*1i - b^3)) - log(tan(c + d*x) + 1i)/(2*d*(3*a
*b^2 + a^2*b*3i - a^3 - b^3*1i)) - (a^3*log(a + b*tan(c + d*x))*(3*a^4 + 10
*b^4 + 9*a^2*b^2))/(b^4*d*(a^2 + b^2)^3)
```

3.479 $\int \frac{\tan^4(c+dx)}{(a+b\tan(c+dx))^3} \, dx$

Optimal. Leaf size=183

$$\frac{a(a^2-3b^2)\,x}{(a^2+b^2)^3}+\frac{b(3a^2-b^2)\log(\cos(c+dx))}{(a^2+b^2)^3\,d}+\frac{a^2(a^4+3a^2b^2+6b^4)\log(a+b\tan(c+dx))}{b^3\,(a^2+b^2)^3\,d}-\frac{a^2\tan^2(c}{2b\,(a^2+b^2)\,d(a+}$$

```
[Out] a*(a^2-3*b^2)*x/(a^2+b^2)^3+b*(3*a^2-b^2)*ln(cos(d*x+c))/(a^2+b^2)^3/d+a^2*
(a^4+3*a^2*b^2+6*b^4)*ln(a+b*tan(d*x+c))/b^3/(a^2+b^2)^3/d-1/2*a^2*tan(d*x+
c)^2/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^2+a^3*(a^2+3*b^2)/b^3/(a^2+b^2)^2/d/(a+
b*tan(d*x+c))
```

Rubi [A]
time = 0.22, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3646, 3716, 3707, 3698, 31, 3556}

$$-\frac{a^2\tan^2(c+dx)}{2bd\,(a^2+b^2)\,(a+b\tan(c+dx))^2}+\frac{b(3a^2-b^2)\log(\cos(c+dx))}{d\,(a^2+b^2)^3}+\frac{ax(a^2-3b^2)}{(a^2+b^2)^3}+\frac{a^2(a^4+3a^2b^2+6b^4)\log(a+b\tan(c+dx))}{b^3d\,(a^2+b^2)^3}+\frac{a^3(a^2+3b^2)}{b^3d\,(a^2+b^2)^2\,(a+b\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (a*(a^2 - 3*b^2)*x)/(a^2 + b^2)^3 + (b*(3*a^2 - b^2)*Log[Cos[c + d*x]])/((a
^2 + b^2)^3*d) + (a^2*(a^4 + 3*a^2*b^2 + 6*b^4)*Log[a + b*Tan[c + d*x]])/(b
^3*(a^2 + b^2)^3*d) - (a^2*Tan[c + d*x]^2)/(2*b*(a^2 + b^2)*d*(a + b*Tan[c
+ d*x])^2) + (a^3*(a^2 + 3*b^2))/(b^3*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
```

```
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3716

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
 -1]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{a^2\tan^2(c+dx)}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} + \frac{\int \frac{\tan(c+dx)\left(2a^2-2ab\tan(c+dx)+2(a^2+b^2)\tan^2(c+d\right.}{(a+b\tan(c+dx))^2}}{2b\,(a^2+b^2)}$$

$$= -\frac{a^2\tan^2(c+dx)}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} + \frac{a^3(a^2+3b^2)}{b^3\,(a^2+b^2)^2\,d(a+b\tan(c+dx))} + \int \frac{2a^{\cdots}}{}$$

$$= \frac{a(a^2-3b^2)\,x}{(a^2+b^2)^3} - \frac{a^2\tan^2(c+dx)}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} + \frac{a^3(a^2+3b^2)}{b^3\,(a^2+b^2)^2\,d(a+b\tan(c}$$

$$= \frac{a(a^2-3b^2)\,x}{(a^2+b^2)^3} + \frac{b(3a^2-b^2)\log(\cos(c+dx))}{(a^2+b^2)^3\,d} - \frac{a^2\tan^2(c+dx)}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx)}$$

$$= \frac{a(a^2-3b^2)\,x}{(a^2+b^2)^3} + \frac{b(3a^2-b^2)\log(\cos(c+dx))}{(a^2+b^2)^3\,d} + \frac{a^2(a^4+3a^2b^2+6b^4)\log(a+b\,\mathrm{t}}{b^3\,(a^2+b^2)^3\,d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.20, size = 351, normalized size = 1.92

$$\frac{\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(-a^4b^2(a^2+b^2)-2a^2b(a^2+b^2)(a^2+4b^2)\sin(c+dx)(a\cos(c+dx)+b\sin(c+dx))+2ab^3(a^2-3b^2)(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2+2ia^2(a^4+3a^2b^2+6b^4)(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2-2ia^2(a^4+3a^2b^2+6b^4)\mathrm{ArcTan}(\tan(c+dx))(a\cos(c+dx)+b\sin(c+dx))^2-2(a^2+b^2)^3\log(\cos(c+dx))(a\cos(c+dx)+b\sin(c+dx))^2+a^2(a^4+3a^2b^2+6b^4)\log\left((a\cos(c+dx)+b\sin(c+dx))^2\right)(a\cos(c+dx)+b\sin(c+dx))^2\right)}{2b^3(a^2+b^2)^3d(a+b\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])*(-(a^4*b^2*(a^2 + b^2)) -
 2*a^2*b*(a^2 + b^2)*(a^2 + 4*b^2)*Sin[c + d*x]*(a*Cos[c + d*x] + b*Sin[c +
 d*x]) + 2*a*b^3*(a^2 - 3*b^2)*(c + d*x)*(a*Cos[c + d*x] + b*Sin[c + d*x])^
2 + (2*I)*a^2*(a^4 + 3*a^2*b^2 + 6*b^4)*(c + d*x)*(a*Cos[c + d*x] + b*Sin[c
 + d*x])^2 - (2*I)*a^2*(a^4 + 3*a^2*b^2 + 6*b^4)*ArcTan[Tan[c + d*x]]*(a*Co
s[c + d*x] + b*Sin[c + d*x])^2 - 2*(a^2 + b^2)^3*Log[Cos[c + d*x]]*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^2 + a^2*(a^4 + 3*a^2*b^2 + 6*b^4)*Log[(a*Cos[c +
d*x] + b*Sin[c + d*x])^2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2))/(2*b^3*(a^2
 + b^2)^3*d*(a + b*Tan[c + d*x])^3)
```

Maple [A]
time = 0.19, size = 169, normalized size = 0.92

method	result
derivativedivides	$\dfrac{\dfrac{\left(-3a^2b+b^3\right)\ln\left(1+\tan^2(dx+c)\right)}{2}+\left(a^3-3b^2a\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^3}-\dfrac{a^4}{2b^3\left(a^2+b^2\right)(a+b\tan(dx+c))^2}+\dfrac{a^2\left(a^4+3a^2b^2+6b^4\right)\ln(a+b\tan(d}{\left(a^2+b^2\right)^3b^3}$ over d
default	$\dfrac{\dfrac{\left(-3a^2b+b^3\right)\ln\left(1+\tan^2(dx+c)\right)}{2}+\left(a^3-3b^2a\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^3}-\dfrac{a^4}{2b^3\left(a^2+b^2\right)(a+b\tan(dx+c))^2}+\dfrac{a^2\left(a^4+3a^2b^2+6b^4\right)\ln(a+b\tan(d}{\left(a^2+b^2\right)^3b^3}$ over d

norman	$\frac{\frac{\left(a^{2}-3b^{2}\right)a^{3}x}{a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}}+\frac{b^{2}\left(a^{2}-3b^{2}\right)ax\left(\tan^{2}(dx+c)\right)}{a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}}+\frac{a^{2}\left(3a^{4}+7a^{2}b^{2}\right)}{2d\,b^{3}\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}+\frac{2a\left(a^{4}+2a^{2}b^{2}\right)\tan(dx+c)}{d\,b^{2}\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}+\frac{2b\left(a^{2}-3b^{2}\right)a^{2}x\tan(d}{a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+}}{(a+b\tan(dx+c))^{2}}$
risch	$-\frac{x}{3ib\,a^{2}-ib^{3}-a^{3}+3b^{2}a}+\frac{2ix}{b^{3}}+\frac{2ic}{d\,b^{3}}-\frac{2ia^{6}x}{(a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6})b^{3}}-\frac{2ia^{6}c}{(a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6})b^{3}d}-\frac{6ia}{(a^{6}+3a^{4}b^{2}+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a^2+b^2)^3*(1/2*(-3*a^2*b+b^3)*ln(1+tan(d*x+c)^2)+(a^3-3*a*b^2)*arc
tan(tan(d*x+c)))-1/2*a^4/b^3/(a^2+b^2)/(a+b*tan(d*x+c))^2+a^2*(a^4+3*a^2*b^
2+6*b^4)/(a^2+b^2)^3/b^3*ln(a+b*tan(d*x+c))+2*a^3*(a^2+2*b^2)/b^3/(a^2+b^2)
^2/(a+b*tan(d*x+c)))
```

Maxima [A]
time = 0.50, size = 279, normalized size = 1.52

$$\frac{\frac{2\,(a^{3}-3\,ab^{2})(dx+c)}{a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}}+\frac{2\,(a^{6}+3\,a^{4}b^{2}+6\,a^{2}b^{4})\log(b\tan(dx+c)+a)}{a^{6}b^{3}+3\,a^{4}b^{5}+3\,a^{2}b^{7}+b^{9}}-\frac{(3\,a^{2}b-b^{3})\log\left(\tan(dx+c)^{2}+1\right)}{a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}}+\frac{3\,a^{6}+7\,a^{4}b^{2}+4\,(a^{5}b+2\,a^{3}b^{3})\tan(dx+c)}{a^{6}b^{3}+2\,a^{4}b^{5}+a^{2}b^{7}+(a^{4}b^{5}+2\,a^{2}b^{7}+b^{9})\tan(dx+c)^{2}+2\,(a^{5}b^{4}+2\,a^{3}b^{6}+ab^{8})\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^3 - 3*a*b^2)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(a
^6 + 3*a^4*b^2 + 6*a^2*b^4)*log(b*tan(d*x + c) + a)/(a^6*b^3 + 3*a^4*b^5 +
3*a^2*b^7 + b^9) - (3*a^2*b - b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6) + (3*a^6 + 7*a^4*b^2 + 4*(a^5*b + 2*a^3*b^3)*tan(d*x +
c))/(a^6*b^3 + 2*a^4*b^5 + a^2*b^7 + (a^4*b^5 + 2*a^2*b^7 + b^9)*tan(d*x +
c)^2 + 2*(a^5*b^4 + 2*a^3*b^6 + a*b^8)*tan(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(181) = 362$.
time = 1.35, size = 481, normalized size = 2.63

$$\frac{a^{6}b^{2}+7\,a^{4}b^{4}+2\,(a^{5}b^{3}-3\,a^{3}b^{5})dx-(3\,a^{6}b^{2}+9\,a^{4}b^{4}-2\,(a^{3}b^{5}-3\,ab^{7})dx)\tan(dx+c)^{2}+\left(a^{8}+3\,a^{6}b^{2}+6\,a^{4}b^{4}+(a^{6}b^{2}+3\,a^{4}b^{4}+6\,a^{2}b^{6})\tan(dx+c)^{2}+2\,(a^{7}b+3\,a^{5}b^{3}+6\,a^{3}b^{5})\tan(dx+c)\right)\log\left(\frac{b^{2}\tan(dx+c)^{2}+2\,ab\tan(dx+c)+a^{2}}{\tan(dx+c)^{2}+1}\right)-\left(a^{8}+3\,a^{6}b^{2}+3\,a^{4}b^{4}+a^{2}b^{6}+(a^{6}b^{2}+3\,a^{4}b^{4}+3\,a^{2}b^{6}+b^{8})\tan(dx+c)^{2}+2\,(a^{7}b+3\,a^{5}b^{3}+3\,a^{3}b^{5}+ab^{7})\tan(dx+c)\right)\log\left(\frac{1}{\tan(dx+c)^{2}+1}\right)-2\,(a^{7}b+3\,a^{5}b^{3}-4\,a^{3}b^{5}-2\,(a^{4}b^{4}-3\,a^{2}b^{6})dx)\tan(dx+c)}{2\,\left((a^{6}b^{5}+3\,a^{4}b^{7}+3\,a^{2}b^{9}+b^{11})d\tan(dx+c)^{2}+2\,(a^{7}b^{4}+3\,a^{5}b^{6}+3\,a^{3}b^{8}+ab^{10})d\tan(dx+c)+(a^{8}b^{3}+3\,a^{6}b^{5}+3\,a^{4}b^{7}+a^{2}b^{9})d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a^6*b^2 + 7*a^4*b^4 + 2*(a^5*b^3 - 3*a^3*b^5)*d*x - (3*a^6*b^2 + 9*a^4
*b^4 - 2*(a^3*b^5 - 3*a*b^7)*d*x)*tan(d*x + c)^2 + (a^8 + 3*a^6*b^2 + 6*a^4
*b^4 + (a^6*b^2 + 3*a^4*b^4 + 6*a^2*b^6)*tan(d*x + c)^2 + 2*(a^7*b + 3*a^5*
b^3 + 6*a^3*b^5)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c)
 + a^2)/(tan(d*x + c)^2 + 1)) - (a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 + (a
^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*tan(d*x + c)^2 + 2*(a^7*b + 3*a^5*b^3
```

```
 + 3*a^3*b^5 + a*b^7)*tan(d*x + c))*log(1/(tan(d*x + c)^2 + 1)) - 2*(a^7*b
+ 3*a^5*b^3 - 4*a^3*b^5 - 2*(a^4*b^4 - 3*a^2*b^6)*d*x)*tan(d*x + c))/((a^6*
b^5 + 3*a^4*b^7 + 3*a^2*b^9 + b^11)*d*tan(d*x + c)^2 + 2*(a^7*b^4 + 3*a^5*b
^6 + 3*a^3*b^8 + a*b^10)*d*tan(d*x + c) + (a^8*b^3 + 3*a^6*b^5 + 3*a^4*b^7
+ a^2*b^9)*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [A]
time = 1.43, size = 304, normalized size = 1.66

$$\frac{\frac{2\,(a^3-3\,ab^2)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{(3\,a^2b-b^3)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\,(a^6+3\,a^4b^2+6\,a^2b^4)\log(|b\tan(dx+c)+a|)}{a^6b^3+3\,a^4b^5+3\,a^2b^7+b^9}-\frac{3\,a^6b\tan(dx+c)^2+9\,a^4b^3\tan(dx+c)^2+18\,a^2b^5\tan(dx+c)^2+2\,a^7\tan(dx+c)+6\,a^5b^2\tan(dx+c)+28\,a^3b^4\tan(dx+c)-a^6b+11\,a^4b^3}{(a^6b^2+3\,a^4b^4+3\,a^2b^6+b^8)(b\tan(dx+c)+a)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(a^3 - 3*a*b^2)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (3*a
^2*b - b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2
*(a^6 + 3*a^4*b^2 + 6*a^2*b^4)*log(abs(b*tan(d*x + c) + a))/(a^6*b^3 + 3*a^
4*b^5 + 3*a^2*b^7 + b^9) - (3*a^6*b*tan(d*x + c)^2 + 9*a^4*b^3*tan(d*x + c)
^2 + 18*a^2*b^5*tan(d*x + c)^2 + 2*a^7*tan(d*x + c) + 6*a^5*b^2*tan(d*x + c
) + 28*a^3*b^4*tan(d*x + c) - a^6*b + 11*a^4*b^3)/((a^6*b^2 + 3*a^4*b^4 + 3
*a^2*b^6 + b^8)*(b*tan(d*x + c) + a)^2))/d
```

Mupad [B]
time = 4.21, size = 240, normalized size = 1.31

$$\frac{\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)}{2\,d\,\left(-a^3\,1{}\mathrm{i}-3\,a^2\,b+a\,b^2\,3{}\mathrm{i}+b^3\right)}+\frac{\frac{3\,a^6+7\,a^4\,b^2}{2\,b^3\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{2\,a^3\,\tan(c+d\,x)\,\left(a^2+2\,b^2\right)}{b^2\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{d\,\left(a^2+2\,a\,b\,\tan\left(c+d\,x\right)+b^2\,{\tan\left(c+d\,x\right)}^2\right)}+\frac{a^2\,\ln\left(a+b\,\tan(c+d\,x)\right)\,\left(a^4+3\,a^2\,b^2+6\,b^4\right)}{b^3\,d\,{\left(a^2+b^2\right)}^3}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,1{}\mathrm{i}}{2\,d\,\left(-a^3-a^2\,b\,3{}\mathrm{i}+3\,a\,b^2+b^3\,1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + b*tan(c + d*x))^3,x)
```

```
[Out] log(tan(c + d*x) + 1i)/(2*d*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3)) + (log(tan
(c + d*x) - 1i)*1i)/(2*d*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)) + ((3*a^6 + 7
*a^4*b^2)/(2*b^3*(a^4 + b^4 + 2*a^2*b^2)) + (2*a^3*tan(c + d*x)*(a^2 + 2*b^
2))/(b^2*(a^4 + b^4 + 2*a^2*b^2)))/(d*(a^2 + b^2*tan(c + d*x)^2 + 2*a*b*tan
(c + d*x))) + (a^2*log(a + b*tan(c + d*x))*(a^4 + 6*b^4 + 3*a^2*b^2))/(b^3*
d*(a^2 + b^2)^3)
```

3.480 $\int \frac{\tan^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=149

$$-\frac{b(3a^2-b^2)\,x}{(a^2+b^2)^3}+\frac{a(a^2-3b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^3\,d}-\frac{a^2\tan(c+dx)}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx))^2}-\frac{}{2b^2\,(a^2}$$

```
[Out] -b*(3*a^2-b^2)*x/(a^2+b^2)^3+a*(a^2-3*b^2)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a
^2+b^2)^3/d-1/2*a^2*tan(d*x+c)/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^2-1/2*a^2*(a^
2+5*b^2)/b^2/(a^2+b^2)^2/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.16, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3646, 3709, 3612, 3611}

$$-\frac{a^2(a^2+5b^2)}{2b^2d\,(a^2+b^2)^2\,(a+b\tan(c+dx))}-\frac{a^2\tan(c+dx)}{2bd\,(a^2+b^2)\,(a+b\tan(c+dx))^2}+\frac{a(a^2-3b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^3}-\frac{bx(3a^2-b^2)}{(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -((b*(3*a^2 - b^2)*x)/(a^2 + b^2)^3) + (a*(a^2 - 3*b^2)*Log[a*Cos[c + d*x]
+ b*Sin[c + d*x]])/((a^2 + b^2)^3*d) - (a^2*Tan[c + d*x])/(2*b*(a^2 + b^2)*
d*(a + b*Tan[c + d*x])^2) - (a^2*(a^2 + 5*b^2))/(2*b^2*(a^2 + b^2)^2*d*(a +
 b*Tan[c + d*x]))
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
```

```
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx &= -\frac{a^2\tan(c+dx)}{2b\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}+\frac{\int \frac{a^2-2ab\tan(c+dx)+\left(a^2+2b^2\right)\tan^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx}{2b\left(a^2+b^2\right)}\\ &= -\frac{a^2\tan(c+dx)}{2b\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}-\frac{a^2(a^2+5b^2)}{2b^2\left(a^2+b^2\right)^2d(a+b\tan(c+dx))}+\frac{\int \text{–}\\ &= -\frac{b(3a^2-b^2)\,x}{\left(a^2+b^2\right)^3}-\frac{a^2\tan(c+dx)}{2b\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}-\frac{a^2(a^2+5b^2)}{2b^2\left(a^2+b^2\right)^2d(a+b\tan\\ &= -\frac{b(3a^2-b^2)\,x}{\left(a^2+b^2\right)^3}+\frac{a(a^2-3b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{\left(a^2+b^2\right)^3d}-\frac{a^2\text{–}}{2b\left(a^2+b^2\right)\text{–}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.81, size = 269, normalized size = 1.81

$$\frac{\frac{i\log(i-\tan(c+dx))}{(a+ib)^2}+\frac{a\log(i-\tan(c+dx))}{(-ia+b)^3}-\frac{i\log(i+\tan(c+dx))}{(a-ib)^2}+\frac{a\log(i+\tan(c+dx))}{(ia+b)^3}-\frac{4ab\log(a+b\tan(c+dx))}{(a^2+b^2)^2}-\frac{a}{b(a+b\tan(c+dx))^2}-\frac{2\tan(c+dx)}{(a+b\tan(c+dx))^2}+\frac{2b}{(a^2+b^2)(a+b\tan(c+dx))}-\frac{ab\left(\left(-6a^2+2b^2\right)\log(a+b\tan(c+dx))+\frac{\left(a^2+b^2\right)\left(5a^2+b^2+4ab\tan(c+dx)\right)}{(a+b\tan(c+dx))^2}\right)}{(a^2+b^2)^3}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((I*Log[I - Tan[c + d*x]])/(a + I*b)^2 + (a*Log[I - Tan[c + d*x]])/((-I)*a
+ b)^3 - (I*Log[I + Tan[c + d*x]])/(a - I*b)^2 + (a*Log[I + Tan[c + d*x]])/
```

```
(I*a + b)^3 - (4*a*b*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^2 - a/(b*(a + b*T
an[c + d*x])^2) - (2*Tan[c + d*x])/(a + b*Tan[c + d*x])^2 + (2*b)/((a^2 + b
^2)*(a + b*Tan[c + d*x])) - (a*b*((-6*a^2 + 2*b^2)*Log[a + b*Tan[c + d*x]]
+ ((a^2 + b^2)*(5*a^2 + b^2 + 4*a*b*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2))
/(a^2 + b^2)^3)/(2*b*d)
```

Maple [A]
time = 0.17, size = 158, normalized size = 1.06

method	result
derivativedivides	$\frac{\frac{\frac{\left(-a^{3}+3b^{2}a\right)\ln\left(1+\tan^{2}(dx+c)\right)}{2}+\left(-3a^{2}b+b^{3}\right)\arctan(\tan(dx+c))}{\left(a^{2}+b^{2}\right)^{3}}+\frac{a\left(a^{2}-3b^{2}\right)\ln(a+b\tan(dx+c))}{\left(a^{2}+b^{2}\right)^{3}}-\frac{a^{2}\left(a^{2}+3b^{2}\right)}{\left(a^{2}+b^{2}\right)^{2}b^{2}(a+b\tan(dx+c))}}{d}$
default	$\frac{\frac{\frac{\left(-a^{3}+3b^{2}a\right)\ln\left(1+\tan^{2}(dx+c)\right)}{2}+\left(-3a^{2}b+b^{3}\right)\arctan(\tan(dx+c))}{\left(a^{2}+b^{2}\right)^{3}}+\frac{a\left(a^{2}-3b^{2}\right)\ln(a+b\tan(dx+c))}{\left(a^{2}+b^{2}\right)^{3}}-\frac{a^{2}\left(a^{2}+3b^{2}\right)}{\left(a^{2}+b^{2}\right)^{2}b^{2}(a+b\tan(dx+c))}}{d}$
norman	$\frac{\frac{\left(a^{4}+3a^{2}b^{2}\right)\left(\tan^{2}(dx+c)\right)}{2a\left(a^{4}+2a^{2}b^{2}+b^{4}\right)d}-\frac{a^{3}}{\left(a^{4}+2a^{2}b^{2}+b^{4}\right)d}-\frac{b^{3}\left(3a^{2}-b^{2}\right)x\left(\tan^{2}(dx+c)\right)}{\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\left(a^{2}+b^{2}\right)}-\frac{\left(3a^{2}-b^{2}\right)a^{2}bx}{\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\left(a^{2}+b^{2}\right)}-\frac{2b^{2}\left(3a^{2}-b^{2}\right)ax\tan(d}{\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\left(a^{2}-\right.}}{(a+b\tan(dx+c))^{2}}$
risch	$-\frac{ix}{3ib\,a^{2}-ib^{3}-a^{3}+3b^{2}a}-\frac{2ia^{3}x}{a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}}+\frac{6ia\,b^{2}x}{a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}}-\frac{2ia^{3}c}{d(a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6})}+\frac{}{d(a^{6}+3a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a^2+b^2)^3*(1/2*(-a^3+3*a*b^2)*ln(1+tan(d*x+c)^2)+(-3*a^2*b+b^3)*ar
ctan(tan(d*x+c)))+a*(a^2-3*b^2)/(a^2+b^2)^3*ln(a+b*tan(d*x+c))-a^2*(a^2+3*b
^2)/(a^2+b^2)^2/b^2/(a+b*tan(d*x+c))+1/2*a^3/b^2/(a^2+b^2)/(a+b*tan(d*x+c))
^2)
```

Maxima [A]
time = 0.51, size = 262, normalized size = 1.76

$$-\frac{\frac{2\,(3\,a^{2}b-b^{3})(dx+c)}{a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}}-\frac{2\,(a^{3}-3\,ab^{2})\log(b\tan(dx+c)+a)}{a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}}+\frac{(a^{3}-3\,ab^{2})\log\left(\tan(dx+c)^{2}+1\right)}{a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}}+\frac{a^{5}+5\,a^{3}b^{2}+2\,(a^{4}b+3\,a^{2}b^{3})\tan(dx+c)}{a^{6}b^{2}+2\,a^{4}b^{4}+a^{2}b^{6}+(a^{4}b^{4}+2\,a^{2}b^{6}+b^{8})\tan(dx+c)^{2}+2\,(a^{5}b^{3}+2\,a^{3}b^{5}+ab^{7})\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(3*a^2*b - b^3)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 2*(
a^3 - 3*a*b^2)*log(b*tan(d*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
+ (a^3 - 3*a*b^2)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6) + (a^5 + 5*a^3*b^2 + 2*(a^4*b + 3*a^2*b^3)*tan(d*x + c))/(a^6*b^2 + 2*a^
4*b^4 + a^2*b^6 + (a^4*b^4 + 2*a^2*b^6 + b^8)*tan(d*x + c)^2 + 2*(a^5*b^3 +
 2*a^3*b^5 + a*b^7)*tan(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(145) = 290$.

time = 0.84, size = 317, normalized size = 2.13

$$\frac{a^5 - 5\,a^3b^2 - 2\,(3\,a^4b - a^2b^3)dx + (a^5 + 7\,a^3b^2 - 2\,(3\,a^2b^3 - b^5)dx)\tan(dx+c)^2 + \left(a^5 - 3\,a^3b^2 + (a^3b^2 - 3\,ab^4)\tan(dx+c)^2 + 2\,(a^4b - 3\,a^2b^3)\tan(dx+c)\right)\log\left(\frac{b^2\tan(dx+c)^2 + 2\,ab\tan(dx+c) + a^2}{\tan(dx+c)^2 + 1}\right) + 2\,(3\,a^4b - 3\,a^2b^3 - 2\,(3\,a^3b^2 - ab^4)dx)\tan(dx+c)}{2\left((a^6b^2 + 3\,a^4b^4 + 3\,a^2b^6 + b^8)d\tan(dx+c)^2 + 2\,(a^7b + 3\,a^5b^3 + 3\,a^3b^5 + ab^7)d\tan(dx+c) + (a^8 + 3\,a^6b^2 + 3\,a^4b^4 + a^2b^6)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a^5 - 5*a^3*b^2 - 2*(3*a^4*b - a^2*b^3)*d*x + (a^5 + 7*a^3*b^2 - 2*(3*
a^2*b^3 - b^5)*d*x)*tan(d*x + c)^2 + (a^5 - 3*a^3*b^2 + (a^3*b^2 - 3*a*b^4)
*tan(d*x + c)^2 + 2*(a^4*b - 3*a^2*b^3)*tan(d*x + c))*log((b^2*tan(d*x + c)
^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) + 2*(3*a^4*b - 3*a^2*b
^3 - 2*(3*a^3*b^2 - a*b^4)*d*x)*tan(d*x + c))/((a^6*b^2 + 3*a^4*b^4 + 3*a^2
*b^6 + b^8)*d*tan(d*x + c)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*
tan(d*x + c) + (a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*d)
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [A]

time = 0.92, size = 282, normalized size = 1.89

$$-\frac{\frac{2\,(3\,a^2b - b^3)(dx+c)}{a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6} + \frac{(a^3 - 3\,ab^2)\log\left(\tan(dx+c)^2 + 1\right)}{a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6} - \frac{2\,(a^3b - 3\,ab^3)\log(|b\tan(dx+c) + a|)}{a^6b + 3\,a^4b^3 + 3\,a^2b^5 + b^7} + \frac{3\,a^3b^4\tan(dx+c)^2 - 9\,ab^6\tan(dx+c)^2 + 2\,a^6b\tan(dx+c) + 14\,a^4b^3\tan(dx+c) - 12\,a^2b^5\tan(dx+c) + a^7 + 9\,a^5b^2 - 4\,a^3b^4}{(a^6b^2 + 3\,a^4b^4 + 3\,a^2b^6 + b^8)(b\tan(dx+c) + a)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*(3*a^2*b - b^3)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (a^
3 - 3*a*b^2)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) -
2*(a^3*b - 3*a*b^3)*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2
*b^5 + b^7) + (3*a^3*b^4*tan(d*x + c)^2 - 9*a*b^6*tan(d*x + c)^2 + 2*a^6*b*
tan(d*x + c) + 14*a^4*b^3*tan(d*x + c) - 12*a^2*b^5*tan(d*x + c) + a^7 + 9*
a^5*b^2 - 4*a^3*b^4)/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*(b*tan(d*x +
c) + a)^2))/d
```

Mupad [B]
time = 4.12, size = 236, normalized size = 1.58

$$\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)}{2\,d\,\left(-a^3+a^2\,b\,3\mathrm{i}+3\,a\,b^2-b^3\,1\mathrm{i}\right)}+\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{a}{{\left(a^2+b^2\right)}^2}-\frac{4\,a\,b^2}{{\left(a^2+b^2\right)}^3}\right)}{d}-\frac{\frac{a\,\left(a^4+5\,a^2\,b^2\right)}{2\,b^2\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{\tan(c+d\,x)\,\left(a^4+3\,a^2\,b^2\right)}{b\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{d\,\left(a^2+2\,a\,b\tan\left(c+d\,x\right)+b^2\tan{\left(c+d\,x\right)}^2\right)}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(-a^3\,1\mathrm{i}+3\,a^2\,b+a\,b^2\,3\mathrm{i}-b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + b*tan(c + d*x))^3,x)
```

```
[Out] (log(tan(c + d*x) - 1i)*1i)/(2*d*(a*b^2*3i + 3*a^2*b - a^3*1i - b^3)) + log
(tan(c + d*x) + 1i)/(2*d*(3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)) + (log(a + b*
tan(c + d*x))*(a/(a^2 + b^2)^2 - (4*a*b^2)/(a^2 + b^2)^3))/d - ((a*(a^4 + 5
*a^2*b^2))/(2*b^2*(a^4 + b^4 + 2*a^2*b^2)) + (tan(c + d*x)*(a^4 + 3*a^2*b^2
))/(b*(a^4 + b^4 + 2*a^2*b^2)))/(d*(a^2 + b^2*tan(c + d*x)^2 + 2*a*b*tan(c
+ d*x)))
```

3.481 $\int \frac{\tan^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=129

$$-\frac{a(a^2-3b^2)\,x}{(a^2+b^2)^3}-\frac{b(3a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^3\,d}-\frac{a^2}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx))^2}+\frac{}{(a^2+b^2}$$

```
[Out] -a*(a^2-3*b^2)*x/(a^2+b^2)^3-b*(3*a^2-b^2)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a
^2+b^2)^3/d-1/2*a^2/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^2+2*a*b/(a^2+b^2)^2/d/(a
+b*tan(d*x+c))
```

Rubi [A]
time = 0.13, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3623, 3610, 3612, 3611}

$$-\frac{a^2}{2bd\,(a^2+b^2)\,(a+b\tan(c+dx))^2}+\frac{2ab}{d\,(a^2+b^2)^2\,(a+b\tan(c+dx))}-\frac{b(3a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^3}-\frac{ax(a^2-3b^2)}{(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -((a*(a^2 - 3*b^2)*x)/(a^2 + b^2)^3) - (b*(3*a^2 - b^2)*Log[a*Cos[c + d*x]
+ b*Sin[c + d*x]])/((a^2 + b^2)^3*d) - a^2/(2*b*(a^2 + b^2)*d*(a + b*Tan[c
+ d*x])^2) + (2*a*b)/((a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
```

```
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx &= -\frac{a^2}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx))^2}+\frac{\int \frac{-a+b\tan(c+dx)}{(a+b\tan(c+dx))^2}\,dx}{a^2+b^2}\\ &= -\frac{a^2}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx))^2}+\frac{2ab}{(a^2+b^2)^2\,d(a+b\tan(c+dx))}+\frac{\int \frac{-a^2}{}}{}\\ &= -\frac{a(a^2-3b^2)\,x}{(a^2+b^2)^3}-\frac{a^2}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx))^2}+\frac{2ab}{(a^2+b^2)^2\,d(a+b\tan(}\\ &= -\frac{a(a^2-3b^2)\,x}{(a^2+b^2)^3}-\frac{b(3a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^3\,d}-\frac{}{2b\,(a^2+b^2}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.04, size = 200, normalized size = 1.55

$$-\frac{-\frac{b^2\tan^3(c+dx)}{(a+b\tan(c+dx))^2}+\frac{b\tan^2(c+dx)}{a+b\tan(c+dx)}+a\left(\frac{(ia+b)^3\log(i-\tan(c+dx))}{(a^2+b^2)^2}+\frac{i(a+ib)\log(i+\tan(c+dx))}{(a-ib)^2}-\frac{2b(-3a^2+b^2)\log(a+b\tan(c+dx))}{(a^2+b^2)^2}+\frac{2a(a-b)(a+b)}{b(a^2+b^2)(a+b\tan(c+dx))}\right)}{2a\,(a^2+b^2)\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -1/2*(-((b^2*Tan[c + d*x]^3)/(a + b*Tan[c + d*x])^2) + (b*Tan[c + d*x]^2)/(
a + b*Tan[c + d*x]) + a*(((I*a + b)^3*Log[I - Tan[c + d*x]])/(a^2 + b^2)^2
+ (I*(a + I*b)*Log[I + Tan[c + d*x]])/(a - I*b)^2 - (2*b*(-3*a^2 + b^2)*Log
[a + b*Tan[c + d*x]])/(a^2 + b^2)^2 + (2*a*(a - b)*(a + b))/(b*(a^2 + b^2)*
(a + b*Tan[c + d*x]))))/(a*(a^2 + b^2)*d)
```

Maple [A]
time = 0.15, size = 150, normalized size = 1.16

method	result
derivativedivides	$\frac{\frac{\left(3a^2b-b^3\right)\ln\left(1+\tan^2(dx+c)\right)}{2}+\left(-a^3+3b^2a\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^3}-\frac{a^2}{2\left(a^2+b^2\right)b(a+b\tan(dx+c))^2}+\frac{2ab}{\left(a^2+b^2\right)^2(a+b\tan(dx+c))}-\frac{b\left(3a\right.}{}$ over d
default	$\frac{\frac{\left(3a^2b-b^3\right)\ln\left(1+\tan^2(dx+c)\right)}{2}+\left(-a^3+3b^2a\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^3}-\frac{a^2}{2\left(a^2+b^2\right)b(a+b\tan(dx+c))^2}+\frac{2ab}{\left(a^2+b^2\right)^2(a+b\tan(dx+c))}-\frac{b\left(3a\right.}{}$ over d
norman	$\frac{\frac{a\left(-a^3+b^2a\right)}{2b\left(a^4+2a^2b^2+b^4\right)d}-\frac{\left(a^2-3b^2\right)a^3x}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}-\frac{b^3\left(\tan^2(dx+c)\right)}{\left(a^4+2a^2b^2+b^4\right)d}-\frac{2b\left(a^2-3b^2\right)a^2x\tan(dx+c)}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}-\frac{b^2\left(a^2-3b^2\right)ax\left(\tan^2(dx+c)\right)}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}}{(a+b\tan(dx+c))^2}-$
risch	$\frac{x}{3ib\,a^2-ib^3-a^3+3b^2a}+\frac{6ia^2bx}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{2ib^3x}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{6ia^2bc}{(a^6+3a^4b^2+3a^2b^4+b^6)d}-\frac{2ib^3}{d(a^6+3a^4b^2+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a^2+b^2)^3*(1/2*(3*a^2*b-b^3)*ln(1+tan(d*x+c)^2)+(-a^3+3*a*b^2)*arc
tan(tan(d*x+c)))-1/2*a^2/(a^2+b^2)/b/(a+b*tan(d*x+c))^2+2*a*b/(a^2+b^2)^2/(
a+b*tan(d*x+c))-b*(3*a^2-b^2)/(a^2+b^2)^3*ln(a+b*tan(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(127) = 254$.
time = 0.53, size = 256, normalized size = 1.98

$$-\frac{\frac{2\left(a^3-3ab^2\right)(dx+c)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{2\left(3a^2b-b^3\right)\log(b\tan(dx+c)+a)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{\left(3a^2b-b^3\right)\log\left(\tan(dx+c)^2+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{4ab^3\tan(dx+c)-a^4+3a^2b^2}{a^6b+2a^4b^3+a^2b^5+\left(a^4b^3+2a^2b^5+b^7\right)\tan(dx+c)^2+2\left(a^5b^2+2a^3b^4+ab^6\right)\tan(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(a^3 - 3*a*b^2)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(
3*a^2*b - b^3)*log(b*tan(d*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
- (3*a^2*b - b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6) - (4*a*b^3*tan(d*x + c) - a^4 + 3*a^2*b^2)/(a^6*b + 2*a^4*b^3 + a^2*b^5
+ (a^4*b^3 + 2*a^2*b^5 + b^7)*tan(d*x + c)^2 + 2*(a^5*b^2 + 2*a^3*b^4 + a*b
^6)*tan(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(127) = 254$.
time = 1.08, size = 326, normalized size = 2.53

$$-\frac{3a^4b-3a^2b^3+2\left(a^5-3a^3b^2\right)dx-\left(a^4b-5a^2b^3-2\left(a^3b^2-3ab^4\right)dx\right)\tan(dx+c)^2+\left(3a^4b-a^2b^3+\left(3a^2b^3-b^5\right)\tan(dx+c)^2+2\left(3a^3b^2-ab^4\right)\tan(dx+c)\right)\log\left(\frac{b^2\tan(dx+c)^2+2ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)-2\left(a^5-3a^3b^2+2ab^4-2\left(a^4b-3a^2b^3\right)dx\right)\tan(dx+c)}{2\left(\left(a^6b^2+3a^4b^4+3a^2b^6+b^8\right)d\tan(dx+c)^2+2\left(a^7b+3a^5b^3+3a^3b^5+ab^7\right)d\tan(dx+c)+\left(a^8+3a^6b^2+3a^4b^4+a^2b^6\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(3*a^4*b - 3*a^2*b^3 + 2*(a^5 - 3*a^3*b^2)*d*x - (a^4*b - 5*a^2*b^3 -
2*(a^3*b^2 - 3*a*b^4)*d*x)*tan(d*x + c)^2 + (3*a^4*b - a^2*b^3 + (3*a^2*b^3
 - b^5)*tan(d*x + c)^2 + 2*(3*a^3*b^2 - a*b^4)*tan(d*x + c))*log((b^2*tan(d
*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) - 2*(a^5 - 3*a^
3*b^2 + 2*a*b^4 - 2*(a^4*b - 3*a^2*b^3)*d*x)*tan(d*x + c))/((a^6*b^2 + 3*a^
4*b^4 + 3*a^2*b^6 + b^8)*d*tan(d*x + c)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^
5 + a*b^7)*d*tan(d*x + c) + (a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(127) = 254.
time = 0.81, size = 263, normalized size = 2.04

$$-\frac{\frac{2\,(a^3-3\,ab^2)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{(3\,a^2b-b^3)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\,(3\,a^2b^2-b^4)\log(|b\tan(dx+c)+a|)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}-\frac{9\,a^2b^4\tan(dx+c)^2-3\,b^6\tan(dx+c)^2+22\,a^3b^3\tan(dx+c)-2\,ab^5\tan(dx+c)-a^6+11\,a^4b^2}{(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)(b\tan(dx+c)+a)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*(a^3 - 3*a*b^2)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (3*
a^2*b - b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) +
2*(3*a^2*b^2 - b^4)*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2
*b^5 + b^7) - (9*a^2*b^4*tan(d*x + c)^2 - 3*b^6*tan(d*x + c)^2 + 22*a^3*b^3
*tan(d*x + c) - 2*a*b^5*tan(d*x + c) - a^6 + 11*a^4*b^2)/((a^6*b + 3*a^4*b^
3 + 3*a^2*b^5 + b^7)*(b*tan(d*x + c) + a)^2))/d
```

Mupad [**B**]
time = 4.09, size = 224, normalized size = 1.74

$$-\frac{\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)}{2\,d\,\left(-a^3\,1{}\mathrm{i}-3\,a^2\,b+a\,b^2\,3{}\mathrm{i}+b^3\right)}-\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{3\,b}{{\left(a^2+b^2\right)}^2}-\frac{4\,b^3}{{\left(a^2+b^2\right)}^3}\right)}{d}-\frac{\frac{a^4-3\,a^2\,b^2}{2\,b\,\left(a^4+2\,a^2\,b^2+b^4\right)}-\frac{2\,a\,b^2\tan(c+d\,x)}{a^4+2\,a^2\,b^2+b^4}}{d\,\left(a^2+2\,a\,b\tan\left(c+d\,x\right)+b^2\tan{\left(c+d\,x\right)}^2\right)}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,1{}\mathrm{i}}{2\,d\,\left(-a^3-a^2\,b\,3{}\mathrm{i}+3\,a\,b^2+b^3\,1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + b*tan(c + d*x))^3,x)
```

```
[Out] - log(tan(c + d*x) + 1i)/(2*d*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3)) - (log(a
 + b*tan(c + d*x))*((3*b)/(a^2 + b^2)^2 - (4*b^3)/(a^2 + b^2)^3))/d - (log(
tan(c + d*x) - 1i)*1i)/(2*d*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)) - ((a^4 -
3*a^2*b^2)/(2*b*(a^4 + b^4 + 2*a^2*b^2)) - (2*a*b^2*tan(c + d*x))/(a^4 + b^
4 + 2*a^2*b^2))/(d*(a^2 + b^2*tan(c + d*x)^2 + 2*a*b*tan(c + d*x)))
```

3.482 $\int \frac{\tan(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=129

$$\frac{b(3a^2-b^2)\,x}{(a^2+b^2)^3}-\frac{a(a^2-3b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^3\,d}+\frac{a}{2\,(a^2+b^2)\,d(a+b\tan(c+dx))^2}+\frac{}{(a^2+b^2)^2}$$

```
[Out] b*(3*a^2-b^2)*x/(a^2+b^2)^3-a*(a^2-3*b^2)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^
2+b^2)^3/d+1/2*a/(a^2+b^2)/d/(a+b*tan(d*x+c))^2+(a^2-b^2)/(a^2+b^2)^2/d/(a+
b*tan(d*x+c))
```

Rubi [A]
time = 0.11, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3610, 3612, 3611}

$$\frac{a}{2d\,(a^2+b^2)\,(a+b\tan(c+dx))^2}+\frac{a^2-b^2}{d\,(a^2+b^2)^2\,(a+b\tan(c+dx))}-\frac{a(a^2-3b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^3}+\frac{bx(3a^2-b^2)}{(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (b*(3*a^2 - b^2)*x)/(a^2 + b^2)^3 - (a*(a^2 - 3*b^2)*Log[a*Cos[c + d*x] + b
*Sin[c + d*x]])/((a^2 + b^2)^3*d) + a/(2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])
^2) + (a^2 - b^2)/((a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
```

```
Q[a*c + b*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan(c+dx)}{(a+b\tan(c+dx))^3}\,dx &= \frac{a}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}+\frac{\int \frac{b+a\tan(c+dx)}{(a+b\tan(c+dx))^2}\,dx}{a^2+b^2}\\
&= \frac{a}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}+\frac{a^2-b^2}{\left(a^2+b^2\right)^2d(a+b\tan(c+dx))}+\frac{\int \frac{2ab+(a^2-\ldots}{a+b\,\mathrm{t}\ldots}}{(a^2\ldots}\\
&= \frac{b(3a^2-b^2)\,x}{\left(a^2+b^2\right)^3}+\frac{a}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}+\frac{a^2-b^2}{\left(a^2+b^2\right)^2d(a+b\tan(c+d\ldots}\\
&= \frac{b(3a^2-b^2)\,x}{\left(a^2+b^2\right)^3}-\frac{a(a^2-3b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{\left(a^2+b^2\right)^3d}+\frac{\ldots}{2\left(a^2+b^2\right)d(a\ldots}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.56, size = 234, normalized size = 1.81

$$
\frac{-\frac{i\log(i-\tan(c+dx))}{(a+ib)^2}+\frac{i\log(i+\tan(c+dx))}{(a-ib)^2}+\frac{4ab\log(a+b\tan(c+dx))}{(a^2+b^2)^2}-\frac{2b}{(a^2+b^2)(a+b\tan(c+dx))}+a\left(\frac{i\log(i-\tan(c+dx))}{(a+ib)^3}-\frac{\log(i+\tan(c+dx))}{(ia+b)^3}+\frac{b\left((-6a^2+2b^2)\log(a+b\tan(c+dx))+\frac{(a^2+b^2)(5a^2+b^2+4ab\tan(c+dx))}{(a+b\tan(c+dx))^2}\right)}{(a^2+b^2)^3}\right)}{2bd}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (((-I)*Log[I - Tan[c + d*x]])/(a + I*b)^2 + (I*Log[I + Tan[c + d*x]])/(a -
I*b)^2 + (4*a*b*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^2 - (2*b)/((a^2 + b^2)
*(a + b*Tan[c + d*x])) + a*((I*Log[I - Tan[c + d*x]])/(a + I*b)^3 - Log[I +
 Tan[c + d*x]]/(I*a + b)^3 + (b*((-6*a^2 + 2*b^2)*Log[a + b*Tan[c + d*x]] +
 ((a^2 + b^2)*(5*a^2 + b^2 + 4*a*b*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2))/
(a^2 + b^2)^3))/(2*b*d)
```

Maple [A]
time = 0.17, size = 147, normalized size = 1.14

method	result
derivativedivides	$\frac{\frac{\frac{\left(a^3-3b^2a\right)\ln\left(1+\tan^2(dx+c)\right)}{2}+\left(3a^2b-b^3\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^3}+\frac{a}{2\left(a^2+b^2\right)(a+b\tan(dx+c))^2}+\frac{a^2-b^2}{\left(a^2+b^2\right)^2(a+b\tan(dx+c))}-\frac{a\left(a^2-3\ldots\right.}{\ldots}}{d}$
default	$\frac{\frac{\frac{\left(a^3-3b^2a\right)\ln\left(1+\tan^2(dx+c)\right)}{2}+\left(3a^2b-b^3\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^3}+\frac{a}{2\left(a^2+b^2\right)(a+b\tan(dx+c))^2}+\frac{a^2-b^2}{\left(a^2+b^2\right)^2(a+b\tan(dx+c))}-\frac{a\left(a^2-3\ldots\right.}{\ldots}}{d}$

norman	$\frac{\frac{(a^2b^2-b^4)\tan(dx+c)}{db(a^4+2a^2b^2+b^4)}+\frac{b^3(3a^2-b^2)x(\tan^2(dx+c))}{(a^4+2a^2b^2+b^4)(a^2+b^2)}+\frac{(3a^2-b^2)a^2bx}{(a^4+2a^2b^2+b^4)(a^2+b^2)}+\frac{a(3a^2b^2-b^4)}{2d\,b^2(a^4+2a^2b^2+b^4)}+\frac{2b^2(3a^2-b^2)ax\tan(dx}{(a^4+2a^2b^2+b^4)(a^2+}}{(a+b\tan(dx+c))^2}$
risch	$\frac{ix}{3ib\,a^2-ib^3-a^3+3b^2a}+\frac{2ia^3x}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{6ia\,b^2x}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{2ia^3c}{d(a^6+3a^4b^2+3a^2b^4+b^6)}-\frac{6i}{d(a^6+3a^4b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a^2+b^2)^3*(1/2*(a^3-3*a*b^2)*ln(1+tan(d*x+c)^2)+(3*a^2*b-b^3)*arct
an(tan(d*x+c)))+1/2*a/(a^2+b^2)/(a+b*tan(d*x+c))^2+(a^2-b^2)/(a^2+b^2)^2/(a
+b*tan(d*x+c))-a*(a^2-3*b^2)/(a^2+b^2)^3*ln(a+b*tan(d*x+c)))
```

Maxima [A]
time = 0.51, size = 253, normalized size = 1.96

$$\frac{\frac{2\,(3\,a^2b-b^3)\,(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{2\,(a^3-3\,ab^2)\log(b\tan(dx+c)+a)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{(a^3-3\,ab^2)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{3\,a^3-ab^2+2\,(a^2b-b^3)\tan(dx+c)}{a^6+2\,a^4b^2+a^2b^4+(a^4b^2+2\,a^2b^4+b^6)\tan(dx+c)^2+2\,(a^5b+2\,a^3b^3+ab^5)\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(3*a^2*b - b^3)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 2*(a
^3 - 3*a*b^2)*log(b*tan(d*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) +
 (a^3 - 3*a*b^2)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
) + (3*a^3 - a*b^2 + 2*(a^2*b - b^3)*tan(d*x + c))/(a^6 + 2*a^4*b^2 + a^2*b
^4 + (a^4*b^2 + 2*a^2*b^4 + b^6)*tan(d*x + c)^2 + 2*(a^5*b + 2*a^3*b^3 + a*
b^5)*tan(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 328 vs. 2(127) = 254.
time = 1.52, size = 328, normalized size = 2.54

$$\frac{5\,a^3b^2-ab^4+2\,(3\,a^4b-a^2b^3)dx-(3\,a^3b^2-3\,ab^4-2\,(3\,a^2b^3-b^5)dx)\tan(dx+c)^2-(a^5-3\,a^3b^2+(a^3b^2-3\,ab^4)\tan(dx+c)^2+2\,(a^4b-3\,a^2b^3)\tan(dx+c))\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)-2\,(2\,a^4b-3\,a^2b^3+b^5-2\,(3\,a^3b^2-ab^4)dx)\tan(dx+c)}{2\,((a^6b^2+3\,a^4b^4+3\,a^2b^6+b^8)d\tan(dx+c)^2+2\,(a^7b+3\,a^5b^3+3\,a^3b^5+ab^7)d\tan(dx+c)+(a^8+3\,a^6b^2+3\,a^4b^4+a^2b^6)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(5*a^3*b^2 - a*b^4 + 2*(3*a^4*b - a^2*b^3)*d*x - (3*a^3*b^2 - 3*a*b^4 -
 2*(3*a^2*b^3 - b^5)*d*x)*tan(d*x + c)^2 - (a^5 - 3*a^3*b^2 + (a^3*b^2 - 3*
a*b^4)*tan(d*x + c)^2 + 2*(a^4*b - 3*a^2*b^3)*tan(d*x + c))*log((b^2*tan(d*
x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) - 2*(2*a^4*b - 3
*a^2*b^3 + b^5 - 2*(3*a^3*b^2 - a*b^4)*d*x)*tan(d*x + c))/((a^6*b^2 + 3*a^4
*b^4 + 3*a^2*b^6 + b^8)*d*tan(d*x + c)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5
 + a*b^7)*d*tan(d*x + c) + (a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(127) = 254.
time = 0.71, size = 275, normalized size = 2.13

$$\frac{\frac{2\,(3\,a^2b-b^3)\,(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{(a^3-3\,ab^2)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{2\,(a^3b-3\,ab^3)\log(|b\tan(dx+c)+a|)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}+\frac{3\,a^3b^2\tan(dx+c)^2-9\,ab^4\tan(dx+c)^2+8\,a^4b\tan(dx+c)-18\,a^2b^3\tan(dx+c)-2\,b^5\tan(dx+c)+6\,a^5-7\,a^3b^2-ab^4}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)(b\tan(dx+c)+a)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(3*a^2*b - b^3)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (a^3
 - 3*a*b^2)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 2
*(a^3*b - 3*a*b^3)*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*
b^5 + b^7) + (3*a^3*b^2*tan(d*x + c)^2 - 9*a*b^4*tan(d*x + c)^2 + 8*a^4*b*t
an(d*x + c) - 18*a^2*b^3*tan(d*x + c) - 2*b^5*tan(d*x + c) + 6*a^5 - 7*a^3*
b^2 - a*b^4)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(b*tan(d*x + c) + a)^2))/
d
```

Mupad [B]
time = 4.01, size = 224, normalized size = 1.74

$$-\frac{\frac{a\,b^2-3\,a^3}{2\,(a^4+2\,a^2\,b^2+b^4)}-\frac{\tan(c+d\,x)\,(a^2\,b-b^3)}{a^4+2\,a^2\,b^2+b^4}}{d\,\left(a^2+2\,a\,b\tan\left(c+d\,x\right)+b^2\tan\left(c+d\,x\right)^2\right)}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)}{2\,d\,\left(-a^3+a^2\,b\,3\mathrm{i}+3\,a\,b^2-b^3\,1\mathrm{i}\right)}-\frac{a\,\ln\left(a+b\tan(c+d\,x)\right)\,(a^2-3\,b^2)}{d\,(a^2+b^2)^3}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(-a^3\,1\mathrm{i}+3\,a^2\,b+a\,b^2\,3\mathrm{i}-b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + b*tan(c + d*x))^3,x)
```

```
[Out] - ((a*b^2 - 3*a^3)/(2*(a^4 + b^4 + 2*a^2*b^2)) - (tan(c + d*x)*(a^2*b - b^3
))/(a^4 + b^4 + 2*a^2*b^2))/(d*(a^2 + b^2*tan(c + d*x)^2 + 2*a*b*tan(c + d*
x))) - (log(tan(c + d*x) - 1i)*1i)/(2*d*(a*b^2*3i + 3*a^2*b - a^3*1i - b^3)
) - log(tan(c + d*x) + 1i)/(2*d*(3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)) - (a*l
og(a + b*tan(c + d*x))*(a^2 - 3*b^2))/(d*(a^2 + b^2)^3)
```

3.483 $\int \frac{1}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=122

$$\frac{a(a^2-3b^2)\,x}{(a^2+b^2)^3}+\frac{b(3a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^3\,d}-\frac{b}{2\,(a^2+b^2)\,d(a+b\tan(c+dx))^2}-\frac{}{(a^2+b^2)^2}$$

```
[Out] a*(a^2-3*b^2)*x/(a^2+b^2)^3+b*(3*a^2-b^2)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^
2+b^2)^3/d-1/2*b/(a^2+b^2)/d/(a+b*tan(d*x+c))^2-2*a*b/(a^2+b^2)^2/d/(a+b*ta
n(d*x+c))
```

Rubi [A]
time = 0.11, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3564, 3610, 3612, 3611}

$$-\frac{2ab}{d\,(a^2+b^2)^2\,(a+b\tan(c+dx))}-\frac{b}{2d\,(a^2+b^2)\,(a+b\tan(c+dx))^2}+\frac{b(3a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^3}+\frac{ax(a^2-3b^2)}{(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^(-3),x]
```

```
[Out] (a*(a^2 - 3*b^2)*x)/(a^2 + b^2)^3 + (b*(3*a^2 - b^2)*Log[a*Cos[c + d*x] + b
*Sin[c + d*x]])/((a^2 + b^2)^3*d) - b/(2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])
^2) - (2*a*b)/((a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\tan(c+dx))^3}\,dx = -\frac{b}{2(a^2+b^2)\,d(a+b\tan(c+dx))^2} + \frac{\int \frac{a-b\tan(c+dx)}{(a+b\tan(c+dx))^2}\,dx}{a^2+b^2}$$

$$= -\frac{b}{2(a^2+b^2)\,d(a+b\tan(c+dx))^2} - \frac{2ab}{(a^2+b^2)^2\,d(a+b\tan(c+dx))} + \frac{\int \frac{a^2-b^2-}{a+b}}{(a}$$

$$= \frac{a(a^2-3b^2)\,x}{(a^2+b^2)^3} - \frac{b}{2(a^2+b^2)\,d(a+b\tan(c+dx))^2} - \frac{2ab}{(a^2+b^2)^2\,d(a+b\tan(c+a}$$

$$= \frac{a(a^2-3b^2)\,x}{(a^2+b^2)^3} + \frac{b(3a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^3\,d} - \frac{}{2(a^2+b^2)\,d(a}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.91, size = 127, normalized size = 1.04

$$\frac{\frac{\log(i-\tan(c+dx))}{(-ia+b)^3} + \frac{\log(i+\tan(c+dx))}{(ia+b)^3} + \frac{b\left((6a^2-2b^2)\log(a+b\tan(c+dx)) - \frac{(a^2+b^2)(5a^2+b^2+4ab\tan(c+dx))}{(a+b\tan(c+dx))^2}\right)}{(a^2+b^2)^3}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^(-3),x]
```

```
[Out] (Log[I - Tan[c + d*x]]/((-I)*a + b)^3 + Log[I + Tan[c + d*x]]/(I*a + b)^3 +
 (b*((6*a^2 - 2*b^2)*Log[a + b*Tan[c + d*x]] - ((a^2 + b^2)*(5*a^2 + b^2 +
4*a*b*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2))/(a^2 + b^2)^3)/(2*d)
```

Maple [A]
time = 0.14, size = 140, normalized size = 1.15

method	result

derivativedivides	$\frac{\frac{\frac{\left(-3a^2b+b^3\right)\ln\left(1+\tan^2(dx+c)\right)}{2}+\left(a^3-3b^2a\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^3}-\frac{b}{2\left(a^2+b^2\right)(a+b\tan(dx+c))^2}+\frac{b\left(3a^2-b^2\right)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^3}-}{d}$
default	$\frac{\frac{\frac{\left(-3a^2b+b^3\right)\ln\left(1+\tan^2(dx+c)\right)}{2}+\left(a^3-3b^2a\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^3}-\frac{b}{2\left(a^2+b^2\right)(a+b\tan(dx+c))^2}+\frac{b\left(3a^2-b^2\right)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^3}-}{d}$
norman	$\frac{\frac{\left(a^2-3b^2\right)a^3x}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}+\frac{b^2\left(a^2-3b^2\right)ax\left(\tan^2(dx+c)\right)}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}+\frac{-5a^2b^3-b^5}{2b^2\left(a^4+2a^2b^2+b^4\right)d}-\frac{2ab^2\tan(dx+c)}{\left(a^4+2a^2b^2+b^4\right)d}+\frac{2b\left(a^2-3b^2\right)a^2x\tan(dx+c}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}}{(a+b\tan(dx+c))^2}$
risch	$-\frac{x}{3ib\,a^2-ib^3-a^3+3b^2a}-\frac{6ia^2bx}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{2ib^3x}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{6ia^2bc}{(a^6+3a^4b^2+3a^2b^4+b^6)d}+\frac{}{d(a^6+3a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a^2+b^2)^3*(1/2*(-3*a^2*b+b^3)*ln(1+tan(d*x+c)^2)+(a^3-3*a*b^2)*arc
tan(tan(d*x+c)))-1/2*b/(a^2+b^2)/(a+b*tan(d*x+c))^2+b*(3*a^2-b^2)/(a^2+b^2)
^3*ln(a+b*tan(d*x+c))-2*a*b/(a^2+b^2)^2/(a+b*tan(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(120) = 240$.

time = 0.52, size = 248, normalized size = 2.03

$$\frac{\frac{2\,(a^3-3\,ab^2)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\,(3\,a^2b-b^3)\log(b\tan(dx+c)+a)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{(3\,a^2b-b^3)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{4\,ab^2\tan(dx+c)+5\,a^2b+b^3}{a^6+2\,a^4b^2+a^2b^4+(a^4b^2+2\,a^2b^4+b^6)\tan(dx+c)^2+2\,(a^5b+2\,a^3b^3+ab^5)\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^3 - 3*a*b^2)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(3
*a^2*b - b^3)*log(b*tan(d*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) -
 (3*a^2*b - b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
) - (4*a*b^2*tan(d*x + c) + 5*a^2*b + b^3)/(a^6 + 2*a^4*b^2 + a^2*b^4 + (a^
4*b^2 + 2*a^2*b^4 + b^6)*tan(d*x + c)^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*tan
(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(120) = 240$.

time = 1.09, size = 321, normalized size = 2.63

$$-\frac{7\,a^2b^3+b^5-2\,(a^5-3\,a^3b^2)dx-(5\,a^2b^3-b^5+2\,(a^3b^2-3\,ab^4)dx)\tan(dx+c)^2-(3\,a^4b-a^2b^3+(3\,a^2b^3-b^5)\tan(dx+c)^2+2\,(3\,a^3b^2-ab^4)\tan(dx+c))\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)-2\,(3\,a^3b^2-3\,ab^4+2\,(a^4b-3\,a^2b^3)dx)\tan(dx+c)}{2\,((a^6b^2+3\,a^4b^4+3\,a^2b^6+b^8)d\tan(dx+c)^2+2\,(a^7b+3\,a^5b^3+3\,a^3b^5+ab^7)d\tan(dx+c)+(a^8+3\,a^6b^2+3\,a^4b^4+a^2b^6)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(7*a^2*b^3 + b^5 - 2*(a^5 - 3*a^3*b^2)*d*x - (5*a^2*b^3 - b^5 + 2*(a^3
*b^2 - 3*a*b^4)*d*x)*tan(d*x + c)^2 - (3*a^4*b - a^2*b^3 + (3*a^2*b^3 - b^5
)*tan(d*x + c)^2 + 2*(3*a^3*b^2 - a*b^4)*tan(d*x + c))*log((b^2*tan(d*x + c
)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) - 2*(3*a^3*b^2 - 3*a*
b^4 + 2*(a^4*b - 3*a^2*b^3)*d*x)*tan(d*x + c))/((a^6*b^2 + 3*a^4*b^4 + 3*a^
2*b^6 + b^8)*d*tan(d*x + c)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d
*tan(d*x + c) + (a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(120) = 240.
time = 0.49, size = 265, normalized size = 2.17

$$\frac{\frac{2\,(a^3-3\,ab^2)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{(3\,a^2b-b^3)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\,(3\,a^2b^2-b^4)\log(|b\tan(dx+c)+a|)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}-\frac{9\,a^2b^3\tan(dx+c)^2-3\,b^5\tan(dx+c)^2+22\,a^3b^2\tan(dx+c)-2\,ab^4\tan(dx+c)+14\,a^4b+3\,a^2b^3+b^5}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)(b\tan(dx+c)+a)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(a^3 - 3*a*b^2)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (3*a
^2*b - b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2
*(3*a^2*b^2 - b^4)*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*
b^5 + b^7) - (9*a^2*b^3*tan(d*x + c)^2 - 3*b^5*tan(d*x + c)^2 + 22*a^3*b^2*
tan(d*x + c) - 2*a*b^4*tan(d*x + c) + 14*a^4*b + 3*a^2*b^3 + b^5)/((a^6 + 3
*a^4*b^2 + 3*a^2*b^4 + b^6)*(b*tan(d*x + c) + a)^2))/d
```

Mupad [B]
time = 4.31, size = 215, normalized size = 1.76

$$\frac{\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)}{2\,d\,\left(-a^3\,1{}\mathrm{i}-3\,a^2\,b+a\,b^2\,3{}\mathrm{i}+b^3\right)}-\frac{\frac{5\,a^2\,b+b^3}{2\,(a^4+2\,a^2\,b^2+b^4)}+\frac{2\,a\,b^2\,\tan(c+d\,x)}{a^4+2\,a^2\,b^2+b^4}}{d\,\left(a^2+2\,a\,b\tan\left(c+d\,x\right)+b^2\tan\left(c+d\,x\right)^2\right)}+\frac{b\,\ln\left(a+b\tan(c+d\,x)\right)\,\left(3\,a^2-b^2\right)}{d\,{\left(a^2+b^2\right)}^3}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,1{}\mathrm{i}}{2\,d\,\left(-a^3-a^2\,b\,3{}\mathrm{i}+3\,a\,b^2+b^3\,1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(c + d*x))^3,x)
```

```
[Out] log(tan(c + d*x) + 1i)/(2*d*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3)) + (log(tan
(c + d*x) - 1i)*1i)/(2*d*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)) - ((5*a^2*b +
 b^3)/(2*(a^4 + b^4 + 2*a^2*b^2)) + (2*a*b^2*tan(c + d*x))/(a^4 + b^4 + 2*a
^2*b^2))/(d*(a^2 + b^2*tan(c + d*x)^2 + 2*a*b*tan(c + d*x))) + (b*log(a + b
*tan(c + d*x))*(3*a^2 - b^2))/(d*(a^2 + b^2)^3)
```

3.484 $\int \frac{\cot(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=168

$$-\frac{b(3a^2-b^2)\,x}{(a^2+b^2)^3}+\frac{\log(\sin(c+dx))}{a^3d}-\frac{b^2(6a^4+3a^2b^2+b^4)\log(a\cos(c+dx)+b\sin(c+dx))}{a^3\,(a^2+b^2)^3\,d}+\frac{\ }{2a\,(a^2+b^2)\,d(a}$$

```
[Out] -b*(3*a^2-b^2)*x/(a^2+b^2)^3+ln(sin(d*x+c))/a^3/d-b^2*(6*a^4+3*a^2*b^2+b^4)
*ln(a*cos(d*x+c)+b*sin(d*x+c))/a^3/(a^2+b^2)^3/d+1/2*b^2/a/(a^2+b^2)/d/(a+b
*tan(d*x+c))^2+b^2*(3*a^2+b^2)/a^2/(a^2+b^2)^2/d/(a+b*tan(d*x+c))
```

Rubi [A]

time = 0.29, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3650, 3730, 3732, 3611, 3556}

$$\frac{\log(\sin(c+dx))}{a^3d}+\frac{b^2(3a^2+b^2)}{a^2d\,(a^2+b^2)^2\,(a+b\tan(c+dx))}+\frac{b^2}{2ad\,(a^2+b^2)\,(a+b\tan(c+dx))^2}-\frac{bx(3a^2-b^2)}{(a^2+b^2)^3}-\frac{b^2(6a^4+3a^2b^2+b^4)\log(a\cos(c+dx)+b\sin(c+dx))}{a^3d\,(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -((b*(3*a^2 - b^2)*x)/(a^2 + b^2)^3) + Log[Sin[c + d*x]]/(a^3*d) - (b^2*(6*
a^4 + 3*a^2*b^2 + b^4)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/(a^3*(a^2 + b^
2)^3*d) + b^2/(2*a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) + (b^2*(3*a^2 + b^
2))/(a^2*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
```

```
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{b^2}{2a\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{\int \frac{\cot(c+dx)\left(2\left(a^2+b^2\right)-2ab\tan(c+dx)+2b^2\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}}{2a\left(a^2+b^2\right)}$$

$$= \frac{b^2}{2a\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{b^2(3a^2+b^2)}{a^2\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))} + \frac{\int \frac{\cot(c}{}}{}$$

$$= -\frac{b(3a^2-b^2)\,x}{\left(a^2+b^2\right)^3} + \frac{b^2}{2a\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{b^2(3a^2+b^2)}{a^2\left(a^2+b^2\right)^2 d(a+b\tan}$$

$$= -\frac{b(3a^2-b^2)\,x}{\left(a^2+b^2\right)^3} + \frac{\log(\sin(c+dx))}{a^3 d} - \frac{b^2(6a^4+3a^2b^2+b^4)\log(a\cos(c+dx)+b}{a^3\left(a^2+b^2\right)^3 d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.38, size = 209, normalized size = 1.24

$$\frac{-\frac{a(a-ib)\log(i-\tan(c+dx))}{(a+ib)^2}+\frac{2(a^2+b^2)\log(\tan(c+dx))}{a^2}-\frac{a(a+ib)\log(i+\tan(c+dx))}{(a-ib)^2}-\frac{2b^2(6a^4+3a^2b^2+b^4)\log(a+b\tan(c+dx))}{a^2(a^2+b^2)^2}+\frac{b^2}{(a+b\tan(c+dx))^2}+\frac{4ab^2}{(a^2+b^2)(a+b\tan(c+dx))}+\frac{2b^2}{a^2+ab\tan(c+dx)}}{2a\left(a^2+b^2\right)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-((a*(a - I*b)*Log[I - Tan[c + d*x]])/(a + I*b)^2) + (2*(a^2 + b^2)*Log[Ta
n[c + d*x]])/a^2 - (a*(a + I*b)*Log[I + Tan[c + d*x]])/(a - I*b)^2 - (2*b^2
*(6*a^4 + 3*a^2*b^2 + b^4)*Log[a + b*Tan[c + d*x]])/(a^2*(a^2 + b^2)^2) + b
^2/(a + b*Tan[c + d*x])^2 + (4*a*b^2)/((a^2 + b^2)*(a + b*Tan[c + d*x])) +
(2*b^2)/(a^2 + a*b*Tan[c + d*x]))/(2*a*(a^2 + b^2)*d)
```

Maple [A]
time = 0.37, size = 182, normalized size = 1.08

method	result
derivativedivides	$\frac{\frac{\ln(\tan(dx+c))}{a^3}+\frac{b^2}{2\left(a^2+b^2\right)a(a+b\tan(dx+c))^2}+\frac{b^2\left(3a^2+b^2\right)}{\left(a^2+b^2\right)^2 a^2(a+b\tan(dx+c))}-\frac{b^2\left(6a^4+3a^2b^2+b^4\right)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^3 a^3}+}{d}$
default	$\frac{\frac{\ln(\tan(dx+c))}{a^3}+\frac{b^2}{2\left(a^2+b^2\right)a(a+b\tan(dx+c))^2}+\frac{b^2\left(3a^2+b^2\right)}{\left(a^2+b^2\right)^2 a^2(a+b\tan(dx+c))}-\frac{b^2\left(6a^4+3a^2b^2+b^4\right)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^3 a^3}+}{d}$
norman	$\frac{-\frac{b^3\left(3a^2-b^2\right)x\left(\tan^2(dx+c)\right)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{\left(3a^2-b^2\right)a^2bx}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{2b^2\left(3a^2-b^2\right)ax\tan(dx+c)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{2b\left(-2a^2b^2-b^4\right)\tan(dx+c)}{d\,a^2\left(a^4+2a^2b^2+b^4\right)}+}{(a+b\tan(dx+c))^2}$
risch	$-\frac{ix}{3ib\,a^2-ib^3-a^3+3b^2a}+\frac{12ia\,b^2x}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{12ia\,b^2c}{d(a^6+3a^4b^2+3a^2b^4+b^6)}+\frac{6ib^4x}{(a^6+3a^4b^2+3a^2b^4+b^6)a}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a^3*ln(tan(d*x+c))+1/2*b^2/(a^2+b^2)/a/(a+b*tan(d*x+c))^2+b^2*(3*a^2
+b^2)/(a^2+b^2)^2/a^2/(a+b*tan(d*x+c))-b^2*(6*a^4+3*a^2*b^2+b^4)/(a^2+b^2)^
3/a^3*ln(a+b*tan(d*x+c))+1/(a^2+b^2)^3*(1/2*(-a^3+3*a*b^2)*ln(1+tan(d*x+c)^
2)+(-3*a^2*b+b^3)*arctan(tan(d*x+c))))
```

Maxima [A]
time = 0.51, size = 290, normalized size = 1.73

$$-\frac{\frac{2\left(3a^2b-b^3\right)(dx+c)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{2\left(6a^4b^2+3a^2b^4+b^6\right)\log(b\tan(dx+c)+a)}{a^9+3a^7b^2+3a^5b^4+a^3b^6}+\frac{\left(a^3-3ab^2\right)\log\left(\tan(dx+c)^2+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{7a^3b^2+3ab^4+2\left(3a^2b^3+b^5\right)\tan(dx+c)}{a^8+2a^6b^2+a^4b^4+\left(a^6b^2+2a^4b^4+a^2b^6\right)\tan(dx+c)^2+2\left(a^7b+2a^5b^3+a^3b^5\right)\tan(dx+c)}-\frac{2\log(\tan(dx+c))}{a^3}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(3*a^2*b - b^3)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(
6*a^4*b^2 + 3*a^2*b^4 + b^6)*log(b*tan(d*x + c) + a)/(a^9 + 3*a^7*b^2 + 3*a
^5*b^4 + a^3*b^6) + (a^3 - 3*a*b^2)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6) - (7*a^3*b^2 + 3*a*b^4 + 2*(3*a^2*b^3 + b^5)*tan(d*x +
 c))/(a^8 + 2*a^6*b^2 + a^4*b^4 + (a^6*b^2 + 2*a^4*b^4 + a^2*b^6)*tan(d*x +
 c)^2 + 2*(a^7*b + 2*a^5*b^3 + a^3*b^5)*tan(d*x + c)) - 2*log(tan(d*x + c))
/a^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(166) = 332$.
time = 1.04, size = 494, normalized size = 2.94

$$\frac{9\,a^4b^4+3\,a^2b^6-2\,(3\,a^7b-a^5b^3)dx-(7\,a^4b^4+a^2b^6+2\,(3\,a^5b^3-a^3b^5)dx)\tan\left(dx+c\right)^2+\left(a^8+3\,a^6b^2+3\,a^4b^4+a^2b^6+(a^6b^2+3\,a^4b^4+3\,a^2b^6+b^8)\tan\left(dx+c\right)^2+2\,(a^7b+3\,a^5b^3+3\,a^3b^5+ab^7)\tan\left(dx+c\right)\right)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)-\left(6\,a^6b^2+3\,a^4b^4+a^2b^6+(6\,a^4b^4+3\,a^2b^6+b^8)\tan\left(dx+c\right)^2+2\,(6\,a^5b^3+3\,a^3b^5+ab^7)\tan\left(dx+c\right)\right)\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)-2\,(4\,a^5b^3-3\,a^3b^5-ab^7+2\,(3\,a^6b^2-a^4b^4)dx)\tan\left(dx+c\right)}{2\left((a^9b^2+3\,a^7b^4+3\,a^5b^6+a^3b^8)d\tan\left(dx+c\right)^2+2\,(a^{10}b+3\,a^8b^3+3\,a^6b^5+a^4b^7)d\tan\left(dx+c\right)+(a^{11}+3\,a^9b^2+3\,a^7b^4+a^5b^6)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(9*a^4*b^4 + 3*a^2*b^6 - 2*(3*a^7*b - a^5*b^3)*d*x - (7*a^4*b^4 + a^2*b
^6 + 2*(3*a^5*b^3 - a^3*b^5)*d*x)*tan(d*x + c)^2 + (a^8 + 3*a^6*b^2 + 3*a^4
*b^4 + a^2*b^6 + (a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*tan(d*x + c)^2 + 2
*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*tan(d*x + c))*log(tan(d*x + c)^2/(
tan(d*x + c)^2 + 1)) - (6*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 + (6*a^4*b^4 + 3*a^
2*b^6 + b^8)*tan(d*x + c)^2 + 2*(6*a^5*b^3 + 3*a^3*b^5 + a*b^7)*tan(d*x + c
))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)
) - 2*(4*a^5*b^3 - 3*a^3*b^5 - a*b^7 + 2*(3*a^6*b^2 - a^4*b^4)*d*x)*tan(d*x
 + c))/((a^9*b^2 + 3*a^7*b^4 + 3*a^5*b^6 + a^3*b^8)*d*tan(d*x + c)^2 + 2*(a
^10*b + 3*a^8*b^3 + 3*a^6*b^5 + a^4*b^7)*d*tan(d*x + c) + (a^11 + 3*a^9*b^2
 + 3*a^7*b^4 + a^5*b^6)*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [A]
time = 0.93, size = 328, normalized size = 1.95

$$-\frac{\frac{2\,(3\,a^2b-b^3)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{(a^3-3\,ab^2)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\,(6\,a^4b^3+3\,a^2b^5+b^7)\log(|b\tan(dx+c)+a|)}{a^9b+3\,a^7b^3+3\,a^5b^5+a^3b^7}-\frac{18\,a^4b^4\tan(dx+c)^2+9\,a^2b^6\tan(dx+c)^2+3\,b^8\tan(dx+c)^2+42\,a^5b^3\tan(dx+c)+26\,a^3b^5\tan(dx+c)+8\,ab^7\tan(dx+c)+25\,a^6b^2+19\,a^4b^4+6\,a^2b^6}{(a^9+3\,a^7b^2+3\,a^5b^4+a^3b^6)(b\tan(dx+c)+a)^2}-\frac{2\,\log(|\tan(dx+c)|)}{a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*(3*a^2*b - b^3)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (a^
3 - 3*a*b^2)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) +
2*(6*a^4*b^3 + 3*a^2*b^5 + b^7)*log(abs(b*tan(d*x + c) + a))/(a^9*b + 3*a^7
*b^3 + 3*a^5*b^5 + a^3*b^7) - (18*a^4*b^4*tan(d*x + c)^2 + 9*a^2*b^6*tan(d*
x + c)^2 + 3*b^8*tan(d*x + c)^2 + 42*a^5*b^3*tan(d*x + c) + 26*a^3*b^5*tan(
d*x + c) + 8*a*b^7*tan(d*x + c) + 25*a^6*b^2 + 19*a^4*b^4 + 6*a^2*b^6)/((a^
9 + 3*a^7*b^2 + 3*a^5*b^4 + a^3*b^6)*(b*tan(d*x + c) + a)^2) - 2*log(abs(ta
n(d*x + c)))/a^3)/d
```

Mupad [B]
time = 4.68, size = 256, normalized size = 1.52

$$\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)}{2\,d\,\left(-a^3+a^2\,b\,3\mathrm{i}+3\,a\,b^2-b^3\,1\mathrm{i}\right)}+\frac{\ln\left(\tan(c+d\,x)\right)}{a^3\,d}+\frac{\frac{7\,a^2\,b^2+3\,b^4}{2\,a\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{\tan(c+d\,x)\,\left(3\,a^2\,b^3+b^5\right)}{a^2\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{d\,\left(a^2+2\,a\,b\tan\left(c+d\,x\right)+b^2\tan\left(c+d\,x\right)^2\right)}-\frac{b^2\,\ln\left(a+b\tan(c+d\,x)\right)\,\left(6\,a^4+3\,a^2\,b^2+b^4\right)}{a^3\,d\,{\left(a^2+b^2\right)}^3}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(-a^3\,1\mathrm{i}+3\,a^2\,b+a\,b^2\,3\mathrm{i}-b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + b*tan(c + d*x))^3,x)
```

```
[Out] (log(tan(c + d*x) - 1i)*1i)/(2*d*(a*b^2*3i + 3*a^2*b - a^3*1i - b^3)) + log
(tan(c + d*x) + 1i)/(2*d*(3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)) + log(tan(c +
 d*x))/(a^3*d) + ((3*b^4 + 7*a^2*b^2)/(2*a*(a^4 + b^4 + 2*a^2*b^2)) + (tan(
c + d*x)*(b^5 + 3*a^2*b^3))/(a^2*(a^4 + b^4 + 2*a^2*b^2)))/(d*(a^2 + b^2*ta
n(c + d*x)^2 + 2*a*b*tan(c + d*x))) - (b^2*log(a + b*tan(c + d*x))*(6*a^4 +
 b^4 + 3*a^2*b^2))/(a^3*d*(a^2 + b^2)^3)
```

3.485 $\int \frac{\cot^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=211

$$-\frac{a(a^2-3b^2)\,x}{(a^2+b^2)^3}-\frac{3b\log(\sin(c+dx))}{a^4d}+\frac{b^3(10a^4+9a^2b^2+3b^4)\log(a\cos(c+dx)+b\sin(c+dx))}{a^4\,(a^2+b^2)^3\,d}-\frac{b}{2a^2\,(a^2+b^2}$$

```
[Out] -a*(a^2-3*b^2)*x/(a^2+b^2)^3-3*b*ln(sin(d*x+c))/a^4/d+b^3*(10*a^4+9*a^2*b^2
+3*b^4)*ln(a*cos(d*x+c)+b*sin(d*x+c))/a^4/(a^2+b^2)^3/d-1/2*b*(2*a^2+3*b^2)
/a^2/(a^2+b^2)/d/(a+b*tan(d*x+c))^2-cot(d*x+c)/a/d/(a+b*tan(d*x+c))^2-b*(a^
4+6*a^2*b^2+3*b^4)/a^3/(a^2+b^2)^2/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.43, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3650, 3730, 3732, 3611, 3556}

$$-\frac{3b\log(\sin(c+dx))}{a^4d}-\frac{b(2a^2+3b^2)}{2a^2d\,(a^2+b^2)\,(a+b\tan(c+dx))^2}-\frac{ax(a^2-3b^2)}{(a^2+b^2)^3}+\frac{b^3(10a^4+9a^2b^2+3b^4)\log(a\cos(c+dx)+b\sin(c+dx))}{a^4d\,(a^2+b^2)^3}-\frac{b(a^4+6a^2b^2+3b^4)}{a^3d\,(a^2+b^2)^2\,(a+b\tan(c+dx))}-\frac{\cot(c+dx)}{ad(a+b\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -((a*(a^2 - 3*b^2)*x)/(a^2 + b^2)^3) - (3*b*Log[Sin[c + d*x]])/(a^4*d) + (b
^3*(10*a^4 + 9*a^2*b^2 + 3*b^4)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/(a^4*
(a^2 + b^2)^3*d) - (b*(2*a^2 + 3*b^2))/(2*a^2*(a^2 + b^2)*d*(a + b*Tan[c +
d*x])^2) - Cot[c + d*x]/(a*d*(a + b*Tan[c + d*x])^2) - (b*(a^4 + 6*a^2*b^2
+ 3*b^4))/(a^3*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
```

```
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{\cot(c+dx)}{ad(a+b\tan(c+dx))^2} - \frac{\int \frac{\cot(c+dx)(3b+a\tan(c+dx)+3b\tan^2(c+dx))}{(a+b\tan(c+dx))^3}\,dx}{a}$$

$$= -\frac{b(2a^2+3b^2)}{2a^2(a^2+b^2)\,d(a+b\tan(c+dx))^2} - \frac{\cot(c+dx)}{ad(a+b\tan(c+dx))^2} - \frac{\int \frac{\cot(c+dx)(6b(a}{}$$

$$= -\frac{b(2a^2+3b^2)}{2a^2(a^2+b^2)\,d(a+b\tan(c+dx))^2} - \frac{\cot(c+dx)}{ad(a+b\tan(c+dx))^2} - \frac{b(a^4+}{a^3(a^2+b^2)^2\,c}$$

$$= -\frac{a(a^2-3b^2)\,x}{(a^2+b^2)^3} - \frac{b(2a^2+3b^2)}{2a^2(a^2+b^2)\,d(a+b\tan(c+dx))^2} - \frac{\cot(c+dx)}{ad(a+b\tan(c+dx))^2}$$

$$= -\frac{a(a^2-3b^2)\,x}{(a^2+b^2)^3} - \frac{3b\log(\sin(c+dx))}{a^4d} + \frac{b^3(10a^4+9a^2b^2+3b^4)\log(a\cos(c+dx}{a^4(a^2+b^2)^3\,d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.93, size = 178, normalized size = 0.84

$$-\frac{\frac{2\cot(c+dx)}{a^3} + \frac{b^5}{a^4(a^2+b^2)(b+a\cot(c+dx))^2} - \frac{2b^4(5a^2+3b^2)}{a^4(a^2+b^2)^2(b+a\cot(c+dx))} + \frac{\log(i-\cot(c+dx))}{(ia+b)^3} + \frac{\log(i+\cot(c+dx))}{(-ia+b)^3} - \frac{2b^3(10a^4+9a^2b^2+3b^4)\log(b+a\cot(c+dx))}{a^4(a^2+b^2)^3}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -1/2*((2*Cot[c + d*x])/a^3 + b^5/(a^4*(a^2 + b^2)*(b + a*Cot[c + d*x])^2) -
 (2*b^4*(5*a^2 + 3*b^2))/(a^4*(a^2 + b^2)^2*(b + a*Cot[c + d*x])) + Log[I -
 Cot[c + d*x]]/(I*a + b)^3 + Log[I + Cot[c + d*x]]/((-I)*a + b)^3 - (2*b^3*
(10*a^4 + 9*a^2*b^2 + 3*b^4)*Log[b + a*Cot[c + d*x]])/(a^4*(a^2 + b^2)^3))/
d
```

Maple [A]
time = 0.38, size = 201, normalized size = 0.95

method	result
derivativedivides	$\frac{-\frac{1}{a^3\tan(dx+c)} - \frac{3b\ln(\tan(dx+c))}{a^4} - \frac{b^3}{2(a^2+b^2)a^2(a+b\tan(dx+c))^2} + \frac{b^3(10a^4+9a^2b^2+3b^4)\ln(a+b\tan(dx+c))}{(a^2+b^2)^3a^4} - \frac{2b^3(2a^2+b^2}{(a^2+b^2)^2a^3(a+b\tan}}{d}$
default	$\frac{-\frac{1}{a^3\tan(dx+c)} - \frac{3b\ln(\tan(dx+c))}{a^4} - \frac{b^3}{2(a^2+b^2)a^2(a+b\tan(dx+c))^2} + \frac{b^3(10a^4+9a^2b^2+3b^4)\ln(a+b\tan(dx+c))}{(a^2+b^2)^3a^4} - \frac{2b^3(2a^2+b^2}{(a^2+b^2)^2a^3(a+b\tan}}{d}$
norman	$\frac{\frac{b(3a^4b+11a^2b^3+6b^5)(\tan^2(dx+c))}{d\,a^3(a^4+2a^2b^2+b^4)} - \frac{1}{da} + \frac{b^2(4a^4b+17a^2b^3+9b^5)(\tan^3(dx+c))}{2d\,a^4(a^4+2a^2b^2+b^4)} - \frac{(a^2-3b^2)a^3x\tan(dx+c)}{(a^4+2a^2b^2+b^4)(a^2+b^2)} - \frac{2b(a^2-3b^2)a^2x(\tan}{(a^4+2a^2b^2+b^4)(}}{\tan(dx+c)(a+b\tan(dx+c))^2}$

risch	$\frac{x}{3ib\,a^2-ib^3-a^3+3b^2a}+\frac{6ibx}{a^4}+\frac{6ibc}{a^4d}-\frac{20ib^3x}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{20ib^3c}{d(a^6+3a^4b^2+3a^2b^4+b^6)}-\frac{18ib^5x}{a^2(a^6+3a^4b^2+3a^2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a^3/tan(d*x+c)-3/a^4*b*ln(tan(d*x+c))-1/2*b^3/(a^2+b^2)/a^2/(a+b*ta
n(d*x+c))^2+b^3*(10*a^4+9*a^2*b^2+3*b^4)/(a^2+b^2)^3/a^4*ln(a+b*tan(d*x+c))
-2*b^3*(2*a^2+b^2)/(a^2+b^2)^2/a^3/(a+b*tan(d*x+c))+1/(a^2+b^2)^3*(1/2*(3*a
^2*b-b^3)*ln(1+tan(d*x+c)^2)+(-a^3+3*a*b^2)*arctan(tan(d*x+c))))
```

Maxima [A]
time = 0.50, size = 349, normalized size = 1.65

$$-\frac{\frac{2\,(a^3-3\,ab^2)\,(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{2\,(10\,a^4b^3+9\,a^2b^5+3\,b^7)\,\log(b\tan(dx+c)+a)}{a^{10}+3\,a^8b^2+3\,a^6b^4+a^4b^6}-\frac{(3\,a^2b-b^3)\,\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\,a^6+4\,a^4b^2+2\,a^2b^4+2\,(a^4b^2+6\,a^2b^4+3\,b^6)\,\tan(dx+c)^2+(4\,a^5b+17\,a^3b^3+9\,ab^5)\,\tan(dx+c)}{(a^7b^2+2\,a^5b^4+a^3b^6)\,\tan(dx+c)^3+2\,(a^8b+2\,a^6b^3+a^4b^5)\,\tan(dx+c)^2+(a^9+2\,a^7b^2+a^5b^4)\,\tan(dx+c)}+\frac{6\,b\log(\tan(dx+c))}{a^4}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(a^3 - 3*a*b^2)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 2*(
10*a^4*b^3 + 9*a^2*b^5 + 3*b^7)*log(b*tan(d*x + c) + a)/(a^10 + 3*a^8*b^2 +
 3*a^6*b^4 + a^4*b^6) - (3*a^2*b - b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^
4*b^2 + 3*a^2*b^4 + b^6) + (2*a^6 + 4*a^4*b^2 + 2*a^2*b^4 + 2*(a^4*b^2 + 6*
a^2*b^4 + 3*b^6)*tan(d*x + c)^2 + (4*a^5*b + 17*a^3*b^3 + 9*a*b^5)*tan(d*x
+ c))/((a^7*b^2 + 2*a^5*b^4 + a^3*b^6)*tan(d*x + c)^3 + 2*(a^8*b + 2*a^6*b^
3 + a^4*b^5)*tan(d*x + c)^2 + (a^9 + 2*a^7*b^2 + a^5*b^4)*tan(d*x + c)) + 6
*b*log(tan(d*x + c))/a^4)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 585 vs. $2(209) = 418$.
time = 1.03, size = 585, normalized size = 2.77

$$-\frac{2\,a^9+6\,a^7b^2+6\,a^5b^4+2\,a^3b^6-(9\,a^4b^5+3\,a^2b^7-2\,(a^7b^2-3\,a^5b^4)dx)\tan(dx+c)^3+2\,(a^7b^2-2\,a^5b^4+6\,a^3b^6+3\,ab^8+2\,(a^8b-3\,a^6b^3)dx)\tan(dx+c)^2+3\,((a^6b^3+3\,a^4b^5+3\,a^2b^7+b^9)\tan(dx+c)^3+2\,(a^7b^2+3\,a^5b^4+3\,a^3b^6+ab^8)\tan(dx+c)^2+(a^8b+3\,a^6b^3+3\,a^4b^5+a^2b^7)\tan(dx+c))\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)-((10\,a^4b^5+9\,a^2b^7+3\,b^9)\tan(dx+c)^3+2\,(10\,a^5b^4+9\,a^3b^6+3\,ab^8)\tan(dx+c)^2+(10\,a^6b^3+9\,a^4b^5+3\,a^2b^7)\tan(dx+c))\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)+(4\,a^8b+12\,a^6b^3+23\,a^4b^5+9\,a^2b^7+2\,(a^9-3\,a^7b^2)dx)\tan(dx+c)}{2\,((a^{10}b^2+3\,a^8b^4+3\,a^6b^6+a^4b^8)d\tan(dx+c)^3+2\,(a^{11}b+3\,a^9b^3+3\,a^7b^5+a^5b^7)d\tan(dx+c)^2+(a^{12}+3\,a^{10}b^2+3\,a^8b^4+a^6b^6)d\tan(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^9 + 6*a^7*b^2 + 6*a^5*b^4 + 2*a^3*b^6 - (9*a^4*b^5 + 3*a^2*b^7 -
2*(a^7*b^2 - 3*a^5*b^4)*d*x)*tan(d*x + c)^3 + 2*(a^7*b^2 - 2*a^5*b^4 + 6*a^
3*b^6 + 3*a*b^8 + 2*(a^8*b - 3*a^6*b^3)*d*x)*tan(d*x + c)^2 + 3*((a^6*b^3 +
 3*a^4*b^5 + 3*a^2*b^7 + b^9)*tan(d*x + c)^3 + 2*(a^7*b^2 + 3*a^5*b^4 + 3*a
^3*b^6 + a*b^8)*tan(d*x + c)^2 + (a^8*b + 3*a^6*b^3 + 3*a^4*b^5 + a^2*b^7)*
tan(d*x + c))*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1)) - ((10*a^4*b^5 + 9*a
^2*b^7 + 3*b^9)*tan(d*x + c)^3 + 2*(10*a^5*b^4 + 9*a^3*b^6 + 3*a*b^8)*tan(d
```

```
*x + c)^2 + (10*a^6*b^3 + 9*a^4*b^5 + 3*a^2*b^7)*tan(d*x + c))*log((b^2*tan
(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) + (4*a^8*b +
12*a^6*b^3 + 23*a^4*b^5 + 9*a^2*b^7 + 2*(a^9 - 3*a^7*b^2)*d*x)*tan(d*x + c)
)/((a^10*b^2 + 3*a^8*b^4 + 3*a^6*b^6 + a^4*b^8)*d*tan(d*x + c)^3 + 2*(a^11*
b + 3*a^9*b^3 + 3*a^7*b^5 + a^5*b^7)*d*tan(d*x + c)^2 + (a^12 + 3*a^10*b^2
+ 3*a^8*b^4 + a^6*b^6)*d*tan(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [A]
time = 1.12, size = 357, normalized size = 1.69

$$-\frac{\frac{2\,(a^3-3\,ab^2)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{(3\,a^2b-b^3)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{2\,(10\,a^4b^4+9\,a^2b^6+3\,b^8)\log(|b\tan(dx+c)+a|)}{a^{10}b+3\,a^8b^3+3\,a^6b^5+a^4b^7}+\frac{30\,a^4b^5\tan(dx+c)^2+27\,a^2b^7\tan(dx+c)^2+9\,b^9\tan(dx+c)^2+68\,a^5b^4\tan(dx+c)+66\,a^3b^6\tan(dx+c)+22\,ab^8\tan(dx+c)+39\,a^6b^3+41\,a^4b^5+14\,a^2b^7}{(a^{10}+3\,a^8b^2+3\,a^6b^4+a^4b^6)(b\tan(dx+c)+a)^2}+\frac{6\,b\log(|\tan(dx+c)|)}{a^4}-\frac{2\,(3\,b\tan(dx+c)-a)}{a^4\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*(a^3 - 3*a*b^2)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (3*
a^2*b - b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) -
2*(10*a^4*b^4 + 9*a^2*b^6 + 3*b^8)*log(abs(b*tan(d*x + c) + a))/(a^10*b + 3
*a^8*b^3 + 3*a^6*b^5 + a^4*b^7) + (30*a^4*b^5*tan(d*x + c)^2 + 27*a^2*b^7*t
an(d*x + c)^2 + 9*b^9*tan(d*x + c)^2 + 68*a^5*b^4*tan(d*x + c) + 66*a^3*b^6
*tan(d*x + c) + 22*a*b^8*tan(d*x + c) + 39*a^6*b^3 + 41*a^4*b^5 + 14*a^2*b^
7)/((a^10 + 3*a^8*b^2 + 3*a^6*b^4 + a^4*b^6)*(b*tan(d*x + c) + a)^2) + 6*b*
log(abs(tan(d*x + c)))/a^4 - 2*(3*b*tan(d*x + c) - a)/(a^4*tan(d*x + c)))/d
```

Mupad [B]
time = 4.80, size = 293, normalized size = 1.39

$$\frac{b^3\,\ln\left(a+b\,\tan(c+d\,x)\right)\,\left(10\,a^4+9\,a^2\,b^2+3\,b^4\right)}{a^4\,d\,{\left(a^2+b^2\right)}^3}-\frac{\frac{1}{a}+\frac{{\tan(c+d\,x)}^2\,\left(a^4\,b^2+6\,a^2\,b^4+3\,b^6\right)}{a^3\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{\tan(c+d\,x)\,\left(4\,a^4\,b+17\,a^2\,b^3+9\,b^5\right)}{2\,a^2\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{d\,\left(a^2\tan\left(c+d\,x\right)+2\,a\,b\tan{\left(c+d\,x\right)}^2+b^2\tan{\left(c+d\,x\right)}^3\right)}-\frac{3\,b\,\ln\left(\tan(c+d\,x)\right)}{a^4\,d}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)}{2\,d\,\left(-a^3\,1\mathrm{i}-3\,a^2\,b+a\,b^2\,3\mathrm{i}+b^3\right)}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(-a^3-a^2\,b\,3\mathrm{i}+3\,a\,b^2+b^3\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + b*tan(c + d*x))^3,x)
```

```
[Out] (b^3*log(a + b*tan(c + d*x))*(10*a^4 + 3*b^4 + 9*a^2*b^2))/(a^4*d*(a^2 + b^
2)^3) - (log(tan(c + d*x) - 1i)*1i)/(2*d*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i
)) - (1/a + (tan(c + d*x)^2*(3*b^6 + 6*a^2*b^4 + a^4*b^2))/(a^3*(a^4 + b^4
+ 2*a^2*b^2)) + (tan(c + d*x)*(4*a^4*b + 9*b^5 + 17*a^2*b^3))/(2*a^2*(a^4 +
 b^4 + 2*a^2*b^2)))/(d*(a^2*tan(c + d*x) + b^2*tan(c + d*x)^3 + 2*a*b*tan(c
 + d*x)^2)) - (3*b*log(tan(c + d*x)))/(a^4*d) - log(tan(c + d*x) + 1i)/(2*d
*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3))
```

3.486 $\int \frac{\tan^6(c+dx)}{(a+b\tan(c+dx))^4}\, dx$

Optimal. Leaf size=315

$$-\frac{(a^4-6a^2b^2+b^4)\,x}{(a^2+b^2)^4}-\frac{4ab(a^2-b^2)\log(\cos(c+dx))}{(a^2+b^2)^4\,d}-\frac{4a^3(a^6+4a^4b^2+6a^2b^4+5b^6)\log(a+b\tan(c+dx))}{b^5\,(a^2+b^2)^4\,d}$$

```
[Out] -(a^4-6*a^2*b^2+b^4)*x/(a^2+b^2)^4-4*a*b*(a^2-b^2)*ln(cos(d*x+c))/(a^2+b^2)
^4/d-4*a^3*(a^6+4*a^4*b^2+6*a^2*b^4+5*b^6)*ln(a+b*tan(d*x+c))/b^5/(a^2+b^2)
^4/d+(4*a^6+12*a^4*b^2+13*a^2*b^4+b^6)*tan(d*x+c)/b^4/(a^2+b^2)^3/d-1/3*a^2
*tan(d*x+c)^4/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^3-1/3*a^2*(2*a^2+5*b^2)*tan(d*
x+c)^3/b^2/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^2-2*a^2*(a^4+3*a^2*b^2+4*b^4)*tan
(d*x+c)^2/b^3/(a^2+b^2)^3/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.61, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3646, 3726, 3728, 3707, 3698, 31, 3556}

$$-\frac{a^2\tan^4(c+dx)}{3bd\,(a^2+b^2)\,(a+b\tan(c+dx))^3}-\frac{a^2(2a^2+5b^2)\tan^3(c+dx)}{3b^2d\,(a^2+b^2)^2\,(a+b\tan(c+dx))^2}-\frac{4ab(a^2-b^2)\log(\cos(c+dx))}{d\,(a^2+b^2)^4}-\frac{x(a^4-6a^2b^2+b^4)}{(a^2+b^2)^4}-\frac{2a^2(a^4+3a^2b^2+4b^4)\tan^2(c+dx)}{b^3d\,(a^2+b^2)^3\,(a+b\tan(c+dx))}+\frac{(4a^6+12a^4b^2+13a^2b^4+b^6)\tan(c+dx)}{b^4d\,(a^2+b^2)^3}-\frac{4a^3(a^6+4a^4b^2+6a^2b^4+5b^6)\log(a+b\tan(c+dx))}{b^5d\,(a^2+b^2)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -(((a^4 - 6*a^2*b^2 + b^4)*x)/(a^2 + b^2)^4) - (4*a*b*(a^2 - b^2)*Log[Cos[c
 + d*x]])/((a^2 + b^2)^4*d) - (4*a^3*(a^6 + 4*a^4*b^2 + 6*a^2*b^4 + 5*b^6)*
Log[a + b*Tan[c + d*x]])/(b^5*(a^2 + b^2)^4*d) + ((4*a^6 + 12*a^4*b^2 + 13*
a^2*b^4 + b^6)*Tan[c + d*x])/(b^4*(a^2 + b^2)^3*d) - (a^2*Tan[c + d*x]^4)/(
3*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^3) - (a^2*(2*a^2 + 5*b^2)*Tan[c + d*
x]^3)/(3*b^2*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^2) - (2*a^2*(a^4 + 3*a^2*
b^2 + 4*b^4)*Tan[c + d*x]^2)/(b^3*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
```

```
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
```

```
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{\tan^6(c+dx)}{(a+b\tan(c+dx))^4}\,dx = -\frac{a^2\tan^4(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^3} + \frac{\int \frac{\tan^3(c+dx)(4a^2-3ab\tan(c+dx)+(4a^2+3b^2)\tan^2(}{(a+b\tan(c+dx))^3}}{3b\,(a^2+b^2)}$$

$$= -\frac{a^2\tan^4(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^3} - \frac{a^2(2a^2+5b^2)\tan^3(c+dx)}{3b^2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))^2} +$$

$$= -\frac{a^2\tan^4(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^3} - \frac{a^2(2a^2+5b^2)\tan^3(c+dx)}{3b^2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))^2} -$$

$$= \frac{(4a^6+12a^4b^2+13a^2b^4+b^6)\tan(c+dx)}{b^4\,(a^2+b^2)^3\,d} - \frac{a^2\tan^4(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^3}$$

$$= -\frac{(a^4-6a^2b^2+b^4)\,x}{(a^2+b^2)^4} + \frac{(4a^6+12a^4b^2+13a^2b^4+b^6)\tan(c+dx)}{b^4\,(a^2+b^2)^3\,d} - \frac{}{3b\,(a^2+}$$

$$= -\frac{(a^4-6a^2b^2+b^4)\,x}{(a^2+b^2)^4} - \frac{4ab(a^2-b^2)\log(\cos(c+dx))}{(a^2+b^2)^4\,d} + \frac{(4a^6+12a^4b^2+13a^2}{b^4\,(a^2+}$$

$$= -\frac{(a^4-6a^2b^2+b^4)\,x}{(a^2+b^2)^4} - \frac{4ab(a^2-b^2)\log(\cos(c+dx))}{(a^2+b^2)^4\,d} - \frac{4a^3(a^6+4a^4b^2+6a^2}{b^5}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.53, size = 1281, normalized size = 4.07

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^6/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] ((-4*I)*(a^16*b^4 - I*a^15*b^5 + 7*a^14*b^6 - (7*I)*a^13*b^7 + 21*a^12*b^8
- (21*I)*a^11*b^9 + 36*a^10*b^10 - (36*I)*a^9*b^11 + 37*a^8*b^12 - (37*I)*a
^7*b^13 + 21*a^6*b^14 - (21*I)*a^5*b^15 + 5*a^4*b^16 - (5*I)*a^3*b^17)*(c +
 d*x)*Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)/((a - I*b)^8*(a +
 I*b)^7*b^9*d*(a + b*Tan[c + d*x])^4) + ((4*I)*(a^9 + 4*a^7*b^2 + 6*a^5*b^4
 + 5*a^3*b^6)*ArcTan[Tan[c + d*x]]*Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c
 + d*x])^4)/(b^5*(a^2 + b^2)^4*d*(a + b*Tan[c + d*x])^4) + (4*a*Log[Cos[c +
```

```
d*x]]*Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)/(b^5*d*(a + b*Ta
n[c + d*x])^4) - (2*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 5*a^3*b^6)*Log[(a*Cos[c
+ d*x] + b*Sin[c + d*x])^2]*Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x]
)^4)/(b^5*(a^2 + b^2)^4*d*(a + b*Tan[c + d*x])^4) + (Sec[c + d*x]^5*(a*Cos[
c + d*x] + b*Sin[c + d*x])*(39*a^10*b + 171*a^8*b^3 + 276*a^6*b^5 + 180*a^4
*b^7 + 45*a^2*b^9 + 9*b^11 - 9*a^7*b^4*(c + d*x) + 45*a^5*b^6*(c + d*x) + 4
5*a^3*b^8*(c + d*x) - 9*a*b^10*(c + d*x) + 12*a^10*b*Cos[2*(c + d*x)] + 32*
a^8*b^3*Cos[2*(c + d*x)] - 16*a^6*b^5*Cos[2*(c + d*x)] - 72*a^4*b^7*Cos[2*(
c + d*x)] - 48*a^2*b^9*Cos[2*(c + d*x)] - 12*b^11*Cos[2*(c + d*x)] - 12*a^7
*b^4*(c + d*x)*Cos[2*(c + d*x)] + 72*a^5*b^6*(c + d*x)*Cos[2*(c + d*x)] - 1
2*a^3*b^8*(c + d*x)*Cos[2*(c + d*x)] - 27*a^10*b*Cos[4*(c + d*x)] - 115*a^8
*b^3*Cos[4*(c + d*x)] - 196*a^6*b^5*Cos[4*(c + d*x)] - 108*a^4*b^7*Cos[4*(c
 + d*x)] + 3*a^2*b^9*Cos[4*(c + d*x)] + 3*b^11*Cos[4*(c + d*x)] - 3*a^7*b^4
*(c + d*x)*Cos[4*(c + d*x)] + 27*a^5*b^6*(c + d*x)*Cos[4*(c + d*x)] - 57*a^
3*b^8*(c + d*x)*Cos[4*(c + d*x)] + 9*a*b^10*(c + d*x)*Cos[4*(c + d*x)] + 24
*a^11*Sin[2*(c + d*x)] + 158*a^9*b^2*Sin[2*(c + d*x)] + 396*a^7*b^4*Sin[2*(
c + d*x)] + 412*a^5*b^6*Sin[2*(c + d*x)] + 168*a^3*b^8*Sin[2*(c + d*x)] + 1
8*a*b^10*Sin[2*(c + d*x)] - 18*a^6*b^5*(c + d*x)*Sin[2*(c + d*x)] + 102*a^4
*b^7*(c + d*x)*Sin[2*(c + d*x)] + 18*a^2*b^9*(c + d*x)*Sin[2*(c + d*x)] - 6
*b^11*(c + d*x)*Sin[2*(c + d*x)] + 12*a^11*Sin[4*(c + d*x)] + 35*a^9*b^2*Si
n[4*(c + d*x)] + 18*a^7*b^4*Sin[4*(c + d*x)] - 74*a^5*b^6*Sin[4*(c + d*x)]
- 78*a^3*b^8*Sin[4*(c + d*x)] - 9*a*b^10*Sin[4*(c + d*x)] - 9*a^6*b^5*(c +
d*x)*Sin[4*(c + d*x)] + 57*a^4*b^7*(c + d*x)*Sin[4*(c + d*x)] - 27*a^2*b^9*
(c + d*x)*Sin[4*(c + d*x)] + 3*b^11*(c + d*x)*Sin[4*(c + d*x)]))/(24*b^4*((
-I)*a + b)^4*(I*a + b)^4*d*(a + b*Tan[c + d*x])^4)
```

Maple [A]

time = 0.28, size = 249, normalized size = 0.79

method	result
derivativedivides	$\frac{\frac{\tan(dx+c)}{b^4}+\frac{\frac{\left(4a^3b-4a\,b^3\right)\ln\left(1+\tan^2(dx+c)\right)}{2}+\left(-a^4+6a^2b^2-b^4\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^4}-\frac{a^6}{3b^5\left(a^2+b^2\right)(a+b\tan(dx+c))^3}-\frac{a^4\left(6a^4+\right.}{b^5\left(a^2+b^2\right)^3}}{d}$
default	$\frac{\frac{\tan(dx+c)}{b^4}+\frac{\frac{\left(4a^3b-4a\,b^3\right)\ln\left(1+\tan^2(dx+c)\right)}{2}+\left(-a^4+6a^2b^2-b^4\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^4}-\frac{a^6}{3b^5\left(a^2+b^2\right)(a+b\tan(dx+c))^3}-\frac{a^4\left(6a^4+\right.}{b^5\left(a^2+b^2\right)^3}}{d}$
norman	$\frac{\frac{\tan^4(dx+c)}{bd}-\frac{a^3\left(22a^7+65a^5b^2+64a^3b^4+9a\,b^6\right)}{3d\,b^5\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}-\frac{\left(a^4-6a^2b^2+b^4\right)a^3x}{\left(a^6+3a^4b^2+3a^2b^4+b^6\right)\left(a^2+b^2\right)}-\frac{b^3\left(a^4-6a^2b^2+b^4\right)x\left(\tan^3(dx+c)\right)}{\left(a^6+3a^4b^2+3a^2b^4+b^6\right)\left(a^2+b^2\right)}-\frac{a\left(12a^7+\right.}{}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^4*tan(d*x+c)+1/(a^2+b^2)^4*(1/2*(4*a^3*b-4*a*b^3)*ln(1+tan(d*x+c)^
2)+(-a^4+6*a^2*b^2-b^4)*arctan(tan(d*x+c)))-1/3/b^5*a^6/(a^2+b^2)/(a+b*tan(
```

```
d*x+c))^3-1/b^5*a^4*(6*a^4+17*a^2*b^2+15*b^4)/(a^2+b^2)^3/(a+b*tan(d*x+c))-
4/b^5*a^3*(a^6+4*a^4*b^2+6*a^2*b^4+5*b^6)/(a^2+b^2)^4*ln(a+b*tan(d*x+c))+1/
b^5*a^5*(2*a^2+3*b^2)/(a^2+b^2)^2/(a+b*tan(d*x+c))^2)
```

Maxima [A]
time = 0.51, size = 444, normalized size = 1.41

$$-\frac{\frac{3\,(a^4-6\,a^2b^2+b^4)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{12\,(a^9+4\,a^7b^2+6\,a^5b^4+5\,a^3b^6)\log(b\tan(dx+c)+a)}{a^8b^5+4\,a^6b^7+6\,a^4b^9+4\,a^2b^{11}+b^{13}}-\frac{6\,(a^3b-ab^3)\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{13\,a^{10}+38\,a^8b^2+37\,a^6b^4+3\,(6\,a^8b^2+17\,a^6b^4+15\,a^4b^6)\tan(dx+c)^2+3\,(10\,a^9b+29\,a^7b^3+27\,a^5b^5)\tan(dx+c)}{a^9b^5+3\,a^7b^7+3\,a^5b^9+a^3b^{11}+(a^6b^8+3\,a^4b^{10}+3\,a^2b^{12}+b^{14})\tan(dx+c)^3+3\,(a^7b^7+3\,a^5b^9+3\,a^3b^{11}+ab^{13})\tan(dx+c)^2+3\,(a^8b^6+3\,a^6b^8+3\,a^4b^{10}+a^2b^{12})\tan(dx+c)}-\frac{3\,\tan(dx+c)}{b^4}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*(a^4 - 6*a^2*b^2 + b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*
a^2*b^6 + b^8) + 12*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 5*a^3*b^6)*log(b*tan(d*x
 + c) + a)/(a^8*b^5 + 4*a^6*b^7 + 6*a^4*b^9 + 4*a^2*b^11 + b^13) - 6*(a^3*b
 - a*b^3)*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6
+ b^8) + (13*a^10 + 38*a^8*b^2 + 37*a^6*b^4 + 3*(6*a^8*b^2 + 17*a^6*b^4 + 1
5*a^4*b^6)*tan(d*x + c)^2 + 3*(10*a^9*b + 29*a^7*b^3 + 27*a^5*b^5)*tan(d*x
+ c))/(a^9*b^5 + 3*a^7*b^7 + 3*a^5*b^9 + a^3*b^11 + (a^6*b^8 + 3*a^4*b^10 +
 3*a^2*b^12 + b^14)*tan(d*x + c)^3 + 3*(a^7*b^7 + 3*a^5*b^9 + 3*a^3*b^11 +
a*b^13)*tan(d*x + c)^2 + 3*(a^8*b^6 + 3*a^6*b^8 + 3*a^4*b^10 + a^2*b^12)*ta
n(d*x + c)) - 3*tan(d*x + c)/b^4)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 886 vs. $2(311) = 622$.
time = 1.15, size = 886, normalized size = 2.81

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(6*a^10*b^2 + 21*a^8*b^4 + 37*a^6*b^6 - 3*(a^8*b^4 + 4*a^6*b^6 + 6*a^4
*b^8 + 4*a^2*b^10 + b^12)*tan(d*x + c)^4 - (22*a^9*b^3 + 81*a^7*b^5 + 108*a
^5*b^7 + 36*a^3*b^9 + 9*a*b^11 - 3*(a^4*b^8 - 6*a^2*b^10 + b^12)*d*x)*tan(d
*x + c)^3 + 3*(a^7*b^5 - 6*a^5*b^7 + a^3*b^9)*d*x - 3*(10*a^10*b^2 + 34*a^8
*b^4 + 40*a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - 3*(a^5*b^7 - 6*a^3*b^9 + a*b^1
1)*d*x)*tan(d*x + c)^2 + 6*(a^12 + 4*a^10*b^2 + 6*a^8*b^4 + 5*a^6*b^6 + (a^
9*b^3 + 4*a^7*b^5 + 6*a^5*b^7 + 5*a^3*b^9)*tan(d*x + c)^3 + 3*(a^10*b^2 + 4
*a^8*b^4 + 6*a^6*b^6 + 5*a^4*b^8)*tan(d*x + c)^2 + 3*(a^11*b + 4*a^9*b^3 +
6*a^7*b^5 + 5*a^5*b^7)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*
x + c) + a^2)/(tan(d*x + c)^2 + 1)) - 6*(a^12 + 4*a^10*b^2 + 6*a^8*b^4 + 4*
a^6*b^6 + a^4*b^8 + (a^9*b^3 + 4*a^7*b^5 + 6*a^5*b^7 + 4*a^3*b^9 + a*b^11)*
tan(d*x + c)^3 + 3*(a^10*b^2 + 4*a^8*b^4 + 6*a^6*b^6 + 4*a^4*b^8 + a^2*b^10
)*tan(d*x + c)^2 + 3*(a^11*b + 4*a^9*b^3 + 6*a^7*b^5 + 4*a^5*b^7 + a^3*b^9)
*tan(d*x + c))*log(1/(tan(d*x + c)^2 + 1)) - 3*(4*a^11*b + 10*a^9*b^3 + 4*a
```

```
^7*b^5 - 23*a^5*b^7 + a^3*b^9 - 3*(a^6*b^6 - 6*a^4*b^8 + a^2*b^10)*d*x)*tan
(d*x + c))/((a^8*b^8 + 4*a^6*b^10 + 6*a^4*b^12 + 4*a^2*b^14 + b^16)*d*tan(d
*x + c)^3 + 3*(a^9*b^7 + 4*a^7*b^9 + 6*a^5*b^11 + 4*a^3*b^13 + a*b^15)*d*ta
n(d*x + c)^2 + 3*(a^10*b^6 + 4*a^8*b^8 + 6*a^6*b^10 + 4*a^4*b^12 + a^2*b^14
)*d*tan(d*x + c) + (a^11*b^5 + 4*a^9*b^7 + 6*a^7*b^9 + 4*a^5*b^11 + a^3*b^1
3)*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+b*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [A]
time = 2.69, size = 472, normalized size = 1.50

$$-\frac{\frac{3\,(a^4-6\,a^2b^2+b^4)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}-\frac{6\,(a^3b-ab^3)\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{12\,(a^9+4\,a^7b^2+6\,a^5b^4+5\,a^3b^6)\log(|b\tan(dx+c)+a|)}{a^8b^5+4\,a^6b^7+6\,a^4b^9+4\,a^2b^{11}+b^{13}}-\frac{22\,a^9b^3\tan(dx+c)^3+88\,a^7b^5\tan(dx+c)^3+132\,a^5b^7\tan(dx+c)^3+110\,a^3b^9\tan(dx+c)^3+48\,a^{10}b^2\tan(dx+c)^2+195\,a^8b^4\tan(dx+c)^2+300\,a^6b^6\tan(dx+c)^2+285\,a^4b^8\tan(dx+c)^2+36\,a^{11}b\tan(dx+c)+147\,a^9b^3\tan(dx+c)+228\,a^7b^5\tan(dx+c)+249\,a^5b^7\tan(dx+c)+9\,a^{12}+37\,a^{10}b^2+57\,a^8b^4+73\,a^6b^6}{(a^8b^5+4\,a^6b^7+6\,a^4b^9+4\,a^2b^{11}+b^{13})(b\tan(dx+c)+a)^3}-\frac{3\tan(dx+c)}{b^4}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(a^4 - 6*a^2*b^2 + b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*
a^2*b^6 + b^8) - 6*(a^3*b - a*b^3)*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2
 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 12*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 5*a^3*b
^6)*log(abs(b*tan(d*x + c) + a))/(a^8*b^5 + 4*a^6*b^7 + 6*a^4*b^9 + 4*a^2*b
^11 + b^13) - (22*a^9*b^3*tan(d*x + c)^3 + 88*a^7*b^5*tan(d*x + c)^3 + 132*
a^5*b^7*tan(d*x + c)^3 + 110*a^3*b^9*tan(d*x + c)^3 + 48*a^10*b^2*tan(d*x +
 c)^2 + 195*a^8*b^4*tan(d*x + c)^2 + 300*a^6*b^6*tan(d*x + c)^2 + 285*a^4*b
^8*tan(d*x + c)^2 + 36*a^11*b*tan(d*x + c) + 147*a^9*b^3*tan(d*x + c) + 228
*a^7*b^5*tan(d*x + c) + 249*a^5*b^7*tan(d*x + c) + 9*a^12 + 37*a^10*b^2 + 5
7*a^8*b^4 + 73*a^6*b^6)/((a^8*b^5 + 4*a^6*b^7 + 6*a^4*b^9 + 4*a^2*b^11 + b^
13)*(b*tan(d*x + c) + a)^3) - 3*tan(d*x + c)/b^4)/d
```

Mupad [B]
time = 4.89, size = 387, normalized size = 1.23

$$\frac{\tan(c+d\,x)}{b^4\,d}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)}{2\,d\,\left(a^4\,1\mathrm{i}-4\,a^3\,b-a^2\,b^2\,6\mathrm{i}+4\,a\,b^3+b^4\,1\mathrm{i}\right)}-\frac{\frac{\tan(c+d\,x)\,\left(10\,a^9+29\,a^7\,b^2+27\,a^5\,b^4\right)}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}+\frac{13\,a^{10}+38\,a^8\,b^2+37\,a^6\,b^4}{3\,b\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)}+\frac{\tan(c+d\,x)^2\,\left(6\,a^8\,b+17\,a^6\,b^3+15\,a^4\,b^5\right)}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}}{d\,\left(a^3\,b^4+3\,a^2\,b^5\tan\left(c+d\,x\right)+3\,a\,b^6\tan\left(c+d\,x\right)^2+b^7\tan\left(c+d\,x\right)^3\right)}-\frac{4\,a^3\,\ln\left(a+b\tan(c+d\,x)\right)\,\left(a^6+4\,a^4\,b^2+6\,a^2\,b^4+5\,b^6\right)}{b^5\,d\,(a^2+b^2)^4}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(a^4-a^3\,b\,4\mathrm{i}-6\,a^2\,b^2+a\,b^3\,4\mathrm{i}+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6/(a + b*tan(c + d*x))^4,x)
```

```
[Out] tan(c + d*x)/(b^4*d) - log(tan(c + d*x) - 1i)/(2*d*(4*a*b^3 - 4*a^3*b + a^4
*1i + b^4*1i - a^2*b^2*6i)) - ((tan(c + d*x)*(10*a^9 + 27*a^5*b^4 + 29*a^7*
b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (13*a^10 + 37*a^6*b^4 + 38*a^8*
b^2)/(3*b*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (tan(c + d*x)^2*(6*a^8*b +
 15*a^4*b^5 + 17*a^6*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))/(d*(a^3*b^4
 + b^7*tan(c + d*x)^3 + 3*a^2*b^5*tan(c + d*x) + 3*a*b^6*tan(c + d*x)^2)) -
 (log(tan(c + d*x) + 1i)*1i)/(2*d*(a*b^3*4i - a^3*b*4i + a^4 + b^4 - 6*a^2*
b^2)) - (4*a^3*log(a + b*tan(c + d*x))*(a^6 + 5*b^6 + 6*a^2*b^4 + 4*a^4*b^2
))/(b^5*d*(a^2 + b^2)^4)
```

3.487 $\int \frac{\tan^5(c+dx)}{(a+b\tan(c+dx))^4}\,dx$

Optimal. Leaf size=256

$$\frac{4ab(a^2-b^2)\,x}{(a^2+b^2)^4}-\frac{(a^4-6a^2b^2+b^4)\log(\cos(c+dx))}{(a^2+b^2)^4\,d}+\frac{a^2(a^6+4a^4b^2+5a^2b^4+10b^6)\log(a+b\tan(c+dx))}{b^4\,(a^2+b^2)^4\,d}-\frac{3}{}$$

```
[Out] 4*a*b*(a^2-b^2)*x/(a^2+b^2)^4-(a^4-6*a^2*b^2+b^4)*ln(cos(d*x+c))/(a^2+b^2)^
4/d+a^2*(a^6+4*a^4*b^2+5*a^2*b^4+10*b^6)*ln(a+b*tan(d*x+c))/b^4/(a^2+b^2)^4
/d-1/3*a^2*tan(d*x+c)^3/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^3-1/2*a^2*(a^2+3*b^2
)*tan(d*x+c)^2/b^2/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^2+a^3*(a^4+3*a^2*b^2+6*b^
4)/b^4/(a^2+b^2)^3/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.39, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3646, 3726, 3716, 3707, 3698, 31, 3556}

$$-\frac{a^2\tan^3(c+dx)}{3bd\,(a^2+b^2)\,(a+b\tan(c+dx))^3}-\frac{a^2(a^2+3b^2)\tan^2(c+dx)}{2b^2d\,(a^2+b^2)^2\,(a+b\tan(c+dx))^2}+\frac{4abx(a^2-b^2)}{(a^2+b^2)^4}-\frac{(a^4-6a^2b^2+b^4)\log(\cos(c+dx))}{d\,(a^2+b^2)^4}+\frac{a^2(a^6+4a^4b^2+5a^2b^4+10b^6)\log(a+b\tan(c+dx))}{b^4d\,(a^2+b^2)^4}+\frac{a^3(a^4+3a^2b^2+6b^4)}{b^4d\,(a^2+b^2)^3\,(a+b\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (4*a*b*(a^2 - b^2)*x)/(a^2 + b^2)^4 - ((a^4 - 6*a^2*b^2 + b^4)*Log[Cos[c +
d*x]])/((a^2 + b^2)^4*d) + (a^2*(a^6 + 4*a^4*b^2 + 5*a^2*b^4 + 10*b^6)*Log[
a + b*Tan[c + d*x]])/(b^4*(a^2 + b^2)^4*d) - (a^2*Tan[c + d*x]^3)/(3*b*(a^2
 + b^2)*d*(a + b*Tan[c + d*x])^3) - (a^2*(a^2 + 3*b^2)*Tan[c + d*x]^2)/(2*b
^2*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^2) + (a^3*(a^4 + 3*a^2*b^2 + 6*b^4)
)/(b^4*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
```

```
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3716

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
 -1]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
```

```
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+b\tan(c+dx))^4}\,dx = -\frac{a^2\tan^3(c+dx)}{3b\left(a^2+b^2\right)d(a+b\tan(c+dx))^3} + \frac{\int \frac{\tan^2(c+dx)\left(3a^2-3ab\tan(c+dx)+3\left(a^2+b^2\right)\tan^2(c+\right.}{(a+b\tan(c+dx))^3}}{3b\left(a^2+b^2\right)}$$

$$= -\frac{a^2\tan^3(c+dx)}{3b\left(a^2+b^2\right)d(a+b\tan(c+dx))^3} - \frac{a^2\left(a^2+3b^2\right)\tan^2(c+dx)}{2b^2\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2} + \int$$

$$= -\frac{a^2\tan^3(c+dx)}{3b\left(a^2+b^2\right)d(a+b\tan(c+dx))^3} - \frac{a^2\left(a^2+3b^2\right)\tan^2(c+dx)}{2b^2\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2} + \frac{}{b^4}$$

$$= \frac{4ab\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^4} - \frac{a^2\tan^3(c+dx)}{3b\left(a^2+b^2\right)d(a+b\tan(c+dx))^3} - \frac{a^2\left(a^2+3b^2\right)\tan^2(c-}{2b^2\left(a^2+b^2\right)^2 d(a+b\tan}$$

$$= \frac{4ab\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^4} - \frac{\left(a^4-6a^2b^2+b^4\right)\log(\cos(c+dx))}{\left(a^2+b^2\right)^4 d} - \frac{a^2\tan^3(c+dx)}{3b\left(a^2+b^2\right)d(a+b\tan(}$$

$$= \frac{4ab\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^4} - \frac{\left(a^4-6a^2b^2+b^4\right)\log(\cos(c+dx))}{\left(a^2+b^2\right)^4 d} + \frac{a^2\left(a^6+4a^4b^2+5a^2b^4+\right.}{b^4\left(a^2\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.41, size = 788, normalized size = 3.08

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -1/6*(a^4*(3*a^2 + 13*b^2)*Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])
^2)/((a - I*b)^3*(a + I*b)^3*b^2*d*(a + b*Tan[c + d*x])^4) + (4*a*(a - b)*b
*(a + b)*(c + d*x)*Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)/((a
- I*b)^4*(a + I*b)^4*d*(a + b*Tan[c + d*x])^4) + ((I*a^15*b^3 + a^14*b^4 +
(7*I)*a^13*b^5 + 7*a^12*b^6 + (20*I)*a^11*b^7 + 20*a^10*b^8 + (38*I)*a^9*b^
9 + 38*a^8*b^10 + (49*I)*a^7*b^11 + 49*a^6*b^12 + (35*I)*a^5*b^13 + 35*a^4*
b^14 + (10*I)*a^3*b^15 + 10*a^2*b^16)*(c + d*x)*Sec[c + d*x]^4*(a*Cos[c + d
*x] + b*Sin[c + d*x])^4)/((a - I*b)^8*(a + I*b)^7*b^7*d*(a + b*Tan[c + d*x]
)^4) - (I*(a^8 + 4*a^6*b^2 + 5*a^4*b^4 + 10*a^2*b^6)*ArcTan[Tan[c + d*x]]*S
ec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)/(b^4*(a^2 + b^2)^4*d*(a
+ b*Tan[c + d*x])^4) - (Log[Cos[c + d*x]]*Sec[c + d*x]^4*(a*Cos[c + d*x] +
b*Sin[c + d*x])^4)/(b^4*d*(a + b*Tan[c + d*x])^4) + ((a^8 + 4*a^6*b^2 + 5*a
^4*b^4 + 10*a^2*b^6)*Log[(a*Cos[c + d*x] + b*Sin[c + d*x])^2]*Sec[c + d*x]^
```

```
4*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)/(2*b^4*(a^2 + b^2)^4*d*(a + b*Tan[c
+ d*x])^4) + (Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^3*(-3*a^6*Si
n[c + d*x] - 11*a^4*b^2*Sin[c + d*x] - 30*a^2*b^4*Sin[c + d*x]))/(3*(a - I*
b)^3*(a + I*b)^3*b^3*d*(a + b*Tan[c + d*x])^4) - (a^4*Sec[c + d*x]^3*(a*Cos
[c + d*x] + b*Sin[c + d*x])*Tan[c + d*x])/(3*(a - I*b)^2*(a + I*b)^2*b*d*(a
 + b*Tan[c + d*x])^4)
```

Maple [A]
time = 0.29, size = 234, normalized size = 0.91 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a^2+b^2)^4*(1/2*(a^4-6*a^2*b^2+b^4)*ln(1+tan(d*x+c)^2)+(4*a^3*b-4*a
*b^3)*arctan(tan(d*x+c)))+a^2*(a^6+4*a^4*b^2+5*a^2*b^4+10*b^6)/(a^2+b^2)^4/
b^4*ln(a+b*tan(d*x+c))-1/2*a^4*(3*a^2+5*b^2)/b^4/(a^2+b^2)^2/(a+b*tan(d*x+c
))^2+1/3*a^5/b^4/(a^2+b^2)/(a+b*tan(d*x+c))^3+a^3*(3*a^4+9*a^2*b^2+10*b^4)/
b^4/(a^2+b^2)^3/(a+b*tan(d*x+c)))
```

Maxima [A]
time = 0.56, size = 433, normalized size = 1.69

$$\frac{\frac{24\,(a^3b-ab^3)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{6\,(a^8+4\,a^6b^2+5\,a^4b^4+10\,a^2b^6)\log\left(b\tan\left(dx+c\right)+a\right)}{a^8b^4+4\,a^6b^6+6\,a^4b^8+4\,a^2b^{10}+b^{12}}+\frac{3\,(a^4-6\,a^2b^2+b^4)\log\left(\tan\left(dx+c\right)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{11\,a^9+34\,a^7b^2+47\,a^5b^4+6\,(3\,a^7b^2+9\,a^5b^4+10\,a^3b^6)\tan\left(dx+c\right)^2+3\,(9\,a^8b+28\,a^6b^3+35\,a^4b^5)\tan\left(dx+c\right)}{a^9b^4+3\,a^7b^6+3\,a^5b^8+a^3b^{10}+(a^6b^7+3\,a^4b^9+3\,a^2b^{11}+b^{13})\tan\left(dx+c\right)^3+3\,(a^7b^6+3\,a^5b^8+3\,a^3b^{10}+ab^{12})\tan\left(dx+c\right)^2+3\,(a^8b^5+3\,a^6b^7+3\,a^4b^9+a^2b^{11})\tan\left(dx+c\right)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/6*(24*(a^3*b - a*b^3)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6
+ b^8) + 6*(a^8 + 4*a^6*b^2 + 5*a^4*b^4 + 10*a^2*b^6)*log(b*tan(d*x + c) +
a)/(a^8*b^4 + 4*a^6*b^6 + 6*a^4*b^8 + 4*a^2*b^10 + b^12) + 3*(a^4 - 6*a^2*b
^2 + b^4)*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6
+ b^8) + (11*a^9 + 34*a^7*b^2 + 47*a^5*b^4 + 6*(3*a^7*b^2 + 9*a^5*b^4 + 10*
a^3*b^6)*tan(d*x + c)^2 + 3*(9*a^8*b + 28*a^6*b^3 + 35*a^4*b^5)*tan(d*x + c
))/(a^9*b^4 + 3*a^7*b^6 + 3*a^5*b^8 + a^3*b^10 + (a^6*b^7 + 3*a^4*b^9 + 3*a
^2*b^11 + b^13)*tan(d*x + c)^3 + 3*(a^7*b^6 + 3*a^5*b^8 + 3*a^3*b^10 + a*b^
12)*tan(d*x + c)^2 + 3*(a^8*b^5 + 3*a^6*b^7 + 3*a^4*b^9 + a^2*b^11)*tan(d*x
 + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 784 vs. $2(252) = 504$.
time = 1.11, size = 784, normalized size = 3.06

$$\frac{3\,a^9b^2+6\,a^7b^4+47\,a^5b^6-(11\,a^8b^3+42\,a^6b^5+75\,a^4b^7-24\,(a^3b^8-ab^{10})dx)\tan\left(dx+c\right)^3+24\,(a^6b^5-a^4b^7)dx-3\,(5\,a^9b^2+18\,a^7b^4+37\,a^5b^6-20\,a^3b^8-24\,(a^4b^7-a^2b^9)dx)\tan\left(dx+c\right)^2+3\,(a^{11}+4\,a^9b^2+5\,a^7b^4+10\,a^5b^6+(a^8b^3+4\,a^6b^5+5\,a^4b^7+10\,a^2b^9)\tan\left(dx+c\right)^3+3\,(a^9b^2+4\,a^7b^4+5\,a^5b^6+10\,a^3b^8)\tan\left(dx+c\right)^2+3\,(a^{10}b+4\,a^8b^3+5\,a^6b^5+10\,a^4b^7)\tan\left(dx+c\right))\log\left(\frac{b^2\tan\left(dx+c\right)^2+2\,ab\tan\left(dx+c\right)+a^2}{\tan\left(dx+c\right)^2+1}\right)-3\,(a^{11}+4\,a^9b^2+6\,a^7b^4+4\,a^5b^6+a^3b^8+(a^8b^3+4\,a^6b^5+6\,a^4b^7+4\,a^2b^9+b^{11})\tan\left(dx+c\right)^3+3\,(a^9b^2+4\,a^7b^4+6\,a^5b^6+4\,a^3b^8+ab^{10})\tan\left(dx+c\right)^2+3\,(a^{10}b+4\,a^8b^3+6\,a^6b^5+4\,a^4b^7+a^2b^9)\tan\left(dx+c\right))\log\left(\frac{1}{\tan\left(dx+c\right)^2+1}\right)-3\,(2\,a^{10}b+5\,a^8b^3+12\,a^6b^5-35\,a^4b^7-24\,(a^5b^6-a^3b^8)dx)\tan\left(dx+c\right)}{6\,((a^8b^7+4\,a^6b^9+6\,a^4b^{11}+4\,a^2b^{13}+b^{15})d\tan\left(dx+c\right)^3+3\,(a^9b^6+4\,a^7b^8+6\,a^5b^{10}+4\,a^3b^{12}+ab^{14})d\tan\left(dx+c\right)^2+3\,(a^{10}b^5+4\,a^8b^7+6\,a^6b^9+4\,a^4b^{11}+a^2b^{13})d\tan\left(dx+c\right)+(a^{11}b^4+4\,a^9b^6+6\,a^7b^8+4\,a^5b^{10}+a^3b^{12})d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^9*b^2 + 6*a^7*b^4 + 47*a^5*b^6 - (11*a^8*b^3 + 42*a^6*b^5 + 75*a^4
*b^7 - 24*(a^3*b^8 - a*b^10)*d*x)*tan(d*x + c)^3 + 24*(a^6*b^5 - a^4*b^7)*d
*x - 3*(5*a^9*b^2 + 18*a^7*b^4 + 37*a^5*b^6 - 20*a^3*b^8 - 24*(a^4*b^7 - a^
2*b^9)*d*x)*tan(d*x + c)^2 + 3*(a^11 + 4*a^9*b^2 + 5*a^7*b^4 + 10*a^5*b^6 +
 (a^8*b^3 + 4*a^6*b^5 + 5*a^4*b^7 + 10*a^2*b^9)*tan(d*x + c)^3 + 3*(a^9*b^2
 + 4*a^7*b^4 + 5*a^5*b^6 + 10*a^3*b^8)*tan(d*x + c)^2 + 3*(a^10*b + 4*a^8*b
^3 + 5*a^6*b^5 + 10*a^4*b^7)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*
tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) - 3*(a^11 + 4*a^9*b^2 + 6*a^7*b^4
 + 4*a^5*b^6 + a^3*b^8 + (a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^1
1)*tan(d*x + c)^3 + 3*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10
)*tan(d*x + c)^2 + 3*(a^10*b + 4*a^8*b^3 + 6*a^6*b^5 + 4*a^4*b^7 + a^2*b^9)
*tan(d*x + c))*log(1/(tan(d*x + c)^2 + 1)) - 3*(2*a^10*b + 5*a^8*b^3 + 12*a
^6*b^5 - 35*a^4*b^7 - 24*(a^5*b^6 - a^3*b^8)*d*x)*tan(d*x + c))/((a^8*b^7 +
 4*a^6*b^9 + 6*a^4*b^11 + 4*a^2*b^13 + b^15)*d*tan(d*x + c)^3 + 3*(a^9*b^6
+ 4*a^7*b^8 + 6*a^5*b^10 + 4*a^3*b^12 + a*b^14)*d*tan(d*x + c)^2 + 3*(a^10*
b^5 + 4*a^8*b^7 + 6*a^6*b^9 + 4*a^4*b^11 + a^2*b^13)*d*tan(d*x + c) + (a^11
*b^4 + 4*a^9*b^6 + 6*a^7*b^8 + 4*a^5*b^10 + a^3*b^12)*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+b*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [A]
time = 2.09, size = 451, normalized size = 1.76

$$\frac{\frac{24\,(a^3b-ab^3)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{3\,(a^4-6\,a^2b^2+b^4)\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{6\,(a^8+4\,a^6b^2+5\,a^4b^4+10\,a^2b^6)\log(|b\tan(dx+c)+a|)}{a^8b^4+4\,a^6b^6+6\,a^4b^8+4\,a^2b^{10}+b^{12}}-\frac{11\,a^8b^2\tan(dx+c)^3+44\,a^6b^4\tan(dx+c)^3+55\,a^4b^6\tan(dx+c)^3+110\,a^2b^8\tan(dx+c)^3+15\,a^9b\tan(dx+c)^2+60\,a^7b^3\tan(dx+c)^2+51\,a^5b^5\tan(dx+c)^2+270\,a^3b^7\tan(dx+c)^2+6\,a^{10}\tan(dx+c)+21\,a^8b^2\tan(dx+c)-24\,a^6b^4\tan(dx+c)+225\,a^4b^6\tan(dx+c)-a^9b-26\,a^7b^3+63\,a^5b^5}{(a^8b^3+4\,a^6b^5+6\,a^4b^7+4\,a^2b^9+b^{11})(b\tan(dx+c)+a)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/6*(24*(a^3*b - a*b^3)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6
+ b^8) + 3*(a^4 - 6*a^2*b^2 + b^4)*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2
 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 6*(a^8 + 4*a^6*b^2 + 5*a^4*b^4 + 10*a^2*b
^6)*log(abs(b*tan(d*x + c) + a))/(a^8*b^4 + 4*a^6*b^6 + 6*a^4*b^8 + 4*a^2*b
^10 + b^12) - (11*a^8*b^2*tan(d*x + c)^3 + 44*a^6*b^4*tan(d*x + c)^3 + 55*a
^4*b^6*tan(d*x + c)^3 + 110*a^2*b^8*tan(d*x + c)^3 + 15*a^9*b*tan(d*x + c)^
2 + 60*a^7*b^3*tan(d*x + c)^2 + 51*a^5*b^5*tan(d*x + c)^2 + 270*a^3*b^7*tan
(d*x + c)^2 + 6*a^10*tan(d*x + c) + 21*a^8*b^2*tan(d*x + c) - 24*a^6*b^4*ta
n(d*x + c) + 225*a^4*b^6*tan(d*x + c) - a^9*b - 26*a^7*b^3 + 63*a^5*b^5)/((
```

```
a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*(b*tan(d*x + c) + a)^3)
)/d
```

Mupad [B]
time = 4.68, size = 373, normalized size = 1.46

$$\frac{\frac{11\,a^9+34\,a^7\,b^2+47\,a^5\,b^4}{6\,b^4\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{\tan(c+d\,x)\,\left(9\,a^8+28\,a^6\,b^2+35\,a^4\,b^4\right)}{2\,b^3\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{a\,\tan(c+d\,x)^2\,\left(3\,a^6+9\,a^4\,b^2+10\,a^2\,b^4\right)}{b^2\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}}{d\,\left(a^3+3\,a^2\,b\tan\left(c+d\,x\right)+3\,a\,b^2\tan\left(c+d\,x\right)^2+b^3\tan\left(c+d\,x\right)^3\right)}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)}{2\,d\,\left(a^4+a^3\,b\,4\mathrm{i}-6\,a^2\,b^2-a\,b^3\,4\mathrm{i}+b^4\right)}+\frac{a^2\,\ln\left(a+b\tan(c+d\,x)\right)\,\left(a^6+4\,a^4\,b^2+5\,a^2\,b^4+10\,b^6\right)}{b^4\,d\,{\left(a^2+b^2\right)}^4}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(a^4\,1\mathrm{i}+4\,a^3\,b-a^2\,b^2\,6\mathrm{i}-4\,a\,b^3+b^4\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + b*tan(c + d*x))^4,x)
```

```
[Out] ((11*a^9 + 47*a^5*b^4 + 34*a^7*b^2)/(6*b^4*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b
^2)) + (tan(c + d*x)*(9*a^8 + 35*a^4*b^4 + 28*a^6*b^2))/(2*b^3*(a^6 + b^6 +
 3*a^2*b^4 + 3*a^4*b^2)) + (a*tan(c + d*x)^2*(3*a^6 + 10*a^2*b^4 + 9*a^4*b^
2))/(b^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))/(d*(a^3 + b^3*tan(c + d*x)^3
 + 3*a*b^2*tan(c + d*x)^2 + 3*a^2*b*tan(c + d*x))) + (log(tan(c + d*x) + 1i
)*1i)/(2*d*(4*a^3*b - 4*a*b^3 + a^4*1i + b^4*1i - a^2*b^2*6i)) + log(tan(c
+ d*x) - 1i)/(2*d*(a^3*b*4i - a*b^3*4i + a^4 + b^4 - 6*a^2*b^2)) + (a^2*log
(a + b*tan(c + d*x))*(a^6 + 10*b^6 + 5*a^2*b^4 + 4*a^4*b^2))/(b^4*d*(a^2 +
b^2)^4)
```

3.488 $\int \frac{\tan^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx$

Optimal. Leaf size=208

$$\frac{(a^4-6a^2b^2+b^4)\,x}{(a^2+b^2)^4}+\frac{4ab(a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^4\,d}-\frac{a^2\tan^2(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^3}+\frac{}{3b^3}$$

```
[Out] (a^4-6*a^2*b^2+b^4)*x/(a^2+b^2)^4+4*a*b*(a^2-b^2)*ln(a*cos(d*x+c)+b*sin(d*x
+c))/(a^2+b^2)^4/d-1/3*a^2*tan(d*x+c)^2/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^3+1/
3*a^3*(a^2+4*b^2)/b^3/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^2-1/3*a^2*(2*a^4+7*a^2
*b^2+17*b^4)/b^3/(a^2+b^2)^3/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.28, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3646, 3716, 3709, 3612, 3611}

$$-\frac{a^2\tan^2(c+dx)}{3bd\,(a^2+b^2)\,(a+b\tan(c+dx))^3}+\frac{4ab(a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^4}+\frac{x(a^4-6a^2b^2+b^4)}{(a^2+b^2)^4}-\frac{a^2(2a^4+7a^2b^2+17b^4)}{3b^3d\,(a^2+b^2)^3\,(a+b\tan(c+dx))}+\frac{a^3(a^2+4b^2)}{3b^3d\,(a^2+b^2)^2\,(a+b\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] ((a^4 - 6*a^2*b^2 + b^4)*x)/(a^2 + b^2)^4 + (4*a*b*(a^2 - b^2)*Log[a*Cos[c
+ d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^4*d) - (a^2*Tan[c + d*x]^2)/(3*b*(a^
2 + b^2)*d*(a + b*Tan[c + d*x])^3) + (a^3*(a^2 + 4*b^2))/(3*b^3*(a^2 + b^2)
^2*d*(a + b*Tan[c + d*x])^2) - (a^2*(2*a^4 + 7*a^2*b^2 + 17*b^4))/(3*b^3*(a
^2 + b^2)^3*d*(a + b*Tan[c + d*x]))
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
```

```
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3716

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
 -1]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx = -\frac{a^2\tan^2(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^3} + \frac{\int \frac{\tan(c+dx)\left(2a^2-3ab\tan(c+dx)+\left(2a^2+3b^2\right)\tan^2(c+\right.}{(a+b\tan(c+dx))^3}}{3b\,(a^2+b^2)}$$

$$= -\frac{a^2\tan^2(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^3} + \frac{a^3(a^2+4b^2)}{3b^3\,(a^2+b^2)^2\,d(a+b\tan(c+dx))^2} + \frac{\int}{}$$

$$= -\frac{a^2\tan^2(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^3} + \frac{a^3(a^2+4b^2)}{3b^3\,(a^2+b^2)^2\,d(a+b\tan(c+dx))^2} - \frac{}{3b}$$

$$= \frac{(a^4-6a^2b^2+b^4)\,x}{(a^2+b^2)^4} - \frac{a^2\tan^2(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^3} + \frac{a^3(a^2+4}{3b^3\,(a^2+b^2)^2\,d(a+}$$

$$= \frac{(a^4-6a^2b^2+b^4)\,x}{(a^2+b^2)^4} + \frac{4ab(a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^4\,d} - \frac{}{3b\,(a^2-}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.62, size = 236, normalized size = 1.13

$$-\frac{\frac{3ib^2\log(i-\tan(c+dx))}{(a+ib)^4} - \frac{3ib^2\log(i+\tan(c+dx))}{(ia+b)^4} - \frac{24a(a-b)b^3(a+b)\log(a+b\tan(c+dx))}{(a^2+b^2)^4} + \frac{2a^4}{b(a^2+b^2)(a+b\tan(c+dx))^3} + \frac{6a\tan(c+dx)}{(a+b\tan(c+dx))^3} + \frac{6b\tan^2(c+dx)}{(a+b\tan(c+dx))^3} + \frac{6ab^3}{(a^2+b^2)^2(a+b\tan(c+dx))^2} - \frac{6b^3(-3a^2+b^2)}{(a^2+b^2)^3(a+b\tan(c+dx))}}{6b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -1/6*(((3*I)*b^2*Log[I - Tan[c + d*x]])/(a + I*b)^4 - ((3*I)*b^2*Log[I + Ta
n[c + d*x]])/(I*a + b)^4 - (24*a*(a - b)*b^3*(a + b)*Log[a + b*Tan[c + d*x]
])/(a^2 + b^2)^4 + (2*a^4)/(b*(a^2 + b^2)*(a + b*Tan[c + d*x])^3) + (6*a*Ta
n[c + d*x])/(a + b*Tan[c + d*x])^3 + (6*b*Tan[c + d*x]^2)/(a + b*Tan[c + d*
x])^3 + (6*a*b^3)/((a^2 + b^2)^2*(a + b*Tan[c + d*x])^2) - (6*b^3*(-3*a^2 +
 b^2))/((a^2 + b^2)^3*(a + b*Tan[c + d*x])))/(b^2*d)
```

Maple [A]
time = 0.18, size = 211, normalized size = 1.01

method	result
derivativedivides	$\frac{\frac{\frac{\left(-4a^3b+4a\,b^3\right)\ln\left(1+\tan^2(dx+c)\right)}{2}+\left(a^4-6a^2b^2+b^4\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^4}-\frac{a^4}{3b^3\left(a^2+b^2\right)(a+b\tan(dx+c))^3}-\frac{a^2\left(a^4+3a^2b^2+6b^4\right)}{\left(a^2+b^2\right)^3b^3(a+b\tan(dx+}}{d}$
default	$\frac{\frac{\frac{\left(-4a^3b+4a\,b^3\right)\ln\left(1+\tan^2(dx+c)\right)}{2}+\left(a^4-6a^2b^2+b^4\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^4}-\frac{a^4}{3b^3\left(a^2+b^2\right)(a+b\tan(dx+c))^3}-\frac{a^2\left(a^4+3a^2b^2+6b^4\right)}{\left(a^2+b^2\right)^3b^3(a+b\tan(dx+}}{d}$
norman	$\frac{\frac{\left(a^4-6a^2b^2+b^4\right)a^3x}{\left(a^6+3a^4b^2+3a^2b^4+b^6\right)\left(a^2+b^2\right)}+\frac{4b^3a^2\left(\tan^2(dx+c)\right)}{\left(a^6+3a^4b^2+3a^2b^4+b^6\right)d}+\frac{b^3\left(a^4-6a^2b^2+b^4\right)x\left(\tan^3(dx+c)\right)}{\left(a^6+3a^4b^2+3a^2b^4+b^6\right)\left(a^2+b^2\right)}+\frac{\left(a^7+3a^5b^2+10a^3b^4\right)\left(\tan^3(dx+\right.}{3a^2\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}}{(a+b\tan(dx+c))^3}$

risch	$-\frac{x}{4ia^3b-4ia\,b^3-a^4+6a^2b^2-b^4}-\frac{8ia^3bx}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}+\frac{8ia\,b^3x}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}-\frac{8i\ldots}{(a^8+4a^6b^2+6a\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a^2+b^2)^4*(1/2*(-4*a^3*b+4*a*b^3)*ln(1+tan(d*x+c)^2)+(a^4-6*a^2*b^
2+b^4)*arctan(tan(d*x+c)))-1/3*a^4/b^3/(a^2+b^2)/(a+b*tan(d*x+c))^3-a^2*(a^
4+3*a^2*b^2+6*b^4)/(a^2+b^2)^3/b^3/(a+b*tan(d*x+c))+4*a*b*(a^2-b^2)/(a^2+b^
2)^4*ln(a+b*tan(d*x+c))+a^3*(a^2+2*b^2)/b^3/(a^2+b^2)^2/(a+b*tan(d*x+c))^2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(202) = 404$.
time = 0.51, size = 410, normalized size = 1.97

$$\frac{\frac{3\,(a^4-6\,a^2b^2+b^4)\,(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{12\,(a^3b-ab^3)\log(b\tan(dx+c)+a)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}-\frac{6\,(a^3b-ab^3)\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}-\frac{a^8+2\,a^6b^2+13\,a^4b^4+3\,(a^6b^2+3\,a^4b^4+6\,a^2b^6)\tan(dx+c)^2+3\,(a^7b+3\,a^5b^3+10\,a^3b^5)\tan(dx+c)}{a^9b^3+3\,a^7b^5+3\,a^5b^7+a^3b^9+(a^6b^6+3\,a^4b^8+3\,a^2b^{10}+b^{12})\tan(dx+c)^3+3\,(a^7b^5+3\,a^5b^7+3\,a^3b^9+ab^{11})\tan(dx+c)^2+3\,(a^8b^4+3\,a^6b^6+3\,a^4b^8+a^2b^{10})\tan(dx+c)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(a^4 - 6*a^2*b^2 + b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a
^2*b^6 + b^8) + 12*(a^3*b - a*b^3)*log(b*tan(d*x + c) + a)/(a^8 + 4*a^6*b^2
 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 6*(a^3*b - a*b^3)*log(tan(d*x + c)^2 + 1)
/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - (a^8 + 2*a^6*b^2 + 13*a^
4*b^4 + 3*(a^6*b^2 + 3*a^4*b^4 + 6*a^2*b^6)*tan(d*x + c)^2 + 3*(a^7*b + 3*a
^5*b^3 + 10*a^3*b^5)*tan(d*x + c))/(a^9*b^3 + 3*a^7*b^5 + 3*a^5*b^7 + a^3*b
^9 + (a^6*b^6 + 3*a^4*b^8 + 3*a^2*b^10 + b^12)*tan(d*x + c)^3 + 3*(a^7*b^5
+ 3*a^5*b^7 + 3*a^3*b^9 + a*b^11)*tan(d*x + c)^2 + 3*(a^8*b^4 + 3*a^6*b^6 +
 3*a^4*b^8 + a^2*b^10)*tan(d*x + c)))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 510 vs. $2(202) = 404$.
time = 1.09, size = 510, normalized size = 2.45

$$\frac{9\,a^6b-13\,a^4b^3+(a^7+3\,a^5b^2+24\,a^3b^4+3\,(a^4b^3-6\,a^2b^5+b^7)dx)\tan(dx+c)^3+3\,(a^7-6\,a^5b^2+a^3b^4)dx-3\,(a^6b-15\,a^4b^3+6\,a^2b^5-3\,(a^5b^2-6\,a^3b^4+ab^6)dx)\tan(dx+c)^2+6\,(a^6b-a^4b^3+(a^3b^4-ab^6)\tan(dx+c)^3+3\,(a^4b^3-a^2b^5)\tan(dx+c)^2+3\,(a^5b^2-a^3b^4)\tan(dx+c))\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)-3\,(a^7-11\,a^5b^2+10\,a^3b^4-3\,(a^6b-6\,a^4b^3+a^2b^5)dx)\tan(dx+c)}{3\,((a^8b^3+4\,a^6b^5+6\,a^4b^7+4\,a^2b^9+b^{11})d\tan(dx+c)^3+3\,(a^9b^2+4\,a^7b^4+6\,a^5b^6+4\,a^3b^8+ab^{10})d\tan(dx+c)^2+3\,(a^{10}b+4\,a^8b^3+6\,a^6b^5+4\,a^4b^7+a^2b^9)d\tan(dx+c)+(a^{11}+4\,a^9b^2+6\,a^7b^4+4\,a^5b^6+a^3b^8)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(9*a^6*b - 13*a^4*b^3 + (a^7 + 3*a^5*b^2 + 24*a^3*b^4 + 3*(a^4*b^3 - 6*
a^2*b^5 + b^7)*d*x)*tan(d*x + c)^3 + 3*(a^7 - 6*a^5*b^2 + a^3*b^4)*d*x - 3*
(a^6*b - 15*a^4*b^3 + 6*a^2*b^5 - 3*(a^5*b^2 - 6*a^3*b^4 + a*b^6)*d*x)*tan(
d*x + c)^2 + 6*(a^6*b - a^4*b^3 + (a^3*b^4 - a*b^6)*tan(d*x + c)^3 + 3*(a^4
*b^3 - a^2*b^5)*tan(d*x + c)^2 + 3*(a^5*b^2 - a^3*b^4)*tan(d*x + c))*log((b
^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) - 3*(a^
```

```
7 - 11*a^5*b^2 + 10*a^3*b^4 - 3*(a^6*b - 6*a^4*b^3 + a^2*b^5)*d*x)*tan(d*x
+ c))/((a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*d*tan(d*x + c)^
3 + 3*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*d*tan(d*x + c)
^2 + 3*(a^10*b + 4*a^8*b^3 + 6*a^6*b^5 + 4*a^4*b^7 + a^2*b^9)*d*tan(d*x + c
) + (a^11 + 4*a^9*b^2 + 6*a^7*b^4 + 4*a^5*b^6 + a^3*b^8)*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+b*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 409 vs. 2(202) = 404.
time = 1.32, size = 409, normalized size = 1.97

$$\frac{\frac{3\,(a^4-6\,a^2b^2+b^4)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}-\frac{6\,(a^3b-ab^3)\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{12\,(a^3b^2-ab^4)\log(|b\tan(dx+c)+a|)}{a^8b+4\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9}-\frac{22\,a^3b^7\tan(dx+c)^3-22\,ab^9\tan(dx+c)^3+3\,a^8b^2\tan(dx+c)^2+12\,a^6b^4\tan(dx+c)^2+93\,a^4b^6\tan(dx+c)^2-48\,a^2b^8\tan(dx+c)^2+3\,a^9b\tan(dx+c)+12\,a^7b^3\tan(dx+c)+105\,a^5b^5\tan(dx+c)-36\,a^3b^7\tan(dx+c)+a^{10}+3\,a^8b^2+37\,a^6b^4-9\,a^4b^6}{(a^8b^3+4\,a^6b^5+6\,a^4b^7+4\,a^2b^9+b^{11})(b\tan(dx+c)+a)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(a^4 - 6*a^2*b^2 + b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a
^2*b^6 + b^8) - 6*(a^3*b - a*b^3)*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2
+ 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 12*(a^3*b^2 - a*b^4)*log(abs(b*tan(d*x + c
) + a))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9) - (22*a^3*b^7*tan
(d*x + c)^3 - 22*a*b^9*tan(d*x + c)^3 + 3*a^8*b^2*tan(d*x + c)^2 + 12*a^6*b
^4*tan(d*x + c)^2 + 93*a^4*b^6*tan(d*x + c)^2 - 48*a^2*b^8*tan(d*x + c)^2 +
 3*a^9*b*tan(d*x + c) + 12*a^7*b^3*tan(d*x + c) + 105*a^5*b^5*tan(d*x + c)
- 36*a^3*b^7*tan(d*x + c) + a^10 + 3*a^8*b^2 + 37*a^6*b^4 - 9*a^4*b^6)/((a^
8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*(b*tan(d*x + c) + a)^3))/
d
```

Mupad [B]
time = 4.64, size = 359, normalized size = 1.73

$$\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{4\,a\,b}{{(a^2+b^2)}^3}-\frac{8\,a\,b^3}{{(a^2+b^2)}^4}\right)}{d}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)}{2\,d\,\left(a^4\,1\mathrm{i}-4\,a^3\,b-a^2\,b^2\,6\mathrm{i}+4\,a\,b^3+b^4\,1\mathrm{i}\right)}-\frac{\frac{a^2\,\left(a^6+2\,a^4\,b^2+13\,a^2\,b^4\right)}{3\,b^3\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{{\tan(c+d\,x)}^2\,\left(a^6+3\,a^4\,b^2+6\,a^2\,b^4\right)}{b\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{a\,\tan(c+d\,x)\,\left(a^6+3\,a^4\,b^2+10\,a^2\,b^4\right)}{b^2\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}}{d\,\left(a^3+3\,a^2\,b\tan\left(c+d\,x\right)+3\,a\,b^2\,{\tan\left(c+d\,x\right)}^2+b^3\,{\tan\left(c+d\,x\right)}^3\right)}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(a^4-a^3\,b\,4\mathrm{i}-6\,a^2\,b^2+a\,b^3\,4\mathrm{i}+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + b*tan(c + d*x))^4,x)
```

```
[Out] (log(a + b*tan(c + d*x))*((4*a*b)/(a^2 + b^2)^3 - (8*a*b^3)/(a^2 + b^2)^4))
/d + log(tan(c + d*x) - 1i)/(2*d*(4*a*b^3 - 4*a^3*b + a^4*1i + b^4*1i - a^2
```

```
*b^2*6i)) + (log(tan(c + d*x) + 1i)*1i)/(2*d*(a*b^3*4i - a^3*b*4i + a^4 + b
^4 - 6*a^2*b^2)) - ((a^2*(a^6 + 13*a^2*b^4 + 2*a^4*b^2))/(3*b^3*(a^6 + b^6
+ 3*a^2*b^4 + 3*a^4*b^2)) + (tan(c + d*x)^2*(a^6 + 6*a^2*b^4 + 3*a^4*b^2))/
(b*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (a*tan(c + d*x)*(a^6 + 10*a^2*b^4
 + 3*a^4*b^2))/(b^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))/(d*(a^3 + b^3*tan
(c + d*x)^3 + 3*a*b^2*tan(c + d*x)^2 + 3*a^2*b*tan(c + d*x)))
```

3.489 $\int \frac{\tan^3(c+dx)}{(a+b\tan(c+dx))^4}\,dx$

Optimal. Leaf size=189

$$-\frac{4ab(a^2-b^2)\,x}{(a^2+b^2)^4}+\frac{(a^4-6a^2b^2+b^4)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^4\,d}-\frac{a^2\tan(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^3}-\ldots$$

```
[Out] -4*a*b*(a^2-b^2)*x/(a^2+b^2)^4+(a^4-6*a^2*b^2+b^4)*ln(a*cos(d*x+c)+b*sin(d*
x+c))/(a^2+b^2)^4/d-1/3*a^2*tan(d*x+c)/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^3-1/6
*a^2*(a^2+7*b^2)/b^2/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^2-a*(a^2-3*b^2)/(a^2+b^
2)^3/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.23, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3646, 3709, 3610, 3612, 3611}

$$-\frac{a^2(a^2+7b^2)}{6b^2d\,(a^2+b^2)^2\,(a+b\tan(c+dx))^2}-\frac{a^2\tan(c+dx)}{3bd\,(a^2+b^2)\,(a+b\tan(c+dx))^3}-\frac{a(a^2-3b^2)}{d\,(a^2+b^2)^3\,(a+b\tan(c+dx))}-\frac{4abx(a^2-b^2)}{(a^2+b^2)^4}+\frac{(a^4-6a^2b^2+b^4)\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (-4*a*b*(a^2 - b^2)*x)/(a^2 + b^2)^4 + ((a^4 - 6*a^2*b^2 + b^4)*Log[a*Cos[c
 + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^4*d) - (a^2*Tan[c + d*x])/(3*b*(a^2
 + b^2)*d*(a + b*Tan[c + d*x])^3) - (a^2*(a^2 + 7*b^2))/(6*b^2*(a^2 + b^2)^
2*d*(a + b*Tan[c + d*x])^2) - (a*(a^2 - 3*b^2))/((a^2 + b^2)^3*d*(a + b*Tan
[c + d*x]))
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
```

```
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{(a+b\tan(c+dx))^4}\,dx = -\frac{a^2\tan(c+dx)}{3b\left(a^2+b^2\right)d(a+b\tan(c+dx))^3}+\frac{\int \frac{a^2-3ab\tan(c+dx)+\left(a^2+3b^2\right)\tan^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx}{3b\left(a^2+b^2\right)}$$

$$= -\frac{a^2\tan(c+dx)}{3b\left(a^2+b^2\right)d(a+b\tan(c+dx))^3}-\frac{a^2(a^2+7b^2)}{6b^2\left(a^2+b^2\right)^2d(a+b\tan(c+dx))^2}+$$

$$= -\frac{a^2\tan(c+dx)}{3b\left(a^2+b^2\right)d(a+b\tan(c+dx))^3}-\frac{a^2(a^2+7b^2)}{6b^2\left(a^2+b^2\right)^2d(a+b\tan(c+dx))^2}-$$

$$= -\frac{4ab(a^2-b^2)\,x}{\left(a^2+b^2\right)^4}-\frac{a^2\tan(c+dx)}{3b\left(a^2+b^2\right)d(a+b\tan(c+dx))^3}-\frac{a^2(a^2+7b}{6b^2\left(a^2+b^2\right)^2d(a+b}$$

$$= -\frac{4ab(a^2-b^2)\,x}{\left(a^2+b^2\right)^4}+\frac{\left(a^4-6a^2b^2+b^4\right)\log(a\cos(c+dx)+b\sin(c+dx))}{\left(a^2+b^2\right)^4d}-\frac{}{3b\,(}$$

Mathematica [C] Result contains complex when optimal does not.

time = 6.24, size = 387, normalized size = 2.05

$$-\frac{\tan(c+dx)}{2bd(a+b\tan(c+dx))^3}-\frac{\frac{a}{3bd(a+b\tan(c+dx))^3}+\frac{2b\left(-\frac{a\left(-\frac{i\log(i-\tan(c+dx))}{2(a+ib)^4}+\frac{i\log(i+\tan(c+dx))}{2(a-ib)^4}+\frac{4a(a-b)b(a+b)\log(a+b\tan(c+dx))}{(a^2+b^2)^4}-\frac{b}{3(a^2+b^2)(a+b\tan(c+dx))^3}-\frac{ab}{(a^2+b^2)^2(a+b\tan(c+dx))^2}-\frac{b(3a^2-b^2)}{(a^2+b^2)^3(a+b\tan(c+dx))}\right)}{b}+\frac{-\frac{\log(i-\tan(c+dx))}{2(ia-b)^3}+\frac{\log(i+\tan(c+dx))}{2(ia+b)^3}+\frac{b(3a^2-b^2)\log(a+b\tan(c+dx))}{(a^2+b^2)^3}-\frac{b}{2(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{2ab}{(a^2+b^2)^2(a+b\tan(c+dx))}}{b}\right)}{d}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -1/2*Tan[c + d*x]/(b*d*(a + b*Tan[c + d*x])^3) - (a/(3*b*d*(a + b*Tan[c + d
*x])^3) + (2*b*(-((a*(((-1/2*I)*Log[I - Tan[c + d*x]])/(a + I*b)^4 + ((I/2)
*Log[I + Tan[c + d*x]])/(a - I*b)^4 + (4*a*(a - b)*b*(a + b)*Log[a + b*Tan[
c + d*x]])/(a^2 + b^2)^4 - b/(3*(a^2 + b^2)*(a + b*Tan[c + d*x])^3) - (a*b)
/((a^2 + b^2)^2*(a + b*Tan[c + d*x])^2) - (b*(3*a^2 - b^2))/((a^2 + b^2)^3*
(a + b*Tan[c + d*x]))))/b) + (-1/2*Log[I - Tan[c + d*x]]/(I*a - b)^3 + Log[
I + Tan[c + d*x]]/(2*(I*a + b)^3) + (b*(3*a^2 - b^2)*Log[a + b*Tan[c + d*x]
])/(a^2 + b^2)^3 - b/(2*(a^2 + b^2)*(a + b*Tan[c + d*x])^2) - (2*a*b)/((a^2
 + b^2)^2*(a + b*Tan[c + d*x])))/b))/d)/(2*b)
```

Maple [A]
time = 0.22, size = 206, normalized size = 1.09

method	result
derivativedivides	$\frac{\frac{\frac{(-a^4+6a^2b^2-b^4)\ln(1+\tan^2(dx+c))}{2}+(-4a^3b+4ab^3)\arctan(\tan(dx+c))}{(a^2+b^2)^4}+\frac{(a^4-6a^2b^2+b^4)\ln(a+b\tan(dx+c))}{(a^2+b^2)^4}-\frac{a(a^2-3b^2}{(a^2+b^2)^3(a+b\tan}}{d}$
default	$\frac{\frac{\frac{(-a^4+6a^2b^2-b^4)\ln(1+\tan^2(dx+c))}{2}+(-4a^3b+4ab^3)\arctan(\tan(dx+c))}{(a^2+b^2)^4}+\frac{(a^4-6a^2b^2+b^4)\ln(a+b\tan(dx+c))}{(a^2+b^2)^4}-\frac{a(a^2-3b^2}{(a^2+b^2)^3(a+b\tan}}{d}$
norman	$\frac{\frac{(a^6+6a^4b^2-3a^2b^4)(\tan^2(dx+c))}{2ad(a^6+3a^4b^2+3a^2b^4+b^6)}+\frac{a(-3a^4b+a^2b^3)}{3db(a^6+3a^4b^2+3a^2b^4+b^6)}+\frac{b(a^6+8a^4b^2-9a^2b^4)(\tan^3(dx+c))}{6a^2d(a^6+3a^4b^2+3a^2b^4+b^6)}-\frac{4(a^2-b^2)a^4bx}{(a^6+3a^4b^2+3a^2b^4+b^6)(a^2}}{(a+b\tan(dx+c))^3}$
risch	$-\frac{ix}{4ia^3b-4iab^3-a^4+6a^2b^2-b^4}-\frac{2ia^4x}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}+\frac{12ia^2b^2x}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}-\frac{2ib^4x}{a^8+4a^6b^2+6a^4b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a^2+b^2)^4*(1/2*(-a^4+6*a^2*b^2-b^4)*ln(1+tan(d*x+c)^2)+(-4*a^3*b+4
*a*b^3)*arctan(tan(d*x+c)))+(a^4-6*a^2*b^2+b^4)/(a^2+b^2)^4*ln(a+b*tan(d*x+
c))-a*(a^2-3*b^2)/(a^2+b^2)^3/(a+b*tan(d*x+c))-1/2*a^2*(a^2+3*b^2)/(a^2+b^2
)^2/b^2/(a+b*tan(d*x+c))^2+1/3*a^3/b^2/(a^2+b^2)/(a+b*tan(d*x+c))^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 402 vs. 2(185) = 370.
time = 0.50, size = 402, normalized size = 2.13

$$-\frac{\frac{24\,(a^3b-ab^3)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}-\frac{6\,(a^4-6\,a^2b^2+b^4)\log(b\tan(dx+c)+a)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{3\,(a^4-6\,a^2b^2+b^4)\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{a^7+14\,a^5b^2-11\,a^3b^4+6\,(a^3b^4-3\,ab^6)\tan(dx+c)^2+3\,(a^6b+8\,a^4b^3-9\,a^2b^5)\tan(dx+c)}{a^9b^2+3\,a^7b^4+3\,a^5b^6+a^3b^8+(a^6b^5+3\,a^4b^7+3\,a^2b^9+b^{11})\tan(dx+c)^3+3\,(a^7b^4+3\,a^5b^6+3\,a^3b^8+ab^{10})\tan(dx+c)^2+3\,(a^8b^3+3\,a^6b^5+3\,a^4b^7+a^2b^9)\tan(dx+c)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/6*(24*(a^3*b - a*b^3)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6
 + b^8) - 6*(a^4 - 6*a^2*b^2 + b^4)*log(b*tan(d*x + c) + a)/(a^8 + 4*a^6*b^
2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 3*(a^4 - 6*a^2*b^2 + b^4)*log(tan(d*x +
c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + (a^7 + 14*a^5*b
^2 - 11*a^3*b^4 + 6*(a^3*b^4 - 3*a*b^6)*tan(d*x + c)^2 + 3*(a^6*b + 8*a^4*b
^3 - 9*a^2*b^5)*tan(d*x + c))/(a^9*b^2 + 3*a^7*b^4 + 3*a^5*b^6 + a^3*b^8 +
(a^6*b^5 + 3*a^4*b^7 + 3*a^2*b^9 + b^11)*tan(d*x + c)^3 + 3*(a^7*b^4 + 3*a^
5*b^6 + 3*a^3*b^8 + a*b^10)*tan(d*x + c)^2 + 3*(a^8*b^3 + 3*a^6*b^5 + 3*a^4
*b^7 + a^2*b^9)*tan(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(185) = 370$.
time = 1.06, size = 526, normalized size = 2.78

$$\frac{3 a^7 - 30 a^5 b^2 + 11 a^3 b^4 + (a^6 b + 18 a^4 b^3 - 27 a^2 b^5 - 24 (a^3 b^4 - a b^6) d x) \tan (d x + c)^3 - 24 (a^6 b - a^4 b^3) d x + 3 (a^7 + 16 a^5 b^2 - 23 a^3 b^4 + 6 a b^6 - 24 (a^4 b^3 - a^2 b^5) d x) \tan (d x + c)^2 + 3 \left(a^7 - 6 a^5 b^2 + a^3 b^4 + (a^4 b^3 - 6 a^2 b^5 + b^7) \tan (d x + c)^3 + 3 (a^5 b^2 - 6 a^3 b^4 + a b^6) \tan (d x + c)^2 + 3 (a^6 b - 6 a^4 b^3 + a^2 b^5) \tan (d x + c)\right) \log \left(\frac{b^2 \tan(d x + c)^2 + 2 a b \tan(d x + c) + a^2}{\tan(d x + c)^2 + 1}\right) + 3 (9 a^6 b - 26 a^4 b^3 + 9 a^2 b^5 - 24 (a^5 b^2 - a^3 b^4) d x) \tan (d x + c)}{6 \left((a^8 b^3 + 4 a^6 b^5 + 6 a^4 b^7 + 4 a^2 b^9 + b^{11}) d \tan (d x + c)^3 + 3 (a^9 b^2 + 4 a^7 b^4 + 6 a^5 b^6 + 4 a^3 b^8 + a b^{10}) d \tan (d x + c)^2 + 3 (a^{10} b + 4 a^8 b^3 + 6 a^6 b^5 + 4 a^4 b^7 + a^2 b^9) d \tan (d x + c) + (a^{11} + 4 a^9 b^2 + 6 a^7 b^4 + 4 a^5 b^6 + a^3 b^8) d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^7 - 30*a^5*b^2 + 11*a^3*b^4 + (a^6*b + 18*a^4*b^3 - 27*a^2*b^5 - 2
4*(a^3*b^4 - a*b^6)*d*x)*tan(d*x + c)^3 - 24*(a^6*b - a^4*b^3)*d*x + 3*(a^7
 + 16*a^5*b^2 - 23*a^3*b^4 + 6*a*b^6 - 24*(a^4*b^3 - a^2*b^5)*d*x)*tan(d*x
+ c)^2 + 3*(a^7 - 6*a^5*b^2 + a^3*b^4 + (a^4*b^3 - 6*a^2*b^5 + b^7)*tan(d*x
 + c)^3 + 3*(a^5*b^2 - 6*a^3*b^4 + a*b^6)*tan(d*x + c)^2 + 3*(a^6*b - 6*a^4
*b^3 + a^2*b^5)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c)
+ a^2)/(tan(d*x + c)^2 + 1)) + 3*(9*a^6*b - 26*a^4*b^3 + 9*a^2*b^5 - 24*(a^
5*b^2 - a^3*b^4)*d*x)*tan(d*x + c))/((a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a
^2*b^9 + b^11)*d*tan(d*x + c)^3 + 3*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^
3*b^8 + a*b^10)*d*tan(d*x + c)^2 + 3*(a^10*b + 4*a^8*b^3 + 6*a^6*b^5 + 4*a^
4*b^7 + a^2*b^9)*d*tan(d*x + c) + (a^11 + 4*a^9*b^2 + 6*a^7*b^4 + 4*a^5*b^6
 + a^3*b^8)*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+b*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 400 vs. 2(185) = 370.
time = 1.10, size = 400, normalized size = 2.12

$$-\frac{\frac{24\,(a^3b-ab^3)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{3\,(a^4-6\,a^2b^2+b^4)\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}-\frac{6\,(a^4b-6\,a^2b^3+b^5)\log(|b\tan(dx+c)+a|)}{a^8b+4\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9}+\frac{11\,a^4b^5\tan(dx+c)^3-66\,a^2b^7\tan(dx+c)^3+11\,b^9\tan(dx+c)^3+39\,a^5b^4\tan(dx+c)^2-210\,a^3b^6\tan(dx+c)^2+15\,ab^8\tan(dx+c)^2+3\,a^8b\tan(dx+c)+60\,a^6b^3\tan(dx+c)-201\,a^4b^5\tan(dx+c)+6\,a^2b^7\tan(dx+c)+a^9+26\,a^7b^2-63\,a^5b^4}{(a^8b^2+4\,a^6b^4+6\,a^4b^6+4\,a^2b^8+b^{10})(b\tan(dx+c)+a)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/6*(24*(a^3*b - a*b^3)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6
 + b^8) + 3*(a^4 - 6*a^2*b^2 + b^4)*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^
2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 6*(a^4*b - 6*a^2*b^3 + b^5)*log(abs(b*ta
n(d*x + c) + a))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9) + (11*a^
4*b^5*tan(d*x + c)^3 - 66*a^2*b^7*tan(d*x + c)^3 + 11*b^9*tan(d*x + c)^3 +
39*a^5*b^4*tan(d*x + c)^2 - 210*a^3*b^6*tan(d*x + c)^2 + 15*a*b^8*tan(d*x +
 c)^2 + 3*a^8*b*tan(d*x + c) + 60*a^6*b^3*tan(d*x + c) - 201*a^4*b^5*tan(d*
x + c) + 6*a^2*b^7*tan(d*x + c) + a^9 + 26*a^7*b^2 - 63*a^5*b^4)/((a^8*b^2
+ 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)*(b*tan(d*x + c) + a)^3))/d
```

Mupad [B]
time = 4.64, size = 359, normalized size = 1.90

$$\frac{\ln\left(a+b\tan(c+d\,x)\right)\left(\frac{1}{(a^2+b^2)^2}-\frac{8\,b^2}{(a^2+b^2)^3}+\frac{8\,b^4}{(a^2+b^2)^4}\right)}{d}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)}{2\,d\,\left(a^4+a^3\,b\,4\mathrm{i}-6\,a^2\,b^2-a\,b^3\,4\mathrm{i}+b^4\right)}-\frac{\frac{a\,(a^6+14\,a^4\,b^2-11\,a^2\,b^4)}{6\,b^2\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)}-\frac{\tan(c+d\,x)^2\,(3\,a\,b^4-a^3\,b^2)}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}+\frac{\tan(c+d\,x)\,(a^6+8\,a^4\,b^2-9\,a^2\,b^4)}{2\,b\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)}}{d\,\left(a^3+3\,a^2\,b\tan\left(c+d\,x\right)+3\,a\,b^2\tan\left(c+d\,x\right)^2+b^3\tan\left(c+d\,x\right)^3\right)}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(a^4\,1\mathrm{i}+4\,a^3\,b-a^2\,b^2\,6\mathrm{i}-4\,a\,b^3+b^4\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + b*tan(c + d*x))^4,x)
```

```
[Out] (log(a + b*tan(c + d*x))*(1/(a^2 + b^2)^2 - (8*b^2)/(a^2 + b^2)^3 + (8*b^4)
/(a^2 + b^2)^4))/d - (log(tan(c + d*x) + 1i)*1i)/(2*d*(4*a^3*b - 4*a*b^3 +
a^4*1i + b^4*1i - a^2*b^2*6i)) - log(tan(c + d*x) - 1i)/(2*d*(a^3*b*4i - a*
b^3*4i + a^4 + b^4 - 6*a^2*b^2)) - ((a*(a^6 - 11*a^2*b^4 + 14*a^4*b^2))/(6*
b^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (tan(c + d*x)^2*(3*a*b^4 - a^3*b
^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (tan(c + d*x)*(a^6 - 9*a^2*b^4 +
 8*a^4*b^2))/(2*b*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))/(d*(a^3 + b^3*tan(c
 + d*x)^3 + 3*a*b^2*tan(c + d*x)^2 + 3*a^2*b*tan(c + d*x)))
```

3.490 $\int \frac{\tan^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx$

Optimal. Leaf size=169

$$-\frac{(a^4-6a^2b^2+b^4)\,x}{(a^2+b^2)^4}-\frac{4ab(a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^4\,d}-\frac{a^2}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^3}+$$

[Out] -(a^4-6*a^2*b^2+b^4)*x/(a^2+b^2)^4-4*a*b*(a^2-b^2)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^4/d-1/3*a^2/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^3+a*b/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^2+b*(3*a^2-b^2)/(a^2+b^2)^3/d/(a+b*tan(d*x+c))

Rubi [A]
time = 0.20, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3623, 3610, 3612, 3611}

$$-\frac{a^2}{3bd\,(a^2+b^2)\,(a+b\tan(c+dx))^3}+\frac{ab}{d\,(a^2+b^2)^2\,(a+b\tan(c+dx))^2}+\frac{b(3a^2-b^2)}{d\,(a^2+b^2)^3\,(a+b\tan(c+dx))}-\frac{4ab(a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^4}-\frac{x(a^4-6a^2b^2+b^4)}{(a^2+b^2)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -(((a^4 - 6*a^2*b^2 + b^4)*x)/(a^2 + b^2)^4) - (4*a*b*(a^2 - b^2)*Log[a*Cos
[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^4*d) - a^2/(3*b*(a^2 + b^2)*d*(a
+ b*Tan[c + d*x])^3) + (a*b)/((a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^2) + (b*
(3*a^2 - b^2))/((a^2 + b^2)^3*d*(a + b*Tan[c + d*x]))
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
```

```
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx = -\frac{a^2}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^3} + \frac{\int \frac{-a+b\tan(c+dx)}{(a+b\tan(c+dx))^3}\,dx}{a^2+b^2}$$

$$= -\frac{a^2}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^3} + \frac{ab}{(a^2+b^2)^2\,d(a+b\tan(c+dx))^2} + \frac{\int \frac{-a^2 \cdots}{(a \cdots}}{\cdots}$$

$$= -\frac{a^2}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^3} + \frac{ab}{(a^2+b^2)^2\,d(a+b\tan(c+dx))^2} + \frac{\cdots}{(a^2+ \cdots}$$

$$= -\frac{(a^4-6a^2b^2+b^4)\,x}{(a^2+b^2)^4} - \frac{a^2}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^3} + \frac{ab}{(a^2+b^2)^2\,d(a+b \cdots}$$

$$= -\frac{(a^4-6a^2b^2+b^4)\,x}{(a^2+b^2)^4} - \frac{4ab(a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^4\,d} - \frac{\cdots}{3b\,(a^2 \cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.21, size = 324, normalized size = 1.92

$$\frac{b^2\tan^3(c+dx)}{3a\,(a^2+b^2)\,d(a+b\tan(c+dx))^3} + \frac{\frac{3a\tan(c+dx)}{d(a+b\tan(c+dx))^2} - \frac{3a\left(\frac{i\log(i-\tan(c+dx))}{(a+ib)^2} - \frac{i\log(i+\tan(c+dx))}{(a-ib)^2} - \frac{4ab\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^2} + \frac{2b}{\left(a^2+b^2\right)(a+b\tan(c+dx))} - 2a\left(\frac{\log(i-\tan(c+dx))}{(ia-b)^3} - \frac{\log(i+\tan(c+dx))}{(ia+b)^3} - \frac{2b\left(3a^2-b^2\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^3} + \frac{b}{\left(a^2+b^2\right)(a+b\tan(c+dx))^2} + \frac{4ab}{\left(a^2+b^2\right)^2(a+b\tan(c+dx))}\right)\right)}{2d}}{3a\,(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (b^2*Tan[c + d*x]^3)/(3*a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^3) + ((3*a*Tan
[c + d*x])/(d*(a + b*Tan[c + d*x])^2) - (3*a*((I*Log[I - Tan[c + d*x]])/(a
+ I*b)^2 - (I*Log[I + Tan[c + d*x]])/(a - I*b)^2 - (4*a*b*Log[a + b*Tan[c +
 d*x]])/(a^2 + b^2)^2 + (2*b)/((a^2 + b^2)*(a + b*Tan[c + d*x])) - 2*a*(Log
[I - Tan[c + d*x]]/(I*a - b)^3 - Log[I + Tan[c + d*x]]/(I*a + b)^3 - (2*b*(
3*a^2 - b^2)*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^3 + b/((a^2 + b^2)*(a + b
```

```
*Tan[c + d*x])^2) + (4*a*b)/((a^2 + b^2)^2*(a + b*Tan[c + d*x])))))/(2*d))/
(3*a*(a^2 + b^2))
```

Maple [A]
time = 0.18, size = 190, normalized size = 1.12

method	result
derivativedivides	$\frac{\frac{\frac{\left(4a^{3}b-4a\,b^{3}\right)\ln\left(1+\tan^{2}(dx+c)\right)}{2}+\left(-a^{4}+6a^{2}b^{2}-b^{4}\right)\arctan(\tan(dx+c))}{\left(a^{2}+b^{2}\right)^{4}}-\frac{a^{2}}{3\left(a^{2}+b^{2}\right)b(a+b\tan(dx+c))^{3}}+\frac{ab}{\left(a^{2}+b^{2}\right)^{2}(a+b\tan(dx+}}{d}$
default	$\frac{\frac{\frac{\left(4a^{3}b-4a\,b^{3}\right)\ln\left(1+\tan^{2}(dx+c)\right)}{2}+\left(-a^{4}+6a^{2}b^{2}-b^{4}\right)\arctan(\tan(dx+c))}{\left(a^{2}+b^{2}\right)^{4}}-\frac{a^{2}}{3\left(a^{2}+b^{2}\right)b(a+b\tan(dx+c))^{3}}+\frac{ab}{\left(a^{2}+b^{2}\right)^{2}(a+b\tan(dx+}}{d}$
norman	$\frac{\frac{\left(3a^{2}b^{4}-b^{6}\right)\left(\tan^{2}(dx+c)\right)}{b\left(a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}\right)d}+\frac{a\left(7a^{2}b^{4}-b^{6}\right)\tan(dx+c)}{b^{2}\left(a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}\right)d}+\frac{a^{2}\left(-a^{4}b^{2}+10a^{2}b^{4}-b^{6}\right)}{3b^{3}\left(a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}\right)d}-\frac{\left(a^{4}-6a^{2}b^{2}+b^{4}\right)a^{3}x}{\left(a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}\right)\left(a^{2}+b^{2}\right)}-\frac{b^{3}(}{(a^{6}}}{(a+b\tan(dx+c))^{3}}$
risch	$\frac{x}{4ia^{3}b-4ia\,b^{3}-a^{4}+6a^{2}b^{2}-b^{4}}+\frac{8ia^{3}bx}{a^{8}+4a^{6}b^{2}+6a^{4}b^{4}+4a^{2}b^{6}+b^{8}}-\frac{8ia\,b^{3}x}{a^{8}+4a^{6}b^{2}+6a^{4}b^{4}+4a^{2}b^{6}+b^{8}}+\frac{8ia^{3}b}{(a^{8}+4a^{6}b^{2}+6a^{4}b^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a^2+b^2)^4*(1/2*(4*a^3*b-4*a*b^3)*ln(1+tan(d*x+c)^2)+(-a^4+6*a^2*b^
2-b^4)*arctan(tan(d*x+c)))-1/3*a^2/(a^2+b^2)/b/(a+b*tan(d*x+c))^3+a*b/(a^2+
b^2)^2/(a+b*tan(d*x+c))^2+b*(3*a^2-b^2)/(a^2+b^2)^3/(a+b*tan(d*x+c))-4*a*b*
(a^2-b^2)/(a^2+b^2)^4*ln(a+b*tan(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 389 vs. 2(167) = 334.
time = 0.50, size = 389, normalized size = 2.30

$$-\frac{\frac{3\left(a^{4}-6a^{2}b^{2}+b^{4}\right)(dx+c)}{a^{8}+4a^{6}b^{2}+6a^{4}b^{4}+4a^{2}b^{6}+b^{8}}+\frac{12\left(a^{3}b-ab^{3}\right)\log(b\tan(dx+c)+a)}{a^{8}+4a^{6}b^{2}+6a^{4}b^{4}+4a^{2}b^{6}+b^{8}}-\frac{6\left(a^{3}b-ab^{3}\right)\log\left(\tan(dx+c)^{2}+1\right)}{a^{8}+4a^{6}b^{2}+6a^{4}b^{4}+4a^{2}b^{6}+b^{8}}+\frac{a^{6}-10a^{4}b^{2}+a^{2}b^{4}-3\left(3a^{2}b^{4}-b^{6}\right)\tan(dx+c)^{2}-3\left(7a^{3}b^{3}-ab^{5}\right)\tan(dx+c)}{a^{9}b+3a^{7}b^{3}+3a^{5}b^{5}+a^{3}b^{7}+\left(a^{6}b^{4}+3a^{4}b^{6}+3a^{2}b^{8}+b^{10}\right)\tan(dx+c)^{3}+3\left(a^{7}b^{3}+3a^{5}b^{5}+3a^{3}b^{7}+ab^{9}\right)\tan(dx+c)^{2}+3\left(a^{8}b^{2}+3a^{6}b^{4}+3a^{4}b^{6}+a^{2}b^{8}\right)\tan(dx+c)}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*(a^4 - 6*a^2*b^2 + b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*
a^2*b^6 + b^8) + 12*(a^3*b - a*b^3)*log(b*tan(d*x + c) + a)/(a^8 + 4*a^6*b^
2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 6*(a^3*b - a*b^3)*log(tan(d*x + c)^2 + 1
)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + (a^6 - 10*a^4*b^2 + a^2
*b^4 - 3*(3*a^2*b^4 - b^6)*tan(d*x + c)^2 - 3*(7*a^3*b^3 - a*b^5)*tan(d*x +
 c))/(a^9*b + 3*a^7*b^3 + 3*a^5*b^5 + a^3*b^7 + (a^6*b^4 + 3*a^4*b^6 + 3*a^
2*b^8 + b^10)*tan(d*x + c)^3 + 3*(a^7*b^3 + 3*a^5*b^5 + 3*a^3*b^7 + a*b^9)*
tan(d*x + c)^2 + 3*(a^8*b^2 + 3*a^6*b^4 + 3*a^4*b^6 + a^2*b^8)*tan(d*x + c)
))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 531 vs. $2(167) = 334$.
time = 1.15, size = 531, normalized size = 3.14

$$-\frac{6 a^{6} b-15 a^{4} b^{3}+a^{2} b^{5}-\left(a^{5} b^{2}-15 a^{3} b^{4}+6 a b^{6}-3\left(a^{4} b^{3}-6 a^{2} b^{5}+b^{7}\right) d x\right) \tan (d x+c)^{3}+3\left(a^{7}-6 a^{5} b^{2}+a^{3} b^{4}\right) d x-3\left(a^{6} b-12 a^{4} b^{3}+8 a^{2} b^{5}-b^{7}-3\left(a^{5} b^{2}-6 a^{3} b^{4}+a b^{6}\right) d x\right) \tan (d x+c)^{2}+6\left(a^{6} b-a^{4} b^{3}+\left(a^{3} b^{4}-a b^{6}\right) \tan (d x+c)^{3}+3\left(a^{4} b^{3}-a^{2} b^{5}\right) \tan (d x+c)^{2}+3\left(a^{5} b^{2}-a^{3} b^{4}\right) \tan (d x+c)\right) \log \left(\frac{b^{2} \tan (d x+c)^{2}+2 a b \tan (d x+c)+a^{2}}{\tan (d x+c)^{2}+1}\right)-3\left(a^{7}-8 a^{5} b^{2}+12 a^{3} b^{4}-a b^{6}-3\left(a^{6} b-6 a^{4} b^{3}+a^{2} b^{5}\right) d x\right) \tan (d x+c)}{3\left(\left(a^{8} b^{3}+4 a^{6} b^{5}+6 a^{4} b^{7}+4 a^{2} b^{9}+b^{11}\right) d \tan (d x+c)^{3}+3\left(a^{9} b^{2}+4 a^{7} b^{4}+6 a^{5} b^{6}+4 a^{3} b^{8}+a b^{10}\right) d \tan (d x+c)^{2}+3\left(a^{10} b+4 a^{8} b^{3}+6 a^{6} b^{5}+4 a^{4} b^{7}+a^{2} b^{9}\right) d \tan (d x+c)+\left(a^{11}+4 a^{9} b^{2}+6 a^{7} b^{4}+4 a^{5} b^{6}+a^{3} b^{8}\right) d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(6*a^6*b - 15*a^4*b^3 + a^2*b^5 - (a^5*b^2 - 15*a^3*b^4 + 6*a*b^6 - 3*
(a^4*b^3 - 6*a^2*b^5 + b^7)*d*x)*tan(d*x + c)^3 + 3*(a^7 - 6*a^5*b^2 + a^3*
b^4)*d*x - 3*(a^6*b - 12*a^4*b^3 + 8*a^2*b^5 - b^7 - 3*(a^5*b^2 - 6*a^3*b^4
 + a*b^6)*d*x)*tan(d*x + c)^2 + 6*(a^6*b - a^4*b^3 + (a^3*b^4 - a*b^6)*tan(
d*x + c)^3 + 3*(a^4*b^3 - a^2*b^5)*tan(d*x + c)^2 + 3*(a^5*b^2 - a^3*b^4)*t
an(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x +
 c)^2 + 1)) - 3*(a^7 - 8*a^5*b^2 + 12*a^3*b^4 - a*b^6 - 3*(a^6*b - 6*a^4*b^
3 + a^2*b^5)*d*x)*tan(d*x + c))/((a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b
^9 + b^11)*d*tan(d*x + c)^3 + 3*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^
8 + a*b^10)*d*tan(d*x + c)^2 + 3*(a^10*b + 4*a^8*b^3 + 6*a^6*b^5 + 4*a^4*b^
7 + a^2*b^9)*d*tan(d*x + c) + (a^11 + 4*a^9*b^2 + 6*a^7*b^4 + 4*a^5*b^6 + a
^3*b^8)*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+b*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(167) = 334$.
time = 1.00, size = 376, normalized size = 2.22

$$-\frac{\frac{3\left(a^{4}-6 a^{2} b^{2}+b^{4}\right)(d x+c)}{a^{8}+4 a^{6} b^{2}+6 a^{4} b^{4}+4 a^{2} b^{6}+b^{8}}-\frac{6\left(a^{3} b-a b^{3}\right) \log \left(\tan (d x+c)^{2}+1\right)}{a^{8}+4 a^{6} b^{2}+6 a^{4} b^{4}+4 a^{2} b^{6}+b^{8}}+\frac{12\left(a^{3} b^{2}-a b^{4}\right) \log (|b \tan (d x+c)+a|)}{a^{8} b+4 a^{6} b^{3}+6 a^{4} b^{5}+4 a^{2} b^{7}+b^{9}}-\frac{22 a^{3} b^{5} \tan (d x+c)^{3}-22 a b^{7} \tan (d x+c)^{3}+75 a^{4} b^{4} \tan (d x+c)^{2}-60 a^{2} b^{6} \tan (d x+c)^{2}-3 b^{8} \tan (d x+c)^{2}+87 a^{5} b^{3} \tan (d x+c)-48 a^{3} b^{5} \tan (d x+c)-3 a b^{7} \tan (d x+c)-a^{8}+31 a^{6} b^{2}-13 a^{4} b^{4}-a^{2} b^{6}}{\left(a^{8} b+4 a^{6} b^{3}+6 a^{4} b^{5}+4 a^{2} b^{7}+b^{9}\right)(b \tan (d x+c)+a)^{3}}}{3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(a^4 - 6*a^2*b^2 + b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*
a^2*b^6 + b^8) - 6*(a^3*b - a*b^3)*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2
 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 12*(a^3*b^2 - a*b^4)*log(abs(b*tan(d*x +
```

```
c) + a))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9) - (22*a^3*b^5*ta
n(d*x + c)^3 - 22*a*b^7*tan(d*x + c)^3 + 75*a^4*b^4*tan(d*x + c)^2 - 60*a^2
*b^6*tan(d*x + c)^2 - 3*b^8*tan(d*x + c)^2 + 87*a^5*b^3*tan(d*x + c) - 48*a
^3*b^5*tan(d*x + c) - 3*a*b^7*tan(d*x + c) - a^8 + 31*a^6*b^2 - 13*a^4*b^4
- a^2*b^6)/((a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*(b*tan(d*x +
c) + a)^3))/d
```

Mupad [B]
time = 4.53, size = 326, normalized size = 1.93

$$-\frac{\frac{a^6-10\,a^4\,b^2+a^2\,b^4}{3\,b\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{{\mathrm{tan}\left(c+d\,x\right)}^2\,\left(b^5-3\,a^2\,b^3\right)}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}+\frac{\mathrm{tan}\left(c+d\,x\right)\,\left(a\,b^4-7\,a^3\,b^2\right)}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}}{d\,\left(a^3+3\,a^2\,b\,\mathrm{tan}\left(c+d\,x\right)+3\,a\,b^2\,{\mathrm{tan}\left(c+d\,x\right)}^2+b^3\,{\mathrm{tan}\left(c+d\,x\right)}^3\right)}-\frac{\ln\left(\mathrm{tan}\left(c+d\,x\right)-\mathrm{i}\right)}{2\,d\,\left(a^4\,1{}\mathrm{i}-4\,a^3\,b-a^2\,b^2\,6{}\mathrm{i}+4\,a\,b^3+b^4\,1{}\mathrm{i}\right)}-\frac{4\,a\,b\,\ln\left(a+b\,\mathrm{tan}\left(c+d\,x\right)\right)\,\left(a^2-b^2\right)}{d\,{\left(a^2+b^2\right)}^4}-\frac{\ln\left(\mathrm{tan}\left(c+d\,x\right)+1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2\,d\,\left(a^4-a^3\,b\,4{}\mathrm{i}-6\,a^2\,b^2+a\,b^3\,4{}\mathrm{i}+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + b*tan(c + d*x))^4,x)
```

```
[Out] - ((a^6 + a^2*b^4 - 10*a^4*b^2)/(3*b*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) +
 (tan(c + d*x)^2*(b^5 - 3*a^2*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (
tan(c + d*x)*(a*b^4 - 7*a^3*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))/(d*(
a^3 + b^3*tan(c + d*x)^3 + 3*a*b^2*tan(c + d*x)^2 + 3*a^2*b*tan(c + d*x)))
- log(tan(c + d*x) - 1i)/(2*d*(4*a*b^3 - 4*a^3*b + a^4*1i + b^4*1i - a^2*b^
2*6i)) - (log(tan(c + d*x) + 1i)*1i)/(2*d*(a*b^3*4i - a^3*b*4i + a^4 + b^4
- 6*a^2*b^2)) - (4*a*b*log(a + b*tan(c + d*x))*(a^2 - b^2))/(d*(a^2 + b^2)^
4)
```

3.491 $\int \frac{\tan(c+dx)}{(a+b\tan(c+dx))^4}\,dx$

Optimal. Leaf size=172

$$\frac{4ab(a^2-b^2)\,x}{(a^2+b^2)^4}-\frac{(a^4-6a^2b^2+b^4)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^4\,d}+\frac{a}{3\,(a^2+b^2)\,d(a+b\tan(c+dx))^3}+\frac{}{2\,(a^2}$$

```
[Out] 4*a*b*(a^2-b^2)*x/(a^2+b^2)^4-(a^4-6*a^2*b^2+b^4)*ln(a*cos(d*x+c)+b*sin(d*x
+c))/(a^2+b^2)^4/d+1/3*a/(a^2+b^2)/d/(a+b*tan(d*x+c))^3+1/2*(a^2-b^2)/(a^2+
b^2)^2/d/(a+b*tan(d*x+c))^2+a*(a^2-3*b^2)/(a^2+b^2)^3/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.17, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3610, 3612, 3611}

$$\frac{a(a^2-3b^2)}{d\,(a^2+b^2)^3\,(a+b\tan(c+dx))}+\frac{a}{3d\,(a^2+b^2)\,(a+b\tan(c+dx))^3}+\frac{a^2-b^2}{2d\,(a^2+b^2)^2\,(a+b\tan(c+dx))^2}+\frac{4abx(a^2-b^2)}{(a^2+b^2)^4}-\frac{(a^4-6a^2b^2+b^4)\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (4*a*b*(a^2 - b^2)*x)/(a^2 + b^2)^4 - ((a^4 - 6*a^2*b^2 + b^4)*Log[a*Cos[c
+ d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^4*d) + a/(3*(a^2 + b^2)*d*(a + b*Tan
[c + d*x])^3) + (a^2 - b^2)/(2*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^2) + (a
*(a^2 - 3*b^2))/((a^2 + b^2)^3*d*(a + b*Tan[c + d*x]))
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
```

```
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \frac{a}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^3} + \frac{\int \frac{b+a\tan(c+dx)}{(a+b\tan(c+dx))^3}\,dx}{a^2+b^2}$$

$$= \frac{a}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^3} + \frac{a^2-b^2}{2\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2} + \frac{\int \frac{2ab+\cdots}{(\cdots}}{\cdots}$$

$$= \frac{a}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^3} + \frac{a^2-b^2}{2\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2} + \frac{\cdots}{(a^2+\cdots}$$

$$= \frac{4ab\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^4} + \frac{a}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^3} + \frac{a^2-b^2}{2\left(a^2+b^2\right)^2 d(a+b\tan(c\cdots}$$

$$= \frac{4ab\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^4} - \frac{\left(a^4-6a^2b^2+b^4\right)\log(a\cos(c+dx)+b\sin(c+dx))}{\left(a^2+b^2\right)^4 d} + \frac{\cdots}{3\left(a^2\cdots\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.24, size = 319, normalized size = 1.85

$$\frac{a\left(\frac{3i\log(i-\tan(c+dx))}{(a+ib)^4}-\frac{3i\log(i+\tan(c+dx))}{(a-ib)^4}-\frac{24a(a-b)b(a+b)\log(a+b\tan(c+dx))}{(a^2+b^2)^4}+\frac{2b}{(a^2+b^2)(a+b\tan(c+dx))^3}+\frac{6ab}{(a^2+b^2)^2(a+b\tan(c+dx))^2}+\frac{6b(3a^2-b^2)}{(a^2+b^2)^3(a+b\tan(c+dx))}\right)}{6bd}-\frac{\frac{\log(i-\tan(c+dx))}{(ia-b)^3}-\frac{\log(i+\tan(c+dx))}{(ia+b)^3}-\frac{2b(3a^2-b^2)\log(a+b\tan(c+dx))}{(a^2+b^2)^3}+\frac{b}{(a^2+b^2)(a+b\tan(c+dx))^2}+\frac{4ab}{(a^2+b^2)^2(a+b\tan(c+dx))}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (a*(((3*I)*Log[I - Tan[c + d*x]])/(a + I*b)^4 - ((3*I)*Log[I + Tan[c + d*x]
])/(a - I*b)^4 - (24*a*(a - b)*b*(a + b)*Log[a + b*Tan[c + d*x]])/(a^2 + b^
2)^4 + (2*b)/((a^2 + b^2)*(a + b*Tan[c + d*x])^3) + (6*a*b)/((a^2 + b^2)^2*
(a + b*Tan[c + d*x])^2) + (6*b*(3*a^2 - b^2))/((a^2 + b^2)^3*(a + b*Tan[c +
 d*x]))))/(6*b*d) - (Log[I - Tan[c + d*x]]/(I*a - b)^3 - Log[I + Tan[c + d*
x]]/(I*a + b)^3 - (2*b*(3*a^2 - b^2)*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^3
 + b/((a^2 + b^2)*(a + b*Tan[c + d*x])^2) + (4*a*b)/((a^2 + b^2)^2*(a + b*T
an[c + d*x])))/(2*b*d)
```

Maple [A]
time = 0.24, size = 191, normalized size = 1.11

method	result

derivativedivides	$\frac{\frac{\frac{\left(a^4-6a^2b^2+b^4\right)\ln\left(1+\tan^2(dx+c)\right)}{2}+\left(4a^3b-4a\,b^3\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^4}+\frac{a}{3\left(a^2+b^2\right)(a+b\tan(dx+c))^3}-\frac{\left(a^4-6a^2b^2+b^4\right)\ln(a+b\tan(}{\left(a^2+b^2\right)^4}}{d}$
default	$\frac{\frac{\frac{\left(a^4-6a^2b^2+b^4\right)\ln\left(1+\tan^2(dx+c)\right)}{2}+\left(4a^3b-4a\,b^3\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^4}+\frac{a}{3\left(a^2+b^2\right)(a+b\tan(dx+c))^3}-\frac{\left(a^4-6a^2b^2+b^4\right)\ln(a+b\tan(}{\left(a^2+b^2\right)^4}}{d}$
norman	$\frac{\frac{a\left(a^2b^3-3b^5\right)\left(\tan^2(dx+c)\right)}{d\left(a^6+3a^4b^2+3a^2b^4+b^6\right)b}+\frac{a\left(11a^4b^3-14b^5a^2-b^7\right)}{6b^3\left(a^6+3a^4b^2+3a^2b^4+b^6\right)d}+\frac{\left(5a^4b^3-12b^5a^2-b^7\right)\tan(dx+c)}{2d\,b^2\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}+\frac{4\left(a^2-b^2\right)a^4bx}{\left(a^6+3a^4b^2+3a^2b^4+b^6\right)\left(a^2+b^2\right)}+}{(a+b\tan(dx+c))^3}$
risch	$\frac{ix}{4ia^3b-4ia\,b^3-a^4+6a^2b^2-b^4}+\frac{2ia^4x}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}-\frac{12ia^2b^2x}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}+\frac{2ib^4x}{a^8+4a^6b^2+6a^4b^4+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a^2+b^2)^4*(1/2*(a^4-6*a^2*b^2+b^4)*ln(1+tan(d*x+c)^2)+(4*a^3*b-4*a
*b^3)*arctan(tan(d*x+c)))+1/3*a/(a^2+b^2)/(a+b*tan(d*x+c))^3-(a^4-6*a^2*b^2
+b^4)/(a^2+b^2)^4*ln(a+b*tan(d*x+c))+1/2*(a^2-b^2)/(a^2+b^2)^2/(a+b*tan(d*x
+c))^2+a*(a^2-3*b^2)/(a^2+b^2)^3/(a+b*tan(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 394 vs. 2(168) = 336.
time = 0.51, size = 394, normalized size = 2.29

$$\frac{\frac{24\left(a^3b-ab^3\right)(dx+c)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}-\frac{6\left(a^4-6a^2b^2+b^4\right)\log(b\tan(dx+c)+a)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}+\frac{3\left(a^4-6a^2b^2+b^4\right)\log\left(\tan(dx+c)^2+1\right)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}+\frac{11a^5-14a^3b^2-ab^4+6\left(a^3b^2-3ab^4\right)\tan(dx+c)^2+3\left(5a^4b-12a^2b^3-b^5\right)\tan(dx+c)}{a^9+3a^7b^2+3a^5b^4+a^3b^6+\left(a^6b^3+3a^4b^5+3a^2b^7+b^9\right)\tan(dx+c)^3+3\left(a^7b^2+3a^5b^4+3a^3b^6+ab^8\right)\tan(dx+c)^2+3\left(a^8b+3a^6b^3+3a^4b^5+a^2b^7\right)\tan(dx+c)}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/6*(24*(a^3*b - a*b^3)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6
+ b^8) - 6*(a^4 - 6*a^2*b^2 + b^4)*log(b*tan(d*x + c) + a)/(a^8 + 4*a^6*b^2
 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 3*(a^4 - 6*a^2*b^2 + b^4)*log(tan(d*x + c
)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + (11*a^5 - 14*a^3
*b^2 - a*b^4 + 6*(a^3*b^2 - 3*a*b^4)*tan(d*x + c)^2 + 3*(5*a^4*b - 12*a^2*b
^3 - b^5)*tan(d*x + c))/(a^9 + 3*a^7*b^2 + 3*a^5*b^4 + a^3*b^6 + (a^6*b^3 +
 3*a^4*b^5 + 3*a^2*b^7 + b^9)*tan(d*x + c)^3 + 3*(a^7*b^2 + 3*a^5*b^4 + 3*a
^3*b^6 + a*b^8)*tan(d*x + c)^2 + 3*(a^8*b + 3*a^6*b^3 + 3*a^4*b^5 + a^2*b^7
)*tan(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 528 vs. 2(168) = 336.
time = 1.27, size = 528, normalized size = 3.07

$$\frac{27a^5b^2-18a^3b^4-ab^6-\left(11a^4b^3-30a^2b^5+3b^7-24\left(a^5b^2-ab^6\right)dx\right)\tan(dx+c)^3+24\left(a^6b-a^4b^3\right)dx-3\left(9a^5b^2-26a^3b^4+9ab^6-24\left(a^4b^3-a^2b^5\right)dx\right)\tan(dx+c)^2-3\left(a^7-6a^5b^2+a^3b^4+\left(a^4b^3-6a^2b^5+b^7\right)\tan(dx+c)^3+3\left(a^5b^2-6a^3b^4+ab^6\right)\tan(dx+c)^2+3\left(a^6b-6a^4b^3+a^2b^5\right)\tan(dx+c)\right)\log\left(\frac{b^2\tan(dx+c)^2+2ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)-3\left(6a^6b-23a^4b^3+16a^2b^5+b^7-24\left(a^5b^2-a^3b^4\right)dx\right)\tan(dx+c)}{6\left(\left(a^8b^3+4a^6b^5+6a^4b^7+4a^2b^9+b^{11}\right)d\tan(dx+c)^3+3\left(a^9b^2+4a^7b^4+6a^5b^6+4a^3b^8+ab^{10}\right)d\tan(dx+c)^2+3\left(a^{10}b+4a^8b^3+6a^6b^5+4a^4b^7+a^2b^9\right)d\tan(dx+c)+\left(a^{11}+4a^9b^2+6a^7b^4+4a^5b^6+a^3b^8\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/6*(27*a^5*b^2 - 18*a^3*b^4 - a*b^6 - (11*a^4*b^3 - 30*a^2*b^5 + 3*b^7 - 2
4*(a^3*b^4 - a*b^6)*d*x)*tan(d*x + c)^3 + 24*(a^6*b - a^4*b^3)*d*x - 3*(9*a
^5*b^2 - 26*a^3*b^4 + 9*a*b^6 - 24*(a^4*b^3 - a^2*b^5)*d*x)*tan(d*x + c)^2
- 3*(a^7 - 6*a^5*b^2 + a^3*b^4 + (a^4*b^3 - 6*a^2*b^5 + b^7)*tan(d*x + c)^3
 + 3*(a^5*b^2 - 6*a^3*b^4 + a*b^6)*tan(d*x + c)^2 + 3*(a^6*b - 6*a^4*b^3 +
a^2*b^5)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/
(tan(d*x + c)^2 + 1)) - 3*(6*a^6*b - 23*a^4*b^3 + 16*a^2*b^5 + b^7 - 24*(a^
5*b^2 - a^3*b^4)*d*x)*tan(d*x + c))/((a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a
^2*b^9 + b^11)*d*tan(d*x + c)^3 + 3*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^
3*b^8 + a*b^10)*d*tan(d*x + c)^2 + 3*(a^10*b + 4*a^8*b^3 + 6*a^6*b^5 + 4*a^
4*b^7 + a^2*b^9)*d*tan(d*x + c) + (a^11 + 4*a^9*b^2 + 6*a^7*b^4 + 4*a^5*b^6
 + a^3*b^8)*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 401 vs. 2(168) = 336.
time = 0.83, size = 401, normalized size = 2.33

$$\frac{\frac{24\,(a^3b-ab^3)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{3\,(a^4-6\,a^2b^2+b^4)\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}-\frac{6\,(a^4b-6\,a^2b^3+b^5)\log(|b\tan(dx+c)+a|)}{a^8b+4\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9}+\frac{11\,a^4b^3\tan(dx+c)^3-66\,a^2b^5\tan(dx+c)^3+11\,b^7\tan(dx+c)^3+39\,a^5b^2\tan(dx+c)^2-210\,a^3b^4\tan(dx+c)^2+15\,ab^6\tan(dx+c)^2+48\,a^6b\tan(dx+c)-219\,a^4b^3\tan(dx+c)-6\,a^2b^5\tan(dx+c)-3\,b^7\tan(dx+c)+22\,a^7-69\,a^5b^2-4\,a^3b^4-ab^6}{(a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8)(b\tan(dx+c)+a)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/6*(24*(a^3*b - a*b^3)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6
+ b^8) + 3*(a^4 - 6*a^2*b^2 + b^4)*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2
 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 6*(a^4*b - 6*a^2*b^3 + b^5)*log(abs(b*tan
(d*x + c) + a))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9) + (11*a^4
*b^3*tan(d*x + c)^3 - 66*a^2*b^5*tan(d*x + c)^3 + 11*b^7*tan(d*x + c)^3 + 3
9*a^5*b^2*tan(d*x + c)^2 - 210*a^3*b^4*tan(d*x + c)^2 + 15*a*b^6*tan(d*x +
c)^2 + 48*a^6*b*tan(d*x + c) - 219*a^4*b^3*tan(d*x + c) - 6*a^2*b^5*tan(d*x
 + c) - 3*b^7*tan(d*x + c) + 22*a^7 - 69*a^5*b^2 - 4*a^3*b^4 - a*b^6)/((a^8
 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(b*tan(d*x + c) + a)^3))/d
```

Mupad [B]
time = 4.45, size = 355, normalized size = 2.06

$$\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)}{2\,d\,\left(a^4+a^3\,b\,4\mathrm{i}-6\,a^2\,b^2-a\,b^3\,4\mathrm{i}+b^4\right)}-\frac{\frac{-11\,a^5+14\,a^3\,b^2+a\,b^4}{6\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{\tan(c+d\,x)\,\left(-5\,a^4\,b+12\,a^2\,b^3+b^5\right)}{2\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{\tan(c+d\,x)^2\,\left(3\,a\,b^4-a^3\,b^2\right)}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}}{d\,\left(a^3+3\,a^2\,b\tan\left(c+d\,x\right)+3\,a\,b^2\tan\left(c+d\,x\right)^2+b^3\tan\left(c+d\,x\right)^3\right)}-\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{1}{{\left(a^2+b^2\right)}^2}-\frac{8\,b^2}{{\left(a^2+b^2\right)}^3}+\frac{8\,b^4}{{\left(a^2+b^2\right)}^4}\right)}{d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(a^4\,1\mathrm{i}+4\,a^3\,b-a^2\,b^2\,6\mathrm{i}-4\,a\,b^3+b^4\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + b*tan(c + d*x))^4,x)
```

```
[Out] (log(tan(c + d*x) + 1i)*1i)/(2*d*(4*a^3*b - 4*a*b^3 + a^4*1i + b^4*1i - a^2
*b^2*6i)) + log(tan(c + d*x) - 1i)/(2*d*(a^3*b*4i - a*b^3*4i + a^4 + b^4 -
6*a^2*b^2)) - ((a*b^4 - 11*a^5 + 14*a^3*b^2)/(6*(a^6 + b^6 + 3*a^2*b^4 + 3*
a^4*b^2)) + (tan(c + d*x)*(b^5 - 5*a^4*b + 12*a^2*b^3))/(2*(a^6 + b^6 + 3*a
^2*b^4 + 3*a^4*b^2)) + (tan(c + d*x)^2*(3*a*b^4 - a^3*b^2))/(a^6 + b^6 + 3*
a^2*b^4 + 3*a^4*b^2))/(d*(a^3 + b^3*tan(c + d*x)^3 + 3*a*b^2*tan(c + d*x)^2
 + 3*a^2*b*tan(c + d*x))) - (log(a + b*tan(c + d*x))*(1/(a^2 + b^2)^2 - (8*
b^2)/(a^2 + b^2)^3 + (8*b^4)/(a^2 + b^2)^4))/d
```

3.492 $\int \frac{1}{(a+b\tan(c+dx))^4}\,dx$

Optimal. Leaf size=165

$$\frac{(a^4-6a^2b^2+b^4)\,x}{(a^2+b^2)^4}+\frac{4ab(a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^4\,d}-\frac{b}{3\,(a^2+b^2)\,d(a+b\tan(c+dx))^3}-\frac{}{(a^2}$$

```
[Out] (a^4-6*a^2*b^2+b^4)*x/(a^2+b^2)^4+4*a*b*(a^2-b^2)*ln(a*cos(d*x+c)+b*sin(d*x
+c))/(a^2+b^2)^4/d-1/3*b/(a^2+b^2)/d/(a+b*tan(d*x+c))^3-a*b/(a^2+b^2)^2/d/(
a+b*tan(d*x+c))^2-b*(3*a^2-b^2)/(a^2+b^2)^3/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.17, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3564, 3610, 3612, 3611}

$$-\frac{b(3a^2-b^2)}{d(a^2+b^2)^3(a+b\tan(c+dx))}-\frac{ab}{d(a^2+b^2)^2(a+b\tan(c+dx))^2}-\frac{b}{3d(a^2+b^2)(a+b\tan(c+dx))^3}+\frac{4ab(a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{d(a^2+b^2)^4}+\frac{x(a^4-6a^2b^2+b^4)}{(a^2+b^2)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^(-4),x]
```

```
[Out] ((a^4 - 6*a^2*b^2 + b^4)*x)/(a^2 + b^2)^4 + (4*a*b*(a^2 - b^2)*Log[a*Cos[c
+ d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^4*d) - b/(3*(a^2 + b^2)*d*(a + b*Tan
[c + d*x])^3) - (a*b)/((a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^2) - (b*(3*a^2
- b^2))/((a^2 + b^2)^3*d*(a + b*Tan[c + d*x]))
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+b\tan(c+dx))^4}\,dx &= -\frac{b}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^3}+\frac{\int \frac{a-b\tan(c+dx)}{(a+b\tan(c+dx))^3}\,dx}{a^2+b^2}\\
&= -\frac{b}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^3}-\frac{ab}{\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2}+\frac{\int \frac{a^2-b^2}{(a+}}{(}\\
&= -\frac{b}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^3}-\frac{ab}{\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2}-\frac{}{(a^2+b}\\
&= \frac{\left(a^4-6a^2b^2+b^4\right)x}{\left(a^2+b^2\right)^4}-\frac{b}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^3}-\frac{ab}{\left(a^2+b^2\right)^2 d(a+b\tan}\\
&= \frac{\left(a^4-6a^2b^2+b^4\right)x}{\left(a^2+b^2\right)^4}+\frac{4ab\left(a^2-b^2\right)\log(a\cos(c+dx)+b\sin(c+dx))}{\left(a^2+b^2\right)^4 d}-\frac{}{3\left(a^2+\right.}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.86, size = 176, normalized size = 1.07

$$\frac{-\frac{3i\log(i-\tan(c+dx))}{(a+ib)^4}+\frac{3i\log(i+\tan(c+dx))}{(a-ib)^4}+\frac{2b\left(12a\left(a^2-b^2\right)\log(a+b\tan(c+dx))-\frac{\left(a^2+b^2\right)\left(13a^4+2a^2b^2+b^4+3ab\left(7a^2-b^2\right)\tan(c+dx)+\left(9a^2b^2-3b^4\right)\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\right)}{\left(a^2+b^2\right)^4}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^(-4),x]
```

```
[Out] (((-3*I)*Log[I - Tan[c + d*x]])/(a + I*b)^4 + ((3*I)*Log[I + Tan[c + d*x]])
/(a - I*b)^4 + (2*b*(12*a*(a^2 - b^2)*Log[a + b*Tan[c + d*x]] - ((a^2 + b^2
)*(13*a^4 + 2*a^2*b^2 + b^4 + 3*a*b*(7*a^2 - b^2)*Tan[c + d*x] + (9*a^2*b^2
 - 3*b^4)*Tan[c + d*x]^2))/(a + b*Tan[c + d*x])^3))/(a^2 + b^2)^4)/(6*d)
```

Maple [A]
time = 0.19, size = 183, normalized size = 1.11

method	result
derivativedivides	$\frac{\frac{\frac{(-4a^3b+4a\,b^3)\ln(1+\tan^2(dx+c))}{2}+(a^4-6a^2b^2+b^4)\arctan(\tan(dx+c))}{(a^2+b^2)^4}-\frac{b}{3(a^2+b^2)(a+b\tan(dx+c))^3}-\frac{b(3a^2-b^2)}{(a^2+b^2)^3(a+b\tan(dx+c}}{d}$
default	$\frac{\frac{\frac{(-4a^3b+4a\,b^3)\ln(1+\tan^2(dx+c))}{2}+(a^4-6a^2b^2+b^4)\arctan(\tan(dx+c))}{(a^2+b^2)^4}-\frac{b}{3(a^2+b^2)(a+b\tan(dx+c))^3}-\frac{b(3a^2-b^2)}{(a^2+b^2)^3(a+b\tan(dx+c}}{d}$
norman	$\frac{\frac{(a^4-6a^2b^2+b^4)a^3x}{(a^6+3a^4b^2+3a^2b^4+b^6)(a^2+b^2)}-\frac{4b^2a^3\tan(dx+c)}{(a^6+3a^4b^2+3a^2b^4+b^6)d}+\frac{b^3(a^4-6a^2b^2+b^4)x(\tan^3(dx+c))}{(a^6+3a^4b^2+3a^2b^4+b^6)(a^2+b^2)}+\frac{-10a^4b^3-3b^5a^2-b^7}{3b^2(a^6+3a^4b^2+3a^2b^4+b^6)d}}{(a+b\tan(dx+c))^3}$
risch	$-\frac{x}{4ia^3b-4ia\,b^3-a^4+6a^2b^2-b^4}-\frac{8ia^3bx}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}+\frac{8ia\,b^3x}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}-\frac{8i}{(a^8+4a^6b^2+6a}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)

[Out] 1/d*(1/(a^2+b^2)^4*(1/2*(-4*a^3*b+4*a*b^3)*ln(1+tan(d*x+c)^2)+(a^4-6*a^2*b^2+b^4)*arctan(tan(d*x+c)))-1/3*b/(a^2+b^2)/(a+b*tan(d*x+c))^3-b*(3*a^2-b^2)/(a^2+b^2)^3/(a+b*tan(d*x+c))-a*b/(a^2+b^2)^2/(a+b*tan(d*x+c))^2+4*a*b*(a^2-b^2)/(a^2+b^2)^4*ln(a+b*tan(d*x+c)))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(163) = 326$.
time = 0.50, size = 385, normalized size = 2.33

$$\frac{\frac{3\,(a^4-6\,a^2b^2+b^4)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{12\,(a^3b-ab^3)\log(b\tan(dx+c)+a)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}-\frac{6\,(a^3b-ab^3)\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}-\frac{13\,a^4b+2\,a^2b^3+b^5+3\,(3\,a^2b^3-b^5)\tan(dx+c)^2+3\,(7\,a^3b^2-ab^4)\tan(dx+c)}{a^9+3\,a^7b^2+3\,a^5b^4+a^3b^6+(a^6b^3+3\,a^4b^5+3\,a^2b^7+b^9)\tan(dx+c)^3+3\,(a^7b^2+3\,a^5b^4+3\,a^3b^6+ab^8)\tan(dx+c)^2+3\,(a^8b+3\,a^6b^3+3\,a^4b^5+a^2b^7)\tan(dx+c)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tan(d*x+c))^4,x, algorithm="maxima")

[Out] 1/3*(3*(a^4 - 6*a^2*b^2 + b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 12*(a^3*b - a*b^3)*log(b*tan(d*x + c) + a)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 6*(a^3*b - a*b^3)*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - (13*a^4*b + 2*a^2*b^3 + b^5 + 3*(3*a^2*b^3 - b^5)*tan(d*x + c)^2 + 3*(7*a^3*b^2 - a*b^4)*tan(d*x + c))/(a^9 + 3*a^7*b^2 + 3*a^5*b^4 + a^3*b^6 + (a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9)*tan(d*x + c)^3 + 3*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*tan(d*x + c)^2 + 3*(a^8*b + 3*a^6*b^3 + 3*a^4*b^5 + a^2*b^7)*tan(d*x + c)))/d

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(163) = 326$.
time = 1.22, size = 511, normalized size = 3.10

$$-\frac{24\,a^4b^3+3\,a^2b^5+b^7-(13\,a^3b^4-9\,ab^6+3\,(a^4b^3-6\,a^2b^5+b^7)dx)\tan(dx+c)^3-3\,(a^7-6\,a^5b^2+a^3b^4)dx-3\,(10\,a^4b^3-11\,a^2b^5+b^7+3\,(a^5b^2-6\,a^3b^4+ab^6)dx)\tan(dx+c)^2-6\,(a^6b-a^4b^3+(a^3b^4-ab^6)\tan(dx+c)^3+3\,(a^4b^3-a^2b^5)\tan(dx+c)^2+3\,(a^5b^2-a^3b^4)\tan(dx+c))\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)-3\,(6\,a^5b^2-15\,a^3b^4+ab^6+3\,(a^6b-6\,a^4b^3+a^2b^5)dx)\tan(dx+c)}{3\,((a^8b^3+4\,a^6b^5+6\,a^4b^7+4\,a^2b^9+b^{11})d\tan(dx+c)^3+3\,(a^9b^2+4\,a^7b^4+6\,a^5b^6+4\,a^3b^8+ab^{10})d\tan(dx+c)^2+3\,(a^{10}b+4\,a^8b^3+6\,a^6b^5+4\,a^4b^7+a^2b^9)d\tan(dx+c)+(a^{11}+4\,a^9b^2+6\,a^7b^4+4\,a^5b^6+a^3b^8)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(24*a^4*b^3 + 3*a^2*b^5 + b^7 - (13*a^3*b^4 - 9*a*b^6 + 3*(a^4*b^3 - 6
*a^2*b^5 + b^7)*d*x)*tan(d*x + c)^3 - 3*(a^7 - 6*a^5*b^2 + a^3*b^4)*d*x - 3
*(10*a^4*b^3 - 11*a^2*b^5 + b^7 + 3*(a^5*b^2 - 6*a^3*b^4 + a*b^6)*d*x)*tan(
d*x + c)^2 - 6*(a^6*b - a^4*b^3 + (a^3*b^4 - a*b^6)*tan(d*x + c)^3 + 3*(a^4
*b^3 - a^2*b^5)*tan(d*x + c)^2 + 3*(a^5*b^2 - a^3*b^4)*tan(d*x + c))*log((b
^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) - 3*(6*
a^5*b^2 - 15*a^3*b^4 + a*b^6 + 3*(a^6*b - 6*a^4*b^3 + a^2*b^5)*d*x)*tan(d*x
 + c))/((a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*d*tan(d*x + c)
^3 + 3*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*d*tan(d*x + c
)^2 + 3*(a^10*b + 4*a^8*b^3 + 6*a^6*b^5 + 4*a^4*b^7 + a^2*b^9)*d*tan(d*x +
c) + (a^11 + 4*a^9*b^2 + 6*a^7*b^4 + 4*a^5*b^6 + a^3*b^8)*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(163) = 326.
time = 0.61, size = 370, normalized size = 2.24

$$\frac{\frac{3\,(a^4-6\,a^2b^2+b^4)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}-\frac{6\,(a^3b-ab^3)\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{12\,(a^3b^2-ab^4)\log(|b\tan(dx+c)+a|)}{a^8b+4\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9}-\frac{22\,a^3b^4\tan(dx+c)^3-22\,ab^6\tan(dx+c)^3+75\,a^4b^3\tan(dx+c)^2-60\,a^2b^5\tan(dx+c)^2-3\,b^7\tan(dx+c)^2+87\,a^5b^2\tan(dx+c)-48\,a^3b^4\tan(dx+c)-3\,ab^6\tan(dx+c)+35\,a^6b-7\,a^4b^3+3\,a^2b^5+b^7}{(a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8)(b\tan(dx+c)+a)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(a^4 - 6*a^2*b^2 + b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a
^2*b^6 + b^8) - 6*(a^3*b - a*b^3)*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2
+ 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 12*(a^3*b^2 - a*b^4)*log(abs(b*tan(d*x + c
) + a))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9) - (22*a^3*b^4*tan
(d*x + c)^3 - 22*a*b^6*tan(d*x + c)^3 + 75*a^4*b^3*tan(d*x + c)^2 - 60*a^2*
b^5*tan(d*x + c)^2 - 3*b^7*tan(d*x + c)^2 + 87*a^5*b^2*tan(d*x + c) - 48*a^
3*b^4*tan(d*x + c) - 3*a*b^6*tan(d*x + c) + 35*a^6*b - 7*a^4*b^3 + 3*a^2*b^
5 + b^7)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(b*tan(d*x + c) +
 a)^3))/d
```

Mupad [B]

time = 4.28, size = 333, normalized size = 2.02

$$\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{4\,a\,b}{{\left(a^2+b^2\right)}^3}-\frac{8\,a\,b^3}{{\left(a^2+b^2\right)}^4}\right)}{d}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)}{2\,d\,\left(a^4\,1\mathrm{i}-4\,a^3\,b-a^2\,b^2\,6\mathrm{i}+4\,a\,b^3+b^4\,1\mathrm{i}\right)}+\frac{\frac{\tan(c+d\,x)^2\,\left(b^5-3\,a^2\,b^3\right)}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}-\frac{13\,a^4\,b+2\,a^2\,b^3+b^5}{3\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{\tan(c+d\,x)\,\left(a\,b^4-7\,a^3\,b^2\right)}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}}{d\,\left(a^3+3\,a^2\,b\tan\left(c+d\,x\right)+3\,a\,b^2\tan\left(c+d\,x\right)^2+b^3\tan\left(c+d\,x\right)^3\right)}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(a^4-a^3\,b\,4\mathrm{i}-6\,a^2\,b^2+a\,b^3\,4\mathrm{i}+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(c + d*x))^4,x)
```

```
[Out] (log(a + b*tan(c + d*x))*((4*a*b)/(a^2 + b^2)^3 - (8*a*b^3)/(a^2 + b^2)^4))
/d + log(tan(c + d*x) - 1i)/(2*d*(4*a*b^3 - 4*a^3*b + a^4*1i + b^4*1i - a^2
*b^2*6i)) + (log(tan(c + d*x) + 1i)*1i)/(2*d*(a*b^3*4i - a^3*b*4i + a^4 + b
^4 - 6*a^2*b^2)) + ((tan(c + d*x)^2*(b^5 - 3*a^2*b^3))/(a^6 + b^6 + 3*a^2*b
^4 + 3*a^4*b^2) - (13*a^4*b + b^5 + 2*a^2*b^3)/(3*(a^6 + b^6 + 3*a^2*b^4 +
3*a^4*b^2)) + (tan(c + d*x)*(a*b^4 - 7*a^3*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3
*a^4*b^2))/(d*(a^3 + b^3*tan(c + d*x)^3 + 3*a*b^2*tan(c + d*x)^2 + 3*a^2*b*
tan(c + d*x)))
```

3.493 $\int \frac{\cot(c+dx)}{(a+b\tan(c+dx))^4}\,dx$

Optimal. Leaf size=226

$$-\frac{4ab(a^2-b^2)\,x}{(a^2+b^2)^4}+\frac{\log(\sin(c+dx))}{a^4d}-\frac{b^2(10a^6+5a^4b^2+4a^2b^4+b^6)\log(a\cos(c+dx)+b\sin(c+dx))}{a^4\,(a^2+b^2)^4\,d}+\frac{}{3a\,(a^2}$$

```
[Out] -4*a*b*(a^2-b^2)*x/(a^2+b^2)^4+ln(sin(d*x+c))/a^4/d-b^2*(10*a^6+5*a^4*b^2+4
*a^2*b^4+b^6)*ln(a*cos(d*x+c)+b*sin(d*x+c))/a^4/(a^2+b^2)^4/d+1/3*b^2/a/(a^
2+b^2)/d/(a+b*tan(d*x+c))^3+1/2*b^2*(3*a^2+b^2)/a^2/(a^2+b^2)^2/d/(a+b*tan(
d*x+c))^2+b^2*(6*a^4+3*a^2*b^2+b^4)/a^3/(a^2+b^2)^3/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.45, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3650, 3730, 3732, 3611, 3556}

$$\frac{\log(\sin(c+dx))}{a^4d}+\frac{b^2(3a^2+b^2)}{2a^2d\,(a^2+b^2)^2\,(a+b\tan(c+dx))^2}+\frac{b^2}{3ad\,(a^2+b^2)\,(a+b\tan(c+dx))^3}-\frac{4abx(a^2-b^2)}{(a^2+b^2)^4}-\frac{b^2(10a^6+5a^4b^2+4a^2b^4+b^6)\log(a\cos(c+dx)+b\sin(c+dx))}{a^4d\,(a^2+b^2)^4}+\frac{b^2(6a^4+3a^2b^2+b^4)}{a^3d\,(a^2+b^2)^3\,(a+b\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (-4*a*b*(a^2 - b^2)*x)/(a^2 + b^2)^4 + Log[Sin[c + d*x]]/(a^4*d) - (b^2*(10
*a^6 + 5*a^4*b^2 + 4*a^2*b^4 + b^6)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/(
a^4*(a^2 + b^2)^4*d) + b^2/(3*a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^3) + (b^
2*(3*a^2 + b^2))/(2*a^2*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^2) + (b^2*(6*a
^4 + 3*a^2*b^2 + b^4))/(a^3*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
```

```
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \frac{b^2}{3a(a^2+b^2)\,d(a+b\tan(c+dx))^3} + \frac{\int \frac{\cot(c+dx)\left(3(a^2+b^2)-3ab\tan(c+dx)+3b^2\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}}{3a(a^2+b^2)}$$

$$= \frac{b^2}{3a(a^2+b^2)\,d(a+b\tan(c+dx))^3} + \frac{b^2(3a^2+b^2)}{2a^2(a^2+b^2)^2\,d(a+b\tan(c+dx))^2} + \frac{\int \mathrm{co}}{}$$

$$= \frac{b^2}{3a(a^2+b^2)\,d(a+b\tan(c+dx))^3} + \frac{b^2(3a^2+b^2)}{2a^2(a^2+b^2)^2\,d(a+b\tan(c+dx))^2} + \frac{}{a^3(}$$

$$= -\frac{4ab(a^2-b^2)\,x}{(a^2+b^2)^4} + \frac{b^2}{3a(a^2+b^2)\,d(a+b\tan(c+dx))^3} + \frac{b^2(3a^2+b^2}{2a^2(a^2+b^2)^2\,d(a+b\tan}$$

$$= -\frac{4ab(a^2-b^2)\,x}{(a^2+b^2)^4} + \frac{\log(\sin(c+dx))}{a^4 d} - \frac{b^2(10a^6+5a^4b^2+4a^2b^4+b^6)\log(a\cos(c}{a^4(a^2+b^2)^4\,d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.07, size = 243, normalized size = 1.08

$$\frac{\frac{3\left(-a^4(a-ib)^4\log(i-\tan(c+dx))+2(a^2+b^2)^4\log(\tan(c+dx))-a^4(a+ib)^4\log(i+\tan(c+dx))-2b^2(10a^6+5a^4b^2+4a^2b^4+b^6)\log(a+b\tan(c+dx))\right)}{a^2(a^2+b^2)^2}+\frac{2ab^2(a^2+b^2)}{(a+b\tan(c+dx))^3}+\frac{3(3a^2b^2+b^4)}{(a+b\tan(c+dx))^2}+\frac{6(6a^4b^2+3a^2b^4+b^6)}{a(a^2+b^2)(a+b\tan(c+dx))}}{6a^2(a^2+b^2)^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] ((3*(-(a^4*(a - I*b)^4*Log[I - Tan[c + d*x]]) + 2*(a^2 + b^2)^4*Log[Tan[c +
 d*x]] - a^4*(a + I*b)^4*Log[I + Tan[c + d*x]] - 2*b^2*(10*a^6 + 5*a^4*b^2
+ 4*a^2*b^4 + b^6)*Log[a + b*Tan[c + d*x]]))/(a^2*(a^2 + b^2)^2) + (2*a*b^2
*(a^2 + b^2))/(a + b*Tan[c + d*x])^3 + (3*(3*a^2*b^2 + b^4))/(a + b*Tan[c +
 d*x])^2 + (6*(6*a^4*b^2 + 3*a^2*b^4 + b^6))/(a*(a^2 + b^2)*(a + b*Tan[c +
d*x])))/(6*a^2*(a^2 + b^2)^2*d)
```

Maple [A]
time = 0.47, size = 246, normalized size = 1.09 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a^4*ln(tan(d*x+c))+1/3*b^2/(a^2+b^2)/a/(a+b*tan(d*x+c))^3+1/2*b^2*(3
*a^2+b^2)/(a^2+b^2)^2/a^2/(a+b*tan(d*x+c))^2+b^2*(6*a^4+3*a^2*b^2+b^4)/(a^2
+b^2)^3/a^3/(a+b*tan(d*x+c))-b^2*(10*a^6+5*a^4*b^2+4*a^2*b^4+b^6)/(a^2+b^2)
^4/a^4*ln(a+b*tan(d*x+c))+1/(a^2+b^2)^4*(1/2*(-a^4+6*a^2*b^2-b^4)*ln(1+tan(
d*x+c)^2)+(-4*a^3*b+4*a*b^3)*arctan(tan(d*x+c))))
```

Maxima [A]

time = 0.51, size = 444, normalized size = 1.96

$$-\frac{\frac{24\,\left(a^{3}b-ab^{3}\right)\left(dx+c\right)}{a^{8}+4\,a^{6}b^{2}+6\,a^{4}b^{4}+4\,a^{2}b^{6}+b^{8}}+\frac{6\,\left(10\,a^{6}b^{2}+5\,a^{4}b^{4}+4\,a^{2}b^{6}+b^{8}\right)\log\left(b\tan\left(dx+c\right)+a\right)}{a^{12}+4\,a^{10}b^{2}+6\,a^{8}b^{4}+4\,a^{6}b^{6}+a^{4}b^{8}}+\frac{3\,\left(a^{4}-6\,a^{2}b^{2}+b^{4}\right)\log\left(\tan\left(dx+c\right)^{2}+1\right)}{a^{8}+4\,a^{6}b^{2}+6\,a^{4}b^{4}+4\,a^{2}b^{6}+b^{8}}-\frac{47\,a^{6}b^{2}+34\,a^{4}b^{4}+11\,a^{2}b^{6}+6\,\left(6\,a^{4}b^{4}+3\,a^{2}b^{6}+b^{8}\right)\tan\left(dx+c\right)^{2}+3\,\left(27\,a^{5}b^{3}+16\,a^{3}b^{5}+5\,ab^{7}\right)\tan\left(dx+c\right)}{a^{12}+3\,a^{10}b^{2}+3\,a^{8}b^{4}+a^{6}b^{6}+\left(a^{9}b^{3}+3\,a^{7}b^{5}+3\,a^{5}b^{7}+a^{3}b^{9}\right)\tan\left(dx+c\right)^{3}+3\,\left(a^{10}b^{2}+3\,a^{8}b^{4}+3\,a^{6}b^{6}+a^{4}b^{8}\right)\tan\left(dx+c\right)^{2}+3\,\left(a^{11}b+3\,a^{9}b^{3}+3\,a^{7}b^{5}+a^{5}b^{7}\right)\tan\left(dx+c\right)}-\frac{6\,\log\left(\tan\left(dx+c\right)\right)}{a^{4}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/6*(24*(a^3*b - a*b^3)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6
 + b^8) + 6*(10*a^6*b^2 + 5*a^4*b^4 + 4*a^2*b^6 + b^8)*log(b*tan(d*x + c) +
 a)/(a^12 + 4*a^10*b^2 + 6*a^8*b^4 + 4*a^6*b^6 + a^4*b^8) + 3*(a^4 - 6*a^2*
b^2 + b^4)*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6
 + b^8) - (47*a^6*b^2 + 34*a^4*b^4 + 11*a^2*b^6 + 6*(6*a^4*b^4 + 3*a^2*b^6
+ b^8)*tan(d*x + c)^2 + 3*(27*a^5*b^3 + 16*a^3*b^5 + 5*a*b^7)*tan(d*x + c))
/(a^12 + 3*a^10*b^2 + 3*a^8*b^4 + a^6*b^6 + (a^9*b^3 + 3*a^7*b^5 + 3*a^5*b^
7 + a^3*b^9)*tan(d*x + c)^3 + 3*(a^10*b^2 + 3*a^8*b^4 + 3*a^6*b^6 + a^4*b^8
)*tan(d*x + c)^2 + 3*(a^11*b + 3*a^9*b^3 + 3*a^7*b^5 + a^5*b^7)*tan(d*x + c
)) - 6*log(tan(d*x + c))/a^4)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 793 vs. $2(222) = 444$.

time = 1.50, size = 793, normalized size = 3.51

$$\frac{75\,a^{7}b^{4}+42\,a^{5}b^{6}+11\,a^{3}b^{8}-\left(47\,a^{6}b^{5}+6\,a^{4}b^{7}+3\,a^{2}b^{9}+24\,\left(a^{7}b^{4}-a^{5}b^{6}\right)dx\right)\tan\left(dx+c\right)^{3}-24\,\left(a^{10}b-a^{8}b^{3}\right)dx-3\,\left(35\,a^{7}b^{4}-12\,a^{5}b^{6}-5\,a^{3}b^{8}-2\,ab^{10}+24\,\left(a^{8}b^{3}-a^{6}b^{5}\right)dx\right)\tan\left(dx+c\right)^{2}+3\,\left(a^{11}+4\,a^{9}b^{2}+6\,a^{7}b^{4}+4\,a^{5}b^{6}+a^{3}b^{8}+\left(a^{8}b^{3}+4\,a^{6}b^{5}+6\,a^{4}b^{7}+4\,a^{2}b^{9}+b^{11}\right)\tan\left(dx+c\right)^{3}+3\,\left(a^{9}b^{2}+4\,a^{7}b^{4}+6\,a^{5}b^{6}+4\,a^{3}b^{8}+ab^{10}\right)\tan\left(dx+c\right)^{2}+3\,\left(a^{10}b+4\,a^{8}b^{3}+6\,a^{6}b^{5}+4\,a^{4}b^{7}+a^{2}b^{9}\right)\tan\left(dx+c\right)\right)\log\left(\frac{\tan\left(dx+c\right)^{2}}{\tan\left(dx+c\right)^{2}+1}\right)-3\,\left(10\,a^{9}b^{2}+5\,a^{7}b^{4}+4\,a^{5}b^{6}+a^{3}b^{8}+\left(10\,a^{6}b^{5}+5\,a^{4}b^{7}+4\,a^{2}b^{9}+b^{11}\right)\tan\left(dx+c\right)^{3}+3\,\left(10\,a^{7}b^{4}+5\,a^{5}b^{6}+4\,a^{3}b^{8}+ab^{10}\right)\tan\left(dx+c\right)^{2}+3\,\left(10\,a^{8}b^{3}+5\,a^{6}b^{5}+4\,a^{4}b^{7}+a^{2}b^{9}\right)\tan\left(dx+c\right)\right)\log\left(\frac{b^{2}\tan\left(dx+c\right)^{2}+2\,ab\tan\left(dx+c\right)+a^{2}}{\tan\left(dx+c\right)^{2}+1}\right)-3\,\left(20\,a^{8}b^{3}-37\,a^{6}b^{5}-18\,a^{4}b^{7}-5\,a^{2}b^{9}+24\,\left(a^{9}b^{2}-a^{7}b^{4}\right)dx\right)\tan\left(dx+c\right)}{6\,\left(\left(a^{12}b^{3}+4\,a^{10}b^{5}+6\,a^{8}b^{7}+4\,a^{6}b^{9}+a^{4}b^{11}\right)d\tan\left(dx+c\right)^{3}+3\,\left(a^{13}b^{2}+4\,a^{11}b^{4}+6\,a^{9}b^{6}+4\,a^{7}b^{8}+a^{5}b^{10}\right)d\tan\left(dx+c\right)^{2}+3\,\left(a^{14}b+4\,a^{12}b^{3}+6\,a^{10}b^{5}+4\,a^{8}b^{7}+a^{6}b^{9}\right)d\tan\left(dx+c\right)+\left(a^{15}+4\,a^{13}b^{2}+6\,a^{11}b^{4}+4\,a^{9}b^{6}+a^{7}b^{8}\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/6*(75*a^7*b^4 + 42*a^5*b^6 + 11*a^3*b^8 - (47*a^6*b^5 + 6*a^4*b^7 + 3*a^2
*b^9 + 24*(a^7*b^4 - a^5*b^6)*d*x)*tan(d*x + c)^3 - 24*(a^10*b - a^8*b^3)*d
*x - 3*(35*a^7*b^4 - 12*a^5*b^6 - 5*a^3*b^8 - 2*a*b^10 + 24*(a^8*b^3 - a^6*
b^5)*d*x)*tan(d*x + c)^2 + 3*(a^11 + 4*a^9*b^2 + 6*a^7*b^4 + 4*a^5*b^6 + a^
3*b^8 + (a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*tan(d*x + c)^3
 + 3*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*tan(d*x + c)^2
+ 3*(a^10*b + 4*a^8*b^3 + 6*a^6*b^5 + 4*a^4*b^7 + a^2*b^9)*tan(d*x + c))*lo
g(tan(d*x + c)^2/(tan(d*x + c)^2 + 1)) - 3*(10*a^9*b^2 + 5*a^7*b^4 + 4*a^5*
b^6 + a^3*b^8 + (10*a^6*b^5 + 5*a^4*b^7 + 4*a^2*b^9 + b^11)*tan(d*x + c)^3
+ 3*(10*a^7*b^4 + 5*a^5*b^6 + 4*a^3*b^8 + a*b^10)*tan(d*x + c)^2 + 3*(10*a^
8*b^3 + 5*a^6*b^5 + 4*a^4*b^7 + a^2*b^9)*tan(d*x + c))*log((b^2*tan(d*x + c
)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) - 3*(20*a^8*b^3 - 37*
a^6*b^5 - 18*a^4*b^7 - 5*a^2*b^9 + 24*(a^9*b^2 - a^7*b^4)*d*x)*tan(d*x + c)
)/((a^12*b^3 + 4*a^10*b^5 + 6*a^8*b^7 + 4*a^6*b^9 + a^4*b^11)*d*tan(d*x + c
)^3 + 3*(a^13*b^2 + 4*a^11*b^4 + 6*a^9*b^6 + 4*a^7*b^8 + a^5*b^10)*d*tan(d*
x + c)^2 + 3*(a^14*b + 4*a^12*b^3 + 6*a^10*b^5 + 4*a^8*b^7 + a^6*b^9)*d*tan
(d*x + c) + (a^15 + 4*a^13*b^2 + 6*a^11*b^4 + 4*a^9*b^6 + a^7*b^8)*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 476 vs. 2(222) = 444.
time = 1.04, size = 476, normalized size = 2.11

$$-\frac{\frac{24\,(a^3b-ab^3)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{3\,(a^4-6\,a^2b^2+b^4)\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{6\,(10\,a^6b^3+5\,a^4b^5+4\,a^2b^7+b^9)\log(|b\tan(dx+c)+a|)}{a^{12}b+4\,a^{10}b^3+6\,a^8b^5+4\,a^6b^7+a^4b^9}-\frac{110\,a^6b^5\tan(dx+c)^3+55\,a^4b^7\tan(dx+c)^3+44\,a^2b^9\tan(dx+c)^3+11\,b^{11}\tan(dx+c)^3+366\,a^7b^4\tan(dx+c)^2+219\,a^5b^6\tan(dx+c)^2+156\,a^3b^8\tan(dx+c)^2+39\,ab^{10}\tan(dx+c)^2+411\,a^8b^3\tan(dx+c)+294\,a^6b^5\tan(dx+c)+195\,a^4b^7\tan(dx+c)+48\,a^2b^9\tan(dx+c)+157\,a^9b^2+136\,a^7b^4+89\,a^5b^6+22\,a^3b^8}{(a^{12}+4\,a^{10}b^2+6\,a^8b^4+4\,a^6b^6+a^4b^8)(b\tan(dx+c)+a)^3}-\frac{6\log(|\tan(dx+c)|)}{a^4}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/6*(24*(a^3*b - a*b^3)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6
 + b^8) + 3*(a^4 - 6*a^2*b^2 + b^4)*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^
2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 6*(10*a^6*b^3 + 5*a^4*b^5 + 4*a^2*b^7 +
b^9)*log(abs(b*tan(d*x + c) + a))/(a^12*b + 4*a^10*b^3 + 6*a^8*b^5 + 4*a^6*
b^7 + a^4*b^9) - (110*a^6*b^5*tan(d*x + c)^3 + 55*a^4*b^7*tan(d*x + c)^3 +
44*a^2*b^9*tan(d*x + c)^3 + 11*b^11*tan(d*x + c)^3 + 366*a^7*b^4*tan(d*x +
c)^2 + 219*a^5*b^6*tan(d*x + c)^2 + 156*a^3*b^8*tan(d*x + c)^2 + 39*a*b^10*
tan(d*x + c)^2 + 411*a^8*b^3*tan(d*x + c) + 294*a^6*b^5*tan(d*x + c) + 195*
a^4*b^7*tan(d*x + c) + 48*a^2*b^9*tan(d*x + c) + 157*a^9*b^2 + 136*a^7*b^4
+ 89*a^5*b^6 + 22*a^3*b^8)/((a^12 + 4*a^10*b^2 + 6*a^8*b^4 + 4*a^6*b^6 + a^
4*b^8)*(b*tan(d*x + c) + a)^3) - 6*log(abs(tan(d*x + c)))/a^4)/d
```

Mupad [B]
time = 4.87, size = 385, normalized size = 1.70

$$\frac{\frac{47\,a^4\,b^2+34\,a^2\,b^4+11\,b^6}{6\,a\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)}+\frac{\tan(c+d\,x)\,(27\,a^4\,b^3+16\,a^2\,b^5+5\,b^7)}{2\,a^2\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)}+\frac{\tan(c+d\,x)^2\,(6\,a^4\,b^4+3\,a^2\,b^6+b^8)}{a^3\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)}}{d\,\left(a^3+3\,a^2\,b\tan\left(c+d\,x\right)+3\,a\,b^2\tan\left(c+d\,x\right)^2+b^3\tan\left(c+d\,x\right)^3\right)}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)}{2\,d\,\left(a^4+a^3\,b\,4\mathrm{i}-6\,a^2\,b^2-a\,b^3\,4\mathrm{i}+b^4\right)}+\frac{\ln\left(\tan(c+d\,x)\right)}{a^4\,d}-\frac{b^2\,\ln\left(a+b\tan(c+d\,x)\right)\,\left(10\,a^6+5\,a^4\,b^2+4\,a^2\,b^4+b^6\right)}{a^4\,d\,(a^2+b^2)^4}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(a^4\,1\mathrm{i}+4\,a^3\,b-a^2\,b^2\,6\mathrm{i}-4\,a\,b^3+b^4\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + b*tan(c + d*x))^4,x)
```

```
[Out] ((11*b^6 + 34*a^2*b^4 + 47*a^4*b^2)/(6*a*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2
)) + (tan(c + d*x)*(5*b^7 + 16*a^2*b^5 + 27*a^4*b^3))/(2*a^2*(a^6 + b^6 + 3
*a^2*b^4 + 3*a^4*b^2)) + (tan(c + d*x)^2*(b^8 + 3*a^2*b^6 + 6*a^4*b^4))/(a^
3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))/(d*(a^3 + b^3*tan(c + d*x)^3 + 3*a*
b^2*tan(c + d*x)^2 + 3*a^2*b*tan(c + d*x))) - (log(tan(c + d*x) + 1i)*1i)/(
2*d*(4*a^3*b - 4*a*b^3 + a^4*1i + b^4*1i - a^2*b^2*6i)) - log(tan(c + d*x)
- 1i)/(2*d*(a^3*b*4i - a*b^3*4i + a^4 + b^4 - 6*a^2*b^2)) + log(tan(c + d*x
))/(a^4*d) - (b^2*log(a + b*tan(c + d*x))*(10*a^6 + b^6 + 4*a^2*b^4 + 5*a^4
*b^2))/(a^4*d*(a^2 + b^2)^4)
```

3.494 $\int \frac{\cot^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx$

Optimal. Leaf size=278

$$-\frac{(a^4-6a^2b^2+b^4)\,x}{(a^2+b^2)^4}-\frac{4b\log(\sin(c+dx))}{a^5d}+\frac{4b^3(5a^6+6a^4b^2+4a^2b^4+b^6)\log(a\cos(c+dx)+b\sin(c+dx))}{a^5\,(a^2+b^2)^4\,d}$$

```
[Out] -(a^4-6*a^2*b^2+b^4)*x/(a^2+b^2)^4-4*b*ln(sin(d*x+c))/a^5/d+4*b^3*(5*a^6+6*
a^4*b^2+4*a^2*b^4+b^6)*ln(a*cos(d*x+c)+b*sin(d*x+c))/a^5/(a^2+b^2)^4/d-1/3*
b*(3*a^2+4*b^2)/a^2/(a^2+b^2)/d/(a+b*tan(d*x+c))^3-cot(d*x+c)/a/d/(a+b*tan(
d*x+c))^3-b*(a^4+4*a^2*b^2+2*b^4)/a^3/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^2-b*(a
^6+13*a^4*b^2+12*a^2*b^4+4*b^6)/a^4/(a^2+b^2)^3/d/(a+b*tan(d*x+c))
```

Rubi [A]

time = 0.58, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3650, 3730, 3732, 3611, 3556}

$$-\frac{4b\log(\sin(c+dx))}{a^5d}-\frac{b(3a^2+4b^2)}{3a^2d\,(a^2+b^2)\,(a+b\tan(c+dx))^3}-\frac{x(a^4-6a^2b^2+b^4)}{(a^2+b^2)^4}-\frac{b(a^6+13a^4b^2+12a^2b^4+4b^6)}{a^4d\,(a^2+b^2)^3\,(a+b\tan(c+dx))}-\frac{b(a^4+4a^2b^2+2b^4)}{a^3d\,(a^2+b^2)^2\,(a+b\tan(c+dx))^2}+\frac{4b^3(5a^6+6a^4b^2+4a^2b^4+b^6)\log(a\cos(c+dx)+b\sin(c+dx))}{a^5d\,(a^2+b^2)^4}-\frac{\cot(c+dx)}{ad(a+b\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -(((a^4 - 6*a^2*b^2 + b^4)*x)/(a^2 + b^2)^4) - (4*b*Log[Sin[c + d*x]])/(a^5
*d) + (4*b^3*(5*a^6 + 6*a^4*b^2 + 4*a^2*b^4 + b^6)*Log[a*Cos[c + d*x] + b*S
in[c + d*x]])/(a^5*(a^2 + b^2)^4*d) - (b*(3*a^2 + 4*b^2))/(3*a^2*(a^2 + b^2
)*d*(a + b*Tan[c + d*x])^3) - Cot[c + d*x]/(a*d*(a + b*Tan[c + d*x])^3) - (
b*(a^4 + 4*a^2*b^2 + 2*b^4))/(a^3*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^2) -
 (b*(a^6 + 13*a^4*b^2 + 12*a^2*b^4 + 4*b^6))/(a^4*(a^2 + b^2)^3*d*(a + b*Ta
n[c + d*x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
```

```
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx = -\frac{\cot(c+dx)}{ad(a+b\tan(c+dx))^3} - \frac{\int \frac{\cot(c+dx)(4b+a\tan(c+dx)+4b\tan^2(c+dx))}{(a+b\tan(c+dx))^4}\,dx}{a}$$

$$= -\frac{b(3a^2+4b^2)}{3a^2(a^2+b^2)\,d(a+b\tan(c+dx))^3} - \frac{\cot(c+dx)}{ad(a+b\tan(c+dx))^3} - \frac{\int \frac{\cot(c+dx)(12}{}}{}$$

$$= -\frac{b(3a^2+4b^2)}{3a^2(a^2+b^2)\,d(a+b\tan(c+dx))^3} - \frac{\cot(c+dx)}{ad(a+b\tan(c+dx))^3} - \frac{b(a^4}{a^3(a^2+b^2)^2}$$

$$= -\frac{b(3a^2+4b^2)}{3a^2(a^2+b^2)\,d(a+b\tan(c+dx))^3} - \frac{\cot(c+dx)}{ad(a+b\tan(c+dx))^3} - \frac{b(a^4}{a^3(a^2+b^2)^2}$$

$$= -\frac{(a^4-6a^2b^2+b^4)\,x}{(a^2+b^2)^4} - \frac{b(3a^2+4b^2)}{3a^2(a^2+b^2)\,d(a+b\tan(c+dx))^3} - \frac{\cot(c+dx)}{ad(a+b\tan(c-}$$

$$= -\frac{(a^4-6a^2b^2+b^4)\,x}{(a^2+b^2)^4} - \frac{4b\log(\sin(c+dx))}{a^5d} + \frac{4b^3(5a^6+6a^4b^2+4a^2b^4+b^6)\,\mathrm{lo}}{a^5(a^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.58, size = 241, normalized size = 0.87

$$-\frac{\frac{\cot(c+dx)}{a^4} - \frac{b^6}{3a^5(a^2+b^2)(b+a\cot(c+dx))^3} + \frac{b^5(3a^2+2b^2)}{a^5(a^2+b^2)^2(b+a\cot(c+dx))^2} - \frac{b^4(15a^4+17a^2b^2+6b^4)}{a^5(a^2+b^2)^3(b+a\cot(c+dx))} + \frac{i\log(i-\cot(c+dx))}{2(a-ib)^4} - \frac{i\log(i+\cot(c+dx))}{2(a+ib)^4} - \frac{4b^3(5a^6+6a^4b^2+4a^2b^4+b^6)\log(b+a\cot(c+dx))}{a^5(a^2+b^2)^4}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -((Cot[c + d*x]/a^4 - b^6/(3*a^5*(a^2 + b^2)*(b + a*Cot[c + d*x])^3) + (b^5
*(3*a^2 + 2*b^2))/(a^5*(a^2 + b^2)^2*(b + a*Cot[c + d*x])^2) - (b^4*(15*a^4
 + 17*a^2*b^2 + 6*b^4))/(a^5*(a^2 + b^2)^3*(b + a*Cot[c + d*x])) + ((I/2)*L
og[I - Cot[c + d*x]])/(a - I*b)^4 - ((I/2)*Log[I + Cot[c + d*x]])/(a + I*b)
^4 - (4*b^3*(5*a^6 + 6*a^4*b^2 + 4*a^2*b^4 + b^6)*Log[b + a*Cot[c + d*x]])/
(a^5*(a^2 + b^2)^4))/d)
```

Maple [A]
time = 0.46, size = 264, normalized size = 0.95

method	result
derivativedivides	$\frac{-\frac{1}{a^4\tan(dx+c)} - \frac{4b\ln(\tan(dx+c))}{a^5} - \frac{b^3}{3(a^2+b^2)a^2(a+b\tan(dx+c))^3} - \frac{b^3(10a^4+9a^2b^2+3b^4)}{(a^2+b^2)^3a^4(a+b\tan(dx+c))} - \frac{b^3(2a^2+b^2)}{(a^2+b^2)^2a^3(a+b\tan(dx+c))}}{d}$

default	$\frac{-\frac{1}{a^4\tan(dx+c)}-\frac{4b\ln(\tan(dx+c))}{a^5}-\frac{b^3}{3(a^2+b^2)a^2(a+b\tan(dx+c))^3}-\frac{b^3(10a^4+9a^2b^2+3b^4)}{(a^2+b^2)^3a^4(a+b\tan(dx+c))}-\frac{b^3(2a^2+b^2)}{(a^2+b^2)^2a^3(a+b\tan(dx+c))^2}}{d}$
norman	$\frac{\frac{b(6a^6b+33a^4b^3+35b^5a^2+12b^7)(\tan^2(dx+c))}{d\,a^3(a^6+3a^4b^2+3a^2b^4+b^6)}+\frac{b^2(8a^6b+51a^4b^3+53b^5a^2+18b^7)(\tan^3(dx+c))}{d\,a^4(a^6+3a^4b^2+3a^2b^4+b^6)}-\frac{1}{da}-\frac{b^3(a^4-6a^2b^2+b^4)x(\tan^4(dx}{(a^6+3a^4b^2+3a^2b^4+b^6)(a^2}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a^4/tan(d*x+c)-4/a^5*b*ln(tan(d*x+c))-1/3*b^3/(a^2+b^2)/a^2/(a+b*ta
n(d*x+c))^3-b^3*(10*a^4+9*a^2*b^2+3*b^4)/(a^2+b^2)^3/a^4/(a+b*tan(d*x+c))-b
^3*(2*a^2+b^2)/(a^2+b^2)^2/a^3/(a+b*tan(d*x+c))^2+4*b^3*(5*a^6+6*a^4*b^2+4*
a^2*b^4+b^6)/(a^2+b^2)^4/a^5*ln(a+b*tan(d*x+c))+1/(a^2+b^2)^4*(1/2*(4*a^3*b
-4*a*b^3)*ln(1+tan(d*x+c)^2)+(-a^4+6*a^2*b^2-b^4)*arctan(tan(d*x+c))))
```

Maxima [A]
time = 0.51, size = 516, normalized size = 1.86

$$-\frac{\frac{3\,(a^4-6\,a^2b^2+b^4)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}-\frac{12\,(5\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9)\log(b\tan(dx+c)+a)}{a^{13}+4\,a^{11}b^2+6\,a^9b^4+4\,a^7b^6+a^5b^8}-\frac{6\,(a^3b-ab^3)\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{3\,a^9+9\,a^7b^2+9\,a^5b^4+3\,a^3b^6+3\,(a^6b^3+13\,a^4b^5+12\,a^2b^7+4\,b^9)\tan(dx+c)^3+3\,(3\,a^7b^2+31\,a^5b^4+30\,a^3b^6+10\,ab^8)\tan(dx+c)^2+(9\,a^8b+64\,a^6b^3+65\,a^4b^5+22\,a^2b^7)\tan(dx+c)}{(a^{10}b^3+3\,a^8b^5+3\,a^6b^7+a^4b^9)\tan(dx+c)^4+3\,(a^{11}b^2+3\,a^9b^4+3\,a^7b^6+a^5b^8)\tan(dx+c)^3+3\,(a^{12}b+3\,a^{10}b^3+3\,a^8b^5+a^6b^7)\tan(dx+c)^2+(a^{13}+3\,a^{11}b^2+3\,a^9b^4+a^7b^6)\tan(dx+c)}+\frac{12\,b\log(\tan(dx+c))}{a^5}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*(a^4 - 6*a^2*b^2 + b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*
a^2*b^6 + b^8) - 12*(5*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*log(b*tan(d*x
 + c) + a)/(a^13 + 4*a^11*b^2 + 6*a^9*b^4 + 4*a^7*b^6 + a^5*b^8) - 6*(a^3*b
 - a*b^3)*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6
+ b^8) + (3*a^9 + 9*a^7*b^2 + 9*a^5*b^4 + 3*a^3*b^6 + 3*(a^6*b^3 + 13*a^4*b
^5 + 12*a^2*b^7 + 4*b^9)*tan(d*x + c)^3 + 3*(3*a^7*b^2 + 31*a^5*b^4 + 30*a^
3*b^6 + 10*a*b^8)*tan(d*x + c)^2 + (9*a^8*b + 64*a^6*b^3 + 65*a^4*b^5 + 22*
a^2*b^7)*tan(d*x + c))/((a^10*b^3 + 3*a^8*b^5 + 3*a^6*b^7 + a^4*b^9)*tan(d*
x + c)^4 + 3*(a^11*b^2 + 3*a^9*b^4 + 3*a^7*b^6 + a^5*b^8)*tan(d*x + c)^3 +
3*(a^12*b + 3*a^10*b^3 + 3*a^8*b^5 + a^6*b^7)*tan(d*x + c)^2 + (a^13 + 3*a^
11*b^2 + 3*a^9*b^4 + a^7*b^6)*tan(d*x + c)) + 12*b*log(tan(d*x + c))/a^5)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(276) = 552$.
time = 1.27, size = 925, normalized size = 3.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^12 + 12*a^10*b^2 + 18*a^8*b^4 + 12*a^6*b^6 + 3*a^4*b^8 - (37*a^6*
b^6 + 21*a^4*b^8 + 6*a^2*b^10 - 3*(a^9*b^3 - 6*a^7*b^5 + a^5*b^7)*d*x)*tan(
d*x + c)^4 + 3*(a^9*b^3 - 23*a^7*b^5 + 4*a^5*b^7 + 10*a^3*b^9 + 4*a*b^11 +
3*(a^10*b^2 - 6*a^8*b^4 + a^6*b^6)*d*x)*tan(d*x + c)^3 + 3*(3*a^10*b^2 - 3*
a^8*b^4 + 40*a^6*b^6 + 34*a^4*b^8 + 10*a^2*b^10 + 3*(a^11*b - 6*a^9*b^3 + a
^7*b^5)*d*x)*tan(d*x + c)^2 + 6*((a^8*b^4 + 4*a^6*b^6 + 6*a^4*b^8 + 4*a^2*b
^10 + b^12)*tan(d*x + c)^4 + 3*(a^9*b^3 + 4*a^7*b^5 + 6*a^5*b^7 + 4*a^3*b^9
 + a*b^11)*tan(d*x + c)^3 + 3*(a^10*b^2 + 4*a^8*b^4 + 6*a^6*b^6 + 4*a^4*b^8
 + a^2*b^10)*tan(d*x + c)^2 + (a^11*b + 4*a^9*b^3 + 6*a^7*b^5 + 4*a^5*b^7 +
 a^3*b^9)*tan(d*x + c))*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1)) - 6*((5*a^
6*b^6 + 6*a^4*b^8 + 4*a^2*b^10 + b^12)*tan(d*x + c)^4 + 3*(5*a^7*b^5 + 6*a^
5*b^7 + 4*a^3*b^9 + a*b^11)*tan(d*x + c)^3 + 3*(5*a^8*b^4 + 6*a^6*b^6 + 4*a
^4*b^8 + a^2*b^10)*tan(d*x + c)^2 + (5*a^9*b^3 + 6*a^7*b^5 + 4*a^5*b^7 + a^
3*b^9)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(t
an(d*x + c)^2 + 1)) + (9*a^11*b + 36*a^9*b^3 + 108*a^7*b^5 + 81*a^5*b^7 + 2
2*a^3*b^9 + 3*(a^12 - 6*a^10*b^2 + a^8*b^4)*d*x)*tan(d*x + c))/((a^13*b^3 +
 4*a^11*b^5 + 6*a^9*b^7 + 4*a^7*b^9 + a^5*b^11)*d*tan(d*x + c)^4 + 3*(a^14*
b^2 + 4*a^12*b^4 + 6*a^10*b^6 + 4*a^8*b^8 + a^6*b^10)*d*tan(d*x + c)^3 + 3*
(a^15*b + 4*a^13*b^3 + 6*a^11*b^5 + 4*a^9*b^7 + a^7*b^9)*d*tan(d*x + c)^2 +
 (a^16 + 4*a^14*b^2 + 6*a^12*b^4 + 4*a^10*b^6 + a^8*b^8)*d*tan(d*x + c))
```

<u>Sympy</u> [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+b*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

<u>Giac</u> [A]
time = 1.12, size = 502, normalized size = 1.81

$$-\frac{\frac{3\,(a^4-6\,a^2b^2+b^4)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}-\frac{6\,(a^3b-ab^3)\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}-\frac{12\,(5\,a^6b^4+6\,a^4b^6+4\,a^2b^8+b^{10})\log(|b\tan(dx+c)+a|)}{a^{13}b+4\,a^{11}b^3+6\,a^9b^5+4\,a^7b^7+a^5b^9}+\frac{110\,a^6b^6\tan(dx+c)^3+132\,a^4b^8\tan(dx+c)^3+88\,a^2b^{10}\tan(dx+c)^3+22\,b^{12}\tan(dx+c)^3+360\,a^7b^5\tan(dx+c)^2+453\,a^5b^7\tan(dx+c)^2+300\,a^3b^9\tan(dx+c)^2+75\,ab^{11}\tan(dx+c)^2+396\,a^8b^4\tan(dx+c)+525\,a^6b^6\tan(dx+c)+348\,a^4b^8\tan(dx+c)+87\,a^2b^{10}\tan(dx+c)+147\,a^9b^3+207\,a^7b^5+139\,a^5b^7+35\,a^3b^9}{(a^{13}+4\,a^{11}b^2+6\,a^9b^4+4\,a^7b^6+a^5b^8)(b\tan(dx+c)+a)^3}+\frac{12\,b\log(|\tan(dx+c)|)}{a^5}-\frac{3\,(4\,b\tan(dx+c)-a)}{a^5\tan(dx+c)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(a^4 - 6*a^2*b^2 + b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*
a^2*b^6 + b^8) - 6*(a^3*b - a*b^3)*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2
 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 12*(5*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b
^10)*log(abs(b*tan(d*x + c) + a))/(a^13*b + 4*a^11*b^3 + 6*a^9*b^5 + 4*a^7*
b^7 + a^5*b^9) + (110*a^6*b^6*tan(d*x + c)^3 + 132*a^4*b^8*tan(d*x + c)^3 +
 88*a^2*b^10*tan(d*x + c)^3 + 22*b^12*tan(d*x + c)^3 + 360*a^7*b^5*tan(d*x
```

```
+ c)^2 + 453*a^5*b^7*tan(d*x + c)^2 + 300*a^3*b^9*tan(d*x + c)^2 + 75*a*b^1
1*tan(d*x + c)^2 + 396*a^8*b^4*tan(d*x + c) + 525*a^6*b^6*tan(d*x + c) + 34
8*a^4*b^8*tan(d*x + c) + 87*a^2*b^10*tan(d*x + c) + 147*a^9*b^3 + 207*a^7*b
^5 + 139*a^5*b^7 + 35*a^3*b^9)/((a^13 + 4*a^11*b^2 + 6*a^9*b^4 + 4*a^7*b^6
+ a^5*b^8)*(b*tan(d*x + c) + a)^3) + 12*b*log(abs(tan(d*x + c)))/a^5 - 3*(4
*b*tan(d*x + c) - a)/(a^5*tan(d*x + c)))/d
```

Mupad [B]
time = 5.55, size = 430, normalized size = 1.55

$$\frac{4\,b^3\,\ln\left(a+b\,\tan(c+d\,x)\right)\,\left(5\,a^6+6\,a^4\,b^2+4\,a^2\,b^4+b^6\right)}{a^5\,d\,{\left(a^2+b^2\right)}^4}-\frac{\frac{1}{a}+\frac{{\tan(c+d\,x)}^3\,\left(a^6\,b^3+13\,a^4\,b^5+12\,a^2\,b^7+4\,b^9\right)}{a^4\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{{\tan(c+d\,x)}^2\,\left(3\,a^6\,b^2+31\,a^4\,b^4+30\,a^2\,b^6+10\,b^8\right)}{a^3\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{\tan(c+d\,x)\,\left(9\,a^6\,b+64\,a^4\,b^3+65\,a^2\,b^5+22\,b^7\right)}{3\,a^2\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}}{d\,\left(a^3\,\tan\left(c+d\,x\right)+3\,a^2\,b\,{\tan\left(c+d\,x\right)}^2+3\,a\,b^2\,{\tan\left(c+d\,x\right)}^3+b^3\,{\tan\left(c+d\,x\right)}^4\right)}-\frac{4\,b\,\ln\left(\tan(c+d\,x)\right)}{a^5\,d}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)}{2\,d\,\left(a^4\,1{}\mathrm{i}-4\,a^3\,b-a^2\,b^2\,6{}\mathrm{i}+4\,a\,b^3+b^4\,1{}\mathrm{i}\right)}-\frac{\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2\,d\,\left(a^4-a^3\,b\,4{}\mathrm{i}-6\,a^2\,b^2+a\,b^3\,4{}\mathrm{i}+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + b*tan(c + d*x))^4,x)
```

```
[Out] (4*b^3*log(a + b*tan(c + d*x))*(5*a^6 + b^6 + 4*a^2*b^4 + 6*a^4*b^2))/(a^5*
d*(a^2 + b^2)^4) - (log(tan(c + d*x) + 1i)*1i)/(2*d*(a*b^3*4i - a^3*b*4i +
a^4 + b^4 - 6*a^2*b^2)) - (1/a + (tan(c + d*x)^3*(4*b^9 + 12*a^2*b^7 + 13*a
^4*b^5 + a^6*b^3))/(a^4*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (tan(c + d*x
)^2*(10*b^8 + 30*a^2*b^6 + 31*a^4*b^4 + 3*a^6*b^2))/(a^3*(a^6 + b^6 + 3*a^2
*b^4 + 3*a^4*b^2)) + (tan(c + d*x)*(9*a^6*b + 22*b^7 + 65*a^2*b^5 + 64*a^4*
b^3))/(3*a^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))/(d*(a^3*tan(c + d*x) + b
^3*tan(c + d*x)^4 + 3*a^2*b*tan(c + d*x)^2 + 3*a*b^2*tan(c + d*x)^3)) - (4*
b*log(tan(c + d*x)))/(a^5*d) - log(tan(c + d*x) - 1i)/(2*d*(4*a*b^3 - 4*a^3
*b + a^4*1i + b^4*1i - a^2*b^2*6i))
```

3.495 $\int \frac{1}{3+5\tan(c+dx)}\,dx$

Optimal. Leaf size=31

$$\frac{3x}{34}+\frac{5\log(3\cos(c+dx)+5\sin(c+dx))}{34d}$$

```
[Out] 3/34*x+5/34*ln(3*cos(d*x+c)+5*sin(d*x+c))/d
```

Rubi [A]
time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {3565, 3611}

$$\frac{5\log(5\sin(c+dx)+3\cos(c+dx))}{34d}+\frac{3x}{34}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 5*Tan[c + d*x])^(-1),x]
```

```
[Out] (3*x)/34 + (5*Log[3*Cos[c + d*x] + 5*Sin[c + d*x]])/(34*d)
```

Rule 3565

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{3+5\tan(c+dx)}\,dx &= \frac{3x}{34}+\frac{5}{34}\int \frac{5-3\tan(c+dx)}{3+5\tan(c+dx)}\,dx\\ &= \frac{3x}{34}+\frac{5\log(3\cos(c+dx)+5\sin(c+dx))}{34d}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.04, size = 65, normalized size = 2.10

$$-\frac{\left(\frac{5}{68}+\frac{3i}{68}\right)\log(i-\tan(c+dx))}{d}-\frac{\left(\frac{5}{68}-\frac{3i}{68}\right)\log(i+\tan(c+dx))}{d}+\frac{5\log(3+5\tan(c+dx))}{34d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 5*Tan[c + d*x])^(-1),x]
```

```
[Out] ((-5/68 - (3*I)/68)*Log[I - Tan[c + d*x]])/d - ((5/68 - (3*I)/68)*Log[I + T
an[c + d*x]])/d + (5*Log[3 + 5*Tan[c + d*x]])/(34*d)
```

Maple [A]
time = 0.08, size = 41, normalized size = 1.32

method	result	size
risch	$\frac{3x}{34}-\frac{5ix}{34}-\frac{5ic}{17d}+\frac{5\ln\left(\mathrm{e}^{2i(dx+c)}-\frac{8}{17}+\frac{15i}{17}\right)}{34d}$	35
norman	$\frac{3x}{34}+\frac{5\ln(3+5\tan(dx+c))}{34d}-\frac{5\ln\left(1+\tan^2(dx+c)\right)}{68d}$	37
derivativedivides	$\frac{-\frac{5\ln\left(1+\tan^2(dx+c)\right)}{68}+\frac{3\arctan(\tan(dx+c))}{34}+\frac{5\ln(3+5\tan(dx+c))}{34}}{d}$	41
default	$\frac{-\frac{5\ln\left(1+\tan^2(dx+c)\right)}{68}+\frac{3\arctan(\tan(dx+c))}{34}+\frac{5\ln(3+5\tan(dx+c))}{34}}{d}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-5/68*ln(1+tan(d*x+c)^2)+3/34*arctan(tan(d*x+c))+5/34*ln(3+5*tan(d*x+c
)))
```

Maxima [A]
time = 0.49, size = 39, normalized size = 1.26

$$\frac{6\,dx+6\,c-5\,\log\left(\tan\left(dx+c\right)^2+1\right)+10\,\log\left(5\,\tan\left(dx+c\right)+3\right)}{68\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/68*(6*d*x + 6*c - 5*log(tan(d*x + c)^2 + 1) + 10*log(5*tan(d*x + c) + 3))
/d
```

Fricas [A]
time = 1.38, size = 46, normalized size = 1.48

$$\frac{6\,dx+5\,\log\left(\frac{25\,\tan\left(dx+c\right)^2+30\,\tan\left(dx+c\right)+9}{\tan\left(dx+c\right)^2+1}\right)}{68\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/68*(6*d*x + 5*log((25*tan(d*x + c)^2 + 30*tan(d*x + c) + 9)/(tan(d*x + c)
^2 + 1)))/d
```

Sympy [A]
time = 0.19, size = 46, normalized size = 1.48

$$\begin{cases} \frac{3x}{34} + \frac{5\log(5\tan(c+dx)+3)}{34d} - \frac{5\log(\tan^2(c+dx)+1)}{68d} & \text{for } d \neq 0 \\ \frac{x}{5\tan(c)+3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*tan(d*x+c)),x)
```

```
[Out] Piecewise((3*x/34 + 5*log(5*tan(c + d*x) + 3)/(34*d) - 5*log(tan(c + d*x)**
2 + 1)/(68*d), Ne(d, 0)), (x/(5*tan(c) + 3), True))
```

Giac [A]
time = 0.48, size = 40, normalized size = 1.29

$$\frac{6\,dx + 6\,c - 5\,\log\left(\tan\left(dx+c\right)^2 + 1\right) + 10\,\log\left(\left|5\,\tan\left(dx+c\right)+3\right|\right)}{68\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/68*(6*d*x + 6*c - 5*log(tan(d*x + c)^2 + 1) + 10*log(abs(5*tan(d*x + c) +
 3)))/d
```

Mupad [B]
time = 4.11, size = 49, normalized size = 1.58

$$\frac{5\,\ln\left(\tan(c+d\,x)+\frac{3}{5}\right)}{34\,d}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-\frac{5}{68}-\frac{3}{68}\mathrm{i}\right)}{d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(-\frac{5}{68}+\frac{3}{68}\mathrm{i}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*tan(c + d*x) + 3),x)
```

```
[Out] (5*log(tan(c + d*x) + 3/5))/(34*d) - (log(tan(c + d*x) + 1i)*(5/68 - 3i/68)
)/d - (log(tan(c + d*x) - 1i)*(5/68 + 3i/68))/d
```

3.496 $\int \frac{1}{(3+5\tan(c+dx))^2}\,dx$

Optimal. Leaf size=50

$$-\frac{4x}{289}+\frac{15\log(3\cos(c+dx)+5\sin(c+dx))}{578d}-\frac{5}{34d(3+5\tan(c+dx))}$$

```
[Out] -4/289*x+15/578*ln(3*cos(d*x+c)+5*sin(d*x+c))/d-5/34/d/(3+5*tan(d*x+c))
```

Rubi [A]
time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3564, 3612, 3611}

$$-\frac{5}{34d(5\tan(c+dx)+3)}+\frac{15\log(5\sin(c+dx)+3\cos(c+dx))}{578d}-\frac{4x}{289}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 5*Tan[c + d*x])^(-2),x]
```

```
[Out] (-4*x)/289 + (15*Log[3*Cos[c + d*x] + 5*Sin[c + d*x]])/(578*d) - 5/(34*d*(3
 + 5*Tan[c + d*x]))
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{1}{(3+5\tan(c+dx))^2}\,dx = -\frac{5}{34d(3+5\tan(c+dx))} + \frac{1}{34}\int \frac{3-5\tan(c+dx)}{3+5\tan(c+dx)}\,dx$$
$$= -\frac{4x}{289} - \frac{5}{34d(3+5\tan(c+dx))} + \frac{15}{578}\int \frac{5-3\tan(c+dx)}{3+5\tan(c+dx)}\,dx$$
$$= -\frac{4x}{289} + \frac{15\log(3\cos(c+dx)+5\sin(c+dx))}{578d} - \frac{5}{34d(3+5\tan(c+dx))}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.28, size = 67, normalized size = 1.34

$$-\frac{(15-8i)\log(i-\tan(c+dx))+(15+8i)\log(i+\tan(c+dx))-30\log(3+5\tan(c+dx))+\frac{170}{3+5\tan(c+dx)}}{1156d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 5*Tan[c + d*x])^(-2),x]
```

```
[Out] -1/1156*((15 - 8*I)*Log[I - Tan[c + d*x]] + (15 + 8*I)*Log[I + Tan[c + d*x]
] - 30*Log[3 + 5*Tan[c + d*x]] + 170/(3 + 5*Tan[c + d*x]))/d
```

Maple [A]
time = 0.08, size = 55, normalized size = 1.10

method	result
derivativedivides	$\frac{-\frac{15\ln\left(1+\tan^2(dx+c)\right)}{1156}-\frac{4\arctan(\tan(dx+c))}{289}-\frac{5}{34(3+5\tan(dx+c))}+\frac{15\ln(3+5\tan(dx+c))}{578}}{d}$
default	$\frac{-\frac{15\ln\left(1+\tan^2(dx+c)\right)}{1156}-\frac{4\arctan(\tan(dx+c))}{289}-\frac{5}{34(3+5\tan(dx+c))}+\frac{15\ln(3+5\tan(dx+c))}{578}}{d}$
norman	$\frac{-\frac{12x}{289}-\frac{20x\tan(dx+c)}{289}+\frac{25\tan(dx+c)}{102d}}{3+5\tan(dx+c)}+\frac{15\ln(3+5\tan(dx+c))}{578d}-\frac{15\ln\left(1+\tan^2(dx+c)\right)}{1156d}$
risch	$-\frac{4x}{289}-\frac{15ix}{578}-\frac{15ic}{289d}-\frac{375}{578d\left(17\,\mathrm{e}^{2i(dx+c)}-8+15i\right)}-\frac{100i}{289d\left(17\,\mathrm{e}^{2i(dx+c)}-8+15i\right)}+\frac{15\ln\left(\mathrm{e}^{2i(dx+c)}-\frac{8}{17}+\frac{15i}{17}\right)}{578d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-15/1156*ln(1+tan(d*x+c)^2)-4/289*arctan(tan(d*x+c))-5/34/(3+5*tan(d*x
+c))+15/578*ln(3+5*tan(d*x+c)))
```

Maxima [A]
time = 0.50, size = 53, normalized size = 1.06

$$-\frac{16\,dx+16\,c+\frac{170}{5\,\tan(dx+c)+3}+15\,\log\left(\tan(dx+c)^2+1\right)-30\,\log\left(5\,\tan(dx+c)+3\right)}{1156\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/1156*(16*d*x + 16*c + 170/(5*tan(d*x + c) + 3) + 15*log(tan(d*x + c)^2 +
 1) - 30*log(5*tan(d*x + c) + 3))/d
```

Fricas [A]
time = 1.02, size = 83, normalized size = 1.66

$$-\frac{48\,dx - 15\,(5\,\tan\left(dx + c\right) + 3)\log\left(\frac{25\,\tan(dx+c)^2 + 30\,\tan(dx+c) + 9}{\tan(dx+c)^2 + 1}\right) + 5\,(16\,dx - 15)\tan\left(dx + c\right) + 125}{1156\,(5\,d\tan\left(dx + c\right) + 3\,d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/1156*(48*d*x - 15*(5*tan(d*x + c) + 3)*log((25*tan(d*x + c)^2 + 30*tan(d
*x + c) + 9)/(tan(d*x + c)^2 + 1)) + 5*(16*d*x - 15)*tan(d*x + c) + 125)/(5
*d*tan(d*x + c) + 3*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(42) = 84$.
time = 0.29, size = 190, normalized size = 3.80

$$\begin{cases} -\frac{80dx\tan(c+dx)}{5780d\tan(c+dx)+3468d} - \frac{48dx}{5780d\tan(c+dx)+3468d} + \frac{150\log(5\tan(c+dx)+3)\tan(c+dx)}{5780d\tan(c+dx)+3468d} + \frac{90\log(5\tan(c+dx)+3)}{5780d\tan(c+dx)+3468d} - \frac{75\log(\tan^2(c+dx)+1)\tan(c+dx)}{5780d\tan(c+dx)+3468d} - \frac{45\log(\tan^2(c+dx)+1)}{5780d\tan(c+dx)+3468d} - \frac{170}{5780d\tan(c+dx)+3468d} & \text{for } d \neq 0 \\ \frac{x}{(5\tan(c)+3)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*tan(d*x+c))**2,x)
```

```
[Out] Piecewise((-80*d*x*tan(c + d*x)/(5780*d*tan(c + d*x) + 3468*d) - 48*d*x/(57
80*d*tan(c + d*x) + 3468*d) + 150*log(5*tan(c + d*x) + 3)*tan(c + d*x)/(578
0*d*tan(c + d*x) + 3468*d) + 90*log(5*tan(c + d*x) + 3)/(5780*d*tan(c + d*x
) + 3468*d) - 75*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(5780*d*tan(c + d*x)
 + 3468*d) - 45*log(tan(c + d*x)**2 + 1)/(5780*d*tan(c + d*x) + 3468*d) - 1
70/(5780*d*tan(c + d*x) + 3468*d), Ne(d, 0)), (x/(5*tan(c) + 3)**2, True))
```

Giac [A]
time = 0.46, size = 64, normalized size = 1.28

$$-\frac{16\,dx + 16\,c + \frac{10\,(15\,\tan(dx+c) + 26)}{5\,\tan(dx+c) + 3} + 15\,\log\left(\tan\left(dx + c\right)^2 + 1\right) - 30\,\log\left(|5\,\tan\left(dx + c\right) + 3|\right)}{1156\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/1156*(16*d*x + 16*c + 10*(15*tan(d*x + c) + 26)/(5*tan(d*x + c) + 3) + 1
5*log(tan(d*x + c)^2 + 1) - 30*log(abs(5*tan(d*x + c) + 3)))/d
```

Mupad [B]
time = 3.95, size = 64, normalized size = 1.28

$$\frac{15\,\ln\left(\tan(c+d\,x)+\frac{3}{5}\right)}{578\,d}-\frac{1}{34\,d\,\left(\tan\left(c+d\,x\right)+\frac{3}{5}\right)}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-\frac{15}{1156}+\frac{2}{289}\mathrm{i}\right)}{d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(-\frac{15}{1156}-\frac{2}{289}\mathrm{i}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*tan(c + d*x) + 3)^2,x)
```

```
[Out] (15*log(tan(c + d*x) + 3/5))/(578*d) - (log(tan(c + d*x) - 1i)*(15/1156 - 2
i/289))/d - (log(tan(c + d*x) + 1i)*(15/1156 + 2i/289))/d - 1/(34*d*(tan(c
+ d*x) + 3/5))
```

3.497 $\int \frac{1}{(3+5\tan(c+dx))^3} dx$

Optimal. Leaf size=69

$$-\frac{99x}{19652}+\frac{5\log(3\cos(c+dx)+5\sin(c+dx))}{19652d}-\frac{5}{68d(3+5\tan(c+dx))^2}-\frac{15}{578d(3+5\tan(c+dx))}$$

```
[Out] -99/19652*x+5/19652*ln(3*cos(d*x+c)+5*sin(d*x+c))/d-5/68/d/(3+5*tan(d*x+c))
^2-15/578/d/(3+5*tan(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3564, 3610, 3612, 3611}

$$-\frac{15}{578d(5\tan(c+dx)+3)}-\frac{5}{68d(5\tan(c+dx)+3)^2}+\frac{5\log(5\sin(c+dx)+3\cos(c+dx))}{19652d}-\frac{99x}{19652}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 5*Tan[c + d*x])^(-3),x]
```

```
[Out] (-99*x)/19652 + (5*Log[3*Cos[c + d*x] + 5*Sin[c + d*x]])/(19652*d) - 5/(68*
d*(3 + 5*Tan[c + d*x])^2) - 15/(578*d*(3 + 5*Tan[c + d*x]))
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{1}{(3+5\tan(c+dx))^3}\,dx = -\frac{5}{68d(3+5\tan(c+dx))^2}+\frac{1}{34}\int \frac{3-5\tan(c+dx)}{(3+5\tan(c+dx))^2}\,dx$$
$$= -\frac{5}{68d(3+5\tan(c+dx))^2}-\frac{15}{578d(3+5\tan(c+dx))}+\frac{\int \frac{-16-30\tan(c+dx)}{3+5\tan(c+dx)}\,dx}{1156}$$
$$= -\frac{99x}{19652}-\frac{5}{68d(3+5\tan(c+dx))^2}-\frac{15}{578d(3+5\tan(c+dx))}+\frac{5\int \frac{5-3\tan(c+}{3+5\tan(c+}}{19652}$$
$$= -\frac{99x}{19652}+\frac{5\log(3\cos(c+dx)+5\sin(c+dx))}{19652d}-\frac{5}{68d(3+5\tan(c+dx))^2}-$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.61, size = 84, normalized size = 1.22

$$\frac{\left(\frac{1}{39304}+\frac{i}{39304}\right)\left((47+52i)\log(i-\tan(c+dx))-(52+47i)\log(i+\tan(c+dx))+(5-5i)\left(\log(3+5\tan(c+dx))-\frac{85(7+6\tan(c+dx))}{(3+5\tan(c+dx))^2}\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 5*Tan[c + d*x])^(-3),x]
[Out] ((1/39304 + I/39304)*((47 + 52*I)*Log[I - Tan[c + d*x]] - (52 + 47*I)*Log[I
 + Tan[c + d*x]] + (5 - 5*I)*(Log[3 + 5*Tan[c + d*x]] - (85*(7 + 6*Tan[c +
d*x]))/(3 + 5*Tan[c + d*x])^2)))/d
```

Maple [A]
time = 0.11, size = 69, normalized size = 1.00

method	result
derivativedivides	$\frac{-\frac{5\ln\left(1+\tan^2(dx+c)\right)}{39304}-\frac{99\arctan(\tan(dx+c))}{19652}-\frac{5}{68(3+5\tan(dx+c))^2}-\frac{15}{578(3+5\tan(dx+c))}+\frac{5\ln(3+5\tan(dx+c))}{19652}}{d}$
default	$\frac{-\frac{5\ln\left(1+\tan^2(dx+c)\right)}{39304}-\frac{99\arctan(\tan(dx+c))}{19652}-\frac{5}{68(3+5\tan(dx+c))^2}-\frac{15}{578(3+5\tan(dx+c))}+\frac{5\ln(3+5\tan(dx+c))}{19652}}{d}$

risch	$-\frac{99x}{19652}-\frac{5ix}{19652}-\frac{5ic}{9826d}+\frac{\left(-\frac{2375}{2083112}-\frac{3675i}{2083112}\right)\left(1802\,{\mathrm e}^{2i(dx+c)}+27+1575i\right)}{d\left(17\,{\mathrm e}^{2i(dx+c)}-8+15i\right)^{2}}+\frac{5\ln\left({\mathrm e}^{2i(dx+c)}-\frac{8}{17}+\frac{15i}{17}\right)}{19652d}$
norman	$\frac{-\frac{891x}{19652}-\frac{1485x\tan(dx+c)}{9826}-\frac{2475x\left(\tan^{2}(dx+c)\right)}{19652}-\frac{175}{1156d}-\frac{75\tan(dx+c)}{578d}}{\left(3+5\tan(dx+c)\right)^{2}}+\frac{5\ln(3+5\tan(dx+c))}{19652d}-\frac{5\ln\left(1+\tan^{2}(dx+c)\right)}{39304d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-5/39304*ln(1+tan(d*x+c)^2)-99/19652*arctan(tan(d*x+c))-5/68/(3+5*tan(
d*x+c))^2-15/578/(3+5*tan(d*x+c))+5/19652*ln(3+5*tan(d*x+c)))
```

Maxima [A]
time = 0.49, size = 73, normalized size = 1.06

$$-\frac{198\,dx+198\,c+\frac{850\,(6\,\tan(dx+c)+7)}{25\,\tan(dx+c)^{2}+30\,\tan(dx+c)+9}+5\,\log\left(\tan\left(dx+c\right)^{2}+1\right)-10\,\log\left(5\,\tan\left(dx+c\right)+3\right)}{39304\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/39304*(198*d*x + 198*c + 850*(6*tan(d*x + c) + 7)/(25*tan(d*x + c)^2 + 3
0*tan(d*x + c) + 9) + 5*log(tan(d*x + c)^2 + 1) - 10*log(5*tan(d*x + c) + 3
))/d
```

Fricas [A]
time = 1.00, size = 120, normalized size = 1.74

$$-\frac{50\,(99\,dx-25)\tan\left(dx+c\right)^{2}+1782\,dx-5\left(25\,\tan\left(dx+c\right)^{2}+30\,\tan\left(dx+c\right)+9\right)\log\left(\frac{25\,\tan(dx+c)^{2}+30\,\tan(dx+c)+9}{\tan(dx+c)^{2}+1}\right)+180\,(33\,dx+20)\tan\left(dx+c\right)+5500}{39304\left(25\,d\tan\left(dx+c\right)^{2}+30\,d\tan\left(dx+c\right)+9\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/39304*(50*(99*d*x - 25)*tan(d*x + c)^2 + 1782*d*x - 5*(25*tan(d*x + c)^2
 + 30*tan(d*x + c) + 9)*log((25*tan(d*x + c)^2 + 30*tan(d*x + c) + 9)/(tan(
d*x + c)^2 + 1)) + 180*(33*d*x + 20)*tan(d*x + c) + 5500)/(25*d*tan(d*x + c
)^2 + 30*d*tan(d*x + c) + 9*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(60) = 120$.
time = 0.38, size = 442, normalized size = 6.41

$$\begin{cases} -\frac{4950dx\tan^{2}(c+dx)}{982600d\tan^{2}(c+dx)+1179120d\tan(c+dx)+353736d}-\frac{5940dx\tan(c+dx)}{982600d\tan^{2}(c+dx)+1179120d\tan(c+dx)+353736d}-\frac{1782dx}{982600d\tan^{2}(c+dx)+1179120d\tan(c+dx)+353736d}+\frac{250\log(5\tan(c+dx)+3)\tan^{2}(c+dx)}{982600d\tan^{2}(c+dx)+1179120d\tan(c+dx)+353736d}+\frac{300\log(5\tan(c+dx)+3)\tan(c+dx)}{982600d\tan^{2}(c+dx)+1179120d\tan(c+dx)+353736d}+\frac{90\log(5\tan(c+dx)+3)}{982600d\tan^{2}(c+dx)+1179120d\tan(c+dx)+353736d}-\frac{125\log(\tan^{2}(c+dx)+1)\tan^{2}(c+dx)}{982600d\tan^{2}(c+dx)+1179120d\tan(c+dx)+353736d}-\frac{150\log(\tan^{2}(c+dx)+1)\tan(c+dx)}{982600d\tan^{2}(c+dx)+1179120d\tan(c+dx)+353736d}-\frac{45\log(\tan^{2}(c+dx)+1)}{982600d\tan^{2}(c+dx)+1179120d\tan(c+dx)+353736d}-\frac{5100\tan(c+dx)}{982600d\tan^{2}(c+dx)+1179120d\tan(c+dx)+353736d}-\frac{5950}{982600d\tan^{2}(c+dx)+1179120d\tan(c+dx)+353736d} & \text{for}\: d \neq 0 \\ \frac{x}{(5\tan(c)+3)^{3}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*tan(d*x+c))**3,x)
```

```
[Out] Piecewise((-4950*d*x*tan(c + d*x)**2/(982600*d*tan(c + d*x)**2 + 1179120*d*
tan(c + d*x) + 353736*d) - 5940*d*x*tan(c + d*x)/(982600*d*tan(c + d*x)**2
+ 1179120*d*tan(c + d*x) + 353736*d) - 1782*d*x/(982600*d*tan(c + d*x)**2 +
 1179120*d*tan(c + d*x) + 353736*d) + 250*log(5*tan(c + d*x) + 3)*tan(c + d
*x)**2/(982600*d*tan(c + d*x)**2 + 1179120*d*tan(c + d*x) + 353736*d) + 300
*log(5*tan(c + d*x) + 3)*tan(c + d*x)/(982600*d*tan(c + d*x)**2 + 1179120*d
*tan(c + d*x) + 353736*d) + 90*log(5*tan(c + d*x) + 3)/(982600*d*tan(c + d*
x)**2 + 1179120*d*tan(c + d*x) + 353736*d) - 125*log(tan(c + d*x)**2 + 1)*t
an(c + d*x)**2/(982600*d*tan(c + d*x)**2 + 1179120*d*tan(c + d*x) + 353736*
d) - 150*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(982600*d*tan(c + d*x)**2 +
1179120*d*tan(c + d*x) + 353736*d) - 45*log(tan(c + d*x)**2 + 1)/(982600*d*
tan(c + d*x)**2 + 1179120*d*tan(c + d*x) + 353736*d) - 5100*tan(c + d*x)/(9
82600*d*tan(c + d*x)**2 + 1179120*d*tan(c + d*x) + 353736*d) - 5950/(982600
*d*tan(c + d*x)**2 + 1179120*d*tan(c + d*x) + 353736*d), Ne(d, 0)), (x/(5*t
an(c) + 3)**3, True))
```

Giac [A]
time = 0.67, size = 74, normalized size = 1.07

$$-\frac{198\,dx+198\,c+\frac{5\left(75\,\tan(dx+c)^2+1110\,\tan(dx+c)+1217\right)}{(5\,\tan(dx+c)+3)^2}+5\,\log\left(\tan\left(dx+c\right)^2+1\right)-10\,\log\left(\left|5\,\tan\left(dx+c\right)+3\right|\right)}{39304\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/39304*(198*d*x + 198*c + 5*(75*tan(d*x + c)^2 + 1110*tan(d*x + c) + 1217
)/(5*tan(d*x + c) + 3)^2 + 5*log(tan(d*x + c)^2 + 1) - 10*log(abs(5*tan(d*x
 + c) + 3)))/d
```

Mupad [B]
time = 4.02, size = 84, normalized size = 1.22

$$\frac{5\,\ln\left(\tan(c+d\,x)+\frac{3}{5}\right)}{19652\,d}-\frac{\frac{3\tan(c+d\,x)}{578}+\frac{7}{1156}}{d\,\left(\tan\left(c+d\,x\right)^2+\frac{6\tan(c+d\,x)}{5}+\frac{9}{25}\right)}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-\frac{5}{39304}+\frac{99}{39304}\mathrm{i}\right)}{d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(-\frac{5}{39304}-\frac{99}{39304}\mathrm{i}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*tan(c + d*x) + 3)^3,x)
```

```
[Out] (5*log(tan(c + d*x) + 3/5))/(19652*d) - (log(tan(c + d*x) + 1i)*(5/39304 +
99i/39304))/d - (log(tan(c + d*x) - 1i)*(5/39304 - 99i/39304))/d - ((3*tan(
c + d*x))/578 + 7/1156)/(d*((6*tan(c + d*x))/5 + tan(c + d*x)^2 + 9/25))
```

3.498 $\int \frac{1}{(3+5\tan(c+dx))^4}\,dx$

Optimal. Leaf size=88

$$-\frac{161x}{334084}-\frac{60\log(3\cos(c+dx)+5\sin(c+dx))}{83521d}-\frac{5}{102d(3+5\tan(c+dx))^3}-\frac{15}{1156d(3+5\tan(c+dx))^2}-\frac{19\ldots}{}$$

```
[Out] -161/334084*x-60/83521*ln(3*cos(d*x+c)+5*sin(d*x+c))/d-5/102/d/(3+5*tan(d*x
+c))^3-15/1156/d/(3+5*tan(d*x+c))^2-5/19652/d/(3+5*tan(d*x+c))
```

Rubi [A]
time = 0.09, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3564, 3610, 3612, 3611}

$$-\frac{5}{19652d(5\tan(c+dx)+3)}-\frac{15}{1156d(5\tan(c+dx)+3)^2}-\frac{5}{102d(5\tan(c+dx)+3)^3}-\frac{60\log(5\sin(c+dx)+3\cos(c+dx))}{83521d}-\frac{161x}{334084}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 5*Tan[c + d*x])^(-4),x]
```

```
[Out] (-161*x)/334084 - (60*Log[3*Cos[c + d*x] + 5*Sin[c + d*x]])/(83521*d) - 5/(
102*d*(3 + 5*Tan[c + d*x])^3) - 15/(1156*d*(3 + 5*Tan[c + d*x])^2) - 5/(196
52*d*(3 + 5*Tan[c + d*x]))
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{1}{(3+5\tan(c+dx))^4}\,dx = -\frac{5}{102d(3+5\tan(c+dx))^3}+\frac{1}{34}\int \frac{3-5\tan(c+dx)}{(3+5\tan(c+dx))^3}\,dx$$
$$= -\frac{5}{102d(3+5\tan(c+dx))^3}-\frac{15}{1156d(3+5\tan(c+dx))^2}+\frac{\int \frac{-16-30\tan(c+dx)}{(3+5\tan(c+dx))^2}\,d}{1156}$$
$$= -\frac{5}{102d(3+5\tan(c+dx))^3}-\frac{15}{1156d(3+5\tan(c+dx))^2}-\frac{5}{19652d(3+5\tan(}$$
$$= -\frac{161x}{334084}-\frac{5}{102d(3+5\tan(c+dx))^3}-\frac{15}{1156d(3+5\tan(c+dx))^2}-\frac{}{19652d}$$
$$= -\frac{161x}{334084}-\frac{60\log(3\cos(c+dx)+5\sin(c+dx))}{83521d}-\frac{5}{102d(3+5\tan(c+dx))^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.18, size = 87, normalized size = 0.99

$$\frac{(720+483i)\log(i-\tan(c+dx))+(720-483i)\log(i+\tan(c+dx))-1440\log(3+5\tan(c+dx))-\frac{170(1064+855\tan(c+dx)+75\tan^2(c+dx))}{(3+5\tan(c+dx))^3}}{2004504d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 5*Tan[c + d*x])^(-4),x]
[Out] ((720 + 483*I)*Log[I - Tan[c + d*x]] + (720 - 483*I)*Log[I + Tan[c + d*x]]
- 1440*Log[3 + 5*Tan[c + d*x]] - (170*(1064 + 855*Tan[c + d*x] + 75*Tan[c +
 d*x]^2))/(3 + 5*Tan[c + d*x])^3)/(2004504*d)
```

Maple [A]
time = 0.10, size = 83, normalized size = 0.94

method	result
derivativedivides	$\frac{\frac{30\ln\left(1+\tan^2(dx+c)\right)}{83521}-\frac{161\arctan(\tan(dx+c))}{334084}-\frac{5}{102(3+5\tan(dx+c))^3}-\frac{15}{1156(3+5\tan(dx+c))^2}-\frac{5}{19652(3+5\tan(dx+c))}-\frac{60\ln(3+5}{8}}{d}$

default	$\frac{\frac{30\ln\left(1+\tan^2(dx+c)\right)}{83521}-\frac{161\arctan(\tan(dx+c))}{334084}-\frac{5}{102(3+5\tan(dx+c))^3}-\frac{15}{1156(3+5\tan(dx+c))^2}-\frac{5}{19652(3+5\tan(dx+c))}-\frac{60\ln(3+5\tan}{835}}{d}$
risch	$-\frac{161x}{334084}+\frac{60ix}{83521}+\frac{120ic}{83521d}+\frac{\left(\frac{875}{75502984}-\frac{5825i}{226508952}\right)\left(391884\,e^{4i(dx+c)}+531675ie^{2i(dx+c)}+114393\,e^{2i(dx+c)}-1165\right.}{d\left(17\,e^{2i(dx+c)}-8+15i\right)^3}$
norman	$\frac{-\frac{4347x}{334084}-\frac{21735x\tan(dx+c)}{334084}-\frac{36225x\left(\tan^2(dx+c)\right)}{334084}-\frac{20125x\left(\tan^3(dx+c)\right)}{334084}+\frac{166250\left(\tan^3(dx+c)\right)}{397953d}+\frac{22325\tan(dx+c)}{58956d}+\frac{131875\left(\tan\right.}{17686}}{(3+5\tan(dx+c))^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(30/83521*ln(1+tan(d*x+c)^2)-161/334084*arctan(tan(d*x+c))-5/102/(3+5*t
an(d*x+c))^3-15/1156/(3+5*tan(d*x+c))^2-5/19652/(3+5*tan(d*x+c))-60/83521*l
n(3+5*tan(d*x+c)))
```

Maxima [A]
time = 0.49, size = 93, normalized size = 1.06

$$-\frac{483\,dx+483\,c+\frac{85\left(75\,\tan(dx+c)^2+855\,\tan(dx+c)+1064\right)}{125\,\tan(dx+c)^3+225\,\tan(dx+c)^2+135\,\tan(dx+c)+27}-360\,\log\left(\tan\left(dx+c\right)^2+1\right)+720\,\log\left(5\,\tan\left(dx+c\right)+3\right)}{1002252\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/1002252*(483*d*x + 483*c + 85*(75*tan(d*x + c)^2 + 855*tan(d*x + c) + 10
64)/(125*tan(d*x + c)^3 + 225*tan(d*x + c)^2 + 135*tan(d*x + c) + 27) - 360
*log(tan(d*x + c)^2 + 1) + 720*log(5*tan(d*x + c) + 3))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(78) = 156.
time = 1.34, size = 157, normalized size = 1.78

$$-\frac{375\,(161\,dx+135)\tan\left(dx+c\right)^3+75\,(1449\,dx+1300)\tan\left(dx+c\right)^2+13041\,dx+360\left(125\,\tan\left(dx+c\right)^3+225\,\tan\left(dx+c\right)^2+135\,\tan\left(dx+c\right)+27\right)\log\left(\frac{25\,\tan(dx+c)^2+30\,\tan(dx+c)+9}{\tan(dx+c)^2+1}\right)+45\,(1449\,dx+2830)\tan\left(dx+c\right)+101375}{1002252\left(125\,d\tan\left(dx+c\right)^3+225\,d\tan\left(dx+c\right)^2+135\,d\tan\left(dx+c\right)+27\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/1002252*(375*(161*d*x + 135)*tan(d*x + c)^3 + 75*(1449*d*x + 1300)*tan(d
*x + c)^2 + 13041*d*x + 360*(125*tan(d*x + c)^3 + 225*tan(d*x + c)^2 + 135*
tan(d*x + c) + 27)*log((25*tan(d*x + c)^2 + 30*tan(d*x + c) + 9)/(tan(d*x +
 c)^2 + 1)) + 45*(1449*d*x + 2830)*tan(d*x + c) + 101375)/(125*d*tan(d*x +
c)^3 + 225*d*tan(d*x + c)^2 + 135*d*tan(d*x + c) + 27*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 790 vs. 2(78) = 156.
time = 0.48, size = 790, normalized size = 8.98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*tan(d*x+c))**4,x)
```

```
[Out] Piecewise((-60375*d*x*tan(c + d*x)**3/(125281500*d*tan(c + d*x)**3 + 225506
700*d*tan(c + d*x)**2 + 135304020*d*tan(c + d*x) + 27060804*d) - 108675*d*x
*tan(c + d*x)**2/(125281500*d*tan(c + d*x)**3 + 225506700*d*tan(c + d*x)**2
 + 135304020*d*tan(c + d*x) + 27060804*d) - 65205*d*x*tan(c + d*x)/(1252815
00*d*tan(c + d*x)**3 + 225506700*d*tan(c + d*x)**2 + 135304020*d*tan(c + d*
x) + 27060804*d) - 13041*d*x/(125281500*d*tan(c + d*x)**3 + 225506700*d*tan
(c + d*x)**2 + 135304020*d*tan(c + d*x) + 27060804*d) - 90000*log(5*tan(c +
 d*x) + 3)*tan(c + d*x)**3/(125281500*d*tan(c + d*x)**3 + 225506700*d*tan(c
 + d*x)**2 + 135304020*d*tan(c + d*x) + 27060804*d) - 162000*log(5*tan(c +
d*x) + 3)*tan(c + d*x)**2/(125281500*d*tan(c + d*x)**3 + 225506700*d*tan(c
+ d*x)**2 + 135304020*d*tan(c + d*x) + 27060804*d) - 97200*log(5*tan(c + d*
x) + 3)*tan(c + d*x)/(125281500*d*tan(c + d*x)**3 + 225506700*d*tan(c + d*x
)**2 + 135304020*d*tan(c + d*x) + 27060804*d) - 19440*log(5*tan(c + d*x) +
3)/(125281500*d*tan(c + d*x)**3 + 225506700*d*tan(c + d*x)**2 + 135304020*d
*tan(c + d*x) + 27060804*d) + 45000*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**
3/(125281500*d*tan(c + d*x)**3 + 225506700*d*tan(c + d*x)**2 + 135304020*d*
tan(c + d*x) + 27060804*d) + 81000*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2
/(125281500*d*tan(c + d*x)**3 + 225506700*d*tan(c + d*x)**2 + 135304020*d*t
an(c + d*x) + 27060804*d) + 48600*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(12
5281500*d*tan(c + d*x)**3 + 225506700*d*tan(c + d*x)**2 + 135304020*d*tan(c
 + d*x) + 27060804*d) + 9720*log(tan(c + d*x)**2 + 1)/(125281500*d*tan(c +
d*x)**3 + 225506700*d*tan(c + d*x)**2 + 135304020*d*tan(c + d*x) + 27060804
*d) - 6375*tan(c + d*x)**2/(125281500*d*tan(c + d*x)**3 + 225506700*d*tan(c
 + d*x)**2 + 135304020*d*tan(c + d*x) + 27060804*d) - 72675*tan(c + d*x)/(1
25281500*d*tan(c + d*x)**3 + 225506700*d*tan(c + d*x)**2 + 135304020*d*tan(
c + d*x) + 27060804*d) - 90440/(125281500*d*tan(c + d*x)**3 + 225506700*d*t
an(c + d*x)**2 + 135304020*d*tan(c + d*x) + 27060804*d), Ne(d, 0)), (x/(5*t
an(c) + 3)**4, True))
```

Giac [A]
time = 0.59, size = 84, normalized size = 0.95

$$-\frac{483\,dx+483\,c-\frac{25\,\left(6600\,\tan\left(dx+c\right)^3+11625\,\tan\left(dx+c\right)^2+4221\,\tan\left(dx+c\right)-2192\right)}{\left(5\,\tan\left(dx+c\right)+3\right)^3}-360\,\log\left(\tan\left(dx+c\right)^2+1\right)+720\,\log\left(\left|5\,\tan\left(dx+c\right)+3\right|\right)}{1002252\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/1002252*(483*d*x + 483*c - 25*(6600*tan(d*x + c)^3 + 11625*tan(d*x + c)^
2 + 4221*tan(d*x + c) - 2192)/(5*tan(d*x + c) + 3)^3 - 360*log(tan(d*x + c)
^2 + 1) + 720*log(abs(5*tan(d*x + c) + 3)))/d
```

Mupad [B]
time = 4.12, size = 104, normalized size = 1.18

$$-\frac{60\,\ln\left(\tan(c+d\,x)+\frac{3}{5}\right)}{83521\,d}-\frac{\frac{\tan(c+d\,x)^2}{19652}+\frac{57\tan(c+d\,x)}{98260}+\frac{266}{368475}}{d\,\left(\tan\left(c+d\,x\right)^3+\frac{9\tan(c+d\,x)^2}{5}+\frac{27\tan(c+d\,x)}{25}+\frac{27}{125}\right)}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(\frac{30}{83521}+\frac{161}{668168}\mathrm{i}\right)}{d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(\frac{30}{83521}-\frac{161}{668168}\mathrm{i}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*tan(c + d*x) + 3)^4,x)
```

```
[Out] (log(tan(c + d*x) - 1i)*(30/83521 + 161i/668168))/d + (log(tan(c + d*x) + 1
i)*(30/83521 - 161i/668168))/d - (60*log(tan(c + d*x) + 3/5))/(83521*d) - (
(57*tan(c + d*x))/98260 + tan(c + d*x)^2/19652 + 266/368475)/(d*((27*tan(c
+ d*x))/25 + (9*tan(c + d*x)^2)/5 + tan(c + d*x)^3 + 27/125))
```

3.499 $\int \frac{1}{5+3\tan(c+dx)}\,dx$

Optimal. Leaf size=31

$$\frac{5x}{34}+\frac{3\log(5\cos(c+dx)+3\sin(c+dx))}{34d}$$

```
[Out] 5/34*x+3/34*ln(5*cos(d*x+c)+3*sin(d*x+c))/d
```

Rubi [A]
time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3565, 3611}

$$\frac{3\log(3\sin(c+dx)+5\cos(c+dx))}{34d}+\frac{5x}{34}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*Tan[c + d*x])^(-1),x]
```

```
[Out] (5*x)/34 + (3*Log[5*Cos[c + d*x] + 3*Sin[c + d*x]])/(34*d)
```

Rule 3565

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{5+3\tan(c+dx)}\,dx &= \frac{5x}{34}+\frac{3}{34}\int \frac{3-5\tan(c+dx)}{5+3\tan(c+dx)}\,dx\\ &= \frac{5x}{34}+\frac{3\log(5\cos(c+dx)+3\sin(c+dx))}{34d}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.04, size = 65, normalized size = 2.10

$$-\frac{\left(\frac{3}{68}+\frac{5i}{68}\right)\log(i-\tan(c+dx))}{d}-\frac{\left(\frac{3}{68}-\frac{5i}{68}\right)\log(i+\tan(c+dx))}{d}+\frac{3\log(5+3\tan(c+dx))}{34d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*Tan[c + d*x])^(-1),x]
```

```
[Out] ((-3/68 - (5*I)/68)*Log[I - Tan[c + d*x]])/d - ((3/68 - (5*I)/68)*Log[I + T
an[c + d*x]])/d + (3*Log[5 + 3*Tan[c + d*x]])/(34*d)
```

Maple [A]
time = 0.07, size = 41, normalized size = 1.32

method	result	size
risch	$\frac{5x}{34}-\frac{3ix}{34}-\frac{3ic}{17d}+\frac{3\ln\left(\mathrm{e}^{2i(dx+c)}+\frac{8}{17}+\frac{15i}{17}\right)}{34d}$	35
norman	$\frac{5x}{34}+\frac{3\ln(5+3\tan(dx+c))}{34d}-\frac{3\ln\left(1+\tan^2(dx+c)\right)}{68d}$	37
derivativedivides	$\frac{-\frac{3\ln\left(1+\tan^2(dx+c)\right)}{68}+\frac{5\arctan(\tan(dx+c))}{34}+\frac{3\ln(5+3\tan(dx+c))}{34}}{d}$	41
default	$\frac{-\frac{3\ln\left(1+\tan^2(dx+c)\right)}{68}+\frac{5\arctan(\tan(dx+c))}{34}+\frac{3\ln(5+3\tan(dx+c))}{34}}{d}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-3/68*ln(1+tan(d*x+c)^2)+5/34*arctan(tan(d*x+c))+3/34*ln(5+3*tan(d*x+c
)))
```

Maxima [A]
time = 0.49, size = 39, normalized size = 1.26

$$\frac{10\,dx+10\,c-3\,\log\left(\tan\left(dx+c\right)^2+1\right)+6\,\log\left(3\,\tan\left(dx+c\right)+5\right)}{68\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/68*(10*d*x + 10*c - 3*log(tan(d*x + c)^2 + 1) + 6*log(3*tan(d*x + c) + 5)
)/d
```

Fricas [A]
time = 0.84, size = 46, normalized size = 1.48

$$\frac{10\,dx+3\,\log\left(\frac{9\,\tan(dx+c)^2+30\,\tan(dx+c)+25}{\tan(dx+c)^2+1}\right)}{68\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/68*(10*d*x + 3*log((9*tan(d*x + c)^2 + 30*tan(d*x + c) + 25)/(tan(d*x + c
)^2 + 1)))/d
```

Sympy [A]
time = 0.18, size = 46, normalized size = 1.48

$$\begin{cases} \frac{5x}{34} + \frac{3\log\left(3\tan\left(c+dx\right)+5\right)}{34d} - \frac{3\log\left(\tan^2\left(c+dx\right)+1\right)}{68d} & \text{for } d \neq 0 \\ \frac{x}{3\tan\left(c\right)+5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*tan(d*x+c)),x)
```

```
[Out] Piecewise((5*x/34 + 3*log(3*tan(c + d*x) + 5)/(34*d) - 3*log(tan(c + d*x)**
2 + 1)/(68*d), Ne(d, 0)), (x/(3*tan(c) + 5), True))
```

Giac [A]
time = 0.48, size = 40, normalized size = 1.29

$$\frac{10\,dx + 10\,c - 3\,\log\left(\tan\left(dx+c\right)^2+1\right) + 6\,\log\left(\left|3\,\tan\left(dx+c\right)+5\right|\right)}{68\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/68*(10*d*x + 10*c - 3*log(tan(d*x + c)^2 + 1) + 6*log(abs(3*tan(d*x + c)
+ 5)))/d
```

Mupad [B]
time = 4.12, size = 49, normalized size = 1.58

$$\frac{3\,\ln\left(\tan(c+d\,x)+\frac{5}{3}\right)}{34\,d} + \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-\frac{3}{68}-\frac{5}{68}\mathrm{i}\right)}{d} + \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(-\frac{3}{68}+\frac{5}{68}\mathrm{i}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*tan(c + d*x) + 5),x)
```

```
[Out] (3*log(tan(c + d*x) + 5/3))/(34*d) - (log(tan(c + d*x) + 1i)*(3/68 - 5i/68)
)/d - (log(tan(c + d*x) - 1i)*(3/68 + 5i/68))/d
```

3.500 $\int \frac{1}{(5+3\tan(c+dx))^2} \, dx$

Optimal. Leaf size=50

$$\frac{4x}{289} + \frac{15\log(5\cos(c+dx) + 3\sin(c+dx))}{578d} - \frac{3}{34d(5+3\tan(c+dx))}$$

```
[Out] 4/289*x+15/578*ln(5*cos(d*x+c)+3*sin(d*x+c))/d-3/34/d/(5+3*tan(d*x+c))
```

Rubi [A]
time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3564, 3612, 3611}

$$-\frac{3}{34d(3\tan(c+dx)+5)} + \frac{15\log(3\sin(c+dx)+5\cos(c+dx))}{578d} + \frac{4x}{289}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*Tan[c + d*x])^(-2),x]
```

```
[Out] (4*x)/289 + (15*Log[5*Cos[c + d*x] + 3*Sin[c + d*x]])/(578*d) - 3/(34*d*(5
+ 3*Tan[c + d*x]))
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{1}{(5+3\tan(c+dx))^2}\,dx = -\frac{3}{34d(5+3\tan(c+dx))} + \frac{1}{34}\int \frac{5-3\tan(c+dx)}{5+3\tan(c+dx)}\,dx$$
$$= \frac{4x}{289} - \frac{3}{34d(5+3\tan(c+dx))} + \frac{15}{578}\int \frac{3-5\tan(c+dx)}{5+3\tan(c+dx)}\,dx$$
$$= \frac{4x}{289} + \frac{15\log(5\cos(c+dx)+3\sin(c+dx))}{578d} - \frac{3}{34d(5+3\tan(c+dx))}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.28, size = 67, normalized size = 1.34

$$-\frac{(15+8i)\log(i-\tan(c+dx)) + (15-8i)\log(i+\tan(c+dx)) - 30\log(5+3\tan(c+dx)) + \frac{102}{5+3\tan(c+dx)}}{1156d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*Tan[c + d*x])^(-2),x]
```

```
[Out] -1/1156*((15 + 8*I)*Log[I - Tan[c + d*x]] + (15 - 8*I)*Log[I + Tan[c + d*x]
] - 30*Log[5 + 3*Tan[c + d*x]] + 102/(5 + 3*Tan[c + d*x]))/d
```

Maple [A]
time = 0.08, size = 55, normalized size = 1.10

method	result
derivativedivides	$\frac{-\frac{15\ln\left(1+\tan^2(dx+c)\right)}{1156}+\frac{4\arctan(\tan(dx+c))}{289}-\frac{3}{34(5+3\tan(dx+c))}+\frac{15\ln(5+3\tan(dx+c))}{578}}{d}$
default	$\frac{-\frac{15\ln\left(1+\tan^2(dx+c)\right)}{1156}+\frac{4\arctan(\tan(dx+c))}{289}-\frac{3}{34(5+3\tan(dx+c))}+\frac{15\ln(5+3\tan(dx+c))}{578}}{d}$
norman	$\frac{\frac{20x}{289}+\frac{12x\tan(dx+c)}{289}-\frac{3}{34d}}{5+3\tan(dx+c)}+\frac{15\ln(5+3\tan(dx+c))}{578d}-\frac{15\ln\left(1+\tan^2(dx+c)\right)}{1156d}$
risch	$\frac{4x}{289}-\frac{15ix}{578}-\frac{15ic}{289d}-\frac{135}{578d\left(17\,e^{2i(dx+c)}+8+15i\right)}+\frac{36i}{289d\left(17\,e^{2i(dx+c)}+8+15i\right)}+\frac{15\ln\left(e^{2i(dx+c)}+\frac{8}{17}+\frac{15i}{17}\right)}{578d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-15/1156*ln(1+tan(d*x+c)^2)+4/289*arctan(tan(d*x+c))-3/34/(5+3*tan(d*x
+c))+15/578*ln(5+3*tan(d*x+c)))
```

Maxima [A]
time = 0.50, size = 53, normalized size = 1.06

$$\frac{16\,dx + 16\,c - \frac{102}{3\,\tan(dx+c)+5} - 15\,\log\left(\tan(dx+c)^2+1\right) + 30\,\log\left(3\,\tan(dx+c)+5\right)}{1156\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/1156*(16*d*x + 16*c - 102/(3*tan(d*x + c) + 5) - 15*log(tan(d*x + c)^2 +
1) + 30*log(3*tan(d*x + c) + 5))/d
```

Fricas [A]
time = 0.98, size = 83, normalized size = 1.66

$$\frac{80\,dx + 15\,(3\,\tan\left(dx+c\right)+5)\log\left(\frac{9\,\tan(dx+c)^2+30\,\tan(dx+c)+25}{\tan(dx+c)^2+1}\right) + 3\,(16\,dx+15)\tan\left(dx+c\right) - 27}{1156\,(3\,d\tan\left(dx+c\right)+5\,d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/1156*(80*d*x + 15*(3*tan(d*x + c) + 5)*log((9*tan(d*x + c)^2 + 30*tan(d*x
 + c) + 25)/(tan(d*x + c)^2 + 1)) + 3*(16*d*x + 15)*tan(d*x + c) - 27)/(3*d
*tan(d*x + c) + 5*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(42) = 84$.
time = 0.28, size = 190, normalized size = 3.80

$$\begin{cases} \frac{48dx\tan\left(c+dx\right)}{3468d\tan\left(c+dx\right)+5780d} + \frac{80dx}{3468d\tan\left(c+dx\right)+5780d} + \frac{90\log\left(3\tan\left(c+dx\right)+5\right)\tan\left(c+dx\right)}{3468d\tan\left(c+dx\right)+5780d} + \frac{150\log\left(3\tan\left(c+dx\right)+5\right)}{3468d\tan\left(c+dx\right)+5780d} - \frac{45\log\left(\tan^2\left(c+dx\right)+1\right)\tan\left(c+dx\right)}{3468d\tan\left(c+dx\right)+5780d} - \frac{75\log\left(\tan^2\left(c+dx\right)+1\right)}{3468d\tan\left(c+dx\right)+5780d} - \frac{102}{3468d\tan\left(c+dx\right)+5780d} & \text{for}\: d \neq 0 \\ \frac{x}{\left(3\tan\left(c\right)+5\right)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*tan(d*x+c))**2,x)
```

```
[Out] Piecewise((48*d*x*tan(c + d*x)/(3468*d*tan(c + d*x) + 5780*d) + 80*d*x/(346
8*d*tan(c + d*x) + 5780*d) + 90*log(3*tan(c + d*x) + 5)*tan(c + d*x)/(3468*
d*tan(c + d*x) + 5780*d) + 150*log(3*tan(c + d*x) + 5)/(3468*d*tan(c + d*x)
 + 5780*d) - 45*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(3468*d*tan(c + d*x)
+ 5780*d) - 75*log(tan(c + d*x)**2 + 1)/(3468*d*tan(c + d*x) + 5780*d) - 10
2/(3468*d*tan(c + d*x) + 5780*d), Ne(d, 0)), (x/(3*tan(c) + 5)**2, True))
```

Giac [A]
time = 0.55, size = 64, normalized size = 1.28

$$\frac{16\,dx + 16\,c - \frac{18\,(5\,\tan(dx+c)+14)}{3\,\tan(dx+c)+5} - 15\,\log\left(\tan\left(dx+c\right)^2+1\right) + 30\,\log\left(|3\,\tan\left(dx+c\right)+5|\right)}{1156\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1156*(16*d*x + 16*c - 18*(5*tan(d*x + c) + 14)/(3*tan(d*x + c) + 5) - 15*
log(tan(d*x + c)^2 + 1) + 30*log(abs(3*tan(d*x + c) + 5)))/d
```

Mupad [B]
time = 4.08, size = 64, normalized size = 1.28

$$\frac{15\ln\left(\tan(c+d\,x)+\frac{5}{3}\right)}{578\,d}-\frac{1}{34\,d\,\left(\tan\left(c+d\,x\right)+\frac{5}{3}\right)}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-\frac{15}{1156}-\frac{2}{289}\mathrm{i}\right)}{d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(-\frac{15}{1156}+\frac{2}{289}\mathrm{i}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*tan(c + d*x) + 5)^2,x)
```

```
[Out] (15*log(tan(c + d*x) + 5/3))/(578*d) - (log(tan(c + d*x) - 1i)*(15/1156 + 2
i/289))/d - (log(tan(c + d*x) + 1i)*(15/1156 - 2i/289))/d - 1/(34*d*(tan(c
+ d*x) + 5/3))
```

3.501 $\int \frac{1}{(5+3\tan(c+dx))^3}\,dx$

Optimal. Leaf size=69

$$-\frac{5x}{19652}+\frac{99\log(5\cos(c+dx)+3\sin(c+dx))}{19652d}-\frac{3}{68d(5+3\tan(c+dx))^2}-\frac{15}{578d(5+3\tan(c+dx))}$$

```
[Out] -5/19652*x+99/19652*ln(5*cos(d*x+c)+3*sin(d*x+c))/d-3/68/d/(5+3*tan(d*x+c))
^2-15/578/d/(5+3*tan(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3564, 3610, 3612, 3611}

$$-\frac{15}{578d(3\tan(c+dx)+5)}-\frac{3}{68d(3\tan(c+dx)+5)^2}+\frac{99\log(3\sin(c+dx)+5\cos(c+dx))}{19652d}-\frac{5x}{19652}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*Tan[c + d*x])^(-3),x]
```

```
[Out] (-5*x)/19652 + (99*Log[5*Cos[c + d*x] + 3*Sin[c + d*x]])/(19652*d) - 3/(68*
d*(5 + 3*Tan[c + d*x])^2) - 15/(578*d*(5 + 3*Tan[c + d*x]))
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{1}{(5+3\tan(c+dx))^3}\,dx = -\frac{3}{68d(5+3\tan(c+dx))^2}+\frac{1}{34}\int \frac{5-3\tan(c+dx)}{(5+3\tan(c+dx))^2}\,dx$$
$$= -\frac{3}{68d(5+3\tan(c+dx))^2}-\frac{15}{578d(5+3\tan(c+dx))}+\frac{\int \frac{16-30\tan(c+dx)}{5+3\tan(c+dx)}\,dx}{1156}$$
$$= -\frac{5x}{19652}-\frac{3}{68d(5+3\tan(c+dx))^2}-\frac{15}{578d(5+3\tan(c+dx))}+\frac{99\int \frac{3-5\tan(}{5+3\tan(}}{1965}$$
$$= -\frac{5x}{19652}+\frac{99\log(5\cos(c+dx)+3\sin(c+dx))}{19652d}-\frac{3}{68d(5+3\tan(c+dx))^2}-$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.71, size = 86, normalized size = 1.25

$$\frac{\left(\frac{1}{39304}+\frac{i}{39304}\right)\left((-47+52i)\log(i-\tan(c+dx))-(52-47i)\log(i+\tan(c+dx))+(3-3i)\left(33\log(5+3\tan(c+dx))-\frac{17(67+30\tan(c+dx))}{(5+3\tan(c+dx))^2}\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*Tan[c + d*x])^(-3),x]
```

```
[Out] ((1/39304 + I/39304)*((-47 + 52*I)*Log[I - Tan[c + d*x]] - (52 - 47*I)*Log[
I + Tan[c + d*x]] + (3 - 3*I)*(33*Log[5 + 3*Tan[c + d*x]] - (17*(67 + 30*Ta
n[c + d*x]))/(5 + 3*Tan[c + d*x])^2)))/d
```

Maple [A]
time = 0.09, size = 69, normalized size = 1.00

method	result
derivativedivides	$\frac{-\frac{99\ln\left(1+\tan^2(dx+c)\right)}{39304}-\frac{5\arctan(\tan(dx+c))}{19652}-\frac{3}{68(5+3\tan(dx+c))^2}-\frac{15}{578(5+3\tan(dx+c))}+\frac{99\ln(5+3\tan(dx+c))}{19652}}{d}$
default	$\frac{-\frac{99\ln\left(1+\tan^2(dx+c)\right)}{39304}-\frac{5\arctan(\tan(dx+c))}{19652}-\frac{3}{68(5+3\tan(dx+c))^2}-\frac{15}{578(5+3\tan(dx+c))}+\frac{99\ln(5+3\tan(dx+c))}{19652}}{d}$

risch	$-\frac{5x}{19652}-\frac{99ix}{19652}-\frac{99ic}{9826d}+\frac{\left(-\frac{2241}{510952}+\frac{675i}{510952}\right)\left(442\,\mathrm{e}^{2i(dx+c)}+275+335i\right)}{d\left(17\,\mathrm{e}^{2i(dx+c)}+8+15i\right)^2}+\frac{99\ln\left(\mathrm{e}^{2i(dx+c)}+\frac{8}{17}+\frac{15i}{17}\right)}{19652d}$	7
norman	$\frac{-\frac{125x}{19652}-\frac{75x\tan(dx+c)}{9826}-\frac{45x\left(\tan^2(dx+c)\right)}{19652}-\frac{201}{1156d}-\frac{45\tan(dx+c)}{578d}}{(5+3\tan(dx+c))^2}+\frac{99\ln(5+3\tan(dx+c))}{19652d}-\frac{99\ln\left(1+\tan^2(dx+c)\right)}{39304d}$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-99/39304*ln(1+tan(d*x+c)^2)-5/19652*arctan(tan(d*x+c))-3/68/(5+3*tan(
d*x+c))^2-15/578/(5+3*tan(d*x+c))+99/19652*ln(5+3*tan(d*x+c)))
```

Maxima [A]
time = 0.50, size = 73, normalized size = 1.06

$$-\frac{10\,dx+10\,c+\frac{102\,(30\,\tan(dx+c)+67)}{9\,\tan(dx+c)^2+30\,\tan(dx+c)+25}+99\,\log\left(\tan\left(dx+c\right)^2+1\right)-198\,\log\left(3\,\tan\left(dx+c\right)+5\right)}{39304\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/39304*(10*d*x + 10*c + 102*(30*tan(d*x + c) + 67)/(9*tan(d*x + c)^2 + 30
*tan(d*x + c) + 25) + 99*log(tan(d*x + c)^2 + 1) - 198*log(3*tan(d*x + c) +
 5))/d
```

Fricas [A]
time = 0.85, size = 120, normalized size = 1.74

$$-\frac{18\,(5\,dx-87)\tan\left(dx+c\right)^2+250\,dx-99\left(9\,\tan\left(dx+c\right)^2+30\,\tan\left(dx+c\right)+25\right)\log\left(\frac{9\,\tan(dx+c)^2+30\,\tan(dx+c)+25}{\tan(dx+c)^2+1}\right)+60\,(5\,dx-36)\tan\left(dx+c\right)+2484}{39304\left(9\,d\tan\left(dx+c\right)^2+30\,d\tan\left(dx+c\right)+25\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/39304*(18*(5*d*x - 87)*tan(d*x + c)^2 + 250*d*x - 99*(9*tan(d*x + c)^2 +
 30*tan(d*x + c) + 25)*log((9*tan(d*x + c)^2 + 30*tan(d*x + c) + 25)/(tan(d
*x + c)^2 + 1)) + 60*(5*d*x - 36)*tan(d*x + c) + 2484)/(9*d*tan(d*x + c)^2
+ 30*d*tan(d*x + c) + 25*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 442 vs. 2(60) = 120.
time = 0.38, size = 442, normalized size = 6.41

$$\begin{cases} -\frac{90dx\tan^2(c+dx)}{353736d\tan^2(c+dx)+1179120d\tan(c+dx)+982600d}-\frac{300dx\tan(c+dx)}{353736d\tan^2(c+dx)+1179120d\tan(c+dx)+982600d}-\frac{250dx}{353736d\tan^2(c+dx)+1179120d\tan(c+dx)+982600d}+\frac{1782\log(3\tan(c+dx)+5)\tan^2(c+dx)}{353736d\tan^2(c+dx)+1179120d\tan(c+dx)+982600d}+\frac{5940\log(3\tan(c+dx)+5)\tan(c+dx)}{353736d\tan^2(c+dx)+1179120d\tan(c+dx)+982600d}+\frac{4950\log(3\tan(c+dx)+5)}{353736d\tan^2(c+dx)+1179120d\tan(c+dx)+982600d}-\frac{891\log(\tan^2(c+dx)+1)\tan^2(c+dx)}{353736d\tan^2(c+dx)+1179120d\tan(c+dx)+982600d}-\frac{2970\log(\tan^2(c+dx)+1)\tan(c+dx)}{353736d\tan^2(c+dx)+1179120d\tan(c+dx)+982600d}-\frac{2475\log(\tan^2(c+dx)+1)}{353736d\tan^2(c+dx)+1179120d\tan(c+dx)+982600d}-\frac{3060\tan(c+dx)}{353736d\tan^2(c+dx)+1179120d\tan(c+dx)+982600d}-\frac{6834}{353736d\tan^2(c+dx)+1179120d\tan(c+dx)+982600d} & \text{for}\ d\neq 0 \\ \frac{x}{(3\tan(c)+5)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*tan(d*x+c))**3,x)
```

```
[Out] Piecewise((-90*d*x*tan(c + d*x)**2/(353736*d*tan(c + d*x)**2 + 1179120*d*ta
n(c + d*x) + 982600*d) - 300*d*x*tan(c + d*x)/(353736*d*tan(c + d*x)**2 + 1
179120*d*tan(c + d*x) + 982600*d) - 250*d*x/(353736*d*tan(c + d*x)**2 + 117
9120*d*tan(c + d*x) + 982600*d) + 1782*log(3*tan(c + d*x) + 5)*tan(c + d*x)
**2/(353736*d*tan(c + d*x)**2 + 1179120*d*tan(c + d*x) + 982600*d) + 5940*l
og(3*tan(c + d*x) + 5)*tan(c + d*x)/(353736*d*tan(c + d*x)**2 + 1179120*d*t
an(c + d*x) + 982600*d) + 4950*log(3*tan(c + d*x) + 5)/(353736*d*tan(c + d*
x)**2 + 1179120*d*tan(c + d*x) + 982600*d) - 891*log(tan(c + d*x)**2 + 1)*t
an(c + d*x)**2/(353736*d*tan(c + d*x)**2 + 1179120*d*tan(c + d*x) + 982600*
d) - 2970*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(353736*d*tan(c + d*x)**2 +
 1179120*d*tan(c + d*x) + 982600*d) - 2475*log(tan(c + d*x)**2 + 1)/(353736
*d*tan(c + d*x)**2 + 1179120*d*tan(c + d*x) + 982600*d) - 3060*tan(c + d*x)
/(353736*d*tan(c + d*x)**2 + 1179120*d*tan(c + d*x) + 982600*d) - 6834/(353
736*d*tan(c + d*x)**2 + 1179120*d*tan(c + d*x) + 982600*d), Ne(d, 0)), (x/(
3*tan(c) + 5)**3, True))
```

Giac [A]
time = 0.62, size = 74, normalized size = 1.07

$$-\frac{10\,dx+10\,c+\frac{3\left(891\,\tan(dx+c)^2+3990\,\tan(dx+c)+4753\right)}{(3\,\tan(dx+c)+5)^2}+99\,\log\left(\tan\left(dx+c\right)^2+1\right)-198\,\log\left(\left|3\,\tan\left(dx+c\right)+5\right|\right)}{39304\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/39304*(10*d*x + 10*c + 3*(891*tan(d*x + c)^2 + 3990*tan(d*x + c) + 4753)
/(3*tan(d*x + c) + 5)^2 + 99*log(tan(d*x + c)^2 + 1) - 198*log(abs(3*tan(d*
x + c) + 5)))/d
```

Mupad [B]
time = 4.09, size = 84, normalized size = 1.22

$$\frac{99\,\ln\left(\tan(c+d\,x)+\frac{5}{3}\right)}{19652\,d}-\frac{\frac{5\,\tan(c+d\,x)}{578}+\frac{67}{3468}}{d\,\left(\tan\left(c+d\,x\right)^2+\frac{10\,\tan(c+d\,x)}{3}+\frac{25}{9}\right)}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-\frac{99}{39304}+\frac{5}{39304}\mathrm{i}\right)}{d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(-\frac{99}{39304}-\frac{5}{39304}\mathrm{i}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*tan(c + d*x) + 5)^3,x)
```

```
[Out] (99*log(tan(c + d*x) + 5/3))/(19652*d) - (log(tan(c + d*x) + 1i)*(99/39304
+ 5i/39304))/d - (log(tan(c + d*x) - 1i)*(99/39304 - 5i/39304))/d - ((5*tan
(c + d*x))/578 + 67/3468)/(d*((10*tan(c + d*x))/3 + tan(c + d*x)^2 + 25/9))
```

3.502 $\int \frac{1}{(5+3\tan(c+dx))^4}\,dx$

Optimal. Leaf size=88

$$-\frac{161x}{334084}+\frac{60\log(5\cos(c+dx)+3\sin(c+dx))}{83521d}-\frac{1}{34d(5+3\tan(c+dx))^3}-\frac{15}{1156d(5+3\tan(c+dx))^2}-\frac{1965}{}$$

```
[Out] -161/334084*x+60/83521*ln(5*cos(d*x+c)+3*sin(d*x+c))/d-1/34/d/(5+3*tan(d*x+
c))^3-15/1156/d/(5+3*tan(d*x+c))^2-99/19652/d/(5+3*tan(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3564, 3610, 3612, 3611}

$$-\frac{99}{19652d(3\tan(c+dx)+5)}-\frac{15}{1156d(3\tan(c+dx)+5)^2}-\frac{1}{34d(3\tan(c+dx)+5)^3}+\frac{60\log(3\sin(c+dx)+5\cos(c+dx))}{83521d}-\frac{161x}{334084}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*Tan[c + d*x])^(-4),x]
```

```
[Out] (-161*x)/334084 + (60*Log[5*Cos[c + d*x] + 3*Sin[c + d*x]])/(83521*d) - 1/(
34*d*(5 + 3*Tan[c + d*x])^3) - 15/(1156*d*(5 + 3*Tan[c + d*x])^2) - 99/(196
52*d*(5 + 3*Tan[c + d*x]))
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(5+3\tan(c+dx))^4}\,dx &= -\frac{1}{34d(5+3\tan(c+dx))^3}+\frac{1}{34}\int \frac{5-3\tan(c+dx)}{(5+3\tan(c+dx))^3}\,dx\\ &= -\frac{1}{34d(5+3\tan(c+dx))^3}-\frac{15}{1156d(5+3\tan(c+dx))^2}+\frac{\int \frac{16-30\tan(c+dx)}{(5+3\tan(c+dx))^2}\,dx}{1156}\\ &= -\frac{1}{34d(5+3\tan(c+dx))^3}-\frac{15}{1156d(5+3\tan(c+dx))^2}-\frac{99}{19652d(5+3\tan(c\dots}\\ &= -\frac{161x}{334084}-\frac{1}{34d(5+3\tan(c+dx))^3}-\frac{15}{1156d(5+3\tan(c+dx))^2}-\frac{\dots}{19652d(\dots}\\ &= -\frac{161x}{334084}+\frac{60\log(5\cos(c+dx)+3\sin(c+dx))}{83521d}-\frac{1}{34d(5+3\tan(c+dx))^3}\dots\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.69, size = 95, normalized size = 1.08

$$-\frac{(240-161i)\log(i-\tan(c+dx))+(240+161i)\log(i+\tan(c+dx))-480\log(5+3\tan(c+dx))+\frac{19652}{(5+3\tan(c+dx))^3}+\frac{8670}{(5+3\tan(c+dx))^2}+\frac{3366}{5+3\tan(c+dx)}}{668168d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*Tan[c + d*x])^(-4),x]
```

```
[Out] -1/668168*((240 - 161*I)*Log[I - Tan[c + d*x]] + (240 + 161*I)*Log[I + Tan[
c + d*x]] - 480*Log[5 + 3*Tan[c + d*x]] + 19652/(5 + 3*Tan[c + d*x])^3 + 86
70/(5 + 3*Tan[c + d*x])^2 + 3366/(5 + 3*Tan[c + d*x]))/d
```

Maple [A]
time = 0.10, size = 83, normalized size = 0.94

method	result
derivativedivides	$\frac{-\frac{30\ln\left(1+\tan^2(dx+c)\right)}{83521}-\frac{161\arctan(\tan(dx+c))}{334084}-\frac{1}{34(5+3\tan(dx+c))^3}-\frac{15}{1156(5+3\tan(dx+c))^2}-\frac{99}{19652(5+3\tan(dx+c))}+\frac{60\ln(5+\dots}{\dots}}{d}$

default	$\frac{-\frac{30\ln\left(1+\tan^2(dx+c)\right)}{83521}-\frac{161\arctan(\tan(dx+c))}{334084}-\frac{1}{34(5+3\tan(dx+c))^3}-\frac{15}{1156(5+3\tan(dx+c))^2}-\frac{99}{19652(5+3\tan(dx+c))}+\frac{60\ln(5+3}{83}}{d}$
risch	$-\frac{161x}{334084}-\frac{60ix}{83521}-\frac{120ic}{83521d}+\frac{\left(-\frac{5535}{48776264}+\frac{351i}{48776264}\right)\left(84388\,e^{4i(dx+c)}+127585ie^{2i(dx+c)}+108987\,e^{2i(dx+c)}-1313\right.}{d\left(17\,e^{2i(dx+c)}+8+15i\right)^3}$
norman	$\frac{-\frac{20125x}{334084}-\frac{36225x\tan(dx+c)}{334084}-\frac{21735x\left(\tan^2(dx+c)\right)}{334084}-\frac{4347x\left(\tan^3(dx+c)\right)}{334084}-\frac{1082}{4913d}-\frac{3735\tan(dx+c)}{19652d}-\frac{891\left(\tan^2(dx+c)\right)}{19652d}}{(5+3\tan(dx+c))^3}+60\ln($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-30/83521*ln(1+tan(d*x+c)^2)-161/334084*arctan(tan(d*x+c))-1/34/(5+3*t
an(d*x+c))^3-15/1156/(5+3*tan(d*x+c))^2-99/19652/(5+3*tan(d*x+c))+60/83521*
ln(5+3*tan(d*x+c)))
```

Maxima [A]
time = 0.49, size = 93, normalized size = 1.06

$$-\frac{161\,dx+161\,c+\frac{17\left(891\,\tan(dx+c)^2+3735\,\tan(dx+c)+4328\right)}{27\,\tan(dx+c)^3+135\,\tan(dx+c)^2+225\,\tan(dx+c)+125}+120\,\log\left(\tan\left(dx+c\right)^2+1\right)-240\,\log\left(3\,\tan\left(dx+c\right)+5\right)}{334084\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/334084*(161*d*x + 161*c + 17*(891*tan(d*x + c)^2 + 3735*tan(d*x + c) + 4
328)/(27*tan(d*x + c)^3 + 135*tan(d*x + c)^2 + 225*tan(d*x + c) + 125) + 12
0*log(tan(d*x + c)^2 + 1) - 240*log(3*tan(d*x + c) + 5))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(78) = 156.
time = 0.98, size = 157, normalized size = 1.78

$$-\frac{27\,(161\,dx-305)\tan\left(dx+c\right)^3+27\,(805\,dx-964)\tan\left(dx+c\right)^2+20125\,dx-120\left(27\,\tan\left(dx+c\right)^3+135\,\tan\left(dx+c\right)^2+225\,\tan\left(dx+c\right)+125\right)\log\left(\frac{9\,\tan(dx+c)^2+30\,\tan(dx+c)+25}{\tan(dx+c)^2+1}\right)+45\,(805\,dx-114)\tan\left(dx+c\right)+35451}{334084\left(27\,d\tan\left(dx+c\right)^3+135\,d\tan\left(dx+c\right)^2+225\,d\tan\left(dx+c\right)+125\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/334084*(27*(161*d*x - 305)*tan(d*x + c)^3 + 27*(805*d*x - 964)*tan(d*x +
 c)^2 + 20125*d*x - 120*(27*tan(d*x + c)^3 + 135*tan(d*x + c)^2 + 225*tan(d
*x + c) + 125)*log((9*tan(d*x + c)^2 + 30*tan(d*x + c) + 25)/(tan(d*x + c)^
2 + 1)) + 45*(805*d*x - 114)*tan(d*x + c) + 35451)/(27*d*tan(d*x + c)^3 + 1
35*d*tan(d*x + c)^2 + 225*d*tan(d*x + c) + 125*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 790 vs. 2(76) = 152.
time = 0.50, size = 790, normalized size = 8.98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*tan(d*x+c))**4,x)
```

```
[Out] Piecewise((-4347*d*x*tan(c + d*x)**3/(9020268*d*tan(c + d*x)**3 + 45101340*
d*tan(c + d*x)**2 + 75168900*d*tan(c + d*x) + 41760500*d) - 21735*d*x*tan(c
 + d*x)**2/(9020268*d*tan(c + d*x)**3 + 45101340*d*tan(c + d*x)**2 + 751689
00*d*tan(c + d*x) + 41760500*d) - 36225*d*x*tan(c + d*x)/(9020268*d*tan(c +
 d*x)**3 + 45101340*d*tan(c + d*x)**2 + 75168900*d*tan(c + d*x) + 41760500*
d) - 20125*d*x/(9020268*d*tan(c + d*x)**3 + 45101340*d*tan(c + d*x)**2 + 75
168900*d*tan(c + d*x) + 41760500*d) + 6480*log(3*tan(c + d*x) + 5)*tan(c +
d*x)**3/(9020268*d*tan(c + d*x)**3 + 45101340*d*tan(c + d*x)**2 + 75168900*
d*tan(c + d*x) + 41760500*d) + 32400*log(3*tan(c + d*x) + 5)*tan(c + d*x)**
2/(9020268*d*tan(c + d*x)**3 + 45101340*d*tan(c + d*x)**2 + 75168900*d*tan(
c + d*x) + 41760500*d) + 54000*log(3*tan(c + d*x) + 5)*tan(c + d*x)/(902026
8*d*tan(c + d*x)**3 + 45101340*d*tan(c + d*x)**2 + 75168900*d*tan(c + d*x)
+ 41760500*d) + 30000*log(3*tan(c + d*x) + 5)/(9020268*d*tan(c + d*x)**3 +
45101340*d*tan(c + d*x)**2 + 75168900*d*tan(c + d*x) + 41760500*d) - 3240*l
og(tan(c + d*x)**2 + 1)*tan(c + d*x)**3/(9020268*d*tan(c + d*x)**3 + 451013
40*d*tan(c + d*x)**2 + 75168900*d*tan(c + d*x) + 41760500*d) - 16200*log(ta
n(c + d*x)**2 + 1)*tan(c + d*x)**2/(9020268*d*tan(c + d*x)**3 + 45101340*d*
tan(c + d*x)**2 + 75168900*d*tan(c + d*x) + 41760500*d) - 27000*log(tan(c +
 d*x)**2 + 1)*tan(c + d*x)/(9020268*d*tan(c + d*x)**3 + 45101340*d*tan(c +
d*x)**2 + 75168900*d*tan(c + d*x) + 41760500*d) - 15000*log(tan(c + d*x)**2
 + 1)/(9020268*d*tan(c + d*x)**3 + 45101340*d*tan(c + d*x)**2 + 75168900*d*
tan(c + d*x) + 41760500*d) - 15147*tan(c + d*x)**2/(9020268*d*tan(c + d*x)*
*3 + 45101340*d*tan(c + d*x)**2 + 75168900*d*tan(c + d*x) + 41760500*d) - 6
3495*tan(c + d*x)/(9020268*d*tan(c + d*x)**3 + 45101340*d*tan(c + d*x)**2 +
 75168900*d*tan(c + d*x) + 41760500*d) - 73576/(9020268*d*tan(c + d*x)**3 +
 45101340*d*tan(c + d*x)**2 + 75168900*d*tan(c + d*x) + 41760500*d), Ne(d,
0)), (x/(3*tan(c) + 5)**4, True))
```

Giac [A]
time = 0.53, size = 83, normalized size = 0.94

$$-\frac{161\,dx + 161\,c + \frac{11880\,\tan\left(dx+c\right)^{3}+74547\,\tan\left(dx+c\right)^{2}+162495\,\tan\left(dx+c\right)+128576}{\left(3\,\tan\left(dx+c\right)+5\right)^{3}} + 120\,\log\left(\tan\left(dx+c\right)^{2}+1\right) - 240\,\log\left(\left|3\,\tan\left(dx+c\right)+5\right|\right)}{334084\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/334084*(161*d*x + 161*c + (11880*tan(d*x + c)^3 + 74547*tan(d*x + c)^2 +
 162495*tan(d*x + c) + 128576)/(3*tan(d*x + c) + 5)^3 + 120*log(tan(d*x + c
)^2 + 1) - 240*log(abs(3*tan(d*x + c) + 5)))/d
```

Mupad [B]

time = 4.07, size = 104, normalized size = 1.18

$$\frac{60\ln\left(\tan(c+dx)+\frac{5}{3}\right)}{83521\,d}-\frac{\frac{33\tan(c+dx)^2}{19652}+\frac{415\tan(c+dx)}{58956}+\frac{1082}{132651}}{d\left(\tan\left(c+dx\right)^3+5\tan\left(c+dx\right)^2+\frac{25\tan(c+dx)}{3}+\frac{125}{27}\right)}+\frac{\ln\left(\tan(c+dx)-\mathrm{i}\right)\left(-\frac{30}{83521}+\frac{161}{668168}\mathrm{i}\right)}{d}+\frac{\ln\left(\tan(c+dx)+1\mathrm{i}\right)\left(-\frac{30}{83521}-\frac{161}{668168}\mathrm{i}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*tan(c + d*x) + 5)^4,x)
```

```
[Out] (60*log(tan(c + d*x) + 5/3))/(83521*d) - (log(tan(c + d*x) + 1i)*(30/83521
+ 161i/668168))/d - (log(tan(c + d*x) - 1i)*(30/83521 - 161i/668168))/d - (
(415*tan(c + d*x))/58956 + (33*tan(c + d*x)^2)/19652 + 1082/132651)/(d*((25
*tan(c + d*x))/3 + 5*tan(c + d*x)^2 + tan(c + d*x)^3 + 125/27))
```

3.503 $\int \tan^4(c+dx)\sqrt{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=456

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}\,d}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}+\sqrt{2}\sqrt{a+b\tan(c$$

```
[Out] 1/2*b*arctanh(((a+(a^2+b^2)^(1/2))^(1/2)-2^(1/2)*(a+b*tan(d*x+c))^(1/2))/(a
-(a^2+b^2)^(1/2))^(1/2))/d*2^(1/2)/(a-(a^2+b^2)^(1/2))^(1/2)-1/2*b*arctanh(
((a+(a^2+b^2)^(1/2))^(1/2)+2^(1/2)*(a+b*tan(d*x+c))^(1/2))/(a-(a^2+b^2)^(1/
2))^(1/2))/d*2^(1/2)/(a-(a^2+b^2)^(1/2))^(1/2)+1/4*b*ln(a+(a^2+b^2)^(1/2)-2
^(1/2)*(a+(a^2+b^2)^(1/2))^(1/2)*(a+b*tan(d*x+c))^(1/2)+b*tan(d*x+c))/d*2^(
1/2)/(a+(a^2+b^2)^(1/2))^(1/2)-1/4*b*ln(a+(a^2+b^2)^(1/2)+2^(1/2)*(a+(a^2+b
^2)^(1/2))^(1/2)*(a+b*tan(d*x+c))^(1/2)+b*tan(d*x+c))/d*2^(1/2)/(a+(a^2+b^2
)^(1/2))^(1/2)+2/105*(8*a^2-35*b^2)*(a+b*tan(d*x+c))^(3/2)/b^3/d-8/35*a*tan
(d*x+c)*(a+b*tan(d*x+c))^(3/2)/b^2/d+2/7*tan(d*x+c)^2*(a+b*tan(d*x+c))^(3/2
)/b/d
```

Rubi [A]

time = 0.54, antiderivative size = 456, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3647, 3728, 3712, 3566, 714, 1143, 648, 632, 212, 642}

$$\frac{b\log\left(-\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\sqrt{a+b\tan(c+dx)}+\sqrt{a^2+b^2}+a+b\tan(c+dx)\right)}{2\sqrt{2}d\sqrt{\sqrt{a^2+b^2}+a}}-\frac{b\log\left(\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\sqrt{a+b\tan(c+dx)}+\sqrt{a^2+b^2}+a+b\tan(c+dx)\right)}{2\sqrt{2}d\sqrt{\sqrt{a^2+b^2}+a}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+b^2}+a}-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}d\sqrt{a-\sqrt{a^2+b^2}}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+b^2}+a}+\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}d\sqrt{a-\sqrt{a^2+b^2}}}+\frac{2(8a^2-35b^2)(a+b\tan(c+dx))^{3/2}}{105b^3d}-\frac{8a\tan(c+dx)(a+b\tan(c+dx))^{3/2}}{35b^2d}+\frac{2\tan^2(c+dx)(a+b\tan(c+dx))^{3/2}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4*Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] (b*ArcTanh[(Sqrt[a + Sqrt[a^2 + b^2]] - Sqrt[2]*Sqrt[a + b*Tan[c + d*x]])/S
qrt[a - Sqrt[a^2 + b^2]]])/(Sqrt[2]*Sqrt[a - Sqrt[a^2 + b^2]]*d) - (b*ArcTa
nh[(Sqrt[a + Sqrt[a^2 + b^2]] + Sqrt[2]*Sqrt[a + b*Tan[c + d*x]])/Sqrt[a -
Sqrt[a^2 + b^2]]])/(Sqrt[2]*Sqrt[a - Sqrt[a^2 + b^2]]*d) + (b*Log[a + Sqrt[
a^2 + b^2] + b*Tan[c + d*x] - Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*Sqrt[a + b*
Tan[c + d*x]]])/(2*Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*d) - (b*Log[a + Sqrt[a
^2 + b^2] + b*Tan[c + d*x] + Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*Sqrt[a + b*T
an[c + d*x]]])/(2*Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*d) + (2*(8*a^2 - 35*b^2
)*(a + b*Tan[c + d*x])^(3/2))/(105*b^3*d) - (8*a*Tan[c + d*x]*(a + b*Tan[c
+ d*x])^(3/2))/(35*b^2*d) + (2*Tan[c + d*x]^2*(a + b*Tan[c + d*x])^(3/2))/(
7*b*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 714

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, S
ubst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x]
 /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1143

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q =
 Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m - 1)/(q
 - r*x + x^2), x], x] - Dist[1/(2*c*r), Int[x^(m - 1)/(q + r*x + x^2), x],
x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 1] && LtQ[m, 3
] && NegQ[b^2 - 4*a*c]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
```

```
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3712

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \tan^4(c+dx)\sqrt{a+b\tan(c+dx)}\,dx = \frac{2\tan^2(c+dx)(a+b\tan(c+dx))^{3/2}}{7bd} + \frac{2\int \tan(c+dx)\sqrt{a+b\tan\cdots}{\cdots}$$

$$= -\frac{8a\tan(c+dx)(a+b\tan(c+dx))^{3/2}}{35b^2d} + \frac{2\tan^2(c+dx)(a+b\tan\cdots}{7bd}$$

$$= \frac{2(8a^2-35b^2)(a+b\tan(c+dx))^{3/2}}{105b^3d} - \frac{8a\tan(c+dx)(a+b\tan(c\cdots}{35b^2d}$$

$$= \frac{2(8a^2-35b^2)(a+b\tan(c+dx))^{3/2}}{105b^3d} - \frac{8a\tan(c+dx)(a+b\tan(c\cdots}{35b^2d}$$

$$= \frac{2(8a^2-35b^2)(a+b\tan(c+dx))^{3/2}}{105b^3d} - \frac{8a\tan(c+dx)(a+b\tan(c\cdots}{35b^2d}$$

$$= \frac{2(8a^2-35b^2)(a+b\tan(c+dx))^{3/2}}{105b^3d} - \frac{8a\tan(c+dx)(a+b\tan(c\cdots}{35b^2d}$$

$$= \frac{2(8a^2-35b^2)(a+b\tan(c+dx))^{3/2}}{105b^3d} - \frac{8a\tan(c+dx)(a+b\tan(c\cdots}{35b^2d}$$

$$= \frac{b\log\left(a+\sqrt{a^2+b^2}+b\tan(c+dx)-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a\cdots}\right.}{2\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\,d}$$

$$= \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}\,d} - \frac{b\tanh\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 19.73, size = 167, normalized size = 0.37

$$\frac{-105i\sqrt{a-ib}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)+105i\sqrt{a+ib}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)+\frac{2\sqrt{a+b\tan(c+dx)}\left(8a^3-38ab^2-2b(2a^2+25b^2)\tan(c+dx)+3b^2\sec^2(c+dx)(a+5b\tan(c+dx))\right)}{b^3}}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4*Sqrt[a + b*Tan[c + d*x]],x]
```

[Out] ((-105*I)*Sqrt[a - I*b]*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] + (105*I)*Sqrt[a + I*b]*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] + (2*Sqrt[a + b*Tan[c + d*x]]*(8*a^3 - 38*a*b^2 - 2*b*(2*a^2 + 25*b^2)*Tan[c + d*x] + 3*b^2*Sec[c + d*x]^2*(a + 5*b*Tan[c + d*x])))/b^3)/(105*d)

Maple [A]
time = 0.43, size = 430, normalized size = 0.94

method	result
derivativedivides	$\frac{\frac{2(a+b\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{4a(a+b\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{2a^2(a+b\tan(dx+c))^{\frac{3}{2}}}{3}-\frac{2b^2(a+b\tan(dx+c))^{\frac{3}{2}}}{3}+2b^4\left(-\frac{\sqrt{2\sqrt{a^2+b^2}+2a}}{}\right.}{}$
default	$\frac{\frac{2(a+b\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{4a(a+b\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{2a^2(a+b\tan(dx+c))^{\frac{3}{2}}}{3}-\frac{2b^2(a+b\tan(dx+c))^{\frac{3}{2}}}{3}+2b^4\left(-\frac{\sqrt{2\sqrt{a^2+b^2}+2a}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*tan(d*x+c))^(1/2)*tan(d*x+c)^4,x,method=_RETURNVERBOSE)

[Out] 2/d/b^3*(1/7*(a+b*tan(d*x+c))^(7/2)-2/5*a*(a+b*tan(d*x+c))^(5/2)+1/3*a^2*(a+b*tan(d*x+c))^(3/2)-1/3*b^2*(a+b*tan(d*x+c))^(3/2)+b^4*(-1/4*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*((a^2+b^2)^(1/2)-a)/b^2*(1/2*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))+1/4*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*((a^2+b^2)^(1/2)-a)/b^2*(1/2*ln(b*tan(d*x+c)+a-(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))))

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^4, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1682 vs. 2(375) = 750.
time = 1.41, size = 1682, normalized size = 3.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/420*(420*sqrt(2)*b^3*d^5*sqrt((a*d^2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/
b^2)*sqrt(b^2/d^4)*((a^2 + b^2)/d^4)^(3/4)*arctan(-(sqrt(2)*b*d^5*sqrt((a*c
os(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt((a*d^2*sqrt((a^2 + b^2)/d^
4) + a^2 + b^2)/b^2)*sqrt(b^2/d^4)*((a^2 + b^2)/d^4)^(3/4) - sqrt(2)*d^5*sq
rt((a*d^2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*sqrt((sqrt(2)*b^3*d^3*sqr
t((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt((a*d^2*sqrt((a^2 + b
^2)/d^4) + a^2 + b^2)/b^2)*((a^2 + b^2)/d^4)^(3/4)*cos(d*x + c) + (a^2*b^2
+ b^4)*d^2*sqrt((a^2 + b^2)/d^4)*cos(d*x + c) + (a^3*b^2 + a*b^4)*cos(d*x +
 c) + (a^2*b^3 + b^5)*sin(d*x + c))/((a^2 + b^2)*cos(d*x + c)))*sqrt(b^2/d^
4)*((a^2 + b^2)/d^4)^(3/4) + (a^2 + b^2)*d^4*sqrt(b^2/d^4)*sqrt((a^2 + b^2)
/d^4) + (a^3 + a*b^2)*d^2*sqrt(b^2/d^4))/(a^2*b^2 + b^4))*cos(d*x + c)^3 +
420*sqrt(2)*b^3*d^5*sqrt((a*d^2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*sqr
t(b^2/d^4)*((a^2 + b^2)/d^4)^(3/4)*arctan(-(sqrt(2)*b*d^5*sqrt((a*cos(d*x +
 c) + b*sin(d*x + c))/cos(d*x + c))*sqrt((a*d^2*sqrt((a^2 + b^2)/d^4) + a^2
 + b^2)/b^2)*sqrt(b^2/d^4)*((a^2 + b^2)/d^4)^(3/4) - sqrt(2)*d^5*sqrt((a*d^
2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*sqrt(-(sqrt(2)*b^3*d^3*sqrt((a*co
s(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt((a*d^2*sqrt((a^2 + b^2)/d^4
) + a^2 + b^2)/b^2)*((a^2 + b^2)/d^4)^(3/4)*cos(d*x + c) - (a^2*b^2 + b^4)*
d^2*sqrt((a^2 + b^2)/d^4)*cos(d*x + c) - (a^3*b^2 + a*b^4)*cos(d*x + c) - (
a^2*b^3 + b^5)*sin(d*x + c))/((a^2 + b^2)*cos(d*x + c)))*sqrt(b^2/d^4)*((a^
2 + b^2)/d^4)^(3/4) - (a^2 + b^2)*d^4*sqrt(b^2/d^4)*sqrt((a^2 + b^2)/d^4) -
 (a^3 + a*b^2)*d^2*sqrt(b^2/d^4))/(a^2*b^2 + b^4))*cos(d*x + c)^3 - 105*sqr
t(2)*(a*b^3*d^3*sqrt((a^2 + b^2)/d^4)*cos(d*x + c)^3 - (a^2*b^3 + b^5)*d*co
s(d*x + c)^3)*sqrt((a*d^2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*((a^2 + b
^2)/d^4)^(1/4)*log((sqrt(2)*b^3*d^3*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/
cos(d*x + c))*sqrt((a*d^2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*((a^2 + b
^2)/d^4)^(3/4)*cos(d*x + c) + (a^2*b^2 + b^4)*d^2*sqrt((a^2 + b^2)/d^4)*cos
```

```
(d*x + c) + (a^3*b^2 + a*b^4)*cos(d*x + c) + (a^2*b^3 + b^5)*sin(d*x + c))/
((a^2 + b^2)*cos(d*x + c))) + 105*sqrt(2)*(a*b^3*d^3*sqrt((a^2 + b^2)/d^4)*
cos(d*x + c)^3 - (a^2*b^3 + b^5)*d*cos(d*x + c)^3)*sqrt((a*d^2*sqrt((a^2 +
b^2)/d^4) + a^2 + b^2)/b^2)*((a^2 + b^2)/d^4)^(1/4)*log(-(sqrt(2)*b^3*d^3*s
qrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt((a*d^2*sqrt((a^2 +
 b^2)/d^4) + a^2 + b^2)/b^2)*((a^2 + b^2)/d^4)^(3/4)*cos(d*x + c) - (a^2*b^
2 + b^4)*d^2*sqrt((a^2 + b^2)/d^4)*cos(d*x + c) - (a^3*b^2 + a*b^4)*cos(d*x
 + c) - (a^2*b^3 + b^5)*sin(d*x + c))/((a^2 + b^2)*cos(d*x + c))) - 8*(2*(4
*a^5 - 15*a^3*b^2 - 19*a*b^4)*cos(d*x + c)^3 + 3*(a^3*b^2 + a*b^4)*cos(d*x
+ c) + (15*a^2*b^3 + 15*b^5 - 2*(2*a^4*b + 27*a^2*b^3 + 25*b^5)*cos(d*x + c
)^2)*sin(d*x + c))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c)))/((
a^2*b^3 + b^5)*d*cos(d*x + c)^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan\left(c+dx\right)}\,\tan^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(1/2)*tan(d*x+c)**4,x)
```

```
[Out] Integral(sqrt(a + b*tan(c + d*x))*tan(c + d*x)**4, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 17.64, size = 371, normalized size = 0.81

$$\sqrt{a+b\tan\left(c+d\,x\right)}\,\left(2\,a\,\left(\frac{4\,a^2}{b^3\,d}-\frac{2\,\left(a^2+b^2\right)}{b^3\,d}\right)-\frac{8\,a^3}{b^3\,d}+\frac{4\,a\,\left(a^2+b^2\right)}{b^3\,d}\right)+\left(\frac{4\,a^2}{3\,b^3\,d}-\frac{2\,\left(a^2+b^2\right)}{3\,b^3\,d}\right)\,{\left(a+b\,\mathrm{tan}\left(c+d\,x\right)\right)}^{3/2}+\frac{2\,{\left(a+b\,\mathrm{tan}\left(c+d\,x\right)\right)}^{7/2}}{7\,b^3\,d}-\frac{4\,a\,{\left(a+b\,\mathrm{tan}\left(c+d\,x\right)\right)}^{5/2}}{5\,b^3\,d}+\mathrm{atan}\left(\frac{d^3\,\left(\frac{16\,\left(b^4-a^2\,b^2\right)\,\sqrt{a+b\,\mathrm{tan}\left(c+d\,x\right)}}{d^2}+\frac{16\,a\,b^2\,\left(a-b\,1{}\mathrm{i}\right)\,\sqrt{a+b\,\mathrm{tan}\left(c+d\,x\right)}}{d^2}\right)\,\sqrt{-\frac{a-b\,1{}\mathrm{i}}{4\,d^2}}\,1{}\mathrm{i}}{8\,\left(a^2\,b^3+b^5\right)}\right)\,\sqrt{-\frac{a-b\,1{}\mathrm{i}}{4\,d^2}}\,2{}\mathrm{i}+\mathrm{atan}\left(\frac{d^3\,\left(\frac{16\,\left(b^4-a^2\,b^2\right)\,\sqrt{a+b\,\mathrm{tan}\left(c+d\,x\right)}}{d^2}+\frac{16\,a\,b^2\,\left(a+b\,1{}\mathrm{i}\right)\,\sqrt{a+b\,\mathrm{tan}\left(c+d\,x\right)}}{d^2}\right)\,\sqrt{-\frac{a+b\,1{}\mathrm{i}}{4\,d^2}}\,1{}\mathrm{i}}{8\,\left(a^2\,b^3+b^5\right)}\right)\,\sqrt{-\frac{a+b\,1{}\mathrm{i}}{4\,d^2}}\,2{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + b*tan(c + d*x))^(1/2),x)
```

```
[Out] atan((d^3*((16*(b^4 - a^2*b^2)*(a + b*tan(c + d*x))^(1/2))/d^2 + (16*a*b^2*
(a - b*1i)*(a + b*tan(c + d*x))^(1/2))/d^2)*(-(a - b*1i)/(4*d^2))^(1/2)*1i)
/(8*(b^5 + a^2*b^3)))*(-(a - b*1i)/(4*d^2))^(1/2)*2i + atan((d^3*((16*(b^4
- a^2*b^2)*(a + b*tan(c + d*x))^(1/2))/d^2 + (16*a*b^2*(a + b*1i)*(a + b*ta
```

```
n(c + d*x))^(1/2))/d^2)*(-(a + b*1i)/(4*d^2))^(1/2)*1i)/(8*(b^5 + a^2*b^3))
)*(-(a + b*1i)/(4*d^2))^(1/2)*2i + (a + b*tan(c + d*x))^(1/2)*(2*a*((4*a^2)
/(b^3*d) - (2*(a^2 + b^2))/(b^3*d)) - (8*a^3)/(b^3*d) + (4*a*(a^2 + b^2))/(
b^3*d)) + ((4*a^2)/(3*b^3*d) - (2*(a^2 + b^2))/(3*b^3*d))*(a + b*tan(c + d*
x))^(3/2) + (2*(a + b*tan(c + d*x))^(7/2))/(7*b^3*d) - (4*a*(a + b*tan(c +
d*x))^(5/2))/(5*b^3*d)
```

3.504 $\int \tan^3(c+dx)\sqrt{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=159

$$\frac{\sqrt{a-ib}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}+\frac{\sqrt{a+ib}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}-\frac{2\sqrt{a+b\tan(c+dx)}}{d}$$

```
[Out] arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))*(a-I*b)^(1/2)/d+arctanh((a+b*
tan(d*x+c))^(1/2)/(a+I*b)^(1/2))*(a+I*b)^(1/2)/d-2*(a+b*tan(d*x+c))^(1/2)/d
-4/15*a*(a+b*tan(d*x+c))^(3/2)/b^2/d+2/5*tan(d*x+c)*(a+b*tan(d*x+c))^(3/2)/
b/d
```

Rubi [A]
time = 0.23, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3647, 3711, 12, 3609, 3620, 3618, 65, 214}

$$-\frac{4a(a+b\tan(c+dx))^{3/2}}{15b^2d}+\frac{2\tan(c+dx)(a+b\tan(c+dx))^{3/2}}{5bd}-\frac{2\sqrt{a+b\tan(c+dx)}}{d}+\frac{\sqrt{a-ib}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}+\frac{\sqrt{a+ib}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3*Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] (Sqrt[a - I*b]*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/d + (Sqrt[a
 + I*b]*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/d - (2*Sqrt[a + b*
Tan[c + d*x]])/d - (4*a*(a + b*Tan[c + d*x])^(3/2))/(15*b^2*d) + (2*Tan[c +
 d*x]*(a + b*Tan[c + d*x])^(3/2))/(5*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \tan^3(c+dx)\sqrt{a+b\tan(c+dx)}\,dx = \frac{2\tan(c+dx)(a+b\tan(c+dx))^{3/2}}{5bd} + \frac{2\int\sqrt{a+b\tan(c+dx)}\,(-$$

$$= -\frac{4a(a+b\tan(c+dx))^{3/2}}{15b^2d} + \frac{2\tan(c+dx)(a+b\tan(c+dx))^{3/2}}{5bd}$$

$$= -\frac{4a(a+b\tan(c+dx))^{3/2}}{15b^2d} + \frac{2\tan(c+dx)(a+b\tan(c+dx))^{3/2}}{5bd}$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{d} - \frac{4a(a+b\tan(c+dx))^{3/2}}{15b^2d} + \frac{2\tan(c+$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{d} - \frac{4a(a+b\tan(c+dx))^{3/2}}{15b^2d} + \frac{2\tan(c+$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{d} - \frac{4a(a+b\tan(c+dx))^{3/2}}{15b^2d} + \frac{2\tan(c+$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{d} - \frac{4a(a+b\tan(c+dx))^{3/2}}{15b^2d} + \frac{2\tan(c+$$

$$= \frac{\sqrt{a-ib}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} + \frac{\sqrt{a+ib}\,\tanh^{-1}\left(\frac{\sqrt{}}{}\right.$$

Mathematica [A]
time = 1.12, size = 140, normalized size = 0.88

$$\frac{15\sqrt{a-ib}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right) + 15\sqrt{a+ib}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right) + \frac{2\sqrt{a+b\tan(c+dx)}\,(-2a^2-15b^2+ab\tan(c+dx)+3b^2\tan^2(c+dx))}{b^2}}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3*Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] (15*Sqrt[a - I*b]*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] + 15*Sqrt
[a + I*b]*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] + (2*Sqrt[a + b*T
an[c + d*x]]*(-2*a^2 - 15*b^2 + a*b*Tan[c + d*x] + 3*b^2*Tan[c + d*x]^2))/b
^2)/(15*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 369 vs. 2(133) = 266.
time = 0.14, size = 370, normalized size = 2.33

method	result

derivativedivides	$$\frac{\frac{2(a+b\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{2a(a+b\tan(dx+c))^{\frac{3}{2}}}{3}-2b^2\sqrt{a+b\tan(dx+c)}+2b^2\left(\frac{\sqrt{2\sqrt{a^2+b^2}+2a}\,\ln\left(b\tan(dx+\right.}{}\right.}{}$$
default	$$\frac{\frac{2(a+b\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{2a(a+b\tan(dx+c))^{\frac{3}{2}}}{3}-2b^2\sqrt{a+b\tan(dx+c)}+2b^2\left(\frac{\sqrt{2\sqrt{a^2+b^2}+2a}\,\ln\left(b\tan(dx+\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(1/2)*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/b^2*(1/5*(a+b*tan(d*x+c))^(5/2)-1/3*a*(a+b*tan(d*x+c))^(3/2)-b^2*(a+b*t
an(d*x+c))^(1/2)+b^2*(1/8*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln(b*tan(d*x+c)+a+(
a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+1/2*((
a^2+b^2)^(1/2)-a)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^
(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-1/8*(2*
(a^2+b^2)^(1/2)+2*a)^(1/2)*ln(-b*tan(d*x+c)-a+(a+b*tan(d*x+c))^(1/2)*(2*(a^
2+b^2)^(1/2)+2*a)^(1/2)-(a^2+b^2)^(1/2))+1/2*(a-(a^2+b^2)^(1/2))/(2*(a^2+b^
2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*
a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1879 vs. 2(129) = 258.
time = 1.58, size = 1879, normalized size = 11.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/60*(60*sqrt(2)*b^2*d^5*sqrt(-(a*d^2*sqrt((a^2 + b^2)/d^4) - a^2 - b^2)/b
^2)*sqrt(b^2/d^4)*((a^2 + b^2)/d^4)^(3/4)*arctan(-((a^2 + b^2)*d^4*sqrt(b^2
/d^4)*sqrt((a^2 + b^2)/d^4) + (a^3 + a*b^2)*d^2*sqrt(b^2/d^4) + sqrt(2)*(d^
7*sqrt(b^2/d^4)*sqrt((a^2 + b^2)/d^4) + a*d^5*sqrt(b^2/d^4))*sqrt((a*cos(d*
x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-(a*d^2*sqrt((a^2 + b^2)/d^4) -
 a^2 - b^2)/b^2)*((a^2 + b^2)/d^4)^(3/4) - sqrt(2)*(d^7*sqrt(b^2/d^4)*sqrt(
(a^2 + b^2)/d^4) + a*d^5*sqrt(b^2/d^4))*sqrt(-(a*d^2*sqrt((a^2 + b^2)/d^4)
- a^2 - b^2)/b^2)*sqrt(((a^2 + b^2)*d^2*sqrt((a^2 + b^2)/d^4)*cos(d*x + c)
+ sqrt(2)*(a*d^3*sqrt((a^2 + b^2)/d^4)*cos(d*x + c) + (a^2 + b^2)*d*cos(d*x
 + c))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-(a*d^2*sq
rt((a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*((a^2 + b^2)/d^4)^(1/4) + (a^3 + a*b^
2)*cos(d*x + c) + (a^2*b + b^3)*sin(d*x + c))/((a^2 + b^2)*cos(d*x + c)))*(
(a^2 + b^2)/d^4)^(3/4))/(a^2*b^2 + b^4))*cos(d*x + c)^2 + 60*sqrt(2)*b^2*d^
5*sqrt(-(a*d^2*sqrt((a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*sqrt(b^2/d^4)*((a^2
+ b^2)/d^4)^(3/4)*arctan(((a^2 + b^2)*d^4*sqrt(b^2/d^4)*sqrt((a^2 + b^2)/d^
4) + (a^3 + a*b^2)*d^2*sqrt(b^2/d^4) - sqrt(2)*(d^7*sqrt(b^2/d^4)*sqrt((a^2
 + b^2)/d^4) + a*d^5*sqrt(b^2/d^4))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/
cos(d*x + c))*sqrt(-(a*d^2*sqrt((a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*((a^2 +
b^2)/d^4)^(3/4) + sqrt(2)*(d^7*sqrt(b^2/d^4)*sqrt((a^2 + b^2)/d^4) + a*d^5*
sqrt(b^2/d^4))*sqrt(-(a*d^2*sqrt((a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*sqrt(((
a^2 + b^2)*d^2*sqrt((a^2 + b^2)/d^4)*cos(d*x + c) - sqrt(2)*(a*d^3*sqrt((a^
2 + b^2)/d^4)*cos(d*x + c) + (a^2 + b^2)*d*cos(d*x + c))*sqrt((a*cos(d*x +
c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-(a*d^2*sqrt((a^2 + b^2)/d^4) - a^2
 - b^2)/b^2)*((a^2 + b^2)/d^4)^(1/4) + (a^3 + a*b^2)*cos(d*x + c) + (a^2*b
+ b^3)*sin(d*x + c))/((a^2 + b^2)*cos(d*x + c)))*((a^2 + b^2)/d^4)^(3/4))/(
a^2*b^2 + b^4))*cos(d*x + c)^2 - 15*sqrt(2)*(a*b^2*d^3*sqrt((a^2 + b^2)/d^4
)*cos(d*x + c)^2 + (a^2*b^2 + b^4)*d*cos(d*x + c)^2)*sqrt(-(a*d^2*sqrt((a^2
 + b^2)/d^4) - a^2 - b^2)/b^2)*((a^2 + b^2)/d^4)^(1/4)*log(((a^2 + b^2)*d^2
*sqrt((a^2 + b^2)/d^4)*cos(d*x + c) + sqrt(2)*(a*d^3*sqrt((a^2 + b^2)/d^4)*
cos(d*x + c) + (a^2 + b^2)*d*cos(d*x + c))*sqrt((a*cos(d*x + c) + b*sin(d*x
 + c))/cos(d*x + c))*sqrt(-(a*d^2*sqrt((a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*(
(a^2 + b^2)/d^4)^(1/4) + (a^3 + a*b^2)*cos(d*x + c) + (a^2*b + b^3)*sin(d*x
 + c))/((a^2 + b^2)*cos(d*x + c))) + 15*sqrt(2)*(a*b^2*d^3*sqrt((a^2 + b^2)
/d^4)*cos(d*x + c)^2 + (a^2*b^2 + b^4)*d*cos(d*x + c)^2)*sqrt(-(a*d^2*sqrt(
(a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*((a^2 + b^2)/d^4)^(1/4)*log(((a^2 + b^2)
*d^2*sqrt((a^2 + b^2)/d^4)*cos(d*x + c) - sqrt(2)*(a*d^3*sqrt((a^2 + b^2)/d
^4)*cos(d*x + c) + (a^2 + b^2)*d*cos(d*x + c))*sqrt((a*cos(d*x + c) + b*sin
(d*x + c))/cos(d*x + c))*sqrt(-(a*d^2*sqrt((a^2 + b^2)/d^4) - a^2 - b^2)/b^
2)*((a^2 + b^2)/d^4)^(1/4) + (a^3 + a*b^2)*cos(d*x + c) + (a^2*b + b^3)*sin
(d*x + c))/((a^2 + b^2)*cos(d*x + c))) - 8*(3*a^2*b^2 + 3*b^4 - 2*(a^4 + 10
*a^2*b^2 + 9*b^4)*cos(d*x + c)^2 + (a^3*b + a*b^3)*cos(d*x + c)*sin(d*x + c
))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c)))/((a^2*b^2 + b^4)*d
*cos(d*x + c)^2)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan(c+dx)}\,\tan^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(1/2)*tan(d*x+c)**3,x)
```

```
[Out] Integral(sqrt(a + b*tan(c + d*x))*tan(c + d*x)**3, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 8.49, size = 354, normalized size = 2.23

$$\left(\frac{2\,a^2}{b^2\,d}-\frac{2\,\left(a^2+b^2\right)}{b^2\,d}\right)\,\sqrt{a+b\tan\left(c+d\,x\right)}+\frac{2\,{\left(a+b\tan(c+d\,x)\right)}^{5/2}}{5\,b^2\,d}-\frac{2\,a\,{\left(a+b\tan(c+d\,x)\right)}^{3/2}}{3\,b^2\,d}+\mathrm{atan}\left(\frac{b^4\,\sqrt{\frac{a}{4\,d^2}-\frac{b\,1\mathrm{i}}{4\,d^2}}\,\sqrt{a+b\tan\left(c+d\,x\right)}\,32\mathrm{i}}{\frac{b^5\,16\mathrm{i}}{d}+\frac{a^2\,b^3\,16\mathrm{i}}{d}}+\frac{32\,a\,b^3\,\sqrt{\frac{a}{4\,d^2}-\frac{b\,1\mathrm{i}}{4\,d^2}}\,\sqrt{a+b\tan\left(c+d\,x\right)}}{\frac{b^5\,16\mathrm{i}}{d}+\frac{a^2\,b^3\,16\mathrm{i}}{d}}\right)\,\sqrt{\frac{a-b\,1\mathrm{i}}{4\,d^2}}\,2\mathrm{i}-\mathrm{atan}\left(\frac{b^4\,\sqrt{\frac{a}{4\,d^2}+\frac{b\,1\mathrm{i}}{4\,d^2}}\,\sqrt{a+b\tan\left(c+d\,x\right)}\,32\mathrm{i}}{\frac{b^5\,16\mathrm{i}}{d}+\frac{a^2\,b^3\,16\mathrm{i}}{d}}-\frac{32\,a\,b^3\,\sqrt{\frac{a}{4\,d^2}+\frac{b\,1\mathrm{i}}{4\,d^2}}\,\sqrt{a+b\tan\left(c+d\,x\right)}}{\frac{b^5\,16\mathrm{i}}{d}+\frac{a^2\,b^3\,16\mathrm{i}}{d}}\right)\,\sqrt{\frac{a+b\,1\mathrm{i}}{4\,d^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + b*tan(c + d*x))^(1/2),x)
```

```
[Out] ((2*a^2)/(b^2*d) - (2*(a^2 + b^2))/(b^2*d))*(a + b*tan(c + d*x))^(1/2) + at
an((b^4*(a/(4*d^2) - (b*1i)/(4*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2)*32i)/
((b^5*16i)/d + (a^2*b^3*16i)/d) + (32*a*b^3*(a/(4*d^2) - (b*1i)/(4*d^2))^(1
/2)*(a + b*tan(c + d*x))^(1/2))/((b^5*16i)/d + (a^2*b^3*16i)/d))*((a - b*1i
)/(4*d^2))^(1/2)*2i - atan((b^4*(a/(4*d^2) + (b*1i)/(4*d^2))^(1/2)*(a + b*t
an(c + d*x))^(1/2)*32i)/((b^5*16i)/d + (a^2*b^3*16i)/d) - (32*a*b^3*(a/(4*d
^2) + (b*1i)/(4*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2))/((b^5*16i)/d + (a^2
*b^3*16i)/d))*((a + b*1i)/(4*d^2))^(1/2)*2i + (2*(a + b*tan(c + d*x))^(5/2)
)/(5*b^2*d) - (2*a*(a + b*tan(c + d*x))^(3/2))/(3*b^2*d)
```

3.505 $\int \tan^2(c+dx)\sqrt{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=382

$$-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}\,d}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}+\sqrt{2}\sqrt{a+b\tan(}{\sqrt{a-\sqrt{a^2+b^2}}}\right.}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}\,d}$$

```
[Out] -1/2*b*arctanh(((a+(a^2+b^2)^(1/2))^(1/2)-2^(1/2)*(a+b*tan(d*x+c))^(1/2))/(
a-(a^2+b^2)^(1/2))^(1/2))/d*2^(1/2)/(a-(a^2+b^2)^(1/2))^(1/2)+1/2*b*arctanh
(((a+(a^2+b^2)^(1/2))^(1/2)+2^(1/2)*(a+b*tan(d*x+c))^(1/2))/(a-(a^2+b^2)^(1
/2))^(1/2))/d*2^(1/2)/(a-(a^2+b^2)^(1/2))^(1/2)-1/4*b*ln(a+(a^2+b^2)^(1/2)-
2^(1/2)*(a+(a^2+b^2)^(1/2))^(1/2)*(a+b*tan(d*x+c))^(1/2)+b*tan(d*x+c))/d*2^
(1/2)/(a+(a^2+b^2)^(1/2))^(1/2)+1/4*b*ln(a+(a^2+b^2)^(1/2)+2^(1/2)*(a+(a^2+
b^2)^(1/2))^(1/2)*(a+b*tan(d*x+c))^(1/2)+b*tan(d*x+c))/d*2^(1/2)/(a+(a^2+b^
2)^(1/2))^(1/2)+2/3*(a+b*tan(d*x+c))^(3/2)/b/d
```

Rubi [A]

time = 0.26, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3624, 3566, 714, 1143, 648, 632, 212, 642}

$$-\frac{b\log\left(-\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\sqrt{a+b\tan(c+dx)}+\sqrt{a^2+b^2}+a+b\tan(c+dx)\right)}{2\sqrt{2}d\sqrt{\sqrt{a^2+b^2}+a}}+\frac{b\log\left(\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\sqrt{a+b\tan(c+dx)}+\sqrt{a^2+b^2}+a+b\tan(c+dx)\right)}{2\sqrt{2}d\sqrt{\sqrt{a^2+b^2}+a}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+b^2}+a}-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}d\sqrt{a-\sqrt{a^2+b^2}}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+b^2}+a}+\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}d\sqrt{a-\sqrt{a^2+b^2}}}+\frac{2(a+b\tan(c+dx))^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2*Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] -((b*ArcTanh[(Sqrt[a + Sqrt[a^2 + b^2]] - Sqrt[2]*Sqrt[a + b*Tan[c + d*x]])
/Sqrt[a - Sqrt[a^2 + b^2]]])/(Sqrt[2]*Sqrt[a - Sqrt[a^2 + b^2]]*d)) + (b*Ar
cTanh[(Sqrt[a + Sqrt[a^2 + b^2]] + Sqrt[2]*Sqrt[a + b*Tan[c + d*x]])/Sqrt[a
 - Sqrt[a^2 + b^2]]])/(Sqrt[2]*Sqrt[a - Sqrt[a^2 + b^2]]*d) - (b*Log[a + Sq
rt[a^2 + b^2] + b*Tan[c + d*x] - Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*Sqrt[a +
 b*Tan[c + d*x]]])/(2*Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*d) + (b*Log[a + Sqr
t[a^2 + b^2] + b*Tan[c + d*x] + Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*Sqrt[a +
b*Tan[c + d*x]]])/(2*Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*d) + (2*(a + b*Tan[c
 + d*x])^(3/2))/(3*b*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 714

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, S
ubst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x]
 /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1143

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q =
 Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m - 1)/(q
 - r*x + x^2), x], x] - Dist[1/(2*c*r), Int[x^(m - 1)/(q + r*x + x^2), x],
x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 1] && LtQ[m, 3
] && NegQ[b^2 - 4*a*c]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \tan^2(c+dx)\sqrt{a+b\tan(c+dx)}\,dx = \frac{2(a+b\tan(c+dx))^{3/2}}{3bd} - \int \sqrt{a+b\tan(c+dx)}\,dx$$

$$= \frac{2(a+b\tan(c+dx))^{3/2}}{3bd} - \frac{b\text{Subst}\left(\int \frac{\sqrt{a+x}}{b^2+x^2}\,dx, x, b\tan(c+dx\right.}{d}$$

$$= \frac{2(a+b\tan(c+dx))^{3/2}}{3bd} - \frac{(2b)\text{Subst}\left(\int \frac{x^2}{a^2+b^2-2ax^2+x^4}\,dx, x, \sqrt{a+\right.}{d}$$

$$= \frac{2(a+b\tan(c+dx))^{3/2}}{3bd} - \frac{b\text{Subst}\left(\int \frac{x}{\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2\right.}{\sqrt{2}\sqrt{a+}$$

$$= \frac{2(a+b\tan(c+dx))^{3/2}}{3bd} - \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2\right.}{2}$$

$$= -\frac{b\log\left(a+\sqrt{a^2+b^2}+b\tan(c+dx)-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\right.}{2\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\,d}$$

$$= -\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}\,d} + \frac{b\tanh}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.27, size = 115, normalized size = 0.30

$$\frac{i\sqrt{a-ib}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{i\sqrt{a+ib}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d} + \frac{2(a+b\tan(c+dx))^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2*Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] (I*Sqrt[a - I*b]*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/d - (I*Sq
rt[a + I*b]*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/d + (2*(a + b*
Tan[c + d*x])^(3/2))/(3*b*d)
```

Maple [A]
time = 0.15, size = 385, normalized size = 1.01

method	result
derivativedivides	$\frac{\frac{2(a+b\tan(dx+c))^{\frac{3}{2}}}{3}-2b^2\left(-\frac{\sqrt{2\sqrt{a^2+b^2}+2a}\,\left(\sqrt{a^2+b^2}-a\right)\left(\frac{\ln\left(b\tan(dx+c)+a+\sqrt{a+b\tan(dx+c)}\right.}{}\right.}{}\right.}{}$
default	$\frac{\frac{2(a+b\tan(dx+c))^{\frac{3}{2}}}{3}-2b^2\left(-\frac{\sqrt{2\sqrt{a^2+b^2}+2a}\,\left(\sqrt{a^2+b^2}-a\right)\left(\frac{\ln\left(b\tan(dx+c)+a+\sqrt{a+b\tan(dx+c)}\right.}{}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(1/2)*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/b*(1/3*(a+b*tan(d*x+c))^(3/2)-b^2*(-1/4*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(
(a^2+b^2)^(1/2)-a)/b^2*(1/2*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^
2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)/(2*(
a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/
2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))+1/4*(2*(a^2+b^2)^(1/2)+2*a)^
(1/2)*((a^2+b^2)^(1/2)-a)/b^2*(1/2*ln(-b*tan(d*x+c)-a+(a+b*tan(d*x+c))^(1/2
)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-(a^2+b^2)^(1/2))-(2*(a^2+b^2)^(1/2)+2*a)^(1
/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2
+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1586 vs. $2(309) = 618$.
time = 0.97, size = 1586, normalized size = 4.15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(12*sqrt(2)*b*d^5*sqrt((a*d^2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*
sqrt(b^2/d^4)*((a^2 + b^2)/d^4)^(3/4)*arctan(-(sqrt(2)*b*d^5*sqrt((a*cos(d*
x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt((a*d^2*sqrt((a^2 + b^2)/d^4) +
a^2 + b^2)/b^2)*sqrt(b^2/d^4)*((a^2 + b^2)/d^4)^(3/4) - sqrt(2)*d^5*sqrt((a
*d^2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*sqrt((sqrt(2)*b^3*d^3*sqrt((a*
cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt((a*d^2*sqrt((a^2 + b^2)/d
^4) + a^2 + b^2)/b^2)*((a^2 + b^2)/d^4)^(3/4)*cos(d*x + c) + (a^2*b^2 + b^4
)*d^2*sqrt((a^2 + b^2)/d^4)*cos(d*x + c) + (a^3*b^2 + a*b^4)*cos(d*x + c) +
 (a^2*b^3 + b^5)*sin(d*x + c))/((a^2 + b^2)*cos(d*x + c)))*sqrt(b^2/d^4)*((
a^2 + b^2)/d^4)^(3/4) + (a^2 + b^2)*d^4*sqrt(b^2/d^4)*sqrt((a^2 + b^2)/d^4)
 + (a^3 + a*b^2)*d^2*sqrt(b^2/d^4))/(a^2*b^2 + b^4))*cos(d*x + c) + 12*sqrt
(2)*b*d^5*sqrt((a*d^2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*sqrt(b^2/d^4)
*((a^2 + b^2)/d^4)^(3/4)*arctan(-(sqrt(2)*b*d^5*sqrt((a*cos(d*x + c) + b*si
n(d*x + c))/cos(d*x + c))*sqrt((a*d^2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/b^
2)*sqrt(b^2/d^4)*((a^2 + b^2)/d^4)^(3/4) - sqrt(2)*d^5*sqrt((a*d^2*sqrt((a^
2 + b^2)/d^4) + a^2 + b^2)/b^2)*sqrt(-(sqrt(2)*b^3*d^3*sqrt((a*cos(d*x + c)
 + b*sin(d*x + c))/cos(d*x + c))*sqrt((a*d^2*sqrt((a^2 + b^2)/d^4) + a^2 +
b^2)/b^2)*((a^2 + b^2)/d^4)^(3/4)*cos(d*x + c) - (a^2*b^2 + b^4)*d^2*sqrt((
a^2 + b^2)/d^4)*cos(d*x + c) - (a^3*b^2 + a*b^4)*cos(d*x + c) - (a^2*b^3 +
b^5)*sin(d*x + c))/((a^2 + b^2)*cos(d*x + c)))*sqrt(b^2/d^4)*((a^2 + b^2)/d
^4)^(3/4) - (a^2 + b^2)*d^4*sqrt(b^2/d^4)*sqrt((a^2 + b^2)/d^4) - (a^3 + a*
b^2)*d^2*sqrt(b^2/d^4))/(a^2*b^2 + b^4))*cos(d*x + c) - 3*sqrt(2)*(a*b*d^3*
sqrt((a^2 + b^2)/d^4)*cos(d*x + c) - (a^2*b + b^3)*d*cos(d*x + c))*sqrt((a*
d^2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*((a^2 + b^2)/d^4)^(1/4)*log((sq
rt(2)*b^3*d^3*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt((a*
d^2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*((a^2 + b^2)/d^4)^(3/4)*cos(d*x
 + c) + (a^2*b^2 + b^4)*d^2*sqrt((a^2 + b^2)/d^4)*cos(d*x + c) + (a^3*b^2 +
 a*b^4)*cos(d*x + c) + (a^2*b^3 + b^5)*sin(d*x + c))/((a^2 + b^2)*cos(d*x +
 c))) + 3*sqrt(2)*(a*b*d^3*sqrt((a^2 + b^2)/d^4)*cos(d*x + c) - (a^2*b + b^
3)*d*cos(d*x + c))*sqrt((a*d^2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*((a^
2 + b^2)/d^4)^(1/4)*log(-(sqrt(2)*b^3*d^3*sqrt((a*cos(d*x + c) + b*sin(d*x
+ c))/cos(d*x + c))*sqrt((a*d^2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*((a
^2 + b^2)/d^4)^(3/4)*cos(d*x + c) - (a^2*b^2 + b^4)*d^2*sqrt((a^2 + b^2)/d^
4)*cos(d*x + c) - (a^3*b^2 + a*b^4)*cos(d*x + c) - (a^2*b^3 + b^5)*sin(d*x
```

```
+ c))/((a^2 + b^2)*cos(d*x + c))) + 8*((a^3 + a*b^2)*cos(d*x + c) + (a^2*b
+ b^3)*sin(d*x + c))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c)))/
((a^2*b + b^3)*d*cos(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\tan{\left(c + dx \right)}}\,\tan^{2}{\left(c + dx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(1/2)*tan(d*x+c)**2,x)
```

```
[Out] Integral(sqrt(a + b*tan(c + d*x))*tan(c + d*x)**2, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 5.47, size = 231, normalized size = 0.60

$$\operatorname{atanh}\left(\frac{d^3\sqrt{-\frac{a-b\,1{}\mathrm{i}}{d^2}}\left(\frac{16\left(b^4-a^2b^2\right)\sqrt{a+b\tan\left(c+d\,x\right)}}{d^2}+\frac{16\,a\,b^2\left(a-b\,1{}\mathrm{i}\right)\sqrt{a+b\tan\left(c+d\,x\right)}}{d^2}\right)}{16\left(a^2b^3+b^5\right)}\right)\sqrt{-\frac{a-b\,1{}\mathrm{i}}{d^2}}+\operatorname{atanh}\left(\frac{d^3\sqrt{-\frac{a+b\,1{}\mathrm{i}}{d^2}}\left(\frac{16\left(b^4-a^2b^2\right)\sqrt{a+b\tan\left(c+d\,x\right)}}{d^2}+\frac{16\,a\,b^2\left(a+b\,1{}\mathrm{i}\right)\sqrt{a+b\tan\left(c+d\,x\right)}}{d^2}\right)}{16\left(a^2b^3+b^5\right)}\right)\sqrt{-\frac{a+b\,1{}\mathrm{i}}{d^2}}+\frac{2\left(a+b\tan\left(c+d\,x\right)\right)^{3/2}}{3\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + b*tan(c + d*x))^(1/2),x)
```

```
[Out] atanh((d^3*(-(a - b*1i)/d^2)^(1/2)*((16*(b^4 - a^2*b^2)*(a + b*tan(c + d*x)
)^(1/2))/d^2 + (16*a*b^2*(a - b*1i)*(a + b*tan(c + d*x))^(1/2))/d^2))/(16*(
b^5 + a^2*b^3)))*(-(a - b*1i)/d^2)^(1/2) + atanh((d^3*(-(a + b*1i)/d^2)^(1/
2)*((16*(b^4 - a^2*b^2)*(a + b*tan(c + d*x))^(1/2))/d^2 + (16*a*b^2*(a + b*
1i)*(a + b*tan(c + d*x))^(1/2))/d^2))/(16*(b^5 + a^2*b^3)))*(-(a + b*1i)/d^
2)^(1/2) + (2*(a + b*tan(c + d*x))^(3/2))/(3*b*d)
```

3.506 $\int \tan(c+dx)\sqrt{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=106

$$-\frac{\sqrt{a-ib}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}-\frac{\sqrt{a+ib}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}+\frac{2\sqrt{a+b\tan(c+dx)}}{d}$$

```
[Out] -arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))*(a-I*b)^(1/2)/d-arctanh((a+b
*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))*(a+I*b)^(1/2)/d+2*(a+b*tan(d*x+c))^(1/2)/
d
```

Rubi [A]
time = 0.12, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3609, 3620, 3618, 65, 214}

$$\frac{2\sqrt{a+b\tan(c+dx)}}{d}-\frac{\sqrt{a-ib}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}-\frac{\sqrt{a+ib}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]*Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] -((Sqrt[a - I*b]*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/d) - (Sqr
t[a + I*b]*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/d + (2*Sqrt[a +
 b*Tan[c + d*x]])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
```

```
0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \tan(c+dx)\sqrt{a+b\tan(c+dx)}\,dx = \frac{2\sqrt{a+b\tan(c+dx)}}{d} + \int \frac{-b+a\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$$

$$= \frac{2\sqrt{a+b\tan(c+dx)}}{d} + \frac{1}{2}(-ia-b)\int \frac{1+i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx + \frac{1}{2}$$

$$= \frac{2\sqrt{a+b\tan(c+dx)}}{d} + \frac{(a-ib)\mathrm{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a-ibx}}\,dx, x, i\,t\right.}{2d}$$

$$= \frac{2\sqrt{a+b\tan(c+dx)}}{d} - \frac{(ia-b)\mathrm{Subst}\left(\int \frac{1}{-1+\frac{ia}{b}-\frac{ix^2}{b}}\,dx, x, \sqrt{a+b}\right.}{bd}$$

$$= -\frac{\sqrt{a-ib}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{\sqrt{a+ib}\,\tanh^{-1}\left(\frac{\sqrt{}}{}\right.}{d}$$

Mathematica [A]
time = 0.11, size = 100, normalized size = 0.94

$$-\frac{\sqrt{a-ib}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)+\sqrt{a+ib}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)-2\sqrt{a+b\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]*Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] -((Sqrt[a - I*b]*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] + Sqrt[a +
 I*b]*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] - 2*Sqrt[a + b*Tan[c
+ d*x]])/d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(88) = 176$.
time = 0.12, size = 327, normalized size = 3.08

method	result
derivativedivides	$\frac{2\sqrt{a+b\tan(dx+c)}-\frac{\sqrt{2\sqrt{a^2+b^2}+2a}\,\ln\left(b\tan(dx+c)+a+\sqrt{a+b\tan(dx+c)}\,\sqrt{2\sqrt{a^2}\right.}{4}}{}$
default	$\frac{2\sqrt{a+b\tan(dx+c)}-\frac{\sqrt{2\sqrt{a^2+b^2}+2a}\,\ln\left(b\tan(dx+c)+a+\sqrt{a+b\tan(dx+c)}\,\sqrt{2\sqrt{a^2}\right.}{4}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(1/2)*tan(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(a+b*tan(d*x+c))^(1/2)-1/4*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln(b*tan(d*
x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2)
)+(a-(a^2+b^2)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+
c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))+1/4
*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln(-b*tan(d*x+c)-a+(a+b*tan(d*x+c))^(1/2)*(2
*(a^2+b^2)^(1/2)+2*a)^(1/2)-(a^2+b^2)^(1/2))+((a^2+b^2)^(1/2)-a)/(2*(a^2+b^
2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*
a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1740 vs. 2(84) = 168.
time = 1.21, size = 1740, normalized size = 16.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*d^5*sqrt(-(a*d^2*sqrt((a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*sqr
t(b^2/d^4)*((a^2 + b^2)/d^4)^(3/4)*arctan(-((a^2 + b^2)*d^4*sqrt(b^2/d^4)*s
qrt((a^2 + b^2)/d^4) + (a^3 + a*b^2)*d^2*sqrt(b^2/d^4) + sqrt(2)*(d^7*sqrt(
b^2/d^4)*sqrt((a^2 + b^2)/d^4) + a*d^5*sqrt(b^2/d^4))*sqrt((a*cos(d*x + c)
+ b*sin(d*x + c))/cos(d*x + c))*sqrt(-(a*d^2*sqrt((a^2 + b^2)/d^4) - a^2 -
b^2)/b^2)*((a^2 + b^2)/d^4)^(3/4) - sqrt(2)*(d^7*sqrt(b^2/d^4)*sqrt((a^2 +
b^2)/d^4) + a*d^5*sqrt(b^2/d^4))*sqrt(-(a*d^2*sqrt((a^2 + b^2)/d^4) - a^2 -
 b^2)/b^2)*sqrt(((a^2 + b^2)*d^2*sqrt((a^2 + b^2)/d^4)*cos(d*x + c) + sqrt(
2)*(a*d^3*sqrt((a^2 + b^2)/d^4)*cos(d*x + c) + (a^2 + b^2)*d*cos(d*x + c))*
sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-(a*d^2*sqrt((a^2
 + b^2)/d^4) - a^2 - b^2)/b^2)*((a^2 + b^2)/d^4)^(1/4) + (a^3 + a*b^2)*cos(
d*x + c) + (a^2*b + b^3)*sin(d*x + c))/((a^2 + b^2)*cos(d*x + c)))*((a^2 +
b^2)/d^4)^(3/4))/(a^2*b^2 + b^4)) + 4*sqrt(2)*d^5*sqrt(-(a*d^2*sqrt((a^2 +
b^2)/d^4) - a^2 - b^2)/b^2)*sqrt(b^2/d^4)*((a^2 + b^2)/d^4)^(3/4)*arctan(((
a^2 + b^2)*d^4*sqrt(b^2/d^4)*sqrt((a^2 + b^2)/d^4) + (a^3 + a*b^2)*d^2*sqrt
(b^2/d^4) - sqrt(2)*(d^7*sqrt(b^2/d^4)*sqrt((a^2 + b^2)/d^4) + a*d^5*sqrt(b
^2/d^4))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-(a*d^2*
sqrt((a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*((a^2 + b^2)/d^4)^(3/4) + sqrt(2)*(
d^7*sqrt(b^2/d^4)*sqrt((a^2 + b^2)/d^4) + a*d^5*sqrt(b^2/d^4))*sqrt(-(a*d^2
*sqrt((a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*sqrt(((a^2 + b^2)*d^2*sqrt((a^2 +
b^2)/d^4)*cos(d*x + c) - sqrt(2)*(a*d^3*sqrt((a^2 + b^2)/d^4)*cos(d*x + c)
+ (a^2 + b^2)*d*cos(d*x + c))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*
x + c))*sqrt(-(a*d^2*sqrt((a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*((a^2 + b^2)/d
^4)^(1/4) + (a^3 + a*b^2)*cos(d*x + c) + (a^2*b + b^3)*sin(d*x + c))/((a^2
+ b^2)*cos(d*x + c)))*((a^2 + b^2)/d^4)^(3/4))/(a^2*b^2 + b^4)) - sqrt(2)*(
a*d^3*sqrt((a^2 + b^2)/d^4) + (a^2 + b^2)*d)*sqrt(-(a*d^2*sqrt((a^2 + b^2)/
d^4) - a^2 - b^2)/b^2)*((a^2 + b^2)/d^4)^(1/4)*log(((a^2 + b^2)*d^2*sqrt((a
^2 + b^2)/d^4)*cos(d*x + c) + sqrt(2)*(a*d^3*sqrt((a^2 + b^2)/d^4)*cos(d*x
+ c) + (a^2 + b^2)*d*cos(d*x + c))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/c
os(d*x + c))*sqrt(-(a*d^2*sqrt((a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*((a^2 + b
^2)/d^4)^(1/4) + (a^3 + a*b^2)*cos(d*x + c) + (a^2*b + b^3)*sin(d*x + c))/(
(a^2 + b^2)*cos(d*x + c))) + sqrt(2)*(a*d^3*sqrt((a^2 + b^2)/d^4) + (a^2 +
b^2)*d)*sqrt(-(a*d^2*sqrt((a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*((a^2 + b^2)/d
^4)^(1/4)*log(((a^2 + b^2)*d^2*sqrt((a^2 + b^2)/d^4)*cos(d*x + c) - sqrt(2)
*(a*d^3*sqrt((a^2 + b^2)/d^4)*cos(d*x + c) + (a^2 + b^2)*d*cos(d*x + c))*sq
rt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-(a*d^2*sqrt((a^2 +
```

```
b^2)/d^4) - a^2 - b^2)/b^2)*((a^2 + b^2)/d^4)^(1/4) + (a^3 + a*b^2)*cos(d*
x + c) + (a^2*b + b^3)*sin(d*x + c))/((a^2 + b^2)*cos(d*x + c))) + 8*(a^2 +
b^2)*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c)))/((a^2 + b^2)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan\left(c+dx\right)}\,\tan\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(1/2)*tan(d*x+c),x)
```

```
[Out] Integral(sqrt(a + b*tan(c + d*x))*tan(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(d*x + c) + a)*tan(d*x + c), x)
```

Mupad [B]
time = 4.79, size = 290, normalized size = 2.74

$$\frac{2\,\sqrt{a+b\tan\left(c+d\,x\right)}}{d}-\mathrm{atan}\left(\frac{b^4\,\sqrt{\frac{a}{4\,d^2}-\frac{b\,1\mathrm{i}}{4\,d^2}}\,\sqrt{a+b\tan\left(c+d\,x\right)}\,32\mathrm{i}}{\frac{b^5\,16\mathrm{i}}{d}+\frac{a^2\,b^3\,16\mathrm{i}}{d}}+\frac{32\,a\,b^3\,\sqrt{\frac{a}{4\,d^2}-\frac{b\,1\mathrm{i}}{4\,d^2}}\,\sqrt{a+b\tan\left(c+d\,x\right)}}{\frac{b^5\,16\mathrm{i}}{d}+\frac{a^2\,b^3\,16\mathrm{i}}{d}}\right)\,\sqrt{\frac{a-b\,1\mathrm{i}}{4\,d^2}}\,2\mathrm{i}+\mathrm{atan}\left(\frac{b^4\,\sqrt{\frac{a}{4\,d^2}+\frac{b\,1\mathrm{i}}{4\,d^2}}\,\sqrt{a+b\tan\left(c+d\,x\right)}\,32\mathrm{i}}{\frac{b^5\,16\mathrm{i}}{d}+\frac{a^2\,b^3\,16\mathrm{i}}{d}}-\frac{32\,a\,b^3\,\sqrt{\frac{a}{4\,d^2}+\frac{b\,1\mathrm{i}}{4\,d^2}}\,\sqrt{a+b\tan\left(c+d\,x\right)}}{\frac{b^5\,16\mathrm{i}}{d}+\frac{a^2\,b^3\,16\mathrm{i}}{d}}\right)\,\sqrt{\frac{a+b\,1\mathrm{i}}{4\,d^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + b*tan(c + d*x))^(1/2),x)
```

```
[Out] (2*(a + b*tan(c + d*x))^(1/2))/d - atan((b^4*(a/(4*d^2) - (b*1i)/(4*d^2))^(
1/2)*(a + b*tan(c + d*x))^(1/2)*32i)/((b^5*16i)/d + (a^2*b^3*16i)/d) + (32*
a*b^3*(a/(4*d^2) - (b*1i)/(4*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2))/((b^5*
16i)/d + (a^2*b^3*16i)/d))*((a - b*1i)/(4*d^2))^(1/2)*2i + atan((b^4*(a/(4*
d^2) + (b*1i)/(4*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2)*32i)/((b^5*16i)/d +
(a^2*b^3*16i)/d) - (32*a*b^3*(a/(4*d^2) + (b*1i)/(4*d^2))^(1/2)*(a + b*tan
(c + d*x))^(1/2))/((b^5*16i)/d + (a^2*b^3*16i)/d))*((a + b*1i)/(4*d^2))^(1/
2)*2i
```

3.507 $\int \sqrt{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=358

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}\,d}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}+\sqrt{2}\sqrt{a+b\tan(c+}{\sqrt{a-\sqrt{a^2+b^2}}}\right.}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}\,d}$$

```
[Out] 1/2*b*arctanh(((a+(a^2+b^2)^(1/2))^(1/2)-2^(1/2)*(a+b*tan(d*x+c))^(1/2))/(a
-(a^2+b^2)^(1/2))^(1/2))/d*2^(1/2)/(a-(a^2+b^2)^(1/2))^(1/2)-1/2*b*arctanh(
((a+(a^2+b^2)^(1/2))^(1/2)+2^(1/2)*(a+b*tan(d*x+c))^(1/2))/(a-(a^2+b^2)^(1/
2))^(1/2))/d*2^(1/2)/(a-(a^2+b^2)^(1/2))^(1/2)+1/4*b*ln(a+(a^2+b^2)^(1/2)-2
^(1/2)*(a+(a^2+b^2)^(1/2))^(1/2)*(a+b*tan(d*x+c))^(1/2)+b*tan(d*x+c))/d*2^(
1/2)/(a+(a^2+b^2)^(1/2))^(1/2)-1/4*b*ln(a+(a^2+b^2)^(1/2)+2^(1/2)*(a+(a^2+b
^2)^(1/2))^(1/2)*(a+b*tan(d*x+c))^(1/2)+b*tan(d*x+c))/d*2^(1/2)/(a+(a^2+b^2
)^(1/2))^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3566, 714, 1143, 648, 632, 212, 642}

$$\frac{b\log\left(-\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\sqrt{a+b\tan(c+dx)}+\sqrt{a^2+b^2}+a+b\tan(c+dx)\right)}{2\sqrt{2}d\sqrt{\sqrt{a^2+b^2}+a}}-\frac{b\log\left(\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\sqrt{a+b\tan(c+dx)}+\sqrt{a^2+b^2}+a+b\tan(c+dx)\right)}{2\sqrt{2}d\sqrt{\sqrt{a^2+b^2}+a}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+b^2}+a}-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}d\sqrt{a-\sqrt{a^2+b^2}}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+b^2}+a}+\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}d\sqrt{a-\sqrt{a^2+b^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] (b*ArcTanh[(Sqrt[a + Sqrt[a^2 + b^2]] - Sqrt[2]*Sqrt[a + b*Tan[c + d*x]])/S
qrt[a - Sqrt[a^2 + b^2]]])/(Sqrt[2]*Sqrt[a - Sqrt[a^2 + b^2]]*d) - (b*ArcTa
nh[(Sqrt[a + Sqrt[a^2 + b^2]] + Sqrt[2]*Sqrt[a + b*Tan[c + d*x]])/Sqrt[a -
Sqrt[a^2 + b^2]]])/(Sqrt[2]*Sqrt[a - Sqrt[a^2 + b^2]]*d) + (b*Log[a + Sqrt[
a^2 + b^2] + b*Tan[c + d*x] - Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*Sqrt[a + b*
Tan[c + d*x]]])/(2*Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*d) - (b*Log[a + Sqrt[a
^2 + b^2] + b*Tan[c + d*x] + Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*Sqrt[a + b*T
an[c + d*x]]])/(2*Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 714

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, S
ubst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x]
 /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1143

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q =
 Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m - 1)/(q
 - r*x + x^2), x], x] - Dist[1/(2*c*r), Int[x^(m - 1)/(q + r*x + x^2), x],
x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 1] && LtQ[m, 3
] && NegQ[b^2 - 4*a*c]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\tan(c+dx)}\,dx = \frac{b\text{Subst}\left(\int \frac{\sqrt{a+x}}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{(2b)\text{Subst}\left(\int \frac{x^2}{a^2+b^2-2ax^2+x^4}\,dx, x, \sqrt{a+b\tan(c+dx)}\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \frac{x}{\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\,x+x^2}\,dx, x, \sqrt{a+b\tan(c+dx)}\right)}{\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\,d}$$

$$= \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\,x+x^2}\,dx, x, \sqrt{a+b\tan(c+dx)}\right)}{2d}$$

$$= \frac{b\log\left(a+\sqrt{a^2+b^2}+b\tan(c+dx)-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\tan(c+d}\right.}{2\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\,d}$$

$$= \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}\,d} - \frac{b\tanh^{-1}\left(\sqrt{a+}\right.}{\sqrt{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.03, size = 87, normalized size = 0.24

$$-\frac{i\left(\sqrt{a-ib}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)-\sqrt{a+ib}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] ((-I)*(Sqrt[a - I*b]*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] - Sqrt
[a + I*b]*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]]))/d
```

Maple [A]
time = 0.51, size = 363, normalized size = 1.01

method	result
derivativedivides	$\frac{2b\left(-\frac{\sqrt{2\sqrt{a^2+b^2}+2a}\,\left(\sqrt{a^2+b^2}-a\right)\left(\frac{\ln\left(b\tan(dx+c)+a+\sqrt{a+b\tan(dx+c)}\,\sqrt{2\sqrt{a^2+b}\right.}{2}\right.}{}\right.}{}$
default	$\frac{2b\left(-\frac{\sqrt{2\sqrt{a^2+b^2}+2a}\,\left(\sqrt{a^2+b^2}-a\right)\left(\frac{\ln\left(b\tan(dx+c)+a+\sqrt{a+b\tan(dx+c)}\,\sqrt{2\sqrt{a^2+b}\right.}{2}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*b*(-1/4*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*((a^2+b^2)^(1/2)-a)/b^2*(1/2*ln(b
*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2
)^(1/2))-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan
((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2
)-2*a)^(1/2)))+1/4*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*((a^2+b^2)^(1/2)-a)/b^2*(1
/2*ln(-b*tan(d*x+c)-a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-
(a^2+b^2)^(1/2))-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2
)*arctan((-2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+
b^2)^(1/2)-2*a)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1465 vs. $2(289) = 578$.
time = 1.40, size = 1465, normalized size = 4.09

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*d^4*sqrt((a*d^2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*sqr
t(b^2/d^4)*((a^2 + b^2)/d^4)^(3/4)*arctan(-(sqrt(2)*b*d^5*sqrt((a*cos(d*x +
 c) + b*sin(d*x + c))/cos(d*x + c))*sqrt((a*d^2*sqrt((a^2 + b^2)/d^4) + a^2
 + b^2)/b^2)*sqrt(b^2/d^4)*((a^2 + b^2)/d^4)^(3/4) - sqrt(2)*d^5*sqrt((a*d^
2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*sqrt((sqrt(2)*b^3*d^3*sqrt((a*cos
(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt((a*d^2*sqrt((a^2 + b^2)/d^4)
 + a^2 + b^2)/b^2)*((a^2 + b^2)/d^4)^(3/4)*cos(d*x + c) + (a^2*b^2 + b^4)*d
^2*sqrt((a^2 + b^2)/d^4)*cos(d*x + c) + (a^3*b^2 + a*b^4)*cos(d*x + c) + (a
^2*b^3 + b^5)*sin(d*x + c))/((a^2 + b^2)*cos(d*x + c)))*sqrt(b^2/d^4)*((a^2
 + b^2)/d^4)^(3/4) + (a^2 + b^2)*d^4*sqrt(b^2/d^4)*sqrt((a^2 + b^2)/d^4) +
(a^3 + a*b^2)*d^2*sqrt(b^2/d^4))/(a^2*b^2 + b^4)) + 4*sqrt(2)*d^4*sqrt((a*d
^2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*sqrt(b^2/d^4)*((a^2 + b^2)/d^4)^
(3/4)*arctan(-(sqrt(2)*b*d^5*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x
 + c))*sqrt((a*d^2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*sqrt(b^2/d^4)*((
a^2 + b^2)/d^4)^(3/4) - sqrt(2)*d^5*sqrt((a*d^2*sqrt((a^2 + b^2)/d^4) + a^2
 + b^2)/b^2)*sqrt(-(sqrt(2)*b^3*d^3*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/
cos(d*x + c))*sqrt((a*d^2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*((a^2 + b
^2)/d^4)^(3/4)*cos(d*x + c) - (a^2*b^2 + b^4)*d^2*sqrt((a^2 + b^2)/d^4)*cos
(d*x + c) - (a^3*b^2 + a*b^4)*cos(d*x + c) - (a^2*b^3 + b^5)*sin(d*x + c))/
((a^2 + b^2)*cos(d*x + c)))*sqrt(b^2/d^4)*((a^2 + b^2)/d^4)^(3/4) - (a^2 +
b^2)*d^4*sqrt(b^2/d^4)*sqrt((a^2 + b^2)/d^4) - (a^3 + a*b^2)*d^2*sqrt(b^2/d
^4))/(a^2*b^2 + b^4)) - sqrt(2)*(a*d^2*sqrt((a^2 + b^2)/d^4) - a^2 - b^2)*s
qrt((a*d^2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*((a^2 + b^2)/d^4)^(1/4)*
log((sqrt(2)*b^3*d^3*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*s
qrt((a*d^2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*((a^2 + b^2)/d^4)^(3/4)*
cos(d*x + c) + (a^2*b^2 + b^4)*d^2*sqrt((a^2 + b^2)/d^4)*cos(d*x + c) + (a^
3*b^2 + a*b^4)*cos(d*x + c) + (a^2*b^3 + b^5)*sin(d*x + c))/((a^2 + b^2)*co
s(d*x + c))) + sqrt(2)*(a*d^2*sqrt((a^2 + b^2)/d^4) - a^2 - b^2)*sqrt((a*d^
2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*((a^2 + b^2)/d^4)^(1/4)*log(-(sqr
t(2)*b^3*d^3*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt((a*d
^2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*((a^2 + b^2)/d^4)^(3/4)*cos(d*x
+ c) - (a^2*b^2 + b^4)*d^2*sqrt((a^2 + b^2)/d^4)*cos(d*x + c) - (a^3*b^2 +
a*b^4)*cos(d*x + c) - (a^2*b^3 + b^5)*sin(d*x + c))/((a^2 + b^2)*cos(d*x +
c))))/(a^2 + b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \tan{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(c + d*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 4.18, size = 213, normalized size = 0.59

$$-\operatorname{atanh}\left(\frac{d^3\sqrt{-\frac{a-b\,1\mathrm{i}}{d^2}}\left(\frac{16\,(b^4-a^2\,b^2)\,\sqrt{a+b\tan\left(c+d\,x\right)}}{d^2}+\frac{16\,a\,b^2\,(a-b\,1\mathrm{i})\,\sqrt{a+b\tan\left(c+d\,x\right)}}{d^2}\right)}{16\,(a^2\,b^3+b^5)}\right)\sqrt{-\frac{a-b\,1\mathrm{i}}{d^2}}-\operatorname{atanh}\left(\frac{d^3\sqrt{-\frac{a+b\,1\mathrm{i}}{d^2}}\left(\frac{16\,(b^4-a^2\,b^2)\,\sqrt{a+b\tan\left(c+d\,x\right)}}{d^2}+\frac{16\,a\,b^2\,(a+b\,1\mathrm{i})\,\sqrt{a+b\tan\left(c+d\,x\right)}}{d^2}\right)}{16\,(a^2\,b^3+b^5)}\right)\sqrt{-\frac{a+b\,1\mathrm{i}}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^(1/2),x)
```

```
[Out] - atanh((d^3*(-(a - b*1i)/d^2)^(1/2)*((16*(b^4 - a^2*b^2)*(a + b*tan(c + d*
x))^(1/2))/d^2 + (16*a*b^2*(a - b*1i)*(a + b*tan(c + d*x))^(1/2))/d^2))/(16
*(b^5 + a^2*b^3)))*(-(a - b*1i)/d^2)^(1/2) - atanh((d^3*(-(a + b*1i)/d^2)^(
1/2)*((16*(b^4 - a^2*b^2)*(a + b*tan(c + d*x))^(1/2))/d^2 + (16*a*b^2*(a +
b*1i)*(a + b*tan(c + d*x))^(1/2))/d^2))/(16*(b^5 + a^2*b^3)))*(-(a + b*1i)/
d^2)^(1/2)
```

3.508 $\int \cot(c+dx)\sqrt{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=116

$$-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{\sqrt{a-ib}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}+\frac{\sqrt{a+ib}\tanh^{-1}\left(\frac{\sqrt{a}}{}\right.}{d}$$

```
[Out] -2*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d+arctanh((a+b*tan(d*x+c
))^(1/2)/(a-I*b)^(1/2))*(a-I*b)^(1/2)/d+arctanh((a+b*tan(d*x+c))^(1/2)/(a+I
*b)^(1/2))*(a+I*b)^(1/2)/d
```

Rubi [A]
time = 0.20, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3653, 3620, 3618, 65, 214, 3715}

$$-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{\sqrt{a-ib}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}+\frac{\sqrt{a+ib}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/d + (Sqrt[a - I*b]*A
rcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/d + (Sqrt[a + I*b]*ArcTanh[
Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3653

```
Int[Sqrt[(a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[Simp[a*c + b*d + (b*c -
a*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x]], x], x] - Dist[d*((b*c - a*d
)/(c^2 + d^2)), Int[(1 + Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]]*(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \cot(c+dx)\sqrt{a+b\tan(c+dx)}\,dx = a\int \frac{\cot(c+dx)\left(1+\tan^2(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}\,dx + \int \frac{b-a\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,d$$

$$= \frac{1}{2}(-ia+b)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx + \frac{1}{2}(ia+b)\int \frac{1+i\tan(}{\sqrt{a+b\tan(}}$$

$$= -\frac{(a-ib)\mathrm{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a-ibx}}\,dx, x, i\tan(c+dx)\right)}{2d} - \frac{(a+i}{}$$

$$= -\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{(ia-b)\mathrm{Subst}\left(\int \frac{1}{-1+\frac{ia}{b}}\right.}{}$$

$$= -\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{\sqrt{a-ib}\,\tanh^{-1}\left(\frac{\sqrt{a}}{}\right.}{d}$$

Mathematica [A]

time = 0.13, size = 111, normalized size = 0.96

$$\frac{-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)+\sqrt{a-ib}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)+\sqrt{a+ib}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]] + Sqrt[a - I*b]*ArcTa
nh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] + Sqrt[a + I*b]*ArcTanh[Sqrt[a +
 b*Tan[c + d*x]]/Sqrt[a + I*b]])/d
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 7.79, size = 15730, normalized size = 135.60

method	result	size
default	Expression too large to display	15730

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(d*x + c) + a)*cot(d*x + c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1755 vs. 2(90) = 180.
time = 1.62, size = 3585, normalized size = 30.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(2)*d^5*sqrt(-(a*d^2*sqrt((a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*s
qrt(b^2/d^4)*((a^2 + b^2)/d^4)^(3/4)*arctan(-((a^2 + b^2)*d^4*sqrt(b^2/d^4)
*sqrt((a^2 + b^2)/d^4) + (a^3 + a*b^2)*d^2*sqrt(b^2/d^4) + sqrt(2)*(d^7*sqr
```

t(b^2/d^4)*sqrt((a^2 + b^2)/d^4) + a*d^5*sqrt(b^2/d^4))*sqrt((a*cos(d*x + c
) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-(a*d^2*sqrt((a^2 + b^2)/d^4) - a^2
- b^2)/b^2)*((a^2 + b^2)/d^4)^(3/4) - sqrt(2)*(d^7*sqrt(b^2/d^4)*sqrt((a^2
+ b^2)/d^4) + a*d^5*sqrt(b^2/d^4))*sqrt(-(a*d^2*sqrt((a^2 + b^2)/d^4) - a^2
- b^2)/b^2)*sqrt(((a^2 + b^2)*d^2*sqrt((a^2 + b^2)/d^4)*cos(d*x + c) + sqr
t(2)*(a*d^3*sqrt((a^2 + b^2)/d^4)*cos(d*x + c) + (a^2 + b^2)*d*cos(d*x + c)
)*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-(a*d^2*sqrt((a
^2 + b^2)/d^4) - a^2 - b^2)/b^2)*((a^2 + b^2)/d^4)^(1/4) + (a^3 + a*b^2)*co
s(d*x + c) + (a^2*b + b^3)*sin(d*x + c))/((a^2 + b^2)*cos(d*x + c)))*((a^2
+ b^2)/d^4)^(3/4))/(a^2*b^2 + b^4)) + 4*sqrt(2)*d^5*sqrt(-(a*d^2*sqrt((a^2
+ b^2)/d^4) - a^2 - b^2)/b^2)*sqrt(b^2/d^4)*((a^2 + b^2)/d^4)^(3/4)*arctan(
((a^2 + b^2)*d^4*sqrt(b^2/d^4)*sqrt((a^2 + b^2)/d^4) + (a^3 + a*b^2)*d^2*sq
rt(b^2/d^4) - sqrt(2)*(d^7*sqrt(b^2/d^4)*sqrt((a^2 + b^2)/d^4) + a*d^5*sqrt
(b^2/d^4))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-(a*d^
2*sqrt((a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*((a^2 + b^2)/d^4)^(3/4) + sqrt(2)
*(d^7*sqrt(b^2/d^4)*sqrt((a^2 + b^2)/d^4) + a*d^5*sqrt(b^2/d^4))*sqrt(-(a*d
^2*sqrt((a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*sqrt(((a^2 + b^2)*d^2*sqrt((a^2
+ b^2)/d^4)*cos(d*x + c) - sqrt(2)*(a*d^3*sqrt((a^2 + b^2)/d^4)*cos(d*x + c
) + (a^2 + b^2)*d*cos(d*x + c))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(
d*x + c))*sqrt(-(a*d^2*sqrt((a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*((a^2 + b^2)
/d^4)^(1/4) + (a^3 + a*b^2)*cos(d*x + c) + (a^2*b + b^3)*sin(d*x + c))/((a^
2 + b^2)*cos(d*x + c)))*((a^2 + b^2)/d^4)^(3/4))/(a^2*b^2 + b^4)) - sqrt(2)
*(a*d^3*sqrt((a^2 + b^2)/d^4) + (a^2 + b^2)*d)*sqrt(-(a*d^2*sqrt((a^2 + b^2
)/d^4) - a^2 - b^2)/b^2)*((a^2 + b^2)/d^4)^(1/4)*log(((a^2 + b^2)*d^2*sqrt(
(a^2 + b^2)/d^4)*cos(d*x + c) + sqrt(2)*(a*d^3*sqrt((a^2 + b^2)/d^4)*cos(d*
x + c) + (a^2 + b^2)*d*cos(d*x + c))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))
/cos(d*x + c))*sqrt(-(a*d^2*sqrt((a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*((a^2 +
b^2)/d^4)^(1/4) + (a^3 + a*b^2)*cos(d*x + c) + (a^2*b + b^3)*sin(d*x + c))
/((a^2 + b^2)*cos(d*x + c))) + sqrt(2)*(a*d^3*sqrt((a^2 + b^2)/d^4) + (a^2
+ b^2)*d)*sqrt(-(a*d^2*sqrt((a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*((a^2 + b^2)
/d^4)^(1/4)*log(((a^2 + b^2)*d^2*sqrt((a^2 + b^2)/d^4)*cos(d*x + c) - sqrt(
2)*(a*d^3*sqrt((a^2 + b^2)/d^4)*cos(d*x + c) + (a^2 + b^2)*d*cos(d*x + c))*
sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-(a*d^2*sqrt((a^2
+ b^2)/d^4) - a^2 - b^2)/b^2)*((a^2 + b^2)/d^4)^(1/4) + (a^3 + a*b^2)*cos(
d*x + c) + (a^2*b + b^3)*sin(d*x + c))/((a^2 + b^2)*cos(d*x + c))) - 2*(a^2
+ b^2)*sqrt(a)*log(-(8*a*b*cos(d*x + c)*sin(d*x + c) + (8*a^2 - b^2)*cos(d
x + c)^2 + b^2 - 4(2*a*cos(d*x + c)^2 + b*cos(d*x + c)*sin(d*x + c))*sqrt
(a)*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c)))/(cos(d*x + c)^2 -
1)))/((a^2 + b^2)*d), -1/4*(4*sqrt(2)*d^5*sqrt(-(a*d^2*sqrt((a^2 + b^2)/d^
4) - a^2 - b^2)/b^2)*sqrt(b^2/d^4)*((a^2 + b^2)/d^4)^(3/4)*arctan(-((a^2 +
b^2)*d^4*sqrt(b^2/d^4)*sqrt((a^2 + b^2)/d^4) + (a^3 + a*b^2)*d^2*sqrt(b^2/d
^4) + sqrt(2)*(d^7*sqrt(b^2/d^4)*sqrt((a^2 + b^2)/d^4) + a*d^5*sqrt(b^2/d^4
))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-(a*d^2*sqrt((
a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*((a^2 + b^2)/d^4)^(3/4) - sqrt(2)*(d^7*sq
rt(b^2/d^4)*sqrt((a^2 + b^2)/d^4) + a*d^5*sqrt(b^2/d^4))*sqrt(-(a*d^2*sqrt(

```
(a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*sqrt(((a^2 + b^2)*d^2*sqrt((a^2 + b^2)/d
^4)*cos(d*x + c) + sqrt(2)*(a*d^3*sqrt((a^2 + b^2)/d^4)*cos(d*x + c) + (a^2
 + b^2)*d*cos(d*x + c))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c)
)*sqrt(-(a*d^2*sqrt((a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*((a^2 + b^2)/d^4)^(1
/4) + (a^3 + a*b^2)*cos(d*x + c) + (a^2*b + b^3)*sin(d*x + c))/((a^2 + b^2)
*cos(d*x + c)))*((a^2 + b^2)/d^4)^(3/4))/(a^2*b^2 + b^4)) + 4*sqrt(2)*d^5*s
qrt(-(a*d^2*sqrt((a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*sqrt(b^2/d^4)*((a^2 + b
^2)/d^4)^(3/4)*arctan(((a^2 + b^2)*d^4*sqrt(b^2/d^4)*sqrt((a^2 + b^2)/d^4)
+ (a^3 + a*b^2)*d^2*sqrt(b^2/d^4) - sqrt(2)*(d^7*sqrt(b^2/d^4)*sqrt((a^2 +
b^2)/d^4) + a*d^5*sqrt(b^2/d^4))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos
(d*x + c))*sqrt(-(a*d^2*sqrt((a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*((a^2 + b^2
)/d^4)^(3/4) + sqrt(2)*(d^7*sqrt(b^2/d^4)*sqrt((a^2 + b^2)/d^4) + a*d^5*sqr
t(b^2/d^4))*sqrt(-(a*d^2*sqrt((a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*sqrt(((a^2
 + b^2)*d^2*sqrt((a^2 + b^2)/d^4)*cos(d*x + c) - sqrt(2)*(a*d^3*sqrt((a^2 +
 b^2)/d^4)*cos(d*x + c) + (a^2 + b^2)*d*cos(d*x + c))*sqrt((a*cos(d*x + c)
+ b*sin(d*x + c))/cos(d*x + c))*sqrt(-(a*d^2*sqrt((a^2 + b^2)/d^4) - a^2 -
b^2)/b^2)*((a^2 + b^2)/d^4)^(1/4) + (a^3 + a*b^2)*cos(d*x + c) + (a^2*b + b
^3)*sin(d*x + c))/((a^2 + b^2)*cos(d*x + c)))*((a^2 + b^2)/d^4)^(3/4))/(a^2
*b^2 + b^4)) - sqrt(2)*(a*d^3*sqrt((a^2 + b^2)/...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan(c+dx)}\,\cot(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(c + d*x))*cot(c + d*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 4.06, size = 682, normalized size = 5.88

$$-\frac{2\sqrt{a}\,\mathrm{atanh}\left(\frac{64\sqrt{a}\,b^{12}\sqrt{a+b\tan(c+d\,x)}}{576\,a^5\,b^8+640\,a^3\,b^{10}+64\,a\,b^{12}}+\frac{640\,a^{5/2}\,b^{10}\sqrt{a+b\tan(c+d\,x)}}{576\,a^5\,b^8+640\,a^3\,b^{10}+64\,a\,b^{12}}+\frac{576\,a^{9/2}\,b^8\sqrt{a+b\tan(c+d\,x)}}{576\,a^5\,b^8+640\,a^3\,b^{10}+64\,a\,b^{12}}\right)}{d}-\mathrm{atan}\left(-\frac{32\,a\,b^{11}\sqrt{\frac{a}{4\,d^2}-\frac{b\,1\mathrm{i}}{4\,d^2}}\,\sqrt{a+b\tan(c+d\,x)}}{\frac{16\,a\,b^{12}}{d}-\frac{a^2\,b^{11}\,48\mathrm{i}}{d}+\frac{16\,a^3\,b^{10}}{d}-\frac{a^4\,b^9\,48\mathrm{i}}{d}}+\frac{a^2\,b^{10}\sqrt{\frac{a}{4\,d^2}-\frac{b\,1\mathrm{i}}{4\,d^2}}\,\sqrt{a+b\tan(c+d\,x)}\,128\mathrm{i}}{\frac{16\,a\,b^{12}}{d}-\frac{a^2\,b^{11}\,48\mathrm{i}}{d}+\frac{16\,a^3\,b^{10}}{d}-\frac{a^4\,b^9\,48\mathrm{i}}{d}}+\frac{96\,a^3\,b^9\sqrt{\frac{a}{4\,d^2}-\frac{b\,1\mathrm{i}}{4\,d^2}}\,\sqrt{a+b\tan(c+d\,x)}}{\frac{16\,a\,b^{12}}{d}-\frac{a^2\,b^{11}\,48\mathrm{i}}{d}+\frac{16\,a^3\,b^{10}}{d}-\frac{a^4\,b^9\,48\mathrm{i}}{d}}\right)\sqrt{\frac{a-b\,1\mathrm{i}}{4\,d^2}}\,2\mathrm{i}-\mathrm{atan}\left(\frac{32\,a\,b^{11}\sqrt{\frac{a}{4\,d^2}+\frac{b\,1\mathrm{i}}{4\,d^2}}\,\sqrt{a+b\tan(c+d\,x)}}{\frac{16\,a\,b^{12}}{d}+\frac{a^2\,b^{11}\,48\mathrm{i}}{d}+\frac{16\,a^3\,b^{10}}{d}+\frac{a^4\,b^9\,48\mathrm{i}}{d}}+\frac{a^2\,b^{10}\sqrt{\frac{a}{4\,d^2}+\frac{b\,1\mathrm{i}}{4\,d^2}}\,\sqrt{a+b\tan(c+d\,x)}\,128\mathrm{i}}{\frac{16\,a\,b^{12}}{d}+\frac{a^2\,b^{11}\,48\mathrm{i}}{d}+\frac{16\,a^3\,b^{10}}{d}+\frac{a^4\,b^9\,48\mathrm{i}}{d}}-\frac{96\,a^3\,b^9\sqrt{\frac{a}{4\,d^2}+\frac{b\,1\mathrm{i}}{4\,d^2}}\,\sqrt{a+b\tan(c+d\,x)}}{\frac{16\,a\,b^{12}}{d}+\frac{a^2\,b^{11}\,48\mathrm{i}}{d}+\frac{16\,a^3\,b^{10}}{d}+\frac{a^4\,b^9\,48\mathrm{i}}{d}}\right)\sqrt{\frac{a+b\,1\mathrm{i}}{4\,d^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + b*tan(c + d*x))^(1/2),x)
```

```
[Out] - atan((a^2*b^10*(a/(4*d^2) - (b*1i)/(4*d^2))^(1/2)*(a + b*tan(c + d*x))^(1
/2)*128i)/((16*a*b^12)/d - (a^2*b^11*48i)/d + (16*a^3*b^10)/d - (a^4*b^9*48
i)/d) - (32*a*b^11*(a/(4*d^2) - (b*1i)/(4*d^2))^(1/2)*(a + b*tan(c + d*x))^
(1/2))/((16*a*b^12)/d - (a^2*b^11*48i)/d + (16*a^3*b^10)/d - (a^4*b^9*48i)/
d) + (96*a^3*b^9*(a/(4*d^2) - (b*1i)/(4*d^2))^(1/2)*(a + b*tan(c + d*x))^(1
/2))/((16*a*b^12)/d - (a^2*b^11*48i)/d + (16*a^3*b^10)/d - (a^4*b^9*48i)/d)
)*((a - b*1i)/(4*d^2))^(1/2)*2i - atan((32*a*b^11*(a/(4*d^2) + (b*1i)/(4*d^
2))^(1/2)*(a + b*tan(c + d*x))^(1/2))/((16*a*b^12)/d + (a^2*b^11*48i)/d + (
16*a^3*b^10)/d + (a^4*b^9*48i)/d) + (a^2*b^10*(a/(4*d^2) + (b*1i)/(4*d^2))^
(1/2)*(a + b*tan(c + d*x))^(1/2)*128i)/((16*a*b^12)/d + (a^2*b^11*48i)/d +
(16*a^3*b^10)/d + (a^4*b^9*48i)/d) - (96*a^3*b^9*(a/(4*d^2) + (b*1i)/(4*d^2
))^(1/2)*(a + b*tan(c + d*x))^(1/2))/((16*a*b^12)/d + (a^2*b^11*48i)/d + (1
6*a^3*b^10)/d + (a^4*b^9*48i)/d))*((a + b*1i)/(4*d^2))^(1/2)*2i - (2*a^(1/2
)*atanh((64*a^(1/2)*b^12*(a + b*tan(c + d*x))^(1/2))/(64*a*b^12 + 640*a^3*b
^10 + 576*a^5*b^8) + (640*a^(5/2)*b^10*(a + b*tan(c + d*x))^(1/2))/(64*a*b^
12 + 640*a^3*b^10 + 576*a^5*b^8) + (576*a^(9/2)*b^8*(a + b*tan(c + d*x))^(1
/2))/(64*a*b^12 + 640*a^3*b^10 + 576*a^5*b^8)))/d
```

3.509 $\int \cot^2(c+dx)\sqrt{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=415

$$-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}\,d}+\frac{b\tanh^{-1}\left(\right.}{}$$

```
[Out] -b*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)-1/2*b*arctanh(((a+(a^2
+b^2)^(1/2))^(1/2)-2^(1/2)*(a+b*tan(d*x+c))^(1/2))/(a-(a^2+b^2)^(1/2))^(1/2
))/d*2^(1/2)/(a-(a^2+b^2)^(1/2))^(1/2)+1/2*b*arctanh(((a+(a^2+b^2)^(1/2))^(
1/2)+2^(1/2)*(a+b*tan(d*x+c))^(1/2))/(a-(a^2+b^2)^(1/2))^(1/2))/d*2^(1/2)/(
a-(a^2+b^2)^(1/2))^(1/2)-1/4*b*ln(a+(a^2+b^2)^(1/2)-2^(1/2)*(a+(a^2+b^2)^(1
/2))^(1/2)*(a+b*tan(d*x+c))^(1/2)+b*tan(d*x+c))/d*2^(1/2)/(a+(a^2+b^2)^(1/2
))^(1/2)+1/4*b*ln(a+(a^2+b^2)^(1/2)+2^(1/2)*(a+(a^2+b^2)^(1/2))^(1/2)*(a+b*
tan(d*x+c))^(1/2)+b*tan(d*x+c))/d*2^(1/2)/(a+(a^2+b^2)^(1/2))^(1/2)-cot(d*x
+c)*(a+b*tan(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.41, antiderivative size = 415, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$,
Rules used = {3649, 3734, 3566, 714, 1143, 648, 632, 212, 642, 3715, 65, 214}

$$-\frac{b\log\left(-\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\sqrt{a+b\tan(c+dx)}+\sqrt{a^2+b^2}+a+b\tan(c+dx)\right)}{2\sqrt{2}d\sqrt{\sqrt{a^2+b^2}+a}}+\frac{b\log\left(\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\sqrt{a+b\tan(c+dx)}+\sqrt{a^2+b^2}+a+b\tan(c+dx)\right)}{2\sqrt{2}d\sqrt{\sqrt{a^2+b^2}+a}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+b^2}+a}-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}d\sqrt{a-\sqrt{a^2+b^2}}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+b^2}+a}+\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}d\sqrt{a-\sqrt{a^2+b^2}}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}-\frac{\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] -((b*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d)) - (b*ArcTanh[(
Sqrt[a + Sqrt[a^2 + b^2]] - Sqrt[2]*Sqrt[a + b*Tan[c + d*x]])/Sqrt[a - Sqrt
[a^2 + b^2]]])/(Sqrt[2]*Sqrt[a - Sqrt[a^2 + b^2]]*d) + (b*ArcTanh[(Sqrt[a +
 Sqrt[a^2 + b^2]] + Sqrt[2]*Sqrt[a + b*Tan[c + d*x]])/Sqrt[a - Sqrt[a^2 + b
^2]]])/(Sqrt[2]*Sqrt[a - Sqrt[a^2 + b^2]]*d) - (b*Log[a + Sqrt[a^2 + b^2] +
 b*Tan[c + d*x] - Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*Sqrt[a + b*Tan[c + d*x]
]])/(2*Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*d) + (b*Log[a + Sqrt[a^2 + b^2] +
b*Tan[c + d*x] + Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*Sqrt[a + b*Tan[c + d*x]]
])/(2*Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*d) - (Cot[c + d*x]*Sqrt[a + b*Tan[c
 + d*x]])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 714

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, S
ubst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x]
 /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1143

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q =
 Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m - 1)/(q
 - r*x + x^2), x], x] - Dist[1/(2*c*r), Int[x^(m - 1)/(q + r*x + x^2), x],
x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 1] && LtQ[m, 3
] && NegQ[b^2 - 4*a*c]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \cot^2(c+dx)\sqrt{a+b\tan(c+dx)}\,dx = -\frac{\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d} - \int \frac{\cot(c+dx)\left(-\frac{b}{2}+a\tan(\right.}{\sqrt{a+b\,\mathrm{t}}}$$

$$= -\frac{\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d} + \frac{1}{2}b\int \frac{\cot(c+dx)\left(1+\tan^2(\right.}{\sqrt{a+b\tan(c+}}$$

$$= -\frac{\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d} + \frac{b\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x,\right.}{2d}$$

$$= -\frac{\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d} + \frac{\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a-}\right.}{d}$$

$$= -\frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d} - \frac{\cot(c+dx)\sqrt{a+b\tan(c+}}{d}$$

$$= -\frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d} - \frac{\cot(c+dx)\sqrt{a+b\tan(c+}}{d}$$

$$= -\frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d} - \frac{b\log\left(a+\sqrt{a^2+b^2}+b\,\mathrm{t}\right.}{}$$

$$= -\frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}}{\sqrt{}}\right.}{\sqrt{2}\sqrt{a-}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.77, size = 139, normalized size = 0.33

$$-\frac{\frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}} - i\sqrt{a-ib}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right) + i\sqrt{a+ib}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right) + \cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] -(((b*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/Sqrt[a] - I*Sqrt[a - I*b]*
ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] + I*Sqrt[a + I*b]*ArcTanh[S
qrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] + Cot[c + d*x]*Sqrt[a + b*Tan[c + d*
x]])/d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.86, size = 26754, normalized size = 64.47

method	result	size
default	Expression too large to display	26754

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(d*x + c) + a)*cot(d*x + c)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1732 vs. 2(338) = 676.
time = 2.76, size = 3539, normalized size = 8.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(2)*(a*d^5*cos(d*x + c)^2 - a*d^5)*sqrt((a*d^2*sqrt((a^2 + b^2)
/d^4) + a^2 + b^2)/b^2)*sqrt(b^2/d^4)*((a^2 + b^2)/d^4)^(3/4)*arctan(-(sqrt
(2)*b*d^5*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt((a*d^2*
sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*sqrt(b^2/d^4)*((a^2 + b^2)/d^4)^(3/
4) - sqrt(2)*d^5*sqrt((a*d^2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*sqrt((
sqrt(2)*b^3*d^3*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt((
a*d^2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*((a^2 + b^2)/d^4)^(3/4)*cos(d
*x + c) + (a^2*b^2 + b^4)*d^2*sqrt((a^2 + b^2)/d^4)*cos(d*x + c) + (a^3*b^2
 + a*b^4)*cos(d*x + c) + (a^2*b^3 + b^5)*sin(d*x + c))/((a^2 + b^2)*cos(d*x
 + c)))*sqrt(b^2/d^4)*((a^2 + b^2)/d^4)^(3/4) + (a^2 + b^2)*d^4*sqrt(b^2/d^
4)*sqrt((a^2 + b^2)/d^4) + (a^3 + a*b^2)*d^2*sqrt(b^2/d^4))/(a^2*b^2 + b^4)
```

```
) + 4*sqrt(2)*(a*d^5*cos(d*x + c)^2 - a*d^5)*sqrt((a*d^2*sqrt((a^2 + b^2)/d
^4) + a^2 + b^2)/b^2)*sqrt(b^2/d^4)*((a^2 + b^2)/d^4)^(3/4)*arctan(-(sqrt(2
)*b*d^5*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt((a*d^2*sq
rt((a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*sqrt(b^2/d^4)*((a^2 + b^2)/d^4)^(3/4)
 - sqrt(2)*d^5*sqrt((a*d^2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*sqrt(-(s
qrt(2)*b^3*d^3*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt((a
*d^2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*((a^2 + b^2)/d^4)^(3/4)*cos(d*
x + c) - (a^2*b^2 + b^4)*d^2*sqrt((a^2 + b^2)/d^4)*cos(d*x + c) - (a^3*b^2
+ a*b^4)*cos(d*x + c) - (a^2*b^3 + b^5)*sin(d*x + c))/((a^2 + b^2)*cos(d*x
+ c)))*sqrt(b^2/d^4)*((a^2 + b^2)/d^4)^(3/4) - (a^2 + b^2)*d^4*sqrt(b^2/d^4
)*sqrt((a^2 + b^2)/d^4) - (a^3 + a*b^2)*d^2*sqrt(b^2/d^4))/(a^2*b^2 + b^4))
 + 4*(a^3 + a*b^2)*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*cos
(d*x + c)*sin(d*x + c) + sqrt(2)*((a^3 + a*b^2)*d*cos(d*x + c)^2 - (a^3 + a
*b^2)*d - (a^2*d^3*cos(d*x + c)^2 - a^2*d^3)*sqrt((a^2 + b^2)/d^4))*sqrt((a
*d^2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*((a^2 + b^2)/d^4)^(1/4)*log((s
qrt(2)*b^3*d^3*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt((a
*d^2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*((a^2 + b^2)/d^4)^(3/4)*cos(d*
x + c) + (a^2*b^2 + b^4)*d^2*sqrt((a^2 + b^2)/d^4)*cos(d*x + c) + (a^3*b^2
+ a*b^4)*cos(d*x + c) + (a^2*b^3 + b^5)*sin(d*x + c))/((a^2 + b^2)*cos(d*x
+ c))) - sqrt(2)*((a^3 + a*b^2)*d*cos(d*x + c)^2 - (a^3 + a*b^2)*d - (a^2*d
^3*cos(d*x + c)^2 - a^2*d^3)*sqrt((a^2 + b^2)/d^4))*sqrt((a*d^2*sqrt((a^2 +
 b^2)/d^4) + a^2 + b^2)/b^2)*((a^2 + b^2)/d^4)^(1/4)*log(-(sqrt(2)*b^3*d^3*
sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt((a*d^2*sqrt((a^2
+ b^2)/d^4) + a^2 + b^2)/b^2)*((a^2 + b^2)/d^4)^(3/4)*cos(d*x + c) - (a^2*b
^2 + b^4)*d^2*sqrt((a^2 + b^2)/d^4)*cos(d*x + c) - (a^3*b^2 + a*b^4)*cos(d*
x + c) - (a^2*b^3 + b^5)*sin(d*x + c))/((a^2 + b^2)*cos(d*x + c))) - (a^2*b
 + b^3 - (a^2*b + b^3)*cos(d*x + c)^2)*sqrt(a)*log(-(8*a*b*cos(d*x + c)*sin
(d*x + c) + (8*a^2 - b^2)*cos(d*x + c)^2 + b^2 - 4*(2*a*cos(d*x + c)^2 + b*
cos(d*x + c)*sin(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/c
os(d*x + c)))/(cos(d*x + c)^2 - 1)))/((a^3 + a*b^2)*d*cos(d*x + c)^2 - (a^3
 + a*b^2)*d), 1/4*(4*sqrt(2)*(a*d^5*cos(d*x + c)^2 - a*d^5)*sqrt((a*d^2*sqr
t((a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*sqrt(b^2/d^4)*((a^2 + b^2)/d^4)^(3/4)*
arctan(-(sqrt(2)*b*d^5*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))
*sqrt((a*d^2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*sqrt(b^2/d^4)*((a^2 +
b^2)/d^4)^(3/4) - sqrt(2)*d^5*sqrt((a*d^2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2
)/b^2)*sqrt((sqrt(2)*b^3*d^3*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x
 + c))*sqrt((a*d^2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*((a^2 + b^2)/d^4
)^(3/4)*cos(d*x + c) + (a^2*b^2 + b^4)*d^2*sqrt((a^2 + b^2)/d^4)*cos(d*x +
c) + (a^3*b^2 + a*b^4)*cos(d*x + c) + (a^2*b^3 + b^5)*sin(d*x + c))/((a^2 +
 b^2)*cos(d*x + c)))*sqrt(b^2/d^4)*((a^2 + b^2)/d^4)^(3/4) + (a^2 + b^2)*d^
4*sqrt(b^2/d^4)*sqrt((a^2 + b^2)/d^4) + (a^3 + a*b^2)*d^2*sqrt(b^2/d^4))/(a
^2*b^2 + b^4)) + 4*sqrt(2)*(a*d^5*cos(d*x + c)^2 - a*d^5)*sqrt((a*d^2*sqrt(
(a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*sqrt(b^2/d^4)*((a^2 + b^2)/d^4)^(3/4)*ar
ctan(-(sqrt(2)*b*d^5*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*s
qrt((a*d^2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*sqrt(b^2/d^4)*((a^2 + b^
```

```
2)/d^4)^(3/4) - sqrt(2)*d^5*sqrt((a*d^2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/
b^2)*sqrt(-(sqrt(2)*b^3*d^3*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x
+ c))*sqrt((a*d^2*sqrt((a^2 + b^2)/d^4) + a^2 + b^2)/b^2)*((a^2 + b^2)/d^4)
^(3/4)*cos(d*x + c) - (a^2*b^2 + b^4)*d^2*sqrt((a^2 + b^2)/d^4)*cos(d*x + c
) - (a^3*b^2 + a*b^4)*cos(d*x + c) - (a^2*b^3 + b^5)*sin(d*x + c))/((a^2 +
b^2)*cos(d*x + c)))*sqrt(b^2/d^4)*((a^2 + b^2)/d^4)^(3/4) - (a^2 + b^2)*d^4
*sqrt(b^2/d^4)*sqrt((a^2 + b^2)/d^4) - (a^3 + a*b^2)*d^2*sqrt(b^2/d^4))/(a^
2*b^2 + b^4)) + 4*(a^3 + a*b^2)*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(
d*x + c))*cos(d*x + c)*sin(d*x + c) + sqrt(2)*((a^3 + a*b^2)*d*cos(d*x + c)
^2 - (a^3 + a*b^2)*d - (a^2*d^3*cos(d*x + c)^2 - a^2*d^3)*sqrt((a^2 + b^2)/
d^4))*sqrt((a*d^2*sqrt((a^2 + b^2)/d^4) + a^2 +...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan\left(c+dx\right)}\,\cot^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(c + d*x))*cot(c + d*x)**2, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 4.04, size = 832, normalized size = 2.00

$$\mathrm{atan}\left(-\frac{b^{12}\sqrt{-\frac{a}{4d^2}-\frac{b\,1\mathrm{i}}{4d^2}}\,\sqrt{a+b\tan\left(c+dx\right)}\,32\mathrm{i}}{\frac{16b^{13}}{d}+\frac{32a^2b^{11}}{d}+\frac{16a^4b^9}{d}+\frac{ab^{12}16\mathrm{i}}{d}+\frac{a^3b^{10}16\mathrm{i}}{d}}+\frac{64ab^{11}\sqrt{-\frac{a}{4d^2}-\frac{b\,1\mathrm{i}}{4d^2}}\,\sqrt{a+b\tan\left(c+dx\right)}}{\frac{16b^{13}}{d}+\frac{32a^2b^{11}}{d}+\frac{16a^4b^9}{d}+\frac{ab^{12}16\mathrm{i}}{d}+\frac{a^3b^{10}16\mathrm{i}}{d}}+\frac{32a^3b^9\sqrt{-\frac{a}{4d^2}-\frac{b\,1\mathrm{i}}{4d^2}}\,\sqrt{a+b\tan\left(c+dx\right)}}{\frac{16b^{13}}{d}+\frac{32a^2b^{11}}{d}+\frac{16a^4b^9}{d}+\frac{ab^{12}16\mathrm{i}}{d}+\frac{a^3b^{10}16\mathrm{i}}{d}}\right)\sqrt{-\frac{a+b\,1\mathrm{i}}{4d^2}}\,2\mathrm{i}-\mathrm{atan}\left(\frac{b^{12}\sqrt{-\frac{a}{4d^2}+\frac{b\,1\mathrm{i}}{4d^2}}\,\sqrt{a+b\tan\left(c+dx\right)}\,32\mathrm{i}}{\frac{16b^{13}}{d}+\frac{32a^2b^{11}}{d}+\frac{16a^4b^9}{d}-\frac{ab^{12}16\mathrm{i}}{d}-\frac{a^3b^{10}16\mathrm{i}}{d}}+\frac{64ab^{11}\sqrt{-\frac{a}{4d^2}+\frac{b\,1\mathrm{i}}{4d^2}}\,\sqrt{a+b\tan\left(c+dx\right)}}{\frac{16b^{13}}{d}+\frac{32a^2b^{11}}{d}+\frac{16a^4b^9}{d}-\frac{ab^{12}16\mathrm{i}}{d}-\frac{a^3b^{10}16\mathrm{i}}{d}}+\frac{32a^3b^9\sqrt{-\frac{a}{4d^2}+\frac{b\,1\mathrm{i}}{4d^2}}\,\sqrt{a+b\tan\left(c+dx\right)}}{\frac{16b^{13}}{d}+\frac{32a^2b^{11}}{d}+\frac{16a^4b^9}{d}-\frac{ab^{12}16\mathrm{i}}{d}-\frac{a^3b^{10}16\mathrm{i}}{d}}\right)\sqrt{-\frac{a-b\,1\mathrm{i}}{4d^2}}\,2\mathrm{i}+\frac{b\sqrt{a+b\tan\left(c+dx\right)}}{a\,d-d\left(a+b\tan\left(c+dx\right)\right)}+\frac{b\,\mathrm{atan}\left(\frac{b^{13}\sqrt{a+b\tan\left(c+dx\right)}\,128\mathrm{i}}{\sqrt{a}\left(128b^{13}+128a^2b^{11}+32a^4b^9+\frac{32b^{15}}{a^2}\right)}+\frac{a^{3/2}b^{11}\sqrt{a+b\tan\left(c+dx\right)}\,128\mathrm{i}}{128b^{13}+128a^2b^{11}+32a^4b^9+\frac{32b^{15}}{a^2}}+\frac{a^{7/2}b^9\sqrt{a+b\tan\left(c+dx\right)}\,32\mathrm{i}}{128b^{13}+128a^2b^{11}+32a^4b^9+\frac{32b^{15}}{a^2}}+\frac{b^{15}\sqrt{a+b\tan\left(c+dx\right)}\,32\mathrm{i}}{a^{5/2}\left(128b^{13}+128a^2b^{11}+32a^4b^9+\frac{32b^{15}}{a^2}\right)}\right)1\mathrm{i}}{\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + b*tan(c + d*x))^(1/2),x)
```

```
[Out] atan((64*a*b^11*(- a/(4*d^2) - (b*1i)/(4*d^2))^(1/2)*(a + b*tan(c + d*x))^(
1/2))/((16*b^13)/d + (a*b^12*16i)/d + (32*a^2*b^11)/d + (a^3*b^10*16i)/d +
(16*a^4*b^9)/d) - (b^12*(- a/(4*d^2) - (b*1i)/(4*d^2))^(1/2)*(a + b*tan(c +
 d*x))^(1/2)*32i)/((16*b^13)/d + (a*b^12*16i)/d + (32*a^2*b^11)/d + (a^3*b^
10*16i)/d + (16*a^4*b^9)/d) + (32*a^3*b^9*(- a/(4*d^2) - (b*1i)/(4*d^2))^(1
```

```
/2)*(a + b*tan(c + d*x))^(1/2))/((16*b^13)/d + (a*b^12*16i)/d + (32*a^2*b^1
1)/d + (a^3*b^10*16i)/d + (16*a^4*b^9)/d))*(-(a + b*1i)/(4*d^2))^(1/2)*2i -
 atan((b^12*((b*1i)/(4*d^2) - a/(4*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2)*3
2i)/((16*b^13)/d - (a*b^12*16i)/d + (32*a^2*b^11)/d - (a^3*b^10*16i)/d + (1
6*a^4*b^9)/d) + (64*a*b^11*((b*1i)/(4*d^2) - a/(4*d^2))^(1/2)*(a + b*tan(c
+ d*x))^(1/2))/((16*b^13)/d - (a*b^12*16i)/d + (32*a^2*b^11)/d - (a^3*b^10*
16i)/d + (16*a^4*b^9)/d) + (32*a^3*b^9*((b*1i)/(4*d^2) - a/(4*d^2))^(1/2)*(
a + b*tan(c + d*x))^(1/2))/((16*b^13)/d - (a*b^12*16i)/d + (32*a^2*b^11)/d
- (a^3*b^10*16i)/d + (16*a^4*b^9)/d))*(-(a - b*1i)/(4*d^2))^(1/2)*2i + (b*(
a + b*tan(c + d*x))^(1/2))/(a*d - d*(a + b*tan(c + d*x))) + (b*atan((b^13*(
a + b*tan(c + d*x))^(1/2)*128i)/(a^(1/2)*(128*b^13 + 128*a^2*b^11 + 32*a^4*
b^9 + (32*b^15)/a^2)) + (a^(3/2)*b^11*(a + b*tan(c + d*x))^(1/2)*128i)/(128
*b^13 + 128*a^2*b^11 + 32*a^4*b^9 + (32*b^15)/a^2) + (a^(7/2)*b^9*(a + b*ta
n(c + d*x))^(1/2)*32i)/(128*b^13 + 128*a^2*b^11 + 32*a^4*b^9 + (32*b^15)/a^
2) + (b^15*(a + b*tan(c + d*x))^(1/2)*32i)/(a^(5/2)*(128*b^13 + 128*a^2*b^1
1 + 32*a^4*b^9 + (32*b^15)/a^2)))*1i)/(a^(1/2)*d)
```

3.510 $\int \cot^3(c+dx)\sqrt{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=189

$$\frac{(8a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{3/2}d}-\frac{\sqrt{a-ib}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}-\frac{\sqrt{a+ib}\tanh^{-1}\left(\right.}{}$$

```
[Out] 1/4*(8*a^2+b^2)*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-arctanh((
a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))*(a-I*b)^(1/2)/d-arctanh((a+b*tan(d*x+c
))^(1/2)/(a+I*b)^(1/2))*(a+I*b)^(1/2)/d-1/4*b*cot(d*x+c)*(a+b*tan(d*x+c))^(
1/2)/a/d-1/2*cot(d*x+c)^2*(a+b*tan(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.42, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3649, 3730, 3734, 3620, 3618, 65, 214, 3715}

$$\frac{(8a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{3/2}d}-\frac{\sqrt{a-ib}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}-\frac{\sqrt{a+ib}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}-\frac{\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}-\frac{b\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] ((8*a^2 + b^2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/(4*a^(3/2)*d) - (
Sqrt[a - I*b]*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/d - (Sqrt[a
+ I*b]*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/d - (b*Cot[c + d*x]
*Sqrt[a + b*Tan[c + d*x]])/(4*a*d) - (Cot[c + d*x]^2*Sqrt[a + b*Tan[c + d*x
]])/(2*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
```

```
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
```

```
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \cot^3(c+dx)\sqrt{a+b\tan(c+dx)}\,dx = -\frac{\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d} - \frac{1}{2}\int \frac{\cot^2(c+dx)\left(-\frac{b}{2}+2a\,t\right.}{\sqrt{a+}}$$

$$= -\frac{b\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{4ad} - \frac{\cot^2(c+dx)\sqrt{a+b\tan(c+}}{2d}$$

$$= -\frac{b\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{4ad} - \frac{\cot^2(c+dx)\sqrt{a+b\tan(c+}}{2d}$$

$$= -\frac{b\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{4ad} - \frac{\cot^2(c+dx)\sqrt{a+b\tan(c+}}{2d}$$

$$= -\frac{b\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{4ad} - \frac{\cot^2(c+dx)\sqrt{a+b\tan(c+}}{2d}$$

$$= \frac{(8a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{3/2}d} - \frac{b\cot(c+dx)\sqrt{a+b}}{4ad}$$

$$= \frac{(8a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{3/2}d} - \frac{\sqrt{a-ib}\,\tanh^{-1}\left(\frac{\sqrt{}}{}\right.}{d}$$

Mathematica [A]
time = 1.18, size = 166, normalized size = 0.88

$$\frac{(8a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)-\sqrt{a}\left(4a\sqrt{a-ib}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)+4a\sqrt{a+ib}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)+\cot(c+dx)(b+2a\cot(c+dx))\sqrt{a+b\tan(c+dx)}\right)}{4a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] ((8*a^2 + b^2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]] - Sqrt[a]*(4*a*Sqr
t[a - I*b]*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] + 4*a*Sqrt[a + I
```

```
*b]*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] + Cot[c + d*x]*(b + 2*a
*Cot[c + d*x])*Sqrt[a + b*Tan[c + d*x]]))/(4*a^(3/2)*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 1.14, size = 45074, normalized size = 238.49

method	result	size
default	Expression too large to display	45074

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*tan(d*x + c) + a)*cot(d*x + c)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2026 vs. 2(153) = 306.
time = 2.06, size = 4130, normalized size = 21.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
[Out] [1/16*(16*sqrt(2)*(a^2*d^5*cos(d*x + c)^2 - a^2*d^5)*sqrt(-(a*d^2*sqrt((a^2
 + b^2)/d^4) - a^2 - b^2)/b^2)*sqrt(b^2/d^4)*((a^2 + b^2)/d^4)^(3/4)*arctan
(-((a^2 + b^2)*d^4*sqrt(b^2/d^4)*sqrt((a^2 + b^2)/d^4) + (a^3 + a*b^2)*d^2*
sqrt(b^2/d^4) + sqrt(2)*(d^7*sqrt(b^2/d^4)*sqrt((a^2 + b^2)/d^4) + a*d^5*sq
rt(b^2/d^4))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-(a*
d^2*sqrt((a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*((a^2 + b^2)/d^4)^(3/4) - sqrt(
2)*(d^7*sqrt(b^2/d^4)*sqrt((a^2 + b^2)/d^4) + a*d^5*sqrt(b^2/d^4))*sqrt(-(a
*d^2*sqrt((a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*sqrt(((a^2 + b^2)*d^2*sqrt((a^
2 + b^2)/d^4)*cos(d*x + c) + sqrt(2)*(a*d^3*sqrt((a^2 + b^2)/d^4)*cos(d*x +
 c) + (a^2 + b^2)*d*cos(d*x + c))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/co
s(d*x + c))*sqrt(-(a*d^2*sqrt((a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*((a^2 + b^
2)/d^4)^(1/4) + (a^3 + a*b^2)*cos(d*x + c) + (a^2*b + b^3)*sin(d*x + c))/((
a^2 + b^2)*cos(d*x + c)))*((a^2 + b^2)/d^4)^(3/4))/(a^2*b^2 + b^4)) + 16*sq
```

```
rt(2)*(a^2*d^5*cos(d*x + c)^2 - a^2*d^5)*sqrt(-(a*d^2*sqrt((a^2 + b^2)/d^4)
 - a^2 - b^2)/b^2)*sqrt(b^2/d^4)*((a^2 + b^2)/d^4)^(3/4)*arctan(((a^2 + b^2
)*d^4*sqrt(b^2/d^4)*sqrt((a^2 + b^2)/d^4) + (a^3 + a*b^2)*d^2*sqrt(b^2/d^4)
 - sqrt(2)*(d^7*sqrt(b^2/d^4)*sqrt((a^2 + b^2)/d^4) + a*d^5*sqrt(b^2/d^4))*
sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-(a*d^2*sqrt((a^2
 + b^2)/d^4) - a^2 - b^2)/b^2)*((a^2 + b^2)/d^4)^(3/4) + sqrt(2)*(d^7*sqrt(
b^2/d^4)*sqrt((a^2 + b^2)/d^4) + a*d^5*sqrt(b^2/d^4))*sqrt(-(a*d^2*sqrt((a^
2 + b^2)/d^4) - a^2 - b^2)/b^2)*sqrt(((a^2 + b^2)*d^2*sqrt((a^2 + b^2)/d^4)
*cos(d*x + c) - sqrt(2)*(a*d^3*sqrt((a^2 + b^2)/d^4)*cos(d*x + c) + (a^2 +
b^2)*d*cos(d*x + c))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*s
qrt(-(a*d^2*sqrt((a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*((a^2 + b^2)/d^4)^(1/4)
 + (a^3 + a*b^2)*cos(d*x + c) + (a^2*b + b^3)*sin(d*x + c))/((a^2 + b^2)*co
s(d*x + c)))*((a^2 + b^2)/d^4)^(3/4))/(a^2*b^2 + b^4)) - 4*sqrt(2)*((a^4 +
a^2*b^2)*d*cos(d*x + c)^2 - (a^4 + a^2*b^2)*d + (a^3*d^3*cos(d*x + c)^2 - a
^3*d^3)*sqrt((a^2 + b^2)/d^4))*sqrt(-(a*d^2*sqrt((a^2 + b^2)/d^4) - a^2 - b
^2)/b^2)*((a^2 + b^2)/d^4)^(1/4)*log(((a^2 + b^2)*d^2*sqrt((a^2 + b^2)/d^4)
*cos(d*x + c) + sqrt(2)*(a*d^3*sqrt((a^2 + b^2)/d^4)*cos(d*x + c) + (a^2 +
b^2)*d*cos(d*x + c))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*s
qrt(-(a*d^2*sqrt((a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*((a^2 + b^2)/d^4)^(1/4)
 + (a^3 + a*b^2)*cos(d*x + c) + (a^2*b + b^3)*sin(d*x + c))/((a^2 + b^2)*co
s(d*x + c))) + 4*sqrt(2)*((a^4 + a^2*b^2)*d*cos(d*x + c)^2 - (a^4 + a^2*b^2
)*d + (a^3*d^3*cos(d*x + c)^2 - a^3*d^3)*sqrt((a^2 + b^2)/d^4))*sqrt(-(a*d^
2*sqrt((a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*((a^2 + b^2)/d^4)^(1/4)*log(((a^2
 + b^2)*d^2*sqrt((a^2 + b^2)/d^4)*cos(d*x + c) - sqrt(2)*(a*d^3*sqrt((a^2 +
 b^2)/d^4)*cos(d*x + c) + (a^2 + b^2)*d*cos(d*x + c))*sqrt((a*cos(d*x + c)
+ b*sin(d*x + c))/cos(d*x + c))*sqrt(-(a*d^2*sqrt((a^2 + b^2)/d^4) - a^2 -
b^2)/b^2)*((a^2 + b^2)/d^4)^(1/4) + (a^3 + a*b^2)*cos(d*x + c) + (a^2*b + b
^3)*sin(d*x + c))/((a^2 + b^2)*cos(d*x + c))) - (8*a^4 + 9*a^2*b^2 + b^4 -
(8*a^4 + 9*a^2*b^2 + b^4)*cos(d*x + c)^2)*sqrt(a)*log(-(8*a*b*cos(d*x + c)*
sin(d*x + c) + (8*a^2 - b^2)*cos(d*x + c)^2 + b^2 + 4*(2*a*cos(d*x + c)^2 +
 b*cos(d*x + c)*sin(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b*sin(d*x + c)
)/cos(d*x + c)))/(cos(d*x + c)^2 - 1)) + 4*(2*(a^4 + a^2*b^2)*cos(d*x + c)^
2 + (a^3*b + a*b^3)*cos(d*x + c)*sin(d*x + c))*sqrt((a*cos(d*x + c) + b*sin
(d*x + c))/cos(d*x + c)))/((a^4 + a^2*b^2)*d*cos(d*x + c)^2 - (a^4 + a^2*b^
2)*d), 1/4*(4*sqrt(2)*(a^2*d^5*cos(d*x + c)^2 - a^2*d^5)*sqrt(-(a*d^2*sqrt(
(a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*sqrt(b^2/d^4)*((a^2 + b^2)/d^4)^(3/4)*ar
ctan(-((a^2 + b^2)*d^4*sqrt(b^2/d^4)*sqrt((a^2 + b^2)/d^4) + (a^3 + a*b^2)*
d^2*sqrt(b^2/d^4) + sqrt(2)*(d^7*sqrt(b^2/d^4)*sqrt((a^2 + b^2)/d^4) + a*d^
5*sqrt(b^2/d^4))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(
-(a*d^2*sqrt((a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*((a^2 + b^2)/d^4)^(3/4) - s
qrt(2)*(d^7*sqrt(b^2/d^4)*sqrt((a^2 + b^2)/d^4) + a*d^5*sqrt(b^2/d^4))*sqrt
(-(a*d^2*sqrt((a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*sqrt(((a^2 + b^2)*d^2*sqrt
((a^2 + b^2)/d^4)*cos(d*x + c) + sqrt(2)*(a*d^3*sqrt((a^2 + b^2)/d^4)*cos(d
*x + c) + (a^2 + b^2)*d*cos(d*x + c))*sqrt((a*cos(d*x + c) + b*sin(d*x + c)
)/cos(d*x + c))*sqrt(-(a*d^2*sqrt((a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*((a^2
```

```
+ b^2)/d^4)^(1/4) + (a^3 + a*b^2)*cos(d*x + c) + (a^2*b + b^3)*sin(d*x + c)
)/((a^2 + b^2)*cos(d*x + c)))*((a^2 + b^2)/d^4)^(3/4))/(a^2*b^2 + b^4)) + 4
*sqrt(2)*(a^2*d^5*cos(d*x + c)^2 - a^2*d^5)*sqrt(-(a*d^2*sqrt((a^2 + b^2)/d
^4) - a^2 - b^2)/b^2)*sqrt(b^2/d^4)*((a^2 + b^2)/d^4)^(3/4)*arctan(((a^2 +
b^2)*d^4*sqrt(b^2/d^4)*sqrt((a^2 + b^2)/d^4) + (a^3 + a*b^2)*d^2*sqrt(b^2/d
^4) - sqrt(2)*(d^7*sqrt(b^2/d^4)*sqrt((a^2 + b^2)/d^4) + a*d^5*sqrt(b^2/d^4
))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-(a*d^2*sqrt((
a^2 + b^2)/d^4) - a^2 - b^2)/b^2)*((a^2 + b^2)/d^4)^(3/4) + sqrt(2)*(d^7*sq
rt(b^2/d^4)*sqrt((a^2 + b^2)/d^4) + a*d^5*sqrt(...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan{(c+dx)}}\,\cot^3{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(c + d*x))*cot(c + d*x)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 4.83, size = 1910, normalized size = 10.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + b*tan(c + d*x))^(1/2),x)
```

```
[Out] atan((b^14*(a/(4*d^2) + (b*1i)/(4*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2)*2i
)/((b^15*1i)/d + (a^2*b^13*17i)/d + (16*a^3*b^12)/d + (a^4*b^11*64i)/d + (1
6*a^5*b^10)/d + (a^6*b^9*48i)/d) - (2*b^13*(a/(4*d^2) + (b*1i)/(4*d^2))^(1/
2)*(a + b*tan(c + d*x))^(1/2))/((a*b^13*17i)/d + (16*a^2*b^12)/d + (a^3*b^1
```

1*64i)/d + (16*a^4*b^10)/d + (a^5*b^9*48i)/d + (b^15*1i)/(a*d)) + (b^12*(a/(4*d^2) + (b*1i)/(4*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2)*32i)/((b^13*17i)/d + (16*a*b^12)/d + (a^2*b^11*64i)/d + (16*a^3*b^10)/d + (a^4*b^9*48i)/d + (b^15*1i)/(a^2*d)) + (a^2*b^10*(a/(4*d^2) + (b*1i)/(4*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2)*128i)/((b^13*17i)/d + (16*a*b^12)/d + (a^2*b^11*64i)/d + (16*a^3*b^10)/d + (a^4*b^9*48i)/d + (b^15*1i)/(a^2*d)) - (96*a^3*b^9*(a/(4*d^2) + (b*1i)/(4*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2))/((b^13*17i)/d + (16*a*b^12)/d + (a^2*b^11*64i)/d + (16*a^3*b^10)/d + (a^4*b^9*48i)/d + (b^15*1i)/(a^2*d)))*((a + b*1i)/(4*d^2))^(1/2)*2i - atan((b^14*(a/(4*d^2) - (b*1i)/(4*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2)*2i)/((b^15*1i)/d + (a^2*b^13*17i)/d - (16*a^3*b^12)/d + (a^4*b^11*64i)/d - (16*a^5*b^10)/d + (a^6*b^9*48i)/d) + (2*b^13*(a/(4*d^2) - (b*1i)/(4*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2))/((a*b^13*17i)/d - (16*a^2*b^12)/d + (a^3*b^11*64i)/d - (16*a^4*b^10)/d + (a^5*b^9*48i)/d + (b^15*1i)/(a*d)) + (b^12*(a/(4*d^2) - (b*1i)/(4*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2)*32i)/((b^13*17i)/d - (16*a*b^12)/d + (a^2*b^11*64i)/d - (16*a^3*b^10)/d + (a^4*b^9*48i)/d + (b^15*1i)/(a^2*d)) + (a^2*b^10*(a/(4*d^2) - (b*1i)/(4*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2)*128i)/((b^13*17i)/d - (16*a*b^12)/d + (a^2*b^11*64i)/d - (16*a^3*b^10)/d + (a^4*b^9*48i)/d + (b^15*1i)/(a^2*d)) + (96*a^3*b^9*(a/(4*d^2) - (b*1i)/(4*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2))/((b^13*17i)/d - (16*a*b^12)/d + (a^2*b^11*64i)/d - (16*a^3*b^10)/d + (a^4*b^9*48i)/d + (b^15*1i)/(a^2*d)))*((a - b*1i)/(4*d^2))^(1/2)*2i - ((b^2*(a + b*tan(c + d*x))^(1/2))/4 + (b^2*(a + b*tan(c + d*x))^(3/2))/(4*a))/(d*(a + b*tan(c + d*x))^2 + a^2*d - 2*a*d*(a + b*tan(c + d*x))) + (log(- (7*a*b^14 + 63*a^3*b^12 + 56*a^5*b^10)/(2*a^2*d^5) - ((a^2 + b^2/8)*(((a + b*tan(c + d*x))^(1/2)*(17*a^2*b^12 - b^14 + 16*a^4*b^10 + 96*a^6*b^8))/(a^2*d^4) + ((a^2 + b^2/8)*((4*b^14*d^2 + 4*a^2*b^12*d^2 - 192*a^4*b^10*d^2 - 192*a^6*b^8*d^2)/(2*a^2*d^5) - ((a^2 + b^2/8)*(((a + b*tan(c + d*x))^(1/2)*(4*a*b^12*d^2 + 256*a^3*b^10*d^2 + 576*a^5*b^8*d^2))/(a^2*d^4) - ((a^2 + b^2/8)*((128*a*b^12*d^4 + 896*a^3*b^10*d^4 + 768*a^5*b^8*d^4)/(2*a^2*d^5) + ((a^2 + b^2/8)*(512*a^2*b^10*d^4 + 768*a^4*b^8*d^4)*(a + b*tan(c + d*x))^(1/2))/(a^2*d^5*(a^3)^(1/2))))/(d*(a^3)^(1/2))))/(d*(a^3)^(1/2))))/(d*(a^3)^(1/2))))/(d*(a^3)^(1/2)))*(a^2 + b^2/8))/(d*(a^3)^(1/2)) - (log(((8*a^2 + b^2)*(((a + b*tan(c + d*x))^(1/2)*(17*a^2*b^12 - b^14 + 16*a^4*b^10 + 96*a^6*b^8))/(a^2*d^4) - ((8*a^2 + b^2)*((4*b^14*d^2 + 4*a^2*b^12*d^2 - 192*a^4*b^10*d^2 - 192*a^6*b^8*d^2)/(2*a^2*d^5) + ((8*a^2 + b^2)*(((a + b*tan(c + d*x))^(1/2)*(4*a*b^12*d^2 + 256*a^3*b^10*d^2 + 576*a^5*b^8*d^2))/(a^2*d^4) + ((8*a^2 + b^2)*((128*a*b^12*d^4 + 896*a^3*b^10*d^4 + 768*a^5*b^8*d^4)/(2*a^2*d^5) - ((8*a^2 + b^2)*(512*a^2*b^10*d^4 + 768*a^4*b^8*d^4)*(a + b*tan(c + d*x))^(1/2))/(8*a^2*d^5*(a^3)^(1/2))))/(8*d*(a^3)^(1/2))))/(8*d*(a^3)^(1/2))))/(8*d*(a^3)^(1/2))))/(8*d*(a^3)^(1/2)) - (7*a*b^14 + 63*a^3*b^12 + 56*a^5*b^10)/(2*a^2*d^5))*(8*a^2 + b^2))/(8*d*(a^3)^(1/2))

3.511 $\int \tan^4(c+dx)(a+b\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=209

$$-\frac{i(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}+\frac{i(a+ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}+\frac{2b\sqrt{a+b\tan}}{d}$$

```
[Out] -I*(a-I*b)^(3/2)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d+I*(a+I*b)^
(3/2)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d+2*b*(a+b*tan(d*x+c))^
(1/2)/d+2/315*(8*a^2-63*b^2)*(a+b*tan(d*x+c))^(5/2)/b^3/d-8/63*a*tan(d*x+c)
*(a+b*tan(d*x+c))^(5/2)/b^2/d+2/9*tan(d*x+c)^2*(a+b*tan(d*x+c))^(5/2)/b/d
```

Rubi [A]

time = 0.31, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3647, 3728, 3712, 3563, 3620, 3618, 65, 214}

$$\frac{2(8a^2-63b^2)(a+b\tan(c+dx))^{5/2}}{315b^3d}-\frac{8a\tan(c+dx)(a+b\tan(c+dx))^{5/2}}{63b^2d}+\frac{2\tan^2(c+dx)(a+b\tan(c+dx))^{5/2}}{9bd}+\frac{2b\sqrt{a+b\tan(c+dx)}}{d}-\frac{i(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}+\frac{i(a+ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4*(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-I)*(a - I*b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/d +
(I*(a + I*b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/d + (2*
b*Sqrt[a + b*Tan[c + d*x]])/d + (2*(8*a^2 - 63*b^2)*(a + b*Tan[c + d*x])^(5
/2))/(315*b^3*d) - (8*a*Tan[c + d*x]*(a + b*Tan[c + d*x])^(5/2))/(63*b^2*d)
 + (2*Tan[c + d*x]^2*(a + b*Tan[c + d*x])^(5/2))/(9*b*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3563

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
```

```
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3712

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
```

```
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \tan^4(c+dx)(a+b\tan(c+dx))^{3/2}\,dx = \frac{2\tan^2(c+dx)(a+b\tan(c+dx))^{5/2}}{9bd} + \frac{2\int \tan(c+dx)(a+b\tan$$

$$= -\frac{8a\tan(c+dx)(a+b\tan(c+dx))^{5/2}}{63b^2d} + \frac{2\tan^2(c+dx)(a+b\,\mathrm{ta}}{9bd}$$

$$= \frac{2(8a^2-63b^2)(a+b\tan(c+dx))^{5/2}}{315b^3d} - \frac{8a\tan(c+dx)(a+b\tan}{63b^2d}$$

$$= \frac{2b\sqrt{a+b\tan(c+dx)}}{d} + \frac{2(8a^2-63b^2)(a+b\tan(c+dx))^{5/2}}{315b^3d}$$

$$= \frac{2b\sqrt{a+b\tan(c+dx)}}{d} + \frac{2(8a^2-63b^2)(a+b\tan(c+dx))^{5/2}}{315b^3d}$$

$$= \frac{2b\sqrt{a+b\tan(c+dx)}}{d} + \frac{2(8a^2-63b^2)(a+b\tan(c+dx))^{5/2}}{315b^3d}$$

$$= \frac{2b\sqrt{a+b\tan(c+dx)}}{d} + \frac{2(8a^2-63b^2)(a+b\tan(c+dx))^{5/2}}{315b^3d}$$

$$= -\frac{i(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} + \frac{i(a+ib)^{3/2}\tan}{}$$

Mathematica [A]

time = 4.73, size = 338, normalized size = 1.62

$$\frac{\cos^2(c+dx)\left(-315i(a^2-b^2)\left(\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}}\right)(a+b\tan(c+dx))^2-630ab\left(\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}}\right)(a+b\tan(c+dx))^2+\frac{2(a+b\tan(c+dx))^{5/2}\left(8a^4-66a^2b^2+413b^4+35b^4\sec^4(c+dx)-4ab\left(a^2+44b^2\right)\tan(c+dx)+b^2\sec^2(c+dx)\left(3a^2-133b^2+50ab\tan(c+dx)\right)\right)}{b^3}\right)}{315d(a\cos(c+dx)+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4*(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] (Cos[c + d*x]^2*((-315*I)*(a^2 - b^2)*(ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqr
t[a - I*b]]/Sqrt[a - I*b] - ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]]
/Sqrt[a + I*b])*(a + b*Tan[c + d*x])^2 - 630*a*b*(ArcTanh[Sqrt[a + b*Tan[c
+ d*x]]/Sqrt[a - I*b]]/Sqrt[a - I*b] + ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqr
t[a + I*b]]/Sqrt[a + I*b])*(a + b*Tan[c + d*x])^2 + (2*(a + b*Tan[c + d*x])
```

```
^(5/2)*(8*a^4 - 66*a^2*b^2 + 413*b^4 + 35*b^4*Sec[c + d*x]^4 - 4*a*b*(a^2 +
 44*b^2)*Tan[c + d*x] + b^2*Sec[c + d*x]^2*(3*a^2 - 133*b^2 + 50*a*b*Tan[c
+ d*x])))/b^3))/(315*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 711 vs. $2(177) = 354$.

time = 0.17, size = 712, normalized size = 3.41

method	result
derivativedivides	$\frac{\frac{2(a+b\tan(dx+c))^{\frac{9}{2}}}{9}-\frac{4a(a+b\tan(dx+c))^{\frac{7}{2}}}{7}+\frac{2a^2(a+b\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{2b^2(a+b\tan(dx+c))^{\frac{5}{2}}}{5}+2b^4\sqrt{a+b\tan(dx+c)}-2}{}$
default	$\frac{\frac{2(a+b\tan(dx+c))^{\frac{9}{2}}}{9}-\frac{4a(a+b\tan(dx+c))^{\frac{7}{2}}}{7}+\frac{2a^2(a+b\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{2b^2(a+b\tan(dx+c))^{\frac{5}{2}}}{5}+2b^4\sqrt{a+b\tan(dx+c)}-2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4*(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/b^3*(1/9*(a+b*tan(d*x+c))^(9/2)-2/7*a*(a+b*tan(d*x+c))^(7/2)+1/5*a^2*(a
+b*tan(d*x+c))^(5/2)-1/5*b^2*(a+b*tan(d*x+c))^(5/2)+b^4*(a+b*tan(d*x+c))^(1
/2)-b^4*(1/4/b^2*(1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a-(2*(
a^2+b^2)^(1/2)+2*a)^(1/2)*a^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^2)*ln(b*tan(d
*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2
))+2*(2*(a^2+b^2)^(1/2)*b^2-1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1
/2)*a-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^2)*
(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b
*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(
1/2)))+1/4/b^2*(-1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a-(2*(a
^2+b^2)^(1/2)+2*a)^(1/2)*a^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^2)*ln(-b*tan(d
*x+c)-a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-(a^2+b^2)^(1/2
))+2*(-2*(a^2+b^2)^(1/2)*b^2+1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(
1/2)*a-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^2)
```

```
*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a
+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)
^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4*(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^(3/2)*tan(d*x + c)^4, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4429 vs. $2(171) = 342$.
time = 1.61, size = 4429, normalized size = 21.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4*(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
[Out] 1/1260*(1260*sqrt(2)*b^3*d^5*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^3
 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 -
 6*a^2*b^4 + b^6))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(3/4)*sqrt((9*
a^4*b^2 - 6*a^2*b^4 + b^6)/d^4)*arctan(((3*a^10 + 11*a^8*b^2 + 14*a^6*b^4 +
 6*a^4*b^6 - a^2*b^8 - b^10)*d^4*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d
^4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + (3*a^13 + 14*a^11*b^2 + 25*a^
9*b^4 + 20*a^7*b^6 + 5*a^5*b^8 - 2*a^3*b^10 - a*b^12)*d^2*sqrt((9*a^4*b^2 -
 6*a^2*b^4 + b^6)/d^4) + sqrt(2)*((3*a^4*b + 2*a^2*b^3 - b^5)*d^7*sqrt((a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4
) + 2*(3*a^7*b + 5*a^5*b^3 + a^3*b^5 - a*b^7)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b
^4 + b^6)/d^4))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^3 - 3*a*b^2)*d
^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 +
b^6))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*((a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6)/d^4)^(3/4) + sqrt(2)*(d^7*sqrt((a^6 + 3*a^4*b^2 + 3*a^
2*b^4 + b^6)/d^4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + 2*(a^3 + a*b^2)
*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + b^6 + (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d
^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt(((9*a^8*b^2 + 12*a^6*b^4 - 2*a^4*b
^6 - 4*a^2*b^8 + b^10)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*co
s(d*x + c) + sqrt(2)*(2*(9*a^5*b^3 - 6*a^3*b^5 + a*b^7)*d^3*sqrt((a^6 + 3*a
^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*cos(d*x + c) + (9*a^8*b^3 + 12*a^6*b^5 - 2*a
^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^
4 + b^6 + (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)
```

)/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos
(d*x + c))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(1/4) + (9*a^11*b^2 +
21*a^9*b^4 + 10*a^7*b^6 - 6*a^5*b^8 - 3*a^3*b^10 + a*b^12)*cos(d*x + c) + (
9*a^10*b^3 + 21*a^8*b^5 + 10*a^6*b^7 - 6*a^4*b^9 - 3*a^2*b^11 + b^13)*sin(d
*x + c))/((a^2 + b^2)*cos(d*x + c)))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d
^4)^(3/4))/(9*a^14*b^2 + 39*a^12*b^4 + 61*a^10*b^6 + 35*a^8*b^8 - 5*a^6*b^1
0 - 11*a^4*b^12 - a^2*b^14 + b^16))*cos(d*x + c)^4 + 1260*sqrt(2)*b^3*d^5*s
qrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*
a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*((a^6 + 3*a
^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(3/4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4
)*arctan(-((3*a^10 + 11*a^8*b^2 + 14*a^6*b^4 + 6*a^4*b^6 - a^2*b^8 - b^10)*
d^4*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*sqrt((9*a^4*b^2 - 6*a^2*b
^4 + b^6)/d^4) + (3*a^13 + 14*a^11*b^2 + 25*a^9*b^4 + 20*a^7*b^6 + 5*a^5*b^
8 - 2*a^3*b^10 - a*b^12)*d^2*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) - sqrt
(2)*((3*a^4*b + 2*a^2*b^3 - b^5)*d^7*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6)/d^4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + 2*(3*a^7*b + 5*a^5*b^3 +
a^3*b^5 - a*b^7)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4))*sqrt((a^6 + 3
*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*
a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) +
b*sin(d*x + c))/cos(d*x + c))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(3/
4) - sqrt(2)*(d^7*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*sqrt((9*a^4
*b^2 - 6*a^2*b^4 + b^6)/d^4) + 2*(a^3 + a*b^2)*d^5*sqrt((9*a^4*b^2 - 6*a^2*
b^4 + b^6)/d^4))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^3 - 3*a*b^2)*
d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 +
 b^6))*sqrt(((9*a^8*b^2 + 12*a^6*b^4 - 2*a^4*b^6 - 4*a^2*b^8 + b^10)*d^2*sq
rt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*cos(d*x + c) - sqrt(2)*(2*(9*a^
5*b^3 - 6*a^3*b^5 + a*b^7)*d^3*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4
)*cos(d*x + c) + (9*a^8*b^3 + 12*a^6*b^5 - 2*a^4*b^7 - 4*a^2*b^9 + b^11)*d*
cos(d*x + c))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^3 - 3*a*b^2)*d^2
*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^
6))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*((a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6)/d^4)^(1/4) + (9*a^11*b^2 + 21*a^9*b^4 + 10*a^7*b^6 - 6*a
^5*b^8 - 3*a^3*b^10 + a*b^12)*cos(d*x + c) + (9*a^10*b^3 + 21*a^8*b^5 + 10*
a^6*b^7 - 6*a^4*b^9 - 3*a^2*b^11 + b^13)*sin(d*x + c))/((a^2 + b^2)*cos(d*x
 + c)))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(3/4))/(9*a^14*b^2 + 39*a
^12*b^4 + 61*a^10*b^6 + 35*a^8*b^8 - 5*a^6*b^10 - 11*a^4*b^12 - a^2*b^14 +
b^16))*cos(d*x + c)^4 + 315*sqrt(2)*((a^3*b^3 - 3*a*b^5)*d^3*sqrt((a^6 + 3*
a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*cos(d*x + c)^4 - (a^6*b^3 + 3*a^4*b^5 + 3*a
^2*b^7 + b^9)*d*cos(d*x + c)^4)*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (
a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^
2 - 6*a^2*b^4 + b^6))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(1/4)*log((
(9*a^8*b^2 + 12*a^6*b^4 - 2*a^4*b^6 - 4*a^2*b^8 + b^10)*d^2*sqrt((a^6 + 3*a
^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*cos(d*x + c) + sqrt(2)*(2*(9*a^5*b^3 - 6*a^3
*b^5 + a*b^7)*d^3*sqrt((a^6 + 3*a^4*b^2 + 3*a^2...

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan (c+dx))^{\frac{3}{2}}\tan ^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4*(a+b*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**(3/2)*tan(c + d*x)**4, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4*(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 41.58, size = 1355, normalized size = 6.48

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + b*tan(c + d*x))^(3/2),x)
```

```
[Out] (a + b*tan(c + d*x))^(1/2)*(2*a*(2*a*((4*a^2)/(b^3*d) - (2*(a^2 + b^2))/(b^
3*d)) - (8*a^3)/(b^3*d) + (4*a*(a^2 + b^2))/(b^3*d)) - ((4*a^2)/(b^3*d) - (
2*(a^2 + b^2))/(b^3*d))*(a^2 + b^2) + (2*a^4)/(b^3*d)) - atan((b^6*(a + b*t
an(c + d*x))^(1/2)*((b^3*1i)/(4*d^2) - a^3/(4*d^2) + (3*a*b^2)/(4*d^2) - (a
^2*b*3i)/(4*d^2))^(1/2)*32i)/((a^2*b^6*32i)/d - (16*a*b^7)/d - (b^8*16i)/d
+ (32*a^3*b^5)/d + (a^4*b^4*48i)/d + (48*a^5*b^3)/d) - (32*a*b^5*(a + b*tan
(c + d*x))^(1/2)*((b^3*1i)/(4*d^2) - a^3/(4*d^2) + (3*a*b^2)/(4*d^2) - (a^2
*b*3i)/(4*d^2))^(1/2))/((a^2*b^6*32i)/d - (16*a*b^7)/d - (b^8*16i)/d + (32*
a^3*b^5)/d + (a^4*b^4*48i)/d + (48*a^5*b^3)/d) - (a^2*b^4*(a + b*tan(c + d*
x))^(1/2)*((b^3*1i)/(4*d^2) - a^3/(4*d^2) + (3*a*b^2)/(4*d^2) - (a^2*b*3i)/
(4*d^2))^(1/2)*96i)/((a^2*b^6*32i)/d - (16*a*b^7)/d - (b^8*16i)/d + (32*a^3
*b^5)/d + (a^4*b^4*48i)/d + (48*a^5*b^3)/d) + (96*a^3*b^3*(a + b*tan(c + d*
x))^(1/2)*((b^3*1i)/(4*d^2) - a^3/(4*d^2) + (3*a*b^2)/(4*d^2) - (a^2*b*3i)/
(4*d^2))^(1/2))/((a^2*b^6*32i)/d - (16*a*b^7)/d - (b^8*16i)/d + (32*a^3*b^5
)/d + (a^4*b^4*48i)/d + (48*a^5*b^3)/d))*((3*a*b^2 - a^2*b*3i - a^3 + b^3*1
i)/(4*d^2))^(1/2)*2i - atan((b^6*(a + b*tan(c + d*x))^(1/2)*((3*a*b^2)/(4*d
^2) - (b^3*1i)/(4*d^2) - a^3/(4*d^2) + (a^2*b*3i)/(4*d^2))^(1/2)*32i)/((b^8
```

```
*16i)/d - (16*a*b^7)/d - (a^2*b^6*32i)/d + (32*a^3*b^5)/d - (a^4*b^4*48i)/d
 + (48*a^5*b^3)/d) + (32*a*b^5*(a + b*tan(c + d*x))^(1/2)*((3*a*b^2)/(4*d^2
) - (b^3*1i)/(4*d^2) - a^3/(4*d^2) + (a^2*b*3i)/(4*d^2))^(1/2))/((b^8*16i)/
d - (16*a*b^7)/d - (a^2*b^6*32i)/d + (32*a^3*b^5)/d - (a^4*b^4*48i)/d + (48
*a^5*b^3)/d) - (a^2*b^4*(a + b*tan(c + d*x))^(1/2)*((3*a*b^2)/(4*d^2) - (b^
3*1i)/(4*d^2) - a^3/(4*d^2) + (a^2*b*3i)/(4*d^2))^(1/2)*96i)/((b^8*16i)/d -
 (16*a*b^7)/d - (a^2*b^6*32i)/d + (32*a^3*b^5)/d - (a^4*b^4*48i)/d + (48*a^
5*b^3)/d) - (96*a^3*b^3*(a + b*tan(c + d*x))^(1/2)*((3*a*b^2)/(4*d^2) - (b^
3*1i)/(4*d^2) - a^3/(4*d^2) + (a^2*b*3i)/(4*d^2))^(1/2))/((b^8*16i)/d - (16
*a*b^7)/d - (a^2*b^6*32i)/d + (32*a^3*b^5)/d - (a^4*b^4*48i)/d + (48*a^5*b^
3)/d))*((3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)/(4*d^2))^(1/2)*2i + (a + b*tan(
c + d*x))^(3/2)*((2*a*((4*a^2)/(b^3*d) - (2*(a^2 + b^2))/(b^3*d)))/3 - (8*a
^3)/(3*b^3*d) + (4*a*(a^2 + b^2))/(3*b^3*d)) + ((4*a^2)/(5*b^3*d) - (2*(a^2
 + b^2))/(5*b^3*d))*(a + b*tan(c + d*x))^(5/2) + (2*(a + b*tan(c + d*x))^(9
/2))/(9*b^3*d) - (4*a*(a + b*tan(c + d*x))^(7/2))/(7*b^3*d)
```

3.512 $\int \tan^3(c+dx)(a+b\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=181

$$\frac{(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}+\frac{(a+ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}-\frac{2a\sqrt{a+b\tan(c+dx)}}{d}$$

```
[Out] (a-I*b)^(3/2)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d+(a+I*b)^(3/2)
*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d-2*a*(a+b*tan(d*x+c))^(1/2)
/d-2/3*(a+b*tan(d*x+c))^(3/2)/d-4/35*a*(a+b*tan(d*x+c))^(5/2)/b^2/d+2/7*tan
(d*x+c)*(a+b*tan(d*x+c))^(5/2)/b/d
```

Rubi [A]
time = 0.26, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3647, 3711, 12, 3609, 3620, 3618, 65, 214}

$$-\frac{4a(a+b\tan(c+dx))^{5/2}}{35b^2d}+\frac{2\tan(c+dx)(a+b\tan(c+dx))^{5/2}}{7bd}-\frac{2(a+b\tan(c+dx))^{3/2}}{3d}-\frac{2a\sqrt{a+b\tan(c+dx)}}{d}+\frac{(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}+\frac{(a+ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3*(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((a - I*b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/d + ((a +
 I*b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/d - (2*a*Sqrt[
a + b*Tan[c + d*x]])/d - (2*(a + b*Tan[c + d*x])^(3/2))/(3*d) - (4*a*(a + b
*Tan[c + d*x])^(5/2))/(35*b^2*d) + (2*Tan[c + d*x]*(a + b*Tan[c + d*x])^(5/
2))/(7*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \tan^3(c+dx)(a+b\tan(c+dx))^{3/2}\,dx &= \frac{2\tan(c+dx)(a+b\tan(c+dx))^{5/2}}{7bd} + \frac{2\int(a+b\tan(c+dx))^{3/2} \cdots}{\cdots} \\
&= -\frac{4a(a+b\tan(c+dx))^{5/2}}{35b^2d} + \frac{2\tan(c+dx)(a+b\tan(c+dx))^{5/\cdots}}{7bd} \\
&= -\frac{4a(a+b\tan(c+dx))^{5/2}}{35b^2d} + \frac{2\tan(c+dx)(a+b\tan(c+dx))^{5/\cdots}}{7bd} \\
&= -\frac{2(a+b\tan(c+dx))^{3/2}}{3d} - \frac{4a(a+b\tan(c+dx))^{5/2}}{35b^2d} + \frac{2\tan(c\cdots}{\cdots} \\
&= -\frac{2a\sqrt{a+b\tan(c+dx)}}{d} - \frac{2(a+b\tan(c+dx))^{3/2}}{3d} - \frac{4a(a+b\cdots}{\cdots} \\
&= -\frac{2a\sqrt{a+b\tan(c+dx)}}{d} - \frac{2(a+b\tan(c+dx))^{3/2}}{3d} - \frac{4a(a+b\cdots}{\cdots} \\
&= -\frac{2a\sqrt{a+b\tan(c+dx)}}{d} - \frac{2(a+b\tan(c+dx))^{3/2}}{3d} - \frac{4a(a+b\cdots}{\cdots} \\
&= -\frac{2a\sqrt{a+b\tan(c+dx)}}{d} - \frac{2(a+b\tan(c+dx))^{3/2}}{3d} - \frac{4a(a+b\cdots}{\cdots} \\
&= \frac{(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} + \frac{(a+ib)^{3/2}\tanh^{-1}\cdots}{\cdots}
\end{aligned}
$$

Mathematica [A]
time = 1.68, size = 170, normalized size = 0.94

$$\frac{(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}+\frac{(a+ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}+\frac{2\sqrt{a+b\tan(c+dx)}\,(-2a(3a^2+70b^2)+b(3a^2-35b^2)\tan(c+dx)+24ab^2\tan^2(c+dx)+15b^3\tan^3(c+dx))}{105b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3*(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((a - I*b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/d + ((a +
 I*b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/d + (2*Sqrt[a
+ b*Tan[c + d*x]]*(-2*a*(3*a^2 + 70*b^2) + b*(3*a^2 - 35*b^2)*Tan[c + d*x]
+ 24*a*b^2*Tan[c + d*x]^2 + 15*b^3*Tan[c + d*x]^3))/(105*b^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 587 vs. $2(151) = 302$.
time = 0.14, size = 588, normalized size = 3.25

method	result
derivativedivides	$\frac{\frac{2(a+b\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{2a(a+b\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{2b^2(a+b\tan(dx+c))^{\frac{3}{2}}}{3}-2a\,b^2\sqrt{a+b\tan\left(dx+c\right)}+2b^2\left(\frac{\left(-\sqrt{2\sqrt{a^2 -}}\right.}{}\right.}{}$
default	$\frac{\frac{2(a+b\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{2a(a+b\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{2b^2(a+b\tan(dx+c))^{\frac{3}{2}}}{3}-2a\,b^2\sqrt{a+b\tan\left(dx+c\right)}+2b^2\left(\frac{\left(-\sqrt{2\sqrt{a^2 -}}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3*(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/b^2*(1/7*(a+b*tan(d*x+c))^(7/2)-1/5*a*(a+b*tan(d*x+c))^(5/2)-1/3*b^2*(a
+b*tan(d*x+c))^(3/2)-a*b^2*(a+b*tan(d*x+c))^(1/2)+b^2*(1/8*(-(2*(a^2+b^2)^(
1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)+2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a)*ln(b*tan
(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1
/2))+1/2*(2*(a^2+b^2)^(1/2)*a-1/2*(-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)
^(1/2)+2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2
*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(
1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-1/8*(-(2*(a^2+b^2)^(1/2)+2*
a)^(1/2)*(a^2+b^2)^(1/2)+2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a)*ln(-b*tan(d*x+c
)-a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-(a^2+b^2)^(1/2))+1
/2*(-2*(a^2+b^2)^(1/2)*a+1/2*(-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2
)+2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2
+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)
+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(3/2)*tan(d*x + c)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4373 vs. $2(147) = 294$.

time = 2.36, size = 4373, normalized size = 24.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/420*(420*sqrt(2)*b^2*d^5*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^3 -
 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6
*a^2*b^4 + b^6))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(3/4)*sqrt((9*a^
4*b^2 - 6*a^2*b^4 + b^6)/d^4)*arctan(((3*a^10 + 11*a^8*b^2 + 14*a^6*b^4 + 6
*a^4*b^6 - a^2*b^8 - b^10)*d^4*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4
)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + (3*a^13 + 14*a^11*b^2 + 25*a^9*
b^4 + 20*a^7*b^6 + 5*a^5*b^8 - 2*a^3*b^10 - a*b^12)*d^2*sqrt((9*a^4*b^2 - 6
*a^2*b^4 + b^6)/d^4) + sqrt(2)*((3*a^5 + 2*a^3*b^2 - a*b^4)*d^7*sqrt((a^6 +
 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4)
+ (3*a^8 + 2*a^6*b^2 - 4*a^4*b^4 - 2*a^2*b^6 + b^8)*d^5*sqrt((9*a^4*b^2 - 6
*a^2*b^4 + b^6)/d^4))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^3 - 3*a*
b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*
b^4 + b^6))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*((a^6 + 3*
a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(3/4) + sqrt(2)*(a*d^7*sqrt((a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6)/d^4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + (a^4 -
b^4)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4))*sqrt((a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6 - (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt(((9*a^8 + 12*a^6*b^2 - 2*a^4*b
^4 - 4*a^2*b^6 + b^8)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*cos
(d*x + c) + sqrt(2)*((9*a^6 - 15*a^4*b^2 + 7*a^2*b^4 - b^6)*d^3*sqrt((a^6 +
 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*cos(d*x + c) + (9*a^9 + 12*a^7*b^2 - 2*a
^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b
^4 + b^6 - (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4
))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/co
s(d*x + c))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(1/4) + (9*a^11 + 21*
a^9*b^2 + 10*a^7*b^4 - 6*a^5*b^6 - 3*a^3*b^8 + a*b^10)*cos(d*x + c) + (9*a^
10*b + 21*a^8*b^3 + 10*a^6*b^5 - 6*a^4*b^7 - 3*a^2*b^9 + b^11)*sin(d*x + c)
)/((a^2 + b^2)*cos(d*x + c)))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(3/
4))/(9*a^14*b^2 + 39*a^12*b^4 + 61*a^10*b^6 + 35*a^8*b^8 - 5*a^6*b^10 - 11*
a^4*b^12 - a^2*b^14 + b^16))*cos(d*x + c)^3 + 420*sqrt(2)*b^2*d^5*sqrt((a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*((a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6)/d^4)^(3/4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4)*arctan
(-((3*a^10 + 11*a^8*b^2 + 14*a^6*b^4 + 6*a^4*b^6 - a^2*b^8 - b^10)*d^4*sqrt
((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6
)/d^4) + (3*a^13 + 14*a^11*b^2 + 25*a^9*b^4 + 20*a^7*b^6 + 5*a^5*b^8 - 2*a^
```

```
3*b^10 - a*b^12)*d^2*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) - sqrt(2)*((3*
a^5 + 2*a^3*b^2 - a*b^4)*d^7*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*
sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + (3*a^8 + 2*a^6*b^2 - 4*a^4*b^4 -
2*a^2*b^6 + b^8)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4))*sqrt((a^6 + 3
*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*
a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) +
b*sin(d*x + c))/cos(d*x + c))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(3/
4) - sqrt(2)*(a*d^7*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*sqrt((9*a
^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + (a^4 - b^4)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^
4 + b^6)/d^4))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^3 - 3*a*b^2)*d^
2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b
^6))*sqrt(((9*a^8 + 12*a^6*b^2 - 2*a^4*b^4 - 4*a^2*b^6 + b^8)*d^2*sqrt((a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*cos(d*x + c) - sqrt(2)*((9*a^6 - 15*a^
4*b^2 + 7*a^2*b^4 - b^6)*d^3*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*
cos(d*x + c) + (9*a^9 + 12*a^7*b^2 - 2*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d
*x + c))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^3 - 3*a*b^2)*d^2*sqrt
((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*s
qrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*((a^6 + 3*a^4*b^2 + 3*a
^2*b^4 + b^6)/d^4)^(1/4) + (9*a^11 + 21*a^9*b^2 + 10*a^7*b^4 - 6*a^5*b^6 -
3*a^3*b^8 + a*b^10)*cos(d*x + c) + (9*a^10*b + 21*a^8*b^3 + 10*a^6*b^5 - 6*
a^4*b^7 - 3*a^2*b^9 + b^11)*sin(d*x + c))/((a^2 + b^2)*cos(d*x + c)))*((a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(3/4))/(9*a^14*b^2 + 39*a^12*b^4 + 61*
a^10*b^6 + 35*a^8*b^8 - 5*a^6*b^10 - 11*a^4*b^12 - a^2*b^14 + b^16))*cos(d*
x + c)^3 + 105*sqrt(2)*((a^3*b^2 - 3*a*b^4)*d^3*sqrt((a^6 + 3*a^4*b^2 + 3*a
^2*b^4 + b^6)/d^4)*cos(d*x + c)^3 + (a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)
*d*cos(d*x + c)^3)*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^3 - 3*a*b^2
)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4
 + b^6))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(1/4)*log(((9*a^8 + 12*a
^6*b^2 - 2*a^4*b^4 - 4*a^2*b^6 + b^8)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6)/d^4)*cos(d*x + c) + sqrt(2)*((9*a^6 - 15*a^4*b^2 + 7*a^2*b^4 - b^6)
*d^3*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan{(c + dx)})^{\frac{3}{2}} \tan^{3}{(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3*(a+b*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**(3/2)*tan(c + d*x)**3, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 20.21, size = 1229, normalized size = 6.79

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + b*tan(c + d*x))^(3/2),x)
```

```
[Out] atan((b^6*(a + b*tan(c + d*x))^(1/2)*(a^3/(4*d^2) - (b^3*1i)/(4*d^2) - (3*a
*b^2)/(4*d^2) + (a^2*b*3i)/(4*d^2))^(1/2)*32i)/((16*b^8)/d - (a*b^7*16i)/d
- (32*a^2*b^6)/d + (a^3*b^5*32i)/d - (48*a^4*b^4)/d + (a^5*b^3*48i)/d) - (3
2*a*b^5*(a + b*tan(c + d*x))^(1/2)*(a^3/(4*d^2) - (b^3*1i)/(4*d^2) - (3*a*b
^2)/(4*d^2) + (a^2*b*3i)/(4*d^2))^(1/2))/((16*b^8)/d - (a*b^7*16i)/d - (32*
a^2*b^6)/d + (a^3*b^5*32i)/d - (48*a^4*b^4)/d + (a^5*b^3*48i)/d) - (a^2*b^4
*(a + b*tan(c + d*x))^(1/2)*(a^3/(4*d^2) - (b^3*1i)/(4*d^2) - (3*a*b^2)/(4*
d^2) + (a^2*b*3i)/(4*d^2))^(1/2)*96i)/((16*b^8)/d - (a*b^7*16i)/d - (32*a^2
*b^6)/d + (a^3*b^5*32i)/d - (48*a^4*b^4)/d + (a^5*b^3*48i)/d) + (96*a^3*b^3
*(a + b*tan(c + d*x))^(1/2)*(a^3/(4*d^2) - (b^3*1i)/(4*d^2) - (3*a*b^2)/(4*
d^2) + (a^2*b*3i)/(4*d^2))^(1/2))/((16*b^8)/d - (a*b^7*16i)/d - (32*a^2*b^6
)/d + (a^3*b^5*32i)/d - (48*a^4*b^4)/d + (a^5*b^3*48i)/d))*(-(3*a*b^2 - a^2
*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2)*2i - atan((b^6*(a + b*tan(c + d*x))^(1
/2)*(a^3/(4*d^2) + (b^3*1i)/(4*d^2) - (3*a*b^2)/(4*d^2) - (a^2*b*3i)/(4*d^2
))^(1/2)*32i)/((32*a^2*b^6)/d - (a*b^7*16i)/d - (16*b^8)/d + (a^3*b^5*32i)/
d + (48*a^4*b^4)/d + (a^5*b^3*48i)/d) + (32*a*b^5*(a + b*tan(c + d*x))^(1/2
)*(a^3/(4*d^2) + (b^3*1i)/(4*d^2) - (3*a*b^2)/(4*d^2) - (a^2*b*3i)/(4*d^2))
^(1/2))/((32*a^2*b^6)/d - (a*b^7*16i)/d - (16*b^8)/d + (a^3*b^5*32i)/d + (4
8*a^4*b^4)/d + (a^5*b^3*48i)/d) - (a^2*b^4*(a + b*tan(c + d*x))^(1/2)*(a^3/
(4*d^2) + (b^3*1i)/(4*d^2) - (3*a*b^2)/(4*d^2) - (a^2*b*3i)/(4*d^2))^(1/2)*
96i)/((32*a^2*b^6)/d - (a*b^7*16i)/d - (16*b^8)/d + (a^3*b^5*32i)/d + (48*a
^4*b^4)/d + (a^5*b^3*48i)/d) - (96*a^3*b^3*(a + b*tan(c + d*x))^(1/2)*(a^3/
(4*d^2) + (b^3*1i)/(4*d^2) - (3*a*b^2)/(4*d^2) - (a^2*b*3i)/(4*d^2))^(1/2))
/((32*a^2*b^6)/d - (a*b^7*16i)/d - (16*b^8)/d + (a^3*b^5*32i)/d + (48*a^4*b
^4)/d + (a^5*b^3*48i)/d))*(-(3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)/(4*d^2))^(1
/2)*2i + (a + b*tan(c + d*x))^(1/2)*(2*a*((2*a^2)/(b^2*d) - (2*(a^2 + b^2))
/(b^2*d)) - (2*a^3)/(b^2*d) + (2*a*(a^2 + b^2))/(b^2*d)) + ((2*a^2)/(3*b^2*
d) - (2*(a^2 + b^2))/(3*b^2*d))*(a + b*tan(c + d*x))^(3/2) + (2*(a + b*tan(
c + d*x))^(7/2))/(7*b^2*d) - (2*a*(a + b*tan(c + d*x))^(5/2))/(5*b^2*d)
```

3.513 $\int \tan^2(c+dx)(a+b\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=135

$$\frac{i(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}-\frac{i(a+ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}-\frac{2b\sqrt{a+b\tan(c+}}{d}$$

[Out] I*(a-I*b)^(3/2)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d-I*(a+I*b)^(3/2)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d-2*b*(a+b*tan(d*x+c))^(1/2)/d+2/5*(a+b*tan(d*x+c))^(5/2)/b/d

Rubi [A]
time = 0.16, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3624, 3563, 3620, 3618, 65, 214}

$$\frac{2(a+b\tan(c+dx))^{5/2}}{5bd}-\frac{2b\sqrt{a+b\tan(c+dx)}}{d}+\frac{i(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}-\frac{i(a+ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d*x]^2*(a + b*Tan[c + d*x])^(3/2),x]

[Out] (I*(a - I*b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/d - (I*(a + I*b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/d - (2*b*Sqrt[a + b*Tan[c + d*x]])/d + (2*(a + b*Tan[c + d*x])^(5/2))/(5*b*d)

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3563

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^{3/2}\,dx = \frac{2(a+b\tan(c+dx))^{5/2}}{5bd} - \int (a+b\tan(c+dx))^{3/2}\,dx$$

$$= -\frac{2b\sqrt{a+b\tan(c+dx)}}{d} + \frac{2(a+b\tan(c+dx))^{5/2}}{5bd} - \int \frac{a^2-b}{\sqrt{a}}$$

$$= -\frac{2b\sqrt{a+b\tan(c+dx)}}{d} + \frac{2(a+b\tan(c+dx))^{5/2}}{5bd} - \frac{1}{2}(a-ib)$$

$$= -\frac{2b\sqrt{a+b\tan(c+dx)}}{d} + \frac{2(a+b\tan(c+dx))^{5/2}}{5bd} - \frac{(i(a-ib)}{}$$

$$= -\frac{2b\sqrt{a+b\tan(c+dx)}}{d} + \frac{2(a+b\tan(c+dx))^{5/2}}{5bd} + \frac{(a-ib)^2}{}$$

$$= \frac{i(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{i(a+ib)^{3/2}\tanh}{}$$

Mathematica [A]

time = 1.12, size = 158, normalized size = 1.17

$$\frac{\frac{2(a+b\tan(c+dx))^{5/2}}{b}+5(ia+b)\left(\sqrt{a-ib}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)-\sqrt{a+b\tan(c+dx)}\right)+5i(a+ib)\left(-\sqrt{a+ib}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)+\sqrt{a+b\tan(c+dx)}\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2*(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((2*(a + b*Tan[c + d*x])^(5/2))/b + 5*(I*a + b)*(Sqrt[a - I*b]*ArcTanh[Sqrt
[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] - Sqrt[a + b*Tan[c + d*x]]) + (5*I)*(a
+ I*b)*(-(Sqrt[a + I*b]*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]]) +
Sqrt[a + b*Tan[c + d*x]]))/(5*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 662 vs. $2(111) = 222$.
time = 0.13, size = 663, normalized size = 4.91

method	result
derivativedivides	$\frac{\frac{2(a+b\tan(dx+c))^{\frac{5}{2}}}{5}-2b^2\sqrt{a+b\tan(dx+c)}+2b^2\left(\frac{\left(\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a-\sqrt{2\sqrt{a^2+\cdots}}\right.}{\cdots}\right.}{\cdots}$
default	$\frac{\frac{2(a+b\tan(dx+c))^{\frac{5}{2}}}{5}-2b^2\sqrt{a+b\tan(dx+c)}+2b^2\left(\frac{\left(\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a-\sqrt{2\sqrt{a^2+\cdots}}\right.}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2*(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/b*(1/5*(a+b*tan(d*x+c))^(5/2)-b^2*(a+b*tan(d*x+c))^(1/2)+b^2*(1/4/b^2*(
1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a-(2*(a^2+b^2)^(1/2)+2*a
)^(1/2)*a^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^2)*ln(b*tan(d*x+c)+a+(a+b*tan(d
*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*(2*(a^2+b^2)^
(1/2)*b^2-1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a-(2*(a^2+b^2)
```

```
^(1/2)+2*a)^(1/2)*a^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^2)*(2*(a^2+b^2)^(1/2)
+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)
+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))+1/4/b^2*(-1
/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a-(2*(a^2+b^2)^(1/2)+2*a)
^(1/2)*a^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^2)*ln(-b*tan(d*x+c)-a+(a+b*tan(d
*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-(a^2+b^2)^(1/2))+2*(-2*(a^2+b^2)
^(1/2)*b^2+1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a-(2*(a^2+b^2
)^(1/2)+2*a)^(1/2)*a^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^2)*(2*(a^2+b^2)^(1/2
)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*tan(d*x+c))^(1/
2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(3/2)*tan(d*x + c)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4304 vs. $2(105) = 210$.
time = 1.62, size = 4304, normalized size = 31.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/20*(20*sqrt(2)*b*d^5*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^3 - 3*
a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^
2*b^4 + b^6))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(3/4)*sqrt((9*a^4*b
^2 - 6*a^2*b^4 + b^6)/d^4)*arctan(((3*a^10 + 11*a^8*b^2 + 14*a^6*b^4 + 6*a^
4*b^6 - a^2*b^8 - b^10)*d^4*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*s
qrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + (3*a^13 + 14*a^11*b^2 + 25*a^9*b^4
 + 20*a^7*b^6 + 5*a^5*b^8 - 2*a^3*b^10 - a*b^12)*d^2*sqrt((9*a^4*b^2 - 6*a^
2*b^4 + b^6)/d^4) + sqrt(2)*((3*a^4*b + 2*a^2*b^3 - b^5)*d^7*sqrt((a^6 + 3*
a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + 2
*(3*a^7*b + 5*a^5*b^3 + a^3*b^5 - a*b^7)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 +
b^6)/d^4))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^3 - 3*a*b^2)*d^2*sq
rt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))
*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*((a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)/d^4)^(3/4) + sqrt(2)*(d^7*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6)/d^4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + 2*(a^3 + a*b^2)*d^5*
sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4
```

```
+ b^6 + (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/
(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt(((9*a^8*b^2 + 12*a^6*b^4 - 2*a^4*b^6 -
4*a^2*b^8 + b^10)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*cos(d*x
 + c) + sqrt(2)*(2*(9*a^5*b^3 - 6*a^3*b^5 + a*b^7)*d^3*sqrt((a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6)/d^4)*cos(d*x + c) + (9*a^8*b^3 + 12*a^6*b^5 - 2*a^4*b^
7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6 + (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*
a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x
+ c))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(1/4) + (9*a^11*b^2 + 21*a^
9*b^4 + 10*a^7*b^6 - 6*a^5*b^8 - 3*a^3*b^10 + a*b^12)*cos(d*x + c) + (9*a^1
0*b^3 + 21*a^8*b^5 + 10*a^6*b^7 - 6*a^4*b^9 - 3*a^2*b^11 + b^13)*sin(d*x +
c))/((a^2 + b^2)*cos(d*x + c)))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(
3/4))/(9*a^14*b^2 + 39*a^12*b^4 + 61*a^10*b^6 + 35*a^8*b^8 - 5*a^6*b^10 - 1
1*a^4*b^12 - a^2*b^14 + b^16))*cos(d*x + c)^2 + 20*sqrt(2)*b*d^5*sqrt((a^6
+ 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*((a^6 + 3*a^4*b^2 +
3*a^2*b^4 + b^6)/d^4)^(3/4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4)*arctan(
-((3*a^10 + 11*a^8*b^2 + 14*a^6*b^4 + 6*a^4*b^6 - a^2*b^8 - b^10)*d^4*sqrt(
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)
/d^4) + (3*a^13 + 14*a^11*b^2 + 25*a^9*b^4 + 20*a^7*b^6 + 5*a^5*b^8 - 2*a^3
*b^10 - a*b^12)*d^2*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) - sqrt(2)*((3*a
^4*b + 2*a^2*b^3 - b^5)*d^7*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*s
qrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + 2*(3*a^7*b + 5*a^5*b^3 + a^3*b^5 -
 a*b^7)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4))*sqrt((a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6 + (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
 b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) + b*sin(d*x
 + c))/cos(d*x + c))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(3/4) - sqrt
(2)*(d^7*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*sqrt((9*a^4*b^2 - 6*
a^2*b^4 + b^6)/d^4) + 2*(a^3 + a*b^2)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6
)/d^4))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^3 - 3*a*b^2)*d^2*sqrt(
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sq
rt(((9*a^8*b^2 + 12*a^6*b^4 - 2*a^4*b^6 - 4*a^2*b^8 + b^10)*d^2*sqrt((a^6 +
 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*cos(d*x + c) - sqrt(2)*(2*(9*a^5*b^3 - 6
*a^3*b^5 + a*b^7)*d^3*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*cos(d*x
 + c) + (9*a^8*b^3 + 12*a^6*b^5 - 2*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x +
 c))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^3 - 3*a*b^2)*d^2*sqrt((a^
6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt(
(a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*((a^6 + 3*a^4*b^2 + 3*a^2*b
^4 + b^6)/d^4)^(1/4) + (9*a^11*b^2 + 21*a^9*b^4 + 10*a^7*b^6 - 6*a^5*b^8 -
3*a^3*b^10 + a*b^12)*cos(d*x + c) + (9*a^10*b^3 + 21*a^8*b^5 + 10*a^6*b^7 -
 6*a^4*b^9 - 3*a^2*b^11 + b^13)*sin(d*x + c))/((a^2 + b^2)*cos(d*x + c)))*(
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(3/4))/(9*a^14*b^2 + 39*a^12*b^4 +
 61*a^10*b^6 + 35*a^8*b^8 - 5*a^6*b^10 - 11*a^4*b^12 - a^2*b^14 + b^16))*co
s(d*x + c)^2 + 5*sqrt(2)*((a^3*b - 3*a*b^3)*d^3*sqrt((a^6 + 3*a^4*b^2 + 3*a
^2*b^4 + b^6)/d^4)*cos(d*x + c)^2 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*d
```

```
*cos(d*x + c)^2)*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^3 - 3*a*b^2)*
d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 +
 b^6))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(1/4)*log(((9*a^8*b^2 + 12
*a^6*b^4 - 2*a^4*b^6 - 4*a^2*b^8 + b^10)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*
b^4 + b^6)/d^4)*cos(d*x + c) + sqrt(2)*(2*(9*a^5*b^3 - 6*a^3*b^5 + a*b^7)*d
^3*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan (c + dx))^{\frac{3}{2}} \tan^2 (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2*(a+b*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**(3/2)*tan(c + d*x)**2, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 9.32, size = 1141, normalized size = 8.45

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + b*tan(c + d*x))^(3/2),x)
```

```
[Out] atan((b^6*(a + b*tan(c + d*x))^(1/2)*((3*a*b^2)/(4*d^2) - (b^3*1i)/(4*d^2)
- a^3/(4*d^2) + (a^2*b*3i)/(4*d^2))^(1/2)*32i)/((b^8*16i)/d - (16*a*b^7)/d
- (a^2*b^6*32i)/d + (32*a^3*b^5)/d - (a^4*b^4*48i)/d + (48*a^5*b^3)/d) + (3
2*a*b^5*(a + b*tan(c + d*x))^(1/2)*((3*a*b^2)/(4*d^2) - (b^3*1i)/(4*d^2) -
a^3/(4*d^2) + (a^2*b*3i)/(4*d^2))^(1/2))/((b^8*16i)/d - (16*a*b^7)/d - (a^2
*b^6*32i)/d + (32*a^3*b^5)/d - (a^4*b^4*48i)/d + (48*a^5*b^3)/d) - (a^2*b^4
*(a + b*tan(c + d*x))^(1/2)*((3*a*b^2)/(4*d^2) - (b^3*1i)/(4*d^2) - a^3/(4*
d^2) + (a^2*b*3i)/(4*d^2))^(1/2)*96i)/((b^8*16i)/d - (16*a*b^7)/d - (a^2*b^
6*32i)/d + (32*a^3*b^5)/d - (a^4*b^4*48i)/d + (48*a^5*b^3)/d) - (96*a^3*b^3
*(a + b*tan(c + d*x))^(1/2)*((3*a*b^2)/(4*d^2) - (b^3*1i)/(4*d^2) - a^3/(4*
```

d^2) + (a^2*b*3i)/(4*d^2))^(1/2))/((b^8*16i)/d - (16*a*b^7)/d - (a^2*b^6*32i)/d + (32*a^3*b^5)/d - (a^4*b^4*48i)/d + (48*a^5*b^3)/d))*((3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)/(4*d^2))^(1/2)*2i + atan((b^6*(a + b*tan(c + d*x))^(1/2)*((b^3*1i)/(4*d^2) - a^3/(4*d^2) + (3*a*b^2)/(4*d^2) - (a^2*b*3i)/(4*d^2))^(1/2)*32i)/((a^2*b^6*32i)/d - (16*a*b^7)/d - (b^8*16i)/d + (32*a^3*b^5)/d + (a^4*b^4*48i)/d + (48*a^5*b^3)/d) - (32*a*b^5*(a + b*tan(c + d*x))^(1/2)*((b^3*1i)/(4*d^2) - a^3/(4*d^2) + (3*a*b^2)/(4*d^2) - (a^2*b*3i)/(4*d^2))^(1/2))/((a^2*b^6*32i)/d - (16*a*b^7)/d - (b^8*16i)/d + (32*a^3*b^5)/d + (a^4*b^4*48i)/d + (48*a^5*b^3)/d) - (a^2*b^4*(a + b*tan(c + d*x))^(1/2)*((b^3*1i)/(4*d^2) - a^3/(4*d^2) + (3*a*b^2)/(4*d^2) - (a^2*b*3i)/(4*d^2))^(1/2)*96i)/((a^2*b^6*32i)/d - (16*a*b^7)/d - (b^8*16i)/d + (32*a^3*b^5)/d + (a^4*b^4*48i)/d + (48*a^5*b^3)/d) + (96*a^3*b^3*(a + b*tan(c + d*x))^(1/2)*((b^3*1i)/(4*d^2) - a^3/(4*d^2) + (3*a*b^2)/(4*d^2) - (a^2*b*3i)/(4*d^2))^(1/2))/((a^2*b^6*32i)/d - (16*a*b^7)/d - (b^8*16i)/d + (32*a^3*b^5)/d + (a^4*b^4*48i)/d + (48*a^5*b^3)/d))*((3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2)*2i + ((2*a^2)/(b*d) - (2*(a^2 + b^2))/(b*d))*(a + b*tan(c + d*x))^(1/2) + (2*(a + b*tan(c + d*x))^(5/2))/(5*b*d)

3.514 $\int \tan(c+dx)(a+b\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=128

$$-\frac{(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}-\frac{(a+ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}+\frac{2a\sqrt{a+b\tan(}}{d}$$

```
[Out] -(a-I*b)^(3/2)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d-(a+I*b)^(3/2
)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d+2*a*(a+b*tan(d*x+c))^(1/2
)/d+2/3*(a+b*tan(d*x+c))^(3/2)/d
```

Rubi [A]
time = 0.15, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3609, 3620, 3618, 65, 214}

$$\frac{2(a+b\tan(c+dx))^{3/2}}{3d}+\frac{2a\sqrt{a+b\tan(c+dx)}}{d}-\frac{(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}-\frac{(a+ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]*(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] -(((a - I*b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/d) - ((
a + I*b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/d + (2*a*Sq
rt[a + b*Tan[c + d*x]])/d + (2*(a + b*Tan[c + d*x])^(3/2))/(3*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
```

```
0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \tan(c+dx)(a+b\tan(c+dx))^{3/2}\,dx = \frac{2(a+b\tan(c+dx))^{3/2}}{3d} + \int (-b+a\tan(c+dx))\sqrt{a+b\tan(c+\ldots}$$

$$= \frac{2a\sqrt{a+b\tan(c+dx)}}{d} + \frac{2(a+b\tan(c+dx))^{3/2}}{3d} + \int \frac{-2ab+(a\ldots}{\sqrt{a+\ldots}}$$

$$= \frac{2a\sqrt{a+b\tan(c+dx)}}{d} + \frac{2(a+b\tan(c+dx))^{3/2}}{3d} - \frac{1}{2}\left(i(a-ib)^2\right)\ldots$$

$$= \frac{2a\sqrt{a+b\tan(c+dx)}}{d} + \frac{2(a+b\tan(c+dx))^{3/2}}{3d} + \frac{(a-ib)^2\mathrm{Subs}\ldots}{\ldots}$$

$$= \frac{2a\sqrt{a+b\tan(c+dx)}}{d} + \frac{2(a+b\tan(c+dx))^{3/2}}{3d} + \frac{\left(i(a-ib)^2\right)\mathrm{S}\ldots}{\ldots}$$

$$= -\frac{(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{(a+ib)^{3/2}\tanh^{-1}\ldots}{\ldots}$$

Mathematica [A]
time = 0.34, size = 116, normalized size = 0.91

$$\frac{-3(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)-3(a+ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)+2\sqrt{a+b\tan(c+dx)}\,(4a+b\tan(c+dx))}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]*(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] (-3*(a - I*b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] - 3*(a
+ I*b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] + 2*Sqrt[a + b
*Tan[c + d*x]]*(4*a + b*Tan[c + d*x]))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 537 vs. $2(106) = 212$.
time = 0.12, size = 538, normalized size = 4.20

method	result
derivativedivides	$\frac{\frac{2(a+b\tan(dx+c))^{\frac{3}{2}}}{3}+2a\sqrt{a+b\tan(dx+c)}+\frac{\left(\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}-2\sqrt{2\sqrt{a^2+b^2}}\right.}{}}{}$
default	$\frac{\frac{2(a+b\tan(dx+c))^{\frac{3}{2}}}{3}+2a\sqrt{a+b\tan(dx+c)}+\frac{\left(\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}-2\sqrt{2\sqrt{a^2+b^2}}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)*(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/3*(a+b*tan(d*x+c))^(3/2)+2*a*(a+b*tan(d*x+c))^(1/2)+1/4*((2*(a^2+b^2
)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)-2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a)*ln(b*
tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)
^(1/2))+(-2*(a^2+b^2)^(1/2)*a-1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^
(1/2)-2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*
(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1
/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-1/4*((2*(a^2+b^2)^(1/2)+2*a)
^(1/2)*(a^2+b^2)^(1/2)-2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a)*ln(-b*tan(d*x+c)-
a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-(a^2+b^2)^(1/2))+(2*
(a^2+b^2)^(1/2)*a+1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)-2*(2*(
a^2+b^2)^(1/2)+2*a)^(1/2)*a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1
/2)-2*a)^(1/2)*arctan((-2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1
/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4234 vs. $2(102) = 204$.

time = 1.85, size = 4234, normalized size = 33.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/12*(12*sqrt(2)*d^5*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^3 - 3*a*
b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*
b^4 + b^6))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(3/4)*sqrt((9*a^4*b^2
 - 6*a^2*b^4 + b^6)/d^4)*arctan(((3*a^10 + 11*a^8*b^2 + 14*a^6*b^4 + 6*a^4*
b^6 - a^2*b^8 - b^10)*d^4*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*sqr
t((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + (3*a^13 + 14*a^11*b^2 + 25*a^9*b^4 +
 20*a^7*b^6 + 5*a^5*b^8 - 2*a^3*b^10 - a*b^12)*d^2*sqrt((9*a^4*b^2 - 6*a^2*
b^4 + b^6)/d^4) + sqrt(2)*((3*a^5 + 2*a^3*b^2 - a*b^4)*d^7*sqrt((a^6 + 3*a^
4*b^2 + 3*a^2*b^4 + b^6)/d^4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + (3*
a^8 + 2*a^6*b^2 - 4*a^4*b^4 - 2*a^2*b^6 + b^8)*d^5*sqrt((9*a^4*b^2 - 6*a^2*
b^4 + b^6)/d^4))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^3 - 3*a*b^2)*
d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 +
 b^6))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*((a^6 + 3*a^4*b
^2 + 3*a^2*b^4 + b^6)/d^4)^(3/4) + sqrt(2)*(a*d^7*sqrt((a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)/d^4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + (a^4 - b^4)*
d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*
b^4 + b^6 - (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^
4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt(((9*a^8 + 12*a^6*b^2 - 2*a^4*b^4 -
4*a^2*b^6 + b^8)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*cos(d*x
+ c) + sqrt(2)*((9*a^6 - 15*a^4*b^2 + 7*a^2*b^4 - b^6)*d^3*sqrt((a^6 + 3*a^
4*b^2 + 3*a^2*b^4 + b^6)/d^4)*cos(d*x + c) + (9*a^9 + 12*a^7*b^2 - 2*a^5*b^
4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
b^6 - (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9
*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x
 + c))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(1/4) + (9*a^11 + 21*a^9*b
^2 + 10*a^7*b^4 - 6*a^5*b^6 - 3*a^3*b^8 + a*b^10)*cos(d*x + c) + (9*a^10*b
+ 21*a^8*b^3 + 10*a^6*b^5 - 6*a^4*b^7 - 3*a^2*b^9 + b^11)*sin(d*x + c))/((a
^2 + b^2)*cos(d*x + c)))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(3/4))/(
9*a^14*b^2 + 39*a^12*b^4 + 61*a^10*b^6 + 35*a^8*b^8 - 5*a^6*b^10 - 11*a^4*b
^12 - a^2*b^14 + b^16))*cos(d*x + c) + 12*sqrt(2)*d^5*sqrt((a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6 - (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4
```

```
+ b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
 b^6)/d^4)^(3/4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4)*arctan(-((3*a^10 +
 11*a^8*b^2 + 14*a^6*b^4 + 6*a^4*b^6 - a^2*b^8 - b^10)*d^4*sqrt((a^6 + 3*a^
4*b^2 + 3*a^2*b^4 + b^6)/d^4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + (3*
a^13 + 14*a^11*b^2 + 25*a^9*b^4 + 20*a^7*b^6 + 5*a^5*b^8 - 2*a^3*b^10 - a*b
^12)*d^2*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) - sqrt(2)*((3*a^5 + 2*a^3*
b^2 - a*b^4)*d^7*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*sqrt((9*a^4*
b^2 - 6*a^2*b^4 + b^6)/d^4) + (3*a^8 + 2*a^6*b^2 - 4*a^4*b^4 - 2*a^2*b^6 +
b^8)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4))*sqrt((a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6 - (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) + b*sin(d*x +
c))/cos(d*x + c))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(3/4) - sqrt(2)
*(a*d^7*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*sqrt((9*a^4*b^2 - 6*a
^2*b^4 + b^6)/d^4) + (a^4 - b^4)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4
))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^3 - 3*a*b^2)*d^2*sqrt((a^6
+ 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt(((
9*a^8 + 12*a^6*b^2 - 2*a^4*b^4 - 4*a^2*b^6 + b^8)*d^2*sqrt((a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6)/d^4)*cos(d*x + c) - sqrt(2)*((9*a^6 - 15*a^4*b^2 + 7*a^
2*b^4 - b^6)*d^3*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*cos(d*x + c)
 + (9*a^9 + 12*a^7*b^2 - 2*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c))*sqr
t((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^
4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d
*x + c) + b*sin(d*x + c))/cos(d*x + c))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
)/d^4)^(1/4) + (9*a^11 + 21*a^9*b^2 + 10*a^7*b^4 - 6*a^5*b^6 - 3*a^3*b^8 +
a*b^10)*cos(d*x + c) + (9*a^10*b + 21*a^8*b^3 + 10*a^6*b^5 - 6*a^4*b^7 - 3*
a^2*b^9 + b^11)*sin(d*x + c))/((a^2 + b^2)*cos(d*x + c)))*((a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6)/d^4)^(3/4))/(9*a^14*b^2 + 39*a^12*b^4 + 61*a^10*b^6 + 3
5*a^8*b^8 - 5*a^6*b^10 - 11*a^4*b^12 - a^2*b^14 + b^16))*cos(d*x + c) + 3*s
qrt(2)*((a^3 - 3*a*b^2)*d^3*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*c
os(d*x + c) + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d*cos(d*x + c))*sqrt((a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*((a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6)/d^4)^(1/4)*log(((9*a^8 + 12*a^6*b^2 - 2*a^4*b^4 - 4*a^2*b
^6 + b^8)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*cos(d*x + c) +
sqrt(2)*((9*a^6 - 15*a^4*b^2 + 7*a^2*b^4 - b^6)*d^3*sqrt((a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6)/d^4)*cos(d*x + c) + (9*a^9 + ...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx))^{\frac{3}{2}} \tan(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))**(3/2),x)
```

[Out] Integral((a + b*tan(c + d*x))**(3/2)*tan(c + d*x), x)

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 6.82, size = 1112, normalized size = 8.69

$$\frac{2\,{\left(a+b\,\mathrm{tan}\left(c+d\,x\right)\right)}^{3/2}}{3\,d}+\frac{2\,a\,\sqrt{a+b\,\mathrm{tan}\left(c+d\,x\right)}}{d}-\mathrm{atan}\left(\frac{b^6\,\sqrt{a+b\,\mathrm{tan}\left(c+d\,x\right)}\,\sqrt{\frac{a^3}{4\,d^2}-\frac{b^3\,1{}\mathrm{i}}{4\,d^2}-\frac{3\,a\,b^2}{4\,d^2}+\frac{a^2\,b\,3{}\mathrm{i}}{4\,d^2}}\,32{}\mathrm{i}}{\frac{16\,b^8}{d}-\frac{a\,b^7\,16{}\mathrm{i}}{d}-\frac{32\,a^2\,b^6}{d}+\frac{a^3\,b^5\,32{}\mathrm{i}}{d}-\frac{48\,a^4\,b^4}{d}+\frac{a^5\,b^3\,48{}\mathrm{i}}{d}}-\frac{32\,a\,b^5\,\sqrt{a+b\,\mathrm{tan}\left(c+d\,x\right)}\,\sqrt{\frac{a^3}{4\,d^2}-\frac{b^3\,1{}\mathrm{i}}{4\,d^2}-\frac{3\,a\,b^2}{4\,d^2}+\frac{a^2\,b\,3{}\mathrm{i}}{4\,d^2}}}{\frac{16\,b^8}{d}-\frac{a\,b^7\,16{}\mathrm{i}}{d}-\frac{32\,a^2\,b^6}{d}+\frac{a^3\,b^5\,32{}\mathrm{i}}{d}-\frac{48\,a^4\,b^4}{d}+\frac{a^5\,b^3\,48{}\mathrm{i}}{d}}-\frac{a^2\,b^4\,\sqrt{a+b\,\mathrm{tan}\left(c+d\,x\right)}\,\sqrt{\frac{a^3}{4\,d^2}-\frac{b^3\,1{}\mathrm{i}}{4\,d^2}-\frac{3\,a\,b^2}{4\,d^2}+\frac{a^2\,b\,3{}\mathrm{i}}{4\,d^2}}\,96{}\mathrm{i}}{\frac{16\,b^8}{d}-\frac{a\,b^7\,16{}\mathrm{i}}{d}-\frac{32\,a^2\,b^6}{d}+\frac{a^3\,b^5\,32{}\mathrm{i}}{d}-\frac{48\,a^4\,b^4}{d}+\frac{a^5\,b^3\,48{}\mathrm{i}}{d}}+\frac{96\,a^3\,b^3\,\sqrt{a+b\,\mathrm{tan}\left(c+d\,x\right)}\,\sqrt{\frac{a^3}{4\,d^2}-\frac{b^3\,1{}\mathrm{i}}{4\,d^2}-\frac{3\,a\,b^2}{4\,d^2}+\frac{a^2\,b\,3{}\mathrm{i}}{4\,d^2}}}{\frac{16\,b^8}{d}-\frac{a\,b^7\,16{}\mathrm{i}}{d}-\frac{32\,a^2\,b^6}{d}+\frac{a^3\,b^5\,32{}\mathrm{i}}{d}-\frac{48\,a^4\,b^4}{d}+\frac{a^5\,b^3\,48{}\mathrm{i}}{d}}\right)\,\sqrt{-\frac{-a^3-a^2\,b\,3{}\mathrm{i}+3\,a\,b^2+b^3\,1{}\mathrm{i}}{4\,d^2}}\,2{}\mathrm{i}+\mathrm{atan}\left(\frac{b^6\,\sqrt{a+b\,\mathrm{tan}\left(c+d\,x\right)}\,\sqrt{\frac{a^3}{4\,d^2}+\frac{b^3\,1{}\mathrm{i}}{4\,d^2}-\frac{3\,a\,b^2}{4\,d^2}-\frac{a^2\,b\,3{}\mathrm{i}}{4\,d^2}}\,32{}\mathrm{i}}{\frac{32\,a^2\,b^6}{d}-\frac{a\,b^7\,16{}\mathrm{i}}{d}-\frac{16\,b^8}{d}+\frac{a^3\,b^5\,32{}\mathrm{i}}{d}+\frac{48\,a^4\,b^4}{d}+\frac{a^5\,b^3\,48{}\mathrm{i}}{d}}+\frac{32\,a\,b^5\,\sqrt{a+b\,\mathrm{tan}\left(c+d\,x\right)}\,\sqrt{\frac{a^3}{4\,d^2}+\frac{b^3\,1{}\mathrm{i}}{4\,d^2}-\frac{3\,a\,b^2}{4\,d^2}-\frac{a^2\,b\,3{}\mathrm{i}}{4\,d^2}}}{\frac{32\,a^2\,b^6}{d}-\frac{a\,b^7\,16{}\mathrm{i}}{d}-\frac{16\,b^8}{d}+\frac{a^3\,b^5\,32{}\mathrm{i}}{d}+\frac{48\,a^4\,b^4}{d}+\frac{a^5\,b^3\,48{}\mathrm{i}}{d}}-\frac{a^2\,b^4\,\sqrt{a+b\,\mathrm{tan}\left(c+d\,x\right)}\,\sqrt{\frac{a^3}{4\,d^2}+\frac{b^3\,1{}\mathrm{i}}{4\,d^2}-\frac{3\,a\,b^2}{4\,d^2}-\frac{a^2\,b\,3{}\mathrm{i}}{4\,d^2}}\,96{}\mathrm{i}}{\frac{32\,a^2\,b^6}{d}-\frac{a\,b^7\,16{}\mathrm{i}}{d}-\frac{16\,b^8}{d}+\frac{a^3\,b^5\,32{}\mathrm{i}}{d}+\frac{48\,a^4\,b^4}{d}+\frac{a^5\,b^3\,48{}\mathrm{i}}{d}}-\frac{96\,a^3\,b^3\,\sqrt{a+b\,\mathrm{tan}\left(c+d\,x\right)}\,\sqrt{\frac{a^3}{4\,d^2}+\frac{b^3\,1{}\mathrm{i}}{4\,d^2}-\frac{3\,a\,b^2}{4\,d^2}-\frac{a^2\,b\,3{}\mathrm{i}}{4\,d^2}}}{\frac{32\,a^2\,b^6}{d}-\frac{a\,b^7\,16{}\mathrm{i}}{d}-\frac{16\,b^8}{d}+\frac{a^3\,b^5\,32{}\mathrm{i}}{d}+\frac{48\,a^4\,b^4}{d}+\frac{a^5\,b^3\,48{}\mathrm{i}}{d}}\right)\,\sqrt{-\frac{-a^3+a^2\,b\,3{}\mathrm{i}+3\,a\,b^2-b^3\,1{}\mathrm{i}}{4\,d^2}}\,2{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + b*tan(c + d*x))^(3/2),x)
```

```
[Out] atan((b^6*(a + b*tan(c + d*x))^(1/2)*(a^3/(4*d^2) + (b^3*1i)/(4*d^2) - (3*a
*b^2)/(4*d^2) - (a^2*b*3i)/(4*d^2))^(1/2)*32i)/((32*a^2*b^6)/d - (a*b^7*16i
)/d - (16*b^8)/d + (a^3*b^5*32i)/d + (48*a^4*b^4)/d + (a^5*b^3*48i)/d) + (3
2*a*b^5*(a + b*tan(c + d*x))^(1/2)*(a^3/(4*d^2) + (b^3*1i)/(4*d^2) - (3*a*b
^2)/(4*d^2) - (a^2*b*3i)/(4*d^2))^(1/2))/((32*a^2*b^6)/d - (a*b^7*16i)/d -
(16*b^8)/d + (a^3*b^5*32i)/d + (48*a^4*b^4)/d + (a^5*b^3*48i)/d) - (a^2*b^4
*(a + b*tan(c + d*x))^(1/2)*(a^3/(4*d^2) + (b^3*1i)/(4*d^2) - (3*a*b^2)/(4*
d^2) - (a^2*b*3i)/(4*d^2))^(1/2)*96i)/((32*a^2*b^6)/d - (a*b^7*16i)/d - (16
*b^8)/d + (a^3*b^5*32i)/d + (48*a^4*b^4)/d + (a^5*b^3*48i)/d) - (96*a^3*b^3
*(a + b*tan(c + d*x))^(1/2)*(a^3/(4*d^2) + (b^3*1i)/(4*d^2) - (3*a*b^2)/(4*
d^2) - (a^2*b*3i)/(4*d^2))^(1/2))/((32*a^2*b^6)/d - (a*b^7*16i)/d - (16*b^8
)/d + (a^3*b^5*32i)/d + (48*a^4*b^4)/d + (a^5*b^3*48i)/d))*(-(3*a*b^2 + a^2
*b*3i - a^3 - b^3*1i)/(4*d^2))^(1/2)*2i - atan((b^6*(a + b*tan(c + d*x))^(1
/2)*(a^3/(4*d^2) - (b^3*1i)/(4*d^2) - (3*a*b^2)/(4*d^2) + (a^2*b*3i)/(4*d^2
))^(1/2)*32i)/((16*b^8)/d - (a*b^7*16i)/d - (32*a^2*b^6)/d + (a^3*b^5*32i)/
d - (48*a^4*b^4)/d + (a^5*b^3*48i)/d) - (32*a*b^5*(a + b*tan(c + d*x))^(1/2
)*(a^3/(4*d^2) - (b^3*1i)/(4*d^2) - (3*a*b^2)/(4*d^2) + (a^2*b*3i)/(4*d^2))
^(1/2))/((16*b^8)/d - (a*b^7*16i)/d - (32*a^2*b^6)/d + (a^3*b^5*32i)/d - (4
8*a^4*b^4)/d + (a^5*b^3*48i)/d) - (a^2*b^4*(a + b*tan(c + d*x))^(1/2)*(a^3/
(4*d^2) - (b^3*1i)/(4*d^2) - (3*a*b^2)/(4*d^2) + (a^2*b*3i)/(4*d^2))^(1/2)*
96i)/((16*b^8)/d - (a*b^7*16i)/d - (32*a^2*b^6)/d + (a^3*b^5*32i)/d - (48*a
^4*b^4)/d + (a^5*b^3*48i)/d) + (96*a^3*b^3*(a + b*tan(c + d*x))^(1/2)*(a^3/
(4*d^2) - (b^3*1i)/(4*d^2) - (3*a*b^2)/(4*d^2) + (a^2*b*3i)/(4*d^2))^(1/2))
/((16*b^8)/d - (a*b^7*16i)/d - (32*a^2*b^6)/d + (a^3*b^5*32i)/d - (48*a^4*b
^4)/d + (a^5*b^3*48i)/d))*(-(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1
/2)*2i + (2*(a + b*tan(c + d*x))^(3/2))/(3*d) + (2*a*(a + b*tan(c + d*x))^(
1/2))/d
```

3.515 $\int (a+b\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=111

$$-\frac{i(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}+\frac{i(a+ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}+\frac{2b\sqrt{a+b\tan(c+dx)}}{d}$$

```
[Out] -I*(a-I*b)^(3/2)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d+I*(a+I*b)^
(3/2)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d+2*b*(a+b*tan(d*x+c))^
(1/2)/d
```

Rubi [A]
time = 0.12, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3563, 3620, 3618, 65, 214}

$$\frac{2b\sqrt{a+b\tan(c+dx)}}{d}-\frac{i(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}+\frac{i(a+ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-I)*(a - I*b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/d +
(I*(a + I*b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/d + (2*
b*Sqrt[a + b*Tan[c + d*x]])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3563

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (a+b\tan(c+dx))^{3/2}\,dx &= \frac{2b\sqrt{a+b\tan(c+dx)}}{d}+\int \frac{a^2-b^2+2ab\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx\\ &= \frac{2b\sqrt{a+b\tan(c+dx)}}{d}+\frac{1}{2}(a-ib)^2\int \frac{1+i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx+\frac{1}{2}(a+ib)^2 \\ &= \frac{2b\sqrt{a+b\tan(c+dx)}}{d}+\frac{\left(i(a-ib)^2\right)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a-ibx}}\,dx,x,i\tan(c\right.}{2d} \\ &= \frac{2b\sqrt{a+b\tan(c+dx)}}{d}-\frac{(a-ib)^2\text{Subst}\left(\int \frac{1}{-1-\frac{ia}{b}+\frac{ix^2}{b}}\,dx,x,\sqrt{a+b\tan(c+d\right.}{bd} \\ &= -\frac{i(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}+\frac{i(a+ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{}}\right.}{d}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 106, normalized size = 0.95

$$\frac{-i(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)+i(a+ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)+2b\sqrt{a+b\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-I)*(a - I*b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] + I*(
a + I*b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] + 2*b*Sqrt[a
 + b*Tan[c + d*x]])/d
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 640 vs. $2(91) = 182$.
time = 0.12, size = 641, normalized size = 5.77

method	result
derivativedivides	$2b\left(\sqrt{a+b\tan(dx+c)}+\frac{\left(-\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a+\sqrt{2\sqrt{a^2+b^2}+2a}\,a^2-\sqrt{2}\right.}{}\right.$
default	$2b\left(\sqrt{a+b\tan(dx+c)}+\frac{\left(-\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a+\sqrt{2\sqrt{a^2+b^2}+2a}\,a^2-\sqrt{2}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*b*((a+b*tan(d*x+c))^(1/2)+1/4/b^2*(1/2*(-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*
(a^2+b^2)^(1/2)*a+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2-(2*(a^2+b^2)^(1/2)+2*a)
^(1/2)*b^2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a
)^(1/2)+(a^2+b^2)^(1/2))+2*(-2*(a^2+b^2)^(1/2)*b^2-1/2*(-(2*(a^2+b^2)^(1/2)
+2*a)^(1/2)*(a^2+b^2)^(1/2)*a+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2-(2*(a^2+b^2
)^(1/2)+2*a)^(1/2)*b^2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2
*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(
2*(a^2+b^2)^(1/2)-2*a)^(1/2)))+1/4/b^2*(-1/2*(-(2*(a^2+b^2)^(1/2)+2*a)^(1/2
)*(a^2+b^2)^(1/2)*a+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2-(2*(a^2+b^2)^(1/2)+2*
a)^(1/2)*b^2)*ln(-b*tan(d*x+c)-a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+
2*a)^(1/2)-(a^2+b^2)^(1/2))+2*(2*(a^2+b^2)^(1/2)*b^2+1/2*(-(2*(a^2+b^2)^(1/
2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2-(2*(a^2+b
^2)^(1/2)+2*a)^(1/2)*b^2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)
```

```
-2*a)^(1/2)*arctan((-2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)
)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4150 vs. $2(85) = 170$.
time = 1.66, size = 4150, normalized size = 37.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*d^5*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^3 - 3*a*b^2
)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4
 + b^6))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(3/4)*sqrt((9*a^4*b^2 -
6*a^2*b^4 + b^6)/d^4)*arctan(((3*a^10 + 11*a^8*b^2 + 14*a^6*b^4 + 6*a^4*b^6
 - a^2*b^8 - b^10)*d^4*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*sqrt((
9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + (3*a^13 + 14*a^11*b^2 + 25*a^9*b^4 + 20
*a^7*b^6 + 5*a^5*b^8 - 2*a^3*b^10 - a*b^12)*d^2*sqrt((9*a^4*b^2 - 6*a^2*b^4
 + b^6)/d^4) + sqrt(2)*((3*a^4*b + 2*a^2*b^3 - b^5)*d^7*sqrt((a^6 + 3*a^4*b
^2 + 3*a^2*b^4 + b^6)/d^4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + 2*(3*a
^7*b + 5*a^5*b^3 + a^3*b^5 - a*b^7)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/
d^4))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^3 - 3*a*b^2)*d^2*sqrt((a
^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt
((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*((a^6 + 3*a^4*b^2 + 3*a^2*
b^4 + b^6)/d^4)^(3/4) + sqrt(2)*(d^7*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6)/d^4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + 2*(a^3 + a*b^2)*d^5*sqrt(
(9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
 + (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^
4*b^2 - 6*a^2*b^4 + b^6))*sqrt(((9*a^8*b^2 + 12*a^6*b^4 - 2*a^4*b^6 - 4*a^2
*b^8 + b^10)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*cos(d*x + c)
 + sqrt(2)*(2*(9*a^5*b^3 - 6*a^3*b^5 + a*b^7)*d^3*sqrt((a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)/d^4)*cos(d*x + c) + (9*a^8*b^3 + 12*a^6*b^5 - 2*a^4*b^7 - 4
```

```
*a^2*b^9 + b^11)*d*cos(d*x + c))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 +
(a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b
^2 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))
*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(1/4) + (9*a^11*b^2 + 21*a^9*b^4
 + 10*a^7*b^6 - 6*a^5*b^8 - 3*a^3*b^10 + a*b^12)*cos(d*x + c) + (9*a^10*b^3
 + 21*a^8*b^5 + 10*a^6*b^7 - 6*a^4*b^9 - 3*a^2*b^11 + b^13)*sin(d*x + c))/(
(a^2 + b^2)*cos(d*x + c)))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(3/4))
/(9*a^14*b^2 + 39*a^12*b^4 + 61*a^10*b^6 + 35*a^8*b^8 - 5*a^6*b^10 - 11*a^4
*b^12 - a^2*b^14 + b^16)) + 4*sqrt(2)*d^5*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6 + (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))
/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(
3/4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4)*arctan(-((3*a^10 + 11*a^8*b^2
+ 14*a^6*b^4 + 6*a^4*b^6 - a^2*b^8 - b^10)*d^4*sqrt((a^6 + 3*a^4*b^2 + 3*a^
2*b^4 + b^6)/d^4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + (3*a^13 + 14*a^
11*b^2 + 25*a^9*b^4 + 20*a^7*b^6 + 5*a^5*b^8 - 2*a^3*b^10 - a*b^12)*d^2*sqr
t((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) - sqrt(2)*((3*a^4*b + 2*a^2*b^3 - b^5)
*d^7*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*sqrt((9*a^4*b^2 - 6*a^2*
b^4 + b^6)/d^4) + 2*(3*a^7*b + 5*a^5*b^3 + a^3*b^5 - a*b^7)*d^5*sqrt((9*a^4
*b^2 - 6*a^2*b^4 + b^6)/d^4))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^
3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2
- 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*((
a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(3/4) - sqrt(2)*(d^7*sqrt((a^6 + 3*
a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + 2
*(a^3 + a*b^2)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4))*sqrt((a^6 + 3*a
^4*b^2 + 3*a^2*b^4 + b^6 + (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^
2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt(((9*a^8*b^2 + 12*a^6
*b^4 - 2*a^4*b^6 - 4*a^2*b^8 + b^10)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4
+ b^6)/d^4)*cos(d*x + c) - sqrt(2)*(2*(9*a^5*b^3 - 6*a^3*b^5 + a*b^7)*d^3*s
qrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*cos(d*x + c) + (9*a^8*b^3 + 12
*a^6*b^5 - 2*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c))*sqrt((a^6 + 3*a^4*
b^2 + 3*a^2*b^4 + b^6 + (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b
^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) + b*sin
(d*x + c))/cos(d*x + c))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(1/4) +
(9*a^11*b^2 + 21*a^9*b^4 + 10*a^7*b^6 - 6*a^5*b^8 - 3*a^3*b^10 + a*b^12)*co
s(d*x + c) + (9*a^10*b^3 + 21*a^8*b^5 + 10*a^6*b^7 - 6*a^4*b^9 - 3*a^2*b^11
 + b^13)*sin(d*x + c))/((a^2 + b^2)*cos(d*x + c)))*((a^6 + 3*a^4*b^2 + 3*a^
2*b^4 + b^6)/d^4)^(3/4))/(9*a^14*b^2 + 39*a^12*b^4 + 61*a^10*b^6 + 35*a^8*b
^8 - 5*a^6*b^10 - 11*a^4*b^12 - a^2*b^14 + b^16)) + sqrt(2)*((a^3 - 3*a*b^2
)*d^3*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4) - (a^6 + 3*a^4*b^2 + 3*
a^2*b^4 + b^6)*d)*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^3 - 3*a*b^2)
*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4
+ b^6))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(1/4)*log(((9*a^8*b^2 + 1
2*a^6*b^4 - 2*a^4*b^6 - 4*a^2*b^8 + b^10)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + b^6)/d^4)*cos(d*x + c) + sqrt(2)*(2*(9*a^5*b^3 - 6*a^3*b^5 + a*b^7)*
d^3*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*cos(d*x + c) + (9*a^8*b^3
```

```
+ 12*a^6*b^5 - 2*a^4*b^7 - 4*a^2*b^9 + b^11)*d...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan{(c+dx)})^{\frac{3}{2}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(3/2),x)
[Out] Integral((a + b*tan(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^(3/2), x)
```

Mupad [B]
time = 5.43, size = 1099, normalized size = 9.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^(3/2),x)
[Out] (2*b*(a + b*tan(c + d*x))^(1/2))/d - atan((b^6*(a + b*tan(c + d*x))^(1/2)*(
(b^3*1i)/(4*d^2) - a^3/(4*d^2) + (3*a*b^2)/(4*d^2) - (a^2*b*3i)/(4*d^2))^(1
/2)*32i)/((a^2*b^6*32i)/d - (16*a*b^7)/d - (b^8*16i)/d + (32*a^3*b^5)/d + (
a^4*b^4*48i)/d + (48*a^5*b^3)/d) - (32*a*b^5*(a + b*tan(c + d*x))^(1/2)*((b
^3*1i)/(4*d^2) - a^3/(4*d^2) + (3*a*b^2)/(4*d^2) - (a^2*b*3i)/(4*d^2))^(1/2
))/((a^2*b^6*32i)/d - (16*a*b^7)/d - (b^8*16i)/d + (32*a^3*b^5)/d + (a^4*b^
4*48i)/d + (48*a^5*b^3)/d) - (a^2*b^4*(a + b*tan(c + d*x))^(1/2)*((b^3*1i)/
(4*d^2) - a^3/(4*d^2) + (3*a*b^2)/(4*d^2) - (a^2*b*3i)/(4*d^2))^(1/2)*96i)/
((a^2*b^6*32i)/d - (16*a*b^7)/d - (b^8*16i)/d + (32*a^3*b^5)/d + (a^4*b^4*4
8i)/d + (48*a^5*b^3)/d) + (96*a^3*b^3*(a + b*tan(c + d*x))^(1/2)*((b^3*1i)/
(4*d^2) - a^3/(4*d^2) + (3*a*b^2)/(4*d^2) - (a^2*b*3i)/(4*d^2))^(1/2))/((a^
2*b^6*32i)/d - (16*a*b^7)/d - (b^8*16i)/d + (32*a^3*b^5)/d + (a^4*b^4*48i)/
d + (48*a^5*b^3)/d))*((3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2)*2i
 - atan((b^6*(a + b*tan(c + d*x))^(1/2)*((3*a*b^2)/(4*d^2) - (b^3*1i)/(4*d^
2) - a^3/(4*d^2) + (a^2*b*3i)/(4*d^2))^(1/2)*32i)/((b^8*16i)/d - (16*a*b^7)
```

/d - (a^2*b^6*32i)/d + (32*a^3*b^5)/d - (a^4*b^4*48i)/d + (48*a^5*b^3)/d) +
 (32*a*b^5*(a + b*tan(c + d*x))^(1/2)*((3*a*b^2)/(4*d^2) - (b^3*1i)/(4*d^2)
 - a^3/(4*d^2) + (a^2*b*3i)/(4*d^2))^(1/2))/((b^8*16i)/d - (16*a*b^7)/d - (
a^2*b^6*32i)/d + (32*a^3*b^5)/d - (a^4*b^4*48i)/d + (48*a^5*b^3)/d) - (a^2*
b^4*(a + b*tan(c + d*x))^(1/2)*((3*a*b^2)/(4*d^2) - (b^3*1i)/(4*d^2) - a^3/
(4*d^2) + (a^2*b*3i)/(4*d^2))^(1/2)*96i)/((b^8*16i)/d - (16*a*b^7)/d - (a^2
*b^6*32i)/d + (32*a^3*b^5)/d - (a^4*b^4*48i)/d + (48*a^5*b^3)/d) - (96*a^3*
b^3*(a + b*tan(c + d*x))^(1/2)*((3*a*b^2)/(4*d^2) - (b^3*1i)/(4*d^2) - a^3/
(4*d^2) + (a^2*b*3i)/(4*d^2))^(1/2))/((b^8*16i)/d - (16*a*b^7)/d - (a^2*b^6
*32i)/d + (32*a^3*b^5)/d - (a^4*b^4*48i)/d + (48*a^5*b^3)/d))*((3*a*b^2 + a
^2*b*3i - a^3 - b^3*1i)/(4*d^2))^(1/2)*2i

3.516 $\int \cot(c+dx)(a+b\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=116

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}+\frac{(a+ib)^{3/2}\tanh^{-1}\left(\sqrt{\cdots}\right.}{d}$$

```
[Out] -2*a^(3/2)*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/d+(a-I*b)^(3/2)*arctanh(
(a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d+(a+I*b)^(3/2)*arctanh((a+b*tan(d*x+
c))^(1/2)/(a+I*b)^(1/2))/d
```

Rubi [A]
time = 0.21, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3654, 3620, 3618, 65, 214, 3715}

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}+\frac{(a+ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^(3/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/d + ((a - I*b)^(3/2)
*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/d + ((a + I*b)^(3/2)*ArcT
anh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3654

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[Simp[a^2*c - b^2*c + 2
*a*b*d + (2*a*b*c - a^2*d + b^2*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x]
], x], x] + Dist[(b*c - a*d)^2/(c^2 + d^2), Int[(1 + Tan[e + f*x]^2)/(Sqrt[
a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \cot(c+dx)(a+b\tan(c+dx))^{3/2}\,dx = a^2\int \frac{\cot(c+dx)\left(1+\tan^2(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}\,dx + \int \frac{2ab-(a^2-b^2)\tan(}{\sqrt{a+b\tan(c+}}$$

$$= \frac{1}{2}\left(i(a-ib)^2\right)\int \frac{1+i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx - \frac{1}{2}\left(i(a+ib)^2\right)\int \frac{1-}{\sqrt{a}}$$

$$= -\frac{(a-ib)^2\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a-ibx}}\,dx, x, i\tan(c+dx)\right)}{2d} - \frac{(a+}{}$$

$$= -\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{\left(i(a-ib)^2\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{}}{}\right.}{d}$$

Mathematica [A]

time = 0.22, size = 111, normalized size = 0.96

$$\frac{-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)+(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)+(a+ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^(3/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]] + (a - I*b)^(3/2)*Arc
Tanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] + (a + I*b)^(3/2)*ArcTanh[Sqrt
[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/d
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.85, size = 22251, normalized size = 191.82

method	result	size
default	Expression too large to display	22251

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(3/2)*cot(d*x + c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4163 vs. 2(90) = 180.
time = 3.72, size = 8401, normalized size = 72.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(2)*d^5*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^3 - 3*a*b^
2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^
4 + b^6))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(3/4)*sqrt((9*a^4*b^2 -
```

6*a^2*b^4 + b^6)/d^4)*arctan(((3*a^10 + 11*a^8*b^2 + 14*a^6*b^4 + 6*a^4*b^
6 - a^2*b^8 - b^10)*d^4*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*sqrt(
(9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + (3*a^13 + 14*a^11*b^2 + 25*a^9*b^4 + 2
0*a^7*b^6 + 5*a^5*b^8 - 2*a^3*b^10 - a*b^12)*d^2*sqrt((9*a^4*b^2 - 6*a^2*b^
4 + b^6)/d^4) + sqrt(2)*((3*a^5 + 2*a^3*b^2 - a*b^4)*d^7*sqrt((a^6 + 3*a^4*
b^2 + 3*a^2*b^4 + b^6)/d^4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + (3*a^
8 + 2*a^6*b^2 - 4*a^4*b^4 - 2*a^2*b^6 + b^8)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^
4 + b^6)/d^4))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^3 - 3*a*b^2)*d^
2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b
^6))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*((a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6)/d^4)^(3/4) + sqrt(2)*(a*d^7*sqrt((a^6 + 3*a^4*b^2 + 3*a
^2*b^4 + b^6)/d^4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + (a^4 - b^4)*d^
5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^
4 + b^6 - (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)
)/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt(((9*a^8 + 12*a^6*b^2 - 2*a^4*b^4 - 4*
a^2*b^6 + b^8)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*cos(d*x +
c) + sqrt(2)*((9*a^6 - 15*a^4*b^2 + 7*a^2*b^4 - b^6)*d^3*sqrt((a^6 + 3*a^4*
b^2 + 3*a^2*b^4 + b^6)/d^4)*cos(d*x + c) + (9*a^9 + 12*a^7*b^2 - 2*a^5*b^4
 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6 - (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a
^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x +
 c))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(1/4) + (9*a^11 + 21*a^9*b^2
 + 10*a^7*b^4 - 6*a^5*b^6 - 3*a^3*b^8 + a*b^10)*cos(d*x + c) + (9*a^10*b +
21*a^8*b^3 + 10*a^6*b^5 - 6*a^4*b^7 - 3*a^2*b^9 + b^11)*sin(d*x + c))/((a^2
 + b^2)*cos(d*x + c)))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(3/4))/(9*
a^14*b^2 + 39*a^12*b^4 + 61*a^10*b^6 + 35*a^8*b^8 - 5*a^6*b^10 - 11*a^4*b^1
2 - a^2*b^14 + b^16)) + 4*sqrt(2)*d^5*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6 - (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*
a^4*b^2 - 6*a^2*b^4 + b^6))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(3/4)
*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4)*arctan(-((3*a^10 + 11*a^8*b^2 + 14
*a^6*b^4 + 6*a^4*b^6 - a^2*b^8 - b^10)*d^4*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^
4 + b^6)/d^4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + (3*a^13 + 14*a^11*b
^2 + 25*a^9*b^4 + 20*a^7*b^6 + 5*a^5*b^8 - 2*a^3*b^10 - a*b^12)*d^2*sqrt((9
*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) - sqrt(2)*((3*a^5 + 2*a^3*b^2 - a*b^4)*d^7
*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*sqrt((9*a^4*b^2 - 6*a^2*b^4
 + b^6)/d^4) + (3*a^8 + 2*a^6*b^2 - 4*a^4*b^4 - 2*a^2*b^6 + b^8)*d^5*sqrt((9
*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 -
 (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*
b^2 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c)
)*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(3/4) - sqrt(2)*(a*d^7*sqrt((a^
6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^
4) + (a^4 - b^4)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4))*sqrt((a^6 + 3
*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*
a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt(((9*a^8 + 12*a^6*b
^2 - 2*a^4*b^4 - 4*a^2*b^6 + b^8)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b

```
^6)/d^4)*cos(d*x + c) - sqrt(2)*((9*a^6 - 15*a^4*b^2 + 7*a^2*b^4 - b^6)*d^3
*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*cos(d*x + c) + (9*a^9 + 12*a
^7*b^2 - 2*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c))*sqrt((a^6 + 3*a^4*b
^2 + 3*a^2*b^4 + b^6 - (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^
4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) + b*sin(
d*x + c))/cos(d*x + c))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(1/4) + (
9*a^11 + 21*a^9*b^2 + 10*a^7*b^4 - 6*a^5*b^6 - 3*a^3*b^8 + a*b^10)*cos(d*x
+ c) + (9*a^10*b + 21*a^8*b^3 + 10*a^6*b^5 - 6*a^4*b^7 - 3*a^2*b^9 + b^11)*
sin(d*x + c))/((a^2 + b^2)*cos(d*x + c)))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6)/d^4)^(3/4))/(9*a^14*b^2 + 39*a^12*b^4 + 61*a^10*b^6 + 35*a^8*b^8 - 5*a^
6*b^10 - 11*a^4*b^12 - a^2*b^14 + b^16)) + sqrt(2)*((a^3 - 3*a*b^2)*d^3*sqr
t((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4) + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
 b^6)*d)*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^3 - 3*a*b^2)*d^2*sqrt
((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*(
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(1/4)*log(((9*a^8 + 12*a^6*b^2 - 2
*a^4*b^4 - 4*a^2*b^6 + b^8)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^
4)*cos(d*x + c) + sqrt(2)*((9*a^6 - 15*a^4*b^2 + 7*a^2*b^4 - b^6)*d^3*sqrt(
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*cos(d*x + c) + (9*a^9 + 12*a^7*b^2
 - 2*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan{(c + dx)})^{\frac{3}{2}} \cot{(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**(3/2)*cot(c + d*x), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 4.67, size = 2260, normalized size = 19.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d*x)*(a + b*tan(c + d*x))^(3/2),x)

[Out] - atan((32*a*b^15*(a + b*tan(c + d*x))^(1/2)*(a^3/(4*d^2) - (b^3*1i)/(4*d^2) - (3*a*b^2)/(4*d^2) + (a^2*b*3i)/(4*d^2))^(1/2))/((a*b^17*16i)/d + (64*a^2*b^16)/d - (a^3*b^15*128i)/d - (128*a^4*b^14)/d + (a^5*b^13*96i)/d - (192*a^6*b^12)/d + (a^7*b^11*384i)/d + (a^9*b^9*144i)/d) - (a^2*b^14*(a + b*tan(c + d*x))^(1/2)*(a^3/(4*d^2) - (b^3*1i)/(4*d^2) - (3*a*b^2)/(4*d^2) + (a^2*b*3i)/(4*d^2))^(1/2)*64i)/((a*b^17*16i)/d + (64*a^2*b^16)/d - (a^3*b^15*128i)/d - (128*a^4*b^14)/d + (a^5*b^13*96i)/d - (192*a^6*b^12)/d + (a^7*b^11*384i)/d + (a^9*b^9*144i)/d) - (96*a^3*b^13*(a + b*tan(c + d*x))^(1/2)*(a^3/(4*d^2) - (b^3*1i)/(4*d^2) - (3*a*b^2)/(4*d^2) + (a^2*b*3i)/(4*d^2))^(1/2))/((a*b^17*16i)/d + (64*a^2*b^16)/d - (a^3*b^15*128i)/d - (128*a^4*b^14)/d + (a^5*b^13*96i)/d - (192*a^6*b^12)/d + (a^7*b^11*384i)/d + (a^9*b^9*144i)/d) + (96*a^5*b^11*(a + b*tan(c + d*x))^(1/2)*(a^3/(4*d^2) - (b^3*1i)/(4*d^2) - (3*a*b^2)/(4*d^2) + (a^2*b*3i)/(4*d^2))^(1/2))/((a*b^17*16i)/d + (64*a^2*b^16)/d - (a^3*b^15*128i)/d - (128*a^4*b^14)/d + (a^5*b^13*96i)/d - (192*a^6*b^12)/d + (a^7*b^11*384i)/d + (a^9*b^9*144i)/d) + (a^6*b^10*(a + b*tan(c + d*x))^(1/2)*(a^3/(4*d^2) - (b^3*1i)/(4*d^2) - (3*a*b^2)/(4*d^2) + (a^2*b*3i)/(4*d^2))^(1/2)*576i)/((a*b^17*16i)/d + (64*a^2*b^16)/d - (a^3*b^15*128i)/d - (128*a^4*b^14)/d + (a^5*b^13*96i)/d - (192*a^6*b^12)/d + (a^7*b^11*384i)/d + (a^9*b^9*144i)/d) - (288*a^7*b^9*(a + b*tan(c + d*x))^(1/2)*(a^3/(4*d^2) - (b^3*1i)/(4*d^2) - (3*a*b^2)/(4*d^2) + (a^2*b*3i)/(4*d^2))^(1/2))/((a*b^17*16i)/d + (64*a^2*b^16)/d - (a^3*b^15*128i)/d - (128*a^4*b^14)/d + (a^5*b^13*96i)/d - (192*a^6*b^12)/d + (a^7*b^11*384i)/d + (a^9*b^9*144i)/d))*(-(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2)*2i - atan((32*a*b^15*(a + b*tan(c + d*x))^(1/2)*(a^3/(4*d^2) + (b^3*1i)/(4*d^2) - (3*a*b^2)/(4*d^2) - (a^2*b*3i)/(4*d^2))^(1/2))/((a*b^17*16i)/d - (64*a^2*b^16)/d - (a^3*b^15*128i)/d + (128*a^4*b^14)/d + (a^5*b^13*96i)/d + (192*a^6*b^12)/d + (a^7*b^11*384i)/d + (a^9*b^9*144i)/d) + (a^2*b^14*(a + b*tan(c + d*x))^(1/2)*(a^3/(4*d^2) + (b^3*1i)/(4*d^2) - (3*a*b^2)/(4*d^2) - (a^2*b*3i)/(4*d^2))^(1/2)*64i)/((a*b^17*16i)/d - (64*a^2*b^16)/d - (a^3*b^15*128i)/d + (128*a^4*b^14)/d + (a^5*b^13*96i)/d + (192*a^6*b^12)/d + (a^7*b^11*384i)/d + (a^9*b^9*144i)/d) - (96*a^3*b^13*(a + b*tan(c + d*x))^(1/2)*(a^3/(4*d^2) + (b^3*1i)/(4*d^2) - (3*a*b^2)/(4*d^2) - (a^2*b*3i)/(4*d^2))^(1/2))/((a*b^17*16i)/d - (64*a^2*b^16)/d - (a^3*b^15*128i)/d + (128*a^4*b^14)/d + (a^5*b^13*96i)/d + (192*a^6*b^12)/d + (a^7*b^11*384i)/d + (a^9*b^9*144i)/d) + (96*a^5*b^11*(a + b*tan(c + d*x))^(1/2)*(a^3/(4*d^2) + (b^3*1i)/(4*d^2) - (3*a*b^2)/(4*d^2) - (a^2*b*3i)/(4*d^2))^(1/2))/((a*b^17*16i)/d - (64*a^2*b^16)/d - (a^3*b^15*128i)/d + (128*a^4*b^14)/d + (a^5*b^13*96i)/d + (192*a^6*b^12)/d + (a^7*b^11*384i)/d + (a^9*b^9*144i)/d) - (a^6*b^10*(a + b*tan(c + d*x))^(1/2)*(a^3/(4*d^2) + (b^3*1i)/(4*d^2) - (3*a*b^2)/(4*d^2) - (a^2*b*3i)/(4*d^2))^(1/2)*576i)/((a*b^17*16i)/d - (64*a^2*b^16)/d - (a^3*b^15*128i)/d + (128*a^4*b^14)/d + (a^5*b^13*96i)/d + (192*a^6*b^12)/d + (a^7*b^11*384i)/d + (a^9*b^9*144i)/d) - (288*a^7*b^9*(a + b*tan(c + d*x))^(1/2)*(a^3/(4*d^2) + (b^3*1i)/(4*d^2) - (3*a*b^2)/(4*d^2) - (a^2*b*3i)/(4*d^2))^(1/2))/((a*b^17*16i)/d - (

64*a^2*b^16)/d - (a^3*b^15*128i)/d + (128*a^4*b^14)/d + (a^5*b^13*96i)/d + (192*a^6*b^12)/d + (a^7*b^11*384i)/d + (a^9*b^9*144i)/d))*(-(3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)/(4*d^2))^(1/2)*2i - (2*atanh((64*b^16*(a^3)^(1/2)*(a + b*tan(c + d*x))^(1/2))/(64*a^2*b^16 + 256*a^4*b^14 + 1920*a^6*b^12 + 2304*a^8*b^10 + 576*a^10*b^8) + (256*a^2*b^14*(a^3)^(1/2)*(a + b*tan(c + d*x))^(1/2))/(64*a^2*b^16 + 256*a^4*b^14 + 1920*a^6*b^12 + 2304*a^8*b^10 + 576*a^10*b^8) + (1920*a^4*b^12*(a^3)^(1/2)*(a + b*tan(c + d*x))^(1/2))/(64*a^2*b^16 + 256*a^4*b^14 + 1920*a^6*b^12 + 2304*a^8*b^10 + 576*a^10*b^8) + (2304*a^6*b^10*(a^3)^(1/2)*(a + b*tan(c + d*x))^(1/2))/(64*a^2*b^16 + 256*a^4*b^14 + 1920*a^6*b^12 + 2304*a^8*b^10 + 576*a^10*b^8) + (576*a^8*b^8*(a^3)^(1/2)*(a + b*tan(c + d*x))^(1/2))/(64*a^2*b^16 + 256*a^4*b^14 + 1920*a^6*b^12 + 2304*a^8*b^10 + 576*a^10*b^8))*(a^3)^(1/2))/d

3.517 $\int \cot^2(c+dx)(a+b\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=149

$$-\frac{3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{i(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}-\frac{i(a+ib)^{3/2}\tanh^{-1}}{}$$

[Out] I*(a-I*b)^(3/2)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d-I*(a+I*b)^(3/2)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d-3*b*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-a*cot(d*x+c)*(a+b*tan(d*x+c))^(1/2)/d

Rubi [A]
time = 0.30, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3648, 3734, 3620, 3618, 65, 214, 3715}

$$\frac{i(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}-\frac{3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{i(a+ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}-\frac{a\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] (-3*Sqrt[a]*b*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/d + (I*(a - I*b)^(
3/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/d - (I*(a + I*b)^(3/2
)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/d - (a*Cot[c + d*x]*Sqrt
[a + b*Tan[c + d*x]])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
```

```
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^{3/2}\,dx = -\frac{a\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d} - \int \frac{\cot(c+dx)\left(-\frac{3ab}{2}+(\right.}{}$$

$$= -\frac{a\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d} + \frac{1}{2}(3ab)\int \frac{\cot(c+dx)(1}{\sqrt{a+b\,\mathrm{ta}}}$$

$$= -\frac{a\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d} - \frac{1}{2}(a-ib)^2\int \frac{1+i\tan(}{\sqrt{a+b\tan}}$$

$$= -\frac{a\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d} + \frac{(3a)\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,\right.}{d}$$

$$= -\frac{3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{a\cot(c+dx)\sqrt{a+}}{d}$$

$$= -\frac{3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{i(a-ib)^{3/2}\tanh^{-1}\left(\right.}{}$$

Mathematica [A]
time = 0.40, size = 186, normalized size = 1.25

$$\frac{-3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)+\sqrt{a-ib}\,(ia+b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)-ia\sqrt{a+ib}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)+\sqrt{a+ib}\,b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)-a\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] (-3*Sqrt[a]*b*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]] + Sqrt[a - I*b]*(I*
a + b)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] - I*a*Sqrt[a + I*b]*
ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] + Sqrt[a + I*b]*b*ArcTanh[S
qrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] - a*Cot[c + d*x]*Sqrt[a + b*Tan[c +
d*x]])/d
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 1.09, size = 35890, normalized size = 240.87

method	result	size
default	Expression too large to display	35890

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4449 vs. $2(117) = 234$.
time = 2.27, size = 8973, normalized size = 60.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(2)*(d^5*cos(d*x + c)^2 - d^5)*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b
^4 + b^6 + (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4
))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)
^(3/4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4)*arctan(((3*a^10 + 11*a^8*b^2
 + 14*a^6*b^4 + 6*a^4*b^6 - a^2*b^8 - b^10)*d^4*sqrt((a^6 + 3*a^4*b^2 + 3*a
^2*b^4 + b^6)/d^4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + (3*a^13 + 14*a
^11*b^2 + 25*a^9*b^4 + 20*a^7*b^6 + 5*a^5*b^8 - 2*a^3*b^10 - a*b^12)*d^2*sq
rt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + sqrt(2)*((3*a^4*b + 2*a^2*b^3 - b^5
)*d^7*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*sqrt((9*a^4*b^2 - 6*a^2
*b^4 + b^6)/d^4) + 2*(3*a^7*b + 5*a^5*b^3 + a^3*b^5 - a*b^7)*d^5*sqrt((9*a^
4*b^2 - 6*a^2*b^4 + b^6)/d^4))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a
^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2
 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*(
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(3/4) + sqrt(2)*(d^7*sqrt((a^6 + 3
*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) +
2*(a^3 + a*b^2)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4))*sqrt((a^6 + 3*
a^4*b^2 + 3*a^2*b^4 + b^6 + (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a
^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt(((9*a^8*b^2 + 12*a^
6*b^4 - 2*a^4*b^6 - 4*a^2*b^8 + b^10)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6)/d^4)*cos(d*x + c) + sqrt(2)*(2*(9*a^5*b^3 - 6*a^3*b^5 + a*b^7)*d^3*
sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*cos(d*x + c) + (9*a^8*b^3 + 1
2*a^6*b^5 - 2*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c))*sqrt((a^6 + 3*a^4
*b^2 + 3*a^2*b^4 + b^6 + (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*
b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) + b*si
n(d*x + c))/cos(d*x + c))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(1/4) +
```

```
(9*a^11*b^2 + 21*a^9*b^4 + 10*a^7*b^6 - 6*a^5*b^8 - 3*a^3*b^10 + a*b^12)*c
os(d*x + c) + (9*a^10*b^3 + 21*a^8*b^5 + 10*a^6*b^7 - 6*a^4*b^9 - 3*a^2*b^1
1 + b^13)*sin(d*x + c))/((a^2 + b^2)*cos(d*x + c)))*((a^6 + 3*a^4*b^2 + 3*a
^2*b^4 + b^6)/d^4)^(3/4))/(9*a^14*b^2 + 39*a^12*b^4 + 61*a^10*b^6 + 35*a^8*
b^8 - 5*a^6*b^10 - 11*a^4*b^12 - a^2*b^14 + b^16)) + 4*sqrt(2)*(d^5*cos(d*x
 + c)^2 - d^5)*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^3 - 3*a*b^2)*d^
2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b
^6))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(3/4)*sqrt((9*a^4*b^2 - 6*a^
2*b^4 + b^6)/d^4)*arctan(-((3*a^10 + 11*a^8*b^2 + 14*a^6*b^4 + 6*a^4*b^6 -
a^2*b^8 - b^10)*d^4*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*sqrt((9*a
^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + (3*a^13 + 14*a^11*b^2 + 25*a^9*b^4 + 20*a^
7*b^6 + 5*a^5*b^8 - 2*a^3*b^10 - a*b^12)*d^2*sqrt((9*a^4*b^2 - 6*a^2*b^4 +
b^6)/d^4) - sqrt(2)*((3*a^4*b + 2*a^2*b^3 - b^5)*d^7*sqrt((a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6)/d^4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + 2*(3*a^7*
b + 5*a^5*b^3 + a^3*b^5 - a*b^7)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4
))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^3 - 3*a*b^2)*d^2*sqrt((a^6
+ 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((a
*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6)/d^4)^(3/4) - sqrt(2)*(d^7*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/
d^4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + 2*(a^3 + a*b^2)*d^5*sqrt((9*
a^4*b^2 - 6*a^2*b^4 + b^6)/d^4))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 +
(a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b
^2 - 6*a^2*b^4 + b^6))*sqrt(((9*a^8*b^2 + 12*a^6*b^4 - 2*a^4*b^6 - 4*a^2*b^
8 + b^10)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*cos(d*x + c) -
sqrt(2)*(2*(9*a^5*b^3 - 6*a^3*b^5 + a*b^7)*d^3*sqrt((a^6 + 3*a^4*b^2 + 3*a^
2*b^4 + b^6)/d^4)*cos(d*x + c) + (9*a^8*b^3 + 12*a^6*b^5 - 2*a^4*b^7 - 4*a^
2*b^9 + b^11)*d*cos(d*x + c))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^
3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2
- 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*((
a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(1/4) + (9*a^11*b^2 + 21*a^9*b^4 +
10*a^7*b^6 - 6*a^5*b^8 - 3*a^3*b^10 + a*b^12)*cos(d*x + c) + (9*a^10*b^3 +
21*a^8*b^5 + 10*a^6*b^7 - 6*a^4*b^9 - 3*a^2*b^11 + b^13)*sin(d*x + c))/((a^
2 + b^2)*cos(d*x + c)))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(3/4))/(9
*a^14*b^2 + 39*a^12*b^4 + 61*a^10*b^6 + 35*a^8*b^8 - 5*a^6*b^10 - 11*a^4*b^
12 - a^2*b^14 + b^16)) - 4*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*sqrt((a*co
s(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - sqrt
(2)*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d*cos(d*x + c)^2 - (a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6)*d - ((a^3 - 3*a*b^2)*d^3*cos(d*x + c)^2 - (a^3 - 3*a*b
^2)*d^3)*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))*sqrt((a^6 + 3*a^4*b
^2 + 3*a^2*b^4 + b^6 + (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^
4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6)/d^4)^(1/4)*log(((9*a^8*b^2 + 12*a^6*b^4...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))^{\frac{3}{2}}\cot^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**(3/2)*cot(c + d*x)**2, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 4.74, size = 2500, normalized size = 16.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + b*tan(c + d*x))^(3/2),x)
```

```
[Out] (a*b*(a + b*tan(c + d*x))^(1/2))/(a*d - d*(a + b*tan(c + d*x))) - atan(((((
((16*(40*a*b^11*d^4 + 40*a^3*b^9*d^4))/d^5 - (16*(32*b^10*d^4 + 48*a^2*b^8*
d^4)*(a + b*tan(c + d*x))^(1/2)*((3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)/(4*d^2
))^(1/2))/d^4)*((3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)/(4*d^2))^(1/2) + (16*(a
 + b*tan(c + d*x))^(1/2)*(44*a*b^12*d^2 + 92*a^3*b^10*d^2 - 20*a^5*b^8*d^2)
)/d^4)*((3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)/(4*d^2))^(1/2) - (16*(50*a^2*b^
13*d^2 + 22*a^4*b^11*d^2 - 28*a^6*b^9*d^2))/d^5)*((3*a*b^2 + a^2*b*3i - a^3
 - b^3*1i)/(4*d^2))^(1/2) - (16*(a + b*tan(c + d*x))^(1/2)*(2*b^16 - a^2*b^
14 + 66*a^4*b^12 - a^6*b^10 + 2*a^8*b^8))/d^4)*((3*a*b^2 + a^2*b*3i - a^3 -
 b^3*1i)/(4*d^2))^(1/2)*1i - (((((16*(40*a*b^11*d^4 + 40*a^3*b^9*d^4))/d^5
+ (16*(32*b^10*d^4 + 48*a^2*b^8*d^4)*(a + b*tan(c + d*x))^(1/2)*((3*a*b^2 +
 a^2*b*3i - a^3 - b^3*1i)/(4*d^2))^(1/2))/d^4)*((3*a*b^2 + a^2*b*3i - a^3 -
 b^3*1i)/(4*d^2))^(1/2) - (16*(a + b*tan(c + d*x))^(1/2)*(44*a*b^12*d^2 + 9
2*a^3*b^10*d^2 - 20*a^5*b^8*d^2))/d^4)*((3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)
/(4*d^2))^(1/2) - (16*(50*a^2*b^13*d^2 + 22*a^4*b^11*d^2 - 28*a^6*b^9*d^2))
/d^5)*((3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)/(4*d^2))^(1/2) + (16*(a + b*tan(
c + d*x))^(1/2)*(2*b^16 - a^2*b^14 + 66*a^4*b^12 - a^6*b^10 + 2*a^8*b^8))/d
^4)*((3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)/(4*d^2))^(1/2)*1i)/((32*(3*a*b^17
```

+ 3*a^3*b^15 + 3*a^7*b^11 + 3*a^9*b^9))/d^5 + (((((16*(40*a*b^11*d^4 + 40*a^3*b^9*d^4))/d^5 - (16*(32*b^10*d^4 + 48*a^2*b^8*d^4)*(a + b*tan(c + d*x))^(1/2)*((3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)/(4*d^2))^(1/2))/d^4)*((3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)/(4*d^2))^(1/2) + (16*(a + b*tan(c + d*x))^(1/2)*(44*a*b^12*d^2 + 92*a^3*b^10*d^2 - 20*a^5*b^8*d^2))/d^4)*((3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)/(4*d^2))^(1/2) - (16*(50*a^2*b^13*d^2 + 22*a^4*b^11*d^2 - 28*a^6*b^9*d^2))/d^5)*((3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)/(4*d^2))^(1/2) - (16*(a + b*tan(c + d*x))^(1/2)*(2*b^16 - a^2*b^14 + 66*a^4*b^12 - a^6*b^10 + 2*a^8*b^8))/d^4)*((3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)/(4*d^2))^(1/2) + (((((16*(40*a*b^11*d^4 + 40*a^3*b^9*d^4))/d^5 + (16*(32*b^10*d^4 + 48*a^2*b^8*d^4)*(a + b*tan(c + d*x))^(1/2)*((3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)/(4*d^2))^(1/2))/d^4)*((3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)/(4*d^2))^(1/2) - (16*(a + b*tan(c + d*x))^(1/2)*(44*a*b^12*d^2 + 92*a^3*b^10*d^2 - 20*a^5*b^8*d^2))/d^4)*((3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)/(4*d^2))^(1/2) - (16*(50*a^2*b^13*d^2 + 22*a^4*b^11*d^2 - 28*a^6*b^9*d^2))/d^5)*((3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)/(4*d^2))^(1/2) + (16*(a + b*tan(c + d*x))^(1/2)*(2*b^16 - a^2*b^14 + 66*a^4*b^12 - a^6*b^10 + 2*a^8*b^8))/d^4)*((3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)/(4*d^2))^(1/2)))*((3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)/(4*d^2))^(1/2)*2i - atan(((((((16*(40*a*b^11*d^4 + 40*a^3*b^9*d^4))/d^5 - (16*(32*b^10*d^4 + 48*a^2*b^8*d^4)*(a + b*tan(c + d*x))^(1/2)*((3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2))/d^4)*((3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2) + (16*(a + b*tan(c + d*x))^(1/2)*(44*a*b^12*d^2 + 92*a^3*b^10*d^2 - 20*a^5*b^8*d^2))/d^4)*((3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2) - (16*(50*a^2*b^13*d^2 + 22*a^4*b^11*d^2 - 28*a^6*b^9*d^2))/d^5)*((3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2) - (16*(a + b*tan(c + d*x))^(1/2)*(2*b^16 - a^2*b^14 + 66*a^4*b^12 - a^6*b^10 + 2*a^8*b^8))/d^4)*((3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2)*1i - (((((16*(40*a*b^11*d^4 + 40*a^3*b^9*d^4))/d^5 + (16*(32*b^10*d^4 + 48*a^2*b^8*d^4)*(a + b*tan(c + d*x))^(1/2)*((3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2))/d^4)*((3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2) - (16*(a + b*tan(c + d*x))^(1/2)*(44*a*b^12*d^2 + 92*a^3*b^10*d^2 - 20*a^5*b^8*d^2))/d^4)*((3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2) - (16*(50*a^2*b^13*d^2 + 22*a^4*b^11*d^2 - 28*a^6*b^9*d^2))/d^5)*((3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2) + (16*(a + b*tan(c + d*x))^(1/2)*(2*b^16 - a^2*b^14 + 66*a^4*b^12 - a^6*b^10 + 2*a^8*b^8))/d^4)*((3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2)*1i)/((32*(3*a*b^17 + 3*a^3*b^15 + 3*a^7*b^11 + 3*a^9*b^9))/d^5 + (((((16*(40*a*b^11*d^4 + 40*a^3*b^9*d^4))/d^5 - (16*(32*b^10*d^4 + 48*a^2*b^8*d^4)*(a + b*tan(c + d*x))^(1/2)*((3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2))/d^4)*((3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2) + (16*(a + b*tan(c + d*x))^(1/2)*(44*a*b^12*d^2 + 92*a^3*b^10*d^2 - 20*a^5*b^8*d^2))/d^4)*((3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2) - (16*(50*a^2*b^13*d^2 + 22*a^4*b^11*d^2 - 28*a^6*b^9*d^2))/d^5)*((3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2) - (16*(a + b*tan(c + d*x))^(1/2)*(2*b^16 - a^2*b^14 + 66*a^4*b^12 - a^6*b^10 + 2*a^8*b^8))/d^4)*((3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2) + (((((16*(40*a*b^11*d^4 + 40*a^3*b^9*d^4))/d^5 + (16*(32

```
*b^10*d^4 + 48*a^2*b^8*d^4)*(a + b*tan(c + d*x))^(1/2)*((3*a*b^2 - a^2*b*3i
 - a^3 + b^3*1i)/(4*d^2))^(1/2))/d^4)*((3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/
(4*d^2))^(1/2) - (16*(a + b*tan(c + d*x))^(1/2)*(44*a*b^12*d^2 + 92*a^3*b^1
0*d^2 - 20*a^5*b^8*d^2))/d^4)*((3*a*b^2 - a^2*b...
```

3.518 $\int \cot^3(c+dx)(a+b\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=189

$$\frac{(8a^2-3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{4\sqrt{a}\,d}-\frac{(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}-\frac{(a+ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}-\ldots$$

```
[Out] -(a-I*b)^(3/2)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d-(a+I*b)^(3/2
)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d+1/4*(8*a^2-3*b^2)*arctanh
((a+b*tan(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)-5/4*b*cot(d*x+c)*(a+b*tan(d*x+c)
)^(1/2)/d-1/2*a*cot(d*x+c)^2*(a+b*tan(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.47, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3648, 3730, 3734, 3620, 3618, 65, 214, 3715}

$$\frac{(8a^2-3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{4\sqrt{a}\,d}-\frac{(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}-\frac{(a+ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}-\frac{a\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}-\frac{5b\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((8*a^2 - 3*b^2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/(4*Sqrt[a]*d) -
 ((a - I*b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/d - ((a
+ I*b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/d - (5*b*Cot[
c + d*x]*Sqrt[a + b*Tan[c + d*x]])/(4*d) - (a*Cot[c + d*x]^2*Sqrt[a + b*Tan
[c + d*x]])/(2*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
```

```
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^{3/2}\,dx = -\frac{a\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d} - \frac{1}{2}\int \frac{\cot^2(c+dx)\left(-\frac{5ab}{2}\right.}{}$$

$$= -\frac{5b\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{4d} - \frac{a\cot^2(c+dx)\sqrt{a+b\tan}}{2d}$$

$$= -\frac{5b\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{4d} - \frac{a\cot^2(c+dx)\sqrt{a+b\tan}}{2d}$$

$$= -\frac{5b\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{4d} - \frac{a\cot^2(c+dx)\sqrt{a+b\tan}}{2d}$$

$$= -\frac{5b\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{4d} - \frac{a\cot^2(c+dx)\sqrt{a+b\tan}}{2d}$$

$$= \frac{(8a^2-3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{4\sqrt{a}\,d} - \frac{5b\cot(c+dx)\sqrt{}}{4}$$

$$= \frac{(8a^2-3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{4\sqrt{a}\,d} - \frac{(a-ib)^{3/2}\tanh^{-}}{}$$

Mathematica [A]
time = 1.59, size = 168, normalized size = 0.89

$$\frac{(8a^2-3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right) - \sqrt{a}\left(4(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right) + 4(a+ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right) + \cot(c+dx)(5b+2a\cot(c+dx))\sqrt{a+b\tan(c+dx)}\right)}{4\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((8*a^2 - 3*b^2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]] - Sqrt[a]*(4*(a
- I*b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] + 4*(a + I*b)^
(3/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] + Cot[c + d*x]*(5*b +
 2*a*Cot[c + d*x])*Sqrt[a + b*Tan[c + d*x]]))/(4*Sqrt[a]*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 1.19, size = 54149, normalized size = 286.50

method	result	size
default	Expression too large to display	54149

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4533 vs. $2(153) = 306$.
time = 2.61, size = 9142, normalized size = 48.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(16*sqrt(2)*(a*d^5*cos(d*x + c)^2 - a*d^5)*sqrt((a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6 - (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
)/d^4)^(3/4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4)*arctan(((3*a^10 + 11*a
^8*b^2 + 14*a^6*b^4 + 6*a^4*b^6 - a^2*b^8 - b^10)*d^4*sqrt((a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6)/d^4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + (3*a^13
+ 14*a^11*b^2 + 25*a^9*b^4 + 20*a^7*b^6 + 5*a^5*b^8 - 2*a^3*b^10 - a*b^12)*
d^2*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + sqrt(2)*((3*a^5 + 2*a^3*b^2 -
 a*b^4)*d^7*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*sqrt((9*a^4*b^2 -
 6*a^2*b^4 + b^6)/d^4) + (3*a^8 + 2*a^6*b^2 - 4*a^4*b^4 - 2*a^2*b^6 + b^8)*
d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*
```

b^4 + b^6 - (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(3/4) + sqrt(2)*(a*d^7*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + (a^4 - b^4)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt(((9*a^8 + 12*a^6*b^2 - 2*a^4*b^4 - 4*a^2*b^6 + b^8)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*cos(d*x + c) + sqrt(2)*((9*a^6 - 15*a^4*b^2 + 7*a^2*b^4 - b^6)*d^3*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*cos(d*x + c) + (9*a^9 + 12*a^7*b^2 - 2*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(1/4) + (9*a^11 + 21*a^9*b^2 + 10*a^7*b^4 - 6*a^5*b^6 - 3*a^3*b^8 + a*b^10)*cos(d*x + c) + (9*a^10*b + 21*a^8*b^3 + 10*a^6*b^5 - 6*a^4*b^7 - 3*a^2*b^9 + b^11)*sin(d*x + c))/((a^2 + b^2)*cos(d*x + c)))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(3/4))/(9*a^14*b^2 + 39*a^12*b^4 + 61*a^10*b^6 + 35*a^8*b^8 - 5*a^6*b^10 - 11*a^4*b^12 - a^2*b^14 + b^16)) + 16*sqrt(2)*(a*d^5*cos(d*x + c)^2 - a*d^5)*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(3/4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4)*arctan(-((3*a^10 + 11*a^8*b^2 + 14*a^6*b^4 + 6*a^4*b^6 - a^2*b^8 - b^10)*d^4*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + (3*a^13 + 14*a^11*b^2 + 25*a^9*b^4 + 20*a^7*b^6 + 5*a^5*b^8 - 2*a^3*b^10 - a*b^12)*d^2*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) - sqrt(2)*((3*a^5 + 2*a^3*b^2 - a*b^4)*d^7*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + (3*a^8 + 2*a^6*b^2 - 4*a^4*b^4 - 2*a^2*b^6 + b^8)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(3/4) - sqrt(2)*(a*d^7*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4) + (a^4 - b^4)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/d^4))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt(((9*a^8 + 12*a^6*b^2 - 2*a^4*b^4 - 4*a^2*b^6 + b^8)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*cos(d*x + c) - sqrt(2)*((9*a^6 - 15*a^4*b^2 + 7*a^2*b^4 - b^6)*d^3*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)*cos(d*x + c) + (9*a^9 + 12*a^7*b^2 - 2*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^3 - 3*a*b^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(1/4) + (9*a^11 + 21*a^9*b^2 + 10*a^7*b^4 - 6*a^5*b^6 - 3*a^3*b^8 + a*b^10)*cos(d*x + c) + (9*a^10*b

```
+ 21*a^8*b^3 + 10*a^6*b^5 - 6*a^4*b^7 - 3*a^2*b^9 + b^11)*sin(d*x + c))/((a
^2 + b^2)*cos(d*x + c)))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(3/4))/(
9*a^14*b^2 + 39*a^12*b^4 + 61*a^10*b^6 + 35*a^8*b^8 - 5*a^6*b^10 - 11*a^4*b
^12 - a^2*b^14 + b^16)) + 4*sqrt(2)*((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*
d*cos(d*x + c)^2 - (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*d + ((a^4 - 3*a^2*
b^2)*d^3*cos(d*x + c)^2 - (a^4 - 3*a^2*b^2)*d^3)*sqrt((a^6 + 3*a^4*b^2 + 3*
a^2*b^4 + b^6)/d^4))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^3 - 3*a*b
^2)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4))/(9*a^4*b^2 - 6*a^2*b
^4 + b^6))*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/d^4)^(1/4)*log(((9*a^8 + 12
*a^6*b^2 - 2*a^4*b^4 - 4*a^2*b^6 + b^8)*d^2*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b
^4 + b^6)/d^4)*cos(d*x + c) + sqrt(2)*((9*a^6 -...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tan (c + dx))^{\frac{3}{2}} \cot^3 (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**(3/2)*cot(c + d*x)**3, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 4.88, size = 2500, normalized size = 13.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + b*tan(c + d*x))^(3/2),x)
```

```
[Out] - atan(((((((384*a^2*b^10*d^4 - 384*b^12*d^4 + 768*a^4*b^8*d^4)/(2*d^5) + (
(512*b^10*d^4 + 768*a^2*b^8*d^4)*(a + b*tan(c + d*x))^(1/2)*(-(3*a*b^2 - a^
2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2))/d^4)*(-(3*a*b^2 - a^2*b*3i - a^3 + b
^3*1i)/(4*d^2))^(1/2) + ((a + b*tan(c + d*x))^(1/2)*(668*a*b^12*d^2 + 1088*
a^3*b^10*d^2 - 576*a^5*b^8*d^2))/d^4)*(-(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)
```

/(4*d^2))^(1/2) - (604*a*b^14*d^2 - 932*a^3*b^12*d^2 - 1344*a^5*b^10*d^2 + 192*a^7*b^8*d^2)/(2*d^5))*(-(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2) + ((a + b*tan(c + d*x))^(1/2)*(41*b^16 + 26*a^2*b^14 + 553*a^4*b^12 - 304*a^6*b^10 + 96*a^8*b^8))/d^4)*(-(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2)*1i - (((((384*a^2*b^10*d^4 - 384*b^12*d^4 + 768*a^4*b^8*d^4)/(2*d^5) - ((512*b^10*d^4 + 768*a^2*b^8*d^4)*(a + b*tan(c + d*x))^(1/2)*(-(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2))/d^4)*(-(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2) - ((a + b*tan(c + d*x))^(1/2)*(668*a*b^12*d^2 + 1088*a^3*b^10*d^2 - 576*a^5*b^8*d^2))/d^4)*(-(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2) - (604*a*b^14*d^2 - 932*a^3*b^12*d^2 - 1344*a^5*b^10*d^2 + 192*a^7*b^8*d^2)/(2*d^5))*(-(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2) - ((a + b*tan(c + d*x))^(1/2)*(41*b^16 + 26*a^2*b^14 + 553*a^4*b^12 - 304*a^6*b^10 + 96*a^8*b^8))/d^4)*(-(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2)*1i)/((119*a^4*b^14 - 71*a^2*b^16 - 15*b^18 + 391*a^6*b^12 + 216*a^8*b^10)/d^5 + (((((384*a^2*b^10*d^4 - 384*b^12*d^4 + 768*a^4*b^8*d^4)/(2*d^5) + ((512*b^10*d^4 + 768*a^2*b^8*d^4)*(a + b*tan(c + d*x))^(1/2)*(-(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2))/d^4)*(-(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2) + ((a + b*tan(c + d*x))^(1/2)*(668*a*b^12*d^2 + 1088*a^3*b^10*d^2 - 576*a^5*b^8*d^2))/d^4)*(-(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2) - (604*a*b^14*d^2 - 932*a^3*b^12*d^2 - 1344*a^5*b^10*d^2 + 192*a^7*b^8*d^2)/(2*d^5))*(-(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2) + ((a + b*tan(c + d*x))^(1/2)*(41*b^16 + 26*a^2*b^14 + 553*a^4*b^12 - 304*a^6*b^10 + 96*a^8*b^8))/d^4)*(-(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2) + (((((384*a^2*b^10*d^4 - 384*b^12*d^4 + 768*a^4*b^8*d^4)/(2*d^5) - ((512*b^10*d^4 + 768*a^2*b^8*d^4)*(a + b*tan(c + d*x))^(1/2)*(-(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2))/d^4)*(-(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2) - ((a + b*tan(c + d*x))^(1/2)*(668*a*b^12*d^2 + 1088*a^3*b^10*d^2 - 576*a^5*b^8*d^2))/d^4)*(-(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2) - (604*a*b^14*d^2 - 932*a^3*b^12*d^2 - 1344*a^5*b^10*d^2 + 192*a^7*b^8*d^2)/(2*d^5))*(-(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2) - ((a + b*tan(c + d*x))^(1/2)*(41*b^16 + 26*a^2*b^14 + 553*a^4*b^12 - 304*a^6*b^10 + 96*a^8*b^8))/d^4)*(-(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2)))*(-(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)/(4*d^2))^(1/2)*2i - atan(((((((384*a^2*b^10*d^4 - 384*b^12*d^4 + 768*a^4*b^8*d^4)/(2*d^5) + ((512*b^10*d^4 + 768*a^2*b^8*d^4)*(a + b*tan(c + d*x))^(1/2)*(-(3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)/(4*d^2))^(1/2))/d^4)*(-(3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)/(4*d^2))^(1/2) + ((a + b*tan(c + d*x))^(1/2)*(668*a*b^12*d^2 + 1088*a^3*b^10*d^2 - 576*a^5*b^8*d^2))/d^4)*(-(3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)/(4*d^2))^(1/2) - (604*a*b^14*d^2 - 932*a^3*b^12*d^2 - 1344*a^5*b^10*d^2 + 192*a^7*b^8*d^2)/(2*d^5))*(-(3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)/(4*d^2))^(1/2) + ((a + b*tan(c + d*x))^(1/2)*(41*b^16 + 26*a^2*b^14 + 553*a^4*b^12 - 304*a^6*b^10 + 96*a^8*b^8))/d^4)*(-(3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)/(4*d^2))^(1/2)*1i - (((((384*a^2*b^10*d^4 - 384*b^12*d^4 + 768*a^4*b^8*d^4)/(2*d^5) - ((512*b^10*d^4 + 768*a^2*b^8*d^4)*(a + b*tan(c + d*x))^(1/2)*(-(3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)/(4*d^2))^(1/2))/d^4)*(-(3*a*b^2 +

a^2*b*3i - a^3 - b^3*1i)/(4*d^2))^(1/2) - ((a + b*tan(c + d*x))^(1/2)*(668*
a*b^12*d^2 + 1088*a^3*b^10*d^2 - 576*a^5*b^8*d^2))/d^4)*(-(3*a*b^2 + a^2*b*
3i - a^3 - b^3*1i)/(4*d^2))^(1/2) - (604*a*b^14*d^2 - 932*a^3*b^12*d^2 - 13
44*a^5*b^10*d^2 + 192*a^7*b^8*d^2)/(2*d^5))*(-(3*a*b^2 + a^2*b*3i - a^3 - b
^3*1i)/(4*d^2))^(1/2) - ((a + b*tan(c + d*x))^(1/2)*(41*b^16 + 26*a^2*b^14
+ 553*a^4*b^12 - 304*a^6*b^10 + 96*a^8*b^8))/d^4)*(-(3*a*b^2 + a^2*b*3i - a
^3 - b^3*1i)/(4*d^2))^(1/2)*1i)/((119*a^4*b^14 - 71*a^2*b^16 - 15*b^18 + 39
1*a^6*b^12 + 216*a^8*b^10)/d^5 + (((((384*a^2*b^10*d^4 - 384*b^12*d^4 + 768
*a^4*b^8*d^4)/(2*d^5) + ((512*b^10*d^4 + 768*a^2*b^8*d^4)*(a + b*tan(c + d*
x))^(1/2)*(-(3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)/(4*d^2))^(1/2))/d^4)*(-(3*a
*b^2 + a^2*b*3i - a^3 - b^3*1i)/(4*d^2))^(1/2) + ((a + b*tan(c + d*x))^(1/2
)*(668*a*b^12*d^2 + 1088*a^3*b^10*d^2 - 576*a^5*b^8*d^2))/d^4)*(-(3*a*b^2 +
 a^2*b*3i - a^3 - b^3*1i)/(4*d^2))^(1/2) - (604*a*b^14*d^2 - 932*a^3*b^12*d
^2 - 1344*a^5*b^10*d^2 + 192*a^7*b^8*d^2)/(2*d^5))*(-(3*a*b^2 + a^2*b*3i -
a^3 - b^3*1i)/(4*d^2))^(1/2) + ((a + b*tan(c + d*x))^(1/2)*(41*b^16 + 26*a^
2*b^14 + 553*a^4*b^12 - 304*a^6*b^10 + 96*a^8*b^8))/d^4)*(-(3*a*b^2 + a^2*b
*3i - a^3 - b^3*1i)/(4*d^2))^(1/2) + (((((384*a...

3.519 $\int \tan^3(c+dx)(a+b\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=211

$$\frac{(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}+\frac{(a+ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}-\frac{2(a^2-b^2)\sqrt{a+}}{d}$$

```
[Out] (a-I*b)^(5/2)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d+(a+I*b)^(5/2)
*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d-2*(a^2-b^2)*(a+b*tan(d*x+c
))^(1/2)/d-2/3*a*(a+b*tan(d*x+c))^(3/2)/d-2/5*(a+b*tan(d*x+c))^(5/2)/d-4/63
*a*(a+b*tan(d*x+c))^(7/2)/b^2/d+2/9*tan(d*x+c)*(a+b*tan(d*x+c))^(7/2)/b/d
```

Rubi [A]

time = 0.32, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3647, 3711, 12, 3609, 3620, 3618, 65, 214}

$$-\frac{2(a^2-b^2)\sqrt{a+b\tan(c+dx)}}{d}-\frac{4a(a+b\tan(c+dx))^{7/2}}{63b^2d}+\frac{2\tan(c+dx)(a+b\tan(c+dx))^{7/2}}{9bd}-\frac{2(a+b\tan(c+dx))^{5/2}}{5d}-\frac{2a(a+b\tan(c+dx))^{3/2}}{3d}+\frac{(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}+\frac{(a+ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3*(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((a - I*b)^(5/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/d + ((a +
 I*b)^(5/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/d - (2*(a^2 -
b^2)*Sqrt[a + b*Tan[c + d*x]])/d - (2*a*(a + b*Tan[c + d*x])^(3/2))/(3*d) -
 (2*(a + b*Tan[c + d*x])^(5/2))/(5*d) - (4*a*(a + b*Tan[c + d*x])^(7/2))/(6
3*b^2*d) + (2*Tan[c + d*x]*(a + b*Tan[c + d*x])^(7/2))/(9*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \tan^3(c+dx)(a+b\tan(c+dx))^{5/2}\,dx = \frac{2\tan(c+dx)(a+b\tan(c+dx))^{7/2}}{9bd} + \frac{2\int(a+b\tan(c+dx))^{5/2}}{\ldots}$$

$$= -\frac{4a(a+b\tan(c+dx))^{7/2}}{63b^2d} + \frac{2\tan(c+dx)(a+b\tan(c+dx))^{7/\ldots}}{9bd}$$

$$= -\frac{4a(a+b\tan(c+dx))^{7/2}}{63b^2d} + \frac{2\tan(c+dx)(a+b\tan(c+dx))^{7/\ldots}}{9bd}$$

$$= -\frac{2(a+b\tan(c+dx))^{5/2}}{5d} - \frac{4a(a+b\tan(c+dx))^{7/2}}{63b^2d} + \frac{2\tan(c\ldots}{\ldots}$$

$$= -\frac{2a(a+b\tan(c+dx))^{3/2}}{3d} - \frac{2(a+b\tan(c+dx))^{5/2}}{5d} - \frac{4a(a+\ldots}{\ldots}$$

$$= -\frac{2(a^2-b^2)\sqrt{a+b\tan(c+dx)}}{d} - \frac{2a(a+b\tan(c+dx))^{3/2}}{3d} - \ldots$$

$$= -\frac{2(a^2-b^2)\sqrt{a+b\tan(c+dx)}}{d} - \frac{2a(a+b\tan(c+dx))^{3/2}}{3d} - \ldots$$

$$= -\frac{2(a^2-b^2)\sqrt{a+b\tan(c+dx)}}{d} - \frac{2a(a+b\tan(c+dx))^{3/2}}{3d} - \ldots$$

$$= -\frac{2(a^2-b^2)\sqrt{a+b\tan(c+dx)}}{d} - \frac{2a(a+b\tan(c+dx))^{3/2}}{3d} - \ldots$$

$$= \frac{(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} + \frac{(a+ib)^{5/2}\tanh^{-1}\ldots}{\ldots}$$

Mathematica [A]
time = 3.79, size = 198, normalized size = 0.94

$$\frac{(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} + \frac{(a+ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d} + \frac{2\sqrt{a+b\tan(c+dx)}\,(-10a^4-483a^2b^2+315b^4+ab(5a^2-231b^2)\tan(c+dx)+(75a^2b^2-63b^4)\tan^2(c+dx)+95ab^3\tan^3(c+dx)+35b^4\tan^4(c+dx))}{315b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3*(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((a - I*b)^(5/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/d + ((a +
 I*b)^(5/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/d + (2*Sqrt[a
+ b*Tan[c + d*x]]*(-10*a^4 - 483*a^2*b^2 + 315*b^4 + a*b*(5*a^2 - 231*b^2)*
Tan[c + d*x] + (75*a^2*b^2 - 63*b^4)*Tan[c + d*x]^2 + 95*a*b^3*Tan[c + d*x]
^3 + 35*b^4*Tan[c + d*x]^4))/(315*b^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 755 vs. $2(177) = 354$.
time = 0.14, size = 756, normalized size = 3.58

method	result
derivativedivides	$\frac{\frac{2(a+b\tan(dx+c))^{\frac{9}{2}}}{9}-\frac{2a(a+b\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{2b^2(a+b\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{2a\,b^2(a+b\tan(dx+c))^{\frac{3}{2}}}{3}-2a^2b^2\sqrt{a+b\tan\left(dx+c\right)}}{}$
default	$\frac{\frac{2(a+b\tan(dx+c))^{\frac{9}{2}}}{9}-\frac{2a(a+b\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{2b^2(a+b\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{2a\,b^2(a+b\tan(dx+c))^{\frac{3}{2}}}{3}-2a^2b^2\sqrt{a+b\tan\left(dx+c\right)}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3*(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/b^2*(1/9*(a+b*tan(d*x+c))^(9/2)-1/7*a*(a+b*tan(d*x+c))^(7/2)-1/5*b^2*(a
+b*tan(d*x+c))^(5/2)-1/3*a*b^2*(a+b*tan(d*x+c))^(3/2)-a^2*b^2*(a+b*tan(d*x+
c))^(1/2)+b^4*(a+b*tan(d*x+c))^(1/2)+b^2*(1/8*(-2*(2*(a^2+b^2)^(1/2)+2*a)^(
1/2)*(a^2+b^2)^(1/2)*a+3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2-(2*(a^2+b^2)^(1/
2)+2*a)^(1/2)*b^2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1
/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+1/2*(2*(a^2+b^2)^(1/2)*a^2-2*(a^2+b^2)^(1/2
)*b^2-1/2*(-2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a+3*(2*(a^2+b^2
)^(1/2)+2*a)^(1/2)*a^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^2)*(2*(a^2+b^2)^(1/2
)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2
)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-1/8*(-2*(2*
(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a+3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2
)*a^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^2)*ln(-b*tan(d*x+c)-a+(a+b*tan(d*x+c)
)^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-(a^2+b^2)^(1/2))+1/2*(-2*(a^2+b^2)^(1
/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+1/2*(-2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^
2)^(1/2)*a+3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2
)*b^2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(
(-2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2
)-2*a)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(5/2)*tan(d*x + c)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6847 vs. $2(173) = 346$.
time = 6.31, size = 6847, normalized size = 32.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/1260*(1260*sqrt(2)*b^2*d^5*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b
^6 + 5*a^2*b^8 + b^10 - (a^5 - 10*a^3*b^2 + 5*a*b^4)*d^2*sqrt((a^10 + 5*a^8
*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4))/(25*a^8*b^2 - 100*
a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*((a^10 + 5*a^8*b^2 + 10*a^6*b^4
 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)^(3/4)*sqrt((25*a^8*b^2 - 100*a^6*b^4
 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4)*arctan(((5*a^18 + 25*a^16*b^2 + 36
*a^14*b^4 - 28*a^12*b^6 - 154*a^10*b^8 - 210*a^8*b^10 - 140*a^6*b^12 - 44*a
^4*b^14 - 3*a^2*b^16 + b^18)*d^4*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a
^4*b^6 + 5*a^2*b^8 + b^10)/d^4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^
6 - 20*a^2*b^8 + b^10)/d^4) + (5*a^23 + 35*a^21*b^2 + 91*a^19*b^4 + 69*a^17
*b^6 - 174*a^15*b^8 - 546*a^13*b^10 - 714*a^11*b^12 - 534*a^9*b^14 - 231*a^
7*b^16 - 49*a^5*b^18 - a^3*b^20 + a*b^22)*d^2*sqrt((25*a^8*b^2 - 100*a^6*b^
4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4) + sqrt(2)*((5*a^10 - 5*a^8*b^2 -
14*a^6*b^4 + 6*a^4*b^6 + 9*a^2*b^8 - b^10)*d^7*sqrt((a^10 + 5*a^8*b^2 + 10*
a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)*sqrt((25*a^8*b^2 - 100*a^6*b^
4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4) + (5*a^15 - 5*a^13*b^2 - 39*a^11*
b^4 - 9*a^9*b^6 + 79*a^7*b^8 + 81*a^5*b^10 + 19*a^3*b^12 - 3*a*b^14)*d^5*sq
rt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4))*sqrt(
(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 - (a^5 - 10*
a^3*b^2 + 5*a*b^4)*d^2*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5
*a^2*b^8 + b^10)/d^4))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8
 + b^10))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*((a^10 + 5*a
^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)^(3/4) - sqrt(2)*(
(a^2 - b^2)*d^7*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^
8 + b^10)/d^4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 +
b^10)/d^4) + (a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^6)*d^5*sqrt((25*a^8*b^2 - 1
00*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4))*sqrt((a^10 + 5*a^8*b^2
+ 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 - (a^5 - 10*a^3*b^2 + 5*a*b^4)
*d^2*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d
^4))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*sqrt(((2
5*a^14 - 25*a^12*b^2 - 115*a^10*b^4 + 35*a^8*b^6 + 171*a^6*b^8 + 53*a^4*b^1
0 - 17*a^2*b^12 + b^14)*d^2*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^
6 + 5*a^2*b^8 + b^10)/d^4)*cos(d*x + c) + sqrt(2)*((25*a^11 - 175*a^9*b^2 +
```

```
410*a^7*b^4 - 350*a^5*b^6 + 61*a^3*b^8 - 3*a*b^10)*d^3*sqrt((a^10 + 5*a^8*
b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)*cos(d*x + c) + (25*a
^16 - 50*a^14*b^2 - 90*a^12*b^4 + 150*a^10*b^6 + 136*a^8*b^8 - 118*a^6*b^10
 - 70*a^4*b^12 + 18*a^2*b^14 - b^16)*d*cos(d*x + c))*sqrt((a^10 + 5*a^8*b^2
 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 - (a^5 - 10*a^3*b^2 + 5*a*b^4
)*d^2*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/
d^4))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*sqrt((a
*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*((a^10 + 5*a^8*b^2 + 10*a^6*b
^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)^(1/4) + (25*a^19 + 25*a^17*b^2 - 1
40*a^15*b^4 - 220*a^13*b^6 + 126*a^11*b^8 + 430*a^9*b^10 + 260*a^7*b^12 + 2
0*a^5*b^14 - 15*a^3*b^16 + a*b^18)*cos(d*x + c) + (25*a^18*b + 25*a^16*b^3
- 140*a^14*b^5 - 220*a^12*b^7 + 126*a^10*b^9 + 430*a^8*b^11 + 260*a^6*b^13
+ 20*a^4*b^15 - 15*a^2*b^17 + b^19)*sin(d*x + c))/((a^2 + b^2)*cos(d*x + c)
))*((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)^(3
/4))/(25*a^26*b^2 + 125*a^24*b^4 + 110*a^22*b^6 - 530*a^20*b^8 - 1469*a^18*
b^10 - 921*a^16*b^12 + 1716*a^14*b^14 + 3924*a^12*b^16 + 3471*a^10*b^18 + 1
531*a^8*b^20 + 254*a^6*b^22 - 34*a^4*b^24 - 11*a^2*b^26 + b^28))*cos(d*x +
c)^4 + 1260*sqrt(2)*b^2*d^5*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^
6 + 5*a^2*b^8 + b^10 - (a^5 - 10*a^3*b^2 + 5*a*b^4)*d^2*sqrt((a^10 + 5*a^8*
b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4))/(25*a^8*b^2 - 100*a
^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*((a^10 + 5*a^8*b^2 + 10*a^6*b^4
+ 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)^(3/4)*sqrt((25*a^8*b^2 - 100*a^6*b^4
+ 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4)*arctan(-((5*a^18 + 25*a^16*b^2 + 36
*a^14*b^4 - 28*a^12*b^6 - 154*a^10*b^8 - 210*a^8*b^10 - 140*a^6*b^12 - 44*a
^4*b^14 - 3*a^2*b^16 + b^18)*d^4*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a
^4*b^6 + 5*a^2*b^8 + b^10)/d^4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^
6 - 20*a^2*b^8 + b^10)/d^4) + (5*a^23 + 35*a^21*b^2 + 91*a^19*b^4 + 69*a^17
*b^6 - 174*a^15*b^8 - 546*a^13*b^10 - 714*a^11*b^12 - 534*a^9*b^14 - 231*a^
7*b^16 - 49*a^5*b^18 - a^3*b^20 + a*b^22)*d^2*sqrt((25*a^8*b^2 - 100*a^6*b^
4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4) - sqrt(2)*((5*a^10 - 5*a^8*b^2 -
14*a^6*b^4 + 6*a^4*b^6 + 9*a^2*b^8 - b^10)*d^7*sqrt((a^10 + 5*a^8*b^2 + 10*
a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)*sqrt((25*a^8*b^2 - 100*a^6*b^
4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4) + (5*a^15 - 5*a^13*b^2 - 39*a^11*
b^4 - 9*a^9*b^6 + 79*a^7*b^8 + 81*a^5*b^10 + 19...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx))^{\frac{5}{2}} \tan^3(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3*(a+b*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**(5/2)*tan(c + d*x)**3, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 47.86, size = 2384, normalized size = 11.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + b*tan(c + d*x))^(5/2),x)
```

```
[Out] (a + b*tan(c + d*x))^(3/2)*((2*a*((2*a^2)/(b^2*d) - (2*(a^2 + b^2))/(b^2*d)
))/3 - (2*a^3)/(3*b^2*d) + (2*a*(a^2 + b^2))/(3*b^2*d)) - atan(((((8*(4*b^6
*d^2 - 4*a^4*b^2*d^2))/d^3 - 64*a*b^2*(a + b*tan(c + d*x))^(1/2)*((5*a*b^4
+ a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2))*((5*a
*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2) -
 (16*(a + b*tan(c + d*x))^(1/2)*(b^8 - 15*a^2*b^6 + 15*a^4*b^4 - a^6*b^2))/
d^2)*((5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2
))^(1/2)*1i - (((8*(4*b^6*d^2 - 4*a^4*b^2*d^2))/d^3 + 64*a*b^2*(a + b*tan(c
 + d*x))^(1/2)*((5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b
^2)/(4*d^2))^(1/2))*((5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*
a^3*b^2)/(4*d^2))^(1/2) + (16*(a + b*tan(c + d*x))^(1/2)*(b^8 - 15*a^2*b^6
+ 15*a^4*b^4 - a^6*b^2))/d^2)*((5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3
*10i - 10*a^3*b^2)/(4*d^2))^(1/2)*1i)/((((8*(4*b^6*d^2 - 4*a^4*b^2*d^2))/d^
3 - 64*a*b^2*(a + b*tan(c + d*x))^(1/2)*((5*a*b^4 + a^4*b*5i + a^5 + b^5*1i
 - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2))*((5*a*b^4 + a^4*b*5i + a^5 + b
^5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2) - (16*(a + b*tan(c + d*x))
^(1/2)*(b^8 - 15*a^2*b^6 + 15*a^4*b^4 - a^6*b^2))/d^2)*((5*a*b^4 + a^4*b*5i
 + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2) + (((8*(4*b^6*d^
2 - 4*a^4*b^2*d^2))/d^3 + 64*a*b^2*(a + b*tan(c + d*x))^(1/2)*((5*a*b^4 + a
^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2))*((5*a*b^
4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2) + (1
6*(a + b*tan(c + d*x))^(1/2)*(b^8 - 15*a^2*b^6 + 15*a^4*b^4 - a^6*b^2))/d^2
)*((5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^
(1/2) - (16*(3*a*b^10 + 8*a^3*b^8 + 6*a^5*b^6 - a^9*b^2))/d^3))*((5*a*b^4 +
 a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2)*2i - at
an(((((8*(4*b^6*d^2 - 4*a^4*b^2*d^2))/d^3 - 64*a*b^2*(a + b*tan(c + d*x))^(
1/2)*((5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4*d^2
```

```
))^(1/2))*((5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(
4*d^2))^(1/2) - (16*(a + b*tan(c + d*x))^(1/2)*(b^8 - 15*a^2*b^6 + 15*a^4*b
^4 - a^6*b^2))/d^2)*((5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*
a^3*b^2)/(4*d^2))^(1/2)*1i - (((8*(4*b^6*d^2 - 4*a^4*b^2*d^2))/d^3 + 64*a*b
^2*(a + b*tan(c + d*x))^(1/2)*((5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3
*10i - 10*a^3*b^2)/(4*d^2))^(1/2))*((5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^
2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2) + (16*(a + b*tan(c + d*x))^(1/2)*(b^
8 - 15*a^2*b^6 + 15*a^4*b^4 - a^6*b^2))/d^2)*((5*a*b^4 - a^4*b*5i + a^5 - b
^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2)*1i)/((((8*(4*b^6*d^2 - 4*a
^4*b^2*d^2))/d^3 - 64*a*b^2*(a + b*tan(c + d*x))^(1/2)*((5*a*b^4 - a^4*b*5i
 + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2))*((5*a*b^4 - a^4
*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2) - (16*(a +
b*tan(c + d*x))^(1/2)*(b^8 - 15*a^2*b^6 + 15*a^4*b^4 - a^6*b^2))/d^2)*((5*a
*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2) +
 (((8*(4*b^6*d^2 - 4*a^4*b^2*d^2))/d^3 + 64*a*b^2*(a + b*tan(c + d*x))^(1/2
)*((5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^
(1/2))*((5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4*d
^2))^(1/2) + (16*(a + b*tan(c + d*x))^(1/2)*(b^8 - 15*a^2*b^6 + 15*a^4*b^4
- a^6*b^2))/d^2)*((5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3
*b^2)/(4*d^2))^(1/2) - (16*(3*a*b^10 + 8*a^3*b^8 + 6*a^5*b^6 - a^9*b^2))/d^
3))*((5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4*d^2)
)^(1/2)*2i + ((2*a^2)/(5*b^2*d) - (2*(a^2 + b^2))/(5*b^2*d))*(a + b*tan(c +
 d*x))^(5/2) - (((2*a^2)/(b^2*d) - (2*(a^2 + b^2))/(b^2*d))*(a^2 + b^2) - 2
*a*(2*a*((2*a^2)/(b^2*d) - (2*(a^2 + b^2))/(b^2*d)) - (2*a^3)/(b^2*d) + (2*
a*(a^2 + b^2))/(b^2*d)))*(a + b*tan(c + d*x))^(1/2) + (2*(a + b*tan(c + d*x
))^(9/2))/(9*b^2*d) - (2*a*(a + b*tan(c + d*x))^(7/2))/(7*b^2*d)
```

3.520 $\int \tan^2(c+dx)(a+b\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=158

$$\frac{i(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}-\frac{i(a+ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}-\frac{4ab\sqrt{a+b\tan}}{d}$$

```
[Out] I*(a-I*b)^(5/2)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d-I*(a+I*b)^(
5/2)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d-4*a*b*(a+b*tan(d*x+c))
^(1/2)/d-2/3*b*(a+b*tan(d*x+c))^(3/2)/d+2/7*(a+b*tan(d*x+c))^(7/2)/b/d
```

Rubi [A]
time = 0.21, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3624, 3563, 3609, 3620, 3618, 65, 214}

$$\frac{2(a+b\tan(c+dx))^{7/2}}{7bd}-\frac{2b(a+b\tan(c+dx))^{3/2}}{3d}-\frac{4ab\sqrt{a+b\tan(c+dx)}}{d}+\frac{i(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}-\frac{i(a+ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2*(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] (I*(a - I*b)^(5/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/d - (I*
(a + I*b)^(5/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/d - (4*a*b
*Sqrt[a + b*Tan[c + d*x]])/d - (2*b*(a + b*Tan[c + d*x])^(3/2))/(3*d) + (2*
(a + b*Tan[c + d*x])^(7/2))/(7*b*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3563

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^{5/2}\,dx = \frac{2(a+b\tan(c+dx))^{7/2}}{7bd} - \int (a+b\tan(c+dx))^{5/2}\,dx$$

$$= -\frac{2b(a+b\tan(c+dx))^{3/2}}{3d} + \frac{2(a+b\tan(c+dx))^{7/2}}{7bd} - \int \sqrt{a+$$

$$= -\frac{4ab\sqrt{a+b\tan(c+dx)}}{d} - \frac{2b(a+b\tan(c+dx))^{3/2}}{3d} + \frac{2(a+b$$

$$= -\frac{4ab\sqrt{a+b\tan(c+dx)}}{d} - \frac{2b(a+b\tan(c+dx))^{3/2}}{3d} + \frac{2(a+b$$

$$= -\frac{4ab\sqrt{a+b\tan(c+dx)}}{d} - \frac{2b(a+b\tan(c+dx))^{3/2}}{3d} + \frac{2(a+b$$

$$= -\frac{4ab\sqrt{a+b\tan(c+dx)}}{d} - \frac{2b(a+b\tan(c+dx))^{3/2}}{3d} + \frac{2(a+b$$

$$= \frac{i(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{i(a+ib)^{5/2}\tanh$$

Mathematica [A]
time = 2.24, size = 205, normalized size = 1.30

$$\frac{21i(a-ib)^{5/2}b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right) - 21i(a+ib)^{5/2}b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right) + \frac{1}{2}\sec^3(c+dx)\left(3a(3a^2-46b^2)\cos(c+dx) + (3a^3-58ab^2)\cos(3(c+dx)) - 2b(-9a^2+4b^2+(-9a^2+10b^2)\cos(2(c+dx)))\sin(c+dx)\right)\sqrt{a+b\tan(c+dx)}}{21bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2*(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((21*I)*(a - I*b)^(5/2)*b*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] -
 (21*I)*(a + I*b)^(5/2)*b*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] +
 (Sec[c + d*x]^3*(3*a*(3*a^2 - 46*b^2)*Cos[c + d*x] + (3*a^3 - 58*a*b^2)*Co
s[3*(c + d*x)] - 2*b*(-9*a^2 + 4*b^2 + (-9*a^2 + 10*b^2)*Cos[2*(c + d*x)])*
Sin[c + d*x])*Sqrt[a + b*Tan[c + d*x]])/2)/(21*b*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 822 vs. $2(130) = 260$.
time = 0.15, size = 823, normalized size = 5.21

method	result

derivativedivides	$\frac{\frac{2(a+b\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{2b^2(a+b\tan(dx+c))^{\frac{3}{2}}}{3}-4a\,b^2\sqrt{a+b\tan(dx+c)}+2b^2\left(\frac{\left(\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+\cdots}\right.}{\cdots}\right.}{\cdots}$
default	$\frac{\frac{2(a+b\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{2b^2(a+b\tan(dx+c))^{\frac{3}{2}}}{3}-4a\,b^2\sqrt{a+b\tan(dx+c)}+2b^2\left(\frac{\left(\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+\cdots}\right.}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2*(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/b*(1/7*(a+b*tan(d*x+c))^(7/2)-1/3*b^2*(a+b*tan(d*x+c))^(3/2)-2*a*b^2*(a
+b*tan(d*x+c))^(1/2)+b^2*(1/4/b^2*(1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+
b^2)^(1/2)*a^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^2-(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)*a^3+3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^2)*ln(b*tan(d*x
+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))
+2*(4*(a^2+b^2)^(1/2)*a*b^2-1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1
/2)*a^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^2-(2*(a^2+b^2)^(1/2
)+2*a)^(1/2)*a^3+3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^2)*(2*(a^2+b^2)^(1/2)+
2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+
(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))+1/4/b^2*(-1/
2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2-(2*(a^2+b^2)^(1/2)+2*a
)^(1/2)*(a^2+b^2)^(1/2)*b^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+3*(2*(a^2+b^2
)^(1/2)+2*a)^(1/2)*a*b^2)*ln(-b*tan(d*x+c)-a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2
+b^2)^(1/2)+2*a)^(1/2)-(a^2+b^2)^(1/2))+2*(-4*(a^2+b^2)^(1/2)*a*b^2+1/2*((2
*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/
2)*(a^2+b^2)^(1/2)*b^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+3*(2*(a^2+b^2)^(1/
2)+2*a)^(1/2)*a*b^2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)
^(1/2)*arctan((-2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*
(a^2+b^2)^(1/2)-2*a)^(1/2)))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(5/2)*tan(d*x + c)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6733 vs. $2(124) = 248$.
time = 5.55, size = 6733, normalized size = 42.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/84*(84*sqrt(2)*b*d^5*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5
*a^2*b^8 + b^10 + (a^5 - 10*a^3*b^2 + 5*a*b^4)*d^2*sqrt((a^10 + 5*a^8*b^2 +
 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4))/(25*a^8*b^2 - 100*a^6*b^
4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*
a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)^(3/4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110
*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4)*arctan(-((5*a^18 + 25*a^16*b^2 + 36*a^14
*b^4 - 28*a^12*b^6 - 154*a^10*b^8 - 210*a^8*b^10 - 140*a^6*b^12 - 44*a^4*b^
14 - 3*a^2*b^16 + b^18)*d^4*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^
6 + 5*a^2*b^8 + b^10)/d^4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 2
0*a^2*b^8 + b^10)/d^4) + (5*a^23 + 35*a^21*b^2 + 91*a^19*b^4 + 69*a^17*b^6
- 174*a^15*b^8 - 546*a^13*b^10 - 714*a^11*b^12 - 534*a^9*b^14 - 231*a^7*b^1
6 - 49*a^5*b^18 - a^3*b^20 + a*b^22)*d^2*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 1
10*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4) + sqrt(2)*(2*(5*a^9*b - 14*a^5*b^5 - 8
*a^3*b^7 + a*b^9)*d^7*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*
a^2*b^8 + b^10)/d^4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*
b^8 + b^10)/d^4) + (15*a^14*b + 25*a^12*b^3 - 37*a^10*b^5 - 99*a^8*b^7 - 51
*a^6*b^9 + 11*a^4*b^11 + 9*a^2*b^13 - b^15)*d^5*sqrt((25*a^8*b^2 - 100*a^6*
b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4))*sqrt((a^10 + 5*a^8*b^2 + 10*a^
6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 + (a^5 - 10*a^3*b^2 + 5*a*b^4)*d^2*sq
rt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4))/(2
5*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*sqrt((a*cos(d*x
 + c) + b*sin(d*x + c))/cos(d*x + c))*((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*
a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)^(3/4) - sqrt(2)*(2*a*d^7*sqrt((a^10 + 5*a^
8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)*sqrt((25*a^8*b^2 -
 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4) + (3*a^6 + 5*a^4*b^2 +
 a^2*b^4 - b^6)*d^5*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b
^8 + b^10)/d^4))*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b
```

```
^8 + b^10 + (a^5 - 10*a^3*b^2 + 5*a*b^4)*d^2*sqrt((a^10 + 5*a^8*b^2 + 10*a^
6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4))/(25*a^8*b^2 - 100*a^6*b^4 + 11
0*a^4*b^6 - 20*a^2*b^8 + b^10))*sqrt(((25*a^14*b^2 - 25*a^12*b^4 - 115*a^10
*b^6 + 35*a^8*b^8 + 171*a^6*b^10 + 53*a^4*b^12 - 17*a^2*b^14 + b^16)*d^2*sq
rt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)*cos
(d*x + c) + sqrt(2)*((75*a^10*b^3 - 325*a^8*b^5 + 430*a^6*b^7 - 170*a^4*b^9
 + 23*a^2*b^11 - b^13)*d^3*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6
 + 5*a^2*b^8 + b^10)/d^4)*cos(d*x + c) + 2*(25*a^15*b^3 - 25*a^13*b^5 - 115
*a^11*b^7 + 35*a^9*b^9 + 171*a^7*b^11 + 53*a^5*b^13 - 17*a^3*b^15 + a*b^17)
*d*cos(d*x + c))*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b
^8 + b^10 + (a^5 - 10*a^3*b^2 + 5*a*b^4)*d^2*sqrt((a^10 + 5*a^8*b^2 + 10*a^
6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4))/(25*a^8*b^2 - 100*a^6*b^4 + 11
0*a^4*b^6 - 20*a^2*b^8 + b^10))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(
d*x + c))*((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/
d^4)^(1/4) + (25*a^19*b^2 + 25*a^17*b^4 - 140*a^15*b^6 - 220*a^13*b^8 + 126
*a^11*b^10 + 430*a^9*b^12 + 260*a^7*b^14 + 20*a^5*b^16 - 15*a^3*b^18 + a*b^
20)*cos(d*x + c) + (25*a^18*b^3 + 25*a^16*b^5 - 140*a^14*b^7 - 220*a^12*b^9
 + 126*a^10*b^11 + 430*a^8*b^13 + 260*a^6*b^15 + 20*a^4*b^17 - 15*a^2*b^19
+ b^21)*sin(d*x + c))/((a^2 + b^2)*cos(d*x + c)))*((a^10 + 5*a^8*b^2 + 10*a
^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)^(3/4))/(25*a^26*b^2 + 125*a^24
*b^4 + 110*a^22*b^6 - 530*a^20*b^8 - 1469*a^18*b^10 - 921*a^16*b^12 + 1716*
a^14*b^14 + 3924*a^12*b^16 + 3471*a^10*b^18 + 1531*a^8*b^20 + 254*a^6*b^22
- 34*a^4*b^24 - 11*a^2*b^26 + b^28))*cos(d*x + c)^3 + 84*sqrt(2)*b*d^5*sqrt
((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 + (a^5 - 10
*a^3*b^2 + 5*a*b^4)*d^2*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 +
5*a^2*b^8 + b^10)/d^4))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^
8 + b^10))*((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)
/d^4)^(3/4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^1
0)/d^4)*arctan(((5*a^18 + 25*a^16*b^2 + 36*a^14*b^4 - 28*a^12*b^6 - 154*a^1
0*b^8 - 210*a^8*b^10 - 140*a^6*b^12 - 44*a^4*b^14 - 3*a^2*b^16 + b^18)*d^4*
sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)*s
qrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4) + (5*
a^23 + 35*a^21*b^2 + 91*a^19*b^4 + 69*a^17*b^6 - 174*a^15*b^8 - 546*a^13*b^
10 - 714*a^11*b^12 - 534*a^9*b^14 - 231*a^7*b^16 - 49*a^5*b^18 - a^3*b^20 +
 a*b^22)*d^2*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^
10)/d^4) - sqrt(2)*(2*(5*a^9*b - 14*a^5*b^5 - 8*a^3*b^7 + a*b^9)*d^7*sqrt((
a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)*sqrt((2
5*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4) + (15*a^14*
b + 25*a^12*b^3 - 37*a^10*b^5 - 99*a^8*b^7 - 51*a^6*b^9 + 11*a^4*b^11 + 9*a
^2*b^13 - b^15)*d^5*sqrt((25*a^8*b^2 - 100*a^6*...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx))^{\frac{5}{2}} \tan^2(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2*(a+b*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**(5/2)*tan(c + d*x)**2, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 25.00, size = 2165, normalized size = 13.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + b*tan(c + d*x))^(5/2),x)
```

```
[Out] atan(((((8*(8*a*b^5*d^2 + 8*a^3*b^3*d^2))/d^3 - 64*a*b^2*(a + b*tan(c + d*x
))^(1/2)*(-(5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(
4*d^2))^(1/2))*(-(5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*
b^2)/(4*d^2))^(1/2) + (16*(a + b*tan(c + d*x))^(1/2)*(b^8 - 15*a^2*b^6 + 15
*a^4*b^4 - a^6*b^2))/d^2)*(-(5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10
i - 10*a^3*b^2)/(4*d^2))^(1/2)*1i - (((8*(8*a*b^5*d^2 + 8*a^3*b^3*d^2))/d^3
 + 64*a*b^2*(a + b*tan(c + d*x))^(1/2)*(-(5*a*b^4 - a^4*b*5i + a^5 - b^5*1i
 + a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2))*(-(5*a*b^4 - a^4*b*5i + a^5 -
b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2) - (16*(a + b*tan(c + d*x)
)^(1/2)*(b^8 - 15*a^2*b^6 + 15*a^4*b^4 - a^6*b^2))/d^2)*(-(5*a*b^4 - a^4*b*
5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2)*1i)/((((8*(8*a
*b^5*d^2 + 8*a^3*b^3*d^2))/d^3 - 64*a*b^2*(a + b*tan(c + d*x))^(1/2)*(-(5*a
*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2))*
(-(5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(
1/2) + (16*(a + b*tan(c + d*x))^(1/2)*(b^8 - 15*a^2*b^6 + 15*a^4*b^4 - a^6*
b^2))/d^2)*(-(5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)
/(4*d^2))^(1/2) - (16*(6*a^4*b^7 - b^11 + 8*a^6*b^5 + 3*a^8*b^3))/d^3 + (((
8*(8*a*b^5*d^2 + 8*a^3*b^3*d^2))/d^3 + 64*a*b^2*(a + b*tan(c + d*x))^(1/2)*
(-(5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(
1/2))*(-(5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4*d
^2))^(1/2) - (16*(a + b*tan(c + d*x))^(1/2)*(b^8 - 15*a^2*b^6 + 15*a^4*b^4
- a^6*b^2))/d^2)*(-(5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^
```

```
3*b^2)/(4*d^2))^(1/2)))*(-(5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i
- 10*a^3*b^2)/(4*d^2))^(1/2)*2i + atan(((((8*(8*a*b^5*d^2 + 8*a^3*b^3*d^2))
/d^3 - 64*a*b^2*(a + b*tan(c + d*x))^(1/2)*(-(5*a*b^4 + a^4*b*5i + a^5 + b^
5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2))*(-(5*a*b^4 + a^4*b*5i + a^
5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2) + (16*(a + b*tan(c +
d*x))^(1/2)*(b^8 - 15*a^2*b^6 + 15*a^4*b^4 - a^6*b^2))/d^2)*(-(5*a*b^4 + a^
4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2)*1i - (((8*
(8*a*b^5*d^2 + 8*a^3*b^3*d^2))/d^3 + 64*a*b^2*(a + b*tan(c + d*x))^(1/2)*(-
(5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/
2))*(-(5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2
))^(1/2) - (16*(a + b*tan(c + d*x))^(1/2)*(b^8 - 15*a^2*b^6 + 15*a^4*b^4 -
a^6*b^2))/d^2)*(-(5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*
b^2)/(4*d^2))^(1/2)*1i)/((((8*(8*a*b^5*d^2 + 8*a^3*b^3*d^2))/d^3 - 64*a*b^2
*(a + b*tan(c + d*x))^(1/2)*(-(5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*
10i - 10*a^3*b^2)/(4*d^2))^(1/2))*(-(5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^
2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2) + (16*(a + b*tan(c + d*x))^(1/2)*(b^
8 - 15*a^2*b^6 + 15*a^4*b^4 - a^6*b^2))/d^2)*(-(5*a*b^4 + a^4*b*5i + a^5 +
b^5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2) - (16*(6*a^4*b^7 - b^11 +
 8*a^6*b^5 + 3*a^8*b^3))/d^3 + (((8*(8*a*b^5*d^2 + 8*a^3*b^3*d^2))/d^3 + 64
*a*b^2*(a + b*tan(c + d*x))^(1/2)*(-(5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^
2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2))*(-(5*a*b^4 + a^4*b*5i + a^5 + b^5*1
i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2) - (16*(a + b*tan(c + d*x))^(1/
2)*(b^8 - 15*a^2*b^6 + 15*a^4*b^4 - a^6*b^2))/d^2)*(-(5*a*b^4 + a^4*b*5i +
a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2)))*(-(5*a*b^4 + a^4*
b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2)*2i + ((2*a^2
)/(3*b*d) - (2*(a^2 + b^2))/(3*b*d))*(a + b*tan(c + d*x))^(3/2) + (2*(a + b
*tan(c + d*x))^(7/2))/(7*b*d) + 2*a*((2*a^2)/(b*d) - (2*(a^2 + b^2))/(b*d))
*(a + b*tan(c + d*x))^(1/2)
```

3.521 $\int \tan(c+dx)(a+b\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=158

$$-\frac{(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}-\frac{(a+ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}+\frac{2(a^2-b^2)\sqrt{a}}{}$$

```
[Out] -(a-I*b)^(5/2)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d-(a+I*b)^(5/2
)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d+2*(a^2-b^2)*(a+b*tan(d*x+
c))^(1/2)/d+2/3*a*(a+b*tan(d*x+c))^(3/2)/d+2/5*(a+b*tan(d*x+c))^(5/2)/d
```

Rubi [A]
time = 0.20, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3609, 3620, 3618, 65, 214}

$$\frac{2(a^2-b^2)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2(a+b\tan(c+dx))^{5/2}}{5d}+\frac{2a(a+b\tan(c+dx))^{3/2}}{3d}-\frac{(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}-\frac{(a+ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]*(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] -(((a - I*b)^(5/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/d) - ((
a + I*b)^(5/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/d + (2*(a^2
 - b^2)*Sqrt[a + b*Tan[c + d*x]])/d + (2*a*(a + b*Tan[c + d*x])^(3/2))/(3*d
) + (2*(a + b*Tan[c + d*x])^(5/2))/(5*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
```

```
0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \tan(c+dx)(a+b\tan(c+dx))^{5/2}\,dx = \frac{2(a+b\tan(c+dx))^{5/2}}{5d} + \int(-b+a\tan(c+dx))(a+b\tan(c+dx)$$

$$= \frac{2a(a+b\tan(c+dx))^{3/2}}{3d} + \frac{2(a+b\tan(c+dx))^{5/2}}{5d} + \int\sqrt{a+b\,\mathrm{ta}$$

$$= \frac{2(a^2-b^2)\sqrt{a+b\tan(c+dx)}}{d} + \frac{2a(a+b\tan(c+dx))^{3/2}}{3d} + \frac{2(a}{}$$

$$= \frac{2(a^2-b^2)\sqrt{a+b\tan(c+dx)}}{d} + \frac{2a(a+b\tan(c+dx))^{3/2}}{3d} + \frac{2(a}{}$$

$$= \frac{2(a^2-b^2)\sqrt{a+b\tan(c+dx)}}{d} + \frac{2a(a+b\tan(c+dx))^{3/2}}{3d} + \frac{2(a}{}$$

$$= \frac{2(a^2-b^2)\sqrt{a+b\tan(c+dx)}}{d} + \frac{2a(a+b\tan(c+dx))^{3/2}}{3d} + \frac{2(a}{}$$

$$= -\frac{(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{(a+ib)^{5/2}\tanh^{-1}}{}$$

Mathematica [A]
time = 0.76, size = 138, normalized size = 0.87

$$\frac{-15(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)-15(a+ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)+2\sqrt{a+b\tan(c+dx)}\,(23a^2-15b^2+11ab\tan(c+dx)+3b^2\tan^2(c+dx))}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]*(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] (-15*(a - I*b)^(5/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] - 15*(
a + I*b)^(5/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] + 2*Sqrt[a +
 b*Tan[c + d*x]]*(23*a^2 - 15*b^2 + 11*a*b*Tan[c + d*x] + 3*b^2*Tan[c + d*x
]^2))/(15*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 703 vs. $2(132) = 264$.

time = 0.14, size = 704, normalized size = 4.46

method	result
derivativedivides	$\frac{\frac{2(a+b\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{2a(a+b\tan(dx+c))^{\frac{3}{2}}}{3}+2a^2\sqrt{a+b\tan(dx+c)}-2b^2\sqrt{a+b\tan(dx+c)}+\frac{\left(2\sqrt{2\cdots\right.}{\cdots}}{\cdots}$
default	$\frac{\frac{2(a+b\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{2a(a+b\tan(dx+c))^{\frac{3}{2}}}{3}+2a^2\sqrt{a+b\tan(dx+c)}-2b^2\sqrt{a+b\tan(dx+c)}+\frac{\left(2\sqrt{2\cdots\right.}{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)*(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/5*(a+b*tan(d*x+c))^(5/2)+2/3*a*(a+b*tan(d*x+c))^(3/2)+2*a^2*(a+b*tan
(d*x+c))^(1/2)-2*b^2*(a+b*tan(d*x+c))^(1/2)+1/4*(2*(2*(a^2+b^2)^(1/2)+2*a)^
(1/2)*(a^2+b^2)^(1/2)*a-3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2+(2*(a^2+b^2)^(1
/2)+2*a)^(1/2)*b^2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(
1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+(-2*(a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*
b^2-1/2*(2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a-3*(2*(a^2+b^2)^(
1/2)+2*a)^(1/2)*a^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^2)*(2*(a^2+b^2)^(1/2)+2
*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(
2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-1/4*(2*(2*(a^2
+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a-3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^
2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^2)*ln(-b*tan(d*x+c)-a+(a+b*tan(d*x+c))^(1
/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-(a^2+b^2)^(1/2))+(2*(a^2+b^2)^(1/2)*a^2-2
*(a^2+b^2)^(1/2)*b^2+1/2*(2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a
-3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^2)*(2*
(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*t
an(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/
2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6683 vs. 2(128) = 256.
time = 5.71, size = 6683, normalized size = 42.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/60*(60*sqrt(2)*d^5*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*
a^2*b^8 + b^10 - (a^5 - 10*a^3*b^2 + 5*a*b^4)*d^2*sqrt((a^10 + 5*a^8*b^2 +
10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4))/(25*a^8*b^2 - 100*a^6*b^4
 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a
^4*b^6 + 5*a^2*b^8 + b^10)/d^4)^(3/4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*
a^4*b^6 - 20*a^2*b^8 + b^10)/d^4)*arctan(((5*a^18 + 25*a^16*b^2 + 36*a^14*b
^4 - 28*a^12*b^6 - 154*a^10*b^8 - 210*a^8*b^10 - 140*a^6*b^12 - 44*a^4*b^14
 - 3*a^2*b^16 + b^18)*d^4*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6
+ 5*a^2*b^8 + b^10)/d^4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*
a^2*b^8 + b^10)/d^4) + (5*a^23 + 35*a^21*b^2 + 91*a^19*b^4 + 69*a^17*b^6 -
174*a^15*b^8 - 546*a^13*b^10 - 714*a^11*b^12 - 534*a^9*b^14 - 231*a^7*b^16
- 49*a^5*b^18 - a^3*b^20 + a*b^22)*d^2*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110
*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4) + sqrt(2)*((5*a^10 - 5*a^8*b^2 - 14*a^6*
b^4 + 6*a^4*b^6 + 9*a^2*b^8 - b^10)*d^7*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4
 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110
*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4) + (5*a^15 - 5*a^13*b^2 - 39*a^11*b^4 - 9
*a^9*b^6 + 79*a^7*b^8 + 81*a^5*b^10 + 19*a^3*b^12 - 3*a*b^14)*d^5*sqrt((25*
a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4))*sqrt((a^10 +
 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 - (a^5 - 10*a^3*b^2
 + 5*a*b^4)*d^2*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^
8 + b^10)/d^4))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10
))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*((a^10 + 5*a^8*b^2
+ 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)^(3/4) - sqrt(2)*((a^2 -
```

b^2)*d^7*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^1
0)/d^4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d
^4) + (a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^6)*d^5*sqrt((25*a^8*b^2 - 100*a^6*
b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4))*sqrt((a^10 + 5*a^8*b^2 + 10*a^
6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 - (a^5 - 10*a^3*b^2 + 5*a*b^4)*d^2*sq
rt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4))/(2
5*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*sqrt(((25*a^14
- 25*a^12*b^2 - 115*a^10*b^4 + 35*a^8*b^6 + 171*a^6*b^8 + 53*a^4*b^10 - 17*
a^2*b^12 + b^14)*d^2*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a
^2*b^8 + b^10)/d^4)*cos(d*x + c) + sqrt(2)*((25*a^11 - 175*a^9*b^2 + 410*a^
7*b^4 - 350*a^5*b^6 + 61*a^3*b^8 - 3*a*b^10)*d^3*sqrt((a^10 + 5*a^8*b^2 + 1
0*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)*cos(d*x + c) + (25*a^16 - 5
0*a^14*b^2 - 90*a^12*b^4 + 150*a^10*b^6 + 136*a^8*b^8 - 118*a^6*b^10 - 70*a
^4*b^12 + 18*a^2*b^14 - b^16)*d*cos(d*x + c))*sqrt((a^10 + 5*a^8*b^2 + 10*a
^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 - (a^5 - 10*a^3*b^2 + 5*a*b^4)*d^2*s
qrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4))/(
25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*sqrt((a*cos(d*
x + c) + b*sin(d*x + c))/cos(d*x + c))*((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10
*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)^(1/4) + (25*a^19 + 25*a^17*b^2 - 140*a^15
*b^4 - 220*a^13*b^6 + 126*a^11*b^8 + 430*a^9*b^10 + 260*a^7*b^12 + 20*a^5*b
^14 - 15*a^3*b^16 + a*b^18)*cos(d*x + c) + (25*a^18*b + 25*a^16*b^3 - 140*a
^14*b^5 - 220*a^12*b^7 + 126*a^10*b^9 + 430*a^8*b^11 + 260*a^6*b^13 + 20*a^
4*b^15 - 15*a^2*b^17 + b^19)*sin(d*x + c))/((a^2 + b^2)*cos(d*x + c)))*((a^
10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)^(3/4))/(2
5*a^26*b^2 + 125*a^24*b^4 + 110*a^22*b^6 - 530*a^20*b^8 - 1469*a^18*b^10 -
921*a^16*b^12 + 1716*a^14*b^14 + 3924*a^12*b^16 + 3471*a^10*b^18 + 1531*a^8
*b^20 + 254*a^6*b^22 - 34*a^4*b^24 - 11*a^2*b^26 + b^28))*cos(d*x + c)^2 +
60*sqrt(2)*d^5*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8
 + b^10 - (a^5 - 10*a^3*b^2 + 5*a*b^4)*d^2*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*
b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4))/(25*a^8*b^2 - 100*a^6*b^4 + 110*
a^4*b^6 - 20*a^2*b^8 + b^10))*((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6
+ 5*a^2*b^8 + b^10)/d^4)^(3/4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6
 - 20*a^2*b^8 + b^10)/d^4)*arctan(-((5*a^18 + 25*a^16*b^2 + 36*a^14*b^4 - 2
8*a^12*b^6 - 154*a^10*b^8 - 210*a^8*b^10 - 140*a^6*b^12 - 44*a^4*b^14 - 3*a
^2*b^16 + b^18)*d^4*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^
2*b^8 + b^10)/d^4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^
8 + b^10)/d^4) + (5*a^23 + 35*a^21*b^2 + 91*a^19*b^4 + 69*a^17*b^6 - 174*a^
15*b^8 - 546*a^13*b^10 - 714*a^11*b^12 - 534*a^9*b^14 - 231*a^7*b^16 - 49*a
^5*b^18 - a^3*b^20 + a*b^22)*d^2*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b
^6 - 20*a^2*b^8 + b^10)/d^4) - sqrt(2)*((5*a^10 - 5*a^8*b^2 - 14*a^6*b^4 +
6*a^4*b^6 + 9*a^2*b^8 - b^10)*d^7*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*
a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b
^6 - 20*a^2*b^8 + b^10)/d^4) + (5*a^15 - 5*a^13*b^2 - 39*a^11*b^4 - 9*a^9*b
^6 + 79*a^7*b^8 + 81*a^5*b^10 + 19*a^3*b^12 - 3...

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan{(c + dx)})^{\frac{5}{2}} \tan{(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))**(5/2),x)
[Out] Integral((a + b*tan(c + d*x))**(5/2)*tan(c + d*x), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [B]
time = 11.83, size = 2191, normalized size = 13.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + b*tan(c + d*x))^(5/2),x)
[Out] atan(((((8*(4*b^6*d^2 - 4*a^4*b^2*d^2))/d^3 - 64*a*b^2*(a + b*tan(c + d*x))
^(1/2)*((5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4*d
^2))^(1/2))*((5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)
/(4*d^2))^(1/2) - (16*(a + b*tan(c + d*x))^(1/2)*(b^8 - 15*a^2*b^6 + 15*a^4
*b^4 - a^6*b^2))/d^2)*((5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 1
0*a^3*b^2)/(4*d^2))^(1/2)*1i - (((8*(4*b^6*d^2 - 4*a^4*b^2*d^2))/d^3 + 64*a
*b^2*(a + b*tan(c + d*x))^(1/2)*((5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b
^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2))*((5*a*b^4 - a^4*b*5i + a^5 - b^5*1i +
a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2) + (16*(a + b*tan(c + d*x))^(1/2)*(
b^8 - 15*a^2*b^6 + 15*a^4*b^4 - a^6*b^2))/d^2)*((5*a*b^4 - a^4*b*5i + a^5 -
 b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2)*1i)/((((8*(4*b^6*d^2 - 4
*a^4*b^2*d^2))/d^3 - 64*a*b^2*(a + b*tan(c + d*x))^(1/2)*((5*a*b^4 - a^4*b*
5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2))*((5*a*b^4 - a
^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2) - (16*(a
+ b*tan(c + d*x))^(1/2)*(b^8 - 15*a^2*b^6 + 15*a^4*b^4 - a^6*b^2))/d^2)*((5
*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2)
 + (((8*(4*b^6*d^2 - 4*a^4*b^2*d^2))/d^3 + 64*a*b^2*(a + b*tan(c + d*x))^(1
```

/2)*((5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4*d^2)
)^(1/2))*((5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4
d^2))^(1/2) + (16(a + b*tan(c + d*x))^(1/2)*(b^8 - 15*a^2*b^6 + 15*a^4*b^
4 - a^6*b^2))/d^2)*((5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a
^3*b^2)/(4*d^2))^(1/2) - (16*(3*a*b^10 + 8*a^3*b^8 + 6*a^5*b^6 - a^9*b^2))/
d^3))*((5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4*d^
2))^(1/2)*2i + atan(((((8*(4*b^6*d^2 - 4*a^4*b^2*d^2))/d^3 - 64*a*b^2*(a +
b*tan(c + d*x))^(1/2)*((5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 1
0*a^3*b^2)/(4*d^2))^(1/2))*((5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10
i - 10*a^3*b^2)/(4*d^2))^(1/2) - (16*(a + b*tan(c + d*x))^(1/2)*(b^8 - 15*a
^2*b^6 + 15*a^4*b^4 - a^6*b^2))/d^2)*((5*a*b^4 + a^4*b*5i + a^5 + b^5*1i -
a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2)*1i - (((8*(4*b^6*d^2 - 4*a^4*b^2*d
^2))/d^3 + 64*a*b^2*(a + b*tan(c + d*x))^(1/2)*((5*a*b^4 + a^4*b*5i + a^5 +
b^5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2))*((5*a*b^4 + a^4*b*5i +
a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2) + (16*(a + b*tan(c
+ d*x))^(1/2)*(b^8 - 15*a^2*b^6 + 15*a^4*b^4 - a^6*b^2))/d^2)*((5*a*b^4 + a
^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2)*1i)/((((8
*(4*b^6*d^2 - 4*a^4*b^2*d^2))/d^3 - 64*a*b^2*(a + b*tan(c + d*x))^(1/2)*((5
*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2)
)*((5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^
(1/2) - (16*(a + b*tan(c + d*x))^(1/2)*(b^8 - 15*a^2*b^6 + 15*a^4*b^4 - a^6
b^2))/d^2)((5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)
/(4*d^2))^(1/2) + (((8*(4*b^6*d^2 - 4*a^4*b^2*d^2))/d^3 + 64*a*b^2*(a + b*t
an(c + d*x))^(1/2)*((5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a
^3*b^2)/(4*d^2))^(1/2))*((5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i -
10*a^3*b^2)/(4*d^2))^(1/2) + (16*(a + b*tan(c + d*x))^(1/2)*(b^8 - 15*a^2*
b^6 + 15*a^4*b^4 - a^6*b^2))/d^2)*((5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2
*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2) - (16*(3*a*b^10 + 8*a^3*b^8 + 6*a^5*b
^6 - a^9*b^2))/d^3))*((5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10
*a^3*b^2)/(4*d^2))^(1/2)*2i + (2*(a + b*tan(c + d*x))^(5/2))/(5*d) - ((2*(a
^2 + b^2))/d - (4*a^2)/d)*(a + b*tan(c + d*x))^(1/2) + (2*a*(a + b*tan(c +
d*x))^(3/2))/(3*d)

3.522 $\int (a+b\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=134

$$-\frac{i(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}+\frac{i(a+ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}+\frac{4ab\sqrt{a+b\tan}}{d}$$

```
[Out] -I*(a-I*b)^(5/2)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d+I*(a+I*b)^
(5/2)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d+4*a*b*(a+b*tan(d*x+c)
)^(1/2)/d+2/3*b*(a+b*tan(d*x+c))^(3/2)/d
```

Rubi [A]
time = 0.17, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3563, 3609, 3620, 3618, 65, 214}

$$\frac{2b(a+b\tan(c+dx))^{3/2}}{3d}+\frac{4ab\sqrt{a+b\tan(c+dx)}}{d}-\frac{i(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}+\frac{i(a+ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-I)*(a - I*b)^(5/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/d +
(I*(a + I*b)^(5/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/d + (4*
a*b*Sqrt[a + b*Tan[c + d*x]])/d + (2*b*(a + b*Tan[c + d*x])^(3/2))/(3*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3563

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (a+b\tan(c+dx))^{5/2}\,dx = \frac{2b(a+b\tan(c+dx))^{3/2}}{3d} + \int \sqrt{a+b\tan(c+dx)}\,\left(a^2-b^2+2ab\tan(c+dx\right.$$

$$= \frac{4ab\sqrt{a+b\tan(c+dx)}}{d} + \frac{2b(a+b\tan(c+dx))^{3/2}}{3d} + \int \frac{a(a^2-3b^2)+b(3a^2}{\sqrt{a+b\tan}}$$

$$= \frac{4ab\sqrt{a+b\tan(c+dx)}}{d} + \frac{2b(a+b\tan(c+dx))^{3/2}}{3d} + \frac{1}{2}(a-ib)^3 \int \frac{1+i\,\mathrm{t}}{\sqrt{a+b}}$$

$$= \frac{4ab\sqrt{a+b\tan(c+dx)}}{d} + \frac{2b(a+b\tan(c+dx))^{3/2}}{3d} + \frac{(ia-b)^3\mathrm{Subst}\left(\int \frac{}{(-1-}\right.}{}$$

$$= \frac{4ab\sqrt{a+b\tan(c+dx)}}{d} + \frac{2b(a+b\tan(c+dx))^{3/2}}{3d} - \frac{(a-ib)^3\mathrm{Subst}\left(\int \frac{}{-1-}\right.}{}$$

$$= -\frac{i(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} + \frac{i(a+ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+}}{}\right.}{d}$$

Mathematica [A]

time = 0.42, size = 121, normalized size = 0.90

$$\frac{-3i(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)+3i(a+ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)+2b\sqrt{a+b\tan(c+dx)}\,(7a+b\tan(c+dx))}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-3*I)*(a - I*b)^(5/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] + (
3*I)*(a + I*b)^(5/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] + 2*b*
Sqrt[a + b*Tan[c + d*x]]*(7*a + b*Tan[c + d*x]))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 791 vs. $2(110) = 220$.
time = 0.12, size = 792, normalized size = 5.91

method	result
derivativedivides	$2b\left(\frac{(a+b\tan(dx+c))^{\frac{3}{2}}}{3}+2a\sqrt{a+b\tan(dx+c)}+\frac{\left(-\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a^2+\sqrt{2\sqrt{a^2+\cdots}}\right.}{\cdots}\right.$
default	$2b\left(\frac{(a+b\tan(dx+c))^{\frac{3}{2}}}{3}+2a\sqrt{a+b\tan(dx+c)}+\frac{\left(-\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a^2+\sqrt{2\sqrt{a^2+\cdots}}\right.}{\cdots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*b*(1/3*(a+b*tan(d*x+c))^(3/2)+2*a*(a+b*tan(d*x+c))^(1/2)+1/4/b^2*(1/2*(
-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2+(2*(a^2+b^2)^(1/2)+2*a)^
(1/2)*(a^2+b^2)^(1/2)*b^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-3*(2*(a^2+b^2)^
(1/2)+2*a)^(1/2)*a*b^2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*(-4*(a^2+b^2)^(1/2)*a*b^2-1/2*(-(2*(
```

```
a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)
*(a^2+b^2)^(1/2)*b^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-3*(2*(a^2+b^2)^(1/2)
+2*a)^(1/2)*a*b^2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(
1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^
2+b^2)^(1/2)-2*a)^(1/2)))+1/4/b^2*(-1/2*(-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^
2+b^2)^(1/2)*a^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^2+(2*(a^2+
b^2)^(1/2)+2*a)^(1/2)*a^3-3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^2)*ln(-b*tan(
d*x+c)-a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-(a^2+b^2)^(1/
2))+2*(4*(a^2+b^2)^(1/2)*a*b^2+1/2*(-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2
)^(1/2)*a^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^2+(2*(a^2+b^2)^
(1/2)+2*a)^(1/2)*a^3-3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^2)*(2*(a^2+b^2)^(1
/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*tan(d*x+c))^(
1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6582 vs. $2(104) = 208$.
time = 5.96, size = 6582, normalized size = 49.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(12*sqrt(2)*d^5*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*
a^2*b^8 + b^10 + (a^5 - 10*a^3*b^2 + 5*a*b^4)*d^2*sqrt((a^10 + 5*a^8*b^2 +
10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4))/(25*a^8*b^2 - 100*a^6*b^4
 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a
^4*b^6 + 5*a^2*b^8 + b^10)/d^4)^(3/4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*
a^4*b^6 - 20*a^2*b^8 + b^10)/d^4)*arctan(-((5*a^18 + 25*a^16*b^2 + 36*a^14*
b^4 - 28*a^12*b^6 - 154*a^10*b^8 - 210*a^8*b^10 - 140*a^6*b^12 - 44*a^4*b^1
4 - 3*a^2*b^16 + b^18)*d^4*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6
 + 5*a^2*b^8 + b^10)/d^4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20
*a^2*b^8 + b^10)/d^4) + (5*a^23 + 35*a^21*b^2 + 91*a^19*b^4 + 69*a^17*b^6 -
```

174*a^15*b^8 - 546*a^13*b^10 - 714*a^11*b^12 - 534*a^9*b^14 - 231*a^7*b^16 - 49*a^5*b^18 - a^3*b^20 + a*b^22)*d^2*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4) + sqrt(2)*(2*(5*a^9*b - 14*a^5*b^5 - 8*a^3*b^7 + a*b^9)*d^7*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4) + (15*a^14*b + 25*a^12*b^3 - 37*a^10*b^5 - 99*a^8*b^7 - 51*a^6*b^9 + 11*a^4*b^11 + 9*a^2*b^13 - b^15)*d^5*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4))*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 + (a^5 - 10*a^3*b^2 + 5*a*b^4)*d^2*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)^(3/4) - sqrt(2)*(2*a*d^7*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4) + (3*a^6 + 5*a^4*b^2 + a^2*b^4 - b^6)*d^5*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4))*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 + (a^5 - 10*a^3*b^2 + 5*a*b^4)*d^2*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*sqrt(((25*a^14*b^2 - 25*a^12*b^4 - 115*a^10*b^6 + 35*a^8*b^8 + 171*a^6*b^10 + 53*a^4*b^12 - 17*a^2*b^14 + b^16)*d^2*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)*cos(d*x + c) + sqrt(2)*((75*a^10*b^3 - 325*a^8*b^5 + 430*a^6*b^7 - 170*a^4*b^9 + 23*a^2*b^11 - b^13)*d^3*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)*cos(d*x + c) + 2*(25*a^15*b^3 - 25*a^13*b^5 - 115*a^11*b^7 + 35*a^9*b^9 + 171*a^7*b^11 + 53*a^5*b^13 - 17*a^3*b^15 + a*b^17)*d*cos(d*x + c))*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 + (a^5 - 10*a^3*b^2 + 5*a*b^4)*d^2*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)^(1/4) + (25*a^19*b^2 + 25*a^17*b^4 - 140*a^15*b^6 - 220*a^13*b^8 + 126*a^11*b^10 + 430*a^9*b^12 + 260*a^7*b^14 + 20*a^5*b^16 - 15*a^3*b^18 + a*b^20)*cos(d*x + c) + (25*a^18*b^3 + 25*a^16*b^5 - 140*a^14*b^7 - 220*a^12*b^9 + 126*a^10*b^11 + 430*a^8*b^13 + 260*a^6*b^15 + 20*a^4*b^17 - 15*a^2*b^19 + b^21)*sin(d*x + c))/((a^2 + b^2)*cos(d*x + c)))*((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)^(3/4))/(25*a^26*b^2 + 125*a^24*b^4 + 110*a^22*b^6 - 530*a^20*b^8 - 1469*a^18*b^10 - 921*a^16*b^12 + 1716*a^14*b^14 + 3924*a^12*b^16 + 3471*a^10*b^18 + 1531*a^8*b^20 + 254*a^6*b^22 - 34*a^4*b^24 - 11*a^2*b^26 + b^28))*cos(d*x + c) + 12*sqrt(2)*d^5*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 + (a^5 - 10*a^3*b^2 + 5*a*b^4)*d^2*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)^(3/4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^

```
4)*arctan(((5*a^18 + 25*a^16*b^2 + 36*a^14*b^4 - 28*a^12*b^6 - 154*a^10*b^8
 - 210*a^8*b^10 - 140*a^6*b^12 - 44*a^4*b^14 - 3*a^2*b^16 + b^18)*d^4*sqrt(
(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)*sqrt((
25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4) + (5*a^23
+ 35*a^21*b^2 + 91*a^19*b^4 + 69*a^17*b^6 - 174*a^15*b^8 - 546*a^13*b^10 -
714*a^11*b^12 - 534*a^9*b^14 - 231*a^7*b^16 - 49*a^5*b^18 - a^3*b^20 + a*b^
22)*d^2*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d
^4) - sqrt(2)*(2*(5*a^9*b - 14*a^5*b^5 - 8*a^3*b^7 + a*b^9)*d^7*sqrt((a^10
+ 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)*sqrt((25*a^8
*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4) + (15*a^14*b + 2
5*a^12*b^3 - 37*a^10*b^5 - 99*a^8*b^7 - 51*a^6*b^9 + 11*a^4*b^11 + 9*a^2*b^
13 - b^15)*d^5*sqrt((25*a^8*b^2 - 100*a^6*b^4 +...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx))^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**(5/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 6.83, size = 2100, normalized size = 15.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^(5/2),x)
```

```
[Out] (2*b*(a + b*tan(c + d*x))^(3/2))/(3*d) - atan(((((8*(8*a*b^5*d^2 + 8*a^3*b^
3*d^2))/d^3 - 64*a*b^2*(a + b*tan(c + d*x))^(1/2)*(-(5*a*b^4 + a^4*b*5i + a
^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2))*(-(5*a*b^4 + a^4*b*
5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2) + (16*(a + b*t
```

an(c + d*x))^(1/2)*(b^8 - 15*a^2*b^6 + 15*a^4*b^4 - a^6*b^2))/d^2)*(-(5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2)*1i - (((8*(8*a*b^5*d^2 + 8*a^3*b^3*d^2))/d^3 + 64*a*b^2*(a + b*tan(c + d*x))^(1/2)*(-(5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2))*(-(5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2) - (16*(a + b*tan(c + d*x))^(1/2)*(b^8 - 15*a^2*b^6 + 15*a^4*b^4 - a^6*b^2))/d^2)*(-(5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2)*1i)/((((8*(8*a*b^5*d^2 + 8*a^3*b^3*d^2))/d^3 - 64*a*b^2*(a + b*tan(c + d*x))^(1/2)*(-(5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2))*(-(5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2) + (16*(a + b*tan(c + d*x))^(1/2)*(b^8 - 15*a^2*b^6 + 15*a^4*b^4 - a^6*b^2))/d^2)*(-(5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2) - (16*(6*a^4*b^7 - b^11 + 8*a^6*b^5 + 3*a^8*b^3))/d^3 + (((8*(8*a*b^5*d^2 + 8*a^3*b^3*d^2))/d^3 + 64*a*b^2*(a + b*tan(c + d*x))^(1/2)*(-(5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2))*(-(5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2) - (16*(a + b*tan(c + d*x))^(1/2)*(b^8 - 15*a^2*b^6 + 15*a^4*b^4 - a^6*b^2))/d^2)*(-(5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2)))*(-(5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2)*2i - atan(((((8*(8*a*b^5*d^2 + 8*a^3*b^3*d^2))/d^3 - 64*a*b^2*(a + b*tan(c + d*x))^(1/2)*(-(5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2))*(-(5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2) + (16*(a + b*tan(c + d*x))^(1/2)*(b^8 - 15*a^2*b^6 + 15*a^4*b^4 - a^6*b^2))/d^2)*(-(5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2)*1i - (((8*(8*a*b^5*d^2 + 8*a^3*b^3*d^2))/d^3 + 64*a*b^2*(a + b*tan(c + d*x))^(1/2)*(-(5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2))*(-(5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2) - (16*(a + b*tan(c + d*x))^(1/2)*(b^8 - 15*a^2*b^6 + 15*a^4*b^4 - a^6*b^2))/d^2)*(-(5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2)*1i)/((((8*(8*a*b^5*d^2 + 8*a^3*b^3*d^2))/d^3 - 64*a*b^2*(a + b*tan(c + d*x))^(1/2)*(-(5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2))*(-(5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2) + (16*(a + b*tan(c + d*x))^(1/2)*(b^8 - 15*a^2*b^6 + 15*a^4*b^4 - a^6*b^2))/d^2)*(-(5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2) - (16*(6*a^4*b^7 - b^11 + 8*a^6*b^5 + 3*a^8*b^3))/d^3 + (((8*(8*a*b^5*d^2 + 8*a^3*b^3*d^2))/d^3 + 64*a*b^2*(a + b*tan(c + d*x))^(1/2)*(-(5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2))*(-(5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2) - (16*(a + b*tan(c + d*x))^(1/2)*(b^8 - 15*a^2*b^6 + 15*a^4*b^4 - a^6*b^2))/d^2)*(-(5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2)))*(-(5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)/(4*d^2))^(1/2)*2i + (4*a*b*(a + b*tan(c + d*x))^(1/2))/d

3.523 $\int \cot(c+dx)(a+b\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=138

$$-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}+\frac{(a+ib)^{5/2}\tanh^{-1}\left(\right.}{}$$

```
[Out] -2*a^(5/2)*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/d+(a-I*b)^(5/2)*arctanh(
(a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d+(a+I*b)^(5/2)*arctanh((a+b*tan(d*x+
c))^(1/2)/(a+I*b)^(1/2))/d+2*b^2*(a+b*tan(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.34, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3647, 3734, 3620, 3618, 65, 214, 3715}

$$-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{2b^2\sqrt{a+b\tan(c+dx)}}{d}+\frac{(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}+\frac{(a+ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^(5/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/d + ((a - I*b)^(5/2)
*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/d + ((a + I*b)^(5/2)*ArcT
anh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/d + (2*b^2*Sqrt[a + b*Tan[c +
d*x]])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
```

```
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)(a+b\tan(c+dx))^{5/2}\,dx &= \frac{2b^2\sqrt{a+b\tan(c+dx)}}{d} + 2\int \frac{\cot(c+dx)\left(\frac{a^3}{2}+\frac{1}{2}b(3a^2-b^2)\,\mathrm{ta}\right.}{\sqrt{a+b\tan}} \\ &= \frac{2b^2\sqrt{a+b\tan(c+dx)}}{d} + 2\int \frac{\frac{1}{2}b(3a^2-b^2)-\frac{1}{2}a(a^2-3b^2)\tan(}{\sqrt{a+b\tan(c+dx)}} \\ &= \frac{2b^2\sqrt{a+b\tan(c+dx)}}{d} + \frac{1}{2}(ia-b)^3\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx \\ &= \frac{2b^2\sqrt{a+b\tan(c+dx)}}{d} - \frac{(a-ib)^3\mathrm{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a-ibx}}\,dx\right.}{2d} \\ &= -\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2b^2\sqrt{a+b\tan(c+dx}}{d} \\ &= -\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{}}{}\right.}{d}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 220, normalized size = 1.59

$$\frac{-2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)+(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)+a^2\sqrt{a+ib}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)+2ia\sqrt{a+ib}\,b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)-\sqrt{a+ib}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)+2b^2\sqrt{a+b\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^(5/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]] + (a - I*b)^(5/2)*Arc
Tanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] + a^2*Sqrt[a + I*b]*ArcTanh[Sq
rt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] + (2*I)*a*Sqrt[a + I*b]*b*ArcTanh[Sqr
t[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] - Sqrt[a + I*b]*b^2*ArcTanh[Sqrt[a + b
*Tan[c + d*x]]/Sqrt[a + I*b]] + 2*b^2*Sqrt[a + b*Tan[c + d*x]])/d
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 1.46, size = 28373, normalized size = 205.60

method	result	size
default	Expression too large to display	28373

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

[Out] result too large to display

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(5/2)*cot(d*x + c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6594 vs. 2(110) = 220.
time = 9.11, size = 13263, normalized size = 96.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(2)*d^5*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^
2*b^8 + b^10 - (a^5 - 10*a^3*b^2 + 5*a*b^4)*d^2*sqrt((a^10 + 5*a^8*b^2 + 10
*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4))/(25*a^8*b^2 - 100*a^6*b^4 +
 110*a^4*b^6 - 20*a^2*b^8 + b^10))*((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4
*b^6 + 5*a^2*b^8 + b^10)/d^4)^(3/4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^
4*b^6 - 20*a^2*b^8 + b^10)/d^4)*arctan(((5*a^18 + 25*a^16*b^2 + 36*a^14*b^4
 - 28*a^12*b^6 - 154*a^10*b^8 - 210*a^8*b^10 - 140*a^6*b^12 - 44*a^4*b^14 -
 3*a^2*b^16 + b^18)*d^4*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 +
5*a^2*b^8 + b^10)/d^4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^
2*b^8 + b^10)/d^4) + (5*a^23 + 35*a^21*b^2 + 91*a^19*b^4 + 69*a^17*b^6 - 17
4*a^15*b^8 - 546*a^13*b^10 - 714*a^11*b^12 - 534*a^9*b^14 - 231*a^7*b^16 -
49*a^5*b^18 - a^3*b^20 + a*b^22)*d^2*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a
^4*b^6 - 20*a^2*b^8 + b^10)/d^4) + sqrt(2)*((5*a^10 - 5*a^8*b^2 - 14*a^6*b^
4 + 6*a^4*b^6 + 9*a^2*b^8 - b^10)*d^7*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 +
 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a
^4*b^6 - 20*a^2*b^8 + b^10)/d^4) + (5*a^15 - 5*a^13*b^2 - 39*a^11*b^4 - 9*a
^9*b^6 + 79*a^7*b^8 + 81*a^5*b^10 + 19*a^3*b^12 - 3*a*b^14)*d^5*sqrt((25*a^
8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4))*sqrt((a^10 + 5
*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 - (a^5 - 10*a^3*b^2 +
 5*a*b^4)*d^2*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8
+ b^10)/d^4))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))
*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*((a^10 + 5*a^8*b^2 +
10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)^(3/4) - sqrt(2)*((a^2 - b^
2)*d^7*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)
/d^4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4
```

```
) + (a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^6)*d^5*sqrt((25*a^8*b^2 - 100*a^6*b^
4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4))*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*
b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 - (a^5 - 10*a^3*b^2 + 5*a*b^4)*d^2*sqrt
((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4))/(25*
a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*sqrt(((25*a^14 -
25*a^12*b^2 - 115*a^10*b^4 + 35*a^8*b^6 + 171*a^6*b^8 + 53*a^4*b^10 - 17*a^
2*b^12 + b^14)*d^2*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2
*b^8 + b^10)/d^4)*cos(d*x + c) + sqrt(2)*((25*a^11 - 175*a^9*b^2 + 410*a^7*
b^4 - 350*a^5*b^6 + 61*a^3*b^8 - 3*a*b^10)*d^3*sqrt((a^10 + 5*a^8*b^2 + 10*
a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)*cos(d*x + c) + (25*a^16 - 50*
a^14*b^2 - 90*a^12*b^4 + 150*a^10*b^6 + 136*a^8*b^8 - 118*a^6*b^10 - 70*a^4
*b^12 + 18*a^2*b^14 - b^16)*d*cos(d*x + c))*sqrt((a^10 + 5*a^8*b^2 + 10*a^6
*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 - (a^5 - 10*a^3*b^2 + 5*a*b^4)*d^2*sqr
t((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4))/(25
*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*sqrt((a*cos(d*x
+ c) + b*sin(d*x + c))/cos(d*x + c))*((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a
^4*b^6 + 5*a^2*b^8 + b^10)/d^4)^(1/4) + (25*a^19 + 25*a^17*b^2 - 140*a^15*b
^4 - 220*a^13*b^6 + 126*a^11*b^8 + 430*a^9*b^10 + 260*a^7*b^12 + 20*a^5*b^1
4 - 15*a^3*b^16 + a*b^18)*cos(d*x + c) + (25*a^18*b + 25*a^16*b^3 - 140*a^1
4*b^5 - 220*a^12*b^7 + 126*a^10*b^9 + 430*a^8*b^11 + 260*a^6*b^13 + 20*a^4*
b^15 - 15*a^2*b^17 + b^19)*sin(d*x + c))/((a^2 + b^2)*cos(d*x + c)))*((a^10
 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)^(3/4))/(25*
a^26*b^2 + 125*a^24*b^4 + 110*a^22*b^6 - 530*a^20*b^8 - 1469*a^18*b^10 - 92
1*a^16*b^12 + 1716*a^14*b^14 + 3924*a^12*b^16 + 3471*a^10*b^18 + 1531*a^8*b
^20 + 254*a^6*b^22 - 34*a^4*b^24 - 11*a^2*b^26 + b^28)) + 4*sqrt(2)*d^5*sqr
t((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 - (a^5 - 1
0*a^3*b^2 + 5*a*b^4)*d^2*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 +
 5*a^2*b^8 + b^10)/d^4))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b
^8 + b^10))*((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10
)/d^4)^(3/4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^
10)/d^4)*arctan(-((5*a^18 + 25*a^16*b^2 + 36*a^14*b^4 - 28*a^12*b^6 - 154*a
^10*b^8 - 210*a^8*b^10 - 140*a^6*b^12 - 44*a^4*b^14 - 3*a^2*b^16 + b^18)*d^
4*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)
*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4) + (
5*a^23 + 35*a^21*b^2 + 91*a^19*b^4 + 69*a^17*b^6 - 174*a^15*b^8 - 546*a^13*
b^10 - 714*a^11*b^12 - 534*a^9*b^14 - 231*a^7*b^16 - 49*a^5*b^18 - a^3*b^20
 + a*b^22)*d^2*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 +
b^10)/d^4) - sqrt(2)*((5*a^10 - 5*a^8*b^2 - 14*a^6*b^4 + 6*a^4*b^6 + 9*a^2*
b^8 - b^10)*d^7*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^
8 + b^10)/d^4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 +
b^10)/d^4) + (5*a^15 - 5*a^13*b^2 - 39*a^11*b^4 - 9*a^9*b^6 + 79*a^7*b^8 +
81*a^5*b^10 + 19*a^3*b^12 - 3*a*b^14)*d^5*sqrt(...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx))^{\frac{5}{2}} \cot(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**(5/2)*cot(c + d*x), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 10.74, size = 2500, normalized size = 18.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + b*tan(c + d*x))^(5/2),x)
```

```
[Out] log(((((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^5*d^2 + 5*a*b^4*d^
2 - 10*a^3*b^2*d^2)/d^4)^(1/2)*(((((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^
(1/2) + a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b^2*d^2)/d^4)^(1/2)*(((((-b^2*d^4*(5
*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b^2*d^2)
/d^4)^(1/2)*(((((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^5*d^2 + 5
*a*b^4*d^2 - 10*a^3*b^2*d^2)/d^4)^(1/2)*((128*a*b^8*(3*a^4 + b^4 + 4*a^2*b^
2))/d - 128*b^8*(((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^5*d^2 +
 5*a*b^4*d^2 - 10*a^3*b^2*d^2)/d^4)^(1/2)*(3*a^2 + 2*b^2)*(a + b*tan(c + d*
x))^(1/2)))/2 + (64*a*b^8*(a + b*tan(c + d*x))^(1/2)*(9*a^6 + 19*b^6 - 5*a^
2*b^4 - 51*a^4*b^2))/d^2))/2 - (96*a^2*b^8*(a^8 + b^8 + 16*a^2*b^6 - 10*a^4
*b^4 - 24*a^6*b^2))/d^3))/2 - (32*b^8*(a + b*tan(c + d*x))^(1/2)*(3*a^12 +
b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 18*a^6*b^6 + 45*a^8*b^4 - 24*a^10*b^2))/d^
4))/2 + (32*a^3*b^10*(a^2 + b^2)^3*(6*a^4 + b^4 + 3*a^2*b^2))/d^5)*((20*a^2
*b^8*d^4 - b^10*d^4 - 110*a^4*b^6*d^4 + 100*a^6*b^4*d^4 - 25*a^8*b^2*d^4)^(
1/2)/(4*d^4) + a^5/(4*d^2) + (5*a*b^4)/(4*d^2) - (5*a^3*b^2)/(2*d^2))^(1/2)
 - log(((((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^5*d^2 + 5*a*b^4
*d^2 - 10*a^3*b^2*d^2)/d^4)^(1/2)*(((((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^
2)^(1/2) + a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b^2*d^2)/d^4)^(1/2)*(((((-b^2*d^4
```

```
*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b^2*d
^2)/d^4)^(1/2)*(((((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^5*d^2
+ 5*a*b^4*d^2 - 10*a^3*b^2*d^2)/d^4)^(1/2)*((128*a*b^8*(3*a^4 + b^4 + 4*a^2
*b^2))/d + 128*b^8*(((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^5*d^
2 + 5*a*b^4*d^2 - 10*a^3*b^2*d^2)/d^4)^(1/2)*(3*a^2 + 2*b^2)*(a + b*tan(c +
 d*x))^(1/2)))/2 - (64*a*b^8*(a + b*tan(c + d*x))^(1/2)*(9*a^6 + 19*b^6 - 5
*a^2*b^4 - 51*a^4*b^2))/d^2))/2 - (96*a^2*b^8*(a^8 + b^8 + 16*a^2*b^6 - 10*
a^4*b^4 - 24*a^6*b^2))/d^3))/2 + (32*b^8*(a + b*tan(c + d*x))^(1/2)*(3*a^12
 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 18*a^6*b^6 + 45*a^8*b^4 - 24*a^10*b^2))
/d^4))/2 + (32*a^3*b^10*(a^2 + b^2)^3*(6*a^4 + b^4 + 3*a^2*b^2))/d^5)*(((20
*a^2*b^8*d^4 - b^10*d^4 - 110*a^4*b^6*d^4 + 100*a^6*b^4*d^4 - 25*a^8*b^2*d^
4)^(1/2) + a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b^2*d^2)/(4*d^4))^(1/2) - log(((-
((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a^5*d^2 - 5*a*b^4*d^2 + 10
*a^3*b^2*d^2)/d^4)^(1/2)*(((-((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2)
 - a^5*d^2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/d^4)^(1/2)*(((-((-b^2*d^4*(5*a^4
 + b^4 - 10*a^2*b^2)^2)^(1/2) - a^5*d^2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/d^4
)^(1/2)*(((-((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a^5*d^2 - 5*a*
b^4*d^2 + 10*a^3*b^2*d^2)/d^4)^(1/2)*((128*a*b^8*(3*a^4 + b^4 + 4*a^2*b^2))
/d + 128*b^8*(-((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a^5*d^2 - 5
*a*b^4*d^2 + 10*a^3*b^2*d^2)/d^4)^(1/2)*(3*a^2 + 2*b^2)*(a + b*tan(c + d*x)
)^(1/2)))/2 - (64*a*b^8*(a + b*tan(c + d*x))^(1/2)*(9*a^6 + 19*b^6 - 5*a^2*
b^4 - 51*a^4*b^2))/d^2))/2 - (96*a^2*b^8*(a^8 + b^8 + 16*a^2*b^6 - 10*a^4*b
^4 - 24*a^6*b^2))/d^3))/2 + (32*b^8*(a + b*tan(c + d*x))^(1/2)*(3*a^12 + b^
12 + 6*a^2*b^10 + 15*a^4*b^8 + 18*a^6*b^6 + 45*a^8*b^4 - 24*a^10*b^2))/d^4)
)/2 + (32*a^3*b^10*(a^2 + b^2)^3*(6*a^4 + b^4 + 3*a^2*b^2))/d^5)*(-((20*a^2
*b^8*d^4 - b^10*d^4 - 110*a^4*b^6*d^4 + 100*a^6*b^4*d^4 - 25*a^8*b^2*d^4)^(
1/2) - a^5*d^2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/(4*d^4))^(1/2) + log(((-((-b
^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a^5*d^2 - 5*a*b^4*d^2 + 10*a^3
*b^2*d^2)/d^4)^(1/2)*(((-((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a
^5*d^2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/d^4)^(1/2)*(((-((-b^2*d^4*(5*a^4 + b
^4 - 10*a^2*b^2)^2)^(1/2) - a^5*d^2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/d^4)^(1
/2)*(((-((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a^5*d^2 - 5*a*b^4*
d^2 + 10*a^3*b^2*d^2)/d^4)^(1/2)*((128*a*b^8*(3*a^4 + b^4 + 4*a^2*b^2))/d -
 128*b^8*(-((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a^5*d^2 - 5*a*b
^4*d^2 + 10*a^3*b^2*d^2)/d^4)^(1/2)*(3*a^2 + 2*b^2)*(a + b*tan(c + d*x))^(1
/2)))/2 + (64*a*b^8*(a + b*tan(c + d*x))^(1/2)*(9*a^6 + 19*b^6 - 5*a^2*b^4
- 51*a^4*b^2))/d^2))/2 - (96*a^2*b^8*(a^8 + b^8 + 16*a^2*b^6 - 10*a^4*b^4 -
 24*a^6*b^2))/d^3))/2 - (32*b^8*(a + b*tan(c + d*x))^(1/2)*(3*a^12 + b^12 +
 6*a^2*b^10 + 15*a^4*b^8 + 18*a^6*b^6 + 45*a^8*b^4 - 24*a^10*b^2))/d^4))/2
+ (32*a^3*b^10*(a^2 + b^2)^3*(6*a^4 + b^4 + 3*a^2*b^2))/d^5)*(a^5/(4*d^2) -
 (20*a^2*b^8*d^4 - b^10*d^4 - 110*a^4*b^6*d^4 + 100*a^6*b^4*d^4 - 25*a^8*b^
2*d^4)^(1/2)/(4*d^4) + (5*a*b^4)/(4*d^2) - (5*a^3*b^2)/(2*d^2))^(1/2) + (2*
b^2*(a + b*tan(c + d*x))^(1/2))/d + (atan((b^20*(a^5)^(1/2)*(a + b*tan(c +
d*x))^(1/2)*64i)/(64*a^3*b^20 + 384*a^5*b^18 + 960*a^7*b^16 - 1280*a^9*b^14
 + 3520*a^11*b^12 + 6016*a^13*b^10 + 576*a^15*b^8) + (a^2*b^18*(a^5)^(1/2)*
```

```
(a + b*tan(c + d*x))^(1/2)*384i)/(64*a^3*b^20 + 384*a^5*b^18 + 960*a^7*b^16
 - 1280*a^9*b^14 + 3520*a^11*b^12 + 6016*a^13*b^10 + 576*a^15*b^8) + (a^4*b
^16*(a^5)^(1/2)*(a + b*tan(c + d*x))^(1/2)*960i)/(64*a^3*b^20 + 384*a^5*b^1
8 + 960*a^7*b^16 - 1280*a^9*b^14 + 3520*a^11*b^...
```

3.524 $\int \cot^2(c+dx)(a+b\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=151

$$-\frac{5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{i(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}-\frac{i(a+ib)^{5/2}\tanh^{-1}}{}$$

[Out] -5*a^(3/2)*b*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/d+I*(a-I*b)^(5/2)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d-I*(a+I*b)^(5/2)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d-a^2*cot(d*x+c)*(a+b*tan(d*x+c))^(1/2)/d

Rubi [A]
time = 0.35, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3646, 3734, 3620, 3618, 65, 214, 3715}

$$-\frac{5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{a^2\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+\frac{i(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}-\frac{i(a+ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^(5/2),x]

[Out] (-5*a^(3/2)*b*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/d + (I*(a - I*b)^(5/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/d - (I*(a + I*b)^(5/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/d - (a^2*Cot[c + d*x]*Sqrt[a + b*Tan[c + d*x]])/d

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^{5/2}\,dx = -\frac{a^2\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d} + \int \frac{\cot(c+dx)\left(\frac{5a^2b}{2} - a\right.}{}$$

$$= -\frac{a^2\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d} + \frac{1}{2}(5a^2b)\int \frac{\cot(c+dx)\,(}{\sqrt{a+b}}$$

$$= -\frac{a^2\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d} - \frac{1}{2}(a-ib)^3\int \frac{1+i\tan}{\sqrt{a+b\tan}}$$

$$= -\frac{a^2\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d} + \frac{(5a^2)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx\right.}{}$$

$$= -\frac{5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{a^2\cot(c+dx)\sqrt{a+}}{d}$$

$$= -\frac{5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{i(a-ib)^{5/2}\tanh^{-1}\left(\right.}{}$$

Mathematica [A]
time = 0.54, size = 233, normalized size = 1.54

$$\frac{-5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right) + i(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right) - ia^2\sqrt{a+ib}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right) + 2a\sqrt{a+ib}\,b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right) + i\sqrt{a+ib}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right) - a^2\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] (-5*a^(3/2)*b*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]] + I*(a - I*b)^(5/2)
*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] - I*a^2*Sqrt[a + I*b]*ArcT
anh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] + 2*a*Sqrt[a + I*b]*b*ArcTanh[S
qrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] + I*Sqrt[a + I*b]*b^2*ArcTanh[Sqrt[a
 + b*Tan[c + d*x]]/Sqrt[a + I*b]] - a^2*Cot[c + d*x]*Sqrt[a + b*Tan[c + d*x
]])/d
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 1.10, size = 45710, normalized size = 302.72

method	result	size
default	Expression too large to display	45710

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^2*(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] result too large to display

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((b*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^2, x)

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6877 vs. $2(119) = 238$.
time = 10.30, size = 13829, normalized size = 91.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")

[Out] [1/4*(4*sqrt(2)*(d^5*cos(d*x + c)^2 - d^5)*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 + (a^5 - 10*a^3*b^2 + 5*a*b^4)*d^2*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)^(3/4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4)*arctan(-((5*a^18 + 25*a^16*b^2 + 36*a^14*b^4 - 28*a^12*b^6 - 154*a^10*b^8 - 210*a^8*b^10 - 140*a^6*b^12 - 44*a^4*b^14 - 3*a^2*b^16 + b^18)*d^4*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4) + (5*a^23 + 35*a^21*b^2 + 91*a^19*b^4 + 69*a^17*b^6 - 174*a^15*b^8 - 546*a^13*b^10 - 714*a^11*b^12 - 534*a^9*b^14 - 231*a^7*b^16 - 49*a^5*b^18 - a^3*b^20 + a*b^22)*d^2*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4) + sqrt(2)*(2*(5*a^9*b - 14*a^5*b^5 - 8*a^3*b^7 + a*b^9)*d^7*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4) + (15*a^14*b + 25*a^12*b^3 - 37*a^10*b^5 - 99*a^8*b^7 - 51*a^6*b^9 + 11*a^4*b^11 + 9*a^2*b^13 - b^15)*d^5*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4))*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 + (a^5 - 10*a^3*b^2 + 5*a*b^4)*d^2*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)^(3/4) - sqrt(2)*(2*a*d

^7*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4
)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4) +
(3*a^6 + 5*a^4*b^2 + a^2*b^4 - b^6)*d^5*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 11
0*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4))*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 +
10*a^4*b^6 + 5*a^2*b^8 + b^10 + (a^5 - 10*a^3*b^2 + 5*a*b^4)*d^2*sqrt((a^10
 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4))/(25*a^8*b^
2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*sqrt(((25*a^14*b^2 - 25
*a^12*b^4 - 115*a^10*b^6 + 35*a^8*b^8 + 171*a^6*b^10 + 53*a^4*b^12 - 17*a^2
*b^14 + b^16)*d^2*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*
b^8 + b^10)/d^4)*cos(d*x + c) + sqrt(2)*((75*a^10*b^3 - 325*a^8*b^5 + 430*a
^6*b^7 - 170*a^4*b^9 + 23*a^2*b^11 - b^13)*d^3*sqrt((a^10 + 5*a^8*b^2 + 10*
a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)*cos(d*x + c) + 2*(25*a^15*b^3
 - 25*a^13*b^5 - 115*a^11*b^7 + 35*a^9*b^9 + 171*a^7*b^11 + 53*a^5*b^13 - 1
7*a^3*b^15 + a*b^17)*d*cos(d*x + c))*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 +
10*a^4*b^6 + 5*a^2*b^8 + b^10 + (a^5 - 10*a^3*b^2 + 5*a*b^4)*d^2*sqrt((a^10
 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4))/(25*a^8*b^
2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*sqrt((a*cos(d*x + c) +
b*sin(d*x + c))/cos(d*x + c))*((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6
+ 5*a^2*b^8 + b^10)/d^4)^(1/4) + (25*a^19*b^2 + 25*a^17*b^4 - 140*a^15*b^6
- 220*a^13*b^8 + 126*a^11*b^10 + 430*a^9*b^12 + 260*a^7*b^14 + 20*a^5*b^16
- 15*a^3*b^18 + a*b^20)*cos(d*x + c) + (25*a^18*b^3 + 25*a^16*b^5 - 140*a^1
4*b^7 - 220*a^12*b^9 + 126*a^10*b^11 + 430*a^8*b^13 + 260*a^6*b^15 + 20*a^4
*b^17 - 15*a^2*b^19 + b^21)*sin(d*x + c))/((a^2 + b^2)*cos(d*x + c)))*((a^1
0 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)^(3/4))/(25
*a^26*b^2 + 125*a^24*b^4 + 110*a^22*b^6 - 530*a^20*b^8 - 1469*a^18*b^10 - 9
21*a^16*b^12 + 1716*a^14*b^14 + 3924*a^12*b^16 + 3471*a^10*b^18 + 1531*a^8*
b^20 + 254*a^6*b^22 - 34*a^4*b^24 - 11*a^2*b^26 + b^28)) + 4*sqrt(2)*(d^5*c
os(d*x + c)^2 - d^5)*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a
^2*b^8 + b^10 + (a^5 - 10*a^3*b^2 + 5*a*b^4)*d^2*sqrt((a^10 + 5*a^8*b^2 + 1
0*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4))/(25*a^8*b^2 - 100*a^6*b^4
+ 110*a^4*b^6 - 20*a^2*b^8 + b^10))*((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^
4*b^6 + 5*a^2*b^8 + b^10)/d^4)^(3/4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a
^4*b^6 - 20*a^2*b^8 + b^10)/d^4)*arctan(((5*a^18 + 25*a^16*b^2 + 36*a^14*b^
4 - 28*a^12*b^6 - 154*a^10*b^8 - 210*a^8*b^10 - 140*a^6*b^12 - 44*a^4*b^14
- 3*a^2*b^16 + b^18)*d^4*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 +
 5*a^2*b^8 + b^10)/d^4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a
^2*b^8 + b^10)/d^4) + (5*a^23 + 35*a^21*b^2 + 91*a^19*b^4 + 69*a^17*b^6 - 1
74*a^15*b^8 - 546*a^13*b^10 - 714*a^11*b^12 - 534*a^9*b^14 - 231*a^7*b^16 -
 49*a^5*b^18 - a^3*b^20 + a*b^22)*d^2*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*
a^4*b^6 - 20*a^2*b^8 + b^10)/d^4) - sqrt(2)*(2*(5*a^9*b - 14*a^5*b^5 - 8*a^
3*b^7 + a*b^9)*d^7*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2
*b^8 + b^10)/d^4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8
 + b^10)/d^4) + (15*a^14*b + 25*a^12*b^3 - 37*a^10*b^5 - 99*a^8*b^7 - 51*a^
6*b^9 + 11*a^4*b^11 + 9*a^2*b^13 - b^15)*d^5*sq...

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 10.36, size = 2500, normalized size = 16.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + b*tan(c + d*x))^(5/2),x)
```

```
[Out] log(((((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a^5*d^2 - 5*a*b^4*d^
2 + 10*a^3*b^2*d^2)/d^4)^(1/2)*(((((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^
(1/2) - a^5*d^2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/d^4)^(1/2)*(((((-b^2*d^4*(5
*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a^5*d^2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)
/d^4)^(1/2)*(((((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a^5*d^2 - 5
*a*b^4*d^2 + 10*a^3*b^2*d^2)/d^4)^(1/2)*((1024*a^2*b^9*(a^2 + b^2))/d - 128
*b^8*(((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a^5*d^2 - 5*a*b^4*d^
2 + 10*a^3*b^2*d^2)/d^4)^(1/2)*(3*a^2 + 2*b^2)*(a + b*tan(c + d*x))^(1/2)))
/2 - (64*a*b^8*(a + b*tan(c + d*x))^(1/2)*(5*a^6 + 19*b^6 - 5*a^2*b^4 - 76*
a^4*b^2))/d^2))/2 - (32*a*b^9*(b^8 - 23*a^8 - 100*a^2*b^6 + 44*a^4*b^4 + 12
2*a^6*b^2))/d^3))/2 - (16*b^8*(a + b*tan(c + d*x))^(1/2)*(2*a^12 + 2*b^12 +
 12*a^2*b^10 + 55*a^4*b^8 - 335*a^6*b^6 + 405*a^8*b^4 - 13*a^10*b^2))/d^4))
/2 + (40*a^2*b^9*(a^2 + b^2)^3*(2*a^6 + 2*b^6 + 11*a^2*b^4 - 9*a^4*b^2))/d^
5)*((20*a^2*b^8*d^4 - b^10*d^4 - 110*a^4*b^6*d^4 + 100*a^6*b^4*d^4 - 25*a^8
*b^2*d^4)^(1/2)/(4*d^4) - a^5/(4*d^2) - (5*a*b^4)/(4*d^2) + (5*a^3*b^2)/(2*
d^2))^(1/2) - log(((((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a^5*d^
2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/d^4)^(1/2)*(((((-b^2*d^4*(5*a^4 + b^4 - 1
```

0*a^2*b^2)^2)^(1/2) - a^5*d^2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/d^4)^(1/2)*((
(((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a^5*d^2 - 5*a*b^4*d^2 + 1
0*a^3*b^2*d^2)/d^4)^(1/2)*(((((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2)
 - a^5*d^2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/d^4)^(1/2)*((1024*a^2*b^9*(a^2 +
 b^2))/d + 128*b^8*(((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a^5*d^
2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/d^4)^(1/2)*(3*a^2 + 2*b^2)*(a + b*tan(c +
 d*x))^(1/2)))/2 + (64*a*b^8*(a + b*tan(c + d*x))^(1/2)*(5*a^6 + 19*b^6 - 5
*a^2*b^4 - 76*a^4*b^2))/d^2))/2 - (32*a*b^9*(b^8 - 23*a^8 - 100*a^2*b^6 + 4
4*a^4*b^4 + 122*a^6*b^2))/d^3))/2 + (16*b^8*(a + b*tan(c + d*x))^(1/2)*(2*a
^12 + 2*b^12 + 12*a^2*b^10 + 55*a^4*b^8 - 335*a^6*b^6 + 405*a^8*b^4 - 13*a^
10*b^2))/d^4))/2 + (40*a^2*b^9*(a^2 + b^2)^3*(2*a^6 + 2*b^6 + 11*a^2*b^4 -
9*a^4*b^2))/d^5)*(((20*a^2*b^8*d^4 - b^10*d^4 - 110*a^4*b^6*d^4 + 100*a^6*b
^4*d^4 - 25*a^8*b^2*d^4)^(1/2) - a^5*d^2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/(4
*d^4))^(1/2) - log(((-((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^5*
d^2 + 5*a*b^4*d^2 - 10*a^3*b^2*d^2)/d^4)^(1/2)*(((-((-b^2*d^4*(5*a^4 + b^4
 - 10*a^2*b^2)^2)^(1/2) + a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b^2*d^2)/d^4)^(1/2)
*(((-((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^5*d^2 + 5*a*b^4*d^2
 - 10*a^3*b^2*d^2)/d^4)^(1/2)*(((-((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^
(1/2) + a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b^2*d^2)/d^4)^(1/2)*((1024*a^2*b^9*(
a^2 + b^2))/d + 128*b^8*(-((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) +
a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b^2*d^2)/d^4)^(1/2)*(3*a^2 + 2*b^2)*(a + b*t
an(c + d*x))^(1/2)))/2 + (64*a*b^8*(a + b*tan(c + d*x))^(1/2)*(5*a^6 + 19*b
^6 - 5*a^2*b^4 - 76*a^4*b^2))/d^2))/2 - (32*a*b^9*(b^8 - 23*a^8 - 100*a^2*b
^6 + 44*a^4*b^4 + 122*a^6*b^2))/d^3))/2 + (16*b^8*(a + b*tan(c + d*x))^(1/2
)*(2*a^12 + 2*b^12 + 12*a^2*b^10 + 55*a^4*b^8 - 335*a^6*b^6 + 405*a^8*b^4 -
 13*a^10*b^2))/d^4))/2 + (40*a^2*b^9*(a^2 + b^2)^3*(2*a^6 + 2*b^6 + 11*a^2*
b^4 - 9*a^4*b^2))/d^5)*(-((20*a^2*b^8*d^4 - b^10*d^4 - 110*a^4*b^6*d^4 + 10
0*a^6*b^4*d^4 - 25*a^8*b^2*d^4)^(1/2) + a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b^2*
d^2)/(4*d^4))^(1/2) + log(((-((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2)
 + a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b^2*d^2)/d^4)^(1/2)*(((-((-b^2*d^4*(5*a^4
 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b^2*d^2)/d^4
)^(1/2)*(((-((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^5*d^2 + 5*a*
b^4*d^2 - 10*a^3*b^2*d^2)/d^4)^(1/2)*(((-((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b
^2)^2)^(1/2) + a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b^2*d^2)/d^4)^(1/2)*((1024*a^
2*b^9*(a^2 + b^2))/d - 128*b^8*(-((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(
1/2) + a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b^2*d^2)/d^4)^(1/2)*(3*a^2 + 2*b^2)*(
a + b*tan(c + d*x))^(1/2)))/2 - (64*a*b^8*(a + b*tan(c + d*x))^(1/2)*(5*a^6
 + 19*b^6 - 5*a^2*b^4 - 76*a^4*b^2))/d^2))/2 - (32*a*b^9*(b^8 - 23*a^8 - 10
0*a^2*b^6 + 44*a^4*b^4 + 122*a^6*b^2))/d^3))/2 - (16*b^8*(a + b*tan(c + d*x
))^(1/2)*(2*a^12 + 2*b^12 + 12*a^2*b^10 + 55*a^4*b^8 - 335*a^6*b^6 + 405*a^
8*b^4 - 13*a^10*b^2))/d^4))/2 + (40*a^2*b^9*(a^2 + b^2)^3*(2*a^6 + 2*b^6 +
11*a^2*b^4 - 9*a^4*b^2))/d^5)*((5*a^3*b^2)/(2*d^2) - a^5/(4*d^2) - (5*a*b^4
)/(4*d^2) - (20*a^2*b^8*d^4 - b^10*d^4 - 110*a^4*b^6*d^4 + 100*a^6*b^4*d^4
 - 25*a^8*b^2*d^4)^(1/2)/(4*d^4))^(1/2) + (b*atan((b^21*(a^3)^(1/2)*(a + b*t
an(c + d*x))^(1/2)*160i)/(160*a^2*b^21 + 960*a^4*b^19 + 42400*a^6*b^17 + 63

```
200*a^8*b^15 + 30400*a^10*b^13 + 8960*a^12*b^11 + 160*a^14*b^9) + (a^2*b^19
*(a^3)^(1/2)*(a + b*tan(c + d*x))^(1/2)*960i)/(160*a^2*b^21 + 960*a^4*b^19
+ 42400*a^6*b^17 + 63200*a^8*b^15 + 30400*a^10*b^13 + 8960*a^12*b^11 + 160*
a^14*b^9) + (a^4*b^17*(a^3)^(1/2)*(a + b*tan(c + d*x))^(1/2)*42400i)/(160*a
^2*b^21 + 960*a^4*b^19 + 42400*a^6*b^17 + 63200...
```

3.525 $\int \cot^3(c+dx)(a+b\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=192

$$\frac{\sqrt{a}\,(8a^2-15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{4d}-\frac{(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}-\frac{(a+ib)^{5/2}}{}$$

```
[Out] -(a-I*b)^(5/2)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d-(a+I*b)^(5/2
)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d+1/4*(8*a^2-15*b^2)*arctan
h((a+b*tan(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-9/4*a*b*cot(d*x+c)*(a+b*tan(d*x
+c))^(1/2)/d-1/2*a^2*cot(d*x+c)^2*(a+b*tan(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.53, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3646, 3730, 3734, 3620, 3618, 65, 214, 3715}

$$\frac{\sqrt{a}\,(8a^2-15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{4d}-\frac{a^2\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}-\frac{(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}-\frac{(a+ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}-\frac{9ab\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[a]*(8*a^2 - 15*b^2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/(4*d)
- ((a - I*b)^(5/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/d - ((a
 + I*b)^(5/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/d - (9*a*b*C
ot[c + d*x]*Sqrt[a + b*Tan[c + d*x]])/(4*d) - (a^2*Cot[c + d*x]^2*Sqrt[a +
b*Tan[c + d*x]])/(2*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
```

```
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^{5/2}\,dx = -\frac{a^2\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d} + \frac{1}{2}\int \frac{\cot^2(c+dx)\left(\frac{9a^2b}{2}\right.}{}$$

$$= -\frac{9ab\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{4d} - \frac{a^2\cot^2(c+dx)\sqrt{a+b}}{2d}$$

$$= -\frac{9ab\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{4d} - \frac{a^2\cot^2(c+dx)\sqrt{a+b}}{2d}$$

$$= -\frac{9ab\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{4d} - \frac{a^2\cot^2(c+dx)\sqrt{a+b}}{2d}$$

$$= -\frac{9ab\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{4d} - \frac{a^2\cot^2(c+dx)\sqrt{a+b}}{2d}$$

$$= \frac{\sqrt{a}\,(8a^2-15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{4d} - \frac{9ab\cot(c+}{}$$

$$= \frac{\sqrt{a}\,(8a^2-15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{4d} - \frac{(a-ib)^{5/2}\,\mathrm{t}}{}$$

Mathematica [A]

time = 1.47, size = 268, normalized size = 1.40

$$-\frac{-\sqrt{a}\,(8a^2-15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)+4(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)+4a^2\sqrt{a+ib}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)+8ia\sqrt{a+ib}\,b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)-4\sqrt{a+ib}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)+9ab\cot(c+dx)\sqrt{a+b\tan(c+dx)}+2a^2\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] -1/4*(-(Sqrt[a]*(8*a^2 - 15*b^2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])
 + 4*(a - I*b)^(5/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] + 4*a^
2*Sqrt[a + I*b]*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] + (8*I)*a*S
qrt[a + I*b]*b*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] - 4*Sqrt[a +
 I*b]*b^2*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] + 9*a*b*Cot[c + d
*x]*Sqrt[a + b*Tan[c + d*x]] + 2*a^2*Cot[c + d*x]^2*Sqrt[a + b*Tan[c + d*x]
])/d
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 1.49, size = 66439, normalized size = 346.04

method	result	size
default	Expression too large to display	66439

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6977 vs. $2(156) = 312$.
time = 10.55, size = 14030, normalized size = 73.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(16*sqrt(2)*(d^5*cos(d*x + c)^2 - d^5)*sqrt((a^10 + 5*a^8*b^2 + 10*a
^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 - (a^5 - 10*a^3*b^2 + 5*a*b^4)*d^2*s
qrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4))/(
25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*((a^10 + 5*a^8
*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)^(3/4)*sqrt((25*a^8*
b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4)*arctan(((5*a^18 +
 25*a^16*b^2 + 36*a^14*b^4 - 28*a^12*b^6 - 154*a^10*b^8 - 210*a^8*b^10 - 14
0*a^6*b^12 - 44*a^4*b^14 - 3*a^2*b^16 + b^18)*d^4*sqrt((a^10 + 5*a^8*b^2 +
```

10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)*sqrt((25*a^8*b^2 - 100*a^6
*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4) + (5*a^23 + 35*a^21*b^2 + 91*a
^19*b^4 + 69*a^17*b^6 - 174*a^15*b^8 - 546*a^13*b^10 - 714*a^11*b^12 - 534*
a^9*b^14 - 231*a^7*b^16 - 49*a^5*b^18 - a^3*b^20 + a*b^22)*d^2*sqrt((25*a^8
*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4) + sqrt(2)*((5*a^
10 - 5*a^8*b^2 - 14*a^6*b^4 + 6*a^4*b^6 + 9*a^2*b^8 - b^10)*d^7*sqrt((a^10
+ 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)*sqrt((25*a^8
*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4) + (5*a^15 - 5*a^
13*b^2 - 39*a^11*b^4 - 9*a^9*b^6 + 79*a^7*b^8 + 81*a^5*b^10 + 19*a^3*b^12 -
3*a*b^14)*d^5*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 +
b^10)/d^4))*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 +
b^10 - (a^5 - 10*a^3*b^2 + 5*a*b^4)*d^2*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4
+ 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4
*b^6 - 20*a^2*b^8 + b^10))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x +
c))*((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)^
(3/4) - sqrt(2)*((a^2 - b^2)*d^7*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a
^4*b^6 + 5*a^2*b^8 + b^10)/d^4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^
6 - 20*a^2*b^8 + b^10)/d^4) + (a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^6)*d^5*sqr
t((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4))*sqrt((
a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 - (a^5 - 10*a
^3*b^2 + 5*a*b^4)*d^2*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*
a^2*b^8 + b^10)/d^4))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8
+ b^10))*sqrt(((25*a^14 - 25*a^12*b^2 - 115*a^10*b^4 + 35*a^8*b^6 + 171*a^6
*b^8 + 53*a^4*b^10 - 17*a^2*b^12 + b^14)*d^2*sqrt((a^10 + 5*a^8*b^2 + 10*a^
6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)*cos(d*x + c) + sqrt(2)*((25*a^1
1 - 175*a^9*b^2 + 410*a^7*b^4 - 350*a^5*b^6 + 61*a^3*b^8 - 3*a*b^10)*d^3*sq
rt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)*cos
(d*x + c) + (25*a^16 - 50*a^14*b^2 - 90*a^12*b^4 + 150*a^10*b^6 + 136*a^8*b
^8 - 118*a^6*b^10 - 70*a^4*b^12 + 18*a^2*b^14 - b^16)*d*cos(d*x + c))*sqrt(
(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 - (a^5 - 10*
a^3*b^2 + 5*a*b^4)*d^2*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5
*a^2*b^8 + b^10)/d^4))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8
+ b^10))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*((a^10 + 5*a
^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)^(1/4) + (25*a^19
+ 25*a^17*b^2 - 140*a^15*b^4 - 220*a^13*b^6 + 126*a^11*b^8 + 430*a^9*b^10 +
260*a^7*b^12 + 20*a^5*b^14 - 15*a^3*b^16 + a*b^18)*cos(d*x + c) + (25*a^18
*b + 25*a^16*b^3 - 140*a^14*b^5 - 220*a^12*b^7 + 126*a^10*b^9 + 430*a^8*b^1
1 + 260*a^6*b^13 + 20*a^4*b^15 - 15*a^2*b^17 + b^19)*sin(d*x + c))/((a^2 +
b^2)*cos(d*x + c)))*((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^
8 + b^10)/d^4)^(3/4))/(25*a^26*b^2 + 125*a^24*b^4 + 110*a^22*b^6 - 530*a^20
*b^8 - 1469*a^18*b^10 - 921*a^16*b^12 + 1716*a^14*b^14 + 3924*a^12*b^16 + 3
471*a^10*b^18 + 1531*a^8*b^20 + 254*a^6*b^22 - 34*a^4*b^24 - 11*a^2*b^26 +
b^28)) + 16*sqrt(2)*(d^5*cos(d*x + c)^2 - d^5)*sqrt((a^10 + 5*a^8*b^2 + 10*
a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 - (a^5 - 10*a^3*b^2 + 5*a*b^4)*d^2*
sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4))/

```
(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*((a^10 + 5*a^
8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)^(3/4)*sqrt((25*a^8
*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4)*arctan(-((5*a^18
 + 25*a^16*b^2 + 36*a^14*b^4 - 28*a^12*b^6 - 154*a^10*b^8 - 210*a^8*b^10 -
140*a^6*b^12 - 44*a^4*b^14 - 3*a^2*b^16 + b^18)*d^4*sqrt((a^10 + 5*a^8*b^2
+ 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)*sqrt((25*a^8*b^2 - 100*a
^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4) + (5*a^23 + 35*a^21*b^2 + 91
*a^19*b^4 + 69*a^17*b^6 - 174*a^15*b^8 - 546*a^13*b^10 - 714*a^11*b^12 - 53
4*a^9*b^14 - 231*a^7*b^16 - 49*a^5*b^18 - a^3*b^20 + a*b^22)*d^2*sqrt((25*a
^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4) - sqrt(2)*((5*
a^10 - 5*a^8*b^2 - 14*a^6*b^4 + 6*a^4*b^6 + 9*a^2*b^8 - b^10)*d^7*sqrt((a^1
0 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)*sqrt((25*a
^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4) + (5*a^15 - 5*
a^13*b^2 - 39*a^11*b^4 - 9*a^9*b^6 + 79*a^7*b^8...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 10.78, size = 2500, normalized size = 13.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + b*tan(c + d*x))^(5/2),x)
```

```
[Out] log(- ((((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^5*d^2 + 5*a*b^4*
d^2 - 10*a^3*b^2*d^2)/d^4)^(1/2)*(((((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2
)^(1/2) + a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b^2*d^2)/d^4)^(1/2)*(((((-b^2*d^4*
```

(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b^2*d^
2)/d^4)^(1/2)*(((((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^5*d^2 +
5*a*b^4*d^2 - 10*a^3*b^2*d^2)/d^4)^(1/2)*(128*b^8*(((-b^2*d^4*(5*a^4 + b^4
- 10*a^2*b^2)^2)^(1/2) + a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b^2*d^2)/d^4)^(1/2
)*(3*a^2 + 2*b^2)*(a + b*tan(c + d*x))^(1/2) - (64*a*b^8*(13*b^4 - 6*a^4 +
7*a^2*b^2))/d))/2 - (4*a*b^8*(a + b*tan(c + d*x))^(1/2)*(144*a^6 + 304*b^6
+ 145*a^2*b^4 - 1056*a^4*b^2))/d^2))/2 - (6*a^2*b^8*(16*a^8 - 309*b^8 + 576
*a^2*b^6 + 485*a^4*b^4 - 384*a^6*b^2))/d^3))/2 + (b^8*(a + b*tan(c + d*x))^
(1/2)*(96*a^12 + 32*b^12 - 33*a^2*b^10 + 4095*a^4*b^8 - 6399*a^6*b^6 + 5265
*a^8*b^4 - 1008*a^10*b^2))/d^4))/2 - (a*b^10*(a^2 + b^2)^3*(504*a^6 - 120*b
^6 + 379*a^2*b^4 - 1113*a^4*b^2))/(2*d^5))*((20*a^2*b^8*d^4 - b^10*d^4 - 11
0*a^4*b^6*d^4 + 100*a^6*b^4*d^4 - 25*a^8*b^2*d^4)^(1/2)/(4*d^4) + a^5/(4*d^
2) + (5*a*b^4)/(4*d^2) - (5*a^3*b^2)/(2*d^2))^(1/2) - log(((((-b^2*d^4*(5*a
^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b^2*d^2)/d
^4)^(1/2)*(((((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^5*d^2 + 5*a
*b^4*d^2 - 10*a^3*b^2*d^2)/d^4)^(1/2)*(((((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b
^2)^2)^(1/2) + a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b^2*d^2)/d^4)^(1/2)*(((((-b^2
d^4(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b
^2*d^2)/d^4)^(1/2)*(128*b^8*(((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2)
+ a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b^2*d^2)/d^4)^(1/2)*(3*a^2 + 2*b^2)*(a +
b*tan(c + d*x))^(1/2) + (64*a*b^8*(13*b^4 - 6*a^4 + 7*a^2*b^2))/d))/2 - (4*
a*b^8*(a + b*tan(c + d*x))^(1/2)*(144*a^6 + 304*b^6 + 145*a^2*b^4 - 1056*a^
4*b^2))/d^2))/2 + (6*a^2*b^8*(16*a^8 - 309*b^8 + 576*a^2*b^6 + 485*a^4*b^4
- 384*a^6*b^2))/d^3))/2 + (b^8*(a + b*tan(c + d*x))^(1/2)*(96*a^12 + 32*b^1
2 - 33*a^2*b^10 + 4095*a^4*b^8 - 6399*a^6*b^6 + 5265*a^8*b^4 - 1008*a^10*b^
2))/d^4))/2 - (a*b^10*(a^2 + b^2)^3*(504*a^6 - 120*b^6 + 379*a^2*b^4 - 1113
*a^4*b^2))/(2*d^5))*(((20*a^2*b^8*d^4 - b^10*d^4 - 110*a^4*b^6*d^4 + 100*a^
6*b^4*d^4 - 25*a^8*b^2*d^4)^(1/2) + a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b^2*d^2)
/(4*d^4))^(1/2) - log(((-((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a
^5*d^2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/d^4)^(1/2)*(((-((-b^2*d^4*(5*a^4 + b
^4 - 10*a^2*b^2)^2)^(1/2) - a^5*d^2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/d^4)^(1
/2)*(((-((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a^5*d^2 - 5*a*b^4*
d^2 + 10*a^3*b^2*d^2)/d^4)^(1/2)*(((-((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^
2)^(1/2) - a^5*d^2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/d^4)^(1/2)*(128*b^8*(-((
-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a^5*d^2 - 5*a*b^4*d^2 + 10*a
^3*b^2*d^2)/d^4)^(1/2)*(3*a^2 + 2*b^2)*(a + b*tan(c + d*x))^(1/2) + (64*a*b
^8*(13*b^4 - 6*a^4 + 7*a^2*b^2))/d))/2 - (4*a*b^8*(a + b*tan(c + d*x))^(1/2
)*(144*a^6 + 304*b^6 + 145*a^2*b^4 - 1056*a^4*b^2))/d^2))/2 + (6*a^2*b^8*(1
6*a^8 - 309*b^8 + 576*a^2*b^6 + 485*a^4*b^4 - 384*a^6*b^2))/d^3))/2 + (b^8*
(a + b*tan(c + d*x))^(1/2)*(96*a^12 + 32*b^12 - 33*a^2*b^10 + 4095*a^4*b^8
- 6399*a^6*b^6 + 5265*a^8*b^4 - 1008*a^10*b^2))/d^4))/2 - (a*b^10*(a^2 + b^
2)^3*(504*a^6 - 120*b^6 + 379*a^2*b^4 - 1113*a^4*b^2))/(2*d^5))*(-((20*a^2*
b^8*d^4 - b^10*d^4 - 110*a^4*b^6*d^4 + 100*a^6*b^4*d^4 - 25*a^8*b^2*d^4)^(1
/2) - a^5*d^2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/(4*d^4))^(1/2) + log(- ((-((-
b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a^5*d^2 - 5*a*b^4*d^2 + 10*a^

3*b^2*d^2)/d^4)^(1/2)*(((-((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a^5*d^2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/d^4)^(1/2)*(((-((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a^5*d^2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/d^4)^(1/2)*(((-((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a^5*d^2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/d^4)^(1/2)*(128*b^8*(-((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a^5*d^2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/d^4)^(1/2)*(3*a^2 + 2*b^2)*(a + b*tan(c + d*x))^(1/2) - (64*a*b^8*(13*b^4 - 6*a^4 + 7*a^2*b^2))/d))/2 - (4*a*b^8*(a + b*tan(c + d*x))^(1/2)*(144*a^6 + 304*b^6 + 145*a^2*b^4 - 1056*a^4*b^2))/d^2))/2 - (6*a^2*b^8*(16*a^8 - 309*b^8 + 576*a^2*b^6 + 485*a^4*b^4 - 384*a^6*b^2))/d^3))/2 + (b^8*(a + b*tan(c + d*x))^(1/2)*(96*a^12 + 32*b^12 - 33*a^2*b^10 + 4095*a^4*b^8 - 6399*a^6*b^6 + 5265*a^8*b^4 - 1008*a^10*b^2))/d^4))/2 - (a*b^10*(a^2 + b^2)^3*(504*a^6 - 120*b^6 + 379*a^2*b^4 - 1113*a^4*b^2))/(2*d^5))*(a^5/(4*d^2) - (20*a^2*b^8*d^4 - b^10*d^4 - 110*a^4*b^6*d^4 + 100*a^6*b^4*d^4 - 25*a^8*b^2*d^4)^(1/2)/(4*d^4) + (5*a*b^4)/(4*d^2) - (5*a^3*b^2)/(2*d^2))^(1/2) + ((7*a^2*b^2*(a + b*tan(c + d*x))^(1/2))/4 - (9*a*b^2*(a + b*tan(c + d*x))^(3/2))/4)/(d*(a + b*tan(c + d*x))^2 + a^2*d - 2*a*d*(a + b*tan(c + d*x))) + (a^(1/2)*atan(((a^(1/2)*(a^2 - (15*b^2)/8)*(((a + b*tan(c + d*x))^(1/2)*(32*b^20 - 33*a^2*b^18 + 4095*a^4*b^16 - 6399*a^6*b^14 + 5265*a^8*b^12 - 1008*...

3.526 $\int \cot^4(c+dx)(a+b\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=237

$$\frac{5b(8a^2-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{8\sqrt{a}\,d}-\frac{i(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}+\frac{i(a+ib)^{5/2}\tan}{}$$

```
[Out] -I*(a-I*b)^(5/2)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d+I*(a+I*b)^
(5/2)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d+5/8*b*(8*a^2-b^2)*arc
tanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)+1/8*(8*a^2-11*b^2)*cot(d*x+c
)*(a+b*tan(d*x+c))^(1/2)/d-13/12*a*b*cot(d*x+c)^2*(a+b*tan(d*x+c))^(1/2)/d-
1/3*a^2*cot(d*x+c)^3*(a+b*tan(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.71, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3646, 3730, 3734, 3620, 3618, 65, 214, 3715}

$$\frac{5b(8a^2-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{8\sqrt{a}\,d}+\frac{(8a^2-11b^2)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{8d}-\frac{a^2\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}-\frac{i(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}+\frac{i(a+ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}-\frac{13ab\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{12d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] (5*b*(8*a^2 - b^2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/(8*Sqrt[a]*d)
 - (I*(a - I*b)^(5/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/d +
(I*(a + I*b)^(5/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/d + ((8
*a^2 - 11*b^2)*Cot[c + d*x]*Sqrt[a + b*Tan[c + d*x]])/(8*d) - (13*a*b*Cot[c
 + d*x]^2*Sqrt[a + b*Tan[c + d*x]])/(12*d) - (a^2*Cot[c + d*x]^3*Sqrt[a + b
*Tan[c + d*x]])/(3*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
```

```
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^{5/2}\,dx = -\frac{a^2\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d} + \frac{1}{3}\int \frac{\cot^3(c+dx)\left(\frac{13a^2}{2}\right.}{}$$

$$= -\frac{13ab\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{12d} - \frac{a^2\cot^3(c+dx)\sqrt{a+}}{3d}$$

$$= \frac{(8a^2-11b^2)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{8d} - \frac{13ab\cot^2(c+d}{}$$

$$= \frac{(8a^2-11b^2)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{8d} - \frac{13ab\cot^2(c+d}{}$$

$$= \frac{(8a^2-11b^2)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{8d} - \frac{13ab\cot^2(c+d}{}$$

$$= \frac{(8a^2-11b^2)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{8d} - \frac{13ab\cot^2(c+d}{}$$

$$= \frac{5b(8a^2-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{8\sqrt{a}\,d} + \frac{(8a^2-11b^2)\cot}{}$$

$$= \frac{5b(8a^2-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{8\sqrt{a}\,d} - \frac{i(a-ib)^{5/2}\tanh}{}$$

Mathematica [A]

time = 3.73, size = 185, normalized size = 0.78

$$-\frac{\frac{15b(-8a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}}+24i(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)-24i(a+ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)+\cot(c+dx)\left(-24a^2+33b^2+26ab\cot(c+dx)+8a^2\cot^2(c+dx)\right)\sqrt{a+b\tan(c+dx)}}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] -1/24*((15*b*(-8*a^2 + b^2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/Sqrt
[a] + (24*I)*(a - I*b)^(5/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]
] - (24*I)*(a + I*b)^(5/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]]
+ Cot[c + d*x]*(-24*a^2 + 33*b^2 + 26*a*b*Cot[c + d*x] + 8*a^2*Cot[c + d*x]
^2)*Sqrt[a + b*Tan[c + d*x]])/d
```

Maple [**C**] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 1.64, size = 88284, normalized size = 372.51

method	result	size
default	Expression too large to display	88284

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 7410 vs. 2(193) = 386.
time = 9.74, size = 14895, normalized size = 62.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(96*sqrt(2)*(a*d^5*cos(d*x + c)^4 - 2*a*d^5*cos(d*x + c)^2 + a*d^5)*
sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 + (a^5
```

- 10*a^3*b^2 + 5*a*b^4)*d^2*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^
6 + 5*a^2*b^8 + b^10)/d^4))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^
2*b^8 + b^10))*((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b
^10)/d^4)^(3/4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 +
 b^10)/d^4)*arctan(-((5*a^18 + 25*a^16*b^2 + 36*a^14*b^4 - 28*a^12*b^6 - 15
4*a^10*b^8 - 210*a^8*b^10 - 140*a^6*b^12 - 44*a^4*b^14 - 3*a^2*b^16 + b^18)
*d^4*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d
^4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4)
+ (5*a^23 + 35*a^21*b^2 + 91*a^19*b^4 + 69*a^17*b^6 - 174*a^15*b^8 - 546*a^
13*b^10 - 714*a^11*b^12 - 534*a^9*b^14 - 231*a^7*b^16 - 49*a^5*b^18 - a^3*b
^20 + a*b^22)*d^2*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8
 + b^10)/d^4) + sqrt(2)*(2*(5*a^9*b - 14*a^5*b^5 - 8*a^3*b^7 + a*b^9)*d^7*s
qrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)*sq
rt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4) + (15*
a^14*b + 25*a^12*b^3 - 37*a^10*b^5 - 99*a^8*b^7 - 51*a^6*b^9 + 11*a^4*b^11
+ 9*a^2*b^13 - b^15)*d^5*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*
a^2*b^8 + b^10)/d^4))*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*
a^2*b^8 + b^10 + (a^5 - 10*a^3*b^2 + 5*a*b^4)*d^2*sqrt((a^10 + 5*a^8*b^2 +
10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4))/(25*a^8*b^2 - 100*a^6*b^4
 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))
/cos(d*x + c))*((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b
^10)/d^4)^(3/4) - sqrt(2)*(2*a*d^7*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10
*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*
b^6 - 20*a^2*b^8 + b^10)/d^4) + (3*a^6 + 5*a^4*b^2 + a^2*b^4 - b^6)*d^5*sqr
t((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4))*sqrt((
a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 + (a^5 - 10*a
^3*b^2 + 5*a*b^4)*d^2*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*
a^2*b^8 + b^10)/d^4))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8
+ b^10))*sqrt(((25*a^14*b^2 - 25*a^12*b^4 - 115*a^10*b^6 + 35*a^8*b^8 + 171
*a^6*b^10 + 53*a^4*b^12 - 17*a^2*b^14 + b^16)*d^2*sqrt((a^10 + 5*a^8*b^2 +
10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)*cos(d*x + c) + sqrt(2)*((7
5*a^10*b^3 - 325*a^8*b^5 + 430*a^6*b^7 - 170*a^4*b^9 + 23*a^2*b^11 - b^13)*
d^3*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^
4)*cos(d*x + c) + 2*(25*a^15*b^3 - 25*a^13*b^5 - 115*a^11*b^7 + 35*a^9*b^9
+ 171*a^7*b^11 + 53*a^5*b^13 - 17*a^3*b^15 + a*b^17)*d*cos(d*x + c))*sqrt((
a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 + (a^5 - 10*a
^3*b^2 + 5*a*b^4)*d^2*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*
a^2*b^8 + b^10)/d^4))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8
+ b^10))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*((a^10 + 5*a^
8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4)^(1/4) + (25*a^19*b
^2 + 25*a^17*b^4 - 140*a^15*b^6 - 220*a^13*b^8 + 126*a^11*b^10 + 430*a^9*b^
12 + 260*a^7*b^14 + 20*a^5*b^16 - 15*a^3*b^18 + a*b^20)*cos(d*x + c) + (25*
a^18*b^3 + 25*a^16*b^5 - 140*a^14*b^7 - 220*a^12*b^9 + 126*a^10*b^11 + 430*
a^8*b^13 + 260*a^6*b^15 + 20*a^4*b^17 - 15*a^2*b^19 + b^21)*sin(d*x + c))/(
(a^2 + b^2)*cos(d*x + c)))*((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5

```
*a^2*b^8 + b^10)/d^4)^(3/4))/(25*a^26*b^2 + 125*a^24*b^4 + 110*a^22*b^6 - 5
30*a^20*b^8 - 1469*a^18*b^10 - 921*a^16*b^12 + 1716*a^14*b^14 + 3924*a^12*b
^16 + 3471*a^10*b^18 + 1531*a^8*b^20 + 254*a^6*b^22 - 34*a^4*b^24 - 11*a^2*
b^26 + b^28)) + 96*sqrt(2)*(a*d^5*cos(d*x + c)^4 - 2*a*d^5*cos(d*x + c)^2 +
 a*d^5)*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10
 + (a^5 - 10*a^3*b^2 + 5*a*b^4)*d^2*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 1
0*a^4*b^6 + 5*a^2*b^8 + b^10)/d^4))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6
 - 20*a^2*b^8 + b^10))*((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2
*b^8 + b^10)/d^4)^(3/4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a
^2*b^8 + b^10)/d^4)*arctan(((5*a^18 + 25*a^16*b^2 + 36*a^14*b^4 - 28*a^12*b
^6 - 154*a^10*b^8 - 210*a^8*b^10 - 140*a^6*b^12 - 44*a^4*b^14 - 3*a^2*b^16
+ b^18)*d^4*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 +
b^10)/d^4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10
)/d^4) + (5*a^23 + 35*a^21*b^2 + 91*a^19*b^4 + 69*a^17*b^6 - 174*a^15*b^8 -
 546*a^13*b^10 - 714*a^11*b^12 - 534*a^9*b^14 - 231*a^7*b^16 - 49*a^5*b^18
- a^3*b^20 + a*b^22)*d^2*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*
a^2*b^8 + b^10)/d^4) - sqrt(2)*(2*(5*a^9*b - 14*a^5*b^5 - 8*a^3*b^7 + a*b^9
)*d^7*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)/
d^4)*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/d^4)
 + (15*a^14*b + 25*a^12*b^3 - 37*a^10*b^5 - 99*...
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*tan(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**B**]
time = 10.91, size = 2500, normalized size = 10.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + b*tan(c + d*x))^(5/2),x)
```

```
[Out] (((5*a*b^3)/3 - 2*a^3*b)*(a + b*tan(c + d*x))^(3/2) + (a^4*b - (5*a^2*b^3)/
8)*(a + b*tan(c + d*x))^(1/2) + (a^2*b - (11*b^3)/8)*(a + b*tan(c + d*x))^(
5/2))/(d*(a + b*tan(c + d*x))^3 - a^3*d - 3*a*d*(a + b*tan(c + d*x))^2 + 3*
a^2*d*(a + b*tan(c + d*x))) - log((5*b^9*(a^2 + b^2)^3*(11*b^8 - 128*a^8 +
15*a^2*b^6 - 896*a^4*b^4 + 592*a^6*b^2))/(8*d^5) - ((-((-b^2*d^4*(5*a^4 + b
^4 - 10*a^2*b^2)^2)^(1/2) + a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b^2*d^2)/d^4)^(1
/2)*(((-((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^5*d^2 + 5*a*b^4*
d^2 - 10*a^3*b^2*d^2)/d^4)^(1/2)*(((-((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^
2)^(1/2) + a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b^2*d^2)/d^4)^(1/2)*(((-((-b^2*d^
4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b^2*
d^2)/d^4)^(1/2)*((32*b^9*(32*a^4 - 5*b^4 + 27*a^2*b^2))/d - 128*b^8*(-((-b^
2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*
b^2*d^2)/d^4)^(1/2)*(3*a^2 + 2*b^2)*(a + b*tan(c + d*x))^(1/2)))/2 - (a*b^8
*(a + b*tan(c + d*x))^(1/2)*(320*a^6 + 1191*b^6 + 80*a^2*b^4 - 4864*a^4*b^2
))/d^2))/2 - (a*b^9*(407*b^8 - 736*a^8 - 3225*a^2*b^6 + 1088*a^4*b^4 + 3984
*a^6*b^2))/d^3))/2 - (b^8*(a + b*tan(c + d*x))^(1/2)*(128*a^12 + 153*b^12 -
 7*a^2*b^10 + 9895*a^4*b^8 - 27465*a^6*b^6 + 26320*a^8*b^4 - 832*a^10*b^2))
/(4*d^4)))/2)*(-((20*a^2*b^8*d^4 - b^10*d^4 - 110*a^4*b^6*d^4 + 100*a^6*b^4
*d^4 - 25*a^8*b^2*d^4)^(1/2) + a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b^2*d^2)/(4*d
^4))^(1/2) - log((5*b^9*(a^2 + b^2)^3*(11*b^8 - 128*a^8 + 15*a^2*b^6 - 896*
a^4*b^4 + 592*a^6*b^2))/(8*d^5) - ((((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2
)^(1/2) - a^5*d^2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/d^4)^(1/2)*(((((-b^2*d^4*
(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a^5*d^2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^
2)/d^4)^(1/2)*(((((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a^5*d^2 -
 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/d^4)^(1/2)*(((((-b^2*d^4*(5*a^4 + b^4 - 10*a
^2*b^2)^2)^(1/2) - a^5*d^2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/d^4)^(1/2)*((32*
b^9*(32*a^4 - 5*b^4 + 27*a^2*b^2))/d - 128*b^8*(((-b^2*d^4*(5*a^4 + b^4 - 1
0*a^2*b^2)^2)^(1/2) - a^5*d^2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/d^4)^(1/2)*(3
*a^2 + 2*b^2)*(a + b*tan(c + d*x))^(1/2)))/2 - (a*b^8*(a + b*tan(c + d*x))^
(1/2)*(320*a^6 + 1191*b^6 + 80*a^2*b^4 - 4864*a^4*b^2))/d^2))/2 - (a*b^9*(4
07*b^8 - 736*a^8 - 3225*a^2*b^6 + 1088*a^4*b^4 + 3984*a^6*b^2))/d^3))/2 - (
b^8*(a + b*tan(c + d*x))^(1/2)*(128*a^12 + 153*b^12 - 7*a^2*b^10 + 9895*a^4
*b^8 - 27465*a^6*b^6 + 26320*a^8*b^4 - 832*a^10*b^2))/(4*d^4)))/2)*(((20*a^
2*b^8*d^4 - b^10*d^4 - 110*a^4*b^6*d^4 + 100*a^6*b^4*d^4 - 25*a^8*b^2*d^4)^
(1/2) - a^5*d^2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/(4*d^4))^(1/2) + log((5*b^9
*(a^2 + b^2)^3*(11*b^8 - 128*a^8 + 15*a^2*b^6 - 896*a^4*b^4 + 592*a^6*b^2))
/(8*d^5) - ((((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a^5*d^2 - 5*a
*b^4*d^2 + 10*a^3*b^2*d^2)/d^4)^(1/2)*(((((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b
^2)^2)^(1/2) - a^5*d^2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/d^4)^(1/2)*(((((-b^2
*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a^5*d^2 - 5*a*b^4*d^2 + 10*a^3*b
^2*d^2)/d^4)^(1/2)*(((((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a^5*
d^2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/d^4)^(1/2)*((32*b^9*(32*a^4 - 5*b^4 + 2
7*a^2*b^2))/d + 128*b^8*(((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a
```

^5*d^2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/d^4)^(1/2)*(3*a^2 + 2*b^2)*(a + b*tan(c + d*x))^(1/2)))/2 + (a*b^8*(a + b*tan(c + d*x))^(1/2)*(320*a^6 + 1191*b^6 + 80*a^2*b^4 - 4864*a^4*b^2))/d^2))/2 - (a*b^9*(407*b^8 - 736*a^8 - 3225*a^2*b^6 + 1088*a^4*b^4 + 3984*a^6*b^2))/d^3))/2 + (b^8*(a + b*tan(c + d*x))^(1/2)*(128*a^12 + 153*b^12 - 7*a^2*b^10 + 9895*a^4*b^8 - 27465*a^6*b^6 + 26320*a^8*b^4 - 832*a^10*b^2))/(4*d^4)))/2)*((20*a^2*b^8*d^4 - b^10*d^4 - 110*a^4*b^6*d^4 + 100*a^6*b^4*d^4 - 25*a^8*b^2*d^4)^(1/2)/(4*d^4) - a^5/(4*d^2) - (5*a*b^4)/(4*d^2) + (5*a^3*b^2)/(2*d^2))^(1/2) + log((5*b^9*(a^2 + b^2)^3*(11*b^8 - 128*a^8 + 15*a^2*b^6 - 896*a^4*b^4 + 592*a^6*b^2))/(8*d^5) - ((-((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b^2*d^2)/d^4)^(1/2)*(((-((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b^2*d^2)/d^4)^(1/2)*(((-((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b^2*d^2)/d^4)^(1/2)*(((-((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b^2*d^2)/d^4)^(1/2)*((32*b^9*(32*a^4 - 5*b^4 + 27*a^2*b^2))/d + 128*b^8*(-((-b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b^2*d^2)/d^4)^(1/2)*(3*a^2 + 2*b^2)*(a + b*tan(c + d*x))^(1/2)))/2 + (a*b^8*(a + b*tan(c + d*x))^(1/2)*(320*a^6 + 1191*b^6 + 80*a^2*b^4 - 4864*a^4*b^2))/d^2))/2 - (a*b^9*(407*b^8 - 736*a^8 - 3225*a^2*b^6 + 1088*a^4*b^4 + 3984*a^6*b^2))/d^3))/2 + (b^8*(a + b*tan(c + d*x))^(1/2)*(128*a^12 + 153*b^12 - 7*a^2*b^10 + 9895*a^4*b^8 - 27465*a^6*b^6 + 26320*a^8*b^4 - 832*a^10*b^2))/(4*d^4)))/2)*((5*a^3*b^2)/(2*d^2) - a^5/(4*d^2) - (5*a*b^4)/(4*d^2) - (20*a^2*b^8*d^4 - b^10*d^4 - 110*a^4*b^6*d^4 + 100*a^6*b^4*d^4 - 25*a^8*b^2*d^4)^(1/2)/(4*d^4))^(1/2)...

3.527 $\int (a+b\tan(c+dx))^{7/2}\,dx$

Optimal. Leaf size=167

$$-\frac{i(a-ib)^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}+\frac{i(a+ib)^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}+\frac{2b(3a^2-b^2)}{}$$

```
[Out] -I*(a-I*b)^(7/2)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d+I*(a+I*b)^
(7/2)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d+2*b*(3*a^2-b^2)*(a+b*
tan(d*x+c))^(1/2)/d+4/3*a*b*(a+b*tan(d*x+c))^(3/2)/d+2/5*b*(a+b*tan(d*x+c))
^(5/2)/d
```

Rubi [A]
time = 0.24, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3563, 3609, 3620, 3618, 65, 214}

$$\frac{2b(3a^2-b^2)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2b(a+b\tan(c+dx))^{5/2}}{5d}+\frac{4ab(a+b\tan(c+dx))^{3/2}}{3d}-\frac{i(a-ib)^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}+\frac{i(a+ib)^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((-I)*(a - I*b)^(7/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/d +
(I*(a + I*b)^(7/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/d + (2*
b*(3*a^2 - b^2)*Sqrt[a + b*Tan[c + d*x]])/d + (4*a*b*(a + b*Tan[c + d*x])^(
3/2))/(3*d) + (2*b*(a + b*Tan[c + d*x])^(5/2))/(5*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3563

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
```

```
+ b^2, 0] && GtQ[n, 1]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (a+b\tan(c+dx))^{7/2}\,dx = \frac{2b(a+b\tan(c+dx))^{5/2}}{5d} + \int (a+b\tan(c+dx))^{3/2}\left(a^2-b^2+2ab\tan(c+dx\right.$$

$$= \frac{4ab(a+b\tan(c+dx))^{3/2}}{3d} + \frac{2b(a+b\tan(c+dx))^{5/2}}{5d} + \int \sqrt{a+b\tan(c+dx}$$

$$= \frac{2b(3a^2-b^2)\sqrt{a+b\tan(c+dx)}}{d} + \frac{4ab(a+b\tan(c+dx))^{3/2}}{3d} + \frac{2b(a+b\tan}{5}$$

$$= \frac{2b(3a^2-b^2)\sqrt{a+b\tan(c+dx)}}{d} + \frac{4ab(a+b\tan(c+dx))^{3/2}}{3d} + \frac{2b(a+b\tan}{5}$$

$$= \frac{2b(3a^2-b^2)\sqrt{a+b\tan(c+dx)}}{d} + \frac{4ab(a+b\tan(c+dx))^{3/2}}{3d} + \frac{2b(a+b\tan}{5}$$

$$= \frac{2b(3a^2-b^2)\sqrt{a+b\tan(c+dx)}}{d} + \frac{4ab(a+b\tan(c+dx))^{3/2}}{3d} + \frac{2b(a+b\tan}{5}$$

$$= -\frac{i(a-ib)^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} + \frac{i(a+ib)^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+}}{\sqrt{}}\right.}{d}$$

Mathematica [A]
time = 0.99, size = 143, normalized size = 0.86

$$\frac{-15i(a-ib)^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)+15i(a+ib)^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)+2b\sqrt{a+b\tan(c+dx)}\,(58a^2-15b^2+16ab\tan(c+dx)+3b^2\tan^2(c+dx))}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((-15*I)*(a - I*b)^(7/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] +
(15*I)*(a + I*b)^(7/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] + 2*
b*Sqrt[a + b*Tan[c + d*x]]*(58*a^2 - 15*b^2 + 16*a*b*Tan[c + d*x] + 3*b^2*T
an[c + d*x]^2))/(15*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 957 vs. 2(139) = 278.
time = 0.15, size = 958, normalized size = 5.74

method	result

derivativedivides	$2b\left(\frac{(a+b\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{2a(a+b\tan(dx+c))^{\frac{3}{2}}}{3}+3a^2\sqrt{a+b\tan(dx+c)}-b^2\sqrt{a+b\tan(dx+c)}+\frac{\left(-\sqrt{2\right.}}{}\right.$
default	$2b\left(\frac{(a+b\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{2a(a+b\tan(dx+c))^{\frac{3}{2}}}{3}+3a^2\sqrt{a+b\tan(dx+c)}-b^2\sqrt{a+b\tan(dx+c)}+\frac{\left(-\sqrt{2\right.}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*b*(1/5*(a+b*tan(d*x+c))^(5/2)+2/3*a*(a+b*tan(d*x+c))^(3/2)+3*a^2*(a+b*t
an(d*x+c))^(1/2)-b^2*(a+b*tan(d*x+c))^(1/2)+1/4/b^2*(1/2*(-(2*(a^2+b^2)^(1/
2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3+3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)
^(1/2)*a*b^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4-6*(2*(a^2+b^2)^(1/2)+2*a)^(1
/2)*a^2*b^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)*ln(b*tan(d*x+c)+a+(a+b*tan(d
*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*(-6*(a^2+b^2)
^(1/2)*a^2*b^2+2*b^4*(a^2+b^2)^(1/2)-1/2*(-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a
^2+b^2)^(1/2)*a^3+3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^2+(2*
(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4-6*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b^2+(2*(
a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^
(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(
1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))+1/4/b^2*(-1/2*(-(2*(a^2+b^2)^(1/2)+2*
a)^(1/2)*(a^2+b^2)^(1/2)*a^3+3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2
)*a*b^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4-6*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a
^2*b^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)*ln(-b*tan(d*x+c)-a+(a+b*tan(d*x+c
))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-(a^2+b^2)^(1/2))+2*(6*(a^2+b^2)^(1/2
)*a^2*b^2-2*b^4*(a^2+b^2)^(1/2)+1/2*(-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^
2)^(1/2)*a^3+3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^2+(2*(a^2+
b^2)^(1/2)+2*a)^(1/2)*a^4-6*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b^2+(2*(a^2+b
^2)^(1/2)+2*a)^(1/2)*b^4)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)
-2*a)^(1/2)*arctan((-2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)
```

```
)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 9021 vs. 2(133) = 266.
time = 11.83, size = 9021, normalized size = 54.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/60*(60*sqrt(2)*d^5*sqrt((a^14 + 7*a^12*b^2 + 21*a^10*b^4 + 35*a^8*b^6 +
35*a^6*b^8 + 21*a^4*b^10 + 7*a^2*b^12 + b^14 + (a^7 - 21*a^5*b^2 + 35*a^3*b
^4 - 7*a*b^6)*d^2*sqrt((a^14 + 7*a^12*b^2 + 21*a^10*b^4 + 35*a^8*b^6 + 35*a
^6*b^8 + 21*a^4*b^10 + 7*a^2*b^12 + b^14)/d^4))/(49*a^12*b^2 - 490*a^10*b^4
 + 1519*a^8*b^6 - 1484*a^6*b^8 + 511*a^4*b^10 - 42*a^2*b^12 + b^14))*((a^14
 + 7*a^12*b^2 + 21*a^10*b^4 + 35*a^8*b^6 + 35*a^6*b^8 + 21*a^4*b^10 + 7*a^2
*b^12 + b^14)/d^4)^(3/4)*sqrt((49*a^12*b^2 - 490*a^10*b^4 + 1519*a^8*b^6 -
1484*a^6*b^8 + 511*a^4*b^10 - 42*a^2*b^12 + b^14)/d^4)*arctan(-((7*a^26 + 3
5*a^24*b^2 - 14*a^22*b^4 - 526*a^20*b^6 - 1795*a^18*b^8 - 3111*a^16*b^10 -
3060*a^14*b^12 - 1428*a^12*b^14 + 273*a^10*b^16 + 805*a^8*b^18 + 482*a^6*b^
20 + 130*a^4*b^22 + 11*a^2*b^24 - b^26)*d^4*sqrt((a^14 + 7*a^12*b^2 + 21*a^
10*b^4 + 35*a^8*b^6 + 35*a^6*b^8 + 21*a^4*b^10 + 7*a^2*b^12 + b^14)/d^4)*sq
rt((49*a^12*b^2 - 490*a^10*b^4 + 1519*a^8*b^6 - 1484*a^6*b^8 + 511*a^4*b^10
 - 42*a^2*b^12 + b^14)/d^4) + (7*a^33 + 56*a^31*b^2 + 112*a^29*b^4 - 456*a^
27*b^6 - 3380*a^25*b^8 - 10088*a^23*b^10 - 18304*a^21*b^12 - 21736*a^19*b^1
4 - 16302*a^17*b^16 - 5720*a^15*b^18 + 2288*a^13*b^20 + 4264*a^11*b^22 + 26
52*a^9*b^24 + 904*a^7*b^26 + 160*a^5*b^28 + 8*a^3*b^30 - a*b^32)*d^2*sqrt((
49*a^12*b^2 - 490*a^10*b^4 + 1519*a^8*b^6 - 1484*a^6*b^8 + 511*a^4*b^10 - 4
2*a^2*b^12 + b^14)/d^4) + sqrt(2)*((21*a^14*b - 49*a^12*b^3 - 175*a^10*b^5
- 45*a^8*b^7 + 111*a^6*b^9 + 29*a^4*b^11 - 21*a^2*b^13 + b^15)*d^7*sqrt((a^
14 + 7*a^12*b^2 + 21*a^10*b^4 + 35*a^8*b^6 + 35*a^6*b^8 + 21*a^4*b^10 + 7*a
^2*b^12 + b^14)/d^4)*sqrt((49*a^12*b^2 - 490*a^10*b^4 + 1519*a^8*b^6 - 1484
```

*a^6*b^8 + 511*a^4*b^10 - 42*a^2*b^12 + b^14)/d^4) + 4*(7*a^21*b - 91*a^17*b^5 - 176*a^15*b^7 - 26*a^13*b^9 + 208*a^11*b^11 + 170*a^9*b^13 - 16*a^7*b^15 - 61*a^5*b^17 - 16*a^3*b^19 + a*b^21)*d^5*sqrt((49*a^12*b^2 - 490*a^10*b^4 + 1519*a^8*b^6 - 1484*a^6*b^8 + 511*a^4*b^10 - 42*a^2*b^12 + b^14)/d^4))*sqrt((a^14 + 7*a^12*b^2 + 21*a^10*b^4 + 35*a^8*b^6 + 35*a^6*b^8 + 21*a^4*b^10 + 7*a^2*b^12 + b^14 + (a^7 - 21*a^5*b^2 + 35*a^3*b^4 - 7*a*b^6)*d^2*sqrt((a^14 + 7*a^12*b^2 + 21*a^10*b^4 + 35*a^8*b^6 + 35*a^6*b^8 + 21*a^4*b^10 + 7*a^2*b^12 + b^14)/d^4))/(49*a^12*b^2 - 490*a^10*b^4 + 1519*a^8*b^6 - 1484*a^6*b^8 + 511*a^4*b^10 - 42*a^2*b^12 + b^14))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*((a^14 + 7*a^12*b^2 + 21*a^10*b^4 + 35*a^8*b^6 + 35*a^6*b^8 + 21*a^4*b^10 + 7*a^2*b^12 + b^14)/d^4)^(3/4) + sqrt(2)*((3*a^2 - b^2)*d^7*sqrt((a^14 + 7*a^12*b^2 + 21*a^10*b^4 + 35*a^8*b^6 + 35*a^6*b^8 + 21*a^4*b^10 + 7*a^2*b^12 + b^14)/d^4)*sqrt((49*a^12*b^2 - 490*a^10*b^4 + 1519*a^8*b^6 - 1484*a^6*b^8 + 511*a^4*b^10 - 42*a^2*b^12 + b^14)/d^4) + 4*(a^9 + 2*a^7*b^2 - 2*a^3*b^6 - a*b^8)*d^5*sqrt((49*a^12*b^2 - 490*a^10*b^4 + 1519*a^8*b^6 - 1484*a^6*b^8 + 511*a^4*b^10 - 42*a^2*b^12 + b^14)/d^4))*sqrt((a^14 + 7*a^12*b^2 + 21*a^10*b^4 + 35*a^8*b^6 + 35*a^6*b^8 + 21*a^4*b^10 + 7*a^2*b^12 + b^14 + (a^7 - 21*a^5*b^2 + 35*a^3*b^4 - 7*a*b^6)*d^2*sqrt((a^14 + 7*a^12*b^2 + 21*a^10*b^4 + 35*a^8*b^6 + 35*a^6*b^8 + 21*a^4*b^10 + 7*a^2*b^12 + b^14)/d^4))/(49*a^12*b^2 - 490*a^10*b^4 + 1519*a^8*b^6 - 1484*a^6*b^8 + 511*a^4*b^10 - 42*a^2*b^12 + b^14))*sqrt(((49*a^20*b^2 - 294*a^18*b^4 - 147*a^16*b^6 + 1848*a^14*b^8 + 1778*a^12*b^10 - 1316*a^10*b^12 - 1518*a^8*b^14 + 312*a^6*b^16 + 349*a^4*b^18 - 38*a^2*b^20 + b^22)*d^2*sqrt((a^14 + 7*a^12*b^2 + 21*a^10*b^4 + 35*a^8*b^6 + 35*a^6*b^8 + 21*a^4*b^10 + 7*a^2*b^12 + b^14)/d^4)*cos(d*x + c) + sqrt(2)*(4*(49*a^15*b^3 - 539*a^13*b^5 + 2009*a^11*b^7 - 3003*a^9*b^9 + 1995*a^7*b^11 - 553*a^5*b^13 + 43*a^3*b^15 - a*b^17)*d^3*sqrt((a^14 + 7*a^12*b^2 + 21*a^10*b^4 + 35*a^8*b^6 + 35*a^6*b^8 + 21*a^4*b^10 + 7*a^2*b^12 + b^14)/d^4)*cos(d*x + c) + (147*a^22*b^3 - 931*a^20*b^5 - 147*a^18*b^7 + 5691*a^16*b^9 + 3486*a^14*b^11 - 5726*a^12*b^13 - 3238*a^10*b^15 + 2454*a^8*b^17 + 735*a^6*b^19 - 463*a^4*b^21 + 41*a^2*b^23 - b^25)*d*cos(d*x + c))*sqrt((a^14 + 7*a^12*b^2 + 21*a^10*b^4 + 35*a^8*b^6 + 35*a^6*b^8 + 21*a^4*b^10 + 7*a^2*b^12 + b^14 + (a^7 - 21*a^5*b^2 + 35*a^3*b^4 - 7*a*b^6)*d^2*sqrt((a^14 + 7*a^12*b^2 + 21*a^10*b^4 + 35*a^8*b^6 + 35*a^6*b^8 + 21*a^4*b^10 + 7*a^2*b^12 + b^14)/d^4))/(49*a^12*b^2 - 490*a^10*b^4 + 1519*a^8*b^6 - 1484*a^6*b^8 + 511*a^4*b^10 - 42*a^2*b^12 + b^14))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*((a^14 + 7*a^12*b^2 + 21*a^10*b^4 + 35*a^8*b^6 + 35*a^6*b^8 + 21*a^4*b^10 + 7*a^2*b^12 + b^14)/d^4)^(1/4) + (49*a^27*b^2 - 147*a^25*b^4 - 882*a^23*b^6 + 574*a^21*b^8 + 6587*a^19*b^10 + 9415*a^17*b^12 + 1716*a^15*b^14 - 6412*a^13*b^16 - 4585*a^11*b^18 + 427*a^9*b^20 + 1246*a^7*b^22 + 238*a^5*b^24 - 35*a^3*b^26 + a*b^28)*cos(d*x + c) + (49*a^26*b^3 - 147*a^24*b^5 - 882*a^22*b^7 + 574*a^20*b^9 + 6587*a^18*b^11 + 9415*a^16*b^13 + 1716*a^14*b^15 - 6412*a^12*b^17 - 4585*a^10*b^19 + 427*a^8*b^21 + 1246*a^6*b^23 + 238*a^4*b^25 - 35*a^2*b^27 + b^29)*sin(d*x + c))/((a^2 + b^2)*cos(d*x + c)))*((a^14 + 7*a^12*b^2 + 21*a^10*b^4 + 35*a^8*b^6 + 35*a^6*b^8 + 21*a^4*b^10 + 7*a^2...

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(7/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**(7/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 11.47, size = 2862, normalized size = 17.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^(7/2),x)
```

```
[Out] ((8*a^2*b)/d - (2*b*(a^2 + b^2))/d)*(a + b*tan(c + d*x))^(1/2) - atan(((((3
2*(2*a^2*b^5*d^2 - b^7*d^2 + 3*a^4*b^3*d^2))/d^3 - 64*a*b^2*(a + b*tan(c +
d*x))^(1/2)*((7*a*b^6 - a^6*b*7i - a^7 + b^7*1i - a^2*b^5*21i - 35*a^3*b^4
+ a^4*b^3*35i + 21*a^5*b^2)/(4*d^2))^(1/2))*((7*a*b^6 - a^6*b*7i - a^7 + b^
7*1i - a^2*b^5*21i - 35*a^3*b^4 + a^4*b^3*35i + 21*a^5*b^2)/(4*d^2))^(1/2)
- (16*(a + b*tan(c + d*x))^(1/2)*(b^10 - 28*a^2*b^8 + 70*a^4*b^6 - 28*a^6*b
^4 + a^8*b^2))/d^2)*((7*a*b^6 - a^6*b*7i - a^7 + b^7*1i - a^2*b^5*21i - 35*
a^3*b^4 + a^4*b^3*35i + 21*a^5*b^2)/(4*d^2))^(1/2)*1i - (((32*(2*a^2*b^5*d^
2 - b^7*d^2 + 3*a^4*b^3*d^2))/d^3 + 64*a*b^2*(a + b*tan(c + d*x))^(1/2)*((7
*a*b^6 - a^6*b*7i - a^7 + b^7*1i - a^2*b^5*21i - 35*a^3*b^4 + a^4*b^3*35i +
 21*a^5*b^2)/(4*d^2))^(1/2))*((7*a*b^6 - a^6*b*7i - a^7 + b^7*1i - a^2*b^5*
21i - 35*a^3*b^4 + a^4*b^3*35i + 21*a^5*b^2)/(4*d^2))^(1/2) + (16*(a + b*ta
n(c + d*x))^(1/2)*(b^10 - 28*a^2*b^8 + 70*a^4*b^6 - 28*a^6*b^4 + a^8*b^2))/
d^2)*((7*a*b^6 - a^6*b*7i - a^7 + b^7*1i - a^2*b^5*21i - 35*a^3*b^4 + a^4*b
^3*35i + 21*a^5*b^2)/(4*d^2))^(1/2)*1i)/((((32*(2*a^2*b^5*d^2 - b^7*d^2 + 3
*a^4*b^3*d^2))/d^3 - 64*a*b^2*(a + b*tan(c + d*x))^(1/2)*((7*a*b^6 - a^6*b*
7i - a^7 + b^7*1i - a^2*b^5*21i - 35*a^3*b^4 + a^4*b^3*35i + 21*a^5*b^2)/(4
```

d^2))^(1/2))((7*a*b^6 - a^6*b*7i - a^7 + b^7*1i - a^2*b^5*21i - 35*a^3*b^4 + a^4*b^3*35i + 21*a^5*b^2)/(4*d^2))^(1/2) - (16*(a + b*tan(c + d*x))^(1/2)*(b^10 - 28*a^2*b^8 + 70*a^4*b^6 - 28*a^6*b^4 + a^8*b^2))/d^2)*((7*a*b^6 - a^6*b*7i - a^7 + b^7*1i - a^2*b^5*21i - 35*a^3*b^4 + a^4*b^3*35i + 21*a^5*b^2)/(4*d^2))^(1/2) + (((32*(2*a^2*b^5*d^2 - b^7*d^2 + 3*a^4*b^3*d^2))/d^3 + 64*a*b^2*(a + b*tan(c + d*x))^(1/2)*((7*a*b^6 - a^6*b*7i - a^7 + b^7*1i - a^2*b^5*21i - 35*a^3*b^4 + a^4*b^3*35i + 21*a^5*b^2)/(4*d^2))^(1/2))*((7*a*b^6 - a^6*b*7i - a^7 + b^7*1i - a^2*b^5*21i - 35*a^3*b^4 + a^4*b^3*35i + 21*a^5*b^2)/(4*d^2))^(1/2) + (16*(a + b*tan(c + d*x))^(1/2)*(b^10 - 28*a^2*b^8 + 70*a^4*b^6 - 28*a^6*b^4 + a^8*b^2))/d^2)*((7*a*b^6 - a^6*b*7i - a^7 + b^7*1i - a^2*b^5*21i - 35*a^3*b^4 + a^4*b^3*35i + 21*a^5*b^2)/(4*d^2))^(1/2) + (64*(a*b^13 + 3*a^3*b^11 + 2*a^5*b^9 - 2*a^7*b^7 - 3*a^9*b^5 - a^11*b^3))/d^3))*((7*a*b^6 - a^6*b*7i - a^7 + b^7*1i - a^2*b^5*21i - 35*a^3*b^4 + a^4*b^3*35i + 21*a^5*b^2)/(4*d^2))^(1/2)*2i - atan(((((32*(2*a^2*b^5*d^2 - b^7*d^2 + 3*a^4*b^3*d^2))/d^3 - 64*a*b^2*(a + b*tan(c + d*x))^(1/2)*((7*a*b^6 + a^6*b*7i - a^7 - b^7*1i + a^2*b^5*21i - 35*a^3*b^4 - a^4*b^3*35i + 21*a^5*b^2)/(4*d^2))^(1/2))*((7*a*b^6 + a^6*b*7i - a^7 - b^7*1i + a^2*b^5*21i - 35*a^3*b^4 - a^4*b^3*35i + 21*a^5*b^2)/(4*d^2))^(1/2) - (16*(a + b*tan(c + d*x))^(1/2)*(b^10 - 28*a^2*b^8 + 70*a^4*b^6 - 28*a^6*b^4 + a^8*b^2))/d^2)*((7*a*b^6 + a^6*b*7i - a^7 - b^7*1i + a^2*b^5*21i - 35*a^3*b^4 - a^4*b^3*35i + 21*a^5*b^2)/(4*d^2))^(1/2)*1i - (((32*(2*a^2*b^5*d^2 - b^7*d^2 + 3*a^4*b^3*d^2))/d^3 + 64*a*b^2*(a + b*tan(c + d*x))^(1/2)*((7*a*b^6 + a^6*b*7i - a^7 - b^7*1i + a^2*b^5*21i - 35*a^3*b^4 - a^4*b^3*35i + 21*a^5*b^2)/(4*d^2))^(1/2))*((7*a*b^6 + a^6*b*7i - a^7 - b^7*1i + a^2*b^5*21i - 35*a^3*b^4 - a^4*b^3*35i + 21*a^5*b^2)/(4*d^2))^(1/2) + (16*(a + b*tan(c + d*x))^(1/2)*(b^10 - 28*a^2*b^8 + 70*a^4*b^6 - 28*a^6*b^4 + a^8*b^2))/d^2)*((7*a*b^6 + a^6*b*7i - a^7 - b^7*1i + a^2*b^5*21i - 35*a^3*b^4 - a^4*b^3*35i + 21*a^5*b^2)/(4*d^2))^(1/2)*1i)/((((32*(2*a^2*b^5*d^2 - b^7*d^2 + 3*a^4*b^3*d^2))/d^3 - 64*a*b^2*(a + b*tan(c + d*x))^(1/2)*((7*a*b^6 + a^6*b*7i - a^7 - b^7*1i + a^2*b^5*21i - 35*a^3*b^4 - a^4*b^3*35i + 21*a^5*b^2)/(4*d^2))^(1/2))*((7*a*b^6 + a^6*b*7i - a^7 - b^7*1i + a^2*b^5*21i - 35*a^3*b^4 - a^4*b^3*35i + 21*a^5*b^2)/(4*d^2))^(1/2) - (16*(a + b*tan(c + d*x))^(1/2)*(b^10 - 28*a^2*b^8 + 70*a^4*b^6 - 28*a^6*b^4 + a^8*b^2))/d^2)*((7*a*b^6 + a^6*b*7i - a^7 - b^7*1i + a^2*b^5*21i - 35*a^3*b^4 - a^4*b^3*35i + 21*a^5*b^2)/(4*d^2))^(1/2) + (((32*(2*a^2*b^5*d^2 - b^7*d^2 + 3*a^4*b^3*d^2))/d^3 + 64*a*b^2*(a + b*tan(c + d*x))^(1/2)*((7*a*b^6 + a^6*b*7i - a^7 - b^7*1i + a^2*b^5*21i - 35*a^3*b^4 - a^4*b^3*35i + 21*a^5*b^2)/(4*d^2))^(1/2))*((7*a*b^6 + a^6*b*7i - a^7 - b^7*1i + a^2*b^5*21i - 35*a^3*b^4 - a^4*b^3*35i + 21*a^5*b^2)/(4*d^2))^(1/2) + (16*(a + b*tan(c + d*x))^(1/2)*(b^10 - 28*a^2*b^8 + 70*a^4*b^6 - 28*a^6*b^4 + a^8*b^2))/d^2)*((7*a*b^6 + a^6*b*7i - a^7 - b^7*1i + a^2*b^5*21i - 35*a^3*b^4 - a^4*b^3*35i + 21*a^5*b^2)/(4*d^2))^(1/2) + (64*(a*b^13 + 3*a^3*b^11 + 2*a^5*b^9 - 2*a^7*b^7 - 3*a^9*b^5 - a^11*b^3))/d^3))*((7*a*b^6 + a^6*b*7i - a^7 - b^7*1i + a^2*b^5*21i - 35*a^3*b^4 - a^4*b^3*35i + 21*a^5*b^2)/(4*d^2))^(1/2)*2i + (2*b*(a + b*tan(c + d*x))^(5/2))/(5*d) + (4*a*b*(a + b*tan(c + d*x))^(3/2))/(3*d)

3.528 $\int \frac{\tan^5(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal. Leaf size=229

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}\,d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}\,d}-\frac{4a(24a^2-35b^2)\sqrt{a+b\tan(c+dx)}}{105b^4d}$$

```
[Out] -arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d/(a-I*b)^(1/2)-arctanh((a+b
*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d/(a+I*b)^(1/2)-4/105*a*(24*a^2-35*b^2)*(
a+b*tan(d*x+c))^(1/2)/b^4/d+2/105*(24*a^2-35*b^2)*(a+b*tan(d*x+c))^(1/2)*ta
n(d*x+c)/b^3/d-12/35*a*(a+b*tan(d*x+c))^(1/2)*tan(d*x+c)^2/b^2/d+2/7*(a+b*t
an(d*x+c))^(1/2)*tan(d*x+c)^3/b/d
```

Rubi [A]

time = 0.38, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3647, 3728, 3729, 3711, 12, 3620, 3618, 65, 214}

$$-\frac{4a(24a^2-35b^2)\sqrt{a+b\tan(c+dx)}}{105b^4d}+\frac{2(24a^2-35b^2)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{105b^3d}-\frac{12a\tan^2(c+dx)\sqrt{a+b\tan(c+dx)}}{35b^2d}+\frac{2\tan^3(c+dx)\sqrt{a+b\tan(c+dx)}}{7bd}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]]/(Sqrt[a - I*b]*d)) - ArcT
anh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]]/(Sqrt[a + I*b]*d) - (4*a*(24*a^
2 - 35*b^2)*Sqrt[a + b*Tan[c + d*x]])/(105*b^4*d) + (2*(24*a^2 - 35*b^2)*Ta
n[c + d*x]*Sqrt[a + b*Tan[c + d*x]])/(105*b^3*d) - (12*a*Tan[c + d*x]^2*Sqr
t[a + b*Tan[c + d*x]])/(35*b^2*d) + (2*Tan[c + d*x]^3*Sqrt[a + b*Tan[c + d*
x]])/(7*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
```

```
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3729

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m + n +
1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*
Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*(b*c*m + a*d*(n + 1)) + d*(A*b -
 b*C)*(m + n + 1)*Tan[e + f*x] - C*m*(b*c - a*d)*Tan[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 +
 b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[
m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx = \frac{2\tan^3(c+dx)\sqrt{a+b\tan(c+dx)}}{7bd} + \frac{2\int \frac{\tan^2(c+dx)\left(-3a-\frac{7}{2}b\tan(c+dx)-3a\tan^2(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}}{7b}$$

$$= -\frac{12a\tan^2(c+dx)\sqrt{a+b\tan(c+dx)}}{35b^2d} + \frac{2\tan^3(c+dx)\sqrt{a+b\tan(c+dx)}}{7bd}$$

$$= \frac{2(24a^2-35b^2)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{105b^3d} - \frac{12a\tan^2(c+dx)\sqrt{a+b\tan}}{35b^2d}$$

$$= -\frac{4a(24a^2-35b^2)\sqrt{a+b\tan(c+dx)}}{105b^4d} + \frac{2(24a^2-35b^2)\tan(c+dx)\sqrt{a+b\,t}}{105b^3d}$$

$$= -\frac{4a(24a^2-35b^2)\sqrt{a+b\tan(c+dx)}}{105b^4d} + \frac{2(24a^2-35b^2)\tan(c+dx)\sqrt{a+b\,t}}{105b^3d}$$

$$= -\frac{4a(24a^2-35b^2)\sqrt{a+b\tan(c+dx)}}{105b^4d} + \frac{2(24a^2-35b^2)\tan(c+dx)\sqrt{a+b\,t}}{105b^3d}$$

$$= -\frac{4a(24a^2-35b^2)\sqrt{a+b\tan(c+dx)}}{105b^4d} + \frac{2(24a^2-35b^2)\tan(c+dx)\sqrt{a+b\,t}}{105b^3d}$$

$$= -\frac{4a(24a^2-35b^2)\sqrt{a+b\tan(c+dx)}}{105b^4d} + \frac{2(24a^2-35b^2)\tan(c+dx)\sqrt{a+b\,t}}{105b^3d}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}\,d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}\,d} - \frac{4a(24}{}$$

Mathematica [A]
time = 4.18, size = 162, normalized size = 0.71

$$-\frac{\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}} + \frac{2\sqrt{a+b\tan(c+dx)}\,(48a^3-88ab^2+(-24a^2b+50b^3)\tan(c+dx)+3b^2\sec^2(c+dx)(6a-5b\tan(c+dx)))}{105b^4}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] -((ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]]/Sqrt[a - I*b] + ArcTanh[
Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]]/Sqrt[a + I*b] + (2*Sqrt[a + b*Tan[c
 + d*x]]*(48*a^3 - 88*a*b^2 + (-24*a^2*b + 50*b^3)*Tan[c + d*x] + 3*b^2*Sec
[c + d*x]^2*(6*a - 5*b*Tan[c + d*x])))/(105*b^4))/d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(197) = 394$.
time = 0.18, size = 445, normalized size = 1.94

method	result
derivativedivides	$\frac{\frac{2(a+b\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{6a(a+b\tan(dx+c))^{\frac{5}{2}}}{5}+2a^2(a+b\tan(dx+c))^{\frac{3}{2}}-\frac{2b^2(a+b\tan(dx+c))^{\frac{3}{2}}}{3}-2a^3\sqrt{a+b\tan(dx+c)}}{}$
default	$\frac{\frac{2(a+b\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{6a(a+b\tan(dx+c))^{\frac{5}{2}}}{5}+2a^2(a+b\tan(dx+c))^{\frac{3}{2}}-\frac{2b^2(a+b\tan(dx+c))^{\frac{3}{2}}}{3}-2a^3\sqrt{a+b\tan(dx+c)}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/b^4*(1/7*(a+b*tan(d*x+c))^(7/2)-3/5*a*(a+b*tan(d*x+c))^(5/2)+a^2*(a+b*t
an(d*x+c))^(3/2)-1/3*b^2*(a+b*tan(d*x+c))^(3/2)-a^3*(a+b*tan(d*x+c))^(1/2)+
a*b^2*(a+b*tan(d*x+c))^(1/2)-b^4*(1/4/(a^2+b^2)^(1/2)*(1/2*(2*(a^2+b^2)^(1/
2)+2*a)^(1/2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2
*a)^(1/2)+(a^2+b^2)^(1/2))+2*(a-(a^2+b^2)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1
/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2
+b^2)^(1/2)-2*a)^(1/2)))+1/4/(a^2+b^2)^(1/2)*(-1/2*(2*(a^2+b^2)^(1/2)+2*a)^
(1/2)*ln(-b*tan(d*x+c)-a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/
2)-(a^2+b^2)^(1/2))+2*((a^2+b^2)^(1/2)-a)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arc
tan((-2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^
(1/2)-2*a)^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^5/sqrt(b*tan(d*x + c) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2309 vs. $2(193) = 386$.
time = 0.98, size = 2309, normalized size = 10.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/420*(420*sqrt(2)*(a^2*b^4 + b^6)*d^5*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a
^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*(
1/((a^2 + b^2)*d^4))^(3/4)*arctan(-((a^4 + 2*a^2*b^2 + b^4)*d^4*sqrt(b^2/((
a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(1/((a^2 + b^2)*d^4)) + (a^3 + a*b^2)*d^2*
sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - sqrt(2)*((a^5 + 2*a^3*b^2 + a*b^4
)*d^7*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(1/((a^2 + b^2)*d^4)) + (
a^4 + 2*a^2*b^2 + b^4)*d^5*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4)))*sqrt(((
a^2 + b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) + sqrt(2)*((a^2 + b^2
)*d^3*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) + a*d*cos(d*x + c))*sqrt((a*co
s(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/
((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(1/4) + a*cos(d*
x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^
2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(3/4) + sqrt(2)*((a^
5 + 2*a^3*b^2 + a*b^4)*d^7*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(1/(
(a^2 + b^2)*d^4)) + (a^4 + 2*a^2*b^2 + b^4)*d^5*sqrt(b^2/((a^4 + 2*a^2*b^2
+ b^4)*d^4)))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-((
a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2
)*d^4))^(3/4))/b^2)*cos(d*x + c)^3 + 420*sqrt(2)*(a^2*b^4 + b^6)*d^5*sqrt(-
((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*sqrt(b^2/((a
^4 + 2*a^2*b^2 + b^4)*d^4))*(1/((a^2 + b^2)*d^4))^(3/4)*arctan(((a^4 + 2*a^
2*b^2 + b^4)*d^4*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(1/((a^2 + b^2
)*d^4)) + (a^3 + a*b^2)*d^2*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + sqrt(
2)*((a^5 + 2*a^3*b^2 + a*b^4)*d^7*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*s
qrt(1/((a^2 + b^2)*d^4)) + (a^4 + 2*a^2*b^2 + b^4)*d^5*sqrt(b^2/((a^4 + 2*a
^2*b^2 + b^4)*d^4)))*sqrt(((a^2 + b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4))*cos(d*
x + c) - sqrt(2)*((a^2 + b^2)*d^3*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) +
a*d*cos(d*x + c))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt
(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 +
 b^2)*d^4))^(1/4) + a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-((
a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2
)*d^4))^(3/4) - sqrt(2)*((a^5 + 2*a^3*b^2 + a*b^4)*d^7*sqrt(b^2/((a^4 + 2*a
^2*b^2 + b^4)*d^4))*sqrt(1/((a^2 + b^2)*d^4)) + (a^4 + 2*a^2*b^2 + b^4)*d^5
*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4)))*sqrt((a*cos(d*x + c) + b*sin(d*x
+ c))/cos(d*x + c))*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^
2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(3/4))/b^2)*cos(d*x + c)^3 + 105*sqrt(2
)*(a*b^4*d^3*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c)^3 + b^4*d*cos(d*x + c)^
```

```
3)*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/
((a^2 + b^2)*d^4))^(1/4)*log(((a^2 + b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4))*cos
(d*x + c) + sqrt(2)*((a^2 + b^2)*d^3*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c)
 + a*d*cos(d*x + c))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*s
qrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^
2 + b^2)*d^4))^(1/4) + a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c)) - 105
*sqrt(2)*(a*b^4*d^3*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c)^3 + b^4*d*cos(d*
x + c)^3)*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b
^2)*(1/((a^2 + b^2)*d^4))^(1/4)*log(((a^2 + b^2)*d^2*sqrt(1/((a^2 + b^2)*d^
4))*cos(d*x + c) - sqrt(2)*((a^2 + b^2)*d^3*sqrt(1/((a^2 + b^2)*d^4))*cos(d
*x + c) + a*d*cos(d*x + c))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x
+ c))*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*
(1/((a^2 + b^2)*d^4))^(1/4) + a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c)
) + 8*(18*a*b^2*cos(d*x + c) + 8*(6*a^3 - 11*a*b^2)*cos(d*x + c)^3 - (15*b^
3 + 2*(12*a^2*b - 25*b^3)*cos(d*x + c)^2)*sin(d*x + c))*sqrt((a*cos(d*x + c
) + b*sin(d*x + c))/cos(d*x + c)))/(b^4*d*cos(d*x + c)^3)
```

<u>Sympy</u> [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+b*tan(d*x+c))**(1/2),x)
[Out] Integral(tan(c + d*x)**5/sqrt(a + b*tan(c + d*x)), x)
```

<u>Giac</u> [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] Timed out
```

<u>Mupad</u> [B]
time = 11.48, size = 938, normalized size = 4.10

$$\sqrt{a+b\tan(c+dx)}\left(2a\left(\frac{8a^2}{b^4d}-\frac{2(a^2+b^2)}{b^4d}\right)-\frac{20a^3}{b^4d}+\frac{6a(a^2+b^2)}{b^4d}\right)+\left(\frac{8a^2}{3b^4d}-\frac{2(a^2+b^2)}{3b^4d}\right)(a+b\tan(c+dx))^{3/2}+\frac{2(a+b\tan(c+dx))^{7/2}}{7b^4d}-\frac{6a(a+b\tan(c+dx))^{5/2}}{5b^4d}-\mathrm{atan}\left(-\frac{b^2\sqrt{\frac{a}{4a^2d^2+4b^2d^2}-\frac{b\,1\mathrm{i}}{4a^2d^2+4b^2d^2}}\sqrt{a+b\tan(c+dx)}\,32\mathrm{i}}{\frac{16b^2}{d}-\frac{64a^2b^2d^2}{4a^2d^3+4b^2d^3}+\frac{ab^3d^2\,64\mathrm{i}}{4a^2d^3+4b^2d^3}}+\frac{a^2b^2\sqrt{\frac{a}{4a^2d^2+4b^2d^2}-\frac{b\,1\mathrm{i}}{4a^2d^2+4b^2d^2}}\sqrt{a+b\tan(c+dx)}\,128\mathrm{i}}{\frac{64b^4}{d}+\frac{64a^2b^2}{d}-\frac{256a^2b^4d^2}{4a^2d^3+4b^2d^3}+\frac{a^3b^3d^2\,256\mathrm{i}}{4a^2d^3+4b^2d^3}-\frac{256a^4b^2d^2}{4a^2d^3+4b^2d^3}+\frac{ab^5d^2\,256\mathrm{i}}{4a^2d^3+4b^2d^3}}+\frac{128ab^3\sqrt{\frac{a}{4a^2d^2+4b^2d^2}-\frac{b\,1\mathrm{i}}{4a^2d^2+4b^2d^2}}\sqrt{a+b\tan(c+dx)}}{\frac{64b^4}{d}+\frac{64a^2b^2}{d}-\frac{256a^2b^4d^2}{4a^2d^3+4b^2d^3}+\frac{a^3b^3d^2\,256\mathrm{i}}{4a^2d^3+4b^2d^3}-\frac{256a^4b^2d^2}{4a^2d^3+4b^2d^3}+\frac{ab^5d^2\,256\mathrm{i}}{4a^2d^3+4b^2d^3}}\right)\sqrt{\frac{a-b\,1\mathrm{i}}{4a^2d^2+4b^2d^2}}\,2\mathrm{i}+\mathrm{atan}\left(\frac{b^2\sqrt{\frac{1}{ad^2-bd^2\,1\mathrm{i}}}\sqrt{a+b\tan(c+dx)}\,16\mathrm{i}}{\frac{16b^2}{d}-\frac{16ab^2d^2}{ad^3-bd^3\,1\mathrm{i}}}+\frac{ab^2\sqrt{\frac{1}{ad^2-bd^2\,1\mathrm{i}}}\sqrt{a+b\tan(c+dx)}\,16\mathrm{i}}{\frac{b^3\,16\mathrm{i}}{d}-\frac{16ab^2}{d}-\frac{ab^3d^2\,16\mathrm{i}}{ad^3-bd^3\,1\mathrm{i}}+\frac{16a^2b^2d^2}{ad^3-bd^3\,1\mathrm{i}}}\right)\sqrt{\frac{1}{ad^2-bd^2\,1\mathrm{i}}}\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + b*tan(c + d*x))^(1/2),x)
```

[Out] (a + b*tan(c + d*x))^(1/2)*(2*a*((8*a^2)/(b^4*d) - (2*(a^2 + b^2))/(b^4*d)) - (20*a^3)/(b^4*d) + (6*a*(a^2 + b^2))/(b^4*d)) - atan((a^2*b^2*(a/(4*a^2*d^2 + 4*b^2*d^2) - (b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2)*128i)/((64*b^4)/d + (64*a^2*b^2)/d - (256*a^2*b^4*d^2)/(4*a^2*d^3 + 4*b^2*d^3) + (a^3*b^3*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) - (256*a^4*b^2*d^2)/(4*a^2*d^3 + 4*b^2*d^3) + (a*b^5*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3)) - (b^2*(a/(4*a^2*d^2 + 4*b^2*d^2) - (b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2)*32i)/((16*b^2)/d - (64*a^2*b^2*d^2)/(4*a^2*d^3 + 4*b^2*d^3) + (a*b^3*d^2*64i)/(4*a^2*d^3 + 4*b^2*d^3)) + (128*a*b^3*(a/(4*a^2*d^2 + 4*b^2*d^2) - (b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2))/((64*b^4)/d + (64*a^2*b^2)/d - (256*a^2*b^4*d^2)/(4*a^2*d^3 + 4*b^2*d^3) + (a^3*b^3*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) - (256*a^4*b^2*d^2)/(4*a^2*d^3 + 4*b^2*d^3) + (a*b^5*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3)))*((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*2i + ((8*a^2)/(3*b^4*d) - (2*(a^2 + b^2))/(3*b^4*d))*(a + b*tan(c + d*x))^(3/2) + atan((b^2*(1/(a*d^2 - b*d^2*1i))^(1/2)*(a + b*tan(c + d*x))^(1/2)*16i)/((16*b^2)/d - (16*a*b^2*d^2)/(a*d^3 - b*d^3*1i)) + (a*b^2*(1/(a*d^2 - b*d^2*1i))^(1/2)*(a + b*tan(c + d*x))^(1/2)*16i)/((b^3*16i)/d - (16*a*b^2)/d - (a*b^3*d^2*16i)/(a*d^3 - b*d^3*1i) + (16*a^2*b^2*d^2)/(a*d^3 - b*d^3*1i)))*(1/(a*d^2 - b*d^2*1i))^(1/2)*1i + (2*(a + b*tan(c + d*x))^(7/2))/(7*b^4*d) - (6*a*(a + b*tan(c + d*x))^(5/2))/(5*b^4*d)

3.529 $\int \frac{\tan^4(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal. Leaf size=500

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a^2+b^2}\sqrt{a-\sqrt{a^2+b^2}}\,d}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}+\sqrt{2}\sqrt{a+b\tan(c}{\sqrt{a-\sqrt{a^2+b^2}}}\right.}{\sqrt{2}\sqrt{a^2+b^2}\sqrt{a-\sqrt{a^2+b^2}}\,d}$$

```
[Out] 1/2*b*arctanh(((a+(a^2+b^2)^(1/2))^(1/2)-2^(1/2)*(a+b*tan(d*x+c))^(1/2))/(a
-(a^2+b^2)^(1/2))^(1/2))/d*2^(1/2)/(a^2+b^2)^(1/2)/(a-(a^2+b^2)^(1/2))^(1/2
)-1/2*b*arctanh(((a+(a^2+b^2)^(1/2))^(1/2)+2^(1/2)*(a+b*tan(d*x+c))^(1/2))/
(a-(a^2+b^2)^(1/2))^(1/2))/d*2^(1/2)/(a^2+b^2)^(1/2)/(a-(a^2+b^2)^(1/2))^(1
/2)-1/4*b*ln(a+(a^2+b^2)^(1/2)-2^(1/2)*(a+(a^2+b^2)^(1/2))^(1/2)*(a+b*tan(d
*x+c))^(1/2)+b*tan(d*x+c))/d*2^(1/2)/(a^2+b^2)^(1/2)/(a+(a^2+b^2)^(1/2))^(1
/2)+1/4*b*ln(a+(a^2+b^2)^(1/2)+2^(1/2)*(a+(a^2+b^2)^(1/2))^(1/2)*(a+b*tan(d
*x+c))^(1/2)+b*tan(d*x+c))/d*2^(1/2)/(a^2+b^2)^(1/2)/(a+(a^2+b^2)^(1/2))^(1
/2)+2/15*(8*a^2-15*b^2)*(a+b*tan(d*x+c))^(1/2)/b^3/d-8/15*a*(a+b*tan(d*x+c)
)^(1/2)*tan(d*x+c)/b^2/d+2/5*(a+b*tan(d*x+c))^(1/2)*tan(d*x+c)^2/b/d
```

Rubi [A]
time = 0.57, antiderivative size = 500, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3647, 3728, 3712, 3566, 722, 1108, 648, 632, 212, 642}

$$-\frac{b\log\left(-\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\sqrt{a+b\tan(c+dx)}+\sqrt{a^2+b^2}+a+b\tan(c+dx)\right)}{2\sqrt{2}d\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+a}}+\frac{b\log\left(\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\sqrt{a+b\tan(c+dx)}+\sqrt{a^2+b^2}+a+b\tan(c+dx)\right)}{2\sqrt{2}d\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+a}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+b^2}+a}-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}d\sqrt{a^2+b^2}\sqrt{a-\sqrt{a^2+b^2}}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+b^2}+a}+\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}d\sqrt{a^2+b^2}\sqrt{a-\sqrt{a^2+b^2}}}+\frac{2(8a^2-15b^2)\sqrt{a+b\tan(c+dx)}}{15b^3d}-\frac{8a\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{15b^2d}+\frac{2\tan^2(c+dx)\sqrt{a+b\tan(c+dx)}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] (b*ArcTanh[(Sqrt[a + Sqrt[a^2 + b^2]] - Sqrt[2]*Sqrt[a + b*Tan[c + d*x]])/S
qrt[a - Sqrt[a^2 + b^2]]])/(Sqrt[2]*Sqrt[a^2 + b^2]*Sqrt[a - Sqrt[a^2 + b^2
]]*d) - (b*ArcTanh[(Sqrt[a + Sqrt[a^2 + b^2]] + Sqrt[2]*Sqrt[a + b*Tan[c +
d*x]])/Sqrt[a - Sqrt[a^2 + b^2]]])/(Sqrt[2]*Sqrt[a^2 + b^2]*Sqrt[a - Sqrt[a
^2 + b^2]]*d) - (b*Log[a + Sqrt[a^2 + b^2] + b*Tan[c + d*x] - Sqrt[2]*Sqrt[
a + Sqrt[a^2 + b^2]]*Sqrt[a + b*Tan[c + d*x]]])/(2*Sqrt[2]*Sqrt[a^2 + b^2]*
Sqrt[a + Sqrt[a^2 + b^2]]*d) + (b*Log[a + Sqrt[a^2 + b^2] + b*Tan[c + d*x]
+ Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*Sqrt[a + b*Tan[c + d*x]]])/(2*Sqrt[2]*S
qrt[a^2 + b^2]*Sqrt[a + Sqrt[a^2 + b^2]]*d) + (2*(8*a^2 - 15*b^2)*Sqrt[a +
b*Tan[c + d*x]])/(15*b^3*d) - (8*a*Tan[c + d*x]*Sqrt[a + b*Tan[c + d*x]])/(
15*b^2*d) + (2*Tan[c + d*x]^2*Sqrt[a + b*Tan[c + d*x]])/(5*b*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 722

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
```

```
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3712

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx = \frac{2\tan^2(c+dx)\sqrt{a+b\tan(c+dx)}}{5bd} + \frac{2\int \frac{\tan(c+dx)\left(-2a-\frac{5}{2}b\tan(c+dx)-2a\tan^2(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}}{5b}$$

$$= -\frac{8a\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{15b^2d} + \frac{2\tan^2(c+dx)\sqrt{a+b\tan(c+dx)}}{5bd} +$$

$$= \frac{2(8a^2-15b^2)\sqrt{a+b\tan(c+dx)}}{15b^3d} - \frac{8a\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{15b^2d} + \frac{2}{}$$

$$= \frac{2(8a^2-15b^2)\sqrt{a+b\tan(c+dx)}}{15b^3d} - \frac{8a\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{15b^2d} + \frac{2}{}$$

$$= \frac{2(8a^2-15b^2)\sqrt{a+b\tan(c+dx)}}{15b^3d} - \frac{8a\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{15b^2d} + \frac{2}{}$$

$$= \frac{2(8a^2-15b^2)\sqrt{a+b\tan(c+dx)}}{15b^3d} - \frac{8a\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{15b^2d} + \frac{2}{}$$

$$= \frac{2(8a^2-15b^2)\sqrt{a+b\tan(c+dx)}}{15b^3d} - \frac{8a\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{15b^2d} + \frac{2}{}$$

$$= -\frac{b\log\left(a+\sqrt{a^2+b^2}+b\tan(c+dx)-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\tan(c+\right.}{2\sqrt{2}\sqrt{a^2+b^2}\sqrt{a+\sqrt{a^2+b^2}}\,d}$$

$$= \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a^2+b^2}\sqrt{a-\sqrt{a^2+b^2}}\,d} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+}}{}\right.}{\sqrt{2}\sqrt{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 17.48, size = 144, normalized size = 0.29

$$\frac{-\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}}+\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}}+\frac{2\sqrt{a+b\tan(c+dx)}\,(8a^2-18b^2+3b^2\sec^2(c+dx)-4ab\tan(c+dx))}{15b^3}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] (((-I)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/Sqrt[a - I*b] + (I*
ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/Sqrt[a + I*b] + (2*Sqrt[a
+ b*Tan[c + d*x]]*(8*a^2 - 18*b^2 + 3*b^2*Sec[c + d*x]^2 - 4*a*b*Tan[c + d*
x]))/(15*b^3))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 845 vs. $2(409) = 818$.

time = 0.14, size = 846, normalized size = 1.69

method	result
derivativedivides	$\frac{\frac{2(a+b\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{4a(a+b\tan(dx+c))^{\frac{3}{2}}}{3}+2a^{2}\sqrt{a+b\tan(dx+c)}-2b^{2}\sqrt{a+b\tan(dx+c)}+2b^{4}\left(\ldots\right.}{\ldots}$
default	$\frac{\frac{2(a+b\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{4a(a+b\tan(dx+c))^{\frac{3}{2}}}{3}+2a^{2}\sqrt{a+b\tan(dx+c)}-2b^{2}\sqrt{a+b\tan(dx+c)}+2b^{4}\left(\ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/b^3*(1/5*(a+b*tan(d*x+c))^(5/2)-2/3*a*(a+b*tan(d*x+c))^(3/2)+a^2*(a+b*t
an(d*x+c))^(1/2)-b^2*(a+b*tan(d*x+c))^(1/2)+b^4*(1/4/b^2/(a^2+b^2)^(3/2)*(1
/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2+(2*(a^2+b^2)^(1/2)+2*
a)^(1/2)*(a^2+b^2)^(1/2)*b^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-(2*(a^2+b^2)
^(1/2)+2*a)^(1/2)*a*b^2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b
^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*(2*a^2*b^2+2*b^4-1/2*((2*(a^2+b^2)^
(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2
)^(1/2)*b^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)
*a*b^2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan
```

```
((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2
)-2*a)^(1/2)))+1/4/b^2/(a^2+b^2)^(3/2)*(-1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)
*(a^2+b^2)^(1/2)*a^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^2-(2*(
a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^2)*ln(-b*ta
n(d*x+c)-a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-(a^2+b^2)^(
1/2))+2*(-2*a^2*b^2-2*b^4+1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2
)*a^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^2-(2*(a^2+b^2)^(1/2)+
2*a)^(1/2)*a^3-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^2)*(2*(a^2+b^2)^(1/2)+2*a)
^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*tan(d*x+c))^(1/2)+(2*
(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^4/sqrt(b*tan(d*x + c) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1886 vs. 2(411) = 822.
time = 0.96, size = 1886, normalized size = 3.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/60*(60*sqrt(2)*(a^2*b^3 + b^5)*d^5*sqrt(((a^3 + a*b^2)*d^2*sqrt(1/((a^2
+ b^2)*d^4)) + a^2 + b^2)/b^2)*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*(1/(
(a^2 + b^2)*d^4))^(3/4)*arctan((sqrt(2)*(a^4 + 2*a^2*b^2 + b^4)*d^7*sqrt((s
qrt(2)*b^3*d*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(((a^
3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) + a^2 + b^2)/b^2)*(1/((a^2 + b^2)*
d^4))^(1/4)*cos(d*x + c) + (a^2*b^2 + b^4)*d^2*sqrt(1/((a^2 + b^2)*d^4))*co
s(d*x + c) + a*b^2*cos(d*x + c) + b^3*sin(d*x + c))/cos(d*x + c))*sqrt(((a^
3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) + a^2 + b^2)/b^2)*sqrt(b^2/((a^4 +
 2*a^2*b^2 + b^4)*d^4))*(1/((a^2 + b^2)*d^4))^(5/4) - sqrt(2)*(a^4*b + 2*a^
2*b^3 + b^5)*d^7*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(
((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) + a^2 + b^2)/b^2)*sqrt(b^2/((a
^4 + 2*a^2*b^2 + b^4)*d^4))*(1/((a^2 + b^2)*d^4))^(5/4) - (a^4 + 2*a^2*b^2
+ b^4)*d^4*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(1/((a^2 + b^2)*d^4)
) - (a^3 + a*b^2)*d^2*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/b^2)*cos(d*x
 + c)^2 + 60*sqrt(2)*(a^2*b^3 + b^5)*d^5*sqrt(((a^3 + a*b^2)*d^2*sqrt(1/((a
^2 + b^2)*d^4)) + a^2 + b^2)/b^2)*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*(
```

```
1/((a^2 + b^2)*d^4))^(3/4)*arctan((sqrt(2)*(a^4 + 2*a^2*b^2 + b^4)*d^7*sqrt
(-(sqrt(2)*b^3*d*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(
((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) + a^2 + b^2)/b^2)*(1/((a^2 + b
^2)*d^4))^(1/4)*cos(d*x + c) - (a^2*b^2 + b^4)*d^2*sqrt(1/((a^2 + b^2)*d^4)
)*cos(d*x + c) - a*b^2*cos(d*x + c) - b^3*sin(d*x + c))/cos(d*x + c))*sqrt(
((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) + a^2 + b^2)/b^2)*sqrt(b^2/((a
^4 + 2*a^2*b^2 + b^4)*d^4))*(1/((a^2 + b^2)*d^4))^(5/4) - sqrt(2)*(a^4*b +
2*a^2*b^3 + b^5)*d^7*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*s
qrt(((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) + a^2 + b^2)/b^2)*sqrt(b^2
/((a^4 + 2*a^2*b^2 + b^4)*d^4))*(1/((a^2 + b^2)*d^4))^(5/4) + (a^4 + 2*a^2*
b^2 + b^4)*d^4*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(1/((a^2 + b^2)*
d^4)) + (a^3 + a*b^2)*d^2*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/b^2)*cos
(d*x + c)^2 + 15*sqrt(2)*(a*b^3*d^3*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c)^
2 - b^3*d*cos(d*x + c)^2)*sqrt(((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4))
 + a^2 + b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(1/4)*log((sqrt(2)*b^3*d*sqrt((a*c
os(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(((a^3 + a*b^2)*d^2*sqrt(1/
((a^2 + b^2)*d^4)) + a^2 + b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(1/4)*cos(d*x +
c) + (a^2*b^2 + b^4)*d^2*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) + a*b^2*cos
(d*x + c) + b^3*sin(d*x + c))/cos(d*x + c)) - 15*sqrt(2)*(a*b^3*d^3*sqrt(1/
((a^2 + b^2)*d^4))*cos(d*x + c)^2 - b^3*d*cos(d*x + c)^2)*sqrt(((a^3 + a*b^
2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) + a^2 + b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(1
/4)*log(-(sqrt(2)*b^3*d*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c)
)*sqrt(((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) + a^2 + b^2)/b^2)*(1/((
a^2 + b^2)*d^4))^(1/4)*cos(d*x + c) - (a^2*b^2 + b^4)*d^2*sqrt(1/((a^2 + b^
2)*d^4))*cos(d*x + c) - a*b^2*cos(d*x + c) - b^3*sin(d*x + c))/cos(d*x + c)
) + 8*(4*a*b*cos(d*x + c)*sin(d*x + c) - 2*(4*a^2 - 9*b^2)*cos(d*x + c)^2 -
 3*b^2)*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c)))/(b^3*d*cos(d*
x + c)^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+b*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**4/sqrt(a + b*tan(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 7.94, size = 791, normalized size = 1.58

$$\left(\frac{4\,a^2}{b^3\,d}-\frac{2\,\left(a^2+b^2\right)}{b^3\,d}\right)\,\sqrt{a+b\tan\left(c+d\,x\right)}+\frac{\ln\left(16\,b^2\,\sqrt{a+b\tan\left(c+d\,x\right)}+16\,b^3\,d\,\sqrt{-\frac{1}{d^2\,\left(a-b\,1\mathrm{i}\right)}}-\frac{16\,a\,b^2\,\sqrt{a+b\tan\left(c+d\,x\right)}}{a-b\,1\mathrm{i}}\right)\,\sqrt{-\frac{1}{a\,d^2-b\,d^2\,1\mathrm{i}}}}{2}-\ln\left(-16\,b^2\,\sqrt{a+b\tan\left(c+d\,x\right)}+16\,b^3\,d\,\sqrt{-\frac{1}{d^2\,\left(a-b\,1\mathrm{i}\right)}}+\frac{16\,a\,b^2\,\sqrt{a+b\tan\left(c+d\,x\right)}}{a-b\,1\mathrm{i}}\right)\,\sqrt{-\frac{1}{4\,\left(a\,d^2-b\,d^2\,1\mathrm{i}\right)}}+\frac{2\,{\left(a+b\tan(c+d\,x)\right)}^{5/2}}{5\,b^3\,d}-\frac{4\,a\,{\left(a+b\tan(c+d\,x)\right)}^{3/2}}{3\,b^3\,d}+\mathrm{atan}\left(-\frac{b^2\,\sqrt{-\frac{a}{4\,a^2\,d^2+4\,b^2\,d^2}+\frac{b\,1\mathrm{i}}{4\,a^2\,d^2+4\,b^2\,d^2}}\,\sqrt{a+b\tan\left(c+d\,x\right)}\,32\mathrm{i}}{-\frac{64\,a\,b^3\,d^2}{4\,a^2\,d^3+4\,b^2\,d^3}+\frac{b^4\,d^2\,64\mathrm{i}}{4\,a^2\,d^3+4\,b^2\,d^3}}+\frac{128\,a\,b^3\,\sqrt{-\frac{a}{4\,a^2\,d^2+4\,b^2\,d^2}+\frac{b\,1\mathrm{i}}{4\,a^2\,d^2+4\,b^2\,d^2}}\,\sqrt{a+b\tan\left(c+d\,x\right)}}{-\frac{256\,a^3\,b^3\,d^2}{4\,a^2\,d^3+4\,b^2\,d^3}-\frac{256\,a\,b^5\,d^2}{4\,a^2\,d^3+4\,b^2\,d^3}+\frac{b^6\,d^2\,256\mathrm{i}}{4\,a^2\,d^3+4\,b^2\,d^3}+\frac{a^2\,b^4\,d^2\,256\mathrm{i}}{4\,a^2\,d^3+4\,b^2\,d^3}}+\frac{a^2\,b^2\,\sqrt{-\frac{a}{4\,a^2\,d^2+4\,b^2\,d^2}+\frac{b\,1\mathrm{i}}{4\,a^2\,d^2+4\,b^2\,d^2}}\,\sqrt{a+b\tan\left(c+d\,x\right)}\,128\mathrm{i}}{-\frac{256\,a^3\,b^3\,d^2}{4\,a^2\,d^3+4\,b^2\,d^3}-\frac{256\,a\,b^5\,d^2}{4\,a^2\,d^3+4\,b^2\,d^3}+\frac{b^6\,d^2\,256\mathrm{i}}{4\,a^2\,d^3+4\,b^2\,d^3}+\frac{a^2\,b^4\,d^2\,256\mathrm{i}}{4\,a^2\,d^3+4\,b^2\,d^3}}\right)\,\sqrt{-\frac{a-b\,1\mathrm{i}}{4\,a^2\,d^2+4\,b^2\,d^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + b*tan(c + d*x))^(1/2),x)
```

```
[Out] ((4*a^2)/(b^3*d) - (2*(a^2 + b^2))/(b^3*d))*(a + b*tan(c + d*x))^(1/2) + (l
og(16*b^2*(a + b*tan(c + d*x))^(1/2) + 16*b^3*d*(-1/(d^2*(a - b*1i)))^(1/2)
 - (16*a*b^2*(a + b*tan(c + d*x))^(1/2))/(a - b*1i))*(-1/(a*d^2 - b*d^2*1i)
)^(1/2))/2 - log(16*b^3*d*(-1/(d^2*(a - b*1i)))^(1/2) - 16*b^2*(a + b*tan(c
 + d*x))^(1/2) + (16*a*b^2*(a + b*tan(c + d*x))^(1/2))/(a - b*1i))*(-1/(4*(
a*d^2 - b*d^2*1i)))^(1/2) + atan((128*a*b^3*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2)
 - a/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2))/((b^6*d^2*2
56i)/(4*a^2*d^3 + 4*b^2*d^3) + (a^2*b^4*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) -
 (256*a^3*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3) - (256*a*b^5*d^2)/(4*a^2*d^3 + 4
*b^2*d^3)) - (b^2*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^
2))^(1/2)*(a + b*tan(c + d*x))^(1/2)*32i)/((b^4*d^2*64i)/(4*a^2*d^3 + 4*b^2
*d^3) - (64*a*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3)) + (a^2*b^2*((b*1i)/(4*a^2*d
^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/
2)*128i)/((b^6*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) + (a^2*b^4*d^2*256i)/(4*a^
2*d^3 + 4*b^2*d^3) - (256*a^3*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3) - (256*a*b^5
*d^2)/(4*a^2*d^3 + 4*b^2*d^3)))*(-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)
*2i + (2*(a + b*tan(c + d*x))^(5/2))/(5*b^3*d) - (4*a*(a + b*tan(c + d*x))^
(3/2))/(3*b^3*d)
```

3.530 $\int \frac{\tan^3(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal. Leaf size=140

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}\,d}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}\,d}-\frac{4a\sqrt{a+b\tan(c+dx)}}{3b^2d}+\frac{2\tan(c+dx)}{}$$

```
[Out] arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d/(a-I*b)^(1/2)+arctanh((a+b*
tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d/(a+I*b)^(1/2)-4/3*a*(a+b*tan(d*x+c))^(1/
2)/b^2/d+2/3*(a+b*tan(d*x+c))^(1/2)*tan(d*x+c)/b/d
```

Rubi [A]

time = 0.18, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3647, 3711, 12, 3620, 3618, 65, 214}

$$-\frac{4a\sqrt{a+b\tan(c+dx)}}{3b^2d}+\frac{2\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]]/(Sqrt[a - I*b]*d) + ArcTanh
[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]]/(Sqrt[a + I*b]*d) - (4*a*Sqrt[a +
b*Tan[c + d*x]])/(3*b^2*d) + (2*Tan[c + d*x]*Sqrt[a + b*Tan[c + d*x]])/(3*b
*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx = \frac{2\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd} + \frac{2\int \frac{-a-\frac{3}{2}b\tan(c+dx)-a\tan^2(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx}{3b}$$

$$= -\frac{4a\sqrt{a+b\tan(c+dx)}}{3b^2d} + \frac{2\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd} + \frac{2\int -\frac{}{2\sqrt{a}}}{}$$

$$= -\frac{4a\sqrt{a+b\tan(c+dx)}}{3b^2d} + \frac{2\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd} - \int \frac{\tan(}{\sqrt{a+b}}$$

$$= -\frac{4a\sqrt{a+b\tan(c+dx)}}{3b^2d} + \frac{2\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd} - \frac{1}{2}i\int \frac{1-}{\sqrt{a+}}$$

$$= -\frac{4a\sqrt{a+b\tan(c+dx)}}{3b^2d} + \frac{2\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd} - \frac{\text{Subst}\left(\int -\right.}{}$$

$$= -\frac{4a\sqrt{a+b\tan(c+dx)}}{3b^2d} + \frac{2\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd} + \frac{i\text{Subst}\left(\int \right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}\,d} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}\,d} - \frac{4a\sqrt{a}}{}$$

Mathematica [**A**]
time = 0.77, size = 159, normalized size = 1.14

$$\frac{3\sqrt{a-ib}\,(a+ib)b^2\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right) - (a-ib)\left(-3\sqrt{a+ib}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right) + 2(a+ib)(2a-b\tan(c+dx))\sqrt{a+b\tan(c+dx)}\right)}{3b^2(a^2+b^2)\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] (3*Sqrt[a - I*b]*(a + I*b)*b^2*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*
b]] - (a - I*b)*(-3*Sqrt[a + I*b]*b^2*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt
[a + I*b]] + 2*(a + I*b)*(2*a - b*Tan[c + d*x])*Sqrt[a + b*Tan[c + d*x]]))/
(3*b^2*(a^2 + b^2)*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 376 vs. 2(116) = 232.
time = 0.14, size = 377, normalized size = 2.69

method	result

derivativedivides	$\frac{\frac{2(a+b\tan(dx+c))^{\frac{3}{2}}}{3}-2a\sqrt{a+b\tan(dx+c)}+2b^2\left(\frac{\frac{\sqrt{2\sqrt{a^2+b^2}+2a}\,\ln\left(b\tan(dx+c)+a+\sqrt{a+b\tan\right.}{2}}{}\right.}{}$
default	$\frac{\frac{2(a+b\tan(dx+c))^{\frac{3}{2}}}{3}-2a\sqrt{a+b\tan(dx+c)}+2b^2\left(\frac{\frac{\sqrt{2\sqrt{a^2+b^2}+2a}\,\ln\left(b\tan(dx+c)+a+\sqrt{a+b\tan\right.}{2}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)^3/(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/d/b^2*(1/3*(a+b*tan(d*x+c))^(3/2)-a*(a+b*tan(d*x+c))^(1/2)+b^2*(1/4/(a^2+b^2)^(1/2)*(1/2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*(a-(a^2+b^2)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))+1/4/(a^2+b^2)^(1/2)*(-1/2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln(-b*tan(d*x+c)-a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-(a^2+b^2)^(1/2))+2*((a^2+b^2)^(1/2)-a)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^3/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(tan(d*x + c)^3/sqrt(b*tan(d*x + c) + a), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2245 vs. 2(112) = 224.
time = 0.96, size = 2245, normalized size = 16.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^3/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 1/12*(12*sqrt(2)*(a^2*b^2 + b^4)*d^5*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*(1/((a^2 + b^2)*d^4))^(3/4)*arctan(-((a^4 + 2*a^2*b^2 + b^4)*d^4*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(1/((a^2 + b^2)*d^4)) + (a^3 + a*b^2)*d^2*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - sqrt(2)*((a^5 + 2*a^3*b^2 + a*b^4)*d^7*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(1/((a^2 + b^2)*d^4)) + (a^4 + 2*a^2*b^2 + b^4)*d^5*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4)))*sqrt(((a^2 + b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) + sqrt(2)*((a^2 + b^2)*d^3*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) + a*d*cos(d*x + c))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(1/4) + a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(3/4) + sqrt(2)*((a^5 + 2*a^3*b^2 + a*b^4)*d^7*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(1/((a^2 + b^2)*d^4)) + (a^4 + 2*a^2*b^2 + b^4)*d^5*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4)))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(3/4))/b^2)*cos(d*x + c) + 12*sqrt(2)*(a^2*b^2 + b^4)*d^5*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*(1/((a^2 + b^2)*d^4))^(3/4)*arctan(((a^4 + 2*a^2*b^2 + b^4)*d^4*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(1/((a^2 + b^2)*d^4)) + (a^3 + a*b^2)*d^2*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + sqrt(2)*((a^5 + 2*a^3*b^2 + a*b^4)*d^7*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(1/((a^2 + b^2)*d^4)) + (a^4 + 2*a^2*b^2 + b^4)*d^5*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4)))*sqrt(((a^2 + b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) - sqrt(2)*((a^2 + b^2)*d^3*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) + a*d*cos(d*x + c))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(1/4) + a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(3/4) - sqrt(2)*((a^5 + 2*a^3*b^2 + a*b^4)*d^7*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(1/((a^2 + b^2)*d^4)) + (a^4 + 2*a^2*b^2 + b^4)*d^5*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4)))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(3/4))/b^2)*cos(d*x + c) + 3*sqrt(2)*(a*b^2*d^3*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) + b^2*d*cos(d*x + c))*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(1/4)*log(((a^2 + b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) + sqrt(2)*((a^2 + b^2)*d^3*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) + a*d*cos(d*x + c))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4))

```
^(1/4) + a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c)) - 3*sqrt(2)*(a*b^2*
d^3*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) + b^2*d*cos(d*x + c))*sqrt(-((a^
3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*
d^4))^(1/4)*log(((a^2 + b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) - s
qrt(2)*((a^2 + b^2)*d^3*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) + a*d*cos(d*
x + c))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-((a^3 +
a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4)
)^(1/4) + a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c)) - 8*(2*a*cos(d*x +
 c) - b*sin(d*x + c))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c)))
/(b^2*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+b*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**3/sqrt(a + b*tan(c + d*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 4.99, size = 827, normalized size = 5.91

$$\frac{2\,{\left(a+b\tan(c+d\,x)\right)}^{3/2}}{3\,b^2\,d}-\frac{2\,a\,\sqrt{a+b\tan(c+d\,x)}}{b^2\,d}+\mathrm{atan}\left(-\frac{b^2\,\sqrt{\frac{a}{4\,a^2\,d^2+4\,b^2\,d^2}-\frac{b\,1{}\mathrm{i}}{4\,a^2\,d^2+4\,b^2\,d^2}}\,\sqrt{a+b\tan(c+d\,x)}\,32{}\mathrm{i}}{\frac{16\,b^2}{d}-\frac{64\,a^2\,b^2\,d^2}{4\,a^2\,d^3+4\,b^2\,d^3}+\frac{a\,b^3\,d^2\,64{}\mathrm{i}}{4\,a^2\,d^3+4\,b^2\,d^3}}+\frac{a^2\,b^2\,\sqrt{\frac{a}{4\,a^2\,d^2+4\,b^2\,d^2}-\frac{b\,1{}\mathrm{i}}{4\,a^2\,d^2+4\,b^2\,d^2}}\,\sqrt{a+b\tan(c+d\,x)}\,128{}\mathrm{i}}{\frac{64\,b^4}{d}+\frac{64\,a^2\,b^2}{d}-\frac{256\,a^2\,b^4\,d^2}{4\,a^2\,d^3+4\,b^2\,d^3}+\frac{a^3\,b^3\,d^2\,256{}\mathrm{i}}{4\,a^2\,d^3+4\,b^2\,d^3}-\frac{256\,a^4\,b^2\,d^2}{4\,a^2\,d^3+4\,b^2\,d^3}+\frac{a\,b^5\,d^2\,256{}\mathrm{i}}{4\,a^2\,d^3+4\,b^2\,d^3}}+\frac{128\,a\,b^3\,\sqrt{\frac{a}{4\,a^2\,d^2+4\,b^2\,d^2}-\frac{b\,1{}\mathrm{i}}{4\,a^2\,d^2+4\,b^2\,d^2}}\,\sqrt{a+b\tan(c+d\,x)}}{\frac{64\,b^4}{d}+\frac{64\,a^2\,b^2}{d}-\frac{256\,a^2\,b^4\,d^2}{4\,a^2\,d^3+4\,b^2\,d^3}+\frac{a^3\,b^3\,d^2\,256{}\mathrm{i}}{4\,a^2\,d^3+4\,b^2\,d^3}-\frac{256\,a^4\,b^2\,d^2}{4\,a^2\,d^3+4\,b^2\,d^3}+\frac{a\,b^5\,d^2\,256{}\mathrm{i}}{4\,a^2\,d^3+4\,b^2\,d^3}}\right)\,\sqrt{\frac{a-b\,1{}\mathrm{i}}{4\,a^2\,d^2+4\,b^2\,d^2}}\,2{}\mathrm{i}-\mathrm{atan}\left(\frac{b^2\,\sqrt{\frac{1}{a\,d^2-b\,d^2\,1{}\mathrm{i}}}\,\sqrt{a+b\tan(c+d\,x)}\,16{}\mathrm{i}}{\frac{16\,b^2}{d}-\frac{16\,a\,b^2\,d^2}{a\,d^3-b\,d^3\,1{}\mathrm{i}}}+\frac{a\,b^2\,\sqrt{\frac{1}{a\,d^2-b\,d^2\,1{}\mathrm{i}}}\,\sqrt{a+b\tan(c+d\,x)}\,16{}\mathrm{i}}{\frac{b^3\,16{}\mathrm{i}}{d}-\frac{16\,a\,b^2}{d}-\frac{a\,b^3\,d^2\,16{}\mathrm{i}}{a\,d^3-b\,d^3\,1{}\mathrm{i}}+\frac{16\,a^2\,b^2\,d^2}{a\,d^3-b\,d^3\,1{}\mathrm{i}}}\right)\,\sqrt{\frac{1}{a\,d^2-b\,d^2\,1{}\mathrm{i}}}\,1{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + b*tan(c + d*x))^(1/2),x)
```

```
[Out] atan((a^2*b^2*(a/(4*a^2*d^2 + 4*b^2*d^2) - (b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^
(1/2)*(a + b*tan(c + d*x))^(1/2)*128i)/((64*b^4)/d + (64*a^2*b^2)/d - (256*
a^2*b^4*d^2)/(4*a^2*d^3 + 4*b^2*d^3) + (a^3*b^3*d^2*256i)/(4*a^2*d^3 + 4*b^
2*d^3) - (256*a^4*b^2*d^2)/(4*a^2*d^3 + 4*b^2*d^3) + (a*b^5*d^2*256i)/(4*a^
2*d^3 + 4*b^2*d^3)) - (b^2*(a/(4*a^2*d^2 + 4*b^2*d^2) - (b*1i)/(4*a^2*d^2 +
 4*b^2*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2)*32i)/((16*b^2)/d - (64*a^2*b^
```

```
2*d^2)/(4*a^2*d^3 + 4*b^2*d^3) + (a*b^3*d^2*64i)/(4*a^2*d^3 + 4*b^2*d^3)) +
 (128*a*b^3*(a/(4*a^2*d^2 + 4*b^2*d^2) - (b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1
/2)*(a + b*tan(c + d*x))^(1/2))/((64*b^4)/d + (64*a^2*b^2)/d - (256*a^2*b^4
*d^2)/(4*a^2*d^3 + 4*b^2*d^3) + (a^3*b^3*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3)
- (256*a^4*b^2*d^2)/(4*a^2*d^3 + 4*b^2*d^3) + (a*b^5*d^2*256i)/(4*a^2*d^3 +
 4*b^2*d^3)))*((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*2i - atan((b^2*(1/
(a*d^2 - b*d^2*1i))^(1/2)*(a + b*tan(c + d*x))^(1/2)*16i)/((16*b^2)/d - (16
*a*b^2*d^2)/(a*d^3 - b*d^3*1i)) + (a*b^2*(1/(a*d^2 - b*d^2*1i))^(1/2)*(a +
b*tan(c + d*x))^(1/2)*16i)/((b^3*16i)/d - (16*a*b^2)/d - (a*b^3*d^2*16i)/(a
*d^3 - b*d^3*1i) + (16*a^2*b^2*d^2)/(a*d^3 - b*d^3*1i)))*(1/(a*d^2 - b*d^2*
1i))^(1/2)*1i + (2*(a + b*tan(c + d*x))^(3/2))/(3*b^2*d) - (2*a*(a + b*tan(
c + d*x))^(1/2))/(b^2*d)
```

3.531 $\int \frac{\tan^2(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal. Leaf size=424

$$-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a^2+b^2}\sqrt{a-\sqrt{a^2+b^2}}\,d}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}+\sqrt{2}\sqrt{a+b\tan(c}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a^2+b^2}\sqrt{a-\sqrt{a^2+b^2}}\,d}$$

```
[Out] -1/2*b*arctanh(((a+(a^2+b^2)^(1/2))^(1/2)-2^(1/2)*(a+b*tan(d*x+c))^(1/2))/(
a-(a^2+b^2)^(1/2))^(1/2))/d*2^(1/2)/(a^2+b^2)^(1/2)/(a-(a^2+b^2)^(1/2))^(1/
2)+1/2*b*arctanh(((a+(a^2+b^2)^(1/2))^(1/2)+2^(1/2)*(a+b*tan(d*x+c))^(1/2))
/(a-(a^2+b^2)^(1/2))^(1/2))/d*2^(1/2)/(a^2+b^2)^(1/2)/(a-(a^2+b^2)^(1/2))^(
1/2)+1/4*b*ln(a+(a^2+b^2)^(1/2)-2^(1/2)*(a+(a^2+b^2)^(1/2))^(1/2)*(a+b*tan(
d*x+c))^(1/2)+b*tan(d*x+c))/d*2^(1/2)/(a^2+b^2)^(1/2)/(a+(a^2+b^2)^(1/2))^(
1/2)-1/4*b*ln(a+(a^2+b^2)^(1/2)+2^(1/2)*(a+(a^2+b^2)^(1/2))^(1/2)*(a+b*tan(
d*x+c))^(1/2)+b*tan(d*x+c))/d*2^(1/2)/(a^2+b^2)^(1/2)/(a+(a^2+b^2)^(1/2))^(
1/2)+2*(a+b*tan(d*x+c))^(1/2)/b/d
```

Rubi [A]
time = 0.28, antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3624, 3566, 722, 1108, 648, 632, 212, 642}

$$\frac{b\log\left(-\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\sqrt{a+b\tan(c+dx)}+\sqrt{a^2+b^2}+a+b\tan(c+dx)\right)}{2\sqrt{2}\,d\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+a}}-\frac{b\log\left(\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\sqrt{a+b\tan(c+dx)}+\sqrt{a^2+b^2}+a+b\tan(c+dx)\right)}{2\sqrt{2}\,d\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+a}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+b^2}+a}-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\,d\sqrt{a^2+b^2}\sqrt{a-\sqrt{a^2+b^2}}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+b^2}+a}+\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\,d\sqrt{a^2+b^2}\sqrt{a-\sqrt{a^2+b^2}}}+\frac{2\sqrt{a+b\tan(c+dx)}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] -((b*ArcTanh[(Sqrt[a + Sqrt[a^2 + b^2]] - Sqrt[2]*Sqrt[a + b*Tan[c + d*x]])
/Sqrt[a - Sqrt[a^2 + b^2]]])/(Sqrt[2]*Sqrt[a^2 + b^2]*Sqrt[a - Sqrt[a^2 + b
^2]]*d)) + (b*ArcTanh[(Sqrt[a + Sqrt[a^2 + b^2]] + Sqrt[2]*Sqrt[a + b*Tan[c
 + d*x]])/Sqrt[a - Sqrt[a^2 + b^2]]])/(Sqrt[2]*Sqrt[a^2 + b^2]*Sqrt[a - Sqr
t[a^2 + b^2]]*d) + (b*Log[a + Sqrt[a^2 + b^2] + b*Tan[c + d*x] - Sqrt[2]*Sq
rt[a + Sqrt[a^2 + b^2]]*Sqrt[a + b*Tan[c + d*x]]])/(2*Sqrt[2]*Sqrt[a^2 + b^
2]*Sqrt[a + Sqrt[a^2 + b^2]]*d) - (b*Log[a + Sqrt[a^2 + b^2] + b*Tan[c + d*
x] + Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*Sqrt[a + b*Tan[c + d*x]]])/(2*Sqrt[2
]*Sqrt[a^2 + b^2]*Sqrt[a + Sqrt[a^2 + b^2]]*d) + (2*Sqrt[a + b*Tan[c + d*x]
])/(b*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 722

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
```

```
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx = \frac{2\sqrt{a+b\tan(c+dx)}}{bd} - \int \frac{1}{\sqrt{a+b\tan(c+dx)}}\,dx$$

$$= \frac{2\sqrt{a+b\tan(c+dx)}}{bd} - \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+x}\,(b^2+x^2)}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{2\sqrt{a+b\tan(c+dx)}}{bd} - \frac{(2b)\text{Subst}\left(\int \frac{1}{a^2+b^2-2ax^2+x^4}\,dx, x, \sqrt{a+b\tan(c+dx)}\right.}{d}$$

$$= \frac{2\sqrt{a+b\tan(c+dx)}}{bd} - \frac{b\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}-x}{\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\,x+x^2}\,dx, x\right.}{\sqrt{2}\sqrt{a^2+b^2}\sqrt{a+\sqrt{a^2+b^2}}}$$

$$= \frac{2\sqrt{a+b\tan(c+dx)}}{bd} - \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\,x+x^2}\,dx, x\right.}{2\sqrt{a^2+b^2}\,d}$$

$$= \frac{b\log\left(a+\sqrt{a^2+b^2}+b\tan(c+dx)-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\tan(c+\right.}{2\sqrt{2}\sqrt{a^2+b^2}\sqrt{a+\sqrt{a^2+b^2}}\,d}$$

$$= -\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a^2+b^2}\sqrt{a-\sqrt{a^2+b^2}}\,d} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a-}}{}\right.}{\sqrt{2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.21, size = 108, normalized size = 0.25

$$\frac{\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}} - \frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}} + \frac{2\sqrt{a+b\tan(c+dx)}}{b}}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d*x]^2/Sqrt[a + b*Tan[c + d*x]],x]

[Out] ((I*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/Sqrt[a - I*b] - (I*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/Sqrt[a + I*b] + (2*Sqrt[a + b*Tan[c + d*x]])/b)/d

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 796 vs. $2(343) = 686$.
time = 0.14, size = 797, normalized size = 1.88

method	result
derivativedivides	$\dfrac{2\sqrt{a+b\tan\left(dx+c\right)}-2b^{2}\left(\dfrac{\left(\sqrt{2\sqrt{a^{2}+b^{2}}+2a}\,\sqrt{a^{2}+b^{2}}\,a^{2}+\sqrt{2\sqrt{a^{2}+b^{2}}+2a}\,\sqrt{a^{2}\cdots}\right.}{\cdots}\right.}{\cdots}$
default	$\dfrac{2\sqrt{a+b\tan\left(dx+c\right)}-2b^{2}\left(\dfrac{\left(\sqrt{2\sqrt{a^{2}+b^{2}}+2a}\,\sqrt{a^{2}+b^{2}}\,a^{2}+\sqrt{2\sqrt{a^{2}+b^{2}}+2a}\,\sqrt{a^{2}\cdots}\right.}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)^2/(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/d/b*((a+b*tan(d*x+c))^(1/2)-b^2*(1/4/b^2/(a^2+b^2)^(3/2)*(1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*(2*a^2*b^2+2*b^4-1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))+1/4/b^2/(a^2+b^2)^(3/2)*(-1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^2)*ln(-b*tan(d*x+c)-a+(a+

```
b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-(a^2+b^2)^(1/2))+2*(-2*a^
2*b^2-2*b^4+1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2+(2*(a^2+
b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3
-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^
2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2
)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^2/sqrt(b*tan(d*x + c) + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1774 vs. 2(345) = 690.
time = 0.88, size = 1774, normalized size = 4.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*(a^2*b + b^3)*d^5*sqrt(((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2
)*d^4)) + a^2 + b^2)/b^2)*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*(1/((a^2
+ b^2)*d^4))^(3/4)*arctan((sqrt(2)*(a^4 + 2*a^2*b^2 + b^4)*d^7*sqrt((sqrt(2
)*b^3*d*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(((a^3 + a
*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) + a^2 + b^2)/b^2)*(1/((a^2 + b^2)*d^4))
^(1/4)*cos(d*x + c) + (a^2*b^2 + b^4)*d^2*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x
 + c) + a*b^2*cos(d*x + c) + b^3*sin(d*x + c))/cos(d*x + c))*sqrt(((a^3 + a
*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) + a^2 + b^2)/b^2)*sqrt(b^2/((a^4 + 2*a^
2*b^2 + b^4)*d^4))*(1/((a^2 + b^2)*d^4))^(5/4) - sqrt(2)*(a^4*b + 2*a^2*b^3
 + b^5)*d^7*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(((a^3
 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) + a^2 + b^2)/b^2)*sqrt(b^2/((a^4 +
2*a^2*b^2 + b^4)*d^4))*(1/((a^2 + b^2)*d^4))^(5/4) - (a^4 + 2*a^2*b^2 + b^4
)*d^4*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(1/((a^2 + b^2)*d^4)) - (
a^3 + a*b^2)*d^2*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/b^2) + 4*sqrt(2)*
(a^2*b + b^3)*d^5*sqrt(((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) + a^2 +
 b^2)/b^2)*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*(1/((a^2 + b^2)*d^4))^(3
/4)*arctan((sqrt(2)*(a^4 + 2*a^2*b^2 + b^4)*d^7*sqrt(-(sqrt(2)*b^3*d*sqrt((
a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(((a^3 + a*b^2)*d^2*sqrt
(1/((a^2 + b^2)*d^4)) + a^2 + b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(1/4)*cos(d*x
 + c) - (a^2*b^2 + b^4)*d^2*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) - a*b^2*
```

```
cos(d*x + c) - b^3*sin(d*x + c))/cos(d*x + c))*sqrt(((a^3 + a*b^2)*d^2*sqrt
(1/((a^2 + b^2)*d^4)) + a^2 + b^2)/b^2)*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d
^4))*(1/((a^2 + b^2)*d^4))^(5/4) - sqrt(2)*(a^4*b + 2*a^2*b^3 + b^5)*d^7*sq
rt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(((a^3 + a*b^2)*d^2*
sqrt(1/((a^2 + b^2)*d^4)) + a^2 + b^2)/b^2)*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^
4)*d^4))*(1/((a^2 + b^2)*d^4))^(5/4) + (a^4 + 2*a^2*b^2 + b^4)*d^4*sqrt(b^2
/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(1/((a^2 + b^2)*d^4)) + (a^3 + a*b^2)*d
^2*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/b^2) + sqrt(2)*(a*b*d^3*sqrt(1/
((a^2 + b^2)*d^4)) - b*d)*sqrt(((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4))
 + a^2 + b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(1/4)*log((sqrt(2)*b^3*d*sqrt((a*c
os(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(((a^3 + a*b^2)*d^2*sqrt(1/
((a^2 + b^2)*d^4)) + a^2 + b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(1/4)*cos(d*x +
c) + (a^2*b^2 + b^4)*d^2*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) + a*b^2*cos
(d*x + c) + b^3*sin(d*x + c))/cos(d*x + c)) - sqrt(2)*(a*b*d^3*sqrt(1/((a^2
 + b^2)*d^4)) - b*d)*sqrt(((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) + a^
2 + b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(1/4)*log(-(sqrt(2)*b^3*d*sqrt((a*cos(d
*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(((a^3 + a*b^2)*d^2*sqrt(1/((a^
2 + b^2)*d^4)) + a^2 + b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(1/4)*cos(d*x + c) -
 (a^2*b^2 + b^4)*d^2*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) - a*b^2*cos(d*x
 + c) - b^3*sin(d*x + c))/cos(d*x + c)) + 8*sqrt((a*cos(d*x + c) + b*sin(d*
x + c))/cos(d*x + c)))/(b*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+b*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**2/sqrt(a + b*tan(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 4.86, size = 730, normalized size = 1.72

$$\frac{\ln\left(-16\,b^2\,\sqrt{a+b\tan(c+d\,x)}+16\,b^3\,d\,\sqrt{-\frac{1}{d^2\,(a-b\,1\mathrm{i})}}+\frac{16\,a\,b^2\,\sqrt{a+b\tan(c+d\,x)}}{a-b\,1\mathrm{i}}\right)\sqrt{-\frac{1}{a\,d^2-b\,d^2\,1\mathrm{i}}}}{2}-\ln\left(16\,b^2\,\sqrt{a+b\tan(c+d\,x)}+16\,b^3\,d\,\sqrt{-\frac{1}{d^2\,(a-b\,1\mathrm{i})}}-\frac{16\,a\,b^2\,\sqrt{a+b\tan(c+d\,x)}}{a-b\,1\mathrm{i}}\right)\sqrt{-\frac{1}{4\,(a\,d^2-b\,d^2\,1\mathrm{i})}}+\frac{2\,\sqrt{a+b\tan(c+d\,x)}}{b\,d}-\mathrm{atan}\left(-\frac{b^2\,\sqrt{-\frac{a}{4\,a^2\,d^2+4\,b^2\,d^2}+\frac{b\,1\mathrm{i}}{4\,a^2\,d^2+4\,b^2\,d^2}}\,\sqrt{a+b\tan(c+d\,x)}\,32\mathrm{i}}{-\frac{64\,a\,b^3\,d^2}{4\,a^2\,d^3+4\,b^2\,d^3}+\frac{b^4\,d^2\,64\mathrm{i}}{4\,a^2\,d^3+4\,b^2\,d^3}}+\frac{128\,a\,b^3\,\sqrt{-\frac{a}{4\,a^2\,d^2+4\,b^2\,d^2}+\frac{b\,1\mathrm{i}}{4\,a^2\,d^2+4\,b^2\,d^2}}\,\sqrt{a+b\tan(c+d\,x)}}{-\frac{256\,a^3\,b^3\,d^2}{4\,a^2\,d^3+4\,b^2\,d^3}-\frac{256\,a\,b^5\,d^2}{4\,a^2\,d^3+4\,b^2\,d^3}+\frac{b^6\,d^2\,256\mathrm{i}}{4\,a^2\,d^3+4\,b^2\,d^3}+\frac{a^2\,b^4\,d^2\,256\mathrm{i}}{4\,a^2\,d^3+4\,b^2\,d^3}}+\frac{a^2\,b^2\,\sqrt{-\frac{a}{4\,a^2\,d^2+4\,b^2\,d^2}+\frac{b\,1\mathrm{i}}{4\,a^2\,d^2+4\,b^2\,d^2}}\,\sqrt{a+b\tan(c+d\,x)}\,128\mathrm{i}}{-\frac{256\,a^3\,b^3\,d^2}{4\,a^2\,d^3+4\,b^2\,d^3}-\frac{256\,a\,b^5\,d^2}{4\,a^2\,d^3+4\,b^2\,d^3}+\frac{b^6\,d^2\,256\mathrm{i}}{4\,a^2\,d^3+4\,b^2\,d^3}+\frac{a^2\,b^4\,d^2\,256\mathrm{i}}{4\,a^2\,d^3+4\,b^2\,d^3}}\right)\sqrt{-\frac{a-b\,1\mathrm{i}}{4\,a^2\,d^2+4\,b^2\,d^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d*x)^2/(a + b*tan(c + d*x))^(1/2),x)

[Out] (log(16*b^3*d*(-1/(d^2*(a - b*1i)))^(1/2) - 16*b^2*(a + b*tan(c + d*x))^(1/2) + (16*a*b^2*(a + b*tan(c + d*x))^(1/2))/(a - b*1i))*(-1/(a*d^2 - b*d^2*1i))^(1/2))/2 - log(16*b^2*(a + b*tan(c + d*x))^(1/2) + 16*b^3*d*(-1/(d^2*(a - b*1i)))^(1/2) - (16*a*b^2*(a + b*tan(c + d*x))^(1/2))/(a - b*1i))*(-1/(4*(a*d^2 - b*d^2*1i)))^(1/2) - atan((128*a*b^3*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2))/((b^6*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) + (a^2*b^4*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) - (256*a^3*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3) - (256*a*b^5*d^2)/(4*a^2*d^3 + 4*b^2*d^3)) - (b^2*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2)*32i)/((b^4*d^2*64i)/(4*a^2*d^3 + 4*b^2*d^3) - (64*a*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3)) + (a^2*b^2*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2)*128i)/((b^6*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) + (a^2*b^4*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) - (256*a^3*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3) - (256*a*b^5*d^2)/(4*a^2*d^3 + 4*b^2*d^3)))*(-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*2i + (2*(a + b*tan(c + d*x))^(1/2))/(b*d)

3.532 $\int \frac{\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal. Leaf size=87

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}\,d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}\,d}$$

```
[Out] -arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d/(a-I*b)^(1/2)-arctanh((a+b
*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d/(a+I*b)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3620, 3618, 65, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]]/(Sqrt[a - I*b]*d)) - ArcT
anh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]]/(Sqrt[a + I*b]*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx = \frac{1}{2}i\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx - \frac{1}{2}i\int \frac{1+i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a-ibx}}\,dx, x, i\tan(c+dx)\right)}{2d} + \frac{\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+ibx}}\,d\right.}{2d}$$

$$= -\frac{i\text{Subst}\left(\int \frac{1}{-1+\frac{ia}{b}-\frac{ix^2}{b}}\,dx, x, \sqrt{a+b\tan(c+dx)}\right)}{bd} + \frac{i\text{Subst}\left(\int \frac{1}{-1-\frac{ia}{b}+\frac{ix^2}{b}}\,dx,\right.}{b}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}\,d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}\,d}$$

Mathematica [A]
time = 0.07, size = 107, normalized size = 1.23

$$\frac{\sqrt{a-ib}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(-a+ib)d} + \frac{\sqrt{a+ib}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(-a-ib)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] (Sqrt[a - I*b]*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/((-a + I*b)
*d) + (Sqrt[a + I*b]*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/((-a
- I*b)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(71) = 142$.
time = 0.62, size = 339, normalized size = 3.90

method	result

derivativedivides	$-\frac{\frac{\sqrt{2\sqrt{a^2+b^2}+2a}\,\ln\left(b\tan(dx+c)+a+\sqrt{a+b\tan\left(dx+c\right)}\,\sqrt{2\sqrt{a^2+b^2}+2a}+\sqrt{a^2+b^2}\right.}{2}}{2\sqrt{a^2+b^2}}$
default	$-\frac{\frac{\sqrt{2\sqrt{a^2+b^2}+2a}\,\ln\left(b\tan(dx+c)+a+\sqrt{a+b\tan\left(dx+c\right)}\,\sqrt{2\sqrt{a^2+b^2}+2a}+\sqrt{a^2+b^2}\right.}{2}}{2\sqrt{a^2+b^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/(a^2+b^2)^(1/2)*(-1/2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln(b*tan(d*x+c
)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2
*((a^2+b^2)^(1/2)-a)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c
))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))+1/2
/(a^2+b^2)^(1/2)*(1/2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln(-b*tan(d*x+c)-a+(a+b
*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-(a^2+b^2)^(1/2))+2*(a-(a^2
+b^2)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*tan(d*x+c))^(1/2
)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2122 vs. 2(67) = 134.
time = 1.25, size = 2122, normalized size = 24.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*(a^2 + b^2)*d^4*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4))
 - a^2 - b^2)/b^2)*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*(1/((a^2 + b^2)*
d^4))^(3/4)*arctan(-((a^4 + 2*a^2*b^2 + b^4)*d^4*sqrt(b^2/((a^4 + 2*a^2*b^2
 + b^4)*d^4))*sqrt(1/((a^2 + b^2)*d^4)) + (a^3 + a*b^2)*d^2*sqrt(b^2/((a^4
+ 2*a^2*b^2 + b^4)*d^4)) - sqrt(2)*((a^5 + 2*a^3*b^2 + a*b^4)*d^7*sqrt(b^2/
((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(1/((a^2 + b^2)*d^4)) + (a^4 + 2*a^2*b^2
 + b^4)*d^5*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4)))*sqrt(((a^2 + b^2)*d^2*
sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) + sqrt(2)*((a^2 + b^2)*d^3*sqrt(1/((
a^2 + b^2)*d^4))*cos(d*x + c) + a*d*cos(d*x + c))*sqrt((a*cos(d*x + c) + b*
sin(d*x + c))/cos(d*x + c))*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^
4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(1/4) + a*cos(d*x + c) + b*sin(
d*x + c))/cos(d*x + c))*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4))
- a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(3/4) + sqrt(2)*((a^5 + 2*a^3*b^2 +
 a*b^4)*d^7*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(1/((a^2 + b^2)*d^4
)) + (a^4 + 2*a^2*b^2 + b^4)*d^5*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4)))*s
qrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-((a^3 + a*b^2)*d^
2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(3/4))/
b^2) - sqrt(2)*(a^2 + b^2)*d^4*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)
*d^4)) - a^2 - b^2)/b^2)*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*(1/((a^2 +
 b^2)*d^4))^(3/4)*arctan(((a^4 + 2*a^2*b^2 + b^4)*d^4*sqrt(b^2/((a^4 + 2*a^
2*b^2 + b^4)*d^4))*sqrt(1/((a^2 + b^2)*d^4)) + (a^3 + a*b^2)*d^2*sqrt(b^2/(
(a^4 + 2*a^2*b^2 + b^4)*d^4)) + sqrt(2)*((a^5 + 2*a^3*b^2 + a*b^4)*d^7*sqrt
(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(1/((a^2 + b^2)*d^4)) + (a^4 + 2*a^
2*b^2 + b^4)*d^5*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4)))*sqrt(((a^2 + b^2)
*d^2*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) - sqrt(2)*((a^2 + b^2)*d^3*sqrt
(1/((a^2 + b^2)*d^4))*cos(d*x + c) + a*d*cos(d*x + c))*sqrt((a*cos(d*x + c)
 + b*sin(d*x + c))/cos(d*x + c))*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^
2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(1/4) + a*cos(d*x + c) + b
*sin(d*x + c))/cos(d*x + c))*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d
^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(3/4) - sqrt(2)*((a^5 + 2*a^3*
b^2 + a*b^4)*d^7*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(1/((a^2 + b^2
)*d^4)) + (a^4 + 2*a^2*b^2 + b^4)*d^5*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4
)))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-((a^3 + a*b^
2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(3
/4))/b^2) - 1/4*sqrt(2)*(a*d^2*sqrt(1/((a^2 + b^2)*d^4)) + 1)*sqrt(-((a^3 +
 a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4
))^(1/4)*log(((a^2 + b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) + sqrt
(2)*((a^2 + b^2)*d^3*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) + a*d*cos(d*x +
 c))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-((a^3 + a*b
^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(
1/4) + a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c)) + 1/4*sqrt(2)*(a*d^2*
sqrt(1/((a^2 + b^2)*d^4)) + 1)*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)
*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(1/4)*log(((a^2 + b^2)*d^2*s
qrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) - sqrt(2)*((a^2 + b^2)*d^3*sqrt(1/((a
^2 + b^2)*d^4))*cos(d*x + c) + a*d*cos(d*x + c))*sqrt((a*cos(d*x + c) + b*s
```

```
in(d*x + c))/cos(d*x + c))*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4
)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(1/4) + a*cos(d*x + c) + b*sin(d
*x + c))/cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(c + dx)}{\sqrt{a + b\tan(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*tan(d*x+c))**(1/2),x)
[Out] Integral(tan(c + d*x)/sqrt(a + b*tan(c + d*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [B]
time = 4.75, size = 781, normalized size = 8.98

$$-2\,\mathrm{atanh}\left(\frac{32\,b^2\,\sqrt{\frac{a}{4\,a^2\,d^2+4\,b^2\,d^2}-\frac{b\,1{}\mathrm{i}}{4\,a^2\,d^2+4\,b^2\,d^2}}\,\sqrt{a+b\,\tan(c+d\,x)}}{\frac{16\,b^2}{d}-\frac{64\,a^2\,b^2\,d^2}{4\,a^2\,d^3+4\,b^2\,d^3}+\frac{a\,b^3\,d^2\,64{}\mathrm{i}}{4\,a^2\,d^3+4\,b^2\,d^3}}-\frac{128\,a^2\,b^2\,\sqrt{\frac{a}{4\,a^2\,d^2+4\,b^2\,d^2}-\frac{b\,1{}\mathrm{i}}{4\,a^2\,d^2+4\,b^2\,d^2}}\,\sqrt{a+b\,\tan(c+d\,x)}}{\frac{64\,b^4}{d}+\frac{64\,a^2\,b^2}{d}-\frac{256\,a^2\,b^4\,d^2}{4\,a^2\,d^3+4\,b^2\,d^3}+\frac{a^3\,b^3\,d^2\,256{}\mathrm{i}}{4\,a^2\,d^3+4\,b^2\,d^3}-\frac{256\,a^4\,b^2\,d^2}{4\,a^2\,d^3+4\,b^2\,d^3}+\frac{a\,b^5\,d^2\,256{}\mathrm{i}}{4\,a^2\,d^3+4\,b^2\,d^3}}+\frac{a\,b^3\,\sqrt{\frac{a}{4\,a^2\,d^2+4\,b^2\,d^2}-\frac{b\,1{}\mathrm{i}}{4\,a^2\,d^2+4\,b^2\,d^2}}\,\sqrt{a+b\,\tan(c+d\,x)}\,128{}\mathrm{i}}{\frac{64\,b^4}{d}+\frac{64\,a^2\,b^2}{d}-\frac{256\,a^2\,b^4\,d^2}{4\,a^2\,d^3+4\,b^2\,d^3}+\frac{a^3\,b^3\,d^2\,256{}\mathrm{i}}{4\,a^2\,d^3+4\,b^2\,d^3}-\frac{256\,a^4\,b^2\,d^2}{4\,a^2\,d^3+4\,b^2\,d^3}+\frac{a\,b^5\,d^2\,256{}\mathrm{i}}{4\,a^2\,d^3+4\,b^2\,d^3}}\right)\,\sqrt{\frac{a-b\,1{}\mathrm{i}}{4\,a^2\,d^2+4\,b^2\,d^2}}-\mathrm{atanh}\left(\frac{16\,b^2\,\sqrt{\frac{1}{a\,d^2-b\,d^2\,1{}\mathrm{i}}}\,\sqrt{a+b\,\tan(c+d\,x)}}{\frac{16\,b^2}{d}-\frac{16\,a\,b^2\,d^2}{a\,d^3-b\,d^3\,1{}\mathrm{i}}}+\frac{16\,a\,b^2\,\sqrt{\frac{1}{a\,d^2-b\,d^2\,1{}\mathrm{i}}}\,\sqrt{a+b\,\tan(c+d\,x)}}{\frac{b^3\,16{}\mathrm{i}}{d}-\frac{16\,a\,b^2}{d}-\frac{a\,b^3\,d^2\,16{}\mathrm{i}}{a\,d^3-b\,d^3\,1{}\mathrm{i}}+\frac{16\,a^2\,b^2\,d^2}{a\,d^3-b\,d^3\,1{}\mathrm{i}}}\right)\,\sqrt{\frac{1}{a\,d^2-b\,d^2\,1{}\mathrm{i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + b*tan(c + d*x))^(1/2),x)
[Out] - 2*atanh((32*b^2*(a/(4*a^2*d^2 + 4*b^2*d^2) - (b*1i)/(4*a^2*d^2 + 4*b^2*d^
2))^(1/2)*(a + b*tan(c + d*x))^(1/2))/((16*b^2)/d - (64*a^2*b^2*d^2)/(4*a^2
*d^3 + 4*b^2*d^3) + (a*b^3*d^2*64i)/(4*a^2*d^3 + 4*b^2*d^3)) - (128*a^2*b^2
*(a/(4*a^2*d^2 + 4*b^2*d^2) - (b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(a + b*
tan(c + d*x))^(1/2))/((64*b^4)/d + (64*a^2*b^2)/d - (256*a^2*b^4*d^2)/(4*a^
2*d^3 + 4*b^2*d^3) + (a^3*b^3*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) - (256*a^4*
b^2*d^2)/(4*a^2*d^3 + 4*b^2*d^3) + (a*b^5*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3)
) + (a*b^3*(a/(4*a^2*d^2 + 4*b^2*d^2) - (b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/
2)*(a + b*tan(c + d*x))^(1/2)*128i)/((64*b^4)/d + (64*a^2*b^2)/d - (256*a^2
*b^4*d^2)/(4*a^2*d^3 + 4*b^2*d^3) + (a^3*b^3*d^2*256i)/(4*a^2*d^3 + 4*b^2*d
^3) - (256*a^4*b^2*d^2)/(4*a^2*d^3 + 4*b^2*d^3) + (a*b^5*d^2*256i)/(4*a^2*d
^3 + 4*b^2*d^3)))*((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2) - atanh((16*b^
2*(1/(a*d^2 - b*d^2*1i))^(1/2)*(a + b*tan(c + d*x))^(1/2))/((16*b^2)/d - (1
6*a*b^2*d^2)/(a*d^3 - b*d^3*1i)) + (16*a*b^2*(1/(a*d^2 - b*d^2*1i))^(1/2)*(
a + b*tan(c + d*x))^(1/2))/((b^3*16i)/d - (16*a*b^2)/d - (a*b^3*d^2*16i)/(a
*d^3 - b*d^3*1i) + (16*a^2*b^2*d^2)/(a*d^3 - b*d^3*1i)))*(1/(a*d^2 - b*d^2*
1i))^(1/2)
```

3.533 $\int \frac{1}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal. Leaf size=402

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a^2+b^2}\sqrt{a-\sqrt{a^2+b^2}}\,d}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}+\sqrt{2}\sqrt{a+b\tan(c+}{\sqrt{a-\sqrt{a^2+b^2}}}\right.}{\sqrt{2}\sqrt{a^2+b^2}\sqrt{a-\sqrt{a^2+b^2}}\,d}$$

```
[Out] 1/2*b*arctanh(((a+(a^2+b^2)^(1/2))^(1/2)-2^(1/2)*(a+b*tan(d*x+c))^(1/2))/(a
-(a^2+b^2)^(1/2))^(1/2))/d*2^(1/2)/(a^2+b^2)^(1/2)/(a-(a^2+b^2)^(1/2))^(1/2
)-1/2*b*arctanh(((a+(a^2+b^2)^(1/2))^(1/2)+2^(1/2)*(a+b*tan(d*x+c))^(1/2))/
(a-(a^2+b^2)^(1/2))^(1/2))/d*2^(1/2)/(a^2+b^2)^(1/2)/(a-(a^2+b^2)^(1/2))^(1
/2)-1/4*b*ln(a+(a^2+b^2)^(1/2)-2^(1/2)*(a+(a^2+b^2)^(1/2))^(1/2)*(a+b*tan(d
*x+c))^(1/2)+b*tan(d*x+c))/d*2^(1/2)/(a^2+b^2)^(1/2)/(a+(a^2+b^2)^(1/2))^(1
/2)+1/4*b*ln(a+(a^2+b^2)^(1/2)+2^(1/2)*(a+(a^2+b^2)^(1/2))^(1/2)*(a+b*tan(d
*x+c))^(1/2)+b*tan(d*x+c))/d*2^(1/2)/(a^2+b^2)^(1/2)/(a+(a^2+b^2)^(1/2))^(1
/2)
```

Rubi [A]

time = 0.25, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3566, 722, 1108, 648, 632, 212, 642}

$$-\frac{b\log\left(-\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\sqrt{a+b\tan(c+dx)}+\sqrt{a^2+b^2}+a+b\tan(c+dx)\right)}{2\sqrt{2}d\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+a}}+\frac{b\log\left(\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\sqrt{a+b\tan(c+dx)}+\sqrt{a^2+b^2}+a+b\tan(c+dx)\right)}{2\sqrt{2}d\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+a}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+b^2}+a}-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}d\sqrt{a^2+b^2}\sqrt{a-\sqrt{a^2+b^2}}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+b^2}+a}+\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}d\sqrt{a^2+b^2}\sqrt{a-\sqrt{a^2+b^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] (b*ArcTanh[(Sqrt[a + Sqrt[a^2 + b^2]] - Sqrt[2]*Sqrt[a + b*Tan[c + d*x]])/S
qrt[a - Sqrt[a^2 + b^2]]])/(Sqrt[2]*Sqrt[a^2 + b^2]*Sqrt[a - Sqrt[a^2 + b^2
]]*d) - (b*ArcTanh[(Sqrt[a + Sqrt[a^2 + b^2]] + Sqrt[2]*Sqrt[a + b*Tan[c +
d*x]])/Sqrt[a - Sqrt[a^2 + b^2]]])/(Sqrt[2]*Sqrt[a^2 + b^2]*Sqrt[a - Sqrt[a
^2 + b^2]]*d) - (b*Log[a + Sqrt[a^2 + b^2] + b*Tan[c + d*x] - Sqrt[2]*Sqrt[
a + Sqrt[a^2 + b^2]]*Sqrt[a + b*Tan[c + d*x]]])/(2*Sqrt[2]*Sqrt[a^2 + b^2]*
Sqrt[a + Sqrt[a^2 + b^2]]*d) + (b*Log[a + Sqrt[a^2 + b^2] + b*Tan[c + d*x]
+ Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*Sqrt[a + b*Tan[c + d*x]]])/(2*Sqrt[2]*S
qrt[a^2 + b^2]*Sqrt[a + Sqrt[a^2 + b^2]]*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 722

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\tan(c+dx)}}\,dx = \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+x}(b^2+x^2)}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{(2b)\text{Subst}\left(\int \frac{1}{a^2+b^2-2ax^2+x^4}\,dx, x, \sqrt{a+b\tan(c+dx)}\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}-x}{\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}x+x^2}\,dx, x, \sqrt{a+b\tan(c+dx)}\right)}{\sqrt{2}\sqrt{a^2+b^2}\sqrt{a+\sqrt{a^2+b^2}}\,d}$$

$$= \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}x+x^2}\,dx, x, \sqrt{a+b\tan(c+dx)}\right)}{2\sqrt{a^2+b^2}\,d}$$

$$= -\frac{b\log\left(a+\sqrt{a^2+b^2}+b\tan(c+dx)-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\tan(c+\right.}{2\sqrt{2}\sqrt{a^2+b^2}\sqrt{a+\sqrt{a^2+b^2}}\,d}$$

$$= \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a^2+b^2}\sqrt{a-\sqrt{a^2+b^2}}\,d} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+}}{\right.}}{\sqrt{2}\sqrt{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.03, size = 87, normalized size = 0.22

$$-\frac{i\left(\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] ((-I)*(ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]]/Sqrt[a - I*b] - ArcT
anh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]]/Sqrt[a + I*b]))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 776 vs. $2(323) = 646$.

time = 0.15, size = 777, normalized size = 1.93

method	result
derivativedivides	$2b\left(\frac{\left(\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a^2+\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,b^2-\sqrt{2\sqrt{a^2+b^2}+\cdots}\right.}{\cdots}\right.$
default	$2b\left(\frac{\left(\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a^2+\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,b^2-\sqrt{2\sqrt{a^2+b^2}+\cdots}\right.}{\cdots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*b*(1/4/b^2/(a^2+b^2)^(3/2)*(1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2
)^(1/2)*a^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^2-(2*(a^2+b^2)^
(1/2)+2*a)^(1/2)*a^3-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^2)*ln(b*tan(d*x+c)+a
+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*(2
*a^2*b^2+2*b^4-1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2+(2*(a
^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*
a^3-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*
(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1
/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))+1/4/b^2/(a^2+b^2)^(3/2)*(-1
/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2+(2*(a^2+b^2)^(1/2)+2*
a)^(1/2)*(a^2+b^2)^(1/2)*b^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-(2*(a^2+b^2)
^(1/2)+2*a)^(1/2)*a*b^2)*ln(-b*tan(d*x+c)-a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+
b^2)^(1/2)+2*a)^(1/2)-(a^2+b^2)^(1/2))+2*(-2*a^2*b^2-2*b^4+1/2*((2*(a^2+b^2
)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b
^2)^(1/2)*b^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-(2*(a^2+b^2)^(1/2)+2*a)^(1/
2)*a*b^2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arct
```

```
an((-2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(
1/2)-2*a)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1725 vs. 2(325) = 650.
time = 1.47, size = 1725, normalized size = 4.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*(a^2 + b^2)*d^4*sqrt(((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4))
+ a^2 + b^2)/b^2)*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*(1/((a^2 + b^2)*d
^4))^(3/4)*arctan((sqrt(2)*(a^4 + 2*a^2*b^2 + b^4)*d^7*sqrt((sqrt(2)*b^3*d*
sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(((a^3 + a*b^2)*d^
2*sqrt(1/((a^2 + b^2)*d^4)) + a^2 + b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(1/4)*c
os(d*x + c) + (a^2*b^2 + b^4)*d^2*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) +
a*b^2*cos(d*x + c) + b^3*sin(d*x + c))/cos(d*x + c))*sqrt(((a^3 + a*b^2)*d^
2*sqrt(1/((a^2 + b^2)*d^4)) + a^2 + b^2)/b^2)*sqrt(b^2/((a^4 + 2*a^2*b^2 +
b^4)*d^4))*(1/((a^2 + b^2)*d^4))^(5/4) - sqrt(2)*(a^4*b + 2*a^2*b^3 + b^5)*
d^7*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(((a^3 + a*b^2
)*d^2*sqrt(1/((a^2 + b^2)*d^4)) + a^2 + b^2)/b^2)*sqrt(b^2/((a^4 + 2*a^2*b^
2 + b^4)*d^4))*(1/((a^2 + b^2)*d^4))^(5/4) - (a^4 + 2*a^2*b^2 + b^4)*d^4*sq
rt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(1/((a^2 + b^2)*d^4)) - (a^3 + a*
b^2)*d^2*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/b^2) - sqrt(2)*(a^2 + b^2
)*d^4*sqrt(((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) + a^2 + b^2)/b^2)*s
qrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*(1/((a^2 + b^2)*d^4))^(3/4)*arctan((
sqrt(2)*(a^4 + 2*a^2*b^2 + b^4)*d^7*sqrt(-(sqrt(2)*b^3*d*sqrt((a*cos(d*x +
c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b
^2)*d^4)) + a^2 + b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(1/4)*cos(d*x + c) - (a^2
*b^2 + b^4)*d^2*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) - a*b^2*cos(d*x + c)
 - b^3*sin(d*x + c))/cos(d*x + c))*sqrt(((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b
```

```
^2)*d^4)) + a^2 + b^2)/b^2)*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*(1/((a^
2 + b^2)*d^4))^(5/4) - sqrt(2)*(a^4*b + 2*a^2*b^3 + b^5)*d^7*sqrt((a*cos(d*
x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(((a^3 + a*b^2)*d^2*sqrt(1/((a^2
 + b^2)*d^4)) + a^2 + b^2)/b^2)*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*(1/
((a^2 + b^2)*d^4))^(5/4) + (a^4 + 2*a^2*b^2 + b^4)*d^4*sqrt(b^2/((a^4 + 2*a
^2*b^2 + b^4)*d^4))*sqrt(1/((a^2 + b^2)*d^4)) + (a^3 + a*b^2)*d^2*sqrt(b^2/
((a^4 + 2*a^2*b^2 + b^4)*d^4)))/b^2) - 1/4*sqrt(2)*(a*d^2*sqrt(1/((a^2 + b^
2)*d^4)) - 1)*sqrt(((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) + a^2 + b^2
)/b^2)*(1/((a^2 + b^2)*d^4))^(1/4)*log((sqrt(2)*b^3*d*sqrt((a*cos(d*x + c)
+ b*sin(d*x + c))/cos(d*x + c))*sqrt(((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)
*d^4)) + a^2 + b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(1/4)*cos(d*x + c) + (a^2*b^
2 + b^4)*d^2*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) + a*b^2*cos(d*x + c) +
b^3*sin(d*x + c))/cos(d*x + c)) + 1/4*sqrt(2)*(a*d^2*sqrt(1/((a^2 + b^2)*d^
4)) - 1)*sqrt(((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) + a^2 + b^2)/b^2
)*(1/((a^2 + b^2)*d^4))^(1/4)*log(-(sqrt(2)*b^3*d*sqrt((a*cos(d*x + c) + b*
sin(d*x + c))/cos(d*x + c))*sqrt(((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4
)) + a^2 + b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(1/4)*cos(d*x + c) - (a^2*b^2 +
b^4)*d^2*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) - a*b^2*cos(d*x + c) - b^3*
sin(d*x + c))/cos(d*x + c))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\tan{\left(c + dx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*tan(c + d*x)), x)
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]

time = 5.14, size = 708, normalized size = 1.76

$$\frac{\ln\left(16\,b^2\,\sqrt{a+b\,\tan\left(c+d\,x\right)}+16\,b^3\,d\,\sqrt{-\frac{1}{d^2\,\left(a-b\,1{}\mathrm{i}\right)}}-\frac{16\,a\,b^2\,\sqrt{a+b\,\tan\left(c+d\,x\right)}}{a-b\,1{}\mathrm{i}}\right)\,\sqrt{-\frac{1}{a\,d^2-b\,d^2\,1{}\mathrm{i}}}}{2}-\ln\left(-16\,b^2\,\sqrt{a+b\,\tan\left(c+d\,x\right)}+16\,b^3\,d\,\sqrt{-\frac{1}{d^2\,\left(a-b\,1{}\mathrm{i}\right)}}+\frac{16\,a\,b^2\,\sqrt{a+b\,\tan\left(c+d\,x\right)}}{a-b\,1{}\mathrm{i}}\right)\,\sqrt{-\frac{1}{4\,\left(a\,d^2-b\,d^2\,1{}\mathrm{i}\right)}}+2\,\mathrm{atanh}\left(\frac{32\,b^2\,\sqrt{-\frac{a}{4\,a^2\,d^2+4\,b^2\,d^2}+\frac{b\,1{}\mathrm{i}}{4\,a^2\,d^2+4\,b^2\,d^2}}\,\sqrt{a+b\,\tan\left(c+d\,x\right)}}{-\frac{64\,a\,b^3\,d^2}{4\,a^2\,d^3+4\,b^2\,d^3}+\frac{b^4\,d^2\,64{}\mathrm{i}}{4\,a^2\,d^3+4\,b^2\,d^3}}+\frac{a\,b^3\,\sqrt{-\frac{a}{4\,a^2\,d^2+4\,b^2\,d^2}+\frac{b\,1{}\mathrm{i}}{4\,a^2\,d^2+4\,b^2\,d^2}}\,\sqrt{a+b\,\tan\left(c+d\,x\right)}\,128{}\mathrm{i}}{-\frac{256\,a^3\,b^3\,d^2}{4\,a^2\,d^3+4\,b^2\,d^3}-\frac{256\,a\,b^5\,d^2}{4\,a^2\,d^3+4\,b^2\,d^3}+\frac{b^6\,d^2\,256{}\mathrm{i}}{4\,a^2\,d^3+4\,b^2\,d^3}+\frac{a^2\,b^4\,d^2\,256{}\mathrm{i}}{4\,a^2\,d^3+4\,b^2\,d^3}}-\frac{128\,a^2\,b^2\,\sqrt{-\frac{a}{4\,a^2\,d^2+4\,b^2\,d^2}+\frac{b\,1{}\mathrm{i}}{4\,a^2\,d^2+4\,b^2\,d^2}}\,\sqrt{a+b\,\tan\left(c+d\,x\right)}}{-\frac{256\,a^3\,b^3\,d^2}{4\,a^2\,d^3+4\,b^2\,d^3}-\frac{256\,a\,b^5\,d^2}{4\,a^2\,d^3+4\,b^2\,d^3}+\frac{b^6\,d^2\,256{}\mathrm{i}}{4\,a^2\,d^3+4\,b^2\,d^3}+\frac{a^2\,b^4\,d^2\,256{}\mathrm{i}}{4\,a^2\,d^3+4\,b^2\,d^3}}\right)\,\sqrt{-\frac{a-b\,1{}\mathrm{i}}{4\,a^2\,d^2+4\,b^2\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(c + d*x))^(1/2),x)
```

```
[Out] (log(16*b^2*(a + b*tan(c + d*x))^(1/2) + 16*b^3*d*(-1/(d^2*(a - b*1i)))^(1/
2) - (16*a*b^2*(a + b*tan(c + d*x))^(1/2))/(a - b*1i))*(-1/(a*d^2 - b*d^2*1
i))^(1/2))/2 - log(16*b^3*d*(-1/(d^2*(a - b*1i)))^(1/2) - 16*b^2*(a + b*tan
(c + d*x))^(1/2) + (16*a*b^2*(a + b*tan(c + d*x))^(1/2))/(a - b*1i))*(-1/(4
*(a*d^2 - b*d^2*1i)))^(1/2) + 2*atanh((32*b^2*((b*1i)/(4*a^2*d^2 + 4*b^2*d^
2) - a/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2))/((b^4*d^2
*64i)/(4*a^2*d^3 + 4*b^2*d^3) - (64*a*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3)) + (
a*b^3*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(a
 + b*tan(c + d*x))^(1/2)*128i)/((b^6*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) + (a
^2*b^4*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) - (256*a^3*b^3*d^2)/(4*a^2*d^3 + 4
*b^2*d^3) - (256*a*b^5*d^2)/(4*a^2*d^3 + 4*b^2*d^3)) - (128*a^2*b^2*((b*1i)
/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(a + b*tan(c +
d*x))^(1/2))/((b^6*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) + (a^2*b^4*d^2*256i)/(
4*a^2*d^3 + 4*b^2*d^3) - (256*a^3*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3) - (256*a
*b^5*d^2)/(4*a^2*d^3 + 4*b^2*d^3)))*(-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(
1/2)
```

3.534 $\int \frac{\cot(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal. Leaf size=116

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}\,d}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}\,d}$$

```
[Out] -2*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)+arctanh((a+b*tan(d*x+c
))^(1/2)/(a-I*b)^(1/2))/d/(a-I*b)^(1/2)+arctanh((a+b*tan(d*x+c))^(1/2)/(a+I
*b)^(1/2))/d/(a+I*b)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3655, 3620, 3618, 65, 214, 3715}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) + ArcTanh[Sqrt[a
 + b*Tan[c + d*x]]/Sqrt[a - I*b]]/(Sqrt[a - I*b]*d) + ArcTanh[Sqrt[a + b*Ta
n[c + d*x]]/Sqrt[a + I*b]]/(Sqrt[a + I*b]*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
```

```
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)/((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[(a + b*Tan[e + f*x])^m*
(c - d*Tan[e + f*x]), x], x] + Dist[d^2/(c^2 + d^2), Int[(a + b*Tan[e + f*x
])^m*((1 + Tan[e + f*x]^2)/(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx = -\int \frac{\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx + \int \frac{\cot(c+dx)\left(1+\tan^2(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}\,dx$$

$$= -\left(\frac{1}{2}i\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx\right) + \frac{1}{2}i\int \frac{1+i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx + \frac{\text{Subst}}{}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a-ibx}}\,dx, x, i\tan(c+dx)\right)}{2d} - \frac{\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+ibx}}\right.}{2}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d} + \frac{i\text{Subst}\left(\int \frac{1}{-1+\frac{ia}{b}-\frac{ix^2}{b}}\,dx, x, \sqrt{a+b\tan}\right.}{bd}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}\,d} + \frac{\tan}{}$$

Mathematica [A]
time = 0.16, size = 111, normalized size = 0.96

$$\frac{-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] ((-2*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/Sqrt[a] + ArcTanh[Sqrt[a +
b*Tan[c + d*x]]/Sqrt[a - I*b]]/Sqrt[a - I*b] + ArcTanh[Sqrt[a + b*Tan[c + d
*x]]/Sqrt[a + I*b]]/Sqrt[a + I*b])/d
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 1.03, size = 20194, normalized size = 174.09

method	result	size
default	Expression too large to display	20194

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)/sqrt(b*tan(d*x + c) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2186 vs. 2(90) = 180.
time = 1.43, size = 4447, normalized size = 38.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(2)*(a^3 + a*b^2)*d^5*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b
^2)*d^4)) - a^2 - b^2)/b^2)*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*(1/((a^
2 + b^2)*d^4))^(3/4)*arctan(-((a^4 + 2*a^2*b^2 + b^4)*d^4*sqrt(b^2/((a^4 +
2*a^2*b^2 + b^4)*d^4))*sqrt(1/((a^2 + b^2)*d^4)) + (a^3 + a*b^2)*d^2*sqrt(b
^2/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - sqrt(2)*((a^5 + 2*a^3*b^2 + a*b^4)*d^7*
sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(1/((a^2 + b^2)*d^4)) + (a^4 +
2*a^2*b^2 + b^4)*d^5*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4)))*sqrt(((a^2 +
b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) + sqrt(2)*((a^2 + b^2)*d^3*
sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) + a*d*cos(d*x + c))*sqrt((a*cos(d*x
+ c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2
+ b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(1/4) + a*cos(d*x + c)
 + b*sin(d*x + c))/cos(d*x + c))*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^
2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(3/4) + sqrt(2)*((a^5 + 2*
a^3*b^2 + a*b^4)*d^7*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(1/((a^2 +
 b^2)*d^4)) + (a^4 + 2*a^2*b^2 + b^4)*d^5*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)
*d^4)))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-((a^3 +
a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4)
)^(3/4))/b^2) + 4*sqrt(2)*(a^3 + a*b^2)*d^5*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1
/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4
))*(1/((a^2 + b^2)*d^4))^(3/4)*arctan(((a^4 + 2*a^2*b^2 + b^4)*d^4*sqrt(b^2
/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(1/((a^2 + b^2)*d^4)) + (a^3 + a*b^2)*d
^2*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + sqrt(2)*((a^5 + 2*a^3*b^2 + a*
b^4)*d^7*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(1/((a^2 + b^2)*d^4))
+ (a^4 + 2*a^2*b^2 + b^4)*d^5*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4)))*sqrt
(((a^2 + b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) - sqrt(2)*((a^2 +
b^2)*d^3*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) + a*d*cos(d*x + c))*sqrt((a
```

```
*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-((a^3 + a*b^2)*d^2*sqrt
(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(1/4) + a*cos
(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/(
(a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(3/4) - sqrt(2)*(
(a^5 + 2*a^3*b^2 + a*b^4)*d^7*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(
1/((a^2 + b^2)*d^4)) + (a^4 + 2*a^2*b^2 + b^4)*d^5*sqrt(b^2/((a^4 + 2*a^2*b
^2 + b^4)*d^4)))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(
-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 +
b^2)*d^4))^(3/4))/b^2) + sqrt(2)*(a^2*d^3*sqrt(1/((a^2 + b^2)*d^4)) + a*d)*
sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a
^2 + b^2)*d^4))^(1/4)*log(((a^2 + b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4))*cos(d*
x + c) + sqrt(2)*((a^2 + b^2)*d^3*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) +
a*d*cos(d*x + c))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt
(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 +
 b^2)*d^4))^(1/4) + a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c)) - sqrt(2
)*(a^2*d^3*sqrt(1/((a^2 + b^2)*d^4)) + a*d)*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1
/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(1/4)*log(((a^2
 + b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) - sqrt(2)*((a^2 + b^2)*d
^3*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) + a*d*cos(d*x + c))*sqrt((a*cos(d
*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a
^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(1/4) + a*cos(d*x +
 c) + b*sin(d*x + c))/cos(d*x + c)) + 2*sqrt(a)*log(-(8*a*b*cos(d*x + c)*si
n(d*x + c) + (8*a^2 - b^2)*cos(d*x + c)^2 + b^2 - 4*(2*a*cos(d*x + c)^2 + b
*cos(d*x + c)*sin(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/
cos(d*x + c)))/(cos(d*x + c)^2 - 1)))/(a*d), 1/4*(4*sqrt(2)*(a^3 + a*b^2)*d
^5*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*sqr
t(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*(1/((a^2 + b^2)*d^4))^(3/4)*arctan(-((
a^4 + 2*a^2*b^2 + b^4)*d^4*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(1/(
(a^2 + b^2)*d^4)) + (a^3 + a*b^2)*d^2*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4
)) - sqrt(2)*((a^5 + 2*a^3*b^2 + a*b^4)*d^7*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^
4)*d^4))*sqrt(1/((a^2 + b^2)*d^4)) + (a^4 + 2*a^2*b^2 + b^4)*d^5*sqrt(b^2/(
(a^4 + 2*a^2*b^2 + b^4)*d^4)))*sqrt(((a^2 + b^2)*d^2*sqrt(1/((a^2 + b^2)*d^
4))*cos(d*x + c) + sqrt(2)*((a^2 + b^2)*d^3*sqrt(1/((a^2 + b^2)*d^4))*cos(d
*x + c) + a*d*cos(d*x + c))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x
+ c))*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*
(1/((a^2 + b^2)*d^4))^(1/4) + a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c)
)*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/(
(a^2 + b^2)*d^4))^(3/4) + sqrt(2)*((a^5 + 2*a^3*b^2 + a*b^4)*d^7*sqrt(b^2/(
(a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(1/((a^2 + b^2)*d^4)) + (a^4 + 2*a^2*b^2
+ b^4)*d^5*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4)))*sqrt((a*cos(d*x + c) +
b*sin(d*x + c))/cos(d*x + c))*sqrt(-((a^3 + a*b...
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)/sqrt(a + b*tan(c + d*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 4.64, size = 2028, normalized size = 17.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + b*tan(c + d*x))^(1/2),x)
```

```
[Out] - atan(((((((((1/(a*d^2 - b*d^2*1i))^(1/2)*((32*(16*b^10*d^2 + 12*a^2*b^8*
d^2))/d^3 - (16*(1/(a*d^2 - b*d^2*1i))^(1/2)*(16*b^10*d^4 + 24*a^2*b^8*d^4)
*(a + b*tan(c + d*x))^(1/2))/d^4))/2 + (576*a*b^8*(a + b*tan(c + d*x))^(1/2
))/d^2)*(1/(a*d^2 - b*d^2*1i))^(1/2))/2 - (96*a*b^8)/d^3)*(1/(a*d^2 - b*d^2
*1i))^(1/2))/2 - (96*b^8*(a + b*tan(c + d*x))^(1/2))/d^4)*(1/(a*d^2 - b*d^2
*1i))^(1/2)*1i)/2 - (((((((1/(a*d^2 - b*d^2*1i))^(1/2)*((32*(16*b^10*d^2 +
 12*a^2*b^8*d^2))/d^3 + (16*(1/(a*d^2 - b*d^2*1i))^(1/2)*(16*b^10*d^4 + 24*
a^2*b^8*d^4)*(a + b*tan(c + d*x))^(1/2))/d^4))/2 - (576*a*b^8*(a + b*tan(c
+ d*x))^(1/2))/d^2)*(1/(a*d^2 - b*d^2*1i))^(1/2))/2 - (96*a*b^8)/d^3)*(1/(a
*d^2 - b*d^2*1i))^(1/2))/2 + (96*b^8*(a + b*tan(c + d*x))^(1/2))/d^4)*(1/(a
*d^2 - b*d^2*1i))^(1/2)*1i)/2)/((((((((1/(a*d^2 - b*d^2*1i))^(1/2)*((32*(1
6*b^10*d^2 + 12*a^2*b^8*d^2))/d^3 - (16*(1/(a*d^2 - b*d^2*1i))^(1/2)*(16*b^
10*d^4 + 24*a^2*b^8*d^4)*(a + b*tan(c + d*x))^(1/2))/d^4))/2 + (576*a*b^8*(
a + b*tan(c + d*x))^(1/2))/d^2)*(1/(a*d^2 - b*d^2*1i))^(1/2))/2 - (96*a*b^8
```

)/d^3)*(1/(a*d^2 - b*d^2*1i))^(1/2))/2 - (96*b^8*(a + b*tan(c + d*x))^(1/2)
)/d^4)*(1/(a*d^2 - b*d^2*1i))^(1/2))/2 + ((((((((1/(a*d^2 - b*d^2*1i))^(1/2
)*((32*(16*b^10*d^2 + 12*a^2*b^8*d^2))/d^3 + (16*(1/(a*d^2 - b*d^2*1i))^(1/
2)*(16*b^10*d^4 + 24*a^2*b^8*d^4)*(a + b*tan(c + d*x))^(1/2))/d^4))/2 - (57
6*a*b^8*(a + b*tan(c + d*x))^(1/2))/d^2)*(1/(a*d^2 - b*d^2*1i))^(1/2))/2 -
(96*a*b^8)/d^3)*(1/(a*d^2 - b*d^2*1i))^(1/2))/2 + (96*b^8*(a + b*tan(c + d*
x))^(1/2))/d^4)*(1/(a*d^2 - b*d^2*1i))^(1/2))/2))*(1/(a*d^2 - b*d^2*1i))^(1
/2)*1i - atan(((((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(((a - b*1i)/(4*
a^2*d^2 + 4*b^2*d^2))^(1/2)*(((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*((3
2*(16*b^10*d^2 + 12*a^2*b^8*d^2))/d^3 - (32*((a - b*1i)/(4*a^2*d^2 + 4*b^2*
d^2))^(1/2)*(16*b^10*d^4 + 24*a^2*b^8*d^4)*(a + b*tan(c + d*x))^(1/2))/d^4)
 + (576*a*b^8*(a + b*tan(c + d*x))^(1/2))/d^2) - (96*a*b^8)/d^3) - (96*b^8*
(a + b*tan(c + d*x))^(1/2))/d^4)*((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)
*1i - (((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(((a - b*1i)/(4*a^2*d^2 +
 4*b^2*d^2))^(1/2)*(((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*((32*(16*b^1
0*d^2 + 12*a^2*b^8*d^2))/d^3 + (32*((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/
2)*(16*b^10*d^4 + 24*a^2*b^8*d^4)*(a + b*tan(c + d*x))^(1/2))/d^4) - (576*a
b^8(a + b*tan(c + d*x))^(1/2))/d^2) - (96*a*b^8)/d^3) + (96*b^8*(a + b*ta
n(c + d*x))^(1/2))/d^4)*((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*1i)/((((
a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^
2))^(1/2)*(((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*((32*(16*b^10*d^2 + 1
2*a^2*b^8*d^2))/d^3 - (32*((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(16*b^
10*d^4 + 24*a^2*b^8*d^4)*(a + b*tan(c + d*x))^(1/2))/d^4) + (576*a*b^8*(a +
 b*tan(c + d*x))^(1/2))/d^2) - (96*a*b^8)/d^3) - (96*b^8*(a + b*tan(c + d*x
))^(1/2))/d^4)*((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2) + (((a - b*1i)/(4
*a^2*d^2 + 4*b^2*d^2))^(1/2)*(((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*((
(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*((32*(16*b^10*d^2 + 12*a^2*b^8*d^
2))/d^3 + (32*((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(16*b^10*d^4 + 24*
a^2*b^8*d^4)*(a + b*tan(c + d*x))^(1/2))/d^4) - (576*a*b^8*(a + b*tan(c + d
*x))^(1/2))/d^2) - (96*a*b^8)/d^3) + (96*b^8*(a + b*tan(c + d*x))^(1/2))/d^
4)*((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)))*((a - b*1i)/(4*a^2*d^2 + 4*
b^2*d^2))^(1/2)*2i - (2*atanh((576*b^8*(a + b*tan(c + d*x))^(1/2))/(a^(1/2)
*(576*b^8 + (1024*b^10)/a^2)) + (1024*b^10*(a + b*tan(c + d*x))^(1/2))/(a^(
5/2)*(576*b^8 + (1024*b^10)/a^2))))/(a^(1/2)*d)

3.535 $\int \frac{\cot^2(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal. Leaf size=461

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a^2+b^2}\sqrt{a-\sqrt{a^2+b^2}}\,d}+\frac{b\tanh^{-1}\left(\sqrt{}\right.}{\sqrt{}}$$

```
[Out] b*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-1/2*b*arctanh(((a+(a^2+
b^2)^(1/2))^(1/2)-2^(1/2)*(a+b*tan(d*x+c))^(1/2))/(a-(a^2+b^2)^(1/2))^(1/2)
)/d*2^(1/2)/(a^2+b^2)^(1/2)/(a-(a^2+b^2)^(1/2))^(1/2)+1/2*b*arctanh(((a+(a^
2+b^2)^(1/2))^(1/2)+2^(1/2)*(a+b*tan(d*x+c))^(1/2))/(a-(a^2+b^2)^(1/2))^(1/
2))/d*2^(1/2)/(a^2+b^2)^(1/2)/(a-(a^2+b^2)^(1/2))^(1/2)+1/4*b*ln(a+(a^2+b^2
)^(1/2)-2^(1/2)*(a+(a^2+b^2)^(1/2))^(1/2)*(a+b*tan(d*x+c))^(1/2)+b*tan(d*x+
c))/d*2^(1/2)/(a^2+b^2)^(1/2)/(a+(a^2+b^2)^(1/2))^(1/2)-1/4*b*ln(a+(a^2+b^2
)^(1/2)+2^(1/2)*(a+(a^2+b^2)^(1/2))^(1/2)*(a+b*tan(d*x+c))^(1/2)+b*tan(d*x+
c))/d*2^(1/2)/(a^2+b^2)^(1/2)/(a+(a^2+b^2)^(1/2))^(1/2)-cot(d*x+c)*(a+b*tan
(d*x+c))^(1/2)/a/d
```

Rubi [A]

time = 0.50, antiderivative size = 461, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3650, 3734, 12, 3566, 722, 1108, 648, 632, 212, 642, 3715, 65, 214}

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{b\log\left(-\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\sqrt{a+b\tan(c+dx)}+\sqrt{a^2+b^2}+a+b\tan(c+dx)\right)}{2\sqrt{2}d\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+a}}-\frac{b\log\left(\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\sqrt{a+b\tan(c+dx)}+\sqrt{a^2+b^2}+a+b\tan(c+dx)\right)}{2\sqrt{2}d\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+a}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+b^2}+a}-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}d\sqrt{a^2+b^2}\sqrt{a-\sqrt{a^2+b^2}}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+b^2}+a}+\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}d\sqrt{a^2+b^2}\sqrt{a-\sqrt{a^2+b^2}}}-\frac{\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] (b*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - (b*ArcTanh[(Sqr
t[a + Sqrt[a^2 + b^2]] - Sqrt[2]*Sqrt[a + b*Tan[c + d*x]])/Sqrt[a - Sqrt[a^
2 + b^2]]])/(Sqrt[2]*Sqrt[a^2 + b^2]*Sqrt[a - Sqrt[a^2 + b^2]]*d) + (b*ArcT
anh[(Sqrt[a + Sqrt[a^2 + b^2]] + Sqrt[2]*Sqrt[a + b*Tan[c + d*x]])/Sqrt[a -
 Sqrt[a^2 + b^2]]])/(Sqrt[2]*Sqrt[a^2 + b^2]*Sqrt[a - Sqrt[a^2 + b^2]]*d) +
 (b*Log[a + Sqrt[a^2 + b^2] + b*Tan[c + d*x] - Sqrt[2]*Sqrt[a + Sqrt[a^2 +
b^2]]*Sqrt[a + b*Tan[c + d*x]]])/(2*Sqrt[2]*Sqrt[a^2 + b^2]*Sqrt[a + Sqrt[a
^2 + b^2]]*d) - (b*Log[a + Sqrt[a^2 + b^2] + b*Tan[c + d*x] + Sqrt[2]*Sqrt[
a + Sqrt[a^2 + b^2]]*Sqrt[a + b*Tan[c + d*x]]])/(2*Sqrt[2]*Sqrt[a^2 + b^2]*
Sqrt[a + Sqrt[a^2 + b^2]]*d) - (Cot[c + d*x]*Sqrt[a + b*Tan[c + d*x]])/(a*d
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 722

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx = -\frac{\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad} - \frac{\int \frac{\cot(c+dx)\left(\frac{b}{2}+a\tan(c+dx)+\frac{1}{2}b\tan^2(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}\,dx}{a}$$

$$= -\frac{\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad} - \frac{\int \frac{a}{\sqrt{a+b\tan(c+dx)}}\,dx}{a} - \frac{b\int \frac{\cot(c+d}{\sqrt{a+}}$$

$$= -\frac{\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad} - \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \tan(c+dx)\right)}{2ad}$$

$$= -\frac{\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tan(c+}\right.}{ad}$$

$$= \frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad} - \frac{(2b)\text{S}}{}$$

$$= \frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad} - \frac{b\text{Sub}}{}$$

$$= \frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad} - \frac{b\text{Sub}}{}$$

$$= \frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{b\log\left(a+\sqrt{a^2+b^2}+b\tan(c+dx)-\right.}{2\sqrt{2}\sqrt{a^2+b^2}}$$

$$= \frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+}}{\sqrt{a-\sqrt{a^2+b^2}}}\right.}{\sqrt{2}\sqrt{a^2+b^2}\sqrt{a-\sqrt{a^2+}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.84, size = 142, normalized size = 0.31

$$\frac{\frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}} - \frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}} - \frac{\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{a}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] ((b*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/a^(3/2) + (I*ArcTanh[Sqrt[a
+ b*Tan[c + d*x]]/Sqrt[a - I*b]])/Sqrt[a - I*b] - (I*ArcTanh[Sqrt[a + b*Tan
[c + d*x]]/Sqrt[a + I*b]])/Sqrt[a + I*b] - (Cot[c + d*x]*Sqrt[a + b*Tan[c +
 d*x]])/a)/d
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 1.06, size = 39033, normalized size = 84.67

method	result	size
default	Expression too large to display	39033

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^2/sqrt(b*tan(d*x + c) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1988 vs. 2(376) = 752.
time = 2.35, size = 4050, normalized size = 8.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(2)*((a^4 + a^2*b^2)*d^5*cos(d*x + c)^2 - (a^4 + a^2*b^2)*d^5)*
sqrt(((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) + a^2 + b^2)/b^2)*sqrt(b^
2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*(1/((a^2 + b^2)*d^4))^(3/4)*arctan((sqrt(2
)*(a^4 + 2*a^2*b^2 + b^4)*d^7*sqrt((sqrt(2)*b^3*d*sqrt((a*cos(d*x + c) + b*
sin(d*x + c))/cos(d*x + c))*sqrt(((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4
)) + a^2 + b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(1/4)*cos(d*x + c) + (a^2*b^2 +
b^4)*d^2*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) + a*b^2*cos(d*x + c) + b^3*
```

sin(d*x + c))/cos(d*x + c))*sqrt(((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4
)) + a^2 + b^2)/b^2)*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*(1/((a^2 + b^2
)*d^4))^(5/4) - sqrt(2)*(a^4*b + 2*a^2*b^3 + b^5)*d^7*sqrt((a*cos(d*x + c)
+ b*sin(d*x + c))/cos(d*x + c))*sqrt(((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)
*d^4)) + a^2 + b^2)/b^2)*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*(1/((a^2 +
b^2)*d^4))^(5/4) - (a^4 + 2*a^2*b^2 + b^4)*d^4*sqrt(b^2/((a^4 + 2*a^2*b^2
+ b^4)*d^4))*sqrt(1/((a^2 + b^2)*d^4)) - (a^3 + a*b^2)*d^2*sqrt(b^2/((a^4 +
2*a^2*b^2 + b^4)*d^4)))/b^2) + 4*sqrt(2)*((a^4 + a^2*b^2)*d^5*cos(d*x + c)
^2 - (a^4 + a^2*b^2)*d^5)*sqrt(((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4))
+ a^2 + b^2)/b^2)*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*(1/((a^2 + b^2)*
d^4))^(3/4)*arctan((sqrt(2)*(a^4 + 2*a^2*b^2 + b^4)*d^7*sqrt(-(sqrt(2)*b^3*
d*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(((a^3 + a*b^2)*
d^2*sqrt(1/((a^2 + b^2)*d^4)) + a^2 + b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(1/4)
*cos(d*x + c) - (a^2*b^2 + b^4)*d^2*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c)
- a*b^2*cos(d*x + c) - b^3*sin(d*x + c))/cos(d*x + c))*sqrt(((a^3 + a*b^2)*
d^2*sqrt(1/((a^2 + b^2)*d^4)) + a^2 + b^2)/b^2)*sqrt(b^2/((a^4 + 2*a^2*b^2
+ b^4)*d^4))*(1/((a^2 + b^2)*d^4))^(5/4) - sqrt(2)*(a^4*b + 2*a^2*b^3 + b^5
)*d^7*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(((a^3 + a*b
^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) + a^2 + b^2)/b^2)*sqrt(b^2/((a^4 + 2*a^2*
b^2 + b^4)*d^4))*(1/((a^2 + b^2)*d^4))^(5/4) + (a^4 + 2*a^2*b^2 + b^4)*d^4*
sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(1/((a^2 + b^2)*d^4)) + (a^3 +
a*b^2)*d^2*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/b^2) + 4*a*sqrt((a*cos(
d*x + c) + b*sin(d*x + c))/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - sqrt(2
)*(a^2*d*cos(d*x + c)^2 - a^2*d - (a^3*d^3*cos(d*x + c)^2 - a^3*d^3)*sqrt(1
/((a^2 + b^2)*d^4)))*sqrt(((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) + a^
2 + b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(1/4)*log((sqrt(2)*b^3*d*sqrt((a*cos(d*
x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(((a^3 + a*b^2)*d^2*sqrt(1/((a^2
+ b^2)*d^4)) + a^2 + b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(1/4)*cos(d*x + c) +
(a^2*b^2 + b^4)*d^2*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) + a*b^2*cos(d*x
+ c) + b^3*sin(d*x + c))/cos(d*x + c)) + sqrt(2)*(a^2*d*cos(d*x + c)^2 - a^
2*d - (a^3*d^3*cos(d*x + c)^2 - a^3*d^3)*sqrt(1/((a^2 + b^2)*d^4)))*sqrt(((
a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) + a^2 + b^2)/b^2)*(1/((a^2 + b^2
)*d^4))^(1/4)*log(-(sqrt(2)*b^3*d*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/co
s(d*x + c))*sqrt(((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) + a^2 + b^2)/
b^2)*(1/((a^2 + b^2)*d^4))^(1/4)*cos(d*x + c) - (a^2*b^2 + b^4)*d^2*sqrt(1/
((a^2 + b^2)*d^4))*cos(d*x + c) - a*b^2*cos(d*x + c) - b^3*sin(d*x + c))/co
s(d*x + c)) + (b*cos(d*x + c)^2 - b)*sqrt(a)*log(-(8*a*b*cos(d*x + c)*sin(d
*x + c) + (8*a^2 - b^2)*cos(d*x + c)^2 + b^2 + 4*(2*a*cos(d*x + c)^2 + b*co
s(d*x + c)*sin(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos
(d*x + c)))/(cos(d*x + c)^2 - 1)))/(a^2*d*cos(d*x + c)^2 - a^2*d), 1/4*(4*s
qrt(2)*((a^4 + a^2*b^2)*d^5*cos(d*x + c)^2 - (a^4 + a^2*b^2)*d^5)*sqrt(((a^
3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) + a^2 + b^2)/b^2)*sqrt(b^2/((a^4 +
2*a^2*b^2 + b^4)*d^4))*(1/((a^2 + b^2)*d^4))^(3/4)*arctan((sqrt(2)*(a^4 +
2*a^2*b^2 + b^4)*d^7*sqrt((sqrt(2)*b^3*d*sqrt((a*cos(d*x + c) + b*sin(d*x +
c))/cos(d*x + c))*sqrt(((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) + a^2

```
+ b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(1/4)*cos(d*x + c) + (a^2*b^2 + b^4)*d^2*
sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) + a*b^2*cos(d*x + c) + b^3*sin(d*x +
 c))/cos(d*x + c))*sqrt(((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) + a^2
+ b^2)/b^2)*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*(1/((a^2 + b^2)*d^4))^(
5/4) - sqrt(2)*(a^4*b + 2*a^2*b^3 + b^5)*d^7*sqrt((a*cos(d*x + c) + b*sin(d
*x + c))/cos(d*x + c))*sqrt(((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) +
a^2 + b^2)/b^2)*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*(1/((a^2 + b^2)*d^4
))^(5/4) - (a^4 + 2*a^2*b^2 + b^4)*d^4*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^
4))*sqrt(1/((a^2 + b^2)*d^4)) - (a^3 + a*b^2)*d^2*sqrt(b^2/((a^4 + 2*a^2*b^
2 + b^4)*d^4)))/b^2) + 4*sqrt(2)*((a^4 + a^2*b^2)*d^5*cos(d*x + c)^2 - (a^4
 + a^2*b^2)*d^5)*sqrt(((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) + a^2 +
b^2)/b^2)*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*(1/((a^2 + b^2)*d^4))^(3/
4)*arctan((sqrt(2)*(a^4 + 2*a^2*b^2 + b^4)*d^7*sqrt(-(sqrt(2)*b^3*d*sqrt((a
*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(((a^3 + a*b^2)*d^2*sqrt(
1/((a^2 + b^2)*d^4)) + a^2 + b^2)/b^2)*(1/((a^2...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+b*tan(d*x+c))**(1/2),x)
[Out] Integral(cot(c + d*x)**2/sqrt(a + b*tan(c + d*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [B]
time = 7.26, size = 2145, normalized size = 4.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + b*tan(c + d*x))^(1/2),x)
[Out] atan((((-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*((((16*(16*a*b^11*d^4 +
8*a^3*b^9*d^4))/(a^2*d^5) - (16*(-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)
```

*(32*a^2*b^10*d^4 + 48*a^4*b^8*d^4)*(a + b*tan(c + d*x))^(1/2))/(a^2*d^4))*(-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2) + (16*(4*a*b^10*d^2 - 20*a^3*b^8*d^2)*(a + b*tan(c + d*x))^(1/2))/(a^2*d^4))*(-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2) + (16*(2*b^11*d^2 + 8*a^2*b^9*d^2))/(a^2*d^5)) + (16*(b^10 - 2*a^2*b^8)*(a + b*tan(c + d*x))^(1/2))/(a^2*d^4))*(-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*1i - ((-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*((((16*(16*a*b^11*d^4 + 8*a^3*b^9*d^4))/(a^2*d^5) + (16*(-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(32*a^2*b^10*d^4 + 48*a^4*b^8*d^4)*(a + b*tan(c + d*x))^(1/2))/(a^2*d^4))*(-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2) - (16*(4*a*b^10*d^2 - 20*a^3*b^8*d^2)*(a + b*tan(c + d*x))^(1/2))/(a^2*d^4))*(-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2) + (16*(2*b^11*d^2 + 8*a^2*b^9*d^2))/(a^2*d^5)) - (16*(b^10 - 2*a^2*b^8)*(a + b*tan(c + d*x))^(1/2))/(a^2*d^4))*(-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*1i)/(((-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*((((16*(16*a*b^11*d^4 + 8*a^3*b^9*d^4))/(a^2*d^5) - (16*(-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(32*a^2*b^10*d^4 + 48*a^4*b^8*d^4)*(a + b*tan(c + d*x))^(1/2))/(a^2*d^4))*(-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2) + (16*(4*a*b^10*d^2 - 20*a^3*b^8*d^2)*(a + b*tan(c + d*x))^(1/2))/(a^2*d^4))*(-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2) + (16*(2*b^11*d^2 + 8*a^2*b^9*d^2))/(a^2*d^5)) + (16*(b^10 - 2*a^2*b^8)*(a + b*tan(c + d*x))^(1/2))/(a^2*d^4))*(-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2) + ((-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*((((16*(16*a*b^11*d^4 + 8*a^3*b^9*d^4))/(a^2*d^5) + (16*(-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(32*a^2*b^10*d^4 + 48*a^4*b^8*d^4)*(a + b*tan(c + d*x))^(1/2))/(a^2*d^4))*(-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2) - (16*(4*a*b^10*d^2 - 20*a^3*b^8*d^2)*(a + b*tan(c + d*x))^(1/2))/(a^2*d^4))*(-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2) + (16*(2*b^11*d^2 + 8*a^2*b^9*d^2))/(a^2*d^5)) - (16*(b^10 - 2*a^2*b^8)*(a + b*tan(c + d*x))^(1/2))/(a^2*d^4))*(-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2) + (32*b^9)/(a*d^5)))*(-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*2i + (log(-((-1/(d^2*(a - b*1i)))^(1/2)*(((-1/(d^2*(a - b*1i)))^(1/2)*(((-1/(d^2*(a - b*1i)))^(1/2)*(((-1/(d^2*(a - b*1i)))^(1/2)*(128*b^8*(-1/(d^2*(a - b*1i)))^(1/2)*(3*a^2 + 2*b^2)*(a + b*tan(c + d*x))^(1/2) + (128*b^9*(a^2 + 2*b^2))/(a*d)))/2 + (64*b^8*(5*a^2 - b^2)*(a + b*tan(c + d*x))^(1/2))/(a*d^2)))/2 + (32*b^9*(4*a^2 + b^2))/(a^2*d^3)))/2 - (16*(b^10 - 2*a^2*b^8)*(a + b*tan(c + d*x))^(1/2))/(a^2*d^4)))/2 - (16*b^9)/(a*d^5))*(-1/(a*d^2 - b*d^2*1i))^(1/2))/2 - log(((-1/(d^2*(a - b*1i)))^(1/2)*(((-1/(d^2*(a - b*1i)))^(1/2)*(((-1/(d^2*(a - b*1i)))^(1/2)*(((-1/(d^2*(a - b*1i)))^(1/2)*(128*b^8*(-1/(d^2*(a - b*1i)))^(1/2)*(3*a^2 + 2*b^2)*(a + b*tan(c + d*x))^(1/2) - (128*b^9*(a^2 + 2*b^2))/(a*d)))/2 + (64*b^8*(5*a^2 - b^2)*(a + b*tan(c + d*x))^(1/2))/(a*d^2)))/2 - (32*b^9*(4*a^2 + b^2))/(a^2*d^3)))/2 - (16*(b^10 - 2*a^2*b^8)*(a + b*tan(c + d*x))^(1/2))/(a^2*d^4)))/2 - (16*b^9)/(a*d^5))*(-1/(4*(a*d^2 - b*d^2*1i)))^(1/2) + (b*(a + b*tan(c + d*x))^(1/2))/(a*(a*d - d*(a + b*tan(c + d*x)))) - (b*atan((b^11*(a + b*tan(c + d*x))^(1/2)*64i)/((a^3)^(1/2)*(32*a*b^9 + (64*b^11)/a + (32*b^13)/a^3 + (32*b^15)/a^5)) + (b^13*(a + b*tan(c + d*x))^(1/2)*32i)/((a^3)^(1/2)*(64*a*b^11 + 32*a^3*b^9 + (32*b^13)/a + (32*b^15)/a^3)) + (b^15*(a + b*tan(c + d*x))^(1/2)*32i)/((a^3)^(1/2)*(3

```
2*a*b^13 + 64*a^3*b^11 + 32*a^5*b^9 + (32*b^15)/a)) + (b^9*(a + b*tan(c + d
*x))^(1/2)*32i)/((a^3)^(1/2)*((32*b^9)/a + (64*b^11)/a^3 + (32*b^13)/a^5 +
(32*b^15)/a^7)))*1i)/(d*(a^3)^(1/2))
```

3.536 $\int \frac{\cot^3(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal. Leaf size=194

$$\frac{(8a^2-3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{5/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}\,d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}\,d}$$

```
[Out] 1/4*(8*a^2-3*b^2)*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d-arctanh
((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d/(a-I*b)^(1/2)-arctanh((a+b*tan(d*x
+c))^(1/2)/(a+I*b)^(1/2))/d/(a+I*b)^(1/2)+3/4*b*cot(d*x+c)*(a+b*tan(d*x+c))
^(1/2)/a^2/d-1/2*cot(d*x+c)^2*(a+b*tan(d*x+c))^(1/2)/a/d
```

Rubi [A]

time = 0.39, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3650, 3730, 3735, 12, 3620, 3618, 65, 214, 3715}

$$\frac{3b\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{4a^2d}+\frac{(8a^2-3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{5/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}-\frac{\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] ((8*a^2 - 3*b^2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/(4*a^(5/2)*d) -
 ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]]/(Sqrt[a - I*b]*d) - ArcTan
h[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]]/(Sqrt[a + I*b]*d) + (3*b*Cot[c +
d*x]*Sqrt[a + b*Tan[c + d*x]])/(4*a^2*d) - (Cot[c + d*x]^2*Sqrt[a + b*Tan[c
 + d*x]])/(2*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
```

```
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3735

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dis
t[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*Simp[a*(A - C) - (A*b - b*C)*Ta
n[e + f*x], x], x], x] + Dist[(A*b^2 + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e
 + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a,
 b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0] &&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx = -\frac{\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2ad} - \frac{\int \frac{\cot^2(c+dx)\left(\frac{3b}{2}+2a\tan(c+dx)+\frac{3}{2}b\tan^2(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}\,dx}{2a}$$

$$= \frac{3b\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{4a^2d} - \frac{\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2ad} + \frac{\int \text{co}}{}$$

$$= \frac{3b\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{4a^2d} - \frac{\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2ad} + \frac{\int -}{\sqrt{}}$$

$$= \frac{3b\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{4a^2d} - \frac{\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2ad} - \frac{(8}{}$$

$$= \frac{3b\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{4a^2d} - \frac{\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2ad} + \frac{1}{2}i\int$$

$$= \frac{(8a^2-3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{5/2}d} + \frac{3b\cot(c+dx)\sqrt{a+b\tan(c+d}}{4a^2d}$$

$$= \frac{(8a^2-3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{5/2}d} + \frac{3b\cot(c+dx)\sqrt{a+b\tan(c+d}}{4a^2d}$$

$$= \frac{(8a^2-3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{5/2}d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}\,d}$$

Mathematica [A]
time = 3.28, size = 203, normalized size = 1.05

$$\frac{\frac{(8a^2-3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{4a^2\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{\sqrt{a-\sqrt{-b^2}}} - \frac{4a^2\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+\sqrt{-b^2}}}\right)}{\sqrt{a+\sqrt{-b^2}}} + 3b\cot(c+dx)\sqrt{a+b\tan(c+dx)} - 2a\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] (((8*a^2 - 3*b^2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/Sqrt[a] - (4*a
^2*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - Sqrt[-b^2]]])/Sqrt[a - Sqrt[-b
^2]] - (4*a^2*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + Sqrt[-b^2]]])/Sqrt[
a + Sqrt[-b^2]] + 3*b*Cot[c + d*x]*Sqrt[a + b*Tan[c + d*x]] - 2*a*Cot[c + d
*x]^2*Sqrt[a + b*Tan[c + d*x]])/(4*a^2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 1.27, size = 58397, normalized size = 301.02

method	result	size
default	Expression too large to display	58397

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(cot(d*x + c)^3/sqrt(b*tan(d*x + c) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2416 vs. 2(158) = 316.
time = 1.67, size = 4908, normalized size = 25.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
[Out] [-1/16*(16*sqrt(2)*((a^5 + a^3*b^2)*d^5*cos(d*x + c)^2 - (a^5 + a^3*b^2)*d^
5)*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*sqr
t(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*(1/((a^2 + b^2)*d^4))^(3/4)*arctan(-((
a^4 + 2*a^2*b^2 + b^4)*d^4*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(1/(
(a^2 + b^2)*d^4)) + (a^3 + a*b^2)*d^2*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4
)) - sqrt(2)*((a^5 + 2*a^3*b^2 + a*b^4)*d^7*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^
4)*d^4))*sqrt(1/((a^2 + b^2)*d^4)) + (a^4 + 2*a^2*b^2 + b^4)*d^5*sqrt(b^2/(
(a^4 + 2*a^2*b^2 + b^4)*d^4)))*sqrt(((a^2 + b^2)*d^2*sqrt(1/((a^2 + b^2)*d^
4))*cos(d*x + c) + sqrt(2)*((a^2 + b^2)*d^3*sqrt(1/((a^2 + b^2)*d^4))*cos(d
*x + c) + a*d*cos(d*x + c))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x
+ c))*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*
(1/((a^2 + b^2)*d^4))^(1/4) + a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c)
)*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/(
(a^2 + b^2)*d^4))^(3/4) + sqrt(2)*((a^5 + 2*a^3*b^2 + a*b^4)*d^7*sqrt(b^2/(
(a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(1/((a^2 + b^2)*d^4)) + (a^4 + 2*a^2*b^2
```

+ b^4)*d^5*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4)))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(3/4))/b^2) + 16*sqrt(2)*((a^5 + a^3*b^2)*d^5*cos(d*x + c)^2 - (a^5 + a^3*b^2)*d^5)*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*(1/((a^2 + b^2)*d^4))^(3/4)*arctan(((a^4 + 2*a^2*b^2 + b^4)*d^4*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(1/((a^2 + b^2)*d^4)) + (a^3 + a*b^2)*d^2*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + sqrt(2)*((a^5 + 2*a^3*b^2 + a*b^4)*d^7*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(1/((a^2 + b^2)*d^4)) + (a^4 + 2*a^2*b^2 + b^4)*d^5*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4)))*sqrt(((a^2 + b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) - sqrt(2)*((a^2 + b^2)*d^3*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) + a*d*cos(d*x + c))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(1/4) + a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(3/4) - sqrt(2)*((a^5 + 2*a^3*b^2 + a*b^4)*d^7*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(1/((a^2 + b^2)*d^4)) + (a^4 + 2*a^2*b^2 + b^4)*d^5*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4)))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(3/4))/b^2) + 4*sqrt(2)*(a^3*d*cos(d*x + c)^2 - a^3*d + (a^4*d^3*cos(d*x + c)^2 - a^4*d^3)*sqrt(1/((a^2 + b^2)*d^4)))*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(1/4)*log(((a^2 + b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) + sqrt(2)*((a^2 + b^2)*d^3*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) + a*d*cos(d*x + c))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(1/4) + a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c)) - 4*sqrt(2)*(a^3*d*cos(d*x + c)^2 - a^3*d + (a^4*d^3*cos(d*x + c)^2 - a^4*d^3)*sqrt(1/((a^2 + b^2)*d^4)))*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(1/4)*log(((a^2 + b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) - sqrt(2)*((a^2 + b^2)*d^3*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) + a*d*cos(d*x + c))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*(1/((a^2 + b^2)*d^4))^(1/4) + a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c)) + ((8*a^2 - 3*b^2)*cos(d*x + c)^2 - 8*a^2 + 3*b^2)*sqrt(a)*log(-(8*a*b*cos(d*x + c)*sin(d*x + c) + (8*a^2 - b^2)*cos(d*x + c)^2 + b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c)*sin(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c)))/(cos(d*x + c)^2 - 1)) - 4*(2*a^2*cos(d*x + c)^2 - 3*a*b*cos(d*x + c)*sin(d*x + c))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c)))/(a^3*d*cos(d*x + c)^2 - a^3*d), -1/4*(4*sqrt(2)*((a^5 + a^3*b^2)*d^5*cos(d*x + c)^2 - (a^5 + a^3*b^2)*d^5)*sqrt(-((a^3 + a*b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*(1/((a^2 + b^2)*d^4))^(3/4)*arctan(-((a^4 + 2*a^2*b^2 + b^4)*d^4*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(1/((a^2 + b^2)*d^4)) + (a^3 + a

```
*b^2)*d^2*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - sqrt(2)*((a^5 + 2*a^3*b
^2 + a*b^4)*d^7*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(1/((a^2 + b^2)
*d^4)) + (a^4 + 2*a^2*b^2 + b^4)*d^5*sqrt(b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4)
))*sqrt(((a^2 + b^2)*d^2*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) + sqrt(2)*(
(a^2 + b^2)*d^3*sqrt(1/((a^2 + b^2)*d^4))*cos(d*x + c) + a*d*cos(d*x + c))*
sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt(-((a^3 + a*b^2)*d
^2*sqrt(1/((a^2 + b^2)*d^4)) - a^2 - b^2)/b^2)*...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+b*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)**3/sqrt(a + b*tan(c + d*x)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**B**]
time = 0.59, size = 2500, normalized size = 12.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + b*tan(c + d*x))^(1/2),x)
```

```
[Out] atan(((((((((((320*a^4*b^10*d^4 - 192*a^2*b^12*d^4 + 384*a^6*b^8*d^4)/(2*a^4
*d^5) - ((512*a^4*b^10*d^4 + 768*a^6*b^8*d^4)*(1/(a*d^2 - b*d^2*1i))^(1/2)*
(a + b*tan(c + d*x))^(1/2))/(4*a^4*d^4))*(1/(a*d^2 - b*d^2*1i))^(1/2))/2 +
((a + b*tan(c + d*x))^(1/2)*(36*a*b^12*d^2 - 192*a^3*b^10*d^2 + 576*a^5*b^8
*d^2))/(2*a^4*d^4))*(1/(a*d^2 - b*d^2*1i))^(1/2))/2 + (18*a*b^12*d^2 - 96*a
^5*b^8*d^2)/(2*a^4*d^5))*(1/(a*d^2 - b*d^2*1i))^(1/2))/2 - ((a + b*tan(c +
```

d*x))^(1/2)*(9*b^12 - 48*a^2*b^10 + 96*a^4*b^8))/(2*a^4*d^4))*(1/(a*d^2 - b*d^2*1i))^(1/2)*1i - ((((((((320*a^4*b^10*d^4 - 192*a^2*b^12*d^4 + 384*a^6*b^8*d^4)/(2*a^4*d^5) + ((512*a^4*b^10*d^4 + 768*a^6*b^8*d^4)*(1/(a*d^2 - b*d^2*1i))^(1/2)*(a + b*tan(c + d*x))^(1/2))/(4*a^4*d^4))*(1/(a*d^2 - b*d^2*1i))^(1/2))/2 - ((a + b*tan(c + d*x))^(1/2)*(36*a*b^12*d^2 - 192*a^3*b^10*d^2 + 576*a^5*b^8*d^2))/(2*a^4*d^4))*(1/(a*d^2 - b*d^2*1i))^(1/2))/2 + (18*a*b^12*d^2 - 96*a^5*b^8*d^2)/(2*a^4*d^5))*(1/(a*d^2 - b*d^2*1i))^(1/2))/2 + ((a + b*tan(c + d*x))^(1/2)*(9*b^12 - 48*a^2*b^10 + 96*a^4*b^8))/(2*a^4*d^4))*(1/(a*d^2 - b*d^2*1i))^(1/2)*1i)/(((((((((320*a^4*b^10*d^4 - 192*a^2*b^12*d^4 + 384*a^6*b^8*d^4)/(2*a^4*d^5) - ((512*a^4*b^10*d^4 + 768*a^6*b^8*d^4)*(1/(a*d^2 - b*d^2*1i))^(1/2)*(a + b*tan(c + d*x))^(1/2))/(4*a^4*d^4))*(1/(a*d^2 - b*d^2*1i))^(1/2))/2 + ((a + b*tan(c + d*x))^(1/2)*(36*a*b^12*d^2 - 192*a^3*b^10*d^2 + 576*a^5*b^8*d^2))/(2*a^4*d^4))*(1/(a*d^2 - b*d^2*1i))^(1/2))/2 + (18*a*b^12*d^2 - 96*a^5*b^8*d^2)/(2*a^4*d^5))*(1/(a*d^2 - b*d^2*1i))^(1/2))/2 - ((a + b*tan(c + d*x))^(1/2)*(9*b^12 - 48*a^2*b^10 + 96*a^4*b^8))/(2*a^4*d^4))*(1/(a*d^2 - b*d^2*1i))^(1/2) + ((((((((320*a^4*b^10*d^4 - 192*a^2*b^12*d^4 + 384*a^6*b^8*d^4)/(2*a^4*d^5) + ((512*a^4*b^10*d^4 + 768*a^6*b^8*d^4)*(1/(a*d^2 - b*d^2*1i))^(1/2)*(a + b*tan(c + d*x))^(1/2))/(4*a^4*d^4))*(1/(a*d^2 - b*d^2*1i))^(1/2))/2 - ((a + b*tan(c + d*x))^(1/2)*(36*a*b^12*d^2 - 192*a^3*b^10*d^2 + 576*a^5*b^8*d^2))/(2*a^4*d^4))*(1/(a*d^2 - b*d^2*1i))^(1/2))/2 + (18*a*b^12*d^2 - 96*a^5*b^8*d^2)/(2*a^4*d^5))*(1/(a*d^2 - b*d^2*1i))^(1/2))/2 + ((a + b*tan(c + d*x))^(1/2)*(9*b^12 - 48*a^2*b^10 + 96*a^4*b^8))/(2*a^4*d^4))*(1/(a*d^2 - b*d^2*1i))^(1/2) - (9*b^12 - 24*a^2*b^10)/(a^4*d^5)))*(1/(a*d^2 - b*d^2*1i))^(1/2)*1i - atan((((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(((320*a^4*b^10*d^4 - 192*a^2*b^12*d^4 + 384*a^6*b^8*d^4)/(a^4*d^5) + (((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(512*a^4*b^10*d^4 + 768*a^6*b^8*d^4)*(a + b*tan(c + d*x))^(1/2))/(a^4*d^4))*((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2) - ((a + b*tan(c + d*x))^(1/2)*(36*a*b^12*d^2 - 192*a^3*b^10*d^2 + 576*a^5*b^8*d^2))/(a^4*d^4)) + (18*a*b^12*d^2 - 96*a^5*b^8*d^2)/(a^4*d^5)) + ((a + b*tan(c + d*x))^(1/2)*(9*b^12 - 48*a^2*b^10 + 96*a^4*b^8))/(a^4*d^4))*1i - ((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(((320*a^4*b^10*d^4 - 192*a^2*b^12*d^4 + 384*a^6*b^8*d^4)/(a^4*d^5) - (((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(512*a^4*b^10*d^4 + 768*a^6*b^8*d^4)*(a + b*tan(c + d*x))^(1/2))/(a^4*d^4))*((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2) + ((a + b*tan(c + d*x))^(1/2)*(36*a*b^12*d^2 - 192*a^3*b^10*d^2 + 576*a^5*b^8*d^2))/(a^4*d^4)) + (18*a*b^12*d^2 - 96*a^5*b^8*d^2)/(a^4*d^5)) - ((a + b*tan(c + d*x))^(1/2)*(9*b^12 - 48*a^2*b^10 + 96*a^4*b^8))/(a^4*d^4))*1i)/(((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(((320*a^4*b^10*d^4 - 192*a^2*b^12*d^4 + 384*a^6*b^8*d^4)/(a^4*d^5) + (((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(512*a^4*b^10*d^4 + 768*a^6*b^8*d^4)*(a + b*tan(c + d*x))^(1/2))/(a^4*d^4))*((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2) - ((a + b*tan(c + d*x))^(1/2)*(36*a*b^12*d^2 - 192

```
*a^3*b^10*d^2 + 576*a^5*b^8*d^2))/(a^4*d^4)) + (18*a*b^12*d^2 - 96*a^5*b^8*
d^2)/(a^4*d^5)) + ((a + b*tan(c + d*x))^(1/2)*(9*b^12 - 48*a^2*b^10 + 96*a^
4*b^8))/(a^4*d^4)) + ((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(((a - b*1i
)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2
)*(((320*a^4*b^10*d^4 - 192*a^2*b^12*d^4 + 384*a^6*b^8*d^4)/(a^4*d^5) - (((
a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(512*a^4*b^10*d^4 + 768*a^6*b^8*d^
4)*(a + b*tan(c + d*x))^(1/2))/(a^4*d^4))*((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^
2))^(1/2) + ((a + b*tan(c + d*x))^(1/2)*(36*a*b^12*d^2 - 192*a^3*b^10*d^2 +
 576*a^5*b^8*d^2))/(a^4*d^4)) + (18*a*b^12*d^2 - 96*a^5*b^8*d^2)/(a^4*d^5))
 - ((a + b*tan(c + d*x))^(1/2)*(9*b^12 - 48*a^2*b^10 + 96*a^4*b^8))/(a^4*d^
4)) - (9*b^12 - 24*a^2*b^10)/(a^4*d^5)))*((a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2
))^(1/2)*2i - ((5*b^2*(a + b*tan(c + d*x))^(1/2))/(4*a) - (3*b^2*(a + b*tan
(c + d*x))^(3/2))/(4*a^2))/(d*(a + b*tan(c + d*x))^2 + a^2*d - 2*a*d*(a + b
*tan(c + d*x))) - (atan((b^16*(a + b*tan(c + d*x))^(1/2)*1755i)/(4*(a^5)^(1
/2)*(696*a^2*b^12 - 927*b^14 + 344*a^4*b^10 - 576*a^6*b^8 + (1755*b^16)/(4*
a^2) - (2997*b^18)/(32*a^4) + (243*b^20)/(32*a^...
```

3.537 $\int \frac{\tan^5(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=282

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d}-\frac{2a^2\tan^3(c+dx)}{b\left(a^2+b^2\right)d\sqrt{a+b\tan(c+dx)}}+2(1$$

```
[Out] -arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(3/2)/d-arctanh((a+b
*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(3/2)/d+2/5*(16*a^4+6*a^2*b^2-5*b
^4)*(a+b*tan(d*x+c))^(1/2)/b^4/(a^2+b^2)/d-2/5*a*(8*a^2+3*b^2)*(a+b*tan(d*x
+c))^(1/2)*tan(d*x+c)/b^3/(a^2+b^2)/d+2/5*(6*a^2+b^2)*(a+b*tan(d*x+c))^(1/2
)*tan(d*x+c)^2/b^2/(a^2+b^2)/d-2*a^2*tan(d*x+c)^3/b/(a^2+b^2)/d/(a+b*tan(d*
x+c))^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3646, 3728, 3711, 3620, 3618, 65, 214}

$$-\frac{2a^2\tan^3(c+dx)}{bd\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{2\left(6a^2+b^2\right)\tan^2(c+dx)\sqrt{a+b\tan(c+dx)}}{5b^2d\left(a^2+b^2\right)}-\frac{2a\left(8a^2+3b^2\right)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{5b^3d\left(a^2+b^2\right)}+\frac{2\left(16a^4+6a^2b^2-5b^4\right)\sqrt{a+b\tan(c+dx)}}{5b^4d\left(a^2+b^2\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]]/((a - I*b)^(3/2)*d)) - Ar
cTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]]/((a + I*b)^(3/2)*d) - (2*a^2*
Tan[c + d*x]^3)/(b*(a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]]) + (2*(16*a^4 + 6
*a^2*b^2 - 5*b^4)*Sqrt[a + b*Tan[c + d*x]])/(5*b^4*(a^2 + b^2)*d) - (2*a*(8
*a^2 + 3*b^2)*Tan[c + d*x]*Sqrt[a + b*Tan[c + d*x]])/(5*b^3*(a^2 + b^2)*d)
+ (2*(6*a^2 + b^2)*Tan[c + d*x]^2*Sqrt[a + b*Tan[c + d*x]])/(5*b^2*(a^2 + b
^2)*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
```

```
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx = -\frac{2a^2\tan^3(c+dx)}{b\left(a^2+b^2\right)d\sqrt{a+b\tan(c+dx)}} + \frac{2\int \frac{\tan^2(c+dx)\left(3a^2-\frac{1}{2}ab\tan(c+dx)+\frac{1}{2}\left(6a^2+b^2\right)\text{ta}}{\sqrt{a+b\tan(c+dx)}}}{b\left(a^2+b^2\right)}$$

$$= -\frac{2a^2\tan^3(c+dx)}{b\left(a^2+b^2\right)d\sqrt{a+b\tan(c+dx)}} + \frac{2(6a^2+b^2)\tan^2(c+dx)\sqrt{a+b\tan(c+}}{5b^2\left(a^2+b^2\right)d}$$

$$= -\frac{2a^2\tan^3(c+dx)}{b\left(a^2+b^2\right)d\sqrt{a+b\tan(c+dx)}} - \frac{2a(8a^2+3b^2)\tan(c+dx)\sqrt{a+b\tan(c}}{5b^3\left(a^2+b^2\right)d}$$

$$= -\frac{2a^2\tan^3(c+dx)}{b\left(a^2+b^2\right)d\sqrt{a+b\tan(c+dx)}} + \frac{2(16a^4+6a^2b^2-5b^4)\sqrt{a+b\tan(c+d}}{5b^4\left(a^2+b^2\right)d}$$

$$= -\frac{2a^2\tan^3(c+dx)}{b\left(a^2+b^2\right)d\sqrt{a+b\tan(c+dx)}} + \frac{2(16a^4+6a^2b^2-5b^4)\sqrt{a+b\tan(c+d}}{5b^4\left(a^2+b^2\right)d}$$

$$= -\frac{2a^2\tan^3(c+dx)}{b\left(a^2+b^2\right)d\sqrt{a+b\tan(c+dx)}} + \frac{2(16a^4+6a^2b^2-5b^4)\sqrt{a+b\tan(c+d}}{5b^4\left(a^2+b^2\right)d}$$

$$= -\frac{2a^2\tan^3(c+dx)}{b\left(a^2+b^2\right)d\sqrt{a+b\tan(c+dx)}} + \frac{2(16a^4+6a^2b^2-5b^4)\sqrt{a+b\tan(c+d}}{5b^4\left(a^2+b^2\right)d}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d} - \frac{}{b\,(a^2}$$

Mathematica [A]

time = 6.00, size = 213, normalized size = 0.76

$$\frac{-\frac{5\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}} - \frac{5\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}} + \frac{2\left(16a^5+5a^3b^2-6ab^4+2b\left(4a^4+a^2b^2-3b^4\right)\tan(c+dx)-3ab^2\left(a^2+b^2\right)\tan^2(c+dx)+b^2\left(a^2+b^2\right)\sec^2(c+dx)(a+b\tan(c+dx))\right)}{b^4\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + b*Tan[c + d*x])^(3/2),x]
```

[Out] ((-5*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/(a - I*b)^(3/2) - (5*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/(a + I*b)^(3/2) + (2*(16*a^5 + 5*a^3*b^2 - 6*a*b^4 + 2*b*(4*a^4 + a^2*b^2 - 3*b^4)*Tan[c + d*x] - 3*a*b^2*(a^2 + b^2)*Tan[c + d*x]^2 + b^2*(a^2 + b^2)*Sec[c + d*x]^2*(a + b*Tan[c + d*x])))/(b^4*(a^2 + b^2)*Sqrt[a + b*Tan[c + d*x]]))/(5*d)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 877 vs. $2(252) = 504$.

time = 0.17, size = 878, normalized size = 3.11

method	result
derivativedivides	$\frac{\frac{2(a+b\tan(dx+c))^{\frac{5}{2}}}{5}-2a(a+b\tan(dx+c))^{\frac{3}{2}}+6a^2\sqrt{a+b\tan(dx+c)}-2b^2\sqrt{a+b\tan(dx+c)}-\ldots 2b^4 \ldots}{\ldots}$
default	$\frac{\frac{2(a+b\tan(dx+c))^{\frac{5}{2}}}{5}-2a(a+b\tan(dx+c))^{\frac{3}{2}}+6a^2\sqrt{a+b\tan(dx+c)}-2b^2\sqrt{a+b\tan(dx+c)}-\ldots 2b^4 \ldots}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)^5/(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/d/b^4*(1/5*(a+b*tan(d*x+c))^(5/2)-a*(a+b*tan(d*x+c))^(3/2)+3*a^2*(a+b*tan(d*x+c))^(1/2)-b^2*(a+b*tan(d*x+c))^(1/2)-b^4/(a^2+b^2)*(1/4/(a^2+b^2)^(3/2)*(1/2*(-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^2+2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*(2*a^4-2*b^4-1/2*(-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^2+2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1

```
/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2
+b^2)^(1/2)-2*a)^(1/2)))+1/4/(a^2+b^2)^(3/2)*(-1/2*(-(2*(a^2+b^2)^(1/2)+2*a
)^(1/2)*(a^2+b^2)^(1/2)*a^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b
^2+2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^
2)*ln(-b*tan(d*x+c)-a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-
(a^2+b^2)^(1/2))+2*(-2*a^4+2*b^4+1/2*(-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b
^2)^(1/2)*a^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^2+2*(2*(a^2+b
^2)^(1/2)+2*a)^(1/2)*a^3+2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^2)*(2*(a^2+b^2
)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*tan(d*x+c
))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))))+a^
5/(a^2+b^2)/(a+b*tan(d*x+c))^(1/2))
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5852 vs. $2(248) = 496$.
time = 1.16, size = 5852, normalized size = 20.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/20*(20*sqrt(2)*((a^10*b^4 + 3*a^8*b^6 + 2*a^6*b^8 - 2*a^4*b^10 - 3*a^2*b
^12 - b^14)*d^5*cos(d*x + c)^4 + 2*(a^9*b^5 + 4*a^7*b^7 + 6*a^5*b^9 + 4*a^3
*b^11 + a*b^13)*d^5*cos(d*x + c)^3*sin(d*x + c) + (a^8*b^6 + 4*a^6*b^8 + 6*
a^4*b^10 + 4*a^2*b^12 + b^14)*d^5*cos(d*x + c)^2)*sqrt((a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6 - (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 +
 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((9
*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 +
 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6)*d^4))^(3/4)*arctan(-((3*a^12 + 14*a^10*b^2 + 25*a^8*b^4 + 20*a^6*b^6 +
5*a^4*b^8 - 2*a^2*b^10 - b^12)*d^4*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^1
2 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*
d^4))*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (3*a^9 + 8*a^7*b^
2 + 6*a^5*b^4 - a*b^8)*d^2*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^
10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) +
sqrt(2)*((a^14 + 5*a^12*b^2 + 9*a^10*b^4 + 5*a^8*b^6 - 5*a^6*b^8 - 9*a^4*b^
```

10 - 5*a^2*b^12 - b^14)*d^7*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (a^11 + 5*a^9*b^2 + 10*a^7*b^4 + 10*a^5*b^6 + 5*a^3*b^8 + a*b^10)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt(((9*a^8 + 12*a^6*b^2 - 2*a^4*b^4 - 4*a^2*b^6 + b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))*cos(d*x + c) + sqrt(2)*((9*a^9 + 12*a^7*b^2 - 2*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d^3*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))*cos(d*x + c) + (9*a^6 - 15*a^4*b^2 + 7*a^2*b^4 - b^6)*d*cos(d*x + c))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^(1/4) + (9*a^5 - 6*a^3*b^2 + a*b^4)*cos(d*x + c) + (9*a^4*b - 6*a^2*b^3 + b^5)*sin(d*x + c))/cos(d*x + c))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^(3/4) + sqrt(2)*((3*a^16 + 14*a^14*b^2 + 22*a^12*b^4 + 6*a^10*b^6 - 20*a^8*b^8 - 22*a^6*b^10 - 6*a^4*b^12 + 2*a^2*b^14 + b^16)*d^7*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (3*a^13 + 14*a^11*b^2 + 25*a^9*b^4 + 20*a^7*b^6 + 5*a^5*b^8 - 2*a^3*b^10 - a*b^12)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^(3/4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6)) + 20*sqrt(2)*((a^10*b^4 + 3*a^8*b^6 + 2*a^6*b^8 - 2*a^4*b^10 - 3*a^2*b^12 - b^14)*d^5*cos(d*x + c)^4 + 2*(a^9*b^5 + 4*a^7*b^7 + 6*a^5*b^9 + 4*a^3*b^11 + a*b^13)*d^5*cos(d*x + c)^3*sin(d*x + c) + (a^8*b^6 + 4*a^6*b^8 + 6*a^4*b^10 + 4*a^2*b^12 + b^14)*d^5*cos(d*x + c)^2)*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^(3/4)*arctan(((3*a^12 + 14*a^10*b^2 + 25*a^8*b^4 + 20*a^6*b^6 + 5*a^4*b^8 - 2*a^2*b^10 - b^12)*d^4*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (3*a^9 + 8*a^7*b^2 + 6*a^5*b^4 - a*b^8)*d^2*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) - sqrt(2)*((a^14 + 5*a^12*b^2 + 9*a^10*b^4 + 5*a^8*b^6 - 5*a^6*b^8 - 9*a^4*b^10 - 5*a^2*b^12 - b^14)*d^7*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^

```
4))*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (a^11 + 5*a^9*b^2 +
 10*a^7*b^4 + 10*a^5*b^6 + 5*a^3*b^8 + a*b^10)*d^5*sqrt((9*a^4*b^2 - 6*a^2*
b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a
^2*b^10 + b^12)*d^4)))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^9 - 6*a
^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt(((9*a^8 + 12*a^6*b^2 - 2*a^4*
b^4 - 4*a^2*b^6 + b^8)*d^2*sqrt(1/((a^6 + 3*a^4...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5(c+dx)}{(a+b\tan(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+b*tan(d*x+c))**(3/2),x)
[Out] Integral(tan(c + d*x)**5/(a + b*tan(c + d*x))**(3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [B]
time = 14.85, size = 2930, normalized size = 10.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + b*tan(c + d*x))^(3/2),x)
[Out] ((8*a^2)/(b^4*d) - (2*(a^2 + b^2))/(b^4*d))*(a + b*tan(c + d*x))^(1/2) - at
an(((1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(((1/(a^3
*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(32*a^6*b^6*d^4 - 48
*a^2*b^10*d^4 - 32*a^4*b^8*d^4 - 16*b^12*d^4 + 48*a^8*b^4*d^4 + 16*a^10*b^2
*d^4 + ((1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(a +
b*tan(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 +
 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5))/4))/2 + ((a + b*tan(
c + d*x))^(1/2)*(16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2
*d^3))/2)*1i + (1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2
```

)*(((1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(16*b^12*d^4 + 48*a^2*b^10*d^4 + 32*a^4*b^8*d^4 - 32*a^6*b^6*d^4 - 48*a^8*b^4*d^4 - 16*a^10*b^2*d^4 + ((1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5))/4))/2 + ((a + b*tan(c + d*x))^(1/2)*(16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))/2)*1i)/((1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(((1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(16*b^12*d^4 + 48*a^2*b^10*d^4 + 32*a^4*b^8*d^4 - 32*a^6*b^6*d^4 - 48*a^8*b^4*d^4 - 16*a^10*b^2*d^4 + ((1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5))/4))/2 + ((a + b*tan(c + d*x))^(1/2)*(16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))/2) - (1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(((1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(32*a^6*b^6*d^4 - 48*a^2*b^10*d^4 - 32*a^4*b^8*d^4 - 16*b^12*d^4 + 48*a^8*b^4*d^4 + 16*a^10*b^2*d^4 + ((1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5))/4))/2 + ((a + b*tan(c + d*x))^(1/2)*(16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))/2) + 16*a*b^8*d^2 + 48*a^3*b^6*d^2 + 48*a^5*b^4*d^2 + 16*a^7*b^2*d^2))*(1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*1i - atan((((1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*((1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5) - 32*b^12*d^4 - 96*a^2*b^10*d^4 - 64*a^4*b^8*d^4 + 64*a^6*b^6*d^4 + 96*a^8*b^4*d^4 + 32*a^10*b^2*d^4) + (a + b*tan(c + d*x))^(1/2)*(16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))*(1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*1i + ((1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(32*b^12*d^4 + (1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5) + 96*a^2*b^10*d^4 + 64*a^4*b^8*d^4 - 64*a^6*b^6*d^4 - 96*a^8*b^4*d^4 - 32*a^10*b^2*d^4) + (a + b*tan(c + d*x))^(1/2)*(16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))*(1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*1i)/(((1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(32*b^12*d^4 + (1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5) + 96*a^2*b^10*d^4 + 64*a^4*b^8*d^4 - 64*a^6*b^6*d^4 - 96*a^8*b^4*d^4 - 32*a^10*b^2*d^4) + (a + b*tan(c + d*x))^(1/2)*(16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))*(1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2) - ((1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*((1i/(4*(a^3

```
*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(a + b*tan(c + d*x)
)^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d
^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5) - 32*b^12*d^4 - 96*a^2*b^10*d^4 - 6
4*a^4*b^8*d^4 + 64*a^6*b^6*d^4 + 96*a^8*b^4*d^4 + 32*a^10*b^2*d^4) + (a + b
*tan(c + d*x))^(1/2)*(16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^
8*b^2*d^3))*(1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1
/2) + 16*a*b^8*d^2 + 48*a^3*b^6*d^2 + 48*a^5*b^4*d^2 + 16*a^7*b^2*d^2))*(1i
/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*2i + (2*(a
+ b*tan(c + d*x))^(5/2))/(5*b^4*d) - (2*a*(a + b*tan(c + d*x))^(3/2))/(b^4*
d) + (2*a^5)/(b^4*d*(a^2 + b^2)*(a + b*tan(c + d*x))^(1/2))
```

3.538 $\int \frac{\tan^4(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=226

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d}+\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d}-\frac{2a^2\tan^2(c+dx)}{b\left(a^2+b^2\right)d\sqrt{a+b\tan(c+dx)}}-$$

```
[Out] -I*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(3/2)/d+I*arctanh(
(a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(3/2)/d-2/3*a*(8*a^2+5*b^2)*(
a+b*tan(d*x+c))^(1/2)/b^3/(a^2+b^2)/d+2/3*(4*a^2+b^2)*(a+b*tan(d*x+c))^(1/2
)*tan(d*x+c)/b^2/(a^2+b^2)/d-2*a^2*tan(d*x+c)^2/b/(a^2+b^2)/d/(a+b*tan(d*x+
c))^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3646, 3728, 3711, 3620, 3618, 65, 214}

$$-\frac{2a^2\tan^2(c+dx)}{bd\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{2\left(4a^2+b^2\right)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3b^2d\left(a^2+b^2\right)}-\frac{2a\left(8a^2+5b^2\right)\sqrt{a+b\tan(c+dx)}}{3b^3d\left(a^2+b^2\right)}-\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{3/2}}+\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-I)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/((a - I*b)^(3/2)*d)
+ (I*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/((a + I*b)^(3/2)*d) -
 (2*a^2*Tan[c + d*x]^2)/(b*(a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]]) - (2*a*(
8*a^2 + 5*b^2)*Sqrt[a + b*Tan[c + d*x]])/(3*b^3*(a^2 + b^2)*d) + (2*(4*a^2
+ b^2)*Tan[c + d*x]*Sqrt[a + b*Tan[c + d*x]])/(3*b^2*(a^2 + b^2)*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
```

, 0] && NeQ[a, 0])))

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx = -\frac{2a^2\tan^2(c+dx)}{b\,(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} + \frac{2\int \frac{\tan(c+dx)\left(2a^2-\frac{1}{2}ab\tan(c+dx)+\frac{1}{2}(4a^2+b^2)\tan\right.}{\sqrt{a+b\tan(c+dx)}}}{b\,(a^2+b^2)}$$

$$= -\frac{2a^2\tan^2(c+dx)}{b\,(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} + \frac{2(4a^2+b^2)\tan(c+dx)\sqrt{a+b\tan(c}}{3b^2\,(a^2+b^2)\,d}$$

$$= -\frac{2a^2\tan^2(c+dx)}{b\,(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} - \frac{2a(8a^2+5b^2)\sqrt{a+b\tan(c+dx)}}{3b^3\,(a^2+b^2)\,d} +$$

$$= -\frac{2a^2\tan^2(c+dx)}{b\,(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} - \frac{2a(8a^2+5b^2)\sqrt{a+b\tan(c+dx)}}{3b^3\,(a^2+b^2)\,d} +$$

$$= -\frac{2a^2\tan^2(c+dx)}{b\,(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} - \frac{2a(8a^2+5b^2)\sqrt{a+b\tan(c+dx)}}{3b^3\,(a^2+b^2)\,d} +$$

$$= -\frac{2a^2\tan^2(c+dx)}{b\,(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} - \frac{2a(8a^2+5b^2)\sqrt{a+b\tan(c+dx)}}{3b^3\,(a^2+b^2)\,d} +$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d} + \frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d} - \frac{\ }{b}$$

Mathematica [A]
time = 5.37, size = 171, normalized size = 0.76

$$\frac{-\frac{3i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}} + \frac{3i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}} - \frac{2(8a^4+5a^2b^2+4ab(a^2+b^2)\tan(c+dx)-b^2(a^2+b^2)\tan^2(c+dx))}{b^3(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((-3*I)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/(a - I*b)^(3/2) +
 ((3*I)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/(a + I*b)^(3/2) -
(2*(8*a^4 + 5*a^2*b^2 + 4*a*b*(a^2 + b^2)*Tan[c + d*x] - b^2*(a^2 + b^2)*Ta
n[c + d*x]^2))/(b^3*(a^2 + b^2)*Sqrt[a + b*Tan[c + d*x]]))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 845 vs. $2(198) = 396$.

time = 0.16, size = 846, normalized size = 3.74

method	result
derivativedivides	$\frac{\frac{2(a+b\tan(dx+c))^{\frac{3}{2}}}{3}-4a\sqrt{a+b\tan(dx+c)}+\frac{2b^4\left(\frac{\left(\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a^3+\sqrt{2\sqrt{a^2+}\right.}{}\right.}{}}{}$
default	$\frac{\frac{2(a+b\tan(dx+c))^{\frac{3}{2}}}{3}-4a\sqrt{a+b\tan(dx+c)}+\frac{2b^4\left(\frac{\left(\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a^3+\sqrt{2\sqrt{a^2+}\right.}{}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/b^3*(1/3*(a+b*tan(d*x+c))^(3/2)-2*a*(a+b*tan(d*x+c))^(1/2)+b^4/(a^2+b^2
)*(1/4/b^2/(a^2+b^2)^(3/2)*(1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1
/2)*a^3+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^2-(2*(a^2+b^2)^(1
/2)+2*a)^(1/2)*a^4+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)*ln(b*tan(d*x+c)+a+(a+
b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*(4*a^3
*b^2+4*a*b^4-1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3+(2*(a^2
+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*
a^4+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a
^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2
)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))+1/4/b^2/(a^2+b^2)^(3/2)*(-1/2
*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3+(2*(a^2+b^2)^(1/2)+2*a)
^(1/2)*(a^2+b^2)^(1/2)*a*b^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4+(2*(a^2+b^2)
^(1/2)+2*a)^(1/2)*b^4)*ln(-b*tan(d*x+c)-a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)-(a^2+b^2)^(1/2))+2*(-4*a^3*b^2-4*a*b^4+1/2*((2*(a^2+b^2
)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b
```

```
^2)^(1/2)*a*b^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4+(2*(a^2+b^2)^(1/2)+2*a)^(
1/2)*b^4)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arct
an((-2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(
1/2)-2*a)^(1/2))))-a^4/(a^2+b^2)/(a+b*tan(d*x+c))^(1/2))
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5793 vs. $2(192) = 384$.
time = 1.41, size = 5793, normalized size = 25.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*(12*sqrt(2)*((a^10*b^3 + 3*a^8*b^5 + 2*a^6*b^7 - 2*a^4*b^9 - 3*a^2*b^1
1 - b^13)*d^5*cos(d*x + c)^3 + 2*(a^9*b^4 + 4*a^7*b^6 + 6*a^5*b^8 + 4*a^3*b
^10 + a*b^12)*d^5*cos(d*x + c)^2*sin(d*x + c) + (a^8*b^5 + 4*a^6*b^7 + 6*a^
4*b^9 + 4*a^2*b^11 + b^13)*d^5*cos(d*x + c))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*
b^4 + b^6 + (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^
4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((9*a^4*
b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a
^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d
^4))^(3/4)*arctan(((3*a^12 + 14*a^10*b^2 + 25*a^8*b^4 + 20*a^6*b^6 + 5*a^4*
b^8 - 2*a^2*b^10 - b^12)*d^4*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*
a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*
sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (3*a^9 + 8*a^7*b^2 + 6*
a^5*b^4 - a*b^8)*d^2*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2
 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) + sqrt(2
)*(2*(a^13 + 6*a^11*b^2 + 15*a^9*b^4 + 20*a^7*b^6 + 15*a^5*b^8 + 6*a^3*b^10
 + a*b^12)*d^7*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*
a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt(1/((a^6 +
 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a
^4*b^6 + 5*a^2*b^8 + b^10)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 +
6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)
))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^9 - 6*a^5*b^4 - 8*a^3*b^6 -
 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2
```

- 6*a^2*b^4 + b^6))*sqrt(((9*a^8*b^2 + 12*a^6*b^4 - 2*a^4*b^6 - 4*a^2*b^8 + b^10)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))*cos(d*x + c) + sqrt(2)*((9*a^8*b^3 + 12*a^6*b^5 - 2*a^4*b^7 - 4*a^2*b^9 + b^11)*d^3*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))*cos(d*x + c) + 2*(9*a^5*b^3 - 6*a^3*b^5 + a*b^7)*d*cos(d*x + c))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^(1/4) + (9*a^5*b^2 - 6*a^3*b^4 + a*b^6)*cos(d*x + c) + (9*a^4*b^3 - 6*a^2*b^5 + b^7)*sin(d*x + c))/cos(d*x + c))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^(3/4) + sqrt(2)*(2*(3*a^15*b + 17*a^13*b^3 + 39*a^11*b^5 + 45*a^9*b^7 + 25*a^7*b^9 + 3*a^5*b^11 - 3*a^3*b^13 - a*b^15)*d^7*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (3*a^12*b + 14*a^10*b^3 + 25*a^8*b^5 + 20*a^6*b^7 + 5*a^4*b^9 - 2*a^2*b^11 - b^13)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^(3/4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6)) + 12*sqrt(2)*((a^10*b^3 + 3*a^8*b^5 + 2*a^6*b^7 - 2*a^4*b^9 - 3*a^2*b^11 - b^13)*d^5*cos(d*x + c)^3 + 2*(a^9*b^4 + 4*a^7*b^6 + 6*a^5*b^8 + 4*a^3*b^10 + a*b^12)*d^5*cos(d*x + c)^2*sin(d*x + c) + (a^8*b^5 + 4*a^6*b^7 + 6*a^4*b^9 + 4*a^2*b^11 + b^13)*d^5*cos(d*x + c))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^(3/4)*arctan(-((3*a^12 + 14*a^10*b^2 + 25*a^8*b^4 + 20*a^6*b^6 + 5*a^4*b^8 - 2*a^2*b^10 - b^12)*d^4*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (3*a^9 + 8*a^7*b^2 + 6*a^5*b^4 - a*b^8)*d^2*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) - sqrt(2)*(2*(a^13 + 6*a^11*b^2 + 15*a^9*b^4 + 20*a^7*b^6 + 15*a^5*b^8 + 6*a^3*b^10 + a*b^12)*d^7*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt(((9*a^8*b^2 + 12*a^6*b^4 - 2*a^4*b^6 - 4*a^2*b^8 + b^10)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^...

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(c+dx)}{(a+b\tan(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+b*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**4/(a + b*tan(c + d*x))**(3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 8.30, size = 2282, normalized size = 10.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + b*tan(c + d*x))^(3/2),x)
```

```
[Out] (log(8*b^9*d^2 - ((((-1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i)
)^(1/2)*(64*a*b^11*d^4 + ((-1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d
^2*3i))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5
+ 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5))/2
 + 256*a^3*b^9*d^4 + 384*a^5*b^7*d^4 + 256*a^7*b^5*d^4 + 64*a^9*b^3*d^4))/2
 - (a + b*tan(c + d*x))^(1/2)*(16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^
3 - 16*a^8*b^2*d^3))*(-1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i
))^(1/2))/2 + 24*a^2*b^7*d^2 + 24*a^4*b^5*d^2 + 8*a^6*b^3*d^2)*(-1/(a^3*d^2
 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2))/2 - log(8*b^9*d^2 - ((-
1/(4*(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i)))^(1/2)*(64*a*b^11
*d^4 - (-1/(4*(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i)))^(1/2)*(
a + b*tan(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d
^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5) + 256*a^3*b^9*d^4
 + 384*a^5*b^7*d^4 + 256*a^7*b^5*d^4 + 64*a^9*b^3*d^4) + (a + b*tan(c + d*x
))^(1/2)*(16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))*
(-1/(4*(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i)))^(1/2) + 24*a^2
```

```
*b^7*d^2 + 24*a^4*b^5*d^2 + 8*a^6*b^3*d^2)*(-1/(4*(a^3*d^2 + b^3*d^2*1i - 3
*a*b^2*d^2 - a^2*b*d^2*3i)))^(1/2) + atan((((-1i/(4*(a^3*d^2*1i + b^3*d^2 -
 a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(64*a*b^11*d^4 + (-1i/(4*(a^3*d^2*1i +
 b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(
64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*
a^9*b^4*d^5 + 64*a^11*b^2*d^5) + 256*a^3*b^9*d^4 + 384*a^5*b^7*d^4 + 256*a^
7*b^5*d^4 + 64*a^9*b^3*d^4) - (a + b*tan(c + d*x))^(1/2)*(16*b^10*d^3 + 32*
a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))*(-1i/(4*(a^3*d^2*1i + b^3*d
^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*1i - ((-1i/(4*(a^3*d^2*1i + b^3*d^
2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(64*a*b^11*d^4 - (-1i/(4*(a^3*d^2*1
i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(a + b*tan(c + d*x))^(1/2
)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 3
20*a^9*b^4*d^5 + 64*a^11*b^2*d^5) + 256*a^3*b^9*d^4 + 384*a^5*b^7*d^4 + 256
*a^7*b^5*d^4 + 64*a^9*b^3*d^4) + (a + b*tan(c + d*x))^(1/2)*(16*b^10*d^3 +
32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))*(-1i/(4*(a^3*d^2*1i + b^
3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*1i)/(16*b^9*d^2 - ((-1i/(4*(a^3
*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(64*a*b^11*d^4 - (-
1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(a + b*ta
n(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640
*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5) + 256*a^3*b^9*d^4 + 384*a
^5*b^7*d^4 + 256*a^7*b^5*d^4 + 64*a^9*b^3*d^4) + (a + b*tan(c + d*x))^(1/2)
*(16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))*(-1i/(4*
(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2) - ((-1i/(4*(a^3
*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(64*a*b^11*d^4 + (-
1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(a + b*ta
n(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640
*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5) + 256*a^3*b^9*d^4 + 384*a
^5*b^7*d^4 + 256*a^7*b^5*d^4 + 64*a^9*b^3*d^4) - (a + b*tan(c + d*x))^(1/2)
*(16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))*(-1i/(4*
(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2) + 48*a^2*b^7*d^
2 + 48*a^4*b^5*d^2 + 16*a^6*b^3*d^2))*(-1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2
*d^2*3i - 3*a^2*b*d^2)))^(1/2)*2i + (2*(a + b*tan(c + d*x))^(3/2))/(3*b^3*d
) - (4*a*(a + b*tan(c + d*x))^(1/2))/(b^3*d) - (2*a^4)/(b^3*d*(a^2 + b^2)*(
a + b*tan(c + d*x))^(1/2))
```

3.539 $\int \frac{\tan^3(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=165

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d}-\frac{2a^2\tan(c+dx)}{b\left(a^2+b^2\right)d\sqrt{a+b\tan(c+dx)}}+\frac{2(2\ldots}{}$$

```
[Out] arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(3/2)/d+arctanh((a+b*
tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(3/2)/d+2*(2*a^2+b^2)*(a+b*tan(d*x
+c))^(1/2)/b^2/(a^2+b^2)/d-2*a^2*tan(d*x+c)/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^
(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3646, 3711, 3620, 3618, 65, 214}

$$-\frac{2a^2\tan(c+dx)}{bd\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{2\left(2a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}{b^2d\left(a^2+b^2\right)}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]]/((a - I*b)^(3/2)*d) + ArcTa
nh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]]/((a + I*b)^(3/2)*d) - (2*a^2*Tan
[c + d*x])/(b*(a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]]) + (2*(2*a^2 + b^2)*Sq
rt[a + b*Tan[c + d*x]])/(b^2*(a^2 + b^2)*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
```

```
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx = -\frac{2a^2\tan(c+dx)}{b\,(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} + \frac{2\int \frac{a^2-\frac{1}{2}ab\tan(c+dx)+\frac{1}{2}(2a^2+b^2)\tan^2(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,d}{b\,(a^2+b^2)}$$

$$= -\frac{2a^2\tan(c+dx)}{b\,(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} + \frac{2(2a^2+b^2)\sqrt{a+b\tan(c+dx)}}{b^2\,(a^2+b^2)\,d} + \frac{2\int}{}$$

$$= -\frac{2a^2\tan(c+dx)}{b\,(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} + \frac{2(2a^2+b^2)\sqrt{a+b\tan(c+dx)}}{b^2\,(a^2+b^2)\,d} + \frac{\int}{}$$

$$= -\frac{2a^2\tan(c+dx)}{b\,(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} + \frac{2(2a^2+b^2)\sqrt{a+b\tan(c+dx)}}{b^2\,(a^2+b^2)\,d} - \frac{\text{Su}}{}$$

$$= -\frac{2a^2\tan(c+dx)}{b\,(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} + \frac{2(2a^2+b^2)\sqrt{a+b\tan(c+dx)}}{b^2\,(a^2+b^2)\,d} - \frac{\text{Su}}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d} - \frac{}{b\,(a^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.54, size = 243, normalized size = 1.47

$$\frac{\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}} - \frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}} + \frac{4a}{b\sqrt{a+b\tan(c+dx)}} - \frac{a\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\tan(c+dx)}{a-ib}\right)}{(ia+b)\sqrt{a+b\tan(c+dx)}} + \frac{a\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\tan(c+dx)}{a+ib}\right)}{(ia-b)\sqrt{a+b\tan(c+dx)}} + \frac{2\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((I*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/Sqrt[a - I*b] - (I*Arc
Tanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/Sqrt[a + I*b] + (4*a)/(b*Sqrt
[a + b*Tan[c + d*x]]) - (a*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[c + d
*x])/(a - I*b)])/((I*a + b)*Sqrt[a + b*Tan[c + d*x]]) + (a*Hypergeometric2F
1[-1/2, 1, 1/2, (a + b*Tan[c + d*x])/(a + I*b)])/((I*a - b)*Sqrt[a + b*Tan[
c + d*x]]) + (2*Tan[c + d*x])/Sqrt[a + b*Tan[c + d*x]])/(b*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 825 vs. 2(145) = 290.
time = 0.14, size = 826, normalized size = 5.01

method	result
derivativedivides	$\frac{2\sqrt{a+b\tan(dx+c)}+\frac{2b^2\left(\frac{\left(-\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a^2-\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2}\right.}{}\right.}{}}{}$
default	$\frac{2\sqrt{a+b\tan(dx+c)}+\frac{2b^2\left(\frac{\left(-\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a^2-\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2}\right.}{}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/b^2*((a+b*tan(d*x+c))^(1/2)+b^2/(a^2+b^2)*(1/4/(a^2+b^2)^(3/2)*(1/2*(-(
2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2-(2*(a^2+b^2)^(1/2)+2*a)^(1
/2)*(a^2+b^2)^(1/2)*b^2+2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+2*(2*(a^2+b^2)^
(1/2)+2*a)^(1/2)*a*b^2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*(2*a^4-2*b^4-1/2*(-(2*(a^2+b^2)^(1/2
)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1
/2)*b^2+2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)
*a*b^2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan
((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2
)-2*a)^(1/2)))+1/4/(a^2+b^2)^(3/2)*(-1/2*(-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a
^2+b^2)^(1/2)*a^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^2+2*(2*(a
^2+b^2)^(1/2)+2*a)^(1/2)*a^3+2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^2)*ln(-b*t
an(d*x+c)-a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-(a^2+b^2)^
(1/2))+2*(-2*a^4+2*b^4+1/2*(-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*
a^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^2+2*(2*(a^2+b^2)^(1/2)+
2*a)^(1/2)*a^3+2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^2)*(2*(a^2+b^2)^(1/2)+2*
a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*tan(d*x+c))^(1/2)+(
2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))))+a^3/(a^2+b^2
```

```
)/(a+b*tan(d*x+c))^(1/2))
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5709 vs. $2(141) = 282$.
time = 0.97, size = 5709, normalized size = 34.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*((a^10*b^2 + 3*a^8*b^4 + 2*a^6*b^6 - 2*a^4*b^8 - 3*a^2*b^10
- b^12)*d^5*cos(d*x + c)^2 + 2*(a^9*b^3 + 4*a^7*b^5 + 6*a^5*b^7 + 4*a^3*b^9
 + a*b^11)*d^5*cos(d*x + c)*sin(d*x + c) + (a^8*b^4 + 4*a^6*b^6 + 6*a^4*b^8
 + 4*a^2*b^10 + b^12)*d^5)*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^9 -
 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4
+ b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((9*a^4*b^2 - 6*a^2*b^4 +
b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^1
0 + b^12)*d^4))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^(3/4)*arctan(
-((3*a^12 + 14*a^10*b^2 + 25*a^8*b^4 + 20*a^6*b^6 + 5*a^4*b^8 - 2*a^2*b^10
- b^12)*d^4*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8
*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt(1/((a^6 + 3*
a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (3*a^9 + 8*a^7*b^2 + 6*a^5*b^4 - a*b^8)*
d^2*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 2
0*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) + sqrt(2)*((a^14 + 5*a^12
*b^2 + 9*a^10*b^4 + 5*a^8*b^6 - 5*a^6*b^8 - 9*a^4*b^10 - 5*a^2*b^12 - b^14)
*d^7*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 +
20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt(1/((a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6)*d^4)) + (a^11 + 5*a^9*b^2 + 10*a^7*b^4 + 10*a^5*b^6 + 5
*a^3*b^8 + a*b^10)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b
^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))*sqrt(
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8
)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2
*b^4 + b^6))*sqrt(((9*a^8 + 12*a^6*b^2 - 2*a^4*b^4 - 4*a^2*b^6 + b^8)*d^2*s
qrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))*cos(d*x + c) + sqrt(2)*((9
*a^9 + 12*a^7*b^2 - 2*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d^3*sqrt(1/((a^6 + 3*a^4
```

```
*b^2 + 3*a^2*b^4 + b^6)*d^4))*cos(d*x + c) + (9*a^6 - 15*a^4*b^2 + 7*a^2*b^
4 - b^6)*d*cos(d*x + c))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^9 - 6
*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) + b*sin(d*x
 + c))/cos(d*x + c))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^(1/4) +
(9*a^5 - 6*a^3*b^2 + a*b^4)*cos(d*x + c) + (9*a^4*b - 6*a^2*b^3 + b^5)*sin(
d*x + c))/cos(d*x + c))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^(3/4)
 + sqrt(2)*((3*a^16 + 14*a^14*b^2 + 22*a^12*b^4 + 6*a^10*b^6 - 20*a^8*b^8 -
 22*a^6*b^10 - 6*a^4*b^12 + 2*a^2*b^14 + b^16)*d^7*sqrt((9*a^4*b^2 - 6*a^2*
b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a
^2*b^10 + b^12)*d^4))*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (
3*a^13 + 14*a^11*b^2 + 25*a^9*b^4 + 20*a^7*b^6 + 5*a^5*b^8 - 2*a^3*b^10 - a
*b^12)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*
b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))*sqrt((a^6 + 3*a^4
*b^2 + 3*a^2*b^4 + b^6 - (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1
/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))
*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*(1/((a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6)*d^4))^(3/4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6)) + 4*sqrt(2)*
((a^10*b^2 + 3*a^8*b^4 + 2*a^6*b^6 - 2*a^4*b^8 - 3*a^2*b^10 - b^12)*d^5*cos
(d*x + c)^2 + 2*(a^9*b^3 + 4*a^7*b^5 + 6*a^5*b^7 + 4*a^3*b^9 + a*b^11)*d^5*
cos(d*x + c)*sin(d*x + c) + (a^8*b^4 + 4*a^6*b^6 + 6*a^4*b^8 + 4*a^2*b^10 +
 b^12)*d^5)*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^9 - 6*a^5*b^4 - 8*
a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(
9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6
*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))
*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^(3/4)*arctan(((3*a^12 + 14*a
^10*b^2 + 25*a^8*b^4 + 20*a^6*b^6 + 5*a^4*b^8 - 2*a^2*b^10 - b^12)*d^4*sqrt
((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^
6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*
b^4 + b^6)*d^4)) + (3*a^9 + 8*a^7*b^2 + 6*a^5*b^4 - a*b^8)*d^2*sqrt((9*a^4*
b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a
^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) - sqrt(2)*((a^14 + 5*a^12*b^2 + 9*a^10*b^
4 + 5*a^8*b^6 - 5*a^6*b^8 - 9*a^4*b^10 - 5*a^2*b^12 - b^14)*d^7*sqrt((9*a^4
*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*
a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6)*d^4)) + (a^11 + 5*a^9*b^2 + 10*a^7*b^4 + 10*a^5*b^6 + 5*a^3*b^8 + a*b^1
0)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4
+ 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))*sqrt((a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6 - (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a
^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqr
t(((9*a^8 + 12*a^6*b^2 - 2*a^4*b^4 - 4*a^2*b^6 + b^8)*d^2*sqrt(1/((a^6 + 3*
a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))*cos(d*x + c) -...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(c+dx)}{(a+b\tan(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+b*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**3/(a + b*tan(c + d*x))**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 6.02, size = 2869, normalized size = 17.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + b*tan(c + d*x))^(3/2),x)
```

```
[Out] atan((((1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(
(1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(a + b*t
an(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 64
0*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5) - 32*b^12*d^4 - 96*a^2*b
^10*d^4 - 64*a^4*b^8*d^4 + 64*a^6*b^6*d^4 + 96*a^8*b^4*d^4 + 32*a^10*b^2*d^
4) + (a + b*tan(c + d*x))^(1/2)*(16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*
d^3 - 16*a^8*b^2*d^3))*(1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*
b*d^2)))^(1/2)*1i + ((1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*
d^2)))^(1/2)*(32*b^12*d^4 + (1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3
*a^2*b*d^2)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^1
0*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d
^5) + 96*a^2*b^10*d^4 + 64*a^4*b^8*d^4 - 64*a^6*b^6*d^4 - 96*a^8*b^4*d^4 -
32*a^10*b^2*d^4) + (a + b*tan(c + d*x))^(1/2)*(16*b^10*d^3 + 32*a^2*b^8*d^3
 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))*(1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d
^2*3i - 3*a^2*b*d^2)))^(1/2)*1i)/(((1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2
*3i - 3*a^2*b*d^2)))^(1/2)*(32*b^12*d^4 + (1i/(4*(a^3*d^2*1i + b^3*d^2 - a*
b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^12*d^5
```

+ 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5) + 96*a^2*b^10*d^4 + 64*a^4*b^8*d^4 - 64*a^6*b^6*d^4 - 96*a^8*b^4*d^4 - 32*a^10*b^2*d^4) + (a + b*tan(c + d*x))^(1/2)*(16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))*(1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2) - ((1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*((1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5) - 32*b^12*d^4 - 96*a^2*b^10*d^4 - 64*a^4*b^8*d^4 + 64*a^6*b^6*d^4 + 96*a^8*b^4*d^4 + 32*a^10*b^2*d^4) + (a + b*tan(c + d*x))^(1/2)*(16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))*(1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2) + 16*a*b^8*d^2 + 48*a^3*b^6*d^2 + 48*a^5*b^4*d^2 + 16*a^7*b^2*d^2))*(1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*2i + atan(((1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(((1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(32*a^6*b^6*d^4 - 48*a^2*b^10*d^4 - 32*a^4*b^8*d^4 - 16*b^12*d^4 + 48*a^8*b^4*d^4 + 16*a^10*b^2*d^4 + ((1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5))/4))/2 + ((a + b*tan(c + d*x))^(1/2)*(16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))/2)*1i + (1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(((1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(16*b^12*d^4 + 48*a^2*b^10*d^4 + 32*a^4*b^8*d^4 - 32*a^6*b^6*d^4 - 48*a^8*b^4*d^4 - 16*a^10*b^2*d^4 + ((1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5))/4))/2 + ((a + b*tan(c + d*x))^(1/2)*(16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))/2)*1i)/((1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(((1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(16*b^12*d^4 + 48*a^2*b^10*d^4 + 32*a^4*b^8*d^4 - 32*a^6*b^6*d^4 - 48*a^8*b^4*d^4 - 16*a^10*b^2*d^4 + ((1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5))/4))/2 + ((a + b*tan(c + d*x))^(1/2)*(16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))/2) - (1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(((1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(32*a^6*b^6*d^4 - 48*a^2*b^10*d^4 - 32*a^4*b^8*d^4 - 16*b^12*d^4 + 48*a^8*b^4*d^4 + 16*a^10*b^2*d^4 + ((1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5))/4))/2 + ((a + b*tan(c + d*x))^(1/2)*(16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))/2) + 16*a*b^8*d^2 + 48*a^3*b^6*d^2 + 48*a^5*b^4*d^2 + 16*a^7*b^2*d^2))*(1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*1i + (2*(a + b*tan(c + d*x))^(1/2))/(b^2*d) + (2*a^3)/(b^2*d*(a^2 + b^2)*(a + b*tan(

```
c + d*x))^(1/2))
```

3.540 $\int \frac{\tan^2(c+dx)}{(a+b\tan(c+dx))^{3/2}} dx$

Optimal. Leaf size=125

$$\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d}-\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d}-\frac{2a^2}{b\left(a^2+b^2\right)d\sqrt{a+b\tan(c+dx)}}$$

```
[Out] I*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(3/2)/d-I*arctanh((
a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(3/2)/d-2*a^2/b/(a^2+b^2)/d/(a
+b*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3623, 3620, 3618, 65, 214}

$$-\frac{2a^2}{bd\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{3/2}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] (I*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/((a - I*b)^(3/2)*d) - (
I*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/((a + I*b)^(3/2)*d) - (2
*a^2)/(b*(a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
```

```
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^2(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx &= -\frac{2a^2}{b\,(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}}+\frac{\int \frac{-a+b\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx}{a^2+b^2}\\
&= -\frac{2a^2}{b\,(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}}-\frac{\int \frac{1+i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx}{2(a-ib)}-\frac{\int \frac{1-}{\sqrt{a+}}\\
&= -\frac{2a^2}{b\,(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}}-\frac{\mathrm{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+ibx}}\,dx,x,-i\tan\right.}{2(ia-b)d}\\
&= -\frac{2a^2}{b\,(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}}+\frac{\mathrm{Subst}\left(\int \frac{1}{-1-\frac{ia}{b}+\frac{ix^2}{b}}\,dx,x,\sqrt{a+b\tan(\right.}{(a-ib)bd}\\
&= \frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d}-\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d}-\frac{}{b\,(}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.16, size = 119, normalized size = 0.95

$$\frac{b(-ia+b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\tan(c+dx)}{a-ib}\right)-(a-ib)\left(2a+2ib-ib\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\tan(c+dx)}{a+ib}\right)\right)}{b\,(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] (b*((-I)*a + b)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[c + d*x])/(a - I
*b)] - (a - I*b)*(2*a + (2*I)*b - I*b*Hypergeometric2F1[-1/2, 1, 1/2, (a +
b*Tan[c + d*x])/(a + I*b)]))/(b*(a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 817 vs. $2(105) = 210$.
time = 0.12, size = 818, normalized size = 6.54

method	result
derivativedivides	$-\frac{2b^2\left(\frac{\left(\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a^3+\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,ab^2-\sqrt{2\sqrt{a^2+b^2}}\cdots\right.}{\cdots}\right.}{\cdots}$
default	$-\frac{2b^2\left(\frac{\left(\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a^3+\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,ab^2-\sqrt{2\sqrt{a^2+b^2}}\cdots\right.}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/b*(-b^2/(a^2+b^2)*(1/4/b^2/(a^2+b^2)^(3/2)*(1/2*((2*(a^2+b^2)^(1/2)+2*a
)^(1/2)*(a^2+b^2)^(1/2)*a^3+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a
*b^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)*l
```

```
n(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+
b^2)^(1/2))+2*(4*a^3*b^2+4*a*b^4-1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^
2)^(1/2)*a^3+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^2-(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)*a^4+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)*(2*(a^2+b^2)^(1/
2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/
2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))+1/4/b^2/(
a^2+b^2)^(3/2)*(-1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3+(2*
(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^2-(2*(a^2+b^2)^(1/2)+2*a)^(1
/2)*a^4+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)*ln(-b*tan(d*x+c)-a+(a+b*tan(d*x+
c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-(a^2+b^2)^(1/2))+2*(-4*a^3*b^2-4*a*
b^4+1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3+(2*(a^2+b^2)^(1/
2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4+(2*(a
^2+b^2)^(1/2)+2*a)^(1/2)*b^4)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(
1/2)-2*a)^(1/2)*arctan((-2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(
1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))))-a^2/(a^2+b^2)/(a+b*tan(d*x+c))^(1/2)
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^2/(b*tan(d*x + c) + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5654 vs. $2(99) = 198$.
time = 1.37, size = 5654, normalized size = 45.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*((a^10*b + 3*a^8*b^3 + 2*a^6*b^5 - 2*a^4*b^7 - 3*a^2*b^9 -
b^11)*d^5*cos(d*x + c)^2 + 2*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 +
 a*b^10)*d^5*cos(d*x + c)*sin(d*x + c) + (a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 +
 4*a^2*b^9 + b^11)*d^5)*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^9 - 6*
a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6
)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 +
 b^12)*d^4))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^(3/4)*arctan(((3
*a^12 + 14*a^10*b^2 + 25*a^8*b^4 + 20*a^6*b^6 + 5*a^4*b^8 - 2*a^2*b^10 - b^
12)*d^4*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4
```

+ 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (3*a^9 + 8*a^7*b^2 + 6*a^5*b^4 - a*b^8)*d^2*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) + sqrt(2)*(2*(a^13 + 6*a^11*b^2 + 15*a^9*b^4 + 20*a^7*b^6 + 15*a^5*b^8 + 6*a^3*b^10 + a*b^12)*d^7*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt(((9*a^8*b^2 + 12*a^6*b^4 - 2*a^4*b^6 - 4*a^2*b^8 + b^10)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))*cos(d*x + c) + sqrt(2)*((9*a^8*b^3 + 12*a^6*b^5 - 2*a^4*b^7 - 4*a^2*b^9 + b^11)*d^3*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))*cos(d*x + c) + 2*(9*a^5*b^3 - 6*a^3*b^5 + a*b^7)*d*cos(d*x + c))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^(1/4) + (9*a^5*b^2 - 6*a^3*b^4 + a*b^6)*cos(d*x + c) + (9*a^4*b^3 - 6*a^2*b^5 + b^7)*sin(d*x + c))/cos(d*x + c))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^(3/4) + sqrt(2)*(2*(3*a^15*b + 17*a^13*b^3 + 39*a^11*b^5 + 45*a^9*b^7 + 25*a^7*b^9 + 3*a^5*b^11 - 3*a^3*b^13 - a*b^15)*d^7*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (3*a^12*b + 14*a^10*b^3 + 25*a^8*b^5 + 20*a^6*b^7 + 5*a^4*b^9 - 2*a^2*b^11 - b^13)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^(3/4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6)) + 4*sqrt(2)*((a^10*b + 3*a^8*b^3 + 2*a^6*b^5 - 2*a^4*b^7 - 3*a^2*b^9 - b^11)*d^5*cos(d*x + c)^2 + 2*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*d^5*cos(d*x + c)*sin(d*x + c) + (a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*d^5)*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^(3/4)*arctan(-((3*a^12 + 14*a^10*b^2 + 25*a^8*b^4 + 20*a^6*b^6 + 5*a^4*b^8 - 2*a^2*b^10 - b^12)*d^4*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (3*a^9 + 8*a^7*b^2 + 6*a^5*b^4 - a*b^8)*d^2*sqrt((9*a^4*b^2 - 6*a^2*b^4

```
 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*
b^10 + b^12)*d^4)) - sqrt(2)*(2*(a^13 + 6*a^11*b^2 + 15*a^9*b^4 + 20*a^7*b^
6 + 15*a^5*b^8 + 6*a^3*b^10 + a*b^12)*d^7*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6
)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 +
 b^12)*d^4))*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (a^10 + 5*
a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^5*sqrt((9*a^4*b^2 -
 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^
8 + 6*a^2*b^10 + b^12)*d^4)))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^
9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b
^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt(((9*a^8*b^2 + 12*a^6*b
^4 - 2*a^4*b^6 - 4*a^2*b^8 + b^10)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6)*d^4))*cos(d*x + c) - sqrt(2)*((9*a^8*b^...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(c+dx)}{(a+b\tan(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+b*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**2/(a + b*tan(c + d*x))**(3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 5.63, size = 2239, normalized size = 17.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + b*tan(c + d*x))^(3/2),x)
```

```
[Out] (log(((((-1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(64*
a*b^11*d^4 - ((-1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2
)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^
8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5))/2 + 256*a^3*b
```

^9*d^4 + 384*a^5*b^7*d^4 + 256*a^7*b^5*d^4 + 64*a^9*b^3*d^4))/2 + (a + b*ta
n(c + d*x))^(1/2)*(16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b
^2*d^3))*(-1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2))/2
- 8*b^9*d^2 - 24*a^2*b^7*d^2 - 24*a^4*b^5*d^2 - 8*a^6*b^3*d^2)*(-1/(a^3*d^2
 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2))/2 - log(((-1/(4*(a^3*d^
2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i)))^(1/2)*(64*a*b^11*d^4 + (-1/(
4*(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i)))^(1/2)*(a + b*tan(c
+ d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7
*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5) + 256*a^3*b^9*d^4 + 384*a^5*b
^7*d^4 + 256*a^7*b^5*d^4 + 64*a^9*b^3*d^4) - (a + b*tan(c + d*x))^(1/2)*(16
*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))*(-1/(4*(a^3*
d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i)))^(1/2) - 8*b^9*d^2 - 24*a^2
*b^7*d^2 - 24*a^4*b^5*d^2 - 8*a^6*b^3*d^2)*(-1/(4*(a^3*d^2 + b^3*d^2*1i - 3
*a*b^2*d^2 - a^2*b*d^2*3i)))^(1/2) - atan((((-1i/(4*(a^3*d^2*1i + b^3*d^2 -
 a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(64*a*b^11*d^4 + (-1i/(4*(a^3*d^2*1i +
 b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(
64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*
a^9*b^4*d^5 + 64*a^11*b^2*d^5) + 256*a^3*b^9*d^4 + 384*a^5*b^7*d^4 + 256*a^
7*b^5*d^4 + 64*a^9*b^3*d^4) - (a + b*tan(c + d*x))^(1/2)*(16*b^10*d^3 + 32*
a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))*(-1i/(4*(a^3*d^2*1i + b^3*d
^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*1i - ((-1i/(4*(a^3*d^2*1i + b^3*d^
2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(64*a*b^11*d^4 - (-1i/(4*(a^3*d^2*1
i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(a + b*tan(c + d*x))^(1/2
)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 3
20*a^9*b^4*d^5 + 64*a^11*b^2*d^5) + 256*a^3*b^9*d^4 + 384*a^5*b^7*d^4 + 256
*a^7*b^5*d^4 + 64*a^9*b^3*d^4) + (a + b*tan(c + d*x))^(1/2)*(16*b^10*d^3 +
32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))*(-1i/(4*(a^3*d^2*1i + b^
3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*1i)/(16*b^9*d^2 - ((-1i/(4*(a^3
*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(64*a*b^11*d^4 - (-
1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(a + b*ta
n(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640
*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5) + 256*a^3*b^9*d^4 + 384*a
^5*b^7*d^4 + 256*a^7*b^5*d^4 + 64*a^9*b^3*d^4) + (a + b*tan(c + d*x))^(1/2)
*(16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))*(-1i/(4*
(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2) - ((-1i/(4*(a^3
*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(64*a*b^11*d^4 + (-
1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(a + b*ta
n(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640
*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5) + 256*a^3*b^9*d^4 + 384*a
^5*b^7*d^4 + 256*a^7*b^5*d^4 + 64*a^9*b^3*d^4) - (a + b*tan(c + d*x))^(1/2)
*(16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))*(-1i/(4*
(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2) + 48*a^2*b^7*d^
2 + 48*a^4*b^5*d^2 + 16*a^6*b^3*d^2))*(-1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2
*d^2*3i - 3*a^2*b*d^2)))^(1/2)*2i - (2*a^2)/(b*d*(a^2 + b^2)*(a + b*tan(c +
 d*x))^(1/2))

3.541 $\int \frac{\tan(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=116

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d}+\frac{2a}{(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}}$$

```
[Out] -arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(3/2)/d-arctanh((a+b
*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(3/2)/d+2*a/(a^2+b^2)/d/(a+b*tan(
d*x+c))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3610, 3620, 3618, 65, 214}

$$\frac{2a}{d\,(a^2+b^2)\sqrt{a+b\tan(c+dx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]]/((a - I*b)^(3/2)*d)) - Ar
cTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]]/((a + I*b)^(3/2)*d) + (2*a)/(
(a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
```

```
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx = \frac{2a}{(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} + \frac{\int \frac{b+a\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx}{a^2+b^2}$$

$$= \frac{2a}{(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} - \frac{\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx}{2(ia-b)} + \frac{\int \frac{1+i\tan(}{\sqrt{a+b\tan}}}{2(ia}$$

$$= \frac{2a}{(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a-ibx}}\,dx, x, i\tan(c+d\right.}{2(a-ib)d}$$

$$= \frac{2a}{(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{-1+\frac{ia}{b}-\frac{ix^2}{b}}\,dx, x, \sqrt{a+b\tan(c+d}\right.}{(ia-b)bd}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d} + \frac{}{(a^2+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.17, size = 100, normalized size = 0.86

$$\frac{(a+ib)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\tan(c+dx)}{a-ib}\right)+(a-ib)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\tan(c+dx)}{a+ib}\right)}{(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((a + I*b)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[c + d*x])/(a - I*b)]
+ (a - I*b)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[c + d*x])/(a + I*b)]
)/((a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 798 vs. $2(98) = 196$.
time = 0.13, size = 799, normalized size = 6.89

method	result
derivativedivides	$\left(\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a^2+\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,b^2-2\sqrt{2\sqrt{a^2+b^2}+2\right.$
default	$\left(\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a^2+\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,b^2-2\sqrt{2\sqrt{a^2+b^2}+2\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/(a^2+b^2)*(1/4/(a^2+b^2)^(3/2)*(1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(
a^2+b^2)^(1/2)*a^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^2-2*(2*(
a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^2)*ln(b*t
an(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^
(1/2))+2*(-2*a^4+2*b^4-1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a
^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^2-2*(2*(a^2+b^2)^(1/2)+2
*a)^(1/2)*a^3-2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^2)*(2*(a^2+b^2)^(1/2)+2*a
)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*
(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))+1/4/(a^2+b^2)^(
3/2)*(-1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2+(2*(a^2+b^2)^
(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^2-2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-2*
(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^2)*ln(-b*tan(d*x+c)-a+(a+b*tan(d*x+c))^(1
/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-(a^2+b^2)^(1/2))+2*(2*a^4-2*b^4+1/2*((2*(
a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)
```

```
*(a^2+b^2)^(1/2)*b^2-2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-2*(2*(a^2+b^2)^(1/
2)+2*a)^(1/2)*a*b^2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)
^(1/2)*arctan((-2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*
(a^2+b^2)^(1/2)-2*a)^(1/2))))+2*a/(a^2+b^2)/(a+b*tan(d*x+c))^(1/2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5638 vs. 2(94) = 188.
time = 1.52, size = 5638, normalized size = 48.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*((a^10 + 3*a^8*b^2 + 2*a^6*b^4 - 2*a^4*b^6 - 3*a^2*b^8 - b^
10)*d^5*cos(d*x + c)^2 + 2*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b
^9)*d^5*cos(d*x + c)*sin(d*x + c) + (a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^
2*b^8 + b^10)*d^5)*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^9 - 6*a^5*b
^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d
^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a
^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12
)*d^4))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^(3/4)*arctan(-((3*a^1
2 + 14*a^10*b^2 + 25*a^8*b^4 + 20*a^6*b^6 + 5*a^4*b^8 - 2*a^2*b^10 - b^12)*
d^4*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 2
0*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt(1/((a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6)*d^4)) + (3*a^9 + 8*a^7*b^2 + 6*a^5*b^4 - a*b^8)*d^2*sqrt
((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^
6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) + sqrt(2)*((a^14 + 5*a^12*b^2 + 9
*a^10*b^4 + 5*a^8*b^6 - 5*a^6*b^8 - 9*a^4*b^10 - 5*a^2*b^12 - b^14)*d^7*sqr
t((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b
^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + b^6)*d^4)) + (a^11 + 5*a^9*b^2 + 10*a^7*b^4 + 10*a^5*b^6 + 5*a^3*b^8
 + a*b^10)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*
```

a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt(((9*a^8 + 12*a^6*b^2 - 2*a^4*b^4 - 4*a^2*b^6 + b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))*cos(d*x + c) + sqrt(2)*((9*a^9 + 12*a^7*b^2 - 2*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d^3*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))*cos(d*x + c) + (9*a^6 - 15*a^4*b^2 + 7*a^2*b^4 - b^6)*d*cos(d*x + c))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^(1/4) + (9*a^5 - 6*a^3*b^2 + a*b^4)*cos(d*x + c) + (9*a^4*b - 6*a^2*b^3 + b^5)*sin(d*x + c))/cos(d*x + c))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^(3/4) + sqrt(2)*((3*a^16 + 14*a^14*b^2 + 22*a^12*b^4 + 6*a^10*b^6 - 20*a^8*b^8 - 22*a^6*b^10 - 6*a^4*b^12 + 2*a^2*b^14 + b^16)*d^7*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (3*a^13 + 14*a^11*b^2 + 25*a^9*b^4 + 20*a^7*b^6 + 5*a^5*b^8 - 2*a^3*b^10 - a*b^12)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^(3/4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6)) + 4*sqrt(2)*((a^10 + 3*a^8*b^2 + 2*a^6*b^4 - 2*a^4*b^6 - 3*a^2*b^8 - b^10)*d^5*cos(d*x + c)^2 + 2*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*d^5*cos(d*x + c)*sin(d*x + c) + (a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)*d^5)*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^(3/4)*arctan(((3*a^12 + 14*a^10*b^2 + 25*a^8*b^4 + 20*a^6*b^6 + 5*a^4*b^8 - 2*a^2*b^10 - b^12)*d^4*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (3*a^9 + 8*a^7*b^2 + 6*a^5*b^4 - a*b^8)*d^2*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) - sqrt(2)*((a^14 + 5*a^12*b^2 + 9*a^10*b^4 + 5*a^8*b^6 - 5*a^6*b^8 - 9*a^4*b^10 - 5*a^2*b^12 - b^14)*d^7*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (a^11 + 5*a^9*b^2 + 10*a^7*b^4 + 10*a^5*b^6 + 5*a^3*b^8 + a*b^10)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 +

```
3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt(((9*a^8 + 12*a^
6*b^2 - 2*a^4*b^4 - 4*a^2*b^6 + b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b
^4 + b^6)*d^4))*cos(d*x + c) - sqrt(2)*((9*a^9 ...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(c+dx)}{(a+b\tan(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)/(a + b*tan(c + d*x))**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 6.30, size = 2844, normalized size = 24.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + b*tan(c + d*x))^(3/2),x)
```

```
[Out] (2*a)/(d*(a^2 + b^2)*(a + b*tan(c + d*x))^(1/2)) - atan(((1/(a^3*d^2 + b^3*
d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(((1/(a^3*d^2 + b^3*d^2*1i - 3*
a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(32*a^6*b^6*d^4 - 48*a^2*b^10*d^4 - 32*a^4
*b^8*d^4 - 16*b^12*d^4 + 48*a^8*b^4*d^4 + 16*a^10*b^2*d^4 + ((1/(a^3*d^2 +
b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(a + b*tan(c + d*x))^(1/2)*
(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320
*a^9*b^4*d^5 + 64*a^11*b^2*d^5))/4))/2 + ((a + b*tan(c + d*x))^(1/2)*(16*b^
10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))/2)*1i + (1/(a^3
*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(((1/(a^3*d^2 + b^3*
d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(16*b^12*d^4 + 48*a^2*b^10*d^4
+ 32*a^4*b^8*d^4 - 32*a^6*b^6*d^4 - 48*a^8*b^4*d^4 - 16*a^10*b^2*d^4 + ((1/
(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(a + b*tan(c + d
*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^
```

6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5))/4))/2 + ((a + b*tan(c + d*x))^(1/2)*(16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))/2)*1i)/((1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(((1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(16*b^12*d^4 + 48*a^2*b^10*d^4 + 32*a^4*b^8*d^4 - 32*a^6*b^6*d^4 - 48*a^8*b^4*d^4 - 16*a^10*b^2*d^4 + ((1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5))/4))/2 + ((a + b*tan(c + d*x))^(1/2)*(16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))/2) - (1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(((1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(32*a^6*b^6*d^4 - 48*a^2*b^10*d^4 - 32*a^4*b^8*d^4 - 16*b^12*d^4 + 48*a^8*b^4*d^4 + 16*a^10*b^2*d^4 + ((1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5))/4))/2 + ((a + b*tan(c + d*x))^(1/2)*(16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))/2) + 16*a*b^8*d^2 + 48*a^3*b^6*d^2 + 48*a^5*b^4*d^2 + 16*a^7*b^2*d^2))*(1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2)*1i - atan((((1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*((1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5) - 32*b^12*d^4 - 96*a^2*b^10*d^4 - 64*a^4*b^8*d^4 + 64*a^6*b^6*d^4 + 96*a^8*b^4*d^4 + 32*a^10*b^2*d^4) + (a + b*tan(c + d*x))^(1/2)*(16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))*(1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*1i + ((1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(32*b^12*d^4 + (1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5) + 96*a^2*b^10*d^4 + 64*a^4*b^8*d^4 - 64*a^6*b^6*d^4 - 96*a^8*b^4*d^4 - 32*a^10*b^2*d^4) + (a + b*tan(c + d*x))^(1/2)*(16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))*(1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*1i)/(((1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(32*b^12*d^4 + (1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5) + 96*a^2*b^10*d^4 + 64*a^4*b^8*d^4 - 64*a^6*b^6*d^4 - 96*a^8*b^4*d^4 - 32*a^10*b^2*d^4) + (a + b*tan(c + d*x))^(1/2)*(16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))*(1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2) - ((1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*((1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5) - 32*b^12*d^4 - 96*a^2*b^10*d^4 - 64*a^4*b^8*d^4 + 64*a^6*b^6*d^4 + 96*a^8*b^4*d^4 + 32*a^10*b^2*d^4) + (a + b*tan(c + d*x))^(1/2)*(

```
16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))*(1i/(4*(a^
3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2) + 16*a*b^8*d^2 + 4
8*a^3*b^6*d^2 + 48*a^5*b^4*d^2 + 16*a^7*b^2*d^2))*(1i/(4*(a^3*d^2*1i + b^3*
d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*2i
```

3.542 $\int \frac{1}{(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=120

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d}+\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d}-\frac{2b}{(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}}$$

```
[Out] -I*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(3/2)/d+I*arctanh(
(a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(3/2)/d-2*b/(a^2+b^2)/d/(a+b*
tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3564, 3620, 3618, 65, 214}

$$-\frac{2b}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{3/2}}+\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^(-3/2),x]
```

```
[Out] ((-I)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/((a - I*b)^(3/2)*d)
+ (I*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/((a + I*b)^(3/2)*d) -
 (2*b)/((a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+b\tan(c+dx))^{3/2}}\,dx &= -\frac{2b}{(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}}+\frac{\int \frac{a-b\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx}{a^2+b^2}\\
&= -\frac{2b}{(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}}+\frac{\int \frac{1+i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx}{2(a-ib)}+\frac{\int \frac{1-i\,\mathrm{ta}}{\sqrt{a+b\,\mathrm{t}}}}{2(a}\\
&= -\frac{2b}{(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}}+\frac{\mathrm{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+ibx}}\,dx,x,-i\tan(c\right.}{2(ia-b)d}\\
&= -\frac{2b}{(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}}-\frac{\mathrm{Subst}\left(\int \frac{1}{-1-\frac{ia}{b}+\frac{ix^2}{b}}\,dx,x,\sqrt{a+b\tan(c+}\right.}{(a-ib)bd}\\
&= -\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d}+\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d}-\frac{}{(a}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.11, size = 105, normalized size = 0.88

$$
\frac{i(a+ib)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\tan(c+dx)}{a-ib}\right)+(-ia-b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\tan(c+dx)}{a+ib}\right)}{(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^(-3/2),x]
```

```
[Out] (I*(a + I*b)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[c + d*x])/(a - I*b)
] + ((-I)*a - b)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[c + d*x])/(a +
I*b)])/((a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(100) = 200$.
time = 0.10, size = 809, normalized size = 6.74

method	result
derivativedivides	$\frac{2b\left(\frac{\left(\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a^3+\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a\,b^2-\sqrt{2\sqrt{a^2+b^2}}\right.}{}\right.}{}$
default	$\frac{2b\left(\frac{\left(\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a^3+\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a\,b^2-\sqrt{2\sqrt{a^2+b^2}}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*b*(1/(a^2+b^2)*(1/4/b^2/(a^2+b^2)^(3/2)*(1/2*((2*(a^2+b^2)^(1/2)+2*a)^(
1/2)*(a^2+b^2)^(1/2)*a^3+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^
2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)*ln(b
*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2
)^(1/2))+2*(4*a^3*b^2+4*a*b^4-1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^
(1/2)*a^3+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^2-(2*(a^2+b^2)^
```

```
(1/2)+2*a)^(1/2)*a^4+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)*(2*(a^2+b^2)^(1/2)+
2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+
(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))+1/4/b^2/(a^2
+b^2)^(3/2)*(-1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3+(2*(a^
2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)
*a^4+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)*ln(-b*tan(d*x+c)-a+(a+b*tan(d*x+c))
^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-(a^2+b^2)^(1/2))+2*(-4*a^3*b^2-4*a*b^4
+1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3+(2*(a^2+b^2)^(1/2)+
2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4+(2*(a^2+
b^2)^(1/2)+2*a)^(1/2)*b^4)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2
)-2*a)^(1/2)*arctan((-2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2
))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))))-1/(a^2+b^2)/(a+b*tan(d*x+c))^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5624 vs. $2(94) = 188$.
time = 1.26, size = 5624, normalized size = 46.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*((a^10 + 3*a^8*b^2 + 2*a^6*b^4 - 2*a^4*b^6 - 3*a^2*b^8 - b^1
0)*d^5*cos(d*x + c)^2 + 2*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^
9)*d^5*cos(d*x + c)*sin(d*x + c) + (a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2
*b^8 + b^10)*d^5)*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^9 - 6*a^5*b^
4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^
4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^
12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)
*d^4))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^(3/4)*arctan(((3*a^12
+ 14*a^10*b^2 + 25*a^8*b^4 + 20*a^6*b^6 + 5*a^4*b^8 - 2*a^2*b^10 - b^12)*d^
4*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*
a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt(1/((a^6 + 3*a^4*b^2 +
```

3*a^2*b^4 + b^6)*d^4)) + (3*a^9 + 8*a^7*b^2 + 6*a^5*b^4 - a*b^8)*d^2*sqrt((
9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6
+ 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) + sqrt(2)*(2*(a^13 + 6*a^11*b^2 + 1
5*a^9*b^4 + 20*a^7*b^6 + 15*a^5*b^8 + 6*a^3*b^10 + a*b^12)*d^7*sqrt((9*a^4*
b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a
^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6)*d^4)) + (a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*
d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 2
0*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))*sqrt((a^6 + 3*a^4*b^2 +
3*a^2*b^4 + b^6 + (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6
+ 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((
(9*a^8*b^2 + 12*a^6*b^4 - 2*a^4*b^6 - 4*a^2*b^8 + b^10)*d^2*sqrt(1/((a^6 +
3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))*cos(d*x + c) + sqrt(2)*((9*a^8*b^3 + 12*
a^6*b^5 - 2*a^4*b^7 - 4*a^2*b^9 + b^11)*d^3*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^
2*b^4 + b^6)*d^4))*cos(d*x + c) + 2*(9*a^5*b^3 - 6*a^3*b^5 + a*b^7)*d*cos(d
*x + c))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^9 - 6*a^5*b^4 - 8*a^3
*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a
^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x +
 c))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^(1/4) + (9*a^5*b^2 - 6*a
^3*b^4 + a*b^6)*cos(d*x + c) + (9*a^4*b^3 - 6*a^2*b^5 + b^7)*sin(d*x + c))/
cos(d*x + c))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^(3/4) + sqrt(2)
(2(3*a^15*b + 17*a^13*b^3 + 39*a^11*b^5 + 45*a^9*b^7 + 25*a^7*b^9 + 3*a^5
*b^11 - 3*a^3*b^13 - a*b^15)*d^7*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12
+ 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^
4))*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (3*a^12*b + 14*a^10
*b^3 + 25*a^8*b^5 + 20*a^6*b^7 + 5*a^4*b^9 - 2*a^2*b^11 - b^13)*d^5*sqrt((9
*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 +
 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
 b^6 + (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x +
 c) + b*sin(d*x + c))/cos(d*x + c))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
*d^4))^(3/4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6)) + 4*sqrt(2)*((a^10 + 3*a^8*b^2
 + 2*a^6*b^4 - 2*a^4*b^6 - 3*a^2*b^8 - b^10)*d^5*cos(d*x + c)^2 + 2*(a^9*b
+ 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*d^5*cos(d*x + c)*sin(d*x + c)
+ (a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)*d^5)*sqrt((a^6 + 3*a
^4*b^2 + 3*a^2*b^4 + b^6 + (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt
(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6
))*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20
*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*(1/((a^6 + 3*a^4*b^2 + 3*a
^2*b^4 + b^6)*d^4))^(3/4)*arctan(-((3*a^12 + 14*a^10*b^2 + 25*a^8*b^4 + 20*
a^6*b^6 + 5*a^4*b^8 - 2*a^2*b^10 - b^12)*d^4*sqrt((9*a^4*b^2 - 6*a^2*b^4 +
b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^1
0 + b^12)*d^4))*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (3*a^9
+ 8*a^7*b^2 + 6*a^5*b^4 - a*b^8)*d^2*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a
^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12

```
)*d^4)) - sqrt(2)*(2*(a^13 + 6*a^11*b^2 + 15*a^9*b^4 + 20*a^7*b^6 + 15*a^5*
b^8 + 6*a^3*b^10 + a*b^12)*d^7*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 +
6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)
)*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (a^10 + 5*a^8*b^2 + 1
0*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4
+ b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b
^10 + b^12)*d^4)))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^9 - 6*a^5*b
^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d
^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt(((9*a^8*b^2 + 12*a^6*b^4 - 2*a^4*
b^6 - 4*a^2*b^8 + b^10)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4
))*cos(d*x + c) - sqrt(2)*((9*a^8*b^3 + 12*a^6*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\tan{(c+dx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**(-3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 5.92, size = 2236, normalized size = 18.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(c + d*x))^(3/2),x)
```

```
[Out] (log(8*b^9*d^2 - ((((-1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i)
)^(1/2)*(64*a*b^11*d^4 + ((-1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d
^2*3i))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5
+ 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5))/2
 + 256*a^3*b^9*d^4 + 384*a^5*b^7*d^4 + 256*a^7*b^5*d^4 + 64*a^9*b^3*d^4))/2
```

- (a + b*tan(c + d*x))^(1/2)*(16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))*(-1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2))/2 + 24*a^2*b^7*d^2 + 24*a^4*b^5*d^2 + 8*a^6*b^3*d^2)*(-1/(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i))^(1/2))/2 - log(8*b^9*d^2 - ((-1/(4*(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i)))^(1/2)*(64*a*b^11*d^4 - (-1/(4*(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5) + 256*a^3*b^9*d^4 + 384*a^5*b^7*d^4 + 256*a^7*b^5*d^4 + 64*a^9*b^3*d^4) + (a + b*tan(c + d*x))^(1/2)*(16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))*(-1/(4*(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i)))^(1/2) + 24*a^2*b^7*d^2 + 24*a^4*b^5*d^2 + 8*a^6*b^3*d^2)*(-1/(4*(a^3*d^2 + b^3*d^2*1i - 3*a*b^2*d^2 - a^2*b*d^2*3i)))^(1/2) + atan((((-1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(64*a*b^11*d^4 + (-1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5) + 256*a^3*b^9*d^4 + 384*a^5*b^7*d^4 + 256*a^7*b^5*d^4 + 64*a^9*b^3*d^4) - (a + b*tan(c + d*x))^(1/2)*(16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))*(-1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*1i - ((-1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(64*a*b^11*d^4 - (-1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5) + 256*a^3*b^9*d^4 + 384*a^5*b^7*d^4 + 256*a^7*b^5*d^4 + 64*a^9*b^3*d^4) + (a + b*tan(c + d*x))^(1/2)*(16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))*(-1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*1i)/(16*b^9*d^2 - ((-1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(64*a*b^11*d^4 - (-1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5) + 256*a^3*b^9*d^4 + 384*a^5*b^7*d^4 + 256*a^7*b^5*d^4 + 64*a^9*b^3*d^4) + (a + b*tan(c + d*x))^(1/2)*(16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))*(-1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2) - ((-1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(64*a*b^11*d^4 + (-1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5) + 256*a^3*b^9*d^4 + 384*a^5*b^7*d^4 + 256*a^7*b^5*d^4 + 64*a^9*b^3*d^4) - (a + b*tan(c + d*x))^(1/2)*(16*b^10*d^3 + 32*a^2*b^8*d^3 - 32*a^6*b^4*d^3 - 16*a^8*b^2*d^3))*(-1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2) + 48*a^2*b^7*d^2 + 48*a^4*b^5*d^2 + 16*a^6*b^3*d^2))*(-1i/(4*(a^3*d^2*1i + b^3*d^2 - a*b^2*d^2*3i - 3*a^2*b*d^2)))^(1/2)*2i - (2*b)/(d*(a^2 + b^2)*(a + b*tan(c + d*x))^(1/2))

3.543 $\int \frac{\cot(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=150

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d}+\frac{}{a(}$$

```
[Out] -2*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+arctanh((a+b*tan(d*x+c
))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(3/2)/d+arctanh((a+b*tan(d*x+c))^(1/2)/(a+I
*b)^(1/2))/(a+I*b)^(3/2)/d+2*b^2/a/(a^2+b^2)/d/(a+b*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3650, 3734, 3620, 3618, 65, 214, 3715}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2b^2}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + ArcTanh[Sqrt[a
 + b*Tan[c + d*x]]/Sqrt[a - I*b]]/((a - I*b)^(3/2)*d) + ArcTanh[Sqrt[a + b*
Tan[c + d*x]]/Sqrt[a + I*b]]/((a + I*b)^(3/2)*d) + (2*b^2)/(a*(a^2 + b^2)*d
*Sqrt[a + b*Tan[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
```

```
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx = \frac{2b^2}{a(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} + \frac{2\int \frac{\cot(c+dx)\left(\frac{1}{2}(a^2+b^2)-\frac{1}{2}ab\tan(c+dx)+\frac{1}{2}b^2\tan^2(c\cdots\right.}{\sqrt{a+b\tan(c+dx)}}}{a(a^2+b^2)}$$

$$= \frac{2b^2}{a(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} + \frac{\int \frac{\cot(c+dx)(1+\tan^2(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx}{a} + \frac{2\int \frac{-\frac{ab}{2}-\cdots}{\sqrt{a+b\cdots}}}{a(\cdots}$$

$$= \frac{2b^2}{a(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} + \frac{\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx}{2(ia-b)} - \frac{\int \frac{1+i\tan\cdots}{\sqrt{a+b\tan\cdots}}}{2(i\cdots}$$

$$= \frac{2b^2}{a(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a-ibx}}\,dx, x, i\tan(c+\cdots\right.}{2(a-ib)d}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2b^2}{a(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} - \frac{\text{Su}\cdots}{\cdots}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d} + \frac{\tan\cdots}{\cdots}$$

Mathematica [A]
time = 1.21, size = 165, normalized size = 1.10

$$\frac{-\frac{2(a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{a(a+ib)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}} + \frac{a(a-ib)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}} + \frac{2b^2}{\sqrt{a+b\tan(c+dx)}}}{a(a^2+b^2)\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-2*(a^2 + b^2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/Sqrt[a] + (a*(a
 + I*b)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/Sqrt[a - I*b] + (a
*(a - I*b)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/Sqrt[a + I*b] +
 (2*b^2)/Sqrt[a + b*Tan[c + d*x]])/(a*(a^2 + b^2)*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 1.74, size = 39358, normalized size = 262.39

method	result	size
default	Expression too large to display	39358

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)/(b*tan(d*x + c) + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5795 vs. $2(122) = 244$.
time = 2.08, size = 11665, normalized size = 77.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(2)*((a^12 + 3*a^10*b^2 + 2*a^8*b^4 - 2*a^6*b^6 - 3*a^4*b^8 - a
^2*b^10)*d^5*cos(d*x + c)^2 + 2*(a^11*b + 4*a^9*b^3 + 6*a^7*b^5 + 4*a^5*b^7
 + a^3*b^9)*d^5*cos(d*x + c)*sin(d*x + c) + (a^10*b^2 + 4*a^8*b^4 + 6*a^6*b
^6 + 4*a^4*b^8 + a^2*b^10)*d^5)*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (
a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((9*a^4*b^2 - 6*a^2*b
^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^
2*b^10 + b^12)*d^4))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^(3/4)*ar
ctan(-((3*a^12 + 14*a^10*b^2 + 25*a^8*b^4 + 20*a^6*b^6 + 5*a^4*b^8 - 2*a^2*
b^10 - b^12)*d^4*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 1
5*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt(1/((a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (3*a^9 + 8*a^7*b^2 + 6*a^5*b^4 - a*
b^8)*d^2*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^
4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) + sqrt(2)*((a^14 + 5
*a^12*b^2 + 9*a^10*b^4 + 5*a^8*b^6 - 5*a^6*b^8 - 9*a^4*b^10 - 5*a^2*b^12 -
b^14)*d^7*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b
^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt(1/((a^6 + 3*a^
4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (a^11 + 5*a^9*b^2 + 10*a^7*b^4 + 10*a^5*b^
6 + 5*a^3*b^8 + a*b^10)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a
```

^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))*
sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*
a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 -
6*a^2*b^4 + b^6))*sqrt(((9*a^8 + 12*a^6*b^2 - 2*a^4*b^4 - 4*a^2*b^6 + b^8)*
d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))*cos(d*x + c) + sqrt(2
)*((9*a^9 + 12*a^7*b^2 - 2*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d^3*sqrt(1/((a^6 +
3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))*cos(d*x + c) + (9*a^6 - 15*a^4*b^2 + 7*a
^2*b^4 - b^6)*d*cos(d*x + c))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^
9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b
^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) + b*si
n(d*x + c))/cos(d*x + c))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^(1/
4) + (9*a^5 - 6*a^3*b^2 + a*b^4)*cos(d*x + c) + (9*a^4*b - 6*a^2*b^3 + b^5)
*sin(d*x + c))/cos(d*x + c))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^
(3/4) + sqrt(2)*((3*a^16 + 14*a^14*b^2 + 22*a^12*b^4 + 6*a^10*b^6 - 20*a^8*
b^8 - 22*a^6*b^10 - 6*a^4*b^12 + 2*a^2*b^14 + b^16)*d^7*sqrt((9*a^4*b^2 - 6
*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8
+ 6*a^2*b^10 + b^12)*d^4))*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)
) + (3*a^13 + 14*a^11*b^2 + 25*a^9*b^4 + 20*a^7*b^6 + 5*a^5*b^8 - 2*a^3*b^1
0 - a*b^12)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15
*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))*sqrt((a^6 +
3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*s
qrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 +
b^6))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*(1/((a^6 + 3*a^4
*b^2 + 3*a^2*b^4 + b^6)*d^4))^(3/4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6)) + 4*sqr
t(2)*((a^12 + 3*a^10*b^2 + 2*a^8*b^4 - 2*a^6*b^6 - 3*a^4*b^8 - a^2*b^10)*d^
5*cos(d*x + c)^2 + 2*(a^11*b + 4*a^9*b^3 + 6*a^7*b^5 + 4*a^5*b^7 + a^3*b^9)
*d^5*cos(d*x + c)*sin(d*x + c) + (a^10*b^2 + 4*a^8*b^4 + 6*a^6*b^6 + 4*a^4*
b^8 + a^2*b^10)*d^5)*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^9 - 6*a^5
*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/(
(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^
12)*d^4))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^(3/4)*arctan(((3*a^
12 + 14*a^10*b^2 + 25*a^8*b^4 + 20*a^6*b^6 + 5*a^4*b^8 - 2*a^2*b^10 - b^12)
*d^4*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 +
20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt(1/((a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6)*d^4)) + (3*a^9 + 8*a^7*b^2 + 6*a^5*b^4 - a*b^8)*d^2*sqr
t((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b
^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) - sqrt(2)*((a^14 + 5*a^12*b^2 +
9*a^10*b^4 + 5*a^8*b^6 - 5*a^6*b^8 - 9*a^4*b^10 - 5*a^2*b^12 - b^14)*d^7*sq
rt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*
b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^
2*b^4 + b^6)*d^4)) + (a^11 + 5*a^9*b^2 + 10*a^7*b^4 + 10*a^5*b^6 + 5*a^3*b^
8 + a*b^10)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15
*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))*sqrt((a^6 +
3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*s

```
qrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 +
b^6))*sqrt(((9*a^8 + 12*a^6*b^2 - 2*a^4*b^4 - 4*a^2*b^6 + b^8)*d^2*sqrt(1/(
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))*cos(d...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(c+dx)}{(a+b\tan(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*tan(d*x+c))**(3/2),x)
[Out] Integral(cot(c + d*x)/(a + b*tan(c + d*x))**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**B**]
time = 4.44, size = 2500, normalized size = 16.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + b*tan(c + d*x))^(3/2),x)
[Out] log(((((8*a^3*d^2 - 24*a*b^2*d^2)^2/64 - b^6*d^4 - a^6*d^4 - 3*a^2*b^4*d^4
- 3*a^4*b^2*d^4)^(1/2) + a^3*d^2 - 3*a*b^2*d^2)/(4*(a^6*d^4 + b^6*d^4 + 3*a
^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2)*((((((8*a^3*d^2 - 24*a*b^2*d^2)^2/64 -
b^6*d^4 - a^6*d^4 - 3*a^2*b^4*d^4 - 3*a^4*b^2*d^4)^(1/2) + a^3*d^2 - 3*a*b^
2*d^2)/(4*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2)*(512*
a^8*b^28*d^8 - (a + b*tan(c + d*x))^(1/2)*((((8*a^3*d^2 - 24*a*b^2*d^2)^2/6
4 - b^6*d^4 - a^6*d^4 - 3*a^2*b^4*d^4 - 3*a^4*b^2*d^4)^(1/2) + a^3*d^2 - 3*
a*b^2*d^2)/(4*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2)*(
512*a^9*b^28*d^9 + 5376*a^11*b^26*d^9 + 25344*a^13*b^24*d^9 + 70656*a^15*b^
22*d^9 + 129024*a^17*b^20*d^9 + 161280*a^19*b^18*d^9 + 139776*a^21*b^16*d^9
 + 82944*a^23*b^14*d^9 + 32256*a^25*b^12*d^9 + 7424*a^27*b^10*d^9 + 768*a^2
9*b^8*d^9) + 5248*a^10*b^26*d^8 + 23936*a^12*b^24*d^8 + 64000*a^14*b^22*d^8
 + 111104*a^16*b^20*d^8 + 130816*a^18*b^18*d^8 + 105728*a^20*b^16*d^8 + 578
```

56*a^22*b^14*d^8 + 20480*a^24*b^12*d^8 + 4224*a^26*b^10*d^8 + 384*a^28*b^8*d^8) + (a + b*tan(c + d*x))^(1/2)*(256*a^8*b^26*d^7 + 1472*a^10*b^24*d^7 + 3712*a^12*b^22*d^7 + 6272*a^14*b^20*d^7 + 9856*a^16*b^18*d^7 + 14336*a^18*b^16*d^7 + 15232*a^20*b^14*d^7 + 10112*a^22*b^12*d^7 + 3712*a^24*b^10*d^7 + 576*a^26*b^8*d^7))*((((8*a^3*d^2 - 24*a*b^2*d^2)^2/64 - b^6*d^4 - a^6*d^4 - 3*a^2*b^4*d^4 - 3*a^4*b^2*d^4)^(1/2) + a^3*d^2 - 3*a*b^2*d^2)/(4*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2) - 128*a^7*b^26*d^6 - 128*a^9*b^24*d^6 + 2592*a^11*b^22*d^6 + 10976*a^13*b^20*d^6 + 20384*a^15*b^18*d^6 + 20832*a^17*b^16*d^6 + 11872*a^19*b^14*d^6 + 3232*a^21*b^12*d^6 + 96*a^23*b^10*d^6 - 96*a^25*b^8*d^6) - (a + b*tan(c + d*x))^(1/2)*(1120*a^15*b^16*d^5 - 352*a^9*b^22*d^5 - 672*a^11*b^20*d^5 - 224*a^13*b^18*d^5 - 64*a^7*b^24*d^5 + 2016*a^17*b^14*d^5 + 1568*a^19*b^12*d^5 + 608*a^21*b^10*d^5 + 96*a^23*b^8*d^5))*((((8*a^3*d^2 - 24*a*b^2*d^2)^2/64 - b^6*d^4 - a^6*d^4 - 3*a^2*b^4*d^4 - 3*a^4*b^2*d^4)^(1/2) + a^3*d^2 - 3*a*b^2*d^2)/(4*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2) + log((-(((8*a^3*d^2 - 24*a*b^2*d^2)^2/64 - b^6*d^4 - a^6*d^4 - 3*a^2*b^4*d^4 - 3*a^4*b^2*d^4)^(1/2) - a^3*d^2 + 3*a*b^2*d^2)/(4*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2)*(((-(((8*a^3*d^2 - 24*a*b^2*d^2)^2/64 - b^6*d^4 - a^6*d^4 - 3*a^2*b^4*d^4 - 3*a^4*b^2*d^4)^(1/2) - a^3*d^2 + 3*a*b^2*d^2)/(4*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2)*(512*a^8*b^28*d^8 - (a + b*tan(c + d*x))^(1/2)*(-(((8*a^3*d^2 - 24*a*b^2*d^2)^2/64 - b^6*d^4 - a^6*d^4 - 3*a^2*b^4*d^4 - 3*a^4*b^2*d^4)^(1/2) - a^3*d^2 + 3*a*b^2*d^2)/(4*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2)*(512*a^9*b^28*d^9 + 5376*a^11*b^26*d^9 + 25344*a^13*b^24*d^9 + 70656*a^15*b^22*d^9 + 129024*a^17*b^20*d^9 + 161280*a^19*b^18*d^9 + 139776*a^21*b^16*d^9 + 82944*a^23*b^14*d^9 + 32256*a^25*b^12*d^9 + 7424*a^27*b^10*d^9 + 768*a^29*b^8*d^9) + 5248*a^10*b^26*d^8 + 23936*a^12*b^24*d^8 + 64000*a^14*b^22*d^8 + 111104*a^16*b^20*d^8 + 130816*a^18*b^18*d^8 + 105728*a^20*b^16*d^8 + 57856*a^22*b^14*d^8 + 20480*a^24*b^12*d^8 + 4224*a^26*b^10*d^8 + 384*a^28*b^8*d^8) + (a + b*tan(c + d*x))^(1/2)*(256*a^8*b^26*d^7 + 1472*a^10*b^24*d^7 + 3712*a^12*b^22*d^7 + 6272*a^14*b^20*d^7 + 9856*a^16*b^18*d^7 + 14336*a^18*b^16*d^7 + 15232*a^20*b^14*d^7 + 10112*a^22*b^12*d^7 + 3712*a^24*b^10*d^7 + 576*a^26*b^8*d^7))*(-(((8*a^3*d^2 - 24*a*b^2*d^2)^2/64 - b^6*d^4 - a^6*d^4 - 3*a^2*b^4*d^4 - 3*a^4*b^2*d^4)^(1/2) - a^3*d^2 + 3*a*b^2*d^2)/(4*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2) - 128*a^7*b^26*d^6 - 128*a^9*b^24*d^6 + 2592*a^11*b^22*d^6 + 10976*a^13*b^20*d^6 + 20384*a^15*b^18*d^6 + 20832*a^17*b^16*d^6 + 11872*a^19*b^14*d^6 + 3232*a^21*b^12*d^6 + 96*a^23*b^10*d^6 - 96*a^25*b^8*d^6) - (a + b*tan(c + d*x))^(1/2)*(1120*a^15*b^16*d^5 - 352*a^9*b^22*d^5 - 672*a^11*b^20*d^5 - 224*a^13*b^18*d^5 - 64*a^7*b^24*d^5 + 2016*a^17*b^14*d^5 + 1568*a^19*b^12*d^5 + 608*a^21*b^10*d^5 + 96*a^23*b^8*d^5))*(-(((8*a^3*d^2 - 24*a*b^2*d^2)^2/64 - b^6*d^4 - a^6*d^4 - 3*a^2*b^4*d^4 - 3*a^4*b^2*d^4)^(1/2) - a^3*d^2 + 3*a*b^2*d^2)/(4*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2) - log(((a^3*d^2 + (6*a^2*b^4*d^4 - b^6*d^4 - 9*a^4*b^2*d^4)^(1/2) - 3*a*b^2*d^2)/(4*a^6*d^4 + 4*b^6*d^4 + 12*a^2*b^4*d^4 + 12*a^4*b^2*d^4))^(1/2)*(((a^3*d^2 + (6*a^2*b^4*d^4 - b^6*d^4 - 9*a^4*b^2*d^4

```
)^(1/2) - 3*a*b^2*d^2)/(4*a^6*d^4 + 4*b^6*d^4 + 12*a^2*b^4*d^4 + 12*a^4*b^2
*d^4))^(1/2)*(((a^3*d^2 + (6*a^2*b^4*d^4 - b^6*d^4 - 9*a^4*b^2*d^4)^(1/2) -
 3*a*b^2*d^2)/(4*a^6*d^4 + 4*b^6*d^4 + 12*a^2*b^4*d^4 + 12*a^4*b^2*d^4))^(1
/2)*(((a^3*d^2 + (6*a^2*b^4*d^4 - b^6*d^4 - 9*a^4*b^2*d^4)^(1/2) - 3*a*b^2*
d^2)/(4*a^6*d^4 + 4*b^6*d^4 + 12*a^2*b^4*d^4 + 12*a^4*b^2*d^4))^(1/2)*(a +
b*tan(c + d*x))^(1/2)*(512*a^9*b^28*d^9 + 5376*a^11*b^26*d^9 + 25344*a^13*b
^24*d^9 + 70656*a^15*b^22*d^9 + 129024*a^17*b^20*d^9 + 161280*a^19*b^18*d^9
 + 139776*a^21*b^16*d^9 + 82944*a^23*b^14*d^9 + 32256*a^25*b^12*d^9 + 7424*
a^27*b^10*d^9 + 768*a^29*b^8*d^9) + 512*a^8*b^2...
```

3.544 $\int \frac{\cot^2(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=192

$$\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d}+\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d}-\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d}-$$

```
[Out] 3*b*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d+I*arctanh((a+b*tan(d*
x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(3/2)/d-I*arctanh((a+b*tan(d*x+c))^(1/2)
/(a+I*b)^(1/2))/(a+I*b)^(3/2)/d-b*(a^2+3*b^2)/a^2/(a^2+b^2)/d/(a+b*tan(d*x+
c))^(1/2)-cot(d*x+c)/a/d/(a+b*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3650, 3730, 3734, 3620, 3618, 65, 214, 3715}

$$\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d}-\frac{b\left(a^2+3b^2\right)}{a^2d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{3/2}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{3/2}}-\frac{\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] (3*b*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/(a^(5/2)*d) + (I*ArcTanh[Sq
rt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/((a - I*b)^(3/2)*d) - (I*ArcTanh[Sqr
t[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/((a + I*b)^(3/2)*d) - (b*(a^2 + 3*b^2
))/(a^2*(a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]]) - Cot[c + d*x]/(a*d*Sqrt[a
+ b*Tan[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
```

```
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx = -\frac{\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}} - \frac{\int \frac{\cot(c+dx)\left(\frac{3b}{2}+a\tan(c+dx)+\frac{3}{2}b\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^{3/2}}\,dx}{a}$$

$$= -\frac{b(a^2+3b^2)}{a^2(a^2+b^2)d\sqrt{a+b\tan(c+dx)}} - \frac{\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}} - \frac{2\int \frac{\cot(c+dx)(\cdots}{\cdots}}{\cdots}$$

$$= -\frac{b(a^2+3b^2)}{a^2(a^2+b^2)d\sqrt{a+b\tan(c+dx)}} - \frac{\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}} - \frac{(3b)\int \frac{\cot(c+\cdots}{\sqrt{a-\cdots}}}{\cdots}$$

$$= -\frac{b(a^2+3b^2)}{a^2(a^2+b^2)d\sqrt{a+b\tan(c+dx)}} - \frac{\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}} - \frac{\int \frac{1+i\,\mathrm{ta}\cdots}{\sqrt{a+b\,\mathrm{ta}\cdots}}}{2(a\cdots}$$

$$= -\frac{b(a^2+3b^2)}{a^2(a^2+b^2)d\sqrt{a+b\tan(c+dx)}} - \frac{\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}} - \frac{3\mathrm{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{3b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} - \frac{b(a^2+3b^2)}{a^2(a^2+b^2)d\sqrt{a+b\tan(c+dx)}} - \frac{\cdots}{aa\cdots}$$

$$= \frac{3b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d} - \frac{i\,\mathrm{ta}\cdots}{\cdots}$$

Mathematica [A]
time = 3.51, size = 184, normalized size = 0.96

$$-\frac{-\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{ia^2\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}} + \frac{ia^2\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}} + \frac{b(a^2+3b^2)}{(a^2+b^2)\sqrt{a+b\tan(c+dx)}} + \frac{a\cot(c+dx)}{\sqrt{a+b\tan(c+dx)}}}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] -(((-3*b*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/Sqrt[a] - (I*a^2*ArcTan
h[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/(a - I*b)^(3/2) + (I*a^2*ArcTanh
[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/(a + I*b)^(3/2) + (b*(a^2 + 3*b^2
))/((a^2 + b^2)*Sqrt[a + b*Tan[c + d*x]]) + (a*Cot[c + d*x])/Sqrt[a + b*Tan
[c + d*x]])/(a^2*d))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 1.55, size = 60231, normalized size = 313.70

method	result	size
default	Expression too large to display	60231

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^2/(b*tan(d*x + c) + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6454 vs. $2(158) = 316$.
time = 2.55, size = 12983, normalized size = 67.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(2)*((a^13 + 3*a^11*b^2 + 2*a^9*b^4 - 2*a^7*b^6 - 3*a^5*b^8 -
a^3*b^10)*d^5*cos(d*x + c)^4 - (a^13 + 2*a^11*b^2 - 2*a^9*b^4 - 8*a^7*b^6 -
 7*a^5*b^8 - 2*a^3*b^10)*d^5*cos(d*x + c)^2 - (a^11*b^2 + 4*a^9*b^4 + 6*a^7
*b^6 + 4*a^5*b^8 + a^3*b^10)*d^5 + 2*((a^12*b + 4*a^10*b^3 + 6*a^8*b^5 + 4*
a^6*b^7 + a^4*b^9)*d^5*cos(d*x + c)^3 - (a^12*b + 4*a^10*b^3 + 6*a^8*b^5 +
4*a^6*b^7 + a^4*b^9)*d^5*cos(d*x + c))*sin(d*x + c))*sqrt((a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6 + (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^
6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt
```

((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^
6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4
+ b^6)*d^4))^(3/4)*arctan(((3*a^12 + 14*a^10*b^2 + 25*a^8*b^4 + 20*a^6*b^6
+ 5*a^4*b^8 - 2*a^2*b^10 - b^12)*d^4*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a
^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12
)*d^4))*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (3*a^9 + 8*a^7*
b^2 + 6*a^5*b^4 - a*b^8)*d^2*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*
a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))
+ sqrt(2)*(2*(a^13 + 6*a^11*b^2 + 15*a^9*b^4 + 20*a^7*b^6 + 15*a^5*b^8 + 6*
a^3*b^10 + a*b^12)*d^7*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b
^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt(1
/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (a^10 + 5*a^8*b^2 + 10*a^6*b^
4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/(
(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^
12)*d^4)))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^9 - 6*a^5*b^4 - 8*a
^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9
*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt(((9*a^8*b^2 + 12*a^6*b^4 - 2*a^4*b^6 - 4*
a^2*b^8 + b^10)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))*cos(d
x + c) + sqrt(2)((9*a^8*b^3 + 12*a^6*b^5 - 2*a^4*b^7 - 4*a^2*b^9 + b^11)*
d^3*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))*cos(d*x + c) + 2*(9*a
^5*b^3 - 6*a^3*b^5 + a*b^7)*d*cos(d*x + c))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b
^4 + b^6 + (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4
*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d
*x + c) + b*sin(d*x + c))/cos(d*x + c))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
b^6)*d^4))^(1/4) + (9*a^5*b^2 - 6*a^3*b^4 + a*b^6)*cos(d*x + c) + (9*a^4*b^
3 - 6*a^2*b^5 + b^7)*sin(d*x + c))/cos(d*x + c))*(1/((a^6 + 3*a^4*b^2 + 3*a
^2*b^4 + b^6)*d^4))^(3/4) + sqrt(2)*(2*(3*a^15*b + 17*a^13*b^3 + 39*a^11*b^
5 + 45*a^9*b^7 + 25*a^7*b^9 + 3*a^5*b^11 - 3*a^3*b^13 - a*b^15)*d^7*sqrt((9
*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 +
 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6)*d^4)) + (3*a^12*b + 14*a^10*b^3 + 25*a^8*b^5 + 20*a^6*b^7 + 5*a^4*b
^9 - 2*a^2*b^11 - b^13)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a
^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))*
sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*
a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 -
6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*(1/(
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^(3/4))/(9*a^4*b^2 - 6*a^2*b^4 + b
^6)) + 4*sqrt(2)*((a^13 + 3*a^11*b^2 + 2*a^9*b^4 - 2*a^7*b^6 - 3*a^5*b^8 -
a^3*b^10)*d^5*cos(d*x + c)^4 - (a^13 + 2*a^11*b^2 - 2*a^9*b^4 - 8*a^7*b^6 -
 7*a^5*b^8 - 2*a^3*b^10)*d^5*cos(d*x + c)^2 - (a^11*b^2 + 4*a^9*b^4 + 6*a^7
*b^6 + 4*a^5*b^8 + a^3*b^10)*d^5 + 2*((a^12*b + 4*a^10*b^3 + 6*a^8*b^5 + 4*
a^6*b^7 + a^4*b^9)*d^5*cos(d*x + c)^3 - (a^12*b + 4*a^10*b^3 + 6*a^8*b^5 +
4*a^6*b^7 + a^4*b^9)*d^5*cos(d*x + c))*sin(d*x + c))*sqrt((a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6 + (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^
6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt

```
((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^
6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4
+ b^6)*d^4))^(3/4)*arctan(-((3*a^12 + 14*a^10*b^2 + 25*a^8*b^4 + 20*a^6*b^6
 + 5*a^4*b^8 - 2*a^2*b^10 - b^12)*d^4*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((
a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^1
2)*d^4))*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (3*a^9 + 8*a^7
*b^2 + 6*a^5*b^4 - a*b^8)*d^2*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6
*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))
 - sqrt(2)*(2*(a^13 + 6*a^11*b^2 + 15*a^9*b^4 + 20*a^7*b^6 + 15*a^5*b^8 + 6
*a^3*b^10 + a*b^12)*d^7*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*
b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt(
1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (a^10 + 5*a^8*b^2 + 10*a^6*b
^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/
((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 +...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(c+dx)}{(a+b\tan(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+b*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(cot(c + d*x)**2/(a + b*tan(c + d*x))**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 4.56, size = 2500, normalized size = 13.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + b*tan(c + d*x))^(3/2),x)
```

```
[Out] log(72*a^14*b^25*d^4 - ((a + b*tan(c + d*x))^(1/2)*(144*a^14*b^26*d^5 + 864
*a^16*b^24*d^5 + 2048*a^18*b^22*d^5 + 2240*a^20*b^20*d^5 + 672*a^22*b^18*d^
```

5 - 896*a^24*b^16*d^5 - 896*a^26*b^14*d^5 - 192*a^28*b^12*d^5 + 80*a^30*b^10*d^5 + 32*a^32*b^8*d^5) + (-(((8*a^3*d^2 - 24*a*b^2*d^2)^2/64 - b^6*d^4 - a^6*d^4 - 3*a^2*b^4*d^4 - 3*a^4*b^2*d^4)^(1/2) + a^3*d^2 - 3*a*b^2*d^2)/(4*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2)*(576*a^15*b^27*d^6 - ((a + b*tan(c + d*x))^(1/2)*(576*a^15*b^28*d^7 + 5184*a^17*b^26*d^7 + 21568*a^19*b^24*d^7 + 53888*a^21*b^22*d^7 + 87808*a^23*b^20*d^7 + 94976*a^25*b^18*d^7 + 66304*a^27*b^16*d^7 + 27008*a^29*b^14*d^7 + 4288*a^31*b^12*d^7 - 832*a^33*b^10*d^7 - 320*a^35*b^8*d^7) - (-(((8*a^3*d^2 - 24*a*b^2*d^2)^2/64 - b^6*d^4 - a^6*d^4 - 3*a^2*b^4*d^4 - 3*a^4*b^2*d^4)^(1/2) + a^3*d^2 - 3*a*b^2*d^2)/(4*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2)*((a + b*tan(c + d*x))^(1/2)*(-(((8*a^3*d^2 - 24*a*b^2*d^2)^2/64 - b^6*d^4 - a^6*d^4 - 3*a^2*b^4*d^4 - 3*a^4*b^2*d^4)^(1/2) + a^3*d^2 - 3*a*b^2*d^2)/(4*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2)*(512*a^18*b^28*d^9 + 5376*a^20*b^26*d^9 + 25344*a^22*b^24*d^9 + 70656*a^24*b^22*d^9 + 129024*a^26*b^20*d^9 + 161280*a^28*b^18*d^9 + 139776*a^30*b^16*d^9 + 82944*a^32*b^14*d^9 + 32256*a^34*b^12*d^9 + 7424*a^36*b^10*d^9 + 768*a^38*b^8*d^9) + 768*a^16*b^29*d^8 + 7680*a^18*b^27*d^8 + 34304*a^20*b^25*d^8 + 90112*a^22*b^23*d^8 + 154112*a^24*b^21*d^8 + 179200*a^26*b^19*d^8 + 143360*a^28*b^17*d^8 + 77824*a^30*b^15*d^8 + 27392*a^32*b^13*d^8 + 5632*a^34*b^11*d^8 + 512*a^36*b^9*d^8))*(-(((8*a^3*d^2 - 24*a*b^2*d^2)^2/64 - b^6*d^4 - a^6*d^4 - 3*a^2*b^4*d^4 - 3*a^4*b^2*d^4)^(1/2) + a^3*d^2 - 3*a*b^2*d^2)/(4*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2) + 3456*a^17*b^25*d^6 + 8480*a^19*b^23*d^6 + 10976*a^21*b^21*d^6 + 8736*a^23*b^19*d^6 + 6496*a^25*b^17*d^6 + 6496*a^27*b^15*d^6 + 5280*a^29*b^13*d^6 + 2336*a^31*b^11*d^6 + 416*a^33*b^9*d^6))*(-(((8*a^3*d^2 - 24*a*b^2*d^2)^2/64 - b^6*d^4 - a^6*d^4 - 3*a^2*b^4*d^4 - 3*a^4*b^2*d^4)^(1/2) + a^3*d^2 - 3*a*b^2*d^2)/(4*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2) + 456*a^16*b^23*d^4 + 1176*a^18*b^21*d^4 + 1512*a^20*b^19*d^4 + 840*a^22*b^17*d^4 - 168*a^24*b^15*d^4 - 504*a^26*b^13*d^4 - 264*a^28*b^11*d^4 - 48*a^30*b^9*d^4)*(-(((8*a^3*d^2 - 24*a*b^2*d^2)^2/64 - b^6*d^4 - a^6*d^4 - 3*a^2*b^4*d^4 - 3*a^4*b^2*d^4)^(1/2) + a^3*d^2 - 3*a*b^2*d^2)/(4*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2) + log(72*a^14*b^25*d^4 - ((a + b*tan(c + d*x))^(1/2)*(144*a^14*b^26*d^5 + 864*a^16*b^24*d^5 + 2048*a^18*b^22*d^5 + 2240*a^20*b^20*d^5 + 672*a^22*b^18*d^5 - 896*a^24*b^16*d^5 - 896*a^26*b^14*d^5 - 192*a^28*b^12*d^5 + 80*a^30*b^10*d^5 + 32*a^32*b^8*d^5) + ((((8*a^3*d^2 - 24*a*b^2*d^2)^2/64 - b^6*d^4 - a^6*d^4 - 3*a^2*b^4*d^4 - 3*a^4*b^2*d^4)^(1/2) - a^3*d^2 + 3*a*b^2*d^2)/(4*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2)*(576*a^15*b^27*d^6 - ((a + b*tan(c + d*x))^(1/2)*(576*a^15*b^28*d^7 + 5184*a^17*b^26*d^7 + 21568*a^19*b^24*d^7 + 53888*a^21*b^22*d^7 + 87808*a^23*b^20*d^7 + 94976*a^25*b^18*d^7 + 66304*a^27*b^16*d^7 + 27008*a^29*b^14*d^7 + 4288*a^31*b^12*d^7 - 832*a^33*b^10*d^7 - 320*a^35*b^8*d^7) - ((((8*a^3*d^2 - 24*a*b^2*d^2)^2/64 - b^6*d^4 - a^6*d^4 - 3*a^2*b^4*d^4 - 3*a^4*b^2*d^4)^(1/2) - a^3*d^2 + 3*a*b^2*d^2)/(4*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2)*((a + b*tan(c + d*x))^(1/2)*((((8*a^3*d^2 - 24*a*b^2*d^2)^2/64 - b^6*d^4 - a^6*d^4 - 3*a^2*b^4*d^4 - 3*a^4*b^2*d^4)^(1/2) - a^3*d^2

+ 3*a*b^2*d^2)/(4*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2)*(512*a^18*b^28*d^9 + 5376*a^20*b^26*d^9 + 25344*a^22*b^24*d^9 + 70656*a^24*b^22*d^9 + 129024*a^26*b^20*d^9 + 161280*a^28*b^18*d^9 + 139776*a^30*b^16*d^9 + 82944*a^32*b^14*d^9 + 32256*a^34*b^12*d^9 + 7424*a^36*b^10*d^9 + 768*a^38*b^8*d^9) + 768*a^16*b^29*d^8 + 7680*a^18*b^27*d^8 + 34304*a^20*b^25*d^8 + 90112*a^22*b^23*d^8 + 154112*a^24*b^21*d^8 + 179200*a^26*b^19*d^8 + 143360*a^28*b^17*d^8 + 77824*a^30*b^15*d^8 + 27392*a^32*b^13*d^8 + 5632*a^34*b^11*d^8 + 512*a^36*b^9*d^8))*((((8*a^3*d^2 - 24*a*b^2*d^2)^2/64 - b^6*d^4 - a^6*d^4 - 3*a^2*b^4*d^4 - 3*a^4*b^2*d^4)^(1/2) - a^3*d^2 + 3*a*b^2*d^2)/(4*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2) + 3456*a^17*b^25*d^6 + 8480*a^19*b^23*d^6 + 10976*a^21*b^21*d^6 + 8736*a^23*b^19*d^6 + 6496*a^25*b^17*d^6 + 6496*a^27*b^15*d^6 + 5280*a^29*b^13*d^6 + 2336*a^31*b^11*d^6 + 416*a^33*b^9*d^6))*((((8*a^3*d^2 - 24*a*b^2*d^2)^2/64 - b^6*d^4 - a^6*d^4 - 3*a^2*b^4*d^4 - 3*a^4*b^2*d^4)^(1/2) - a^3*d^2 + 3*a*b^2*d^2)/(4*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2) + 456*a^16*b^23*d^4 + 1176*a^18*b^21*d^4 + 1512*a^20*b^19*d^4 + 840*a^22*b^17*d^4 - 168*a^24*b^15*d^4 - 504*a^26*b^13*d^4 - 264*a^28*b^11*d^4 - 48*a^30*b^9*d^4)*((((8*a^3*d^2 - 24*a*b^2*d^2)^2/64 - b^6*d^4 - a^6*d^4 - 3*a^2*b^4*d^4 - 3*a^4*b^2*d^4)^(1/2) - a^3*d^2 + 3*a*b^2*d^2)/(4*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2) - log(72*a^14*b^25*...

3.545 $\int \frac{\cot^3(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=241

$$\frac{(8a^2-15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{7/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+d}}{\sqrt{a+ib}}\right.}{(a+ib)^{3/2}d}$$

```
[Out] 1/4*(8*a^2-15*b^2)*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/a^(7/2)/d-arctan
h((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(3/2)/d-arctanh((a+b*tan(d*
x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(3/2)/d+1/4*b^2*(7*a^2+15*b^2)/a^3/(a^2+
b^2)/d/(a+b*tan(d*x+c))^(1/2)+5/4*b*cot(d*x+c)/a^2/d/(a+b*tan(d*x+c))^(1/2)
-1/2*cot(d*x+c)^2/a/d/(a+b*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.61, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3650, 3730, 3731, 3734, 3620, 3618, 65, 214, 3715}

$$\frac{5b\cot(c+dx)}{4a^2d\sqrt{a+b\tan(c+dx)}}+\frac{(8a^2-15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{7/2}d}+\frac{b^2(7a^2+15b^2)}{4a^3d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{3/2}}-\frac{\cot^2(c+dx)}{2ad\sqrt{a+b\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((8*a^2 - 15*b^2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/(4*a^(7/2)*d)
- ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]]/((a - I*b)^(3/2)*d) - Arc
Tanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]]/((a + I*b)^(3/2)*d) + (b^2*(7*
a^2 + 15*b^2))/(4*a^3*(a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]]) + (5*b*Cot[c
+ d*x])/(4*a^2*d*Sqrt[a + b*Tan[c + d*x]]) - Cot[c + d*x]^2/(2*a*d*Sqrt[a +
 b*Tan[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3731

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)
*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp
[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - a*C*(b*c*(m + 1) + a*d*(n
+ 1)) - (m + 1)*(b*c - a*d)*(A*b - b*C)*Tan[e + f*x] - d*(A*b^2 + a^2*C)*(m
 + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m,
 -1] &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx = -\frac{\cot^2(c+dx)}{2ad\sqrt{a+b\tan(c+dx)}} - \frac{\int \frac{\cot^2(c+dx)\left(\frac{5b}{2}+2a\tan(c+dx)+\frac{5}{2}b\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^{3/2}}\,dx}{2a}$$

$$= \frac{5b\cot(c+dx)}{4a^2d\sqrt{a+b\tan(c+dx)}} - \frac{\cot^2(c+dx)}{2ad\sqrt{a+b\tan(c+dx)}} + \frac{\int \frac{\cot(c+dx)(\frac{1}{4}(-8a^2+15}{(a+b\tan(}}{2}$$

$$= \frac{b^2(7a^2+15b^2)}{4a^3(a^2+b^2)d\sqrt{a+b\tan(c+dx)}} + \frac{5b\cot(c+dx)}{4a^2d\sqrt{a+b\tan(c+dx)}} - \frac{\cot}{2ad\sqrt{a+}}$$

$$= \frac{b^2(7a^2+15b^2)}{4a^3(a^2+b^2)d\sqrt{a+b\tan(c+dx)}} + \frac{5b\cot(c+dx)}{4a^2d\sqrt{a+b\tan(c+dx)}} - \frac{\cot}{2ad\sqrt{a+}}$$

$$= \frac{b^2(7a^2+15b^2)}{4a^3(a^2+b^2)d\sqrt{a+b\tan(c+dx)}} + \frac{5b\cot(c+dx)}{4a^2d\sqrt{a+b\tan(c+dx)}} - \frac{\cot}{2ad\sqrt{a+}}$$

$$= \frac{b^2(7a^2+15b^2)}{4a^3(a^2+b^2)d\sqrt{a+b\tan(c+dx)}} + \frac{5b\cot(c+dx)}{4a^2d\sqrt{a+b\tan(c+dx)}} - \frac{\cot}{2ad\sqrt{a+}}$$

$$= \frac{(8a^2-15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{7/2}d} + \frac{b^2(7a^2+15b^2)}{4a^3(a^2+b^2)d\sqrt{a+b\tan(c}}$$

$$= \frac{(8a^2-15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{7/2}d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx}}{\sqrt{a-ib}}\right.}{(a-ib)^{3/2}d}$$

Mathematica [A]
time = 5.72, size = 259, normalized size = 1.07

$$\frac{\frac{(8a^2-15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{-\frac{4a^3\left(a+\sqrt{-b^2}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{\sqrt{a-\sqrt{-b^2}}} + \frac{4a^3\left(-a+\sqrt{-b^2}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+\sqrt{-b^2}}}\right)}{\sqrt{a+\sqrt{-b^2}}} + \frac{7a^2b^2+15b^4+5ab\left(a^2+b^2\right)\cot(c+dx)-2a^2\left(a^2+b^2\right)\cot^2(c+dx)}{\sqrt{a+b\tan(c+dx)}}}{a(a^2+b^2)}}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((8*a^2 - 15*b^2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/a^(3/2) + ((-
4*a^3*(a + Sqrt[-b^2])*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - Sqrt[-b^2]
]])/Sqrt[a - Sqrt[-b^2]] + (4*a^3*(-a + Sqrt[-b^2])*ArcTanh[Sqrt[a + b*Tan[
c + d*x]]/Sqrt[a + Sqrt[-b^2]]])/Sqrt[a + Sqrt[-b^2]] + (7*a^2*b^2 + 15*b^4
```

```
 + 5*a*b*(a^2 + b^2)*Cot[c + d*x] - 2*a^2*(a^2 + b^2)*Cot[c + d*x]^2)/Sqrt[
a + b*Tan[c + d*x]])/(a*(a^2 + b^2)))/(4*a^2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 2.26, size = 93020, normalized size = 385.98

method	result	size
default	Expression too large to display	93020

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6540 vs. 2(201) = 402.
time = 2.18, size = 13156, normalized size = 54.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(16*sqrt(2)*((a^14 + 3*a^12*b^2 + 2*a^10*b^4 - 2*a^8*b^6 - 3*a^6*b^8
 - a^4*b^10)*d^5*cos(d*x + c)^4 - (a^14 + 2*a^12*b^2 - 2*a^10*b^4 - 8*a^8*b
^6 - 7*a^6*b^8 - 2*a^4*b^10)*d^5*cos(d*x + c)^2 - (a^12*b^2 + 4*a^10*b^4 +
6*a^8*b^6 + 4*a^6*b^8 + a^4*b^10)*d^5 + 2*((a^13*b + 4*a^11*b^3 + 6*a^9*b^5
 + 4*a^7*b^7 + a^5*b^9)*d^5*cos(d*x + c)^3 - (a^13*b + 4*a^11*b^3 + 6*a^9*b
^5 + 4*a^7*b^7 + a^5*b^9)*d^5*cos(d*x + c))*sin(d*x + c))*sqrt((a^6 + 3*a^4
*b^2 + 3*a^2*b^4 + b^6 - (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1
/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))
*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a
^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*(1/((a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + b^6)*d^4))^(3/4)*arctan(-((3*a^12 + 14*a^10*b^2 + 25*a^8*b^4 + 20*a^
6*b^6 + 5*a^4*b^8 - 2*a^2*b^10 - b^12)*d^4*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^
6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10
```

+ b^12)*d^4))*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (3*a^9 + 8*a^7*b^2 + 6*a^5*b^4 - a*b^8)*d^2*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) + sqrt(2)*((a^14 + 5*a^12*b^2 + 9*a^10*b^4 + 5*a^8*b^6 - 5*a^6*b^8 - 9*a^4*b^10 - 5*a^2*b^12 - b^14)*d^7*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (a^11 + 5*a^9*b^2 + 10*a^7*b^4 + 10*a^5*b^6 + 5*a^3*b^8 + a*b^10)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt(((9*a^8 + 12*a^6*b^2 - 2*a^4*b^4 - 4*a^2*b^6 + b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))*cos(d*x + c) + sqrt(2)*((9*a^9 + 12*a^7*b^2 - 2*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d^3*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))*cos(d*x + c) + (9*a^6 - 15*a^4*b^2 + 7*a^2*b^4 - b^6)*d*cos(d*x + c))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^(1/4) + (9*a^5 - 6*a^3*b^2 + a*b^4)*cos(d*x + c) + (9*a^4*b - 6*a^2*b^3 + b^5)*sin(d*x + c))/cos(d*x + c))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^(3/4) + sqrt(2)*((3*a^16 + 14*a^14*b^2 + 22*a^12*b^4 + 6*a^10*b^6 - 20*a^8*b^8 - 22*a^6*b^10 - 6*a^4*b^12 + 2*a^2*b^14 + b^16)*d^7*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (3*a^13 + 14*a^11*b^2 + 25*a^9*b^4 + 20*a^7*b^6 + 5*a^5*b^8 - 2*a^3*b^10 - a*b^12)*d^5*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^(3/4))/(9*a^4*b^2 - 6*a^2*b^4 + b^6)) + 16*sqrt(2)*((a^14 + 3*a^12*b^2 + 2*a^10*b^4 - 2*a^8*b^6 - 3*a^6*b^8 - a^4*b^10)*d^5*cos(d*x + c)^4 - (a^14 + 2*a^12*b^2 - 2*a^10*b^4 - 8*a^8*b^6 - 7*a^6*b^8 - 2*a^4*b^10)*d^5*cos(d*x + c)^2 - (a^12*b^2 + 4*a^10*b^4 + 6*a^8*b^6 + 4*a^6*b^8 + a^4*b^10)*d^5 + 2*((a^13*b + 4*a^11*b^3 + 6*a^9*b^5 + 4*a^7*b^7 + a^5*b^9)*d^5*cos(d*x + c)^3 - (a^13*b + 4*a^11*b^3 + 6*a^9*b^5 + 4*a^7*b^7 + a^5*b^9)*d^5*cos(d*x + c))*sin(d*x + c))*sqrt((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 - (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^(3/4)*arctan(((3*a^12 + 14*a^10*b^2 + 25*a^8*b^4 + 20*a^6*b^6 + 5*a^4*b^8 - 2*a^2*b^10 - b^12)*d^4*sqrt((9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 +

```
6*a^2*b^10 + b^12)*d^4))*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))
 + (3*a^9 + 8*a^7*b^2 + 6*a^5*b^4 - a*b^8)*d^2*sqrt((9*a^4*b^2 - 6*a^2*b^4
+ b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b
^10 + b^12)*d^4)) - sqrt(2)*((a^14 + 5*a^12*b^2 + 9*a^10*b^4 + 5*a^8*b^6 -
5*a^6*b^8 - 9*a^4*b^10 - 5*a^2*b^12 - b^14)*d^7*sqrt((9*a^4*b^2 - 6*a^2*b^4
 + b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*
b^10 + b^12)*d^4))*sqrt(1/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (a^1
1 + 5*a^9*b^2 + 10*a^7*b^4 + 10*a^5*b^6 + 5*a^3*b^8 + a*b^10)*d^5*sqrt((9*a
^4*b^2 - 6*a^2*b^4 + b^6)/((a^12 + 6*a^10*b^2 +...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(c+dx)}{(a+b\tan(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+b*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(cot(c + d*x)**3/(a + b*tan(c + d*x))**(3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 4.64, size = 2500, normalized size = 10.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + b*tan(c + d*x))^(3/2),x)
```

```
[Out] ((2*b^4)/(a*(a^2 + b^2)) - ((25*b^4 + 9*a^2*b^2)*(a + b*tan(c + d*x)))/(4*a
^2*(a^2 + b^2)) + (b^2*(7*a^2 + 15*b^2)*(a + b*tan(c + d*x))^2)/(4*a^3*(a^2
 + b^2)))/(d*(a + b*tan(c + d*x))^(5/2) - 2*a*d*(a + b*tan(c + d*x))^(3/2)
+ a^2*d*(a + b*tan(c + d*x))^(1/2)) + log((((((8*a^3*d^2 - 24*a*b^2*d^2)^2/
64 - b^6*d^4 - a^6*d^4 - 3*a^2*b^4*d^4 - 3*a^4*b^2*d^4)^(1/2) + a^3*d^2 - 3
*a*b^2*d^2)/(4*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2)*
((((((8*a^3*d^2 - 24*a*b^2*d^2)^2/64 - b^6*d^4 - a^6*d^4 - 3*a^2*b^4*d^4 -
```

3*a^4*b^2*d^4)^(1/2) + a^3*d^2 - 3*a*b^2*d^2)/(4*(a^6*d^4 + b^6*d^4 + 3*a^2
*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2)*(251658240*a^24*b^30*d^8 - (a + b*tan(c +
d*x))^(1/2)*((((8*a^3*d^2 - 24*a*b^2*d^2)^2/64 - b^6*d^4 - a^6*d^4 - 3*a^2
*b^4*d^4 - 3*a^4*b^2*d^4)^(1/2) + a^3*d^2 - 3*a*b^2*d^2)/(4*(a^6*d^4 + b^6*
d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2)*(134217728*a^27*b^28*d^9 + 140
9286144*a^29*b^26*d^9 + 6643777536*a^31*b^24*d^9 + 18522046464*a^33*b^22*d^
9 + 33822867456*a^35*b^20*d^9 + 42278584320*a^37*b^18*d^9 + 36641439744*a^3
9*b^16*d^9 + 21743271936*a^41*b^14*d^9 + 8455716864*a^43*b^12*d^9 + 1946157
056*a^45*b^10*d^9 + 201326592*a^47*b^8*d^9) + 2382364672*a^26*b^28*d^8 + 99
48889088*a^28*b^26*d^8 + 23924310016*a^30*b^24*d^8 + 36071014400*a^32*b^22*
d^8 + 34292629504*a^34*b^20*d^8 + 18555600896*a^36*b^18*d^8 + 2483027968*a^
38*b^16*d^8 - 3841982464*a^40*b^14*d^8 - 2852126720*a^42*b^12*d^8 - 8556380
16*a^44*b^10*d^8 - 100663296*a^46*b^8*d^8) + (a + b*tan(c + d*x))^(1/2)*(23
5929600*a^22*b^30*d^7 + 1871708160*a^24*b^28*d^7 + 6295650304*a^26*b^26*d^7
+ 11144265728*a^28*b^24*d^7 + 9560915968*a^30*b^22*d^7 - 337641472*a^32*b^
20*d^7 - 9307160576*a^34*b^18*d^7 - 8887730176*a^36*b^16*d^7 - 2943352832*a
^38*b^14*d^7 + 621805568*a^40*b^12*d^7 + 721420288*a^42*b^10*d^7 + 15099494
4*a^44*b^8*d^7))*((((8*a^3*d^2 - 24*a*b^2*d^2)^2/64 - b^6*d^4 - a^6*d^4 - 3
*a^2*b^4*d^4 - 3*a^4*b^2*d^4)^(1/2) + a^3*d^2 - 3*a*b^2*d^2)/(4*(a^6*d^4 +
b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2) + 117964800*a^21*b^30*d^6
+ 699924480*a^23*b^28*d^6 + 1889533952*a^25*b^26*d^6 + 3336568832*a^27*b^24
*d^6 + 4495245312*a^29*b^22*d^6 + 4279238656*a^31*b^20*d^6 + 1923088384*a^3
3*b^18*d^6 - 773849088*a^35*b^16*d^6 - 1421344768*a^37*b^14*d^6 - 587726848
*a^39*b^12*d^6 - 25165824*a^41*b^10*d^6 + 25165824*a^43*b^8*d^6) - (a + b*t
an(c + d*x))^(1/2)*(704643072*a^29*b^20*d^5 - 290979840*a^23*b^26*d^5 - 465
043456*a^25*b^24*d^5 - 37224448*a^27*b^22*d^5 - 58982400*a^21*b^28*d^5 + 76
7033344*a^31*b^18*d^5 + 238551040*a^33*b^16*d^5 + 1572864*a^35*b^14*d^5 + 9
2536832*a^37*b^12*d^5 + 96468992*a^39*b^10*d^5 + 25165824*a^41*b^8*d^5))*((
((8*a^3*d^2 - 24*a*b^2*d^2)^2/64 - b^6*d^4 - a^6*d^4 - 3*a^2*b^4*d^4 - 3*a^
4*b^2*d^4)^(1/2) + a^3*d^2 - 3*a*b^2*d^2)/(4*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4
*d^4 + 3*a^4*b^2*d^4)))^(1/2) + 29491200*a^22*b^26*d^4 + 190709760*a^24*b^2
4*d^4 + 509214720*a^26*b^22*d^4 + 701890560*a^28*b^20*d^4 + 481689600*a^30*
b^18*d^4 + 68812800*a^32*b^16*d^4 - 123863040*a^34*b^14*d^4 - 80609280*a^36
*b^12*d^4 - 15728640*a^38*b^10*d^4)*((((8*a^3*d^2 - 24*a*b^2*d^2)^2/64 - b^
6*d^4 - a^6*d^4 - 3*a^2*b^4*d^4 - 3*a^4*b^2*d^4)^(1/2) + a^3*d^2 - 3*a*b^2*
d^2)/(4*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2) + log((
(-(((8*a^3*d^2 - 24*a*b^2*d^2)^2/64 - b^6*d^4 - a^6*d^4 - 3*a^2*b^4*d^4 - 3
*a^4*b^2*d^4)^(1/2) - a^3*d^2 + 3*a*b^2*d^2)/(4*(a^6*d^4 + b^6*d^4 + 3*a^2*
b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2)*(((-(((8*a^3*d^2 - 24*a*b^2*d^2)^2/64 - b^
6*d^4 - a^6*d^4 - 3*a^2*b^4*d^4 - 3*a^4*b^2*d^4)^(1/2) - a^3*d^2 + 3*a*b^2*
d^2)/(4*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2)*(251658
240*a^24*b^30*d^8 - (a + b*tan(c + d*x))^(1/2)*(-(((8*a^3*d^2 - 24*a*b^2*d^
2)^2/64 - b^6*d^4 - a^6*d^4 - 3*a^2*b^4*d^4 - 3*a^4*b^2*d^4)^(1/2) - a^3*d^
2 + 3*a*b^2*d^2)/(4*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(
1/2)*(134217728*a^27*b^28*d^9 + 1409286144*a^29*b^26*d^9 + 6643777536*a^31*

b^24*d^9 + 18522046464*a^33*b^22*d^9 + 33822867456*a^35*b^20*d^9 + 42278584
320*a^37*b^18*d^9 + 36641439744*a^39*b^16*d^9 + 21743271936*a^41*b^14*d^9 +
 8455716864*a^43*b^12*d^9 + 1946157056*a^45*b^10*d^9 + 201326592*a^47*b^8*d
^9) + 2382364672*a^26*b^28*d^8 + 9948889088*a^28*b^26*d^8 + 23924310016*a^3
0*b^24*d^8 + 36071014400*a^32*b^22*d^8 + 34292629504*a^34*b^20*d^8 + 185556
00896*a^36*b^18*d^8 + 2483027968*a^38*b^16*d^8 - 3841982464*a^40*b^14*d^8 -
 2852126720*a^42*b^12*d^8 - 855638016*a^44*b^10*d^8 - 100663296*a^46*b^8*d^
8) + (a + b*tan(c + d*x))^(1/2)*(235929600*a^22*b^30*d^7 + 1871708160*a^24*
b^28*d^7 + 6295650304*a^26*b^26*d^7 + 11144265728*a^28*b^24*d^7 + 956091596
8*a^30*b^22*d^7 - 337641472*a^32*b^20*d^7 - 9307160576*a^34*b^18*d^7 - 8887
730176*a^36*b^16*d^7 - 2943352832*a^38*b^14*d^7 + 621805568*a^40*b^12*d^7 +
 721420288*a^42*b^10*d^7 + 150994944*a^44*b^8*d^7))*(-(((8*a^3*d^2 - 24*a*b
^2*d^2)^2/64 - b^6*d^4 - a^6*d^4 - 3*a^2*b^4*d^4 - 3*a^4*b^2*d^4)^(1/2) - a
^3*d^2 + 3*a*b^2*d^2)/(4*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4
)))^(1/2) + 117964800*a^21*b^30*d^6 + 699924480...

3.546 $\int \frac{\tan^5(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=291

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d}-\frac{2a^2\tan^3(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}}-$$

```
[Out] -arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(5/2)/d-arctanh((a+b
*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(5/2)/d-4/3*a*(8*a^4+15*a^2*b^2+4
*b^4)*(a+b*tan(d*x+c))^(1/2)/b^4/(a^2+b^2)^2/d+2/3*(8*a^4+15*a^2*b^2+b^4)*(
a+b*tan(d*x+c))^(1/2)*tan(d*x+c)/b^3/(a^2+b^2)^2/d-4*a^2*(a^2+2*b^2)*tan(d*
x+c)^2/b^2/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^(1/2)-2/3*a^2*tan(d*x+c)^3/b/(a^2
+b^2)/d/(a+b*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.55, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3646, 3726, 3728, 3711, 3620, 3618, 65, 214}

$$-\frac{2a^2\tan^3(c+dx)}{3bd\,(a^2+b^2)\,(a+b\tan(c+dx))^{3/2}}-\frac{4a^2(a^2+2b^2)\tan^2(c+dx)}{b^2d\,(a^2+b^2)^2\sqrt{a+b\tan(c+dx)}}-\frac{4a(8a^4+15a^2b^2+4b^4)\sqrt{a+b\tan(c+dx)}}{3b^4d\,(a^2+b^2)^2}+\frac{2(8a^4+15a^2b^2+b^4)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3b^3d\,(a^2+b^2)^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]]/((a - I*b)^(5/2)*d)) - Ar
cTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]]/((a + I*b)^(5/2)*d) - (2*a^2*
Tan[c + d*x]^3)/(3*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^(3/2)) - (4*a^2*(a^
2 + 2*b^2)*Tan[c + d*x]^2)/(b^2*(a^2 + b^2)^2*d*Sqrt[a + b*Tan[c + d*x]]) -
 (4*a*(8*a^4 + 15*a^2*b^2 + 4*b^4)*Sqrt[a + b*Tan[c + d*x]])/(3*b^4*(a^2 +
b^2)^2*d) + (2*(8*a^4 + 15*a^2*b^2 + b^4)*Tan[c + d*x]*Sqrt[a + b*Tan[c + d
*x]])/(3*b^3*(a^2 + b^2)^2*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx = -\frac{2a^2\tan^3(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} + \frac{2\int \frac{\tan^2(c+dx)\left(3a^2-\frac{3}{2}ab\tan(c+dx)+\frac{3}{2}\left(2a^2+b^2\right)\right.}{(a+b\tan(c+dx))^{3/2}}}{3b\,(a^2+b^2)}$$

$$= -\frac{2a^2\tan^3(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{4a^2(a^2+2b^2)\tan^2(c+dx)}{b^2\,(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}} +$$

$$= -\frac{2a^2\tan^3(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{4a^2(a^2+2b^2)\tan^2(c+dx)}{b^2\,(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}} +$$

$$= -\frac{2a^2\tan^3(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{4a^2(a^2+2b^2)\tan^2(c+dx)}{b^2\,(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}} -$$

$$= -\frac{2a^2\tan^3(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{4a^2(a^2+2b^2)\tan^2(c+dx)}{b^2\,(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}} -$$

$$= -\frac{2a^2\tan^3(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{4a^2(a^2+2b^2)\tan^2(c+dx)}{b^2\,(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}} -$$

$$= -\frac{2a^2\tan^3(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{4a^2(a^2+2b^2)\tan^2(c+dx)}{b^2\,(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}} -$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d} - \frac{}{3b\,(a}$$

Mathematica [A]
time = 6.86, size = 556, normalized size = 1.91

$$\frac{\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\left(-\frac{2a(16a^4+31a^2b^2+8b^4)}{3(a-ib)^2(a+ib)^2b^4}+\frac{2a^5}{3(a-ib)^2(a+ib)^2b^2(a\cos(c+dx)+b\sin(c+dx))^2}+\frac{2(7a^5\sin(c+dx)+15a^3b^2\sin(c+dx))}{3(a-ib)^2(a+ib)^2b^3(a\cos(c+dx)+b\sin(c+dx))}+\frac{2\tan(c+dx)}{3b^3}\right)}{d(a+b\tan(c+dx))^{5/2}}+\frac{\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^{5/2}\left(-\frac{2iab\left(\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}}\right)\sqrt{a+b\tan(c+dx)}}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b\sin(c+dx)}}-\frac{(a^2-b^2)\left(\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}}\right)\sqrt{a+b\tan(c+dx)}}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b\sin(c+dx)}}\right)}{(a-ib)^2(a+ib)^2d(a+b\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] (Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^3*((-2*a*(16*a^4 + 31*a^2
*b^2 + 8*b^4))/(3*(a - I*b)^2*(a + I*b)^2*b^4) + (2*a^5)/(3*(a - I*b)^2*(a
+ I*b)^2*b^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) + (2*(7*a^5*Sin[c + d*x]
+ 15*a^3*b^2*Sin[c + d*x]))/(3*(a - I*b)^2*(a + I*b)^2*b^3*(a*Cos[c + d*x]
+ b*Sin[c + d*x])) + (2*Tan[c + d*x])/(3*b^3)))/(d*(a + b*Tan[c + d*x])^(5/
```

```
2)) + (Sec[c + d*x]^(5/2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^(5/2)*(((-2*I)*
a*b*(ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]]/Sqrt[a - I*b] - ArcTan
h[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]]/Sqrt[a + I*b])*Sqrt[a + b*Tan[c +
 d*x]])/(Sqrt[Sec[c + d*x]]*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]]) - ((a^2
- b^2)*(ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]]/Sqrt[a - I*b] + Arc
Tanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]]/Sqrt[a + I*b])*Sqrt[a + b*Tan[
c + d*x]])/(Sqrt[Sec[c + d*x]]*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]])))/((a
 - I*b)^2*(a + I*b)^2*d*(a + b*Tan[c + d*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 999 vs. $2(261) = 522$.

time = 0.22, size = 1000, normalized size = 3.44

method	result
derivativedivides	$\frac{\frac{2(a+b\tan(dx+c))^{\frac{3}{2}}}{3}-6a\sqrt{a+b\tan(dx+c)}-\frac{2b^4\left(\frac{\left(-2\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a^3-2\sqrt{2\sqrt{a}\cdots}\right.}{\cdots}\right.}{\cdots}}{\cdots}$
default	$\frac{\frac{2(a+b\tan(dx+c))^{\frac{3}{2}}}{3}-6a\sqrt{a+b\tan(dx+c)}-\frac{2b^4\left(\frac{\left(-2\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a^3-2\sqrt{2\sqrt{a}\cdots}\right.}{\cdots}\right.}{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/b^4*(1/3*(a+b*tan(d*x+c))^(3/2)-3*a*(a+b*tan(d*x+c))^(1/2)-b^4/(a^2+b^2
)^2*(1/4/(a^2+b^2)^(3/2)*(1/2*(-2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(
1/2)*a^3-2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^2+3*(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)*a^4+2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b^2-(2*(a^2+b^2
)^(1/2)+2*a)^(1/2)*b^4)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*(2*a^5-4*a^3*b^2-6*a*b^4-1/2*(-2*(2*
```

```
(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3-2*(2*(a^2+b^2)^(1/2)+2*a)^(1
/2)*(a^2+b^2)^(1/2)*a*b^2+3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4+2*(2*(a^2+b^2
)^(1/2)+2*a)^(1/2)*a^2*b^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)*(2*(a^2+b^2)^
(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^
(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))+1/4/(a
^2+b^2)^(3/2)*(-1/2*(-2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3-2
*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^2+3*(2*(a^2+b^2)^(1/2)+2
*a)^(1/2)*a^4+2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b^2-(2*(a^2+b^2)^(1/2)+2*
a)^(1/2)*b^4)*ln(-b*tan(d*x+c)-a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+
2*a)^(1/2)-(a^2+b^2)^(1/2))+2*(-2*a^5+4*a^3*b^2+6*a*b^4+1/2*(-2*(2*(a^2+b^2
)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3-2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2
+b^2)^(1/2)*a*b^2+3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4+2*(2*(a^2+b^2)^(1/2)+
2*a)^(1/2)*a^2*b^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)*(2*(a^2+b^2)^(1/2)+2*
a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*tan(d*x+c))^(1/2)+(
2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))))-a^4*(3*a^2+5
*b^2)/(a^2+b^2)^2/(a+b*tan(d*x+c))^(1/2)+1/3*a^5/(a^2+b^2)/(a+b*tan(d*x+c))
^(3/2))
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 10005 vs. $2(257) = 514$.
time = 1.91, size = 10005, normalized size = 34.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(12*sqrt(2)*((a^18*b^4 + a^16*b^6 - 20*a^14*b^8 - 84*a^12*b^10 - 154*a
^10*b^12 - 154*a^8*b^14 - 84*a^6*b^16 - 20*a^4*b^18 + a^2*b^20 + b^22)*d^5*
cos(d*x + c)^5 + 2*(3*a^16*b^6 + 20*a^14*b^8 + 56*a^12*b^10 + 84*a^10*b^12
+ 70*a^8*b^14 + 28*a^6*b^16 - 4*a^2*b^20 - b^22)*d^5*cos(d*x + c)^3 + (a^14
*b^8 + 7*a^12*b^10 + 21*a^10*b^12 + 35*a^8*b^14 + 35*a^6*b^16 + 21*a^4*b^18
 + 7*a^2*b^20 + b^22)*d^5*cos(d*x + c) + 4*((a^17*b^5 + 6*a^15*b^7 + 14*a^1
3*b^9 + 14*a^11*b^11 - 14*a^7*b^15 - 14*a^5*b^17 - 6*a^3*b^19 - a*b^21)*d^5
*cos(d*x + c)^4 + (a^15*b^7 + 7*a^13*b^9 + 21*a^11*b^11 + 35*a^9*b^13 + 35*
a^7*b^15 + 21*a^5*b^17 + 7*a^3*b^19 + a*b^21)*d^5*cos(d*x + c)^2)*sin(d*x +
```

c))*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 - (a^15 - 5*a^13*b^2 - 35*a^11*b^4 - 65*a^9*b^6 - 45*a^7*b^8 + a^5*b^10 + 15*a^3*b^12 + 5*a*b^14)*d^2*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4)))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b^20)*d^4))*(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4))^(3/4)*arctan(((5*a^20 + 30*a^18*b^2 + 61*a^16*b^4 + 8*a^14*b^6 - 182*a^12*b^8 - 364*a^10*b^10 - 350*a^8*b^12 - 184*a^6*b^14 - 47*a^4*b^16 - 2*a^2*b^18 + b^20)*d^4*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b^20)*d^4))*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4)) + (5*a^15 + 15*a^13*b^2 + a^11*b^4 - 45*a^9*b^6 - 65*a^7*b^8 - 35*a^5*b^10 - 5*a^3*b^12 + a*b^14)*d^2*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b^20)*d^4)) - sqrt(2)*((a^23 + 7*a^21*b^2 + 15*a^19*b^4 - 15*a^17*b^6 - 150*a^15*b^8 - 378*a^13*b^10 - 546*a^11*b^12 - 510*a^9*b^14 - 315*a^7*b^16 - 125*a^5*b^18 - 29*a^3*b^20 - 3*a*b^22)*d^7*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b^20)*d^4))*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4)) + (a^18 + 7*a^16*b^2 + 20*a^14*b^4 + 28*a^12*b^6 + 14*a^10*b^8 - 14*a^8*b^10 - 28*a^6*b^12 - 20*a^4*b^14 - 7*a^2*b^16 - b^18)*d^5*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b^20)*d^4)))*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 - (a^15 - 5*a^13*b^2 - 35*a^11*b^4 - 65*a^9*b^6 - 45*a^7*b^8 + a^5*b^10 + 15*a^3*b^12 + 5*a*b^14)*d^2*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4)))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*sqrt(((25*a^14 - 25*a^12*b^2 - 115*a^10*b^4 + 35*a^8*b^6 + 171*a^6*b^8 + 53*a^4*b^10 - 17*a^2*b^12 + b^14)*d^2*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4))*cos(d*x + c) + sqrt(2)*((25*a^16 - 50*a^14*b^2 - 90*a^12*b^4 + 150*a^10*b^6 + 136*a^8*b^8 - 118*a^6*b^10 - 70*a^4*b^12 + 18*a^2*b^14 - b^16)*d^3*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4))*cos(d*x + c) + (25*a^11 - 175*a^9*b^2 + 410*a^7*b^4 - 350*a^5*b^6 + 61*a^3*b^8 - 3*a*b^10)*d*cos(d*x + c))*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 - (a^15 - 5*a^13*b^2 - 35*a^11*b^4 - 65*a^9*b^6 - 45*a^7*b^8 + a^5*b^10 + 15*a^3*b^12 + 5*a*b^14)*d^2*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4)))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*sqrt((a*cos(d*x +

```
c) + b*sin(d*x + c))/cos(d*x + c))*(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10
*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4))^(1/4) + (25*a^9 - 100*a^7*b^2 + 110*a^5*
b^4 - 20*a^3*b^6 + a*b^8)*cos(d*x + c) + (25*a^8*b - 100*a^6*b^3 + 110*a^4*
b^5 - 20*a^2*b^7 + b^9)*sin(d*x + c))/cos(d*x + c))*(1/((a^10 + 5*a^8*b^2 +
 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4))^(3/4) + sqrt(2)*((5*a^27
 + 25*a^25*b^2 + 6*a^23*b^4 - 218*a^21*b^6 - 585*a^19*b^8 - 405*a^17*b^10 +
 900*a^15*b^12 + 2532*a^13*b^14 + 2979*a^11*b^16 + 2015*a^9*b^18 + 790*a^7*
b^20 + 150*a^5*b^22 + a^3*b^24 - 3*a*b^26)*d^7*sqrt((25*a^8*b^2 - 100*a^6*b
^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 +
120*a^14*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 +
 45*a^4*b^16 + 10*a^2*b^18 + b^20)*d^4))*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6
*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4)) + (5*a^22 + 25*a^20*b^2 + 31*a^
18*b^4 - 53*a^16*b^6 - 190*a^14*b^8 - 182*a^12*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5(c+dx)}{(a+b\tan(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+b*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(tan(c + d*x)**5/(a + b*tan(c + d*x))**(5/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 14.33, size = 2500, normalized size = 8.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + b*tan(c + d*x))^(5/2),x)
```

```
[Out] atan(((((1/(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2
*10i - 10*a^3*b^2*d^2))^(1/2)*(48*a*b^20*d^4 - ((1/(a^5*d^2 - b^5*d^2*1i +
5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2))^(1/2)*(a +
```

b*tan(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5))/4 + 368*a^3*b^18*d^4 + 1216*a^5*b^16*d^4 + 2240*a^7*b^14*d^4 + 2464*a^9*b^12*d^4 + 1568*a^11*b^10*d^4 + 448*a^13*b^8*d^4 - 64*a^15*b^6*d^4 - 80*a^17*b^4*d^4 - 16*a^19*b^2*d^4))/2 - ((a + b*tan(c + d*x))^(1/2)*(320*a^4*b^14*d^3 - 16*b^18*d^3 + 1024*a^6*b^12*d^3 + 1440*a^8*b^10*d^3 + 1024*a^10*b^8*d^3 + 320*a^12*b^6*d^3 - 16*a^16*b^2*d^3))/2)*(1/(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2))^(1/2)*1i - (((1/(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2))^(1/2)*(48*a*b^20*d^4 + ((1/(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5))/4 + 368*a^3*b^18*d^4 + 1216*a^5*b^16*d^4 + 2240*a^7*b^14*d^4 + 2464*a^9*b^12*d^4 + 1568*a^11*b^10*d^4 + 448*a^13*b^8*d^4 - 64*a^15*b^6*d^4 - 80*a^17*b^4*d^4 - 16*a^19*b^2*d^4))/2 + ((a + b*tan(c + d*x))^(1/2)*(320*a^4*b^14*d^3 - 16*b^18*d^3 + 1024*a^6*b^12*d^3 + 1440*a^8*b^10*d^3 + 1024*a^10*b^8*d^3 + 320*a^12*b^6*d^3 - 16*a^16*b^2*d^3))/2)*(1/(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2))^(1/2)*1i)/((((1/(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2))^(1/2)*(48*a*b^20*d^4 - ((1/(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5))/4 + 368*a^3*b^18*d^4 + 1216*a^5*b^16*d^4 + 2240*a^7*b^14*d^4 + 2464*a^9*b^12*d^4 + 1568*a^11*b^10*d^4 + 448*a^13*b^8*d^4 - 64*a^15*b^6*d^4 - 80*a^17*b^4*d^4 - 16*a^19*b^2*d^4))/2 - ((a + b*tan(c + d*x))^(1/2)*(320*a^4*b^14*d^3 - 16*b^18*d^3 + 1024*a^6*b^12*d^3 + 1440*a^8*b^10*d^3 + 1024*a^10*b^8*d^3 + 320*a^12*b^6*d^3 - 16*a^16*b^2*d^3))/2)*(1/(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2))^(1/2) + (((1/(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2))^(1/2)*(48*a*b^20*d^4 + ((1/(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5))/4 + 368*a^3*b^18*d^4 + 1216*a^5*b^16*d^4 + 2240*a^7*b^14*d^4 + 2464*a^9*b^12*d^4 + 1568*a^11*b^10*d^4 + 448*a^13*b^8*d^4 - 64*a^15*b^6*d^4 - 80*a^17*b^4*d^4 - 16*a^19*b^2*d^4))/2 + ((a + b*tan(c + d*x))^(1/2)*(320*a^4*b^14*d^3 - 16*b^18*d^3 + 1024*a^6*b^12*d^3 + 1440*a^8*b^10*d^3 + 1024*a^10*b^8*d^3 + 320*a^12*b^6*d^3 - 16*a^16*b^2*d^3))/2)*(1/(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^

2*d^2))^(1/2) - 16*b^16*d^2 - 80*a^2*b^14*d^2 - 144*a^4*b^12*d^2 - 80*a^6*b^10*d^2 + 80*a^8*b^8*d^2 + 144*a^10*b^6*d^2 + 80*a^12*b^4*d^2 + 16*a^14*b^2*d^2))*(1/(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2))^(1/2)*1i - atan(((1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*((1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*(96*a*b^20*d^4 + 736*a^3*b^18*d^4 + 2432*a^5*b^16*d^4 + 4480*a^7*b^14*d^4 + 4928*a^9*b^12*d^4 + 3136*a^11*b^10*d^4 + 896*a^13*b^8*d^4 - 128*a^15*b^6*d^4 - 160*a^17*b^4*d^4 - 32*a^19*b^2*d^4 + (1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5)) + (a + b*tan(c + d*x))^(1/2)*(320*a^4*b^14*d^3 - 16*b^18*d^3 + 1024*a^6*b^12*d^3 + 1440*a^8*b^10*d^3 + 1024*a^10*b^8*d^3 + 320*a^12*b^6*d^3 - 16*a^16*b^2*d^3))*1i - (1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*((1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*(96*a*b^20*d^4 + 7...

3.547 $\int \frac{\tan^4(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=226

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d}+\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d}-\frac{2a^2\tan^2(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}}$$

```
[Out] -I*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(5/2)/d+I*arctanh(
(a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(5/2)/d+4/3*a^3*(2*a^2+5*b^2)
/b^3/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^(1/2)+2/3*(4*a^2+3*b^2)*(a+b*tan(d*x+c)
)^(1/2)/b^3/(a^2+b^2)/d-2/3*a^2*tan(d*x+c)^2/b/(a^2+b^2)/d/(a+b*tan(d*x+c))
^(3/2)
```

Rubi [A]
time = 0.36, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3646, 3716, 3711, 3620, 3618, 65, 214}

$$-\frac{2a^2\tan^2(c+dx)}{3bd\,(a^2+b^2)\,(a+b\tan(c+dx))^{3/2}}+\frac{2(4a^2+3b^2)\sqrt{a+b\tan(c+dx)}}{3b^3d\,(a^2+b^2)}+\frac{4a^3(2a^2+5b^2)}{3b^3d\,(a^2+b^2)^2\sqrt{a+b\tan(c+dx)}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{5/2}}+\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-I)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/((a - I*b)^(5/2)*d)
+ (I*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/((a + I*b)^(5/2)*d) -
 (2*a^2*Tan[c + d*x]^2)/(3*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^(3/2)) + (4
*a^3*(2*a^2 + 5*b^2))/(3*b^3*(a^2 + b^2)^2*d*Sqrt[a + b*Tan[c + d*x]]) + (2
*(4*a^2 + 3*b^2)*Sqrt[a + b*Tan[c + d*x]])/(3*b^3*(a^2 + b^2)*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3716

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
 -1]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx = -\frac{2a^2\tan^2(c+dx)}{3b\left(a^2+b^2\right)d(a+b\tan(c+dx))^{3/2}} + \frac{2\int \frac{\tan(c+dx)\left(2a^2-\frac{3}{2}ab\tan(c+dx)+\frac{1}{2}\left(4a^2+3b\right.\right.}{(a+b\tan(c+dx))^{3/2}}}{3b\left(a^2+b^2\right)}$$

$$= -\frac{2a^2\tan^2(c+dx)}{3b\left(a^2+b^2\right)d(a+b\tan(c+dx))^{3/2}} + \frac{4a^3\left(2a^2+5b^2\right)}{3b^3\left(a^2+b^2\right)^2 d\sqrt{a+b\tan(c+dx)}}$$

$$= -\frac{2a^2\tan^2(c+dx)}{3b\left(a^2+b^2\right)d(a+b\tan(c+dx))^{3/2}} + \frac{4a^3\left(2a^2+5b^2\right)}{3b^3\left(a^2+b^2\right)^2 d\sqrt{a+b\tan(c+dx)}}$$

$$= -\frac{2a^2\tan^2(c+dx)}{3b\left(a^2+b^2\right)d(a+b\tan(c+dx))^{3/2}} + \frac{4a^3\left(2a^2+5b^2\right)}{3b^3\left(a^2+b^2\right)^2 d\sqrt{a+b\tan(c+dx)}}$$

$$= -\frac{2a^2\tan^2(c+dx)}{3b\left(a^2+b^2\right)d(a+b\tan(c+dx))^{3/2}} + \frac{4a^3\left(2a^2+5b^2\right)}{3b^3\left(a^2+b^2\right)^2 d\sqrt{a+b\tan(c+dx)}}$$

$$= -\frac{2a^2\tan^2(c+dx)}{3b\left(a^2+b^2\right)d(a+b\tan(c+dx))^{3/2}} + \frac{4a^3\left(2a^2+5b^2\right)}{3b^3\left(a^2+b^2\right)^2 d\sqrt{a+b\tan(c+dx)}}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d} + \frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d} - 3$$

Mathematica [A]
time = 5.33, size = 216, normalized size = 0.96

$$\frac{-\frac{3i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}} + \frac{3i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}} + \frac{2\left(8a^6+18a^4b^2+3a^2b^4-a^4b^2\sec^2(c+dx)+6ab\left(2a^4+4a^2b^2+b^4\right)\tan(c+dx)+b^2\left(4a^4+6a^2b^2+3b^4\right)\tan^2(c+dx)\right)}{b^3\left(a^2+b^2\right)^2(a+b\tan(c+dx))^{3/2}}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((-3*I)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/(a - I*b)^(5/2) +
 ((3*I)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/(a + I*b)^(5/2) +
(2*(8*a^6 + 18*a^4*b^2 + 3*a^2*b^4 - a^4*b^2*Sec[c + d*x]^2 + 6*a*b*(2*a^4
+ 4*a^2*b^2 + b^4)*Tan[c + d*x] + b^2*(4*a^4 + 6*a^2*b^2 + 3*b^4)*Tan[c + d
*x]^2))/(b^3*(a^2 + b^2)^2*(a + b*Tan[c + d*x])^(3/2)))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 985 vs. $2(196) = 392$.
time = 0.15, size = 986, normalized size = 4.36

method	result
derivativedivides	$\frac{2\sqrt{a+b\tan(dx+c)}+\frac{2b^4\left(\frac{\left(\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a^4-\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+\cdots}\right.}{\cdots}\right.}{\cdots}}{\cdots}$
default	$\frac{2\sqrt{a+b\tan(dx+c)}+\frac{2b^4\left(\frac{\left(\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a^4-\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+\cdots}\right.}{\cdots}\right.}{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/b^3*((a+b*tan(d*x+c))^(1/2)+b^4/(a^2+b^2)^2*(1/4/b^2/(a^2+b^2)^(3/2)*(1
/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^4-(2*(a^2+b^2)^(1/2)+2*
a)^(1/2)*(a^2+b^2)^(1/2)*b^4-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^5+2*(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)*a^3*b^2+3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^4)*ln(b*tan
(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1
/2))+2*(6*a^4*b^2+4*a^2*b^4-2*b^6-1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b
^2)^(1/2)*a^4-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^4-(2*(a^2+b^2
)^(1/2)+2*a)^(1/2)*a^5+2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3*b^2+3*(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)*a*b^4)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2
)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)
)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))+1/4/b^2/(a^2+b^2)^(3/2)*(-1/2*((2*(a^2+b^
2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^4-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+
b^2)^(1/2)*b^4-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^5+2*(2*(a^2+b^2)^(1/2)+2*a)^
(1/2)*a^3*b^2+3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^4)*ln(-b*tan(d*x+c)-a+(a+
b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-(a^2+b^2)^(1/2))+2*(-6*a^
4*b^2-4*a^2*b^4+2*b^6+1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^
4-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^4-(2*(a^2+b^2)^(1/2)+2*a)
^(1/2)*a^5+2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3*b^2+3*(2*(a^2+b^2)^(1/2)+2*a
```

```
)^(1/2)*a*b^4)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)
*arctan((-2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b
^2)^(1/2)-2*a)^(1/2))))-1/3*a^4/(a^2+b^2)/(a+b*tan(d*x+c))^(3/2)+2*a^3*(a^2
+2*b^2)/(a^2+b^2)^2/(a+b*tan(d*x+c))^(1/2))
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 9896 vs. $2(190) = 380$.
time = 1.57, size = 9896, normalized size = 43.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(12*sqrt(2)*((a^18*b^3 + a^16*b^5 - 20*a^14*b^7 - 84*a^12*b^9 - 154*a^
10*b^11 - 154*a^8*b^13 - 84*a^6*b^15 - 20*a^4*b^17 + a^2*b^19 + b^21)*d^5*c
os(d*x + c)^4 + 2*(3*a^16*b^5 + 20*a^14*b^7 + 56*a^12*b^9 + 84*a^10*b^11 +
70*a^8*b^13 + 28*a^6*b^15 - 4*a^2*b^19 - b^21)*d^5*cos(d*x + c)^2 + (a^14*b
^7 + 7*a^12*b^9 + 21*a^10*b^11 + 35*a^8*b^13 + 35*a^6*b^15 + 21*a^4*b^17 +
7*a^2*b^19 + b^21)*d^5 + 4*((a^17*b^4 + 6*a^15*b^6 + 14*a^13*b^8 + 14*a^11*
b^10 - 14*a^7*b^14 - 14*a^5*b^16 - 6*a^3*b^18 - a*b^20)*d^5*cos(d*x + c)^3
+ (a^15*b^6 + 7*a^13*b^8 + 21*a^11*b^10 + 35*a^9*b^12 + 35*a^7*b^14 + 21*a^
5*b^16 + 7*a^3*b^18 + a*b^20)*d^5*cos(d*x + c))*sin(d*x + c))*sqrt((a^10 +
5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 + (a^15 - 5*a^13*b^2
 - 35*a^11*b^4 - 65*a^9*b^6 - 45*a^7*b^8 + a^5*b^10 + 15*a^3*b^12 + 5*a*b^1
4)*d^2*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^
10)*d^4)))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*sq
rt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^20 + 10
*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210
*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b^20)*d^4))*(1/((a^1
0 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4))^(3/4)*arc
tan(((5*a^20 + 30*a^18*b^2 + 61*a^16*b^4 + 8*a^14*b^6 - 182*a^12*b^8 - 364*
a^10*b^10 - 350*a^8*b^12 - 184*a^6*b^14 - 47*a^4*b^16 - 2*a^2*b^18 + b^20)*
d^4*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^2
0 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 + 252*a^10*b^10
 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b^20)*d^4))*sq
```

rt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4)) + (5*a^15 + 15*a^13*b^2 + a^11*b^4 - 45*a^9*b^6 - 65*a^7*b^8 - 35*a^5*b^10 - 5*a^3*b^12 + a*b^14)*d^2*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b^20)*d^4)) - sqrt(2)*((3*a^22 + 29*a^20*b^2 + 125*a^18*b^4 + 315*a^16*b^6 + 510*a^14*b^8 + 546*a^12*b^10 + 378*a^10*b^12 + 150*a^8*b^14 + 15*a^6*b^16 - 15*a^4*b^18 - 7*a^2*b^20 - b^22)*d^7*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b^20)*d^4))*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4)) + 2*(a^17 + 8*a^15*b^2 + 28*a^13*b^4 + 56*a^11*b^6 + 70*a^9*b^8 + 56*a^7*b^10 + 28*a^5*b^12 + 8*a^3*b^14 + a*b^16)*d^5*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b^20)*d^4)))*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 + (a^15 - 5*a^13*b^2 - 35*a^11*b^4 - 65*a^9*b^6 - 45*a^7*b^8 + a^5*b^10 + 15*a^3*b^12 + 5*a*b^14)*d^2*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4)))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*sqrt(((25*a^14*b^2 - 25*a^12*b^4 - 115*a^10*b^6 + 35*a^8*b^8 + 171*a^6*b^10 + 53*a^4*b^12 - 17*a^2*b^14 + b^16)*d^2*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4))*cos(d*x + c) + sqrt(2)*(2*(25*a^15*b^3 - 25*a^13*b^5 - 115*a^11*b^7 + 35*a^9*b^9 + 171*a^7*b^11 + 53*a^5*b^13 - 17*a^3*b^15 + a*b^17)*d^3*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4))*cos(d*x + c) + (75*a^10*b^3 - 325*a^8*b^5 + 430*a^6*b^7 - 170*a^4*b^9 + 23*a^2*b^11 - b^13)*d*cos(d*x + c))*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 + (a^15 - 5*a^13*b^2 - 35*a^11*b^4 - 65*a^9*b^6 - 45*a^7*b^8 + a^5*b^10 + 15*a^3*b^12 + 5*a*b^14)*d^2*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4)))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4))^(1/4) + (25*a^9*b^2 - 100*a^7*b^4 + 110*a^5*b^6 - 20*a^3*b^8 + a*b^10)*cos(d*x + c) + (25*a^8*b^3 - 100*a^6*b^5 + 110*a^4*b^7 - 20*a^2*b^9 + b^11)*sin(d*x + c))/cos(d*x + c))*(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4))^(3/4) + sqrt(2)*((15*a^26*b + 115*a^24*b^3 + 338*a^22*b^5 + 354*a^20*b^7 - 475*a^18*b^9 - 2055*a^16*b^11 - 3060*a^14*b^13 - 2484*a^12*b^15 - 1047*a^10*b^17 - 75*a^8*b^19 + 130*a^6*b^21 + 50*a^4*b^23 + 3*a^2*b^25 - b^27)*d^7*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b^20)*d^4))*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4)) + 2*(5*a^21*b + 30*a^19*b^3 + 61*a^17*b^5 + 8*a^15*b^7 - 182*a^13*b^9 - 364*a^11*b^11 - 350*a^9*b^1...

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(c+dx)}{(a+b\tan(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+b*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(tan(c + d*x)**4/(a + b*tan(c + d*x))**(5/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 11.24, size = 2500, normalized size = 11.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + b*tan(c + d*x))^(5/2),x)
```

```
[Out] (log(16*a*b^15*d^2 - ((-1/(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5
i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2))^(1/2)*(((-1/(a^5*d^2 - b^5*d^2*1i +
5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2))^(1/2)*(896*
a^6*b^15*d^4 - 160*a^2*b^19*d^4 - 128*a^4*b^17*d^4 - 32*b^21*d^4 + 3136*a^8
*b^13*d^4 + 4928*a^10*b^11*d^4 + 4480*a^12*b^9*d^4 + 2432*a^14*b^7*d^4 + 73
6*a^16*b^5*d^4 + 96*a^18*b^3*d^4 + ((-1/(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2
 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2))^(1/2)*(a + b*tan(c + d
*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7
*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5
+ 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^
5))/2))/2 - (a + b*tan(c + d*x))^(1/2)*(320*a^4*b^14*d^3 - 16*b^18*d^3 + 10
24*a^6*b^12*d^3 + 1440*a^8*b^10*d^3 + 1024*a^10*b^8*d^3 + 320*a^12*b^6*d^3
- 16*a^16*b^2*d^3)))/2 + 96*a^3*b^13*d^2 + 240*a^5*b^11*d^2 + 320*a^7*b^9*d
^2 + 240*a^9*b^7*d^2 + 96*a^11*b^5*d^2 + 16*a^13*b^3*d^2)*(-1/(a^5*d^2 - b^
5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2))^
(1/2))/2 - log(16*a*b^15*d^2 - ((-1/(4*(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2
```

- a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2)))^(1/2)*(896*a^6*b^15*d^
4 - 160*a^2*b^19*d^4 - 128*a^4*b^17*d^4 - 32*b^21*d^4 + 3136*a^8*b^13*d^4 +
4928*a^10*b^11*d^4 + 4480*a^12*b^9*d^4 + 2432*a^14*b^7*d^4 + 736*a^16*b^5*
d^4 + 96*a^18*b^3*d^4 - (-1/(4*(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*
d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2)))^(1/2)*(a + b*tan(c + d*x))^(1/
2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^
5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a
^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5)) + (a
+ b*tan(c + d*x))^(1/2)*(320*a^4*b^14*d^3 - 16*b^18*d^3 + 1024*a^6*b^12*d^
3 + 1440*a^8*b^10*d^3 + 1024*a^10*b^8*d^3 + 320*a^12*b^6*d^3 - 16*a^16*b^2*
d^3))*(-1/(4*(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d
^2*10i - 10*a^3*b^2*d^2)))^(1/2) + 96*a^3*b^13*d^2 + 240*a^5*b^11*d^2 + 320
*a^7*b^9*d^2 + 240*a^9*b^7*d^2 + 96*a^11*b^5*d^2 + 16*a^13*b^3*d^2)*(-1/(4*
(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a
^3*b^2*d^2)))^(1/2) + atan((((-1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i -
5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*(896*a^6*b^15*d^4
- 160*a^2*b^19*d^4 - 128*a^4*b^17*d^4 - 32*b^21*d^4 + 3136*a^8*b^13*d^4 + 4
928*a^10*b^11*d^4 + 4480*a^12*b^9*d^4 + 2432*a^14*b^7*d^4 + 736*a^16*b^5*d^
4 + 96*a^18*b^3*d^4 + (-1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i - 5*a^4*
b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*(a + b*tan(c + d*x))^(1/2
)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5
+ 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^
15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5)) - (a
+ b*tan(c + d*x))^(1/2)*(320*a^4*b^14*d^3 - 16*b^18*d^3 + 1024*a^6*b^12*d^3
+ 1440*a^8*b^10*d^3 + 1024*a^10*b^8*d^3 + 320*a^12*b^6*d^3 - 16*a^16*b^2*d
^3))*(-1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^
3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*1i - ((-1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^
4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*(896*a^6
*b^15*d^4 - 160*a^2*b^19*d^4 - 128*a^4*b^17*d^4 - 32*b^21*d^4 + 3136*a^8*b^
13*d^4 + 4928*a^10*b^11*d^4 + 4480*a^12*b^9*d^4 + 2432*a^14*b^7*d^4 + 736*a
^16*b^5*d^4 + 96*a^18*b^3*d^4 - (-1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5
i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*(a + b*tan(c +
d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^
7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5
+ 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d
^5)) + (a + b*tan(c + d*x))^(1/2)*(320*a^4*b^14*d^3 - 16*b^18*d^3 + 1024*a^
6*b^12*d^3 + 1440*a^8*b^10*d^3 + 1024*a^10*b^8*d^3 + 320*a^12*b^6*d^3 - 16*
a^16*b^2*d^3))*(-1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 +
10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*1i)/(32*a*b^15*d^2 - ((-1i/(4*(a
^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2
*d^2*10i)))^(1/2)*(896*a^6*b^15*d^4 - 160*a^2*b^19*d^4 - 128*a^4*b^17*d^4 -
32*b^21*d^4 + 3136*a^8*b^13*d^4 + 4928*a^10*b^11*d^4 + 4480*a^12*b^9*d^4 +
2432*a^14*b^7*d^4 + 736*a^16*b^5*d^4 + 96*a^18*b^3*d^4 - (-1i/(4*(a^5*d^2*
1i - b^5*d^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10
i)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2

```
880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12
*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^
19*b^4*d^5 + 64*a^21*b^2*d^5)) + (a + b*tan(c + d*x))^(1/2)*(320*a^4*b^14*d
^3 - 16*b^18*d^3 + 1024*a^6*b^12*d^3 + 1440*a^8*b^10*d^3 + 1024*a^10*b^8*d^
3 + 320*a^12*b^6*d^3 - 16*a^16*b^2*d^3))*(-1i/(4*(a^5*d^2*1i - b^5*d^2 + a*
b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3...
```

3.548 $\int \frac{\tan^3(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=172

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d}-\frac{2a^2\tan(c+dx)}{3b\left(a^2+b^2\right)d(a+b\tan(c+dx))^{3/2}}-\frac{}{3b^2}$$

```
[Out] arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(5/2)/d+arctanh((a+b*
tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(5/2)/d-4/3*a^2*(a^2+4*b^2)/b^2/(a
^2+b^2)^2/d/(a+b*tan(d*x+c))^(1/2)-2/3*a^2*tan(d*x+c)/b/(a^2+b^2)/d/(a+b*ta
n(d*x+c))^(3/2)
```

Rubi [A]
time = 0.26, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3646, 3709, 3620, 3618, 65, 214}

$$-\frac{4a^2\left(a^2+4b^2\right)}{3b^2d\left(a^2+b^2\right)^2\sqrt{a+b\tan(c+dx)}}-\frac{2a^2\tan(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{5/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]]/((a - I*b)^(5/2)*d) + ArcTa
nh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]]/((a + I*b)^(5/2)*d) - (2*a^2*Tan
[c + d*x])/(3*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^(3/2)) - (4*a^2*(a^2 + 4
*b^2))/(3*b^2*(a^2 + b^2)^2*d*Sqrt[a + b*Tan[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
```

```
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx = -\frac{2a^2\tan(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} + \frac{2\int \frac{a^2-\frac{3}{2}ab\tan(c+dx)+\frac{1}{2}(2a^2+3b^2)\tan^2(c+dx)}{(a+b\tan(c+dx))^{3/2}}$$

$$= -\frac{2a^2\tan(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{4a^2(a^2+4b^2)}{3b^2\,(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}}$$

$$= -\frac{2a^2\tan(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{4a^2(a^2+4b^2)}{3b^2\,(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}}$$

$$= -\frac{2a^2\tan(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{4a^2(a^2+4b^2)}{3b^2\,(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}}$$

$$= -\frac{2a^2\tan(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{4a^2(a^2+4b^2)}{3b^2\,(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d} - \frac{}{3b\,(a^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.52, size = 220, normalized size = 1.28

$$\frac{-\frac{4a}{b}-\frac{a\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\tan(c+dx)}{a-ib}\right)}{ia+b}+\frac{a\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\tan(c+dx)}{a+ib}\right)}{ia-b}-6\tan(c+dx)+\frac{3\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\tan(c+dx)}{a-ib}\right)(a+b\tan(c+dx))}{ia+b}+\frac{3i\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\tan(c+dx)}{a+ib}\right)(a+b\tan(c+dx))}{a+ib}}{3bd(a+b\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-4*a)/b - (a*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[c + d*x])/(a - I
*b)])/(I*a + b) + (a*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[c + d*x])/
(a + I*b)])/(I*a - b) - 6*Tan[c + d*x] + (3*Hypergeometric2F1[-1/2, 1, 1/2,
 (a + b*Tan[c + d*x])/(a - I*b)]*(a + b*Tan[c + d*x]))/(I*a + b) + ((3*I)*H
ypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[c + d*x])/(a + I*b)]*(a + b*Tan[c
 + d*x]))/(a + I*b))/(3*b*d*(a + b*Tan[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 967 vs. 2(148) = 296.
time = 0.13, size = 968, normalized size = 5.63

method	result
derivativedivides	$2b^2\left(\frac{\left(-2\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a^3-2\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a\,b^2+3\sqrt{2\sqrt{a^2+}\right.}{}\right.$
default	$2b^2\left(\frac{\left(-2\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a^3-2\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a\,b^2+3\sqrt{2\sqrt{a^2+}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/b^2*(b^2/(a^2+b^2)^2*(1/4/(a^2+b^2)^(3/2)*(1/2*(-2*(2*(a^2+b^2)^(1/2)+2
*a)^(1/2)*(a^2+b^2)^(1/2)*a^3-2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/
2)*a*b^2+3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4+2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2
)*a^2*b^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x
+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*(2*a^5-4*a^3*b^
2-6*a*b^4-1/2*(-2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3-2*(2*(a
^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^2+3*(2*(a^2+b^2)^(1/2)+2*a)^(1
/2)*a^4+2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/
2)*b^4)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan
((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2
)-2*a)^(1/2)))+1/4/(a^2+b^2)^(3/2)*(-1/2*(-2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*
(a^2+b^2)^(1/2)*a^3-2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^2+3
*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4+2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b^2-
(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)*ln(-b*tan(d*x+c)-a+(a+b*tan(d*x+c))^(1/2
)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-(a^2+b^2)^(1/2))+2*(-2*a^5+4*a^3*b^2+6*a*b^
4+1/2*(-2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3-2*(2*(a^2+b^2)^
(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^2+3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4+
2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)*
```

(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))))-a^2*(a^2+3*b^2)/(a^2+b^2)^2/(a+b*tan(d*x+c))^(1/2)+1/3*a^3/(a^2+b^2)/(a+b*tan(d*x+c))^(3/2))

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^3/(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")

[Out] Timed out

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 9834 vs. $2(144) = 288$.
time = 1.61, size = 9834, normalized size = 57.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^3/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")

[Out] -1/12*(12*sqrt(2)*((a^18*b^2 + a^16*b^4 - 20*a^14*b^6 - 84*a^12*b^8 - 154*a^10*b^10 - 154*a^8*b^12 - 84*a^6*b^14 - 20*a^4*b^16 + a^2*b^18 + b^20)*d^5*cos(d*x + c)^4 + 2*(3*a^16*b^4 + 20*a^14*b^6 + 56*a^12*b^8 + 84*a^10*b^10 + 70*a^8*b^12 + 28*a^6*b^14 - 4*a^2*b^18 - b^20)*d^5*cos(d*x + c)^2 + (a^14*b^6 + 7*a^12*b^8 + 21*a^10*b^10 + 35*a^8*b^12 + 35*a^6*b^14 + 21*a^4*b^16 + 7*a^2*b^18 + b^20)*d^5 + 4*((a^17*b^3 + 6*a^15*b^5 + 14*a^13*b^7 + 14*a^11*b^9 - 14*a^7*b^13 - 14*a^5*b^15 - 6*a^3*b^17 - a*b^19)*d^5*cos(d*x + c)^3 + (a^15*b^5 + 7*a^13*b^7 + 21*a^11*b^9 + 35*a^9*b^11 + 35*a^7*b^13 + 21*a^5*b^15 + 7*a^3*b^17 + a*b^19)*d^5*cos(d*x + c))*sin(d*x + c))*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 - (a^15 - 5*a^13*b^2 - 35*a^11*b^4 - 65*a^9*b^6 - 45*a^7*b^8 + a^5*b^10 + 15*a^3*b^12 + 5*a*b^14)*d^2*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4)))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b^20)*d^4))*(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4))^(3/4)*arctan(((5*a^20 + 30*a^18*b^2 + 61*a^16*b^4 + 8*a^14*b^6 - 182*a^12*b^8 - 364*a^10*b^10 - 350*a^8*b^12 - 184*a^6*b^14 - 47*a^4*b^16 - 2*a^2*b^18 + b^20)*d^4*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b^20)*d^4))*sqr

t(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4))
+ (5*a^15 + 15*a^13*b^2 + a^11*b^4 - 45*a^9*b^6 - 65*a^7*b^8 - 35*a^5*b^10
- 5*a^3*b^12 + a*b^14)*d^2*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 2
0*a^2*b^8 + b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a
^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^
2*b^18 + b^20)*d^4)) - sqrt(2)*((a^23 + 7*a^21*b^2 + 15*a^19*b^4 - 15*a^17*
b^6 - 150*a^15*b^8 - 378*a^13*b^10 - 546*a^11*b^12 - 510*a^9*b^14 - 315*a^7
*b^16 - 125*a^5*b^18 - 29*a^3*b^20 - 3*a*b^22)*d^7*sqrt((25*a^8*b^2 - 100*a
^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^
4 + 120*a^14*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^
14 + 45*a^4*b^16 + 10*a^2*b^18 + b^20)*d^4))*sqrt(1/((a^10 + 5*a^8*b^2 + 10
*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4)) + (a^18 + 7*a^16*b^2 + 20*a
^14*b^4 + 28*a^12*b^6 + 14*a^10*b^8 - 14*a^8*b^10 - 28*a^6*b^12 - 20*a^4*b^
14 - 7*a^2*b^16 - b^18)*d^5*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 -
20*a^2*b^8 + b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*
a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a
^2*b^18 + b^20)*d^4)))*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5
*a^2*b^8 + b^10 - (a^15 - 5*a^13*b^2 - 35*a^11*b^4 - 65*a^9*b^6 - 45*a^7*b^
8 + a^5*b^10 + 15*a^3*b^12 + 5*a*b^14)*d^2*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a
^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4)))/(25*a^8*b^2 - 100*a^6*b^4 +
110*a^4*b^6 - 20*a^2*b^8 + b^10))*sqrt(((25*a^14 - 25*a^12*b^2 - 115*a^10*b
^4 + 35*a^8*b^6 + 171*a^6*b^8 + 53*a^4*b^10 - 17*a^2*b^12 + b^14)*d^2*sqrt(
1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4))*co
s(d*x + c) + sqrt(2)*((25*a^16 - 50*a^14*b^2 - 90*a^12*b^4 + 150*a^10*b^6 +
136*a^8*b^8 - 118*a^6*b^10 - 70*a^4*b^12 + 18*a^2*b^14 - b^16)*d^3*sqrt(1/
((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4))*cos(
d*x + c) + (25*a^11 - 175*a^9*b^2 + 410*a^7*b^4 - 350*a^5*b^6 + 61*a^3*b^8
- 3*a*b^10)*d*cos(d*x + c))*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^
6 + 5*a^2*b^8 + b^10 - (a^15 - 5*a^13*b^2 - 35*a^11*b^4 - 65*a^9*b^6 - 45*a
^7*b^8 + a^5*b^10 + 15*a^3*b^12 + 5*a*b^14)*d^2*sqrt(1/((a^10 + 5*a^8*b^2 +
10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4)))/(25*a^8*b^2 - 100*a^6*b
^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*sqrt((a*cos(d*x + c) + b*sin(d*x + c
))/cos(d*x + c))*(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^
8 + b^10)*d^4))^(1/4) + (25*a^9 - 100*a^7*b^2 + 110*a^5*b^4 - 20*a^3*b^6 +
a*b^8)*cos(d*x + c) + (25*a^8*b - 100*a^6*b^3 + 110*a^4*b^5 - 20*a^2*b^7 +
b^9)*sin(d*x + c))/cos(d*x + c))*(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^
4*b^6 + 5*a^2*b^8 + b^10)*d^4))^(3/4) + sqrt(2)*((5*a^27 + 25*a^25*b^2 + 6*
a^23*b^4 - 218*a^21*b^6 - 585*a^19*b^8 - 405*a^17*b^10 + 900*a^15*b^12 + 25
32*a^13*b^14 + 2979*a^11*b^16 + 2015*a^9*b^18 + 790*a^7*b^20 + 150*a^5*b^22
+ a^3*b^24 - 3*a*b^26)*d^7*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 -
20*a^2*b^8 + b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*
a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a
^2*b^18 + b^20)*d^4))*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 +
5*a^2*b^8 + b^10)*d^4)) + (5*a^22 + 25*a^20*b^2 + 31*a^18*b^4 - 53*a^16*b^
6 - 190*a^14*b^8 - 182*a^12*b^10 + 14*a^10*b^12...

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(c+dx)}{(a+b\tan(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+b*tan(d*x+c))**(5/2),x)
[Out] Integral(tan(c + d*x)**3/(a + b*tan(c + d*x))**(5/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [B]
time = 9.45, size = 2500, normalized size = 14.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + b*tan(c + d*x))^(5/2),x)
[Out] atan(((1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^
3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*((1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*
5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*(96*a*b^20*d^4
 + 736*a^3*b^18*d^4 + 2432*a^5*b^16*d^4 + 4480*a^7*b^14*d^4 + 4928*a^9*b^12
*d^4 + 3136*a^11*b^10*d^4 + 896*a^13*b^8*d^4 - 128*a^15*b^6*d^4 - 160*a^17*
b^4*d^4 - 32*a^19*b^2*d^4 + (1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i - 5
*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*(a + b*tan(c + d*x))
^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^1
6*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 76
80*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5))
+ (a + b*tan(c + d*x))^(1/2)*(320*a^4*b^14*d^3 - 16*b^18*d^3 + 1024*a^6*b^1
2*d^3 + 1440*a^8*b^10*d^3 + 1024*a^10*b^8*d^3 + 320*a^12*b^6*d^3 - 16*a^16*
b^2*d^3))*1i - (1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 +
10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*((1i/(4*(a^5*d^2*1i - b^5*d^2 + a
*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*(96*a
*b^20*d^4 + 736*a^3*b^18*d^4 + 2432*a^5*b^16*d^4 + 4480*a^7*b^14*d^4 + 4928
```

*a^9*b^12*d^4 + 3136*a^11*b^10*d^4 + 896*a^13*b^8*d^4 - 128*a^15*b^6*d^4 -
160*a^17*b^4*d^4 - 32*a^19*b^2*d^4 - (1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d
^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*(a + b*tan(
c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 768
0*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10
*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b
^2*d^5)) - (a + b*tan(c + d*x))^(1/2)*(320*a^4*b^14*d^3 - 16*b^18*d^3 + 102
4*a^6*b^12*d^3 + 1440*a^8*b^10*d^3 + 1024*a^10*b^8*d^3 + 320*a^12*b^6*d^3 -
 16*a^16*b^2*d^3))*1i)/((1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i - 5*a^4
*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*((1i/(4*(a^5*d^2*1i - b^
5*d^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1
/2)*(96*a*b^20*d^4 + 736*a^3*b^18*d^4 + 2432*a^5*b^16*d^4 + 4480*a^7*b^14*d
^4 + 4928*a^9*b^12*d^4 + 3136*a^11*b^10*d^4 + 896*a^13*b^8*d^4 - 128*a^15*b
^6*d^4 - 160*a^17*b^4*d^4 - 32*a^19*b^2*d^4 + (1i/(4*(a^5*d^2*1i - b^5*d^2
+ a*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*(a
 + b*tan(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*
d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*
a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 +
64*a^21*b^2*d^5)) + (a + b*tan(c + d*x))^(1/2)*(320*a^4*b^14*d^3 - 16*b^18*
d^3 + 1024*a^6*b^12*d^3 + 1440*a^8*b^10*d^3 + 1024*a^10*b^8*d^3 + 320*a^12*
b^6*d^3 - 16*a^16*b^2*d^3)) + (1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i -
 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*((1i/(4*(a^5*d^2*1
i - b^5*d^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i
)))^(1/2)*(96*a*b^20*d^4 + 736*a^3*b^18*d^4 + 2432*a^5*b^16*d^4 + 4480*a^7*
b^14*d^4 + 4928*a^9*b^12*d^4 + 3136*a^11*b^10*d^4 + 896*a^13*b^8*d^4 - 128*
a^15*b^6*d^4 - 160*a^17*b^4*d^4 - 32*a^19*b^2*d^4 - (1i/(4*(a^5*d^2*1i - b^
5*d^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1
/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5
*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 +
13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*
d^5 + 64*a^21*b^2*d^5)) - (a + b*tan(c + d*x))^(1/2)*(320*a^4*b^14*d^3 - 16
*b^18*d^3 + 1024*a^6*b^12*d^3 + 1440*a^8*b^10*d^3 + 1024*a^10*b^8*d^3 + 320
*a^12*b^6*d^3 - 16*a^16*b^2*d^3)) - 16*b^16*d^2 - 80*a^2*b^14*d^2 - 144*a^4
*b^12*d^2 - 80*a^6*b^10*d^2 + 80*a^8*b^8*d^2 + 144*a^10*b^6*d^2 + 80*a^12*b
^4*d^2 + 16*a^14*b^2*d^2))*(1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i - 5*
a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*2i - atan(((((1/(a^5*
d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^
2*d^2))^(1/2)*(48*a*b^20*d^4 - ((1/(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^
4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2))^(1/2)*(a + b*tan(c + d*x))^
(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16
*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 768
0*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5))/4
 + 368*a^3*b^18*d^4 + 1216*a^5*b^16*d^4 + 2240*a^7*b^14*d^4 + 2464*a^9*b^12
*d^4 + 1568*a^11*b^10*d^4 + 448*a^13*b^8*d^4 - 64*a^15*b^6*d^4 - 80*a^17*b^
4*d^4 - 16*a^19*b^2*d^4))/2 - ((a + b*tan(c + d*x))^(1/2)*(320*a^4*b^14*d^3

```
 - 16*b^18*d^3 + 1024*a^6*b^12*d^3 + 1440*a^8*b^10*d^3 + 1024*a^10*b^8*d^3
+ 320*a^12*b^6*d^3 - 16*a^16*b^2*d^3))/2)*(1/(a^5*d^2 - b^5*d^2*1i + 5*a*b^
4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2))^(1/2)*1i - (((1/(
a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^
3*b^2*d^2))^(1/2)*(48*a*b^20*d^4 + ((1/(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2
- a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d...
```

3.549 $\int \frac{\tan^2(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=157

$$\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d}-\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d}-\frac{2a^2}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}}-$$

```
[Out] I*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(5/2)/d-I*arctanh((
a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(5/2)/d+4*a*b/(a^2+b^2)^2/d/(a
+b*tan(d*x+c))^(1/2)-2/3*a^2/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.22, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3623, 3610, 3620, 3618, 65, 214}

$$-\frac{2a^2}{3bd\,(a^2+b^2)\,(a+b\tan(c+dx))^{3/2}}+\frac{4ab}{d\,(a^2+b^2)^2\,\sqrt{a+b\tan(c+dx)}}+\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{5/2}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] (I*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/((a - I*b)^(5/2)*d) - (
I*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/((a + I*b)^(5/2)*d) - (2
*a^2)/(3*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^(3/2)) + (4*a*b)/((a^2 + b^2)
^2*d*Sqrt[a + b*Tan[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
```

```
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx = -\frac{2a^2}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} + \frac{\int \frac{-a+b\tan(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx}{a^2+b^2}$$

$$= -\frac{2a^2}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} + \frac{4ab}{(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}} +$$

$$= -\frac{2a^2}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} + \frac{4ab}{(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}} -$$

$$= -\frac{2a^2}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} + \frac{4ab}{(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}} -$$

$$= -\frac{2a^2}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} + \frac{4ab}{(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}} +$$

$$= \frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d} - \frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d} - \frac{}{3b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.18, size = 122, normalized size = 0.78

$$\frac{b(-ia+b)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\tan(c+dx)}{a-ib}\right) - (a-ib)\left(2a+2ib-ib\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\tan(c+dx)}{a+ib}\right)\right)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] (b*((-I)*a + b)*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[c + d*x])/(a - 
I*b)] - (a - I*b)*(2*a + (2*I)*b - I*b*Hypergeometric2F1[-3/2, 1, -1/2, (a 
+ b*Tan[c + d*x])/(a + I*b)]))/(3*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^(3/2
))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 966 vs. 2(133) = 266.
time = 0.15, size = 967, normalized size = 6.16

method	result

derivativedivides	$-\frac{2b^2\left(\frac{\left(\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a^4-\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,b^4-\sqrt{2\sqrt{a^2+b^2}}\right.}{}\right.}{}$
default	$-\frac{2b^2\left(\frac{\left(\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a^4-\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,b^4-\sqrt{2\sqrt{a^2+b^2}}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/b*(-b^2/(a^2+b^2)^2*(1/4/b^2/(a^2+b^2)^(3/2)*(1/2*((2*(a^2+b^2)^(1/2)+2
*a)^(1/2)*(a^2+b^2)^(1/2)*a^4-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)
*b^4-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^5+2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3*
b^2+3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^4)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c
))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*(6*a^4*b^2+4*a^2*
b^4-2*b^6-1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^4-(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^4-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^5+2
*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3*b^2+3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^
4)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(
a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a
)^(1/2)))+1/4/b^2/(a^2+b^2)^(3/2)*(-1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2
+b^2)^(1/2)*a^4-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^4-(2*(a^2+b
^2)^(1/2)+2*a)^(1/2)*a^5+2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3*b^2+3*(2*(a^2+
b^2)^(1/2)+2*a)^(1/2)*a*b^4)*ln(-b*tan(d*x+c)-a+(a+b*tan(d*x+c))^(1/2)*(2*(
a^2+b^2)^(1/2)+2*a)^(1/2)-(a^2+b^2)^(1/2))+2*(-6*a^4*b^2-4*a^2*b^4+2*b^6+1/
2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^4-(2*(a^2+b^2)^(1/2)+2*a
)^(1/2)*(a^2+b^2)^(1/2)*b^4-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^5+2*(2*(a^2+b^2
)^(1/2)+2*a)^(1/2)*a^3*b^2+3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^4)*(2*(a^2+b
^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*tan(d*x
```

```
+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))))-
1/3*a^2/(a^2+b^2)/(a+b*tan(d*x+c))^(3/2)+2*a*b^2/(a^2+b^2)^2/(a+b*tan(d*x+c
))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(tan(d*x + c)^2/(b*tan(d*x + c) + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 9804 vs. 2(127) = 254.
time = 1.61, size = 9804, normalized size = 62.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
[Out] -1/12*(12*sqrt(2)*((a^18*b + a^16*b^3 - 20*a^14*b^5 - 84*a^12*b^7 - 154*a^1
0*b^9 - 154*a^8*b^11 - 84*a^6*b^13 - 20*a^4*b^15 + a^2*b^17 + b^19)*d^5*cos
(d*x + c)^4 + 2*(3*a^16*b^3 + 20*a^14*b^5 + 56*a^12*b^7 + 84*a^10*b^9 + 70*
a^8*b^11 + 28*a^6*b^13 - 4*a^2*b^17 - b^19)*d^5*cos(d*x + c)^2 + (a^14*b^5
+ 7*a^12*b^7 + 21*a^10*b^9 + 35*a^8*b^11 + 35*a^6*b^13 + 21*a^4*b^15 + 7*a^
2*b^17 + b^19)*d^5 + 4*((a^17*b^2 + 6*a^15*b^4 + 14*a^13*b^6 + 14*a^11*b^8
- 14*a^7*b^12 - 14*a^5*b^14 - 6*a^3*b^16 - a*b^18)*d^5*cos(d*x + c)^3 + (a^
15*b^4 + 7*a^13*b^6 + 21*a^11*b^8 + 35*a^9*b^10 + 35*a^7*b^12 + 21*a^5*b^14
 + 7*a^3*b^16 + a*b^18)*d^5*cos(d*x + c))*sin(d*x + c))*sqrt((a^10 + 5*a^8*
b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 + (a^15 - 5*a^13*b^2 - 35*
a^11*b^4 - 65*a^9*b^6 - 45*a^7*b^8 + a^5*b^10 + 15*a^3*b^12 + 5*a*b^14)*d^2
*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^
4)))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*sqrt((25
*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^20 + 10*a^18*
b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b
^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b^20)*d^4))*(1/((a^10 + 5*
a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4))^(3/4)*arctan(((
5*a^20 + 30*a^18*b^2 + 61*a^16*b^4 + 8*a^14*b^6 - 182*a^12*b^8 - 364*a^10*b
^10 - 350*a^8*b^12 - 184*a^6*b^14 - 47*a^4*b^16 - 2*a^2*b^18 + b^20)*d^4*sq
rt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^20 + 10
*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210
*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b^20)*d^4))*sqrt(1/(
(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4)) + (5*
```

a^15 + 15*a^13*b^2 + a^11*b^4 - 45*a^9*b^6 - 65*a^7*b^8 - 35*a^5*b^10 - 5*a
^3*b^12 + a*b^14)*d^2*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2
*b^8 + b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b
^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^1
8 + b^20)*d^4)) - sqrt(2)*((3*a^22 + 29*a^20*b^2 + 125*a^18*b^4 + 315*a^16*
b^6 + 510*a^14*b^8 + 546*a^12*b^10 + 378*a^10*b^12 + 150*a^8*b^14 + 15*a^6*
b^16 - 15*a^4*b^18 - 7*a^2*b^20 - b^22)*d^7*sqrt((25*a^8*b^2 - 100*a^6*b^4
+ 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120
*a^14*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45
*a^4*b^16 + 10*a^2*b^18 + b^20)*d^4))*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^
4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4)) + 2*(a^17 + 8*a^15*b^2 + 28*a^13*b
^4 + 56*a^11*b^6 + 70*a^9*b^8 + 56*a^7*b^10 + 28*a^5*b^12 + 8*a^3*b^14 + a*
b^16)*d^5*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)
/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 + 252*a^1
0*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b^20)*d^
4)))*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 +
(a^15 - 5*a^13*b^2 - 35*a^11*b^4 - 65*a^9*b^6 - 45*a^7*b^8 + a^5*b^10 + 15*
a^3*b^12 + 5*a*b^14)*d^2*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^
6 + 5*a^2*b^8 + b^10)*d^4)))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a
^2*b^8 + b^10))*sqrt(((25*a^14*b^2 - 25*a^12*b^4 - 115*a^10*b^6 + 35*a^8*b^
8 + 171*a^6*b^10 + 53*a^4*b^12 - 17*a^2*b^14 + b^16)*d^2*sqrt(1/((a^10 + 5*
a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4))*cos(d*x + c) +
sqrt(2)*(2*(25*a^15*b^3 - 25*a^13*b^5 - 115*a^11*b^7 + 35*a^9*b^9 + 171*a^7
*b^11 + 53*a^5*b^13 - 17*a^3*b^15 + a*b^17)*d^3*sqrt(1/((a^10 + 5*a^8*b^2 +
 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4))*cos(d*x + c) + (75*a^10*
b^3 - 325*a^8*b^5 + 430*a^6*b^7 - 170*a^4*b^9 + 23*a^2*b^11 - b^13)*d*cos(d
*x + c))*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^1
0 + (a^15 - 5*a^13*b^2 - 35*a^11*b^4 - 65*a^9*b^6 - 45*a^7*b^8 + a^5*b^10 +
 15*a^3*b^12 + 5*a*b^14)*d^2*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^
4*b^6 + 5*a^2*b^8 + b^10)*d^4)))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 -
20*a^2*b^8 + b^10))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*(1
/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4))^(1/
4) + (25*a^9*b^2 - 100*a^7*b^4 + 110*a^5*b^6 - 20*a^3*b^8 + a*b^10)*cos(d*x
 + c) + (25*a^8*b^3 - 100*a^6*b^5 + 110*a^4*b^7 - 20*a^2*b^9 + b^11)*sin(d*
x + c))/cos(d*x + c))*(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a
^2*b^8 + b^10)*d^4))^(3/4) + sqrt(2)*((15*a^26*b + 115*a^24*b^3 + 338*a^22*
b^5 + 354*a^20*b^7 - 475*a^18*b^9 - 2055*a^16*b^11 - 3060*a^14*b^13 - 2484*
a^12*b^15 - 1047*a^10*b^17 - 75*a^8*b^19 + 130*a^6*b^21 + 50*a^4*b^23 + 3*a
^2*b^25 - b^27)*d^7*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b
^8 + b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8
 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18
+ b^20)*d^4))*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b
^8 + b^10)*d^4)) + 2*(5*a^21*b + 30*a^19*b^3 + 61*a^17*b^5 + 8*a^15*b^7 - 1
82*a^13*b^9 - 364*a^11*b^11 - 350*a^9*b^13 - 18...

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(c+dx)}{(a+b\tan(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+b*tan(d*x+c))**(5/2),x)
[Out] Integral(tan(c + d*x)**2/(a + b*tan(c + d*x))**(5/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**B**]
time = 8.46, size = 2500, normalized size = 15.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + b*tan(c + d*x))^(5/2),x)
[Out] (log(((-1/(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*
10i - 10*a^3*b^2*d^2))^(1/2)*(((-1/(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^
4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2))^(1/2)*(896*a^6*b^15*d^4 - 1
60*a^2*b^19*d^4 - 128*a^4*b^17*d^4 - 32*b^21*d^4 + 3136*a^8*b^13*d^4 + 4928
*a^10*b^11*d^4 + 4480*a^12*b^9*d^4 + 2432*a^14*b^7*d^4 + 736*a^16*b^5*d^4 +
 96*a^18*b^3*d^4 - ((-1/(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i
+ a^2*b^3*d^2*10i - 10*a^3*b^2*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a
*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 1344
0*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*
d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5))/2))/2 + (a +
 b*tan(c + d*x))^(1/2)*(320*a^4*b^14*d^3 - 16*b^18*d^3 + 1024*a^6*b^12*d^3
+ 1440*a^8*b^10*d^3 + 1024*a^10*b^8*d^3 + 320*a^12*b^6*d^3 - 16*a^16*b^2*d^
3)))/2 - 16*a*b^15*d^2 - 96*a^3*b^13*d^2 - 240*a^5*b^11*d^2 - 320*a^7*b^9*d
^2 - 240*a^9*b^7*d^2 - 96*a^11*b^5*d^2 - 16*a^13*b^3*d^2)*(-1/(a^5*d^2 - b^
5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2))^
(1/2))/2 - log(((-1/(4*(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i +
```

a^2*b^3*d^2*10i - 10*a^3*b^2*d^2)))^(1/2)*(896*a^6*b^15*d^4 - 160*a^2*b^19*d^4 - 128*a^4*b^17*d^4 - 32*b^21*d^4 + 3136*a^8*b^13*d^4 + 4928*a^10*b^11*d^4 + 4480*a^12*b^9*d^4 + 2432*a^14*b^7*d^4 + 736*a^16*b^5*d^4 + 96*a^18*b^3*d^4 + (-1/(4*(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5)) - (a + b*tan(c + d*x))^(1/2)*(320*a^4*b^14*d^3 - 16*b^18*d^3 + 1024*a^6*b^12*d^3 + 1440*a^8*b^10*d^3 + 1024*a^10*b^8*d^3 + 320*a^12*b^6*d^3 - 16*a^16*b^2*d^3))*(-1/(4*(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2)))^(1/2) - 16*a*b^15*d^2 - 96*a^3*b^13*d^2 - 240*a^5*b^11*d^2 - 320*a^7*b^9*d^2 - 240*a^9*b^7*d^2 - 96*a^11*b^5*d^2 - 16*a^13*b^3*d^2)*(-1/(4*(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2)))^(1/2) - atan((((-1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*(896*a^6*b^15*d^4 - 160*a^2*b^19*d^4 - 128*a^4*b^17*d^4 - 32*b^21*d^4 + 3136*a^8*b^13*d^4 + 4928*a^10*b^11*d^4 + 4480*a^12*b^9*d^4 + 2432*a^14*b^7*d^4 + 736*a^16*b^5*d^4 + 96*a^18*b^3*d^4 + (-1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5)) - (a + b*tan(c + d*x))^(1/2)*(320*a^4*b^14*d^3 - 16*b^18*d^3 + 1024*a^6*b^12*d^3 + 1440*a^8*b^10*d^3 + 1024*a^10*b^8*d^3 + 320*a^12*b^6*d^3 - 16*a^16*b^2*d^3))*(-1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*1i - ((-1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*(896*a^6*b^15*d^4 - 160*a^2*b^19*d^4 - 128*a^4*b^17*d^4 - 32*b^21*d^4 + 3136*a^8*b^13*d^4 + 4928*a^10*b^11*d^4 + 4480*a^12*b^9*d^4 + 2432*a^14*b^7*d^4 + 736*a^16*b^5*d^4 + 96*a^18*b^3*d^4 - (-1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5)) + (a + b*tan(c + d*x))^(1/2)*(320*a^4*b^14*d^3 - 16*b^18*d^3 + 1024*a^6*b^12*d^3 + 1440*a^8*b^10*d^3 + 1024*a^10*b^8*d^3 + 320*a^12*b^6*d^3 - 16*a^16*b^2*d^3))*(-1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*1i)/(32*a*b^15*d^2 - ((-1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*(896*a^6*b^15*d^4 - 160*a^2*b^19*d^4 - 128*a^4*b^17*d^4 - 32*b^21*d^4 + 3136*a^8*b^13*d^4 + 4928*a^10*b^11*d^4 + 4480*a^12*b^9*d^4 + 2432*a^14*b^7*d^4 + 736*a^16*b^5*d^4 + 96*a^18*b^3*d^4 - (-1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2

```
880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12
*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^
19*b^4*d^5 + 64*a^21*b^2*d^5)) + (a + b*tan(c + d*x))^(1/2)*(320*a^4*b^14*d
^3 - 16*b^18*d^3 + 1024*a^6*b^12*d^3 + 1440*a^8*b^10*d^3 + 1024*a^10*b^8*d^
3 + 320*a^12*b^6*d^3 - 16*a^16*b^2*d^3))*(-1i/(4*(a^5*d^2*1i - b^5*d^2 + a*
b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3...
```

3.550 $\int \frac{\tan(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=155

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d}+\frac{2a}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^{3/2}}+\frac{}{(a}$$

```
[Out] -arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(5/2)/d-arctanh((a+b
*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(5/2)/d+2*(a^2-b^2)/(a^2+b^2)^2/d
/(a+b*tan(d*x+c))^(1/2)+2/3*a/(a^2+b^2)/d/(a+b*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.19, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3610, 3620, 3618, 65, 214}

$$\frac{2a}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{2\left(a^2-b^2\right)}{d\left(a^2+b^2\right)^2\sqrt{a+b\tan(c+dx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]]/((a - I*b)^(5/2)*d)) - Ar
cTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]]/((a + I*b)^(5/2)*d) + (2*a)/(
3*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^(3/2)) + (2*(a^2 - b^2))/((a^2 + b^2)^
2*d*Sqrt[a + b*Tan[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
```

```
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx = \frac{2a}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^{3/2}} + \frac{\int \frac{b+a\tan(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx}{a^2+b^2}$$

$$= \frac{2a}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^{3/2}} + \frac{2(a^2-b^2)}{\left(a^2+b^2\right)^2 d\sqrt{a+b\tan(c+dx)}} + \frac{\int -\cdots}{\cdots}$$

$$= \frac{2a}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^{3/2}} + \frac{2(a^2-b^2)}{\left(a^2+b^2\right)^2 d\sqrt{a+b\tan(c+dx)}} - \frac{i\int \cdots}{\cdots}$$

$$= \frac{2a}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^{3/2}} + \frac{2(a^2-b^2)}{\left(a^2+b^2\right)^2 d\sqrt{a+b\tan(c+dx)}} + \frac{\mathrm{Sub}\cdots}{\cdots}$$

$$= \frac{2a}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^{3/2}} + \frac{2(a^2-b^2)}{\left(a^2+b^2\right)^2 d\sqrt{a+b\tan(c+dx)}} + \frac{i\mathrm{Su}\cdots}{\cdots}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d} + \frac{\cdots}{3\,(a\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.16, size = 103, normalized size = 0.66

$$\frac{(a+ib)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\tan(c+dx)}{a-ib}\right)+(a-ib)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\tan(c+dx)}{a+ib}\right)}{3\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((a + I*b)*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[c + d*x])/(a - I*b)]
 + (a - I*b)*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[c + d*x])/(a + I*b
)])/(3*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 952 vs. 2(133) = 266.
time = 0.13, size = 953, normalized size = 6.15

method	result
derivativedivides	$\left(2\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a^3+2\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a\,b^2-3\sqrt{2\sqrt{a^2+b^2}+\right.$
default	$\left(2\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a^3+2\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a\,b^2-3\sqrt{2\sqrt{a^2+b^2}+\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/(a^2+b^2)^2*(1/4/(a^2+b^2)^(3/2)*(1/2*(2*(2*(a^2+b^2)^(1/2)+2*a)^(1/
2)*(a^2+b^2)^(1/2)*a^3+2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^
2-3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4-2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b
^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1
/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*(-2*a^5+4*a^3*b^2+6*a*
b^4-1/2*(2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3+2*(2*(a^2+b^2)
^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^2-3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4
-2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)
*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+
```

```
b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^
(1/2)))+1/4/(a^2+b^2)^(3/2)*(-1/2*(2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2
)^(1/2)*a^3+2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^2-3*(2*(a^2
+b^2)^(1/2)+2*a)^(1/2)*a^4-2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b^2+(2*(a^2+
b^2)^(1/2)+2*a)^(1/2)*b^4)*ln(-b*tan(d*x+c)-a+(a+b*tan(d*x+c))^(1/2)*(2*(a^
2+b^2)^(1/2)+2*a)^(1/2)-(a^2+b^2)^(1/2))+2*(2*a^5-4*a^3*b^2-6*a*b^4+1/2*(2*
(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3+2*(2*(a^2+b^2)^(1/2)+2*a)
^(1/2)*(a^2+b^2)^(1/2)*a*b^2-3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4-2*(2*(a^2+
b^2)^(1/2)+2*a)^(1/2)*a^2*b^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)*(2*(a^2+b^
2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*tan(d*x+
c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))))-2
/(a^2+b^2)^2*(-a^2+b^2)/(a+b*tan(d*x+c))^(1/2)+2/3*a/(a^2+b^2)/(a+b*tan(d*x
+c))^(3/2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 9790 vs. $2(129) = 258$.
time = 1.84, size = 9790, normalized size = 63.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(12*sqrt(2)*((a^18 + a^16*b^2 - 20*a^14*b^4 - 84*a^12*b^6 - 154*a^10*b
^8 - 154*a^8*b^10 - 84*a^6*b^12 - 20*a^4*b^14 + a^2*b^16 + b^18)*d^5*cos(d*
x + c)^4 + 2*(3*a^16*b^2 + 20*a^14*b^4 + 56*a^12*b^6 + 84*a^10*b^8 + 70*a^8
*b^10 + 28*a^6*b^12 - 4*a^2*b^16 - b^18)*d^5*cos(d*x + c)^2 + (a^14*b^4 + 7
*a^12*b^6 + 21*a^10*b^8 + 35*a^8*b^10 + 35*a^6*b^12 + 21*a^4*b^14 + 7*a^2*b
^16 + b^18)*d^5 + 4*((a^17*b + 6*a^15*b^3 + 14*a^13*b^5 + 14*a^11*b^7 - 14*
a^7*b^11 - 14*a^5*b^13 - 6*a^3*b^15 - a*b^17)*d^5*cos(d*x + c)^3 + (a^15*b^
3 + 7*a^13*b^5 + 21*a^11*b^7 + 35*a^9*b^9 + 35*a^7*b^11 + 21*a^5*b^13 + 7*a
^3*b^15 + a*b^17)*d^5*cos(d*x + c))*sin(d*x + c))*sqrt((a^10 + 5*a^8*b^2 +
10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 - (a^15 - 5*a^13*b^2 - 35*a^11*b
```

^4 - 65*a^9*b^6 - 45*a^7*b^8 + a^5*b^10 + 15*a^3*b^12 + 5*a*b^14)*d^2*sqrt(
1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4)))/(
25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*sqrt((25*a^8*b
^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^20 + 10*a^18*b^2 +
45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 +
120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b^20)*d^4))*(1/((a^10 + 5*a^8*b^
2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4))^(3/4)*arctan(((5*a^20
+ 30*a^18*b^2 + 61*a^16*b^4 + 8*a^14*b^6 - 182*a^12*b^8 - 364*a^10*b^10 -
350*a^8*b^12 - 184*a^6*b^14 - 47*a^4*b^16 - 2*a^2*b^18 + b^20)*d^4*sqrt((25
*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^20 + 10*a^18*
b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b
^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b^20)*d^4))*sqrt(1/((a^10
+ 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4)) + (5*a^15 +
15*a^13*b^2 + a^11*b^4 - 45*a^9*b^6 - 65*a^7*b^8 - 35*a^5*b^10 - 5*a^3*b^1
2 + a*b^14)*d^2*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 +
b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 + 2
52*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b^
20)*d^4)) - sqrt(2)*((a^23 + 7*a^21*b^2 + 15*a^19*b^4 - 15*a^17*b^6 - 150*a
^15*b^8 - 378*a^13*b^10 - 546*a^11*b^12 - 510*a^9*b^14 - 315*a^7*b^16 - 125
*a^5*b^18 - 29*a^3*b^20 - 3*a*b^22)*d^7*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 11
0*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^1
4*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4
*b^16 + 10*a^2*b^18 + b^20)*d^4))*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 +
10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4)) + (a^18 + 7*a^16*b^2 + 20*a^14*b^4 + 2
8*a^12*b^6 + 14*a^10*b^8 - 14*a^8*b^10 - 28*a^6*b^12 - 20*a^4*b^14 - 7*a^2*
b^16 - b^18)*d^5*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8
+ b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 +
252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b
^20)*d^4)))*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 +
b^10 - (a^15 - 5*a^13*b^2 - 35*a^11*b^4 - 65*a^9*b^6 - 45*a^7*b^8 + a^5*b^1
0 + 15*a^3*b^12 + 5*a*b^14)*d^2*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10
*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4)))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6
- 20*a^2*b^8 + b^10))*sqrt(((25*a^14 - 25*a^12*b^2 - 115*a^10*b^4 + 35*a^8
*b^6 + 171*a^6*b^8 + 53*a^4*b^10 - 17*a^2*b^12 + b^14)*d^2*sqrt(1/((a^10 +
5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4))*cos(d*x + c)
+ sqrt(2)*((25*a^16 - 50*a^14*b^2 - 90*a^12*b^4 + 150*a^10*b^6 + 136*a^8*b^
8 - 118*a^6*b^10 - 70*a^4*b^12 + 18*a^2*b^14 - b^16)*d^3*sqrt(1/((a^10 + 5*
a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4))*cos(d*x + c) +
(25*a^11 - 175*a^9*b^2 + 410*a^7*b^4 - 350*a^5*b^6 + 61*a^3*b^8 - 3*a*b^10)
*d*cos(d*x + c))*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b
^8 + b^10 - (a^15 - 5*a^13*b^2 - 35*a^11*b^4 - 65*a^9*b^6 - 45*a^7*b^8 + a^
5*b^10 + 15*a^3*b^12 + 5*a*b^14)*d^2*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4
+ 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4)))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^
4*b^6 - 20*a^2*b^8 + b^10))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x
+ c))*(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d

```
^4))^(1/4) + (25*a^9 - 100*a^7*b^2 + 110*a^5*b^4 - 20*a^3*b^6 + a*b^8)*cos(
d*x + c) + (25*a^8*b - 100*a^6*b^3 + 110*a^4*b^5 - 20*a^2*b^7 + b^9)*sin(d*
x + c))/cos(d*x + c))*(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a
^2*b^8 + b^10)*d^4))^(3/4) + sqrt(2)*((5*a^27 + 25*a^25*b^2 + 6*a^23*b^4 -
218*a^21*b^6 - 585*a^19*b^8 - 405*a^17*b^10 + 900*a^15*b^12 + 2532*a^13*b^1
4 + 2979*a^11*b^16 + 2015*a^9*b^18 + 790*a^7*b^20 + 150*a^5*b^22 + a^3*b^24
 - 3*a*b^26)*d^7*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8
+ b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 +
252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b
^20)*d^4))*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8
+ b^10)*d^4)) + (5*a^22 + 25*a^20*b^2 + 31*a^18*b^4 - 53*a^16*b^6 - 190*a^1
4*b^8 - 182*a^12*b^10 + 14*a^10*b^12 + 166*a^8*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(c+dx)}{(a+b\tan(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(tan(c + d*x)/(a + b*tan(c + d*x))**(5/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 9.63, size = 2500, normalized size = 16.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + b*tan(c + d*x))^(5/2),x)
```

```
[Out] atan(((((1/(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2
*10i - 10*a^3*b^2*d^2))^(1/2)*(48*a*b^20*d^4 - ((1/(a^5*d^2 - b^5*d^2*1i +
5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2))^(1/2)*(a +
b*tan(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5
```

+ 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^1
3*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*
a^21*b^2*d^5))/4 + 368*a^3*b^18*d^4 + 1216*a^5*b^16*d^4 + 2240*a^7*b^14*d^4
+ 2464*a^9*b^12*d^4 + 1568*a^11*b^10*d^4 + 448*a^13*b^8*d^4 - 64*a^15*b^6*
d^4 - 80*a^17*b^4*d^4 - 16*a^19*b^2*d^4))/2 - ((a + b*tan(c + d*x))^(1/2)*(
320*a^4*b^14*d^3 - 16*b^18*d^3 + 1024*a^6*b^12*d^3 + 1440*a^8*b^10*d^3 + 10
24*a^10*b^8*d^3 + 320*a^12*b^6*d^3 - 16*a^16*b^2*d^3))/2)*(1/(a^5*d^2 - b^5
*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2))^(
1/2)*1i - (((1/(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3
*d^2*10i - 10*a^3*b^2*d^2))^(1/2)*(48*a*b^20*d^4 + ((1/(a^5*d^2 - b^5*d^2*1
i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2))^(1/2)*(
a + b*tan(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18
*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440
*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 +
64*a^21*b^2*d^5))/4 + 368*a^3*b^18*d^4 + 1216*a^5*b^16*d^4 + 2240*a^7*b^14
*d^4 + 2464*a^9*b^12*d^4 + 1568*a^11*b^10*d^4 + 448*a^13*b^8*d^4 - 64*a^15*
b^6*d^4 - 80*a^17*b^4*d^4 - 16*a^19*b^2*d^4))/2 + ((a + b*tan(c + d*x))^(1/
2)*(320*a^4*b^14*d^3 - 16*b^18*d^3 + 1024*a^6*b^12*d^3 + 1440*a^8*b^10*d^3
+ 1024*a^10*b^8*d^3 + 320*a^12*b^6*d^3 - 16*a^16*b^2*d^3))/2)*(1/(a^5*d^2 -
b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2
))^(1/2)*1i)/((((1/(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2
*b^3*d^2*10i - 10*a^3*b^2*d^2))^(1/2)*(48*a*b^20*d^4 - ((1/(a^5*d^2 - b^5*d
^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2))^(1/
2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*
b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 1
3440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d
^5 + 64*a^21*b^2*d^5))/4 + 368*a^3*b^18*d^4 + 1216*a^5*b^16*d^4 + 2240*a^7*
b^14*d^4 + 2464*a^9*b^12*d^4 + 1568*a^11*b^10*d^4 + 448*a^13*b^8*d^4 - 64*a
^15*b^6*d^4 - 80*a^17*b^4*d^4 - 16*a^19*b^2*d^4))/2 - ((a + b*tan(c + d*x))
^(1/2)*(320*a^4*b^14*d^3 - 16*b^18*d^3 + 1024*a^6*b^12*d^3 + 1440*a^8*b^10*
d^3 + 1024*a^10*b^8*d^3 + 320*a^12*b^6*d^3 - 16*a^16*b^2*d^3))/2)*(1/(a^5*d
^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2
*d^2))^(1/2) + (((1/(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^
2*b^3*d^2*10i - 10*a^3*b^2*d^2))^(1/2)*(48*a*b^20*d^4 + ((1/(a^5*d^2 - b^5*
d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2))^(1
/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5
*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 +
13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*
d^5 + 64*a^21*b^2*d^5))/4 + 368*a^3*b^18*d^4 + 1216*a^5*b^16*d^4 + 2240*a^7
*b^14*d^4 + 2464*a^9*b^12*d^4 + 1568*a^11*b^10*d^4 + 448*a^13*b^8*d^4 - 64*
a^15*b^6*d^4 - 80*a^17*b^4*d^4 - 16*a^19*b^2*d^4))/2 + ((a + b*tan(c + d*x)
)^(1/2)*(320*a^4*b^14*d^3 - 16*b^18*d^3 + 1024*a^6*b^12*d^3 + 1440*a^8*b^10
*d^3 + 1024*a^10*b^8*d^3 + 320*a^12*b^6*d^3 - 16*a^16*b^2*d^3))/2)*(1/(a^5*
d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^
2*d^2))^(1/2) - 16*b^16*d^2 - 80*a^2*b^14*d^2 - 144*a^4*b^12*d^2 - 80*a^6*b

```
^10*d^2 + 80*a^8*b^8*d^2 + 144*a^10*b^6*d^2 + 80*a^12*b^4*d^2 + 16*a^14*b^2
*d^2))*(1/(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*
10i - 10*a^3*b^2*d^2))^(1/2)*1i - atan(((1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^
4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*((1i/(4*
(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b
^2*d^2*10i)))^(1/2)*(96*a*b^20*d^4 + 736*a^3*b^18*d^4 + 2432*a^5*b^16*d^4 +
 4480*a^7*b^14*d^4 + 4928*a^9*b^12*d^4 + 3136*a^11*b^10*d^4 + 896*a^13*b^8*
d^4 - 128*a^15*b^6*d^4 - 160*a^17*b^4*d^4 - 32*a^19*b^2*d^4 + (1i/(4*(a^5*d
^2*1i - b^5*d^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2
*10i)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5
+ 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b
^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640
*a^19*b^4*d^5 + 64*a^21*b^2*d^5)) + (a + b*tan(c + d*x))^(1/2)*(320*a^4*b^1
4*d^3 - 16*b^18*d^3 + 1024*a^6*b^12*d^3 + 1440*a^8*b^10*d^3 + 1024*a^10*b^8
*d^3 + 320*a^12*b^6*d^3 - 16*a^16*b^2*d^3))*1i - (1i/(4*(a^5*d^2*1i - b^5*d
^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)
*((1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^
2 - a^3*b^2*d^2*10i)))^(1/2)*(96*a*b^20*d^4 + 7...
```

3.551 $\int \frac{1}{(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=152

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d}+\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d}-\frac{2b}{3(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}}-$$

```
[Out] -I*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(5/2)/d+I*arctanh(
(a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(5/2)/d-4*a*b/(a^2+b^2)^2/d/(
a+b*tan(d*x+c))^(1/2)-2/3*b/(a^2+b^2)/d/(a+b*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.18, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3564, 3610, 3620, 3618, 65, 214}

$$-\frac{4ab}{d\left(a^2+b^2\right)^2\sqrt{a+b\tan(c+dx)}}-\frac{2b}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{5/2}}+\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^(-5/2),x]
```

```
[Out] ((-I)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/((a - I*b)^(5/2)*d)
+ (I*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/((a + I*b)^(5/2)*d) -
 (2*b)/(3*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^(3/2)) - (4*a*b)/((a^2 + b^2)^
2*d*Sqrt[a + b*Tan[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(a+b\tan(c+dx))^{5/2}}\,dx = -\frac{2b}{3(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} + \frac{\int \frac{a-b\tan(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx}{a^2+b^2}$$

$$= -\frac{2b}{3(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{4ab}{(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}} + \frac{\int \cdots}{\cdots}$$

$$= -\frac{2b}{3(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{4ab}{(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}} + \frac{\int \cdots}{\cdots}$$

$$= -\frac{2b}{3(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{4ab}{(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}} + \frac{i\mathrm{Su}\cdots}{\cdots}$$

$$= -\frac{2b}{3(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{4ab}{(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}} - \frac{\mathrm{Su}\cdots}{\cdots}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d} + \frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d} - \frac{\cdots}{3(\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.10, size = 108, normalized size = 0.71

$$\frac{i(a+ib)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\tan(c+dx)}{a-ib}\right) + (-ia-b)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\tan(c+dx)}{a+ib}\right)}{3(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^(-5/2),x]
```

```
[Out] (I*(a + I*b)*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[c + d*x])/(a - I*b
)] + ((-I)*a - b)*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[c + d*x])/(a
+ I*b)])/(3*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 954 vs. 2(128) = 256.
time = 0.11, size = 955, normalized size = 6.28

method	result

derivativedivides	$2b\left(\frac{\left(\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a^4-\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,b^4-\sqrt{2\sqrt{a^2+b^2}+\cdots\right.}{\cdots}\right.$
default	$2b\left(\frac{\left(\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a^4-\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,b^4-\sqrt{2\sqrt{a^2+b^2}+\cdots\right.}{\cdots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*b*(1/(a^2+b^2)^2*(1/4/b^2/(a^2+b^2)^(3/2)*(1/2*((2*(a^2+b^2)^(1/2)+2*a)
^(1/2)*(a^2+b^2)^(1/2)*a^4-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^
4-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^5+2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3*b^2
+3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^4)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^
(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*(6*a^4*b^2+4*a^2*b^4
-2*b^6-1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^4-(2*(a^2+b^2)^
(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^4-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^5+2*(2
*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3*b^2+3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^4)*
(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b
*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(
1/2)))+1/4/b^2/(a^2+b^2)^(3/2)*(-1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^
2)^(1/2)*a^4-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^4-(2*(a^2+b^2)
^(1/2)+2*a)^(1/2)*a^5+2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3*b^2+3*(2*(a^2+b^2
)^(1/2)+2*a)^(1/2)*a*b^4)*ln(-b*tan(d*x+c)-a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2
+b^2)^(1/2)+2*a)^(1/2)-(a^2+b^2)^(1/2))+2*(-6*a^4*b^2-4*a^2*b^4+2*b^6+1/2*(
(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^4-(2*(a^2+b^2)^(1/2)+2*a)^(
1/2)*(a^2+b^2)^(1/2)*b^4-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^5+2*(2*(a^2+b^2)^(
```

```
1/2)+2*a)^(1/2)*a^3*b^2+3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^4)*(2*(a^2+b^2)
^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*tan(d*x+c)
)^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))-1/3
/(a^2+b^2)/(a+b*tan(d*x+c))^(3/2)-2/(a^2+b^2)^2*a/(a+b*tan(d*x+c))^(1/2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 9767 vs. $2(122) = 244$.
time = 2.03, size = 9767, normalized size = 64.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(12*sqrt(2)*((a^18 + a^16*b^2 - 20*a^14*b^4 - 84*a^12*b^6 - 154*a^10*b
^8 - 154*a^8*b^10 - 84*a^6*b^12 - 20*a^4*b^14 + a^2*b^16 + b^18)*d^5*cos(d*
x + c)^4 + 2*(3*a^16*b^2 + 20*a^14*b^4 + 56*a^12*b^6 + 84*a^10*b^8 + 70*a^8
*b^10 + 28*a^6*b^12 - 4*a^2*b^16 - b^18)*d^5*cos(d*x + c)^2 + (a^14*b^4 + 7
*a^12*b^6 + 21*a^10*b^8 + 35*a^8*b^10 + 35*a^6*b^12 + 21*a^4*b^14 + 7*a^2*b
^16 + b^18)*d^5 + 4*((a^17*b + 6*a^15*b^3 + 14*a^13*b^5 + 14*a^11*b^7 - 14*
a^7*b^11 - 14*a^5*b^13 - 6*a^3*b^15 - a*b^17)*d^5*cos(d*x + c)^3 + (a^15*b^
3 + 7*a^13*b^5 + 21*a^11*b^7 + 35*a^9*b^9 + 35*a^7*b^11 + 21*a^5*b^13 + 7*a
^3*b^15 + a*b^17)*d^5*cos(d*x + c))*sin(d*x + c))*sqrt((a^10 + 5*a^8*b^2 +
10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 + (a^15 - 5*a^13*b^2 - 35*a^11*b
^4 - 65*a^9*b^6 - 45*a^7*b^8 + a^5*b^10 + 15*a^3*b^12 + 5*a*b^14)*d^2*sqrt(
1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4)))/(
25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*sqrt((25*a^8*b
^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^20 + 10*a^18*b^2 +
45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 +
120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b^20)*d^4))*(1/((a^10 + 5*a^8*b^
2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4))^(3/4)*arctan(((5*a^20
 + 30*a^18*b^2 + 61*a^16*b^4 + 8*a^14*b^6 - 182*a^12*b^8 - 364*a^10*b^10 -
350*a^8*b^12 - 184*a^6*b^14 - 47*a^4*b^16 - 2*a^2*b^18 + b^20)*d^4*sqrt((25
```

*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^20 + 10*a^18*
b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b
^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b^20)*d^4))*sqrt(1/((a^10
+ 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4)) + (5*a^15 +
15*a^13*b^2 + a^11*b^4 - 45*a^9*b^6 - 65*a^7*b^8 - 35*a^5*b^10 - 5*a^3*b^1
2 + a*b^14)*d^2*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 +
b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 + 2
52*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b^
20)*d^4)) - sqrt(2)*((3*a^22 + 29*a^20*b^2 + 125*a^18*b^4 + 315*a^16*b^6 +
510*a^14*b^8 + 546*a^12*b^10 + 378*a^10*b^12 + 150*a^8*b^14 + 15*a^6*b^16 -
15*a^4*b^18 - 7*a^2*b^20 - b^22)*d^7*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*
a^4*b^6 - 20*a^2*b^8 + b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*
b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b
^16 + 10*a^2*b^18 + b^20)*d^4))*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10
*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4)) + 2*(a^17 + 8*a^15*b^2 + 28*a^13*b^4 + 5
6*a^11*b^6 + 70*a^9*b^8 + 56*a^7*b^10 + 28*a^5*b^12 + 8*a^3*b^14 + a*b^16)*
d^5*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^2
0 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 + 252*a^10*b^10
+ 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b^20)*d^4)))*s
qrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 + (a^15
- 5*a^13*b^2 - 35*a^11*b^4 - 65*a^9*b^6 - 45*a^7*b^8 + a^5*b^10 + 15*a^3*b^
12 + 5*a*b^14)*d^2*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*
a^2*b^8 + b^10)*d^4)))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8
+ b^10))*sqrt(((25*a^14*b^2 - 25*a^12*b^4 - 115*a^10*b^6 + 35*a^8*b^8 + 17
1*a^6*b^10 + 53*a^4*b^12 - 17*a^2*b^14 + b^16)*d^2*sqrt(1/((a^10 + 5*a^8*b^
2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4))*cos(d*x + c) + sqrt(2
)*(2*(25*a^15*b^3 - 25*a^13*b^5 - 115*a^11*b^7 + 35*a^9*b^9 + 171*a^7*b^11
+ 53*a^5*b^13 - 17*a^3*b^15 + a*b^17)*d^3*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^
6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4))*cos(d*x + c) + (75*a^10*b^3 -
325*a^8*b^5 + 430*a^6*b^7 - 170*a^4*b^9 + 23*a^2*b^11 - b^13)*d*cos(d*x + c
))*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 + (a
^15 - 5*a^13*b^2 - 35*a^11*b^4 - 65*a^9*b^6 - 45*a^7*b^8 + a^5*b^10 + 15*a^
3*b^12 + 5*a*b^14)*d^2*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6
+ 5*a^2*b^8 + b^10)*d^4)))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2
*b^8 + b^10))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*(1/((a^1
0 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4))^(1/4) + (
25*a^9*b^2 - 100*a^7*b^4 + 110*a^5*b^6 - 20*a^3*b^8 + a*b^10)*cos(d*x + c)
+ (25*a^8*b^3 - 100*a^6*b^5 + 110*a^4*b^7 - 20*a^2*b^9 + b^11)*sin(d*x + c)
)/cos(d*x + c))*(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8
+ b^10)*d^4))^(3/4) + sqrt(2)*((15*a^26*b + 115*a^24*b^3 + 338*a^22*b^5 +
354*a^20*b^7 - 475*a^18*b^9 - 2055*a^16*b^11 - 3060*a^14*b^13 - 2484*a^12*b
^15 - 1047*a^10*b^17 - 75*a^8*b^19 + 130*a^6*b^21 + 50*a^4*b^23 + 3*a^2*b^2
5 - b^27)*d^7*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b
^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 + 252
*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b^20

```
)*d^4))*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b
^10)*d^4)) + 2*(5*a^21*b + 30*a^19*b^3 + 61*a^17*b^5 + 8*a^15*b^7 - 182*a^1
3*b^9 - 364*a^11*b^11 - 350*a^9*b^13 - 184*a^7*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\tan(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**(-5/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 8.86, size = 2500, normalized size = 16.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(c + d*x))^(5/2),x)
```

```
[Out] (log(16*a*b^15*d^2 - ((-1/(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5
i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2))^(1/2)*(((-1/(a^5*d^2 - b^5*d^2*1i +
5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2))^(1/2)*(896*
a^6*b^15*d^4 - 160*a^2*b^19*d^4 - 128*a^4*b^17*d^4 - 32*b^21*d^4 + 3136*a^8
*b^13*d^4 + 4928*a^10*b^11*d^4 + 4480*a^12*b^9*d^4 + 2432*a^14*b^7*d^4 + 73
6*a^16*b^5*d^4 + 96*a^18*b^3*d^4 + ((-1/(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2
 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2))^(1/2)*(a + b*tan(c + d
*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7
*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5
+ 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^
5))/2))/2 - (a + b*tan(c + d*x))^(1/2)*(320*a^4*b^14*d^3 - 16*b^18*d^3 + 10
24*a^6*b^12*d^3 + 1440*a^8*b^10*d^3 + 1024*a^10*b^8*d^3 + 320*a^12*b^6*d^3
- 16*a^16*b^2*d^3)))/2 + 96*a^3*b^13*d^2 + 240*a^5*b^11*d^2 + 320*a^7*b^9*d
```

^2 + 240*a^9*b^7*d^2 + 96*a^11*b^5*d^2 + 16*a^13*b^3*d^2)*(-1/(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2))^(1/2))/2 - log(16*a*b^15*d^2 - ((-1/(4*(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2)))^(1/2)*(896*a^6*b^15*d^4 - 160*a^2*b^19*d^4 - 128*a^4*b^17*d^4 - 32*b^21*d^4 + 3136*a^8*b^13*d^4 + 4928*a^10*b^11*d^4 + 4480*a^12*b^9*d^4 + 2432*a^14*b^7*d^4 + 736*a^16*b^5*d^4 + 96*a^18*b^3*d^4 - (-1/(4*(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5)) + (a + b*tan(c + d*x))^(1/2)*(320*a^4*b^14*d^3 - 16*b^18*d^3 + 1024*a^6*b^12*d^3 + 1440*a^8*b^10*d^3 + 1024*a^10*b^8*d^3 + 320*a^12*b^6*d^3 - 16*a^16*b^2*d^3))*(-1/(4*(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2)))^(1/2) + 96*a^3*b^13*d^2 + 240*a^5*b^11*d^2 + 320*a^7*b^9*d^2 + 240*a^9*b^7*d^2 + 96*a^11*b^5*d^2 + 16*a^13*b^3*d^2)*(-1/(4*(a^5*d^2 - b^5*d^2*1i + 5*a*b^4*d^2 - a^4*b*d^2*5i + a^2*b^3*d^2*10i - 10*a^3*b^2*d^2)))^(1/2) + atan((((-1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*(896*a^6*b^15*d^4 - 160*a^2*b^19*d^4 - 128*a^4*b^17*d^4 - 32*b^21*d^4 + 3136*a^8*b^13*d^4 + 4928*a^10*b^11*d^4 + 4480*a^12*b^9*d^4 + 2432*a^14*b^7*d^4 + 736*a^16*b^5*d^4 + 96*a^18*b^3*d^4 + (-1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5)) - (a + b*tan(c + d*x))^(1/2)*(320*a^4*b^14*d^3 - 16*b^18*d^3 + 1024*a^6*b^12*d^3 + 1440*a^8*b^10*d^3 + 1024*a^10*b^8*d^3 + 320*a^12*b^6*d^3 - 16*a^16*b^2*d^3))*(-1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*1i - ((-1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*(896*a^6*b^15*d^4 - 160*a^2*b^19*d^4 - 128*a^4*b^17*d^4 - 32*b^21*d^4 + 3136*a^8*b^13*d^4 + 4928*a^10*b^11*d^4 + 4480*a^12*b^9*d^4 + 2432*a^14*b^7*d^4 + 736*a^16*b^5*d^4 + 96*a^18*b^3*d^4 - (-1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5)) + (a + b*tan(c + d*x))^(1/2)*(320*a^4*b^14*d^3 - 16*b^18*d^3 + 1024*a^6*b^12*d^3 + 1440*a^8*b^10*d^3 + 1024*a^10*b^8*d^3 + 320*a^12*b^6*d^3 - 16*a^16*b^2*d^3))*(-1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*1i)/(32*a*b^15*d^2 - ((-1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*(896*a^6*b^15*d^4 - 160*a^2*b^19*d^4 - 128*a^4*b^17*d^4 - 32*b^21*d^4 + 3136*a^8*b^13*d^4 + 4928*a^10*b^11*d^4 + 4480*a^12*b^9*d^4 +

2432*a^14*b^7*d^4 + 736*a^16*b^5*d^4 + 96*a^18*b^3*d^4 - (-1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3*b^2*d^2*10i)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5)) + (a + b*tan(c + d*x))^(1/2)*(320*a^4*b^14*d^3 - 16*b^18*d^3 + 1024*a^6*b^12*d^3 + 1440*a^8*b^10*d^3 + 1024*a^10*b^8*d^3 + 320*a^12*b^6*d^3 - 16*a^16*b^2*d^3))*(-1i/(4*(a^5*d^2*1i - b^5*d^2 + a*b^4*d^2*5i - 5*a^4*b*d^2 + 10*a^2*b^3*d^2 - a^3...

3.552 $\int \frac{\cot(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=195

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d}+$$

```
[Out] -2*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d+arctanh((a+b*tan(d*x+c
))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(5/2)/d+arctanh((a+b*tan(d*x+c))^(1/2)/(a+I
*b)^(1/2))/(a+I*b)^(5/2)/d+2*b^2*(3*a^2+b^2)/a^2/(a^2+b^2)^2/d/(a+b*tan(d*x
+c))^(1/2)+2/3*b^2/a/(a^2+b^2)/d/(a+b*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.51, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3650, 3730, 3734, 3620, 3618, 65, 214, 3715}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d}+\frac{2b^2(3a^2+b^2)}{a^2d(a^2+b^2)^2\sqrt{a+b\tan(c+dx)}}+\frac{2b^2}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{5/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/(a^(5/2)*d) + ArcTanh[Sqrt[a
 + b*Tan[c + d*x]]/Sqrt[a - I*b]]/((a - I*b)^(5/2)*d) + ArcTanh[Sqrt[a + b*
Tan[c + d*x]]/Sqrt[a + I*b]]/((a + I*b)^(5/2)*d) + (2*b^2)/(3*a*(a^2 + b^2)
*d*(a + b*Tan[c + d*x])^(3/2)) + (2*b^2*(3*a^2 + b^2))/(a^2*(a^2 + b^2)^2*d
*Sqrt[a + b*Tan[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
```

```
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx = \frac{2b^2}{3a\left(a^2+b^2\right)d(a+b\tan(c+dx))^{3/2}} + \frac{2\int \frac{\cot(c+dx)\left(\frac{3}{2}\left(a^2+b^2\right)-\frac{3}{2}ab\tan(c+dx)+\frac{3}{2}b^2\,\text{ta}}{(a+b\tan(c+dx))^{3/2}}}{3a\left(a^2+b^2\right)}$$

$$= \frac{2b^2}{3a\left(a^2+b^2\right)d(a+b\tan(c+dx))^{3/2}} + \frac{2b^2(3a^2+b^2)}{a^2\left(a^2+b^2\right)^2 d\sqrt{a+b\tan(c+dx)}} +$$

$$= \frac{2b^2}{3a\left(a^2+b^2\right)d(a+b\tan(c+dx))^{3/2}} + \frac{2b^2(3a^2+b^2)}{a^2\left(a^2+b^2\right)^2 d\sqrt{a+b\tan(c+dx)}} +$$

$$= \frac{2b^2}{3a\left(a^2+b^2\right)d(a+b\tan(c+dx))^{3/2}} + \frac{2b^2(3a^2+b^2)}{a^2\left(a^2+b^2\right)^2 d\sqrt{a+b\tan(c+dx)}} +$$

$$= \frac{2b^2}{3a\left(a^2+b^2\right)d(a+b\tan(c+dx))^{3/2}} + \frac{2b^2(3a^2+b^2)}{a^2\left(a^2+b^2\right)^2 d\sqrt{a+b\tan(c+dx)}} -$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{2b^2}{3a\left(a^2+b^2\right)d(a+b\tan(c+dx))^{3/2}} +$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d} + \text{ta}$$

Mathematica [A]
time = 2.25, size = 237, normalized size = 1.22

$$\frac{2\left(\frac{-\frac{3\left(a^2+b^2\right)^2\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{3a^2(a+ib)^2\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{2\sqrt{a-ib}}+\frac{3a^2(a-ib)^2\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{2\sqrt{a+ib}}}{a\left(a^2+b^2\right)}+\frac{b^2}{(a+b\tan(c+dx))^{3/2}}+\frac{3b^2\left(3a^2+b^2\right)}{a\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}\right)}{3a\left(a^2+b^2\right)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] (2*(((-3*(a^2 + b^2)^2*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/Sqrt[a] +
 (3*a^2*(a + I*b)^2*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/(2*Sqr
t[a - I*b]) + (3*a^2*(a - I*b)^2*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a +
I*b]])/(2*Sqrt[a + I*b]))/(a*(a^2 + b^2)) + b^2/(a + b*Tan[c + d*x])^(3/2)
+ (3*b^2*(3*a^2 + b^2))/(a*(a^2 + b^2)*Sqrt[a + b*Tan[c + d*x]])))/(3*a*(a^
2 + b^2)*d)
```

Maple [**C**] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 2.85, size = 115830, normalized size = 594.00

method	result	size
default	Expression too large to display	115830

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)/(b*tan(d*x + c) + a)^(5/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 10106 vs. 2(163) = 326.
time = 3.59, size = 20287, normalized size = 104.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(12*sqrt(2)*((a^21 + a^19*b^2 - 20*a^17*b^4 - 84*a^15*b^6 - 154*a^13
*b^8 - 154*a^11*b^10 - 84*a^9*b^12 - 20*a^7*b^14 + a^5*b^16 + a^3*b^18)*d^5
*cos(d*x + c)^4 + 2*(3*a^19*b^2 + 20*a^17*b^4 + 56*a^15*b^6 + 84*a^13*b^8 +
 70*a^11*b^10 + 28*a^9*b^12 - 4*a^5*b^16 - a^3*b^18)*d^5*cos(d*x + c)^2 + (
a^17*b^4 + 7*a^15*b^6 + 21*a^13*b^8 + 35*a^11*b^10 + 35*a^9*b^12 + 21*a^7*b
```

^14 + 7*a^5*b^16 + a^3*b^18)*d^5 + 4*((a^20*b + 6*a^18*b^3 + 14*a^16*b^5 + 14*a^14*b^7 - 14*a^10*b^11 - 14*a^8*b^13 - 6*a^6*b^15 - a^4*b^17)*d^5*cos(d*x + c)^3 + (a^18*b^3 + 7*a^16*b^5 + 21*a^14*b^7 + 35*a^12*b^9 + 35*a^10*b^11 + 21*a^8*b^13 + 7*a^6*b^15 + a^4*b^17)*d^5*cos(d*x + c))*sin(d*x + c))*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 - (a^15 - 5*a^13*b^2 - 35*a^11*b^4 - 65*a^9*b^6 - 45*a^7*b^8 + a^5*b^10 + 15*a^3*b^12 + 5*a*b^14)*d^2*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4)))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b^20)*d^4))*(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4))^(3/4)*arctan(((5*a^20 + 30*a^18*b^2 + 61*a^16*b^4 + 8*a^14*b^6 - 182*a^12*b^8 - 364*a^10*b^10 - 350*a^8*b^12 - 184*a^6*b^14 - 47*a^4*b^16 - 2*a^2*b^18 + b^20)*d^4*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b^20)*d^4))*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4)) + (5*a^15 + 15*a^13*b^2 + a^11*b^4 - 45*a^9*b^6 - 65*a^7*b^8 - 35*a^5*b^10 - 5*a^3*b^12 + a*b^14)*d^2*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b^20)*d^4)) - sqrt(2)*((a^23 + 7*a^21*b^2 + 15*a^19*b^4 - 15*a^17*b^6 - 150*a^15*b^8 - 378*a^13*b^10 - 546*a^11*b^12 - 510*a^9*b^14 - 315*a^7*b^16 - 125*a^5*b^18 - 29*a^3*b^20 - 3*a*b^22)*d^7*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b^20)*d^4))*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4)) + (a^18 + 7*a^16*b^2 + 20*a^14*b^4 + 28*a^12*b^6 + 14*a^10*b^8 - 14*a^8*b^10 - 28*a^6*b^12 - 20*a^4*b^14 - 7*a^2*b^16 - b^18)*d^5*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b^20)*d^4)))*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 - (a^15 - 5*a^13*b^2 - 35*a^11*b^4 - 65*a^9*b^6 - 45*a^7*b^8 + a^5*b^10 + 15*a^3*b^12 + 5*a*b^14)*d^2*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4)))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*sqrt(((25*a^14 - 25*a^12*b^2 - 115*a^10*b^4 + 35*a^8*b^6 + 171*a^6*b^8 + 53*a^4*b^10 - 17*a^2*b^12 + b^14)*d^2*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4))*cos(d*x + c) + sqrt(2)*((25*a^16 - 50*a^14*b^2 - 90*a^12*b^4 + 150*a^10*b^6 + 136*a^8*b^8 - 118*a^6*b^10 - 70*a^4*b^12 + 18*a^2*b^14 - b^16)*d^3*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4))*cos(d*x + c) + (25*a^11 - 175*a^9*b^2 + 410*a^7*b^4 - 350*a^5*b^6 + 61*a^3*b^8 - 3*a*b^10)*d*cos(d*x + c))*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^

```
4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10 - (a^15 - 5*a^13*b^2 - 35*a^11*b^4 - 65*a
^9*b^6 - 45*a^7*b^8 + a^5*b^10 + 15*a^3*b^12 + 5*a*b^14)*d^2*sqrt(1/((a^10
+ 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4)))/(25*a^8*b^
2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*sqrt((a*cos(d*x + c) +
b*sin(d*x + c))/cos(d*x + c))*(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b
^6 + 5*a^2*b^8 + b^10)*d^4))^(1/4) + (25*a^9 - 100*a^7*b^2 + 110*a^5*b^4 -
20*a^3*b^6 + a*b^8)*cos(d*x + c) + (25*a^8*b - 100*a^6*b^3 + 110*a^4*b^5 -
20*a^2*b^7 + b^9)*sin(d*x + c))/cos(d*x + c))*(1/((a^10 + 5*a^8*b^2 + 10*a^
6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4))^(3/4) + sqrt(2)*((5*a^27 + 25*
a^25*b^2 + 6*a^23*b^4 - 218*a^21*b^6 - 585*a^19*b^8 - 405*a^17*b^10 + 900*a
^15*b^12 + 2532*a^13*b^14 + 2979*a^11*b^16 + 2015*a^9*b^18 + 790*a^7*b^20 +
 150*a^5*b^22 + a^3*b^24 - 3*a*b^26)*d^7*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 1
10*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^
14*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^
4*b^16 + 10*a^2*b^18 + b^20)*d^4))*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 +
 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4)) + (5*a^22 + 25*a^20*b^2 + 31*a^18*b^4
 - 53*a^16*b^6 - 190*a^14*b^8 - 182*a^12*b^10 +...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(c+dx)}{(a+b\tan(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(cot(c + d*x)/(a + b*tan(c + d*x))**(5/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 4.89, size = 2500, normalized size = 12.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + b*tan(c + d*x))^(5/2),x)
```

[Out] (log((((a + b*tan(c + d*x))^(1/2)*(10304*a^20*b^34*d^5 - 512*a^16*b^38*d^5
- 544*a^18*b^36*d^5 - 64*a^14*b^40*d^5 + 66976*a^22*b^32*d^5 + 221312*a^24*
b^30*d^5 + 480480*a^26*b^28*d^5 + 741312*a^28*b^26*d^5 + 837408*a^30*b^24*d
^5 + 695552*a^32*b^22*d^5 + 416416*a^34*b^20*d^5 + 168896*a^36*b^18*d^5 + 3
7856*a^38*b^16*d^5 - 896*a^40*b^14*d^5 - 3424*a^42*b^12*d^5 - 960*a^44*b^10
*d^5 - 96*a^46*b^8*d^5) - ((((20*a^2*b^8*d^4 - b^10*d^4 - 110*a^4*b^6*d^4 +
100*a^6*b^4*d^4 - 25*a^8*b^2*d^4)^(1/2) + a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b
^2*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*
d^4 + 5*a^8*b^2*d^4))^(1/2)*(384*a^15*b^42*d^6 - ((((((20*a^2*b^8*d^4 - b^1
0*d^4 - 110*a^4*b^6*d^4 + 100*a^6*b^4*d^4 - 25*a^8*b^2*d^4)^(1/2) + a^5*d^2
+ 5*a*b^4*d^2 - 10*a^3*b^2*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*
a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(512*a^16*b^46*d^8 - (
(((20*a^2*b^8*d^4 - b^10*d^4 - 110*a^4*b^6*d^4 + 100*a^6*b^4*d^4 - 25*a^8*b
^2*d^4)^(1/2) + a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b^2*d^2)/(a^10*d^4 + b^10*d^
4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)
*(a + b*tan(c + d*x))^(1/2)*(512*a^18*b^46*d^9 + 9984*a^20*b^44*d^9 + 92160
*a^22*b^42*d^9 + 535296*a^24*b^40*d^9 + 2193408*a^26*b^38*d^9 + 6736896*a^2
8*b^36*d^9 + 16084992*a^30*b^34*d^9 + 30551040*a^32*b^32*d^9 + 46844928*a^3
4*b^30*d^9 + 58499584*a^36*b^28*d^9 + 59744256*a^38*b^26*d^9 + 49900032*a^4
0*b^24*d^9 + 33945600*a^42*b^22*d^9 + 18643968*a^44*b^20*d^9 + 8146944*a^46
*b^18*d^9 + 2767872*a^48*b^16*d^9 + 705024*a^50*b^14*d^9 + 126720*a^52*b^12
*d^9 + 14336*a^54*b^10*d^9 + 768*a^56*b^8*d^9))/2 + 9728*a^18*b^44*d^8 + 87
936*a^20*b^42*d^8 + 502144*a^22*b^40*d^8 + 2028544*a^24*b^38*d^8 + 6153216*
a^26*b^36*d^8 + 14518784*a^28*b^34*d^8 + 27243008*a^30*b^32*d^8 + 41213952*
a^32*b^30*d^8 + 50665472*a^34*b^28*d^8 + 50775296*a^36*b^26*d^8 + 41443584*
a^38*b^24*d^8 + 27409408*a^40*b^22*d^8 + 14543872*a^42*b^20*d^8 + 6093312*a
^44*b^18*d^8 + 1966592*a^46*b^16*d^8 + 470528*a^48*b^14*d^8 + 78336*a^50*b^
12*d^8 + 8064*a^52*b^10*d^8 + 384*a^54*b^8*d^8))/2 + (a + b*tan(c + d*x))^(
1/2)*(256*a^15*b^44*d^7 + 4608*a^17*b^42*d^7 + 40512*a^19*b^40*d^7 + 224768
*a^21*b^38*d^7 + 864768*a^23*b^36*d^7 + 2419200*a^25*b^34*d^7 + 5055232*a^2
7*b^32*d^7 + 8007168*a^29*b^30*d^7 + 9664512*a^31*b^28*d^7 + 8859136*a^33*b
^26*d^7 + 6095232*a^35*b^24*d^7 + 3095040*a^37*b^22*d^7 + 1164800*a^39*b^20
*d^7 + 376320*a^41*b^18*d^7 + 154368*a^43*b^16*d^7 + 76288*a^45*b^14*d^7 +
28416*a^47*b^12*d^7 + 6144*a^49*b^10*d^7 + 576*a^51*b^8*d^7))*(((20*a^2*b^8
*d^4 - b^10*d^4 - 110*a^4*b^6*d^4 + 100*a^6*b^4*d^4 - 25*a^8*b^2*d^4)^(1/2)
+ a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b^2*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8
*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2))/2 + 7296*a^
17*b^40*d^6 + 59424*a^19*b^38*d^6 + 280992*a^21*b^36*d^6 + 866208*a^23*b^34
*d^6 + 1825824*a^25*b^32*d^6 + 2629536*a^27*b^30*d^6 + 2374944*a^29*b^28*d^
6 + 727584*a^31*b^26*d^6 - 1413984*a^33*b^24*d^6 - 2649504*a^35*b^22*d^6 -
2454816*a^37*b^20*d^6 - 1476384*a^39*b^18*d^6 - 597408*a^41*b^16*d^6 - 1561
92*a^43*b^14*d^6 - 22944*a^45*b^12*d^6 - 1056*a^47*b^10*d^6 + 96*a^49*b^8*d
^6))/2)*(((20*a^2*b^8*d^4 - b^10*d^4 - 110*a^4*b^6*d^4 + 100*a^6*b^4*d^4 -
25*a^8*b^2*d^4)^(1/2) + a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b^2*d^2)/(a^10*d^4 +
b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4

))^(1/2))/2 + 32*a^14*b^38*d^4 + 448*a^16*b^36*d^4 + 2912*a^18*b^34*d^4 + 1
1648*a^20*b^32*d^4 + 32032*a^22*b^30*d^4 + 64064*a^24*b^28*d^4 + 96096*a^26
*b^26*d^4 + 109824*a^28*b^24*d^4 + 96096*a^30*b^22*d^4 + 64064*a^32*b^20*d^
4 + 32032*a^34*b^18*d^4 + 11648*a^36*b^16*d^4 + 2912*a^38*b^14*d^4 + 448*a^
40*b^12*d^4 + 32*a^42*b^10*d^4)*(((20*a^2*b^8*d^4 - b^10*d^4 - 110*a^4*b^6*
d^4 + 100*a^6*b^4*d^4 - 25*a^8*b^2*d^4)^(1/2) + a^5*d^2 + 5*a*b^4*d^2 - 10*
a^3*b^2*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6
*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2))/2 + (log((((a + b*tan(c + d*x))^(1/2)*(10
304*a^20*b^34*d^5 - 512*a^16*b^38*d^5 - 544*a^18*b^36*d^5 - 64*a^14*b^40*d^
5 + 66976*a^22*b^32*d^5 + 221312*a^24*b^30*d^5 + 480480*a^26*b^28*d^5 + 741
312*a^28*b^26*d^5 + 837408*a^30*b^24*d^5 + 695552*a^32*b^22*d^5 + 416416*a^
34*b^20*d^5 + 168896*a^36*b^18*d^5 + 37856*a^38*b^16*d^5 - 896*a^40*b^14*d^
5 - 3424*a^42*b^12*d^5 - 960*a^44*b^10*d^5 - 96*a^46*b^8*d^5) - ((-((20*a^2
*b^8*d^4 - b^10*d^4 - 110*a^4*b^6*d^4 + 100*a^6*b^4*d^4 - 25*a^8*b^2*d^4)^(
1/2) - a^5*d^2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2
*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(384*a^1
5*b^42*d^6 - ((((-((20*a^2*b^8*d^4 - b^10*d^4 - 110*a^4*b^6*d^4 + 100*a^6*b
^4*d^4 - 25*a^8*b^2*d^4)^(1/2) - a^5*d^2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/(a
^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^
8*b^2*d^4))^(1/2)*(512*a^16*b^46*d^8 - ((-((20*a^2*b^8*d^4 - b^10*d^4 - 110
*a^4*b^6*d^4 + 100*a^6*b^4*d^4 - 25*a^8*b^2*d^4)^(1/2) - a^5*d^2 - 5*a*b^4*
d^2 + 10*a^3*b^2*d^2)/(a^10*d^4 + b^10*d^4 + 5*...

3.553 $\int \frac{\cot^2(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=245

$$\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{7/2}d}+\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d}-\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d}-$$

```
[Out] 5*b*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/a^(7/2)/d+I*arctanh((a+b*tan(d*
x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(5/2)/d-I*arctanh((a+b*tan(d*x+c))^(1/2)
/(a+I*b)^(1/2))/(a+I*b)^(5/2)/d-b*(a^4+10*a^2*b^2+5*b^4)/a^3/(a^2+b^2)^2/d/
(a+b*tan(d*x+c))^(1/2)-1/3*b*(3*a^2+5*b^2)/a^2/(a^2+b^2)/d/(a+b*tan(d*x+c))
^(3/2)-cot(d*x+c)/a/d/(a+b*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.65, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3650, 3730, 3734, 3620, 3618, 65, 214, 3715}

$$\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{7/2}d}-\frac{b(3a^2+5b^2)}{3a^2d(a^2+b^2)(a+b\tan(c+dx))^{3/2}}-\frac{b(a^4+10a^2b^2+5b^4)}{a^3d(a^2+b^2)^2\sqrt{a+b\tan(c+dx)}}+\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{5/2}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{5/2}}-\frac{\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] (5*b*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/(a^(7/2)*d) + (I*ArcTanh[Sq
rt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/((a - I*b)^(5/2)*d) - (I*ArcTanh[Sqr
t[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/((a + I*b)^(5/2)*d) - (b*(3*a^2 + 5*b
^2))/(3*a^2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^(3/2)) - Cot[c + d*x]/(a*d*(
a + b*Tan[c + d*x])^(3/2)) - (b*(a^4 + 10*a^2*b^2 + 5*b^4))/(a^3*(a^2 + b^2
)^2*d*Sqrt[a + b*Tan[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx = -\frac{\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}} - \frac{\int \frac{\cot(c+dx)\left(\frac{5b}{2}+a\tan(c+dx)+\frac{5}{2}b\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^{5/2}}\,dx}{a}$$

$$= -\frac{b(3a^2+5b^2)}{3a^2(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}} - \frac{2\int \frac{\cot(}{}}{}$$

$$= -\frac{b(3a^2+5b^2)}{3a^2(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}} - \frac{}{a^3(a^2}$$

$$= -\frac{b(3a^2+5b^2)}{3a^2(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}} - \frac{}{a^3(a^2}$$

$$= -\frac{b(3a^2+5b^2)}{3a^2(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}} - \frac{}{a^3(a^2}$$

$$= -\frac{b(3a^2+5b^2)}{3a^2(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}} - \frac{}{a^3(a^2}$$

$$= \frac{5b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{7/2}d} - \frac{b(3a^2+5b^2)}{3a^2(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} -$$

$$= \frac{5b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{7/2}d} + \frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d} - \frac{i}{}$$

Mathematica [A]

time = 4.88, size = 232, normalized size = 0.95

$$-\frac{-\frac{15b\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}}-3ia^2\left(\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}}\right)+\frac{b(3a^2+5b^2)}{(a^2+b^2)(a+b\tan(c+dx))^{3/2}}+\frac{3a\cot(c+dx)}{(a+b\tan(c+dx))^{3/2}}+\frac{3b(a^4+10a^2b^2+5b^4)}{a(a^2+b^2)^2\sqrt{a+b\tan(c+dx)}}}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] -1/3*((-15*b*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/a^(3/2) - (3*I)*a^2
*(ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]]/(a - I*b)^(5/2) - ArcTanh
[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]]/(a + I*b)^(5/2)) + (b*(3*a^2 + 5*b
^2))/((a^2 + b^2)*(a + b*Tan[c + d*x])^(3/2)) + (3*a*Cot[c + d*x])/(a + b*T
an[c + d*x])^(3/2) + (3*b*(a^4 + 10*a^2*b^2 + 5*b^4))/(a*(a^2 + b^2)^2*Sqrt
[a + b*Tan[c + d*x]]))/(a^2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 4.66, size = 175534, normalized size = 716.47

method	result	size
default	Expression too large to display	175534

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^2/(b*tan(d*x + c) + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11132 vs. 2(207) = 414.
time = 4.46, size = 22339, normalized size = 91.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(12*sqrt(2)*((a^22 + a^20*b^2 - 20*a^18*b^4 - 84*a^16*b^6 - 154*a^14
*b^8 - 154*a^12*b^10 - 84*a^10*b^12 - 20*a^8*b^14 + a^6*b^16 + a^4*b^18)*d^
5*cos(d*x + c)^6 - (a^22 - 5*a^20*b^2 - 60*a^18*b^4 - 196*a^16*b^6 - 322*a^
14*b^8 - 294*a^12*b^10 - 140*a^10*b^12 - 20*a^8*b^14 + 9*a^6*b^16 + 3*a^4*b
^18)*d^5*cos(d*x + c)^4 - 3*(2*a^20*b^2 + 13*a^18*b^4 + 35*a^16*b^6 + 49*a^
14*b^8 + 35*a^12*b^10 + 7*a^10*b^12 - 7*a^8*b^14 - 5*a^6*b^16 - a^4*b^18)*d
^5*cos(d*x + c)^2 - (a^18*b^4 + 7*a^16*b^6 + 21*a^14*b^8 + 35*a^12*b^10 + 3
5*a^10*b^12 + 21*a^8*b^14 + 7*a^6*b^16 + a^4*b^18)*d^5 + 4*((a^21*b + 6*a^1
9*b^3 + 14*a^17*b^5 + 14*a^15*b^7 - 14*a^11*b^11 - 14*a^9*b^13 - 6*a^7*b^15
 - a^5*b^17)*d^5*cos(d*x + c)^5 - (a^21*b + 5*a^19*b^3 + 7*a^17*b^5 - 7*a^1
5*b^7 - 35*a^13*b^9 - 49*a^11*b^11 - 35*a^9*b^13 - 13*a^7*b^15 - 2*a^5*b^17
)*d^5*cos(d*x + c)^3 - (a^19*b^3 + 7*a^17*b^5 + 21*a^15*b^7 + 35*a^13*b^9 +
 35*a^11*b^11 + 21*a^9*b^13 + 7*a^7*b^15 + a^5*b^17)*d^5*cos(d*x + c))*sin(
d*x + c))*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^
10 + (a^15 - 5*a^13*b^2 - 35*a^11*b^4 - 65*a^9*b^6 - 45*a^7*b^8 + a^5*b^10
+ 15*a^3*b^12 + 5*a*b^14)*d^2*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a
^4*b^6 + 5*a^2*b^8 + b^10)*d^4)))/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 -
 20*a^2*b^8 + b^10))*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*
b^8 + b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^
8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18
 + b^20)*d^4))*(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8
+ b^10)*d^4))^(3/4)*arctan(((5*a^20 + 30*a^18*b^2 + 61*a^16*b^4 + 8*a^14*b^
6 - 182*a^12*b^8 - 364*a^10*b^10 - 350*a^8*b^12 - 184*a^6*b^14 - 47*a^4*b^1
6 - 2*a^2*b^18 + b^20)*d^4*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 2
0*a^2*b^8 + b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a
^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^
2*b^18 + b^20)*d^4))*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 +
5*a^2*b^8 + b^10)*d^4)) + (5*a^15 + 15*a^13*b^2 + a^11*b^4 - 45*a^9*b^6 - 6
5*a^7*b^8 - 35*a^5*b^10 - 5*a^3*b^12 + a*b^14)*d^2*sqrt((25*a^8*b^2 - 100*a
^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^
4 + 120*a^14*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^
14 + 45*a^4*b^16 + 10*a^2*b^18 + b^20)*d^4)) - sqrt(2)*((3*a^22 + 29*a^20*b
^2 + 125*a^18*b^4 + 315*a^16*b^6 + 510*a^14*b^8 + 546*a^12*b^10 + 378*a^10*
b^12 + 150*a^8*b^14 + 15*a^6*b^16 - 15*a^4*b^18 - 7*a^2*b^20 - b^22)*d^7*sq
rt((25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^20 + 10
*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210
*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b^20)*d^4))*sqrt(1/(
(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4)) + 2*(
a^17 + 8*a^15*b^2 + 28*a^13*b^4 + 56*a^11*b^6 + 70*a^9*b^8 + 56*a^7*b^10 +
28*a^5*b^12 + 8*a^3*b^14 + a*b^16)*d^5*sqrt((25*a^8*b^2 - 100*a^6*b^4 + 110
*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14
*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*
b^16 + 10*a^2*b^18 + b^20)*d^4)))*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*
a^4*b^6 + 5*a^2*b^8 + b^10 + (a^15 - 5*a^13*b^2 - 35*a^11*b^4 - 65*a^9*b^6
- 45*a^7*b^8 + a^5*b^10 + 15*a^3*b^12 + 5*a*b^14)*d^2*sqrt(1/((a^10 + 5*a^8
```

```
*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4)))/(25*a^8*b^2 - 100
*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*sqrt(((25*a^14*b^2 - 25*a^12*b
^4 - 115*a^10*b^6 + 35*a^8*b^8 + 171*a^6*b^10 + 53*a^4*b^12 - 17*a^2*b^14 +
 b^16)*d^2*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8
+ b^10)*d^4))*cos(d*x + c) + sqrt(2)*(2*(25*a^15*b^3 - 25*a^13*b^5 - 115*a^
11*b^7 + 35*a^9*b^9 + 171*a^7*b^11 + 53*a^5*b^13 - 17*a^3*b^15 + a*b^17)*d^
3*sqrt(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d
^4))*cos(d*x + c) + (75*a^10*b^3 - 325*a^8*b^5 + 430*a^6*b^7 - 170*a^4*b^9
+ 23*a^2*b^11 - b^13)*d*cos(d*x + c))*sqrt((a^10 + 5*a^8*b^2 + 10*a^6*b^4 +
 10*a^4*b^6 + 5*a^2*b^8 + b^10 + (a^15 - 5*a^13*b^2 - 35*a^11*b^4 - 65*a^9*
b^6 - 45*a^7*b^8 + a^5*b^10 + 15*a^3*b^12 + 5*a*b^14)*d^2*sqrt(1/((a^10 + 5
*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4)))/(25*a^8*b^2 -
 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10))*sqrt((a*cos(d*x + c) + b*s
in(d*x + c))/cos(d*x + c))*(1/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6
+ 5*a^2*b^8 + b^10)*d^4))^(1/4) + (25*a^9*b^2 - 100*a^7*b^4 + 110*a^5*b^6 -
 20*a^3*b^8 + a*b^10)*cos(d*x + c) + (25*a^8*b^3 - 100*a^6*b^5 + 110*a^4*b^
7 - 20*a^2*b^9 + b^11)*sin(d*x + c))/cos(d*x + c))*(1/((a^10 + 5*a^8*b^2 +
10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^4))^(3/4) + sqrt(2)*((15*a^26
*b + 115*a^24*b^3 + 338*a^22*b^5 + 354*a^20*b^7 - 475*a^18*b^9 - 2055*a^16*
b^11 - 3060*a^14*b^13 - 2484*a^12*b^15 - 1047*a^10*b^17 - 75*a^8*b^19 + 130
*a^6*b^21 + 50*a^4*b^23 + 3*a^2*b^25 - b^27)*d^7*sqrt((25*a^8*b^2 - 100*a^6
*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^20 ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(c+dx)}{(a+b\tan(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+b*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(cot(c + d*x)**2/(a + b*tan(c + d*x))**(5/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 4.95, size = 2500, normalized size = 10.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d*x)^2/(a + b*tan(c + d*x))^(5/2),x)

[Out] ((2*b^3)/(3*a*(a^2 + b^2)) + (2*b^3*(11*a^2 + 5*b^2)*(a + b*tan(c + d*x)))/(3*(a*b^2 + a^3)^2) - (b*(a + b*tan(c + d*x))^2*(a^4 + 5*b^4 + 10*a^2*b^2))/(a^3*(a^2 + b^2)^2))/(d*(a + b*tan(c + d*x))^(5/2) - a*d*(a + b*tan(c + d*x))^(3/2)) + (log(400*a^22*b^39*d^4 - ((((((20*a^2*b^8*d^4 - b^10*d^4 - 110*a^4*b^6*d^4 + 100*a^6*b^4*d^4 - 25*a^8*b^2*d^4)^(1/2) - a^5*d^2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(90304*a^29*b^37*d^6 - 800*a^21*b^45*d^6 - 10400*a^23*b^43*d^6 - 54400*a^25*b^41*d^6 - 121600*a^27*b^39*d^6 - (((a + b*tan(c + d*x))^(1/2)*(1600*a^22*b^46*d^7 + 28800*a^24*b^44*d^7 + 244800*a^26*b^42*d^7 + 1304256*a^28*b^40*d^7 + 4880128*a^30*b^38*d^7 + 13627392*a^32*b^36*d^7 + 29476608*a^34*b^34*d^7 + 50615552*a^36*b^32*d^7 + 70152576*a^38*b^30*d^7 + 79329536*a^40*b^28*d^7 + 73600384*a^42*b^26*d^7 + 56025216*a^44*b^24*d^7 + 34754304*a^46*b^22*d^7 + 17296384*a^48*b^20*d^7 + 6713088*a^50*b^18*d^7 + 1934592*a^52*b^16*d^7 + 377408*a^54*b^14*d^7 + 39552*a^56*b^12*d^7 + 64*a^58*b^10*d^7 - 320*a^60*b^8*d^7) - ((((20*a^2*b^8*d^4 - b^10*d^4 - 110*a^4*b^6*d^4 + 100*a^6*b^4*d^4 - 25*a^8*b^2*d^4)^(1/2) - a^5*d^2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(((((20*a^2*b^8*d^4 - b^10*d^4 - 110*a^4*b^6*d^4 + 100*a^6*b^4*d^4 - 25*a^8*b^2*d^4)^(1/2) - a^5*d^2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(512*a^27*b^46*d^9 + 9984*a^29*b^44*d^9 + 92160*a^31*b^42*d^9 + 535296*a^33*b^40*d^9 + 2193408*a^35*b^38*d^9 + 6736896*a^37*b^36*d^9 + 16084992*a^39*b^34*d^9 + 30551040*a^41*b^32*d^9 + 46844928*a^43*b^30*d^9 + 58499584*a^45*b^28*d^9 + 59744256*a^47*b^26*d^9 + 49900032*a^49*b^24*d^9 + 33945600*a^51*b^22*d^9 + 18643968*a^53*b^20*d^9 + 8146944*a^55*b^18*d^9 + 2767872*a^57*b^16*d^9 + 705024*a^59*b^14*d^9 + 126720*a^61*b^12*d^9 + 14336*a^63*b^10*d^9 + 768*a^65*b^8*d^9))/2 + 1280*a^24*b^47*d^8 + 24320*a^26*b^45*d^8 + 219008*a^28*b^43*d^8 + 1241984*a^30*b^41*d^8 + 4970496*a^32*b^39*d^8 + 14909440*a^34*b^37*d^8 + 34746880*a^36*b^35*d^8 + 64356864*a^38*b^33*d^8 + 96092672*a^40*b^31*d^8 + 116633088*a^42*b^29*d^8 + 115498240*a^44*b^27*d^8 + 93267200*a^46*b^25*d^8 + 61128704*a^48*b^23*d^8 + 32212992*a^50*b^21*d^8 + 13439488*a^52*b^19*d^8 + 4334080*a^54*b^17*d^8 + 1040640*a^56*b^15*d^8 + 174848*a^58*b^13*d^8 + 18304*a^60*b^11*d^8 + 896*a^62*b^9*d^8))/2)*(((20*a^2*b^8*d^4 - b^10*d^4 - 110*a^4*b^6*d^4 + 100*a^6*b^4*d^4 - 25*a^8*b^2*d^4)^(1/2) - a^5*d^2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2))/2 + 1465856*a^31*b^35*d^6 + 5014464*a^33*b^33*d^6 + 10323456*a^35*b^31*d^6 + 14661504*a^37*b^29*d^6 + 14908608*a^39*b^27*d^6 + 10808512*a^41*b^25*d^6 + 5328128*a^43*b^23*d^6 + 1531712*a^45*b^21*d^6 + 87808*a^47*b^19*d^6 - 85696*a^49*b^17*d^6 - 6144*a^51*b^15*d^6 + 15264*a^53*b^13*d^6 + 5856*a^55*b^11*d^6 +

704*a^57*b^9*d^6))/2 + (a + b*tan(c + d*x))^(1/2)*(67232*a^27*b^36*d^5 - 3200*a^23*b^40*d^5 - 3200*a^25*b^38*d^5 - 400*a^21*b^42*d^5 + 437248*a^29*b^34*d^5 + 1458912*a^31*b^32*d^5 + 3214848*a^33*b^30*d^5 + 5065632*a^35*b^28*d^5 + 5898464*a^37*b^26*d^5 + 5129696*a^39*b^24*d^5 + 3313024*a^41*b^22*d^5 + 1552096*a^43*b^20*d^5 + 500864*a^45*b^18*d^5 + 99232*a^47*b^16*d^5 + 8448*a^49*b^14*d^5 - 288*a^51*b^12*d^5 + 48*a^53*b^10*d^5 + 32*a^55*b^8*d^5))*(((20*a^2*b^8*d^4 - b^10*d^4 - 110*a^4*b^6*d^4 + 100*a^6*b^4*d^4 - 25*a^8*b^2*d^4)^(1/2) - a^5*d^2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2))/2 + 5520*a^24*b^37*d^4 + 35280*a^26*b^35*d^4 + 138320*a^28*b^33*d^4 + 371280*a^30*b^31*d^4 + 720720*a^32*b^29*d^4 + 1041040*a^34*b^27*d^4 + 1132560*a^36*b^25*d^4 + 926640*a^38*b^23*d^4 + 560560*a^40*b^21*d^4 + 240240*a^42*b^19*d^4 + 65520*a^44*b^17*d^4 + 7280*a^46*b^15*d^4 - 1680*a^48*b^13*d^4 - 720*a^50*b^11*d^4 - 80*a^52*b^9*d^4)*(((20*a^2*b^8*d^4 - b^10*d^4 - 110*a^4*b^6*d^4 + 100*a^6*b^4*d^4 - 25*a^8*b^2*d^4)^(1/2) - a^5*d^2 - 5*a*b^4*d^2 + 10*a^3*b^2*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2))/2 + (log(400*a^22*b^39*d^4 - ((((-((20*a^2*b^8*d^4 - b^10*d^4 - 110*a^4*b^6*d^4 + 100*a^6*b^4*d^4 - 25*a^8*b^2*d^4)^(1/2) + a^5*d^2 + 5*a*b^4*d^2 - 10*a^3*b^2*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(90304*a^29*b^37*d^6 - 800*a^21*b^45*d^6 - 10400*a^23*b^43*d^6 - 54400*a^25*b^41*d^6 - 121600*a^27*b^39*d^6 - (((a + b*tan(c + d*x))^(1/2)*(1600*a^22*b^46*d^7 + 28800*a^24*b^44*d^7 + 244800*a^26*b^42*d^7 + 1304256*a^28*b^40*d^7 + 4880128*a^30*b^38*d^7 + 13627392*a^32*b^36*d^7 + 29476608*a^34*b^34*d^7 + 50615552*a^36*b^32*d^7 + 70152576*a^38*b^30*d^7 + 79329536*a^40*b^28*d^7 + 73600384*a^42*b^26*d^7 + 56025216*a^44*b^24*d^7...

3.554 $\int \frac{1}{(a+b\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=194

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{7/2}d}+\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{7/2}d}-\frac{2b}{5\left(a^2+b^2\right)d(a+b\tan(c+dx))^{5/2}}$$

```
[Out] -I*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(7/2)/d+I*arctanh(
(a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(7/2)/d-2*b*(3*a^2-b^2)/(a^2+
b^2)^3/d/(a+b*tan(d*x+c))^(1/2)-2/5*b/(a^2+b^2)/d/(a+b*tan(d*x+c))^(5/2)-4/
3*a*b/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.29, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3564, 3610, 3620, 3618, 65, 214}

$$-\frac{2b(3a^2-b^2)}{d\left(a^2+b^2\right)^3\sqrt{a+b\tan(c+dx)}}-\frac{4ab}{3d\left(a^2+b^2\right)^2(a+b\tan(c+dx))^{3/2}}-\frac{2b}{5d\left(a^2+b^2\right)(a+b\tan(c+dx))^{5/2}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{7/2}}+\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^(-7/2),x]
```

```
[Out] ((-I)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/((a - I*b)^(7/2)*d)
+ (I*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/((a + I*b)^(7/2)*d) -
 (2*b)/(5*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^(5/2)) - (4*a*b)/(3*(a^2 + b^2
)^2*d*(a + b*Tan[c + d*x])^(3/2)) - (2*b*(3*a^2 - b^2))/((a^2 + b^2)^3*d*Sq
rt[a + b*Tan[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
```

```
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(a+b\tan(c+dx))^{7/2}}\,dx = -\frac{2b}{5\left(a^2+b^2\right)d(a+b\tan(c+dx))^{5/2}} + \frac{\int \frac{a-b\tan(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx}{a^2+b^2}$$

$$= -\frac{2b}{5\left(a^2+b^2\right)d(a+b\tan(c+dx))^{5/2}} - \frac{4ab}{3\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^{3/2}} +$$

$$= -\frac{2b}{5\left(a^2+b^2\right)d(a+b\tan(c+dx))^{5/2}} - \frac{4ab}{3\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^{3/2}} -$$

$$= -\frac{2b}{5\left(a^2+b^2\right)d(a+b\tan(c+dx))^{5/2}} - \frac{4ab}{3\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^{3/2}} -$$

$$= -\frac{2b}{5\left(a^2+b^2\right)d(a+b\tan(c+dx))^{5/2}} - \frac{4ab}{3\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^{3/2}} -$$

$$= -\frac{2b}{5\left(a^2+b^2\right)d(a+b\tan(c+dx))^{5/2}} - \frac{4ab}{3\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^{3/2}} -$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{7/2}d} + \frac{i\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{7/2}d} - \frac{}{5}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.26, size = 108, normalized size = 0.56

$$\frac{i(a+ib)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\frac{a+b\tan(c+dx)}{a-ib}\right)+(-ia-b)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\frac{a+b\tan(c+dx)}{a+ib}\right)}{5\left(a^2+b^2\right)d(a+b\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^(-7/2),x]
```

```
[Out] (I*(a + I*b)*Hypergeometric2F1[-5/2, 1, -3/2, (a + b*Tan[c + d*x])/(a - I*b
)] + ((-I)*a - b)*Hypergeometric2F1[-5/2, 1, -3/2, (a + b*Tan[c + d*x])/(a
+ I*b)])/(5*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1210 vs. 2(166) = 332.
time = 0.14, size = 1211, normalized size = 6.24 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*b*(1/(a^2+b^2)^3*(1/4/b^2/(a^2+b^2)^(3/2)*(1/2*((2*(a^2+b^2)^(1/2)+2*a)
^(1/2)*(a^2+b^2)^(1/2)*a^5-2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*
a^3*b^2-3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^4-(2*(a^2+b^2)^
(1/2)+2*a)^(1/2)*a^6+5*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4*b^2+5*(2*(a^2+b^2)
^(1/2)+2*a)^(1/2)*a^2*b^4-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^6)*ln(b*tan(d*x+c
)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2
*(8*a^5*b^2-8*a*b^6-1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^5-
2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3*b^2-3*(2*(a^2+b^2)^(1/2
)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^4-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^6+5*(2*(
a^2+b^2)^(1/2)+2*a)^(1/2)*a^4*b^2+5*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b^4-(
2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^6)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^
2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a
)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))+1/4/b^2/(a^2+b^2)^(3/2)*(-1/2*((2*
(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^5-2*(2*(a^2+b^2)^(1/2)+2*a)^(1
/2)*(a^2+b^2)^(1/2)*a^3*b^2-3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)
*a*b^4-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^6+5*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^
4*b^2+5*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b^4-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)
*b^6)*ln(-b*tan(d*x+c)-a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/
2)-(a^2+b^2)^(1/2))+2*(-8*a^5*b^2+8*a*b^6+1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2
)*(a^2+b^2)^(1/2)*a^5-2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3*b
^2-3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^4-(2*(a^2+b^2)^(1/2)
+2*a)^(1/2)*a^6+5*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4*b^2+5*(2*(a^2+b^2)^(1/2
)+2*a)^(1/2)*a^2*b^4-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^6)*(2*(a^2+b^2)^(1/2)+
2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*tan(d*x+c))^(1/2)
+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))))-1/5/(a^2+b
^2)/(a+b*tan(d*x+c))^(5/2)-(3*a^2-b^2)/(a^2+b^2)^3/(a+b*tan(d*x+c))^(1/2)-2
/3/(a^2+b^2)^2*a/(a+b*tan(d*x+c))^(3/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 14477 vs. $2(160) = 320$.
time = 3.22, size = 14477, normalized size = 74.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tan(d*x+c))^(7/2),x, algorithm="fricas")

[Out] -1/60*(60*sqrt(2)*((a^26 - 5*a^24*b^2 - 90*a^22*b^4 - 406*a^20*b^6 - 925*a^18*b^8 - 1143*a^16*b^10 - 540*a^14*b^12 + 540*a^12*b^14 + 1143*a^10*b^16 + 925*a^8*b^18 + 406*a^6*b^20 + 90*a^4*b^22 + 5*a^2*b^24 - b^26)*d^5*cos(d*x + c)^6 + 3*(5*a^24*b^2 + 40*a^22*b^4 + 126*a^20*b^6 + 160*a^18*b^8 - 105*a^16*b^10 - 720*a^14*b^12 - 1260*a^12*b^14 - 1248*a^10*b^16 - 765*a^8*b^18 - 280*a^6*b^20 - 50*a^4*b^22 + b^26)*d^5*cos(d*x + c)^4 + 3*(5*a^22*b^4 + 49*a^20*b^6 + 215*a^18*b^8 + 555*a^16*b^10 + 930*a^14*b^12 + 1050*a^12*b^14 + 798*a^10*b^16 + 390*a^8*b^18 + 105*a^6*b^20 + 5*a^4*b^22 - 5*a^2*b^24 - b^26)*d^5*cos(d*x + c)^2 + (a^20*b^6 + 10*a^18*b^8 + 45*a^16*b^10 + 120*a^14*b^12 + 210*a^12*b^14 + 252*a^10*b^16 + 210*a^8*b^18 + 120*a^6*b^20 + 45*a^4*b^22 + 10*a^2*b^24 + b^26)*d^5 + 2*((3*a^25*b + 20*a^23*b^3 + 38*a^21*b^5 - 60*a^19*b^7 - 435*a^17*b^9 - 984*a^15*b^11 - 1260*a^13*b^13 - 984*a^11*b^15 - 435*a^9*b^17 - 60*a^7*b^19 + 38*a^5*b^21 + 20*a^3*b^23 + 3*a*b^25)*d^5*cos(d*x + c)^5 + 2*(5*a^23*b^3 + 47*a^21*b^5 + 195*a^19*b^7 + 465*a^17*b^9 + 690*a^15*b^11 + 630*a^13*b^13 + 294*a^11*b^15 - 30*a^9*b^17 - 135*a^7*b^19 - 85*a^5*b^21 - 25*a^3*b^23 - 3*a*b^25)*d^5*cos(d*x + c)^3 + 3*(a^21*b^5 + 10*a^19*b^7 + 45*a^17*b^9 + 120*a^15*b^11 + 210*a^13*b^13 + 252*a^11*b^15 + 210*a^9*b^17 + 120*a^7*b^19 + 45*a^5*b^21 + 10*a^3*b^23 + a*b^25)*d^5*cos(d*x + c))*sin(d*x + c))*sqrt((a^14 + 7*a^12*b^2 + 21*a^10*b^4 + 35*a^8*b^6 + 35*a^6*b^8 + 21*a^4*b^10 + 7*a^2*b^12 + b^14 + (a^21 - 14*a^19*b^2 - 91*a^17*b^4 - 168*a^15*b^6 - 14*a^13*b^8 + 364*a^11*b^10 + 546*a^9*b^12 + 344*a^7*b^14 + 77*a^5*b^16 - 14*a^3*b^18 - 7*a*b^20)*d^2*sqrt(1/((a^14 + 7*a^12*b^2 + 21*a^10*b^4 + 35*a^8*b^6 + 35*a^6*b^8 + 21*a^4*b^10 + 7*a^2*b^12 + b^14)*d^4)))/(49*a^12*b^2 - 490*a^10*b^4 + 1519*a^8*b^6 - 1484*a^6*b^8 + 511*a^4*b^10 - 42*a^2*b^12 + b^14))*sqrt((49*a^12*b^2 - 490*a^10*b^4 + 1519*a^8*b^6 - 1484*a^6*b^8 + 511*a^4*b^10 - 42*a^2*b^12 + b^14)/((a^28 + 14*a^26*b^2 + 91*a^24*b^4 + 364*a^22*b^6 + 1001*a^20*b^8 + 2002*a^18*b^10 + 3003*a^16*b^12 + 3432*a^14*b^14 + 3003*a^12*b^16 + 2002*a^10*b^18 + 1001*a^8*b^20 + 364*a^6*b^22 + 91*a^4*b^24 + 14*a^2*b^26 + b^28)*d^4))*(1/((a^14 + 7*a^12*b^2 + 21*a^10*b^4 + 35*a^8*b^6 + 35*a^6*b^8 + 21*a^4*b^10 + 7*a^2*b^12 + b^14)*d^4))^(3/4)*arctan(-((7*a^28 + 42*a^26*b^2 + 21*a^24*b^4 - 540*a^22*b^6 - 2321*a^20*b^8 - 4906*a^18*b^10 - 6171*a^16*b^12 - 4488*a^14*b^14 - 1155*a^12*b^16 + 1078*a^10*b^18 + 1287*a^8*b^20 + 612*a^6*b^22 + 141*a^4*b^24 + 10*a^2*b^26 - b^28)*d^4*sqrt((49*a^12*b^2 - 490*a^10*b^4 + 1519*a^8*b^6 - 1484*a^6*b^8 + 511*a^4*b^10 - 42*a^2*b^12 + b^14)/((a^28 + 14*a^26*b^2 + 91*a^24*b^4 + 364*a^22*b^6 + 1001*a^20*b^8 + 2002*a^18*b^10 + 3003*a^16*b^12 + 3432*a^14*b^14 + 3003*a^12*b^16 + 2002*a^10*b^18 + 1001*a^8*b^20 + 364*a^6*b^22 + 91*a^4*b^24 + 14*a^2*b^26 + b^28)*d^4))*sqrt(1/((a^14 + 7*a^12*b^2 + 21*a^10*b^4 + 35*a^8*b^6 + 35*a^6*b^8 + 21*a^4*b^10 + 7*a^2*b^12 + b^14)*d^4)) + (7*a^21 + 14*a^19*b^2 - 77*a^17*b^4 - 344*a^15*b^6 - 546*a^13*b^8 - 364*a^11*b^10 + 14*a^9*b^12 + 168*a^7*b^14 + 91*a^5*b^16 + 14*a^3*b^18 -

```
a*b^20)*d^2*sqrt((49*a^12*b^2 - 490*a^10*b^4 + 1519*a^8*b^6 - 1484*a^6*b^8
 + 511*a^4*b^10 - 42*a^2*b^12 + b^14)/((a^28 + 14*a^26*b^2 + 91*a^24*b^4 +
364*a^22*b^6 + 1001*a^20*b^8 + 2002*a^18*b^10 + 3003*a^16*b^12 + 3432*a^14*
b^14 + 3003*a^12*b^16 + 2002*a^10*b^18 + 1001*a^8*b^20 + 364*a^6*b^22 + 91*
a^4*b^24 + 14*a^2*b^26 + b^28)*d^4)) + sqrt(2)*(4*(a^31 + 13*a^29*b^2 + 77*
a^27*b^4 + 273*a^25*b^6 + 637*a^23*b^8 + 1001*a^21*b^10 + 1001*a^19*b^12 +
429*a^17*b^14 - 429*a^15*b^16 - 1001*a^13*b^18 - 1001*a^11*b^20 - 637*a^9*b
^22 - 273*a^7*b^24 - 77*a^5*b^26 - 13*a^3*b^28 - a*b^30)*d^7*sqrt((49*a^12*
b^2 - 490*a^10*b^4 + 1519*a^8*b^6 - 1484*a^6*b^8 + 511*a^4*b^10 - 42*a^2*b^
12 + b^14)/((a^28 + 14*a^26*b^2 + 91*a^24*b^4 + 364*a^22*b^6 + 1001*a^20*b^
8 + 2002*a^18*b^10 + 3003*a^16*b^12 + 3432*a^14*b^14 + 3003*a^12*b^16 + 200
2*a^10*b^18 + 1001*a^8*b^20 + 364*a^6*b^22 + 91*a^4*b^24 + 14*a^2*b^26 + b^
28)*d^4))*sqrt(1/((a^14 + 7*a^12*b^2 + 21*a^10*b^4 + 35*a^8*b^6 + 35*a^6*b^
8 + 21*a^4*b^10 + 7*a^2*b^12 + b^14)*d^4)) + (3*a^24 + 32*a^22*b^2 + 154*a^
20*b^4 + 440*a^18*b^6 + 825*a^16*b^8 + 1056*a^14*b^10 + 924*a^12*b^12 + 528
*a^10*b^14 + 165*a^8*b^16 - 22*a^4*b^20 - 8*a^2*b^22 - b^24)*d^5*sqrt((49*a
^12*b^2 - 490*a^10*b^4 + 1519*a^8*b^6 - 1484*a^6*b^8 + 511*a^4*b^10 - 42*a^
2*b^12 + b^14)/((a^28 + 14*a^26*b^2 + 91*a^24*b^4 + 364*a^22*b^6 + 1001*a^2
0*b^8 + 2002*a^18*b^10 + 3003*a^16*b^12 + 3432*a^14*b^14 + 3003*a^12*b^16 +
 2002*a^10*b^18 + 1001*a^8*b^20 + 364*a^6*b^22 + 91*a^4*b^24 + 14*a^2*b^26
+ b^28)*d^4)))*sqrt((a^14 + 7*a^12*b^2 + 21*a^10*b^4 + 35*a^8*b^6 + 35*a^6*
b^8 + 21*a^4*b^10 + 7*a^2*b^12 + b^14 + (a^21 - 14*a^19*b^2 - 91*a^17*b^4 -
 168*a^15*b^6 - 14*a^13*b^8 + 364*a^11*b^10 + 546*a^9*b^12 + 344*a^7*b^14 +
 77*a^5*b^16 - 14*a^3*b^18 - 7*a*b^20)*d^2*sqrt(1/((a^14 + 7*a^12*b^2 + 21*
a^10*b^4 + 35*a^8*b^6 + 35*a^6*b^8 + 21*a^4*b^10 + 7*a^2*b^12 + b^14)*d^4))
)/(49*a^12*b^2 - 490*a^10*b^4 + 1519*a^8*b^6 - ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\tan(c+dx))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))**(7/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**(-7/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^(7/2),x, algorithm="giac")
```

[Out] Timed out

Mupad [B]
time = 20.89, size = 2500, normalized size = 12.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*tan(c + d*x))^(7/2),x)

[Out] (log((((a + b*tan(c + d*x))^(1/2)*(16*b^26*d^3 - 96*a^2*b^24*d^3 - 1344*a^4*b^22*d^3 - 5152*a^6*b^20*d^3 - 9648*a^8*b^18*d^3 - 8640*a^10*b^16*d^3 + 8640*a^14*b^12*d^3 + 9648*a^16*b^10*d^3 + 5152*a^18*b^8*d^3 + 1344*a^20*b^6*d^3 + 96*a^22*b^4*d^3 - 16*a^24*b^2*d^3) - ((-1/(a^7*d^2 + b^7*d^2*1i - 7*a*b^6*d^2 - a^6*b*d^2*7i - a^2*b^5*d^2*21i + 35*a^3*b^4*d^2 + a^4*b^3*d^2*35i - 21*a^5*b^2*d^2))^(1/2)*(((-1/(a^7*d^2 + b^7*d^2*1i - 7*a*b^6*d^2 - a^6*b*d^2*7i - a^2*b^5*d^2*21i + 35*a^3*b^4*d^2 + a^4*b^3*d^2*35i - 21*a^5*b^2*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^32*d^5 + 960*a^3*b^30*d^5 + 6720*a^5*b^28*d^5 + 29120*a^7*b^26*d^5 + 87360*a^9*b^24*d^5 + 192192*a^11*b^22*d^5 + 320320*a^13*b^20*d^5 + 411840*a^15*b^18*d^5 + 411840*a^17*b^16*d^5 + 320320*a^19*b^14*d^5 + 192192*a^21*b^12*d^5 + 87360*a^23*b^10*d^5 + 29120*a^25*b^8*d^5 + 6720*a^27*b^6*d^5 + 960*a^29*b^4*d^5 + 64*a^31*b^2*d^5))/2 - 128*a*b^29*d^4 - 1408*a^3*b^27*d^4 - 6912*a^5*b^25*d^4 - 19712*a^7*b^23*d^4 - 35200*a^9*b^21*d^4 - 38016*a^11*b^19*d^4 - 16896*a^13*b^17*d^4 + 16896*a^15*b^15*d^4 + 38016*a^17*b^13*d^4 + 35200*a^19*b^11*d^4 + 19712*a^21*b^9*d^4 + 6912*a^23*b^7*d^4 + 1408*a^25*b^5*d^4 + 128*a^27*b^3*d^4))/2)*(-1/(a^7*d^2 + b^7*d^2*1i - 7*a*b^6*d^2 - a^6*b*d^2*7i - a^2*b^5*d^2*21i + 35*a^3*b^4*d^2 + a^4*b^3*d^2*35i - 21*a^5*b^2*d^2))^(1/2))/2 - 8*b^23*d^2 - 48*a^2*b^21*d^2 - 72*a^4*b^19*d^2 + 192*a^6*b^17*d^2 + 1008*a^8*b^15*d^2 + 2016*a^10*b^13*d^2 + 2352*a^12*b^11*d^2 + 1728*a^14*b^9*d^2 + 792*a^16*b^7*d^2 + 208*a^18*b^5*d^2 + 24*a^20*b^3*d^2)*(-1/(a^7*d^2 + b^7*d^2*1i - 7*a*b^6*d^2 - a^6*b*d^2*7i - a^2*b^5*d^2*21i + 35*a^3*b^4*d^2 + a^4*b^3*d^2*35i - 21*a^5*b^2*d^2))^(1/2))/2 - log(192*a^6*b^17*d^2 - 8*b^23*d^2 - 48*a^2*b^21*d^2 - 72*a^4*b^19*d^2 - (-1/(4*(a^7*d^2 + b^7*d^2*1i - 7*a*b^6*d^2 - a^6*b*d^2*7i - a^2*b^5*d^2*21i + 35*a^3*b^4*d^2 + a^4*b^3*d^2*35i - 21*a^5*b^2*d^2)))^(1/2)*((a + b*tan(c + d*x))^(1/2)*(16*b^26*d^3 - 96*a^2*b^24*d^3 - 1344*a^4*b^22*d^3 - 5152*a^6*b^20*d^3 - 9648*a^8*b^18*d^3 - 8640*a^10*b^16*d^3 + 8640*a^14*b^12*d^3 + 9648*a^16*b^10*d^3 + 5152*a^18*b^8*d^3 + 1344*a^20*b^6*d^3 + 96*a^22*b^4*d^3 - 16*a^24*b^2*d^3) - (-1/(4*(a^7*d^2 + b^7*d^2*1i - 7*a*b^6*d^2 - a^6*b*d^2*7i - a^2*b^5*d^2*21i + 35*a^3*b^4*d^2 + a^4*b^3*d^2*35i - 21*a^5*b^2*d^2)))^(1/2)*(128*a*b^29*d^4 + (-1/(4*(a^7*d^2 + b^7*d^2*1i - 7*a*b^6*d^2 - a^6*b*d^2*7i - a^2*b^5*d^2*21i + 35*a^3*b^4*d^2 + a^4*b^3*d^2*35i - 21*a^5*b^2*d^2)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^32*d^5 + 960*a^3*b^30*d^5 + 6720*a^5*b^28*d^5 + 29120*a^7*b^26*d^5 + 87360*a^9*b^24*d^5 + 192192*a^11*b^22*d^5 + 320320*a^13*b^20*d^5 + 411840*a^15*b^18 *d^5 + 411840*a^17*b^16*d^5 + 320320*a^19*b^14*d^5 + 192192*a^21*b^12*d^5 + 87360*a^23*b^10*d^5 + 29120*a^25*b^8*d^5 + 6720*a^27*b^6*d^5 + 960*a^29*b^4*d^5 + 64*a^31*b^2*d^5) + 1408*a^3*b^27*d^4 + 6912*a^5*b^25*d^4 + 19712*a^7*b^23*d^4 + 35200*a^9*b^21*d^4 + 38016*a^11*b^19*d^4 + 16896*a^13*b^17*d^4 - 16896*a^15*b^15*d^4 - 38016*a^17*b^13*d^4 - 35200*a^19*b^11*d^4 - 19712*a^21*b^9*d^4 - 6912*a^23*b^7*d^4 - 1408*a^25*b^5*d^4 - 128*a^27*b^3*d^4)) + 1008*a^8*b^15*d^2 + 2016*a^10*b^13*d^2 + 2352*a^12*b^11*d^2 + 1728*a^14*b^9*d^2 + 792*a^16*b^7*d^2 + 208*a^18*b^5*d^2 + 24*a^20*b^3*d^2)*(-1/(4*(a^7*d^2 + b^7*d^2*1i - 7*a*b^6*d^2 - a^6*b*d^2*7i - a^2*b^5*d^2*21i + 35*a^3*b^4*d^2 + a^4*b^3*d^2*35i - 21*a^5*b^2*d^2)))^(1/2) + atan(-((-1i/(4*(a^7*d^2*1i + b^7*d^2 - a*b^6*d^2*7i - 7*a^6*b*d^2 - 21*a^2*b^5*d^2 + a^3*b^4*d^2*35i + 35*a^4*b^3*d^2 - a^5*b^2*d^2*21i)))^(1/2)*((a + b*tan(c + d*x))^(1/2)*(16*b^26*d^3 - 96*a^2*b^24*d^3 - 1344*a^4*b^22*d^3 - 5152*a^6*b^20*d^3 - 9648*a^8*b^18*d^3 - 8640*a^10*b^16*d^3 + 8640*a^14*b^12*d^3 + 9648*a^16*b^10*d^3 + 5152*a^18*b^8*d^3 + 1344*a^20*b^6*d^3 + 96*a^22*b^4*d^3 - 16*a^24*b^2*d^3) - (-1i/(4*(a^7*d^2*1i + b^7*d^2 - a*b^6*d^2*7i - 7*a^6*b*d^2 - 21*a^2*b^5*d^2 + a^3*b^4*d^2*35i + 35*a^4*b^3*d^2 - a^5*b^2*d^2*21i)))^(1/2)*((-1i/(4*(a^7*d^2*1i + b^7*d^2 - a*b^6*d^2*7i - 7*a^6*b*d^2 - 21*a^2*b^5*d^2 + a^3*b^4*d^2*35i + 35*a^4*b^3*d^2 - a^5*b^2*d^2*21i)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^32*d^5 + 960*a^3*b^30*d^5 + 6720*a^5*b^28*d^5 + 29120*a^7*b^26*d^5 + 87360*a^9*b^24*d^5 + 192192*a^11*b^22*d^5 + 320320*a^13*b^20*d^5 + 411840*a^15*b^18*d^5 + 411840*a^17*b^16*d^5 + 320320*a^19*b^14*d^5 + 192192*a^21*b^12*d^5 + 87360*a^23*b^10*d^5 + 29120*a^25*b^8*d^5 + 6720*a^27*b^6*d^5 + 960*a^29*b^4*d^5 + 64*a^31*b^2*d^5) - 128*a*b^29*d^4 - 1408*a^3*b^27*d^4 - 6912*a^5*b^25*d^4 - 19712*a^7*b^23*d^4 - 35200*a^9*b^21*d^4 - 38016*a^11*b^19*d^4 - 16896*a^13*b^17*d^4 + 16896*a^15*b^15*d^4 + 38016*a^17*b^13*d^4 + 35200*a^19*b^11*d^4 + 19712*a^21*b^9*d^4 + 6912*a^23*b^7*d^4 + 1408*a^25*b^5*d^4 + 128*a^27*b^3*d^4))*1i + (-1i/(4*(a^7*d^2*1i + b^7*d^2 - a*b^6*d^2*7i - 7*a^6*b*d^2 - 21*a^2*b^5*d^2 + a^3*b^4*d^2*35i + 35*a^4*b^3*d^2 - a^5*b^2*d^2*21i)))^(1/2)*((a + b*tan(c + d*x))^(1/2)*(16*b^26*d^3 - 96*a^2*b^24*d^3 - 1344*a^4*b^22*d^3 - 5152*a^6*b^20*d^3 - 9648*a^8*b^18*d^3 - 8640*a^10*b^16*d^3 + 8640*a^14*b^12*d^3 + 964...

3.555 $\int \tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=202

$$\frac{(a-b)\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a-b)\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a+b)\log\left(1-\sqrt{2}\right.}{}$$

```
[Out] -1/2*(a-b)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)-1/2*(a-b)*arctan(1
+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)-1/4*(a+b)*ln(1-2^(1/2)*tan(d*x+c)^(1/2
)+tan(d*x+c))/d*2^(1/2)+1/4*(a+b)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))
/d*2^(1/2)-2*b*tan(d*x+c)^(1/2)/d+2/3*a*tan(d*x+c)^(3/2)/d+2/5*b*tan(d*x+c)
^(5/2)/d
```

Rubi [A]
time = 0.12, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a-b)\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a-b)\mathrm{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d}-\frac{(a+b)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{(a+b)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{2a\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{2b\tan^{\frac{5}{2}}(c+dx)}{5d}-\frac{2b\sqrt{\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x]),x]
```

```
[Out] ((a - b)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) - ((a - b)*Arc
Tan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) - ((a + b)*Log[1 - Sqrt[2]
*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) + ((a + b)*Log[1 + Sqrt[
2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) - (2*b*Sqrt[Tan[c + d*
x]])/d + (2*a*Tan[c + d*x]^(3/2))/(3*d) + (2*b*Tan[c + d*x]^(5/2))/(5*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))\,dx = \frac{2b\tan^{\frac{5}{2}}(c+dx)}{5d} + \int \tan^{\frac{3}{2}}(c+dx)(-b+a\tan(c+dx))\,dx$$

$$= \frac{2a\tan^{\frac{3}{2}}(c+dx)}{3d} + \frac{2b\tan^{\frac{5}{2}}(c+dx)}{5d} + \int \sqrt{\tan(c+dx)}\,(-a-b\,\text{ta}$$

$$= -\frac{2b\sqrt{\tan(c+dx)}}{d} + \frac{2a\tan^{\frac{3}{2}}(c+dx)}{3d} + \frac{2b\tan^{\frac{5}{2}}(c+dx)}{5d} + \int \frac{b-}{\sqrt{}}$$

$$= -\frac{2b\sqrt{\tan(c+dx)}}{d} + \frac{2a\tan^{\frac{3}{2}}(c+dx)}{3d} + \frac{2b\tan^{\frac{5}{2}}(c+dx)}{5d} + \frac{2\text{Sub}}{}$$

$$= -\frac{2b\sqrt{\tan(c+dx)}}{d} + \frac{2a\tan^{\frac{3}{2}}(c+dx)}{3d} + \frac{2b\tan^{\frac{5}{2}}(c+dx)}{5d} - \frac{(a-}{}$$

$$= -\frac{2b\sqrt{\tan(c+dx)}}{d} + \frac{2a\tan^{\frac{3}{2}}(c+dx)}{3d} + \frac{2b\tan^{\frac{5}{2}}(c+dx)}{5d} - \frac{(a-}{}$$

$$= -\frac{(a+b)\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}\,d} + \frac{(a+b)\,\text{lo}}{}$$

$$= \frac{(a-b)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d} - \frac{(a-b)\tan^{-1}\left(1+\sqrt{2}\right.}{\sqrt{2}\,d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.62, size = 106, normalized size = 0.52

$$\frac{-15\sqrt[4]{-1}\,(ia+b)\,\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+15(-1)^{3/4}(a+ib)\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+2\sqrt{\tan(c+dx)}\,(-15b+5a\tan(c+dx)+3b\tan^2(c+dx))}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x]),x]
```

```
[Out] (-15*(-1)^(1/4)*(I*a + b)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + 15*(-1)^(
3/4)*(a + I*b)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + 2*Sqrt[Tan[c + d*x]
]*(-15*b + 5*a*Tan[c + d*x] + 3*b*Tan[c + d*x]^2))/(15*d)
```

Maple [A]
time = 0.07, size = 211, normalized size = 1.04

method	result

derivativedivides	$\frac{\frac{2b\left(\tan^{\frac{5}{2}}(dx+c)\right)}{5}+\frac{2a\left(\tan^{\frac{3}{2}}(dx+c)\right)}{3}-2b\left(\sqrt{\tan}\,(dx+c)\right)+\frac{b\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}\,(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}\,(dx+c)\right)+\tan(dx+c)}\right)+2\arctan}{}}{}$
default	$\frac{\frac{2b\left(\tan^{\frac{5}{2}}(dx+c)\right)}{5}+\frac{2a\left(\tan^{\frac{3}{2}}(dx+c)\right)}{3}-2b\left(\sqrt{\tan}\,(dx+c)\right)+\frac{b\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}\,(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}\,(dx+c)\right)+\tan(dx+c)}\right)+2\arctan}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/5*b*tan(d*x+c)^(5/2)+2/3*a*tan(d*x+c)^(3/2)-2*b*tan(d*x+c)^(1/2)+1/4
*b*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c
)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/
2)*tan(d*x+c)^(1/2)))-1/4*a*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x
+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)
^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))))
```

Maxima [A]
time = 0.49, size = 158, normalized size = 0.78

$$\frac{24\,b\tan\left(dx+c\right)^{\frac{5}{2}}-30\,\sqrt{2}\left(a-b\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(dx+c\right)}\right)\right)-30\,\sqrt{2}\left(a-b\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(dx+c\right)}\right)\right)+15\,\sqrt{2}\left(a+b\right)\log\left(\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)-15\,\sqrt{2}\left(a+b\right)\log\left(-\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)+40\,a\tan\left(dx+c\right)^{\frac{3}{2}}-120\,b\sqrt{\tan\left(dx+c\right)}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(24*b*tan(d*x + c)^(5/2) - 30*sqrt(2)*(a - b)*arctan(1/2*sqrt(2)*(sqrt
(2) + 2*sqrt(tan(d*x + c)))) - 30*sqrt(2)*(a - b)*arctan(-1/2*sqrt(2)*(sqrt
(2) - 2*sqrt(tan(d*x + c)))) + 15*sqrt(2)*(a + b)*log(sqrt(2)*sqrt(tan(d*x
+ c)) + tan(d*x + c) + 1) - 15*sqrt(2)*(a + b)*log(-sqrt(2)*sqrt(tan(d*x +
c)) + tan(d*x + c) + 1) + 40*a*tan(d*x + c)^(3/2) - 120*b*sqrt(tan(d*x + c)
))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2847 vs. 2(164) = 328.
time = 1.40, size = 2847, normalized size = 14.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

[Out] -1/60*(60*sqrt(2)*d^5*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4)*arctan(((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^4*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4) + sqrt(2)*(b*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4) + (a^3 + a*b^2)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4))*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) + sqrt(2)*((a^5 - 2*a^3*b^2 + a*b^4)*d^3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) + (a^6*b - a^4*b^3 - a^2*b^5 + b^7)*d*cos(d*x + c))*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4) + (a^8 - 2*a^4*b^4 + b^8)*sin(d*x + c))/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4) + sqrt(2)*((a^4*b - b^5)*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4) + (a^7 + a^5*b^2 - a^3*b^4 - a*b^6)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4))*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4))/(a^12 + 2*a^10*b^2 - a^8*b^4 - 4*a^6*b^6 - a^4*b^8 + 2*a^2*b^10 + b^12))*cos(d*x + c)^2 + 60*sqrt(2)*d^5*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4)*arctan(-((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^4*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4) - sqrt(2)*(b*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4) + (a^3 + a*b^2)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4))*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) - sqrt(2)*((a^5 - 2*a^3*b^2 + a*b^4)*d^3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) + (a^6*b - a^4*b^3 - a^2*b^5 + b^7)*d*cos(d*x + c))*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4) + (a^8 - 2*a^4*b^4 + b^8)*sin(d*x + c))/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4) - sqrt(2)*((a^4*b - b^5)*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4) + (a^7 + a^5*b^2 - a^3*b^4 - a*b^6)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4))*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4))/(a^12 + 2*a^10*b^2 - a^8*b^4 - 4*a^6*b^6 - a^4*b^8 + 2*a^2*b^10 + b^12))*cos(d*x + c)^2 - 15*sqrt(2)*(2*a*b*d^3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d*cos(d*x + c)^2)*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4)*log(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) + sqrt(2)*((a^5 - 2*a^3*b^2 + a*b^4)*d^3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) + (a^6*b - a^

```
4*b^3 - a^2*b^5 + b^7)*d*cos(d*x + c))*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b
^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d
*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4) + (a^8 - 2*a^4*b^
4 + b^8)*sin(d*x + c))/cos(d*x + c)) + 15*sqrt(2)*(2*a*b*d^3*sqrt((a^4 + 2*
a^2*b^2 + b^4)/d^4)*cos(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d*cos(d*x + c)
^2)*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 -
b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4)*log(((a^6
 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x +
 c) - sqrt(2)*((a^5 - 2*a^3*b^2 + a*b^4)*d^3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d
^4)*cos(d*x + c) + (a^6*b - a^4*b^3 - a^2*b^5 + b^7)*d*cos(d*x + c))*sqrt(-
(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4
- 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4
)/d^4)^(1/4) + (a^8 - 2*a^4*b^4 + b^8)*sin(d*x + c))/cos(d*x + c)) - 8*(3*a
^4*b + 6*a^2*b^3 + 3*b^5 - 18*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^2 + 5*
(a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c)*sin(d*x + c))*sqrt(sin(d*x + c)/cos(
d*x + c)))/((a^4 + 2*a^2*b^2 + b^4)*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx))\tan^{\frac{5}{2}}(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(5/2)*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*tan(c + d*x)**(5/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 5.94, size = 130, normalized size = 0.64

$$\frac{2\,a\,\tan(c+d\,x)^{3/2}}{3\,d}-\frac{2\,b\,\sqrt{\tan(c+d\,x)}}{d}+\frac{2\,b\,\tan(c+d\,x)^{5/2}}{5\,d}-\frac{(-1)^{1/4}\,a\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{\tan(c+d\,x)}\right)}{d}+\frac{(-1)^{1/4}\,a\,\mathrm{atanh}\left((-1)^{1/4}\,\sqrt{\tan(c+d\,x)}\right)}{d}-\frac{(-1)^{1/4}\,b\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{\tan(c+d\,x)}\right)\,1{}\mathrm{i}}{d}-\frac{(-1)^{1/4}\,b\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{\tan(c+d\,x)}\,1{}\mathrm{i}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(5/2)*(a + b*tan(c + d*x)),x)
```

```
[Out] (2*a*tan(c + d*x)^(3/2))/(3*d) - (2*b*tan(c + d*x)^(1/2))/d + (2*b*tan(c +
d*x)^(5/2))/(5*d) - ((-1)^(1/4)*a*atan((-1)^(1/4)*tan(c + d*x)^(1/2)))/d +
((-1)^(1/4)*a*atanh((-1)^(1/4)*tan(c + d*x)^(1/2)))/d - ((-1)^(1/4)*b*atan(
(-1)^(1/4)*tan(c + d*x)^(1/2))*1i)/d - ((-1)^(1/4)*b*atan((-1)^(1/4)*tan(c
+ d*x)^(1/2)*1i))/d
```

3.556 $\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=184

$$\frac{(a+b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a+b)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a-b)\log\left(1-\sqrt{2}\sqrt{\right.}}{}$$

[Out] -1/2*(a+b)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)-1/2*(a+b)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)+1/4*(a-b)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)-1/4*(a-b)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)+2*a*tan(d*x+c)^(1/2)/d+2/3*b*tan(d*x+c)^(3/2)/d

Rubi [A]

time = 0.10, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a+b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a+b)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d}+\frac{(a-b)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{(a-b)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{2a\sqrt{\tan(c+dx)}}{d}+\frac{2b\tan^{\frac{3}{2}}(c+dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x]),x]

[Out] ((a + b)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) - ((a + b)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) + ((a - b)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) - ((a - b)*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) + (2*a*Sqrt[Tan[c + d*x]])/d + (2*b*Tan[c + d*x]^(3/2))/(3*d)

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))\,dx = \frac{2b\tan^{\frac{3}{2}}(c+dx)}{3d} + \int \sqrt{\tan(c+dx)}\,(-b+a\tan(c+dx))\,dx$$
$$= \frac{2a\sqrt{\tan(c+dx)}}{d} + \frac{2b\tan^{\frac{3}{2}}(c+dx)}{3d} + \int \frac{-a-b\tan(c+dx)}{\sqrt{\tan(c+dx)}}\,dx$$
$$= \frac{2a\sqrt{\tan(c+dx)}}{d} + \frac{2b\tan^{\frac{3}{2}}(c+dx)}{3d} + \frac{2\text{Subst}\left(\int \frac{-a-bx^2}{1+x^4}\,dx, x, \sqrt{\text{ta}\right.}{d}$$
$$= \frac{2a\sqrt{\tan(c+dx)}}{d} + \frac{2b\tan^{\frac{3}{2}}(c+dx)}{3d} - \frac{(a-b)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x,\right.}{d}$$
$$= \frac{2a\sqrt{\tan(c+dx)}}{d} + \frac{2b\tan^{\frac{3}{2}}(c+dx)}{3d} + \frac{(a-b)\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}\,x-x}\right.}{2\sqrt{2}}$$
$$= \frac{(a-b)\log\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}\,d} - \frac{(a-b)\log\left(\right.}{}$$
$$= \frac{(a+b)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d} - \frac{(a+b)\tan^{-1}\left(1+\sqrt{2}\right.}{\sqrt{2}\,d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.26, size = 94, normalized size = 0.51

$$\frac{3\sqrt[4]{-1}\,(a-ib)\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+3\sqrt[4]{-1}\,(a+ib)\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+2\sqrt{\tan(c+dx)}\,(3a+b\tan(c+dx))}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x]),x]
```

```
[Out] (3*(-1)^(1/4)*(a - I*b)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + 3*(-1)^(1/4
)*(a + I*b)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + 2*Sqrt[Tan[c + d*x]]*(
3*a + b*Tan[c + d*x]))/(3*d)
```

Maple [A]
time = 0.06, size = 200, normalized size = 1.09

method	result
derivativedivides	$\frac{\frac{2b\left(\tan^{\frac{3}{2}}(dx+c)\right)}{3}+2a\left(\sqrt{\tan}(dx+c)\right)-\frac{a\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan}\right.\right.\right.}{4}}{}$

default	$\frac{\frac{2b\left(\tan^{\frac{3}{2}}(dx+c)\right)}{3}+2a\left(\sqrt{\tan}\,(dx+c)\right)-\frac{a\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}\,(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}\,(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\mathrm{t}\cdots}\right.\right.\right.}{4}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/3*b*tan(d*x+c)^(3/2)+2*a*tan(d*x+c)^(1/2)-1/4*a*2^(1/2)*(ln((1+2^(1/
2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*
arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))-1
/4*b*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x
+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(
1/2)*tan(d*x+c)^(1/2))))
```

Maxima [A]
time = 0.49, size = 147, normalized size = 0.80

$$-\frac{6\sqrt{2}(a+b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+6\sqrt{2}(a+b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)+3\sqrt{2}(a-b)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)-3\sqrt{2}(a-b)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)-8b\tan(dx+c)^{\frac{3}{2}}-24a\sqrt{\tan(dx+c)}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(6*sqrt(2)*(a + b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c))
)) + 6*sqrt(2)*(a + b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))
) + 3*sqrt(2)*(a - b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) -
3*sqrt(2)*(a - b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - 8*b
*tan(d*x + c)^(3/2) - 24*a*sqrt(tan(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2757 vs. $2(150) = 300$.
time = 1.63, size = 2757, normalized size = 14.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(12*sqrt(2)*d^5*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a
^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4
)^(3/4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4)*arctan(-((a^8 + 2*a^6*b^2 - 2*a^2
*b^6 - b^8)*d^4*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b
^4)/d^4) - sqrt(2)*(a*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a
^2*b^2 + b^4)/d^4) - (a^2*b + b^3)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4))*s
qrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(
```

a^4 - 2*a^2*b^2 + b^4))*sqrt(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) + sqrt(2)*((a^4*b - 2*a^2*b^3 + b^5)*d^3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) - (a^7 - a^5*b^2 - a^3*b^4 + a*b^6)*d*cos(d*x + c))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4) + (a^8 - 2*a^4*b^4 + b^8)*sin(d*x + c))/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4) - sqrt(2)*((a^5 - a*b^4)*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4) - (a^6*b + a^4*b^3 - a^2*b^5 - b^7)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4))/(a^12 + 2*a^10*b^2 - a^8*b^4 - 4*a^6*b^6 - a^4*b^8 + 2*a^2*b^10 + b^12))*cos(d*x + c) + 12*sqrt(2)*d^5*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4)*arctan(((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^4*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4) + sqrt(2)*(a*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4) - (a^2*b + b^3)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) - sqrt(2)*((a^4*b - 2*a^2*b^3 + b^5)*d^3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) - (a^7 - a^5*b^2 - a^3*b^4 + a*b^6)*d*cos(d*x + c))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4) + (a^8 - 2*a^4*b^4 + b^8)*sin(d*x + c))/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4) + sqrt(2)*((a^5 - a*b^4)*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4) - (a^6*b + a^4*b^3 - a^2*b^5 - b^7)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4))/(a^12 + 2*a^10*b^2 - a^8*b^4 - 4*a^6*b^6 - a^4*b^8 + 2*a^2*b^10 + b^12))*cos(d*x + c) + 3*sqrt(2)*(2*a*b*d^3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) - (a^4 + 2*a^2*b^2 + b^4)*d*cos(d*x + c))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4)*log(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) + sqrt(2)*((a^4*b - 2*a^2*b^3 + b^5)*d^3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) - (a^7 - a^5*b^2 - a^3*b^4 + a*b^6)*d*cos(d*x + c))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4) + (a^8 - 2*a^4*b^4 + b^8)*sin(d*x + c))/cos(d*x + c)) - 3*sqrt(2)*(2*a*b*d^3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) - (a^4 + 2*a^2*b^2 + b^4)*d*cos(d*x + c))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4)*log(((a^6 - a^4*b^2 - a^2*b^4 + b^

```
6)*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) - sqrt(2)*((a^4*b - 2
*a^2*b^3 + b^5)*d^3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) - (a^7 -
 a^5*b^2 - a^3*b^4 + a*b^6)*d*cos(d*x + c))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a
^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(s
in(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4) + (a^8 - 2*a^
4*b^4 + b^8)*sin(d*x + c))/cos(d*x + c)) - 8*(3*(a^5 + 2*a^3*b^2 + a*b^4)*c
os(d*x + c) + (a^4*b + 2*a^2*b^3 + b^5)*sin(d*x + c))*sqrt(sin(d*x + c)/cos
(d*x + c)))/((a^4 + 2*a^2*b^2 + b^4)*d*cos(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan\left(c + dx\right)\right)\tan^{\frac{3}{2}}\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(3/2)*(a+b*tan(d*x+c)),x)
[Out] Integral((a + b*tan(c + d*x))*tan(c + d*x)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c)),x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)*tan(d*x + c)^(3/2), x)
```

Mupad [**B**]
time = 5.06, size = 114, normalized size = 0.62

$$\frac{2a\sqrt{\tan\left(c+dx\right)}}{d}+\frac{2b\tan\left(c+dx\right)^{3/2}}{3d}-\frac{(-1)^{1/4}\,b\,\mathrm{atan}\left((-1)^{1/4}\sqrt{\tan\left(c+dx\right)}\right)}{d}+\frac{(-1)^{1/4}\,b\,\mathrm{atanh}\left((-1)^{1/4}\sqrt{\tan\left(c+dx\right)}\right)}{d}+\frac{(-1)^{1/4}\,a\,\mathrm{atan}\left((-1)^{1/4}\sqrt{\tan\left(c+dx\right)}\right)\,1\mathrm{i}}{d}+\frac{(-1)^{1/4}\,a\,\mathrm{atanh}\left((-1)^{1/4}\sqrt{\tan\left(c+dx\right)}\right)\,1\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x)),x)
[Out] (2*a*tan(c + d*x)^(1/2))/d + (2*b*tan(c + d*x)^(3/2))/(3*d) + ((-1)^(1/4)*a
*atan((-1)^(1/4)*tan(c + d*x)^(1/2))*1i)/d + ((-1)^(1/4)*a*atanh((-1)^(1/4)
*tan(c + d*x)^(1/2))*1i)/d - ((-1)^(1/4)*b*atan((-1)^(1/4)*tan(c + d*x)^(1/
2)))/d + ((-1)^(1/4)*b*atanh((-1)^(1/4)*tan(c + d*x)^(1/2)))/d
```

3.557 $\int \sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=166

$$-\frac{(a-b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a-b)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a+b)\log\left(1-\sqrt{2}\right.}{}$$

```
[Out] 1/2*(a-b)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)+1/2*(a-b)*arctan(1+
2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)+1/4*(a+b)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)
+tan(d*x+c))/d*2^(1/2)-1/4*(a+b)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/
d*2^(1/2)+2*b*tan(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.08, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{(a-b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a-b)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d}+\frac{(a+b)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{(a+b)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{2b\sqrt{\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x]),x]
```

```
[Out] -(((a - b)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d)) + ((a - b)*
ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) + ((a + b)*Log[1 - Sqrt
[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) - ((a + b)*Log[1 + Sq
rt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) + (2*b*Sqrt[Tan[c +
 d*x]])/d
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))\,dx = \frac{2b\sqrt{\tan(c+dx)}}{d} + \int \frac{-b+a\tan(c+dx)}{\sqrt{\tan(c+dx)}}\,dx$$

$$= \frac{2b\sqrt{\tan(c+dx)}}{d} + \frac{2\text{Subst}\left(\int \frac{-b+ax^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d}$$

$$= \frac{2b\sqrt{\tan(c+dx)}}{d} + \frac{(a-b)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d}$$

$$= \frac{2b\sqrt{\tan(c+dx)}}{d} + \frac{(a-b)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{\tan(c+\right.}{2d}$$

$$= \frac{(a+b)\log\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}\,d} - \frac{(a+b)\log}{}$$

$$= -\frac{(a-b)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d} + \frac{(a-b)\tan^{-1}\left(1+\right.}{\sqrt{2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.08, size = 79, normalized size = 0.48

$$\frac{\sqrt[4]{-1}\,(ia+b)\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right) - (-1)^{3/4}(a+ib)\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right) + 2b\sqrt{\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x]),x]
```

```
[Out] ((-1)^(1/4)*(I*a + b)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] - (-1)^(3/4)*(a
 + I*b)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + 2*b*Sqrt[Tan[c + d*x]])/d
```

Maple [A]
time = 0.05, size = 189, normalized size = 1.14

method	result
derivativedivides	$\frac{2b\left(\sqrt{\tan}\,(dx+c)\right) - \frac{b\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}\,(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}\,(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan}\,(dx+c)\right)\right)+2\,\text{arcta}\right.}{4}}{}$
default	$\frac{2b\left(\sqrt{\tan}\,(dx+c)\right) - \frac{b\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}\,(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}\,(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan}\,(dx+c)\right)\right)+2\,\text{arcta}\right.}{4}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b*tan(d*x+c)^(1/2)-1/4*b*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan
(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*
x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*a*2^(1/2)*(ln((1-2^(
1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+
2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))
)
```

Maxima [A]
time = 0.51, size = 135, normalized size = 0.81

$$\frac{2\sqrt{2}(a-b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}(a-b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-\sqrt{2}(a+b)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+\sqrt{2}(a+b)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+8b\sqrt{\tan(dx+c)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*(a - b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c))))
 + 2*sqrt(2)*(a - b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c))))
- sqrt(2)*(a + b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt
(2)*(a + b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 8*b*sqrt(
tan(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2729 vs. $2(136) = 272$.
time = 1.47, size = 2729, normalized size = 16.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*d^5*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4
 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^
(3/4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4)*arctan(((a^8 + 2*a^6*b^2 - 2*a^2*b^
6 - b^8)*d^4*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)
/d^4) + sqrt(2)*(b*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*
b^2 + b^4)/d^4) + (a^3 + a*b^2)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4))*sqrt
(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^
4 - 2*a^2*b^2 + b^4))*sqrt(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt((a^4 +
 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) + sqrt(2)*((a^5 - 2*a^3*b^2 + a*b^4)*d^
3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) + (a^6*b - a^4*b^3 - a^2*b
```

^5 + b^7)*d*cos(d*x + c))*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4
) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d
x + c))((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4) + (a^8 - 2*a^4*b^4 + b^8)*sin(
d*x + c))/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4) + sqrt(2)*((a^4
*b - b^5)*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4
)/d^4) + (a^7 + a^5*b^2 - a^3*b^4 - a*b^6)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)
/d^4))*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2
 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2*
a^2*b^2 + b^4)/d^4)^(3/4))/(a^12 + 2*a^10*b^2 - a^8*b^4 - 4*a^6*b^6 - a^4*b
^8 + 2*a^2*b^10 + b^12)) + 4*sqrt(2)*d^5*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2
*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4 +
2*a^2*b^2 + b^4)/d^4)^(3/4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4)*arctan(-((a^8
 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^4*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt(
(a^4 - 2*a^2*b^2 + b^4)/d^4) - sqrt(2)*(b*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/
d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4) + (a^3 + a*b^2)*d^5*sqrt((a^4 - 2*a^
2*b^2 + b^4)/d^4))*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4
 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(((a^6 - a^4*b^2 - a^2*b^4
 + b^6)*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) - sqrt(2)*((a^5
- 2*a^3*b^2 + a*b^4)*d^3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) + (
a^6*b - a^4*b^3 - a^2*b^5 + b^7)*d*cos(d*x + c))*sqrt(-(2*a*b*d^2*sqrt((a^4
 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*
sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4) + (a^8
- 2*a^4*b^4 + b^8)*sin(d*x + c))/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4
)^(3/4) - sqrt(2)*((a^4*b - b^5)*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt
((a^4 - 2*a^2*b^2 + b^4)/d^4) + (a^7 + a^5*b^2 - a^3*b^4 - a*b^6)*d^5*sqrt(
(a^4 - 2*a^2*b^2 + b^4)/d^4))*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)
/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/c
os(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4))/(a^12 + 2*a^10*b^2 - a^8*
b^4 - 4*a^6*b^6 - a^4*b^8 + 2*a^2*b^10 + b^12)) - sqrt(2)*(2*a*b*d^3*sqrt((
a^4 + 2*a^2*b^2 + b^4)/d^4) + (a^4 + 2*a^2*b^2 + b^4)*d)*sqrt(-(2*a*b*d^2*s
qrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2
+ b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4)*log(((a^6 - a^4*b^2 - a^2*b^4 +
 b^6)*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) + sqrt(2)*((a^5 -
2*a^3*b^2 + a*b^4)*d^3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) + (a^
6*b - a^4*b^3 - a^2*b^5 + b^7)*d*cos(d*x + c))*sqrt(-(2*a*b*d^2*sqrt((a^4 +
 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*sq
rt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4) + (a^8 -
2*a^4*b^4 + b^8)*sin(d*x + c))/cos(d*x + c)) + sqrt(2)*(2*a*b*d^3*sqrt((a^4
 + 2*a^2*b^2 + b^4)/d^4) + (a^4 + 2*a^2*b^2 + b^4)*d)*sqrt(-(2*a*b*d^2*sqrt
((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b
^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4)*log(((a^6 - a^4*b^2 - a^2*b^4 + b^
6)*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) - sqrt(2)*((a^5 - 2*a
^3*b^2 + a*b^4)*d^3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) + (a^6*b
 - a^4*b^3 - a^2*b^5 + b^7)*d*cos(d*x + c))*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*
a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(

```
sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4) + (a^8 - 2*a
^4*b^4 + b^8)*sin(d*x + c))/cos(d*x + c)) + 8*(a^4*b + 2*a^2*b^3 + b^5)*sqr
t(sin(d*x + c)/cos(d*x + c)))/((a^4 + 2*a^2*b^2 + b^4)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan (c+dx)) \sqrt{\tan (c+dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/2)*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*sqrt(tan(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)*sqrt(tan(d*x + c)), x)
```

Mupad [B]
time = 4.57, size = 153, normalized size = 0.92

$$\frac{2\,b\,\sqrt{\tan\left(c+d\,x\right)}}{d}+\frac{\sqrt{2}\,a\,\left(\ln\left(\sqrt{2}\,\sqrt{\tan\left(c+d\,x\right)}-\tan(c+d\,x)-1\right)-\ln\left(\tan(c+d\,x)+\sqrt{2}\,\sqrt{\tan\left(c+d\,x\right)}+1\right)\right)}{4\,d}+\frac{\sqrt{2}\,a\,\left(\mathrm{atan}\left(\sqrt{2}\,\sqrt{\tan\left(c+d\,x\right)}-1\right)+\mathrm{atan}\left(\sqrt{2}\,\sqrt{\tan\left(c+d\,x\right)}+1\right)\right)}{2\,d}+\frac{{\left(-1\right)}^{1/4}\,b\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,\sqrt{\tan\left(c+d\,x\right)}\right)\,1{}\mathrm{i}}{d}+\frac{{\left(-1\right)}^{1/4}\,b\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,\sqrt{\tan\left(c+d\,x\right)}\right)\,1{}\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)*(a + b*tan(c + d*x)),x)
```

```
[Out] (2*b*tan(c + d*x)^(1/2))/d + (2^(1/2)*a*(log(2^(1/2)*tan(c + d*x)^(1/2) - t
an(c + d*x) - 1) - log(tan(c + d*x) + 2^(1/2)*tan(c + d*x)^(1/2) + 1)))/(4*
d) + ((-1)^(1/4)*b*atan((-1)^(1/4)*tan(c + d*x)^(1/2))*1i)/d + ((-1)^(1/4)*
b*atanh((-1)^(1/4)*tan(c + d*x)^(1/2))*1i)/d + (2^(1/2)*a*(atan(2^(1/2)*tan
(c + d*x)^(1/2) - 1) + atan(2^(1/2)*tan(c + d*x)^(1/2) + 1)))/(2*d)
```

3.558 $\int \frac{a+b\tan(c+dx)}{\sqrt{\tan(c+dx)}}\,dx$

Optimal. Leaf size=150

$$-\frac{(a+b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a+b)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a-b)\log\left(1-\sqrt{2}\right.}{}$$

```
[Out] 1/2*(a+b)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)+1/2*(a+b)*arctan(1+
2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)-1/4*(a-b)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)
+tan(d*x+c))/d*2^(1/2)+1/4*(a-b)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/
d*2^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{(a+b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a+b)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d}-\frac{(a-b)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{(a-b)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])/Sqrt[Tan[c + d*x]],x]
```

```
[Out] -(((a + b)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d)) + ((a + b)*
ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) - ((a - b)*Log[1 - Sqrt
[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) + ((a - b)*Log[1 + Sq
rt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{a+b\tan(c+dx)}{\sqrt{\tan(c+dx)}}\,dx = \frac{2\text{Subst}\left(\int \frac{a+bx^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d}$$

$$= \frac{(a-b)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d} + \frac{(a+b)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\tan(c\right.}}{d}$$

$$= -\frac{(a-b)\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{(a-b)\text{Subst}\left(\int \frac{\sqrt{2}-}{-1+\sqrt{2}}\right.}{2}$$

$$= -\frac{(a-b)\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d} + \frac{(a-b)\log\left(1+\sqrt{2}\sqrt{\text{ta}}\right.}{2}$$

$$= -\frac{(a+b)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} + \frac{(a+b)\tan^{-1}\left(1+\sqrt{2}\sqrt{\tan(c+dx}\right.}{\sqrt{2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.04, size = 61, normalized size = 0.41

$$-\frac{\sqrt[4]{-1}\left((a-ib)\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+(a+ib)\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])/Sqrt[Tan[c + d*x]],x]
```

```
[Out] -(((-1)^(1/4)*((a - I*b)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + (a + I*b)*
ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]))/d)
```

Maple [A]
time = 0.09, size = 178, normalized size = 1.19

method	result
derivativedivides	$\frac{a\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)\right)+2\arctan\left(-1+\sqrt{2}\left(\sqrt{\tan}\right.\right.\right.}{4}$
default	$\frac{a\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)\right)+2\arctan\left(-1+\sqrt{2}\left(\sqrt{\tan}\right.\right.\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*tan(d*x+c))/tan(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/d*(1/4*a*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*b*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))))

Maxima [A]
time = 0.49, size = 124, normalized size = 0.83

$$\frac{2\sqrt{2}\,(a+b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(dx+c\right)}\right)\right)+2\sqrt{2}\,(a+b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(dx+c\right)}\right)\right)+\sqrt{2}\,(a-b)\log\left(\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)-\sqrt{2}\,(a-b)\log\left(-\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorithm="maxima")

[Out] 1/4*(2*sqrt(2)*(a + b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*(a + b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)*(a - b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*(a - b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/d

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2640 vs. $2(122) = 244$.
time = 0.74, size = 2640, normalized size = 17.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorithm="fricas")

[Out] 1/4*(4*sqrt(2)*d^4*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4)*arctan(-((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^4*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4) - sqrt(2)*(a*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4) - (a^2*b + b^3)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) + sqrt(2)*((a^4*b - 2*a^2*b^3 + b^5)*d^3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) - (a^7 - a^5*b^2 - a^3*b^4 + a*b^6)*d*cos(d*x + c))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4) + (a^8 - 2*a^4*b^4 + b^8)*sin(d*x + c))/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4) - sqrt(2)*((a^5 - a*b^4)*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4) - (a^6*b + a^4*b^3 - a^2*b^5 - b^7)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/d

^4))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4))/(a^12 + 2*a^10*b^2 - a^8*b^4 - 4*a^6*b^6 - a^4*b^8 + 2*a^2*b^10 + b^12)) + 4*sqrt(2)*d^4*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4)*arctan(((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^4*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4) + sqrt(2)*(a*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4) - (a^2*b + b^3)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) - sqrt(2)*((a^4*b - 2*a^2*b^3 + b^5)*d^3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) - (a^7 - a^5*b^2 - a^3*b^4 + a*b^6)*d*cos(d*x + c))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4) + (a^8 - 2*a^4*b^4 + b^8)*sin(d*x + c))/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4) + sqrt(2)*((a^5 - a*b^4)*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4) - (a^6*b + a^4*b^3 - a^2*b^5 - b^7)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4))/(a^12 + 2*a^10*b^2 - a^8*b^4 - 4*a^6*b^6 - a^4*b^8 + 2*a^2*b^10 + b^12)) + sqrt(2)*(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4)*log(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) + sqrt(2)*((a^4*b - 2*a^2*b^3 + b^5)*d^3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) - (a^7 - a^5*b^2 - a^3*b^4 + a*b^6)*d*cos(d*x + c))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4) + (a^8 - 2*a^4*b^4 + b^8)*sin(d*x + c))/cos(d*x + c)) - sqrt(2)*(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4)*log(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) - sqrt(2)*((a^4*b - 2*a^2*b^3 + b^5)*d^3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) - (a^7 - a^5*b^2 - a^3*b^4 + a*b^6)*d*cos(d*x + c))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4) + (a^8 - 2*a^4*b^4 + b^8)*sin(d*x + c))/cos(d*x + c)))/(a^4 + 2*a^2*b^2 + b^4)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\tan(c + dx)}{\sqrt{\tan(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/tan(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))/sqrt(tan(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)/sqrt(tan(d*x + c)), x)
```

Mupad [B]
time = 4.37, size = 141, normalized size = 0.94

$$\frac{\sqrt{2}\, b\left(\ln\left(\sqrt{2}\,\sqrt{\tan(c+d\,x)} - \tan(c+d\,x) - 1\right) - \ln\left(\tan(c+d\,x) + \sqrt{2}\,\sqrt{\tan(c+d\,x)} + 1\right)\right)}{4\,d} + \frac{\sqrt{2}\, b\left(\mathrm{atan}\left(\sqrt{2}\,\sqrt{\tan(c+d\,x)} - 1\right) + \mathrm{atan}\left(\sqrt{2}\,\sqrt{\tan(c+d\,x)} + 1\right)\right)}{2\,d} - \frac{(-1)^{1/4}\,a\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{\tan(c+d\,x)}\right)\,1\mathrm{i}}{d} - \frac{(-1)^{1/4}\,a\,\mathrm{atanh}\left((-1)^{1/4}\,\sqrt{\tan(c+d\,x)}\right)\,1\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))/tan(c + d*x)^(1/2),x)
```

```
[Out] (2^(1/2)*b*(log(2^(1/2)*tan(c + d*x)^(1/2) - tan(c + d*x) - 1) - log(tan(c
+ d*x) + 2^(1/2)*tan(c + d*x)^(1/2) + 1)))/(4*d) - ((-1)^(1/4)*a*atan((-1)^
(1/4)*tan(c + d*x)^(1/2))*1i)/d - ((-1)^(1/4)*a*atanh((-1)^(1/4)*tan(c + d*
x)^(1/2))*1i)/d + (2^(1/2)*b*(atan(2^(1/2)*tan(c + d*x)^(1/2) - 1) + atan(2
^(1/2)*tan(c + d*x)^(1/2) + 1)))/(2*d)
```

3.559 $\int \frac{a+b\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=166

$$\frac{(a-b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a-b)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a+b)\log\left(1-\sqrt{2}\sqrt{\right.}$$

```
[Out] -1/2*(a-b)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)-1/2*(a-b)*arctan(1
+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)-1/4*(a+b)*ln(1-2^(1/2)*tan(d*x+c)^(1/2
)+tan(d*x+c))/d*2^(1/2)+1/4*(a+b)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))
/d*2^(1/2)-2*a/d/tan(d*x+c)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a-b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a-b)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d}-\frac{(a+b)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{(a+b)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{2a}{d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])/Tan[c + d*x]^(3/2),x]
```

```
[Out] ((a - b)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) - ((a - b)*Arc
Tan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) - ((a + b)*Log[1 - Sqrt[2]
*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) + ((a + b)*Log[1 + Sqrt[
2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) - (2*a)/(d*Sqrt[Tan[c
+ d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{a+b\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2a}{d\sqrt{\tan(c+dx)}} + \int \frac{b-a\tan(c+dx)}{\sqrt{\tan(c+dx)}}\,dx$$

$$= -\frac{2a}{d\sqrt{\tan(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{b-ax^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d}$$

$$= -\frac{2a}{d\sqrt{\tan(c+dx)}} - \frac{(a-b)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d} + \frac{(a+b)\text{Subst}}{}$$

$$= -\frac{2a}{d\sqrt{\tan(c+dx)}} - \frac{(a-b)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2d} - \frac{(a-b}{}$$

$$= -\frac{(a+b)\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}\,d} + \frac{(a+b)\log\left(1+\sqrt{2}\sqrt{\text{ta}}\right.}{2\sqrt{}}$$

$$= \frac{(a-b)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d} - \frac{(a-b)\tan^{-1}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.11, size = 69, normalized size = 0.42

$$\frac{-\left((a+ib)\,_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-i\tan(c+dx)\right)\right)-(a-ib)\,_2F_1\left(-\frac{1}{2},1;\frac{1}{2};i\tan(c+dx)\right)}{d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])/Tan[c + d*x]^(3/2),x]
[Out] (-((a + I*b)*Hypergeometric2F1[-1/2, 1, 1/2, (-I)*Tan[c + d*x]]) - (a - I*b
)*Hypergeometric2F1[-1/2, 1, 1/2, I*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]])
```

Maple [A]
time = 0.05, size = 189, normalized size = 1.14

method	result
derivativedivides	$-\frac{2a}{\sqrt{\tan(dx+c)}}+\frac{b\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)\right)+2\,\text{arc}\right.}{4}$

default	$\frac{-\frac{2a}{\sqrt{\tan(dx+c)}}+\frac{b\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)\right)+2 a}{4}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))/tan(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*a/tan(d*x+c)^(1/2)+1/4*b*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+ta
n(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d
*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))-1/4*a*2^(1/2)*(ln((1-2^
(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))
+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))
))
```

Maxima [A]
time = 0.50, size = 135, normalized size = 0.81

$$-\frac{2\sqrt{2}(a-b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}(a-b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-\sqrt{2}(a+b)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+\sqrt{2}(a+b)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+\frac{8a}{\sqrt{\tan(dx+c)}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/tan(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*(a - b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))
) + 2*sqrt(2)*(a - b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c))))
 - sqrt(2)*(a + b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqr
t(2)*(a + b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 8*a/sqrt
(tan(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2883 vs. $2(136) = 272$.
time = 0.96, size = 2883, normalized size = 17.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/tan(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*(d^5*cos(d*x + c)^2 - d^5)*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a
^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4
+ 2*a^2*b^2 + b^4)/d^4)^(3/4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4)*arctan(((a^
8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^4*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt
((a^4 - 2*a^2*b^2 + b^4)/d^4) + sqrt(2)*(b*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)
/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4) + (a^3 + a*b^2)*d^5*sqrt((a^4 - 2*a
```

^2*b^2 + b^4)/d^4))*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^
4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(((a^6 - a^4*b^2 - a^2*b^
4 + b^6)*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) + sqrt(2)*((a^5
 - 2*a^3*b^2 + a*b^4)*d^3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) +
(a^6*b - a^4*b^3 - a^2*b^5 + b^7)*d*cos(d*x + c))*sqrt(-(2*a*b*d^2*sqrt((a^
4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))
*sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4) + (a^8
 - 2*a^4*b^4 + b^8)*sin(d*x + c))/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^
4)^(3/4) + sqrt(2)*((a^4*b - b^5)*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqr
t((a^4 - 2*a^2*b^2 + b^4)/d^4) + (a^7 + a^5*b^2 - a^3*b^4 - a*b^6)*d^5*sqrt
((a^4 - 2*a^2*b^2 + b^4)/d^4))*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4
)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/
cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4))/(a^12 + 2*a^10*b^2 - a^8
*b^4 - 4*a^6*b^6 - a^4*b^8 + 2*a^2*b^10 + b^12)) + 4*sqrt(2)*(d^5*cos(d*x +
 c)^2 - d^5)*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a
^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4)*
sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4)*arctan(-((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b
^8)*d^4*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4)
 - sqrt(2)*(b*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 +
 b^4)/d^4) + (a^3 + a*b^2)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4))*sqrt(-(2*
a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2
*a^2*b^2 + b^4))*sqrt(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt((a^4 + 2*a^
2*b^2 + b^4)/d^4)*cos(d*x + c) - sqrt(2)*((a^5 - 2*a^3*b^2 + a*b^4)*d^3*sqr
t((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) + (a^6*b - a^4*b^3 - a^2*b^5 +
b^7)*d*cos(d*x + c))*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a
^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x +
c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4) + (a^8 - 2*a^4*b^4 + b^8)*sin(d*x +
 c))/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4) - sqrt(2)*((a^4*b -
b^5)*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4
) + (a^7 + a^5*b^2 - a^3*b^4 - a*b^6)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4)
)*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^
4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b
^2 + b^4)/d^4)^(3/4))/(a^12 + 2*a^10*b^2 - a^8*b^4 - 4*a^6*b^6 - a^4*b^8 +
2*a^2*b^10 + b^12)) - 8*(a^5 + 2*a^3*b^2 + a*b^4)*sqrt(sin(d*x + c)/cos(d*x
 + c))*cos(d*x + c)*sin(d*x + c) - sqrt(2)*((a^4 + 2*a^2*b^2 + b^4)*d*cos(d
*x + c)^2 - (a^4 + 2*a^2*b^2 + b^4)*d + 2*(a*b*d^3*cos(d*x + c)^2 - a*b*d^3
)*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4))*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2
 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4 + 2*a^
2*b^2 + b^4)/d^4)^(1/4)*log(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt((a^4
+ 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) + sqrt(2)*((a^5 - 2*a^3*b^2 + a*b^4)*d
^3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) + (a^6*b - a^4*b^3 - a^2*
b^5 + b^7)*d*cos(d*x + c))*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^
4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(
d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4) + (a^8 - 2*a^4*b^4 + b^8)*sin
(d*x + c))/cos(d*x + c)) + sqrt(2)*((a^4 + 2*a^2*b^2 + b^4)*d*cos(d*x + c)^

```
2 - (a^4 + 2*a^2*b^2 + b^4)*d + 2*(a*b*d^3*cos(d*x + c)^2 - a*b*d^3)*sqrt((
a^4 + 2*a^2*b^2 + b^4)/d^4))*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/
d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4 + 2*a^2*b^2 +
b^4)/d^4)^(1/4)*log(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt((a^4 + 2*a^2*
b^2 + b^4)/d^4)*cos(d*x + c) - sqrt(2)*((a^5 - 2*a^3*b^2 + a*b^4)*d^3*sqrt(
(a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) + (a^6*b - a^4*b^3 - a^2*b^5 + b^
7)*d*cos(d*x + c))*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4
 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c)
)*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4) + (a^8 - 2*a^4*b^4 + b^8)*sin(d*x + c
))/cos(d*x + c)))/((a^4 + 2*a^2*b^2 + b^4)*d*cos(d*x + c)^2 - (a^4 + 2*a^2*
b^2 + b^4)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\tan{\left(c + dx \right)}}{\tan^{\frac{3}{2}}{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/tan(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))/tan(c + d*x)**(3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/tan(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 4.54, size = 102, normalized size = 0.61

$$\frac{(-1)^{1/4}\,a\,\mathrm{atanh}\left((-1)^{1/4}\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d}-\frac{(-1)^{1/4}\,a\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d}-\frac{2\,a}{d\,\sqrt{\tan\left(c+d\,x\right)}}-\frac{(-1)^{1/4}\,b\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{\tan\left(c+d\,x\right)}\right)\,1{}\mathrm{i}}{d}-\frac{(-1)^{1/4}\,b\,\mathrm{atanh}\left((-1)^{1/4}\,\sqrt{\tan\left(c+d\,x\right)}\right)\,1{}\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))/tan(c + d*x)^(3/2),x)
```

```
[Out] ((-1)^(1/4)*a*atanh((-1)^(1/4)*tan(c + d*x)^(1/2)))/d - ((-1)^(1/4)*a*atan(
(-1)^(1/4)*tan(c + d*x)^(1/2)))/d - (2*a)/(d*tan(c + d*x)^(1/2)) - ((-1)^(1
/4)*b*atan((-1)^(1/4)*tan(c + d*x)^(1/2))*1i)/d - ((-1)^(1/4)*b*atanh((-1)^
(1/4)*tan(c + d*x)^(1/2))*1i)/d
```

3.560 $\int \frac{a+b\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=184

$$\frac{(a+b)\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a+b)\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a-b)\log\left(1-\sqrt{2}\,\sqrt{\right.}}{}$$

```
[Out] -1/2*(a+b)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)-1/2*(a+b)*arctan(1
+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)+1/4*(a-b)*ln(1-2^(1/2)*tan(d*x+c)^(1/2
)+tan(d*x+c))/d*2^(1/2)-1/4*(a-b)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))
/d*2^(1/2)-2*b/d/tan(d*x+c)^(1/2)-2/3*a/d/tan(d*x+c)^(3/2)
```

Rubi [A]
time = 0.10, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a+b)\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a+b)\text{ArcTan}\left(\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d}+\frac{(a-b)\log\left(\tan(c+dx)-\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{(a-b)\log\left(\tan(c+dx)+\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{2a}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2b}{d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])/Tan[c + d*x]^(5/2),x]
```

```
[Out] ((a + b)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) - ((a + b)*Arc
Tan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) + ((a - b)*Log[1 - Sqrt[2]
*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) - ((a - b)*Log[1 + Sqrt[
2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) - (2*a)/(3*d*Tan[c + d
*x]^(3/2)) - (2*b)/(d*Sqrt[Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{a+b\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)}\,dx = -\frac{2a}{3d\tan^{\frac{3}{2}}(c+dx)} + \int \frac{b-a\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$
$$= -\frac{2a}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{2b}{d\sqrt{\tan(c+dx)}} + \int \frac{-a-b\tan(c+dx)}{\sqrt{\tan(c+dx)}}\,dx$$
$$= -\frac{2a}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{2b}{d\sqrt{\tan(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{-a-bx^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d}$$
$$= -\frac{2a}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{2b}{d\sqrt{\tan(c+dx)}} - \frac{(a-b)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right.}{d}$$
$$= -\frac{2a}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{2b}{d\sqrt{\tan(c+dx)}} + \frac{(a-b)\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan}\right.}{2\sqrt{2}d}$$
$$= \frac{(a-b)\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d} - \frac{(a-b)\log\left(1+\sqrt{2}\sqrt{\tan}\right.}{2\sqrt{2}}$$
$$= \frac{(a+b)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} - \frac{(a+b)\tan^{-1}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.11, size = 72, normalized size = 0.39

$$\frac{-\left((a+ib)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-i\tan(c+dx)\right)\right)-(a-ib)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};i\tan(c+dx)\right)}{3d\tan^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])/Tan[c + d*x]^(5/2),x]
```

```
[Out] (-((a + I*b)*Hypergeometric2F1[-3/2, 1, -1/2, (-I)*Tan[c + d*x]]) - (a - I*
b)*Hypergeometric2F1[-3/2, 1, -1/2, I*Tan[c + d*x]])/(3*d*Tan[c + d*x]^(3/2
))
```

Maple [A]
time = 0.05, size = 200, normalized size = 1.09

method	result

derivativedivides	$-\frac{2a}{3\tan(dx+c)^{\frac{3}{2}}}-\frac{2b}{\sqrt{\tan(dx+c)}}-\frac{a\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan}\right.\right.\right.}{4}$
default	$-\frac{2a}{3\tan(dx+c)^{\frac{3}{2}}}-\frac{2b}{\sqrt{\tan(dx+c)}}-\frac{a\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan}\right.\right.\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))/tan(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/3*a/tan(d*x+c)^(3/2)-2*b/tan(d*x+c)^(1/2)-1/4*a*2^(1/2)*(ln((1+2^(1
/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2
*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))-
1/4*b*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*
x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^
(1/2)*tan(d*x+c)^(1/2))))
```

Maxima [A]
time = 0.49, size = 146, normalized size = 0.79

$$-\frac{6\sqrt{2}(a+b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+6\sqrt{2}(a+b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)+3\sqrt{2}(a-b)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)-3\sqrt{2}(a-b)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+\frac{8(3b\tan(dx+c)+a)}{\tan(dx+c)^{\frac{3}{2}}}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/tan(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12*(6*sqrt(2)*(a + b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c))
)) + 6*sqrt(2)*(a + b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))
) + 3*sqrt(2)*(a - b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) -
3*sqrt(2)*(a - b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 8*(
3*b*tan(d*x + c) + a)/tan(d*x + c)^(3/2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2861 vs. $2(150) = 300$.
time = 1.22, size = 2861, normalized size = 15.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/tan(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(12*sqrt(2)*(d^5*cos(d*x + c)^2 - d^5)*sqrt((2*a*b*d^2*sqrt((a^4 + 2*
a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4
```

+ 2*a^2*b^2 + b^4)/d^4)^(3/4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4)*arctan(-((
a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^4*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sq
rt((a^4 - 2*a^2*b^2 + b^4)/d^4) - sqrt(2)*(a*d^7*sqrt((a^4 + 2*a^2*b^2 + b^
4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4) - (a^2*b + b^3)*d^5*sqrt((a^4 - 2
*a^2*b^2 + b^4)/d^4))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a
^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(((a^6 - a^4*b^2 - a^2*b
^4 + b^6)*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) + sqrt(2)*((a^
4*b - 2*a^2*b^3 + b^5)*d^3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) -
(a^7 - a^5*b^2 - a^3*b^4 + a*b^6)*d*cos(d*x + c))*sqrt((2*a*b*d^2*sqrt((a^
4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))
*sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4) + (a^8
- 2*a^4*b^4 + b^8)*sin(d*x + c))/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^
4)^(3/4) - sqrt(2)*((a^5 - a*b^4)*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqr
t((a^4 - 2*a^2*b^2 + b^4)/d^4) - (a^6*b + a^4*b^3 - a^2*b^5 - b^7)*d^5*sqrt
((a^4 - 2*a^2*b^2 + b^4)/d^4))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)
/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/c
os(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4))/(a^12 + 2*a^10*b^2 - a^8*
b^4 - 4*a^6*b^6 - a^4*b^8 + 2*a^2*b^10 + b^12)) + 12*sqrt(2)*(d^5*cos(d*x +
c)^2 - d^5)*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^
2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4)*s
qrt((a^4 - 2*a^2*b^2 + b^4)/d^4)*arctan(((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8
)*d^4*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4) +
sqrt(2)*(a*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b
^4)/d^4) - (a^2*b + b^3)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4))*sqrt((2*a*b
*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^
2*b^2 + b^4))*sqrt(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt((a^4 + 2*a^2*b
^2 + b^4)/d^4)*cos(d*x + c) - sqrt(2)*((a^4*b - 2*a^2*b^3 + b^5)*d^3*sqrt((
a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) - (a^7 - a^5*b^2 - a^3*b^4 + a*b^6
)*d*cos(d*x + c))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 +
2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*
((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4) + (a^8 - 2*a^4*b^4 + b^8)*sin(d*x + c))
/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4) + sqrt(2)*((a^5 - a*b^4)
*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4) -
(a^6*b + a^4*b^3 - a^2*b^5 - b^7)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4))*sq
rt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a
^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 +
b^4)/d^4)^(3/4))/(a^12 + 2*a^10*b^2 - a^8*b^4 - 4*a^6*b^6 - a^4*b^8 + 2*a^2
*b^10 + b^12)) - 3*sqrt(2)*((a^4 + 2*a^2*b^2 + b^4)*d*cos(d*x + c)^2 - (a^4
+ 2*a^2*b^2 + b^4)*d - 2*(a*b*d^3*cos(d*x + c)^2 - a*b*d^3)*sqrt((a^4 + 2*
a^2*b^2 + b^4)/d^4))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^
4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)
^(1/4)*log(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4
)/d^4)*cos(d*x + c) + sqrt(2)*((a^4*b - 2*a^2*b^3 + b^5)*d^3*sqrt((a^4 + 2*
a^2*b^2 + b^4)/d^4)*cos(d*x + c) - (a^7 - a^5*b^2 - a^3*b^4 + a*b^6)*d*cos(
d*x + c))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b

```
^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 +
2*a^2*b^2 + b^4)/d^4)^(1/4) + (a^8 - 2*a^4*b^4 + b^8)*sin(d*x + c))/cos(d*x
 + c)) + 3*sqrt(2)*((a^4 + 2*a^2*b^2 + b^4)*d*cos(d*x + c)^2 - (a^4 + 2*a^2
*b^2 + b^4)*d - 2*(a*b*d^3*cos(d*x + c)^2 - a*b*d^3)*sqrt((a^4 + 2*a^2*b^2
+ b^4)/d^4))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^
2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4)*l
og(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*c
os(d*x + c) - sqrt(2)*((a^4*b - 2*a^2*b^3 + b^5)*d^3*sqrt((a^4 + 2*a^2*b^2
+ b^4)/d^4)*cos(d*x + c) - (a^7 - a^5*b^2 - a^3*b^4 + a*b^6)*d*cos(d*x + c)
)*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4
)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^
2 + b^4)/d^4)^(1/4) + (a^8 - 2*a^4*b^4 + b^8)*sin(d*x + c))/cos(d*x + c)) -
 8*((a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c)^2 + 3*(a^4*b + 2*a^2*b^3 + b^5)*
cos(d*x + c)*sin(d*x + c))*sqrt(sin(d*x + c)/cos(d*x + c)))/((a^4 + 2*a^2*b
^2 + b^4)*d*cos(d*x + c)^2 - (a^4 + 2*a^2*b^2 + b^4)*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\tan{\left(c + dx \right)}}{\tan^{\frac{5}{2}}{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/tan(d*x+c)**(5/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))/tan(c + d*x)**(5/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/tan(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 5.13, size = 114, normalized size = 0.62

$$\frac{(-1)^{1/4}\,b\,\mathrm{atanh}\left((-1)^{1/4}\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d}-\frac{2\,b}{d\,\sqrt{\tan\left(c+d\,x\right)}}-\frac{(-1)^{1/4}\,b\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d}-\frac{2\,a}{3\,d\,{\tan\left(c+d\,x\right)}^{3/2}}+\frac{(-1)^{1/4}\,a\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{\tan\left(c+d\,x\right)}\right)\,1{}\mathrm{i}}{d}+\frac{(-1)^{1/4}\,a\,\mathrm{atanh}\left((-1)^{1/4}\,\sqrt{\tan\left(c+d\,x\right)}\right)\,1{}\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))/tan(c + d*x)^(5/2),x)
```

```
[Out] ((-1)^(1/4)*a*atan((-1)^(1/4)*tan(c + d*x)^(1/2))*1i)/d - (2*b)/(d*tan(c +
d*x)^(1/2)) - (2*a)/(3*d*tan(c + d*x)^(3/2)) + ((-1)^(1/4)*a*atanh((-1)^(1/
4)*tan(c + d*x)^(1/2))*1i)/d - ((-1)^(1/4)*b*atan((-1)^(1/4)*tan(c + d*x)^(
1/2)))/d + ((-1)^(1/4)*b*atanh((-1)^(1/4)*tan(c + d*x)^(1/2)))/d
```

3.561 $\int \frac{a+b\tan(c+dx)}{\tan^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=202

$$-\frac{(a-b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a-b)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a+b)\log\left(1-\sqrt{2}\right.}{}$$

[Out] 1/2*(a-b)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)+1/2*(a-b)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)+1/4*(a+b)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)-1/4*(a+b)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)+2*a/d/tan(d*x+c)^(1/2)-2/5*a/d/tan(d*x+c)^(5/2)-2/3*b/d/tan(d*x+c)^(3/2)

Rubi [A]
time = 0.11, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{(a-b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a-b)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d}+\frac{(a+b)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{(a+b)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{2a}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{2a}{d\sqrt{\tan(c+dx)}}-\frac{2b}{3d\tan^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Tan[c + d*x])/Tan[c + d*x]^(7/2),x]

[Out] -(((a - b)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d)) + ((a - b)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) + ((a + b)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) - ((a + b)*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) - (2*a)/(5*d*Tan[c + d*x]^(5/2)) - (2*b)/(3*d*Tan[c + d*x]^(3/2)) + (2*a)/(d*Sqrt[Tan[c + d*x]])

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{a+b\tan(c+dx)}{\tan^{\frac{7}{2}}(c+dx)}\,dx = -\frac{2a}{5d\tan^{\frac{5}{2}}(c+dx)} + \int \frac{b-a\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$
$$= -\frac{2a}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{2b}{3d\tan^{\frac{3}{2}}(c+dx)} + \int \frac{-a-b\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$
$$= -\frac{2a}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{2b}{3d\tan^{\frac{3}{2}}(c+dx)} + \frac{2a}{d\sqrt{\tan(c+dx)}} + \int \frac{-b+a\tan(c+dx)}{\sqrt{\tan(c+dx)}}$$
$$= -\frac{2a}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{2b}{3d\tan^{\frac{3}{2}}(c+dx)} + \frac{2a}{d\sqrt{\tan(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{-b+ax^2}{1+x^4}\,dx, x\right.}{d}$$
$$= -\frac{2a}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{2b}{3d\tan^{\frac{3}{2}}(c+dx)} + \frac{2a}{d\sqrt{\tan(c+dx)}} + \frac{(a-b)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\right.}{}$$
$$= -\frac{2a}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{2b}{3d\tan^{\frac{3}{2}}(c+dx)} + \frac{2a}{d\sqrt{\tan(c+dx)}} + \frac{(a-b)\text{Subst}\left(\int \frac{}{1-\sqrt{2}}\right.}{}$$
$$= \frac{(a+b)\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}\,d} - \frac{(a+b)\log\left(1+\sqrt{2}\sqrt{\tan}\right.}{2\sqrt{2}}$$
$$= -\frac{(a-b)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d} + \frac{(a-b)\tan^{-1}\left(1+\sqrt{2}\sqrt{\tan(c+dx}\right.}{\sqrt{2}\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.11, size = 72, normalized size = 0.36

$$\frac{-\left((a+ib)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-i\tan(c+dx)\right)\right)-(a-ib)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};i\tan(c+dx)\right)}{5d\tan^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])/Tan[c + d*x]^(7/2),x]
```

```
[Out] (-((a + I*b)*Hypergeometric2F1[-5/2, 1, -3/2, (-I)*Tan[c + d*x]]) - (a - I*
b)*Hypergeometric2F1[-5/2, 1, -3/2, I*Tan[c + d*x]])/(5*d*Tan[c + d*x]^(5/2
))
```

Maple [A]
time = 0.05, size = 211, normalized size = 1.04

method	result

derivativedivides	$\frac{-\frac{2a}{5\tan(dx+c)^{\frac{5}{2}}}-\frac{2b}{3\tan(dx+c)^{\frac{3}{2}}}+\frac{2a}{\sqrt{\tan(dx+c)}}-\frac{b\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\left(1\right.\right.}{}}{}$
default	$\frac{-\frac{2a}{5\tan(dx+c)^{\frac{5}{2}}}-\frac{2b}{3\tan(dx+c)^{\frac{3}{2}}}+\frac{2a}{\sqrt{\tan(dx+c)}}-\frac{b\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\left(1\right.\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))/tan(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/5*a/tan(d*x+c)^(5/2)-2/3*b/tan(d*x+c)^(3/2)+2*a/tan(d*x+c)^(1/2)-1/
4*b*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+
c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1
/2)*tan(d*x+c)^(1/2)))+1/4*a*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*
x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c
)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))))
```

Maxima [A]
time = 0.50, size = 159, normalized size = 0.79

$$\frac{30\sqrt{2}(a-b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+30\sqrt{2}(a-b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-15\sqrt{2}(a+b)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+15\sqrt{2}(a+b)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+\frac{8\left(15a\tan(dx+c)^2-5b\tan(dx+c)-3a\right)}{\tan(dx+c)^{\frac{5}{2}}}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/tan(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/60*(30*sqrt(2)*(a - b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c))
)) + 30*sqrt(2)*(a - b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c))
)) - 15*sqrt(2)*(a + b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1)
+ 15*sqrt(2)*(a + b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) +
8*(15*a*tan(d*x + c)^2 - 5*b*tan(d*x + c) - 3*a)/tan(d*x + c)^(5/2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3096 vs. $2(164) = 328$.
time = 1.67, size = 3096, normalized size = 15.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/tan(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/60*(60*sqrt(2)*(d^5*cos(d*x + c)^4 - 2*d^5*cos(d*x + c)^2 + d^5)*sqrt(-(2
*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 -
```

2*a^2*b^2 + b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4)*sqrt((a^4 - 2*a^2*b^2
 + b^4)/d^4)*arctan(((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^4*sqrt((a^4 + 2*
a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4) + sqrt(2)*(b*d^7*sqrt
((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4) + (a^3 + a*
b^2)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4))*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*
a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(
((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(
d*x + c) + sqrt(2)*((a^5 - 2*a^3*b^2 + a*b^4)*d^3*sqrt((a^4 + 2*a^2*b^2 + b
^4)/d^4)*cos(d*x + c) + (a^6*b - a^4*b^3 - a^2*b^5 + b^7)*d*cos(d*x + c))*s
qrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/
(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2
 + b^4)/d^4)^(1/4) + (a^8 - 2*a^4*b^4 + b^8)*sin(d*x + c))/cos(d*x + c))*((a
^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4) + sqrt(2)*((a^4*b - b^5)*d^7*sqrt((a^4 + 2
*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4) + (a^7 + a^5*b^2 - a
^3*b^4 - a*b^6)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4))*sqrt(-(2*a*b*d^2*sqr
t((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 +
b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4))/
(a^12 + 2*a^10*b^2 - a^8*b^4 - 4*a^6*b^6 - a^4*b^8 + 2*a^2*b^10 + b^12)) +
60*sqrt(2)*(d^5*cos(d*x + c)^4 - 2*d^5*cos(d*x + c)^2 + d^5)*sqrt(-(2*a*b*d
^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*
b^2 + b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4)*sqrt((a^4 - 2*a^2*b^2 + b^4
)/d^4)*arctan(-((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^4*sqrt((a^4 + 2*a^2*b
^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4) - sqrt(2)*(b*d^7*sqrt((a^4
 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4) + (a^3 + a*b^2)*
d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4))*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b
^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(((a^6
 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x +
 c) - sqrt(2)*((a^5 - 2*a^3*b^2 + a*b^4)*d^3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d
^4)*cos(d*x + c) + (a^6*b - a^4*b^3 - a^2*b^5 + b^7)*d*cos(d*x + c))*sqrt(-
(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4
 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4
)/d^4)^(1/4) + (a^8 - 2*a^4*b^4 + b^8)*sin(d*x + c))/cos(d*x + c))*((a^4 +
2*a^2*b^2 + b^4)/d^4)^(3/4) - sqrt(2)*((a^4*b - b^5)*d^7*sqrt((a^4 + 2*a^2*
b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4) + (a^7 + a^5*b^2 - a^3*b^
4 - a*b^6)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4))*sqrt(-(2*a*b*d^2*sqrt((a^
4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))
*sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4))/(a^12
 + 2*a^10*b^2 - a^8*b^4 - 4*a^6*b^6 - a^4*b^8 + 2*a^2*b^10 + b^12)) - 15*sq
rt(2)*((a^4 + 2*a^2*b^2 + b^4)*d*cos(d*x + c)^4 - 2*(a^4 + 2*a^2*b^2 + b^4)
*d*cos(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d + 2*(a*b*d^3*cos(d*x + c)^4 -
 2*a*b*d^3*cos(d*x + c)^2 + a*b*d^3)*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4))*sqr
t(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a
^4 - 2*a^2*b^2 + b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4)*log(((a^6 - a^4*
b^2 - a^2*b^4 + b^6)*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) + s
qrt(2)*((a^5 - 2*a^3*b^2 + a*b^4)*d^3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos

```
(d*x + c) + (a^6*b - a^4*b^3 - a^2*b^5 + b^7)*d*cos(d*x + c))*sqrt(-(2*a*b*
d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2
*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^
(1/4) + (a^8 - 2*a^4*b^4 + b^8)*sin(d*x + c))/cos(d*x + c)) + 15*sqrt(2)*((
a^4 + 2*a^2*b^2 + b^4)*d*cos(d*x + c)^4 - 2*(a^4 + 2*a^2*b^2 + b^4)*d*cos(d
*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d + 2*(a*b*d^3*cos(d*x + c)^4 - 2*a*b*d
^3*cos(d*x + c)^2 + a*b*d^3)*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4))*sqrt(-(2*a*
b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a
^2*b^2 + b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4)*log(((a^6 - a^4*b^2 - a^
2*b^4 + b^6)*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) - sqrt(2)*(
(a^5 - 2*a^3*b^2 + a*b^4)*d^3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c
) + (a^6*b - a^4*b^3 - a^2*b^5 + b^7)*d*cos(d*x + c))*sqrt(-(2*a*b*d^2*sqrt
((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b
^4))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4) +
(a^8 - 2*a^4*b^4 + b^8)*sin(d*x + c))/cos(d*x + c)) + 8*(5*(a^4*b + 2*a^2*b
^3 + b^5)*cos(d*x + c)^4 - 5*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^2 - 3*(
6*(a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c)^3 - 5*(a^5 + 2*a^3*b^2 + a*b^4)*co
s(d*x + c))*sin(d*x + c))*sqrt(sin(d*x + c)/cos(d*x + c)))/((a^4 + 2*a^2*b^
2 + b^4)*d*cos(d*x + c)^4 - 2*(a^4 + 2*a^2*b^2 + b^4)*d*cos(d*x + c)^2 + (a
^4 + 2*a^2*b^2 + b^4)*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\tan{\left(c + dx \right)}}{\tan^{\frac{7}{2}}{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/tan(d*x+c)**(7/2),x)
[Out] Integral((a + b*tan(c + d*x))/tan(c + d*x)**(7/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/tan(d*x+c)^(7/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**B**]
time = 5.89, size = 128, normalized size = 0.63

$$\frac{(-1)^{1/4}\,a\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d}-\frac{\frac{2\,a}{5}-2\,a\,\tan(c+d\,x)^2}{d\,\tan\left(c+d\,x\right)^{5/2}}-\frac{2\,b}{3\,d\,\tan\left(c+d\,x\right)^{3/2}}-\frac{(-1)^{1/4}\,a\,\mathrm{atanh}\left((-1)^{1/4}\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d}+\frac{(-1)^{1/4}\,b\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{\tan\left(c+d\,x\right)}\right)\,1{}\mathrm{i}}{d}+\frac{(-1)^{1/4}\,b\,\mathrm{atanh}\left((-1)^{1/4}\,\sqrt{\tan\left(c+d\,x\right)}\right)\,1{}\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))/tan(c + d*x)^(7/2),x)
```

```
[Out] ((-1)^(1/4)*a*atan((-1)^(1/4)*tan(c + d*x)^(1/2)))/d - ((2*a)/5 - 2*a*tan(c
 + d*x)^2)/(d*tan(c + d*x)^(5/2)) - (2*b)/(3*d*tan(c + d*x)^(3/2)) - ((-1)^
(1/4)*a*atanh((-1)^(1/4)*tan(c + d*x)^(1/2)))/d + ((-1)^(1/4)*b*atan((-1)^(
1/4)*tan(c + d*x)^(1/2))*1i)/d + ((-1)^(1/4)*b*atanh((-1)^(1/4)*tan(c + d*x
)^(1/2))*1i)/d
```

3.562 $\int \tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=268

$$\frac{(a^2-2ab-b^2)\operatorname{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a^2-2ab-b^2)\operatorname{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a^2}{}$$

```
[Out] -1/2*(a^2-2*a*b-b^2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)-1/2*(a^2
-2*a*b-b^2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)-1/4*(a^2+2*a*b-b^2
)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)+1/4*(a^2+2*a*b-b^2)*l
n(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)-4*a*b*tan(d*x+c)^(1/2)/d
+2/3*(a^2-b^2)*tan(d*x+c)^(3/2)/d+4/5*a*b*tan(d*x+c)^(5/2)/d+2/7*b^2*tan(d*
x+c)^(7/2)/d
```

Rubi [A]
time = 0.18, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3624, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a^2-2ab-b^2)\operatorname{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a^2-2ab-b^2)\operatorname{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d}+\frac{2(a^2-b^2)\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{(a^2+2ab-b^2)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{(a^2+2ab-b^2)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{4ab\tan^{\frac{5}{2}}(c+dx)}{5d}-\frac{4ab\sqrt{\tan(c+dx)}}{d}+\frac{2b^2\tan^{\frac{7}{2}}(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((a^2 - 2*a*b - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) -
((a^2 - 2*a*b - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) -
((a^2 + 2*a*b - b^2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2
*Sqrt[2]*d) + ((a^2 + 2*a*b - b^2)*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan
[c + d*x]])/(2*Sqrt[2]*d) - (4*a*b*Sqrt[Tan[c + d*x]])/d + (2*(a^2 - b^2)*T
an[c + d*x]^(3/2))/(3*d) + (4*a*b*Tan[c + d*x]^(5/2))/(5*d) + (2*b^2*Tan[c
+ d*x]^(7/2))/(7*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
```

```
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{2b^2\tan^{\frac{7}{2}}(c+dx)}{7d} + \int \tan^{\frac{5}{2}}(c+dx)\left(a^2-b^2+2ab\tan(c+dx)\right)\,c$$

$$= \frac{4ab\tan^{\frac{5}{2}}(c+dx)}{5d} + \frac{2b^2\tan^{\frac{7}{2}}(c+dx)}{7d} + \int \tan^{\frac{3}{2}}(c+dx)\left(-2ab+\right.$$

$$= \frac{2(a^2-b^2)\tan^{\frac{3}{2}}(c+dx)}{3d} + \frac{4ab\tan^{\frac{5}{2}}(c+dx)}{5d} + \frac{2b^2\tan^{\frac{7}{2}}(c+dx)}{7d}$$

$$= -\frac{4ab\sqrt{\tan(c+dx)}}{d} + \frac{2(a^2-b^2)\tan^{\frac{3}{2}}(c+dx)}{3d} + \frac{4ab\tan^{\frac{5}{2}}(c+a}{5d}$$

$$= -\frac{4ab\sqrt{\tan(c+dx)}}{d} + \frac{2(a^2-b^2)\tan^{\frac{3}{2}}(c+dx)}{3d} + \frac{4ab\tan^{\frac{5}{2}}(c+a}{5d}$$

$$= -\frac{4ab\sqrt{\tan(c+dx)}}{d} + \frac{2(a^2-b^2)\tan^{\frac{3}{2}}(c+dx)}{3d} + \frac{4ab\tan^{\frac{5}{2}}(c+a}{5d}$$

$$= -\frac{4ab\sqrt{\tan(c+dx)}}{d} + \frac{2(a^2-b^2)\tan^{\frac{3}{2}}(c+dx)}{3d} + \frac{4ab\tan^{\frac{5}{2}}(c+a}{5d}$$

$$= -\frac{(a^2+2ab-b^2)\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}\,d} +$$

$$= \frac{(a^2-2ab-b^2)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d} - \frac{(a^2-2ab-b}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.13, size = 133, normalized size = 0.50

$$\frac{-105(-1)^{3/4}(a-ib)^2\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+105(-1)^{3/4}(a+ib)^2\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+2\sqrt{\tan(c+dx)}\left(-210ab+35(a^2-b^2)\tan(c+dx)+42ab\tan^2(c+dx)+15b^2\tan^3(c+dx)\right)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-105*(-1)^(3/4)*(a - I*b)^2*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + 105*(-
1)^(3/4)*(a + I*b)^2*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + 2*Sqrt[Tan[c
+ d*x]]*(-210*a*b + 35*(a^2 - b^2)*Tan[c + d*x] + 42*a*b*Tan[c + d*x]^2 + 1
5*b^2*Tan[c + d*x]^3))/(105*d)
```

Maple [A]
time = 0.07, size = 250, normalized size = 0.93

method	result
derivativedivides	$\frac{\frac{2b^2\left(\tan^{\frac{7}{2}}(dx+c)\right)}{7}+\frac{4ab\left(\tan^{\frac{5}{2}}(dx+c)\right)}{5}+\frac{2a^2\left(\tan^{\frac{3}{2}}(dx+c)\right)}{3}-\frac{2b^2\left(\tan^{\frac{3}{2}}(dx+c)\right)}{3}-4ab\left(\sqrt{\tan}(dx+c)\right)+\frac{ab\sqrt{2}\left(\ln\left(\frac{1+\sqrt{}}{1-\sqrt{}}\right.\right.}{}}{}$
default	$\frac{\frac{2b^2\left(\tan^{\frac{7}{2}}(dx+c)\right)}{7}+\frac{4ab\left(\tan^{\frac{5}{2}}(dx+c)\right)}{5}+\frac{2a^2\left(\tan^{\frac{3}{2}}(dx+c)\right)}{3}-\frac{2b^2\left(\tan^{\frac{3}{2}}(dx+c)\right)}{3}-4ab\left(\sqrt{\tan}(dx+c)\right)+\frac{ab\sqrt{2}\left(\ln\left(\frac{1+\sqrt{}}{1-\sqrt{}}\right.\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/7*b^2*tan(d*x+c)^(7/2)+4/5*a*b*tan(d*x+c)^(5/2)+2/3*a^2*tan(d*x+c)^(
3/2)-2/3*b^2*tan(d*x+c)^(3/2)-4*a*b*tan(d*x+c)^(1/2)+1/2*a*b*2^(1/2)*(ln((1
+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c
)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/
2)))+1/4*(-a^2+b^2)*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+
2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+
2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))))
```

Maxima [A]
time = 0.49, size = 217, normalized size = 0.81

$$\frac{120\,b^{2}\tan\left(dx+c\right)^{\frac{7}{2}}+336\,ab\tan\left(dx+c\right)^{\frac{5}{2}}-210\,\sqrt{2}\left(a^{2}-2\,ab-b^{2}\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\sqrt{\tan\left(dx+c\right)}\right)\right)-210\,\sqrt{2}\left(a^{2}-2\,ab-b^{2}\right)\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\sqrt{\tan\left(dx+c\right)}\right)\right)+105\,\sqrt{2}\left(a^{2}+2\,ab-b^{2}\right)\log\left(\sqrt{2}\,\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)-105\,\sqrt{2}\left(a^{2}+2\,ab-b^{2}\right)\log\left(-\sqrt{2}\,\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)-1680\,ab\sqrt{\tan\left(dx+c\right)}+280\left(a^{2}-b^{2}\right)\tan\left(dx+c\right)^{\frac{3}{2}}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/420*(120*b^2*tan(d*x + c)^(7/2) + 336*a*b*tan(d*x + c)^(5/2) - 210*sqrt(2
)*(a^2 - 2*a*b - b^2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c))))
- 210*sqrt(2)*(a^2 - 2*a*b - b^2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan
(d*x + c)))) + 105*sqrt(2)*(a^2 + 2*a*b - b^2)*log(sqrt(2)*sqrt(tan(d*x + c
)) + tan(d*x + c) + 1) - 105*sqrt(2)*(a^2 + 2*a*b - b^2)*log(-sqrt(2)*sqrt(
tan(d*x + c)) + tan(d*x + c) + 1) - 1680*a*b*sqrt(tan(d*x + c)) + 280*(a^2
- b^2)*tan(d*x + c)^(3/2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5080 vs. $2(226) = 452$.
time = 1.75, size = 5080, normalized size = 18.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/420*(420*sqrt(2)*d^5*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8
- 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8
)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*((a^8 + 4*a^6*b
^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(3/4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^
4*b^4 - 12*a^2*b^6 + b^8)/d^4)*arctan(-((a^16 - 20*a^12*b^4 - 64*a^10*b^6 -
 90*a^8*b^8 - 64*a^6*b^10 - 20*a^4*b^12 + b^16)*d^4*sqrt((a^8 + 4*a^6*b^2 +
 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12
*a^2*b^6 + b^8)/d^4) - sqrt(2)*(2*a*b*d^7*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4
 + 4*a^2*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 +
 b^8)/d^4) + (a^6 + a^4*b^2 - a^2*b^4 - b^6)*d^5*sqrt((a^8 - 12*a^6*b^2 + 3
8*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a
^2*b^6 + b^8 - 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*
a^2*b^6 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sq
rt(((a^12 - 10*a^10*b^2 + 15*a^8*b^4 + 52*a^6*b^6 + 15*a^4*b^8 - 10*a^2*b^1
0 + b^12)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*cos
(d*x + c) + sqrt(2)*((a^10 - 13*a^8*b^2 + 50*a^6*b^4 - 50*a^4*b^6 + 13*a^2*
b^8 - b^10)*d^3*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*c
os(d*x + c) + 2*(a^13*b - 10*a^11*b^3 + 15*a^9*b^5 + 52*a^7*b^7 + 15*a^5*b^
9 - 10*a^3*b^11 + a*b^13)*d*cos(d*x + c))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4
 + 4*a^2*b^6 + b^8 - 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^
4 + 4*a^2*b^6 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^
8))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b
^6 + b^8)/d^4)^(1/4) + (a^16 - 8*a^14*b^2 - 4*a^12*b^4 + 72*a^10*b^6 + 134*
a^8*b^8 + 72*a^6*b^10 - 4*a^4*b^12 - 8*a^2*b^14 + b^16)*sin(d*x + c))/cos(d
*x + c))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(3/4) + sqrt
(2)*(2*(a^9*b - 4*a^7*b^3 - 10*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d^7*sqrt((a^8 +
 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*
a^4*b^4 - 12*a^2*b^6 + b^8)/d^4) + (a^14 - 3*a^12*b^2 - 15*a^10*b^4 - 11*a^
8*b^6 + 11*a^6*b^8 + 15*a^4*b^10 + 3*a^2*b^12 - b^14)*d^5*sqrt((a^8 - 12*a^
6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*
b^4 + 4*a^2*b^6 + b^8 - 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4
*b^4 + 4*a^2*b^6 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 +
 b^8))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^
2*b^6 + b^8)/d^4)^(3/4))/(a^24 - 4*a^22*b^2 - 30*a^20*b^4 + 12*a^18*b^6 + 3
67*a^16*b^8 + 1016*a^14*b^10 + 1372*a^12*b^12 + 1016*a^10*b^14 + 367*a^8*b^
16 + 12*a^6*b^18 - 30*a^4*b^20 - 4*a^2*b^22 + b^24))*cos(d*x + c)^3 + 420*s
qrt(2)*d^5*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 - 4*(a^3*b -
 a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4))/(a^8
 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*((a^8 + 4*a^6*b^2 + 6*a^4*b
^4 + 4*a^2*b^6 + b^8)/d^4)^(3/4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a
^2*b^6 + b^8)/d^4)*arctan(((a^16 - 20*a^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 -
```

```
 64*a^6*b^10 - 20*a^4*b^12 + b^16)*d^4*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
4*a^2*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^
8)/d^4) + sqrt(2)*(2*a*b*d^7*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6
+ b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4) +
(a^6 + a^4*b^2 - a^2*b^4 - b^6)*d^5*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 1
2*a^2*b^6 + b^8)/d^4))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8
- 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8
)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(((a^12 - 1
0*a^10*b^2 + 15*a^8*b^4 + 52*a^6*b^6 + 15*a^4*b^8 - 10*a^2*b^10 + b^12)*d^2
*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*cos(d*x + c) - s
qrt(2)*((a^10 - 13*a^8*b^2 + 50*a^6*b^4 - 50*a^4*b^6 + 13*a^2*b^8 - b^10)*d
^3*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*cos(d*x + c) +
 2*(a^13*b - 10*a^11*b^3 + 15*a^9*b^5 + 52*a^7*b^7 + 15*a^5*b^9 - 10*a^3*b^
11 + a*b^13)*d*cos(d*x + c))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6
+ b^8 - 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6
 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(sin(
d*x + c)/cos(d*x + c))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4
)^(1/4) + (a^16 - 8*a^14*b^2 - 4*a^12*b^4 + 72*a^10*b^6 + 134*a^8*b^8 + 72*
a^6*b^10 - 4*a^4*b^12 - 8*a^2*b^14 + b^16)*sin(d*x + c))/cos(d*x + c))*((a^
8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(3/4) - sqrt(2)*(2*(a^9*b
 - 4*a^7*b^3 - 10*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d^7*sqrt((a^8 + 4*a^6*b^2 +
6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*
a^2*b^6 + b^8)/d^4) + (a^14 - 3*a^12*b^2 - 15*a^10*b^4 - 11*a^8*b^6 + 11*a^
6*b^8 + 15*a^4*b^10 + 3*a^2*b^12 - b^14)*d^5*sqrt((a^8 - 12*a^6*b^2 + 38*a^
4*b^4 - 12*a^2*b^6 + b^8)/d^4))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b
^6 + b^8 - 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tan (c + dx))^2 \tan^{\frac{5}{2}} (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(5/2)*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*tan(c + d*x)**(5/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

[Out] Timed out

Mupad [B]
time = 6.99, size = 995, normalized size = 3.71

$$\tan(c+dx)^{3/2}\left(\frac{2a^2}{3d}-\frac{2b^2}{3d}\right)+\frac{2b^2\tan(c+dx)^{7/2}}{7d}-\frac{4ab\sqrt{\tan(c+dx)}}{d}+\frac{4ab\tan(c+dx)^{5/2}}{5d}-\operatorname{atan}\left(\frac{a^4\sqrt{\tan(c+dx)}\sqrt{\frac{a^3b}{d^2}-\frac{b^4\,1i}{4d^2}-\frac{ab^3}{d^2}-\frac{a^4\,1i}{4d^2}+\frac{a^2b^2\,3i}{2d^2}}\,32i}{\frac{16a^6}{d}-\frac{16b^6}{d}+\frac{112a^2b^4}{d}-\frac{112a^4b^2}{d}+\frac{ab^5\,32i}{d}+\frac{a^5b\,32i}{d}-\frac{a^3b^3\,192i}{d}}+\frac{b^4\sqrt{\tan(c+dx)}\sqrt{\frac{a^3b}{d^2}-\frac{b^4\,1i}{4d^2}-\frac{ab^3}{d^2}-\frac{a^4\,1i}{4d^2}+\frac{a^2b^2\,3i}{2d^2}}\,32i}{\frac{16a^6}{d}-\frac{16b^6}{d}+\frac{112a^2b^4}{d}-\frac{112a^4b^2}{d}+\frac{ab^5\,32i}{d}+\frac{a^5b\,32i}{d}-\frac{a^3b^3\,192i}{d}}-\frac{a^2b^2\sqrt{\tan(c+dx)}\sqrt{\frac{a^3b}{d^2}-\frac{b^4\,1i}{4d^2}-\frac{ab^3}{d^2}-\frac{a^4\,1i}{4d^2}+\frac{a^2b^2\,3i}{2d^2}}\,192i}{\frac{16a^6}{d}-\frac{16b^6}{d}+\frac{112a^2b^4}{d}-\frac{112a^4b^2}{d}+\frac{ab^5\,32i}{d}+\frac{a^5b\,32i}{d}-\frac{a^3b^3\,192i}{d}}\right)\sqrt{-\frac{a^4\,1i-4a^3b-a^2b^2\,6i+4ab^3+b^4\,1i}{4d^2}}\,2i+\operatorname{atan}\left(\frac{a^4\sqrt{\tan(c+dx)}\sqrt{\frac{a^4\,1i}{4d^2}+\frac{b^4\,1i}{4d^2}-\frac{ab^3}{d^2}+\frac{a^3b}{d^2}-\frac{a^2b^2\,3i}{2d^2}}\,32i}{\frac{16b^6}{d}-\frac{16a^6}{d}-\frac{112a^2b^4}{d}+\frac{112a^4b^2}{d}+\frac{ab^5\,32i}{d}+\frac{a^5b\,32i}{d}-\frac{a^3b^3\,192i}{d}}+\frac{b^4\sqrt{\tan(c+dx)}\sqrt{\frac{a^4\,1i}{4d^2}+\frac{b^4\,1i}{4d^2}-\frac{ab^3}{d^2}+\frac{a^3b}{d^2}-\frac{a^2b^2\,3i}{2d^2}}\,32i}{\frac{16b^6}{d}-\frac{16a^6}{d}-\frac{112a^2b^4}{d}+\frac{112a^4b^2}{d}+\frac{ab^5\,32i}{d}+\frac{a^5b\,32i}{d}-\frac{a^3b^3\,192i}{d}}-\frac{a^2b^2\sqrt{\tan(c+dx)}\sqrt{\frac{a^4\,1i}{4d^2}+\frac{b^4\,1i}{4d^2}-\frac{ab^3}{d^2}+\frac{a^3b}{d^2}-\frac{a^2b^2\,3i}{2d^2}}\,192i}{\frac{16b^6}{d}-\frac{16a^6}{d}-\frac{112a^2b^4}{d}+\frac{112a^4b^2}{d}+\frac{ab^5\,32i}{d}+\frac{a^5b\,32i}{d}-\frac{a^3b^3\,192i}{d}}\right)\sqrt{\frac{a^4\,1i+4a^3b-a^2b^2\,6i-4ab^3+b^4\,1i}{4d^2}}\,2i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(5/2)*(a + b*tan(c + d*x))^2,x)
```

```
[Out] tan(c + d*x)^(3/2)*((2*a^2)/(3*d) - (2*b^2)/(3*d)) - atan((a^4*tan(c + d*x)
^(1/2)*((a^3*b)/d^2 - (b^4*1i)/(4*d^2) - (a*b^3)/d^2 - (a^4*1i)/(4*d^2) + (
a^2*b^2*3i)/(2*d^2))^(1/2)*32i)/((16*a^6)/d - (16*b^6)/d + (a*b^5*32i)/d +
(a^5*b*32i)/d + (112*a^2*b^4)/d - (a^3*b^3*192i)/d - (112*a^4*b^2)/d) + (b^
4*tan(c + d*x)^(1/2)*((a^3*b)/d^2 - (b^4*1i)/(4*d^2) - (a*b^3)/d^2 - (a^4*1
i)/(4*d^2) + (a^2*b^2*3i)/(2*d^2))^(1/2)*32i)/((16*a^6)/d - (16*b^6)/d + (a
*b^5*32i)/d + (a^5*b*32i)/d + (112*a^2*b^4)/d - (a^3*b^3*192i)/d - (112*a^4
*b^2)/d) - (a^2*b^2*tan(c + d*x)^(1/2)*((a^3*b)/d^2 - (b^4*1i)/(4*d^2) - (a
*b^3)/d^2 - (a^4*1i)/(4*d^2) + (a^2*b^2*3i)/(2*d^2))^(1/2)*192i)/((16*a^6)/
d - (16*b^6)/d + (a*b^5*32i)/d + (a^5*b*32i)/d + (112*a^2*b^4)/d - (a^3*b^3
*192i)/d - (112*a^4*b^2)/d))*(-(4*a*b^3 - 4*a^3*b + a^4*1i + b^4*1i - a^2*b
^2*6i)/(4*d^2))^(1/2)*2i + atan((a^4*tan(c + d*x)^(1/2)*((a^4*1i)/(4*d^2) +
 (b^4*1i)/(4*d^2) - (a*b^3)/d^2 + (a^3*b)/d^2 - (a^2*b^2*3i)/(2*d^2))^(1/2)
*32i)/((16*b^6)/d - (16*a^6)/d + (a*b^5*32i)/d + (a^5*b*32i)/d - (112*a^2*b
^4)/d - (a^3*b^3*192i)/d + (112*a^4*b^2)/d) + (b^4*tan(c + d*x)^(1/2)*((a^4
*1i)/(4*d^2) + (b^4*1i)/(4*d^2) - (a*b^3)/d^2 + (a^3*b)/d^2 - (a^2*b^2*3i)/
(2*d^2))^(1/2)*32i)/((16*b^6)/d - (16*a^6)/d + (a*b^5*32i)/d + (a^5*b*32i)/
d - (112*a^2*b^4)/d - (a^3*b^3*192i)/d + (112*a^4*b^2)/d) - (a^2*b^2*tan(c
+ d*x)^(1/2)*((a^4*1i)/(4*d^2) + (b^4*1i)/(4*d^2) - (a*b^3)/d^2 + (a^3*b)/d
^2 - (a^2*b^2*3i)/(2*d^2))^(1/2)*192i)/((16*b^6)/d - (16*a^6)/d + (a*b^5*32
i)/d + (a^5*b*32i)/d - (112*a^2*b^4)/d - (a^3*b^3*192i)/d + (112*a^4*b^2)/d
))*((4*a^3*b - 4*a*b^3 + a^4*1i + b^4*1i - a^2*b^2*6i)/(4*d^2))^(1/2)*2i +
(2*b^2*tan(c + d*x)^(7/2))/(7*d) - (4*a*b*tan(c + d*x)^(1/2))/d + (4*a*b*ta
n(c + d*x)^(5/2))/(5*d)
```

3.563 $\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=249

$$\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a^2 -}{}$$

```
[Out] -1/2*(a^2+2*a*b-b^2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)-1/2*(a^2
+2*a*b-b^2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)+1/4*(a^2-2*a*b-b^2
)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)-1/4*(a^2-2*a*b-b^2)*l
n(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)+2*(a^2-b^2)*tan(d*x+c)^(
1/2)/d+4/3*a*b*tan(d*x+c)^(3/2)/d+2/5*b^2*tan(d*x+c)^(5/2)/d
```

Rubi [A]

time = 0.16, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3624, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d}+\frac{2(a^2-b^2)\sqrt{\tan(c+dx)}}{d}+\frac{(a^2-2ab-b^2)\log\left(\tan(c+dx)-\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{(a^2-2ab-b^2)\log\left(\tan(c+dx)+\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{4ab\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{2b^2\tan^{\frac{5}{2}}(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((a^2 + 2*a*b - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) -
((a^2 + 2*a*b - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) +
((a^2 - 2*a*b - b^2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2
*Sqrt[2]*d) - ((a^2 - 2*a*b - b^2)*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan
[c + d*x]])/(2*Sqrt[2]*d) + (2*(a^2 - b^2)*Sqrt[Tan[c + d*x]])/d + (4*a*b*T
an[c + d*x]^(3/2))/(3*d) + (2*b^2*Tan[c + d*x]^(5/2))/(5*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{2b^2\tan^{\frac{5}{2}}(c+dx)}{5d} + \int \tan^{\frac{3}{2}}(c+dx)\left(a^2-b^2+2ab\tan(c+dx)\right)\,dx$$

$$= \frac{4ab\tan^{\frac{3}{2}}(c+dx)}{3d} + \frac{2b^2\tan^{\frac{5}{2}}(c+dx)}{5d} + \int \sqrt{\tan(c+dx)}\,(-2ab+$$

$$= \frac{2(a^2-b^2)\sqrt{\tan(c+dx)}}{d} + \frac{4ab\tan^{\frac{3}{2}}(c+dx)}{3d} + \frac{2b^2\tan^{\frac{5}{2}}(c+dx)}{5d}$$

$$= \frac{2(a^2-b^2)\sqrt{\tan(c+dx)}}{d} + \frac{4ab\tan^{\frac{3}{2}}(c+dx)}{3d} + \frac{2b^2\tan^{\frac{5}{2}}(c+dx)}{5d}$$

$$= \frac{2(a^2-b^2)\sqrt{\tan(c+dx)}}{d} + \frac{4ab\tan^{\frac{3}{2}}(c+dx)}{3d} + \frac{2b^2\tan^{\frac{5}{2}}(c+dx)}{5d}$$

$$= \frac{2(a^2-b^2)\sqrt{\tan(c+dx)}}{d} + \frac{4ab\tan^{\frac{3}{2}}(c+dx)}{3d} + \frac{2b^2\tan^{\frac{5}{2}}(c+dx)}{5d}$$

$$= \frac{(a^2-2ab-b^2)\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}\,d} - \frac{(a^2}{}$$

$$= \frac{(a^2+2ab-b^2)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d} - \frac{(a^2+2ab-b^2)}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.73, size = 120, normalized size = 0.48

$$\frac{15\sqrt[4]{-1}\,(a-ib)^2\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+15\sqrt[4]{-1}\,(a+ib)^2\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+2\sqrt{\tan(c+dx)}\,(15a^2-15b^2+10ab\tan(c+dx)+3b^2\tan^2(c+dx))}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (15*(-1)^(1/4)*(a - I*b)^2*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + 15*(-1)^
(1/4)*(a + I*b)^2*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + 2*Sqrt[Tan[c + d
*x]]*(15*a^2 - 15*b^2 + 10*a*b*Tan[c + d*x] + 3*b^2*Tan[c + d*x]^2))/(15*d)
```

Maple [A]
time = 0.05, size = 238, normalized size = 0.96

method	result

derivativedivides	$\frac{\frac{2b^2\left(\tan^{\frac{5}{2}}(dx+c)\right)}{5}+\frac{4ab\left(\tan^{\frac{3}{2}}(dx+c)\right)}{3}+2a^2\left(\sqrt{\tan}(dx+c)\right)-2b^2\left(\sqrt{\tan}(dx+c)\right)+\frac{(-a^2+b^2)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}}{1-\sqrt{2}}\right.\right.}{}}{}$
default	$\frac{\frac{2b^2\left(\tan^{\frac{5}{2}}(dx+c)\right)}{5}+\frac{4ab\left(\tan^{\frac{3}{2}}(dx+c)\right)}{3}+2a^2\left(\sqrt{\tan}(dx+c)\right)-2b^2\left(\sqrt{\tan}(dx+c)\right)+\frac{(-a^2+b^2)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}}{1-\sqrt{2}}\right.\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/5*b^2*tan(d*x+c)^(5/2)+4/3*a*b*tan(d*x+c)^(3/2)+2*a^2*tan(d*x+c)^(1/
2)-2*b^2*tan(d*x+c)^(1/2)+1/4*(-a^2+b^2)*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^
(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/
2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))-1/2*a*b*2^(1/2)
*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+ta
n(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x
+c)^(1/2))))
```

Maxima [A]
time = 0.52, size = 205, normalized size = 0.82

$$\frac{24\,b^2\tan(dx+c)^{\frac{5}{2}}+80\,ab\tan(dx+c)^{\frac{3}{2}}-30\sqrt{2}\,(a^2+2\,ab-b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)-30\sqrt{2}\,(a^2+2\,ab-b^2)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-15\sqrt{2}\,(a^2-2\,ab-b^2)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+15\sqrt{2}\,(a^2-2\,ab-b^2)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+120\,(a^2-b^2)\sqrt{\tan(dx+c)}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/60*(24*b^2*tan(d*x + c)^(5/2) + 80*a*b*tan(d*x + c)^(3/2) - 30*sqrt(2)*(a
^2 + 2*a*b - b^2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) - 30
*sqrt(2)*(a^2 + 2*a*b - b^2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x
+ c)))) - 15*sqrt(2)*(a^2 - 2*a*b - b^2)*log(sqrt(2)*sqrt(tan(d*x + c)) + t
an(d*x + c) + 1) + 15*sqrt(2)*(a^2 - 2*a*b - b^2)*log(-sqrt(2)*sqrt(tan(d*x
 + c)) + tan(d*x + c) + 1) + 120*(a^2 - b^2)*sqrt(tan(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5043 vs. $2(211) = 422$.
time = 1.08, size = 5043, normalized size = 20.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

[Out] -1/60*(60*sqrt(2)*d^5*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(3/4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4)*arctan(-((a^16 - 20*a^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 - 64*a^6*b^10 - 20*a^4*b^12 + b^16)*d^4*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4) + sqrt(2)*((a^2 - b^2)*d^7*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4) - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^5*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(((a^12 - 10*a^10*b^2 + 15*a^8*b^4 + 52*a^6*b^6 + 15*a^4*b^8 - 10*a^2*b^10 + b^12)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*cos(d*x + c) + sqrt(2)*(2*(a^9*b - 12*a^7*b^3 + 38*a^5*b^5 - 12*a^3*b^7 + a*b^9)*d^3*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*cos(d*x + c) - (a^14 - 11*a^12*b^2 + 25*a^10*b^4 + 37*a^8*b^6 - 37*a^6*b^8 - 25*a^4*b^10 + 11*a^2*b^12 - b^14)*d*cos(d*x + c))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(1/4) + (a^16 - 8*a^14*b^2 - 4*a^12*b^4 + 72*a^10*b^6 + 134*a^8*b^8 + 72*a^6*b^10 - 4*a^4*b^12 - 8*a^2*b^14 + b^16)*sin(d*x + c))/cos(d*x + c))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(3/4) - sqrt(2)*((a^10 - 5*a^8*b^2 - 6*a^6*b^4 + 6*a^4*b^6 + 5*a^2*b^8 - b^10)*d^7*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4) - 2*(a^13*b - 2*a^11*b^3 - 17*a^9*b^5 - 28*a^7*b^7 - 17*a^5*b^9 - 2*a^3*b^11 + a*b^13)*d^5*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(3/4))/(a^24 - 4*a^22*b^2 - 30*a^20*b^4 + 12*a^18*b^6 + 367*a^16*b^8 + 1016*a^14*b^10 + 1372*a^12*b^12 + 1016*a^10*b^14 + 367*a^8*b^16 + 12*a^6*b^18 - 30*a^4*b^20 - 4*a^2*b^22 + b^24))*cos(d*x + c)^2 + 60*sqrt(2)*d^5*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(3/4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4)*arctan(((a^16 - 20*a^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 - 64*a^6*b^10 - 20*a^4*b^12 + b^16)*d^4*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4) - sqrt(2)*((a^2 - b^2)*d^7*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d

```
^4) - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^5*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4
 - 12*a^2*b^6 + b^8)/d^4))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 +
b^8 + 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 +
 b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(((a^12
 - 10*a^10*b^2 + 15*a^8*b^4 + 52*a^6*b^6 + 15*a^4*b^8 - 10*a^2*b^10 + b^12)
*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*cos(d*x + c)
 - sqrt(2)*(2*(a^9*b - 12*a^7*b^3 + 38*a^5*b^5 - 12*a^3*b^7 + a*b^9)*d^3*sq
rt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*cos(d*x + c) - (a^1
4 - 11*a^12*b^2 + 25*a^10*b^4 + 37*a^8*b^6 - 37*a^6*b^8 - 25*a^4*b^10 + 11*
a^2*b^12 - b^14)*d*cos(d*x + c))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*
b^6 + b^8 + 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2
*b^6 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(
sin(d*x + c)/cos(d*x + c))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)
/d^4)^(1/4) + (a^16 - 8*a^14*b^2 - 4*a^12*b^4 + 72*a^10*b^6 + 134*a^8*b^8 +
 72*a^6*b^10 - 4*a^4*b^12 - 8*a^2*b^14 + b^16)*sin(d*x + c))/cos(d*x + c))*
((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(3/4) + sqrt(2)*((a^1
0 - 5*a^8*b^2 - 6*a^6*b^4 + 6*a^4*b^6 + 5*a^2*b^8 - b^10)*d^7*sqrt((a^8 + 4
*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^
4*b^4 - 12*a^2*b^6 + b^8)/d^4) - 2*(a^13*b - 2*a^11*b^3 - 17*a^9*b^5 - 28*a
^7*b^7 - 17*a^5*b^9 - 2*a^3*b^11 + a*b^13)*d^5*sqrt((a^8 - 12*a^6*b^2 + 38*
a^4*b^4 - 12*a^2*b^6 + b^8)/d^4))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2
*b^6 + b^8 + 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + ...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan{(c + dx)})^2 \tan^{\frac{3}{2}}{(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(3/2)*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*tan(c + d*x)**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 5.76, size = 986, normalized size = 3.96

$$\sqrt{\tan(c+dx)}\left(\frac{2a^2}{d}-\frac{2b^2}{d}\right)+\frac{2b^2\tan(c+dx)^{5/2}}{5d}+\frac{4ab\tan(c+dx)^{3/2}}{3d}-\mathrm{atan}\left(\frac{a^4\sqrt{\tan(c+dx)}\sqrt{\frac{ab^3}{d^2}-\frac{b^4 1i}{4d^2}-\frac{a^4 1i}{4d^2}-\frac{a^3b}{d^2}+\frac{a^2b^2 3i}{2d^2}}\,32i}{\frac{a^6 16i}{d}-\frac{b^6 16i}{d}+\frac{32ab^5}{d}+\frac{32a^5b}{d}+\frac{a^2b^4 112i}{d}-\frac{192a^3b^3}{d}-\frac{a^4b^2 112i}{d}}+\frac{b^4\sqrt{\tan(c+dx)}\sqrt{\frac{ab^3}{d^2}-\frac{b^4 1i}{4d^2}-\frac{a^4 1i}{4d^2}-\frac{a^3b}{d^2}+\frac{a^2b^2 3i}{2d^2}}\,32i}{\frac{a^6 16i}{d}-\frac{b^6 16i}{d}+\frac{32ab^5}{d}+\frac{32a^5b}{d}+\frac{a^2b^4 112i}{d}-\frac{192a^3b^3}{d}-\frac{a^4b^2 112i}{d}}-\frac{a^2b^2\sqrt{\tan(c+dx)}\sqrt{\frac{ab^3}{d^2}-\frac{b^4 1i}{4d^2}-\frac{a^4 1i}{4d^2}-\frac{a^3b}{d^2}+\frac{a^2b^2 3i}{2d^2}}\,192i}{\frac{a^6 16i}{d}-\frac{b^6 16i}{d}+\frac{32ab^5}{d}+\frac{32a^5b}{d}+\frac{a^2b^4 112i}{d}-\frac{192a^3b^3}{d}-\frac{a^4b^2 112i}{d}}\right)\sqrt{-\frac{a^4 1i+4a^3b-a^2b^2 6i-4ab^3+b^4 1i}{4d^2}}\,2i-\mathrm{atan}\left(\frac{a^4\sqrt{\tan(c+dx)}\sqrt{\frac{a^4 1i}{4d^2}+\frac{b^4 1i}{4d^2}+\frac{ab^3}{d^2}-\frac{a^3b}{d^2}-\frac{a^2b^2 3i}{2d^2}}\,32i}{\frac{32ab^5}{d}+\frac{b^6 16i}{d}-\frac{a^6 16i}{d}+\frac{32a^5b}{d}-\frac{a^2b^4 112i}{d}-\frac{192a^3b^3}{d}+\frac{a^4b^2 112i}{d}}+\frac{b^4\sqrt{\tan(c+dx)}\sqrt{\frac{a^4 1i}{4d^2}+\frac{b^4 1i}{4d^2}+\frac{ab^3}{d^2}-\frac{a^3b}{d^2}-\frac{a^2b^2 3i}{2d^2}}\,32i}{\frac{32ab^5}{d}+\frac{b^6 16i}{d}-\frac{a^6 16i}{d}+\frac{32a^5b}{d}-\frac{a^2b^4 112i}{d}-\frac{192a^3b^3}{d}+\frac{a^4b^2 112i}{d}}-\frac{a^2b^2\sqrt{\tan(c+dx)}\sqrt{\frac{a^4 1i}{4d^2}+\frac{b^4 1i}{4d^2}+\frac{ab^3}{d^2}-\frac{a^3b}{d^2}-\frac{a^2b^2 3i}{2d^2}}\,192i}{\frac{32ab^5}{d}+\frac{b^6 16i}{d}-\frac{a^6 16i}{d}+\frac{32a^5b}{d}-\frac{a^2b^4 112i}{d}-\frac{192a^3b^3}{d}+\frac{a^4b^2 112i}{d}}\right)\sqrt{\frac{a^4 1i-4a^3b-a^2b^2 6i+4ab^3+b^4 1i}{4d^2}}\,2i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x))^2,x)
```

```
[Out] tan(c + d*x)^(1/2)*((2*a^2)/d - (2*b^2)/d) - atan((a^4*tan(c + d*x)^(1/2)*(
(a*b^3)/d^2 - (b^4*1i)/(4*d^2) - (a^4*1i)/(4*d^2) - (a^3*b)/d^2 + (a^2*b^2*
3i)/(2*d^2))^(1/2)*32i)/((a^6*16i)/d - (b^6*16i)/d + (32*a*b^5)/d + (32*a^5
*b)/d + (a^2*b^4*112i)/d - (192*a^3*b^3)/d - (a^4*b^2*112i)/d) + (b^4*tan(c
 + d*x)^(1/2)*((a*b^3)/d^2 - (b^4*1i)/(4*d^2) - (a^4*1i)/(4*d^2) - (a^3*b)/
d^2 + (a^2*b^2*3i)/(2*d^2))^(1/2)*32i)/((a^6*16i)/d - (b^6*16i)/d + (32*a*b
^5)/d + (32*a^5*b)/d + (a^2*b^4*112i)/d - (192*a^3*b^3)/d - (a^4*b^2*112i)/
d) - (a^2*b^2*tan(c + d*x)^(1/2)*((a*b^3)/d^2 - (b^4*1i)/(4*d^2) - (a^4*1i)
/(4*d^2) - (a^3*b)/d^2 + (a^2*b^2*3i)/(2*d^2))^(1/2)*192i)/((a^6*16i)/d - (
b^6*16i)/d + (32*a*b^5)/d + (32*a^5*b)/d + (a^2*b^4*112i)/d - (192*a^3*b^3)
/d - (a^4*b^2*112i)/d))*(-(4*a^3*b - 4*a*b^3 + a^4*1i + b^4*1i - a^2*b^2*6i
)/(4*d^2))^(1/2)*2i - atan((a^4*tan(c + d*x)^(1/2)*((a^4*1i)/(4*d^2) + (b^4
*1i)/(4*d^2) + (a*b^3)/d^2 - (a^3*b)/d^2 - (a^2*b^2*3i)/(2*d^2))^(1/2)*32i)
/((b^6*16i)/d - (a^6*16i)/d + (32*a*b^5)/d + (32*a^5*b)/d - (a^2*b^4*112i)/
d - (192*a^3*b^3)/d + (a^4*b^2*112i)/d) + (b^4*tan(c + d*x)^(1/2)*((a^4*1i)
/(4*d^2) + (b^4*1i)/(4*d^2) + (a*b^3)/d^2 - (a^3*b)/d^2 - (a^2*b^2*3i)/(2*d
^2))^(1/2)*32i)/((b^6*16i)/d - (a^6*16i)/d + (32*a*b^5)/d + (32*a^5*b)/d -
(a^2*b^4*112i)/d - (192*a^3*b^3)/d + (a^4*b^2*112i)/d) - (a^2*b^2*tan(c + d
*x)^(1/2)*((a^4*1i)/(4*d^2) + (b^4*1i)/(4*d^2) + (a*b^3)/d^2 - (a^3*b)/d^2
- (a^2*b^2*3i)/(2*d^2))^(1/2)*192i)/((b^6*16i)/d - (a^6*16i)/d + (32*a*b^5)
/d + (32*a^5*b)/d - (a^2*b^4*112i)/d - (192*a^3*b^3)/d + (a^4*b^2*112i)/d))
*((4*a*b^3 - 4*a^3*b + a^4*1i + b^4*1i - a^2*b^2*6i)/(4*d^2))^(1/2)*2i + (2
*b^2*tan(c + d*x)^(5/2))/(5*d) + (4*a*b*tan(c + d*x)^(3/2))/(3*d)
```

3.564 $\int \sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=223

$$-\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(\cdots}{}$$

```
[Out] 1/2*(a^2-2*a*b-b^2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)+1/2*(a^2-
2*a*b-b^2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)+1/4*(a^2+2*a*b-b^2)
*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)-1/4*(a^2+2*a*b-b^2)*ln
(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)+4*a*b*tan(d*x+c)^(1/2)/d+
2/3*b^2*tan(d*x+c)^(3/2)/d
```

Rubi [A]

time = 0.13, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3624, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d}+\frac{(a^2+2ab-b^2)\log\left(\tan(c+dx)-\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{(a^2+2ab-b^2)\log\left(\tan(c+dx)+\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{4ab\sqrt{\tan(c+dx)}}{d}+\frac{2b^2\tan^{\frac{3}{2}}(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((a^2 - 2*a*b - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d))
 + ((a^2 - 2*a*b - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d)
 + ((a^2 + 2*a*b - b^2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])
/(2*Sqrt[2]*d) - ((a^2 + 2*a*b - b^2)*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] +
Tan[c + d*x]])/(2*Sqrt[2]*d) + (4*a*b*Sqrt[Tan[c + d*x]])/d + (2*b^2*Tan[c
+ d*x]^(3/2))/(3*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^2\,dx = \frac{2b^2\tan^{\frac{3}{2}}(c+dx)}{3d} + \int \sqrt{\tan(c+dx)}\,(a^2-b^2+2ab\tan(c+dx$$

$$= \frac{4ab\sqrt{\tan(c+dx)}}{d} + \frac{2b^2\tan^{\frac{3}{2}}(c+dx)}{3d} + \int \frac{-2ab+(a^2-b^2)\,t}{\sqrt{\tan(c+}}$$

$$= \frac{4ab\sqrt{\tan(c+dx)}}{d} + \frac{2b^2\tan^{\frac{3}{2}}(c+dx)}{3d} + \frac{2\text{Subst}\left(\int \frac{-2ab+(a^2-b^2}{1+x^4}\right.}{}$$

$$= \frac{4ab\sqrt{\tan(c+dx)}}{d} + \frac{2b^2\tan^{\frac{3}{2}}(c+dx)}{3d} + \frac{(a^2-2ab-b^2)\,\text{Subst}}{}$$

$$= \frac{4ab\sqrt{\tan(c+dx)}}{d} + \frac{2b^2\tan^{\frac{3}{2}}(c+dx)}{3d} + \frac{(a^2-2ab-b^2)\,\text{Subst}}{}$$

$$= \frac{(a^2+2ab-b^2)\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}\,d} -$$

$$= -\frac{(a^2-2ab-b^2)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d} + \frac{(a^2-2ab}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.29, size = 99, normalized size = 0.44

$$\frac{3(-1)^{3/4}(a-ib)^2\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)-3(-1)^{3/4}(a+ib)^2\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+2b\sqrt{\tan(c+dx)}\,(6a+b\tan(c+dx))}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^2,x]
[Out] (3*(-1)^(3/4)*(a - I*b)^2*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] - 3*(-1)^(3
/4)*(a + I*b)^2*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + 2*b*Sqrt[Tan[c + d
*x]]*(6*a + b*Tan[c + d*x]))/(3*d)
```

Maple [A]
time = 0.06, size = 212, normalized size = 0.95

method	result
derivativedivides	$\frac{\frac{2b^2\left(\tan^{\frac{3}{2}}(dx+c)\right)}{3}+4ab\left(\sqrt{\tan}(dx+c)\right)-\frac{ab\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\right.\right.\right.}{2}}{}$

default	$\frac{\frac{2b^2\left(\tan^{\frac{3}{2}}(dx+c)\right)}{3}+4ab\left(\sqrt{\tan}(dx+c)\right)-\frac{ab\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{t\right.\right.}{2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/3*b^2*tan(d*x+c)^(3/2)+4*a*b*tan(d*x+c)^(1/2)-1/2*a*b*2^(1/2)*(ln((1
+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c
)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/
2)))+1/4*(a^2-b^2)*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2
^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2
*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))))
```

Maxima [A]
time = 0.50, size = 186, normalized size = 0.83

$$\frac{8\,b^2\tan(dx+c)^{\frac{3}{2}}+6\sqrt{2}\,(a^2-2\,ab-b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+6\sqrt{2}\,(a^2-2\,ab-b^2)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-3\sqrt{2}\,(a^2+2\,ab-b^2)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+3\sqrt{2}\,(a^2+2\,ab-b^2)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+48\,ab\sqrt{\tan(dx+c)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/12*(8*b^2*tan(d*x + c)^(3/2) + 6*sqrt(2)*(a^2 - 2*a*b - b^2)*arctan(1/2*s
qrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 6*sqrt(2)*(a^2 - 2*a*b - b^2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - 3*sqrt(2)*(a^2 + 2*a*
b - b^2)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 3*sqrt(2)*(a^
2 + 2*a*b - b^2)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 48*a
*b*sqrt(tan(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4967 vs. $2(189) = 378$.
time = 2.70, size = 4967, normalized size = 22.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/12*(12*sqrt(2)*d^5*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 -
 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)
/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*((a^8 + 4*a^6*b^
2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(3/4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4
*b^4 - 12*a^2*b^6 + b^8)/d^4)*arctan(-((a^16 - 20*a^12*b^4 - 64*a^10*b^6 -
90*a^8*b^8 - 64*a^6*b^10 - 20*a^4*b^12 + b^16)*d^4*sqrt((a^8 + 4*a^6*b^2 +
```

6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4) - sqrt(2)*(2*a*b*d^7*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4) + (a^6 + a^4*b^2 - a^2*b^4 - b^6)*d^5*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 - 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(((a^12 - 10*a^10*b^2 + 15*a^8*b^4 + 52*a^6*b^6 + 15*a^4*b^8 - 10*a^2*b^10 + b^12)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*cos(d*x + c) + sqrt(2)*((a^10 - 13*a^8*b^2 + 50*a^6*b^4 - 50*a^4*b^6 + 13*a^2*b^8 - b^10)*d^3*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*cos(d*x + c) + 2*(a^13*b - 10*a^11*b^3 + 15*a^9*b^5 + 52*a^7*b^7 + 15*a^5*b^9 - 10*a^3*b^11 + a*b^13)*d*cos(d*x + c))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 - 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(1/4) + (a^16 - 8*a^14*b^2 - 4*a^12*b^4 + 72*a^10*b^6 + 134*a^8*b^8 + 72*a^6*b^10 - 4*a^4*b^12 - 8*a^2*b^14 + b^16)*sin(d*x + c))/cos(d*x + c))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(3/4) + sqrt(2)*(2*(a^9*b - 4*a^7*b^3 - 10*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d^7*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4) + (a^14 - 3*a^12*b^2 - 15*a^10*b^4 - 11*a^8*b^6 + 11*a^6*b^8 + 15*a^4*b^10 + 3*a^2*b^12 - b^14)*d^5*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 - 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(3/4))/(a^24 - 4*a^22*b^2 - 30*a^20*b^4 + 12*a^18*b^6 + 367*a^16*b^8 + 1016*a^14*b^10 + 1372*a^12*b^12 + 1016*a^10*b^14 + 367*a^8*b^16 + 12*a^6*b^18 - 30*a^4*b^20 - 4*a^2*b^22 + b^24))*cos(d*x + c) + 12*sqrt(2)*d^5*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 - 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(3/4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4)*arctan(((a^16 - 20*a^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 - 64*a^6*b^10 - 20*a^4*b^12 + b^16)*d^4*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4) + sqrt(2)*(2*a*b*d^7*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4) + (a^6 + a^4*b^2 - a^2*b^4 - b^6)*d^5*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 - 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(((a^12 - 10*a^10*b^2 + 15*a^8*b^4 + 52*a^6*b^6 + 15*a^4*b^8 - 10*a^2*b^10 + b^12)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*cos(d*x + c) - sqrt(

```
2)*((a^10 - 13*a^8*b^2 + 50*a^6*b^4 - 50*a^4*b^6 + 13*a^2*b^8 - b^10)*d^3*s
qrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*cos(d*x + c) + 2*(
a^13*b - 10*a^11*b^3 + 15*a^9*b^5 + 52*a^7*b^7 + 15*a^5*b^9 - 10*a^3*b^11 +
 a*b^13)*d*cos(d*x + c))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^
8 - 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b
^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(sin(d*x
+ c)/cos(d*x + c))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(1
/4) + (a^16 - 8*a^14*b^2 - 4*a^12*b^4 + 72*a^10*b^6 + 134*a^8*b^8 + 72*a^6*
b^10 - 4*a^4*b^12 - 8*a^2*b^14 + b^16)*sin(d*x + c))/cos(d*x + c))*((a^8 +
4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(3/4) - sqrt(2)*(2*(a^9*b - 4
*a^7*b^3 - 10*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d^7*sqrt((a^8 + 4*a^6*b^2 + 6*a^
4*b^4 + 4*a^2*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*
b^6 + b^8)/d^4) + (a^14 - 3*a^12*b^2 - 15*a^10*b^4 - 11*a^8*b^6 + 11*a^6*b^
8 + 15*a^4*b^10 + 3*a^2*b^12 - b^14)*d^5*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^
4 - 12*a^2*b^6 + b^8)/d^4))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 +
 b^8 - 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tan (c + dx))^2 \sqrt{\tan (c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/2)*(a+b*tan(d*x+c))**2,x)
[Out] Integral((a + b*tan(c + d*x))**2*sqrt(tan(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^2,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^2*sqrt(tan(d*x + c)), x)
```

Mupad [B]
time = 4.87, size = 954, normalized size = 4.28

$$\frac{2 b^2 \tan(c+d x)^{3/2}}{3 d}+\frac{4 a b \sqrt{\tan(c+d x)}}{d}+\mathrm{atan}\left(\frac{a^4 \sqrt{\tan(c+d x)} \sqrt{\frac{a^3 b}{d^2}-\frac{b^4 1i}{4 d^2}-\frac{a b^3}{d^2}-\frac{a^4 1i}{4 d^2}+\frac{a^2 b^2 3i}{2 d^2}}\, 32i}{\frac{16 a^6}{d}-\frac{16 b^6}{d}+\frac{112 a^2 b^4}{d}-\frac{112 a^4 b^2}{d}+\frac{a b^5 32i}{d}+\frac{a^5 b 32i}{d}-\frac{a^3 b^3 192i}{d}}+\frac{b^4 \sqrt{\tan(c+d x)} \sqrt{\frac{a^3 b}{d^2}-\frac{b^4 1i}{4 d^2}-\frac{a b^3}{d^2}-\frac{a^4 1i}{4 d^2}+\frac{a^2 b^2 3i}{2 d^2}}\, 32i}{\frac{16 a^6}{d}-\frac{16 b^6}{d}+\frac{112 a^2 b^4}{d}-\frac{112 a^4 b^2}{d}+\frac{a b^5 32i}{d}+\frac{a^5 b 32i}{d}-\frac{a^3 b^3 192i}{d}}-\frac{a^2 b^2 \sqrt{\tan(c+d x)} \sqrt{\frac{a^3 b}{d^2}-\frac{b^4 1i}{4 d^2}-\frac{a b^3}{d^2}-\frac{a^4 1i}{4 d^2}+\frac{a^2 b^2 3i}{2 d^2}}\, 192i}{\frac{16 a^6}{d}-\frac{16 b^6}{d}+\frac{112 a^2 b^4}{d}-\frac{112 a^4 b^2}{d}+\frac{a b^5 32i}{d}+\frac{a^5 b 32i}{d}-\frac{a^3 b^3 192i}{d}}\right) \sqrt{-\frac{a^4 1i-4 a^3 b-a^2 b^2 6i+4 a b^3+b^4 1i}{4 d^2}}\, 2i-\mathrm{atan}\left(\frac{a^4 \sqrt{\tan(c+d x)} \sqrt{\frac{a^4 1i}{4 d^2}+\frac{b^4 1i}{4 d^2}-\frac{a b^3}{d^2}+\frac{a^3 b}{d^2}-\frac{a^2 b^2 3i}{2 d^2}}\, 32i}{\frac{16 b^6}{d}-\frac{16 a^6}{d}-\frac{112 a^2 b^4}{d}+\frac{112 a^4 b^2}{d}+\frac{a b^5 32i}{d}+\frac{a^5 b 32i}{d}-\frac{a^3 b^3 192i}{d}}+\frac{b^4 \sqrt{\tan(c+d x)} \sqrt{\frac{a^4 1i}{4 d^2}+\frac{b^4 1i}{4 d^2}-\frac{a b^3}{d^2}+\frac{a^3 b}{d^2}-\frac{a^2 b^2 3i}{2 d^2}}\, 32i}{\frac{16 b^6}{d}-\frac{16 a^6}{d}-\frac{112 a^2 b^4}{d}+\frac{112 a^4 b^2}{d}+\frac{a b^5 32i}{d}+\frac{a^5 b 32i}{d}-\frac{a^3 b^3 192i}{d}}-\frac{a^2 b^2 \sqrt{\tan(c+d x)} \sqrt{\frac{a^4 1i}{4 d^2}+\frac{b^4 1i}{4 d^2}-\frac{a b^3}{d^2}+\frac{a^3 b}{d^2}-\frac{a^2 b^2 3i}{2 d^2}}\, 192i}{\frac{16 b^6}{d}-\frac{16 a^6}{d}-\frac{112 a^2 b^4}{d}+\frac{112 a^4 b^2}{d}+\frac{a b^5 32i}{d}+\frac{a^5 b 32i}{d}-\frac{a^3 b^3 192i}{d}}\right) \sqrt{\frac{a^4 1i+4 a^3 b-a^2 b^2 6i-4 a b^3+b^4 1i}{4 d^2}}\, 2i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)*(a + b*tan(c + d*x))^2,x)
```

```
[Out] atan((a^4*tan(c + d*x)^(1/2)*((a^3*b)/d^2 - (b^4*1i)/(4*d^2) - (a*b^3)/d^2
- (a^4*1i)/(4*d^2) + (a^2*b^2*3i)/(2*d^2))^(1/2)*32i)/((16*a^6)/d - (16*b^6
)/d + (a*b^5*32i)/d + (a^5*b*32i)/d + (112*a^2*b^4)/d - (a^3*b^3*192i)/d -
(112*a^4*b^2)/d) + (b^4*tan(c + d*x)^(1/2)*((a^3*b)/d^2 - (b^4*1i)/(4*d^2)
- (a*b^3)/d^2 - (a^4*1i)/(4*d^2) + (a^2*b^2*3i)/(2*d^2))^(1/2)*32i)/((16*a^
6)/d - (16*b^6)/d + (a*b^5*32i)/d + (a^5*b*32i)/d + (112*a^2*b^4)/d - (a^3*
b^3*192i)/d - (112*a^4*b^2)/d) - (a^2*b^2*tan(c + d*x)^(1/2)*((a^3*b)/d^2 -
 (b^4*1i)/(4*d^2) - (a*b^3)/d^2 - (a^4*1i)/(4*d^2) + (a^2*b^2*3i)/(2*d^2))^
(1/2)*192i)/((16*a^6)/d - (16*b^6)/d + (a*b^5*32i)/d + (a^5*b*32i)/d + (112
*a^2*b^4)/d - (a^3*b^3*192i)/d - (112*a^4*b^2)/d))*(-(4*a*b^3 - 4*a^3*b + a
^4*1i + b^4*1i - a^2*b^2*6i)/(4*d^2))^(1/2)*2i - atan((a^4*tan(c + d*x)^(1/
2)*((a^4*1i)/(4*d^2) + (b^4*1i)/(4*d^2) - (a*b^3)/d^2 + (a^3*b)/d^2 - (a^2*
b^2*3i)/(2*d^2))^(1/2)*32i)/((16*b^6)/d - (16*a^6)/d + (a*b^5*32i)/d + (a^5
*b*32i)/d - (112*a^2*b^4)/d - (a^3*b^3*192i)/d + (112*a^4*b^2)/d) + (b^4*ta
n(c + d*x)^(1/2)*((a^4*1i)/(4*d^2) + (b^4*1i)/(4*d^2) - (a*b^3)/d^2 + (a^3*
b)/d^2 - (a^2*b^2*3i)/(2*d^2))^(1/2)*32i)/((16*b^6)/d - (16*a^6)/d + (a*b^5
*32i)/d + (a^5*b*32i)/d - (112*a^2*b^4)/d - (a^3*b^3*192i)/d + (112*a^4*b^2
)/d) - (a^2*b^2*tan(c + d*x)^(1/2)*((a^4*1i)/(4*d^2) + (b^4*1i)/(4*d^2) - (
a*b^3)/d^2 + (a^3*b)/d^2 - (a^2*b^2*3i)/(2*d^2))^(1/2)*192i)/((16*b^6)/d -
(16*a^6)/d + (a*b^5*32i)/d + (a^5*b*32i)/d - (112*a^2*b^4)/d - (a^3*b^3*192
i)/d + (112*a^4*b^2)/d))*((4*a^3*b - 4*a*b^3 + a^4*1i + b^4*1i - a^2*b^2*6i
)/(4*d^2))^(1/2)*2i + (2*b^2*tan(c + d*x)^(3/2))/(3*d) + (4*a*b*tan(c + d*x
)^(1/2))/d
```

3.565 $\int \frac{(a+b\tan(c+dx))^2}{\sqrt{\tan(c+dx)}}\,dx$

Optimal. Leaf size=204

$$-\frac{(a^2+2ab-b^2)\,\mathrm{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a^2+2ab-b^2)\,\mathrm{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a^2$$

```
[Out] 1/2*(a^2+2*a*b-b^2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)+1/2*(a^2+
2*a*b-b^2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)-1/4*(a^2-2*a*b-b^2)
*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)+1/4*(a^2-2*a*b-b^2)*ln
(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)+2*b^2*tan(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.11, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3624, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{(a^2+2ab-b^2)\,\mathrm{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a^2+2ab-b^2)\,\mathrm{ArcTan}\left(\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d}-\frac{(a^2-2ab-b^2)\log\left(\tan(c+dx)-\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{(a^2-2ab-b^2)\log\left(\tan(c+dx)+\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{2b^2\sqrt{\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^2/Sqrt[Tan[c + d*x]],x]
```

```
[Out] -(((a^2 + 2*a*b - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d))
 + ((a^2 + 2*a*b - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d)
 - ((a^2 - 2*a*b - b^2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])
/(2*Sqrt[2]*d) + ((a^2 - 2*a*b - b^2)*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] +
Tan[c + d*x]])/(2*Sqrt[2]*d) + (2*b^2*Sqrt[Tan[c + d*x]])/d
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \frac{(a+b\tan(c+dx))^2}{\sqrt{\tan(c+dx)}}\,dx = \frac{2b^2\sqrt{\tan(c+dx)}}{d} + \int \frac{a^2-b^2+2ab\tan(c+dx)}{\sqrt{\tan(c+dx)}}\,dx$$
$$= \frac{2b^2\sqrt{\tan(c+dx)}}{d} + \frac{2\text{Subst}\left(\int \frac{a^2-b^2+2abx^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d}$$
$$= \frac{2b^2\sqrt{\tan(c+dx)}}{d} + \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d} + \frac{(a}{}$$
$$= \frac{2b^2\sqrt{\tan(c+dx)}}{d} - \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(c+dx}\right.}{2\sqrt{2}d}$$
$$= -\frac{(a^2-2ab-b^2)\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d} + \frac{(a^2-2ab-b}{}$$
$$= -\frac{(a^2+2ab-b^2)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} + \frac{(a^2+2ab-b^2)\tan^{-1}\left(1-\right.}{\sqrt{2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.12, size = 85, normalized size = 0.42

$$-\frac{\sqrt[4]{-1}(a-ib)^2\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+\sqrt[4]{-1}(a+ib)^2\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)-2b^2\sqrt{\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^2/Sqrt[Tan[c + d*x]],x]
```

```
[Out] -(((-1)^(1/4)*(a - I*b)^2*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + (-1)^(1/4
)*(a + I*b)^2*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] - 2*b^2*Sqrt[Tan[c + d
*x]])/d)
```

Maple [A]
time = 0.06, size = 200, normalized size = 0.98

method	result
derivativedivides	$2b^2\left(\sqrt{\tan}(dx+c)\right)+\frac{(a^2-b^2)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)\right)\right.}{4}$

default	$\frac{2b^2\left(\sqrt{\tan}\,(dx+c)\right)+\frac{(a^2-b^2)\sqrt{2}\,\left(\ln\left(\frac{1+\sqrt{2}\,\left(\sqrt{\tan}\,(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\,\left(\sqrt{\tan}\,(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{2}\,\left(\sqrt{\tan}\,(dx+c)\right.\right.\right.}{4}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^2/tan(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b^2*tan(d*x+c)^(1/2)+1/4*(a^2-b^2)*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)
^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1
/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/2*a*b*2^(1/2
)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+t
an(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*
x+c)^(1/2))))
```

Maxima [A]
time = 0.50, size = 173, normalized size = 0.85

$$\frac{2\sqrt{2}\,(a^2+2\,ab-b^2)\arctan\left(\frac{1}{2}\sqrt{2}\,\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}\,(a^2+2\,ab-b^2)\arctan\left(-\frac{1}{2}\sqrt{2}\,\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)+\sqrt{2}\,(a^2-2\,ab-b^2)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)-\sqrt{2}\,(a^2-2\,ab-b^2)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+8\,b^2\sqrt{\tan(dx+c)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^2/tan(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*(a^2 + 2*a*b - b^2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan
(d*x + c)))) + 2*sqrt(2)*(a^2 + 2*a*b - b^2)*arctan(-1/2*sqrt(2)*(sqrt(2) -
 2*sqrt(tan(d*x + c)))) + sqrt(2)*(a^2 - 2*a*b - b^2)*log(sqrt(2)*sqrt(tan(
d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*(a^2 - 2*a*b - b^2)*log(-sqrt(2)*sq
rt(tan(d*x + c)) + tan(d*x + c) + 1) + 8*b^2*sqrt(tan(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4881 vs. $2(174) = 348$.
time = 1.07, size = 4881, normalized size = 23.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^2/tan(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*d^5*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 4*
(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^
4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*((a^8 + 4*a^6*b^2 +
 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(3/4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^
4 - 12*a^2*b^6 + b^8)/d^4)*arctan(-((a^16 - 20*a^12*b^4 - 64*a^10*b^6 - 90*
a^8*b^8 - 64*a^6*b^10 - 20*a^4*b^12 + b^16)*d^4*sqrt((a^8 + 4*a^6*b^2 + 6*a
^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2
```

b^6 + b^8)/d^4) + sqrt(2)((a^2 - b^2)*d^7*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b
^4 + 4*a^2*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6
 + b^8)/d^4) - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^5*sqrt((a^8 - 12*a^6*b^2 + 3
8*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a
^2*b^6 + b^8 + 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*
a^2*b^6 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sq
rt(((a^12 - 10*a^10*b^2 + 15*a^8*b^4 + 52*a^6*b^6 + 15*a^4*b^8 - 10*a^2*b^1
0 + b^12)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*cos
(d*x + c) + sqrt(2)*(2*(a^9*b - 12*a^7*b^3 + 38*a^5*b^5 - 12*a^3*b^7 + a*b^
9)*d^3*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*cos(d*x +
c) - (a^14 - 11*a^12*b^2 + 25*a^10*b^4 + 37*a^8*b^6 - 37*a^6*b^8 - 25*a^4*b
^10 + 11*a^2*b^12 - b^14)*d*cos(d*x + c))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4
 + 4*a^2*b^6 + b^8 + 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^
4 + 4*a^2*b^6 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^
8))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b
^6 + b^8)/d^4)^(1/4) + (a^16 - 8*a^14*b^2 - 4*a^12*b^4 + 72*a^10*b^6 + 134*
a^8*b^8 + 72*a^6*b^10 - 4*a^4*b^12 - 8*a^2*b^14 + b^16)*sin(d*x + c))/cos(d
x + c))((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(3/4) - sqrt
(2)*((a^10 - 5*a^8*b^2 - 6*a^6*b^4 + 6*a^4*b^6 + 5*a^2*b^8 - b^10)*d^7*sqrt
((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a^6*b^
2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4) - 2*(a^13*b - 2*a^11*b^3 - 17*a^9*b
^5 - 28*a^7*b^7 - 17*a^5*b^9 - 2*a^3*b^11 + a*b^13)*d^5*sqrt((a^8 - 12*a^6*
b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^
4 + 4*a^2*b^6 + b^8 + 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b
^4 + 4*a^2*b^6 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b
^8))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*
b^6 + b^8)/d^4)^(3/4))/(a^24 - 4*a^22*b^2 - 30*a^20*b^4 + 12*a^18*b^6 + 367
*a^16*b^8 + 1016*a^14*b^10 + 1372*a^12*b^12 + 1016*a^10*b^14 + 367*a^8*b^16
 + 12*a^6*b^18 - 30*a^4*b^20 - 4*a^2*b^22 + b^24)) + 4*sqrt(2)*d^5*sqrt((a^
8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 4*(a^3*b - a*b^3)*d^2*sqrt((a
^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*
a^4*b^4 - 12*a^2*b^6 + b^8))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^
8)/d^4)^(3/4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4)*
arctan(((a^16 - 20*a^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 - 64*a^6*b^10 - 20*a
^4*b^12 + b^16)*d^4*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^
4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4) - sqrt(2)*(
(a^2 - b^2)*d^7*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*s
qrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4) - 2*(a^5*b + 2*
a^3*b^3 + a*b^5)*d^5*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8
)/d^4))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 4*(a^3*b - a*
b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4))/(a^8 -
12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(((a^12 - 10*a^10*b^2 + 15
*a^8*b^4 + 52*a^6*b^6 + 15*a^4*b^8 - 10*a^2*b^10 + b^12)*d^2*sqrt((a^8 + 4*
a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*cos(d*x + c) - sqrt(2)*(2*(a^9*
b - 12*a^7*b^3 + 38*a^5*b^5 - 12*a^3*b^7 + a*b^9)*d^3*sqrt((a^8 + 4*a^6*b^2

```
 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*cos(d*x + c) - (a^14 - 11*a^12*b^2 + 2
5*a^10*b^4 + 37*a^8*b^6 - 37*a^6*b^8 - 25*a^4*b^10 + 11*a^2*b^12 - b^14)*d*
cos(d*x + c))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 4*(a^3*
b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4))/(
a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(sin(d*x + c)/cos(d*
x + c))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(1/4) + (a^16
 - 8*a^14*b^2 - 4*a^12*b^4 + 72*a^10*b^6 + 134*a^8*b^8 + 72*a^6*b^10 - 4*a^
4*b^12 - 8*a^2*b^14 + b^16)*sin(d*x + c))/cos(d*x + c))*((a^8 + 4*a^6*b^2 +
 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(3/4) + sqrt(2)*((a^10 - 5*a^8*b^2 - 6*a
^6*b^4 + 6*a^4*b^6 + 5*a^2*b^8 - b^10)*d^7*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^
4 + 4*a^2*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6
+ b^8)/d^4) - 2*(a^13*b - 2*a^11*b^3 - 17*a^9*b^5 - 28*a^7*b^7 - 17*a^5*b^9
 - 2*a^3*b^11 + a*b^13)*d^5*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^
6 + b^8)/d^4))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 4*(a^3
*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(c+dx))^2}{\sqrt{\tan(c+dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**2/tan(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2/sqrt(tan(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^2/tan(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 4.54, size = 937, normalized size = 4.59

$$\frac{2\,b^2\,\sqrt{\tan(c+d\,x)}}{d}-2\,\mathrm{atanh}\left(\frac{32\,a^4\,\sqrt{\tan(c+d\,x)}\,\sqrt{\frac{a^4\,1\mathrm{i}}{4\,d^2}+\frac{b^4\,1\mathrm{i}}{4\,d^2}+\frac{a\,b^3}{d^2}-\frac{a^3\,b}{d^2}-\frac{a^2\,b^2\,3\mathrm{i}}{2\,d^2}}}{\frac{32\,a\,b^5}{d}+\frac{b^6\,16\mathrm{i}}{d}-\frac{a^6\,16\mathrm{i}}{d}+\frac{32\,a^5\,b}{d}-\frac{a^2\,b^4\,112\mathrm{i}}{d}-\frac{192\,a^3\,b^3}{d}+\frac{a^4\,b^2\,112\mathrm{i}}{d}}+\frac{32\,b^4\,\sqrt{\tan(c+d\,x)}\,\sqrt{\frac{a^4\,1\mathrm{i}}{4\,d^2}+\frac{b^4\,1\mathrm{i}}{4\,d^2}+\frac{a\,b^3}{d^2}-\frac{a^3\,b}{d^2}-\frac{a^2\,b^2\,3\mathrm{i}}{2\,d^2}}}{\frac{32\,a\,b^5}{d}+\frac{b^6\,16\mathrm{i}}{d}-\frac{a^6\,16\mathrm{i}}{d}+\frac{32\,a^5\,b}{d}-\frac{a^2\,b^4\,112\mathrm{i}}{d}-\frac{192\,a^3\,b^3}{d}+\frac{a^4\,b^2\,112\mathrm{i}}{d}}-\frac{192\,a^2\,b^2\,\sqrt{\tan(c+d\,x)}\,\sqrt{\frac{a^4\,1\mathrm{i}}{4\,d^2}+\frac{b^4\,1\mathrm{i}}{4\,d^2}+\frac{a\,b^3}{d^2}-\frac{a^3\,b}{d^2}-\frac{a^2\,b^2\,3\mathrm{i}}{2\,d^2}}}{\frac{32\,a\,b^5}{d}+\frac{b^6\,16\mathrm{i}}{d}-\frac{a^6\,16\mathrm{i}}{d}+\frac{32\,a^5\,b}{d}-\frac{a^2\,b^4\,112\mathrm{i}}{d}-\frac{192\,a^3\,b^3}{d}+\frac{a^4\,b^2\,112\mathrm{i}}{d}}\right)\,\sqrt{\frac{a^4\,1\mathrm{i}-4\,a^3\,b-a^2\,b^2\,6\mathrm{i}+4\,a\,b^3+b^4\,1\mathrm{i}}{4\,d^2}}-2\,\mathrm{atanh}\left(\frac{32\,a^4\,\sqrt{\tan(c+d\,x)}\,\sqrt{\frac{a\,b^3}{d^2}-\frac{b^4\,1\mathrm{i}}{4\,d^2}-\frac{a^4\,1\mathrm{i}}{4\,d^2}-\frac{a^3\,b}{d^2}+\frac{a^2\,b^2\,3\mathrm{i}}{2\,d^2}}}{\frac{a^6\,16\mathrm{i}}{d}-\frac{b^6\,16\mathrm{i}}{d}+\frac{32\,a\,b^5}{d}+\frac{32\,a^5\,b}{d}+\frac{a^2\,b^4\,112\mathrm{i}}{d}-\frac{192\,a^3\,b^3}{d}-\frac{a^4\,b^2\,112\mathrm{i}}{d}}+\frac{32\,b^4\,\sqrt{\tan(c+d\,x)}\,\sqrt{\frac{a\,b^3}{d^2}-\frac{b^4\,1\mathrm{i}}{4\,d^2}-\frac{a^4\,1\mathrm{i}}{4\,d^2}-\frac{a^3\,b}{d^2}+\frac{a^2\,b^2\,3\mathrm{i}}{2\,d^2}}}{\frac{a^6\,16\mathrm{i}}{d}-\frac{b^6\,16\mathrm{i}}{d}+\frac{32\,a\,b^5}{d}+\frac{32\,a^5\,b}{d}+\frac{a^2\,b^4\,112\mathrm{i}}{d}-\frac{192\,a^3\,b^3}{d}-\frac{a^4\,b^2\,112\mathrm{i}}{d}}-\frac{192\,a^2\,b^2\,\sqrt{\tan(c+d\,x)}\,\sqrt{\frac{a\,b^3}{d^2}-\frac{b^4\,1\mathrm{i}}{4\,d^2}-\frac{a^4\,1\mathrm{i}}{4\,d^2}-\frac{a^3\,b}{d^2}+\frac{a^2\,b^2\,3\mathrm{i}}{2\,d^2}}}{\frac{a^6\,16\mathrm{i}}{d}-\frac{b^6\,16\mathrm{i}}{d}+\frac{32\,a\,b^5}{d}+\frac{32\,a^5\,b}{d}+\frac{a^2\,b^4\,112\mathrm{i}}{d}-\frac{192\,a^3\,b^3}{d}-\frac{a^4\,b^2\,112\mathrm{i}}{d}}\right)\,\sqrt{-\frac{a^4\,1\mathrm{i}+4\,a^3\,b-a^2\,b^2\,6\mathrm{i}-4\,a\,b^3+b^4\,1\mathrm{i}}{4\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^2/tan(c + d*x)^(1/2),x)
```

```
[Out] (2*b^2*tan(c + d*x)^(1/2))/d - 2*atanh((32*a^4*tan(c + d*x)^(1/2)*((a^4*1i)
/(4*d^2) + (b^4*1i)/(4*d^2) + (a*b^3)/d^2 - (a^3*b)/d^2 - (a^2*b^2*3i)/(2*d
```

```
^2))^(1/2))/((b^6*16i)/d - (a^6*16i)/d + (32*a*b^5)/d + (32*a^5*b)/d - (a^2
*b^4*112i)/d - (192*a^3*b^3)/d + (a^4*b^2*112i)/d) + (32*b^4*tan(c + d*x)^(
1/2)*((a^4*1i)/(4*d^2) + (b^4*1i)/(4*d^2) + (a*b^3)/d^2 - (a^3*b)/d^2 - (a^
2*b^2*3i)/(2*d^2))^(1/2))/((b^6*16i)/d - (a^6*16i)/d + (32*a*b^5)/d + (32*a
^5*b)/d - (a^2*b^4*112i)/d - (192*a^3*b^3)/d + (a^4*b^2*112i)/d) - (192*a^2
*b^2*tan(c + d*x)^(1/2)*((a^4*1i)/(4*d^2) + (b^4*1i)/(4*d^2) + (a*b^3)/d^2
- (a^3*b)/d^2 - (a^2*b^2*3i)/(2*d^2))^(1/2))/((b^6*16i)/d - (a^6*16i)/d + (
32*a*b^5)/d + (32*a^5*b)/d - (a^2*b^4*112i)/d - (192*a^3*b^3)/d + (a^4*b^2*
112i)/d))*((4*a*b^3 - 4*a^3*b + a^4*1i + b^4*1i - a^2*b^2*6i)/(4*d^2))^(1/2
) - 2*atanh((32*a^4*tan(c + d*x)^(1/2)*((a*b^3)/d^2 - (b^4*1i)/(4*d^2) - (a
^4*1i)/(4*d^2) - (a^3*b)/d^2 + (a^2*b^2*3i)/(2*d^2))^(1/2))/((a^6*16i)/d -
(b^6*16i)/d + (32*a*b^5)/d + (32*a^5*b)/d + (a^2*b^4*112i)/d - (192*a^3*b^3
)/d - (a^4*b^2*112i)/d) + (32*b^4*tan(c + d*x)^(1/2)*((a*b^3)/d^2 - (b^4*1i
)/(4*d^2) - (a^4*1i)/(4*d^2) - (a^3*b)/d^2 + (a^2*b^2*3i)/(2*d^2))^(1/2))/(
(a^6*16i)/d - (b^6*16i)/d + (32*a*b^5)/d + (32*a^5*b)/d + (a^2*b^4*112i)/d
- (192*a^3*b^3)/d - (a^4*b^2*112i)/d) - (192*a^2*b^2*tan(c + d*x)^(1/2)*((a
*b^3)/d^2 - (b^4*1i)/(4*d^2) - (a^4*1i)/(4*d^2) - (a^3*b)/d^2 + (a^2*b^2*3i
)/(2*d^2))^(1/2))/((a^6*16i)/d - (b^6*16i)/d + (32*a*b^5)/d + (32*a^5*b)/d
+ (a^2*b^4*112i)/d - (192*a^3*b^3)/d - (a^4*b^2*112i)/d))*(-(4*a^3*b - 4*a*
b^3 + a^4*1i + b^4*1i - a^2*b^2*6i)/(4*d^2))^(1/2)
```

3.566 $\int \frac{(a+b\tan(c+dx))^2}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=204

$$\frac{(a^2-2ab-b^2)\,\mathrm{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a^2-2ab-b^2)\,\mathrm{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a^2}{}$$

```
[Out] -1/2*(a^2-2*a*b-b^2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)-1/2*(a^2
-2*a*b-b^2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)-1/4*(a^2+2*a*b-b^2
)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)+1/4*(a^2+2*a*b-b^2)*l
n(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)-2*a^2/d/tan(d*x+c)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3623, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a^2-2ab-b^2)\,\mathrm{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a^2-2ab-b^2)\,\mathrm{ArcTan}\left(\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d}-\frac{(a^2+2ab-b^2)\log\left(\tan(c+dx)-\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{(a^2+2ab-b^2)\log\left(\tan(c+dx)+\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{2a^2}{d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^2/Tan[c + d*x]^(3/2),x]
```

```
[Out] ((a^2 - 2*a*b - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) -
((a^2 - 2*a*b - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) -
((a^2 + 2*a*b - b^2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2
*Sqrt[2]*d) + ((a^2 + 2*a*b - b^2)*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan
[c + d*x]])/(2*Sqrt[2]*d) - (2*a^2)/(d*Sqrt[Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+b\tan(c+dx))^2}{\tan^{\frac{3}{2}}(c+dx)}\,dx &= -\frac{2a^2}{d\sqrt{\tan(c+dx)}} + \int \frac{2ab-(a^2-b^2)\tan(c+dx)}{\sqrt{\tan(c+dx)}}\,dx \\
&= -\frac{2a^2}{d\sqrt{\tan(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{2ab+(-a^2+b^2)x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d} \\
&= -\frac{2a^2}{d\sqrt{\tan(c+dx)}} - \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d} + \\
&= -\frac{2a^2}{d\sqrt{\tan(c+dx)}} - \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right.}{2d} \\
&= -\frac{(a^2+2ab-b^2)\log\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}\,d} + \frac{(a^2+2ab-}{} \\
&= \frac{(a^2-2ab-b^2)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d} - \frac{(a^2-2ab-b^2)\tan^{-1}\left(1+\right.}{\sqrt{2}\,d}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.02, size = 166, normalized size = 0.81

$$
-\frac{\sqrt{2}\,ab\left(2\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)-2\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}\right)+\log\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}+\tan(c+dx)\right)-\log\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\right)+\frac{4b^2}{\sqrt{\tan(c+dx)}}+\frac{4(a-b)(a+b)\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\tan^2(c+dx)\right)}{\sqrt{\tan(c+dx)}}}{2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^2/Tan[c + d*x]^(3/2),x]
```

```
[Out] -1/2*(Sqrt[2]*a*b*(2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 2*ArcTan[1 +
Sqrt[2]*Sqrt[Tan[c + d*x]]] + Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]) + (4*b^2)/Sqrt[
Tan[c + d*x]] + (4*(a - b)*(a + b)*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[c +
 d*x]^2])/Sqrt[Tan[c + d*x]])/d
```

Maple [A]
time = 0.06, size = 200, normalized size = 0.98

method	result

derivativedivides	$\frac{-\frac{2a^2}{\sqrt{\tan(dx+c)}}+\frac{ab\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)\right)+2\,\mathrm{ar}}{2}}{}$
default	$\frac{-\frac{2a^2}{\sqrt{\tan(dx+c)}}+\frac{ab\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)\right)+2\,\mathrm{ar}}{2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^2/tan(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*a^2/tan(d*x+c)^(1/2)+1/2*a*b*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2
)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*t
an(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(-a^2+b^2)*2^(1
/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)
+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(
d*x+c)^(1/2))))
```

Maxima [A]
time = 0.53, size = 173, normalized size = 0.85

$$-\frac{2\sqrt{2}(a^2-2ab-b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}(a^2-2ab-b^2)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-\sqrt{2}(a^2+2ab-b^2)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+\sqrt{2}(a^2+2ab-b^2)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+\frac{8a^2}{\sqrt{\tan(dx+c)}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^2/tan(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*(a^2 - 2*a*b - b^2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(ta
n(d*x + c)))) + 2*sqrt(2)*(a^2 - 2*a*b - b^2)*arctan(-1/2*sqrt(2)*(sqrt(2)
- 2*sqrt(tan(d*x + c)))) - sqrt(2)*(a^2 + 2*a*b - b^2)*log(sqrt(2)*sqrt(tan
(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*(a^2 + 2*a*b - b^2)*log(-sqrt(2)*s
qrt(tan(d*x + c)) + tan(d*x + c) + 1) + 8*a^2/sqrt(tan(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5093 vs. $2(174) = 348$.
time = 2.14, size = 5093, normalized size = 24.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^2/tan(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*(d^5*cos(d*x + c)^2 - d^5)*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4
 + 4*a^2*b^6 + b^8 - 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^
```

4 + 4*a^2*b^6 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(3/4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4)*arctan(-((a^16 - 20*a^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 - 64*a^6*b^10 - 20*a^4*b^12 + b^16)*d^4*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4) - sqrt(2)*(2*a*b*d^7*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4) + (a^6 + a^4*b^2 - a^2*b^4 - b^6)*d^5*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 - 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(((a^12 - 10*a^10*b^2 + 15*a^8*b^4 + 52*a^6*b^6 + 15*a^4*b^8 - 10*a^2*b^10 + b^12)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*cos(d*x + c) + sqrt(2)*((a^10 - 13*a^8*b^2 + 50*a^6*b^4 - 50*a^4*b^6 + 13*a^2*b^8 - b^10)*d^3*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*cos(d*x + c) + 2*(a^13*b - 10*a^11*b^3 + 15*a^9*b^5 + 52*a^7*b^7 + 15*a^5*b^9 - 10*a^3*b^11 + a*b^13)*d*cos(d*x + c))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 - 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(1/4) + (a^16 - 8*a^14*b^2 - 4*a^12*b^4 + 72*a^10*b^6 + 134*a^8*b^8 + 72*a^6*b^10 - 4*a^4*b^12 - 8*a^2*b^14 + b^16)*sin(d*x + c))/cos(d*x + c))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(3/4) + sqrt(2)*(2*(a^9*b - 4*a^7*b^3 - 10*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d^7*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4) + (a^14 - 3*a^12*b^2 - 15*a^10*b^4 - 11*a^8*b^6 + 11*a^6*b^8 + 15*a^4*b^10 + 3*a^2*b^12 - b^14)*d^5*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 - 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(3/4))/(a^24 - 4*a^22*b^2 - 30*a^20*b^4 + 12*a^18*b^6 + 367*a^16*b^8 + 1016*a^14*b^10 + 1372*a^12*b^12 + 1016*a^10*b^14 + 367*a^8*b^16 + 12*a^6*b^18 - 30*a^4*b^20 - 4*a^2*b^22 + b^24)) + 4*sqrt(2)*(d^5*cos(d*x + c)^2 - d^5)*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 - 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(3/4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4)*arctan(((a^16 - 20*a^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 - 64*a^6*b^10 - 20*a^4*b^12 + b^16)*d^4*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4) + sqrt(2)*(2*a*b*d^7*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4) + (a^6 + a^4*b^2 - a^2*b^4 - b^6)*d^5*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4))*sqrt((a^8 + 4*a^6*b^2 + 6*a

```
^4*b^4 + 4*a^2*b^6 + b^8 - 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*
a^4*b^4 + 4*a^2*b^6 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^
6 + b^8))*sqrt(((a^12 - 10*a^10*b^2 + 15*a^8*b^4 + 52*a^6*b^6 + 15*a^4*b^8
- 10*a^2*b^10 + b^12)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b
^8)/d^4)*cos(d*x + c) - sqrt(2)*((a^10 - 13*a^8*b^2 + 50*a^6*b^4 - 50*a^4*b
^6 + 13*a^2*b^8 - b^10)*d^3*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 +
 b^8)/d^4)*cos(d*x + c) + 2*(a^13*b - 10*a^11*b^3 + 15*a^9*b^5 + 52*a^7*b^7
 + 15*a^5*b^9 - 10*a^3*b^11 + a*b^13)*d*cos(d*x + c))*sqrt((a^8 + 4*a^6*b^2
 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 - 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^
2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*
a^2*b^6 + b^8))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^8 + 4*a^6*b^2 + 6*a^4*b
^4 + 4*a^2*b^6 + b^8)/d^4)^(1/4) + (a^16 - 8*a^14*b^2 - 4*a^12*b^4 + 72*a^1
0*b^6 + 134*a^8*b^8 + 72*a^6*b^10 - 4*a^4*b^12 - 8*a^2*b^14 + b^16)*sin(d*x
 + c))/cos(d*x + c))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^
(3/4) - sqrt(2)*(2*(a^9*b - 4*a^7*b^3 - 10*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d^7
*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a
^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4) + (a^14 - 3*a^12*b^2 - 15*a^10
*b^4 - 11*a^8*b^6 + 11*a^6*b^8 + 15*a^4*b^10 + 3*a^2*b^12 - b^14)*d^5*sqrt(
(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4))*sqrt((a^8 + 4*a^6*
b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 - 4*(a^3*b - ...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan{(c+dx)})^2}{\tan^{\frac{3}{2}}{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**2/tan(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2/tan(c + d*x)**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^2/tan(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 4.49, size = 949, normalized size = 4.65

$$2\,\mathrm{atanh}\left(\frac{32\,a^4\,d^3\,\sqrt{\tan(c+d\,x)}\,\sqrt{\frac{a^3\,b}{d^2}-\frac{b^4\,1{}\mathrm{i}}{4\,d^2}-\frac{a\,b^3}{d^2}-\frac{a^4\,1{}\mathrm{i}}{4\,d^2}+\frac{a^2\,b^2\,3{}\mathrm{i}}{2\,d^2}}}{16\,a^6\,d^2+a^5\,b\,d^2\,32{}\mathrm{i}-112\,a^4\,b^2\,d^2-a^3\,b^3\,d^2\,192{}\mathrm{i}+112\,a^2\,b^4\,d^2+a\,b^5\,d^2\,32{}\mathrm{i}-16\,b^6\,d^2}+\frac{32\,b^4\,d^3\,\sqrt{\tan(c+d\,x)}\,\sqrt{\frac{a^3\,b}{d^2}-\frac{b^4\,1{}\mathrm{i}}{4\,d^2}-\frac{a\,b^3}{d^2}-\frac{a^4\,1{}\mathrm{i}}{4\,d^2}+\frac{a^2\,b^2\,3{}\mathrm{i}}{2\,d^2}}}{16\,a^6\,d^2+a^5\,b\,d^2\,32{}\mathrm{i}-112\,a^4\,b^2\,d^2-a^3\,b^3\,d^2\,192{}\mathrm{i}+112\,a^2\,b^4\,d^2+a\,b^5\,d^2\,32{}\mathrm{i}-16\,b^6\,d^2}-\frac{192\,a^2\,b^2\,d^3\,\sqrt{\tan(c+d\,x)}\,\sqrt{\frac{a^3\,b}{d^2}-\frac{b^4\,1{}\mathrm{i}}{4\,d^2}-\frac{a\,b^3}{d^2}-\frac{a^4\,1{}\mathrm{i}}{4\,d^2}+\frac{a^2\,b^2\,3{}\mathrm{i}}{2\,d^2}}}{16\,a^6\,d^2+a^5\,b\,d^2\,32{}\mathrm{i}-112\,a^4\,b^2\,d^2-a^3\,b^3\,d^2\,192{}\mathrm{i}+112\,a^2\,b^4\,d^2+a\,b^5\,d^2\,32{}\mathrm{i}-16\,b^6\,d^2}\right)\sqrt{-\frac{a^4\,1{}\mathrm{i}-4\,a^3\,b-a^2\,b^2\,6{}\mathrm{i}+4\,a\,b^3+b^4\,1{}\mathrm{i}}{4\,d^2}}-2\,\mathrm{atanh}\left(\frac{32\,a^4\,d^3\,\sqrt{\tan(c+d\,x)}\,\sqrt{\frac{a^4\,1{}\mathrm{i}}{4\,d^2}+\frac{b^4\,1{}\mathrm{i}}{4\,d^2}-\frac{a\,b^3}{d^2}+\frac{a^3\,b}{d^2}-\frac{a^2\,b^2\,3{}\mathrm{i}}{2\,d^2}}}{-16\,a^6\,d^2+a^5\,b\,d^2\,32{}\mathrm{i}+112\,a^4\,b^2\,d^2-a^3\,b^3\,d^2\,192{}\mathrm{i}-112\,a^2\,b^4\,d^2+a\,b^5\,d^2\,32{}\mathrm{i}+16\,b^6\,d^2}+\frac{32\,b^4\,d^3\,\sqrt{\tan(c+d\,x)}\,\sqrt{\frac{a^4\,1{}\mathrm{i}}{4\,d^2}+\frac{b^4\,1{}\mathrm{i}}{4\,d^2}-\frac{a\,b^3}{d^2}+\frac{a^3\,b}{d^2}-\frac{a^2\,b^2\,3{}\mathrm{i}}{2\,d^2}}}{-16\,a^6\,d^2+a^5\,b\,d^2\,32{}\mathrm{i}+112\,a^4\,b^2\,d^2-a^3\,b^3\,d^2\,192{}\mathrm{i}-112\,a^2\,b^4\,d^2+a\,b^5\,d^2\,32{}\mathrm{i}+16\,b^6\,d^2}-\frac{192\,a^2\,b^2\,d^3\,\sqrt{\tan(c+d\,x)}\,\sqrt{\frac{a^4\,1{}\mathrm{i}}{4\,d^2}+\frac{b^4\,1{}\mathrm{i}}{4\,d^2}-\frac{a\,b^3}{d^2}+\frac{a^3\,b}{d^2}-\frac{a^2\,b^2\,3{}\mathrm{i}}{2\,d^2}}}{-16\,a^6\,d^2+a^5\,b\,d^2\,32{}\mathrm{i}+112\,a^4\,b^2\,d^2-a^3\,b^3\,d^2\,192{}\mathrm{i}-112\,a^2\,b^4\,d^2+a\,b^5\,d^2\,32{}\mathrm{i}+16\,b^6\,d^2}\right)\sqrt{\frac{a^4\,1{}\mathrm{i}+4\,a^3\,b-a^2\,b^2\,6{}\mathrm{i}-4\,a\,b^3+b^4\,1{}\mathrm{i}}{4\,d^2}}-\frac{2\,a^2}{d\,\sqrt{\tan(c+d\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^2/tan(c + d*x)^(3/2),x)
```

```
[Out] 2*atanh((32*a^4*d^3*tan(c + d*x)^(1/2)*((a^3*b)/d^2 - (b^4*1i)/(4*d^2) - (a
*b^3)/d^2 - (a^4*1i)/(4*d^2) + (a^2*b^2*3i)/(2*d^2))^(1/2))/(16*a^6*d^2 - 1
6*b^6*d^2 + a*b^5*d^2*32i + a^5*b*d^2*32i + 112*a^2*b^4*d^2 - a^3*b^3*d^2*1
92i - 112*a^4*b^2*d^2) + (32*b^4*d^3*tan(c + d*x)^(1/2)*((a^3*b)/d^2 - (b^4
*1i)/(4*d^2) - (a*b^3)/d^2 - (a^4*1i)/(4*d^2) + (a^2*b^2*3i)/(2*d^2))^(1/2)
)/(16*a^6*d^2 - 16*b^6*d^2 + a*b^5*d^2*32i + a^5*b*d^2*32i + 112*a^2*b^4*d^
2 - a^3*b^3*d^2*192i - 112*a^4*b^2*d^2) - (192*a^2*b^2*d^3*tan(c + d*x)^(1/
2)*((a^3*b)/d^2 - (b^4*1i)/(4*d^2) - (a*b^3)/d^2 - (a^4*1i)/(4*d^2) + (a^2*
b^2*3i)/(2*d^2))^(1/2))/(16*a^6*d^2 - 16*b^6*d^2 + a*b^5*d^2*32i + a^5*b*d^
2*32i + 112*a^2*b^4*d^2 - a^3*b^3*d^2*192i - 112*a^4*b^2*d^2))*(-(4*a*b^3 -
 4*a^3*b + a^4*1i + b^4*1i - a^2*b^2*6i)/(4*d^2))^(1/2) - 2*atanh((32*a^4*d
^3*tan(c + d*x)^(1/2)*((a^4*1i)/(4*d^2) + (b^4*1i)/(4*d^2) - (a*b^3)/d^2 +
(a^3*b)/d^2 - (a^2*b^2*3i)/(2*d^2))^(1/2))/(16*b^6*d^2 - 16*a^6*d^2 + a*b^5
*d^2*32i + a^5*b*d^2*32i - 112*a^2*b^4*d^2 - a^3*b^3*d^2*192i + 112*a^4*b^2
*d^2) + (32*b^4*d^3*tan(c + d*x)^(1/2)*((a^4*1i)/(4*d^2) + (b^4*1i)/(4*d^2)
 - (a*b^3)/d^2 + (a^3*b)/d^2 - (a^2*b^2*3i)/(2*d^2))^(1/2))/(16*b^6*d^2 - 1
6*a^6*d^2 + a*b^5*d^2*32i + a^5*b*d^2*32i - 112*a^2*b^4*d^2 - a^3*b^3*d^2*1
92i + 112*a^4*b^2*d^2) - (192*a^2*b^2*d^3*tan(c + d*x)^(1/2)*((a^4*1i)/(4*d
^2) + (b^4*1i)/(4*d^2) - (a*b^3)/d^2 + (a^3*b)/d^2 - (a^2*b^2*3i)/(2*d^2))^
(1/2))/(16*b^6*d^2 - 16*a^6*d^2 + a*b^5*d^2*32i + a^5*b*d^2*32i - 112*a^2*b
^4*d^2 - a^3*b^3*d^2*192i + 112*a^4*b^2*d^2))*((4*a^3*b - 4*a*b^3 + a^4*1i
+ b^4*1i - a^2*b^2*6i)/(4*d^2))^(1/2) - (2*a^2)/(d*tan(c + d*x)^(1/2))
```

3.567 $\int \frac{(a+b\tan(c+dx))^2}{\tan^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=223

$$\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a^2-$$

```
[Out] -1/2*(a^2+2*a*b-b^2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)-1/2*(a^2
+2*a*b-b^2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)+1/4*(a^2-2*a*b-b^2
)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)-1/4*(a^2-2*a*b-b^2)*l
n(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)-4*a*b/d/tan(d*x+c)^(1/2)
-2/3*a^2/d/tan(d*x+c)^(3/2)
```

Rubi [A]
time = 0.13, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3623, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d}+\frac{(a^2-2ab-b^2)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{(a^2-2ab-b^2)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{2a^2}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{4ab}{d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^2/Tan[c + d*x]^(5/2),x]
```

```
[Out] ((a^2 + 2*a*b - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) -
((a^2 + 2*a*b - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) +
((a^2 - 2*a*b - b^2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2
*Sqrt[2]*d) - ((a^2 - 2*a*b - b^2)*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan
[c + d*x]])/(2*Sqrt[2]*d) - (2*a^2)/(3*d*Tan[c + d*x]^(3/2)) - (4*a*b)/(d*S
qrt[Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
```

```
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(c+dx))^2}{\tan^{\frac{5}{2}}(c+dx)}\,dx = -\frac{2a^2}{3d\tan^{\frac{3}{2}}(c+dx)} + \int \frac{2ab-(a^2-b^2)\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

$$= -\frac{2a^2}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{4ab}{d\sqrt{\tan(c+dx)}} + \int \frac{-a^2+b^2-2ab\tan(c+dx)}{\sqrt{\tan(c+dx)}}\,dx$$

$$= -\frac{2a^2}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{4ab}{d\sqrt{\tan(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{-a^2+b^2-2abx^2}{1+x^4}\,dx, x, \sqrt{\tan(c-}\right.}{d}$$

$$= -\frac{2a^2}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{4ab}{d\sqrt{\tan(c+dx)}} - \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{}\right.}{d}$$

$$= -\frac{2a^2}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{4ab}{d\sqrt{\tan(c+dx)}} + \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\right.}{2\sqrt{2}d}$$

$$= \frac{(a^2-2ab-b^2)\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d} - \frac{(a^2-2ab-b^2)}{}$$

$$= \frac{(a^2+2ab-b^2)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} - \frac{(a^2+2ab-b^2)\tan^{-1}\left(1+\right.}{\sqrt{2}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.22, size = 77, normalized size = 0.35

$$-\frac{2\left((a^2-b^2)\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\tan^2(c+dx)\right)+b\left(b+6a\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\tan^2(c+dx)\right)\tan(c+dx)\right)\right)}{3d\tan^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^2/Tan[c + d*x]^(5/2),x]
```

```
[Out] (-2*((a^2 - b^2)*Hypergeometric2F1[-3/4, 1, 1/4, -Tan[c + d*x]^2] + b*(b +
6*a*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[c + d*x]^2]*Tan[c + d*x])))/(3*d*T
an[c + d*x]^(3/2))
```

Maple [A]
time = 0.05, size = 212, normalized size = 0.95

method	result
derivativedivides	$\frac{-\frac{2a^2}{3\tan(dx+c)^{\frac{3}{2}}}-\frac{4ab}{\sqrt{\tan(dx+c)}}+\frac{(-a^2+b^2)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{2}\right.}{4}}{}$
default	$\frac{-\frac{2a^2}{3\tan(dx+c)^{\frac{3}{2}}}-\frac{4ab}{\sqrt{\tan(dx+c)}}+\frac{(-a^2+b^2)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{2}\right.}{4}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^2/tan(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/3*a^2/tan(d*x+c)^(3/2)-4*a*b/tan(d*x+c)^(1/2)+1/4*(-a^2+b^2)*2^(1/2
)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+t
an(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*
x+c)^(1/2)))-1/2*a*b*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1
+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))
+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))))
```

Maxima [A]
time = 0.49, size = 185, normalized size = 0.83

$$-\frac{6\sqrt{2}(a^2+2ab-b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+6\sqrt{2}(a^2+2ab-b^2)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)+3\sqrt{2}(a^2-2ab-b^2)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)-3\sqrt{2}(a^2-2ab-b^2)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+\frac{8(6ab\tan(dx+c)+a^2)}{\tan(dx+c)^{\frac{3}{2}}}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^2/tan(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12*(6*sqrt(2)*(a^2 + 2*a*b - b^2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(t
an(d*x + c)))) + 6*sqrt(2)*(a^2 + 2*a*b - b^2)*arctan(-1/2*sqrt(2)*(sqrt(2)
 - 2*sqrt(tan(d*x + c)))) + 3*sqrt(2)*(a^2 - 2*a*b - b^2)*log(sqrt(2)*sqrt(
tan(d*x + c)) + tan(d*x + c) + 1) - 3*sqrt(2)*(a^2 - 2*a*b - b^2)*log(-sqrt
(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 8*(6*a*b*tan(d*x + c) + a^2)/t
an(d*x + c)^(3/2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5149 vs. $2(189) = 378$.
time = 1.41, size = 5149, normalized size = 23.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^2/tan(d*x+c)^(5/2),x, algorithm="fricas")
```

[Out] -1/12*(12*sqrt(2)*(d^5*cos(d*x + c)^2 - d^5)*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(3/4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4)*arctan(-((a^16 - 20*a^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 - 64*a^6*b^10 - 20*a^4*b^12 + b^16)*d^4*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4) + sqrt(2)*((a^2 - b^2)*d^7*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4) - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^5*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(((a^12 - 10*a^10*b^2 + 15*a^8*b^4 + 52*a^6*b^6 + 15*a^4*b^8 - 10*a^2*b^10 + b^12)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*cos(d*x + c) + sqrt(2)*(2*(a^9*b - 12*a^7*b^3 + 38*a^5*b^5 - 12*a^3*b^7 + a*b^9)*d^3*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*cos(d*x + c) - (a^14 - 11*a^12*b^2 + 25*a^10*b^4 + 37*a^8*b^6 - 37*a^6*b^8 - 25*a^4*b^10 + 11*a^2*b^12 - b^14)*d*cos(d*x + c))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(1/4) + (a^16 - 8*a^14*b^2 - 4*a^12*b^4 + 72*a^10*b^6 + 134*a^8*b^8 + 72*a^6*b^10 - 4*a^4*b^12 - 8*a^2*b^14 + b^16)*sin(d*x + c))/cos(d*x + c))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(3/4) - sqrt(2)*((a^10 - 5*a^8*b^2 - 6*a^6*b^4 + 6*a^4*b^6 + 5*a^2*b^8 - b^10)*d^7*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4) - 2*(a^13*b - 2*a^11*b^3 - 17*a^9*b^5 - 28*a^7*b^7 - 17*a^5*b^9 - 2*a^3*b^11 + a*b^13)*d^5*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(3/4))/(a^24 - 4*a^22*b^2 - 30*a^20*b^4 + 12*a^18*b^6 + 367*a^16*b^8 + 1016*a^14*b^10 + 1372*a^12*b^12 + 1016*a^10*b^14 + 367*a^8*b^16 + 12*a^6*b^18 - 30*a^4*b^20 - 4*a^2*b^22 + b^24)) + 12*sqrt(2)*(d^5*cos(d*x + c)^2 - d^5)*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(3/4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4)*arctan(((a^16 - 20*a^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 - 64*a^6*b^10 - 20*a^4*b^12 + b^16)*d^4*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4) - sqrt(2)*((a^2 - b^2)*d^7*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 3

```
8*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4) - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^5*sqrt
((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4))*sqrt((a^8 + 4*a^6
*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^
6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 -
 12*a^2*b^6 + b^8))*sqrt(((a^12 - 10*a^10*b^2 + 15*a^8*b^4 + 52*a^6*b^6 + 1
5*a^4*b^8 - 10*a^2*b^10 + b^12)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a
^2*b^6 + b^8)/d^4)*cos(d*x + c) - sqrt(2)*(2*(a^9*b - 12*a^7*b^3 + 38*a^5*b
^5 - 12*a^3*b^7 + a*b^9)*d^3*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6
+ b^8)/d^4)*cos(d*x + c) - (a^14 - 11*a^12*b^2 + 25*a^10*b^4 + 37*a^8*b^6 -
 37*a^6*b^8 - 25*a^4*b^10 + 11*a^2*b^12 - b^14)*d*cos(d*x + c))*sqrt((a^8 +
 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 4*(a^3*b - a*b^3)*d^2*sqrt((a^8
+ 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4
*b^4 - 12*a^2*b^6 + b^8))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^8 + 4*a^6*b^2
 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(1/4) + (a^16 - 8*a^14*b^2 - 4*a^12*b^
4 + 72*a^10*b^6 + 134*a^8*b^8 + 72*a^6*b^10 - 4*a^4*b^12 - 8*a^2*b^14 + b^1
6)*sin(d*x + c))/cos(d*x + c))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 +
b^8)/d^4)^(3/4) + sqrt(2)*((a^10 - 5*a^8*b^2 - 6*a^6*b^4 + 6*a^4*b^6 + 5*a^
2*b^8 - b^10)*d^7*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)
*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4) - 2*(a^13*b -
 2*a^11*b^3 - 17*a^9*b^5 - 28*a^7*b^7 - 17*a^5*b^9 - 2*a^3*b^11 + a*b^13)*d
^5*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4))*sqrt((a^8
+ 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 4*(...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(c+dx))^2}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**2/tan(d*x+c)**(5/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2/tan(c + d*x)**(5/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^2/tan(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 5.07, size = 968, normalized size = 4.34

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tan(c + d*x))^2/tan(c + d*x)^(5/2),x)

[Out] 2*atanh((32*a^4*d^3*tan(c + d*x)^(1/2)*((a*b^3)/d^2 - (b^4*1i)/(4*d^2) - (a^4*1i)/(4*d^2) - (a^3*b)/d^2 + (a^2*b^2*3i)/(2*d^2))^(1/2))/(a^6*d^2*16i - b^6*d^2*16i + 32*a*b^5*d^2 + 32*a^5*b*d^2 + a^2*b^4*d^2*112i - 192*a^3*b^3*d^2 - a^4*b^2*d^2*112i) + (32*b^4*d^3*tan(c + d*x)^(1/2)*((a*b^3)/d^2 - (b^4*1i)/(4*d^2) - (a^4*1i)/(4*d^2) - (a^3*b)/d^2 + (a^2*b^2*3i)/(2*d^2))^(1/2))/(a^6*d^2*16i - b^6*d^2*16i + 32*a*b^5*d^2 + 32*a^5*b*d^2 + a^2*b^4*d^2*112i - 192*a^3*b^3*d^2 - a^4*b^2*d^2*112i) - (192*a^2*b^2*d^3*tan(c + d*x)^(1/2)*((a*b^3)/d^2 - (b^4*1i)/(4*d^2) - (a^4*1i)/(4*d^2) - (a^3*b)/d^2 + (a^2*b^2*3i)/(2*d^2))^(1/2))/(a^6*d^2*16i - b^6*d^2*16i + 32*a*b^5*d^2 + 32*a^5*b*d^2 + a^2*b^4*d^2*112i - 192*a^3*b^3*d^2 - a^4*b^2*d^2*112i))*(-(4*a^3*b - 4*a*b^3 + a^4*1i + b^4*1i - a^2*b^2*6i)/(4*d^2))^(1/2) + 2*atanh((32*a^4*d^3*tan(c + d*x)^(1/2)*((a^4*1i)/(4*d^2) + (b^4*1i)/(4*d^2) + (a*b^3)/d^2 - (a^3*b)/d^2 - (a^2*b^2*3i)/(2*d^2))^(1/2))/(b^6*d^2*16i - a^6*d^2*16i + 32*a*b^5*d^2 + 32*a^5*b*d^2 - a^2*b^4*d^2*112i - 192*a^3*b^3*d^2 + a^4*b^2*d^2*112i) + (32*b^4*d^3*tan(c + d*x)^(1/2)*((a^4*1i)/(4*d^2) + (b^4*1i)/(4*d^2) + (a*b^3)/d^2 - (a^3*b)/d^2 - (a^2*b^2*3i)/(2*d^2))^(1/2))/(b^6*d^2*16i - a^6*d^2*16i + 32*a*b^5*d^2 + 32*a^5*b*d^2 - a^2*b^4*d^2*112i - 192*a^3*b^3*d^2 + a^4*b^2*d^2*112i) - (192*a^2*b^2*d^3*tan(c + d*x)^(1/2)*((a^4*1i)/(4*d^2) + (b^4*1i)/(4*d^2) + (a*b^3)/d^2 - (a^3*b)/d^2 - (a^2*b^2*3i)/(2*d^2))^(1/2))/(b^6*d^2*16i - a^6*d^2*16i + 32*a*b^5*d^2 + 32*a^5*b*d^2 - a^2*b^4*d^2*112i - 192*a^3*b^3*d^2 + a^4*b^2*d^2*112i))*((4*a*b^3 - 4*a^3*b + a^4*1i + b^4*1i - a^2*b^2*6i)/(4*d^2))^(1/2) - ((2*a^2)/3 + 4*a*b*tan(c + d*x))/(d*tan(c + d*x)^(3/2))

3.568 $\int \frac{(a+b\tan(c+dx))^2}{\tan^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=249

$$-\frac{(a^2-2ab-b^2)\,\mathrm{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a^2-2ab-b^2)\,\mathrm{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(\cdots}{\cdots}$$

```
[Out] 1/2*(a^2-2*a*b-b^2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)+1/2*(a^2-
2*a*b-b^2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)+1/4*(a^2+2*a*b-b^2)
*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)-1/4*(a^2+2*a*b-b^2)*ln
(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)+2*(a^2-b^2)/d/tan(d*x+c)^
(1/2)-2/5*a^2/d/tan(d*x+c)^(5/2)-4/3*a*b/d/tan(d*x+c)^(3/2)
```

Rubi [A]

time = 0.15, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3623, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{(a^2-2ab-b^2)\,\mathrm{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a^2-2ab-b^2)\,\mathrm{ArcTan}\left(\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d}+\frac{2(a^2-b^2)}{d\sqrt{\tan(c+dx)}}+\frac{(a^2+2ab-b^2)\log\left(\tan(c+dx)-\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{(a^2+2ab-b^2)\log\left(\tan(c+dx)+\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{2a^2}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{4ab}{3d\tan^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^2/Tan[c + d*x]^(7/2),x]
```

```
[Out] -(((a^2 - 2*a*b - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d))
 + ((a^2 - 2*a*b - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d)
 + ((a^2 + 2*a*b - b^2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])
/(2*Sqrt[2]*d) - ((a^2 + 2*a*b - b^2)*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] +
Tan[c + d*x]])/(2*Sqrt[2]*d) - (2*a^2)/(5*d*Tan[c + d*x]^(5/2)) - (4*a*b)/(
3*d*Tan[c + d*x]^(3/2)) + (2*(a^2 - b^2))/(d*Sqrt[Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
```

```
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(c+dx))^2}{\tan^{\frac{7}{2}}(c+dx)}\,dx = -\frac{2a^2}{5d\tan^{\frac{5}{2}}(c+dx)} + \int \frac{2ab-(a^2-b^2)\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$

$$= -\frac{2a^2}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{4ab}{3d\tan^{\frac{3}{2}}(c+dx)} + \int \frac{-a^2+b^2-2ab\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

$$= -\frac{2a^2}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{4ab}{3d\tan^{\frac{3}{2}}(c+dx)} + \frac{2(a^2-b^2)}{d\sqrt{\tan(c+dx)}} + \int \frac{-2ab+(a^2-}{\sqrt{\tan(c}}$$

$$= -\frac{2a^2}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{4ab}{3d\tan^{\frac{3}{2}}(c+dx)} + \frac{2(a^2-b^2)}{d\sqrt{\tan(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{-2ab+(a}{1+}\right.}{}$$

$$= -\frac{2a^2}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{4ab}{3d\tan^{\frac{3}{2}}(c+dx)} + \frac{2(a^2-b^2)}{d\sqrt{\tan(c+dx)}} + \frac{(a^2-2ab-b^2)\,\text{Su}}{}$$

$$= -\frac{2a^2}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{4ab}{3d\tan^{\frac{3}{2}}(c+dx)} + \frac{2(a^2-b^2)}{d\sqrt{\tan(c+dx)}} + \frac{(a^2-2ab-b^2)\,\text{Su}}{}$$

$$= \frac{(a^2+2ab-b^2)\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}\,d} - \frac{(a^2+2ab-b^2}{}$$

$$= -\frac{(a^2-2ab-b^2)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d} + \frac{(a^2-2ab-b^2)\tan^{-1}\left(1\right.}{\sqrt{2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.22, size = 81, normalized size = 0.33

$$\frac{-6(a^2-b^2)\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\tan^2(c+dx)\right)-2b\left(3b+10a\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\tan^2(c+dx)\right)\tan(c+dx)\right)}{15d\tan^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^2/Tan[c + d*x]^(7/2),x]
```

```
[Out] (-6*(a^2 - b^2)*Hypergeometric2F1[-5/4, 1, -1/4, -Tan[c + d*x]^2] - 2*b*(3*
b + 10*a*Hypergeometric2F1[-3/4, 1, 1/4, -Tan[c + d*x]^2]*Tan[c + d*x]))/(1
5*d*Tan[c + d*x]^(5/2))
```

Maple [A]
time = 0.08, size = 231, normalized size = 0.93

method	result
derivativedivides	$\frac{-\frac{2a^2}{5\tan(dx+c)^{\frac{5}{2}}}-\frac{2(-a^2+b^2)}{\sqrt{\tan(dx+c)}}-\frac{4ab}{3\tan(dx+c)^{\frac{3}{2}}}-\frac{ab\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\left(1+\right.\right.}{}}{}$
default	$\frac{-\frac{2a^2}{5\tan(dx+c)^{\frac{5}{2}}}-\frac{2(-a^2+b^2)}{\sqrt{\tan(dx+c)}}-\frac{4ab}{3\tan(dx+c)^{\frac{3}{2}}}-\frac{ab\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\left(1+\right.\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^2/tan(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/5*a^2/tan(d*x+c)^(5/2)-2*(-a^2+b^2)/tan(d*x+c)^(1/2)-4/3*a*b/tan(d*
x+c)^(3/2)-1/2*a*b*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2
^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2
*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(a^2-b^2)*2^(1/2)*(ln((1-2^(1/2)*
tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arc
tan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))))
```

Maxima [A]
time = 0.52, size = 206, normalized size = 0.83

$$\frac{30\sqrt{2}(a^2-2ab-b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+30\sqrt{2}(a^2-2ab-b^2)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-15\sqrt{2}(a^2+2ab-b^2)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+15\sqrt{2}(a^2+2ab-b^2)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)-\frac{8\left(10ab\tan(dx+c)-15(a^2-b^2)\tan(dx+c)^2+3a^2\right)}{\tan(dx+c)^{\frac{5}{2}}}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^2/tan(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/60*(30*sqrt(2)*(a^2 - 2*a*b - b^2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(t
an(d*x + c)))) + 30*sqrt(2)*(a^2 - 2*a*b - b^2)*arctan(-1/2*sqrt(2)*(sqrt(2
) - 2*sqrt(tan(d*x + c)))) - 15*sqrt(2)*(a^2 + 2*a*b - b^2)*log(sqrt(2)*sqr
t(tan(d*x + c)) + tan(d*x + c) + 1) + 15*sqrt(2)*(a^2 + 2*a*b - b^2)*log(-s
qrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - 8*(10*a*b*tan(d*x + c) - 15
*(a^2 - b^2)*tan(d*x + c)^2 + 3*a^2)/tan(d*x + c)^(5/2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5441 vs. $2(211) = 422$.
time = 1.85, size = 5441, normalized size = 21.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^2/tan(d*x+c)^(7/2),x, algorithm="fricas")

[Out] -1/60*(60*sqrt(2)*(d^5*cos(d*x + c)^4 - 2*d^5*cos(d*x + c)^2 + d^5)*sqrt((a
^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 - 4*(a^3*b - a*b^3)*d^2*sqrt((
a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38
*a^4*b^4 - 12*a^2*b^6 + b^8))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b
^8)/d^4)^(3/4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4)
*arctan(-((a^16 - 20*a^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 - 64*a^6*b^10 - 20
*a^4*b^12 + b^16)*d^4*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/
d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4) - sqrt(2)
*(2*a*b*d^7*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*sqrt(
(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4) + (a^6 + a^4*b^2 -
a^2*b^4 - b^6)*d^5*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/
d^4))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 - 4*(a^3*b - a*b^
3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4))/(a^8 - 12
*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(((a^12 - 10*a^10*b^2 + 15*a
^8*b^4 + 52*a^6*b^6 + 15*a^4*b^8 - 10*a^2*b^10 + b^12)*d^2*sqrt((a^8 + 4*a^
6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*cos(d*x + c) + sqrt(2)*((a^10 - 1
3*a^8*b^2 + 50*a^6*b^4 - 50*a^4*b^6 + 13*a^2*b^8 - b^10)*d^3*sqrt((a^8 + 4*
a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*cos(d*x + c) + 2*(a^13*b - 10*a
^11*b^3 + 15*a^9*b^5 + 52*a^7*b^7 + 15*a^5*b^9 - 10*a^3*b^11 + a*b^13)*d*co
s(d*x + c))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 - 4*(a^3*b
- a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4))/(a^
8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(sin(d*x + c)/cos(d*x
+ c))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(1/4) + (a^16 -
 8*a^14*b^2 - 4*a^12*b^4 + 72*a^10*b^6 + 134*a^8*b^8 + 72*a^6*b^10 - 4*a^4*
b^12 - 8*a^2*b^14 + b^16)*sin(d*x + c))/cos(d*x + c))*((a^8 + 4*a^6*b^2 + 6
*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(3/4) + sqrt(2)*(2*(a^9*b - 4*a^7*b^3 - 10
*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d^7*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2
*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^
4) + (a^14 - 3*a^12*b^2 - 15*a^10*b^4 - 11*a^8*b^6 + 11*a^6*b^8 + 15*a^4*b^
10 + 3*a^2*b^12 - b^14)*d^5*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^
6 + b^8)/d^4))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 - 4*(a^3
*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4))/
(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(sin(d*x + c)/cos(d
*x + c))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(3/4))/(a^24
 - 4*a^22*b^2 - 30*a^20*b^4 + 12*a^18*b^6 + 367*a^16*b^8 + 1016*a^14*b^10 +
 1372*a^12*b^12 + 1016*a^10*b^14 + 367*a^8*b^16 + 12*a^6*b^18 - 30*a^4*b^20
 - 4*a^2*b^22 + b^24)) + 60*sqrt(2)*(d^5*cos(d*x + c)^4 - 2*d^5*cos(d*x + c
)^2 + d^5)*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 - 4*(a^3*b -
 a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4))/(a^8
 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*((a^8 + 4*a^6*b^2 + 6*a^4*b
^4 + 4*a^2*b^6 + b^8)/d^4)^(3/4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a
^2*b^6 + b^8)/d^4)*arctan(((a^16 - 20*a^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 -
 64*a^6*b^10 - 20*a^4*b^12 + b^16)*d^4*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
```

```
4*a^2*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^
8)/d^4) + sqrt(2)*(2*a*b*d^7*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6
+ b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^4) +
(a^6 + a^4*b^2 - a^2*b^4 - b^6)*d^5*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 1
2*a^2*b^6 + b^8)/d^4))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8
- 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8
)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(((a^12 - 1
0*a^10*b^2 + 15*a^8*b^4 + 52*a^6*b^6 + 15*a^4*b^8 - 10*a^2*b^10 + b^12)*d^2
*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*cos(d*x + c) - s
qrt(2)*((a^10 - 13*a^8*b^2 + 50*a^6*b^4 - 50*a^4*b^6 + 13*a^2*b^8 - b^10)*d
^3*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*cos(d*x + c) +
 2*(a^13*b - 10*a^11*b^3 + 15*a^9*b^5 + 52*a^7*b^7 + 15*a^5*b^9 - 10*a^3*b^
11 + a*b^13)*d*cos(d*x + c))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6
+ b^8 - 4*(a^3*b - a*b^3)*d^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6
 + b^8)/d^4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(sin(
d*x + c)/cos(d*x + c))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4
)^(1/4) + (a^16 - 8*a^14*b^2 - 4*a^12*b^4 + 72*a^10*b^6 + 134*a^8*b^8 + 72*
a^6*b^10 - 4*a^4*b^12 - 8*a^2*b^14 + b^16)*sin(d*x + c))/cos(d*x + c))*((a^
8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)^(3/4) - sqrt(2)*(2*(a^9*b
 - 4*a^7*b^3 - 10*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d^7*sqrt((a^8 + 4*a^6*b^2 +
6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^4)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*
a^2*b^6 + b^8)/d^4) + (a^14 - 3*a^12*b^2 - 15*a^10*b^4 - 11*a^8*b^6 + 11*a^
6*b^8 + 15*a^4*b^10 + 3*a^2*b^12 - b^14)*d^5*sqrt((a^8 - 12*a^6*b^2 + 38*a^
4*b^4 - 12*a^2*b^6 + b^8)/d^4))*sqrt((a^8 + 4*a...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan(c + dx))^2}{\tan^{\frac{7}{2}}(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**2/tan(d*x+c)**(7/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2/tan(c + d*x)**(7/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^2/tan(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 5.69, size = 983, normalized size = 3.95

$$-2\,\mathrm{atanh}\left(\frac{32\,a^4\,d^3\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{a^3\,b}{d^2}-\frac{b^4\,1{}\mathrm{i}}{4\,d^2}-\frac{a\,b^3}{d^2}-\frac{a^4\,1{}\mathrm{i}}{4\,d^2}+\frac{a^2\,b^2\,3{}\mathrm{i}}{2\,d^2}}}{16\,a^6\,d^2+a^5\,b\,d^2\,32{}\mathrm{i}-112\,a^4\,b^2\,d^2-a^3\,b^3\,d^2\,192{}\mathrm{i}+112\,a^2\,b^4\,d^2+a\,b^5\,d^2\,32{}\mathrm{i}-16\,b^6\,d^2}+\frac{32\,b^4\,d^3\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{a^3\,b}{d^2}-\frac{b^4\,1{}\mathrm{i}}{4\,d^2}-\frac{a\,b^3}{d^2}-\frac{a^4\,1{}\mathrm{i}}{4\,d^2}+\frac{a^2\,b^2\,3{}\mathrm{i}}{2\,d^2}}}{16\,a^6\,d^2+a^5\,b\,d^2\,32{}\mathrm{i}-112\,a^4\,b^2\,d^2-a^3\,b^3\,d^2\,192{}\mathrm{i}+112\,a^2\,b^4\,d^2+a\,b^5\,d^2\,32{}\mathrm{i}-16\,b^6\,d^2}-\frac{192\,a^2\,b^2\,d^3\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{a^3\,b}{d^2}-\frac{b^4\,1{}\mathrm{i}}{4\,d^2}-\frac{a\,b^3}{d^2}-\frac{a^4\,1{}\mathrm{i}}{4\,d^2}+\frac{a^2\,b^2\,3{}\mathrm{i}}{2\,d^2}}}{16\,a^6\,d^2+a^5\,b\,d^2\,32{}\mathrm{i}-112\,a^4\,b^2\,d^2-a^3\,b^3\,d^2\,192{}\mathrm{i}+112\,a^2\,b^4\,d^2+a\,b^5\,d^2\,32{}\mathrm{i}-16\,b^6\,d^2}\right)\,\sqrt{-\frac{a^4\,1{}\mathrm{i}-4\,a^3\,b-a^2\,b^2\,6{}\mathrm{i}+4\,a\,b^3+b^4\,1{}\mathrm{i}}{4\,d^2}}+2\,\mathrm{atanh}\left(\frac{32\,a^4\,d^3\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{a^4\,1{}\mathrm{i}}{4\,d^2}+\frac{b^4\,1{}\mathrm{i}}{4\,d^2}-\frac{a\,b^3}{d^2}+\frac{a^3\,b}{d^2}-\frac{a^2\,b^2\,3{}\mathrm{i}}{2\,d^2}}}{-16\,a^6\,d^2+a^5\,b\,d^2\,32{}\mathrm{i}+112\,a^4\,b^2\,d^2-a^3\,b^3\,d^2\,192{}\mathrm{i}-112\,a^2\,b^4\,d^2+a\,b^5\,d^2\,32{}\mathrm{i}+16\,b^6\,d^2}+\frac{32\,b^4\,d^3\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{a^4\,1{}\mathrm{i}}{4\,d^2}+\frac{b^4\,1{}\mathrm{i}}{4\,d^2}-\frac{a\,b^3}{d^2}+\frac{a^3\,b}{d^2}-\frac{a^2\,b^2\,3{}\mathrm{i}}{2\,d^2}}}{-16\,a^6\,d^2+a^5\,b\,d^2\,32{}\mathrm{i}+112\,a^4\,b^2\,d^2-a^3\,b^3\,d^2\,192{}\mathrm{i}-112\,a^2\,b^4\,d^2+a\,b^5\,d^2\,32{}\mathrm{i}+16\,b^6\,d^2}-\frac{192\,a^2\,b^2\,d^3\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{a^4\,1{}\mathrm{i}}{4\,d^2}+\frac{b^4\,1{}\mathrm{i}}{4\,d^2}-\frac{a\,b^3}{d^2}+\frac{a^3\,b}{d^2}-\frac{a^2\,b^2\,3{}\mathrm{i}}{2\,d^2}}}{-16\,a^6\,d^2+a^5\,b\,d^2\,32{}\mathrm{i}+112\,a^4\,b^2\,d^2-a^3\,b^3\,d^2\,192{}\mathrm{i}-112\,a^2\,b^4\,d^2+a\,b^5\,d^2\,32{}\mathrm{i}+16\,b^6\,d^2}\right)\,\sqrt{\frac{a^4\,1{}\mathrm{i}+4\,a^3\,b-a^2\,b^2\,6{}\mathrm{i}-4\,a\,b^3+b^4\,1{}\mathrm{i}}{4\,d^2}}-\frac{\frac{2\,a^2}{5}-{\tan\left(c+d\,x\right)}^2\,\left(2\,a^2-2\,b^2\right)+\frac{4\,a\,b\,\tan\left(c+d\,x\right)}{3}}{d\,{\tan\left(c+d\,x\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^2/tan(c + d*x)^(7/2),x)
```

```
[Out] 2*atanh((32*a^4*d^3*tan(c + d*x)^(1/2)*((a^4*1i)/(4*d^2) + (b^4*1i)/(4*d^2)
 - (a*b^3)/d^2 + (a^3*b)/d^2 - (a^2*b^2*3i)/(2*d^2))^(1/2))/(16*b^6*d^2 - 1
6*a^6*d^2 + a*b^5*d^2*32i + a^5*b*d^2*32i - 112*a^2*b^4*d^2 - a^3*b^3*d^2*1
92i + 112*a^4*b^2*d^2) + (32*b^4*d^3*tan(c + d*x)^(1/2)*((a^4*1i)/(4*d^2) +
 (b^4*1i)/(4*d^2) - (a*b^3)/d^2 + (a^3*b)/d^2 - (a^2*b^2*3i)/(2*d^2))^(1/2)
)/(16*b^6*d^2 - 16*a^6*d^2 + a*b^5*d^2*32i + a^5*b*d^2*32i - 112*a^2*b^4*d^
2 - a^3*b^3*d^2*192i + 112*a^4*b^2*d^2) - (192*a^2*b^2*d^3*tan(c + d*x)^(1/
2)*((a^4*1i)/(4*d^2) + (b^4*1i)/(4*d^2) - (a*b^3)/d^2 + (a^3*b)/d^2 - (a^2*
b^2*3i)/(2*d^2))^(1/2))/(16*b^6*d^2 - 16*a^6*d^2 + a*b^5*d^2*32i + a^5*b*d^
2*32i - 112*a^2*b^4*d^2 - a^3*b^3*d^2*192i + 112*a^4*b^2*d^2))*((4*a^3*b -
4*a*b^3 + a^4*1i + b^4*1i - a^2*b^2*6i)/(4*d^2))^(1/2) - 2*atanh((32*a^4*d^
3*tan(c + d*x)^(1/2)*((a^3*b)/d^2 - (b^4*1i)/(4*d^2) - (a*b^3)/d^2 - (a^4*1
i)/(4*d^2) + (a^2*b^2*3i)/(2*d^2))^(1/2))/(16*a^6*d^2 - 16*b^6*d^2 + a*b^5*
d^2*32i + a^5*b*d^2*32i + 112*a^2*b^4*d^2 - a^3*b^3*d^2*192i - 112*a^4*b^2*
d^2) + (32*b^4*d^3*tan(c + d*x)^(1/2)*((a^3*b)/d^2 - (b^4*1i)/(4*d^2) - (a*
b^3)/d^2 - (a^4*1i)/(4*d^2) + (a^2*b^2*3i)/(2*d^2))^(1/2))/(16*a^6*d^2 - 16
*b^6*d^2 + a*b^5*d^2*32i + a^5*b*d^2*32i + 112*a^2*b^4*d^2 - a^3*b^3*d^2*19
2i - 112*a^4*b^2*d^2) - (192*a^2*b^2*d^3*tan(c + d*x)^(1/2)*((a^3*b)/d^2 -
(b^4*1i)/(4*d^2) - (a*b^3)/d^2 - (a^4*1i)/(4*d^2) + (a^2*b^2*3i)/(2*d^2))^(
1/2))/(16*a^6*d^2 - 16*b^6*d^2 + a*b^5*d^2*32i + a^5*b*d^2*32i + 112*a^2*b^
4*d^2 - a^3*b^3*d^2*192i - 112*a^4*b^2*d^2))*(-(4*a*b^3 - 4*a^3*b + a^4*1i
+ b^4*1i - a^2*b^2*6i)/(4*d^2))^(1/2) - ((2*a^2)/5 - tan(c + d*x)^2*(2*a^2
- 2*b^2) + (4*a*b*tan(c + d*x))/3)/(d*tan(c + d*x)^(5/2))
```

3.569 $\int \tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=328

$$\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}$$

```
[Out] -1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)-1/
2*(a+b)*(a^2-4*a*b+b^2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)-1/4*(a
-b)*(a^2+4*a*b+b^2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)+1/4
*(a-b)*(a^2+4*a*b+b^2)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)-
2*b*(3*a^2-b^2)*tan(d*x+c)^(1/2)/d+2/3*a*(a^2-3*b^2)*tan(d*x+c)^(3/2)/d+2/5
*b*(3*a^2-b^2)*tan(d*x+c)^(5/2)/d+40/63*a*b^2*tan(d*x+c)^(7/2)/d+2/9*b^2*ta
n(d*x+c)^(7/2)*(a+b*tan(d*x+c))/d
```

Rubi [A]
time = 0.31, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3647, 3711, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d}+\frac{2b\left(3a^2-b^2\right)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{2a\left(a^2-3b^2\right)\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2b\left(3a^2-b^2\right)\sqrt{\tan(c+dx)}}{d}-\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{2b^2\tan^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))}{9d}+\frac{40ab^2\tan^{\frac{7}{2}}(c+dx)}{63d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[
2]*d) - ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]
)/(Sqrt[2]*d) - ((a - b)*(a^2 + 4*a*b + b^2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d
*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) + ((a - b)*(a^2 + 4*a*b + b^2)*Log[1 +
Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) - (2*b*(3*a^2 - b
^2)*Sqrt[Tan[c + d*x]])/d + (2*a*(a^2 - 3*b^2)*Tan[c + d*x]^(3/2))/(3*d) +
(2*b*(3*a^2 - b^2)*Tan[c + d*x]^(5/2))/(5*d) + (40*a*b^2*Tan[c + d*x]^(7/2)
)/(63*d) + (2*b^2*Tan[c + d*x]^(7/2)*(a + b*Tan[c + d*x]))/(9*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{2b^2\tan^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))}{9d} + \frac{2}{9}\int \tan^{\frac{5}{2}}(c+dx)\left(\frac{1}{2}a(9a\right.$$

$$= \frac{40ab^2\tan^{\frac{7}{2}}(c+dx)}{63d} + \frac{2b^2\tan^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))}{9d} + \frac{2}{9}\int$$

$$= \frac{2b(3a^2-b^2)\tan^{\frac{5}{2}}(c+dx)}{5d} + \frac{40ab^2\tan^{\frac{7}{2}}(c+dx)}{63d} + \frac{2b^2\tan^{\frac{7}{2}}(c+}{}$$

$$= \frac{2a(a^2-3b^2)\tan^{\frac{3}{2}}(c+dx)}{3d} + \frac{2b(3a^2-b^2)\tan^{\frac{5}{2}}(c+dx)}{5d} + \frac{40ab^2\,\mathrm{t}}{}$$

$$= -\frac{2b(3a^2-b^2)\sqrt{\tan(c+dx)}}{d} + \frac{2a(a^2-3b^2)\tan^{\frac{3}{2}}(c+dx)}{3d} + \frac{2b(}{}$$

$$= -\frac{2b(3a^2-b^2)\sqrt{\tan(c+dx)}}{d} + \frac{2a(a^2-3b^2)\tan^{\frac{3}{2}}(c+dx)}{3d} + \frac{2b(}{}$$

$$= -\frac{2b(3a^2-b^2)\sqrt{\tan(c+dx)}}{d} + \frac{2a(a^2-3b^2)\tan^{\frac{3}{2}}(c+dx)}{3d} + \frac{2b(}{}$$

$$= -\frac{2b(3a^2-b^2)\sqrt{\tan(c+dx)}}{d} + \frac{2a(a^2-3b^2)\tan^{\frac{3}{2}}(c+dx)}{3d} + \frac{2b(}{}$$

$$= -\frac{(a-b)(a^2+4ab+b^2)\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+\right.}{2\sqrt{2}\,d}$$

$$= \frac{(a+b)(a^2-4ab+b^2)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d} - \frac{(a+b}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.05, size = 164, normalized size = 0.50

$$\frac{-315(-1)^{3/4}(a-ib)^3\mathrm{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+315(-1)^{3/4}(a+ib)^3\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+2\sqrt{\tan(c+dx)}\left(315b(-3a^2+b^2)+105a(a^2-3b^2)\tan(c+dx)-63b(-3a^2+b^2)\tan^2(c+dx)+135ab^2\tan^3(c+dx)+35b^3\tan^4(c+dx)\right)}{315d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-315*(-1)^(3/4)*(a - I*b)^3*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + 315*(-
1)^(3/4)*(a + I*b)^3*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + 2*Sqrt[Tan[c
+ d*x]]*(315*b*(-3*a^2 + b^2) + 105*a*(a^2 - 3*b^2)*Tan[c + d*x] - 63*b*(-3
*a^2 + b^2)*Tan[c + d*x]^2 + 135*a*b^2*Tan[c + d*x]^3 + 35*b^3*Tan[c + d*x]
^4))/(315*d)
```

Maple [A]
time = 0.06, size = 308, normalized size = 0.94

method	result
derivativedivides	$\frac{\frac{2b^3\left(\tan^{\frac{9}{2}}(dx+c)\right)}{9}+\frac{6b^2a\left(\tan^{\frac{7}{2}}(dx+c)\right)}{7}+\frac{6a^2b\left(\tan^{\frac{5}{2}}(dx+c)\right)}{5}-\frac{2b^3\left(\tan^{\frac{5}{2}}(dx+c)\right)}{5}+\frac{2a^3\left(\tan^{\frac{3}{2}}(dx+c)\right)}{3}-2a\,b^2\left(\tan^{\frac{3}{2}}(dx+c)\right)}{}$
default	$\frac{\frac{2b^3\left(\tan^{\frac{9}{2}}(dx+c)\right)}{9}+\frac{6b^2a\left(\tan^{\frac{7}{2}}(dx+c)\right)}{7}+\frac{6a^2b\left(\tan^{\frac{5}{2}}(dx+c)\right)}{5}-\frac{2b^3\left(\tan^{\frac{5}{2}}(dx+c)\right)}{5}+\frac{2a^3\left(\tan^{\frac{3}{2}}(dx+c)\right)}{3}-2a\,b^2\left(\tan^{\frac{3}{2}}(dx+c)\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/9*b^3*tan(d*x+c)^(9/2)+6/7*b^2*a*tan(d*x+c)^(7/2)+6/5*a^2*b*tan(d*x+
c)^(5/2)-2/5*b^3*tan(d*x+c)^(5/2)+2/3*a^3*tan(d*x+c)^(3/2)-2*a*b^2*tan(d*x+
c)^(3/2)-6*a^2*b*tan(d*x+c)^(1/2)+2*b^3*tan(d*x+c)^(1/2)+1/4*(3*a^2*b-b^3)*
2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(
1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*
tan(d*x+c)^(1/2)))+1/4*(-a^3+3*a*b^2)*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/
2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*
tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))))
```

Maxima [A]
time = 0.51, size = 280, normalized size = 0.85

$$\frac{280\,b^3\tan(dx+c)^{\frac{9}{2}}+1080\,ab^2\tan(dx+c)^{\frac{7}{2}}+504\,(3\,a^2b-b^3)\tan(dx+c)^{\frac{5}{2}}-630\,\sqrt{2}\,(a^3-3\,a^2b-3\,ab^2+b^3)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\sqrt{\tan(dx+c)}\right)\right)-630\,\sqrt{2}\,(a^3-3\,a^2b-3\,ab^2+b^3)\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\sqrt{\tan(dx+c)}\right)\right)+315\,\sqrt{2}\,(a^3+3\,a^2b-3\,ab^2-b^3)\log\left(\sqrt{2}\,\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)-315\,\sqrt{2}\,(a^3+3\,a^2b-3\,ab^2-b^3)\log\left(-\sqrt{2}\,\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+840\,(a^3-3\,ab^2)\tan(dx+c)^{\frac{3}{2}}-2520\,(3\,a^2b-b^3)\sqrt{\tan(dx+c)}}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/1260*(280*b^3*tan(d*x + c)^(9/2) + 1080*a*b^2*tan(d*x + c)^(7/2) + 504*(3
*a^2*b - b^3)*tan(d*x + c)^(5/2) - 630*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b
^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) - 630*sqrt(2)*(a^3
 - 3*a^2*b - 3*a*b^2 + b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x +
 c)))) + 315*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*log(sqrt(2)*sqrt(tan(d
*x + c)) + tan(d*x + c) + 1) - 315*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*
log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 840*(a^3 - 3*a*b^2)*t
an(d*x + c)^(3/2) - 2520*(3*a^2*b - b^3)*sqrt(tan(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7332 vs. $2(282) = 564$.
time = 3.16, size = 7332, normalized size = 22.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))^3,x, algorithm="fricas")

[Out] 1/1260*(1260*sqrt(2)*d^5*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 - 2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)^(3/4)*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4)*arctan(-((a^24 - 6*a^22*b^2 - 84*a^20*b^4 - 322*a^18*b^6 - 603*a^16*b^8 - 540*a^14*b^10 + 540*a^10*b^14 + 603*a^8*b^16 + 322*a^6*b^18 + 84*a^4*b^20 + 6*a^2*b^22 - b^24)*d^4*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4) + sqrt(2)*((3*a^2*b - b^3)*d^7*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4) + (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^5*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4))*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 - 2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*sqrt(((a^18 - 27*a^16*b^2 + 168*a^14*b^4 + 224*a^12*b^6 - 366*a^10*b^8 - 366*a^8*b^10 + 224*a^6*b^12 + 168*a^4*b^14 - 27*a^2*b^16 + b^18)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)*cos(d*x + c) + sqrt(2)*((a^15 - 33*a^13*b^2 + 345*a^11*b^4 - 1217*a^9*b^6 + 1611*a^7*b^8 - 795*a^5*b^10 + 91*a^3*b^12 - 3*a*b^14)*d^3*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)*cos(d*x + c) + (3*a^20*b - 82*a^18*b^3 + 531*a^16*b^5 + 504*a^14*b^7 - 1322*a^12*b^9 - 732*a^10*b^11 + 1038*a^8*b^13 + 280*a^6*b^15 - 249*a^4*b^17 + 30*a^2*b^19 - b^21)*d*cos(d*x + c))*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 - 2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)^(1/4) + (a^24 - 24*a^22*b^2 + 90*a^20*b^4 + 648*a^18*b^6 + 783*a^16*b^8 - 624*a^14*b^10 - 1748*a^12*b^12 - 624*a^10*b^14 + 783*a^8*b^16 + 648*a^6*b^18 + 90*a^4*b^20 - 24*a^2*b^22 + b^24)*sin(d*x + c))/cos(d*x + c))*((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)^(3/4) + sqrt(2)*((3*a^14*b - 37*a^12*b^3 - 69*a^10*b^5 + 27*a^8*b^7 + 81*a^6*b^9 + 9*a^4*b^11 - 15*a^2*b^13 + b^15)*d^7*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^1

```
2)/d^4)*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8
- 30*a^2*b^10 + b^12)/d^4) + (a^21 - 12*a^19*b^2 - 33*a^17*b^4 + 64*a^15*b^
6 + 282*a^13*b^8 + 264*a^11*b^10 - 82*a^9*b^12 - 288*a^7*b^14 - 171*a^5*b^1
6 - 28*a^3*b^18 + 3*a*b^20)*d^5*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 45
2*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4))*sqrt((a^12 + 6*a^10*b^2
 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 - 2*(3*a^5*b -
10*a^3*b^3 + 3*a*b^5)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6
 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4
- 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*sqrt(sin(d*x + c)/cos(d*
x + c))*((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*
b^10 + b^12)/d^4)^(3/4))/(a^36 - 18*a^34*b^2 - 39*a^32*b^4 + 848*a^30*b^6 +
 5556*a^28*b^8 + 15240*a^26*b^10 + 20420*a^24*b^12 + 5424*a^22*b^14 - 25938
*a^20*b^16 - 42988*a^18*b^18 - 25938*a^16*b^20 + 5424*a^14*b^22 + 20420*a^1
2*b^24 + 15240*a^10*b^26 + 5556*a^8*b^28 + 848*a^6*b^30 - 39*a^4*b^32 - 18*
a^2*b^34 + b^36))*cos(d*x + c)^4 + 1260*sqrt(2)*d^5*sqrt((a^12 + 6*a^10*b^2
 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 - 2*(3*a^5*b -
10*a^3*b^3 + 3*a*b^5)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6
 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4
- 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*((a^12 + 6*a^10*b^2 + 15
*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)^(3/4)*sqrt((a^
12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 +
b^12)/d^4)*arctan(((a^24 - 6*a^22*b^2 - 84*a^20*b^4 - 322*a^18*b^6 - 603*a^
16*b^8 - 540*a^14*b^10 + 540*a^10*b^14 + 603*a^8*b^16 + 322*a^6*b^18 + 84*a
^4*b^20 + 6*a^2*b^22 - b^24)*d^4*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*
a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)*sqrt((a^12 - 30*a^10*b^2 + 2
55*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tan (c + dx))^3 \tan^{\frac{5}{2}} (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(5/2)*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*tan(c + d*x)**(5/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

[Out] Timed out

Mupad [B]
time = 10.12, size = 1796, normalized size = 5.48

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(5/2)*(a + b*tan(c + d*x))^3,x)
```

```
[Out] tan(c + d*x)^(3/2)*((2*a^3)/(3*d) - (2*a*b^2)/d) + tan(c + d*x)^(1/2)*((2*b
^3)/d - (6*a^2*b)/d) - tan(c + d*x)^(5/2)*((2*b^3)/(5*d) - (6*a^2*b)/(5*d))
 + atan((((8*(4*b^3*d^2 - 12*a^2*b*d^2)*((6*a*b^5 + 6*a^5*b - a^6*1i + b^6*
1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*d^2))^(1/2))/d^3 - (16*tan(
c + d*x)^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*((6*a*b^5 + 6*a^
5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*d^2))^(1
/2)*1i - ((8*(4*b^3*d^2 - 12*a^2*b*d^2)*((6*a*b^5 + 6*a^5*b - a^6*1i + b^6*
1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*d^2))^(1/2))/d^3 + (16*tan(
c + d*x)^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*((6*a*b^5 + 6*a^
5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*d^2))^(1
/2)*1i)/(((8*(4*b^3*d^2 - 12*a^2*b*d^2)*((6*a*b^5 + 6*a^5*b - a^6*1i + b^6*
1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*d^2))^(1/2))/d^3 - (16*tan(
c + d*x)^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*((6*a*b^5 + 6*a^
5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*d^2))^(1
/2) - (16*(3*a*b^8 - a^9 + 8*a^3*b^6 + 6*a^5*b^4))/d^3 + ((8*(4*b^3*d^2 - 1
2*a^2*b*d^2)*((6*a*b^5 + 6*a^5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^3*b
^3 + a^4*b^2*15i)/(4*d^2))^(1/2))/d^3 + (16*tan(c + d*x)^(1/2)*(a^6 - b^6 +
 15*a^2*b^4 - 15*a^4*b^2))/d^2)*((6*a*b^5 + 6*a^5*b - a^6*1i + b^6*1i - a^2
*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*d^2))^(1/2)))*((6*a*b^5 + 6*a^5*b -
 a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*d^2))^(1/2)*2
i + atan((((8*(4*b^3*d^2 - 12*a^2*b*d^2)*((6*a*b^5 + 6*a^5*b + a^6*1i - b^6
*1i + a^2*b^4*15i - 20*a^3*b^3 - a^4*b^2*15i)/(4*d^2))^(1/2))/d^3 - (16*tan
(c + d*x)^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*((6*a*b^5 + 6*a
^5*b + a^6*1i - b^6*1i + a^2*b^4*15i - 20*a^3*b^3 - a^4*b^2*15i)/(4*d^2))^(
1/2)*1i - ((8*(4*b^3*d^2 - 12*a^2*b*d^2)*((6*a*b^5 + 6*a^5*b + a^6*1i - b^6
*1i + a^2*b^4*15i - 20*a^3*b^3 - a^4*b^2*15i)/(4*d^2))^(1/2))/d^3 + (16*tan
(c + d*x)^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*((6*a*b^5 + 6*a
^5*b + a^6*1i - b^6*1i + a^2*b^4*15i - 20*a^3*b^3 - a^4*b^2*15i)/(4*d^2))^(
1/2)*1i)/(((8*(4*b^3*d^2 - 12*a^2*b*d^2)*((6*a*b^5 + 6*a^5*b + a^6*1i - b^6
*1i + a^2*b^4*15i - 20*a^3*b^3 - a^4*b^2*15i)/(4*d^2))^(1/2))/d^3 - (16*tan
(c + d*x)^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*((6*a*b^5 + 6*a
^5*b + a^6*1i - b^6*1i + a^2*b^4*15i - 20*a^3*b^3 - a^4*b^2*15i)/(4*d^2))^(
1/2) - (16*(3*a*b^8 - a^9 + 8*a^3*b^6 + 6*a^5*b^4))/d^3 + ((8*(4*b^3*d^2 -
12*a^2*b*d^2)*((6*a*b^5 + 6*a^5*b + a^6*1i - b^6*1i + a^2*b^4*15i - 20*a^3*
b^3 - a^4*b^2*15i)/(4*d^2))^(1/2))/d^3 + (16*tan(c + d*x)^(1/2)*(a^6 - b^6
```

```
+ 15*a^2*b^4 - 15*a^4*b^2))/d^2)*((6*a*b^5 + 6*a^5*b + a^6*1i - b^6*1i + a^
2*b^4*15i - 20*a^3*b^3 - a^4*b^2*15i)/(4*d^2))^(1/2)))*((6*a*b^5 + 6*a^5*b
+ a^6*1i - b^6*1i + a^2*b^4*15i - 20*a^3*b^3 - a^4*b^2*15i)/(4*d^2))^(1/2)*
2i + (2*b^3*tan(c + d*x)^(9/2))/(9*d) + (6*a*b^2*tan(c + d*x)^(7/2))/(7*d)
```

3.570 $\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=299

$$\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}$$

```
[Out] -1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)-1/
2*(a-b)*(a^2+4*a*b+b^2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)+1/4*(a
+b)*(a^2-4*a*b+b^2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)-1/4
*(a+b)*(a^2-4*a*b+b^2)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)+
2*a*(a^2-3*b^2)*tan(d*x+c)^(1/2)/d+2/3*b*(3*a^2-b^2)*tan(d*x+c)^(3/2)/d+32/
35*a*b^2*tan(d*x+c)^(5/2)/d+2/7*b^2*tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))/d
```

Rubi [A]
time = 0.27, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3647, 3711, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d}+\frac{2b\left(3a^2-b^2\right)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{2a\left(a^2-3b^2\right)\sqrt{\tan(c+dx)}}{d}+\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{2b^2\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}{7d}+\frac{32ab^2\tan^{\frac{5}{2}}(c+dx)}{35d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[
2]*d) - ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]
)/(Sqrt[2]*d) + ((a + b)*(a^2 - 4*a*b + b^2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d
*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) - ((a + b)*(a^2 - 4*a*b + b^2)*Log[1 +
Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) + (2*a*(a^2 - 3*b
^2)*Sqrt[Tan[c + d*x]])/d + (2*b*(3*a^2 - b^2)*Tan[c + d*x]^(3/2))/(3*d) +
(32*a*b^2*Tan[c + d*x]^(5/2))/(35*d) + (2*b^2*Tan[c + d*x]^(5/2)*(a + b*Tan
[c + d*x]))/(7*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
```

```
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{2b^2\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}{7d}+\frac{2}{7}\int \tan^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}a(7a\right.$$

$$= \frac{32ab^2\tan^{\frac{5}{2}}(c+dx)}{35d}+\frac{2b^2\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}{7d}+\frac{2}{7}\int$$

$$= \frac{2b(3a^2-b^2)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{32ab^2\tan^{\frac{5}{2}}(c+dx)}{35d}+\frac{2b^2\tan^{\frac{5}{2}}(c+}{}$$

$$= \frac{2a(a^2-3b^2)\sqrt{\tan(c+dx)}}{d}+\frac{2b(3a^2-b^2)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{32ab}{}$$

$$= \frac{2a(a^2-3b^2)\sqrt{\tan(c+dx)}}{d}+\frac{2b(3a^2-b^2)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{32ab}{}$$

$$= \frac{2a(a^2-3b^2)\sqrt{\tan(c+dx)}}{d}+\frac{2b(3a^2-b^2)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{32ab}{}$$

$$= \frac{2a(a^2-3b^2)\sqrt{\tan(c+dx)}}{d}+\frac{2b(3a^2-b^2)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{32ab}{}$$

$$= \frac{(a+b)(a^2-4ab+b^2)\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx\right.}{2\sqrt{2}\,d}$$

$$= \frac{(a-b)(a^2+4ab+b^2)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a-b}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.41, size = 144, normalized size = 0.48

$$\frac{105\sqrt[4]{-1}\,(a-ib)^3\mathrm{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+105\sqrt[4]{-1}\,(a+ib)^3\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+2\sqrt{\tan(c+dx)}\,(105(a^3-3ab^2)-35b(-3a^2+b^2)\tan(c+dx)+63ab^2\tan^2(c+dx)+15b^3\tan^3(c+dx))}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (105*(-1)^(1/4)*(a - I*b)^3*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + 105*(-1
)^(1/4)*(a + I*b)^3*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + 2*Sqrt[Tan[c +
 d*x]]*(105*(a^3 - 3*a*b^2) - 35*b*(-3*a^2 + b^2)*Tan[c + d*x] + 63*a*b^2*T
an[c + d*x]^2 + 15*b^3*Tan[c + d*x]^3))/(105*d)
```

Maple [A]
time = 0.05, size = 279, normalized size = 0.93

method	result
derivativedivides	$\frac{\frac{2b^3\left(\tan^{\frac{7}{2}}(dx+c)\right)}{7}+\frac{6b^2a\left(\tan^{\frac{5}{2}}(dx+c)\right)}{5}+2a^2b\left(\tan^{\frac{3}{2}}(dx+c)\right)-\frac{2b^3\left(\tan^{\frac{3}{2}}(dx+c)\right)}{3}+2a^3\left(\sqrt{\tan}(dx+c)\right)-6b^2a\left(\sqrt{\tan}\right.}{}$
default	$\frac{\frac{2b^3\left(\tan^{\frac{7}{2}}(dx+c)\right)}{7}+\frac{6b^2a\left(\tan^{\frac{5}{2}}(dx+c)\right)}{5}+2a^2b\left(\tan^{\frac{3}{2}}(dx+c)\right)-\frac{2b^3\left(\tan^{\frac{3}{2}}(dx+c)\right)}{3}+2a^3\left(\sqrt{\tan}(dx+c)\right)-6b^2a\left(\sqrt{\tan}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/7*b^3*tan(d*x+c)^(7/2)+6/5*b^2*a*tan(d*x+c)^(5/2)+2*a^2*b*tan(d*x+c)
^(3/2)-2/3*b^3*tan(d*x+c)^(3/2)+2*a^3*tan(d*x+c)^(1/2)-6*b^2*a*tan(d*x+c)^(
1/2)+1/4*(-a^3+3*a*b^2)*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))
/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/
2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(-3*a^2*b+b^3)*2^(1/2)*(ln((
1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+
c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1
/2))))
```

Maxima [A]
time = 0.50, size = 258, normalized size = 0.86

$$\frac{120\,b^3\tan(dx+c)^{\frac{7}{2}}+504\,ab^2\tan(dx+c)^{\frac{5}{2}}-210\,\sqrt{2}\,(a^3+3\,a^2b-3\,ab^2-b^3)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\sqrt{\tan(dx+c)}\right)\right)-210\,\sqrt{2}\,(a^3+3\,a^2b-3\,ab^2-b^3)\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\sqrt{\tan(dx+c)}\right)\right)-105\,\sqrt{2}\,(a^3-3\,a^2b-3\,ab^2+b^3)\log\left(\sqrt{2}\,\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+105\,\sqrt{2}\,(a^3-3\,a^2b-3\,ab^2+b^3)\log\left(-\sqrt{2}\,\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+280\,(3\,a^2b-b^3)\tan(dx+c)^{\frac{3}{2}}+840\,(a^3-3\,ab^2)\sqrt{\tan(dx+c)}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/420*(120*b^3*tan(d*x + c)^(7/2) + 504*a*b^2*tan(d*x + c)^(5/2) - 210*sqrt
(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(ta
n(d*x + c)))) - 210*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*arctan(-1/2*sqr
t(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - 105*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b
^2 + b^3)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 105*sqrt(2)*
(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x +
 c) + 1) + 280*(3*a^2*b - b^3)*tan(d*x + c)^(3/2) + 840*(a^3 - 3*a*b^2)*sqr
t(tan(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7272 vs. $2(257) = 514$.
time = 3.30, size = 7272, normalized size = 24.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/420*(420*sqrt(2)*d^5*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 +
15*a^4*b^8 + 6*a^2*b^10 + b^12 + 2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*d^2*sqr
t((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 +
b^12)/d^4))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 -
 30*a^2*b^10 + b^12))*((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^
4*b^8 + 6*a^2*b^10 + b^12)/d^4)^(3/4)*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^
4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4)*arctan(((a^24 - 6*
a^22*b^2 - 84*a^20*b^4 - 322*a^18*b^6 - 603*a^16*b^8 - 540*a^14*b^10 + 540*
a^10*b^14 + 603*a^8*b^16 + 322*a^6*b^18 + 84*a^4*b^20 + 6*a^2*b^22 - b^24)*
d^4*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*
b^10 + b^12)/d^4)*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 25
5*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4) - sqrt(2)*((a^3 - 3*a*b^2)*d^7*sqrt((a
^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12
)/d^4)*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 -
 30*a^2*b^10 + b^12)/d^4) - (3*a^8*b + 8*a^6*b^3 + 6*a^4*b^5 - b^9)*d^5*sqr
t((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^
10 + b^12)/d^4))*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4
*b^8 + 6*a^2*b^10 + b^12 + 2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*d^2*sqrt((a^1
2 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/
d^4))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^
2*b^10 + b^12))*sqrt(((a^18 - 27*a^16*b^2 + 168*a^14*b^4 + 224*a^12*b^6 - 3
66*a^10*b^8 - 366*a^8*b^10 + 224*a^6*b^12 + 168*a^4*b^14 - 27*a^2*b^16 + b^
18)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*
a^2*b^10 + b^12)/d^4)*cos(d*x + c) + sqrt(2)*((3*a^14*b - 91*a^12*b^3 + 795
*a^10*b^5 - 1611*a^8*b^7 + 1217*a^6*b^9 - 345*a^4*b^11 + 33*a^2*b^13 - b^15
)*d^3*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^
```

2*b^10 + b^12)/d^4)*cos(d*x + c) - (a^21 - 30*a^19*b^2 + 249*a^17*b^4 - 280*a^15*b^6 - 1038*a^13*b^8 + 732*a^11*b^10 + 1322*a^9*b^12 - 504*a^7*b^14 - 531*a^5*b^16 + 82*a^3*b^18 - 3*a*b^20)*d*cos(d*x + c))*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 + 2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)^(1/4) + (a^24 - 24*a^22*b^2 + 90*a^20*b^4 + 648*a^18*b^6 + 783*a^16*b^8 - 624*a^14*b^10 - 1748*a^12*b^12 - 624*a^10*b^14 + 783*a^8*b^16 + 648*a^6*b^18 + 90*a^4*b^20 - 24*a^2*b^22 + b^24)*sin(d*x + c))/cos(d*x + c))*((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)^(3/4) - sqrt(2)*((a^15 - 15*a^13*b^2 + 9*a^11*b^4 + 81*a^9*b^6 + 27*a^7*b^8 - 69*a^5*b^10 - 37*a^3*b^12 + 3*a*b^14)*d^7*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4) - (3*a^20*b - 28*a^18*b^3 - 171*a^16*b^5 - 288*a^14*b^7 - 82*a^12*b^9 + 264*a^10*b^11 + 282*a^8*b^13 + 64*a^6*b^15 - 33*a^4*b^17 - 12*a^2*b^19 + b^21)*d^5*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4))*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 + 2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)^(3/4))/(a^36 - 18*a^34*b^2 - 39*a^32*b^4 + 848*a^30*b^6 + 5556*a^28*b^8 + 15240*a^26*b^10 + 20420*a^24*b^12 + 5424*a^22*b^14 - 25938*a^20*b^16 - 42988*a^18*b^18 - 25938*a^16*b^20 + 5424*a^14*b^22 + 20420*a^12*b^24 + 15240*a^10*b^26 + 5556*a^8*b^28 + 848*a^6*b^30 - 39*a^4*b^32 - 18*a^2*b^34 + b^36))*cos(d*x + c)^3 + 420*sqrt(2)*d^5*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 + 2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)^(3/4)*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4)*arctan(-((a^24 - 6*a^22*b^2 - 84*a^20*b^4 - 322*a^18*b^6 - 603*a^16*b^8 - 540*a^14*b^10 + 540*a^10*b^14 + 603*a^8*b^16 + 322*a^6*b^18 + 84*a^4*b^20 + 6*a^2*b^22 - b^24)*d^4*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^...

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))^3\tan^{\frac{3}{2}}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(3/2)*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*tan(c + d*x)**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 8.24, size = 1729, normalized size = 5.78

$$\sqrt{\tan(c+dx)}\left(\frac{2a^3}{d}-\frac{6ab^2}{d}\right)-\tan(c+dx)^{3/2}\left(\frac{2b^3}{3d}-\frac{2a^2b}{d}\right)+\frac{2b^3\tan(c+dx)^{7/2}}{7d}+\frac{6ab^2\tan(c+dx)^{5/2}}{5d}-\mathrm{atan}\left(\ldots\right)$$ [illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x))^3,x)
```

```
[Out] tan(c + d*x)^(1/2)*((2*a^3)/d - (6*a*b^2)/d) - tan(c + d*x)^(3/2)*((2*b^3)/
(3*d) - (2*a^2*b)/d) - atan((((8*(4*a^3*d^2 - 12*a*b^2*d^2)*(-(6*a*b^5 + 6*
a^5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*d^2))^
(1/2))/d^3 - (16*tan(c + d*x)^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/
d^2)*(-(6*a*b^5 + 6*a^5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^3*b^3 + a^
4*b^2*15i)/(4*d^2))^(1/2)*1i - ((8*(4*a^3*d^2 - 12*a*b^2*d^2)*(-(6*a*b^5 +
6*a^5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*d^2)
)^(1/2))/d^3 + (16*tan(c + d*x)^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2)
)/d^2)*(-(6*a*b^5 + 6*a^5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^3*b^3 +
a^4*b^2*15i)/(4*d^2))^(1/2)*1i)/((16*(3*a^8*b - b^9 + 6*a^4*b^5 + 8*a^6*b^3
))/d^3 + ((8*(4*a^3*d^2 - 12*a*b^2*d^2)*(-(6*a*b^5 + 6*a^5*b - a^6*1i + b^6
*1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*d^2))^(1/2))/d^3 - (16*tan
(c + d*x)^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*(-(6*a*b^5 + 6*
a^5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*d^2))^
(1/2) + ((8*(4*a^3*d^2 - 12*a*b^2*d^2)*(-(6*a*b^5 + 6*a^5*b - a^6*1i + b^6*
1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*d^2))^(1/2))/d^3 + (16*tan(
c + d*x)^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*(-(6*a*b^5 + 6*a
^5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*d^2))^(
```

1/2)))*(-(6*a*b^5 + 6*a^5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*d^2))^(1/2)*2i - atan((((8*(4*a^3*d^2 - 12*a*b^2*d^2)*(-(6*a*b^5 + 6*a^5*b + a^6*1i - b^6*1i + a^2*b^4*15i - 20*a^3*b^3 - a^4*b^2*15i)/(4*d^2))^(1/2))/d^3 - (16*tan(c + d*x)^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*(-(6*a*b^5 + 6*a^5*b + a^6*1i - b^6*1i + a^2*b^4*15i - 20*a^3*b^3 - a^4*b^2*15i)/(4*d^2))^(1/2)*1i - ((8*(4*a^3*d^2 - 12*a*b^2*d^2)*(-(6*a*b^5 + 6*a^5*b + a^6*1i - b^6*1i + a^2*b^4*15i - 20*a^3*b^3 - a^4*b^2*15i)/(4*d^2))^(1/2))/d^3 + (16*tan(c + d*x)^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*(-(6*a*b^5 + 6*a^5*b + a^6*1i - b^6*1i + a^2*b^4*15i - 20*a^3*b^3 - a^4*b^2*15i)/(4*d^2))^(1/2)*1i)/((16*(3*a^8*b - b^9 + 6*a^4*b^5 + 8*a^6*b^3))/d^3 + ((8*(4*a^3*d^2 - 12*a*b^2*d^2)*(-(6*a*b^5 + 6*a^5*b + a^6*1i - b^6*1i + a^2*b^4*15i - 20*a^3*b^3 - a^4*b^2*15i)/(4*d^2))^(1/2))/d^3 - (16*tan(c + d*x)^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*(-(6*a*b^5 + 6*a^5*b + a^6*1i - b^6*1i + a^2*b^4*15i - 20*a^3*b^3 - a^4*b^2*15i)/(4*d^2))^(1/2) + ((8*(4*a^3*d^2 - 12*a*b^2*d^2)*(-(6*a*b^5 + 6*a^5*b + a^6*1i - b^6*1i + a^2*b^4*15i - 20*a^3*b^3 - a^4*b^2*15i)/(4*d^2))^(1/2))/d^3 + (16*tan(c + d*x)^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*(-(6*a*b^5 + 6*a^5*b + a^6*1i - b^6*1i + a^2*b^4*15i - 20*a^3*b^3 - a^4*b^2*15i)/(4*d^2))^(1/2)))*(-(6*a*b^5 + 6*a^5*b + a^6*1i - b^6*1i + a^2*b^4*15i - 20*a^3*b^3 - a^4*b^2*15i)/(4*d^2))^(1/2)*2i + (2*b^3*tan(c + d*x)^(7/2))/(7*d) + (6*a*b^2*tan(c + d*x)^(5/2))/(5*d)

3.571 $\int \sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=272

$$-\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan\right.}}{\sqrt{2}\,d}$$

```
[Out] 1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)+1/2
*(a+b)*(a^2-4*a*b+b^2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)+1/4*(a-
b)*(a^2+4*a*b+b^2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)-1/4*
(a-b)*(a^2+4*a*b+b^2)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)+2
*b*(3*a^2-b^2)*tan(d*x+c)^(1/2)/d+8/5*a*b^2*tan(d*x+c)^(3/2)/d+2/5*b^2*tan(
d*x+c)^(3/2)*(a+b*tan(d*x+c))/d
```

Rubi [A]
time = 0.25, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3647, 3711, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d}+\frac{2b\left(3a^2-b^2\right)\sqrt{\tan(c+dx)}}{d}+\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\tan(c+dx)-\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\tan(c+dx)+\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{8ab^2\tan^{\frac{3}{2}}(c+dx)}{5d}+\frac{2b^2\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -(((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqr
t[2]*d)) + ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x
]]])/(Sqrt[2]*d) + ((a - b)*(a^2 + 4*a*b + b^2)*Log[1 - Sqrt[2]*Sqrt[Tan[c
+ d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) - ((a - b)*(a^2 + 4*a*b + b^2)*Log[1
 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) + (2*b*(3*a^2
- b^2)*Sqrt[Tan[c + d*x]])/d + (8*a*b^2*Tan[c + d*x]^(3/2))/(5*d) + (2*b^2*
Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x]))/(5*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
```

```
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^3\,dx = \frac{2b^2\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}{5d} + \frac{2}{5}\int \sqrt{\tan(c+dx)}\,\Big(\frac{1}{2}c$$

$$= \frac{8ab^2\tan^{\frac{3}{2}}(c+dx)}{5d} + \frac{2b^2\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}{5d} + \frac{2}{5}$$

$$= \frac{2b(3a^2-b^2)\sqrt{\tan(c+dx)}}{d} + \frac{8ab^2\tan^{\frac{3}{2}}(c+dx)}{5d} + \frac{2b^2\tan^{\frac{3}{2}}(c}{}$$

$$= \frac{2b(3a^2-b^2)\sqrt{\tan(c+dx)}}{d} + \frac{8ab^2\tan^{\frac{3}{2}}(c+dx)}{5d} + \frac{2b^2\tan^{\frac{3}{2}}(c}{}$$

$$= \frac{2b(3a^2-b^2)\sqrt{\tan(c+dx)}}{d} + \frac{8ab^2\tan^{\frac{3}{2}}(c+dx)}{5d} + \frac{2b^2\tan^{\frac{3}{2}}(c}{}$$

$$= \frac{2b(3a^2-b^2)\sqrt{\tan(c+dx)}}{d} + \frac{8ab^2\tan^{\frac{3}{2}}(c+dx)}{5d} + \frac{2b^2\tan^{\frac{3}{2}}(c}{}$$

$$= \frac{(a-b)(a^2+4ab+b^2)\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+\right.}{2\sqrt{2}\,d}$$

$$= -\frac{(a+b)(a^2-4ab+b^2)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d} + \frac{(a}{}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.83, size = 120, normalized size = 0.44

$$\frac{-5\sqrt[4]{-1}\,(ia+b)^3\operatorname{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)-5(-1)^{3/4}(a+ib)^3\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+2b\sqrt{\tan(c+dx)}\,(15a^2-5b^2+5ab\tan(c+dx)+b^2\tan^2(c+dx))}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-5*(-1)^(1/4)*(I*a + b)^3*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] - 5*(-1)^(
3/4)*(a + I*b)^3*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + 2*b*Sqrt[Tan[c +
d*x]]*(15*a^2 - 5*b^2 + 5*a*b*Tan[c + d*x] + b^2*Tan[c + d*x]^2))/(5*d)
```

Maple [A]
time = 0.06, size = 250, normalized size = 0.92

method	result
derivativedivides	$\frac{\frac{2b^3\left(\tan^{\frac{5}{2}}(dx+c)\right)}{5}+2a\,b^2\left(\tan^{\frac{3}{2}}(dx+c)\right)+6a^2b\left(\sqrt{\tan}\,(dx+c)\right)-2b^3\left(\sqrt{\tan}\,(dx+c)\right)+\frac{\left(-3a^2b+b^3\right)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{}}{1-\sqrt{}}\right.\right.}{}}{}$ [truncated]
default	$\frac{\frac{2b^3\left(\tan^{\frac{5}{2}}(dx+c)\right)}{5}+2a\,b^2\left(\tan^{\frac{3}{2}}(dx+c)\right)+6a^2b\left(\sqrt{\tan}\,(dx+c)\right)-2b^3\left(\sqrt{\tan}\,(dx+c)\right)+\frac{\left(-3a^2b+b^3\right)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{}}{1-\sqrt{}}\right.\right.}{}}{}$ [truncated]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/5*b^3*tan(d*x+c)^(5/2)+2*a*b^2*tan(d*x+c)^(3/2)+6*a^2*b*tan(d*x+c)^(
1/2)-2*b^3*tan(d*x+c)^(1/2)+1/4*(-3*a^2*b+b^3)*2^(1/2)*(ln((1+2^(1/2)*tan(d
*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1
+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(a^3-
3*a*b^2)*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan
(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1
+2^(1/2)*tan(d*x+c)^(1/2))))
```

Maxima [A]
time = 0.51, size = 238, normalized size = 0.88

$$\frac{8\,b^3\tan(dx+c)^{\frac{5}{2}}+40\,ab^2\tan(dx+c)^{\frac{3}{2}}+10\,\sqrt{2}\,(a^3-3\,a^2b-3\,ab^2+b^3)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+10\,\sqrt{2}\,(a^3-3\,a^2b-3\,ab^2+b^3)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-5\,\sqrt{2}\,(a^3+3\,a^2b-3\,ab^2-b^3)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+5\,\sqrt{2}\,(a^3+3\,a^2b-3\,ab^2-b^3)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+40\,(3\,a^2b-b^3)\sqrt{\tan(dx+c)}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/20*(8*b^3*tan(d*x + c)^(5/2) + 40*a*b^2*tan(d*x + c)^(3/2) + 10*sqrt(2)*(
a^3 - 3*a^2*b - 3*a*b^2 + b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x
```

```
 + c)))) + 10*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*arctan(-1/2*sqrt(2)*(
sqrt(2) - 2*sqrt(tan(d*x + c)))) - 5*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3
)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 5*sqrt(2)*(a^3 + 3*a
^2*b - 3*a*b^2 - b^3)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) +
 40*(3*a^2*b - b^3)*sqrt(tan(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7179 vs. $2(234) = 468$.
time = 3.74, size = 7179, normalized size = 26.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/20*(20*sqrt(2)*d^5*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 1
5*a^4*b^8 + 6*a^2*b^10 + b^12 - 2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*d^2*sqrt
((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b
^12)/d^4))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 -
30*a^2*b^10 + b^12))*((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4
*b^8 + 6*a^2*b^10 + b^12)/d^4)^(3/4)*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4
 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4)*arctan(-((a^24 - 6*
a^22*b^2 - 84*a^20*b^4 - 322*a^18*b^6 - 603*a^16*b^8 - 540*a^14*b^10 + 540*
a^10*b^14 + 603*a^8*b^16 + 322*a^6*b^18 + 84*a^4*b^20 + 6*a^2*b^22 - b^24)*
d^4*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*
b^10 + b^12)/d^4)*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 25
5*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4) + sqrt(2)*((3*a^2*b - b^3)*d^7*sqrt((a
^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12
)/d^4)*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 -
 30*a^2*b^10 + b^12)/d^4) + (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^5*sqr
t((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^
10 + b^12)/d^4))*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4
*b^8 + 6*a^2*b^10 + b^12 - 2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*d^2*sqrt((a^1
2 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/
d^4))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^
2*b^10 + b^12))*sqrt(((a^18 - 27*a^16*b^2 + 168*a^14*b^4 + 224*a^12*b^6 - 3
66*a^10*b^8 - 366*a^8*b^10 + 224*a^6*b^12 + 168*a^4*b^14 - 27*a^2*b^16 + b^
18)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*
a^2*b^10 + b^12)/d^4)*cos(d*x + c) + sqrt(2)*((a^15 - 33*a^13*b^2 + 345*a^1
1*b^4 - 1217*a^9*b^6 + 1611*a^7*b^8 - 795*a^5*b^10 + 91*a^3*b^12 - 3*a*b^14
)*d^3*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^
2*b^10 + b^12)/d^4)*cos(d*x + c) + (3*a^20*b - 82*a^18*b^3 + 531*a^16*b^5 +
 504*a^14*b^7 - 1322*a^12*b^9 - 732*a^10*b^11 + 1038*a^8*b^13 + 280*a^6*b^1
5 - 249*a^4*b^17 + 30*a^2*b^19 - b^21)*d*cos(d*x + c))*sqrt((a^12 + 6*a^10*
b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 - 2*(3*a^5*b
```

```
 - 10*a^3*b^3 + 3*a*b^5)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*
b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4))/(a^12 - 30*a^10*b^2 + 255*a^8*b
^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*sqrt(sin(d*x + c)/cos
(d*x + c))*((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a
^2*b^10 + b^12)/d^4)^(1/4) + (a^24 - 24*a^22*b^2 + 90*a^20*b^4 + 648*a^18*b
^6 + 783*a^16*b^8 - 624*a^14*b^10 - 1748*a^12*b^12 - 624*a^10*b^14 + 783*a^
8*b^16 + 648*a^6*b^18 + 90*a^4*b^20 - 24*a^2*b^22 + b^24)*sin(d*x + c))/cos
(d*x + c))*((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a
^2*b^10 + b^12)/d^4)^(3/4) + sqrt(2)*((3*a^14*b - 37*a^12*b^3 - 69*a^10*b^5
 + 27*a^8*b^7 + 81*a^6*b^9 + 9*a^4*b^11 - 15*a^2*b^13 + b^15)*d^7*sqrt((a^1
2 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/
d^4)*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 3
0*a^2*b^10 + b^12)/d^4) + (a^21 - 12*a^19*b^2 - 33*a^17*b^4 + 64*a^15*b^6 +
 282*a^13*b^8 + 264*a^11*b^10 - 82*a^9*b^12 - 288*a^7*b^14 - 171*a^5*b^16 -
 28*a^3*b^18 + 3*a*b^20)*d^5*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a
^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4))*sqrt((a^12 + 6*a^10*b^2 +
15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 - 2*(3*a^5*b - 10*
a^3*b^3 + 3*a*b^5)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 +
15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 4
52*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*sqrt(sin(d*x + c)/cos(d*x +
 c))*((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^1
0 + b^12)/d^4)^(3/4))/(a^36 - 18*a^34*b^2 - 39*a^32*b^4 + 848*a^30*b^6 + 55
56*a^28*b^8 + 15240*a^26*b^10 + 20420*a^24*b^12 + 5424*a^22*b^14 - 25938*a^
20*b^16 - 42988*a^18*b^18 - 25938*a^16*b^20 + 5424*a^14*b^22 + 20420*a^12*b
^24 + 15240*a^10*b^26 + 5556*a^8*b^28 + 848*a^6*b^30 - 39*a^4*b^32 - 18*a^2
*b^34 + b^36))*cos(d*x + c)^2 + 20*sqrt(2)*d^5*sqrt((a^12 + 6*a^10*b^2 + 15
*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 - 2*(3*a^5*b - 10*a^
3*b^3 + 3*a*b^5)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15
*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452
*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*((a^12 + 6*a^10*b^2 + 15*a^8*
b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)^(3/4)*sqrt((a^12 -
30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)
/d^4)*arctan(((a^24 - 6*a^22*b^2 - 84*a^20*b^4 - 322*a^18*b^6 - 603*a^16*b^
8 - 540*a^14*b^10 + 540*a^10*b^14 + 603*a^8*b^16 + 322*a^6*b^18 + 84*a^4*b^
20 + 6*a^2*b^22 - b^24)*d^4*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b
^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)*sqrt((a^12 - 30*a^10*b^2 + 255*a^
8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx))^3 \sqrt{\tan(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/2)*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*sqrt(tan(c + d*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 6.09, size = 1742, normalized size = 6.40

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)*(a + b*tan(c + d*x))^3,x)
```

```
[Out] (2*b^3*tan(c + d*x)^(5/2))/(5*d) - atan((((8*(4*b^3*d^2 - 12*a^2*b*d^2)*((6
*a*b^5 + 6*a^5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i
)/(4*d^2))^(1/2))/d^3 - (16*tan(c + d*x)^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15
*a^4*b^2))/d^2)*((6*a*b^5 + 6*a^5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^
3*b^3 + a^4*b^2*15i)/(4*d^2))^(1/2)*1i - ((8*(4*b^3*d^2 - 12*a^2*b*d^2)*((6
*a*b^5 + 6*a^5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i
)/(4*d^2))^(1/2))/d^3 + (16*tan(c + d*x)^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15
*a^4*b^2))/d^2)*((6*a*b^5 + 6*a^5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^
3*b^3 + a^4*b^2*15i)/(4*d^2))^(1/2)*1i)/(((8*(4*b^3*d^2 - 12*a^2*b*d^2)*((6
*a*b^5 + 6*a^5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i
)/(4*d^2))^(1/2))/d^3 - (16*tan(c + d*x)^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15
*a^4*b^2))/d^2)*((6*a*b^5 + 6*a^5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^
3*b^3 + a^4*b^2*15i)/(4*d^2))^(1/2) - (16*(3*a*b^8 - a^9 + 8*a^3*b^6 + 6*a^
5*b^4))/d^3 + ((8*(4*b^3*d^2 - 12*a^2*b*d^2)*((6*a*b^5 + 6*a^5*b - a^6*1i +
 b^6*1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*d^2))^(1/2))/d^3 + (16
*tan(c + d*x)^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*((6*a*b^5 +
 6*a^5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*d^2
))^(1/2)))*((6*a*b^5 + 6*a^5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^3*b^3
 + a^4*b^2*15i)/(4*d^2))^(1/2)*2i - atan((((8*(4*b^3*d^2 - 12*a^2*b*d^2)*((
6*a*b^5 + 6*a^5*b + a^6*1i - b^6*1i + a^2*b^4*15i - 20*a^3*b^3 - a^4*b^2*15
i)/(4*d^2))^(1/2))/d^3 - (16*tan(c + d*x)^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 1
5*a^4*b^2))/d^2)*((6*a*b^5 + 6*a^5*b + a^6*1i - b^6*1i + a^2*b^4*15i - 20*a
^3*b^3 - a^4*b^2*15i)/(4*d^2))^(1/2)*1i - ((8*(4*b^3*d^2 - 12*a^2*b*d^2)*((
6*a*b^5 + 6*a^5*b + a^6*1i - b^6*1i + a^2*b^4*15i - 20*a^3*b^3 - a^4*b^2*15
i)/(4*d^2))^(1/2))/d^3 + (16*tan(c + d*x)^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 1
5*a^4*b^2))/d^2)*((6*a*b^5 + 6*a^5*b + a^6*1i - b^6*1i + a^2*b^4*15i - 20*a
```

```
^3*b^3 - a^4*b^2*15i)/(4*d^2))^(1/2)*1i)/(((8*(4*b^3*d^2 - 12*a^2*b*d^2)*((
6*a*b^5 + 6*a^5*b + a^6*1i - b^6*1i + a^2*b^4*15i - 20*a^3*b^3 - a^4*b^2*15
i)/(4*d^2))^(1/2))/d^3 - (16*tan(c + d*x)^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 1
5*a^4*b^2))/d^2)*((6*a*b^5 + 6*a^5*b + a^6*1i - b^6*1i + a^2*b^4*15i - 20*a
^3*b^3 - a^4*b^2*15i)/(4*d^2))^(1/2) - (16*(3*a*b^8 - a^9 + 8*a^3*b^6 + 6*a
^5*b^4))/d^3 + ((8*(4*b^3*d^2 - 12*a^2*b*d^2)*((6*a*b^5 + 6*a^5*b + a^6*1i
- b^6*1i + a^2*b^4*15i - 20*a^3*b^3 - a^4*b^2*15i)/(4*d^2))^(1/2))/d^3 + (1
6*tan(c + d*x)^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*((6*a*b^5
+ 6*a^5*b + a^6*1i - b^6*1i + a^2*b^4*15i - 20*a^3*b^3 - a^4*b^2*15i)/(4*d^
2))^(1/2)))*((6*a*b^5 + 6*a^5*b + a^6*1i - b^6*1i + a^2*b^4*15i - 20*a^3*b^
3 - a^4*b^2*15i)/(4*d^2))^(1/2)*2i - tan(c + d*x)^(1/2)*((2*b^3)/d - (6*a^2
*b)/d) + (2*a*b^2*tan(c + d*x)^(3/2))/d
```

3.572 $\int \frac{(a+b\tan(c+dx))^3}{\sqrt{\tan(c+dx)}}\,dx$

Optimal. Leaf size=245

$$-\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan\right.}}{\sqrt{2}\,d}$$

```
[Out] 1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)+1/2
*(a-b)*(a^2+4*a*b+b^2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)-1/4*(a+
b)*(a^2-4*a*b+b^2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)+1/4*
(a+b)*(a^2-4*a*b+b^2)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)+1
6/3*a*b^2*tan(d*x+c)^(1/2)/d+2/3*b^2*tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))/d
```

Rubi [A]
time = 0.21, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3647, 3711, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d}-\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{2b^2\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))}{3d}+\frac{16ab^2\sqrt{\tan(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^3/Sqrt[Tan[c + d*x]],x]
```

```
[Out] -(((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqr
t[2]*d)) + ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x
]]])/(Sqrt[2]*d) - ((a + b)*(a^2 - 4*a*b + b^2)*Log[1 - Sqrt[2]*Sqrt[Tan[c
+ d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) + ((a + b)*(a^2 - 4*a*b + b^2)*Log[1
 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) + (16*a*b^2*Sq
rt[Tan[c + d*x]])/(3*d) + (2*b^2*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x]))/(
3*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\tan(c+dx))^3}{\sqrt{\tan(c+dx)}}\,dx = \frac{2b^2\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))}{3d} + \frac{2}{3}\int \frac{\frac{1}{2}a(3a^2-b^2)+\frac{3}{2}b(3a^2-b^2)\tan}{\sqrt{\tan(c-}}$$

$$= \frac{16ab^2\sqrt{\tan(c+dx)}}{3d} + \frac{2b^2\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))}{3d} + \frac{2}{3}\int \frac{\frac{3}{2}a(a^2-}{}$$

$$= \frac{16ab^2\sqrt{\tan(c+dx)}}{3d} + \frac{2b^2\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))}{3d} + \frac{4\text{Subst}\Big(\int \frac{3}{2}a}{}$$

$$= \frac{16ab^2\sqrt{\tan(c+dx)}}{3d} + \frac{2b^2\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))}{3d} + \frac{((a+b)\,(a^2-}{}$$

$$= \frac{16ab^2\sqrt{\tan(c+dx)}}{3d} + \frac{2b^2\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))}{3d} - \frac{((a+b)\,(a^2-}{}$$

$$= -\frac{(a+b)\,(a^2-4ab+b^2)\log\Big(1-\sqrt{2}\,\sqrt{\tan(c+dx)}+\tan(c+dx)\Big)}{2\sqrt{2}\,d} + \frac{(a+}{}$$

$$= -\frac{(a-b)\,(a^2+4ab+b^2)\tan^{-1}\Big(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\Big)}{\sqrt{2}\,d} + \frac{(a-b)\,(a^2+4ab}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.36, size = 101, normalized size = 0.41

$$\frac{-3\sqrt[4]{-1}\,(a-ib)^3\text{ArcTan}\Big((-1)^{3/4}\sqrt{\tan(c+dx)}\Big) - 3\sqrt[4]{-1}\,(a+ib)^3\tanh^{-1}\Big((-1)^{3/4}\sqrt{\tan(c+dx)}\Big) + 2b^2\sqrt{\tan(c+dx)}\,(9a+b\tan(c+dx))}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^3/Sqrt[Tan[c + d*x]],x]
```

```
[Out] (-3*(-1)^(1/4)*(a - I*b)^3*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] - 3*(-1)^(
1/4)*(a + I*b)^3*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + 2*b^2*Sqrt[Tan[c
+ d*x]]*(9*a + b*Tan[c + d*x]))/(3*d)
```

Maple [A]
time = 0.05, size = 225, normalized size = 0.92

method	result
derivativedivides	$\frac{\frac{2b^3\left(\tan^{\frac{3}{2}}(dx+c)\right)}{3}+6b^2a\left(\sqrt{\tan}\,(dx+c)\right)+\frac{\left(a^3-3b^2a\right)\sqrt{2}\,\left(\ln\left(\frac{1+\sqrt{2}\,\left(\sqrt{\tan}\,(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\,\left(\sqrt{\tan}\,(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{\right.\right.}{4}}{}$
default	$\frac{\frac{2b^3\left(\tan^{\frac{3}{2}}(dx+c)\right)}{3}+6b^2a\left(\sqrt{\tan}\,(dx+c)\right)+\frac{\left(a^3-3b^2a\right)\sqrt{2}\,\left(\ln\left(\frac{1+\sqrt{2}\,\left(\sqrt{\tan}\,(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\,\left(\sqrt{\tan}\,(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{\right.\right.}{4}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^3/tan(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/3*b^3*tan(d*x+c)^(3/2)+6*b^2*a*tan(d*x+c)^(1/2)+1/4*(a^3-3*a*b^2)*2^
(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/
2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*ta
n(d*x+c)^(1/2)))+1/4*(3*a^2*b-b^3)*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+
tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan
(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))))
```

Maxima [A]
time = 0.50, size = 216, normalized size = 0.88

$$\frac{8\,b^3\tan\left(dx+c\right)^{\frac{3}{2}}+72\,ab^2\sqrt{\tan\left(dx+c\right)}+6\,\sqrt{2}\left(a^3+3\,a^2b-3\,ab^2-b^3\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\sqrt{\tan\left(dx+c\right)}\right)\right)+6\,\sqrt{2}\left(a^3+3\,a^2b-3\,ab^2-b^3\right)\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\sqrt{\tan\left(dx+c\right)}\right)\right)+3\,\sqrt{2}\left(a^3-3\,a^2b-3\,ab^2+b^3\right)\log\left(\sqrt{2}\,\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)-3\,\sqrt{2}\left(a^3-3\,a^2b-3\,ab^2+b^3\right)\log\left(-\sqrt{2}\,\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^3/tan(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(8*b^3*tan(d*x + c)^(3/2) + 72*a*b^2*sqrt(tan(d*x + c)) + 6*sqrt(2)*(a
^3 + 3*a^2*b - 3*a*b^2 - b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x
+ c)))) + 6*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*arctan(-1/2*sqrt(2)*(sq
rt(2) - 2*sqrt(tan(d*x + c)))) + 3*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*
log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - 3*sqrt(2)*(a^3 - 3*a^2
*b - 3*a*b^2 + b^3)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7117 vs. 2(209) = 418.
time = 3.32, size = 7117, normalized size = 29.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(d*x+c))^3/tan(d*x+c)^(1/2),x, algorithm="fricas")

[Out] -1/12*(12*sqrt(2)*d^5*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 + 2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)^(3/4)*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4)*arctan(((a^24 - 6*a^22*b^2 - 84*a^20*b^4 - 322*a^18*b^6 - 603*a^16*b^8 - 540*a^14*b^10 + 540*a^10*b^14 + 603*a^8*b^16 + 322*a^6*b^18 + 84*a^4*b^20 + 6*a^2*b^22 - b^24)*d^4*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4) - sqrt(2)*((a^3 - 3*a*b^2)*d^7*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4) - (3*a^8*b + 8*a^6*b^3 + 6*a^4*b^5 - b^9)*d^5*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4))*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 + 2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*sqrt(((a^18 - 27*a^16*b^2 + 168*a^14*b^4 + 224*a^12*b^6 - 366*a^10*b^8 - 366*a^8*b^10 + 224*a^6*b^12 + 168*a^4*b^14 - 27*a^2*b^16 + b^18)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)*cos(d*x + c) + sqrt(2)*((3*a^14*b - 91*a^12*b^3 + 795*a^10*b^5 - 1611*a^8*b^7 + 1217*a^6*b^9 - 345*a^4*b^11 + 33*a^2*b^13 - b^15)*d^3*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)*cos(d*x + c) - (a^21 - 30*a^19*b^2 + 249*a^17*b^4 - 280*a^15*b^6 - 1038*a^13*b^8 + 732*a^11*b^10 + 1322*a^9*b^12 - 504*a^7*b^14 - 531*a^5*b^16 + 82*a^3*b^18 - 3*a*b^20)*d*cos(d*x + c))*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 + 2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)^(1/4) + (a^24 - 24*a^22*b^2 + 90*a^20*b^4 + 648*a^18*b^6 + 783*a^16*b^8 - 624*a^14*b^10 - 1748*a^12*b^12 - 624*a^10*b^14 + 783*a^8*b^16 + 648*a^6*b^18 + 90*a^4*b^20 - 24*a^2*b^22 + b^24)*sin(d*x + c))/cos(d*x + c))*((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)^(3/4) - sqrt(2)*((a^15 - 15*a^13*b^2 + 9*a^11*b^4 + 81*a^9*b^6 + 27*a^7*b^8 - 69*a^5*b^10 - 37*a^3*b^12 + 3*a*b^14)*d^7*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4) - (3*a^20*b - 28*a^18*b^3 - 171*a^16*b^5 - 288*a^14*

```
b^7 - 82*a^12*b^9 + 264*a^10*b^11 + 282*a^8*b^13 + 64*a^6*b^15 - 33*a^4*b^1
7 - 12*a^2*b^19 + b^21)*d^5*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^
6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4))*sqrt((a^12 + 6*a^10*b^2 + 1
5*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 + 2*(3*a^5*b - 10*a
^3*b^3 + 3*a*b^5)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 1
5*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 45
2*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*sqrt(sin(d*x + c)/cos(d*x +
c))*((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10
 + b^12)/d^4)^(3/4))/(a^36 - 18*a^34*b^2 - 39*a^32*b^4 + 848*a^30*b^6 + 555
6*a^28*b^8 + 15240*a^26*b^10 + 20420*a^24*b^12 + 5424*a^22*b^14 - 25938*a^2
0*b^16 - 42988*a^18*b^18 - 25938*a^16*b^20 + 5424*a^14*b^22 + 20420*a^12*b^
24 + 15240*a^10*b^26 + 5556*a^8*b^28 + 848*a^6*b^30 - 39*a^4*b^32 - 18*a^2*
b^34 + b^36))*cos(d*x + c) + 12*sqrt(2)*d^5*sqrt((a^12 + 6*a^10*b^2 + 15*a^
8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 + 2*(3*a^5*b - 10*a^3*b
^3 + 3*a*b^5)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^
4*b^8 + 6*a^2*b^10 + b^12)/d^4))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^
6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*((a^12 + 6*a^10*b^2 + 15*a^8*b^4
 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)^(3/4)*sqrt((a^12 - 30*
a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^
4)*arctan(-((a^24 - 6*a^22*b^2 - 84*a^20*b^4 - 322*a^18*b^6 - 603*a^16*b^8
- 540*a^14*b^10 + 540*a^10*b^14 + 603*a^8*b^16 + 322*a^6*b^18 + 84*a^4*b^20
 + 6*a^2*b^22 - b^24)*d^4*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6
 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*
b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 +...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan{(c + dx)})^3}{\sqrt{\tan{(c + dx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**3/tan(d*x+c)**(1/2),x)
[Out] Integral((a + b*tan(c + d*x))**3/sqrt(tan(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^3/tan(d*x+c)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [B]

time = 4.97, size = 1674, normalized size = 6.83

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^3/tan(c + d*x)^(1/2),x)
```

```
[Out] atan((((8*(4*a^3*d^2 - 12*a*b^2*d^2)*(-(6*a*b^5 + 6*a^5*b - a^6*1i + b^6*1i
 - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*d^2))^(1/2))/d^3 - (16*tan(c
+ d*x)^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*(-(6*a*b^5 + 6*a^5
*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*d^2))^(1/
2)*1i - ((8*(4*a^3*d^2 - 12*a*b^2*d^2)*(-(6*a*b^5 + 6*a^5*b - a^6*1i + b^6*
1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*d^2))^(1/2))/d^3 + (16*tan(
c + d*x)^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*(-(6*a*b^5 + 6*a
^5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*d^2))^(
1/2)*1i)/((16*(3*a^8*b - b^9 + 6*a^4*b^5 + 8*a^6*b^3))/d^3 + ((8*(4*a^3*d^2
 - 12*a*b^2*d^2)*(-(6*a*b^5 + 6*a^5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*
a^3*b^3 + a^4*b^2*15i)/(4*d^2))^(1/2))/d^3 - (16*tan(c + d*x)^(1/2)*(a^6 -
b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*(-(6*a*b^5 + 6*a^5*b - a^6*1i + b^6*1i
 - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*d^2))^(1/2) + ((8*(4*a^3*d^2
- 12*a*b^2*d^2)*(-(6*a*b^5 + 6*a^5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*a
^3*b^3 + a^4*b^2*15i)/(4*d^2))^(1/2))/d^3 + (16*tan(c + d*x)^(1/2)*(a^6 - b
^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*(-(6*a*b^5 + 6*a^5*b - a^6*1i + b^6*1i
- a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*d^2))^(1/2)))*(-(6*a*b^5 + 6*a
^5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*d^2))^(
1/2)*2i + atan((((8*(4*a^3*d^2 - 12*a*b^2*d^2)*(-(6*a*b^5 + 6*a^5*b + a^6*1
i - b^6*1i + a^2*b^4*15i - 20*a^3*b^3 - a^4*b^2*15i)/(4*d^2))^(1/2))/d^3 -
(16*tan(c + d*x)^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*(-(6*a*b
^5 + 6*a^5*b + a^6*1i - b^6*1i + a^2*b^4*15i - 20*a^3*b^3 - a^4*b^2*15i)/(4
*d^2))^(1/2)*1i - ((8*(4*a^3*d^2 - 12*a*b^2*d^2)*(-(6*a*b^5 + 6*a^5*b + a^6
*1i - b^6*1i + a^2*b^4*15i - 20*a^3*b^3 - a^4*b^2*15i)/(4*d^2))^(1/2))/d^3
+ (16*tan(c + d*x)^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*(-(6*a
*b^5 + 6*a^5*b + a^6*1i - b^6*1i + a^2*b^4*15i - 20*a^3*b^3 - a^4*b^2*15i)/
(4*d^2))^(1/2)*1i)/((16*(3*a^8*b - b^9 + 6*a^4*b^5 + 8*a^6*b^3))/d^3 + ((8*
(4*a^3*d^2 - 12*a*b^2*d^2)*(-(6*a*b^5 + 6*a^5*b + a^6*1i - b^6*1i + a^2*b^4
*15i - 20*a^3*b^3 - a^4*b^2*15i)/(4*d^2))^(1/2))/d^3 - (16*tan(c + d*x)^(1/
2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*(-(6*a*b^5 + 6*a^5*b + a^6*1
i - b^6*1i + a^2*b^4*15i - 20*a^3*b^3 - a^4*b^2*15i)/(4*d^2))^(1/2) + ((8*(
4*a^3*d^2 - 12*a*b^2*d^2)*(-(6*a*b^5 + 6*a^5*b + a^6*1i - b^6*1i + a^2*b^4*
15i - 20*a^3*b^3 - a^4*b^2*15i)/(4*d^2))^(1/2))/d^3 + (16*tan(c + d*x)^(1/2
)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*(-(6*a*b^5 + 6*a^5*b + a^6*1i
 - b^6*1i + a^2*b^4*15i - 20*a^3*b^3 - a^4*b^2*15i)/(4*d^2))^(1/2)))*(-(6*a
*b^5 + 6*a^5*b + a^6*1i - b^6*1i + a^2*b^4*15i - 20*a^3*b^3 - a^4*b^2*15i)/
(4*d^2))^(1/2)*2i + (2*b^3*tan(c + d*x)^(3/2))/(3*d) + (6*a*b^2*tan(c + d*x
)^(1/2))/d
```

3.573 $\int \frac{(a+b\tan(c+dx))^3}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=245

$$\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right.}{\sqrt{2}\,d}$$

```
[Out] -1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)-1/
2*(a+b)*(a^2-4*a*b+b^2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)-1/4*(a
-b)*(a^2+4*a*b+b^2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)+1/4
*(a-b)*(a^2+4*a*b+b^2)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)+
2*b*(a^2+b^2)*tan(d*x+c)^(1/2)/d-2*a^2*(a+b*tan(d*x+c))/d/tan(d*x+c)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3646, 3711, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d}+\frac{2b\left(a^2+b^2\right)\sqrt{\tan(c+dx)}}{d}-\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{2a^2(a+b\tan(c+dx))}{d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^3/Tan[c + d*x]^(3/2),x]
```

```
[Out] ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[
2]*d) - ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]
)/(Sqrt[2]*d) - ((a - b)*(a^2 + 4*a*b + b^2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d
*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) + ((a - b)*(a^2 + 4*a*b + b^2)*Log[1 +
Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) + (2*b*(a^2 + b^2
)*Sqrt[Tan[c + d*x]])/d - (2*a^2*(a + b*Tan[c + d*x]))/(d*Sqrt[Tan[c + d*x]
])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
```

```
n, -1] && IntegerQ[2*m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\tan(c+dx))^3}{\tan^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2a^2(a+b\tan(c+dx))}{d\sqrt{\tan(c+dx)}} + 2\int \frac{2a^2b-\frac{1}{2}a(a^2-3b^2)\tan(c+dx)+\frac{1}{2}b(a^2+b^2)\,t}{\sqrt{\tan(c+dx)}}$$

$$= \frac{2b(a^2+b^2)\sqrt{\tan(c+dx)}}{d} - \frac{2a^2(a+b\tan(c+dx))}{d\sqrt{\tan(c+dx)}} + 2\int \frac{\frac{1}{2}b(3a^2-b^2)-\frac{1}{2}a(}{\sqrt{\tan(}}$$

$$= \frac{2b(a^2+b^2)\sqrt{\tan(c+dx)}}{d} - \frac{2a^2(a+b\tan(c+dx))}{d\sqrt{\tan(c+dx)}} + \frac{4\text{Subst}\left(\int \frac{\frac{1}{2}b(3a^2-b^2)-\frac{1}{2}a}{1+x^4}\right.}{}$$

$$= \frac{2b(a^2+b^2)\sqrt{\tan(c+dx)}}{d} - \frac{2a^2(a+b\tan(c+dx))}{d\sqrt{\tan(c+dx)}} - \frac{((a+b)\,(a^2-4ab+b^2)}{}$$

$$= \frac{2b(a^2+b^2)\sqrt{\tan(c+dx)}}{d} - \frac{2a^2(a+b\tan(c+dx))}{d\sqrt{\tan(c+dx)}} - \frac{((a+b)\,(a^2-4ab+b^2)}{}$$

$$= -\frac{(a-b)\,(a^2+4ab+b^2)\log\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}\,d} + \frac{(a-b}{}$$

$$= \frac{(a+b)\,(a^2-4ab+b^2)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d} - \frac{(a+b)\,(a^2-4ab+b}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.98, size = 192, normalized size = 0.78

$$\frac{-32ab^2-8a(a^2-3b^2)\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\tan^2(c+dx)\right)+\sqrt{2}\,b(-3a^2+b^2)\left(2\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)-2\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}\right)+\log\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}+\tan(c+dx)\right)-\log\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\right)\sqrt{\tan(c+dx)}+8b^2(a+b\tan(c+dx))}{4d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^3/Tan[c + d*x]^(3/2),x]
```

```
[Out] (-32*a*b^2 - 8*a*(a^2 - 3*b^2)*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[c + d*x
]^2] + Sqrt[2]*b*(-3*a^2 + b^2)*(2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] -
 2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + Log[1 - Sqrt[2]*Sqrt[Tan[c + d*
x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])*S
qrt[Tan[c + d*x]] + 8*b^2*(a + b*Tan[c + d*x]))/(4*d*Sqrt[Tan[c + d*x]])
```

Maple [A]
time = 0.06, size = 226, normalized size = 0.92

method	result
derivativedivides	$\frac{2b^3\left(\sqrt{\tan}(dx+c)\right)-\frac{2a^3}{\sqrt{\tan(dx+c)}}+\frac{(3a^2b-b^3)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}\right)+2\arctan(\right.}{4}}{}$
default	$\frac{2b^3\left(\sqrt{\tan}(dx+c)\right)-\frac{2a^3}{\sqrt{\tan(dx+c)}}+\frac{(3a^2b-b^3)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}\right)+2\arctan(\right.}{4}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^3/tan(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b^3*tan(d*x+c)^(1/2)-2*a^3/tan(d*x+c)^(1/2)+1/4*(3*a^2*b-b^3)*2^(1/2
)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+t
an(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*
x+c)^(1/2)))+1/4*(-a^3+3*a*b^2)*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan
(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*
x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))))
```

Maxima [A]
time = 0.49, size = 214, normalized size = 0.87

$$\frac{8\,b^3\sqrt{\tan(dx+c)} - 2\sqrt{2}\,(a^3-3\,a^2b-3\,ab^2+b^3)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right) - 2\sqrt{2}\,(a^3-3\,a^2b-3\,ab^2+b^3)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right) + \sqrt{2}\,(a^3+3\,a^2b-3\,ab^2-b^3)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right) - \sqrt{2}\,(a^3+3\,a^2b-3\,ab^2-b^3)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right) - \frac{8\,a^3}{\sqrt{\tan(dx+c)}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^3/tan(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(8*b^3*sqrt(tan(d*x + c)) - 2*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*a
rctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) - 2*sqrt(2)*(a^3 - 3*a^
2*b - 3*a*b^2 + b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c))))
+ sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*log(sqrt(2)*sqrt(tan(d*x + c)) +
tan(d*x + c) + 1) - sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*log(-sqrt(2)*sq
rt(tan(d*x + c)) + tan(d*x + c) + 1) - 8*a^3/sqrt(tan(d*x + c)))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 7395 vs. $2(213) = 426$.
time = 3.56, size = 7395, normalized size = 30.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^3/tan(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*(d^5*cos(d*x + c)^2 - d^5)*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*
b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 - 2*(3*a^5*b - 10*a^3*b^3
 + 3*a*b^5)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*
b^8 + 6*a^2*b^10 + b^12)/d^4))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*
b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*((a^12 + 6*a^10*b^2 + 15*a^8*b^4 +
 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)^(3/4)*sqrt((a^12 - 30*a^
10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4)
*arctan(-((a^24 - 6*a^22*b^2 - 84*a^20*b^4 - 322*a^18*b^6 - 603*a^16*b^8 -
540*a^14*b^10 + 540*a^10*b^14 + 603*a^8*b^16 + 322*a^6*b^18 + 84*a^4*b^20 +
 6*a^2*b^22 - b^24)*d^4*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 +
 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^
4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4) + sqrt(2)*((3*a^2*
b - b^3)*d^7*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8
 + 6*a^2*b^10 + b^12)/d^4)*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6
*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4) + (a^9 - 6*a^5*b^4 - 8*a^3*b^
6 - 3*a*b^8)*d^5*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255
*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4))*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 +
 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 - 2*(3*a^5*b - 10*a^3*b^3 + 3*
a*b^5)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 +
 6*a^2*b^10 + b^12)/d^4))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 +
 255*a^4*b^8 - 30*a^2*b^10 + b^12))*sqrt(((a^18 - 27*a^16*b^2 + 168*a^14*b^
4 + 224*a^12*b^6 - 366*a^10*b^8 - 366*a^8*b^10 + 224*a^6*b^12 + 168*a^4*b^1
4 - 27*a^2*b^16 + b^18)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b
^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)*cos(d*x + c) + sqrt(2)*((a^15 - 3
3*a^13*b^2 + 345*a^11*b^4 - 1217*a^9*b^6 + 1611*a^7*b^8 - 795*a^5*b^10 + 91
*a^3*b^12 - 3*a*b^14)*d^3*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6
 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)*cos(d*x + c) + (3*a^20*b - 82*a^18*
b^3 + 531*a^16*b^5 + 504*a^14*b^7 - 1322*a^12*b^9 - 732*a^10*b^11 + 1038*a^
8*b^13 + 280*a^6*b^15 - 249*a^4*b^17 + 30*a^2*b^19 - b^21)*d*cos(d*x + c))*
sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10
 + b^12 - 2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*d^2*sqrt((a^12 + 6*a^10*b^2 +
15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4))/(a^12 - 30*
a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*s
qrt(sin(d*x + c)/cos(d*x + c))*((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^
6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)^(1/4) + (a^24 - 24*a^22*b^2 + 90*a
^20*b^4 + 648*a^18*b^6 + 783*a^16*b^8 - 624*a^14*b^10 - 1748*a^12*b^12 - 62
```

```
4*a^10*b^14 + 783*a^8*b^16 + 648*a^6*b^18 + 90*a^4*b^20 - 24*a^2*b^22 + b^2
4)*sin(d*x + c))/cos(d*x + c))*((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^
6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)^(3/4) + sqrt(2)*((3*a^14*b - 37*a^
12*b^3 - 69*a^10*b^5 + 27*a^8*b^7 + 81*a^6*b^9 + 9*a^4*b^11 - 15*a^2*b^13 +
 b^15)*d^7*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 +
 6*a^2*b^10 + b^12)/d^4)*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b
^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4) + (a^21 - 12*a^19*b^2 - 33*a^17
*b^4 + 64*a^15*b^6 + 282*a^13*b^8 + 264*a^11*b^10 - 82*a^9*b^12 - 288*a^7*b
^14 - 171*a^5*b^16 - 28*a^3*b^18 + 3*a*b^20)*d^5*sqrt((a^12 - 30*a^10*b^2 +
 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4))*sqrt((
a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^1
2 - 2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8
*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4))/(a^12 - 30*a^10*b
^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*sqrt(si
n(d*x + c)/cos(d*x + c))*((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15
*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)^(3/4))/(a^36 - 18*a^34*b^2 - 39*a^32*b^4
 + 848*a^30*b^6 + 5556*a^28*b^8 + 15240*a^26*b^10 + 20420*a^24*b^12 + 5424*
a^22*b^14 - 25938*a^20*b^16 - 42988*a^18*b^18 - 25938*a^16*b^20 + 5424*a^14
*b^22 + 20420*a^12*b^24 + 15240*a^10*b^26 + 5556*a^8*b^28 + 848*a^6*b^30 -
39*a^4*b^32 - 18*a^2*b^34 + b^36)) + 4*sqrt(2)*(d^5*cos(d*x + c)^2 - d^5)*s
qrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10
+ b^12 - 2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*d^2*sqrt((a^12 + 6*a^10*b^2 + 1
5*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4))/(a^12 - 30*a
^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*((
a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^1
2)/d^4)^(3/4)*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^
4*b^8 - 30*a^2*b^10 + b^12)/d^4)*arctan(((a^24 - 6*a^22*b^2 - 84*a^20*b^4 -
 322*a^18*b^6 - 603*a^16*b^8 - 540*a^14*b^10 + 540*a^10*b^14 + 603*a^8*b^16
 + 322*a^6*b^18 + 84*a^4*b^20 + 6*a^2*b^22 - b^24)*d^4*sqrt((a^12 + 6*a^10*
b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)*sqrt((
a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 ...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan(c + dx))^3}{\tan^{\frac{3}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**3/tan(d*x+c)**(3/2),x)
[Out] Integral((a + b*tan(c + d*x))**3/tan(c + d*x)**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^3/tan(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 4.66, size = 1767, normalized size = 7.21

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^3/tan(c + d*x)^(3/2),x)
```

```
[Out] (2*b^3*tan(c + d*x)^(1/2))/d - atan((a^6*d^3*tan(c + d*x)^(1/2)*((a^6*1i)/(
4*d^2) - (b^6*1i)/(4*d^2) + (3*a*b^5)/(2*d^2) + (3*a^5*b)/(2*d^2) + (a^2*b^
4*15i)/(4*d^2) - (5*a^3*b^3)/d^2 - (a^4*b^2*15i)/(4*d^2))^(1/2)*32i)/(16*a^
9*d^2 - b^9*d^2*16i + 48*a*b^8*d^2 - a^8*b*d^2*48i + a^2*b^7*d^2*288i - 736
*a^3*b^6*d^2 - a^4*b^5*d^2*960i + 960*a^5*b^4*d^2 + a^6*b^3*d^2*736i - 288*
a^7*b^2*d^2) - (b^6*d^3*tan(c + d*x)^(1/2)*((a^6*1i)/(4*d^2) - (b^6*1i)/(4*
d^2) + (3*a*b^5)/(2*d^2) + (3*a^5*b)/(2*d^2) + (a^2*b^4*15i)/(4*d^2) - (5*a
^3*b^3)/d^2 - (a^4*b^2*15i)/(4*d^2))^(1/2)*32i)/(16*a^9*d^2 - b^9*d^2*16i +
 48*a*b^8*d^2 - a^8*b*d^2*48i + a^2*b^7*d^2*288i - 736*a^3*b^6*d^2 - a^4*b^
5*d^2*960i + 960*a^5*b^4*d^2 + a^6*b^3*d^2*736i - 288*a^7*b^2*d^2) + (a^2*b
^4*d^3*tan(c + d*x)^(1/2)*((a^6*1i)/(4*d^2) - (b^6*1i)/(4*d^2) + (3*a*b^5)/
(2*d^2) + (3*a^5*b)/(2*d^2) + (a^2*b^4*15i)/(4*d^2) - (5*a^3*b^3)/d^2 - (a^
4*b^2*15i)/(4*d^2))^(1/2)*480i)/(16*a^9*d^2 - b^9*d^2*16i + 48*a*b^8*d^2 -
a^8*b*d^2*48i + a^2*b^7*d^2*288i - 736*a^3*b^6*d^2 - a^4*b^5*d^2*960i + 960
*a^5*b^4*d^2 + a^6*b^3*d^2*736i - 288*a^7*b^2*d^2) - (a^4*b^2*d^3*tan(c + d
*x)^(1/2)*((a^6*1i)/(4*d^2) - (b^6*1i)/(4*d^2) + (3*a*b^5)/(2*d^2) + (3*a^5
*b)/(2*d^2) + (a^2*b^4*15i)/(4*d^2) - (5*a^3*b^3)/d^2 - (a^4*b^2*15i)/(4*d^
2))^(1/2)*480i)/(16*a^9*d^2 - b^9*d^2*16i + 48*a*b^8*d^2 - a^8*b*d^2*48i +
a^2*b^7*d^2*288i - 736*a^3*b^6*d^2 - a^4*b^5*d^2*960i + 960*a^5*b^4*d^2 + a
^6*b^3*d^2*736i - 288*a^7*b^2*d^2))*((6*a*b^5 + 6*a^5*b + a^6*1i - b^6*1i +
 a^2*b^4*15i - 20*a^3*b^3 - a^4*b^2*15i)/(4*d^2))^(1/2)*2i - (2*a^3)/(d*tan
(c + d*x)^(1/2)) - atan((a^6*d^3*tan(c + d*x)^(1/2)*((b^6*1i)/(4*d^2) - (a^
6*1i)/(4*d^2) + (3*a*b^5)/(2*d^2) + (3*a^5*b)/(2*d^2) - (a^2*b^4*15i)/(4*d^
2) - (5*a^3*b^3)/d^2 + (a^4*b^2*15i)/(4*d^2))^(1/2)*32i)/(16*a^9*d^2 + b^9*
d^2*16i + 48*a*b^8*d^2 + a^8*b*d^2*48i - a^2*b^7*d^2*288i - 736*a^3*b^6*d^2
 + a^4*b^5*d^2*960i + 960*a^5*b^4*d^2 - a^6*b^3*d^2*736i - 288*a^7*b^2*d^2)
 - (b^6*d^3*tan(c + d*x)^(1/2)*((b^6*1i)/(4*d^2) - (a^6*1i)/(4*d^2) + (3*a*
b^5)/(2*d^2) + (3*a^5*b)/(2*d^2) - (a^2*b^4*15i)/(4*d^2) - (5*a^3*b^3)/d^2
+ (a^4*b^2*15i)/(4*d^2))^(1/2)*32i)/(16*a^9*d^2 + b^9*d^2*16i + 48*a*b^8*d^
2 + a^8*b*d^2*48i - a^2*b^7*d^2*288i - 736*a^3*b^6*d^2 + a^4*b^5*d^2*960i +
 960*a^5*b^4*d^2 - a^6*b^3*d^2*736i - 288*a^7*b^2*d^2) + (a^2*b^4*d^3*tan(c
 + d*x)^(1/2)*((b^6*1i)/(4*d^2) - (a^6*1i)/(4*d^2) + (3*a*b^5)/(2*d^2) + (3
```

*a^5*b)/(2*d^2) - (a^2*b^4*15i)/(4*d^2) - (5*a^3*b^3)/d^2 + (a^4*b^2*15i)/(4*d^2))^(1/2)*480i)/(16*a^9*d^2 + b^9*d^2*16i + 48*a*b^8*d^2 + a^8*b*d^2*48i - a^2*b^7*d^2*288i - 736*a^3*b^6*d^2 + a^4*b^5*d^2*960i + 960*a^5*b^4*d^2 - a^6*b^3*d^2*736i - 288*a^7*b^2*d^2) - (a^4*b^2*d^3*tan(c + d*x)^(1/2)*((b^6*1i)/(4*d^2) - (a^6*1i)/(4*d^2) + (3*a*b^5)/(2*d^2) + (3*a^5*b)/(2*d^2) - (a^2*b^4*15i)/(4*d^2) - (5*a^3*b^3)/d^2 + (a^4*b^2*15i)/(4*d^2))^(1/2)*480i)/(16*a^9*d^2 + b^9*d^2*16i + 48*a*b^8*d^2 + a^8*b*d^2*48i - a^2*b^7*d^2*288i - 736*a^3*b^6*d^2 + a^4*b^5*d^2*960i + 960*a^5*b^4*d^2 - a^6*b^3*d^2*736i - 288*a^7*b^2*d^2))*((6*a*b^5 + 6*a^5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*d^2))^(1/2)*2i

3.574 $\int \frac{(a+b\tan(c+dx))^3}{\tan^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=245

$$\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c\right.}{\sqrt{2}\,d}$$

```
[Out] -1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)-1/
2*(a-b)*(a^2+4*a*b+b^2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)+1/4*(a
+b)*(a^2-4*a*b+b^2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)-1/4
*(a+b)*(a^2-4*a*b+b^2)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)-
16/3*a^2*b/d/tan(d*x+c)^(1/2)-2/3*a^2*(a+b*tan(d*x+c))/d/tan(d*x+c)^(3/2)
```

Rubi [A]
time = 0.22, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3646, 3709, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d}+\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{2a^2(a+b\tan(c+dx))}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{16a^2b}{3d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^3/Tan[c + d*x]^(5/2),x]
```

```
[Out] ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[
2]*d) - ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]
)/(Sqrt[2]*d) + ((a + b)*(a^2 - 4*a*b + b^2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d
*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) - ((a + b)*(a^2 - 4*a*b + b^2)*Log[1 +
Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) - (16*a^2*b)/(3*d
*Sqrt[Tan[c + d*x]]) - (2*a^2*(a + b*Tan[c + d*x]))/(3*d*Tan[c + d*x]^(3/2)
)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
```

```
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(c+dx))^3}{\tan^{\frac{5}{2}}(c+dx)}\,dx = -\frac{2a^2(a+b\tan(c+dx))}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{2}{3}\int \frac{4a^2b-\frac{3}{2}a(a^2-3b^2)\tan(c+dx)-\frac{1}{2}b(a^2-3b^2}{\tan^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{16a^2b}{3d\sqrt{\tan(c+dx)}}-\frac{2a^2(a+b\tan(c+dx))}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{2}{3}\int \frac{-\frac{3}{2}a(a^2-3b^2)-\frac{3}{2}b(3a^2}{\sqrt{\tan(c+d}}$$

$$= -\frac{16a^2b}{3d\sqrt{\tan(c+dx)}}-\frac{2a^2(a+b\tan(c+dx))}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{4\text{Subst}\left(\int \frac{-\frac{3}{2}a(a^2-3b^2)-\frac{3}{2}b(3a^2-b^2}{1+x^4}\right.}{3d}$$

$$= -\frac{16a^2b}{3d\sqrt{\tan(c+dx)}}-\frac{2a^2(a+b\tan(c+dx))}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{((a+b)(a^2-4ab+b^2))\,\text{Subst}}{}$$

$$= -\frac{16a^2b}{3d\sqrt{\tan(c+dx)}}-\frac{2a^2(a+b\tan(c+dx))}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{((a+b)(a^2-4ab+b^2))\,\text{Subst}}{}$$

$$= \frac{(a+b)(a^2-4ab+b^2)\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}\,d}-\frac{(a+b)(}{}$$

$$= \frac{(a-b)(a^2+4ab+b^2)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a-b)(a^2+4ab+}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.31, size = 91, normalized size = 0.37

$$\frac{-(a+ib)^3\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-i\tan(c+dx)\right)-(a-ib)^3\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};i\tan(c+dx)\right)-6b^2(a+b\tan(c+dx))}{3d\tan^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Tan[c + d*x])^3/Tan[c + d*x]^(5/2),x]

[Out] (-((a + I*b)^3*Hypergeometric2F1[-3/2, 1, -1/2, (-I)*Tan[c + d*x]]) - (a - I*b)^3*Hypergeometric2F1[-3/2, 1, -1/2, I*Tan[c + d*x]] - 6*b^2*(a + b*Tan[c + d*x]))/(3*d*Tan[c + d*x]^(3/2))

Maple [A]
time = 0.06, size = 225, normalized size = 0.92

method	result
derivativedivides	$-\frac{2a^3}{3\tan(dx+c)^{\frac{3}{2}}}-\frac{6a^2b}{\sqrt{\tan(dx+c)}}+\frac{(-a^3+3b^2a)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{\right.}\right.}{4}$
default	$-\frac{2a^3}{3\tan(dx+c)^{\frac{3}{2}}}-\frac{6a^2b}{\sqrt{\tan(dx+c)}}+\frac{(-a^3+3b^2a)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{\right.}\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*tan(d*x+c))^3/tan(d*x+c)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/d*(-2/3*a^3/tan(d*x+c)^(3/2)-6*a^2*b/tan(d*x+c)^(1/2)+1/4*(-a^3+3*a*b^2)*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(-3*a^2*b+b^3)*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))))

Maxima [A]
time = 0.50, size = 215, normalized size = 0.88

$$-\frac{6\sqrt{2}(a^3+3a^2b-3ab^2-b^3)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+6\sqrt{2}(a^3+3a^2b-3ab^2-b^3)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)+3\sqrt{2}(a^3-3a^2b-3ab^2+b^3)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)-3\sqrt{2}(a^3-3a^2b-3ab^2+b^3)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+\frac{8(9a^2b\tan(dx+c)+a^3)}{\tan(dx+c)^{\frac{3}{2}}}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(d*x+c))^3/tan(d*x+c)^(5/2),x, algorithm="maxima")

[Out] -1/12*(6*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 6*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + 3*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - 3*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 8*(9*a^2*b*tan(d*x + c) + a^3)/tan(d*x + c)^(3/2))/d

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7355 vs. $2(209) = 418$.
time = 2.94, size = 7355, normalized size = 30.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^3/tan(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(12*sqrt(2)*(d^5*cos(d*x + c)^2 - d^5)*sqrt((a^12 + 6*a^10*b^2 + 15*a^
8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 + 2*(3*a^5*b - 10*a^3*b
^3 + 3*a*b^5)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^
4*b^8 + 6*a^2*b^10 + b^12)/d^4))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^
6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*((a^12 + 6*a^10*b^2 + 15*a^8*b^4
 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)^(3/4)*sqrt((a^12 - 30*
a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^
4)*arctan(((a^24 - 6*a^22*b^2 - 84*a^20*b^4 - 322*a^18*b^6 - 603*a^16*b^8 -
 540*a^14*b^10 + 540*a^10*b^14 + 603*a^8*b^16 + 322*a^6*b^18 + 84*a^4*b^20
+ 6*a^2*b^22 - b^24)*d^4*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6
+ 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b
^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4) - sqrt(2)*((a^3 -
 3*a*b^2)*d^7*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^
8 + 6*a^2*b^10 + b^12)/d^4)*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^
6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4) - (3*a^8*b + 8*a^6*b^3 + 6*a
^4*b^5 - b^9)*d^5*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 25
5*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4))*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4
+ 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 + 2*(3*a^5*b - 10*a^3*b^3 + 3
*a*b^5)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8
+ 6*a^2*b^10 + b^12)/d^4))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6
+ 255*a^4*b^8 - 30*a^2*b^10 + b^12))*sqrt(((a^18 - 27*a^16*b^2 + 168*a^14*b
^4 + 224*a^12*b^6 - 366*a^10*b^8 - 366*a^8*b^10 + 224*a^6*b^12 + 168*a^4*b^
14 - 27*a^2*b^16 + b^18)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*
b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)*cos(d*x + c) + sqrt(2)*((3*a^14*
b - 91*a^12*b^3 + 795*a^10*b^5 - 1611*a^8*b^7 + 1217*a^6*b^9 - 345*a^4*b^11
 + 33*a^2*b^13 - b^15)*d^3*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^
6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)*cos(d*x + c) - (a^21 - 30*a^19*b^2
 + 249*a^17*b^4 - 280*a^15*b^6 - 1038*a^13*b^8 + 732*a^11*b^10 + 1322*a^9*b
^12 - 504*a^7*b^14 - 531*a^5*b^16 + 82*a^3*b^18 - 3*a*b^20)*d*cos(d*x + c))
*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^1
0 + b^12 + 2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*d^2*sqrt((a^12 + 6*a^10*b^2 +
 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4))/(a^12 - 30
*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*
sqrt(sin(d*x + c)/cos(d*x + c))*((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b
^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)^(1/4) + (a^24 - 24*a^22*b^2 + 90*
a^20*b^4 + 648*a^18*b^6 + 783*a^16*b^8 - 624*a^14*b^10 - 1748*a^12*b^12 - 6
```

```
24*a^10*b^14 + 783*a^8*b^16 + 648*a^6*b^18 + 90*a^4*b^20 - 24*a^2*b^22 + b^
24)*sin(d*x + c))/cos(d*x + c))*((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b
^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)^(3/4) - sqrt(2)*((a^15 - 15*a^13*
b^2 + 9*a^11*b^4 + 81*a^9*b^6 + 27*a^7*b^8 - 69*a^5*b^10 - 37*a^3*b^12 + 3*
a*b^14)*d^7*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8
+ 6*a^2*b^10 + b^12)/d^4)*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*
b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4) - (3*a^20*b - 28*a^18*b^3 - 17
1*a^16*b^5 - 288*a^14*b^7 - 82*a^12*b^9 + 264*a^10*b^11 + 282*a^8*b^13 + 64
*a^6*b^15 - 33*a^4*b^17 - 12*a^2*b^19 + b^21)*d^5*sqrt((a^12 - 30*a^10*b^2
+ 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4))*sqrt(
(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^
12 + 2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^
8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4))/(a^12 - 30*a^10*
b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*sqrt(s
in(d*x + c)/cos(d*x + c))*((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 1
5*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)^(3/4))/(a^36 - 18*a^34*b^2 - 39*a^32*b^
4 + 848*a^30*b^6 + 5556*a^28*b^8 + 15240*a^26*b^10 + 20420*a^24*b^12 + 5424
*a^22*b^14 - 25938*a^20*b^16 - 42988*a^18*b^18 - 25938*a^16*b^20 + 5424*a^1
4*b^22 + 20420*a^12*b^24 + 15240*a^10*b^26 + 5556*a^8*b^28 + 848*a^6*b^30 -
 39*a^4*b^32 - 18*a^2*b^34 + b^36)) + 12*sqrt(2)*(d^5*cos(d*x + c)^2 - d^5)
*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^1
0 + b^12 + 2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*d^2*sqrt((a^12 + 6*a^10*b^2 +
 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4))/(a^12 - 30
*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*
((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b
^12)/d^4)^(3/4)*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*
a^4*b^8 - 30*a^2*b^10 + b^12)/d^4)*arctan(-((a^24 - 6*a^22*b^2 - 84*a^20*b^
4 - 322*a^18*b^6 - 603*a^16*b^8 - 540*a^14*b^10 + 540*a^10*b^14 + 603*a^8*b
^16 + 322*a^6*b^18 + 84*a^4*b^20 + 6*a^2*b^22 - b^24)*d^4*sqrt((a^12 + 6*a^
10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)*sqr
t((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan{(c + dx)})^3}{\tan^{\frac{5}{2}}{(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**3/tan(d*x+c)**(5/2),x)
[Out] Integral((a + b*tan(c + d*x))**3/tan(c + d*x)**(5/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^3/tan(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 5.02, size = 1752, normalized size = 7.15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^3/tan(c + d*x)^(5/2),x)
```

```
[Out] 2*atanh((32*a^6*d^3*tan(c + d*x)^(1/2)*((b^6*1i)/(4*d^2) - (a^6*1i)/(4*d^2)
 - (3*a*b^5)/(2*d^2) - (3*a^5*b)/(2*d^2) - (a^2*b^4*15i)/(4*d^2) + (5*a^3*b
^3)/d^2 + (a^4*b^2*15i)/(4*d^2))^(1/2))/(a^9*d^2*16i + 16*b^9*d^2 + a*b^8*d
^2*48i + 48*a^8*b*d^2 - 288*a^2*b^7*d^2 - a^3*b^6*d^2*736i + 960*a^4*b^5*d^
2 + a^5*b^4*d^2*960i - 736*a^6*b^3*d^2 - a^7*b^2*d^2*288i) - (32*b^6*d^3*ta
n(c + d*x)^(1/2)*((b^6*1i)/(4*d^2) - (a^6*1i)/(4*d^2) - (3*a*b^5)/(2*d^2) -
 (3*a^5*b)/(2*d^2) - (a^2*b^4*15i)/(4*d^2) + (5*a^3*b^3)/d^2 + (a^4*b^2*15i
)/(4*d^2))^(1/2))/(a^9*d^2*16i + 16*b^9*d^2 + a*b^8*d^2*48i + 48*a^8*b*d^2
- 288*a^2*b^7*d^2 - a^3*b^6*d^2*736i + 960*a^4*b^5*d^2 + a^5*b^4*d^2*960i -
 736*a^6*b^3*d^2 - a^7*b^2*d^2*288i) + (480*a^2*b^4*d^3*tan(c + d*x)^(1/2)*
((b^6*1i)/(4*d^2) - (a^6*1i)/(4*d^2) - (3*a*b^5)/(2*d^2) - (3*a^5*b)/(2*d^2
) - (a^2*b^4*15i)/(4*d^2) + (5*a^3*b^3)/d^2 + (a^4*b^2*15i)/(4*d^2))^(1/2))
/(a^9*d^2*16i + 16*b^9*d^2 + a*b^8*d^2*48i + 48*a^8*b*d^2 - 288*a^2*b^7*d^2
 - a^3*b^6*d^2*736i + 960*a^4*b^5*d^2 + a^5*b^4*d^2*960i - 736*a^6*b^3*d^2
- a^7*b^2*d^2*288i) - (480*a^4*b^2*d^3*tan(c + d*x)^(1/2)*((b^6*1i)/(4*d^2)
 - (a^6*1i)/(4*d^2) - (3*a*b^5)/(2*d^2) - (3*a^5*b)/(2*d^2) - (a^2*b^4*15i)
/(4*d^2) + (5*a^3*b^3)/d^2 + (a^4*b^2*15i)/(4*d^2))^(1/2))/(a^9*d^2*16i + 1
6*b^9*d^2 + a*b^8*d^2*48i + 48*a^8*b*d^2 - 288*a^2*b^7*d^2 - a^3*b^6*d^2*73
6i + 960*a^4*b^5*d^2 + a^5*b^4*d^2*960i - 736*a^6*b^3*d^2 - a^7*b^2*d^2*288
i))*(-(6*a*b^5 + 6*a^5*b + a^6*1i - b^6*1i + a^2*b^4*15i - 20*a^3*b^3 - a^4
*b^2*15i)/(4*d^2))^(1/2) - 2*atanh((32*a^6*d^3*tan(c + d*x)^(1/2)*((a^6*1i)
/(4*d^2) - (b^6*1i)/(4*d^2) - (3*a*b^5)/(2*d^2) - (3*a^5*b)/(2*d^2) + (a^2*
b^4*15i)/(4*d^2) + (5*a^3*b^3)/d^2 - (a^4*b^2*15i)/(4*d^2))^(1/2))/(a^9*d^2
*16i - 16*b^9*d^2 + a*b^8*d^2*48i - 48*a^8*b*d^2 + 288*a^2*b^7*d^2 - a^3*b^
6*d^2*736i - 960*a^4*b^5*d^2 + a^5*b^4*d^2*960i + 736*a^6*b^3*d^2 - a^7*b^2
*d^2*288i) - (32*b^6*d^3*tan(c + d*x)^(1/2)*((a^6*1i)/(4*d^2) - (b^6*1i)/(4
*d^2) - (3*a*b^5)/(2*d^2) - (3*a^5*b)/(2*d^2) + (a^2*b^4*15i)/(4*d^2) + (5*
a^3*b^3)/d^2 - (a^4*b^2*15i)/(4*d^2))^(1/2))/(a^9*d^2*16i - 16*b^9*d^2 + a*
b^8*d^2*48i - 48*a^8*b*d^2 + 288*a^2*b^7*d^2 - a^3*b^6*d^2*736i - 960*a^4*b
^5*d^2 + a^5*b^4*d^2*960i + 736*a^6*b^3*d^2 - a^7*b^2*d^2*288i) + (480*a^2*
b^4*d^3*tan(c + d*x)^(1/2)*((a^6*1i)/(4*d^2) - (b^6*1i)/(4*d^2) - (3*a*b^5)
/(2*d^2) - (3*a^5*b)/(2*d^2) + (a^2*b^4*15i)/(4*d^2) + (5*a^3*b^3)/d^2 - (a
```

^4*b^2*15i)/(4*d^2))^(1/2))/(a^9*d^2*16i - 16*b^9*d^2 + a*b^8*d^2*48i - 48*a^8*b*d^2 + 288*a^2*b^7*d^2 - a^3*b^6*d^2*736i - 960*a^4*b^5*d^2 + a^5*b^4*d^2*960i + 736*a^6*b^3*d^2 - a^7*b^2*d^2*288i) - (480*a^4*b^2*d^3*tan(c + d*x)^(1/2)*((a^6*1i)/(4*d^2) - (b^6*1i)/(4*d^2) - (3*a*b^5)/(2*d^2) - (3*a^5*b)/(2*d^2) + (a^2*b^4*15i)/(4*d^2) + (5*a^3*b^3)/d^2 - (a^4*b^2*15i)/(4*d^2))^(1/2))/(a^9*d^2*16i - 16*b^9*d^2 + a*b^8*d^2*48i - 48*a^8*b*d^2 + 288*a^2*b^7*d^2 - a^3*b^6*d^2*736i - 960*a^4*b^5*d^2 + a^5*b^4*d^2*960i + 736*a^6*b^3*d^2 - a^7*b^2*d^2*288i))*(-(6*a*b^5 + 6*a^5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*d^2))^(1/2) - ((2*a^3)/3 + 6*a^2*b*tan(c + d*x))/(d*tan(c + d*x)^(3/2))

3.575 $\int \frac{(a+b\tan(c+dx))^3}{\tan^{\frac{7}{2}}(c+dx)} \, dx$

Optimal. Leaf size=270

$$-\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(\right.}{\sqrt{2}\,d}$$

```
[Out] 1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)+1/2
*(a+b)*(a^2-4*a*b+b^2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)+1/4*(a-
b)*(a^2+4*a*b+b^2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)-1/4*
(a-b)*(a^2+4*a*b+b^2)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)+2
*a*(a^2-3*b^2)/d/tan(d*x+c)^(1/2)-8/5*a^2*b/d/tan(d*x+c)^(3/2)-2/5*a^2*(a+b
*tan(d*x+c))/d/tan(d*x+c)^(5/2)
```

Rubi [A]
time = 0.25, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3646, 3709, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d}+\frac{2a\left(a^2-3b^2\right)}{d\sqrt{\tan(c+dx)}}+\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{2a^2(a+b\tan(c+dx))}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{8a^2b}{5d\tan^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^3/Tan[c + d*x]^(7/2),x]
```

```
[Out] -(((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqr
t[2]*d)) + ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x
]]])/(Sqrt[2]*d) + ((a - b)*(a^2 + 4*a*b + b^2)*Log[1 - Sqrt[2]*Sqrt[Tan[c
+ d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) - ((a - b)*(a^2 + 4*a*b + b^2)*Log[1
 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) - (8*a^2*b)/(5
*d*Tan[c + d*x]^(3/2)) + (2*a*(a^2 - 3*b^2))/(d*Sqrt[Tan[c + d*x]]) - (2*a^
2*(a + b*Tan[c + d*x]))/(5*d*Tan[c + d*x]^(5/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
```

```
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(c+dx))^3}{\tan^{\frac{7}{2}}(c+dx)}\,dx = -\frac{2a^2(a+b\tan(c+dx))}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{2}{5}\int \frac{6a^2b-\frac{5}{2}a(a^2-3b^2)\tan(c+dx)-\frac{1}{2}b(3a^2-5b}{\tan^{\frac{5}{2}}(c+dx)}$$

$$= -\frac{8a^2b}{5d\tan^{\frac{3}{2}}(c+dx)}-\frac{2a^2(a+b\tan(c+dx))}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{2}{5}\int \frac{-\frac{5}{2}a(a^2-3b^2)-\frac{5}{2}b(3a^2-}{\tan^{\frac{3}{2}}(c+dx}$$

$$= -\frac{8a^2b}{5d\tan^{\frac{3}{2}}(c+dx)}+\frac{2a(a^2-3b^2)}{d\sqrt{\tan(c+dx)}}-\frac{2a^2(a+b\tan(c+dx))}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{2}{5}\int \frac{-\frac{5}{2}b(3a}{}$$

$$= -\frac{8a^2b}{5d\tan^{\frac{3}{2}}(c+dx)}+\frac{2a(a^2-3b^2)}{d\sqrt{\tan(c+dx)}}-\frac{2a^2(a+b\tan(c+dx))}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{4\text{Subst}\left(\int -\right.}{}$$

$$= -\frac{8a^2b}{5d\tan^{\frac{3}{2}}(c+dx)}+\frac{2a(a^2-3b^2)}{d\sqrt{\tan(c+dx)}}-\frac{2a^2(a+b\tan(c+dx))}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{((a+b)\,(a^2}{}$$

$$= -\frac{8a^2b}{5d\tan^{\frac{3}{2}}(c+dx)}+\frac{2a(a^2-3b^2)}{d\sqrt{\tan(c+dx)}}-\frac{2a^2(a+b\tan(c+dx))}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{((a+b)\,(a^2}{}$$

$$= \frac{(a-b)\,(a^2+4ab+b^2)\log\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}\,d}-\frac{(a-b)\,(}{}$$

$$= -\frac{(a+b)\,(a^2-4ab+b^2)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a+b)\,(a^2-4ab+}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.52, size = 103, normalized size = 0.38

$$-\frac{2\left(3a(a^2-3b^2)\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\tan^2(c+dx)\right)+b\left(5(3a^2-b^2)\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\tan^2(c+dx)\right)\tan(c+dx)+b(9a+5b\tan(c+dx))\right)\right)}{15d\tan^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^3/Tan[c + d*x]^(7/2),x]
```

```
[Out] (-2*(3*a*(a^2 - 3*b^2)*Hypergeometric2F1[-5/4, 1, -1/4, -Tan[c + d*x]^2] +
b*(5*(3*a^2 - b^2)*Hypergeometric2F1[-3/4, 1, 1/4, -Tan[c + d*x]^2]*Tan[c +
 d*x] + b*(9*a + 5*b*Tan[c + d*x]))))/(15*d*Tan[c + d*x]^(5/2))
```

Maple [A]
time = 0.06, size = 243, normalized size = 0.90

method	result
derivativedivides	$\frac{-\frac{2a^3}{5\tan(dx+c)^{\frac{5}{2}}}+\frac{2a\left(a^2-3b^2\right)}{\sqrt{\tan\left(dx+c\right)}}-\frac{2a^2b}{\tan(dx+c)^{\frac{3}{2}}}+\frac{\left(-3a^2b+b^3\right)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}\,(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}\,(dx+c)\right)+\tan(dx+c)}\right)+2}{}}{}$
default	$\frac{-\frac{2a^3}{5\tan(dx+c)^{\frac{5}{2}}}+\frac{2a\left(a^2-3b^2\right)}{\sqrt{\tan\left(dx+c\right)}}-\frac{2a^2b}{\tan(dx+c)^{\frac{3}{2}}}+\frac{\left(-3a^2b+b^3\right)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}\,(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}\,(dx+c)\right)+\tan(dx+c)}\right)+2}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^3/tan(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/5*a^3/tan(d*x+c)^(5/2)+2*a*(a^2-3*b^2)/tan(d*x+c)^(1/2)-2*a^2*b/tan
(d*x+c)^(3/2)+1/4*(-3*a^2*b+b^3)*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+ta
n(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d
*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(a^3-3*a*b^2)*2^(1/
2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+
tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d
*x+c)^(1/2))))
```

Maxima [A]
time = 0.51, size = 235, normalized size = 0.87

$$\frac{10\sqrt{2}(a^3-3a^2b-3ab^2+b^3)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(dx+c\right)}\right)\right)+10\sqrt{2}(a^3-3a^2b-3ab^2+b^3)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(dx+c\right)}\right)\right)-5\sqrt{2}(a^3+3a^2b-3ab^2-b^3)\log\left(\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)+5\sqrt{2}(a^3+3a^2b-3ab^2-b^3)\log\left(-\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)-\frac{8\left(5a^2b\tan(dx+c)+a^3-5\left(a^3-3ab^2\right)\tan(dx+c)^2\right)}{\tan(dx+c)^{\frac{5}{2}}}}{20d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^3/tan(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/20*(10*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*arctan(1/2*sqrt(2)*(sqrt(2
) + 2*sqrt(tan(d*x + c)))) + 10*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*arc
tan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - 5*sqrt(2)*(a^3 + 3*a^2
*b - 3*a*b^2 - b^3)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 5*
sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*log(-sqrt(2)*sqrt(tan(d*x + c)) + t
an(d*x + c) + 1) - 8*(5*a^2*b*tan(d*x + c) + a^3 - 5*(a^3 - 3*a*b^2)*tan(d*
x + c)^2)/tan(d*x + c)^(5/2))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 7742 vs. $2(232) = 464$.
time = 3.33, size = 7742, normalized size = 28.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^3/tan(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/20*(20*sqrt(2)*(d^5*cos(d*x + c)^4 - 2*d^5*cos(d*x + c)^2 + d^5)*sqrt((a
^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12
 - 2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*
b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4))/(a^12 - 30*a^10*b^
2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*((a^12 +
 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4
)^(3/4)*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8
- 30*a^2*b^10 + b^12)/d^4)*arctan(-((a^24 - 6*a^22*b^2 - 84*a^20*b^4 - 322*
a^18*b^6 - 603*a^16*b^8 - 540*a^14*b^10 + 540*a^10*b^14 + 603*a^8*b^16 + 32
2*a^6*b^18 + 84*a^4*b^20 + 6*a^2*b^22 - b^24)*d^4*sqrt((a^12 + 6*a^10*b^2 +
 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)*sqrt((a^12
- 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^1
2)/d^4) + sqrt(2)*((3*a^2*b - b^3)*d^7*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4
 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)*sqrt((a^12 - 30*a^10*b
^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4) + (
a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^5*sqrt((a^12 - 30*a^10*b^2 + 255*a
^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4))*sqrt((a^12 +
 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 - 2*
(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 +
 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4))/(a^12 - 30*a^10*b^2 + 2
55*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*sqrt(((a^18 -
 27*a^16*b^2 + 168*a^14*b^4 + 224*a^12*b^6 - 366*a^10*b^8 - 366*a^8*b^10 +
224*a^6*b^12 + 168*a^4*b^14 - 27*a^2*b^16 + b^18)*d^2*sqrt((a^12 + 6*a^10*b
^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)*cos(d*x
 + c) + sqrt(2)*((a^15 - 33*a^13*b^2 + 345*a^11*b^4 - 1217*a^9*b^6 + 1611*a
^7*b^8 - 795*a^5*b^10 + 91*a^3*b^12 - 3*a*b^14)*d^3*sqrt((a^12 + 6*a^10*b^2
 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)*cos(d*x +
```

```
c) + (3*a^20*b - 82*a^18*b^3 + 531*a^16*b^5 + 504*a^14*b^7 - 1322*a^12*b^9
 - 732*a^10*b^11 + 1038*a^8*b^13 + 280*a^6*b^15 - 249*a^4*b^17 + 30*a^2*b^1
9 - b^21)*d*cos(d*x + c))*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6
 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 - 2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*d^2*
sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10
 + b^12)/d^4))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^
8 - 30*a^2*b^10 + b^12))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^12 + 6*a^10*b^
2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)^(1/4) +
(a^24 - 24*a^22*b^2 + 90*a^20*b^4 + 648*a^18*b^6 + 783*a^16*b^8 - 624*a^14*
b^10 - 1748*a^12*b^12 - 624*a^10*b^14 + 783*a^8*b^16 + 648*a^6*b^18 + 90*a^
4*b^20 - 24*a^2*b^22 + b^24)*sin(d*x + c))/cos(d*x + c))*((a^12 + 6*a^10*b^
2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)^(3/4) +
sqrt(2)*((3*a^14*b - 37*a^12*b^3 - 69*a^10*b^5 + 27*a^8*b^7 + 81*a^6*b^9 +
9*a^4*b^11 - 15*a^2*b^13 + b^15)*d^7*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 +
 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)*sqrt((a^12 - 30*a^10*b^2
 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4) + (a^
21 - 12*a^19*b^2 - 33*a^17*b^4 + 64*a^15*b^6 + 282*a^13*b^8 + 264*a^11*b^10
 - 82*a^9*b^12 - 288*a^7*b^14 - 171*a^5*b^16 - 28*a^3*b^18 + 3*a*b^20)*d^5*
sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2
*b^10 + b^12)/d^4))*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*
a^4*b^8 + 6*a^2*b^10 + b^12 - 2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*d^2*sqrt((
a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^1
2)/d^4))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30
*a^2*b^10 + b^12))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^12 + 6*a^10*b^2 + 15
*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)^(3/4))/(a^36 -
 18*a^34*b^2 - 39*a^32*b^4 + 848*a^30*b^6 + 5556*a^28*b^8 + 15240*a^26*b^10
 + 20420*a^24*b^12 + 5424*a^22*b^14 - 25938*a^20*b^16 - 42988*a^18*b^18 - 2
5938*a^16*b^20 + 5424*a^14*b^22 + 20420*a^12*b^24 + 15240*a^10*b^26 + 5556*
a^8*b^28 + 848*a^6*b^30 - 39*a^4*b^32 - 18*a^2*b^34 + b^36)) + 20*sqrt(2)*(
d^5*cos(d*x + c)^4 - 2*d^5*cos(d*x + c)^2 + d^5)*sqrt((a^12 + 6*a^10*b^2 +
15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 - 2*(3*a^5*b - 10*
a^3*b^3 + 3*a*b^5)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 +
15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 4
52*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*((a^12 + 6*a^10*b^2 + 15*a^
8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)^(3/4)*sqrt((a^12
- 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^1
2)/d^4)*arctan(((a^24 - 6*a^22*b^2 - 84*a^20*b^4 - 322*a^18*b^6 - 603*a^16*
b^8 - 540*a^14*b^10 + 540*a^10*b^14 + 603*a^8*b^16 + 322*a^6*b^18 + 84*a^4*
b^20 + 6*a^2*b^22 - b^24)*d^4*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6
*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)*sqr...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan(c + dx))^3}{\tan^{\frac{7}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**3/tan(d*x+c)**(7/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3/tan(c + d*x)**(7/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^3/tan(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 6.27, size = 1777, normalized size = 6.58

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^3/tan(c + d*x)^(7/2),x)
```

```
[Out] - 2*atanh((32*a^6*d^3*tan(c + d*x)^(1/2)*((b^6*1i)/(4*d^2) - (a^6*1i)/(4*d^
2) + (3*a*b^5)/(2*d^2) + (3*a^5*b)/(2*d^2) - (a^2*b^4*15i)/(4*d^2) - (5*a^3
*b^3)/d^2 + (a^4*b^2*15i)/(4*d^2))^(1/2))/(16*a^9*d^2 + b^9*d^2*16i + 48*a*
b^8*d^2 + a^8*b*d^2*48i - a^2*b^7*d^2*288i - 736*a^3*b^6*d^2 + a^4*b^5*d^2*
960i + 960*a^5*b^4*d^2 - a^6*b^3*d^2*736i - 288*a^7*b^2*d^2) - (32*b^6*d^3*
tan(c + d*x)^(1/2)*((b^6*1i)/(4*d^2) - (a^6*1i)/(4*d^2) + (3*a*b^5)/(2*d^2)
 + (3*a^5*b)/(2*d^2) - (a^2*b^4*15i)/(4*d^2) - (5*a^3*b^3)/d^2 + (a^4*b^2*1
5i)/(4*d^2))^(1/2))/(16*a^9*d^2 + b^9*d^2*16i + 48*a*b^8*d^2 + a^8*b*d^2*48
i - a^2*b^7*d^2*288i - 736*a^3*b^6*d^2 + a^4*b^5*d^2*960i + 960*a^5*b^4*d^2
 - a^6*b^3*d^2*736i - 288*a^7*b^2*d^2) + (480*a^2*b^4*d^3*tan(c + d*x)^(1/2
)*((b^6*1i)/(4*d^2) - (a^6*1i)/(4*d^2) + (3*a*b^5)/(2*d^2) + (3*a^5*b)/(2*d
^2) - (a^2*b^4*15i)/(4*d^2) - (5*a^3*b^3)/d^2 + (a^4*b^2*15i)/(4*d^2))^(1/2
))/(16*a^9*d^2 + b^9*d^2*16i + 48*a*b^8*d^2 + a^8*b*d^2*48i - a^2*b^7*d^2*2
88i - 736*a^3*b^6*d^2 + a^4*b^5*d^2*960i + 960*a^5*b^4*d^2 - a^6*b^3*d^2*73
6i - 288*a^7*b^2*d^2) - (480*a^4*b^2*d^3*tan(c + d*x)^(1/2)*((b^6*1i)/(4*d^
2) - (a^6*1i)/(4*d^2) + (3*a*b^5)/(2*d^2) + (3*a^5*b)/(2*d^2) - (a^2*b^4*15
i)/(4*d^2) - (5*a^3*b^3)/d^2 + (a^4*b^2*15i)/(4*d^2))^(1/2))/(16*a^9*d^2 +
```

b^9*d^2*16i + 48*a*b^8*d^2 + a^8*b*d^2*48i - a^2*b^7*d^2*288i - 736*a^3*b^6*d^2 + a^4*b^5*d^2*960i + 960*a^5*b^4*d^2 - a^6*b^3*d^2*736i - 288*a^7*b^2*d^2))*((6*a*b^5 + 6*a^5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*d^2))^(1/2) - 2*atanh((32*a^6*d^3*tan(c + d*x)^(1/2)*((a^6*1i)/(4*d^2) - (b^6*1i)/(4*d^2) + (3*a*b^5)/(2*d^2) + (3*a^5*b)/(2*d^2) + (a^2*b^4*15i)/(4*d^2) - (5*a^3*b^3)/d^2 - (a^4*b^2*15i)/(4*d^2))^(1/2))/(16*a^9*d^2 - b^9*d^2*16i + 48*a*b^8*d^2 - a^8*b*d^2*48i + a^2*b^7*d^2*288i - 736*a^3*b^6*d^2 - a^4*b^5*d^2*960i + 960*a^5*b^4*d^2 + a^6*b^3*d^2*736i - 288*a^7*b^2*d^2) - (32*b^6*d^3*tan(c + d*x)^(1/2)*((a^6*1i)/(4*d^2) - (b^6*1i)/(4*d^2) + (3*a*b^5)/(2*d^2) + (3*a^5*b)/(2*d^2) + (a^2*b^4*15i)/(4*d^2) - (5*a^3*b^3)/d^2 - (a^4*b^2*15i)/(4*d^2))^(1/2))/(16*a^9*d^2 - b^9*d^2*16i + 48*a*b^8*d^2 - a^8*b*d^2*48i + a^2*b^7*d^2*288i - 736*a^3*b^6*d^2 - a^4*b^5*d^2*960i + 960*a^5*b^4*d^2 + a^6*b^3*d^2*736i - 288*a^7*b^2*d^2) + (480*a^2*b^4*d^3*tan(c + d*x)^(1/2)*((a^6*1i)/(4*d^2) - (b^6*1i)/(4*d^2) + (3*a*b^5)/(2*d^2) + (3*a^5*b)/(2*d^2) + (a^2*b^4*15i)/(4*d^2) - (5*a^3*b^3)/d^2 - (a^4*b^2*15i)/(4*d^2))^(1/2))/(16*a^9*d^2 - b^9*d^2*16i + 48*a*b^8*d^2 - a^8*b*d^2*48i + a^2*b^7*d^2*288i - 736*a^3*b^6*d^2 - a^4*b^5*d^2*960i + 960*a^5*b^4*d^2 + a^6*b^3*d^2*736i - 288*a^7*b^2*d^2) - (480*a^4*b^2*d^3*tan(c + d*x)^(1/2)*((a^6*1i)/(4*d^2) - (b^6*1i)/(4*d^2) + (3*a*b^5)/(2*d^2) + (3*a^5*b)/(2*d^2) + (a^2*b^4*15i)/(4*d^2) - (5*a^3*b^3)/d^2 - (a^4*b^2*15i)/(4*d^2))^(1/2))/(16*a^9*d^2 - b^9*d^2*16i + 48*a*b^8*d^2 - a^8*b*d^2*48i + a^2*b^7*d^2*288i - 736*a^3*b^6*d^2 - a^4*b^5*d^2*960i + 960*a^5*b^4*d^2 + a^6*b^3*d^2*736i - 288*a^7*b^2*d^2))*((6*a*b^5 + 6*a^5*b + a^6*1i - b^6*1i + a^2*b^4*15i - 20*a^3*b^3 - a^4*b^2*15i)/(4*d^2))^(1/2) - (tan(c + d*x)^2*(6*a*b^2 - 2*a^3) + (2*a^3)/5 + 2*a^2*b*tan(c + d*x))/(d*tan(c + d*x)^(5/2))

3.576 $\int \frac{(a+b\tan(c+dx))^3}{\tan^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=299

$$-\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(\right.}{\sqrt{2}\,d}$$

```
[Out] 1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)+1/2
*(a-b)*(a^2+4*a*b+b^2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)-1/4*(a+
b)*(a^2-4*a*b+b^2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)+1/4*
(a+b)*(a^2-4*a*b+b^2)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)+2
*b*(3*a^2-b^2)/d/tan(d*x+c)^(1/2)-32/35*a^2*b/d/tan(d*x+c)^(5/2)+2/3*a*(a^2
-3*b^2)/d/tan(d*x+c)^(3/2)-2/7*a^2*(a+b*tan(d*x+c))/d/tan(d*x+c)^(7/2)
```

Rubi [A]
time = 0.27, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3646, 3709, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d}+\frac{2a\left(a^2-3b^2\right)}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{2b\left(3a^2-b^2\right)}{d\sqrt{\tan(c+dx)}}-\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{2a^2(a+b\tan(c+dx))}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{32a^2b}{35d\tan^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^3/Tan[c + d*x]^(9/2),x]
```

```
[Out] -(((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqr
t[2]*d)) + ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x
]]])/(Sqrt[2]*d) - ((a + b)*(a^2 - 4*a*b + b^2)*Log[1 - Sqrt[2]*Sqrt[Tan[c
+ d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) + ((a + b)*(a^2 - 4*a*b + b^2)*Log[1
 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) - (32*a^2*b)/(
35*d*Tan[c + d*x]^(5/2)) + (2*a*(a^2 - 3*b^2))/(3*d*Tan[c + d*x]^(3/2)) + (
2*b*(3*a^2 - b^2))/(d*Sqrt[Tan[c + d*x]]) - (2*a^2*(a + b*Tan[c + d*x]))/(7
*d*Tan[c + d*x]^(7/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(c+dx))^3}{\tan^{\frac{9}{2}}(c+dx)}\,dx = -\frac{2a^2(a+b\tan(c+dx))}{7d\tan^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{8a^2b - \frac{7}{2}a(a^2-3b^2)\tan(c+dx) - \frac{1}{2}b(5a^2 - }{\tan^{\frac{7}{2}}(c+dx)}$$

$$= -\frac{32a^2b}{35d\tan^{\frac{5}{2}}(c+dx)} - \frac{2a^2(a+b\tan(c+dx))}{7d\tan^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{-\frac{7}{2}a(a^2-3b^2) - \frac{7}{2}b(3a^2}{\tan^{\frac{5}{2}}(c+}$$

$$= -\frac{32a^2b}{35d\tan^{\frac{5}{2}}(c+dx)} + \frac{2a(a^2-3b^2)}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{2a^2(a+b\tan(c+dx))}{7d\tan^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{-\frac{7}{2}b(}{}$$

$$= -\frac{32a^2b}{35d\tan^{\frac{5}{2}}(c+dx)} + \frac{2a(a^2-3b^2)}{3d\tan^{\frac{3}{2}}(c+dx)} + \frac{2b(3a^2-b^2)}{d\sqrt{\tan(c+dx)}} - \frac{2a^2(a+b\tan(c}{7d\tan^{\frac{7}{2}}(c+}$$

$$= -\frac{32a^2b}{35d\tan^{\frac{5}{2}}(c+dx)} + \frac{2a(a^2-3b^2)}{3d\tan^{\frac{3}{2}}(c+dx)} + \frac{2b(3a^2-b^2)}{d\sqrt{\tan(c+dx)}} - \frac{2a^2(a+b\tan(c}{7d\tan^{\frac{7}{2}}(c+}$$

$$= -\frac{32a^2b}{35d\tan^{\frac{5}{2}}(c+dx)} + \frac{2a(a^2-3b^2)}{3d\tan^{\frac{3}{2}}(c+dx)} + \frac{2b(3a^2-b^2)}{d\sqrt{\tan(c+dx)}} - \frac{2a^2(a+b\tan(c}{7d\tan^{\frac{7}{2}}(c+}$$

$$= -\frac{32a^2b}{35d\tan^{\frac{5}{2}}(c+dx)} + \frac{2a(a^2-3b^2)}{3d\tan^{\frac{3}{2}}(c+dx)} + \frac{2b(3a^2-b^2)}{d\sqrt{\tan(c+dx)}} - \frac{2a^2(a+b\tan(c}{7d\tan^{\frac{7}{2}}(c+}$$

$$= -\frac{(a+b)(a^2-4ab+b^2)\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}\,d} + \frac{(a+}{}$$

$$= -\frac{(a-b)(a^2+4ab+b^2)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d} + \frac{(a-b)(a^2+4ab}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.60, size = 103, normalized size = 0.34

$$-\frac{2\left(5a(a^2-3b^2)\,{}_2F_1\left(-\frac{7}{4},1;-\frac{3}{4};-\tan^2(c+dx)\right)+b\left(7(3a^2-b^2)\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\tan^2(c+dx)\right)\tan(c+dx)+b(15a+7b\tan(c+dx))\right)\right)}{35d\tan^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^3/Tan[c + d*x]^(9/2),x]
```

```
[Out] (-2*(5*a*(a^2 - 3*b^2)*Hypergeometric2F1[-7/4, 1, -3/4, -Tan[c + d*x]^2] +
b*(7*(3*a^2 - b^2)*Hypergeometric2F1[-5/4, 1, -1/4, -Tan[c + d*x]^2]*Tan[c
+ d*x] + b*(15*a + 7*b*Tan[c + d*x]))))/(35*d*Tan[c + d*x]^(7/2))
```

Maple [A]
time = 0.06, size = 267, normalized size = 0.89

method	result
derivativedivides	$\frac{-\frac{2a^3}{7\tan(dx+c)^{\frac{7}{2}}}+\frac{2a\left(a^2-3b^2\right)}{3\tan(dx+c)^{\frac{3}{2}}}-\frac{6a^2b}{5\tan(dx+c)^{\frac{5}{2}}}+\frac{2b\left(3a^2-b^2\right)}{\sqrt{\tan(dx+c)}}+\frac{\left(a^3-3b^2a\right)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+t}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+t}\right.\right.}{}}{}$
default	$\frac{-\frac{2a^3}{7\tan(dx+c)^{\frac{7}{2}}}+\frac{2a\left(a^2-3b^2\right)}{3\tan(dx+c)^{\frac{3}{2}}}-\frac{6a^2b}{5\tan(dx+c)^{\frac{5}{2}}}+\frac{2b\left(3a^2-b^2\right)}{\sqrt{\tan(dx+c)}}+\frac{\left(a^3-3b^2a\right)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+t}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+t}\right.\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^3/tan(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/7*a^3/tan(d*x+c)^(7/2)+2/3*a*(a^2-3*b^2)/tan(d*x+c)^(3/2)-6/5*a^2*b
/tan(d*x+c)^(5/2)+2*b*(3*a^2-b^2)/tan(d*x+c)^(1/2)+1/4*(a^3-3*a*b^2)*2^(1/2
)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+t
an(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*
x+c)^(1/2)))+1/4*(3*a^2*b-b^3)*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(
d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x
+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))))
```

Maxima [A]
time = 0.50, size = 259, normalized size = 0.87

$$\frac{210\sqrt{2}\left(a^3+3a^2b-3ab^2-b^3\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+210\sqrt{2}\left(a^3+3a^2b-3ab^2-b^3\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)+105\sqrt{2}\left(a^3-3a^2b-3ab^2+b^3\right)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)-105\sqrt{2}\left(a^3-3a^2b-3ab^2+b^3\right)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)-\frac{8\left(63a^2b\tan(dx+c)-105\left(3a^2b-b^3\right)\tan(dx+c)^3+15a^3-35\left(a^3-3ab^2\right)\tan(dx+c)^2\right)}{\tan(dx+c)^{\frac{7}{2}}}}{420d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^3/tan(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/420*(210*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*arctan(1/2*sqrt(2)*(sqrt
(2) + 2*sqrt(tan(d*x + c)))) + 210*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*
arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + 105*sqrt(2)*(a^3 -
3*a^2*b - 3*a*b^2 + b^3)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1)
 - 105*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*log(-sqrt(2)*sqrt(tan(d*x +
c)) + tan(d*x + c) + 1) - 8*(63*a^2*b*tan(d*x + c) - 105*(3*a^2*b - b^3)*ta
n(d*x + c)^3 + 15*a^3 - 35*(a^3 - 3*a*b^2)*tan(d*x + c)^2)/tan(d*x + c)^(7/
2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7771 vs. $2(257) = 514$.
time = 3.20, size = 7771, normalized size = 25.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^3/tan(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/420*(420*sqrt(2)*(d^5*cos(d*x + c)^4 - 2*d^5*cos(d*x + c)^2 + d^5)*sqrt(
(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^
12 + 2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^
8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4))/(a^12 - 30*a^10*
b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*((a^12
 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d
^4)^(3/4)*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^
8 - 30*a^2*b^10 + b^12)/d^4)*arctan(((a^24 - 6*a^22*b^2 - 84*a^20*b^4 - 322
*a^18*b^6 - 603*a^16*b^8 - 540*a^14*b^10 + 540*a^10*b^14 + 603*a^8*b^16 + 3
22*a^6*b^18 + 84*a^4*b^20 + 6*a^2*b^22 - b^24)*d^4*sqrt((a^12 + 6*a^10*b^2
+ 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)*sqrt((a^12
 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^
12)/d^4) - sqrt(2)*((a^3 - 3*a*b^2)*d^7*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^
4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)*sqrt((a^12 - 30*a^10*
b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4) -
(3*a^8*b + 8*a^6*b^3 + 6*a^4*b^5 - b^9)*d^5*sqrt((a^12 - 30*a^10*b^2 + 255*
a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4))*sqrt((a^12
+ 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 + 2
*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4
+ 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4))/(a^12 - 30*a^10*b^2 +
255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*sqrt(((a^18
- 27*a^16*b^2 + 168*a^14*b^4 + 224*a^12*b^6 - 366*a^10*b^8 - 366*a^8*b^10 +
 224*a^6*b^12 + 168*a^4*b^14 - 27*a^2*b^16 + b^18)*d^2*sqrt((a^12 + 6*a^10*
b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)*cos(d*
x + c) + sqrt(2)*((3*a^14*b - 91*a^12*b^3 + 795*a^10*b^5 - 1611*a^8*b^7 + 1
217*a^6*b^9 - 345*a^4*b^11 + 33*a^2*b^13 - b^15)*d^3*sqrt((a^12 + 6*a^10*b^
2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)*cos(d*x
+ c) - (a^21 - 30*a^19*b^2 + 249*a^17*b^4 - 280*a^15*b^6 - 1038*a^13*b^8 +
732*a^11*b^10 + 1322*a^9*b^12 - 504*a^7*b^14 - 531*a^5*b^16 + 82*a^3*b^18 -
 3*a*b^20)*d*cos(d*x + c))*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^
6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 + 2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*d^2
*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^1
0 + b^12)/d^4))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b
^8 - 30*a^2*b^10 + b^12))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^12 + 6*a^10*b
^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)^(1/4) +
 (a^24 - 24*a^22*b^2 + 90*a^20*b^4 + 648*a^18*b^6 + 783*a^16*b^8 - 624*a^14
*b^10 - 1748*a^12*b^12 - 624*a^10*b^14 + 783*a^8*b^16 + 648*a^6*b^18 + 90*a
^4*b^20 - 24*a^2*b^22 + b^24)*sin(d*x + c))/cos(d*x + c))*((a^12 + 6*a^10*b
^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)^(3/4) -
 sqrt(2)*((a^15 - 15*a^13*b^2 + 9*a^11*b^4 + 81*a^9*b^6 + 27*a^7*b^8 - 69*a
^5*b^10 - 37*a^3*b^12 + 3*a*b^14)*d^7*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4
```

```
+ 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)*sqrt((a^12 - 30*a^10*b^
2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4) - (3
*a^20*b - 28*a^18*b^3 - 171*a^16*b^5 - 288*a^14*b^7 - 82*a^12*b^9 + 264*a^1
0*b^11 + 282*a^8*b^13 + 64*a^6*b^15 - 33*a^4*b^17 - 12*a^2*b^19 + b^21)*d^5
*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^
2*b^10 + b^12)/d^4))*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15
*a^4*b^8 + 6*a^2*b^10 + b^12 + 2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*d^2*sqrt(
(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^
12)/d^4))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 3
0*a^2*b^10 + b^12))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^12 + 6*a^10*b^2 + 1
5*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)^(3/4))/(a^36
- 18*a^34*b^2 - 39*a^32*b^4 + 848*a^30*b^6 + 5556*a^28*b^8 + 15240*a^26*b^1
0 + 20420*a^24*b^12 + 5424*a^22*b^14 - 25938*a^20*b^16 - 42988*a^18*b^18 -
25938*a^16*b^20 + 5424*a^14*b^22 + 20420*a^12*b^24 + 15240*a^10*b^26 + 5556
*a^8*b^28 + 848*a^6*b^30 - 39*a^4*b^32 - 18*a^2*b^34 + b^36)) + 420*sqrt(2)
*(d^5*cos(d*x + c)^4 - 2*d^5*cos(d*x + c)^2 + d^5)*sqrt((a^12 + 6*a^10*b^2
+ 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 + 2*(3*a^5*b - 1
0*a^3*b^3 + 3*a*b^5)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6
+ 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 -
 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*((a^12 + 6*a^10*b^2 + 15*
a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)^(3/4)*sqrt((a^1
2 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b
^12)/d^4)*arctan(-((a^24 - 6*a^22*b^2 - 84*a^20*b^4 - 322*a^18*b^6 - 603*a^
16*b^8 - 540*a^14*b^10 + 540*a^10*b^14 + 603*a^8*b^16 + 322*a^6*b^18 + 84*a
^4*b^20 + 6*a^2*b^22 - b^24)*d^4*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*
a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)*...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan(c + dx))^3}{\tan^{\frac{9}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**3/tan(d*x+c)**(9/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3/tan(c + d*x)**(9/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^3/tan(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 8.40, size = 1795, normalized size = 6.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^3/tan(c + d*x)^(9/2),x)
[Out] 2*atanh((32*a^6*d^3*tan(c + d*x)^(1/2)*((a^6*1i)/(4*d^2) - (b^6*1i)/(4*d^2)
 - (3*a*b^5)/(2*d^2) - (3*a^5*b)/(2*d^2) + (a^2*b^4*15i)/(4*d^2) + (5*a^3*b
^3)/d^2 - (a^4*b^2*15i)/(4*d^2))^(1/2))/(a^9*d^2*16i - 16*b^9*d^2 + a*b^8*d
^2*48i - 48*a^8*b*d^2 + 288*a^2*b^7*d^2 - a^3*b^6*d^2*736i - 960*a^4*b^5*d^
2 + a^5*b^4*d^2*960i + 736*a^6*b^3*d^2 - a^7*b^2*d^2*288i) - (32*b^6*d^3*ta
n(c + d*x)^(1/2)*((a^6*1i)/(4*d^2) - (b^6*1i)/(4*d^2) - (3*a*b^5)/(2*d^2) -
 (3*a^5*b)/(2*d^2) + (a^2*b^4*15i)/(4*d^2) + (5*a^3*b^3)/d^2 - (a^4*b^2*15i
)/(4*d^2))^(1/2))/(a^9*d^2*16i - 16*b^9*d^2 + a*b^8*d^2*48i - 48*a^8*b*d^2
+ 288*a^2*b^7*d^2 - a^3*b^6*d^2*736i - 960*a^4*b^5*d^2 + a^5*b^4*d^2*960i +
 736*a^6*b^3*d^2 - a^7*b^2*d^2*288i) + (480*a^2*b^4*d^3*tan(c + d*x)^(1/2)*
((a^6*1i)/(4*d^2) - (b^6*1i)/(4*d^2) - (3*a*b^5)/(2*d^2) - (3*a^5*b)/(2*d^2
) + (a^2*b^4*15i)/(4*d^2) + (5*a^3*b^3)/d^2 - (a^4*b^2*15i)/(4*d^2))^(1/2))
/(a^9*d^2*16i - 16*b^9*d^2 + a*b^8*d^2*48i - 48*a^8*b*d^2 + 288*a^2*b^7*d^2
 - a^3*b^6*d^2*736i - 960*a^4*b^5*d^2 + a^5*b^4*d^2*960i + 736*a^6*b^3*d^2
- a^7*b^2*d^2*288i) - (480*a^4*b^2*d^3*tan(c + d*x)^(1/2)*((a^6*1i)/(4*d^2)
 - (b^6*1i)/(4*d^2) - (3*a*b^5)/(2*d^2) - (3*a^5*b)/(2*d^2) + (a^2*b^4*15i)
/(4*d^2) + (5*a^3*b^3)/d^2 - (a^4*b^2*15i)/(4*d^2))^(1/2))/(a^9*d^2*16i - 1
6*b^9*d^2 + a*b^8*d^2*48i - 48*a^8*b*d^2 + 288*a^2*b^7*d^2 - a^3*b^6*d^2*73
6i - 960*a^4*b^5*d^2 + a^5*b^4*d^2*960i + 736*a^6*b^3*d^2 - a^7*b^2*d^2*288
i))*(-(6*a*b^5 + 6*a^5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^3*b^3 + a^4
*b^2*15i)/(4*d^2))^(1/2) - 2*atanh((32*a^6*d^3*tan(c + d*x)^(1/2)*((b^6*1i)
/(4*d^2) - (a^6*1i)/(4*d^2) - (3*a*b^5)/(2*d^2) - (3*a^5*b)/(2*d^2) - (a^2*
b^4*15i)/(4*d^2) + (5*a^3*b^3)/d^2 + (a^4*b^2*15i)/(4*d^2))^(1/2))/(a^9*d^2
*16i + 16*b^9*d^2 + a*b^8*d^2*48i + 48*a^8*b*d^2 - 288*a^2*b^7*d^2 - a^3*b^
6*d^2*736i + 960*a^4*b^5*d^2 + a^5*b^4*d^2*960i - 736*a^6*b^3*d^2 - a^7*b^2
*d^2*288i) - (32*b^6*d^3*tan(c + d*x)^(1/2)*((b^6*1i)/(4*d^2) - (a^6*1i)/(4
*d^2) - (3*a*b^5)/(2*d^2) - (3*a^5*b)/(2*d^2) - (a^2*b^4*15i)/(4*d^2) + (5*
a^3*b^3)/d^2 + (a^4*b^2*15i)/(4*d^2))^(1/2))/(a^9*d^2*16i + 16*b^9*d^2 + a*
b^8*d^2*48i + 48*a^8*b*d^2 - 288*a^2*b^7*d^2 - a^3*b^6*d^2*736i + 960*a^4*b
^5*d^2 + a^5*b^4*d^2*960i - 736*a^6*b^3*d^2 - a^7*b^2*d^2*288i) + (480*a^2*
b^4*d^3*tan(c + d*x)^(1/2)*((b^6*1i)/(4*d^2) - (a^6*1i)/(4*d^2) - (3*a*b^5)
/(2*d^2) - (3*a^5*b)/(2*d^2) - (a^2*b^4*15i)/(4*d^2) + (5*a^3*b^3)/d^2 + (a
^4*b^2*15i)/(4*d^2))^(1/2))/(a^9*d^2*16i + 16*b^9*d^2 + a*b^8*d^2*48i + 48*
a^8*b*d^2 - 288*a^2*b^7*d^2 - a^3*b^6*d^2*736i + 960*a^4*b^5*d^2 + a^5*b^4*
d^2*960i - 736*a^6*b^3*d^2 - a^7*b^2*d^2*288i) - (480*a^4*b^2*d^3*tan(c + d
```

```
*x)^(1/2)*((b^6*1i)/(4*d^2) - (a^6*1i)/(4*d^2) - (3*a*b^5)/(2*d^2) - (3*a^5
*b)/(2*d^2) - (a^2*b^4*15i)/(4*d^2) + (5*a^3*b^3)/d^2 + (a^4*b^2*15i)/(4*d^
2))^(1/2))/(a^9*d^2*16i + 16*b^9*d^2 + a*b^8*d^2*48i + 48*a^8*b*d^2 - 288*a
^2*b^7*d^2 - a^3*b^6*d^2*736i + 960*a^4*b^5*d^2 + a^5*b^4*d^2*960i - 736*a^
6*b^3*d^2 - a^7*b^2*d^2*288i))*(-(6*a*b^5 + 6*a^5*b + a^6*1i - b^6*1i + a^2
*b^4*15i - 20*a^3*b^3 - a^4*b^2*15i)/(4*d^2))^(1/2) - (tan(c + d*x)^2*(2*a*
b^2 - (2*a^3)/3) - tan(c + d*x)^3*(6*a^2*b - 2*b^3) + (2*a^3)/7 + (6*a^2*b*
tan(c + d*x))/5)/(d*tan(c + d*x)^(7/2))
```

3.577 $\int \frac{(a+b\tan(c+dx))^3}{\tan^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=326

$$\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(c}\right.}{\sqrt{2}\,d}$$

```
[Out] -1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)-1/
2*(a+b)*(a^2-4*a*b+b^2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)-1/4*(a
-b)*(a^2+4*a*b+b^2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)+1/4
*(a-b)*(a^2+4*a*b+b^2)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)-
2*a*(a^2-3*b^2)/d/tan(d*x+c)^(1/2)-40/63*a^2*b/d/tan(d*x+c)^(7/2)+2/5*a*(a^
2-3*b^2)/d/tan(d*x+c)^(5/2)+2/3*b*(3*a^2-b^2)/d/tan(d*x+c)^(3/2)-2/9*a^2*(a
+b*tan(d*x+c))/d/tan(d*x+c)^(9/2)
```

Rubi [A]
time = 0.30, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3646, 3709, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d}+\frac{2a(a^2-3b^2)}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{2b(3a^2-b^2)}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2a(a^2-3b^2)}{d\sqrt{\tan(c+dx)}}-\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\tan(c+dx)-\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\tan(c+dx)+\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{2a^2(a+b\tan(c+dx))}{9d\tan^{\frac{9}{2}}(c+dx)}-\frac{40a^2b}{63d\tan^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^3/Tan[c + d*x]^(11/2),x]
```

```
[Out] ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[
2]*d) - ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]
)/(Sqrt[2]*d) - ((a - b)*(a^2 + 4*a*b + b^2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d
*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) + ((a - b)*(a^2 + 4*a*b + b^2)*Log[1 +
Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) - (40*a^2*b)/(63*
d*Tan[c + d*x]^(7/2)) + (2*a*(a^2 - 3*b^2))/(5*d*Tan[c + d*x]^(5/2)) + (2*b
*(3*a^2 - b^2))/(3*d*Tan[c + d*x]^(3/2)) - (2*a*(a^2 - 3*b^2))/(d*Sqrt[Tan[
c + d*x]]) - (2*a^2*(a + b*Tan[c + d*x]))/(9*d*Tan[c + d*x]^(9/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(c+dx))^3}{\tan^{\frac{11}{2}}(c+dx)}\,dx = -\frac{2a^2(a+b\tan(c+dx))}{9d\tan^{\frac{9}{2}}(c+dx)} + \frac{2}{9}\int \frac{10a^2b - \frac{9}{2}a(a^2-3b^2)\tan(c+dx) - \frac{1}{2}b(7a^2-9}{\tan^{\frac{9}{2}}(c+dx)}$$

$$= -\frac{40a^2b}{63d\tan^{\frac{7}{2}}(c+dx)} - \frac{2a^2(a+b\tan(c+dx))}{9d\tan^{\frac{9}{2}}(c+dx)} + \frac{2}{9}\int \frac{-\frac{9}{2}a(a^2-3b^2) - \frac{9}{2}b(3a^2-}{\tan^{\frac{7}{2}}(c+dx}$$

$$= -\frac{40a^2b}{63d\tan^{\frac{7}{2}}(c+dx)} + \frac{2a(a^2-3b^2)}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{2a^2(a+b\tan(c+dx))}{9d\tan^{\frac{9}{2}}(c+dx)} + \frac{2}{9}\int \frac{-\frac{9}{2}b(3a}{}$$

$$= -\frac{40a^2b}{63d\tan^{\frac{7}{2}}(c+dx)} + \frac{2a(a^2-3b^2)}{5d\tan^{\frac{5}{2}}(c+dx)} + \frac{2b(3a^2-b^2)}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{2a^2(a+b\tan(c+}{9d\tan^{\frac{9}{2}}(c+dx}$$

$$= -\frac{40a^2b}{63d\tan^{\frac{7}{2}}(c+dx)} + \frac{2a(a^2-3b^2)}{5d\tan^{\frac{5}{2}}(c+dx)} + \frac{2b(3a^2-b^2)}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{2a(a^2-3b^2)}{d\sqrt{\tan(c+dx)}} -$$

$$= -\frac{40a^2b}{63d\tan^{\frac{7}{2}}(c+dx)} + \frac{2a(a^2-3b^2)}{5d\tan^{\frac{5}{2}}(c+dx)} + \frac{2b(3a^2-b^2)}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{2a(a^2-3b^2)}{d\sqrt{\tan(c+dx)}} -$$

$$= -\frac{40a^2b}{63d\tan^{\frac{7}{2}}(c+dx)} + \frac{2a(a^2-3b^2)}{5d\tan^{\frac{5}{2}}(c+dx)} + \frac{2b(3a^2-b^2)}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{2a(a^2-3b^2)}{d\sqrt{\tan(c+dx)}} -$$

$$= -\frac{40a^2b}{63d\tan^{\frac{7}{2}}(c+dx)} + \frac{2a(a^2-3b^2)}{5d\tan^{\frac{5}{2}}(c+dx)} + \frac{2b(3a^2-b^2)}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{2a(a^2-3b^2)}{d\sqrt{\tan(c+dx)}} -$$

$$= -\frac{(a-b)(a^2+4ab+b^2)\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}\,d} + \frac{(a-b)}{}$$

$$= \frac{(a+b)(a^2-4ab+b^2)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d} - \frac{(a+b)(a^2-4ab+b}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.71, size = 104, normalized size = 0.32

$$-\frac{2\left(7a(a^2-3b^2)\,{}_2F_1\left(-\frac{9}{4},1;-\frac{5}{4};-\tan^2(c+dx)\right)+3b\left(3(3a^2-b^2)\,{}_2F_1\left(-\frac{7}{4},1;-\frac{3}{4};-\tan^2(c+dx)\right)\tan(c+dx)+b(7a+3b\tan(c+dx))\right)\right)}{63d\tan^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^3/Tan[c + d*x]^(11/2),x]
```

```
[Out] (-2*(7*a*(a^2 - 3*b^2)*Hypergeometric2F1[-9/4, 1, -5/4, -Tan[c + d*x]^2] +
3*b*(3*(3*a^2 - b^2)*Hypergeometric2F1[-7/4, 1, -3/4, -Tan[c + d*x]^2]*Tan[
c + d*x] + b*(7*a + 3*b*Tan[c + d*x]))))/(63*d*Tan[c + d*x]^(9/2))
```

Maple [A]
time = 0.06, size = 289, normalized size = 0.89

method	result
derivativedivides	$\frac{-\frac{2a^3}{9\tan(dx+c)^{\frac{9}{2}}}-\frac{2a\left(a^2-3b^2\right)}{\sqrt{\tan\left(dx+c\right)}}+\frac{2a\left(a^2-3b^2\right)}{5\tan(dx+c)^{\frac{5}{2}}}-\frac{6a^2b}{7\tan(dx+c)^{\frac{7}{2}}}+\frac{2b\left(3a^2-b^2\right)}{3\tan(dx+c)^{\frac{3}{2}}}+\frac{\left(3a^2b-b^3\right)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\right.}\right.}{1-\sqrt{2}\left(\sqrt{\right.}\right.}\right.\right.}{}}{}$
default	$\frac{-\frac{2a^3}{9\tan(dx+c)^{\frac{9}{2}}}-\frac{2a\left(a^2-3b^2\right)}{\sqrt{\tan\left(dx+c\right)}}+\frac{2a\left(a^2-3b^2\right)}{5\tan(dx+c)^{\frac{5}{2}}}-\frac{6a^2b}{7\tan(dx+c)^{\frac{7}{2}}}+\frac{2b\left(3a^2-b^2\right)}{3\tan(dx+c)^{\frac{3}{2}}}+\frac{\left(3a^2b-b^3\right)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\right.}\right.}{1-\sqrt{2}\left(\sqrt{\right.}\right.}\right.\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^3/tan(d*x+c)^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/9*a^3/tan(d*x+c)^(9/2)-2*a*(a^2-3*b^2)/tan(d*x+c)^(1/2)+2/5*a*(a^2-
3*b^2)/tan(d*x+c)^(5/2)-6/7*a^2*b/tan(d*x+c)^(7/2)+2/3*b*(3*a^2-b^2)/tan(d*
x+c)^(3/2)+1/4*(3*a^2*b-b^3)*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*
x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c
)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(-a^3+3*a*b^2)*2^(1/2)*
(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan
(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+
c)^(1/2))))
```

Maxima [A]
time = 0.50, size = 279, normalized size = 0.86

$$-\frac{630\sqrt{2}\left(a^3-3a^2b-3ab^2+b^3\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(dx+c\right)}\right)\right)+630\sqrt{2}\left(a^3-3a^2b-3ab^2+b^3\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(dx+c\right)}\right)\right)-315\sqrt{2}\left(a^3+3a^2b-3ab^2-b^3\right)\log\left(\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)+315\sqrt{2}\left(a^3+3a^2b-3ab^2-b^3\right)\log\left(-\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)+\frac{8\left(315\left(a^3-3ab^2\right)\tan(dx+c)^4+135a^2b\tan(dx+c)-105\left(3a^2b-b^3\right)\tan(dx+c)^3+35a^3-63\left(a^3-3ab^2\right)\tan(dx+c)^2\right)}{\tan(dx+c)^{\frac{9}{2}}}}{1260d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^3/tan(d*x+c)^(11/2),x, algorithm="maxima")
```

```
[Out] -1/1260*(630*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*arctan(1/2*sqrt(2)*(sq
rt(2) + 2*sqrt(tan(d*x + c)))) + 630*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3
)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - 315*sqrt(2)*(a^3
+ 3*a^2*b - 3*a*b^2 - b^3)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) +
1) + 315*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*log(-sqrt(2)*sqrt(tan(d*x
+ c)) + tan(d*x + c) + 1) + 8*(315*(a^3 - 3*a*b^2)*tan(d*x + c)^4 + 135*a^2
*b*tan(d*x + c) - 105*(3*a^2*b - b^3)*tan(d*x + c)^3 + 35*a^3 - 63*(a^3 - 3
*a*b^2)*tan(d*x + c)^2)/tan(d*x + c)^(9/2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 8178 vs. $2(280) = 560$.

time = 3.48, size = 8178, normalized size = 25.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^3/tan(d*x+c)^(11/2),x, algorithm="fricas")
[Out] 1/1260*(1260*sqrt(2)*(d^5*cos(d*x + c)^6 - 3*d^5*cos(d*x + c)^4 + 3*d^5*cos
(d*x + c)^2 - d^5)*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a
^4*b^8 + 6*a^2*b^10 + b^12 - 2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*d^2*sqrt((a
^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12
)/d^4))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*
a^2*b^10 + b^12))*((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^
8 + 6*a^2*b^10 + b^12)/d^4)^(3/4)*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 -
452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4)*arctan(-((a^24 - 6*a^2
2*b^2 - 84*a^20*b^4 - 322*a^18*b^6 - 603*a^16*b^8 - 540*a^14*b^10 + 540*a^1
0*b^14 + 603*a^8*b^16 + 322*a^6*b^18 + 84*a^4*b^20 + 6*a^2*b^22 - b^24)*d^4
*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^1
0 + b^12)/d^4)*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a
^4*b^8 - 30*a^2*b^10 + b^12)/d^4) + sqrt(2)*((3*a^2*b - b^3)*d^7*sqrt((a^12
 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d
^4)*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30
*a^2*b^10 + b^12)/d^4) + (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^5*sqrt((
a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10
+ b^12)/d^4))*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^
8 + 6*a^2*b^10 + b^12 - 2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*d^2*sqrt((a^12 +
 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4
))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b
^10 + b^12))*sqrt(((a^18 - 27*a^16*b^2 + 168*a^14*b^4 + 224*a^12*b^6 - 366*
a^10*b^8 - 366*a^8*b^10 + 224*a^6*b^12 + 168*a^4*b^14 - 27*a^2*b^16 + b^18)
*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2
*b^10 + b^12)/d^4)*cos(d*x + c) + sqrt(2)*((a^15 - 33*a^13*b^2 + 345*a^11*b
^4 - 1217*a^9*b^6 + 1611*a^7*b^8 - 795*a^5*b^10 + 91*a^3*b^12 - 3*a*b^14)*d
^3*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b
^10 + b^12)/d^4)*cos(d*x + c) + (3*a^20*b - 82*a^18*b^3 + 531*a^16*b^5 + 50
4*a^14*b^7 - 1322*a^12*b^9 - 732*a^10*b^11 + 1038*a^8*b^13 + 280*a^6*b^15 -
 249*a^4*b^17 + 30*a^2*b^19 - b^21)*d*cos(d*x + c))*sqrt((a^12 + 6*a^10*b^2
 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 - 2*(3*a^5*b -
10*a^3*b^3 + 3*a*b^5)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6
 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4
- 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*sqrt(sin(d*x + c)/cos(d*
x + c))*((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*
b^10 + b^12)/d^4)^(1/4) + (a^24 - 24*a^22*b^2 + 90*a^20*b^4 + 648*a^18*b^6
+ 783*a^16*b^8 - 624*a^14*b^10 - 1748*a^12*b^12 - 624*a^10*b^14 + 783*a^8*b
^16 + 648*a^6*b^18 + 90*a^4*b^20 - 24*a^2*b^22 + b^24)*sin(d*x + c))/cos(d*
```

```
x + c))*((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*
b^10 + b^12)/d^4)^(3/4) + sqrt(2)*((3*a^14*b - 37*a^12*b^3 - 69*a^10*b^5 +
27*a^8*b^7 + 81*a^6*b^9 + 9*a^4*b^11 - 15*a^2*b^13 + b^15)*d^7*sqrt((a^12 +
 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)/d^4
)*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a
^2*b^10 + b^12)/d^4) + (a^21 - 12*a^19*b^2 - 33*a^17*b^4 + 64*a^15*b^6 + 28
2*a^13*b^8 + 264*a^11*b^10 - 82*a^9*b^12 - 288*a^7*b^14 - 171*a^5*b^16 - 28
*a^3*b^18 + 3*a*b^20)*d^5*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*
b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4))*sqrt((a^12 + 6*a^10*b^2 + 15*
a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 - 2*(3*a^5*b - 10*a^3
*b^3 + 3*a*b^5)*d^2*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*
a^4*b^8 + 6*a^2*b^10 + b^12)/d^4))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*
a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*sqrt(sin(d*x + c)/cos(d*x + c)
)*((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 +
 b^12)/d^4)^(3/4))/(a^36 - 18*a^34*b^2 - 39*a^32*b^4 + 848*a^30*b^6 + 5556*
a^28*b^8 + 15240*a^26*b^10 + 20420*a^24*b^12 + 5424*a^22*b^14 - 25938*a^20*
b^16 - 42988*a^18*b^18 - 25938*a^16*b^20 + 5424*a^14*b^22 + 20420*a^12*b^24
 + 15240*a^10*b^26 + 5556*a^8*b^28 + 848*a^6*b^30 - 39*a^4*b^32 - 18*a^2*b^
34 + b^36)) + 1260*sqrt(2)*(d^5*cos(d*x + c)^6 - 3*d^5*cos(d*x + c)^4 + 3*d
^5*cos(d*x + c)^2 - d^5)*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6
+ 15*a^4*b^8 + 6*a^2*b^10 + b^12 - 2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*d^2*s
qrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10
+ b^12)/d^4))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8
 - 30*a^2*b^10 + b^12))*((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*
a^4*b^8 + 6*a^2*b^10 + b^12)/d^4)^(3/4)*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*
b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/d^4)*arctan(((a^24 -
6*a^22*b^2 - 84*a^20*b^4 - 322*a^18*b^6 - 603*a^16*b^8 - 540*a^14*b^10 + 54
0*a^10*b^14 + 603*a^8*b^16 + 322*a^6*b^18 + 84*a^4*b^20 + 6*a^2*b^22 - b^24
)*d^4*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan{(c + dx)})^3}{\tan^{\frac{11}{2}}{(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**3/tan(d*x+c)**(11/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3/tan(c + d*x)**(11/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^3/tan(d*x+c)^(11/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 10.10, size = 1821, normalized size = 5.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^3/tan(c + d*x)^(11/2),x)
```

```
[Out] 2*atanh((32*a^6*d^3*tan(c + d*x)^(1/2)*((b^6*1i)/(4*d^2) - (a^6*1i)/(4*d^2)
 + (3*a*b^5)/(2*d^2) + (3*a^5*b)/(2*d^2) - (a^2*b^4*15i)/(4*d^2) - (5*a^3*b
^3)/d^2 + (a^4*b^2*15i)/(4*d^2))^(1/2))/(16*a^9*d^2 + b^9*d^2*16i + 48*a*b^
8*d^2 + a^8*b*d^2*48i - a^2*b^7*d^2*288i - 736*a^3*b^6*d^2 + a^4*b^5*d^2*96
0i + 960*a^5*b^4*d^2 - a^6*b^3*d^2*736i - 288*a^7*b^2*d^2) - (32*b^6*d^3*ta
n(c + d*x)^(1/2)*((b^6*1i)/(4*d^2) - (a^6*1i)/(4*d^2) + (3*a*b^5)/(2*d^2) +
 (3*a^5*b)/(2*d^2) - (a^2*b^4*15i)/(4*d^2) - (5*a^3*b^3)/d^2 + (a^4*b^2*15i
)/(4*d^2))^(1/2))/(16*a^9*d^2 + b^9*d^2*16i + 48*a*b^8*d^2 + a^8*b*d^2*48i
- a^2*b^7*d^2*288i - 736*a^3*b^6*d^2 + a^4*b^5*d^2*960i + 960*a^5*b^4*d^2 -
 a^6*b^3*d^2*736i - 288*a^7*b^2*d^2) + (480*a^2*b^4*d^3*tan(c + d*x)^(1/2)*
((b^6*1i)/(4*d^2) - (a^6*1i)/(4*d^2) + (3*a*b^5)/(2*d^2) + (3*a^5*b)/(2*d^2
) - (a^2*b^4*15i)/(4*d^2) - (5*a^3*b^3)/d^2 + (a^4*b^2*15i)/(4*d^2))^(1/2))
/(16*a^9*d^2 + b^9*d^2*16i + 48*a*b^8*d^2 + a^8*b*d^2*48i - a^2*b^7*d^2*288
i - 736*a^3*b^6*d^2 + a^4*b^5*d^2*960i + 960*a^5*b^4*d^2 - a^6*b^3*d^2*736i
 - 288*a^7*b^2*d^2) - (480*a^4*b^2*d^3*tan(c + d*x)^(1/2)*((b^6*1i)/(4*d^2)
 - (a^6*1i)/(4*d^2) + (3*a*b^5)/(2*d^2) + (3*a^5*b)/(2*d^2) - (a^2*b^4*15i)
/(4*d^2) - (5*a^3*b^3)/d^2 + (a^4*b^2*15i)/(4*d^2))^(1/2))/(16*a^9*d^2 + b^
9*d^2*16i + 48*a*b^8*d^2 + a^8*b*d^2*48i - a^2*b^7*d^2*288i - 736*a^3*b^6*d
^2 + a^4*b^5*d^2*960i + 960*a^5*b^4*d^2 - a^6*b^3*d^2*736i - 288*a^7*b^2*d^
2))*((6*a*b^5 + 6*a^5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*
b^2*15i)/(4*d^2))^(1/2) + 2*atanh((32*a^6*d^3*tan(c + d*x)^(1/2)*((a^6*1i)/
(4*d^2) - (b^6*1i)/(4*d^2) + (3*a*b^5)/(2*d^2) + (3*a^5*b)/(2*d^2) + (a^2*b
^4*15i)/(4*d^2) - (5*a^3*b^3)/d^2 - (a^4*b^2*15i)/(4*d^2))^(1/2))/(16*a^9*d
^2 - b^9*d^2*16i + 48*a*b^8*d^2 - a^8*b*d^2*48i + a^2*b^7*d^2*288i - 736*a^
3*b^6*d^2 - a^4*b^5*d^2*960i + 960*a^5*b^4*d^2 + a^6*b^3*d^2*736i - 288*a^7
*b^2*d^2) - (32*b^6*d^3*tan(c + d*x)^(1/2)*((a^6*1i)/(4*d^2) - (b^6*1i)/(4*
d^2) + (3*a*b^5)/(2*d^2) + (3*a^5*b)/(2*d^2) + (a^2*b^4*15i)/(4*d^2) - (5*a
^3*b^3)/d^2 - (a^4*b^2*15i)/(4*d^2))^(1/2))/(16*a^9*d^2 - b^9*d^2*16i + 48*
a*b^8*d^2 - a^8*b*d^2*48i + a^2*b^7*d^2*288i - 736*a^3*b^6*d^2 - a^4*b^5*d^
2*960i + 960*a^5*b^4*d^2 + a^6*b^3*d^2*736i - 288*a^7*b^2*d^2) + (480*a^2*b
^4*d^3*tan(c + d*x)^(1/2)*((a^6*1i)/(4*d^2) - (b^6*1i)/(4*d^2) + (3*a*b^5)/
(2*d^2) + (3*a^5*b)/(2*d^2) + (a^2*b^4*15i)/(4*d^2) - (5*a^3*b^3)/d^2 - (a^
```

4*b^2*15i)/(4*d^2))^(1/2))/(16*a^9*d^2 - b^9*d^2*16i + 48*a*b^8*d^2 - a^8*b
*d^2*48i + a^2*b^7*d^2*288i - 736*a^3*b^6*d^2 - a^4*b^5*d^2*960i + 960*a^5*
b^4*d^2 + a^6*b^3*d^2*736i - 288*a^7*b^2*d^2) - (480*a^4*b^2*d^3*tan(c + d*
x)^(1/2)*((a^6*1i)/(4*d^2) - (b^6*1i)/(4*d^2) + (3*a*b^5)/(2*d^2) + (3*a^5*
b)/(2*d^2) + (a^2*b^4*15i)/(4*d^2) - (5*a^3*b^3)/d^2 - (a^4*b^2*15i)/(4*d^2
))^(1/2))/(16*a^9*d^2 - b^9*d^2*16i + 48*a*b^8*d^2 - a^8*b*d^2*48i + a^2*b^
7*d^2*288i - 736*a^3*b^6*d^2 - a^4*b^5*d^2*960i + 960*a^5*b^4*d^2 + a^6*b^3
*d^2*736i - 288*a^7*b^2*d^2))*((6*a*b^5 + 6*a^5*b + a^6*1i - b^6*1i + a^2*b
^4*15i - 20*a^3*b^3 - a^4*b^2*15i)/(4*d^2))^(1/2) - (tan(c + d*x)^2*((6*a*b
^2)/5 - (2*a^3)/5) - tan(c + d*x)^4*(6*a*b^2 - 2*a^3) - tan(c + d*x)^3*(2*a
^2*b - (2*b^3)/3) + (2*a^3)/9 + (6*a^2*b*tan(c + d*x))/7)/(d*tan(c + d*x)^(
9/2))

3.578 $\int \frac{a+b\tan(c+dx)}{\sqrt{\tan(c+dx)}}\,dx$

Optimal. Leaf size=150

$$-\frac{(a+b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a+b)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a-b)\log\left(1-\sqrt{2}\right.}{}$$

```
[Out] 1/2*(a+b)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)+1/2*(a+b)*arctan(1+
2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)-1/4*(a-b)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)
+tan(d*x+c))/d*2^(1/2)+1/4*(a-b)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/
d*2^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{(a+b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a+b)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d}-\frac{(a-b)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{(a-b)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])/Sqrt[Tan[c + d*x]],x]
```

```
[Out] -(((a + b)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d)) + ((a + b)*
ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) - ((a - b)*Log[1 - Sqrt
[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) + ((a - b)*Log[1 + Sq
rt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{a+b\tan(c+dx)}{\sqrt{\tan(c+dx)}}\,dx = \frac{2\text{Subst}\left(\int \frac{a+bx^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d}$$

$$= \frac{(a-b)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d} + \frac{(a+b)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\tan(c}\right.}{d}$$

$$= -\frac{(a-b)\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{(a-b)\text{Subst}\left(\int \frac{\sqrt{2}-}{-1+\sqrt{2}}\right.}{2}$$

$$= -\frac{(a-b)\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d} + \frac{(a-b)\log\left(1+\sqrt{2}\sqrt{\text{ta}}\right.}{2\sqrt{}}$$

$$= -\frac{(a+b)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} + \frac{(a+b)\tan^{-1}\left(1+\sqrt{2}\sqrt{\tan(c+dx}\right.}{\sqrt{2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.05, size = 61, normalized size = 0.41

$$-\frac{\sqrt[4]{-1}\left((a-ib)\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+(a+ib)\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])/Sqrt[Tan[c + d*x]],x]
[Out] -(((-1)^(1/4)*((a - I*b)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + (a + I*b)*
ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]))/d)
```

Maple [A]
time = 0.00, size = 178, normalized size = 1.19

method	result
derivativedivides	$\frac{a\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)\right)+2\arctan\left(-1+\sqrt{2}\left(\sqrt{\tan}\right.\right.\right.}{4}$
default	$\frac{a\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)\right)+2\arctan\left(-1+\sqrt{2}\left(\sqrt{\tan}\right.\right.\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))/tan(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*a*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*t
an(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(
-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*b*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)
+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*ta
n(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))))
```

Maxima [A]
time = 0.49, size = 124, normalized size = 0.83

$$\frac{2\sqrt{2}\,(a+b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}\,(a+b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)+\sqrt{2}\,(a-b)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)-\sqrt{2}\,(a-b)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*(a + b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c))))
 + 2*sqrt(2)*(a + b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c))))
+ sqrt(2)*(a - b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt
(2)*(a - b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2640 vs. $2(122) = 244$.
time = 1.32, size = 2640, normalized size = 17.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*d^4*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4
+ 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(
3/4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4)*arctan(-((a^8 + 2*a^6*b^2 - 2*a^2*b^
6 - b^8)*d^4*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)
/d^4) - sqrt(2)*(a*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*
b^2 + b^4)/d^4) - (a^2*b + b^3)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4))*sqrt
((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4
 - 2*a^2*b^2 + b^4))*sqrt(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt((a^4 +
2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) + sqrt(2)*((a^4*b - 2*a^2*b^3 + b^5)*d^3
*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) - (a^7 - a^5*b^2 - a^3*b^4
+ a*b^6)*d*cos(d*x + c))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)
+ a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x
 + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4) + (a^8 - 2*a^4*b^4 + b^8)*sin(d*
x + c))/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4) - sqrt(2)*((a^5 -
 a*b^4)*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/
d^4) - (a^6*b + a^4*b^3 - a^2*b^5 - b^7)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/d
```

^4))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 +
b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2
*b^2 + b^4)/d^4)^(3/4))/(a^12 + 2*a^10*b^2 - a^8*b^4 - 4*a^6*b^6 - a^4*b^8
+ 2*a^2*b^10 + b^12)) + 4*sqrt(2)*d^4*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2
+ b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4 + 2*a^
2*b^2 + b^4)/d^4)^(3/4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4)*arctan(((a^8 + 2*
a^6*b^2 - 2*a^2*b^6 - b^8)*d^4*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4
- 2*a^2*b^2 + b^4)/d^4) + sqrt(2)*(a*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*
sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4) - (a^2*b + b^3)*d^5*sqrt((a^4 - 2*a^2*b^2
+ b^4)/d^4))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a
^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(((a^6 - a^4*b^2 - a^2*b^4 + b^6
)*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) - sqrt(2)*((a^4*b - 2*
a^2*b^3 + b^5)*d^3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) - (a^7 -
a^5*b^2 - a^3*b^4 + a*b^6)*d*cos(d*x + c))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^
2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(si
n(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4) + (a^8 - 2*a^4
*b^4 + b^8)*sin(d*x + c))/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4)
+ sqrt(2)*((a^5 - a*b^4)*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 -
2*a^2*b^2 + b^4)/d^4) - (a^6*b + a^4*b^3 - a^2*b^5 - b^7)*d^5*sqrt((a^4 -
2*a^2*b^2 + b^4)/d^4))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) +
a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x +
c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4))/(a^12 + 2*a^10*b^2 - a^8*b^4 - 4*
a^6*b^6 - a^4*b^8 + 2*a^2*b^10 + b^12)) + sqrt(2)*(2*a*b*d^2*sqrt((a^4 + 2*
a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)*sqrt((2*a*b*d^2*sqrt((a^4 + 2*
a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4
+ 2*a^2*b^2 + b^4)/d^4)^(1/4)*log(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqr
t((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) + sqrt(2)*((a^4*b - 2*a^2*b^3 +
b^5)*d^3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) - (a^7 - a^5*b^2 -
a^3*b^4 + a*b^6)*d*cos(d*x + c))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b
^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c
)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4) + (a^8 - 2*a^4*b^4 + b^
8)*sin(d*x + c))/cos(d*x + c)) - sqrt(2)*(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 +
b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 +
b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4 + 2*a^2*
b^2 + b^4)/d^4)^(1/4)*log(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt((a^4 +
2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) - sqrt(2)*((a^4*b - 2*a^2*b^3 + b^5)*d^3
*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) - (a^7 - a^5*b^2 - a^3*b^4
+ a*b^6)*d*cos(d*x + c))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)
+ a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x
+ c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4) + (a^8 - 2*a^4*b^4 + b^8)*sin(d*
x + c))/cos(d*x + c)))/(a^4 + 2*a^2*b^2 + b^4)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \tan (c + dx)}{\sqrt{\tan (c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/tan(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))/sqrt(tan(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 0.00, size = 141, normalized size = 0.94

$$\frac{\sqrt{2}\, b \left(\ln\left(\sqrt{2}\, \sqrt{\tan (c + d\,x)} - \tan(c + d\,x) - 1\right) - \ln\left(\tan(c + d\,x) + \sqrt{2}\, \sqrt{\tan (c + d\,x)} + 1\right)\right)}{4\,d} + \frac{\sqrt{2}\, b \left(\mathrm{atan}\left(\sqrt{2}\, \sqrt{\tan (c + d\,x)} - 1\right) + \mathrm{atan}\left(\sqrt{2}\, \sqrt{\tan (c + d\,x)} + 1\right)\right)}{2\,d} - \frac{(-1)^{1/4}\, a\, \mathrm{atan}\left((-1)^{1/4}\, \sqrt{\tan (c + d\,x)}\right)\, 1\mathrm{i}}{d} - \frac{(-1)^{1/4}\, a\, \mathrm{atanh}\left((-1)^{1/4}\, \sqrt{\tan (c + d\,x)}\right)\, 1\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))/tan(c + d*x)^(1/2),x)
```

```
[Out] (2^(1/2)*b*(log(2^(1/2)*tan(c + d*x)^(1/2) - tan(c + d*x) - 1) - log(tan(c
+ d*x) + 2^(1/2)*tan(c + d*x)^(1/2) + 1)))/(4*d) - ((-1)^(1/4)*a*atan((-1)^
(1/4)*tan(c + d*x)^(1/2))*1i)/d - ((-1)^(1/4)*a*atanh((-1)^(1/4)*tan(c + d*
x)^(1/2))*1i)/d + (2^(1/2)*b*(atan(2^(1/2)*tan(c + d*x)^(1/2) - 1) + atan(2
^(1/2)*tan(c + d*x)^(1/2) + 1)))/(2*d)
```

3.579 $\int \frac{a+b\tan(c+dx)}{\sqrt{-\tan(c+dx)}}\,dx$

Optimal. Leaf size=162

$$\frac{(a-b)\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{-\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a-b)\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{-\tan(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a+b)\log\left(1-\right.}{}$$

```
[Out] -1/2*(a-b)*arctan(-1+2^(1/2)*(-tan(d*x+c))^(1/2))/d*2^(1/2)-1/2*(a-b)*arcta
n(1+2^(1/2)*(-tan(d*x+c))^(1/2))/d*2^(1/2)+1/4*(a+b)*ln(1-2^(1/2)*(-tan(d*x
+c))^(1/2)-tan(d*x+c))/d*2^(1/2)-1/4*(a+b)*ln(1+2^(1/2)*(-tan(d*x+c))^(1/2)
-tan(d*x+c))/d*2^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a-b)\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{-\tan(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a-b)\mathrm{ArcTan}\left(\sqrt{2}\sqrt{-\tan(c+dx)}+1\right)}{\sqrt{2}\,d}+\frac{(a+b)\log\left(-\tan(c+dx)-\sqrt{2}\sqrt{-\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{(a+b)\log\left(-\tan(c+dx)+\sqrt{2}\sqrt{-\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])/Sqrt[-Tan[c + d*x]],x]
```

```
[Out] ((a - b)*ArcTan[1 - Sqrt[2]*Sqrt[-Tan[c + d*x]]])/(Sqrt[2]*d) - ((a - b)*Ar
cTan[1 + Sqrt[2]*Sqrt[-Tan[c + d*x]]])/(Sqrt[2]*d) + ((a + b)*Log[1 - Sqrt[
2]*Sqrt[-Tan[c + d*x]] - Tan[c + d*x]])/(2*Sqrt[2]*d) - ((a + b)*Log[1 + Sq
rt[2]*Sqrt[-Tan[c + d*x]] - Tan[c + d*x]])/(2*Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{a+b\tan(c+dx)}{\sqrt{-\tan(c+dx)}}\,dx = \frac{2\text{Subst}\left(\int \frac{-a+bx^2}{1+x^4}\,dx, x, \sqrt{-\tan(c+dx)}\right)}{d}$$
$$= -\frac{(a-b)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{-\tan(c+dx)}\right)}{d} - \frac{(a+b)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{-\right.}{d}$$
$$= -\frac{(a-b)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{-\tan(c+dx)}\right)}{2d} - \frac{(a-b)\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\right.}{}$$
$$= \frac{(a+b)\log\left(1-\sqrt{2}\sqrt{-\tan(c+dx)}-\tan(c+dx)\right)}{2\sqrt{2}d} - \frac{(a+b)\log\left(1+\sqrt{2}\sqrt{-\right.}{2}$$
$$= \frac{(a-b)\tan^{-1}\left(1-\sqrt{2}\sqrt{-\tan(c+dx)}\right)}{\sqrt{2}d} - \frac{(a-b)\tan^{-1}\left(1+\sqrt{2}\sqrt{-\tan(c+\right.}{\sqrt{2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.16, size = 82, normalized size = 0.51

$$\frac{\sqrt[4]{-1}\left((a-ib)\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+(a+ib)\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\right)\tan^{\frac{3}{2}}(c+dx)}{d(-\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])/Sqrt[-Tan[c + d*x]],x]
```

```
[Out] ((-1)^(1/4)*((a - I*b)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + (a + I*b)*Ar
cTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])*Tan[c + d*x]^(3/2))/(d*(-Tan[c + d*x]
)^(3/2))
```

Maple [A]
time = 0.12, size = 202, normalized size = 1.25

method	result
derivativedivides	$-\frac{a\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\sqrt{-\tan(dx+c)}-\tan(dx+c)}{1-\sqrt{2}\sqrt{-\tan(dx+c)}-\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{2}\sqrt{-\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{}\right.\right.}{4}$
default	$-\frac{a\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\sqrt{-\tan(dx+c)}-\tan(dx+c)}{1-\sqrt{2}\sqrt{-\tan(dx+c)}-\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{2}\sqrt{-\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{}\right.\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))/(-tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4*a*2^(1/2)*(ln((1+2^(1/2)*(-tan(d*x+c))^(1/2)-tan(d*x+c))/(1-2^(1/
2)*(-tan(d*x+c))^(1/2)-tan(d*x+c)))+2*arctan(1+2^(1/2)*(-tan(d*x+c))^(1/2))
+2*arctan(-1+2^(1/2)*(-tan(d*x+c))^(1/2)))+1/4*b*2^(1/2)*(ln((1-2^(1/2)*(-t
an(d*x+c))^(1/2)-tan(d*x+c))/(1+2^(1/2)*(-tan(d*x+c))^(1/2)-tan(d*x+c)))+2*
arctan(1+2^(1/2)*(-tan(d*x+c))^(1/2))+2*arctan(-1+2^(1/2)*(-tan(d*x+c))^(1/
2))))
```

Maxima [**A**]
time = 0.49, size = 136, normalized size = 0.84

$$-\frac{2\sqrt{2}(a-b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{-\tan(dx+c)}\right)\right)+2\sqrt{2}(a-b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{-\tan(dx+c)}\right)\right)+\sqrt{2}(a+b)\log\left(\sqrt{2}\sqrt{-\tan(dx+c)}-\tan(dx+c)+1\right)-\sqrt{2}(a+b)\log\left(-\sqrt{2}\sqrt{-\tan(dx+c)}-\tan(dx+c)+1\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/(-tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*(a - b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(-tan(d*x + c))
)) + 2*sqrt(2)*(a - b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(-tan(d*x + c))
)) + sqrt(2)*(a + b)*log(sqrt(2)*sqrt(-tan(d*x + c)) - tan(d*x + c) + 1) -
sqrt(2)*(a + b)*log(-sqrt(2)*sqrt(-tan(d*x + c)) - tan(d*x + c) + 1))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2694 vs. 2(134) = 268.
time = 0.99, size = 2694, normalized size = 16.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/(-tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*d^4*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^
4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)
^(3/4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4)*arctan(((a^8 + 2*a^6*b^2 - 2*a^2*b
^6 - b^8)*d^4*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4
)/d^4) + sqrt(2)*(a*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2
*b^2 + b^4)/d^4) + (a^2*b + b^3)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4))*sqr
t(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a
^4 - 2*a^2*b^2 + b^4))*sqrt(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt((a^4
+ 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) + sqrt(2)*((a^4*b - 2*a^2*b^3 + b^5)*d
^3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) + (a^7 - a^5*b^2 - a^3*b^
4 + a*b^6)*d*cos(d*x + c))*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^
4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(-sin(d*x + c)/cos
(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4) - (a^8 - 2*a^4*b^4 + b^8)*si
n(d*x + c))/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4) + sqrt(2)*((a
^5 - a*b^4)*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b
```

```
^4)/d^4) + (a^6*b + a^4*b^3 - a^2*b^5 - b^7)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^
4)/d^4))*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b
^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(-sin(d*x + c)/cos(d*x + c))*((a^4 +
 2*a^2*b^2 + b^4)/d^4)^(3/4))/(a^12 + 2*a^10*b^2 - a^8*b^4 - 4*a^6*b^6 - a^
4*b^8 + 2*a^2*b^10 + b^12)) + 4*sqrt(2)*d^4*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*
a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4
 + 2*a^2*b^2 + b^4)/d^4)^(3/4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4)*arctan(-((
a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^4*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sq
rt((a^4 - 2*a^2*b^2 + b^4)/d^4) - sqrt(2)*(a*d^7*sqrt((a^4 + 2*a^2*b^2 + b^
4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4) + (a^2*b + b^3)*d^5*sqrt((a^4 - 2
*a^2*b^2 + b^4)/d^4))*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) -
a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(((a^6 - a^4*b^2 - a^2*
b^4 + b^6)*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) - sqrt(2)*((a
^4*b - 2*a^2*b^3 + b^5)*d^3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c)
+ (a^7 - a^5*b^2 - a^3*b^4 + a*b^6)*d*cos(d*x + c))*sqrt(-(2*a*b*d^2*sqrt((
a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4
))*sqrt(-sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4) - (
a^8 - 2*a^4*b^4 + b^8)*sin(d*x + c))/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)
/d^4)^(3/4) - sqrt(2)*((a^5 - a*b^4)*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*
sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4) + (a^6*b + a^4*b^3 - a^2*b^5 - b^7)*d^5*s
qrt((a^4 - 2*a^2*b^2 + b^4)/d^4))*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 +
b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(-sin(d*x +
 c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4))/(a^12 + 2*a^10*b^2 -
 a^8*b^4 - 4*a^6*b^6 - a^4*b^8 + 2*a^2*b^10 + b^12)) + sqrt(2)*(2*a*b*d^2*s
qrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)*sqrt(-(2*a*b*d^2*
sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2
 + b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4)*log(((a^6 - a^4*b^2 - a^2*b^4
+ b^6)*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) + sqrt(2)*((a^4*b
 - 2*a^2*b^3 + b^5)*d^3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) + (a
^7 - a^5*b^2 - a^3*b^4 + a*b^6)*d*cos(d*x + c))*sqrt(-(2*a*b*d^2*sqrt((a^4
+ 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*s
qrt(-sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4) - (a^8
- 2*a^4*b^4 + b^8)*sin(d*x + c))/cos(d*x + c)) - sqrt(2)*(2*a*b*d^2*sqrt((a
^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)*sqrt(-(2*a*b*d^2*sqrt((
a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4
))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4)*log(((a^6 - a^4*b^2 - a^2*b^4 + b^6)
*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) - sqrt(2)*((a^4*b - 2*a
^2*b^3 + b^5)*d^3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) + (a^7 - a
^5*b^2 - a^3*b^4 + a*b^6)*d*cos(d*x + c))*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^
2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(-s
in(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4) - (a^8 - 2*a^
4*b^4 + b^8)*sin(d*x + c))/cos(d*x + c)))/(a^4 + 2*a^2*b^2 + b^4)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\tan(c + dx)}{\sqrt{-\tan(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/(-tan(d*x+c))**(1/2),x)
[Out] Integral((a + b*tan(c + d*x))/sqrt(-tan(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/(-tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)/sqrt(-tan(d*x + c)), x)
```

Mupad [B]
time = 4.59, size = 94, normalized size = 0.58

$$\frac{(-1)^{1/4}\,b\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{-\tan(c+d\,x)}\right)}{d}-\frac{(-1)^{1/4}\,b\,\mathrm{atanh}\left((-1)^{1/4}\,\sqrt{-\tan(c+d\,x)}\right)}{d}+\frac{(-1)^{1/4}\,a\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{-\tan(c+d\,x)}\right)\,1\mathrm{i}}{d}+\frac{(-1)^{1/4}\,a\,\mathrm{atanh}\left((-1)^{1/4}\,\sqrt{-\tan(c+d\,x)}\right)\,1\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))/(-tan(c + d*x))^(1/2),x)
[Out] ((-1)^(1/4)*a*atan((-1)^(1/4)*(-tan(c + d*x))^(1/2))*1i)/d + ((-1)^(1/4)*a*
atanh((-1)^(1/4)*(-tan(c + d*x))^(1/2))*1i)/d + ((-1)^(1/4)*b*atan((-1)^(1/
4)*(-tan(c + d*x))^(1/2)))/d - ((-1)^(1/4)*b*atanh((-1)^(1/4)*(-tan(c + d*x
))^(1/2)))/d
```

3.580 $\int \frac{a+b\tan(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx$

Optimal. Leaf size=208

$$-\frac{(a+b)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}}+\frac{(a+b)\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}}-\frac{(a-b)\log\left(\sqrt{e}\right.}{}$$

```
[Out] -1/2*(a+b)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)/e^(1/2)
+1/2*(a+b)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)/e^(1/2)
-1/4*(a-b)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d*2^
(1/2)/e^(1/2)+1/4*(a-b)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan
(d*x+c))/d*2^(1/2)/e^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{(a+b)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}}+\frac{(a+b)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,d\sqrt{e}}-\frac{(a-b)\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\sqrt{e}}+\frac{(a-b)\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])/Sqrt[e*Tan[c + d*x]],x]
```

```
[Out] -(((a + b)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*S
qrt[e])) + ((a + b)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sq
rt[2]*d*Sqrt[e]) - ((a - b)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sq
rt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*Sqrt[e]) + ((a - b)*Log[Sqrt[e] + Sqrt[e]
*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*Sqrt[e])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{a+b\tan(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx = \frac{2\text{Subst}\left(\int \frac{ae+bx^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{d}$$

$$= \frac{(a-b)\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{d} + \frac{(a+b)\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\,\text{t}\right.}{d}$$

$$= \frac{(a+b)\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}\,x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2d} + \frac{(a+b)\text{Subst}\left(\int \frac{}{e+\sqrt{2}\right.}{}$$

$$= -\frac{(a-b)\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,d\sqrt{e}} + \frac{(a-b)\log\left(\sqrt{e}\right.}{}$$

$$= -\frac{(a+b)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}} + \frac{(a+b)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c-}}{\sqrt{e}}\right.}{\sqrt{2}\,d\sqrt{e}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.09, size = 83, normalized size = 0.40

$$-\frac{\sqrt[4]{-1}\left((a-ib)\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+(a+ib)\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\right)\sqrt{\tan(c+dx)}}{d\sqrt{e\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])/Sqrt[e*Tan[c + d*x]],x]
[Out] -(((-1)^(1/4)*((a - I*b)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + (a + I*b)*
ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])*Sqrt[Tan[c + d*x]])/(d*Sqrt[e*Tan[c
 + d*x]]))
```

Maple [A]
time = 0.23, size = 273, normalized size = 1.31

method	result
derivativedivides	$\frac{\frac{a(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\tan(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\tan(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)\right.}{4e}}{}$

default	$\frac{\frac{a\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\tan(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\tan\left(dx+c\right)}\sqrt{2}+\sqrt{e^{2}}}{e\tan(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\tan\left(dx+c\right)}\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\tan\left(dx+c\right)}}{\left(e^{2}\right)^{\frac{1}{4}}}+\right.\right.}{4e}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))/(e*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*a/e*(e^2)^(1/4)*2^(1/2)*(ln((e*tan(d*x+c)+(e^2)^(1/4)*(e*tan(d*x+c
))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*tan(d*x+c)-(e^2)^(1/4)*(e*tan(d*x+c))^(1/2
)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)+1
)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)+1))+1/4*b/(e^2)^(1/4)*
2^(1/2)*(ln((e*tan(d*x+c)-(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1
/2))/(e*tan(d*x+c)+(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2
*arctan(2^(1/2)/(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)
^(1/4)*(e*tan(d*x+c))^(1/2)+1)))
```

Maxima [A]
time = 0.50, size = 126, normalized size = 0.61

$$\frac{\left(2\sqrt{2}(a+b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(dx+c\right)}\right)\right)+2\sqrt{2}(a+b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(dx+c\right)}\right)\right)+\sqrt{2}(a-b)\log\left(\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)-\sqrt{2}(a-b)\log\left(-\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)\right)e^{\left(-\frac{1}{2}\right)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*(a + b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c))))
 + 2*sqrt(2)*(a + b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c))))
+ sqrt(2)*(a - b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt
(2)*(a - b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))*e^(-1/2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2696 vs. $2(143) = 286$.
time = 1.06, size = 2696, normalized size = 12.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*d^4*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4
+ 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(
3/4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4)*arctan(-((a^8 + 2*a^6*b^2 - 2*a^2*b^
6 - b^8)*d^4*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)
/d^4) - sqrt(2)*(a*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*
b^2 + b^4)/d^4)*e^(3/2) - (a^2*b + b^3)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^
```

```
4)*e^(3/2))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2
*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*
d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) + sqrt(2)*((a^4*b - 2*a^
2*b^3 + b^5)*d^3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c)*e^(1/2) - (
a^7 - a^5*b^2 - a^3*b^4 + a*b^6)*d*cos(d*x + c)*e^(1/2))*sqrt((2*a*b*d^2*sq
rt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 +
 b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4)*
e^(-1/2) + (a^8 - 2*a^4*b^4 + b^8)*sin(d*x + c))/cos(d*x + c))*((a^4 + 2*a^
2*b^2 + b^4)/d^4)^(3/4)*e^(-3/2) - sqrt(2)*((a^5 - a*b^4)*d^7*sqrt((a^4 + 2
*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4)*e^(3/2) - (a^6*b + a
^4*b^3 - a^2*b^5 - b^7)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4)*e^(3/2))*sqrt
((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4
 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^
4)/d^4)^(3/4)*e^(-3/2))/(a^12 + 2*a^10*b^2 - a^8*b^4 - 4*a^6*b^6 - a^4*b^8
+ 2*a^2*b^10 + b^12))*e^(-1/2) + 4*sqrt(2)*d^4*sqrt((2*a*b*d^2*sqrt((a^4 +
2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a
^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4)*arctan((
(a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^4*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*s
qrt((a^4 - 2*a^2*b^2 + b^4)/d^4) + sqrt(2)*(a*d^7*sqrt((a^4 + 2*a^2*b^2 + b
^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4)*e^(3/2) - (a^2*b + b^3)*d^5*sqrt
((a^4 - 2*a^2*b^2 + b^4)/d^4)*e^(3/2))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^
2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(((a^6
- a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x +
c) - sqrt(2)*((a^4*b - 2*a^2*b^3 + b^5)*d^3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^
4)*cos(d*x + c)*e^(1/2) - (a^7 - a^5*b^2 - a^3*b^4 + a*b^6)*d*cos(d*x + c)*
e^(1/2))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^
2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2
*a^2*b^2 + b^4)/d^4)^(1/4)*e^(-1/2) + (a^8 - 2*a^4*b^4 + b^8)*sin(d*x + c))
/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4)*e^(-3/2) + sqrt(2)*((a^5
 - a*b^4)*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4
)/d^4)*e^(3/2) - (a^6*b + a^4*b^3 - a^2*b^5 - b^7)*d^5*sqrt((a^4 - 2*a^2*b^
2 + b^4)/d^4)*e^(3/2))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) +
a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x +
 c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4)*e^(-3/2))/(a^12 + 2*a^10*b^2 - a^8
*b^4 - 4*a^6*b^6 - a^4*b^8 + 2*a^2*b^10 + b^12))*e^(-1/2) + sqrt(2)*(2*a*b*
d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)*sqrt((2*a*b*
d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2
*b^2 + b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4)*e^(-1/2)*log(((a^6 - a^4*b
^2 - a^2*b^4 + b^6)*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c) + sq
rt(2)*((a^4*b - 2*a^2*b^3 + b^5)*d^3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(
d*x + c)*e^(1/2) - (a^7 - a^5*b^2 - a^3*b^4 + a*b^6)*d*cos(d*x + c)*e^(1/2)
)*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4
)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*((a^4 + 2*a^2*b^
2 + b^4)/d^4)^(1/4)*e^(-1/2) + (a^8 - 2*a^4*b^4 + b^8)*sin(d*x + c))/cos(d*
x + c)) - sqrt(2)*(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^
```

```
2*b^2 - b^4)*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4 + 2*a^
2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4)*e
^(-1/2)*log(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt((a^4 + 2*a^2*b^2 + b^
4)/d^4)*cos(d*x + c) - sqrt(2)*((a^4*b - 2*a^2*b^3 + b^5)*d^3*sqrt((a^4 + 2
*a^2*b^2 + b^4)/d^4)*cos(d*x + c)*e^(1/2) - (a^7 - a^5*b^2 - a^3*b^4 + a*b^
6)*d*cos(d*x + c)*e^(1/2))*sqrt((2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4
) + a^4 + 2*a^2*b^2 + b^4)/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d
*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4)*e^(-1/2) + (a^8 - 2*a^4*b^4 +
b^8)*sin(d*x + c))/cos(d*x + c)))/(a^4 + 2*a^2*b^2 + b^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\tan(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/(e*tan(d*x+c))**(1/2),x)
[Out] Integral((a + b*tan(c + d*x))/sqrt(e*tan(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**B**]
time = 4.78, size = 120, normalized size = 0.58

$$\frac{(-1)^{1/4}\,b\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\tan(c+d\,x)}}{\sqrt{e}}\right)}{d\,\sqrt{e}}-\frac{(-1)^{1/4}\,b\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{e\tan(c+d\,x)}}{\sqrt{e}}\right)}{d\,\sqrt{e}}-\frac{(-1)^{1/4}\,a\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\tan(c+d\,x)}}{\sqrt{e}}\right)\,1\mathrm{i}}{d\,\sqrt{e}}-\frac{(-1)^{1/4}\,a\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{e\tan(c+d\,x)}}{\sqrt{e}}\right)\,1\mathrm{i}}{d\,\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))/(e*tan(c + d*x))^(1/2),x)
[Out] ((-1)^(1/4)*b*atan(((-1)^(1/4)*(e*tan(c + d*x))^(1/2))/e^(1/2)))/(d*e^(1/2
) - ((-1)^(1/4)*a*atanh(((-1)^(1/4)*(e*tan(c + d*x))^(1/2))/e^(1/2))*1i)/(d
*e^(1/2)) - ((-1)^(1/4)*a*atan(((-1)^(1/4)*(e*tan(c + d*x))^(1/2))/e^(1/2))
*1i)/(d*e^(1/2)) - ((-1)^(1/4)*b*atanh(((-1)^(1/4)*(e*tan(c + d*x))^(1/2))/
e^(1/2)))/(d*e^(1/2))
```

3.581 $\int \frac{a+b\tan(c+dx)}{\sqrt{-e\tan(c+dx)}}\,dx$

Optimal. Leaf size=214

$$\frac{(a-b)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{-e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}}-\frac{(a-b)\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{-e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}}+\frac{(a+b)\log\left(\right.}{}$$

```
[Out] 1/2*(a-b)*arctan(1-2^(1/2)*(-e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)/e^(1/2)
-1/2*(a-b)*arctan(1+2^(1/2)*(-e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)/e^(1/2
)+1/4*(a+b)*ln(e^(1/2)-2^(1/2)*(-e*tan(d*x+c))^(1/2)-e^(1/2)*tan(d*x+c))/d*
2^(1/2)/e^(1/2)-1/4*(a+b)*ln(e^(1/2)+2^(1/2)*(-e*tan(d*x+c))^(1/2)-e^(1/2)*
tan(d*x+c))/d*2^(1/2)/e^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a-b)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{-e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}}-\frac{(a-b)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{-e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,d\sqrt{e}}+\frac{(a+b)\log\left(-\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{-e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\sqrt{e}}-\frac{(a+b)\log\left(-\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{-e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])/Sqrt[-(e*Tan[c + d*x])],x]
```

```
[Out] ((a - b)*ArcTan[1 - (Sqrt[2]*Sqrt[-(e*Tan[c + d*x])])/Sqrt[e]])/(Sqrt[2]*d*
Sqrt[e]) - ((a - b)*ArcTan[1 + (Sqrt[2]*Sqrt[-(e*Tan[c + d*x])])/Sqrt[e]])/
(Sqrt[2]*d*Sqrt[e]) + ((a + b)*Log[Sqrt[e] - Sqrt[e]*Tan[c + d*x] - Sqrt[2]
*Sqrt[-(e*Tan[c + d*x])]])/(2*Sqrt[2]*d*Sqrt[e]) - ((a + b)*Log[Sqrt[e] - S
qrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[-(e*Tan[c + d*x])]])/(2*Sqrt[2]*d*Sqrt[e
])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{a+b\tan(c+dx)}{\sqrt{-e\tan(c+dx)}}\,dx = \frac{2\text{Subst}\left(\int \frac{-ae+bx^2}{e^2+x^4}\,dx, x, \sqrt{-e\tan(c+dx)}\right)}{d}$$

$$= -\frac{(a-b)\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{-e\tan(c+dx)}\right)}{d} - \frac{(a+b)\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x,\right.}{d}$$

$$= -\frac{(a-b)\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}\,x+x^2}\,dx, x, \sqrt{-e\tan(c+dx)}\right)}{2d} - \frac{(a-b)\text{Subst}\left(\int\right.}{}$$

$$= \frac{(a+b)\log\left(\sqrt{e}-\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{-e\tan(c+dx)}\right)}{2\sqrt{2}\,d\sqrt{e}} - \frac{(a+b)\log\left(\sqrt{\right.}}{}$$

$$= \frac{(a-b)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{-e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}} - \frac{(a-b)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{-e\tan}}{\sqrt{e}}\right.}{\sqrt{2}\,d\sqrt{e}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.17, size = 84, normalized size = 0.39

$$\frac{\sqrt[4]{-1}\,e\left((a-ib)\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+(a+ib)\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\right)\tan^{\frac{3}{2}}(c+dx)}{d(-e\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])/Sqrt[-(e*Tan[c + d*x])],x]
```

```
[Out] ((-1)^(1/4)*e*((a - I*b)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + (a + I*b)*
ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])*Tan[c + d*x]^(3/2))/(d*(-(e*Tan[c +
 d*x]))^(3/2))
```

Maple [A]
time = 0.18, size = 285, normalized size = 1.33

method	result
derivativedivides	$$-\frac{\frac{a(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{-e\tan(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{-e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}{-e\tan(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{-e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{-e\tan(dx+}}{(e^2)^{\frac{1}{4}}}\right.\right.}{4e}}{}$$

default	$-\frac{a\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{-e\tan(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{-e\tan\left(dx+c\right)}\,\sqrt{2}+\sqrt{e^{2}}}{-e\tan(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{-e\tan\left(dx+c\right)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{-e\tan\left(dx+\right.}}{\left(e^{2}\right)^{\frac{1}{4}}}\right.\right.}{4e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))/(-e*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4*a/e*(e^2)^(1/4)*2^(1/2)*(ln((-e*tan(d*x+c)+(e^2)^(1/4)*(-e*tan(d*
x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(-e*tan(d*x+c)-(e^2)^(1/4)*(-e*tan(d*x+c))
^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(-e*tan(d*x+c))^(
1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(-e*tan(d*x+c))^(1/2)+1))+1/4*b/(e^2)
^(1/4)*2^(1/2)*(ln((-e*tan(d*x+c)-(e^2)^(1/4)*(-e*tan(d*x+c))^(1/2)*2^(1/2)
+(e^2)^(1/2))/(-e*tan(d*x+c)+(e^2)^(1/4)*(-e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2
)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(-e*tan(d*x+c))^(1/2)+1)-2*arctan(-2
^(1/2)/(e^2)^(1/4)*(-e*tan(d*x+c))^(1/2)+1)))
```

Maxima [A]
time = 0.51, size = 172, normalized size = 0.80

$$-\frac{2\,\sqrt{2}\,(a-b)\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(\sqrt{2}\,e^{\frac{1}{2}}+2\,\sqrt{-e\tan\left(dx+c\right)}\right)e^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{1}{2}\right)}+2\,\sqrt{2}\,(a-b)\arctan\left(-\frac{1}{2}\,\sqrt{2}\,\left(\sqrt{2}\,e^{\frac{1}{2}}-2\,\sqrt{-e\tan\left(dx+c\right)}\right)e^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{1}{2}\right)}+\sqrt{2}\,(a+b)e^{\left(-\frac{1}{2}\right)}\log\left(\sqrt{2}\,\sqrt{-e\tan\left(dx+c\right)}\,e^{\frac{1}{2}}-e\tan\left(dx+c\right)+e\right)-\sqrt{2}\,(a+b)e^{\left(-\frac{1}{2}\right)}\log\left(-\sqrt{2}\,\sqrt{-e\tan\left(dx+c\right)}\,e^{\frac{1}{2}}-e\tan\left(dx+c\right)+e\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/(-e*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*(a - b)*arctan(1/2*sqrt(2)*(sqrt(2)*e^(1/2) + 2*sqrt(-e*tan
(d*x + c)))*e^(-1/2))*e^(-1/2) + 2*sqrt(2)*(a - b)*arctan(-1/2*sqrt(2)*(sqr
t(2)*e^(1/2) - 2*sqrt(-e*tan(d*x + c)))*e^(-1/2))*e^(-1/2) + sqrt(2)*(a + b
)*e^(-1/2)*log(sqrt(2)*sqrt(-e*tan(d*x + c))*e^(1/2) - e*tan(d*x + c) + e)
- sqrt(2)*(a + b)*e^(-1/2)*log(-sqrt(2)*sqrt(-e*tan(d*x + c))*e^(1/2) - e*t
an(d*x + c) + e))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2778 vs. 2(161) = 322.
time = 1.38, size = 2778, normalized size = 12.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/(-e*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*d^4*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^
4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)
^(3/4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4)*arctan(((a^8 + 2*a^6*b^2 - 2*a^2*b
^6 - b^8)*d^4*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4
```

)/d^4) + sqrt(2)*((a^5 - a*b^4)*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt(
(a^4 - 2*a^2*b^2 + b^4)/d^4)*e + (a^6*b + a^4*b^3 - a^2*b^5 - b^7)*d^5*sqrt
((a^4 - 2*a^2*b^2 + b^4)/d^4)*e)*sqrt(-e*sin(d*x + c)/cos(d*x + c))*sqrt(-(
2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 -
2*a^2*b^2 + b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4)*e^(-3/2) + sqrt(2)*(
a*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4)*e
+ (a^2*b + b^3)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4)*e)*sqrt(-(2*a*b*d^2*
sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2
+ b^4))*sqrt(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt((a^4 + 2*a^2*b^2 +
b^4)/d^4)*cos(d*x + c)*e + sqrt(2)*((a^4*b - 2*a^2*b^3 + b^5)*d^3*sqrt((a^4
+ 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c)*e + (a^7 - a^5*b^2 - a^3*b^4 + a*b^6)
*d*cos(d*x + c)*e)*sqrt(-e*sin(d*x + c)/cos(d*x + c))*sqrt(-(2*a*b*d^2*sqrt
((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b
^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4)*e^(-1/2) - (a^8 - 2*a^4*b^4 + b^8)
*e*sin(d*x + c))/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4)*e^(-3/2)
)/(a^12 + 2*a^10*b^2 - a^8*b^4 - 4*a^6*b^6 - a^4*b^8 + 2*a^2*b^10 + b^12))*
e^(-1/2) + 4*sqrt(2)*d^4*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)
- a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4 + 2*a^2*b^2 + b^4)
/d^4)^(3/4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4)*arctan(-((a^8 + 2*a^6*b^2 - 2
*a^2*b^6 - b^8)*d^4*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2
+ b^4)/d^4) - sqrt(2)*((a^5 - a*b^4)*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)
*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4)*e + (a^6*b + a^4*b^3 - a^2*b^5 - b^7)*d^
5*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4)*e)*sqrt(-e*sin(d*x + c)/cos(d*x + c))*s
qrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/
(a^4 - 2*a^2*b^2 + b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4)*e^(-3/2) - sqr
t(2)*(a*d^7*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*sqrt((a^4 - 2*a^2*b^2 + b^4)/
d^4)*e + (a^2*b + b^3)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/d^4)*e)*sqrt(-(2*a*
b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a
^2*b^2 + b^4))*sqrt(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt((a^4 + 2*a^2*
b^2 + b^4)/d^4)*cos(d*x + c)*e - sqrt(2)*((a^4*b - 2*a^2*b^3 + b^5)*d^3*sqr
t((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c)*e + (a^7 - a^5*b^2 - a^3*b^4 +
a*b^6)*d*cos(d*x + c)*e)*sqrt(-e*sin(d*x + c)/cos(d*x + c))*sqrt(-(2*a*b*d^
2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b
^2 + b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4)*e^(-1/2) - (a^8 - 2*a^4*b^4
+ b^8)*e*sin(d*x + c))/cos(d*x + c))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(3/4)*e^
(-3/2))/(a^12 + 2*a^10*b^2 - a^8*b^4 - 4*a^6*b^6 - a^4*b^8 + 2*a^2*b^10 + b
^12))*e^(-1/2) + sqrt(2)*(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) + a^4
+ 2*a^2*b^2 + b^4)*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^
4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)
^(1/4)*e^(-1/2)*log(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt((a^4 + 2*a^2*
b^2 + b^4)/d^4)*cos(d*x + c)*e + sqrt(2)*((a^4*b - 2*a^2*b^3 + b^5)*d^3*sqr
t((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c)*e + (a^7 - a^5*b^2 - a^3*b^4 +
a*b^6)*d*cos(d*x + c)*e)*sqrt(-e*sin(d*x + c)/cos(d*x + c))*sqrt(-(2*a*b*d^
2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b
^2 + b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4)*e^(-1/2) - (a^8 - 2*a^4*b^4

```
+ b^8)*e*sin(d*x + c))/cos(d*x + c)) - sqrt(2)*(2*a*b*d^2*sqrt((a^4 + 2*a^2
*b^2 + b^4)/d^4) + a^4 + 2*a^2*b^2 + b^4)*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^
2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4 +
 2*a^2*b^2 + b^4)/d^4)^(1/4)*e^(-1/2)*log(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*
d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c)*e - sqrt(2)*((a^4*b - 2*
a^2*b^3 + b^5)*d^3*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4)*cos(d*x + c)*e + (a^7
- a^5*b^2 - a^3*b^4 + a*b^6)*d*cos(d*x + c)*e)*sqrt(-e*sin(d*x + c)/cos(d*x
 + c))*sqrt(-(2*a*b*d^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/d^4) - a^4 - 2*a^2*b^2
 - b^4)/(a^4 - 2*a^2*b^2 + b^4))*((a^4 + 2*a^2*b^2 + b^4)/d^4)^(1/4)*e^(-1/
2) - (a^8 - 2*a^4*b^4 + b^8)*e*sin(d*x + c))/cos(d*x + c)))/(a^4 + 2*a^2*b^
2 + b^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\tan{\left(c + dx \right)}}{\sqrt{- e\tan{\left(c + dx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/(-e*tan(d*x+c))**(1/2),x)
[Out] Integral((a + b*tan(c + d*x))/sqrt(-e*tan(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/(-e*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)/sqrt(-e*tan(d*x + c)), x)
```

Mupad [B]
time = 4.71, size = 122, normalized size = 0.57

$$\frac{(-1)^{1/4}\, b\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{-e\tan(c+d\,x)}}{\sqrt{e}}\right)}{d\sqrt{e}}-\frac{(-1)^{1/4}\, b\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{-e\tan(c+d\,x)}}{\sqrt{e}}\right)}{d\sqrt{e}}+\frac{(-1)^{1/4}\, a\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{-e\tan(c+d\,x)}}{\sqrt{e}}\right)\,1{}\mathrm{i}}{d\sqrt{e}}+\frac{(-1)^{1/4}\, a\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{-e\tan(c+d\,x)}}{\sqrt{e}}\right)\,1{}\mathrm{i}}{d\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))/(-e*tan(c + d*x))^(1/2),x)
[Out] ((-1)^(1/4)*a*atan(((-1)^(1/4)*(-e*tan(c + d*x))^(1/2))/e^(1/2))*1i)/(d*e^(
1/2)) + ((-1)^(1/4)*a*atanh(((-1)^(1/4)*(-e*tan(c + d*x))^(1/2))/e^(1/2))*1
i)/(d*e^(1/2)) + ((-1)^(1/4)*b*atan(((-1)^(1/4)*(-e*tan(c + d*x))^(1/2))/e^
(1/2)))/(d*e^(1/2)) - ((-1)^(1/4)*b*atanh(((-1)^(1/4)*(-e*tan(c + d*x))^(1/
2))/e^(1/2)))/(d*e^(1/2))
```

3.582 $\int \frac{\tan^{\frac{9}{2}}(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=300

$$-\frac{(a+b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)d}+\frac{(a+b)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)d}-\frac{2a^{9/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{}}{}\right.}{b^{7/2}\left(a^2+\right.}$$

```
[Out] -2*a^(9/2)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/b^(7/2)/(a^2+b^2)/d+1/2
*(a+b)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)+1/2*(a+b)*ar
ctan(1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)+1/4*(a-b)*ln(1-2^(1/2)
*tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)/d*2^(1/2)-1/4*(a-b)*ln(1+2^(1/2)*ta
n(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)/d*2^(1/2)+2*(a^2-b^2)*tan(d*x+c)^(1/2)
/b^3/d-2/3*a*tan(d*x+c)^(3/2)/b^2/d+2/5*tan(d*x+c)^(5/2)/b/d
```

Rubi [A]
time = 0.56, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3647, 3728, 3729, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{(a+b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\left(a^2+b^2\right)}+\frac{(a+b)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d\left(a^2+b^2\right)}+\frac{(a-b)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d\left(a^2+b^2\right)}-\frac{(a-b)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d\left(a^2+b^2\right)}+\frac{2\left(a^2-b^2\right)\sqrt{\tan(c+dx)}}{b^3d}-\frac{2a^{9/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{7/2}d\left(a^2+b^2\right)}-\frac{2a\tan^{\frac{3}{2}}(c+dx)}{3b^2d}+\frac{2\tan^{\frac{5}{2}}(c+dx)}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(9/2)/(a + b*Tan[c + d*x]),x]
```

```
[Out] -(((a + b)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)*d))
 + ((a + b)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)*d)
 - (2*a^(9/2)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(b^(7/2)*(a^2 +
 b^2)*d) + ((a - b)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*
Sqrt[2]*(a^2 + b^2)*d) - ((a - b)*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[
c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)*d) + (2*(a^2 - b^2)*Sqrt[Tan[c + d*x]])/(
b^3*d) - (2*a*Tan[c + d*x]^(3/2))/(3*b^2*d) + (2*Tan[c + d*x]^(5/2))/(5*b*d
)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
```

```
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3729

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m + n +
1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*
Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*(b*c*m + a*d*(n + 1)) + d*(A*b -
 b*C)*(m + n + 1)*Tan[e + f*x] - C*m*(b*c - a*d)*Tan[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 +
 b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[
```

```
m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\tan^{\frac{9}{2}}(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{2\tan^{\frac{5}{2}}(c+dx)}{5bd} + \frac{2\int \frac{\tan^{\frac{3}{2}}(c+dx)\left(-\frac{5a}{2}-\frac{5}{2}b\tan(c+dx)-\frac{5}{2}a\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx}{5b}$$

$$= -\frac{2a\tan^{\frac{3}{2}}(c+dx)}{3b^2d} + \frac{2\tan^{\frac{5}{2}}(c+dx)}{5bd} + \frac{4\int \frac{\sqrt{\tan(c+dx)}\left(\frac{15a^2}{4}+\frac{15}{4}(a^2-b^2)\tan^2(c+dx)\right)}{a+b\tan(c+dx)}}{15b^2}$$

$$= \frac{2(a^2-b^2)\sqrt{\tan(c+dx)}}{b^3d} - \frac{2a\tan^{\frac{3}{2}}(c+dx)}{3b^2d} + \frac{2\tan^{\frac{5}{2}}(c+dx)}{5bd} + \frac{8\int \frac{-\frac{15}{8}a(a^2-b^2)+}{\sqrt{\mathrm{ta}}}}{}$$

$$= \frac{2(a^2-b^2)\sqrt{\tan(c+dx)}}{b^3d} - \frac{2a\tan^{\frac{3}{2}}(c+dx)}{3b^2d} + \frac{2\tan^{\frac{5}{2}}(c+dx)}{5bd} + \frac{8\int \frac{\frac{15b^4}{8}+\frac{15}{8}ab^3\tan}{\sqrt{\tan(c+}}}{15b^3(a^2+}$$

$$= \frac{2(a^2-b^2)\sqrt{\tan(c+dx)}}{b^3d} - \frac{2a\tan^{\frac{3}{2}}(c+dx)}{3b^2d} + \frac{2\tan^{\frac{5}{2}}(c+dx)}{5bd} + \frac{16\text{Subst}\left(\int \frac{15b^4}{8}\right.}{}$$

$$= \frac{2(a^2-b^2)\sqrt{\tan(c+dx)}}{b^3d} - \frac{2a\tan^{\frac{3}{2}}(c+dx)}{3b^2d} + \frac{2\tan^{\frac{5}{2}}(c+dx)}{5bd} - \frac{(a-b)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2a^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{7/2}(a^2+b^2)d} + \frac{2(a^2-b^2)\sqrt{\tan(c+dx)}}{b^3d} - \frac{2a\tan^{\frac{3}{2}}(c+}{3b^2d}$$

$$= -\frac{2a^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{7/2}(a^2+b^2)d} + \frac{(a-b)\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan\right.}{2\sqrt{2}(a^2+b^2)d}$$

$$= -\frac{(a+b)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}(a^2+b^2)d} + \frac{(a+b)\tan^{-1}\left(1+\sqrt{2}\sqrt{\tan(c+dx}\right.}{\sqrt{2}(a^2+b^2)d}$$

Mathematica [A]
time = 1.86, size = 243, normalized size = 0.81

$$-\frac{\frac{15\left(2\sqrt{2}b^{7/2}(a+b)\left(\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\right)+8a^{9/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)+\sqrt{2}b^{7/2}(-a+b)\left(\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)-\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\right)+8\sqrt{b}(-a^2+b^2)(a^2+b^2)\sqrt{\tan(c+dx)}\right)}{b^{3/2}(a^2+b^2)}-40a\tan^{\frac{3}{2}}(c+dx)+24b\tan^{\frac{5}{2}}(c+dx)}{60b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(9/2)/(a + b*Tan[c + d*x]),x]
```

```
[Out] ((-15*(2*Sqrt[2]*b^(7/2)*(a + b)*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] -
ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]) + 8*a^(9/2)*ArcTan[(Sqrt[b]*Sqrt[Ta
n[c + d*x]])/Sqrt[a]] + Sqrt[2]*b^(7/2)*(-a + b)*(Log[1 - Sqrt[2]*Sqrt[Tan[
```

```
c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*
x]]) + 8*Sqrt[b]*(-a^2 + b^2)*(a^2 + b^2)*Sqrt[Tan[c + d*x]]))/(b^(3/2)*(a^
2 + b^2)) - 40*a*Tan[c + d*x]^(3/2) + 24*b*Tan[c + d*x]^(5/2))/(60*b^2*d)
```

Maple [A]
time = 0.15, size = 284, normalized size = 0.95

method	result
derivativedivides	$\frac{\frac{\frac{2b^2\left(\tan^{\frac{5}{2}}(dx+c)\right)}{5}-\frac{2ab\left(\tan^{\frac{3}{2}}(dx+c)\right)}{3}+2a^2\left(\sqrt{\tan}(dx+c)\right)-2b^2\left(\sqrt{\tan}(dx+c)\right)}{b^3}-\frac{2a^5\arctan\left(\frac{b\left(\sqrt{\tan}(dx+c)\right)}{\sqrt{ab}}\right)}{b^3(a^2+b^2)\sqrt{ab}}-}{}$
default	$\frac{\frac{\frac{2b^2\left(\tan^{\frac{5}{2}}(dx+c)\right)}{5}-\frac{2ab\left(\tan^{\frac{3}{2}}(dx+c)\right)}{3}+2a^2\left(\sqrt{\tan}(dx+c)\right)-2b^2\left(\sqrt{\tan}(dx+c)\right)}{b^3}-\frac{2a^5\arctan\left(\frac{b\left(\sqrt{\tan}(dx+c)\right)}{\sqrt{ab}}\right)}{b^3(a^2+b^2)\sqrt{ab}}-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(9/2)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b^3*(1/5*b^2*tan(d*x+c)^(5/2)-1/3*a*b*tan(d*x+c)^(3/2)+a^2*tan(d*x+c
)^(1/2)-b^2*tan(d*x+c)^(1/2))-2/b^3*a^5/(a^2+b^2)/(a*b)^(1/2)*arctan(b*tan(
d*x+c)^(1/2)/(a*b)^(1/2))+2/(a^2+b^2)*(1/8*b*2^(1/2)*(ln((1+2^(1/2)*tan(d*x
+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2
^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/8*a*2^(1/
2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+
tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d
*x+c)^(1/2)))))
```

Maxima [A]
time = 0.49, size = 225, normalized size = 0.75

$$-\frac{\frac{120\,a^5\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^2b^3+b^5)\sqrt{ab}}-\frac{15\left(2\sqrt{2}(a+b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}(a+b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-\sqrt{2}(a-b)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+\sqrt{2}(a-b)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)\right)}{a^2+b^2}-\frac{8\left(3b^2\tan(dx+c)^{\frac{5}{2}}-5ab\tan(dx+c)^{\frac{3}{2}}+15(a^2-b^2)\sqrt{\tan(dx+c)}\right)}{b^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(9/2)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(120*a^5*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^2*b^3 + b^5)*sqrt
(a*b)) - 15*(2*sqrt(2)*(a + b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x
 + c)))) + 2*sqrt(2)*(a + b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x
+ c)))) - sqrt(2)*(a - b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1
) + sqrt(2)*(a - b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a
^2 + b^2) - 8*(3*b^2*tan(d*x + c)^(5/2) - 5*a*b*tan(d*x + c)^(3/2) + 15*(a^
2 - b^2)*sqrt(tan(d*x + c)))/b^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3679 vs. $2(252) = 504$.

time = 9.64, size = 7470, normalized size = 24.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(9/2)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/60*(60*sqrt(2)*(a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9)*d^5*sqrt((a^4 +
2*a^2*b^2 + b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^
2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8
 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(1/((a^4 + 2*a^2*b^2 + b^
4)*d^4))^(3/4)*arctan(-((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^4*sqrt((a^4 -
 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sq
rt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - sqrt(2)*((a^8*b + 4*a^6*b^3 + 6*a^4*b
^5 + 4*a^2*b^7 + b^9)*d^7*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 +
6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) -
(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^
8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))*sqrt((a^4 + 2*a^2*b^2 +
 b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^
4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqr
t(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) + sqrt(2)*((a^7 - a^5*b^2 -
 a^3*b^4 + a*b^6)*d^3*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) -
(a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c))*sqrt((a^4 + 2*a^2*b^2 + b^4 + 2*(
a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4
- 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 +
b^4)*d^4))^(1/4) + (a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c))/cos(d*x + c))*(1/(
(a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4) - sqrt(2)*((a^10*b + 3*a^8*b^3 + 2*a^6*
b^5 - 2*a^4*b^7 - 3*a^2*b^9 - b^11)*d^7*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8
+ 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 +
 b^4)*d^4)) - (a^9 + 2*a^7*b^2 - 2*a^3*b^6 - a*b^8)*d^5*sqrt((a^4 - 2*a^2*b
^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))*sqrt((a^4
 + 2*a^2*b^2 + b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2
*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))
*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4))/(a^4 - 2*a^2*b^2 + b^4))*cos(d*x
+ c)^2 + 60*sqrt(2)*(a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9)*d^5*sqrt((a^4 +
 2*a^2*b^2 + b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b
^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^
8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(1/((a^4 + 2*a^2*b^2 + b
^4)*d^4))^(3/4)*arctan(((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^4*sqrt((a^4 -
 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sq
rt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + sqrt(2)*((a^8*b + 4*a^6*b^3 + 6*a^4*b
^5 + 4*a^2*b^7 + b^9)*d^7*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 +
6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) -
```

```
(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^
8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))*sqrt((a^4 + 2*a^2*b^2 +
 b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^
4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqr
t(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) - sqrt(2)*((a^7 - a^5*b^2 -
 a^3*b^4 + a*b^6)*d^3*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) -
(a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c))*sqrt((a^4 + 2*a^2*b^2 + b^4 + 2*(
a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4
- 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 +
b^4)*d^4))^(1/4) + (a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c))/cos(d*x + c))*(1/(
(a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4) + sqrt(2)*((a^10*b + 3*a^8*b^3 + 2*a^6*
b^5 - 2*a^4*b^7 - 3*a^2*b^9 - b^11)*d^7*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8
+ 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 +
 b^4)*d^4)) - (a^9 + 2*a^7*b^2 - 2*a^3*b^6 - a*b^8)*d^5*sqrt((a^4 - 2*a^2*b
^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))*sqrt((a^4
 + 2*a^2*b^2 + b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2
*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))
*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4))/(a^4 - 2*a^2*b^2 + b^4))*cos(d*x
+ c)^2 - 30*a^4*sqrt(-a/b)*cos(d*x + c)^2*log(-(6*a*b*cos(d*x + c)*sin(d*x
+ c) - (a^2 - b^2)*cos(d*x + c)^2 - b^2 - 4*(a*b*cos(d*x + c)^2 - b^2*cos(d
*x + c)*sin(d*x + c))*sqrt(-a/b)*sqrt(sin(d*x + c)/cos(d*x + c)))/(2*a*b*co
s(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - 15*sqrt(2)*(
2*(a^3*b^4 + a*b^6)*d^3*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c)^
2 - (a^2*b^3 + b^5)*d*cos(d*x + c)^2)*sqrt((a^4 + 2*a^2*b^2 + b^4 + 2*(a^5*
b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*
a^2*b^2 + b^4))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(1/4)*log(((a^6 - a^4*b^2
 - a^2*b^4 + b^6)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) +
sqrt(2)*((a^7 - a^5*b^2 - a^3*b^4 + a*b^6)*d^3*sqrt(1/((a^4 + 2*a^2*b^2 + b
^4)*d^4))*cos(d*x + c) - (a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c))*sqrt((a^
4 + 2*a^2*b^2 + b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^
2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sq...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{9}{2}}(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(9/2)/(a+b*tan(d*x+c)),x)
[Out] Integral(tan(c + d*x)**(9/2)/(a + b*tan(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^(9/2)/(a+b*tan(d*x+c)),x, algorithm="giac")

[Out] Timed out

Mupad [B]
time = 10.44, size = 2500, normalized size = 8.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d*x)^(9/2)/(a + b*tan(c + d*x)),x)

[Out] (log((((((((((256*b^3*tan(c + d*x)^(1/2)*(a^2 - b^2)*(a^2 + b^2)^2*(1i/(d^2*(a*1i - b)^2))^(1/2) - (128*a*b*(4*a^2 - b^2)*(a^2 + b^2)^2)/d)*(1i/(d^2*(a*1i - b)^2))^(1/2))/2 + (64*a*tan(c + d*x)^(1/2)*(8*a^10 - 7*b^10 + 2*a^2*b^8 + a^4*b^6))/(b^4*d^2))*(1i/(d^2*(a*1i - b)^2))^(1/2))/2 - (32*a^2*(12*a^8 + b^8 + a^2*b^6 + 16*a^4*b^4 - 16*a^6*b^2))/(b^4*d^3))*(1i/(d^2*(a*1i - b)^2))^(1/2))/2 + (32*tan(c + d*x)^(1/2)*(2*a^10 - b^10))/(b^5*d^4))*(1i/(d^2*(a*1i - b)^2))^(1/2))/2 - (32*a^5*(a^4 + b^4 - a^2*b^2))/(b^5*d^5))*(-1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2))/2 - tan(c + d*x)^(1/2)*(2/(b*d) - (2*a^2)/(b^3*d)) - log(- ((((((((256*b^3*tan(c + d*x)^(1/2)*(a^2 - b^2)*(a^2 + b^2)^2*(1i/(d^2*(a*1i - b)^2))^(1/2) + (128*a*b*(4*a^2 - b^2)*(a^2 + b^2)^2)/d)*(1i/(d^2*(a*1i - b)^2))^(1/2))/2 + (64*a*tan(c + d*x)^(1/2)*(8*a^10 - 7*b^10 + 2*a^2*b^8 + a^4*b^6))/(b^4*d^2))*(1i/(d^2*(a*1i - b)^2))^(1/2))/2 + (32*a^2*(12*a^8 + b^8 + a^2*b^6 + 16*a^4*b^4 - 16*a^6*b^2))/(b^4*d^3))*(1i/(d^2*(a*1i - b)^2))^(1/2))/2 + (32*tan(c + d*x)^(1/2)*(2*a^10 - b^10))/(b^5*d^4))*(1i/(d^2*(a*1i - b)^2))^(1/2))/2 - (32*a^5*(a^4 + b^4 - a^2*b^2))/(b^5*d^5))*(-1/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) + atan((((((-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*((32*(8*a^3*b^10*d^4 - 4*a*b^12*d^4 + 28*a^5*b^8*d^4 + 16*a^7*b^6*d^4))/(b^5*d^5) - (32*tan(c + d*x)^(1/2)*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^14*d^4 + 16*a^2*b^12*d^4 - 16*a^4*b^10*d^4 - 16*a^6*b^8*d^4))/(b^5*d^4)) + (32*tan(c + d*x)^(1/2)*(16*a^11*b*d^2 - 14*a*b^11*d^2 + 4*a^3*b^9*d^2 + 2*a^5*b^7*d^2))/(b^5*d^4))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*(12*a^10*b*d^2 + a^2*b^9*d^2 + a^4*b^7*d^2 + 16*a^6*b^5*d^2 - 16*a^8*b^3*d^2))/(b^5*d^5))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*tan(c + d*x)^(1/2)*(2*a^10 - b^10))/(b^5*d^4))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*1i - ((((-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*((32*(8*a^3*b^10*d^4 - 4*a*b^12*d^4 + 28*a^5*b^8*d^4 + 16*a^7*b^6*d^4))/(b^5*d^5) + (32*tan(c + d*x)^(1/2)*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^14*d^4 + 16*a^2*b^12*d^4 - 16*a^4*b^10*d^4 - 16*a^6*b^8*d^4))/(b^5*d^4)) - (32*tan(c + d*x)^(1/2)*(16*a^11*b*d^2 - 14*a*b^11*d^2 + 4*a^3*b^9*d^2 + 2*a^5*b^7*d^2))/(b^5*d^4))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*(12*a^10*b*d^2 + a^2*b^9*d^2 + a^4*b^7*d^2 + 16*a^6*b^5*d^2 - 1

6*a^8*b^3*d^2))/(b^5*d^5))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*tan(c + d*x)^(1/2)*(2*a^10 - b^10))/(b^5*d^4))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*1i)/(((((-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*((32*(8*a^3*b^10*d^4 - 4*a*b^12*d^4 + 28*a^5*b^8*d^4 + 16*a^7*b^6*d^4))/(b^5*d^5) - (32*tan(c + d*x)^(1/2)*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^14*d^4 + 16*a^2*b^12*d^4 - 16*a^4*b^10*d^4 - 16*a^6*b^8*d^4))/(b^5*d^4)) + (32*tan(c + d*x)^(1/2)*(16*a^11*b*d^2 - 14*a*b^11*d^2 + 4*a^3*b^9*d^2 + 2*a^5*b^7*d^2))/(b^5*d^4))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*(12*a^10*b*d^2 + a^2*b^9*d^2 + a^4*b^7*d^2 + 16*a^6*b^5*d^2 - 16*a^8*b^3*d^2))/(b^5*d^5))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*tan(c + d*x)^(1/2)*(2*a^10 - b^10))/(b^5*d^4))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + ((((-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*((32*(8*a^3*b^10*d^4 - 4*a*b^12*d^4 + 28*a^5*b^8*d^4 + 16*a^7*b^6*d^4))/(b^5*d^5) + (32*tan(c + d*x)^(1/2)*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^14*d^4 + 16*a^2*b^12*d^4 - 16*a^4*b^10*d^4 - 16*a^6*b^8*d^4))/(b^5*d^4)) - (32*tan(c + d*x)^(1/2)*(16*a^11*b*d^2 - 14*a*b^11*d^2 + 4*a^3*b^9*d^2 + 2*a^5*b^7*d^2))/(b^5*d^4))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*(12*a^10*b*d^2 + a^2*b^9*d^2 + a^4*b^7*d^2 + 16*a^6*b^5*d^2 - 16*a^8*b^3*d^2))/(b^5*d^5))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*tan(c + d*x)^(1/2)*(2*a^10 - b^10))/(b^5*d^4))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (64*(a^9 + a^5*b^4 - a^7*b^2))/(b^5*d^5)))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*2i + (2*tan(c + d*x)^(5/2))/(5*b*d) - (2*a*tan(c + d*x)^(3/2))/(3*b^2*d) + (atan((((-a^9*b^7)^(1/2)*((32*tan(c + d*x)^(1/2)*(2*a^10 - b^10))/(b^5*d^4) + (((32*(12*a^10*b*d^2 + a^2*b^9*d^2 + a^4*b^7*d^2 + 16*a^6*b^5*d^2 - 16*a^8*b^3*d^2))/(b^5*d^5) + ((-a^9*b^7)^(1/2)*((32*tan(c + d*x)^(1/2)*(16*a^11*b*d^2 - 14*a*b^11*d^2 + 4*a^3*b^9*d^2 + 2*a^5*b^7*d^2))/(b^5*d^4) + (((32*(8*a^3*b^10*d^4 - 4*a*b^12*d^4 + 28*a^5*b^8*d^4 + 16*a^7*b^6*d^4))/(b^5*d^5) - (32*tan(c + d*x)^(1/2)*(-a^9*b^7)^(1/2)*(16*b^14*d^4 + 16*a^2*b^12*d^4 - 16*a^4*b^10*d^4 - 16*a^6*b^8*d^4))/(b^12*d^5*(a^2 + b^2)))*(-a^9*b^7)^(1/2))/(b^7*d*(a^2 + b^2))))/(b^7*d*(a^2 + b^2)))*(-a^9*b^7)^(1/2))/(b^7*d*(a^2 + b^2)))*1i)/(b^7*d*(a^2 + b^2)) + ((-a^9*b^7)^(1/2)*((32*tan(c + d*x)^(1/2)*(2*a^10 - b^10))/(b^5*d^4) - (((32*(12*a^10*b*d^2 + a^2*b^9*d^2 + a^4*b^7*d^2 + 16*a^6*b^5*d^2 - 16*a^8*b^3*d^2))/(b^5*d^5) - ((-a^9*b^7)^(1/2)*((32*tan(c + d*x)^(1/2)*(16*a^11*b*d^2 - 14*a*b^11*d...

3.583 $\int \frac{\tan^{\frac{7}{2}}(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=271

$$-\frac{(a-b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)d}+\frac{(a-b)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)d}+\frac{2a^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{}}{}\right.}{b^{5/2}\left(a^2+\right.}$$

```
[Out] 2*a^(7/2)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/b^(5/2)/(a^2+b^2)/d+1/2*
(a-b)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)+1/2*(a-b)*arc
tan(1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)-1/4*(a+b)*ln(1-2^(1/2)*
tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)/d*2^(1/2)+1/4*(a+b)*ln(1+2^(1/2)*tan
(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)/d*2^(1/2)-2*a*tan(d*x+c)^(1/2)/b^2/d+2/
3*tan(d*x+c)^(3/2)/b/d
```

Rubi [A]
time = 0.38, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3647, 3728, 3735, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{(a-b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d\left(a^2+b^2\right)}+\frac{(a-b)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d\left(a^2+b^2\right)}-\frac{(a+b)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)}+\frac{(a+b)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)}+\frac{2a^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{5/2}d\left(a^2+b^2\right)}-\frac{2a\sqrt{\tan(c+dx)}}{b^2d}+\frac{2\tan^{\frac{3}{2}}(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(7/2)/(a + b*Tan[c + d*x]),x]
```

```
[Out] -(((a - b)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)*d))
 + ((a - b)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)*d)
 + (2*a^(7/2)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(b^(5/2)*(a^2 +
 b^2)*d) - ((a + b)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*
Sqrt[2]*(a^2 + b^2)*d) + ((a + b)*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[
c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)*d) - (2*a*Sqrt[Tan[c + d*x]])/(b^2*d) + (
2*Tan[c + d*x]^(3/2))/(3*b*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
```

```
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3735

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dis
t[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*Simp[a*(A - C) - (A*b - b*C)*Ta
n[e + f*x], x], x], x] + Dist[(A*b^2 + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e
 + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a,
 b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0] &&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\tan^{\frac{7}{2}}(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{2\tan^{\frac{3}{2}}(c+dx)}{3bd} + \frac{2\int \frac{\sqrt{\tan(c+dx)}\left(-\frac{3a}{2}-\frac{3}{2}b\tan(c+dx)-\frac{3}{2}a\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx}{3b}$$

$$= -\frac{2a\sqrt{\tan(c+dx)}}{b^2d} + \frac{2\tan^{\frac{3}{2}}(c+dx)}{3bd} + \frac{4\int \frac{\frac{3a^2}{4}+\frac{3}{4}(a^2-b^2)\tan^2(c+dx)}{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))}\,dx}{3b^2}$$

$$= -\frac{2a\sqrt{\tan(c+dx)}}{b^2d} + \frac{2\tan^{\frac{3}{2}}(c+dx)}{3bd} + \frac{4\int \frac{\frac{3ab^2}{4}-\frac{3}{4}b^3\tan(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{3b^2(a^2+b^2)} + \frac{a^4\int \frac{\ldots}{\sqrt{\tan(c\ldots}}}{b^{\ldots}}$$

$$= -\frac{2a\sqrt{\tan(c+dx)}}{b^2d} + \frac{2\tan^{\frac{3}{2}}(c+dx)}{3bd} + \frac{8\text{Subst}\left(\int \frac{\frac{3ab^2}{4}-\frac{3b^3x^2}{4}}{1+x^4}\,dx, x, \sqrt{\tan(c+d\ldots}\right.}{3b^2(a^2+b^2)d}$$

$$= -\frac{2a\sqrt{\tan(c+dx)}}{b^2d} + \frac{2\tan^{\frac{3}{2}}(c+dx)}{3bd} + \frac{(a-b)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+d\ldots}\right.}{(a^2+b^2)d}$$

$$= \frac{2a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{5/2}(a^2+b^2)d} - \frac{2a\sqrt{\tan(c+dx)}}{b^2d} + \frac{2\tan^{\frac{3}{2}}(c+dx)}{3bd} + \frac{(a\ldots}{\ldots}$$

$$= \frac{2a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{5/2}(a^2+b^2)d} - \frac{(a+b)\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan\ldots\right.}{2\sqrt{2}(a^2+b^2)d}$$

$$= -\frac{(a-b)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}(a^2+b^2)d} + \frac{(a-b)\tan^{-1}\left(1+\sqrt{2}\sqrt{\tan(c+d\ldots}\right.}{\sqrt{2}(a^2+b^2)d}$$

Mathematica [A]
time = 0.88, size = 222, normalized size = 0.82

$$\frac{-\frac{6\sqrt{2}(a-b)b^2\left(\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\right)}{a^2+b^2}+\frac{24a^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b}(a^2+b^2)}-\frac{3\sqrt{2}b^2(a+b)\left(\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)-\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\right)}{a^2+b^2}-24a\sqrt{\tan(c+dx)}+8b\tan^{\frac{3}{2}}(c+dx)}{12b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(7/2)/(a + b*Tan[c + d*x]),x]
```

```
[Out] ((-6*Sqrt[2]*(a - b)*b^2*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - ArcTan[1
 + Sqrt[2]*Sqrt[Tan[c + d*x]]]))/(a^2 + b^2) + (24*a^(7/2)*ArcTan[(Sqrt[b]*
Sqrt[Tan[c + d*x]])/Sqrt[a]])/(Sqrt[b]*(a^2 + b^2)) - (3*Sqrt[2]*b^2*(a + b
)*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqr
t[Tan[c + d*x]] + Tan[c + d*x]]))/(a^2 + b^2) - 24*a*Sqrt[Tan[c + d*x]] + 8
*b*Tan[c + d*x]^(3/2))/(12*b^2*d)
```

Maple [A]
time = 0.15, size = 255, normalized size = 0.94

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{b\left(\tan^{\frac{3}{2}}(dx+c)\right)}{3}+a\left(\sqrt{\tan}(dx+c)\right)\right)}{b^2}+\frac{2a^4\arctan\left(\frac{b\left(\sqrt{\tan}(dx+c)\right)}{\sqrt{ab}}\right)}{b^2(a^2+b^2)\sqrt{ab}}+\frac{a\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)}\right.\right.}{}}{}$
default	$\frac{-\frac{2\left(-\frac{b\left(\tan^{\frac{3}{2}}(dx+c)\right)}{3}+a\left(\sqrt{\tan}(dx+c)\right)\right)}{b^2}+\frac{2a^4\arctan\left(\frac{b\left(\sqrt{\tan}(dx+c)\right)}{\sqrt{ab}}\right)}{b^2(a^2+b^2)\sqrt{ab}}+\frac{a\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)}\right.\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(7/2)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b^2*(-1/3*b*tan(d*x+c)^(3/2)+a*tan(d*x+c)^(1/2))+2/b^2*a^4/(a^2+b^2
)/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))+2/(a^2+b^2)*(1/8*a*2^(
1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2
)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan
(d*x+c)^(1/2)))-1/8*b*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(
1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2)
)+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))))
```

Maxima [A]
time = 0.49, size = 202, normalized size = 0.75

$$\frac{\frac{24a^4\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^2b^2+b^4)\sqrt{ab}}+\frac{3\left(2\sqrt{2}(a-b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}(a-b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)+\sqrt{2}(a+b)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)-\sqrt{2}(a+b)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)\right)}{a^2+b^2}+\frac{8\left(b\tan(dx+c)^{\frac{3}{2}}-3a\sqrt{\tan(dx+c)}\right)}{b^2}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(7/2)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(24*a^4*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^2*b^2 + b^4)*sqrt(a
*b)) + 3*(2*sqrt(2)*(a - b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x +
c)))) + 2*sqrt(2)*(a - b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c
)))) + sqrt(2)*(a + b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) -
 sqrt(2)*(a + b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a^2
+ b^2) + 8*(b*tan(d*x + c)^(3/2) - 3*a*sqrt(tan(d*x + c)))/b^2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3622 vs. 2(227) = 454.
time = 9.30, size = 7356, normalized size = 27.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^(7/2)/(a+b*tan(d*x+c)),x, algorithm="fricas")

[Out] [1/12*(12*sqrt(2)*(a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d^5*sqrt((a^4 + 2*a^2*b^2 + b^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4)*arctan(((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^4*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + sqrt(2)*((a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*d^7*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))*sqrt((a^4 + 2*a^2*b^2 + b^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) + sqrt(2)*((a^6*b - a^4*b^3 - a^2*b^5 + b^7)*d^3*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c))*sqrt((a^4 + 2*a^2*b^2 + b^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(1/4) + (a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c))/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4) + sqrt(2)*((a^11 + 3*a^9*b^2 + 2*a^7*b^4 - 2*a^5*b^6 - 3*a^3*b^8 - a*b^10)*d^7*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + (a^8*b + 2*a^6*b^3 - 2*a^2*b^7 - b^9)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))*sqrt((a^4 + 2*a^2*b^2 + b^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4))/(a^4 - 2*a^2*b^2 + b^4))*cos(d*x + c) + 12*sqrt(2)*(a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d^5*sqrt((a^4 + 2*a^2*b^2 + b^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4)*arctan(-((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^4*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - sqrt(2)*((a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*d^7*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))*sqrt((a^4 + 2*a^2*b^2 + b^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) - sqrt(2)*((a^6*b - a^4*b^3 -

```
a^2*b^5 + b^7)*d^3*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) + (a^
5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c))*sqrt((a^4 + 2*a^2*b^2 + b^4 - 2*(a^5
*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2
*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4
)*d^4))^(1/4) + (a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c))/cos(d*x + c))*(1/((a^
4 + 2*a^2*b^2 + b^4)*d^4))^(3/4) - sqrt(2)*((a^11 + 3*a^9*b^2 + 2*a^7*b^4 -
 2*a^5*b^6 - 3*a^3*b^8 - a*b^10)*d^7*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4
*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^
4)*d^4)) + (a^8*b + 2*a^6*b^3 - 2*a^2*b^7 - b^9)*d^5*sqrt((a^4 - 2*a^2*b^2
+ b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))*sqrt((a^4 +
2*a^2*b^2 + b^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^
2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*(1
/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4))/(a^4 - 2*a^2*b^2 + b^4))*cos(d*x + c
) + 6*a^3*sqrt(-a/b)*cos(d*x + c)*log(-(6*a*b*cos(d*x + c)*sin(d*x + c) - (
a^2 - b^2)*cos(d*x + c)^2 - b^2 + 4*(a*b*cos(d*x + c)^2 - b^2*cos(d*x + c)*
sin(d*x + c))*sqrt(-a/b)*sqrt(sin(d*x + c)/cos(d*x + c)))/(2*a*b*cos(d*x +
c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) + 3*sqrt(2)*(2*(a^3*b^
3 + a*b^5)*d^3*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) + (a^2*b^
2 + b^4)*d*cos(d*x + c))*sqrt((a^4 + 2*a^2*b^2 + b^4 - 2*(a^5*b + 2*a^3*b^3
 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4
))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(1/4)*log(((a^6 - a^4*b^2 - a^2*b^4 +
b^6)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) + sqrt(2)*((a^6
*b - a^4*b^3 - a^2*b^5 + b^7)*d^3*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos
(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c))*sqrt((a^4 + 2*a^2*b^2
 + b^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*
d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{7}{2}}(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(7/2)/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**(7/2)/(a + b*tan(c + d*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(7/2)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

[Out] Timed out

Mupad [B]
time = 6.18, size = 2500, normalized size = 9.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d*x)^(7/2)/(a + b*tan(c + d*x)),x)

[Out] (2*tan(c + d*x)^(3/2))/(3*b*d) - atan((((((1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*(((1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*((((32*(12*a^2*b^9*d^4 + 24*a^4*b^7*d^4 + 12*a^6*b^5*d^4))/(b^3*d^5) - (16*tan(c + d*x)^(1/2)*(1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*(16*b^12*d^4 + 16*a^2*b^10*d^4 - 16*a^4*b^8*d^4 - 16*a^6*b^6*d^4))/(b^3*d^4))*(1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2))/2 + (32*tan(c + d*x)^(1/2)*(14*a*b^9*d^2 + 16*a^9*b*d^2 - 4*a^3*b^7*d^2 - 2*a^5*b^5*d^2))/(b^3*d^4)))/2 - (32*(4*a^9*d^2 + a*b^8*d^2 + a^3*b^6*d^2 + 16*a^5*b^4*d^2 - 16*a^7*b^2*d^2))/(b^3*d^5)))/2 - (32*tan(c + d*x)^(1/2)*(2*a^8 + b^8))/(b^3*d^4))*(1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*1i)/2 - ((((1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*(((1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*((((32*(12*a^2*b^9*d^4 + 24*a^4*b^7*d^4 + 12*a^6*b^5*d^4))/(b^3*d^5) + (16*tan(c + d*x)^(1/2)*(1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*(16*b^12*d^4 + 16*a^2*b^10*d^4 - 16*a^4*b^8*d^4 - 16*a^6*b^6*d^4))/(b^3*d^4))*(1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2))/2 - (32*tan(c + d*x)^(1/2)*(14*a*b^9*d^2 + 16*a^9*b*d^2 - 4*a^3*b^7*d^2 - 2*a^5*b^5*d^2))/(b^3*d^4)))/2 - (32*(4*a^9*d^2 + a*b^8*d^2 + a^3*b^6*d^2 + 16*a^5*b^4*d^2 - 16*a^7*b^2*d^2))/(b^3*d^5)))/2 + (32*tan(c + d*x)^(1/2)*(2*a^8 + b^8))/(b^3*d^4))*(1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*1i)/2)/(((((1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*(((1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*((((32*(12*a^2*b^9*d^4 + 24*a^4*b^7*d^4 + 12*a^6*b^5*d^4))/(b^3*d^5) - (16*tan(c + d*x)^(1/2)*(1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*(16*b^12*d^4 + 16*a^2*b^10*d^4 - 16*a^4*b^8*d^4 - 16*a^6*b^6*d^4))/(b^3*d^4))*(1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2))/2 + (32*tan(c + d*x)^(1/2)*(14*a*b^9*d^2 + 16*a^9*b*d^2 - 4*a^3*b^7*d^2 - 2*a^5*b^5*d^2))/(b^3*d^4)))/2 - (32*(4*a^9*d^2 + a*b^8*d^2 + a^3*b^6*d^2 + 16*a^5*b^4*d^2 - 16*a^7*b^2*d^2))/(b^3*d^5)))/2 - (32*tan(c + d*x)^(1/2)*(2*a^8 + b^8))/(b^3*d^4))*(1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2))/2 + ((((1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*(((1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*((((32*(12*a^2*b^9*d^4 + 24*a^4*b^7*d^4 + 12*a^6*b^5*d^4))/(b^3*d^5) + (16*tan(c + d*x)^(1/2)*(1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*(16*b^12*d^4 + 16*a^2*b^10*d^4 - 16*a^4*b^8*d^4 - 16*a^6*b^6*d^4))/(b^3*d^4))*(1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2))/2 - (32*tan(c + d*x)^(1/2)*(14*a*b^9*d^2 + 16*a^9*b*d^2 - 4*a^3*b^7*d^2 - 2*a^5*b^5*d^2))/(b^3*d^4)))/2 - (32*(4*a^9*d^2 + a*b^8*d^2 + a^3*b^6*d^2 + 16*a^5*b^4*d^2 - 16*a^7*b^2*d^2))/(b^3*d^5)))/2

+ (32*tan(c + d*x)^(1/2)*(2*a^8 + b^8))/(b^3*d^4))*(1/(b^2*d^2*1i - a^2*d^2
*1i + 2*a*b*d^2))^(1/2))/2 - (64*(a^6*b - a^4*b^3))/(b^3*d^5)))*(1/(b^2*d^2
*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*1i - atan((((((32*(12*a^2*b^9*d^4 + 2
4*a^4*b^7*d^4 + 12*a^6*b^5*d^4))/(b^3*d^5) - (32*tan(c + d*x)^(1/2)*(1i/(4*
(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^12*d^4 + 16*a^2*b^10*d^4 - 1
6*a^4*b^8*d^4 - 16*a^6*b^6*d^4))/(b^3*d^4))*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b
*d^2*2i)))^(1/2) + (32*tan(c + d*x)^(1/2)*(14*a*b^9*d^2 + 16*a^9*b*d^2 - 4*
a^3*b^7*d^2 - 2*a^5*b^5*d^2))/(b^3*d^4))*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^
2*2i)))^(1/2) - (32*(4*a^9*d^2 + a*b^8*d^2 + a^3*b^6*d^2 + 16*a^5*b^4*d^2 -
 16*a^7*b^2*d^2))/(b^3*d^5))*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2
) - (32*tan(c + d*x)^(1/2)*(2*a^8 + b^8))/(b^3*d^4))*(1i/(4*(b^2*d^2 - a^2*
d^2 + a*b*d^2*2i)))^(1/2)*1i - (((((32*(12*a^2*b^9*d^4 + 24*a^4*b^7*d^4 + 1
2*a^6*b^5*d^4))/(b^3*d^5) + (32*tan(c + d*x)^(1/2)*(1i/(4*(b^2*d^2 - a^2*d^
2 + a*b*d^2*2i)))^(1/2)*(16*b^12*d^4 + 16*a^2*b^10*d^4 - 16*a^4*b^8*d^4 - 1
6*a^6*b^6*d^4))/(b^3*d^4))*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)
- (32*tan(c + d*x)^(1/2)*(14*a*b^9*d^2 + 16*a^9*b*d^2 - 4*a^3*b^7*d^2 - 2*a
^5*b^5*d^2))/(b^3*d^4))*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (
32*(4*a^9*d^2 + a*b^8*d^2 + a^3*b^6*d^2 + 16*a^5*b^4*d^2 - 16*a^7*b^2*d^2))
/(b^3*d^5))*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*tan(c + d
x)^(1/2)(2*a^8 + b^8))/(b^3*d^4))*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)
))^(1/2)*1i)/((((((32*(12*a^2*b^9*d^4 + 24*a^4*b^7*d^4 + 12*a^6*b^5*d^4))/(
b^3*d^5) - (32*tan(c + d*x)^(1/2)*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))
^(1/2)*(16*b^12*d^4 + 16*a^2*b^10*d^4 - 16*a^4*b^8*d^4 - 16*a^6*b^6*d^4))/(
b^3*d^4))*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*tan(c + d*x
)^(1/2)*(14*a*b^9*d^2 + 16*a^9*b*d^2 - 4*a^3*b^7*d^2 - 2*a^5*b^5*d^2))/(b^3
d^4))(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*(4*a^9*d^2 + a
*b^8*d^2 + a^3*b^6*d^2 + 16*a^5*b^4*d^2 - 16*a^7*b^2*d^2))/(b^3*d^5))*(1i/(
4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*tan(c + d*x)^(1/2)*(2*a^8
+ b^8))/(b^3*d^4))*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (((((3
2*(12*a^2*b^9*d^4 + 24*a^4*b^7*d^4 + 12*a^6*b^5*d^4))/(b^3*d^5) + (32*tan(c
 + d*x)^(1/2)*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^12*d^4
+ 16*a^2*b^10*d^4 - 16*a^4*b^8*d^4 - 16*a^6*b^6...

3.584 $\int \frac{\tan^{\frac{5}{2}}(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=250

$$\frac{(a+b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}-\frac{(a+b)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}-\frac{2a^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{}}{}\right.}{b^{3/2}\,(a^2+}$$

```
[Out] -2*a^(5/2)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/b^(3/2)/(a^2+b^2)/d-1/2
*(a+b)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)-1/2*(a+b)*ar
ctan(1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)-1/4*(a-b)*ln(1-2^(1/2)
*tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)/d*2^(1/2)+1/4*(a-b)*ln(1+2^(1/2)*ta
n(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)/d*2^(1/2)+2*tan(d*x+c)^(1/2)/b/d
```

Rubi [A]
time = 0.28, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {3647, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(a+b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d\,(a^2+b^2)}-\frac{(a+b)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d\,(a^2+b^2)}-\frac{(a-b)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)}+\frac{(a-b)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)}-\frac{2a^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{3/2}d\,(a^2+b^2)}+\frac{2\sqrt{\tan(c+dx)}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(5/2)/(a + b*Tan[c + d*x]),x]
```

```
[Out] ((a + b)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)*d) -
((a + b)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)*d) -
(2*a^(5/2)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(b^(3/2)*(a^2 + b^
2)*d) - ((a - b)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqr
t[2]*(a^2 + b^2)*d) + ((a - b)*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
 d*x]])/(2*Sqrt[2]*(a^2 + b^2)*d) + (2*Sqrt[Tan[c + d*x]])/(b*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{2\sqrt{\tan(c+dx)}}{bd} + \frac{2\int \frac{-\frac{a}{2}-\frac{1}{2}b\tan(c+dx)-\frac{1}{2}a\tan^2(c+dx)}{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))}\,dx}{b}$$

$$= \frac{2\sqrt{\tan(c+dx)}}{bd} + \frac{2\int \frac{-\frac{b^2}{2}-\frac{1}{2}ab\tan(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{b(a^2+b^2)} - \frac{a^3\int \frac{1+\tan^2(c+dx)}{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))}\,dx}{b(a^2+b^2)}$$

$$= \frac{2\sqrt{\tan(c+dx)}}{bd} + \frac{4\text{Subst}\left(\int \frac{-\frac{b^2}{2}-\frac{1}{2}abx^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{b(a^2+b^2)\,d} - \frac{a^3\text{Subst}\left(\int \frac{}{\sqrt{x}}\right.}{}$$

$$= \frac{2\sqrt{\tan(c+dx)}}{bd} + \frac{(a-b)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{(a^2+b^2)\,d} - \frac{(2a^3)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{3/2}(a^2+b^2)\,d} + \frac{2\sqrt{\tan(c+dx)}}{bd} - \frac{(a-b)\text{Subst}\left(\int \frac{\sqrt{2}}{-1-\sqrt{}}\right.}{2\sqrt{2}}$$

$$= -\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{3/2}(a^2+b^2)\,d} - \frac{(a-b)\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan\right.}{2\sqrt{2}\,(a^2+b^2)\,d}$$

$$= \frac{(a+b)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} - \frac{(a+b)\tan^{-1}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.18, size = 155, normalized size = 0.62

$$\frac{\sqrt[4]{-1}\,b^{3/2}(-ia+b)\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right) - 2a^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right) + \sqrt[4]{-1}\,b^{3/2}(ia+b)\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right) + 2a^2\sqrt{b}\sqrt{\tan(c+dx)} + 2b^{5/2}\sqrt{\tan(c+dx)}}{b^{3/2}(a^2+b^2)\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(5/2)/(a + b*Tan[c + d*x]),x]
```

```
[Out] ((-1)^(1/4)*b^(3/2)*((-I)*a + b)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] - 2*
a^(5/2)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]] + (-1)^(1/4)*b^(3/2)*(
I*a + b)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + 2*a^2*Sqrt[b]*Sqrt[Tan[c
+ d*x]] + 2*b^(5/2)*Sqrt[Tan[c + d*x]])/(b^(3/2)*(a^2 + b^2)*d)
```

Maple [A]
time = 0.13, size = 241, normalized size = 0.96

method	result

derivativedivides	$\frac{\frac{2\left(\sqrt{\tan}\,(dx+c)\right)}{b}-\frac{2a^3\arctan\left(\frac{b\left(\sqrt{\tan}\,(dx+c)\right)}{\sqrt{ab}}\right)}{b(a^2+b^2)\sqrt{ab}}+\frac{-b\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}\,(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}\,(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\right.}{}}{}$
default	$\frac{\frac{2\left(\sqrt{\tan}\,(dx+c)\right)}{b}-\frac{2a^3\arctan\left(\frac{b\left(\sqrt{\tan}\,(dx+c)\right)}{\sqrt{ab}}\right)}{b(a^2+b^2)\sqrt{ab}}+\frac{-b\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}\,(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}\,(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b*tan(d*x+c)^(1/2)-2/b*a^3/(a^2+b^2)/(a*b)^(1/2)*arctan(b*tan(d*x+c)
^(1/2)/(a*b)^(1/2))+2/(a^2+b^2)*(-1/8*b*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(
1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2
)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))-1/8*a*2^(1/2)*(l
n((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d
*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)
^(1/2)))))
```

Maxima [A]
time = 0.51, size = 185, normalized size = 0.74

$$-\frac{\frac{8a^3\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^2b+b^3)\sqrt{ab}}+\frac{2\sqrt{2}(a+b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}(a+b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-\sqrt{2}(a-b)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+\sqrt{2}(a-b)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)}{a^2+b^2}-\frac{8\sqrt{\tan(dx+c)}}{b}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(8*a^3*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^2*b + b^3)*sqrt(a*b)
) + (2*sqrt(2)*(a + b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c))))
 + 2*sqrt(2)*(a + b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c))))
- sqrt(2)*(a - b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt
(2)*(a - b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a^2 + b^2
) - 8*sqrt(tan(d*x + c))/b)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3543 vs. $2(210) = 420$.
time = 8.28, size = 7198, normalized size = 28.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x, algorithm="fricas")

[Out] [1/4*(4*sqrt(2)*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*d^5*sqrt((a^4 + 2*a^2*b^2 + b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4)*arctan(-((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^4*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - sqrt(2)*((a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*d^7*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))*sqrt((a^4 + 2*a^2*b^2 + b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) + sqrt(2)*((a^7 - a^5*b^2 - a^3*b^4 + a*b^6)*d^3*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) - (a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c))*sqrt((a^4 + 2*a^2*b^2 + b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(1/4) + (a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c))/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4) - sqrt(2)*((a^10*b + 3*a^8*b^3 + 2*a^6*b^5 - 2*a^4*b^7 - 3*a^2*b^9 - b^11)*d^7*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - (a^9 + 2*a^7*b^2 - 2*a^3*b^6 - a*b^8)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))*sqrt((a^4 + 2*a^2*b^2 + b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4))/(a^4 - 2*a^2*b^2 + b^4)) + 4*sqrt(2)*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*d^5*sqrt((a^4 + 2*a^2*b^2 + b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4)*arctan(((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^4*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + sqrt(2)*((a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*d^7*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))*sqrt((a^4 + 2*a^2*b^2 + b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) - sqrt(2)*((a^7 - a^5*b^2 - a^3*b^4 + a*b^6)*d^3*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) - (a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c))*sqrt((a^4 + 2*a^2*b^2 + b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b

```
^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqr
t(sin(d*x + c)/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(1/4) + (a^4
 - 2*a^2*b^2 + b^4)*sin(d*x + c))/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4)
*d^4))^(3/4) + sqrt(2)*((a^10*b + 3*a^8*b^3 + 2*a^6*b^5 - 2*a^4*b^7 - 3*a^2
*b^9 - b^11)*d^7*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4
 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - (a^9 + 2*
a^7*b^2 - 2*a^3*b^6 - a*b^8)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6
*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))*sqrt((a^4 + 2*a^2*b^2 + b^4 + 2*
(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4
 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 +
 b^4)*d^4))^(3/4))/(a^4 - 2*a^2*b^2 + b^4)) + 2*a^2*sqrt(-a/b)*log(-(6*a*b*
cos(d*x + c)*sin(d*x + c) - (a^2 - b^2)*cos(d*x + c)^2 - b^2 - 4*(a*b*cos(d
*x + c)^2 - b^2*cos(d*x + c)*sin(d*x + c))*sqrt(-a/b)*sqrt(sin(d*x + c)/cos
(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 +
 b^2)) - sqrt(2)*(2*(a^3*b^2 + a*b^4)*d^3*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d
^4)) - (a^2*b + b^3)*d)*sqrt((a^4 + 2*a^2*b^2 + b^4 + 2*(a^5*b + 2*a^3*b^3
+ a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4)
)*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(1/4)*log(((a^6 - a^4*b^2 - a^2*b^4 + b
^6)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) + sqrt(2)*((a^7
- a^5*b^2 - a^3*b^4 + a*b^6)*d^3*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(
d*x + c) - (a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c))*sqrt((a^4 + 2*a^2*b^2
+ b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d
^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a^4 + 2
*a^2*b^2 + b^4)*d^4))^(1/4) + (a^4 - 2*a^2*b^2 ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(5/2)/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**(5/2)/(a + b*tan(c + d*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 7.62, size = 2500, normalized size = 10.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(5/2)/(a + b*tan(c + d*x)),x)
```

```
[Out] (log((32*a^3*(a^2 - b^2))/(b*d^5) - ((((((((128*a*b^3*(a^2 + b^2)^2)/d - 2
56*b^3*tan(c + d*x)^(1/2)*(a^2 - b^2)*(a^2 + b^2)^2*(1i/(d^2*(a*1i - b)^2))
^(1/2))*(1i/(d^2*(a*1i - b)^2))^(1/2))/2 - (64*a*tan(c + d*x)^(1/2)*(8*a^6
- 7*b^6 + 2*a^2*b^4 + a^4*b^2))/d^2)*(1i/(d^2*(a*1i - b)^2))^(1/2))/2 - (32
*a^2*(12*a^4 + b^4 - 15*a^2*b^2))/d^3)*(1i/(d^2*(a*1i - b)^2))^(1/2))/2 - (
32*tan(c + d*x)^(1/2)*(2*a^6 - b^6))/(b*d^4))*(1i/(d^2*(a*1i - b)^2))^(1/2)
)/2)*(-1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2))/2 - log((32*a^3*(a^2
 - b^2))/(b*d^5) - ((((((((128*a*b^3*(a^2 + b^2)^2)/d + 256*b^3*tan(c + d*
x)^(1/2)*(a^2 - b^2)*(a^2 + b^2)^2*(1i/(d^2*(a*1i - b)^2))^(1/2))*(1i/(d^2*
(a*1i - b)^2))^(1/2))/2 + (64*a*tan(c + d*x)^(1/2)*(8*a^6 - 7*b^6 + 2*a^2*b
^4 + a^4*b^2))/d^2)*(1i/(d^2*(a*1i - b)^2))^(1/2))/2 - (32*a^2*(12*a^4 + b^
4 - 15*a^2*b^2))/d^3)*(1i/(d^2*(a*1i - b)^2))^(1/2))/2 + (32*tan(c + d*x)^(
1/2)*(2*a^6 - b^6))/(b*d^4))*(1i/(d^2*(a*1i - b)^2))^(1/2))/2)*(-1/(4*(b^2*
d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) + atan((((-1i/(4*(b^2*d^2 - a^2*d^
2 + a*b*d^2*2i)))^(1/2)*((((32*(4*a*b^8*d^4 + 8*a^3*b^6*d^4 + 4*a^5*b^4*d^4
))/(b*d^5) - (32*tan(c + d*x)^(1/2)*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i
)))^(1/2)*(16*b^10*d^4 + 16*a^2*b^8*d^4 - 16*a^4*b^6*d^4 - 16*a^6*b^4*d^4))
/(b*d^4))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*tan(c + d*
x)^(1/2)*(16*a^7*b*d^2 - 14*a*b^7*d^2 + 4*a^3*b^5*d^2 + 2*a^5*b^3*d^2))/(b*
d^4))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*(12*a^6*b*d^2
+ a^2*b^5*d^2 - 15*a^4*b^3*d^2))/(b*d^5)) + (32*tan(c + d*x)^(1/2)*(2*a^6 -
 b^6))/(b*d^4))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*1i - ((-1i
/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*((((32*(4*a*b^8*d^4 + 8*a^3*b^
6*d^4 + 4*a^5*b^4*d^4))/(b*d^5) + (32*tan(c + d*x)^(1/2)*(-1i/(4*(b^2*d^2 -
 a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^10*d^4 + 16*a^2*b^8*d^4 - 16*a^4*b^6*d
^4 - 16*a^6*b^4*d^4))/(b*d^4))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(
1/2) - (32*tan(c + d*x)^(1/2)*(16*a^7*b*d^2 - 14*a*b^7*d^2 + 4*a^3*b^5*d^2
+ 2*a^5*b^3*d^2))/(b*d^4))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)
 - (32*(12*a^6*b*d^2 + a^2*b^5*d^2 - 15*a^4*b^3*d^2))/(b*d^5)) - (32*tan(c
+ d*x)^(1/2)*(2*a^6 - b^6))/(b*d^4))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2
i)))^(1/2)*1i)/(((-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*((((32*(4
*a*b^8*d^4 + 8*a^3*b^6*d^4 + 4*a^5*b^4*d^4))/(b*d^5) - (32*tan(c + d*x)^(1/
2)*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^10*d^4 + 16*a^2*b
^8*d^4 - 16*a^4*b^6*d^4 - 16*a^6*b^4*d^4))/(b*d^4))*(-1i/(4*(b^2*d^2 - a^2*
d^2 + a*b*d^2*2i)))^(1/2) + (32*tan(c + d*x)^(1/2)*(16*a^7*b*d^2 - 14*a*b^7
*d^2 + 4*a^3*b^5*d^2 + 2*a^5*b^3*d^2))/(b*d^4))*(-1i/(4*(b^2*d^2 - a^2*d^2
+ a*b*d^2*2i)))^(1/2) - (32*(12*a^6*b*d^2 + a^2*b^5*d^2 - 15*a^4*b^3*d^2))/
```

(b*d^5)) + (32*tan(c + d*x)^(1/2)*(2*a^6 - b^6))/(b*d^4))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + ((-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*((((32*(4*a*b^8*d^4 + 8*a^3*b^6*d^4 + 4*a^5*b^4*d^4))/(b*d^5) + (32*tan(c + d*x)^(1/2)*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^10*d^4 + 16*a^2*b^8*d^4 - 16*a^4*b^6*d^4 - 16*a^6*b^4*d^4))/(b*d^4))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*tan(c + d*x)^(1/2)*(16*a^7*b*d^2 - 14*a*b^7*d^2 + 4*a^3*b^5*d^2 + 2*a^5*b^3*d^2))/(b*d^4))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*(12*a^6*b*d^2 + a^2*b^5*d^2 - 15*a^4*b^3*d^2))/(b*d^5)) - (32*tan(c + d*x)^(1/2)*(2*a^6 - b^6))/(b*d^4))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (64*(a^5 - a^3*b^2))/(b*d^5)))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*2i + (2*tan(c + d*x)^(1/2))/(b*d) - (atan((((-a^5*b^3)^(1/2)*((32*tan(c + d*x)^(1/2)*(2*a^6 - b^6))/(b*d^4) + ((-a^5*b^3)^(1/2)*((32*(12*a^6*b*d^2 + a^2*b^5*d^2 - 15*a^4*b^3*d^2))/(b*d^5) + (((32*tan(c + d*x)^(1/2)*(16*a^7*b*d^2 - 14*a*b^7*d^2 + 4*a^3*b^5*d^2 + 2*a^5*b^3*d^2))/(b*d^4) - (((32*(4*a*b^8*d^4 + 8*a^3*b^6*d^4 + 4*a^5*b^4*d^4))/(b*d^5) + (32*tan(c + d*x)^(1/2)*(-a^5*b^3)^(1/2)*(16*b^10*d^4 + 16*a^2*b^8*d^4 - 16*a^4*b^6*d^4 - 16*a^6*b^4*d^4))/(b^4*d^5*(a^2 + b^2)))*(-a^5*b^3)^(1/2))/(b^3*d*(a^2 + b^2)))*(-a^5*b^3)^(1/2))/(b^3*d*(a^2 + b^2))))/(b^3*d*(a^2 + b^2)))*1i)/(b^3*d*(a^2 + b^2)) + ((-a^5*b^3)^(1/2)*((32*tan(c + d*x)^(1/2)*(2*a^6 - b^6))/(b*d^4) - ((-a^5*b^3)^(1/2)*((32*(12*a^6*b*d^2 + a^2*b^5*d^2 - 15*a^4*b^3*d^2))/(b*d^5) - (((32*tan(c + d*x)^(1/2)*(16*a^7*b*d^2 - 14*a*b^7*d^2 + 4*a^3*b^5*d^2 + 2*a^5*b^3*d^2))/(b*d^4) + (((32*(4*a*b^8*d^4 + 8*a^3*b^6*d^4 + 4*a^5*b^4*d^4))/(b*d^5) - (32*tan(c + d*x)^(1/2)*(-a^5*b^3)^(1/2)*(16*b^10*d^4 + 16*a^2*b^8*d^4 - 16*a^4*b^6*d^4 - 16*a^6*b^4*d^4))/(b^4*d^5*(a^2 + b^2)))*(-a^5*b^3)^(1/2))/(b^3*d*(a^2 + b^2)))*(-a^5*b^3)^(1/2))/(b^3*d*(a^2 + b^2))))/(b^3*d*(a^2 + b^2)))*1i)/(b^3*d*(a^2 + b^2)))/((64*(a^5 - a^3*b^2))/(b*d^5) + ((-a^5*b^3)^(1/2)*((32*tan(c + d*x)^(1/2)*(2*a^6 - b^6))/(b*d^4) + ((-a^5*b^3)^(1/2)*((32*(12*a^6*b*d^2 + a^2*b^5*d^2 - 15*a^4*b^3*d^2))/(b*d^5) + (((32*tan(c + d*x)^(1/2)*(16*a^7*b*d^2 - 14*a*b^7*d^2 + 4*a^3*b^5*d^...

3.585 $\int \frac{\tan^{\frac{3}{2}}(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=232

$$\frac{(a-b)\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}-\frac{(a-b)\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}+\frac{2a^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{t}}{}\right.}{\sqrt{b}\,(a^2+b}$$

```
[Out] -1/2*(a-b)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)-1/2*(a-b
)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)+1/4*(a+b)*ln(1-2^(
1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)/d*2^(1/2)-1/4*(a+b)*ln(1+2^(1/2
)*tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)/d*2^(1/2)+2*a^(3/2)*arctan(b^(1/2)
*tan(d*x+c)^(1/2)/a^(1/2))/(a^2+b^2)/d/b^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3654, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(a-b)\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d\,(a^2+b^2)}-\frac{(a-b)\text{ArcTan}\left(\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d\,(a^2+b^2)}+\frac{(a+b)\log\left(\tan(c+dx)-\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)}-\frac{(a+b)\log\left(\tan(c+dx)+\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)}+\frac{2a^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b}\,d\,(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(3/2)/(a + b*Tan[c + d*x]),x]
```

```
[Out] ((a - b)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)*d) -
((a - b)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)*d) +
(2*a^(3/2)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(Sqrt[b]*(a^2 + b^
2)*d) + ((a + b)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqr
t[2]*(a^2 + b^2)*d) - ((a + b)*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
 d*x]])/(2*Sqrt[2]*(a^2 + b^2)*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3654

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[Simp[a^2*c - b^2*c + 2
*a*b*d + (2*a*b*c - a^2*d + b^2*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x]
], x], x] + Dist[(b*c - a*d)^2/(c^2 + d^2), Int[(1 + Tan[e + f*x]^2)/(Sqrt[
a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\int \frac{-a+b\tan(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{a^2+b^2} + \frac{a^2\int \frac{1+\tan^2(c+dx)}{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))}\,dx}{a^2+b^2}$$

$$= \frac{2\text{Subst}\left(\int \frac{-a+bx^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{(a^2+b^2)\,d} + \frac{a^2\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx, x, \tan(c+dx\right.}{(a^2+b^2)\,d}$$

$$= \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{(a^2+b^2)\,d} - \frac{(a-b)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\tan(c}\right.}{(a^2+b^2)\,d}$$

$$= \frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b}\,(a^2+b^2)\,d} - \frac{(a-b)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{\tan(c+}\right.}{2\,(a^2+b^2)\,d}$$

$$= \frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b}\,(a^2+b^2)\,d} + \frac{(a+b)\log\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}+\tan(c\right.}{2\sqrt{2}\,(a^2+b^2)\,d}$$

$$= \frac{(a-b)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} - \frac{(a-b)\tan^{-1}\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}\right.}{\sqrt{2}\,(a^2+b^2)\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.29, size = 227, normalized size = 0.98

$$\frac{3a\left(2\sqrt{2}\,\sqrt{b}\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)-2\sqrt{2}\,\sqrt{b}\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}\right)+8\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)+\sqrt{2}\,\sqrt{b}\,\log\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}+\tan(c+dx)\right)-\sqrt{2}\,\sqrt{b}\,\log\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\right)+8b^{3/2}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{12\sqrt{b}\,(a^2+b^2)\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(3/2)/(a + b*Tan[c + d*x]),x]
```

```
[Out] (3*a*(2*Sqrt[2]*Sqrt[b]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 2*Sqrt[2]*
Sqrt[b]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + 8*Sqrt[a]*ArcTan[(Sqrt[b]*
Sqrt[Tan[c + d*x]])/Sqrt[a]] + Sqrt[2]*Sqrt[b]*Log[1 - Sqrt[2]*Sqrt[Tan[c +
 d*x]] + Tan[c + d*x]] - Sqrt[2]*Sqrt[b]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]
 + Tan[c + d*x]]) + 8*b^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d*x]^
2]*Tan[c + d*x]^(3/2))/(12*Sqrt[b]*(a^2 + b^2)*d)
```

Maple [A]
time = 0.12, size = 225, normalized size = 0.97

method	result
derivativedivides	$\frac{\frac{2a^2 \arctan\left(\frac{b\left(\sqrt{\tan}(dx+c)\right)}{\sqrt{ab}}\right)}{(a^2+b^2)\sqrt{ab}}+\frac{-\frac{a\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan}(\right.\right.}{4}}{}}{}$
default	$\frac{\frac{2a^2 \arctan\left(\frac{b\left(\sqrt{\tan}(dx+c)\right)}{\sqrt{ab}}\right)}{(a^2+b^2)\sqrt{ab}}+\frac{-\frac{a\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan}(\right.\right.}{4}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*a^2/(a^2+b^2)/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))+2/(
a^2+b^2)*(-1/8*a*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(
1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*a
rctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/8*b*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)
^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1
/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))))
```

Maxima [A]
time = 0.50, size = 171, normalized size = 0.74

$$\frac{\frac{8\,a^2\arctan\left(\frac{b\sqrt{\tan\left(dx+c\right)}}{\sqrt{ab}}\right)}{(a^2+b^2)\sqrt{ab}}-\frac{2\sqrt{2}\,(a-b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(dx+c\right)}\right)\right)+2\sqrt{2}\,(a-b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(dx+c\right)}\right)\right)+\sqrt{2}\,(a+b)\log\left(\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan(dx+c)+1\right)-\sqrt{2}\,(a+b)\log\left(-\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan(dx+c)+1\right)}{a^2+b^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(8*a^2*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^2 + b^2)*sqrt(a*b)) -
 (2*sqrt(2)*(a - b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) +
```

```
2*sqrt(2)*(a - b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + s
qrt(2)*(a + b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)
*(a + b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a^2 + b^2))/
d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3491 vs. $2(194) = 388$.
time = 8.68, size = 7094, normalized size = 30.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(2)*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^5*sqrt((a^4 + 2*a^2*
b^2 + b^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^
4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a
^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4
))^(3/4)*arctan(((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^4*sqrt((a^4 - 2*a^2*
b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((
a^4 + 2*a^2*b^2 + b^4)*d^4)) + sqrt(2)*((a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^
3*b^6 + a*b^8)*d^7*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b
^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + (a^6*b
+ 3*a^4*b^3 + 3*a^2*b^5 + b^7)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a
^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))*sqrt((a^4 + 2*a^2*b^2 + b^4 -
2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a
^4 - 2*a^2*b^2 + b^4))*sqrt(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt(1/((a
^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) + sqrt(2)*((a^6*b - a^4*b^3 - a^2*
b^5 + b^7)*d^3*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) + (a^5 -
2*a^3*b^2 + a*b^4)*d*cos(d*x + c))*sqrt((a^4 + 2*a^2*b^2 + b^4 - 2*(a^5*b +
 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2
*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^
4))^(1/4) + (a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c))/cos(d*x + c))*(1/((a^4 +
2*a^2*b^2 + b^4)*d^4))^(3/4) + sqrt(2)*((a^11 + 3*a^9*b^2 + 2*a^7*b^4 - 2*a
^5*b^6 - 3*a^3*b^8 - a*b^10)*d^7*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6
*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d
^4)) + (a^8*b + 2*a^6*b^3 - 2*a^2*b^7 - b^9)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^
4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))*sqrt((a^4 + 2*a^
2*b^2 + b^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 +
b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a
^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4))/(a^4 - 2*a^2*b^2 + b^4)) + 4*sqrt(2)*(a^
6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^5*sqrt((a^4 + 2*a^2*b^2 + b^4 - 2*(a^5*b
 + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a
^2*b^2 + b^4))*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
 4*a^2*b^6 + b^8)*d^4))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4)*arctan(-((a
```

```
^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^4*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 +
4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b
^4)*d^4)) - sqrt(2)*((a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*d^7*
sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^
8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + (a^6*b + 3*a^4*b^3 + 3*a^2
*b^5 + b^7)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4
+ 4*a^2*b^6 + b^8)*d^4)))*sqrt((a^4 + 2*a^2*b^2 + b^4 - 2*(a^5*b + 2*a^3*b^
3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^
4))*sqrt(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^
4)*d^4))*cos(d*x + c) - sqrt(2)*((a^6*b - a^4*b^3 - a^2*b^5 + b^7)*d^3*sqrt
(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4)*
d*cos(d*x + c))*sqrt((a^4 + 2*a^2*b^2 + b^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)
*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(s
in(d*x + c)/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(1/4) + (a^4 -
2*a^2*b^2 + b^4)*sin(d*x + c))/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^
4))^(3/4) - sqrt(2)*((a^11 + 3*a^9*b^2 + 2*a^7*b^4 - 2*a^5*b^6 - 3*a^3*b^8
- a*b^10)*d^7*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + (a^8*b + 2*a
^6*b^3 - 2*a^2*b^7 - b^9)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^
2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))*sqrt((a^4 + 2*a^2*b^2 + b^4 - 2*(a^
5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 -
2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^
4)*d^4))^(3/4))/(a^4 - 2*a^2*b^2 + b^4)) + sqrt(2)*(2*(a^3*b + a*b^3)*d^3*s
qrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + (a^2 + b^2)*d)*sqrt((a^4 + 2*a^2*b^2
 + b^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*
d^4)))/(a^4 - 2*a^2*b^2 + b^4))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(1/4)*log
(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))
*cos(d*x + c) + sqrt(2)*((a^6*b - a^4*b^3 - a^2*b^5 + b^7)*d^3*sqrt(1/((a^4
 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*
x + c))*sqrt((a^4 + 2*a^2*b^2 + b^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqr
t(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x +
 c)/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(1/4) + (a^4 - 2*a^2*b^
2 + b^4)*sin(d*x + c))/cos(d*x + c)) - sqrt(2)*(2*(a^3*b + a*b^3)*d^3*sqrt(
1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + (a^2 + b^2)*d)*sqrt((a^4 + 2*a^2*b^2 + b
^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)
))/(a^4 - 2*a^2*b^2 + b^4))*(1/((a^4 + 2*a^2*b^...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(3/2)/(a+b*tan(d*x+c)),x)
```

[Out] Integral(tan(c + d*x)**(3/2)/(a + b*tan(c + d*x)), x)

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x, algorithm="giac")

[Out] Timed out

Mupad [**B**]
time = 5.26, size = 2500, normalized size = 10.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d*x)^(3/2)/(a + b*tan(c + d*x)),x)

[Out] - atan(((((1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(((32*(4*a^2*b^6*d^4 + 8*a^4*b^4*d^4 + 4*a^6*b^2*d^4))/d^5 - (32*tan(c + d*x)^(1/2)*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^9*d^4 + 16*a^2*b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/d^4)*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*tan(c + d*x)^(1/2)*(14*a*b^6*d^2 - 4*a^3*b^4*d^2 + 14*a^5*b^2*d^2))/d^4) + (32*(a*b^5*d^2 + 4*a^5*b*d^2 - 15*a^3*b^3*d^2))/d^5)*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*tan(c + d*x)^(1/2)*(2*a^4*b + b^5))/d^4)*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*1i - (((1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(((32*(4*a^2*b^6*d^4 + 8*a^4*b^4*d^4 + 4*a^6*b^2*d^4))/d^5 + (32*tan(c + d*x)^(1/2)*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^9*d^4 + 16*a^2*b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/d^4)*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*tan(c + d*x)^(1/2)*(14*a*b^6*d^2 - 4*a^3*b^4*d^2 + 14*a^5*b^2*d^2))/d^4) + (32*(a*b^5*d^2 + 4*a^5*b*d^2 - 15*a^3*b^3*d^2))/d^5)*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*tan(c + d*x)^(1/2)*(2*a^4*b + b^5))/d^4)*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*1i)/((((1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(((32*(4*a^2*b^6*d^4 + 8*a^4*b^4*d^4 + 4*a^6*b^2*d^4))/d^5 - (32*tan(c + d*x)^(1/2)*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^9*d^4 + 16*a^2*b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/d^4)*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*tan(c + d*x)^(1/2)*(14*a*b^6*d^2 - 4*a^3*b^4*d^2 + 14*a^5*b^2*d^2))/d^4) + (32*(a*b^5*d^2 + 4*a^5*b*d^2 - 15*a^3*b^3*d^2))/d^5)*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*tan(c + d*x)^(1/2)*(2*a^4*b + b^5))/d^4)*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (((1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(((32*(4*a^2*b^6*d^4 + 8*a^4*b^4*d^4 + 4*a^6*b^2*d^4))/d^5 + (32*tan(c + d*x)^(1/2)*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^9*d^4 + 16* a^2*b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/d^4)*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*tan(c + d*x)^(1/2)*(14*a*b^6*d^2 - 4*a^3*b^4*d^2 + 14*a^5*b^2*d^2))/d^4) + (32*(a*b^5*d^2 + 4*a^5*b*d^2 - 15*a^3*b^3*d^2))/d^5)*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*tan(c + d*x)^(1/2)*(2*a^4*b + b^5))/d^4)*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (64*a^2*b^2)/d^5))*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*2i - atan((((1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*(((((((1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*((32*(4*a^2*b^6*d^4 + 8*a^4*b^4*d^4 + 4*a^6*b^2*d^4))/d^5 - (16*tan(c + d*x)^(1/2)*(1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*(16*b^9*d^4 + 16*a^2*b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/d^4))/2 + (32*tan(c + d*x)^(1/2)*(14*a*b^6*d^2 - 4*a^3*b^4*d^2 + 14*a^5*b^2*d^2))/d^4)*(1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2))/2 + (32*(a*b^5*d^2 + 4*a^5*b*d^2 - 15*a^3*b^3*d^2))/d^5)*(1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2))/2 - (32*tan(c + d*x)^(1/2)*(2*a^4*b + b^5))/d^4)*1i)/2 - ((1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*(((((((1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*((32*(4*a^2*b^6*d^4 + 8*a^4*b^4*d^4 + 4*a^6*b^2*d^4))/d^5 + (16*tan(c + d*x)^(1/2)*(1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*(16*b^9*d^4 + 16*a^2*b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/d^4))/2 - (32*tan(c + d*x)^(1/2)*(14*a*b^6*d^2 - 4*a^3*b^4*d^2 + 14*a^5*b^2*d^2))/d^4)*(1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2))/2 + (32*(a*b^5*d^2 + 4*a^5*b*d^2 - 15*a^3*b^3*d^2))/d^5)*(1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2))/2 + (32*tan(c + d*x)^(1/2)*(2*a^4*b + b^5))/d^4)*1i)/2)/(((1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*(((((((1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*((32*(4*a^2*b^6*d^4 + 8*a^4*b^4*d^4 + 4*a^6*b^2*d^4))/d^5 - (16*tan(c + d*x)^(1/2)*(1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*(16*b^9*d^4 + 16*a^2*b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/d^4))/2 + (32*tan(c + d*x)^(1/2)*(14*a*b^6*d^2 - 4*a^3*b^4*d^2 + 14*a^5*b^2*d^2))/d^4)*(1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2))/2 + (32*(a*b^5*d^2 + 4*a^5*b*d^2 - 15*a^3*b^3*d^2))/d^5)*(1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2))/2 - (32*tan(c + d*x)^(1/2)*(2*a^4*b + b^5))/d^4))/2 + ((1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*(((((((1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*((32*(4*a^2*b^6*d^4 + 8*a^4*b^4*d^4 + 4*a^6*b^2*d^4))/d^5 + (16*tan(c + d*x)^(1/2)*(1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*(16*b^9*d^4 + 16*a^2*b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/d^4))/2 - (32*tan(c + d*x)^(1/2)*(14*a*b^6*d^2 - 4*a^3*b^4*d^2 + 14*a^5*b^2*d^2))/d^4)*(1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2))/2 + (32*(a*b^5*d^2 + 4*a^5*b*d^2 - 15*a^3*b^3*d^2))/d^5)*(1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2))/2 + (32*tan(c + d*x)^(1/2)*(2*a^4*b + b^5))/d^4))/2 + (64*a^2*b^2)/d^5))*(1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*1i - (atan((((-a^3*b)^(1/2)*((((32*(a*b^5*d^2 + 4*a^5*b*d^2 - 15*a^3*b^3*d^2))/d^5 + ((-a^3*b)^(1/2)*((((32*(4*a^2*b^6*d^4 + 8*a^4*b^4*d...

3.586 $\int \frac{\sqrt{\tan(c+dx)}}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=232

$$-\frac{(a+b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)d}+\frac{(a+b)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)d}-\frac{2\sqrt{a}\sqrt{b}\text{ArcTan}\left(\right.}{\left(a^2+\right.}$$

```
[Out] 1/2*(a+b)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)+1/2*(a+b)
*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)+1/4*(a-b)*ln(1-2^(1
/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)/d*2^(1/2)-1/4*(a-b)*ln(1+2^(1/2)
*tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)/d*2^(1/2)-2*arctan(b^(1/2)*tan(d*x+
c)^(1/2)/a^(1/2))*a^(1/2)*b^(1/2)/(a^2+b^2)/d
```

Rubi [A]
time = 0.16, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3653, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{(a+b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d\left(a^2+b^2\right)}+\frac{(a+b)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d\left(a^2+b^2\right)}-\frac{2\sqrt{a}\sqrt{b}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{d\left(a^2+b^2\right)}+\frac{(a-b)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)}-\frac{(a-b)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[c + d*x]]/(a + b*Tan[c + d*x]),x]
```

```
[Out] -(((a + b)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)*d))
 + ((a + b)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)*d)
 - (2*Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/((a^2 +
 b^2)*d) + ((a - b)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*
Sqrt[2]*(a^2 + b^2)*d) - ((a - b)*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[
c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3653

```
Int[Sqrt[(a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[Simp[a*c + b*d + (b*c -
a*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x]], x], x] - Dist[d*((b*c - a*d
)/(c^2 + d^2)), Int[(1 + Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]]*(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{\sqrt{\tan(c+dx)}}{a+b\tan(c+dx)}\,dx = \frac{\int \frac{b+a\tan(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{a^2+b^2} - \frac{(ab)\int \frac{1+\tan^2(c+dx)}{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))}\,dx}{a^2+b^2}$$

$$= \frac{2\text{Subst}\left(\int \frac{b+ax^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{(a^2+b^2)\,d} - \frac{(ab)\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx, x, \tan(c+d\ldots\right.}{(a^2+b^2)\,d}$$

$$= -\frac{(a-b)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{(a^2+b^2)\,d} - \frac{(2ab)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{\tan(\ldots}\right.}{(a^2+b^2)\,d}$$

$$= -\frac{2\sqrt{a}\,\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{(a^2+b^2)\,d} + \frac{(a-b)\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}\,x-x^2}\,dx, x, \sqrt{\ldots}\right.}{2\sqrt{2}\,(a^2+b^2)\,d}$$

$$= -\frac{2\sqrt{a}\,\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{(a^2+b^2)\,d} + \frac{(a-b)\log\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}+\ldots\right.}{2\sqrt{2}\,(a^2+b^2)\,d}$$

$$= -\frac{(a+b)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} + \frac{(a+b)\tan^{-1}\left(1+\sqrt{2}\,\sqrt{\tan(c+dx\ldots}\right.}{\sqrt{2}\,(a^2+b^2)\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.18, size = 204, normalized size = 0.88

$$\frac{-6\sqrt{2}\,b\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)+6\sqrt{2}\,b\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}\right)-24\sqrt{a}\,\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)-3\sqrt{2}\,b\log\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}+\tan(c+dx)\right)+3\sqrt{2}\,b\log\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}+\tan(c+dx)\right)+8a\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{12\,(a^2+b^2)\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tan[c + d*x]]/(a + b*Tan[c + d*x]),x]
```

```
[Out] (-6*Sqrt[2]*b*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] + 6*Sqrt[2]*b*ArcTan[1
 + Sqrt[2]*Sqrt[Tan[c + d*x]]] - 24*Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[Ta
n[c + d*x]])/Sqrt[a]] - 3*Sqrt[2]*b*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Ta
n[c + d*x]] + 3*Sqrt[2]*b*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]
] + 8*a*Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d*x]^2]*Tan[c + d*x]^(3/2))
/(12*(a^2 + b^2)*d)
```

Maple [A]
time = 0.16, size = 224, normalized size = 0.97

method	result
derivativedivides	$\frac{-\frac{2ba\arctan\left(\frac{b\left(\sqrt{\tan}(dx+c)\right)}{\sqrt{ab}}\right)}{(a^2+b^2)\sqrt{ab}}+\frac{b\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan}(d\right.\right.\right.}{4}}{}$
default	$\frac{-\frac{2ba\arctan\left(\frac{b\left(\sqrt{\tan}(dx+c)\right)}{\sqrt{ab}}\right)}{(a^2+b^2)\sqrt{ab}}+\frac{b\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan}(d\right.\right.\right.}{4}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*b*a/(a^2+b^2)/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))+2/
(a^2+b^2)*(1/8*b*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(
1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*a
rctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/8*a*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)
^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1
/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))))
```

Maxima [A]
time = 0.49, size = 170, normalized size = 0.73

$$-\frac{\frac{8\,ab\arctan\left(\frac{b\sqrt{\tan\left(dx+c\right)}}{\sqrt{ab}}\right)}{(a^2+b^2)\sqrt{ab}}-\frac{2\sqrt{2}(a+b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(dx+c\right)}\right)\right)+2\sqrt{2}(a+b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(dx+c\right)}\right)\right)-\sqrt{2}(a-b)\log\left(\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan(dx+c)+1\right)+\sqrt{2}(a-b)\log\left(-\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan(dx+c)+1\right)}{a^2+b^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(8*a*b*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^2 + b^2)*sqrt(a*b))
- (2*sqrt(2)*(a + b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c))) +
```

```
 2*sqrt(2)*(a + b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) -
sqrt(2)*(a - b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2
)*(a - b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a^2 + b^2))
/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3600 vs. $2(194) = 388$.
time = 11.53, size = 7205, normalized size = 31.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(2)*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^5*sqrt((a^4 + 2*a^2*
b^2 + b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^
4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a
^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4
))^(3/4)*arctan(-((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^4*sqrt((a^4 - 2*a^2
*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/(
(a^4 + 2*a^2*b^2 + b^4)*d^4)) - sqrt(2)*((a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4
*a^2*b^7 + b^9)*d^7*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*
b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - (a^7 +
 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*
a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))*sqrt((a^4 + 2*a^2*b^2 + b^4 +
 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(
a^4 - 2*a^2*b^2 + b^4))*sqrt(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt(1/((
a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) + sqrt(2)*((a^7 - a^5*b^2 - a^3*b
^4 + a*b^6)*d^3*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) - (a^4*b
 - 2*a^2*b^3 + b^5)*d*cos(d*x + c))*sqrt((a^4 + 2*a^2*b^2 + b^4 + 2*(a^5*b
+ 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^
2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4)*d
^4))^(1/4) + (a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c))/cos(d*x + c))*(1/((a^4 +
 2*a^2*b^2 + b^4)*d^4))^(3/4) - sqrt(2)*((a^10*b + 3*a^8*b^3 + 2*a^6*b^5 -
2*a^4*b^7 - 3*a^2*b^9 - b^11)*d^7*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^
6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*
d^4)) - (a^9 + 2*a^7*b^2 - 2*a^3*b^6 - a*b^8)*d^5*sqrt((a^4 - 2*a^2*b^2 + b
^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))*sqrt((a^4 + 2*a
^2*b^2 + b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 +
 b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((
a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4))/(a^4 - 2*a^2*b^2 + b^4)) + 4*sqrt(2)*(a
^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^5*sqrt((a^4 + 2*a^2*b^2 + b^4 + 2*(a^5*
b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*
a^2*b^2 + b^4))*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4
+ 4*a^2*b^6 + b^8)*d^4))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4)*arctan(((a
```

```
^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^4*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 +
4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b
^4)*d^4)) + sqrt(2)*((a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*d^7*
sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^
8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - (a^7 + 3*a^5*b^2 + 3*a^3*b
^4 + a*b^6)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4
+ 4*a^2*b^6 + b^8)*d^4)))*sqrt((a^4 + 2*a^2*b^2 + b^4 + 2*(a^5*b + 2*a^3*b^
3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^
4))*sqrt(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^
4)*d^4))*cos(d*x + c) - sqrt(2)*((a^7 - a^5*b^2 - a^3*b^4 + a*b^6)*d^3*sqrt
(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) - (a^4*b - 2*a^2*b^3 + b^5)*
d*cos(d*x + c))*sqrt((a^4 + 2*a^2*b^2 + b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)
*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(s
in(d*x + c)/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(1/4) + (a^4 -
2*a^2*b^2 + b^4)*sin(d*x + c))/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^
4))^(3/4) + sqrt(2)*((a^10*b + 3*a^8*b^3 + 2*a^6*b^5 - 2*a^4*b^7 - 3*a^2*b^
9 - b^11)*d^7*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - (a^9 + 2*a^7
*b^2 - 2*a^3*b^6 - a*b^8)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^
2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))*sqrt((a^4 + 2*a^2*b^2 + b^4 + 2*(a^
5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 -
2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^
4)*d^4))^(3/4))/(a^4 - 2*a^2*b^2 + b^4)) - sqrt(2)*(2*(a^3*b + a*b^3)*d^3*s
qrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - (a^2 + b^2)*d)*sqrt((a^4 + 2*a^2*b^2
 + b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*
d^4)))/(a^4 - 2*a^2*b^2 + b^4))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(1/4)*log
(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))
*cos(d*x + c) + sqrt(2)*((a^7 - a^5*b^2 - a^3*b^4 + a*b^6)*d^3*sqrt(1/((a^4
 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) - (a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*
x + c))*sqrt((a^4 + 2*a^2*b^2 + b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqr
t(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x +
 c)/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(1/4) + (a^4 - 2*a^2*b^
2 + b^4)*sin(d*x + c))/cos(d*x + c)) + sqrt(2)*(2*(a^3*b + a*b^3)*d^3*sqrt(
1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - (a^2 + b^2)*d)*sqrt((a^4 + 2*a^2*b^2 + b
^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)
))/(a^4 - 2*a^2*b^2 + b^4))*(1/((a^4 + 2*a^2*b^2 + b^...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\tan(c + dx)}}{a + b\tan(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/2)/(a+b*tan(d*x+c)),x)
```

[Out] Integral(sqrt(tan(c + d*x))/(a + b*tan(c + d*x)), x)

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x, algorithm="giac")

[Out] integrate(sqrt(tan(d*x + c))/(b*tan(d*x + c) + a), x)

Mupad [**B**]
time = 6.72, size = 2500, normalized size = 10.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d*x)^(1/2)/(a + b*tan(c + d*x)),x)

[Out] atan(((((((32*(12*a*b^7*d^4 + 24*a^3*b^5*d^4 + 12*a^5*b^3*d^4))/d^5 - (32*tan(c + d*x)^(1/2)*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^9*d^4 + 16*a^2*b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/d^4)*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*tan(c + d*x)^(1/2)*(20*a^3*b^4*d^2 - 14*a*b^6*d^2 + 2*a^5*b^2*d^2))/d^4)*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*(13*a^2*b^4*d^2 + a^4*b^2*d^2))/d^5)*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*tan(c + d*x)^(1/2)*(b^5 - 2*a^2*b^3))/d^4)*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*1i - (((((32*(12*a*b^7*d^4 + 24*a^3*b^5*d^4 + 12*a^5*b^3*d^4))/d^5 + (32*tan(c + d*x)^(1/2)*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^9*d^4 + 16*a^2*b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/d^4)*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*tan(c + d*x)^(1/2)*(20*a^3*b^4*d^2 - 14*a*b^6*d^2 + 2*a^5*b^2*d^2))/d^4)*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*(13*a^2*b^4*d^2 + a^4*b^2*d^2))/d^5)*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*tan(c + d*x)^(1/2)*(b^5 - 2*a^2*b^3))/d^4)*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*1i)/((((((32*(12*a*b^7*d^4 + 24*a^3*b^5*d^4 + 12*a^5*b^3*d^4))/d^5 - (32*tan(c + d*x)^(1/2)*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^9*d^4 + 16*a^2*b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/d^4)*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*tan(c + d*x)^(1/2)*(20*a^3*b^4*d^2 - 14*a*b^6*d^2 + 2*a^5*b^2*d^2))/d^4)*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*(13*a^2*b^4*d^2 + a^4*b^2*d^2))/d^5)*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*tan(c + d*x)^(1/2)*(b^5 - 2*a^2*b^3))/d^4)*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (((((32*(12*a*b^7*d^4 + 24*a^3*b^5*d^4 + 12*a^5*b^3*d^4))/d^5 + (32*tan(c + d*x)^(1/2)*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^9*d^4 + 16*a^2*b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/d^4)*(-1i/(4

```
*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*tan(c + d*x)^(1/2)*(20*a^3*
b^4*d^2 - 14*a*b^6*d^2 + 2*a^5*b^2*d^2))/d^4)*(-1i/(4*(b^2*d^2 - a^2*d^2 +
a*b*d^2*2i)))^(1/2) + (32*(13*a^2*b^4*d^2 + a^4*b^2*d^2))/d^5)*(-1i/(4*(b^2
*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*tan(c + d*x)^(1/2)*(b^5 - 2*a^2*
b^3))/d^4)*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (64*a*b^3)/d^
5))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*2i + (log(- ((((((((3
84*a*b^3*(a^2 + b^2)^2)/d - 256*b^3*tan(c + d*x)^(1/2)*(a^2 - b^2)*(a^2 + b
^2)^2*(1i/(d^2*(a*1i - b)^2))^(1/2))*(1i/(d^2*(a*1i - b)^2))^(1/2))/2 - (64
*a*b^2*tan(c + d*x)^(1/2)*(a^4 - 7*b^4 + 10*a^2*b^2))/d^2)*(1i/(d^2*(a*1i -
 b)^2))^(1/2))/2 + (32*a^2*b^2*(a^2 + 13*b^2))/d^3)*(1i/(d^2*(a*1i - b)^2))
^(1/2))/2 + (32*tan(c + d*x)^(1/2)*(b^5 - 2*a^2*b^3))/d^4)*(1i/(d^2*(a*1i -
 b)^2))^(1/2))/2 - (32*a*b^3)/d^5)*(-1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2
))^(1/2))/2 - log(- ((((((((384*a*b^3*(a^2 + b^2)^2)/d + 256*b^3*tan(c + d
*x)^(1/2)*(a^2 - b^2)*(a^2 + b^2)^2*(1i/(d^2*(a*1i - b)^2))^(1/2))*(1i/(d^2
*(a*1i - b)^2))^(1/2))/2 + (64*a*b^2*tan(c + d*x)^(1/2)*(a^4 - 7*b^4 + 10*a
^2*b^2))/d^2)*(1i/(d^2*(a*1i - b)^2))^(1/2))/2 + (32*a^2*b^2*(a^2 + 13*b^2)
)/d^3)*(1i/(d^2*(a*1i - b)^2))^(1/2))/2 - (32*tan(c + d*x)^(1/2)*(b^5 - 2*a
^2*b^3))/d^4)*(1i/(d^2*(a*1i - b)^2))^(1/2))/2 - (32*a*b^3)/d^5)*(-1/(4*(b^
2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) - (atan((((-a*b)^(1/2)*((32*tan(
c + d*x)^(1/2)*(b^5 - 2*a^2*b^3))/d^4 - ((-a*b)^(1/2)*((32*(13*a^2*b^4*d^2
+ a^4*b^2*d^2))/d^5 + ((-a*b)^(1/2)*((32*tan(c + d*x)^(1/2)*(20*a^3*b^4*d^2
 - 14*a*b^6*d^2 + 2*a^5*b^2*d^2))/d^4 + ((-a*b)^(1/2)*((32*(12*a*b^7*d^4 +
24*a^3*b^5*d^4 + 12*a^5*b^3*d^4))/d^5 - (32*tan(c + d*x)^(1/2)*(-a*b)^(1/2)
*(16*b^9*d^4 + 16*a^2*b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/(d^5*(a^2
 + b^2))))/(d*(a^2 + b^2))))/(d*(a^2 + b^2))))/(d*(a^2 + b^2)))*1i)/(d*(a^2
 + b^2)) + ((-a*b)^(1/2)*((32*tan(c + d*x)^(1/2)*(b^5 - 2*a^2*b^3))/d^4 + (
(-a*b)^(1/2)*((32*(13*a^2*b^4*d^2 + a^4*b^2*d^2))/d^5 - ((-a*b)^(1/2)*((32*
tan(c + d*x)^(1/2)*(20*a^3*b^4*d^2 - 14*a*b^6*d^2 + 2*a^5*b^2*d^2))/d^4 - (
(-a*b)^(1/2)*((32*(12*a*b^7*d^4 + 24*a^3*b^5*d^4 + 12*a^5*b^3*d^4))/d^5 + (
32*tan(c + d*x)^(1/2)*(-a*b)^(1/2)*(16*b^9*d^4 + 16*a^2*b^7*d^4 - 16*a^4*b^
5*d^4 - 16*a^6*b^3*d^4))/(d^5*(a^2 + b^2))))/(d*(a^2 + b^2))))/(d*(a^2 + b^
2))))/(d*(a^2 + b^2)))*1i)/(d*(a^2 + b^2)))/((64*a*b^3)/d^5 - ((-a*b)^(1/2)
*((32*tan(c + d*x)^(1/2)*(b^5 - 2*a^2*b^3))/d^4 - ((-a*b)^(1/2)*((32*(13*a^
2*b^4*d^2 + a^4*b^2*d^2))/d^5 + ((-a*b)^(1/2)*((32*tan(c + d*x)^(1/2)*(20*a
^3*b^4*d^2 - 14*a*b^6*d^2 + 2*a^5*b^2*d^2))/d^4 + ((-a*b)^(1/2)*((32*(12*a*
b^7*d^4 + 24*a^3*b^5*d^4 + 12*a^5*b^3*d^4))/d^5 - (32*tan(c + d*x)^(1/2)*(-
a*b)^(1/2)*(16*b^9*d^4 + 16*a^2*b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4))
/(d^5*(a^2 + b^2))))/(d*(a^2 + b^2))))/(d*(a^2 + b^2))))/(d*(a^2 + b^2))))/
(d*(a^2 + b^2)) + ((-a*b)^(1/2)*((32*tan(c + d*x)^(1/2)*(b^5 - 2*a^2*b^3))/
d^4 + ((-a*b)^(1/2)*((32*(13*a^2*b^4*d^2 + a^4*b^2*d^2))/d^5 - ((-a*b)^(1/2
)*((32*tan(c + d*x)^(1/2)*(20*a^3*b^4*d^2 - 14*...
```

3.587 $\int \frac{1}{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))}\,dx$

Optimal. Leaf size=232

$$-\frac{(a-b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)d}+\frac{(a-b)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)d}+\frac{2b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{}}{}\right.}{\sqrt{a}\left(a^2+\right.}$$

```
[Out] 1/2*(a-b)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)+1/2*(a-b)
*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)-1/4*(a+b)*ln(1-2^(1
/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)/d*2^(1/2)+1/4*(a+b)*ln(1+2^(1/2)
*tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)/d*2^(1/2)+2*b^(3/2)*arctan(b^(1/2)*
tan(d*x+c)^(1/2)/a^(1/2))/(a^2+b^2)/d/a^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3655, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{(a-b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d\left(a^2+b^2\right)}+\frac{(a-b)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d\left(a^2+b^2\right)}+\frac{2b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d\left(a^2+b^2\right)}-\frac{(a+b)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)}+\frac{(a+b)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])),x]
```

```
[Out] -(((a - b)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)*d))
 + ((a - b)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)*d)
 + (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(Sqrt[a]*(a^2 +
 b^2)*d) - ((a + b)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*
Sqrt[2]*(a^2 + b^2)*d) + ((a + b)*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[
c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)/((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[(a + b*Tan[e + f*x])^m*
(c - d*Tan[e + f*x]), x], x] + Dist[d^2/(c^2 + d^2), Int[(a + b*Tan[e + f*x
])^m*((1 + Tan[e + f*x]^2)/(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{1}{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))}\,dx = \frac{\int \frac{a-b\tan(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{a^2+b^2} + \frac{b^2\int \frac{1+\tan^2(c+dx)}{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))}\,dx}{a^2+b^2}$$

$$= \frac{2\text{Subst}\left(\int \frac{a-bx^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{(a^2+b^2)\,d} + \frac{b^2\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(a+bx)}\,d \ldots\right.}{(a^2+b^2 \ldots}$$

$$= \frac{(a-b)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{(a^2+b^2)\,d} + \frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{a+b \ldots}\right.}{(a \ldots}$$

$$= \frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,(a^2+b^2)\,d} + \frac{(a-b)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2 \ldots}\right.}{2\,(a^2+ \ldots}$$

$$= \frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,(a^2+b^2)\,d} - \frac{(a+b)\log\left(1-\sqrt{2}\,\sqrt{\tan(\ldots}\right.}{2\sqrt{2}\,(a^2 \ldots}$$

$$= -\frac{(a-b)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} + \frac{(a-b)\tan^{-1}\left(1+ \ldots\right.}{\sqrt{2}\,(a^2 \ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.19, size = 225, normalized size = 0.97

$$\frac{-6\sqrt{2}\,a^{3/2}\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)+6\sqrt{2}\,a^{3/2}\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}\right)+24b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)-3\sqrt{2}\,a^{3/2}\log\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}+\tan(c+dx)\right)+3\sqrt{2}\,a^{3/2}\log\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}+\tan(c+dx)\right)-8\sqrt{a}\,b\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{12\sqrt{a}\,(a^2+b^2)\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])),x]
```

```
[Out] (-6*Sqrt[2]*a^(3/2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] + 6*Sqrt[2]*a^(3
/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + 24*b^(3/2)*ArcTan[(Sqrt[b]*Sqr
t[Tan[c + d*x]])/Sqrt[a]] - 3*Sqrt[2]*a^(3/2)*Log[1 - Sqrt[2]*Sqrt[Tan[c +
d*x]] + Tan[c + d*x]] + 3*Sqrt[2]*a^(3/2)*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]
] + Tan[c + d*x]] - 8*Sqrt[a]*b*Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d*x
]^2]*Tan[c + d*x]^(3/2))/(12*Sqrt[a]*(a^2 + b^2)*d)
```

Maple [A]
time = 0.12, size = 225, normalized size = 0.97

method	result
derivativedivides	$\frac{\frac{2b^2 \arctan\left(\frac{b\left(\sqrt{\tan}\,(dx+c)\right)}{\sqrt{ab}}\right)}{(a^2+b^2)\sqrt{ab}}+\frac{a\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}\,(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}\,(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan}\,(dx\right.\right.}{4}}{}$
default	$\frac{\frac{2b^2 \arctan\left(\frac{b\left(\sqrt{\tan}\,(dx+c)\right)}{\sqrt{ab}}\right)}{(a^2+b^2)\sqrt{ab}}+\frac{a\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}\,(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}\,(dx+c)\right)+\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan}\,(dx\right.\right.}{4}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b^2/(a^2+b^2)/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))+2/(
a^2+b^2)*(1/8*a*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1
/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*ar
ctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))-1/8*b*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^
(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/
2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))))
```

Maxima [A]
time = 0.50, size = 170, normalized size = 0.73

$$\frac{\frac{8\,b^2\arctan\left(\frac{b\sqrt{\tan\left(dx+c\right)}}{\sqrt{ab}}\right)}{(a^2+b^2)\sqrt{ab}}+\frac{2\sqrt{2}\,(a-b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(dx+c\right)}\right)\right)+2\sqrt{2}\,(a-b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(dx+c\right)}\right)\right)+\sqrt{2}\,(a+b)\log\left(\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan(dx+c)+1\right)-\sqrt{2}\,(a+b)\log\left(-\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan(dx+c)+1\right)}{a^2+b^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(8*b^2*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^2 + b^2)*sqrt(a*b)) +
 (2*sqrt(2)*(a - b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) +
```

```
2*sqrt(2)*(a - b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + s
qrt(2)*(a + b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)
*(a + b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a^2 + b^2))/
d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3599 vs. $2(194) = 388$.
time = 10.98, size = 7202, normalized size = 31.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(2)*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^5*sqrt((a^4 + 2*a^2*b
^2 + b^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4
)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^
6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)
)^(3/4)*arctan(((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^4*sqrt((a^4 - 2*a^2*b
^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a
^4 + 2*a^2*b^2 + b^4)*d^4)) + sqrt(2)*((a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3
*b^6 + a*b^8)*d^7*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^
4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + (a^6*b +
 3*a^4*b^3 + 3*a^2*b^5 + b^7)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^
6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))*sqrt((a^4 + 2*a^2*b^2 + b^4 - 2
*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^
4 - 2*a^2*b^2 + b^4))*sqrt(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt(1/((a^
4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) + sqrt(2)*((a^6*b - a^4*b^3 - a^2*b
^5 + b^7)*d^3*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) + (a^5 - 2
*a^3*b^2 + a*b^4)*d*cos(d*x + c))*sqrt((a^4 + 2*a^2*b^2 + b^4 - 2*(a^5*b +
2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*
b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4
))^(1/4) + (a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c))/cos(d*x + c))*(1/((a^4 + 2
*a^2*b^2 + b^4)*d^4))^(3/4) + sqrt(2)*((a^11 + 3*a^9*b^2 + 2*a^7*b^4 - 2*a^
5*b^6 - 3*a^3*b^8 - a*b^10)*d^7*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*
b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^
4)) + (a^8*b + 2*a^6*b^3 - 2*a^2*b^7 - b^9)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4
)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))*sqrt((a^4 + 2*a^2
*b^2 + b^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b
^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a^
4 + 2*a^2*b^2 + b^4)*d^4))^(3/4))/(a^4 - 2*a^2*b^2 + b^4)) + 4*sqrt(2)*(a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^5*sqrt((a^4 + 2*a^2*b^2 + b^4 - 2*(a^5*b
+ 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^
2*b^2 + b^4))*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
4*a^2*b^6 + b^8)*d^4))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4)*arctan(-((a^
```

```
8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^4*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4
*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^
4)*d^4)) - sqrt(2)*((a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*d^7*s
qrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8
)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + (a^6*b + 3*a^4*b^3 + 3*a^2*
b^5 + b^7)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
 4*a^2*b^6 + b^8)*d^4)))*sqrt((a^4 + 2*a^2*b^2 + b^4 - 2*(a^5*b + 2*a^3*b^3
 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4
))*sqrt(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4
)*d^4))*cos(d*x + c) - sqrt(2)*((a^6*b - a^4*b^3 - a^2*b^5 + b^7)*d^3*sqrt(
1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d
*cos(d*x + c))*sqrt((a^4 + 2*a^2*b^2 + b^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*
d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(si
n(d*x + c)/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(1/4) + (a^4 - 2
*a^2*b^2 + b^4)*sin(d*x + c))/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4
))^(3/4) - sqrt(2)*((a^11 + 3*a^9*b^2 + 2*a^7*b^4 - 2*a^5*b^6 - 3*a^3*b^8 -
 a*b^10)*d^7*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4
*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + (a^8*b + 2*a^
6*b^3 - 2*a^2*b^7 - b^9)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2
 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))*sqrt((a^4 + 2*a^2*b^2 + b^4 - 2*(a^5
*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2
*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4
)*d^4))^(3/4))/(a^4 - 2*a^2*b^2 + b^4)) + sqrt(2)*(2*(a^3*b + a*b^3)*d^3*sq
rt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + (a^2 + b^2)*d)*sqrt((a^4 + 2*a^2*b^2
+ b^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d
^4)))/(a^4 - 2*a^2*b^2 + b^4))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(1/4)*log(
((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*
cos(d*x + c) + sqrt(2)*((a^6*b - a^4*b^3 - a^2*b^5 + b^7)*d^3*sqrt(1/((a^4
+ 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x
 + c))*sqrt((a^4 + 2*a^2*b^2 + b^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt
(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x +
c)/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(1/4) + (a^4 - 2*a^2*b^2
 + b^4)*sin(d*x + c))/cos(d*x + c)) - sqrt(2)*(2*(a^3*b + a*b^3)*d^3*sqrt(1
/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + (a^2 + b^2)*d)*sqrt((a^4 + 2*a^2*b^2 + b^
4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))
)/(a^4 - 2*a^2*b^2 + b^4))*(1/((a^4 + 2*a^2*b^2...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\tan{(c + dx)})\sqrt{\tan{(c + dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(1/2)/(a+b*tan(d*x+c)),x)
```

[Out] Integral(1/((a + b*tan(c + d*x))*sqrt(tan(c + d*x))), x)

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x, algorithm="giac")

[Out] Timed out

Mupad [**B**]
time = 5.31, size = 2500, normalized size = 10.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(tan(c + d*x)^(1/2)*(a + b*tan(c + d*x))),x)

[Out] - atan(((((1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(((32*(16*b^8*d^2 + 28*a^2*b^6*d^2 + 8*a^4*b^4*d^2 - 4*a^6*b^2*d^2))/d^3 - (32*tan(c + d*x)^(1/2)*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^9*d^4 + 16*a^2*b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/d^4)*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*tan(c + d*x)^(1/2)*(4*a^3*b^4*d^2 - 30*a*b^6*d^2 + 2*a^5*b^2*d^2))/d^4) - (32*(5*a*b^5 + a^3*b^3))/d^3)*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (96*b^5*tan(c + d*x)^(1/2))/d^4)*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*1i - (((1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(((32*(16*b^8*d^2 + 28*a^2*b^6*d^2 + 8*a^4*b^4*d^2 - 4*a^6*b^2*d^2))/d^3 + (32*tan(c + d*x)^(1/2)*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^9*d^4 + 16*a^2*b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/d^4)*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*tan(c + d*x)^(1/2)*(4*a^3*b^4*d^2 - 30*a*b^6*d^2 + 2*a^5*b^2*d^2))/d^4) - (32*(5*a*b^5 + a^3*b^3))/d^3)*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (96*b^5*tan(c + d*x)^(1/2))/d^4)*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*1i)/((((1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(((32*(16*b^8*d^2 + 28*a^2*b^6*d^2 + 8*a^4*b^4*d^2 - 4*a^6*b^2*d^2))/d^3 - (32*tan(c + d*x)^(1/2)*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^9*d^4 + 16*a^2*b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/d^4)*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*tan(c + d*x)^(1/2)*(4*a^3*b^4*d^2 - 30*a*b^6*d^2 + 2*a^5*b^2*d^2))/d^4) - (32*(5*a*b^5 + a^3*b^3))/d^3)*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (96*b^5*tan(c + d*x)^(1/2))/d^4)*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (((1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(((32*(16*b^8*d^2 + 28*a^2*b^6*d^2 + 8*a^4*b^4*d^2 - 4*a^6*b^2*d^2))/d^3 + (32*tan(c + d*x)^(1/2)*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^9*d^4 + 16*a^2*b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/d^4)

```
*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*tan(c + d*x)^(1/2)*(
4*a^3*b^4*d^2 - 30*a*b^6*d^2 + 2*a^5*b^2*d^2))/d^4) - (32*(5*a*b^5 + a^3*b^
3))/d^3)*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (96*b^5*tan(c +
d*x)^(1/2))/d^4)*(1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)))*(1i/(4*(
b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*2i - atan((((1/(b^2*d^2*1i - a^2*d^
2*1i + 2*a*b*d^2))^(1/2)*(((1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*
(((((1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*((32*(16*b^8*d^2 + 28*a
^2*b^6*d^2 + 8*a^4*b^4*d^2 - 4*a^6*b^2*d^2))/d^3 - (16*tan(c + d*x)^(1/2)*(
1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*(16*b^9*d^4 + 16*a^2*b^7*d^4
 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/d^4))/2 - (32*tan(c + d*x)^(1/2)*(4*a^
3*b^4*d^2 - 30*a*b^6*d^2 + 2*a^5*b^2*d^2))/d^4)*(1/(b^2*d^2*1i - a^2*d^2*1i
 + 2*a*b*d^2))^(1/2))/2 - (32*(5*a*b^5 + a^3*b^3))/d^3))/2 - (96*b^5*tan(c
+ d*x)^(1/2))/d^4)*1i)/2 - ((1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)
*(((1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*(((((1/(b^2*d^2*1i - a^2
*d^2*1i + 2*a*b*d^2))^(1/2)*((32*(16*b^8*d^2 + 28*a^2*b^6*d^2 + 8*a^4*b^4*d
^2 - 4*a^6*b^2*d^2))/d^3 + (16*tan(c + d*x)^(1/2)*(1/(b^2*d^2*1i - a^2*d^2*
1i + 2*a*b*d^2))^(1/2)*(16*b^9*d^4 + 16*a^2*b^7*d^4 - 16*a^4*b^5*d^4 - 16*a
^6*b^3*d^4))/d^4))/2 + (32*tan(c + d*x)^(1/2)*(4*a^3*b^4*d^2 - 30*a*b^6*d^2
 + 2*a^5*b^2*d^2))/d^4)*(1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2))/2
- (32*(5*a*b^5 + a^3*b^3))/d^3))/2 + (96*b^5*tan(c + d*x)^(1/2))/d^4)*1i)/2
)/(((1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*(((1/(b^2*d^2*1i - a^2*
d^2*1i + 2*a*b*d^2))^(1/2)*(((((1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1
/2)*((32*(16*b^8*d^2 + 28*a^2*b^6*d^2 + 8*a^4*b^4*d^2 - 4*a^6*b^2*d^2))/d^3
 - (16*tan(c + d*x)^(1/2)*(1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*(
16*b^9*d^4 + 16*a^2*b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/d^4))/2 - (
32*tan(c + d*x)^(1/2)*(4*a^3*b^4*d^2 - 30*a*b^6*d^2 + 2*a^5*b^2*d^2))/d^4)*
(1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2))/2 - (32*(5*a*b^5 + a^3*b^3
))/d^3))/2 - (96*b^5*tan(c + d*x)^(1/2))/d^4))/2 + ((1/(b^2*d^2*1i - a^2*d^
2*1i + 2*a*b*d^2))^(1/2)*(((1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*
(((((1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*((32*(16*b^8*d^2 + 28*a
^2*b^6*d^2 + 8*a^4*b^4*d^2 - 4*a^6*b^2*d^2))/d^3 + (16*tan(c + d*x)^(1/2)*(
1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*(16*b^9*d^4 + 16*a^2*b^7*d^4
 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/d^4))/2 + (32*tan(c + d*x)^(1/2)*(4*a^
3*b^4*d^2 - 30*a*b^6*d^2 + 2*a^5*b^2*d^2))/d^4)*(1/(b^2*d^2*1i - a^2*d^2*1i
 + 2*a*b*d^2))^(1/2))/2 - (32*(5*a*b^5 + a^3*b^3))/d^3))/2 + (96*b^5*tan(c
+ d*x)^(1/2))/d^4))/2))*(1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*1i
- (atan((((-a*b^3)^(1/2)*((((32*(5*a*b^5 + a^3*b^3))/d^3 - ((-a*b^3)^(1/2)*
(((-a*b^3)^(1/2)*((32*(16*b^8*d^2 + 28*a^2*b^6*d^2 + 8*a^4*b^4*d^2 - 4*a^6*
b^2*d^2))/d^3 - (32*tan(c + d*x)^(1/2)*(-a*b^3)^(1/2)*(16*b^9*d^4 + 16*a^2*
b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/(d^4*(a^3*d + a*b^2*d))))/(a^3*
d + a*b^2*d) - (32*tan(c + d*x)^(1/2)*(4*a^3*b^...
```

3.588 $\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx$

Optimal. Leaf size=250

$$\frac{(a+b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}-\frac{(a+b)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}-\frac{2b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\text{t}}}{}\right.}{a^{3/2}\,(a^2+b}$$

```
[Out] -2*b^(5/2)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/a^(3/2)/(a^2+b^2)/d-1/2
*(a+b)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)-1/2*(a+b)*ar
ctan(1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)-1/4*(a-b)*ln(1-2^(1/2)
*tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)/d*2^(1/2)+1/4*(a-b)*ln(1+2^(1/2)*ta
n(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)/d*2^(1/2)-2/a/d/tan(d*x+c)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3650, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(a+b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d\,(a^2+b^2)}-\frac{(a+b)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d\,(a^2+b^2)}-\frac{(a-b)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)}+\frac{(a-b)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)}-\frac{2b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d\,(a^2+b^2)}-\frac{2}{ad\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])),x]
```

```
[Out] ((a + b)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)*d) -
((a + b)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)*d) -
(2*b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(a^(3/2)*(a^2 + b^
2)*d) - ((a - b)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqr
t[2]*(a^2 + b^2)*d) + ((a - b)*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
 d*x]])/(2*Sqrt[2]*(a^2 + b^2)*d) - 2/(a*d*Sqrt[Tan[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = -\frac{2}{ad\sqrt{\tan(c+dx)}} - \frac{2\int \frac{\frac{b}{2}+\frac{1}{2}a\tan(c+dx)+\frac{1}{2}b\tan^2(c+dx)}{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))}\,dx}{a}$$

$$= -\frac{2}{ad\sqrt{\tan(c+dx)}} - \frac{2\int \frac{\frac{ab}{2}+\frac{1}{2}a^2\tan(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{a\left(a^2+b^2\right)} - \frac{b^3\int \frac{1+\tan}{\sqrt{\tan(c+d}}}{a\left(a^2\right.}$$

$$= -\frac{2}{ad\sqrt{\tan(c+dx)}} - \frac{4\mathrm{Subst}\left(\int \frac{\frac{ab}{2}+\frac{a^2x^2}{2}}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{a\left(a^2+b^2\right)d}$$

$$= -\frac{2}{ad\sqrt{\tan(c+dx)}} + \frac{(a-b)\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{\left(a^2+b^2\right)d}$$

$$= -\frac{2b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}\left(a^2+b^2\right)d} - \frac{2}{ad\sqrt{\tan(c+dx)}} - \frac{(a-}{}$$

$$= -\frac{2b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}\left(a^2+b^2\right)d} - \frac{(a-b)\log\left(1-\sqrt{2}\sqrt{\tan}\right.}{2\sqrt{2}\left(a\right.}$$

$$= \frac{(a+b)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)d} - \frac{(a+b)\tan^{-1}\left(1+\sqrt{2}\right.}{\sqrt{2}\left(a^2+\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.55, size = 131, normalized size = 0.52

$$\frac{-(-1)^{3/4}(a+ib)\mathrm{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right) - \frac{2b^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}} + \sqrt[4]{-1}\,(ia+b)\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right) - \frac{2(a^2+b^2)}{a\sqrt{\tan(c+dx)}}}{\left(a^2+b^2\right)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])),x]
```

```
[Out] (-((-1)^(3/4)*(a + I*b)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]) - (2*b^(5/2)
*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/a^(3/2) + (-1)^(1/4)*(I*a +
b)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] - (2*(a^2 + b^2))/(a*Sqrt[Tan[c +
 d*x]]))/((a^2 + b^2)*d)
```

Maple [A]
time = 0.11, size = 241, normalized size = 0.96

method	result
derivativedivides	$\frac{-\frac{2b^3\arctan\left(\frac{b\left(\sqrt{\tan}(dx+c)\right)}{\sqrt{ab}}\right)}{a(a^2+b^2)\sqrt{ab}}-\frac{2}{a\sqrt{\tan(dx+c)}}+\frac{-\frac{b\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}\right)+2\,\mathrm{arct}}{}}{}}{}$
default	$\frac{-\frac{2b^3\arctan\left(\frac{b\left(\sqrt{\tan}(dx+c)\right)}{\sqrt{ab}}\right)}{a(a^2+b^2)\sqrt{ab}}-\frac{2}{a\sqrt{\tan(dx+c)}}+\frac{-\frac{b\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)+\tan(dx+c)}\right)+2\,\mathrm{arct}}{}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/a*b^3/(a^2+b^2)/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))-
2/a/tan(d*x+c)^(1/2)+2/(a^2+b^2)*(-1/8*b*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^
(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/
2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))-1/8*a*2^(1/2)*(
ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(
d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c
)^(1/2)))))
```

Maxima [A]
time = 0.50, size = 185, normalized size = 0.74

$$-\frac{\frac{8b^3\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^3+ab^2)\sqrt{ab}}+\frac{2\sqrt{2}(a+b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}(a+b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-\sqrt{2}(a-b)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+\sqrt{2}(a-b)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)}{a^2+b^2}+\frac{8}{a\sqrt{\tan(dx+c)}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(8*b^3*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^3 + a*b^2)*sqrt(a*b)
) + (2*sqrt(2)*(a + b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c))))
 + 2*sqrt(2)*(a + b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c))))
- sqrt(2)*(a - b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt
(2)*(a - b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a^2 + b^2
) + 8/(a*sqrt(tan(d*x + c))))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3872 vs. $2(210) = 420$.
time = 10.96, size = 7748, normalized size = 30.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x, algorithm="fricas")

[Out] [1/4*(4*sqrt(2)*((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*d^5*cos(d*x + c)^2 - (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*d^5)*sqrt((a^4 + 2*a^2*b^2 + b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4)*arctan(-((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^4*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - sqrt(2)*((a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*d^7*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))*sqrt((a^4 + 2*a^2*b^2 + b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) + sqrt(2)*((a^7 - a^5*b^2 - a^3*b^4 + a*b^6)*d^3*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) - (a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c))*sqrt((a^4 + 2*a^2*b^2 + b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(1/4) + (a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c))/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4) - sqrt(2)*((a^10*b + 3*a^8*b^3 + 2*a^6*b^5 - 2*a^4*b^7 - 3*a^2*b^9 - b^11)*d^7*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - (a^9 + 2*a^7*b^2 - 2*a^3*b^6 - a*b^8)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))*sqrt((a^4 + 2*a^2*b^2 + b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4))/(a^4 - 2*a^2*b^2 + b^4)) + 4*sqrt(2)*((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*d^5*cos(d*x + c)^2 - (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*d^5)*sqrt((a^4 + 2*a^2*b^2 + b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4)*arctan(((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^4*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + sqrt(2)*((a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*d^7*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))*sqrt((a^4 + 2*a^2*b^2 + b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(((a^6 - a^4*b^2

```
- a^2*b^4 + b^6)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) - s
qrt(2)*((a^7 - a^5*b^2 - a^3*b^4 + a*b^6)*d^3*sqrt(1/((a^4 + 2*a^2*b^2 + b^
4)*d^4))*cos(d*x + c) - (a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c))*sqrt((a^4
 + 2*a^2*b^2 + b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2
*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))
*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(1/4) + (a^4 - 2*a^2*b^2 + b^4)*sin(d*x
+ c))/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4) + sqrt(2)*((a^1
0*b + 3*a^8*b^3 + 2*a^6*b^5 - 2*a^4*b^7 - 3*a^2*b^9 - b^11)*d^7*sqrt((a^4 -
 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sq
rt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - (a^9 + 2*a^7*b^2 - 2*a^3*b^6 - a*b^8)
*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6
 + b^8)*d^4)))*sqrt((a^4 + 2*a^2*b^2 + b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*
d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(si
n(d*x + c)/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4))/(a^4 - 2*
a^2*b^2 + b^4)) + 8*(a^2 + b^2)*sqrt(sin(d*x + c)/cos(d*x + c))*cos(d*x + c
)*sin(d*x + c) + sqrt(2)*((a^3 + a*b^2)*d*cos(d*x + c)^2 - (a^3 + a*b^2)*d
- 2*((a^4*b + a^2*b^3)*d^3*cos(d*x + c)^2 - (a^4*b + a^2*b^3)*d^3)*sqrt(1/(
(a^4 + 2*a^2*b^2 + b^4)*d^4)))*sqrt((a^4 + 2*a^2*b^2 + b^4 + 2*(a^5*b + 2*a
^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2
 + b^4))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(1/4)*log(((a^6 - a^4*b^2 - a^2*
b^4 + b^6)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) + sqrt(2)
*((a^7 - a^5*b^2 - a^3*b^4 + a*b^6)*d^3*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4
))*cos(d*x + c) - (a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c))*sqrt((a^4 + 2*a
^2*b^2 + b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 +
 b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((
a^4 + 2*a^2*b^2 + b^4)*d^4))^(1/4) + (a^4 - 2*a...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\tan(c+dx))\tan^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(3/2)/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(1/((a + b*tan(c + d*x))*tan(c + d*x)**(3/2)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

[Out] Timed out

Mupad [B]
time = 5.33, size = 2500, normalized size = 10.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x))),x)

[Out] atan(((-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*((-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*((tan(c + d*x)^(1/2)*(512*a^8*b^8*d^7 - 448*a^10*b^6*d^7 + 128*a^12*b^4*d^7 + 64*a^14*b^2*d^7) - (-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(tan(c + d*x)^(1/2)*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(512*a^9*b^9*d^9 + 512*a^11*b^7*d^9 - 512*a^13*b^5*d^9 - 512*a^15*b^3*d^9) - 512*a^8*b^9*d^8 - 640*a^10*b^7*d^8 + 256*a^12*b^5*d^8 + 384*a^14*b^3*d^8))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + 128*a^7*b^8*d^6 - 32*a^11*b^4*d^6 - 32*a^13*b^2*d^6) + tan(c + d*x)^(1/2)*(64*a^7*b^7*d^5 - 32*a^9*b^5*d^5))*1i + (-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*((-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*((tan(c + d*x)^(1/2)*(512*a^8*b^8*d^7 - 448*a^10*b^6*d^7 + 128*a^12*b^4*d^7 + 64*a^14*b^2*d^7) - (-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(tan(c + d*x)^(1/2)*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(512*a^9*b^9*d^9 + 512*a^11*b^7*d^9 - 512*a^13*b^5*d^9 - 512*a^15*b^3*d^9) + 512*a^8*b^9*d^8 + 640*a^10*b^7*d^8 - 256*a^12*b^5*d^8 - 384*a^14*b^3*d^8))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - 128*a^7*b^8*d^6 + 32*a^11*b^4*d^6 + 32*a^13*b^2*d^6) + tan(c + d*x)^(1/2)*(64*a^7*b^7*d^5 - 32*a^9*b^5*d^5))*1i)/((-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*((-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*((tan(c + d*x)^(1/2)*(512*a^8*b^8*d^7 - 448*a^10*b^6*d^7 + 128*a^12*b^4*d^7 + 64*a^14*b^2*d^7) - (-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(tan(c + d*x)^(1/2)*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(512*a^9*b^9*d^9 + 512*a^11*b^7*d^9 - 512*a^13*b^5*d^9 - 512*a^15*b^3*d^9) - 512*a^8*b^9*d^8 - 640*a^10*b^7*d^8 + 256*a^12*b^5*d^8 + 384*a^14*b^3*d^8))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + 128*a^7*b^8*d^6 - 32*a^11*b^4*d^6 - 32*a^13*b^2*d^6) + tan(c + d*x)^(1/2)*(64*a^7*b^7*d^5 - 32*a^9*b^5*d^5)) - (-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*((-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*((tan(c + d*x)^(1/2)*(512*a^8*b^8*d^7 - 448*a^10*b^6*d^7 + 128*a^12*b^4*d^7 + 64*a^14*b^2*d^7) - (-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(tan(c + d*x)^(1/2)*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(512*a^9*b^9*d^9 + 512*a^11*b^7*d^9 - 512*a^13*b^5*d^9 - 512*a^15*b^3*d^9) + 512*a^8*b^9*d^8 + 640*a^10*b^7*d^8 - 256*a^12*b^5*d^8 - 384*a^14*b^3*d^8))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - 128*a^7*b^8*d^6 + 32*a^11*b^4*d^6 + 32*a^13*b^2*d^6) + tan(c + d*x)^(1/2)*(64*a^7*b^7*d^5 - 32*a^9*b^5*d^5))))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*2i + (log(((-1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)

)^(1/2)*(((tan(c + d*x)^(1/2)*(512*a^8*b^8*d^7 - 448*a^10*b^6*d^7 + 128*a^1
2*b^4*d^7 + 64*a^14*b^2*d^7) - ((-1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^
(1/2)*((tan(c + d*x)^(1/2)*(-1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)
*(512*a^9*b^9*d^9 + 512*a^11*b^7*d^9 - 512*a^13*b^5*d^9 - 512*a^15*b^3*d^9)
)/2 + 512*a^8*b^9*d^8 + 640*a^10*b^7*d^8 - 256*a^12*b^5*d^8 - 384*a^14*b^3*
d^8))/2)*(-1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2))/2 - 128*a^7*b^8*
d^6 + 32*a^11*b^4*d^6 + 32*a^13*b^2*d^6))/2 + tan(c + d*x)^(1/2)*(64*a^7*b^
7*d^5 - 32*a^9*b^5*d^5))*(-1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2))/
2 - log((-1/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2)*((tan(c + d*x)
^(1/2)*(512*a^8*b^8*d^7 - 448*a^10*b^6*d^7 + 128*a^12*b^4*d^7 + 64*a^14*b^2
d^7) - (-1/(4(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2)*(tan(c + d*x)^
(1/2)*(-1/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2)*(512*a^9*b^9*d^9
 + 512*a^11*b^7*d^9 - 512*a^13*b^5*d^9 - 512*a^15*b^3*d^9) - 512*a^8*b^9*d^
8 - 640*a^10*b^7*d^8 + 256*a^12*b^5*d^8 + 384*a^14*b^3*d^8))*(-1/(4*(b^2*d^
2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) + 128*a^7*b^8*d^6 - 32*a^11*b^4*d^6
- 32*a^13*b^2*d^6) + tan(c + d*x)^(1/2)*(64*a^7*b^7*d^5 - 32*a^9*b^5*d^5))*
(-1/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) - 2/(a*d*tan(c + d*x)^
(1/2)) - (atan((((tan(c + d*x)^(1/2)*(64*a^7*b^7*d^5 - 32*a^9*b^5*d^5) + ((
-a^3*b^5)^(1/2)*(32*a^11*b^4*d^6 - 128*a^7*b^8*d^6 + 32*a^13*b^2*d^6 + ((ta
n(c + d*x)^(1/2)*(512*a^8*b^8*d^7 - 448*a^10*b^6*d^7 + 128*a^12*b^4*d^7 + 6
4*a^14*b^2*d^7) - ((-a^3*b^5)^(1/2)*(512*a^8*b^9*d^8 + 640*a^10*b^7*d^8 - 2
56*a^12*b^5*d^8 - 384*a^14*b^3*d^8 + (tan(c + d*x)^(1/2)*(-a^3*b^5)^(1/2)*(
512*a^9*b^9*d^9 + 512*a^11*b^7*d^9 - 512*a^13*b^5*d^9 - 512*a^15*b^3*d^9))/
(a^3*d*(a^2 + b^2))))/(a^3*d*(a^2 + b^2)))*(-a^3*b^5)^(1/2))/(a^3*d*(a^2 +
b^2))))/(a^3*d*(a^2 + b^2)))*(-a^3*b^5)^(1/2)*1i)/(a^3*d*(a^2 + b^2)) + ((t
an(c + d*x)^(1/2)*(64*a^7*b^7*d^5 - 32*a^9*b^5*d^5) + ((-a^3*b^5)^(1/2)*(12
8*a^7*b^8*d^6 - 32*a^11*b^4*d^6 - 32*a^13*b^2*d^6 + ((tan(c + d*x)^(1/2)*(5
12*a^8*b^8*d^7 - 448*a^10*b^6*d^7 + 128*a^12*b^4*d^7 + 64*a^14*b^2*d^7) - (
(-a^3*b^5)^(1/2)*(256*a^12*b^5*d^8 - 640*a^10*b^7*d^8 - 512*a^8*b^9*d^8 + 3
84*a^14*b^3*d^8 + (tan(c + d*x)^(1/2)*(-a^3*b^5)^(1/2)*(512*a^9*b^9*d^9 + 5
12*a^11*b^7*d^9 - 512*a^13*b^5*d^9 - 512*a^15*b^3*d^9))/(a^3*d*(a^2 + b^2))
))/(a^3*d*(a^2 + b^2)))*(-a^3*b^5)^(1/2))/(a^3*...

3.589 $\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))} dx$

Optimal. Leaf size=271

$$\frac{(a-b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}(a^2+b^2)d}-\frac{(a-b)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}(a^2+b^2)d}+\frac{2b^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\ldots}}{\ldots}\right)}{a^{5/2}(a^2+\ldots}$$

```
[Out] 2*b^(7/2)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/a^(5/2)/(a^2+b^2)/d-1/2*
(a-b)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)-1/2*(a-b)*arc
tan(1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)+1/4*(a+b)*ln(1-2^(1/2)*
tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)/d*2^(1/2)-1/4*(a+b)*ln(1+2^(1/2)*tan
(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)/d*2^(1/2)+2*b/a^2/d/tan(d*x+c)^(1/2)-2/
3/a/d/tan(d*x+c)^(3/2)
```

Rubi [A]
time = 0.40, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3650, 3730, 3735, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(a-b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d(a^2+b^2)}-\frac{(a-b)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d(a^2+b^2)}+\frac{(a+b)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d(a^2+b^2)}-\frac{(a+b)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d(a^2+b^2)}+\frac{2b}{a^2d\sqrt{\tan(c+dx)}}+\frac{2b^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d(a^2+b^2)}-\frac{2}{3ad\tan^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])),x]
```

```
[Out] ((a - b)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)*d) -
((a - b)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)*d) +
(2*b^(7/2)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(a^(5/2)*(a^2 + b^
2)*d) + ((a + b)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqr
t[2]*(a^2 + b^2)*d) - ((a + b)*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
 d*x]])/(2*Sqrt[2]*(a^2 + b^2)*d) - 2/(3*a*d*Tan[c + d*x]^(3/2)) + (2*b)/(a
^2*d*Sqrt[Tan[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
```

```
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3735

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dis
t[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*Simp[a*(A - C) - (A*b - b*C)*Ta
n[e + f*x], x], x], x] + Dist[(A*b^2 + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e
 + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a,
 b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0] &&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = -\frac{2}{3ad\tan^{\frac{3}{2}}(c+dx)} - \frac{2\int \frac{\frac{3b}{2}+\frac{3}{2}a\tan(c+dx)+\frac{3}{2}b\tan^2(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx}{3a}$$

$$= -\frac{2}{3ad\tan^{\frac{3}{2}}(c+dx)} + \frac{2b}{a^2d\sqrt{\tan(c+dx)}} + \frac{4\int \frac{-\frac{3}{4}(a^2-b^2)+\frac{3}{4}b^2\tan^2}{\sqrt{\tan(c+dx)}\,(a+b}}{3a^2}$$

$$= -\frac{2}{3ad\tan^{\frac{3}{2}}(c+dx)} + \frac{2b}{a^2d\sqrt{\tan(c+dx)}} + \frac{4\int \frac{-\frac{3a^3}{4}+\frac{3}{4}a^2b\tan(c+dx)}{\sqrt{\tan(c+dx)}}\,d}{3a^2(a^2+b^2)}$$

$$= -\frac{2}{3ad\tan^{\frac{3}{2}}(c+dx)} + \frac{2b}{a^2d\sqrt{\tan(c+dx)}} + \frac{8\text{Subst}\left(\int \frac{-\frac{3a^3}{4}+\frac{3}{4}a^2bx^2}{1+x^4}\right.}{3a^2(a^2}$$

$$= -\frac{2}{3ad\tan^{\frac{3}{2}}(c+dx)} + \frac{2b}{a^2d\sqrt{\tan(c+dx)}} - \frac{(a-b)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx\right.}{(a^2+}$$

$$= \frac{2b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}(a^2+b^2)d} - \frac{2}{3ad\tan^{\frac{3}{2}}(c+dx)} + \frac{2}{a^2d\sqrt{\tan}}$$

$$= \frac{2b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}(a^2+b^2)d} + \frac{(a+b)\log\left(1-\sqrt{2}\sqrt{\tan(c}\right.}{2\sqrt{2}(a^2+}$$

$$= \frac{(a-b)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}(a^2+b^2)d} - \frac{(a-b)\tan^{-1}\left(1+\sqrt{2}\right.}{\sqrt{2}(a^2+b}$$

Mathematica [A]

time = 1.89, size = 222, normalized size = 0.82

$$\frac{\frac{6\sqrt{2}a^2(a-b)\left(\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\right)}{a^2+b^2}+\frac{24b^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}(a^2+b^2)}+\frac{3\sqrt{2}a^2(a+b)\left(\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)-\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\right)}{a^2+b^2}-\frac{8a}{\tan^{\frac{3}{2}}(c+dx)}+\frac{24b}{\sqrt{\tan(c+dx)}}}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])),x]
```

```
[Out] ((6*Sqrt[2]*a^2*(a - b)*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - ArcTan[1
+ Sqrt[2]*Sqrt[Tan[c + d*x]]]))/(a^2 + b^2) + (24*b^(7/2)*ArcTan[(Sqrt[b]*S
qrt[Tan[c + d*x]])/Sqrt[a]])/(Sqrt[a]*(a^2 + b^2)) + (3*Sqrt[2]*a^2*(a + b)
*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt
```

```
[Tan[c + d*x]] + Tan[c + d*x]]))/(a^2 + b^2) - (8*a)/Tan[c + d*x]^(3/2) + (
24*b)/Sqrt[Tan[c + d*x]])/(12*a^2*d)
```

Maple [A]
time = 0.12, size = 255, normalized size = 0.94

method	result
derivativedivides	$-\frac{2}{3a\tan(dx+c)^{\frac{3}{2}}}+\frac{2b}{a^2\sqrt{\tan(dx+c)}}+\frac{2b^4\arctan\left(\frac{b\left(\sqrt{\tan}(dx+c)\right)}{\sqrt{ab}}\right)}{a^2(a^2+b^2)\sqrt{ab}}+\frac{-a\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)\cdots}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)\cdots}\right)\cdots\right.}{\cdots}$
default	$-\frac{2}{3a\tan(dx+c)^{\frac{3}{2}}}+\frac{2b}{a^2\sqrt{\tan(dx+c)}}+\frac{2b^4\arctan\left(\frac{b\left(\sqrt{\tan}(dx+c)\right)}{\sqrt{ab}}\right)}{a^2(a^2+b^2)\sqrt{ab}}+\frac{-a\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)\cdots}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)\cdots}\right)\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/3/a/tan(d*x+c)^(3/2)+2*b/a^2/tan(d*x+c)^(1/2)+2/a^2*b^4/(a^2+b^2)/(
a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))+2/(a^2+b^2)*(-1/8*a*2^(1/
2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+
tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d
*x+c)^(1/2)))+1/8*b*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+
2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+
2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))))
```

Maxima [A]
time = 0.51, size = 201, normalized size = 0.74

$$\frac{\frac{24\,b^4\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^4+a^2b^2)\sqrt{ab}}-\frac{3\left(2\sqrt{2}(a-b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}(a-b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)+\sqrt{2}(a+b)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)-\sqrt{2}(a+b)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)\right)}{a^2+b^2}+\frac{8\,(3\,b\tan(dx+c)-a)}{a^2\tan(dx+c)^{\frac{3}{2}}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(24*b^4*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^4 + a^2*b^2)*sqrt(a
*b)) - 3*(2*sqrt(2)*(a - b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x +
c)))) + 2*sqrt(2)*(a - b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c
)))) + sqrt(2)*(a + b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) -
 sqrt(2)*(a + b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a^2
+ b^2) + 8*(3*b*tan(d*x + c) - a)/(a^2*tan(d*x + c)^(3/2)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3908 vs. $2(227) = 454$.

time = 11.56, size = 7820, normalized size = 28.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/12*(12*sqrt(2)*((a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*d^5*cos(d*x + c
)^2 - (a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*d^5)*sqrt((a^4 + 2*a^2*b^2 +
b^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4
)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2
 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/
4)*arctan(((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^4*sqrt((a^4 - 2*a^2*b^2 +
b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 +
2*a^2*b^2 + b^4)*d^4)) + sqrt(2)*((a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6
+ a*b^8)*d^7*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4
*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + (a^6*b + 3*a^
4*b^3 + 3*a^2*b^5 + b^7)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2
 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))*sqrt((a^4 + 2*a^2*b^2 + b^4 - 2*(a^5
*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2
*a^2*b^2 + b^4))*sqrt(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt(1/((a^4 + 2
*a^2*b^2 + b^4)*d^4))*cos(d*x + c) + sqrt(2)*((a^6*b - a^4*b^3 - a^2*b^5 +
b^7)*d^3*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) + (a^5 - 2*a^3*
b^2 + a*b^4)*d*cos(d*x + c))*sqrt((a^4 + 2*a^2*b^2 + b^4 - 2*(a^5*b + 2*a^3
*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 +
 b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(1
/4) + (a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c))/cos(d*x + c))*(1/((a^4 + 2*a^2*
b^2 + b^4)*d^4))^(3/4) + sqrt(2)*((a^11 + 3*a^9*b^2 + 2*a^7*b^4 - 2*a^5*b^6
 - 3*a^3*b^8 - a*b^10)*d^7*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 +
 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) +
 (a^8*b + 2*a^6*b^3 - 2*a^2*b^7 - b^9)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a
^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))*sqrt((a^4 + 2*a^2*b^2
+ b^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d
^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a^4 + 2
*a^2*b^2 + b^4)*d^4))^(3/4))/(a^4 - 2*a^2*b^2 + b^4)) + 12*sqrt(2)*((a^8 +
3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*d^5*cos(d*x + c)^2 - (a^8 + 3*a^6*b^2 + 3*
a^4*b^4 + a^2*b^6)*d^5)*sqrt((a^4 + 2*a^2*b^2 + b^4 - 2*(a^5*b + 2*a^3*b^3
+ a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4)
)*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 +
b^8)*d^4))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4)*arctan(-((a^8 + 2*a^6*b^
2 - 2*a^2*b^6 - b^8)*d^4*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6
*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - s
qrt(2)*((a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*d^7*sqrt((a^4 - 2
*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt
(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*d
```

```
^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 +
 b^8)*d^4)))*sqrt((a^4 + 2*a^2*b^2 + b^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^
2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(((a^
6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(
d*x + c) - sqrt(2)*((a^6*b - a^4*b^3 - a^2*b^5 + b^7)*d^3*sqrt(1/((a^4 + 2*
a^2*b^2 + b^4)*d^4))*cos(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c
))*sqrt((a^4 + 2*a^2*b^2 + b^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/(
(a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/c
os(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(1/4) + (a^4 - 2*a^2*b^2 + b
^4)*sin(d*x + c))/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4) - s
qrt(2)*((a^11 + 3*a^9*b^2 + 2*a^7*b^4 - 2*a^5*b^6 - 3*a^3*b^8 - a*b^10)*d^7
*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b
^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + (a^8*b + 2*a^6*b^3 - 2*a^
2*b^7 - b^9)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4
 + 4*a^2*b^6 + b^8)*d^4)))*sqrt((a^4 + 2*a^2*b^2 + b^4 - 2*(a^5*b + 2*a^3*b
^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b
^4))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4
))/(a^4 - 2*a^2*b^2 + b^4)) + 3*sqrt(2)*((a^4 + a^2*b^2)*d*cos(d*x + c)^2 -
 (a^4 + a^2*b^2)*d + 2*((a^5*b + a^3*b^3)*d^3*cos(d*x + c)^2 - (a^5*b + a^3
*b^3)*d^3)*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))*sqrt((a^4 + 2*a^2*b^2 + b
^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)
))/(a^4 - 2*a^2*b^2 + b^4))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(1/4)*log(((a
^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos
(d*x + c) + sqrt(2)*((a^6*b - a^4*b^3 - a^2*b^5 + b^7)*d^3*sqrt(1/((a^4 + 2
*a^2*b^2 + b^4)*d^4))*cos(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x +
c))*sqrt((a^4 + 2*a^2*b^2 + b^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/
((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/
cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(1/4) + (a^4 - 2*a^2*b^2 +
b^4)*sin(d*x + c))/cos(d*x + c)) - 3*sqrt(2)*((...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\tan(c + dx))\tan^{\frac{5}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(5/2)/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(1/((a + b*tan(c + d*x))*tan(c + d*x)**(5/2)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 5.90, size = 2500, normalized size = 9.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(5/2)*(a + b*tan(c + d*x))),x)
```

```
[Out] (atan((((-a^5*b^7)^(1/2)*(tan(c + d*x)^(1/2)*(64*a^14*b^9*d^5 + 32*a^18*b^5
*d^5) - ((-a^5*b^7)^(1/2)*(32*a^19*b^5*d^6 - 384*a^15*b^9*d^6 + 32*a^21*b^3
*d^6 + ((tan(c + d*x)^(1/2)*(512*a^15*b^10*d^7 + 448*a^19*b^6*d^7 - 128*a^2
1*b^4*d^7 - 64*a^23*b^2*d^7) + ((-a^5*b^7)^(1/2)*(512*a^16*b^10*d^8 + 512*a
^18*b^8*d^8 - 384*a^20*b^6*d^8 - 256*a^22*b^4*d^8 + 128*a^24*b^2*d^8 - (tan
(c + d*x)^(1/2)*(-a^5*b^7)^(1/2)*(512*a^18*b^9*d^9 + 512*a^20*b^7*d^9 - 512
*a^22*b^5*d^9 - 512*a^24*b^3*d^9))/(a^5*d*(a^2 + b^2))))/(a^5*d*(a^2 + b^2)
))*(-a^5*b^7)^(1/2))/(a^5*d*(a^2 + b^2))))/(a^5*d*(a^2 + b^2)))*1i)/(a^5*d*
(a^2 + b^2)) + ((-a^5*b^7)^(1/2)*(tan(c + d*x)^(1/2)*(64*a^14*b^9*d^5 + 32*
a^18*b^5*d^5) - ((-a^5*b^7)^(1/2)*(384*a^15*b^9*d^6 - 32*a^19*b^5*d^6 - 32*
a^21*b^3*d^6 + ((tan(c + d*x)^(1/2)*(512*a^15*b^10*d^7 + 448*a^19*b^6*d^7 -
 128*a^21*b^4*d^7 - 64*a^23*b^2*d^7) - ((-a^5*b^7)^(1/2)*(512*a^16*b^10*d^8
 + 512*a^18*b^8*d^8 - 384*a^20*b^6*d^8 - 256*a^22*b^4*d^8 + 128*a^24*b^2*d^
8 + (tan(c + d*x)^(1/2)*(-a^5*b^7)^(1/2)*(512*a^18*b^9*d^9 + 512*a^20*b^7*d
^9 - 512*a^22*b^5*d^9 - 512*a^24*b^3*d^9))/(a^5*d*(a^2 + b^2))))/(a^5*d*(a^
2 + b^2)))*(-a^5*b^7)^(1/2))/(a^5*d*(a^2 + b^2))))/(a^5*d*(a^2 + b^2)))*1i)
/(a^5*d*(a^2 + b^2)))/(64*a^14*b^8*d^4 - ((-a^5*b^7)^(1/2)*(tan(c + d*x)^(1
/2)*(64*a^14*b^9*d^5 + 32*a^18*b^5*d^5) - ((-a^5*b^7)^(1/2)*(32*a^19*b^5*d^
6 - 384*a^15*b^9*d^6 + 32*a^21*b^3*d^6 + ((tan(c + d*x)^(1/2)*(512*a^15*b^1
0*d^7 + 448*a^19*b^6*d^7 - 128*a^21*b^4*d^7 - 64*a^23*b^2*d^7) + ((-a^5*b^7
)^(1/2)*(512*a^16*b^10*d^8 + 512*a^18*b^8*d^8 - 384*a^20*b^6*d^8 - 256*a^22
*b^4*d^8 + 128*a^24*b^2*d^8 - (tan(c + d*x)^(1/2)*(-a^5*b^7)^(1/2)*(512*a^1
8*b^9*d^9 + 512*a^20*b^7*d^9 - 512*a^22*b^5*d^9 - 512*a^24*b^3*d^9))/(a^5*d
*(a^2 + b^2))))/(a^5*d*(a^2 + b^2)))*(-a^5*b^7)^(1/2))/(a^5*d*(a^2 + b^2)))
)/(a^5*d*(a^2 + b^2))))/(a^5*d*(a^2 + b^2)) + ((-a^5*b^7)^(1/2)*(tan(c + d*
x)^(1/2)*(64*a^14*b^9*d^5 + 32*a^18*b^5*d^5) - ((-a^5*b^7)^(1/2)*(384*a^15*
b^9*d^6 - 32*a^19*b^5*d^6 - 32*a^21*b^3*d^6 + ((tan(c + d*x)^(1/2)*(512*a^1
5*b^10*d^7 + 448*a^19*b^6*d^7 - 128*a^21*b^4*d^7 - 64*a^23*b^2*d^7) - ((-a^
5*b^7)^(1/2)*(512*a^16*b^10*d^8 + 512*a^18*b^8*d^8 - 384*a^20*b^6*d^8 - 256
*a^22*b^4*d^8 + 128*a^24*b^2*d^8 + (tan(c + d*x)^(1/2)*(-a^5*b^7)^(1/2)*(51
2*a^18*b^9*d^9 + 512*a^20*b^7*d^9 - 512*a^22*b^5*d^9 - 512*a^24*b^3*d^9))/(
a^5*d*(a^2 + b^2))))/(a^5*d*(a^2 + b^2)))*(-a^5*b^7)^(1/2))/(a^5*d*(a^2 + b
^2))))/(a^5*d*(a^2 + b^2))))/(a^5*d*(a^2 + b^2))))*(-a^5*b^7)^(1/2)*2i)/(a^
5*d*(a^2 + b^2)) - atan(((((1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*
```

((((tan(c + d*x)^(1/2)*(512*a^15*b^10*d^7 + 448*a^19*b^6*d^7 - 128*a^21*b^4
*d^7 - 64*a^23*b^2*d^7))/2 + ((1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/
2)*(256*a^16*b^10*d^8 + 256*a^18*b^8*d^8 - 192*a^20*b^6*d^8 - 128*a^22*b^4*
d^8 + 64*a^24*b^2*d^8 - (tan(c + d*x)^(1/2)*(1/(b^2*d^2*1i - a^2*d^2*1i + 2
*a*b*d^2))^(1/2)*(512*a^18*b^9*d^9 + 512*a^20*b^7*d^9 - 512*a^22*b^5*d^9 -
512*a^24*b^3*d^9))/4))/2)*(1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2))/
2 - 192*a^15*b^9*d^6 + 16*a^19*b^5*d^6 + 16*a^21*b^3*d^6))/2 - (tan(c + d*x
)^(1/2)*(64*a^14*b^9*d^5 + 32*a^18*b^5*d^5))/2)*(1/(b^2*d^2*1i - a^2*d^2*1i
 + 2*a*b*d^2))^(1/2)*1i + (((1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)
*((((tan(c + d*x)^(1/2)*(512*a^15*b^10*d^7 + 448*a^19*b^6*d^7 - 128*a^21*b^
4*d^7 - 64*a^23*b^2*d^7))/2 - ((1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1
/2)*(256*a^16*b^10*d^8 + 256*a^18*b^8*d^8 - 192*a^20*b^6*d^8 - 128*a^22*b^4
*d^8 + 64*a^24*b^2*d^8 + (tan(c + d*x)^(1/2)*(1/(b^2*d^2*1i - a^2*d^2*1i +
2*a*b*d^2))^(1/2)*(512*a^18*b^9*d^9 + 512*a^20*b^7*d^9 - 512*a^22*b^5*d^9 -
 512*a^24*b^3*d^9))/4))/2)*(1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2))
/2 + 192*a^15*b^9*d^6 - 16*a^19*b^5*d^6 - 16*a^21*b^3*d^6))/2 - (tan(c + d*
x)^(1/2)*(64*a^14*b^9*d^5 + 32*a^18*b^5*d^5))/2)*(1/(b^2*d^2*1i - a^2*d^2*1
i + 2*a*b*d^2))^(1/2)*1i)/((((1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2
)*((((tan(c + d*x)^(1/2)*(512*a^15*b^10*d^7 + 448*a^19*b^6*d^7 - 128*a^21*b
^4*d^7 - 64*a^23*b^2*d^7))/2 + ((1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(
1/2)*(256*a^16*b^10*d^8 + 256*a^18*b^8*d^8 - 192*a^20*b^6*d^8 - 128*a^22*b^
4*d^8 + 64*a^24*b^2*d^8 - (tan(c + d*x)^(1/2)*(1/(b^2*d^2*1i - a^2*d^2*1i +
 2*a*b*d^2))^(1/2)*(512*a^18*b^9*d^9 + 512*a^20*b^7*d^9 - 512*a^22*b^5*d^9
- 512*a^24*b^3*d^9))/4))/2)*(1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)
)/2 - 192*a^15*b^9*d^6 + 16*a^19*b^5*d^6 + 16*a^21*b^3*d^6))/2 - (tan(c + d
x)^(1/2)(64*a^14*b^9*d^5 + 32*a^18*b^5*d^5))/2)*(1/(b^2*d^2*1i - a^2*d^2*
1i + 2*a*b*d^2))^(1/2) - (((1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*
((((tan(c + d*x)^(1/2)*(512*a^15*b^10*d^7 + 448*a^19*b^6*d^7 - 128*a^21*b^4
*d^7 - 64*a^23*b^2*d^7))/2 - ((1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/
2)*(256*a^16*b^10*d^8 + 256*a^18*b^8*d^8 - 192*a^20*b^6*d^8 - 128*a^22*b^4*
d^8 + 64*a^24*b^2*d^8 + (tan(c + d*x)^(1/2)*(1/(b^2*d^2*1i - a^2*d^2*1i + 2
*a*b*d^2))^(1/2)*(512*a^18*b^9*d^9 + 512*a^20*b^7*d^9 - 512*a^22*b^5*d^9 -
512*a^24*b^3*d^9))/4))/2)*(1/(b^2*d^2*1i - a^2*...

3.590 $\int \frac{1}{\tan^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))}\,dx$

Optimal. Leaf size=300

$$-\frac{(a+b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}+\frac{(a+b)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}-\frac{2b^{9/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{}}{}\right.}{a^{7/2}\,(a^2+}$$

```
[Out] -2*b^(9/2)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/a^(7/2)/(a^2+b^2)/d+1/2
*(a+b)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)+1/2*(a+b)*ar
ctan(1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)+1/4*(a-b)*ln(1-2^(1/2)
*tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)/d*2^(1/2)-1/4*(a-b)*ln(1+2^(1/2)*ta
n(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)/d*2^(1/2)+2*(a^2-b^2)/a^3/d/tan(d*x+c)
^(1/2)-2/5/a/d/tan(d*x+c)^(5/2)+2/3*b/a^2/d/tan(d*x+c)^(3/2)
```

Rubi [A]
time = 0.54, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3650, 3730, 3731, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{(a+b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d\,(a^2+b^2)}+\frac{(a+b)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d\,(a^2+b^2)}+\frac{(a-b)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)}-\frac{(a-b)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)}+\frac{2b}{3a^2d\tan^{\frac{3}{2}}(c+dx)}-\frac{2b^{9/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^{7/2}d\,(a^2+b^2)}+\frac{2(a^2-b^2)}{a^3d\sqrt{\tan(c+dx)}}-\frac{2}{5ad\tan^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(7/2)*(a + b*Tan[c + d*x])),x]
```

```
[Out] -(((a + b)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)*d))
 + ((a + b)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)*d)
 - (2*b^(9/2)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(a^(7/2)*(a^2 +
 b^2)*d) + ((a - b)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*
Sqrt[2]*(a^2 + b^2)*d) - ((a - b)*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[
c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)*d) - 2/(5*a*d*Tan[c + d*x]^(5/2)) + (2*b)
/(3*a^2*d*Tan[c + d*x]^(3/2)) + (2*(a^2 - b^2))/(a^3*d*Sqrt[Tan[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
```

```
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3731

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)
*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp
[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - a*C*(b*c*(m + 1) + a*d*(n
+ 1)) - (m + 1)*(b*c - a*d)*(A*b - b*C)*Tan[e + f*x] - d*(A*b^2 + a^2*C)*(m
 + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, n},
```

```
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m,
 -1] &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\tan^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = -\frac{2}{5ad\tan^{\frac{5}{2}}(c+dx)} - \frac{2\int \frac{\frac{5b}{2}+\frac{5}{2}a\tan(c+dx)+\frac{5}{2}b\tan^2(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx}{5a}$$

$$= -\frac{2}{5ad\tan^{\frac{5}{2}}(c+dx)} + \frac{2b}{3a^2d\tan^{\frac{3}{2}}(c+dx)} + \frac{4\int \frac{-\frac{15}{4}(a^2-b^2)+\frac{15}{4}b^2\tan^2(c}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+}}{15a^2}$$

$$= -\frac{2}{5ad\tan^{\frac{5}{2}}(c+dx)} + \frac{2b}{3a^2d\tan^{\frac{3}{2}}(c+dx)} + \frac{2(a^2-b^2)}{a^3d\sqrt{\tan(c+dx)}} - \frac{8}{-}$$

$$= -\frac{2}{5ad\tan^{\frac{5}{2}}(c+dx)} + \frac{2b}{3a^2d\tan^{\frac{3}{2}}(c+dx)} + \frac{2(a^2-b^2)}{a^3d\sqrt{\tan(c+dx)}} - \frac{8}{-}$$

$$= -\frac{2}{5ad\tan^{\frac{5}{2}}(c+dx)} + \frac{2b}{3a^2d\tan^{\frac{3}{2}}(c+dx)} + \frac{2(a^2-b^2)}{a^3d\sqrt{\tan(c+dx)}} - \frac{1}{-}$$

$$= -\frac{2}{5ad\tan^{\frac{5}{2}}(c+dx)} + \frac{2b}{3a^2d\tan^{\frac{3}{2}}(c+dx)} + \frac{2(a^2-b^2)}{a^3d\sqrt{\tan(c+dx)}} - \frac{(}{-}$$

$$= -\frac{2b^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^{7/2}(a^2+b^2)d} - \frac{2}{5ad\tan^{\frac{5}{2}}(c+dx)} + \frac{}{3a^2d\,\mathrm{ta}}$$

$$= -\frac{2b^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^{7/2}(a^2+b^2)d} + \frac{(a-b)\log\left(1-\sqrt{2}\sqrt{\tan(}\right.}{2\sqrt{2}(a^2}$$

$$= -\frac{(a+b)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}(a^2+b^2)d} + \frac{(a+b)\tan^{-1}\left(1+\sqrt{2}\right.}{\sqrt{2}(a^2+}$$

Mathematica [A]
time = 3.77, size = 248, normalized size = 0.83

$$\frac{-15\left(\frac{2\sqrt{2}a^2(a+b)\left(\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\right)}{a^2+b^2}+\frac{8b^{9/2}\mathrm{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}(a^2+b^2)}-\frac{\sqrt{2}a^2(a-b)\left(\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)-\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\right)}{a^2+b^2}-\frac{8(a-b)(a+b)}{a\sqrt{\tan(c+dx)}}\right)-\frac{24a}{\tan^{\frac{5}{2}}(c+dx)}+\frac{40b}{\tan^{\frac{3}{2}}(c+dx)}}{60a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Tan[c + d*x]^(7/2)*(a + b*Tan[c + d*x])),x]
```

```
[Out] (-15*((2*Sqrt[2]*a^2*(a + b)*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - ArcT
an[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]))/(a^2 + b^2) + (8*b^(9/2)*ArcTan[(Sqrt[
```

```
b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(a^(3/2)*(a^2 + b^2)) - (Sqrt[2]*a^2*(a -
b)*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sq
rt[Tan[c + d*x]] + Tan[c + d*x]]))/(a^2 + b^2) - (8*(a - b)*(a + b))/(a*Sqr
t[Tan[c + d*x]])) - (24*a)/Tan[c + d*x]^(5/2) + (40*b)/Tan[c + d*x]^(3/2))/
(60*a^2*d)
```

Maple [A]
time = 0.12, size = 277, normalized size = 0.92

method	result
derivativedivides	$\frac{-\frac{2}{5a\tan(dx+c)^{\frac{5}{2}}}-\frac{2(-a^2+b^2)}{a^3\sqrt{\tan(dx+c)}}+\frac{2b}{3a^2\tan(dx+c)^{\frac{3}{2}}}-\frac{2b^5\arctan\left(\frac{b(\sqrt{\tan}(dx+c))}{\sqrt{ab}}\right)}{a^3(a^2+b^2)\sqrt{ab}}+\frac{b\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\right.}{1-\sqrt{2}\left(\right.}\right.\right.}{}}{}$
default	$\frac{-\frac{2}{5a\tan(dx+c)^{\frac{5}{2}}}-\frac{2(-a^2+b^2)}{a^3\sqrt{\tan(dx+c)}}+\frac{2b}{3a^2\tan(dx+c)^{\frac{3}{2}}}-\frac{2b^5\arctan\left(\frac{b(\sqrt{\tan}(dx+c))}{\sqrt{ab}}\right)}{a^3(a^2+b^2)\sqrt{ab}}+\frac{b\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\right.}{1-\sqrt{2}\left(\right.}\right.\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(7/2)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/5/a/tan(d*x+c)^(5/2)-2*(-a^2+b^2)/a^3/tan(d*x+c)^(1/2)+2/3*b/a^2/ta
n(d*x+c)^(3/2)-2/a^3*b^5/(a^2+b^2)/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a
*b)^(1/2))+2/(a^2+b^2)*(1/8*b*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d
*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+
c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/8*a*2^(1/2)*(ln((1-2^(1/
2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*
arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))))
```

Maxima [A]
time = 0.51, size = 223, normalized size = 0.74

$$-\frac{\frac{120\,b^5\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^5+a^3b^2)\sqrt{ab}}-\frac{15\left(2\sqrt{2}(a+b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}(a+b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-\sqrt{2}(a-b)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+\sqrt{2}(a-b)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)\right)}{a^2+b^2}-\frac{8\left(5ab\tan(dx+c)+15(a^2-b^2)\tan(dx+c)^2-3a^2\right)}{a^3\tan(dx+c)^{\frac{5}{2}}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(7/2)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(120*b^5*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^5 + a^3*b^2)*sqrt
(a*b)) - 15*(2*sqrt(2)*(a + b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x
 + c)))) + 2*sqrt(2)*(a + b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x
+ c)))) - sqrt(2)*(a - b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1
) + sqrt(2)*(a - b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a
```

```
^2 + b^2) - 8*(5*a*b*tan(d*x + c) + 15*(a^2 - b^2)*tan(d*x + c)^2 - 3*a^2)/
(a^3*tan(d*x + c)^(5/2)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4172 vs. $2(252) = 504$.
time = 10.65, size = 8348, normalized size = 27.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(7/2)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/60*(60*sqrt(2)*((a^9 + 3*a^7*b^2 + 3*a^5*b^4 + a^3*b^6)*d^5*cos(d*x + c
)^4 - 2*(a^9 + 3*a^7*b^2 + 3*a^5*b^4 + a^3*b^6)*d^5*cos(d*x + c)^2 + (a^9 +
 3*a^7*b^2 + 3*a^5*b^4 + a^3*b^6)*d^5)*sqrt((a^4 + 2*a^2*b^2 + b^4 + 2*(a^5
*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2
*a^2*b^2 + b^4))*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4
 + 4*a^2*b^6 + b^8)*d^4))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4)*arctan(-(
(a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^4*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8
+ 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 +
 b^4)*d^4)) - sqrt(2)*((a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*d^
7*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 +
b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - (a^7 + 3*a^5*b^2 + 3*a^3
*b^4 + a*b^6)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^
4 + 4*a^2*b^6 + b^8)*d^4)))*sqrt((a^4 + 2*a^2*b^2 + b^4 + 2*(a^5*b + 2*a^3*
b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 +
b^4))*sqrt(((a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 +
b^4)*d^4))*cos(d*x + c) + sqrt(2)*((a^7 - a^5*b^2 - a^3*b^4 + a*b^6)*d^3*sq
rt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) - (a^4*b - 2*a^2*b^3 + b^5
)*d*cos(d*x + c))*sqrt((a^4 + 2*a^2*b^2 + b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^
5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt
(sin(d*x + c)/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(1/4) + (a^4
- 2*a^2*b^2 + b^4)*sin(d*x + c))/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4)*
d^4))^(3/4) - sqrt(2)*((a^10*b + 3*a^8*b^3 + 2*a^6*b^5 - 2*a^4*b^7 - 3*a^2*
b^9 - b^11)*d^7*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4
+ 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - (a^9 + 2*a
^7*b^2 - 2*a^3*b^6 - a*b^8)*d^5*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*
b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))*sqrt((a^4 + 2*a^2*b^2 + b^4 + 2*(
a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4
- 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a^4 + 2*a^2*b^2 +
b^4)*d^4))^(3/4))/(a^4 - 2*a^2*b^2 + b^4)) + 60*sqrt(2)*((a^9 + 3*a^7*b^2 +
 3*a^5*b^4 + a^3*b^6)*d^5*cos(d*x + c)^4 - 2*(a^9 + 3*a^7*b^2 + 3*a^5*b^4 +
 a^3*b^6)*d^5*cos(d*x + c)^2 + (a^9 + 3*a^7*b^2 + 3*a^5*b^4 + a^3*b^6)*d^5)
*sqrt((a^4 + 2*a^2*b^2 + b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a
^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt((a^4 - 2*a^2*b^2
```

```
 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(1/((a^4 + 2
*a^2*b^2 + b^4)*d^4))^(3/4)*arctan(((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^4
*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b
^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + sqrt(2)*((a^8*b + 4*a^6*b
^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*d^7*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 +
4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 + 2*a^2*b^2 + b
^4)*d^4)) - (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*d^5*sqrt((a^4 - 2*a^2*b^2
 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))*sqrt((a^4 +
 2*a^2*b^2 + b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b
^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(((a^6 - a^4*b^2 - a^2*b^4 +
b^6)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) - sqrt(2)*((a^7
 - a^5*b^2 - a^3*b^4 + a*b^6)*d^3*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos
(d*x + c) - (a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c))*sqrt((a^4 + 2*a^2*b^2
 + b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*
d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a^4 +
2*a^2*b^2 + b^4)*d^4))^(1/4) + (a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c))/cos(d*
x + c))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4) + sqrt(2)*((a^10*b + 3*a^8*
b^3 + 2*a^6*b^5 - 2*a^4*b^7 - 3*a^2*b^9 - b^11)*d^7*sqrt((a^4 - 2*a^2*b^2 +
 b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt(1/((a^4 +
 2*a^2*b^2 + b^4)*d^4)) - (a^9 + 2*a^7*b^2 - 2*a^3*b^6 - a*b^8)*d^5*sqrt((a
^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)
))*sqrt((a^4 + 2*a^2*b^2 + b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(1/(
(a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2 + b^4))*sqrt(sin(d*x + c)/c
os(d*x + c))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4))/(a^4 - 2*a^2*b^2 + b^
4)) + 15*sqrt(2)*((a^5 + a^3*b^2)*d*cos(d*x + c)^4 - 2*(a^5 + a^3*b^2)*d*co
s(d*x + c)^2 + (a^5 + a^3*b^2)*d - 2*((a^6*b + a^4*b^3)*d^3*cos(d*x + c)^4
- 2*(a^6*b + a^4*b^3)*d^3*cos(d*x + c)^2 + (a^6*b + a^4*b^3)*d^3)*sqrt(1/((
a^4 + 2*a^2*b^2 + b^4)*d^4)))*sqrt((a^4 + 2*a^2*b^2 + b^4 + 2*(a^5*b + 2*a^
3*b^3 + a*b^5)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(a^4 - 2*a^2*b^2
+ b^4))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(1/4)*log(((a^6 - a^4*b^2 - a^2*b
^4 + b^6)*d^2*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) + sqrt(2)*
((a^7 - a^5*b^2 - a^3*b^4 + a*b^6)*d^3*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^4)
)*cos(d*x + c) - (a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c))*sqrt((a^4 + 2*a^
2*b^2 + b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\tan{(c+dx)})\tan^{\frac{7}{2}}{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(7/2)/(a+b*tan(d*x+c)),x)
[Out] Integral(1/((a + b*tan(c + d*x))*tan(c + d*x)**(7/2)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(7/2)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 6.91, size = 2500, normalized size = 8.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(7/2)*(a + b*tan(c + d*x))),x)
```

```
[Out] atan(((tan(c + d*x)^(1/2)*(64*a^21*b^11*d^5 - 32*a^27*b^5*d^5) + (-1i/(4*(b
^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*((-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^
2*2i)))^(1/2)*(tan(c + d*x)^(1/2)*(512*a^22*b^12*d^7 - 448*a^28*b^6*d^7 + 1
28*a^30*b^4*d^7 + 64*a^32*b^2*d^7) + (-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2
i)))^(1/2)*(512*a^24*b^11*d^8 - tan(c + d*x)^(1/2)*(-1i/(4*(b^2*d^2 - a^2*d
^2 + a*b*d^2*2i)))^(1/2)*(512*a^27*b^9*d^9 + 512*a^29*b^7*d^9 - 512*a^31*b^
5*d^9 - 512*a^33*b^3*d^9) + 512*a^26*b^9*d^8 - 128*a^28*b^7*d^8 + 256*a^30*
b^5*d^8 + 384*a^32*b^3*d^8)) - 128*a^21*b^12*d^6 + 512*a^23*b^10*d^6 + 32*a
^29*b^4*d^6 + 32*a^31*b^2*d^6))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^
(1/2)*1i + (tan(c + d*x)^(1/2)*(64*a^21*b^11*d^5 - 32*a^27*b^5*d^5) - (-1i/
(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(512*a^23*b^10*d^6 - 128*a^21*b
^12*d^6 - (-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(tan(c + d*x)^(1
/2)*(512*a^22*b^12*d^7 - 448*a^28*b^6*d^7 + 128*a^30*b^4*d^7 + 64*a^32*b^2*
d^7) - (-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(tan(c + d*x)^(1/2)
*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(512*a^27*b^9*d^9 + 512*a
^29*b^7*d^9 - 512*a^31*b^5*d^9 - 512*a^33*b^3*d^9) + 512*a^24*b^11*d^8 + 51
2*a^26*b^9*d^8 - 128*a^28*b^7*d^8 + 256*a^30*b^5*d^8 + 384*a^32*b^3*d^8)) +
 32*a^29*b^4*d^6 + 32*a^31*b^2*d^6))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2
i)))^(1/2)*1i)/((tan(c + d*x)^(1/2)*(64*a^21*b^11*d^5 - 32*a^27*b^5*d^5) +
(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*((-1i/(4*(b^2*d^2 - a^2*d^
2 + a*b*d^2*2i)))^(1/2)*(tan(c + d*x)^(1/2)*(512*a^22*b^12*d^7 - 448*a^28*b
^6*d^7 + 128*a^30*b^4*d^7 + 64*a^32*b^2*d^7) + (-1i/(4*(b^2*d^2 - a^2*d^2 +
 a*b*d^2*2i)))^(1/2)*(512*a^24*b^11*d^8 - tan(c + d*x)^(1/2)*(-1i/(4*(b^2*d
^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(512*a^27*b^9*d^9 + 512*a^29*b^7*d^9 - 5
12*a^31*b^5*d^9 - 512*a^33*b^3*d^9) + 512*a^26*b^9*d^8 - 128*a^28*b^7*d^8 +
 256*a^30*b^5*d^8 + 384*a^32*b^3*d^8)) - 128*a^21*b^12*d^6 + 512*a^23*b^10*
d^6 + 32*a^29*b^4*d^6 + 32*a^31*b^2*d^6))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*
```

```
d^2*2i)))^(1/2) - (tan(c + d*x)^(1/2)*(64*a^21*b^11*d^5 - 32*a^27*b^5*d^5)
- (-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(512*a^23*b^10*d^6 - 128
*a^21*b^12*d^6 - (-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(tan(c +
d*x)^(1/2)*(512*a^22*b^12*d^7 - 448*a^28*b^6*d^7 + 128*a^30*b^4*d^7 + 64*a^
32*b^2*d^7) - (-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(tan(c + d*x
)^(1/2)*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(512*a^27*b^9*d^9
+ 512*a^29*b^7*d^9 - 512*a^31*b^5*d^9 - 512*a^33*b^3*d^9) + 512*a^24*b^11*d
^8 + 512*a^26*b^9*d^8 - 128*a^28*b^7*d^8 + 256*a^30*b^5*d^8 + 384*a^32*b^3*
d^8)) + 32*a^29*b^4*d^6 + 32*a^31*b^2*d^6))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*
b*d^2*2i)))^(1/2) + 64*a^22*b^9*d^4))*(-1i/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*
2i)))^(1/2)*2i + (log(((tan(c + d*x)^(1/2)*(64*a^21*b^11*d^5 - 32*a^27*b^5*
d^5) - ((-1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*(512*a^23*b^10*d^6
 - 128*a^21*b^12*d^6 - ((-1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*(t
an(c + d*x)^(1/2)*(512*a^22*b^12*d^7 - 448*a^28*b^6*d^7 + 128*a^30*b^4*d^7
+ 64*a^32*b^2*d^7) - ((-1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*((ta
n(c + d*x)^(1/2)*(-1/(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2)*(512*a^27
*b^9*d^9 + 512*a^29*b^7*d^9 - 512*a^31*b^5*d^9 - 512*a^33*b^3*d^9))/2 + 512
*a^24*b^11*d^8 + 512*a^26*b^9*d^8 - 128*a^28*b^7*d^8 + 256*a^30*b^5*d^8 + 3
84*a^32*b^3*d^8))/2))/2 + 32*a^29*b^4*d^6 + 32*a^31*b^2*d^6))/2)*(-1/(b^2*d
^2*1i - a^2*d^2*1i + 2*a*b*d^2))^(1/2))/2 - 32*a^22*b^9*d^4)*(-1/(b^2*d^2*1
i - a^2*d^2*1i + 2*a*b*d^2))^(1/2))/2 - log(- (tan(c + d*x)^(1/2)*(64*a^21*
b^11*d^5 - 32*a^27*b^5*d^5) + (-1/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))
)^(1/2)*((-1/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2)*(tan(c + d*x)
^(1/2)*(512*a^22*b^12*d^7 - 448*a^28*b^6*d^7 + 128*a^30*b^4*d^7 + 64*a^32*b
^2*d^7) + (-1/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2)*(512*a^24*b^
11*d^8 - tan(c + d*x)^(1/2)*(-1/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^
(1/2)*(512*a^27*b^9*d^9 + 512*a^29*b^7*d^9 - 512*a^31*b^5*d^9 - 512*a^33*b^
3*d^9) + 512*a^26*b^9*d^8 - 128*a^28*b^7*d^8 + 256*a^30*b^5*d^8 + 384*a^32*
b^3*d^8)) - 128*a^21*b^12*d^6 + 512*a^23*b^10*d^6 + 32*a^29*b^4*d^6 + 32*a^
31*b^2*d^6))*(-1/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) - 32*a^22
*b^9*d^4)*(-1/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) + ((2*tan(c
+ d*x)^2*(a^2 - b^2))/a^3 - 2/(5*a) + (2*b*tan(c + d*x))/(3*a^2))/(d*tan(c
+ d*x)^(5/2)) + (atan((((tan(c + d*x)^(1/2)*(64*a^21*b^11*d^5 - 32*a^27*b^5
*d^5) + ((-a^7*b^9)^(1/2)*(512*a^23*b^10*d^6 - 128*a^21*b^12*d^6 + 32*a^29*
b^4*d^6 + 32*a^31*b^2*d^6 + ((tan(c + d*x)^(1/2)*(512*a^22*b^12*d^7 - 448*a
^28*b^6*d^7 + 128*a^30*b^4*d^7 + 64*a^32*b^2*d^7) + ((-a^7*b^9)^(1/2)*(512*
a^24*b^11*d^8 + 512*a^26*b^9*d^8 - 128*a^28*b^7*d^8 + 256*a^30*b^5*d^8 + 38
4*a^32*b^3*d^8 - (tan(c + d*x)^(1/2)*(-a^7*b^9)^(1/2)*(512*a^27*b^9*d^9 + 5
12*a^29*b^7*d^9 - 512*a^31*b^5*d^9 - 512*a^33*b^3*d^9))/(a^7*d*(a^2 + b^2))
))/(a^7*d*(a^2 + b^2)))*(-a^7*b^9)^(1/2))/(a^7*...
```

3.591 $\int \frac{\tan^{\frac{9}{2}}(c+dx)}{(a+b\tan(c+dx))^2} dx$

Optimal. Leaf size=399

$$-\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}+\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}+\frac{a^{7/}}{}$$

```
[Out] a^(7/2)*(5*a^2+9*b^2)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/b^(7/2)/(a^2
+b^2)^2/d+1/2*(a^2+2*a*b-b^2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)
^2/d*2^(1/2)+1/2*(a^2+2*a*b-b^2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^
2)^2/d*2^(1/2)+1/4*(a^2-2*a*b-b^2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)
)/(a^2+b^2)^2/d*2^(1/2)-1/4*(a^2-2*a*b-b^2)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+t
an(d*x+c))/(a^2+b^2)^2/d*2^(1/2)-a*(5*a^2+4*b^2)*tan(d*x+c)^(1/2)/b^3/(a^2+
b^2)/d+1/3*(5*a^2+2*b^2)*tan(d*x+c)^(3/2)/b^2/(a^2+b^2)/d-a^2*tan(d*x+c)^(5
/2)/b/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.68, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3646, 3728, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}-\frac{a^2\tan^{\frac{5}{2}}(c+dx)}{bd\,(a^2+b^2)\,(a+b\tan(c+dx))}+\frac{(5a^2+2b^2)\tan^{\frac{3}{2}}(c+dx)}{3b^2d\,(a^2+b^2)}+\frac{(a^2-2ab-b^2)\log\left(\tan(c+dx)-\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}-\frac{(a^2-2ab-b^2)\log\left(\tan(c+dx)+\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}-\frac{a(5a^2+4b^2)\sqrt{\tan(c+dx)}}{b^3d\,(a^2+b^2)}+\frac{a^{7/2}(5a^2+9b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{7/2}d\,(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(9/2)/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((a^2 + 2*a*b - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*(a^
2 + b^2)^2*d)) + ((a^2 + 2*a*b - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]
])/(Sqrt[2]*(a^2 + b^2)^2*d) + (a^(7/2)*(5*a^2 + 9*b^2)*ArcTan[(Sqrt[b]*Sqr
t[Tan[c + d*x]])/Sqrt[a]])/(b^(7/2)*(a^2 + b^2)^2*d) + ((a^2 - 2*a*b - b^2)
*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)
^2*d) - ((a^2 - 2*a*b - b^2)*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d
*x]])/(2*Sqrt[2]*(a^2 + b^2)^2*d) - (a*(5*a^2 + 4*b^2)*Sqrt[Tan[c + d*x]])/
(b^3*(a^2 + b^2)*d) + ((5*a^2 + 2*b^2)*Tan[c + d*x]^(3/2))/(3*b^2*(a^2 + b^
2)*d) - (a^2*Tan[c + d*x]^(5/2))/(b*(a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
```

```
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\tan^{\frac{9}{2}}(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{a^2\tan^{\frac{5}{2}}(c+dx)}{b\,(a^2+b^2)\,d(a+b\tan(c+dx))} + \frac{\int \frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{5a^2}{2}-ab\tan(c+dx)+\frac{1}{2}(5a^2+2b^2)\tan^2(c}{a+b\tan(c+dx)}}{b\,(a^2+b^2)}$$

$$= \frac{(5a^2+2b^2)\tan^{\frac{3}{2}}(c+dx)}{3b^2\,(a^2+b^2)\,d} - \frac{a^2\tan^{\frac{5}{2}}(c+dx)}{b\,(a^2+b^2)\,d(a+b\tan(c+dx))} + \frac{2\int \sqrt{\tan(c+}}{}$$

$$= -\frac{a(5a^2+4b^2)\sqrt{\tan(c+dx)}}{b^3\,(a^2+b^2)\,d} + \frac{(5a^2+2b^2)\tan^{\frac{3}{2}}(c+dx)}{3b^2\,(a^2+b^2)\,d} - \frac{a^2\tan^{\frac{5}{2}}(}{b\,(a^2+b^2)\,d(a+}$$

$$= -\frac{a(5a^2+4b^2)\sqrt{\tan(c+dx)}}{b^3\,(a^2+b^2)\,d} + \frac{(5a^2+2b^2)\tan^{\frac{3}{2}}(c+dx)}{3b^2\,(a^2+b^2)\,d} - \frac{a^2\tan^{\frac{5}{2}}(}{b\,(a^2+b^2)\,d(a+}$$

$$= -\frac{a(5a^2+4b^2)\sqrt{\tan(c+dx)}}{b^3\,(a^2+b^2)\,d} + \frac{(5a^2+2b^2)\tan^{\frac{3}{2}}(c+dx)}{3b^2\,(a^2+b^2)\,d} - \frac{a^2\tan^{\frac{5}{2}}(}{b\,(a^2+b^2)\,d(a+}$$

$$= -\frac{a(5a^2+4b^2)\sqrt{\tan(c+dx)}}{b^3\,(a^2+b^2)\,d} + \frac{(5a^2+2b^2)\tan^{\frac{3}{2}}(c+dx)}{3b^2\,(a^2+b^2)\,d} - \frac{a^2\tan^{\frac{5}{2}}(}{b\,(a^2+b^2)\,d(a+}$$

$$= \frac{a^{7/2}(5a^2+9b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{7/2}\,(a^2+b^2)^2\,d} - \frac{a(5a^2+4b^2)\sqrt{\tan(c+dx)}}{b^3\,(a^2+b^2)\,d} -$$

$$= \frac{a^{7/2}(5a^2+9b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{7/2}\,(a^2+b^2)^2\,d} + \frac{(a^2-2ab-b^2)\log\left(1-\sqrt{2}\right.}{2\sqrt{2}\,(a}$$

$$= -\frac{(a^2+2ab-b^2)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} + \frac{(a^2+2ab-b^2)\tan^{-1}\left(1\right.}{\sqrt{2}\,(a^2}$$

Mathematica [C] Result contains complex when optimal does not.

time = 1.91, size = 442, normalized size = 1.11

$$\frac{b^2\tan^{\frac{11}{2}}(c+dx)}{a(a^2+b^2)d(a+b\tan(c+dx))}+\frac{-\frac{b\tan^{\frac{9}{2}}(c+dx)}{d}+\frac{2\left(\frac{9ab\tan^{\frac{7}{2}}(c+dx)}{2d}+\frac{2\left(-\frac{63a^2b\tan^{\frac{5}{2}}(c+dx)}{4d}+\frac{2\left(\frac{2\left(\frac{2\left(\frac{2\left(-\frac{945}{32}a^3b^6+\frac{945}{64}a^3b^4(5a^2+4b^2)+\frac{945}{64}a^3b^2(5a^4+4a^2b^2-2b^4)\right)\mathrm{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}(a^2+b^2)d}+\frac{-\frac{\sqrt[4]{-1}\left(\frac{945a^2b^6}{16}-\frac{945}{32}ia(a-b)b^5(a+b)\right)\mathrm{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d}-\frac{\sqrt[4]{-1}\left(\frac{945a^2b^6}{16}+\frac{945}{32}ia(a-b)b^5(a+b)\right)\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d}}{a^2+b^2}\right)}{b}-\frac{945a^2b(5a^2+4b^2)\sqrt{\tan(c+dx)}}{16d}\right)}{3b}+\frac{105ab(5a^2+2b^2)\tan^{\frac{3}{2}}(c+dx)}{8d}\right)}{5b}\right)}{7b}\right)}{9b}}{a(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(9/2)/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (b^2*Tan[c + d*x]^(11/2))/(a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])) + (-((b*Ta
n[c + d*x]^(9/2))/d) + (2*((9*a*b*Tan[c + d*x]^(7/2))/(2*d) + (2*((-63*a^2*
b*Tan[c + d*x]^(5/2))/(4*d) + (2*((2*((2*((2*((-945*a^3*b^6)/32 + (945*a^3*
b^4*(5*a^2 + 4*b^2))/64 + (945*a^3*b^2*(5*a^4 + 4*a^2*b^2 - 2*b^4))/64)*Arc
Tan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*(a^2 + b^2)*d)
 + (-(((-1)^(1/4)*((945*a^2*b^6)/16 - ((945*I)/32)*a*(a - b)*b^5*(a + b))*Ar
cTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])/d) - ((-1)^(1/4)*((945*a^2*b^6)/16 + (
(945*I)/32)*a*(a - b)*b^5*(a + b))*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])/
d)/(a^2 + b^2)))/b - (945*a^2*b*(5*a^2 + 4*b^2)*Sqrt[Tan[c + d*x]])/(16*d))
)/(3*b) + (105*a*b*(5*a^2 + 2*b^2)*Tan[c + d*x]^(3/2))/(8*d)))/(5*b)))/(7*b
)))/(9*b))/(a*(a^2 + b^2))
```

Maple [A]
time = 0.14, size = 311, normalized size = 0.78

method	result

derivativedivides	$\frac{-\frac{2\left(-\frac{b\left(\tan^{\frac{3}{2}}(dx+c)\right)}{3}+2a\left(\sqrt{\tan}(dx+c)\right)\right)}{b^3}+\frac{2a^4\left(\frac{\left(-\frac{a^2}{2}-\frac{b^2}{2}\right)\left(\sqrt{\tan}(dx+c)\right)}{a+b\tan(dx+c)}+\frac{\left(5a^2+9b^2\right)\arctan\left(\frac{b\left(\sqrt{\tan}(dx+\cdots\right)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\cdots\right)}{b^3\left(a^2+b^2\right)^2}\cdots}{\cdots}$
default	$\frac{-\frac{2\left(-\frac{b\left(\tan^{\frac{3}{2}}(dx+c)\right)}{3}+2a\left(\sqrt{\tan}(dx+c)\right)\right)}{b^3}+\frac{2a^4\left(\frac{\left(-\frac{a^2}{2}-\frac{b^2}{2}\right)\left(\sqrt{\tan}(dx+c)\right)}{a+b\tan(dx+c)}+\frac{\left(5a^2+9b^2\right)\arctan\left(\frac{b\left(\sqrt{\tan}(dx+\cdots\right)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\cdots\right)}{b^3\left(a^2+b^2\right)^2}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(9/2)/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b^3*(-1/3*b*tan(d*x+c)^(3/2)+2*a*tan(d*x+c)^(1/2))+2*a^4/b^3/(a^2+b
^2)^2*((-1/2*a^2-1/2*b^2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))+1/2*(5*a^2+9*b^
2)/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2)))+2/(a^2+b^2)^2*(1/4*a
*b*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c
)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/
2)*tan(d*x+c)^(1/2)))+1/8*(a^2-b^2)*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)
+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*ta
n(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))))
```

Maxima [A]

time = 0.51, size = 311, normalized size = 0.78

$$-\frac{\frac{12\,a^4\sqrt{\tan(dx+c)}}{a^3b^3+ab^5+(a^2b^4+b^6)\tan(dx+c)}-\frac{12\,(5\,a^6+9\,a^4b^2)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^4b^3+2\,a^2b^5+b^7)\sqrt{ab}}-\frac{3\left(2\sqrt{2}\,(a^2+2\,ab-b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}\,(a^2+2\,ab-b^2)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-\sqrt{2}\,(a^2-2\,ab-b^2)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+\sqrt{2}\,(a^2-2\,ab-b^2)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)\right)}{a^4+2\,a^2b^2+b^4}-\frac{8\left(b\tan(dx+c)^{\frac{3}{2}}-6\,a\sqrt{\tan(dx+c)}\right)}{b^3}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(9/2)/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/12*(12*a^4*sqrt(tan(d*x + c))/(a^3*b^3 + a*b^5 + (a^2*b^4 + b^6)*tan(d*x
 + c)) - 12*(5*a^6 + 9*a^4*b^2)*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^
4*b^3 + 2*a^2*b^5 + b^7)*sqrt(a*b)) - 3*(2*sqrt(2)*(a^2 + 2*a*b - b^2)*arct
an(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*(a^2 + 2*a*b -
 b^2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*(a^2
- 2*a*b - b^2)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)
*(a^2 - 2*a*b - b^2)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(
a^4 + 2*a^2*b^2 + b^4) - 8*(b*tan(d*x + c)^(3/2) - 6*a*sqrt(tan(d*x + c)))/
b^3)/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 7246 vs. $2(353) = 706$.
time = 14.90, size = 14604, normalized size = 36.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(9/2)/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/12*(12*sqrt(2)*((a^14*b^3 + 5*a^12*b^5 + 9*a^10*b^7 + 5*a^8*b^9 - 5*a^6*
b^11 - 9*a^4*b^13 - 5*a^2*b^15 - b^17)*d^5*cos(d*x + c)^3 + 2*(a^13*b^4 + 6
*a^11*b^6 + 15*a^9*b^8 + 20*a^7*b^10 + 15*a^5*b^12 + 6*a^3*b^14 + a*b^16)*d
^5*cos(d*x + c)^2*sin(d*x + c) + (a^12*b^5 + 6*a^10*b^7 + 15*a^8*b^9 + 20*a
^6*b^11 + 15*a^4*b^13 + 6*a^2*b^15 + b^17)*d^5*cos(d*x + c))*sqrt((a^8 + 4*
a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 -
 2*a^5*b^7 - 3*a^3*b^9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
 4*a^2*b^6 + b^8)*d^4)))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)
)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^16 + 8*a^14*b
^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8
*a^2*b^14 + b^16)*d^4))*(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)
*d^4))^(3/4)*arctan(((a^16 - 20*a^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 - 64*a^
6*b^10 - 20*a^4*b^12 + b^16)*d^4*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a
^2*b^6 + b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8
+ 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4))*sqrt(1/((a^8 + 4*a^6
*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)) + sqrt(2)*(2*(a^17*b + 8*a^15*b^3
 + 28*a^13*b^5 + 56*a^11*b^7 + 70*a^9*b^9 + 56*a^7*b^11 + 28*a^5*b^13 + 8*a
^3*b^15 + a*b^17)*d^7*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^
8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^
10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4))*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^
4*b^4 + 4*a^2*b^6 + b^8)*d^4)) - (a^14 + 5*a^12*b^2 + 9*a^10*b^4 + 5*a^8*b^
6 - 5*a^6*b^8 - 9*a^4*b^10 - 5*a^2*b^12 - b^14)*d^5*sqrt((a^8 - 12*a^6*b^2
+ 38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^1
0*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4)))*
sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 4*(a^11*b + 3*a^9*b^3
 + 2*a^7*b^5 - 2*a^5*b^7 - 3*a^3*b^9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b^2
 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*
a^2*b^6 + b^8))*sqrt(((a^12 - 10*a^10*b^2 + 15*a^8*b^4 + 52*a^6*b^6 + 15*a^
4*b^8 - 10*a^2*b^10 + b^12)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^
2*b^6 + b^8)*d^4))*cos(d*x + c) + sqrt(2)*((a^14 - 11*a^12*b^2 + 25*a^10*b^
4 + 37*a^8*b^6 - 37*a^6*b^8 - 25*a^4*b^10 + 11*a^2*b^12 - b^14)*d^3*sqrt(1/
((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*cos(d*x + c) - 2*(a^
9*b - 12*a^7*b^3 + 38*a^5*b^5 - 12*a^3*b^7 + a*b^9)*d*cos(d*x + c))*sqrt((a
^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 4*(a^11*b + 3*a^9*b^3 + 2*a^
7*b^5 - 2*a^5*b^7 - 3*a^3*b^9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^
4*b^4 + 4*a^2*b^6 + b^8)*d^4)))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6
```

```
 + b^8))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
 4*a^2*b^6 + b^8)*d^4))^(1/4) + (a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6
 + b^8)*sin(d*x + c))/cos(d*x + c))*(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^
2*b^6 + b^8)*d^4))^(3/4) - sqrt(2)*(2*(a^21*b + 2*a^19*b^3 - 19*a^17*b^5 -
104*a^15*b^7 - 238*a^13*b^9 - 308*a^11*b^11 - 238*a^9*b^13 - 104*a^7*b^15 -
 19*a^5*b^17 + 2*a^3*b^19 + a*b^21)*d^7*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4
 - 12*a^2*b^6 + b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a
^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4))*sqrt(1/((a^8
+ 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)) - (a^18 - a^16*b^2 - 20*a^
14*b^4 - 44*a^12*b^6 - 26*a^10*b^8 + 26*a^8*b^10 + 44*a^6*b^12 + 20*a^4*b^1
4 + a^2*b^16 - b^18)*d^5*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 +
 b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6
*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4)))*sqrt((a^8 + 4*a^6*b^2 + 6*a
^4*b^4 + 4*a^2*b^6 + b^8 + 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 - 2*a^5*b^7 -
3*a^3*b^9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 +
b^8)*d^4)))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(sin(d*
x + c)/cos(d*x + c))*(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^
4))^(3/4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)) + 12*sqrt(2)
*((a^14*b^3 + 5*a^12*b^5 + 9*a^10*b^7 + 5*a^8*b^9 - 5*a^6*b^11 - 9*a^4*b^13
 - 5*a^2*b^15 - b^17)*d^5*cos(d*x + c)^3 + 2*(a^13*b^4 + 6*a^11*b^6 + 15*a^
9*b^8 + 20*a^7*b^10 + 15*a^5*b^12 + 6*a^3*b^14 + a*b^16)*d^5*cos(d*x + c)^2
*sin(d*x + c) + (a^12*b^5 + 6*a^10*b^7 + 15*a^8*b^9 + 20*a^6*b^11 + 15*a^4*
b^13 + 6*a^2*b^15 + b^17)*d^5*cos(d*x + c))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b
^4 + 4*a^2*b^6 + b^8 + 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 - 2*a^5*b^7 - 3*a^
3*b^9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)
*d^4)))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt((a^8 - 12*
a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4
+ 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)
*d^4))*(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))^(3/4)*arct
an(-((a^16 - 20*a^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 - 64*a^6*b^10 - 20*a^4*
b^12 + b^16)*d^4*sqrt((a^8 - 12*a^6*b^2 + 38*a^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(9/2)/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(9/2)/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 9.32, size = 2500, normalized size = 6.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(9/2)/(a + b*tan(c + d*x))^2,x)
```

```
[Out] (2*tan(c + d*x)^(3/2))/(3*b^2*d) - atan((((((1/(a^4*d^2*1i + b^4*d^2*1i + 4
*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^(1/2)*(((1/(a^4*d^2*1i + b^4*d^
2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^(1/2)*((((8*(64*a^2*b^1
8*d^4 + 480*a^4*b^16*d^4 + 1440*a^6*b^14*d^4 + 2240*a^8*b^12*d^4 + 1920*a^1
0*b^10*d^4 + 864*a^12*b^8*d^4 + 160*a^14*b^6*d^4))/(b^13*d^5 + 4*a^2*b^11*d
^5 + 6*a^4*b^9*d^5 + 4*a^6*b^7*d^5 + a^8*b^5*d^5) - (8*tan(c + d*x)^(1/2)*(
1/(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^(
1/2)*(32*b^22*d^4 + 160*a^2*b^20*d^4 + 288*a^4*b^18*d^4 + 160*a^6*b^16*d^4
 - 160*a^8*b^14*d^4 - 288*a^10*b^12*d^4 - 160*a^12*b^10*d^4 - 32*a^14*b^8*d^
4))/(b^13*d^4 + 4*a^2*b^11*d^4 + 6*a^4*b^9*d^4 + 4*a^6*b^7*d^4 + a^8*b^5*d^
4))*(1/(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6
i))^(1/2))/2 + (16*tan(c + d*x)^(1/2)*(60*a*b^17*d^2 + 200*a^17*b*d^2 + 52*
a^3*b^15*d^2 - 72*a^5*b^13*d^2 - 56*a^7*b^11*d^2 + 660*a^9*b^9*d^2 + 2020*a
^11*b^7*d^2 + 2288*a^13*b^5*d^2 + 1120*a^15*b^3*d^2))/(b^13*d^4 + 4*a^2*b^1
1*d^4 + 6*a^4*b^9*d^4 + 4*a^6*b^7*d^4 + a^8*b^5*d^4)))/2 - (8*(4*a*b^15*d^2
 + 400*a^15*b*d^2 + 16*a^3*b^13*d^2 + 600*a^5*b^11*d^2 - 240*a^7*b^9*d^2 -
1612*a^9*b^7*d^2 - 144*a^11*b^5*d^2 + 1040*a^13*b^3*d^2))/(b^13*d^5 + 4*a^2
*b^11*d^5 + 6*a^4*b^9*d^5 + 4*a^6*b^7*d^5 + a^8*b^5*d^5)))/2 - (16*tan(c +
d*x)^(1/2)*(2*b^14 - 25*a^14 + 4*a^2*b^12 + 2*a^4*b^10 + 81*a^8*b^6 + 9*a^1
0*b^4 - 65*a^12*b^2))/(b^13*d^4 + 4*a^2*b^11*d^4 + 6*a^4*b^9*d^4 + 4*a^6*b^
7*d^4 + a^8*b^5*d^4))*(1/(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d
^2 - a^2*b^2*d^2*6i))^(1/2)*1i)/2 - ((((1/(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^
3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^(1/2)*(((1/(a^4*d^2*1i + b^4*d^2*1i
+ 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^(1/2)*((((8*(64*a^2*b^18*d^4
 + 480*a^4*b^16*d^4 + 1440*a^6*b^14*d^4 + 2240*a^8*b^12*d^4 + 1920*a^10*b^1
0*d^4 + 864*a^12*b^8*d^4 + 160*a^14*b^6*d^4))/(b^13*d^5 + 4*a^2*b^11*d^5 +
6*a^4*b^9*d^5 + 4*a^6*b^7*d^5 + a^8*b^5*d^5) + (8*tan(c + d*x)^(1/2)*(1/(a^
4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^(1/2)*
(32*b^22*d^4 + 160*a^2*b^20*d^4 + 288*a^4*b^18*d^4 + 160*a^6*b^16*d^4 - 160
*a^8*b^14*d^4 - 288*a^10*b^12*d^4 - 160*a^12*b^10*d^4 - 32*a^14*b^8*d^4))/(
b^13*d^4 + 4*a^2*b^11*d^4 + 6*a^4*b^9*d^4 + 4*a^6*b^7*d^4 + a^8*b^5*d^4))*(
1/(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^(
```

1/2))/2 - (16*tan(c + d*x)^(1/2)*(60*a*b^17*d^2 + 200*a^17*b*d^2 + 52*a^3*b^15*d^2 - 72*a^5*b^13*d^2 - 56*a^7*b^11*d^2 + 660*a^9*b^9*d^2 + 2020*a^11*b^7*d^2 + 2288*a^13*b^5*d^2 + 1120*a^15*b^3*d^2))/(b^13*d^4 + 4*a^2*b^11*d^4 + 6*a^4*b^9*d^4 + 4*a^6*b^7*d^4 + a^8*b^5*d^4)))/2 - (8*(4*a*b^15*d^2 + 400*a^15*b*d^2 + 16*a^3*b^13*d^2 + 600*a^5*b^11*d^2 - 240*a^7*b^9*d^2 - 1612*a^9*b^7*d^2 - 144*a^11*b^5*d^2 + 1040*a^13*b^3*d^2))/(b^13*d^5 + 4*a^2*b^11*d^5 + 6*a^4*b^9*d^5 + 4*a^6*b^7*d^5 + a^8*b^5*d^5)))/2 + (16*tan(c + d*x)^(1/2)*(2*b^14 - 25*a^14 + 4*a^2*b^12 + 2*a^4*b^10 + 81*a^8*b^6 + 9*a^10*b^4 - 65*a^12*b^2))/(b^13*d^4 + 4*a^2*b^11*d^4 + 6*a^4*b^9*d^4 + 4*a^6*b^7*d^4 + a^8*b^5*d^4))*(1/(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^(1/2)*1i)/2)/(((((1/(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^(1/2)*(((1/(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^(1/2)*((((8*(64*a^2*b^18*d^4 + 480*a^4*b^16*d^4 + 1440*a^6*b^14*d^4 + 2240*a^8*b^12*d^4 + 1920*a^10*b^10*d^4 + 864*a^12*b^8*d^4 + 160*a^14*b^6*d^4))/(b^13*d^5 + 4*a^2*b^11*d^5 + 6*a^4*b^9*d^5 + 4*a^6*b^7*d^5 + a^8*b^5*d^5) - (8*tan(c + d*x)^(1/2)*(1/(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^(1/2)*(32*b^22*d^4 + 160*a^2*b^20*d^4 + 288*a^4*b^18*d^4 + 160*a^6*b^16*d^4 - 160*a^8*b^14*d^4 - 288*a^10*b^12*d^4 - 160*a^12*b^10*d^4 - 32*a^14*b^8*d^4))/(b^13*d^4 + 4*a^2*b^11*d^4 + 6*a^4*b^9*d^4 + 4*a^6*b^7*d^4 + a^8*b^5*d^4))*(1/(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^(1/2))/2 + (16*tan(c + d*x)^(1/2)*(60*a*b^17*d^2 + 200*a^17*b*d^2 + 52*a^3*b^15*d^2 - 72*a^5*b^13*d^2 - 56*a^7*b^11*d^2 + 660*a^9*b^9*d^2 + 2020*a^11*b^7*d^2 + 2288*a^13*b^5*d^2 + 1120*a^15*b^3*d^2))/(b^13*d^4 + 4*a^2*b^11*d^4 + 6*a^4*b^9*d^4 + 4*a^6*b^7*d^4 + a^8*b^5*d^4)))/2 - (8*(4*a*b^15*d^2 + 400*a^15*b*d^2 + 16*a^3*b^13*d^2 + 600*a^5*b^11*d^2 - 240*a^7*b^9*d^2 - 1612*a^9*b^7*d^2 - 144*a^11*b^5*d^2 + 1040*a^13*b^3*d^2))/(b^13*d^5 + 4*a^2*b^11*d^5 + 6*a^4*b^9*d^5 + 4*a^6*b^7*d^5 + a^8*b^5*d^5)))/2 - (16*tan(c + d*x)^(1/2)*(2*b^14 - 25*a^14 + 4*a^2*b^12 + 2*a^4*b^10 + 81*a^8*b^6 + 9*a^10*b^4 - 65*a^12*b^2))/(b^13*d^4 + 4*a^2*b^11*d^4 + 6*a^4*b^9*d^4 + 4*a^6*b^7*d^4 + a^8*b^5*d^4))*(1/(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^(1/2))/2 - (16*(25*a^12 - 18*a^4*b^8 - 10*a^6*b^6 + 81*a^8*b^4 + 90*a^10*b^2))/(b^13*d^5 + 4*a^2*b^11*d^5 + 6*a^4*b^9*d^5 + 4*a^6*b^7*d^5 + a^8*b^5*d^5) + ((((1/(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^(1/2)*(((1/(a^4*d^2*1i + b^...

3.592 $\int \frac{\tan^{\frac{7}{2}}(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=358

$$-\frac{(a^2-2ab-b^2)\,\mathrm{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}+\frac{(a^2-2ab-b^2)\,\mathrm{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}-\frac{a^{5/}}{}$$

```
[Out] -a^(5/2)*(3*a^2+7*b^2)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/b^(5/2)/(a^
2+b^2)^2/d+1/2*(a^2-2*a*b-b^2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2
)^2/d*2^(1/2)+1/2*(a^2-2*a*b-b^2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b
^2)^2/d*2^(1/2)-1/4*(a^2+2*a*b-b^2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c
))/(a^2+b^2)^2/d*2^(1/2)+1/4*(a^2+2*a*b-b^2)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+
tan(d*x+c))/(a^2+b^2)^2/d*2^(1/2)+(3*a^2+2*b^2)*tan(d*x+c)^(1/2)/b^2/(a^2+b
^2)/d-a^2*tan(d*x+c)^(3/2)/b/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.50, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3646, 3728, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{(a^2-2ab-b^2)\,\mathrm{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{(a^2-2ab-b^2)\,\mathrm{ArcTan}\left(\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}-\frac{a^2\tan^{\frac{3}{2}}(c+dx)}{bd\,(a^2+b^2)\,(a+b\tan(c+dx))}+\frac{(3a^2+2b^2)\,\sqrt{\tan(c+dx)}}{b^2d\,(a^2+b^2)}-\frac{(a^2+2ab-b^2)\log\left(\tan(c+dx)-\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{(a^2+2ab-b^2)\log\left(\tan(c+dx)+\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}-\frac{a^{5/2}(3a^2+7b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{5/2}d\,(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(7/2)/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((a^2 - 2*a*b - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*(a^
2 + b^2)^2*d)) + ((a^2 - 2*a*b - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]
])/(Sqrt[2]*(a^2 + b^2)^2*d) - (a^(5/2)*(3*a^2 + 7*b^2)*ArcTan[(Sqrt[b]*Sqr
t[Tan[c + d*x]])/Sqrt[a]])/(b^(5/2)*(a^2 + b^2)^2*d) - ((a^2 + 2*a*b - b^2)
*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)
^2*d) + ((a^2 + 2*a*b - b^2)*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d
*x]])/(2*Sqrt[2]*(a^2 + b^2)^2*d) + ((3*a^2 + 2*b^2)*Sqrt[Tan[c + d*x]])/(b
^2*(a^2 + b^2)*d) - (a^2*Tan[c + d*x]^(3/2))/(b*(a^2 + b^2)*d*(a + b*Tan[c
+ d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
```

```
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\tan^{\frac{7}{2}}(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{a^2\tan^{\frac{3}{2}}(c+dx)}{b\,(a^2+b^2)\,d(a+b\tan(c+dx))} + \frac{\int \frac{\sqrt{\tan(c+dx)}\,\left(\frac{3a^2}{2}-ab\tan(c+dx)+\frac{1}{2}(3a^2+2b^2}{a+b\tan(c+dx)}}{b\,(a^2+b^2)}$$

$$= \frac{(3a^2+2b^2)\sqrt{\tan(c+dx)}}{b^2(a^2+b^2)\,d} - \frac{a^2\tan^{\frac{3}{2}}(c+dx)}{b\,(a^2+b^2)\,d(a+b\tan(c+dx))} + \frac{2\int \frac{-\frac{1}{4}a(3a^2+2}{\sqrt{}}}{}$$

$$= \frac{(3a^2+2b^2)\sqrt{\tan(c+dx)}}{b^2(a^2+b^2)\,d} - \frac{a^2\tan^{\frac{3}{2}}(c+dx)}{b\,(a^2+b^2)\,d(a+b\tan(c+dx))} + \frac{2\int \frac{\frac{1}{2}b^2(a^2-b^2)}{\sqrt{\tan(}}}{b^2\,(a^2}$$

$$= \frac{(3a^2+2b^2)\sqrt{\tan(c+dx)}}{b^2(a^2+b^2)\,d} - \frac{a^2\tan^{\frac{3}{2}}(c+dx)}{b\,(a^2+b^2)\,d(a+b\tan(c+dx))} + \frac{4\text{Subst}\left(\int \frac{1}{2}b^2\right.}{}$$

$$= \frac{(3a^2+2b^2)\sqrt{\tan(c+dx)}}{b^2(a^2+b^2)\,d} - \frac{a^2\tan^{\frac{3}{2}}(c+dx)}{b\,(a^2+b^2)\,d(a+b\tan(c+dx))} + \frac{(a^2-2ab-b}{}$$

$$= -\frac{a^{5/2}(3a^2+7b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{5/2}\,(a^2+b^2)^2\,d} + \frac{(3a^2+2b^2)\sqrt{\tan(c+dx)}}{b^2\,(a^2+b^2)\,d}$$

$$= -\frac{a^{5/2}(3a^2+7b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{5/2}\,(a^2+b^2)^2\,d} - \frac{(a^2+2ab-b^2)\log\left(1-\sqrt{2}\right.}{2\sqrt{2}\,(}$$

$$= -\frac{(a^2-2ab-b^2)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} + \frac{(a^2-2ab-b^2)\tan^{-1}\left(1\right.}{\sqrt{2}\,(a^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.22, size = 375, normalized size = 1.05

$$\frac{-3a^{11/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)-7a^{7/2}b^2\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)+3a^5\sqrt{b}\,\sqrt{\tan(c+dx)}+5a^3b^{5/2}\sqrt{\tan(c+dx)}+2ab^{9/2}\sqrt{\tan(c+dx)}-3a^{9/2}b\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\tan(c+dx)-7a^{5/2}b^3\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\tan(c+dx)+2a^4b^{3/2}\tan^{\frac{3}{2}}(c+dx)+4a^2b^{7/2}\tan^{\frac{3}{2}}(c+dx)+2b^{11/2}\tan^{\frac{3}{2}}(c+dx)+\sqrt[4]{-1}\,b^{5/2}(-ia+b)^2\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)(a+b\tan(c+dx))+\sqrt[4]{-1}\,b^{5/2}(ia+b)^2\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)(a+b\tan(c+dx))}{b^{5/2}\,(a^2+b^2)^2\,d(a+b\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(7/2)/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-3*a^(11/2)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]] - 7*a^(7/2)*b^2*A
rcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]] + 3*a^5*Sqrt[b]*Sqrt[Tan[c + d*
```

```
x]] + 5*a^3*b^(5/2)*Sqrt[Tan[c + d*x]] + 2*a*b^(9/2)*Sqrt[Tan[c + d*x]] - 3
*a^(9/2)*b*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Tan[c + d*x] - 7*a^
(5/2)*b^3*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Tan[c + d*x] + 2*a^4
*b^(3/2)*Tan[c + d*x]^(3/2) + 4*a^2*b^(7/2)*Tan[c + d*x]^(3/2) + 2*b^(11/2)
*Tan[c + d*x]^(3/2) + (-1)^(1/4)*b^(5/2)*((-I)*a + b)^2*ArcTan[(-1)^(3/4)*S
qrt[Tan[c + d*x]]]*(a + b*Tan[c + d*x]) + (-1)^(1/4)*b^(5/2)*(I*a + b)^2*Ar
cTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*(a + b*Tan[c + d*x]))/(b^(5/2)*(a^2 +
b^2)^2*d*(a + b*Tan[c + d*x]))
```

Maple [A]

time = 0.14, size = 296, normalized size = 0.83

method	result
derivativedivides	$\frac{\frac{2\left(\sqrt{\tan}\,(dx+c)\right)}{b^2}-\frac{2a^3\left(\frac{\left(-\frac{a^2}{2}-\frac{b^2}{2}\right)\left(\sqrt{\tan}\,(dx+c)\right)}{a+b\tan(dx+c)}+\frac{\left(3a^2+7b^2\right)\arctan\left(\frac{b\left(\sqrt{\tan}\,(dx+c)\right)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{\left(a^2+b^2\right)^2 b^2}+\frac{\left(a^2-b^2\right)\sqrt{2}\,\Big(\ln}{}}{}$
default	$\frac{\frac{2\left(\sqrt{\tan}\,(dx+c)\right)}{b^2}-\frac{2a^3\left(\frac{\left(-\frac{a^2}{2}-\frac{b^2}{2}\right)\left(\sqrt{\tan}\,(dx+c)\right)}{a+b\tan(dx+c)}+\frac{\left(3a^2+7b^2\right)\arctan\left(\frac{b\left(\sqrt{\tan}\,(dx+c)\right)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{\left(a^2+b^2\right)^2 b^2}+\frac{\left(a^2-b^2\right)\sqrt{2}\,\Big(\ln}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(7/2)/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b^2*tan(d*x+c)^(1/2)-2*a^3/(a^2+b^2)^2/b^2*((-1/2*a^2-1/2*b^2)*tan(d
*x+c)^(1/2)/(a+b*tan(d*x+c))+1/2*(3*a^2+7*b^2)/(a*b)^(1/2)*arctan(b*tan(d*x
+c)^(1/2)/(a*b)^(1/2)))+2/(a^2+b^2)^2*(1/8*(a^2-b^2)*2^(1/2)*(ln((1+2^(1/2)
*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*ar
ctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))-1/4
*a*b*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x
+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(
1/2)*tan(d*x+c)^(1/2)))))
```

Maxima [A]

time = 0.51, size = 296, normalized size = 0.83

$$\frac{\frac{4a^3\sqrt{\tan(dx+c)}}{a^3b^2+ab^4+(a^2b^3+b^5)\tan(dx+c)}-\frac{4(3a^5+7a^3b^2)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^4b^2+2a^2b^4+b^6)\sqrt{ab}}+\frac{2\sqrt{2}(a^2-2ab-b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}(a^2-2ab-b^2)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)+\sqrt{2}(a^2+2ab-b^2)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)-\sqrt{2}(a^2+2ab-b^2)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)}{a^4+2a^2b^2+b^4}+\frac{8\sqrt{\tan(dx+c)}}{b^2}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(7/2)/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(4*a^3*sqrt(tan(d*x + c))/(a^3*b^2 + a*b^4 + (a^2*b^3 + b^5)*tan(d*x +
c)) - 4*(3*a^5 + 7*a^3*b^2)*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^4*b^
2 + 2*a^2*b^4 + b^6)*sqrt(a*b)) + (2*sqrt(2)*(a^2 - 2*a*b - b^2)*arctan(1/2
*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*(a^2 - 2*a*b - b^2)*
arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)*(a^2 + 2*a*
b - b^2)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*(a^2
+ 2*a*b - b^2)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a^4 +
2*a^2*b^2 + b^4) + 8*sqrt(tan(d*x + c))/b^2)/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 7100 vs. $2(316) = 632$.
time = 13.87, size = 14311, normalized size = 39.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(7/2)/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(2)*((a^14*b^2 + 5*a^12*b^4 + 9*a^10*b^6 + 5*a^8*b^8 - 5*a^6*b^
10 - 9*a^4*b^12 - 5*a^2*b^14 - b^16)*d^5*cos(d*x + c)^2 + 2*(a^13*b^3 + 6*a
^11*b^5 + 15*a^9*b^7 + 20*a^7*b^9 + 15*a^5*b^11 + 6*a^3*b^13 + a*b^15)*d^5*
cos(d*x + c)*sin(d*x + c) + (a^12*b^4 + 6*a^10*b^6 + 15*a^8*b^8 + 20*a^6*b^
10 + 15*a^4*b^12 + 6*a^2*b^14 + b^16)*d^5)*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^
4 + 4*a^2*b^6 + b^8 - 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 - 2*a^5*b^7 - 3*a^3
*b^9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*
d^4)))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt((a^8 - 12*a
^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 +
 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*
d^4))*(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))^(3/4)*arcta
n(((a^16 - 20*a^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 - 64*a^6*b^10 - 20*a^4*b^
12 + b^16)*d^4*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^
16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28
*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4))*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
 4*a^2*b^6 + b^8)*d^4)) - sqrt(2)*((a^18 + 7*a^16*b^2 + 20*a^14*b^4 + 28*a^
12*b^6 + 14*a^10*b^8 - 14*a^8*b^10 - 28*a^6*b^12 - 20*a^4*b^14 - 7*a^2*b^16
 - b^18)*d^7*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^16
 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a
^4*b^12 + 8*a^2*b^14 + b^16)*d^4))*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4
*a^2*b^6 + b^8)*d^4)) + 2*(a^13*b + 6*a^11*b^3 + 15*a^9*b^5 + 20*a^7*b^7 +
15*a^5*b^9 + 6*a^3*b^11 + a*b^13)*d^5*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 -
 12*a^2*b^6 + b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8
*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4)))*sqrt((a^8 + 4*
a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 - 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 -
 2*a^5*b^7 - 3*a^3*b^9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
```

```
4*a^2*b^6 + b^8)*d^4)))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)
)*sqrt(((a^12 - 10*a^10*b^2 + 15*a^8*b^4 + 52*a^6*b^6 + 15*a^4*b^8 - 10*a^2
*b^10 + b^12)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d
^4))*cos(d*x + c) + sqrt(2)*(2*(a^13*b - 10*a^11*b^3 + 15*a^9*b^5 + 52*a^7*
b^7 + 15*a^5*b^9 - 10*a^3*b^11 + a*b^13)*d^3*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a
^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*cos(d*x + c) + (a^10 - 13*a^8*b^2 + 50*a^6*
b^4 - 50*a^4*b^6 + 13*a^2*b^8 - b^10)*d*cos(d*x + c))*sqrt((a^8 + 4*a^6*b^2
 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 - 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 - 2*a^5*
b^7 - 3*a^3*b^9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*
b^6 + b^8)*d^4)))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(
sin(d*x + c)/cos(d*x + c))*(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b
^8)*d^4))^(1/4) + (a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*sin(d*
x + c))/cos(d*x + c))*(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d
^4))^(3/4) + sqrt(2)*((a^22 + a^20*b^2 - 21*a^18*b^4 - 85*a^16*b^6 - 134*a^
14*b^8 - 70*a^12*b^10 + 70*a^10*b^12 + 134*a^8*b^14 + 85*a^6*b^16 + 21*a^4*
b^18 - a^2*b^20 - b^22)*d^7*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^
6 + b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*
a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4))*sqrt(1/((a^8 + 4*a^6*b^2
+ 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)) + 2*(a^17*b - 20*a^13*b^5 - 64*a^11*b^
7 - 90*a^9*b^9 - 64*a^7*b^11 - 20*a^5*b^13 + a*b^17)*d^5*sqrt((a^8 - 12*a^6
*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 5
6*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^
4)))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 - 4*(a^11*b + 3*a^
9*b^3 + 2*a^7*b^5 - 2*a^5*b^7 - 3*a^3*b^9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^
6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4
- 12*a^2*b^6 + b^8))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a^8 + 4*a^6*b^2 +
 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))^(3/4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 -
 12*a^2*b^6 + b^8)) + 4*sqrt(2)*((a^14*b^2 + 5*a^12*b^4 + 9*a^10*b^6 + 5*a^
8*b^8 - 5*a^6*b^10 - 9*a^4*b^12 - 5*a^2*b^14 - b^16)*d^5*cos(d*x + c)^2 + 2
*(a^13*b^3 + 6*a^11*b^5 + 15*a^9*b^7 + 20*a^7*b^9 + 15*a^5*b^11 + 6*a^3*b^1
3 + a*b^15)*d^5*cos(d*x + c)*sin(d*x + c) + (a^12*b^4 + 6*a^10*b^6 + 15*a^8
*b^8 + 20*a^6*b^10 + 15*a^4*b^12 + 6*a^2*b^14 + b^16)*d^5)*sqrt((a^8 + 4*a^
6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 - 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 - 2
*a^5*b^7 - 3*a^3*b^9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4
*a^2*b^6 + b^8)*d^4)))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*
sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^16 + 8*a^14*b^2
 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a
^2*b^14 + b^16)*d^4))*(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d
^4))^(3/4)*arctan(-((a^16 - 20*a^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 - 64*a^6
*b^10 - 20*a^4*b^12 + b^16)*d^4*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^
2*b^6 + b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)**(7/2)/(a+b*tan(d*x+c))**2,x)

[Out] Timed out

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^(7/2)/(a+b*tan(d*x+c))^2,x, algorithm="giac")

[Out] Timed out

Mupad [**B**]
time = 12.62, size = 2500, normalized size = 6.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d*x)^(7/2)/(a + b*tan(c + d*x))^2,x)

```
[Out] atan(((-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*
d^2)))^(1/2)*((-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*
a^2*b^2*d^2)))^(1/2)*((16*(30*a^6*b^8*d^2 - 224*a^4*b^10*d^2 - 18*a^14*d^2
+ 600*a^8*b^6*d^2 + 388*a^10*b^4*d^2 + 24*a^12*b^2*d^2))/(b^11*d^5 + 4*a^2*
b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5) - (-1i/(4*(a^4*d^2 +
 b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((-1i/(4*(a
^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((1
6*(8*a*b^17*d^4 + 96*a^3*b^15*d^4 + 360*a^5*b^13*d^4 + 640*a^7*b^11*d^4 + 6
00*a^9*b^9*d^4 + 288*a^11*b^7*d^4 + 56*a^13*b^5*d^4))/(b^11*d^5 + 4*a^2*b^9
*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5) - (16*tan(c + d*x)^(1/2
)*(-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)
))^(1/2)*(32*b^20*d^4 + 160*a^2*b^18*d^4 + 288*a^4*b^16*d^4 + 160*a^6*b^14*
d^4 - 160*a^8*b^12*d^4 - 288*a^10*b^10*d^4 - 160*a^12*b^8*d^4 - 32*a^14*b^6
*d^4))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*
d^4)) + (16*tan(c + d*x)^(1/2)*(72*a^15*b*d^2 - 60*a*b^15*d^2 - 52*a^3*b^13
*d^2 + 72*a^5*b^11*d^2 + 448*a^7*b^9*d^2 + 1108*a^9*b^7*d^2 + 1132*a^11*b^5
*d^2 + 480*a^13*b^3*d^2))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6
*b^5*d^4 + a^8*b^3*d^4))) + (16*tan(c + d*x)^(1/2)*(9*a^12 + 2*b^12 + 4*a^2
*b^10 + 2*a^4*b^8 - 49*a^6*b^6 + 7*a^8*b^4 + 33*a^10*b^2))/(b^11*d^4 + 4*a^
2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))*1i - (-1i/(4*(a^4
*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((-1i
/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/
```

2)*((16*(30*a^6*b^8*d^2 - 224*a^4*b^10*d^2 - 18*a^14*d^2 + 600*a^8*b^6*d^2 + 388*a^10*b^4*d^2 + 24*a^12*b^2*d^2))/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5) - (-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((16*(8*a*b^17*d^4 + 96*a^3*b^15*d^4 + 360*a^5*b^13*d^4 + 640*a^7*b^11*d^4 + 600*a^9*b^9*d^4 + 288*a^11*b^7*d^4 + 56*a^13*b^5*d^4))/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5) + (16*tan(c + d*x)^(1/2)*(-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(32*b^20*d^4 + 160*a^2*b^18*d^4 + 288*a^4*b^16*d^4 + 160*a^6*b^14*d^4 - 160*a^8*b^12*d^4 - 288*a^10*b^10*d^4 - 160*a^12*b^8*d^4 - 32*a^14*b^6*d^4))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4)) - (16*tan(c + d*x)^(1/2)*(72*a^15*b*d^2 - 60*a*b^15*d^2 - 52*a^3*b^13*d^2 + 72*a^5*b^11*d^2 + 448*a^7*b^9*d^2 + 1108*a^9*b^7*d^2 + 1132*a^11*b^5*d^2 + 480*a^13*b^3*d^2))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))) - (16*tan(c + d*x)^(1/2)*(9*a^12 + 2*b^12 + 4*a^2*b^10 + 2*a^4*b^8 - 49*a^6*b^6 + 7*a^8*b^4 + 33*a^10*b^2))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))*1i)/((-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((16*(30*a^6*b^8*d^2 - 224*a^4*b^10*d^2 - 18*a^14*d^2 + 600*a^8*b^6*d^2 + 388*a^10*b^4*d^2 + 24*a^12*b^2*d^2))/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5) - (-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((16*(8*a*b^17*d^4 + 96*a^3*b^15*d^4 + 360*a^5*b^13*d^4 + 640*a^7*b^11*d^4 + 600*a^9*b^9*d^4 + 288*a^11*b^7*d^4 + 56*a^13*b^5*d^4))/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5) - (16*tan(c + d*x)^(1/2)*(-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(32*b^20*d^4 + 160*a^2*b^18*d^4 + 288*a^4*b^16*d^4 + 160*a^6*b^14*d^4 - 160*a^8*b^12*d^4 - 288*a^10*b^10*d^4 - 160*a^12*b^8*d^4 - 32*a^14*b^6*d^4))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4)) + (16*tan(c + d*x)^(1/2)*(72*a^15*b*d^2 - 60*a*b^15*d^2 - 52*a^3*b^13*d^2 + 72*a^5*b^11*d^2 + 448*a^7*b^9*d^2 + 1108*a^9*b^7*d^2 + 1132*a^11*b^5*d^2 + 480*a^13*b^3*d^2))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))) + (16*tan(c + d*x)^(1/2)*(9*a^12 + 2*b^12 + 4*a^2*b^10 + 2*a^4*b^8 - 49*a^6*b^6 + 7*a^8*b^4 + 33*a^10*b^2))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4)) + (-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((16*(30*a^6*b^8*d^2 - 224*a^4*b^10*d^2 - 18*a^14*d^2 + 600*a^8*b^6*d^2 + 388*a^10*b^4*d^2 + 24*a^12*b^2*d^2))/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5) - (-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((16*(8*a*b^17*d^4 + 96*a^3*b^15...

3.593 $\int \frac{\tan^{\frac{5}{2}}(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=318

$$\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}-\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}+\frac{a^{3/}}{}$$

```
[Out] a^(3/2)*(a^2+5*b^2)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/b^(3/2)/(a^2+b
^2)^2/d-1/2*(a^2+2*a*b-b^2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)^2
/d*2^(1/2)-1/2*(a^2+2*a*b-b^2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)
^2/d*2^(1/2)-1/4*(a^2-2*a*b-b^2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/
(a^2+b^2)^2/d*2^(1/2)+1/4*(a^2-2*a*b-b^2)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan
(d*x+c))/(a^2+b^2)^2/d*2^(1/2)-a^2*tan(d*x+c)^(1/2)/b/(a^2+b^2)/d/(a+b*tan(
d*x+c))
```

Rubi [A]
time = 0.33, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3646, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}-\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}-\frac{a^2\sqrt{\tan(c+dx)}}{bd\,(a^2+b^2)\,(a+b\tan(c+dx))}-\frac{(a^2-2ab-b^2)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{(a^2-2ab-b^2)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{a^{3/2}(a^2+5b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{3/2}d\,(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(5/2)/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((a^2 + 2*a*b - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*(a^2
 + b^2)^2*d) - ((a^2 + 2*a*b - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/
(Sqrt[2]*(a^2 + b^2)^2*d) + (a^(3/2)*(a^2 + 5*b^2)*ArcTan[(Sqrt[b]*Sqrt[Tan
[c + d*x]])/Sqrt[a]])/(b^(3/2)*(a^2 + b^2)^2*d) - ((a^2 - 2*a*b - b^2)*Log[
1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^2*d)
 + ((a^2 - 2*a*b - b^2)*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])
/(2*Sqrt[2]*(a^2 + b^2)^2*d) - (a^2*Sqrt[Tan[c + d*x]])/(b*(a^2 + b^2)*d*(a
 + b*Tan[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{a^2\sqrt{\tan(c+dx)}}{b\,(a^2+b^2)\,d(a+b\tan(c+dx))} + \frac{\int \frac{\frac{a^2}{2}-ab\tan(c+dx)+\frac{1}{2}(a^2+2b^2)\tan^2(c+dx)}{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))}\,dx}{b\,(a^2+b^2)}$$

$$= -\frac{a^2\sqrt{\tan(c+dx)}}{b\,(a^2+b^2)\,d(a+b\tan(c+dx))} + \frac{\int \frac{-2ab^2-b(a^2-b^2)\tan(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{b\,(a^2+b^2)^2} + \frac{(a^2(a^2+5b^2)}{}$$

$$= -\frac{a^2\sqrt{\tan(c+dx)}}{b\,(a^2+b^2)\,d(a+b\tan(c+dx))} + \frac{2\text{Subst}\left(\int \frac{-2ab^2-b(a^2-b^2)x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+}\right.}{b\,(a^2+b^2)^2\,d}$$

$$= -\frac{a^2\sqrt{\tan(c+dx)}}{b\,(a^2+b^2)\,d(a+b\tan(c+dx))} + \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(}\right.}{(a^2+b^2)^2\,d}$$

$$= \frac{a^{3/2}(a^2+5b^2)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{3/2}\,(a^2+b^2)^2\,d} - \frac{a^2\sqrt{\tan(c+dx)}}{b\,(a^2+b^2)\,d(a+b\tan(c+dx))}$$

$$= \frac{a^{3/2}(a^2+5b^2)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{3/2}\,(a^2+b^2)^2\,d} - \frac{(a^2-2ab-b^2)\log\left(1-\sqrt{2}\sqrt{t\right.}{2\sqrt{2}\,(a^2+}$$

$$= \frac{(a^2+2ab-b^2)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} - \frac{(a^2+2ab-b^2)\tan^{-1}\left(1+\right.}{\sqrt{2}\,(a^2+b}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.72, size = 157, normalized size = 0.49

$$\frac{(-1)^{3/4}(-ia+b)^2\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+\frac{a^{3/2}(a^2+5b^2)\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{3/2}}+(-1)^{3/4}(a-ib)^2\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)-\frac{a^2(a^2+b^2)\sqrt{\tan(c+dx)}}{b(a+b\tan(c+dx))}}{(a^2+b^2)^2\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(5/2)/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((-1)^(3/4)*((-I)*a + b)^2*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + (a^(3/2)
*(a^2 + 5*b^2)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/b^(3/2) + (-1)
^(3/4)*(a - I*b)^2*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] - (a^2*(a^2 + b^2
)*Sqrt[Tan[c + d*x]])/(b*(a + b*Tan[c + d*x])))/((a^2 + b^2)^2*d)
```

Maple [A]
time = 0.13, size = 281, normalized size = 0.88

method	result
derivativedivides	$\frac{2a^2\left(-\frac{(a^2+b^2)\left(\sqrt{\tan}(dx+c)\right)}{2b(a+b\tan(dx+c))}+\frac{(a^2+5b^2)\arctan\left(\frac{b\left(\sqrt{\tan}(dx+c)\right)}{\sqrt{ab}}\right)}{2b\sqrt{ab}}\right)}{(a^2+b^2)^2}+\frac{-ab\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)}\right)\right)}{}$
default	$\frac{2a^2\left(-\frac{(a^2+b^2)\left(\sqrt{\tan}(dx+c)\right)}{2b(a+b\tan(dx+c))}+\frac{(a^2+5b^2)\arctan\left(\frac{b\left(\sqrt{\tan}(dx+c)\right)}{\sqrt{ab}}\right)}{2b\sqrt{ab}}\right)}{(a^2+b^2)^2}+\frac{-ab\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)}\right)\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*a^2/(a^2+b^2)^2*(-1/2*(a^2+b^2)/b*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))+
1/2*(a^2+5*b^2)/b/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2)))+2/(a^
2+b^2)^2*(-1/4*a*b*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2
^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2
*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/8*(-a^2+b^2)*2^(1/2)*(ln((1-2^(1/2)
*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*ar
ctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))))
```

Maxima [A]

time = 0.50, size = 277, normalized size = 0.87

$$-\frac{\frac{4a^2\sqrt{\tan(dx+c)}}{a^3b+ab^3+(a^2b^2+b^4)\tan(dx+c)}-\frac{4(a^4+5a^2b^2)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^4b+2a^2b^3+b^5)\sqrt{ab}}+\frac{2\sqrt{2}(a^2+2ab-b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}(a^2+2ab-b^2)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-\sqrt{2}(a^2-2ab-b^2)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+\sqrt{2}(a^2-2ab-b^2)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)}{a^4+2a^2b^2+b^4}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(4*a^2*sqrt(tan(d*x + c))/(a^3*b + a*b^3 + (a^2*b^2 + b^4)*tan(d*x + c
)) - 4*(a^4 + 5*a^2*b^2)*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^4*b + 2
*a^2*b^3 + b^5)*sqrt(a*b)) + (2*sqrt(2)*(a^2 + 2*a*b - b^2)*arctan(1/2*sqrt
(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*(a^2 + 2*a*b - b^2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*(a^2 - 2*a*b - b
^2)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*(a^2 - 2*a
*b - b^2)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a^4 + 2*a^2
*b^2 + b^4))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 7073 vs. $2(278) = 556$.
time = 11.67, size = 14258, normalized size = 44.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(2)*((a^14*b + 5*a^12*b^3 + 9*a^10*b^5 + 5*a^8*b^7 - 5*a^6*b^9
 - 9*a^4*b^11 - 5*a^2*b^13 - b^15)*d^5*cos(d*x + c)^2 + 2*(a^13*b^2 + 6*a^1
1*b^4 + 15*a^9*b^6 + 20*a^7*b^8 + 15*a^5*b^10 + 6*a^3*b^12 + a*b^14)*d^5*co
s(d*x + c)*sin(d*x + c) + (a^12*b^3 + 6*a^10*b^5 + 15*a^8*b^7 + 20*a^6*b^9
+ 15*a^4*b^11 + 6*a^2*b^13 + b^15)*d^5)*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
 4*a^2*b^6 + b^8 + 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 - 2*a^5*b^7 - 3*a^3*b^
9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4
)))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt((a^8 - 12*a^6*
b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56
*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4
))*(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))^(3/4)*arctan((
(a^16 - 20*a^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 - 64*a^6*b^10 - 20*a^4*b^12
+ b^16)*d^4*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^16
+ 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^
4*b^12 + 8*a^2*b^14 + b^16)*d^4))*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*
a^2*b^6 + b^8)*d^4)) + sqrt(2)*(2*(a^17*b + 8*a^15*b^3 + 28*a^13*b^5 + 56*a
^11*b^7 + 70*a^9*b^9 + 56*a^7*b^11 + 28*a^5*b^13 + 8*a^3*b^15 + a*b^17)*d^7
*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^16 + 8*a^14*b^
2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*
a^2*b^14 + b^16)*d^4))*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b
^8)*d^4)) - (a^14 + 5*a^12*b^2 + 9*a^10*b^4 + 5*a^8*b^6 - 5*a^6*b^8 - 9*a^4
*b^10 - 5*a^2*b^12 - b^14)*d^5*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2
*b^6 + b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 +
56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4)))*sqrt((a^8 + 4*a^6*b^2
 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 - 2*a^5*
b^7 - 3*a^3*b^9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*
b^6 + b^8)*d^4)))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(
((a^12 - 10*a^10*b^2 + 15*a^8*b^4 + 52*a^6*b^6 + 15*a^4*b^8 - 10*a^2*b^10 +
 b^12)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*co
s(d*x + c) + sqrt(2)*((a^14 - 11*a^12*b^2 + 25*a^10*b^4 + 37*a^8*b^6 - 37*a
^6*b^8 - 25*a^4*b^10 + 11*a^2*b^12 - b^14)*d^3*sqrt(1/((a^8 + 4*a^6*b^2 + 6
*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*cos(d*x + c) - 2*(a^9*b - 12*a^7*b^3 + 38
*a^5*b^5 - 12*a^3*b^7 + a*b^9)*d*cos(d*x + c))*sqrt((a^8 + 4*a^6*b^2 + 6*a^
4*b^4 + 4*a^2*b^6 + b^8 + 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 - 2*a^5*b^7 - 3
*a^3*b^9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b
^8)*d^4)))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(sin(d*x
```

```
 + c)/cos(d*x + c))*(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4
))^(1/4) + (a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*sin(d*x + c))
/cos(d*x + c))*(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))^(3
/4) - sqrt(2)*(2*(a^21*b + 2*a^19*b^3 - 19*a^17*b^5 - 104*a^15*b^7 - 238*a^
13*b^9 - 308*a^11*b^11 - 238*a^9*b^13 - 104*a^7*b^15 - 19*a^5*b^17 + 2*a^3*
b^19 + a*b^21)*d^7*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/
((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10
+ 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4))*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b
^4 + 4*a^2*b^6 + b^8)*d^4)) - (a^18 - a^16*b^2 - 20*a^14*b^4 - 44*a^12*b^6
- 26*a^10*b^8 + 26*a^8*b^10 + 44*a^6*b^12 + 20*a^4*b^14 + a^2*b^16 - b^18)*
d^5*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^16 + 8*a^14
*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 +
 8*a^2*b^14 + b^16)*d^4)))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 +
b^8 + 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 - 2*a^5*b^7 - 3*a^3*b^9 - a*b^11)*d
^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))/(a^8 - 12
*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(sin(d*x + c)/cos(d*x + c))*
(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))^(3/4))/(a^8 - 12*
a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)) + 4*sqrt(2)*((a^14*b + 5*a^12*b^3
 + 9*a^10*b^5 + 5*a^8*b^7 - 5*a^6*b^9 - 9*a^4*b^11 - 5*a^2*b^13 - b^15)*d^5
*cos(d*x + c)^2 + 2*(a^13*b^2 + 6*a^11*b^4 + 15*a^9*b^6 + 20*a^7*b^8 + 15*a
^5*b^10 + 6*a^3*b^12 + a*b^14)*d^5*cos(d*x + c)*sin(d*x + c) + (a^12*b^3 +
6*a^10*b^5 + 15*a^8*b^7 + 20*a^6*b^9 + 15*a^4*b^11 + 6*a^2*b^13 + b^15)*d^5
)*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 4*(a^11*b + 3*a^9*b
^3 + 2*a^7*b^5 - 2*a^5*b^7 - 3*a^3*b^9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b
^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 1
2*a^2*b^6 + b^8))*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(
(a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 +
 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4))*(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
4*a^2*b^6 + b^8)*d^4))^(3/4)*arctan(-((a^16 - 20*a^12*b^4 - 64*a^10*b^6 - 9
0*a^8*b^8 - 64*a^6*b^10 - 20*a^4*b^12 + b^16)*d^4*sqrt((a^8 - 12*a^6*b^2 +
38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^16 + 8*a^14*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(5/2)/(a+b*tan(d*x+c))**2,x)
[Out] Integral(tan(c + d*x)**(5/2)/(a + b*tan(c + d*x))**2, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^2,x, algorithm="giac")

[Out] Timed out

Mupad [B]
time = 6.76, size = 2500, normalized size = 7.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d*x)^(5/2)/(a + b*tan(c + d*x))^2,x)

[Out] - atan(((1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2 *d^2)))^(1/2)*((1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6* a^2*b^2*d^2)))^(1/2)*((1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2* 4i - 6*a^2*b^2*d^2)))^(1/2)*(((8*(96*a^2*b^14*d^4 + 480*a^4*b^12*d^4 + 960* a^6*b^10*d^4 + 960*a^8*b^8*d^4 + 480*a^10*b^6*d^4 + 96*a^12*b^4*d^4))/(b^9* d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5) - (16*tan(c + d*x)^(1/2)*(1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6* a^2*b^2*d^2)))^(1/2)*(32*b^18*d^4 + 160*a^2*b^16*d^4 + 288*a^4*b^14*d^4 + 1 60*a^6*b^12*d^4 - 160*a^8*b^10*d^4 - 288*a^10*b^8*d^4 - 160*a^12*b^6*d^4 - 32*a^14*b^4*d^4))/(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4* a^6*b^3*d^4))*(1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a ^2*b^2*d^2)))^(1/2) + (16*tan(c + d*x)^(1/2)*(60*a*b^13*d^2 + 8*a^13*b*d^2 + 52*a^3*b^11*d^2 + 128*a^5*b^9*d^2 + 424*a^7*b^7*d^2 + 380*a^9*b^5*d^2 + 1 00*a^11*b^3*d^2))/(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4* a^6*b^3*d^4)) + (8*(4*a*b^11*d^2 + 16*a^11*b*d^2 - 304*a^3*b^9*d^2 - 120*a^ 5*b^7*d^2 + 320*a^7*b^5*d^2 + 148*a^9*b^3*d^2))/(b^9*d^5 + a^8*b*d^5 + 4*a^ 2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5)) + (16*tan(c + d*x)^(1/2)*(a^10 - 2*b^10 - 4*a^2*b^8 - 27*a^4*b^6 + 15*a^6*b^4 + 9*a^8*b^2))/(b^9*d^4 + a^8 *b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4))*1i - (1i/(4*(a^4*d ^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((1i/(4 *(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)* ((1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2))) ^(1/2)*(((8*(96*a^2*b^14*d^4 + 480*a^4*b^12*d^4 + 960*a^6*b^10*d^4 + 960*a^ 8*b^8*d^4 + 480*a^10*b^6*d^4 + 96*a^12*b^4*d^4))/(b^9*d^5 + a^8*b*d^5 + 4*a ^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5) + (16*tan(c + d*x)^(1/2)*(1i/(4 *(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)* (32*b^18*d^4 + 160*a^2*b^16*d^4 + 288*a^4*b^14*d^4 + 160*a^6*b^12*d^4 - 160 *a^8*b^10*d^4 - 288*a^10*b^8*d^4 - 160*a^12*b^6*d^4 - 32*a^14*b^4*d^4))/(b^ 9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4))*(1i/(4* (a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) - (16*tan(c + d*x)^(1/2)*(60*a*b^13*d^2 + 8*a^13*b*d^2 + 52*a^3*b^11*d^2 + 1 28*a^5*b^9*d^2 + 424*a^7*b^7*d^2 + 380*a^9*b^5*d^2 + 100*a^11*b^3*d^2))/(b^

9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4)) + (8*(4
*a*b^11*d^2 + 16*a^11*b*d^2 - 304*a^3*b^9*d^2 - 120*a^5*b^7*d^2 + 320*a^7*b
^5*d^2 + 148*a^9*b^3*d^2))/(b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5
*d^5 + 4*a^6*b^3*d^5)) - (16*tan(c + d*x)^(1/2)*(a^10 - 2*b^10 - 4*a^2*b^8
- 27*a^4*b^6 + 15*a^6*b^4 + 9*a^8*b^2))/(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^
4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4))*1i)/((1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*
d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((1i/(4*(a^4*d^2 + b^4*d^2 +
a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((1i/(4*(a^4*d^2 + b^
4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(((8*(96*a^2*b
^14*d^4 + 480*a^4*b^12*d^4 + 960*a^6*b^10*d^4 + 960*a^8*b^8*d^4 + 480*a^10*
b^6*d^4 + 96*a^12*b^4*d^4))/(b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^
5*d^5 + 4*a^6*b^3*d^5) - (16*tan(c + d*x)^(1/2)*(1i/(4*(a^4*d^2 + b^4*d^2 +
a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(32*b^18*d^4 + 160*a^
2*b^16*d^4 + 288*a^4*b^14*d^4 + 160*a^6*b^12*d^4 - 160*a^8*b^10*d^4 - 288*a
^10*b^8*d^4 - 160*a^12*b^6*d^4 - 32*a^14*b^4*d^4))/(b^9*d^4 + a^8*b*d^4 + 4
*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4))*(1i/(4*(a^4*d^2 + b^4*d^2 +
a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) + (16*tan(c + d*x)^(1/
2)*(60*a*b^13*d^2 + 8*a^13*b*d^2 + 52*a^3*b^11*d^2 + 128*a^5*b^9*d^2 + 424*
a^7*b^7*d^2 + 380*a^9*b^5*d^2 + 100*a^11*b^3*d^2))/(b^9*d^4 + a^8*b*d^4 + 4
*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4)) + (8*(4*a*b^11*d^2 + 16*a^11
*b*d^2 - 304*a^3*b^9*d^2 - 120*a^5*b^7*d^2 + 320*a^7*b^5*d^2 + 148*a^9*b^3*
d^2))/(b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5)
) + (16*tan(c + d*x)^(1/2)*(a^10 - 2*b^10 - 4*a^2*b^8 - 27*a^4*b^6 + 15*a^6
*b^4 + 9*a^8*b^2))/(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4
*a^6*b^3*d^4)) + (1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i -
6*a^2*b^2*d^2)))^(1/2)*((1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^
2*4i - 6*a^2*b^2*d^2)))^(1/2)*((1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a
^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(((8*(96*a^2*b^14*d^4 + 480*a^4*b^12*d
^4 + 960*a^6*b^10*d^4 + 960*a^8*b^8*d^4 + 480*a^10*b^6*d^4 + 96*a^12*b^4*d^
4))/(b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5) +
(16*tan(c + d*x)^(1/2)*(1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^
2*4i - 6*a^2*b^2*d^2)))^(1/2)*(32*b^18*d^4 + 160*a^2*b^16*d^4 + 288*a^4*b^1
4*d^4 + 160*a^6*b^12*d^4 - 160*a^8*b^10*d^4 - 288*a^10*b^8*d^4 - 160*a^12*b
^6*d^4 - 32*a^14*b^4*d^4))/(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5
*d^4 + 4*a^6*b^3*d^4))*(1i/(4*(a^4*d^2 + b^4*d^...

3.594 $\int \frac{\tan^{\frac{3}{2}}(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=312

$$\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}-\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}+\frac{\sqrt{a}}{}$$

```
[Out] -1/2*(a^2-2*a*b-b^2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)^2/d*2^(1
/2)-1/2*(a^2-2*a*b-b^2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)^2/d*2^
(1/2)+1/4*(a^2+2*a*b-b^2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^
2)^2/d*2^(1/2)-1/4*(a^2+2*a*b-b^2)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)
)/(a^2+b^2)^2/d*2^(1/2)+(a^2-3*b^2)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2)
)*a^(1/2)/(a^2+b^2)^2/d/b^(1/2)+a*tan(d*x+c)^(1/2)/(a^2+b^2)/d/(a+b*tan(d*x
+c))
```

Rubi [A]
time = 0.30, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3648, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{\sqrt{a}\,(a^2-3b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b}\,d\,(a^2+b^2)^2}+\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}-\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{a\sqrt{\tan(c+dx)}}{d\,(a^2+b^2)\,(a+b\tan(c+dx))}+\frac{(a^2+2ab-b^2)\log\left(\tan(c+dx)-\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}-\frac{(a^2+2ab-b^2)\log\left(\tan(c+dx)+\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(3/2)/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((a^2 - 2*a*b - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*(a^2
+ b^2)^2*d) - ((a^2 - 2*a*b - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/
(Sqrt[2]*(a^2 + b^2)^2*d) + (Sqrt[a]*(a^2 - 3*b^2)*ArcTan[(Sqrt[b]*Sqrt[Tan
[c + d*x]])/Sqrt[a]])/(Sqrt[b]*(a^2 + b^2)^2*d) + ((a^2 + 2*a*b - b^2)*Log[
1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^2*d)
 - ((a^2 + 2*a*b - b^2)*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])
/(2*Sqrt[2]*(a^2 + b^2)^2*d) + (a*Sqrt[Tan[c + d*x]])/((a^2 + b^2)*d*(a + b
*Tan[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{a\sqrt{\tan(c+dx)}}{(a^2+b^2)\,d(a+b\tan(c+dx))} - \frac{\int \frac{\frac{a}{2}-b\tan(c+dx)-\frac{1}{2}a\tan^2(c+dx)}{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))}\,dx}{a^2+b^2}$$

$$= \frac{a\sqrt{\tan(c+dx)}}{(a^2+b^2)\,d(a+b\tan(c+dx))} - \frac{\int \frac{a^2-b^2-2ab\tan(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{(a^2+b^2)^2} + \frac{(a(a^2-3b^2))\int \frac{}{\sqrt{t}}}{2}$$

$$= \frac{a\sqrt{\tan(c+dx)}}{(a^2+b^2)\,d(a+b\tan(c+dx))} - \frac{2\text{Subst}\left(\int \frac{a^2-b^2-2abx^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{(a^2+b^2)^2\,d}$$

$$= \frac{a\sqrt{\tan(c+dx)}}{(a^2+b^2)\,d(a+b\tan(c+dx))} + \frac{(a(a^2-3b^2))\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{\tan(c+}\right.}{(a^2+b^2)^2\,d}$$

$$= \frac{\sqrt{a}\,(a^2-3b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b}\,(a^2+b^2)^2\,d} + \frac{a\sqrt{\tan(c+dx)}}{(a^2+b^2)\,d(a+b\tan(c+dx))}$$

$$= \frac{\sqrt{a}\,(a^2-3b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b}\,(a^2+b^2)^2\,d} + \frac{(a^2+2ab-b^2)\log\left(1-\sqrt{2}\,\sqrt{}\right.}{2\sqrt{2}\,(a^2}$$

$$= \frac{(a^2-2ab-b^2)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} - \frac{(a^2-2ab-b^2)\tan^{-1}\left(1+\right.}{\sqrt{2}\,(a^2+}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.30, size = 151, normalized size = 0.48

$$\frac{\sqrt[4]{-1}\,(a+ib)^2\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right) + \frac{\sqrt{a}\,(a^2-3b^2)\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b}} + \sqrt[4]{-1}\,(a-ib)^2\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right) + \frac{a(a^2+b^2)\sqrt{\tan(c+dx)}}{a+b\tan(c+dx)}}{(a^2+b^2)^2\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(3/2)/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((-1)^(1/4)*(a + I*b)^2*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + (Sqrt[a]*(a
^2 - 3*b^2)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/Sqrt[b] + (-1)^(1
/4)*(a - I*b)^2*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + (a*(a^2 + b^2)*Sqr
t[Tan[c + d*x]])/(a + b*Tan[c + d*x]))/((a^2 + b^2)^2*d)
```

Maple [A]
time = 0.12, size = 276, normalized size = 0.88

method	result
derivativedivides	$\frac{2a\left(\frac{\left(\frac{a^2}{2}+\frac{b^2}{2}\right)\left(\sqrt{\tan}\,(dx+c)\right)}{a+b\tan(dx+c)}+\frac{\left(a^2-3b^2\right)\arctan\left(\frac{b\left(\sqrt{\tan}\,(dx+c)\right)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{\left(a^2+b^2\right)^2}+\frac{\left(-a^2+b^2\right)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}\,(dx\right.}{1-\sqrt{2}\left(\sqrt{\tan}\,(dx\right.}\right.\right.}{}$
default	$\frac{2a\left(\frac{\left(\frac{a^2}{2}+\frac{b^2}{2}\right)\left(\sqrt{\tan}\,(dx+c)\right)}{a+b\tan(dx+c)}+\frac{\left(a^2-3b^2\right)\arctan\left(\frac{b\left(\sqrt{\tan}\,(dx+c)\right)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{\left(a^2+b^2\right)^2}+\frac{\left(-a^2+b^2\right)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}\,(dx\right.}{1-\sqrt{2}\left(\sqrt{\tan}\,(dx\right.}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*a/(a^2+b^2)^2*((1/2*a^2+1/2*b^2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))+1
/2*(a^2-3*b^2)/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2)))+2/(a^2+b
^2)^2*(1/8*(-a^2+b^2)*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(
1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2)
)+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*a*b*2^(1/2)*(ln((1-2^(1/2)*tan
(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan
(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))))
```

Maxima [A]

time = 0.51, size = 268, normalized size = 0.86

$$\frac{\frac{4\left(a^3-3ab^2\right)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{\left(a^4+2a^2b^2+b^4\right)\sqrt{ab}}+\frac{4a\sqrt{\tan(dx+c)}}{a^3+ab^2+\left(a^2b+b^3\right)\tan(dx+c)}-\frac{2\sqrt{2}\left(a^2-2ab-b^2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}\left(a^2-2ab-b^2\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)+\sqrt{2}\left(a^2+2ab-b^2\right)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)-\sqrt{2}\left(a^2+2ab-b^2\right)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)}{a^4+2a^2b^2+b^4}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(4*(a^3 - 3*a*b^2)*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^4 + 2*a^2
*b^2 + b^4)*sqrt(a*b)) + 4*a*sqrt(tan(d*x + c))/(a^3 + a*b^2 + (a^2*b + b^3
)*tan(d*x + c)) - (2*sqrt(2)*(a^2 - 2*a*b - b^2)*arctan(1/2*sqrt(2)*(sqrt(2
) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*(a^2 - 2*a*b - b^2)*arctan(-1/2*sqrt
(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)*(a^2 + 2*a*b - b^2)*log(sqr
t(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*(a^2 + 2*a*b - b^2)*l
og(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a^4 + 2*a^2*b^2 + b^4)
)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7007 vs. $2(272) = 544$.
time = 12.67, size = 14125, normalized size = 45.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(2)*((a^14 + 5*a^12*b^2 + 9*a^10*b^4 + 5*a^8*b^6 - 5*a^6*b^8 -
 9*a^4*b^10 - 5*a^2*b^12 - b^14)*d^5*cos(d*x + c)^2 + 2*(a^13*b + 6*a^11*b^
3 + 15*a^9*b^5 + 20*a^7*b^7 + 15*a^5*b^9 + 6*a^3*b^11 + a*b^13)*d^5*cos(d*x
 + c)*sin(d*x + c) + (a^12*b^2 + 6*a^10*b^4 + 15*a^8*b^6 + 20*a^6*b^8 + 15*
a^4*b^10 + 6*a^2*b^12 + b^14)*d^5)*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^
2*b^6 + b^8 - 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 - 2*a^5*b^7 - 3*a^3*b^9 - a
*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))/(
a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt((a^8 - 12*a^6*b^2 +
 38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10
*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4))*(1
/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))^(3/4)*arctan(((a^16
 - 20*a^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 - 64*a^6*b^10 - 20*a^4*b^12 + b^1
6)*d^4*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^16 + 8*a
^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^1
2 + 8*a^2*b^14 + b^16)*d^4))*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b
^6 + b^8)*d^4)) - sqrt(2)*((a^18 + 7*a^16*b^2 + 20*a^14*b^4 + 28*a^12*b^6 +
 14*a^10*b^8 - 14*a^8*b^10 - 28*a^6*b^12 - 20*a^4*b^14 - 7*a^2*b^16 - b^18)
*d^7*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^16 + 8*a^1
4*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12
+ 8*a^2*b^14 + b^16)*d^4))*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6
 + b^8)*d^4)) + 2*(a^13*b + 6*a^11*b^3 + 15*a^9*b^5 + 20*a^7*b^7 + 15*a^5*b
^9 + 6*a^3*b^11 + a*b^13)*d^5*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*
b^6 + b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 5
6*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4)))*sqrt((a^8 + 4*a^6*b^2
+ 6*a^4*b^4 + 4*a^2*b^6 + b^8 - 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 - 2*a^5*b
^7 - 3*a^3*b^9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b
^6 + b^8)*d^4)))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt((
(a^12 - 10*a^10*b^2 + 15*a^8*b^4 + 52*a^6*b^6 + 15*a^4*b^8 - 10*a^2*b^10 +
b^12)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*cos
(d*x + c) + sqrt(2)*(2*(a^13*b - 10*a^11*b^3 + 15*a^9*b^5 + 52*a^7*b^7 + 15
*a^5*b^9 - 10*a^3*b^11 + a*b^13)*d^3*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
 4*a^2*b^6 + b^8)*d^4))*cos(d*x + c) + (a^10 - 13*a^8*b^2 + 50*a^6*b^4 - 50
*a^4*b^6 + 13*a^2*b^8 - b^10)*d*cos(d*x + c))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4
*b^4 + 4*a^2*b^6 + b^8 - 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 - 2*a^5*b^7 - 3*
a^3*b^9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^
8)*d^4)))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(sin(d*x
```

```
+ c)/cos(d*x + c))*(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)
)^(1/4) + (a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*sin(d*x + c))/
cos(d*x + c))*(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))^(3/
4) + sqrt(2)*((a^22 + a^20*b^2 - 21*a^18*b^4 - 85*a^16*b^6 - 134*a^14*b^8 -
 70*a^12*b^10 + 70*a^10*b^12 + 134*a^8*b^14 + 85*a^6*b^16 + 21*a^4*b^18 - a
^2*b^20 - b^22)*d^7*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)
/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10
 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4))*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*
b^4 + 4*a^2*b^6 + b^8)*d^4)) + 2*(a^17*b - 20*a^13*b^5 - 64*a^11*b^7 - 90*a
^9*b^9 - 64*a^7*b^11 - 20*a^5*b^13 + a*b^17)*d^5*sqrt((a^8 - 12*a^6*b^2 + 3
8*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b
^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4)))*sqr
t((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 - 4*(a^11*b + 3*a^9*b^3 +
2*a^7*b^5 - 2*a^5*b^7 - 3*a^3*b^9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 +
6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2
*b^6 + b^8))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a^8 + 4*a^6*b^2 + 6*a^4*b
^4 + 4*a^2*b^6 + b^8)*d^4))^(3/4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*
b^6 + b^8)) + 4*sqrt(2)*((a^14 + 5*a^12*b^2 + 9*a^10*b^4 + 5*a^8*b^6 - 5*a^
6*b^8 - 9*a^4*b^10 - 5*a^2*b^12 - b^14)*d^5*cos(d*x + c)^2 + 2*(a^13*b + 6*
a^11*b^3 + 15*a^9*b^5 + 20*a^7*b^7 + 15*a^5*b^9 + 6*a^3*b^11 + a*b^13)*d^5*
cos(d*x + c)*sin(d*x + c) + (a^12*b^2 + 6*a^10*b^4 + 15*a^8*b^6 + 20*a^6*b^
8 + 15*a^4*b^10 + 6*a^2*b^12 + b^14)*d^5)*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4
 + 4*a^2*b^6 + b^8 - 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 - 2*a^5*b^7 - 3*a^3*
b^9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d
^4)))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt((a^8 - 12*a^
6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 +
56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d
^4))*(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))^(3/4)*arctan
(-((a^16 - 20*a^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 - 64*a^6*b^10 - 20*a^4*b^
12 + b^16)*d^4*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^
16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 7...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(3/2)/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**(3/2)/(a + b*tan(c + d*x))**2, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x, algorithm="giac")

[Out] Timed out

Mupad [B]
time = 11.57, size = 2500, normalized size = 8.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d*x)^(3/2)/(a + b*tan(c + d*x))^2,x)

[Out] atan(((-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((16*(2*a^10*b*d^2 - 78*a^2*b^9*d^2 + 8*a^4*b^7*d^2 + 60*a^6*b^5*d^2 - 24*a^8*b^3*d^2))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) - (-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((16*(40*a*b^14*d^4 + 192*a^3*b^12*d^4 + 360*a^5*b^10*d^4 + 320*a^7*b^8*d^4 + 120*a^9*b^6*d^4 - 8*a^13*b^2*d^4))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) - (16*tan(c + d*x)^(1/2)*(-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(32*b^17*d^4 + 160*a^2*b^15*d^4 + 288*a^4*b^13*d^4 + 160*a^6*b^11*d^4 - 160*a^8*b^9*d^4 - 288*a^10*b^7*d^4 - 160*a^12*b^5*d^4 - 32*a^14*b^3*d^4))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4)) + (16*tan(c + d*x)^(1/2)*(20*a^3*b^10*d^2 - 60*a*b^12*d^2 + 168*a^5*b^8*d^2 + 40*a^7*b^6*d^2 - 44*a^9*b^4*d^2 + 4*a^11*b^2*d^2))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))) + (16*tan(c + d*x)^(1/2)*(a^8*b + 2*b^9 - 5*a^2*b^7 + 17*a^4*b^5 - 7*a^6*b^3))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*1i - (-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((16*(2*a^10*b*d^2 - 78*a^2*b^9*d^2 + 8*a^4*b^7*d^2 + 60*a^6*b^5*d^2 - 24*a^8*b^3*d^2))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) - (-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((16*(40*a*b^14*d^4 + 192*a^3*b^12*d^4 + 360*a^5*b^10*d^4 + 320*a^7*b^8*d^4 + 120*a^9*b^6*d^4 - 8*a^13*b^2*d^4))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) + (16*tan(c + d*x)^(1/2)*(-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(32*b^17*d^4 + 160*a^2*b^15*d^4 + 288*a^4*b^13*d^4 + 160*a^6*b^11*d^4 - 160*a^8*b^9*d^4 - 288*a^10*b^7*d^4 - 160*a^12*b^5*d^4 - 32*a^14*b^3*d^4))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4)) - (16*tan(c + d*x)^(1/2)*(20*a^3*b^10*d^2 - 60*a*b^12*d^2 +

168*a^5*b^8*d^2 + 40*a^7*b^6*d^2 - 44*a^9*b^4*d^2 + 4*a^11*b^2*d^2))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))) - (16*tan(c + d*x)^(1/2)*(a^8*b + 2*b^9 - 5*a^2*b^7 + 17*a^4*b^5 - 7*a^6*b^3))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*1i)/((-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((16*(2*a^10*b*d^2 - 78*a^2*b^9*d^2 + 8*a^4*b^7*d^2 + 60*a^6*b^5*d^2 - 24*a^8*b^3*d^2))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) - (-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((16*(40*a*b^14*d^4 + 192*a^3*b^12*d^4 + 360*a^5*b^10*d^4 + 320*a^7*b^8*d^4 + 120*a^9*b^6*d^4 - 8*a^13*b^2*d^4))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) - (16*tan(c + d*x)^(1/2)*(-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(32*b^17*d^4 + 160*a^2*b^15*d^4 + 288*a^4*b^13*d^4 + 160*a^6*b^11*d^4 - 160*a^8*b^9*d^4 - 288*a^10*b^7*d^4 - 160*a^12*b^5*d^4 - 32*a^14*b^3*d^4))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4)) + (16*tan(c + d*x)^(1/2)*(20*a^3*b^10*d^2 - 60*a*b^12*d^2 + 168*a^5*b^8*d^2 + 40*a^7*b^6*d^2 - 44*a^9*b^4*d^2 + 4*a^11*b^2*d^2))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))) + (16*tan(c + d*x)^(1/2)*(a^8*b + 2*b^9 - 5*a^2*b^7 + 17*a^4*b^5 - 7*a^6*b^3))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4)) + (-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((16*(2*a^10*b*d^2 - 78*a^2*b^9*d^2 + 8*a^4*b^7*d^2 + 60*a^6*b^5*d^2 - 24*a^8*b^3*d^2))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) - (-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((16*(40*a*b^14*d^4 + 192*a^3*b^12*d^4 + 360*a^5*b^10*d^4 + 320*a^7*b^8*d^4 + 120*a^9*b^6*d^4 - 8*a^13*b^2*d^4))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) + (16*tan(c + d*x)^(1/2)*(-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(32*b^17*d^4 + 160*a^2*b^15*d^4 + 288*a^4*b^13*d^4 + 160*a^6*b^11*d^4 - 160*a^8*b^9*d^4 - 288*a^10*b^7*d^4 - 160*a^12*b^5*d^4 - 32*a^14*b^3*d^4))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*...

3.595 $\int \frac{\sqrt{\tan(c+dx)}}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=316

$$-\frac{(a^2+2ab-b^2)\,\mathrm{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}+\frac{(a^2+2ab-b^2)\,\mathrm{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}-\cdots$$

```
[Out] 1/2*(a^2+2*a*b-b^2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)^2/d*2^(1/
2)+1/2*(a^2+2*a*b-b^2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)^2/d*2^(
1/2)+1/4*(a^2-2*a*b-b^2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2
)^2/d*2^(1/2)-1/4*(a^2-2*a*b-b^2)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))
/(a^2+b^2)^2/d*2^(1/2)-(3*a^2-b^2)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))
*b^(1/2)/(a^2+b^2)^2/d/a^(1/2)-b*tan(d*x+c)^(1/2)/(a^2+b^2)/d/(a+b*tan(d*x+
c))
```

Rubi [A]
time = 0.29, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3649, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{\sqrt{b}\,(3a^2-b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d\,(a^2+b^2)^2}-\frac{(a^2+2ab-b^2)\,\mathrm{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{(a^2+2ab-b^2)\,\mathrm{ArcTan}\left(\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}-\frac{b\sqrt{\tan(c+dx)}}{d\,(a^2+b^2)\,(a+b\tan(c+dx))}+\frac{(a^2-2ab-b^2)\log\left(\tan(c+dx)-\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}-\frac{(a^2-2ab-b^2)\log\left(\tan(c+dx)+\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[c + d*x]]/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((a^2 + 2*a*b - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*(a^
2 + b^2)^2*d)) + ((a^2 + 2*a*b - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]
])/(Sqrt[2]*(a^2 + b^2)^2*d) - (Sqrt[b]*(3*a^2 - b^2)*ArcTan[(Sqrt[b]*Sqrt[
Tan[c + d*x]])/Sqrt[a]])/(Sqrt[a]*(a^2 + b^2)^2*d) + ((a^2 - 2*a*b - b^2)*L
og[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^2
*d) - ((a^2 - 2*a*b - b^2)*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x
]])/(2*Sqrt[2]*(a^2 + b^2)^2*d) - (b*Sqrt[Tan[c + d*x]])/((a^2 + b^2)*d*(a
+ b*Tan[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{\tan(c+dx)}}{(a+b\tan(c+dx))^2}\,dx = -\frac{b\sqrt{\tan(c+dx)}}{(a^2+b^2)\,d(a+b\tan(c+dx))} - \frac{\int \frac{-\frac{b}{2}-a\tan(c+dx)+\frac{1}{2}b\tan^2(c+dx)}{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))}\,dx}{a^2+b^2}$$

$$= -\frac{b\sqrt{\tan(c+dx)}}{(a^2+b^2)\,d(a+b\tan(c+dx))} - \frac{\int \frac{-2ab-(a^2-b^2)\tan(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{(a^2+b^2)^2} - \frac{(b(3a^2-b^2))\int \cdots}{}$$

$$= -\frac{b\sqrt{\tan(c+dx)}}{(a^2+b^2)\,d(a+b\tan(c+dx))} - \frac{2\text{Subst}\left(\int \frac{-2ab+(-a^2+b^2)x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+d\cdots}\right.}{(a^2+b^2)^2\,d}$$

$$= -\frac{b\sqrt{\tan(c+dx)}}{(a^2+b^2)\,d(a+b\tan(c+dx))} - \frac{(b(3a^2-b^2))\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{\tan(c+\cdots}\right.}{(a^2+b^2)^2\,d}$$

$$= -\frac{\sqrt{b}\,(3a^2-b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,(a^2+b^2)^2\,d} - \frac{b\sqrt{\tan(c+dx)}}{(a^2+b^2)\,d(a+b\tan(c+dx))}$$

$$= -\frac{\sqrt{b}\,(3a^2-b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,(a^2+b^2)^2\,d} + \frac{(a^2-2ab-b^2)\log\left(1-\sqrt{2}\,\cdots\right.}{2\sqrt{2}\,(a^2\cdots}$$

$$= -\frac{(a^2+2ab-b^2)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} + \frac{(a^2+2ab-b^2)\tan^{-1}\left(1-\cdots\right.}{\sqrt{2}\,(a^2+\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.73, size = 182, normalized size = 0.58

$$\frac{-\frac{\sqrt{a}\,\sqrt{b}\,(3a^2-b^2)\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^2+b^2} + \frac{(-1)^{3/4}a\left((a+ib)^2\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)-(a-ib)^2\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\right)}{a^2+b^2} - b\sqrt{\tan(c+dx)} + \frac{b^2\tan^{\frac{3}{2}}(c+dx)}{a+b\tan(c+dx)}}{a\,(a^2+b^2)\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tan[c + d*x]]/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-((Sqrt[a]*Sqrt[b]*(3*a^2 - b^2)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[
a]])/(a^2 + b^2)) + ((-1)^(3/4)*a*((a + I*b)^2*ArcTan[(-1)^(3/4)*Sqrt[Tan[c
 + d*x]]] - (a - I*b)^2*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]))/(a^2 + b^2
) - b*Sqrt[Tan[c + d*x]] + (b^2*Tan[c + d*x]^(3/2))/(a + b*Tan[c + d*x]))/(
a*(a^2 + b^2)*d)
```

Maple [A]
time = 0.13, size = 278, normalized size = 0.88

method	result
derivativedivides	$-\frac{2b\left(\frac{\left(\frac{a^2}{2}+\frac{b^2}{2}\right)\left(\sqrt{\tan}(dx+c)\right)}{a+b\tan(dx+c)}+\frac{\left(3a^2-b^2\right)\arctan\left(\frac{b\left(\sqrt{\tan}(dx+c)\right)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{\left(a^2+b^2\right)^2}+\frac{ab\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)\cdots}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)\cdots}\right)\cdots\right)}{\cdots}$
default	$-\frac{2b\left(\frac{\left(\frac{a^2}{2}+\frac{b^2}{2}\right)\left(\sqrt{\tan}(dx+c)\right)}{a+b\tan(dx+c)}+\frac{\left(3a^2-b^2\right)\arctan\left(\frac{b\left(\sqrt{\tan}(dx+c)\right)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{\left(a^2+b^2\right)^2}+\frac{ab\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)\cdots}{1-\sqrt{2}\left(\sqrt{\tan}(dx+c)\right)\cdots}\right)\cdots\right)}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*b/(a^2+b^2)^2*((1/2*a^2+1/2*b^2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))+
1/2*(3*a^2-b^2)/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2)))+2/(a^2+
b^2)^2*(1/4*a*b*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1
/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*ar
ctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/8*(a^2-b^2)*2^(1/2)*(ln((1-2^(1/2)*tan
(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan
(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))))
```

Maxima [A]

time = 0.51, size = 270, normalized size = 0.85

$$-\frac{\frac{4\left(3a^2b-b^3\right)\arctan\left(\frac{b\sqrt{\tan\left(dx+c\right)}}{\sqrt{ab}}\right)}{\left(a^4+2a^2b^2+b^4\right)\sqrt{ab}}+\frac{4b\sqrt{\tan\left(dx+c\right)}}{a^3+ab^2+\left(a^2b+b^3\right)\tan\left(dx+c\right)}-\frac{2\sqrt{2}\left(a^2+2ab-b^2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(dx+c\right)}\right)\right)+2\sqrt{2}\left(a^2+2ab-b^2\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(dx+c\right)}\right)\right)-\sqrt{2}\left(a^2-2ab-b^2\right)\log\left(\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)+\sqrt{2}\left(a^2-2ab-b^2\right)\log\left(-\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)}{a^4+2a^2b^2+b^4}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(4*(3*a^2*b - b^3)*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^4 + 2*a^
2*b^2 + b^4)*sqrt(a*b)) + 4*b*sqrt(tan(d*x + c))/(a^3 + a*b^2 + (a^2*b + b^
3)*tan(d*x + c)) - (2*sqrt(2)*(a^2 + 2*a*b - b^2)*arctan(1/2*sqrt(2)*(sqrt(
2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*(a^2 + 2*a*b - b^2)*arctan(-1/2*sqr
t(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*(a^2 - 2*a*b - b^2)*log(sq
rt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*(a^2 - 2*a*b - b^2)*
log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a^4 + 2*a^2*b^2 + b^4
))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7153 vs. $2(276) = 552$.

time = 10.15, size = 14310, normalized size = 45.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(2)*((a^14 + 5*a^12*b^2 + 9*a^10*b^4 + 5*a^8*b^6 - 5*a^6*b^8 -
9*a^4*b^10 - 5*a^2*b^12 - b^14)*d^5*cos(d*x + c)^2 + 2*(a^13*b + 6*a^11*b^3
 + 15*a^9*b^5 + 20*a^7*b^7 + 15*a^5*b^9 + 6*a^3*b^11 + a*b^13)*d^5*cos(d*x
+ c)*sin(d*x + c) + (a^12*b^2 + 6*a^10*b^4 + 15*a^8*b^6 + 20*a^6*b^8 + 15*a
^4*b^10 + 6*a^2*b^12 + b^14)*d^5)*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2
*b^6 + b^8 + 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 - 2*a^5*b^7 - 3*a^3*b^9 - a*
b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))/(a
^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt((a^8 - 12*a^6*b^2 +
38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*
b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4))*(1/
((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))^(3/4)*arctan(((a^16
- 20*a^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 - 64*a^6*b^10 - 20*a^4*b^12 + b^16
)*d^4*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^16 + 8*a^
14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12
 + 8*a^2*b^14 + b^16)*d^4))*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^
6 + b^8)*d^4)) + sqrt(2)*(2*(a^17*b + 8*a^15*b^3 + 28*a^13*b^5 + 56*a^11*b^
7 + 70*a^9*b^9 + 56*a^7*b^11 + 28*a^5*b^13 + 8*a^3*b^15 + a*b^17)*d^7*sqrt(
(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^16 + 8*a^14*b^2 + 28
*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^
14 + b^16)*d^4))*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^
4)) - (a^14 + 5*a^12*b^2 + 9*a^10*b^4 + 5*a^8*b^6 - 5*a^6*b^8 - 9*a^4*b^10
- 5*a^2*b^12 - b^14)*d^5*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 +
 b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6
*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4)))*sqrt((a^8 + 4*a^6*b^2 + 6*a
^4*b^4 + 4*a^2*b^6 + b^8 + 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 - 2*a^5*b^7 -
3*a^3*b^9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 +
b^8)*d^4)))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(((a^12
 - 10*a^10*b^2 + 15*a^8*b^4 + 52*a^6*b^6 + 15*a^4*b^8 - 10*a^2*b^10 + b^12)
*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*cos(d*x
+ c) + sqrt(2)*((a^14 - 11*a^12*b^2 + 25*a^10*b^4 + 37*a^8*b^6 - 37*a^6*b^8
 - 25*a^4*b^10 + 11*a^2*b^12 - b^14)*d^3*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b
^4 + 4*a^2*b^6 + b^8)*d^4))*cos(d*x + c) - 2*(a^9*b - 12*a^7*b^3 + 38*a^5*b
^5 - 12*a^3*b^7 + a*b^9)*d*cos(d*x + c))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4
+ 4*a^2*b^6 + b^8 + 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 - 2*a^5*b^7 - 3*a^3*b
^9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^
4)))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(sin(d*x + c)/
```

```
cos(d*x + c))*(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))^(1/
4) + (a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*sin(d*x + c))/cos(d
*x + c))*(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))^(3/4) -
sqrt(2)*(2*(a^21*b + 2*a^19*b^3 - 19*a^17*b^5 - 104*a^15*b^7 - 238*a^13*b^9
 - 308*a^11*b^11 - 238*a^9*b^13 - 104*a^7*b^15 - 19*a^5*b^17 + 2*a^3*b^19 +
 a*b^21)*d^7*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^16
 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a
^4*b^12 + 8*a^2*b^14 + b^16)*d^4))*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4
*a^2*b^6 + b^8)*d^4)) - (a^18 - a^16*b^2 - 20*a^14*b^4 - 44*a^12*b^6 - 26*a
^10*b^8 + 26*a^8*b^10 + 44*a^6*b^12 + 20*a^4*b^14 + a^2*b^16 - b^18)*d^5*sq
rt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^16 + 8*a^14*b^2 +
 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2
*b^14 + b^16)*d^4)))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 +
4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 - 2*a^5*b^7 - 3*a^3*b^9 - a*b^11)*d^2*sqr
t(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))/(a^8 - 12*a^6*b
^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a
^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))^(3/4))/(a^8 - 12*a^6*b^
2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)) + 4*sqrt(2)*((a^14 + 5*a^12*b^2 + 9*a^1
0*b^4 + 5*a^8*b^6 - 5*a^6*b^8 - 9*a^4*b^10 - 5*a^2*b^12 - b^14)*d^5*cos(d*x
 + c)^2 + 2*(a^13*b + 6*a^11*b^3 + 15*a^9*b^5 + 20*a^7*b^7 + 15*a^5*b^9 + 6
*a^3*b^11 + a*b^13)*d^5*cos(d*x + c)*sin(d*x + c) + (a^12*b^2 + 6*a^10*b^4
+ 15*a^8*b^6 + 20*a^6*b^8 + 15*a^4*b^10 + 6*a^2*b^12 + b^14)*d^5)*sqrt((a^8
 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 4*(a^11*b + 3*a^9*b^3 + 2*a^7*
b^5 - 2*a^5*b^7 - 3*a^3*b^9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*
b^4 + 4*a^2*b^6 + b^8)*d^4)))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 +
 b^8))*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^16 + 8*a
^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^1
2 + 8*a^2*b^14 + b^16)*d^4))*(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 +
 b^8)*d^4))^(3/4)*arctan(-((a^16 - 20*a^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 -
 64*a^6*b^10 - 20*a^4*b^12 + b^16)*d^4*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4
- 12*a^2*b^6 + b^8)/((a^16 + 8*a^14*b^2 + 28*a^...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\tan(c+dx)}}{(a+b\tan(c+dx))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/2)/(a+b*tan(d*x+c))**2,x)
[Out] Integral(sqrt(tan(c + d*x))/(a + b*tan(c + d*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(tan(d*x + c))/(b*tan(d*x + c) + a)^2, x)
```

Mupad [B]
time = 6.80, size = 2500, normalized size = 7.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)/(a + b*tan(c + d*x))^2,x)
```

```
[Out] atan((((((8*(52*a*b^10*d^2 - 128*a^3*b^8*d^2 - 24*a^5*b^6*d^2 + 160*a^7*b^
4*d^2 + 4*a^9*b^2*d^2))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5
+ 4*a^6*b^2*d^5) + (((((8*(320*a^6*b^9*d^4 - 96*a^2*b^13*d^4 - 32*b^15*d^4
+ 480*a^8*b^7*d^4 + 288*a^10*b^5*d^4 + 64*a^12*b^3*d^4))/(a^8*d^5 + b^8*d^5
 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) - (8*tan(c + d*x)^(1/2)*(
1/(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^(
1/2)*(32*b^17*d^4 + 160*a^2*b^15*d^4 + 288*a^4*b^13*d^4 + 160*a^6*b^11*d^4
- 160*a^8*b^9*d^4 - 288*a^10*b^7*d^4 - 160*a^12*b^5*d^4 - 32*a^14*b^3*d^4))
/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(1/(a
^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^(1/2)
)/2 + (16*tan(c + d*x)^(1/2)*(68*a*b^12*d^2 + 20*a^3*b^10*d^2 - 88*a^5*b^8*
d^2 + 40*a^7*b^6*d^2 + 84*a^9*b^4*d^2 + 4*a^11*b^2*d^2))/(a^8*d^4 + b^8*d^4
 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(1/(a^4*d^2*1i + b^4*d^2
*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^(1/2))/2)*(1/(a^4*d^2*1i
 + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^(1/2))/2 - (16
*tan(c + d*x)^(1/2)*(3*b^9 - 3*a^2*b^7 + 17*a^4*b^5 - 9*a^6*b^3))/(a^8*d^4
+ b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(1/(a^4*d^2*1i
+ b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^(1/2)*1i)/2 - (
((((8*(52*a*b^10*d^2 - 128*a^3*b^8*d^2 - 24*a^5*b^6*d^2 + 160*a^7*b^4*d^2 +
 4*a^9*b^2*d^2))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6
*b^2*d^5) + (((((8*(320*a^6*b^9*d^4 - 96*a^2*b^13*d^4 - 32*b^15*d^4 + 480*a
^8*b^7*d^4 + 288*a^10*b^5*d^4 + 64*a^12*b^3*d^4))/(a^8*d^5 + b^8*d^5 + 4*a^
2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) + (8*tan(c + d*x)^(1/2)*(1/(a^4*
d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^(1/2)*(3
2*b^17*d^4 + 160*a^2*b^15*d^4 + 288*a^4*b^13*d^4 + 160*a^6*b^11*d^4 - 160*a
^8*b^9*d^4 - 288*a^10*b^7*d^4 - 160*a^12*b^5*d^4 - 32*a^14*b^3*d^4))/(a^8*d
^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(1/(a^4*d^2*
1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^(1/2))/2 - (
16*tan(c + d*x)^(1/2)*(68*a*b^12*d^2 + 20*a^3*b^10*d^2 - 88*a^5*b^8*d^2 + 4
0*a^7*b^6*d^2 + 84*a^9*b^4*d^2 + 4*a^11*b^2*d^2))/(a^8*d^4 + b^8*d^4 + 4*a^
2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(1/(a^4*d^2*1i + b^4*d^2*1i + 4
*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^(1/2))/2)*(1/(a^4*d^2*1i + b^4*
```

d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^(1/2))/2 + (16*tan(c + d*x)^(1/2)*(3*b^9 - 3*a^2*b^7 + 17*a^4*b^5 - 9*a^6*b^3))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(1/(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^(1/2)*1i)/2)/((((((8*(52*a*b^10*d^2 - 128*a^3*b^8*d^2 - 24*a^5*b^6*d^2 + 160*a^7*b^4*d^2 + 4*a^9*b^2*d^2))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) + (((((8*(320*a^6*b^9*d^4 - 96*a^2*b^13*d^4 - 32*b^15*d^4 + 480*a^8*b^7*d^4 + 288*a^10*b^5*d^4 + 64*a^12*b^3*d^4))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) - (8*tan(c + d*x)^(1/2)*(1/(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^(1/2)*(32*b^17*d^4 + 160*a^2*b^15*d^4 + 288*a^4*b^13*d^4 + 160*a^6*b^11*d^4 - 160*a^8*b^9*d^4 - 288*a^10*b^7*d^4 - 160*a^12*b^5*d^4 - 32*a^14*b^3*d^4))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(1/(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^(1/2))/2 + (16*tan(c + d*x)^(1/2)*(68*a*b^12*d^2 + 20*a^3*b^10*d^2 - 88*a^5*b^8*d^2 + 40*a^7*b^6*d^2 + 84*a^9*b^4*d^2 + 4*a^11*b^2*d^2))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(1/(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^(1/2))/2)*(1/(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^(1/2))/2 - (16*tan(c + d*x)^(1/2)*(3*b^9 - 3*a^2*b^7 + 17*a^4*b^5 - 9*a^6*b^3))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(1/(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^(1/2))/2 + (((((8*(52*a*b^10*d^2 - 128*a^3*b^8*d^2 - 24*a^5*b^6*d^2 + 160*a^7*b^4*d^2 + 4*a^9*b^2*d^2))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) + (((((8*(320*a^6*b^9*d^4 - 96*a^2*b^13*d^4 - 32*b^15*d^4 + 480*a^8*b^7*d^4 + 288*a^10*b^5*d^4 + 64*a^12*b^3*d^4))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) + (8*tan(c + d*x)^(1/2)*(1/(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^(1/2)*(32*b^17*d^4 + 160*a^2*b^15*d^4 + 288*a^4*b^13*d^4 + 160*a^6*b^11*d^4 - 160*a^8*b^9*d^4 - 288*a^10*b^7*d^4 - 160*a^12*b^5*d^4 - 32*a^14*b^3*d^4))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(1/(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^(1/2))/2 - (16*tan(c + d*x)^(1/2)*(68*a*b^12*d^2 + 20*a^3*b^10*d^2 - 88*a^5*b^8*d^2 + 40*a^7*b^6*d^2 + 84*a^9*b^4*d^2 + 4*a^11*b^2*d^2))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(1/(a^4*d^2*1i + b^4...

3.596 $\int \frac{1}{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=317

$$-\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}+\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}+\frac{b^{3/}}{}$$

```
[Out] b^(3/2)*(5*a^2+b^2)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/a^(3/2)/(a^2+b
^2)^2/d+1/2*(a^2-2*a*b-b^2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)^2
/d*2^(1/2)+1/2*(a^2-2*a*b-b^2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)
^2/d*2^(1/2)-1/4*(a^2+2*a*b-b^2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/
(a^2+b^2)^2/d*2^(1/2)+1/4*(a^2+2*a*b-b^2)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan
(d*x+c))/(a^2+b^2)^2/d*2^(1/2)+b^2*tan(d*x+c)^(1/2)/a/(a^2+b^2)/d/(a+b*tan(
d*x+c))
```

Rubi [A]
time = 0.34, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3650, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{b^2\sqrt{\tan(c+dx)}}{ad\,(a^2+b^2)\,(a+b\tan(c+dx))}-\frac{(a^2+2ab-b^2)\log\left(\tan(c+dx)-\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{(a^2+2ab-b^2)\log\left(\tan(c+dx)+\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{b^{3/2}(5a^2+b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d\,(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^2),x]
```

```
[Out] -(((a^2 - 2*a*b - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*(a^
2 + b^2)^2*d)) + ((a^2 - 2*a*b - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]
])/(Sqrt[2]*(a^2 + b^2)^2*d) + (b^(3/2)*(5*a^2 + b^2)*ArcTan[(Sqrt[b]*Sqrt[
Tan[c + d*x]])/Sqrt[a]])/(a^(3/2)*(a^2 + b^2)^2*d) - ((a^2 + 2*a*b - b^2)*L
og[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^2
*d) + ((a^2 + 2*a*b - b^2)*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x
]])/(2*Sqrt[2]*(a^2 + b^2)^2*d) + (b^2*Sqrt[Tan[c + d*x]])/(a*(a^2 + b^2)*d
*(a + b*Tan[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^2}\,dx = \frac{b^2\sqrt{\tan(c+dx)}}{a\,(a^2+b^2)\,d(a+b\tan(c+dx))} + \frac{\int \frac{\frac{1}{2}(2a^2+b^2)-ab\tan(c+dx)+\frac{1}{2}b^2\tan\ldots}{\sqrt{\tan(c+dx)}\,(a+b\tan(c\ldots}}{a\,(a^2+b^2)}$$

$$= \frac{b^2\sqrt{\tan(c+dx)}}{a\,(a^2+b^2)\,d(a+b\tan(c+dx))} + \frac{\int \frac{a(a^2-b^2)-2a^2b\tan(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{a\,(a^2+b^2)^2} + \ldots$$

$$= \frac{b^2\sqrt{\tan(c+dx)}}{a\,(a^2+b^2)\,d(a+b\tan(c+dx))} + \frac{2\text{Subst}\left(\int \frac{a(a^2-b^2)-2a^2bx^2}{1+x^4}\,dx,\ldots\right.}{a\,(a^2+b^2)\ldots}$$

$$= \frac{b^2\sqrt{\tan(c+dx)}}{a\,(a^2+b^2)\,d(a+b\tan(c+dx))} + \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{1+\ldots}{1+\ldots}\right.}{(a^2+b\ldots}$$

$$= \frac{b^{3/2}(5a^2+b^2)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}\,(a^2+b^2)^2\,d} + \frac{b^2\sqrt{\tan(\ldots}}{a\,(a^2+b^2)\,d(a+\ldots}$$

$$= \frac{b^{3/2}(5a^2+b^2)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}\,(a^2+b^2)^2\,d} - \frac{(a^2+2ab-b^2)\ldots}{\ldots}$$

$$= -\frac{(a^2-2ab-b^2)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} + \frac{(a^2-2ab\ldots}{\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.64, size = 166, normalized size = 0.52

$$\frac{\frac{b^{3/2}(5a^2+b^2)\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,(a^2+b^2)} - \frac{\sqrt[4]{-1}\,a\left((a+ib)^2\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+(a-ib)^2\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\right)}{a^2+b^2} + \frac{b^2\sqrt{\tan(c+dx)}}{a+b\tan(c+dx)}}{a\,(a^2+b^2)\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^2),x]
```

```
[Out] ((b^(3/2)*(5*a^2 + b^2)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(Sqrt
[a]*(a^2 + b^2)) - ((-1)^(1/4)*a*((a + I*b)^2*ArcTan[(-1)^(3/4)*Sqrt[Tan[c
+ d*x]]] + (a - I*b)^2*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]))/(a^2 + b^2)
 + (b^2*Sqrt[Tan[c + d*x]])/(a + b*Tan[c + d*x]))/(a*(a^2 + b^2)*d)
```

Maple [A]
time = 0.16, size = 281, normalized size = 0.89

method	result
derivativedivides	$\frac{2b^2\left(\frac{\left(a^2+b^2\right)\left(\sqrt{\tan}\,(dx+c)\right)}{2a(a+b\tan(dx+c))}+\frac{\left(5a^2+b^2\right)\arctan\left(\frac{b\left(\sqrt{\tan}\,(dx+c)\right)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{\left(a^2+b^2\right)^2}+\frac{\left(a^2-b^2\right)\sqrt{2}\,\left(\ln\left(\frac{1+\sqrt{2}\,\left(\sqrt{\tan}\,(dx+c\right.}{1-\sqrt{2}\,\left(\sqrt{\tan}\,(dx+c\right.}\right.\right.}{}$
default	$\frac{2b^2\left(\frac{\left(a^2+b^2\right)\left(\sqrt{\tan}\,(dx+c)\right)}{2a(a+b\tan(dx+c))}+\frac{\left(5a^2+b^2\right)\arctan\left(\frac{b\left(\sqrt{\tan}\,(dx+c)\right)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{\left(a^2+b^2\right)^2}+\frac{\left(a^2-b^2\right)\sqrt{2}\,\left(\ln\left(\frac{1+\sqrt{2}\,\left(\sqrt{\tan}\,(dx+c\right.}{1-\sqrt{2}\,\left(\sqrt{\tan}\,(dx+c\right.}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b^2/(a^2+b^2)^2*(1/2*(a^2+b^2)/a*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))+1
/2*(5*a^2+b^2)/a/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2)))+2/(a^2
+b^2)^2*(1/8*(a^2-b^2)*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/
(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2
))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))-1/4*a*b*2^(1/2)*(ln((1-2^(1/2)*ta
n(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arcta
n(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))))
```

Maxima [A]

time = 0.51, size = 277, normalized size = 0.87

$$\frac{\frac{4b^2\sqrt{\tan(dx+c)}}{a^4+a^2b^2+(a^3b+ab^3)\tan(dx+c)}+\frac{4(5a^2b^2+b^4)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^5+2a^3b^2+ab^4)\sqrt{ab}}+\frac{2\sqrt{2}(a^2-2ab-b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}(a^2-2ab-b^2)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)+\sqrt{2}(a^2+2ab-b^2)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)-\sqrt{2}(a^2+2ab-b^2)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)}{a^4+2a^2b^2+b^4}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(4*b^2*sqrt(tan(d*x + c))/(a^4 + a^2*b^2 + (a^3*b + a*b^3)*tan(d*x + c)
) + 4*(5*a^2*b^2 + b^4)*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^5 + 2*a^
3*b^2 + a*b^4)*sqrt(a*b)) + (2*sqrt(2)*(a^2 - 2*a*b - b^2)*arctan(1/2*sqrt(
2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*(a^2 - 2*a*b - b^2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)*(a^2 + 2*a*b - b^
2)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*(a^2 + 2*a*
b - b^2)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a^4 + 2*a^2*
b^2 + b^4))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7154 vs. $2(277) = 554$.

time = 15.38, size = 14311, normalized size = 45.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(2)*((a^15 + 5*a^13*b^2 + 9*a^11*b^4 + 5*a^9*b^6 - 5*a^7*b^8 -
9*a^5*b^10 - 5*a^3*b^12 - a*b^14)*d^5*cos(d*x + c)^2 + 2*(a^14*b + 6*a^12*b
^3 + 15*a^10*b^5 + 20*a^8*b^7 + 15*a^6*b^9 + 6*a^4*b^11 + a^2*b^13)*d^5*cos
(d*x + c)*sin(d*x + c) + (a^13*b^2 + 6*a^11*b^4 + 15*a^9*b^6 + 20*a^7*b^8 +
 15*a^5*b^10 + 6*a^3*b^12 + a*b^14)*d^5)*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4
+ 4*a^2*b^6 + b^8 - 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 - 2*a^5*b^7 - 3*a^3*b
^9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^
4)))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt((a^8 - 12*a^6
*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 5
6*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^
4))*(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))^(3/4)*arctan(
((a^16 - 20*a^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 - 64*a^6*b^10 - 20*a^4*b^12
 + b^16)*d^4*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^16
 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a
^4*b^12 + 8*a^2*b^14 + b^16)*d^4))*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4
*a^2*b^6 + b^8)*d^4)) - sqrt(2)*((a^18 + 7*a^16*b^2 + 20*a^14*b^4 + 28*a^12
*b^6 + 14*a^10*b^8 - 14*a^8*b^10 - 28*a^6*b^12 - 20*a^4*b^14 - 7*a^2*b^16 -
 b^18)*d^7*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^16 +
 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4
*b^12 + 8*a^2*b^14 + b^16)*d^4))*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a
^2*b^6 + b^8)*d^4)) + 2*(a^13*b + 6*a^11*b^3 + 15*a^9*b^5 + 20*a^7*b^7 + 15
*a^5*b^9 + 6*a^3*b^11 + a*b^13)*d^5*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 1
2*a^2*b^6 + b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b
^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4)))*sqrt((a^8 + 4*a^
6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 - 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 - 2
*a^5*b^7 - 3*a^3*b^9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4
*a^2*b^6 + b^8)*d^4)))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*
sqrt(((a^12 - 10*a^10*b^2 + 15*a^8*b^4 + 52*a^6*b^6 + 15*a^4*b^8 - 10*a^2*b
^10 + b^12)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4
))*cos(d*x + c) + sqrt(2)*(2*(a^13*b - 10*a^11*b^3 + 15*a^9*b^5 + 52*a^7*b^
7 + 15*a^5*b^9 - 10*a^3*b^11 + a*b^13)*d^3*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4
*b^4 + 4*a^2*b^6 + b^8)*d^4))*cos(d*x + c) + (a^10 - 13*a^8*b^2 + 50*a^6*b^
4 - 50*a^4*b^6 + 13*a^2*b^8 - b^10)*d*cos(d*x + c))*sqrt((a^8 + 4*a^6*b^2 +
 6*a^4*b^4 + 4*a^2*b^6 + b^8 - 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 - 2*a^5*b^
7 - 3*a^3*b^9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^
6 + b^8)*d^4)))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(si
```

```
n(d*x + c)/cos(d*x + c))*(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8
)*d^4))^(1/4) + (a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*sin(d*x
+ c))/cos(d*x + c))*(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4
))^(3/4) + sqrt(2)*((a^22 + a^20*b^2 - 21*a^18*b^4 - 85*a^16*b^6 - 134*a^14
*b^8 - 70*a^12*b^10 + 70*a^10*b^12 + 134*a^8*b^14 + 85*a^6*b^16 + 21*a^4*b^
18 - a^2*b^20 - b^22)*d^7*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6
+ b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^
6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4))*sqrt(1/((a^8 + 4*a^6*b^2 +
6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)) + 2*(a^17*b - 20*a^13*b^5 - 64*a^11*b^7
- 90*a^9*b^9 - 64*a^7*b^11 - 20*a^5*b^13 + a*b^17)*d^5*sqrt((a^8 - 12*a^6*b
^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*
a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4)
))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 - 4*(a^11*b + 3*a^9*
b^3 + 2*a^7*b^5 - 2*a^5*b^7 - 3*a^3*b^9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*
b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 -
12*a^2*b^6 + b^8))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a^8 + 4*a^6*b^2 + 6
*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))^(3/4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 1
2*a^2*b^6 + b^8)) + 4*sqrt(2)*((a^15 + 5*a^13*b^2 + 9*a^11*b^4 + 5*a^9*b^6
- 5*a^7*b^8 - 9*a^5*b^10 - 5*a^3*b^12 - a*b^14)*d^5*cos(d*x + c)^2 + 2*(a^1
4*b + 6*a^12*b^3 + 15*a^10*b^5 + 20*a^8*b^7 + 15*a^6*b^9 + 6*a^4*b^11 + a^2
*b^13)*d^5*cos(d*x + c)*sin(d*x + c) + (a^13*b^2 + 6*a^11*b^4 + 15*a^9*b^6
+ 20*a^7*b^8 + 15*a^5*b^10 + 6*a^3*b^12 + a*b^14)*d^5)*sqrt((a^8 + 4*a^6*b^
2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 - 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 - 2*a^5
*b^7 - 3*a^3*b^9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2
*b^6 + b^8)*d^4)))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt
((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^16 + 8*a^14*b^2 + 2
8*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b
^14 + b^16)*d^4))*(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))
^(3/4)*arctan(-((a^16 - 20*a^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 - 64*a^6*b^1
0 - 20*a^4*b^12 + b^16)*d^4*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^
6 + b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56...
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\tan(c+dx))^2\sqrt{\tan(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(1/2)/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(1/((a + b*tan(c + d*x))**2*sqrt(tan(c + d*x))), x)
```

Giac [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 11.26, size = 2500, normalized size = 7.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(1/2)*(a + b*tan(c + d*x))^2),x)
```

```
[Out] (log(((((((((((128*b^2*(2*b^6 - a^6 + 9*a^2*b^4 + 6*a^4*b^2))/(a*d) + 256*b^
3*tan(c + d*x)^(1/2)*(a^2 - b^2)*(a^2 + b^2)^2*(1i/(d^2*(a*1i - b)^4))^(1/2
))*(1i/(d^2*(a*1i - b)^4))^(1/2))/2 + (64*b^2*tan(c + d*x)^(1/2)*(2*b^8 - a
^8 + 5*a^2*b^6 + 67*a^4*b^4 - a^6*b^2))/(a*d^2*(a^2 + b^2)^2))*(1i/(d^2*(a*
1i - b)^4))^(1/2))/2 - (32*b^5*(25*a^6 + b^6 - 13*a^2*b^4 - 85*a^4*b^2))/(a
^2*d^3*(a^2 + b^2)^3))*(1i/(d^2*(a*1i - b)^4))^(1/2))/2 + (16*b^5*tan(c + d
*x)^(1/2)*(b^6 - 27*a^6 + 7*a^2*b^4 + 11*a^4*b^2))/(a^2*d^4*(a^2 + b^2)^4))
*(1i/(d^2*(a*1i - b)^4))^(1/2))/2 + (16*b^6*(5*a^2 + b^2))/(a*d^5*(a^2 + b^
2)^4))*(-1/(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d
^2*6i))^(1/2))/2 - atan(((-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*
d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i
- a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3
*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((-1i/(4*(a^4*d^2 + b^4*d^2
 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((16*(16*a*b^16*d^4
 + 136*a^3*b^14*d^4 + 432*a^5*b^12*d^4 + 680*a^7*b^10*d^4 + 560*a^9*b^8*d^4
 + 216*a^11*b^6*d^4 + 16*a^13*b^4*d^4 - 8*a^15*b^2*d^4))/(a^10*d^5 + a^2*b^
8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5) - (16*tan(c + d*x)^(
1/2)*(-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d
^2)))^(1/2)*(32*a^2*b^17*d^4 + 160*a^4*b^15*d^4 + 288*a^6*b^13*d^4 + 160*a^
8*b^11*d^4 - 160*a^10*b^9*d^4 - 288*a^12*b^7*d^4 - 160*a^14*b^5*d^4 - 32*a^
16*b^3*d^4))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^
8*b^2*d^4)) + (16*tan(c + d*x)^(1/2)*(8*a*b^14*d^2 + 36*a^3*b^12*d^2 + 316*
a^5*b^10*d^2 + 552*a^7*b^8*d^2 + 256*a^9*b^6*d^2 - 12*a^11*b^4*d^2 - 4*a^13
*b^2*d^2))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*
b^2*d^4)) + (16*(24*a^2*b^11*d^2 - 2*b^13*d^2 + 196*a^4*b^9*d^2 + 120*a^6*b
^7*d^2 - 50*a^8*b^5*d^2))/(a^10*d^5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b
^4*d^5 + 4*a^8*b^2*d^5)) + (16*tan(c + d*x)^(1/2)*(b^11 + 7*a^2*b^9 + 11*a^
4*b^7 - 27*a^6*b^5))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^
4 + 4*a^8*b^2*d^4))*1i - (-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*
d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i
- a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3
*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((-1i/(4*(a^4*d^2 + b^4*d^2
```

+ a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((16*(16*a*b^16*d^4 + 136*a^3*b^14*d^4 + 432*a^5*b^12*d^4 + 680*a^7*b^10*d^4 + 560*a^9*b^8*d^4 + 216*a^11*b^6*d^4 + 16*a^13*b^4*d^4 - 8*a^15*b^2*d^4))/(a^10*d^5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5) + (16*tan(c + d*x)^(1/2)*(-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(32*a^2*b^17*d^4 + 160*a^4*b^15*d^4 + 288*a^6*b^13*d^4 + 160*a^8*b^11*d^4 - 160*a^10*b^9*d^4 - 288*a^12*b^7*d^4 - 160*a^14*b^5*d^4 - 32*a^16*b^3*d^4))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4)) - (16*tan(c + d*x)^(1/2)*(8*a*b^14*d^2 + 36*a^3*b^12*d^2 + 316*a^5*b^10*d^2 + 552*a^7*b^8*d^2 + 256*a^9*b^6*d^2 - 12*a^11*b^4*d^2 - 4*a^13*b^2*d^2))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4)) + (16*(24*a^2*b^11*d^2 - 2*b^13*d^2 + 196*a^4*b^9*d^2 + 120*a^6*b^7*d^2 - 50*a^8*b^5*d^2))/(a^10*d^5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5)) - (16*tan(c + d*x)^(1/2)*(b^11 + 7*a^2*b^9 + 11*a^4*b^7 - 27*a^6*b^5))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4))*1i)/((32*(a*b^8 + 5*a^3*b^6))/(a^10*d^5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5) + (-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((16*(16*a*b^16*d^4 + 136*a^3*b^14*d^4 + 432*a^5*b^12*d^4 + 680*a^7*b^10*d^4 + 560*a^9*b^8*d^4 + 216*a^11*b^6*d^4 + 16*a^13*b^4*d^4 - 8*a^15*b^2*d^4))/(a^10*d^5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5) - (16*tan(c + d*x)^(1/2)*(-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(32*a^2*b^17*d^4 + 160*a^4*b^15*d^4 + 288*a^6*b^13*d^4 + 160*a^8*b^11*d^4 - 160*a^10*b^9*d^4 - 288*a^12*b^7*d^4 - 160*a^14*b^5*d^4 - 32*a^16*b^3*d^4))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4)) + (16*tan(c + d*x)^(1/2)*(8*a*b^14*d^2 + 36*a^3*b^12*d^2 + 316*a^5*b^10*d^2 + 552*a^7*b^8*d^2 + 256*a^9*b^6*d^2 - 12*a^11*b^4*d^2 - 4*a^13*b^2*d^2))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4)) + (16*(24*a^2*b^11*d^2 - 2*b^13*d^2 + 196*a^4*b^9*d^2 + 120*a^6*b^7*d^2 - 50*a^8*b^5*d^2))/(a^10*d^5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5)) + (16*tan(c + d*x)^(1/2)*(b^11 + 7*a^2*b^9 + 11*a^4*b^7 - 27*a^6*b^5))/(...

3.597 $\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=358

$$\frac{(a^2+2ab-b^2)\operatorname{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2 d}-\frac{(a^2+2ab-b^2)\operatorname{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2 d}-\frac{b^{5/2}}{}$$

```
[Out] -b^(5/2)*(7*a^2+3*b^2)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/a^(5/2)/(a^
2+b^2)^2/d-1/2*(a^2+2*a*b-b^2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2
)^2/d*2^(1/2)-1/2*(a^2+2*a*b-b^2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b
^2)^2/d*2^(1/2)-1/4*(a^2-2*a*b-b^2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c
))/(a^2+b^2)^2/d*2^(1/2)+1/4*(a^2-2*a*b-b^2)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+
tan(d*x+c))/(a^2+b^2)^2/d*2^(1/2)+(-2*a^2-3*b^2)/a^2/(a^2+b^2)/d/tan(d*x+c)
^(1/2)+b^2/a/(a^2+b^2)/d/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.50, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3650, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(a^2+2ab-b^2)\operatorname{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}-\frac{(a^2+2ab-b^2)\operatorname{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{b^2}{ad\,(a^2+b^2)\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))}-\frac{2a^2+3b^2}{a^2d\,(a^2+b^2)\sqrt{\tan(c+dx)}}-\frac{(a^2-2ab-b^2)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{(a^2-2ab-b^2)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}-\frac{b^{5/2}(7a^2+3b^2)\operatorname{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d\,(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^2),x]
```

```
[Out] ((a^2 + 2*a*b - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*(a^2
+ b^2)^2*d) - ((a^2 + 2*a*b - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/
(Sqrt[2]*(a^2 + b^2)^2*d) - (b^(5/2)*(7*a^2 + 3*b^2)*ArcTan[(Sqrt[b]*Sqrt[T
an[c + d*x]])/Sqrt[a]])/(a^(5/2)*(a^2 + b^2)^2*d) - ((a^2 - 2*a*b - b^2)*Lo
g[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^2*
d) + ((a^2 - 2*a*b - b^2)*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]
])/(2*Sqrt[2]*(a^2 + b^2)^2*d) - (2*a^2 + 3*b^2)/(a^2*(a^2 + b^2)*d*Sqrt[Ta
n[c + d*x]]) + b^2/(a*(a^2 + b^2)*d*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])
)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
```

```
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx = \frac{b^2}{a\left(a^2+b^2\right)d\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))} + \frac{\int \frac{\frac{1}{2}\left(2a^2+3b^2\right)-ab\tan}{\tan^{\frac{3}{2}}(c+dx}}{a\,(}$$

$$= -\frac{2a^2+3b^2}{a^2\left(a^2+b^2\right)d\sqrt{\tan(c+dx)}} + \frac{b^2}{a\left(a^2+b^2\right)d\sqrt{\tan(c+dx)}\,(a+}$$

$$= -\frac{2a^2+3b^2}{a^2\left(a^2+b^2\right)d\sqrt{\tan(c+dx)}} + \frac{b^2}{a\left(a^2+b^2\right)d\sqrt{\tan(c+dx)}\,(a+}$$

$$= -\frac{2a^2+3b^2}{a^2\left(a^2+b^2\right)d\sqrt{\tan(c+dx)}} + \frac{b^2}{a\left(a^2+b^2\right)d\sqrt{\tan(c+dx)}\,(a+}$$

$$= -\frac{2a^2+3b^2}{a^2\left(a^2+b^2\right)d\sqrt{\tan(c+dx)}} + \frac{b^2}{a\left(a^2+b^2\right)d\sqrt{\tan(c+dx)}\,(a+}$$

$$= -\frac{b^{5/2}\left(7a^2+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}\left(a^2+b^2\right)^2 d} - \frac{2a^2+3}{a^2\left(a^2+b^2\right)d\sqrt{t}}$$

$$= -\frac{b^{5/2}\left(7a^2+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}\left(a^2+b^2\right)^2 d} - \frac{\left(a^2-2ab-b^2\right)\mathrm{l}}{}$$

$$= \frac{\left(a^2+2ab-b^2\right)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)^2 d} - \frac{\left(a^2+2ab-b^2\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.83, size = 195, normalized size = 0.54

$$-\frac{\frac{b^{5/2}\left(7a^2+3b^2\right)\mathrm{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}\left(a^2+b^2\right)}+\frac{(-1)^{3/4}a\left((a+ib)^2\mathrm{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)-(a-ib)^2\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\right)}{a^2+b^2}+\frac{2a^2+3b^2}{a\sqrt{\tan(c+dx)}}-\frac{b^2}{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))}}{a\left(a^2+b^2\right)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^2),x]
```

```
[Out] -(((b^(5/2)*(7*a^2 + 3*b^2)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(
a^(3/2)*(a^2 + b^2)) + ((-1)^(3/4)*a*((a + I*b)^2*ArcTan[(-1)^(3/4)*Sqrt[Ta
n[c + d*x]]] - (a - I*b)^2*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]))/(a^2 +
b^2) + (2*a^2 + 3*b^2)/(a*Sqrt[Tan[c + d*x]]) - b^2/(Sqrt[Tan[c + d*x]]*(a
+ b*Tan[c + d*x])))/(a*(a^2 + b^2)*d))
```

Maple [A]

time = 0.12, size = 296, normalized size = 0.83

method	result
derivativedivides	$-\frac{2}{a^{2}\sqrt{\tan\left(dx+c\right)}}-\frac{2b^{3}\left(\frac{\left(\frac{a^{2}}{2}+\frac{b^{2}}{2}\right)\left(\sqrt{\tan}\left(dx+c\right)\right)}{a+b\tan\left(dx+c\right)}+\frac{\left(7a^{2}+3b^{2}\right)\arctan\left(\frac{b\left(\sqrt{\tan}\left(dx+c\right)\right)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{\left(a^{2}+b^{2}\right)^{2}a^{2}}+\frac{-\frac{ab\sqrt{2}\left(\cdots\right.}{\cdots}}{\cdots}$
default	$-\frac{2}{a^{2}\sqrt{\tan\left(dx+c\right)}}-\frac{2b^{3}\left(\frac{\left(\frac{a^{2}}{2}+\frac{b^{2}}{2}\right)\left(\sqrt{\tan}\left(dx+c\right)\right)}{a+b\tan\left(dx+c\right)}+\frac{\left(7a^{2}+3b^{2}\right)\arctan\left(\frac{b\left(\sqrt{\tan}\left(dx+c\right)\right)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{\left(a^{2}+b^{2}\right)^{2}a^{2}}+\frac{-\frac{ab\sqrt{2}\left(\cdots\right.}{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/a^2/tan(d*x+c)^(1/2)-2*b^3/(a^2+b^2)^2/a^2*((1/2*a^2+1/2*b^2)*tan(d
*x+c)^(1/2)/(a+b*tan(d*x+c))+1/2*(7*a^2+3*b^2)/(a*b)^(1/2)*arctan(b*tan(d*x
+c)^(1/2)/(a*b)^(1/2)))+2/(a^2+b^2)^2*(-1/4*a*b*2^(1/2)*(ln((1+2^(1/2)*tan(
d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(
1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/8*(-a^
2+b^2)*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d
*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2
^(1/2)*tan(d*x+c)^(1/2)))))
```

Maxima [A]

time = 0.50, size = 315, normalized size = 0.88

$$-\frac{\frac{4\left(7a^{2}b^{3}+3b^{5}\right)\arctan\left(\frac{b\sqrt{\tan\left(dx+c\right)}}{\sqrt{ab}}\right)}{\left(a^{6}+2a^{4}b^{2}+a^{2}b^{4}\right)\sqrt{ab}}+\frac{2\sqrt{2}\left(a^{2}+2ab-b^{2}\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(dx+c\right)}\right)\right)+2\sqrt{2}\left(a^{2}+2ab-b^{2}\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(dx+c\right)}\right)\right)-\sqrt{2}\left(a^{2}-2ab-b^{2}\right)\log\left(\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)+\sqrt{2}\left(a^{2}-2ab-b^{2}\right)\log\left(-\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)}{a^{4}+2a^{2}b^{2}+b^{4}}+\frac{4\left(2a^{3}+2ab^{2}+\left(2a^{2}b+3b^{3}\right)\tan\left(dx+c\right)\right)}{\left(a^{4}b+a^{2}b^{3}\right)\tan\left(dx+c\right)^{\frac{3}{2}}+\left(a^{5}+a^{3}b^{2}\right)\sqrt{\tan\left(dx+c\right)}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(4*(7*a^2*b^3 + 3*b^5)*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^6 +
2*a^4*b^2 + a^2*b^4)*sqrt(a*b)) + (2*sqrt(2)*(a^2 + 2*a*b - b^2)*arctan(1/2
*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*(a^2 + 2*a*b - b^2)*
arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*(a^2 - 2*a*
b - b^2)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*(a^2
- 2*a*b - b^2)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a^4 +
2*a^2*b^2 + b^4) + 4*(2*a^3 + 2*a*b^2 + (2*a^2*b + 3*b^3)*tan(d*x + c))/((a
^4*b + a^2*b^3)*tan(d*x + c)^(3/2) + (a^5 + a^3*b^2)*sqrt(tan(d*x + c))))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 8039 vs. $2(316) = 632$.
time = 10.99, size = 16081, normalized size = 44.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(2)*((a^16 + 5*a^14*b^2 + 9*a^12*b^4 + 5*a^10*b^6 - 5*a^8*b^8
- 9*a^6*b^10 - 5*a^4*b^12 - a^2*b^14)*d^5*cos(d*x + c)^4 - (a^16 + 4*a^14*b
^2 + 3*a^12*b^4 - 10*a^10*b^6 - 25*a^8*b^8 - 24*a^6*b^10 - 11*a^4*b^12 - 2*
a^2*b^14)*d^5*cos(d*x + c)^2 - (a^14*b^2 + 6*a^12*b^4 + 15*a^10*b^6 + 20*a^
8*b^8 + 15*a^6*b^10 + 6*a^4*b^12 + a^2*b^14)*d^5 + 2*((a^15*b + 6*a^13*b^3
+ 15*a^11*b^5 + 20*a^9*b^7 + 15*a^7*b^9 + 6*a^5*b^11 + a^3*b^13)*d^5*cos(d*
x + c)^3 - (a^15*b + 6*a^13*b^3 + 15*a^11*b^5 + 20*a^9*b^7 + 15*a^7*b^9 + 6
*a^5*b^11 + a^3*b^13)*d^5*cos(d*x + c))*sin(d*x + c))*sqrt((a^8 + 4*a^6*b^2
 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 - 2*a^5*
b^7 - 3*a^3*b^9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*
b^6 + b^8)*d^4)))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(
(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^16 + 8*a^14*b^2 + 28
*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^
14 + b^16)*d^4))*(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))^
(3/4)*arctan(((a^16 - 20*a^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 - 64*a^6*b^10
- 20*a^4*b^12 + b^16)*d^4*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6
+ b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^
6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4))*sqrt(1/((a^8 + 4*a^6*b^2 +
6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)) + sqrt(2)*(2*(a^17*b + 8*a^15*b^3 + 28*a
^13*b^5 + 56*a^11*b^7 + 70*a^9*b^9 + 56*a^7*b^11 + 28*a^5*b^13 + 8*a^3*b^15
 + a*b^17)*d^7*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^
16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28
*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4))*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
 4*a^2*b^6 + b^8)*d^4)) - (a^14 + 5*a^12*b^2 + 9*a^10*b^4 + 5*a^8*b^6 - 5*a
^6*b^8 - 9*a^4*b^10 - 5*a^2*b^12 - b^14)*d^5*sqrt((a^8 - 12*a^6*b^2 + 38*a^
4*b^4 - 12*a^2*b^6 + b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 +
 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4)))*sqrt((a
```

```
^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 4*(a^11*b + 3*a^9*b^3 + 2*a^
7*b^5 - 2*a^5*b^7 - 3*a^3*b^9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^
4*b^4 + 4*a^2*b^6 + b^8)*d^4)))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6
 + b^8))*sqrt(((a^12 - 10*a^10*b^2 + 15*a^8*b^4 + 52*a^6*b^6 + 15*a^4*b^8 -
 10*a^2*b^10 + b^12)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 +
 b^8)*d^4))*cos(d*x + c) + sqrt(2)*((a^14 - 11*a^12*b^2 + 25*a^10*b^4 + 37*
a^8*b^6 - 37*a^6*b^8 - 25*a^4*b^10 + 11*a^2*b^12 - b^14)*d^3*sqrt(1/((a^8 +
 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*cos(d*x + c) - 2*(a^9*b - 1
2*a^7*b^3 + 38*a^5*b^5 - 12*a^3*b^7 + a*b^9)*d*cos(d*x + c))*sqrt((a^8 + 4*
a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 -
 2*a^5*b^7 - 3*a^3*b^9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
 4*a^2*b^6 + b^8)*d^4)))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)
)*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*
b^6 + b^8)*d^4))^(1/4) + (a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)
*sin(d*x + c))/cos(d*x + c))*(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 +
 b^8)*d^4))^(3/4) - sqrt(2)*(2*(a^21*b + 2*a^19*b^3 - 19*a^17*b^5 - 104*a^1
5*b^7 - 238*a^13*b^9 - 308*a^11*b^11 - 238*a^9*b^13 - 104*a^7*b^15 - 19*a^5
*b^17 + 2*a^3*b^19 + a*b^21)*d^7*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a
^2*b^6 + b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8
+ 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4))*sqrt(1/((a^8 + 4*a^6
*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)) - (a^18 - a^16*b^2 - 20*a^14*b^4
- 44*a^12*b^6 - 26*a^10*b^8 + 26*a^8*b^10 + 44*a^6*b^12 + 20*a^4*b^14 + a^2
*b^16 - b^18)*d^5*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(
(a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 +
 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4)))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4
+ 4*a^2*b^6 + b^8 + 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 - 2*a^5*b^7 - 3*a^3*b
^9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^
4)))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(sin(d*x + c)/
cos(d*x + c))*(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))^(3/
4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)) + 4*sqrt(2)*((a^16
+ 5*a^14*b^2 + 9*a^12*b^4 + 5*a^10*b^6 - 5*a^8*b^8 - 9*a^6*b^10 - 5*a^4*b^1
2 - a^2*b^14)*d^5*cos(d*x + c)^4 - (a^16 + 4*a^14*b^2 + 3*a^12*b^4 - 10*a^1
0*b^6 - 25*a^8*b^8 - 24*a^6*b^10 - 11*a^4*b^12 - 2*a^2*b^14)*d^5*cos(d*x +
c)^2 - (a^14*b^2 + 6*a^12*b^4 + 15*a^10*b^6 + 20*a^8*b^8 + 15*a^6*b^10 + 6*
a^4*b^12 + a^2*b^14)*d^5 + 2*((a^15*b + 6*a^13*b^3 + 15*a^11*b^5 + 20*a^9*b
^7 + 15*a^7*b^9 + 6*a^5*b^11 + a^3*b^13)*d^5*cos(d*x + c)^3 - (a^15*b + 6*a
^13*b^3 + 15*a^11*b^5 + 20*a^9*b^7 + 15*a^7*b^9 + 6*a^5*b^11 + a^3*b^13)*d^
5*cos(d*x + c))*sin(d*x + c))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6
 + b^8 + 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 - 2*a^5*b^7 - 3*a^3*b^9 - a*b^11
)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\tan(c+dx))^2\tan^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(3/2)/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(1/((a + b*tan(c + d*x))**2*tan(c + d*x)**(3/2)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 7.76, size = 2500, normalized size = 6.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x))^2),x)
```

```
[Out] atan(((1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d
^2)))^(1/2)*(tan(c + d*x)^(1/2)*(144*a^14*b^23*d^5 + 1248*a^16*b^21*d^5 + 4
224*a^18*b^19*d^5 + 6720*a^20*b^17*d^5 + 3872*a^22*b^15*d^5 - 2816*a^24*b^1
3*d^5 - 5632*a^26*b^11*d^5 - 3136*a^28*b^9*d^5 - 560*a^30*b^7*d^5 + 32*a^32
*b^5*d^5) + (1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2
*b^2*d^2)))^(1/2)*(26496*a^25*b^14*d^6 - 1152*a^15*b^24*d^6 - 8448*a^17*b^2
2*d^6 - 23776*a^19*b^20*d^6 - 29664*a^21*b^18*d^6 - 6528*a^23*b^16*d^6 - (1
i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1
/2)*((1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^
2)))^(1/2)*(768*a^16*b^27*d^8 + 8704*a^18*b^25*d^8 + 44288*a^20*b^23*d^8 +
133120*a^22*b^21*d^8 + 261120*a^24*b^19*d^8 + 347136*a^26*b^17*d^8 + 311808
*a^28*b^15*d^8 + 178176*a^30*b^13*d^8 + 49920*a^32*b^11*d^8 - 7680*a^34*b^9
*d^8 - 12032*a^36*b^7*d^8 - 4096*a^38*b^5*d^8 - 512*a^40*b^3*d^8 - tan(c +
d*x)^(1/2)*(1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*
b^2*d^2)))^(1/2)*(512*a^18*b^27*d^9 + 5120*a^20*b^25*d^9 + 22528*a^22*b^23*
d^9 + 56320*a^24*b^21*d^9 + 84480*a^26*b^19*d^9 + 67584*a^28*b^17*d^9 - 675
84*a^32*b^13*d^9 - 84480*a^34*b^11*d^9 - 56320*a^36*b^9*d^9 - 22528*a^38*b^
7*d^9 - 5120*a^40*b^5*d^9 - 512*a^42*b^3*d^9)) + tan(c + d*x)^(1/2)*(1152*a
^15*b^26*d^7 + 13440*a^17*b^24*d^7 + 69056*a^19*b^22*d^7 + 202752*a^21*b^20
*d^7 + 372800*a^23*b^18*d^7 + 443136*a^25*b^16*d^7 + 337792*a^27*b^14*d^7 +
 156160*a^29*b^12*d^7 + 37632*a^31*b^10*d^7 + 3200*a^33*b^8*d^7 + 704*a^35*
b^6*d^7 + 512*a^37*b^4*d^7 + 64*a^39*b^2*d^7)) + 33984*a^27*b^12*d^6 + 1862
```

4*a^29*b^10*d^6 + 5376*a^31*b^8*d^6 + 1152*a^33*b^6*d^6 + 288*a^35*b^4*d^6 + 32*a^37*b^2*d^6))*1i + (1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(tan(c + d*x)^(1/2)*(144*a^14*b^23*d^5 + 1248*a^16*b^21*d^5 + 4224*a^18*b^19*d^5 + 6720*a^20*b^17*d^5 + 3872*a^22*b^15*d^5 - 2816*a^24*b^13*d^5 - 5632*a^26*b^11*d^5 - 3136*a^28*b^9*d^5 - 560*a^30*b^7*d^5 + 32*a^32*b^5*d^5) - (1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(26496*a^25*b^14*d^6 - 1152*a^15*b^24*d^6 - 8448*a^17*b^22*d^6 - 23776*a^19*b^20*d^6 - 29664*a^21*b^18*d^6 - 6528*a^23*b^16*d^6 - (1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(768*a^16*b^27*d^8 + 8704*a^18*b^25*d^8 + 44288*a^20*b^23*d^8 + 133120*a^22*b^21*d^8 + 261120*a^24*b^19*d^8 + 347136*a^26*b^17*d^8 + 311808*a^28*b^15*d^8 + 178176*a^30*b^13*d^8 + 49920*a^32*b^11*d^8 - 7680*a^34*b^9*d^8 - 12032*a^36*b^7*d^8 - 4096*a^38*b^5*d^8 - 512*a^40*b^3*d^8 + tan(c + d*x)^(1/2)*(1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(512*a^18*b^27*d^9 + 5120*a^20*b^25*d^9 + 22528*a^22*b^23*d^9 + 56320*a^24*b^21*d^9 + 84480*a^26*b^19*d^9 + 67584*a^28*b^17*d^9 - 67584*a^32*b^13*d^9 - 84480*a^34*b^11*d^9 - 56320*a^36*b^9*d^9 - 22528*a^38*b^7*d^9 - 5120*a^40*b^5*d^9 - 512*a^42*b^3*d^9)) - tan(c + d*x)^(1/2)*(1152*a^15*b^26*d^7 + 13440*a^17*b^24*d^7 + 69056*a^19*b^22*d^7 + 202752*a^21*b^20*d^7 + 372800*a^23*b^18*d^7 + 443136*a^25*b^16*d^7 + 337792*a^27*b^14*d^7 + 156160*a^29*b^12*d^7 + 37632*a^31*b^10*d^7 + 3200*a^33*b^8*d^7 + 704*a^35*b^6*d^7 + 512*a^37*b^4*d^7 + 64*a^39*b^2*d^7)) + 33984*a^27*b^12*d^6 + 18624*a^29*b^10*d^6 + 5376*a^31*b^8*d^6 + 1152*a^33*b^6*d^6 + 288*a^35*b^4*d^6 + 32*a^37*b^2*d^6))*1i)/((1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(tan(c + d*x)^(1/2)*(144*a^14*b^23*d^5 + 1248*a^16*b^21*d^5 + 4224*a^18*b^19*d^5 + 6720*a^20*b^17*d^5 + 3872*a^22*b^15*d^5 - 2816*a^24*b^13*d^5 - 5632*a^26*b^11*d^5 - 3136*a^28*b^9*d^5 - 560*a^30*b^7*d^5 + 32*a^32*b^5*d^5) + (1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(26496*a^25*b^14*d^6 - 1152*a^15*b^24*d^6 - 8448*a^17*b^22*d^6 - 23776*a^19*b^20*d^6 - 29664*a^21*b^18*d^6 - 6528*a^23*b^16*d^6 - (1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(768*a^16*b^27*d^8 + 8704*a^18*b^25*d^8 + 44288*a^20*b^23*d^8 + 133120*a^22*b^21*d^8 + 261120*a^24*b^19*d^8 + 347136*a^26*b^17*d^8 + 311808*a^28*b^15*d^8 + 178176*a^30*b^13*d^8 + 49920*a^32*b^11*d^8 - 7680*a^34*b^9*d^8 - 12032*a^36*b^7*d^8 - 4096*a^38*b^5*d^8 - 512*a^40*b^3*d^8 - tan(c + d*x)^(1/2)*(1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(512*a^18*b^27*d^9 + 5120*a^20*b^25*d^9 + 22528*a^22*b^23*d^9 + 56320*a^24*b^21*d^9 + 84480*a^26*b^19*d^9 + 67584*a^28*b^17*d^9 - 67584*a^32*b^13*d^9 - 84480*a^34*b^11*d^9 - 56320*a^36*b^9*d^9 - 22528*a^38*b^7*d^9 - 5120*a^40*b^5*d^9 - 512*a^42*b^3*d^9)) + tan(c + d*x)^(1/2)*(1152*a^15*b^26*d^7 + 13440*a^17*b^24*d^7 + 69056*a^19*b^22*d^7 + 202752*a^21*b^20*d^7 + 372800*a^23*b^18*d^7 + 443136*a^25*b^16*d^7 + 337792*a^27*b^14*d^7 + 156160*a...

3.598 $\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=397

$$\frac{(a^2-2ab-b^2)\,\mathrm{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}-\frac{(a^2-2ab-b^2)\,\mathrm{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}+\frac{b^{7/2}(}{}$$

```
[Out] b^(7/2)*(9*a^2+5*b^2)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/a^(7/2)/(a^2
+b^2)^2/d-1/2*(a^2-2*a*b-b^2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)
^2/d*2^(1/2)-1/2*(a^2-2*a*b-b^2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^
2)^2/d*2^(1/2)+1/4*(a^2+2*a*b-b^2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)
)/(a^2+b^2)^2/d*2^(1/2)-1/4*(a^2+2*a*b-b^2)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+t
an(d*x+c))/(a^2+b^2)^2/d*2^(1/2)+b*(4*a^2+5*b^2)/a^3/(a^2+b^2)/d/tan(d*x+c)
^(1/2)+1/3*(-2*a^2-5*b^2)/a^2/(a^2+b^2)/d/tan(d*x+c)^(3/2)+b^2/a/(a^2+b^2)/
d/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.68, antiderivative size = 397, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3650, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(a^2-2ab-b^2)\,\mathrm{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}-\frac{(a^2-2ab-b^2)\,\mathrm{ArcTan}\left(\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{b^2}{ad\,(a^2+b^2)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}-\frac{2a^2+5b^2}{3a^2d\,(a^2+b^2)\tan^{\frac{3}{2}}(c+dx)}+\frac{(a^2+2ab-b^2)\log\left(\tan(c+dx)-\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}-\frac{(a^2+2ab-b^2)\log\left(\tan(c+dx)+\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{b^{7/2}(9a^2+5b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^{7/2}d\,(a^2+b^2)^2}+\frac{b(4a^2+5b^2)}{a^3d\,(a^2+b^2)\,\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^2),x]
```

```
[Out] ((a^2 - 2*a*b - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*(a^2
+ b^2)^2*d) - ((a^2 - 2*a*b - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/
(Sqrt[2]*(a^2 + b^2)^2*d) + (b^(7/2)*(9*a^2 + 5*b^2)*ArcTan[(Sqrt[b]*Sqrt[T
an[c + d*x]])/Sqrt[a]])/(a^(7/2)*(a^2 + b^2)^2*d) + ((a^2 + 2*a*b - b^2)*Lo
g[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^2*
d) - ((a^2 + 2*a*b - b^2)*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]
])/(2*Sqrt[2]*(a^2 + b^2)^2*d) - (2*a^2 + 5*b^2)/(3*a^2*(a^2 + b^2)*d*Tan[c
 + d*x]^(3/2)) + (b*(4*a^2 + 5*b^2))/(a^3*(a^2 + b^2)*d*Sqrt[Tan[c + d*x]])
 + b^2/(a*(a^2 + b^2)*d*Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
```

```
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx = \frac{b^2}{a\left(a^2+b^2\right)d\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))} + \frac{\int \frac{\frac{1}{2}\left(2a^2+5b^2\right)-ab\tan}{\tan^{\frac{5}{2}}(c+dx}}{a(}$$

$$= -\frac{2a^2+5b^2}{3a^2\left(a^2+b^2\right)d\tan^{\frac{3}{2}}(c+dx)} + \frac{b^2}{a\left(a^2+b^2\right)d\tan^{\frac{3}{2}}(c+dx)(a+b}$$

$$= -\frac{2a^2+5b^2}{3a^2\left(a^2+b^2\right)d\tan^{\frac{3}{2}}(c+dx)} + \frac{b\left(4a^2+5b^2\right)}{a^3\left(a^2+b^2\right)d\sqrt{\tan(c+dx)}} +$$

$$= -\frac{2a^2+5b^2}{3a^2\left(a^2+b^2\right)d\tan^{\frac{3}{2}}(c+dx)} + \frac{b\left(4a^2+5b^2\right)}{a^3\left(a^2+b^2\right)d\sqrt{\tan(c+dx)}} +$$

$$= -\frac{2a^2+5b^2}{3a^2\left(a^2+b^2\right)d\tan^{\frac{3}{2}}(c+dx)} + \frac{b\left(4a^2+5b^2\right)}{a^3\left(a^2+b^2\right)d\sqrt{\tan(c+dx)}} +$$

$$= -\frac{2a^2+5b^2}{3a^2\left(a^2+b^2\right)d\tan^{\frac{3}{2}}(c+dx)} + \frac{b\left(4a^2+5b^2\right)}{a^3\left(a^2+b^2\right)d\sqrt{\tan(c+dx)}} +$$

$$= \frac{b^{7/2}\left(9a^2+5b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^{7/2}\left(a^2+b^2\right)^2 d} - \frac{2a^2+5}{3a^2\left(a^2+b^2\right)d\,\mathrm{ta}}$$

$$= \frac{b^{7/2}\left(9a^2+5b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^{7/2}\left(a^2+b^2\right)^2 d} + \frac{\left(a^2+2ab-b^2\right)\mathrm{lo}}{}$$

$$= \frac{\left(a^2-2ab-b^2\right)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)^2 d} - \frac{\left(a^2-2ab-b\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.50, size = 230, normalized size = 0.58

$$\frac{\frac{3\left(\sqrt[4]{-1}\,a^{7/2}(a+ib)^2\,\mathrm{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+b^{7/2}\left(9a^2+5b^2\right)\mathrm{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)+\sqrt[4]{-1}\,a^{7/2}(a-ib)^2\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\right)}{a^{5/2}\left(a^2+b^2\right)} - \frac{2a^2+5b^2}{a\tan^{\frac{3}{2}}(c+dx)} + \frac{3b\left(4a^2+5b^2\right)}{a^2\sqrt{\tan(c+dx)}} + \frac{3b^2}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}}{3a\left(a^2+b^2\right)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^2),x]
```

```
[Out] ((3*((-1)^(1/4)*a^(7/2)*(a + I*b)^2*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] +
 b^(7/2)*(9*a^2 + 5*b^2)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]] + (-1
)^(1/4)*a^(7/2)*(a - I*b)^2*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]))/(a^(5/
2)*(a^2 + b^2)) - (2*a^2 + 5*b^2)/(a*Tan[c + d*x]^(3/2)) + (3*b*(4*a^2 + 5*
b^2))/(a^2*Sqrt[Tan[c + d*x]]) + (3*b^2)/(Tan[c + d*x]^(3/2)*(a + b*Tan[c +
 d*x])))/(3*a*(a^2 + b^2)*d)
```

Maple [A]

time = 0.12, size = 310, normalized size = 0.78

method	result
derivativedivides	$\frac{-\frac{2}{3a^2\tan(dx+c)^{\frac{3}{2}}}+\frac{4b}{a^3\sqrt{\tan(dx+c)}}+\frac{2b^4\left(\frac{\left(\frac{a^2}{2}+\frac{b^2}{2}\right)\left(\sqrt{\tan}(dx+c)\right)}{a+b\tan(dx+c)}+\frac{\left(9a^2+5b^2\right)\arctan\left(\frac{b\left(\sqrt{\tan}(dx+c)\right)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right.}{a^3\left(a^2+b^2\right)^2}}{}$
default	$\frac{-\frac{2}{3a^2\tan(dx+c)^{\frac{3}{2}}}+\frac{4b}{a^3\sqrt{\tan(dx+c)}}+\frac{2b^4\left(\frac{\left(\frac{a^2}{2}+\frac{b^2}{2}\right)\left(\sqrt{\tan}(dx+c)\right)}{a+b\tan(dx+c)}+\frac{\left(9a^2+5b^2\right)\arctan\left(\frac{b\left(\sqrt{\tan}(dx+c)\right)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right.}{a^3\left(a^2+b^2\right)^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/3/a^2/tan(d*x+c)^(3/2)+4/a^3*b/tan(d*x+c)^(1/2)+2*b^4/a^3/(a^2+b^2)
^2*((1/2*a^2+1/2*b^2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))+1/2*(9*a^2+5*b^2)/(
a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2)))+2/(a^2+b^2)^2*(1/8*(-a^2
+b^2)*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*
x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^
(1/2)*tan(d*x+c)^(1/2)))+1/4*a*b*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+ta
n(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d
*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))))
```

Maxima [A]

time = 0.50, size = 342, normalized size = 0.86

$$\frac{\frac{12\left(9a^2b^4+5b^6\right)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{\left(a^7+2a^5b^2+a^3b^4\right)\sqrt{ab}}-\frac{4\left(2a^4+2a^2b^2-3\left(4a^2b^2+5b^4\right)\tan(dx+c)^2-10\left(a^3b+ab^3\right)\tan(dx+c)\right)}{\left(a^5b+a^3b^3\right)\tan(dx+c)^{\frac{5}{2}}+\left(a^6+a^4b^2\right)\tan(dx+c)^{\frac{3}{2}}}-\frac{3\left(2\sqrt{2}\left(a^2-2ab-b^2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}\left(a^2-2ab-b^2\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)+\sqrt{2}\left(a^2+2ab-b^2\right)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)-\sqrt{2}\left(a^2+2ab-b^2\right)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)\right)}{a^4+2a^2b^2+b^4}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/12*(12*(9*a^2*b^4 + 5*b^6)*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^7 +
 2*a^5*b^2 + a^3*b^4)*sqrt(a*b)) - 4*(2*a^4 + 2*a^2*b^2 - 3*(4*a^2*b^2 + 5*
b^4)*tan(d*x + c)^2 - 10*(a^3*b + a*b^3)*tan(d*x + c))/((a^5*b + a^3*b^3)*t
an(d*x + c)^(5/2) + (a^6 + a^4*b^2)*tan(d*x + c)^(3/2)) - 3*(2*sqrt(2)*(a^2
 - 2*a*b - b^2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sq
rt(2)*(a^2 - 2*a*b - b^2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c
)))) + sqrt(2)*(a^2 + 2*a*b - b^2)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x
 + c) + 1) - sqrt(2)*(a^2 + 2*a*b - b^2)*log(-sqrt(2)*sqrt(tan(d*x + c)) +
tan(d*x + c) + 1))/(a^4 + 2*a^2*b^2 + b^4))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 8015 vs. $2(351) = 702$.
time = 12.86, size = 16034, normalized size = 40.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(12*sqrt(2)*((a^17 + 5*a^15*b^2 + 9*a^13*b^4 + 5*a^11*b^6 - 5*a^9*b^
8 - 9*a^7*b^10 - 5*a^5*b^12 - a^3*b^14)*d^5*cos(d*x + c)^4 - (a^17 + 4*a^15
*b^2 + 3*a^13*b^4 - 10*a^11*b^6 - 25*a^9*b^8 - 24*a^7*b^10 - 11*a^5*b^12 -
2*a^3*b^14)*d^5*cos(d*x + c)^2 - (a^15*b^2 + 6*a^13*b^4 + 15*a^11*b^6 + 20*
a^9*b^8 + 15*a^7*b^10 + 6*a^5*b^12 + a^3*b^14)*d^5 + 2*((a^16*b + 6*a^14*b^
3 + 15*a^12*b^5 + 20*a^10*b^7 + 15*a^8*b^9 + 6*a^6*b^11 + a^4*b^13)*d^5*cos
(d*x + c)^3 - (a^16*b + 6*a^14*b^3 + 15*a^12*b^5 + 20*a^10*b^7 + 15*a^8*b^9
 + 6*a^6*b^11 + a^4*b^13)*d^5*cos(d*x + c))*sin(d*x + c))*sqrt((a^8 + 4*a^6
*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 - 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 - 2*
a^5*b^7 - 3*a^3*b^9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*
a^2*b^6 + b^8)*d^4)))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*s
qrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^16 + 8*a^14*b^2
+ 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^
2*b^14 + b^16)*d^4))*(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^
4))^(3/4)*arctan(((a^16 - 20*a^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 - 64*a^6*b
^10 - 20*a^4*b^12 + b^16)*d^4*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*
b^6 + b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 5
6*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4))*sqrt(1/((a^8 + 4*a^6*b^
2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)) - sqrt(2)*((a^18 + 7*a^16*b^2 + 20*a
^14*b^4 + 28*a^12*b^6 + 14*a^10*b^8 - 14*a^8*b^10 - 28*a^6*b^12 - 20*a^4*b^
14 - 7*a^2*b^16 - b^18)*d^7*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^
6 + b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*
a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4))*sqrt(1/((a^8 + 4*a^6*b^2
```

+ 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)) + 2*(a^13*b + 6*a^11*b^3 + 15*a^9*b^5 + 20*a^7*b^7 + 15*a^5*b^9 + 6*a^3*b^11 + a*b^13)*d^5*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4))) *sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 - 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 - 2*a^5*b^7 - 3*a^3*b^9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(((a^12 - 10*a^10*b^2 + 15*a^8*b^4 + 52*a^6*b^6 + 15*a^4*b^8 - 10*a^2*b^10 + b^12)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*cos(d*x + c) + sqrt(2)*(2*(a^13*b - 10*a^11*b^3 + 15*a^9*b^5 + 52*a^7*b^7 + 15*a^5*b^9 - 10*a^3*b^11 + a*b^13)*d^3*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*cos(d*x + c) + (a^10 - 13*a^8*b^2 + 50*a^6*b^4 - 50*a^4*b^6 + 13*a^2*b^8 - b^10)*d*cos(d*x + c))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 - 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 - 2*a^5*b^7 - 3*a^3*b^9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))^(1/4) + (a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*sin(d*x + c))/cos(d*x + c))*(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))^(3/4) + sqrt(2)*((a^22 + a^20*b^2 - 21*a^18*b^4 - 85*a^16*b^6 - 134*a^14*b^8 - 70*a^12*b^10 + 70*a^10*b^12 + 134*a^8*b^14 + 85*a^6*b^16 + 21*a^4*b^18 - a^2*b^20 - b^22)*d^7*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4))*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)) + 2*(a^17*b - 20*a^13*b^5 - 64*a^11*b^7 - 90*a^9*b^9 - 64*a^7*b^11 - 20*a^5*b^13 + a*b^17)*d^5*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4)))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 - 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 - 2*a^5*b^7 - 3*a^3*b^9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))^(3/4))/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)) + 12*sqrt(2)*((a^17 + 5*a^15*b^2 + 9*a^13*b^4 + 5*a^11*b^6 - 5*a^9*b^8 - 9*a^7*b^10 - 5*a^5*b^12 - a^3*b^14)*d^5*cos(d*x + c)^4 - (a^17 + 4*a^15*b^2 + 3*a^13*b^4 - 10*a^11*b^6 - 25*a^9*b^8 - 24*a^7*b^10 - 11*a^5*b^12 - 2*a^3*b^14)*d^5*cos(d*x + c)^2 - (a^15*b^2 + 6*a^13*b^4 + 15*a^11*b^6 + 20*a^9*b^8 + 15*a^7*b^10 + 6*a^5*b^12 + a^3*b^14)*d^5 + 2*((a^16*b + 6*a^14*b^3 + 15*a^12*b^5 + 20*a^10*b^7 + 15*a^8*b^9 + 6*a^6*b^11 + a^4*b^13)*d^5*cos(d*x + c)^3 - (a^16*b + 6*a^14*b^3 + 15*a^12*b^5 + 20*a^10*b^7 + 15*a^8*b^9 + 6*a^6*b^11 + a^4*b^13)*d^5*cos(d*x + c))*sin(d*x + c))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 - 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 - 2*a^5*b^7 - 3*a^3*b^9 - a*b^11)*d^2*sqrt(1/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4...

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\tan(c+dx))^2\tan^{\frac{5}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(5/2)/(a+b*tan(d*x+c))**2,x)
[Out] Integral(1/((a + b*tan(c + d*x))**2*tan(c + d*x)**(5/2)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^2,x, algorithm="giac")
[Out] Timed out
```

Mupad [B]
time = 9.64, size = 2500, normalized size = 6.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(5/2)*(a + b*tan(c + d*x))^2),x)
[Out] ((10*b*tan(c + d*x))/(3*a^2) - 2/(3*a) + (tan(c + d*x)^2*(5*b^4 + 4*a^2*b^2
))/(a^3*(a^2 + b^2)))/(a*d*tan(c + d*x)^(3/2) + b*d*tan(c + d*x)^(5/2)) - a
tan(((-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d
^2)))^(1/2)*((-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a
^2*b^2*d^2)))^(1/2)*((-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*
4i - 6*a^2*b^2*d^2)))^(1/2)*(tan(c + d*x)^(1/2)*(3200*a^22*b^28*d^7 + 33920
*a^24*b^26*d^7 + 158208*a^26*b^24*d^7 + 425536*a^28*b^22*d^7 + 727296*a^30*
b^20*d^7 + 820672*a^32*b^18*d^7 + 615936*a^34*b^16*d^7 + 304256*a^36*b^14*d
^7 + 98432*a^38*b^12*d^7 + 22016*a^40*b^10*d^7 + 3072*a^42*b^8*d^7 - 704*a^
44*b^6*d^7 - 512*a^46*b^4*d^7 - 64*a^48*b^2*d^7) - (-1i/(4*(a^4*d^2 + b^4*d
^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(1280*a^24*b^28*d
^8 + 13824*a^26*b^26*d^8 + 66944*a^28*b^24*d^8 + 190848*a^30*b^22*d^8 + 352
640*a^32*b^20*d^8 + 435840*a^34*b^18*d^8 + 354048*a^36*b^16*d^8 + 169728*a^
38*b^14*d^8 + 24576*a^40*b^12*d^8 - 21760*a^42*b^10*d^8 - 13440*a^44*b^8*d^
8 - 2176*a^46*b^6*d^8 + 384*a^48*b^4*d^8 + 128*a^50*b^2*d^8 + tan(c + d*x)^
(1/2)*(-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*
```

d^2)))^(1/2)*(512*a^27*b^27*d^9 + 5120*a^29*b^25*d^9 + 22528*a^31*b^23*d^9 + 56320*a^33*b^21*d^9 + 84480*a^35*b^19*d^9 + 67584*a^37*b^17*d^9 - 67584*a^41*b^13*d^9 - 84480*a^43*b^11*d^9 - 56320*a^45*b^9*d^9 - 22528*a^47*b^7*d^9 - 5120*a^49*b^5*d^9 - 512*a^51*b^3*d^9))) - 800*a^21*b^27*d^6 - 2080*a^23*b^25*d^6 + 12928*a^25*b^23*d^6 + 78464*a^27*b^21*d^6 + 183616*a^29*b^19*d^6 + 238400*a^31*b^17*d^6 + 184960*a^33*b^15*d^6 + 84608*a^35*b^13*d^6 + 20704*a^37*b^11*d^6 + 2016*a^39*b^9*d^6) - tan(c + d*x)^(1/2)*(9472*a^31*b^15*d^5 - 3040*a^23*b^23*d^5 - 9056*a^25*b^21*d^5 - 12352*a^27*b^19*d^5 - 4256*a^29*b^17*d^5 - 400*a^21*b^25*d^5 + 13760*a^33*b^13*d^5 + 7744*a^35*b^11*d^5 + 1968*a^37*b^9*d^5 + 224*a^39*b^7*d^5 + 32*a^41*b^5*d^5))*1i - (-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(12928*a^25*b^23*d^6 - 800*a^21*b^27*d^6 - 2080*a^23*b^25*d^6 - (-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(tan(c + d*x)^(1/2)*(3200*a^22*b^28*d^7 + 33920*a^24*b^26*d^7 + 158208*a^26*b^24*d^7 + 425536*a^28*b^22*d^7 + 727296*a^30*b^20*d^7 + 820672*a^32*b^18*d^7 + 615936*a^34*b^16*d^7 + 304256*a^36*b^14*d^7 + 98432*a^38*b^12*d^7 + 22016*a^40*b^10*d^7 + 3072*a^42*b^8*d^7 - 704*a^44*b^6*d^7 - 512*a^46*b^4*d^7 - 64*a^48*b^2*d^7) + (-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(1280*a^24*b^28*d^8 + 13824*a^26*b^26*d^8 + 66944*a^28*b^24*d^8 + 190848*a^30*b^22*d^8 + 352640*a^32*b^20*d^8 + 435840*a^34*b^18*d^8 + 354048*a^36*b^16*d^8 + 169728*a^38*b^14*d^8 + 24576*a^40*b^12*d^8 - 21760*a^42*b^10*d^8 - 13440*a^44*b^8*d^8 - 2176*a^46*b^6*d^8 + 384*a^48*b^4*d^8 + 128*a^50*b^2*d^8 - tan(c + d*x)^(1/2)*(-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(512*a^27*b^27*d^9 + 5120*a^29*b^25*d^9 + 22528*a^31*b^23*d^9 + 56320*a^33*b^21*d^9 + 84480*a^35*b^19*d^9 + 67584*a^37*b^17*d^9 - 67584*a^41*b^13*d^9 - 84480*a^43*b^11*d^9 - 56320*a^45*b^9*d^9 - 22528*a^47*b^7*d^9 - 5120*a^49*b^5*d^9 - 512*a^51*b^3*d^9))) + 78464*a^27*b^21*d^6 + 183616*a^29*b^19*d^6 + 238400*a^31*b^17*d^6 + 184960*a^33*b^15*d^6 + 84608*a^35*b^13*d^6 + 20704*a^37*b^11*d^6 + 2016*a^39*b^9*d^6) + tan(c + d*x)^(1/2)*(9472*a^31*b^15*d^5 - 3040*a^23*b^23*d^5 - 9056*a^25*b^21*d^5 - 12352*a^27*b^19*d^5 - 4256*a^29*b^17*d^5 - 400*a^21*b^25*d^5 + 13760*a^33*b^13*d^5 + 7744*a^35*b^11*d^5 + 1968*a^37*b^9*d^5 + 224*a^39*b^7*d^5 + 32*a^41*b^5*d^5))*1i)/(160*a^24*b^20*d^4 - (-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*((-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(12928*a^25*b^23*d^6 - 800*a^21*b^27*d^6 - 2080*a^23*b^25*d^6 - (-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(tan(c + d*x)^(1/2)*(3200*a^22*b^28*d^7 + 33920*a^24*b^26*d^7 + 158208*a^26*b^24*d^7 + 425536*a^28*b^22*d^7 + 727296*a^30*b^20*d^7 + 820672*a^32*b^18*d^7 + 615936*a^34*b^16*d^7 + 304256*a^36*b^14*d^7 + 98432*a^38*b^12*d^7 + 22016*a^40*b^10*d^7 + 3072*a^42*b^8*d^7 - 704*a^44*b^6*d^7 - 512*a^46*b^4*d^7 - 64*a^48*b^2*d^7) + (-1i/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(1280*a^24*b^28*d^8 + 13824*a^26*b^26*d^8 + 66944*a^28*b^24*d^8 + 190848*a^30*b^22*d^8 + 352640*a^32*b^20*d^8 +

435840*a^34*b^18*d^8 + 354048*a^36*b^16*d^8 + 169728*a^38*b^14*d^8 + 24576*
a^40*b^12*d^8 - 21760*a^42*b^10*d^8 - 13440*a^44*b^8*d^8 - 2176*a^46*b^6*d^
8 + 384*a^48*b^4*d^8 + 128*a^50*b^2*d^8 - tan(c + d*x)^(1/2)*(-1i/(4*(a^4*d
^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(512*a^
27*b^27*d^9 + 5120*a^29*b^25*d^9 + 22528*a^31*b^23*d^9 + 56320*a^33*b^21*d^
9 + 84480*a^35*b^19*d^9 + 67584*a^37*b^17*d^9 -...

3.599 $\int \frac{\tan^{\frac{11}{2}}(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=493

$$\frac{(a+b)\left(a^2-4ab+b^2\right)\operatorname{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,\left(a^2+b^2\right)^3 d}-\frac{(a+b)\left(a^2-4ab+b^2\right)\operatorname{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(c\right.}}{\sqrt{2}\,\left(a^2+b^2\right)^3 d}$$

```
[Out] 1/4*a^(7/2)*(35*a^4+102*a^2*b^2+99*b^4)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(
1/2))/b^(9/2)/(a^2+b^2)^3/d-1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(-1+2^(1/2)*tan
(d*x+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(1+2^(
1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)+1/4*(a-b)*(a^2+4*a*b+b^2)*ln(1
-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)^3/d*2^(1/2)-1/4*(a-b)*(a^2+
4*a*b+b^2)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)^3/d*2^(1/2)-
1/4*a*(35*a^4+67*a^2*b^2+24*b^4)*tan(d*x+c)^(1/2)/b^4/(a^2+b^2)^2/d+1/12*(3
5*a^4+67*a^2*b^2+8*b^4)*tan(d*x+c)^(3/2)/b^3/(a^2+b^2)^2/d-1/2*a^2*tan(d*x+
c)^(7/2)/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^2-1/4*a^2*(7*a^2+15*b^2)*tan(d*x+c)
^(5/2)/b^2/(a^2+b^2)^2/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.98, antiderivative size = 493, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3646, 3726, 3728, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(a+b)\left(a^2-4ab+b^2\right)\operatorname{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d\left(a^2+b^2\right)^3}-\frac{(a+b)\left(a^2-4ab+b^2\right)\operatorname{ArcTan}\left(\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d\left(a^2+b^2\right)^3}-\frac{a^2\left(7a^2+15b^2\right)\tan^{\frac{5}{2}}(c+dx)}{4b^2d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}-\frac{a^2\tan^{\frac{7}{2}}(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}+\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\tan(c+dx)-\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)^3}-\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\tan(c+dx)+\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)^3}-\frac{a\left(35a^4+67a^2b^2+24b^4\right)\sqrt{\tan(c+dx)}}{4b^4d\left(a^2+b^2\right)^2}+\frac{\left(35a^4+67a^2b^2+8b^4\right)\tan^{\frac{3}{2}}(c+dx)}{12b^3d\left(a^2+b^2\right)^2}+\frac{a^{7/2}\left(35a^4+102a^2b^2+99b^4\right)\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{4b^{9/2}d\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(11/2)/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[
2]*(a^2 + b^2)^3*d) - ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 + Sqrt[2]*Sqrt[
Tan[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)^3*d) + (a^(7/2)*(35*a^4 + 102*a^2*b^2
+ 99*b^4)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(4*b^(9/2)*(a^2 + b
^2)^3*d) + ((a - b)*(a^2 + 4*a*b + b^2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
+ Tan[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^3*d) - ((a - b)*(a^2 + 4*a*b + b^2)
*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)
^3*d) - (a*(35*a^4 + 67*a^2*b^2 + 24*b^4)*Sqrt[Tan[c + d*x]])/(4*b^4*(a^2 +
 b^2)^2*d) + ((35*a^4 + 67*a^2*b^2 + 8*b^4)*Tan[c + d*x]^(3/2))/(12*b^3*(a^
2 + b^2)^2*d) - (a^2*Tan[c + d*x]^(7/2))/(2*b*(a^2 + b^2)*d*(a + b*Tan[c +
d*x])^2) - (a^2*(7*a^2 + 15*b^2)*Tan[c + d*x]^(5/2))/(4*b^2*(a^2 + b^2)^2*d
*(a + b*Tan[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ [b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & & (LtQ[a, 0] || LtQ[b, 0])

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 631

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1176

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e /(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] & & EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1179

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 1182

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D

```
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\tan^{\frac{11}{2}}(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{a^2\tan^{\frac{7}{2}}(c+dx)}{2b(a^2+b^2)\,d(a+b\tan(c+dx))^2} + \frac{\int \frac{\tan^{\frac{5}{2}}(c+dx)\left(\frac{7a^2}{2}-2ab\tan(c+dx)+\frac{1}{2}(7a^2+4b^2)\tan^2\right.}{(a+b\tan(c+dx))^2}}{2b(a^2+b^2)}$$

$$= -\frac{a^2\tan^{\frac{7}{2}}(c+dx)}{2b(a^2+b^2)\,d(a+b\tan(c+dx))^2} - \frac{a^2(7a^2+15b^2)\tan^{\frac{5}{2}}(c+dx)}{4b^2(a^2+b^2)^2\,d(a+b\tan(c+dx))} + \frac{\int \text{ta}}{}$$

$$= \frac{(35a^4+67a^2b^2+8b^4)\tan^{\frac{3}{2}}(c+dx)}{12b^3(a^2+b^2)^2\,d} - \frac{a^2\tan^{\frac{7}{2}}(c+dx)}{2b(a^2+b^2)\,d(a+b\tan(c+dx))^2} - \frac{a}{4b^2}$$

$$= -\frac{a(35a^4+67a^2b^2+24b^4)\sqrt{\tan(c+dx)}}{4b^4(a^2+b^2)^2\,d} + \frac{(35a^4+67a^2b^2+8b^4)\tan^{\frac{3}{2}}(c+dx}{12b^3(a^2+b^2)^2\,d}$$

$$= -\frac{a(35a^4+67a^2b^2+24b^4)\sqrt{\tan(c+dx)}}{4b^4(a^2+b^2)^2\,d} + \frac{(35a^4+67a^2b^2+8b^4)\tan^{\frac{3}{2}}(c+dx}{12b^3(a^2+b^2)^2\,d}$$

$$= -\frac{a(35a^4+67a^2b^2+24b^4)\sqrt{\tan(c+dx)}}{4b^4(a^2+b^2)^2\,d} + \frac{(35a^4+67a^2b^2+8b^4)\tan^{\frac{3}{2}}(c+dx}{12b^3(a^2+b^2)^2\,d}$$

$$= -\frac{a(35a^4+67a^2b^2+24b^4)\sqrt{\tan(c+dx)}}{4b^4(a^2+b^2)^2\,d} + \frac{(35a^4+67a^2b^2+8b^4)\tan^{\frac{3}{2}}(c+dx}{12b^3(a^2+b^2)^2\,d}$$

$$= \frac{a^{7/2}(35a^4+102a^2b^2+99b^4)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{4b^{9/2}(a^2+b^2)^3\,d} - \frac{a(35a^4+67a^2b^2+}{4b^4(a}$$

$$= \frac{a^{7/2}(35a^4+102a^2b^2+99b^4)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{4b^{9/2}(a^2+b^2)^3\,d} + \frac{(a-b)(a^2+4ab-}{}$$

$$= \frac{(a+b)(a^2-4ab+b^2)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}(a^2+b^2)^3\,d} - \frac{(a+b)(a^2-4ab+}{}$$

Mathematica [C] Result contains complex when optimal does not.

time = 6.38, size = 723, normalized size = 1.47

$$\frac{b^2\tan^{\frac{11}{2}}(c+dx)}{2a\left(a^2+b^2\right)d\left(a+b\tan(c+dx)\right)^2}+\frac{-\frac{b\tan^{\frac{11}{2}}(c+dx)}{d(a+b\tan(c+dx))}+\frac{2\left(\frac{9ab\tan^{\frac{9}{2}}(c+dx)}{2d(a+b\tan(c+dx))}+\frac{2\left(-\frac{63a^2b\tan^{\frac{7}{2}}(c+dx)}{4d(a+b\tan(c+dx))}+\frac{2\left(\frac{105ab\left(7a^2+4b^2\right)\tan^{\frac{5}{2}}(c+dx)}{8d(a+b\tan(c+dx))}+\frac{2\left(-\frac{315a^2b\left(35a^2+32b^2\right)\tan^{\frac{3}{2}}(c+dx)}{16d(a+b\tan(c+dx))}+\frac{2\left(-\frac{945ab\left(35a^4+32a^2b^2-4b^4\right)\sqrt{\tan(c+dx)}}{32d(a+b\tan(c+dx))}-\frac{2\left(\frac{2\left(\frac{945a^4b^8}{16}-\frac{[illegible]}{[illegible]}\right)\operatorname{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\left(a^2+b^2\right)d}+\frac{\sqrt[4]{-1}\,([illegible])\operatorname{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{[illegible]}+\frac{\sqrt[4]{-1}\,([illegible])\operatorname{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{[illegible]}+\frac{[illegible]\sqrt{\tan(c+dx)}}{a\left(a^2+b^2\right)d(a+b\tan(c+dx))}\right)}{b}\right)}{b}\right)}{3b}\right)}{5b}\right)}{7b}\right)}{9b}}{2a\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(11/2)/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (b^2*Tan[c + d*x]^(13/2))/(2*a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) + (-((
b*Tan[c + d*x]^(11/2))/(d*(a + b*Tan[c + d*x]))) + (2*((9*a*b*Tan[c + d*x]^
(9/2))/(2*d*(a + b*Tan[c + d*x])) + (2*((-63*a^2*b*Tan[c + d*x]^(7/2))/(4*d
*(a + b*Tan[c + d*x])) + (2*((105*a*b*(7*a^2 + 4*b^2)*Tan[c + d*x]^(5/2))/(
8*d*(a + b*Tan[c + d*x])) + (2*((-315*a^2*b*(35*a^2 + 32*b^2)*Tan[c + d*x]^
(3/2))/(16*d*(a + b*Tan[c + d*x])) + (2*((-945*a*b*(35*a^4 + 32*a^2*b^2 - 4
*b^4)*Sqrt[Tan[c + d*x]])/(32*d*(a + b*Tan[c + d*x])) - (2*(((2*((945*a^4*b
^8)/16 - (945*a^4*b^4*(35*a^4 + 67*a^2*b^2 + 24*b^4))/256 - (945*a^4*b^2*(3
5*a^6 + 67*a^4*b^2 + 32*a^2*b^4 - 8*b^6))/256)*ArcTan[(Sqrt[b]*Sqrt[Tan[c +
 d*x]])/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*(a^2 + b^2)*d) + (-(((-1)^(1/4)*((945*a^
3*b^6*(a^2 - 3*b^2))/32 + ((945*I)/32)*a^2*b^7*(3*a^2 - b^2))*ArcTan[(-1)^(
3/4)*Sqrt[Tan[c + d*x]]])/d) - ((-1)^(1/4)*((945*a^3*b^6*(a^2 - 3*b^2))/32
- ((945*I)/32)*a^2*b^7*(3*a^2 - b^2))*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]
])/d)/(a^2 + b^2))/(a*(a^2 + b^2)) + (((-945*a^2*b^4*(35*a^4 + 32*a^2*b^2 -
 4*b^4))/128 - a*((-945*a*b^8)/32 + (945*a^5*b^2*(35*a^2 + 32*b^2))/128))*S
qrt[Tan[c + d*x]])/(a*(a^2 + b^2)*d*(a + b*Tan[c + d*x]))))/b))/b))/(3*b)))
/(5*b)))/(7*b)))/(9*b))/(2*a*(a^2 + b^2))
```

Maple [A]
time = 0.19, size = 373, normalized size = 0.76

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{b\left(\tan^{\frac{3}{2}}(dx+c)\right)}{3}+3a\left(\sqrt{\tan}(dx+c)\right)\right)}{b^4}+\frac{2a^4\left(\frac{\left(-\frac{13}{8}a^4b-\frac{17}{4}a^2b^3-\frac{21}{8}b^5\right)\left(\tan^{\frac{3}{2}}(dx+c)\right)-\frac{a\left(11a^4+30a^2b^2+19b^4\right)\left(\sqrt{t\cdots}\right.}{8}}{(a+b\tan(dx+c))^2}\cdots\right.}{b^4\left(a^2+b^2\cdots\right.}}{\cdots}$
default	$\frac{-\frac{2\left(-\frac{b\left(\tan^{\frac{3}{2}}(dx+c)\right)}{3}+3a\left(\sqrt{\tan}(dx+c)\right)\right)}{b^4}+\frac{2a^4\left(\frac{\left(-\frac{13}{8}a^4b-\frac{17}{4}a^2b^3-\frac{21}{8}b^5\right)\left(\tan^{\frac{3}{2}}(dx+c)\right)-\frac{a\left(11a^4+30a^2b^2+19b^4\right)\left(\sqrt{t\cdots}\right.}{8}}{(a+b\tan(dx+c))^2}\cdots\right.}{b^4\left(a^2+b^2\cdots\right.}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(11/2)/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b^4*(-1/3*b*tan(d*x+c)^(3/2)+3*a*tan(d*x+c)^(1/2))+2*a^4/b^4/(a^2+b
^2)^3*(((-13/8*a^4*b-17/4*a^2*b^3-21/8*b^5)*tan(d*x+c)^(3/2)-1/8*a*(11*a^4+
30*a^2*b^2+19*b^4)*tan(d*x+c)^(1/2))/(a+b*tan(d*x+c))^2+1/8*(35*a^4+102*a^2
*b^2+99*b^4)/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2)))+2/(a^2+b^2
)^3*(1/8*(-a^3+3*a*b^2)*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))
```

```
/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/
2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/8*(3*a^2*b-b^3)*2^(1/2)*(ln((1
-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c
)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/
2)))))
```

Maxima [A]
time = 0.49, size = 449, normalized size = 0.91

$$\frac{\frac{3\,(35\,a^8+102\,a^6b^2+99\,a^4b^4)\arctan\left(\frac{b\sqrt{\tan\left(dx+c\right)}}{\sqrt{ab}}\right)}{(a^6b^4+3\,a^4b^6+3\,a^2b^8+b^{10})\sqrt{ab}}-\frac{3\left(2\sqrt{2}\,(a^3-3\,a^2b-3\,ab^2+b^3)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(dx+c\right)}\right)\right)+2\sqrt{2}\,(a^3-3\,a^2b-3\,ab^2+b^3)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(dx+c\right)}\right)\right)+\sqrt{2}\,(a^3+3\,a^2b-3\,ab^2-b^3)\log\left(\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan(dx+c)+1\right)-\sqrt{2}\,(a^3+3\,a^2b-3\,ab^2-b^3)\log\left(-\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan(dx+c)+1\right)\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{3\left((13\,a^6b+21\,a^4b^3)\tan(dx+c)^{\frac{3}{2}}+(11\,a^7+19\,a^5b^2)\sqrt{\tan\left(dx+c\right)}\right)}{a^6b^4+2\,a^4b^6+a^2b^8+(a^4b^6+2\,a^2b^8+b^{10})\tan(dx+c)^2+2\,(a^5b^5+2\,a^3b^7+ab^9)\tan(dx+c)}+\frac{8\left(b\tan(dx+c)^{\frac{3}{2}}-9\,a\sqrt{\tan\left(dx+c\right)}\right)}{b^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(11/2)/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/12*(3*(35*a^8 + 102*a^6*b^2 + 99*a^4*b^4)*arctan(b*sqrt(tan(d*x + c))/sqr
t(a*b))/((a^6*b^4 + 3*a^4*b^6 + 3*a^2*b^8 + b^10)*sqrt(a*b)) - 3*(2*sqrt(2)
*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d
*x + c)))) + 2*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*arctan(-1/2*sqrt(2)*
(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)
*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*(a^3 + 3*a^2*
b - 3*a*b^2 - b^3)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a^
6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 3*((13*a^6*b + 21*a^4*b^3)*tan(d*x + c)^
(3/2) + (11*a^7 + 19*a^5*b^2)*sqrt(tan(d*x + c)))/(a^6*b^4 + 2*a^4*b^6 + a^
2*b^8 + (a^4*b^6 + 2*a^2*b^8 + b^10)*tan(d*x + c)^2 + 2*(a^5*b^5 + 2*a^3*b^
7 + a*b^9)*tan(d*x + c)) + 8*(b*tan(d*x + c)^(3/2) - 9*a*sqrt(tan(d*x + c))
)/b^4)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11465 vs. $2(437) = 874$.
time = 15.24, size = 23042, normalized size = 46.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(11/2)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/48*(48*sqrt(2)*((a^22*b^4 + 3*a^20*b^6 - 17*a^18*b^8 - 123*a^16*b^10 - 3
42*a^14*b^12 - 546*a^12*b^14 - 546*a^10*b^16 - 342*a^8*b^18 - 123*a^6*b^20
- 17*a^4*b^22 + 3*a^2*b^24 + b^26)*d^5*cos(d*x + c)^5 + 2*(3*a^20*b^6 + 26*
a^18*b^8 + 99*a^16*b^10 + 216*a^14*b^12 + 294*a^12*b^14 + 252*a^10*b^16 + 1
26*a^8*b^18 + 24*a^6*b^20 - 9*a^4*b^22 - 6*a^2*b^24 - b^26)*d^5*cos(d*x + c
)^3 + (a^18*b^8 + 9*a^16*b^10 + 36*a^14*b^12 + 84*a^12*b^14 + 126*a^10*b^16
 + 126*a^8*b^18 + 84*a^6*b^20 + 36*a^4*b^22 + 9*a^2*b^24 + b^26)*d^5*cos(d*
x + c) + 4*((a^21*b^5 + 8*a^19*b^7 + 27*a^17*b^9 + 48*a^15*b^11 + 42*a^13*b
^13 - 42*a^9*b^17 - 48*a^7*b^19 - 27*a^5*b^21 - 8*a^3*b^23 - a*b^25)*d^5*co
s(d*x + c)^4 + (a^19*b^7 + 9*a^17*b^9 + 36*a^15*b^11 + 84*a^13*b^13 + 126*a
```

^11*b^15 + 126*a^9*b^17 + 84*a^7*b^19 + 36*a^5*b^21 + 9*a^3*b^23 + a*b^25)*
d^5*cos(d*x + c)^2)*sin(d*x + c))*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20
*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 - 2*(3*a^17*b + 8*a^15*b^3 - 12*a
^13*b^5 - 72*a^11*b^7 - 110*a^9*b^9 - 72*a^7*b^11 - 12*a^5*b^13 + 8*a^3*b^1
5 + 3*a*b^17)*d^2*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15
*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 45
2*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*sqrt((a^12 - 30*a^10*b^2 + 2
55*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 12*a^
22*b^2 + 66*a^20*b^4 + 220*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 924*a^
12*b^12 + 792*a^10*b^14 + 495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 12*a^
2*b^22 + b^24)*d^4))*(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*
a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))^(3/4)*arctan(-((a^24 - 6*a^22*b^2 - 84*a
^20*b^4 - 322*a^18*b^6 - 603*a^16*b^8 - 540*a^14*b^10 + 540*a^10*b^14 + 603
*a^8*b^16 + 322*a^6*b^18 + 84*a^4*b^20 + 6*a^2*b^22 - b^24)*d^4*sqrt((a^12
- 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^1
2)/((a^24 + 12*a^22*b^2 + 66*a^20*b^4 + 220*a^18*b^6 + 495*a^16*b^8 + 792*a
^14*b^10 + 924*a^12*b^12 + 792*a^10*b^14 + 495*a^8*b^16 + 220*a^6*b^18 + 66
*a^4*b^20 + 12*a^2*b^22 + b^24)*d^4))*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b
^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) + sqrt(2)*((a^27 +
9*a^25*b^2 + 30*a^23*b^4 + 22*a^21*b^6 - 165*a^19*b^8 - 693*a^17*b^10 - 145
2*a^15*b^12 - 1980*a^13*b^14 - 1881*a^11*b^16 - 1265*a^9*b^18 - 594*a^7*b^2
0 - 186*a^5*b^22 - 35*a^3*b^24 - 3*a*b^26)*d^7*sqrt((a^12 - 30*a^10*b^2 + 2
55*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 12*a^
22*b^2 + 66*a^20*b^4 + 220*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 924*a^
12*b^12 + 792*a^10*b^14 + 495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 12*a^
2*b^22 + b^24)*d^4))*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 +
15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) + (3*a^20*b + 26*a^18*b^3 + 99*a^16*
b^5 + 216*a^14*b^7 + 294*a^12*b^9 + 252*a^10*b^11 + 126*a^8*b^13 + 24*a^6*b
^15 - 9*a^4*b^17 - 6*a^2*b^19 - b^21)*d^5*sqrt((a^12 - 30*a^10*b^2 + 255*a^
8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 12*a^22*b^
2 + 66*a^20*b^4 + 220*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 924*a^12*b^
12 + 792*a^10*b^14 + 495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 12*a^2*b^2
2 + b^24)*d^4)))*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4
*b^8 + 6*a^2*b^10 + b^12 - 2*(3*a^17*b + 8*a^15*b^3 - 12*a^13*b^5 - 72*a^11
*b^7 - 110*a^9*b^9 - 72*a^7*b^11 - 12*a^5*b^13 + 8*a^3*b^15 + 3*a*b^17)*d^2
*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*
b^10 + b^12)*d^4)))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a
^4*b^8 - 30*a^2*b^10 + b^12))*sqrt(((a^18 - 27*a^16*b^2 + 168*a^14*b^4 + 22
4*a^12*b^6 - 366*a^10*b^8 - 366*a^8*b^10 + 224*a^6*b^12 + 168*a^4*b^14 - 27
*a^2*b^16 + b^18)*d^2*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6
+ 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*cos(d*x + c) + sqrt(2)*((3*a^20*b -
82*a^18*b^3 + 531*a^16*b^5 + 504*a^14*b^7 - 1322*a^12*b^9 - 732*a^10*b^11
+ 1038*a^8*b^13 + 280*a^6*b^15 - 249*a^4*b^17 + 30*a^2*b^19 - b^21)*d^3*sqr
t(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10
+ b^12)*d^4))*cos(d*x + c) + (a^15 - 33*a^13*b^2 + 345*a^11*b^4 - 1217*a^9

```
*b^6 + 1611*a^7*b^8 - 795*a^5*b^10 + 91*a^3*b^12 - 3*a*b^14)*d*cos(d*x + c)
)*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^
10 + b^12 - 2*(3*a^17*b + 8*a^15*b^3 - 12*a^13*b^5 - 72*a^11*b^7 - 110*a^9*
b^9 - 72*a^7*b^11 - 12*a^5*b^13 + 8*a^3*b^15 + 3*a*b^17)*d^2*sqrt(1/((a^12
+ 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^
4)))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2
*b^10 + b^12))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a^12 + 6*a^10*b^2 + 15*
a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))^(1/4) + (a^12
- 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^1
2)*sin(d*x + c))/cos(d*x + c))*(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6
*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))^(3/4) + sqrt(2)*((a^33 - 6*a^3
1*b^2 - 90*a^29*b^4 - 294*a^27*b^6 - 54*a^25*b^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(11/2)/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(11/2)/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 20.45, size = 2500, normalized size = 5.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(11/2)/(a + b*tan(c + d*x))^3,x)
```

```
[Out] (2*tan(c + d*x)^(3/2))/(3*b^3*d) - atan((((((32*a*b^22*d^2 - 2450*a^23*d^2
+ 192*a^3*b^20*d^2 + 19488*a^5*b^18*d^2 - 24128*a^7*b^16*d^2 - 180858*a^9*b
^14*d^2 - 146126*a^11*b^12*d^2 + 208974*a^13*b^10*d^2 + 452586*a^15*b^8*d^2
 + 330770*a^17*b^6*d^2 + 106102*a^19*b^4*d^2 + 7770*a^21*b^2*d^2)/(2*(b^23*
d^5 + 8*a^2*b^21*d^5 + 28*a^4*b^19*d^5 + 56*a^6*b^17*d^5 + 70*a^8*b^15*d^5
```

+ 56*a^10*b^13*d^5 + 28*a^12*b^11*d^5 + 8*a^14*b^9*d^5 + a^16*b^7*d^5)) - (
((((640*a^2*b^27*d^4 + 9536*a^4*b^25*d^4 + 55488*a^6*b^23*d^4 + 177408*a^8*
b^21*d^4 + 354816*a^10*b^19*d^4 + 470400*a^12*b^17*d^4 + 422016*a^14*b^15*d
^4 + 254208*a^16*b^13*d^4 + 98688*a^18*b^11*d^4 + 22336*a^20*b^9*d^4 + 2240
*a^22*b^7*d^4)/(2*(b^23*d^5 + 8*a^2*b^21*d^5 + 28*a^4*b^19*d^5 + 56*a^6*b^1
7*d^5 + 70*a^8*b^15*d^5 + 56*a^10*b^13*d^5 + 28*a^12*b^11*d^5 + 8*a^14*b^9*
d^5 + a^16*b^7*d^5)) - (tan(c + d*x)^(1/2)*(1/(b^6*d^2*1i - a^6*d^2*1i + 6*
a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15
i))^(1/2)*(512*b^32*d^4 + 4608*a^2*b^30*d^4 + 17920*a^4*b^28*d^4 + 38400*a^
6*b^26*d^4 + 46080*a^8*b^24*d^4 + 21504*a^10*b^22*d^4 - 21504*a^12*b^20*d^4
 - 46080*a^14*b^18*d^4 - 38400*a^16*b^16*d^4 - 17920*a^18*b^14*d^4 - 4608*a
^20*b^12*d^4 - 512*a^22*b^10*d^4))/(4*(b^23*d^4 + 8*a^2*b^21*d^4 + 28*a^4*b
^19*d^4 + 56*a^6*b^17*d^4 + 70*a^8*b^15*d^4 + 56*a^10*b^13*d^4 + 28*a^12*b^
11*d^4 + 8*a^14*b^9*d^4 + a^16*b^7*d^4)))*(1/(b^6*d^2*1i - a^6*d^2*1i + 6*a
*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i
))^(1/2))/2 + (tan(c + d*x)^(1/2)*(1472*a*b^25*d^2 + 9800*a^25*b*d^2 + 1024
*a^3*b^23*d^2 - 8448*a^5*b^21*d^2 - 14336*a^7*b^19*d^2 + 74440*a^9*b^17*d^2
 + 480320*a^11*b^15*d^2 + 1258208*a^13*b^13*d^2 + 1894848*a^15*b^11*d^2 + 1
794928*a^17*b^9*d^2 + 1098176*a^19*b^7*d^2 + 425952*a^21*b^5*d^2 + 96320*a^
23*b^3*d^2))/(2*(b^23*d^4 + 8*a^2*b^21*d^4 + 28*a^4*b^19*d^4 + 56*a^6*b^17*
d^4 + 70*a^8*b^15*d^4 + 56*a^10*b^13*d^4 + 28*a^12*b^11*d^4 + 8*a^14*b^9*d^
4 + a^16*b^7*d^4)))*(1/(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2
 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i))^(1/2))/2)*(1/(b^6*d
^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b
^3*d^2 + a^4*b^2*d^2*15i))^(1/2))/2 + (tan(c + d*x)^(1/2)*(1225*a^20 + 32*b
^20 + 128*a^2*b^18 + 192*a^4*b^16 + 128*a^6*b^14 + 9833*a^8*b^12 - 38610*a^
10*b^10 - 94041*a^12*b^8 - 76668*a^14*b^6 - 24281*a^16*b^4 - 210*a^18*b^2))
/(2*(b^23*d^4 + 8*a^2*b^21*d^4 + 28*a^4*b^19*d^4 + 56*a^6*b^17*d^4 + 70*a^8
*b^15*d^4 + 56*a^10*b^13*d^4 + 28*a^12*b^11*d^4 + 8*a^14*b^9*d^4 + a^16*b^7
d^4)))(1/(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d
^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i))^(1/2)*1i - ((((32*a*b^22*d^2 -
2450*a^23*d^2 + 192*a^3*b^20*d^2 + 19488*a^5*b^18*d^2 - 24128*a^7*b^16*d^2
 - 180858*a^9*b^14*d^2 - 146126*a^11*b^12*d^2 + 208974*a^13*b^10*d^2 + 45258
6*a^15*b^8*d^2 + 330770*a^17*b^6*d^2 + 106102*a^19*b^4*d^2 + 7770*a^21*b^2*
d^2)/(2*(b^23*d^5 + 8*a^2*b^21*d^5 + 28*a^4*b^19*d^5 + 56*a^6*b^17*d^5 + 70
*a^8*b^15*d^5 + 56*a^10*b^13*d^5 + 28*a^12*b^11*d^5 + 8*a^14*b^9*d^5 + a^16
*b^7*d^5)) - (((((640*a^2*b^27*d^4 + 9536*a^4*b^25*d^4 + 55488*a^6*b^23*d^4
 + 177408*a^8*b^21*d^4 + 354816*a^10*b^19*d^4 + 470400*a^12*b^17*d^4 + 4220
16*a^14*b^15*d^4 + 254208*a^16*b^13*d^4 + 98688*a^18*b^11*d^4 + 22336*a^20*
b^9*d^4 + 2240*a^22*b^7*d^4)/(2*(b^23*d^5 + 8*a^2*b^21*d^5 + 28*a^4*b^19*d^
5 + 56*a^6*b^17*d^5 + 70*a^8*b^15*d^5 + 56*a^10*b^13*d^5 + 28*a^12*b^11*d^5
 + 8*a^14*b^9*d^5 + a^16*b^7*d^5)) + (tan(c + d*x)^(1/2)*(1/(b^6*d^2*1i - a
^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 +
a^4*b^2*d^2*15i))^(1/2)*(512*b^32*d^4 + 4608*a^2*b^30*d^4 + 17920*a^4*b^28*
d^4 + 38400*a^6*b^26*d^4 + 46080*a^8*b^24*d^4 + 21504*a^10*b^22*d^4 - 21504

*a^12*b^20*d^4 - 46080*a^14*b^18*d^4 - 38400*a^16*b^16*d^4 - 17920*a^18*b^14*d^4 - 4608*a^20*b^12*d^4 - 512*a^22*b^10*d^4))/(4*(b^23*d^4 + 8*a^2*b^21*d^4 + 28*a^4*b^19*d^4 + 56*a^6*b^17*d^4 + 70*a^8*b^15*d^4 + 56*a^10*b^13*d^4 + 28*a^12*b^11*d^4 + 8*a^14*b^9*d^4 + a^16*b^7*d^4)))*(1/(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i))^(1/2))/2 - (tan(c + d*x)^(1/2)*(1472*a*b^25*d^2 + 9800*a^25*b*d^2 + 1024*a^3*b^23*d^2 - 8448*a^5*b^21*d^2 - 14336*a^7*b^19*d^2 + 74440*a^9*b^17*d^2 + 480320*a^11*b^15*d^2 + 1258208*a^13*b^13*d^2 + 1894848*a^15*b^11*d^2 + 1794928*a^17*b^9*d^2 + 1098176*a^19*b^7*d^2 + 425952*a^21*b^5*d^2 + 96320*a^23*b^3*d^2))/(2*(b^23*d^4 + 8*a^2*b^21*d^4 + 28*a^4*b^19*d^4 + 56*a^6*b^17*d^4 + 70*a^8*b^15*d^4 + 56*a^10*b^13*d^4 + 28*a^12*b^11*d^4 + 8*a^14*b^9*d^4 + a^16*b^7*d^4)))*(1/(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i))^(1/2))/2)*(1/(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i))^(1/2))/2 - (tan(c + d*x)^(1/2)*(1225*a^20 + 32*b^20 + 128*a^2*b^18 + 192*a^4*b^16 + 128*a^6*b^14 + 9833*a^8*b^12 - 38610*a^10*b^10 - 94041*a^12*b^8 - 76668*...

3.600 $\int \frac{\tan^{\frac{9}{2}}(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=444

$$-\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,\left(a^2+b^2\right)^3 d}+\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(\right.}{\sqrt{2}\,\left(a^2+b^2\right)^3 d}$$

```
[Out] -1/4*a^(5/2)*(15*a^4+46*a^2*b^2+63*b^4)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(
1/2))/b^(7/2)/(a^2+b^2)^3/d+1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(-1+2^(1/2)*tan
(d*x+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)+1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(1+2^(
1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)+1/4*(a+b)*(a^2-4*a*b+b^2)*ln(1
-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)^3/d*2^(1/2)-1/4*(a+b)*(a^2-
4*a*b+b^2)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)^3/d*2^(1/2)+
1/4*(15*a^4+31*a^2*b^2+8*b^4)*tan(d*x+c)^(1/2)/b^3/(a^2+b^2)^2/d-1/2*a^2*ta
n(d*x+c)^(5/2)/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^2-1/4*a^2*(5*a^2+13*b^2)*tan(
d*x+c)^(3/2)/b^2/(a^2+b^2)^2/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.74, antiderivative size = 444, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3646, 3726, 3728, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d\left(a^2+b^2\right)^3}+\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d\left(a^2+b^2\right)^3}-\frac{a^2\left(5a^2+13b^2\right)\tan^{\frac{3}{2}}(c+dx)}{4b^2d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}-\frac{a^2\tan^{\frac{5}{2}}(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}+\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\tan(c+dx)-\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)^3}-\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\tan(c+dx)+\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)^3}+\frac{\left(15a^4+31a^2b^2+8b^4\right)\sqrt{\tan(c+dx)}}{4b^3d\left(a^2+b^2\right)^2}-\frac{a^{5/2}\left(15a^4+46a^2b^2+63b^4\right)\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{4b^{7/2}d\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(9/2)/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -(((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqr
t[2]*(a^2 + b^2)^3*d)) + ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 + Sqrt[2]*Sq
rt[Tan[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)^3*d) - (a^(5/2)*(15*a^4 + 46*a^2*b^
2 + 63*b^4)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(4*b^(7/2)*(a^2 +
 b^2)^3*d) + ((a + b)*(a^2 - 4*a*b + b^2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]
] + Tan[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^3*d) - ((a + b)*(a^2 - 4*a*b + b^
2)*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*(a^2 + b^
2)^3*d) + ((15*a^4 + 31*a^2*b^2 + 8*b^4)*Sqrt[Tan[c + d*x]])/(4*b^3*(a^2 +
b^2)^2*d) - (a^2*Tan[c + d*x]^(5/2))/(2*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x]
)^2) - (a^2*(5*a^2 + 13*b^2)*Tan[c + d*x]^(3/2))/(4*b^2*(a^2 + b^2)^2*d*(a
+ b*Tan[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
```

```
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
```

```
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\tan^{\frac{9}{2}}(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{a^2\tan^{\frac{5}{2}}(c+dx)}{2b\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{\int \frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{5a^2}{2}-2ab\tan(c+dx)+\frac{1}{2}\left(5a^2+4b^2\right)\tan^2 \cdots\right)}{(a+b\tan(c+dx))^2}}{2b\left(a^2+b^2\right)}$$

$$= -\frac{a^2\tan^{\frac{5}{2}}(c+dx)}{2b\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{a^2\left(5a^2+13b^2\right)\tan^{\frac{3}{2}}(c+dx)}{4b^2\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))} + \frac{\int \cdots}{\cdots}$$

$$= \frac{\left(15a^4+31a^2b^2+8b^4\right)\sqrt{\tan(c+dx)}}{4b^3\left(a^2+b^2\right)^2 d} - \frac{a^2\tan^{\frac{5}{2}}(c+dx)}{2b\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{\cdots}{4b\cdots}$$

$$= \frac{\left(15a^4+31a^2b^2+8b^4\right)\sqrt{\tan(c+dx)}}{4b^3\left(a^2+b^2\right)^2 d} - \frac{a^2\tan^{\frac{5}{2}}(c+dx)}{2b\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{\cdots}{4b\cdots}$$

$$= \frac{\left(15a^4+31a^2b^2+8b^4\right)\sqrt{\tan(c+dx)}}{4b^3\left(a^2+b^2\right)^2 d} - \frac{a^2\tan^{\frac{5}{2}}(c+dx)}{2b\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{\cdots}{4b\cdots}$$

$$= \frac{\left(15a^4+31a^2b^2+8b^4\right)\sqrt{\tan(c+dx)}}{4b^3\left(a^2+b^2\right)^2 d} - \frac{a^2\tan^{\frac{5}{2}}(c+dx)}{2b\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{\cdots}{4b\cdots}$$

$$= -\frac{a^{5/2}\left(15a^4+46a^2b^2+63b^4\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{4b^{7/2}\left(a^2+b^2\right)^3 d} + \frac{\left(15a^4+31a^2b^2+\cdots\right.}{4b^3\left(a^2\cdots\right.}$$

$$= -\frac{a^{5/2}\left(15a^4+46a^2b^2+63b^4\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{4b^{7/2}\left(a^2+b^2\right)^3 d} + \frac{(a+b)\left(a^2-4ab\cdots\right.}{\cdots}$$

$$= -\frac{(a-b)\left(a^2+4ab+b^2\right)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 d} + \frac{(a-b)\left(a^2+4ab+\cdots\right.}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.35, size = 403, normalized size = 0.91

$$\frac{2b^2\tan^{\frac{11}{2}}(c+dx) + \frac{2a^2(15a^2+16b^2)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}{b^3} + \frac{2a(5a^2+4b^2)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}{b^2} - \frac{2a^2\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}{b} + 2a\tan^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx)) - 2b\tan^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx)) - \frac{a(a+b\tan(c+dx))\left(a\sqrt{b}(a^2+b^2)(15a^4+31a^2b^2+24b^4)\sqrt{\tan(c+dx)} + \left(-4(-1)^{3/4}(a+ib)^3b^{7/2}\operatorname{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right) + a^{5/2}(15a^4+46a^2b^2+63b^4)\operatorname{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right) - 4\sqrt[4]{-1}b^{7/2}(ia+b)^3\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\right)(a+b\tan(c+dx))\right)}{b^{7/2}(a^2+b^2)^2}}{4a\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(9/2)/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (2*b^2*Tan[c + d*x]^(11/2) + (2*a^2*(15*a^2 + 16*b^2)*Sqrt[Tan[c + d*x]]*(a
 + b*Tan[c + d*x]))/b^3 + (2*a*(5*a^2 + 4*b^2)*Tan[c + d*x]^(3/2)*(a + b*Ta
n[c + d*x]))/b^2 - (2*a^2*Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x]))/b + 2*a*
```

```
Tan[c + d*x]^(7/2)*(a + b*Tan[c + d*x]) - 2*b*Tan[c + d*x]^(9/2)*(a + b*Tan
[c + d*x]) - (a*(a + b*Tan[c + d*x])*(a*Sqrt[b]*(a^2 + b^2)*(15*a^4 + 31*a^
2*b^2 + 24*b^4)*Sqrt[Tan[c + d*x]] + (-4*(-1)^(3/4)*(a + I*b)^3*b^(7/2)*Arc
Tan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + a^(5/2)*(15*a^4 + 46*a^2*b^2 + 63*b^4)
*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]] - 4*(-1)^(1/4)*b^(7/2)*(I*a +
 b)^3*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])*(a + b*Tan[c + d*x])))/(b^(7/
2)*(a^2 + b^2)^2))/(4*a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2)
```

Maple [A]
time = 0.15, size = 356, normalized size = 0.80

method	result
derivativedivides	$\frac{\frac{2\left(\sqrt{\tan}(dx+c)\right)}{b^3}-\frac{2a^3\left(\frac{\left(-\frac{9}{8}a^4b-\frac{13}{4}a^2b^3-\frac{17}{8}b^5\right)\left(\tan^{\frac{3}{2}}(dx+c)\right)-\frac{a(7a^4+22a^2b^2+15b^4)\left(\sqrt{\tan}(dx+c)\right)}{8}}{(a+b\tan(dx+c))^2}+\frac{(15a^4+46a^2\cdots}{\cdots}\cdots\right)}{b^3(a^2+b^2)^3}\cdots}{\cdots}$
default	$\frac{\frac{2\left(\sqrt{\tan}(dx+c)\right)}{b^3}-\frac{2a^3\left(\frac{\left(-\frac{9}{8}a^4b-\frac{13}{4}a^2b^3-\frac{17}{8}b^5\right)\left(\tan^{\frac{3}{2}}(dx+c)\right)-\frac{a(7a^4+22a^2b^2+15b^4)\left(\sqrt{\tan}(dx+c)\right)}{8}}{(a+b\tan(dx+c))^2}+\frac{(15a^4+46a^2\cdots}{\cdots}\cdots\right)}{b^3(a^2+b^2)^3}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(9/2)/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b^3*tan(d*x+c)^(1/2)-2*a^3/b^3/(a^2+b^2)^3*(((-9/8*a^4*b-13/4*a^2*b^
3-17/8*b^5)*tan(d*x+c)^(3/2)-1/8*a*(7*a^4+22*a^2*b^2+15*b^4)*tan(d*x+c)^(1/
2))/(a+b*tan(d*x+c))^2+1/8*(15*a^4+46*a^2*b^2+63*b^4)/(a*b)^(1/2)*arctan(b*
tan(d*x+c)^(1/2)/(a*b)^(1/2)))+2/(a^2+b^2)^3*(1/8*(3*a^2*b-b^3)*2^(1/2)*(ln
((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*
x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^
(1/2)))+1/8*(a^3-3*a*b^2)*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c
))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(
1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))))
```

Maxima [A]
time = 0.50, size = 434, normalized size = 0.98

$$-\frac{\frac{(15a^7+46a^5b^2+63a^3b^4)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^6b^3+3a^4b^5+3a^2b^7+b^9)\sqrt{ab}}-\frac{2\sqrt{2}(a^3+3a^2b-3ab^2-b^3)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}(a^3+3a^2b-3ab^2-b^3)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-\sqrt{2}(a^3-3a^2b-3ab^2+b^3)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+\sqrt{2}(a^3-3a^2b-3ab^2+b^3)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{(9a^5b+17a^3b^3)\tan(dx+c)^{\frac{3}{2}}+(7a^6+15a^4b^2)\sqrt{\tan(dx+c)}}{a^6b^3+2a^4b^5+a^2b^7+(a^4b^5+2a^2b^7+b^9)\tan(dx+c)^2+2(a^5b^4+2a^3b^6+ab^8)\tan(dx+c)}-\frac{8\sqrt{\tan(dx+c)}}{b^3}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(9/2)/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*((15*a^7 + 46*a^5*b^2 + 63*a^3*b^4)*arctan(b*sqrt(tan(d*x + c))/sqrt(a
*b))/((a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9)*sqrt(a*b)) - (2*sqrt(2)*(a^3
+ 3*a^2*b - 3*a*b^2 - b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c
)))) + 2*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*arctan(-1/2*sqrt(2)*(sqrt(
2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*log(s
qrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*(a^3 - 3*a^2*b - 3*
a*b^2 + b^3)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a^6 + 3*
a^4*b^2 + 3*a^2*b^4 + b^6) - ((9*a^5*b + 17*a^3*b^3)*tan(d*x + c)^(3/2) + (
7*a^6 + 15*a^4*b^2)*sqrt(tan(d*x + c)))/(a^6*b^3 + 2*a^4*b^5 + a^2*b^7 + (a
^4*b^5 + 2*a^2*b^7 + b^9)*tan(d*x + c)^2 + 2*(a^5*b^4 + 2*a^3*b^6 + a*b^8)*
tan(d*x + c)) - 8*sqrt(tan(d*x + c))/b^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11339 vs. 2(392) = 784.
time = 18.47, size = 22791, normalized size = 51.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(9/2)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*sqrt(2)*((a^22*b^3 + 3*a^20*b^5 - 17*a^18*b^7 - 123*a^16*b^9 - 34
2*a^14*b^11 - 546*a^12*b^13 - 546*a^10*b^15 - 342*a^8*b^17 - 123*a^6*b^19 -
 17*a^4*b^21 + 3*a^2*b^23 + b^25)*d^5*cos(d*x + c)^4 + 2*(3*a^20*b^5 + 26*a
^18*b^7 + 99*a^16*b^9 + 216*a^14*b^11 + 294*a^12*b^13 + 252*a^10*b^15 + 126
*a^8*b^17 + 24*a^6*b^19 - 9*a^4*b^21 - 6*a^2*b^23 - b^25)*d^5*cos(d*x + c)^
2 + (a^18*b^7 + 9*a^16*b^9 + 36*a^14*b^11 + 84*a^12*b^13 + 126*a^10*b^15 +
126*a^8*b^17 + 84*a^6*b^19 + 36*a^4*b^21 + 9*a^2*b^23 + b^25)*d^5 + 4*((a^2
1*b^4 + 8*a^19*b^6 + 27*a^17*b^8 + 48*a^15*b^10 + 42*a^13*b^12 - 42*a^9*b^1
6 - 48*a^7*b^18 - 27*a^5*b^20 - 8*a^3*b^22 - a*b^24)*d^5*cos(d*x + c)^3 + (
a^19*b^6 + 9*a^17*b^8 + 36*a^15*b^10 + 84*a^13*b^12 + 126*a^11*b^14 + 126*a
^9*b^16 + 84*a^7*b^18 + 36*a^5*b^20 + 9*a^3*b^22 + a*b^24)*d^5*cos(d*x + c)
)*sin(d*x + c))*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*
b^8 + 6*a^2*b^10 + b^12 + 2*(3*a^17*b + 8*a^15*b^3 - 12*a^13*b^5 - 72*a^11*
b^7 - 110*a^9*b^9 - 72*a^7*b^11 - 12*a^5*b^13 + 8*a^3*b^15 + 3*a*b^17)*d^2*
sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b
^10 + b^12)*d^4)))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^
4*b^8 - 30*a^2*b^10 + b^12))*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a
^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 12*a^22*b^2 + 66*a^20*b
^4 + 220*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 924*a^12*b^12 + 792*a^10
*b^14 + 495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 12*a^2*b^22 + b^24)*d^4
))*(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^
10 + b^12)*d^4))^(3/4)*arctan(((a^24 - 6*a^22*b^2 - 84*a^20*b^4 - 322*a^18*
b^6 - 603*a^16*b^8 - 540*a^14*b^10 + 540*a^10*b^14 + 603*a^8*b^16 + 322*a^6
```

*b^18 + 84*a^4*b^20 + 6*a^2*b^22 - b^24)*d^4*sqrt((a^12 - 30*a^10*b^2 + 255
*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 12*a^22
*b^2 + 66*a^20*b^4 + 220*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 924*a^12
*b^12 + 792*a^10*b^14 + 495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 12*a^2*
b^22 + b^24)*d^4))*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 1
5*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) - sqrt(2)*((3*a^26*b + 35*a^24*b^3 + 1
86*a^22*b^5 + 594*a^20*b^7 + 1265*a^18*b^9 + 1881*a^16*b^11 + 1980*a^14*b^1
3 + 1452*a^12*b^15 + 693*a^10*b^17 + 165*a^8*b^19 - 22*a^6*b^21 - 30*a^4*b^
23 - 9*a^2*b^25 - b^27)*d^7*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^
6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 12*a^22*b^2 + 66*a^20*b^
4 + 220*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 924*a^12*b^12 + 792*a^10*
b^14 + 495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 12*a^2*b^22 + b^24)*d^4)
)*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2
*b^10 + b^12)*d^4)) - (a^21 + 6*a^19*b^2 + 9*a^17*b^4 - 24*a^15*b^6 - 126*a
^13*b^8 - 252*a^11*b^10 - 294*a^9*b^12 - 216*a^7*b^14 - 99*a^5*b^16 - 26*a^
3*b^18 - 3*a*b^20)*d^5*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6
 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 12*a^22*b^2 + 66*a^20*b^4 + 2
20*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 924*a^12*b^12 + 792*a^10*b^14
+ 495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 12*a^2*b^22 + b^24)*d^4)))*sq
rt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 +
 b^12 + 2*(3*a^17*b + 8*a^15*b^3 - 12*a^13*b^5 - 72*a^11*b^7 - 110*a^9*b^9
- 72*a^7*b^11 - 12*a^5*b^13 + 8*a^3*b^15 + 3*a*b^17)*d^2*sqrt(1/((a^12 + 6*
a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))
/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^1
0 + b^12))*sqrt(((a^18 - 27*a^16*b^2 + 168*a^14*b^4 + 224*a^12*b^6 - 366*a^
10*b^8 - 366*a^8*b^10 + 224*a^6*b^12 + 168*a^4*b^14 - 27*a^2*b^16 + b^18)*d
^2*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^
2*b^10 + b^12)*d^4))*cos(d*x + c) + sqrt(2)*((a^21 - 30*a^19*b^2 + 249*a^17
*b^4 - 280*a^15*b^6 - 1038*a^13*b^8 + 732*a^11*b^10 + 1322*a^9*b^12 - 504*a
^7*b^14 - 531*a^5*b^16 + 82*a^3*b^18 - 3*a*b^20)*d^3*sqrt(1/((a^12 + 6*a^10
*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*cos(
d*x + c) - (3*a^14*b - 91*a^12*b^3 + 795*a^10*b^5 - 1611*a^8*b^7 + 1217*a^6
*b^9 - 345*a^4*b^11 + 33*a^2*b^13 - b^15)*d*cos(d*x + c))*sqrt((a^12 + 6*a^
10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 + 2*(3*a^
17*b + 8*a^15*b^3 - 12*a^13*b^5 - 72*a^11*b^7 - 110*a^9*b^9 - 72*a^7*b^11 -
 12*a^5*b^13 + 8*a^3*b^15 + 3*a*b^17)*d^2*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a
^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))/(a^12 - 30*a^1
0*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*sqrt
(sin(d*x + c)/cos(d*x + c))*(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^
6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))^(1/4) + (a^12 - 30*a^10*b^2 + 255
*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)*sin(d*x + c))/co
s(d*x + c))*(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 +
 6*a^2*b^10 + b^12)*d^4))^(3/4) - sqrt(2)*((3*a^32*b - 10*a^30*b^3 - 294*a^
28*b^5 - 1674*a^26*b^7 - 4890*a^24*b^9 - 8370*a...

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(9/2)/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(9/2)/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 20.80, size = 2500, normalized size = 5.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(9/2)/(a + b*tan(c + d*x))^3,x)
```

```
[Out] (log(- ((((((((((64*a*b*(15*a^4 + 2*b^4 + 41*a^2*b^2))/d - 256*b^3*tan(c + d
*x)^(1/2)*(a^2 - b^2)*(a^2 + b^2)^2*(1i/(d^2*(a*1i - b)^6))^(1/2))*(1i/(d^2
*(a*1i - b)^6))^(1/2))/2 - (8*a*tan(c + d*x)^(1/2)*(225*a^14 - 184*b^14 + 6
08*a^2*b^12 - 272*a^4*b^10 + 3937*a^6*b^8 + 5804*a^8*b^6 + 4006*a^10*b^4 +
1380*a^12*b^2))/(b^4*d^2*(a^2 + b^2)^4))*(1i/(d^2*(a*1i - b)^6))^(1/2))/2 +
 (2*a^2*(1125*a^14 + 16*b^14 + 6112*a^2*b^12 - 17727*a^4*b^10 - 23239*a^6*b
^8 - 11174*a^8*b^6 + 2930*a^10*b^4 + 3525*a^12*b^2))/(b^4*d^3*(a^2 + b^2)^6
))*(1i/(d^2*(a*1i - b)^6))^(1/2))/2 + (tan(c + d*x)^(1/2)*(32*b^18 - 225*a^
18 + 128*a^2*b^16 + 192*a^4*b^14 - 3841*a^6*b^12 + 18050*a^8*b^10 + 26801*a
^10*b^8 + 16860*a^12*b^6 + 4049*a^14*b^4 - 30*a^16*b^2))/(b^5*d^4*(a^2 + b^
2)^8))*(1i/(d^2*(a*1i - b)^6))^(1/2))/2 - (a^3*(225*a^12 + 504*b^12 + 872*a
^2*b^10 + 4457*a^4*b^8 + 5916*a^6*b^6 + 4006*a^8*b^4 + 1380*a^10*b^2))/(2*b
^5*d^5*(a^2 + b^2)^8))*(-1/(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b
*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i))^(1/2))/2 - log(
- ((((((((((64*a*b*(15*a^4 + 2*b^4 + 41*a^2*b^2))/d + 256*b^3*tan(c + d*x)^(
1/2)*(a^2 - b^2)*(a^2 + b^2)^2*(1i/(d^2*(a*1i - b)^6))^(1/2))*(1i/(d^2*(a*1
i - b)^6))^(1/2))/2 + (8*a*tan(c + d*x)^(1/2)*(225*a^14 - 184*b^14 + 608*a^
```

2*b^12 - 272*a^4*b^10 + 3937*a^6*b^8 + 5804*a^8*b^6 + 4006*a^10*b^4 + 1380*
a^12*b^2))/(b^4*d^2*(a^2 + b^2)^4))*(1i/(d^2*(a*1i - b)^6))^(1/2))/2 + (2*a
^2*(1125*a^14 + 16*b^14 + 6112*a^2*b^12 - 17727*a^4*b^10 - 23239*a^6*b^8 -
11174*a^8*b^6 + 2930*a^10*b^4 + 3525*a^12*b^2))/(b^4*d^3*(a^2 + b^2)^6))*(1
i/(d^2*(a*1i - b)^6))^(1/2))/2 - (tan(c + d*x)^(1/2)*(32*b^18 - 225*a^18 +
128*a^2*b^16 + 192*a^4*b^14 - 3841*a^6*b^12 + 18050*a^8*b^10 + 26801*a^10*b
^8 + 16860*a^12*b^6 + 4049*a^14*b^4 - 30*a^16*b^2))/(b^5*d^4*(a^2 + b^2)^8)
)*(1i/(d^2*(a*1i - b)^6))^(1/2))/2 - (a^3*(225*a^12 + 504*b^12 + 872*a^2*b^
10 + 4457*a^4*b^8 + 5916*a^6*b^6 + 4006*a^8*b^4 + 1380*a^10*b^2))/(2*b^5*d^
5*(a^2 + b^2)^8))*(-1/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d
^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) - atan(((-
1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a
^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*((-1i/(4*(b^6*d^2 - a^6*d^2 + a*b^
5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2
)))^(1/2)*((2250*a^20*b*d^2 + 32*a^2*b^19*d^2 + 12288*a^4*b^17*d^2 - 10974*
a^6*b^15*d^2 - 105162*a^8*b^13*d^2 - 150758*a^10*b^11*d^2 - 85314*a^12*b^9*
d^2 - 3578*a^14*b^7*d^2 + 22210*a^16*b^5*d^2 + 11550*a^18*b^3*d^2)/(b^21*d^
5 + 8*a^2*b^19*d^5 + 28*a^4*b^17*d^5 + 56*a^6*b^15*d^5 + 70*a^8*b^13*d^5 +
56*a^10*b^11*d^5 + 28*a^12*b^9*d^5 + 8*a^14*b^7*d^5 + a^16*b^5*d^5) + (-1i/
(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*
b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*((-1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d
^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))
^(1/2)*((128*a*b^26*d^4 + 3648*a^3*b^24*d^4 + 25536*a^5*b^22*d^4 + 88320*a^
7*b^20*d^4 + 182784*a^9*b^18*d^4 + 244608*a^11*b^16*d^4 + 217728*a^13*b^14*
d^4 + 128256*a^15*b^12*d^4 + 48000*a^17*b^10*d^4 + 10304*a^19*b^8*d^4 + 960
*a^21*b^6*d^4)/(b^21*d^5 + 8*a^2*b^19*d^5 + 28*a^4*b^17*d^5 + 56*a^6*b^15*d
^5 + 70*a^8*b^13*d^5 + 56*a^10*b^11*d^5 + 28*a^12*b^9*d^5 + 8*a^14*b^7*d^5
+ a^16*b^5*d^5) + (tan(c + d*x)^(1/2)*(-1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^
2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^
(1/2)*(512*b^30*d^4 + 4608*a^2*b^28*d^4 + 17920*a^4*b^26*d^4 + 38400*a^6*b^
24*d^4 + 46080*a^8*b^22*d^4 + 21504*a^10*b^20*d^4 - 21504*a^12*b^18*d^4 - 4
6080*a^14*b^16*d^4 - 38400*a^16*b^14*d^4 - 17920*a^18*b^12*d^4 - 4608*a^20*
b^10*d^4 - 512*a^22*b^8*d^4))/(b^21*d^4 + 8*a^2*b^19*d^4 + 28*a^4*b^17*d^4
+ 56*a^6*b^15*d^4 + 70*a^8*b^13*d^4 + 56*a^10*b^11*d^4 + 28*a^12*b^9*d^4 +
8*a^14*b^7*d^4 + a^16*b^5*d^4)) - (tan(c + d*x)^(1/2)*(1800*a^23*b*d^2 - 14
72*a*b^23*d^2 - 1024*a^3*b^21*d^2 + 8448*a^5*b^19*d^2 + 46088*a^7*b^17*d^2
+ 177344*a^9*b^15*d^2 + 402912*a^11*b^13*d^2 + 541632*a^13*b^11*d^2 + 45547
2*a^15*b^9*d^2 + 248064*a^17*b^7*d^2 + 87008*a^19*b^5*d^2 + 18240*a^21*b^3*
d^2))/(b^21*d^4 + 8*a^2*b^19*d^4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4 + 70*a
^8*b^13*d^4 + 56*a^10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a^16*b^
5*d^4))) + (tan(c + d*x)^(1/2)*(32*b^18 - 225*a^18 + 128*a^2*b^16 + 192*a^4
*b^14 - 3841*a^6*b^12 + 18050*a^8*b^10 + 26801*a^10*b^8 + 16860*a^12*b^6 +
4049*a^14*b^4 - 30*a^16*b^2))/(b^21*d^4 + 8*a^2*b^19*d^4 + 28*a^4*b^17*d^4
+ 56*a^6*b^15*d^4 + 70*a^8*b^13*d^4 + 56*a^10*b^11*d^4 + 28*a^12*b^9*d^4 +
8*a^14*b^7*d^4 + a^16*b^5*d^4))*1i - (-1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2

```
*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(
1/2)*((-1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4
*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*((2250*a^20*b*d^2 + 32*a^2
*b^19*d^2 + 12288*a^4*b^17*d^2 - 10974*a^6*b^15*d^2 - 105162*a^8*b^13*d^2 -
 150758*a^10*b^11*d^2 - 85314*a^12*b^9*d^2 - 35...
```

3.601 $\int \frac{\tan^{\frac{7}{2}}(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=396

$$-\frac{(a+b)\left(a^2-4ab+b^2\right)\operatorname{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,\left(a^2+b^2\right)^3 d}+\frac{(a+b)\left(a^2-4ab+b^2\right)\operatorname{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan}\right.}{\sqrt{2}\,\left(a^2+b^2\right)^3 d}$$

```
[Out] 1/4*a^(3/2)*(3*a^4+6*a^2*b^2+35*b^4)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2
))/b^(5/2)/(a^2+b^2)^3/d+1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(-1+2^(1/2)*tan(d*
x+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)+1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(1+2^(1/2
)*tan(d*x+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/4*(a-b)*(a^2+4*a*b+b^2)*ln(1-2^
(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)^3/d*2^(1/2)+1/4*(a-b)*(a^2+4*a
*b+b^2)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)^3/d*2^(1/2)-1/2
*a^2*tan(d*x+c)^(3/2)/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^2-1/4*a^2*(3*a^2+11*b^
2)*tan(d*x+c)^(1/2)/b^2/(a^2+b^2)^2/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.57, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3646, 3726, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{(a+b)\left(a^2-4ab+b^2\right)\operatorname{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d\left(a^2+b^2\right)^3}+\frac{(a+b)\left(a^2-4ab+b^2\right)\operatorname{ArcTan}\left(\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d\left(a^2+b^2\right)^3}-\frac{a^2\tan^{\frac{3}{2}}(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{a^2\left(3a^2+11b^2\right)\sqrt{\tan(c+dx)}}{4b^2d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}-\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\tan(c+dx)-\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)^3}+\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\tan(c+dx)+\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)^3}+\frac{a^{3/2}\left(3a^4+6a^2b^2+35b^4\right)\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{4b^{5/2}d\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(7/2)/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -(((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqr
t[2]*(a^2 + b^2)^3*d)) + ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 + Sqrt[2]*Sq
rt[Tan[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)^3*d) + (a^(3/2)*(3*a^4 + 6*a^2*b^2
+ 35*b^4)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(4*b^(5/2)*(a^2 + b
^2)^3*d) - ((a - b)*(a^2 + 4*a*b + b^2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
+ Tan[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^3*d) + ((a - b)*(a^2 + 4*a*b + b^2)
*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)
^3*d) - (a^2*Tan[c + d*x]^(3/2))/(2*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2)
 - (a^2*(3*a^2 + 11*b^2)*Sqrt[Tan[c + d*x]])/(4*b^2*(a^2 + b^2)^2*d*(a + b*
Tan[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
```

```
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
```

```
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\tan^{\frac{7}{2}}(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{a^2\tan^{\frac{3}{2}}(c+dx)}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} + \frac{\int \frac{\sqrt{\tan(c+dx)}\,\left(\frac{3a^2}{2}-2ab\tan(c+dx)+\frac{1}{2}(3a^2+4\right.}{(a+b\tan(c+dx))^2}}{2b\,(a^2+b^2)}$$

$$= -\frac{a^2\tan^{\frac{3}{2}}(c+dx)}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} - \frac{a^2(3a^2+11b^2)\sqrt{\tan(c+dx)}}{4b^2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))} + \frac{\int \frac{1}{4}}{}$$

$$= -\frac{a^2\tan^{\frac{3}{2}}(c+dx)}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} - \frac{a^2(3a^2+11b^2)\sqrt{\tan(c+dx)}}{4b^2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))} + \frac{\int \frac{2}{}}{}$$

$$= -\frac{a^2\tan^{\frac{3}{2}}(c+dx)}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} - \frac{a^2(3a^2+11b^2)\sqrt{\tan(c+dx)}}{4b^2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))} + \frac{\mathrm{Sul}}{}$$

$$= -\frac{a^2\tan^{\frac{3}{2}}(c+dx)}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} - \frac{a^2(3a^2+11b^2)\sqrt{\tan(c+dx)}}{4b^2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))} + \frac{((a}{}$$

$$= \frac{a^{3/2}(3a^4+6a^2b^2+35b^4)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{4b^{5/2}\,(a^2+b^2)^3\,d} - \frac{a^2\tan^{\frac{3}{2}}(c+c}{2b\,(a^2+b^2)\,d(a+b\,\mathrm{ta}}$$

$$= \frac{a^{3/2}(3a^4+6a^2b^2+35b^4)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{4b^{5/2}\,(a^2+b^2)^3\,d} - \frac{(a-b)\,(a^2+4ab+b^2}{}$$

$$= -\frac{(a+b)\,(a^2-4ab+b^2)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d} + \frac{(a+b)\,(a^2-4ab+}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.92, size = 373, normalized size = 0.94

$$\frac{2b^{9/2}\tan^{\frac{9}{2}}(c+dx)-2b^{7/2}\tan^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))-\frac{a(a+b\tan(c+dx))\left(2\sqrt{b}\,(a^2+b^2)^2(3a^2+4b^2)\sqrt{\tan(c+dx)}-\sqrt{b}\,(a^2+b^2)(3a^4+3a^2b^2+8b^4)\sqrt{\tan(c+dx)}+2ab^{3/2}(a^2+b^2)^2\tan^{\frac{3}{2}}(c+dx)-2b^{5/2}(a^2+b^2)^2\tan^{\frac{5}{2}}(c+dx)-\left(-4\sqrt[4]{-1}\,(a+ib)^3b^{5/2}\mathrm{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+a^{3/2}(3a^4+6a^2b^2+35b^4)\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)-4\sqrt[4]{-1}\,(a-ib)^3b^{5/2}\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\right)(a+b\tan(c+dx))\right)}{(a^2+b^2)^2}}{4ab^{5/2}\,(a^2+b^2)\,d(a+b\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(7/2)/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (2*b^(9/2)*Tan[c + d*x]^(9/2) - 2*b^(7/2)*Tan[c + d*x]^(7/2)*(a + b*Tan[c +
 d*x]) - (a*(a + b*Tan[c + d*x])*(2*Sqrt[b]*(a^2 + b^2)^2*(3*a^2 + 4*b^2)*S
qrt[Tan[c + d*x]] - Sqrt[b]*(a^2 + b^2)*(3*a^4 + 3*a^2*b^2 + 8*b^4)*Sqrt[Ta
n[c + d*x]] + 2*a*b^(3/2)*(a^2 + b^2)^2*Tan[c + d*x]^(3/2) - 2*b^(5/2)*(a^2
 + b^2)^2*Tan[c + d*x]^(5/2) - (-4*(-1)^(1/4)*(a + I*b)^3*b^(5/2)*ArcTan[(-
1)^(3/4)*Sqrt[Tan[c + d*x]]] + a^(3/2)*(3*a^4 + 6*a^2*b^2 + 35*b^4)*ArcTan[
(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]] - 4*(-1)^(1/4)*(a - I*b)^3*b^(5/2)*Ar
cTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])*(a + b*Tan[c + d*x])))/(a^2 + b^2)^2)
/(4*a*b^(5/2)*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2)
```

Maple [A]
time = 0.17, size = 347, normalized size = 0.88

method	result
derivativedivides	$\frac{2a^2\left(\frac{-\frac{(5a^4+18a^2b^2+13b^4)\left(\tan^{\frac{3}{2}}(dx+c)\right)}{8b}-\frac{a(3a^4+14a^2b^2+11b^4)\left(\sqrt{\tan}(dx+c)\right)}{8b^2}}{(a+b\tan(dx+c))^2}+\frac{(3a^4+6a^2b^2+35b^4)\arctan\left(\frac{b\left(\sqrt{\tan \ldots}\right.}{\sqrt{\ldots}}\right)}{8b^2\sqrt{ab}}\ldots\right)}{(a^2+b^2)^3}\ldots$
default	$\frac{2a^2\left(\frac{-\frac{(5a^4+18a^2b^2+13b^4)\left(\tan^{\frac{3}{2}}(dx+c)\right)}{8b}-\frac{a(3a^4+14a^2b^2+11b^4)\left(\sqrt{\tan}(dx+c)\right)}{8b^2}}{(a+b\tan(dx+c))^2}+\frac{(3a^4+6a^2b^2+35b^4)\arctan\left(\frac{b\left(\sqrt{\tan \ldots}\right.}{\sqrt{\ldots}}\right)}{8b^2\sqrt{ab}}\ldots\right)}{(a^2+b^2)^3}\ldots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(7/2)/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*a^2/(a^2+b^2)^3*((-1/8*(5*a^4+18*a^2*b^2+13*b^4)/b*tan(d*x+c)^(3/2)-
1/8*a*(3*a^4+14*a^2*b^2+11*b^4)/b^2*tan(d*x+c)^(1/2))/(a+b*tan(d*x+c))^2+1/
8*(3*a^4+6*a^2*b^2+35*b^4)/b^2/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^
(1/2)))+2/(a^2+b^2)^3*(1/8*(a^3-3*a*b^2)*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^
(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/
2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/8*(-3*a^2*b+b
^3)*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+
c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1
/2)*tan(d*x+c)^(1/2)))))
```

Maxima [A]

time = 0.50, size = 420, normalized size = 1.06

$$\frac{\frac{(3a^6+6a^4b^2+35a^2b^4)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^6b^2+3a^4b^4+3a^2b^6+b^8)\sqrt{ab}}+\frac{2\sqrt{2}\,(a^3-3a^2b-3ab^2+b^3)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}\,(a^3-3a^2b-3ab^2+b^3)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)+\sqrt{2}\,(a^3+3a^2b-3ab^2-b^3)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)-\sqrt{2}\,(a^3+3a^2b-3ab^2-b^3)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{(5a^4b+13a^2b^3)\tan(dx+c)^{\frac{3}{2}}+(3a^5+11a^3b^2)\sqrt{\tan(dx+c)}}{a^6b^2+2a^4b^4+a^2b^6+(a^4b^4+2a^2b^6+b^8)\tan(dx+c)^2+2(a^5b^3+2a^3b^5+ab^7)\tan(dx+c)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(7/2)/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*((3*a^6 + 6*a^4*b^2 + 35*a^2*b^4)*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b)
)/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*sqrt(a*b)) + (2*sqrt(2)*(a^3 - 3
*a^2*b - 3*a*b^2 + b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))
) + 2*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*arctan(-1/2*sqrt(2)*(sqrt(2)
- 2*sqrt(tan(d*x + c)))) + sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*log(sqrt
(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*(a^3 + 3*a^2*b - 3*a*b
^2 - b^3)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a^6 + 3*a^4
*b^2 + 3*a^2*b^4 + b^6) - ((5*a^4*b + 13*a^2*b^3)*tan(d*x + c)^(3/2) + (3*a
^5 + 11*a^3*b^2)*sqrt(tan(d*x + c)))/(a^6*b^2 + 2*a^4*b^4 + a^2*b^6 + (a^4*
b^4 + 2*a^2*b^6 + b^8)*tan(d*x + c)^2 + 2*(a^5*b^3 + 2*a^3*b^5 + a*b^7)*tan
(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11265 vs. $2(348) = 696$.
time = 14.42, size = 22644, normalized size = 57.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(7/2)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*sqrt(2)*((a^22*b^2 + 3*a^20*b^4 - 17*a^18*b^6 - 123*a^16*b^8 - 3
42*a^14*b^10 - 546*a^12*b^12 - 546*a^10*b^14 - 342*a^8*b^16 - 123*a^6*b^18
- 17*a^4*b^20 + 3*a^2*b^22 + b^24)*d^5*cos(d*x + c)^4 + 2*(3*a^20*b^4 + 26*
a^18*b^6 + 99*a^16*b^8 + 216*a^14*b^10 + 294*a^12*b^12 + 252*a^10*b^14 + 12
6*a^8*b^16 + 24*a^6*b^18 - 9*a^4*b^20 - 6*a^2*b^22 - b^24)*d^5*cos(d*x + c)
^2 + (a^18*b^6 + 9*a^16*b^8 + 36*a^14*b^10 + 84*a^12*b^12 + 126*a^10*b^14 +
 126*a^8*b^16 + 84*a^6*b^18 + 36*a^4*b^20 + 9*a^2*b^22 + b^24)*d^5 + 4*((a^
21*b^3 + 8*a^19*b^5 + 27*a^17*b^7 + 48*a^15*b^9 + 42*a^13*b^11 - 42*a^9*b^1
5 - 48*a^7*b^17 - 27*a^5*b^19 - 8*a^3*b^21 - a*b^23)*d^5*cos(d*x + c)^3 + (
a^19*b^5 + 9*a^17*b^7 + 36*a^15*b^9 + 84*a^13*b^11 + 126*a^11*b^13 + 126*a^
9*b^15 + 84*a^7*b^17 + 36*a^5*b^19 + 9*a^3*b^21 + a*b^23)*d^5*cos(d*x + c))
*sin(d*x + c))*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b
^8 + 6*a^2*b^10 + b^12 - 2*(3*a^17*b + 8*a^15*b^3 - 12*a^13*b^5 - 72*a^11*b
^7 - 110*a^9*b^9 - 72*a^7*b^11 - 12*a^5*b^13 + 8*a^3*b^15 + 3*a*b^17)*d^2*s
qrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^
10 + b^12)*d^4)))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4
*b^8 - 30*a^2*b^10 + b^12))*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^
```

6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 12*a^22*b^2 + 66*a^20*b^
4 + 220*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 924*a^12*b^12 + 792*a^10*
b^14 + 495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 12*a^2*b^22 + b^24)*d^4)
)*(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^1
0 + b^12)*d^4))^(3/4)*arctan(-((a^24 - 6*a^22*b^2 - 84*a^20*b^4 - 322*a^18*
b^6 - 603*a^16*b^8 - 540*a^14*b^10 + 540*a^10*b^14 + 603*a^8*b^16 + 322*a^6
*b^18 + 84*a^4*b^20 + 6*a^2*b^22 - b^24)*d^4*sqrt((a^12 - 30*a^10*b^2 + 255
*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 12*a^22
*b^2 + 66*a^20*b^4 + 220*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 924*a^12
*b^12 + 792*a^10*b^14 + 495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 12*a^2*
b^22 + b^24)*d^4))*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 1
5*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) + sqrt(2)*((a^27 + 9*a^25*b^2 + 30*a^2
3*b^4 + 22*a^21*b^6 - 165*a^19*b^8 - 693*a^17*b^10 - 1452*a^15*b^12 - 1980*
a^13*b^14 - 1881*a^11*b^16 - 1265*a^9*b^18 - 594*a^7*b^20 - 186*a^5*b^22 -
35*a^3*b^24 - 3*a*b^26)*d^7*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^
6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 12*a^22*b^2 + 66*a^20*b^
4 + 220*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 924*a^12*b^12 + 792*a^10*
b^14 + 495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 12*a^2*b^22 + b^24)*d^4)
)*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2
*b^10 + b^12)*d^4)) + (3*a^20*b + 26*a^18*b^3 + 99*a^16*b^5 + 216*a^14*b^7
+ 294*a^12*b^9 + 252*a^10*b^11 + 126*a^8*b^13 + 24*a^6*b^15 - 9*a^4*b^17 -
6*a^2*b^19 - b^21)*d^5*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6
 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 12*a^22*b^2 + 66*a^20*b^4 + 2
20*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 924*a^12*b^12 + 792*a^10*b^14
+ 495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 12*a^2*b^22 + b^24)*d^4)))*sq
rt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 +
 b^12 - 2*(3*a^17*b + 8*a^15*b^3 - 12*a^13*b^5 - 72*a^11*b^7 - 110*a^9*b^9
- 72*a^7*b^11 - 12*a^5*b^13 + 8*a^3*b^15 + 3*a*b^17)*d^2*sqrt(1/((a^12 + 6*
a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))
/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^1
0 + b^12))*sqrt(((a^18 - 27*a^16*b^2 + 168*a^14*b^4 + 224*a^12*b^6 - 366*a^
10*b^8 - 366*a^8*b^10 + 224*a^6*b^12 + 168*a^4*b^14 - 27*a^2*b^16 + b^18)*d
^2*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^
2*b^10 + b^12)*d^4))*cos(d*x + c) + sqrt(2)*((3*a^20*b - 82*a^18*b^3 + 531*
a^16*b^5 + 504*a^14*b^7 - 1322*a^12*b^9 - 732*a^10*b^11 + 1038*a^8*b^13 + 2
80*a^6*b^15 - 249*a^4*b^17 + 30*a^2*b^19 - b^21)*d^3*sqrt(1/((a^12 + 6*a^10
*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*cos(
d*x + c) + (a^15 - 33*a^13*b^2 + 345*a^11*b^4 - 1217*a^9*b^6 + 1611*a^7*b^8
 - 795*a^5*b^10 + 91*a^3*b^12 - 3*a*b^14)*d*cos(d*x + c))*sqrt((a^12 + 6*a^
10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 - 2*(3*a^
17*b + 8*a^15*b^3 - 12*a^13*b^5 - 72*a^11*b^7 - 110*a^9*b^9 - 72*a^7*b^11 -
 12*a^5*b^13 + 8*a^3*b^15 + 3*a*b^17)*d^2*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a
^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))/(a^12 - 30*a^1
0*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*sqrt
(sin(d*x + c)/cos(d*x + c))*(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^

```
6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))^(1/4) + (a^12 - 30*a^10*b^2 + 255
*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)*sin(d*x + c))/co
s(d*x + c))*(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 +
 6*a^2*b^10 + b^12)*d^4))^(3/4) + sqrt(2)*((a^33 - 6*a^31*b^2 - 90*a^29*b^4
 - 294*a^27*b^6 - 54*a^25*b^8 + 2082*a^23*b^10 ...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{7}{2}}(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(7/2)/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**(7/2)/(a + b*tan(c + d*x))**3, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(7/2)/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 9.84, size = 2500, normalized size = 6.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(7/2)/(a + b*tan(c + d*x))^3,x)
```

```
[Out] atan((((1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4
*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*((1i/(4*(b^6*d^2 - a^6*d^2
 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*
b^2*d^2)))^(1/2)*((1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i -
 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*((1600*a^2*b^23
*d^4 + 12864*a^4*b^21*d^4 + 45312*a^6*b^19*d^4 + 91392*a^8*b^17*d^4 + 11558
4*a^10*b^15*d^4 + 94080*a^12*b^13*d^4 + 48384*a^14*b^11*d^4 + 14592*a^16*b^
9*d^4 + 2112*a^18*b^7*d^4 + 64*a^20*b^5*d^4)/(b^19*d^5 + 8*a^2*b^17*d^5 + 2
8*a^4*b^15*d^5 + 56*a^6*b^13*d^5 + 70*a^8*b^11*d^5 + 56*a^10*b^9*d^5 + 28*a
^12*b^7*d^5 + 8*a^14*b^5*d^5 + a^16*b^3*d^5) + (tan(c + d*x)^(1/2)*(1i/(4*(
b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*
```

d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*(512*b^28*d^4 + 4608*a^2*b^26*d^4 + 17920*a^4*b^24*d^4 + 38400*a^6*b^22*d^4 + 46080*a^8*b^20*d^4 + 21504*a^10*b^18*d^4 - 21504*a^12*b^16*d^4 - 46080*a^14*b^14*d^4 - 38400*a^16*b^12*d^4 - 17920*a^18*b^10*d^4 - 4608*a^20*b^8*d^4 - 512*a^22*b^6*d^4))/(b^19*d^4 + 8*a^2*b^17*d^4 + 28*a^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*d^4 + 28*a^12*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*d^4)) - (tan(c + d*x)^(1/2)*(1472*a*b^21*d^2 + 72*a^21*b*d^2 + 1024*a^3*b^19*d^2 + 1352*a^5*b^17*d^2 + 28224*a^7*b^15*d^2 + 70240*a^9*b^13*d^2 + 72640*a^11*b^11*d^2 + 39088*a^13*b^9*d^2 + 13248*a^15*b^7*d^2 + 3488*a^17*b^5*d^2 + 576*a^19*b^3*d^2))/(b^19*d^4 + 8*a^2*b^17*d^4 + 28*a^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*d^4 + 28*a^12*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*d^4)) + (32*a*b^18*d^2 - 18*a^19*d^2 - 6528*a^3*b^16*d^2 + 2758*a^5*b^14*d^2 + 26482*a^7*b^12*d^2 + 21582*a^9*b^10*d^2 + 7594*a^11*b^8*d^2 + 3314*a^13*b^6*d^2 + 246*a^15*b^4*d^2 + 90*a^17*b^2*d^2)/(b^19*d^5 + 8*a^2*b^17*d^5 + 28*a^4*b^15*d^5 + 56*a^6*b^13*d^5 + 70*a^8*b^11*d^5 + 56*a^10*b^9*d^5 + 28*a^12*b^7*d^5 + 8*a^14*b^5*d^5 + a^16*b^3*d^5)) + (tan(c + d*x)^(1/2)*(9*a^16 + 32*b^16 + 128*a^2*b^14 + 1417*a^4*b^12 - 6802*a^6*b^10 - 1017*a^8*b^8 - 1020*a^10*b^6 + 39*a^12*b^4 - 18*a^14*b^2))/(b^19*d^4 + 8*a^2*b^17*d^4 + 28*a^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*d^4 + 28*a^12*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*d^4))*(1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*1i - ((1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*((1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*((1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*((1600*a^2*b^23*d^4 + 12864*a^4*b^21*d^4 + 45312*a^6*b^19*d^4 + 91392*a^8*b^17*d^4 + 115584*a^10*b^15*d^4 + 94080*a^12*b^13*d^4 + 48384*a^14*b^11*d^4 + 14592*a^16*b^9*d^4 + 2112*a^18*b^7*d^4 + 64*a^20*b^5*d^4)/(b^19*d^5 + 8*a^2*b^17*d^5 + 28*a^4*b^15*d^5 + 56*a^6*b^13*d^5 + 70*a^8*b^11*d^5 + 56*a^10*b^9*d^5 + 28*a^12*b^7*d^5 + 8*a^14*b^5*d^5 + a^16*b^3*d^5) - (tan(c + d*x)^(1/2)*(1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*(512*b^28*d^4 + 4608*a^2*b^26*d^4 + 17920*a^4*b^24*d^4 + 38400*a^6*b^22*d^4 + 46080*a^8*b^20*d^4 + 21504*a^10*b^18*d^4 - 21504*a^12*b^16*d^4 - 46080*a^14*b^14*d^4 - 38400*a^16*b^12*d^4 - 17920*a^18*b^10*d^4 - 4608*a^20*b^8*d^4 - 512*a^22*b^6*d^4))/(b^19*d^4 + 8*a^2*b^17*d^4 + 28*a^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*d^4 + 28*a^12*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*d^4)) + (tan(c + d*x)^(1/2)*(1472*a*b^21*d^2 + 72*a^21*b*d^2 + 1024*a^3*b^19*d^2 + 1352*a^5*b^17*d^2 + 28224*a^7*b^15*d^2 + 70240*a^9*b^13*d^2 + 72640*a^11*b^11*d^2 + 39088*a^13*b^9*d^2 + 13248*a^15*b^7*d^2 + 3488*a^17*b^5*d^2 + 576*a^19*b^3*d^2))/(b^19*d^4 + 8*a^2*b^17*d^4 + 28*a^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*d^4 + 28*a^12*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*d^4)) + (32*a*b^18*d^2 - 18*a^19*d^2 - 6528*a^3*b^16*d^2 + 2758*a^5*b^14*d^2 + 26482*a^7*b^12*d^2 + 21582*a^9*b^10*d^2 + 7594*a^11*b^8*d^2 + 3314*a

```
^13*b^6*d^2 + 246*a^15*b^4*d^2 + 90*a^17*b^2*d^2)/(b^19*d^5 + 8*a^2*b^17*d^
5 + 28*a^4*b^15*d^5 + 56*a^6*b^13*d^5 + 70*a^8*b^11*d^5 + 56*a^10*b^9*d^5 +
 28*a^12*b^7*d^5 + 8*a^14*b^5*d^5 + a^16*b^3*d^5)) - (tan(c + d*x)^(1/2)*(9
*a^16 + 32*b^16 + 128*a^2*b^14 + 1417*a^4*b^12 - 6802*a^6*b^10 - 1017*a^8*b
^8 - 1020*a^10*b^6 + 39*a^12*b^4 - 18*a^14*b^2))/(b^19*d^4 + 8*a^2*b^17*d^4
 + 28*a^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*d^4 +
28*a^12*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*d^4))*(1i/(4*(b^6*d^2 - a^6*d^2
 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*
b^2*d^2)))^(1/2)*1i)/(((1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2
*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*((1i/(4*(b
^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i ...
```

3.602 $\int \frac{\tan^{\frac{5}{2}}(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=390

$$\frac{(a-b)(a^2+4ab+b^2)\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3 d}-\frac{(a-b)(a^2+4ab+b^2)\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(\ldots}$$

```
[Out] -1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)^3/
d*2^(1/2)-1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2
+b^2)^3/d*2^(1/2)-1/4*(a+b)*(a^2-4*a*b+b^2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+t
an(d*x+c))/(a^2+b^2)^3/d*2^(1/2)+1/4*(a+b)*(a^2-4*a*b+b^2)*ln(1+2^(1/2)*tan
(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)^3/d*2^(1/2)+1/4*(a^4+18*a^2*b^2-15*b^4)
*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))*a^(1/2)/b^(3/2)/(a^2+b^2)^3/d-1/2
*a^2*tan(d*x+c)^(1/2)/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^2+1/4*a*(a^2+9*b^2)*ta
n(d*x+c)^(1/2)/b/(a^2+b^2)^2/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.59, antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3646, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(a-b)(a^2+4ab+b^2)\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d\,(a^2+b^2)^3}-\frac{(a-b)(a^2+4ab+b^2)\,\mathrm{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d\,(a^2+b^2)^3}-\frac{a^2\sqrt{\tan(c+dx)}}{2bd\,(a^2+b^2)\,(a+b\tan(c+dx))^2}+\frac{a(a^2+9b^2)\sqrt{\tan(c+dx)}}{4bd\,(a^2+b^2)^2\,(a+b\tan(c+dx))}-\frac{(a+b)(a^2-4ab+b^2)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)^3}+\frac{(a+b)(a^2-4ab+b^2)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)^3}+\frac{\sqrt{a}\,(a^4+18a^2b^2-15b^4)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{4b^{3/2}d\,(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(5/2)/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[
2]*(a^2 + b^2)^3*d) - ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 + Sqrt[2]*Sqrt[
Tan[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)^3*d) + (Sqrt[a]*(a^4 + 18*a^2*b^2 - 15
*b^4)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(4*b^(3/2)*(a^2 + b^2)^
3*d) - ((a + b)*(a^2 - 4*a*b + b^2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Ta
n[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^3*d) + ((a + b)*(a^2 - 4*a*b + b^2)*Log
[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^3*d
) - (a^2*Sqrt[Tan[c + d*x]])/(2*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) + (
a*(a^2 + 9*b^2)*Sqrt[Tan[c + d*x]])/(4*b*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x
]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
```

```
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
```

```
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{a^2\sqrt{\tan(c+dx)}}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} + \frac{\int \frac{\frac{a^2}{2}-2ab\tan(c+dx)+\frac{1}{2}(a^2+4b^2)\tan^2(c+dx)}{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^2}\,dx}{2b\,(a^2+b^2)}$$

$$= -\frac{a^2\sqrt{\tan(c+dx)}}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} + \frac{a(a^2+9b^2)\sqrt{\tan(c+dx)}}{4b\,(a^2+b^2)^2\,d(a+b\tan(c+dx))} + \frac{\int \frac{1}{4}a \cdots}{\cdots}$$

$$= -\frac{a^2\sqrt{\tan(c+dx)}}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} + \frac{a(a^2+9b^2)\sqrt{\tan(c+dx)}}{4b\,(a^2+b^2)^2\,d(a+b\tan(c+dx))} + \frac{\int \frac{-2\cdots}{\cdots}}{\cdots}$$

$$= -\frac{a^2\sqrt{\tan(c+dx)}}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} + \frac{a(a^2+9b^2)\sqrt{\tan(c+dx)}}{4b\,(a^2+b^2)^2\,d(a+b\tan(c+dx))} + \frac{\text{Subs}\cdots}{\cdots}$$

$$= -\frac{a^2\sqrt{\tan(c+dx)}}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} + \frac{a(a^2+9b^2)\sqrt{\tan(c+dx)}}{4b\,(a^2+b^2)^2\,d(a+b\tan(c+dx))} + \frac{((a\cdots}{\cdots}$$

$$= \frac{\sqrt{a}\,(a^4+18a^2b^2-15b^4)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{4b^{3/2}\,(a^2+b^2)^3\,d} - \frac{a^2\sqrt{\tan(c+\cdots}}{2b\,(a^2+b^2)\,d(a+b\tan\cdots}$$

$$= \frac{\sqrt{a}\,(a^4+18a^2b^2-15b^4)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{4b^{3/2}\,(a^2+b^2)^3\,d} - \frac{(a+b)(a^2-4ab+b^2\cdots}{\cdots}$$

$$= \frac{(a-b)(a^2+4ab+b^2)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d} - \frac{(a-b)(a^2+4ab+b\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.26, size = 325, normalized size = 0.83

$$\frac{2b^{7/2}\tan^{\frac{7}{2}}(c+dx)-2b^{5/2}\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))-\frac{a(a+b\tan(c+dx))\left(2a\sqrt{b}(a^2+b^2)^2\sqrt{\tan(c+dx)}-a\sqrt{b}(a^2+b^2)(a^2+9b^2)\sqrt{\tan(c+dx)}-2b^{3/2}(a^2+b^2)^2\tan^{\frac{3}{2}}(c+dx)-\left(-4(-1)^{3/4}(a+ib)^3b^{3/2}\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+\sqrt{a}(a^4+18a^2b^2-15b^4)\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)-4\sqrt[4]{-1}\,b^{3/2}(ia+b)^3\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\right)(a+b\tan(c+dx))\right)}{(a^2+b^2)^2}}{4ab^{3/2}(a^2+b^2)\,d(a+b\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(5/2)/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (2*b^(7/2)*Tan[c + d*x]^(7/2) - 2*b^(5/2)*Tan[c + d*x]^(5/2)*(a + b*Tan[c +
 d*x]) - (a*(a + b*Tan[c + d*x])*(2*a*Sqrt[b]*(a^2 + b^2)^2*Sqrt[Tan[c + d*
x]] - a*Sqrt[b]*(a^2 + b^2)*(a^2 + 9*b^2)*Sqrt[Tan[c + d*x]] - 2*b^(3/2)*(a
^2 + b^2)^2*Tan[c + d*x]^(3/2) - (-4*(-1)^(3/4)*(a + I*b)^3*b^(3/2)*ArcTan[
(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + Sqrt[a]*(a^4 + 18*a^2*b^2 - 15*b^4)*ArcTan
[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]] - 4*(-1)^(1/4)*b^(3/2)*(I*a + b)^3*A
rcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])*(a + b*Tan[c + d*x])))/(a^2 + b^2)^2
)/(4*a*b^(3/2)*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2)
```

Maple [A]

time = 0.17, size = 339, normalized size = 0.87

method	result
derivativedivides	$\frac{2a\left(\frac{\left(\frac{1}{8}a^4+\frac{5}{4}a^2b^2+\frac{9}{8}b^4\right)\left(\tan^{\frac{3}{2}}(dx+c)\right)-\frac{a\left(a^4-6a^2b^2-7b^4\right)\left(\sqrt{\tan}(dx+c)\right)}{8b}}{(a+b\tan(dx+c))^2}+\frac{\left(a^4+18a^2b^2-15b^4\right)\arctan\left(\frac{b\left(\sqrt{\tan}(dx+\right.}{\sqrt{ab}}\right.}{8b\sqrt{ab}}\right.}{\left(a^2+b^2\right)^3}$
default	$\frac{2a\left(\frac{\left(\frac{1}{8}a^4+\frac{5}{4}a^2b^2+\frac{9}{8}b^4\right)\left(\tan^{\frac{3}{2}}(dx+c)\right)-\frac{a\left(a^4-6a^2b^2-7b^4\right)\left(\sqrt{\tan}(dx+c)\right)}{8b}}{(a+b\tan(dx+c))^2}+\frac{\left(a^4+18a^2b^2-15b^4\right)\arctan\left(\frac{b\left(\sqrt{\tan}(dx+\right.}{\sqrt{ab}}\right.}{8b\sqrt{ab}}\right.}{\left(a^2+b^2\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*a/(a^2+b^2)^3*(((1/8*a^4+5/4*a^2*b^2+9/8*b^4)*tan(d*x+c)^(3/2)-1/8*a
*(a^4-6*a^2*b^2-7*b^4)/b*tan(d*x+c)^(1/2))/(a+b*tan(d*x+c))^2+1/8*(a^4+18*a
^2*b^2-15*b^4)/b/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2)))+2/(a^2
+b^2)^3*(1/8*(-3*a^2*b+b^3)*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x
+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)
^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/8*(-a^3+3*a*b^2)*2^(1/2)*(
ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(
d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c
)^(1/2)))))
```

Maxima [A]

time = 0.51, size = 408, normalized size = 1.05

$$\frac{\frac{\left(a^5+18a^3b^2-15ab^4\right)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{\left(a^6b+3a^4b^3+3a^2b^5+b^7\right)\sqrt{ab}}-\frac{2\sqrt{2}\left(a^3+3a^2b-3ab^2-b^3\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}\left(a^3+3a^2b-3ab^2-b^3\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-\sqrt{2}\left(a^3-3a^2b-3ab^2+b^3\right)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+\sqrt{2}\left(a^3-3a^2b-3ab^2+b^3\right)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{\left(a^3b+9ab^3\right)\tan(dx+c)^{\frac{3}{2}}-\left(a^4-7a^2b^2\right)\sqrt{\tan(dx+c)}}{a^6b+2a^4b^3+a^2b^5+\left(a^4b^3+2a^2b^5+b^7\right)\tan(dx+c)^2+2\left(a^5b^2+2a^3b^4+ab^6\right)\tan(dx+c)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*((a^5 + 18*a^3*b^2 - 15*a*b^4)*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/(
(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sqrt(a*b)) - (2*sqrt(2)*(a^3 + 3*a^2*
b - 3*a*b^2 - b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2
*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*s
qrt(tan(d*x + c)))) - sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*log(sqrt(2)*s
qrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 +
b^3)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6) + ((a^3*b + 9*a*b^3)*tan(d*x + c)^(3/2) - (a^4 - 7*a^2*b
^2)*sqrt(tan(d*x + c)))/(a^6*b + 2*a^4*b^3 + a^2*b^5 + (a^4*b^3 + 2*a^2*b^5
 + b^7)*tan(d*x + c)^2 + 2*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*tan(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11245 vs. $2(342) = 684$.
time = 13.35, size = 22603, normalized size = 57.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*sqrt(2)*((a^22*b + 3*a^20*b^3 - 17*a^18*b^5 - 123*a^16*b^7 - 342
*a^14*b^9 - 546*a^12*b^11 - 546*a^10*b^13 - 342*a^8*b^15 - 123*a^6*b^17 - 1
7*a^4*b^19 + 3*a^2*b^21 + b^23)*d^5*cos(d*x + c)^4 + 2*(3*a^20*b^3 + 26*a^1
8*b^5 + 99*a^16*b^7 + 216*a^14*b^9 + 294*a^12*b^11 + 252*a^10*b^13 + 126*a^
8*b^15 + 24*a^6*b^17 - 9*a^4*b^19 - 6*a^2*b^21 - b^23)*d^5*cos(d*x + c)^2 +
 (a^18*b^5 + 9*a^16*b^7 + 36*a^14*b^9 + 84*a^12*b^11 + 126*a^10*b^13 + 126*
a^8*b^15 + 84*a^6*b^17 + 36*a^4*b^19 + 9*a^2*b^21 + b^23)*d^5 + 4*((a^21*b^
2 + 8*a^19*b^4 + 27*a^17*b^6 + 48*a^15*b^8 + 42*a^13*b^10 - 42*a^9*b^14 - 4
8*a^7*b^16 - 27*a^5*b^18 - 8*a^3*b^20 - a*b^22)*d^5*cos(d*x + c)^3 + (a^19*
b^4 + 9*a^17*b^6 + 36*a^15*b^8 + 84*a^13*b^10 + 126*a^11*b^12 + 126*a^9*b^1
4 + 84*a^7*b^16 + 36*a^5*b^18 + 9*a^3*b^20 + a*b^22)*d^5*cos(d*x + c))*sin(
d*x + c))*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 +
6*a^2*b^10 + b^12 + 2*(3*a^17*b + 8*a^15*b^3 - 12*a^13*b^5 - 72*a^11*b^7 -
110*a^9*b^9 - 72*a^7*b^11 - 12*a^5*b^13 + 8*a^3*b^15 + 3*a*b^17)*d^2*sqrt(1
/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 +
b^12)*d^4)))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8
- 30*a^2*b^10 + b^12))*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6
 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 12*a^22*b^2 + 66*a^20*b^4 + 2
20*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 924*a^12*b^12 + 792*a^10*b^14
+ 495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 12*a^2*b^22 + b^24)*d^4))*(1/
((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b
^12)*d^4))^(3/4)*arctan(((a^24 - 6*a^22*b^2 - 84*a^20*b^4 - 322*a^18*b^6 -
```

603*a^16*b^8 - 540*a^14*b^10 + 540*a^10*b^14 + 603*a^8*b^16 + 322*a^6*b^18 + 84*a^4*b^20 + 6*a^2*b^22 - b^24)*d^4*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 12*a^22*b^2 + 66*a^20*b^4 + 220*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 924*a^12*b^12 + 792*a^10*b^14 + 495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 12*a^2*b^22 + b^24)*d^4))*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) - sqrt(2)*((3*a^26*b + 35*a^24*b^3 + 186*a^22*b^5 + 594*a^20*b^7 + 1265*a^18*b^9 + 1881*a^16*b^11 + 1980*a^14*b^13 + 1452*a^12*b^15 + 693*a^10*b^17 + 165*a^8*b^19 - 22*a^6*b^21 - 30*a^4*b^23 - 9*a^2*b^25 - b^27)*d^7*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 12*a^22*b^2 + 66*a^20*b^4 + 220*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 924*a^12*b^12 + 792*a^10*b^14 + 495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 12*a^2*b^22 + b^24)*d^4))*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) - (a^21 + 6*a^19*b^2 + 9*a^17*b^4 - 24*a^15*b^6 - 126*a^13*b^8 - 252*a^11*b^10 - 294*a^9*b^12 - 216*a^7*b^14 - 99*a^5*b^16 - 26*a^3*b^18 - 3*a*b^20)*d^5*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 12*a^22*b^2 + 66*a^20*b^4 + 220*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 924*a^12*b^12 + 792*a^10*b^14 + 495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 12*a^2*b^22 + b^24)*d^4)))*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 + 2*(3*a^17*b + 8*a^15*b^3 - 12*a^13*b^5 - 72*a^11*b^7 - 110*a^9*b^9 - 72*a^7*b^11 - 12*a^5*b^13 + 8*a^3*b^15 + 3*a*b^17)*d^2*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*sqrt(((a^18 - 27*a^16*b^2 + 168*a^14*b^4 + 224*a^12*b^6 - 366*a^10*b^8 - 366*a^8*b^10 + 224*a^6*b^12 + 168*a^4*b^14 - 27*a^2*b^16 + b^18)*d^2*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*cos(d*x + c) + sqrt(2)*((a^21 - 30*a^19*b^2 + 249*a^17*b^4 - 280*a^15*b^6 - 1038*a^13*b^8 + 732*a^11*b^10 + 1322*a^9*b^12 - 504*a^7*b^14 - 531*a^5*b^16 + 82*a^3*b^18 - 3*a*b^20)*d^3*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*cos(d*x + c) - (3*a^14*b - 91*a^12*b^3 + 795*a^10*b^5 - 1611*a^8*b^7 + 1217*a^6*b^9 - 345*a^4*b^11 + 33*a^2*b^13 - b^15)*d*cos(d*x + c))*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 + 2*(3*a^17*b + 8*a^15*b^3 - 12*a^13*b^5 - 72*a^11*b^7 - 110*a^9*b^9 - 72*a^7*b^11 - 12*a^5*b^13 + 8*a^3*b^15 + 3*a*b^17)*d^2*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))^(1/4) + (a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)*sin(d*x + c))/cos(d*x + c))*(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))^(3/4) - sqrt(2)*((3*a^32*b - 10*a^30*b^3 - 294*a^28*b^5 - 1674*a^26*b^7 - 4890*a^24*b^9 - 8370*a^22*b^...

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(5/2)/(a+b*tan(d*x+c))**3,x)
[Out] Integral(tan(c + d*x)**(5/2)/(a + b*tan(c + d*x))**3, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^3,x, algorithm="giac")
[Out] Timed out
```

Mupad [B]
time = 20.84, size = 2500, normalized size = 6.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(5/2)/(a + b*tan(c + d*x))^3,x)
[Out] atan(((-1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4
*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*((-1i/(4*(b^6*d^2 - a^6*d^
2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4
*b^2*d^2)))^(1/2)*((-1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i
 - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*(((832*a*b^22
*d^4 + 5952*a^3*b^20*d^4 + 17664*a^5*b^18*d^4 + 26880*a^7*b^16*d^4 + 18816*
a^9*b^14*d^4 - 2688*a^11*b^12*d^4 - 16128*a^13*b^10*d^4 - 13056*a^15*b^8*d^
4 - 4800*a^17*b^6*d^4 - 704*a^19*b^4*d^4)/(b^17*d^5 + a^16*b*d^5 + 8*a^2*b^
15*d^5 + 28*a^4*b^13*d^5 + 56*a^6*b^11*d^5 + 70*a^8*b^9*d^5 + 56*a^10*b^7*d
^5 + 28*a^12*b^5*d^5 + 8*a^14*b^3*d^5) - (tan(c + d*x)^(1/2)*(-1i/(4*(b^6*d
^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*2
0i + 15*a^4*b^2*d^2)))^(1/2)*(512*b^26*d^4 + 4608*a^2*b^24*d^4 + 17920*a^4*
b^22*d^4 + 38400*a^6*b^20*d^4 + 46080*a^8*b^18*d^4 + 21504*a^10*b^16*d^4 -
21504*a^12*b^14*d^4 - 46080*a^14*b^12*d^4 - 38400*a^16*b^10*d^4 - 17920*a^1
8*b^8*d^4 - 4608*a^20*b^6*d^4 - 512*a^22*b^4*d^4))/(b^17*d^4 + a^16*b*d^4 +
 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*a
```

^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4))*(-1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) + (tan(c + d*x)^(1/2)*(8*a^19*b*d^2 - 1472*a*b^19*d^2 + 776*a^3*b^17*d^2 + 11328*a^5*b^15*d^2 + 10208*a^7*b^13*d^2 - 5056*a^9*b^11*d^2 - 5328*a^11*b^9*d^2 + 4032*a^13*b^7*d^2 + 3552*a^15*b^5*d^2 + 384*a^17*b^3*d^2))/(b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4)) - (10*a^16*b*d^2 - 2398*a^2*b^15*d^2 + 5238*a^4*b^13*d^2 + 7386*a^6*b^11*d^2 - 8322*a^8*b^9*d^2 - 5498*a^10*b^7*d^2 + 2946*a^12*b^5*d^2 + 382*a^14*b^3*d^2)/(b^17*d^5 + a^16*b*d^5 + 8*a^2*b^15*d^5 + 28*a^4*b^13*d^5 + 56*a^6*b^11*d^5 + 70*a^8*b^9*d^5 + 56*a^10*b^7*d^5 + 28*a^12*b^5*d^5 + 8*a^14*b^3*d^5)) + (tan(c + d*x)^(1/2)*(a^14 - 32*b^14 + 97*a^2*b^12 - 2082*a^4*b^10 + 3631*a^6*b^8 - 2300*a^8*b^6 + 79*a^10*b^4 + 30*a^12*b^2))/(b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4))*1i - (-1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*((-1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*((-1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*(((832*a*b^22*d^4 + 5952*a^3*b^20*d^4 + 17664*a^5*b^18*d^4 + 26880*a^7*b^16*d^4 + 18816*a^9*b^14*d^4 - 2688*a^11*b^12*d^4 - 16128*a^13*b^10*d^4 - 13056*a^15*b^8*d^4 - 4800*a^17*b^6*d^4 - 704*a^19*b^4*d^4)/(b^17*d^5 + a^16*b*d^5 + 8*a^2*b^15*d^5 + 28*a^4*b^13*d^5 + 56*a^6*b^11*d^5 + 70*a^8*b^9*d^5 + 56*a^10*b^7*d^5 + 28*a^12*b^5*d^5 + 8*a^14*b^3*d^5) + (tan(c + d*x)^(1/2)*(-1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*(512*b^26*d^4 + 4608*a^2*b^24*d^4 + 17920*a^4*b^22*d^4 + 38400*a^6*b^20*d^4 + 46080*a^8*b^18*d^4 + 21504*a^10*b^16*d^4 - 21504*a^12*b^14*d^4 - 46080*a^14*b^12*d^4 - 38400*a^16*b^10*d^4 - 17920*a^18*b^8*d^4 - 4608*a^20*b^6*d^4 - 512*a^22*b^4*d^4))/(b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4))*(-1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) - (tan(c + d*x)^(1/2)*(8*a^19*b*d^2 - 1472*a*b^19*d^2 + 776*a^3*b^17*d^2 + 11328*a^5*b^15*d^2 + 10208*a^7*b^13*d^2 - 5056*a^9*b^11*d^2 - 5328*a^11*b^9*d^2 + 4032*a^13*b^7*d^2 + 3552*a^15*b^5*d^2 + 384*a^17*b^3*d^2))/(b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4)) - (10*a^16*b*d^2 - 2398*a^2*b^15*d^2 + 5238*a^4*b^13*d^2 + 7386*a^6*b^11*d^2 - 8322*a^8*b^9*d^2 - 5498*a^10*b^7*d^2 + 2946*a^12*b^5*d^2 + 382*a^14*b^3*d^2)/(b^17*d^5 + a^16*b*d^5 + 8*a^2*b^15*d^5 + 28*a^4*b^13*d^5 + 56*a^6*b^11*d^5 + 70*a^8*b^9*d^5 + 56*a^10*b^7*d^5 + 28*a^12*b^5*d^5 + 8*a^14*b^3*d^5)) - (tan(c + d*x)^(1/2)*(a^14 - 32*b^14 + 97*a^2*b^12 - 2082*a^4*b^10 + 3631*a^6*b^8 - 2300*a^8*b^6 + 79*a^10*b^4 + 30*a^12*b^2))/(b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*a^10

```
*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4))*1i)/((-1i/(4*(b^6*d^2 - a^6*d
^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^
4*b^2*d^2)))^(1/2)*((-1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6
i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*((-1i/(4*(b^
6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^
2*20i + 15*a^4*b^2*d^2)))^(1/2)*(((832*a*b^22*d...
```

3.603 $\int \frac{\tan^{\frac{3}{2}}(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=385

$$\frac{(a+b)\left(a^2-4ab+b^2\right)\mathrm{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,\left(a^2+b^2\right)^3 d}-\frac{(a+b)\left(a^2-4ab+b^2\right)\mathrm{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,\left(a^2+b^2\right)^3 d}$$

```
[Out] -1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)^3/
d*2^(1/2)-1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2
+b^2)^3/d*2^(1/2)+1/4*(a-b)*(a^2+4*a*b+b^2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+t
an(d*x+c))/(a^2+b^2)^3/d*2^(1/2)-1/4*(a-b)*(a^2+4*a*b+b^2)*ln(1+2^(1/2)*tan
(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)^3/d*2^(1/2)+1/4*(3*a^4-26*a^2*b^2+3*b^4
)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/(a^2+b^2)^3/d/a^(1/2)/b^(1/2)+1/
2*a*tan(d*x+c)^(1/2)/(a^2+b^2)/d/(a+b*tan(d*x+c))^2+1/4*(3*a^2-5*b^2)*tan(d
*x+c)^(1/2)/(a^2+b^2)^2/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.54, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3648, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(a+b)\left(a^2-4ab+b^2\right)\mathrm{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d\left(a^2+b^2\right)^3}-\frac{(a+b)\left(a^2-4ab+b^2\right)\mathrm{ArcTan}\left(\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d\left(a^2+b^2\right)^3}+\frac{a\sqrt{\tan(c+dx)}}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}+\frac{\left(3a^2-5b^2\right)\sqrt{\tan(c+dx)}}{4d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}+\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\tan(c+dx)-\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)^3}-\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\tan(c+dx)+\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)^3}+\frac{\left(3a^4-26a^2b^2+3b^4\right)\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{4\sqrt{a}\,\sqrt{b}\,d\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(3/2)/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[
2]*(a^2 + b^2)^3*d) - ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 + Sqrt[2]*Sqrt[
Tan[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)^3*d) + ((3*a^4 - 26*a^2*b^2 + 3*b^4)*A
rcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(4*Sqrt[a]*Sqrt[b]*(a^2 + b^2)
^3*d) + ((a - b)*(a^2 + 4*a*b + b^2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + T
an[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^3*d) - ((a - b)*(a^2 + 4*a*b + b^2)*Lo
g[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^3*
d) + (a*Sqrt[Tan[c + d*x]])/(2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) + ((3*
a^2 - 5*b^2)*Sqrt[Tan[c + d*x]])/(4*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
```

```
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{a\sqrt{\tan(c+dx)}}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{\int \frac{\frac{a}{2}-2b\tan(c+dx)-\frac{3}{2}a\tan^2(c+dx)}{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^2}\,dx}{2\left(a^2+b^2\right)}$$

$$= \frac{a\sqrt{\tan(c+dx)}}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{\left(3a^2-5b^2\right)\sqrt{\tan(c+dx)}}{4\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))} - \frac{\int \frac{1}{4}a(5a^2 \ldots}{\ldots}$$

$$= \frac{a\sqrt{\tan(c+dx)}}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{\left(3a^2-5b^2\right)\sqrt{\tan(c+dx)}}{4\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))} - \frac{\int 2a^2(a^2 \ldots}{\ldots}$$

$$= \frac{a\sqrt{\tan(c+dx)}}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{\left(3a^2-5b^2\right)\sqrt{\tan(c+dx)}}{4\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))} - \frac{\text{Subst}\Big(\ldots}{\ldots}$$

$$= \frac{a\sqrt{\tan(c+dx)}}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{\left(3a^2-5b^2\right)\sqrt{\tan(c+dx)}}{4\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))} - \frac{((a+b) \ldots}{\ldots}$$

$$= \frac{\left(3a^4-26a^2b^2+3b^4\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{4\sqrt{a}\sqrt{b}\left(a^2+b^2\right)^3 d} + \frac{a\sqrt{\tan(c+dx)}}{2\left(a^2+b^2\right)d(a+b\tan(c+ \ldots}$$

$$= \frac{\left(3a^4-26a^2b^2+3b^4\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{4\sqrt{a}\sqrt{b}\left(a^2+b^2\right)^3 d} + \frac{(a-b)\left(a^2+4ab+b^2\right)\log \ldots}{2 \ldots}$$

$$= \frac{(a+b)\left(a^2-4ab+b^2\right)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 d} - \frac{(a+b)\left(a^2-4ab+ \ldots}{\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.78, size = 326, normalized size = 0.85

$$\frac{b^{9/2}\tan^{\frac{5}{2}}(c+dx)-b^{7/2}\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))+\frac{(a+b\tan(c+dx))\left(a^{5/2}b^{5/2}\left(a^2+b^2\right)^2\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^{5/2}b^{5/2}\left(3a^2-5b^2\right)\left(a^2+b^2\right)\sqrt{\tan(c+dx)}+\left(4\sqrt[4]{-1}\,a^{5/2}(a+ib)^3b^{5/2}\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+a^2b^2\left(3a^4-26a^2b^2+3b^4\right)\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)+4\sqrt[4]{-1}\,a^{5/2}(a-ib)^3b^{5/2}\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\right)(a+b\tan(c+dx))\right)\right)}{a^{3/2}\left(a^2+b^2\right)^2}}{2ab^{5/2}\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(3/2)/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (b^(9/2)*Tan[c + d*x]^(5/2) - b^(7/2)*Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x
]) + ((a + b*Tan[c + d*x])*(a^(5/2)*b^(5/2)*(a^2 + b^2)^2*Sqrt[Tan[c + d*x]
] + (a^(5/2)*b^(5/2)*(3*a^2 - 5*b^2)*(a^2 + b^2)*Sqrt[Tan[c + d*x]] + (4*(-
1)^(1/4)*a^(5/2)*(a + I*b)^3*b^(5/2)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]
+ a^2*b^2*(3*a^4 - 26*a^2*b^2 + 3*b^4)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/
Sqrt[a]] + 4*(-1)^(1/4)*a^(5/2)*(a - I*b)^3*b^(5/2)*ArcTanh[(-1)^(3/4)*Sqrt
[Tan[c + d*x]]])*(a + b*Tan[c + d*x]))/2))/(a^(3/2)*(a^2 + b^2)^2))/(2*a*b^
(5/2)*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2)
```

Maple [A]
time = 0.16, size = 339, normalized size = 0.88

method	result
derivativedivides	$\frac{\frac{2\left(\left(\frac{3}{8}a^4b-\frac{1}{4}a^2b^3-\frac{5}{8}b^5\right)\left(\tan^{\frac{3}{2}}(dx+c)\right)+\frac{a\left(5a^4+2a^2b^2-3b^4\right)\left(\sqrt{\tan}(dx+c)\right)}{8}\right)}{(a+b\tan(dx+c))^2}+\frac{\left(3a^4-26a^2b^2+3b^4\right)\arctan\left(\frac{b\left(\sqrt{\tan}(d\right.}{\sqrt{ab}}\right.}{4\sqrt{ab}}}{\left(a^2+b^2\right)^3}$
default	$\frac{\frac{2\left(\left(\frac{3}{8}a^4b-\frac{1}{4}a^2b^3-\frac{5}{8}b^5\right)\left(\tan^{\frac{3}{2}}(dx+c)\right)+\frac{a\left(5a^4+2a^2b^2-3b^4\right)\left(\sqrt{\tan}(dx+c)\right)}{8}\right)}{(a+b\tan(dx+c))^2}+\frac{\left(3a^4-26a^2b^2+3b^4\right)\arctan\left(\frac{b\left(\sqrt{\tan}(d\right.}{\sqrt{ab}}\right.}{4\sqrt{ab}}}{\left(a^2+b^2\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/(a^2+b^2)^3*(((3/8*a^4*b-1/4*a^2*b^3-5/8*b^5)*tan(d*x+c)^(3/2)+1/8*a
*(5*a^4+2*a^2*b^2-3*b^4)*tan(d*x+c)^(1/2))/(a+b*tan(d*x+c))^2+1/8*(3*a^4-26
*a^2*b^2+3*b^4)/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2)))+2/(a^2+
b^2)^3*(1/8*(-a^3+3*a*b^2)*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+
c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^
(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/8*(3*a^2*b-b^3)*2^(1/2)*(ln
((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*
x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^
(1/2)))))
```

Maxima [A]
time = 0.50, size = 402, normalized size = 1.04

$$\frac{\frac{\left(3a^4-26a^2b^2+3b^4\right)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{\left(a^6+3a^4b^2+3a^2b^4+b^6\right)\sqrt{ab}}-\frac{2\sqrt{2}\left(a^3-3a^2b-3ab^2+b^3\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}\left(a^3-3a^2b-3ab^2+b^3\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)+\sqrt{2}\left(a^3+3a^2b-3ab^2-b^3\right)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)-\sqrt{2}\left(a^3+3a^2b-3ab^2-b^3\right)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{\left(3a^2b-5b^3\right)\tan(dx+c)^{\frac{3}{2}}+\left(5a^3-3ab^2\right)\sqrt{\tan(dx+c)}}{a^6+2a^4b^2+a^2b^4+\left(a^4b^2+2a^2b^4+b^6\right)\tan(dx+c)^2+2\left(a^5b+2a^3b^3+ab^5\right)\tan(dx+c)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*((3*a^4 - 26*a^2*b^2 + 3*b^4)*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((
a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a*b)) - (2*sqrt(2)*(a^3 - 3*a^2*b -
 3*a*b^2 + b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sq
rt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt
(tan(d*x + c)))) + sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*log(sqrt(2)*sqrt
(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3
)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6) + ((3*a^2*b - 5*b^3)*tan(d*x + c)^(3/2) + (5*a^3 - 3*a*b^2)
*sqrt(tan(d*x + c)))/(a^6 + 2*a^4*b^2 + a^2*b^4 + (a^4*b^2 + 2*a^2*b^4 + b^
6)*tan(d*x + c)^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*tan(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11404 vs. $2(337) = 674$.
time = 12.89, size = 22816, normalized size = 59.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*sqrt(2)*((a^23*b + 3*a^21*b^3 - 17*a^19*b^5 - 123*a^17*b^7 - 342*
a^15*b^9 - 546*a^13*b^11 - 546*a^11*b^13 - 342*a^9*b^15 - 123*a^7*b^17 - 17
*a^5*b^19 + 3*a^3*b^21 + a*b^23)*d^5*cos(d*x + c)^4 + 2*(3*a^21*b^3 + 26*a^
19*b^5 + 99*a^17*b^7 + 216*a^15*b^9 + 294*a^13*b^11 + 252*a^11*b^13 + 126*a
^9*b^15 + 24*a^7*b^17 - 9*a^5*b^19 - 6*a^3*b^21 - a*b^23)*d^5*cos(d*x + c)^
2 + (a^19*b^5 + 9*a^17*b^7 + 36*a^15*b^9 + 84*a^13*b^11 + 126*a^11*b^13 + 1
26*a^9*b^15 + 84*a^7*b^17 + 36*a^5*b^19 + 9*a^3*b^21 + a*b^23)*d^5 + 4*((a^
22*b^2 + 8*a^20*b^4 + 27*a^18*b^6 + 48*a^16*b^8 + 42*a^14*b^10 - 42*a^10*b^
14 - 48*a^8*b^16 - 27*a^6*b^18 - 8*a^4*b^20 - a^2*b^22)*d^5*cos(d*x + c)^3
+ (a^20*b^4 + 9*a^18*b^6 + 36*a^16*b^8 + 84*a^14*b^10 + 126*a^12*b^12 + 126
*a^10*b^14 + 84*a^8*b^16 + 36*a^6*b^18 + 9*a^4*b^20 + a^2*b^22)*d^5*cos(d*x
 + c))*sin(d*x + c))*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15
*a^4*b^8 + 6*a^2*b^10 + b^12 - 2*(3*a^17*b + 8*a^15*b^3 - 12*a^13*b^5 - 72*
a^11*b^7 - 110*a^9*b^9 - 72*a^7*b^11 - 12*a^5*b^13 + 8*a^3*b^15 + 3*a*b^17)
*d^2*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*
a^2*b^10 + b^12)*d^4)))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 2
55*a^4*b^8 - 30*a^2*b^10 + b^12))*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 -
452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 12*a^22*b^2 + 66*a
^20*b^4 + 220*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 924*a^12*b^12 + 792
*a^10*b^14 + 495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 12*a^2*b^22 + b^24
)*d^4))*(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a
^2*b^10 + b^12)*d^4))^(3/4)*arctan(-((a^24 - 6*a^22*b^2 - 84*a^20*b^4 - 322
*a^18*b^6 - 603*a^16*b^8 - 540*a^14*b^10 + 540*a^10*b^14 + 603*a^8*b^16 + 3
22*a^6*b^18 + 84*a^4*b^20 + 6*a^2*b^22 - b^24)*d^4*sqrt((a^12 - 30*a^10*b^2
 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 1
```

```
2*a^22*b^2 + 66*a^20*b^4 + 220*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 92
4*a^12*b^12 + 792*a^10*b^14 + 495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 1
2*a^2*b^22 + b^24)*d^4))*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b
^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) + sqrt(2)*((a^27 + 9*a^25*b^2 +
30*a^23*b^4 + 22*a^21*b^6 - 165*a^19*b^8 - 693*a^17*b^10 - 1452*a^15*b^12 -
 1980*a^13*b^14 - 1881*a^11*b^16 - 1265*a^9*b^18 - 594*a^7*b^20 - 186*a^5*b
^22 - 35*a^3*b^24 - 3*a*b^26)*d^7*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 -
452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 12*a^22*b^2 + 66*a
^20*b^4 + 220*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 924*a^12*b^12 + 792
*a^10*b^14 + 495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 12*a^2*b^22 + b^24
)*d^4))*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 +
 6*a^2*b^10 + b^12)*d^4)) + (3*a^20*b + 26*a^18*b^3 + 99*a^16*b^5 + 216*a^1
4*b^7 + 294*a^12*b^9 + 252*a^10*b^11 + 126*a^8*b^13 + 24*a^6*b^15 - 9*a^4*b
^17 - 6*a^2*b^19 - b^21)*d^5*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a
^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 12*a^22*b^2 + 66*a^20*b
^4 + 220*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 924*a^12*b^12 + 792*a^10
*b^14 + 495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 12*a^2*b^22 + b^24)*d^4
)))*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*
b^10 + b^12 - 2*(3*a^17*b + 8*a^15*b^3 - 12*a^13*b^5 - 72*a^11*b^7 - 110*a^
9*b^9 - 72*a^7*b^11 - 12*a^5*b^13 + 8*a^3*b^15 + 3*a*b^17)*d^2*sqrt(1/((a^1
2 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*
d^4)))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a
^2*b^10 + b^12))*sqrt(((a^18 - 27*a^16*b^2 + 168*a^14*b^4 + 224*a^12*b^6 -
366*a^10*b^8 - 366*a^8*b^10 + 224*a^6*b^12 + 168*a^4*b^14 - 27*a^2*b^16 + b
^18)*d^2*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8
+ 6*a^2*b^10 + b^12)*d^4))*cos(d*x + c) + sqrt(2)*((3*a^20*b - 82*a^18*b^3
+ 531*a^16*b^5 + 504*a^14*b^7 - 1322*a^12*b^9 - 732*a^10*b^11 + 1038*a^8*b^
13 + 280*a^6*b^15 - 249*a^4*b^17 + 30*a^2*b^19 - b^21)*d^3*sqrt(1/((a^12 +
6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)
)*cos(d*x + c) + (a^15 - 33*a^13*b^2 + 345*a^11*b^4 - 1217*a^9*b^6 + 1611*a
^7*b^8 - 795*a^5*b^10 + 91*a^3*b^12 - 3*a*b^14)*d*cos(d*x + c))*sqrt((a^12
+ 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 - 2
*(3*a^17*b + 8*a^15*b^3 - 12*a^13*b^5 - 72*a^11*b^7 - 110*a^9*b^9 - 72*a^7*
b^11 - 12*a^5*b^13 + 8*a^3*b^15 + 3*a*b^17)*d^2*sqrt(1/((a^12 + 6*a^10*b^2
+ 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))/(a^12 -
30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)
)*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*
a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))^(1/4) + (a^12 - 30*a^10*b^2
 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)*sin(d*x +
c))/cos(d*x + c))*(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4
*b^8 + 6*a^2*b^10 + b^12)*d^4))^(3/4) + sqrt(2)*((a^33 - 6*a^31*b^2 - 90*a^
29*b^4 - 294*a^27*b^6 - 54*a^25*b^8 + 2082*a^23...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(3/2)/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**(3/2)/(a + b*tan(c + d*x))**3, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 9.16, size = 2500, normalized size = 6.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(3/2)/(a + b*tan(c + d*x))^3,x)
```

```
[Out] atan((((((518*a*b^15*d^2 - 18*a^15*b*d^2 - 4494*a^3*b^13*d^2 + 3022*a^5*b^1
1*d^2 + 17194*a^7*b^9*d^2 + 5298*a^9*b^7*d^2 - 3338*a^11*b^5*d^2 + 506*a^13
*b^3*d^2)/(2*(a^16*d^5 + b^16*d^5 + 8*a^2*b^14*d^5 + 28*a^4*b^12*d^5 + 56*a
^6*b^10*d^5 + 70*a^8*b^8*d^5 + 56*a^10*b^6*d^5 + 28*a^12*b^4*d^5 + 8*a^14*b
^2*d^5)) + (((((4224*a^4*b^18*d^4 - 320*a^2*b^20*d^4 - 192*b^22*d^4 + 22272
*a^6*b^16*d^4 + 51072*a^8*b^14*d^4 + 67200*a^10*b^12*d^4 + 53760*a^12*b^10*
d^4 + 25344*a^14*b^8*d^4 + 5952*a^16*b^6*d^4 + 192*a^18*b^4*d^4 - 128*a^20*
b^2*d^4)/(2*(a^16*d^5 + b^16*d^5 + 8*a^2*b^14*d^5 + 28*a^4*b^12*d^5 + 56*a^
6*b^10*d^5 + 70*a^8*b^8*d^5 + 56*a^10*b^6*d^5 + 28*a^12*b^4*d^5 + 8*a^14*b^
2*d^5)) - (tan(c + d*x)^(1/2)*(1/(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6
*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i))^(1/2)*(51
2*b^25*d^4 + 4608*a^2*b^23*d^4 + 17920*a^4*b^21*d^4 + 38400*a^6*b^19*d^4 +
46080*a^8*b^17*d^4 + 21504*a^10*b^15*d^4 - 21504*a^12*b^13*d^4 - 46080*a^14
*b^11*d^4 - 38400*a^16*b^9*d^4 - 17920*a^18*b^7*d^4 - 4608*a^20*b^5*d^4 - 5
12*a^22*b^3*d^4))/(4*(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^12*d^
4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d^4 +
8*a^14*b^2*d^4)))*(1/(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 -
```

a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i))^(1/2))/2 + (tan(c + d*x)^(1/2)*(1544*a*b^18*d^2 + 64*a^3*b^16*d^2 - 7456*a^5*b^14*d^2 - 576*a^7*b^12*d^2 + 19504*a^9*b^10*d^2 + 18880*a^11*b^8*d^2 + 3808*a^13*b^6*d^2 - 960*a^15*b^4*d^2 + 8*a^17*b^2*d^2))/(2*(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d^4 + 8*a^14*b^2*d^4)))*(1/(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i))^(1/2))/2)*(1/(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i))^(1/2))/2 - (tan(c + d*x)^(1/2)*(9*a^12*b + 41*b^13 - 82*a^2*b^11 + 1831*a^4*b^9 - 4348*a^6*b^7 + 1671*a^8*b^5 - 210*a^10*b^3))/(2*(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d^4 + 8*a^14*b^2*d^4)))*(1/(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i))^(1/2)*1i - ((((518*a*b^15*d^2 - 18*a^15*b*d^2 - 4494*a^3*b^13*d^2 + 3022*a^5*b^11*d^2 + 17194*a^7*b^9*d^2 + 5298*a^9*b^7*d^2 - 3338*a^11*b^5*d^2 + 506*a^13*b^3*d^2)/(2*(a^16*d^5 + b^16*d^5 + 8*a^2*b^14*d^5 + 28*a^4*b^12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 + 56*a^10*b^6*d^5 + 28*a^12*b^4*d^5 + 8*a^14*b^2*d^5)) + (((((4224*a^4*b^18*d^4 - 320*a^2*b^20*d^4 - 192*b^22*d^4 + 22272*a^6*b^16*d^4 + 51072*a^8*b^14*d^4 + 67200*a^10*b^12*d^4 + 53760*a^12*b^10*d^4 + 25344*a^14*b^8*d^4 + 5952*a^16*b^6*d^4 + 192*a^18*b^4*d^4 - 128*a^20*b^2*d^4)/(2*(a^16*d^5 + b^16*d^5 + 8*a^2*b^14*d^5 + 28*a^4*b^12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 + 56*a^10*b^6*d^5 + 28*a^12*b^4*d^5 + 8*a^14*b^2*d^5)) + (tan(c + d*x)^(1/2)*(1/(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i))^(1/2)*(512*b^25*d^4 + 4608*a^2*b^23*d^4 + 17920*a^4*b^21*d^4 + 38400*a^6*b^19*d^4 + 46080*a^8*b^17*d^4 + 21504*a^10*b^15*d^4 - 21504*a^12*b^13*d^4 - 46080*a^14*b^11*d^4 - 38400*a^16*b^9*d^4 - 17920*a^18*b^7*d^4 - 4608*a^20*b^5*d^4 - 512*a^22*b^3*d^4))/(4*(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d^4 + 8*a^14*b^2*d^4)))*(1/(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i))^(1/2))/2 - (tan(c + d*x)^(1/2)*(1544*a*b^18*d^2 + 64*a^3*b^16*d^2 - 7456*a^5*b^14*d^2 - 576*a^7*b^12*d^2 + 19504*a^9*b^10*d^2 + 18880*a^11*b^8*d^2 + 3808*a^13*b^6*d^2 - 960*a^15*b^4*d^2 + 8*a^17*b^2*d^2))/(2*(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d^4 + 8*a^14*b^2*d^4)))*(1/(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i))^(1/2))/2)*(1/(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i))^(1/2))/2 + (tan(c + d*x)^(1/2)*(9*a^12*b + 41*b^13 - 82*a^2*b^11 + 1831*a^4*b^9 - 4348*a^6*b^7 + 1671*a^8*b^5 - 210*a^10*b^3))/(2*(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d^4 + 8*a^14*b^2*d^4)))*(1/(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i))^(1/2)*1i)/(((((518*a*b^15*d^2 - 18*a^

```
15*b*d^2 - 4494*a^3*b^13*d^2 + 3022*a^5*b^11*d^2 + 17194*a^7*b^9*d^2 + 5298
*a^9*b^7*d^2 - 3338*a^11*b^5*d^2 + 506*a^13*b^3*d^2)/(2*(a^16*d^5 + b^16*d^
5 + 8*a^2*b^14*d^5 + 28*a^4*b^12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 + 5
6*a^10*b^6*d^5 + 28*a^12*b^4*d^5 + 8*a^14*b^2*d^5)) + (((((4224*a^4*b^18*d^
4 - 320*a^2*b^20*d^4 - 192*b^22*d^4 + 22272*a^6...
```

3.604 $\int \frac{\sqrt{\tan(c+dx)}}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=389

$$-\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 d}+\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan\right.}}{\sqrt{2}\left(a^2+b^2\right)^3 d}$$

```
[Out] 1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)^3/d
*2^(1/2)+1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+
b^2)^3/d*2^(1/2)+1/4*(a+b)*(a^2-4*a*b+b^2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+ta
n(d*x+c))/(a^2+b^2)^3/d*2^(1/2)-1/4*(a+b)*(a^2-4*a*b+b^2)*ln(1+2^(1/2)*tan(
d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)^3/d*2^(1/2)-1/4*(15*a^4-18*a^2*b^2-b^4)*
arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))*b^(1/2)/a^(3/2)/(a^2+b^2)^3/d-1/2*
b*tan(d*x+c)^(1/2)/(a^2+b^2)/d/(a+b*tan(d*x+c))^2-1/4*b*(7*a^2-b^2)*tan(d*x
+c)^(1/2)/a/(a^2+b^2)^2/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.53, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3649, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\left(a^2+b^2\right)^3}+\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d\left(a^2+b^2\right)^3}-\frac{b\left(7a^2-b^2\right)\sqrt{\tan(c+dx)}}{4ad\left(a^2+b^2\right)^2(a+b\tan(c+dx))}-\frac{b\sqrt{\tan(c+dx)}}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}+\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d\left(a^2+b^2\right)^3}-\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d\left(a^2+b^2\right)^3}-\frac{\sqrt{b}\left(15a^4-18a^2b^2-b^4\right)\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{3/2}d\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[c + d*x]]/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -(((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqr
t[2]*(a^2 + b^2)^3*d)) + ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 + Sqrt[2]*Sq
rt[Tan[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)^3*d) - (Sqrt[b]*(15*a^4 - 18*a^2*b^
2 - b^4)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(4*a^(3/2)*(a^2 + b^
2)^3*d) + ((a + b)*(a^2 - 4*a*b + b^2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] +
 Tan[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^3*d) - ((a + b)*(a^2 - 4*a*b + b^2)*
Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^
3*d) - (b*Sqrt[Tan[c + d*x]])/(2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) - (b
*(7*a^2 - b^2)*Sqrt[Tan[c + d*x]])/(4*a*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x]
))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
```

```
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
```

```
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{\tan(c+dx)}}{(a+b\tan(c+dx))^3}\,dx = -\frac{b\sqrt{\tan(c+dx)}}{2(a^2+b^2)d(a+b\tan(c+dx))^2} - \frac{\int \frac{-\frac{b}{2}-2a\tan(c+dx)+\frac{3}{2}b\tan^2(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}\,dx}{2(a^2+b^2)}$$

$$= -\frac{b\sqrt{\tan(c+dx)}}{2(a^2+b^2)d(a+b\tan(c+dx))^2} - \frac{b(7a^2-b^2)\sqrt{\tan(c+dx)}}{4a(a^2+b^2)^2d(a+b\tan(c+dx))} - \frac{\int \frac{-\frac{1}{4}\cdots}{\cdots}}{\cdots}$$

$$= -\frac{b\sqrt{\tan(c+dx)}}{2(a^2+b^2)d(a+b\tan(c+dx))^2} - \frac{b(7a^2-b^2)\sqrt{\tan(c+dx)}}{4a(a^2+b^2)^2d(a+b\tan(c+dx))} - \frac{\int \frac{-2a\cdots}{\cdots}}{\cdots}$$

$$= -\frac{b\sqrt{\tan(c+dx)}}{2(a^2+b^2)d(a+b\tan(c+dx))^2} - \frac{b(7a^2-b^2)\sqrt{\tan(c+dx)}}{4a(a^2+b^2)^2d(a+b\tan(c+dx))} - \frac{\text{Subs}\cdots}{\cdots}$$

$$= -\frac{b\sqrt{\tan(c+dx)}}{2(a^2+b^2)d(a+b\tan(c+dx))^2} - \frac{b(7a^2-b^2)\sqrt{\tan(c+dx)}}{4a(a^2+b^2)^2d(a+b\tan(c+dx))} - \frac{((a+\cdots}{\cdots}$$

$$= -\frac{\sqrt{b}(15a^4-18a^2b^2-b^4)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{3/2}(a^2+b^2)^3d} - \frac{b\sqrt{\tan(c+\cdots}}{2(a^2+b^2)d(a+b\,\text{t}\cdots}$$

$$= -\frac{\sqrt{b}(15a^4-18a^2b^2-b^4)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{3/2}(a^2+b^2)^3d} + \frac{(a+b)(a^2-4ab+\cdots}{\cdots}$$

$$= -\frac{(a-b)(a^2+4ab+b^2)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}(a^2+b^2)^3d} + \frac{(a-b)(a^2+4ab+\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.52, size = 259, normalized size = 0.67

$$-\frac{-\frac{b^{3/2}(-15a^4+18a^2b^2+b^4)\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}(a^2+b^2)^2} + \frac{4\sqrt[4]{-1}ab\left((ia-b)^3\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)-(ia+b)^3\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\right)}{(a^2+b^2)^2} - \frac{2b^3\tan^{\frac{3}{2}}(c+dx)}{(a+b\tan(c+dx))^2} + \frac{2b^2\sqrt{\tan(c+dx)}}{a+b\tan(c+dx)} + \frac{(7a^2b^2-b^4)\sqrt{\tan(c+dx)}}{(a^2+b^2)(a+b\tan(c+dx))}}{4ab(a^2+b^2)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tan[c + d*x]]/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -1/4*(-((b^(3/2)*(-15*a^4 + 18*a^2*b^2 + b^4)*ArcTan[(Sqrt[b]*Sqrt[Tan[c +
d*x]])/Sqrt[a]])/(Sqrt[a]*(a^2 + b^2)^2)) + (4*(-1)^(1/4)*a*b*((I*a - b)^3*
ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] - (I*a + b)^3*ArcTanh[(-1)^(3/4)*Sqrt
[Tan[c + d*x]]]))/(a^2 + b^2)^2 - (2*b^3*Tan[c + d*x]^(3/2))/(a + b*Tan[c +
 d*x])^2 + (2*b^2*Sqrt[Tan[c + d*x]])/(a + b*Tan[c + d*x]) + ((7*a^2*b^2 -
b^4)*Sqrt[Tan[c + d*x]])/((a^2 + b^2)*(a + b*Tan[c + d*x])))/(a*b*(a^2 + b^
2)*d)
```

Maple [A]

time = 0.17, size = 343, normalized size = 0.88

method	result
derivativedivides	$-\frac{2b\left(\frac{\frac{b\left(7a^4+6a^2b^2-b^4\right)\left(\tan^{\frac{3}{2}}(dx+c)\right)}{8a}+\left(\frac{9}{8}a^4+\frac{5}{4}a^2b^2+\frac{1}{8}b^4\right)\left(\sqrt{\tan}(dx+c)\right)}{(a+b\tan(dx+c))^2}+\frac{\left(15a^4-18a^2b^2-b^4\right)\arctan\left(\frac{b\left(\sqrt{\tan}(dx\right.}{\sqrt{ab}}\right.}{8a\sqrt{ab}}\right.}{\left(a^2+b^2\right)^3}$
default	$-\frac{2b\left(\frac{\frac{b\left(7a^4+6a^2b^2-b^4\right)\left(\tan^{\frac{3}{2}}(dx+c)\right)}{8a}+\left(\frac{9}{8}a^4+\frac{5}{4}a^2b^2+\frac{1}{8}b^4\right)\left(\sqrt{\tan}(dx+c)\right)}{(a+b\tan(dx+c))^2}+\frac{\left(15a^4-18a^2b^2-b^4\right)\arctan\left(\frac{b\left(\sqrt{\tan}(dx\right.}{\sqrt{ab}}\right.}{8a\sqrt{ab}}\right.}{\left(a^2+b^2\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*b/(a^2+b^2)^3*((1/8*b*(7*a^4+6*a^2*b^2-b^4)/a*tan(d*x+c)^(3/2)+(9/8
*a^4+5/4*a^2*b^2+1/8*b^4)*tan(d*x+c)^(1/2))/(a+b*tan(d*x+c))^2+1/8*(15*a^4-
18*a^2*b^2-b^4)/a/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2)))+2/(a^
2+b^2)^3*(1/8*(3*a^2*b-b^3)*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x
+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)
^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/8*(a^3-3*a*b^2)*2^(1/2)*(l
n((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d
*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)
^(1/2)))))
```

Maxima [A]

time = 0.50, size = 411, normalized size = 1.06

$$-\frac{\frac{\left(15a^4b-18a^2b^3-b^5\right)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{\left(a^7+3a^5b^2+3a^3b^4+ab^6\right)\sqrt{ab}}-\frac{2\sqrt{2}\left(a^3+3a^2b-3ab^2-b^3\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}\left(a^3+3a^2b-3ab^2-b^3\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-\sqrt{2}\left(a^3-3a^2b-3ab^2+b^3\right)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+\sqrt{2}\left(a^3-3a^2b-3ab^2+b^3\right)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{\left(7a^2b^2-b^4\right)\tan(dx+c)^{\frac{3}{2}}+\left(9a^3b+ab^3\right)\sqrt{\tan(dx+c)}}{a^7+2a^5b^2+a^3b^4+\left(a^5b^2+2a^3b^4+ab^6\right)\tan(dx+c)^2+2\left(a^6b+2a^4b^3+a^2b^5\right)\tan(dx+c)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*((15*a^4*b - 18*a^2*b^3 - b^5)*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/
((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*sqrt(a*b)) - (2*sqrt(2)*(a^3 + 3*a^2
*b - 3*a*b^2 - b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) +
2*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*
sqrt(tan(d*x + c)))) - sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*log(sqrt(2)*
sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 +
 b^3)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6) + ((7*a^2*b^2 - b^4)*tan(d*x + c)^(3/2) + (9*a^3*b + a*
b^3)*sqrt(tan(d*x + c)))/(a^7 + 2*a^5*b^2 + a^3*b^4 + (a^5*b^2 + 2*a^3*b^4
+ a*b^6)*tan(d*x + c)^2 + 2*(a^6*b + 2*a^4*b^3 + a^2*b^5)*tan(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11386 vs. $2(341) = 682$.
time = 14.06, size = 22777, normalized size = 58.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*sqrt(2)*((a^23 + 3*a^21*b^2 - 17*a^19*b^4 - 123*a^17*b^6 - 342*a^
15*b^8 - 546*a^13*b^10 - 546*a^11*b^12 - 342*a^9*b^14 - 123*a^7*b^16 - 17*a
^5*b^18 + 3*a^3*b^20 + a*b^22)*d^5*cos(d*x + c)^4 + 2*(3*a^21*b^2 + 26*a^19
*b^4 + 99*a^17*b^6 + 216*a^15*b^8 + 294*a^13*b^10 + 252*a^11*b^12 + 126*a^9
*b^14 + 24*a^7*b^16 - 9*a^5*b^18 - 6*a^3*b^20 - a*b^22)*d^5*cos(d*x + c)^2
+ (a^19*b^4 + 9*a^17*b^6 + 36*a^15*b^8 + 84*a^13*b^10 + 126*a^11*b^12 + 126
*a^9*b^14 + 84*a^7*b^16 + 36*a^5*b^18 + 9*a^3*b^20 + a*b^22)*d^5 + 4*((a^22
*b + 8*a^20*b^3 + 27*a^18*b^5 + 48*a^16*b^7 + 42*a^14*b^9 - 42*a^10*b^13 -
48*a^8*b^15 - 27*a^6*b^17 - 8*a^4*b^19 - a^2*b^21)*d^5*cos(d*x + c)^3 + (a^
20*b^3 + 9*a^18*b^5 + 36*a^16*b^7 + 84*a^14*b^9 + 126*a^12*b^11 + 126*a^10*
b^13 + 84*a^8*b^15 + 36*a^6*b^17 + 9*a^4*b^19 + a^2*b^21)*d^5*cos(d*x + c))
*sin(d*x + c))*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b
^8 + 6*a^2*b^10 + b^12 + 2*(3*a^17*b + 8*a^15*b^3 - 12*a^13*b^5 - 72*a^11*b
^7 - 110*a^9*b^9 - 72*a^7*b^11 - 12*a^5*b^13 + 8*a^3*b^15 + 3*a*b^17)*d^2*s
qrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^
10 + b^12)*d^4)))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4
*b^8 - 30*a^2*b^10 + b^12))*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^
6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 12*a^22*b^2 + 66*a^20*b^
4 + 220*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 924*a^12*b^12 + 792*a^10*
b^14 + 495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 12*a^2*b^22 + b^24)*d^4)
)*(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^1
0 + b^12)*d^4))^(3/4)*arctan(((a^24 - 6*a^22*b^2 - 84*a^20*b^4 - 322*a^18*b
```

^6 - 603*a^16*b^8 - 540*a^14*b^10 + 540*a^10*b^14 + 603*a^8*b^16 + 322*a^6*
b^18 + 84*a^4*b^20 + 6*a^2*b^22 - b^24)*d^4*sqrt((a^12 - 30*a^10*b^2 + 255*
a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 12*a^22*
b^2 + 66*a^20*b^4 + 220*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 924*a^12*
b^12 + 792*a^10*b^14 + 495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 12*a^2*b
^22 + b^24)*d^4))*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15
*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) - sqrt(2)*((3*a^26*b + 35*a^24*b^3 + 18
6*a^22*b^5 + 594*a^20*b^7 + 1265*a^18*b^9 + 1881*a^16*b^11 + 1980*a^14*b^13
 + 1452*a^12*b^15 + 693*a^10*b^17 + 165*a^8*b^19 - 22*a^6*b^21 - 30*a^4*b^2
3 - 9*a^2*b^25 - b^27)*d^7*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6
*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 12*a^22*b^2 + 66*a^20*b^4
 + 220*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 924*a^12*b^12 + 792*a^10*b
^14 + 495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 12*a^2*b^22 + b^24)*d^4))
*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*
b^10 + b^12)*d^4)) - (a^21 + 6*a^19*b^2 + 9*a^17*b^4 - 24*a^15*b^6 - 126*a^
13*b^8 - 252*a^11*b^10 - 294*a^9*b^12 - 216*a^7*b^14 - 99*a^5*b^16 - 26*a^3
*b^18 - 3*a*b^20)*d^5*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6
+ 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 12*a^22*b^2 + 66*a^20*b^4 + 22
0*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 924*a^12*b^12 + 792*a^10*b^14 +
 495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 12*a^2*b^22 + b^24)*d^4)))*sqr
t((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 +
b^12 + 2*(3*a^17*b + 8*a^15*b^3 - 12*a^13*b^5 - 72*a^11*b^7 - 110*a^9*b^9 -
 72*a^7*b^11 - 12*a^5*b^13 + 8*a^3*b^15 + 3*a*b^17)*d^2*sqrt(1/((a^12 + 6*a
^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))/
(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10
 + b^12))*sqrt(((a^18 - 27*a^16*b^2 + 168*a^14*b^4 + 224*a^12*b^6 - 366*a^1
0*b^8 - 366*a^8*b^10 + 224*a^6*b^12 + 168*a^4*b^14 - 27*a^2*b^16 + b^18)*d^
2*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2
*b^10 + b^12)*d^4))*cos(d*x + c) + sqrt(2)*((a^21 - 30*a^19*b^2 + 249*a^17*
b^4 - 280*a^15*b^6 - 1038*a^13*b^8 + 732*a^11*b^10 + 1322*a^9*b^12 - 504*a^
7*b^14 - 531*a^5*b^16 + 82*a^3*b^18 - 3*a*b^20)*d^3*sqrt(1/((a^12 + 6*a^10*
b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*cos(d
*x + c) - (3*a^14*b - 91*a^12*b^3 + 795*a^10*b^5 - 1611*a^8*b^7 + 1217*a^6*
b^9 - 345*a^4*b^11 + 33*a^2*b^13 - b^15)*d*cos(d*x + c))*sqrt((a^12 + 6*a^1
0*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 + 2*(3*a^1
7*b + 8*a^15*b^3 - 12*a^13*b^5 - 72*a^11*b^7 - 110*a^9*b^9 - 72*a^7*b^11 -
12*a^5*b^13 + 8*a^3*b^15 + 3*a*b^17)*d^2*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^
8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))/(a^12 - 30*a^10
*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*sqrt(
sin(d*x + c)/cos(d*x + c))*(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6
 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))^(1/4) + (a^12 - 30*a^10*b^2 + 255*
a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)*sin(d*x + c))/cos
(d*x + c))*(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 +
6*a^2*b^10 + b^12)*d^4))^(3/4) - sqrt(2)*((3*a^32*b - 10*a^30*b^3 - 294*a^2
8*b^5 - 1674*a^26*b^7 - 4890*a^24*b^9 - 8370*a^...

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\tan(c+dx)}}{(a+b\tan(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/2)/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(sqrt(tan(c + d*x))/(a + b*tan(c + d*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(tan(d*x + c))/(b*tan(d*x + c) + a)^3, x)
```

Mupad [B]
time = 16.56, size = 2500, normalized size = 6.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)/(a + b*tan(c + d*x))^3,x)
```

```
[Out] atan(((-1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4
*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*((-1i/(4*(b^6*d^2 - a^6*d^
2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4
*b^2*d^2)))^(1/2)*(((-1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6
i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*((64*a*b^23*
d^4 + 1472*a^3*b^21*d^4 + 8832*a^5*b^19*d^4 + 25344*a^7*b^17*d^4 + 40320*a^
9*b^15*d^4 + 34944*a^11*b^13*d^4 + 10752*a^13*b^11*d^4 - 8448*a^15*b^9*d^4
- 10176*a^17*b^7*d^4 - 4160*a^19*b^5*d^4 - 640*a^21*b^3*d^4)/(a^18*d^5 + a^
2*b^16*d^5 + 8*a^4*b^14*d^5 + 28*a^6*b^12*d^5 + 56*a^8*b^10*d^5 + 70*a^10*b
^8*d^5 + 56*a^12*b^6*d^5 + 28*a^14*b^4*d^5 + 8*a^16*b^2*d^5) + (tan(c + d*x
)^(1/2)*(-1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b
^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*(512*a^2*b^25*d^4 + 4608
*a^4*b^23*d^4 + 17920*a^6*b^21*d^4 + 38400*a^8*b^19*d^4 + 46080*a^10*b^17*d
^4 + 21504*a^12*b^15*d^4 - 21504*a^14*b^13*d^4 - 46080*a^16*b^11*d^4 - 3840
0*a^18*b^9*d^4 - 17920*a^20*b^7*d^4 - 4608*a^22*b^5*d^4 - 512*a^24*b^3*d^4)
)/(a^18*d^4 + a^2*b^16*d^4 + 8*a^4*b^14*d^4 + 28*a^6*b^12*d^4 + 56*a^8*b^10
```

*d^4 + 70*a^10*b^8*d^4 + 56*a^12*b^6*d^4 + 28*a^14*b^4*d^4 + 8*a^16*b^2*d^4
)) - (tan(c + d*x)^(1/2)*(8*a*b^20*d^2 - 1152*a^3*b^18*d^2 + 2528*a^5*b^16*
d^2 + 15296*a^7*b^14*d^2 + 14128*a^9*b^12*d^2 - 5056*a^11*b^10*d^2 - 9248*a
^13*b^8*d^2 + 64*a^15*b^6*d^2 + 1800*a^17*b^4*d^2 + 64*a^19*b^2*d^2))/(a^18
*d^4 + a^2*b^16*d^4 + 8*a^4*b^14*d^4 + 28*a^6*b^12*d^4 + 56*a^8*b^10*d^4 +
70*a^10*b^8*d^4 + 56*a^12*b^6*d^4 + 28*a^14*b^4*d^4 + 8*a^16*b^2*d^4))*(-1i
/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3
*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) - (2*b^18*d^2 - 138*a^2*b^16*d^2 - 3
046*a^4*b^14*d^2 + 4862*a^6*b^12*d^2 + 9222*a^8*b^10*d^2 - 5246*a^10*b^8*d^
2 - 4290*a^12*b^6*d^2 + 2442*a^14*b^4*d^2 + 32*a^16*b^2*d^2)/(a^18*d^5 + a^
2*b^16*d^5 + 8*a^4*b^14*d^5 + 28*a^6*b^12*d^5 + 56*a^8*b^10*d^5 + 70*a^10*b
^8*d^5 + 56*a^12*b^6*d^5 + 28*a^14*b^4*d^5 + 8*a^16*b^2*d^5)) + (tan(c + d*
x)^(1/2)*(2*a^2*b^13 - b^15 + 49*a^4*b^11 + 2460*a^6*b^9 - 3631*a^8*b^7 + 1
922*a^10*b^5 - 225*a^12*b^3))/(a^18*d^4 + a^2*b^16*d^4 + 8*a^4*b^14*d^4 + 2
8*a^6*b^12*d^4 + 56*a^8*b^10*d^4 + 70*a^10*b^8*d^4 + 56*a^12*b^6*d^4 + 28*a
^14*b^4*d^4 + 8*a^16*b^2*d^4))*1i - (-1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*
6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1
/2)*((-1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*
d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*(((-1i/(4*(b^6*d^2 - a^6*d^
2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4
*b^2*d^2)))^(1/2)*((64*a*b^23*d^4 + 1472*a^3*b^21*d^4 + 8832*a^5*b^19*d^4 +
25344*a^7*b^17*d^4 + 40320*a^9*b^15*d^4 + 34944*a^11*b^13*d^4 + 10752*a^13
*b^11*d^4 - 8448*a^15*b^9*d^4 - 10176*a^17*b^7*d^4 - 4160*a^19*b^5*d^4 - 64
0*a^21*b^3*d^4)/(a^18*d^5 + a^2*b^16*d^5 + 8*a^4*b^14*d^5 + 28*a^6*b^12*d^5
+ 56*a^8*b^10*d^5 + 70*a^10*b^8*d^5 + 56*a^12*b^6*d^5 + 28*a^14*b^4*d^5 +
8*a^16*b^2*d^5) - (tan(c + d*x)^(1/2)*(-1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^
2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^
(1/2)*(512*a^2*b^25*d^4 + 4608*a^4*b^23*d^4 + 17920*a^6*b^21*d^4 + 38400*a^
8*b^19*d^4 + 46080*a^10*b^17*d^4 + 21504*a^12*b^15*d^4 - 21504*a^14*b^13*d^
4 - 46080*a^16*b^11*d^4 - 38400*a^18*b^9*d^4 - 17920*a^20*b^7*d^4 - 4608*a^
22*b^5*d^4 - 512*a^24*b^3*d^4))/(a^18*d^4 + a^2*b^16*d^4 + 8*a^4*b^14*d^4 +
28*a^6*b^12*d^4 + 56*a^8*b^10*d^4 + 70*a^10*b^8*d^4 + 56*a^12*b^6*d^4 + 28
*a^14*b^4*d^4 + 8*a^16*b^2*d^4)) + (tan(c + d*x)^(1/2)*(8*a*b^20*d^2 - 1152
*a^3*b^18*d^2 + 2528*a^5*b^16*d^2 + 15296*a^7*b^14*d^2 + 14128*a^9*b^12*d^2
- 5056*a^11*b^10*d^2 - 9248*a^13*b^8*d^2 + 64*a^15*b^6*d^2 + 1800*a^17*b^4
*d^2 + 64*a^19*b^2*d^2))/(a^18*d^4 + a^2*b^16*d^4 + 8*a^4*b^14*d^4 + 28*a^6
*b^12*d^4 + 56*a^8*b^10*d^4 + 70*a^10*b^8*d^4 + 56*a^12*b^6*d^4 + 28*a^14*b
^4*d^4 + 8*a^16*b^2*d^4))*(-1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b
*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) - (2*b
^18*d^2 - 138*a^2*b^16*d^2 - 3046*a^4*b^14*d^2 + 4862*a^6*b^12*d^2 + 9222*a
^8*b^10*d^2 - 5246*a^10*b^8*d^2 - 4290*a^12*b^6*d^2 + 2442*a^14*b^4*d^2 + 3
2*a^16*b^2*d^2)/(a^18*d^5 + a^2*b^16*d^5 + 8*a^4*b^14*d^5 + 28*a^6*b^12*d^5
+ 56*a^8*b^10*d^5 + 70*a^10*b^8*d^5 + 56*a^12*b^6*d^5 + 28*a^14*b^4*d^5 +
8*a^16*b^2*d^5)) - (tan(c + d*x)^(1/2)*(2*a^2*b^13 - b^15 + 49*a^4*b^11 + 2
460*a^6*b^9 - 3631*a^8*b^7 + 1922*a^10*b^5 - 225*a^12*b^3))/(a^18*d^4 + a^2

```
*b^16*d^4 + 8*a^4*b^14*d^4 + 28*a^6*b^12*d^4 + 56*a^8*b^10*d^4 + 70*a^10*b^
8*d^4 + 56*a^12*b^6*d^4 + 28*a^14*b^4*d^4 + 8*a^16*b^2*d^4))*1i)/((7*a*b^11
 + 116*a^3*b^9 - 270*a^5*b^7 + 420*a^7*b^5 - 225*a^9*b^3)/(a^18*d^5 + a^2*b
^16*d^5 + 8*a^4*b^14*d^5 + 28*a^6*b^12*d^5 + 56*a^8*b^10*d^5 + 70*a^10*b^8*
d^5 + 56*a^12*b^6*d^5 + 28*a^14*b^4*d^5 + 8*a^16*b^2*d^5) + (-1i/(4*(b^6*d^
2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*...
```

3.605 $\int \frac{1}{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=396

$$-\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,\left(a^2+b^2\right)^3 d}+\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan\right.}$$

```
[Out] 1/4*b^(3/2)*(35*a^4+6*a^2*b^2+3*b^4)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2
))/a^(5/2)/(a^2+b^2)^3/d+1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(-1+2^(1/2)*tan(d*
x+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)+1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(1+2^(1/2
)*tan(d*x+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/4*(a-b)*(a^2+4*a*b+b^2)*ln(1-2^
(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)^3/d*2^(1/2)+1/4*(a-b)*(a^2+4*a
*b+b^2)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)^3/d*2^(1/2)+1/2
*b^2*tan(d*x+c)^(1/2)/a/(a^2+b^2)/d/(a+b*tan(d*x+c))^2+1/4*b^2*(11*a^2+3*b^
2)*tan(d*x+c)^(1/2)/a^2/(a^2+b^2)^2/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.57, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3650, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d\left(a^2+b^2\right)^3}+\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d\left(a^2+b^2\right)^3}+\frac{b^2\left(11a^2+3b^2\right)\sqrt{\tan(c+dx)}}{4a^2d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}+\frac{b^2\sqrt{\tan(c+dx)}}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\tan(c+dx)-\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)^3}+\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\tan(c+dx)+\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)^3}+\frac{b^{3/2}\left(35a^4+6a^2b^2+3b^4\right)\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{5/2}d\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^3),x]
```

```
[Out] -(((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqr
t[2]*(a^2 + b^2)^3*d)) + ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 + Sqrt[2]*Sq
rt[Tan[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)^3*d) + (b^(3/2)*(35*a^4 + 6*a^2*b^2
 + 3*b^4)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(4*a^(5/2)*(a^2 + b
^2)^3*d) - ((a - b)*(a^2 + 4*a*b + b^2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
+ Tan[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^3*d) + ((a - b)*(a^2 + 4*a*b + b^2)
*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)
^3*d) + (b^2*Sqrt[Tan[c + d*x]])/(2*a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2)
 + (b^2*(11*a^2 + 3*b^2)*Sqrt[Tan[c + d*x]])/(4*a^2*(a^2 + b^2)^2*d*(a + b*
Tan[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
```

```
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
```

```
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^3}\,dx = \frac{b^2\sqrt{\tan(c+dx)}}{2a\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} + \frac{\int \frac{\frac{1}{2}(4a^2+3b^2)-2ab\tan(c+dx)+\frac{3}{2}b^2 \ldots}{\sqrt{\tan(c+dx)}\,(a+b\tan(\ldots}}{2a\,(a^2+b^2)}$$

$$= \frac{b^2\sqrt{\tan(c+dx)}}{2a\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} + \frac{b^2(11a^2+3b^2)\sqrt{\tan(c+ \ldots}}{4a^2\,(a^2+b^2)^2\,d(a+b\tan(c \ldots}$$

$$= \frac{b^2\sqrt{\tan(c+dx)}}{2a\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} + \frac{b^2(11a^2+3b^2)\sqrt{\tan(c+ \ldots}}{4a^2\,(a^2+b^2)^2\,d(a+b\tan(c \ldots}$$

$$= \frac{b^2\sqrt{\tan(c+dx)}}{2a\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} + \frac{b^2(11a^2+3b^2)\sqrt{\tan(c+ \ldots}}{4a^2\,(a^2+b^2)^2\,d(a+b\tan(c \ldots}$$

$$= \frac{b^2\sqrt{\tan(c+dx)}}{2a\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} + \frac{b^2(11a^2+3b^2)\sqrt{\tan(c+ \ldots}}{4a^2\,(a^2+b^2)^2\,d(a+b\tan(c \ldots}$$

$$= \frac{b^{3/2}(35a^4+6a^2b^2+3b^4)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{5/2}\,(a^2+b^2)^3\,d} + \frac{\ldots}{2a\,(a^2 \ldots}$$

$$= \frac{b^{3/2}(35a^4+6a^2b^2+3b^4)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{5/2}\,(a^2+b^2)^3\,d} - \frac{(a-b) \ldots}{\ldots}$$

$$= -\frac{(a+b)\,(a^2-4ab+b^2)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d} + \frac{(a \ldots}{\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.86, size = 235, normalized size = 0.59

$$\frac{\frac{-4\sqrt[4]{-1}\,a^{5/2}(a+ib)^3\,\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+b^{3/2}(35a^4+6a^2b^2+3b^4)\,\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)-4\sqrt[4]{-1}\,a^{5/2}(a-ib)^3\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{a^{3/2}(a^2+b^2)^2}+\frac{2b^2\sqrt{\tan(c+dx)}}{(a+b\tan(c+dx))^2}+\frac{(11a^2b^2+3b^4)\sqrt{\tan(c+dx)}}{a(a^2+b^2)(a+b\tan(c+dx))}}{4a\,(a^2+b^2)\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^3),x]
```

```
[Out] ((-4*(-1)^(1/4)*a^(5/2)*(a + I*b)^3*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] +
 b^(3/2)*(35*a^4 + 6*a^2*b^2 + 3*b^4)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/S
qrt[a]] - 4*(-1)^(1/4)*a^(5/2)*(a - I*b)^3*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c +
d*x]]])/(a^(3/2)*(a^2 + b^2)^2) + (2*b^2*Sqrt[Tan[c + d*x]])/(a + b*Tan[c +
 d*x])^2 + ((11*a^2*b^2 + 3*b^4)*Sqrt[Tan[c + d*x]])/(a*(a^2 + b^2)*(a + b*
Tan[c + d*x])))/(4*a*(a^2 + b^2)*d)
```

Maple [A]
time = 0.16, size = 347, normalized size = 0.88

method	result
derivativedivides	$\frac{2b^2\left(\frac{\frac{b\left(11a^4+14a^2b^2+3b^4\right)\left(\tan^{\frac{3}{2}}(dx+c)\right)}{8a^2}+\frac{\left(13a^4+18a^2b^2+5b^4\right)\left(\sqrt{\tan}(dx+c)\right)}{8a}}{(a+b\tan(dx+c))^2}+\frac{\left(35a^4+6a^2b^2+3b^4\right)\arctan\left(\frac{b\left(\sqrt{\tan}\cdots\right.}{\sqrt{a\cdots}}\right.}{8a^2\sqrt{ab}}\cdots\right.}{\left(a^2+b^2\right)^3}\cdots$
default	$\frac{2b^2\left(\frac{\frac{b\left(11a^4+14a^2b^2+3b^4\right)\left(\tan^{\frac{3}{2}}(dx+c)\right)}{8a^2}+\frac{\left(13a^4+18a^2b^2+5b^4\right)\left(\sqrt{\tan}(dx+c)\right)}{8a}}{(a+b\tan(dx+c))^2}+\frac{\left(35a^4+6a^2b^2+3b^4\right)\arctan\left(\frac{b\left(\sqrt{\tan}\cdots\right.}{\sqrt{a\cdots}}\right.}{8a^2\sqrt{ab}}\cdots\right.}{\left(a^2+b^2\right)^3}\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b^2/(a^2+b^2)^3*((1/8*b*(11*a^4+14*a^2*b^2+3*b^4)/a^2*tan(d*x+c)^(3/
2)+1/8*(13*a^4+18*a^2*b^2+5*b^4)/a*tan(d*x+c)^(1/2))/(a+b*tan(d*x+c))^2+1/8
*(35*a^4+6*a^2*b^2+3*b^4)/a^2/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(
1/2)))+2/(a^2+b^2)^3*(1/8*(a^3-3*a*b^2)*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(
1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2
)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/8*(-3*a^2*b+b^
3)*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c
)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/
2)*tan(d*x+c)^(1/2)))))
```

Maxima [A]
time = 0.52, size = 419, normalized size = 1.06

$$\frac{\frac{\left(35a^4b^2+6a^2b^4+3b^6\right)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{\left(a^8+3a^6b^2+3a^4b^4+a^2b^6\right)\sqrt{ab}}+\frac{2\sqrt{2}\left(a^3-3a^2b-3ab^2+b^3\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}\left(a^3-3a^2b-3ab^2+b^3\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)+\sqrt{2}\left(a^3+3a^2b-3ab^2-b^3\right)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)-\sqrt{2}\left(a^3+3a^2b-3ab^2-b^3\right)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{\left(11a^2b^3+3b^5\right)\tan(dx+c)^{\frac{3}{2}}+\left(13a^3b^2+5ab^4\right)\sqrt{\tan(dx+c)}}{a^8+2a^6b^2+a^4b^4+\left(a^6b^2+2a^4b^4+a^2b^6\right)\tan(dx+c)^2+2\left(a^7b+2a^5b^3+a^3b^5\right)\tan(dx+c)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*((35*a^4*b^2 + 6*a^2*b^4 + 3*b^6)*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b)
)/((a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*sqrt(a*b)) + (2*sqrt(2)*(a^3 - 3
*a^2*b - 3*a*b^2 + b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))
) + 2*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*arctan(-1/2*sqrt(2)*(sqrt(2)
- 2*sqrt(tan(d*x + c)))) + sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*log(sqrt
(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*(a^3 + 3*a^2*b - 3*a*b
^2 - b^3)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a^6 + 3*a^4
*b^2 + 3*a^2*b^4 + b^6) + ((11*a^2*b^3 + 3*b^5)*tan(d*x + c)^(3/2) + (13*a^
3*b^2 + 5*a*b^4)*sqrt(tan(d*x + c)))/(a^8 + 2*a^6*b^2 + a^4*b^4 + (a^6*b^2
+ 2*a^4*b^4 + a^2*b^6)*tan(d*x + c)^2 + 2*(a^7*b + 2*a^5*b^3 + a^3*b^5)*tan
(d*x + c)))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 11419 vs. $2(348) = 696$.
time = 12.89, size = 22843, normalized size = 57.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*sqrt(2)*((a^24 + 3*a^22*b^2 - 17*a^20*b^4 - 123*a^18*b^6 - 342*a
^16*b^8 - 546*a^14*b^10 - 546*a^12*b^12 - 342*a^10*b^14 - 123*a^8*b^16 - 17
*a^6*b^18 + 3*a^4*b^20 + a^2*b^22)*d^5*cos(d*x + c)^4 + 2*(3*a^22*b^2 + 26*
a^20*b^4 + 99*a^18*b^6 + 216*a^16*b^8 + 294*a^14*b^10 + 252*a^12*b^12 + 126
*a^10*b^14 + 24*a^8*b^16 - 9*a^6*b^18 - 6*a^4*b^20 - a^2*b^22)*d^5*cos(d*x
+ c)^2 + (a^20*b^4 + 9*a^18*b^6 + 36*a^16*b^8 + 84*a^14*b^10 + 126*a^12*b^1
2 + 126*a^10*b^14 + 84*a^8*b^16 + 36*a^6*b^18 + 9*a^4*b^20 + a^2*b^22)*d^5
+ 4*((a^23*b + 8*a^21*b^3 + 27*a^19*b^5 + 48*a^17*b^7 + 42*a^15*b^9 - 42*a^
11*b^13 - 48*a^9*b^15 - 27*a^7*b^17 - 8*a^5*b^19 - a^3*b^21)*d^5*cos(d*x +
c)^3 + (a^21*b^3 + 9*a^19*b^5 + 36*a^17*b^7 + 84*a^15*b^9 + 126*a^13*b^11 +
 126*a^11*b^13 + 84*a^9*b^15 + 36*a^7*b^17 + 9*a^5*b^19 + a^3*b^21)*d^5*cos
(d*x + c))*sin(d*x + c))*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6
+ 15*a^4*b^8 + 6*a^2*b^10 + b^12 - 2*(3*a^17*b + 8*a^15*b^3 - 12*a^13*b^5 -
 72*a^11*b^7 - 110*a^9*b^9 - 72*a^7*b^11 - 12*a^5*b^13 + 8*a^3*b^15 + 3*a*b
^17)*d^2*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8
+ 6*a^2*b^10 + b^12)*d^4)))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6
 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^
4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 12*a^22*b^2 +
66*a^20*b^4 + 220*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 924*a^12*b^12 +
 792*a^10*b^14 + 495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 12*a^2*b^22 +
b^24)*d^4))*(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 +
```

6*a^2*b^10 + b^12)*d^4))^(3/4)*arctan(-((a^24 - 6*a^22*b^2 - 84*a^20*b^4 - 322*a^18*b^6 - 603*a^16*b^8 - 540*a^14*b^10 + 540*a^10*b^14 + 603*a^8*b^16 + 322*a^6*b^18 + 84*a^4*b^20 + 6*a^2*b^22 - b^24)*d^4*sqrt((a^12 - 30*a^10 *b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 12*a^22*b^2 + 66*a^20*b^4 + 220*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 924*a^12*b^12 + 792*a^10*b^14 + 495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 12*a^2*b^22 + b^24)*d^4))*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a ^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) + sqrt(2)*((a^27 + 9*a^25*b^ 2 + 30*a^23*b^4 + 22*a^21*b^6 - 165*a^19*b^8 - 693*a^17*b^10 - 1452*a^15*b^ 12 - 1980*a^13*b^14 - 1881*a^11*b^16 - 1265*a^9*b^18 - 594*a^7*b^20 - 186*a ^5*b^22 - 35*a^3*b^24 - 3*a*b^26)*d^7*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^ 4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 12*a^22*b^2 + 66*a^20*b^4 + 220*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 924*a^12*b^12 + 792*a^10*b^14 + 495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 12*a^2*b^22 + b^24)*d^4))*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b ^8 + 6*a^2*b^10 + b^12)*d^4)) + (3*a^20*b + 26*a^18*b^3 + 99*a^16*b^5 + 216 *a^14*b^7 + 294*a^12*b^9 + 252*a^10*b^11 + 126*a^8*b^13 + 24*a^6*b^15 - 9*a ^4*b^17 - 6*a^2*b^19 - b^21)*d^5*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 4 52*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 12*a^22*b^2 + 66*a^ 20*b^4 + 220*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 924*a^12*b^12 + 792* a^10*b^14 + 495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 12*a^2*b^22 + b^24) *d^4)))*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6* a^2*b^10 + b^12 - 2*(3*a^17*b + 8*a^15*b^3 - 12*a^13*b^5 - 72*a^11*b^7 - 11 0*a^9*b^9 - 72*a^7*b^11 - 12*a^5*b^13 + 8*a^3*b^15 + 3*a*b^17)*d^2*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^ 12)*d^4)))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*sqrt(((a^18 - 27*a^16*b^2 + 168*a^14*b^4 + 224*a^12*b^ 6 - 366*a^10*b^8 - 366*a^8*b^10 + 224*a^6*b^12 + 168*a^4*b^14 - 27*a^2*b^16 + b^18)*d^2*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4* b^8 + 6*a^2*b^10 + b^12)*d^4))*cos(d*x + c) + sqrt(2)*((3*a^20*b - 82*a^18* b^3 + 531*a^16*b^5 + 504*a^14*b^7 - 1322*a^12*b^9 - 732*a^10*b^11 + 1038*a^ 8*b^13 + 280*a^6*b^15 - 249*a^4*b^17 + 30*a^2*b^19 - b^21)*d^3*sqrt(1/((a^1 2 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)* d^4))*cos(d*x + c) + (a^15 - 33*a^13*b^2 + 345*a^11*b^4 - 1217*a^9*b^6 + 16 11*a^7*b^8 - 795*a^5*b^10 + 91*a^3*b^12 - 3*a*b^14)*d*cos(d*x + c))*sqrt((a ^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 - 2*(3*a^17*b + 8*a^15*b^3 - 12*a^13*b^5 - 72*a^11*b^7 - 110*a^9*b^9 - 72* a^7*b^11 - 12*a^5*b^13 + 8*a^3*b^15 + 3*a*b^17)*d^2*sqrt(1/((a^12 + 6*a^10* b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))/(a^1 2 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b ^12))*sqrt(sin(d*x + c)/cos(d*x + c))*(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))^(1/4) + (a^12 - 30*a^10 *b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)*sin(d* x + c))/cos(d*x + c))*(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15 *a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))^(3/4) + sqrt(2)*((a^33 - 6*a^31*b^2 - 9

```
0*a^29*b^4 - 294*a^27*b^6 - 54*a^25*b^8 + 2082*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\tan{(c + dx)})^3 \sqrt{\tan{(c + dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(1/2)/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(1/((a + b*tan(c + d*x))**3*sqrt(tan(c + d*x))), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 9.64, size = 2500, normalized size = 6.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(1/2)*(a + b*tan(c + d*x))^3),x)
```

```
[Out] ((tan(c + d*x)^(1/2)*(5*b^4 + 13*a^2*b^2))/(4*a*(a^4 + b^4 + 2*a^2*b^2)) +
(b*tan(c + d*x)^(3/2)*(3*b^4 + 11*a^2*b^2))/(4*a^2*(a^4 + b^4 + 2*a^2*b^2))
)/(a^2*d + b^2*d*tan(c + d*x)^2 + 2*a*b*d*tan(c + d*x)) - atan((((1i/(4*(b^
6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^
2*20i + 15*a^4*b^2*d^2)))^(1/2)*(((1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i
+ a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)
*((192*a^2*b^24*d^4 + 1728*a^4*b^22*d^4 + 8320*a^6*b^20*d^4 + 27264*a^8*b^1
8*d^4 + 62592*a^10*b^16*d^4 + 99456*a^12*b^14*d^4 + 107520*a^14*b^12*d^4 +
76800*a^16*b^10*d^4 + 33984*a^18*b^8*d^4 + 7872*a^20*b^6*d^4 + 384*a^22*b^4
*d^4 - 128*a^24*b^2*d^4)/(a^20*d^5 + a^4*b^16*d^5 + 8*a^6*b^14*d^5 + 28*a^8
*b^12*d^5 + 56*a^10*b^10*d^5 + 70*a^12*b^8*d^5 + 56*a^14*b^6*d^5 + 28*a^16*
b^4*d^5 + 8*a^18*b^2*d^5) - (tan(c + d*x)^(1/2)*(1i/(4*(b^6*d^2 - a^6*d^2 +
 a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^
2*d^2)))^(1/2)*(512*a^4*b^25*d^4 + 4608*a^6*b^23*d^4 + 17920*a^8*b^21*d^4 +
 38400*a^10*b^19*d^4 + 46080*a^12*b^17*d^4 + 21504*a^14*b^15*d^4 - 21504*a^
```

16*b^13*d^4 - 46080*a^18*b^11*d^4 - 38400*a^20*b^9*d^4 - 17920*a^22*b^7*d^4 - 4608*a^24*b^5*d^4 - 512*a^26*b^3*d^4))/(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^8*b^12*d^4 + 56*a^10*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16*b^4*d^4 + 8*a^18*b^2*d^4)) + (tan(c + d*x)^(1/2)*(72*a*b^22*d^2 + 576*a^3*b^20*d^2 + 5024*a^5*b^18*d^2 + 14272*a^7*b^16*d^2 + 27824*a^9*b^14*d^2 + 53184*a^11*b^12*d^2 + 70240*a^13*b^10*d^2 + 47680*a^15*b^8*d^2 + 12616*a^17*b^6*d^2 - 64*a^21*b^2*d^2))/(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^8*b^12*d^4 + 56*a^10*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16*b^4*d^4 + 8*a^18*b^2*d^4))*(1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) - (90*a*b^19*d^2 + 846*a^3*b^17*d^2 + 1714*a^5*b^15*d^2 + 3606*a^7*b^13*d^2 - 14578*a^9*b^11*d^2 - 34486*a^11*b^9*d^2 - 14970*a^13*b^7*d^2 + 2258*a^15*b^5*d^2 - 32*a^17*b^3*d^2)/(a^20*d^5 + a^4*b^16*d^5 + 8*a^6*b^14*d^5 + 28*a^8*b^12*d^5 + 56*a^10*b^10*d^5 + 70*a^12*b^8*d^5 + 56*a^14*b^6*d^5 + 28*a^16*b^4*d^5 + 8*a^18*b^2*d^5)) + (tan(c + d*x)^(1/2)*(18*a^2*b^15 - 9*b^17 - 71*a^4*b^13 + 892*a^6*b^11 + 857*a^8*b^9 + 6802*a^10*b^7 - 1257*a^12*b^5))/(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^8*b^12*d^4 + 56*a^10*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16*b^4*d^4 + 8*a^18*b^2*d^4))*(1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*1i - ((1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*(((1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*((192*a^2*b^24*d^4 + 1728*a^4*b^22*d^4 + 8320*a^6*b^20*d^4 + 27264*a^8*b^18*d^4 + 62592*a^10*b^16*d^4 + 99456*a^12*b^14*d^4 + 107520*a^14*b^12*d^4 + 76800*a^16*b^10*d^4 + 33984*a^18*b^8*d^4 + 7872*a^20*b^6*d^4 + 384*a^22*b^4*d^4 - 128*a^24*b^2*d^4)/(a^20*d^5 + a^4*b^16*d^5 + 8*a^6*b^14*d^5 + 28*a^8*b^12*d^5 + 56*a^10*b^10*d^5 + 70*a^12*b^8*d^5 + 56*a^14*b^6*d^5 + 28*a^16*b^4*d^5 + 8*a^18*b^2*d^5) + (tan(c + d*x)^(1/2)*(1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*(512*a^4*b^25*d^4 + 4608*a^6*b^23*d^4 + 17920*a^8*b^21*d^4 + 38400*a^10*b^19*d^4 + 46080*a^12*b^17*d^4 + 21504*a^14*b^15*d^4 - 21504*a^16*b^13*d^4 - 46080*a^18*b^11*d^4 - 38400*a^20*b^9*d^4 - 17920*a^22*b^7*d^4 - 4608*a^24*b^5*d^4 - 512*a^26*b^3*d^4))/(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^8*b^12*d^4 + 56*a^10*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16*b^4*d^4 + 8*a^18*b^2*d^4)) - (tan(c + d*x)^(1/2)*(72*a*b^22*d^2 + 576*a^3*b^20*d^2 + 5024*a^5*b^18*d^2 + 14272*a^7*b^16*d^2 + 27824*a^9*b^14*d^2 + 53184*a^11*b^12*d^2 + 70240*a^13*b^10*d^2 + 47680*a^15*b^8*d^2 + 12616*a^17*b^6*d^2 - 64*a^21*b^2*d^2))/(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^8*b^12*d^4 + 56*a^10*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16*b^4*d^4 + 8*a^18*b^2*d^4))*(1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) - (90*a*b^19*d^2 + 846*a^3*b^17*d^2 + 1714*a^5*b^15*d^2 + 3606*a^7*b^13*d^2 - 14578*a^9*b^11*d^2 - 34486*a^11*b^9*d^2 - 14970*a^13*b^7*d^2 + 2258*a^15*b^5*d^2 - 32*a^17*b^3*d^2)/(a^20*d^5 + a^4*b^16*d^5 + 8*a^6*b^14

```
*d^5 + 28*a^8*b^12*d^5 + 56*a^10*b^10*d^5 + 70*a^12*b^8*d^5 + 56*a^14*b^6*d
^5 + 28*a^16*b^4*d^5 + 8*a^18*b^2*d^5)) - (tan(c + d*x)^(1/2)*(18*a^2*b^15
- 9*b^17 - 71*a^4*b^13 + 892*a^6*b^11 + 857*a^8*b^9 + 6802*a^10*b^7 - 1257*
a^12*b^5))/(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^8*b^12*d^4 + 56
*a^10*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16*b^4*d^4 + 8*a^
18*b^2*d^4))*(1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a
^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2))...
```

3.606 $\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=444

$$\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,\left(a^2+b^2\right)^3 d}-\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(c}\right.}{\sqrt{2}\,\left(a^2+b^2\right)^3 d}$$

```
[Out] -1/4*b^(5/2)*(63*a^4+46*a^2*b^2+15*b^4)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(
1/2))/a^(7/2)/(a^2+b^2)^3/d-1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(-1+2^(1/2)*tan
(d*x+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(1+2^(
1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/4*(a+b)*(a^2-4*a*b+b^2)*ln(1
-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)^3/d*2^(1/2)+1/4*(a+b)*(a^2-
4*a*b+b^2)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)^3/d*2^(1/2)+
1/4*(-8*a^4-31*a^2*b^2-15*b^4)/a^3/(a^2+b^2)^2/d/tan(d*x+c)^(1/2)+1/2*b^2/a
/(a^2+b^2)/d/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2+1/4*b^2*(13*a^2+5*b^2)/a^2
/(a^2+b^2)^2/d/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.74, antiderivative size = 444, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3650, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d\left(a^2+b^2\right)^3}-\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d\left(a^2+b^2\right)^3}+\frac{b^2\left(13a^2+5b^2\right)}{4a^2d\left(a^2+b^2\right)^2\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))}+\frac{b^2}{2ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^2}-\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\tan(c+dx)-\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)^3}+\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\tan(c+dx)+\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)^3}-\frac{b^{5/2}\left(63a^4+46a^2b^2+15b^4\right)\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{7/2}d\left(a^2+b^2\right)^3}-\frac{8a^4+31a^2b^2+15b^4}{4a^3d\left(a^2+b^2\right)^2\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^3),x]
```

```
[Out] ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[
2]*(a^2 + b^2)^3*d) - ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 + Sqrt[2]*Sqrt[
Tan[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)^3*d) - (b^(5/2)*(63*a^4 + 46*a^2*b^2 +
 15*b^4)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(4*a^(7/2)*(a^2 + b^
2)^3*d) - ((a + b)*(a^2 - 4*a*b + b^2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] +
 Tan[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^3*d) + ((a + b)*(a^2 - 4*a*b + b^2)*
Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^
3*d) - (8*a^4 + 31*a^2*b^2 + 15*b^4)/(4*a^3*(a^2 + b^2)^2*d*Sqrt[Tan[c + d*
x]]) + b^2/(2*a*(a^2 + b^2)*d*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^2) +
(b^2*(13*a^2 + 5*b^2))/(4*a^2*(a^2 + b^2)^2*d*Sqrt[Tan[c + d*x]]*(a + b*Tan
[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
```

```
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
```

```
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx = \frac{b^2}{2a\left(a^2+b^2\right)d\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^2} + \frac{\int \frac{\frac{1}{2}\left(4a^2+5b^2\right)-2a}{\tan^{\frac{3}{2}}(c+}}{2}$$

$$= \frac{b^2}{2a\left(a^2+b^2\right)d\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^2} + \frac{}{4a^2\left(a^2+b^2\right)^2}$$

$$= -\frac{8a^4+31a^2b^2+15b^4}{4a^3\left(a^2+b^2\right)^2 d\sqrt{\tan(c+dx)}} + \frac{b^2}{2a\left(a^2+b^2\right)d\sqrt{\tan(c+dx)}\,(a}$$

$$= -\frac{8a^4+31a^2b^2+15b^4}{4a^3\left(a^2+b^2\right)^2 d\sqrt{\tan(c+dx)}} + \frac{b^2}{2a\left(a^2+b^2\right)d\sqrt{\tan(c+dx)}\,(a}$$

$$= -\frac{8a^4+31a^2b^2+15b^4}{4a^3\left(a^2+b^2\right)^2 d\sqrt{\tan(c+dx)}} + \frac{b^2}{2a\left(a^2+b^2\right)d\sqrt{\tan(c+dx)}\,(a}$$

$$= -\frac{8a^4+31a^2b^2+15b^4}{4a^3\left(a^2+b^2\right)^2 d\sqrt{\tan(c+dx)}} + \frac{b^2}{2a\left(a^2+b^2\right)d\sqrt{\tan(c+dx)}\,(a}$$

$$= -\frac{b^{5/2}\left(63a^4+46a^2b^2+15b^4\right)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{7/2}\left(a^2+b^2\right)^3 d} - \frac{}{4a^3\,(}$$

$$= -\frac{b^{5/2}\left(63a^4+46a^2b^2+15b^4\right)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{7/2}\left(a^2+b^2\right)^3 d} - \frac{(a+}{}$$

$$= \frac{(a-b)\left(a^2+4ab+b^2\right)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,\left(a^2+b^2\right)^3 d} - \frac{(a-b)}{}$$

Mathematica [C] Result contains complex when optimal does not.

time = 4.17, size = 358, normalized size = 0.81

$$\frac{\frac{-8a^{13/2}-39a^{9/2}b^2-46a^{5/2}b^4-15\sqrt{a}\,b^6-4(-1)^{3/4}a^{7/2}(a+ib)^3\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\sqrt{\tan(c+dx)}-63a^4b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{\tan(c+dx)}-46a^2b^{9/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{\tan(c+dx)}-15b^{13/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{\tan(c+dx)}-4\sqrt[4]{-1}\,a^{7/2}(ia+b)^3\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\sqrt{\tan(c+dx)}}{a^{5/2}(a^2+b^2)^2}+\frac{2b^2}{(a+b\tan(c+dx))^2}+\frac{13a^2b^2+5b^4}{a(a^2+b^2)(a+b\tan(c+dx))}}{4a\left(a^2+b^2\right)d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^3),x]
```

```
[Out] ((-8*a^(13/2) - 39*a^(9/2)*b^2 - 46*a^(5/2)*b^4 - 15*Sqrt[a]*b^6 - 4*(-1)^(
3/4)*a^(7/2)*(a + I*b)^3*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*Sqrt[Tan[c +
 d*x]] - 63*a^4*b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[T
an[c + d*x]] - 46*a^2*b^(9/2)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*
Sqrt[Tan[c + d*x]] - 15*b^(13/2)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a
]]*Sqrt[Tan[c + d*x]] - 4*(-1)^(1/4)*a^(7/2)*(I*a + b)^3*ArcTanh[(-1)^(3/4)
*Sqrt[Tan[c + d*x]]]*Sqrt[Tan[c + d*x]])/(a^(5/2)*(a^2 + b^2)^2) + (2*b^2)/
(a + b*Tan[c + d*x])^2 + (13*a^2*b^2 + 5*b^4)/(a*(a^2 + b^2)*(a + b*Tan[c +
 d*x])))/(4*a*(a^2 + b^2)*d*Sqrt[Tan[c + d*x]])
```

Maple [A]

time = 0.15, size = 356, normalized size = 0.80

method	result
derivativedivides	$-\frac{2}{a^3\sqrt{\tan(dx+c)}}-\frac{2b^3\left(\frac{\left(\frac{15}{8}a^4b+\frac{11}{4}a^2b^3+\frac{7}{8}b^5\right)\left(\tan^{\frac{3}{2}}(dx+c)\right)+\frac{a\left(17a^4+26a^2b^2+9b^4\right)\left(\sqrt{\tan}(dx+c)\right)}{8}}{(a+b\tan(dx+c))^2}+\frac{(63a^4+ \cdots}{\cdots}\right)}{\left(a^2+b^2\right)^3a^3}\ \cdots$
default	$-\frac{2}{a^3\sqrt{\tan(dx+c)}}-\frac{2b^3\left(\frac{\left(\frac{15}{8}a^4b+\frac{11}{4}a^2b^3+\frac{7}{8}b^5\right)\left(\tan^{\frac{3}{2}}(dx+c)\right)+\frac{a\left(17a^4+26a^2b^2+9b^4\right)\left(\sqrt{\tan}(dx+c)\right)}{8}}{(a+b\tan(dx+c))^2}+\frac{(63a^4+ \cdots}{\cdots}\right)}{\left(a^2+b^2\right)^3a^3}\ \cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/a^3/tan(d*x+c)^(1/2)-2*b^3/(a^2+b^2)^3/a^3*(((15/8*a^4*b+11/4*a^2*b
^3+7/8*b^5)*tan(d*x+c)^(3/2)+1/8*a*(17*a^4+26*a^2*b^2+9*b^4)*tan(d*x+c)^(1/
2))/(a+b*tan(d*x+c))^2+1/8*(63*a^4+46*a^2*b^2+15*b^4)/(a*b)^(1/2)*arctan(b*
tan(d*x+c)^(1/2)/(a*b)^(1/2)))+2/(a^2+b^2)^3*(1/8*(-3*a^2*b+b^3)*2^(1/2)*(l
n((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d
*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)
```

```
^(1/2)))+1/8*(-a^3+3*a*b^2)*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x
+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)
^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))))
```

Maxima [A]
time = 0.51, size = 464, normalized size = 1.05

$$-\frac{\frac{(63\,a^4b^3+46\,a^2b^5+15\,b^7)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^9+3\,a^7b^2+3\,a^5b^4+a^3b^6)\sqrt{ab}}+\frac{8\,a^6+16\,a^4b^2+8\,a^2b^4+(8\,a^4b^2+31\,a^2b^4+15\,b^6)\tan(dx+c)^2+(16\,a^5b+49\,a^3b^3+25\,ab^5)\tan(dx+c)}{(a^7b^2+2\,a^5b^4+a^3b^6)\tan(dx+c)^{\frac{5}{2}}+2\,(a^8b+2\,a^6b^3+a^4b^5)\tan(dx+c)^{\frac{3}{2}}+(a^9+2\,a^7b^2+a^5b^4)\sqrt{\tan(dx+c)}}+\frac{2\sqrt{2}\,(a^3+3\,a^2b-3\,ab^2-b^3)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}\,(a^3+3\,a^2b-3\,ab^2-b^3)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-\sqrt{2}\,(a^3-3\,a^2b-3\,ab^2+b^3)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+\sqrt{2}\,(a^3-3\,a^2b-3\,ab^2+b^3)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*((63*a^4*b^3 + 46*a^2*b^5 + 15*b^7)*arctan(b*sqrt(tan(d*x + c))/sqrt(a
*b))/((a^9 + 3*a^7*b^2 + 3*a^5*b^4 + a^3*b^6)*sqrt(a*b)) + (8*a^6 + 16*a^4*
b^2 + 8*a^2*b^4 + (8*a^4*b^2 + 31*a^2*b^4 + 15*b^6)*tan(d*x + c)^2 + (16*a^
5*b + 49*a^3*b^3 + 25*a*b^5)*tan(d*x + c))/((a^7*b^2 + 2*a^5*b^4 + a^3*b^6)
*tan(d*x + c)^(5/2) + 2*(a^8*b + 2*a^6*b^3 + a^4*b^5)*tan(d*x + c)^(3/2) +
(a^9 + 2*a^7*b^2 + a^5*b^4)*sqrt(tan(d*x + c))) + (2*sqrt(2)*(a^3 + 3*a^2*b
 - 3*a*b^2 - b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*
sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sq
rt(tan(d*x + c)))) - sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*log(sqrt(2)*sq
rt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b
^3)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 12694 vs. 2(392) = 784.
time = 19.74, size = 25393, normalized size = 57.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*sqrt(2)*((a^25 + 3*a^23*b^2 - 17*a^21*b^4 - 123*a^19*b^6 - 342*a
^17*b^8 - 546*a^15*b^10 - 546*a^13*b^12 - 342*a^11*b^14 - 123*a^9*b^16 - 17
*a^7*b^18 + 3*a^5*b^20 + a^3*b^22)*d^5*cos(d*x + c)^6 - (a^25 - 3*a^23*b^2
- 69*a^21*b^4 - 321*a^19*b^6 - 774*a^17*b^8 - 1134*a^15*b^10 - 1050*a^13*b^
12 - 594*a^11*b^14 - 171*a^9*b^16 + a^7*b^18 + 15*a^5*b^20 + 3*a^3*b^22)*d^
5*cos(d*x + c)^4 - 3*(2*a^23*b^2 + 17*a^21*b^4 + 63*a^19*b^6 + 132*a^17*b^8
 + 168*a^15*b^10 + 126*a^13*b^12 + 42*a^11*b^14 - 12*a^9*b^16 - 18*a^7*b^18
 - 7*a^5*b^20 - a^3*b^22)*d^5*cos(d*x + c)^2 - (a^21*b^4 + 9*a^19*b^6 + 36*
a^17*b^8 + 84*a^15*b^10 + 126*a^13*b^12 + 126*a^11*b^14 + 84*a^9*b^16 + 36*
a^7*b^18 + 9*a^5*b^20 + a^3*b^22)*d^5 + 4*((a^24*b + 8*a^22*b^3 + 27*a^20*b
^5 + 48*a^18*b^7 + 42*a^16*b^9 - 42*a^12*b^13 - 48*a^10*b^15 - 27*a^8*b^17
- 8*a^6*b^19 - a^4*b^21)*d^5*cos(d*x + c)^5 - (a^24*b + 7*a^22*b^3 + 18*a^2
```

0*b^5 + 12*a^18*b^7 - 42*a^16*b^9 - 126*a^14*b^11 - 168*a^12*b^13 - 132*a^1
0*b^15 - 63*a^8*b^17 - 17*a^6*b^19 - 2*a^4*b^21)*d^5*cos(d*x + c)^3 - (a^22
*b^3 + 9*a^20*b^5 + 36*a^18*b^7 + 84*a^16*b^9 + 126*a^14*b^11 + 126*a^12*b^
13 + 84*a^10*b^15 + 36*a^8*b^17 + 9*a^6*b^19 + a^4*b^21)*d^5*cos(d*x + c))*
sin(d*x + c))*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^
8 + 6*a^2*b^10 + b^12 + 2*(3*a^17*b + 8*a^15*b^3 - 12*a^13*b^5 - 72*a^11*b^
7 - 110*a^9*b^9 - 72*a^7*b^11 - 12*a^5*b^13 + 8*a^3*b^15 + 3*a*b^17)*d^2*sq
rt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^1
0 + b^12)*d^4)))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*
b^8 - 30*a^2*b^10 + b^12))*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6
*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 12*a^22*b^2 + 66*a^20*b^4
 + 220*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 924*a^12*b^12 + 792*a^10*b
^14 + 495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 12*a^2*b^22 + b^24)*d^4))
*(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10
 + b^12)*d^4))^(3/4)*arctan(((a^24 - 6*a^22*b^2 - 84*a^20*b^4 - 322*a^18*b^
6 - 603*a^16*b^8 - 540*a^14*b^10 + 540*a^10*b^14 + 603*a^8*b^16 + 322*a^6*b
^18 + 84*a^4*b^20 + 6*a^2*b^22 - b^24)*d^4*sqrt((a^12 - 30*a^10*b^2 + 255*a
^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 12*a^22*b
^2 + 66*a^20*b^4 + 220*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 924*a^12*b
^12 + 792*a^10*b^14 + 495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 12*a^2*b^
22 + b^24)*d^4))*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*
a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) - sqrt(2)*((3*a^26*b + 35*a^24*b^3 + 186
*a^22*b^5 + 594*a^20*b^7 + 1265*a^18*b^9 + 1881*a^16*b^11 + 1980*a^14*b^13
+ 1452*a^12*b^15 + 693*a^10*b^17 + 165*a^8*b^19 - 22*a^6*b^21 - 30*a^4*b^23
 - 9*a^2*b^25 - b^27)*d^7*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*
b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 12*a^22*b^2 + 66*a^20*b^4
+ 220*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 924*a^12*b^12 + 792*a^10*b^
14 + 495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 12*a^2*b^22 + b^24)*d^4))*
sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b
^10 + b^12)*d^4)) - (a^21 + 6*a^19*b^2 + 9*a^17*b^4 - 24*a^15*b^6 - 126*a^1
3*b^8 - 252*a^11*b^10 - 294*a^9*b^12 - 216*a^7*b^14 - 99*a^5*b^16 - 26*a^3*
b^18 - 3*a*b^20)*d^5*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 +
 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 12*a^22*b^2 + 66*a^20*b^4 + 220
*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 924*a^12*b^12 + 792*a^10*b^14 +
495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 12*a^2*b^22 + b^24)*d^4)))*sqrt
((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b
^12 + 2*(3*a^17*b + 8*a^15*b^3 - 12*a^13*b^5 - 72*a^11*b^7 - 110*a^9*b^9 -
72*a^7*b^11 - 12*a^5*b^13 + 8*a^3*b^15 + 3*a*b^17)*d^2*sqrt(1/((a^12 + 6*a^
10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))/(
a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10
+ b^12))*sqrt(((a^18 - 27*a^16*b^2 + 168*a^14*b^4 + 224*a^12*b^6 - 366*a^10
*b^8 - 366*a^8*b^10 + 224*a^6*b^12 + 168*a^4*b^14 - 27*a^2*b^16 + b^18)*d^2
*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*
b^10 + b^12)*d^4))*cos(d*x + c) + sqrt(2)*((a^21 - 30*a^19*b^2 + 249*a^17*b
^4 - 280*a^15*b^6 - 1038*a^13*b^8 + 732*a^11*b^10 + 1322*a^9*b^12 - 504*a^7

```
*b^14 - 531*a^5*b^16 + 82*a^3*b^18 - 3*a*b^20)*d^3*sqrt(1/((a^12 + 6*a^10*b
^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*cos(d*
x + c) - (3*a^14*b - 91*a^12*b^3 + 795*a^10*b^5 - 1611*a^8*b^7 + 1217*a^6*b
^9 - 345*a^4*b^11 + 33*a^2*b^13 - b^15)*d*cos(d*x + c))*sqrt((a^12 + 6*a^10
*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 + 2*(3*a^17
*b + 8*a^15*b^3 - 12*a^13*b^5 - 72*a^11*b^7 - 110*a^9*b^9 - 72*a^7*b^11 - 1
2*a^5*b^13 + 8*a^3*b^15 + 3*a*b^17)*d^2*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8
*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))/(a^12 - 30*a^10*
b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*sqrt(s
in(d*x + c)/cos(d*x + c))*(1/((a^12 + 6*a^10*b^...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\tan(c+dx))^3\tan^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(3/2)/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(1/((a + b*tan(c + d*x))**3*tan(c + d*x)**(3/2)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 13.71, size = 2500, normalized size = 5.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x))^3),x)
```

```
[Out] (log(29491200*a^22*b^35*d^4 - ((((-1/(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2
 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i))^(1/2)
*(((((-1/(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2
*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i))^(1/2)*(251658240*a^24*b^45*d^8 +
5049942016*a^26*b^43*d^8 + 48368713728*a^28*b^41*d^8 + 293819383808*a^30*b^
39*d^8 + 1268458192896*a^32*b^37*d^8 + 4132731617280*a^34*b^35*d^8 + 105311
```

92700928*a^36*b^33*d^8 + 21462823993344*a^38*b^31*d^8 + 35469618315264*a^40
*b^29*d^8 + 47896904859648*a^42*b^27*d^8 + 52983958077440*a^44*b^25*d^8 + 4
7896904859648*a^46*b^23*d^8 + 35090285461504*a^48*b^21*d^8 + 20487396655104
*a^50*b^19*d^8 + 9230622916608*a^52*b^17*d^8 + 2994733056000*a^54*b^15*d^8
+ 565576728576*a^56*b^13*d^8 - 18572378112*a^58*b^11*d^8 - 50281316352*a^60
*b^9*d^8 - 16089350144*a^62*b^7*d^8 - 2516582400*a^64*b^5*d^8 - 167772160*a
^66*b^3*d^8 + (tan(c + d*x)^(1/2)*(-1/(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^
2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i))^(1/2
)*(134217728*a^27*b^45*d^9 + 2550136832*a^29*b^43*d^9 + 22817013760*a^31*b^
41*d^9 + 127506841600*a^33*b^39*d^9 + 497276682240*a^35*b^37*d^9 + 14306267
62752*a^37*b^35*d^9 + 3121367482368*a^39*b^33*d^9 + 5202279137280*a^41*b^31
*d^9 + 6502848921600*a^43*b^29*d^9 + 5635802398720*a^45*b^27*d^9 + 22543209
59488*a^47*b^25*d^9 - 2254320959488*a^49*b^23*d^9 - 5635802398720*a^51*b^21
*d^9 - 6502848921600*a^53*b^19*d^9 - 5202279137280*a^55*b^17*d^9 - 31213674
82368*a^57*b^15*d^9 - 1430626762752*a^59*b^13*d^9 - 497276682240*a^61*b^11*
d^9 - 127506841600*a^63*b^9*d^9 - 22817013760*a^65*b^7*d^9 - 2550136832*a^6
7*b^5*d^9 - 134217728*a^69*b^3*d^9))/2))/2 - tan(c + d*x)^(1/2)*(471859200*
a^22*b^44*d^7 + 9500098560*a^24*b^42*d^7 + 91857354752*a^26*b^40*d^7 + 5645
02986752*a^28*b^38*d^7 + 2464648527872*a^30*b^36*d^7 + 8104469069824*a^32*b
^34*d^7 + 20769933361152*a^34*b^32*d^7 + 42351565209600*a^36*b^30*d^7 + 695
34945902592*a^38*b^28*d^7 + 92434029608960*a^40*b^26*d^7 + 99508717355008*a
^42*b^24*d^7 + 86342935511040*a^44*b^22*d^7 + 59767095558144*a^46*b^20*d^7
+ 32432589897728*a^48*b^18*d^7 + 13411815522304*a^50*b^16*d^7 + 40304577085
44*a^52*b^14*d^7 + 805425905664*a^54*b^12*d^7 + 86608183296*a^56*b^10*d^7 +
1612709888*a^58*b^8*d^7 + 16777216*a^60*b^6*d^7 + 167772160*a^62*b^4*d^7 +
16777216*a^64*b^2*d^7))*(-1/(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5
*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i))^(1/2))/2 - 11
7964800*a^21*b^42*d^6 - 841482240*a^23*b^40*d^6 + 3829399552*a^25*b^38*d^6
+ 78068580352*a^27*b^36*d^6 + 497438162944*a^29*b^34*d^6 + 1899895980032*a^
31*b^32*d^6 + 4972695519232*a^33*b^30*d^6 + 9371195015168*a^35*b^28*d^6 + 1
2890720436224*a^37*b^26*d^6 + 12726089809920*a^39*b^24*d^6 + 8366961197056*
a^41*b^22*d^6 + 2597662490624*a^43*b^20*d^6 - 1171836108800*a^45*b^18*d^6 -
1986881650688*a^47*b^16*d^6 - 1237583921152*a^49*b^14*d^6 - 449507753984*a
^51*b^12*d^6 - 97476149248*a^53*b^10*d^6 - 11931222016*a^55*b^8*d^6 - 10066
32960*a^57*b^6*d^6 - 134217728*a^59*b^4*d^6 - 8388608*a^61*b^2*d^6))/2 + ta
n(c + d*x)^(1/2)*(7610564608*a^27*b^33*d^5 - 597688320*a^23*b^37*d^5 - 1671
430144*a^25*b^35*d^5 - 58982400*a^21*b^39*d^5 + 85774565376*a^29*b^31*d^5 +
385487994880*a^31*b^29*d^5 + 1104303620096*a^33*b^27*d^5 + 2240523796480*a
^35*b^25*d^5 + 3345249468416*a^37*b^23*d^5 + 3717287903232*a^39*b^21*d^5 +
3053967114240*a^41*b^19*d^5 + 1807474491392*a^43*b^17*d^5 + 726513221632*a^
45*b^15*d^5 + 170768990208*a^47*b^13*d^5 + 10492051456*a^49*b^11*d^5 - 4917
821440*a^51*b^9*d^5 - 923009024*a^53*b^7*d^5 + 8388608*a^55*b^5*d^5))*(-1/(
b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*
a^3*b^3*d^2 + a^4*b^2*d^2*15i))^(1/2))/2 + 460062720*a^24*b^33*d^4 + 343972
2496*a^26*b^31*d^4 + 16227237888*a^28*b^29*d^4 + 53669396480*a^30*b^27*d^4

+ 131031367680*a^32*b^25*d^4 + 242529730560*a^34*b^23*d^4 + 344454070272*a^36*b^21*d^4 + 375993532416*a^38*b^19*d^4 + 313043189760*a^40*b^17*d^4 + 195253370880*a^42*b^15*d^4 + 88318935040*a^44*b^13*d^4 + 27352498176*a^46*b^11*d^4 + 5187043328*a^48*b^9*d^4 + 454164480*a^50*b^7*d^4)*(-1/(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i))^(1/2))/2 - log(29491200*a^22*b^35*d^4 - ((-1/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2)*(((-1/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2)*(251658240*a^24*b^45*d^8 + 5049942016*a^26*b^43*d^8 + 48368713728*a^28*b^41*d^8 + 293819383808*a^30*b^39*d^8 + 1268458192896*a^32*b^37*d^8 + 4132731617280*a^34*b^35*d^8 + 10531192700928*a^36*b^33*d^8 + 21462823993344*a^38*b^31*d^8 + 35469618315264*a^40*b^29*d^8 + 47896904859648*a^42*b^27*d^8 + 52983958077440*a^44*b^25*d^8 + 47896904859648*a^46*b^23*d^8 + 35090285461504*a^48*b^21*d^8 + 20487396655104*a^50*b^19*d^8 + 9230622916608*a^52*b^17*d^8 + 2994733056000*a^54*b^15*d^8 + 565576728576...

3.607 $\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=493

$$\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,\left(a^2+b^2\right)^3 d}-\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(\right.}{\sqrt{2}\,\left(a^2+b^2\right)^3 d}$$

```
[Out] 1/4*b^(7/2)*(99*a^4+102*a^2*b^2+35*b^4)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(
1/2))/a^(9/2)/(a^2+b^2)^3/d-1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(-1+2^(1/2)*tan
(d*x+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(1+2^(
1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)+1/4*(a-b)*(a^2+4*a*b+b^2)*ln(1
-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)^3/d*2^(1/2)-1/4*(a-b)*(a^2+
4*a*b+b^2)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)^3/d*2^(1/2)+
1/4*b*(24*a^4+67*a^2*b^2+35*b^4)/a^4/(a^2+b^2)^2/d/tan(d*x+c)^(1/2)+1/12*(-
8*a^4-67*a^2*b^2-35*b^4)/a^3/(a^2+b^2)^2/d/tan(d*x+c)^(3/2)+1/2*b^2/a/(a^2+
b^2)/d/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2+1/4*b^2*(15*a^2+7*b^2)/a^2/(a^2+
b^2)^2/d/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.96, antiderivative size = 493, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3650, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d\left(a^2+b^2\right)^3}-\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d\left(a^2+b^2\right)^3}+\frac{b^2\left(15a^2+7b^2\right)}{4a^2d\left(a^2+b^2\right)^2\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}+\frac{b^2}{2ad\left(a^2+b^2\right)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}+\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\tan(c+dx)-\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)^3}-\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\tan(c+dx)+\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)^3}+\frac{b\left(24a^4+67a^2b^2+35b^4\right)}{4a^4d\left(a^2+b^2\right)^2\sqrt{\tan(c+dx)}}+\frac{b^{7/2}\left(99a^4+102a^2b^2+35b^4\right)\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{9/2}d\left(a^2+b^2\right)^3}-\frac{8a^4+67a^2b^2+35b^4}{12a^3d\left(a^2+b^2\right)^2\tan^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^3),x]
```

```
[Out] ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[
2]*(a^2 + b^2)^3*d) - ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 + Sqrt[2]*Sqrt[
Tan[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)^3*d) + (b^(7/2)*(99*a^4 + 102*a^2*b^2
+ 35*b^4)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(4*a^(9/2)*(a^2 + b
^2)^3*d) + ((a - b)*(a^2 + 4*a*b + b^2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
+ Tan[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^3*d) - ((a - b)*(a^2 + 4*a*b + b^2)
*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)
^3*d) - (8*a^4 + 67*a^2*b^2 + 35*b^4)/(12*a^3*(a^2 + b^2)^2*d*Tan[c + d*x]^
(3/2)) + (b*(24*a^4 + 67*a^2*b^2 + 35*b^4))/(4*a^4*(a^2 + b^2)^2*d*Sqrt[Tan
[c + d*x]]) + b^2/(2*a*(a^2 + b^2)*d*Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x]
)^2) + (b^2*(15*a^2 + 7*b^2))/(4*a^2*(a^2 + b^2)^2*d*Tan[c + d*x]^(3/2)*(a
+ b*Tan[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 631

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1176

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1179

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 1182

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D

```
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx = \frac{b^2}{2a\left(a^2+b^2\right)d\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2} + \frac{\int \frac{\frac{1}{2}\left(4a^2+7b^2\right)-2a \cdots}{\tan^{\frac{5}{2}}(c+\cdots}}{2 \cdots}$$

$$= \frac{b^2}{2a\left(a^2+b^2\right)d\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2} + \frac{\cdots}{4a^2\left(a^2+b^2\right)^2 \cdots}$$

$$= -\frac{8a^4+67a^2b^2+35b^4}{12a^3\left(a^2+b^2\right)^2 d\tan^{\frac{3}{2}}(c+dx)} + \frac{b^2}{2a\left(a^2+b^2\right)d\tan^{\frac{3}{2}}(c+dx)(a \cdots}$$

$$= -\frac{8a^4+67a^2b^2+35b^4}{12a^3\left(a^2+b^2\right)^2 d\tan^{\frac{3}{2}}(c+dx)} + \frac{b(24a^4+67a^2b^2+35b^4)}{4a^4\left(a^2+b^2\right)^2 d\sqrt{\tan(c+dx)} \cdots}$$

$$= -\frac{8a^4+67a^2b^2+35b^4}{12a^3\left(a^2+b^2\right)^2 d\tan^{\frac{3}{2}}(c+dx)} + \frac{b(24a^4+67a^2b^2+35b^4)}{4a^4\left(a^2+b^2\right)^2 d\sqrt{\tan(c+dx)} \cdots}$$

$$= -\frac{8a^4+67a^2b^2+35b^4}{12a^3\left(a^2+b^2\right)^2 d\tan^{\frac{3}{2}}(c+dx)} + \frac{b(24a^4+67a^2b^2+35b^4)}{4a^4\left(a^2+b^2\right)^2 d\sqrt{\tan(c+dx)} \cdots}$$

$$= -\frac{8a^4+67a^2b^2+35b^4}{12a^3\left(a^2+b^2\right)^2 d\tan^{\frac{3}{2}}(c+dx)} + \frac{b(24a^4+67a^2b^2+35b^4)}{4a^4\left(a^2+b^2\right)^2 d\sqrt{\tan(c+dx)} \cdots}$$

$$= \frac{b^{7/2}(99a^4+102a^2b^2+35b^4)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{9/2}\left(a^2+b^2\right)^3 d} - \frac{\cdots}{12a \cdots}$$

$$= \frac{b^{7/2}(99a^4+102a^2b^2+35b^4)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{9/2}\left(a^2+b^2\right)^3 d} + \frac{(a - \cdots}{\cdots}$$

$$= \frac{(a+b)\left(a^2-4ab+b^2\right)\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 d} - \frac{(a+b \cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.20, size = 495, normalized size = 1.00

$$\frac{b^2}{2a\left(a^2+b^2\right)d\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2} + \frac{-\frac{2\left(-\frac{2\left(\frac{2\left(-3a^6b^2+\frac{3}{16}a^2b^2\left(24a^4+67a^2b^2+35b^4\right)-\frac{3}{16}b^2\left(8a^6-32a^4b^2-67a^2b^4-35b^6\right)\right)\mathrm{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\left(a^2+b^2\right)d}+\frac{-\frac{\sqrt[4]{-1}\left(-\frac{3}{2}a^5\left(a^2-3b^2\right)-\frac{3}{2}ia^4b\left(3a^2-b^2\right)\right)\mathrm{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d}-\frac{\sqrt[4]{-1}\left(-\frac{3}{2}a^5\left(a^2-3b^2\right)+\frac{3}{2}ia^4b\left(3a^2-b^2\right)\right)\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d}}{a^2+b^2}\right)}{a}-\frac{3b\left(24a^4+67a^2b^2+35b^4\right)}{4ad\sqrt{\tan(c+dx)}}\right)}{3a}-\frac{8a^4+67a^2b^2+35b^4}{6ad\tan^{\frac{3}{2}}(c+dx)}}{a\left(a^2+b^2\right)}+\frac{\frac{11a^2b^2}{2}+\frac{1}{2}b^2\left(4a^2+7b^2\right)}{a\left(a^2+b^2\right)d\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}}{2a\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^3),x]

[Out] b^2/(2*a*(a^2 + b^2)*d*Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^2) + (((-2*((-2*((2*(-3*a^6*b^2 + (3*a^2*b^2*(24*a^4 + 67*a^2*b^2 + 35*b^4))/16 - (3*b^2*(8*a^6 - 32*a^4*b^2 - 67*a^2*b^4 - 35*b^6))/16)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*(a^2 + b^2)*d) + (-(((-1)^(1/4)*((-3*a^5*(a^2 - 3*b^2))/2 - ((3*I)/2)*a^4*b*(3*a^2 - b^2))*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])/d) - ((-1)^(1/4)*((-3*a^5*(a^2 - 3*b^2))/2 + ((3*I)/2)*a^4*b*(3*a^2 - b^2))*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])/d)/(a^2 + b^2)))/a - (3*b*(24*a^4 + 67*a^2*b^2 + 35*b^4))/(4*a*d*Sqrt[Tan[c + d*x]])))/(3*a) - (8*a^4 + 67*a^2*b^2 + 35*b^4)/(6*a*d*Tan[c + d*x]^(3/2)))/(a*(a^2 + b^2)) + ((11*a^2*b^2)/2 + (b^2*(4*a^2 + 7*b^2))/2)/(a*(a^2 + b^2)*d*Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])))/(2*a*(a^2 + b^2))

Maple [A]

time = 0.16, size = 372, normalized size = 0.75

method	result
derivativedivides	$$\frac{-\frac{2}{3a^3\tan(dx+c)^{\frac{3}{2}}}+\frac{6b}{a^4\sqrt{\tan(dx+c)}}+\frac{2b^4\left(\frac{\left(\frac{19}{8}a^4b+\frac{15}{4}a^2b^3+\frac{11}{8}b^5\right)\left(\tan^{\frac{3}{2}}(dx+c)\right)+\frac{a\left(21a^4+34a^2b^2+13b^4\right)\left(\sqrt{\tan}\right.}{8}}{(a+b\tan(dx+c))^2}\right.}{a^4\left(a^2+b^2\right)^3}}{}$$
default	$$\frac{-\frac{2}{3a^3\tan(dx+c)^{\frac{3}{2}}}+\frac{6b}{a^4\sqrt{\tan(dx+c)}}+\frac{2b^4\left(\frac{\left(\frac{19}{8}a^4b+\frac{15}{4}a^2b^3+\frac{11}{8}b^5\right)\left(\tan^{\frac{3}{2}}(dx+c)\right)+\frac{a\left(21a^4+34a^2b^2+13b^4\right)\left(\sqrt{\tan}\right.}{8}}{(a+b\tan(dx+c))^2}\right.}{a^4\left(a^2+b^2\right)^3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(-2/3/a^3/tan(d*x+c)^(3/2)+6/a^4*b/tan(d*x+c)^(1/2)+2*b^4/a^4/(a^2+b^2)^3*(((19/8*a^4*b+15/4*a^2*b^3+11/8*b^5)*tan(d*x+c)^(3/2)+1/8*a*(21*a^4+34*a^2*b^2+13*b^4)*tan(d*x+c)^(1/2))/(a+b*tan(d*x+c))^2+1/8*(99*a^4+102*a^2*b^2+35*b^4)/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2)))+2/(a^2+b^2)^3*(1/8*(-a^3+3*a*b^2)*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/8*(3*a^2*b-b^3)*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))))

Maxima [A]
time = 0.51, size = 502, normalized size = 1.02

$$\frac{\frac{3\,(99\,a^4b^4+102\,a^2b^6+35\,b^8)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^{10}+3\,a^8b^2+3\,a^6b^4+a^4b^6)\sqrt{ab}}-\frac{8\,a^7+16\,a^5b^2+8\,a^3b^4-3\,(24\,a^4b^3+67\,a^2b^5+35\,b^7)\tan(dx+c)^3-(136\,a^5b^2+335\,a^3b^4+175\,ab^6)\tan(dx+c)^2-56\,(a^6b+2\,a^4b^3+a^2b^5)\tan(dx+c)}{(a^8b^2+2\,a^6b^4+a^4b^6)\tan(dx+c)^{\frac{7}{2}}+2\,(a^9b+2\,a^7b^3+a^5b^5)\tan(dx+c)^{\frac{5}{2}}+(a^{10}+2\,a^8b^2+a^6b^4)\tan(dx+c)^{\frac{3}{2}}}-\frac{3\left(2\sqrt{2}\,(a^3-3\,a^2b-3\,ab^2+b^3)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}\,(a^3-3\,a^2b-3\,ab^2+b^3)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)+\sqrt{2}\,(a^3+3\,a^2b-3\,ab^2-b^3)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)-\sqrt{2}\,(a^3+3\,a^2b-3\,ab^2-b^3)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/12*(3*(99*a^4*b^4 + 102*a^2*b^6 + 35*b^8)*arctan(b*sqrt(tan(d*x + c))/sqr
t(a*b))/((a^10 + 3*a^8*b^2 + 3*a^6*b^4 + a^4*b^6)*sqrt(a*b)) - (8*a^7 + 16*
a^5*b^2 + 8*a^3*b^4 - 3*(24*a^4*b^3 + 67*a^2*b^5 + 35*b^7)*tan(d*x + c)^3 -
 (136*a^5*b^2 + 335*a^3*b^4 + 175*a*b^6)*tan(d*x + c)^2 - 56*(a^6*b + 2*a^4
*b^3 + a^2*b^5)*tan(d*x + c))/((a^8*b^2 + 2*a^6*b^4 + a^4*b^6)*tan(d*x + c)
^(7/2) + 2*(a^9*b + 2*a^7*b^3 + a^5*b^5)*tan(d*x + c)^(5/2) + (a^10 + 2*a^8
*b^2 + a^6*b^4)*tan(d*x + c)^(3/2)) - 3*(2*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2
 + b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*(a
^3 - 3*a^2*b - 3*a*b^2 + b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x
 + c)))) + sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*log(sqrt(2)*sqrt(tan(d*x
 + c)) + tan(d*x + c) + 1) - sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*log(-s
qrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 12698 vs. $2(437) = 874$.
time = 14.82, size = 25401, normalized size = 51.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/48*(48*sqrt(2)*((a^26 + 3*a^24*b^2 - 17*a^22*b^4 - 123*a^20*b^6 - 342*a^
18*b^8 - 546*a^16*b^10 - 546*a^14*b^12 - 342*a^12*b^14 - 123*a^10*b^16 - 17
*a^8*b^18 + 3*a^6*b^20 + a^4*b^22)*d^5*cos(d*x + c)^6 - (a^26 - 3*a^24*b^2
- 69*a^22*b^4 - 321*a^20*b^6 - 774*a^18*b^8 - 1134*a^16*b^10 - 1050*a^14*b^
12 - 594*a^12*b^14 - 171*a^10*b^16 + a^8*b^18 + 15*a^6*b^20 + 3*a^4*b^22)*d
^5*cos(d*x + c)^4 - 3*(2*a^24*b^2 + 17*a^22*b^4 + 63*a^20*b^6 + 132*a^18*b^
8 + 168*a^16*b^10 + 126*a^14*b^12 + 42*a^12*b^14 - 12*a^10*b^16 - 18*a^8*b^
18 - 7*a^6*b^20 - a^4*b^22)*d^5*cos(d*x + c)^2 - (a^22*b^4 + 9*a^20*b^6 + 3
6*a^18*b^8 + 84*a^16*b^10 + 126*a^14*b^12 + 126*a^12*b^14 + 84*a^10*b^16 +
36*a^8*b^18 + 9*a^6*b^20 + a^4*b^22)*d^5 + 4*((a^25*b + 8*a^23*b^3 + 27*a^2
1*b^5 + 48*a^19*b^7 + 42*a^17*b^9 - 42*a^13*b^13 - 48*a^11*b^15 - 27*a^9*b^
17 - 8*a^7*b^19 - a^5*b^21)*d^5*cos(d*x + c)^5 - (a^25*b + 7*a^23*b^3 + 18*
a^21*b^5 + 12*a^19*b^7 - 42*a^17*b^9 - 126*a^15*b^11 - 168*a^13*b^13 - 132*
a^11*b^15 - 63*a^9*b^17 - 17*a^7*b^19 - 2*a^5*b^21)*d^5*cos(d*x + c)^3 - (a
```

^23*b^3 + 9*a^21*b^5 + 36*a^19*b^7 + 84*a^17*b^9 + 126*a^15*b^11 + 126*a^13
*b^13 + 84*a^11*b^15 + 36*a^9*b^17 + 9*a^7*b^19 + a^5*b^21)*d^5*cos(d*x + c
))*sin(d*x + c))*sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4
*b^8 + 6*a^2*b^10 + b^12 - 2*(3*a^17*b + 8*a^15*b^3 - 12*a^13*b^5 - 72*a^11
*b^7 - 110*a^9*b^9 - 72*a^7*b^11 - 12*a^5*b^13 + 8*a^3*b^15 + 3*a*b^17)*d^2
*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*
b^10 + b^12)*d^4)))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a
^4*b^8 - 30*a^2*b^10 + b^12))*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*
a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 12*a^22*b^2 + 66*a^20*
b^4 + 220*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 924*a^12*b^12 + 792*a^1
0*b^14 + 495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 12*a^2*b^22 + b^24)*d^
4))*(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b
^10 + b^12)*d^4))^(3/4)*arctan(-((a^24 - 6*a^22*b^2 - 84*a^20*b^4 - 322*a^1
8*b^6 - 603*a^16*b^8 - 540*a^14*b^10 + 540*a^10*b^14 + 603*a^8*b^16 + 322*a
^6*b^18 + 84*a^4*b^20 + 6*a^2*b^22 - b^24)*d^4*sqrt((a^12 - 30*a^10*b^2 + 2
55*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 12*a^
22*b^2 + 66*a^20*b^4 + 220*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 924*a^
12*b^12 + 792*a^10*b^14 + 495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 12*a^
2*b^22 + b^24)*d^4))*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 +
15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) + sqrt(2)*((a^27 + 9*a^25*b^2 + 30*a
^23*b^4 + 22*a^21*b^6 - 165*a^19*b^8 - 693*a^17*b^10 - 1452*a^15*b^12 - 198
0*a^13*b^14 - 1881*a^11*b^16 - 1265*a^9*b^18 - 594*a^7*b^20 - 186*a^5*b^22
- 35*a^3*b^24 - 3*a*b^26)*d^7*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*
a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 12*a^22*b^2 + 66*a^20*
b^4 + 220*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 924*a^12*b^12 + 792*a^1
0*b^14 + 495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 12*a^2*b^22 + b^24)*d^
4))*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a
^2*b^10 + b^12)*d^4)) + (3*a^20*b + 26*a^18*b^3 + 99*a^16*b^5 + 216*a^14*b^
7 + 294*a^12*b^9 + 252*a^10*b^11 + 126*a^8*b^13 + 24*a^6*b^15 - 9*a^4*b^17
- 6*a^2*b^19 - b^21)*d^5*sqrt((a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b
^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/((a^24 + 12*a^22*b^2 + 66*a^20*b^4 +
220*a^18*b^6 + 495*a^16*b^8 + 792*a^14*b^10 + 924*a^12*b^12 + 792*a^10*b^1
4 + 495*a^8*b^16 + 220*a^6*b^18 + 66*a^4*b^20 + 12*a^2*b^22 + b^24)*d^4)))*
sqrt((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10
+ b^12 - 2*(3*a^17*b + 8*a^15*b^3 - 12*a^13*b^5 - 72*a^11*b^7 - 110*a^9*b^
9 - 72*a^7*b^11 - 12*a^5*b^13 + 8*a^3*b^15 + 3*a*b^17)*d^2*sqrt(1/((a^12 +
6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)
))/(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b
^10 + b^12))*sqrt(((a^18 - 27*a^16*b^2 + 168*a^14*b^4 + 224*a^12*b^6 - 366*
a^10*b^8 - 366*a^8*b^10 + 224*a^6*b^12 + 168*a^4*b^14 - 27*a^2*b^16 + b^18)
*d^2*sqrt(1/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*
a^2*b^10 + b^12)*d^4))*cos(d*x + c) + sqrt(2)*((3*a^20*b - 82*a^18*b^3 + 53
1*a^16*b^5 + 504*a^14*b^7 - 1322*a^12*b^9 - 732*a^10*b^11 + 1038*a^8*b^13 +
280*a^6*b^15 - 249*a^4*b^17 + 30*a^2*b^19 - b^21)*d^3*sqrt(1/((a^12 + 6*a^
10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*co

```
s(d*x + c) + (a^15 - 33*a^13*b^2 + 345*a^11*b^4 - 1217*a^9*b^6 + 1611*a^7*b
^8 - 795*a^5*b^10 + 91*a^3*b^12 - 3*a*b^14)*d*cos(d*x + c))*sqrt((a^12 + 6*
a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12 - 2*(3*
a^17*b + 8*a^15*b^3 - 12*a^13*b^5 - 72*a^11*b^7 - 110*a^9*b^9 - 72*a^7*b^11
 - 12*a^5*b^13 + 8*a^3*b^15 + 3*a*b^17)*d^2*sqrt(1/((a^12 + 6*a^10*b^2 + 15
*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))/(a^12 - 30*a
^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12))*sq
rt(sin(d*x + c)/cos(d*x + c))*(1/((a^12 + 6*a^1...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\tan(c+dx))^3 \tan^{\frac{5}{2}}(c+dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(5/2)/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(1/((a + b*tan(c + d*x))**3*tan(c + d*x)**(5/2)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 18.88, size = 2500, normalized size = 5.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(5/2)*(a + b*tan(c + d*x))^3),x)
```

```
[Out] atan(((tan(c + d*x)^(1/2)*(47691333632*a^34*b^35*d^5 - 3156213760*a^30*b^39
*d^5 - 7535067136*a^32*b^37*d^5 - 321126400*a^28*b^41*d^5 + 451224272896*a^
36*b^33*d^5 + 1855390220288*a^38*b^31*d^5 + 4902111674368*a^40*b^29*d^5 + 9
182617010176*a^42*b^27*d^5 + 12661071282176*a^44*b^25*d^5 + 12996430528512*
a^46*b^23*d^5 + 9861255397376*a^48*b^21*d^5 + 5375636013056*a^50*b^19*d^5 +
 1966525644800*a^52*b^17*d^5 + 396976193536*a^54*b^15*d^5 + 1851785216*a^56
*b^13*d^5 - 18624806912*a^58*b^11*d^5 - 3332636672*a^60*b^9*d^5 - 117440512
*a^62*b^7*d^5 - 8388608*a^64*b^5*d^5) + (1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d
```

^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))
^(1/2)*((tan(c + d*x)^(1/2)*(2569011200*a^29*b^46*d^7 + 50939822080*a^31*b^
44*d^7 + 479763365888*a^33*b^42*d^7 + 2849006157824*a^35*b^40*d^7 + 1194392
6562816*a^37*b^38*d^7 + 37510046547968*a^39*b^36*d^7 + 91385554272256*a^41*
b^34*d^7 + 176470173417472*a^43*b^32*d^7 + 273612095356928*a^45*b^30*d^7 +
342917730271232*a^47*b^28*d^7 + 347997439262720*a^49*b^26*d^7 + 28513016165
1712*a^51*b^24*d^7 + 187202969534464*a^53*b^22*d^7 + 97245760323584*a^55*b^
20*d^7 + 39238359842816*a^57*b^18*d^7 + 12009902964736*a^59*b^16*d^7 + 2725
695717376*a^61*b^14*d^7 + 460706545664*a^63*b^12*d^7 + 62631444480*a^65*b^1
0*d^7 + 6710886400*a^67*b^8*d^7 - 16777216*a^69*b^6*d^7 - 167772160*a^71*b^
4*d^7 - 16777216*a^73*b^2*d^7) - (1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i +
a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*
(587202560*a^32*b^46*d^8 + 11693719552*a^34*b^44*d^8 + 110612185088*a^36*b^
42*d^8 + 660435107840*a^38*b^40*d^8 + 2789614813184*a^40*b^38*d^8 + 8853034
893312*a^42*b^36*d^8 + 21878664069120*a^44*b^34*d^8 + 43052282413056*a^46*b
^32*d^8 + 68374033858560*a^48*b^30*d^8 + 88257920499712*a^50*b^28*d^8 + 927
16364988416*a^52*b^26*d^8 + 78893818052608*a^54*b^24*d^8 + 53688433377280*a
^56*b^22*d^8 + 28464224665600*a^58*b^20*d^8 + 11116449103872*a^60*b^18*d^8
+ 2734619099136*a^62*b^16*d^8 + 110377304064*a^64*b^14*d^8 - 221308256256*a
^66*b^12*d^8 - 95546245120*a^68*b^10*d^8 - 18303942656*a^70*b^8*d^8 - 97307
8528*a^72*b^6*d^8 + 234881024*a^74*b^4*d^8 + 33554432*a^76*b^2*d^8 + tan(c
+ d*x)^(1/2)*(1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a
^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*(134217728*a^36*b^45
*d^9 + 2550136832*a^38*b^43*d^9 + 22817013760*a^40*b^41*d^9 + 127506841600*
a^42*b^39*d^9 + 497276682240*a^44*b^37*d^9 + 1430626762752*a^46*b^35*d^9 +
3121367482368*a^48*b^33*d^9 + 5202279137280*a^50*b^31*d^9 + 6502848921600*a
^52*b^29*d^9 + 5635802398720*a^54*b^27*d^9 + 2254320959488*a^56*b^25*d^9 -
2254320959488*a^58*b^23*d^9 - 5635802398720*a^60*b^21*d^9 - 6502848921600*a
^62*b^19*d^9 - 5202279137280*a^64*b^17*d^9 - 3121367482368*a^66*b^15*d^9 -
1430626762752*a^68*b^13*d^9 - 497276682240*a^70*b^11*d^9 - 127506841600*a^7
2*b^9*d^9 - 22817013760*a^74*b^7*d^9 - 2550136832*a^76*b^5*d^9 - 134217728*
a^78*b^3*d^9)))*(1i/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 1
5*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) - 3211264000*a^29
*b^43*d^6 - 47618457600*a^31*b^41*d^6 - 318746132480*a^33*b^39*d^6 - 124835
4893824*a^35*b^37*d^6 - 3038318166016*a^37*b^35*d^6 - 4139457183744*a^39*b^
33*d^6 - 252148973568*a^41*b^31*d^6 + 12756182892544*a^43*b^29*d^6 + 325756
25101312*a^45*b^27*d^6 + 48725922676736*a^47*b^25*d^6 + 50814978621440*a^49
*b^23*d^6 + 38616415862784*a^51*b^21*d^6 + 21485550829568*a^53*b^19*d^6 + 8
584215658496*a^55*b^17*d^6 + 2355709870080*a^57*b^15*d^6 + 411347976192*a^5
9*b^13*d^6 + 42875748352*a^61*b^11*d^6 + 4541906944*a^63*b^9*d^6 + 10066329
60*a^65*b^7*d^6 + 134217728*a^67*b^5*d^6 + 8388608*a^69*b^3*d^6))*(1i/(4*(b
^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d
^2*20i + 15*a^4*b^2*d^2)))^(1/2)*1i + (tan(c + d*x)^(1/2)*(47691333632*a^34
*b^35*d^5 - 3156213760*a^30*b^39*d^5 - 7535067136*a^32*b^37*d^5 - 321126400
*a^28*b^41*d^5 + 451224272896*a^36*b^33*d^5 + 1855390220288*a^38*b^31*d^5 +

4902111674368*a^40*b^29*d^5 + 9182617010176*a^42*b^27*d^5 + 12661071282176
*a^44*b^25*d^5 + 12996430528512*a^46*b^23*d^5 + 9861255397376*a^48*b^21*d^5
 + 5375636013056*a^50*b^19*d^5 + 1966525644800*a^52*b^17*d^5 + 396976193536
*a^54*b^15*d^5 + 1851785216*a^56*b^13*d^5 - 18624806912*a^58*b^11*d^5 - 333
2636672*a^60*b^9*d^5 - 117440512*a^62*b^7*d^5 - 8388608*a^64*b^5*d^5) - (1i
/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3
*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*(12756182892544*a^43*b^29*d^6 - 3211
264000*a^29*b^43*d^6 - 47618457600*a^31*b^41*d^6 - 318746132480*a^33*b^39*d
^6 - 1248354893824*a^35*b^37*d^6 - 3038318166016*a^37*b^35*d^6 - 4139457183
744*a^39*b^33*d^6 - 252148973568*a^41*b^31*d^6 - (tan(c + d*x)^(1/2)*(25690
11200*a^29*b^46*d^7 + 50939822080*a^31*b^44*d^7 + 479763365888*a^33*b^42*d^
7 + 2849006157824*a^35*b^40*d^7 + 11943926562816*a^37*b^38*d^7 + 3751004654
7968*a^39*b^36*d^7 + 91385554272256*a^41*b^34*d^7 + 176470173417472*a^43*b^
32*d^7 + 273612095356928*a^45*b^30*d^7 + 342917...

3.608 $\int \tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=231

$$-\frac{\sqrt{ia-b}\,\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{(a^2+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{4b^{3/2}d}+\frac{\sqrt{ia+b}\,\tanh^{-1}}{}$$

```
[Out] -1/4*(a^2+8*b^2)*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/b
^(3/2)/d-arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*(I*a
-b)^(1/2)/d+arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*
(I*a+b)^(1/2)/d-1/4*a*tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)/b/d+1/2*tan(d
*x+c)^(1/2)*(a+b*tan(d*x+c))^(3/2)/b/d
```

Rubi [A]
time = 1.12, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3647, 3728, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$-\frac{(a^2+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{4b^{3/2}d}-\frac{\sqrt{-b+ia}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{3/2}}{2bd}-\frac{a\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{4bd}+\frac{\sqrt{b+ia}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(5/2)*Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] -((Sqrt[I*a - b]*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
 + d*x]]])/d) - ((a^2 + 8*b^2)*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a
+ b*Tan[c + d*x]]])/(4*b^(3/2)*d) + (Sqrt[I*a + b]*ArcTanh[(Sqrt[I*a + b]*S
qrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/d - (a*Sqrt[Tan[c + d*x]]*Sqr
t[a + b*Tan[c + d*x]])/(4*b*d) + (Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(
3/2))/(2*b*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
```

```
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}\,dx = \frac{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{3/2}}{2bd} + \frac{\int \frac{\sqrt{a+b\tan(c+dx)}\,(\cdots}{\sqrt{\tan\cdots}}}{\cdots}$$

$$= -\frac{a\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{4bd} + \frac{\sqrt{\tan(c+dx)}\,(a+b\,\mathrm{t}\cdots}{2bd}$$

$$= -\frac{a\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{4bd} + \frac{\sqrt{\tan(c+dx)}\,(a+b\,\mathrm{t}\cdots}{2bd}$$

$$= -\frac{a\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{4bd} + \frac{\sqrt{\tan(c+dx)}\,(a+b\,\mathrm{t}\cdots}{2bd}$$

$$= -\frac{a\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{4bd} + \frac{\sqrt{\tan(c+dx)}\,(a+b\,\mathrm{t}\cdots}{2bd}$$

$$= -\frac{a\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{4bd} + \frac{\sqrt{\tan(c+dx)}\,(a+b\,\mathrm{t}\cdots}{2bd}$$

$$= -\frac{a\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{4bd} + \frac{\sqrt{\tan(c+dx)}\,(a+b\,\mathrm{t}\cdots}{2bd}$$

$$= -\frac{(a^2+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{4b^{3/2}d} - \frac{a\sqrt{\tan(c+dx)}\cdots}{\cdots}$$

$$= -\frac{\sqrt{ia-b}\,\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{(a^2+8b^2)\tanh\cdots}{\cdots}$$

Mathematica [A]
time = 1.90, size = 282, normalized size = 1.22

$$\frac{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{3/2}}{2bd} + \frac{-\frac{a\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{2d} + \frac{-2\sqrt[4]{-1}\,\sqrt{-a+ib}\,b\,\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right) - 2\sqrt[4]{-1}\,\sqrt{a+ib}\,b\,\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right) - \frac{\sqrt{a}\,(a^2+8b^2)\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{1+\frac{b\tan(c+dx)}{a}}}{2\sqrt{b}\,\sqrt{a+b\tan(c+dx)}}}{d}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(5/2)*Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] (Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(3/2))/(2*b*d) + (-1/2*(a*Sqrt[Tan
[c + d*x]]*Sqrt[a + b*Tan[c + d*x]])/d + (-2*(-1)^(1/4)*Sqrt[-a + I*b]*b*Ar
cTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]
```

```
]] - 2*(-1)^(1/4)*Sqrt[a + I*b]*b*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan
[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] - (Sqrt[a]*(a^2 + 8*b^2)*ArcSinh[(Sqr
t[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a])/(2*Sqrt[b]*
Sqrt[a + b*Tan[c + d*x]]))/d)/(2*b)
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 1.26, size = 1092020, normalized size = 4727.36

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(1/2),x)
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^(5/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\tan{(c + dx)}}\tan^{\frac{5}{2}}{(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(5/2)*(a+b*tan(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*tan(c + d*x))*tan(c + d*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c+d\,x)^{5/2}\,\sqrt{a+b\tan\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(5/2)*(a + b*tan(c + d*x))^(1/2),x)
[Out] int(tan(c + d*x)^(5/2)*(a + b*tan(c + d*x))^(1/2), x)
```

3.609 $\int \tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=184

$$\frac{i\sqrt{ia-b}\,\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{b}\,d}+\frac{i\sqrt{ia+b}\,\tanh^{-1}\left(\frac{\sqrt{ia+}}{\sqrt{a}}\right.}{d}$$

```
[Out] I*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*(I*a-b)^(1/
2)/d+a*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/d/b^(1/2)+I
*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*(I*a+b)^(1/
2)/d+tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.49, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3651, 3736, 6857, 65, 223, 212, 924, 95, 211, 214}

$$\frac{i\sqrt{-b+ia}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{d}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{b}\,d}+\frac{i\sqrt{b+ia}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(3/2)*Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] (I*Sqrt[I*a - b]*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
 + d*x]]])/d + (a*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d
*x]]])/(Sqrt[b]*d) + (I*Sqrt[I*a + b]*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d
*x]])/Sqrt[a + b*Tan[c + d*x]]])/d + (Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c +
 d*x]])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 3651

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m - 1)*((c +
d*Tan[e + f*x])^n/(f*(m + n - 1))), x] + Dist[1/(m + n - 1), Int[(a + b*Tan
[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a^2*c*(m + n - 1) - b*
(b*c*(m - 1) + a*d*n) + (2*a*b*c + a^2*d - b^2*d)*(m + n - 1)*Tan[e + f*x]
+ b*(b*c*n + a*d*(2*m + n - 2))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
 0] && GtQ[m, 1] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
```

```
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}\,dx = \frac{\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{d} + \int \frac{-\frac{a}{2} - b\tan(c+dx) + \frac{1}{2}\ldots}{\sqrt{\tan(c+dx)}\,\sqrt{a+b\ldots}}$$

$$= \frac{\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{d} + \frac{\text{Subst}\left(\int \frac{-\frac{a}{2}-bx+\frac{ax^2}{2}}{\sqrt{x}\,\sqrt{a+bx}\,(1+\ldots}\right.}{d}$$

$$= \frac{\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{d} + \frac{\text{Subst}\left(\int \left(\frac{a}{2\sqrt{x}\,\sqrt{a+bx}}\ldots\right.\right.}{\ldots}$$

$$= \frac{\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{d} - \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{\sqrt{x}\,(1+x^2)}\,dx, x, \text{t}\ldots\right.}{d}$$

$$= \frac{\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{d} - \frac{\text{Subst}\left(\int \left(\frac{ia-b}{2(i-x)\sqrt{x}\,\sqrt{a+\ldots}}\right.\right.}{\ldots}$$

$$= \frac{\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{d} + \frac{a\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\ldots}{\sqrt{a\ldots}}\right.}{d}$$

$$= \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{b}\,d} + \frac{\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c\ldots}}{d}$$

$$= \frac{i\sqrt{ia-b}\,\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\ldots}}{\sqrt{a+b\ldots}}\right.}{\sqrt{b}\ldots}$$

Mathematica [A]

time = 1.56, size = 217, normalized size = 1.18

$$\frac{-(-1)^{3/4}\sqrt{-a+ib}\,\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+(-1)^{3/4}\sqrt{a+ib}\,\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}+\frac{a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{1+\frac{b\tan(c+dx)}{a}}}{\sqrt{b}\,\sqrt{a+b\tan(c+dx)}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(3/2)*Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] (-((-1)^(3/4)*Sqrt[-a + I*b]*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c +
 d*x]])/Sqrt[a + b*Tan[c + d*x]]]) + (-1)^(3/4)*Sqrt[a + I*b]*ArcTan[((-1)^
(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + Sqrt[Ta
n[c + d*x]]*Sqrt[a + b*Tan[c + d*x]] + (a^(3/2)*ArcSinh[(Sqrt[b]*Sqrt[Tan[c
 + d*x]])/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a])/(Sqrt[b]*Sqrt[a + b*Tan[c
+ d*x]]))/d
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 0.56, size = 1091177, normalized size = 5930.31

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^(3/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \tan{\left(c + dx \right)}} \tan^{\frac{3}{2}}{\left(c + dx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(3/2)*(a+b*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(c + d*x))*tan(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^{3/2}\,\sqrt{a+b\tan\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(1/2),x)
[Out] int(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(1/2), x)
```

3.610 $\int \sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=151

$$\frac{\sqrt{ia-b}\,\mathrm{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{\sqrt{ia+b}\,\tanh^{-1}\left(\frac{\sqrt{\ldots}}{\ldots}\right.}{\ldots}$$

```
[Out] arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*(I*a-b)^(1/2)
/d+2*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*b^(1/2)/d-arc
tanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*(I*a+b)^(1/2)/d
```

Rubi [A]
time = 0.36, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3656, 920, 65, 223, 212, 6857, 95, 211, 214}

$$\frac{\sqrt{-b+ia}\,\mathrm{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{\sqrt{b+ia}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] (Sqrt[I*a - b]*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c +
 d*x]]])/d + (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan
[c + d*x]]])/d - (Sqrt[I*a + b]*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/
Sqrt[a + b*Tan[c + d*x]]])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 920

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Dist[e*(g/c), Int[(d + e*x)^(m - 1)*(f + g*x)^(n - 1), x
], x] + Dist[1/c, Int[Simp[c*d*f - a*e*g + (c*e*f + c*d*g)*x, x]*(d + e*x)^
(m - 1)*((f + g*x)^(n - 1)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[m, 0
] && GtQ[n, 0]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{x}\sqrt{a+bx}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{-b+ax}{\sqrt{x}\sqrt{a+bx}\,(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d} + \frac{b\text{Subst}\left(\int \right.}{}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{-a-ib}{2(i-x)\sqrt{x}\sqrt{a+bx}} + \frac{a-ib}{2\sqrt{x}\,(i+x)\sqrt{a+bx}}\right) dx, x, \text{ta}\right.}{d}$$

$$= \frac{(a-ib)\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(i+x)\sqrt{a+bx}}\,dx, x, \tan(c+dx)\right)}{2d} - \frac{(a-}{}$$

$$= \frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{(a-ib)\text{Subst}\left(\int \frac{}{i-(-}\right.}{}$$

$$= \frac{\sqrt{ia-b}\,\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{2\sqrt{b}\,\tanh^{-1}\left(\right.}{}$$

Mathematica [A]
time = 0.56, size = 189, normalized size = 1.25

$$\frac{\sqrt[4]{-1}\left(\sqrt{-a+ib}\,\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+\sqrt{a+ib}\,\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\right)+\frac{2\sqrt{a}\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{1+\frac{b\tan(c+dx)}{a}}}{\sqrt{a+b\tan(c+dx)}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] ((-1)^(1/4)*(Sqrt[-a + I*b]*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c +
d*x]])/Sqrt[a + b*Tan[c + d*x]]] + Sqrt[a + I*b]*ArcTan[((-1)^(1/4)*Sqrt[a
+ I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]) + (2*Sqrt[a]*Sqrt[b]*
ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a])
/Sqrt[a + b*Tan[c + d*x]])/d
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 0.44, size = 1089481, normalized size = 7215.11

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(d*x + c) + a)*sqrt(tan(d*x + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \tan{\left(c + dx \right)}} \sqrt{\tan{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/2)*(a+b*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(c + d*x))*sqrt(tan(c + d*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 6.91, size = 286, normalized size = 1.89

$$\frac{4\sqrt{b}\,\operatorname{atanh}\left(\frac{\sqrt{b}\,\sqrt{\tan\left(c+d\,x\right)}}{\sqrt{a+b\tan\left(c+d\,x\right)}-\sqrt{a}}\right)}{d}+\operatorname{atan}\left(\frac{2\left(d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{a+b\tan\left(c+d\,x\right)}\,\sqrt{-\frac{-b+a\,1\mathrm{i}}{4\,d^2}}\,1\mathrm{i}-\sqrt{a}\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{-\frac{-b+a\,1\mathrm{i}}{4\,d^2}}\,1\mathrm{i}\right)}{a+b\tan\left(c+d\,x\right)-\sqrt{a}\,\sqrt{a+b\tan\left(c+d\,x\right)}}\right)\sqrt{-\frac{-b+a\,1\mathrm{i}}{4\,d^2}}\,2\mathrm{i}+\operatorname{atan}\left(\frac{2\left(\sqrt{a}\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{b+a\,1\mathrm{i}}{4\,d^2}}-d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{a+b\tan\left(c+d\,x\right)}\,\sqrt{\frac{b+a\,1\mathrm{i}}{4\,d^2}}\right)}{a\,1\mathrm{i}+b\tan\left(c+d\,x\right)\,1\mathrm{i}-\sqrt{a}\,\sqrt{a+b\tan\left(c+d\,x\right)}\,1\mathrm{i}}\right)\sqrt{\frac{b+a\,1\mathrm{i}}{4\,d^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(1/2),x)
```

```
[Out] atan((2*(d*tan(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(1/2)*(-(a*1i - b)/(4*d^
2))^(1/2)*1i - a^(1/2)*d*tan(c + d*x)^(1/2)*(-(a*1i - b)/(4*d^2))^(1/2)*1i)
)/(a + b*tan(c + d*x) - a^(1/2)*(a + b*tan(c + d*x))^(1/2)))*(-(a*1i - b)/(
4*d^2))^(1/2)*2i + atan((2*(a^(1/2)*d*tan(c + d*x)^(1/2)*((a*1i + b)/(4*d^2
))^(1/2) - d*tan(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(1/2)*((a*1i + b)/(4*d
^2))^(1/2)))/(a*1i + b*tan(c + d*x)*1i - a^(1/2)*(a + b*tan(c + d*x))^(1/2)
*1i))*((a*1i + b)/(4*d^2))^(1/2)*2i + (4*b^(1/2)*atanh((b^(1/2)*tan(c + d*x
)^(1/2))/((a + b*tan(c + d*x))^(1/2) - a^(1/2))))/d
```

3.611 $\int \frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{\tan(c+dx)}}\,dx$

Optimal. Leaf size=115

$$-\frac{i\sqrt{ia-b}\,\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{i\sqrt{ia+b}\,\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

```
[Out] -I*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*(I*a-b)^(1
/2)/d-I*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*(I*a
+b)^(1/2)/d
```

Rubi [A]
time = 0.10, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3656, 924, 95, 211, 214}

$$-\frac{i\sqrt{-b+ia}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{i\sqrt{b+ia}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Tan[c + d*x]]/Sqrt[Tan[c + d*x]],x]
```

```
[Out] ((-I)*Sqrt[I*a - b]*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Ta
n[c + d*x]]])/d - (I*Sqrt[I*a + b]*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]
])/Sqrt[a + b*Tan[c + d*x]]])/d
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{\tan(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{\sqrt{x}\,(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{ia-b}{2(i-x)\sqrt{x}\,\sqrt{a+bx}} + \frac{ia+b}{2\sqrt{x}\,(i+x)\sqrt{a+bx}}\right) dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{(ia-b)\text{Subst}\left(\int \frac{1}{(i-x)\sqrt{x}\,\sqrt{a+bx}}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{(ia+b)\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{(ia-b)\text{Subst}\left(\int \frac{1}{i-(a+ib)x^2}\,dx, x, \frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{(ia+b)\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{i\sqrt{ia-b}\,\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{i\sqrt{ia+b}\,\tanh^{-1}\left(\frac{\sqrt{ia+b}\cdots}{\sqrt{a+\cdots}}\right.}{d}$$

Mathematica [A]
time = 0.10, size = 123, normalized size = 1.07

$$\frac{(-1)^{3/4}\left(\sqrt{-a+ib}\,\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)-\sqrt{a+ib}\,\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Tan[c + d*x]]/Sqrt[Tan[c + d*x]],x]
```

```
[Out] ((-1)^(3/4)*(Sqrt[-a + I*b]*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c +
d*x]])/Sqrt[a + b*Tan[c + d*x]]] - Sqrt[a + I*b]*ArcTan[((-1)^(1/4)*Sqrt[a
+ I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]))/d
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 0.46, size = 1085178, normalized size = 9436.33

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/2),x)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*tan(d*x + c) + a)/sqrt(tan(d*x + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \tan{\left(c + dx \right)}}}{\sqrt{\tan{\left(c + dx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(1/2)/tan(d*x+c)**(1/2),x)
[Out] Integral(sqrt(a + b*tan(c + d*x))/sqrt(tan(c + d*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 6.08, size = 224, normalized size = 1.95

$$-\mathrm{atan}\left(\frac{\sqrt{a}\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{-\frac{b+a\,1{}\mathrm{i}}{d^2}}-d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{-\frac{b+a\,1{}\mathrm{i}}{d^2}}\,\sqrt{a+b\,\tan\left(c+d\,x\right)}}{a+b\,\tan\left(c+d\,x\right)-\sqrt{a}\,\sqrt{a+b\,\tan\left(c+d\,x\right)}}\right)\,\sqrt{-\frac{b+a\,1{}\mathrm{i}}{d^2}}\,1{}\mathrm{i}-\mathrm{atan}\left(\frac{d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{-b+a\,1{}\mathrm{i}}{d^2}}\,\sqrt{a+b\,\tan\left(c+d\,x\right)}-\sqrt{a}\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{-b+a\,1{}\mathrm{i}}{d^2}}}{a+b\,\tan\left(c+d\,x\right)-\sqrt{a}\,\sqrt{a+b\,\tan\left(c+d\,x\right)}}\right)\,\sqrt{\frac{-b+a\,1{}\mathrm{i}}{d^2}}\,1{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^(1/2)/tan(c + d*x)^(1/2),x)
```

```
[Out] - atan((a^(1/2)*d*tan(c + d*x)^(1/2)*(-(a*1i + b)/d^2)^(1/2) - d*tan(c + d*
x)^(1/2)*(-(a*1i + b)/d^2)^(1/2)*(a + b*tan(c + d*x))^(1/2))/(a + b*tan(c +
 d*x) - a^(1/2)*(a + b*tan(c + d*x))^(1/2)))*(-(a*1i + b)/d^2)^(1/2)*1i - a
tan((d*tan(c + d*x)^(1/2)*((a*1i - b)/d^2)^(1/2)*(a + b*tan(c + d*x))^(1/2)
 - a^(1/2)*d*tan(c + d*x)^(1/2)*((a*1i - b)/d^2)^(1/2))/(a + b*tan(c + d*x)
 - a^(1/2)*(a + b*tan(c + d*x))^(1/2)))*((a*1i - b)/d^2)^(1/2)*1i
```

3.612 $\int \frac{\sqrt{a+b\tan(c+dx)}}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=139

$$-\frac{\sqrt{ia-b}\,\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{\sqrt{ia+b}\,\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{2\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

```
[Out] -arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*(I*a-b)^(1/2
)/d+arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*(I*a+b)^
(1/2)/d-2*(a+b*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3649, 3697, 3696, 95, 209, 212}

$$-\frac{\sqrt{-b+ia}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{2\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}+\frac{\sqrt{b+ia}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Tan[c + d*x]]/Tan[c + d*x]^(3/2),x]
```

```
[Out] -((Sqrt[I*a - b]*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
 + d*x]]])/d) + (Sqrt[I*a + b]*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/S
qrt[a + b*Tan[c + d*x]]])/d - (2*Sqrt[a + b*Tan[c + d*x]])/(d*Sqrt[Tan[c +
d*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\tan(c+dx)}}{\tan^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}} - 2\int \frac{-\frac{b}{2}+\frac{1}{2}a\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\,dx$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}} - \frac{1}{2}(-ia-b)\int \frac{1+i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}} - \frac{(ia-b)\text{Subst}\left(\int \frac{1}{(1+ix)\sqrt{x}\sqrt{a+bx}}\,dx, x, \tan(c+\right.}{2d}$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}} - \frac{(ia-b)\text{Subst}\left(\int \frac{1}{1-(-ia+b)x^2}\,dx, x, \frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+}}\right.}{d}$$

$$= -\frac{\sqrt{ia-b}\,\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{\sqrt{ia+b}\,\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{}}{\sqrt{a+b\tan}}\right.}{d}$$

Mathematica [**A**]
time = 0.39, size = 154, normalized size = 1.11

$$-\frac{\sqrt[4]{-1}\sqrt{-a+ib}\,\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+\sqrt[4]{-1}\sqrt{a+ib}\,\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+\frac{2\sqrt{a+b\tan(c+dx)}}{\sqrt{\tan(c+dx)}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Tan[c + d*x]]/Tan[c + d*x]^(3/2),x]
[Out] -(((-1)^(1/4)*Sqrt[-a + I*b]*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c +
 d*x]])/Sqrt[a + b*Tan[c + d*x]]] + (-1)^(1/4)*Sqrt[a + I*b]*ArcTan[((-1)^(
1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + (2*Sqrt[
a + b*Tan[c + d*x]])/Sqrt[Tan[c + d*x]])/d)
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 0.46, size = 1089777, normalized size = 7840.12

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(3/2),x)
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(d*x + c) + a)/tan(d*x + c)^(3/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\tan{(c + dx)}}}{\tan^{\frac{3}{2}}{(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(1/2)/tan(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(c + d*x))/tan(c + d*x)**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + b\tan\left(c + d\,x\right)}}{{\tan\left(c + d\,x\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^(1/2)/tan(c + d*x)^(3/2),x)
```

```
[Out] int((a + b*tan(c + d*x))^(1/2)/tan(c + d*x)^(3/2), x)
```

3.613 $\int \frac{\sqrt{a+b\tan(c+dx)}}{\tan^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=181

$$\frac{i\sqrt{ia-b}\,\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{i\sqrt{ia+b}\,\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{2\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

```
[Out] I*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*(I*a-b)^(1/
2)/d+I*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*(I*a+
b)^(1/2)/d-2/3*b*(a+b*tan(d*x+c))^(1/2)/a/d/tan(d*x+c)^(1/2)-2/3*(a+b*tan(d
*x+c))^(1/2)/d/tan(d*x+c)^(3/2)
```

Rubi [A]
time = 0.24, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3649, 3730, 21, 3656, 924, 95, 211, 214}

$$\frac{i\sqrt{-b+ia}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{2\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2b\sqrt{a+b\tan(c+dx)}}{3ad\sqrt{\tan(c+dx)}}+\frac{i\sqrt{b+ia}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Tan[c + d*x]]/Tan[c + d*x]^(5/2),x]
```

```
[Out] (I*Sqrt[I*a - b]*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
 + d*x]]])/d + (I*Sqrt[I*a + b]*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/
Sqrt[a + b*Tan[c + d*x]]])/d - (2*Sqrt[a + b*Tan[c + d*x]])/(3*d*Tan[c + d*
x]^(3/2)) - (2*b*Sqrt[a + b*Tan[c + d*x]])/(3*a*d*Sqrt[Tan[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
```

```
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{\sqrt{a+b\tan(c+dx)}}{\tan^{\frac{5}{2}}(c+dx)}\,dx = -\frac{2\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{2}{3}\int \frac{-\frac{b}{2}+\frac{3}{2}a\tan(c+dx)+b\tan^2(c+dx)}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{2b\sqrt{a+b\tan(c+dx)}}{3ad\sqrt{\tan(c+dx)}} + \frac{4\int \frac{-\frac{3a^2}{4}-\frac{3}{4}ab\tan(}{\sqrt{\tan(c+dx)}\sqrt{a+}}}{3a}$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{2b\sqrt{a+b\tan(c+dx)}}{3ad\sqrt{\tan(c+dx)}} - \int \frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{\tan(c+dx)}}\,d$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{2b\sqrt{a+b\tan(c+dx)}}{3ad\sqrt{\tan(c+dx)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{\sqrt{x}\,(1+x^2)}\,dx, x\right.}{d}$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{2b\sqrt{a+b\tan(c+dx)}}{3ad\sqrt{\tan(c+dx)}} - \frac{\text{Subst}\left(\int \left(\frac{ia-b}{2(i-x)\sqrt{x}\sqrt{a}}\right.\right.}{}$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{2b\sqrt{a+b\tan(c+dx)}}{3ad\sqrt{\tan(c+dx)}} - \frac{(ia-b)\text{Subst}\left(\int \frac{}{(i-x)\sqrt{x}}\right.}{}$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{2b\sqrt{a+b\tan(c+dx)}}{3ad\sqrt{\tan(c+dx)}} - \frac{(ia-b)\text{Subst}\left(\int \frac{1}{i-(a+ib)x}\right.}{}$$

$$= \frac{i\sqrt{ia-b}\,\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{i\sqrt{ia+b}\,\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{}}{\sqrt{a+b\tan}}\right.}{d}$$

Mathematica [A]
time = 0.60, size = 161, normalized size = 0.89

$$\frac{-3(-1)^{3/4}\sqrt{-a+ib}\,\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+3(-1)^{3/4}\sqrt{a+ib}\,\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)-\frac{2(a+b\tan(c+dx))^{3/2}}{a\tan^{\frac{3}{2}}(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Tan[c + d*x]]/Tan[c + d*x]^(5/2),x]
```

```
[Out] (-3*(-1)^(3/4)*Sqrt[-a + I*b]*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c
+ d*x]])/Sqrt[a + b*Tan[c + d*x]]] + 3*(-1)^(3/4)*Sqrt[a + I*b]*ArcTan[((-1
)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] - (2*(a
 + b*Tan[c + d*x])^(3/2))/(a*Tan[c + d*x]^(3/2)))/(3*d)
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 1.08, size = 1090997, normalized size = 6027.61

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(d*x + c) + a)/tan(d*x + c)^(5/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \tan{\left(c + dx \right)}}}{\tan^{\frac{5}{2}}{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(1/2)/tan(d*x+c)**(5/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(c + d*x))/tan(c + d*x)**(5/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\tan\left(c+d\,x\right)}}{{\tan\left(c+d\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^(1/2)/tan(c + d*x)^(5/2),x)
```

```
[Out] int((a + b*tan(c + d*x))^(1/2)/tan(c + d*x)^(5/2), x)
```

3.614 $\int \frac{\sqrt{a+b\tan(c+dx)}}{\tan^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=221

$$\frac{\sqrt{ia-b}\,\mathrm{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{\sqrt{ia+b}\,\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{2\sqrt{a+b\tan(}}{5d\tan^{\frac{5}{2}}(c}$$

```
[Out] arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*(I*a-b)^(1/2)
/d-arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*(I*a+b)^(
1/2)/d+2/15*(15*a^2+2*b^2)*(a+b*tan(d*x+c))^(1/2)/a^2/d/tan(d*x+c)^(1/2)-2/
5*(a+b*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(5/2)-2/15*b*(a+b*tan(d*x+c))^(1/2)/a
/d/tan(d*x+c)^(3/2)
```

Rubi [A]
time = 0.54, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3649, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(15a^2+2b^2)\sqrt{a+b\tan(c+dx)}}{15a^2d\sqrt{\tan(c+dx)}}+\frac{\sqrt{-b+ia}\,\mathrm{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{2b\sqrt{a+b\tan(c+dx)}}{15ad\tan^{\frac{3}{2}}(c+dx)}-\frac{2\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{\sqrt{b+ia}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Tan[c + d*x]]/Tan[c + d*x]^(7/2),x]
```

```
[Out] (Sqrt[I*a - b]*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c +
 d*x]]])/d - (Sqrt[I*a + b]*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt
[a + b*Tan[c + d*x]]])/d - (2*Sqrt[a + b*Tan[c + d*x]])/(5*d*Tan[c + d*x]^(
5/2)) - (2*b*Sqrt[a + b*Tan[c + d*x]])/(15*a*d*Tan[c + d*x]^(3/2)) + (2*(15
*a^2 + 2*b^2)*Sqrt[a + b*Tan[c + d*x]])/(15*a^2*d*Sqrt[Tan[c + d*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
```

```
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{\sqrt{a+b\tan(c+dx)}}{\tan^{\frac{7}{2}}(c+dx)}\,dx = -\frac{2\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{b}{2}+\frac{5}{2}a\tan(c+dx)+2b\tan^2(c+dx)}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{2b\sqrt{a+b\tan(c+dx)}}{15ad\tan^{\frac{3}{2}}(c+dx)} + \frac{4\int \frac{\frac{1}{4}(-15a^2-2b^2)-\frac{15}{4}ab\tan(}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+}}}{15a}$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{2b\sqrt{a+b\tan(c+dx)}}{15ad\tan^{\frac{3}{2}}(c+dx)} + \frac{2(15a^2+2b^2)\sqrt{a+b\,\mathrm{ta}}}{15a^2d\sqrt{\tan(c+}}$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{2b\sqrt{a+b\tan(c+dx)}}{15ad\tan^{\frac{3}{2}}(c+dx)} + \frac{2(15a^2+2b^2)\sqrt{a+b\,\mathrm{ta}}}{15a^2d\sqrt{\tan(c+}}$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{2b\sqrt{a+b\tan(c+dx)}}{15ad\tan^{\frac{3}{2}}(c+dx)} + \frac{2(15a^2+2b^2)\sqrt{a+b\,\mathrm{ta}}}{15a^2d\sqrt{\tan(c+}}$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{2b\sqrt{a+b\tan(c+dx)}}{15ad\tan^{\frac{3}{2}}(c+dx)} + \frac{2(15a^2+2b^2)\sqrt{a+b\,\mathrm{ta}}}{15a^2d\sqrt{\tan(c+}}$$

$$= \frac{\sqrt{ia-b}\,\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{\sqrt{ia+b}\,\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{}}{\sqrt{a+b\,\mathrm{ta}}}\right.}{d}$$

Mathematica [A]

time = 1.50, size = 197, normalized size = 0.89

$$\frac{15\sqrt[4]{-1}\,\sqrt{-a+ib}\,\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+15\sqrt[4]{-1}\,\sqrt{a+ib}\,\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+\frac{2\sqrt{a+b\tan(c+dx)}\,(-3a^2-ab\tan(c+dx)+(15a^2+2b^2)\tan^2(c+dx))}{a^2\tan^{\frac{5}{2}}(c+dx)}}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Tan[c + d*x]]/Tan[c + d*x]^(7/2),x]
```

```
[Out] (15*(-1)^(1/4)*Sqrt[-a + I*b]*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c
+ d*x]])/Sqrt[a + b*Tan[c + d*x]]] + 15*(-1)^(1/4)*Sqrt[a + I*b]*ArcTan[((-
1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + (2*S
qrt[a + b*Tan[c + d*x]]*(-3*a^2 - a*b*Tan[c + d*x] + (15*a^2 + 2*b^2)*Tan[c
 + d*x]^2))/(a^2*Tan[c + d*x]^(5/2)))/(15*d)
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 0.40, size = 1092009, normalized size = 4941.22

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(7/2),x)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(d*x + c) + a)/tan(d*x + c)^(7/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\tan\left(c + dx\right)}}{\tan^{\frac{7}{2}}\left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(1/2)/tan(d*x+c)**(7/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(c + d*x))/tan(c + d*x)**(7/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\tan\left(c+d\,x\right)}}{{\tan\left(c+d\,x\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^(1/2)/tan(c + d*x)^(7/2),x)
```

```
[Out] int((a + b*tan(c + d*x))^(1/2)/tan(c + d*x)^(7/2), x)
```

3.615 $\int \tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=280

$$\frac{i(ia-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{a(a^2+24b^2)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{8b^{3/2}d}-\frac{i(ia+b)^{3/2}\tan$$

```
[Out] I*(I*a-b)^(3/2)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2
))/d-1/8*a*(a^2+24*b^2)*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(
1/2))/b^(3/2)/d-I*(I*a+b)^(3/2)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b
*tan(d*x+c))^(1/2))/d-1/8*(a^2+8*b^2)*tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/
2)/b/d-1/12*a*tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(3/2)/b/d+1/3*tan(d*x+c)^(1
/2)*(a+b*tan(d*x+c))^(5/2)/b/d
```

Rubi [A]

time = 1.42, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3647, 3728, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$-\frac{(a^2+8b^2)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{8bd}-\frac{a(a^2+24b^2)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{8b^{3/2}d}+\frac{i(-b+ia)^{3/2}\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}{3bd}-\frac{a\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{12bd}-\frac{i(b+ia)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] (I*(I*a - b)^(3/2)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan
[c + d*x]]])/d - (a*(a^2 + 24*b^2)*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqr
t[a + b*Tan[c + d*x]]])/(8*b^(3/2)*d) - (I*(I*a + b)^(3/2)*ArcTanh[(Sqrt[I*
a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/d - ((a^2 + 8*b^2)*Sq
rt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]])/(8*b*d) - (a*Sqrt[Tan[c + d*x]]*
(a + b*Tan[c + d*x])^(3/2))/(12*b*d) + (Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d
*x])^(5/2))/(3*b*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
```

```
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
```

```
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}\,dx = \frac{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{5/2}}{3bd} + \frac{\int \frac{(a+b\tan(c+dx))^{3/2}\left(-\frac{a}{2}-3b\right.}{\sqrt{\tan(c}}}{3}$$

$$= -\frac{a\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{3/2}}{12bd} + \frac{\sqrt{\tan(c+dx)}\,(a+}{3bd}$$

$$= -\frac{(a^2+8b^2)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{8bd} - \frac{a\sqrt{\tan(c+}}{}$$

$$= -\frac{(a^2+8b^2)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{8bd} - \frac{a\sqrt{\tan(c+}}{}$$

$$= -\frac{(a^2+8b^2)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{8bd} - \frac{a\sqrt{\tan(c+}}{}$$

$$= -\frac{(a^2+8b^2)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{8bd} - \frac{a\sqrt{\tan(c+}}{}$$

$$= -\frac{(a^2+8b^2)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{8bd} - \frac{a\sqrt{\tan(c+}}{}$$

$$= -\frac{(a^2+8b^2)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{8bd} - \frac{a\sqrt{\tan(c+}}{}$$

$$= -\frac{a(a^2+24b^2)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{8b^{3/2}d} - \frac{(a^2+8b^2)\sqrt{}}{}$$

$$= \frac{i(ia-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{a(a^2+24b^2)\,\mathrm{ta}}{}$$

<u>Mathematica</u> [A]
time = 3.90, size = 313, normalized size = 1.12

$$\frac{-3a^{3/2}(a^2+24b^2)\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{1+\frac{b\tan(c+dx)}{a}}+\sqrt{b}\left(24\sqrt[4]{-1}(-a+ib)^{3/2}b\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{a+b\tan(c+dx)}-24\sqrt[4]{-1}(a+ib)^{3/2}b\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{a+b\tan(c+dx)}+\sqrt{\tan(c+dx)}(3(a^3-8ab^2)+b(17a^2-24b^2)\tan(c+dx)+22ab^2\tan^2(c+dx)+8b^3\tan^3(c+dx))\right)}{24b^{3/2}d\sqrt{a+b\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] (-3*a^(3/2)*(a^2 + 24*b^2)*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sq
rt[1 + (b*Tan[c + d*x])/a] + Sqrt[b]*(24*(-1)^(1/4)*(-a + I*b)^(3/2)*b*ArcT
```

```
an[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]
*Sqrt[a + b*Tan[c + d*x]] - 24*(-1)^(1/4)*(a + I*b)^(3/2)*b*ArcTan[((-1)^(1
/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[a + b*
Tan[c + d*x]] + Sqrt[Tan[c + d*x]]*(3*(a^3 - 8*a*b^2) + b*(17*a^2 - 24*b^2)
*Tan[c + d*x] + 22*a*b^2*Tan[c + d*x]^2 + 8*b^3*Tan[c + d*x]^3)))/(24*b^(3/
2)*d*Sqrt[a + b*Tan[c + d*x]])
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 0.76, size = 1347974, normalized size = 4814.19

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(3/2)*tan(d*x + c)^(5/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(5/2)*(a+b*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c+d\,x)^{5/2}\,(a+b\tan(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(5/2)*(a + b*tan(c + d*x))^(3/2),x)
[Out] int(tan(c + d*x)^(5/2)*(a + b*tan(c + d*x))^(3/2), x)
```

3.616 $\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=226

$$\frac{(ia-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{(3a^2-8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{4\sqrt{b}\,d}+\frac{(ia+b)^{3/2}\tan$$

```
[Out] (I*a-b)^(3/2)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))
/d+(I*a+b)^(3/2)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1
/2))/d+1/4*(3*a^2-8*b^2)*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^
(1/2))/d/b^(1/2)+3/4*a*tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)/d+1/2*tan(d*
x+c)^(1/2)*(a+b*tan(d*x+c))^(3/2)/d
```

Rubi [A]
time = 1.09, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3651, 3728, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{(3a^2-8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{4\sqrt{b}\,d}+\frac{(-b+ia)^{3/2}\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{3/2}}{2d}+\frac{3a\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{4d}+\frac{(b+ia)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((I*a - b)^(3/2)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
 + d*x]]])/d + ((3*a^2 - 8*b^2)*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a
 + b*Tan[c + d*x]]])/(4*Sqrt[b]*d) + ((I*a + b)^(3/2)*ArcTanh[(Sqrt[I*a + b
]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/d + (3*a*Sqrt[Tan[c + d*x]
]*Sqrt[a + b*Tan[c + d*x]])/(4*d) + (Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x]
)^(3/2))/(2*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3651

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m - 1)*((c +
d*Tan[e + f*x])^n/(f*(m + n - 1))), x] + Dist[1/(m + n - 1), Int[(a + b*Tan
[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a^2*c*(m + n - 1) - b*
(b*c*(m - 1) + a*d*n) + (2*a*b*c + a^2*d - b^2*d)*(m + n - 1)*Tan[e + f*x]
+ b*(b*c*n + a*d*(2*m + n - 2))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
 0] && GtQ[m, 1] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}\,dx = \frac{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{3/2}}{2d} + \frac{1}{2}\int \frac{\sqrt{a+b\tan(c+dx}}{}$$

$$= \frac{3a\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{4d} + \frac{\sqrt{\tan(c+dx)}\,(a+b}{2d}$$

$$= \frac{3a\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{4d} + \frac{\sqrt{\tan(c+dx)}\,(a+b}{2d}$$

$$= \frac{3a\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{4d} + \frac{\sqrt{\tan(c+dx)}\,(a+b}{2d}$$

$$= \frac{3a\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{4d} + \frac{\sqrt{\tan(c+dx)}\,(a+b}{2d}$$

$$= \frac{3a\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{4d} + \frac{\sqrt{\tan(c+dx)}\,(a+b}{2d}$$

$$= \frac{3a\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{4d} + \frac{\sqrt{\tan(c+dx)}\,(a+b}{2d}$$

$$= \frac{(3a^2-8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{4\sqrt{b}\,d} + \frac{3a\sqrt{\tan(c+dx)}}{}$$

$$= \frac{(ia-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{(3a^2-8b^2)\tan}{}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 850 vs. $2(226) = 452$.
time = 6.12, size = 850, normalized size = 3.76

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] (2*a*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]]*(1 + (b*Tan[c + d*x])/a)^2
*((3*Sqrt[a]*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(8*Sqrt[b]*Sqrt
[Tan[c + d*x]]*(1 + (b*Tan[c + d*x])/a)^(5/2)) + (3/(2*(1 + (b*Tan[c + d*x]
)/a)^2) + (1 + (b*Tan[c + d*x])/a)^(-1))/4) - I*(-2*b*Sqrt[Tan[c + d*x]]*Sq
```

```
rt[a + b*Tan[c + d*x]]*(1 + (b*Tan[c + d*x])/a)*((Sqrt[a]*ArcSinh[(Sqrt[b]*
Sqrt[Tan[c + d*x]])/Sqrt[a]])/(2*Sqrt[b]*Sqrt[Tan[c + d*x]]*(1 + (b*Tan[c +
 d*x])/a)^(3/2)) + 1/(2*(1 + (b*Tan[c + d*x])/a))) - (-a - I*b)*((2*(-1)^(1
/4)*(-a - I*b)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a
+ b*Tan[c + d*x]]])/Sqrt[a + I*b] - (2*Sqrt[a]*Sqrt[b]*ArcSinh[(Sqrt[b]*Sqr
t[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a])/Sqrt[a + b*Tan[c +
d*x]])))/(2*d) + (2*a*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]]*(1 + (b*T
an[c + d*x])/a)^2*((3*Sqrt[a]*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]
)/(8*Sqrt[b]*Sqrt[Tan[c + d*x]]*(1 + (b*Tan[c + d*x])/a)^(5/2)) + (3/(2*(1
+ (b*Tan[c + d*x])/a)^2) + (1 + (b*Tan[c + d*x])/a)^(-1))/4) - I*(2*b*Sqrt[
Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]]*(1 + (b*Tan[c + d*x])/a)*((Sqrt[a]*A
rcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(2*Sqrt[b]*Sqrt[Tan[c + d*x]]
*(1 + (b*Tan[c + d*x])/a)^(3/2)) + 1/(2*(1 + (b*Tan[c + d*x])/a))) - (-a +
I*b)*(2*(-1)^(1/4)*Sqrt[-a + I*b]*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Ta
n[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + (2*Sqrt[a]*Sqrt[b]*ArcSinh[(Sqrt[b
]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a])/Sqrt[a + b*Tan
[c + d*x]])))/(2*d)
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 1.18, size = 1346578, normalized size = 5958.31

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(3/2),x)
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^(3/2)*tan(d*x + c)^(3/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

[Out] Timed out

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))^{\frac{3}{2}}\tan^{\frac{3}{2}}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(3/2)*(a+b*tan(d*x+c))**(3/2),x)
[Out] Integral((a + b*tan(c + d*x))**(3/2)*tan(c + d*x)**(3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c+d\,x)^{3/2}\,(a+b\tan(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(3/2),x)
[Out] int(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(3/2), x)
```

3.617 $\int \sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=186

$$-\frac{i(ia-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{3a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{i(ia+b)^{3/2}\tanh}{}$$

```
[Out] -I*(I*a-b)^(3/2)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/
2))/d+I*(I*a+b)^(3/2)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c
))^(1/2))/d+3*a*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*b^
(1/2)/d+b*tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.89, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3651, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$-\frac{i(-b+ia)^{3/2}\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{b\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}+\frac{3a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{i(b+ia)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-I)*(I*a - b)^(3/2)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*
Tan[c + d*x]]])/d + (3*a*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[
a + b*Tan[c + d*x]]])/d + (I*(I*a + b)^(3/2)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Ta
n[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/d + (b*Sqrt[Tan[c + d*x]]*Sqrt[a +
b*Tan[c + d*x]])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3651

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m - 1)*((c +
d*Tan[e + f*x])^n/(f*(m + n - 1))), x] + Dist[1/(m + n - 1), Int[(a + b*Tan
[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a^2*c*(m + n - 1) - b*
(b*c*(m - 1) + a*d*n) + (2*a*b*c + a^2*d - b^2*d)*(m + n - 1)*Tan[e + f*x]
+ b*(b*c*n + a*d*(2*m + n - 2))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
 0] && GtQ[m, 1] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

```
[n, 0]
```

Rubi steps

$$\int \sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{3/2}\,dx = \frac{b\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{d} + \int \frac{-\frac{ab}{2}+(a^2-b^2)\tan}{\sqrt{\tan(c+dx)}}$$

$$= \frac{b\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{d} + \frac{\text{Subst}\left(\int \frac{-\frac{ab}{2}+(a^2-b^2)x}{\sqrt{x}\,\sqrt{a+bx}}\right.}{}$$

$$= \frac{b\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{d} + \frac{\text{Subst}\left(\int \left(\frac{3ab}{2\sqrt{x}\,\sqrt{a+}}\right.\right.}{}$$

$$= \frac{b\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{d} - \frac{\text{Subst}\left(\int \frac{2ab-(a^2-b}{\sqrt{x}\,\sqrt{a+bx}}\right.}{}$$

$$= \frac{b\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{d} - \frac{\text{Subst}\left(\int \left(\frac{a^2+2iab}{2(i-x)\sqrt{x}\,\sqrt{}}\right.\right.}{}$$

$$= \frac{b\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{d} + \frac{(a-ib)^2\text{Subst}\left(\int \frac{}{\sqrt{x}}\right.}{}$$

$$= \frac{3a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{b\sqrt{\tan(c+dx)}\,\sqrt{a}}{d}$$

$$= -\frac{i(ia-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{3a\sqrt{b}\,\tanh}{}$$

Mathematica [A]
time = 2.55, size = 219, normalized size = 1.18

$$\frac{-\sqrt[4]{-1}\,(-a+ib)^{3/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+\sqrt[4]{-1}\,(a+ib)^{3/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+b\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}+\frac{3\sqrt{a}\,\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{a+b\tan(c+dx)}}{\sqrt{1+\frac{b\tan(c+dx)}{a}}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] (-((-1)^(1/4)*(-a + I*b)^(3/2)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c
 + d*x]])/Sqrt[a + b*Tan[c + d*x]]]) + (-1)^(1/4)*(a + I*b)^(3/2)*ArcTan[((
-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + b*S
```

```
qrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]] + (3*Sqrt[a]*Sqrt[b]*ArcSinh[(Sq
rt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[a + b*Tan[c + d*x]])/Sqrt[1 + (b*Ta
n[c + d*x])/a])/d
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 1.09, size = 1345543, normalized size = 7234.10

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(3/2),x)
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^(3/2)*sqrt(tan(d*x + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tan (c + dx))^{\frac{3}{2}} \sqrt{\tan (c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/2)*(a+b*tan(d*x+c))**(3/2),x)
[Out] Integral((a + b*tan(c + d*x))**(3/2)*sqrt(tan(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(3/2)*sqrt(tan(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\tan\left(c+d\,x\right)}\,\left(a+b\tan(c+d\,x)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(3/2),x)
```

```
[Out] int(tan(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(3/2), x)
```

3.618 $\int \frac{(a+b\tan(c+dx))^{3/2}}{\sqrt{\tan(c+dx)}}\,dx$

Optimal. Leaf size=152

$$-\frac{(ia-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{(ia+b)^{3/2}\tanh^{-1}}{}$$

```
[Out] -(I*a-b)^(3/2)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2)
)/d+2*b^(3/2)*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/d-(I
*a+b)^(3/2)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/
d
```

Rubi [A]
time = 0.43, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3656, 924, 65, 223, 212, 6857, 95, 211, 214}

$$-\frac{(-b+ia)^{3/2}\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{(b+ia)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^(3/2)/Sqrt[Tan[c + d*x]],x]
```

```
[Out] -(((I*a - b)^(3/2)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan
[c + d*x]]])/d) + (2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a +
b*Tan[c + d*x]]])/d - ((I*a + b)^(3/2)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c +
d*x]])/Sqrt[a + b*Tan[c + d*x]]])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(c+dx))^{3/2}}{\sqrt{\tan(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{\sqrt{x}\,(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b^2}{\sqrt{x}\,\sqrt{a+bx}} + \frac{a^2-b^2+2abx}{\sqrt{x}\,\sqrt{a+bx}\,(1+x^2)}\right) dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{a^2-b^2+2abx}{\sqrt{x}\,\sqrt{a+bx}\,(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d} + \frac{b^2\text{Subst}\left(\int \frac{1}{\sqrt{x}\,\sqrt{a+b}\cdots}\right.}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{-2ab+i(a^2-b^2)}{2(i-x)\sqrt{x}\,\sqrt{a+bx}} + \frac{2ab+i(a^2-b^2)}{2\sqrt{x}\,(i+x)\sqrt{a+bx}}\right) dx, x, \tan(c+dx)\right)}{d} \cdots$$

$$= \frac{(i(a-ib)^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(i+x)\sqrt{a+bx}}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{(i(a+ib)^2)\,\text{S}\cdots}{}$$

$$= \frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{(i(a-ib)^2)\,\text{Subst}\left(\int \frac{1}{i-(-a+ib)x^2}\,dx, \cdots\right.}{d}$$

$$= -\frac{(ia-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan\cdots}}{\sqrt{a+b\tan\cdots}}\right.}{d}$$

Mathematica [A]
time = 0.80, size = 203, normalized size = 1.34

$$\frac{\sqrt[4]{-1}\left(\sqrt{-a+ib}\,(ia+b)\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right) + \sqrt{a+ib}\,(-ia+b)\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\right) + \frac{2\sqrt{a}\,b^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{1+\frac{b\tan(c+dx)}{a}}}{\sqrt{a+b\tan(c+dx)}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^(3/2)/Sqrt[Tan[c + d*x]],x]
```

```
[Out] ((-1)^(1/4)*(Sqrt[-a + I*b]*(I*a + b)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqr
t[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + Sqrt[a + I*b]*((-I)*a + b)*Arc
Tan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]
) + (2*Sqrt[a]*b^(3/2)*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1
 + (b*Tan[c + d*x])/a])/Sqrt[a + b*Tan[c + d*x]])/d
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 1.10, size = 1343730, normalized size = 8840.33

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(3/2)/tan(d*x+c)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(3/2)/tan(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(3/2)/sqrt(tan(d*x + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(3/2)/tan(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan(c + dx))^{\frac{3}{2}}}{\sqrt{\tan(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(3/2)/tan(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**(3/2)/sqrt(tan(c + d*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(3/2)/tan(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\tan(c+d\,x))^{3/2}}{\sqrt{\tan\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^(3/2)/tan(c + d*x)^(1/2),x)
[Out] int((a + b*tan(c + d*x))^(3/2)/tan(c + d*x)^(1/2), x)
```

3.619 $\int \frac{(a+b\tan(c+dx))^{3/2}}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=145

$$\frac{i(ia-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{i(ia+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{2a\sqrt{a+b}}{d\sqrt{\tan}}$$

```
[Out] I*(I*a-b)^(3/2)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2
))/d-I*(I*a+b)^(3/2)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)
)^(1/2))/d-2*a*(a+b*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3648, 3697, 3696, 95, 209, 212}

$$\frac{i(-b+ia)^{3/2}\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{2a\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{i(b+ia)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^(3/2)/Tan[c + d*x]^(3/2),x]
```

```
[Out] (I*(I*a - b)^(3/2)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan
[c + d*x]]])/d - (I*(I*a + b)^(3/2)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x
]])/Sqrt[a + b*Tan[c + d*x]]])/d - (2*a*Sqrt[a + b*Tan[c + d*x]])/(d*Sqrt[T
an[c + d*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(c+dx))^{3/2}}{\tan^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2a\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}} - 2\int \frac{-ab+\frac{1}{2}(a^2-b^2)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\,dx$$

$$= -\frac{2a\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}} + \frac{1}{2}(i(a-ib)^2)\int \frac{1+i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+a}}$$

$$= -\frac{2a\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}} + \frac{(i(a-ib)^2)\,\text{Subst}\left(\int \frac{1}{(1-ix)\sqrt{x}\sqrt{a+bx}}\,dx, x, \text{ta}\right.}{2d}$$

$$= -\frac{2a\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}} + \frac{(i(a-ib)^2)\,\text{Subst}\left(\int \frac{1}{1-(ia+b)x^2}\,dx, x, \frac{\sqrt{\tan(c-}}{\sqrt{a+b\tan}}\right.}{d}$$

$$= \frac{i(ia-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{i(ia+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ia+}}{\sqrt{a+}}\right.}{d}$$

Mathematica [A]
time = 0.29, size = 175, normalized size = 1.21

$$\frac{\sqrt[4]{-1}(-a+ib)^{3/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\tan(c+dx)} - \sqrt[4]{-1}(a+ib)^{3/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\tan(c+dx)} - 2a\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^(3/2)/Tan[c + d*x]^(3/2),x]
```

```
[Out] ((-1)^(1/4)*(-a + I*b)^(3/2)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c +
 d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Tan[c + d*x]] - (-1)^(1/4)*(a + I*b)
^(3/2)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[
c + d*x]]]*Sqrt[Tan[c + d*x]] - 2*a*Sqrt[a + b*Tan[c + d*x]])/(d*Sqrt[Tan[c
 + d*x]])
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 1.65, size = 1344189, normalized size = 9270.27

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(3/2)/tan(d*x+c)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(3/2)/tan(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^(3/2)/tan(d*x + c)^(3/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(3/2)/tan(d*x+c)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan{(c + dx)})^{\frac{3}{2}}}{\tan^{\frac{3}{2}}{(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(3/2)/tan(d*x+c)**(3/2),x)
[Out] Integral((a + b*tan(c + d*x))**(3/2)/tan(c + d*x)**(3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(3/2)/tan(d*x+c)^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\tan(c + d\,x))^{3/2}}{\tan{(c + d\,x)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^(3/2)/tan(c + d*x)^(3/2),x)
[Out] int((a + b*tan(c + d*x))^(3/2)/tan(c + d*x)^(3/2), x)
```

3.620 $\int \frac{(a+b\tan(c+dx))^{3/2}}{\tan^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=173

$$\frac{(ia-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{(ia+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{2a\sqrt{a+b\,\mathrm{t}}}{3d\tan^{\frac{3}{2}}(}$$

```
[Out] (I*a-b)^(3/2)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))
/d+(I*a+b)^(3/2)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1
/2))/d-8/3*b*(a+b*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(1/2)-2/3*a*(a+b*tan(d*x+c
))^(1/2)/d/tan(d*x+c)^(3/2)
```

Rubi [A]
time = 0.44, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3648, 3730, 3697, 3696, 95, 209, 212}

$$\frac{(-b+ia)^{3/2}\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{2a\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{8b\sqrt{a+b\tan(c+dx)}}{3d\sqrt{\tan(c+dx)}}+\frac{(b+ia)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^(3/2)/Tan[c + d*x]^(5/2),x]
```

```
[Out] ((I*a - b)^(3/2)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
 + d*x]]])/d + ((I*a + b)^(3/2)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/
Sqrt[a + b*Tan[c + d*x]]])/d - (2*a*Sqrt[a + b*Tan[c + d*x]])/(3*d*Tan[c +
d*x]^(3/2)) - (8*b*Sqrt[a + b*Tan[c + d*x]])/(3*d*Sqrt[Tan[c + d*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
```

```
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{(a+b\tan(c+dx))^{3/2}}{\tan^{\frac{5}{2}}(c+dx)}\,dx = -\frac{2a\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{2}{3}\int \frac{-2ab+\frac{3}{2}(a^2-b^2)\tan(c+dx)+ab\tan^2(c+}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}$$

$$= -\frac{2a\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{8b\sqrt{a+b\tan(c+dx)}}{3d\sqrt{\tan(c+dx)}} + \frac{4\int \frac{-\frac{3}{4}a(a^2-b^2)-\frac{3}{2}}{\sqrt{\tan(c+dx)}\sqrt{a}}}{3a}$$

$$= -\frac{2a\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{8b\sqrt{a+b\tan(c+dx)}}{3d\sqrt{\tan(c+dx)}} - \frac{1}{2}(a-ib)^2\int \frac{}{\sqrt{\tan(c}}$$

$$= -\frac{2a\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{8b\sqrt{a+b\tan(c+dx)}}{3d\sqrt{\tan(c+dx)}} - \frac{(a-ib)^2\text{Subst}\left(\int \frac{}{(1-ia}\right.}{}$$

$$= -\frac{2a\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{8b\sqrt{a+b\tan(c+dx)}}{3d\sqrt{\tan(c+dx)}} - \frac{(a-ib)^2\text{Subst}\left(\int \frac{}{1-(ia}\right.}{}$$

$$= \frac{(ia-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{(ia+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ia+b}}{\sqrt{a+b}}\right.}{d}$$

Mathematica [A]
time = 0.87, size = 176, normalized size = 1.02

$$\frac{-3\sqrt[4]{-1}\sqrt{-a+ib}\,(ia+b)\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+3(-1)^{3/4}(a+ib)^{3/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)-\frac{2\sqrt{a+b\tan(c+dx)}\,(a+4b\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^(3/2)/Tan[c + d*x]^(5/2),x]
```

```
[Out] (-3*(-1)^(1/4)*Sqrt[-a + I*b]*(I*a + b)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*S
qrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + 3*(-1)^(3/4)*(a + I*b)^(3/2)
*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*
x]]] - (2*Sqrt[a + b*Tan[c + d*x]]*(a + 4*b*Tan[c + d*x]))/Tan[c + d*x]^(3/
2))/(3*d)
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 1.38, size = 1345207, normalized size = 7775.76

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(3/2)/tan(d*x+c)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(3/2)/tan(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(3/2)/tan(d*x + c)^(5/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(3/2)/tan(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan{(c+dx)})^{\frac{3}{2}}}{\tan^{\frac{5}{2}}{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(3/2)/tan(d*x+c)**(5/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**(3/2)/tan(c + d*x)**(5/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(3/2)/tan(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\tan(c+d\,x))^{3/2}}{\tan\left(c+d\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^(3/2)/tan(c + d*x)^(5/2),x)
[Out] int((a + b*tan(c + d*x))^(3/2)/tan(c + d*x)^(5/2), x)
```

3.621 $\int \frac{(a+b\tan(c+dx))^{3/2}}{\tan^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=224

$$-\frac{i(ia-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{i(ia+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{2a\sqrt{a}}{5d}$$

```
[Out] -I*(I*a-b)^(3/2)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/
2))/d+I*(I*a+b)^(3/2)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c
))^(1/2))/d+2/5*(5*a^2-b^2)*(a+b*tan(d*x+c))^(1/2)/a/d/tan(d*x+c)^(1/2)-2/5
*a*(a+b*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(5/2)-4/5*b*(a+b*tan(d*x+c))^(1/2)/d
/tan(d*x+c)^(3/2)
```

Rubi [A]
time = 0.61, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3648, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(5a^2-b^2)\sqrt{a+b\tan(c+dx)}}{5ad\sqrt{\tan(c+dx)}}-\frac{i(-b+ia)^{3/2}\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{4b\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{3}{2}}(c+dx)}-\frac{2a\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{i(b+ia)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^(3/2)/Tan[c + d*x]^(7/2),x]
```

```
[Out] ((-I)*(I*a - b)^(3/2)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*
Tan[c + d*x]]])/d + (I*(I*a + b)^(3/2)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c +
d*x]])/Sqrt[a + b*Tan[c + d*x]]])/d - (2*a*Sqrt[a + b*Tan[c + d*x]])/(5*d*T
an[c + d*x]^(5/2)) - (4*b*Sqrt[a + b*Tan[c + d*x]])/(5*d*Tan[c + d*x]^(3/2)
) + (2*(5*a^2 - b^2)*Sqrt[a + b*Tan[c + d*x]])/(5*a*d*Sqrt[Tan[c + d*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
```

```
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{(a+b\tan(c+dx))^{3/2}}{\tan^{\frac{7}{2}}(c+dx)}\,dx = -\frac{2a\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{2}{5}\int \frac{-3ab+\frac{5}{2}(a^2-b^2)\tan(c+dx)+2ab\tan^2(}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}$$

$$= -\frac{2a\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{4b\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{3}{2}}(c+dx)} + \frac{4\int \frac{-\frac{3}{4}a(5a^2-b^2)-\frac{15}{2}a^2b\,\mathrm{t}}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a}}}{1}$$

$$= -\frac{2a\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{4b\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{3}{2}}(c+dx)} + \frac{2(5a^2-b^2)\sqrt{a+b\,\mathrm{ta}}}{5ad\sqrt{\tan(c+}}$$

$$= -\frac{2a\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{4b\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{3}{2}}(c+dx)} + \frac{2(5a^2-b^2)\sqrt{a+b\,\mathrm{ta}}}{5ad\sqrt{\tan(c+}}$$

$$= -\frac{2a\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{4b\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{3}{2}}(c+dx)} + \frac{2(5a^2-b^2)\sqrt{a+b\,\mathrm{ta}}}{5ad\sqrt{\tan(c+}}$$

$$= -\frac{2a\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{4b\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{3}{2}}(c+dx)} + \frac{2(5a^2-b^2)\sqrt{a+b\,\mathrm{ta}}}{5ad\sqrt{\tan(c+}}$$

$$= -\frac{i(ia-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{i(ia+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ia}}{\sqrt{}}\right.}{d}$$

Mathematica [A]
time = 1.69, size = 197, normalized size = 0.88

$$\frac{-5\sqrt[4]{-1}\,(-a+ib)^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+5\sqrt[4]{-1}\,(a+ib)^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+\frac{2\sqrt{a+b\tan(c+dx)}\,(-a^2-2ab\tan(c+dx)+(5a^2-b^2)\tan^2(c+dx))}{a\tan^{\frac{5}{2}}(c+dx)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^(3/2)/Tan[c + d*x]^(7/2),x]
```

```
[Out] (-5*(-1)^(1/4)*(-a + I*b)^(3/2)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[
c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + 5*(-1)^(1/4)*(a + I*b)^(3/2)*ArcTan[
((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + (
2*Sqrt[a + b*Tan[c + d*x]]*(-a^2 - 2*a*b*Tan[c + d*x] + (5*a^2 - b^2)*Tan[c
 + d*x]^2))/(a*Tan[c + d*x]^(5/2)))/(5*d)
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 1.25, size = 1346038, normalized size = 6009.10

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(3/2)/tan(d*x+c)^(7/2),x)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(3/2)/tan(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(3/2)/tan(d*x + c)^(7/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(3/2)/tan(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \tan{(c + dx)})^{\frac{3}{2}}}{\tan^{\frac{7}{2}}{(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(3/2)/tan(d*x+c)**(7/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**(3/2)/tan(c + d*x)**(7/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(3/2)/tan(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\tan(c+d\,x))^{3/2}}{\tan\left(c+d\,x\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^(3/2)/tan(c + d*x)^(7/2),x)
```

```
[Out] int((a + b*tan(c + d*x))^(3/2)/tan(c + d*x)^(7/2), x)
```

3.622 $\int \frac{(a+b\tan(c+dx))^{3/2}}{\tan^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=266

$$-\frac{(ia-b)^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{(ia+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{2a\sqrt{a+b}}{7d\tan}$$

```
[Out] -(I*a-b)^(3/2)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2)
)/d-(I*a+b)^(3/2)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(
1/2))/d+4/105*b*(70*a^2+3*b^2)*(a+b*tan(d*x+c))^(1/2)/a^2/d/tan(d*x+c)^(1/2
)-2/7*a*(a+b*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(7/2)-16/35*b*(a+b*tan(d*x+c))^
(1/2)/d/tan(d*x+c)^(5/2)+2/105*(35*a^2-3*b^2)*(a+b*tan(d*x+c))^(1/2)/a/d/ta
n(d*x+c)^(3/2)
```

Rubi [A]
time = 0.77, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3648, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(35a^2-3b^2)\sqrt{a+b\tan(c+dx)}}{105ad\tan^{\frac{3}{2}}(c+dx)}+\frac{4b(70a^2+3b^2)\sqrt{a+b\tan(c+dx)}}{105a^2d\sqrt{\tan(c+dx)}}-\frac{(-b+ia)^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{16b\sqrt{a+b\tan(c+dx)}}{35d\tan^{\frac{5}{2}}(c+dx)}-\frac{2a\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{(b+ia)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^(3/2)/Tan[c + d*x]^(9/2),x]
```

```
[Out] -(((I*a - b)^(3/2)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan
[c + d*x]]])/d) - ((I*a + b)^(3/2)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]
])/Sqrt[a + b*Tan[c + d*x]]])/d - (2*a*Sqrt[a + b*Tan[c + d*x]])/(7*d*Tan[c
 + d*x]^(7/2)) - (16*b*Sqrt[a + b*Tan[c + d*x]])/(35*d*Tan[c + d*x]^(5/2))
+ (2*(35*a^2 - 3*b^2)*Sqrt[a + b*Tan[c + d*x]])/(105*a*d*Tan[c + d*x]^(3/2)
) + (4*b*(70*a^2 + 3*b^2)*Sqrt[a + b*Tan[c + d*x]])/(105*a^2*d*Sqrt[Tan[c +
 d*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
```

```
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{(a+b\tan(c+dx))^{3/2}}{\tan^{\frac{9}{2}}(c+dx)}\,dx = -\frac{2a\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)} - \frac{2}{7}\int \frac{-4ab+\frac{7}{2}(a^2-b^2)\tan(c+dx)+3ab\tan^2(c}{\tan^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}$$

$$= -\frac{2a\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)} - \frac{16b\sqrt{a+b\tan(c+dx)}}{35d\tan^{\frac{5}{2}}(c+dx)} + \frac{4\int \frac{-\frac{1}{4}a(35a^2-3b^2)-\frac{35}{2}a^2b}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a}}}{}$$

$$= -\frac{2a\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)} - \frac{16b\sqrt{a+b\tan(c+dx)}}{35d\tan^{\frac{5}{2}}(c+dx)} + \frac{2(35a^2-3b^2)\sqrt{a+b}}{105ad\tan^{\frac{3}{2}}(c}$$

$$= -\frac{2a\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)} - \frac{16b\sqrt{a+b\tan(c+dx)}}{35d\tan^{\frac{5}{2}}(c+dx)} + \frac{2(35a^2-3b^2)\sqrt{a+b}}{105ad\tan^{\frac{3}{2}}(c}$$

$$= -\frac{2a\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)} - \frac{16b\sqrt{a+b\tan(c+dx)}}{35d\tan^{\frac{5}{2}}(c+dx)} + \frac{2(35a^2-3b^2)\sqrt{a+b}}{105ad\tan^{\frac{3}{2}}(c}$$

$$= -\frac{2a\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)} - \frac{16b\sqrt{a+b\tan(c+dx)}}{35d\tan^{\frac{5}{2}}(c+dx)} + \frac{2(35a^2-3b^2)\sqrt{a+b}}{105ad\tan^{\frac{3}{2}}(c}$$

$$= -\frac{2a\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)} - \frac{16b\sqrt{a+b\tan(c+dx)}}{35d\tan^{\frac{5}{2}}(c+dx)} + \frac{2(35a^2-3b^2)\sqrt{a+b}}{105ad\tan^{\frac{3}{2}}(c}$$

$$= -\frac{(ia-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{(ia+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ia+}}{\sqrt{a+}}\right.}{d}$$

Mathematica [A]
time = 3.02, size = 229, normalized size = 0.86

$$\frac{105\sqrt[4]{-1}\sqrt{-a+ib}\,(ia+b)\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)-105(-1)^{3/4}(a+ib)^{3/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+\frac{2\sqrt{a+b\tan(c+dx)}\left(-15a^3-24a^2b\tan(c+dx)+a(35a^2-3b^2)\tan^2(c+dx)+2b(70a^2+3b^2)\tan^3(c+dx)\right)}{a^2\tan^{\frac{7}{2}}(c+dx)}}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^(3/2)/Tan[c + d*x]^(9/2),x]
```

```
[Out] (105*(-1)^(1/4)*Sqrt[-a + I*b]*(I*a + b)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*
Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] - 105*(-1)^(3/4)*(a + I*b)^(3
/2)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c +
 d*x]]] + (2*Sqrt[a + b*Tan[c + d*x]]*(-15*a^3 - 24*a^2*b*Tan[c + d*x] + a*
(35*a^2 - 3*b^2)*Tan[c + d*x]^2 + 2*b*(70*a^2 + 3*b^2)*Tan[c + d*x]^3))/(a^
2*Tan[c + d*x]^(7/2)))/(105*d)
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 0.44, size = 1346975, normalized size = 5063.82

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(3/2)/tan(d*x+c)^(9/2),x)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(3/2)/tan(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(3/2)/tan(d*x + c)^(9/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(3/2)/tan(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(3/2)/tan(d*x+c)**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(3/2)/tan(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\tan(c+d\,x))^{3/2}}{\tan\left(c+d\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^(3/2)/tan(c + d*x)^(9/2),x)
```

```
[Out] int((a + b*tan(c + d*x))^(3/2)/tan(c + d*x)^(9/2), x)
```

3.623 $\int \tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=332

$$\frac{(ia-b)^{5/2}\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{(5a^4+240a^2b^2-128b^4)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{64b^{3/2}d}-$$

```
[Out] (I*a-b)^(5/2)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))
/d-1/64*(5*a^4+240*a^2*b^2-128*b^4)*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*t
an(d*x+c))^(1/2))/b^(3/2)/d-(I*a+b)^(5/2)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^
(1/2)/(a+b*tan(d*x+c))^(1/2))/d-1/64*a*(5*a^2+112*b^2)*tan(d*x+c)^(1/2)*(a+
b*tan(d*x+c))^(1/2)/b/d-1/96*(5*a^2+48*b^2)*tan(d*x+c)^(1/2)*(a+b*tan(d*x+c
))^(3/2)/b/d-1/24*a*tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(5/2)/b/d+1/4*tan(d*x
+c)^(1/2)*(a+b*tan(d*x+c))^(7/2)/b/d
```

Rubi [A]

time = 1.93, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3647, 3728, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$-\frac{(5a^2+48b^2)\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{3/2}}{96bd}-\frac{a(5a^2+112b^2)\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{64bd}-\frac{(5a^4+240a^2b^2-128b^4)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{64b^{3/2}d}+\frac{(-b+ia)^{5/2}\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{7/2}}{4bd}-\frac{a\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{5/2}}{24bd}-\frac{(b+ia)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((I*a - b)^(5/2)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
 + d*x]]])/d - ((5*a^4 + 240*a^2*b^2 - 128*b^4)*ArcTanh[(Sqrt[b]*Sqrt[Tan[c
 + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(64*b^(3/2)*d) - ((I*a + b)^(5/2)*ArcT
anh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/d - (a*(5
*a^2 + 112*b^2)*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]])/(64*b*d) - ((5
*a^2 + 48*b^2)*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(3/2))/(96*b*d) - (a
*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(5/2))/(24*b*d) + (Sqrt[Tan[c + d*
x]]*(a + b*Tan[c + d*x])^(7/2))/(4*b*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
```

```
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}\,dx = \frac{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{7/2}}{4bd} + \frac{\int \frac{(a+b\tan(c+dx))^{5/2}\left(-\frac{a}{2}-4b\tan\cdots\right.}{\sqrt{\tan(c\cdots}}}{4b\cdots}$$

$$= -\frac{a\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{5/2}}{24bd} + \frac{\sqrt{\tan(c+dx)}\,(a+b\cdots}{4bd}$$

$$= -\frac{(5a^2+48b^2)\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{3/2}}{96bd} - \frac{a\sqrt{\tan(c\cdots}}{}$$

$$= -\frac{a(5a^2+112b^2)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{64bd} - \frac{(5a^2+4\cdots}{}$$

$$= -\frac{a(5a^2+112b^2)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{64bd} - \frac{(5a^2+4\cdots}{}$$

$$= -\frac{a(5a^2+112b^2)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{64bd} - \frac{(5a^2+4\cdots}{}$$

$$= -\frac{a(5a^2+112b^2)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{64bd} - \frac{(5a^2+4\cdots}{}$$

$$= -\frac{a(5a^2+112b^2)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{64bd} - \frac{(5a^2+4\cdots}{}$$

$$= -\frac{a(5a^2+112b^2)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{64bd} - \frac{(5a^2+4\cdots}{}$$

$$= -\frac{(5a^4+240a^2b^2-128b^4)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{64b^{3/2}d} - \frac{a(5\cdots}{}$$

$$= \frac{(ia-b)^{5/2}\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{(5a^4+240a^2b^2-\cdots}{}$$

Mathematica [A]
time = 3.42, size = 349, normalized size = 1.05

$$-\frac{192\sqrt[4]{-1}(-a+ib)^{5/2}b\,\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+192\sqrt[4]{-1}(a+ib)^{5/2}b\,\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+3a(5a^2+112b^2)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}+2(5a^2+48b^2)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}+8a\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}-48\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{7/2}+\frac{3\sqrt{a}(5a^4+240a^2b^2-128b^4)\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{1+\frac{b\tan(c+dx)}{a}}}{\sqrt{b}\sqrt{a+b\tan(c+dx)}}}{192bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] -1/192*(192*(-1)^(1/4)*(-a + I*b)^(5/2)*b*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]
*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + 192*(-1)^(1/4)*(a + I*b)^(
5/2)*b*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[
c + d*x]]] + 3*a*(5*a^2 + 112*b^2)*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*
x]] + 2*(5*a^2 + 48*b^2)*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(3/2) + 8*
a*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(5/2) - 48*Sqrt[Tan[c + d*x]]*(a
+ b*Tan[c + d*x])^(7/2) + (3*Sqrt[a]*(5*a^4 + 240*a^2*b^2 - 128*b^4)*ArcSin
h[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a])/(Sqrt
[b]*Sqrt[a + b*Tan[c + d*x]]))/(b*d)
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 0.50, size = 1347722, normalized size = 4059.40

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(5/2)*tan(d*x + c)^(5/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(5/2)*(a+b*tan(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c+d\,x)^{5/2}\,{\left(a+b\tan(c+d\,x)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(5/2)*(a + b*tan(c + d*x))^(5/2),x)
```

```
[Out] int(tan(c + d*x)^(5/2)*(a + b*tan(c + d*x))^(5/2), x)
```

3.624 $\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=277

$$-\frac{i(ia-b)^{5/2}\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{5a(a^2-8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{8\sqrt{b}\,d}-\frac{i(ia+b)}{}$$

```
[Out] -I*(I*a-b)^(5/2)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/
2))/d-I*(I*a+b)^(5/2)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c
))^(1/2))/d+5/8*a*(a^2-8*b^2)*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x
+c))^(1/2))/d/b^(1/2)+1/8*(11*a^2-8*b^2)*tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^
(1/2)/d+13/12*a*b*(a+b*tan(d*x+c))^(1/2)*tan(d*x+c)^(3/2)/d+1/3*b^2*(a+b*ta
n(d*x+c))^(1/2)*tan(d*x+c)^(5/2)/d
```

Rubi [A]
time = 1.57, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3647, 3728, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{(11a^2-8b^2)\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{8d}+\frac{5a(a^2-8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{8\sqrt{b}\,d}-\frac{i(-b+ia)^{5/2}\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{b^2\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}+\frac{13ab\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{12d}-\frac{i(b+ia)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-I)*(I*a - b)^(5/2)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*
Tan[c + d*x]]])/d + (5*a*(a^2 - 8*b^2)*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])
/Sqrt[a + b*Tan[c + d*x]]])/(8*Sqrt[b]*d) - (I*(I*a + b)^(5/2)*ArcTanh[(Sqr
t[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/d + ((11*a^2 - 8*
b^2)*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]])/(8*d) + (13*a*b*Tan[c + d
*x]^(3/2)*Sqrt[a + b*Tan[c + d*x]])/(12*d) + (b^2*Tan[c + d*x]^(5/2)*Sqrt[a
 + b*Tan[c + d*x]])/(3*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
```

```
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
```

```
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}\,dx = \frac{b^2\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{3d} + \frac{1}{3}\int \frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}a(6a\right.}{}$$

$$= \frac{13ab\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{12d} + \frac{b^2\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b}}{3d}$$

$$= \frac{(11a^2-8b^2)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{8d} + \frac{13ab\tan^{\frac{3}{2}}(c}{}$$

$$= \frac{(11a^2-8b^2)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{8d} + \frac{13ab\tan^{\frac{3}{2}}(c}{}$$

$$= \frac{(11a^2-8b^2)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{8d} + \frac{13ab\tan^{\frac{3}{2}}(c}{}$$

$$= \frac{(11a^2-8b^2)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{8d} + \frac{13ab\tan^{\frac{3}{2}}(c}{}$$

$$= \frac{(11a^2-8b^2)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{8d} + \frac{13ab\tan^{\frac{3}{2}}(c}{}$$

$$= \frac{(11a^2-8b^2)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{8d} + \frac{13ab\tan^{\frac{3}{2}}(c}{}$$

$$= \frac{5a(a^2-8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{8\sqrt{b}\,d} + \frac{(11a^2-8b^2)\sqrt{t}}{}$$

$$= -\frac{i(ia-b)^{5/2}\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{5a(a^2-8b^2)\,t}{}$$

Mathematica [A]

time = 3.20, size = 300, normalized size = 1.08

$$\frac{-24(-1)^{3/4}(-a+ib)^{5/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+24(-1)^{3/4}(a+ib)^{5/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+3(11a^2-8b^2)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}+26ab\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}+8b^2\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}+\frac{15a^{3/2}(a^2-8b^2)\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{1+\frac{b\tan(c+dx)}{a}}}{\sqrt{b}\sqrt{a+b\tan(c+dx)}}}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] (-24*(-1)^(3/4)*(-a + I*b)^(5/2)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan
[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + 24*(-1)^(3/4)*(a + I*b)^(5/2)*ArcTa
```

```
n[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] +
 3*(11*a^2 - 8*b^2)*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]] + 26*a*b*Ta
n[c + d*x]^(3/2)*Sqrt[a + b*Tan[c + d*x]] + 8*b^2*Tan[c + d*x]^(5/2)*Sqrt[a
 + b*Tan[c + d*x]] + (15*a^(3/2)*(a^2 - 8*b^2)*ArcSinh[(Sqrt[b]*Sqrt[Tan[c
+ d*x]])/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a])/(Sqrt[b]*Sqrt[a + b*Tan[c +
 d*x]]))/(24*d)
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 0.48, size = 1310426, normalized size = 4730.78

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(5/2)*tan(d*x + c)^(3/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(3/2)*(a+b*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c+d\,x)^{3/2}\,\left(a+b\tan(c+d\,x)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(5/2),x)
```

```
[Out] int(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(5/2), x)
```

3.625 $\int \sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=231

$$-\frac{(ia-b)^{5/2}\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{\sqrt{b}\,(15a^2-8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{4d}+\frac{(ia+}{}$$

```
[Out] -(I*a-b)^(5/2)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2)
)/d+(I*a+b)^(5/2)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(
1/2))/d+1/4*(15*a^2-8*b^2)*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)
)^(1/2))*b^(1/2)/d+9/4*a*b*tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)/d+1/2*b^
2*(a+b*tan(d*x+c))^(1/2)*tan(d*x+c)^(3/2)/d
```

Rubi [A]
time = 1.51, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3647, 3728, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{\sqrt{b}\,(15a^2-8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{4d}-\frac{(-b+ia)^{5/2}\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{b^2\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}+\frac{9ab\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{4d}+\frac{(b+ia)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] -(((I*a - b)^(5/2)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan
[c + d*x]]])/d) + (Sqrt[b]*(15*a^2 - 8*b^2)*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d
*x]])/Sqrt[a + b*Tan[c + d*x]]])/(4*d) + ((I*a + b)^(5/2)*ArcTanh[(Sqrt[I*a
 + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/d + (9*a*b*Sqrt[Tan[c
+ d*x]]*Sqrt[a + b*Tan[c + d*x]])/(4*d) + (b^2*Tan[c + d*x]^(3/2)*Sqrt[a +
b*Tan[c + d*x]])/(2*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
```

```
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{5/2}\,dx = \frac{b^2\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{2d} + \frac{1}{2}\int \frac{\sqrt{\tan(c+dx)}\,\left(\frac{1}{2}c\right.}{}$$

$$= \frac{9ab\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{4d} + \frac{b^2\tan^{\frac{3}{2}}(c+dx)\sqrt{a\,+}}{2d}$$

$$= \frac{9ab\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{4d} + \frac{b^2\tan^{\frac{3}{2}}(c+dx)\sqrt{a\,+}}{2d}$$

$$= \frac{9ab\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{4d} + \frac{b^2\tan^{\frac{3}{2}}(c+dx)\sqrt{a\,+}}{2d}$$

$$= \frac{9ab\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{4d} + \frac{b^2\tan^{\frac{3}{2}}(c+dx)\sqrt{a\,+}}{2d}$$

$$= \frac{9ab\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{4d} + \frac{b^2\tan^{\frac{3}{2}}(c+dx)\sqrt{a\,+}}{2d}$$

$$= \frac{9ab\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{4d} + \frac{b^2\tan^{\frac{3}{2}}(c+dx)\sqrt{a\,+}}{2d}$$

$$= \frac{\sqrt{b}\,(15a^2-8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{4d} + \frac{9ab\sqrt{\tan(}}{}$$

$$= -\frac{(ia-b)^{5/2}\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{\sqrt{b}\,(15a^2\,-}{}$$

Mathematica [A]
time = 2.02, size = 264, normalized size = 1.14

$$\frac{4\sqrt[4]{-1}\,(-a+ib)^{5/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+4\sqrt[4]{-1}\,(a+ib)^{5/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+9ab\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}+2b^2\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}+\frac{\sqrt{a}\,\sqrt{b}\,(15a^2-8b^2)\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{1+\frac{b\tan(c+dx)}{a}}}{\sqrt{a+b\tan(c+dx)}}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] (4*(-1)^(1/4)*(-a + I*b)^(5/2)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c
 + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + 4*(-1)^(1/4)*(a + I*b)^(5/2)*ArcTan[(
(-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + 9*
a*b*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]] + 2*b^2*Tan[c + d*x]^(3/2)*
```

```
Sqrt[a + b*Tan[c + d*x]] + (Sqrt[a]*Sqrt[b]*(15*a^2 - 8*b^2)*ArcSinh[(Sqrt[
b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a])/Sqrt[a + b*Ta
n[c + d*x]])/(4*d)
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 0.31, size = 1345303, normalized size = 5823.82

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(5/2),x)
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^(5/2)*sqrt(tan(d*x + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx))^{\frac{5}{2}} \sqrt{\tan(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/2)*(a+b*tan(d*x+c))**(5/2),x)
[Out] Integral((a + b*tan(c + d*x))**(5/2)*sqrt(tan(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\tan\left(c+d\,x\right)}\,\left(a+b\tan(c+d\,x)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(5/2),x)
```

```
[Out] int(tan(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(5/2), x)
```

3.626 $\int \frac{(a+b\tan(c+dx))^{5/2}}{\sqrt{\tan(c+dx)}}\,dx$

Optimal. Leaf size=188

$$\frac{i(ia-b)^{5/2}\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{i(ia+b)^{5/2}\tanh^{-1}}{}$$

```
[Out] I*(I*a-b)^(5/2)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2
))/d+5*a*b^(3/2)*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/d
+I*(I*a+b)^(5/2)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1
/2))/d+b^2*tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)/d
```

Rubi [A]
time = 1.03, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3647, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{i(-b+ia)^{5/2}\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{b^2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}+\frac{i(b+ia)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^(5/2)/Sqrt[Tan[c + d*x]],x]
```

```
[Out] (I*(I*a - b)^(5/2)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan
[c + d*x]]])/d + (5*a*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a +
 b*Tan[c + d*x]]])/d + (I*(I*a + b)^(5/2)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c
 + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/d + (b^2*Sqrt[Tan[c + d*x]]*Sqrt[a + b
*Tan[c + d*x]])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

[n, 0]

Rubi steps

$$\int \frac{(a+b\tan(c+dx))^{5/2}}{\sqrt{\tan(c+dx)}}\,dx = \frac{b^2\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{d} + \int \frac{\frac{1}{2}a(2a^2-b^2)+b(3a^2-b^2)\tan(c}{\sqrt{\tan(c+dx)}\,\sqrt{a-}}$$

$$= \frac{b^2\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{d} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(2a^2-b^2)+b(3a^2-b^2)x+\frac{5}{2}ab^2x}{\sqrt{x}\,\sqrt{a+bx}\,(1+x^2)}\right.}{d}$$

$$= \frac{b^2\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{d} + \frac{\text{Subst}\left(\int \left(\frac{5ab^2}{2\sqrt{x}\,\sqrt{a+bx}} + \frac{a^3-3}{\sqrt{x}\,\sqrt{}}\right.\right.}{}$$

$$= \frac{b^2\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{d} + \frac{\text{Subst}\left(\int \frac{a^3-3ab^2+b(3a^2-b^2)x}{\sqrt{x}\,\sqrt{a+bx}\,(1+x^2)}\,dx, x,\right.}{d}$$

$$= \frac{b^2\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{d} + \frac{\text{Subst}\left(\int \left(\frac{-b(3a^2-b^2)+i(a^3-3ab^2)}{2(i-x)\sqrt{x}\,\sqrt{a+bx}} + \frac{b}{2}\right.\right.}{}$$

$$= \frac{b^2\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{d} - \frac{(ia-b)^3\text{Subst}\left(\int \frac{1}{(i-x)\sqrt{x}\,\sqrt{a+bx}}\right.}{2d}$$

$$= \frac{5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{b^2\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+}}{d}$$

$$= \frac{i(ia-b)^{5/2}\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan}}{\sqrt{a+b\tan}}\right.}{d}$$

Mathematica [A]
time = 0.83, size = 221, normalized size = 1.18

$$\frac{(-1)^{3/4}(-a+ib)^{5/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right) - (-1)^{3/4}(a+ib)^{5/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right) + b^2\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)} + \frac{5a^{3/2}b^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{1+\frac{b\tan(c+dx)}{a}}}{\sqrt{a+b\tan(c+dx)}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^(5/2)/Sqrt[Tan[c + d*x]],x]
```

```
[Out] ((-1)^(3/4)*(-a + I*b)^(5/2)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c +
 d*x]])/Sqrt[a + b*Tan[c + d*x]]] - (-1)^(3/4)*(a + I*b)^(5/2)*ArcTan[((-1)
^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + b^2*Sq
```

```
rt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]] + (5*a^(3/2)*b^(3/2)*ArcSinh[(Sqr
t[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a])/Sqrt[a + b*
Tan[c + d*x]])/d
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 0.32, size = 1308215, normalized size = 6958.59

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(5/2)/tan(d*x+c)^(1/2),x)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/2)/tan(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^(5/2)/sqrt(tan(d*x + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/2)/tan(d*x+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(c+dx))^{\frac{5}{2}}}{\sqrt{\tan(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(5/2)/tan(d*x+c)**(1/2),x)
[Out] Integral((a + b*tan(c + d*x))**(5/2)/sqrt(tan(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/2)/tan(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\tan(c+d\,x))^{5/2}}{\sqrt{\tan\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^(5/2)/tan(c + d*x)^(1/2),x)
```

```
[Out] int((a + b*tan(c + d*x))^(5/2)/tan(c + d*x)^(1/2), x)
```

3.627 $\int \frac{(a+b\tan(c+dx))^{5/2}}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=183

$$\frac{(ia-b)^{5/2}\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{(ia+b)^{5/2}\tanh^{-1}\left(\right.}{}$$

```
[Out] (I*a-b)^(5/2)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))
/d+2*b^(5/2)*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/d-(I*
a+b)^(5/2)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/d
-2*a^2*(a+b*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(1/2)
```

Rubi [A]

time = 1.03, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3646, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$-\frac{2a^2\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}+\frac{(-b+ia)^{5/2}\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{(b+ia)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^(5/2)/Tan[c + d*x]^(3/2),x]
```

```
[Out] ((I*a - b)^(5/2)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
 + d*x]]])/d + (2*b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*T
an[c + d*x]]])/d - ((I*a + b)^(5/2)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x
]])/Sqrt[a + b*Tan[c + d*x]]])/d - (2*a^2*Sqrt[a + b*Tan[c + d*x]])/(d*Sqrt
[Tan[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(c+dx))^{5/2}}{\tan^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2a^2\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}} + 2\int \frac{\frac{3a^2b}{2}-\frac{1}{2}a(a^2-3b^2)\tan(c+dx)+\frac{1}{2}b^3\tan^2(\cdots}{\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}$$

$$= -\frac{2a^2\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{\frac{3a^2b}{2}-\frac{1}{2}a(a^2-3b^2)x+\frac{b^3x^2}{2}}{\sqrt{x}\,\sqrt{a+bx}\,(1+x^2)}\,dx, x, \tan(c+dx\cdots}{d}$$

$$= -\frac{2a^2\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}} + \frac{2\text{Subst}\left(\int \left(\frac{b^3}{2\sqrt{x}\,\sqrt{a+bx}} + \frac{3a^2b-b^3-a(a^2-3b^2)\cdots}{2\sqrt{x}\,\sqrt{a+bx}\,(1+\cdots}\right.\right.}{d}$$

$$= -\frac{2a^2\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}} + \frac{\text{Subst}\left(\int \frac{3a^2b-b^3-a(a^2-3b^2)x}{\sqrt{x}\,\sqrt{a+bx}\,(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{2a^2\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}} + \frac{\text{Subst}\left(\int \left(\frac{a(a^2-3b^2)+i(3a^2b-b^3)}{2(i-x)\sqrt{x}\,\sqrt{a+bx}} + \frac{-a(a^2-3b^2)+i(3a^2\cdots}{2\sqrt{x}\,(i+x)\sqrt{a+\cdots}}\right.\right.}{d}$$

$$= -\frac{2a^2\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}} - \frac{(a-ib)^3\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(i+x)\sqrt{a+bx}}\,dx, x, \tan(\cdots}{2d}$$

$$= \frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2a^2\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}} - \frac{(a-ib)^3\cdots}{\cdots}$$

$$= \frac{(ia-b)^{5/2}\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+\cdots}}{\sqrt{a+b\tan(c\cdots}}\right.}{d}$$

Mathematica [A]
time = 2.29, size = 244, normalized size = 1.33

$$\frac{\frac{2\sqrt{a}\,b^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{1+\frac{b\tan(c+dx)}{a}}}{\sqrt{a+b\tan(c+dx)}} - \frac{\sqrt[4]{-1}\,(-a+ib)^{5/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\tan(c+dx)}+\sqrt[4]{-1}\,(a+ib)^{5/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\tan(c+dx)}+2a^2\sqrt{a+b\tan(c+dx)}}{\sqrt{\tan(c+dx)}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^(5/2)/Tan[c + d*x]^(3/2),x]
```

```
[Out] ((2*Sqrt[a]*b^(5/2)*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 +
(b*Tan[c + d*x])/a])/Sqrt[a + b*Tan[c + d*x]] - ((-1)^(1/4)*(-a + I*b)^(5/2
)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c +
d*x]]]*Sqrt[Tan[c + d*x]] + (-1)^(1/4)*(a + I*b)^(5/2)*ArcTan[((-1)^(1/4)*S
qrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Tan[c + d*x
]] + 2*a^2*Sqrt[a + b*Tan[c + d*x]])/Sqrt[Tan[c + d*x]])/d
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 18.49, size = 27882, normalized size = 152.36

method	result	size
default	Expression too large to display	27882

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(5/2)/tan(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/2)/tan(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(5/2)/tan(d*x + c)^(3/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/2)/tan(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(c+dx))^{\frac{5}{2}}}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(5/2)/tan(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**(5/2)/tan(c + d*x)**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/2)/tan(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\tan(c+d\,x))^{5/2}}{\tan\left(c+d\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^(5/2)/tan(c + d*x)^(3/2),x)
```

```
[Out] int((a + b*tan(c + d*x))^(5/2)/tan(c + d*x)^(3/2), x)
```

3.628 $\int \frac{(a+b\tan(c+dx))^{5/2}}{\tan^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=182

$$-\frac{i(ia-b)^{5/2}\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{i(ia+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{2a^2\sqrt{a}}{3d}$$

```
[Out] -I*(I*a-b)^(5/2)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/
2))/d-I*(I*a+b)^(5/2)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c
))^(1/2))/d-14/3*a*b*(a+b*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(1/2)-2/3*a^2*(a+b
*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(3/2)
```

Rubi [A]

time = 0.51, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3646, 3730, 3697, 3696, 95, 209, 212}

$$-\frac{2a^2\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{i(-b+ia)^{5/2}\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{14ab\sqrt{a+b\tan(c+dx)}}{3d\sqrt{\tan(c+dx)}}-\frac{i(b+ia)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^(5/2)/Tan[c + d*x]^(5/2),x]
```

```
[Out] ((-I)*(I*a - b)^(5/2)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*
Tan[c + d*x]]])/d - (I*(I*a + b)^(5/2)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c +
d*x]])/Sqrt[a + b*Tan[c + d*x]]])/d - (2*a^2*Sqrt[a + b*Tan[c + d*x]])/(3*d
*Tan[c + d*x]^(3/2)) - (14*a*b*Sqrt[a + b*Tan[c + d*x]])/(3*d*Sqrt[Tan[c +
d*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
```

```
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+b\tan(c+dx))^{5/2}}{\tan^{\frac{5}{2}}(c+dx)}\,dx &= -\frac{2a^2\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\frac{7a^2b}{2} - \frac{3}{2}a(a^2-3b^2)\tan(c+dx) - \frac{1}{2}b(2a^2}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+}} \\
&= -\frac{2a^2\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{14ab\sqrt{a+b\tan(c+dx)}}{3d\sqrt{\tan(c+dx)}} - \frac{4\int \frac{\frac{3}{4}a^2(a^2-3b^2)+}{\sqrt{\tan(c+dx)}}}{} \\
&= -\frac{2a^2\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{14ab\sqrt{a+b\tan(c+dx)}}{3d\sqrt{\tan(c+dx)}} - \frac{1}{2}(a-ib)^3\int \frac{}{\sqrt{\text{ta}}} \\
&= -\frac{2a^2\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{14ab\sqrt{a+b\tan(c+dx)}}{3d\sqrt{\tan(c+dx)}} - \frac{(a-ib)^3\text{Subst}\Big(\int}{} \\
&= -\frac{2a^2\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{14ab\sqrt{a+b\tan(c+dx)}}{3d\sqrt{\tan(c+dx)}} - \frac{(a-ib)^3\text{Subst}\Big(\int}{} \\
&= -\frac{i(ia-b)^{5/2}\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{i(ia+b)^{5/2}\tanh^{-1}\Big(\frac{\sqrt{ia}}{\sqrt{}}}{d}
\end{aligned}$$

Mathematica [A]
time = 0.91, size = 170, normalized size = 0.93

$$\frac{-3(-1)^{3/4}(-a+ib)^{5/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+3(-1)^{3/4}(a+ib)^{5/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)-\frac{2a\sqrt{a+b\tan(c+dx)}(a+7b\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^(5/2)/Tan[c + d*x]^(5/2),x]
```

```
[Out] (-3*(-1)^(3/4)*(-a + I*b)^(5/2)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[
c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + 3*(-1)^(3/4)*(a + I*b)^(5/2)*ArcTan[
((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] - (
2*a*Sqrt[a + b*Tan[c + d*x]]*(a + 7*b*Tan[c + d*x]))/Tan[c + d*x]^(3/2))/(3
*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.79, size = 16922, normalized size = 92.98

method	result	size
default	Expression too large to display	16922

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(5/2)/tan(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/2)/tan(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(5/2)/tan(d*x + c)^(5/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/2)/tan(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan{(c + dx)})^{\frac{5}{2}}}{\tan^{\frac{5}{2}}{(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(5/2)/tan(d*x+c)**(5/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**(5/2)/tan(c + d*x)**(5/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/2)/tan(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\tan(c+d\,x))^{5/2}}{\tan\left(c+d\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^(5/2)/tan(c + d*x)^(5/2),x)
```

```
[Out] int((a + b*tan(c + d*x))^(5/2)/tan(c + d*x)^(5/2), x)
```

3.629 $\int \frac{(a+b\tan(c+dx))^{5/2}}{\tan^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=219

$$-\frac{(ia-b)^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{(ia+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{2a^2\sqrt{a+}}{5d\tan}$$

```
[Out] -(I*a-b)^(5/2)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2)
)/d+(I*a+b)^(5/2)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(
1/2))/d+2/15*(15*a^2-23*b^2)*(a+b*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(1/2)-2/5*
a^2*(a+b*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(5/2)-22/15*a*b*(a+b*tan(d*x+c))^(1
/2)/d/tan(d*x+c)^(3/2)
```

Rubi [A]

time = 0.68, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3646, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(15a^2-23b^2)\sqrt{a+b\tan(c+dx)}}{15d\sqrt{\tan(c+dx)}}-\frac{2a^2\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{(-b+ia)^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{22ab\sqrt{a+b\tan(c+dx)}}{15d\tan^{\frac{3}{2}}(c+dx)}+\frac{(b+ia)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^(5/2)/Tan[c + d*x]^(7/2),x]
```

```
[Out] -(((I*a - b)^(5/2)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan
[c + d*x]]])/d) + ((I*a + b)^(5/2)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]
])/Sqrt[a + b*Tan[c + d*x]]])/d - (2*a^2*Sqrt[a + b*Tan[c + d*x]])/(5*d*Tan
[c + d*x]^(5/2)) - (22*a*b*Sqrt[a + b*Tan[c + d*x]])/(15*d*Tan[c + d*x]^(3/
2)) + (2*(15*a^2 - 23*b^2)*Sqrt[a + b*Tan[c + d*x]])/(15*d*Sqrt[Tan[c + d*x
]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
```

```
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{(a+b\tan(c+dx))^{5/2}}{\tan^{\frac{7}{2}}(c+dx)}\,dx = -\frac{2a^2\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{11a^2b}{2} - \frac{5}{2}a(a^2-3b^2)\tan(c+dx) - \frac{1}{2}b(4a^2 \cdots}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+ \cdots}}$$

$$= -\frac{2a^2\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{22ab\sqrt{a+b\tan(c+dx)}}{15d\tan^{\frac{3}{2}}(c+dx)} - \frac{4\int \frac{\frac{1}{4}a^2(15a^2-23b^2)+\frac{15}{4}\cdots}{\tan^{\frac{3}{2}}(c+\cdots}}{\cdots}$$

$$= -\frac{2a^2\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{22ab\sqrt{a+b\tan(c+dx)}}{15d\tan^{\frac{3}{2}}(c+dx)} + \frac{2(15a^2-23b^2)\sqrt{a \cdots}}{15d\sqrt{\tan(\cdots}}$$

$$= -\frac{2a^2\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{22ab\sqrt{a+b\tan(c+dx)}}{15d\tan^{\frac{3}{2}}(c+dx)} + \frac{2(15a^2-23b^2)\sqrt{a \cdots}}{15d\sqrt{\tan(\cdots}}$$

$$= -\frac{2a^2\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{22ab\sqrt{a+b\tan(c+dx)}}{15d\tan^{\frac{3}{2}}(c+dx)} + \frac{2(15a^2-23b^2)\sqrt{a \cdots}}{15d\sqrt{\tan(\cdots}}$$

$$= -\frac{2a^2\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{22ab\sqrt{a+b\tan(c+dx)}}{15d\tan^{\frac{3}{2}}(c+dx)} + \frac{2(15a^2-23b^2)\sqrt{a \cdots}}{15d\sqrt{\tan(\cdots}}$$

$$= -\frac{(ia-b)^{5/2}\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{(ia+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{ia+\cdots}}{\sqrt{a+\cdots}}\right.}{d}$$

Mathematica [A]
time = 1.72, size = 194, normalized size = 0.89

$$\frac{15\sqrt[4]{-1}(-a+ib)^{5/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right) + 15\sqrt[4]{-1}(a+ib)^{5/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right) + \frac{2\sqrt{a+b\tan(c+dx)}(-3a^2-11ab\tan(c+dx)+(15a^2-23b^2)\tan^2(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^(5/2)/Tan[c + d*x]^(7/2),x]
```

```
[Out] (15*(-1)^(1/4)*(-a + I*b)^(5/2)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[
c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + 15*(-1)^(1/4)*(a + I*b)^(5/2)*ArcTan
[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] +
(2*Sqrt[a + b*Tan[c + d*x]]*(-3*a^2 - 11*a*b*Tan[c + d*x] + (15*a^2 - 23*b^
2)*Tan[c + d*x]^2))/Tan[c + d*x]^(5/2))/(15*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.80, size = 33401, normalized size = 152.52

method	result	size
default	Expression too large to display	33401

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(5/2)/tan(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/2)/tan(d*x+c)^(7/2),x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^(5/2)/tan(d*x + c)^(7/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/2)/tan(d*x+c)^(7/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(5/2)/tan(d*x+c)**(7/2),x)
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/2)/tan(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\tan(c+d\,x))^{5/2}}{\tan\left(c+d\,x\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^(5/2)/tan(c + d*x)^(7/2),x)
```

```
[Out] int((a + b*tan(c + d*x))^(5/2)/tan(c + d*x)^(7/2), x)
```

3.630 $\int \frac{(a+b\tan(c+dx))^{5/2}}{\tan^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=270

$$\frac{i(ia-b)^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{i(ia+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{2a^2\sqrt{a-}}{7d\,\mathrm{ta}}$$

```
[Out] I*(I*a-b)^(5/2)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2
))/d+I*(I*a+b)^(5/2)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)
)^(1/2))/d+2/21*b*(49*a^2-3*b^2)*(a+b*tan(d*x+c))^(1/2)/a/d/tan(d*x+c)^(1/2
)-2/7*a^2*(a+b*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(7/2)-6/7*a*b*(a+b*tan(d*x+c)
)^(1/2)/d/tan(d*x+c)^(5/2)+2/21*(7*a^2-9*b^2)*(a+b*tan(d*x+c))^(1/2)/d/tan(
d*x+c)^(3/2)
```

Rubi [A]

time = 0.88, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3646, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(7a^2-9b^2)\sqrt{a+b\tan(c+dx)}}{21d\tan^{\frac{3}{2}}(c+dx)}+\frac{2b(49a^2-3b^2)\sqrt{a+b\tan(c+dx)}}{21ad\sqrt{\tan(c+dx)}}-\frac{2a^2\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+\frac{i(-b+ia)^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{6ab\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{5}{2}}(c+dx)}+\frac{i(b+ia)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^(5/2)/Tan[c + d*x]^(9/2),x]
```

```
[Out] (I*(I*a - b)^(5/2)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan
[c + d*x]]])/d + (I*(I*a + b)^(5/2)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x
]])/Sqrt[a + b*Tan[c + d*x]]])/d - (2*a^2*Sqrt[a + b*Tan[c + d*x]])/(7*d*Ta
n[c + d*x]^(7/2)) - (6*a*b*Sqrt[a + b*Tan[c + d*x]])/(7*d*Tan[c + d*x]^(5/2
)) + (2*(7*a^2 - 9*b^2)*Sqrt[a + b*Tan[c + d*x]])/(21*d*Tan[c + d*x]^(3/2))
 + (2*b*(49*a^2 - 3*b^2)*Sqrt[a + b*Tan[c + d*x]])/(21*a*d*Sqrt[Tan[c + d*x
]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
```

```
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{(a+b\tan(c+dx))^{5/2}}{\tan^{\frac{9}{2}}(c+dx)}\,dx = -\frac{2a^2\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{\frac{15a^2b}{2} - \frac{7}{2}a(a^2-3b^2)\tan(c+dx) - \frac{1}{2}b(6a^2}{\tan^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c}}$$

$$= -\frac{2a^2\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)} - \frac{6ab\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{5}{2}}(c+dx)} - \frac{4\int \frac{\frac{5}{4}a^2(7a^2-9b^2)+\frac{35}{4}a}{\tan^{\frac{5}{2}}(c+}}{}$$

$$= -\frac{2a^2\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)} - \frac{6ab\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{5}{2}}(c+dx)} + \frac{2(7a^2-9b^2)\sqrt{a+}}{21d\tan^{\frac{3}{2}}(c}$$

$$= -\frac{2a^2\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)} - \frac{6ab\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{5}{2}}(c+dx)} + \frac{2(7a^2-9b^2)\sqrt{a+}}{21d\tan^{\frac{3}{2}}(c}$$

$$= -\frac{2a^2\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)} - \frac{6ab\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{5}{2}}(c+dx)} + \frac{2(7a^2-9b^2)\sqrt{a+}}{21d\tan^{\frac{3}{2}}(c}$$

$$= -\frac{2a^2\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)} - \frac{6ab\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{5}{2}}(c+dx)} + \frac{2(7a^2-9b^2)\sqrt{a+}}{21d\tan^{\frac{3}{2}}(c}$$

$$= -\frac{2a^2\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)} - \frac{6ab\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{5}{2}}(c+dx)} + \frac{2(7a^2-9b^2)\sqrt{a+}}{21d\tan^{\frac{3}{2}}(c}$$

$$= \frac{i(ia-b)^{5/2}\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{i(ia+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{ia+}}{\sqrt{a}}\right.}{d}$$

Mathematica [A]
time = 3.69, size = 249, normalized size = 0.92

$$-\frac{-42(-1)^{3/4}(-a+ib)^{5/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+42(-1)^{3/4}(a+ib)^{5/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+\frac{\sec^3(c+dx)\left(a(2a^2+9b^2)\cos(c+dx)+(10a^3-9ab^2)\cos(3(c+dx))+2b(-40a^2+3b^2+(58a^2-3b^2)\cos(2(c+dx)))\sin(c+dx)\right)\sqrt{a+b\tan(c+dx)}}{a\tan^{\frac{7}{2}}(c+dx)}}{42d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^(5/2)/Tan[c + d*x]^(9/2),x]
```

```
[Out] -1/42*(-42*(-1)^(3/4)*(-a + I*b)^(5/2)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sq
rt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + 42*(-1)^(3/4)*(a + I*b)^(5/2)
*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*
x]]] + (Sec[c + d*x]^3*(a*(2*a^2 + 9*b^2)*Cos[c + d*x] + (10*a^3 - 9*a*b^2)
*Cos[3*(c + d*x)] + 2*b*(-40*a^2 + 3*b^2 + (58*a^2 - 3*b^2)*Cos[2*(c + d*x)
])*Sin[c + d*x])*Sqrt[a + b*Tan[c + d*x]])/(a*Tan[c + d*x]^(7/2)))/d
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.89, size = 34250, normalized size = 126.85

method	result	size
default	Expression too large to display	34250

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(5/2)/tan(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/2)/tan(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(5/2)/tan(d*x + c)^(9/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/2)/tan(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(5/2)/tan(d*x+c)**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/2)/tan(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\tan(c+d\,x))^{5/2}}{\tan\left(c+d\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^(5/2)/tan(c + d*x)^(9/2),x)
```

```
[Out] int((a + b*tan(c + d*x))^(5/2)/tan(c + d*x)^(9/2), x)
```

3.631 $\int \frac{(a+b\tan(c+dx))^{5/2}}{\tan^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=318

$$\frac{(ia-b)^{5/2}\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{(ia+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{2a^2\sqrt{a+b}}{9d\tan^{\frac{9}{2}}}$$

```
[Out] (I*a-b)^(5/2)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))
/d-(I*a+b)^(5/2)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1
/2))/d-2/315*(315*a^4-483*a^2*b^2-10*b^4)*(a+b*tan(d*x+c))^(1/2)/a^2/d/tan(
d*x+c)^(1/2)-2/9*a^2*(a+b*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(9/2)-38/63*a*b*(a
+b*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(7/2)+2/105*(21*a^2-25*b^2)*(a+b*tan(d*x+
c))^(1/2)/d/tan(d*x+c)^(5/2)+2/315*b*(231*a^2-5*b^2)*(a+b*tan(d*x+c))^(1/2)
/a/d/tan(d*x+c)^(3/2)
```

Rubi [A]
time = 1.07, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$,
Rules used = {3646, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2b(231a^2-5b^2)\sqrt{a+b\tan(c+dx)}}{315ad\tan^{\frac{3}{2}}(c+dx)}+\frac{2(21a^2-25b^2)\sqrt{a+b\tan(c+dx)}}{105d\tan^{\frac{5}{2}}(c+dx)}-\frac{2a^2\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}-\frac{2(315a^4-483a^2b^2-10b^4)\sqrt{a+b\tan(c+dx)}}{315a^2d\sqrt{\tan(c+dx)}}+\frac{(-b+ia)^{5/2}\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{38ab\sqrt{a+b\tan(c+dx)}}{63d\tan^{\frac{7}{2}}(c+dx)}-\frac{(b+ia)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^(5/2)/Tan[c + d*x]^(11/2),x]
```

```
[Out] ((I*a - b)^(5/2)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
 + d*x]]])/d - ((I*a + b)^(5/2)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/
Sqrt[a + b*Tan[c + d*x]]])/d - (2*a^2*Sqrt[a + b*Tan[c + d*x]])/(9*d*Tan[c
+ d*x]^(9/2)) - (38*a*b*Sqrt[a + b*Tan[c + d*x]])/(63*d*Tan[c + d*x]^(7/2))
 + (2*(21*a^2 - 25*b^2)*Sqrt[a + b*Tan[c + d*x]])/(105*d*Tan[c + d*x]^(5/2)
) + (2*b*(231*a^2 - 5*b^2)*Sqrt[a + b*Tan[c + d*x]])/(315*a*d*Tan[c + d*x]^
(3/2)) - (2*(315*a^4 - 483*a^2*b^2 - 10*b^4)*Sqrt[a + b*Tan[c + d*x]])/(315
*a^2*d*Sqrt[Tan[c + d*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
```

```
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{(a+b\tan(c+dx))^{5/2}}{\tan^{\frac{11}{2}}(c+dx)}\,dx = -\frac{2a^2\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)} + \frac{2}{9}\int \frac{\frac{19a^2b}{2} - \frac{9}{2}a(a^2-3b^2)\tan(c+dx) - \frac{1}{2}b(8a^2 \cdots}{\tan^{\frac{9}{2}}(c+dx)\sqrt{a+b\tan(c+ \cdots}}$$

$$= -\frac{2a^2\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)} - \frac{38ab\sqrt{a+b\tan(c+dx)}}{63d\tan^{\frac{7}{2}}(c+dx)} - \frac{4\int \frac{\frac{3}{4}a^2(21a^2-25b^2)+\frac{63}{4}\cdots}{\tan^{\frac{7}{2}}(c+\cdots}}{\cdots}$$

$$= -\frac{2a^2\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)} - \frac{38ab\sqrt{a+b\tan(c+dx)}}{63d\tan^{\frac{7}{2}}(c+dx)} + \frac{2(21a^2-25b^2)\sqrt{a \cdots}}{105d\tan^{\frac{5}{2}}(\cdots}$$

$$= -\frac{2a^2\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)} - \frac{38ab\sqrt{a+b\tan(c+dx)}}{63d\tan^{\frac{7}{2}}(c+dx)} + \frac{2(21a^2-25b^2)\sqrt{a \cdots}}{105d\tan^{\frac{5}{2}}(\cdots}$$

$$= -\frac{2a^2\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)} - \frac{38ab\sqrt{a+b\tan(c+dx)}}{63d\tan^{\frac{7}{2}}(c+dx)} + \frac{2(21a^2-25b^2)\sqrt{a \cdots}}{105d\tan^{\frac{5}{2}}(\cdots}$$

$$= -\frac{2a^2\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)} - \frac{38ab\sqrt{a+b\tan(c+dx)}}{63d\tan^{\frac{7}{2}}(c+dx)} + \frac{2(21a^2-25b^2)\sqrt{a \cdots}}{105d\tan^{\frac{5}{2}}(\cdots}$$

$$= -\frac{2a^2\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)} - \frac{38ab\sqrt{a+b\tan(c+dx)}}{63d\tan^{\frac{7}{2}}(c+dx)} + \frac{2(21a^2-25b^2)\sqrt{a \cdots}}{105d\tan^{\frac{5}{2}}(\cdots}$$

$$= -\frac{2a^2\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)} - \frac{38ab\sqrt{a+b\tan(c+dx)}}{63d\tan^{\frac{7}{2}}(c+dx)} + \frac{2(21a^2-25b^2)\sqrt{a \cdots}}{105d\tan^{\frac{5}{2}}(\cdots}$$

$$= \frac{(ia-b)^{5/2}\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{(ia+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{ia+b}\cdots}{\sqrt{a+b\cdots}}\right.}{d}$$

Mathematica [A]

time = 4.22, size = 300, normalized size = 0.94

$$\frac{-1260\sqrt[4]{-1}\,(-a+ib)^{5/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)-1260\sqrt[4]{-1}\,(a+ib)^{5/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+\frac{\sec^4(c+dx)\left(-987a^4+1374a^2b^2+30b^4+4\left(280a^4-483a^2b^2-10b^4\right)\cos(2(c+dx))+\left(-413a^4+558a^2b^2+10b^4\right)\cos(4(c+dx))+272a^3b\sin(2(c+dx))-10ab^3\sin(2(c+dx))-326a^3b\sin(4(c+dx))+5ab^3\sin(4(c+dx))\right)\sqrt{a+b\tan(c+dx)}}{a^2\tan^{\frac{9}{2}}(c+dx)}}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^(5/2)/Tan[c + d*x]^(11/2),x]
```

```
[Out] (-1260*(-1)^(1/4)*(-a + I*b)^(5/2)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[T
an[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] - 1260*(-1)^(1/4)*(a + I*b)^(5/2)*A
rcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]
]] + (Sec[c + d*x]^4*(-987*a^4 + 1374*a^2*b^2 + 30*b^4 + 4*(280*a^4 - 483*a
^2*b^2 - 10*b^4)*Cos[2*(c + d*x)] + (-413*a^4 + 558*a^2*b^2 + 10*b^4)*Cos[4
*(c + d*x)] + 272*a^3*b*Sin[2*(c + d*x)] - 10*a*b^3*Sin[2*(c + d*x)] - 326*
a^3*b*Sin[4*(c + d*x)] + 5*a*b^3*Sin[4*(c + d*x)])*Sqrt[a + b*Tan[c + d*x]]
)/(a^2*Tan[c + d*x]^(9/2)))/(1260*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.90, size = 50542, normalized size = 158.94

method	result	size
default	Expression too large to display	50542

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(5/2)/tan(d*x+c)^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/2)/tan(d*x+c)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(5/2)/tan(d*x + c)^(11/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/2)/tan(d*x+c)^(11/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(5/2)/tan(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/2)/tan(d*x+c)^(11/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\tan(c+d\,x))^{5/2}}{\tan\left(c+d\,x\right)^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^(5/2)/tan(c + d*x)^(11/2),x)
```

```
[Out] int((a + b*tan(c + d*x))^(5/2)/tan(c + d*x)^(11/2), x)
```

3.632 $\int \frac{\tan^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal. Leaf size=232

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia-b}\,d}+\frac{(3a^2-8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{4b^{5/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia+b}\,d}$$

```
[Out] 1/4*(3*a^2-8*b^2)*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/
b^(5/2)/d+arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/d/(
I*a-b)^(1/2)+arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))
/d/(I*a+b)^(1/2)-3/4*a*tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)/b^2/d+1/2*(a
+b*tan(d*x+c))^(1/2)*tan(d*x+c)^(3/2)/b/d
```

Rubi [A]

time = 0.72, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3647, 3728, 3737, 6857, 65, 223, 212, 926, 95, 211, 214}

$$\frac{(3a^2-8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{4b^{5/2}d}+\frac{\operatorname{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}-\frac{3a\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{4b^2d}+\frac{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{2bd}+\frac{\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(7/2)/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]/(Sqrt[I
*a - b]*d) + ((3*a^2 - 8*b^2)*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a +
 b*Tan[c + d*x]]])/(4*b^(5/2)*d) + ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]
])/Sqrt[a + b*Tan[c + d*x]]]/(Sqrt[I*a + b]*d) - (3*a*Sqrt[Tan[c + d*x]]*Sq
rt[a + b*Tan[c + d*x]])/(4*b^2*d) + (Tan[c + d*x]^(3/2)*Sqrt[a + b*Tan[c +
d*x]])/(2*b*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
```

```
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
```

```
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3737

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b*ff*x)
^m*(c + d*ff*x)^n*((A + C*ff^2*x^2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff
], x]] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\tan^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx = \frac{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{2bd} + \frac{\int \frac{\sqrt{\tan(c+dx)}\left(-\frac{3a}{2}-2b\tan(c+dx)-\frac{3}{2}a\tan^2(c-\right.}{\sqrt{a+b\tan(c+dx)}}}{2b}$$

$$= -\frac{3a\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{4b^2d} + \frac{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{2bd}$$

$$= -\frac{3a\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{4b^2d} + \frac{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{2bd}$$

$$= -\frac{3a\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{4b^2d} + \frac{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{2bd}$$

$$= -\frac{3a\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{4b^2d} + \frac{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{2bd}$$

$$= -\frac{3a\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{4b^2d} + \frac{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{2bd}$$

$$= -\frac{3a\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{4b^2d} + \frac{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{2bd}$$

$$= \frac{(3a^2-8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{4b^{5/2}d} - \frac{3a\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c}{4b^2d}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia-b}\,d} + \frac{(3a^2-8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right.}{4b^{5/2}d}$$

Mathematica [A]
time = 4.18, size = 270, normalized size = 1.16

$$\frac{-\frac{4(-1)^{3/4}b^2\,\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}} - \frac{4(-1)^{3/4}b^2\,\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{a+ib}} - 3a\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)} + 2b\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)} + \frac{\sqrt{a}(3a^2-8b^2)\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{1+\frac{b\tan(c+dx)}{a}}}{\sqrt{b}\sqrt{a+b\tan(c+dx)}}}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(7/2)/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] ((-4*(-1)^(3/4)*b^2*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/S
qrt[a + b*Tan[c + d*x]]])/Sqrt[-a + I*b] - (4*(-1)^(3/4)*b^2*ArcTan[((-1)^(
1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[a +
```

```
I*b] - 3*a*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]] + 2*b*Tan[c + d*x]^(
3/2)*Sqrt[a + b*Tan[c + d*x]] + (Sqrt[a]*(3*a^2 - 8*b^2)*ArcSinh[(Sqrt[b]*S
qrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a])/(Sqrt[b]*Sqrt[a +
 b*Tan[c + d*x]]))/(4*b^2*d)
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 1.03, size = 945945, normalized size = 4077.35

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(7/2)/(a+b*tan(d*x+c))^(1/2),x)
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(7/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(tan(d*x + c)^(7/2)/sqrt(b*tan(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(7/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(7/2)/(a+b*tan(d*x+c))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(7/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(tan(d*x + c)^(7/2)/sqrt(b*tan(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(c+d\,x)^{7/2}}{\sqrt{a+b\tan\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(7/2)/(a + b*tan(c + d*x))^(1/2),x)
[Out] int(tan(c + d*x)^(7/2)/(a + b*tan(c + d*x))^(1/2), x)
```

3.633 $\int \frac{\tan^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal. Leaf size=188

$$-\frac{i\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia-b}\,d}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{b^{3/2}d}+\frac{i\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia+b}\,d}$$

```
[Out] -a*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/b^(3/2)/d-I*arc
tan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/d/(I*a-b)^(1/2)+
I*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/d/(I*a+b)^
(1/2)+tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)/b/d
```

Rubi [A]
time = 0.49, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3647, 3736, 6857, 65, 223, 212, 924, 95, 211, 214}

$$-\frac{i\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{b^{3/2}d}+\frac{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{bd}+\frac{i\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(5/2)/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] ((-I)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/
(Sqrt[I*a - b]*d) - (a*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[
c + d*x]]])/(b^(3/2)*d) + (I*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqr
t[a + b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d) + (Sqrt[Tan[c + d*x]]*Sqrt[a + b*
Tan[c + d*x]])/(b*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx = \frac{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{bd} + \frac{\int \frac{-\frac{a}{2}-b\tan(c+dx)-\frac{1}{2}a\tan^2(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}}{b}$$

$$= \frac{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{bd} + \frac{\text{Subst}\left(\int \frac{-\frac{a}{2}-bx-\frac{ax^2}{2}}{\sqrt{x}\sqrt{a+bx}\,(1+x^2)}\,dx, x, \tan\right.}{bd}$$

$$= \frac{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{bd} + \frac{\text{Subst}\left(\int \left(-\frac{a}{2\sqrt{x}\sqrt{a+bx}} - \frac{b\sqrt{}}{\sqrt{a+}}\right.\right.}{bd}$$

$$= \frac{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{bd} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{a+bx}\,(1+x^2)}\,dx, x, \tan(c\right.}{d}$$

$$= \frac{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{bd} - \frac{\text{Subst}\left(\int \left(-\frac{1}{2(i-x)\sqrt{x}\sqrt{a+bx}} + \frac{}{2\sqrt{}}\right.\right.}{}$$

$$= \frac{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{bd} + \frac{\text{Subst}\left(\int \frac{1}{(i-x)\sqrt{x}\sqrt{a+bx}}\,dx, x, \tan\right.}{2d}$$

$$= -\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{b^{3/2}d} + \frac{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{bd} -$$

$$= -\frac{i\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia-b}\,d} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{b^{3/2}d} +$$

Mathematica [A]

time = 1.98, size = 221, normalized size = 1.18

$$\frac{\frac{\sqrt[4]{-1}\,\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}}-\frac{\sqrt[4]{-1}\,\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{a+ib}}+\frac{\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{b}-\frac{a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{1+\frac{b\tan(c+dx)}{a}}}{b^{3/2}\sqrt{a+b\tan(c+dx)}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(5/2)/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] (((-1)^(1/4)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a +
 b*Tan[c + d*x]]])/Sqrt[-a + I*b] - ((-1)^(1/4)*ArcTan[((-1)^(1/4)*Sqrt[a +
 I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[a + I*b] + (Sqrt[
Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]])/b - (a^(3/2)*ArcSinh[(Sqrt[b]*Sqrt[
Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a])/(b^(3/2)*Sqrt[a + b*T
an[c + d*x]]))/d
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 1.17, size = 945569, normalized size = 5029.62

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^(5/2)/sqrt(b*tan(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{5}{2}}(c + dx)}{\sqrt{a + b\tan(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(5/2)/(a+b*tan(d*x+c))**(1/2),x)
[Out] Integral(tan(c + d*x)**(5/2)/sqrt(a + b*tan(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(tan(d*x + c)^(5/2)/sqrt(b*tan(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\tan(c + d\,x)}^{5/2}}{\sqrt{a + b\tan(c + d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(5/2)/(a + b*tan(c + d*x))^(1/2),x)
[Out] int(tan(c + d*x)^(5/2)/(a + b*tan(c + d*x))^(1/2), x)
```

3.634 $\int \frac{\tan^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal. Leaf size=152

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia-b}\,d}+\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{b}\,d}-\frac{\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia+b}\,d}$$

```
[Out] -arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/d/(I*a-b)^(1
/2)+2*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/d/b^(1/2)-ar
ctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/d/(I*a+b)^(1/2
)
```

Rubi [A]
time = 0.13, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3656, 924, 65, 223, 212, 926, 95, 211, 214}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{b}\,d}-\frac{\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(3/2)/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] -(ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]/(Sqrt
[I*a - b]*d)) + (2*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c +
d*x]]])/(Sqrt[b]*d) - ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b
*Tan[c + d*x]]]/(Sqrt[I*a + b]*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^{3/2}}{\sqrt{a+bx}\,(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{\sqrt{x}\sqrt{a+bx}} - \frac{1}{\sqrt{x}\sqrt{a+bx}\,(1+x^2)}\right) dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx, x, \tan(c+dx)\right)}{d} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{a+bx}\,(1+x^2)}\,dx\right.}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{i}{2(i-x)\sqrt{x}\sqrt{a+bx}} + \frac{i}{2\sqrt{x}\,(i+x)\sqrt{a+bx}}\right) dx, x, \tan(c+dx)\right)}{d} +$$

$$= -\frac{i\text{Subst}\left(\int \frac{1}{(i-x)\sqrt{x}\sqrt{a+bx}}\,dx, x, \tan(c+dx)\right)}{2d} - \frac{i\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(i+x)\sqrt{a+}\right.}{2}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{b}\,d} - \frac{i\text{Subst}\left(\int \frac{1}{i-(-a+ib)x^2}\,dx, x, \frac{\sqrt{\tan(c+}}{\sqrt{a+b\tan(c}}\right.}{d}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia-b}\,d} + \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{b}\,d} - \text{ta}$$

Mathematica [A]
time = 0.81, size = 189, normalized size = 1.24

$$\frac{(-1)^{3/4}\left(\frac{\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}} + \frac{\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{a+ib}}\right) + \frac{2\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{1+\frac{b\tan(c+dx)}{a}}}{\sqrt{b}\sqrt{a+b\tan(c+dx)}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(3/2)/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] ((-1)^(3/4)*(ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a +
 b*Tan[c + d*x]]]/Sqrt[-a + I*b] + ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Ta
n[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]/Sqrt[a + I*b]) + (2*Sqrt[a]*ArcSinh[
(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a])/(Sqrt[b
]*Sqrt[a + b*Tan[c + d*x]]))/d
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 0.55, size = 943433, normalized size = 6206.80

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^(3/2)/sqrt(b*tan(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(3/2)/(a+b*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**(3/2)/sqrt(a + b*tan(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^(3/2)/sqrt(b*tan(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^{3/2}}{\sqrt{a+b\tan\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(3/2)/(a + b*tan(c + d*x))^(1/2),x)
[Out] int(tan(c + d*x)^(3/2)/(a + b*tan(c + d*x))^(1/2), x)
```

3.635 $\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal. Leaf size=115

$$\frac{i\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia-b}\,d}-\frac{i\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia+b}\,d}$$

```
[Out] I*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/d/(I*a-b)^(
1/2)-I*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/d/(I*
a+b)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3656, 924, 95, 211, 214}

$$\frac{i\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[c + d*x]]/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] (I*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sq
rt[I*a - b]*d) - (I*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*T
an[c + d*x]]])/(Sqrt[I*a + b]*d)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{a+bx}\,(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{2(i-x)\sqrt{x}\sqrt{a+bx}} + \frac{1}{2\sqrt{x}\,(i+x)\sqrt{a+bx}}\right) dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{(i-x)\sqrt{x}\sqrt{a+bx}}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(i+x)\sqrt{a+b}}\right.}{2d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{i-(-a+ib)x^2}\,dx, x, \frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{\text{Subst}\left(\int \frac{1}{i-(a+ib)x^2}\,dx, x, \right.}{d}$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia-b}\,d} - \frac{i\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia+b}\,d}$$

Mathematica [A]
time = 0.10, size = 123, normalized size = 1.07

$$\frac{\sqrt[4]{-1}\left(-\frac{\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}} + \frac{\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{a+ib}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tan[c + d*x]]/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] ((-1)^(1/4)*(-(ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a
 + b*Tan[c + d*x]]]/Sqrt[-a + I*b]) + ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt
[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]/Sqrt[a + I*b]))/d
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 0.65, size = 940502, normalized size = 8178.28

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(tan(d*x + c))/sqrt(b*tan(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\tan{\left(c + dx \right)}}}{\sqrt{a + b\tan{\left(c + dx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/2)/(a+b*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(tan(c + d*x))/sqrt(a + b*tan(c + d*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 43.19, size = 2500, normalized size = 21.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)/(a + b*tan(c + d*x))^(1/2),x)
```

```
[Out] (log(((-1/(d^2*(a*1i + b)))^(1/2)*(((-1/(d^2*(a*1i + b)))^(1/2)*(((-1/(d^2*
(a*1i + b)))^(1/2)*(((-1/(d^2*(a*1i + b)))^(1/2)*(((-1/(d^2*(a*1i + b)))^(1
/2)*(536870912*a^8*b^16*(-1/(d^2*(a*1i + b)))^(1/2)*(8*a^2 + 7*b^2)*(8*a^2
+ 7*b^2 - (17*b^3*tan(c + d*x))/((a + b*tan(c + d*x))^(1/2) - a^(1/2))^2 -
(16*a^2*b*tan(c + d*x))/((a + b*tan(c + d*x))^(1/2) - a^(1/2))^2) + (107374
1824*a^7*b^17*tan(c + d*x)^(1/2)*(48*a^4 + 5*b^4 + 52*a^2*b^2))/(d*((a + b*
tan(c + d*x))^(1/2) - a^(1/2)))))/2 + (268435456*a^6*b^17*(144*a^4 - b^4 +
112*a^2*b^2))/d^2 + (268435456*a^6*b^16*tan(c + d*x)*(256*a^6 + b^6 - 270*a
^2*b^4 - 32*a^4*b^2))/(d^2*((a + b*tan(c + d*x))^(1/2) - a^(1/2))^2)))/2 +
(536870912*a^7*b^18*tan(c + d*x)^(1/2)*(48*a^2 + 5*b^2))/(d^3*((a + b*tan(c
 + d*x))^(1/2) - a^(1/2)))))/2 + (134217728*a^6*b^16*(32*a^4 - b^4 + 40*a^2
*b^2))/d^4 + (134217728*a^6*b^17*tan(c + d*x)*(192*a^4 + b^4 - 92*a^2*b^2))
/(d^4*((a + b*tan(c + d*x))^(1/2) - a^(1/2))^2)))/2 + (67108864*a^7*b^17*ta
n(c + d*x)^(1/2)*(48*a^2 + 5*b^2))/(d^5*((a + b*tan(c + d*x))^(1/2) - a^(1/
2)))))/2 + (16777216*a^6*b^17*(16*a^2 - b^2))/d^6 + (16777216*a^6*b^16*tan(
c + d*x)*(16*a^2 - b^2)^2)/(d^6*((a + b*tan(c + d*x))^(1/2) - a^(1/2))^2))*
(-1/(a*d^2*1i + b*d^2))^(1/2))/2 - log(((-1/(d^2*(a*1i + b)))^(1/2)*(((-1/(
d^2*(a*1i + b)))^(1/2)*(((-1/(d^2*(a*1i + b)))^(1/2)*(((-1/(d^2*(a*1i + b))
)^(1/2)*(((-1/(d^2*(a*1i + b)))^(1/2)*(536870912*a^8*b^16*(-1/(d^2*(a*1i +
b)))^(1/2)*(8*a^2 + 7*b^2)*(8*a^2 + 7*b^2 - (17*b^3*tan(c + d*x))/((a + b*t
an(c + d*x))^(1/2) - a^(1/2))^2 - (16*a^2*b*tan(c + d*x))/((a + b*tan(c + d
*x))^(1/2) - a^(1/2))^2) - (1073741824*a^7*b^17*tan(c + d*x)^(1/2)*(48*a^4
+ 5*b^4 + 52*a^2*b^2))/(d*((a + b*tan(c + d*x))^(1/2) - a^(1/2)))))/2 + (26
8435456*a^6*b^17*(144*a^4 - b^4 + 112*a^2*b^2))/d^2 + (268435456*a^6*b^16*t
an(c + d*x)*(256*a^6 + b^6 - 270*a^2*b^4 - 32*a^4*b^2))/(d^2*((a + b*tan(c
+ d*x))^(1/2) - a^(1/2))^2)))/2 - (536870912*a^7*b^18*tan(c + d*x)^(1/2)*(4
```

8*a^2 + 5*b^2))/(d^3*((a + b*tan(c + d*x))^(1/2) - a^(1/2)))))/2 + (1342177
28*a^6*b^16*(32*a^4 - b^4 + 40*a^2*b^2))/d^4 + (134217728*a^6*b^17*tan(c +
d*x)*(192*a^4 + b^4 - 92*a^2*b^2))/(d^4*((a + b*tan(c + d*x))^(1/2) - a^(1/
2))^2)))/2 - (67108864*a^7*b^17*tan(c + d*x)^(1/2)*(48*a^2 + 5*b^2))/(d^5*(
(a + b*tan(c + d*x))^(1/2) - a^(1/2)))))/2 + (16777216*a^6*b^17*(16*a^2 - b
^2))/d^6 + (16777216*a^6*b^16*tan(c + d*x)*(16*a^2 - b^2)^2)/(d^6*((a + b*t
an(c + d*x))^(1/2) - a^(1/2))^2))*(-1/(4*(a*d^2*1i + b*d^2)))^(1/2) + atan(
((-(a*1i + b)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*((-(a*1i + b)/(4*a^2*d^2 + 4*b
^2*d^2))^(1/2)*((-(a*1i + b)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*((-(a*1i + b)/(
4*a^2*d^2 + 4*b^2*d^2))^(1/2)*((-(a*1i + b)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*
((-(a*1i + b)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*((16777216*(3136*a^8*b^20*d^6
+ 7168*a^10*b^18*d^6 + 4096*a^12*b^16*d^6))/d^6 - (16777216*tan(c + d*x)*(7
616*a^8*b^21*d^6 + 15872*a^10*b^19*d^6 + 8192*a^12*b^17*d^6))/(d^6*((a + b*
tan(c + d*x))^(1/2) - a^(1/2))^2)) - (67108864*tan(c + d*x)^(1/2)*(80*a^7*b
^21*d^4 + 832*a^9*b^19*d^4 + 768*a^11*b^17*d^4))/(d^5*((a + b*tan(c + d*x))
^(1/2) - a^(1/2)))) + (16777216*(1792*a^8*b^19*d^4 - 16*a^6*b^21*d^4 + 2304
*a^10*b^17*d^4))/d^6 + (16777216*tan(c + d*x)*(16*a^6*b^22*d^4 - 4320*a^8*b
^20*d^4 - 512*a^10*b^18*d^4 + 4096*a^12*b^16*d^4))/(d^6*((a + b*tan(c + d*x
))^(1/2) - a^(1/2))^2)) - (67108864*tan(c + d*x)^(1/2)*(40*a^7*b^20*d^2 + 3
84*a^9*b^18*d^2))/(d^5*((a + b*tan(c + d*x))^(1/2) - a^(1/2)))) + (16777216
*(320*a^8*b^18*d^2 - 8*a^6*b^20*d^2 + 256*a^10*b^16*d^2))/d^6 + (16777216*t
an(c + d*x)*(8*a^6*b^21*d^2 - 736*a^8*b^19*d^2 + 1536*a^10*b^17*d^2))/(d^6*
((a + b*tan(c + d*x))^(1/2) - a^(1/2))^2)) - (67108864*tan(c + d*x)^(1/2)*(
5*a^7*b^19 + 48*a^9*b^17))/(d^5*((a + b*tan(c + d*x))^(1/2) - a^(1/2))))*1i
 - (-(a*1i + b)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*((-(a*1i + b)/(4*a^2*d^2 + 4
*b^2*d^2))^(1/2)*((-(a*1i + b)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*((-(a*1i + b)
/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*((-(a*1i + b)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2
)*((-(a*1i + b)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*((16777216*(3136*a^8*b^20*d^
6 + 7168*a^10*b^18*d^6 + 4096*a^12*b^16*d^6))/d^6 - (16777216*tan(c + d*x)*
(7616*a^8*b^21*d^6 + 15872*a^10*b^19*d^6 + 8192*a^12*b^17*d^6))/(d^6*((a +
b*tan(c + d*x))^(1/2) - a^(1/2))^2)) + (67108864*tan(c + d*x)^(1/2)*(80*a^7
*b^21*d^4 + 832*a^9*b^19*d^4 + 768*a^11*b^17*d^4))/(d^5*((a + b*tan(c + d*x
))^(1/2) - a^(1/2)))) + (16777216*(1792*a^8*b^19*d^4 - 16*a^6*b^21*d^4 + 23
04*a^10*b^17*d^4))/d^6 + (16777216*tan(c + d*x)*(16*a^6*b^22*d^4 - 4320*a^8
*b^20*d^4 - 512*a^10*b^18*d^4 + 4096*a^12*b^16*d^4))/(d^6*((a + b*tan(c + d
*x))^(1/2) - a^(1/2))^2)) + (67108864*tan(c + d*x)^(1/2)*(40*a^7*b^20*d^2 +
 384*a^9*b^18*d^2))/(d^5*((a + b*tan(c + d*x))^(1/2) - a^(1/2)))) + (167772
16*(320*a^8*b^18*d^2 - 8*a^6*b^20*d^2 + 256*a^10*b^16*d^2))/d^6 + (16777216
*tan(c + d*x)*(8*a^6*b^21*d^2 - 736*a^8*b^19*d^2 + 1536*a^10*b^17*d^2))/(d^
6*((a + b*tan(c + d*x))^(1/2) - a^(1/2))^2)) + (67108864*tan(c + d*x)^(1/2)
*(5*a^7*b^19 + 48*a^9*b^17))/(d^5*((a + b*tan(c...

3.636 $\int \frac{1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal. Leaf size=109

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia-b}\,d}+\frac{\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia+b}\,d}$$

```
[Out] arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/d/(I*a-b)^(1/
2)+arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/d/(I*a+b)
^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3656, 926, 95, 211, 214}

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]]),x]
```

```
[Out] ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]/(Sqrt[I
*a - b]*d) + ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c +
d*x]]]/(Sqrt[I*a + b]*d)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{a+bx}(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{i}{2(i-x)\sqrt{x}\sqrt{a+bx}} + \frac{i}{2\sqrt{x}(i+x)\sqrt{a+bx}}\right)dx, x, \text{t}\right.}{d}$$

$$= \frac{i\text{Subst}\left(\int \frac{1}{(i-x)\sqrt{x}\sqrt{a+bx}}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{i\text{Subst}\left(\int\right.}{}$$

$$= \frac{i\text{Subst}\left(\int \frac{1}{i-(-a+ib)x^2}\,dx, x, \frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{i\text{Subst}\left(\right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia-b}\,d} + \frac{\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan}}{\sqrt{a+b\tan(c}}\right.}{\sqrt{ia+b}\,d}$$

Mathematica [A]
time = 0.10, size = 124, normalized size = 1.14

$$\frac{(-1)^{3/4}\left(-\frac{\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}} - \frac{\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{a+ib}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]]),x]
```

```
[Out] ((-1)^(3/4)*(-(ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a
 + b*Tan[c + d*x]]]/Sqrt[-a + I*b]) - ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt
[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]/Sqrt[a + I*b]))/d
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 0.70, size = 940263, normalized size = 8626.27

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*tan(d*x + c) + a)*sqrt(tan(d*x + c))), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\tan\left(c+dx\right)}\,\sqrt{\tan\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(1/2)/(a+b*tan(d*x+c))**(1/2),x)
```

[Out] Integral(1/(sqrt(a + b*tan(c + d*x))*sqrt(tan(c + d*x))), x)

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")

[Out] Timed out

Mupad [B]
time = 13.30, size = 1716, normalized size = 15.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(tan(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(1/2)),x)

[Out] atan(((400*b^6*d^3*tan(c + d*x)^(1/2)*((a*1i + b)/(4*a^2*d^2 + 4*b^2*d^2))^(3/2))/((a + b*tan(c + d*x))^(1/2) - a^(1/2)) - (48*b^5*d*tan(c + d*x)^(1/2)*((a*1i + b)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2))/((a + b*tan(c + d*x))^(1/2) - a^(1/2)) - (a^7*d^5*tan(c + d*x)^(1/2)*((a*1i + b)/(4*a^2*d^2 + 4*b^2*d^2))^(5/2)*4096i)/((a + b*tan(c + d*x))^(1/2) - a^(1/2)) - (a^5*d*tan(c + d*x)^(1/2)*((a*1i + b)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*256i)/((a + b*tan(c + d*x))^(1/2) - a^(1/2)) - (832*b^7*d^5*tan(c + d*x)^(1/2)*((a*1i + b)/(4*a^2*d^2 + 4*b^2*d^2))^(5/2))/((a + b*tan(c + d*x))^(1/2) - a^(1/2)) - (112*a^2*b^3*d*tan(c + d*x)^(1/2)*((a*1i + b)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2))/((a + b*tan(c + d*x))^(1/2) - a^(1/2)) + (a^3*b^2*d*tan(c + d*x)^(1/2)*((a*1i + b)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*720i)/((a + b*tan(c + d*x))^(1/2) - a^(1/2)) - (a*b^5*d^3*tan(c + d*x)^(1/2)*((a*1i + b)/(4*a^2*d^2 + 4*b^2*d^2))^(3/2)*1200i)/((a + b*tan(c + d*x))^(1/2) - a^(1/2)) + (a^5*b*d^3*tan(c + d*x)^(1/2)*((a*1i + b)/(4*a^2*d^2 + 4*b^2*d^2))^(3/2)*2048i)/((a + b*tan(c + d*x))^(1/2) - a^(1/2)) + (a*b^6*d^5*tan(c + d*x)^(1/2)*((a*1i + b)/(4*a^2*d^2 + 4*b^2*d^2))^(5/2)*2496i)/((a + b*tan(c + d*x))^(1/2) - a^(1/2)) + (12288*a^6*b*d^5*tan(c + d*x)^(1/2)*((a*1i + b)/(4*a^2*d^2 + 4*b^2*d^2))^(5/2))/((a + b*tan(c + d*x))^(1/2) - a^(1/2)) + (848*a^2*b^4*d^3*tan(c + d*x)^(1/2)*((a*1i + b)/(4*a^2*d^2 + 4*b^2*d^2))^(3/2))/((a + b*tan(c + d*x))^(1/2) - a^(1/2)) - (a^3*b^3*d^3*tan(c + d*x)^(1/2)*((a*1i + b)/(4*a^2*d^2 + 4*b^2*d^2))^(3/2)*5744i)/((a + b*tan(c + d*x))^(1/2) - a^(1/2)) - (6144*a^4*b^2*d^3*tan(c + d*x)^(1/2)*((a*1i + b)/(4*a^2*d^2 + 4*b^2*d^2))^(3/2))/((a + b*tan(c + d*x))^(1/2) - a^(1/2)) - (2368*a^2*b^5*d^5*tan(c + d*x)^(1/2)*((a*1i + b)/(4*a^2*d^2 + 4*b^2*d^2))^(5/2))/((a + b*tan(c + d*x))^(1/2) - a^(1/2)) + (a^3*b^4*d^5*tan(c + d*x)^(1/2)*((a*1i + b)/(4*a^2*d^2 + 4*b^2*d^2))^(5/2)*13760i)/((a + b*tan(c + d*x))^(1/2) - a^(1/2)) + (10496*a^4*b^3*d^5*tan(c +

```
 d*x)^(1/2)*((a*1i + b)/(4*a^2*d^2 + 4*b^2*d^2))^(5/2))/((a + b*tan(c + d*x
))^(1/2) - a^(1/2)) + (a^5*b^2*d^5*tan(c + d*x)^(1/2)*((a*1i + b)/(4*a^2*d^
2 + 4*b^2*d^2))^(5/2)*7424i)/((a + b*tan(c + d*x))^(1/2) - a^(1/2)) + (a*b^
4*d*tan(c + d*x)^(1/2)*((a*1i + b)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*144i)/((a
 + b*tan(c + d*x))^(1/2) - a^(1/2)) + (768*a^4*b*d*tan(c + d*x)^(1/2)*((a*1
i + b)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2))/((a + b*tan(c + d*x))^(1/2) - a^(1/2
)))/(a*b^3 + (a*b^4*tan(c + d*x))/((a + b*tan(c + d*x))^(1/2) - a^(1/2))^2)
)*((a*1i + b)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*2i - atan(((2*a^3*d^5*tan(c +
d*x)^(1/2)*(-(a*1i - b)/(a^2*d^2 + b^2*d^2))^(5/2))/((a + b*tan(c + d*x))^(
1/2) - a^(1/2)) + (b^3*d^5*tan(c + d*x)^(1/2)*(-(a*1i - b)/(a^2*d^2 + b^2*d
^2))^(5/2)*2i)/((a + b*tan(c + d*x))^(1/2) - a^(1/2)) - (6*a*b^2*d^5*tan(c
+ d*x)^(1/2)*(-(a*1i - b)/(a^2*d^2 + b^2*d^2))^(5/2))/((a + b*tan(c + d*x))
^(1/2) - a^(1/2)) - (a^2*b*d^5*tan(c + d*x)^(1/2)*(-(a*1i - b)/(a^2*d^2 + b
^2*d^2))^(5/2)*6i)/((a + b*tan(c + d*x))^(1/2) - a^(1/2)))/((b*tan(c + d*x)
)/((a + b*tan(c + d*x))^(1/2) - a^(1/2))^2 + 1))*(-(a*1i - b)/(a^2*d^2 + b^
2*d^2))^(1/2)*1i
```

3.637 $\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal. Leaf size=147

$$-\frac{i\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia-b}\,d}+\frac{i\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia+b}\,d}-\frac{2\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}$$

```
[Out] -I*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/d/(I*a-b)^
(1/2)+I*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/d/(I
*a+b)^(1/2)-2*(a+b*tan(d*x+c))^(1/2)/a/d/tan(d*x+c)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3650, 12, 3656, 924, 95, 211, 214}

$$-\frac{i\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}-\frac{2\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{i\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(3/2)*Sqrt[a + b*Tan[c + d*x]]),x]
```

```
[Out] ((-I)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/
(Sqrt[I*a - b]*d) + (I*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a +
b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d) - (2*Sqrt[a + b*Tan[c + d*x]])/(a*d*Sqr
t[Tan[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx = -\frac{2\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}} - \frac{2\int \frac{a\sqrt{\tan(c+dx)}}{2\sqrt{a+b\tan(c+dx)}}\,dx}{a}$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}} - \int \frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\,dx$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{a+bx}\,(1+x^2)}\,dx, x, \tan(c\right.}{d}$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}} - \frac{\text{Subst}\left(\int \left(-\frac{1}{2(i-x)\sqrt{x}\sqrt{a+bx}} + \frac{}{2}\right.\right.}{}$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{(i-x)\sqrt{x}\sqrt{a+bx}}\,dx, x, \tan\right.}{2d}$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{i-(-a+ib)x^2}\,dx, x, \frac{\sqrt{\tan(c}}{\sqrt{a+b\tan}}\right.}{d}$$

$$= -\frac{i\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia-b}\,d} + \frac{i\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan}}{\sqrt{a+b\tan}}\right.}{\sqrt{ia+b}\,d}$$

Mathematica [A]
time = 0.42, size = 157, normalized size = 1.07

$$\frac{\frac{\sqrt[4]{-1}\,\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}} - \frac{\sqrt[4]{-1}\,\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{a+ib}} - \frac{2\sqrt{a+b\tan(c+dx)}}{a\sqrt{\tan(c+dx)}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Tan[c + d*x]^(3/2)*Sqrt[a + b*Tan[c + d*x]]),x]
```

```
[Out] (((-1)^(1/4)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a +
 b*Tan[c + d*x]]])/Sqrt[-a + I*b] - ((-1)^(1/4)*ArcTan[((-1)^(1/4)*Sqrt[a +
 I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[a + I*b] - (2*Sqr
t[a + b*Tan[c + d*x]])/(a*Sqrt[Tan[c + d*x]]))/d
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 0.64, size = 944864, normalized size = 6427.65

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^(3/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\tan{\left(c + dx \right)}}\tan^{\frac{3}{2}}{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(3/2)/(a+b*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*tan(c + d*x))*tan(c + d*x)**(3/2)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\tan\left(c+d\,x\right)^{3/2}\,\sqrt{a+b\tan\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(1/2)),x)
[Out] int(1/(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(1/2)), x)
```

3.638 $\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal. Leaf size=180

$$-\frac{\operatorname{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia-b}\,d}-\frac{\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia+b}\,d}-\frac{2\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}+\frac{4b\sqrt{a\cdots}}{3a^2d\cdots}$$

```
[Out] -arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/d/(I*a-b)^(1
/2)-arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/d/(I*a+b
)^(1/2)+4/3*b*(a+b*tan(d*x+c))^(1/2)/a^2/d/tan(d*x+c)^(1/2)-2/3*(a+b*tan(d*
x+c))^(1/2)/a/d/tan(d*x+c)^(3/2)
```

Rubi [A]

time = 0.26, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3650, 3730, 12, 3656, 926, 95, 211, 214}

$$\frac{4b\sqrt{a+b\tan(c+dx)}}{3a^2d\sqrt{\tan(c+dx)}}-\frac{\operatorname{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}-\frac{2\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}-\frac{\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(5/2)*Sqrt[a + b*Tan[c + d*x]]),x]
```

```
[Out] -(ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]/(Sqrt
[I*a - b]*d)) - ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
 + d*x]]]/(Sqrt[I*a + b]*d) - (2*Sqrt[a + b*Tan[c + d*x]])/(3*a*d*Tan[c + d
*x]^(3/2)) + (4*b*Sqrt[a + b*Tan[c + d*x]])/(3*a^2*d*Sqrt[Tan[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
```

```
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx = -\frac{2\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)} - \frac{2\int \frac{b+\frac{3}{2}a\tan(c+dx)+b\tan^2(c+dx)}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx}{3a}$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)} + \frac{4b\sqrt{a+b\tan(c+dx)}}{3a^2d\sqrt{\tan(c+dx)}} + \frac{4\int -\frac{}{4\sqrt{\tan}}}{}$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)} + \frac{4b\sqrt{a+b\tan(c+dx)}}{3a^2d\sqrt{\tan(c+dx)}} - \int \frac{}{\sqrt{\tan(c}}$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)} + \frac{4b\sqrt{a+b\tan(c+dx)}}{3a^2d\sqrt{\tan(c+dx)}} - \frac{\text{Subst}\left(\int \frac{}{\sqrt{}}\right.}{}$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)} + \frac{4b\sqrt{a+b\tan(c+dx)}}{3a^2d\sqrt{\tan(c+dx)}} - \frac{\text{Subst}\left(\int \left(\right.\right.}{}$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)} + \frac{4b\sqrt{a+b\tan(c+dx)}}{3a^2d\sqrt{\tan(c+dx)}} - \frac{i\text{Subst}\left(\int \frac{}{(i}\right.}{}$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)} + \frac{4b\sqrt{a+b\tan(c+dx)}}{3a^2d\sqrt{\tan(c+dx)}} - \frac{i\text{Subst}\left(\int \frac{}{i-}\right.}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia-b}\,d} - \frac{\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c}}{\sqrt{a+b\tan(c+}}\right.}{\sqrt{ia+b}\,d}$$

Mathematica [A]
time = 1.62, size = 172, normalized size = 0.96

$$\frac{\frac{3(-1)^{3/4}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}}+\frac{3(-1)^{3/4}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{a+ib}}-\frac{2(a-2b\tan(c+dx))\sqrt{a+b\tan(c+dx)}}{a^2\tan^{\frac{3}{2}}(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Tan[c + d*x]^(5/2)*Sqrt[a + b*Tan[c + d*x]]),x]
```

```
[Out] ((3*(-1)^(3/4)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a
 + b*Tan[c + d*x]]])/Sqrt[-a + I*b] + (3*(-1)^(3/4)*ArcTan[((-1)^(1/4)*Sqrt
[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[a + I*b] - (2
*(a - 2*b*Tan[c + d*x])*Sqrt[a + b*Tan[c + d*x]])/(a^2*Tan[c + d*x]^(3/2)))
/(3*d)
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 0.57, size = 945613, normalized size = 5253.41

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^(5/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \tan{\left(c + dx \right)}} \tan^{\frac{5}{2}}{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(5/2)/(a+b*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*tan(c + d*x))*tan(c + d*x)**(5/2)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\tan\left(c+d\,x\right)^{5/2}\,\sqrt{a+b\tan\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(5/2)*(a + b*tan(c + d*x))^(1/2)),x)
[Out] int(1/(tan(c + d*x)^(5/2)*(a + b*tan(c + d*x))^(1/2)), x)
```

3.639 $\int \frac{1}{\tan^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal. Leaf size=229

$$\frac{i\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia-b}\,d}-\frac{i\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia+b}\,d}-\frac{2\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}+\frac{8b\sqrt{c}}{15a}$$

```
[Out] I*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/d/(I*a-b)^(
1/2)-I*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/d/(I*
a+b)^(1/2)+2/15*(15*a^2-8*b^2)*(a+b*tan(d*x+c))^(1/2)/a^3/d/tan(d*x+c)^(1/2
)-2/5*(a+b*tan(d*x+c))^(1/2)/a/d/tan(d*x+c)^(5/2)+8/15*b*(a+b*tan(d*x+c))^(
1/2)/a^2/d/tan(d*x+c)^(3/2)
```

Rubi [A]

time = 0.38, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3650, 3730, 3731, 12, 3656, 924, 95, 211, 214}

$$\frac{8b\sqrt{a+b\tan(c+dx)}}{15a^2d\tan^{\frac{3}{2}}(c+dx)}+\frac{2(15a^2-8b^2)\sqrt{a+b\tan(c+dx)}}{15a^3d\sqrt{\tan(c+dx)}}+\frac{i\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}-\frac{2\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}-\frac{i\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(7/2)*Sqrt[a + b*Tan[c + d*x]]),x]
```

```
[Out] (I*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sq
rt[I*a - b]*d) - (I*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*T
an[c + d*x]]])/(Sqrt[I*a + b]*d) - (2*Sqrt[a + b*Tan[c + d*x]])/(5*a*d*Tan[
c + d*x]^(5/2)) + (8*b*Sqrt[a + b*Tan[c + d*x]])/(15*a^2*d*Tan[c + d*x]^(3/
2)) + (2*(15*a^2 - 8*b^2)*Sqrt[a + b*Tan[c + d*x]])/(15*a^3*d*Sqrt[Tan[c +
d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
```

```
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3731

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)
*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp
[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - a*C*(b*c*(m + 1) + a*d*(n
+ 1)) - (m + 1)*(b*c - a*d)*(A*b - b*C)*Tan[e + f*x] - d*(A*b^2 + a^2*C)*(m
 + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m,
 -1] &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{\tan^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx = -\frac{2\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)} - \frac{2\int \frac{2b+\frac{5}{2}a\tan(c+dx)+2b\tan^2(c+dx)}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{5a}$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)} + \frac{8b\sqrt{a+b\tan(c+dx)}}{15a^2d\tan^{\frac{3}{2}}(c+dx)} + \frac{4\int \frac{\frac{1}{4}(-15a}{\tan^{\frac{3}{2}}(c+dx}}{}$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)} + \frac{8b\sqrt{a+b\tan(c+dx)}}{15a^2d\tan^{\frac{3}{2}}(c+dx)} + \frac{2(15a^2-8b}{15a^3d}$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)} + \frac{8b\sqrt{a+b\tan(c+dx)}}{15a^2d\tan^{\frac{3}{2}}(c+dx)} + \frac{2(15a^2-8b}{15a^3d}$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)} + \frac{8b\sqrt{a+b\tan(c+dx)}}{15a^2d\tan^{\frac{3}{2}}(c+dx)} + \frac{2(15a^2-8b}{15a^3d}$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)} + \frac{8b\sqrt{a+b\tan(c+dx)}}{15a^2d\tan^{\frac{3}{2}}(c+dx)} + \frac{2(15a^2-8b}{15a^3d}$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)} + \frac{8b\sqrt{a+b\tan(c+dx)}}{15a^2d\tan^{\frac{3}{2}}(c+dx)} + \frac{2(15a^2-8b}{15a^3d}$$

$$= -\frac{2\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)} + \frac{8b\sqrt{a+b\tan(c+dx)}}{15a^2d\tan^{\frac{3}{2}}(c+dx)} + \frac{2(15a^2-8b}{15a^3d}$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia-b}\,d} - \frac{i\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c}}{\sqrt{a+b\tan(c}}\right.}{\sqrt{ia+b}\,d}$$

Mathematica [A]
time = 2.80, size = 197, normalized size = 0.86

$$\frac{-\frac{15\sqrt[4]{-1}\,\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}} + \frac{15\sqrt[4]{-1}\,\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{a+ib}} + \frac{2\sqrt{a+b\tan(c+dx)}\,(-3a^2+4ab\tan(c+dx)+(15a^2-8b^2)\tan^2(c+dx))}{a^3\tan^{\frac{5}{2}}(c+dx)}}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Tan[c + d*x]^(7/2)*Sqrt[a + b*Tan[c + d*x]]),x]
[Out] ((-15*(-1)^(1/4)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt
[a + b*Tan[c + d*x]]])/Sqrt[-a + I*b] + (15*(-1)^(1/4)*ArcTan[((-1)^(1/4)*S
```

```
qrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[a + I*b] +
 (2*Sqrt[a + b*Tan[c + d*x]]*(-3*a^2 + 4*a*b*Tan[c + d*x] + (15*a^2 - 8*b^2
)*Tan[c + d*x]^2))/(a^3*Tan[c + d*x]^(5/2)))/(15*d)
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 0.62, size = 947049, normalized size = 4135.59

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(7/2)/(a+b*tan(d*x+c))^(1/2),x)
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(7/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^(7/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(7/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\tan{\left(c + dx \right)}}\tan^{\frac{7}{2}}{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(7/2)/(a+b*tan(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(a + b*tan(c + d*x))*tan(c + d*x)**(7/2)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(7/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\tan(c+d\,x)^{7/2}\,\sqrt{a+b\tan(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(7/2)*(a + b*tan(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(tan(c + d*x)^(7/2)*(a + b*tan(c + d*x))^(1/2)), x)
```

3.640 $\int \frac{\tan^{\frac{7}{2}}(c+dx)}{(a+b\tan(c+dx))^{3/2}} \, dx$

Optimal. Leaf size=250

$$\frac{i\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{3/2}d}-\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{b^{5/2}d}+\frac{i\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{3/2}d}$$

```
[Out] I*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a-b)^(3/
2)/d-3*a*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/b^(5/2)/d
+I*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a+b)^(
3/2)/d+(3*a^2+b^2)*tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)/b^2/(a^2+b^2)/d-
2*a^2*tan(d*x+c)^(3/2)/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^(1/2)
```

Rubi [A]

time = 1.16, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3646, 3728, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$-\frac{2a^2\tan^{\frac{3}{2}}(c+dx)}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{(3a^2+b^2)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{b^2d(a^2+b^2)}+\frac{i\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{3/2}}-\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{b^{5/2}d}+\frac{i\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(7/2)/(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] (I*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/((I
*a - b)^(3/2)*d) - (3*a*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan
[c + d*x]]])/(b^(5/2)*d) + (I*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sq
rt[a + b*Tan[c + d*x]]])/((I*a + b)^(3/2)*d) - (2*a^2*Tan[c + d*x]^(3/2))/(
b*(a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]]) + ((3*a^2 + b^2)*Sqrt[Tan[c + d*x
]]*Sqrt[a + b*Tan[c + d*x]])/(b^2*(a^2 + b^2)*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
```

```
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
```

```
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\tan^{\frac{7}{2}}(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx = -\frac{2a^2\tan^{\frac{3}{2}}(c+dx)}{b\left(a^2+b^2\right)d\sqrt{a+b\tan(c+dx)}} + \frac{2\int \frac{\sqrt{\tan(c+dx)}\left(\frac{3a^2}{2}-\frac{1}{2}ab\tan(c+dx)+\frac{1}{2}(3a\right.}{\sqrt{a+b\tan(c+dx)}}}{b\left(a^2+b^2\right)}$$

$$= -\frac{2a^2\tan^{\frac{3}{2}}(c+dx)}{b\left(a^2+b^2\right)d\sqrt{a+b\tan(c+dx)}} + \frac{\left(3a^2+b^2\right)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c}{b^2\left(a^2+b^2\right)d}$$

$$= -\frac{2a^2\tan^{\frac{3}{2}}(c+dx)}{b\left(a^2+b^2\right)d\sqrt{a+b\tan(c+dx)}} + \frac{\left(3a^2+b^2\right)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c}{b^2\left(a^2+b^2\right)d}$$

$$= -\frac{2a^2\tan^{\frac{3}{2}}(c+dx)}{b\left(a^2+b^2\right)d\sqrt{a+b\tan(c+dx)}} + \frac{\left(3a^2+b^2\right)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c}{b^2\left(a^2+b^2\right)d}$$

$$= -\frac{2a^2\tan^{\frac{3}{2}}(c+dx)}{b\left(a^2+b^2\right)d\sqrt{a+b\tan(c+dx)}} + \frac{\left(3a^2+b^2\right)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c}{b^2\left(a^2+b^2\right)d}$$

$$= -\frac{2a^2\tan^{\frac{3}{2}}(c+dx)}{b\left(a^2+b^2\right)d\sqrt{a+b\tan(c+dx)}} + \frac{\left(3a^2+b^2\right)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c}{b^2\left(a^2+b^2\right)d}$$

$$= -\frac{2a^2\tan^{\frac{3}{2}}(c+dx)}{b\left(a^2+b^2\right)d\sqrt{a+b\tan(c+dx)}} + \frac{\left(3a^2+b^2\right)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c}{b^2\left(a^2+b^2\right)d}$$

$$= -\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{b^{5/2}d} - \frac{2a^2\tan^{\frac{3}{2}}(c+dx)}{b\left(a^2+b^2\right)d\sqrt{a+b\tan(c+dx)}} + \frac{(3}{}$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{3/2}d} - \frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{b^{5/2}d} + \frac{i}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 3.18, size = 270, normalized size = 1.08

$$\frac{\frac{5(-1)^{3/4}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(-a+ib)^{3/2}} - \frac{5(-1)^{3/4}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(a+ib)^{3/2}} - \frac{5\sqrt{\tan(c+dx)}}{(a-ib)\sqrt{a+b\tan(c+dx)}} - \frac{5\sqrt{\tan(c+dx)}}{(a+ib)\sqrt{a+b\tan(c+dx)}} + \frac{2\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};-\frac{b\tan(c+dx)}{a}\right)\tan^{\frac{5}{2}}(c+dx)\sqrt{1+\frac{b\tan(c+dx)}{a}}}{a\sqrt{a+b\tan(c+dx)}}}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(7/2)/(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((5*(-1)^(3/4)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a
 + b*Tan[c + d*x]]])/(-a + I*b)^(3/2) - (5*(-1)^(3/4)*ArcTan[((-1)^(1/4)*Sq
rt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(a + I*b)^(3/2)
- (5*Sqrt[Tan[c + d*x]])/((a - I*b)*Sqrt[a + b*Tan[c + d*x]]) - (5*Sqrt[Tan
[c + d*x]])/((a + I*b)*Sqrt[a + b*Tan[c + d*x]]) + (2*Hypergeometric2F1[3/2
, 5/2, 7/2, -((b*Tan[c + d*x])/a)]*Tan[c + d*x]^(5/2)*Sqrt[1 + (b*Tan[c + d
*x])/a])/(a*Sqrt[a + b*Tan[c + d*x]]))/(5*d)
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 2.18, size = 764550, normalized size = 3058.20

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(7/2)/(a+b*tan(d*x+c))^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(7/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^(7/2)/(b*tan(d*x + c) + a)^(3/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(7/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(7/2)/(a+b*tan(d*x+c))**(3/2),x)
```

[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(7/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(c+d\,x)^{7/2}}{\left(a+b\tan\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(7/2)/(a + b*tan(c + d*x))^(3/2),x)
```

```
[Out] int(tan(c + d*x)^(7/2)/(a + b*tan(c + d*x))^(3/2), x)
```

3.641 $\int \frac{\tan^{\frac{5}{2}}(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=195

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{3/2}d}+\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{b^{3/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{3/2}d}$$

```
[Out] arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a-b)^(3/2)
/d+2*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/b^(3/2)/d-arc
tanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a+b)^(3/2)/d
-2*a^2*tan(d*x+c)^(1/2)/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^(1/2)
```

Rubi [A]

time = 0.89, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3646, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$-\frac{2a^2\sqrt{\tan(c+dx)}}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\operatorname{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{3/2}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{b^{3/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(5/2)/(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]/((I*a -
 b)^(3/2)*d) + (2*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d
*x]]])/(b^(3/2)*d) - ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*
Tan[c + d*x]]]/((I*a + b)^(3/2)*d) - (2*a^2*Sqrt[Tan[c + d*x]])/(b*(a^2 + b
^2)*d*Sqrt[a + b*Tan[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx = -\frac{2a^2\sqrt{\tan(c+dx)}}{b\left(a^2+b^2\right)d\sqrt{a+b\tan(c+dx)}} + \frac{2\int \frac{\frac{a^2}{2}-\frac{1}{2}ab\tan(c+dx)+\frac{1}{2}\left(a^2+b^2\right)\tan^2(c+dx)}{\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}}{b\left(a^2+b^2\right)}$$

$$= -\frac{2a^2\sqrt{\tan(c+dx)}}{b\left(a^2+b^2\right)d\sqrt{a+b\tan(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{\frac{a^2}{2}-\frac{abx}{2}+\frac{1}{2}\left(a^2+b^2\right)x^2}{\sqrt{x}\,\sqrt{a+bx}\,(1+x^2)}\,dx, x, \text{ta}\right.}{b\left(a^2+b^2\right)d}$$

$$= -\frac{2a^2\sqrt{\tan(c+dx)}}{b\left(a^2+b^2\right)d\sqrt{a+b\tan(c+dx)}} + \frac{2\text{Subst}\left(\int \left(\frac{a^2+b^2}{2\sqrt{x}\,\sqrt{a+bx}} - \frac{}{2\sqrt{x}\,\sqrt{}}\right.\right.}{b\left(a^2+\right.}$$

$$= -\frac{2a^2\sqrt{\tan(c+dx)}}{b\left(a^2+b^2\right)d\sqrt{a+b\tan(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx, x, \tan(c+\right.}{bd}$$

$$= -\frac{2a^2\sqrt{\tan(c+dx)}}{b\left(a^2+b^2\right)d\sqrt{a+b\tan(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{\tan(c+}\right.}{bd}$$

$$= -\frac{2a^2\sqrt{\tan(c+dx)}}{b\left(a^2+b^2\right)d\sqrt{a+b\tan(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(i+x)\sqrt{a+bx}}\,dx, x, \tan\right.}{2(a-ib)d}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{b^{3/2}d} - \frac{2a^2\sqrt{\tan(c+dx)}}{b\left(a^2+b^2\right)d\sqrt{a+b\tan(c+dx)}} - \frac{\text{Su}}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{3/2}d} + \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{b^{3/2}d} - \frac{\text{ta}}{}$$

Mathematica [A]
time = 1.26, size = 321, normalized size = 1.65

$$-\frac{2\sqrt{a}\,\sqrt{-a+ib}\,\sqrt{a+ib}\left(a^2+b^2\right)\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{1+\frac{b\tan(c+dx)}{a}}+\sqrt{b}\left(\sqrt[4]{-1}\,(a+ib)^{3/2}b\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{a+b\tan(c+dx)}+\sqrt{-a+ib}\left(-2a^2\sqrt{a+ib}\,\sqrt{\tan(c+dx)}-\sqrt[4]{-1}\,(a-ib)b\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{a+b\tan(c+dx)}\right)\right)}{(-a+ib)^{3/2}(a+ib)^{3/2}b^{3/2}d\sqrt{a+b\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(5/2)/(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] -((2*Sqrt[a]*Sqrt[-a + I*b]*Sqrt[a + I*b]*(a^2 + b^2)*ArcSinh[(Sqrt[b]*Sqrt
[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a] + Sqrt[b]*((-1)^(1/4)
*(a + I*b)^(3/2)*b*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sq
rt[a + b*Tan[c + d*x]]]*Sqrt[a + b*Tan[c + d*x]] + Sqrt[-a + I*b]*(-2*a^2*S
qrt[a + I*b]*Sqrt[Tan[c + d*x]] - (-1)^(1/4)*(a - I*b)*b*ArcTan[((-1)^(1/4)
*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[a + b*Tan
[c + d*x]])))/((-a + I*b)^(3/2)*(a + I*b)^(3/2)*b^(3/2)*d*Sqrt[a + b*Tan[c
+ d*x]]))
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 0.97, size = 798950, normalized size = 4097.18

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^(5/2)/(b*tan(d*x + c) + a)^(3/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)}{(a+b\tan(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(5/2)/(a+b*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**(5/2)/(a + b*tan(c + d*x))**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^{5/2}}{\left(a+b\tan\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(5/2)/(a + b*tan(c + d*x))^(3/2),x)
```

```
[Out] int(tan(c + d*x)^(5/2)/(a + b*tan(c + d*x))^(3/2), x)
```

3.642 $\int \frac{\tan^{\frac{3}{2}}(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=154

$$-\frac{i\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{3/2}d}-\frac{i\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{3/2}d}+\frac{2a\sqrt{\tan(c+dx)}}{(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}}$$

```
[Out] -I*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a-b)^(3
/2)/d-I*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a
+b)^(3/2)/d+2*a*tan(d*x+c)^(1/2)/(a^2+b^2)/d/(a+b*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3648, 3697, 3696, 95, 209, 212}

$$\frac{2a\sqrt{\tan(c+dx)}}{d\,(a^2+b^2)\,\sqrt{a+b\tan(c+dx)}}-\frac{i\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{3/2}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(3/2)/(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-I)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/
((I*a - b)^(3/2)*d) - (I*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a
+ b*Tan[c + d*x]]])/((I*a + b)^(3/2)*d) + (2*a*Sqrt[Tan[c + d*x]])/((a^2 +
b^2)*d*Sqrt[a + b*Tan[c + d*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rubi steps

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx = \frac{2a\sqrt{\tan(c+dx)}}{(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} - \frac{2\int \frac{\frac{a}{2}-\frac{1}{2}b\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\,dx}{a^2+b^2}$$

$$= \frac{2a\sqrt{\tan(c+dx)}}{(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} - \frac{\int \frac{1+i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\,dx}{2(a-ib)} -$$

$$= \frac{2a\sqrt{\tan(c+dx)}}{(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{(1-ix)\sqrt{x}\sqrt{a+bx}}\,dx, x, \tan(c+\right.}{2(a-ib)d}$$

$$= \frac{2a\sqrt{\tan(c+dx)}}{(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{1-(ia+b)x^2}\,dx, x, \frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+d}}\right.}{(a-ib)d}$$

$$= -\frac{i\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{3/2}d} - \frac{i\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{3/2}d}$$

Mathematica [A]
time = 1.29, size = 182, normalized size = 1.18

$$\frac{-\frac{(-1)^{3/4}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(-a+ib)^{3/2}} + \frac{\frac{\sqrt[4]{-1}(ia+b)\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(a+ib)^{3/2}} + \frac{2a\sqrt{\tan(c+dx)}}{(a+ib)\sqrt{a+b\tan(c+dx)}}}{a-ib}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(3/2)/(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] (-(((-1)^(3/4)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a
 + b*Tan[c + d*x]]])/(-a + I*b)^(3/2)) + (((-1)^(1/4)*(I*a + b)*ArcTan[((-1
)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(a + I
*b)^(3/2) + (2*a*Sqrt[Tan[c + d*x]])/((a + I*b)*Sqrt[a + b*Tan[c + d*x]]))/
(a - I*b))/d
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 2.29, size = 762478, normalized size = 4951.16

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x)
```

[Out] result too large to display

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(tan(d*x + c)^(3/2)/(b*tan(d*x + c) + a)^(3/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{3}{2}}(c + dx)}{(a + b\tan(c + dx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(3/2)/(a+b*tan(d*x+c))**(3/2),x)
[Out] Integral(tan(c + d*x)**(3/2)/(a + b*tan(c + d*x))**(3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c + d\,x)^{3/2}}{(a + b\tan(c + d\,x))^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(3/2)/(a + b*tan(c + d*x))^(3/2),x)
[Out] int(tan(c + d*x)^(3/2)/(a + b*tan(c + d*x))^(3/2), x)
```

3.643 $\int \frac{\sqrt{\tan(c+dx)}}{(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=149

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{3/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{3/2}d}-\frac{2b\sqrt{\tan(c+dx)}}{(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}}$$

```
[Out] -arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a-b)^(3/2
)/d+arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a+b)^
(3/2)/d-2*b*tan(d*x+c)^(1/2)/(a^2+b^2)/d/(a+b*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3649, 3697, 3696, 95, 209, 212}

$$-\frac{2b\sqrt{\tan(c+dx)}}{d\,(a^2+b^2)\,\sqrt{a+b\tan(c+dx)}}-\frac{\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[c + d*x]]/(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] -(ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]/((I*a
 - b)^(3/2)*d)) + ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan
[c + d*x]]]/((I*a + b)^(3/2)*d) - (2*b*Sqrt[Tan[c + d*x]])/((a^2 + b^2)*d*S
qrt[a + b*Tan[c + d*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\tan(c+dx)}}{(a+b\tan(c+dx))^{3/2}}\,dx = -\frac{2b\sqrt{\tan(c+dx)}}{(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} - \frac{2\int \frac{-\frac{b}{2}-\frac{1}{2}a\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\,d}{a^2+b^2}$$

$$= -\frac{2b\sqrt{\tan(c+dx)}}{(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} - \frac{\int \frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\,dx}{2(ia-b)}$$

$$= -\frac{2b\sqrt{\tan(c+dx)}}{(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{(1+ix)\sqrt{x}\sqrt{a+bx}}\,dx, x, \tan(c\right.}{2(ia-b)d}$$

$$= -\frac{2b\sqrt{\tan(c+dx)}}{(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{1-(-ia+b)x^2}\,dx, x, \frac{\sqrt{\tan(c+}}{\sqrt{a+b\tan(c}}\right.}{(ia-b)d}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{3/2}d} + \frac{\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{3/2}d} -$$

Mathematica [A]
time = 1.24, size = 163, normalized size = 1.09

$$\frac{\frac{\sqrt[4]{-1}\,\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(-a+ib)^{3/2}} + \frac{\sqrt[4]{-1}\,\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(a+ib)^{3/2}} - \frac{2b\sqrt{\tan(c+dx)}}{(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tan[c + d*x]]/(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((-1)^(1/4)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a +
 b*Tan[c + d*x]]])/(-a + I*b)^(3/2) + ((-1)^(1/4)*ArcTan[((-1)^(1/4)*Sqrt[a
 + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(a + I*b)^(3/2) - (2
*b*Sqrt[Tan[c + d*x]])/((a^2 + b^2)*Sqrt[a + b*Tan[c + d*x]]))/d
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 1.00, size = 798605, normalized size = 5359.77

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(tan(d*x + c))/(b*tan(d*x + c) + a)^(3/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\tan(c+dx)}}{(a+b\tan(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/2)/(a+b*tan(d*x+c))**(3/2),x)
[Out] Integral(sqrt(tan(c + d*x))/(a + b*tan(c + d*x))**(3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\tan\left(c+d\,x\right)}}{{\left(a+b\,\tan\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)/(a + b*tan(c + d*x))^(3/2),x)
[Out] int(tan(c + d*x)^(1/2)/(a + b*tan(c + d*x))^(3/2), x)
```

3.644 $\int \frac{1}{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=159

$$\frac{i\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{3/2}d}+\frac{i\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{3/2}d}+\frac{2b^2\sqrt{\tan(c+dx)}}{a\,(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}}$$

```
[Out] I*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a-b)^(3/
2)/d+I*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a+
b)^(3/2)/d+2*b^2*tan(d*x+c)^(1/2)/a/(a^2+b^2)/d/(a+b*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3650, 3697, 3696, 95, 209, 212}

$$\frac{2b^2\sqrt{\tan(c+dx)}}{ad\,(a^2+b^2)\,\sqrt{a+b\tan(c+dx)}}+\frac{i\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{3/2}}+\frac{i\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(3/2)),x]
```

```
[Out] (I*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/((I
*a - b)^(3/2)*d) + (I*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b
*Tan[c + d*x]]])/((I*a + b)^(3/2)*d) + (2*b^2*Sqrt[Tan[c + d*x]])/(a*(a^2 +
 b^2)*d*Sqrt[a + b*Tan[c + d*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{3/2}}\,dx = \frac{2b^2\sqrt{\tan(c+dx)}}{a\,(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} + \frac{2\int \frac{\frac{a^2}{2}-\frac{1}{2}ab\tan(c+}{\sqrt{\tan(c+dx)}\,\sqrt{a+b}}}{a\,(a^2+b^2)}$$

$$= \frac{2b^2\sqrt{\tan(c+dx)}}{a\,(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} + \frac{\int \frac{1+i\tan(c+dx)}{\sqrt{\tan(c+dx)}\,\sqrt{a+b\,t}}}{2(a-ib)}$$

$$= \frac{2b^2\sqrt{\tan(c+dx)}}{a\,(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1-ix)\sqrt{x}\,\sqrt{a+b}}\right.}{2(a-i}$$

$$= \frac{2b^2\sqrt{\tan(c+dx)}}{a\,(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-(ia+b)x^2}\,dx, x, \right.}{(a-ib}$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{3/2}d} + \frac{i\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{ta}}{\sqrt{a+b\tan}}\right.}{(ia+b)^{3/2}d}$$

Mathematica [A]
time = 0.95, size = 183, normalized size = 1.15

$$-\frac{\frac{(-1)^{3/4}(a+ib)\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}} + \frac{\sqrt[4]{-1}\,(ia+b)\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{a+ib}} - \frac{2b^2\sqrt{\tan(c+dx)}}{a\sqrt{a+b\tan(c+dx)}}}{(a^2+b^2)\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(3/2)),x]
```

```
[Out] -((((-1)^(3/4)*(a + I*b)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x
]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[-a + I*b] + ((-1)^(1/4)*(I*a + b)*ArcTa
n[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/
Sqrt[a + I*b] - (2*b^2*Sqrt[Tan[c + d*x]])/(a*Sqrt[a + b*Tan[c + d*x]]))/((
a^2 + b^2)*d))
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 1.02, size = 762494, normalized size = 4795.56

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*tan(d*x + c) + a)^(3/2)*sqrt(tan(d*x + c))), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\tan(c+dx))^{\frac{3}{2}}\sqrt{\tan(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(1/2)/(a+b*tan(d*x+c))**(3/2),x)
[Out] Integral(1/((a + b*tan(c + d*x))**(3/2)*sqrt(tan(c + d*x))), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\tan(c+d\,x)}\,(a+b\tan(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(3/2)),x)
[Out] int(1/(tan(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(3/2)), x)
```

3.645 $\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=193

$$\frac{\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{3/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{3/2}d}-\frac{2}{ad\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+}$$

```
[Out] arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a-b)^(3/2)
/d-arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a+b)^(
3/2)/d-2/a/d/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2)-2*b*(a^2+2*b^2)*tan(d*
x+c)^(1/2)/a^2/(a^2+b^2)/d/(a+b*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3650, 3730, 3697, 3696, 95, 209, 212}

$$-\frac{2b(a^2+2b^2)\sqrt{\tan(c+dx)}}{a^2d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{3/2}}-\frac{2}{ad\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(3/2)),x]
```

```
[Out] ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]/((I*a -
 b)^(3/2)*d) - ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
+ d*x]]]/((I*a + b)^(3/2)*d) - 2/(a*d*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c +
 d*x]]) - (2*b*(a^2 + 2*b^2)*Sqrt[Tan[c + d*x]])/(a^2*(a^2 + b^2)*d*Sqrt[a
+ b*Tan[c + d*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
```

```
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx = -\frac{2}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}} - \frac{2\int \frac{b+\frac{1}{2}a\tan(c+dx)+}{\sqrt{\tan(c+dx)}(a}}{a}$$

$$= -\frac{2}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}} - \frac{2b(a^2+2b^2)\sqrt{\tan}}{a^2(a^2+b^2)\,d\sqrt{a+b}}$$

$$= -\frac{2}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}} - \frac{2b(a^2+2b^2)\sqrt{\tan}}{a^2(a^2+b^2)\,d\sqrt{a+b}}$$

$$= -\frac{2}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}} - \frac{2b(a^2+2b^2)\sqrt{\tan}}{a^2(a^2+b^2)\,d\sqrt{a+b}}$$

$$= -\frac{2}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}} - \frac{2b(a^2+2b^2)\sqrt{\tan}}{a^2(a^2+b^2)\,d\sqrt{a+b}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{3/2}d} - \frac{\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c}}{\sqrt{a+b\tan(c+}}\right)}{(ia+b)^{3/2}d}$$

Mathematica [A]
time = 4.44, size = 202, normalized size = 1.05

$$-\frac{\frac{\sqrt[4]{-1}\,\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(-a+ib)^{3/2}} + \frac{\frac{\sqrt[4]{-1}\,(a-ib)\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{a+ib}} + \frac{2\left(a\left(a^2+b^2\right)+b\left(a^2+2b^2\right)\tan(c+dx)\right)}{a^2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}}{a^2+b^2}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(3/2)),x]
```

```
[Out] -((((-1)^(1/4)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a
 + b*Tan[c + d*x]]])/(-a + I*b)^(3/2) + (((-1)^(1/4)*(a - I*b)*ArcTan[((-1)
^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[a
+ I*b] + (2*(a*(a^2 + b^2) + b*(a^2 + 2*b^2)*Tan[c + d*x]))/(a^2*Sqrt[Tan[c
 + d*x]]*Sqrt[a + b*Tan[c + d*x]]))/(a^2 + b^2))/d)
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 0.98, size = 799513, normalized size = 4142.55

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x)
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*tan(d*x + c) + a)^(3/2)*tan(d*x + c)^(3/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\tan\left(c + dx\right)\right)^{\frac{3}{2}}\tan^{\frac{3}{2}}\left(c + dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(3/2)/(a+b*tan(d*x+c))**(3/2),x)
[Out] Integral(1/((a + b*tan(c + d*x))**(3/2)*tan(c + d*x)**(3/2)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\tan\left(c+d\,x\right)^{3/2}\,\left(a+b\tan\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(3/2)),x)
[Out] int(1/(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(3/2)), x)
```

3.646 $\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=241

$$-\frac{i\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{3/2}d}-\frac{i\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{3/2}d}-\frac{2}{3ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}$$

```
[Out] -I*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a-b)^(3
/2)/d-I*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a
+b)^(3/2)/d+8/3*b/a^2/d/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2)+2/3*b^2*(5*
a^2+8*b^2)*tan(d*x+c)^(1/2)/a^3/(a^2+b^2)/d/(a+b*tan(d*x+c))^(1/2)-2/3/a/d/
(a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(3/2)
```

Rubi [A]
time = 0.59, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3650, 3730, 3731, 3697, 3696, 95, 209, 212}

$$\frac{8b}{3a^2d\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}+\frac{2b^2(5a^2+8b^2)\sqrt{\tan(c+dx)}}{3a^3d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}-\frac{i\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{3/2}}-\frac{2}{3ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^(3/2)),x]
```

```
[Out] ((-I)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/
((I*a - b)^(3/2)*d) - (I*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a
+ b*Tan[c + d*x]]])/((I*a + b)^(3/2)*d) - 2/(3*a*d*Tan[c + d*x]^(3/2)*Sqrt[
a + b*Tan[c + d*x]]) + (8*b)/(3*a^2*d*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c +
 d*x]]) + (2*b^2*(5*a^2 + 8*b^2)*Sqrt[Tan[c + d*x]])/(3*a^3*(a^2 + b^2)*d*S
qrt[a + b*Tan[c + d*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
```

```
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3731

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)
*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp
[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - a*C*(b*c*(m + 1) + a*d*(n
+ 1)) - (m + 1)*(b*c - a*d)*(A*b - b*C)*Tan[e + f*x] - d*(A*b^2 + a^2*C)*(m
 + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m,
 -1] && !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx = -\frac{2}{3ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}} - \frac{2\int \frac{2b+\frac{3}{2}a\tan(c+dx)+2b}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan}}{3a}$$

$$= -\frac{2}{3ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}} + \frac{8}{3a^2d\sqrt{\tan(c+dx)}}$$

$$= -\frac{2}{3ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}} + \frac{8}{3a^2d\sqrt{\tan(c+dx)}}$$

$$= -\frac{2}{3ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}} + \frac{8}{3a^2d\sqrt{\tan(c+dx)}}$$

$$= -\frac{2}{3ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}} + \frac{8}{3a^2d\sqrt{\tan(c+dx)}}$$

$$= -\frac{2}{3ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}} + \frac{8}{3a^2d\sqrt{\tan(c+dx)}}$$

$$= -\frac{i\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{3/2}d} - \frac{i\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{t}}{\sqrt{a+b\tan}}\right)}{(ia+b)^{3/2}}$$

Mathematica [A]
time = 5.72, size = 223, normalized size = 0.93

$$\frac{-\frac{3(-1)^{3/4}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(-a+ib)^{3/2}}+\frac{3(-1)^{3/4}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(a+ib)^{3/2}}+\frac{-2a^2(a^2+b^2)+8ab(a^2+b^2)\tan(c+dx)+2b^2(5a^2+8b^2)\tan^2(c+dx)}{a^3(a^2+b^2)\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^(3/2)),x]
```

```
[Out] ((-3*(-1)^(3/4)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[
a + b*Tan[c + d*x]]])/(-a + I*b)^(3/2) + (3*(-1)^(3/4)*ArcTan[((-1)^(1/4)*S
qrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(a + I*b)^(3/2)
 + (-2*a^2*(a^2 + b^2) + 8*a*b*(a^2 + b^2)*Tan[c + d*x] + 2*b^2*(5*a^2 + 8*
b^2)*Tan[c + d*x]^2)/(a^3*(a^2 + b^2)*Tan[c + d*x]^(3/2)*Sqrt[a + b*Tan[c +
 d*x]]))/(3*d)
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 1.01, size = 764302, normalized size = 3171.38

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*tan(d*x + c) + a)^(3/2)*tan(d*x + c)^(5/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\tan(c+dx))^{\frac{3}{2}}\tan^{\frac{5}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(5/2)/(a+b*tan(d*x+c))**(3/2),x)
[Out] Integral(1/((a + b*tan(c + d*x))**(3/2)*tan(c + d*x)**(5/2)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\tan(c+d\,x)^{5/2}\,(a+b\tan(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(5/2)*(a + b*tan(c + d*x))^(3/2)),x)
[Out] int(1/(tan(c + d*x)^(5/2)*(a + b*tan(c + d*x))^(3/2)), x)
```

3.647 $\int \frac{\tan^{\frac{9}{2}}(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=317

$$-\frac{i\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{5/2}d}-\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{b^{7/2}d}+\frac{i\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{5/2}d}$$

```
[Out] -I*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a-b)^(5
/2)/d-5*a*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/b^(7/2)/
d+I*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a+b)^
(5/2)/d+(5*a^4+10*a^2*b^2+b^4)*tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)/b^3/
(a^2+b^2)^2/d-2/3*a^2*(5*a^2+11*b^2)*tan(d*x+c)^(3/2)/b^2/(a^2+b^2)^2/d/(a+
b*tan(d*x+c))^(1/2)-2/3*a^2*tan(d*x+c)^(5/2)/b/(a^2+b^2)/d/(a+b*tan(d*x+c))
^(3/2)
```

Rubi [A]

time = 1.59, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3646, 3726, 3728, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$-\frac{2a^2\tan^{\frac{5}{2}}(c+dx)}{3bd(a^2+b^2)(a+b\tan(c+dx))^{3/2}}-\frac{2a^2(5a^2+11b^2)\tan^{\frac{3}{2}}(c+dx)}{3b^2d(a^2+b^2)^2\sqrt{a+b\tan(c+dx)}}+\frac{(5a^4+10a^2b^2+b^4)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{b^3d(a^2+b^2)^2}-\frac{i\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{5/2}}-\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{b^{7/2}d}+\frac{i\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(9/2)/(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-I)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/
((I*a - b)^(5/2)*d) - (5*a*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*
Tan[c + d*x]]])/(b^(7/2)*d) + (I*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])
/Sqrt[a + b*Tan[c + d*x]]])/((I*a + b)^(5/2)*d) - (2*a^2*Tan[c + d*x]^(5/2)
)/(3*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^(3/2)) - (2*a^2*(5*a^2 + 11*b^2)*
Tan[c + d*x]^(3/2))/(3*b^2*(a^2 + b^2)^2*d*Sqrt[a + b*Tan[c + d*x]]) + ((5*
a^4 + 10*a^2*b^2 + b^4)*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]])/(b^3*(
a^2 + b^2)^2*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
```

```
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\tan^{\frac{9}{2}}(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx = -\frac{2a^2\tan^{\frac{5}{2}}(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} + \frac{2\int \frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{5a^2}{2}-\frac{3}{2}ab\tan(c+dx)+\frac{1}{2}(5a^2+}{(a+b\tan(c+dx))^{3/2}}}{3b\,(a^2+b^2)}$$

$$= -\frac{2a^2\tan^{\frac{5}{2}}(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{2a^2(5a^2+11b^2)\tan^{\frac{3}{2}}(c+dx)}{3b^2\,(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}}$$

$$= -\frac{2a^2\tan^{\frac{5}{2}}(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{2a^2(5a^2+11b^2)\tan^{\frac{3}{2}}(c+dx)}{3b^2\,(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}}$$

$$= -\frac{2a^2\tan^{\frac{5}{2}}(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{2a^2(5a^2+11b^2)\tan^{\frac{3}{2}}(c+dx)}{3b^2\,(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}}$$

$$= -\frac{2a^2\tan^{\frac{5}{2}}(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{2a^2(5a^2+11b^2)\tan^{\frac{3}{2}}(c+dx)}{3b^2\,(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}}$$

$$= -\frac{2a^2\tan^{\frac{5}{2}}(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{2a^2(5a^2+11b^2)\tan^{\frac{3}{2}}(c+dx)}{3b^2\,(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}}$$

$$= -\frac{2a^2\tan^{\frac{5}{2}}(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{2a^2(5a^2+11b^2)\tan^{\frac{3}{2}}(c+dx)}{3b^2\,(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}}$$

$$= -\frac{2a^2\tan^{\frac{5}{2}}(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{2a^2(5a^2+11b^2)\tan^{\frac{3}{2}}(c+dx)}{3b^2\,(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}}$$

$$= -\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{b^{7/2}d} - \frac{2a^2\tan^{\frac{5}{2}}(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}}$$

$$= -\frac{i\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{5/2}d} - \frac{5a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{b^{7/2}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.17, size = 407, normalized size = 1.28

$$\frac{\sqrt[4]{-1}\,\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(a-ib)(-a+ib)^{3/2}d} - \frac{\sqrt[4]{-1}\,\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(-a-ib)(a+ib)^{3/2}d} + \frac{\tan^{\frac{3}{2}}(c+dx)}{3(-a+ib)d(a+b\tan(c+dx))^{3/2}} - \frac{\tan^{\frac{3}{2}}(c+dx)}{3(a+ib)d(a+b\tan(c+dx))^{3/2}} - \frac{i\sqrt{\tan(c+dx)}}{(a-ib)(-a+ib)d\sqrt{a+b\tan(c+dx)}} + \frac{i\sqrt{\tan(c+dx)}}{(-a-ib)(a+ib)d\sqrt{a+b\tan(c+dx)}} + \frac{2\,{}_2F_1\left(\frac{5}{2},\frac{7}{2};\frac{9}{2};-\frac{b\tan(c+dx)}{a}\right)\tan^{\frac{7}{2}}(c+dx)\sqrt{1+\frac{b\tan(c+dx)}{a}}}{7a^2d\sqrt{a+b\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(9/2)/(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-1)^(1/4)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a +
b*Tan[c + d*x]]])/((a - I*b)*(-a + I*b)^(3/2)*d) - ((-1)^(1/4)*ArcTan[((-1)
^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/((-a -
I*b)*(a + I*b)^(3/2)*d) + Tan[c + d*x]^(3/2)/(3*(-a + I*b)*d*(a + b*Tan[c +
 d*x])^(3/2)) - Tan[c + d*x]^(3/2)/(3*(a + I*b)*d*(a + b*Tan[c + d*x])^(3/2
)) - (I*Sqrt[Tan[c + d*x]])/((a - I*b)*(-a + I*b)*d*Sqrt[a + b*Tan[c + d*x]
]) + (I*Sqrt[Tan[c + d*x]])/((-a - I*b)*(a + I*b)*d*Sqrt[a + b*Tan[c + d*x]
]) + (2*Hypergeometric2F1[5/2, 7/2, 9/2, -((b*Tan[c + d*x])/a)]*Tan[c + d*x
]^(7/2)*Sqrt[1 + (b*Tan[c + d*x])/a])/(7*a^2*d*Sqrt[a + b*Tan[c + d*x]])
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 1.32, size = 1493684, normalized size = 4711.94

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(9/2)/(a+b*tan(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(9/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^(9/2)/(b*tan(d*x + c) + a)^(5/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(9/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(9/2)/(a+b*tan(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7317 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(9/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(c+d\,x)^{9/2}}{\left(a+b\tan\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(9/2)/(a + b*tan(c + d*x))^(5/2),x)
```

```
[Out] int(tan(c + d*x)^(9/2)/(a + b*tan(c + d*x))^(5/2), x)
```

3.648 $\int \frac{\tan^{\frac{7}{2}}(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=251

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{5/2}d}+\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{b^{5/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{5/2}d}$$

```
[Out] -arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a-b)^(5/2
)/d+2*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/b^(5/2)/d-ar
ctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a+b)^(5/2)/
d-2*a^2*(a^2+3*b^2)*tan(d*x+c)^(1/2)/b^2/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^(1/
2)-2/3*a^2*tan(d*x+c)^(3/2)/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 1.32, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3646, 3726, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$-\frac{2a^2\tan^{\frac{3}{2}}(c+dx)}{3bd\,(a^2+b^2)\,(a+b\tan(c+dx))^{3/2}}-\frac{2a^2(a^2+3b^2)\,\sqrt{\tan(c+dx)}}{b^2d\,(a^2+b^2)^2\,\sqrt{a+b\tan(c+dx)}}-\frac{\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{5/2}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{b^{5/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(7/2)/(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] -(ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]/((I*a
 - b)^(5/2)*d)) + (2*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
+ d*x]]])/(b^(5/2)*d) - ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a +
 b*Tan[c + d*x]]]/((I*a + b)^(5/2)*d) - (2*a^2*Tan[c + d*x]^(3/2))/(3*b*(a^
2 + b^2)*d*(a + b*Tan[c + d*x])^(3/2)) - (2*a^2*(a^2 + 3*b^2)*Sqrt[Tan[c +
d*x]])/(b^2*(a^2 + b^2)^2*d*Sqrt[a + b*Tan[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
```

```
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
```

```
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\tan^{\frac{7}{2}}(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx = -\frac{2a^2\tan^{\frac{3}{2}}(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} + \frac{2\int \frac{\sqrt{\tan(c+dx)}\left(\frac{3a^2}{2}-\frac{3}{2}ab\tan(c+dx)+\right.}{(a+b\tan(c+dx))^{3/2}}}{3b\,(a^2+b^2)}$$

$$= -\frac{2a^2\tan^{\frac{3}{2}}(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{2a^2(a^2+3b^2)\sqrt{\tan(c+dx)}}{b^2\,(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}}$$

$$= -\frac{2a^2\tan^{\frac{3}{2}}(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{2a^2(a^2+3b^2)\sqrt{\tan(c+dx)}}{b^2\,(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}}$$

$$= -\frac{2a^2\tan^{\frac{3}{2}}(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{2a^2(a^2+3b^2)\sqrt{\tan(c+dx)}}{b^2\,(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}}$$

$$= -\frac{2a^2\tan^{\frac{3}{2}}(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{2a^2(a^2+3b^2)\sqrt{\tan(c+dx)}}{b^2\,(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}}$$

$$= -\frac{2a^2\tan^{\frac{3}{2}}(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{2a^2(a^2+3b^2)\sqrt{\tan(c+dx)}}{b^2\,(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}}$$

$$= -\frac{2a^2\tan^{\frac{3}{2}}(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{2a^2(a^2+3b^2)\sqrt{\tan(c+dx)}}{b^2\,(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{b^{5/2}d} - \frac{2a^2\tan^{\frac{3}{2}}(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} -$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{5/2}d} + \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{b^{5/2}d} -$$

<u>Mathematica</u> [A]

time = 6.24, size = 468, normalized size = 1.86

$$-\frac{\sqrt[4]{-1}\,\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(-a+ib)^{3/2}(ia+b)d} + \frac{\sqrt[4]{-1}\,\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)(a+ib)^{3/2}d} - \frac{i\tan^{\frac{3}{2}}(c+dx)}{3(a-ib)d(a+b\tan(c+dx))^{3/2}} + \frac{i\tan^{\frac{3}{2}}(c+dx)}{3(a+ib)d(a+b\tan(c+dx))^{3/2}} - \frac{2\tan^{\frac{3}{2}}(c+dx)}{3bd(a+b\tan(c+dx))^{3/2}} - \frac{i\sqrt{\tan(c+dx)}}{(ia-b)(a+ib)d\sqrt{a+b\tan(c+dx)}} - \frac{2\sqrt{\tan(c+dx)}}{b^2d\sqrt{a+b\tan(c+dx)}} - \frac{i\sqrt{\tan(c+dx)}}{(a-ib)(ia+b)d\sqrt{a+b\tan(c+dx)}} + \frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{a+b\tan(c+dx)}}{\sqrt{a}\,b^{5/2}d\sqrt{1+\frac{b\tan(c+dx)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(7/2)/(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] -(((-1)^(1/4)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a
+ b*Tan[c + d*x]]])/((-a + I*b)^(3/2)*(I*a + b)*d)) + ((-1)^(1/4)*ArcTan[((
```

```
-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/((I*
a - b)*(a + I*b)^(3/2)*d) - ((I/3)*Tan[c + d*x]^(3/2))/((a - I*b)*d*(a + b*
Tan[c + d*x])^(3/2)) + ((I/3)*Tan[c + d*x]^(3/2))/((a + I*b)*d*(a + b*Tan[c
 + d*x])^(3/2)) - (2*Tan[c + d*x]^(3/2))/(3*b*d*(a + b*Tan[c + d*x])^(3/2))
 - (I*Sqrt[Tan[c + d*x]])/((I*a - b)*(a + I*b)*d*Sqrt[a + b*Tan[c + d*x]])
- (2*Sqrt[Tan[c + d*x]])/(b^2*d*Sqrt[a + b*Tan[c + d*x]]) - (I*Sqrt[Tan[c +
 d*x]])/((a - I*b)*(I*a + b)*d*Sqrt[a + b*Tan[c + d*x]]) + (2*ArcSinh[(Sqrt
[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[a + b*Tan[c + d*x]])/(Sqrt[a]*b^(5/2)
*d*Sqrt[1 + (b*Tan[c + d*x])/a])
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 1.26, size = 1491834, normalized size = 5943.56

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(7/2)/(a+b*tan(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(7/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^(7/2)/(b*tan(d*x + c) + a)^(5/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(7/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(7/2)/(a+b*tan(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(7/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(c+d\,x)^{7/2}}{\left(a+b\tan\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(7/2)/(a + b*tan(c + d*x))^(5/2),x)
```

```
[Out] int(tan(c + d*x)^(7/2)/(a + b*tan(c + d*x))^(5/2), x)
```

3.649 $\int \frac{\tan^{\frac{5}{2}}(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=214

$$\frac{i\mathrm{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{5/2}d}-\frac{i\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{5/2}d}-\frac{2a^2\sqrt{\tan(c+dx)}}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx)}$$

```
[Out] I*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a-b)^(5/
2)/d-I*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a+
b)^(5/2)/d+2/3*a*(a^2+7*b^2)*tan(d*x+c)^(1/2)/b/(a^2+b^2)^2/d/(a+b*tan(d*x+
c))^(1/2)-2/3*a^2*tan(d*x+c)^(1/2)/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.52, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3646, 3730, 3697, 3696, 95, 209, 212}

$$-\frac{2a^2\sqrt{\tan(c+dx)}}{3bd\,(a^2+b^2)\,(a+b\tan(c+dx))^{3/2}}+\frac{2a(a^2+7b^2)\sqrt{\tan(c+dx)}}{3bd\,(a^2+b^2)^2\sqrt{a+b\tan(c+dx)}}+\frac{i\mathrm{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{5/2}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(5/2)/(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] (I*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/((I
*a - b)^(5/2)*d) - (I*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b
*Tan[c + d*x]]])/((I*a + b)^(5/2)*d) - (2*a^2*Sqrt[Tan[c + d*x]])/(3*b*(a^2
 + b^2)*d*(a + b*Tan[c + d*x])^(3/2)) + (2*a*(a^2 + 7*b^2)*Sqrt[Tan[c + d*x
]])/(3*b*(a^2 + b^2)^2*d*Sqrt[a + b*Tan[c + d*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
```

```
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx = -\frac{2a^2\sqrt{\tan(c+dx)}}{3b(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} + \frac{2\int \frac{\frac{a^2}{2}-\frac{3}{2}ab\tan(c+dx)+\frac{1}{2}(a^2+3b^2)\tan^2(c+dx)}{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{3/2}}}{3b(a^2+b^2)}$$

$$= -\frac{2a^2\sqrt{\tan(c+dx)}}{3b(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} + \frac{2a(a^2+7b^2)\sqrt{\tan(c+dx)}}{3b(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}} +$$

$$= -\frac{2a^2\sqrt{\tan(c+dx)}}{3b(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} + \frac{2a(a^2+7b^2)\sqrt{\tan(c+dx)}}{3b(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}} +$$

$$= -\frac{2a^2\sqrt{\tan(c+dx)}}{3b(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} + \frac{2a(a^2+7b^2)\sqrt{\tan(c+dx)}}{3b(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}} +$$

$$= -\frac{2a^2\sqrt{\tan(c+dx)}}{3b(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} + \frac{2a(a^2+7b^2)\sqrt{\tan(c+dx)}}{3b(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}} +$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{5/2}d} - \frac{i\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{5/2}d} -$$

Mathematica [A]
time = 3.42, size = 189, normalized size = 0.88

$$\frac{\frac{3\sqrt[4]{-1}\,\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(-a+ib)^{5/2}} - \frac{3\sqrt[4]{-1}\,\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(a+ib)^{5/2}} + \frac{2a\sqrt{\tan(c+dx)}\,(6ab+(a^2+7b^2)\tan(c+dx))}{(a^2+b^2)^2(a+b\tan(c+dx))^{3/2}}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(5/2)/(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((3*(-1)^(1/4)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a
 + b*Tan[c + d*x]]])/(-a + I*b)^(5/2) - (3*(-1)^(1/4)*ArcTan[((-1)^(1/4)*Sq
rt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(a + I*b)^(5/2)
+ (2*a*Sqrt[Tan[c + d*x]]*(6*a*b + (a^2 + 7*b^2)*Tan[c + d*x]))/((a^2 + b^2
)^2*(a + b*Tan[c + d*x])^(3/2)))/(3*d)
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 1.32, size = 1489186, normalized size = 6958.81

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^(5/2)/(b*tan(d*x + c) + a)^(5/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)}{(a+b\tan(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(5/2)/(a+b*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(tan(c + d*x)**(5/2)/(a + b*tan(c + d*x))**(5/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(c+d\,x)^{5/2}}{{\left(a+b\,\tan\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(5/2)/(a + b*tan(c + d*x))^(5/2),x)
[Out] int(tan(c + d*x)^(5/2)/(a + b*tan(c + d*x))^(5/2), x)
```

3.650 $\int \frac{\tan^{\frac{3}{2}}(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=199

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{5/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{5/2}d}+\frac{2a\sqrt{\tan(c+dx)}}{3\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}}$$

```
[Out] arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a-b)^(5/2)
/d+arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a+b)^(
5/2)/d+4/3*(a^2-2*b^2)*tan(d*x+c)^(1/2)/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^(1/2
)+2/3*a*tan(d*x+c)^(1/2)/(a^2+b^2)/d/(a+b*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.45, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3648, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2a\sqrt{\tan(c+dx)}}{3d\,(a^2+b^2)\,(a+b\tan(c+dx))^{3/2}}+\frac{4(a^2-2b^2)\,\sqrt{\tan(c+dx)}}{3d\,(a^2+b^2)^2\,\sqrt{a+b\tan(c+dx)}}+\frac{\operatorname{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{5/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(3/2)/(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]/((I*a -
 b)^(5/2)*d) + ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
+ d*x]]]/((I*a + b)^(5/2)*d) + (2*a*Sqrt[Tan[c + d*x]])/(3*(a^2 + b^2)*d*(a
 + b*Tan[c + d*x])^(3/2)) + (4*(a^2 - 2*b^2)*Sqrt[Tan[c + d*x]])/(3*(a^2 +
b^2)^2*d*Sqrt[a + b*Tan[c + d*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
```

```
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx = \frac{2a\sqrt{\tan(c+dx)}}{3(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{2\int \frac{\frac{a}{2}-\frac{3}{2}b\tan(c+dx)-a\tan^2(c+dx)}{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{3/2}}\,dx}{3(a^2+b^2)}$$

$$= \frac{2a\sqrt{\tan(c+dx)}}{3(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} + \frac{4(a^2-2b^2)\sqrt{\tan(c+dx)}}{3(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}} - \frac{4}{}$$

$$= \frac{2a\sqrt{\tan(c+dx)}}{3(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} + \frac{4(a^2-2b^2)\sqrt{\tan(c+dx)}}{3(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}} - \frac{\int}{}$$

$$= \frac{2a\sqrt{\tan(c+dx)}}{3(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} + \frac{4(a^2-2b^2)\sqrt{\tan(c+dx)}}{3(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}} - \frac{S}{}$$

$$= \frac{2a\sqrt{\tan(c+dx)}}{3(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} + \frac{4(a^2-2b^2)\sqrt{\tan(c+dx)}}{3(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}} - \frac{S}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{5/2}d} + \frac{\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{5/2}d} +$$

Mathematica [A]
time = 3.04, size = 198, normalized size = 0.99

$$\frac{\frac{3(-1)^{3/4}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(-a+ib)^{5/2}} + \frac{3(-1)^{3/4}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(a+ib)^{5/2}} + \frac{2\sqrt{\tan(c+dx)}\,(3a(a^2-b^2)+2b(a^2-2b^2)\tan(c+dx))}{(a^2+b^2)^2(a+b\tan(c+dx))^{3/2}}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(3/2)/(a + b*Tan[c + d*x])^(5/2),x]
[Out] ((3*(-1)^(3/4)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a
 + b*Tan[c + d*x]]])/(-a + I*b)^(5/2) + (3*(-1)^(3/4)*ArcTan[((-1)^(1/4)*Sq
rt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(a + I*b)^(5/2)
+ (2*Sqrt[Tan[c + d*x]]*(3*a*(a^2 - b^2) + 2*b*(a^2 - 2*b^2)*Tan[c + d*x]))
/((a^2 + b^2)^2*(a + b*Tan[c + d*x])^(3/2)))/(3*d)
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 1.30, size = 1488901, normalized size = 7481.91

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^(3/2)/(b*tan(d*x + c) + a)^(5/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{3}{2}}(c + dx)}{(a + b\tan(c + dx))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(3/2)/(a+b*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(tan(c + d*x)**(3/2)/(a + b*tan(c + d*x))**(5/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^{3/2}}{\left(a+b\tan\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(3/2)/(a + b*tan(c + d*x))^(5/2),x)
[Out] int(tan(c + d*x)^(3/2)/(a + b*tan(c + d*x))^(5/2), x)
```

3.651 $\int \frac{\sqrt{\tan(c+dx)}}{(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=211

$$-\frac{i\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{5/2}d}+\frac{i\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{5/2}d}-\frac{2b\sqrt{\tan(c+dx)}}{3\left(a^2+b^2\right)d(a+b\tan(c+dx}$$

```
[Out] -I*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a-b)^(5
/2)/d+I*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a
+b)^(5/2)/d-2/3*b*(5*a^2-b^2)*tan(d*x+c)^(1/2)/a/(a^2+b^2)^2/d/(a+b*tan(d*x
+c))^(1/2)-2/3*b*tan(d*x+c)^(1/2)/(a^2+b^2)/d/(a+b*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.47, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3649, 3730, 3697, 3696, 95, 209, 212}

$$-\frac{2b(5a^2-b^2)\sqrt{\tan(c+dx)}}{3ad\left(a^2+b^2\right)^2\sqrt{a+b\tan(c+dx)}}-\frac{2b\sqrt{\tan(c+dx)}}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{i\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{5/2}}+\frac{i\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[c + d*x]]/(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-I)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/
((I*a - b)^(5/2)*d) + (I*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a
+ b*Tan[c + d*x]]])/((I*a + b)^(5/2)*d) - (2*b*Sqrt[Tan[c + d*x]])/(3*(a^2
+ b^2)*d*(a + b*Tan[c + d*x])^(3/2)) - (2*b*(5*a^2 - b^2)*Sqrt[Tan[c + d*x]
])/(3*a*(a^2 + b^2)^2*d*Sqrt[a + b*Tan[c + d*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
```

```
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{\sqrt{\tan(c+dx)}}{(a+b\tan(c+dx))^{5/2}}\,dx = -\frac{2b\sqrt{\tan(c+dx)}}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{b}{2}-\frac{3}{2}a\tan(c+dx)+b\tan^2(c+dx)}{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{3/2}}\,dx}{3\left(a^2+b^2\right)}$$

$$= -\frac{2b\sqrt{\tan(c+dx)}}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^{3/2}} - \frac{2b(5a^2-b^2)\sqrt{\tan(c+dx)}}{3a\left(a^2+b^2\right)^2 d\sqrt{a+b\tan(c+dx)}} -$$

$$= -\frac{2b\sqrt{\tan(c+dx)}}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^{3/2}} - \frac{2b(5a^2-b^2)\sqrt{\tan(c+dx)}}{3a\left(a^2+b^2\right)^2 d\sqrt{a+b\tan(c+dx)}} -$$

$$= -\frac{2b\sqrt{\tan(c+dx)}}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^{3/2}} - \frac{2b(5a^2-b^2)\sqrt{\tan(c+dx)}}{3a\left(a^2+b^2\right)^2 d\sqrt{a+b\tan(c+dx)}} -$$

$$= -\frac{2b\sqrt{\tan(c+dx)}}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^{3/2}} - \frac{2b(5a^2-b^2)\sqrt{\tan(c+dx)}}{3a\left(a^2+b^2\right)^2 d\sqrt{a+b\tan(c+dx)}} -$$

$$= -\frac{i\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{5/2}d} + \frac{i\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{5/2}d}$$

Mathematica [A]
time = 3.87, size = 194, normalized size = 0.92

$$\frac{-\frac{3\sqrt[4]{-1}\,\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(-a+ib)^{5/2}} + \frac{3\sqrt[4]{-1}\,\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(a+ib)^{5/2}} + \frac{2b\sqrt{\tan(c+dx)}\left(-6a^3+\left(-5a^2b+b^3\right)\tan(c+dx)\right)}{a\left(a^2+b^2\right)^2(a+b\tan(c+dx))^{3/2}}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tan[c + d*x]]/(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-3*(-1)^(1/4)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[
a + b*Tan[c + d*x]]])/(-a + I*b)^(5/2) + (3*(-1)^(1/4)*ArcTan[((-1)^(1/4)*S
qrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(a + I*b)^(5/2)
 + (2*b*Sqrt[Tan[c + d*x]]*(-6*a^3 + (-5*a^2*b + b^3)*Tan[c + d*x]))/(a*(a^
2 + b^2)^2*(a + b*Tan[c + d*x])^(3/2)))/(3*d)
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 1.30, size = 1489210, normalized size = 7057.87

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(tan(d*x + c))/(b*tan(d*x + c) + a)^(5/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\tan(c+dx)}}{(a+b\tan(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/2)/(a+b*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(sqrt(tan(c + d*x))/(a + b*tan(c + d*x))**(5/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\tan \left(c+d\,x\right)}}{{\left(a+b\,\tan \left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)/(a + b*tan(c + d*x))^(5/2),x)
[Out] int(tan(c + d*x)^(1/2)/(a + b*tan(c + d*x))^(5/2), x)
```

3.652 $\int \frac{1}{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=212

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{5/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{5/2}d}+\frac{2b^2\sqrt{\tan(c+dx)}}{3a\,(a^2+b^2)\,d(a+b\tan(c+d$$

```
[Out] -arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a-b)^(5/2
)/d-arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a+b)^
(5/2)/d+4/3*b^2*(4*a^2+b^2)*tan(d*x+c)^(1/2)/a^2/(a^2+b^2)^2/d/(a+b*tan(d*x
+c))^(1/2)+2/3*b^2*tan(d*x+c)^(1/2)/a/(a^2+b^2)/d/(a+b*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.50, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3650, 3730, 3697, 3696, 95, 209, 212}

$$\frac{4b^2(4a^2+b^2)\sqrt{\tan(c+dx)}}{3a^2d\,(a^2+b^2)^2\sqrt{a+b\tan(c+dx)}}+\frac{2b^2\sqrt{\tan(c+dx)}}{3ad\,(a^2+b^2)\,(a+b\tan(c+dx))^{3/2}}-\frac{\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(5/2)),x]
```

```
[Out] -(ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]/((I*a
 - b)^(5/2)*d)) - ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan
[c + d*x]]]/((I*a + b)^(5/2)*d) + (2*b^2*Sqrt[Tan[c + d*x]])/(3*a*(a^2 + b^
2)*d*(a + b*Tan[c + d*x])^(3/2)) + (4*b^2*(4*a^2 + b^2)*Sqrt[Tan[c + d*x]])
/(3*a^2*(a^2 + b^2)^2*d*Sqrt[a + b*Tan[c + d*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
```

```
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{5/2}}\,dx = \frac{2b^2\sqrt{\tan(c+dx)}}{3a\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} + \frac{2\int \frac{\frac{1}{2}(3a^2+2b^2)-\frac{3}{2}ab\tan(c+\ldots}{\sqrt{\tan(c+dx)}\,(a\ldots}}{3a\,(a^2+\ldots}$$

$$= \frac{2b^2\sqrt{\tan(c+dx)}}{3a\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} + \frac{4b^2(4a^2+b^2)\,\sqrt{\tan\ldots}}{3a^2\,(a^2+b^2)^2\,d\sqrt{a+\ldots}}$$

$$= \frac{2b^2\sqrt{\tan(c+dx)}}{3a\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} + \frac{4b^2(4a^2+b^2)\,\sqrt{\tan\ldots}}{3a^2\,(a^2+b^2)^2\,d\sqrt{a+\ldots}}$$

$$= \frac{2b^2\sqrt{\tan(c+dx)}}{3a\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} + \frac{4b^2(4a^2+b^2)\,\sqrt{\tan\ldots}}{3a^2\,(a^2+b^2)^2\,d\sqrt{a+\ldots}}$$

$$= \frac{2b^2\sqrt{\tan(c+dx)}}{3a\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} + \frac{4b^2(4a^2+b^2)\,\sqrt{\tan\ldots}}{3a^2\,(a^2+b^2)^2\,d\sqrt{a+\ldots}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{5/2}d} - \frac{\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\ldots}}{\sqrt{a+b\tan\ldots}}\right)}{(ia+b)^{5/\ldots}}$$

Mathematica [A]
time = 1.45, size = 235, normalized size = 1.11

$$\frac{-3(-1)^{3/4}\left(\frac{(a+ib)^2\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}}+\frac{(a-ib)^2\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{a+ib}}\right)+\frac{2b^2(a^2+b^2)\sqrt{\tan(c+dx)}}{a(a+b\tan(c+dx))^{3/2}}+\frac{4b^2(4a^2+b^2)\sqrt{\tan(c+dx)}}{a^2\sqrt{a+b\tan(c+dx)}}}{3\,(a^2+b^2)^2\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(5/2)),x]
```

```
[Out] (-3*(-1)^(3/4)*(((a + I*b)^2*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c +
 d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[-a + I*b] + ((a - I*b)^2*ArcTan[((-
1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[
a + I*b]) + (2*b^2*(a^2 + b^2)*Sqrt[Tan[c + d*x]])/(a*(a + b*Tan[c + d*x])^
(3/2)) + (4*b^2*(4*a^2 + b^2)*Sqrt[Tan[c + d*x]])/(a^2*Sqrt[a + b*Tan[c + d
*x]]))/(3*(a^2 + b^2)^2*d)
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 1.24, size = 1488925, normalized size = 7023.23

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(5/2),x)
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(1/((b*tan(d*x + c) + a)^(5/2)*sqrt(tan(d*x + c))), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\tan(c+dx))^{\frac{5}{2}}\sqrt{\tan(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(1/2)/(a+b*tan(d*x+c))**(5/2),x)
[Out] Integral(1/((a + b*tan(c + d*x))**(5/2)*sqrt(tan(c + d*x))), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{\tan\left(c+d\,x\right)}\,{\left(a+b\,\tan\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(tan(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(5/2)), x)
```

3.653 $\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=265

$$\frac{i\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{5/2}d}-\frac{i\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{5/2}d}-\frac{2}{ad\sqrt{\tan(c+dx)}\,(a+b\tan(c+$$

```
[Out] I*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a-b)^(5/
2)/d-I*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a+
b)^(5/2)/d-2/3*b*(3*a^4+17*a^2*b^2+8*b^4)*tan(d*x+c)^(1/2)/a^3/(a^2+b^2)^2/
d/(a+b*tan(d*x+c))^(1/2)-2/a/d/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2)-2/3*
b*(3*a^2+4*b^2)*tan(d*x+c)^(1/2)/a^2/(a^2+b^2)/d/(a+b*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.67, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3650, 3730, 3697, 3696, 95, 209, 212}

$$-\frac{2b(3a^2+4b^2)\sqrt{\tan(c+dx)}}{3a^2d(a^2+b^2)(a+b\tan(c+dx))^{3/2}}-\frac{2b(3a^4+17a^2b^2+8b^4)\sqrt{\tan(c+dx)}}{3a^3d(a^2+b^2)^2\sqrt{a+b\tan(c+dx)}}+\frac{i\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{5/2}}-\frac{2}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(5/2)),x]
```

```
[Out] (I*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/((I
*a - b)^(5/2)*d) - (I*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b
*Tan[c + d*x]]])/((I*a + b)^(5/2)*d) - 2/(a*d*Sqrt[Tan[c + d*x]]*(a + b*Tan
[c + d*x])^(3/2)) - (2*b*(3*a^2 + 4*b^2)*Sqrt[Tan[c + d*x]])/(3*a^2*(a^2 +
b^2)*d*(a + b*Tan[c + d*x])^(3/2)) - (2*b*(3*a^4 + 17*a^2*b^2 + 8*b^4)*Sqrt
[Tan[c + d*x]])/(3*a^3*(a^2 + b^2)^2*d*Sqrt[a + b*Tan[c + d*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
```

```
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx = -\frac{2}{ad\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{3/2}} - \frac{2\int \frac{2b+\frac{1}{2}a\tan(c+dx)\cdots}{\sqrt{\tan(c+dx)}\,(\cdots}}{a\cdots}$$

$$= -\frac{2}{ad\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{3/2}} - \frac{2b(3a^2+4b^2)\,\sqrt{\cdots}}{3a^2\,(a^2+b^2)\,d(a+\cdots}$$

$$= -\frac{2}{ad\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{3/2}} - \frac{2b(3a^2+4b^2)\,\sqrt{\cdots}}{3a^2\,(a^2+b^2)\,d(a+\cdots}$$

$$= -\frac{2}{ad\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{3/2}} - \frac{2b(3a^2+4b^2)\,\sqrt{\cdots}}{3a^2\,(a^2+b^2)\,d(a+\cdots}$$

$$= -\frac{2}{ad\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{3/2}} - \frac{2b(3a^2+4b^2)\,\sqrt{\cdots}}{3a^2\,(a^2+b^2)\,d(a+\cdots}$$

$$= -\frac{2}{ad\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{3/2}} - \frac{2b(3a^2+4b^2)\,\sqrt{\cdots}}{3a^2\,(a^2+b^2)\,d(a+\cdots}$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{5/2}d} - \frac{i\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan\cdots}}{\sqrt{a+b\tan(c\cdots}}\right)}{(ia+b)^{5/2}d}$$

Mathematica [A]
time = 2.78, size = 294, normalized size = 1.11

$$-\frac{-\frac{3\sqrt[4]{-1}\,a^2\left(\frac{(a+ib)^2\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}}-\frac{(a-ib)^2\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{a+ib}}\right)}{(a^2+b^2)^2}+\frac{6a}{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{3/2}}+\frac{2b(3a^2+4b^2)\,\sqrt{\tan(c+dx)}}{(a^2+b^2)(a+b\tan(c+dx))^{3/2}}+\frac{2b(3a^4+17a^2b^2+8b^4)\,\sqrt{\tan(c+dx)}}{a(a^2+b^2)^2\,\sqrt{a+b\tan(c+dx)}}}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(5/2)),x]
```

```
[Out] -1/3*((-3*(-1)^(1/4)*a^2*(((a + I*b)^2*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sq
rt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[-a + I*b] - ((a - I*b)^2*
```

```
ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x
]]])/Sqrt[a + I*b]))/(a^2 + b^2)^2 + (6*a)/(Sqrt[Tan[c + d*x]]*(a + b*Tan[c
 + d*x])^(3/2)) + (2*b*(3*a^2 + 4*b^2)*Sqrt[Tan[c + d*x]])/((a^2 + b^2)*(a
+ b*Tan[c + d*x])^(3/2)) + (2*b*(3*a^4 + 17*a^2*b^2 + 8*b^4)*Sqrt[Tan[c + d
*x]])/(a*(a^2 + b^2)^2*Sqrt[a + b*Tan[c + d*x]]))/(a^2*d)
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 1.28, size = 1490722, normalized size = 5625.37

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*tan(d*x + c) + a)^(5/2)*tan(d*x + c)^(3/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\tan{(c+dx)})^{\frac{5}{2}}\tan^{\frac{3}{2}}{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(3/2)/(a+b*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(1/((a + b*tan(c + d*x))**(5/2)*tan(c + d*x)**(3/2)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\tan\left(c+d\,x\right)^{3/2}\,{\left(a+b\tan\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(5/2)),x)
[Out] int(1/(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(5/2)), x)
```

3.654 $\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=298

$$\frac{\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{5/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{5/2}d}-\frac{2}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+$$

```
[Out] arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a-b)^(5/2)
/d+arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a+b)^(
5/2)/d+4/3*b^2*(4*a^4+15*a^2*b^2+8*b^4)*tan(d*x+c)^(1/2)/a^4/(a^2+b^2)^2/d/
(a+b*tan(d*x+c))^(1/2)+4*b/a^2/d/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2)+2/
3*b^2*(7*a^2+8*b^2)*tan(d*x+c)^(1/2)/a^3/(a^2+b^2)/d/(a+b*tan(d*x+c))^(3/2)
-2/3/a/d/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.80, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3650, 3730, 3731, 3697, 3696, 95, 209, 212}

$$\frac{4b}{a^2d\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{3/2}}+\frac{4b^2(4a^4+15a^2b^2+8b^4)\sqrt{\tan(c+dx)}}{3a^4d\,(a^2+b^2)^2\sqrt{a+b\tan(c+dx)}}+\frac{2b^2(7a^2+8b^2)\sqrt{\tan(c+dx)}}{3a^3d\,(a^2+b^2)\,(a+b\tan(c+dx))^{3/2}}+\frac{\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{5/2}}-\frac{2}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^(5/2)),x]
```

```
[Out] ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]/((I*a -
 b)^(5/2)*d) + ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
+ d*x]]]/((I*a + b)^(5/2)*d) - 2/(3*a*d*Tan[c + d*x]^(3/2)*(a + b*Tan[c + d
*x])^(3/2)) + (4*b)/(a^2*d*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(3/2)) +
 (2*b^2*(7*a^2 + 8*b^2)*Sqrt[Tan[c + d*x]])/(3*a^3*(a^2 + b^2)*d*(a + b*Tan
[c + d*x])^(3/2)) + (4*b^2*(4*a^4 + 15*a^2*b^2 + 8*b^4)*Sqrt[Tan[c + d*x]])
/(3*a^4*(a^2 + b^2)^2*d*Sqrt[a + b*Tan[c + d*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
```

```
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3731

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)
*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp
[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - a*C*(b*c*(m + 1) + a*d*(n
+ 1)) - (m + 1)*(b*c - a*d)*(A*b - b*C)*Tan[e + f*x] - d*(A*b^2 + a^2*C)*(m
 + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m,
 -1] &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx = -\frac{2}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}} - \frac{2\int \frac{3b+\frac{3}{2}a\tan(c+dx)+3b\cdots}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(\cdots}}{3a}$$

$$= -\frac{2}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}} + \frac{4\cdots}{a^2d\sqrt{\tan(c+dx)}\,(a\cdots}$$

$$= -\frac{2}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}} + \frac{4\cdots}{a^2d\sqrt{\tan(c+dx)}\,(a\cdots}$$

$$= -\frac{2}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}} + \frac{4\cdots}{a^2d\sqrt{\tan(c+dx)}\,(a\cdots}$$

$$= -\frac{2}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}} + \frac{4\cdots}{a^2d\sqrt{\tan(c+dx)}\,(a\cdots}$$

$$= -\frac{2}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}} + \frac{4\cdots}{a^2d\sqrt{\tan(c+dx)}\,(a\cdots}$$

$$= -\frac{2}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}} + \frac{4\cdots}{a^2d\sqrt{\tan(c+dx)}\,(a\cdots}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{5/2}d} + \frac{\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c\cdots}}{\sqrt{a+b\tan(c+\cdots}}\right)}{(ia+b)^{5/2}d}$$

Mathematica [A]
time = 6.29, size = 483, normalized size = 1.62

$$-\frac{2}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}} - \frac{2\left(-\frac{6b}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}} - \frac{3\left(\frac{ab\sqrt{\tan(c+dx)}}{3(ia-b)(a+b\tan(c+dx))^{3/2}} + \frac{16b^2\sqrt{\tan(c+dx)}}{3a(a+b\tan(c+dx))^{3/2}} - \frac{ab\sqrt{\tan(c+dx)}}{3(ia+b)(a+b\tan(c+dx))^{3/2}} + \frac{32b^2\sqrt{\tan(c+dx)}}{3a^2\sqrt{a+b\tan(c+dx)}} - \frac{-\frac{3\sqrt[4]{-1}a^2\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(-a+ib)^{3/2}} + \frac{(5a-2ib)b\sqrt{\tan(c+dx)}}{(a-ib)\sqrt{a+b\tan(c+dx)}}}{3(ia+b)} + \frac{-\frac{3\sqrt[4]{-1}a^2\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(a+ib)^{3/2}} + \frac{(5a+2ib)b\sqrt{\tan(c+dx)}}{(a+ib)\sqrt{a+b\tan(c+dx)}}}{3(ia-b)}\right)}{2ad}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^(5/2)),x]
```

```
[Out] -2/(3*a*d*Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(3/2)) - (2*((-6*b)/(a*d*
Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(3/2)) - (3*((a*b*Sqrt[Tan[c + d*x]
])/(3*(I*a - b)*(a + b*Tan[c + d*x])^(3/2)) + (16*b^2*Sqrt[Tan[c + d*x]])/(
```

```
3*a*(a + b*Tan[c + d*x])^(3/2)) - (a*b*Sqrt[Tan[c + d*x]])/(3*(I*a + b)*(a
+ b*Tan[c + d*x])^(3/2)) + (32*b^2*Sqrt[Tan[c + d*x]])/(3*a^2*Sqrt[a + b*Ta
n[c + d*x]]) - ((-3*(-1)^(1/4)*a^2*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[T
an[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(-a + I*b)^(3/2) + ((5*a - (2*I)*b
)*b*Sqrt[Tan[c + d*x]])/((a - I*b)*Sqrt[a + b*Tan[c + d*x]]))/(3*(I*a + b))
 + ((-3*(-1)^(1/4)*a^2*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])
/Sqrt[a + b*Tan[c + d*x]]])/(a + I*b)^(3/2) + ((5*a + (2*I)*b)*b*Sqrt[Tan[c
 + d*x]])/((a + I*b)*Sqrt[a + b*Tan[c + d*x]]))/(3*(I*a - b))))/(2*a*d)))/(
3*a)
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 3.45, size = 1491406, normalized size = 5004.72

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(5/2),x)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(1/((b*tan(d*x + c) + a)^(5/2)*tan(d*x + c)^(5/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\tan\left(c + dx\right)\right)^{\frac{5}{2}}\tan^{\frac{5}{2}}\left(c + dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(5/2)/(a+b*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(1/((a + b*tan(c + d*x))**(5/2)*tan(c + d*x)**(5/2)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\tan\left(c+d\,x\right)^{5/2}\,{\left(a+b\,\tan\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(5/2)*(a + b*tan(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(tan(c + d*x)^(5/2)*(a + b*tan(c + d*x))^(5/2)), x)
```

3.655 $\int \frac{1}{\sqrt{\tan(c+dx)}\sqrt{2+3\tan(c+dx)}}\,dx$

Optimal. Leaf size=89

$$\frac{\tanh^{-1}\left(\frac{\sqrt{3-2i}\sqrt{\tan(c+dx)}}{\sqrt{2+3\tan(c+dx)}}\right)}{\sqrt{3-2i}\,d}+\frac{\tanh^{-1}\left(\frac{\sqrt{3+2i}\sqrt{\tan(c+dx)}}{\sqrt{2+3\tan(c+dx)}}\right)}{\sqrt{3+2i}\,d}$$

```
[Out] arctanh((3-2*I)^(1/2)*tan(d*x+c)^(1/2)/(2+3*tan(d*x+c))^(1/2))/d/(3-2*I)^(1
/2)+arctanh((3+2*I)^(1/2)*tan(d*x+c)^(1/2)/(2+3*tan(d*x+c))^(1/2))/d/(3+2*I
)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3656, 926, 95, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{3-2i}\sqrt{\tan(c+dx)}}{\sqrt{3\tan(c+dx)+2}}\right)}{\sqrt{3-2i}\,d}+\frac{\tanh^{-1}\left(\frac{\sqrt{3+2i}\sqrt{\tan(c+dx)}}{\sqrt{3\tan(c+dx)+2}}\right)}{\sqrt{3+2i}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Tan[c + d*x]]*Sqrt[2 + 3*Tan[c + d*x]]),x]
```

```
[Out] ArcTanh[(Sqrt[3 - 2*I]*Sqrt[Tan[c + d*x]])/Sqrt[2 + 3*Tan[c + d*x]]]/(Sqrt[
3 - 2*I]*d) + ArcTanh[(Sqrt[3 + 2*I]*Sqrt[Tan[c + d*x]])/Sqrt[2 + 3*Tan[c +
 d*x]]]/(Sqrt[3 + 2*I]*d)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
```

```
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\tan(c+dx)}\sqrt{2+3\tan(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{2+3x}\,(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{i}{2(i-x)\sqrt{x}\sqrt{2+3x}} + \frac{i}{2\sqrt{x}\,(i+x)\sqrt{2+3x}}\right) dx, x, \text{ta}\right.}{d}$$

$$= \frac{i\text{Subst}\left(\int \frac{1}{(i-x)\sqrt{x}\sqrt{2+3x}}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{i\text{Subst}\left(\int\right.}{}$$

$$= \frac{i\text{Subst}\left(\int \frac{1}{i-(2+3i)x^2}\,dx, x, \frac{\sqrt{\tan(c+dx)}}{\sqrt{2+3\tan(c+dx)}}\right)}{d} + \frac{i\text{Subst}\left(\int\right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{3-2i}\sqrt{\tan(c+dx)}}{\sqrt{2+3\tan(c+dx)}}\right)}{\sqrt{3-2i}\,d} + \frac{\tanh^{-1}\left(\frac{\sqrt{3+2i}\sqrt{\tan}}{\sqrt{2+3\tan(c}}\right.}{\sqrt{3+2i}\,d}$$

Mathematica [A]
time = 0.19, size = 89, normalized size = 1.00

$$\frac{\text{ArcTan}\left(\frac{\sqrt{-3+2i}\sqrt{\tan(c+dx)}}{\sqrt{2+3\tan(c+dx)}}\right)}{\sqrt{-3+2i}\,d} + \frac{\tanh^{-1}\left(\frac{\sqrt{3+2i}\sqrt{\tan(c+dx)}}{\sqrt{2+3\tan(c+dx)}}\right)}{\sqrt{3+2i}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Tan[c + d*x]]*Sqrt[2 + 3*Tan[c + d*x]]),x]
```

```
[Out] ArcTan[(Sqrt[-3 + 2*I]*Sqrt[Tan[c + d*x]])/Sqrt[2 + 3*Tan[c + d*x]]]/(Sqrt[
-3 + 2*I]*d) + ArcTanh[(Sqrt[3 + 2*I]*Sqrt[Tan[c + d*x]])/Sqrt[2 + 3*Tan[c
+ d*x]]]/(Sqrt[3 + 2*I]*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(73) = 146$.
time = 2.95, size = 480, normalized size = 5.39

method	result
derivativedivides	$\frac{\sqrt{\frac{\tan(dx+c)(2+3\tan(dx+c))}{\left(\sqrt{13}-3+2\tan(dx+c)\right)^2}}\left(\sqrt{13}-3+2\tan(dx+c)\right)\left(3\sqrt{13}\,\sqrt{2\sqrt{13}+6}\,\arctan\left(\frac{\sqrt{2\sqrt{13}-}}{}\right.\right.}{}$
default	$\frac{\sqrt{\frac{\tan(dx+c)(2+3\tan(dx+c))}{\left(\sqrt{13}-3+2\tan(dx+c)\right)^2}}\left(\sqrt{13}-3+2\tan(dx+c)\right)\left(3\sqrt{13}\,\sqrt{2\sqrt{13}+6}\,\arctan\left(\frac{\sqrt{2\sqrt{13}-}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(1/2)/(2+3*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(tan(d*x+c)*(2+3*tan(d*x+c))/(13^(1/2)-3+2*tan(d*x+c))^2)^(1/2)*(13^(
1/2)-3+2*tan(d*x+c))*(3*13^(1/2)*(2*13^(1/2)+6)^(1/2)*arctan(1/416*(2*13^(1
/2)-6)^(1/2)*((11*13^(1/2)-39)*tan(d*x+c)*(39+11*13^(1/2))*(2+3*tan(d*x+c))
/(13^(1/2)-3+2*tan(d*x+c))^2)^(1/2)*(3*13^(1/2)+11)*(13^(1/2)+3-2*tan(d*x+c
))*(11*13^(1/2)-39)*(13^(1/2)-3+2*tan(d*x+c))/tan(d*x+c)/(2+3*tan(d*x+c)))*
(2*13^(1/2)-6)^(1/2)-11*(2*13^(1/2)+6)^(1/2)*arctan(1/416*(2*13^(1/2)-6)^(1
/2)*((11*13^(1/2)-39)*tan(d*x+c)*(39+11*13^(1/2))*(2+3*tan(d*x+c))/(13^(1/2
)-3+2*tan(d*x+c))^2)^(1/2)*(3*13^(1/2)+11)*(13^(1/2)+3-2*tan(d*x+c))*(11*13
^(1/2)-39)*(13^(1/2)-3+2*tan(d*x+c))/tan(d*x+c)/(2+3*tan(d*x+c)))*(2*13^(1/
2)-6)^(1/2)+4*arctanh(4*13^(1/2)*(tan(d*x+c)*(2+3*tan(d*x+c))/(13^(1/2)-3+2
*tan(d*x+c))^2)^(1/2)/(26*13^(1/2)+78)^(1/2))*13^(1/2)-12*arctanh(4*13^(1/2
)*(tan(d*x+c)*(2+3*tan(d*x+c))/(13^(1/2)-3+2*tan(d*x+c))^2)^(1/2)/(26*13^(1
/2)+78)^(1/2)))/tan(d*x+c)^(1/2)/(2+3*tan(d*x+c))^(1/2)/(2*13^(1/2)+6)^(1/2
)/(11*13^(1/2)-39)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/2)/(2+3*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/2)/(2+3*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\tan\left(c+dx\right)+2}\,\sqrt{\tan\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(1/2)/(2+3*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(3*tan(c + d*x) + 2)*sqrt(tan(c + d*x))), x)
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(65) = 130$.
time = 0.57, size = 489, normalized size = 5.49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/2)/(2+3*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/156*sqrt(3)*((3*I + 2)*sqrt(6*sqrt(13) - 18)*(-2*I/(sqrt(13) - 3) + 1)*l
og((120*I + 40)*sqrt(13)*(sqrt(3)*sqrt(tan(d*x + c)) - sqrt(3*tan(d*x + c)
+ 2))^2 + (432*I + 144)*(sqrt(3)*sqrt(tan(d*x + c)) - sqrt(3*tan(d*x + c) +
 2))^2 + 80*sqrt(13)*sqrt(15*sqrt(13) + 54) - 800*sqrt(13) - (16*I - 288)*s
qrt(15*sqrt(13) + 54) - 2880) - (3*I + 2)*sqrt(6*sqrt(13) - 18)*(-2*I/(sqrt
(13) - 3) + 1)*log((120*I + 40)*sqrt(13)*(sqrt(3)*sqrt(tan(d*x + c)) - sqrt
(3*tan(d*x + c) + 2))^2 + (432*I + 144)*(sqrt(3)*sqrt(tan(d*x + c)) - sqrt(
3*tan(d*x + c) + 2))^2 - 80*sqrt(13)*sqrt(15*sqrt(13) + 54) - 800*sqrt(13)
+ (16*I - 288)*sqrt(15*sqrt(13) + 54) - 2880) + (2*I + 3)*sqrt(6*sqrt(13) +
 18)*(-2*I/(sqrt(13) + 3) + 1)*log(8*sqrt(13)*(sqrt(3)*sqrt(tan(d*x + c)) -
 sqrt(3*tan(d*x + c) + 2))^2 - 24*(sqrt(3)*sqrt(tan(d*x + c)) - sqrt(3*tan(
d*x + c) + 2))^2 + 8*sqrt(13)*sqrt(6*sqrt(13) - 18) - (48*I + 16)*sqrt(13)
```

```
+ (16*I - 24)*sqrt(6*sqrt(13) - 18) + 144*I + 48) - (2*I + 3)*sqrt(6*sqrt(1
3) + 18)*(-2*I/(sqrt(13) + 3) + 1)*log(8*sqrt(13)*(sqrt(3)*sqrt(tan(d*x + c
)) - sqrt(3*tan(d*x + c) + 2))^2 - 24*(sqrt(3)*sqrt(tan(d*x + c)) - sqrt(3*
tan(d*x + c) + 2))^2 - 8*sqrt(13)*sqrt(6*sqrt(13) - 18) - (48*I + 16)*sqrt(
13) - (16*I - 24)*sqrt(6*sqrt(13) - 18) + 144*I + 48))/d
```

Mupad [B]
time = 6.12, size = 207, normalized size = 2.33

$$-\mathrm{atan}\left(\frac{\sqrt{2}\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{\frac{3}{52}-\frac{1}{26}\mathrm{i}}{d^2}}\,\left(4-6\mathrm{i}\right)+d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{\frac{3}{52}-\frac{1}{26}\mathrm{i}}{d^2}}\,\sqrt{3\tan\left(c+d\,x\right)+2}\,\left(-4+6\mathrm{i}\right)}{3\tan\left(c+d\,x\right)-\sqrt{2}\,\sqrt{3\tan\left(c+d\,x\right)+2}+2}\right)\sqrt{\frac{\frac{3}{52}-\frac{1}{26}\mathrm{i}}{d^2}}\,2\mathrm{i}+\mathrm{atan}\left(\frac{\sqrt{2}\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{\frac{3}{52}+\frac{1}{26}\mathrm{i}}{d^2}}\,\left(4+6\mathrm{i}\right)+d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{\frac{3}{52}+\frac{1}{26}\mathrm{i}}{d^2}}\,\sqrt{3\tan\left(c+d\,x\right)+2}\,\left(-4-6\mathrm{i}\right)}{3\tan\left(c+d\,x\right)-\sqrt{2}\,\sqrt{3\tan\left(c+d\,x\right)+2}+2}\right)\sqrt{\frac{\frac{3}{52}+\frac{1}{26}\mathrm{i}}{d^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(1/2)*(3*tan(c + d*x) + 2)^(1/2)),x)
```

```
[Out] atan((2^(1/2)*d*tan(c + d*x)^(1/2)*((3/52 + 1i/26)/d^2)^(1/2)*(4 + 6i) - d*
tan(c + d*x)^(1/2)*((3/52 + 1i/26)/d^2)^(1/2)*(3*tan(c + d*x) + 2)^(1/2)*(4
 + 6i))/(3*tan(c + d*x) - 2^(1/2)*(3*tan(c + d*x) + 2)^(1/2) + 2))*((3/52 +
 1i/26)/d^2)^(1/2)*2i - atan((2^(1/2)*d*tan(c + d*x)^(1/2)*((3/52 - 1i/26)/
d^2)^(1/2)*(4 - 6i) - d*tan(c + d*x)^(1/2)*((3/52 - 1i/26)/d^2)^(1/2)*(3*ta
n(c + d*x) + 2)^(1/2)*(4 - 6i))/(3*tan(c + d*x) - 2^(1/2)*(3*tan(c + d*x) +
 2)^(1/2) + 2))*((3/52 - 1i/26)/d^2)^(1/2)*2i
```

3.656 $\int \frac{1}{\sqrt{\tan(c+dx)}\sqrt{-2+3\tan(c+dx)}}\,dx$

Optimal. Leaf size=89

$$\frac{\tanh^{-1}\left(\frac{\sqrt{3-2i}\sqrt{\tan(c+dx)}}{\sqrt{-2+3\tan(c+dx)}}\right)}{\sqrt{3-2i}\,d}+\frac{\tanh^{-1}\left(\frac{\sqrt{3+2i}\sqrt{\tan(c+dx)}}{\sqrt{-2+3\tan(c+dx)}}\right)}{\sqrt{3+2i}\,d}$$

```
[Out] arctanh((3-2*I)^(1/2)*tan(d*x+c)^(1/2)/(-2+3*tan(d*x+c))^(1/2))/d/(3-2*I)^(
1/2)+arctanh((3+2*I)^(1/2)*tan(d*x+c)^(1/2)/(-2+3*tan(d*x+c))^(1/2))/d/(3+2
*I)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3656, 926, 95, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{3-2i}\sqrt{\tan(c+dx)}}{\sqrt{3\tan(c+dx)-2}}\right)}{\sqrt{3-2i}\,d}+\frac{\tanh^{-1}\left(\frac{\sqrt{3+2i}\sqrt{\tan(c+dx)}}{\sqrt{3\tan(c+dx)-2}}\right)}{\sqrt{3+2i}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Tan[c + d*x]]*Sqrt[-2 + 3*Tan[c + d*x]]),x]
```

```
[Out] ArcTanh[(Sqrt[3 - 2*I]*Sqrt[Tan[c + d*x]])/Sqrt[-2 + 3*Tan[c + d*x]]]/(Sqrt
[3 - 2*I]*d) + ArcTanh[(Sqrt[3 + 2*I]*Sqrt[Tan[c + d*x]])/Sqrt[-2 + 3*Tan[c
 + d*x]]]/(Sqrt[3 + 2*I]*d)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
```

```
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\tan(c+dx)}\sqrt{-2+3\tan(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{-2+3x}\,(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{i}{2(i-x)\sqrt{x}\sqrt{-2+3x}} + \frac{i}{2\sqrt{x}\,(i+x)\sqrt{-2+3x}}\right) d\right.}{d}$$

$$= \frac{i\text{Subst}\left(\int \frac{1}{(i-x)\sqrt{x}\sqrt{-2+3x}}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{i\text{Subs}}{}$$

$$= \frac{i\text{Subst}\left(\int \frac{1}{i-(2+3i)x^2}\,dx, x, \frac{\sqrt{\tan(c+dx)}}{\sqrt{-2+3\tan(c+dx)}}\right)}{d} + \frac{i\text{Subs}}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{3-2i}\sqrt{\tan(c+dx)}}{\sqrt{-2+3\tan(c+dx)}}\right)}{\sqrt{3-2i}\,d} + \frac{\tanh^{-1}\left(\frac{\sqrt{3+2i}\sqrt{}}{\sqrt{-2+3t}}\right.}{\sqrt{3+2i}}$$

Mathematica [A]
time = 0.15, size = 89, normalized size = 1.00

$$\frac{\text{ArcTan}\left(\frac{\sqrt{-3+2i}\sqrt{\tan(c+dx)}}{\sqrt{-2+3\tan(c+dx)}}\right)}{\sqrt{-3+2i}\,d} + \frac{\tanh^{-1}\left(\frac{\sqrt{3+2i}\sqrt{\tan(c+dx)}}{\sqrt{-2+3\tan(c+dx)}}\right)}{\sqrt{3+2i}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Tan[c + d*x]]*Sqrt[-2 + 3*Tan[c + d*x]]),x]
[Out] ArcTan[(Sqrt[-3 + 2*I]*Sqrt[Tan[c + d*x]])/Sqrt[-2 + 3*Tan[c + d*x]]]/(Sqrt
[-3 + 2*I]*d) + ArcTanh[(Sqrt[3 + 2*I]*Sqrt[Tan[c + d*x]])/Sqrt[-2 + 3*Tan[
c + d*x]]]/(Sqrt[3 + 2*I]*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(73) = 146$.

time = 1.87, size = 480, normalized size = 5.39

method	result
derivativedivides	$-\frac{\sqrt{\frac{\tan(dx+c)(-2+3\tan(dx+c))}{\left(\sqrt{13}-3-2\tan(dx+c)\right)^2}}\left(\sqrt{13}-3-2\tan(dx+c)\right)\left(3\sqrt{2\sqrt{13}-6}\,\sqrt{13}\,\sqrt{2\sqrt{13}+6}\,\text{arcta}\right.}{}$
default	$-\frac{\sqrt{\frac{\tan(dx+c)(-2+3\tan(dx+c))}{\left(\sqrt{13}-3-2\tan(dx+c)\right)^2}}\left(\sqrt{13}-3-2\tan(dx+c)\right)\left(3\sqrt{2\sqrt{13}-6}\,\sqrt{13}\,\sqrt{2\sqrt{13}+6}\,\text{arcta}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(1/2)/(-2+3*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(tan(d*x+c)*(-2+3*tan(d*x+c))/(13^(1/2)-3-2*tan(d*x+c))^2)^(1/2)*(13
^(1/2)-3-2*tan(d*x+c))*(3*(2*13^(1/2)-6)^(1/2)*13^(1/2)*(2*13^(1/2)+6)^(1/2
)*arctan(1/416*(2*13^(1/2)-6)^(1/2)*((11*13^(1/2)-39)*tan(d*x+c)*(39+11*13^
(1/2))*(-2+3*tan(d*x+c))/(13^(1/2)-3-2*tan(d*x+c))^2)^(1/2)*(3*13^(1/2)+11)
*(13^(1/2)+3+2*tan(d*x+c))*(11*13^(1/2)-39)*(13^(1/2)-3-2*tan(d*x+c))/tan(d
*x+c)/(-2+3*tan(d*x+c)))-11*(2*13^(1/2)-6)^(1/2)*(2*13^(1/2)+6)^(1/2)*arcta
n(1/416*(2*13^(1/2)-6)^(1/2)*((11*13^(1/2)-39)*tan(d*x+c)*(39+11*13^(1/2))*
(-2+3*tan(d*x+c))/(13^(1/2)-3-2*tan(d*x+c))^2)^(1/2)*(3*13^(1/2)+11)*(13^(1
/2)+3+2*tan(d*x+c))*(11*13^(1/2)-39)*(13^(1/2)-3-2*tan(d*x+c))/tan(d*x+c)/(
-2+3*tan(d*x+c)))+4*arctanh(4*13^(1/2)*(tan(d*x+c)*(-2+3*tan(d*x+c))/(13^(1
/2)-3-2*tan(d*x+c))^2)^(1/2)/(26*13^(1/2)+78)^(1/2))*13^(1/2)-12*arctanh(4*
13^(1/2)*(tan(d*x+c)*(-2+3*tan(d*x+c))/(13^(1/2)-3-2*tan(d*x+c))^2)^(1/2)/(
26*13^(1/2)+78)^(1/2)))/tan(d*x+c)^(1/2)/(-2+3*tan(d*x+c))^(1/2)/(2*13^(1/2
)+6)^(1/2)/(11*13^(1/2)-39)
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/2)/(-2+3*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/2)/(-2+3*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\tan\left(c+dx\right)-2}\,\sqrt{\tan\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(1/2)/(-2+3*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(3*tan(c + d*x) - 2)*sqrt(tan(c + d*x))), x)
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 489 vs. 2(65) = 130.
time = 0.52, size = 489, normalized size = 5.49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/2)/(-2+3*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/156*sqrt(3)*(-(3*I + 2)*sqrt(6*sqrt(13) - 18)*(-2*I/(sqrt(13) - 3) + 1)*
log((120*I + 40)*sqrt(13)*(sqrt(3)*sqrt(tan(d*x + c)) - sqrt(3*tan(d*x + c)
 - 2))^2 + (432*I + 144)*(sqrt(3)*sqrt(tan(d*x + c)) - sqrt(3*tan(d*x + c)
- 2))^2 + 80*sqrt(13)*sqrt(15*sqrt(13) + 54) + 800*sqrt(13) - (16*I - 288)*
sqrt(15*sqrt(13) + 54) + 2880) + (3*I + 2)*sqrt(6*sqrt(13) - 18)*(-2*I/(sqr
t(13) - 3) + 1)*log((120*I + 40)*sqrt(13)*(sqrt(3)*sqrt(tan(d*x + c)) - sqr
t(3*tan(d*x + c) - 2))^2 + (432*I + 144)*(sqrt(3)*sqrt(tan(d*x + c)) - sqrt
(3*tan(d*x + c) - 2))^2 - 80*sqrt(13)*sqrt(15*sqrt(13) + 54) + 800*sqrt(13)
 + (16*I - 288)*sqrt(15*sqrt(13) + 54) + 2880) - (2*I + 3)*sqrt(6*sqrt(13)
+ 18)*(-2*I/(sqrt(13) + 3) + 1)*log(8*sqrt(13)*(sqrt(3)*sqrt(tan(d*x + c))
- sqrt(3*tan(d*x + c) - 2))^2 - 24*(sqrt(3)*sqrt(tan(d*x + c)) - sqrt(3*tan
(d*x + c) - 2))^2 + 8*sqrt(13)*sqrt(6*sqrt(13) - 18) + (48*I + 16)*sqrt(13)
```

```
 + (16*I - 24)*sqrt(6*sqrt(13) - 18) - 144*I - 48) + (2*I + 3)*sqrt(6*sqrt(
13) + 18)*(-2*I/(sqrt(13) + 3) + 1)*log(8*sqrt(13)*(sqrt(3)*sqrt(tan(d*x +
c)) - sqrt(3*tan(d*x + c) - 2))^2 - 24*(sqrt(3)*sqrt(tan(d*x + c)) - sqrt(3
*tan(d*x + c) - 2))^2 - 8*sqrt(13)*sqrt(6*sqrt(13) - 18) + (48*I + 16)*sqrt
(13) - (16*I - 24)*sqrt(6*sqrt(13) - 18) - 144*I - 48))/d
```

Mupad [B]
time = 6.21, size = 89, normalized size = 1.00

$$2\,\mathrm{atanh}\left(\frac{2\,d\,\sqrt{\frac{\frac{3}{52}-\frac{1}{26}\mathrm{i}}{d^2}}\,\sqrt{3\tan\left(c+d\,x\right)-2}}{\sqrt{\tan\left(c+d\,x\right)}}\right)\,\sqrt{\frac{\frac{3}{52}-\frac{1}{26}\mathrm{i}}{d^2}}+2\,\mathrm{atanh}\left(\frac{2\,d\,\sqrt{\frac{\frac{3}{52}+\frac{1}{26}\mathrm{i}}{d^2}}\,\sqrt{3\tan\left(c+d\,x\right)-2}}{\sqrt{\tan\left(c+d\,x\right)}}\right)\,\sqrt{\frac{\frac{3}{52}+\frac{1}{26}\mathrm{i}}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(1/2)*(3*tan(c + d*x) - 2)^(1/2)),x)
```

```
[Out] 2*atanh((2*d*((3/52 - 1i/26)/d^2)^(1/2)*(3*tan(c + d*x) - 2)^(1/2))/tan(c +
 d*x)^(1/2))*((3/52 - 1i/26)/d^2)^(1/2) + 2*atanh((2*d*((3/52 + 1i/26)/d^2)
^(1/2)*(3*tan(c + d*x) - 2)^(1/2))/tan(c + d*x)^(1/2))*((3/52 + 1i/26)/d^2)
^(1/2)
```

3.657 $\int \frac{1}{\sqrt{2-3\tan(c+dx)}\sqrt{\tan(c+dx)}}\,dx$

Optimal. Leaf size=89

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{3-2i}\sqrt{\tan(c+dx)}}{\sqrt{2-3\tan(c+dx)}}\right)}{\sqrt{3-2i}\,d}+\frac{\operatorname{ArcTan}\left(\frac{\sqrt{3+2i}\sqrt{\tan(c+dx)}}{\sqrt{2-3\tan(c+dx)}}\right)}{\sqrt{3+2i}\,d}$$

```
[Out] arctan((3-2*I)^(1/2)*tan(d*x+c)^(1/2)/(2-3*tan(d*x+c))^(1/2))/d/(3-2*I)^(1/
2)+arctan((3+2*I)^(1/2)*tan(d*x+c)^(1/2)/(2-3*tan(d*x+c))^(1/2))/d/(3+2*I)^
(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3656, 926, 95, 211}

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{3-2i}\sqrt{\tan(c+dx)}}{\sqrt{2-3\tan(c+dx)}}\right)}{\sqrt{3-2i}\,d}+\frac{\operatorname{ArcTan}\left(\frac{\sqrt{3+2i}\sqrt{\tan(c+dx)}}{\sqrt{2-3\tan(c+dx)}}\right)}{\sqrt{3+2i}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*Tan[c + d*x]]*Sqrt[Tan[c + d*x]]),x]
```

```
[Out] ArcTan[(Sqrt[3 - 2*I]*Sqrt[Tan[c + d*x]])/Sqrt[2 - 3*Tan[c + d*x]]]/(Sqrt[3
 - 2*I]*d) + ArcTan[(Sqrt[3 + 2*I]*Sqrt[Tan[c + d*x]])/Sqrt[2 - 3*Tan[c + d
*x]]]/(Sqrt[3 + 2*I]*d)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
```

```
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
& !IntegerQ[m] && !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3\tan(c+dx)}\sqrt{\tan(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2-3x}\sqrt{x}(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{i}{2\sqrt{2-3x}(i-x)\sqrt{x}} + \frac{i}{2\sqrt{2-3x}\sqrt{x}(i+x)}\right)dx, x, \text{ta}$$

$$= \frac{i\text{Subst}\left(\int \frac{1}{\sqrt{2-3x}(i-x)\sqrt{x}}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{i\text{Subst}\left(\int\right.}{}$$

$$= \frac{i\text{Subst}\left(\int \frac{1}{i-(2-3i)x^2}\,dx, x, \frac{\sqrt{\tan(c+dx)}}{\sqrt{2-3\tan(c+dx)}}\right)}{d} + \frac{i\text{Subst}\left(\int\right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{3-2i}\sqrt{\tan(c+dx)}}{\sqrt{2-3\tan(c+dx)}}\right)}{\sqrt{3-2i}\,d} + \frac{\tan^{-1}\left(\frac{\sqrt{3+2i}\sqrt{\tan(c}{\sqrt{2-3\tan(c+}\right.}{\sqrt{3+2i}\,d}$$

Mathematica [A]
time = 0.18, size = 101, normalized size = 1.13

$$\frac{-\sqrt{3+2i}\,\text{ArcTan}\left(\frac{\sqrt{\frac{3}{13}+\frac{2i}{13}}\sqrt{2-3\tan(c+dx)}}{\sqrt{\tan(c+dx)}}\right)+\sqrt{-3+2i}\,\tanh^{-1}\left(\frac{\sqrt{-\frac{3}{13}+\frac{2i}{13}}\sqrt{2-3\tan(c+dx)}}{\sqrt{\tan(c+dx)}}\right)}{\sqrt{13}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*Tan[c + d*x]]*Sqrt[Tan[c + d*x]]),x]
```

```
[Out] (-(Sqrt[3 + 2*I]*ArcTan[(Sqrt[3/13 + (2*I)/13]*Sqrt[2 - 3*Tan[c + d*x]])/Sq
rt[Tan[c + d*x]]]) + Sqrt[-3 + 2*I]*ArcTanh[(Sqrt[-3/13 + (2*I)/13]*Sqrt[2
- 3*Tan[c + d*x]])/Sqrt[Tan[c + d*x]]])/(Sqrt[13]*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(73) = 146$.
time = 0.68, size = 435, normalized size = 4.89

method	result
derivativedivides	$\frac{\sqrt{2-3\tan(dx+c)}\,\sqrt{-\frac{\tan(dx+c)(-2+3\tan(dx+c))}{\left(\sqrt{13}-3-2\tan(dx+c)\right)^2}}\,\left(\sqrt{13}-3-2\tan(dx+c)\right)\left(3\sqrt{13}\,\sqrt{2\sqrt{13}\right.}{}$
default	$\frac{\sqrt{2-3\tan(dx+c)}\,\sqrt{-\frac{\tan(dx+c)(-2+3\tan(dx+c))}{\left(\sqrt{13}-3-2\tan(dx+c)\right)^2}}\,\left(\sqrt{13}-3-2\tan(dx+c)\right)\left(3\sqrt{13}\,\sqrt{2\sqrt{13}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2-3*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(2-3*tan(d*x+c))^(1/2)*(-tan(d*x+c)*(-2+3*tan(d*x+c))/(13^(1/2)-3-2*t
an(d*x+c))^2)^(1/2)*(13^(1/2)-3-2*tan(d*x+c))*(3*13^(1/2)*(2*13^(1/2)+6)^(1
/2)*arctanh(1/52*(13^(1/2)+3)*(13^(1/2)+3+2*tan(d*x+c))*(11*13^(1/2)-39)/(1
3^(1/2)-3-2*tan(d*x+c))/(2*13^(1/2)-6)^(1/2)*13^(1/2)/(-tan(d*x+c)*(-2+3*ta
n(d*x+c))/(13^(1/2)-3-2*tan(d*x+c))^2)^(1/2))*(2*13^(1/2)-6)^(1/2)-11*(2*13
^(1/2)+6)^(1/2)*arctanh(1/52*(13^(1/2)+3)*(13^(1/2)+3+2*tan(d*x+c))*(11*13^
(1/2)-39)/(13^(1/2)-3-2*tan(d*x+c))/(2*13^(1/2)-6)^(1/2)*13^(1/2)/(-tan(d*x
+c)*(-2+3*tan(d*x+c))/(13^(1/2)-3-2*tan(d*x+c))^2)^(1/2))*(2*13^(1/2)-6)^(1
/2)-4*arctan(4*13^(1/2)*(-tan(d*x+c)*(-2+3*tan(d*x+c))/(13^(1/2)-3-2*tan(d*
x+c))^2)^(1/2)/(26*13^(1/2)+78)^(1/2))*13^(1/2)+12*arctan(4*13^(1/2)*(-tan(
d*x+c)*(-2+3*tan(d*x+c))/(13^(1/2)-3-2*tan(d*x+c))^2)^(1/2)/(26*13^(1/2)+78
)^(1/2)))/tan(d*x+c)^(1/2)/(-2+3*tan(d*x+c))/(2*13^(1/2)+6)^(1/2)/(11*13^(1
/2)-39)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2 - 3\tan{(c + dx)}}\,\sqrt{\tan{(c + dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*tan(d*x+c))**(1/2)/tan(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2 - 3*tan(c + d*x))*sqrt(tan(c + d*x))), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1061 vs. $2(65) = 130$.
time = 0.69, size = 1061, normalized size = 11.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/105456*(2*sqrt(3)*(9/13)^(3/4)*d^2*(78*sqrt(13) + 338)^(3/2) + 156*sqrt(
3)*(9/13)^(3/4)*d^2*sqrt(78*sqrt(13) + 338)*(3*sqrt(13) - 13) - 234*sqrt(3)
*(9/13)^(3/4)*d*(3*sqrt(13) + 13)*sqrt(-78*sqrt(13) + 338)*abs(d) + 3*sqrt(
3)*(9/13)^(3/4)*d*(-78*sqrt(13) + 338)^(3/2)*abs(d) - 1352*sqrt(3)*(9/13)^(
1/4)*d^2*sqrt(78*sqrt(13) + 338) + 2028*sqrt(3)*(9/13)^(1/4)*d*sqrt(-78*sqr
t(13) + 338)*abs(d))*arctan(13/18*(9/13)^(3/4)*(2*(9/13)^(1/4)*sqrt(-3/26*s
qrt(13) + 1/2) + (sqrt(3)*sqrt(tan(d*x + c)) - sqrt(2))/sqrt(-3*tan(d*x + c
) + 2) - sqrt(-3*tan(d*x + c) + 2)/(sqrt(3)*sqrt(tan(d*x + c)) - sqrt(2)))/
sqrt(3/26*sqrt(13) + 1/2))/d^3 - 1/105456*(2*sqrt(3)*(9/13)^(3/4)*d^2*(78*s
qrt(13) + 338)^(3/2) + 156*sqrt(3)*(9/13)^(3/4)*d^2*sqrt(78*sqrt(13) + 338)
```

```
*(3*sqrt(13) - 13) - 234*sqrt(3)*(9/13)^(3/4)*d*(3*sqrt(13) + 13)*sqrt(-78*
sqrt(13) + 338)*abs(d) + 3*sqrt(3)*(9/13)^(3/4)*d*(-78*sqrt(13) + 338)^(3/2
)*abs(d) - 1352*sqrt(3)*(9/13)^(1/4)*d^2*sqrt(78*sqrt(13) + 338) + 2028*sqr
t(3)*(9/13)^(1/4)*d*sqrt(-78*sqrt(13) + 338)*abs(d))*arctan(-13/18*(9/13)^(
3/4)*(2*(9/13)^(1/4)*sqrt(-3/26*sqrt(13) + 1/2) - (sqrt(3)*sqrt(tan(d*x + c
)) - sqrt(2))/sqrt(-3*tan(d*x + c) + 2) + sqrt(-3*tan(d*x + c) + 2)/(sqrt(3
)*sqrt(tan(d*x + c)) - sqrt(2)))/sqrt(3/26*sqrt(13) + 1/2))/d^3 - 1/210912*
(156*sqrt(3)*(9/13)^(3/4)*d^2*(3*sqrt(13) + 13)*sqrt(-78*sqrt(13) + 338) -
2*sqrt(3)*(9/13)^(3/4)*d^2*(-78*sqrt(13) + 338)^(3/2) + 3*sqrt(3)*(9/13)^(3
/4)*d*(78*sqrt(13) + 338)^(3/2)*abs(d) + 234*sqrt(3)*(9/13)^(3/4)*d*sqrt(78
*sqrt(13) + 338)*(3*sqrt(13) - 13)*abs(d) - 1352*sqrt(3)*(9/13)^(1/4)*d^2*s
qrt(-78*sqrt(13) + 338) - 2028*sqrt(3)*(9/13)^(1/4)*d*sqrt(78*sqrt(13) + 33
8)*abs(d))*log(((sqrt(3)*sqrt(tan(d*x + c)) - sqrt(2))/sqrt(-3*tan(d*x + c)
 + 2) - sqrt(-3*tan(d*x + c) + 2)/(sqrt(3)*sqrt(tan(d*x + c)) - sqrt(2)))^2
 + 4*(9/13)^(1/4)*sqrt(-3/26*sqrt(13) + 1/2)*((sqrt(3)*sqrt(tan(d*x + c)) -
 sqrt(2))/sqrt(-3*tan(d*x + c) + 2) - sqrt(-3*tan(d*x + c) + 2)/(sqrt(3)*sq
rt(tan(d*x + c)) - sqrt(2))) + 12*sqrt(1/13))/d^3 + 1/210912*(156*sqrt(3)*(
9/13)^(3/4)*d^2*(3*sqrt(13) + 13)*sqrt(-78*sqrt(13) + 338) - 2*sqrt(3)*(9/1
3)^(3/4)*d^2*(-78*sqrt(13) + 338)^(3/2) + 3*sqrt(3)*(9/13)^(3/4)*d*(78*sqrt
(13) + 338)^(3/2)*abs(d) + 234*sqrt(3)*(9/13)^(3/4)*d*sqrt(78*sqrt(13) + 33
8)*(3*sqrt(13) - 13)*abs(d) - 1352*sqrt(3)*(9/13)^(1/4)*d^2*sqrt(-78*sqrt(1
3) + 338) - 2028*sqrt(3)*(9/13)^(1/4)*d*sqrt(78*sqrt(13) + 338)*abs(d))*log
(((sqrt(3)*sqrt(tan(d*x + c)) - sqrt(2))/sqrt(-3*tan(d*x + c) + 2) - sqrt(-
3*tan(d*x + c) + 2)/(sqrt(3)*sqrt(tan(d*x + c)) - sqrt(2)))^2 - 4*(9/13)^(1
/4)*sqrt(-3/26*sqrt(13) + 1/2)*((sqrt(3)*sqrt(tan(d*x + c)) - sqrt(2))/sqrt
(-3*tan(d*x + c) + 2) - sqrt(-3*tan(d*x + c) + 2)/(sqrt(3)*sqrt(tan(d*x + c
)) - sqrt(2))) + 12*sqrt(1/13))/d^3
```

Mupad [B]
time = 6.33, size = 205, normalized size = 2.30

$$\mathrm{atan}\left(\frac{\sqrt{2}\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{-\frac{3}{52}-\frac{1}{26}\mathrm{i}}{d^2}}\,\left(4+6\mathrm{i}\right)+d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{-\frac{3}{52}-\frac{1}{26}\mathrm{i}}{d^2}}\,\sqrt{2-3\tan\left(c+d\,x\right)}\,\left(-4-6\mathrm{i}\right)}{3\tan\left(c+d\,x\right)+\sqrt{2}\,\sqrt{2-3\tan\left(c+d\,x\right)}-2}\right)\sqrt{\frac{-\frac{3}{52}-\frac{1}{26}\mathrm{i}}{d^2}}\,2\mathrm{i}-\mathrm{atan}\left(\frac{\sqrt{2}\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{-\frac{3}{52}+\frac{1}{26}\mathrm{i}}{d^2}}\,\left(4-6\mathrm{i}\right)+d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{-\frac{3}{52}+\frac{1}{26}\mathrm{i}}{d^2}}\,\sqrt{2-3\tan\left(c+d\,x\right)}\,\left(-4+6\mathrm{i}\right)}{3\tan\left(c+d\,x\right)+\sqrt{2}\,\sqrt{2-3\tan\left(c+d\,x\right)}-2}\right)\sqrt{\frac{-\frac{3}{52}+\frac{1}{26}\mathrm{i}}{d^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(1/2)*(2 - 3*tan(c + d*x))^(1/2)),x)
```

```
[Out] atan((2^(1/2)*d*tan(c + d*x)^(1/2)*((- 3/52 - 1i/26)/d^2)^(1/2)*(4 + 6i) -
d*tan(c + d*x)^(1/2)*((- 3/52 - 1i/26)/d^2)^(1/2)*(2 - 3*tan(c + d*x))^(1/2
)*(4 + 6i))/(3*tan(c + d*x) + 2^(1/2)*(2 - 3*tan(c + d*x))^(1/2) - 2))*((-
3/52 - 1i/26)/d^2)^(1/2)*2i - atan((2^(1/2)*d*tan(c + d*x)^(1/2)*((- 3/52 +
 1i/26)/d^2)^(1/2)*(4 - 6i) - d*tan(c + d*x)^(1/2)*((- 3/52 + 1i/26)/d^2)^(
1/2)*(2 - 3*tan(c + d*x))^(1/2)*(4 - 6i))/(3*tan(c + d*x) + 2^(1/2)*(2 - 3*
tan(c + d*x))^(1/2) - 2))*((- 3/52 + 1i/26)/d^2)^(1/2)*2i
```

3.658 $\int \frac{1}{\sqrt{-2-3\tan(c+dx)}\sqrt{\tan(c+dx)}}\,dx$

Optimal. Leaf size=89

$$\frac{\text{ArcTan}\left(\frac{\sqrt{3-2i}\sqrt{\tan(c+dx)}}{\sqrt{-2-3\tan(c+dx)}}\right)}{\sqrt{3-2i}\,d}+\frac{\text{ArcTan}\left(\frac{\sqrt{3+2i}\sqrt{\tan(c+dx)}}{\sqrt{-2-3\tan(c+dx)}}\right)}{\sqrt{3+2i}\,d}$$

```
[Out] arctan((3-2*I)^(1/2)*tan(d*x+c)^(1/2)/(-2-3*tan(d*x+c))^(1/2))/d/(3-2*I)^(1
/2)+arctan((3+2*I)^(1/2)*tan(d*x+c)^(1/2)/(-2-3*tan(d*x+c))^(1/2))/d/(3+2*I
)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3656, 926, 95, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{3-2i}\sqrt{\tan(c+dx)}}{\sqrt{-3\tan(c+dx)-2}}\right)}{\sqrt{3-2i}\,d}+\frac{\text{ArcTan}\left(\frac{\sqrt{3+2i}\sqrt{\tan(c+dx)}}{\sqrt{-3\tan(c+dx)-2}}\right)}{\sqrt{3+2i}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-2 - 3*Tan[c + d*x]]*Sqrt[Tan[c + d*x]]),x]
```

```
[Out] ArcTan[(Sqrt[3 - 2*I]*Sqrt[Tan[c + d*x]])/Sqrt[-2 - 3*Tan[c + d*x]]]/(Sqrt[
3 - 2*I]*d) + ArcTan[(Sqrt[3 + 2*I]*Sqrt[Tan[c + d*x]])/Sqrt[-2 - 3*Tan[c +
 d*x]]]/(Sqrt[3 + 2*I]*d)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
```

```
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-3\tan(c+dx)}\sqrt{\tan(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{-2-3x}\sqrt{x}(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{i}{2\sqrt{-2-3x}(i-x)\sqrt{x}} + \frac{i}{2\sqrt{-2-3x}\sqrt{x}(i+x)}\right)\, c\right.}{d}$$

$$= \frac{i\text{Subst}\left(\int \frac{1}{\sqrt{-2-3x}(i-x)\sqrt{x}}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{i\text{Subs}}{}$$

$$= \frac{i\text{Subst}\left(\int \frac{1}{i-(2-3i)x^2}\,dx, x, \frac{\sqrt{\tan(c+dx)}}{\sqrt{-2-3\tan(c+dx)}}\right)}{d} + \frac{i\text{Subs}}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{3-2i}\sqrt{\tan(c+dx)}}{\sqrt{-2-3\tan(c+dx)}}\right)}{\sqrt{3-2i}\,d} + \frac{\tan^{-1}\left(\frac{\sqrt{3+2i}\sqrt{\tan}}{\sqrt{-2-3\tan}}\right.}{\sqrt{3+2i}\,d}$$

Mathematica [A]
time = 0.15, size = 101, normalized size = 1.13

$$\frac{-\sqrt{3+2i}\,\text{ArcTan}\left(\frac{\sqrt{\frac{3}{13}+\frac{2i}{13}}\sqrt{-2-3\tan(c+dx)}}{\sqrt{\tan(c+dx)}}\right)+\sqrt{-3+2i}\,\tanh^{-1}\left(\frac{\sqrt{-\frac{3}{13}+\frac{2i}{13}}\sqrt{-2-3\tan(c+dx)}}{\sqrt{\tan(c+dx)}}\right)}{\sqrt{13}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-2 - 3*Tan[c + d*x]]*Sqrt[Tan[c + d*x]]),x]
```

```
[Out] (-(Sqrt[3 + 2*I]*ArcTan[(Sqrt[3/13 + (2*I)/13]*Sqrt[-2 - 3*Tan[c + d*x]])/S
qrt[Tan[c + d*x]]]) + Sqrt[-3 + 2*I]*ArcTanh[(Sqrt[-3/13 + (2*I)/13]*Sqrt[-
2 - 3*Tan[c + d*x]])/Sqrt[Tan[c + d*x]]])/(Sqrt[13]*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(73) = 146$.
time = 0.72, size = 435, normalized size = 4.89

method	result
derivativedivides	$-\frac{\sqrt{-2-3\tan(dx+c)}\,\sqrt{-\frac{\tan(dx+c)(2+3\tan(dx+c))}{\left(\sqrt{13}-3+2\tan(dx+c)\right)^2}}\left(\sqrt{13}-3+2\tan(dx+c)\right)\left(3\sqrt{2\sqrt{13}-6}\right.}{}$
default	$-\frac{\sqrt{-2-3\tan(dx+c)}\,\sqrt{-\frac{\tan(dx+c)(2+3\tan(dx+c))}{\left(\sqrt{13}-3+2\tan(dx+c)\right)^2}}\left(\sqrt{13}-3+2\tan(dx+c)\right)\left(3\sqrt{2\sqrt{13}-6}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2-3*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(-2-3*tan(d*x+c))^(1/2)*(-tan(d*x+c)*(2+3*tan(d*x+c))/(13^(1/2)-3+2*
tan(d*x+c))^2)^(1/2)*(13^(1/2)-3+2*tan(d*x+c))*(3*(2*13^(1/2)-6)^(1/2)*13^(
1/2)*(2*13^(1/2)+6)^(1/2)*arctanh(1/52*(13^(1/2)+3)*(13^(1/2)+3-2*tan(d*x+c
))*(11*13^(1/2)-39)/(13^(1/2)-3+2*tan(d*x+c))/(2*13^(1/2)-6)^(1/2)*13^(1/2)
/(-tan(d*x+c)*(2+3*tan(d*x+c))/(13^(1/2)-3+2*tan(d*x+c))^2)^(1/2))-11*(2*13
^(1/2)-6)^(1/2)*(2*13^(1/2)+6)^(1/2)*arctanh(1/52*(13^(1/2)+3)*(13^(1/2)+3-
2*tan(d*x+c))*(11*13^(1/2)-39)/(13^(1/2)-3+2*tan(d*x+c))/(2*13^(1/2)-6)^(1/
2)*13^(1/2)/(-tan(d*x+c)*(2+3*tan(d*x+c))/(13^(1/2)-3+2*tan(d*x+c))^2)^(1/2
))-4*arctan(4*13^(1/2)*(-tan(d*x+c)*(2+3*tan(d*x+c))/(13^(1/2)-3+2*tan(d*x+
c))^2)^(1/2)/(26*13^(1/2)+78)^(1/2))*13^(1/2)+12*arctan(4*13^(1/2)*(-tan(d*
x+c)*(2+3*tan(d*x+c))/(13^(1/2)-3+2*tan(d*x+c))^2)^(1/2)/(26*13^(1/2)+78)^(
1/2)))/tan(d*x+c)^(1/2)/(2+3*tan(d*x+c))/(2*13^(1/2)+6)^(1/2)/(11*13^(1/2)-
39)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\tan{(c+dx)}-2}\,\sqrt{\tan{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*tan(d*x+c))**(1/2)/tan(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-3*tan(c + d*x) - 2)*sqrt(tan(c + d*x))), x)
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(65) = 130.
time = 0.62, size = 269, normalized size = 3.02

$$-\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{13}\left(\sqrt{3}\sqrt{-\tan(dx+c)}-\sqrt{-3\tan(dx+c)-2}\right)^2+3\left(\sqrt{3}\sqrt{-\tan(dx+c)}-\sqrt{-3\tan(dx+c)-2}\right)^2-(6i-2)\sqrt{13}-18i+6}{\sqrt{13}\sqrt{6\sqrt{13}+18}+(2i+3)\sqrt{6\sqrt{13}+18}}\right)}{d\sqrt{6\sqrt{13}+18}\left(\frac{2i}{\sqrt{13}+3}+1\right)}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{13}\left(\sqrt{3}\sqrt{-\tan(dx+c)}-\sqrt{-3\tan(dx+c)-2}\right)^2+3\left(\sqrt{3}\sqrt{-\tan(dx+c)}-\sqrt{-3\tan(dx+c)-2}\right)^2+(6i+2)\sqrt{13}+18i+6}{\sqrt{13}\sqrt{6\sqrt{13}+18}-(2i-3)\sqrt{6\sqrt{13}+18}}\right)}{d\sqrt{6\sqrt{13}+18}\left(-\frac{2i}{\sqrt{13}+3}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(3)*arctan((sqrt(13)*(sqrt(3)*sqrt(-tan(d*x + c)) - sqrt(-3*tan(d*x
+ c) - 2))^2 + 3*(sqrt(3)*sqrt(-tan(d*x + c)) - sqrt(-3*tan(d*x + c) - 2))^
2 - (6*I - 2)*sqrt(13) - 18*I + 6)/(sqrt(13)*sqrt(6*sqrt(13) + 18) + (2*I +
 3)*sqrt(6*sqrt(13) + 18)))/(d*sqrt(6*sqrt(13) + 18)*(2*I/(sqrt(13) + 3) +
1)) + 2*sqrt(3)*arctan((sqrt(13)*(sqrt(3)*sqrt(-tan(d*x + c)) - sqrt(-3*tan
(d*x + c) - 2))^2 + 3*(sqrt(3)*sqrt(-tan(d*x + c)) - sqrt(-3*tan(d*x + c) -
 2))^2 + (6*I + 2)*sqrt(13) + 18*I + 6)/(sqrt(13)*sqrt(6*sqrt(13) + 18) - (
2*I - 3)*sqrt(6*sqrt(13) + 18)))/(d*sqrt(6*sqrt(13) + 18)*(-2*I/(sqrt(13) +
 3) + 1))
```

Mupad [B]
time = 6.08, size = 129, normalized size = 1.45

$$-2\,\mathrm{atanh}\left(\frac{4\,d\,\sqrt{\frac{-\frac{3}{52}-\frac{1}{26}\mathrm{i}}{d^2}}+6\,d\tan(c+d\,x)\,\sqrt{\frac{-\frac{3}{52}-\frac{1}{26}\mathrm{i}}{d^2}}}{\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{-3\tan\left(c+d\,x\right)-2}}\right)\sqrt{\frac{-\frac{3}{52}-\frac{1}{26}\mathrm{i}}{d^2}}-2\,\mathrm{atanh}\left(\frac{4\,d\,\sqrt{\frac{-\frac{3}{52}+\frac{1}{26}\mathrm{i}}{d^2}}+6\,d\tan(c+d\,x)\,\sqrt{\frac{-\frac{3}{52}+\frac{1}{26}\mathrm{i}}{d^2}}}{\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{-3\tan\left(c+d\,x\right)-2}}\right)\sqrt{\frac{-\frac{3}{52}+\frac{1}{26}\mathrm{i}}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(1/2)*(- 3*tan(c + d*x) - 2)^(1/2)),x)
```

```
[Out] - 2*atanh((4*d*((- 3/52 - 1i/26)/d^2)^(1/2) + 6*d*tan(c + d*x)*((- 3/52 - 1
i/26)/d^2)^(1/2))/(tan(c + d*x)^(1/2)*(- 3*tan(c + d*x) - 2)^(1/2)))*((- 3/
52 - 1i/26)/d^2)^(1/2) - 2*atanh((4*d*((- 3/52 + 1i/26)/d^2)^(1/2) + 6*d*ta
n(c + d*x)*((- 3/52 + 1i/26)/d^2)^(1/2))/(tan(c + d*x)^(1/2)*(- 3*tan(c + d
*x) - 2)^(1/2)))*((- 3/52 + 1i/26)/d^2)^(1/2)
```

3.659 $\int \frac{1}{\sqrt{\tan(c+dx)}\sqrt{3+2\tan(c+dx)}}\,dx$

Optimal. Leaf size=89

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2-3i}\sqrt{\tan(c+dx)}}{\sqrt{3+2\tan(c+dx)}}\right)}{\sqrt{2-3i}\,d}+\frac{\tanh^{-1}\left(\frac{\sqrt{2+3i}\sqrt{\tan(c+dx)}}{\sqrt{3+2\tan(c+dx)}}\right)}{\sqrt{2+3i}\,d}$$

```
[Out] arctanh((2-3*I)^(1/2)*tan(d*x+c)^(1/2)/(3+2*tan(d*x+c))^(1/2))/d/(2-3*I)^(1
/2)+arctanh((2+3*I)^(1/2)*tan(d*x+c)^(1/2)/(3+2*tan(d*x+c))^(1/2))/d/(2+3*I
)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3656, 926, 95, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2-3i}\sqrt{\tan(c+dx)}}{\sqrt{2\tan(c+dx)+3}}\right)}{\sqrt{2-3i}\,d}+\frac{\tanh^{-1}\left(\frac{\sqrt{2+3i}\sqrt{\tan(c+dx)}}{\sqrt{2\tan(c+dx)+3}}\right)}{\sqrt{2+3i}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Tan[c + d*x]]*Sqrt[3 + 2*Tan[c + d*x]]),x]
```

```
[Out] ArcTanh[(Sqrt[2 - 3*I]*Sqrt[Tan[c + d*x]])/Sqrt[3 + 2*Tan[c + d*x]]]/(Sqrt[
2 - 3*I]*d) + ArcTanh[(Sqrt[2 + 3*I]*Sqrt[Tan[c + d*x]])/Sqrt[3 + 2*Tan[c +
 d*x]]]/(Sqrt[2 + 3*I]*d)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
```

```
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\tan(c+dx)}\sqrt{3+2\tan(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{3+2x}\,(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{i}{2(i-x)\sqrt{x}\sqrt{3+2x}} + \frac{i}{2\sqrt{x}\,(i+x)\sqrt{3+2x}}\right) dx, x, \text{ta}$$

$$= \frac{i\text{Subst}\left(\int \frac{1}{(i-x)\sqrt{x}\sqrt{3+2x}}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{i\text{Subst}\left(\int \right.}{}$$

$$= \frac{i\text{Subst}\left(\int \frac{1}{i-(3+2i)x^2}\,dx, x, \frac{\sqrt{\tan(c+dx)}}{\sqrt{3+2\tan(c+dx)}}\right)}{d} + \frac{i\text{Subst}\left(\int \right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{2-3i}\sqrt{\tan(c+dx)}}{\sqrt{3+2\tan(c+dx)}}\right)}{\sqrt{2-3i}\,d} + \frac{\tanh^{-1}\left(\frac{\sqrt{2+3i}\sqrt{\tan}}{\sqrt{3+2\tan(c}}\right.}{\sqrt{2+3i}\,d}$$

Mathematica [A]
time = 0.19, size = 89, normalized size = 1.00

$$\frac{\text{ArcTan}\left(\frac{\sqrt{-2+3i}\sqrt{\tan(c+dx)}}{\sqrt{3+2\tan(c+dx)}}\right)}{\sqrt{-2+3i}\,d} + \frac{\tanh^{-1}\left(\frac{\sqrt{2+3i}\sqrt{\tan(c+dx)}}{\sqrt{3+2\tan(c+dx)}}\right)}{\sqrt{2+3i}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Tan[c + d*x]]*Sqrt[3 + 2*Tan[c + d*x]]),x]
[Out] ArcTan[(Sqrt[-2 + 3*I]*Sqrt[Tan[c + d*x]])/Sqrt[3 + 2*Tan[c + d*x]]]/(Sqrt[
-2 + 3*I]*d) + ArcTanh[(Sqrt[2 + 3*I]*Sqrt[Tan[c + d*x]])/Sqrt[3 + 2*Tan[c
+ d*x]]]/(Sqrt[2 + 3*I]*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(73) = 146$.
time = 1.17, size = 480, normalized size = 5.39

method	result
derivativedivides	$\frac{\sqrt{\frac{\tan(dx+c)(3+2\tan(dx+c))}{\left(\sqrt{13}-2+3\tan(dx+c)\right)^2}}\left(\sqrt{13}-2+3\tan(dx+c)\right)\left(4\sqrt{13}\,\sqrt{2\sqrt{13}+4}\,\arctan\left(\frac{\sqrt{\frac{\left(17\sqrt{13}-\right.}{}}}{}\right.\right.}{}$
default	$\frac{\sqrt{\frac{\tan(dx+c)(3+2\tan(dx+c))}{\left(\sqrt{13}-2+3\tan(dx+c)\right)^2}}\left(\sqrt{13}-2+3\tan(dx+c)\right)\left(4\sqrt{13}\,\sqrt{2\sqrt{13}+4}\,\arctan\left(\frac{\sqrt{\frac{\left(17\sqrt{13}-\right.}{}}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(1/2)/(3+2*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(tan(d*x+c)*(3+2*tan(d*x+c))/(13^(1/2)-2+3*tan(d*x+c))^2)^(1/2)*(13^(
1/2)-2+3*tan(d*x+c))*(4*13^(1/2)*(2*13^(1/2)+4)^(1/2)*arctan(1/56862*((17*1
3^(1/2)-52)*tan(d*x+c)*(3+2*tan(d*x+c))*(52+17*13^(1/2))/(13^(1/2)-2+3*tan(
d*x+c))^2)^(1/2)*(-4+2*13^(1/2))^(1/2)*(4*13^(1/2)+17)*(13^(1/2)+2-3*tan(d*
x+c))*(17*13^(1/2)-52)*(13^(1/2)-2+3*tan(d*x+c))/tan(d*x+c)/(3+2*tan(d*x+c)
))*(-4+2*13^(1/2))^(1/2)-17*(2*13^(1/2)+4)^(1/2)*arctan(1/56862*((17*13^(1/
2)-52)*tan(d*x+c)*(3+2*tan(d*x+c))*(52+17*13^(1/2))/(13^(1/2)-2+3*tan(d*x+c
))^2)^(1/2)*(-4+2*13^(1/2))^(1/2)*(4*13^(1/2)+17)*(13^(1/2)+2-3*tan(d*x+c))
*(17*13^(1/2)-52)*(13^(1/2)-2+3*tan(d*x+c))/tan(d*x+c)/(3+2*tan(d*x+c)))*(-
4+2*13^(1/2))^(1/2)+18*arctanh(6*13^(1/2)*(tan(d*x+c)*(3+2*tan(d*x+c))/(13^
(1/2)-2+3*tan(d*x+c))^2)^(1/2)/(26*13^(1/2)+52)^(1/2))*13^(1/2)-36*arctanh(
6*13^(1/2)*(tan(d*x+c)*(3+2*tan(d*x+c))/(13^(1/2)-2+3*tan(d*x+c))^2)^(1/2)/
(26*13^(1/2)+52)^(1/2)))/tan(d*x+c)^(1/2)/(3+2*tan(d*x+c))^(1/2)/(2*13^(1/2
)+4)^(1/2)/(17*13^(1/2)-52)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/2)/(3+2*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/2)/(3+2*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\tan(c+dx)+3}\,\sqrt{\tan(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(1/2)/(3+2*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2*tan(c + d*x) + 3)*sqrt(tan(c + d*x))), x)
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 479 vs. 2(65) = 130.
time = 0.53, size = 479, normalized size = 5.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/2)/(3+2*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/52*sqrt(2)*(sqrt(sqrt(13) - 2)*((9*I - 6)/(sqrt(13) - 2) - 2*I - 3)*log((
915*I + 1098)*sqrt(13)*(sqrt(2)*sqrt(tan(d*x + c)) - sqrt(2*tan(d*x + c) +
3))^2 + (2370*I + 2844)*(sqrt(2)*sqrt(tan(d*x + c)) - sqrt(2*tan(d*x + c) +
 3))^2 + 366*sqrt(13)*sqrt(61*sqrt(13) + 158) + (1647*I - 6954)*sqrt(13) -
(918*I - 948)*sqrt(61*sqrt(13) + 158) + 4266*I - 18012) - sqrt(sqrt(13) - 2
)*((9*I - 6)/(sqrt(13) - 2) - 2*I - 3)*log((915*I + 1098)*sqrt(13)*(sqrt(2)
*sqrt(tan(d*x + c)) - sqrt(2*tan(d*x + c) + 3))^2 + (2370*I + 2844)*(sqrt(2
)*sqrt(tan(d*x + c)) - sqrt(2*tan(d*x + c) + 3))^2 - 366*sqrt(13)*sqrt(61*s
qrt(13) + 158) + (1647*I - 6954)*sqrt(13) + (918*I - 948)*sqrt(61*sqrt(13)
+ 158) + 4266*I - 18012) - sqrt(sqrt(13) + 2)*((6*I - 9)/(sqrt(13) + 2) - 3
*I - 2)*log((90*I + 45)*sqrt(13)*(sqrt(2)*sqrt(tan(d*x + c)) - sqrt(2*tan(d
*x + c) + 3))^2 - (108*I + 54)*(sqrt(2)*sqrt(tan(d*x + c)) - sqrt(2*tan(d*x
```

```
 + c) + 3))^2 + 90*sqrt(13)*sqrt(5*sqrt(13) - 6) - (450*I - 225)*sqrt(13) -
 (306*I + 108)*sqrt(5*sqrt(13) - 6) + 540*I - 270) + sqrt(sqrt(13) + 2)*((6
*I - 9)/(sqrt(13) + 2) - 3*I - 2)*log((90*I + 45)*sqrt(13)*(sqrt(2)*sqrt(ta
n(d*x + c)) - sqrt(2*tan(d*x + c) + 3))^2 - (108*I + 54)*(sqrt(2)*sqrt(tan(
d*x + c)) - sqrt(2*tan(d*x + c) + 3))^2 - 90*sqrt(13)*sqrt(5*sqrt(13) - 6)
- (450*I - 225)*sqrt(13) + (306*I + 108)*sqrt(5*sqrt(13) - 6) + 540*I - 270
))/d
```

Mupad [B]
time = 6.44, size = 204, normalized size = 2.29

$$\mathrm{atan}\left(\frac{\sqrt{3}\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{\frac{1}{26}+\frac{3}{52}\mathrm{i}}{d^2}}\,\left(6+4\mathrm{i}\right)+d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{\frac{1}{26}+\frac{3}{52}\mathrm{i}}{d^2}}\,\sqrt{2\tan\left(c+d\,x\right)+3}\,\left(-6-4\mathrm{i}\right)}{2\tan\left(c+d\,x\right)-\sqrt{3}\,\sqrt{2\tan\left(c+d\,x\right)+3}+3}\right)\sqrt{\frac{\frac{1}{26}+\frac{3}{52}\mathrm{i}}{d^2}}\,2\mathrm{i}-\mathrm{atan}\left(\frac{\sqrt{3}\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{\frac{1}{26}-\frac{3}{52}\mathrm{i}}{d^2}}\,\left(6-4\mathrm{i}\right)+d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{\frac{1}{26}-\frac{3}{52}\mathrm{i}}{d^2}}\,\sqrt{2\tan\left(c+d\,x\right)+3}\,\left(-6+4\mathrm{i}\right)}{2\tan\left(c+d\,x\right)-\sqrt{6\tan\left(c+d\,x\right)+9}+3}\right)\sqrt{\frac{\frac{1}{26}-\frac{3}{52}\mathrm{i}}{d^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(1/2)*(2*tan(c + d*x) + 3)^(1/2)),x)
```

```
[Out] atan((3^(1/2)*d*tan(c + d*x)^(1/2)*((1/26 + 3i/52)/d^2)^(1/2)*(6 + 4i) - d*
tan(c + d*x)^(1/2)*((1/26 + 3i/52)/d^2)^(1/2)*(2*tan(c + d*x) + 3)^(1/2)*(6
 + 4i))/(2*tan(c + d*x) - 3^(1/2)*(2*tan(c + d*x) + 3)^(1/2) + 3))*((1/26 +
 3i/52)/d^2)^(1/2)*2i - atan((3^(1/2)*d*tan(c + d*x)^(1/2)*((1/26 - 3i/52)/
d^2)^(1/2)*(6 - 4i) - d*tan(c + d*x)^(1/2)*((1/26 - 3i/52)/d^2)^(1/2)*(2*ta
n(c + d*x) + 3)^(1/2)*(6 - 4i))/(2*tan(c + d*x) - (6*tan(c + d*x) + 9)^(1/2
) + 3))*((1/26 - 3i/52)/d^2)^(1/2)*2i
```

3.660 $\int \frac{1}{\sqrt{3-2\tan(c+dx)}\sqrt{\tan(c+dx)}}\,dx$

Optimal. Leaf size=89

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2-3i}\sqrt{\tan(c+dx)}}{\sqrt{3-2\tan(c+dx)}}\right)}{\sqrt{2-3i}\,d}+\frac{\text{ArcTan}\left(\frac{\sqrt{2+3i}\sqrt{\tan(c+dx)}}{\sqrt{3-2\tan(c+dx)}}\right)}{\sqrt{2+3i}\,d}$$

```
[Out] arctan((2-3*I)^(1/2)*tan(d*x+c)^(1/2)/(3-2*tan(d*x+c))^(1/2))/d/(2-3*I)^(1/
2)+arctan((2+3*I)^(1/2)*tan(d*x+c)^(1/2)/(3-2*tan(d*x+c))^(1/2))/d/(2+3*I)^
(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3656, 926, 95, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2-3i}\sqrt{\tan(c+dx)}}{\sqrt{3-2\tan(c+dx)}}\right)}{\sqrt{2-3i}\,d}+\frac{\text{ArcTan}\left(\frac{\sqrt{2+3i}\sqrt{\tan(c+dx)}}{\sqrt{3-2\tan(c+dx)}}\right)}{\sqrt{2+3i}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - 2*Tan[c + d*x]]*Sqrt[Tan[c + d*x]]),x]
```

```
[Out] ArcTan[(Sqrt[2 - 3*I]*Sqrt[Tan[c + d*x]])/Sqrt[3 - 2*Tan[c + d*x]]]/(Sqrt[2
 - 3*I]*d) + ArcTan[(Sqrt[2 + 3*I]*Sqrt[Tan[c + d*x]])/Sqrt[3 - 2*Tan[c + d
*x]]]/(Sqrt[2 + 3*I]*d)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
```

```
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
& !IntegerQ[m] && !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-2\tan(c+dx)}\sqrt{\tan(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{3-2x}\sqrt{x}\,(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{i}{2\sqrt{3-2x}\,(i-x)\sqrt{x}} + \frac{i}{2\sqrt{3-2x}\sqrt{x}\,(i+x)}\right) dx, x, \text{t}\right.}{d}$$

$$= \frac{i\text{Subst}\left(\int \frac{1}{\sqrt{3-2x}\,(i-x)\sqrt{x}}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{i\text{Subst}\left(\int\right.}{}$$

$$= \frac{i\text{Subst}\left(\int \frac{1}{i-(3-2i)x^2}\,dx, x, \frac{\sqrt{\tan(c+dx)}}{\sqrt{3-2\tan(c+dx)}}\right)}{d} + \frac{i\text{Subst}\left(\int\right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{2-3i}\sqrt{\tan(c+dx)}}{\sqrt{3-2\tan(c+dx)}}\right)}{\sqrt{2-3i}\,d} + \frac{\tan^{-1}\left(\frac{\sqrt{2+3i}\sqrt{\tan(\right.}{\sqrt{3-2\tan(c}}}{\sqrt{2+3i}\,d}$$

Mathematica [A]
time = 0.18, size = 101, normalized size = 1.13

$$\frac{-\sqrt{2+3i}\,\text{ArcTan}\left(\frac{\sqrt{\frac{2}{13}+\frac{3i}{13}}\sqrt{3-2\tan(c+dx)}}{\sqrt{\tan(c+dx)}}\right) + \sqrt{-2+3i}\,\tanh^{-1}\left(\frac{\sqrt{-\frac{2}{13}+\frac{3i}{13}}\sqrt{3-2\tan(c+dx)}}{\sqrt{\tan(c+dx)}}\right)}{\sqrt{13}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - 2*Tan[c + d*x]]*Sqrt[Tan[c + d*x]]),x]
```

```
[Out] (-(Sqrt[2 + 3*I]*ArcTan[(Sqrt[2/13 + (3*I)/13]*Sqrt[3 - 2*Tan[c + d*x]])/Sq
rt[Tan[c + d*x]]]) + Sqrt[-2 + 3*I]*ArcTanh[(Sqrt[-2/13 + (3*I)/13]*Sqrt[3
- 2*Tan[c + d*x]])/Sqrt[Tan[c + d*x]]])/(Sqrt[13]*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(73) = 146$.
time = 0.82, size = 435, normalized size = 4.89

method	result
derivativedivides	$\frac{\sqrt{3-2\tan(dx+c)}\,\sqrt{-\frac{\tan(dx+c)(-3+2\tan(dx+c))}{\left(\sqrt{13}-2-3\tan(dx+c)\right)^2}}\left(\sqrt{13}-2-3\tan(dx+c)\right)\left(4\sqrt{-4+2\sqrt{13}}\right.}{}$
default	$\frac{\sqrt{3-2\tan(dx+c)}\,\sqrt{-\frac{\tan(dx+c)(-3+2\tan(dx+c))}{\left(\sqrt{13}-2-3\tan(dx+c)\right)^2}}\left(\sqrt{13}-2-3\tan(dx+c)\right)\left(4\sqrt{-4+2\sqrt{13}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-2*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(3-2*tan(d*x+c))^(1/2)*(-tan(d*x+c)*(-3+2*tan(d*x+c))/(13^(1/2)-2-3*t
an(d*x+c))^2)^(1/2)*(13^(1/2)-2-3*tan(d*x+c))*(4*(-4+2*13^(1/2))^(1/2)*13^(
1/2)*(2*13^(1/2)+4)^(1/2)*arctanh(1/351*(2+13^(1/2))*(13^(1/2)+2+3*tan(d*x+
c))*(17*13^(1/2)-52)/(-4+2*13^(1/2))^(1/2)/(13^(1/2)-2-3*tan(d*x+c))*13^(1/
2)/(-tan(d*x+c)*(-3+2*tan(d*x+c))/(13^(1/2)-2-3*tan(d*x+c))^2)^(1/2))-17*(-
4+2*13^(1/2))^(1/2)*(2*13^(1/2)+4)^(1/2)*arctanh(1/351*(2+13^(1/2))*(13^(1/
2)+2+3*tan(d*x+c))*(17*13^(1/2)-52)/(-4+2*13^(1/2))^(1/2)/(13^(1/2)-2-3*tan
(d*x+c))*13^(1/2)/(-tan(d*x+c)*(-3+2*tan(d*x+c))/(13^(1/2)-2-3*tan(d*x+c))^
2)^(1/2))-18*arctan(6*13^(1/2)*(-tan(d*x+c)*(-3+2*tan(d*x+c))/(13^(1/2)-2-3
*tan(d*x+c))^2)^(1/2)/(26*13^(1/2)+52)^(1/2))*13^(1/2)+36*arctan(6*13^(1/2)
*(-tan(d*x+c)*(-3+2*tan(d*x+c))/(13^(1/2)-2-3*tan(d*x+c))^2)^(1/2)/(26*13^(
1/2)+52)^(1/2)))/tan(d*x+c)^(1/2)/(2*13^(1/2)+4)^(1/2)/(-3+2*tan(d*x+c))/(1
7*13^(1/2)-52)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3-2\tan{(c+dx)}}\,\sqrt{\tan{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*tan(d*x+c))**(1/2)/tan(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(3 - 2*tan(c + d*x))*sqrt(tan(c + d*x))), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1061 vs. 2(65) = 130.
time = 0.69, size = 1061, normalized size = 11.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/105456*(3*sqrt(2)*(4/13)^(3/4)*d^2*(52*sqrt(13) + 338)^(3/2) + 234*sqrt(
2)*(4/13)^(3/4)*d^2*sqrt(52*sqrt(13) + 338)*(2*sqrt(13) - 13) - 156*sqrt(2)
*(4/13)^(3/4)*d*(2*sqrt(13) + 13)*sqrt(-52*sqrt(13) + 338)*abs(d) + 2*sqrt(
2)*(4/13)^(3/4)*d*(-52*sqrt(13) + 338)^(3/2)*abs(d) - 2028*sqrt(2)*(4/13)^(
1/4)*d^2*sqrt(52*sqrt(13) + 338) + 1352*sqrt(2)*(4/13)^(1/4)*d*sqrt(-52*sqr
t(13) + 338)*abs(d))*arctan(13/8*(4/13)^(3/4)*(2*(4/13)^(1/4)*sqrt(-1/13*sq
rt(13) + 1/2) + (sqrt(2)*sqrt(tan(d*x + c)) - sqrt(3))/sqrt(-2*tan(d*x + c)
 + 3) - sqrt(-2*tan(d*x + c) + 3)/(sqrt(2)*sqrt(tan(d*x + c)) - sqrt(3)))/s
qrt(1/13*sqrt(13) + 1/2))/d^3 - 1/105456*(3*sqrt(2)*(4/13)^(3/4)*d^2*(52*sq
rt(13) + 338)^(3/2) + 234*sqrt(2)*(4/13)^(3/4)*d^2*sqrt(52*sqrt(13) + 338)*
```

```
(2*sqrt(13) - 13) - 156*sqrt(2)*(4/13)^(3/4)*d*(2*sqrt(13) + 13)*sqrt(-52*s
qrt(13) + 338)*abs(d) + 2*sqrt(2)*(4/13)^(3/4)*d*(-52*sqrt(13) + 338)^(3/2)
*abs(d) - 2028*sqrt(2)*(4/13)^(1/4)*d^2*sqrt(52*sqrt(13) + 338) + 1352*sqrt
(2)*(4/13)^(1/4)*d*sqrt(-52*sqrt(13) + 338)*abs(d))*arctan(-13/8*(4/13)^(3/
4)*(2*(4/13)^(1/4)*sqrt(-1/13*sqrt(13) + 1/2) - (sqrt(2)*sqrt(tan(d*x + c))
 - sqrt(3))/sqrt(-2*tan(d*x + c) + 3) + sqrt(-2*tan(d*x + c) + 3)/(sqrt(2)*
sqrt(tan(d*x + c)) - sqrt(3)))/sqrt(1/13*sqrt(13) + 1/2))/d^3 - 1/210912*(2
34*sqrt(2)*(4/13)^(3/4)*d^2*(2*sqrt(13) + 13)*sqrt(-52*sqrt(13) + 338) - 3*
sqrt(2)*(4/13)^(3/4)*d^2*(-52*sqrt(13) + 338)^(3/2) + 2*sqrt(2)*(4/13)^(3/4
)*d*(52*sqrt(13) + 338)^(3/2)*abs(d) + 156*sqrt(2)*(4/13)^(3/4)*d*sqrt(52*s
qrt(13) + 338)*(2*sqrt(13) - 13)*abs(d) - 2028*sqrt(2)*(4/13)^(1/4)*d^2*sqr
t(-52*sqrt(13) + 338) - 1352*sqrt(2)*(4/13)^(1/4)*d*sqrt(52*sqrt(13) + 338)
*abs(d))*log(((sqrt(2)*sqrt(tan(d*x + c)) - sqrt(3))/sqrt(-2*tan(d*x + c) +
 3) - sqrt(-2*tan(d*x + c) + 3)/(sqrt(2)*sqrt(tan(d*x + c)) - sqrt(3)))^2 +
 4*(4/13)^(1/4)*sqrt(-1/13*sqrt(13) + 1/2)*((sqrt(2)*sqrt(tan(d*x + c)) - s
qrt(3))/sqrt(-2*tan(d*x + c) + 3) - sqrt(-2*tan(d*x + c) + 3)/(sqrt(2)*sqrt
(tan(d*x + c)) - sqrt(3))) + 8*sqrt(1/13))/d^3 + 1/210912*(234*sqrt(2)*(4/1
3)^(3/4)*d^2*(2*sqrt(13) + 13)*sqrt(-52*sqrt(13) + 338) - 3*sqrt(2)*(4/13)^
(3/4)*d^2*(-52*sqrt(13) + 338)^(3/2) + 2*sqrt(2)*(4/13)^(3/4)*d*(52*sqrt(13
) + 338)^(3/2)*abs(d) + 156*sqrt(2)*(4/13)^(3/4)*d*sqrt(52*sqrt(13) + 338)*
(2*sqrt(13) - 13)*abs(d) - 2028*sqrt(2)*(4/13)^(1/4)*d^2*sqrt(-52*sqrt(13)
+ 338) - 1352*sqrt(2)*(4/13)^(1/4)*d*sqrt(52*sqrt(13) + 338)*abs(d))*log(((
sqrt(2)*sqrt(tan(d*x + c)) - sqrt(3))/sqrt(-2*tan(d*x + c) + 3) - sqrt(-2*t
an(d*x + c) + 3)/(sqrt(2)*sqrt(tan(d*x + c)) - sqrt(3)))^2 - 4*(4/13)^(1/4)
*sqrt(-1/13*sqrt(13) + 1/2)*((sqrt(2)*sqrt(tan(d*x + c)) - sqrt(3))/sqrt(-2
*tan(d*x + c) + 3) - sqrt(-2*tan(d*x + c) + 3)/(sqrt(2)*sqrt(tan(d*x + c))
- sqrt(3))) + 8*sqrt(1/13))/d^3
```

Mupad [B]

time = 6.36, size = 201, normalized size = 2.26

$$-\mathrm{atan}\left(\frac{\sqrt{3}\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{-\frac{1}{26}+\frac{3}{52}\mathrm{i}}{d^2}}\,\left(6-4\mathrm{i}\right)+d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{-\frac{1}{26}+\frac{3}{52}\mathrm{i}}{d^2}}\,\sqrt{3-2\tan\left(c+d\,x\right)}\,\left(-6+4\mathrm{i}\right)}{2\tan\left(c+d\,x\right)+\sqrt{3}\,\sqrt{3-2\tan\left(c+d\,x\right)}-3}\right)\sqrt{\frac{-\frac{1}{26}+\frac{3}{52}\mathrm{i}}{d^2}}\,2\mathrm{i}+\mathrm{atan}\left(\frac{\sqrt{3}\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{-\frac{1}{26}-\frac{3}{52}\mathrm{i}}{d^2}}\,\left(6+4\mathrm{i}\right)+d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{-\frac{1}{26}-\frac{3}{52}\mathrm{i}}{d^2}}\,\sqrt{3-2\tan\left(c+d\,x\right)}\,\left(-6-4\mathrm{i}\right)}{2\tan\left(c+d\,x\right)+\sqrt{9-6\tan\left(c+d\,x\right)}-3}\right)\sqrt{\frac{-\frac{1}{26}-\frac{3}{52}\mathrm{i}}{d^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(1/2)*(3 - 2*tan(c + d*x))^(1/2)),x)
```

```
[Out] atan((3^(1/2)*d*tan(c + d*x)^(1/2)*((- 1/26 - 3i/52)/d^2)^(1/2)*(6 + 4i) -
d*tan(c + d*x)^(1/2)*((- 1/26 - 3i/52)/d^2)^(1/2)*(3 - 2*tan(c + d*x))^(1/2
)*(6 + 4i))/(2*tan(c + d*x) + (9 - 6*tan(c + d*x))^(1/2) - 3))*((- 1/26 - 3
i/52)/d^2)^(1/2)*2i - atan((3^(1/2)*d*tan(c + d*x)^(1/2)*((- 1/26 + 3i/52)/
d^2)^(1/2)*(6 - 4i) - d*tan(c + d*x)^(1/2)*((- 1/26 + 3i/52)/d^2)^(1/2)*(3
- 2*tan(c + d*x))^(1/2)*(6 - 4i))/(2*tan(c + d*x) + 3^(1/2)*(3 - 2*tan(c +
d*x))^(1/2) - 3))*((- 1/26 + 3i/52)/d^2)^(1/2)*2i
```

3.661 $\int \frac{1}{\sqrt{\tan(c+dx)}\sqrt{-3+2\tan(c+dx)}}\,dx$

Optimal. Leaf size=89

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2-3i}\sqrt{\tan(c+dx)}}{\sqrt{-3+2\tan(c+dx)}}\right)}{\sqrt{2-3i}\,d}+\frac{\tanh^{-1}\left(\frac{\sqrt{2+3i}\sqrt{\tan(c+dx)}}{\sqrt{-3+2\tan(c+dx)}}\right)}{\sqrt{2+3i}\,d}$$

```
[Out] arctanh((2-3*I)^(1/2)*tan(d*x+c)^(1/2)/(-3+2*tan(d*x+c))^(1/2))/d/(2-3*I)^(
1/2)+arctanh((2+3*I)^(1/2)*tan(d*x+c)^(1/2)/(-3+2*tan(d*x+c))^(1/2))/d/(2+3
*I)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3656, 926, 95, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2-3i}\sqrt{\tan(c+dx)}}{\sqrt{2\tan(c+dx)-3}}\right)}{\sqrt{2-3i}\,d}+\frac{\tanh^{-1}\left(\frac{\sqrt{2+3i}\sqrt{\tan(c+dx)}}{\sqrt{2\tan(c+dx)-3}}\right)}{\sqrt{2+3i}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Tan[c + d*x]]*Sqrt[-3 + 2*Tan[c + d*x]]),x]
```

```
[Out] ArcTanh[(Sqrt[2 - 3*I]*Sqrt[Tan[c + d*x]])/Sqrt[-3 + 2*Tan[c + d*x]]]/(Sqrt
[2 - 3*I]*d) + ArcTanh[(Sqrt[2 + 3*I]*Sqrt[Tan[c + d*x]])/Sqrt[-3 + 2*Tan[c
 + d*x]]]/(Sqrt[2 + 3*I]*d)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
```

```
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\tan(c+dx)}\sqrt{-3+2\tan(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{-3+2x}\,(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{i}{2(i-x)\sqrt{x}\sqrt{-3+2x}} + \frac{i}{2\sqrt{x}\,(i+x)\sqrt{-3+2x}}\right) dx$$

$$= \frac{i\text{Subst}\left(\int \frac{1}{(i-x)\sqrt{x}\sqrt{-3+2x}}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{i\text{Subst}\left(\right.}{}$$

$$= \frac{i\text{Subst}\left(\int \frac{1}{i-(3+2i)x^2}\,dx, x, \frac{\sqrt{\tan(c+dx)}}{\sqrt{-3+2\tan(c+dx)}}\right)}{d} + \frac{i\text{Subst}\left(\right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{2-3i}\sqrt{\tan(c+dx)}}{\sqrt{-3+2\tan(c+dx)}}\right)}{\sqrt{2-3i}\,d} + \frac{\tanh^{-1}\left(\frac{\sqrt{2+3i}\sqrt{\text{ta}}}{\sqrt{-3+2\,\text{ta}}}\right.}{\sqrt{2+3i}\,d}$$

Mathematica [A]
time = 0.15, size = 89, normalized size = 1.00

$$\frac{\text{ArcTan}\left(\frac{\sqrt{-2+3i}\sqrt{\tan(c+dx)}}{\sqrt{-3+2\tan(c+dx)}}\right)}{\sqrt{-2+3i}\,d} + \frac{\tanh^{-1}\left(\frac{\sqrt{2+3i}\sqrt{\tan(c+dx)}}{\sqrt{-3+2\tan(c+dx)}}\right)}{\sqrt{2+3i}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Tan[c + d*x]]*Sqrt[-3 + 2*Tan[c + d*x]]),x]
```

```
[Out] ArcTan[(Sqrt[-2 + 3*I]*Sqrt[Tan[c + d*x]])/Sqrt[-3 + 2*Tan[c + d*x]]]/(Sqrt
[-2 + 3*I]*d) + ArcTanh[(Sqrt[2 + 3*I]*Sqrt[Tan[c + d*x]])/Sqrt[-3 + 2*Tan[
c + d*x]]]/(Sqrt[2 + 3*I]*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(73) = 146$.

time = 1.21, size = 480, normalized size = 5.39

method	result
derivativedivides	$-\frac{\sqrt{\frac{\tan(dx+c)(-3+2\tan(dx+c))}{\left(\sqrt{13}-2-3\tan(dx+c)\right)^2}}\left(\sqrt{13}-2-3\tan(dx+c)\right)\left(4\sqrt{13}\,\sqrt{2\sqrt{13}+4}\,\sqrt{-4+2\sqrt{13}}\right.}{}$
default	$-\frac{\sqrt{\frac{\tan(dx+c)(-3+2\tan(dx+c))}{\left(\sqrt{13}-2-3\tan(dx+c)\right)^2}}\left(\sqrt{13}-2-3\tan(dx+c)\right)\left(4\sqrt{13}\,\sqrt{2\sqrt{13}+4}\,\sqrt{-4+2\sqrt{13}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(1/2)/(-3+2*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(tan(d*x+c)*(-3+2*tan(d*x+c))/(13^(1/2)-2-3*tan(d*x+c))^2)^(1/2)*(13
^(1/2)-2-3*tan(d*x+c))*(4*13^(1/2)*(2*13^(1/2)+4)^(1/2)*(-4+2*13^(1/2))^(1/
2)*arctan(1/56862*(-4+2*13^(1/2))^(1/2)*((17*13^(1/2)-52)*tan(d*x+c)*(-3+2*
tan(d*x+c))*(52+17*13^(1/2))/(13^(1/2)-2-3*tan(d*x+c))^2)^(1/2)*(4*13^(1/2)
+17)*(13^(1/2)+2+3*tan(d*x+c))*(17*13^(1/2)-52)*(13^(1/2)-2-3*tan(d*x+c))/t
an(d*x+c)/(-3+2*tan(d*x+c)))-17*(2*13^(1/2)+4)^(1/2)*(-4+2*13^(1/2))^(1/2)*
arctan(1/56862*(-4+2*13^(1/2))^(1/2)*((17*13^(1/2)-52)*tan(d*x+c)*(-3+2*tan
(d*x+c))*(52+17*13^(1/2))/(13^(1/2)-2-3*tan(d*x+c))^2)^(1/2)*(4*13^(1/2)+17
)*(13^(1/2)+2+3*tan(d*x+c))*(17*13^(1/2)-52)*(13^(1/2)-2-3*tan(d*x+c))/tan(
d*x+c)/(-3+2*tan(d*x+c)))+18*arctanh(6*13^(1/2)*(tan(d*x+c)*(-3+2*tan(d*x+c
))/(13^(1/2)-2-3*tan(d*x+c))^2)^(1/2)/(26*13^(1/2)+52)^(1/2))*13^(1/2)-36*a
rctanh(6*13^(1/2)*(tan(d*x+c)*(-3+2*tan(d*x+c))/(13^(1/2)-2-3*tan(d*x+c))^2
)^(1/2)/(26*13^(1/2)+52)^(1/2)))/tan(d*x+c)^(1/2)/(-3+2*tan(d*x+c))^(1/2)/(
2*13^(1/2)+4)^(1/2)/(17*13^(1/2)-52)
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/2)/(-3+2*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/2)/(-3+2*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\tan(c+dx)-3}\,\sqrt{\tan(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(1/2)/(-3+2*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2*tan(c + d*x) - 3)*sqrt(tan(c + d*x))), x)
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 479 vs. 2(65) = 130.
time = 0.50, size = 479, normalized size = 5.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/2)/(-3+2*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/52*sqrt(2)*(sqrt(sqrt(13) - 2)*((9*I - 6)/(sqrt(13) - 2) - 2*I - 3)*log(
(915*I + 1098)*sqrt(13)*(sqrt(2)*sqrt(tan(d*x + c)) - sqrt(2*tan(d*x + c) -
 3))^2 + (2370*I + 2844)*(sqrt(2)*sqrt(tan(d*x + c)) - sqrt(2*tan(d*x + c)
- 3))^2 + 366*sqrt(13)*sqrt(61*sqrt(13) + 158) - (1647*I - 6954)*sqrt(13) -
 (918*I - 948)*sqrt(61*sqrt(13) + 158) - 4266*I + 18012) - sqrt(sqrt(13) -
2)*((9*I - 6)/(sqrt(13) - 2) - 2*I - 3)*log((915*I + 1098)*sqrt(13)*(sqrt(2
)*sqrt(tan(d*x + c)) - sqrt(2*tan(d*x + c) - 3))^2 + (2370*I + 2844)*(sqrt(
2)*sqrt(tan(d*x + c)) - sqrt(2*tan(d*x + c) - 3))^2 - 366*sqrt(13)*sqrt(61*
sqrt(13) + 158) - (1647*I - 6954)*sqrt(13) + (918*I - 948)*sqrt(61*sqrt(13)
 + 158) - 4266*I + 18012) - sqrt(sqrt(13) + 2)*((6*I - 9)/(sqrt(13) + 2) -
3*I - 2)*log((90*I + 45)*sqrt(13)*(sqrt(2)*sqrt(tan(d*x + c)) - sqrt(2*tan(
d*x + c) - 3))^2 - (108*I + 54)*(sqrt(2)*sqrt(tan(d*x + c)) - sqrt(2*tan(d*
```

```
x + c) - 3))^2 + 90*sqrt(13)*sqrt(5*sqrt(13) - 6) + (450*I - 225)*sqrt(13)
- (306*I + 108)*sqrt(5*sqrt(13) - 6) - 540*I + 270) + sqrt(sqrt(13) + 2)*((
6*I - 9)/(sqrt(13) + 2) - 3*I - 2)*log((90*I + 45)*sqrt(13)*(sqrt(2)*sqrt(t
an(d*x + c)) - sqrt(2*tan(d*x + c) - 3))^2 - (108*I + 54)*(sqrt(2)*sqrt(tan
(d*x + c)) - sqrt(2*tan(d*x + c) - 3))^2 - 90*sqrt(13)*sqrt(5*sqrt(13) - 6)
 + (450*I - 225)*sqrt(13) + (306*I + 108)*sqrt(5*sqrt(13) - 6) - 540*I + 27
0))/d
```

Mupad [B]
time = 6.09, size = 89, normalized size = 1.00

$$2\,\mathrm{atanh}\left(\frac{2\,d\,\sqrt{\frac{\frac{1}{26}-\frac{3}{52}\mathrm{i}}{d^2}}\,\sqrt{2\tan\left(c+d\,x\right)-3}}{\sqrt{\tan\left(c+d\,x\right)}}\right)\,\sqrt{\frac{\frac{1}{26}-\frac{3}{52}\mathrm{i}}{d^2}}+2\,\mathrm{atanh}\left(\frac{2\,d\,\sqrt{\frac{\frac{1}{26}+\frac{3}{52}\mathrm{i}}{d^2}}\,\sqrt{2\tan\left(c+d\,x\right)-3}}{\sqrt{\tan\left(c+d\,x\right)}}\right)\,\sqrt{\frac{\frac{1}{26}+\frac{3}{52}\mathrm{i}}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(1/2)*(2*tan(c + d*x) - 3)^(1/2)),x)
```

```
[Out] 2*atanh((2*d*((1/26 - 3i/52)/d^2)^(1/2)*(2*tan(c + d*x) - 3)^(1/2))/tan(c +
 d*x)^(1/2))*((1/26 - 3i/52)/d^2)^(1/2) + 2*atanh((2*d*((1/26 + 3i/52)/d^2)
^(1/2)*(2*tan(c + d*x) - 3)^(1/2))/tan(c + d*x)^(1/2))*((1/26 + 3i/52)/d^2)
^(1/2)
```

3.662 $\int \frac{1}{\sqrt{-3-2\tan(c+dx)}\,\sqrt{\tan(c+dx)}}\,dx$

Optimal. Leaf size=89

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2-3i}\,\sqrt{\tan(c+dx)}}{\sqrt{-3-2\tan(c+dx)}}\right)}{\sqrt{2-3i}\,d}+\frac{\text{ArcTan}\left(\frac{\sqrt{2+3i}\,\sqrt{\tan(c+dx)}}{\sqrt{-3-2\tan(c+dx)}}\right)}{\sqrt{2+3i}\,d}$$

```
[Out] arctan((2-3*I)^(1/2)*tan(d*x+c)^(1/2)/(-3-2*tan(d*x+c))^(1/2))/d/(2-3*I)^(1
/2)+arctan((2+3*I)^(1/2)*tan(d*x+c)^(1/2)/(-3-2*tan(d*x+c))^(1/2))/d/(2+3*I
)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3656, 926, 95, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2-3i}\,\sqrt{\tan(c+dx)}}{\sqrt{-2\tan(c+dx)-3}}\right)}{\sqrt{2-3i}\,d}+\frac{\text{ArcTan}\left(\frac{\sqrt{2+3i}\,\sqrt{\tan(c+dx)}}{\sqrt{-2\tan(c+dx)-3}}\right)}{\sqrt{2+3i}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 - 2*Tan[c + d*x]]*Sqrt[Tan[c + d*x]]),x]
```

```
[Out] ArcTan[(Sqrt[2 - 3*I]*Sqrt[Tan[c + d*x]])/Sqrt[-3 - 2*Tan[c + d*x]]]/(Sqrt[
2 - 3*I]*d) + ArcTan[(Sqrt[2 + 3*I]*Sqrt[Tan[c + d*x]])/Sqrt[-3 - 2*Tan[c +
 d*x]]]/(Sqrt[2 + 3*I]*d)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
```

```
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-2\tan(c+dx)}\sqrt{\tan(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{-3-2x}\sqrt{x}\,(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{i}{2\sqrt{-3-2x}\,(i-x)\sqrt{x}} + \frac{i}{2\sqrt{-3-2x}\sqrt{x}\,(i+x)}\right) c\right.}{d}$$

$$= \frac{i\text{Subst}\left(\int \frac{1}{\sqrt{-3-2x}\,(i-x)\sqrt{x}}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{i\text{Subs}}{}$$

$$= \frac{i\text{Subst}\left(\int \frac{1}{i-(3-2i)x^2}\,dx, x, \frac{\sqrt{\tan(c+dx)}}{\sqrt{-3-2\tan(c+dx)}}\right)}{d} + \frac{i\text{Subs}}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{2-3i}\sqrt{\tan(c+dx)}}{\sqrt{-3-2\tan(c+dx)}}\right)}{\sqrt{2-3i}\,d} + \frac{\tan^{-1}\left(\frac{\sqrt{2+3i}\sqrt{\tan}}{\sqrt{-3-2\tan}}\right.}{\sqrt{2+3i}\,d}$$

Mathematica [A]
time = 0.16, size = 101, normalized size = 1.13

$$\frac{-\sqrt{2+3i}\,\text{ArcTan}\left(\frac{\sqrt{\frac{2}{13}+\frac{3i}{13}}\sqrt{-3-2\tan(c+dx)}}{\sqrt{\tan(c+dx)}}\right) + \sqrt{-2+3i}\,\tanh^{-1}\left(\frac{\sqrt{-\frac{2}{13}+\frac{3i}{13}}\sqrt{-3-2\tan(c+dx)}}{\sqrt{\tan(c+dx)}}\right)}{\sqrt{13}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 - 2*Tan[c + d*x]]*Sqrt[Tan[c + d*x]]),x]
```

```
[Out] (-(Sqrt[2 + 3*I]*ArcTan[(Sqrt[2/13 + (3*I)/13]*Sqrt[-3 - 2*Tan[c + d*x]])/S
qrt[Tan[c + d*x]]]) + Sqrt[-2 + 3*I]*ArcTanh[(Sqrt[-2/13 + (3*I)/13]*Sqrt[-
3 - 2*Tan[c + d*x]])/Sqrt[Tan[c + d*x]]])/(Sqrt[13]*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(73) = 146$.
time = 0.67, size = 435, normalized size = 4.89

method	result
derivativedivides	$-\frac{\sqrt{-3-2\tan(dx+c)}\,\sqrt{-\frac{\tan(dx+c)(3+2\tan(dx+c))}{\left(\sqrt{13}-2+3\tan(dx+c)\right)^2}}\,\left(\sqrt{13}-2+3\tan(dx+c)\right)\left(4\sqrt{13}\,\sqrt{2\sqrt{13}\cdots\right.}{\cdots}$
default	$-\frac{\sqrt{-3-2\tan(dx+c)}\,\sqrt{-\frac{\tan(dx+c)(3+2\tan(dx+c))}{\left(\sqrt{13}-2+3\tan(dx+c)\right)^2}}\,\left(\sqrt{13}-2+3\tan(dx+c)\right)\left(4\sqrt{13}\,\sqrt{2\sqrt{13}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3-2*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(-3-2*tan(d*x+c))^(1/2)*(-tan(d*x+c)*(3+2*tan(d*x+c))/(13^(1/2)-2+3*
tan(d*x+c))^2)^(1/2)*(13^(1/2)-2+3*tan(d*x+c))*(4*13^(1/2)*(2*13^(1/2)+4)^(
1/2)*arctanh(1/351*(2+13^(1/2))*(13^(1/2)+2-3*tan(d*x+c))*(17*13^(1/2)-52)/
(-4+2*13^(1/2))^(1/2)/(13^(1/2)-2+3*tan(d*x+c))*13^(1/2)/(-tan(d*x+c)*(3+2*
tan(d*x+c))/(13^(1/2)-2+3*tan(d*x+c))^2)^(1/2))*(-4+2*13^(1/2))^(1/2)-17*(2
*13^(1/2)+4)^(1/2)*arctanh(1/351*(2+13^(1/2))*(13^(1/2)+2-3*tan(d*x+c))*(17
*13^(1/2)-52)/(-4+2*13^(1/2))^(1/2)/(13^(1/2)-2+3*tan(d*x+c))*13^(1/2)/(-ta
n(d*x+c)*(3+2*tan(d*x+c))/(13^(1/2)-2+3*tan(d*x+c))^2)^(1/2))*(-4+2*13^(1/2
))^(1/2)-18*arctan(6*13^(1/2)*(-tan(d*x+c)*(3+2*tan(d*x+c))/(13^(1/2)-2+3*t
an(d*x+c))^2)^(1/2)/(26*13^(1/2)+52)^(1/2))*13^(1/2)+36*arctan(6*13^(1/2)*(
-tan(d*x+c)*(3+2*tan(d*x+c))/(13^(1/2)-2+3*tan(d*x+c))^2)^(1/2)/(26*13^(1/2
)+52)^(1/2)))/tan(d*x+c)^(1/2)/(2*13^(1/2)+4)^(1/2)/(3+2*tan(d*x+c))/(17*13
^(1/2)-52)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-2*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-2*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\tan\left(c+dx\right)-3}\,\sqrt{\tan\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-2*tan(d*x+c))**(1/2)/tan(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-2*tan(c + d*x) - 3)*sqrt(tan(c + d*x))), x)
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 413 vs. 2(65) = 130.
time = 0.52, size = 413, normalized size = 4.64

$$-\frac{(1904i+1536)\sqrt{2}\log\left(3\left(\sqrt{2}\sqrt{-\tan\left(dx+c\right)}-\sqrt{-2\tan\left(dx+c\right)-3}\right)^4+(24i+18)\left(\sqrt{2}\sqrt{-\tan\left(dx+c\right)}-\sqrt{-2\tan\left(dx+c\right)-3}\right)^2+27\right)}{23377\,d}-\frac{(1904i-1536)\sqrt{2}\log\left(3\left(\sqrt{2}\sqrt{-\tan\left(dx+c\right)}-\sqrt{-2\tan\left(dx+c\right)-3}\right)^4-(24i-18)\left(\sqrt{2}\sqrt{-\tan\left(dx+c\right)}-\sqrt{-2\tan\left(dx+c\right)-3}\right)^2+27\right)}{23377\,d}-\frac{(12720i+27456)\sqrt{2}\arctan\left(\frac{\sqrt{13}\left(\sqrt{2}\sqrt{-\tan\left(dx+c\right)}-\sqrt{-2\tan\left(dx+c\right)-3}\right)^2+2\left(\sqrt{2}\sqrt{-\tan\left(dx+c\right)}-\sqrt{-2\tan\left(dx+c\right)-3}\right)^2-(4i-3)\sqrt{13}-8i+6}{2\left(\sqrt{13}\sqrt{\sqrt{13}+2}+(3i+2)\sqrt{\sqrt{13}+2}\right)}\right)}{23377\,d\sqrt{\sqrt{13}+2}\left(\frac{3i}{\sqrt{13}+2}+1\right)}-\frac{(12720i-27456)\sqrt{2}\arctan\left(\frac{\sqrt{13}\left(\sqrt{2}\sqrt{-\tan\left(dx+c\right)}-\sqrt{-2\tan\left(dx+c\right)-3}\right)^2+2\left(\sqrt{2}\sqrt{-\tan\left(dx+c\right)}-\sqrt{-2\tan\left(dx+c\right)-3}\right)^2+(4i+3)\sqrt{13}+8i+6}{2\left(\sqrt{13}\sqrt{\sqrt{13}+2}-(3i-2)\sqrt{\sqrt{13}+2}\right)}\right)}{23377\,d\sqrt{\sqrt{13}+2}\left(-\frac{3i}{\sqrt{13}+2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-2*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -(1904/23377*I + 1536/23377)*sqrt(2)*log(3*(sqrt(2)*sqrt(-tan(d*x + c)) - s
qrt(-2*tan(d*x + c) - 3))^4 + (24*I + 18)*(sqrt(2)*sqrt(-tan(d*x + c)) - sq
rt(-2*tan(d*x + c) - 3))^2 + 27)/d - (1904/23377*I - 1536/23377)*sqrt(2)*lo
g(3*(sqrt(2)*sqrt(-tan(d*x + c)) - sqrt(-2*tan(d*x + c) - 3))^4 - (24*I - 1
8)*(sqrt(2)*sqrt(-tan(d*x + c)) - sqrt(-2*tan(d*x + c) - 3))^2 + 27)/d - (1
2720/23377*I + 27456/23377)*sqrt(2)*arctan(1/2*(sqrt(13)*(sqrt(2)*sqrt(-tan
(d*x + c)) - sqrt(-2*tan(d*x + c) - 3))^2 + 2*(sqrt(2)*sqrt(-tan(d*x + c))
- sqrt(-2*tan(d*x + c) - 3))^2 - (4*I - 3)*sqrt(13) - 8*I + 6)/(sqrt(13)*sq
rt(sqrt(13) + 2) + (3*I + 2)*sqrt(sqrt(13) + 2)))/(d*sqrt(sqrt(13) + 2)*(3*
I/(sqrt(13) + 2) + 1)) - (12720/23377*I - 27456/23377)*sqrt(2)*arctan(1/2*(
```

```
sqrt(13)*(sqrt(2)*sqrt(-tan(d*x + c)) - sqrt(-2*tan(d*x + c) - 3))^2 + 2*(s
qrt(2)*sqrt(-tan(d*x + c)) - sqrt(-2*tan(d*x + c) - 3))^2 + (4*I + 3)*sqrt(
13) + 8*I + 6)/(sqrt(13)*sqrt(sqrt(13) + 2) - (3*I - 2)*sqrt(sqrt(13) + 2))
)/(d*sqrt(sqrt(13) + 2)*(-3*I/(sqrt(13) + 2) + 1))
```

Mupad [B]

time = 6.53, size = 219, normalized size = 2.46

$$-\mathrm{atan}\left(\frac{\sqrt{\frac{-\frac{1}{26}-\frac{3}{52}\mathrm{i}}{d^2}}\,\left(-\sqrt{\tan\left(c+d\,x\right)}+\frac{\sqrt{2}\,\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(59001680335459079474380800+1416040328051017907385139\mathrm{2i}\right)}{\sqrt{-2\,\tan\left(c+d\,x\right)-3}\,\left(\frac{17700504100637723842314 24-737521004193238493429760\mathrm{i}}{d}+\frac{{\left(-\sqrt{\tan\left(c+d\,x\right)}+\frac{\sqrt{2}\,\sqrt{3}\,1\mathrm{i}}{2}\right)}^2\,\left(354010082012754476846284 8-1475042008386476986859520\mathrm{i}\right)}{d\,\left(2\,\tan\left(c+d\,x\right)+3\right)}\right)}\right)\,\sqrt{\frac{-\frac{1}{26}-\frac{3}{52}\mathrm{i}}{d^2}}\,2\mathrm{i}+\mathrm{atan}\left(\frac{\sqrt{\frac{-\frac{1}{26}+\frac{3}{52}\mathrm{i}}{d^2}}\,\left(-\sqrt{\tan\left(c+d\,x\right)}+\frac{\sqrt{2}\,\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(5900168033545907947438080-1416040328051017907385139 2\mathrm{i}\right)}{\sqrt{-2\,\tan\left(c+d\,x\right)-3}\,\left(\frac{1770050410063772384231424+737521004193238493429760\mathrm{i}}{d}+\frac{{\left(-\sqrt{\tan\left(c+d\,x\right)}+\frac{\sqrt{2}\,\sqrt{3}\,1\mathrm{i}}{2}\right)}^2\,\left(3540100820127544768462848+1475042008386476986859520\mathrm{i}\right)}{d\,\left(2\,\tan\left(c+d\,x\right)+3\right)}\right)}\right)\,\sqrt{\frac{-\frac{1}{26}+\frac{3}{52}\mathrm{i}}{d^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(1/2)*(- 2*tan(c + d*x) - 3)^(1/2)),x)
```

```
[Out] atan((((- 1/26 + 3i/52)/d^2)^(1/2)*((2^(1/2)*3^(1/2)*1i)/2 - tan(c + d*x)^(
1/2))*(5900168033545907947438080 - 14160403280510179073851392i))/((- 2*tan(
c + d*x) - 3)^(1/2)*((1770050410063772384231424 + 737521004193238493429760i
)/d + (((2^(1/2)*3^(1/2)*1i)/2 - tan(c + d*x)^(1/2))^2*(3540100820127544768
462848 + 1475042008386476986859520i))/(d*(2*tan(c + d*x) + 3)))))*((- 1/26
+ 3i/52)/d^2)^(1/2)*2i - atan((((- 1/26 - 3i/52)/d^2)^(1/2)*((2^(1/2)*3^(1/
2)*1i)/2 - tan(c + d*x)^(1/2))*(5900168033545907947438080 + 141604032805101
79073851392i))/((- 2*tan(c + d*x) - 3)^(1/2)*((1770050410063772384231424 -
737521004193238493429760i)/d + (((2^(1/2)*3^(1/2)*1i)/2 - tan(c + d*x)^(1/2
))^2*(3540100820127544768462848 - 1475042008386476986859520i))/(d*(2*tan(c
+ d*x) + 3)))))*((- 1/26 - 3i/52)/d^2)^(1/2)*2i
```

3.663 $\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{2+3\tan(c+dx)}}\,dx$

Optimal. Leaf size=95

$$\frac{i\tanh^{-1}\left(\frac{\sqrt{3-2i}\,\sqrt{\tan(c+dx)}}{\sqrt{2+3\tan(c+dx)}}\right)}{\sqrt{3-2i}\,d}-\frac{i\tanh^{-1}\left(\frac{\sqrt{3+2i}\,\sqrt{\tan(c+dx)}}{\sqrt{2+3\tan(c+dx)}}\right)}{\sqrt{3+2i}\,d}$$

```
[Out] I*arctanh((3-2*I)^(1/2)*tan(d*x+c)^(1/2)/(2+3*tan(d*x+c))^(1/2))/d/(3-2*I)^
(1/2)-I*arctanh((3+2*I)^(1/2)*tan(d*x+c)^(1/2)/(2+3*tan(d*x+c))^(1/2))/d/(3
+2*I)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3656, 924, 95, 214}

$$\frac{i\tanh^{-1}\left(\frac{\sqrt{3-2i}\,\sqrt{\tan(c+dx)}}{\sqrt{3\tan(c+dx)+2}}\right)}{\sqrt{3-2i}\,d}-\frac{i\tanh^{-1}\left(\frac{\sqrt{3+2i}\,\sqrt{\tan(c+dx)}}{\sqrt{3\tan(c+dx)+2}}\right)}{\sqrt{3+2i}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[c + d*x]]/Sqrt[2 + 3*Tan[c + d*x]],x]
```

```
[Out] (I*ArcTanh[(Sqrt[3 - 2*I]*Sqrt[Tan[c + d*x]])/Sqrt[2 + 3*Tan[c + d*x]]])/(S
qrt[3 - 2*I]*d) - (I*ArcTanh[(Sqrt[3 + 2*I]*Sqrt[Tan[c + d*x]])/Sqrt[2 + 3*
Tan[c + d*x]]])/(Sqrt[3 + 2*I]*d)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
```

```
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{2+3\tan(c+dx)}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{2+3x}\,(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(-\frac{1}{2(i-x)\sqrt{x}\sqrt{2+3x}} + \frac{1}{2\sqrt{x}\,(i+x)\sqrt{2+3x}}\right)dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{1}{(i-x)\sqrt{x}\sqrt{2+3x}}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{x}\,(i+x)\sqrt{2+3}}\right.}{2d}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{1}{i-(2+3i)x^2}\,dx, x, \frac{\sqrt{\tan(c+dx)}}{\sqrt{2+3\tan(c+dx)}}\right)}{d} + \frac{\operatorname{Subst}\left(\int \frac{1}{i+(2-3i)x^2}\,dx, x,\right.}{d}$$

$$= \frac{i\tanh^{-1}\left(\frac{\sqrt{3-2i}\sqrt{\tan(c+dx)}}{\sqrt{2+3\tan(c+dx)}}\right)}{\sqrt{3-2i}\,d} - \frac{i\tanh^{-1}\left(\frac{\sqrt{3+2i}\sqrt{\tan(c+dx)}}{\sqrt{2+3\tan(c+dx)}}\right)}{\sqrt{3+2i}\,d}$$

Mathematica [A]
time = 0.10, size = 95, normalized size = 1.00

$$\frac{i\operatorname{ArcTan}\left(\frac{\sqrt{-3+2i}\sqrt{\tan(c+dx)}}{\sqrt{2+3\tan(c+dx)}}\right)}{\sqrt{-3+2i}\,d} - \frac{i\tanh^{-1}\left(\frac{\sqrt{3+2i}\sqrt{\tan(c+dx)}}{\sqrt{2+3\tan(c+dx)}}\right)}{\sqrt{3+2i}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tan[c + d*x]]/Sqrt[2 + 3*Tan[c + d*x]],x]
```

```
[Out] (I*ArcTan[(Sqrt[-3 + 2*I]*Sqrt[Tan[c + d*x]])/Sqrt[2 + 3*Tan[c + d*x]]])/(S
qrt[-3 + 2*I]*d) - (I*ArcTanh[(Sqrt[3 + 2*I]*Sqrt[Tan[c + d*x]])/Sqrt[2 + 3
*Tan[c + d*x]]])/(Sqrt[3 + 2*I]*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(77) = 154$.
time = 0.88, size = 479, normalized size = 5.04

method	result
derivativedivides	$-\frac{\sqrt{\frac{\tan(dx+c)(2+3\tan(dx+c))}{\left(\sqrt{13}-3+2\tan(dx+c)\right)^2}}\left(\sqrt{13}-3+2\tan(dx+c)\right)\left(\sqrt{13}\,\sqrt{2\sqrt{13}+6}\,\arctan\left(\frac{\sqrt{2\sqrt{13}-}}{}\right.\right.}{}$
default	$-\frac{\sqrt{\frac{\tan(dx+c)(2+3\tan(dx+c))}{\left(\sqrt{13}-3+2\tan(dx+c)\right)^2}}\left(\sqrt{13}-3+2\tan(dx+c)\right)\left(\sqrt{13}\,\sqrt{2\sqrt{13}+6}\,\arctan\left(\frac{\sqrt{2\sqrt{13}-}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)/(2+3*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(tan(d*x+c)*(2+3*tan(d*x+c))/(13^(1/2)-3+2*tan(d*x+c))^2)^(1/2)*(13^
(1/2)-3+2*tan(d*x+c))*(13^(1/2)*(2*13^(1/2)+6)^(1/2)*arctan(1/416*(2*13^(1/
2)-6)^(1/2)*((11*13^(1/2)-39)*tan(d*x+c)*(39+11*13^(1/2))*(2+3*tan(d*x+c))/
(13^(1/2)-3+2*tan(d*x+c))^2)^(1/2)*(3*13^(1/2)+11)*(13^(1/2)+3-2*tan(d*x+c)
)*(11*13^(1/2)-39)*(13^(1/2)-3+2*tan(d*x+c))/tan(d*x+c)/(2+3*tan(d*x+c)))*(
2*13^(1/2)-6)^(1/2)-3*(2*13^(1/2)+6)^(1/2)*arctan(1/416*(2*13^(1/2)-6)^(1/2
)*((11*13^(1/2)-39)*tan(d*x+c)*(39+11*13^(1/2))*(2+3*tan(d*x+c))/(13^(1/2)-
3+2*tan(d*x+c))^2)^(1/2)*(3*13^(1/2)+11)*(13^(1/2)+3-2*tan(d*x+c))*(11*13^(
1/2)-39)*(13^(1/2)-3+2*tan(d*x+c))/tan(d*x+c)/(2+3*tan(d*x+c)))*(2*13^(1/2)
-6)^(1/2)-12*arctanh(4*13^(1/2)*(tan(d*x+c)*(2+3*tan(d*x+c))/(13^(1/2)-3+2*
tan(d*x+c))^2)^(1/2)/(26*13^(1/2)+78)^(1/2))*13^(1/2)+44*arctanh(4*13^(1/2)
*(tan(d*x+c)*(2+3*tan(d*x+c))/(13^(1/2)-3+2*tan(d*x+c))^2)^(1/2)/(26*13^(1/
2)+78)^(1/2)))/tan(d*x+c)^(1/2)/(2+3*tan(d*x+c))^(1/2)/(2*13^(1/2)+6)^(1/2)
/(11*13^(1/2)-39)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(2+3*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(tan(d*x + c))/sqrt(3*tan(d*x + c) + 2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(2+3*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{3\tan(c+dx)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/2)/(2+3*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(tan(c + d*x))/sqrt(3*tan(c + d*x) + 2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(2+3*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, choosing root of [1,0,-702,-
2704,70473] at parameters values [0]Unable to find common minimal polynomia
l Erro
```

Mupad [**B**]
time = 5.57, size = 207, normalized size = 2.18

$$\mathrm{atan}\left(\frac{\sqrt{2}\,d\,\sqrt{\tan(c+d\,x)}\,\sqrt{\frac{-\frac{3}{52}-\frac{1}{26}\mathrm{i}}{d^2}}\,(6-4\mathrm{i})+d\,\sqrt{\tan(c+d\,x)}\,\sqrt{\frac{-\frac{3}{52}-\frac{1}{26}\mathrm{i}}{d^2}}\,\sqrt{3\tan(c+d\,x)+2}\,(-6+4\mathrm{i})}{3\tan(c+d\,x)-\sqrt{2}\,\sqrt{3\tan(c+d\,x)+2}+2}\right)\sqrt{\frac{-\frac{3}{52}-\frac{1}{26}\mathrm{i}}{d^2}}\,2\mathrm{i}-\mathrm{atan}\left(\frac{\sqrt{2}\,d\,\sqrt{\tan(c+d\,x)}\,\sqrt{\frac{-\frac{3}{52}+\frac{1}{26}\mathrm{i}}{d^2}}\,(6+4\mathrm{i})+d\,\sqrt{\tan(c+d\,x)}\,\sqrt{\frac{-\frac{3}{52}+\frac{1}{26}\mathrm{i}}{d^2}}\,\sqrt{3\tan(c+d\,x)+2}\,(-6-4\mathrm{i})}{3\tan(c+d\,x)-\sqrt{2}\,\sqrt{3\tan(c+d\,x)+2}+2}\right)\sqrt{\frac{-\frac{3}{52}+\frac{1}{26}\mathrm{i}}{d^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)/(3*tan(c + d*x) + 2)^(1/2),x)
```

```
[Out] atan((2^(1/2)*d*tan(c + d*x)^(1/2)*((- 3/52 - 1i/26)/d^2)^(1/2)*(6 - 4i) -
d*tan(c + d*x)^(1/2)*((- 3/52 - 1i/26)/d^2)^(1/2)*(3*tan(c + d*x) + 2)^(1/2
)*(6 - 4i))/(3*tan(c + d*x) - 2^(1/2)*(3*tan(c + d*x) + 2)^(1/2) + 2))*((-
3/52 - 1i/26)/d^2)^(1/2)*2i - atan((2^(1/2)*d*tan(c + d*x)^(1/2)*((- 3/52 +
 1i/26)/d^2)^(1/2)*(6 + 4i) - d*tan(c + d*x)^(1/2)*((- 3/52 + 1i/26)/d^2)^(
1/2)*(3*tan(c + d*x) + 2)^(1/2)*(6 + 4i))/(3*tan(c + d*x) - 2^(1/2)*(3*tan(
c + d*x) + 2)^(1/2) + 2))*((- 3/52 + 1i/26)/d^2)^(1/2)*2i
```

3.664 $\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{-2+3\tan(c+dx)}}\,dx$

Optimal. Leaf size=95

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{3-2i}\,\sqrt{\tan(c+dx)}}{\sqrt{-2+3\tan(c+dx)}}\right)}{\sqrt{3-2i}\,d}+\frac{i\tanh^{-1}\left(\frac{\sqrt{3+2i}\,\sqrt{\tan(c+dx)}}{\sqrt{-2+3\tan(c+dx)}}\right)}{\sqrt{3+2i}\,d}$$

```
[Out] -I*arctanh((3-2*I)^(1/2)*tan(d*x+c)^(1/2)/(-2+3*tan(d*x+c))^(1/2))/d/(3-2*I
)^(1/2)+I*arctanh((3+2*I)^(1/2)*tan(d*x+c)^(1/2)/(-2+3*tan(d*x+c))^(1/2))/d
/(3+2*I)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3656, 924, 95, 214}

$$\frac{i\tanh^{-1}\left(\frac{\sqrt{3+2i}\,\sqrt{\tan(c+dx)}}{\sqrt{3\tan(c+dx)-2}}\right)}{\sqrt{3+2i}\,d}-\frac{i\tanh^{-1}\left(\frac{\sqrt{3-2i}\,\sqrt{\tan(c+dx)}}{\sqrt{3\tan(c+dx)-2}}\right)}{\sqrt{3-2i}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[c + d*x]]/Sqrt[-2 + 3*Tan[c + d*x]],x]
```

```
[Out] ((-I)*ArcTanh[(Sqrt[3 - 2*I]*Sqrt[Tan[c + d*x]])/Sqrt[-2 + 3*Tan[c + d*x]]]
)/(Sqrt[3 - 2*I]*d) + (I*ArcTanh[(Sqrt[3 + 2*I]*Sqrt[Tan[c + d*x]])/Sqrt[-2
 + 3*Tan[c + d*x]]])/(Sqrt[3 + 2*I]*d)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
```

```
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{-2+3\tan(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{-2+3x}\,(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{2(i-x)\sqrt{x}\sqrt{-2+3x}} + \frac{1}{2\sqrt{x}\,(i+x)\sqrt{-2+3x}}\right) dx, x, \tan(c+$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{(i-x)\sqrt{x}\sqrt{-2+3x}}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(i+x)\sqrt{}}\right.}{}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{i-(2+3i)x^2}\,dx, x, \frac{\sqrt{\tan(c+dx)}}{\sqrt{-2+3\tan(c+dx)}}\right)}{d} - \frac{\text{Subst}\left(\int \frac{1}{i+(2-3i)x^2}\,dx\right.}{}$$
$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{3-2i}\sqrt{\tan(c+dx)}}{\sqrt{-2+3\tan(c+dx)}}\right)}{\sqrt{3-2i}\,d} + \frac{i\tanh^{-1}\left(\frac{\sqrt{3+2i}\sqrt{\tan(c+dx}}{\sqrt{-2+3\tan(c+dx)}}\right.}{\sqrt{3+2i}\,d}$$

Mathematica [A]
time = 0.07, size = 95, normalized size = 1.00

$$-\frac{i\text{ArcTan}\left(\frac{\sqrt{-3+2i}\sqrt{\tan(c+dx)}}{\sqrt{-2+3\tan(c+dx)}}\right)}{\sqrt{-3+2i}\,d} + \frac{i\tanh^{-1}\left(\frac{\sqrt{3+2i}\sqrt{\tan(c+dx)}}{\sqrt{-2+3\tan(c+dx)}}\right)}{\sqrt{3+2i}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tan[c + d*x]]/Sqrt[-2 + 3*Tan[c + d*x]],x]
```

```
[Out] ((-I)*ArcTan[(Sqrt[-3 + 2*I]*Sqrt[Tan[c + d*x]])/Sqrt[-2 + 3*Tan[c + d*x]]]
)/(Sqrt[-3 + 2*I]*d) + (I*ArcTanh[(Sqrt[3 + 2*I]*Sqrt[Tan[c + d*x]])/Sqrt[-
2 + 3*Tan[c + d*x]]])/(Sqrt[3 + 2*I]*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(77) = 154$.

time = 0.85, size = 479, normalized size = 5.04

method	result
derivativedivides	$-\sqrt{\frac{\tan(dx+c)(-2+3\tan(dx+c))}{\left(\sqrt{13}-3-2\tan(dx+c)\right)^2}}\left(\sqrt{13}-3-2\tan(dx+c)\right)\left(\sqrt{2\sqrt{13}-6}\,\arctan\left(\sqrt{2\sqrt{13}-6}\sqrt{\cdots}\right.\right.$
default	$-\sqrt{\frac{\tan(dx+c)(-2+3\tan(dx+c))}{\left(\sqrt{13}-3-2\tan(dx+c)\right)^2}}\left(\sqrt{13}-3-2\tan(dx+c)\right)\left(\sqrt{2\sqrt{13}-6}\,\arctan\left(\sqrt{2\sqrt{13}-6}\sqrt{\cdots}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)/(-2+3*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(tan(d*x+c)*(-2+3*tan(d*x+c))/(13^(1/2)-3-2*tan(d*x+c))^2)^(1/2)*(13
^(1/2)-3-2*tan(d*x+c))*((2*13^(1/2)-6)^(1/2)*arctan(1/416*(2*13^(1/2)-6)^(1
/2)*((11*13^(1/2)-39)*tan(d*x+c)*(39+11*13^(1/2))*(-2+3*tan(d*x+c))/(13^(1/
2)-3-2*tan(d*x+c))^2)^(1/2)*(3*13^(1/2)+11)*(13^(1/2)+3+2*tan(d*x+c))*(11*1
3^(1/2)-39)*(13^(1/2)-3-2*tan(d*x+c))/tan(d*x+c)/(-2+3*tan(d*x+c)))*13^(1/2
)*(2*13^(1/2)+6)^(1/2)-3*(2*13^(1/2)-6)^(1/2)*arctan(1/416*(2*13^(1/2)-6)^(
1/2)*((11*13^(1/2)-39)*tan(d*x+c)*(39+11*13^(1/2))*(-2+3*tan(d*x+c))/(13^(1
/2)-3-2*tan(d*x+c))^2)^(1/2)*(3*13^(1/2)+11)*(13^(1/2)+3+2*tan(d*x+c))*(11*
13^(1/2)-39)*(13^(1/2)-3-2*tan(d*x+c))/tan(d*x+c)/(-2+3*tan(d*x+c)))*(2*13^
(1/2)+6)^(1/2)-12*arctanh(4*13^(1/2)*(tan(d*x+c)*(-2+3*tan(d*x+c))/(13^(1/2
)-3-2*tan(d*x+c))^2)^(1/2)/(26*13^(1/2)+78)^(1/2))*13^(1/2)+44*arctanh(4*13
^(1/2)*(tan(d*x+c)*(-2+3*tan(d*x+c))/(13^(1/2)-3-2*tan(d*x+c))^2)^(1/2)/(26
*13^(1/2)+78)^(1/2)))/tan(d*x+c)^(1/2)/(-2+3*tan(d*x+c))^(1/2)/(2*13^(1/2)+
6)^(1/2)/(11*13^(1/2)-39)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(-2+3*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(tan(d*x + c))/sqrt(3*tan(d*x + c) - 2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(-2+3*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\tan\left(c+dx\right)}}{\sqrt{3\tan\left(c+dx\right)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/2)/(-2+3*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(tan(c + d*x))/sqrt(3*tan(c + d*x) - 2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(-2+3*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, choosing root of [1,0,-702,-
2704,70473] at parameters values [0]Invalid _EXT in replace_ext Error: Bad
Argume
```

Mupad [**B**]
time = 5.64, size = 191, normalized size = 2.01

$$-\mathrm{atan}\left(\frac{12\,d\,\sqrt{\frac{-\frac{3}{52}-\frac{1}{26}\mathrm{i}}{d^2}}\,\left(\frac{\sqrt{2}\,\sqrt{3}}{3}-\sqrt{\tan\left(c+d\,x\right)}\right)}{\sqrt{3\tan\left(c+d\,x\right)-2}\,\left(\frac{3\,{\left(\frac{\sqrt{2}\,\sqrt{3}}{3}-\sqrt{\tan\left(c+d\,x\right)}\right)}^2}{3\tan(c+d\,x)-2}+1\right)}\right)\,\sqrt{\frac{-\frac{3}{52}-\frac{1}{26}\mathrm{i}}{d^2}}\,2\mathrm{i}+\mathrm{atan}\left(\frac{12\,d\,\sqrt{\frac{-\frac{3}{52}+\frac{1}{26}\mathrm{i}}{d^2}}\,\left(\frac{\sqrt{2}\,\sqrt{3}}{3}-\sqrt{\tan\left(c+d\,x\right)}\right)}{\sqrt{3\tan\left(c+d\,x\right)-2}\,\left(\frac{3\,{\left(\frac{\sqrt{2}\,\sqrt{3}}{3}-\sqrt{\tan\left(c+d\,x\right)}\right)}^2}{3\tan(c+d\,x)-2}+1\right)}\right)\,\sqrt{\frac{-\frac{3}{52}+\frac{1}{26}\mathrm{i}}{d^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)/(3*tan(c + d*x) - 2)^(1/2),x)
```

```
[Out] atan((12*d*((- 3/52 + 1i/26)/d^2)^(1/2)*((2^(1/2)*3^(1/2))/3 - tan(c + d*x)
^(1/2)))/((3*tan(c + d*x) - 2)^(1/2)*((3*((2^(1/2)*3^(1/2))/3 - tan(c + d*x
)^(1/2))^2)/(3*tan(c + d*x) - 2) + 1)))*((- 3/52 + 1i/26)/d^2)^(1/2)*2i - a
tan((12*d*((- 3/52 - 1i/26)/d^2)^(1/2)*((2^(1/2)*3^(1/2))/3 - tan(c + d*x)^
(1/2)))/((3*tan(c + d*x) - 2)^(1/2)*((3*((2^(1/2)*3^(1/2))/3 - tan(c + d*x)
^(1/2))^2)/(3*tan(c + d*x) - 2) + 1)))*((- 3/52 - 1i/26)/d^2)^(1/2)*2i
```

3.665 $\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{2-3\tan(c+dx)}}\,dx$

Optimal. Leaf size=95

$$-\frac{i\text{ArcTan}\left(\frac{\sqrt{3-2i}\sqrt{\tan(c+dx)}}{\sqrt{2-3\tan(c+dx)}}\right)}{\sqrt{3-2i}\,d}+\frac{i\text{ArcTan}\left(\frac{\sqrt{3+2i}\sqrt{\tan(c+dx)}}{\sqrt{2-3\tan(c+dx)}}\right)}{\sqrt{3+2i}\,d}$$

```
[Out] -I*arctan((3-2*I)^(1/2)*tan(d*x+c)^(1/2)/(2-3*tan(d*x+c))^(1/2))/d/(3-2*I)^
(1/2)+I*arctan((3+2*I)^(1/2)*tan(d*x+c)^(1/2)/(2-3*tan(d*x+c))^(1/2))/d/(3+
2*I)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3656, 924, 95, 211}

$$\frac{i\text{ArcTan}\left(\frac{\sqrt{3+2i}\sqrt{\tan(c+dx)}}{\sqrt{2-3\tan(c+dx)}}\right)}{\sqrt{3+2i}\,d}-\frac{i\text{ArcTan}\left(\frac{\sqrt{3-2i}\sqrt{\tan(c+dx)}}{\sqrt{2-3\tan(c+dx)}}\right)}{\sqrt{3-2i}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[c + d*x]]/Sqrt[2 - 3*Tan[c + d*x]],x]
```

```
[Out] ((-I)*ArcTan[(Sqrt[3 - 2*I]*Sqrt[Tan[c + d*x]])/Sqrt[2 - 3*Tan[c + d*x]]])/
(Sqrt[3 - 2*I]*d) + (I*ArcTan[(Sqrt[3 + 2*I]*Sqrt[Tan[c + d*x]])/Sqrt[2 - 3
*Tan[c + d*x]]])/(Sqrt[3 + 2*I]*d)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
```

```
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{2-3\tan(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{2-3x}\,(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{2\sqrt{2-3x}\,(i-x)\sqrt{x}} + \frac{1}{2\sqrt{2-3x}\,\sqrt{x}\,(i+x)}\right) dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{2-3x}\,(i-x)\sqrt{x}}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2-3x}\,\sqrt{x}\,(i\cdots}\right.}{2d}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{i-(2-3i)x^2}\,dx, x, \frac{\sqrt{\tan(c+dx)}}{\sqrt{2-3\tan(c+dx)}}\right)}{d} + \frac{\text{Subst}\left(\int \frac{1}{i+(2+3i)x^2}\,dx, x, \cdots\right.}{d}$$

$$= -\frac{i\tan^{-1}\left(\frac{\sqrt{3-2i}\,\sqrt{\tan(c+dx)}}{\sqrt{2-3\tan(c+dx)}}\right)}{\sqrt{3-2i}\,d} + \frac{i\tan^{-1}\left(\frac{\sqrt{3+2i}\,\sqrt{\tan(c+dx)}}{\sqrt{2-3\tan(c+dx)}}\right)}{\sqrt{3+2i}\,d}$$

Mathematica [A]
time = 0.07, size = 103, normalized size = 1.08

$$\frac{i\left(\sqrt{3+2i}\,\text{ArcTan}\left(\frac{\sqrt{\frac{3}{13}+\frac{2i}{13}}\,\sqrt{2-3\tan(c+dx)}}{\sqrt{\tan(c+dx)}}\right)+\sqrt{-3+2i}\,\tanh^{-1}\left(\frac{\sqrt{-\frac{3}{13}+\frac{2i}{13}}\,\sqrt{2-3\tan(c+dx)}}{\sqrt{\tan(c+dx)}}\right)\right)}{\sqrt{13}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tan[c + d*x]]/Sqrt[2 - 3*Tan[c + d*x]],x]
```

```
[Out] (I*(Sqrt[3 + 2*I]*ArcTan[(Sqrt[3/13 + (2*I)/13]*Sqrt[2 - 3*Tan[c + d*x]])/S
qrt[Tan[c + d*x]]] + Sqrt[-3 + 2*I]*ArcTanh[(Sqrt[-3/13 + (2*I)/13]*Sqrt[2
- 3*Tan[c + d*x]])/Sqrt[Tan[c + d*x]]]))/(Sqrt[13]*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(77) = 154$.
time = 0.67, size = 434, normalized size = 4.57

method	result
derivativedivides	$\frac{\sqrt{2-3\tan\left(dx+c\right)}\,\sqrt{-\frac{\tan(dx+c)(-2+3\tan(dx+c))}{\left(\sqrt{13}-3-2\tan(dx+c)\right)^{2}}}\left(\sqrt{13}-3-2\tan(dx+c)\right)\left(\sqrt{13}\,\sqrt{2\sqrt{13}-\right.}{}$
default	$\frac{\sqrt{2-3\tan\left(dx+c\right)}\,\sqrt{-\frac{\tan(dx+c)(-2+3\tan(dx+c))}{\left(\sqrt{13}-3-2\tan(dx+c)\right)^{2}}}\left(\sqrt{13}-3-2\tan(dx+c)\right)\left(\sqrt{13}\,\sqrt{2\sqrt{13}-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)/(2-3*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(2-3*tan(d*x+c))^(1/2)*(-tan(d*x+c)*(-2+3*tan(d*x+c))/(13^(1/2)-3-2*t
an(d*x+c))^2)^(1/2)*(13^(1/2)-3-2*tan(d*x+c))*(13^(1/2)*(2*13^(1/2)+6)^(1/2
)*arctanh(1/52*(13^(1/2)+3)*(13^(1/2)+3+2*tan(d*x+c))*(11*13^(1/2)-39)/(13^
(1/2)-3-2*tan(d*x+c))/(2*13^(1/2)-6)^(1/2)*13^(1/2)/(-tan(d*x+c)*(-2+3*tan(
d*x+c))/(13^(1/2)-3-2*tan(d*x+c))^2)^(1/2))*(2*13^(1/2)-6)^(1/2)-3*(2*13^(1
/2)+6)^(1/2)*arctanh(1/52*(13^(1/2)+3)*(13^(1/2)+3+2*tan(d*x+c))*(11*13^(1/
2)-39)/(13^(1/2)-3-2*tan(d*x+c))/(2*13^(1/2)-6)^(1/2)*13^(1/2)/(-tan(d*x+c)
*(-2+3*tan(d*x+c))/(13^(1/2)-3-2*tan(d*x+c))^2)^(1/2))*(2*13^(1/2)-6)^(1/2)
+12*arctan(4*13^(1/2)*(-tan(d*x+c)*(-2+3*tan(d*x+c))/(13^(1/2)-3-2*tan(d*x+
c))^2)^(1/2)/(26*13^(1/2)+78)^(1/2))*13^(1/2)-44*arctan(4*13^(1/2)*(-tan(d*
x+c)*(-2+3*tan(d*x+c))/(13^(1/2)-3-2*tan(d*x+c))^2)^(1/2)/(26*13^(1/2)+78)^
(1/2)))/tan(d*x+c)^(1/2)/(-2+3*tan(d*x+c))/(2*13^(1/2)+6)^(1/2)/(11*13^(1/2
)-39)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(2-3*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(tan(d*x + c))/sqrt(-3*tan(d*x + c) + 2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(2-3*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{2-3\tan(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/2)/(2-3*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(tan(c + d*x))/sqrt(2 - 3*tan(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 641 vs. 2(67) = 134.
time = 0.94, size = 641, normalized size = 6.75

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(2-3*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2028*sqrt(3)*(2*(2*d^2*sqrt(1014*sqrt(13) - 702) - 3*d*sqrt(1014*sqrt(13
) + 702)*abs(d))*arctan(13/18*(9/13)^(3/4)*(2*(9/13)^(1/4)*sqrt(-3/26*sqrt(
13) + 1/2) + (sqrt(3)*sqrt(tan(d*x + c)) - sqrt(2))/sqrt(-3*tan(d*x + c) +
2) - sqrt(-3*tan(d*x + c) + 2)/(sqrt(3)*sqrt(tan(d*x + c)) - sqrt(2)))/sqrt
(3/26*sqrt(13) + 1/2))/d^3 + 2*(2*d^2*sqrt(1014*sqrt(13) - 702) - 3*d*sqrt(
1014*sqrt(13) + 702)*abs(d))*arctan(-13/18*(9/13)^(3/4)*(2*(9/13)^(1/4)*sqr
t(-3/26*sqrt(13) + 1/2) - (sqrt(3)*sqrt(tan(d*x + c)) - sqrt(2))/sqrt(-3*ta
n(d*x + c) + 2) + sqrt(-3*tan(d*x + c) + 2)/(sqrt(3)*sqrt(tan(d*x + c)) - s
qrt(2)))/sqrt(3/26*sqrt(13) + 1/2))/d^3 + (2*d^2*sqrt(1014*sqrt(13) + 702)
+ 3*d*sqrt(1014*sqrt(13) - 702)*abs(d))*log(((sqrt(3)*sqrt(tan(d*x + c)) -
```

```
sqrt(2))/sqrt(-3*tan(d*x + c) + 2) - sqrt(-3*tan(d*x + c) + 2)/(sqrt(3)*sqr
t(tan(d*x + c)) - sqrt(2)))^2 + 4*(9/13)^(1/4)*sqrt(-3/26*sqrt(13) + 1/2)*(
(sqrt(3)*sqrt(tan(d*x + c)) - sqrt(2))/sqrt(-3*tan(d*x + c) + 2) - sqrt(-3*
tan(d*x + c) + 2)/(sqrt(3)*sqrt(tan(d*x + c)) - sqrt(2))) + 12*sqrt(1/13))/
d^3 - (2*d^2*sqrt(1014*sqrt(13) + 702) + 3*d*sqrt(1014*sqrt(13) - 702)*abs(
d))*log(((sqrt(3)*sqrt(tan(d*x + c)) - sqrt(2))/sqrt(-3*tan(d*x + c) + 2) -
 sqrt(-3*tan(d*x + c) + 2)/(sqrt(3)*sqrt(tan(d*x + c)) - sqrt(2)))^2 - 4*(9
/13)^(1/4)*sqrt(-3/26*sqrt(13) + 1/2)*((sqrt(3)*sqrt(tan(d*x + c)) - sqrt(2
))/sqrt(-3*tan(d*x + c) + 2) - sqrt(-3*tan(d*x + c) + 2)/(sqrt(3)*sqrt(tan(
d*x + c)) - sqrt(2))) + 12*sqrt(1/13))/d^3)
```

Mupad [B]
time = 5.43, size = 205, normalized size = 2.16

$$\mathrm{atan}\left(\frac{\sqrt{2}\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{\frac{3}{52}-\frac{1}{26}\mathrm{i}}{d^2}}\,\left(6+4\mathrm{i}\right)+d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{\frac{3}{52}-\frac{1}{26}\mathrm{i}}{d^2}}\,\sqrt{2-3\tan\left(c+d\,x\right)}\,\left(-6-4\mathrm{i}\right)}{3\tan\left(c+d\,x\right)+\sqrt{2}\,\sqrt{2-3\tan\left(c+d\,x\right)}-2}\right)\,\sqrt{\frac{\frac{3}{52}-\frac{1}{26}\mathrm{i}}{d^2}}\,2\mathrm{i}-\mathrm{atan}\left(\frac{\sqrt{2}\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{\frac{3}{52}+\frac{1}{26}\mathrm{i}}{d^2}}\,\left(6-4\mathrm{i}\right)+d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{\frac{3}{52}+\frac{1}{26}\mathrm{i}}{d^2}}\,\sqrt{2-3\tan\left(c+d\,x\right)}\,\left(-6+4\mathrm{i}\right)}{3\tan\left(c+d\,x\right)+\sqrt{2}\,\sqrt{2-3\tan\left(c+d\,x\right)}-2}\right)\,\sqrt{\frac{\frac{3}{52}+\frac{1}{26}\mathrm{i}}{d^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)/(2 - 3*tan(c + d*x))^(1/2),x)
```

```
[Out] atan((2^(1/2)*d*tan(c + d*x)^(1/2)*((3/52 - 1i/26)/d^2)^(1/2)*(6 + 4i) - d*
tan(c + d*x)^(1/2)*((3/52 - 1i/26)/d^2)^(1/2)*(2 - 3*tan(c + d*x))^(1/2)*(6
 + 4i))/(3*tan(c + d*x) + 2^(1/2)*(2 - 3*tan(c + d*x))^(1/2) - 2))*((3/52 -
 1i/26)/d^2)^(1/2)*2i - atan((2^(1/2)*d*tan(c + d*x)^(1/2)*((3/52 + 1i/26)/
d^2)^(1/2)*(6 - 4i) - d*tan(c + d*x)^(1/2)*((3/52 + 1i/26)/d^2)^(1/2)*(2 -
3*tan(c + d*x))^(1/2)*(6 - 4i))/(3*tan(c + d*x) + 2^(1/2)*(2 - 3*tan(c + d*
x))^(1/2) - 2))*((3/52 + 1i/26)/d^2)^(1/2)*2i
```

3.666 $\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{-2-3\tan(c+dx)}}\,dx$

Optimal. Leaf size=95

$$\frac{i\text{ArcTan}\left(\frac{\sqrt{3-2i}\,\sqrt{\tan(c+dx)}}{\sqrt{-2-3\tan(c+dx)}}\right)}{\sqrt{3-2i}\,d}-\frac{i\text{ArcTan}\left(\frac{\sqrt{3+2i}\,\sqrt{\tan(c+dx)}}{\sqrt{-2-3\tan(c+dx)}}\right)}{\sqrt{3+2i}\,d}$$

```
[Out] I*arctan((3-2*I)^(1/2)*tan(d*x+c)^(1/2)/(-2-3*tan(d*x+c))^(1/2))/d/(3-2*I)^
(1/2)-I*arctan((3+2*I)^(1/2)*tan(d*x+c)^(1/2)/(-2-3*tan(d*x+c))^(1/2))/d/(3
+2*I)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3656, 924, 95, 211}

$$\frac{i\text{ArcTan}\left(\frac{\sqrt{3-2i}\,\sqrt{\tan(c+dx)}}{\sqrt{-3\tan(c+dx)-2}}\right)}{\sqrt{3-2i}\,d}-\frac{i\text{ArcTan}\left(\frac{\sqrt{3+2i}\,\sqrt{\tan(c+dx)}}{\sqrt{-3\tan(c+dx)-2}}\right)}{\sqrt{3+2i}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[c + d*x]]/Sqrt[-2 - 3*Tan[c + d*x]],x]
```

```
[Out] (I*ArcTan[(Sqrt[3 - 2*I]*Sqrt[Tan[c + d*x]])/Sqrt[-2 - 3*Tan[c + d*x]]])/(S
qrt[3 - 2*I]*d) - (I*ArcTan[(Sqrt[3 + 2*I]*Sqrt[Tan[c + d*x]])/Sqrt[-2 - 3*
Tan[c + d*x]]])/(Sqrt[3 + 2*I]*d)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
```

```
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{-2-3\tan(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{-2-3x}\,(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{2\sqrt{-2-3x}\,(i-x)\sqrt{x}} + \frac{1}{2\sqrt{-2-3x}\,\sqrt{x}\,(i+x)}\right) dx, x, \tan(c+\right.}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{-2-3x}\,(i-x)\sqrt{x}}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{-2-3x}\right.}{}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{i-(2-3i)x^2}\,dx, x, \frac{\sqrt{\tan(c+dx)}}{\sqrt{-2-3\tan(c+dx)}}\right)}{d} - \frac{\text{Subst}\left(\int \frac{1}{i+(2+3i)x^2}\,dx\right.}{}$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{3-2i}\,\sqrt{\tan(c+dx)}}{\sqrt{-2-3\tan(c+dx)}}\right)}{\sqrt{3-2i}\,d} - \frac{i\tan^{-1}\left(\frac{\sqrt{3+2i}\,\sqrt{\tan(c+dx)}}{\sqrt{-2-3\tan(c+dx)}}\right)}{\sqrt{3+2i}\,d}$$

Mathematica [A]
time = 0.07, size = 103, normalized size = 1.08

$$-\frac{i\left(\sqrt{3+2i}\,\text{ArcTan}\left(\frac{\sqrt{\frac{3}{13}+\frac{2i}{13}}\,\sqrt{-2-3\tan(c+dx)}}{\sqrt{\tan(c+dx)}}\right)+\sqrt{-3+2i}\,\tanh^{-1}\left(\frac{\sqrt{-\frac{3}{13}+\frac{2i}{13}}\,\sqrt{-2-3\tan(c+dx)}}{\sqrt{\tan(c+dx)}}\right)\right)}{\sqrt{13}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tan[c + d*x]]/Sqrt[-2 - 3*Tan[c + d*x]],x]
```

```
[Out] ((-I)*(Sqrt[3 + 2*I]*ArcTan[(Sqrt[3/13 + (2*I)/13]*Sqrt[-2 - 3*Tan[c + d*x]
])/Sqrt[Tan[c + d*x]]] + Sqrt[-3 + 2*I]*ArcTanh[(Sqrt[-3/13 + (2*I)/13]*Sqr
t[-2 - 3*Tan[c + d*x]])/Sqrt[Tan[c + d*x]]]))/(Sqrt[13]*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(77) = 154$.

time = 0.68, size = 434, normalized size = 4.57

method	result
derivativedivides	$\frac{\sqrt{-2-3\tan(dx+c)}\,\sqrt{-\frac{\tan(dx+c)(2+3\tan(dx+c))}{\left(\sqrt{13}-3+2\tan(dx+c)\right)^2}}\left(\sqrt{13}-3+2\tan(dx+c)\right)\left(\sqrt{2\sqrt{13}-6}\,\sqrt{\ }\right.}{}$
default	$\frac{\sqrt{-2-3\tan(dx+c)}\,\sqrt{-\frac{\tan(dx+c)(2+3\tan(dx+c))}{\left(\sqrt{13}-3+2\tan(dx+c)\right)^2}}\left(\sqrt{13}-3+2\tan(dx+c)\right)\left(\sqrt{2\sqrt{13}-6}\,\sqrt{\ }\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)/(-2-3*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(-2-3*tan(d*x+c))^(1/2)*(-tan(d*x+c)*(2+3*tan(d*x+c))/(13^(1/2)-3+2*t
an(d*x+c))^2)^(1/2)*(13^(1/2)-3+2*tan(d*x+c))*((2*13^(1/2)-6)^(1/2)*13^(1/2
)*(2*13^(1/2)+6)^(1/2)*arctanh(1/52*(13^(1/2)+3)*(13^(1/2)+3-2*tan(d*x+c))*
(11*13^(1/2)-39)/(13^(1/2)-3+2*tan(d*x+c))/(2*13^(1/2)-6)^(1/2)*13^(1/2)/(-
tan(d*x+c)*(2+3*tan(d*x+c))/(13^(1/2)-3+2*tan(d*x+c))^2)^(1/2))-3*(2*13^(1/
2)-6)^(1/2)*(2*13^(1/2)+6)^(1/2)*arctanh(1/52*(13^(1/2)+3)*(13^(1/2)+3-2*ta
n(d*x+c))*(11*13^(1/2)-39)/(13^(1/2)-3+2*tan(d*x+c))/(2*13^(1/2)-6)^(1/2)*1
3^(1/2)/(-tan(d*x+c)*(2+3*tan(d*x+c))/(13^(1/2)-3+2*tan(d*x+c))^2)^(1/2))+1
2*arctan(4*13^(1/2)*(-tan(d*x+c)*(2+3*tan(d*x+c))/(13^(1/2)-3+2*tan(d*x+c))
^2)^(1/2)/(26*13^(1/2)+78)^(1/2))*13^(1/2)-44*arctan(4*13^(1/2)*(-tan(d*x+c
)*(2+3*tan(d*x+c))/(13^(1/2)-3+2*tan(d*x+c))^2)^(1/2)/(26*13^(1/2)+78)^(1/2
)))/tan(d*x+c)^(1/2)/(2+3*tan(d*x+c))/(2*13^(1/2)+6)^(1/2)/(11*13^(1/2)-39)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(-2-3*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(tan(d*x + c))/sqrt(-3*tan(d*x + c) - 2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(-2-3*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\tan{\left(c + dx \right)}}}{\sqrt{-3\tan{\left(c + dx \right)} - 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/2)/(-2-3*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(tan(c + d*x))/sqrt(-3*tan(c + d*x) - 2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(-2-3*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, choosing root of [1,0,-702,-
2704,70473] at parameters values [0]Invalid _EXT in replace_ext Error: Bad
Argume
```

Mupad [**B**]
time = 5.59, size = 189, normalized size = 1.99

$$-\mathrm{atan}\left(\frac{12\,d\,\sqrt{\frac{\frac{3}{52}+\frac{1}{26}\mathrm{i}}{d^2}}\,\left(-\sqrt{\tan\left(c+d\,x\right)}+\frac{\sqrt{2}\,\sqrt{3}\,1\mathrm{i}}{3}\right)}{\sqrt{-3\,\tan\left(c+d\,x\right)-2}\,\left(\frac{3\,{\left(-\sqrt{\tan\left(c+d\,x\right)}+\frac{\sqrt{2}\,\sqrt{3}\,1\mathrm{i}}{3}\right)}^2}{3\,\tan\left(c+d\,x\right)+2}+1\right)}\right)\,\sqrt{\frac{\frac{3}{52}+\frac{1}{26}\mathrm{i}}{d^2}}\,2\mathrm{i}+\mathrm{atan}\left(\frac{12\,d\,\sqrt{\frac{\frac{3}{52}-\frac{1}{26}\mathrm{i}}{d^2}}\,\left(-\sqrt{\tan\left(c+d\,x\right)}+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}{\sqrt{-3\,\tan\left(c+d\,x\right)-2}\,\left(\frac{3\,{\left(-\sqrt{\tan\left(c+d\,x\right)}+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}^2}{3\,\tan\left(c+d\,x\right)+2}+1\right)}\right)\,\sqrt{\frac{\frac{3}{52}-\frac{1}{26}\mathrm{i}}{d^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)/(- 3*tan(c + d*x) - 2)^(1/2),x)
```

```
[Out] atan((12*d*((3/52 - 1i/26)/d^2)^(1/2)*((6^(1/2)*1i)/3 - tan(c + d*x)^(1/2))
)/((- 3*tan(c + d*x) - 2)^(1/2)*((3*((6^(1/2)*1i)/3 - tan(c + d*x)^(1/2))^2
)/(3*tan(c + d*x) + 2) + 1)))*((3/52 - 1i/26)/d^2)^(1/2)*2i - atan((12*d*((
3/52 + 1i/26)/d^2)^(1/2)*((2^(1/2)*3^(1/2)*1i)/3 - tan(c + d*x)^(1/2)))/((-
 3*tan(c + d*x) - 2)^(1/2)*((3*((2^(1/2)*3^(1/2)*1i)/3 - tan(c + d*x)^(1/2)
)^2)/(3*tan(c + d*x) + 2) + 1)))*((3/52 + 1i/26)/d^2)^(1/2)*2i
```

3.667 $\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{3+2\tan(c+dx)}}\,dx$

Optimal. Leaf size=95

$$\frac{i\tanh^{-1}\left(\frac{\sqrt{2-3i}\sqrt{\tan(c+dx)}}{\sqrt{3+2\tan(c+dx)}}\right)}{\sqrt{2-3i}\,d}-\frac{i\tanh^{-1}\left(\frac{\sqrt{2+3i}\sqrt{\tan(c+dx)}}{\sqrt{3+2\tan(c+dx)}}\right)}{\sqrt{2+3i}\,d}$$

```
[Out] I*arctanh((2-3*I)^(1/2)*tan(d*x+c)^(1/2)/(3+2*tan(d*x+c))^(1/2))/d/(2-3*I)^
(1/2)-I*arctanh((2+3*I)^(1/2)*tan(d*x+c)^(1/2)/(3+2*tan(d*x+c))^(1/2))/d/(2
+3*I)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3656, 924, 95, 214}

$$\frac{i\tanh^{-1}\left(\frac{\sqrt{2-3i}\sqrt{\tan(c+dx)}}{\sqrt{2\tan(c+dx)+3}}\right)}{\sqrt{2-3i}\,d}-\frac{i\tanh^{-1}\left(\frac{\sqrt{2+3i}\sqrt{\tan(c+dx)}}{\sqrt{2\tan(c+dx)+3}}\right)}{\sqrt{2+3i}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[c + d*x]]/Sqrt[3 + 2*Tan[c + d*x]],x]
```

```
[Out] (I*ArcTanh[(Sqrt[2 - 3*I]*Sqrt[Tan[c + d*x]])/Sqrt[3 + 2*Tan[c + d*x]]])/(S
qrt[2 - 3*I]*d) - (I*ArcTanh[(Sqrt[2 + 3*I]*Sqrt[Tan[c + d*x]])/Sqrt[3 + 2*
Tan[c + d*x]]])/(Sqrt[2 + 3*I]*d)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
```

```
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{3+2\tan(c+dx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{3+2x}\,(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{2(i-x)\sqrt{x}\sqrt{3+2x}}+\frac{1}{2\sqrt{x}\,(i+x)\sqrt{3+2x}}\right)dx, x, \tan(c+dx)\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{(i-x)\sqrt{x}\sqrt{3+2x}}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(i+x)\sqrt{3+2\cdots}}\right.}{2d}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{i-(3+2i)x^2}\,dx, x, \frac{\sqrt{\tan(c+dx)}}{\sqrt{3+2\tan(c+dx)}}\right)}{d} + \frac{\text{Subst}\left(\int \frac{1}{i+(3-2i)x^2}\,dx, x, \cdots\right.}{d}\\
&= \frac{i\tanh^{-1}\left(\frac{\sqrt{2-3i}\sqrt{\tan(c+dx)}}{\sqrt{3+2\tan(c+dx)}}\right)}{\sqrt{2-3i}\,d} - \frac{i\tanh^{-1}\left(\frac{\sqrt{2+3i}\sqrt{\tan(c+dx)}}{\sqrt{3+2\tan(c+dx)}}\right)}{\sqrt{2+3i}\,d}
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 95, normalized size = 1.00

$$\frac{i\text{ArcTan}\left(\frac{\sqrt{-2+3i}\sqrt{\tan(c+dx)}}{\sqrt{3+2\tan(c+dx)}}\right)}{\sqrt{-2+3i}\,d} - \frac{i\tanh^{-1}\left(\frac{\sqrt{2+3i}\sqrt{\tan(c+dx)}}{\sqrt{3+2\tan(c+dx)}}\right)}{\sqrt{2+3i}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tan[c + d*x]]/Sqrt[3 + 2*Tan[c + d*x]],x]
```

```
[Out] (I*ArcTan[(Sqrt[-2 + 3*I]*Sqrt[Tan[c + d*x]])/Sqrt[3 + 2*Tan[c + d*x]]])/(S
qrt[-2 + 3*I]*d) - (I*ArcTanh[(Sqrt[2 + 3*I]*Sqrt[Tan[c + d*x]])/Sqrt[3 + 2
*Tan[c + d*x]]])/(Sqrt[2 + 3*I]*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(77) = 154$.

time = 0.87, size = 479, normalized size = 5.04

method	result
derivativedivides	$-\frac{3\sqrt{\frac{\tan(dx+c)(3+2\tan(dx+c))}{\left(\sqrt{13}-2+3\tan(dx+c)\right)^2}}\left(\sqrt{13}-2+3\tan(dx+c)\right)\left(\sqrt{13}\,\sqrt{2\sqrt{13}+4}\,\arctan\left(\sqrt{\frac{\left(17\sqrt{13}\right.}{}}\right.\right.}{}$
default	$-\frac{3\sqrt{\frac{\tan(dx+c)(3+2\tan(dx+c))}{\left(\sqrt{13}-2+3\tan(dx+c)\right)^2}}\left(\sqrt{13}-2+3\tan(dx+c)\right)\left(\sqrt{13}\,\sqrt{2\sqrt{13}+4}\,\arctan\left(\sqrt{\frac{\left(17\sqrt{13}\right.}{}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)/(3+2*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -3/2/d*(tan(d*x+c)*(3+2*tan(d*x+c))/(13^(1/2)-2+3*tan(d*x+c))^2)^(1/2)*(13^
(1/2)-2+3*tan(d*x+c))*(13^(1/2)*(2*13^(1/2)+4)^(1/2)*arctan(1/56862*((17*13
^(1/2)-52)*tan(d*x+c)*(3+2*tan(d*x+c))*(52+17*13^(1/2))/(13^(1/2)-2+3*tan(d
*x+c))^2)^(1/2)*(-4+2*13^(1/2))^(1/2)*(4*13^(1/2)+17)*(13^(1/2)+2-3*tan(d*x
+c))*(17*13^(1/2)-52)*(13^(1/2)-2+3*tan(d*x+c))/tan(d*x+c)/(3+2*tan(d*x+c))
)*(-4+2*13^(1/2))^(1/2)-2*(2*13^(1/2)+4)^(1/2)*arctan(1/56862*((17*13^(1/2)
-52)*tan(d*x+c)*(3+2*tan(d*x+c))*(52+17*13^(1/2))/(13^(1/2)-2+3*tan(d*x+c))
^2)^(1/2)*(-4+2*13^(1/2))^(1/2)*(4*13^(1/2)+17)*(13^(1/2)+2-3*tan(d*x+c))*(
17*13^(1/2)-52)*(13^(1/2)-2+3*tan(d*x+c))/tan(d*x+c)/(3+2*tan(d*x+c)))*(-4+
2*13^(1/2))^(1/2)-8*arctanh(6*13^(1/2)*(tan(d*x+c)*(3+2*tan(d*x+c))/(13^(1/
2)-2+3*tan(d*x+c))^2)^(1/2)/(26*13^(1/2)+52)^(1/2))*13^(1/2)+34*arctanh(6*1
3^(1/2)*(tan(d*x+c)*(3+2*tan(d*x+c))/(13^(1/2)-2+3*tan(d*x+c))^2)^(1/2)/(26
*13^(1/2)+52)^(1/2)))/tan(d*x+c)^(1/2)/(3+2*tan(d*x+c))^(1/2)/(2*13^(1/2)+4
)^(1/2)/(17*13^(1/2)-52)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^(1/2)/(3+2*tan(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(tan(d*x + c))/sqrt(2*tan(d*x + c) + 3), x)

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^(1/2)/(3+2*tan(d*x+c))^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\tan(c + dx)}}{\sqrt{2\tan(c + dx) + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)**(1/2)/(3+2*tan(d*x+c))**(1/2),x)

[Out] Integral(sqrt(tan(c + d*x))/sqrt(2*tan(c + d*x) + 3), x)

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 498 vs. 2(67) = 134.
time = 1.36, size = 498, normalized size = 5.24

$$-\frac{1}{676}\sqrt{2}\left(\frac{3\left(2\sqrt{325\sqrt{13}-1118}\arctan\left(\frac{13\left(\frac{4}{13}\right)^{\frac{3}{4}}\left(\left(\frac{4}{13}\right)^{\frac{1}{4}}\sqrt{\frac{1}{13}\sqrt{13}+\frac{1}{2}}+1\right)}{4\sqrt{-\frac{1}{13}\sqrt{13}+\frac{1}{2}}}\right)+2\sqrt{325\sqrt{13}-1118}\arctan\left(-\frac{13\left(\frac{4}{13}\right)^{\frac{3}{4}}\left(\left(\frac{4}{13}\right)^{\frac{1}{4}}\sqrt{\frac{1}{13}\sqrt{13}+\frac{1}{2}}-1\right)}{4\sqrt{-\frac{1}{13}\sqrt{13}+\frac{1}{2}}}\right)-\sqrt{325\sqrt{13}-1118}\log\left(2\left(\frac{4}{13}\right)^{\frac{1}{4}}\sqrt{\frac{1}{13}\sqrt{13}+\frac{1}{2}}+2\sqrt{\frac{1}{13}}+1\right)+\sqrt{325\sqrt{13}-1118}\log\left(-2\left(\frac{4}{13}\right)^{\frac{1}{4}}\sqrt{\frac{1}{13}\sqrt{13}+\frac{1}{2}}+2\sqrt{\frac{1}{13}}+1\right)\right)}{d}-\frac{2\left(3d^2\sqrt{325\sqrt{13}-1118}-2d\sqrt{325\sqrt{13}-1118}|d|\right)\arctan\left(\frac{13\left(\frac{4}{13}\right)^{\frac{3}{4}}\left(\left(\frac{4}{13}\right)^{\frac{1}{4}}\sqrt{\frac{1}{13}\sqrt{13}+\frac{1}{2}}+\sqrt{-\frac{3}{2\tan(dx+c)+3}+1}\right)}{4\sqrt{-\frac{1}{13}\sqrt{13}+\frac{1}{2}}}\right)}{d^3}-\frac{2\left(3d^2\sqrt{325\sqrt{13}-1118}-2d\sqrt{325\sqrt{13}-1118}|d|\right)\arctan\left(-\frac{13\left(\frac{4}{13}\right)^{\frac{3}{4}}\left(\left(\frac{4}{13}\right)^{\frac{1}{4}}\sqrt{\frac{1}{13}\sqrt{13}+\frac{1}{2}}-\sqrt{-\frac{3}{2\tan(dx+c)+3}+1}\right)}{4\sqrt{-\frac{1}{13}\sqrt{13}+\frac{1}{2}}}\right)}{d^3}+\frac{\left(3d^2\sqrt{325\sqrt{13}-1118}+2d\sqrt{325\sqrt{13}-1118}|d|\right)\log\left(2\left(\frac{4}{13}\right)^{\frac{1}{4}}\sqrt{\frac{1}{13}\sqrt{13}+\frac{1}{2}}\sqrt{-\frac{3}{2\tan(dx+c)+3}+1}+2\sqrt{\frac{1}{13}}-\frac{3}{2\tan(dx+c)+3}+1\right)}{d^3}-\frac{\left(3d^2\sqrt{325\sqrt{13}-1118}+2d\sqrt{325\sqrt{13}-1118}|d|\right)\log\left(-2\left(\frac{4}{13}\right)^{\frac{1}{4}}\sqrt{\frac{1}{13}\sqrt{13}+\frac{1}{2}}\sqrt{-\frac{3}{2\tan(dx+c)+3}+1}+2\sqrt{\frac{1}{13}}-\frac{3}{2\tan(dx+c)+3}+1\right)}{d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^(1/2)/(3+2*tan(d*x+c))^(1/2),x, algorithm="giac")

```
[Out] -1/676*sqrt(2)*(3*(2*sqrt(325*sqrt(13) - 1118)*arctan(13/4*(4/13)^(3/4)*((4
/13)^(1/4)*sqrt(1/13*sqrt(13) + 1/2) + 1)/sqrt(-1/13*sqrt(13) + 1/2)) + 2*s
qrt(325*sqrt(13) - 1118)*arctan(-13/4*(4/13)^(3/4)*((4/13)^(1/4)*sqrt(1/13*
sqrt(13) + 1/2) - 1)/sqrt(-1/13*sqrt(13) + 1/2)) - sqrt(325*sqrt(13) - 1118
)*log(2*(4/13)^(1/4)*sqrt(1/13*sqrt(13) + 1/2) + 2*sqrt(1/13) + 1) + sqrt(3
25*sqrt(13) - 1118)*log(-2*(4/13)^(1/4)*sqrt(1/13*sqrt(13) + 1/2) + 2*sqrt(
1/13) + 1))/d - 2*(3*d^2*sqrt(325*sqrt(13) - 1118) - 2*d*sqrt(325*sqrt(13)
- 1118)*abs(d))*arctan(13/4*(4/13)^(3/4)*((4/13)^(1/4)*sqrt(1/13*sqrt(13) +
 1/2) + sqrt(-3/(2*tan(d*x + c) + 3) + 1))/sqrt(-1/13*sqrt(13) + 1/2))/d^3
- 2*(3*d^2*sqrt(325*sqrt(13) - 1118) - 2*d*sqrt(325*sqrt(13) - 1118)*abs(d)
)*arctan(-13/4*(4/13)^(3/4)*((4/13)^(1/4)*sqrt(1/13*sqrt(13) + 1/2) - sqrt(
-3/(2*tan(d*x + c) + 3) + 1))/sqrt(-1/13*sqrt(13) + 1/2))/d^3 + (3*d^2*sqrt
```

```
(325*sqrt(13) - 1118) + 2*d*sqrt(325*sqrt(13) - 1118)*abs(d))*log(2*(4/13)^
(1/4)*sqrt(1/13*sqrt(13) + 1/2)*sqrt(-3/(2*tan(d*x + c) + 3) + 1) + 2*sqrt(
1/13) - 3/(2*tan(d*x + c) + 3) + 1)/d^3 - (3*d^2*sqrt(325*sqrt(13) - 1118)
+ 2*d*sqrt(325*sqrt(13) - 1118)*abs(d))*log(-2*(4/13)^(1/4)*sqrt(1/13*sqrt(
13) + 1/2)*sqrt(-3/(2*tan(d*x + c) + 3) + 1) + 2*sqrt(1/13) - 3/(2*tan(d*x
+ c) + 3) + 1)/d^3)
```

Mupad [B]
time = 5.61, size = 207, normalized size = 2.18

$$\mathrm{atan}\left(\frac{\sqrt{3}\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{-\frac{1}{26}-\frac{3}{52}\mathrm{i}}{d^2}}\,\left(4-6\mathrm{i}\right)+d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{-\frac{1}{26}-\frac{3}{52}\mathrm{i}}{d^2}}\,\sqrt{2\tan\left(c+d\,x\right)+3}\,\left(-4+6\mathrm{i}\right)}{2\tan\left(c+d\,x\right)-\sqrt{3}\,\sqrt{2\tan\left(c+d\,x\right)+3}+3}\right)\sqrt{\frac{-\frac{1}{26}-\frac{3}{52}\mathrm{i}}{d^2}}\,2\mathrm{i}-\mathrm{atan}\left(\frac{\sqrt{3}\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{-\frac{1}{26}+\frac{3}{52}\mathrm{i}}{d^2}}\,\left(4+6\mathrm{i}\right)+d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{-\frac{1}{26}+\frac{3}{52}\mathrm{i}}{d^2}}\,\sqrt{2\tan\left(c+d\,x\right)+3}\,\left(-4-6\mathrm{i}\right)}{2\tan\left(c+d\,x\right)-\sqrt{3}\,\sqrt{2\tan\left(c+d\,x\right)+3}+3}\right)\sqrt{\frac{-\frac{1}{26}+\frac{3}{52}\mathrm{i}}{d^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)/(2*tan(c + d*x) + 3)^(1/2),x)
```

```
[Out] atan((3^(1/2)*d*tan(c + d*x)^(1/2)*((- 1/26 - 3i/52)/d^2)^(1/2)*(4 - 6i) -
d*tan(c + d*x)^(1/2)*((- 1/26 - 3i/52)/d^2)^(1/2)*(2*tan(c + d*x) + 3)^(1/2
)*(4 - 6i))/(2*tan(c + d*x) - 3^(1/2)*(2*tan(c + d*x) + 3)^(1/2) + 3))*((-
1/26 - 3i/52)/d^2)^(1/2)*2i - atan((3^(1/2)*d*tan(c + d*x)^(1/2)*((- 1/26 +
 3i/52)/d^2)^(1/2)*(4 + 6i) - d*tan(c + d*x)^(1/2)*((- 1/26 + 3i/52)/d^2)^(
1/2)*(2*tan(c + d*x) + 3)^(1/2)*(4 + 6i))/(2*tan(c + d*x) - 3^(1/2)*(2*tan(
c + d*x) + 3)^(1/2) + 3))*((- 1/26 + 3i/52)/d^2)^(1/2)*2i
```

3.668 $\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{3-2\tan(c+dx)}}\,dx$

Optimal. Leaf size=95

$$-\frac{i\text{ArcTan}\left(\frac{\sqrt{2-3i}\,\sqrt{\tan(c+dx)}}{\sqrt{3-2\tan(c+dx)}}\right)}{\sqrt{2-3i}\,d}+\frac{i\text{ArcTan}\left(\frac{\sqrt{2+3i}\,\sqrt{\tan(c+dx)}}{\sqrt{3-2\tan(c+dx)}}\right)}{\sqrt{2+3i}\,d}$$

```
[Out] -I*arctan((2-3*I)^(1/2)*tan(d*x+c)^(1/2)/(3-2*tan(d*x+c))^(1/2))/d/(2-3*I)^
(1/2)+I*arctan((2+3*I)^(1/2)*tan(d*x+c)^(1/2)/(3-2*tan(d*x+c))^(1/2))/d/(2+
3*I)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3656, 924, 95, 211}

$$\frac{i\text{ArcTan}\left(\frac{\sqrt{2+3i}\,\sqrt{\tan(c+dx)}}{\sqrt{3-2\tan(c+dx)}}\right)}{\sqrt{2+3i}\,d}-\frac{i\text{ArcTan}\left(\frac{\sqrt{2-3i}\,\sqrt{\tan(c+dx)}}{\sqrt{3-2\tan(c+dx)}}\right)}{\sqrt{2-3i}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[c + d*x]]/Sqrt[3 - 2*Tan[c + d*x]],x]
```

```
[Out] ((-I)*ArcTan[(Sqrt[2 - 3*I]*Sqrt[Tan[c + d*x]])/Sqrt[3 - 2*Tan[c + d*x]]])/
(Sqrt[2 - 3*I]*d) + (I*ArcTan[(Sqrt[2 + 3*I]*Sqrt[Tan[c + d*x]])/Sqrt[3 - 2
*Tan[c + d*x]]])/(Sqrt[2 + 3*I]*d)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
```

```
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{3-2\tan(c+dx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{3-2x}\,(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{2\sqrt{3-2x}\,(i-x)\sqrt{x}} + \frac{1}{2\sqrt{3-2x}\,\sqrt{x}\,(i+x)}\right) dx, x, \tan(c+dx)\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{3-2x}\,(i-x)\sqrt{x}}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{3-2x}\,\sqrt{x}
\\
&= -\frac{\text{Subst}\left(\int \frac{1}{i-(3-2i)x^2}\,dx, x, \frac{\sqrt{\tan(c+dx)}}{\sqrt{3-2\tan(c+dx)}}\right)}{d} + \frac{\text{Subst}\left(\int \frac{1}{i+(3+2i)x^2}\,dx, x
\\
&= -\frac{i\tan^{-1}\left(\frac{\sqrt{2-3i}\,\sqrt{\tan(c+dx)}}{\sqrt{3-2\tan(c+dx)}}\right)}{\sqrt{2-3i}\,d} + \frac{i\tan^{-1}\left(\frac{\sqrt{2+3i}\,\sqrt{\tan(c+dx)}}{\sqrt{3-2\tan(c+dx)}}\right)}{\sqrt{2+3i}\,d}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 103, normalized size = 1.08

$$\frac{i\left(\sqrt{2+3i}\,\text{ArcTan}\left(\frac{\sqrt{\frac{2}{13}+\frac{3i}{13}}\,\sqrt{3-2\tan(c+dx)}}{\sqrt{\tan(c+dx)}}\right)+\sqrt{-2+3i}\,\tanh^{-1}\left(\frac{\sqrt{-\frac{2}{13}+\frac{3i}{13}}\,\sqrt{3-2\tan(c+dx)}}{\sqrt{\tan(c+dx)}}\right)\right)}{\sqrt{13}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tan[c + d*x]]/Sqrt[3 - 2*Tan[c + d*x]],x]
```

```
[Out] (I*(Sqrt[2 + 3*I]*ArcTan[(Sqrt[2/13 + (3*I)/13]*Sqrt[3 - 2*Tan[c + d*x]])/S
qrt[Tan[c + d*x]]] + Sqrt[-2 + 3*I]*ArcTanh[(Sqrt[-2/13 + (3*I)/13]*Sqrt[3
- 2*Tan[c + d*x]])/Sqrt[Tan[c + d*x]]]))/(Sqrt[13]*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(77) = 154$.
time = 1.03, size = 434, normalized size = 4.57

method	result
derivativedivides	$\frac{3\sqrt{3-2\tan(dx+c)}\,\sqrt{-\frac{\tan(dx+c)(-3+2\tan(dx+c))}{\left(\sqrt{13}-2-3\tan(dx+c)\right)^{2}}}\left(\sqrt{13}-2-3\tan(dx+c)\right)\left(\sqrt{-4+2\sqrt{13}}\right.}{}$
default	$\frac{3\sqrt{3-2\tan(dx+c)}\,\sqrt{-\frac{\tan(dx+c)(-3+2\tan(dx+c))}{\left(\sqrt{13}-2-3\tan(dx+c)\right)^{2}}}\left(\sqrt{13}-2-3\tan(dx+c)\right)\left(\sqrt{-4+2\sqrt{13}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)/(3-2*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/2/d*(3-2*tan(d*x+c))^(1/2)*(-tan(d*x+c)*(-3+2*tan(d*x+c))/(13^(1/2)-2-3*t
an(d*x+c))^2)^(1/2)*(13^(1/2)-2-3*tan(d*x+c))*(arctanh(1/351*(2+13^(1/2))*(
13^(1/2)+2+3*tan(d*x+c))*(17*13^(1/2)-52)/(-4+2*13^(1/2))^(1/2)/(13^(1/2)-2
-3*tan(d*x+c))*13^(1/2)/(-tan(d*x+c)*(-3+2*tan(d*x+c))/(13^(1/2)-2-3*tan(d*
x+c))^2)^(1/2))*(-4+2*13^(1/2))^(1/2)*13^(1/2)*(2*13^(1/2)+4)^(1/2)-2*arcta
nh(1/351*(2+13^(1/2))*(13^(1/2)+2+3*tan(d*x+c))*(17*13^(1/2)-52)/(-4+2*13^(
1/2))^(1/2)/(13^(1/2)-2-3*tan(d*x+c))*13^(1/2)/(-tan(d*x+c)*(-3+2*tan(d*x+c
))/(13^(1/2)-2-3*tan(d*x+c))^2)^(1/2))*(-4+2*13^(1/2))^(1/2)*(2*13^(1/2)+4)
^(1/2)+8*arctan(6*13^(1/2)*(-tan(d*x+c)*(-3+2*tan(d*x+c))/(13^(1/2)-2-3*tan
(d*x+c))^2)^(1/2)/(26*13^(1/2)+52)^(1/2))*13^(1/2)-34*arctan(6*13^(1/2)*(-t
an(d*x+c)*(-3+2*tan(d*x+c))/(13^(1/2)-2-3*tan(d*x+c))^2)^(1/2)/(26*13^(1/2)
+52)^(1/2)))/tan(d*x+c)^(1/2)/(2*13^(1/2)+4)^(1/2)/(-3+2*tan(d*x+c))/(17*13
^(1/2)-52)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(3-2*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(tan(d*x + c))/sqrt(-2*tan(d*x + c) + 3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(3-2*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{3-2\tan(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/2)/(3-2*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(tan(c + d*x))/sqrt(3 - 2*tan(c + d*x)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 641 vs. 2(67) = 134.
time = 0.89, size = 641, normalized size = 6.75

$$-\frac{1}{676}\sqrt{2}\left(\frac{2\left(3d^2\sqrt{169\sqrt{13}-598}-2d\sqrt{169\sqrt{13}+598}\,|d|\right)\arctan\left(\frac{13\left(\frac{4}{13}\right)^{\frac34}\left(2\left(\frac{4}{13}\right)^{\frac14}\sqrt{-\frac{1}{13}\sqrt{13}+\frac12}+\frac{\sqrt{2}\sqrt{\tan(dx+c)}-\sqrt{3}}{\sqrt{-2\tan(dx+c)+3}}-\frac{\sqrt{-2\tan(dx+c)+3}}{\sqrt{2}\sqrt{\tan(dx+c)}-\sqrt{3}}\right)}{8\sqrt{\frac{1}{13}\sqrt{13}+\frac12}}\right)}{d^3}+\frac{2\left(3d^2\sqrt{169\sqrt{13}-598}-2d\sqrt{169\sqrt{13}+598}\,|d|\right)\arctan\left(-\frac{13\left(\frac{4}{13}\right)^{\frac34}\left(2\left(\frac{4}{13}\right)^{\frac14}\sqrt{-\frac{1}{13}\sqrt{13}+\frac12}-\frac{\sqrt{2}\sqrt{\tan(dx+c)}-\sqrt{3}}{\sqrt{-2\tan(dx+c)+3}}+\frac{\sqrt{-2\tan(dx+c)+3}}{\sqrt{2}\sqrt{\tan(dx+c)}-\sqrt{3}}\right)}{8\sqrt{\frac{1}{13}\sqrt{13}+\frac12}}\right)}{d^3}+\frac{\left(3d^2\sqrt{169\sqrt{13}+598}+2d\sqrt{169\sqrt{13}-598}\,|d|\right)\log\left(\left(\frac{\sqrt{2}\sqrt{\tan(dx+c)}-\sqrt{3}}{\sqrt{-2\tan(dx+c)+3}}-\frac{\sqrt{-2\tan(dx+c)+3}}{\sqrt{2}\sqrt{\tan(dx+c)}-\sqrt{3}}\right)^2+4\left(\frac{4}{13}\right)^{\frac14}\sqrt{-\frac{1}{13}\sqrt{13}+\frac12}\left(\frac{\sqrt{2}\sqrt{\tan(dx+c)}-\sqrt{3}}{\sqrt{-2\tan(dx+c)+3}}-\frac{\sqrt{-2\tan(dx+c)+3}}{\sqrt{2}\sqrt{\tan(dx+c)}-\sqrt{3}}\right)+8\sqrt{\frac{1}{13}}\right)}{d^3}-\frac{\left(3d^2\sqrt{169\sqrt{13}+598}+2d\sqrt{169\sqrt{13}-598}\,|d|\right)\log\left(\left(\frac{\sqrt{2}\sqrt{\tan(dx+c)}-\sqrt{3}}{\sqrt{-2\tan(dx+c)+3}}-\frac{\sqrt{-2\tan(dx+c)+3}}{\sqrt{2}\sqrt{\tan(dx+c)}-\sqrt{3}}\right)^2-4\left(\frac{4}{13}\right)^{\frac14}\sqrt{-\frac{1}{13}\sqrt{13}+\frac12}\left(\frac{\sqrt{2}\sqrt{\tan(dx+c)}-\sqrt{3}}{\sqrt{-2\tan(dx+c)+3}}-\frac{\sqrt{-2\tan(dx+c)+3}}{\sqrt{2}\sqrt{\tan(dx+c)}-\sqrt{3}}\right)+8\sqrt{\frac{1}{13}}\right)}{d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(3-2*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/676*sqrt(2)*(2*(3*d^2*sqrt(169*sqrt(13) - 598) - 2*d*sqrt(169*sqrt(13) +
 598)*abs(d))*arctan(13/8*(4/13)^(3/4)*(2*(4/13)^(1/4)*sqrt(-1/13*sqrt(13)
+ 1/2) + (sqrt(2)*sqrt(tan(d*x + c)) - sqrt(3))/sqrt(-2*tan(d*x + c) + 3) -
 sqrt(-2*tan(d*x + c) + 3)/(sqrt(2)*sqrt(tan(d*x + c)) - sqrt(3)))/sqrt(1/1
3*sqrt(13) + 1/2))/d^3 + 2*(3*d^2*sqrt(169*sqrt(13) - 598) - 2*d*sqrt(169*s
qrt(13) + 598)*abs(d))*arctan(-13/8*(4/13)^(3/4)*(2*(4/13)^(1/4)*sqrt(-1/13
*sqrt(13) + 1/2) - (sqrt(2)*sqrt(tan(d*x + c)) - sqrt(3))/sqrt(-2*tan(d*x +
 c) + 3) + sqrt(-2*tan(d*x + c) + 3)/(sqrt(2)*sqrt(tan(d*x + c)) - sqrt(3))
)/sqrt(1/13*sqrt(13) + 1/2))/d^3 + (3*d^2*sqrt(169*sqrt(13) + 598) + 2*d*sq
rt(169*sqrt(13) - 598)*abs(d))*log(((sqrt(2)*sqrt(tan(d*x + c)) - sqrt(3))/
```

```
sqrt(-2*tan(d*x + c) + 3) - sqrt(-2*tan(d*x + c) + 3)/(sqrt(2)*sqrt(tan(d*x
 + c)) - sqrt(3)))^2 + 4*(4/13)^(1/4)*sqrt(-1/13*sqrt(13) + 1/2)*((sqrt(2)*
sqrt(tan(d*x + c)) - sqrt(3))/sqrt(-2*tan(d*x + c) + 3) - sqrt(-2*tan(d*x +
 c) + 3)/(sqrt(2)*sqrt(tan(d*x + c)) - sqrt(3))) + 8*sqrt(1/13))/d^3 - (3*d
^2*sqrt(169*sqrt(13) + 598) + 2*d*sqrt(169*sqrt(13) - 598)*abs(d))*log(((sq
rt(2)*sqrt(tan(d*x + c)) - sqrt(3))/sqrt(-2*tan(d*x + c) + 3) - sqrt(-2*tan
(d*x + c) + 3)/(sqrt(2)*sqrt(tan(d*x + c)) - sqrt(3)))^2 - 4*(4/13)^(1/4)*s
qrt(-1/13*sqrt(13) + 1/2)*((sqrt(2)*sqrt(tan(d*x + c)) - sqrt(3))/sqrt(-2*t
an(d*x + c) + 3) - sqrt(-2*tan(d*x + c) + 3)/(sqrt(2)*sqrt(tan(d*x + c)) -
sqrt(3))) + 8*sqrt(1/13))/d^3)
```

Mupad [B]

time = 5.68, size = 205, normalized size = 2.16

$$\mathrm{atan}\left(\frac{\sqrt{3}\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{\frac{1}{26}-\frac{3}{52}\mathrm{i}}{d^2}}\,\left(4+6\mathrm{i}\right)+d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{\frac{1}{26}-\frac{3}{52}\mathrm{i}}{d^2}}\,\sqrt{3-2\tan\left(c+d\,x\right)}\,\left(-4-6\mathrm{i}\right)}{2\tan\left(c+d\,x\right)+\sqrt{3}\,\sqrt{3-2\tan\left(c+d\,x\right)}-3}\right)\sqrt{\frac{\frac{1}{26}-\frac{3}{52}\mathrm{i}}{d^2}}\,2\mathrm{i}-\mathrm{atan}\left(\frac{\sqrt{3}\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{\frac{1}{26}+\frac{3}{52}\mathrm{i}}{d^2}}\,\left(4-6\mathrm{i}\right)+d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{\frac{1}{26}+\frac{3}{52}\mathrm{i}}{d^2}}\,\sqrt{3-2\tan\left(c+d\,x\right)}\,\left(-4+6\mathrm{i}\right)}{2\tan\left(c+d\,x\right)+\sqrt{3}\,\sqrt{3-2\tan\left(c+d\,x\right)}-3}\right)\sqrt{\frac{\frac{1}{26}+\frac{3}{52}\mathrm{i}}{d^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)/(3 - 2*tan(c + d*x))^(1/2),x)
```

```
[Out] atan((3^(1/2)*d*tan(c + d*x)^(1/2)*((1/26 - 3i/52)/d^2)^(1/2)*(4 + 6i) - d*
tan(c + d*x)^(1/2)*((1/26 - 3i/52)/d^2)^(1/2)*(3 - 2*tan(c + d*x))^(1/2)*(4
 + 6i))/(2*tan(c + d*x) + 3^(1/2)*(3 - 2*tan(c + d*x))^(1/2) - 3))*((1/26 -
 3i/52)/d^2)^(1/2)*2i - atan((3^(1/2)*d*tan(c + d*x)^(1/2)*((1/26 + 3i/52)/
d^2)^(1/2)*(4 - 6i) - d*tan(c + d*x)^(1/2)*((1/26 + 3i/52)/d^2)^(1/2)*(3 -
2*tan(c + d*x))^(1/2)*(4 - 6i))/(2*tan(c + d*x) + 3^(1/2)*(3 - 2*tan(c + d*
x))^(1/2) - 3))*((1/26 + 3i/52)/d^2)^(1/2)*2i
```

3.669 $\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{-3+2\tan(c+dx)}}\,dx$

Optimal. Leaf size=95

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{2-3i}\sqrt{\tan(c+dx)}}{\sqrt{-3+2\tan(c+dx)}}\right)}{\sqrt{2-3i}\,d}+\frac{i\tanh^{-1}\left(\frac{\sqrt{2+3i}\sqrt{\tan(c+dx)}}{\sqrt{-3+2\tan(c+dx)}}\right)}{\sqrt{2+3i}\,d}$$

```
[Out] -I*arctanh((2-3*I)^(1/2)*tan(d*x+c)^(1/2)/(-3+2*tan(d*x+c))^(1/2))/d/(2-3*I
)^(1/2)+I*arctanh((2+3*I)^(1/2)*tan(d*x+c)^(1/2)/(-3+2*tan(d*x+c))^(1/2))/d
/(2+3*I)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3656, 924, 95, 214}

$$\frac{i\tanh^{-1}\left(\frac{\sqrt{2+3i}\sqrt{\tan(c+dx)}}{\sqrt{2\tan(c+dx)-3}}\right)}{\sqrt{2+3i}\,d}-\frac{i\tanh^{-1}\left(\frac{\sqrt{2-3i}\sqrt{\tan(c+dx)}}{\sqrt{2\tan(c+dx)-3}}\right)}{\sqrt{2-3i}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[c + d*x]]/Sqrt[-3 + 2*Tan[c + d*x]],x]
```

```
[Out] ((-I)*ArcTanh[(Sqrt[2 - 3*I]*Sqrt[Tan[c + d*x]])/Sqrt[-3 + 2*Tan[c + d*x]]]
)/(Sqrt[2 - 3*I]*d) + (I*ArcTanh[(Sqrt[2 + 3*I]*Sqrt[Tan[c + d*x]])/Sqrt[-3
 + 2*Tan[c + d*x]]])/(Sqrt[2 + 3*I]*d)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
```

```
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{-3+2\tan(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{-3+2x}\,(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{2(i-x)\sqrt{x}\sqrt{-3+2x}} + \frac{1}{2\sqrt{x}\,(i+x)\sqrt{-3+2x}}\right) dx, x, \tan(c+d$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{(i-x)\sqrt{x}\sqrt{-3+2x}}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(i+x)\sqrt{-}}\right.}{}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{i-(3+2i)x^2}\,dx, x, \frac{\sqrt{\tan(c+dx)}}{\sqrt{-3+2\tan(c+dx)}}\right)}{d} - \frac{\text{Subst}\left(\int \frac{1}{i+(3-2i)x^2}\,dx,\right.}{}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{2-3i}\sqrt{\tan(c+dx)}}{\sqrt{-3+2\tan(c+dx)}}\right)}{\sqrt{2-3i}\,d} + \frac{i\tanh^{-1}\left(\frac{\sqrt{2+3i}\sqrt{\tan(c+dx)}}{\sqrt{-3+2\tan(c+dx)}}\right.}{\sqrt{2+3i}\,d}$$

Mathematica [A]
time = 0.06, size = 95, normalized size = 1.00

$$-\frac{i\text{ArcTan}\left(\frac{\sqrt{-2+3i}\sqrt{\tan(c+dx)}}{\sqrt{-3+2\tan(c+dx)}}\right)}{\sqrt{-2+3i}\,d} + \frac{i\tanh^{-1}\left(\frac{\sqrt{2+3i}\sqrt{\tan(c+dx)}}{\sqrt{-3+2\tan(c+dx)}}\right)}{\sqrt{2+3i}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tan[c + d*x]]/Sqrt[-3 + 2*Tan[c + d*x]],x]
[Out] ((-I)*ArcTan[(Sqrt[-2 + 3*I]*Sqrt[Tan[c + d*x]])/Sqrt[-3 + 2*Tan[c + d*x]]]
)/(Sqrt[-2 + 3*I]*d) + (I*ArcTanh[(Sqrt[2 + 3*I]*Sqrt[Tan[c + d*x]])/Sqrt[-
3 + 2*Tan[c + d*x]]])/(Sqrt[2 + 3*I]*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(77) = 154$.
time = 1.10, size = 479, normalized size = 5.04

method	result
derivativedivides	$-\frac{3\sqrt{\frac{\tan(dx+c)(-3+2\tan(dx+c))}{\left(\sqrt{13}-2-3\tan(dx+c)\right)^{2}}}\left(\sqrt{13}-2-3\tan(dx+c)\right)\left(\sqrt{13}\,\sqrt{2\sqrt{13}+4}\,\sqrt{-4+2\sqrt{13}}\right.}{}$
default	$-\frac{3\sqrt{\frac{\tan(dx+c)(-3+2\tan(dx+c))}{\left(\sqrt{13}-2-3\tan(dx+c)\right)^{2}}}\left(\sqrt{13}-2-3\tan(dx+c)\right)\left(\sqrt{13}\,\sqrt{2\sqrt{13}+4}\,\sqrt{-4+2\sqrt{13}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)/(-3+2*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -3/2/d*(tan(d*x+c)*(-3+2*tan(d*x+c))/(13^(1/2)-2-3*tan(d*x+c))^2)^(1/2)*(13
^(1/2)-2-3*tan(d*x+c))*(13^(1/2)*(2*13^(1/2)+4)^(1/2)*(-4+2*13^(1/2))^(1/2)
*arctan(1/56862*(-4+2*13^(1/2))^(1/2)*((17*13^(1/2)-52)*tan(d*x+c)*(-3+2*ta
n(d*x+c))*(52+17*13^(1/2))/(13^(1/2)-2-3*tan(d*x+c))^2)^(1/2)*(4*13^(1/2)+1
7)*(13^(1/2)+2+3*tan(d*x+c))*(17*13^(1/2)-52)*(13^(1/2)-2-3*tan(d*x+c))/tan
(d*x+c)/(-3+2*tan(d*x+c)))-2*(2*13^(1/2)+4)^(1/2)*(-4+2*13^(1/2))^(1/2)*arc
tan(1/56862*(-4+2*13^(1/2))^(1/2)*((17*13^(1/2)-52)*tan(d*x+c)*(-3+2*tan(d*
x+c))*(52+17*13^(1/2))/(13^(1/2)-2-3*tan(d*x+c))^2)^(1/2)*(4*13^(1/2)+17)*(
13^(1/2)+2+3*tan(d*x+c))*(17*13^(1/2)-52)*(13^(1/2)-2-3*tan(d*x+c))/tan(d*x
+c)/(-3+2*tan(d*x+c)))-8*arctanh(6*13^(1/2)*(tan(d*x+c)*(-3+2*tan(d*x+c))/(
13^(1/2)-2-3*tan(d*x+c))^2)^(1/2)/(26*13^(1/2)+52)^(1/2))*13^(1/2)+34*arcta
nh(6*13^(1/2)*(tan(d*x+c)*(-3+2*tan(d*x+c))/(13^(1/2)-2-3*tan(d*x+c))^2)^(1
/2)/(26*13^(1/2)+52)^(1/2)))/tan(d*x+c)^(1/2)/(-3+2*tan(d*x+c))^(1/2)/(2*13
^(1/2)+4)^(1/2)/(17*13^(1/2)-52)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(-3+2*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(tan(d*x + c))/sqrt(2*tan(d*x + c) - 3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(-3+2*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\tan{\left(c + dx \right)}}}{\sqrt{2 \tan{\left(c + dx \right)} - 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/2)/(-3+2*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(tan(c + d*x))/sqrt(2*tan(c + d*x) - 3), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 498 vs. 2(67) = 134.
time = 1.24, size = 498, normalized size = 5.24

$$\frac{1}{676}\sqrt{2}\left(\frac{3\left(2\sqrt{325\sqrt{13}-1118}\arctan\left(\frac{13\left(\frac{4}{13}\right)^{\frac{3}{4}}\left(\left(\frac{4}{13}\right)^{\frac{1}{4}}\sqrt{\frac{1}{13}\sqrt{13}+\frac{1}{2}}+1\right)}{4\sqrt{-\frac{1}{13}\sqrt{13}+\frac{1}{2}}}\right)+2\sqrt{325\sqrt{13}-1118}\arctan\left(-\frac{13\left(\frac{4}{13}\right)^{\frac{3}{4}}\left(\left(\frac{4}{13}\right)^{\frac{1}{4}}\sqrt{\frac{1}{13}\sqrt{13}+\frac{1}{2}}-1\right)}{4\sqrt{-\frac{1}{13}\sqrt{13}+\frac{1}{2}}}\right)-\sqrt{325\sqrt{13}-1118}\log\left(2\left(\frac{4}{13}\right)^{\frac{1}{4}}\sqrt{\frac{1}{13}\sqrt{13}+\frac{1}{2}}+2\sqrt{\frac{1}{13}}+1\right)+\sqrt{325\sqrt{13}-1118}\log\left(-2\left(\frac{4}{13}\right)^{\frac{1}{4}}\sqrt{\frac{1}{13}\sqrt{13}+\frac{1}{2}}+2\sqrt{\frac{1}{13}}+1\right)\right)}{d}-\frac{2\left(3d^{2}\sqrt{325\sqrt{13}-1118}-2d\sqrt{325\sqrt{13}-1118}|d|\right)\arctan\left(\frac{13\left(\frac{4}{13}\right)^{\frac{3}{4}}\left(\left(\frac{4}{13}\right)^{\frac{1}{4}}\sqrt{\frac{1}{13}\sqrt{13}+\frac{1}{2}}+\sqrt{\frac{3}{2\tan\left(dx+c\right)-3}+1}\right)}{4\sqrt{-\frac{1}{13}\sqrt{13}+\frac{1}{2}}}\right)}{d^{3}}-\frac{2\left(3d^{2}\sqrt{325\sqrt{13}-1118}-2d\sqrt{325\sqrt{13}-1118}|d|\right)\arctan\left(-\frac{13\left(\frac{4}{13}\right)^{\frac{3}{4}}\left(\left(\frac{4}{13}\right)^{\frac{1}{4}}\sqrt{\frac{1}{13}\sqrt{13}+\frac{1}{2}}-\sqrt{\frac{3}{2\tan\left(dx+c\right)-3}+1}\right)}{4\sqrt{-\frac{1}{13}\sqrt{13}+\frac{1}{2}}}\right)}{d^{3}}+\frac{\left(3d^{2}\sqrt{325\sqrt{13}-1118}+2d\sqrt{325\sqrt{13}-1118}|d|\right)\log\left(2\left(\frac{4}{13}\right)^{\frac{1}{4}}\sqrt{\frac{1}{13}\sqrt{13}+\frac{1}{2}}\sqrt{\frac{3}{2\tan\left(dx+c\right)-3}+1}+2\sqrt{\frac{1}{13}}+\frac{3}{2\tan\left(dx+c\right)-3}+1\right)}{d^{3}}-\frac{\left(3d^{2}\sqrt{325\sqrt{13}-1118}+2d\sqrt{325\sqrt{13}-1118}|d|\right)\log\left(-2\left(\frac{4}{13}\right)^{\frac{1}{4}}\sqrt{\frac{1}{13}\sqrt{13}+\frac{1}{2}}\sqrt{\frac{3}{2\tan\left(dx+c\right)-3}+1}+2\sqrt{\frac{1}{13}}+\frac{3}{2\tan\left(dx+c\right)-3}+1\right)}{d^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(-3+2*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/676*sqrt(2)*(3*(2*sqrt(325*sqrt(13) - 1118)*arctan(13/4*(4/13)^(3/4)*((4/
13)^(1/4)*sqrt(1/13*sqrt(13) + 1/2) + 1)/sqrt(-1/13*sqrt(13) + 1/2)) + 2*sq
rt(325*sqrt(13) - 1118)*arctan(-13/4*(4/13)^(3/4)*((4/13)^(1/4)*sqrt(1/13*s
qrt(13) + 1/2) - 1)/sqrt(-1/13*sqrt(13) + 1/2)) - sqrt(325*sqrt(13) - 1118)
*log(2*(4/13)^(1/4)*sqrt(1/13*sqrt(13) + 1/2) + 2*sqrt(1/13) + 1) + sqrt(32
5*sqrt(13) - 1118)*log(-2*(4/13)^(1/4)*sqrt(1/13*sqrt(13) + 1/2) + 2*sqrt(1
/13) + 1))/d - 2*(3*d^2*sqrt(325*sqrt(13) - 1118) - 2*d*sqrt(325*sqrt(13) -
 1118)*abs(d))*arctan(13/4*(4/13)^(3/4)*((4/13)^(1/4)*sqrt(1/13*sqrt(13) +
1/2) + sqrt(3/(2*tan(d*x + c) - 3) + 1))/sqrt(-1/13*sqrt(13) + 1/2))/d^3 -
2*(3*d^2*sqrt(325*sqrt(13) - 1118) - 2*d*sqrt(325*sqrt(13) - 1118)*abs(d))*
arctan(-13/4*(4/13)^(3/4)*((4/13)^(1/4)*sqrt(1/13*sqrt(13) + 1/2) - sqrt(3/
(2*tan(d*x + c) - 3) + 1))/sqrt(-1/13*sqrt(13) + 1/2))/d^3 + (3*d^2*sqrt(32
```

```
5*sqrt(13) - 1118) + 2*d*sqrt(325*sqrt(13) - 1118)*abs(d))*log(2*(4/13)^(1/
4)*sqrt(1/13*sqrt(13) + 1/2)*sqrt(3/(2*tan(d*x + c) - 3) + 1) + 2*sqrt(1/13
) + 3/(2*tan(d*x + c) - 3) + 1)/d^3 - (3*d^2*sqrt(325*sqrt(13) - 1118) + 2*
d*sqrt(325*sqrt(13) - 1118)*abs(d))*log(-2*(4/13)^(1/4)*sqrt(1/13*sqrt(13)
+ 1/2)*sqrt(3/(2*tan(d*x + c) - 3) + 1) + 2*sqrt(1/13) + 3/(2*tan(d*x + c)
- 3) + 1)/d^3)
```

Mupad [B]
time = 5.71, size = 191, normalized size = 2.01

$$-\operatorname{atan}\left(\frac{8\,d\,\sqrt{\frac{-\frac{1}{26}-\frac{3}{52}\mathrm{i}}{d^2}}\,\left(\frac{\sqrt{2}\,\sqrt{3}}{2}-\sqrt{\tan\left(c+d\,x\right)}\right)}{\sqrt{2\tan\left(c+d\,x\right)-3}\,\left(\frac{2\left(\frac{\sqrt{2}\,\sqrt{3}}{2}-\sqrt{\tan\left(c+d\,x\right)}\right)^2}{2\tan\left(c+d\,x\right)-3}+1\right)}\right)\sqrt{\frac{-\frac{1}{26}-\frac{3}{52}\mathrm{i}}{d^2}}\,2\mathrm{i}+\operatorname{atan}\left(\frac{8\,d\,\sqrt{\frac{-\frac{1}{26}+\frac{3}{52}\mathrm{i}}{d^2}}\,\left(\frac{\sqrt{2}\,\sqrt{3}}{2}-\sqrt{\tan\left(c+d\,x\right)}\right)}{\sqrt{2\tan\left(c+d\,x\right)-3}\,\left(\frac{2\left(\frac{\sqrt{2}\,\sqrt{3}}{2}-\sqrt{\tan\left(c+d\,x\right)}\right)^2}{2\tan\left(c+d\,x\right)-3}+1\right)}\right)\sqrt{\frac{-\frac{1}{26}+\frac{3}{52}\mathrm{i}}{d^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)/(2*tan(c + d*x) - 3)^(1/2),x)
```

```
[Out] atan((8*d*((- 1/26 + 3i/52)/d^2)^(1/2)*((2^(1/2)*3^(1/2))/2 - tan(c + d*x)^
(1/2)))/((2*tan(c + d*x) - 3)^(1/2)*((2*((2^(1/2)*3^(1/2))/2 - tan(c + d*x)
^(1/2))^2)/(2*tan(c + d*x) - 3) + 1)))*((- 1/26 + 3i/52)/d^2)^(1/2)*2i - at
an((8*d*((- 1/26 - 3i/52)/d^2)^(1/2)*((2^(1/2)*3^(1/2))/2 - tan(c + d*x)^(1
/2)))/((2*tan(c + d*x) - 3)^(1/2)*((2*((2^(1/2)*3^(1/2))/2 - tan(c + d*x)^(
1/2))^2)/(2*tan(c + d*x) - 3) + 1)))*((- 1/26 - 3i/52)/d^2)^(1/2)*2i
```

3.670 $\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{-3-2\tan(c+dx)}}\,dx$

Optimal. Leaf size=95

$$\frac{i\text{ArcTan}\left(\frac{\sqrt{2-3i}\,\sqrt{\tan(c+dx)}}{\sqrt{-3-2\tan(c+dx)}}\right)}{\sqrt{2-3i}\,d}-\frac{i\text{ArcTan}\left(\frac{\sqrt{2+3i}\,\sqrt{\tan(c+dx)}}{\sqrt{-3-2\tan(c+dx)}}\right)}{\sqrt{2+3i}\,d}$$

```
[Out] I*arctan((2-3*I)^(1/2)*tan(d*x+c)^(1/2)/(-3-2*tan(d*x+c))^(1/2))/d/(2-3*I)^
(1/2)-I*arctan((2+3*I)^(1/2)*tan(d*x+c)^(1/2)/(-3-2*tan(d*x+c))^(1/2))/d/(2
+3*I)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3656, 924, 95, 211}

$$\frac{i\text{ArcTan}\left(\frac{\sqrt{2-3i}\,\sqrt{\tan(c+dx)}}{\sqrt{-2\tan(c+dx)-3}}\right)}{\sqrt{2-3i}\,d}-\frac{i\text{ArcTan}\left(\frac{\sqrt{2+3i}\,\sqrt{\tan(c+dx)}}{\sqrt{-2\tan(c+dx)-3}}\right)}{\sqrt{2+3i}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[c + d*x]]/Sqrt[-3 - 2*Tan[c + d*x]],x]
```

```
[Out] (I*ArcTan[(Sqrt[2 - 3*I]*Sqrt[Tan[c + d*x]])/Sqrt[-3 - 2*Tan[c + d*x]]])/(S
qrt[2 - 3*I]*d) - (I*ArcTan[(Sqrt[2 + 3*I]*Sqrt[Tan[c + d*x]])/Sqrt[-3 - 2*
Tan[c + d*x]]])/(Sqrt[2 + 3*I]*d)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
```

```
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{-3-2\tan(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{-3-2x}\,(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{2\sqrt{-3-2x}\,(i-x)\sqrt{x}} + \frac{1}{2\sqrt{-3-2x}\,\sqrt{x}\,(i+x)}\right) dx, x, \tan(c+\right.}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{-3-2x}\,(i-x)\sqrt{x}}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{-3-2x}\right.}{}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{i-(3-2i)x^2}\,dx, x, \frac{\sqrt{\tan(c+dx)}}{\sqrt{-3-2\tan(c+dx)}}\right)}{d} - \frac{\text{Subst}\left(\int \frac{1}{i+(3+2i)x^2}\,dx\right.}{}$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{2-3i}\,\sqrt{\tan(c+dx)}}{\sqrt{-3-2\tan(c+dx)}}\right)}{\sqrt{2-3i}\,d} - \frac{i\tan^{-1}\left(\frac{\sqrt{2+3i}\,\sqrt{\tan(c+dx)}}{\sqrt{-3-2\tan(c+dx)}}\right)}{\sqrt{2+3i}\,d}$$

Mathematica [A]
time = 0.07, size = 103, normalized size = 1.08

$$-\frac{i\left(\sqrt{2+3i}\,\text{ArcTan}\left(\frac{\sqrt{\frac{2}{13}+\frac{3i}{13}}\,\sqrt{-3-2\tan(c+dx)}}{\sqrt{\tan(c+dx)}}\right)+\sqrt{-2+3i}\,\tanh^{-1}\left(\frac{\sqrt{-\frac{2}{13}+\frac{3i}{13}}\,\sqrt{-3-2\tan(c+dx)}}{\sqrt{\tan(c+dx)}}\right)\right)}{\sqrt{13}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tan[c + d*x]]/Sqrt[-3 - 2*Tan[c + d*x]],x]
```

```
[Out] ((-I)*(Sqrt[2 + 3*I]*ArcTan[(Sqrt[2/13 + (3*I)/13]*Sqrt[-3 - 2*Tan[c + d*x]
])/Sqrt[Tan[c + d*x]]] + Sqrt[-2 + 3*I]*ArcTanh[(Sqrt[-2/13 + (3*I)/13]*Sqr
t[-3 - 2*Tan[c + d*x]])/Sqrt[Tan[c + d*x]]]))/(Sqrt[13]*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(77) = 154$.
time = 0.70, size = 434, normalized size = 4.57

method	result
derivativedivides	$\frac{3\sqrt{-3-2\tan(dx+c)}\,\sqrt{-\frac{\tan(dx+c)(3+2\tan(dx+c))}{\left(\sqrt{13}-2+3\tan(dx+c)\right)^{2}}}\left(\sqrt{13}-2+3\tan(dx+c)\right)\left(\sqrt{13}\,\sqrt{2\sqrt{13}\right.}{}$
default	$\frac{3\sqrt{-3-2\tan(dx+c)}\,\sqrt{-\frac{\tan(dx+c)(3+2\tan(dx+c))}{\left(\sqrt{13}-2+3\tan(dx+c)\right)^{2}}}\left(\sqrt{13}-2+3\tan(dx+c)\right)\left(\sqrt{13}\,\sqrt{2\sqrt{13}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)/(-3-2*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/2/d*(-3-2*tan(d*x+c))^(1/2)*(-tan(d*x+c)*(3+2*tan(d*x+c))/(13^(1/2)-2+3*t
an(d*x+c))^2)^(1/2)*(13^(1/2)-2+3*tan(d*x+c))*(13^(1/2)*(2*13^(1/2)+4)^(1/2
)*arctanh(1/351*(2+13^(1/2))*(13^(1/2)+2-3*tan(d*x+c))*(17*13^(1/2)-52)/(-4
+2*13^(1/2))^(1/2)/(13^(1/2)-2+3*tan(d*x+c))*13^(1/2)/(-tan(d*x+c)*(3+2*tan
(d*x+c))/(13^(1/2)-2+3*tan(d*x+c))^2)^(1/2))*(-4+2*13^(1/2))^(1/2)-2*(2*13^
(1/2)+4)^(1/2)*arctanh(1/351*(2+13^(1/2))*(13^(1/2)+2-3*tan(d*x+c))*(17*13^
(1/2)-52)/(-4+2*13^(1/2))^(1/2)/(13^(1/2)-2+3*tan(d*x+c))*13^(1/2)/(-tan(d*
x+c)*(3+2*tan(d*x+c))/(13^(1/2)-2+3*tan(d*x+c))^2)^(1/2))*(-4+2*13^(1/2))^(
1/2)+8*arctan(6*13^(1/2)*(-tan(d*x+c)*(3+2*tan(d*x+c))/(13^(1/2)-2+3*tan(d*
x+c))^2)^(1/2)/(26*13^(1/2)+52)^(1/2))*13^(1/2)-34*arctan(6*13^(1/2)*(-tan(
d*x+c)*(3+2*tan(d*x+c))/(13^(1/2)-2+3*tan(d*x+c))^2)^(1/2)/(26*13^(1/2)+52)
^(1/2)))/tan(d*x+c)^(1/2)/(2*13^(1/2)+4)^(1/2)/(3+2*tan(d*x+c))/(17*13^(1/2
)-52)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(-3-2*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(tan(d*x + c))/sqrt(-2*tan(d*x + c) - 3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(-3-2*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{-2\tan(c+dx)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/2)/(-3-2*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(tan(c + d*x))/sqrt(-2*tan(c + d*x) - 3), x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(67) = 134$.
time = 0.85, size = 209, normalized size = 2.20

$$-\frac{\sqrt{2}\left(-i\left(\sqrt{2}\sqrt{-\tan(dx+c)}-\sqrt{-2\tan(dx+c)-3}\right)^4-6i\left(\sqrt{2}\sqrt{-\tan(dx+c)}-\sqrt{-2\tan(dx+c)-3}\right)^2-9i\right)\log\left(\left(\sqrt{2}\sqrt{-\tan(dx+c)}-\sqrt{-2\tan(dx+c)-3}\right)^8+12\left(\sqrt{2}\sqrt{-\tan(dx+c)}-\sqrt{-2\tan(dx+c)-3}\right)^6+118\left(\sqrt{2}\sqrt{-\tan(dx+c)}-\sqrt{-2\tan(dx+c)-3}\right)^4+108\left(\sqrt{2}\sqrt{-\tan(dx+c)}-\sqrt{-2\tan(dx+c)-3}\right)^2+81\right)}{27d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)/(-3-2*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/27*sqrt(2)*(-I*(sqrt(2)*sqrt(-tan(d*x + c)) - sqrt(-2*tan(d*x + c) - 3))
^4 - 6*I*(sqrt(2)*sqrt(-tan(d*x + c)) - sqrt(-2*tan(d*x + c) - 3))^2 - 9*I)
*log((sqrt(2)*sqrt(-tan(d*x + c)) - sqrt(-2*tan(d*x + c) - 3))^8 + 12*(sqrt
(2)*sqrt(-tan(d*x + c)) - sqrt(-2*tan(d*x + c) - 3))^6 + 118*(sqrt(2)*sqrt(
-tan(d*x + c)) - sqrt(-2*tan(d*x + c) - 3))^4 + 108*(sqrt(2)*sqrt(-tan(d*x
+ c)) - sqrt(-2*tan(d*x + c) - 3))^2 + 81)/d
```

Mupad [B]
time = 5.70, size = 189, normalized size = 1.99

$$-\mathrm{atan}\left(\frac{8\,d\,\sqrt{\frac{\frac{1}{26}+\frac{3}{52}\mathrm{i}}{d^2}}\,\left(-\sqrt{\tan\left(c+d\,x\right)}+\frac{\sqrt{2}\,\sqrt{3}\,1\mathrm{i}}{2}\right)}{\sqrt{-2\tan\left(c+d\,x\right)-3}\,\left(\frac{2\left(-\sqrt{\tan\left(c+d\,x\right)}+\frac{\sqrt{2}\,\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{2\tan(c+d\,x)+3}+1\right)}\right)\sqrt{\frac{\frac{1}{26}+\frac{3}{52}\mathrm{i}}{d^2}}\,2\mathrm{i}+\mathrm{atan}\left(\frac{8\,d\,\sqrt{\frac{\frac{1}{26}-\frac{3}{52}\mathrm{i}}{d^2}}\,\left(-\sqrt{\tan\left(c+d\,x\right)}+\frac{\sqrt{6}\,1\mathrm{i}}{2}\right)}{\sqrt{-2\tan\left(c+d\,x\right)-3}\,\left(\frac{2\left(-\sqrt{\tan\left(c+d\,x\right)}+\frac{\sqrt{6}\,1\mathrm{i}}{2}\right)^2}{2\tan(c+d\,x)+3}+1\right)}\right)\sqrt{\frac{\frac{1}{26}-\frac{3}{52}\mathrm{i}}{d^2}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)/(- 2*tan(c + d*x) - 3)^(1/2),x)
```

```
[Out] atan((8*d*((1/26 - 3i/52)/d^2)^(1/2)*((6^(1/2)*1i)/2 - tan(c + d*x)^(1/2)))
/((- 2*tan(c + d*x) - 3)^(1/2)*((2*((6^(1/2)*1i)/2 - tan(c + d*x)^(1/2))^2)
/(2*tan(c + d*x) + 3) + 1)))*((1/26 - 3i/52)/d^2)^(1/2)*2i - atan((8*d*((1/
26 + 3i/52)/d^2)^(1/2)*((2^(1/2)*3^(1/2)*1i)/2 - tan(c + d*x)^(1/2)))/((- 2
*tan(c + d*x) - 3)^(1/2)*((2*((2^(1/2)*3^(1/2)*1i)/2 - tan(c + d*x)^(1/2))^
2)/(2*tan(c + d*x) + 3) + 1)))*((1/26 + 3i/52)/d^2)^(1/2)*2i
```

3.671 $\int \frac{\tan^{\frac{5}{3}}(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=466

$$-\frac{b\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{2\left(a^2+b^2\right)d}+\frac{b\text{ArcTan}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{2\left(a^2+b^2\right)d}-\frac{\sqrt{3}\,a^{5/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{}}{\sqrt{3}}\right.}{b^{2/3}\left(a^2+b^2\right)d}$$

```
[Out] 1/2*b*arctan(-3^(1/2)+2*tan(d*x+c)^(1/3))/(a^2+b^2)/d+1/2*b*arctan(3^(1/2)+
2*tan(d*x+c)^(1/3))/(a^2+b^2)/d+b*arctan(tan(d*x+c)^(1/3))/(a^2+b^2)/d-3/2*
a^(5/3)*ln(a^(1/3)+b^(1/3)*tan(d*x+c)^(1/3))/b^(2/3)/(a^2+b^2)/d-1/2*a*ln(1
+tan(d*x+c)^(2/3))/(a^2+b^2)/d+1/2*a^(5/3)*ln(a+b*tan(d*x+c))/b^(2/3)/(a^2+
b^2)/d+1/4*a*ln(1-tan(d*x+c)^(2/3)+tan(d*x+c)^(4/3))/(a^2+b^2)/d-a^(5/3)*ar
ctan(1/3*(a^(1/3)-2*b^(1/3)*tan(d*x+c)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)/b^(2
/3)/(a^2+b^2)/d+1/2*a*arctan(1/3*(1-2*tan(d*x+c)^(2/3))*3^(1/2))*3^(1/2)/(a
^2+b^2)/d+1/4*b*ln(1-3^(1/2)*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3))*3^(1/2)/(a^
2+b^2)/d-1/4*b*ln(1+3^(1/2)*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3))*3^(1/2)/(a^2
+b^2)/d
```

Rubi [A]
time = 0.51, antiderivative size = 466, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 18, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3655, 3609, 3619, 3557, 335, 281, 206, 31, 648, 632, 210, 642, 301, 209, 3715, 52, 58, 631}

$$\frac{\sqrt{3}\,a\text{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)}{2d\left(a^2+b^2\right)}-\frac{b\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{2d\left(a^2+b^2\right)}+\frac{b\text{ArcTan}\left(2\sqrt[3]{\tan(c+dx)}+\sqrt{3}\right)}{2d\left(a^2+b^2\right)}+\frac{b\text{ArcTan}\left(\sqrt[3]{\tan(c+dx)}\right)}{d\left(a^2+b^2\right)}-\frac{a\log\left(\tan^{\frac{2}{3}}(c+dx)+1\right)}{2d\left(a^2+b^2\right)}+\frac{a\log\left(\tan^{\frac{4}{3}}(c+dx)-\tan^{\frac{2}{3}}(c+dx)+1\right)}{4d\left(a^2+b^2\right)}+\frac{\sqrt{3}\,b\log\left(\tan^{\frac{2}{3}}(c+dx)-\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{4d\left(a^2+b^2\right)}-\frac{\sqrt{3}\,b\log\left(\tan^{\frac{2}{3}}(c+dx)+\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{4d\left(a^2+b^2\right)}-\frac{\sqrt{3}\,a^{5/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{2/3}d\left(a^2+b^2\right)}-\frac{3a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{\tan(c+dx)}\right)}{2b^{2/3}d\left(a^2+b^2\right)}+\frac{a^{5/3}\log(a+b\tan(c+dx))}{2b^{2/3}d\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(5/3)/(a + b*Tan[c + d*x]),x]
```

```
[Out] -1/2*(b*ArcTan[Sqrt[3] - 2*Tan[c + d*x]^(1/3)])/((a^2 + b^2)*d) + (b*ArcTan
[Sqrt[3] + 2*Tan[c + d*x]^(1/3)])/(2*(a^2 + b^2)*d) - (Sqrt[3]*a^(5/3)*ArcT
an[(a^(1/3) - 2*b^(1/3)*Tan[c + d*x]^(1/3))/(Sqrt[3]*a^(1/3))])/(b^(2/3)*(a
^2 + b^2)*d) + (Sqrt[3]*a*ArcTan[(1 - 2*Tan[c + d*x]^(2/3))/Sqrt[3]])/(2*(a
^2 + b^2)*d) + (b*ArcTan[Tan[c + d*x]^(1/3)])/((a^2 + b^2)*d) - (3*a^(5/3)*
Log[a^(1/3) + b^(1/3)*Tan[c + d*x]^(1/3)])/(2*b^(2/3)*(a^2 + b^2)*d) - (a*L
og[1 + Tan[c + d*x]^(2/3)])/(2*(a^2 + b^2)*d) + (Sqrt[3]*b*Log[1 - Sqrt[3]*
Tan[c + d*x]^(1/3) + Tan[c + d*x]^(2/3)])/(4*(a^2 + b^2)*d) - (Sqrt[3]*b*Lo
g[1 + Sqrt[3]*Tan[c + d*x]^(1/3) + Tan[c + d*x]^(2/3)])/(4*(a^2 + b^2)*d) +
 (a^(5/3)*Log[a + b*Tan[c + d*x]])/(2*b^(2/3)*(a^2 + b^2)*d) + (a*Log[1 - T
an[c + d*x]^(2/3) + Tan[c + d*x]^(4/3)])/(4*(a^2 + b^2)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)/((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[(a + b*Tan[e + f*x])^m*
(c - d*Tan[e + f*x]), x], x] + Dist[d^2/(c^2 + d^2), Int[(a + b*Tan[e + f*x
])^m*((1 + Tan[e + f*x]^2)/(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{\tan^{\frac{5}{3}}(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\int \tan^{\frac{5}{3}}(c+dx)(a-b\tan(c+dx))\,dx}{a^2+b^2} + \frac{b^2\int \frac{\tan^{\frac{5}{3}}(c+dx)(1+\tan^2(c+dx))}{a+b\tan(c+dx)}\,dx}{a^2+b^2}$$

$$= -\frac{3b\tan^{\frac{5}{3}}(c+dx)}{5(a^2+b^2)d} + \frac{\int \tan^{\frac{2}{3}}(c+dx)(b+a\tan(c+dx))\,dx}{a^2+b^2} + \frac{b^2\text{Subst}\left(\int \frac{x^{5/3}}{a+bx}\,dx,\right.}{(a^2+}$$

$$= \frac{3a\tan^{\frac{2}{3}}(c+dx)}{2(a^2+b^2)d} + \frac{\int \frac{-a+b\tan(c+dx)}{\sqrt[3]{\tan(c+dx)}}\,dx}{a^2+b^2} - \frac{(ab)\text{Subst}\left(\int \frac{x^{2/3}}{a+bx}\,dx, x, \tan(c+dx)\right)}{(a^2+b^2)d}$$

$$= -\frac{a\int \frac{1}{\sqrt[3]{\tan(c+dx)}}\,dx}{a^2+b^2} + \frac{b\int \tan^{\frac{2}{3}}(c+dx)\,dx}{a^2+b^2} + \frac{a^2\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx, x, \text{ta}\right.}{(a^2+b^2)d}$$

$$= \frac{a^{5/3}\log(a+b\tan(c+dx))}{2b^{2/3}(a^2+b^2)d} - \frac{a\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\,(1+x^2)}\,dx, x, \tan(c+dx)\right)}{(a^2+b^2)d} + \frac{(3a^2)\text{S}}{}$$

$$= -\frac{3a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{\tan(c+dx)}\right)}{2b^{2/3}(a^2+b^2)d} + \frac{a^{5/3}\log(a+b\tan(c+dx))}{2b^{2/3}(a^2+b^2)d} - \frac{(3a)\text{Sub}}{}$$

$$= -\frac{\sqrt{3}\,a^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}(a^2+b^2)d} - \frac{3a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{\tan(c+d}\right.}{2b^{2/3}(a^2+b^2)d}$$

$$= -\frac{\sqrt{3}\,a^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}(a^2+b^2)d} + \frac{b\tan^{-1}\left(\sqrt[3]{\tan(c+dx)}\right)}{(a^2+b^2)d} - \frac{3a^{5/3}}{}$$

$$= -\frac{\sqrt{3}\,a^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}(a^2+b^2)d} + \frac{b\tan^{-1}\left(\sqrt[3]{\tan(c+dx)}\right)}{(a^2+b^2)d} - \frac{3a^{5/3}}{}$$

$$= -\frac{b\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{2(a^2+b^2)d} + \frac{b\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{2(a^2+b^2)d} - \frac{\sqrt{3}}{}$$

$$= -\frac{b\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{2(a^2+b^2)d} + \frac{b\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{2(a^2+b^2)d} - \frac{\sqrt{3}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.43, size = 153, normalized size = 0.33

$$\frac{5a\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)-2\log\left(1+\tan^{\frac{2}{3}}(c+dx)\right)+\log\left(1-\tan^{\frac{2}{3}}(c+dx)+\tan^{\frac{4}{3}}(c+dx)\right)\right)+30a\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{b\tan(c+dx)}{a}\right)\tan^{\frac{2}{3}}(c+dx)+12b\,{}_2F_1\left(\frac{5}{6},1;\frac{11}{6};-\tan^2(c+dx)\right)\tan^{\frac{5}{3}}(c+dx)}{20\,(a^2+b^2)\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(5/3)/(a + b*Tan[c + d*x]),x]
```

```
[Out] (5*a*(2*Sqrt[3]*ArcTan[(1 - 2*Tan[c + d*x]^(2/3))/Sqrt[3]] - 2*Log[1 + Tan[
c + d*x]^(2/3)] + Log[1 - Tan[c + d*x]^(2/3) + Tan[c + d*x]^(4/3)]) + 30*a*
Hypergeometric2F1[2/3, 1, 5/3, -((b*Tan[c + d*x])/a)]*Tan[c + d*x]^(2/3) +
12*b*Hypergeometric2F1[5/6, 1, 11/6, -Tan[c + d*x]^2]*Tan[c + d*x]^(5/3))/(
20*(a^2 + b^2)*d)
```

Maple [A]
time = 0.25, size = 341, normalized size = 0.73

method	result
derivativedivides	$\frac{\frac{-\frac{3a\ln\left(1+\tan^{\frac{2}{3}}(dx+c)\right)}{2}+3b\arctan\left(\tan^{\frac{1}{3}}(dx+c)\right)}{3a^2+3b^2}+\frac{-\frac{3\left(-\sqrt{3}\,b-a\right)\ln\left(1-\sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)\right)}{4}-3\left(\sqrt{3}\,a+b\right.}{}}{}$
default	$\frac{\frac{-\frac{3a\ln\left(1+\tan^{\frac{2}{3}}(dx+c)\right)}{2}+3b\arctan\left(\tan^{\frac{1}{3}}(dx+c)\right)}{3a^2+3b^2}+\frac{-\frac{3\left(-\sqrt{3}\,b-a\right)\ln\left(1-\sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)\right)}{4}-3\left(\sqrt{3}\,a+b\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(5/3)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(3/(3*a^2+3*b^2)*(-1/2*a*ln(1+tan(d*x+c)^(2/3))+b*arctan(tan(d*x+c)^(1/
3)))+3/(3*a^2+3*b^2)*(-1/4*(-3^(1/2)*b-a)*ln(1-3^(1/2)*tan(d*x+c)^(1/3)+tan
(d*x+c)^(2/3))-(3^(1/2)*a+b+1/2*(-3^(1/2)*b-a)*3^(1/2))*arctan(-3^(1/2)+2*t
an(d*x+c)^(1/3))+1/4*(-3^(1/2)*b+a)*ln(1+3^(1/2)*tan(d*x+c)^(1/3)+tan(d*x+c
)^(2/3))+(3^(1/2)*a-b-1/2*(-3^(1/2)*b+a)*3^(1/2))*arctan(3^(1/2)+2*tan(d*x+
c)^(1/3)))+3*(-1/3/b/(a/b)^(1/3)*ln(tan(d*x+c)^(1/3)+(a/b)^(1/3))+1/6/b/(a/
b)^(1/3)*ln(tan(d*x+c)^(2/3)-(a/b)^(1/3)*tan(d*x+c)^(1/3)+(a/b)^(2/3))+1/3*
3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*tan(d*x+c)^(1/3)-1)
))*a^2/(a^2+b^2))
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/3)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**C**] Result contains complex when optimal does not.
time = 3.89, size = 73878, normalized size = 158.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/3)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/48*(12*(a^2 + b^2)*d*(sqrt(2*(I*a^2 + I*b^2)*d*(2*a/(a^4*d^3 + 2*a^2*b^2
*d^3 + b^4*d^3) - (3*a^2 - b^2)*a/((a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2)*(a^2
*d + b^2*d)) + a^3/(a^2*d + b^2*d)^3)/b - (3*a^2 - b^2)/(a^4*d^2 + 2*a^2*b^
2*d^2 + b^4*d^2) - 2*(a^2 - b^2)/(a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2) + 2*a^
2/(a^2*d + b^2*d)^2) - a/(a^2*d + b^2*d) + I*b/((a^2 + b^2)*d))*log(1/8*(a^
6 + a^4*b^2 - a^2*b^4 - b^6)*d^3*(sqrt(2*(I*a^2 + I*b^2)*d*(2*a/(a^4*d^3 +
2*a^2*b^2*d^3 + b^4*d^3) - (3*a^2 - b^2)*a/((a^4*d^2 + 2*a^2*b^2*d^2 + b^4*
d^2)*(a^2*d + b^2*d)) + a^3/(a^2*d + b^2*d)^3)/b - (3*a^2 - b^2)/(a^4*d^2 +
 2*a^2*b^2*d^2 + b^4*d^2) - 2*(a^2 - b^2)/(a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^
2) + 2*a^2/(a^2*d + b^2*d)^2) - a/(a^2*d + b^2*d) + I*b/((a^2 + b^2)*d))^3
+ 1/4*(3*a^5 + 2*a^3*b^2 - a*b^4)*d^2*(sqrt(2*(I*a^2 + I*b^2)*d*(2*a/(a^4*d
^3 + 2*a^2*b^2*d^3 + b^4*d^3) - (3*a^2 - b^2)*a/((a^4*d^2 + 2*a^2*b^2*d^2 +
 b^4*d^2)*(a^2*d + b^2*d)) + a^3/(a^2*d + b^2*d)^3)/b - (3*a^2 - b^2)/(a^4*
d^2 + 2*a^2*b^2*d^2 + b^4*d^2) - 2*(a^2 - b^2)/(a^4*d^2 + 2*a^2*b^2*d^2 + b
^4*d^2) + 2*a^2/(a^2*d + b^2*d)^2) - a/(a^2*d + b^2*d) + I*b/((a^2 + b^2)*d
))^2 + 2*a^3 - 2*a*b^2 + 1/2*(3*a^4 - 4*a^2*b^2 + b^4)*d*(sqrt(2*(I*a^2 + I
```

*b^2)*d*(2*a/(a^4*d^3 + 2*a^2*b^2*d^3 + b^4*d^3) - (3*a^2 - b^2)*a/((a^4*d^
2 + 2*a^2*b^2*d^2 + b^4*d^2)*(a^2*d + b^2*d)) + a^3/(a^2*d + b^2*d)^3)/b -
(3*a^2 - b^2)/(a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2) - 2*(a^2 - b^2)/(a^4*d^2
+ 2*a^2*b^2*d^2 + b^4*d^2) + 2*a^2/(a^2*d + b^2*d)^2) - a/(a^2*d + b^2*d) +
I*b/((a^2 + b^2)*d)) - (3*a^2*b - b^3)*tan(d*x + c)^(1/3)) - 2*(2*(1/2)^(2
/3)*(-I*sqrt(3) + 1)*(6*(sqrt(3)*a^3*d*sqrt(-(a^2 - 2*I*a*b - b^2)/((a^4 +
2*a^2*b^2 + b^4)*d^2)) + I*sqrt(3)*a^2*b*d*sqrt(-(a^2 - 2*I*a*b - b^2)/((a^
4 + 2*a^2*b^2 + b^4)*d^2)) + sqrt(3)*a*b^2*d*sqrt(-(a^2 - 2*I*a*b - b^2)/((
a^4 + 2*a^2*b^2 + b^4)*d^2)) + I*sqrt(3)*b^3*d*sqrt(-(a^2 - 2*I*a*b - b^2)/
((a^4 + 2*a^2*b^2 + b^4)*d^2)) + 3*a^2 + 4*I*a*b - b^2)/(a^4*d^2 + 2*a^2*b^
2*d^2 + b^4*d^2) - (sqrt(3)*a^2*d*sqrt(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2
*b^2 + b^4)*d^2)) + sqrt(3)*b^2*d*sqrt(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2
*b^2 + b^4)*d^2)) + 3*a + I*b)^2/(a^2*d + b^2*d)^2)/(27*(3*sqrt(3)*a^4*d^3*
(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))^(3/2) + 3*sqrt(3)*b^
4*d^3*(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))^(3/2) + 2*I*sq
rt(3)*a*b*d*sqrt(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - 7*
sqrt(3)*b^2*d*sqrt(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) +
(6*sqrt(3)*b^2*d^3*(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))^(
3/2) + 7*sqrt(3)*d*sqrt(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2
)))*a^2 + 4*a + 4*I*b)/(a^4*d^3 + 2*a^2*b^2*d^3 + b^4*d^3) + 2*(sqrt(3)*a^2
*d*sqrt(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + sqrt(3)*b^2
*d*sqrt(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + 3*a + I*b)^
3/(a^2*d + b^2*d)^3 - 18*(sqrt(3)*a^3*d*sqrt(-(a^2 - 2*I*a*b - b^2)/((a^4 +
2*a^2*b^2 + b^4)*d^2)) + I*sqrt(3)*a^2*b*d*sqrt(-(a^2 - 2*I*a*b - b^2)/((a
^4 + 2*a^2*b^2 + b^4)*d^2)) + sqrt(3)*a*b^2*d*sqrt(-(a^2 - 2*I*a*b - b^2)/(
(a^4 + 2*a^2*b^2 + b^4)*d^2)) + I*sqrt(3)*b^3*d*sqrt(-(a^2 - 2*I*a*b - b^2)
/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + 3*a^2 + 4*I*a*b - b^2)*(sqrt(3)*a^2*d*sqr
t(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + sqrt(3)*b^2*d*sqr
t(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + 3*a + I*b)/((a^4*
d^2 + 2*a^2*b^2*d^2 + b^4*d^2)*(a^2*d + b^2*d)) + 9*sqrt(-72*sqrt(3)*(I*a^1
0*b + 5*I*a^8*b^3 + 10*I*a^6*b^5 + 10*I*a^4*b^7 + 5*I*a^2*b^9 + I*b^11)*d^5
*(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))^(5/2) + 36*a^6 + 21
6*I*a^5*b - 636*a^4*b^2 - 1104*I*a^3*b^3 + 1180*a^2*b^4 + 728*I*a*b^5 - 196
*b^6 - 32*sqrt(3)*(12*I*a^8*b - 15*a^7*b^2 + 23*I*a^6*b^3 - 45*a^5*b^4 - 3*
I*a^4*b^5 - 45*a^3*b^6 - 27*I*a^2*b^7 - 15*a*b^8 - 13*I*b^9)*d^3*(-(a^2 - 2
*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))^(3/2) - 8*sqrt(3)*(39*I*a^6*b
- 48*a^5*b^2 + I*a^4*b^3 + 24*a^3*b^4 + 21*I*a^2*b^5 + 72*a*b^6 + 59*I*b^7)
*d*sqrt(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - 9*(71*a^8 -
76*I*a^7*b - 44*a^6*b^2 - 108*I*a^5*b^3 - 182*a^4*b^4 + 12*I*a^3*b^5 + 52*
a^2*b^6 + 44*I*a*b^7 + 119*b^8)*(a^2 - 2*I*a*b - b^2)/(a^4 + 2*a^2*b^2 + b^
4) + 18*(49*a^10 - 18*I*a^9*b + 139*a^8*b^2 - 72*I*a^7*b^3 + 66*a^6*b^4 - 1
08*I*a^5*b^5 - 146*a^4*b^6 - 72*I*a^3*b^7 - 179*a^2*b^8 - 18*I*a*b^9 - 57*b
^10)*(a^2 - 2*I*a*b - b^2)^2/(a^4 + 2*a^2*b^2 + b^4)^2 - 279*(a^12 + 6*a^10
*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*(a^2 - 2*I
*a*b - b^2)^3/(a^4 + 2*a^2*b^2 + b^4)^3)/((a^2 + b^2)^3*d^3))^(1/3) - (1/2)

```
^(1/3)*(I*sqrt(3) + 1)*(27*(3*sqrt(3)*a^4*d^3*(-(a^2 - 2*I*a*b - b^2)/((a^4
 + 2*a^2*b^2 + b^4)*d^2))^(3/2) + 3*sqrt(3)*b^4*d^3*(-(a^2 - 2*I*a*b - b^2)
/((a^4 + 2*a^2*b^2 + b^4)*d^2))^(3/2) + 2*I*sqrt(3)*a*b*d*sqrt(-(a^2 - 2*I*
a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - 7*sqrt(3)*b^2*d*sqrt(-(a^2 - 2*
I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + (6*sqrt(3)*b^2*d^3*(-(a^2 - 2
*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))^(3...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{5}{3}}(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(5/3)/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**(5/3)/(a + b*tan(c + d*x)), x)
```

Giac [**A**]
time = 0.71, size = 435, normalized size = 0.93

$$-\frac{a^{2}\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\tan(dx+c)^{\frac{1}{3}}\right|\right)}{a^{3}d+ab^{2}d}+\frac{\left(\sqrt{3}\,a+b\right)\arctan\left(\sqrt{3}+2\,\tan(dx+c)^{\frac{1}{3}}\right)}{2\left(a^{2}d+b^{2}d\right)}-\frac{\left(\sqrt{3}\,a-b\right)\arctan\left(-\sqrt{3}+2\,\tan(dx+c)^{\frac{1}{3}}\right)}{2\left(a^{2}d+b^{2}d\right)}+\frac{b\arctan\left(\tan(dx+c)^{\frac{1}{3}}\right)}{a^{2}d+b^{2}d}+\frac{a\log\left(\tan(dx+c)^{\frac{4}{3}}-\tan(dx+c)^{\frac{2}{3}}+1\right)}{4\left(a^{2}d+b^{2}d\right)}-\frac{3\,b\log\left(\sqrt{3}\,\tan(dx+c)^{\frac{1}{3}}+\tan(dx+c)^{\frac{2}{3}}+1\right)}{4\left(\sqrt{3}\,a^{2}d+\sqrt{3}\,b^{2}d\right)}+\frac{3\,b\log\left(-\sqrt{3}\,\tan(dx+c)^{\frac{1}{3}}+\tan(dx+c)^{\frac{2}{3}}+1\right)}{4\left(\sqrt{3}\,a^{2}d+\sqrt{3}\,b^{2}d\right)}-\frac{a\log\left(\tan(dx+c)^{\frac{2}{3}}+1\right)}{2\left(a^{2}d+b^{2}d\right)}-\frac{3\left(-ab^{2}\right)^{\frac{2}{3}}a\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\tan(dx+c)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\left(\sqrt{3}\,a^{2}b^{2}+\sqrt{3}\,b^{4}\right)d}+\frac{\left(-ab^{2}\right)^{\frac{2}{3}}a\log\left(\left(-\frac{a}{b}\right)^{\frac{2}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\tan(dx+c)^{\frac{1}{3}}+\tan(dx+c)^{\frac{2}{3}}\right)}{2\left(a^{2}b^{2}+b^{4}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/3)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -a^2*(-a/b)^(2/3)*log(abs(-(-a/b)^(1/3) + tan(d*x + c)^(1/3)))/(a^3*d + a*b
^2*d) + 1/2*(sqrt(3)*a + b)*arctan(sqrt(3) + 2*tan(d*x + c)^(1/3))/(a^2*d +
 b^2*d) - 1/2*(sqrt(3)*a - b)*arctan(-sqrt(3) + 2*tan(d*x + c)^(1/3))/(a^2*
d + b^2*d) + b*arctan(tan(d*x + c)^(1/3))/(a^2*d + b^2*d) + 1/4*a*log(tan(d
*x + c)^(4/3) - tan(d*x + c)^(2/3) + 1)/(a^2*d + b^2*d) - 3/4*b*log(sqrt(3)
*tan(d*x + c)^(1/3) + tan(d*x + c)^(2/3) + 1)/(sqrt(3)*a^2*d + sqrt(3)*b^2*
d) + 3/4*b*log(-sqrt(3)*tan(d*x + c)^(1/3) + tan(d*x + c)^(2/3) + 1)/(sqrt(
3)*a^2*d + sqrt(3)*b^2*d) - 1/2*a*log(tan(d*x + c)^(2/3) + 1)/(a^2*d + b^2*
d) - 3*(-a*b^2)^(2/3)*a*arctan(1/3*sqrt(3)*((-a/b)^(1/3) + 2*tan(d*x + c)^(
1/3))/(-a/b)^(1/3))/((sqrt(3)*a^2*b^2 + sqrt(3)*b^4)*d) + 1/2*(-a*b^2)^(2/3
)*a*log((-a/b)^(2/3) + (-a/b)^(1/3)*tan(d*x + c)^(1/3) + tan(d*x + c)^(2/3)
)/((a^2*b^2 + b^4)*d)
```

Mupad [**B**]
time = 12.19, size = 2137, normalized size = 4.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(5/3)/(a + b*tan(c + d*x)),x)
```

```
[Out] symsum(log(root(32*a^2*b^2*d^4*z^4 + 16*b^4*d^4*z^4 + 16*a^4*d^4*z^4 - 16*a
*b^2*d^3*z^3 - 16*a^3*d^3*z^3 - 4*b^2*d^2*z^2 + 12*a^2*d^2*z^2 - 4*a*d*z +
1, z, k)^2*(root(32*a^2*b^2*d^4*z^4 + 16*b^4*d^4*z^4 + 16*a^4*d^4*z^4 - 16*
a*b^2*d^3*z^3 - 16*a^3*d^3*z^3 - 4*b^2*d^2*z^2 + 12*a^2*d^2*z^2 - 4*a*d*z +
 1, z, k)*(root(32*a^2*b^2*d^4*z^4 + 16*b^4*d^4*z^4 + 16*a^4*d^4*z^4 - 16*a
*b^2*d^3*z^3 - 16*a^3*d^3*z^3 - 4*b^2*d^2*z^2 + 12*a^2*d^2*z^2 - 4*a*d*z +
1, z, k)^2*(root(32*a^2*b^2*d^4*z^4 + 16*b^4*d^4*z^4 + 16*a^4*d^4*z^4 - 16*
a*b^2*d^3*z^3 - 16*a^3*d^3*z^3 - 4*b^2*d^2*z^2 + 12*a^2*d^2*z^2 - 4*a*d*z +
 1, z, k)*((6561*tan(c + d*x)^(1/3)*(96*a^4*b^9*d^6 - 8*a^2*b^11*d^6 + 208*
a^6*b^7*d^6 + 96*a^8*b^5*d^6 - 8*a^10*b^3*d^6))/d^8 + (6561*root(32*a^2*b^2
*d^4*z^4 + 16*b^4*d^4*z^4 + 16*a^4*d^4*z^4 - 16*a*b^2*d^3*z^3 - 16*a^3*d^3*
z^3 - 4*b^2*d^2*z^2 + 12*a^2*d^2*z^2 - 4*a*d*z + 1, z, k)^2*(64*a*b^14*d^6
+ 192*a^3*b^12*d^6 + 128*a^5*b^10*d^6 - 128*a^7*b^8*d^6 - 192*a^9*b^6*d^6 -
 64*a^11*b^4*d^6))/d^6) - (6561*(44*a^2*b^10*d^3 + 20*a^4*b^8*d^3 - 28*a^6*
b^6*d^3 + 60*a^8*b^4*d^3 + 64*a^10*b^2*d^3))/d^6) - (6561*tan(c + d*x)^(1/3
)*(8*a^9*b*d^3 + a^3*b^7*d^3 + 50*a^5*b^5*d^3 - 55*a^7*b^3*d^3))/d^8) + (65
61*(a*b^8 + a^3*b^6 + 4*a^7*b^2))/d^6) - (6561*tan(c + d*x)^(1/3)*(a^6*b -
a^4*b^3))/d^8)*root(32*a^2*b^2*d^4*z^4 + 16*b^4*d^4*z^4 + 16*a^4*d^4*z^4 -
16*a*b^2*d^3*z^3 - 16*a^3*d^3*z^3 - 4*b^2*d^2*z^2 + 12*a^2*d^2*z^2 - 4*a*d*
z + 1, z, k), k, 1, 4) - (log(tan(c + d*x)^(1/3) + 1i)*1i)/(2*(a*d*1i - b*d
)) + log(- ((((419904*a*b^4*(a^2 - b^2)*(a^2 + b^2)^4*(-a^5/(b^2*d^3*(a^2 +
 b^2)^3))^(2/3) + (52488*a^2*b^3*tan(c + d*x)^(1/3)*(a^2 + b^2)^2*(a^4 + b^
4 - 14*a^2*b^2))/d^2)*(-a^5/(b^2*d^3*(a^2 + b^2)^3))^(1/3) + (26244*a^2*b^2
*(a^2 + b^2)^2*(16*a^4 + 11*b^4 - 17*a^2*b^2))/d^3)*(-a^5/(b^2*d^3*(a^2 + b
^2)^3))^(2/3) + (6561*a^3*b*tan(c + d*x)^(1/3)*(8*a^6 + b^6 + 50*a^2*b^4 -
55*a^4*b^2))/d^5)*(-a^5/(b^2*d^3*(a^2 + b^2)^3))^(1/3) - (6561*a*b^2*(4*a^6
 + b^6 + a^2*b^4))/d^6)*(-a^5/(b^2*d^3*(a^2 + b^2)^3))^(2/3) - (6561*a^4*b*
tan(c + d*x)^(1/3)*(a^2 - b^2))/d^8)*(-a^5/(b^8*d^3 + 3*a^2*b^6*d^3 + 3*a^4
*b^4*d^3 + a^6*b^2*d^3))^(1/3) - log(tan(c + d*x)^(1/3)*1i + 1)/(2*(a*d - b
*d*1i)) + log(((3^(1/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2 + 1/2)*(((3^(1/2)*1i
)/2 - 1/2)*(419904*a*b^4*((3^(1/2)*1i)/2 + 1/2)*(a^2 - b^2)*(a^2 + b^2)^4*(
-a^5/(b^2*d^3*(a^2 + b^2)^3))^(2/3) - (52488*a^2*b^3*tan(c + d*x)^(1/3)*(a^
2 + b^2)^2*(a^4 + b^4 - 14*a^2*b^2))/d^2)*(-a^5/(b^2*d^3*(a^2 + b^2)^3))^(1
/3) - (26244*a^2*b^2*(a^2 + b^2)^2*(16*a^4 + 11*b^4 - 17*a^2*b^2))/d^3)*(-a
^5/(b^2*d^3*(a^2 + b^2)^3))^(2/3) + (6561*a^3*b*tan(c + d*x)^(1/3)*(8*a^6 +
 b^6 + 50*a^2*b^4 - 55*a^4*b^2))/d^5)*((3^(1/2)*1i)/2 - 1/2)*(-a^5/(b^2*d^3
*(a^2 + b^2)^3))^(1/3) - (6561*a*b^2*(4*a^6 + b^6 + a^2*b^4))/d^6)*(-a^5/(b
^2*d^3*(a^2 + b^2)^3))^(2/3) - (6561*a^4*b*tan(c + d*x)^(1/3)*(a^2 - b^2))/
d^8)*((3^(1/2)*1i)/2 - 1/2)*(-a^5/(b^8*d^3 + 3*a^2*b^6*d^3 + 3*a^4*b^4*d^3
+ a^6*b^2*d^3))^(1/3) - log(- ((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 - 1/
2)*(((3^(1/2)*1i)/2 + 1/2)*(419904*a*b^4*((3^(1/2)*1i)/2 - 1/2)*(a^2 - b^2)
*(a^2 + b^2)^4*(-a^5/(b^2*d^3*(a^2 + b^2)^3))^(2/3) + (52488*a^2*b^3*tan(c
+ d*x)^(1/3)*(a^2 + b^2)^2*(a^4 + b^4 - 14*a^2*b^2))/d^2)*(-a^5/(b^2*d^3*(a
^2 + b^2)^3))^(1/3) - (26244*a^2*b^2*(a^2 + b^2)^2*(16*a^4 + 11*b^4 - 17*a^
2*b^2))/d^3)*(-a^5/(b^2*d^3*(a^2 + b^2)^3))^(2/3) - (6561*a^3*b*tan(c + d*x
```

```
)^(1/3)*(8*a^6 + b^6 + 50*a^2*b^4 - 55*a^4*b^2))/d^5)*((3^(1/2)*1i)/2 + 1/2
)*(-a^5/(b^2*d^3*(a^2 + b^2)^3))^(1/3) - (6561*a*b^2*(4*a^6 + b^6 + a^2*b^4
))/d^6)*(-a^5/(b^2*d^3*(a^2 + b^2)^3))^(2/3) - (6561*a^4*b*tan(c + d*x)^(1/
3)*(a^2 - b^2))/d^8)*((3^(1/2)*1i)/2 + 1/2)*(-a^5/(b^8*d^3 + 3*a^2*b^6*d^3
+ 3*a^4*b^4*d^3 + a^6*b^2*d^3))^(1/3)
```

3.672 $\int \frac{\sqrt[3]{\tan(c+dx)}}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=465

$$-\frac{b\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{2\left(a^2+b^2\right)d}+\frac{b\text{ArcTan}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{2\left(a^2+b^2\right)d}+\frac{\sqrt{3}\sqrt[3]{a}\,b^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}}{\sqrt{}}\right.}{\left(a^2+b^2\right)d}$$

[Out] 1/2*b*arctan(-3^(1/2)+2*tan(d*x+c)^(1/3))/(a^2+b^2)/d+1/2*b*arctan(3^(1/2)+2*tan(d*x+c)^(1/3))/(a^2+b^2)/d+b*arctan(tan(d*x+c)^(1/3))/(a^2+b^2)/d-3/2*a^(1/3)*b^(2/3)*ln(a^(1/3)+b^(1/3)*tan(d*x+c)^(1/3))/(a^2+b^2)/d-1/2*a*ln(1+tan(d*x+c)^(2/3))/(a^2+b^2)/d+1/2*a^(1/3)*b^(2/3)*ln(a+b*tan(d*x+c))/(a^2+b^2)/d+1/4*a*ln(1-tan(d*x+c)^(2/3)+tan(d*x+c)^(4/3))/(a^2+b^2)/d+a^(1/3)*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*tan(d*x+c)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)/(a^2+b^2)/d-1/2*a*arctan(1/3*(1-2*tan(d*x+c)^(2/3))*3^(1/2))*3^(1/2)/(a^2+b^2)/d-1/4*b*ln(1-3^(1/2)*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3))*3^(1/2)/(a^2+b^2)/d+1/4*b*ln(1+3^(1/2)*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3))*3^(1/2)/(a^2+b^2)/d

Rubi [A]
time = 0.39, antiderivative size = 465, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 18, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3655, 3609, 3619, 3557, 335, 215, 648, 632, 210, 642, 209, 281, 298, 31, 3715, 52, 60, 631}

$$-\frac{\sqrt{3}\,a\text{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)}{2d\left(a^2+b^2\right)}-\frac{b\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{2d\left(a^2+b^2\right)}+\frac{b\text{ArcTan}\left(2\sqrt[3]{\tan(c+dx)}+\sqrt{3}\right)}{2d\left(a^2+b^2\right)}+\frac{b\text{ArcTan}\left(\sqrt[3]{\tan(c+dx)}\right)}{d\left(a^2+b^2\right)}+\frac{\sqrt{3}\sqrt[3]{a}\,b^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{d\left(a^2+b^2\right)}-\frac{a\log\left(\tan^{\frac{2}{3}}(c+dx)+1\right)}{2d\left(a^2+b^2\right)}-\frac{\sqrt{3}\,b\log\left(\tan^{\frac{2}{3}}(c+dx)-\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{4d\left(a^2+b^2\right)}+\frac{\sqrt{3}\,b\log\left(\tan^{\frac{2}{3}}(c+dx)+\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{4d\left(a^2+b^2\right)}+\frac{a\log\left(\tan^{\frac{4}{3}}(c+dx)-\tan^{\frac{2}{3}}(c+dx)+1\right)}{4d\left(a^2+b^2\right)}-\frac{3\sqrt[3]{a}\,b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{\tan(c+dx)}\right)}{2d\left(a^2+b^2\right)}+\frac{\sqrt[3]{a}\,b^{2/3}\log(a+b\tan(c+dx))}{2d\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(1/3)/(a + b*Tan[c + d*x]),x]
```

```
[Out] -1/2*(b*ArcTan[Sqrt[3] - 2*Tan[c + d*x]^(1/3)])/((a^2 + b^2)*d) + (b*ArcTan[Sqrt[3] + 2*Tan[c + d*x]^(1/3)])/(2*(a^2 + b^2)*d) + (Sqrt[3]*a^(1/3)*b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Tan[c + d*x]^(1/3))/(Sqrt[3]*a^(1/3))])/((a^2 + b^2)*d) - (Sqrt[3]*a*ArcTan[(1 - 2*Tan[c + d*x]^(2/3))/Sqrt[3]])/(2*(a^2 + b^2)*d) + (b*ArcTan[Tan[c + d*x]^(1/3)])/((a^2 + b^2)*d) - (3*a^(1/3)*b^(2/3)*Log[a^(1/3) + b^(1/3)*Tan[c + d*x]^(1/3)])/(2*(a^2 + b^2)*d) - (a*Log[1 + Tan[c + d*x]^(2/3)])/(2*(a^2 + b^2)*d) - (Sqrt[3]*b*Log[1 - Sqrt[3]*Tan[c + d*x]^(1/3) + Tan[c + d*x]^(2/3)])/(4*(a^2 + b^2)*d) + (Sqrt[3]*b*Log[1 + Sqrt[3]*Tan[c + d*x]^(1/3) + Tan[c + d*x]^(2/3)])/(4*(a^2 + b^2)*d) + (a^(1/3)*b^(2/3)*Log[a + b*Tan[c + d*x]])/(2*(a^2 + b^2)*d) + (a*Log[1 - Tan[c + d*x]^(2/3) + Tan[c + d*x]^(4/3)])/(4*(a^2 + b^2)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)/((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[(a + b*Tan[e + f*x])^m*
(c - d*Tan[e + f*x]), x], x] + Dist[d^2/(c^2 + d^2), Int[(a + b*Tan[e + f*x
])^m*((1 + Tan[e + f*x]^2)/(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{\sqrt[3]{\tan(c+dx)}}{a+b\tan(c+dx)}\,dx = \frac{\int \sqrt[3]{\tan(c+dx)}\,(a-b\tan(c+dx))\,dx}{a^2+b^2} + \frac{b^2\int \frac{\sqrt[3]{\tan(c+dx)}\,(1+\tan^2(c+dx))}{a+b\tan(c+dx)}\,dx}{a^2+b^2}$$

$$= -\frac{3b\sqrt[3]{\tan(c+dx)}}{(a^2+b^2)\,d} + \frac{\int \frac{b+a\tan(c+dx)}{\tan^{\frac{2}{3}}(c+dx)}\,dx}{a^2+b^2} + \frac{b^2\text{Subst}\left(\int \frac{\sqrt[3]{x}}{a+bx}\,dx, x, \tan(c+dx)\right)}{(a^2+b^2)\,d}$$

$$= \frac{a\int \sqrt[3]{\tan(c+dx)}\,dx}{a^2+b^2} + \frac{b\int \frac{1}{\tan^{\frac{2}{3}}(c+dx)}\,dx}{a^2+b^2} - \frac{(ab)\text{Subst}\left(\int \frac{1}{x^{2/3}(a+bx)}\,dx, x, \tan(c+\right.}{(a^2+b^2)\,d}$$

$$= \frac{\sqrt[3]{a}\,b^{2/3}\log(a+b\tan(c+dx))}{2\,(a^2+b^2)\,d} + \frac{a\text{Subst}\left(\int \frac{\sqrt[3]{x}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{(a^2+b^2)\,d} - \frac{\left(3a^{2/3}\sqrt[3]{b}\right.}{}$$

$$= -\frac{3\sqrt[3]{a}\,b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{\tan(c+dx)}\right)}{2\,(a^2+b^2)\,d} + \frac{\sqrt[3]{a}\,b^{2/3}\log(a+b\tan(c+dx))}{2\,(a^2+b^2)\,d} + \frac{(}{}$$

$$= \frac{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{(a^2+b^2)\,d} - \frac{3\sqrt[3]{a}\,b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{\tan(c\right.}{2\,(a^2+b^2)\,d}$$

$$= \frac{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{(a^2+b^2)\,d} + \frac{b\tan^{-1}\left(\sqrt[3]{\tan(c+dx)}\right)}{(a^2+b^2)\,d} - \frac{3\sqrt[3]{a}}{}$$

$$= \frac{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{(a^2+b^2)\,d} + \frac{b\tan^{-1}\left(\sqrt[3]{\tan(c+dx)}\right)}{(a^2+b^2)\,d} - \frac{3\sqrt[3]{a}}{}$$

$$= -\frac{b\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{2\,(a^2+b^2)\,d} + \frac{b\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{2\,(a^2+b^2)\,d} + \frac{\sqrt{3}\,\sqrt[3]{}}{}$$

$$= -\frac{b\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{2\,(a^2+b^2)\,d} + \frac{b\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{2\,(a^2+b^2)\,d} + \frac{\sqrt{3}\,\sqrt[3]{}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.43, size = 204, normalized size = 0.44

$$\frac{2\sqrt[3]{a}\,b^{2/3}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sqrt[3]{\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{\tan(c+dx)}\right)+\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{\tan(c+dx)}+b^{2/3}\tan^{\frac{2}{3}}(c+dx)\right)\right)+12b\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-\tan^2(c+dx)\right)\sqrt[3]{\tan(c+dx)}+3a\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\tan^2(c+dx)\right)\tan^{\frac{4}{3}}(c+dx)}{4\left(a^2+b^2\right)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(1/3)/(a + b*Tan[c + d*x]),x]
```

```
[Out] (2*a^(1/3)*b^(2/3)*(2*Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)*Tan[c + d*x]^(1/3
))/(Sqrt[3]*a^(1/3))] - 2*Log[a^(1/3) + b^(1/3)*Tan[c + d*x]^(1/3)] + Log[a
^(2/3) - a^(1/3)*b^(1/3)*Tan[c + d*x]^(1/3) + b^(2/3)*Tan[c + d*x]^(2/3)])
+ 12*b*Hypergeometric2F1[1/6, 1, 7/6, -Tan[c + d*x]^2]*Tan[c + d*x]^(1/3) +
 3*a*Hypergeometric2F1[2/3, 1, 5/3, -Tan[c + d*x]^2]*Tan[c + d*x]^(4/3))/(4
*(a^2 + b^2)*d)
```

Maple [A]
time = 0.20, size = 328, normalized size = 0.71

method	result
derivativedivides	$\frac{-\frac{3a\ln\left(1+\tan^{\frac{2}{3}}(dx+c)\right)}{2}+3b\arctan\left(\tan^{\frac{1}{3}}(dx+c)\right)}{3a^2+3b^2}-\frac{3\left(\frac{\ln\left(\tan^{\frac{1}{3}}(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\tan^{\frac{2}{3}}(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\tan^{\frac{1}{3}}(dx+c)+\left(\frac{a}{b}\right.\right.}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{a^2+b^2}$
default	$\frac{-\frac{3a\ln\left(1+\tan^{\frac{2}{3}}(dx+c)\right)}{2}+3b\arctan\left(\tan^{\frac{1}{3}}(dx+c)\right)}{3a^2+3b^2}-\frac{3\left(\frac{\ln\left(\tan^{\frac{1}{3}}(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\tan^{\frac{2}{3}}(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\tan^{\frac{1}{3}}(dx+c)+\left(\frac{a}{b}\right.\right.}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{a^2+b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/3)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(3/(3*a^2+3*b^2)*(-1/2*a*ln(1+tan(d*x+c)^(2/3))+b*arctan(tan(d*x+c)^(1/
3)))-3*(1/3/b/(a/b)^(2/3)*ln(tan(d*x+c)^(1/3)+(a/b)^(1/3))-1/6/b/(a/b)^(2/3
)*ln(tan(d*x+c)^(2/3)-(a/b)^(1/3)*tan(d*x+c)^(1/3)+(a/b)^(2/3))+1/3/b/(a/b)
^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*tan(d*x+c)^(1/3)-1)))*a/(a
^2+b^2)*b+3/(3*a^2+3*b^2)*(1/4*(3^(1/2)*b+a)*ln(1+3^(1/2)*tan(d*x+c)^(1/3)+
tan(d*x+c)^(2/3))+(2*b-1/2*(3^(1/2)*b+a)*3^(1/2))*arctan(3^(1/2)+2*tan(d*x+
c)^(1/3))-1/4*(3^(1/2)*b-a)*ln(1-3^(1/2)*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3))
-(-2*b+1/2*(3^(1/2)*b-a)*3^(1/2))*arctan(-3^(1/2)+2*tan(d*x+c)^(1/3))))
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/3)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**C**] Result contains complex when optimal does not.
time = 3.24, size = 74435, normalized size = 160.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/3)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(6*(sqrt(3)*a^3*d*sqrt(-(a^2 - 2*I*
a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + I*sqrt(3)*a^2*b*d*sqrt(-(a^2 -
2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + sqrt(3)*a*b^2*d*sqrt(-(a^2
- 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + I*sqrt(3)*b^3*d*sqrt(-(a^
2 - 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + 3*a^2 + 4*I*a*b - b^2)/
(a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2) - (sqrt(3)*a^2*d*sqrt(-(a^2 - 2*I*a*b -
 b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + sqrt(3)*b^2*d*sqrt(-(a^2 - 2*I*a*b -
 b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + 3*a + I*b)^2/(a^2*d + b^2*d)^2)/(27*
(3*sqrt(3)*a^4*d^3*(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))^(
3/2) + 3*sqrt(3)*b^4*d^3*(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d
^2))^(3/2) + 2*I*sqrt(3)*a*b*d*sqrt(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2*b^
2 + b^4)*d^2)) - 7*sqrt(3)*b^2*d*sqrt(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2*
b^2 + b^4)*d^2)) + (6*sqrt(3)*b^2*d^3*(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2
*b^2 + b^4)*d^2))^(3/2) + 7*sqrt(3)*d*sqrt(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2
*a^2*b^2 + b^4)*d^2)))*a^2 + 4*a + 4*I*b)/(a^4*d^3 + 2*a^2*b^2*d^3 + b^4*d^
3) + 2*(sqrt(3)*a^2*d*sqrt(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*
d^2)) + sqrt(3)*b^2*d*sqrt(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*
```

d^2)) + 3*a + I*b)^3/(a^2*d + b^2*d)^3 - 18*(sqrt(3)*a^3*d*sqrt(-(a^2 - 2*I
*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + I*sqrt(3)*a^2*b*d*sqrt(-(a^2 -
2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + sqrt(3)*a*b^2*d*sqrt(-(a^2
- 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + I*sqrt(3)*b^3*d*sqrt(-(a
^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + 3*a^2 + 4*I*a*b - b^2)
*(sqrt(3)*a^2*d*sqrt(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))
+ sqrt(3)*b^2*d*sqrt(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))
+ 3*a + I*b)/((a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2)*(a^2*d + b^2*d)) + 9*sqrt
(-72*sqrt(3)*(I*a^10*b + 5*I*a^8*b^3 + 10*I*a^6*b^5 + 10*I*a^4*b^7 + 5*I*a^
2*b^9 + I*b^11)*d^5*(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))^
(5/2) + 36*a^6 + 216*I*a^5*b - 636*a^4*b^2 - 1104*I*a^3*b^3 + 1180*a^2*b^4
+ 728*I*a*b^5 - 196*b^6 - 32*sqrt(3)*(12*I*a^8*b - 15*a^7*b^2 + 23*I*a^6*b^
3 - 45*a^5*b^4 - 3*I*a^4*b^5 - 45*a^3*b^6 - 27*I*a^2*b^7 - 15*a*b^8 - 13*I*
b^9)*d^3*(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))^(3/2) - 8*s
qrt(3)*(39*I*a^6*b - 48*a^5*b^2 + I*a^4*b^3 + 24*a^3*b^4 + 21*I*a^2*b^5 + 7
2*a*b^6 + 59*I*b^7)*d*sqrt(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*
d^2)) - 9*(71*a^8 - 76*I*a^7*b - 44*a^6*b^2 - 108*I*a^5*b^3 - 182*a^4*b^4 +
12*I*a^3*b^5 + 52*a^2*b^6 + 44*I*a*b^7 + 119*b^8)*(a^2 - 2*I*a*b - b^2)/(a
^4 + 2*a^2*b^2 + b^4) + 18*(49*a^10 - 18*I*a^9*b + 139*a^8*b^2 - 72*I*a^7*b
^3 + 66*a^6*b^4 - 108*I*a^5*b^5 - 146*a^4*b^6 - 72*I*a^3*b^7 - 179*a^2*b^8
- 18*I*a*b^9 - 57*b^10)*(a^2 - 2*I*a*b - b^2)^2/(a^4 + 2*a^2*b^2 + b^4)^2 -
279*(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10
+ b^12)*(a^2 - 2*I*a*b - b^2)^3/(a^4 + 2*a^2*b^2 + b^4)^3)/((a^2 + b^2)^3*
d^3))^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(27*(3*sqrt(3)*a^4*d^3*(-(a^2 - 2
*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))^(3/2) + 3*sqrt(3)*b^4*d^3*(-(a
^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))^(3/2) + 2*I*sqrt(3)*a*b*
d*sqrt(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - 7*sqrt(3)*b^
2*d*sqrt(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + (6*sqrt(3)
*b^2*d^3*(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))^(3/2) + 7*s
qrt(3)*d*sqrt(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)))*a^2 +
4*a + 4*I*b)/(a^4*d^3 + 2*a^2*b^2*d^3 + b^4*d^3) + 2*(sqrt(3)*a^2*d*sqrt(-(
a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + sqrt(3)*b^2*d*sqrt(-(
a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + 3*a + I*b)^3/(a^2*d +
b^2*d)^3 - 18*(sqrt(3)*a^3*d*sqrt(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2
+ b^4)*d^2)) + I*sqrt(3)*a^2*b*d*sqrt(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2
*b^2 + b^4)*d^2)) + sqrt(3)*a*b^2*d*sqrt(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2*a
^2*b^2 + b^4)*d^2)) + I*sqrt(3)*b^3*d*sqrt(-(a^2 - 2*I*a*b - b^2)/((a^4 + 2
*a^2*b^2 + b^4)*d^2)) + 3*a^2 + 4*I*a*b - b^2)*(sqrt(3)*a^2*d*sqrt(-(a^2 -
2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + sqrt(3)*b^2*d*sqrt(-(a^2 -
2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + 3*a + I*b)/((a^4*d^2 + 2*a^
2*b^2*d^2 + b^4*d^2)*(a^2*d + b^2*d)) + 9*sqrt(-72*sqrt(3)*(I*a^10*b + 5*I*
a^8*b^3 + 10*I*a^6*b^5 + 10*I*a^4*b^7 + 5*I*a^2*b^9 + I*b^11)*d^5*(-(a^2 -
2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))^(5/2) + 36*a^6 + 216*I*a^5*b
- 636*a^4*b^2 - 1104*I*a^3*b^3 + 1180*a^2*b^4 + 728*I*a*b^5 - 196*b^6 - 32*
sqrt(3)*(12*I*a^8*b - 15*a^7*b^2 + 23*I*a^6*b^3 - 45*a^5*b^4 - 3*I*a^4*b^5

```
- 45*a^3*b^6 - 27*I*a^2*b^7 - 15*a*b^8 - 13*I*b^9)*d^3*(-(a^2 - 2*I*a*b - b
^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))^(3/2) - 8*sqrt(3)*(39*I*a^6*b - 48*a^5*b
^2 + I*a^4*b^3 + 24*a^3*b^4 + 21*I*a^2*b^5 + 72*a*b^6 + 59*I*b^7)*d*sqrt(-(
a^2 - 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - 9*(71*a^8 - 76*I*a^7*
b - 44*a^6*b^2 - 108*I*a^5*b^3 - 182*a^4*b^4 + ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\tan(c+dx)}}{a+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/3)/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**(1/3)/(a + b*tan(c + d*x)), x)
```

Giac [A]
time = 0.71, size = 424, normalized size = 0.91

$$\frac{ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\tan(dx+c)^{\frac{1}{3}}\right|\right)}{a^3d+ab^2d}-\frac{\left(\sqrt{3}\,a-b\right)\arctan\left(\sqrt{3}+2\tan(dx+c)^{\frac{1}{3}}\right)}{2\left(a^2d+b^2d\right)}+\frac{\left(\sqrt{3}\,a+b\right)\arctan\left(-\sqrt{3}+2\tan(dx+c)^{\frac{1}{3}}\right)}{2\left(a^2d+b^2d\right)}+\frac{b\arctan\left(\tan(dx+c)^{\frac{1}{3}}\right)}{a^2d+b^2d}+\frac{a\log\left(\tan(dx+c)^{\frac{4}{3}}-\tan(dx+c)^{\frac{2}{3}}+1\right)}{4\left(a^2d+b^2d\right)}+\frac{3b\log\left(\sqrt{3}\tan(dx+c)^{\frac{1}{3}}+\tan(dx+c)^{\frac{2}{3}}+1\right)}{4\left(\sqrt{3}\,a^2d+\sqrt{3}\,b^2d\right)}-\frac{3b\log\left(-\sqrt{3}\tan(dx+c)^{\frac{1}{3}}+\tan(dx+c)^{\frac{2}{3}}+1\right)}{4\left(\sqrt{3}\,a^2d+\sqrt{3}\,b^2d\right)}-\frac{a\log\left(\tan(dx+c)^{\frac{2}{3}}+1\right)}{2\left(a^2d+b^2d\right)}-\frac{3\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\tan(dx+c)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\left(\sqrt{3}\,a^2+\sqrt{3}\,b^2\right)d}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(\left(-\frac{a}{b}\right)^{\frac{2}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\tan(dx+c)^{\frac{1}{3}}+\tan(dx+c)^{\frac{2}{3}}\right)}{2\left(a^2+b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/3)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] a*b*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + tan(d*x + c)^(1/3)))/(a^3*d + a*b^
2*d) - 1/2*(sqrt(3)*a - b)*arctan(sqrt(3) + 2*tan(d*x + c)^(1/3))/(a^2*d +
b^2*d) + 1/2*(sqrt(3)*a + b)*arctan(-sqrt(3) + 2*tan(d*x + c)^(1/3))/(a^2*d
 + b^2*d) + b*arctan(tan(d*x + c)^(1/3))/(a^2*d + b^2*d) + 1/4*a*log(tan(d*
x + c)^(4/3) - tan(d*x + c)^(2/3) + 1)/(a^2*d + b^2*d) + 3/4*b*log(sqrt(3)*
tan(d*x + c)^(1/3) + tan(d*x + c)^(2/3) + 1)/(sqrt(3)*a^2*d + sqrt(3)*b^2*d
) - 3/4*b*log(-sqrt(3)*tan(d*x + c)^(1/3) + tan(d*x + c)^(2/3) + 1)/(sqrt(3
)*a^2*d + sqrt(3)*b^2*d) - 1/2*a*log(tan(d*x + c)^(2/3) + 1)/(a^2*d + b^2*d
) - 3*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*((-a/b)^(1/3) + 2*tan(d*x + c)^(1/3
))/(-a/b)^(1/3))/((sqrt(3)*a^2 + sqrt(3)*b^2)*d) - 1/2*(-a*b^2)^(1/3)*log((
-a/b)^(2/3) + (-a/b)^(1/3)*tan(d*x + c)^(1/3) + tan(d*x + c)^(2/3))/((a^2 +
 b^2)*d)
```

Mupad [B]
time = 11.41, size = 2111, normalized size = 4.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/3)/(a + b*tan(c + d*x)),x)
```

```
[Out] symsum(log(-root(32*a^2*b^2*d^4*z^4 + 16*b^4*d^4*z^4 + 16*a^4*d^4*z^4 - 16*
a*b^2*d^3*z^3 - 16*a^3*d^3*z^3 - 4*b^2*d^2*z^2 + 12*a^2*d^2*z^2 - 4*a*d*z +
 1, z, k)*(root(32*a^2*b^2*d^4*z^4 + 16*b^4*d^4*z^4 + 16*a^4*d^4*z^4 - 16*a
*b^2*d^3*z^3 - 16*a^3*d^3*z^3 - 4*b^2*d^2*z^2 + 12*a^2*d^2*z^2 - 4*a*d*z +
1, z, k)*(root(32*a^2*b^2*d^4*z^4 + 16*b^4*d^4*z^4 + 16*a^4*d^4*z^4 - 16*a*
b^2*d^3*z^3 - 16*a^3*d^3*z^3 - 4*b^2*d^2*z^2 + 12*a^2*d^2*z^2 - 4*a*d*z + 1
, z, k)^2*(root(32*a^2*b^2*d^4*z^4 + 16*b^4*d^4*z^4 + 16*a^4*d^4*z^4 - 16*a
*b^2*d^3*z^3 - 16*a^3*d^3*z^3 - 4*b^2*d^2*z^2 + 12*a^2*d^2*z^2 - 4*a*d*z +
1, z, k)*((6561*(44*a^2*b^10*d^3 + 84*a^4*b^8*d^3 + 36*a^6*b^6*d^3 - 4*a^8*
b^4*d^3))/d^6 + root(32*a^2*b^2*d^4*z^4 + 16*b^4*d^4*z^4 + 16*a^4*d^4*z^4 -
 16*a*b^2*d^3*z^3 - 16*a^3*d^3*z^3 - 4*b^2*d^2*z^2 + 12*a^2*d^2*z^2 - 4*a*d
*z + 1, z, k)^2*((6561*tan(c + d*x)^(1/3)*(64*a*b^13*d^6 + 240*a^3*b^11*d^6
 + 320*a^5*b^9*d^6 + 160*a^7*b^7*d^6 - 16*a^11*b^3*d^6))/d^7 - (6561*root(3
2*a^2*b^2*d^4*z^4 + 16*b^4*d^4*z^4 + 16*a^4*d^4*z^4 - 16*a*b^2*d^3*z^3 - 16
*a^3*d^3*z^3 - 4*b^2*d^2*z^2 + 12*a^2*d^2*z^2 - 4*a*d*z + 1, z, k)*(64*a*b^
14*d^6 + 192*a^3*b^12*d^6 + 128*a^5*b^10*d^6 - 128*a^7*b^8*d^6 - 192*a^9*b^
6*d^6 - 64*a^11*b^4*d^6))/d^6)) - (6561*tan(c + d*x)^(1/3)*(50*a^2*b^9*d^3
 - 58*a^4*b^7*d^3 + 22*a^6*b^5*d^3 + 2*a^8*b^3*d^3))/d^7) - (6561*(a*b^8 + 5
*a^3*b^6))/d^6) + (6561*tan(c + d*x)^(1/3)*(a*b^7 - 2*a^3*b^5))/d^7))*root(
32*a^2*b^2*d^4*z^4 + 16*b^4*d^4*z^4 + 16*a^4*d^4*z^4 - 16*a*b^2*d^3*z^3 - 1
6*a^3*d^3*z^3 - 4*b^2*d^2*z^2 + 12*a^2*d^2*z^2 - 4*a*d*z + 1, z, k), k, 1,
4) - (log(tan(c + d*x)^(1/3) + 1i)*1i)/(2*(a*d*1i - b*d)) - log(tan(c + d*x
)^(1/3)*1i + 1)/(2*(a*d - b*d*1i)) + log(((((419904*a*b^4*(a^2 - b^2)*(a^2
+ b^2)^4*(-(a*b^2)/(d^3*(a^2 + b^2)^3))^(1/3) - (104976*a*b^3*tan(c + d*x)^
(1/3)*(a^2 - 4*b^2)*(a^2 + b^2)^4)/d)*(-(a*b^2)/(d^3*(a^2 + b^2)^3))^(2/3)
- (26244*a^2*b^4*(a^2 - 11*b^2)*(a^2 + b^2)^2)/d^3)*(-(a*b^2)/(d^3*(a^2 + b
^2)^3))^(1/3) - (13122*a^2*b^3*tan(c + d*x)^(1/3)*(a^6 + 25*b^6 - 29*a^2*b^
4 + 11*a^4*b^2))/d^4)*(-(a*b^2)/(d^3*(a^2 + b^2)^3))^(2/3) - (6561*a*b^6*(5
*a^2 + b^2))/d^6)*(-(a*b^2)/(d^3*(a^2 + b^2)^3))^(1/3) - (6561*a*b^5*tan(c
+ d*x)^(1/3)*(2*a^2 - b^2))/d^7)*(-(a*b^2)/(a^6*d^3 + b^6*d^3 + 3*a^2*b^4*d
^3 + 3*a^4*b^2*d^3))^(1/3) + log(((3^(1/2)*1i)/2 - 1/2)*(((3^(1/2)*1i)/2 +
1/2)*(((3^(1/2)*1i)/2 - 1/2)*(((3^(1/2)*1i)/2 + 1/2)*(419904*a*b^4*((3^(1/2
)*1i)/2 - 1/2)*(a^2 - b^2)*(a^2 + b^2)^4*(-(a*b^2)/(d^3*(a^2 + b^2)^3))^(1/
3) - (104976*a*b^3*tan(c + d*x)^(1/3)*(a^2 - 4*b^2)*(a^2 + b^2)^4)/d)*(-(a*
b^2)/(d^3*(a^2 + b^2)^3))^(2/3) + (26244*a^2*b^4*(a^2 - 11*b^2)*(a^2 + b^2)
^2)/d^3)*(-(a*b^2)/(d^3*(a^2 + b^2)^3))^(1/3) + (13122*a^2*b^3*tan(c + d*x)
^(1/3)*(a^6 + 25*b^6 - 29*a^2*b^4 + 11*a^4*b^2))/d^4)*(-(a*b^2)/(d^3*(a^2 +
 b^2)^3))^(2/3) - (6561*a*b^6*(5*a^2 + b^2))/d^6)*(-(a*b^2)/(d^3*(a^2 + b^2
)^3))^(1/3) - (6561*a*b^5*tan(c + d*x)^(1/3)*(2*a^2 - b^2))/d^7)*((3^(1/2)*
1i)/2 - 1/2)*(-(a*b^2)/(a^6*d^3 + b^6*d^3 + 3*a^2*b^4*d^3 + 3*a^4*b^2*d^3))
^(1/3) - log(- ((3^(1/2)*1i)/2 + 1/2)*(((3^(1/2)*1i)/2 - 1/2)*(((3^(1/2)*1i
)/2 + 1/2)*(((3^(1/2)*1i)/2 - 1/2)*(419904*a*b^4*((3^(1/2)*1i)/2 + 1/2)*(a^
2 - b^2)*(a^2 + b^2)^4*(-(a*b^2)/(d^3*(a^2 + b^2)^3))^(1/3) + (104976*a*b^3
*tan(c + d*x)^(1/3)*(a^2 - 4*b^2)*(a^2 + b^2)^4)/d)*(-(a*b^2)/(d^3*(a^2 + b
^2)^3))^(2/3) + (26244*a^2*b^4*(a^2 - 11*b^2)*(a^2 + b^2)^2)/d^3)*(-(a*b^2)
```

```
/(d^3*(a^2 + b^2)^3))^(1/3) - (13122*a^2*b^3*tan(c + d*x)^(1/3)*(a^6 + 25*b
^6 - 29*a^2*b^4 + 11*a^4*b^2))/d^4)*(-(a*b^2)/(d^3*(a^2 + b^2)^3))^(2/3) -
(6561*a*b^6*(5*a^2 + b^2))/d^6)*(-(a*b^2)/(d^3*(a^2 + b^2)^3))^(1/3) - (656
1*a*b^5*tan(c + d*x)^(1/3)*(2*a^2 - b^2))/d^7)*((3^(1/2)*1i)/2 + 1/2)*(-(a*
b^2)/(a^6*d^3 + b^6*d^3 + 3*a^2*b^4*d^3 + 3*a^4*b^2*d^3))^(1/3)
```

3.673 $\int \frac{1}{\sqrt[3]{\tan(c+dx)}\,(a+b\tan(c+dx))}\,dx$

Optimal. Leaf size=467

$$\frac{b\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{2\left(a^2+b^2\right)d}-\frac{b\text{ArcTan}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{2\left(a^2+b^2\right)d}-\frac{\sqrt{3}\,b^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sqrt[3]{\tan}}{\sqrt{3}\,\sqrt[3]{}}\right.}{\sqrt[3]{a}\,\left(a^2+b^2\right)d}$$

```
[Out] -1/2*b*arctan(-3^(1/2)+2*tan(d*x+c)^(1/3))/(a^2+b^2)/d-1/2*b*arctan(3^(1/2)
+2*tan(d*x+c)^(1/3))/(a^2+b^2)/d-b*arctan(tan(d*x+c)^(1/3))/(a^2+b^2)/d-3/2
*b^(4/3)*ln(a^(1/3)+b^(1/3)*tan(d*x+c)^(1/3))/a^(1/3)/(a^2+b^2)/d+1/2*a*ln(
1+tan(d*x+c)^(2/3))/(a^2+b^2)/d+1/2*b^(4/3)*ln(a+b*tan(d*x+c))/a^(1/3)/(a^2
+b^2)/d-1/4*a*ln(1-tan(d*x+c)^(2/3)+tan(d*x+c)^(4/3))/(a^2+b^2)/d-b^(4/3)*a
rctan(1/3*(a^(1/3)-2*b^(1/3)*tan(d*x+c)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)/a^(
1/3)/(a^2+b^2)/d-1/2*a*arctan(1/3*(1-2*tan(d*x+c)^(2/3))*3^(1/2))*3^(1/2)/(
a^2+b^2)/d-1/4*b*ln(1-3^(1/2)*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3))*3^(1/2)/(a
^2+b^2)/d+1/4*b*ln(1+3^(1/2)*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3))*3^(1/2)/(a^
2+b^2)/d
```

Rubi [A]
time = 0.48, antiderivative size = 467, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 16, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {3655, 3619, 3557, 335, 281, 206, 31, 648, 632, 210, 642, 301, 209, 3715, 58, 631}

$$-\frac{\sqrt{3}\,a\text{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)}{2d\left(a^2+b^2\right)}+\frac{b\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{2d\left(a^2+b^2\right)}-\frac{b\text{ArcTan}\left(2\sqrt[3]{\tan(c+dx)}+\sqrt{3}\right)}{2d\left(a^2+b^2\right)}-\frac{b\text{ArcTan}\left(\sqrt[3]{\tan(c+dx)}\right)}{d\left(a^2+b^2\right)}-\frac{\sqrt{3}\,b^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sqrt[3]{\tan(c+dx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt[3]{a}\,d\left(a^2+b^2\right)}-\frac{\sqrt{3}\,b\log\left(\tan^{\frac{2}{3}}(c+dx)-\sqrt{3}\,\sqrt[3]{\tan(c+dx)}+1\right)}{4d\left(a^2+b^2\right)}+\frac{\sqrt{3}\,b\log\left(\tan^{\frac{2}{3}}(c+dx)+\sqrt{3}\,\sqrt[3]{\tan(c+dx)}+1\right)}{4d\left(a^2+b^2\right)}+\frac{a\log\left(\tan^{\frac{2}{3}}(c+dx)+1\right)}{2d\left(a^2+b^2\right)}-\frac{a\log\left(\tan^{\frac{4}{3}}(c+dx)-\tan^{\frac{2}{3}}(c+dx)+1\right)}{4d\left(a^2+b^2\right)}-\frac{3b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{\tan(c+dx)}\right)}{2\sqrt[3]{a}\,d\left(a^2+b^2\right)}+\frac{b^{4/3}\log(a+b\tan(c+dx))}{2\sqrt[3]{a}\,d\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(1/3)*(a + b*Tan[c + d*x])),x]
```

```
[Out] (b*ArcTan[Sqrt[3] - 2*Tan[c + d*x]^(1/3)])/(2*(a^2 + b^2)*d) - (b*ArcTan[Sq
rt[3] + 2*Tan[c + d*x]^(1/3)])/(2*(a^2 + b^2)*d) - (Sqrt[3]*b^(4/3)*ArcTan[
(a^(1/3) - 2*b^(1/3)*Tan[c + d*x]^(1/3))/(Sqrt[3]*a^(1/3))])/(a^(1/3)*(a^2
+ b^2)*d) - (Sqrt[3]*a*ArcTan[(1 - 2*Tan[c + d*x]^(2/3))/Sqrt[3]])/(2*(a^2
+ b^2)*d) - (b*ArcTan[Tan[c + d*x]^(1/3)])/((a^2 + b^2)*d) - (3*b^(4/3)*Log
[a^(1/3) + b^(1/3)*Tan[c + d*x]^(1/3)])/(2*a^(1/3)*(a^2 + b^2)*d) + (a*Log[
1 + Tan[c + d*x]^(2/3)])/(2*(a^2 + b^2)*d) - (Sqrt[3]*b*Log[1 - Sqrt[3]*Tan
[c + d*x]^(1/3) + Tan[c + d*x]^(2/3)])/(4*(a^2 + b^2)*d) + (Sqrt[3]*b*Log[1
 + Sqrt[3]*Tan[c + d*x]^(1/3) + Tan[c + d*x]^(2/3)])/(4*(a^2 + b^2)*d) + (b
^(4/3)*Log[a + b*Tan[c + d*x]])/(2*a^(1/3)*(a^2 + b^2)*d) - (a*Log[1 - Tan[
c + d*x]^(2/3) + Tan[c + d*x]^(4/3)])/(4*(a^2 + b^2)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)/((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[(a + b*Tan[e + f*x])^m*
(c - d*Tan[e + f*x]), x], x] + Dist[d^2/(c^2 + d^2), Int[(a + b*Tan[e + f*x
])^m*((1 + Tan[e + f*x]^2)/(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{\tan(c+dx)}\,(a+b\tan(c+dx))}\,dx = \frac{\int \frac{a-b\tan(c+dx)}{\sqrt[3]{\tan(c+dx)}}\,dx}{a^2+b^2} + \frac{b^2\int \frac{1+\tan^2(c+dx)}{\sqrt[3]{\tan(c+dx)}\,(a+b\tan(c+dx))}\,dx}{a^2+b^2}$$

$$= \frac{a\int \frac{1}{\sqrt[3]{\tan(c+dx)}}\,dx}{a^2+b^2} - \frac{b\int \tan^{\frac{2}{3}}(c+dx)\,dx}{a^2+b^2} + \frac{b^2\text{Subst}\Big(\int \frac{}{\sqrt[3]{x}}$$

$$= \frac{b^{4/3}\log(a+b\tan(c+dx))}{2\sqrt[3]{a}\,(a^2+b^2)\,d} + \frac{a\text{Subst}\Big(\int \frac{1}{\sqrt[3]{x}\,(1+x^2)}\,dx, x, \tan(c-}{(a^2+b^2)\,d}$$

$$= -\frac{3b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{\tan(c+dx)}\right)}{2\sqrt[3]{a}\,(a^2+b^2)\,d} + \frac{b^{4/3}\log(a+b\tan(c+}{2\sqrt[3]{a}\,(a^2+b^2)\,}$$

$$= -\frac{\sqrt{3}\,b^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}\,(a^2+b^2)\,d} - \frac{3b^{4/3}\log\left(\sqrt[3]{a}+\right.}{2\sqrt[3]{a}\,(}$$

$$= -\frac{\sqrt{3}\,b^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}\,(a^2+b^2)\,d} - \frac{b\tan^{-1}\left(\sqrt[3]{\tan(c\right.}{(a^2+b^2)\,}$$

$$= -\frac{\sqrt{3}\,b^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}\,(a^2+b^2)\,d} - \frac{b\tan^{-1}\left(\sqrt[3]{\tan(c\right.}{(a^2+b^2)\,}$$

$$= \frac{b\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{2\,(a^2+b^2)\,d} - \frac{b\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(c-}\right.}{2\,(a^2+b^2)\,d}$$

$$= \frac{b\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{2\,(a^2+b^2)\,d} - \frac{b\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(c-}\right.}{2\,(a^2+b^2)\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.28, size = 162, normalized size = 0.35

$$\frac{30b^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{b\tan(c+dx)}{a}\right)\tan^{\frac{2}{3}}(c+dx)-a\left(5a\left(2\sqrt{3}\,\mathrm{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)-2\log\left(1+\tan^{\frac{2}{3}}(c+dx)\right)+\log\left(1-\tan^{\frac{2}{3}}(c+dx)+\tan^{\frac{4}{3}}(c+dx)\right)\right)+12b\,{}_2F_1\left(\frac{5}{6},1;\frac{11}{6};-\tan^2(c+dx)\right)\tan^{\frac{5}{3}}(c+dx)\right)}{20a\,(a^2+b^2)\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Tan[c + d*x]^(1/3)*(a + b*Tan[c + d*x])),x]
```

```
[Out] (30*b^2*Hypergeometric2F1[2/3, 1, 5/3, -((b*Tan[c + d*x])/a)]*Tan[c + d*x]^
(2/3) - a*(5*a*(2*Sqrt[3]*ArcTan[(1 - 2*Tan[c + d*x]^(2/3))/Sqrt[3]] - 2*Lo
g[1 + Tan[c + d*x]^(2/3)] + Log[1 - Tan[c + d*x]^(2/3) + Tan[c + d*x]^(4/3)
]) + 12*b*Hypergeometric2F1[5/6, 1, 11/6, -Tan[c + d*x]^2]*Tan[c + d*x]^(5/
3)))/(20*a*(a^2 + b^2)*d)
```

Maple [A]
time = 0.20, size = 340, normalized size = 0.73

method	result
derivativedivides	$\frac{\frac{3a\ln\left(1+\tan^{\frac{2}{3}}(dx+c)\right)}{2}-3b\arctan\left(\tan^{\frac{1}{3}}(dx+c)\right)}{3a^2+3b^2}+\frac{-\frac{3\left(\sqrt{3}\,b+a\right)\ln\left(1-\sqrt{3}\,\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)\right)}{4}-3\left(-\sqrt{3}\,a-b+\right.}{}$
default	$\frac{\frac{3a\ln\left(1+\tan^{\frac{2}{3}}(dx+c)\right)}{2}-3b\arctan\left(\tan^{\frac{1}{3}}(dx+c)\right)}{3a^2+3b^2}+\frac{-\frac{3\left(\sqrt{3}\,b+a\right)\ln\left(1-\sqrt{3}\,\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)\right)}{4}-3\left(-\sqrt{3}\,a-b+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(1/3)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(3/(3*a^2+3*b^2)*(1/2*a*ln(1+tan(d*x+c)^(2/3))-b*arctan(tan(d*x+c)^(1/3
)))+3/(3*a^2+3*b^2)*(-1/4*(3^(1/2)*b+a)*ln(1-3^(1/2)*tan(d*x+c)^(1/3)+tan(d
*x+c)^(2/3))-(-3^(1/2)*a-b+1/2*(3^(1/2)*b+a)*3^(1/2))*arctan(-3^(1/2)+2*tan
(d*x+c)^(1/3))+1/4*(3^(1/2)*b-a)*ln(1+3^(1/2)*tan(d*x+c)^(1/3)+tan(d*x+c)^(
2/3))+(-3^(1/2)*a+b-1/2*(3^(1/2)*b-a)*3^(1/2))*arctan(3^(1/2)+2*tan(d*x+c)^
(1/3)))+3*(-1/3/b/(a/b)^(1/3)*ln(tan(d*x+c)^(1/3)+(a/b)^(1/3))+1/6/b/(a/b)^
(1/3)*ln(tan(d*x+c)^(2/3)-(a/b)^(1/3)*tan(d*x+c)^(1/3)+(a/b)^(2/3))+1/3*3^(
1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*tan(d*x+c)^(1/3)-1)))/
(a^2+b^2)*b^2)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/3)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**C**] Result contains complex when optimal does not.
time = 3.91, size = 73744, normalized size = 157.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/3)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/48*(2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(6*(sqrt(3)*a^3*d*sqrt(-(a^2 + 2*I
*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - I*sqrt(3)*a^2*b*d*sqrt(-(a^2 +
 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + sqrt(3)*a*b^2*d*sqrt(-(a^2
 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - I*sqrt(3)*b^3*d*sqrt(-(a
^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - 3*a^2 + 4*I*a*b + b^2)
/(a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2) + (sqrt(3)*a^2*d*sqrt(-(a^2 + 2*I*a*b
- b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + sqrt(3)*b^2*d*sqrt(-(a^2 + 2*I*a*b
- b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - 3*a + I*b)^2/(a^2*d + b^2*d)^2)/(27
*(3*sqrt(3)*a^4*d^3*(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))^
(3/2) + 3*sqrt(3)*b^4*d^3*(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*
d^2))^(3/2) - 2*I*sqrt(3)*a*b*d*sqrt(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b
^2 + b^4)*d^2)) - 7*sqrt(3)*b^2*d*sqrt(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2
*b^2 + b^4)*d^2)) + (6*sqrt(3)*b^2*d^3*(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^
2*b^2 + b^4)*d^2))^(3/2) + 7*sqrt(3)*d*sqrt(-(a^2 + 2*I*a*b - b^2)/((a^4 +
2*a^2*b^2 + b^4)*d^2)))*a^2 - 4*a + 4*I*b)/(a^4*d^3 + 2*a^2*b^2*d^3 + b^4*d
^3) + 2*(sqrt(3)*a^2*d*sqrt(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)
```

*d^2)) + sqrt(3)*b^2*d*sqrt(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)
*d^2)) - 3*a + I*b)^3/(a^2*d + b^2*d)^3 + 18*(sqrt(3)*a^3*d*sqrt(-(a^2 + 2*
I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - I*sqrt(3)*a^2*b*d*sqrt(-(a^2
+ 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + sqrt(3)*a*b^2*d*sqrt(-(a^
2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - I*sqrt(3)*b^3*d*sqrt(-(
a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - 3*a^2 + 4*I*a*b + b^2
)*(sqrt(3)*a^2*d*sqrt(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))
 + sqrt(3)*b^2*d*sqrt(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))
 - 3*a + I*b)/((a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2)*(a^2*d + b^2*d)) + 18*sq
rt(-18*sqrt(3)*(I*a^10*b + 5*I*a^8*b^3 + 10*I*a^6*b^5 + 10*I*a^4*b^7 + 5*I*
a^2*b^9 + I*b^11)*d^5*(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)
)^(5/2) + 9*a^6 - 54*I*a^5*b - 159*a^4*b^2 + 276*I*a^3*b^3 + 295*a^2*b^4 -
182*I*a*b^5 - 49*b^6 - 8*sqrt(3)*(12*I*a^8*b + 15*a^7*b^2 + 23*I*a^6*b^3 +
45*a^5*b^4 - 3*I*a^4*b^5 + 45*a^3*b^6 - 27*I*a^2*b^7 + 15*a*b^8 - 13*I*b^9)
d^3(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))^(3/2) - 2*sqrt(
3)*(39*I*a^6*b + 48*a^5*b^2 + I*a^4*b^3 - 24*a^3*b^4 + 21*I*a^2*b^5 - 72*a*
b^6 + 59*I*b^7)*d*sqrt(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)
) - 9/4*(71*a^8 + 76*I*a^7*b - 44*a^6*b^2 + 108*I*a^5*b^3 - 182*a^4*b^4 - 1
2*I*a^3*b^5 + 52*a^2*b^6 - 44*I*a*b^7 + 119*b^8)*(a^2 + 2*I*a*b - b^2)/(a^4
 + 2*a^2*b^2 + b^4) + 9/2*(49*a^10 + 18*I*a^9*b + 139*a^8*b^2 + 72*I*a^7*b^
3 + 66*a^6*b^4 + 108*I*a^5*b^5 - 146*a^4*b^6 + 72*I*a^3*b^7 - 179*a^2*b^8 +
 18*I*a*b^9 - 57*b^10)*(a^2 + 2*I*a*b - b^2)^2/(a^4 + 2*a^2*b^2 + b^4)^2 -
279/4*(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^1
0 + b^12)*(a^2 + 2*I*a*b - b^2)^3/(a^4 + 2*a^2*b^2 + b^4)^3)/((a^2 + b^2)^3
d^3))^(1/3) + (1/2)^(1/3)(I*sqrt(3) + 1)*(27*(3*sqrt(3)*a^4*d^3*(-(a^2 +
2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))^(3/2) + 3*sqrt(3)*b^4*d^3*(-(
a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))^(3/2) - 2*I*sqrt(3)*a*b
*d*sqrt(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - 7*sqrt(3)*b
^2*d*sqrt(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + (6*sqrt(3
)*b^2*d^3*(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))^(3/2) + 7*
sqrt(3)*d*sqrt(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)))*a^2 -
 4*a + 4*I*b)/(a^4*d^3 + 2*a^2*b^2*d^3 + b^4*d^3) + 2*(sqrt(3)*a^2*d*sqrt(-
(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + sqrt(3)*b^2*d*sqrt(-
(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - 3*a + I*b)^3/(a^2*d
+ b^2*d)^3 + 18*(sqrt(3)*a^3*d*sqrt(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^
2 + b^4)*d^2)) - I*sqrt(3)*a^2*b*d*sqrt(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^
2*b^2 + b^4)*d^2)) + sqrt(3)*a*b^2*d*sqrt(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*
a^2*b^2 + b^4)*d^2)) - I*sqrt(3)*b^3*d*sqrt(-(a^2 + 2*I*a*b - b^2)/((a^4 +
2*a^2*b^2 + b^4)*d^2)) - 3*a^2 + 4*I*a*b + b^2)*(sqrt(3)*a^2*d*sqrt(-(a^2 +
 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + sqrt(3)*b^2*d*sqrt(-(a^2 +
 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - 3*a + I*b)/((a^4*d^2 + 2*a
^2*b^2*d^2 + b^4*d^2)*(a^2*d + b^2*d)) + 18*sqrt(-18*sqrt(3)*(I*a^10*b + 5*
I*a^8*b^3 + 10*I*a^6*b^5 + 10*I*a^4*b^7 + 5*I*a^2*b^9 + I*b^11)*d^5*(-(a^2
+ 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))^(5/2) + 9*a^6 - 54*I*a^5*b
- 159*a^4*b^2 + 276*I*a^3*b^3 + 295*a^2*b^4 - 182*I*a*b^5 - 49*b^6 - 8*sqrt

```
(3)*(12*I*a^8*b + 15*a^7*b^2 + 23*I*a^6*b^3 + 45*a^5*b^4 - 3*I*a^4*b^5 + 45
*a^3*b^6 - 27*I*a^2*b^7 + 15*a*b^8 - 13*I*b^9)*d^3*(-(a^2 + 2*I*a*b - b^2)/
((a^4 + 2*a^2*b^2 + b^4)*d^2))^(3/2) - 2*sqrt(3)*(39*I*a^6*b + 48*a^5*b^2 +
 I*a^4*b^3 - 24*a^3*b^4 + 21*I*a^2*b^5 - 72*a*b^6 + 59*I*b^7)*d*sqrt(-(a^2
+ 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - 9/4*(71*a^8 + 76*I*a^7*b
- 44*a^6*b^2 + 108*I*a^5*b^3 - 182*a^4*b^4 - 12...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\tan{(c + dx)})\sqrt[3]{\tan{(c + dx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(1/3)/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(1/((a + b*tan(c + d*x))*tan(c + d*x)**(1/3)), x)
```

Giac [**A**]
time = 0.69, size = 430, normalized size = 0.92

$$-\frac{b^{2}\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\tan\left(dx+c\right)^{\frac{1}{3}}\right|\right)}{a^{3}d+ab^{2}d}-\frac{\left(\sqrt{3}\,a+b\right)\arctan\left(\sqrt{3}+2\,\tan\left(dx+c\right)^{\frac{1}{3}}\right)}{2\left(a^{2}d+b^{2}d\right)}+\frac{\left(\sqrt{3}\,a-b\right)\arctan\left(-\sqrt{3}+2\,\tan\left(dx+c\right)^{\frac{1}{3}}\right)}{2\left(a^{2}d+b^{2}d\right)}-\frac{b\arctan\left(\tan\left(dx+c\right)^{\frac{1}{3}}\right)}{a^{2}d+b^{2}d}-\frac{a\log\left(\tan\left(dx+c\right)^{\frac{4}{3}}-\tan\left(dx+c\right)^{\frac{2}{3}}+1\right)}{4\left(a^{2}d+b^{2}d\right)}+\frac{3\,b\log\left(\sqrt{3}\,\tan\left(dx+c\right)^{\frac{1}{3}}+\tan\left(dx+c\right)^{\frac{2}{3}}+1\right)}{4\left(\sqrt{3}\,a^{2}d+\sqrt{3}\,b^{2}d\right)}-\frac{3\,b\log\left(-\sqrt{3}\,\tan\left(dx+c\right)^{\frac{1}{3}}+\tan\left(dx+c\right)^{\frac{2}{3}}+1\right)}{4\left(\sqrt{3}\,a^{2}d+\sqrt{3}\,b^{2}d\right)}+\frac{a\log\left(\tan\left(dx+c\right)^{\frac{2}{3}}+1\right)}{2\left(a^{2}d+b^{2}d\right)}-\frac{3\left(-ab^{2}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,\tan\left(dx+c\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\left(\sqrt{3}\,a^{3}+\sqrt{3}\,ab^{2}\right)d}+\frac{\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(\left(-\frac{a}{b}\right)^{\frac{2}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\tan\left(dx+c\right)^{\frac{1}{3}}+\tan\left(dx+c\right)^{\frac{2}{3}}\right)}{2\left(a^{3}+ab^{2}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(1/3)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -b^2*(-a/b)^(2/3)*log(abs(-(-a/b)^(1/3) + tan(d*x + c)^(1/3)))/(a^3*d + a*b
^2*d) - 1/2*(sqrt(3)*a + b)*arctan(sqrt(3) + 2*tan(d*x + c)^(1/3))/(a^2*d +
 b^2*d) + 1/2*(sqrt(3)*a - b)*arctan(-sqrt(3) + 2*tan(d*x + c)^(1/3))/(a^2*
d + b^2*d) - b*arctan(tan(d*x + c)^(1/3))/(a^2*d + b^2*d) - 1/4*a*log(tan(d
*x + c)^(4/3) - tan(d*x + c)^(2/3) + 1)/(a^2*d + b^2*d) + 3/4*b*log(sqrt(3)
*tan(d*x + c)^(1/3) + tan(d*x + c)^(2/3) + 1)/(sqrt(3)*a^2*d + sqrt(3)*b^2*
d) - 3/4*b*log(-sqrt(3)*tan(d*x + c)^(1/3) + tan(d*x + c)^(2/3) + 1)/(sqrt(
3)*a^2*d + sqrt(3)*b^2*d) + 1/2*a*log(tan(d*x + c)^(2/3) + 1)/(a^2*d + b^2*
d) - 3*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*((-a/b)^(1/3) + 2*tan(d*x + c)^(1/
3))/(-a/b)^(1/3))/((sqrt(3)*a^3 + sqrt(3)*a*b^2)*d) + 1/2*(-a*b^2)^(2/3)*lo
g((-a/b)^(2/3) + (-a/b)^(1/3)*tan(d*x + c)^(1/3) + tan(d*x + c)^(2/3))/((a^
3 + a*b^2)*d)
```

Mupad [**B**]
time = 11.67, size = 2050, normalized size = 4.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(1/3)*(a + b*tan(c + d*x))),x)
```

```
[Out] (log(tan(c + d*x)^(1/3) + 1i)*1i)/(2*(a*d*1i - b*d)) + log(tan(c + d*x)^(1/
3)*1i + 1)/(2*(a*d - b*d*1i)) + symsum(log((6561*b^7*tan(c + d*x)^(1/3))/d^
8 - root(32*a^2*b^2*d^4*z^4 + 16*b^4*d^4*z^4 + 16*a^4*d^4*z^4 + 16*a*b^2*d^
3*z^3 + 16*a^3*d^3*z^3 - 4*b^2*d^2*z^2 + 12*a^2*d^2*z^2 + 4*a*d*z + 1, z, k
)^2*(root(32*a^2*b^2*d^4*z^4 + 16*b^4*d^4*z^4 + 16*a^4*d^4*z^4 + 16*a*b^2*d
^3*z^3 + 16*a^3*d^3*z^3 - 4*b^2*d^2*z^2 + 12*a^2*d^2*z^2 + 4*a*d*z + 1, z,
k)*(root(32*a^2*b^2*d^4*z^4 + 16*b^4*d^4*z^4 + 16*a^4*d^4*z^4 + 16*a*b^2*d^
3*z^3 + 16*a^3*d^3*z^3 - 4*b^2*d^2*z^2 + 12*a^2*d^2*z^2 + 4*a*d*z + 1, z, k
)^2*((6561*(20*a^2*b^10*d^3 + 44*a^4*b^8*d^3 + 28*a^6*b^6*d^3 + 4*a^8*b^4*d
^3))/d^6 - root(32*a^2*b^2*d^4*z^4 + 16*b^4*d^4*z^4 + 16*a^4*d^4*z^4 + 16*a
*b^2*d^3*z^3 + 16*a^3*d^3*z^3 - 4*b^2*d^2*z^2 + 12*a^2*d^2*z^2 + 4*a*d*z +
1, z, k)*((6561*tan(c + d*x)^(1/3)*(64*b^13*d^6 + 120*a^2*b^11*d^6 + 96*a^4
*b^9*d^6 + 80*a^6*b^7*d^6 + 32*a^8*b^5*d^6 - 8*a^10*b^3*d^6))/d^8 + (6561*r
oot(32*a^2*b^2*d^4*z^4 + 16*b^4*d^4*z^4 + 16*a^4*d^4*z^4 + 16*a*b^2*d^3*z^3
 + 16*a^3*d^3*z^3 - 4*b^2*d^2*z^2 + 12*a^2*d^2*z^2 + 4*a*d*z + 1, z, k)^2*(
64*a*b^14*d^6 + 192*a^3*b^12*d^6 + 128*a^5*b^10*d^6 - 128*a^7*b^8*d^6 - 192
*a^9*b^6*d^6 - 64*a^11*b^4*d^6))/d^6)) - (6561*tan(c + d*x)^(1/3)*(40*a*b^9
*d^3 - 7*a^3*b^7*d^3 + 2*a^5*b^5*d^3 + a^7*b^3*d^3))/d^8) + (6561*(3*a*b^8
- a^3*b^6))/d^6))*root(32*a^2*b^2*d^4*z^4 + 16*b^4*d^4*z^4 + 16*a^4*d^4*z^4
 + 16*a*b^2*d^3*z^3 + 16*a^3*d^3*z^3 - 4*b^2*d^2*z^2 + 12*a^2*d^2*z^2 + 4*a
*d*z + 1, z, k), k, 1, 4) + log((6561*b^7*tan(c + d*x)^(1/3))/d^8 - ((((419
904*a*b^4*(a^2 - b^2)*(a^2 + b^2)^4*(-b^4/(a*d^3*(a^2 + b^2)^3))^(2/3) + (5
2488*b^3*tan(c + d*x)^(1/3)*(a^2 + b^2)^2*(a^6 - 8*b^6 + a^2*b^4 - 6*a^4*b^
2))/d^2)*(-b^4/(a*d^3*(a^2 + b^2)^3))^(1/3) + (26244*a^2*b^4*(a^2 + 5*b^2)*
(a^2 + b^2)^2)/d^3)*(-b^4/(a*d^3*(a^2 + b^2)^3))^(2/3) - (6561*a*b^3*tan(c
+ d*x)^(1/3)*(a^6 + 40*b^6 - 7*a^2*b^4 + 2*a^4*b^2))/d^5)*(-b^4/(a*d^3*(a^2
 + b^2)^3))^(1/3) - (6561*a*b^6*(a^2 - 3*b^2))/d^6)*(-b^4/(a*d^3*(a^2 + b^2
)^3))^(2/3))*(-b^4/(a^7*d^3 + a*b^6*d^3 + 3*a^3*b^4*d^3 + 3*a^5*b^2*d^3))^(
1/3) - log((6561*b^7*tan(c + d*x)^(1/3))/d^8 - ((3^(1/2)*1i)/2 - 1/2)*(((3^
(1/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2 + 1/2)*((52488*b^3*tan(c + d*x)^(1/3)*
(a^2 + b^2)^2*(a^6 - 8*b^6 + a^2*b^4 - 6*a^4*b^2))/d^2 + 419904*a*b^4*((3^(
1/2)*1i)/2 - 1/2)*(a^2 - b^2)*(a^2 + b^2)^4*(-b^4/(a*d^3*(a^2 + b^2)^3))^(2
/3))*(-b^4/(a*d^3*(a^2 + b^2)^3))^(1/3) - (26244*a^2*b^4*(a^2 + 5*b^2)*(a^2
 + b^2)^2)/d^3)*((3^(1/2)*1i)/2 - 1/2)*(-b^4/(a*d^3*(a^2 + b^2)^3))^(2/3) +
 (6561*a*b^3*tan(c + d*x)^(1/3)*(a^6 + 40*b^6 - 7*a^2*b^4 + 2*a^4*b^2))/d^5
)*(-b^4/(a*d^3*(a^2 + b^2)^3))^(1/3) - (6561*a*b^6*(a^2 - 3*b^2))/d^6)*(-b^
4/(a*d^3*(a^2 + b^2)^3))^(2/3))*((3^(1/2)*1i)/2 + 1/2)*(-b^4/(a^7*d^3 + a*b
^6*d^3 + 3*a^3*b^4*d^3 + 3*a^5*b^2*d^3))^(1/3) + log((6561*b^7*tan(c + d*x)
^(1/3))/d^8 - ((3^(1/2)*1i)/2 + 1/2)*(((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i
)/2 - 1/2)*((52488*b^3*tan(c + d*x)^(1/3)*(a^2 + b^2)^2*(a^6 - 8*b^6 + a^2*
b^4 - 6*a^4*b^2))/d^2 - 419904*a*b^4*((3^(1/2)*1i)/2 + 1/2)*(a^2 - b^2)*(a^
2 + b^2)^4*(-b^4/(a*d^3*(a^2 + b^2)^3))^(2/3))*(-b^4/(a*d^3*(a^2 + b^2)^3))
^(1/3) + (26244*a^2*b^4*(a^2 + 5*b^2)*(a^2 + b^2)^2)/d^3)*((3^(1/2)*1i)/2 +
 1/2)*(-b^4/(a*d^3*(a^2 + b^2)^3))^(2/3) + (6561*a*b^3*tan(c + d*x)^(1/3)*(
a^6 + 40*b^6 - 7*a^2*b^4 + 2*a^4*b^2))/d^5)*(-b^4/(a*d^3*(a^2 + b^2)^3))^(1
```

```
/3) + (6561*a*b^6*(a^2 - 3*b^2))/d^6)*(-b^4/(a*d^3*(a^2 + b^2)^3))^(2/3))*(
(3^(1/2)*1i)/2 - 1/2)*(-b^4/(a^7*d^3 + a*b^6*d^3 + 3*a^3*b^4*d^3 + 3*a^5*b^
2*d^3))^(1/3)
```

3.674 $\int \frac{1}{\tan^{\frac{5}{3}}(c+dx)(a+b\tan(c+dx))}\,dx$

Optimal. Leaf size=525

$$\frac{b\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{2(a^2+b^2)d}-\frac{b\text{ArcTan}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx)}\right)}{2(a^2+b^2)d}+\frac{\sqrt{3}\,b^{8/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sqrt[3]{\tan(}}{\sqrt{3}\,\sqrt[3]{a}}\right.}{a^{5/3}(a^2+b^2)d}$$

```
[Out] -1/2*b*arctan(-3^(1/2)+2*tan(d*x+c)^(1/3))/(a^2+b^2)/d-1/2*b*arctan(3^(1/2)
+2*tan(d*x+c)^(1/3))/(a^2+b^2)/d-b*arctan(tan(d*x+c)^(1/3))/(a^2+b^2)/d-3/2
*b^(8/3)*ln(a^(1/3)+b^(1/3)*tan(d*x+c)^(1/3))/a^(5/3)/(a^2+b^2)/d+1/2*a*ln(
1+tan(d*x+c)^(2/3))/(a^2+b^2)/d+1/2*b^(8/3)*ln(a+b*tan(d*x+c))/a^(5/3)/(a^2
+b^2)/d-1/4*a*ln(1-tan(d*x+c)^(2/3)+tan(d*x+c)^(4/3))/(a^2+b^2)/d+b^(8/3)*a
rctan(1/3*(a^(1/3)-2*b^(1/3)*tan(d*x+c)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)/a^(
5/3)/(a^2+b^2)/d+1/2*a*arctan(1/3*(1-2*tan(d*x+c)^(2/3))*3^(1/2))*3^(1/2)/(
a^2+b^2)/d+1/4*b*ln(1-3^(1/2)*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3))*3^(1/2)/(a
^2+b^2)/d-1/4*b*ln(1+3^(1/2)*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3))*3^(1/2)/(a^
2+b^2)/d-3/2*a/(a^2+b^2)/d/tan(d*x+c)^(2/3)-3/2*b^2/a/(a^2+b^2)/d/tan(d*x+c
)^(2/3)
```

Rubi [A]
time = 0.39, antiderivative size = 525, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 18, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3655, 3610, 3619, 3557, 335, 215, 648, 632, 210, 642, 209, 281, 298, 31, 3715, 53, 60, 631}

$$\frac{\sqrt{3}\,a\text{ArcTan}\left(\frac{1-2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)}{2d(a^2+b^2)}+\frac{b\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{2d(a^2+b^2)}-\frac{b\text{ArcTan}\left(2\sqrt[3]{\tan(c+dx)}+\sqrt{3}\right)}{2d(a^2+b^2)}-\frac{b\text{ArcTan}\left(\sqrt[3]{\tan(c+dx)}\right)}{d(a^2+b^2)}-\frac{3b^2}{2ad(a^2+b^2)\tan^{\frac{2}{3}}(c+dx)}-\frac{3a}{2d(a^2+b^2)\tan^{\frac{2}{3}}(c+dx)}+\frac{\sqrt{3}\,b\log\left(\tan^{\frac{2}{3}}(c+dx)-\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{4d(a^2+b^2)}-\frac{\sqrt{3}\,b\log\left(\tan^{\frac{2}{3}}(c+dx)+\sqrt{3}\sqrt[3]{\tan(c+dx)}+1\right)}{4d(a^2+b^2)}+\frac{a\log\left(\tan^{\frac{2}{3}}(c+dx)+1\right)}{2d(a^2+b^2)}-\frac{a\log\left(\tan^{\frac{4}{3}}(c+dx)-\tan^{\frac{2}{3}}(c+dx)+1\right)}{4d(a^2+b^2)}+\frac{\sqrt{3}\,b^{8/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}d(a^2+b^2)}-\frac{3b^{8/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{\tan(c+dx)}\right)}{2a^{5/3}d(a^2+b^2)}+\frac{b^{8/3}\log(a+b\tan(c+dx))}{2a^{5/3}d(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(5/3)*(a + b*Tan[c + d*x])),x]
```

```
[Out] (b*ArcTan[Sqrt[3] - 2*Tan[c + d*x]^(1/3)])/(2*(a^2 + b^2)*d) - (b*ArcTan[Sq
rt[3] + 2*Tan[c + d*x]^(1/3)])/(2*(a^2 + b^2)*d) + (Sqrt[3]*b^(8/3)*ArcTan[
(a^(1/3) - 2*b^(1/3)*Tan[c + d*x]^(1/3))/(Sqrt[3]*a^(1/3))])/(a^(5/3)*(a^2
+ b^2)*d) + (Sqrt[3]*a*ArcTan[(1 - 2*Tan[c + d*x]^(2/3))/Sqrt[3]])/(2*(a^2
+ b^2)*d) - (b*ArcTan[Tan[c + d*x]^(1/3)])/((a^2 + b^2)*d) - (3*b^(8/3)*Log
[a^(1/3) + b^(1/3)*Tan[c + d*x]^(1/3)])/(2*a^(5/3)*(a^2 + b^2)*d) + (a*Log[
1 + Tan[c + d*x]^(2/3)])/(2*(a^2 + b^2)*d) + (Sqrt[3]*b*Log[1 - Sqrt[3]*Tan
[c + d*x]^(1/3) + Tan[c + d*x]^(2/3)])/(4*(a^2 + b^2)*d) - (Sqrt[3]*b*Log[1
 + Sqrt[3]*Tan[c + d*x]^(1/3) + Tan[c + d*x]^(2/3)])/(4*(a^2 + b^2)*d) + (b
^(8/3)*Log[a + b*Tan[c + d*x]])/(2*a^(5/3)*(a^2 + b^2)*d) - (a*Log[1 - Tan[
c + d*x]^(2/3) + Tan[c + d*x]^(4/3)])/(4*(a^2 + b^2)*d) - (3*a)/(2*(a^2 + b
^2)*d*Tan[c + d*x]^(2/3)) - (3*b^2)/(2*a*(a^2 + b^2)*d*Tan[c + d*x]^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)/((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[(a + b*Tan[e + f*x])^m*
(c - d*Tan[e + f*x]), x], x] + Dist[d^2/(c^2 + d^2), Int[(a + b*Tan[e + f*x
])^m*((1 + Tan[e + f*x]^2)/(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{1}{\tan^{\frac{5}{3}}(c+dx)(a+b\tan(c+dx))}\,dx = \frac{\int \frac{a-b\tan(c+dx)}{\tan^{\frac{5}{3}}(c+dx)}\,dx}{a^2+b^2} + \frac{b^2\int \frac{1+\tan^2(c+dx)}{\tan^{\frac{5}{3}}(c+dx)(a+b\tan(c+dx))}\,dx}{a^2+b^2}$$

$$= -\frac{3a}{2(a^2+b^2)\,d\tan^{\frac{2}{3}}(c+dx)} + \frac{\int \frac{-b-a\tan(c+dx)}{\tan^{\frac{2}{3}}(c+dx)}\,dx}{a^2+b^2} + \frac{b^2\text{Subst}\Big(\int \frac{}{x^{5/3}(}\Big.}{(}$$

$$= -\frac{3a}{2(a^2+b^2)\,d\tan^{\frac{2}{3}}(c+dx)} - \frac{3b^2}{2a(a^2+b^2)\,d\tan^{\frac{2}{3}}(c+dx)} - \frac{a\int \sqrt[3]{}}{}$$

$$= \frac{b^{8/3}\log(a+b\tan(c+dx))}{2a^{5/3}(a^2+b^2)\,d} - \frac{3a}{2(a^2+b^2)\,d\tan^{\frac{2}{3}}(c+dx)} - \frac{}{2a(a^2+}$$

$$= -\frac{3b^{8/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{\tan(c+dx)}\right)}{2a^{5/3}(a^2+b^2)\,d} + \frac{b^{8/3}\log(a+b\tan(c+dx}{2a^{5/3}(a^2+b^2)\,d}$$

$$= \frac{\sqrt{3}\,b^{8/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}(a^2+b^2)\,d} - \frac{3b^{8/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{}\right.}{2a^{5/3}(a^2+}$$

$$= \frac{\sqrt{3}\,b^{8/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}(a^2+b^2)\,d} - \frac{b\tan^{-1}\left(\sqrt[3]{\tan(c+dx}\right.}{(a^2+b^2)\,d}$$

$$= \frac{\sqrt{3}\,b^{8/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}(a^2+b^2)\,d} - \frac{b\tan^{-1}\left(\sqrt[3]{\tan(c+dx}\right.}{(a^2+b^2)\,d}$$

$$= \frac{b\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{2(a^2+b^2)\,d} - \frac{b\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx}\right.}{2(a^2+b^2)\,d}$$

$$= \frac{b\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(c+dx)}\right)}{2(a^2+b^2)\,d} - \frac{b\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\tan(c+dx}\right.}{2(a^2+b^2)\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.25, size = 104, normalized size = 0.20

$$-\frac{3\left(b^2\,{}_2F_1\left(-\frac{2}{3},1;\frac{1}{3};-\frac{b\tan(c+dx)}{a}\right)+a\left(a\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};-\tan^2(c+dx)\right)+2b\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-\tan^2(c+dx)\right)\tan(c+dx)\right)\right)}{2a\left(a^2+b^2\right)d\tan^{\frac{2}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Tan[c + d*x]^(5/3)*(a + b*Tan[c + d*x])),x]
```

```
[Out] (-3*(b^2*Hypergeometric2F1[-2/3, 1, 1/3, -((b*Tan[c + d*x])/a)] + a*(a*Hype
rgeometric2F1[-1/3, 1, 2/3, -Tan[c + d*x]^2] + 2*b*Hypergeometric2F1[1/6, 1
, 7/6, -Tan[c + d*x]^2]*Tan[c + d*x])))/(2*a*(a^2 + b^2)*d*Tan[c + d*x]^(2/
3))
```

Maple [A]
time = 0.20, size = 350, normalized size = 0.67

method	result
derivativedivides	$-\frac{3}{2a\tan(dx+c)^{\frac{2}{3}}}+\frac{-\frac{3\left(-\sqrt{3}\,b+a\right)\ln\left(1-\sqrt{3}\,\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)\right)}{4}-3\left(2b+\frac{\left(-\sqrt{3}\,b+a\right)\sqrt{3}}{2}\right)\arctan\left(-\sqrt{3}\right.}{}$
default	$-\frac{3}{2a\tan(dx+c)^{\frac{2}{3}}}+\frac{-\frac{3\left(-\sqrt{3}\,b+a\right)\ln\left(1-\sqrt{3}\,\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)\right)}{4}-3\left(2b+\frac{\left(-\sqrt{3}\,b+a\right)\sqrt{3}}{2}\right)\arctan\left(-\sqrt{3}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(d*x+c)^(5/3)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-3/2/a/tan(d*x+c)^(2/3)+3/(3*a^2+3*b^2)*(-1/4*(-3^(1/2)*b+a)*ln(1-3^(1
/2)*tan(d*x+c)^(1/3)+tan(d*x+c)^(2/3))-(2*b+1/2*(-3^(1/2)*b+a)*3^(1/2))*arc
tan(-3^(1/2)+2*tan(d*x+c)^(1/3))+1/4*(-3^(1/2)*b-a)*ln(1+3^(1/2)*tan(d*x+c)
^(1/3)+tan(d*x+c)^(2/3))+(-2*b-1/2*(-3^(1/2)*b-a)*3^(1/2))*arctan(3^(1/2)+2
*tan(d*x+c)^(1/3)))+3/(3*a^2+3*b^2)*(1/2*a*ln(1+tan(d*x+c)^(2/3))-b*arctan(
tan(d*x+c)^(1/3)))-3*(1/3/b/(a/b)^(2/3)*ln(tan(d*x+c)^(1/3)+(a/b)^(1/3))-1/
6/b/(a/b)^(2/3)*ln(tan(d*x+c)^(2/3)-(a/b)^(1/3)*tan(d*x+c)^(1/3)+(a/b)^(2/3
))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*tan(d*x+c)^(
1/3)-1)))/a*b^3/(a^2+b^2))
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(5/3)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains complex when optimal does not.
time = 3.66, size = 74494, normalized size = 141.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(5/3)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/48*(12*(a^3 + a*b^2)*d*(2*sqrt(-1/2*(I*a^2 + I*b^2)*d*(2*a/(a^4*d^3 + 2*
a^2*b^2*d^3 + b^4*d^3) - (3*a^2 - b^2)*a/((a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^
2)*(a^2*d + b^2*d)) + a^3/(a^2*d + b^2*d)^3)/b - 1/4*(3*a^2 - b^2)/(a^4*d^2
 + 2*a^2*b^2*d^2 + b^4*d^2) - 1/2*(a^2 - b^2)/(a^4*d^2 + 2*a^2*b^2*d^2 + b^
4*d^2) + 1/2*a^2/(a^2*d + b^2*d)^2) + a/(a^2*d + b^2*d) + I*b/((a^2 + b^2)*
d))*log(1/4*(a^6 + 2*a^4*b^2 + a^2*b^4)*d^3*(2*sqrt(-1/2*(I*a^2 + I*b^2)*d*
(2*a/(a^4*d^3 + 2*a^2*b^2*d^3 + b^4*d^3) - (3*a^2 - b^2)*a/((a^4*d^2 + 2*a^
2*b^2*d^2 + b^4*d^2)*(a^2*d + b^2*d)) + a^3/(a^2*d + b^2*d)^3)/b - 1/4*(3*a
^2 - b^2)/(a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2) - 1/2*(a^2 - b^2)/(a^4*d^2 +
2*a^2*b^2*d^2 + b^4*d^2) + 1/2*a^2/(a^2*d + b^2*d)^2) + a/(a^2*d + b^2*d) +
 I*b/((a^2 + b^2)*d))^3 - 1/4*(3*a^5 + 2*a^3*b^2 - a*b^4)*d^2*(2*sqrt(-1/2*
(I*a^2 + I*b^2)*d*(2*a/(a^4*d^3 + 2*a^2*b^2*d^3 + b^4*d^3) - (3*a^2 - b^2)*
a/((a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2)*(a^2*d + b^2*d)) + a^3/(a^2*d + b^2*
d)^3)/b - 1/4*(3*a^2 - b^2)/(a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2) - 1/2*(a^2
- b^2)/(a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2) + 1/2*a^2/(a^2*d + b^2*d)^2) + a
/(a^2*d + b^2*d) + I*b/((a^2 + b^2)*d))^2 - a^3 - a*b^2 + 1/2*(3*a^4 - 4*a^
```

2*b^2 + b^4)*d*(2*sqrt(-1/2*(I*a^2 + I*b^2)*d*(2*a/(a^4*d^3 + 2*a^2*b^2*d^3 + b^4*d^3) - (3*a^2 - b^2)*a/((a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2)*(a^2*d + b^2*d)) + a^3/(a^2*d + b^2*d)^3)/b - 1/4*(3*a^2 - b^2)/(a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2) - 1/2*(a^2 - b^2)/(a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2) + 1/2*a^2/(a^2*d + b^2*d)^2) + a/(a^2*d + b^2*d) + I*b/((a^2 + b^2)*d)) - (3*a^2*b - b^3)*tan(d*x + c)^(1/3))*tan(d*x + c) + 2*(a^3 + a*b^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(6*(sqrt(3)*a^3*d*sqrt(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - I*sqrt(3)*a^2*b*d*sqrt(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + sqrt(3)*a*b^2*d*sqrt(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - I*sqrt(3)*b^3*d*sqrt(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - 3*a^2 + 4*I*a*b + b^2)/(a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2) + (sqrt(3)*a^2*d*sqrt(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + sqrt(3)*b^2*d*sqrt(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - 3*a + I*b)^2/(a^2*d + b^2*d)^2)/(27*(3*sqrt(3)*a^4*d^3*(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))^(3/2) + 3*sqrt(3)*b^4*d^3*(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))^(3/2) - 2*I*sqrt(3)*a*b*d*sqrt(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - 7*sqrt(3)*b^2*d*sqrt(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + (6*sqrt(3)*b^2*d^3*(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))^(3/2) + 7*sqrt(3)*d*sqrt(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)))*a^2 - 4*a + 4*I*b)/(a^4*d^3 + 2*a^2*b^2*d^3 + b^4*d^3) + 2*(sqrt(3)*a^2*d*sqrt(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + sqrt(3)*b^2*d*sqrt(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - 3*a + I*b)^3/(a^2*d + b^2*d)^3 + 18*(sqrt(3)*a^3*d*sqrt(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - I*sqrt(3)*a^2*b*d*sqrt(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + sqrt(3)*a*b^2*d*sqrt(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - I*sqrt(3)*b^3*d*sqrt(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - 3*a^2 + 4*I*a*b + b^2)*(sqrt(3)*a^2*d*sqrt(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + sqrt(3)*b^2*d*sqrt(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - 3*a + I*b)/((a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2)*(a^2*d + b^2*d)) + 18*sqrt(-18*sqrt(3)*(I*a^10*b + 5*I*a^8*b^3 + 10*I*a^6*b^5 + 10*I*a^4*b^7 + 5*I*a^2*b^9 + I*b^11)*d^5*(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))^(5/2) + 9*a^6 - 54*I*a^5*b - 159*a^4*b^2 + 276*I*a^3*b^3 + 295*a^2*b^4 - 182*I*a*b^5 - 49*b^6 - 8*sqrt(3)*(12*I*a^8*b + 15*a^7*b^2 + 23*I*a^6*b^3 + 45*a^5*b^4 - 3*I*a^4*b^5 + 45*a^3*b^6 - 27*I*a^2*b^7 + 15*a*b^8 - 13*I*b^9)*d^3*(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))^(3/2) - 2*sqrt(3)*(39*I*a^6*b + 48*a^5*b^2 + I*a^4*b^3 - 24*a^3*b^4 + 21*I*a^2*b^5 - 72*a*b^6 + 59*I*b^7)*d*sqrt(-(a^2 + 2*I*a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - 9/4*(71*a^8 + 76*I*a^7*b - 44*a^6*b^2 + 108*I*a^5*b^3 - 182*a^4*b^4 - 12*I*a^3*b^5 + 52*a^2*b^6 - 44*I*a*b^7 + 119*b^8)*(a^2 + 2*I*a*b - b^2)/(a^4 + 2*a^2*b^2 + b^4) + 9/2*(49*a^10 + 18*I*a^9*b + 139*a^8*b^2 + 72*I*a^7*b^3 + 66*a^6*b^4 + 108*I*a^5*b^5 - 146*a^4*b^6 + 72*I*a^3*b^7 - 179*a^2*b^8 + 18*I*a*b^9 - 57*b^10)*(a^2 + 2*I*a*b - b^2)^2/(a^4 + 2*a^2*b^2 + b^4)^2 - 279/4*(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*(a^2 + 2*I*a

```
*b - b^2)^3/(a^4 + 2*a^2*b^2 + b^4)^3)/((a^2 + b^2)^3*d^3))^(1/3) + (1/2)^(
1/3)*(I*sqrt(3) + 1)*(27*(3*sqrt(3)*a^4*d^3*(-(a^2 + 2*I*a*b - b^2)/((a^4 +
 2*a^2*b^2 + b^4)*d^2))^(3/2) + 3*sqrt(3)*b^4*d^3*(-(a^2 + 2*I*a*b - b^2)/(
(a^4 + 2*a^2*b^2 + b^4)*d^2))^(3/2) - 2*I*sqrt(3)*a*b*d*sqrt(-(a^2 + 2*I*a*
b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - 7*sqrt(3)*b^2*d*sqrt(-(a^2 + 2*I*
a*b - b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + (6*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\tan(c+dx))\tan^{\frac{5}{3}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)**(5/3)/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(1/((a + b*tan(c + d*x))*tan(c + d*x)**(5/3)), x)
```

Giac [A]
time = 1.40, size = 457, normalized size = 0.87

$$\frac{b^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\tan(dx+c)^{\frac{1}{3}}\right|\right)}{a^4d+a^2b^2d}-\frac{3\left(-ab^2\right)^{\frac{1}{3}}b^2\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\tan(dx+c)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\left(\sqrt{3}a^4+\sqrt{3}a^2b^2\right)d}-\frac{\left(-ab^2\right)^{\frac{1}{3}}b^2\log\left(\left(-\frac{a}{b}\right)^{\frac{2}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\tan(dx+c)^{\frac{1}{3}}+\tan(dx+c)^{\frac{2}{3}}\right)}{2\left(a^4+a^2b^2\right)d}+\frac{\left(\sqrt{3}a-b\right)\arctan\left(\sqrt{3}+2\tan(dx+c)^{\frac{1}{3}}\right)}{2\left(a^2d+b^2d\right)}-\frac{\left(\sqrt{3}a+b\right)\arctan\left(-\sqrt{3}+2\tan(dx+c)^{\frac{1}{3}}\right)}{2\left(a^2d+b^2d\right)}-\frac{b\arctan\left(\tan(dx+c)^{\frac{1}{3}}\right)}{a^2d+b^2d}-\frac{a\log\left(\tan(dx+c)^{\frac{4}{3}}-\tan(dx+c)^{\frac{2}{3}}+1\right)}{4\left(a^2d+b^2d\right)}-\frac{3b\log\left(\sqrt{3}\tan(dx+c)^{\frac{1}{3}}+\tan(dx+c)^{\frac{2}{3}}+1\right)}{4\left(\sqrt{3}a^2d+\sqrt{3}b^2d\right)}+\frac{3b\log\left(-\sqrt{3}\tan(dx+c)^{\frac{1}{3}}+\tan(dx+c)^{\frac{2}{3}}+1\right)}{4\left(\sqrt{3}a^2d+\sqrt{3}b^2d\right)}+\frac{a\log\left(\tan(dx+c)^{\frac{2}{3}}+1\right)}{2\left(a^2d+b^2d\right)}-\frac{3}{2ad\tan(dx+c)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(d*x+c)^(5/3)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] b^3*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + tan(d*x + c)^(1/3)))/(a^4*d + a^2*
b^2*d) - 3*(-a*b^2)^(1/3)*b^2*arctan(1/3*sqrt(3)*((-a/b)^(1/3) + 2*tan(d*x
+ c)^(1/3))/(-a/b)^(1/3))/((sqrt(3)*a^4 + sqrt(3)*a^2*b^2)*d) - 1/2*(-a*b^2
)^(1/3)*b^2*log((-a/b)^(2/3) + (-a/b)^(1/3)*tan(d*x + c)^(1/3) + tan(d*x +
c)^(2/3))/((a^4 + a^2*b^2)*d) + 1/2*(sqrt(3)*a - b)*arctan(sqrt(3) + 2*tan(
d*x + c)^(1/3))/(a^2*d + b^2*d) - 1/2*(sqrt(3)*a + b)*arctan(-sqrt(3) + 2*t
an(d*x + c)^(1/3))/(a^2*d + b^2*d) - b*arctan(tan(d*x + c)^(1/3))/(a^2*d +
b^2*d) - 1/4*a*log(tan(d*x + c)^(4/3) - tan(d*x + c)^(2/3) + 1)/(a^2*d + b^
2*d) - 3/4*b*log(sqrt(3)*tan(d*x + c)^(1/3) + tan(d*x + c)^(2/3) + 1)/(sqrt
(3)*a^2*d + sqrt(3)*b^2*d) + 3/4*b*log(-sqrt(3)*tan(d*x + c)^(1/3) + tan(d*
x + c)^(2/3) + 1)/(sqrt(3)*a^2*d + sqrt(3)*b^2*d) + 1/2*a*log(tan(d*x + c)^
(2/3) + 1)/(a^2*d + b^2*d) - 3/2/(a*d*tan(d*x + c)^(2/3))
```

Mupad [B]
time = 5.62, size = 2500, normalized size = 4.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(5/3)*(a + b*tan(c + d*x))),x)
```

```
[Out] (log(tan(c + d*x)^(1/3) + 1i)*1i)/(2*(a*d*1i - b*d)) + log(tan(c + d*x)^(1/
3)*1i + 1)/(2*(a*d - b*d*1i)) + symsum(log(root(32*a^2*b^2*d^4*z^4 + 16*b^4
*d^4*z^4 + 16*a^4*d^4*z^4 + 16*a*b^2*d^3*z^3 + 16*a^3*d^3*z^3 - 4*b^2*d^2*z
^2 + 12*a^2*d^2*z^2 + 4*a*d*z + 1, z, k)*(root(32*a^2*b^2*d^4*z^4 + 16*b^4*
d^4*z^4 + 16*a^4*d^4*z^4 + 16*a*b^2*d^3*z^3 + 16*a^3*d^3*z^3 - 4*b^2*d^2*z^
2 + 12*a^2*d^2*z^2 + 4*a*d*z + 1, z, k)*(root(32*a^2*b^2*d^4*z^4 + 16*b^4*d
^4*z^4 + 16*a^4*d^4*z^4 + 16*a*b^2*d^3*z^3 + 16*a^3*d^3*z^3 - 4*b^2*d^2*z^2
 + 12*a^2*d^2*z^2 + 4*a*d*z + 1, z, k)^2*(tan(c + d*x)^(1/3)*(314928*a^13*b
^13*d^11 - 419904*a^15*b^11*d^11 + 118098*a^17*b^9*d^11 + 39366*a^19*b^7*d^
11 + 39366*a^21*b^5*d^11 + 13122*a^23*b^3*d^11) - root(32*a^2*b^2*d^4*z^4 +
 16*b^4*d^4*z^4 + 16*a^4*d^4*z^4 + 16*a*b^2*d^3*z^3 + 16*a^3*d^3*z^3 - 4*b^
2*d^2*z^2 + 12*a^2*d^2*z^2 + 4*a*d*z + 1, z, k)*(root(32*a^2*b^2*d^4*z^4 +
16*b^4*d^4*z^4 + 16*a^4*d^4*z^4 + 16*a*b^2*d^3*z^3 + 16*a^3*d^3*z^3 - 4*b^2
*d^2*z^2 + 12*a^2*d^2*z^2 + 4*a*d*z + 1, z, k)^2*(tan(c + d*x)^(1/3)*(41990
4*a^14*b^15*d^14 + 1259712*a^16*b^13*d^14 + 944784*a^18*b^11*d^14 - 419904*
a^20*b^9*d^14 - 629856*a^22*b^7*d^14 + 104976*a^26*b^3*d^14) - root(32*a^2*
b^2*d^4*z^4 + 16*b^4*d^4*z^4 + 16*a^4*d^4*z^4 + 16*a*b^2*d^3*z^3 + 16*a^3*d
^3*z^3 - 4*b^2*d^2*z^2 + 12*a^2*d^2*z^2 + 4*a*d*z + 1, z, k)*(419904*a^16*b
^14*d^15 + 1259712*a^18*b^12*d^15 + 839808*a^20*b^10*d^15 - 839808*a^22*b^8
*d^15 - 1259712*a^24*b^6*d^15 - 419904*a^26*b^4*d^15)) + 419904*a^13*b^14*d
^12 + 419904*a^15*b^12*d^12 - 288684*a^17*b^10*d^12 - 131220*a^19*b^8*d^12
+ 183708*a^21*b^6*d^12 + 26244*a^23*b^4*d^12)) - 26244*a^12*b^12*d^9 + 6561
*a^16*b^8*d^9 + 6561*a^18*b^6*d^9) + tan(c + d*x)^(1/3)*(13122*a^12*b^11*d^
8 - 6561*a^14*b^9*d^8)))*root(32*a^2*b^2*d^4*z^4 + 16*b^4*d^4*z^4 + 16*a^4*
d^4*z^4 + 16*a*b^2*d^3*z^3 + 16*a^3*d^3*z^3 - 4*b^2*d^2*z^2 + 12*a^2*d^2*z^
2 + 4*a*d*z + 1, z, k), k, 1, 4) + log((-b^8/(a^11*d^3 + a^5*b^6*d^3 + 3*a^
7*b^4*d^3 + 3*a^9*b^2*d^3))^(1/3)*((tan(c + d*x)^(1/3)*(314928*a^13*b^13*d^
11 - 419904*a^15*b^11*d^11 + 118098*a^17*b^9*d^11 + 39366*a^19*b^7*d^11 + 3
9366*a^21*b^5*d^11 + 13122*a^23*b^3*d^11) - (-b^8/(a^11*d^3 + a^5*b^6*d^3 +
 3*a^7*b^4*d^3 + 3*a^9*b^2*d^3))^(1/3)*((tan(c + d*x)^(1/3)*(419904*a^14*b^
15*d^14 + 1259712*a^16*b^13*d^14 + 944784*a^18*b^11*d^14 - 419904*a^20*b^9*
d^14 - 629856*a^22*b^7*d^14 + 104976*a^26*b^3*d^14) - (-b^8/(a^11*d^3 + a^5
*b^6*d^3 + 3*a^7*b^4*d^3 + 3*a^9*b^2*d^3))^(1/3)*(419904*a^16*b^14*d^15 + 1
259712*a^18*b^12*d^15 + 839808*a^20*b^10*d^15 - 839808*a^22*b^8*d^15 - 1259
712*a^24*b^6*d^15 - 419904*a^26*b^4*d^15))*(-b^8/(a^11*d^3 + a^5*b^6*d^3 +
3*a^7*b^4*d^3 + 3*a^9*b^2*d^3))^(2/3) + 419904*a^13*b^14*d^12 + 419904*a^15
*b^12*d^12 - 288684*a^17*b^10*d^12 - 131220*a^19*b^8*d^12 + 183708*a^21*b^6
*d^12 + 26244*a^23*b^4*d^12))*(-b^8/(a^11*d^3 + a^5*b^6*d^3 + 3*a^7*b^4*d^3
 + 3*a^9*b^2*d^3))^(2/3) - 26244*a^12*b^12*d^9 + 6561*a^16*b^8*d^9 + 6561*a
^18*b^6*d^9) + tan(c + d*x)^(1/3)*(13122*a^12*b^11*d^8 - 6561*a^14*b^9*d^8)
)*(-b^8/(a^11*d^3 + a^5*b^6*d^3 + 3*a^7*b^4*d^3 + 3*a^9*b^2*d^3))^(1/3) - l
og(tan(c + d*x)^(1/3)*(13122*a^12*b^11*d^8 - 6561*a^14*b^9*d^8) - ((3^(1/2)
*1i)/2 + 1/2)*(-b^8/(a^11*d^3 + a^5*b^6*d^3 + 3*a^7*b^4*d^3 + 3*a^9*b^2*d^3
))^(1/3)*((tan(c + d*x)^(1/3)*(314928*a^13*b^13*d^11 - 419904*a^15*b^11*d^1
1 + 118098*a^17*b^9*d^11 + 39366*a^19*b^7*d^11 + 39366*a^21*b^5*d^11 + 1312
```

2*a^23*b^3*d^11) + ((3^(1/2)*1i)/2 + 1/2)*(-b^8/(a^11*d^3 + a^5*b^6*d^3 + 3*a^7*b^4*d^3 + 3*a^9*b^2*d^3))^(1/3)*(((3^(1/2)*1i)/2 + 1/2)^2*(tan(c + d*x)^(1/3)*(419904*a^14*b^15*d^14 + 1259712*a^16*b^13*d^14 + 944784*a^18*b^11*d^14 - 419904*a^20*b^9*d^14 - 629856*a^22*b^7*d^14 + 104976*a^26*b^3*d^14) + ((3^(1/2)*1i)/2 + 1/2)*(-b^8/(a^11*d^3 + a^5*b^6*d^3 + 3*a^7*b^4*d^3 + 3*a^9*b^2*d^3))^(1/3)*(419904*a^16*b^14*d^15 + 1259712*a^18*b^12*d^15 + 839808*a^20*b^10*d^15 - 839808*a^22*b^8*d^15 - 1259712*a^24*b^6*d^15 - 419904*a^26*b^4*d^15))*(-b^8/(a^11*d^3 + a^5*b^6*d^3 + 3*a^7*b^4*d^3 + 3*a^9*b^2*d^3))^(2/3) + 419904*a^13*b^14*d^12 + 419904*a^15*b^12*d^12 - 288684*a^17*b^10*d^12 - 131220*a^19*b^8*d^12 + 183708*a^21*b^6*d^12 + 26244*a^23*b^4*d^12))*((3^(1/2)*1i)/2 + 1/2)^2*(-b^8/(a^11*d^3 + a^5*b^6*d^3 + 3*a^7*b^4*d^3 + 3*a^9*b^2*d^3))^(2/3) - 26244*a^12*b^12*d^9 + 6561*a^16*b^8*d^9 + 6561*a^18*b^6*d^9))*((3^(1/2)*1i)/2 + 1/2)*(-b^8/(a^11*d^3 + a^5*b^6*d^3 + 3*a^7*b^4*d^3 + 3*a^9*b^2*d^3))^(1/3) + log(tan(c + d*x)^(1/3)*(13122*a^12*b^11*d^8 - 6561*a^14*b^9*d^8) + ((3^(1/2)*1i)/2 - 1/2)*(-b^8/(a^11*d^3 + a^5*b^6*d^3 + 3*a^7*b^4*d^3 + 3*a^9*b^2*d^3))^(1/3)*((tan(c + d*x)^(1/3)*(314928*a^13*b^13*d^11 - 419904*a^15*b^11*d^11 + 118098*a^17*b^9*d^11 + 39366*a^19*b^7*d^11 + 39366*a^21*b^5*d^11 + 13122*a^23*b^3*d^11) - ((3^(1/2)*1i)/2 - 1/2)*(-b^8/(a^11*d^3 + a^5*b^6*d^3 + 3*a^7*b^4*d^3 + 3*a^9*b^2*d^3))^(1/3)*(((3^(1/2)*1i)/2 - 1/2)^2*(tan(c + d*x)^(1/3)*(419904*a^14*b^15*d^14 + 1259712*a^16*b^13*d^14 + 944784*a^18*b^11*d^14 - 419904*a^20*b^9*d^14 - 629856*a^22*b^7*d^14 + 104976*a^26*b^3*d^14) - ((3^(1/2)*1i)/...

3.675 $\int \frac{\tan^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal. Leaf size=163

$$\frac{3F_1\left(\frac{7}{3};1,\frac{1}{2};\frac{10}{3};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)\tan^{\frac{7}{3}}(c+dx)\sqrt{1+\frac{b\tan(c+dx)}{a}}}{14d\sqrt{a+b\tan(c+dx)}}+\frac{3F_1\left(\frac{7}{3};1,\frac{1}{2};\frac{10}{3};i\tan(c+d\right.}{}$$

```
[Out] 3/14*AppellF1(7/3,1,1/2,10/3,-I*tan(d*x+c),-b*tan(d*x+c)/a)*(1+b*tan(d*x+c)
/a)^(1/2)*tan(d*x+c)^(7/3)/d/(a+b*tan(d*x+c))^(1/2)+3/14*AppellF1(7/3,1,1/2
,10/3,I*tan(d*x+c),-b*tan(d*x+c)/a)*(1+b*tan(d*x+c)/a)^(1/2)*tan(d*x+c)^(7/
3)/d/(a+b*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3656, 926, 129, 525, 524}

$$\frac{3\tan^{\frac{7}{3}}(c+dx)\sqrt{\frac{b\tan(c+dx)}{a}+1}\,F_1\left(\frac{7}{3};1,\frac{1}{2};\frac{10}{3};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)}{14d\sqrt{a+b\tan(c+dx)}}+\frac{3\tan^{\frac{7}{3}}(c+dx)\sqrt{\frac{b\tan(c+dx)}{a}+1}\,F_1\left(\frac{7}{3};1,\frac{1}{2};\frac{10}{3};i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)}{14d\sqrt{a+b\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(4/3)/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] (3*AppellF1[7/3, 1, 1/2, 10/3, (-I)*Tan[c + d*x], -((b*Tan[c + d*x])/a)]*Ta
n[c + d*x]^(7/3)*Sqrt[1 + (b*Tan[c + d*x])/a])/(14*d*Sqrt[a + b*Tan[c + d*x
]]) + (3*AppellF1[7/3, 1, 1/2, 10/3, I*Tan[c + d*x], -((b*Tan[c + d*x])/a)]
*Tan[c + d*x]^(7/3)*Sqrt[1 + (b*Tan[c + d*x])/a])/(14*d*Sqrt[a + b*Tan[c +
d*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{\tan^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^{4/3}}{\sqrt{a+bx}\,(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{ix^{4/3}}{2(i-x)\sqrt{a+bx}} + \frac{ix^{4/3}}{2(i+x)\sqrt{a+bx}}\right) dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{i\text{Subst}\left(\int \frac{x^{4/3}}{(i-x)\sqrt{a+bx}}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{i\text{Subst}\left(\int \frac{x^{4/3}}{(i+x)\sqrt{a+bx}}\,dx, x\right.}{2d}$$

$$= \frac{(3i)\text{Subst}\left(\int \frac{x^6}{(i-x^3)\sqrt{a+bx^3}}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{2d} + \frac{(3i)\text{Subst}\left(\int \frac{x}{(i+x^3)\sqrt{a}}\right.}{}$$

$$= \frac{\left(3i\sqrt{1+\frac{b\tan(c+dx)}{a}}\right)\text{Subst}\left(\int \frac{x^6}{(i-x^3)\sqrt{1+\frac{bx^3}{a}}}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{2d\sqrt{a+b\tan(c+dx)}}$$

$$= \frac{3F_1\left(\frac{7}{3}; 1, \frac{1}{2}; \frac{10}{3}; -i\tan(c+dx), -\frac{b\tan(c+dx)}{a}\right)\tan^{\frac{7}{3}}(c+dx)\sqrt{1+\frac{b\tan(c+dx)}{a}}}{14d\sqrt{a+b\tan(c+dx)}}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 21046 vs. $2(163) = 326$.
time = 36.82, size = 21046, normalized size = 129.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^(4/3)/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [**F**]
time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{4}{3}}(dx+c)}{\sqrt{a+b\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(4/3)/(a+b*tan(d*x+c))^(1/2),x)
```

```
[Out] int(tan(d*x+c)^(4/3)/(a+b*tan(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(4/3)/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(tan(d*x + c)^(4/3)/sqrt(b*tan(d*x + c) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(4/3)/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(4/3)/(a+b*tan(d*x+c))**(1/2),x)
[Out] Integral(tan(c + d*x)**(4/3)/sqrt(a + b*tan(c + d*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(4/3)/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^{4/3}}{\sqrt{a+b\tan(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(4/3)/(a + b*tan(c + d*x))^(1/2),x)
[Out] int(tan(c + d*x)^(4/3)/(a + b*tan(c + d*x))^(1/2), x)
```

3.676 $\int \frac{\tan^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal. Leaf size=163

$$\frac{3F_1\left(\frac{5}{3};1,\frac{1}{2};\frac{8}{3};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)\tan^{\frac{5}{3}}(c+dx)\sqrt{1+\frac{b\tan(c+dx)}{a}}}{10d\sqrt{a+b\tan(c+dx)}}+\frac{3F_1\left(\frac{5}{3};1,\frac{1}{2};\frac{8}{3};i\tan(c+dx)\dots\right.}{10\dots}$$

```
[Out] 3/10*AppellF1(5/3,1,1/2,8/3,-I*tan(d*x+c),-b*tan(d*x+c)/a)*(1+b*tan(d*x+c)/
a)^(1/2)*tan(d*x+c)^(5/3)/d/(a+b*tan(d*x+c))^(1/2)+3/10*AppellF1(5/3,1,1/2,
8/3,I*tan(d*x+c),-b*tan(d*x+c)/a)*(1+b*tan(d*x+c)/a)^(1/2)*tan(d*x+c)^(5/3)
/d/(a+b*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3656, 926, 129, 525, 524}

$$\frac{3\tan^{\frac{5}{3}}(c+dx)\sqrt{\frac{b\tan(c+dx)}{a}+1}\,F_1\left(\frac{5}{3};1,\frac{1}{2};\frac{8}{3};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)}{10d\sqrt{a+b\tan(c+dx)}}+\frac{3\tan^{\frac{5}{3}}(c+dx)\sqrt{\frac{b\tan(c+dx)}{a}+1}\,F_1\left(\frac{5}{3};1,\frac{1}{2};\frac{8}{3};i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)}{10d\sqrt{a+b\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(2/3)/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] (3*AppellF1[5/3, 1, 1/2, 8/3, (-I)*Tan[c + d*x], -((b*Tan[c + d*x])/a)]*Tan
[c + d*x]^(5/3)*Sqrt[1 + (b*Tan[c + d*x])/a])/(10*d*Sqrt[a + b*Tan[c + d*x]
]) + (3*AppellF1[5/3, 1, 1/2, 8/3, I*Tan[c + d*x], -((b*Tan[c + d*x])/a)]*T
an[c + d*x]^(5/3)*Sqrt[1 + (b*Tan[c + d*x])/a])/(10*d*Sqrt[a + b*Tan[c + d*
x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{\tan^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^{2/3}}{\sqrt{a+bx}\,(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{ix^{2/3}}{2(i-x)\sqrt{a+bx}} + \frac{ix^{2/3}}{2(i+x)\sqrt{a+bx}}\right)dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{i\text{Subst}\left(\int \frac{x^{2/3}}{(i-x)\sqrt{a+bx}}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{i\text{Subst}\left(\int \frac{x^{2/3}}{(i+x)\sqrt{a+bx}}\,dx, x\right.}{2d}$$

$$= \frac{(3i)\text{Subst}\left(\int \frac{x^4}{(i-x^3)\sqrt{a+bx^3}}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{2d} + \frac{(3i)\text{Subst}\left(\int \frac{x}{(i+x^3)\sqrt{}}\right.}{}$$

$$= \frac{\left(3i\sqrt{1+\frac{b\tan(c+dx)}{a}}\right)\text{Subst}\left(\int \frac{x^4}{(i-x^3)\sqrt{1+\frac{bx^3}{a}}}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{2d\sqrt{a+b\tan(c+dx)}}$$

$$= \frac{3F_1\left(\frac{5}{3};1,\frac{1}{2};\frac{8}{3};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)\tan^{\frac{5}{3}}(c+dx)\sqrt{1+\frac{b\tan(c+dx)}{a}}}{10d\sqrt{a+b\tan(c+dx)}}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 7362 vs. $2(163) = 326$.
time = 126.53, size = 7362, normalized size = 45.17

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^(2/3)/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [**F**]
time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{2}{3}}(dx+c)}{\sqrt{a+b\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(2/3)/(a+b*tan(d*x+c))^(1/2),x)
```

```
[Out] int(tan(d*x+c)^(2/3)/(a+b*tan(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(2/3)/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(tan(d*x + c)^(2/3)/sqrt(b*tan(d*x + c) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(2/3)/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(2/3)/(a+b*tan(d*x+c))**(1/2),x)
[Out] Integral(tan(c + d*x)**(2/3)/sqrt(a + b*tan(c + d*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(2/3)/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^{2/3}}{\sqrt{a+b\tan(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(2/3)/(a + b*tan(c + d*x))^(1/2),x)
[Out] int(tan(c + d*x)^(2/3)/(a + b*tan(c + d*x))^(1/2), x)
```

3.677 $\int \frac{\sqrt[3]{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal. Leaf size=163

$$\frac{3F_1\left(\frac{4}{3};1,\frac{1}{2};\frac{7}{3};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)\tan^{\frac{4}{3}}(c+dx)\sqrt{1+\frac{b\tan(c+dx)}{a}}}{8d\sqrt{a+b\tan(c+dx)}}+\frac{3F_1\left(\frac{4}{3};1,\frac{1}{2};\frac{7}{3};i\tan(c+dx)\right.}{8}$$

```
[Out] 3/8*AppellF1(4/3,1,1/2,7/3,-I*tan(d*x+c),-b*tan(d*x+c)/a)*(1+b*tan(d*x+c)/a
)^(1/2)*tan(d*x+c)^(4/3)/d/(a+b*tan(d*x+c))^(1/2)+3/8*AppellF1(4/3,1,1/2,7/
3,I*tan(d*x+c),-b*tan(d*x+c)/a)*(1+b*tan(d*x+c)/a)^(1/2)*tan(d*x+c)^(4/3)/d
/(a+b*tan(d*x+c))^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3656, 926, 129, 525, 524}

$$\frac{3\tan^{\frac{4}{3}}(c+dx)\sqrt{\frac{b\tan(c+dx)}{a}+1}\,F_1\left(\frac{4}{3};1,\frac{1}{2};\frac{7}{3};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)}{8d\sqrt{a+b\tan(c+dx)}}+\frac{3\tan^{\frac{4}{3}}(c+dx)\sqrt{\frac{b\tan(c+dx)}{a}+1}\,F_1\left(\frac{4}{3};1,\frac{1}{2};\frac{7}{3};i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)}{8d\sqrt{a+b\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(1/3)/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] (3*AppellF1[4/3, 1, 1/2, 7/3, (-I)*Tan[c + d*x], -((b*Tan[c + d*x])/a)]*Tan
[c + d*x]^(4/3)*Sqrt[1 + (b*Tan[c + d*x])/a])/(8*d*Sqrt[a + b*Tan[c + d*x]]
) + (3*AppellF1[4/3, 1, 1/2, 7/3, I*Tan[c + d*x], -((b*Tan[c + d*x])/a)]*Ta
n[c + d*x]^(4/3)*Sqrt[1 + (b*Tan[c + d*x])/a])/(8*d*Sqrt[a + b*Tan[c + d*x]
])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt[3]{x}}{\sqrt{a+bx}\,(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{i\sqrt[3]{x}}{2(i-x)\sqrt{a+bx}} + \frac{i\sqrt[3]{x}}{2(i+x)\sqrt{a+bx}}\right)\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{i\text{Subst}\left(\int \frac{\sqrt[3]{x}}{(i-x)\sqrt{a+bx}}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{i\text{Subst}\left(\int \frac{\sqrt[3]{x}}{(i+x)\sqrt{a+bx}}\,dx, x\right.}{2d}$$

$$= \frac{(3i)\text{Subst}\left(\int \frac{x^3}{(i-x^3)\sqrt{a+bx^3}}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{2d} + \frac{(3i)\text{Subst}\left(\int \frac{x}{(i+x^3)\sqrt{a}}\right.}{}$$

$$= \frac{\left(3i\sqrt{1+\frac{b\tan(c+dx)}{a}}\right)\text{Subst}\left(\int \frac{x^3}{(i-x^3)\sqrt{1+\frac{bx^3}{a}}}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{2d\sqrt{a+b\tan(c+dx)}}$$

$$= \frac{3F_1\left(\frac{4}{3}; 1, \frac{1}{2}; \frac{7}{3}; -i\tan(c+dx), -\frac{b\tan(c+dx)}{a}\right)\tan^{\frac{4}{3}}(c+dx)\sqrt{1+\frac{b\tan(c+dx)}{a}}}{8d\sqrt{a+b\tan(c+dx)}}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 6316 vs. 2(163) = 326.
time = 34.42, size = 6316, normalized size = 38.75

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^(1/3)/Sqrt[a + b*Tan[c + d*x]],x]
[Out] Result too large to show
```

Maple [**F**]
time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{1}{3}}(dx+c)}{\sqrt{a+b\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/3)/(a+b*tan(d*x+c))^(1/2),x)
```

[Out] int(tan(d*x+c)^(1/3)/(a+b*tan(d*x+c))^(1/2),x)

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/3)/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(tan(d*x + c)^(1/3)/sqrt(b*tan(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/3)/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\tan\left(c+dx\right)}}{\sqrt{a+b\tan\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/3)/(a+b*tan(d*x+c))**(1/2),x)
[Out] Integral(tan(c + d*x)**(1/3)/sqrt(a + b*tan(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/3)/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\tan(c+d\,x)}^{1/3}}{\sqrt{a+b\tan\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/3)/(a + b*tan(c + d*x))^(1/2),x)
[Out] int(tan(c + d*x)^(1/3)/(a + b*tan(c + d*x))^(1/2), x)
```

3.678 $\int \frac{1}{\sqrt[3]{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal. Leaf size=163

$$\frac{3F_1\left(\frac{2}{3};1,\frac{1}{2};\frac{5}{3};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)\tan^{\frac{2}{3}}(c+dx)\sqrt{1+\frac{b\tan(c+dx)}{a}}}{4d\sqrt{a+b\tan(c+dx)}}+\frac{3F_1\left(\frac{2}{3};1,\frac{1}{2};\frac{5}{3};i\tan(c+dx)\right.}{4}$$

```
[Out] 3/4*AppellF1(2/3,1,1/2,5/3,-I*tan(d*x+c),-b*tan(d*x+c)/a)*(1+b*tan(d*x+c)/a
)^(1/2)*tan(d*x+c)^(2/3)/d/(a+b*tan(d*x+c))^(1/2)+3/4*AppellF1(2/3,1,1/2,5/
3,I*tan(d*x+c),-b*tan(d*x+c)/a)*(1+b*tan(d*x+c)/a)^(1/2)*tan(d*x+c)^(2/3)/d
/(a+b*tan(d*x+c))^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3656, 926, 129, 525, 524}

$$\frac{3\tan^{\frac{2}{3}}(c+dx)\sqrt{\frac{b\tan(c+dx)}{a}+1}\,F_1\left(\frac{2}{3};1,\frac{1}{2};\frac{5}{3};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)}{4d\sqrt{a+b\tan(c+dx)}}+\frac{3\tan^{\frac{2}{3}}(c+dx)\sqrt{\frac{b\tan(c+dx)}{a}+1}\,F_1\left(\frac{2}{3};1,\frac{1}{2};\frac{5}{3};i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)}{4d\sqrt{a+b\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(1/3)*Sqrt[a + b*Tan[c + d*x]]),x]
```

```
[Out] (3*AppellF1[2/3, 1, 1/2, 5/3, (-I)*Tan[c + d*x], -((b*Tan[c + d*x])/a)]*Tan
[c + d*x]^(2/3)*Sqrt[1 + (b*Tan[c + d*x])/a])/(4*d*Sqrt[a + b*Tan[c + d*x]]
) + (3*AppellF1[2/3, 1, 1/2, 5/3, I*Tan[c + d*x], -((b*Tan[c + d*x])/a)]*Ta
n[c + d*x]^(2/3)*Sqrt[1 + (b*Tan[c + d*x])/a])/(4*d*Sqrt[a + b*Tan[c + d*x]
])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\,\sqrt{a+bx}\,(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{i}{2(i-x)\sqrt[3]{x}\,\sqrt{a+bx}} + \frac{i}{2\sqrt[3]{x}\,(i+x)\sqrt{a+bx}}\right) dx, x, \text{t}}{d}$$

$$= \frac{i\text{Subst}\left(\int \frac{1}{(i-x)\sqrt[3]{x}\,\sqrt{a+bx}}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{i\text{Subst}\left(\int\right.}{}$$

$$= \frac{(3i)\text{Subst}\left(\int \frac{x}{(i-x^3)\sqrt{a+bx^3}}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{2d} + \frac{(3i)\text{Su}}{}$$

$$= \frac{\left(3i\sqrt{1+\frac{b\tan(c+dx)}{a}}\right)\text{Subst}\left(\int \frac{x}{(i-x^3)\sqrt{1+\frac{bx^3}{a}}}\,dx, x, \sqrt[3]{}\right.}{2d\sqrt{a+b\tan(c+dx)}}$$

$$= \frac{3F_1\left(\frac{2}{3}; 1, \frac{1}{2}; \frac{5}{3}; -i\tan(c+dx), -\frac{b\tan(c+dx)}{a}\right)\tan^{\frac{2}{3}}(c+dx)\sqrt{1+}}{4d\sqrt{a+b\tan(c+dx)}}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 32685 vs. 2(163) = 326.
time = 118.64, size = 32685, normalized size = 200.52

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Tan[c + d*x]^(1/3)*Sqrt[a + b*Tan[c + d*x]]),x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\tan(dx+c)}\,\tan(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/3),x)
```

```
[Out] int(1/(a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/3),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^(1/3)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\tan{(c + dx)}}\,\sqrt[3]{\tan{(c + dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))**(1/2)/tan(d*x+c)**(1/3),x)
[Out] Integral(1/(sqrt(a + b*tan(c + d*x))*tan(c + d*x)**(1/3)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/3),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\tan\left(c+d\,x\right)}^{1/3}\,\sqrt{a+b\,\tan\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(1/3)*(a + b*tan(c + d*x))^(1/2)),x)
[Out] int(1/(tan(c + d*x)^(1/3)*(a + b*tan(c + d*x))^(1/2)), x)
```

3.679 $\int \frac{1}{\tan^{\frac{2}{3}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal. Leaf size=163

$$\frac{3F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)\sqrt[3]{\tan(c+dx)}\sqrt{1+\frac{b\tan(c+dx)}{a}}}{2d\sqrt{a+b\tan(c+dx)}}+\frac{3F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};i\tan(c+d\right.}{}$$

```
[Out] 3/2*AppellF1(1/3,1,1/2,4/3,-I*tan(d*x+c),-b*tan(d*x+c)/a)*(1+b*tan(d*x+c)/a
)^(1/2)*tan(d*x+c)^(1/3)/d/(a+b*tan(d*x+c))^(1/2)+3/2*AppellF1(1/3,1,1/2,4/
3,I*tan(d*x+c),-b*tan(d*x+c)/a)*(1+b*tan(d*x+c)/a)^(1/2)*tan(d*x+c)^(1/3)/d
/(a+b*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3656, 926, 129, 441, 440}

$$\frac{3\sqrt[3]{\tan(c+dx)}\sqrt{\frac{b\tan(c+dx)}{a}+1}\,F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)}{2d\sqrt{a+b\tan(c+dx)}}+\frac{3\sqrt[3]{\tan(c+dx)}\sqrt{\frac{b\tan(c+dx)}{a}+1}\,F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)}{2d\sqrt{a+b\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(2/3)*Sqrt[a + b*Tan[c + d*x]]),x]
```

```
[Out] (3*AppellF1[1/3, 1, 1/2, 4/3, (-I)*Tan[c + d*x], -((b*Tan[c + d*x])/a)]*Tan
[c + d*x]^(1/3)*Sqrt[1 + (b*Tan[c + d*x])/a])/(2*d*Sqrt[a + b*Tan[c + d*x]]
) + (3*AppellF1[1/3, 1, 1/2, 4/3, I*Tan[c + d*x], -((b*Tan[c + d*x])/a)]*Ta
n[c + d*x]^(1/3)*Sqrt[1 + (b*Tan[c + d*x])/a])/(2*d*Sqrt[a + b*Tan[c + d*x]
])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{1}{\tan^{\frac{2}{3}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^{2/3}\sqrt{a+bx}\,(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{i}{2(i-x)x^{2/3}\sqrt{a+bx}} + \frac{i}{2x^{2/3}(i+x)\sqrt{a+bx}}\right)dx, x, \tan(c\right.}{d}$$

$$= \frac{i\text{Subst}\left(\int \frac{1}{(i-x)x^{2/3}\sqrt{a+bx}}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{i\text{Subst}\left(\int \frac{}{x^{2/3}(}\right.}{}$$

$$= \frac{(3i)\text{Subst}\left(\int \frac{1}{(i-x^3)\sqrt{a+bx^3}}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{2d} + \frac{(3i)\text{Subst}}{}$$

$$= \frac{\left(3i\sqrt{1+\frac{b\tan(c+dx)}{a}}\right)\text{Subst}\left(\int \frac{1}{(i-x^3)\sqrt{1+\frac{bx^3}{a}}}\,dx, x, \sqrt[3]{\tan\right.}{2d\sqrt{a+b\tan(c+dx)}}$$

$$= \frac{3F_1\left(\frac{1}{3}; 1, \frac{1}{2}; \frac{4}{3}; -i\tan(c+dx), -\frac{b\tan(c+dx)}{a}\right)\sqrt[3]{\tan(c+dx)}\sqrt{1+}}{2d\sqrt{a+b\tan(c+dx)}}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 6198 vs. $2(163) = 326$.
time = 92.91, size = 6198, normalized size = 38.02

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Tan[c + d*x]^(2/3)*Sqrt[a + b*Tan[c + d*x]]),x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\tan\left(dx+c\right)}\,\tan\left(dx+c\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(2/3),x)
```

```
[Out] int(1/(a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^(2/3)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\tan\left(c+dx\right)}\,\tan^{\frac{2}{3}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))**(1/2)/tan(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/(sqrt(a + b*tan(c + d*x))*tan(c + d*x)**(2/3)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\tan\left(c+d\,x\right)^{2/3}\,\sqrt{a+b\tan\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(2/3)*(a + b*tan(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(tan(c + d*x)^(2/3)*(a + b*tan(c + d*x))^(1/2)), x)
```

3.680 $\int \frac{1}{\tan^{\frac{4}{3}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal. Leaf size=163

$$-\frac{3F_1\left(-\frac{1}{3};1,\frac{1}{2};\frac{2}{3};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)\sqrt{1+\frac{b\tan(c+dx)}{a}}}{2d\sqrt[3]{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}-\frac{3F_1\left(-\frac{1}{3};1,\frac{1}{2};\frac{2}{3};i\tan(c+dx),-\frac{b\tan(}{}\right.}{2d\sqrt[3]{\tan(c+dx)}\sqrt{a+}}$$

```
[Out] -3/2*AppellF1(-1/3,1,1/2,2/3,-I*tan(d*x+c),-b*tan(d*x+c)/a)*(1+b*tan(d*x+c)
/a)^(1/2)/d/(a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/3)-3/2*AppellF1(-1/3,1,1/2
,2/3,I*tan(d*x+c),-b*tan(d*x+c)/a)*(1+b*tan(d*x+c)/a)^(1/2)/d/(a+b*tan(d*x+
c))^(1/2)/tan(d*x+c)^(1/3)
```

Rubi [A]

time = 0.19, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3656, 926, 129, 525, 524}

$$-\frac{3\sqrt{\frac{b\tan(c+dx)}{a}+1}\,F_1\left(-\frac{1}{3};1,\frac{1}{2};\frac{2}{3};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)}{2d\sqrt[3]{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}-\frac{3\sqrt{\frac{b\tan(c+dx)}{a}+1}\,F_1\left(-\frac{1}{3};1,\frac{1}{2};\frac{2}{3};i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)}{2d\sqrt[3]{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Tan[c + d*x]^(4/3)*Sqrt[a + b*Tan[c + d*x]]),x]
```

```
[Out] (-3*AppellF1[-1/3, 1, 1/2, 2/3, (-I)*Tan[c + d*x], -((b*Tan[c + d*x])/a)]*S
qrt[1 + (b*Tan[c + d*x])/a])/(2*d*Tan[c + d*x]^(1/3)*Sqrt[a + b*Tan[c + d*x
]]) - (3*AppellF1[-1/3, 1, 1/2, 2/3, I*Tan[c + d*x], -((b*Tan[c + d*x])/a)]
*Sqrt[1 + (b*Tan[c + d*x])/a])/(2*d*Tan[c + d*x]^(1/3)*Sqrt[a + b*Tan[c + d
*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{1}{\tan^{\frac{4}{3}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^{4/3}\sqrt{a+bx}\,(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{i}{2(i-x)x^{4/3}\sqrt{a+bx}} + \frac{i}{2x^{4/3}(i+x)\sqrt{a+bx}}\right)\,dx, x, \tan(\ldots}{d}$$

$$= \frac{i\text{Subst}\left(\int \frac{1}{(i-x)x^{4/3}\sqrt{a+bx}}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{i\text{Subst}\left(\int \frac{\ldots}{x^{4/\ldots}}\ldots\right.}{\ldots}$$

$$= \frac{(3i)\text{Subst}\left(\int \frac{1}{x^2(i-x^3)\sqrt{a+bx^3}}\,dx, x, \sqrt[3]{\tan(c+dx)}\right)}{2d} + \frac{(3i)\text{Su}\ldots}{\ldots}$$

$$= \frac{\left(3i\sqrt{1+\frac{b\tan(c+dx)}{a}}\right)\text{Subst}\left(\int \frac{1}{x^2(i-x^3)\sqrt{1+\frac{bx^3}{a}}}\,dx, x, \sqrt[3]{\ldots}\right.}{2d\sqrt{a+b\tan(c+dx)}}$$

$$= -\frac{3F_1\left(-\frac{1}{3};1,\frac{1}{2};\frac{2}{3};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)\sqrt{1+\frac{b\tan(c+\ldots}{a}}}{2d\sqrt[3]{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 23355 vs. 2(163) = 326.
time = 179.20, size = 23355, normalized size = 143.28

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Tan[c + d*x]^(4/3)*Sqrt[a + b*Tan[c + d*x]]),x]
[Out] Result too large to show
```

Maple [F]
time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\tan(dx+c)}\,\tan(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(4/3),x)
```

```
[Out] int(1/(a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(4/3),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^(4/3)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(4/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\tan{\left(c + dx \right)}}\tan^{\frac{4}{3}}{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))**(1/2)/tan(d*x+c)**(4/3),x)
[Out] Integral(1/(sqrt(a + b*tan(c + d*x))*tan(c + d*x)**(4/3)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(4/3),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\mathrm{tan}\left(c+d\,x\right)}^{4/3}\,\sqrt{a+b\,\mathrm{tan}\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(c + d*x)^(4/3)*(a + b*tan(c + d*x))^(1/2)),x)
[Out] int(1/(tan(c + d*x)^(4/3)*(a + b*tan(c + d*x))^(1/2)), x)
```

3.681 $\int \tan^4(e+fx)\sqrt[3]{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=525

$$-\frac{1}{4}\sqrt[3]{c-\sqrt{-d^2}}\,x-\frac{1}{4}\sqrt[3]{c+\sqrt{-d^2}}\,x-\frac{\sqrt{3}\sqrt{-d^2}\sqrt[3]{c-\sqrt{-d^2}}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c-\sqrt{-d^2}}}}{\sqrt{3}}\right)}{2df}+\frac{\sqrt{}}{}$$

```
[Out] -1/4*x*(c-(-d^2)^(1/2))^(1/3)+1/4*ln(cos(f*x+e))*(c-(-d^2)^(1/2))^(1/3)*(-d
^2)^(1/2)/d/f+3/4*ln((c-(-d^2)^(1/2))^(1/3)-(c+d*tan(f*x+e))^(1/3))*(c-(-d^
2)^(1/2))^(1/3)*(-d^2)^(1/2)/d/f-1/2*arctan(1/3*(1+2*(c+d*tan(f*x+e))^(1/3)
/(c-(-d^2)^(1/2))^(1/3))*3^(1/2))*3^(1/2)*(c-(-d^2)^(1/2))^(1/3)*(-d^2)^(1/
2)/d/f-1/4*x*(c+(-d^2)^(1/2))^(1/3)-1/4*ln(cos(f*x+e))*(-d^2)^(1/2)*(c+(-d^
2)^(1/2))^(1/3)/d/f-3/4*ln((c+(-d^2)^(1/2))^(1/3)-(c+d*tan(f*x+e))^(1/3))*(
-d^2)^(1/2)*(c+(-d^2)^(1/2))^(1/3)/d/f+1/2*arctan(1/3*(1+2*(c+d*tan(f*x+e))
^(1/3)/(c+(-d^2)^(1/2))^(1/3))*3^(1/2))*3^(1/2)*(-d^2)^(1/2)*(c+(-d^2)^(1/2
))^(1/3)/d/f+3/140*(9*c^2-35*d^2)*(c+d*tan(f*x+e))^(4/3)/d^3/f-9/35*c*tan(f
*x+e)*(c+d*tan(f*x+e))^(4/3)/d^2/f+3/10*tan(f*x+e)^2*(c+d*tan(f*x+e))^(4/3)
/d/f
```

Rubi [A]
time = 0.62, antiderivative size = 525, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3647, 3728, 3712, 3566, 726, 52, 59, 631, 210, 31}

$$-\frac{\sqrt{3}\sqrt{-d^2}\sqrt[3]{c-\sqrt{-d^2}}\,\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c-\sqrt{-d^2}}}+1}{\sqrt{3}}\right)}{2df}+\frac{\sqrt{3}\sqrt{-d^2}\sqrt[3]{c+\sqrt{-d^2}}\,\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c+\sqrt{-d^2}}}+1}{\sqrt{3}}\right)}{2df}+\frac{3(9c^2-35d^2)(c+d\tan(e+fx))^{4/3}}{140d^3f}-\frac{9c\tan(e+fx)(c+d\tan(e+fx))^{4/3}}{35d^2f}+\frac{3\sqrt{-d^2}\sqrt[3]{c-\sqrt{-d^2}}\log\left(\sqrt[3]{c-\sqrt{-d^2}}-\sqrt[3]{c+d\tan(e+fx)}\right)}{4df}-\frac{3\sqrt{-d^2}\sqrt[3]{c+\sqrt{-d^2}}\log\left(\sqrt[3]{c+\sqrt{-d^2}}-\sqrt[3]{c+d\tan(e+fx)}\right)}{4df}+\frac{\sqrt{-d^2}\sqrt[3]{c-\sqrt{-d^2}}\log(\cos(e+fx))}{4df}-\frac{\sqrt{-d^2}\sqrt[3]{c+\sqrt{-d^2}}\log(\cos(e+fx))}{4df}-\frac{1}{4}x\sqrt[3]{c-\sqrt{-d^2}}-\frac{1}{4}x\sqrt[3]{c+\sqrt{-d^2}}+\frac{3\tan^2(e+fx)(c+d\tan(e+fx))^{4/3}}{10df}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4*(c + d*Tan[e + f*x])^(1/3),x]
```

```
[Out] -1/4*((c - Sqrt[-d^2])^(1/3)*x) - ((c + Sqrt[-d^2])^(1/3)*x)/4 - (Sqrt[3]*S
qrt[-d^2]*(c - Sqrt[-d^2])^(1/3)*ArcTan[(1 + (2*(c + d*Tan[e + f*x])^(1/3))
/(c - Sqrt[-d^2])^(1/3))/Sqrt[3]])/(2*d*f) + (Sqrt[3]*Sqrt[-d^2]*(c + Sqrt[
-d^2])^(1/3)*ArcTan[(1 + (2*(c + d*Tan[e + f*x])^(1/3))/(c + Sqrt[-d^2])^(1
/3))/Sqrt[3]])/(2*d*f) + (Sqrt[-d^2]*(c - Sqrt[-d^2])^(1/3)*Log[Cos[e + f*x
]])/(4*d*f) - (Sqrt[-d^2]*(c + Sqrt[-d^2])^(1/3)*Log[Cos[e + f*x]])/(4*d*f)
 + (3*Sqrt[-d^2]*(c - Sqrt[-d^2])^(1/3)*Log[(c - Sqrt[-d^2])^(1/3) - (c + d
*Tan[e + f*x])^(1/3)])/(4*d*f) - (3*Sqrt[-d^2]*(c + Sqrt[-d^2])^(1/3)*Log[(
c + Sqrt[-d^2])^(1/3) - (c + d*Tan[e + f*x])^(1/3)])/(4*d*f) + (3*(9*c^2 -
35*d^2)*(c + d*Tan[e + f*x])^(4/3))/(140*d^3*f) - (9*c*Tan[e + f*x]*(c + d*
Tan[e + f*x])^(4/3))/(35*d^2*f) + (3*Tan[e + f*x]^2*(c + d*Tan[e + f*x])^(4
/3))/(10*d*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 726

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3712

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \tan^4(e+fx)\sqrt[3]{c+d\tan(e+fx)}\,dx = \frac{3\tan^2(e+fx)(c+d\tan(e+fx))^{4/3}}{10df} + \frac{3\int \tan(e+fx)\sqrt[3]{c+d\,\mathrm{ta}}}{}$$

$$= -\frac{9c\tan(e+fx)(c+d\tan(e+fx))^{4/3}}{35d^2f} + \frac{3\tan^2(e+fx)(c+d\tan}{10df}$$

$$= \frac{3(9c^2-35d^2)(c+d\tan(e+fx))^{4/3}}{140d^3f} - \frac{9c\tan(e+fx)(c+d\tan(e}{35d^2f}$$

$$= \frac{3(9c^2-35d^2)(c+d\tan(e+fx))^{4/3}}{140d^3f} - \frac{9c\tan(e+fx)(c+d\tan(e}{35d^2f}$$

$$= \frac{3(9c^2-35d^2)(c+d\tan(e+fx))^{4/3}}{140d^3f} - \frac{9c\tan(e+fx)(c+d\tan(e}{35d^2f}$$

$$= \frac{3(9c^2-35d^2)(c+d\tan(e+fx))^{4/3}}{140d^3f} - \frac{9c\tan(e+fx)(c+d\tan(e}{35d^2f}$$

$$= \frac{3(9c^2-35d^2)(c+d\tan(e+fx))^{4/3}}{140d^3f} - \frac{9c\tan(e+fx)(c+d\tan(e}{35d^2f}$$

$$= -\frac{1}{4}\sqrt[3]{c-\sqrt{-d^2}}\,x - \frac{1}{4}\sqrt[3]{c+\sqrt{-d^2}}\,x + \frac{\sqrt{-d^2}\sqrt[3]{c-\sqrt{-d^2}}\,\mathrm{lo}}{4df}$$

$$= -\frac{1}{4}\sqrt[3]{c-\sqrt{-d^2}}\,x - \frac{1}{4}\sqrt[3]{c+\sqrt{-d^2}}\,x + \frac{\sqrt{-d^2}\sqrt[3]{c-\sqrt{-d^2}}\,\mathrm{lo}}{4df}$$

$$= -\frac{1}{4}\sqrt[3]{c-\sqrt{-d^2}}\,x - \frac{1}{4}\sqrt[3]{c+\sqrt{-d^2}}\,x - \frac{\sqrt{3}\sqrt{-d^2}\sqrt[3]{c-\sqrt{-d^2}}}{}$$

Mathematica [C] Result contains complex when optimal does not.

time = 16.15, size = 371, normalized size = 0.71

$$\frac{-i\sqrt[3]{c-id}\left(2\sqrt{3}\,\mathrm{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c-id}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c-id}-\sqrt[3]{c+d\tan(e+fx)}\right)+\log\left((c-id)^{2/3}+\sqrt[3]{c-id}\,\sqrt[3]{c+d\tan(e+fx)}+(c+d\tan(e+fx))^{2/3}\right)\right)+i\sqrt[3]{c+id}\left(2\sqrt{3}\,\mathrm{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c+id}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c+id}-\sqrt[3]{c+d\tan(e+fx)}\right)+\log\left((c+id)^{2/3}+\sqrt[3]{c+id}\,\sqrt[3]{c+d\tan(e+fx)}+(c+d\tan(e+fx))^{2/3}\right)\right)+\frac{3\sqrt[3]{c+d\tan(e+fx)}\,(9c^3-37cd^2-d(3c^2+49d^2)\tan(e+fx)+2d^2\sec^2(e+fx)(c+7d\tan(e+fx)))}{35d^3}}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4*(c + d*Tan[e + f*x])^(1/3),x]
```

```
[Out] ((-I)*(c - I*d)^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(c + d*Tan[e + f*x])^(1/3))
/(c - I*d)^(1/3))/Sqrt[3]] - 2*Log[(c - I*d)^(1/3) - (c + d*Tan[e + f*x])^(
1/3)] + Log[(c - I*d)^(2/3) + (c - I*d)^(1/3)*(c + d*Tan[e + f*x])^(1/3) +
(c + d*Tan[e + f*x])^(2/3)]) + I*(c + I*d)^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*
(c + d*Tan[e + f*x])^(1/3))/(c + I*d)^(1/3))/Sqrt[3]] - 2*Log[(c + I*d)^(1/
3) - (c + d*Tan[e + f*x])^(1/3)] + Log[(c + I*d)^(2/3) + (c + I*d)^(1/3)*(c
 + d*Tan[e + f*x])^(1/3) + (c + d*Tan[e + f*x])^(2/3)]) + (3*(c + d*Tan[e +
 f*x])^(1/3)*(9*c^3 - 37*c*d^2 - d*(3*c^2 + 49*d^2)*Tan[e + f*x] + 2*d^2*Se
c[e + f*x]^2*(c + 7*d*Tan[e + f*x])))/(35*d^3))/(4*f)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.37, size = 131, normalized size = 0.25

method	result
derivativedivides	$\frac{\frac{3(c+d\tan(fx+e))^{\frac{10}{3}}}{10}-\frac{6c(c+d\tan(fx+e))^{\frac{7}{3}}}{7}+\frac{3c^2(c+d\tan(fx+e))^{\frac{4}{3}}}{4}-\frac{3d^2(c+d\tan(fx+e))^{\frac{4}{3}}}{4}+\frac{d^4\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-2c\_Z^3+\right.}\right.}{}}{f\,d^3}$
default	$\frac{\frac{3(c+d\tan(fx+e))^{\frac{10}{3}}}{10}-\frac{6c(c+d\tan(fx+e))^{\frac{7}{3}}}{7}+\frac{3c^2(c+d\tan(fx+e))^{\frac{4}{3}}}{4}-\frac{3d^2(c+d\tan(fx+e))^{\frac{4}{3}}}{4}+\frac{d^4\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-2c\_Z^3+\right.}\right.}{}}{f\,d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4*(c+d*tan(f*x+e))^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/f/d^3*(1/10*(c+d*tan(f*x+e))^(10/3)-2/7*c*(c+d*tan(f*x+e))^(7/3)+1/4*c^2*
(c+d*tan(f*x+e))^(4/3)-1/4*d^2*(c+d*tan(f*x+e))^(4/3)+1/6*d^4*sum(_R^3/(_R^
5-_R^2*c)*ln((c+d*tan(f*x+e))^(1/3)-_R),_R=RootOf(_Z^6-2*_Z^3*c+c^2+d^2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(c+d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(f*x + e) + c)^(1/3)*tan(f*x + e)^4, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3638 vs. $2(434) = 868$.
time = 1.20, size = 3638, normalized size = 6.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(c+d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/140*(70*d^3*f*((c^2 + d^2)/f^6)^(1/6)*cos(f*x + e)^3*cos(2/3*arctan((f^6*
sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2))*log(2*c*f^
4*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*sqrt(c^2/f^6)*((c^
2 + d^2)/f^6)^(1/6)*sin(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6)
 + d*f^3*sqrt(c^2/f^6))/c^2)) + c^2*f^2*((c^2 + d^2)/f^6)^(1/3) + c^2*((c*c
os(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(2/3)) + 280*d^3*f*((c^2 + d^2)
/f^6)^(1/6)*arctan(-(c*f^8*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))
^(1/3)*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(5/6) - sqrt(2*c*f^4*((c*cos(f*x + e
) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(1/
6)*sin(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2
/f^6))/c^2)) + c^2*f^2*((c^2 + d^2)/f^6)^(1/3) + c^2*((c*cos(f*x + e) + d*s
in(f*x + e))/cos(f*x + e))^(2/3))*f^8*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(5/6)
 + (c^4 + c^2*d^2)*sin(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6)
+ d*f^3*sqrt(c^2/f^6))/c^2)))/((c^4 + c^2*d^2)*cos(2/3*arctan((f^6*sqrt(c^2
/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2))))*cos(f*x + e)^3*s
in(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6
))/c^2)) + 140*(sqrt(3)*d^3*f*((c^2 + d^2)/f^6)^(1/6)*cos(f*x + e)^3*cos(2/
3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^
2)) - d^3*f*((c^2 + d^2)/f^6)^(1/6)*cos(f*x + e)^3*sin(2/3*arctan((f^6*sqrt
(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2)))*arctan(-(2*c*
f^8*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*sqrt(c^2/f^6)*((
c^2 + d^2)/f^6)^(5/6)*cos(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^
6) + d*f^3*sqrt(c^2/f^6))/c^2)) - 2*(sqrt(3)*c*f^8*((c*cos(f*x + e) + d*sin
(f*x + e))/cos(f*x + e))^(1/3)*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(5/6) + 2*(c
^4 + c^2*d^2)*cos(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f
^3*sqrt(c^2/f^6))/c^2)))*sin(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)
/f^6) + d*f^3*sqrt(c^2/f^6))/c^2)) + 2*(sqrt(3)*f^8*sqrt(c^2/f^6)*((c^2 + d
^2)/f^6)^(5/6)*sin(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*
f^3*sqrt(c^2/f^6))/c^2)) - f^8*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(5/6)*cos(2/
3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^
2)))*sqrt(sqrt(3)*c*f^4*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1
/3)*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(1/6)*cos(2/3*arctan((f^6*sqrt(c^2/f^6)
*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2)) - c*f^4*((c*cos(f*x + e
) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(1/
```

```
6)*sin(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2
/f^6))/c^2)) + c^2*f^2*((c^2 + d^2)/f^6)^(1/3) + c^2*((c*cos(f*x + e) + d*s
in(f*x + e))/cos(f*x + e))^(2/3)) + sqrt(3)*(c^4 + c^2*d^2))/(3*c^4 + 3*c^2
*d^2 - 4*(c^4 + c^2*d^2)*cos(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)
/f^6) + d*f^3*sqrt(c^2/f^6))/c^2))^2)) + 140*(sqrt(3)*d^3*f*((c^2 + d^2)/f^
6)^(1/6)*cos(f*x + e)^3*cos(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/
f^6) + d*f^3*sqrt(c^2/f^6))/c^2)) + d^3*f*((c^2 + d^2)/f^6)^(1/6)*cos(f*x +
 e)^3*sin(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(
c^2/f^6))/c^2)))*arctan((2*c*f^8*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x
 + e))^(1/3)*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(5/6)*cos(2/3*arctan((f^6*sqrt
(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2)) + 2*(sqrt(3)*c
*f^8*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*sqrt(c^2/f^6)*(
(c^2 + d^2)/f^6)^(5/6) - 2*(c^4 + c^2*d^2)*cos(2/3*arctan((f^6*sqrt(c^2/f^6
)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2)))*sin(2/3*arctan((f^6*s
qrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2)) - 2*(sqrt(3
)*f^8*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(5/6)*sin(2/3*arctan((f^6*sqrt(c^2/f^
6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2)) + f^8*sqrt(c^2/f^6)*(
(c^2 + d^2)/f^6)^(5/6)*cos(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f
^6) + d*f^3*sqrt(c^2/f^6))/c^2)))*sqrt(-sqrt(3)*c*f^4*((c*cos(f*x + e) + d*
sin(f*x + e))/cos(f*x + e))^(1/3)*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(1/6)*cos
(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))
/c^2)) - c*f^4*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*sqrt(
c^2/f^6)*((c^2 + d^2)/f^6)^(1/6)*sin(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^
2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2)) + c^2*f^2*((c^2 + d^2)/f^6)^(1/3
) + c^2*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(2/3)) - sqrt(3)*(
c^4 + c^2*d^2))/(3*c^4 + 3*c^2*d^2 - 4*(c^4 + c^2*d^2)*cos(2/3*arctan((f^6*
sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2))^2)) - 35*(
sqrt(3)*d^3*f*((c^2 + d^2)/f^6)^(1/6)*cos(f*x + e)^3*sin(2/3*arctan((f^6*sq
rt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2)) + d^3*f*((c^
2 + d^2)/f^6)^(1/6)*cos(f*x + e)^3*cos(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((
c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2)))*log(sqrt(3)*c*f^4*((c*cos(f*x
 + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)
^(1/6)*cos(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{c+d\tan(e+fx)}\,\tan^4(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4*(c+d*tan(f*x+e))**(1/3),x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(1/3)*tan(e + f*x)**4, x)
```

Giac [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(c+d*tan(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 18.88, size = 1015, normalized size = 1.93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(c + d*tan(e + f*x))^(1/3),x)
```

```
[Out] log((c + d*tan(e + f*x))^(1/3) + f*(-(c*1i + d)/f^3)^(1/3)*1i)*(-(c*1i + d)
/(8*f^3))^(1/3) + (c + d*tan(e + f*x))^(1/3)*(2*c*((6*c^2)/(d^3*f) - (3*(c^
2 + d^2))/(d^3*f)) - (12*c^3)/(d^3*f) + (6*c*(c^2 + d^2))/(d^3*f)) + ((3*c^
2)/(2*d^3*f) - (3*(c^2 + d^2))/(4*d^3*f))*(c + d*tan(e + f*x))^(4/3) + log(
d*(c + d*tan(e + f*x))^(1/3)*1i - c*(c + d*tan(e + f*x))^(1/3) + f^4*((c*1i
 - d)/f^3)^(4/3) + 2*d*f*((c*1i - d)/f^3)^(1/3))*((c*1i - d)/(8*f^3))^(1/3)
 - log((((c*1i - d)/f^3)^(1/3)*((3^(1/2)*1i)/2 + 1/2)*((((c*1i - d)/f^3)^(2
/3)*((3^(1/2)*1i)/2 - 1/2)*((3888*d^5*(c^2 + d^2)*(c + d*tan(e + f*x))^(1/3
))/f - 3888*c*d^4*((c*1i - d)/f^3)^(1/3)*((3^(1/2)*1i)/2 + 1/2)*(c^2 + d^2)
))/4 + (1944*c*d^5*(c^2 + d^2))/f^3))/2 - (486*(d^8 - c^4*d^4)*(c + d*tan(e
 + f*x))^(1/3))/f^4)*((3^(1/2)*1i)/2 + 1/2)*((c*1i - d)/(8*f^3))^(1/3) + lo
g((486*(d^8 - c^4*d^4)*(c + d*tan(e + f*x))^(1/3))/f^4 - (((c*1i - d)/f^3)^
(1/3)*((3^(1/2)*1i)/2 - 1/2)*((((c*1i - d)/f^3)^(2/3)*((3^(1/2)*1i)/2 + 1/2
)*((3888*d^5*(c^2 + d^2)*(c + d*tan(e + f*x))^(1/3))/f + 3888*c*d^4*((c*1i
- d)/f^3)^(1/3)*((3^(1/2)*1i)/2 - 1/2)*(c^2 + d^2)))/4 - (1944*c*d^5*(c^2 +
 d^2))/f^3))/2)*((3^(1/2)*1i)/2 - 1/2)*((c*1i - d)/(8*f^3))^(1/3) + (3*(c +
 d*tan(e + f*x))^(10/3))/(10*d^3*f) - log((((3^(1/2)*1i)/2 + 1/2)*((((3888*
d^5*(c^2 + d^2)*(c + d*tan(e + f*x))^(1/3))/f - 3888*c*d^4*((3^(1/2)*1i)/2
+ 1/2)*(-(c*1i + d)/f^3)^(1/3)*(c^2 + d^2))*((3^(1/2)*1i)/2 - 1/2)*(-(c*1i
+ d)/f^3)^(2/3))/4 + (1944*c*d^5*(c^2 + d^2))/f^3)*(-(c*1i + d)/f^3)^(1/3))
/2 - (486*(d^8 - c^4*d^4)*(c + d*tan(e + f*x))^(1/3))/f^4)*((3^(1/2)*1i)/2
+ 1/2)*(-(c*1i + d)/(8*f^3))^(1/3) + log((486*(d^8 - c^4*d^4)*(c + d*tan(e
+ f*x))^(1/3))/f^4 - (((3^(1/2)*1i)/2 - 1/2)*((((3888*d^5*(c^2 + d^2)*(c +
d*tan(e + f*x))^(1/3))/f + 3888*c*d^4*((3^(1/2)*1i)/2 - 1/2)*(-(c*1i + d)/f
^3)^(1/3)*(c^2 + d^2))*((3^(1/2)*1i)/2 + 1/2)*(-(c*1i + d)/f^3)^(2/3))/4 -
(1944*c*d^5*(c^2 + d^2))/f^3)*(-(c*1i + d)/f^3)^(1/3))/2)*((3^(1/2)*1i)/2 -
 1/2)*(-(c*1i + d)/(8*f^3))^(1/3) - (6*c*(c + d*tan(e + f*x))^(7/3))/(7*d^3
*f)
```

3.682 $\int \tan^3(e+fx)\sqrt[3]{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=373

$$-\frac{1}{4}i\sqrt[3]{c-id}\,x+\frac{1}{4}i\sqrt[3]{c+id}\,x+\frac{\sqrt{3}\,\sqrt[3]{c-id}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c-id}}}{\sqrt{3}}\right)}{2f}+\frac{\sqrt{3}\,\sqrt[3]{c+id}\,\text{ArcTan}\left(\frac{1+\ldots}{\ldots}\right)}{2\ldots}$$

```
[Out] -1/4*I*(c-I*d)^(1/3)*x+1/4*I*(c+I*d)^(1/3)*x-1/4*(c-I*d)^(1/3)*ln(cos(f*x+e
))/f-1/4*(c+I*d)^(1/3)*ln(cos(f*x+e))/f-3/4*(c-I*d)^(1/3)*ln((c-I*d)^(1/3)-
(c+d*tan(f*x+e))^(1/3))/f-3/4*(c+I*d)^(1/3)*ln((c+I*d)^(1/3)-(c+d*tan(f*x+e
))^(1/3))/f+1/2*(c-I*d)^(1/3)*arctan(1/3*(1+2*(c+d*tan(f*x+e))^(1/3)/(c-I*d
)^(1/3))*3^(1/2))*3^(1/2)/f+1/2*(c+I*d)^(1/3)*arctan(1/3*(1+2*(c+d*tan(f*x+
e))^(1/3)/(c+I*d)^(1/3))*3^(1/2))*3^(1/2)/f-3*(c+d*tan(f*x+e))^(1/3)/f-9/28
*c*(c+d*tan(f*x+e))^(4/3)/d^2/f+3/7*tan(f*x+e)*(c+d*tan(f*x+e))^(4/3)/d/f
```

Rubi [A]
time = 0.40, antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3647, 3711, 12, 3609, 3620, 3618, 59, 631, 210, 31}

$$\frac{\sqrt{3}\,\sqrt[3]{c-id}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c-id}}}{\sqrt{3}}\right)}{2f}+\frac{\sqrt{3}\,\sqrt[3]{c+id}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c+id}}}{\sqrt{3}}\right)}{2f}-\frac{9c(c+d\tan(e+fx))^{4/3}}{28d^2f}+\frac{3\tan(e+fx)(c+d\tan(e+fx))^{4/3}}{7df}-\frac{3\sqrt[3]{c+d\tan(e+fx)}}{f}-\frac{3\sqrt[3]{c-id}\,\log\left(-\sqrt[3]{c+d\tan(e+fx)}+\sqrt[3]{c-id}\right)}{4f}-\frac{3\sqrt[3]{c+id}\,\log\left(-\sqrt[3]{c+d\tan(e+fx)}+\sqrt[3]{c+id}\right)}{4f}-\frac{\sqrt[3]{c-id}\,\log(\cos(e+fx))}{4f}-\frac{\sqrt[3]{c+id}\,\log(\cos(e+fx))}{4f}-\frac{1}{4}ix\sqrt[3]{c-id}+\frac{1}{4}ix\sqrt[3]{c+id}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3*(c + d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (-1/4*I)*(c - I*d)^(1/3)*x + (I/4)*(c + I*d)^(1/3)*x + (Sqrt[3]*(c - I*d)^(
1/3)*ArcTan[(1 + (2*(c + d*Tan[e + f*x])^(1/3))/(c - I*d)^(1/3))/Sqrt[3]])/
(2*f) + (Sqrt[3]*(c + I*d)^(1/3)*ArcTan[(1 + (2*(c + d*Tan[e + f*x])^(1/3))
/(c + I*d)^(1/3))/Sqrt[3]])/(2*f) - ((c - I*d)^(1/3)*Log[Cos[e + f*x]])/(4*
f) - ((c + I*d)^(1/3)*Log[Cos[e + f*x]])/(4*f) - (3*(c - I*d)^(1/3)*Log[(c
- I*d)^(1/3) - (c + d*Tan[e + f*x])^(1/3)])/(4*f) - (3*(c + I*d)^(1/3)*Log[
(c + I*d)^(1/3) - (c + d*Tan[e + f*x])^(1/3)])/(4*f) - (3*(c + d*Tan[e + f*
x])^(1/3))/f - (9*c*(c + d*Tan[e + f*x])^(4/3))/(28*d^2*f) + (3*Tan[e + f*x
]*(c + d*Tan[e + f*x])^(4/3))/(7*d*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \tan^3(e+fx)\sqrt[3]{c+d\tan(e+fx)}\,dx = \frac{3\tan(e+fx)(c+d\tan(e+fx))^{4/3}}{7df} + \frac{3\int \sqrt[3]{c+d\tan(e+fx)}\,(-$$

$$= -\frac{9c(c+d\tan(e+fx))^{4/3}}{28d^2f} + \frac{3\tan(e+fx)(c+d\tan(e+fx))^{4/3}}{7df}$$

$$= -\frac{9c(c+d\tan(e+fx))^{4/3}}{28d^2f} + \frac{3\tan(e+fx)(c+d\tan(e+fx))^{4/3}}{7df}$$

$$= -\frac{3\sqrt[3]{c+d\tan(e+fx)}}{f} - \frac{9c(c+d\tan(e+fx))^{4/3}}{28d^2f} + \frac{3\tan(e+f}{}$$

$$= -\frac{3\sqrt[3]{c+d\tan(e+fx)}}{f} - \frac{9c(c+d\tan(e+fx))^{4/3}}{28d^2f} + \frac{3\tan(e+f}{}$$

$$= -\frac{3\sqrt[3]{c+d\tan(e+fx)}}{f} - \frac{9c(c+d\tan(e+fx))^{4/3}}{28d^2f} + \frac{3\tan(e+f}{}$$

$$= -\frac{1}{4}i\sqrt[3]{c-id}\,x + \frac{1}{4}i\sqrt[3]{c+id}\,x - \frac{\sqrt[3]{c-id}\,\log(\cos(e+fx))}{4f} - \frac{\sqrt[3]{c}}{}$$

$$= -\frac{1}{4}i\sqrt[3]{c-id}\,x + \frac{1}{4}i\sqrt[3]{c+id}\,x - \frac{\sqrt[3]{c-id}\,\log(\cos(e+fx))}{4f} - \frac{\sqrt[3]{c}}{}$$

$$= -\frac{1}{4}i\sqrt[3]{c-id}\,x + \frac{1}{4}i\sqrt[3]{c+id}\,x + \frac{\sqrt{3}\,\sqrt[3]{c-id}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{c+d}}{\sqrt[3]{}}}{}\right.}{2f}$$

Mathematica [A]
time = 1.09, size = 442, normalized size = 1.18

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3*(c + d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (14*Sqrt[3]*(c - I*d)^(1/3)*d^2*ArcTan[(1 + (2*(c + d*Tan[e + f*x])^(1/3))/
(c - I*d)^(1/3))/Sqrt[3]] + 14*Sqrt[3]*(c + I*d)^(1/3)*d^2*ArcTan[(1 + (2*(
c + d*Tan[e + f*x])^(1/3))/(c + I*d)^(1/3))/Sqrt[3]] - 14*(c - I*d)^(1/3)*d
^2*Log[(c - I*d)^(1/3) - (c + d*Tan[e + f*x])^(1/3)] - 14*(c + I*d)^(1/3)*d
^2*Log[(c + I*d)^(1/3) - (c + d*Tan[e + f*x])^(1/3)] + 7*(c - I*d)^(1/3)*d^
2*Log[(c - I*d)^(2/3) + (c - I*d)^(1/3)*(c + d*Tan[e + f*x])^(1/3) + (c + d
```

```
*Tan[e + f*x])^(2/3)] + 7*(c + I*d)^(1/3)*d^2*Log[(c + I*d)^(2/3) + (c + I*
d)^(1/3)*(c + d*Tan[e + f*x])^(1/3) + (c + d*Tan[e + f*x])^(2/3)] - 9*c^2*(
c + d*Tan[e + f*x])^(1/3) - 84*d^2*(c + d*Tan[e + f*x])^(1/3) + 3*c*d*Tan[e
 + f*x]*(c + d*Tan[e + f*x])^(1/3) + 12*d^2*Tan[e + f*x]^2*(c + d*Tan[e + f
*x])^(1/3))/(28*d^2*f)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.24, size = 124, normalized size = 0.33

method	result
derivativedivides	$\frac{\frac{3(c+d\tan(fx+e))^{\frac{7}{3}}}{7}-\frac{3c(c+d\tan(fx+e))^{\frac{4}{3}}}{4}-3d^2(c+d\tan(fx+e))^{\frac{1}{3}}+\frac{d^2\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-2c\_Z^3+c^2+d^2\right)}\frac{\left(-\_R^3c+c^2\cdots\right.}{\cdots}\right)}{2}}{f\,d^2}$
default	$\frac{\frac{3(c+d\tan(fx+e))^{\frac{7}{3}}}{7}-\frac{3c(c+d\tan(fx+e))^{\frac{4}{3}}}{4}-3d^2(c+d\tan(fx+e))^{\frac{1}{3}}+\frac{d^2\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-2c\_Z^3+c^2+d^2\right)}\frac{\left(-\_R^3c+c^2\cdots\right.}{\cdots}\right)}{2}}{f\,d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3*(c+d*tan(f*x+e))^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/f/d^2*(1/7*(c+d*tan(f*x+e))^(7/3)-1/4*c*(c+d*tan(f*x+e))^(4/3)-d^2*(c+d*t
an(f*x+e))^(1/3)+1/6*d^2*sum((-_R^3*c+c^2+d^2)/(_R^5-_R^2*c)*ln((c+d*tan(f*
x+e))^(1/3)-_R),_R=RootOf(_Z^6-2*_Z^3*c+c^2+d^2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(c+d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(f*x + e) + c)^(1/3)*tan(f*x + e)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3010 vs. 2(291) = 582.
time = 1.17, size = 3010, normalized size = 8.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(c+d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/28*(14*d^2*f*((c^2 + d^2)/f^6)^(1/6)*cos(f*x + e)^2*cos(2/3*arctan((f^6*s
qrt((c^2 + d^2)/f^6) + c*f^3)*sqrt(d^2/f^6)/d^2))*log(2*f*((c*cos(f*x + e)
```

+ d*sin(f*x + e))/cos(f*x + e))^(1/3)*((c^2 + d^2)/f^6)^(1/6)*cos(2/3*arcta
n((f^6*sqrt((c^2 + d^2)/f^6) + c*f^3)*sqrt(d^2/f^6)/d^2)) + f^2*((c^2 + d^2
)/f^6)^(1/3) + ((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(2/3)) - 56
*d^2*f*((c^2 + d^2)/f^6)^(1/6)*arctan((sqrt(2*f*((c*cos(f*x + e) + d*sin(f*
x + e))/cos(f*x + e))^(1/3)*((c^2 + d^2)/f^6)^(1/6)*cos(2/3*arctan((f^6*sqr
t((c^2 + d^2)/f^6) + c*f^3)*sqrt(d^2/f^6)/d^2)) + f^2*((c^2 + d^2)/f^6)^(1/
3) + ((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(2/3))*f^5*((c^2 + d^
2)/f^6)^(5/6) - f^5*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*
((c^2 + d^2)/f^6)^(5/6) - (c^2 + d^2)*cos(2/3*arctan((f^6*sqrt((c^2 + d^2)/
f^6) + c*f^3)*sqrt(d^2/f^6)/d^2)))/((c^2 + d^2)*sin(2/3*arctan((f^6*sqrt((c
^2 + d^2)/f^6) + c*f^3)*sqrt(d^2/f^6)/d^2))))*cos(f*x + e)^2*sin(2/3*arctan
((f^6*sqrt((c^2 + d^2)/f^6) + c*f^3)*sqrt(d^2/f^6)/d^2)) - 28*(sqrt(3)*d^2*
f*((c^2 + d^2)/f^6)^(1/6)*cos(f*x + e)^2*cos(2/3*arctan((f^6*sqrt((c^2 + d^
2)/f^6) + c*f^3)*sqrt(d^2/f^6)/d^2)) - d^2*f*((c^2 + d^2)/f^6)^(1/6)*cos(f*
x + e)^2*sin(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) + c*f^3)*sqrt(d^2/f^6)/d
^2)))*arctan(-(2*sqrt(3)*f^5*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e
))^(1/3)*((c^2 + d^2)/f^6)^(5/6)*cos(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6)
+ c*f^3)*sqrt(d^2/f^6)/d^2)) + 2*(f^5*((c*cos(f*x + e) + d*sin(f*x + e))/co
s(f*x + e))^(1/3)*((c^2 + d^2)/f^6)^(5/6) - 2*(c^2 + d^2)*cos(2/3*arctan((f
^6*sqrt((c^2 + d^2)/f^6) + c*f^3)*sqrt(d^2/f^6)/d^2)))*sin(2/3*arctan((f^6*
sqrt((c^2 + d^2)/f^6) + c*f^3)*sqrt(d^2/f^6)/d^2)) - 2*(sqrt(3)*f^5*((c^2 +
d^2)/f^6)^(5/6)*cos(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) + c*f^3)*sqrt(d^
2/f^6)/d^2)) + f^5*((c^2 + d^2)/f^6)^(5/6)*sin(2/3*arctan((f^6*sqrt((c^2 +
d^2)/f^6) + c*f^3)*sqrt(d^2/f^6)/d^2)))*sqrt(-sqrt(3)*f*((c*cos(f*x + e) +
d*sin(f*x + e))/cos(f*x + e))^(1/3)*((c^2 + d^2)/f^6)^(1/6)*sin(2/3*arctan(
(f^6*sqrt((c^2 + d^2)/f^6) + c*f^3)*sqrt(d^2/f^6)/d^2)) - f*((c*cos(f*x + e
) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*((c^2 + d^2)/f^6)^(1/6)*cos(2/3*arc
tan((f^6*sqrt((c^2 + d^2)/f^6) + c*f^3)*sqrt(d^2/f^6)/d^2)) + f^2*((c^2 + d
^2)/f^6)^(1/3) + ((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(2/3)) -
sqrt(3)*(c^2 + d^2))/(4*(c^2 + d^2)*cos(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^
6) + c*f^3)*sqrt(d^2/f^6)/d^2))^2 - c^2 - d^2)) + 28*(sqrt(3)*d^2*f*((c^2 +
d^2)/f^6)^(1/6)*cos(f*x + e)^2*cos(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) +
c*f^3)*sqrt(d^2/f^6)/d^2)) + d^2*f*((c^2 + d^2)/f^6)^(1/6)*cos(f*x + e)^2*
sin(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) + c*f^3)*sqrt(d^2/f^6)/d^2)))*arc
tan((2*sqrt(3)*f^5*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*(
(c^2 + d^2)/f^6)^(5/6)*cos(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) + c*f^3)*s
qrt(d^2/f^6)/d^2)) - 2*(f^5*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e)
)^(1/3)*((c^2 + d^2)/f^6)^(5/6) - 2*(c^2 + d^2)*cos(2/3*arctan((f^6*sqrt((c
^2 + d^2)/f^6) + c*f^3)*sqrt(d^2/f^6)/d^2)))*sin(2/3*arctan((f^6*sqrt((c^2
+ d^2)/f^6) + c*f^3)*sqrt(d^2/f^6)/d^2)) - 2*(sqrt(3)*f^5*((c^2 + d^2)/f^6)
^(5/6)*cos(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) + c*f^3)*sqrt(d^2/f^6)/d^2
)) - f^5*((c^2 + d^2)/f^6)^(5/6)*sin(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6)
+ c*f^3)*sqrt(d^2/f^6)/d^2)))*sqrt(sqrt(3)*f*((c*cos(f*x + e) + d*sin(f*x +
e))/cos(f*x + e))^(1/3)*((c^2 + d^2)/f^6)^(1/6)*sin(2/3*arctan((f^6*sqrt((
c^2 + d^2)/f^6) + c*f^3)*sqrt(d^2/f^6)/d^2)) - f*((c*cos(f*x + e) + d*sin(f

```
*x + e))/cos(f*x + e))^(1/3)*((c^2 + d^2)/f^6)^(1/6)*cos(2/3*arctan((f^6*sq
rt((c^2 + d^2)/f^6) + c*f^3)*sqrt(d^2/f^6)/d^2)) + f^2*((c^2 + d^2)/f^6)^(1
/3) + ((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(2/3)) - sqrt(3)*(c^
2 + d^2))/(4*(c^2 + d^2)*cos(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) + c*f^3)
*sqrt(d^2/f^6)/d^2))^2 - c^2 - d^2)) + 7*(sqrt(3)*d^2*f*((c^2 + d^2)/f^6)^(
1/6)*cos(f*x + e)^2*sin(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) + c*f^3)*sqrt
(d^2/f^6)/d^2)) - d^2*f*((c^2 + d^2)/f^6)^(1/6)*cos(f*x + e)^2*cos(2/3*arct
an((f^6*sqrt((c^2 + d^2)/f^6) + c*f^3)*sqrt(d^2/f^6)/d^2)))*log(sqrt(3)*f*(
(c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*((c^2 + d^2)/f^6)^(1/
6)*sin(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) + c*f^3)*sqrt(d^2/f^6)/d^2)) -
 f*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*((c^2 + d^2)/f^6)
^(1/6)*cos(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) + c*f^3)*sqrt(d^2/f^6)/d^2
)) + f^2*((c^2 + d^2)/f^6)^(1/3) + ((c*cos(f*x + e) + d*sin(f*x + e))/cos(f
*x + e))^(2/3)) - 7*(sqrt(3)*d^2*f*((c^2 + d^2)/f^6)^(1/6)*cos(f*x + e)^2*s
in(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) + c*f^3)*sqrt(d^2/f^6)/d^2)) + d^2
*f*((c^2 + d^2)/f^6)^(1/6)*cos(f*x + e)^2*cos(2/3*arctan((f^6*sqrt((c^2 + d
^2)/f^6) + c*f^3)*sqrt(d^2/f^6)/d^2)))*log(-sqrt(3)*f*((c*cos(f*x + e) + d*
sin(f*x + e))/cos(f*x + e))^(1/3)*((c^2 + d^2)/f^6)^(1/6)*sin(2/3*arctan((f
^6*sqrt((c^2 + d^2)/f^6) + c*f^3)*sqrt(d^2/f^6)/d^2)) - f*((c*cos(f*x + e)
+ d*sin(f*x + e))/cos(f*x + e))^(1/3)*((c^2 + d...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{c + d \tan{\left(e + f x \right)}} \tan^{3}{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3*(c+d*tan(f*x+e))**(1/3),x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(1/3)*tan(e + f*x)**3, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(c+d*tan(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 11.79, size = 890, normalized size = 2.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(c + d*tan(e + f*x))^(1/3),x)
```

```
[Out] log((c + d*tan(e + f*x))^(1/3) + f*(-(c - d*1i)/f^3)^(1/3))*(-(c - d*1i)/(8
*f^3))^(1/3) + log((c + d*tan(e + f*x))^(1/3) + f*(-(c + d*1i)/f^3)^(1/3))*
(-(c + d*1i)/(8*f^3))^(1/3) + ((3*c^2)/(d^2*f) - (3*(c^2 + d^2))/(d^2*f))*(
c + d*tan(e + f*x))^(1/3) + (3*(c + d*tan(e + f*x))^(7/3))/(7*d^2*f) + log(
(((3^(1/2)*1i)/2 - 1/2)*((972*(d^8 - c^4*d^4))/f^3 + (((3^(1/2)*1i)/2 + 1/2
)*((3888*c*d^4*(c^2 + d^2)*(c + d*tan(e + f*x))^(1/3))/f + 3888*c*d^4*((3^(
1/2)*1i)/2 - 1/2)*(-(c - d*1i)/f^3)^(1/3)*(c^2 + d^2))*(-(c - d*1i)/f^3)^(2
/3))/4)*(-(c - d*1i)/f^3)^(1/3))/2 + (486*(d^8 - c^4*d^4)*(c + d*tan(e + f*
x))^(1/3))/f^4)*((3^(1/2)*1i)/2 - 1/2)*(-(c - d*1i)/(8*f^3))^(1/3) + log(((
(3^(1/2)*1i)/2 - 1/2)*((972*(d^8 - c^4*d^4))/f^3 + (((3^(1/2)*1i)/2 + 1/2)*
((3888*c*d^4*(c^2 + d^2)*(c + d*tan(e + f*x))^(1/3))/f + 3888*c*d^4*((3^(1/
2)*1i)/2 - 1/2)*(-(c + d*1i)/f^3)^(1/3)*(c^2 + d^2))*(-(c + d*1i)/f^3)^(2/3
))/4)*(-(c + d*1i)/f^3)^(1/3))/2 + (486*(d^8 - c^4*d^4)*(c + d*tan(e + f*x)
)^(1/3))/f^4)*((3^(1/2)*1i)/2 - 1/2)*(-(c + d*1i)/(8*f^3))^(1/3) - log((486
*(d^8 - c^4*d^4)*(c + d*tan(e + f*x))^(1/3))/f^4 - (((3^(1/2)*1i)/2 + 1/2)*
((972*(d^8 - c^4*d^4))/f^3 - (((3^(1/2)*1i)/2 - 1/2)*((3888*c*d^4*(c^2 + d^
2)*(c + d*tan(e + f*x))^(1/3))/f - 3888*c*d^4*((3^(1/2)*1i)/2 + 1/2)*(-(c -
 d*1i)/f^3)^(1/3)*(c^2 + d^2))*(-(c - d*1i)/f^3)^(2/3))/4)*(-(c - d*1i)/f^3
)^(1/3))/2)*((3^(1/2)*1i)/2 + 1/2)*(-(c - d*1i)/(8*f^3))^(1/3) - log((486*(
d^8 - c^4*d^4)*(c + d*tan(e + f*x))^(1/3))/f^4 - (((3^(1/2)*1i)/2 + 1/2)*((
972*(d^8 - c^4*d^4))/f^3 - (((3^(1/2)*1i)/2 - 1/2)*((3888*c*d^4*(c^2 + d^2)
*(c + d*tan(e + f*x))^(1/3))/f - 3888*c*d^4*((3^(1/2)*1i)/2 + 1/2)*(-(c + d
*1i)/f^3)^(1/3)*(c^2 + d^2))*(-(c + d*1i)/f^3)^(2/3))/4)*(-(c + d*1i)/f^3)^
(1/3))/2)*((3^(1/2)*1i)/2 + 1/2)*(-(c + d*1i)/(8*f^3))^(1/3) - (3*c*(c + d*
tan(e + f*x))^(4/3))/(4*d^2*f)
```

3.683 $\int \tan^2(e+fx)\sqrt[3]{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=439

$$\frac{1}{4}\sqrt[3]{c-\sqrt{-d^2}}\,x+\frac{1}{4}\sqrt[3]{c+\sqrt{-d^2}}\,x-\frac{\sqrt{3}\,d\sqrt[3]{c-\sqrt{-d^2}}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c-\sqrt{-d^2}}}}{\sqrt{3}}\right)}{2\sqrt{-d^2}\,f}+\frac{\sqrt{3}\,d\sqrt[3]{c-}}{}$$

```
[Out] 1/4*x*(c-(-d^2)^(1/2))^(1/3)+1/4*d*ln(cos(f*x+e))*(c-(-d^2)^(1/2))^(1/3)/f/
(-d^2)^(1/2)+3/4*d*ln((c-(-d^2)^(1/2))^(1/3)-(c+d*tan(f*x+e))^(1/3))*(c-(-d
^2)^(1/2))^(1/3)/f/(-d^2)^(1/2)-1/2*d*arctan(1/3*(1+2*(c+d*tan(f*x+e))^(1/3
)/(c-(-d^2)^(1/2))^(1/3))*3^(1/2))*3^(1/2)*(c-(-d^2)^(1/2))^(1/3)/f/(-d^2)^
(1/2)+1/4*x*(c+(-d^2)^(1/2))^(1/3)-1/4*d*ln(cos(f*x+e))*(c+(-d^2)^(1/2))^(1
/3)/f/(-d^2)^(1/2)-3/4*d*ln((c+(-d^2)^(1/2))^(1/3)-(c+d*tan(f*x+e))^(1/3))*
(c+(-d^2)^(1/2))^(1/3)/f/(-d^2)^(1/2)+1/2*d*arctan(1/3*(1+2*(c+d*tan(f*x+e)
)^(1/3)/(c+(-d^2)^(1/2))^(1/3))*3^(1/2))*3^(1/2)*(c+(-d^2)^(1/2))^(1/3)/f/(
-d^2)^(1/2)+3/4*(c+d*tan(f*x+e))^(4/3)/d/f
```

Rubi [A]
time = 0.25, antiderivative size = 439, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3624, 3566, 726, 52, 59, 631, 210, 31}

$$-\frac{\sqrt{3}\,d\sqrt[3]{c-\sqrt{-d^2}}\,\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c-\sqrt{-d^2}}}+1}{\sqrt{3}}\right)}{2\sqrt{-d^2}\,f}+\frac{\sqrt{3}\,d\sqrt[3]{c+\sqrt{-d^2}}\,\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c+\sqrt{-d^2}}}+1}{\sqrt{3}}\right)}{2\sqrt{-d^2}\,f}+\frac{3d\sqrt[3]{c-\sqrt{-d^2}}\log\left(\sqrt[3]{c-\sqrt{-d^2}}-\sqrt[3]{c+d\tan(e+fx)}\right)}{4\sqrt{-d^2}\,f}-\frac{3d\sqrt[3]{c+\sqrt{-d^2}}\log\left(\sqrt[3]{c+\sqrt{-d^2}}-\sqrt[3]{c+d\tan(e+fx)}\right)}{4\sqrt{-d^2}\,f}+\frac{d\sqrt[3]{c-\sqrt{-d^2}}\log(\cos(e+fx))}{4\sqrt{-d^2}\,f}-\frac{d\sqrt[3]{c+\sqrt{-d^2}}\log(\cos(e+fx))}{4\sqrt{-d^2}\,f}+\frac{1}{4}x\sqrt[3]{c-\sqrt{-d^2}}+\frac{1}{4}x\sqrt[3]{c+\sqrt{-d^2}}+\frac{3(c+d\tan(e+fx))^{4/3}}{4df}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2*(c + d*Tan[e + f*x])^(1/3),x]
```

```
[Out] ((c - Sqrt[-d^2])^(1/3)*x)/4 + ((c + Sqrt[-d^2])^(1/3)*x)/4 - (Sqrt[3]*d*(c
 - Sqrt[-d^2])^(1/3)*ArcTan[(1 + (2*(c + d*Tan[e + f*x])^(1/3))/(c - Sqrt[-
d^2])^(1/3))/Sqrt[3]])/(2*Sqrt[-d^2]*f) + (Sqrt[3]*d*(c + Sqrt[-d^2])^(1/3)
*ArcTan[(1 + (2*(c + d*Tan[e + f*x])^(1/3))/(c + Sqrt[-d^2])^(1/3))/Sqrt[3]
])/(2*Sqrt[-d^2]*f) + (d*(c - Sqrt[-d^2])^(1/3)*Log[Cos[e + f*x]])/(4*Sqrt[
-d^2]*f) - (d*(c + Sqrt[-d^2])^(1/3)*Log[Cos[e + f*x]])/(4*Sqrt[-d^2]*f) +
(3*d*(c - Sqrt[-d^2])^(1/3)*Log[(c - Sqrt[-d^2])^(1/3) - (c + d*Tan[e + f*x
])^(1/3)])/(4*Sqrt[-d^2]*f) - (3*d*(c + Sqrt[-d^2])^(1/3)*Log[(c + Sqrt[-d^
2])^(1/3) - (c + d*Tan[e + f*x])^(1/3)])/(4*Sqrt[-d^2]*f) + (3*(c + d*Tan[e
 + f*x])^(4/3))/(4*d*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 726

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \tan^2(e+fx)\sqrt[3]{c+d\tan(e+fx)}\,dx = \frac{3(c+d\tan(e+fx))^{4/3}}{4df} - \int \sqrt[3]{c+d\tan(e+fx)}\,dx$$

$$= \frac{3(c+d\tan(e+fx))^{4/3}}{4df} - \frac{d\text{Subst}\left(\int \frac{\sqrt[3]{c+x}}{d^2+x^2}\,dx, x, d\tan(e+f$$

$$= \frac{3(c+d\tan(e+fx))^{4/3}}{4df} - \frac{d\text{Subst}\left(\int \left(\frac{\sqrt{-d^2}\sqrt[3]{c+x}}{2d^2\left(\sqrt{-d^2}-x\right)} + \frac{\sqrt{-d^2}}{2d^2\left(\sqrt{}\right.}\right.\right.}{f}$$

$$= \frac{3(c+d\tan(e+fx))^{4/3}}{4df} + \frac{d\text{Subst}\left(\int \frac{\sqrt[3]{c+x}}{\sqrt{-d^2}-x}\,dx, x, d\tan(e+\right.}{2\sqrt{-d^2}\,f}$$

$$= \frac{3(c+d\tan(e+fx))^{4/3}}{4df} + \frac{\left(d\left(c+\sqrt{-d^2}\right)\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d^2}-\right.}\right.}{2\sqrt{-d^2}}$$

$$= \frac{1}{4}\sqrt[3]{c-\sqrt{-d^2}}\,x + \frac{1}{4}\sqrt[3]{c+\sqrt{-d^2}}\,x - \frac{\sqrt{-d^2}\sqrt[3]{c-\sqrt{-d^2}}\,\text{lo}}{4df}$$

$$= \frac{1}{4}\sqrt[3]{c-\sqrt{-d^2}}\,x + \frac{1}{4}\sqrt[3]{c+\sqrt{-d^2}}\,x - \frac{\sqrt{-d^2}\sqrt[3]{c-\sqrt{-d^2}}\,\text{lo}}{4df}$$

$$= \frac{1}{4}\sqrt[3]{c-\sqrt{-d^2}}\,x + \frac{1}{4}\sqrt[3]{c+\sqrt{-d^2}}\,x + \frac{\sqrt{3}\sqrt{-d^2}\sqrt[3]{c-\sqrt{-d^2}}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.82, size = 313, normalized size = 0.71

$$\frac{i\sqrt[3]{c-id}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c-id}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c-id}-\sqrt[3]{c+d\tan(e+fx)}\right)+\log\left((c-id)^{2/3}+\sqrt[3]{c-id}\,\sqrt[3]{c+d\tan(e+fx)}+(c+d\tan(e+fx))^{2/3}\right)\right)-i\sqrt[3]{c+id}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c+id}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c+id}-\sqrt[3]{c+d\tan(e+fx)}\right)+\log\left((c+id)^{2/3}+\sqrt[3]{c+id}\,\sqrt[3]{c+d\tan(e+fx)}+(c+d\tan(e+fx))^{2/3}\right)\right)+\frac{3(c+d\tan(e+fx))^{4/3}}{d}}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2*(c + d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (I*(c - I*d)^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(c + d*Tan[e + f*x])^(1/3))/(c
 - I*d)^(1/3))/Sqrt[3]] - 2*Log[(c - I*d)^(1/3) - (c + d*Tan[e + f*x])^(1/3
)] + Log[(c - I*d)^(2/3) + (c - I*d)^(1/3)*(c + d*Tan[e + f*x])^(1/3) + (c
+ d*Tan[e + f*x])^(2/3)]) - I*(c + I*d)^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(c
+ d*Tan[e + f*x])^(1/3))/(c + I*d)^(1/3))/Sqrt[3]] - 2*Log[(c + I*d)^(1/3)
- (c + d*Tan[e + f*x])^(1/3)] + Log[(c + I*d)^(2/3) + (c + I*d)^(1/3)*(c +
d*Tan[e + f*x])^(1/3) + (c + d*Tan[e + f*x])^(2/3)]) + (3*(c + d*Tan[e + f*
x])^(4/3))/d)/(4*f)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.23, size = 82, normalized size = 0.19

method	result	size
derivativedivides	$\frac{\frac{3(c+d\tan(fx+e))^{\frac{4}{3}}}{4}-\frac{d^2\left(\sum_{\_R=\text{RootOf}\left(\_Z^6-2c\_Z^3+c^2+d^2\right)}\frac{\_R^3\ln\left((c+d\tan(fx+e))^{\frac{1}{3}}-\_R\right)}{\_R^5-\_R^2c}\right)}{2}}{df}$	82
default	$\frac{\frac{3(c+d\tan(fx+e))^{\frac{4}{3}}}{4}-\frac{d^2\left(\sum_{\_R=\text{RootOf}\left(\_Z^6-2c\_Z^3+c^2+d^2\right)}\frac{\_R^3\ln\left((c+d\tan(fx+e))^{\frac{1}{3}}-\_R\right)}{\_R^5-\_R^2c}\right)}{2}}{df}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2*(c+d*tan(f*x+e))^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/f/d*(1/4*(c+d*tan(f*x+e))^(4/3)-1/6*d^2*sum(_R^3/(_R^5-_R^2*c)*ln((c+d*ta
n(f*x+e))^(1/3)-_R),_R=RootOf(_Z^6-2*_Z^3*c+c^2+d^2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(c+d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(f*x + e) + c)^(1/3)*tan(f*x + e)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3577 vs. 2(352) = 704.
time = 1.29, size = 3577, normalized size = 8.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(c+d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*d*f*((c^2 + d^2)/f^6)^(1/6)*cos(f*x + e)*cos(2/3*arctan((f^6*sqrt(c^
2/f^6)*sqrt((c^2 + d^2)/f^6) - d*f^3*sqrt(c^2/f^6))/c^2))*log(2*c*f^4*((c*c
os(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*sqrt(c^2/f^6)*((c^2 + d^2
)/f^6)^(1/6)*sin(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) - d*f^
3*sqrt(c^2/f^6))/c^2)) + c^2*f^2*((c^2 + d^2)/f^6)^(1/3) + c^2*((c*cos(f*x
+ e) + d*sin(f*x + e))/cos(f*x + e))^(2/3)) + 8*d*f*((c^2 + d^2)/f^6)^(1/6)
*arctan(-(c*f^8*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*sqrt
(c^2/f^6)*((c^2 + d^2)/f^6)^(5/6) - sqrt(2*c*f^4*((c*cos(f*x + e) + d*sin(f
*x + e))/cos(f*x + e))^(1/3)*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(1/6)*sin(2/3*
arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) - d*f^3*sqrt(c^2/f^6))/c^2)
) + c^2*f^2*((c^2 + d^2)/f^6)^(1/3) + c^2*((c*cos(f*x + e) + d*sin(f*x + e)
)/cos(f*x + e))^(2/3))*f^8*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(5/6) + (c^4 + c
^2*d^2)*sin(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) - d*f^3*sqr
t(c^2/f^6))/c^2)))/((c^4 + c^2*d^2)*cos(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt(
(c^2 + d^2)/f^6) - d*f^3*sqrt(c^2/f^6))/c^2))))*cos(f*x + e)*sin(2/3*arctan
((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) - d*f^3*sqrt(c^2/f^6))/c^2)) + 4*
(sqrt(3)*d*f*((c^2 + d^2)/f^6)^(1/6)*cos(f*x + e)*cos(2/3*arctan((f^6*sqrt(
c^2/f^6)*sqrt((c^2 + d^2)/f^6) - d*f^3*sqrt(c^2/f^6))/c^2)) - d*f*((c^2 + d
^2)/f^6)^(1/6)*cos(f*x + e)*sin(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d
^2)/f^6) - d*f^3*sqrt(c^2/f^6))/c^2)))*arctan(-(2*c*f^8*((c*cos(f*x + e) +
d*sin(f*x + e))/cos(f*x + e))^(1/3)*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(5/6)*c
os(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) - d*f^3*sqrt(c^2/f^6
))/c^2)) - 2*(sqrt(3)*c*f^8*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e)
)^(1/3)*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(5/6) + 2*(c^4 + c^2*d^2)*cos(2/3*a
rctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) - d*f^3*sqrt(c^2/f^6))/c^2))
)*sin(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) - d*f^3*sqrt(c^2/
f^6))/c^2)) + 2*(sqrt(3)*f^8*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(5/6)*sin(2/3*
arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) - d*f^3*sqrt(c^2/f^6))/c^2)
) - f^8*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(5/6)*cos(2/3*arctan((f^6*sqrt(c^2/
f^6)*sqrt((c^2 + d^2)/f^6) - d*f^3*sqrt(c^2/f^6))/c^2)))*sqrt(sqrt(3)*c*f^4
*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*sqrt(c^2/f^6)*((c^2
 + d^2)/f^6)^(1/6)*cos(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6)
- d*f^3*sqrt(c^2/f^6))/c^2)) - c*f^4*((c*cos(f*x + e) + d*sin(f*x + e))/cos
(f*x + e))^(1/3)*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(1/6)*sin(2/3*arctan((f^6*
sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) - d*f^3*sqrt(c^2/f^6))/c^2)) + c^2*f^2*
((c^2 + d^2)/f^6)^(1/3) + c^2*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x +
```

```
e))^(2/3)) + sqrt(3)*(c^4 + c^2*d^2))/(3*c^4 + 3*c^2*d^2 - 4*(c^4 + c^2*d^2
)*cos(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) - d*f^3*sqrt(c^2/
f^6))/c^2))^2)) + 4*(sqrt(3)*d*f*((c^2 + d^2)/f^6)^(1/6)*cos(f*x + e)*cos(2
/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) - d*f^3*sqrt(c^2/f^6))/c
^2)) + d*f*((c^2 + d^2)/f^6)^(1/6)*cos(f*x + e)*sin(2/3*arctan((f^6*sqrt(c^
2/f^6)*sqrt((c^2 + d^2)/f^6) - d*f^3*sqrt(c^2/f^6))/c^2)))*arctan((2*c*f^8*
((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*sqrt(c^2/f^6)*((c^2
+ d^2)/f^6)^(5/6)*cos(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) -
 d*f^3*sqrt(c^2/f^6))/c^2)) + 2*(sqrt(3)*c*f^8*((c*cos(f*x + e) + d*sin(f*x
 + e))/cos(f*x + e))^(1/3)*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(5/6) - 2*(c^4 +
 c^2*d^2)*cos(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) - d*f^3*s
qrt(c^2/f^6))/c^2)))*sin(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6
) - d*f^3*sqrt(c^2/f^6))/c^2)) - 2*(sqrt(3)*f^8*sqrt(c^2/f^6)*((c^2 + d^2)/
f^6)^(5/6)*sin(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) - d*f^3*
sqrt(c^2/f^6))/c^2)) + f^8*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(5/6)*cos(2/3*ar
ctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) - d*f^3*sqrt(c^2/f^6))/c^2)))
*sqrt(-sqrt(3)*c*f^4*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)
*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(1/6)*cos(2/3*arctan((f^6*sqrt(c^2/f^6)*sq
rt((c^2 + d^2)/f^6) - d*f^3*sqrt(c^2/f^6))/c^2)) - c*f^4*((c*cos(f*x + e) +
 d*sin(f*x + e))/cos(f*x + e))^(1/3)*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(1/6)*
sin(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) - d*f^3*sqrt(c^2/f^
6))/c^2)) + c^2*f^2*((c^2 + d^2)/f^6)^(1/3) + c^2*((c*cos(f*x + e) + d*sin(
f*x + e))/cos(f*x + e))^(2/3)) - sqrt(3)*(c^4 + c^2*d^2))/(3*c^4 + 3*c^2*d^
2 - 4*(c^4 + c^2*d^2)*cos(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^
6) - d*f^3*sqrt(c^2/f^6))/c^2))^2)) - (sqrt(3)*d*f*((c^2 + d^2)/f^6)^(1/6)*
cos(f*x + e)*sin(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) - d*f^
3*sqrt(c^2/f^6))/c^2)) + d*f*((c^2 + d^2)/f^6)^(1/6)*cos(f*x + e)*cos(2/3*a
rctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) - d*f^3*sqrt(c^2/f^6))/c^2))
)*log(sqrt(3)*c*f^4*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*
sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(1/6)*cos(2/3*arctan((f^6*sqrt(c^2/f^6)*sqr
t((c^2 + d^2)/f^6) - d*f^3*sqrt(c^2/f^6))/c^2))...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{c+d\tan(e+fx)}\,\tan^2(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2*(c+d*tan(f*x+e))**(1/3),x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(1/3)*tan(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(c+d*tan(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 8.73, size = 881, normalized size = 2.01

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(c + d*tan(e + f*x))^(1/3),x)
```

```
[Out] log(f*(-(c*1i - d)/f^3)^(1/3)*1i + (c + d*tan(e + f*x))^(1/3))*(-(c*1i - d)
/(8*f^3))^(1/3) + log(c*(c + d*tan(e + f*x))^(1/3) + d*(c + d*tan(e + f*x))
^(1/3)*1i - f^4*((c*1i + d)/f^3)^(4/3) + 2*d*f*((c*1i + d)/f^3)^(1/3))*((c*
1i + d)/(8*f^3))^(1/3) + log(- (486*(d^8 - c^4*d^4)*(c + d*tan(e + f*x))^(1
/3))/f^4 - ((-(c*1i - d)/f^3)^(1/3)*((3^(1/2)*1i)/2 - 1/2)*(((-(c*1i - d)/f
^3)^(2/3)*((3^(1/2)*1i)/2 + 1/2)*((3888*d^5*(c^2 + d^2)*(c + d*tan(e + f*x)
)^(1/3))/f - 3888*c*d^4*(-(c*1i - d)/f^3)^(1/3)*((3^(1/2)*1i)/2 - 1/2)*(c^2
 + d^2)))/4 - (1944*c*d^5*(c^2 + d^2))/f^3))/2)*((3^(1/2)*1i)/2 - 1/2)*(-(c
*1i - d)/(8*f^3))^(1/3) - log((486*(d^8 - c^4*d^4)*(c + d*tan(e + f*x))^(1/
3))/f^4 + ((-(c*1i - d)/f^3)^(1/3)*((3^(1/2)*1i)/2 + 1/2)*(((-(c*1i - d)/f^
3)^(2/3)*((3^(1/2)*1i)/2 - 1/2)*((3888*d^5*(c^2 + d^2)*(c + d*tan(e + f*x))
^(1/3))/f + 3888*c*d^4*(-(c*1i - d)/f^3)^(1/3)*((3^(1/2)*1i)/2 + 1/2)*(c^2
+ d^2)))/4 + (1944*c*d^5*(c^2 + d^2))/f^3))/2)*((3^(1/2)*1i)/2 + 1/2)*(-(c*
1i - d)/(8*f^3))^(1/3) + (3*(c + d*tan(e + f*x))^(4/3))/(4*d*f) + log(- (48
6*(d^8 - c^4*d^4)*(c + d*tan(e + f*x))^(1/3))/f^4 - (((3^(1/2)*1i)/2 - 1/2)
*((((3888*d^5*(c^2 + d^2)*(c + d*tan(e + f*x))^(1/3))/f - 3888*c*d^4*((3^(1
/2)*1i)/2 - 1/2)*((c*1i + d)/f^3)^(1/3)*(c^2 + d^2))*((3^(1/2)*1i)/2 + 1/2)
*((c*1i + d)/f^3)^(2/3))/4 - (1944*c*d^5*(c^2 + d^2))/f^3)*((c*1i + d)/f^3)
^(1/3))/2)*((3^(1/2)*1i)/2 - 1/2)*((c*1i + d)/(8*f^3))^(1/3) - log((486*(d^
8 - c^4*d^4)*(c + d*tan(e + f*x))^(1/3))/f^4 + (((3^(1/2)*1i)/2 + 1/2)*((((
3888*d^5*(c^2 + d^2)*(c + d*tan(e + f*x))^(1/3))/f + 3888*c*d^4*((3^(1/2)*1
i)/2 + 1/2)*((c*1i + d)/f^3)^(1/3)*(c^2 + d^2))*((3^(1/2)*1i)/2 - 1/2)*((c*
1i + d)/f^3)^(2/3))/4 + (1944*c*d^5*(c^2 + d^2))/f^3)*((c*1i + d)/f^3)^(1/3
))/2)*((3^(1/2)*1i)/2 + 1/2)*((c*1i + d)/(8*f^3))^(1/3)
```

3.684 $\int \tan(e+fx)\sqrt[3]{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=318

$$\frac{1}{4}i\sqrt[3]{c-id}\,x-\frac{1}{4}i\sqrt[3]{c+id}\,x-\frac{\sqrt{3}\,\sqrt[3]{c-id}\,\mathrm{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c-id}}}{\sqrt{3}}\right)}{2f}-\frac{\sqrt{3}\,\sqrt[3]{c+id}\,\mathrm{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{}}{}}{}\right)}{2f}$$

```
[Out] 1/4*I*(c-I*d)^(1/3)*x-1/4*I*(c+I*d)^(1/3)*x+1/4*(c-I*d)^(1/3)*ln(cos(f*x+e)
)/f+1/4*(c+I*d)^(1/3)*ln(cos(f*x+e))/f+3/4*(c-I*d)^(1/3)*ln((c-I*d)^(1/3)-(
c+d*tan(f*x+e))^(1/3))/f+3/4*(c+I*d)^(1/3)*ln((c+I*d)^(1/3)-(c+d*tan(f*x+e)
)^(1/3))/f-1/2*(c-I*d)^(1/3)*arctan(1/3*(1+2*(c+d*tan(f*x+e))^(1/3)/(c-I*d)
^(1/3))*3^(1/2))*3^(1/2)/f-1/2*(c+I*d)^(1/3)*arctan(1/3*(1+2*(c+d*tan(f*x+e
))^(1/3)/(c+I*d)^(1/3))*3^(1/2))*3^(1/2)/f+3*(c+d*tan(f*x+e))^(1/3)/f
```

Rubi [A]

time = 0.22, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3609, 3620, 3618, 59, 631, 210, 31}

$$-\frac{\sqrt{3}\,\sqrt[3]{c-id}\,\mathrm{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c-id}}}{\sqrt{3}}\right)}{2f}-\frac{\sqrt{3}\,\sqrt[3]{c+id}\,\mathrm{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c+id}}}{\sqrt{3}}\right)}{2f}+\frac{3\sqrt[3]{c+d\tan(e+fx)}}{f}+\frac{3\sqrt[3]{c-id}\,\log\left(-\sqrt[3]{c+d\tan(e+fx)}+\sqrt[3]{c-id}\right)}{4f}+\frac{3\sqrt[3]{c+id}\,\log\left(-\sqrt[3]{c+d\tan(e+fx)}+\sqrt[3]{c+id}\right)}{4f}+\frac{\sqrt[3]{c-id}\,\log(\cos(e+fx))}{4f}+\frac{\sqrt[3]{c+id}\,\log(\cos(e+fx))}{4f}+\frac{1}{4}ix\sqrt[3]{c-id}-\frac{1}{4}ix\sqrt[3]{c+id}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]*(c + d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (I/4)*(c - I*d)^(1/3)*x - (I/4)*(c + I*d)^(1/3)*x - (Sqrt[3]*(c - I*d)^(1/3
)*ArcTan[(1 + (2*(c + d*Tan[e + f*x])^(1/3))/(c - I*d)^(1/3))/Sqrt[3]])/(2*
f) - (Sqrt[3]*(c + I*d)^(1/3)*ArcTan[(1 + (2*(c + d*Tan[e + f*x])^(1/3))/(c
 + I*d)^(1/3))/Sqrt[3]])/(2*f) + ((c - I*d)^(1/3)*Log[Cos[e + f*x]])/(4*f)
+ ((c + I*d)^(1/3)*Log[Cos[e + f*x]])/(4*f) + (3*(c - I*d)^(1/3)*Log[(c - I
*d)^(1/3) - (c + d*Tan[e + f*x])^(1/3)])/(4*f) + (3*(c + I*d)^(1/3)*Log[(c
+ I*d)^(1/3) - (c + d*Tan[e + f*x])^(1/3)])/(4*f) + (3*(c + d*Tan[e + f*x])
^(1/3))/f
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
```

```
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \tan(e+fx)\sqrt[3]{c+d\tan(e+fx)}\,dx = \frac{3\sqrt[3]{c+d\tan(e+fx)}}{f} + \int \frac{-d+c\tan(e+fx)}{(c+d\tan(e+fx))^{2/3}}\,dx$$

$$= \frac{3\sqrt[3]{c+d\tan(e+fx)}}{f} + \frac{1}{2}(-ic-d)\int \frac{1+i\tan(e+fx)}{(c+d\tan(e+fx))^{2/3}}\,dx$$

$$= \frac{3\sqrt[3]{c+d\tan(e+fx)}}{f} + \frac{(c-id)\text{Subst}\left(\int \frac{1}{(-1+x)(c-idx)^{2/3}}\,dx, x, i\,\text{ta}\right.}{2f}$$

$$= \frac{1}{4}i\sqrt[3]{c-id}\,x - \frac{1}{4}i\sqrt[3]{c+id}\,x + \frac{\sqrt[3]{c-id}\,\log(\cos(e+fx))}{4f} + \frac{\sqrt[3]{c+}}{}$$

$$= \frac{1}{4}i\sqrt[3]{c-id}\,x - \frac{1}{4}i\sqrt[3]{c+id}\,x + \frac{\sqrt[3]{c-id}\,\log(\cos(e+fx))}{4f} + \frac{\sqrt[3]{c+}}{}$$

$$= \frac{1}{4}i\sqrt[3]{c-id}\,x - \frac{1}{4}i\sqrt[3]{c+id}\,x - \frac{\sqrt{3}\,\sqrt[3]{c-id}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{c+d\,\text{ta}}}{\sqrt[3]{c-}}}{\sqrt{3}}\right.}{2f}$$

Mathematica [A]
time = 0.24, size = 346, normalized size = 1.09

$$-\frac{2\sqrt{3}\sqrt[3]{c-id}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c-id}}}{\sqrt{3}}\right)+2\sqrt{3}\sqrt[3]{c+id}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c+id}}}{\sqrt{3}}\right)-2\sqrt[3]{c-id}\,\log\left(\sqrt[3]{c-id}-\sqrt[3]{c+d\tan(e+fx)}\right)-2\sqrt[3]{c+id}\,\log\left(\sqrt[3]{c+id}-\sqrt[3]{c+d\tan(e+fx)}\right)+\sqrt[3]{c-id}\,\log\left((c-id)^{2/3}+\sqrt[3]{c-id}\sqrt[3]{c+d\tan(e+fx)}+(c+d\tan(e+fx))^{2/3}\right)+\sqrt[3]{c+id}\,\log\left((c+id)^{2/3}+\sqrt[3]{c+id}\sqrt[3]{c+d\tan(e+fx)}+(c+d\tan(e+fx))^{2/3}\right)-12\sqrt[3]{c+d\tan(e+fx)}}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]*(c + d*Tan[e + f*x])^(1/3),x]
```

```
[Out] -1/4*(2*Sqrt[3]*(c - I*d)^(1/3)*ArcTan[(1 + (2*(c + d*Tan[e + f*x])^(1/3))/
(c - I*d)^(1/3))/Sqrt[3]] + 2*Sqrt[3]*(c + I*d)^(1/3)*ArcTan[(1 + (2*(c + d
*Tan[e + f*x])^(1/3))/(c + I*d)^(1/3))/Sqrt[3]] - 2*(c - I*d)^(1/3)*Log[(c
- I*d)^(1/3) - (c + d*Tan[e + f*x])^(1/3)] - 2*(c + I*d)^(1/3)*Log[(c + I*d
)^(1/3) - (c + d*Tan[e + f*x])^(1/3)] + (c - I*d)^(1/3)*Log[(c - I*d)^(2/3)
 + (c - I*d)^(1/3)*(c + d*Tan[e + f*x])^(1/3) + (c + d*Tan[e + f*x])^(2/3)]
 + (c + I*d)^(1/3)*Log[(c + I*d)^(2/3) + (c + I*d)^(1/3)*(c + d*Tan[e + f*x
])^(1/3) + (c + d*Tan[e + f*x])^(2/3)] - 12*(c + d*Tan[e + f*x])^(1/3))/f
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.24, size = 88, normalized size = 0.28

method	result	size

derivativedivides	$\frac{3(c+d\tan(fx+e))^{\frac{1}{3}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-2c\_Z^3+c^2+d^2\right)}\frac{\left(\_R^3c-c^2-d^2\right)\ln\left((c+d\tan(fx+e))^{\frac{1}{3}}-\_R\right)}{\_R^5-\_R^2c}\right)}{2}}{f}$	88
default	$\frac{3(c+d\tan(fx+e))^{\frac{1}{3}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-2c\_Z^3+c^2+d^2\right)}\frac{\left(\_R^3c-c^2-d^2\right)\ln\left((c+d\tan(fx+e))^{\frac{1}{3}}-\_R\right)}{\_R^5-\_R^2c}\right)}{2}}{f}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)*(c+d*tan(f*x+e))^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(3*(c+d*tan(f*x+e))^(1/3)+1/2*sum((_R^3*c-c^2-d^2)/(_R^5-_R^2*c)*ln((c+
d*tan(f*x+e))^(1/3)-_R),_R=RootOf(_Z^6-2*_Z^3*c+c^2+d^2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(c+d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(f*x + e) + c)^(1/3)*tan(f*x + e), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2859 vs. $2(241) = 482$.
time = 1.20, size = 2859, normalized size = 8.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(c+d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*f*((c^2 + d^2)/f^6)^(1/6)*cos(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6)
- c*f^3)*sqrt(d^2/f^6)/d^2))*log(-2*f*((c*cos(f*x + e) + d*sin(f*x + e))/co
s(f*x + e))^(1/3)*((c^2 + d^2)/f^6)^(1/6)*cos(2/3*arctan((f^6*sqrt((c^2 + d
^2)/f^6) - c*f^3)*sqrt(d^2/f^6)/d^2)) + f^2*((c^2 + d^2)/f^6)^(1/3) + ((c*c
os(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(2/3)) + 8*f*((c^2 + d^2)/f^6)^
(1/6)*arctan((sqrt(-2*f*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1
/3)*((c^2 + d^2)/f^6)^(1/6)*cos(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) - c*f
^3)*sqrt(d^2/f^6)/d^2)) + f^2*((c^2 + d^2)/f^6)^(1/3) + ((c*cos(f*x + e) +
d*sin(f*x + e))/cos(f*x + e))^(2/3))*f^5*((c^2 + d^2)/f^6)^(5/6) - f^5*((c*
cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*((c^2 + d^2)/f^6)^(5/6)
```

+ (c^2 + d^2)*cos(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) - c*f^3)*sqrt(d^2/f
^6)/d^2)))/((c^2 + d^2)*sin(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) - c*f^3)*
sqrt(d^2/f^6)/d^2))))*sin(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) - c*f^3)*sq
rt(d^2/f^6)/d^2)) + 4*(sqrt(3)*f*((c^2 + d^2)/f^6)^(1/6)*cos(2/3*arctan((f^
6*sqrt((c^2 + d^2)/f^6) - c*f^3)*sqrt(d^2/f^6)/d^2)) - f*((c^2 + d^2)/f^6)^
(1/6)*sin(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) - c*f^3)*sqrt(d^2/f^6)/d^2)
))*arctan(-(2*sqrt(3)*f^5*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^
(1/3)*((c^2 + d^2)/f^6)^(5/6)*cos(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) - c
*f^3)*sqrt(d^2/f^6)/d^2)) + 2*(f^5*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f
x + e))^(1/3)((c^2 + d^2)/f^6)^(5/6) + 2*(c^2 + d^2)*cos(2/3*arctan((f^6*
sqrt((c^2 + d^2)/f^6) - c*f^3)*sqrt(d^2/f^6)/d^2)))*sin(2/3*arctan((f^6*sqr
t((c^2 + d^2)/f^6) - c*f^3)*sqrt(d^2/f^6)/d^2)) - 2*(sqrt(3)*f^5*((c^2 + d^
2)/f^6)^(5/6)*cos(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) - c*f^3)*sqrt(d^2/f
^6)/d^2)) + f^5*((c^2 + d^2)/f^6)^(5/6)*sin(2/3*arctan((f^6*sqrt((c^2 + d^2
)/f^6) - c*f^3)*sqrt(d^2/f^6)/d^2)))*sqrt(sqrt(3)*f*((c*cos(f*x + e) + d*si
n(f*x + e))/cos(f*x + e))^(1/3)*((c^2 + d^2)/f^6)^(1/6)*sin(2/3*arctan((f^6
*sqrt((c^2 + d^2)/f^6) - c*f^3)*sqrt(d^2/f^6)/d^2)) + f*((c*cos(f*x + e) +
d*sin(f*x + e))/cos(f*x + e))^(1/3)*((c^2 + d^2)/f^6)^(1/6)*cos(2/3*arctan(
(f^6*sqrt((c^2 + d^2)/f^6) - c*f^3)*sqrt(d^2/f^6)/d^2)) + f^2*((c^2 + d^2)/
f^6)^(1/3) + ((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(2/3)) + sqrt
(3)*(c^2 + d^2))/(4*(c^2 + d^2)*cos(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) -
 c*f^3)*sqrt(d^2/f^6)/d^2))^2 - c^2 - d^2)) - 4*(sqrt(3)*f*((c^2 + d^2)/f^6
)^(1/6)*cos(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) - c*f^3)*sqrt(d^2/f^6)/d^
2)) + f*((c^2 + d^2)/f^6)^(1/6)*sin(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) -
 c*f^3)*sqrt(d^2/f^6)/d^2)))*arctan((2*sqrt(3)*f^5*((c*cos(f*x + e) + d*sin
(f*x + e))/cos(f*x + e))^(1/3)*((c^2 + d^2)/f^6)^(5/6)*cos(2/3*arctan((f^6*
sqrt((c^2 + d^2)/f^6) - c*f^3)*sqrt(d^2/f^6)/d^2)) - 2*(f^5*((c*cos(f*x + e
) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*((c^2 + d^2)/f^6)^(5/6) + 2*(c^2 +
d^2)*cos(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) - c*f^3)*sqrt(d^2/f^6)/d^2))
)*sin(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) - c*f^3)*sqrt(d^2/f^6)/d^2)) -
2*(sqrt(3)*f^5*((c^2 + d^2)/f^6)^(5/6)*cos(2/3*arctan((f^6*sqrt((c^2 + d^2)
/f^6) - c*f^3)*sqrt(d^2/f^6)/d^2)) - f^5*((c^2 + d^2)/f^6)^(5/6)*sin(2/3*ar
ctan((f^6*sqrt((c^2 + d^2)/f^6) - c*f^3)*sqrt(d^2/f^6)/d^2)))*sqrt(-sqrt(3)
f((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*((c^2 + d^2)/f^6)
^(1/6)*sin(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) - c*f^3)*sqrt(d^2/f^6)/d^2
)) + f*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*((c^2 + d^2)/
f^6)^(1/6)*cos(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) - c*f^3)*sqrt(d^2/f^6)
/d^2)) + f^2*((c^2 + d^2)/f^6)^(1/3) + ((c*cos(f*x + e) + d*sin(f*x + e))/c
os(f*x + e))^(2/3)) + sqrt(3)*(c^2 + d^2))/(4*(c^2 + d^2)*cos(2/3*arctan((f
^6*sqrt((c^2 + d^2)/f^6) - c*f^3)*sqrt(d^2/f^6)/d^2))^2 - c^2 - d^2)) - (sq
rt(3)*f*((c^2 + d^2)/f^6)^(1/6)*sin(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) -
 c*f^3)*sqrt(d^2/f^6)/d^2)) + f*((c^2 + d^2)/f^6)^(1/6)*cos(2/3*arctan((f^6
*sqrt((c^2 + d^2)/f^6) - c*f^3)*sqrt(d^2/f^6)/d^2)))*log(sqrt(3)*f*((c*cos(
f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*((c^2 + d^2)/f^6)^(1/6)*sin(
2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) - c*f^3)*sqrt(d^2/f^6)/d^2)) + f*((c*

```
cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*((c^2 + d^2)/f^6)^(1/6)*
cos(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) - c*f^3)*sqrt(d^2/f^6)/d^2)) + f^
2*((c^2 + d^2)/f^6)^(1/3) + ((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e)
)^(2/3)) + (sqrt(3)*f*((c^2 + d^2)/f^6)^(1/6)*sin(2/3*arctan((f^6*sqrt((c^2
 + d^2)/f^6) - c*f^3)*sqrt(d^2/f^6)/d^2)) - f*((c^2 + d^2)/f^6)^(1/6)*cos(2
/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) - c*f^3)*sqrt(d^2/f^6)/d^2)))*log(-sqr
t(3)*f*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*((c^2 + d^2)/
f^6)^(1/6)*sin(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) - c*f^3)*sqrt(d^2/f^6)
/d^2)) + f*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*((c^2 + d
^2)/f^6)^(1/6)*cos(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) - c*f^3)*sqrt(d^2/
f^6)/d^2)) + f^2*((c^2 + d^2)/f^6)^(1/3) + ((c*cos(f*x + e) + d*sin(f*x + e
))/cos(f*x + e))^(2/3)) + 12*((c*cos(f*x + e) +...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{c+d\tan(e+fx)}\,\tan(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(c+d*tan(f*x+e))**(1/3),x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(1/3)*tan(e + f*x), x)
```

Giac [A]
time = 1.43, size = 18, normalized size = 0.06

$$\frac{3\,(d\tan(fx+e)+c)^{\frac{1}{3}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(c+d*tan(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] 3*(d*tan(f*x + e) + c)^(1/3)/f
```

Mupad [B]
time = 8.08, size = 830, normalized size = 2.61

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(c + d*tan(e + f*x))^(1/3),x)
```

```
[Out] log((c + d*tan(e + f*x))^(1/3) - f*((c - d*1i)/f^3)^(1/3))*((c - d*1i)/(8*f
^3))^(1/3) + log((c + d*tan(e + f*x))^(1/3) - f*((c + d*1i)/f^3)^(1/3))*((c
 + d*1i)/(8*f^3))^(1/3) + (3*(c + d*tan(e + f*x))^(1/3))/f + log((486*(d^8
```

```
- c^4*d^4)*(c + d*tan(e + f*x))^(1/3))/f^4 - (((3^(1/2)*1i)/2 - 1/2)*((972*
(d^8 - c^4*d^4))/f^3 + (((3^(1/2)*1i)/2 + 1/2)*((3888*c*d^4*(c^2 + d^2)*(c
+ d*tan(e + f*x))^(1/3))/f - 3888*c*d^4*((3^(1/2)*1i)/2 - 1/2)*((c - d*1i)/
f^3)^(1/3)*(c^2 + d^2))*((c - d*1i)/f^3)^(2/3))/4)*((c - d*1i)/f^3)^(1/3))/
2)*((3^(1/2)*1i)/2 - 1/2)*((c - d*1i)/(8*f^3))^(1/3) + log((486*(d^8 - c^4*
d^4)*(c + d*tan(e + f*x))^(1/3))/f^4 - (((3^(1/2)*1i)/2 - 1/2)*((972*(d^8 -
 c^4*d^4))/f^3 + (((3^(1/2)*1i)/2 + 1/2)*((3888*c*d^4*(c^2 + d^2)*(c + d*ta
n(e + f*x))^(1/3))/f - 3888*c*d^4*((3^(1/2)*1i)/2 - 1/2)*((c + d*1i)/f^3)^(
1/3)*(c^2 + d^2))*((c + d*1i)/f^3)^(2/3))/4)*((c + d*1i)/f^3)^(1/3))/2)*((3
^(1/2)*1i)/2 - 1/2)*((c + d*1i)/(8*f^3))^(1/3) - log((((3^(1/2)*1i)/2 + 1/2
)*((972*(d^8 - c^4*d^4))/f^3 - (((3^(1/2)*1i)/2 - 1/2)*((3888*c*d^4*(c^2 +
d^2)*(c + d*tan(e + f*x))^(1/3))/f + 3888*c*d^4*((3^(1/2)*1i)/2 + 1/2)*((c
- d*1i)/f^3)^(1/3)*(c^2 + d^2))*((c - d*1i)/f^3)^(2/3))/4)*((c - d*1i)/f^3)
^(1/3))/2 + (486*(d^8 - c^4*d^4)*(c + d*tan(e + f*x))^(1/3))/f^4)*((3^(1/2)
*1i)/2 + 1/2)*((c - d*1i)/(8*f^3))^(1/3) - log((((3^(1/2)*1i)/2 + 1/2)*((97
2*(d^8 - c^4*d^4))/f^3 - (((3^(1/2)*1i)/2 - 1/2)*((3888*c*d^4*(c^2 + d^2)*(
c + d*tan(e + f*x))^(1/3))/f + 3888*c*d^4*((3^(1/2)*1i)/2 + 1/2)*((c + d*1i
)/f^3)^(1/3)*(c^2 + d^2))*((c + d*1i)/f^3)^(2/3))/4)*((c + d*1i)/f^3)^(1/3)
)/2 + (486*(d^8 - c^4*d^4)*(c + d*tan(e + f*x))^(1/3))/f^4)*((3^(1/2)*1i)/2
 + 1/2)*((c + d*1i)/(8*f^3))^(1/3)
```

3.685 $\int \sqrt[3]{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=415

$$-\frac{1}{4}\sqrt[3]{c-\sqrt{-d^2}}\,x-\frac{1}{4}\sqrt[3]{c+\sqrt{-d^2}}\,x+\frac{\sqrt{3}\,d\sqrt[3]{c-\sqrt{-d^2}}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c-\sqrt{-d^2}}}}{\sqrt{3}}\right)}{2\sqrt{-d^2}\,f}-\frac{\sqrt{3}\,d\sqrt[3]{\cdots}}{\cdots}$$

```
[Out] -1/4*x*(c-(-d^2)^(1/2))^(1/3)-1/4*d*ln(cos(f*x+e))*(c-(-d^2)^(1/2))^(1/3)/f
/(-d^2)^(1/2)-3/4*d*ln((c-(-d^2)^(1/2))^(1/3)-(c+d*tan(f*x+e))^(1/3))*(c-(-
d^2)^(1/2))^(1/3)/f/(-d^2)^(1/2)+1/2*d*arctan(1/3*(1+2*(c+d*tan(f*x+e))^(1/
3)/(c-(-d^2)^(1/2))^(1/3))*3^(1/2))*3^(1/2)*(c-(-d^2)^(1/2))^(1/3)/f/(-d^2)
^(1/2)-1/4*x*(c+(-d^2)^(1/2))^(1/3)+1/4*d*ln(cos(f*x+e))*(c+(-d^2)^(1/2))^(
1/3)/f/(-d^2)^(1/2)+3/4*d*ln((c+(-d^2)^(1/2))^(1/3)-(c+d*tan(f*x+e))^(1/3))
*(c+(-d^2)^(1/2))^(1/3)/f/(-d^2)^(1/2)-1/2*d*arctan(1/3*(1+2*(c+d*tan(f*x+e
))^(1/3)/(c+(-d^2)^(1/2))^(1/3))*3^(1/2))*3^(1/2)*(c+(-d^2)^(1/2))^(1/3)/f/
(-d^2)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 415, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3566, 726, 52, 59, 631, 210, 31}

$$\frac{\sqrt{3}\,d\sqrt[3]{c-\sqrt{-d^2}}\,\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c-\sqrt{-d^2}}}+1}{\sqrt{3}}\right)}{2\sqrt{-d^2}\,f}-\frac{\sqrt{3}\,d\sqrt[3]{c+\sqrt{-d^2}}\,\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c+\sqrt{-d^2}}}+1}{\sqrt{3}}\right)}{2\sqrt{-d^2}\,f}-\frac{3d\sqrt[3]{c-\sqrt{-d^2}}\log\left(\sqrt[3]{c-\sqrt{-d^2}}-\sqrt[3]{c+d\tan(e+fx)}\right)}{4\sqrt{-d^2}\,f}+\frac{3d\sqrt[3]{c+\sqrt{-d^2}}\log\left(\sqrt[3]{c+\sqrt{-d^2}}-\sqrt[3]{c+d\tan(e+fx)}\right)}{4\sqrt{-d^2}\,f}-\frac{d\sqrt[3]{c-\sqrt{-d^2}}\log(\cos(e+fx))}{4\sqrt{-d^2}\,f}+\frac{d\sqrt[3]{c+\sqrt{-d^2}}\log(\cos(e+fx))}{4\sqrt{-d^2}\,f}-\frac{1}{4}x\sqrt[3]{c-\sqrt{-d^2}}-\frac{1}{4}x\sqrt[3]{c+\sqrt{-d^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^(1/3),x]
```

```
[Out] -1/4*((c - Sqrt[-d^2])^(1/3)*x) - ((c + Sqrt[-d^2])^(1/3)*x)/4 + (Sqrt[3]*d
*(c - Sqrt[-d^2])^(1/3)*ArcTan[(1 + (2*(c + d*Tan[e + f*x])^(1/3))/(c - Sqr
t[-d^2])^(1/3))/Sqrt[3]])/(2*Sqrt[-d^2]*f) - (Sqrt[3]*d*(c + Sqrt[-d^2])^(1
/3)*ArcTan[(1 + (2*(c + d*Tan[e + f*x])^(1/3))/(c + Sqrt[-d^2])^(1/3))/Sqrt
[3]])/(2*Sqrt[-d^2]*f) - (d*(c - Sqrt[-d^2])^(1/3)*Log[Cos[e + f*x]])/(4*Sq
rt[-d^2]*f) + (d*(c + Sqrt[-d^2])^(1/3)*Log[Cos[e + f*x]])/(4*Sqrt[-d^2]*f)
 - (3*d*(c - Sqrt[-d^2])^(1/3)*Log[(c - Sqrt[-d^2])^(1/3) - (c + d*Tan[e +
f*x])^(1/3)])/(4*Sqrt[-d^2]*f) + (3*d*(c + Sqrt[-d^2])^(1/3)*Log[(c + Sqrt[
-d^2])^(1/3) - (c + d*Tan[e + f*x])^(1/3)])/(4*Sqrt[-d^2]*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 726

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \sqrt[3]{c+d\tan(e+fx)}\,dx=\frac{d\text{Subst}\left(\int \frac{\sqrt[3]{c+x}}{d^2+x^2}\,dx,x,d\tan(e+fx)\right)}{f}$$

$$=\frac{d\text{Subst}\left(\int \left(\frac{\sqrt{-d^2}\sqrt[3]{c+x}}{2d^2\left(\sqrt{-d^2}-x\right)}+\frac{\sqrt{-d^2}\sqrt[3]{c+x}}{2d^2\left(\sqrt{-d^2}+x\right)}\right)dx,x,d\tan(e+fx)\right)}{f}$$

$$=-\frac{d\text{Subst}\left(\int \frac{\sqrt[3]{c+x}}{\sqrt{-d^2}-x}\,dx,x,d\tan(e+fx)\right)}{2\sqrt{-d^2}\,f}-\frac{d\text{Subst}\left(\int \frac{\sqrt[3]{c+x}}{\sqrt{-d^2}+x}\,dx,x,d\,\text{t}\right.}{2\sqrt{-d^2}\,f}$$

$$=-\frac{\left(d\left(c+\sqrt{-d^2}\right)\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d^2}-x\right)(c+x)^{2/3}}\,dx,x,d\tan(e+fx)\right)}{2\sqrt{-d^2}\,f}+\frac{\left(d^2\right.}{}$$

$$=-\frac{1}{4}\sqrt[3]{c-\sqrt{-d^2}}\,x-\frac{1}{4}\sqrt[3]{c+\sqrt{-d^2}}\,x+\frac{\sqrt{-d^2}\sqrt[3]{c-\sqrt{-d^2}}\log(\cos(e+f}{4df}$$

$$=-\frac{1}{4}\sqrt[3]{c-\sqrt{-d^2}}\,x-\frac{1}{4}\sqrt[3]{c+\sqrt{-d^2}}\,x+\frac{\sqrt{-d^2}\sqrt[3]{c-\sqrt{-d^2}}\log(\cos(e+f}{4df}$$

$$=-\frac{1}{4}\sqrt[3]{c-\sqrt{-d^2}}\,x-\frac{1}{4}\sqrt[3]{c+\sqrt{-d^2}}\,x-\frac{\sqrt{3}\sqrt{-d^2}\sqrt[3]{c-\sqrt{-d^2}}\tan^{-1}\left(\right.}{2df}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.22, size = 294, normalized size = 0.71

$$\frac{-i\sqrt[3]{c-id}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c-id}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c-id}-\sqrt[3]{c+d\tan(e+fx)}\right)+\log\left((c-id)^{2/3}+\sqrt[3]{c-id}\sqrt[3]{c+d\tan(e+fx)}+(c+d\tan(e+fx))^{2/3}\right)\right)+i\sqrt[3]{c+id}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c+id}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c+id}-\sqrt[3]{c+d\tan(e+fx)}\right)+\log\left((c+id)^{2/3}+\sqrt[3]{c+id}\sqrt[3]{c+d\tan(e+fx)}+(c+d\tan(e+fx))^{2/3}\right)\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^(1/3),x]
```

```
[Out] ((-I)*(c - I*d)^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(c + d*Tan[e + f*x])^(1/3))
/(c - I*d)^(1/3))/Sqrt[3]] - 2*Log[(c - I*d)^(1/3) - (c + d*Tan[e + f*x])^(
1/3)] + Log[(c - I*d)^(2/3) + (c - I*d)^(1/3)*(c + d*Tan[e + f*x])^(1/3) +
(c + d*Tan[e + f*x])^(2/3)]) + I*(c + I*d)^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*
(c + d*Tan[e + f*x])^(1/3))/(c + I*d)^(1/3))/Sqrt[3]] - 2*Log[(c + I*d)^(1/
3) - (c + d*Tan[e + f*x])^(1/3)] + Log[(c + I*d)^(2/3) + (c + I*d)^(1/3)*(c
 + d*Tan[e + f*x])^(1/3) + (c + d*Tan[e + f*x])^(2/3)]))/(4*f)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.56, size = 60, normalized size = 0.14

method	result	size
derivativedivides	$\frac{d\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-2c\_Z^3+c^2+d^2\right)}\frac{\_R^3\ln\left(\left(c+d\tan(fx+e)\right)^{\frac{1}{3}}-\_R\right)}{\_R^5-\_R^2c}\right)}{2f}$	60
default	$\frac{d\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-2c\_Z^3+c^2+d^2\right)}\frac{\_R^3\ln\left(\left(c+d\tan(fx+e)\right)^{\frac{1}{3}}-\_R\right)}{\_R^5-\_R^2c}\right)}{2f}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*d*sum(_R^3/(_R^5-_R^2*c)*ln((c+d*tan(f*x+e))^(1/3)-_R),_R=RootOf(_Z^6
-2*_Z^3*c+c^2+d^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(f*x + e) + c)^(1/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3383 vs. 2(331) = 662.
time = 1.48, size = 3383, normalized size = 8.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

[Out] 1/2*((c^2 + d^2)/f^6)^(1/6)*cos(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2))*log(2*c*f^4*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(1/6)*sin(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2)) + c^2*f^2*((c^2 + d^2)/f^6)^(1/3) + c^2*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(2/3)) + 2*((c^2 + d^2)/f^6)^(1/6)*arctan(-(c*f^8*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(5/6) - sqrt(2*c*f^4*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(1/6)*sin(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2)) + c^2*f^2*((c^2 + d^2)/f^6)^(1/3) + c^2*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(2/3))*f^8*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(5/6) + (c^4 + c^2*d^2)*sin(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2)))/((c^4 + c^2*d^2)*cos(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2))))*sin(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2)) + (sqrt(3)*((c^2 + d^2)/f^6)^(1/6)*cos(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2)) - ((c^2 + d^2)/f^6)^(1/6)*sin(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2)))*arctan(-(2*c*f^8*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(5/6)*cos(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2)) - 2*(sqrt(3)*c*f^8*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(5/6) + 2*(c^4 + c^2*d^2)*cos(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2)))*sin(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2)) + 2*(sqrt(3)*f^8*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(5/6)*sin(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2)) - f^8*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(5/6)*cos(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2)))*sqrt(sqrt(3)*c*f^4*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(1/6)*cos(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2)) - c*f^4*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(1/6)*sin(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2)) + c^2*f^2*((c^2 + d^2)/f^6)^(1/3) + c^2*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(2/3)) + sqrt(3)*(c^4 + c^2*d^2))/(3*c^4 + 3*c^2*d^2 - 4*(c^4 + c^2*d^2)*cos(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2))^2)) + (sqrt(3)*((c^2 + d^2)/f^6)^(1/6)*cos(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2)) + ((c^2 + d^2)/f^6)^(1/6)*sin(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2)))*arctan((2*c*f^8*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(5/6)*cos(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2)) + 2*(sqrt(3)*c*f^8*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(5/6) - 2*(c^4 + c^2*d^2)*cos(2/3*arctan((f^6*sqrt

```
(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2)))*sin(2/3*arcta
n((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2)) - 2
*(sqrt(3)*f^8*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(5/6)*sin(2/3*arctan((f^6*sqr
t(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2)) + f^8*sqrt(c^
2/f^6)*((c^2 + d^2)/f^6)^(5/6)*cos(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2
+ d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2)))*sqrt(-sqrt(3)*c*f^4*((c*cos(f*x +
 e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(
1/6)*cos(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c
^2/f^6))/c^2)) - c*f^4*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/
3)*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(1/6)*sin(2/3*arctan((f^6*sqrt(c^2/f^6)*
sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2)) + c^2*f^2*((c^2 + d^2)/f
^6)^(1/3) + c^2*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(2/3)) - s
qrt(3)*(c^4 + c^2*d^2))/(3*c^4 + 3*c^2*d^2 - 4*(c^4 + c^2*d^2)*cos(2/3*arct
an((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2))^2)
) - 1/4*(sqrt(3)*((c^2 + d^2)/f^6)^(1/6)*sin(2/3*arctan((f^6*sqrt(c^2/f^6)*
sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2)) + ((c^2 + d^2)/f^6)^(1/6
)*cos(2/3*arctan((f^6*sqrt(c^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/
f^6))/c^2)))*log(sqrt(3)*c*f^4*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x +
 e))^(1/3)*sqrt(c^2/f^6)*((c^2 + d^2)/f^6)^(1/6)*cos(2/3*arctan((f^6*sqrt(c
^2/f^6)*sqrt((c^2 + d^2)/f^6) + d*f^3*sqrt(c^2/f^6))/c^2)) - c*f^4*((c*cos(
f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*sqrt(c^2/f^6)*((c^2 + d^2)/f
^6)^(1/6)*sin(2/3*arctan((f^6*sqrt(c^2/f^6)*sqr...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{c + d \tan{\left(e + f x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/3),x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 6.88, size = 863, normalized size = 2.08

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(1/3),x)
```

```
[Out] log((c + d*tan(e + f*x))^(1/3) + f*(-(c*1i + d)/f^3)^(1/3)*1i)*(-(c*1i + d)
/(8*f^3))^(1/3) + log(d*(c + d*tan(e + f*x))^(1/3)*1i - c*(c + d*tan(e + f*
x))^(1/3) + f^4*((c*1i - d)/f^3)^(4/3) + 2*d*f*((c*1i - d)/f^3)^(1/3))*((c*
1i - d)/(8*f^3))^(1/3) - log((((c*1i - d)/f^3)^(1/3)*((3^(1/2)*1i)/2 + 1/2)
*((((c*1i - d)/f^3)^(2/3)*((3^(1/2)*1i)/2 - 1/2)*((3888*d^5*(c^2 + d^2)*(c
+ d*tan(e + f*x))^(1/3))/f - 3888*c*d^4*((c*1i - d)/f^3)^(1/3)*((3^(1/2)*1i
)/2 + 1/2)*(c^2 + d^2)))/4 + (1944*c*d^5*(c^2 + d^2))/f^3))/2 - (486*(d^8 -
 c^4*d^4)*(c + d*tan(e + f*x))^(1/3))/f^4)*((3^(1/2)*1i)/2 + 1/2)*((c*1i -
d)/(8*f^3))^(1/3) + log((486*(d^8 - c^4*d^4)*(c + d*tan(e + f*x))^(1/3))/f^
4 - (((c*1i - d)/f^3)^(1/3)*((3^(1/2)*1i)/2 - 1/2)*((((c*1i - d)/f^3)^(2/3)
*((3^(1/2)*1i)/2 + 1/2)*((3888*d^5*(c^2 + d^2)*(c + d*tan(e + f*x))^(1/3))/
f + 3888*c*d^4*((c*1i - d)/f^3)^(1/3)*((3^(1/2)*1i)/2 - 1/2)*(c^2 + d^2)))/
4 - (1944*c*d^5*(c^2 + d^2))/f^3))/2)*((3^(1/2)*1i)/2 - 1/2)*((c*1i - d)/(8
*f^3))^(1/3) - log((((3^(1/2)*1i)/2 + 1/2)*((((3888*d^5*(c^2 + d^2)*(c + d*
tan(e + f*x))^(1/3))/f - 3888*c*d^4*((3^(1/2)*1i)/2 + 1/2)*(-(c*1i + d)/f^3
)^(1/3)*(c^2 + d^2))*((3^(1/2)*1i)/2 - 1/2)*(-(c*1i + d)/f^3)^(2/3))/4 + (1
944*c*d^5*(c^2 + d^2))/f^3)*(-(c*1i + d)/f^3)^(1/3))/2 - (486*(d^8 - c^4*d^
4)*(c + d*tan(e + f*x))^(1/3))/f^4)*((3^(1/2)*1i)/2 + 1/2)*(-(c*1i + d)/(8*
f^3))^(1/3) + log((486*(d^8 - c^4*d^4)*(c + d*tan(e + f*x))^(1/3))/f^4 - ((
(3^(1/2)*1i)/2 - 1/2)*((((3888*d^5*(c^2 + d^2)*(c + d*tan(e + f*x))^(1/3))/
f + 3888*c*d^4*((3^(1/2)*1i)/2 - 1/2)*(-(c*1i + d)/f^3)^(1/3)*(c^2 + d^2))*
((3^(1/2)*1i)/2 + 1/2)*(-(c*1i + d)/f^3)^(2/3))/4 - (1944*c*d^5*(c^2 + d^2)
)/f^3)*(-(c*1i + d)/f^3)^(1/3))/2)*((3^(1/2)*1i)/2 - 1/2)*(-(c*1i + d)/(8*f
^3))^(1/3)
```

3.686 $\int \cot(e+fx)\sqrt[3]{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=402

$$-\frac{1}{4}i\sqrt[3]{c-id}\,x+\frac{1}{4}i\sqrt[3]{c+id}\,x-\frac{\sqrt{3}\sqrt[3]{c}\,\text{ArcTan}\left(\frac{\sqrt[3]{c}+2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt{3}\sqrt[3]{c}}\right)}{f}+\frac{\sqrt{3}\sqrt[3]{c-id}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{\cdots}}{\cdots}}{\cdots}\right)}{2f}$$

```
[Out] -1/4*I*(c-I*d)^(1/3)*x+1/4*I*(c+I*d)^(1/3)*x-1/4*(c-I*d)^(1/3)*ln(cos(f*x+e
))/f-1/4*(c+I*d)^(1/3)*ln(cos(f*x+e))/f-1/2*c^(1/3)*ln(tan(f*x+e))/f+3/2*c^
(1/3)*ln(c^(1/3)-(c+d*tan(f*x+e))^(1/3))/f-3/4*(c-I*d)^(1/3)*ln((c-I*d)^(1/
3)-(c+d*tan(f*x+e))^(1/3))/f-3/4*(c+I*d)^(1/3)*ln((c+I*d)^(1/3)-(c+d*tan(f*
x+e))^(1/3))/f-c^(1/3)*arctan(1/3*(c^(1/3)+2*(c+d*tan(f*x+e))^(1/3))/c^(1/3
)*3^(1/2))*3^(1/2)/f+1/2*(c-I*d)^(1/3)*arctan(1/3*(1+2*(c+d*tan(f*x+e))^(1/
3)/(c-I*d)^(1/3))*3^(1/2))*3^(1/2)/f+1/2*(c+I*d)^(1/3)*arctan(1/3*(1+2*(c+d
*tan(f*x+e))^(1/3)/(c+I*d)^(1/3))*3^(1/2))*3^(1/2)/f
```

Rubi [A]
time = 0.36, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3655, 3609, 3620, 3618, 59, 631, 210, 31, 3715, 52}

$$-\frac{\sqrt{3}\sqrt[3]{c}\,\text{ArcTan}\left(\frac{2\sqrt[3]{c+d\tan(e+fx)}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{f}+\frac{\sqrt{3}\sqrt[3]{c-id}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c-id}}}{\sqrt{3}}\right)}{2f}+\frac{\sqrt{3}\sqrt[3]{c+id}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c+id}}}{\sqrt{3}}\right)}{2f}+\frac{3\sqrt[3]{c}\log\left(\sqrt[3]{c}-\sqrt[3]{c+d\tan(e+fx)}\right)}{2f}-\frac{3\sqrt[3]{c-id}\log\left(-\sqrt[3]{c+d\tan(e+fx)}+\sqrt[3]{c-id}\right)}{4f}-\frac{3\sqrt[3]{c+id}\log\left(-\sqrt[3]{c+d\tan(e+fx)}+\sqrt[3]{c+id}\right)}{4f}-\frac{\sqrt[3]{c-id}\log(\cos(e+fx))}{4f}-\frac{\sqrt[3]{c+id}\log(\cos(e+fx))}{4f}-\frac{1}{4}ix\sqrt[3]{c-id}+\frac{1}{4}ix\sqrt[3]{c+id}-\frac{\sqrt[3]{c}\log(\tan(e+fx))}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(c + d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (-1/4*I)*(c - I*d)^(1/3)*x + (I/4)*(c + I*d)^(1/3)*x - (Sqrt[3]*c^(1/3)*Arc
Tan[(c^(1/3) + 2*(c + d*Tan[e + f*x])^(1/3))/(Sqrt[3]*c^(1/3))])/f + (Sqrt[
3]*(c - I*d)^(1/3)*ArcTan[(1 + (2*(c + d*Tan[e + f*x])^(1/3))/(c - I*d)^(1/
3))/Sqrt[3]])/(2*f) + (Sqrt[3]*(c + I*d)^(1/3)*ArcTan[(1 + (2*(c + d*Tan[e
+ f*x])^(1/3))/(c + I*d)^(1/3))/Sqrt[3]])/(2*f) - ((c - I*d)^(1/3)*Log[Cos[
e + f*x]])/(4*f) - ((c + I*d)^(1/3)*Log[Cos[e + f*x]])/(4*f) - (c^(1/3)*Log
[Tan[e + f*x]])/(2*f) + (3*c^(1/3)*Log[c^(1/3) - (c + d*Tan[e + f*x])^(1/3)
])/(2*f) - (3*(c - I*d)^(1/3)*Log[(c - I*d)^(1/3) - (c + d*Tan[e + f*x])^(1
/3)])/(4*f) - (3*(c + I*d)^(1/3)*Log[(c + I*d)^(1/3) - (c + d*Tan[e + f*x])
^(1/3)])/(4*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
```

```
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)/((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[(a + b*Tan[e + f*x])^m*
(c - d*Tan[e + f*x]), x], x] + Dist[d^2/(c^2 + d^2), Int[(a + b*Tan[e + f*x
])^m*((1 + Tan[e + f*x]^2)/(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \cot(e+fx)\sqrt[3]{c+d\tan(e+fx)}\,dx = -\int \tan(e+fx)\sqrt[3]{c+d\tan(e+fx)}\,dx + \int \cot(e+fx)\sqrt[3]{c+d}$$

$$= -\frac{3\sqrt[3]{c+d\tan(e+fx)}}{f} + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{c+dx}}{x}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\left(\frac{1}{2}(-ic-d)\int \frac{1+i\tan(e+fx)}{(c+d\tan(e+fx))^{2/3}}\,dx\right) - \frac{1}{2}(ic-d)\int \frac{1}{(c+}$$

$$= -\frac{\sqrt[3]{c}\,\log(\tan(e+fx))}{2f} - \frac{(3\sqrt[3]{c})\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-x}\,dx, x, \sqrt[3]{c+d\,t}\right.}{2f}$$

$$= -\frac{1}{4}i\sqrt[3]{c-id}\,x + \frac{1}{4}i\sqrt[3]{c+id}\,x - \frac{\sqrt[3]{c-id}\,\log(\cos(e+fx))}{4f} - \frac{\sqrt[3]{c}}{}$$

$$= -\frac{1}{4}i\sqrt[3]{c-id}\,x + \frac{1}{4}i\sqrt[3]{c+id}\,x - \frac{\sqrt{3}\,\sqrt[3]{c}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{c+d\tan(}}{\sqrt[3]{c}}}{\sqrt{3}}\right.}{f}$$

$$= -\frac{1}{4}i\sqrt[3]{c-id}\,x + \frac{1}{4}i\sqrt[3]{c+id}\,x - \frac{\sqrt{3}\,\sqrt[3]{c}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{c+d\tan(}}{\sqrt[3]{c}}}{\sqrt{3}}\right.}{f}$$

Mathematica [A]
time = 0.86, size = 744, normalized size = 1.85

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(c + d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (-4*Sqrt[3]*c^(1/3)*ArcTan[(c^(1/3) + 2*(c + d*Tan[e + f*x])^(1/3))/(Sqrt[3
]*c^(1/3))] + 2*Sqrt[3]*(c - I*d)^(1/3)*ArcTan[(1 + (2*(c + d*Tan[e + f*x])
^(1/3))/(c - I*d)^(1/3))/Sqrt[3]] + (2*Sqrt[3]*c*ArcTan[(1 + (2*(c + d*Tan[
e + f*x])^(1/3))/(c + I*d)^(1/3))/Sqrt[3]])/(c + I*d)^(2/3) + ((2*I)*Sqrt[3
]*d*ArcTan[(1 + (2*(c + d*Tan[e + f*x])^(1/3))/(c + I*d)^(1/3))/Sqrt[3]])/(
c + I*d)^(2/3) + 4*c^(1/3)*Log[c^(1/3) - (c + d*Tan[e + f*x])^(1/3)] - (2*c
*Log[(c - I*d)^(1/3) - (c + d*Tan[e + f*x])^(1/3)])/(c - I*d)^(2/3) + ((2*I
```

```
)*d*Log[(c - I*d)^(1/3) - (c + d*Tan[e + f*x])^(1/3)])/(c - I*d)^(2/3) - (2
*c*Log[(c + I*d)^(1/3) - (c + d*Tan[e + f*x])^(1/3)])/(c + I*d)^(2/3) - ((2
*I)*d*Log[(c + I*d)^(1/3) - (c + d*Tan[e + f*x])^(1/3)])/(c + I*d)^(2/3) -
2*c^(1/3)*Log[c^(2/3) + c^(1/3)*(c + d*Tan[e + f*x])^(1/3) + (c + d*Tan[e +
 f*x])^(2/3)] + (c*Log[(c - I*d)^(2/3) + (c - I*d)^(1/3)*(c + d*Tan[e + f*x
])^(1/3) + (c + d*Tan[e + f*x])^(2/3)])/(c - I*d)^(2/3) - (I*d*Log[(c - I*d
)^(2/3) + (c - I*d)^(1/3)*(c + d*Tan[e + f*x])^(1/3) + (c + d*Tan[e + f*x])
^(2/3)])/(c - I*d)^(2/3) + (c*Log[(c + I*d)^(2/3) + (c + I*d)^(1/3)*(c + d*
Tan[e + f*x])^(1/3) + (c + d*Tan[e + f*x])^(2/3)])/(c + I*d)^(2/3) + (I*d*L
og[(c + I*d)^(2/3) + (c + I*d)^(1/3)*(c + d*Tan[e + f*x])^(1/3) + (c + d*Ta
n[e + f*x])^(2/3)])/(c + I*d)^(2/3))/(4*f)
```

Maple [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int \cot(fx+e)\,(c+d\tan(fx+e))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(c+d*tan(f*x+e))^(1/3),x)
[Out] int(cot(f*x+e)*(c+d*tan(f*x+e))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(c+d*tan(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate((d*tan(f*x + e) + c)^(1/3)*cot(f*x + e), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(c+d*tan(f*x+e))^(1/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{c+d\tan(e+fx)}\,\cot(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(c+d*tan(f*x+e))**(1/3),x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(1/3)*cot(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(c+d*tan(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((d*tan(f*x + e) + c)^(1/3)*cot(f*x + e), x)
```

Mupad [B]
time = 15.56, size = 2133, normalized size = 5.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(c + d*tan(e + f*x))^(1/3),x)
```

```
[Out] log((c + d*tan(e + f*x))^(1/3) - f*(c/f^3)^(1/3))*(c/f^3)^(1/3) + log(((-(c
 - d*1i)/f^3)^(1/3)*(((-(c - d*1i)/f^3)^(2/3)*(((((104976*c*d^14*(-(c - d*1
i)/f^3)^(1/3)*(3*c^4 + 2*d^4 + 5*c^2*d^2) - (104976*c*d^14*(c + d*tan(e + f
*x))^(1/3)*(3*c^4 + 4*d^4 + 7*c^2*d^2))/f)*(-(c - d*1i)/f^3)^(2/3))/4 - (78
732*c^2*d^14*(c^4 - d^4))/f^3)*(-(c - d*1i)/f^3)^(1/3))/2 - (39366*c^2*d^14
*(c + d*tan(e + f*x))^(1/3)*(5*c^4 + 3*d^4 + 8*c^2*d^2))/f^4))/4 + (6561*c*
d^14*(d^6 - 3*c^6 + 7*c^2*d^4 + 3*c^4*d^2))/f^6))/2 - (6561*c*d^14*(c + d*t
an(e + f*x))^(1/3)*(3*c^6 + d^6 + c^2*d^4 + 3*c^4*d^2))/f^7)*(-(c - d*1i)/(
8*f^3))^(1/3) + log(((-(c + d*1i)/f^3)^(1/3)*(((-(c + d*1i)/f^3)^(2/3)*((((
(104976*c*d^14*(-(c + d*1i)/f^3)^(1/3)*(3*c^4 + 2*d^4 + 5*c^2*d^2) - (10497
6*c*d^14*(c + d*tan(e + f*x))^(1/3)*(3*c^4 + 4*d^4 + 7*c^2*d^2))/f)*(-(c +
d*1i)/f^3)^(2/3))/4 - (78732*c^2*d^14*(c^4 - d^4))/f^3)*(-(c + d*1i)/f^3)^(
1/3))/2 - (39366*c^2*d^14*(c + d*tan(e + f*x))^(1/3)*(5*c^4 + 3*d^4 + 8*c^2
*d^2))/f^4))/4 + (6561*c*d^14*(d^6 - 3*c^6 + 7*c^2*d^4 + 3*c^4*d^2))/f^6))/
2 - (6561*c*d^14*(c + d*tan(e + f*x))^(1/3)*(3*c^6 + d^6 + c^2*d^4 + 3*c^4*
d^2))/f^7)*(-(c + d*1i)/(8*f^3))^(1/3) + (log(2*(c + d*tan(e + f*x))^(1/3)
+ f*(c/f^3)^(1/3) - 3^(1/2)*f*(c/f^3)^(1/3)*1i)*(3^(1/2)*1i - 1)*(c/f^3)^(1
/3))/2 - (log(2*(c + d*tan(e + f*x))^(1/3) + f*(c/f^3)^(1/3) + 3^(1/2)*f*(c
/f^3)^(1/3)*1i)*(3^(1/2)*1i + 1)*(c/f^3)^(1/3))/2 + log((((3^(1/2)*1i)/2 -
1/2)*((((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*(-(c - d*1i)/f^3)^
(2/3)*((104976*c*d^14*(c + d*tan(e + f*x))^(1/3)*(3*c^4 + 4*d^4 + 7*c^2*d^2
))/f - 104976*c*d^14*((3^(1/2)*1i)/2 - 1/2)*(-(c - d*1i)/f^3)^(1/3)*(3*c^4
```

```
+ 2*d^4 + 5*c^2*d^2)))/4 - (78732*c^2*d^14*(c^4 - d^4))/f^3)*(-(c - d*1i)/f
^3)^(1/3))/2 - (39366*c^2*d^14*(c + d*tan(e + f*x))^(1/3)*(5*c^4 + 3*d^4 +
8*c^2*d^2))/f^4)*((3^(1/2)*1i)/2 + 1/2)*(-(c - d*1i)/f^3)^(2/3))/4 - (6561*
c*d^14*(d^6 - 3*c^6 + 7*c^2*d^4 + 3*c^4*d^2))/f^6)*(-(c - d*1i)/f^3)^(1/3))
/2 + (6561*c*d^14*(c + d*tan(e + f*x))^(1/3)*(3*c^6 + d^6 + c^2*d^4 + 3*c^4
*d^2))/f^7)*((3^(1/2)*1i)/2 - 1/2)*(-(c - d*1i)/(8*f^3))^(1/3) + log((((3^(
1/2)*1i)/2 - 1/2)*((((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*(-(c
+ d*1i)/f^3)^(2/3)*((104976*c*d^14*(c + d*tan(e + f*x))^(1/3)*(3*c^4 + 4*d^
4 + 7*c^2*d^2))/f - 104976*c*d^14*((3^(1/2)*1i)/2 - 1/2)*(-(c + d*1i)/f^3)^
(1/3)*(3*c^4 + 2*d^4 + 5*c^2*d^2)))/4 - (78732*c^2*d^14*(c^4 - d^4))/f^3)*(
-(c + d*1i)/f^3)^(1/3))/2 - (39366*c^2*d^14*(c + d*tan(e + f*x))^(1/3)*(5*c
^4 + 3*d^4 + 8*c^2*d^2))/f^4)*((3^(1/2)*1i)/2 + 1/2)*(-(c + d*1i)/f^3)^(2/3
))/4 - (6561*c*d^14*(d^6 - 3*c^6 + 7*c^2*d^4 + 3*c^4*d^2))/f^6)*(-(c + d*1i
)/f^3)^(1/3))/2 + (6561*c*d^14*(c + d*tan(e + f*x))^(1/3)*(3*c^6 + d^6 + c^
2*d^4 + 3*c^4*d^2))/f^7)*((3^(1/2)*1i)/2 - 1/2)*(-(c + d*1i)/(8*f^3))^(1/3)
 - log((((3^(1/2)*1i)/2 + 1/2)*((((((3^(1/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2
 - 1/2)*(-(c - d*1i)/f^3)^(2/3)*((104976*c*d^14*(c + d*tan(e + f*x))^(1/3)*
(3*c^4 + 4*d^4 + 7*c^2*d^2))/f + 104976*c*d^14*((3^(1/2)*1i)/2 + 1/2)*(-(c
- d*1i)/f^3)^(1/3)*(3*c^4 + 2*d^4 + 5*c^2*d^2)))/4 + (78732*c^2*d^14*(c^4 -
 d^4))/f^3)*(-(c - d*1i)/f^3)^(1/3))/2 - (39366*c^2*d^14*(c + d*tan(e + f*x
))^(1/3)*(5*c^4 + 3*d^4 + 8*c^2*d^2))/f^4)*((3^(1/2)*1i)/2 - 1/2)*(-(c - d*
1i)/f^3)^(2/3))/4 + (6561*c*d^14*(d^6 - 3*c^6 + 7*c^2*d^4 + 3*c^4*d^2))/f^6
)*(-(c - d*1i)/f^3)^(1/3))/2 + (6561*c*d^14*(c + d*tan(e + f*x))^(1/3)*(3*c
^6 + d^6 + c^2*d^4 + 3*c^4*d^2))/f^7)*((3^(1/2)*1i)/2 + 1/2)*(-(c - d*1i)/(
8*f^3))^(1/3) - log((((3^(1/2)*1i)/2 + 1/2)*((((((3^(1/2)*1i)/2 + 1/2)*((((
3^(1/2)*1i)/2 - 1/2)*(-(c + d*1i)/f^3)^(2/3)*((104976*c*d^14*(c + d*tan(e +
 f*x))^(1/3)*(3*c^4 + 4*d^4 + 7*c^2*d^2))/f + 104976*c*d^14*((3^(1/2)*1i)/2
 + 1/2)*(-(c + d*1i)/f^3)^(1/3)*(3*c^4 + 2*d^4 + 5*c^2*d^2)))/4 + (78732*c^
2*d^14*(c^4 - d^4))/f^3)*(-(c + d*1i)/f^3)^(1/3))/2 - (39366*c^2*d^14*(c +
d*tan(e + f*x))^(1/3)*(5*c^4 + 3*d^4 + 8*c^2*d^2))/f^4)*((3^(1/2)*1i)/2 - 1
/2)*(-(c + d*1i)/f^3)^(2/3))/4 + (6561*c*d^14*(d^6 - 3*c^6 + 7*c^2*d^4 + 3*
c^4*d^2))/f^6)*(-(c + d*1i)/f^3)^(1/3))/2 + (6561*c*d^14*(c + d*tan(e + f*x
))^(1/3)*(3*c^6 + d^6 + c^2*d^4 + 3*c^4*d^2))/f^7)*((3^(1/2)*1i)/2 + 1/2)*(
-(c + d*1i)/(8*f^3))^(1/3)
```

3.687 $\int \cot^2(e+fx)\sqrt[3]{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=546

$$\frac{1}{4}\sqrt[3]{c-\sqrt{-d^2}}\,x+\frac{1}{4}\sqrt[3]{c+\sqrt{-d^2}}\,x-\frac{d\text{ArcTan}\left(\frac{\sqrt[3]{c}+2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}f}-\frac{\sqrt{3}\,d\sqrt[3]{c-\sqrt{-d^2}}\,\text{ArcTan}}{2\sqrt{}}$$

```
[Out] -1/6*d*ln(tan(f*x+e))/c^(2/3)/f+1/2*d*ln(c^(1/3)-(c+d*tan(f*x+e))^(1/3))/c^
(2/3)/f-1/3*d*arctan(1/3*(c^(1/3)+2*(c+d*tan(f*x+e))^(1/3))/c^(1/3)*3^(1/2)
)/c^(2/3)/f*3^(1/2)+1/4*x*(c-(-d^2)^(1/2))^(1/3)+1/4*d*ln(cos(f*x+e))*(c-(-
d^2)^(1/2))^(1/3)/f/(-d^2)^(1/2)+3/4*d*ln((c-(-d^2)^(1/2))^(1/3)-(c+d*tan(f
*x+e))^(1/3))*(c-(-d^2)^(1/2))^(1/3)/f/(-d^2)^(1/2)-1/2*d*arctan(1/3*(1+2*(
c+d*tan(f*x+e))^(1/3)/(c-(-d^2)^(1/2))^(1/3))*3^(1/2))*3^(1/2)*(c-(-d^2)^(1
/2))^(1/3)/f/(-d^2)^(1/2)+1/4*x*(c+(-d^2)^(1/2))^(1/3)-1/4*d*ln(cos(f*x+e))
*(c+(-d^2)^(1/2))^(1/3)/f/(-d^2)^(1/2)-3/4*d*ln((c+(-d^2)^(1/2))^(1/3)-(c+d
*tan(f*x+e))^(1/3))*(c+(-d^2)^(1/2))^(1/3)/f/(-d^2)^(1/2)+1/2*d*arctan(1/3*
(1+2*(c+d*tan(f*x+e))^(1/3)/(c+(-d^2)^(1/2))^(1/3))*3^(1/2))*3^(1/2)*(c+(-d
^2)^(1/2))^(1/3)/f/(-d^2)^(1/2)-cot(f*x+e)*(c+d*tan(f*x+e))^(1/3)/f
```

Rubi [A]
time = 0.45, antiderivative size = 546, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3649, 3734, 3566, 726, 52, 59, 631, 210, 31, 3715}

$$-\frac{d\text{ArcTan}\left(\frac{2\sqrt[3]{c+d\tan(e+fx)}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}f}-\frac{\sqrt{3}\,d\sqrt[3]{c-\sqrt{-d^2}}\,\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c-\sqrt{-d^2}}}+1}{\sqrt{3}}\right)}{2\sqrt{-d^2}\,f}+\frac{\sqrt{3}\,d\sqrt[3]{c+\sqrt{-d^2}}\,\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c+\sqrt{-d^2}}}+1}{\sqrt{3}}\right)}{2\sqrt{-d^2}\,f}-\frac{d\log(\tan(e+fx))}{6c^{2/3}f}+\frac{d\log\left(\sqrt[3]{c}-\sqrt[3]{c+d\tan(e+fx)}\right)}{2c^{2/3}f}+\frac{3d\sqrt[3]{c-\sqrt{-d^2}}\log\left(\sqrt[3]{c-\sqrt{-d^2}}-\sqrt[3]{c+d\tan(e+fx)}\right)}{4\sqrt{-d^2}\,f}-\frac{3d\sqrt[3]{c+\sqrt{-d^2}}\log\left(\sqrt[3]{c+\sqrt{-d^2}}-\sqrt[3]{c+d\tan(e+fx)}\right)}{4\sqrt{-d^2}\,f}+\frac{d\sqrt[3]{c-\sqrt{-d^2}}\log(\cos(e+fx))}{4\sqrt{-d^2}\,f}-\frac{d\sqrt[3]{c+\sqrt{-d^2}}\log(\cos(e+fx))}{4\sqrt{-d^2}\,f}+\frac{1}{4}x\sqrt[3]{c-\sqrt{-d^2}}+\frac{1}{4}x\sqrt[3]{c+\sqrt{-d^2}}-\frac{\cot(e+fx)\sqrt[3]{c+d\tan(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(c + d*Tan[e + f*x])^(1/3),x]
```

```
[Out] ((c - Sqrt[-d^2])^(1/3)*x)/4 + ((c + Sqrt[-d^2])^(1/3)*x)/4 - (d*ArcTan[(c^
(1/3) + 2*(c + d*Tan[e + f*x])^(1/3))/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*
f) - (Sqrt[3]*d*(c - Sqrt[-d^2])^(1/3)*ArcTan[(1 + (2*(c + d*Tan[e + f*x])^
(1/3))/(c - Sqrt[-d^2])^(1/3))/Sqrt[3]])/(2*Sqrt[-d^2]*f) + (Sqrt[3]*d*(c +
 Sqrt[-d^2])^(1/3)*ArcTan[(1 + (2*(c + d*Tan[e + f*x])^(1/3))/(c + Sqrt[-d^
2])^(1/3))/Sqrt[3]])/(2*Sqrt[-d^2]*f) + (d*(c - Sqrt[-d^2])^(1/3)*Log[Cos[e
 + f*x]])/(4*Sqrt[-d^2]*f) - (d*(c + Sqrt[-d^2])^(1/3)*Log[Cos[e + f*x]])/(
4*Sqrt[-d^2]*f) - (d*Log[Tan[e + f*x]])/(6*c^(2/3)*f) + (d*Log[c^(1/3) - (c
 + d*Tan[e + f*x])^(1/3)])/(2*c^(2/3)*f) + (3*d*(c - Sqrt[-d^2])^(1/3)*Log[
(c - Sqrt[-d^2])^(1/3) - (c + d*Tan[e + f*x])^(1/3)])/(4*Sqrt[-d^2]*f) - (3
```

```
*d*(c + Sqrt[-d^2])^(1/3)*Log[(c + Sqrt[-d^2])^(1/3) - (c + d*Tan[e + f*x])
^(1/3)])/(4*Sqrt[-d^2]*f) - (Cot[e + f*x]*(c + d*Tan[e + f*x])^(1/3))/f
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 726

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
```

```
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \cot^2(e+fx)\sqrt[3]{c+d\tan(e+fx)}\,dx = -\frac{\cot(e+fx)\sqrt[3]{c+d\tan(e+fx)}}{f} - \int \frac{\cot(e+fx)\left(-\frac{d}{3}+c\tan(\right.}{(c+d\tan}$$

$$= -\frac{\cot(e+fx)\sqrt[3]{c+d\tan(e+fx)}}{f} + \frac{1}{3}d\int \frac{\cot(e+fx)\left(1+\tan^2(\right.}{(c+d\tan(e+fx}$$

$$= -\frac{\cot(e+fx)\sqrt[3]{c+d\tan(e+fx)}}{f} + \frac{d\text{Subst}\left(\int \frac{1}{x(c+dx)^{2/3}}\,dx, x, \text{ta}\right.}{3f}$$

$$= -\frac{d\log(\tan(e+fx))}{6c^{2/3}f} - \frac{\cot(e+fx)\sqrt[3]{c+d\tan(e+fx)}}{f} - \frac{d\text{Subs}}{}$$

$$= -\frac{d\log(\tan(e+fx))}{6c^{2/3}f} + \frac{d\log\left(\sqrt[3]{c}-\sqrt[3]{c+d\tan(e+fx)}\right)}{2c^{2/3}f} - \text{cot}$$

$$= -\frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}f} - \frac{d\log(\tan(e+fx))}{6c^{2/3}f} + \frac{d\,\text{l}}{}$$

$$= \frac{1}{4}\sqrt[3]{c-\sqrt{-d^2}}\,x + \frac{1}{4}\sqrt[3]{c+\sqrt{-d^2}}\,x - \frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{c+d\tan(}}{\sqrt[3]{c}}}{\sqrt{3}}\right.}{\sqrt{3}\,c^{2/3}f}$$

$$= \frac{1}{4}\sqrt[3]{c-\sqrt{-d^2}}\,x + \frac{1}{4}\sqrt[3]{c+\sqrt{-d^2}}\,x - \frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{c+d\tan(}}{\sqrt[3]{c}}}{\sqrt{3}}\right.}{\sqrt{3}\,c^{2/3}f}$$

$$= \frac{1}{4}\sqrt[3]{c-\sqrt{-d^2}}\,x + \frac{1}{4}\sqrt[3]{c+\sqrt{-d^2}}\,x - \frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{c+d\tan(}}{\sqrt[3]{c}}}{\sqrt{3}}\right.}{\sqrt{3}\,c^{2/3}f}$$

Mathematica [C] Result contains complex when optimal does not.

time = 3.40, size = 464, normalized size = 0.85

$$\frac{\frac{1}{3}\sqrt[3]{c}\,d\log\left(\sqrt[3]{c}-\sqrt[3]{c+d\tan(e+fx)}\right)-\frac{1}{6}\sqrt[3]{c}\,d\left(2\sqrt{3}\,\mathrm{ArcTan}\left(\frac{\sqrt[3]{c}+2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt{3}\sqrt[3]{c}}\right)+\log\left(c^{2/3}+\sqrt[3]{c}\sqrt[3]{c+d\tan(e+fx)}+(c+d\tan(e+fx))^{2/3}\right)\right)+\frac{1}{4}ic\sqrt[3]{c-id}\left(2\sqrt{3}\,\mathrm{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c-id}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c-id}-\sqrt[3]{c+d\tan(e+fx)}\right)+\log\left((c-id)^{2/3}+\sqrt[3]{c-id}\sqrt[3]{c+d\tan(e+fx)}+(c+d\tan(e+fx))^{2/3}\right)\right)-\frac{1}{4}ic\sqrt[3]{c+id}\left(2\sqrt{3}\,\mathrm{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c+id}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c+id}-\sqrt[3]{c+d\tan(e+fx)}\right)+\log\left((c+id)^{2/3}+\sqrt[3]{c+id}\sqrt[3]{c+d\tan(e+fx)}+(c+d\tan(e+fx))^{2/3}\right)\right)+d\sqrt[3]{c+d\tan(e+fx)}-\cot(e+fx)(c+d\tan(e+fx))^{4/3}}{cf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(c + d*Tan[e + f*x])^(1/3),x]
```

```
[Out] ((c^(1/3)*d*Log[c^(1/3) - (c + d*Tan[e + f*x])^(1/3)])/3 - (c^(1/3)*d*(2*Sq
rt[3]*ArcTan[(c^(1/3) + 2*(c + d*Tan[e + f*x])^(1/3))/(Sqrt[3]*c^(1/3))] +
Log[c^(2/3) + c^(1/3)*(c + d*Tan[e + f*x])^(1/3) + (c + d*Tan[e + f*x])^(2/
3)]))/6 + (I/4)*c*(c - I*d)^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(c + d*Tan[e +
f*x])^(1/3))/(c - I*d)^(1/3))/Sqrt[3]] - 2*Log[(c - I*d)^(1/3) - (c + d*Tan
[e + f*x])^(1/3)] + Log[(c - I*d)^(2/3) + (c - I*d)^(1/3)*(c + d*Tan[e + f*
x])^(1/3) + (c + d*Tan[e + f*x])^(2/3)]) - (I/4)*c*(c + I*d)^(1/3)*(2*Sqrt[
3]*ArcTan[(1 + (2*(c + d*Tan[e + f*x])^(1/3))/(c + I*d)^(1/3))/Sqrt[3]] - 2
*Log[(c + I*d)^(1/3) - (c + d*Tan[e + f*x])^(1/3)] + Log[(c + I*d)^(2/3) +
(c + I*d)^(1/3)*(c + d*Tan[e + f*x])^(1/3) + (c + d*Tan[e + f*x])^(2/3)]) +
 d*(c + d*Tan[e + f*x])^(1/3) - Cot[e + f*x]*(c + d*Tan[e + f*x])^(4/3))/(c
*f)
```

Maple [**F**]
time = 0.24, size = 0, normalized size = 0.00

$$\int \left(\cot^2\left(fx+e\right)\right)\left(c+d\tan\left(fx+e\right)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(c+d*tan(f*x+e))^(1/3),x)
```

```
[Out] int(cot(f*x+e)^2*(c+d*tan(f*x+e))^(1/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(c+d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(f*x + e) + c)^(1/3)*cot(f*x + e)^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(c+d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{c + d \tan(e + fx)} \cot^2(e + fx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(c+d*tan(f*x+e))**(1/3),x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(1/3)*cot(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(c+d*tan(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 21.08, size = 2500, normalized size = 4.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(c + d*tan(e + f*x))^(1/3),x)
```

```
[Out] log((((((243*(c + d*tan(e + f*x))^(1/3)*(576*d^19*f^6 + 1584*c^2*d^17*f^6 +
 1008*c^4*d^15*f^6))/f^7 - (243*(1728*c*d^18*f^6 + 4320*c^3*d^16*f^6 + 2592
*c^5*d^14*f^6)*(d^3/(27*c^2*f^3))^(1/3))/f^6)*(d^3/(27*c^2*f^3))^(2/3) + (2
43*(464*c*d^19*f^3 + 1112*c^3*d^17*f^3 + 648*c^5*d^15*f^3))/f^6)*(d^3/(27*c
^2*f^3))^(1/3) - (243*(c + d*tan(e + f*x))^(1/3)*(160*d^20*f^3 + 358*c^2*d^
18*f^3 + 144*c^4*d^16*f^3 - 54*c^6*d^14*f^3))/f^7)*(d^3/(27*c^2*f^3))^(2/3)
 - (243*(35*c*d^20 + 89*c^3*d^18 + 81*c^5*d^16 + 27*c^7*d^14))/f^6)*(d^3/(2
7*c^2*f^3))^(1/3) + (243*(c + d*tan(e + f*x))^(1/3)*(11*d^21 + 27*c^2*d^19
+ 25*c^4*d^17 + 9*c^6*d^15))/f^7)*(d^3/(27*c^2*f^3))^(1/3) + log((243*d^15*
(c + d*tan(e + f*x))^(1/3)*(9*c^6 + 11*d^6 + 27*c^2*d^4 + 25*c^4*d^2))/f^7
- (((c*1i + d)/f^3)^(1/3)*((((((c*1i + d)/f^3)^(1/3)*(((104976*c*d^14*((c*1
i + d)/f^3)^(1/3)*(3*c^4 + 2*d^4 + 5*c^2*d^2) - (34992*d^15*(c + d*tan(e +
f*x))^(1/3)*(7*c^4 + 4*d^4 + 11*c^2*d^2))/f)*((c*1i + d)/f^3)^(2/3))/4 - (1
944*c*d^15*(81*c^4 + 58*d^4 + 139*c^2*d^2))/f^3))/2 + (486*d^14*(c + d*tan(
```

e + f*x))^(1/3)*(80*d^6 - 27*c^6 + 179*c^2*d^4 + 72*c^4*d^2))/f^4)*((c*1i + d)/f^3)^(2/3))/4 + (243*c*d^14*(27*c^6 + 35*d^6 + 89*c^2*d^4 + 81*c^4*d^2))/f^6))/2)*((c*1i + d)/(8*f^3))^(1/3) + log(((-(c*1i - d)/f^3)^(1/3)*(((-(c*1i - d)/f^3)^(2/3)*(((-(c*1i - d)/f^3)^(1/3)*(((-(c*1i - d)/f^3)^(2/3)*(104976*c*d^14*(-(c*1i - d)/f^3)^(1/3)*(3*c^4 + 2*d^4 + 5*c^2*d^2) - (34992*d^15*(c + d*tan(e + f*x))^(1/3)*(7*c^4 + 4*d^4 + 11*c^2*d^2))/f))/4 - (1944*c*d^15*(81*c^4 + 58*d^4 + 139*c^2*d^2))/f^3))/2 + (486*d^14*(c + d*tan(e + f*x))^(1/3)*(80*d^6 - 27*c^6 + 179*c^2*d^4 + 72*c^4*d^2))/f^4))/4 + (243*c*d^14*(27*c^6 + 35*d^6 + 89*c^2*d^4 + 81*c^4*d^2))/f^6))/2 - (243*d^15*(c + d*tan(e + f*x))^(1/3)*(9*c^6 + 11*d^6 + 27*c^2*d^4 + 25*c^4*d^2))/f^7)*(-(c*1i - d)/(8*f^3))^(1/3) + log(- ((-(c*1i - d)/f^3)^(1/3)*((3^(1/2)*1i)/2 - 1/2)*(((-(c*1i - d)/f^3)^(2/3)*((3^(1/2)*1i)/2 + 1/2)*(((-(c*1i - d)/f^3)^(1/3)*((3^(1/2)*1i)/2 - 1/2)*(((-(c*1i - d)/f^3)^(2/3)*((3^(1/2)*1i)/2 + 1/2)*((34992*d^15*(c + d*tan(e + f*x))^(1/3)*(7*c^4 + 4*d^4 + 11*c^2*d^2))/f - 104976*c*d^14*(-(c*1i - d)/f^3)^(1/3)*((3^(1/2)*1i)/2 - 1/2)*(3*c^4 + 2*d^4 + 5*c^2*d^2)))/4 - (1944*c*d^15*(81*c^4 + 58*d^4 + 139*c^2*d^2))/f^3))/2 + (486*d^14*(c + d*tan(e + f*x))^(1/3)*(80*d^6 - 27*c^6 + 179*c^2*d^4 + 72*c^4*d^2))/f^4))/4 - (243*c*d^14*(27*c^6 + 35*d^6 + 89*c^2*d^4 + 81*c^4*d^2))/f^6))/2 - (243*d^15*(c + d*tan(e + f*x))^(1/3)*(9*c^6 + 11*d^6 + 27*c^2*d^4 + 25*c^4*d^2))/f^7)*((3^(1/2)*1i)/2 - 1/2)*(-(c*1i - d)/(8*f^3))^(1/3) - log(((-(c*1i - d)/f^3)^(1/3)*((3^(1/2)*1i)/2 + 1/2)*(((-(c*1i - d)/f^3)^(2/3)*((3^(1/2)*1i)/2 - 1/2)*(((-(c*1i - d)/f^3)^(1/3)*((3^(1/2)*1i)/2 + 1/2)*(((-(c*1i - d)/f^3)^(2/3)*((3^(1/2)*1i)/2 - 1/2)*((34992*d^15*(c + d*tan(e + f*x))^(1/3)*(7*c^4 + 4*d^4 + 11*c^2*d^2))/f + 104976*c*d^14*(-(c*1i - d)/f^3)^(1/3)*((3^(1/2)*1i)/2 + 1/2)*(3*c^4 + 2*d^4 + 5*c^2*d^2)))/4 + (1944*c*d^15*(81*c^4 + 58*d^4 + 139*c^2*d^2))/f^3))/2 + (486*d^14*(c + d*tan(e + f*x))^(1/3)*(80*d^6 - 27*c^6 + 179*c^2*d^4 + 72*c^4*d^2))/f^4))/4 + (243*c*d^14*(27*c^6 + 35*d^6 + 89*c^2*d^4 + 81*c^4*d^2))/f^6))/2 + (243*d^15*(c + d*tan(e + f*x))^(1/3)*(9*c^6 + 11*d^6 + 27*c^2*d^4 + 25*c^4*d^2))/f^7)*((3^(1/2)*1i)/2 + 1/2)*(-(c*1i - d)/(8*f^3))^(1/3) + (d*(c + d*tan(e + f*x))^(1/3))/(c*f - f*(c + d*tan(e + f*x))) + log(- (((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*((c*1i + d)/f^3)^(2/3)*((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*((34992*d^15*(c + d*tan(e + f*x))^(1/3)*(7*c^4 + 4*d^4 + 11*c^2*d^2))/f - 104976*c*d^14*((3^(1/2)*1i)/2 - 1/2)*((c*1i + d)/f^3)^(1/3)*(3*c^4 + 2*d^4 + 5*c^2*d^2))*((c*1i + d)/f^3)^(2/3))/4 - (1944*c*d^15*(81*c^4 + 58*d^4 + 139*c^2*d^2))/f^3)*((c*1i + d)/f^3)^(1/3))/2 + (486*d^14*(c + d*tan(e + f*x))^(1/3)*(80*d^6 - 27*c^6 + 179*c^2*d^4 + 72*c^4*d^2))/f^4))/4 - (243*c*d^14*(27*c^6 + 35*d^6 + 89*c^2*d^4 + 81*c^4*d^2))/f^6)*((c*1i + d)/f^3)^(1/3))/2 - (243*d^15*(c + d*tan(e + f*x))^(1/3)*(9*c^6 + 11*d^6 + 27*c^2*d^4 + 25*c^4*d^2))/f^7)*((3^(1/2)*1i)/2 - 1/2)*((c*1i + d)/(8*f^3))^(1/3) - log(((((3^(1/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*((c*1i + d)/f^3)^(2/3)*((((3^(1/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*((34992*d^15*(c + d*tan(e + f*x))^(1/3)*(7*c^4 + 4*d^4 + 11*c^2*d^2))/f + 104976*c*d^14*((3^(1/2)*1i)/2 + 1/2)*((c*1i + d)/f^3)^(1/3)*(3*c^4 + 2*d^4 + 5*c^2*d^2))*((c*1i + d)/f^3)^(2/3))/4 + (1944*c*d^15*(81*c^4 + 58*d^4 + 139*c^2*d^2))/f^3)*(

(c*1i + d)/f^3)^(1/3))/2 + (486*d^14*(c + d*tan(e + f*x))^(1/3)*(80*d^6 - 2
7*c^6 + 179*c^2*d^4 + 72*c^4*d^2))/f^4))/4 + (243*c*d^14*(27*c^6 + 35*d^6 +
 89*c^2*d^4 + 81*c^4*d^2))/f^6)*((c*1i + d)/f^3)^(1/3))/2 + (243*d^15*(c +
d*tan(e + f*x))^(1/3)*(9*c^6 + 11*d^6 + 27*c^2*d^4 + 25*c^4*d^2))/f^7)*((3^
(1/2)*1i)/2 + 1/2)*((c*1i + d)/(8*f^3))^(1/3) + log(- (((3^(1/2)*1i)/2 - 1/
2)*((((3^(1/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/
2)*((34992*d^15*(c + d*tan(e + f*x))^(1/3)*(7*c^4 + 4*d^4 + 11*c^2*d^2))/f
- 69984*c*d^14*((3^(1/2)*1i)/2 - 1/2)*(3*c^4 + ...

3.688 $\int (a+b\tan(c+dx))^{5/3}\,dx$

Optimal. Leaf size=329

$$-\frac{1}{4}(a-ib)^{5/3}x-\frac{1}{4}(a+ib)^{5/3}x+\frac{i\sqrt{3}\,(a-ib)^{5/3}\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-ib}}}{\sqrt{3}}\right)}{2d}-\frac{i\sqrt{3}\,(a+ib)^{5/3}\text{ArcTan}$$

```
[Out] -1/4*(a-I*b)^(5/3)*x-1/4*(a+I*b)^(5/3)*x+1/4*I*(a-I*b)^(5/3)*ln(cos(d*x+c))
/d-1/4*I*(a+I*b)^(5/3)*ln(cos(d*x+c))/d+3/4*I*(a-I*b)^(5/3)*ln((a-I*b)^(1/3
)-(a+b*tan(d*x+c))^(1/3))/d-3/4*I*(a+I*b)^(5/3)*ln((a+I*b)^(1/3)-(a+b*tan(d
*x+c))^(1/3))/d+1/2*I*(a-I*b)^(5/3)*arctan(1/3*(1+2*(a+b*tan(d*x+c))^(1/3)/
(a-I*b)^(1/3))*3^(1/2))*3^(1/2)/d-1/2*I*(a+I*b)^(5/3)*arctan(1/3*(1+2*(a+b*
tan(d*x+c))^(1/3)/(a+I*b)^(1/3))*3^(1/2))*3^(1/2)/d+3/2*b*(a+b*tan(d*x+c))^
(2/3)/d
```

Rubi [A]
time = 0.26, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3563, 3620, 3618, 57, 631, 210, 31}

$$\frac{i\sqrt{3}\,(a-ib)^{5/3}\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-ib}}}{\sqrt{3}}\right)}{2d}-\frac{i\sqrt{3}\,(a+ib)^{5/3}\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a+ib}}}{\sqrt{3}}\right)}{2d}+\frac{3b(a+b\tan(c+dx))^{2/3}}{2d}+\frac{3i(a-ib)^{5/3}\log\left(-\sqrt[3]{a+b\tan(c+dx)}+\sqrt[3]{a-ib}\right)}{4d}-\frac{3i(a+ib)^{5/3}\log\left(-\sqrt[3]{a+b\tan(c+dx)}+\sqrt[3]{a+ib}\right)}{4d}+\frac{i(a-ib)^{5/3}\log(\cos(c+dx))}{4d}-\frac{i(a+ib)^{5/3}\log(\cos(c+dx))}{4d}-\frac{1}{4}x(a-ib)^{5/3}-\frac{1}{4}x(a+ib)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^(5/3),x]
```

```
[Out] -1/4*((a - I*b)^(5/3)*x) - ((a + I*b)^(5/3)*x)/4 + ((I/2)*Sqrt[3]*(a - I*b)
^(5/3)*ArcTan[(1 + (2*(a + b*Tan[c + d*x])^(1/3))/(a - I*b)^(1/3))/Sqrt[3]]
)/d - ((I/2)*Sqrt[3]*(a + I*b)^(5/3)*ArcTan[(1 + (2*(a + b*Tan[c + d*x])^(1
/3))/(a + I*b)^(1/3))/Sqrt[3]])/d + ((I/4)*(a - I*b)^(5/3)*Log[Cos[c + d*x]
])/d - ((I/4)*(a + I*b)^(5/3)*Log[Cos[c + d*x]])/d + (((3*I)/4)*(a - I*b)^(
5/3)*Log[(a - I*b)^(1/3) - (a + b*Tan[c + d*x])^(1/3)])/d - (((3*I)/4)*(a +
 I*b)^(5/3)*Log[(a + I*b)^(1/3) - (a + b*Tan[c + d*x])^(1/3)])/d + (3*b*(a
+ b*Tan[c + d*x])^(2/3))/(2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
```

```
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3563

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (a+b\tan(c+dx))^{5/3}\,dx = \frac{3b(a+b\tan(c+dx))^{2/3}}{2d} + \int \frac{a^2-b^2+2ab\tan(c+dx)}{\sqrt[3]{a+b\tan(c+dx)}}\,dx$$

$$= \frac{3b(a+b\tan(c+dx))^{2/3}}{2d} + \frac{1}{2}(a-ib)^2 \int \frac{1+i\tan(c+dx)}{\sqrt[3]{a+b\tan(c+dx)}}\,dx + \frac{1}{2}(a+ib)$$

$$= \frac{3b(a+b\tan(c+dx))^{2/3}}{2d} + \frac{(i(a-ib)^2)\,\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt[3]{a-ibx}}\,dx, x, i\tan(\right.}{2d}$$

$$= -\frac{1}{4}(a-ib)^{5/3}x - \frac{1}{4}(a+ib)^{5/3}x + \frac{i(a-ib)^{5/3}\log(\cos(c+dx))}{4d} - \frac{i(a+ib)^{5/3}}{}$$

$$= -\frac{1}{4}(a-ib)^{5/3}x - \frac{1}{4}(a+ib)^{5/3}x + \frac{i(a-ib)^{5/3}\log(\cos(c+dx))}{4d} - \frac{i(a+ib)^{5/3}}{}$$

$$= -\frac{1}{4}(a-ib)^{5/3}x - \frac{1}{4}(a+ib)^{5/3}x + \frac{i\sqrt{3}\,(a-ib)^{5/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c-}}{\sqrt[3]{a-ib}}}{\sqrt{3}}\right.}{2d}$$

Mathematica [A]
time = 1.04, size = 300, normalized size = 0.91

$$\frac{(ia+b)\left(2\sqrt{3}\,(a-ib)^{2/3}\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-ib}}}{\sqrt{3}}\right)-(a-ib)^{2/3}\log(i+\tan(c+dx))+3\left((a-ib)^{2/3}\log\left(\sqrt[3]{a-ib}-\sqrt[3]{a+b\tan(c+dx)}\right)+(a+b\tan(c+dx))^{2/3}\right)\right)+(-ia+b)\left(2\sqrt{3}\,(a+ib)^{2/3}\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a+ib}}}{\sqrt{3}}\right)-(a+ib)^{2/3}\log(i-\tan(c+dx))+3\left((a+ib)^{2/3}\log\left(\sqrt[3]{a+ib}-\sqrt[3]{a+b\tan(c+dx)}\right)+(a+b\tan(c+dx))^{2/3}\right)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^(5/3),x]
```

```
[Out] ((I*a + b)*(2*Sqrt[3]*(a - I*b)^(2/3)*ArcTan[(1 + (2*(a + b*Tan[c + d*x])^(
1/3))/(a - I*b)^(1/3))/Sqrt[3]] - (a - I*b)^(2/3)*Log[I + Tan[c + d*x]] + 3
*((a - I*b)^(2/3)*Log[(a - I*b)^(1/3) - (a + b*Tan[c + d*x])^(1/3)] + (a +
b*Tan[c + d*x])^(2/3))) + ((-I)*a + b)*(2*Sqrt[3]*(a + I*b)^(2/3)*ArcTan[(1
 + (2*(a + b*Tan[c + d*x])^(1/3))/(a + I*b)^(1/3))/Sqrt[3]] - (a + I*b)^(2/
3)*Log[I - Tan[c + d*x]] + 3*((a + I*b)^(2/3)*Log[(a + I*b)^(1/3) - (a + b*
Tan[c + d*x])^(1/3)] + (a + b*Tan[c + d*x])^(2/3))))/(4*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.22, size = 90, normalized size = 0.27

method	result

derivativedivides	$\frac{3b\left(\frac{(a+b\tan(dx+c))^{\frac{2}{3}}}{2}-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-2a\_Z^3+a^2+b^2\right)}\frac{\left(-2a\_R^4+\left(a^2+b^2\right)\_R\right)\ln\left((a+b\tan(dx+c))^{\frac{1}{3}}-\_R\right)}{\_R^5-\_R^2a}\right)}{6}\right)}{d}$
default	$\frac{3b\left(\frac{(a+b\tan(dx+c))^{\frac{2}{3}}}{2}-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-2a\_Z^3+a^2+b^2\right)}\frac{\left(-2a\_R^4+\left(a^2+b^2\right)\_R\right)\ln\left((a+b\tan(dx+c))^{\frac{1}{3}}-\_R\right)}{\_R^5-\_R^2a}\right)}{6}\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(5/3),x,method=_RETURNVERBOSE)
[Out] 3/d*b*(1/2*(a+b*tan(d*x+c))^(2/3)-1/6*sum((-2*a*_R^4+(a^2+b^2)*_R)/(_R^5-_R
^2*a)*ln((a+b*tan(d*x+c))^(1/3)-_R),_R=RootOf(_Z^6-2*_Z^3*a+a^2+b^2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/3),x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^(5/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))^{\frac{5}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(5/3),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**(5/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 10.09, size = 1540, normalized size = 4.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^(5/3),x)
```

```
[Out] log(((-(a*1i + b)^5/d^3)^(2/3)*(((-(a*1i + b)^5/d^3)^(1/3)*(1944*a*b^4*(-(a
*1i + b)^5/d^3)^(2/3)*(a^2 + b^2) + (1944*b^4*(a^2 - b^2)*(a^2 + b^2)^2*(a
+ b*tan(c + d*x))^(1/3))/d^2))/2 + (1944*a*b^5*(3*a^6 + 3*b^6 - 7*a^2*b^4 -
 7*a^4*b^2))/d^3))/4 + (243*b^5*(3*a^2 - b^2)*(a^2 + b^2)^4*(a + b*tan(c +
d*x))^(1/3))/d^5)*(-(a*b^4*5i + 5*a^4*b + a^5*1i + b^5 - 10*a^2*b^3 - a^3*b
^2*10i)/(8*d^3))^(1/3) + log(((((1944*a*b^4*(a^2 + b^2)*(((a + b*1i)^5*1i)/
d^3)^(2/3) + (1944*b^4*(a^2 - b^2)*(a^2 + b^2)^2*(a + b*tan(c + d*x))^(1/3)
)/d^2)*(((a + b*1i)^5*1i)/d^3)^(1/3))/2 + (1944*a*b^5*(3*a^6 + 3*b^6 - 7*a^
2*b^4 - 7*a^4*b^2))/d^3)*(((a + b*1i)^5*1i)/d^3)^(2/3))/4 + (243*b^5*(3*a^2
 - b^2)*(a^2 + b^2)^4*(a + b*tan(c + d*x))^(1/3))/d^5)*((a*b^4*5i - 5*a^4*b
 + a^5*1i - b^5 + 10*a^2*b^3 - a^3*b^2*10i)/(8*d^3))^(1/3) - log((243*b^5*(
3*a^2 - b^2)*(a^2 + b^2)^4*(a + b*tan(c + d*x))^(1/3))/d^5 - (((3^(1/2)*1i)
/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*((1944*b^4*(a^2 - b^2)*(a^2 + b^2)^2*(a
+ b*tan(c + d*x))^(1/3))/d^2 + 1944*a*b^4*((3^(1/2)*1i)/2 - 1/2)*(a^2 + b^2
)*(((a + b*1i)^5*1i)/d^3)^(2/3))*(((a + b*1i)^5*1i)/d^3)^(1/3))/2 - (1944*a
*b^5*(3*a^6 + 3*b^6 - 7*a^2*b^4 - 7*a^4*b^2))/d^3)*(((a + b*1i)^5*1i)/d^3)^
(2/3))/4)*((3^(1/2)*1i)/2 + 1/2)*((a*b^4*5i - 5*a^4*b + a^5*1i - b^5 + 10*a
^2*b^3 - a^3*b^2*10i)/(8*d^3))^(1/3) + log((243*b^5*(3*a^2 - b^2)*(a^2 + b^
2)^4*(a + b*tan(c + d*x))^(1/3))/d^5 - (((3^(1/2)*1i)/2 + 1/2)*((((3^(1/2)*
1i)/2 - 1/2)*((1944*b^4*(a^2 - b^2)*(a^2 + b^2)^2*(a + b*tan(c + d*x))^(1/3
))/d^2 - 1944*a*b^4*((3^(1/2)*1i)/2 + 1/2)*(a^2 + b^2)*(((a + b*1i)^5*1i)/d
^3)^(2/3))*(((a + b*1i)^5*1i)/d^3)^(1/3))/2 + (1944*a*b^5*(3*a^6 + 3*b^6 -
7*a^2*b^4 - 7*a^4*b^2))/d^3)*(((a + b*1i)^5*1i)/d^3)^(2/3))/4)*((3^(1/2)*1i
)/2 - 1/2)*((a*b^4*5i - 5*a^4*b + a^5*1i - b^5 + 10*a^2*b^3 - a^3*b^2*10i)/
```

```
(8*d^3))^(1/3) - log((243*b^5*(3*a^2 - b^2)*(a^2 + b^2)^4*(a + b*tan(c + d*
x))^(1/3))/d^5 - ((-(a*1i + b)^5/d^3)^(2/3)*((3^(1/2)*1i)/2 - 1/2)*(((-(a*1
i + b)^5/d^3)^(1/3)*((3^(1/2)*1i)/2 + 1/2)*((1944*b^4*(a^2 - b^2)*(a^2 + b^
2)^2*(a + b*tan(c + d*x))^(1/3))/d^2 + 1944*a*b^4*(-(a*1i + b)^5/d^3)^(2/3)
*((3^(1/2)*1i)/2 - 1/2)*(a^2 + b^2)))/2 - (1944*a*b^5*(3*a^6 + 3*b^6 - 7*a^
2*b^4 - 7*a^4*b^2))/d^3))/4)*((3^(1/2)*1i)/2 + 1/2)*(-(a*b^4*5i + 5*a^4*b +
 a^5*1i + b^5 - 10*a^2*b^3 - a^3*b^2*10i)/(8*d^3))^(1/3) + log((243*b^5*(3*
a^2 - b^2)*(a^2 + b^2)^4*(a + b*tan(c + d*x))^(1/3))/d^5 - ((-(a*1i + b)^5/
d^3)^(2/3)*((3^(1/2)*1i)/2 + 1/2)*(((-(a*1i + b)^5/d^3)^(1/3)*((3^(1/2)*1i)
/2 - 1/2)*((1944*b^4*(a^2 - b^2)*(a^2 + b^2)^2*(a + b*tan(c + d*x))^(1/3))/
d^2 - 1944*a*b^4*(-(a*1i + b)^5/d^3)^(2/3)*((3^(1/2)*1i)/2 + 1/2)*(a^2 + b^
2)))/2 + (1944*a*b^5*(3*a^6 + 3*b^6 - 7*a^2*b^4 - 7*a^4*b^2))/d^3))/4)*((3^
(1/2)*1i)/2 - 1/2)*(-(a*b^4*5i + 5*a^4*b + a^5*1i + b^5 - 10*a^2*b^3 - a^3*
b^2*10i)/(8*d^3))^(1/3) + (3*b*(a + b*tan(c + d*x))^(2/3))/(2*d)
```

3.689 $\int (a+b\tan(c+dx))^{4/3}\,dx$

Optimal. Leaf size=327

$$-\frac{1}{4}(a-ib)^{4/3}x-\frac{1}{4}(a+ib)^{4/3}x-\frac{i\sqrt{3}\,(a-ib)^{4/3}\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-ib}}}{\sqrt{3}}\right)}{2d}+\frac{i\sqrt{3}\,(a+ib)^{4/3}\text{ArcTa}}{}$$

```
[Out] -1/4*(a-I*b)^(4/3)*x-1/4*(a+I*b)^(4/3)*x+1/4*I*(a-I*b)^(4/3)*ln(cos(d*x+c))
/d-1/4*I*(a+I*b)^(4/3)*ln(cos(d*x+c))/d+3/4*I*(a-I*b)^(4/3)*ln((a-I*b)^(1/3
)-(a+b*tan(d*x+c))^(1/3))/d-3/4*I*(a+I*b)^(4/3)*ln((a+I*b)^(1/3)-(a+b*tan(d
*x+c))^(1/3))/d-1/2*I*(a-I*b)^(4/3)*arctan(1/3*(1+2*(a+b*tan(d*x+c))^(1/3)/
(a-I*b)^(1/3))*3^(1/2))*3^(1/2)/d+1/2*I*(a+I*b)^(4/3)*arctan(1/3*(1+2*(a+b*
tan(d*x+c))^(1/3)/(a+I*b)^(1/3))*3^(1/2))*3^(1/2)/d+3*b*(a+b*tan(d*x+c))^(1
/3)/d
```

Rubi [A]
time = 0.25, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3563, 3620, 3618, 59, 631, 210, 31}

$$-\frac{i\sqrt{3}\,(a-ib)^{4/3}\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-ib}}}{\sqrt{3}}\right)}{2d}+\frac{i\sqrt{3}\,(a+ib)^{4/3}\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a+ib}}}{\sqrt{3}}\right)}{2d}+\frac{3b\sqrt[3]{a+b\tan(c+dx)}}{d}+\frac{3i(a-ib)^{4/3}\log\left(-\sqrt[3]{a+b\tan(c+dx)}+\sqrt[3]{a-ib}\right)}{4d}-\frac{3i(a+ib)^{4/3}\log\left(-\sqrt[3]{a+b\tan(c+dx)}+\sqrt[3]{a+ib}\right)}{4d}+\frac{i(a-ib)^{4/3}\log(\cos(c+dx))}{4d}-\frac{i(a+ib)^{4/3}\log(\cos(c+dx))}{4d}-\frac{1}{4}x(a-ib)^{4/3}-\frac{1}{4}x(a+ib)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^(4/3),x]
```

```
[Out] -1/4*((a - I*b)^(4/3)*x) - ((a + I*b)^(4/3)*x)/4 - ((I/2)*Sqrt[3]*(a - I*b)
^(4/3)*ArcTan[(1 + (2*(a + b*Tan[c + d*x])^(1/3))/(a - I*b)^(1/3))/Sqrt[3]]
)/d + ((I/2)*Sqrt[3]*(a + I*b)^(4/3)*ArcTan[(1 + (2*(a + b*Tan[c + d*x])^(1
/3))/(a + I*b)^(1/3))/Sqrt[3]])/d + ((I/4)*(a - I*b)^(4/3)*Log[Cos[c + d*x]
])/d - ((I/4)*(a + I*b)^(4/3)*Log[Cos[c + d*x]])/d + (((3*I)/4)*(a - I*b)^(
4/3)*Log[(a - I*b)^(1/3) - (a + b*Tan[c + d*x])^(1/3)])/d - (((3*I)/4)*(a +
 I*b)^(4/3)*Log[(a + I*b)^(1/3) - (a + b*Tan[c + d*x])^(1/3)])/d + (3*b*(a
+ b*Tan[c + d*x])^(1/3))/d
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
```

```
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3563

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (a+b\tan(c+dx))^{4/3}\,dx = \frac{3b\sqrt[3]{a+b\tan(c+dx)}}{d} + \int \frac{a^2-b^2+2ab\tan(c+dx)}{(a+b\tan(c+dx))^{2/3}}\,dx$$

$$= \frac{3b\sqrt[3]{a+b\tan(c+dx)}}{d} + \frac{1}{2}(a-ib)^2 \int \frac{1+i\tan(c+dx)}{(a+b\tan(c+dx))^{2/3}}\,dx + \frac{1}{2}(a+ib)$$

$$= \frac{3b\sqrt[3]{a+b\tan(c+dx)}}{d} + \frac{(i(a-ib)^2)\,\mathrm{Subst}\left(\int \frac{1}{(-1+x)(a-ibx)^{2/3}}\,dx, x, i\tan(c+\right.}{2d}$$

$$= -\frac{1}{4}(a-ib)^{4/3}x - \frac{1}{4}(a+ib)^{4/3}x + \frac{i(a-ib)^{4/3}\log(\cos(c+dx))}{4d} - \frac{i(a+ib)^{4/3}}{}$$

$$= -\frac{1}{4}(a-ib)^{4/3}x - \frac{1}{4}(a+ib)^{4/3}x + \frac{i(a-ib)^{4/3}\log(\cos(c+dx))}{4d} - \frac{i(a+ib)^{4/3}}{}$$

$$= -\frac{1}{4}(a-ib)^{4/3}x - \frac{1}{4}(a+ib)^{4/3}x - \frac{i\sqrt{3}\,(a-ib)^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c-}}{\sqrt[3]{a-ib}}}{\sqrt{3}}\right.}{2d}$$

Mathematica [A]
time = 0.82, size = 365, normalized size = 1.12

$$\frac{(ia+b)\left(2\sqrt[3]{a-ib}\log\left(\sqrt[3]{a-ib}-\sqrt[3]{a+b\tan(c+dx)}\right)-\sqrt[3]{a-ib}\left(2\sqrt{3}\,\mathrm{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-ib}}}{\sqrt{3}}\right)+\log\left((a-ib)^{2/3}+\sqrt[3]{a-ib}\sqrt[3]{a+b\tan(c+dx)}+(a+b\tan(c+dx))^{2/3}\right)\right)+6\sqrt[3]{a+b\tan(c+dx)}\right)-(ia-b)\left(2\sqrt[3]{a+ib}\log\left(\sqrt[3]{a+ib}-\sqrt[3]{a+b\tan(c+dx)}\right)-\sqrt[3]{a+ib}\left(2\sqrt{3}\,\mathrm{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a+ib}}}{\sqrt{3}}\right)+\log\left((a+ib)^{2/3}+\sqrt[3]{a+ib}\sqrt[3]{a+b\tan(c+dx)}+(a+b\tan(c+dx))^{2/3}\right)\right)+6\sqrt[3]{a+b\tan(c+dx)}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^(4/3),x]
```

```
[Out] ((I*a + b)*(2*(a - I*b)^(1/3)*Log[(a - I*b)^(1/3) - (a + b*Tan[c + d*x])^(1
/3)] - (a - I*b)^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*Tan[c + d*x])^(1/3)
)/(a - I*b)^(1/3))/Sqrt[3]] + Log[(a - I*b)^(2/3) + (a - I*b)^(1/3)*(a + b*
Tan[c + d*x])^(1/3) + (a + b*Tan[c + d*x])^(2/3)]) + 6*(a + b*Tan[c + d*x])
^(1/3)) - (I*a - b)*(2*(a + I*b)^(1/3)*Log[(a + I*b)^(1/3) - (a + b*Tan[c +
 d*x])^(1/3)] - (a + I*b)^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*Tan[c + d*
x])^(1/3))/(a + I*b)^(1/3))/Sqrt[3]] + Log[(a + I*b)^(2/3) + (a + I*b)^(1/3
)*(a + b*Tan[c + d*x])^(1/3) + (a + b*Tan[c + d*x])^(2/3)]) + 6*(a + b*Tan[
c + d*x])^(1/3)))/(4*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.21, size = 89, normalized size = 0.27

method	result	si

derivativedivides	$\frac{3b\left((a+b\tan(dx+c))^{\frac{1}{3}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-2a\_Z^{3}+a^{2}+b^{2}\right)}\frac{\left(2\_R^{3}a-a^{2}-b^{2}\right)\ln\left((a+b\tan(dx+c))^{\frac{1}{3}}-\_R\right)}{\_R^{5}-\_R^{2}a}\right)}{6}\right)}{d}$	89
default	$\frac{3b\left((a+b\tan(dx+c))^{\frac{1}{3}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-2a\_Z^{3}+a^{2}+b^{2}\right)}\frac{\left(2\_R^{3}a-a^{2}-b^{2}\right)\ln\left((a+b\tan(dx+c))^{\frac{1}{3}}-\_R\right)}{\_R^{5}-\_R^{2}a}\right)}{6}\right)}{d}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(4/3),x,method=_RETURNVERBOSE)
[Out] 3/d*b*((a+b*tan(d*x+c))^(1/3)+1/6*sum((2*_R^3*a-a^2-b^2)/(_R^5-_R^2*a)*ln((
a+b*tan(d*x+c))^(1/3)-_R),_R=RootOf(_Z^6-2*_Z^3*a+a^2+b^2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(4/3),x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^(4/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 16583 vs. $2(235) = 470$.
time = 103.07, size = 16583, normalized size = 50.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(4/3),x, algorithm="fricas")
[Out] 1/4*(2*d*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^6)^(1/6)*cos(2/
3*arctan((d^6*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^6) + 4
*(a^3*b - a*b^3)*d^3)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^
8)/d^6)/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)))*log(2*(a^7 - 5
*a^5*b^2 - 5*a^3*b^4 + a*b^6)*d^4*((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*
x + c))^(1/3)*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^6)^(1/6)*s
```

qrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^6)*sin(2/3*arctan(
(d^6*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^6) + 4*(a^3*b -
a*b^3)*d^3)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^6)/(
a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))) - 2*(a^10*b - 11*a^8*b^
3 + 26*a^6*b^5 + 26*a^4*b^7 - 11*a^2*b^9 + b^11)*d*((a*cos(d*x + c) + b*sin
(d*x + c))/cos(d*x + c))^(1/3)*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 +
b^8)/d^6)^(1/6)*cos(2/3*arctan((d^6*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a
^2*b^6 + b^8)/d^6) + 4*(a^3*b - a*b^3)*d^3)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4
*b^4 - 12*a^2*b^6 + b^8)/d^6)/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 +
b^8))) + (a^10 - 11*a^8*b^2 + 26*a^6*b^4 + 26*a^4*b^6 - 11*a^2*b^8 + b^10)
d^2((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^6)^(1/3) + (a^12 -
10*a^10*b^2 + 15*a^8*b^4 + 52*a^6*b^6 + 15*a^4*b^8 - 10*a^2*b^10 + b^12)*((
a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))^(2/3)) - 8*d*((a^8 + 4*a^6*b
^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^6)^(1/6)*arctan(-((a^7 - 5*a^5*b^2 - 5*
a^3*b^4 + a*b^6)*d^8*((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))^(1/3)
*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^6)^(5/6)*sqrt((a^8 - 12
*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^6)*cos(2/3*arctan((d^6*sqrt((a^
8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^6) + 4*(a^3*b - a*b^3)*d^3)*
sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^6)/(a^8 - 12*a^6*
b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))) - (a^13*b - 2*a^11*b^3 - 17*a^9*b^5
- 28*a^7*b^7 - 17*a^5*b^9 - 2*a^3*b^11 + a*b^13)*d^3*sqrt((a^8 - 12*a^6*b^2
+ 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^6) - ((a^10*b - 11*a^8*b^3 + 26*a^6*b^5
+ 26*a^4*b^7 - 11*a^2*b^9 + b^11)*d^5*((a*cos(d*x + c) + b*sin(d*x + c))/c
os(d*x + c))^(1/3)*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^6)^(5
/6) - (a^18 - 7*a^16*b^2 - 12*a^14*b^4 + 68*a^12*b^6 + 206*a^10*b^8 + 206*a
^8*b^10 + 68*a^6*b^12 - 12*a^4*b^14 - 7*a^2*b^16 + b^18)*cos(2/3*arctan((d^
6*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^6) + 4*(a^3*b - a*
b^3)*d^3)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^6)/(a^8
- 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))))*sin(2/3*arctan((d^6*sqrt(
(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^6) + 4*(a^3*b - a*b^3)*d^
3)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^6)/(a^8 - 12*a
^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))) - (a*d^8*((a^8 + 4*a^6*b^2 + 6*a^
4*b^4 + 4*a^2*b^6 + b^8)/d^6)^(5/6)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 1
2*a^2*b^6 + b^8)/d^6)*cos(2/3*arctan((d^6*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4
+ 4*a^2*b^6 + b^8)/d^6) + 4*(a^3*b - a*b^3)*d^3)*sqrt((a^8 - 12*a^6*b^2 +
38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^6)/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2
*b^6 + b^8))) - (a^4*b - 6*a^2*b^3 + b^5)*d^5*((a^8 + 4*a^6*b^2 + 6*a^4*b^4
+ 4*a^2*b^6 + b^8)/d^6)^(5/6)*sin(2/3*arctan((d^6*sqrt((a^8 + 4*a^6*b^2 +
6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^6) + 4*(a^3*b - a*b^3)*d^3)*sqrt((a^8 - 12*a
^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^6)/(a^8 - 12*a^6*b^2 + 38*a^4*b^4
- 12*a^2*b^6 + b^8))))*sqrt(2*(a^7 - 5*a^5*b^2 - 5*a^3*b^4 + a*b^6)*d^4*((
a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))^(1/3)*((a^8 + 4*a^6*b^2 + 6*
a^4*b^4 + 4*a^2*b^6 + b^8)/d^6)^(1/6)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 -
12*a^2*b^6 + b^8)/d^6)*sin(2/3*arctan((d^6*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b
^4 + 4*a^2*b^6 + b^8)/d^6) + 4*(a^3*b - a*b^3)*d^3)*sqrt((a^8 - 12*a^6*b^2

```
+ 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^6)/(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a
^2*b^6 + b^8))) - 2*(a^10*b - 11*a^8*b^3 + 26*a^6*b^5 + 26*a^4*b^7 - 11*a^2
*b^9 + b^11)*d*((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))^(1/3)*((a^8
 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^6)^(1/6)*cos(2/3*arctan((d^6*
sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^6) + 4*(a^3*b - a*b^
3)*d^3)*sqrt((a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^6)/(a^8 -
 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8))) + (a^10 - 11*a^8*b^2 + 26*a^
6*b^4 + 26*a^4*b^6 - 11*a^2*b^8 + b^10)*d^2*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
 4*a^2*b^6 + b^8)/d^6)^(1/3) + (a^12 - 10*a^10*b^2 + 15*a^8*b^4 + 52*a^6*b^
6 + 15*a^4*b^8 - 10*a^2*b^10 + b^12)*((a*cos(d*x + c) + b*sin(d*x + c))/cos
(d*x + c))^(2/3)))/(a^16*b^2 - 8*a^14*b^4 - 4*a^12*b^6 + 72*a^10*b^8 + 134*
a^8*b^10 + 72*a^6*b^12 - 4*a^4*b^14 - 8*a^2*b^16 + b^18 - (a^18 - 7*a^16*b^
2 - 12*a^14*b^4 + 68*a^12*b^6 + 206*a^10*b^8 + 206*a^8*b^10 + 68*a^6*b^12 -
 12*a^4*b^14 - 7*a^2*b^16 + b^18)*cos(2/3*arctan((d^6*sqrt((a^8 + 4*a^6*b^2
 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/d^6) + 4*(a^3*b - a*b^3)*d^3)*sqrt((a^8 - 1
2*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/d^6)...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx))^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(4/3),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 7.50, size = 924, normalized size = 2.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^(4/3),x)
```

```
[Out] log(a*(a + b*tan(c + d*x))^(1/3) - b*(a + b*tan(c + d*x))^(1/3)*1i + d*(-((
a - b*1i)^4*1i)/d^3)^(1/3)*1i)*(-(4*a^3*b - 4*a*b^3 + a^4*1i + b^4*1i - a^2
*b^2*6i)/(8*d^3))^(1/3) + log(a*(a + b*tan(c + d*x))^(1/3)*1i - b*(a + b*ta
n(c + d*x))^(1/3) + d*(((a*1i - b)^4*1i)/d^3)^(1/3))*((4*a*b^3 - 4*a^3*b +
a^4*1i + b^4*1i - a^2*b^2*6i)/(8*d^3))^(1/3) + log((b^4*((3^(1/2)*1i)/2 - 1
/2)*(a - b*1i)^2*(((a*1i - b)^4*1i)/d^3)^(1/3)*(a*b^4 + a^4*b*1i + a^5 + b^
5*1i - a^2*b^3*6i - 6*a^3*b^2)*486i)/d^3 - (486*b^4*(a^2 + b^2)^2*(a + b*ta
n(c + d*x))^(1/3)*(a^4 + b^4 - 6*a^2*b^2))/d^4)*((3^(1/2)*1i)/2 - 1/2)*((4*
a*b^3 - 4*a^3*b + a^4*1i + b^4*1i - a^2*b^2*6i)/(8*d^3))^(1/3) - log((486*b
^4*(a^2 + b^2)^2*(a + b*tan(c + d*x))^(1/3)*(a^4 + b^4 - 6*a^2*b^2))/d^4 +
(b^4*((3^(1/2)*1i)/2 + 1/2)*(a - b*1i)^2*(((a*1i - b)^4*1i)/d^3)^(1/3)*(a*b
^4 + a^4*b*1i + a^5 + b^5*1i - a^2*b^3*6i - 6*a^3*b^2)*486i)/d^3)*((3^(1/2)
*1i)/2 + 1/2)*((4*a*b^3 - 4*a^3*b + a^4*1i + b^4*1i - a^2*b^2*6i)/(8*d^3))^
(1/3) + (3*b*(a + b*tan(c + d*x))^(1/3))/d + log(- (486*b^4*(a^2 + b^2)^2*(
a + b*tan(c + d*x))^(1/3)*(a^4 + b^4 - 6*a^2*b^2))/d^4 - (486*b^4*((3^(1/2)
*1i)/2 - 1/2)*(a + b*1i)^2*(-((a - b*1i)^4*1i)/d^3)^(1/3)*(a*b^4*1i + a^4*b
 + a^5*1i + b^5 - 6*a^2*b^3 - a^3*b^2*6i))/d^3)*((3^(1/2)*1i)/2 - 1/2)*(-(4
*a^3*b - 4*a*b^3 + a^4*1i + b^4*1i - a^2*b^2*6i)/(8*d^3))^(1/3) - log((486*
b^4*(a^2 + b^2)^2*(a + b*tan(c + d*x))^(1/3)*(a^4 + b^4 - 6*a^2*b^2))/d^4 -
 (486*b^4*((3^(1/2)*1i)/2 + 1/2)*(a + b*1i)^2*(-((a - b*1i)^4*1i)/d^3)^(1/3
)*(a*b^4*1i + a^4*b + a^5*1i + b^5 - 6*a^2*b^3 - a^3*b^2*6i))/d^3)*((3^(1/2
)*1i)/2 + 1/2)*(-(4*a^3*b - 4*a*b^3 + a^4*1i + b^4*1i - a^2*b^2*6i)/(8*d^3)
)^(1/3)
```

3.690 $\int (a+b\tan(c+dx))^{2/3}\,dx$

Optimal. Leaf size=415

$$-\frac{1}{4}\left(a-\sqrt{-b^2}\right)^{2/3}x-\frac{1}{4}\left(a+\sqrt{-b^2}\right)^{2/3}x-\frac{\sqrt{3}\,b\left(a-\sqrt{-b^2}\right)^{2/3}\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-\sqrt{-b^2}}}}{\sqrt{3}}\right)}{2\sqrt{-b^2}\,d}+-$$

```
[Out] -1/4*x*(a-(-b^2)^(1/2))^(2/3)-1/4*b*ln(cos(d*x+c))*(a-(-b^2)^(1/2))^(2/3)/d
/(-b^2)^(1/2)-3/4*b*ln((a-(-b^2)^(1/2))^(1/3)-(a+b*tan(d*x+c))^(1/3))*(a-(-
b^2)^(1/2))^(2/3)/d/(-b^2)^(1/2)-1/2*b*arctan(1/3*(1+2*(a+b*tan(d*x+c))^(1/
3)/(a-(-b^2)^(1/2))^(1/3))*3^(1/2))*3^(1/2)*(a-(-b^2)^(1/2))^(2/3)/d/(-b^2)
^(1/2)-1/4*x*(a+(-b^2)^(1/2))^(2/3)+1/4*b*ln(cos(d*x+c))*(a+(-b^2)^(1/2))^(
2/3)/d/(-b^2)^(1/2)+3/4*b*ln((a+(-b^2)^(1/2))^(1/3)-(a+b*tan(d*x+c))^(1/3))
*(a+(-b^2)^(1/2))^(2/3)/d/(-b^2)^(1/2)+1/2*b*arctan(1/3*(1+2*(a+b*tan(d*x+c
))^(1/3)/(a+(-b^2)^(1/2))^(1/3))*3^(1/2))*3^(1/2)*(a+(-b^2)^(1/2))^(2/3)/d/
(-b^2)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 415, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$,
Rules used = {3566, 726, 52, 57, 631, 210, 31}

$$-\frac{\sqrt{3}\,b\left(a-\sqrt{-b^2}\right)^{2/3}\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-\sqrt{-b^2}}}+1}{\sqrt{3}}\right)}{2\sqrt{-b^2}\,d}+\frac{\sqrt{3}\,b\left(a+\sqrt{-b^2}\right)^{2/3}\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a+\sqrt{-b^2}}}+1}{\sqrt{3}}\right)}{2\sqrt{-b^2}\,d}-\frac{3b\left(a-\sqrt{-b^2}\right)^{2/3}\log\left(\sqrt[3]{a-\sqrt{-b^2}}-\sqrt[3]{a+b\tan(c+dx)}\right)}{4\sqrt{-b^2}\,d}+\frac{3b\left(a+\sqrt{-b^2}\right)^{2/3}\log\left(\sqrt[3]{a+\sqrt{-b^2}}-\sqrt[3]{a+b\tan(c+dx)}\right)}{4\sqrt{-b^2}\,d}-\frac{b\left(a-\sqrt{-b^2}\right)^{2/3}\log(\cos(c+dx))}{4\sqrt{-b^2}\,d}+\frac{b\left(a+\sqrt{-b^2}\right)^{2/3}\log(\cos(c+dx))}{4\sqrt{-b^2}\,d}-\frac{1}{4}x\left(a-\sqrt{-b^2}\right)^{2/3}-\frac{1}{4}x\left(a+\sqrt{-b^2}\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^(2/3),x]
```

```
[Out] -1/4*((a - Sqrt[-b^2])^(2/3)*x) - ((a + Sqrt[-b^2])^(2/3)*x)/4 - (Sqrt[3]*b
*(a - Sqrt[-b^2])^(2/3)*ArcTan[(1 + (2*(a + b*Tan[c + d*x])^(1/3))/(a - Sqr
t[-b^2])^(1/3))/Sqrt[3]])/(2*Sqrt[-b^2]*d) + (Sqrt[3]*b*(a + Sqrt[-b^2])^(2
/3)*ArcTan[(1 + (2*(a + b*Tan[c + d*x])^(1/3))/(a + Sqrt[-b^2])^(1/3))/Sqrt
[3]])/(2*Sqrt[-b^2]*d) - (b*(a - Sqrt[-b^2])^(2/3)*Log[Cos[c + d*x]])/(4*Sq
rt[-b^2]*d) + (b*(a + Sqrt[-b^2])^(2/3)*Log[Cos[c + d*x]])/(4*Sqrt[-b^2]*d)
 - (3*b*(a - Sqrt[-b^2])^(2/3)*Log[(a - Sqrt[-b^2])^(1/3) - (a + b*Tan[c +
d*x])^(1/3)])/(4*Sqrt[-b^2]*d) + (3*b*(a + Sqrt[-b^2])^(2/3)*Log[(a + Sqrt[
-b^2])^(1/3) - (a + b*Tan[c + d*x])^(1/3)])/(4*Sqrt[-b^2]*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 726

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a+b\tan(c+dx))^{2/3}\,dx = \frac{b\text{Subst}\left(\int \frac{(a+x)^{2/3}}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \left(\frac{\sqrt{-b^2}\,(a+x)^{2/3}}{2b^2\left(\sqrt{-b^2}-x\right)} + \frac{\sqrt{-b^2}\,(a+x)^{2/3}}{2b^2\left(\sqrt{-b^2}+x\right)}\right) dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{b\text{Subst}\left(\int \frac{(a+x)^{2/3}}{\sqrt{-b^2}-x}\,dx, x, b\tan(c+dx)\right)}{2\sqrt{-b^2}\,d} - \frac{b\text{Subst}\left(\int \frac{(a+x)^{2/3}}{\sqrt{-b^2}+x}\,dx, x, b\tan\right.}{2\sqrt{-b^2}\,d}$$

$$= -\frac{\left(b\left(a+\sqrt{-b^2}\right)\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-b^2}-x\right)\sqrt[3]{a+x}}\,dx, x, b\tan(c+dx)\right)}{2\sqrt{-b^2}\,d} + \frac{\left(b^2\right.}{}$$

$$= -\frac{1}{4}\left(a-\sqrt{-b^2}\right)^{2/3}x - \frac{1}{4}\left(a+\sqrt{-b^2}\right)^{2/3}x + \frac{\sqrt{-b^2}\left(a-\sqrt{-b^2}\right)^{2/3}\log(\cos}{4bd}$$

$$= -\frac{1}{4}\left(a-\sqrt{-b^2}\right)^{2/3}x - \frac{1}{4}\left(a+\sqrt{-b^2}\right)^{2/3}x + \frac{\sqrt{-b^2}\left(a-\sqrt{-b^2}\right)^{2/3}\log(\cos}{4bd}$$

$$= -\frac{1}{4}\left(a-\sqrt{-b^2}\right)^{2/3}x - \frac{1}{4}\left(a+\sqrt{-b^2}\right)^{2/3}x + \frac{\sqrt{3}\sqrt{-b^2}\left(a-\sqrt{-b^2}\right)^{2/3}\tan}{2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.31, size = 224, normalized size = 0.54

$$\frac{\frac{(ia+b)\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-ib}}}{\sqrt{3}}\right)-\log(i+\tan(c+dx))+3\log\left(\sqrt[3]{a-ib}-\sqrt[3]{a+b\tan(c+dx)}\right)\right)}{\sqrt[3]{a-ib}}+\frac{(-ia+b)\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a+ib}}}{\sqrt{3}}\right)-\log(i-\tan(c+dx))+3\log\left(\sqrt[3]{a+ib}-\sqrt[3]{a+b\tan(c+dx)}\right)\right)}{\sqrt[3]{a+ib}}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^(2/3),x]
```

```
[Out] (((I*a + b)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*Tan[c + d*x])^(1/3))/(a - I*b)
^(1/3))/Sqrt[3]] - Log[I + Tan[c + d*x]] + 3*Log[(a - I*b)^(1/3) - (a + b*T
an[c + d*x])^(1/3)]))/(a - I*b)^(1/3) + (((-I)*a + b)*(2*Sqrt[3]*ArcTan[(1
+ (2*(a + b*Tan[c + d*x])^(1/3))/(a + I*b)^(1/3))/Sqrt[3]] - Log[I - Tan[c
+ d*x]] + 3*Log[(a + I*b)^(1/3) - (a + b*Tan[c + d*x])^(1/3)]))/(a + I*b)^(
1/3))/(4*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.32, size = 60, normalized size = 0.14

method	result	size
derivativedivides	$\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-2a\_Z^3+a^2+b^2\right)}\frac{\_R^4\ln\left((a+b\tan(dx+c))^{\frac{1}{3}}-\_R\right)}{\_R^5-\_R^2a}\right)}{2d}$	60
default	$\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-2a\_Z^3+a^2+b^2\right)}\frac{\_R^4\ln\left((a+b\tan(dx+c))^{\frac{1}{3}}-\_R\right)}{\_R^5-\_R^2a}\right)}{2d}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*b*sum(_R^4/(_R^5-_R^2*a)*ln((a+b*tan(d*x+c))^(1/3)-_R),_R=RootOf(_Z^6
-2*_Z^3*a+a^2+b^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(2/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tan (c + dx))^{\frac{2}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(2/3),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 8.22, size = 1229, normalized size = 2.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^(2/3),x)
```

```
[Out] log((((((1944*b^4*(a^2 + b^2)^2*(a + b*tan(c + d*x))^(1/3))/d^2 + 1944*a*b^
4*(a^2 + b^2)*(-((a - b*1i)^2*1i)/d^3)^(2/3))*(-((a - b*1i)^2*1i)/d^3)^(1/3
))/2 + (972*b^5*(3*a^4 - b^4 + 2*a^2*b^2))/d^3)*(-((a - b*1i)^2*1i)/d^3)^(2
/3))/4 + (486*a*b^5*(a^2 + b^2)^2*(a + b*tan(c + d*x))^(1/3))/d^5)*(-(2*a*b
 + a^2*1i - b^2*1i)/(8*d^3))^(1/3) + log(((((1944*a*b^4*(a^2 + b^2)*(-((a*1
i - b)^2*1i)/d^3)^(2/3) + (1944*b^4*(a^2 + b^2)^2*(a + b*tan(c + d*x))^(1/3
))/d^2)*(-((a*1i - b)^2*1i)/d^3)^(1/3))/2 + (972*b^5*(3*a^4 - b^4 + 2*a^2*b
^2))/d^3)*(-((a*1i - b)^2*1i)/d^3)^(2/3))/4 + (486*a*b^5*(a^2 + b^2)^2*(a +
 b*tan(c + d*x))^(1/3))/d^5)*(-(2*a*b - a^2*1i + b^2*1i)/(8*d^3))^(1/3) - l
og((((3^(1/2)*1i)/2 - 1/2)*((972*b^5*(3*a^4 - b^4 + 2*a^2*b^2))/d^3 - (((3^
(1/2)*1i)/2 + 1/2)*((1944*b^4*(a^2 + b^2)^2*(a + b*tan(c + d*x))^(1/3))/d^2
 + 1944*a*b^4*((3^(1/2)*1i)/2 - 1/2)*(a^2 + b^2)*(-((a - b*1i)^2*1i)/d^3)^(
2/3))*(-((a - b*1i)^2*1i)/d^3)^(1/3))/2)*(-((a - b*1i)^2*1i)/d^3)^(2/3))/4
+ (486*a*b^5*(a^2 + b^2)^2*(a + b*tan(c + d*x))^(1/3))/d^5)*((3^(1/2)*1i)/2
 + 1/2)*(-(2*a*b + a^2*1i - b^2*1i)/(8*d^3))^(1/3) + log((486*a*b^5*(a^2 +
b^2)^2*(a + b*tan(c + d*x))^(1/3))/d^5 - (((3^(1/2)*1i)/2 + 1/2)*((972*b^5*
(3*a^4 - b^4 + 2*a^2*b^2))/d^3 + (((3^(1/2)*1i)/2 - 1/2)*((1944*b^4*(a^2 +
```

```
b^2)^2*(a + b*tan(c + d*x))^(1/3))/d^2 - 1944*a*b^4*((3^(1/2)*1i)/2 + 1/2)*
(a^2 + b^2)*(-((a - b*1i)^2*1i)/d^3)^(2/3))*(-((a - b*1i)^2*1i)/d^3)^(1/3))
/2)*(-((a - b*1i)^2*1i)/d^3)^(2/3))/4)*((3^(1/2)*1i)/2 - 1/2)*(-(2*a*b + a^
2*1i - b^2*1i)/(8*d^3))^(1/3) - log((486*a*b^5*(a^2 + b^2)^2*(a + b*tan(c +
 d*x))^(1/3))/d^5 - (((3^(1/2)*1i)/2 - 1/2)*((((1944*b^4*(a^2 + b^2)^2*(a +
 b*tan(c + d*x))^(1/3))/d^2 + 1944*a*b^4*((3^(1/2)*1i)/2 - 1/2)*(a^2 + b^2)
*(-((a*1i - b)^2*1i)/d^3)^(2/3))*((3^(1/2)*1i)/2 + 1/2)*(-((a*1i - b)^2*1i)
/d^3)^(1/3))/2 - (972*b^5*(3*a^4 - b^4 + 2*a^2*b^2))/d^3)*(-((a*1i - b)^2*1
i)/d^3)^(2/3))/4)*((3^(1/2)*1i)/2 + 1/2)*(-(2*a*b - a^2*1i + b^2*1i)/(8*d^3
))^(1/3) + log((486*a*b^5*(a^2 + b^2)^2*(a + b*tan(c + d*x))^(1/3))/d^5 - (
((3^(1/2)*1i)/2 + 1/2)*((((1944*b^4*(a^2 + b^2)^2*(a + b*tan(c + d*x))^(1/3
))/d^2 - 1944*a*b^4*((3^(1/2)*1i)/2 + 1/2)*(a^2 + b^2)*(-((a*1i - b)^2*1i)/
d^3)^(2/3))*((3^(1/2)*1i)/2 - 1/2)*(-((a*1i - b)^2*1i)/d^3)^(1/3))/2 + (972
*b^5*(3*a^4 - b^4 + 2*a^2*b^2))/d^3)*(-((a*1i - b)^2*1i)/d^3)^(2/3))/4)*((3
^(1/2)*1i)/2 - 1/2)*(-(2*a*b - a^2*1i + b^2*1i)/(8*d^3))^(1/3)
```

3.691 $\int \sqrt[3]{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=415

$$-\frac{1}{4}\sqrt[3]{a-\sqrt{-b^2}}\,x-\frac{1}{4}\sqrt[3]{a+\sqrt{-b^2}}\,x+\frac{\sqrt{3}\,b\sqrt[3]{a-\sqrt{-b^2}}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-\sqrt{-b^2}}}}{\sqrt{3}}\right)}{2\sqrt{-b^2}\,d}-\frac{\sqrt{3}\,b\sqrt[3]{a-}\cdots}{\cdots}$$

```
[Out] -1/4*x*(a-(-b^2)^(1/2))^(1/3)-1/4*b*ln(cos(d*x+c))*(a-(-b^2)^(1/2))^(1/3)/d
/(-b^2)^(1/2)-3/4*b*ln((a-(-b^2)^(1/2))^(1/3)-(a+b*tan(d*x+c))^(1/3))*(a-(-
b^2)^(1/2))^(1/3)/d/(-b^2)^(1/2)+1/2*b*arctan(1/3*(1+2*(a+b*tan(d*x+c))^(1/
3)/(a-(-b^2)^(1/2))^(1/3))*3^(1/2))*3^(1/2)*(a-(-b^2)^(1/2))^(1/3)/d/(-b^2)
^(1/2)-1/4*x*(a+(-b^2)^(1/2))^(1/3)+1/4*b*ln(cos(d*x+c))*(a+(-b^2)^(1/2))^(
1/3)/d/(-b^2)^(1/2)+3/4*b*ln((a+(-b^2)^(1/2))^(1/3)-(a+b*tan(d*x+c))^(1/3))
*(a+(-b^2)^(1/2))^(1/3)/d/(-b^2)^(1/2)-1/2*b*arctan(1/3*(1+2*(a+b*tan(d*x+c
))^(1/3)/(a+(-b^2)^(1/2))^(1/3))*3^(1/2))*3^(1/2)*(a+(-b^2)^(1/2))^(1/3)/d/
(-b^2)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 415, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3566, 726, 52, 59, 631, 210, 31}

$$\frac{\sqrt{3}\,b\sqrt[3]{a-\sqrt{-b^2}}\,\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-\sqrt{-b^2}}}+1}{\sqrt{3}}\right)}{2\sqrt{-b^2}\,d}-\frac{\sqrt{3}\,b\sqrt[3]{a+\sqrt{-b^2}}\,\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a+\sqrt{-b^2}}}+1}{\sqrt{3}}\right)}{2\sqrt{-b^2}\,d}-\frac{3b\sqrt[3]{a-\sqrt{-b^2}}\log\left(\sqrt[3]{a-\sqrt{-b^2}}-\sqrt[3]{a+b\tan(c+dx)}\right)}{4\sqrt{-b^2}\,d}+\frac{3b\sqrt[3]{a+\sqrt{-b^2}}\log\left(\sqrt[3]{a+\sqrt{-b^2}}-\sqrt[3]{a+b\tan(c+dx)}\right)}{4\sqrt{-b^2}\,d}-\frac{b\sqrt[3]{a-\sqrt{-b^2}}\log(\cos(c+dx))}{4\sqrt{-b^2}\,d}+\frac{b\sqrt[3]{a+\sqrt{-b^2}}\log(\cos(c+dx))}{4\sqrt{-b^2}\,d}-\frac{1}{4}x\sqrt[3]{a-\sqrt{-b^2}}-\frac{1}{4}x\sqrt[3]{a+\sqrt{-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^(1/3),x]
```

```
[Out] -1/4*((a - Sqrt[-b^2])^(1/3)*x) - ((a + Sqrt[-b^2])^(1/3)*x)/4 + (Sqrt[3]*b
*(a - Sqrt[-b^2])^(1/3)*ArcTan[(1 + (2*(a + b*Tan[c + d*x])^(1/3))/(a - Sqr
t[-b^2])^(1/3))/Sqrt[3]])/(2*Sqrt[-b^2]*d) - (Sqrt[3]*b*(a + Sqrt[-b^2])^(1
/3)*ArcTan[(1 + (2*(a + b*Tan[c + d*x])^(1/3))/(a + Sqrt[-b^2])^(1/3))/Sqrt
[3]])/(2*Sqrt[-b^2]*d) - (b*(a - Sqrt[-b^2])^(1/3)*Log[Cos[c + d*x]])/(4*Sq
rt[-b^2]*d) + (b*(a + Sqrt[-b^2])^(1/3)*Log[Cos[c + d*x]])/(4*Sqrt[-b^2]*d)
 - (3*b*(a - Sqrt[-b^2])^(1/3)*Log[(a - Sqrt[-b^2])^(1/3) - (a + b*Tan[c +
d*x])^(1/3)])/(4*Sqrt[-b^2]*d) + (3*b*(a + Sqrt[-b^2])^(1/3)*Log[(a + Sqrt[
-b^2])^(1/3) - (a + b*Tan[c + d*x])^(1/3)])/(4*Sqrt[-b^2]*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 726

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \sqrt[3]{a+b\tan(c+dx)}\,dx = \frac{b\text{Subst}\left(\int \frac{\sqrt[3]{a+x}}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \left(\frac{\sqrt{-b^2}\sqrt[3]{a+x}}{2b^2\left(\sqrt{-b^2}-x\right)} + \frac{\sqrt{-b^2}\sqrt[3]{a+x}}{2b^2\left(\sqrt{-b^2}+x\right)}\right) dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{b\text{Subst}\left(\int \frac{\sqrt[3]{a+x}}{\sqrt{-b^2}-x}\,dx, x, b\tan(c+dx)\right)}{2\sqrt{-b^2}\,d} - \frac{b\text{Subst}\left(\int \frac{\sqrt[3]{a+x}}{\sqrt{-b^2}+x}\,dx, x, b\tan(\right.}{2\sqrt{-b^2}\,d}$$

$$= -\frac{\left(b\left(a+\sqrt{-b^2}\right)\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-b^2}-x\right)(a+x)^{2/3}}\,dx, x, b\tan(c+dx)\right)}{2\sqrt{-b^2}\,d} + \frac{\left(b^2 + \right.}{}$$

$$= -\frac{1}{4}\sqrt[3]{a-\sqrt{-b^2}}\,x - \frac{1}{4}\sqrt[3]{a+\sqrt{-b^2}}\,x + \frac{\sqrt{-b^2}\sqrt[3]{a-\sqrt{-b^2}}\,\log(\cos(c+dx}{4bd}$$

$$= -\frac{1}{4}\sqrt[3]{a-\sqrt{-b^2}}\,x - \frac{1}{4}\sqrt[3]{a+\sqrt{-b^2}}\,x + \frac{\sqrt{-b^2}\sqrt[3]{a-\sqrt{-b^2}}\,\log(\cos(c+dx}{4bd}$$

$$= -\frac{1}{4}\sqrt[3]{a-\sqrt{-b^2}}\,x - \frac{1}{4}\sqrt[3]{a+\sqrt{-b^2}}\,x - \frac{\sqrt{3}\sqrt{-b^2}\sqrt[3]{a-\sqrt{-b^2}}\,\tan^{-1}\left(\frac{1+}{}\right.}{2bd}$$

<u>**Mathematica**</u> [C] Result contains complex when optimal does not.
time = 0.28, size = 294, normalized size = 0.71

$$\frac{-i\sqrt[3]{a-ib}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-ib}}}{\sqrt{3}}\right) - 2\log\left(\sqrt[3]{a-ib}-\sqrt[3]{a+b\tan(c+dx)}\right) + \log\left((a-ib)^{2/3}+\sqrt[3]{a-ib}\sqrt[3]{a+b\tan(c+dx)}+(a+b\tan(c+dx))^{2/3}\right)\right) + i\sqrt[3]{a+ib}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a+ib}}}{\sqrt{3}}\right) - 2\log\left(\sqrt[3]{a+ib}-\sqrt[3]{a+b\tan(c+dx)}\right) + \log\left((a+ib)^{2/3}+\sqrt[3]{a+ib}\sqrt[3]{a+b\tan(c+dx)}+(a+b\tan(c+dx))^{2/3}\right)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^(1/3),x]
```

```
[Out] ((-I)*(a - I*b)^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*Tan[c + d*x])^(1/3))
/(a - I*b)^(1/3))/Sqrt[3]] - 2*Log[(a - I*b)^(1/3) - (a + b*Tan[c + d*x])^(
1/3)] + Log[(a - I*b)^(2/3) + (a - I*b)^(1/3)*(a + b*Tan[c + d*x])^(1/3) +
(a + b*Tan[c + d*x])^(2/3)]) + I*(a + I*b)^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*
(a + b*Tan[c + d*x])^(1/3))/(a + I*b)^(1/3))/Sqrt[3]] - 2*Log[(a + I*b)^(1/
3) - (a + b*Tan[c + d*x])^(1/3)] + Log[(a + I*b)^(2/3) + (a + I*b)^(1/3)*(a
 + b*Tan[c + d*x])^(1/3) + (a + b*Tan[c + d*x])^(2/3)]))/(4*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.40, size = 60, normalized size = 0.14

method	result	size
derivativedivides	$\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-2a\_Z^{3}+a^{2}+b^{2}\right)}\frac{\_R^{3}\ln\left(\left(a+b\tan\left(dx+c\right)\right)^{\frac{1}{3}}-\_R\right)}{\_R^{5}-\_R^{2}a}\right)}{2d}$	60
default	$\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-2a\_Z^{3}+a^{2}+b^{2}\right)}\frac{\_R^{3}\ln\left(\left(a+b\tan\left(dx+c\right)\right)^{\frac{1}{3}}-\_R\right)}{\_R^{5}-\_R^{2}a}\right)}{2d}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*b*sum(_R^3/(_R^5-_R^2*a)*ln((a+b*tan(d*x+c))^(1/3)-_R),_R=RootOf(_Z^6
-2*_Z^3*a+a^2+b^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(1/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3320 vs. 2(325) = 650.
time = 1.55, size = 3320, normalized size = 8.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*((a^2 + b^2)/d^6)^(1/6)*cos(2/3*arctan((d^6*sqrt(a^2/d^6)*sqrt((a^2 + b
^2)/d^6) + b*d^3*sqrt(a^2/d^6))/a^2))*log(2*a*d^4*((a*cos(d*x + c) + b*sin(
```

d*x + c))/cos(d*x + c))^(1/3)*sqrt(a^2/d^6)*((a^2 + b^2)/d^6)^(1/6)*sin(2/3
*arctan((d^6*sqrt(a^2/d^6)*sqrt((a^2 + b^2)/d^6) + b*d^3*sqrt(a^2/d^6))/a^2
)) + a^2*d^2*((a^2 + b^2)/d^6)^(1/3) + a^2*((a*cos(d*x + c) + b*sin(d*x + c
))/cos(d*x + c))^(2/3)) + 2*((a^2 + b^2)/d^6)^(1/6)*arctan(-(a*d^8*((a*cos(
d*x + c) + b*sin(d*x + c))/cos(d*x + c))^(1/3)*sqrt(a^2/d^6)*((a^2 + b^2)/d
^6)^(5/6) - sqrt(2*a*d^4*((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))^(
1/3)*sqrt(a^2/d^6)*((a^2 + b^2)/d^6)^(1/6)*sin(2/3*arctan((d^6*sqrt(a^2/d^6
)*sqrt((a^2 + b^2)/d^6) + b*d^3*sqrt(a^2/d^6))/a^2)) + a^2*d^2*((a^2 + b^2)
/d^6)^(1/3) + a^2*((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))^(2/3))*d
^8*sqrt(a^2/d^6)*((a^2 + b^2)/d^6)^(5/6) + (a^4 + a^2*b^2)*sin(2/3*arctan((
d^6*sqrt(a^2/d^6)*sqrt((a^2 + b^2)/d^6) + b*d^3*sqrt(a^2/d^6))/a^2)))/((a^4
 + a^2*b^2)*cos(2/3*arctan((d^6*sqrt(a^2/d^6)*sqrt((a^2 + b^2)/d^6) + b*d^3
*sqrt(a^2/d^6))/a^2))))*sin(2/3*arctan((d^6*sqrt(a^2/d^6)*sqrt((a^2 + b^2)/
d^6) + b*d^3*sqrt(a^2/d^6))/a^2)) + (sqrt(3)*((a^2 + b^2)/d^6)^(1/6)*cos(2/
3*arctan((d^6*sqrt(a^2/d^6)*sqrt((a^2 + b^2)/d^6) + b*d^3*sqrt(a^2/d^6))/a^
2)) - ((a^2 + b^2)/d^6)^(1/6)*sin(2/3*arctan((d^6*sqrt(a^2/d^6)*sqrt((a^2 +
 b^2)/d^6) + b*d^3*sqrt(a^2/d^6))/a^2)))*arctan(-(2*a*d^8*((a*cos(d*x + c)
+ b*sin(d*x + c))/cos(d*x + c))^(1/3)*sqrt(a^2/d^6)*((a^2 + b^2)/d^6)^(5/6)
*cos(2/3*arctan((d^6*sqrt(a^2/d^6)*sqrt((a^2 + b^2)/d^6) + b*d^3*sqrt(a^2/d
^6))/a^2)) - 2*(sqrt(3)*a*d^8*((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x +
c))^(1/3)*sqrt(a^2/d^6)*((a^2 + b^2)/d^6)^(5/6) + 2*(a^4 + a^2*b^2)*cos(2/3
*arctan((d^6*sqrt(a^2/d^6)*sqrt((a^2 + b^2)/d^6) + b*d^3*sqrt(a^2/d^6))/a^2
)))*sin(2/3*arctan((d^6*sqrt(a^2/d^6)*sqrt((a^2 + b^2)/d^6) + b*d^3*sqrt(a^
2/d^6))/a^2)) + 2*(sqrt(3)*d^8*sqrt(a^2/d^6)*((a^2 + b^2)/d^6)^(5/6)*sin(2/
3*arctan((d^6*sqrt(a^2/d^6)*sqrt((a^2 + b^2)/d^6) + b*d^3*sqrt(a^2/d^6))/a^
2)) - d^8*sqrt(a^2/d^6)*((a^2 + b^2)/d^6)^(5/6)*cos(2/3*arctan((d^6*sqrt(a^
2/d^6)*sqrt((a^2 + b^2)/d^6) + b*d^3*sqrt(a^2/d^6))/a^2)))*sqrt(sqrt(3)*a*d
^4*((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))^(1/3)*sqrt(a^2/d^6)*((a
^2 + b^2)/d^6)^(1/6)*cos(2/3*arctan((d^6*sqrt(a^2/d^6)*sqrt((a^2 + b^2)/d^6
) + b*d^3*sqrt(a^2/d^6))/a^2)) - a*d^4*((a*cos(d*x + c) + b*sin(d*x + c))/c
os(d*x + c))^(1/3)*sqrt(a^2/d^6)*((a^2 + b^2)/d^6)^(1/6)*sin(2/3*arctan((d^
6*sqrt(a^2/d^6)*sqrt((a^2 + b^2)/d^6) + b*d^3*sqrt(a^2/d^6))/a^2)) + a^2*d^
2*((a^2 + b^2)/d^6)^(1/3) + a^2*((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x
+ c))^(2/3)) + sqrt(3)*(a^4 + a^2*b^2))/(3*a^4 + 3*a^2*b^2 - 4*(a^4 + a^2*b
^2)*cos(2/3*arctan((d^6*sqrt(a^2/d^6)*sqrt((a^2 + b^2)/d^6) + b*d^3*sqrt(a^
2/d^6))/a^2))^2)) + (sqrt(3)*((a^2 + b^2)/d^6)^(1/6)*cos(2/3*arctan((d^6*sq
rt(a^2/d^6)*sqrt((a^2 + b^2)/d^6) + b*d^3*sqrt(a^2/d^6))/a^2)) + ((a^2 + b^
2)/d^6)^(1/6)*sin(2/3*arctan((d^6*sqrt(a^2/d^6)*sqrt((a^2 + b^2)/d^6) + b*d
^3*sqrt(a^2/d^6))/a^2)))*arctan((2*a*d^8*((a*cos(d*x + c) + b*sin(d*x + c))
/cos(d*x + c))^(1/3)*sqrt(a^2/d^6)*((a^2 + b^2)/d^6)^(5/6)*cos(2/3*arctan((
d^6*sqrt(a^2/d^6)*sqrt((a^2 + b^2)/d^6) + b*d^3*sqrt(a^2/d^6))/a^2)) + 2*(s
qrt(3)*a*d^8*((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))^(1/3)*sqrt(a^
2/d^6)*((a^2 + b^2)/d^6)^(5/6) - 2*(a^4 + a^2*b^2)*cos(2/3*arctan((d^6*sqrt
(a^2/d^6)*sqrt((a^2 + b^2)/d^6) + b*d^3*sqrt(a^2/d^6))/a^2)))*sin(2/3*arcta
n((d^6*sqrt(a^2/d^6)*sqrt((a^2 + b^2)/d^6) + b*d^3*sqrt(a^2/d^6))/a^2)) - 2

```
*(sqrt(3)*d^8*sqrt(a^2/d^6)*((a^2 + b^2)/d^6)^(5/6)*sin(2/3*arctan((d^6*sqr
t(a^2/d^6)*sqrt((a^2 + b^2)/d^6) + b*d^3*sqrt(a^2/d^6))/a^2)) + d^8*sqrt(a^
2/d^6)*((a^2 + b^2)/d^6)^(5/6)*cos(2/3*arctan((d^6*sqrt(a^2/d^6)*sqrt((a^2
+ b^2)/d^6) + b*d^3*sqrt(a^2/d^6))/a^2)))*sqrt(-sqrt(3)*a*d^4*((a*cos(d*x +
 c) + b*sin(d*x + c))/cos(d*x + c))^(1/3)*sqrt(a^2/d^6)*((a^2 + b^2)/d^6)^(
1/6)*cos(2/3*arctan((d^6*sqrt(a^2/d^6)*sqrt((a^2 + b^2)/d^6) + b*d^3*sqrt(a
^2/d^6))/a^2)) - a*d^4*((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))^(1/
3)*sqrt(a^2/d^6)*((a^2 + b^2)/d^6)^(1/6)*sin(2/3*arctan((d^6*sqrt(a^2/d^6)*
sqrt((a^2 + b^2)/d^6) + b*d^3*sqrt(a^2/d^6))/a^2)) + a^2*d^2*((a^2 + b^2)/d
^6)^(1/3) + a^2*((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))^(2/3)) - s
qrt(3)*(a^4 + a^2*b^2))/(3*a^4 + 3*a^2*b^2 - 4*(a^4 + a^2*b^2)*cos(2/3*arct
an((d^6*sqrt(a^2/d^6)*sqrt((a^2 + b^2)/d^6) + b*d^3*sqrt(a^2/d^6))/a^2))^2)
) - 1/4*(sqrt(3)*((a^2 + b^2)/d^6)^(1/6)*sin(2/3*arctan((d^6*sqrt(a^2/d^6)*
sqrt((a^2 + b^2)/d^6) + b*d^3*sqrt(a^2/d^6))/a^2)) + ((a^2 + b^2)/d^6)^(1/6
)*cos(2/3*arctan((d^6*sqrt(a^2/d^6)*sqrt((a^2 + b^2)/d^6) + b*d^3*sqrt(a^2/
d^6))/a^2)))*log(sqrt(3)*a*d^4*((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x +
 c))^(1/3)*sqrt(a^2/d^6)*((a^2 + b^2)/d^6)^(1/6)*cos(2/3*arctan((d^6*sqrt(a
^2/d^6)*sqrt((a^2 + b^2)/d^6) + b*d^3*sqrt(a^2/d^6))/a^2)) - a*d^4*((a*cos(
d*x + c) + b*sin(d*x + c))/cos(d*x + c))^(1/3)*sqrt(a^2/d^6)*((a^2 + b^2)/d
^6)^(1/6)*sin(2/3*arctan((d^6*sqrt(a^2/d^6)*sqr...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{a + b\tan{(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(1/3),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**(1/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [**B**]
time = 7.01, size = 863, normalized size = 2.08

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^(1/3),x)
```

```
[Out] log((a + b*tan(c + d*x))^(1/3) + d*(-(a*1i + b)/d^3)^(1/3)*1i)*(-(a*1i + b)
/(8*d^3))^(1/3) + log(b*(a + b*tan(c + d*x))^(1/3)*1i - a*(a + b*tan(c + d*
x))^(1/3) + d^4*((a*1i - b)/d^3)^(4/3) + 2*b*d*((a*1i - b)/d^3)^(1/3))*((a*
1i - b)/(8*d^3))^(1/3) - log((486*(b^8 - a^4*b^4)*(a + b*tan(c + d*x))^(1/3
))/d^4 - (((((3^(1/2)*1i)/2 - 1/2)*(-(a*1i + b)/d^3)^(2/3)*((3888*b^5*(a^2
+ b^2)*(a + b*tan(c + d*x))^(1/3))/d - 3888*a*b^4*((3^(1/2)*1i)/2 + 1/2)*(-
(a*1i + b)/d^3)^(1/3)*(a^2 + b^2)))/4 + (1944*a*b^5*(a^2 + b^2))/d^3)*((3^(
1/2)*1i)/2 + 1/2)*(-(a*1i + b)/d^3)^(1/3))/2)*((3^(1/2)*1i)/2 + 1/2)*(-(a*1
i + b)/(8*d^3))^(1/3) + log((486*(b^8 - a^4*b^4)*(a + b*tan(c + d*x))^(1/3)
)/d^4 - (((((3^(1/2)*1i)/2 + 1/2)*(-(a*1i + b)/d^3)^(2/3)*((3888*b^5*(a^2 +
 b^2)*(a + b*tan(c + d*x))^(1/3))/d + 3888*a*b^4*((3^(1/2)*1i)/2 - 1/2)*(-(
a*1i + b)/d^3)^(1/3)*(a^2 + b^2)))/4 - (1944*a*b^5*(a^2 + b^2))/d^3)*((3^(1
/2)*1i)/2 - 1/2)*(-(a*1i + b)/d^3)^(1/3))/2)*((3^(1/2)*1i)/2 - 1/2)*(-(a*1i
 + b)/(8*d^3))^(1/3) - log((((a*1i - b)/d^3)^(1/3)*((3^(1/2)*1i)/2 + 1/2)*(
(((a*1i - b)/d^3)^(2/3)*((3^(1/2)*1i)/2 - 1/2)*((3888*b^5*(a^2 + b^2)*(a +
b*tan(c + d*x))^(1/3))/d - 3888*a*b^4*((a*1i - b)/d^3)^(1/3)*((3^(1/2)*1i)/
2 + 1/2)*(a^2 + b^2)))/4 + (1944*a*b^5*(a^2 + b^2))/d^3))/2 - (486*(b^8 - a
^4*b^4)*(a + b*tan(c + d*x))^(1/3))/d^4)*((3^(1/2)*1i)/2 + 1/2)*((a*1i - b)
/(8*d^3))^(1/3) + log((486*(b^8 - a^4*b^4)*(a + b*tan(c + d*x))^(1/3))/d^4
- (((a*1i - b)/d^3)^(1/3)*((3^(1/2)*1i)/2 - 1/2)*((((a*1i - b)/d^3)^(2/3)*(
(3^(1/2)*1i)/2 + 1/2)*((3888*b^5*(a^2 + b^2)*(a + b*tan(c + d*x))^(1/3))/d
+ 3888*a*b^4*((a*1i - b)/d^3)^(1/3)*((3^(1/2)*1i)/2 - 1/2)*(a^2 + b^2)))/4
- (1944*a*b^5*(a^2 + b^2))/d^3))/2)*((3^(1/2)*1i)/2 - 1/2)*((a*1i - b)/(8*d
^3))^(1/3)
```

3.692 $\int \frac{1}{\sqrt[3]{a+b\tan(c+dx)}}\,dx$

Optimal. Leaf size=415

$$-\frac{x}{4\sqrt[3]{a-\sqrt{-b^2}}}-\frac{x}{4\sqrt[3]{a+\sqrt{-b^2}}}-\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-\sqrt{-b^2}}}}{\sqrt{3}}\right)}{2\sqrt{-b^2}\,\sqrt[3]{a-\sqrt{-b^2}}\,d}+\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{a+b}}{\sqrt[3]{a}}}{}\right)}{2\sqrt{-b^2}\,\sqrt[3]{a+\sqrt{}}}$$

```
[Out] -1/4*x/(a-(-b^2)^(1/2))^(1/3)-1/4*b*ln(cos(d*x+c))/d/(a-(-b^2)^(1/2))^(1/3)
/(-b^2)^(1/2)-3/4*b*ln((a-(-b^2)^(1/2))^(1/3)-(a+b*tan(d*x+c))^(1/3))/d/(a-
(-b^2)^(1/2))^(1/3)/(-b^2)^(1/2)-1/2*b*arctan(1/3*(1+2*(a+b*tan(d*x+c))^(1/
3)/(a-(-b^2)^(1/2))^(1/3))*3^(1/2))*3^(1/2)/d/(a-(-b^2)^(1/2))^(1/3)/(-b^2)
^(1/2)-1/4*x/(a+(-b^2)^(1/2))^(1/3)+1/4*b*ln(cos(d*x+c))/d/(-b^2)^(1/2)/(a+
(-b^2)^(1/2))^(1/3)+3/4*b*ln((a+(-b^2)^(1/2))^(1/3)-(a+b*tan(d*x+c))^(1/3))
/d/(-b^2)^(1/2)/(a+(-b^2)^(1/2))^(1/3)+1/2*b*arctan(1/3*(1+2*(a+b*tan(d*x+c
))^(1/3)/(a+(-b^2)^(1/2))^(1/3))*3^(1/2))*3^(1/2)/d/(-b^2)^(1/2)/(a+(-b^2)^
(1/2))^(1/3)
```

Rubi [A]
time = 0.22, antiderivative size = 415, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3566, 726, 57, 631, 210, 31}

$$-\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-\sqrt{-b^2}}}+1}{\sqrt{3}}\right)}{2\sqrt{-b^2}\,d\sqrt[3]{a-\sqrt{-b^2}}}+\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a+\sqrt{-b^2}}}+1}{\sqrt{3}}\right)}{2\sqrt{-b^2}\,d\sqrt[3]{a+\sqrt{-b^2}}}-\frac{3b\log\left(\sqrt[3]{a-\sqrt{-b^2}}-\sqrt[3]{a+b\tan(c+dx)}\right)}{4\sqrt{-b^2}\,d\sqrt[3]{a-\sqrt{-b^2}}}+\frac{3b\log\left(\sqrt[3]{a+\sqrt{-b^2}}-\sqrt[3]{a+b\tan(c+dx)}\right)}{4\sqrt{-b^2}\,d\sqrt[3]{a+\sqrt{-b^2}}}-\frac{b\log(\cos(c+dx))}{4\sqrt{-b^2}\,d\sqrt[3]{a-\sqrt{-b^2}}}+\frac{b\log(\cos(c+dx))}{4\sqrt{-b^2}\,d\sqrt[3]{a+\sqrt{-b^2}}}-\frac{x}{4\sqrt[3]{a-\sqrt{-b^2}}}-\frac{x}{4\sqrt[3]{a+\sqrt{-b^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^(-1/3),x]
```

```
[Out] -1/4*x/(a - Sqrt[-b^2])^(1/3) - x/(4*(a + Sqrt[-b^2])^(1/3)) - (Sqrt[3]*b*A
rcTan[(1 + (2*(a + b*Tan[c + d*x])^(1/3))/(a - Sqrt[-b^2])^(1/3))/Sqrt[3]])
/(2*Sqrt[-b^2]*(a - Sqrt[-b^2])^(1/3)*d) + (Sqrt[3]*b*ArcTan[(1 + (2*(a + b
*Tan[c + d*x])^(1/3))/(a + Sqrt[-b^2])^(1/3))/Sqrt[3]])/(2*Sqrt[-b^2]*(a +
Sqrt[-b^2])^(1/3)*d) - (b*Log[Cos[c + d*x]])/(4*Sqrt[-b^2]*(a - Sqrt[-b^2])
^(1/3)*d) + (b*Log[Cos[c + d*x]])/(4*Sqrt[-b^2]*(a + Sqrt[-b^2])^(1/3)*d) -
 (3*b*Log[(a - Sqrt[-b^2])^(1/3) - (a + b*Tan[c + d*x])^(1/3)])/(4*Sqrt[-b^
2]*(a - Sqrt[-b^2])^(1/3)*d) + (3*b*Log[(a + Sqrt[-b^2])^(1/3) - (a + b*Tan
[c + d*x])^(1/3)])/(4*Sqrt[-b^2]*(a + Sqrt[-b^2])^(1/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 726

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+b\tan(c+dx)}}\,dx = \frac{b\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+x}\,(b^2+x^2)}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \left(\frac{\sqrt{-b^2}}{2b^2\left(\sqrt{-b^2}-x\right)\sqrt[3]{a+x}} + \frac{\sqrt{-b^2}}{2b^2\sqrt[3]{a+x}\left(\sqrt{-b^2}+x\right)}\right) dx, x, b\tan(c$$

$$= -\frac{b\text{Subst}\left(\int \frac{1}{\left(\sqrt{-b^2}-x\right)\sqrt[3]{a+x}}\,dx, x, b\tan(c+dx)\right)}{2\sqrt{-b^2}\,d} - \frac{b\text{Subst}\left(\int \frac{}{\sqrt[3]{a+x}}\right.}{}$$

$$= -\frac{x}{4\sqrt[3]{a-\sqrt{-b^2}}} - \frac{x}{4\sqrt[3]{a+\sqrt{-b^2}}} - \frac{b\log(\cos(c+dx))}{4\sqrt{-b^2}\sqrt[3]{a-\sqrt{-b^2}}\,d} + \frac{b\log(\text{co}}{4\sqrt{-b^2}\sqrt[3]{}}$$

$$= -\frac{x}{4\sqrt[3]{a-\sqrt{-b^2}}} - \frac{x}{4\sqrt[3]{a+\sqrt{-b^2}}} - \frac{b\log(\cos(c+dx))}{4\sqrt{-b^2}\sqrt[3]{a-\sqrt{-b^2}}\,d} + \frac{b\log(\text{co}}{4\sqrt{-b^2}\sqrt[3]{}}$$

$$= -\frac{x}{4\sqrt[3]{a-\sqrt{-b^2}}} - \frac{x}{4\sqrt[3]{a+\sqrt{-b^2}}} - \frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-\sqrt{-b^2}}}}{\sqrt{3}}\right.}{2\sqrt{-b^2}\sqrt[3]{a-\sqrt{-b^2}}\,d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.26, size = 251, normalized size = 0.60

$$\frac{i\left(\frac{2\sqrt{3}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-ib}}}{\sqrt{3}}\right)}{\sqrt[3]{a-ib}} - \frac{2\sqrt{3}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a+ib}}}{\sqrt{3}}\right)}{\sqrt[3]{a+ib}} + \frac{\log(i-\tan(c+dx))}{\sqrt[3]{a+ib}} - \frac{\log(i+\tan(c+dx))}{\sqrt[3]{a-ib}} + \frac{3\log\left(\sqrt[3]{a-ib}-\sqrt[3]{a+b\tan(c+dx)}\right)}{\sqrt[3]{a-ib}} - \frac{3\log\left(\sqrt[3]{a+ib}-\sqrt[3]{a+b\tan(c+dx)}\right)}{\sqrt[3]{a+ib}}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^(-1/3),x]
```

```
[Out] ((I/4)*((2*Sqrt[3]*ArcTan[(1 + (2*(a + b*Tan[c + d*x])^(1/3))/(a - I*b)^(1/
3))/Sqrt[3]])/(a - I*b)^(1/3) - (2*Sqrt[3]*ArcTan[(1 + (2*(a + b*Tan[c + d*
```

```
x])^(1/3))/(a + I*b)^(1/3))/Sqrt[3]])/(a + I*b)^(1/3) + Log[I - Tan[c + d*x
]]/(a + I*b)^(1/3) - Log[I + Tan[c + d*x]]/(a - I*b)^(1/3) + (3*Log[(a - I*
b)^(1/3) - (a + b*Tan[c + d*x])^(1/3)])/(a - I*b)^(1/3) - (3*Log[(a + I*b)^
(1/3) - (a + b*Tan[c + d*x])^(1/3)])/(a + I*b)^(1/3)))/d
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.31, size = 58, normalized size = 0.14

method	result	size
derivativedivides	$\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-2a\_Z^{3}+a^{2}+b^{2}\right)}\frac{\_R\ln\left((a+b\tan(dx+c))^{\frac{1}{3}}-\_R\right)}{\_R^{5}-\_R^{2}a}\right)}{2d}$	58
default	$\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-2a\_Z^{3}+a^{2}+b^{2}\right)}\frac{\_R\ln\left((a+b\tan(dx+c))^{\frac{1}{3}}-\_R\right)}{\_R^{5}-\_R^{2}a}\right)}{2d}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c))^(1/3),x,method=_RETURNVERBOSE)
[Out] 1/2/d*b*sum(_R/(_R^5-_R^2*a)*ln((a+b*tan(d*x+c))^(1/3)-_R),_R=RootOf(_Z^6-2
*_Z^3*a+a^2+b^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^(-1/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^(1/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a+b\tan(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))**(1/3),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**(-1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 7.24, size = 817, normalized size = 1.97

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(c + d*x))^(1/3),x)
```

```
[Out] (log((243*b^5*(a + b*tan(c + d*x))^(1/3))/d^5 + ((1944*b^4*(a^2 - b^2)*(a +
 b*tan(c + d*x))^(1/3))/d^2 + 1944*a*b^4*(1/(d^3*(a*1i + b)))^(2/3)*(a^2 +
b^2))/(8*d^3*(a*1i + b)))*(1/(a*d^3*1i + b*d^3))^(1/3))/2 + log((243*b^5*(a
 + b*tan(c + d*x))^(1/3))/d^5 + (((1944*b^4*(a^2 - b^2)*(a + b*tan(c + d*x)
)^(1/3))/d^2 + 1944*a*b^4*(a^2 + b^2)*((a*1i + b)/(d^3*(a^2 + b^2)))^(2/3))
*(a*1i + b))/(8*d^3*(a^2 + b^2)))*((a*1i + b)/(8*a^2*d^3 + 8*b^2*d^3))^(1/3
) + (log((243*b^5*(a + b*tan(c + d*x))^(1/3))/d^5 + (((1944*b^4*(a^2 - b^2)
*(a + b*tan(c + d*x))^(1/3))/d^2 + 486*a*b^4*(1/(d^3*(a*1i + b)))^(2/3)*(3^
(1/2)*1i - 1)^2*(a^2 + b^2))*(3^(1/2)*1i - 1)^3)/(64*d^3*(a*1i + b)))*(3^(1
/2)*1i - 1)*(1/(a*d^3*1i + b*d^3))^(1/3))/4 - (log((243*b^5*(a + b*tan(c +
d*x))^(1/3))/d^5 - (((1944*b^4*(a^2 - b^2)*(a + b*tan(c + d*x))^(1/3))/d^2
+ 486*a*b^4*(1/(d^3*(a*1i + b)))^(2/3)*(3^(1/2)*1i + 1)^2*(a^2 + b^2))*(3^(
1/2)*1i + 1)^3)/(64*d^3*(a*1i + b)))*(3^(1/2)*1i + 1)*(1/(a*d^3*1i + b*d^3)
)^(1/3))/4 + (log((243*b^5*(a + b*tan(c + d*x))^(1/3))/d^5 + (((1944*b^4*(a
^2 - b^2)*(a + b*tan(c + d*x))^(1/3))/d^2 + 1944*a*b^4*(3^(1/2)*1i - 1)^2*(
a^2 + b^2)*(1i/(8*d^3*(a + b*1i)))^(2/3))*(3^(1/2)*1i - 1)^3*1i)/(64*d^3*(a
 + b*1i)))*(3^(1/2)*1i - 1)*(1i/(8*(a*d^3 + b*d^3*1i)))^(1/3))/2 - (log((24
3*b^5*(a + b*tan(c + d*x))^(1/3))/d^5 - (((1944*b^4*(a^2 - b^2)*(a + b*tan(
c + d*x))^(1/3))/d^2 + 1944*a*b^4*(3^(1/2)*1i + 1)^2*(a^2 + b^2)*(1i/(8*d^3
*(a + b*1i)))^(2/3))*(3^(1/2)*1i + 1)^3*1i)/(64*d^3*(a + b*1i)))*(3^(1/2)*1
i + 1)*(1i/(8*(a*d^3 + b*d^3*1i)))^(1/3))/2
```

3.693 $\int \frac{1}{(a+b\tan(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=415

$$-\frac{x}{4\left(a-\sqrt{-b^2}\right)^{2/3}}-\frac{x}{4\left(a+\sqrt{-b^2}\right)^{2/3}}+\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-\sqrt{-b^2}}}}{\sqrt{3}}\right)}{2\sqrt{-b^2}\left(a-\sqrt{-b^2}\right)^{2/3}d}-\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{a}}{}}{}\right)}{2\sqrt{-b^2}\left(a+\right.}$$

```
[Out] -1/4*x/(a-(-b^2)^(1/2))^(2/3)-1/4*b*ln(cos(d*x+c))/d/(a-(-b^2)^(1/2))^(2/3)
/(-b^2)^(1/2)-3/4*b*ln((a-(-b^2)^(1/2))^(1/3)-(a+b*tan(d*x+c))^(1/3))/d/(a-
(-b^2)^(1/2))^(2/3)/(-b^2)^(1/2)+1/2*b*arctan(1/3*(1+2*(a+b*tan(d*x+c))^(1/
3)/(a-(-b^2)^(1/2))^(1/3))*3^(1/2))*3^(1/2)/d/(a-(-b^2)^(1/2))^(2/3)/(-b^2)
^(1/2)-1/4*x/(a+(-b^2)^(1/2))^(2/3)+1/4*b*ln(cos(d*x+c))/d/(-b^2)^(1/2)/(a+
(-b^2)^(1/2))^(2/3)+3/4*b*ln((a+(-b^2)^(1/2))^(1/3)-(a+b*tan(d*x+c))^(1/3))
/d/(-b^2)^(1/2)/(a+(-b^2)^(1/2))^(2/3)-1/2*b*arctan(1/3*(1+2*(a+b*tan(d*x+c
))^(1/3)/(a+(-b^2)^(1/2))^(1/3))*3^(1/2))*3^(1/2)/d/(-b^2)^(1/2)/(a+(-b^2)^
(1/2))^(2/3)
```

Rubi [A]
time = 0.24, antiderivative size = 415, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3566, 726, 59, 631, 210, 31}

$$\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-\sqrt{-b^2}}}+1}{\sqrt{3}}\right)}{2\sqrt{-b^2}\,d\left(a-\sqrt{-b^2}\right)^{2/3}}-\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a+\sqrt{-b^2}}}+1}{\sqrt{3}}\right)}{2\sqrt{-b^2}\,d\left(a+\sqrt{-b^2}\right)^{2/3}}-\frac{3b\log\left(\sqrt[3]{a-\sqrt{-b^2}}-\sqrt[3]{a+b\tan(c+dx)}\right)}{4\sqrt{-b^2}\,d\left(a-\sqrt{-b^2}\right)^{2/3}}+\frac{3b\log\left(\sqrt[3]{a+\sqrt{-b^2}}-\sqrt[3]{a+b\tan(c+dx)}\right)}{4\sqrt{-b^2}\,d\left(a+\sqrt{-b^2}\right)^{2/3}}-\frac{b\log(\cos(c+dx))}{4\sqrt{-b^2}\,d\left(a-\sqrt{-b^2}\right)^{2/3}}+\frac{b\log(\cos(c+dx))}{4\sqrt{-b^2}\,d\left(a+\sqrt{-b^2}\right)^{2/3}}-\frac{x}{4\left(a-\sqrt{-b^2}\right)^{2/3}}-\frac{x}{4\left(a+\sqrt{-b^2}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^(-2/3),x]
```

```
[Out] -1/4*x/(a - Sqrt[-b^2])^(2/3) - x/(4*(a + Sqrt[-b^2])^(2/3)) + (Sqrt[3]*b*A
rcTan[(1 + (2*(a + b*Tan[c + d*x])^(1/3))/(a - Sqrt[-b^2])^(1/3))/Sqrt[3]])
/(2*Sqrt[-b^2]*(a - Sqrt[-b^2])^(2/3)*d) - (Sqrt[3]*b*ArcTan[(1 + (2*(a + b
*Tan[c + d*x])^(1/3))/(a + Sqrt[-b^2])^(1/3))/Sqrt[3]])/(2*Sqrt[-b^2]*(a +
Sqrt[-b^2])^(2/3)*d) - (b*Log[Cos[c + d*x]])/(4*Sqrt[-b^2]*(a - Sqrt[-b^2])
^(2/3)*d) + (b*Log[Cos[c + d*x]])/(4*Sqrt[-b^2]*(a + Sqrt[-b^2])^(2/3)*d) -
 (3*b*Log[(a - Sqrt[-b^2])^(1/3) - (a + b*Tan[c + d*x])^(1/3)])/(4*Sqrt[-b^
2]*(a - Sqrt[-b^2])^(2/3)*d) + (3*b*Log[(a + Sqrt[-b^2])^(1/3) - (a + b*Tan
[c + d*x])^(1/3)])/(4*Sqrt[-b^2]*(a + Sqrt[-b^2])^(2/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 726

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\tan(c+dx))^{2/3}}\,dx = \frac{b\text{Subst}\left(\int \frac{1}{(a+x)^{2/3}(b^2+x^2)}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \left(\frac{\sqrt{-b^2}}{2b^2\left(\sqrt{-b^2}-x\right)(a+x)^{2/3}} + \frac{\sqrt{-b^2}}{2b^2(a+x)^{2/3}\left(\sqrt{-b^2}+x\right)}\right)\,dx, x, b\tan(c+d\right.}{d}$$

$$= -\frac{b\text{Subst}\left(\int \frac{1}{\left(\sqrt{-b^2}-x\right)(a+x)^{2/3}}\,dx, x, b\tan(c+dx)\right)}{2\sqrt{-b^2}\,d} - \frac{b\text{Subst}\left(\int \frac{1}{(a+x)^{2/3}\left(\sqrt{\right.}\right.}{}$$

$$= -\frac{x}{4\left(a-\sqrt{-b^2}\right)^{2/3}} - \frac{x}{4\left(a+\sqrt{-b^2}\right)^{2/3}} - \frac{b\log(\cos(c+dx))}{4\sqrt{-b^2}\left(a-\sqrt{-b^2}\right)^{2/3}d} + \frac{}{4\sqrt{}}$$

$$= -\frac{x}{4\left(a-\sqrt{-b^2}\right)^{2/3}} - \frac{x}{4\left(a+\sqrt{-b^2}\right)^{2/3}} - \frac{b\log(\cos(c+dx))}{4\sqrt{-b^2}\left(a-\sqrt{-b^2}\right)^{2/3}d} + \frac{}{4\sqrt{}}$$

$$= -\frac{x}{4\left(a-\sqrt{-b^2}\right)^{2/3}} - \frac{x}{4\left(a+\sqrt{-b^2}\right)^{2/3}} + \frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+}}{\sqrt[3]{a-\sqrt{-b^2}}}}{\sqrt{3}}\right.}{2\sqrt{-b^2}\left(a-\sqrt{-b^2}\right)^{2/3}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.40, size = 313, normalized size = 0.75

$$\frac{i\left(\frac{2\log\left(\sqrt[3]{a-ib}-\sqrt[3]{a+b\tan(c+dx)}\right)}{(a-ib)^{2/3}} - \frac{2\log\left(\sqrt[3]{a+ib}-\sqrt[3]{a+b\tan(c+dx)}\right)}{(a+ib)^{2/3}} - \frac{2\sqrt{3}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-ib}}}{\sqrt{3}}\right)+\log\left((a-ib)^{2/3}+\sqrt[3]{a-ib}\sqrt[3]{a+b\tan(c+dx)}+(a+b\tan(c+dx))^{2/3}\right)}{(a-ib)^{2/3}} + \frac{2\sqrt{3}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a+ib}}}{\sqrt{3}}\right)+\log\left((a+ib)^{2/3}+\sqrt[3]{a+ib}\sqrt[3]{a+b\tan(c+dx)}+(a+b\tan(c+dx))^{2/3}\right)}{(a+ib)^{2/3}}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^(-2/3),x]
```

```
[Out] ((I/4)*((2*Log[(a - I*b)^(1/3) - (a + b*Tan[c + d*x])^(1/3)])/(a - I*b)^(2/
3) - (2*Log[(a + I*b)^(1/3) - (a + b*Tan[c + d*x])^(1/3)])/(a + I*b)^(2/3)
```

```
- (2*Sqrt[3]*ArcTan[(1 + (2*(a + b*Tan[c + d*x])^(1/3))/(a - I*b)^(1/3))/Sq
rt[3]] + Log[(a - I*b)^(2/3) + (a - I*b)^(1/3)*(a + b*Tan[c + d*x])^(1/3) +
 (a + b*Tan[c + d*x])^(2/3)])/(a - I*b)^(2/3) + (2*Sqrt[3]*ArcTan[(1 + (2*(
a + b*Tan[c + d*x])^(1/3))/(a + I*b)^(1/3))/Sqrt[3]] + Log[(a + I*b)^(2/3)
+ (a + I*b)^(1/3)*(a + b*Tan[c + d*x])^(1/3) + (a + b*Tan[c + d*x])^(2/3)])
/(a + I*b)^(2/3)))/d
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.28, size = 57, normalized size = 0.14

method	result	size
derivativedivides	$\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-2a\_Z^{3}+a^{2}+b^{2}\right)}\frac{\ln\left(\left(a+b\tan(dx+c)\right)^{\frac{1}{3}}-\_R\right)}{\_R^{5}-\_R^{2}a}\right)}{2d}$	57
default	$\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-2a\_Z^{3}+a^{2}+b^{2}\right)}\frac{\ln\left(\left(a+b\tan(dx+c)\right)^{\frac{1}{3}}-\_R\right)}{\_R^{5}-\_R^{2}a}\right)}{2d}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c))^(2/3),x,method=_RETURNVERBOSE)
[Out] 1/2/d*b*sum(1/(_R^5-_R^2*a)*ln((a+b*tan(d*x+c))^(1/3)-_R),_R=RootOf(_Z^6-2*
_Z^3*a+a^2+b^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^(-2/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 16201 vs. 2(325) = 650.
time = 81.71, size = 16201, normalized size = 39.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^(2/3),x, algorithm="fricas")
[Out] 1/2*(1/((a^4 + 2*a^2*b^2 + b^4)*d^6))^(1/6)*cos(2/3*arctan(((a^8 + 4*a^6*b^
2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^6*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^6))
```

- 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^3)*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4
*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^6))/(a^4 - 2*a^2*b^2 + b^4)))*log
(2*(a^7 + a^5*b^2 - a^3*b^4 - a*b^6)*d^4*((a*cos(d*x + c) + b*sin(d*x + c))
/cos(d*x + c))^(1/3)*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4
*b^4 + 4*a^2*b^6 + b^8)*d^6))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^6))^(1/6)*sin(2
/3*arctan(((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^6*sqrt(1/((a^4
 + 2*a^2*b^2 + b^4)*d^6)) - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^3)*sqrt((a^4 -
2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^6))/(a^
4 - 2*a^2*b^2 + b^4))) + 2*(a^4*b - 2*a^2*b^3 + b^5)*d*((a*cos(d*x + c) + b
*sin(d*x + c))/cos(d*x + c))^(1/3)*(1/((a^4 + 2*a^2*b^2 + b^4)*d^6))^(1/6)*
cos(2/3*arctan(((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^6*sqrt(1/
((a^4 + 2*a^2*b^2 + b^4)*d^6)) - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^3)*sqrt((a
^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^6)
)/(a^4 - 2*a^2*b^2 + b^4))) + (a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*(1/((a^4
+ 2*a^2*b^2 + b^4)*d^6))^(1/3) + (a^4 - 2*a^2*b^2 + b^4)*((a*cos(d*x + c) +
 b*sin(d*x + c))/cos(d*x + c))^(2/3)) + 2*(1/((a^4 + 2*a^2*b^2 + b^4)*d^6))
^(1/6)*arctan(((a^11 + 3*a^9*b^2 + 2*a^7*b^4 - 2*a^5*b^6 - 3*a^3*b^8 - a*b^
10)*d^8*((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))^(1/3)*sqrt((a^4 -
2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^6))*(1/
((a^4 + 2*a^2*b^2 + b^4)*d^6))^(5/6)*cos(2/3*arctan(((a^8 + 4*a^6*b^2 + 6*a
^4*b^4 + 4*a^2*b^6 + b^8)*d^6*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^6)) - 2*(a^
5*b + 2*a^3*b^3 + a*b^5)*d^3)*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^
2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^6))/(a^4 - 2*a^2*b^2 + b^4))) + (a^7*b +
 a^5*b^3 - a^3*b^5 - a*b^7)*d^3*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*
b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^6)) + ((a^8*b - 2*a^4*b^5 + b^9)*d^5*(
(a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))^(1/3)*(1/((a^4 + 2*a^2*b^2
+ b^4)*d^6))^(5/6) + (a^6 - a^4*b^2 - a^2*b^4 + b^6)*cos(2/3*arctan(((a^8 +
 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^6*sqrt(1/((a^4 + 2*a^2*b^2 + b^
4)*d^6)) - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^3)*sqrt((a^4 - 2*a^2*b^2 + b^4)/
((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^6))/(a^4 - 2*a^2*b^2 + b
^4))))*sin(2/3*arctan(((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^6*
sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^6)) - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^3)*
sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^
8)*d^6))/(a^4 - 2*a^2*b^2 + b^4))) + ((a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*
b^6 + a*b^8)*d^8*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4
 + 4*a^2*b^6 + b^8)*d^6))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^6))^(5/6)*cos(2/3*a
rctan(((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^6*sqrt(1/((a^4 + 2
*a^2*b^2 + b^4)*d^6)) - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^3)*sqrt((a^4 - 2*a^
2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^6))/(a^4 -
2*a^2*b^2 + b^4))) + (a^6*b + a^4*b^3 - a^2*b^5 - b^7)*d^5*(1/((a^4 + 2*a^2
*b^2 + b^4)*d^6))^(5/6)*sin(2/3*arctan(((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^
2*b^6 + b^8)*d^6*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^6)) - 2*(a^5*b + 2*a^3*b
^3 + a*b^5)*d^3)*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4
 + 4*a^2*b^6 + b^8)*d^6))/(a^4 - 2*a^2*b^2 + b^4))))*sqrt(2*(a^7 + a^5*b^2
- a^3*b^4 - a*b^6)*d^4*((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))^(1/

```
3)*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 +
 b^8)*d^6))*(1/((a^4 + 2*a^2*b^2 + b^4)*d^6))^(1/6)*sin(2/3*arctan(((a^8 +
4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^6*sqrt(1/((a^4 + 2*a^2*b^2 + b^4
)*d^6)) - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^3)*sqrt((a^4 - 2*a^2*b^2 + b^4)/(
(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^6))/(a^4 - 2*a^2*b^2 + b^
4))) + 2*(a^4*b - 2*a^2*b^3 + b^5)*d*((a*cos(d*x + c) + b*sin(d*x + c))/cos
(d*x + c))^(1/3)*(1/((a^4 + 2*a^2*b^2 + b^4)*d^6))^(1/6)*cos(2/3*arctan(((a
^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^6*sqrt(1/((a^4 + 2*a^2*b^2
+ b^4)*d^6)) - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^3)*sqrt((a^4 - 2*a^2*b^2 + b
^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^6))/(a^4 - 2*a^2*b^2
 + b^4))) + (a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^2*(1/((a^4 + 2*a^2*b^2 + b^4)
*d^6))^(1/3) + (a^4 - 2*a^2*b^2 + b^4)*((a*cos(d*x + c) + b*sin(d*x + c))/c
os(d*x + c))^(2/3)))/(a^4*b^2 - 2*a^2*b^4 + b^6 - (a^6 - a^4*b^2 - a^2*b^4
+ b^6)*cos(2/3*arctan(((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^6*
sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^6)) - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^3)*
sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^
8)*d^6))/(a^4 - 2*a^2*b^2 + b^4)))^2))*sin(2/3*arctan(((a^8 + 4*a^6*b^2 + 6
*a^4*b^4 + 4*a^2*b^6 + b^8)*d^6*sqrt(1/((a^4 + 2*a^2*b^2 + b^4)*d^6)) - 2*(
a^5*b + 2*a^3*b^3 + a*b^5)*d^3)*sqrt((a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*
b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^6))/(a^4 -...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\tan\left(c + dx\right)\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))**(2/3),x)
[Out] Integral((a + b*tan(c + d*x))**(-2/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^(-2/3), x)
```

Mupad [**B**]
time = 8.48, size = 1048, normalized size = 2.53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(c + d*x))^(2/3),x)
```

```
[Out] (log(((-1i/(d^3*(a*1i - b)^2))^(4/3)*(a^2*b^5*d*486i - b^7*d*486i + 486*a^3
*b^4*d - 486*a*b^6*d + (972*a*b^5*(a + b*tan(c + d*x))^(1/3))/(-1i/(d^3*(a*
1i - b)^2))^(1/3)))/d - (486*b^4*(a + b*tan(c + d*x))^(1/3))/d^4)*(1/(b^2*d
^3*1i - a^2*d^3*1i + 2*a*b*d^3))^(1/3))/2 + log((((7776*a*b^5*(a + b*tan(c
+ d*x))^(1/3))/d + 7776*a*b^4*(a^2 + b^2)*(1i/(8*d^3*(a*1i + b)^2))^(1/3))*
(1i/(8*d^3*(a*1i + b)^2))^(2/3) - (972*b^5)/d^3)*(1i/(8*d^3*(a*1i + b)^2))^
(1/3) - (486*b^4*(a + b*tan(c + d*x))^(1/3))/d^4)*(1i/(8*(b^2*d^3 - a^2*d^3
 + a*b*d^3*2i)))^(1/3) + (log((486*b^4*(a + b*tan(c + d*x))^(1/3))/d^4 + ((
3^(1/2)*1i - 1)*((972*b^5)/d^3 - ((3^(1/2)*1i - 1)^2*((7776*a*b^5*(a + b*ta
n(c + d*x))^(1/3))/d + 1944*a*b^4*(3^(1/2)*1i - 1)*(a^2 + b^2)*(-1i/(d^3*(a
*1i - b)^2))^(1/3))*(-1i/(d^3*(a*1i - b)^2))^(2/3))/16)*(-1i/(d^3*(a*1i - b
)^2))^(1/3))/4)*(3^(1/2)*1i - 1)*(1/(b^2*d^3*1i - a^2*d^3*1i + 2*a*b*d^3))^
(1/3))/4 - (log((486*b^4*(a + b*tan(c + d*x))^(1/3))/d^4 - ((3^(1/2)*1i + 1
)*((972*b^5)/d^3 - ((3^(1/2)*1i + 1)^2*((7776*a*b^5*(a + b*tan(c + d*x))^(1
/3))/d - 1944*a*b^4*(3^(1/2)*1i + 1)*(a^2 + b^2)*(-1i/(d^3*(a*1i - b)^2))^(
1/3))*(-1i/(d^3*(a*1i - b)^2))^(2/3))/16)*(-1i/(d^3*(a*1i - b)^2))^(1/3))/4
)*(3^(1/2)*1i + 1)*(1/(b^2*d^3*1i - a^2*d^3*1i + 2*a*b*d^3))^(1/3))/4 + (lo
g((486*b^4*(a + b*tan(c + d*x))^(1/3))/d^4 + ((3^(1/2)*1i - 1)*((972*b^5)/d
^3 - ((3^(1/2)*1i - 1)^2*((7776*a*b^5*(a + b*tan(c + d*x))^(1/3))/d + 3888*
a*b^4*(3^(1/2)*1i - 1)*(a^2 + b^2)*(1i/(8*d^3*(a*1i + b)^2))^(1/3))*(1i/(8*
d^3*(a*1i + b)^2))^(2/3))/4)*(1i/(8*d^3*(a*1i + b)^2))^(1/3))/2)*(3^(1/2)*1
i - 1)*(1i/(8*(b^2*d^3 - a^2*d^3 + a*b*d^3*2i)))^(1/3))/2 - (log((486*b^4*(
a + b*tan(c + d*x))^(1/3))/d^4 - ((3^(1/2)*1i + 1)*((972*b^5)/d^3 - ((3^(1/
2)*1i + 1)^2*((7776*a*b^5*(a + b*tan(c + d*x))^(1/3))/d - 3888*a*b^4*(3^(1/
2)*1i + 1)*(a^2 + b^2)*(1i/(8*d^3*(a*1i + b)^2))^(1/3))*(1i/(8*d^3*(a*1i +
b)^2))^(2/3))/4)*(1i/(8*d^3*(a*1i + b)^2))^(1/3))/2)*(3^(1/2)*1i + 1)*(1i/(
8*(b^2*d^3 - a^2*d^3 + a*b*d^3*2i)))^(1/3))/2
```

3.694 $\int \frac{1}{(a+b\tan(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=336

$$-\frac{x}{4(a-ib)^{4/3}}-\frac{x}{4(a+ib)^{4/3}}+\frac{i\sqrt{3}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-ib}}}{\sqrt{3}}\right)}{2(a-ib)^{4/3}d}-\frac{i\sqrt{3}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a+ib}}}{\sqrt{3}}\right)}{2(a+ib)^{4/3}d}$$

```
[Out] -1/4*x/(a-I*b)^(4/3)-1/4*x/(a+I*b)^(4/3)+1/4*I*ln(cos(d*x+c))/(a-I*b)^(4/3)
/d-1/4*I*ln(cos(d*x+c))/(a+I*b)^(4/3)/d+3/4*I*ln((a-I*b)^(1/3)-(a+b*tan(d*x
+c))^(1/3))/(a-I*b)^(4/3)/d-3/4*I*ln((a+I*b)^(1/3)-(a+b*tan(d*x+c))^(1/3))/
(a+I*b)^(4/3)/d+1/2*I*arctan(1/3*(1+2*(a+b*tan(d*x+c))^(1/3)/(a-I*b)^(1/3))
*3^(1/2))*3^(1/2)/(a-I*b)^(4/3)/d-1/2*I*arctan(1/3*(1+2*(a+b*tan(d*x+c))^(1
/3)/(a+I*b)^(1/3))*3^(1/2))*3^(1/2)/(a+I*b)^(4/3)/d-3*b/(a^2+b^2)/d/(a+b*ta
n(d*x+c))^(1/3)
```

Rubi [A]
time = 0.27, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3564, 3620, 3618, 57, 631, 210, 31}

$$-\frac{3b}{d\left(a^2+b^2\right)\sqrt[3]{a+b\tan(c+dx)}}+\frac{i\sqrt{3}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-ib}}}{\sqrt{3}}\right)}{2d(a-ib)^{4/3}}-\frac{i\sqrt{3}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a+ib}}}{\sqrt{3}}\right)}{2d(a+ib)^{4/3}}+\frac{3i\log\left(-\sqrt[3]{a+b\tan(c+dx)}+\sqrt[3]{a-ib}\right)}{4d(a-ib)^{4/3}}-\frac{3i\log\left(-\sqrt[3]{a+b\tan(c+dx)}+\sqrt[3]{a+ib}\right)}{4d(a+ib)^{4/3}}+\frac{i\log(\cos(c+dx))}{4d(a-ib)^{4/3}}-\frac{i\log(\cos(c+dx))}{4d(a+ib)^{4/3}}-\frac{x}{4(a-ib)^{4/3}}-\frac{x}{4(a+ib)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^(-4/3),x]
```

```
[Out] -1/4*x/(a - I*b)^(4/3) - x/(4*(a + I*b)^(4/3)) + ((I/2)*Sqrt[3]*ArcTan[(1 +
 (2*(a + b*Tan[c + d*x])^(1/3))/(a - I*b)^(1/3))/Sqrt[3]])/((a - I*b)^(4/3)
*d) - ((I/2)*Sqrt[3]*ArcTan[(1 + (2*(a + b*Tan[c + d*x])^(1/3))/(a + I*b)^(
1/3))/Sqrt[3]])/((a + I*b)^(4/3)*d) + ((I/4)*Log[Cos[c + d*x]])/((a - I*b)^
(4/3)*d) - ((I/4)*Log[Cos[c + d*x]])/((a + I*b)^(4/3)*d) + (((3*I)/4)*Log[(
a - I*b)^(1/3) - (a + b*Tan[c + d*x])^(1/3)])/((a - I*b)^(4/3)*d) - (((3*I)
/4)*Log[(a + I*b)^(1/3) - (a + b*Tan[c + d*x])^(1/3)])/((a + I*b)^(4/3)*d)
- (3*b)/((a^2 + b^2)*d*(a + b*Tan[c + d*x])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
```

```
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(a+b\tan(c+dx))^{4/3}}\,dx = -\frac{3b}{(a^2+b^2)\,d\sqrt[3]{a+b\tan(c+dx)}} + \frac{\int \frac{a-b\tan(c+dx)}{\sqrt[3]{a+b\tan(c+dx)}}\,dx}{a^2+b^2}$$

$$= -\frac{3b}{(a^2+b^2)\,d\sqrt[3]{a+b\tan(c+dx)}} + \frac{\int \frac{1+i\tan(c+dx)}{\sqrt[3]{a+b\tan(c+dx)}}\,dx}{2(a-ib)} + \frac{\int \frac{1-i}{\sqrt[3]{a+b}}}{2(}$$

$$= -\frac{3b}{(a^2+b^2)\,d\sqrt[3]{a+b\tan(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt[3]{a+ibx}}\,dx, x, -i\tan(\right.}{2(ia-b)d}$$

$$= -\frac{x}{4(a-ib)^{4/3}} - \frac{x}{4(a+ib)^{4/3}} + \frac{i\log(\cos(c+dx))}{4(a-ib)^{4/3}d} - \frac{i\log(\cos(c+dx))}{4(a+ib)^{4/3}d} - \frac{}{(a}$$

$$= -\frac{x}{4(a-ib)^{4/3}} - \frac{x}{4(a+ib)^{4/3}} + \frac{i\log(\cos(c+dx))}{4(a-ib)^{4/3}d} - \frac{i\log(\cos(c+dx))}{4(a+ib)^{4/3}d} + \frac{3i}{}$$

$$= -\frac{x}{4(a-ib)^{4/3}} - \frac{x}{4(a+ib)^{4/3}} + \frac{i\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-ib}}}{\sqrt{3}}\right)}{2(a-ib)^{4/3}d} - $$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.25, size = 106, normalized size = 0.32

$$\frac{3i\left((a+ib)\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{a+b\tan(c+dx)}{a-ib}\right)-(a-ib)\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{a+b\tan(c+dx)}{a+ib}\right)\right)}{2\left(a^2+b^2\right)d\sqrt[3]{a+b\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^(-4/3),x]
```

```
[Out] (((3*I)/2)*((a + I*b)*Hypergeometric2F1[-1/3, 1, 2/3, (a + b*Tan[c + d*x])/
(a - I*b)] - (a - I*b)*Hypergeometric2F1[-1/3, 1, 2/3, (a + b*Tan[c + d*x])
/(a + I*b)]))/((a^2 + b^2)*d*(a + b*Tan[c + d*x])^(1/3))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.22, size = 102, normalized size = 0.30

method	result

derivativedivides	$\frac{3b\left(-\frac{1}{\left(a^{2}+b^{2}\right)\left(a+b\tan(dx+c)\right)^{\frac{1}{3}}}+\frac{\sum_{\_R=\text{RootOf}\left(\_Z^{6}-2a\_Z^{3}+a^{2}+b^{2}\right)}\frac{\left(-\_R^{4}+2\_Ra\right)\ln\left(\left(a+b\tan(dx+c)\right)^{\frac{1}{3}}-\_R\right)}{\_R^{5}-\_R^{2}a}}{6a^{2}+6b^{2}}\right)}{d}$
default	$\frac{3b\left(-\frac{1}{\left(a^{2}+b^{2}\right)\left(a+b\tan(dx+c)\right)^{\frac{1}{3}}}+\frac{\sum_{\_R=\text{RootOf}\left(\_Z^{6}-2a\_Z^{3}+a^{2}+b^{2}\right)}\frac{\left(-\_R^{4}+2\_Ra\right)\ln\left(\left(a+b\tan(dx+c)\right)^{\frac{1}{3}}-\_R\right)}{\_R^{5}-\_R^{2}a}}{6a^{2}+6b^{2}}\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c))^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d*b*(-1/(a^2+b^2)/(a+b*tan(d*x+c))^(1/3)+1/6/(a^2+b^2)*sum((-_R^4+2*_R*a)
/(_R^5-_R^2*a)*ln((a+b*tan(d*x+c))^(1/3)-_R),_R=RootOf(_Z^6-2*_Z^3*a+a^2+b^
2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(-4/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\tan\left(c + dx\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))**(4/3),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**(-4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 6.59, size = 2500, normalized size = 7.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(c + d*x))^(4/3),x)
```

```
[Out] (log((((a + b*tan(c + d*x))^(1/3)*(38880*a^4*b^18*d^7 - 1944*a^2*b^20*d^7 -
 1944*b^22*d^7 + 163296*a^6*b^16*d^7 + 299376*a^8*b^14*d^7 + 299376*a^10*b^
12*d^7 + 163296*a^12*b^10*d^7 + 38880*a^14*b^8*d^7 - 1944*a^16*b^6*d^7 - 19
44*a^18*b^4*d^7) - ((-1/(a^4*d^3*1i + b^4*d^3*1i + 4*a*b^3*d^3 - 4*a^3*b*d^
3 - a^2*b^2*d^3*6i))^(2/3)*(7776*a*b^24*d^9 + 77760*a^3*b^22*d^9 + 349920*a
^5*b^20*d^9 + 933120*a^7*b^18*d^9 + 1632960*a^9*b^16*d^9 + 1959552*a^11*b^1
4*d^9 + 1632960*a^13*b^12*d^9 + 933120*a^15*b^10*d^9 + 349920*a^17*b^8*d^9
+ 77760*a^19*b^6*d^9 + 7776*a^21*b^4*d^9))/4)*(-1/(a^4*d^3*1i + b^4*d^3*1i
+ 4*a*b^3*d^3 - 4*a^3*b*d^3 - a^2*b^2*d^3*6i))^(1/3))/2 - 972*b^21*d^6 - 38
88*a^2*b^19*d^6 + 27216*a^6*b^15*d^6 + 68040*a^8*b^13*d^6 + 81648*a^10*b^11
*d^6 + 54432*a^12*b^9*d^6 + 19440*a^14*b^7*d^6 + 2916*a^16*b^5*d^6)*(-1/(a^
4*d^3*1i + b^4*d^3*1i + 4*a*b^3*d^3 - 4*a^3*b*d^3 - a^2*b^2*d^3*6i))^(1/3))
/2 + log(((a + b*tan(c + d*x))^(1/3)*(38880*a^4*b^18*d^7 - 1944*a^2*b^20*d^
7 - 1944*b^22*d^7 + 163296*a^6*b^16*d^7 + 299376*a^8*b^14*d^7 + 299376*a^10
*b^12*d^7 + 163296*a^12*b^10*d^7 + 38880*a^14*b^8*d^7 - 1944*a^16*b^6*d^7 -
 1944*a^18*b^4*d^7) - (-1i/(8*(a^4*d^3 + b^4*d^3 + a*b^3*d^3*4i - a^3*b*d^3
*4i - 6*a^2*b^2*d^3)))^(2/3)*(7776*a*b^24*d^9 + 77760*a^3*b^22*d^9 + 349920
*a^5*b^20*d^9 + 933120*a^7*b^18*d^9 + 1632960*a^9*b^16*d^9 + 1959552*a^11*b
^14*d^9 + 1632960*a^13*b^12*d^9 + 933120*a^15*b^10*d^9 + 349920*a^17*b^8*d^
9 + 77760*a^19*b^6*d^9 + 7776*a^21*b^4*d^9))*(-1i/(8*(a^4*d^3 + b^4*d^3 + a
*b^3*d^3*4i - a^3*b*d^3*4i - 6*a^2*b^2*d^3)))^(1/3) - 972*b^21*d^6 - 3888*a
^2*b^19*d^6 + 27216*a^6*b^15*d^6 + 68040*a^8*b^13*d^6 + 81648*a^10*b^11*d^6
```

+ 54432*a^12*b^9*d^6 + 19440*a^14*b^7*d^6 + 2916*a^16*b^5*d^6)*(-1i/(8*(a^4*d^3 + b^4*d^3 + a*b^3*d^3*4i - a^3*b*d^3*4i - 6*a^2*b^2*d^3)))^(1/3) + (log(((-1i/(8*(a^4*d^3 + b^4*d^3 + a*b^3*d^3*4i - a^3*b*d^3*4i - 6*a^2*b^2*d^3)))^(1/3)*(3^(1/2)*1i - 1)*((a + b*tan(c + d*x))^(1/3)*(38880*a^4*b^18*d^7 - 1944*a^2*b^20*d^7 - 1944*b^22*d^7 + 163296*a^6*b^16*d^7 + 299376*a^8*b^14*d^7 + 299376*a^10*b^12*d^7 + 163296*a^12*b^10*d^7 + 38880*a^14*b^8*d^7 - 1944*a^16*b^6*d^7 - 1944*a^18*b^4*d^7) - ((-1i/(8*(a^4*d^3 + b^4*d^3 + a*b^3*d^3*4i - a^3*b*d^3*4i - 6*a^2*b^2*d^3)))^(2/3)*(3^(1/2)*1i - 1)^2*(7776*a*b^24*d^9 + 77760*a^3*b^22*d^9 + 349920*a^5*b^20*d^9 + 933120*a^7*b^18*d^9 + 1632960*a^9*b^16*d^9 + 1959552*a^11*b^14*d^9 + 1632960*a^13*b^12*d^9 + 933120*a^15*b^10*d^9 + 349920*a^17*b^8*d^9 + 77760*a^19*b^6*d^9 + 7776*a^21*b^4*d^9))/4))/2 - 972*b^21*d^6 - 3888*a^2*b^19*d^6 + 27216*a^6*b^15*d^6 + 68040*a^8*b^13*d^6 + 81648*a^10*b^11*d^6 + 54432*a^12*b^9*d^6 + 19440*a^14*b^7*d^6 + 2916*a^16*b^5*d^6)*(-1i/(8*(a^4*d^3 + b^4*d^3 + a*b^3*d^3*4i - a^3*b*d^3*4i - 6*a^2*b^2*d^3)))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(27216*a^6*b^15*d^6 - ((-1i/(8*(a^4*d^3 + b^4*d^3 + a*b^3*d^3*4i - a^3*b*d^3*4i - 6*a^2*b^2*d^3)))^(1/3)*(3^(1/2)*1i + 1)*((a + b*tan(c + d*x))^(1/3)*(38880*a^4*b^18*d^7 - 1944*a^2*b^20*d^7 - 1944*b^22*d^7 + 163296*a^6*b^16*d^7 + 299376*a^8*b^14*d^7 + 299376*a^10*b^12*d^7 + 163296*a^12*b^10*d^7 + 38880*a^14*b^8*d^7 - 1944*a^16*b^6*d^7 - 1944*a^18*b^4*d^7) - ((-1i/(8*(a^4*d^3 + b^4*d^3 + a*b^3*d^3*4i - a^3*b*d^3*4i - 6*a^2*b^2*d^3)))^(2/3)*(3^(1/2)*1i + 1)^2*(7776*a*b^24*d^9 + 77760*a^3*b^22*d^9 + 349920*a^5*b^20*d^9 + 933120*a^7*b^18*d^9 + 1632960*a^9*b^16*d^9 + 1959552*a^11*b^14*d^9 + 1632960*a^13*b^12*d^9 + 933120*a^15*b^10*d^9 + 349920*a^17*b^8*d^9 + 77760*a^19*b^6*d^9 + 7776*a^21*b^4*d^9))/4))/2 - 3888*a^2*b^19*d^6 - 972*b^21*d^6 + 68040*a^8*b^13*d^6 + 81648*a^10*b^11*d^6 + 54432*a^12*b^9*d^6 + 19440*a^14*b^7*d^6 + 2916*a^16*b^5*d^6)*(-1i/(8*(a^4*d^3 + b^4*d^3 + a*b^3*d^3*4i - a^3*b*d^3*4i - 6*a^2*b^2*d^3)))^(1/3)*(3^(1/2)*1i + 1))/2 + (log(((-1/(8*(a^4*d^3*1i + b^4*d^3*1i + 4*a*b^3*d^3 - 4*a^3*b*d^3 - a^2*b^2*d^3*6i)))^(1/3)*(3^(1/2)*1i - 1)*((a + b*tan(c + d*x))^(1/3)*(38880*a^4*b^18*d^7 - 1944*a^2*b^20*d^7 - 1944*b^22*d^7 + 163296*a^6*b^16*d^7 + 299376*a^8*b^14*d^7 + 299376*a^10*b^12*d^7 + 163296*a^12*b^10*d^7 + 38880*a^14*b^8*d^7 - 1944*a^16*b^6*d^7 - 1944*a^18*b^4*d^7) - ((-1/(8*(a^4*d^3*1i + b^4*d^3*1i + 4*a*b^3*d^3 - 4*a^3*b*d^3 - a^2*b^2*d^3*6i)))^(2/3)*(3^(1/2)*1i - 1)^2*(7776*a*b^24*d^9 + 77760*a^3*b^22*d^9 + 349920*a^5*b^20*d^9 + 933120*a^7*b^18*d^9 + 1632960*a^9*b^16*d^9 + 1959552*a^11*b^14*d^9 + 1632960*a^13*b^12*d^9 + 933120*a^15*b^10*d^9 + 349920*a^17*b^8*d^9 + 77760*a^19*b^6*d^9 + 7776*a^21*b^4*d^9))/4))/2 - 972*b^21*d^6 - 3888*a^2*b^19*d^6 + 27216*a^6*b^15*d^6 + 68040*a^8*b^13*d^6 + 81648*a^10*b^11*d^6 + 54432*a^12*b^9*d^6 + 19440*a^14*b^7*d^6 + 2916*a^16*b^5*d^6)*(-1/(8*(a^4*d^3*1i + b^4*d^3*1i + 4*a*b^3*d^3 - 4*a^3*b*d^3 - a^2*b^2*d^3*6i)))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(27216*a^6*b^15*d^6 - ((-1/(8*(a^4*d^3*1i + b^4*d^3*1i + 4*a*b^3*d^3 - 4*a^3*b*d^3 - a^2*b^2*d^3*6i)))^(1/3)*(3^(1/2)*1i + 1)*((a + b*tan(c + d*x))^(1/3)*(38880*a^4*b^18*d^7 - 1944*a^2*b^20*d^7 - 1944*b^22*d^7 + 163296*a^6*b^16*d^7 + 299376*a^8*b^14*d^7 + 299376*a^10*b^12*d^7 + 163296*a^12*b^10*d^7 + 38880*a...

3.695 $\int \frac{1}{(a+b\tan(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=338

$$-\frac{x}{4(a-ib)^{5/3}}-\frac{x}{4(a+ib)^{5/3}}-\frac{i\sqrt{3}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-ib}}}{\sqrt{3}}\right)}{2(a-ib)^{5/3}d}+\frac{i\sqrt{3}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a+ib}}}{\sqrt{3}}\right)}{2(a+ib)^{5/3}d}$$

```
[Out] -1/4*x/(a-I*b)^(5/3)-1/4*x/(a+I*b)^(5/3)+1/4*I*ln(cos(d*x+c))/(a-I*b)^(5/3)
/d-1/4*I*ln(cos(d*x+c))/(a+I*b)^(5/3)/d+3/4*I*ln((a-I*b)^(1/3)-(a+b*tan(d*x
+c))^(1/3))/(a-I*b)^(5/3)/d-3/4*I*ln((a+I*b)^(1/3)-(a+b*tan(d*x+c))^(1/3))/
(a+I*b)^(5/3)/d-1/2*I*arctan(1/3*(1+2*(a+b*tan(d*x+c))^(1/3)/(a-I*b)^(1/3))
*3^(1/2))*3^(1/2)/(a-I*b)^(5/3)/d+1/2*I*arctan(1/3*(1+2*(a+b*tan(d*x+c))^(1
/3)/(a+I*b)^(1/3))*3^(1/2))*3^(1/2)/(a+I*b)^(5/3)/d-3/2*b/(a^2+b^2)/d/(a+b*
tan(d*x+c))^(2/3)
```

Rubi [A]

time = 0.26, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3564, 3620, 3618, 59, 631, 210, 31}

$$-\frac{3b}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^{2/3}}-\frac{i\sqrt{3}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-ib}}}{\sqrt{3}}\right)}{2d(a-ib)^{5/3}}+\frac{i\sqrt{3}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a+ib}}}{\sqrt{3}}\right)}{2d(a+ib)^{5/3}}+\frac{3i\log\left(-\sqrt[3]{a+b\tan(c+dx)}+\sqrt[3]{a-ib}\right)}{4d(a-ib)^{5/3}}-\frac{3i\log\left(-\sqrt[3]{a+b\tan(c+dx)}+\sqrt[3]{a+ib}\right)}{4d(a+ib)^{5/3}}+\frac{i\log(\cos(c+dx))}{4d(a-ib)^{5/3}}-\frac{i\log(\cos(c+dx))}{4d(a+ib)^{5/3}}-\frac{x}{4(a-ib)^{5/3}}-\frac{x}{4(a+ib)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^(-5/3),x]
```

```
[Out] -1/4*x/(a - I*b)^(5/3) - x/(4*(a + I*b)^(5/3)) - ((I/2)*Sqrt[3]*ArcTan[(1 +
 (2*(a + b*Tan[c + d*x])^(1/3))/(a - I*b)^(1/3))/Sqrt[3]])/((a - I*b)^(5/3)
*d) + ((I/2)*Sqrt[3]*ArcTan[(1 + (2*(a + b*Tan[c + d*x])^(1/3))/(a + I*b)^(
1/3))/Sqrt[3]])/((a + I*b)^(5/3)*d) + ((I/4)*Log[Cos[c + d*x]])/((a - I*b)^
(5/3)*d) - ((I/4)*Log[Cos[c + d*x]])/((a + I*b)^(5/3)*d) + (((3*I)/4)*Log[(
a - I*b)^(1/3) - (a + b*Tan[c + d*x])^(1/3)])/((a - I*b)^(5/3)*d) - (((3*I)
/4)*Log[(a + I*b)^(1/3) - (a + b*Tan[c + d*x])^(1/3)])/((a + I*b)^(5/3)*d)
- (3*b)/(2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
```

```
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(a+b\tan(c+dx))^{5/3}}\,dx = -\frac{3b}{2(a^2+b^2)\,d(a+b\tan(c+dx))^{2/3}} + \frac{\int \frac{a-b\tan(c+dx)}{(a+b\tan(c+dx))^{2/3}}\,dx}{a^2+b^2}$$

$$= -\frac{3b}{2(a^2+b^2)\,d(a+b\tan(c+dx))^{2/3}} + \frac{\int \frac{1+i\tan(c+dx)}{(a+b\tan(c+dx))^{2/3}}\,dx}{2(a-ib)} + \frac{\int \frac{1-i\tan(c+d}{(a+b\tan(c+dx}}{2(a+ib}$$

$$= -\frac{3b}{2(a^2+b^2)\,d(a+b\tan(c+dx))^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{(-1+x)(a+ibx)^{2/3}}\,dx, x, -i\tan\right.}{2(ia-b)d}$$

$$= -\frac{x}{4(a-ib)^{5/3}} - \frac{x}{4(a+ib)^{5/3}} + \frac{i\log(\cos(c+dx))}{4(a-ib)^{5/3}d} - \frac{i\log(\cos(c+dx))}{4(a+ib)^{5/3}d} - \frac{}{2(}$$

$$= -\frac{x}{4(a-ib)^{5/3}} - \frac{x}{4(a+ib)^{5/3}} + \frac{i\log(\cos(c+dx))}{4(a-ib)^{5/3}d} - \frac{i\log(\cos(c+dx))}{4(a+ib)^{5/3}d} + \frac{3i}{}$$

$$= -\frac{x}{4(a-ib)^{5/3}} - \frac{x}{4(a+ib)^{5/3}} - \frac{i\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-ib}}}{\sqrt{3}}\right)}{2(a-ib)^{5/3}d} + $$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.16, size = 106, normalized size = 0.31

$$\frac{3i\left((a+ib)\,{}_2F_1\left(-\frac{2}{3},1;\frac{1}{3};\frac{a+b\tan(c+dx)}{a-ib}\right)-(a-ib)\,{}_2F_1\left(-\frac{2}{3},1;\frac{1}{3};\frac{a+b\tan(c+dx)}{a+ib}\right)\right)}{4(a^2+b^2)\,d(a+b\tan(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^(-5/3),x]
```

```
[Out] (((3*I)/4)*((a + I*b)*Hypergeometric2F1[-2/3, 1, 1/3, (a + b*Tan[c + d*x])/
(a - I*b)] - (a - I*b)*Hypergeometric2F1[-2/3, 1, 1/3, (a + b*Tan[c + d*x])
/(a + I*b)]))/((a^2 + b^2)*d*(a + b*Tan[c + d*x])^(2/3))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.21, size = 101, normalized size = 0.30

method	result	s

derivativedivides	$\dfrac{3b\left(-\frac{1}{2\left(a^2+b^2\right)(a+b\tan(dx+c))^{\frac{2}{3}}}+\frac{\sum\limits_{\_R=\mathrm{RootOf}\left(\_Z^6-2a\_Z^3+a^2+b^2\right)}\frac{\left(-\_R^3+2a\right)\ln\left((a+b\tan(dx+c))^{\frac{1}{3}}-\_R\right)}{\_R^5-\_R^2a}}{6a^2+6b^2}\right)}{d}$	10
default	$\dfrac{3b\left(-\frac{1}{2\left(a^2+b^2\right)(a+b\tan(dx+c))^{\frac{2}{3}}}+\frac{\sum\limits_{\_R=\mathrm{RootOf}\left(\_Z^6-2a\_Z^3+a^2+b^2\right)}\frac{\left(-\_R^3+2a\right)\ln\left((a+b\tan(dx+c))^{\frac{1}{3}}-\_R\right)}{\_R^5-\_R^2a}}{6a^2+6b^2}\right)}{d}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c))^(5/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d*b*(-1/2/(a^2+b^2)/(a+b*tan(d*x+c))^(2/3)+1/6/(a^2+b^2)*sum((-_R^3+2*a)/
(_R^5-_R^2*a)*ln((a+b*tan(d*x+c))^(1/3)-_R),_R=RootOf(_Z^6-2*_Z^3*a+a^2+b^2
)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(-5/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\tan(c+dx))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tan(d*x+c))**(5/3),x)

[Out] Integral((a + b*tan(c + d*x))**(-5/3), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tan(d*x+c))^(5/3),x, algorithm="giac")

[Out] undef

Mupad [B]
time = 6.44, size = 2500, normalized size = 7.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*tan(c + d*x))^(5/3),x)

[Out] (log((a + b*tan(c + d*x))^(1/3)*(486*b^18*d^5 + 2430*a^2*b^16*d^5 + 4374*a^4*b^14*d^5 + 2430*a^6*b^12*d^5 - 2430*a^8*b^10*d^5 - 4374*a^10*b^8*d^5 - 2430*a^12*b^6*d^5 - 486*a^14*b^4*d^5) + ((1/(a^5*d^3*1i + b^5*d^3 + a*b^4*d^3*5i + 5*a^4*b*d^3 - 10*a^2*b^3*d^3 - a^3*b^2*d^3*10i))^(1/3)*(((((1/(a^5*d^3*1i + b^5*d^3 + a*b^4*d^3*5i + 5*a^4*b*d^3 - 10*a^2*b^3*d^3 - a^3*b^2*d^3*10i))^(1/3)*(7776*a*b^24*d^9 + 77760*a^3*b^22*d^9 + 349920*a^5*b^20*d^9 + 933120*a^7*b^18*d^9 + 1632960*a^9*b^16*d^9 + 1959552*a^11*b^14*d^9 + 1632960*a^13*b^12*d^9 + 933120*a^15*b^10*d^9 + 349920*a^17*b^8*d^9 + 77760*a^19*b^6*d^9 + 7776*a^21*b^4*d^9))/2 + (a + b*tan(c + d*x))^(1/3)*(108864*a^6*b^17*d^8 - 19440*a^2*b^21*d^8 - 15552*a^4*b^19*d^8 - 3888*b^23*d^8 + 381024*a^8*b^15*d^8 + 598752*a^10*b^13*d^8 + 544320*a^12*b^11*d^8 + 295488*a^14*b^9*d^8 + 89424*a^16*b^7*d^8 + 11664*a^18*b^5*d^8))*(1/(a^5*d^3*1i + b^5*d^3 + a*b^4*d^3*5i + 5*a^4*b*d^3 - 10*a^2*b^3*d^3 - a^3*b^2*d^3*10i))^(2/3))/4 + 3888*a*b^19*d^6 + 19440*a^3*b^17*d^6 + 34992*a^5*b^15*d^6 + 19440*a^7*b^13*d^6 - 19440*a^9*b^11*d^6 - 34992*a^11*b^9*d^6 - 19440*a^13*b^7*d^6 - 3888*a^15*b^5*d^6))/2)*(1/(a^5*d^3*1i + b^5*d^3 + a*b^4*d^3*5i + 5*a^4*b*d^3 - 10*a^2*b^3*d^3 - a^3*b^2*d^3*10i))^(1/3))/2 + log((a + b*tan(c + d*x))^(1/3)*(486*b^18*d^5 + 2430*a^2*b^16*d^5 + 4374*a^4*b^14*d^5 + 2430*a^6*b^12*d^5 - 2430*a^8*b^10*d^5 - 4374*a^10*b^8*d^5 - 2430*a^12*b^6*d^5 - 486*a^14*b^4*d^5) + (1i/(8*(a^5*d^3 + b^5*d^3*1i + 5*a*b^4*d^3 + a^4*b*d^3*5i - a^2*b^3*d^3*10i - 10*a^3*b^2*d^3)))^(1/3)*(((1i/(8*(a^5*d^3 + b^5*d^3*1i + 5*a*b^4*d^3 + a^4*b*d^3*5i - a^2*b^3*d^3*10i - 10*a^3*b^2*d^3)))^(1/3)*(7776*a*b^24*d

^9 + 77760*a^3*b^22*d^9 + 349920*a^5*b^20*d^9 + 933120*a^7*b^18*d^9 + 16329
60*a^9*b^16*d^9 + 1959552*a^11*b^14*d^9 + 1632960*a^13*b^12*d^9 + 933120*a^
15*b^10*d^9 + 349920*a^17*b^8*d^9 + 77760*a^19*b^6*d^9 + 7776*a^21*b^4*d^9)
 + (a + b*tan(c + d*x))^(1/3)*(108864*a^6*b^17*d^8 - 19440*a^2*b^21*d^8 - 1
5552*a^4*b^19*d^8 - 3888*b^23*d^8 + 381024*a^8*b^15*d^8 + 598752*a^10*b^13*
d^8 + 544320*a^12*b^11*d^8 + 295488*a^14*b^9*d^8 + 89424*a^16*b^7*d^8 + 116
64*a^18*b^5*d^8))*(1i/(8*(a^5*d^3 + b^5*d^3*1i + 5*a*b^4*d^3 + a^4*b*d^3*5i
 - a^2*b^3*d^3*10i - 10*a^3*b^2*d^3)))^(2/3) + 3888*a*b^19*d^6 + 19440*a^3*
b^17*d^6 + 34992*a^5*b^15*d^6 + 19440*a^7*b^13*d^6 - 19440*a^9*b^11*d^6 - 3
4992*a^11*b^9*d^6 - 19440*a^13*b^7*d^6 - 3888*a^15*b^5*d^6))*(1i/(8*(a^5*d^
3 + b^5*d^3*1i + 5*a*b^4*d^3 + a^4*b*d^3*5i - a^2*b^3*d^3*10i - 10*a^3*b^2*
d^3)))^(1/3) + (log((a + b*tan(c + d*x))^(1/3)*(486*b^18*d^5 + 2430*a^2*b^1
6*d^5 + 4374*a^4*b^14*d^5 + 2430*a^6*b^12*d^5 - 2430*a^8*b^10*d^5 - 4374*a^
10*b^8*d^5 - 2430*a^12*b^6*d^5 - 486*a^14*b^4*d^5) + ((3^(1/2)*1i - 1)*(1i/
(8*(a^5*d^3 + b^5*d^3*1i + 5*a*b^4*d^3 + a^4*b*d^3*5i - a^2*b^3*d^3*10i - 1
0*a^3*b^2*d^3)))^(1/3)*(3888*a*b^19*d^6 + 19440*a^3*b^17*d^6 + 34992*a^5*b^
15*d^6 + 19440*a^7*b^13*d^6 - 19440*a^9*b^11*d^6 - 34992*a^11*b^9*d^6 - 194
40*a^13*b^7*d^6 - 3888*a^15*b^5*d^6 + ((3^(1/2)*1i - 1)^2*(1i/(8*(a^5*d^3 +
 b^5*d^3*1i + 5*a*b^4*d^3 + a^4*b*d^3*5i - a^2*b^3*d^3*10i - 10*a^3*b^2*d^3
)))^(2/3)*((a + b*tan(c + d*x))^(1/3)*(108864*a^6*b^17*d^8 - 19440*a^2*b^21
*d^8 - 15552*a^4*b^19*d^8 - 3888*b^23*d^8 + 381024*a^8*b^15*d^8 + 598752*a^
10*b^13*d^8 + 544320*a^12*b^11*d^8 + 295488*a^14*b^9*d^8 + 89424*a^16*b^7*d
^8 + 11664*a^18*b^5*d^8) + ((3^(1/2)*1i - 1)*(1i/(8*(a^5*d^3 + b^5*d^3*1i +
 5*a*b^4*d^3 + a^4*b*d^3*5i - a^2*b^3*d^3*10i - 10*a^3*b^2*d^3)))^(1/3)*(77
76*a*b^24*d^9 + 77760*a^3*b^22*d^9 + 349920*a^5*b^20*d^9 + 933120*a^7*b^18*
d^9 + 1632960*a^9*b^16*d^9 + 1959552*a^11*b^14*d^9 + 1632960*a^13*b^12*d^9
+ 933120*a^15*b^10*d^9 + 349920*a^17*b^8*d^9 + 77760*a^19*b^6*d^9 + 7776*a^
21*b^4*d^9))/2))/4))/2)*(3^(1/2)*1i - 1)*(1i/(8*(a^5*d^3 + b^5*d^3*1i + 5*a
*b^4*d^3 + a^4*b*d^3*5i - a^2*b^3*d^3*10i - 10*a^3*b^2*d^3)))^(1/3))/2 - (l
og((a + b*tan(c + d*x))^(1/3)*(486*b^18*d^5 + 2430*a^2*b^16*d^5 + 4374*a^4*
b^14*d^5 + 2430*a^6*b^12*d^5 - 2430*a^8*b^10*d^5 - 4374*a^10*b^8*d^5 - 2430
*a^12*b^6*d^5 - 486*a^14*b^4*d^5) - ((3^(1/2)*1i + 1)*(1i/(8*(a^5*d^3 + b^5
*d^3*1i + 5*a*b^4*d^3 + a^4*b*d^3*5i - a^2*b^3*d^3*10i - 10*a^3*b^2*d^3)))^
(1/3)*(3888*a*b^19*d^6 + 19440*a^3*b^17*d^6 + 34992*a^5*b^15*d^6 + 19440*a^
7*b^13*d^6 - 19440*a^9*b^11*d^6 - 34992*a^11*b^9*d^6 - 19440*a^13*b^7*d^6 -
 3888*a^15*b^5*d^6 + ((3^(1/2)*1i + 1)^2*(1i/(8*(a^5*d^3 + b^5*d^3*1i + 5*a
*b^4*d^3 + a^4*b*d^3*5i - a^2*b^3*d^3*10i - 10*a^3*b^2*d^3)))^(2/3)*((a + b
*tan(c + d*x))^(1/3)*(108864*a^6*b^17*d^8 - 19440*a^2*b^21*d^8 - 15552*a^4*
b^19*d^8 - 3888*b^23*d^8 + 381024*a^8*b^15*d^8 + 598752*a^10*b^13*d^8 + 544
320*a^12*b^11*d^8 + 295488*a^14*b^9*d^8 + 89424*a^16*b^7*d^8 + 11664*a^18*b
^5*d^8) - ((3^(1/2)*1i + 1)*(1i/(8*(a^5*d^3 + b^5*d^3*1i + 5*a*b^4*d^3 + a^
4*b*d^3*5i - a^2*b^3*d^3*10i - 10*a^3*b^2*d^3)))^(1/3)*(7776*a*b^24*d^9 + 7
7760*a^3*b^22*d^9 + 349920*a^5*b^20*d^9 + 933120*a^7*b^18*d^9 + 1632960*a^9
*b^16*d^9 + 1959552*a^11*b^14*d^9 + 1632960*a^13*b^12*d^9 + 933120*a^15*b^1
0*d^9 + 349920*a^17*b^8*d^9 + 77760*a^19*b^6*d^...

3.696 $\int (d\tan(e+fx))^n (a+b\tan(e+fx))^4\,dx$

Optimal. Leaf size=261

$$-\frac{b^2(b^2(3+n)-a^2(17+5n))\,(d\tan(e+fx))^{1+n}}{df(1+n)(3+n)}+\frac{(a^4-6a^2b^2+b^4)\;{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)(d\,\mathrm{ta}}{df(1+n)}$$

```
[Out] -b^2*(b^2*(3+n)-a^2*(17+5*n))*(d*tan(f*x+e))^(1+n)/d/f/(1+n)/(3+n)+(a^4-6*a
^2*b^2+b^4)*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],-tan(f*x+e)^2)*(d*tan(f*x+
e))^(1+n)/d/f/(1+n)+2*a*b^3*(4+n)*tan(f*x+e)*(d*tan(f*x+e))^(1+n)/d/f/(2+n)
/(3+n)+4*a*b*(a^2-b^2)*hypergeom([1, 1+1/2*n],[2+1/2*n],-tan(f*x+e)^2)*(d*t
an(f*x+e))^(2+n)/d^2/f/(2+n)+b^2*(d*tan(f*x+e))^(1+n)*(a+b*tan(f*x+e))^2/d/
f/(3+n)
```

Rubi [A]

time = 0.45, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = $\{3647, 3718, 3711, 3619, 3557, 371\}$

$$\frac{4ab(a^2-b^2)\,(d\tan(e+fx))^{n+2}\,{}_2F_1\left(1,\frac{n+2}{2};\frac{n+4}{2};-\tan^2(e+fx)\right)}{d^2f(n+2)}-\frac{b^2(b^2(n+3)-a^2(5n+17))\,(d\tan(e+fx))^{n+1}}{df(n+1)(n+3)}+\frac{(a^4-6a^2b^2+b^4)\,(d\tan(e+fx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(e+fx)\right)}{df(n+1)}+\frac{2ab^3(n+4)\tan(e+fx)(d\tan(e+fx))^{n+1}}{df(n+2)(n+3)}+\frac{b^2(a+b\tan(e+fx))^2(d\tan(e+fx))^{n+1}}{df(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^n*(a + b*Tan[e + f*x])^4,x]
```

```
[Out] -((b^2*(b^2*(3 + n) - a^2*(17 + 5*n))*(d*Tan[e + f*x])^(1 + n))/(d*f*(1 + n
)*(3 + n))) + ((a^4 - 6*a^2*b^2 + b^4)*Hypergeometric2F1[1, (1 + n)/2, (3 +
 n)/2, -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(1 + n))/(d*f*(1 + n)) + (2*a*b^3*
(4 + n)*Tan[e + f*x]*(d*Tan[e + f*x])^(1 + n))/(d*f*(2 + n)*(3 + n)) + (4*a
*b*(a^2 - b^2)*Hypergeometric2F1[1, (2 + n)/2, (4 + n)/2, -Tan[e + f*x]^2]*
(d*Tan[e + f*x])^(2 + n))/(d^2*f*(2 + n)) + (b^2*(d*Tan[e + f*x])^(1 + n)*(
a + b*Tan[e + f*x])^2)/(d*f*(3 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int (d\tan(e+fx))^n(a+b\tan(e+fx))^4\,dx &= \frac{b^2(d\tan(e+fx))^{1+n}(a+b\tan(e+fx))^2}{df(3+n)} + \frac{\int (d\tan(e+fx))\\ &= \frac{2ab^3(4+n)\tan(e+fx)(d\tan(e+fx))^{1+n}}{df(2+n)(3+n)} + \frac{b^2(d\tan(e+fx\\ &= -\frac{b^2(b^2(3+n)-a^2(17+5n))\,(d\tan(e+fx))^{1+n}}{df(1+n)(3+n)} + \frac{2ab^3(4+\\ &= -\frac{b^2(b^2(3+n)-a^2(17+5n))\,(d\tan(e+fx))^{1+n}}{df(1+n)(3+n)} + \frac{2ab^3(4+\\ &= -\frac{b^2(b^2(3+n)-a^2(17+5n))\,(d\tan(e+fx))^{1+n}}{df(1+n)(3+n)} + \frac{2ab^3(4+\\ &= -\frac{b^2(b^2(3+n)-a^2(17+5n))\,(d\tan(e+fx))^{1+n}}{df(1+n)(3+n)} + \frac{(a^4-6a^2\end{aligned}$$

Mathematica [A]
time = 1.68, size = 191, normalized size = 0.73

$$\frac{\tan(e+fx)(d\tan(e+fx))^n\left(\frac{-b^4(3+n)+a^2b^2(17+5n)}{1+n}+\frac{(a^4-6a^2b^2+b^4)(3+n)\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)}{1+n}+\frac{2ab^3(4+n)\tan(e+fx)}{2+n}+\frac{4a(a-b)b(a+b)(3+n)\,{}_2F_1\left(1,\frac{2+n}{2};\frac{4+n}{2};-\tan^2(e+fx)\right)\tan(e+fx)}{2+n}+b^2(a+b\tan(e+fx))^2\right)}{f(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^n*(a + b*Tan[e + f*x])^4,x]
```

```
[Out] (Tan[e + f*x]*(d*Tan[e + f*x])^n*((-(b^4*(3 + n)) + a^2*b^2*(17 + 5*n))/(1
+ n) + ((a^4 - 6*a^2*b^2 + b^4)*(3 + n)*Hypergeometric2F1[1, (1 + n)/2, (3
+ n)/2, -Tan[e + f*x]^2])/(1 + n) + (2*a*b^3*(4 + n)*Tan[e + f*x])/(2 + n)
+ (4*a*(a - b)*b*(a + b)*(3 + n)*Hypergeometric2F1[1, (2 + n)/2, (4 + n)/2,
 -Tan[e + f*x]^2]*Tan[e + f*x])/(2 + n) + b^2*(a + b*Tan[e + f*x])^2))/(f*(
3 + n))
```

Maple [F]
time = 0.51, size = 0, normalized size = 0.00

$$\int \left(d\tan\left(fx+e\right)\right)^{n}\left(a+b\tan\left(fx+e\right)\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^n*(a+b*tan(f*x+e))^4,x)
```

```
[Out] int((d*tan(f*x+e))^n*(a+b*tan(f*x+e))^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+b*tan(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^4*(d*tan(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+b*tan(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] integral((b^4*tan(f*x + e)^4 + 4*a*b^3*tan(f*x + e)^3 + 6*a^2*b^2*tan(f*x +
 e)^2 + 4*a^3*b*tan(f*x + e) + a^4)*(d*tan(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\tan (e + fx))^n (a + b\tan (e + fx))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**n*(a+b*tan(f*x+e))**4,x)
```

```
[Out] Integral((d*tan(e + f*x))**n*(a + b*tan(e + f*x))**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+b*tan(f*x+e))^4,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)^4*(d*tan(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(d\,\tan(e+f\,x)\right)}^n\,{\left(a+b\,\tan(e+f\,x)\right)}^4 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^n*(a + b*tan(e + f*x))^4,x)
```

```
[Out] int((d*tan(e + f*x))^n*(a + b*tan(e + f*x))^4, x)
```

3.697 $\int (d\tan(e+fx))^n (a+b\tan(e+fx))^3\,dx$

Optimal. Leaf size=198

$$\frac{ab^2(5+2n)(d\tan(e+fx))^{1+n}}{df(1+n)(2+n)}+\frac{a(a^2-3b^2)\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)(d\tan(e+fx))^{1+n}}{df(1+n)}+\frac{b(3a^2-$$

```
[Out] a*b^2*(5+2*n)*(d*tan(f*x+e))^(1+n)/d/f/(1+n)/(2+n)+a*(a^2-3*b^2)*hypergeom(
[1, 1/2+1/2*n],[3/2+1/2*n],-tan(f*x+e)^2)*(d*tan(f*x+e))^(1+n)/d/f/(1+n)+b*
(3*a^2-b^2)*hypergeom([1, 1+1/2*n],[2+1/2*n],-tan(f*x+e)^2)*(d*tan(f*x+e))^
(2+n)/d^2/f/(2+n)+b^2*(d*tan(f*x+e))^(1+n)*(a+b*tan(f*x+e))/d/f/(2+n)
```

Rubi [A]
time = 0.21, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$,
Rules used = {3647, 3711, 3619, 3557, 371}

$$\frac{b(3a^2-b^2)(d\tan(e+fx))^{n+2}\,{}_2F_1\left(1,\frac{n+2}{2};\frac{n+4}{2};-\tan^2(e+fx)\right)}{d^2f(n+2)}+\frac{a(a^2-3b^2)(d\tan(e+fx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(e+fx)\right)}{df(n+1)}+\frac{ab^2(2n+5)(d\tan(e+fx))^{n+1}}{df(n+1)(n+2)}+\frac{b^2(a+b\tan(e+fx))(d\tan(e+fx))^{n+1}}{df(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^n*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (a*b^2*(5 + 2*n)*(d*Tan[e + f*x])^(1 + n))/(d*f*(1 + n)*(2 + n)) + (a*(a^2
- 3*b^2)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[e + f*x]^2]*(d*Tan
[e + f*x])^(1 + n))/(d*f*(1 + n)) + (b*(3*a^2 - b^2)*Hypergeometric2F1[1, (
2 + n)/2, (4 + n)/2, -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(2 + n))/(d^2*f*(2 +
 n)) + (b^2*(d*Tan[e + f*x])^(1 + n)*(a + b*Tan[e + f*x]))/(d*f*(2 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
```

```
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (d\tan(e+fx))^n(a+b\tan(e+fx))^3\,dx &= \frac{b^2(d\tan(e+fx))^{1+n}(a+b\tan(e+fx))}{df(2+n)} + \frac{\int (d\tan(e+fx))^n \,(\\
&= \frac{ab^2(5+2n)(d\tan(e+fx))^{1+n}}{df(1+n)(2+n)} + \frac{b^2(d\tan(e+fx))^{1+n}(a+b\,\mathrm{ta}}{df(2+n)}\\
&= \frac{ab^2(5+2n)(d\tan(e+fx))^{1+n}}{df(1+n)(2+n)} + \frac{b^2(d\tan(e+fx))^{1+n}(a+b\,\mathrm{ta}}{df(2+n)}\\
&= \frac{ab^2(5+2n)(d\tan(e+fx))^{1+n}}{df(1+n)(2+n)} + \frac{b^2(d\tan(e+fx))^{1+n}(a+b\,\mathrm{ta}}{df(2+n)}\\
&= \frac{ab^2(5+2n)(d\tan(e+fx))^{1+n}}{df(1+n)(2+n)} + \frac{a(a^2-3b^2)\ {}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\right.}{df}
\end{aligned}$$

Mathematica [A]
time = 0.80, size = 141, normalized size = 0.71

$$\frac{\tan(e+fx)(d\tan(e+fx))^n\left(a(a^2-3b^2)(2+n)\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)+b\left((3a^2-b^2)(1+n)\,{}_2F_1\left(1,\frac{2+n}{2};\frac{4+n}{2};-\tan^2(e+fx)\right)\tan(e+fx)+b(3a(2+n)+b(1+n)\tan(e+fx))\right)\right)}{f(1+n)(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^n*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (Tan[e + f*x]*(d*Tan[e + f*x])^n*(a*(a^2 - 3*b^2)*(2 + n)*Hypergeometric2F1
[1, (1 + n)/2, (3 + n)/2, -Tan[e + f*x]^2] + b*((3*a^2 - b^2)*(1 + n)*Hyper
geometric2F1[1, (2 + n)/2, (4 + n)/2, -Tan[e + f*x]^2]*Tan[e + f*x] + b*(3*
a*(2 + n) + b*(1 + n)*Tan[e + f*x]))))/(f*(1 + n)*(2 + n))
```

Maple [F]
time = 0.43, size = 0, normalized size = 0.00

$$\int (d\tan (fx+e))^{n} (a+b\tan (fx+e))^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^n*(a+b*tan(f*x+e))^3,x)
```

```
[Out] int((d*tan(f*x+e))^n*(a+b*tan(f*x+e))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^3*(d*tan(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*tan(f*x + e)^3 + 3*a*b^2*tan(f*x + e)^2 + 3*a^2*b*tan(f*x + e
) + a^3)*(d*tan(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\tan (e+fx))^{n} (a+b\tan (e+fx))^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**n*(a+b*tan(f*x+e))**3,x)
```

```
[Out] Integral((d*tan(e + f*x))**n*(a + b*tan(e + f*x))**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+b*tan(f*x+e))^3,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^3*(d*tan(f*x + e))^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\tan(e+f\,x))^n\,(a+b\tan(e+f\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^n*(a + b*tan(e + f*x))^3,x)
[Out] int((d*tan(e + f*x))^n*(a + b*tan(e + f*x))^3, x)
```

3.698 $\int (d\tan(e+fx))^n(a+b\tan(e+fx))^2\,dx$

Optimal. Leaf size=140

$$\frac{b^2(d\tan(e+fx))^{1+n}}{df(1+n)}+\frac{(a^2-b^2)\ {}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)(d\tan(e+fx))^{1+n}}{df(1+n)}+\frac{2ab\,{}_2F_1\left(1,\frac{2+n}{2};\frac{4+n}{2};\right.}{}$$

```
[Out] b^2*(d*tan(f*x+e))^(1+n)/d/f/(1+n)+(a^2-b^2)*hypergeom([1, 1/2+1/2*n],[3/2+
1/2*n],-tan(f*x+e)^2)*(d*tan(f*x+e))^(1+n)/d/f/(1+n)+2*a*b*hypergeom([1, 1+
1/2*n],[2+1/2*n],-tan(f*x+e)^2)*(d*tan(f*x+e))^(2+n)/d^2/f/(2+n)
```

Rubi [A]
time = 0.10, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3624, 3619, 3557, 371}

$$\frac{(a^2-b^2)(d\tan(e+fx))^{n+1}{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(e+fx)\right)}{df(n+1)}+\frac{2ab(d\tan(e+fx))^{n+2}{}_2F_1\left(1,\frac{n+2}{2};\frac{n+4}{2};-\tan^2(e+fx)\right)}{d^2f(n+2)}+\frac{b^2(d\tan(e+fx))^{n+1}}{df(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^n*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (b^2*(d*Tan[e + f*x])^(1 + n))/(d*f*(1 + n)) + ((a^2 - b^2)*Hypergeometric2
F1[1, (1 + n)/2, (3 + n)/2, -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(1 + n))/(d*f
*(1 + n)) + (2*a*b*Hypergeometric2F1[1, (2 + n)/2, (4 + n)/2, -Tan[e + f*x]
^2]*(d*Tan[e + f*x])^(2 + n))/(d^2*f*(2 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] && !IntegerQ[2*m]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int (d\tan(e+fx))^n(a+b\tan(e+fx))^2\,dx = \frac{b^2(d\tan(e+fx))^{1+n}}{df(1+n)} + \int (d\tan(e+fx))^n\left(a^2-b^2+2ab\tan(e\right.$$
$$= \frac{b^2(d\tan(e+fx))^{1+n}}{df(1+n)} + \left(a^2-b^2\right)\int (d\tan(e+fx))^n\,dx + \frac{(2ab}{}$$
$$= \frac{b^2(d\tan(e+fx))^{1+n}}{df(1+n)} + \frac{(2ab)\text{Subst}\left(\int \frac{x^{1+n}}{d^2+x^2}\,dx, x, d\tan(e+fx\right.}{f}$$
$$= \frac{b^2(d\tan(e+fx))^{1+n}}{df(1+n)} + \frac{\left(a^2-b^2\right)\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+f\right.}{df(1+n)}$$

Mathematica [A]
time = 0.42, size = 116, normalized size = 0.83

$$\frac{\tan(e+fx)(d\tan(e+fx))^n\left(\left(a^2-b^2\right)(2+n)\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)+b\left(b(2+n)+2a(1+n)\,{}_2F_1\left(1,\frac{2+n}{2};\frac{4+n}{2};-\tan^2(e+fx)\right)\tan(e+fx)\right)\right)}{f(1+n)(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^n*(a + b*Tan[e + f*x])^2,x]
[Out] (Tan[e + f*x]*(d*Tan[e + f*x])^n*((a^2 - b^2)*(2 + n)*Hypergeometric2F1[1,
(1 + n)/2, (3 + n)/2, -Tan[e + f*x]^2] + b*(b*(2 + n) + 2*a*(1 + n)*Hyperge
ometric2F1[1, (2 + n)/2, (4 + n)/2, -Tan[e + f*x]^2]*Tan[e + f*x])))/(f*(1
+ n)*(2 + n))
```

Maple [F]
time = 0.37, size = 0, normalized size = 0.00

$$\int (d\tan(fx+e))^n\,(a+b\tan(fx+e))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^n*(a+b*tan(f*x+e))^2,x)
[Out] int((d*tan(f*x+e))^n*(a+b*tan(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)^2*(d*tan(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
[Out] integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)*(d*tan(f*x + e))^n
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\tan(e+fx))^n (a+b\tan(e+fx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**n*(a+b*tan(f*x+e))**2,x)
[Out] Integral((d*tan(e + f*x))**n*(a + b*tan(e + f*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+b*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^2*(d*tan(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\tan(e+f\,x))^n\,(a+b\tan(e+f\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^n*(a + b*tan(e + f*x))^2,x)
[Out] int((d*tan(e + f*x))^n*(a + b*tan(e + f*x))^2, x)
```

3.699 $\int (d\tan(e+fx))^n(a+b\tan(e+fx))\,dx$

Optimal. Leaf size=103

$$\frac{a\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)(d\tan(e+fx))^{1+n}}{df(1+n)}+\frac{b\,{}_2F_1\left(1,\frac{2+n}{2};\frac{4+n}{2};-\tan^2(e+fx)\right)(d\tan(e+fx))^{2+n}}{d^2f(2+n)}$$

```
[Out] a*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],-tan(f*x+e)^2)*(d*tan(f*x+e))^(1+n)/
d/f/(1+n)+b*hypergeom([1, 1+1/2*n],[2+1/2*n],-tan(f*x+e)^2)*(d*tan(f*x+e))^
(2+n)/d^2/f/(2+n)
```

Rubi [A]

time = 0.06, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3619, 3557, 371}

$$\frac{a(d\tan(e+fx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(e+fx)\right)}{df(n+1)}+\frac{b(d\tan(e+fx))^{n+2}\,{}_2F_1\left(1,\frac{n+2}{2};\frac{n+4}{2};-\tan^2(e+fx)\right)}{d^2f(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^n*(a + b*Tan[e + f*x]),x]
```

```
[Out] (a*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[e + f*x]^2]*(d*Tan[e + f
*x])^(1 + n))/(d*f*(1 + n)) + (b*Hypergeometric2F1[1, (2 + n)/2, (4 + n)/2,
 -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(2 + n))/(d^2*f*(2 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rubi steps

$$\int (d\tan(e+fx))^n(a+b\tan(e+fx))\,dx = a\int (d\tan(e+fx))^n\,dx + \frac{b\int (d\tan(e+fx))^{1+n}\,dx}{d}$$
$$= \frac{b\text{Subst}\left(\int \frac{x^{1+n}}{d^2+x^2}\,dx, x, d\tan(e+fx)\right)}{f} + \frac{(ad)\text{Subst}\left(\int \frac{x^n}{d^2+x^2}\,dx,\right.}{f}$$
$$= \frac{a\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)(d\tan(e+fx))^{1+n}}{df(1+n)} + \frac{b\,{}_2F_1\left(\right.}{}$$

Mathematica [A]
time = 0.27, size = 99, normalized size = 0.96

$$\frac{\tan(e+fx)(d\tan(e+fx))^n\left(a(2+n)\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)+b(1+n)\,{}_2F_1\left(1,\frac{2+n}{2};\frac{4+n}{2};-\tan^2(e+fx)\right)\tan(e+fx)\right)}{f(1+n)(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^n*(a + b*Tan[e + f*x]),x]
[Out] (Tan[e + f*x]*(d*Tan[e + f*x])^n*(a*(2 + n)*Hypergeometric2F1[1, (1 + n)/2,
 (3 + n)/2, -Tan[e + f*x]^2] + b*(1 + n)*Hypergeometric2F1[1, (2 + n)/2, (4
 + n)/2, -Tan[e + f*x]^2]*Tan[e + f*x]))/(f*(1 + n)*(2 + n))
```

Maple [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int (d\tan(fx+e))^n\,(a+b\tan(fx+e))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^n*(a+b*tan(f*x+e)),x)
[Out] int((d*tan(f*x+e))^n*(a+b*tan(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+b*tan(f*x+e)),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)*(d*tan(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e) + a)*(d*tan(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\tan (e+fx))^n (a+b\tan (e+fx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**n*(a+b*tan(f*x+e)),x)
```

```
[Out] Integral((d*tan(e + f*x))**n*(a + b*tan(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)*(d*tan(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\tan(e+f\,x))^n\,(a+b\tan(e+f\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^n*(a + b*tan(e + f*x)),x)
```

```
[Out] int((d*tan(e + f*x))^n*(a + b*tan(e + f*x)), x)
```

3.700 $\int \frac{(d\tan(e+fx))^n}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=181

$$\frac{a\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)(d\tan(e+fx))^{1+n}}{(a^2+b^2)\,df(1+n)}+\frac{b^2\,{}_2F_1\left(1,1+n;2+n;-\frac{b\tan(e+fx)}{a}\right)(d\tan(e+fx))^{1+n}}{a\,(a^2+b^2)\,df(1+n)}$$

```
[Out] a*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],-tan(f*x+e)^2)*(d*tan(f*x+e))^(1+n)/
(a^2+b^2)/d/f/(1+n)+b^2*hypergeom([1, 1+n],[2+n],-b*tan(f*x+e)/a)*(d*tan(f*
x+e))^(1+n)/a/(a^2+b^2)/d/f/(1+n)-b*hypergeom([1, 1+1/2*n],[2+1/2*n],-tan(f
*x+e)^2)*(d*tan(f*x+e))^(2+n)/(a^2+b^2)/d^2/f/(2+n)
```

Rubi [A]
time = 0.19, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3655, 3619, 3557, 371, 3715, 66}

$$-\frac{b(d\tan(e+fx))^{n+2}\,{}_2F_1\left(1,\frac{n+2}{2};\frac{n+4}{2};-\tan^2(e+fx)\right)}{d^2f(n+2)\,(a^2+b^2)}+\frac{a(d\tan(e+fx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(e+fx)\right)}{df(n+1)\,(a^2+b^2)}+\frac{b^2(d\tan(e+fx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;-\frac{b\tan(e+fx)}{a}\right)}{adf(n+1)\,(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^n/(a + b*Tan[e + f*x]),x]
```

```
[Out] (a*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[e + f*x]^2]*(d*Tan[e + f
*x])^(1 + n))/((a^2 + b^2)*d*f*(1 + n)) + (b^2*Hypergeometric2F1[1, 1 + n,
2 + n, -((b*Tan[e + f*x])/a)]*(d*Tan[e + f*x])^(1 + n))/(a*(a^2 + b^2)*d*f*
(1 + n)) - (b*Hypergeometric2F1[1, (2 + n)/2, (4 + n)/2, -Tan[e + f*x]^2]*(
d*Tan[e + f*x])^(2 + n))/((a^2 + b^2)*d^2*f*(2 + n))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)/((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[(a + b*Tan[e + f*x])^m*
(c - d*Tan[e + f*x]), x], x] + Dist[d^2/(c^2 + d^2), Int[(a + b*Tan[e + f*x
])^m*((1 + Tan[e + f*x]^2)/(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d\tan(e+fx))^n}{a+b\tan(e+fx)}\,dx &= \frac{\int (d\tan(e+fx))^n(a-b\tan(e+fx))\,dx}{a^2+b^2}+\frac{b^2\int \frac{(d\tan(e+fx))^n(1+\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx}{a^2+b^2}\\
&= \frac{a\int (d\tan(e+fx))^n\,dx}{a^2+b^2}-\frac{b\int (d\tan(e+fx))^{1+n}\,dx}{(a^2+b^2)\,d}+\frac{b^2\mathrm{Subst}\left(\int \frac{(dx)^n}{a+bx}\,dx,x,\tan(e\right.}{(a^2+b^2)\,f}\\
&= \frac{b^2\,{}_2F_1\left(1,1+n;2+n;-\frac{b\tan(e+fx)}{a}\right)(d\tan(e+fx))^{1+n}}{a\,(a^2+b^2)\,df(1+n)}-\frac{b\mathrm{Subst}\left(\int \frac{x^{1+n}}{d^2+x^2}\,dx,x,d\right.}{(a^2+b^2)\,f}\\
&= \frac{a\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)(d\tan(e+fx))^{1+n}}{(a^2+b^2)\,df(1+n)}+\frac{b^2\,{}_2F_1\left(1,1+n;2+n;-\right.}{a\,(a^2+}
\end{aligned}$$

Mathematica [A]
time = 0.53, size = 142, normalized size = 0.78

$$\frac{\tan(e+fx)(d\tan(e+fx))^n\left(a^2(2+n)\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)+b\left(b(2+n)\,{}_2F_1\left(1,1+n;2+n;-\frac{b\tan(e+fx)}{a}\right)-a(1+n)\,{}_2F_1\left(1,\frac{2+n}{2};\frac{4+n}{2};-\tan^2(e+fx)\right)\tan(e+fx)\right)\right)}{a\,(a^2+b^2)\,f(1+n)(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^n/(a + b*Tan[e + f*x]),x]
```

```
[Out] (Tan[e + f*x]*(d*Tan[e + f*x])^n*(a^2*(2 + n)*Hypergeometric2F1[1, (1 + n)/
2, (3 + n)/2, -Tan[e + f*x]^2] + b*(b*(2 + n)*Hypergeometric2F1[1, 1 + n, 2
 + n, -((b*Tan[e + f*x])/a)] - a*(1 + n)*Hypergeometric2F1[1, (2 + n)/2, (4
 + n)/2, -Tan[e + f*x]^2]*Tan[e + f*x])))/(a*(a^2 + b^2)*f*(1 + n)*(2 + n))
```

Maple [F]
time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{(d \tan (f x+e))^{n}}{a+b \tan (f x+e)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^n/(a+b*tan(f*x+e)),x)
```

```
[Out] int((d*tan(f*x+e))^n/(a+b*tan(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(f*x + e))^n/(b*tan(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((d*tan(f*x + e))^n/(b*tan(f*x + e) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d \tan (e+f x))^{n}}{a+b \tan (e+f x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**n/(a+b*tan(f*x+e)),x)
```

```
[Out] Integral((d*tan(e + f*x))**n/(a + b*tan(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n/(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*tan(f*x + e))^n/(b*tan(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\tan(e+f\,x))^n}{a+b\tan\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^n/(a + b*tan(e + f*x)),x)
```

```
[Out] int((d*tan(e + f*x))^n/(a + b*tan(e + f*x)), x)
```

3.701 $\int \frac{(d\tan(e+fx))^n}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=252

$$\frac{(a^2-b^2)\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)(d\tan(e+fx))^{1+n}}{(a^2+b^2)^2\,df(1+n)}+\frac{b^2(a^2(2-n)-b^2n)\,{}_2F_1\left(1,1+n;2+n;-\frac{b}{\cdots}\right.}{a^2\,(a^2+b^2)^2\,df(1\cdots}$$

```
[Out] (a^2-b^2)*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],-tan(f*x+e)^2)*(d*tan(f*x+e)
)^(1+n)/(a^2+b^2)^2/d/f/(1+n)+b^2*(a^2*(2-n)-b^2*n)*hypergeom([1, 1+n],[2+n
],-b*tan(f*x+e)/a)*(d*tan(f*x+e))^(1+n)/a^2/(a^2+b^2)^2/d/f/(1+n)-2*a*b*hyp
ergeom([1, 1+1/2*n],[2+1/2*n],-tan(f*x+e)^2)*(d*tan(f*x+e))^(2+n)/(a^2+b^2)
^2/d^2/f/(2+n)+b^2*(d*tan(f*x+e))^(1+n)/a/(a^2+b^2)/d/f/(a+b*tan(f*x+e))
```

Rubi [A]
time = 0.38, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3650, 3734, 3619, 3557, 371, 3715, 66}

$$-\frac{2ab(d\tan(e+fx))^{n+2}\,{}_2F_1\left(1,\frac{n+2}{2};\frac{n+4}{2};-\tan^2(e+fx)\right)}{d^2f(n+2)\left(a^2+b^2\right)^2}+\frac{(a^2-b^2)\,(d\tan(e+fx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(e+fx)\right)}{df(n+1)\left(a^2+b^2\right)^2}+\frac{b^2(a^2(2-n)-b^2n)\,(d\tan(e+fx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;-\frac{b\tan(e+fx)}{a}\right)}{a^2df(n+1)\left(a^2+b^2\right)^2}+\frac{b^2(d\tan(e+fx))^{n+1}}{adf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^n/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] ((a^2 - b^2)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[e + f*x]^2]*(d
*Tan[e + f*x])^(1 + n))/((a^2 + b^2)^2*d*f*(1 + n)) + (b^2*(a^2*(2 - n) - b
^2*n)*Hypergeometric2F1[1, 1 + n, 2 + n, -((b*Tan[e + f*x])/a)]*(d*Tan[e +
f*x])^(1 + n))/(a^2*(a^2 + b^2)^2*d*f*(1 + n)) - (2*a*b*Hypergeometric2F1[1
, (2 + n)/2, (4 + n)/2, -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(2 + n))/((a^2 +
b^2)^2*d^2*f*(2 + n)) + (b^2*(d*Tan[e + f*x])^(1 + n))/(a*(a^2 + b^2)*d*f*(
a + b*Tan[e + f*x]))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^n}{(a+b\tan(e+fx))^2}\,dx = \frac{b^2(d\tan(e+fx))^{1+n}}{a\left(a^2+b^2\right)df(a+b\tan(e+fx))} + \frac{\int \frac{(d\tan(e+fx))^n\left(d\left(a^2-b^2n\right)-abd\tan(e+fx)-b^2dn\tan\right.}{a+b\tan(e+fx)}}{a\left(a^2+b^2\right)d}$$

$$= \frac{b^2(d\tan(e+fx))^{1+n}}{a\left(a^2+b^2\right)df(a+b\tan(e+fx))} + \frac{\int(d\tan(e+fx))^n\left(a\left(a^2-b^2\right)d-2a^2bd\,\mathrm{t}\right.}{a\left(a^2+b^2\right)^2d}$$

$$= \frac{b^2(d\tan(e+fx))^{1+n}}{a\left(a^2+b^2\right)df(a+b\tan(e+fx))} + \frac{\left(a^2-b^2\right)\int(d\tan(e+fx))^n\,dx}{\left(a^2+b^2\right)^2} - \frac{(2ab)\int}{}$$

$$= \frac{b^2\left(a^2(2-n)-b^2n\right)\,{}_2F_1\left(1,1+n;2+n;-\frac{b\tan(e+fx)}{a}\right)(d\tan(e+fx))^{1+n}}{a^2\left(a^2+b^2\right)^2df(1+n)} + \frac{}{a}$$

$$= \frac{\left(a^2-b^2\right)\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)(d\tan(e+fx))^{1+n}}{\left(a^2+b^2\right)^2df(1+n)} + \frac{b^2\left(a^2(2-n)\right.}{}$$

Mathematica [**A**]
time = 2.68, size = 198, normalized size = 0.79

$$\frac{\tan(e+fx)(d\tan(e+fx))^n\left(-\frac{b^2\left(a^2(-2+n)+b^2n\right)\,{}_2F_1\left(1,1+n;2+n;-\frac{b\tan(e+fx)}{a}\right)}{a\left(a^2+b^2\right)(1+n)}+\frac{b^2}{a+b\tan(e+fx)}+\frac{a\left(\frac{\left(a^2-b^2\right)\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)}{1+n}-\frac{2ab\,{}_2F_1\left(1,\frac{2+n}{2};\frac{4+n}{2};-\tan^2(e+fx)\right)\tan(e+fx)}{2+n}\right)}{a^2+b^2}\right)}{a\left(a^2+b^2\right)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^n/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (Tan[e + f*x]*(d*Tan[e + f*x])^n*(-((b^2*(a^2*(-2 + n) + b^2*n)*Hypergeomet
ric2F1[1, 1 + n, 2 + n, -((b*Tan[e + f*x])/a)])/(a*(a^2 + b^2)*(1 + n))) +
b^2/(a + b*Tan[e + f*x]) + (a*(((a^2 - b^2)*Hypergeometric2F1[1, (1 + n)/2,
 (3 + n)/2, -Tan[e + f*x]^2])/(1 + n) - (2*a*b*Hypergeometric2F1[1, (2 + n)
/2, (4 + n)/2, -Tan[e + f*x]^2]*Tan[e + f*x])/(2 + n)))/(a^2 + b^2)))/(a*(a
^2 + b^2)*f)
```

Maple [**F**]
time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{(d\tan(fx+e))^n}{(a+b\tan(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^n/(a+b*tan(f*x+e))^2,x)
```

```
[Out] int((d*tan(f*x+e))^n/(a+b*tan(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((d*tan(f*x + e))^n/(b*tan(f*x + e) + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
[Out] integral((d*tan(f*x + e))^n/(b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\tan{(e+fx)})^n}{(a+b\tan{(e+fx)})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**n/(a+b*tan(f*x+e))**2,x)
[Out] Integral((d*tan(e + f*x))**n/(a + b*tan(e + f*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n/(a+b*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*tan(f*x + e))^n/(b*tan(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\tan(e+f\,x))^n}{(a+b\tan{(e+f\,x)})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^n/(a + b*tan(e + f*x))^2,x)
[Out] int((d*tan(e + f*x))^n/(a + b*tan(e + f*x))^2, x)
```

3.702 $\int \tan^m(c+dx)(a+b\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=175

$$\frac{aF_1\left(1+m;-\frac{3}{2},1;2+m;-\frac{b\tan(c+dx)}{a},-i\tan(c+dx)\right)\tan^{1+m}(c+dx)\sqrt{a+b\tan(c+dx)}}{2d(1+m)\sqrt{1+\frac{b\tan(c+dx)}{a}}}+\frac{aF_1\left(1+m;-\right.}{}$$

[Out] 1/2*a*AppellF1(1+m,1,-3/2,2+m,-I*tan(d*x+c),-b*tan(d*x+c)/a)*(a+b*tan(d*x+c))^(1/2)*tan(d*x+c)^(1+m)/d/(1+m)/(1+b*tan(d*x+c)/a)^(1/2)+1/2*a*AppellF1(1+m,1,-3/2,2+m,I*tan(d*x+c),-b*tan(d*x+c)/a)*(a+b*tan(d*x+c))^(1/2)*tan(d*x+c)^(1+m)/d/(1+m)/(1+b*tan(d*x+c)/a)^(1/2)

Rubi [A]

time = 0.15, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3656, 926, 140, 138}

$$\frac{a\tan^{m+1}(c+dx)\sqrt{a+b\tan(c+dx)}\,F_1\left(m+1;-\frac{3}{2},1;m+2;-\frac{b\tan(c+dx)}{a},-i\tan(c+dx)\right)}{2d(m+1)\sqrt{\frac{b\tan(c+dx)}{a}+1}}+\frac{a\tan^{m+1}(c+dx)\sqrt{a+b\tan(c+dx)}\,F_1\left(m+1;-\frac{3}{2},1;m+2;-\frac{b\tan(c+dx)}{a},i\tan(c+dx)\right)}{2d(m+1)\sqrt{\frac{b\tan(c+dx)}{a}+1}}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d*x]^m*(a + b*Tan[c + d*x])^(3/2),x]

[Out] (a*AppellF1[1 + m, -3/2, 1, 2 + m, -((b*Tan[c + d*x])/a), (-I)*Tan[c + d*x]]*Tan[c + d*x]^(1 + m)*Sqrt[a + b*Tan[c + d*x]])/(2*d*(1 + m)*Sqrt[1 + (b*Tan[c + d*x])/a]) + (a*AppellF1[1 + m, -3/2, 1, 2 + m, -((b*Tan[c + d*x])/a), I*Tan[c + d*x]]*Tan[c + d*x]^(1 + m)*Sqrt[a + b*Tan[c + d*x]])/(2*d*(1 + m)*Sqrt[1 + (b*Tan[c + d*x])/a])

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{x^m(a+bx)^{3/2}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{ix^m(a+bx)^{3/2}}{2(i-x)} + \frac{ix^m(a+bx)^{3/2}}{2(i+x)}\right)\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{i\text{Subst}\left(\int \frac{x^m(a+bx)^{3/2}}{i-x}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{i\text{Subst}\left(\int \frac{x^m(a+bx)^{3/2}}{i+x}\right.}{2d}$$

$$= \frac{\left(ia\sqrt{a+b\tan(c+dx)}\right)\text{Subst}\left(\int \frac{x^m\left(1+\frac{bx}{a}\right)^{3/2}}{i-x}\,dx, x, \tan(c+dx\right.}{2d\sqrt{1+\frac{b\tan(c+dx)}{a}}}$$

$$= \frac{aF_1\left(1+m;-\frac{3}{2},1;2+m;-\frac{b\tan(c+dx)}{a},-i\tan(c+dx)\right)\tan^{1+m}(c}{2d(1+m)\sqrt{1+\frac{b\tan(c+dx)}{a}}}$$

Mathematica [F]
time = 18.93, size = 0, normalized size = 0.00

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^m*(a + b*Tan[c + d*x])^(3/2),x]
[Out] Integrate[Tan[c + d*x]^m*(a + b*Tan[c + d*x])^(3/2), x]
```

Maple [F]
time = 0.39, size = 0, normalized size = 0.00

$$\int \left(\tan^{m}\left(dx+c\right)\right)\left(a+b\tan\left(dx+c\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^m*(a+b*tan(d*x+c))^(3/2),x)
[Out] int(tan(d*x+c)^m*(a+b*tan(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^(3/2)*tan(d*x + c)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral((b*tan(d*x + c) + a)^(3/2)*tan(d*x + c)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan\left(c+dx\right)\right)^{\frac{3}{2}}\tan^{m}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**m*(a+b*tan(d*x+c))**(3/2),x)
[Out] Integral((a + b*tan(c + d*x))**(3/2)*tan(c + d*x)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(3/2)*tan(d*x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^m\,\left(a+b\tan(c+d\,x)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^m*(a + b*tan(c + d*x))^(3/2),x)
```

```
[Out] int(tan(c + d*x)^m*(a + b*tan(c + d*x))^(3/2), x)
```

3.703 $\int \tan^m(c+dx)\sqrt{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=173

$$\frac{F_1\left(1+m;-\frac{1}{2},1;2+m;-\frac{b\tan(c+dx)}{a},-i\tan(c+dx)\right)\tan^{1+m}(c+dx)\sqrt{a+b\tan(c+dx)}}{2d(1+m)\sqrt{1+\frac{b\tan(c+dx)}{a}}}+\frac{F_1\left(1+m;-\frac{1}{2},\right.}{}$$

[Out] 1/2*AppellF1(1+m,1,-1/2,2+m,-I*tan(d*x+c),-b*tan(d*x+c)/a)*(a+b*tan(d*x+c))^(1/2)*tan(d*x+c)^(1+m)/d/(1+m)/(1+b*tan(d*x+c)/a)^(1/2)+1/2*AppellF1(1+m,1,-1/2,2+m,I*tan(d*x+c),-b*tan(d*x+c)/a)*(a+b*tan(d*x+c))^(1/2)*tan(d*x+c)^(1+m)/d/(1+m)/(1+b*tan(d*x+c)/a)^(1/2)

Rubi [A]

time = 0.13, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3656, 926, 140, 138}

$$\frac{\tan^{m+1}(c+dx)\sqrt{a+b\tan(c+dx)}\,F_1\left(m+1;-\frac{1}{2},1;m+2;-\frac{b\tan(c+dx)}{a},-i\tan(c+dx)\right)}{2d(m+1)\sqrt{\frac{b\tan(c+dx)}{a}+1}}+\frac{\tan^{m+1}(c+dx)\sqrt{a+b\tan(c+dx)}\,F_1\left(m+1;-\frac{1}{2},1;m+2;-\frac{b\tan(c+dx)}{a},i\tan(c+dx)\right)}{2d(m+1)\sqrt{\frac{b\tan(c+dx)}{a}+1}}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d*x]^m*Sqrt[a + b*Tan[c + d*x]],x]

[Out] (AppellF1[1 + m, -1/2, 1, 2 + m, -((b*Tan[c + d*x])/a), (-I)*Tan[c + d*x]]*Tan[c + d*x]^(1 + m)*Sqrt[a + b*Tan[c + d*x]])/(2*d*(1 + m)*Sqrt[1 + (b*Tan[c + d*x])/a]) + (AppellF1[1 + m, -1/2, 1, 2 + m, -((b*Tan[c + d*x])/a), I*Tan[c + d*x]]*Tan[c + d*x]^(1 + m)*Sqrt[a + b*Tan[c + d*x]])/(2*d*(1 + m)*Sqrt[1 + (b*Tan[c + d*x])/a])

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \tan^m(c+dx)\sqrt{a+b\tan(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^m\sqrt{a+bx}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(\frac{ix^m\sqrt{a+bx}}{2(i-x)} + \frac{ix^m\sqrt{a+bx}}{2(i+x)}\right)\,dx, x, \tan(c+dx)\right)}{d} \\
&= \frac{i\text{Subst}\left(\int \frac{x^m\sqrt{a+bx}}{i-x}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{i\text{Subst}\left(\int \frac{x^m\sqrt{a+b
\end{aligned}$$

$$\begin{aligned}
&= \frac{\left(i\sqrt{a+b\tan(c+dx)}\right)\text{Subst}\left(\int \frac{x^m\sqrt{1+\frac{bx}{a}}}{i-x}\,dx, x, \tan(c+dx)\right.}{2d\sqrt{1+\frac{b\tan(c+dx)}{a}}} \\
&= \frac{F_1\left(1+m;-\frac{1}{2},1;2+m;-\frac{b\tan(c+dx)}{a},-i\tan(c+dx)\right)\tan^{1+m}(c+}{2d(1+m)\sqrt{1+\frac{b\tan(c+dx)}{a}}}
\end{aligned}$$

Mathematica [F]
time = 0.74, size = 0, normalized size = 0.00

$$\int \tan^m(c+dx)\sqrt{a+b\tan(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^m*Sqrt[a + b*Tan[c + d*x]],x]
[Out] Integrate[Tan[c + d*x]^m*Sqrt[a + b*Tan[c + d*x]], x]
```

Maple [F]
time = 0.36, size = 0, normalized size = 0.00

$$\int \left(\tan^{m}\left(dx+c\right)\right) \sqrt{a+b\tan\left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^m*(a+b*tan(d*x+c))^(1/2),x)
[Out] int(tan(d*x+c)^m*(a+b*tan(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan\left(c+dx\right)} \tan^{m}\left(c+dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**m*(a+b*tan(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*tan(c + d*x))*tan(c + d*x)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^m\,\sqrt{a+b\tan\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^m*(a + b*tan(c + d*x))^(1/2),x)
```

```
[Out] int(tan(c + d*x)^m*(a + b*tan(c + d*x))^(1/2), x)
```

3.704 $\int \frac{\tan^m(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal. Leaf size=173

$$\frac{F_1\left(1+m;\frac{1}{2},1;2+m;-\frac{b\tan(c+dx)}{a},-i\tan(c+dx)\right)\tan^{1+m}(c+dx)\sqrt{1+\frac{b\tan(c+dx)}{a}}}{2d(1+m)\sqrt{a+b\tan(c+dx)}}+\frac{F_1\left(1+m;\frac{1}{2},1;2\right.}{}$$

```
[Out] 1/2*AppellF1(1+m,1,1/2,2+m,-I*tan(d*x+c),-b*tan(d*x+c)/a)*(1+b*tan(d*x+c)/a
)^(1/2)*tan(d*x+c)^(1+m)/d/(1+m)/(a+b*tan(d*x+c))^(1/2)+1/2*AppellF1(1+m,1,
1/2,2+m,I*tan(d*x+c),-b*tan(d*x+c)/a)*(1+b*tan(d*x+c)/a)^(1/2)*tan(d*x+c)^(
1+m)/d/(1+m)/(a+b*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3656, 926, 140, 138}

$$\frac{\tan^{m+1}(c+dx)\sqrt{\frac{b\tan(c+dx)}{a}+1}\,F_1\left(m+1;\frac{1}{2},1;m+2;-\frac{b\tan(c+dx)}{a},-i\tan(c+dx)\right)}{2d(m+1)\sqrt{a+b\tan(c+dx)}}+\frac{\tan^{m+1}(c+dx)\sqrt{\frac{b\tan(c+dx)}{a}+1}\,F_1\left(m+1;\frac{1}{2},1;m+2;-\frac{b\tan(c+dx)}{a},i\tan(c+dx)\right)}{2d(m+1)\sqrt{a+b\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^m/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] (AppellF1[1 + m, 1/2, 1, 2 + m, -((b*Tan[c + d*x])/a), (-I)*Tan[c + d*x]]*T
an[c + d*x]^(1 + m)*Sqrt[1 + (b*Tan[c + d*x])/a])/(2*d*(1 + m)*Sqrt[a + b*T
an[c + d*x]]) + (AppellF1[1 + m, 1/2, 1, 2 + m, -((b*Tan[c + d*x])/a), I*Ta
n[c + d*x]]*Tan[c + d*x]^(1 + m)*Sqrt[1 + (b*Tan[c + d*x])/a])/(2*d*(1 + m)
*Sqrt[a + b*Tan[c + d*x]])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{\tan^m(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^m}{\sqrt{a+bx}\,(1+x^2)}\,dx,x,\tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{ix^m}{2(i-x)\sqrt{a+bx}}+\frac{ix^m}{2(i+x)\sqrt{a+bx}}\right)dx,x,\tan(c+dx)\right)}{d}$$

$$= \frac{i\text{Subst}\left(\int \frac{x^m}{(i-x)\sqrt{a+bx}}\,dx,x,\tan(c+dx)\right)}{2d} + \frac{i\text{Subst}\left(\int \frac{x^m}{(i+x)\sqrt{a+bx}}\,dx,x,\right.}{2d}$$

$$= \frac{\left(i\sqrt{1+\frac{b\tan(c+dx)}{a}}\right)\text{Subst}\left(\int \frac{x^m}{(i-x)\sqrt{1+\frac{bx}{a}}}\,dx,x,\tan(c+dx)\right)}{2d\sqrt{a+b\tan(c+dx)}} + \frac{\left(i\sqrt{\right.}}{}$$

$$= \frac{F_1\left(1+m;\frac{1}{2},1;2+m;-\frac{b\tan(c+dx)}{a},-i\tan(c+dx)\right)\tan^{1+m}(c+dx)\sqrt{1+\frac{b\tan}{}}}{2d(1+m)\sqrt{a+b\tan(c+dx)}}$$

Mathematica [F]
time = 8.26, size = 0, normalized size = 0.00

$$\int \frac{\tan^m(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^m/Sqrt[a + b*Tan[c + d*x]],x]
[Out] Integrate[Tan[c + d*x]^m/Sqrt[a + b*Tan[c + d*x]], x]
```

Maple [F]
time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\tan^{m}(dx+c)}{\sqrt{a+b\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^m/(a+b*tan(d*x+c))^(1/2),x)
[Out] int(tan(d*x+c)^m/(a+b*tan(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(tan(d*x + c)^m/sqrt(b*tan(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(tan(d*x + c)^m/sqrt(b*tan(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{m}(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**m/(a+b*tan(d*x+c))**(1/2),x)
[Out] Integral(tan(c + d*x)**m/sqrt(a + b*tan(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(tan(d*x + c)^m/sqrt(b*tan(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^m}{\sqrt{a+b\tan\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^m/(a + b*tan(c + d*x))^(1/2),x)
[Out] int(tan(c + d*x)^m/(a + b*tan(c + d*x))^(1/2), x)
```

3.705 $\int \frac{\tan^m(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=179

$$\frac{F_1\left(1+m;\frac{3}{2},1;2+m;-\frac{b\tan(c+dx)}{a},-i\tan(c+dx)\right)\tan^{1+m}(c+dx)\sqrt{1+\frac{b\tan(c+dx)}{a}}}{2ad(1+m)\sqrt{a+b\tan(c+dx)}}+\frac{F_1\left(1+m;\frac{3}{2},1;2\right.}{}$$

```
[Out] 1/2*AppellF1(1+m,1,3/2,2+m,-I*tan(d*x+c),-b*tan(d*x+c)/a)*(1+b*tan(d*x+c)/a
)^(1/2)*tan(d*x+c)^(1+m)/a/d/(1+m)/(a+b*tan(d*x+c))^(1/2)+1/2*AppellF1(1+m,
1,3/2,2+m,I*tan(d*x+c),-b*tan(d*x+c)/a)*(1+b*tan(d*x+c)/a)^(1/2)*tan(d*x+c)
^(1+m)/a/d/(1+m)/(a+b*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3656, 926, 140, 138}

$$\frac{\tan^{m+1}(c+dx)\sqrt{\frac{b\tan(c+dx)}{a}+1}\,F_1\left(m+1;\frac{3}{2},1;m+2;-\frac{b\tan(c+dx)}{a},-i\tan(c+dx)\right)}{2ad(m+1)\sqrt{a+b\tan(c+dx)}}+\frac{\tan^{m+1}(c+dx)\sqrt{\frac{b\tan(c+dx)}{a}+1}\,F_1\left(m+1;\frac{3}{2},1;m+2;-\frac{b\tan(c+dx)}{a},i\tan(c+dx)\right)}{2ad(m+1)\sqrt{a+b\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^m/(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] (AppellF1[1 + m, 3/2, 1, 2 + m, -((b*Tan[c + d*x])/a), (-I)*Tan[c + d*x]]*T
an[c + d*x]^(1 + m)*Sqrt[1 + (b*Tan[c + d*x])/a])/(2*a*d*(1 + m)*Sqrt[a + b
*Tan[c + d*x]]) + (AppellF1[1 + m, 3/2, 1, 2 + m, -((b*Tan[c + d*x])/a), I*
Tan[c + d*x]]*Tan[c + d*x]^(1 + m)*Sqrt[1 + (b*Tan[c + d*x])/a])/(2*a*d*(1
+ m)*Sqrt[a + b*Tan[c + d*x]])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^m(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{x^m}{(a+bx)^{3/2}(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(\frac{ix^m}{2(i-x)(a+bx)^{3/2}}+\frac{ix^m}{2(i+x)(a+bx)^{3/2}}\right)dx, x, \tan(c+dx)\right)}{d} \\
&= \frac{i\text{Subst}\left(\int \frac{x^m}{(i-x)(a+bx)^{3/2}}\,dx, x, \tan(c+dx)\right)}{2d}+\frac{i\text{Subst}\left(\int \frac{x^m}{(i+x)(a+bx)^{3/2}}\,dx, x, \tan\right.}{2d} \\
&= \frac{\left(i\sqrt{1+\frac{b\tan(c+dx)}{a}}\right)\text{Subst}\left(\int \frac{x^m}{(i-x)\left(1+\frac{bx}{a}\right)^{3/2}}\,dx, x, \tan(c+dx)\right)}{2ad\sqrt{a+b\tan(c+dx)}}+\frac{\left(i\sqrt{\right.}}{} \\
&= \frac{F_1\left(1+m;\frac{3}{2},1;2+m;-\frac{b\tan(c+dx)}{a},-i\tan(c+dx)\right)\tan^{1+m}(c+dx)\sqrt{1+\frac{b\tan}{}}}{2ad(1+m)\sqrt{a+b\tan(c+dx)}}
\end{aligned}$$

Mathematica [F]
time = 34.73, size = 0, normalized size = 0.00

$$\int \frac{\tan^m(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^m/(a + b*Tan[c + d*x])^(3/2),x]
[Out] Integrate[Tan[c + d*x]^m/(a + b*Tan[c + d*x])^(3/2), x]
```

Maple [F]
time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{\tan^{m}\left(dx+c\right)}{\left(a+b\tan\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^m/(a+b*tan(d*x+c))^(3/2),x)
```

```
[Out] int(tan(d*x+c)^m/(a+b*tan(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^m/(b*tan(d*x + c) + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^m/(b^2*tan(d*x + c)^2 + 2*a*
b*tan(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{m}\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**m/(a+b*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**m/(a + b*tan(c + d*x))**(3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m/(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^m}{\left(a+b\tan\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^m/(a + b*tan(c + d*x))^(3/2),x)
```

```
[Out] int(tan(c + d*x)^m/(a + b*tan(c + d*x))^(3/2), x)
```

3.706 $\int (d\tan(e+fx))^n(a+b\tan(e+fx))^m\,dx$

Optimal. Leaf size=179

$$\frac{F_1\left(1+n;-m,1;2+n;-\frac{b\tan(e+fx)}{a},-i\tan(e+fx)\right)(d\tan(e+fx))^{1+n}(a+b\tan(e+fx))^m\left(1+\frac{b\tan(e+fx)}{a}\right.}{2df(1+n)}$$

```
[Out] 1/2*AppellF1(1+n,1,-m,2+n,-I*tan(f*x+e),-b*tan(f*x+e)/a)*(d*tan(f*x+e))^(1+
n)*(a+b*tan(f*x+e))^m/d/f/(1+n)/((1+b*tan(f*x+e)/a)^m)+1/2*AppellF1(1+n,1,-
m,2+n,I*tan(f*x+e),-b*tan(f*x+e)/a)*(d*tan(f*x+e))^(1+n)*(a+b*tan(f*x+e))^m
/d/f/(1+n)/((1+b*tan(f*x+e)/a)^m)
```

Rubi [A]

time = 0.13, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3656, 926, 140, 138}

$$\frac{(d\tan(e+fx))^{n+1}(a+b\tan(e+fx))^m\left(\frac{b\tan(e+fx)}{a}+1\right)^{-m}F_1\left(n+1;-m,1;n+2;-\frac{b\tan(e+fx)}{a},-i\tan(e+fx)\right)}{2df(n+1)}+\frac{(d\tan(e+fx))^{n+1}(a+b\tan(e+fx))^m\left(\frac{b\tan(e+fx)}{a}+1\right)^{-m}F_1\left(n+1;-m,1;n+2;-\frac{b\tan(e+fx)}{a},i\tan(e+fx)\right)}{2df(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^n*(a + b*Tan[e + f*x])^m,x]
```

```
[Out] (AppellF1[1 + n, -m, 1, 2 + n, -((b*Tan[e + f*x])/a), (-I)*Tan[e + f*x]]*(d
*Tan[e + f*x])^(1 + n)*(a + b*Tan[e + f*x])^m)/(2*d*f*(1 + n)*(1 + (b*Tan[e
 + f*x])/a)^m) + (AppellF1[1 + n, -m, 1, 2 + n, -((b*Tan[e + f*x])/a), I*Ta
n[e + f*x]]*(d*Tan[e + f*x])^(1 + n)*(a + b*Tan[e + f*x])^m)/(2*d*f*(1 + n)
*(1 + (b*Tan[e + f*x])/a)^m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
```

```
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int (d\tan(e+fx))^n(a+b\tan(e+fx))^m\,dx = \frac{\text{Subst}\left(\int \frac{(dx)^n(a+bx)^m}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{i(dx)^n(a+bx)^m}{2(i-x)} + \frac{i(dx)^n(a+bx)^m}{2(i+x)}\right)\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{i\text{Subst}\left(\int \frac{(dx)^n(a+bx)^m}{i-x}\,dx, x, \tan(e+fx)\right)}{2f} + \frac{i\text{Subst}\left(\int \frac{(dx)^n(a+b}{i+x}\right.}{}$$
$$= \frac{\left(i(a+b\tan(e+fx))^m\left(1+\frac{b\tan(e+fx)}{a}\right)^{-m}\right)\text{Subst}\left(\int \frac{(dx)^n\left(1+\right.}{i-x}\right.}{2f}$$
$$= \frac{F_1\left(1+n;-m,1;2+n;-\frac{b\tan(e+fx)}{a}, -i\tan(e+fx)\right)(d\tan(e}{2df(1+n)}$$

Mathematica [F]
time = 1.59, size = 0, normalized size = 0.00

$$\int (d\tan(e+fx))^n(a+b\tan(e+fx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Tan[e + f*x])^n*(a + b*Tan[e + f*x])^m,x]
[Out] Integrate[(d*Tan[e + f*x])^n*(a + b*Tan[e + f*x])^m, x]
```

Maple [F]
time = 0.39, size = 0, normalized size = 0.00

$$\int (d\tan(fx+e))^n\,(a+b\tan(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^n*(a+b*tan(f*x+e))^m,x)
```

```
[Out] int((d*tan(f*x+e))^n*(a+b*tan(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+b*tan(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^m*(d*tan(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+b*tan(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e) + a)^m*(d*tan(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\tan(e+fx))^n (a+b\tan(e+fx))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**n*(a+b*tan(f*x+e))**m,x)
```

```
[Out] Integral((d*tan(e + f*x))**n*(a + b*tan(e + f*x))**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n*(a+b*tan(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)^m*(d*tan(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\tan(e+fx))^n (a+b\tan(e+fx))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^n*(a + b*tan(e + f*x))^m,x)
[Out] int((d*tan(e + f*x))^n*(a + b*tan(e + f*x))^m, x)
```

3.707 $\int \tan^4(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=297

$$\frac{(2a^2-b^2(2+n)(3+n))\,(a+b\tan(c+dx))^{1+n}}{b^3d(1+n)(2+n)(3+n)}-\frac{\sqrt{-b^2}\ {}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)(a+b\tan(c+d$$

```
[Out] (2*a^2-b^2*(2+n)*(3+n))*(a+b*tan(d*x+c))^(1+n)/b^3/d/(1+n)/(2+n)/(3+n)-1/2*
hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)))*(-b^2)^(1/2)*(a
+b*tan(d*x+c))^(1+n)/b/d/(1+n)/(a-(-b^2)^(1/2))+1/2*hypergeom([1, 1+n],[2+n
],(a+b*tan(d*x+c))/(a+(-b^2)^(1/2)))*(-b^2)^(1/2)*(a+b*tan(d*x+c))^(1+n)/b/
d/(1+n)/(a+(-b^2)^(1/2))-2*a*tan(d*x+c)*(a+b*tan(d*x+c))^(1+n)/b^2/d/(2+n)/
(3+n)+tan(d*x+c)^2*(a+b*tan(d*x+c))^(1+n)/b/d/(3+n)
```

Rubi [A]
time = 0.42, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3647, 3728, 3712, 3566, 726, 70}

$$\frac{(2a^2-b^2(n+2)(n+3))\,(a+b\tan(c+dx))^{n+1}}{b^3d(n+1)(n+2)(n+3)}-\frac{\sqrt{-b^2}\,(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)}{2bd(n+1)\left(a-\sqrt{-b^2}\right)}+\frac{\sqrt{-b^2}\,(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)}{2bd(n+1)\left(a+\sqrt{-b^2}\right)}-\frac{2a\tan(c+dx)(a+b\tan(c+dx))^{n+1}}{b^2d(n+2)(n+3)}+\frac{\tan^2(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] ((2*a^2 - b^2*(2 + n)*(3 + n))*(a + b*Tan[c + d*x])^(1 + n))/(b^3*d*(1 + n)
*(2 + n)*(3 + n)) - (Sqrt[-b^2]*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*T
an[c + d*x])/(a - Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(2*b*(a - Sqrt
[-b^2])*d*(1 + n)) + (Sqrt[-b^2]*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*
Tan[c + d*x])/(a + Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(2*b*(a + Sqr
t[-b^2])*d*(1 + n)) - (2*a*Tan[c + d*x]*(a + b*Tan[c + d*x])^(1 + n))/(b^2*
d*(2 + n)*(3 + n)) + (Tan[c + d*x]^2*(a + b*Tan[c + d*x])^(1 + n))/(b*d*(3
+ n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 726

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
```

```
& NeQ[c*d^2 + a*e^2, 0] && !IntegerQ[m]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) && !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3712

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] && !LeQ[m, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$
\begin{aligned}
\int \tan^4(c+dx)(a+b\tan(c+dx))^n\,dx &= \frac{\tan^2(c+dx)(a+b\tan(c+dx))^{1+n}}{bd(3+n)} + \frac{\int \tan(c+dx)(a+b\tan(c-
}{} \\
&= -\frac{2a\tan(c+dx)(a+b\tan(c+dx))^{1+n}}{b^2d(2+n)(3+n)} + \frac{\tan^2(c+dx)(a+b\tan(}{bd(3+n)} \\
&= \frac{\left(2a^2-b^2(2+n)(3+n)\right)(a+b\tan(c+dx))^{1+n}}{b^3d(1+n)(2+n)(3+n)} - \frac{2a\tan(c+dx)}{b^2d(2} \\
&= \frac{\left(2a^2-b^2(2+n)(3+n)\right)(a+b\tan(c+dx))^{1+n}}{b^3d(1+n)(2+n)(3+n)} - \frac{2a\tan(c+dx)}{b^2d(2} \\
&= \frac{\left(2a^2-b^2(2+n)(3+n)\right)(a+b\tan(c+dx))^{1+n}}{b^3d(1+n)(2+n)(3+n)} - \frac{2a\tan(c+dx)}{b^2d(2} \\
&= \frac{\left(2a^2-b^2(2+n)(3+n)\right)(a+b\tan(c+dx))^{1+n}}{b^3d(1+n)(2+n)(3+n)} - \frac{2a\tan(c+dx)}{b^2d(2} \\
&= \frac{\left(2a^2-b^2(2+n)(3+n)\right)(a+b\tan(c+dx))^{1+n}}{b^3d(1+n)(2+n)(3+n)} + \frac{b\,{}_2F_1\left(1,1+n\right.}{2}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.31, size = 249, normalized size = 0.84

$$
-\frac{(a+b\tan(c+dx))^{1+n}\left(2(ia-b)(ia+b)\left(2a^2-b^2(6+5n+n^2)\right)+i(a+ib)b^3(2+n)(3+n)\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a-ib}\right)-b^3(ia+b)(2+n)(3+n)\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a+ib}\right)+4a(a-ib)(a+ib)b(1+n)\tan(c+dx)-2(a-ib)(a+ib)b^2(1+n)(2+n)\tan^2(c+dx)\right)}{2(a-ib)(a+ib)b^3d(1+n)(2+n)(3+n)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] -1/2*((a + b*Tan[c + d*x])^(1 + n)*(2*(I*a - b)*(I*a + b)*(2*a^2 - b^2*(6 +
 5*n + n^2)) + I*(a + I*b)*b^3*(2 + n)*(3 + n)*Hypergeometric2F1[1, 1 + n,
2 + n, (a + b*Tan[c + d*x])/(a - I*b)] - b^3*(I*a + b)*(2 + n)*(3 + n)*Hype
rgeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + I*b)] + 4*a*(a - I
*b)*(a + I*b)*b*(1 + n)*Tan[c + d*x] - 2*(a - I*b)*(a + I*b)*b^2*(1 + n)*(2
 + n)*Tan[c + d*x]^2))/((a - I*b)*(a + I*b)*b^3*d*(1 + n)*(2 + n)*(3 + n))
```

Maple [F]
time = 0.34, size = 0, normalized size = 0.00

$$
\int \left(\tan^4(dx+c)\right)(a+b\tan(dx+c))^n\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(tan(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan{(c+dx)})^n\tan^4{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**n*tan(c + d*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c+d\,x)^4\,(a+b\tan(c+d\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + b*tan(c + d*x))^n,x)
[Out] int(tan(c + d*x)^4*(a + b*tan(c + d*x))^n, x)
```

3.708 $\int \tan^3(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=192

$$-\frac{a(a+b\tan(c+dx))^{1+n}}{b^2d(1+n)(2+n)}+\frac{{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a-ib}\right)(a+b\tan(c+dx))^{1+n}}{2(a-ib)d(1+n)}+\frac{{}_2F_1\left(1,1+n;2+n;\frac{a\cdots}{}\right)}{2(a\cdots}$$

```
[Out] -a*(a+b*tan(d*x+c))^(1+n)/b^2/d/(1+n)/(2+n)+1/2*hypergeom([1, 1+n],[2+n],(a
+b*tan(d*x+c))/(a-I*b))*(a+b*tan(d*x+c))^(1+n)/(a-I*b)/d/(1+n)+1/2*hypergeo
m([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a+I*b))*(a+b*tan(d*x+c))^(1+n)/(a+I*b)/d
/(1+n)+tan(d*x+c)*(a+b*tan(d*x+c))^(1+n)/b/d/(2+n)
```

Rubi [A]
time = 0.20, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3647, 3711, 12, 3620, 3618, 70}

$$-\frac{a(a+b\tan(c+dx))^{n+1}}{b^2d(n+1)(n+2)}+\frac{(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a-ib}\right)}{2d(n+1)(a-ib)}+\frac{(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a+ib}\right)}{2d(n+1)(a+ib)}+\frac{\tan(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] -((a*(a + b*Tan[c + d*x])^(1 + n))/(b^2*d*(1 + n)*(2 + n))) + (Hypergeometr
ic2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - I*b)]*(a + b*Tan[c + d*x])
^(1 + n))/(2*(a - I*b)*d*(1 + n)) + (Hypergeometric2F1[1, 1 + n, 2 + n, (a
+ b*Tan[c + d*x])/(a + I*b)]*(a + b*Tan[c + d*x])^(1 + n))/(2*(a + I*b)*d*(
1 + n)) + (Tan[c + d*x]*(a + b*Tan[c + d*x])^(1 + n))/(b*d*(2 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
```

```
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \tan^3(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{\tan(c+dx)(a+b\tan(c+dx))^{1+n}}{bd(2+n)} + \frac{\int(a+b\tan(c+dx))^n\,(-a-$$
$$= -\frac{a(a+b\tan(c+dx))^{1+n}}{b^2d(1+n)(2+n)} + \frac{\tan(c+dx)(a+b\tan(c+dx))^{1+n}}{bd(2+n)} -$$
$$= -\frac{a(a+b\tan(c+dx))^{1+n}}{b^2d(1+n)(2+n)} + \frac{\tan(c+dx)(a+b\tan(c+dx))^{1+n}}{bd(2+n)} -$$
$$= -\frac{a(a+b\tan(c+dx))^{1+n}}{b^2d(1+n)(2+n)} + \frac{\tan(c+dx)(a+b\tan(c+dx))^{1+n}}{bd(2+n)} -$$
$$= -\frac{a(a+b\tan(c+dx))^{1+n}}{b^2d(1+n)(2+n)} + \frac{\tan(c+dx)(a+b\tan(c+dx))^{1+n}}{bd(2+n)} -$$
$$= -\frac{a(a+b\tan(c+dx))^{1+n}}{b^2d(1+n)(2+n)} + \frac{{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a-ib}\right)(a+}{2(a-ib)d(1+n)}$$

Mathematica [A]
time = 1.12, size = 135, normalized size = 0.70

$$\frac{(a+b\tan(c+dx))^{1+n}\left(\frac{{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a-ib}\right)}{(a-ib)(1+n)}+\frac{{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a+ib}\right)}{(a+ib)(1+n)}+\frac{2\left(-\frac{a}{1+n}+b\tan(c+dx)\right)}{b^2(2+n)}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] ((a + b*Tan[c + d*x])^(1 + n)*(Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Ta
n[c + d*x])/(a - I*b)]/((a - I*b)*(1 + n)) + Hypergeometric2F1[1, 1 + n, 2
+ n, (a + b*Tan[c + d*x])/(a + I*b)]/((a + I*b)*(1 + n)) + (2*(-(a/(1 + n))
 + b*Tan[c + d*x]))/(b^2*(2 + n))))/(2*d)
```

Maple [F]
time = 0.34, size = 0, normalized size = 0.00

$$\int \left(\tan^3(dx+c)\right)(a+b\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(tan(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*tan(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan{(c + dx)})^n \tan^3{(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3*(a+b*tan(d*x+c))**n,x)
[Out] Integral((a + b*tan(c + d*x))**n*tan(c + d*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{tan}(c+d\,x)^3\,{\left(a+b\,\mathrm{tan}(c+d\,x)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + b*tan(c + d*x))^n,x)
[Out] int(tan(c + d*x)^3*(a + b*tan(c + d*x))^n, x)
```

3.709 $\int \tan^2(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=193

$$\frac{(a+b\tan(c+dx))^{1+n}}{bd(1+n)}-\frac{b\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)(a+b\tan(c+dx))^{1+n}}{2\sqrt{-b^2}\left(a-\sqrt{-b^2}\right)d(1+n)}+\frac{b\,{}_2F_1\left(1,1+n;2+n;\frac{a+\cdots}{\cdots}\right.}{2\sqrt{-b^2}\left(\cdots\right.}$$

```
[Out] (a+b*tan(d*x+c))^(1+n)/b/d/(1+n)-1/2*b*hypergeom([1, 1+n],[2+n],(a+b*tan(d*
x+c))/(a-(-b^2)^(1/2)))*(a+b*tan(d*x+c))^(1+n)/d/(1+n)/(a-(-b^2)^(1/2))/(-b
^2)^(1/2)+1/2*b*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a+(-b^2)^(1/2)))
*(a+b*tan(d*x+c))^(1+n)/d/(1+n)/(-b^2)^(1/2)/(a+(-b^2)^(1/2))
```

Rubi [A]
time = 0.14, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3624, 3566, 726, 70}

$$-\frac{b(a+b\tan(c+dx))^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)}{2\sqrt{-b^2}\,d(n+1)\left(a-\sqrt{-b^2}\right)}+\frac{b(a+b\tan(c+dx))^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)}{2\sqrt{-b^2}\,d(n+1)\left(a+\sqrt{-b^2}\right)}+\frac{(a+b\tan(c+dx))^{n+1}}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (a + b*Tan[c + d*x])^(1 + n)/(b*d*(1 + n)) - (b*Hypergeometric2F1[1, 1 + n,
 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n)
)/(2*Sqrt[-b^2]*(a - Sqrt[-b^2])*d*(1 + n)) + (b*Hypergeometric2F1[1, 1 + n
, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n
))/(2*Sqrt[-b^2]*(a + Sqrt[-b^2])*d*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 726

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{(a+b\tan(c+dx))^{1+n}}{bd(1+n)} - \int (a+b\tan(c+dx))^n\,dx$$
$$= \frac{(a+b\tan(c+dx))^{1+n}}{bd(1+n)} - \frac{b\text{Subst}\left(\int \frac{(a+x)^n}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{d}$$
$$= \frac{(a+b\tan(c+dx))^{1+n}}{bd(1+n)} - \frac{b\text{Subst}\left(\int \left(\frac{\sqrt{-b^2}\,(a+x)^n}{2b^2\left(\sqrt{-b^2}-x\right)} + \frac{\sqrt{-b^2}\,(a}{2b^2\left(\sqrt{-b^2}\right.}\right.\right.}{d}$$
$$= \frac{(a+b\tan(c+dx))^{1+n}}{bd(1+n)} + \frac{b\text{Subst}\left(\int \frac{(a+x)^n}{\sqrt{-b^2}-x}\,dx, x, b\tan(c+dx)\right)}{2\sqrt{-b^2}\,d}$$
$$= \frac{(a+b\tan(c+dx))^{1+n}}{bd(1+n)} - \frac{b\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)(a+}{2\sqrt{-b^2}\,\left(a-\sqrt{-b^2}\right)d(1}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.23, size = 138, normalized size = 0.72

$$\frac{\left(i(a+ib)b\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a-ib}\right)+(a-ib)\left(2a+2ib-ib\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a+ib}\right)\right)\right)(a+b\tan(c+dx))^{1+n}}{2b(-ia+b)(ia+b)d(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] ((I*(a + I*b)*b*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a
- I*b)] + (a - I*b)*(2*a + (2*I)*b - I*b*Hypergeometric2F1[1, 1 + n, 2 + n,
```

```
 (a + b*Tan[c + d*x])/(a + I*b)]))*(a + b*Tan[c + d*x])^(1 + n))/(2*b*((-I)
*a + b)*(I*a + b)*d*(1 + n))
```

Maple [F]
time = 0.31, size = 0, normalized size = 0.00

$$\int \left(\tan^2 \left(dx+c\right)\right) \left(a+b\tan \left(dx+c\right)\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
[Out] int(tan(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*tan(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan \left(c+dx\right)\right)^n \tan^2 \left(c+dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2*(a+b*tan(d*x+c))**n,x)
[Out] Integral((a + b*tan(c + d*x))**n*tan(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^2\,\left(a+b\tan(c+d\,x)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + b*tan(c + d*x))^n,x)
```

```
[Out] int(tan(c + d*x)^2*(a + b*tan(c + d*x))^n, x)
```

3.710 $\int \tan(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=127

$$-\frac{{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a-ib}\right)(a+b\tan(c+dx))^{1+n}}{2(a-ib)d(1+n)}-\frac{{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a+ib}\right)(a+b\tan(c+$$

```
[Out] -1/2*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a-I*b))*(a+b*tan(d*x+c))^(1
+n)/(a-I*b)/d/(1+n)-1/2*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a+I*b))*
(a+b*tan(d*x+c))^(1+n)/(a+I*b)/d/(1+n)
```

Rubi [A]
time = 0.09, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3620, 3618, 70}

$$-\frac{(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a-ib}\right)}{2d(n+1)(a-ib)}-\frac{(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a+ib}\right)}{2d(n+1)(a+ib)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] -1/2*(Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - I*b)]*(a
 + b*Tan[c + d*x])^(1 + n))/((a - I*b)*d*(1 + n)) - (Hypergeometric2F1[1, 1
 + n, 2 + n, (a + b*Tan[c + d*x])/(a + I*b)]*(a + b*Tan[c + d*x])^(1 + n))/
(2*(a + I*b)*d*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
```

```
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \tan(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{1}{2}i\int(1-i\tan(c+dx))(a+b\tan(c+dx))^n\,dx - \frac{1}{2}i\int(1+i\tan(\ldots$$
$$= \frac{\text{Subst}\left(\int\frac{(a-ibx)^n}{-1+x}\,dx, x, i\tan(c+dx)\right)}{2d} + \frac{\text{Subst}\left(\int\frac{(a+ibx)^n}{-1+x}\,dx, x, -i\ldots\right.}{2d}$$
$$= -\frac{{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a-ib}\right)(a+b\tan(c+dx))^{1+n}}{2(a-ib)d(1+n)} - \frac{{}_2F_1\left(\ldots\right.}{\ldots}$$

Mathematica [A]
time = 0.19, size = 117, normalized size = 0.92

$$-\frac{\left((a+ib)\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a-ib}\right)+(a-ib)\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a+ib}\right)\right)(a+b\tan(c+dx))^{1+n}}{2(a-ib)(a+ib)d(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] -1/2*(((a + I*b)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a
 - I*b)] + (a - I*b)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x]
)/(a + I*b)])*(a + b*Tan[c + d*x])^(1 + n))/((a - I*b)*(a + I*b)*d*(1 + n))
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int \tan(dx+c)\,(a+b\tan(dx+c))^n\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(tan(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*tan(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n*tan(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx))^n \tan(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**n*tan(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*tan(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c + d\,x)\, (a + b\tan(c + d\,x))^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + b*tan(c + d*x))^n,x)
```

```
[Out] int(tan(c + d*x)*(a + b*tan(c + d*x))^n, x)
```

3.711 $\int (a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=167

$$\frac{b\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)(a+b\tan(c+dx))^{1+n}}{2\sqrt{-b^2}\left(a-\sqrt{-b^2}\right)d(1+n)}-\frac{b\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)(a+b\tan(c+dx))^{1+n}}{2\sqrt{-b^2}\left(a+\sqrt{-b^2}\right)d(1+n)}$$

```
[Out] 1/2*b*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)))*(a+b*tan(
d*x+c))^(1+n)/d/(1+n)/(a-(-b^2)^(1/2))/(-b^2)^(1/2)-1/2*b*hypergeom([1, 1+n
],[2+n],(a+b*tan(d*x+c))/(a+(-b^2)^(1/2)))*(a+b*tan(d*x+c))^(1+n)/d/(1+n)/(
-b^2)^(1/2)/(a+(-b^2)^(1/2))
```

Rubi [A]
time = 0.11, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3566, 726, 70}

$$\frac{b(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)}{2\sqrt{-b^2}\,d(n+1)\left(a-\sqrt{-b^2}\right)}-\frac{b(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)}{2\sqrt{-b^2}\,d(n+1)\left(a+\sqrt{-b^2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (b*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])
]*(a + b*Tan[c + d*x])^(1 + n))/(2*Sqrt[-b^2]*(a - Sqrt[-b^2])*d*(1 + n)) -
 (b*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b^2]
)]*(a + b*Tan[c + d*x])^(1 + n))/(2*Sqrt[-b^2]*(a + Sqrt[-b^2])*d*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 726

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
```

```
d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int (a+b\tan(c+dx))^n\,dx &= \frac{b\text{Subst}\left(\int \frac{(a+x)^n}{b^2+x^2}\,dx,x,b\tan(c+dx)\right)}{d}\\
&= \frac{b\text{Subst}\left(\int \left(\frac{\sqrt{-b^2}\,(a+x)^n}{2b^2\left(\sqrt{-b^2}-x\right)}+\frac{\sqrt{-b^2}\,(a+x)^n}{2b^2\left(\sqrt{-b^2}+x\right)}\right)dx,x,b\tan(c+dx)\right)}{d}\\
&= -\frac{b\text{Subst}\left(\int \frac{(a+x)^n}{\sqrt{-b^2}-x}\,dx,x,b\tan(c+dx)\right)}{2\sqrt{-b^2}\,d}-\frac{b\text{Subst}\left(\int \frac{(a+x)^n}{\sqrt{-b^2}+x}\,dx,x,b\tan(c\right.}{2\sqrt{-b^2}\,d}\\
&= \frac{b\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)(a+b\tan(c+dx))^{1+n}}{2\sqrt{-b^2}\left(a-\sqrt{-b^2}\right)d(1+n)}-\frac{b\,{}_2F_1\left(1,1+n;2+\right.}{2\sqrt{-}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.16, size = 118, normalized size = 0.71

$$\frac{\left((a+ib)\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a-ib}\right)-(a-ib)\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a+ib}\right)\right)(a+b\tan(c+dx))^{1+n}}{2(a+ib)(ia+b)d(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (((a + I*b)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - I*
b)] - (a - I*b)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a
+ I*b)])*(a + b*Tan[c + d*x])^(1 + n))/(2*(a + I*b)*(I*a + b)*d*(1 + n))
```

Maple [F]
time = 0.25, size = 0, normalized size = 0.00

$$\int (a+b\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^n,x)
```

```
[Out] int((a+b*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*tan(d*x + c) + a)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tan{(c + dx)})^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**n,x)
[Out] Integral((a + b*tan(c + d*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\tan(c + d\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^n,x)
[Out] int((a + b*tan(c + d*x))^n, x)
```

3.712 $\int \cot(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=175

$$\frac{{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a-ib}\right)(a+b\tan(c+dx))^{1+n}}{2(a-ib)d(1+n)}+\frac{{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a+ib}\right)(a+b\tan(c+d$$

```
[Out] 1/2*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a-I*b))*(a+b*tan(d*x+c))^(1+
n)/(a-I*b)/d/(1+n)+1/2*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a+I*b))*(
a+b*tan(d*x+c))^(1+n)/(a+I*b)/d/(1+n)-hypergeom([1, 1+n],[2+n],1+b*tan(d*x+
c)/a)*(a+b*tan(d*x+c))^(1+n)/a/d/(1+n)
```

Rubi [A]

time = 0.16, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3655, 3620, 3618, 70, 3715, 67}

$$\frac{(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a-ib}\right)}{2d(n+1)(a-ib)}+\frac{(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a+ib}\right)}{2d(n+1)(a+ib)}-\frac{(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b\tan(c+dx)}{a}+1\right)}{ad(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - I*b)]*(a + b*
Tan[c + d*x])^(1 + n))/(2*(a - I*b)*d*(1 + n)) + (Hypergeometric2F1[1, 1 +
n, 2 + n, (a + b*Tan[c + d*x])/(a + I*b)]*(a + b*Tan[c + d*x])^(1 + n))/(2*
(a + I*b)*d*(1 + n)) - (Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*Tan[c + d
*x])/a]*(a + b*Tan[c + d*x])^(1 + n))/(a*d*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
```

```
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)/((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[(a + b*Tan[e + f*x])^m*
(c - d*Tan[e + f*x]), x], x] + Dist[d^2/(c^2 + d^2), Int[(a + b*Tan[e + f*x
])^m*((1 + Tan[e + f*x]^2)/(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \cot(c+dx)(a+b\tan(c+dx))^n\,dx = -\int \tan(c+dx)(a+b\tan(c+dx))^n\,dx + \int \cot(c+dx)(a+b\tan$$

$$= -\left(\frac{1}{2}i\int (1-i\tan(c+dx))(a+b\tan(c+dx))^n\,dx\right) + \frac{1}{2}i\int (1+i$$

$$= -\frac{{}_2F_1\left(1,1+n;2+n;1+\frac{b\tan(c+dx)}{a}\right)(a+b\tan(c+dx))^{1+n}}{ad(1+n)} - \text{Sul}$$

$$= \frac{{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a-ib}\right)(a+b\tan(c+dx))^{1+n}}{2(a-ib)d(1+n)} + \frac{{}_2F_1\left(1,\right.}{}$$

Mathematica [A]
time = 0.27, size = 154, normalized size = 0.88

$$\frac{\left(a(a+ib)\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a-ib}\right)+(a-ib)\left(a\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a+ib}\right)-2(a+ib)\,{}_2F_1\left(1,1+n;2+n;1+\frac{b\tan(c+dx)}{a}\right)\right)\right)(a+b\tan(c+dx))^{1+n}}{2a(a-ib)(a+ib)d(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] ((a*(a + I*b)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a -
I*b)] + (a - I*b)*(a*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x]
)/(a + I*b)] - 2*(a + I*b)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*Tan[c
+ d*x])/a]))*(a + b*Tan[c + d*x])^(1 + n))/(2*a*(a - I*b)*(a + I*b)*d*(1 +
n))
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int \cot \left(dx+c\right) \left(a+b\tan \left(dx+c\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(cot(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*cot(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n*cot(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan \left(c+dx\right)\right)^{n}\cot \left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**n*cot(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*cot(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)\,\left(a+b\tan(c+d\,x)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + b*tan(c + d*x))^n,x)
```

```
[Out] int(cot(c + d*x)*(a + b*tan(c + d*x))^n, x)
```

3.713 $\int \cot^2(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=245

$$-\frac{\cot(c+dx)(a+b\tan(c+dx))^{1+n}}{ad}-\frac{b\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)(a+b\tan(c+dx))^{1+n}}{2\sqrt{-b^2}\left(a-\sqrt{-b^2}\right)d(1+n)}+\frac{b\,{}_2F_1\left(1,1\right.}{}$$

```
[Out] -cot(d*x+c)*(a+b*tan(d*x+c))^(1+n)/a/d-b*n*hypergeom([1, 1+n],[2+n],1+b*tan
(d*x+c)/a)*(a+b*tan(d*x+c))^(1+n)/a^2/d/(1+n)-1/2*b*hypergeom([1, 1+n],[2+n
],(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)))*(a+b*tan(d*x+c))^(1+n)/d/(1+n)/(a-(-b^
2)^(1/2))/(-b^2)^(1/2)+1/2*b*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a+(
-b^2)^(1/2)))*(a+b*tan(d*x+c))^(1+n)/d/(1+n)/(-b^2)^(1/2)/(a+(-b^2)^(1/2))
```

Rubi [A]
time = 0.32, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3650, 3734, 12, 3566, 726, 70, 3715, 67}

$$-\frac{bn(a+b\tan(c+dx))^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{b\tan(c+dx)}{a}+1\right)}{a^2d(n+1)}-\frac{b(a+b\tan(c+dx))^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)}{2\sqrt{-b^2}\,d(n+1)\left(a-\sqrt{-b^2}\right)}+\frac{b(a+b\tan(c+dx))^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)}{2\sqrt{-b^2}\,d(n+1)\left(a+\sqrt{-b^2}\right)}-\frac{\cot(c+dx)(a+b\tan(c+dx))^{n+1}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] -((Cot[c + d*x]*(a + b*Tan[c + d*x])^(1 + n))/(a*d)) - (b*Hypergeometric2F1
[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])]*(a + b*Tan[c + d*x
])^(1 + n))/(2*Sqrt[-b^2]*(a - Sqrt[-b^2])*d*(1 + n)) + (b*Hypergeometric2F
1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*(a + b*Tan[c + d*
x])^(1 + n))/(2*Sqrt[-b^2]*(a + Sqrt[-b^2])*d*(1 + n)) - (b*n*Hypergeometri
c2F1[1, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a]*(a + b*Tan[c + d*x])^(1 + n))
/(a^2*d*(1 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 726

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
```

```
!GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \cot^2(c+dx)(a+b\tan(c+dx))^n\,dx &= -\frac{\cot(c+dx)(a+b\tan(c+dx))^{1+n}}{ad} - \frac{\int \cot(c+dx)(a+b\tan(c+
\\
&= -\frac{\cot(c+dx)(a+b\tan(c+dx))^{1+n}}{ad} - \frac{\int a(a+b\tan(c+dx))^n\,dx}{a}
\\
&= -\frac{\cot(c+dx)(a+b\tan(c+dx))^{1+n}}{ad} + \frac{(bn)\mathrm{Subst}\left(\int \frac{(a+bx)^n}{x}\,dx, x, \mathrm{t}\right.}{ad}
\\
&= -\frac{\cot(c+dx)(a+b\tan(c+dx))^{1+n}}{ad} - \frac{bn\,{}_2F_1\left(1,1+n;2+n;1+\right.}{a^2}
\\
&= -\frac{\cot(c+dx)(a+b\tan(c+dx))^{1+n}}{ad} - \frac{bn\,{}_2F_1\left(1,1+n;2+n;1+\right.}{a^2}
\\
&= -\frac{\cot(c+dx)(a+b\tan(c+dx))^{1+n}}{ad} - \frac{bn\,{}_2F_1\left(1,1+n;2+n;1+\right.}{a^2}
\\
&= -\frac{\cot(c+dx)(a+b\tan(c+dx))^{1+n}}{ad} - \frac{b\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan}{a-\sqrt{}}\right.}{2\sqrt{-b^2}\,(a-}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.85, size = 190, normalized size = 0.78

$$
-\frac{(b+a\cot(c+dx))\left(a^2(-ia+b)\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a-ib}\right)+(a-ib)\left(ia^2\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a+ib}\right)+2(a+ib)\left(a(1+n)\cot(c+dx)+bn\,{}_2F_1\left(1,1+n;2+n;1+\frac{b\tan(c+dx)}{a}\right)\right)\right)\right)\tan(c+dx)(a+b\tan(c+dx))^n}{2a^2(a-ib)(a+ib)d(1+n)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] -1/2*((b + a*Cot[c + d*x])*(a^2*((-I)*a + b)*Hypergeometric2F1[1, 1 + n, 2
+ n, (a + b*Tan[c + d*x])/(a - I*b)] + (a - I*b)*(I*a^2*Hypergeometric2F1[1
, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + I*b)] + 2*(a + I*b)*(a*(1 + n)*Co
t[c + d*x] + b*n*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a]
)))*Tan[c + d*x]*(a + b*Tan[c + d*x])^n)/(a^2*(a - I*b)*(a + I*b)*d*(1 + n)
)
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$
\int \left(\cot^2(dx+c)\right)(a+b\tan(dx+c))^n\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(cot(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))^n \cot^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**n*cot(c + d*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c+d\,x)^2\,\left(a+b\tan(c+d\,x)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + b*tan(c + d*x))^n,x)
[Out] int(cot(c + d*x)^2*(a + b*tan(c + d*x))^n, x)
```

3.714 $\int \cot^3(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=261

$$\frac{b(1-n)\cot(c+dx)(a+b\tan(c+dx))^{1+n}}{2a^2d}-\frac{\cot^2(c+dx)(a+b\tan(c+dx))^{1+n}}{2ad}-\frac{{}_2F_1\left(1,1+n;2+n;\frac{a+b\,\mathrm{t}}{}\right.}{2(a-}$$

```
[Out] 1/2*b*(1-n)*cot(d*x+c)*(a+b*tan(d*x+c))^(1+n)/a^2/d-1/2*cot(d*x+c)^2*(a+b*t
an(d*x+c))^(1+n)/a/d-1/2*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a-I*b))
*(a+b*tan(d*x+c))^(1+n)/(a-I*b)/d/(1+n)-1/2*hypergeom([1, 1+n],[2+n],(a+b*t
an(d*x+c))/(a+I*b))*(a+b*tan(d*x+c))^(1+n)/(a+I*b)/d/(1+n)+1/2*(2*a^2+b^2*(
1-n)*n)*hypergeom([1, 1+n],[2+n],1+b*tan(d*x+c)/a)*(a+b*tan(d*x+c))^(1+n)/a
^3/d/(1+n)
```

Rubi [A]
time = 0.39, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3650, 3730, 3735, 12, 3620, 3618, 70, 3715, 67}

$$\frac{b(1-n)\cot(c+dx)(a+b\tan(c+dx))^{n+1}}{2a^2d}+\frac{\left(2a^2+b^2(1-n)n\right)(a+b\tan(c+dx))^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{b\tan(c+dx)}{a}+1\right)}{2a^3d(n+1)}-\frac{(a+b\tan(c+dx))^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a-ib}\right)}{2d(n+1)(a-ib)}-\frac{(a+b\tan(c+dx))^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a+ib}\right)}{2d(n+1)(a+ib)}-\frac{\cot^2(c+dx)(a+b\tan(c+dx))^{n+1}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (b*(1 - n)*Cot[c + d*x]*(a + b*Tan[c + d*x])^(1 + n))/(2*a^2*d) - (Cot[c +
d*x]^2*(a + b*Tan[c + d*x])^(1 + n))/(2*a*d) - (Hypergeometric2F1[1, 1 + n,
 2 + n, (a + b*Tan[c + d*x])/(a - I*b)]*(a + b*Tan[c + d*x])^(1 + n))/(2*(a
 - I*b)*d*(1 + n)) - (Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x
])/(a + I*b)]*(a + b*Tan[c + d*x])^(1 + n))/(2*(a + I*b)*d*(1 + n)) + ((2*a
^2 + b^2*(1 - n)*n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])
/a]*(a + b*Tan[c + d*x])^(1 + n))/(2*a^3*d*(1 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
```

```
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3735

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dis
t[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*Simp[a*(A - C) - (A*b - b*C)*Ta
n[e + f*x], x], x], x] + Dist[(A*b^2 + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e
 + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a,
 b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0] &&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^n\,dx = -\frac{\cot^2(c+dx)(a+b\tan(c+dx))^{1+n}}{2ad} - \frac{\int \cot^2(c+dx)(a+b\tan(\ldots}{\ldots}$$

$$= \frac{b(1-n)\cot(c+dx)(a+b\tan(c+dx))^{1+n}}{2a^2d} - \frac{\cot^2(c+dx)(a+b\,t\ldots}{2ad\ldots}$$

$$= \frac{b(1-n)\cot(c+dx)(a+b\tan(c+dx))^{1+n}}{2a^2d} - \frac{\cot^2(c+dx)(a+b\,t\ldots}{2ad\ldots}$$

$$= \frac{b(1-n)\cot(c+dx)(a+b\tan(c+dx))^{1+n}}{2a^2d} - \frac{\cot^2(c+dx)(a+b\,t\ldots}{2ad\ldots}$$

$$= \frac{b(1-n)\cot(c+dx)(a+b\tan(c+dx))^{1+n}}{2a^2d} - \frac{\cot^2(c+dx)(a+b\,t\ldots}{2ad\ldots}$$

$$= \frac{b(1-n)\cot(c+dx)(a+b\tan(c+dx))^{1+n}}{2a^2d} - \frac{\cot^2(c+dx)(a+b\,t\ldots}{2ad\ldots}$$

$$= \frac{b(1-n)\cot(c+dx)(a+b\tan(c+dx))^{1+n}}{2a^2d} - \frac{\cot^2(c+dx)(a+b\,t\ldots}{2ad\ldots}$$

Mathematica [A]
time = 2.04, size = 212, normalized size = 0.81

$$-\frac{(b+a\cot(c+dx))\left(a^3(a+ib)\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a-ib}\right)+(a-ib)\left(a^3\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a+ib}\right)+(a+ib)\left(a(1+n)\cot(c+dx)(b(-1+n)+a\cot(c+dx))+(-2a^2+b^2(-1+n)n)\,{}_2F_1\left(1,1+n;2+n;1+\frac{b\tan(c+dx)}{a}\right)\right)\right)\right)\tan(c+dx)(a+b\tan(c+dx))^n}{2a^3(a-ib)(a+ib)d(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] -1/2*((b + a*Cot[c + d*x])*(a^3*(a + I*b)*Hypergeometric2F1[1, 1 + n, 2 + n
, (a + b*Tan[c + d*x])/(a - I*b)] + (a - I*b)*(a^3*Hypergeometric2F1[1, 1 +
 n, 2 + n, (a + b*Tan[c + d*x])/(a + I*b)] + (a + I*b)*(a*(1 + n)*Cot[c + d
*x]*(b*(-1 + n) + a*Cot[c + d*x]) + (-2*a^2 + b^2*(-1 + n)*n)*Hypergeometri
c2F1[1, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a])))*Tan[c + d*x]*(a + b*Tan[c
+ d*x])^n)/(a^3*(a - I*b)*(a + I*b)*d*(1 + n))
```

Maple [F]
time = 0.35, size = 0, normalized size = 0.00

$$\int \left(\cot^{3}\left(dx+c\right)\right) \left(a+b\tan\left(dx+c\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(cot(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan\left(c+dx\right)\right)^{n}\cot^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**n*cot(c + d*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c+d\,x)^3\,(a+b\tan(c+d\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + b*tan(c + d*x))^n,x)
```

```
[Out] int(cot(c + d*x)^3*(a + b*tan(c + d*x))^n, x)
```

3.715 $\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=159

$$\frac{F_1\left(\frac{5}{2};1,-n;\frac{7}{2};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^n\left(1+\frac{b\tan(c+dx)}{a}\right)^{-n}}{5d}+\frac{F_1\left(\frac{5}{2};1,-\right.}{}$$

```
[Out] 1/5*AppellF1(5/2,1,-n,7/2,-I*tan(d*x+c),-b*tan(d*x+c)/a)*tan(d*x+c)^(5/2)*(
a+b*tan(d*x+c))^n/d/((1+b*tan(d*x+c)/a)^n)+1/5*AppellF1(5/2,1,-n,7/2,I*tan(
d*x+c),-b*tan(d*x+c)/a)*tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))^n/d/((1+b*tan(d*x
+c)/a)^n)
```

Rubi [A]
time = 0.16, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3656, 926, 129, 525, 524}

$$\frac{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}F_1\left(\frac{5}{2};1,-n;\frac{7}{2};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)}{5d}+\frac{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}F_1\left(\frac{5}{2};1,-n;\frac{7}{2};i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (AppellF1[5/2, 1, -n, 7/2, (-I)*Tan[c + d*x], -((b*Tan[c + d*x])/a)]*Tan[c
+ d*x]^(5/2)*(a + b*Tan[c + d*x])^n)/(5*d*(1 + (b*Tan[c + d*x])/a)^n) + (Ap
pellF1[5/2, 1, -n, 7/2, I*Tan[c + d*x], -((b*Tan[c + d*x])/a)]*Tan[c + d*x]
^(5/2)*(a + b*Tan[c + d*x])^n)/(5*d*(1 + (b*Tan[c + d*x])/a)^n)
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
```

```
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{\text{Subst}\left(\int \frac{x^{3/2}(a+bx)^n}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{ix^{3/2}(a+bx)^n}{2(i-x)}+\frac{ix^{3/2}(a+bx)^n}{2(i+x)}\right)dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{i\text{Subst}\left(\int \frac{x^{3/2}(a+bx)^n}{i-x}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{i\text{Subst}\left(\int \frac{x^{3/2}(a+bx)^n}{i+x}\,d\right.}{2d}$$
$$= \frac{i\text{Subst}\left(\int \frac{x^4(a+bx^2)^n}{i-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d} + \frac{i\text{Subst}\left(\int \frac{x^4(a+bx^2)^{n}}{i+x^2}\right.}{}$$
$$= \frac{\left(i(a+b\tan(c+dx))^n\left(1+\frac{b\tan(c+dx)}{a}\right)^{-n}\right)\text{Subst}\left(\int \frac{x^4\left(1+\frac{bx^2}{a}\right)^n}{i-x^2}\right.}{d}$$
$$= \frac{F_1\left(\frac{5}{2};1,-n;\frac{7}{2};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)\tan^{\frac{5}{2}}(c+dx)(a+b\,\text{ta}}{5d}$$

Mathematica [F]
time = 1.48, size = 0, normalized size = 0.00

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] Integrate[Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^n, x]
```

Maple [F]
time = 0.38, size = 0, normalized size = 0.00

$$\int \left(\tan^{\frac{3}{2}}\left(dx+c\right)\right)\left(a+b\tan\left(dx+c\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*tan(d*x + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n*tan(d*x + c)^(3/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan\left(c+dx\right)\right)^{n}\tan^{\frac{3}{2}}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(3/2)*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**n*tan(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^n*tan(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^{3/2}\,{\left(a+b\tan(c+d\,x)\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x))^n,x)
[Out] int(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x))^n, x)
```

3.716 $\int \sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=159

$$\frac{F_1\left(\frac{3}{2};1,-n;\frac{5}{2};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n\left(1+\frac{b\tan(c+dx)}{a}\right)^{-n}}{3d}+\frac{F_1\left(\frac{3}{2};1,-\right.}{}$$

```
[Out] 1/3*AppellF1(3/2,1,-n,5/2,-I*tan(d*x+c),-b*tan(d*x+c)/a)*tan(d*x+c)^(3/2)*(
a+b*tan(d*x+c))^n/d/((1+b*tan(d*x+c)/a)^n)+1/3*AppellF1(3/2,1,-n,5/2,I*tan(
d*x+c),-b*tan(d*x+c)/a)*tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^n/d/((1+b*tan(d*x
+c)/a)^n)
```

Rubi [A]
time = 0.16, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3656, 926, 129, 525, 524}

$$\frac{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}F_1\left(\frac{3}{2};1,-n;\frac{5}{2};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)}{3d}+\frac{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}F_1\left(\frac{3}{2};1,-n;\frac{5}{2};i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (AppellF1[3/2, 1, -n, 5/2, (-I)*Tan[c + d*x], -((b*Tan[c + d*x])/a)]*Tan[c
+ d*x]^(3/2)*(a + b*Tan[c + d*x])^n)/(3*d*(1 + (b*Tan[c + d*x])/a)^n) + (Ap
pellF1[3/2, 1, -n, 5/2, I*Tan[c + d*x], -((b*Tan[c + d*x])/a)]*Tan[c + d*x]
^(3/2)*(a + b*Tan[c + d*x])^n)/(3*d*(1 + (b*Tan[c + d*x])/a)^n)
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
```

```
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^n\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{x}\,(a+bx)^n}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{i\sqrt{x}\,(a+bx)^n}{2(i-x)} + \frac{i\sqrt{x}\,(a+bx)^n}{2(i+x)}\right)\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{i\text{Subst}\left(\int \frac{\sqrt{x}\,(a+bx)^n}{i-x}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{i\text{Subst}\left(\int \frac{\sqrt{x}\,(a+b\cdots}{i+x}\cdots\right.}{\cdots}$$

$$= \frac{i\text{Subst}\left(\int \frac{x^2(a+bx^2)^n}{i-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d} + \frac{i\text{Subst}\left(\int \frac{x^2(a+bx\cdots}{i+x^2}\cdots\right.}{\cdots}$$

$$= \frac{\left(i(a+b\tan(c+dx))^n\left(1+\frac{b\tan(c+dx)}{a}\right)^{-n}\right)\text{Subst}\left(\int \frac{x^2\left(1+\frac{bx^2}{a}\right)\cdots}{i-x^2}\cdots\right.}{d}$$

$$= \frac{F_1\left(\frac{3}{2};1,-n;\frac{5}{2};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)\tan^{\frac{3}{2}}(c+dx)(a+b\cdots}{3d}$$

Mathematica [F]

time = 1.63, size = 0, normalized size = 0.00

$$\int \sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] Integrate[Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^n, x]
```

Maple [F]
time = 0.43, size = 0, normalized size = 0.00

$$\int \left(\sqrt{\tan}\,(dx+c)\right)(a+b\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*sqrt(tan(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n*sqrt(tan(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))^n\sqrt{\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/2)*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**n*sqrt(tan(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*sqrt(tan(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\tan\left(c+d\,x\right)}\,\left(a+b\tan(c+d\,x)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)*(a + b*tan(c + d*x))^n,x)
```

```
[Out] int(tan(c + d*x)^(1/2)*(a + b*tan(c + d*x))^n, x)
```

3.717 $\int \frac{(a+b\tan(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$

Optimal. Leaf size=153

$$\frac{F_1\left(\frac{1}{2};1,-n;\frac{3}{2};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^n\left(1+\frac{b\tan(c+dx)}{a}\right)^{-n}}{d}+\frac{F_1\left(\frac{1}{2};1,\right.}{}$$

```
[Out] AppellF1(1/2,1,-n,3/2,-I*tan(d*x+c),-b*tan(d*x+c)/a)*tan(d*x+c)^(1/2)*(a+b*
tan(d*x+c))^n/d/((1+b*tan(d*x+c)/a)^n)+AppellF1(1/2,1,-n,3/2,I*tan(d*x+c),-
b*tan(d*x+c)/a)*tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^n/d/((1+b*tan(d*x+c)/a)^n
)
```

Rubi [A]

time = 0.14, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3656, 926, 129, 441, 440}

$$\frac{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}F_1\left(\frac{1}{2};1,-n;\frac{3}{2};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)}{d}+\frac{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}F_1\left(\frac{1}{2};1,-n;\frac{3}{2};i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^n/Sqrt[Tan[c + d*x]],x]
```

```
[Out] (AppellF1[1/2, 1, -n, 3/2, (-I)*Tan[c + d*x], -((b*Tan[c + d*x])/a)]*Sqrt[T
an[c + d*x]]*(a + b*Tan[c + d*x])^n)/(d*(1 + (b*Tan[c + d*x])/a)^n) + (Appe
llF1[1/2, 1, -n, 3/2, I*Tan[c + d*x], -((b*Tan[c + d*x])/a)]*Sqrt[Tan[c + d
*x]]*(a + b*Tan[c + d*x])^n)/(d*(1 + (b*Tan[c + d*x])/a)^n)
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
```

```
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx)^n}{\sqrt{x}\,(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{i(a+bx)^n}{2(i-x)\sqrt{x}} + \frac{i(a+bx)^n}{2\sqrt{x}\,(i+x)}\right)dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{i\text{Subst}\left(\int \frac{(a+bx)^n}{(i-x)\sqrt{x}}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{i\text{Subst}\left(\int \frac{(a+bx)^n}{\sqrt{x}\,(i+x)}\,dx, x, \tan(c+d\right.}{2d}$$

$$= \frac{i\text{Subst}\left(\int \frac{(a+bx^2)^n}{i-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d} + \frac{i\text{Subst}\left(\int \frac{(a+bx^2)^n}{i+x^2}\,dx, x, \sqrt{\tan(c\right.}{d}$$

$$= \frac{\left(i(a+b\tan(c+dx))^n\left(1+\frac{b\tan(c+dx)}{a}\right)^{-n}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^n}{i-x^2}\,dx, x, \sqrt{\tan(c\right.}{d}$$

$$= \frac{F_1\left(\frac{1}{2};1,-n;\frac{3}{2};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx)}{d}$$

Mathematica [F]
time = 1.49, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Tan[c + d*x])^n/Sqrt[Tan[c + d*x]],x]
```

```
[Out] Integrate[(a + b*Tan[c + d*x])^n/Sqrt[Tan[c + d*x]], x]
```

Maple [F]
time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(dx+c))^n}{\sqrt{\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^n/tan(d*x+c)^(1/2),x)
```

```
[Out] int((a+b*tan(d*x+c))^n/tan(d*x+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^n/tan(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n/sqrt(tan(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^n/tan(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n/sqrt(tan(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**n/tan(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**n/sqrt(tan(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^n/tan(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^n/sqrt(tan(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\tan(c+d\,x))^n}{\sqrt{\tan\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^n/tan(c + d*x)^(1/2),x)
[Out] int((a + b*tan(c + d*x))^n/tan(c + d*x)^(1/2), x)
```

3.718 $\int \frac{(a+b\tan(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=155

$$-\frac{F_1\left(-\frac{1}{2};1,-n;\frac{1}{2};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)(a+b\tan(c+dx))^n\left(1+\frac{b\tan(c+dx)}{a}\right)^{-n}}{d\sqrt{\tan(c+dx)}}-\frac{F_1\left(-\frac{1}{2};1,-n;\frac{1}{2};i\tan\right.}{}$$

```
[Out] -AppellF1(-1/2,1,-n,1/2,-I*tan(d*x+c),-b*tan(d*x+c)/a)*(a+b*tan(d*x+c))^n/d
/tan(d*x+c)^(1/2)/((1+b*tan(d*x+c)/a)^n)-AppellF1(-1/2,1,-n,1/2,I*tan(d*x+c
),-b*tan(d*x+c)/a)*(a+b*tan(d*x+c))^n/d/tan(d*x+c)^(1/2)/((1+b*tan(d*x+c)/a
)^n)
```

Rubi [A]
time = 0.16, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3656, 926, 129, 525, 524}

$$-\frac{(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}F_1\left(-\frac{1}{2};1,-n;\frac{1}{2};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)}{d\sqrt{\tan(c+dx)}}-\frac{(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}F_1\left(-\frac{1}{2};1,-n;\frac{1}{2};i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)}{d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^n/Tan[c + d*x]^(3/2),x]
```

```
[Out] -((AppellF1[-1/2, 1, -n, 1/2, (-I)*Tan[c + d*x], -((b*Tan[c + d*x])/a)]*(a
+ b*Tan[c + d*x])^n)/(d*Sqrt[Tan[c + d*x]]*(1 + (b*Tan[c + d*x])/a)^n)) - (
AppellF1[-1/2, 1, -n, 1/2, I*Tan[c + d*x], -((b*Tan[c + d*x])/a)]*(a + b*Ta
n[c + d*x])^n)/(d*Sqrt[Tan[c + d*x]]*(1 + (b*Tan[c + d*x])/a)^n)
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+b\tan(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx &= \frac{\mathrm{Subst}\left(\int \frac{(a+bx)^n}{x^{3/2}(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d} \\
&= \frac{\mathrm{Subst}\left(\int \left(\frac{i(a+bx)^n}{2(i-x)x^{3/2}}+\frac{i(a+bx)^n}{2x^{3/2}(i+x)}\right)\,dx, x, \tan(c+dx)\right)}{d} \\
&= \frac{i\mathrm{Subst}\left(\int \frac{(a+bx)^n}{(i-x)x^{3/2}}\,dx, x, \tan(c+dx)\right)}{2d}+\frac{i\mathrm{Subst}\left(\int \frac{(a+bx)^n}{x^{3/2}(i+x)}\,dx, x, \tan(c+dx)\right.}{2d} \\
&= \frac{i\mathrm{Subst}\left(\int \frac{(a+bx^2)^n}{x^2(i-x^2)}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d}+\frac{i\mathrm{Subst}\left(\int \frac{(a+bx^2)^n}{x^2(i+x^2)}\,dx, x, \sqrt{\tan(c\right.}}{d} \\
&= \frac{\left(i(a+b\tan(c+dx))^n\left(1+\frac{b\tan(c+dx)}{a}\right)^{-n}\right)\mathrm{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^n}{x^2(i-x^2)}\,dx, x, \sqrt{\tan(c\right.}}{d} \\
&= -\frac{F_1\left(-\frac{1}{2};1,-n;\frac{1}{2};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)(a+b\tan(c+dx))^n\left(1+\frac{b\tan}{}\right.}{d\sqrt{\tan(c+dx)}}
\end{aligned}$$

Mathematica [F]

time = 1.52, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Tan[c + d*x])^n/Tan[c + d*x]^(3/2),x]
[Out] Integrate[(a + b*Tan[c + d*x])^n/Tan[c + d*x]^(3/2), x]
```

Maple [F]
time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(dx+c))^n}{\tan(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^n/tan(d*x+c)^(3/2),x)
[Out] int((a+b*tan(d*x+c))^n/tan(d*x+c)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^n/tan(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n/tan(d*x + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^n/tan(d*x+c)^(3/2),x, algorithm="fricas")
[Out] integral((b*tan(d*x + c) + a)^n/tan(d*x + c)^(3/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**n/tan(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**n/tan(c + d*x)**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^n/tan(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\tan(c+d\,x))^n}{\tan\left(c+d\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^n/tan(c + d*x)^(3/2),x)
```

```
[Out] int((a + b*tan(c + d*x))^n/tan(c + d*x)^(3/2), x)
```

3.719 $\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=65

$$-\frac{2(-1)^{3/4}a\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}-\frac{2ia\sqrt{\cot(c+dx)}}{d}-\frac{2a\cot^{\frac{3}{2}}(c+dx)}{3d}$$

```
[Out] -2*(-1)^(3/4)*a*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/d-2/3*a*cot(d*x+c)^(3/
2)/d-2*I*a*cot(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.08, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3754, 3609, 3614, 214}

$$-\frac{2a\cot^{\frac{3}{2}}(c+dx)}{3d}-\frac{2ia\sqrt{\cot(c+dx)}}{d}-\frac{2(-1)^{3/4}a\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (-2*(-1)^(3/4)*a*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/d - ((2*I)*a*Sqrt[
Cot[c + d*x]])/d - (2*a*Cot[c + d*x]^(3/2))/(3*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
```

```
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))\,dx = \int \cot^{\frac{3}{2}}(c+dx)(ia+a\cot(c+dx))\,dx$$
$$= -\frac{2a\cot^{\frac{3}{2}}(c+dx)}{3d} + \int \sqrt{\cot(c+dx)}\,(-a+ia\cot(c+dx))\,dx$$
$$= -\frac{2ia\sqrt{\cot(c+dx)}}{d} - \frac{2a\cot^{\frac{3}{2}}(c+dx)}{3d} + \int \frac{-ia-a\cot(c+dx)}{\sqrt{\cot(c+dx)}}$$
$$= -\frac{2ia\sqrt{\cot(c+dx)}}{d} - \frac{2a\cot^{\frac{3}{2}}(c+dx)}{3d} - \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{ia-ax^2}\,d\right.}{d}$$
$$= -\frac{2(-1)^{3/4}a\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d} - \frac{2ia\sqrt{\cot(c+dx}}{d}$$

Mathematica [A]
time = 1.08, size = 116, normalized size = 1.78

$$-\frac{2ae^{-ic}\sqrt{\cot(c+dx)}\,(i+\cot(c+dx))(\cos(dx)-i\sin(dx))\sin(c+dx)\left(3i+\cot(c+dx)-3i\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\right)\sqrt{i\tan(c+dx)}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (-2*a*Sqrt[Cot[c + d*x]]*(I + Cot[c + d*x])*(Cos[d*x] - I*Sin[d*x])*Sin[c +
 d*x]*(3*I + Cot[c + d*x] - (3*I)*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(
1 + E^((2*I)*(c + d*x)))]]*Sqrt[I*Tan[c + d*x]]))/(3*d*E^(I*c))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 13.37, size = 788, normalized size = 12.12

method	result
default	$-\frac{a\left(-3i\sqrt{-\frac{\cos(dx+c)-1-\sin(dx+c)}{\sin(dx+c)}}\sqrt{\frac{\cos(dx+c)-1+\sin(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\text{EllipticPi}\left(\sqrt{-\frac{\cos(dx+c)-1-\sin(}{\sin(dx+c)}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/d*(-3*I*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1
+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticP
i((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(
d*x+c)*sin(d*x+c)-3*I*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d
*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*El
lipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2
))*sin(d*x+c)+3*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-
1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Elliptic
Pi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos
(d*x+c)*sin(d*x+c)-3*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*
x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ell
ipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c
)*sin(d*x+c)+3*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1
+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticP
i((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(
d*x+c)-3*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(co
s(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*sin(d*x+c)+3*I*2^(1/2
)*cos(d*x+c)*sin(d*x+c)+2^(1/2)*cos(d*x+c)^2)*(cos(d*x+c)/sin(d*x+c))^(5/2)
*sin(d*x+c)/cos(d*x+c)^3*2^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(51) = 102$.
time = 0.48, size = 139, normalized size = 2.14

$$-\frac{3\left(-(2i+2)\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-(2i+2)\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-(i-1)\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+(i-1)\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)\right)a+\frac{24ia}{\sqrt{\tan(dx+c)}}+\frac{8a}{\tan(dx+c)^{\frac{3}{2}}}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(3*(-(2*I + 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x +
 c)))) - (2*I + 2)*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x +
c)))) - (I - 1)*sqrt(2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1
) + (I - 1)*sqrt(2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*
a + 24*I*a/sqrt(tan(d*x + c)) + 8*a/tan(d*x + c)^(3/2))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(51) = 102$.
time = 0.98, size = 287, normalized size = 4.42

$$-\frac{3\left(de^{(2idx+2ic)}-d\right)\sqrt{-\frac{4ia^2}{d^2}}\log\left(\frac{\left(\left(ide^{(2idx+2ic)}-id\right)\sqrt{-\frac{4ia^2}{d^2}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}+2iae^{(2idx+2ic)}\right)e^{(-2idx-2ic)}}{a}\right)-3\left(de^{(2idx+2ic)}-d\right)\sqrt{-\frac{4ia^2}{d^2}}\log\left(\frac{\left(\left(-ide^{(2idx+2ic)}+id\right)\sqrt{-\frac{4ia^2}{d^2}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}+2iae^{(2idx+2ic)}\right)e^{(-2idx-2ic)}}{a}\right)+16\left(2iae^{(2idx+2ic)}-ia\right)\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}}{12\left(de^{(2idx+2ic)}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(3*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(-4*I*a^2/d^2)*log(((I*d*e^(2*I*d*
x + 2*I*c) - I*d)*sqrt(-4*I*a^2/d^2)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2
*I*d*x + 2*I*c) - 1)) + 2*I*a*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/a)
- 3*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(-4*I*a^2/d^2)*log(((-I*d*e^(2*I*d*x +
2*I*c) + I*d)*sqrt(-4*I*a^2/d^2)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d
*x + 2*I*c) - 1)) + 2*I*a*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/a) + 16
*(2*I*a*e^(2*I*d*x + 2*I*c) - I*a)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I
*d*x + 2*I*c) - 1)))/(d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-i\cot^{\frac{5}{2}}(c+dx)\right)dx+\int\tan(c+dx)\cot^{\frac{5}{2}}(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(5/2)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] I*a*(Integral(-I*cot(c + d*x)**(5/2), x) + Integral(tan(c + d*x)*cot(c + d*
x)**(5/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)*cot(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\cot(c+d\,x)^{5/2}\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i),x)
```

```
[Out] int(cot(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i), x)
```

3.720 $\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=45

$$-\frac{2\sqrt[4]{-1}\,a\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}-\frac{2a\sqrt{\cot(c+dx)}}{d}$$

```
[Out] -2*(-1)^(1/4)*a*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/d-2*a*cot(d*x+c)^(1/2)
/d
```

Rubi [A]
time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3754, 3609, 3614, 214}

$$-\frac{2a\sqrt{\cot(c+dx)}}{d}-\frac{2\sqrt[4]{-1}\,a\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (-2*(-1)^(1/4)*a*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/d - (2*a*Sqrt[Cot[
c + d*x]])/d
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
```

```
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))\,dx = \int \sqrt{\cot(c+dx)}\,(ia+a\cot(c+dx))\,dx$$
$$= -\frac{2a\sqrt{\cot(c+dx)}}{d} + \int \frac{-a+ia\cot(c+dx)}{\sqrt{\cot(c+dx)}}\,dx$$
$$= -\frac{2a\sqrt{\cot(c+dx)}}{d} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{a+iax^2}\,dx, x, \sqrt{\cot(c+dx)}\right)}{d}$$
$$= -\frac{2\sqrt[4]{-1}\,a\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d} - \frac{2a\sqrt{\cot(c+dx)}}{d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.61, size = 67, normalized size = 1.49

$$\frac{2a\sqrt{\cot(c+dx)}\left(-1+\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\right)\sqrt{i\tan(c+dx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (2*a*Sqrt[Cot[c + d*x]]*(-1 + ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 +
E^((2*I)*(c + d*x)))]]*Sqrt[I*Tan[c + d*x]]))/d
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 13.11, size = 734, normalized size = 16.31

method	result
default	$-\frac{a\left(i\cos(dx+c)\sqrt{-\frac{\cos(dx+c)-1-\sin(dx+c)}{\sin(dx+c)}}\sqrt{\frac{\cos(dx+c)-1+\sin(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\text{EllipticF}\left(\sqrt{-\frac{\cos(dx+c)-}{\sin(}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -a/d*(I*cos(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+
c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellip
```

```
ticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-I*cos(d*x+c
)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/
sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+
c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-cos(d*x+c)*(-(cos
(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin
(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+I*(-(cos(d*x+c)-1-sin(d*x
+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c
))^(1/2),1/2*2^(1/2))-I*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos
(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1
/2))-(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c
))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d
*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+cos(d*x+c)*2^(
1/2))*sin(d*x+c)*(cos(d*x+c)/sin(d*x+c))^(3/2)/cos(d*x+c)^2*2^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.52, size = 127, normalized size = 2.82

$$\frac{\left(-(2i-2)\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-(2i-2)\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+(i+1)\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-(i+1)\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)\right)a-\frac{8a}{\sqrt{\tan(dx+c)}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*((-(2*I - 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c))
)) - (2*I - 2)*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))
) + (I + 1)*sqrt(2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) -
(I + 1)*sqrt(2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a -
8*a/sqrt(tan(d*x + c)))/d
```

Fricas [C] Result contains complex when optimal does not.
time = 0.67, size = 228, normalized size = 5.07

$$\frac{d\sqrt{\frac{4i a^2}{d^2}}\log\left(\frac{\left((de^{(2i dx+2i c)}-d)\sqrt{\frac{4i a^2}{d^2}}\sqrt{\frac{i e^{(2i dx+2i c)}+i}{e^{(2i dx+2i c)}-1}}+2i ae^{(2i dx+2i c)}\right)e^{(-2i dx-2i c)}}{a}\right)-d\sqrt{\frac{4i a^2}{d^2}}\log\left(-\frac{\left((de^{(2i dx+2i c)}-d)\sqrt{\frac{4i a^2}{d^2}}\sqrt{\frac{i e^{(2i dx+2i c)}+i}{e^{(2i dx+2i c)}-1}}-2i ae^{(2i dx+2i c)}\right)e^{(-2i dx-2i c)}}{a}\right)-8a\sqrt{\frac{i e^{(2i dx+2i c)}+i}{e^{(2i dx+2i c)}-1}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(d*sqrt(4*I*a^2/d^2)*log(((d*e^(2*I*d*x + 2*I*c) - d)*sqrt(4*I*a^2/d^2)
*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)) + 2*I*a*e^(2*I
*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/a) - d*sqrt(4*I*a^2/d^2)*log(-((d*e^(2*
I*d*x + 2*I*c) - d)*sqrt(4*I*a^2/d^2)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(
```

```
2*I*d*x + 2*I*c) - 1)) - 2*I*a*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/a)
 - 8*a*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-i\cot^{\frac{3}{2}}\left(c+dx\right)\right)dx+\int\tan\left(c+dx\right)\cot^{\frac{3}{2}}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(3/2)*(a+I*a*tan(d*x+c)),x)
[Out] I*a*(Integral(-I*cot(c + d*x)**(3/2), x) + Integral(tan(c + d*x)*cot(c + d*
x)**(3/2), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)*cot(d*x + c)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cot(c+d\,x)^{3/2}\,\left(a+a\,\tan(c+d\,x)\,1\mathrm{i}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i),x)
[Out] int(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i), x)
```

3.721 $\int \sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=28

$$\frac{2(-1)^{3/4}a\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}$$

```
[Out] 2*(-1)^(3/4)*a*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/d
```

Rubi [A]
time = 0.04, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3754, 3614, 214}

$$\frac{2(-1)^{3/4}a\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (2*(-1)^(3/4)*a*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/d
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))\,dx = \int \frac{ia+a\cot(c+dx)}{\sqrt{\cot(c+dx)}}\,dx$$
$$= -\frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{-ia+ax^2}\,dx, x, \sqrt{\cot(c+dx)}\right)}{d}$$
$$= \frac{2(-1)^{3/4}a\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.53, size = 111, normalized size = 3.96

$$-\frac{2ia\sqrt{\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{\frac{i\left(1+e^{2i(c+dx)}\right)}{-1+e^{2i(c+dx)}}}\,\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-2*I)*a*Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[(
I*(1 + E^((2*I)*(c + d*x))))/(-1 + E^((2*I)*(c + d*x)))]*ArcTanh[Sqrt[(-1 +
 E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])/d
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 12.31, size = 246, normalized size = 8.79

method	result
default	$a\sqrt{\frac{\cos(dx+c)}{\sin(dx+c)}}\,(-1+\cos(dx+c))\sqrt{-\frac{\cos(dx+c)-1-\sin(dx+c)}{\sin(dx+c)}}\sqrt{\frac{\cos(dx+c)-1+\sin(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\Big(i\,\text{EllipticPi}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] a/d*(cos(d*x+c)/sin(d*x+c))^(1/2)*(-1+cos(d*x+c))*(-(cos(d*x+c)-1-sin(d*x+c
))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*(I*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x
+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin
(d*x+c))^(1/2),1/2*2^(1/2))-EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+
c))^(1/2),1/2+1/2*I,1/2*2^(1/2)))/sin(d*x+c)^2/cos(d*x+c)*(cos(d*x+c)+1)^2*
2^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.52, size = 114, normalized size = 4.07

$$\frac{\left(-(2i+2)\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-(2i+2)\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-(i-1)\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+(i-1)\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)\right)a}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(-(2*I + 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))
) - (2*I + 2)*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c))))
 - (I - 1)*sqrt(2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + (
I - 1)*sqrt(2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a/d
```

Fricas [C] Result contains complex when optimal does not.
time = 0.66, size = 191, normalized size = 6.82

$$\frac{1}{4}\sqrt{-\frac{4i a^2}{d^2}}\log\left(\frac{\left(\left(i de^{(2i dx+2i c)}-i d\right)\sqrt{-\frac{4i a^2}{d^2}}\sqrt{\frac{i e^{(2i dx+2i c)}+i}{e^{(2i dx+2i c)}-1}}+2i ae^{(2i dx+2i c)}\right)e^{(-2i dx-2i c)}}{a}\right)-\frac{1}{4}\sqrt{-\frac{4i a^2}{d^2}}\log\left(\frac{\left(\left(-i de^{(2i dx+2i c)}+i d\right)\sqrt{-\frac{4i a^2}{d^2}}\sqrt{\frac{i e^{(2i dx+2i c)}+i}{e^{(2i dx+2i c)}-1}}+2i ae^{(2i dx+2i c)}\right)e^{(-2i dx-2i c)}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-4*I*a^2/d^2)*log(((I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(-4*I*a^2/d
^2)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)) + 2*I*a*e^(
2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/a) - 1/4*sqrt(-4*I*a^2/d^2)*log(((-I
*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(-4*I*a^2/d^2)*sqrt((I*e^(2*I*d*x + 2*I*c
) + I)/(e^(2*I*d*x + 2*I*c) - 1)) + 2*I*a*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x
- 2*I*c)/a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-i\sqrt{\cot(c+dx)}\right)dx+\int\tan(c+dx)\sqrt{\cot(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(1/2)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] I*a*(Integral(-I*sqrt(cot(c + d*x)), x) + Integral(tan(c + d*x)*sqrt(cot(c
+ d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)*sqrt(cot(d*x + c)), x)
```

Mupad [B]
time = 5.41, size = 22, normalized size = 0.79

$$-\frac{2\,{\left(-1\right)}^{1/4}\,a\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,\sqrt{\mathrm{cot}\left(c+d\,x\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i),x)
[Out] -(2*(-1)^(1/4)*a*atan((-1)^(1/4)*cot(c + d*x)^(1/2)))/d
```

3.722 $\int \frac{a+ia\tan(c+dx)}{\sqrt{\cot(c+dx)}}\,dx$

Optimal. Leaf size=47

$$\frac{2\sqrt[4]{-1}\,a\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}+\frac{2ia}{d\sqrt{\cot(c+dx)}}$$

```
[Out] 2*(-1)^(1/4)*a*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/d+2*I*a/d/cot(d*x+c)^(1
/2)
```

Rubi [A]
time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3754, 3610, 3614, 214}

$$\frac{2\sqrt[4]{-1}\,a\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}+\frac{2ia}{d\sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])/Sqrt[Cot[c + d*x]],x]
```

```
[Out] (2*(-1)^(1/4)*a*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/d + ((2*I)*a)/(d*Sq
rt[Cot[c + d*x]])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+ia\tan(c+dx)}{\sqrt{\cot(c+dx)}}\,dx &= \int \frac{ia+a\cot(c+dx)}{\cot^{\frac{3}{2}}(c+dx)}\,dx\\
&= \frac{2ia}{d\sqrt{\cot(c+dx)}}+\int \frac{a-ia\cot(c+dx)}{\sqrt{\cot(c+dx)}}\,dx\\
&= \frac{2ia}{d\sqrt{\cot(c+dx)}}+\frac{(2a^2)\,\mathrm{Subst}\left(\int \frac{1}{-a-iax^2}\,dx,x,\sqrt{\cot(c+dx)}\right)}{d}\\
&= \frac{2\sqrt[4]{-1}\,a\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}+\frac{2ia}{d\sqrt{\cot(c+dx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.77, size = 71, normalized size = 1.51

$$
\frac{a\left(2i-\frac{2i\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\right)}{\sqrt{i\tan(c+dx)}}\right)}{d\sqrt{\cot(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])/Sqrt[Cot[c + d*x]],x]
```

```
[Out] (a*(2*I - ((2*I)*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c +
 d*x)))]])/Sqrt[I*Tan[c + d*x]]))/(d*Sqrt[Cot[c + d*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 26.53, size = 420, normalized size = 8.94

method	result

default	$\frac{a(\cos(dx+c)+1)^2(-1+\cos(dx+c))\left(-i\sin(dx+c)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\sqrt{\frac{\cos(dx+c)-1+\sin(dx+c)}{\sin(dx+c)}}\sqrt{-\frac{\cos(dx+c)-1-\sin(dx+c)}{\sin(dx+c)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))/cot(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))*(-I*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-si
n(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c
))^(1/2),1/2*2^(1/2))+I*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos
(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x
+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2
*I,1/2*2^(1/2))+(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-
1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Elliptic
Pi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin
(d*x+c)+I*cos(d*x+c)*2^(1/2)-I*2^(1/2))/sin(d*x+c)^4/(cos(d*x+c)/sin(d*x+c)
)^(1/2)*2^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(37) = 74$.
time = 0.49, size = 127, normalized size = 2.70

$$-\frac{\left(-(2i-2)\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-(2i-2)\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+(i+1)\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-(i+1)\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)\right)a-8ia\sqrt{\tan(dx+c)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/cot(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*((-(2*I - 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)
))) - (2*I - 2)*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c))
)) + (I + 1)*sqrt(2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) -
 (I + 1)*sqrt(2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a -
 8*I*a*sqrt(tan(d*x + c)))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(37) = 74$.
time = 0.67, size = 278, normalized size = 5.91

$$-\frac{(de^{(2idx+2ic)}+d)\sqrt{\frac{4ia^2}{d^2}}\log\left(\frac{\left((de^{(2idx+2ic)}-d)\sqrt{\frac{4ia^2}{d^2}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}+2iae^{(2idx+2ic)}\right)e^{(-2idx-2ic)}}{a}\right)-(de^{(2idx+2ic)}+d)\sqrt{\frac{4ia^2}{d^2}}\log\left(-\frac{\left((de^{(2idx+2ic)}-d)\sqrt{\frac{4ia^2}{d^2}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}-2iae^{(2idx+2ic)}\right)e^{(-2idx-2ic)}}{a}\right)-8(ae^{(2idx+2ic)}-a)\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}}{4(de^{(2idx+2ic)}+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/cot(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*((d*e^(2*I*d*x + 2*I*c) + d)*sqrt(4*I*a^2/d^2)*log(((d*e^(2*I*d*x + 2*
I*c) - d)*sqrt(4*I*a^2/d^2)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
2*I*c) - 1)) + 2*I*a*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/a) - (d*e^(2
*I*d*x + 2*I*c) + d)*sqrt(4*I*a^2/d^2)*log(-((d*e^(2*I*d*x + 2*I*c) - d)*sq
rt(4*I*a^2/d^2)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))
 - 2*I*a*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/a) - 8*(a*e^(2*I*d*x + 2
*I*c) - a)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))/(d*
e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-\frac{i}{\sqrt{\cot\left(c+dx\right)}}\right)dx+\int\frac{\tan\left(c+dx\right)}{\sqrt{\cot\left(c+dx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/cot(d*x+c)**(1/2),x)
```

```
[Out] I*a*(Integral(-I/sqrt(cot(c + d*x)), x) + Integral(tan(c + d*x)/sqrt(cot(c
+ d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/cot(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)/sqrt(cot(d*x + c)), x)
```

Mupad [B]
time = 5.64, size = 102, normalized size = 2.17

$$\frac{a\,2\mathrm{i}}{d\,\sqrt{\cot\left(c+d\,x\right)}}-\frac{{\left(-1\right)}^{1/4}\,a\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}}{\sqrt{\cot\left(c+d\,x\right)}}\right)}{d}+\frac{{\left(-1\right)}^{1/4}\,a\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,\sqrt{\cot\left(c+d\,x\right)}\right)\,1\mathrm{i}}{d}-\frac{{\left(-1\right)}^{1/4}\,a\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}}{\sqrt{\cot\left(c+d\,x\right)}}\right)}{d}+\frac{{\left(-1\right)}^{1/4}\,a\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,\sqrt{\cot\left(c+d\,x\right)}\right)\,1\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)/cot(c + d*x)^(1/2),x)
```

```
[Out] (a*2i)/(d*cot(c + d*x)^(1/2)) - ((-1)^(1/4)*a*atan((-1)^(1/4)/cot(c + d*x)^
(1/2)))/d + ((-1)^(1/4)*a*atan((-1)^(1/4)*cot(c + d*x)^(1/2))*1i)/d - ((-1)
^(1/4)*a*atanh((-1)^(1/4)/cot(c + d*x)^(1/2)))/d + ((-1)^(1/4)*a*atanh((-1)
^(1/4)*cot(c + d*x)^(1/2))*1i)/d
```

3.723 $\int \frac{a+ia\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=65

$$-\frac{2(-1)^{3/4}a\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}+\frac{2ia}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{2a}{d\sqrt{\cot(c+dx)}}$$

[Out] -2*(-1)^(3/4)*a*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/d+2/3*I*a/d/cot(d*x+c)^(3/2)+2*a/d/cot(d*x+c)^(1/2)

Rubi [A]
time = 0.08, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3754, 3610, 3614, 214}

$$\frac{2ia}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{2a}{d\sqrt{\cot(c+dx)}}-\frac{2(-1)^{3/4}a\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + I*a*Tan[c + d*x])/Cot[c + d*x]^(3/2),x]

[Out] (-2*(-1)^(3/4)*a*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/d + (((2*I)/3)*a)/(d*Cot[c + d*x]^(3/2)) + (2*a)/(d*Sqrt[Cot[c + d*x]])

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int \frac{a+ia\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)}\,dx &= \int \frac{ia+a\cot(c+dx)}{\cot^{\frac{5}{2}}(c+dx)}\,dx\\ &= \frac{2ia}{3d\cot^{\frac{3}{2}}(c+dx)}+\int \frac{a-ia\cot(c+dx)}{\cot^{\frac{3}{2}}(c+dx)}\,dx\\ &= \frac{2ia}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{2a}{d\sqrt{\cot(c+dx)}}+\int \frac{-ia-a\cot(c+dx)}{\sqrt{\cot(c+dx)}}\,dx\\ &= \frac{2ia}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{2a}{d\sqrt{\cot(c+dx)}}-\frac{(2a^2)\operatorname{Subst}\left(\int \frac{1}{ia-ax^2}\,dx,x,\sqrt{\cot(c+dx)}\right.}{d}\\ &= -\frac{2(-1)^{3/4}a\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}+\frac{2ia}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{2a}{d\sqrt{\cot(c+}}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 136 vs. 2(65) = 130.
time = 1.11, size = 136, normalized size = 2.09

$$\frac{ae^{-ic}(i+\cot(c+dx))\sec(c+dx)(i\cos(dx)+\sin(dx))\left(1-\cos(2(c+dx))-3i\sin(2(c+dx))+6\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\right)\cos^2(c+dx)\sqrt{i\tan(c+dx)}\right)}{3d\sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])/Cot[c + d*x]^(3/2),x]
```

```
[Out] (a*(I + Cot[c + d*x])*Sec[c + d*x]*(I*Cos[d*x] + Sin[d*x])*(1 - Cos[2*(c +
d*x)] - (3*I)*Sin[2*(c + d*x)] + 6*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/
(1 + E^((2*I)*(c + d*x)))]]*Cos[c + d*x]^2*Sqrt[I*Tan[c + d*x]]))/(3*d*E^(I
*c)*Sqrt[Cot[c + d*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 25.32, size = 475, normalized size = 7.31

method	result

default	$\frac{a(-1+\cos(dx+c))\left(-3i\sqrt{-\frac{\cos(dx+c)-1-\sin(dx+c)}{\sin(dx+c)}}\ \sqrt{\frac{\cos(dx+c)-1+\sin(dx+c)}{\sin(dx+c)}}\ \sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\ \text{EllipticPi}\left(\sqrt{-\frac{\cos(dx}{}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))/cot(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a/d*(-1+cos(d*x+c))*(-3*I*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)
*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^
(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/
2*2^(1/2))*cos(d*x+c)*sin(d*x+c)+3*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^
(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2
*I,1/2*2^(1/2))*cos(d*x+c)*sin(d*x+c)-3*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x
+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2
*2^(1/2))*cos(d*x+c)*sin(d*x+c)+I*cos(d*x+c)*sin(d*x+c)*2^(1/2)+3*2^(1/2)*c
os(d*x+c)^2-I*sin(d*x+c)*2^(1/2)-3*cos(d*x+c)*2^(1/2))*(cos(d*x+c)+1)^2/(co
s(d*x+c)/sin(d*x+c))^(3/2)/sin(d*x+c)^5*2^(1/2)
```

<u>Maxima</u> [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(51) = 102$.
time $= 0.54$, size $= 142$, normalized size $= 2.18$

$$\frac{8\left(i\,a+\frac{3\,a}{\tan(dx+c)}\right)\tan\left(dx+c\right)^{\frac{3}{2}}-3\left(-(2i+2)\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)-(2i+2)\,\sqrt{2}\,\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)-(i-1)\,\sqrt{2}\,\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)+(i-1)\,\sqrt{2}\,\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)\right)a}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/cot(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(8*(I*a + 3*a/tan(d*x + c))*tan(d*x + c)^(3/2) - 3*(-(2*I + 2)*sqrt(2)
*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) - (2*I + 2)*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - (I - 1)*sqrt(2)*log(
sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + (I - 1)*sqrt(2)*log(-sqr
t(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a)/d
```

<u>Fricas</u> [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(51) = 102$.
time $= 0.83$, size $= 329$, normalized size $= 5.06$

$$-\frac{3\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)\sqrt{-\frac{4i\,a^{2}}{d^{2}}}\log\left(\frac{\left((i\,de^{(2i\,dx+2i\,c)}-i\,d)\sqrt{-\frac{4i\,a^{2}}{d^{2}}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}+2i\,ae^{(2i\,dx+2i\,c)}\right)e^{(-2i\,dx-2i\,c)}}{a}\right)-3\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)\sqrt{-\frac{4i\,a^{2}}{d^{2}}}\log\left(\frac{\left((-i\,de^{(2i\,dx+2i\,c)}+i\,d)\sqrt{-\frac{4i\,a^{2}}{d^{2}}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}+2i\,ae^{(2i\,dx+2i\,c)}\right)e^{(-2i\,dx-2i\,c)}}{a}\right)+16\left(2i\,ae^{(4i\,dx+4i\,c)}-i\,ae^{(2i\,dx+2i\,c)}-i\,a\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}}{12\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/cot(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/12*(3*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-4*I*a^
2/d^2)*log(((I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(-4*I*a^2/d^2)*sqrt((I*e^(2
*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)) + 2*I*a*e^(2*I*d*x + 2*I*c)
)*e^(-2*I*d*x - 2*I*c)/a) - 3*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I
*c) + d)*sqrt(-4*I*a^2/d^2)*log(((-I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(-4*I
*a^2/d^2)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)) + 2*I
*a*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/a) + 16*(2*I*a*e^(4*I*d*x + 4*
I*c) - I*a*e^(2*I*d*x + 2*I*c) - I*a)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(
2*I*d*x + 2*I*c) - 1)))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) +
d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-\frac{i}{\cot^{\frac{3}{2}}(c+dx)}\right)dx+\int\frac{\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/cot(d*x+c)**(3/2),x)
```

```
[Out] I*a*(Integral(-I/cot(c + d*x)**(3/2), x) + Integral(tan(c + d*x)/cot(c + d*
x)**(3/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/cot(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)/cot(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{a+a\tan(c+d\,x)\,1\mathrm{i}}{\cot\left(c+d\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)/cot(c + d*x)^(3/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)/cot(c + d*x)^(3/2), x)
```

3.724 $\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=91

$$\frac{4\sqrt[4]{-1}\,a^2\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}+\frac{4a^2\sqrt{\cot(c+dx)}}{d}-\frac{4ia^2\cot^{\frac{3}{2}}(c+dx)}{3d}-\frac{2a^2\cot^{\frac{5}{2}}(c+dx)}{5d}$$

```
[Out] 4*(-1)^(1/4)*a^2*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/d-4/3*I*a^2*cot(d*x+c
)^(3/2)/d-2/5*a^2*cot(d*x+c)^(5/2)/d+4*a^2*cot(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.12, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3754, 3624, 3609, 3614, 214}

$$-\frac{2a^2\cot^{\frac{5}{2}}(c+dx)}{5d}-\frac{4ia^2\cot^{\frac{3}{2}}(c+dx)}{3d}+\frac{4a^2\sqrt{\cot(c+dx)}}{d}+\frac{4\sqrt[4]{-1}\,a^2\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(7/2)*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (4*(-1)^(1/4)*a^2*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/d + (4*a^2*Sqrt[C
ot[c + d*x]])/d - (((4*I)/3)*a^2*Cot[c + d*x]^(3/2))/d - (2*a^2*Cot[c + d*x
]^(5/2))/(5*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^2\,dx = \int \cot^{\frac{3}{2}}(c+dx)(ia+a\cot(c+dx))^2\,dx$$
$$= -\frac{2a^2\cot^{\frac{5}{2}}(c+dx)}{5d} + \int \cot^{\frac{3}{2}}(c+dx)\left(-2a^2+2ia^2\cot(c+dx)\right)$$
$$= -\frac{4ia^2\cot^{\frac{3}{2}}(c+dx)}{3d} - \frac{2a^2\cot^{\frac{5}{2}}(c+dx)}{5d} + \int \sqrt{\cot(c+dx)}\,(-2$$
$$= \frac{4a^2\sqrt{\cot(c+dx)}}{d} - \frac{4ia^2\cot^{\frac{3}{2}}(c+dx)}{3d} - \frac{2a^2\cot^{\frac{5}{2}}(c+dx)}{5d} + \int$$
$$= \frac{4a^2\sqrt{\cot(c+dx)}}{d} - \frac{4ia^2\cot^{\frac{3}{2}}(c+dx)}{3d} - \frac{2a^2\cot^{\frac{5}{2}}(c+dx)}{5d} + \frac{(8}{}$$
$$= \frac{4\sqrt[4]{-1}\,a^2\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d} + \frac{4a^2\sqrt{\cot(c+dx)}}{d}$$

Mathematica [A]
time = 3.10, size = 104, normalized size = 1.14

$$-\frac{a^2\sqrt{\cot(c+dx)}\left(\csc^2(c+dx)(-27+33\cos(2(c+dx))+10i\sin(2(c+dx)))+60\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\right)\sqrt{i\tan(c+dx)}\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(7/2)*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -1/15*(a^2*Sqrt[Cot[c + d*x]]*(Csc[c + d*x]^2*(-27 + 33*Cos[2*(c + d*x)] +
(10*I)*Sin[2*(c + d*x)]) + 60*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 +
E^((2*I)*(c + d*x)))]]*Sqrt[I*Tan[c + d*x]]))/d
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 13.48, size = 1482, normalized size = 16.29

method	result	size
default	Expression too large to display	1482

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*a^2/d*(-30*I*cos(d*x+c)^3*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/
2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,
1/2*2^(1/2))+30*I*cos(d*x+c)^3*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2
)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))
^(1/2)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))
-30*cos(d*x+c)^3*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)
-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellipti
cPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-30
*I*cos(d*x+c)^2*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-
1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Elliptic
Pi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+30*
I*cos(d*x+c)^2*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1
+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF
((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-30*cos(d*x+c)^2
*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/s
in(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c
)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+30*I*cos(d*x+c)*(-
(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(
d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1
-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-30*I*cos(d*x+c)*(-(co
s(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x
+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(cos(d*x+c)-1-sin
(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+10*I*cos(d*x+c)^2*sin(d*x+c)*2^(1/2
)+30*cos(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-
1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Elliptic
Pi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+30*
I*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/
sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+
c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-30*I*(-(cos(d*x+c
)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1
/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c)
)/sin(d*x+c))^(1/2),1/2*2^(1/2))+33*cos(d*x+c)^3*2^(1/2)+30*(-(cos(d*x+c)-1
-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/
```

```
sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-30*cos(d*x+c)*2^(1/2))*(cos(d*x+c)
/sin(d*x+c))^(7/2)*sin(d*x+c)/cos(d*x+c)^4*2^(1/2)
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(73) = 146$.
time = 0.53, size = 158, normalized size = 1.74

$$-\frac{15\left(-(2i-2)\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)-(2i-2)\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+(i+1)\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)-(i+1)\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)\right)a^2-\frac{120\,a^2}{\sqrt{\tan\left(dx+c\right)}}+\frac{40i\,a^2}{\tan(dx+c)^{\frac{3}{2}}}+\frac{12\,a^2}{\tan(dx+c)^{\frac{5}{2}}}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/30*(15*(-(2*I - 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x
+ c)))) - (2*I - 2)*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x +
 c)))) + (I + 1)*sqrt(2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) +
1) - (I + 1)*sqrt(2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))
*a^2 - 120*a^2/sqrt(tan(d*x + c)) + 40*I*a^2/tan(d*x + c)^(3/2) + 12*a^2/ta
n(d*x + c)^(5/2))/d
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(73) = 146$.
time = 0.79, size = 340, normalized size = 3.74

$$-\frac{15\sqrt{\frac{16i\,a^4}{d^2}}\left(de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{\left(4i\,a^2e^{(2i\,dx+2i\,c)}+\sqrt{\frac{16i\,a^4}{d^2}}\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\right)e^{(-2i\,dx-2i\,c)}}{2\,a^2}\right)-15\sqrt{\frac{16i\,a^4}{d^2}}\left(de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{\left(4i\,a^2e^{(2i\,dx+2i\,c)}-\sqrt{\frac{16i\,a^4}{d^2}}\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\right)e^{(-2i\,dx-2i\,c)}}{2\,a^2}\right)-8\left(43\,a^2e^{(4i\,dx+4i\,c)}-54\,a^2e^{(2i\,dx+2i\,c)}+23\,a^2\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}}{60\left(de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/60*(15*sqrt(16*I*a^4/d^2)*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*
c) + d)*log(1/2*(4*I*a^2*e^(2*I*d*x + 2*I*c) + sqrt(16*I*a^4/d^2)*(d*e^(2*I
*d*x + 2*I*c) - d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) -
1)))*e^(-2*I*d*x - 2*I*c)/a^2) - 15*sqrt(16*I*a^4/d^2)*(d*e^(4*I*d*x + 4*I*
c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*log(1/2*(4*I*a^2*e^(2*I*d*x + 2*I*c) - sq
rt(16*I*a^4/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt((I*e^(2*I*d*x + 2*I*c) +
I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)/a^2) - 8*(43*a^2*e^(4*I
*d*x + 4*I*c) - 54*a^2*e^(2*I*d*x + 2*I*c) + 23*a^2)*sqrt((I*e^(2*I*d*x + 2
*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))/(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*
d*x + 2*I*c) + d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(7/2)*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4847 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^2*cot(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^{7/2}\,\left(a+a\tan(c+d\,x)\,\,1\mathrm{i}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(7/2)*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] int(cot(c + d*x)^(7/2)*(a + a*tan(c + d*x)*1i)^2, x)
```

3.725 $\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=71

$$-\frac{4(-1)^{3/4}a^2\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}-\frac{4ia^2\sqrt{\cot(c+dx)}}{d}-\frac{2a^2\cot^{\frac{3}{2}}(c+dx)}{3d}$$

```
[Out] -4*(-1)^(3/4)*a^2*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/d-2/3*a^2*cot(d*x+c)
^(3/2)/d-4*I*a^2*cot(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.10, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3754, 3624, 3609, 3614, 214}

$$-\frac{2a^2\cot^{\frac{3}{2}}(c+dx)}{3d}-\frac{4ia^2\sqrt{\cot(c+dx)}}{d}-\frac{4(-1)^{3/4}a^2\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (-4*(-1)^(3/4)*a^2*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/d - ((4*I)*a^2*S
qrt[Cot[c + d*x]])/d - (2*a^2*Cot[c + d*x]^(3/2))/(3*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
```

```
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && !LeQ
[m, -1] && !(EqQ[m, 2] && EqQ[a, 0])
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2\,dx = \int \sqrt{\cot(c+dx)}\,(ia+a\cot(c+dx))^2\,dx$$
$$= -\frac{2a^2\cot^{\frac{3}{2}}(c+dx)}{3d} + \int \sqrt{\cot(c+dx)}\,\left(-2a^2+2ia^2\cot(c+dx)\right)$$
$$= -\frac{4ia^2\sqrt{\cot(c+dx)}}{d} - \frac{2a^2\cot^{\frac{3}{2}}(c+dx)}{3d} + \int \frac{-2ia^2-2a^2\cot(c}{\sqrt{\cot(c+dx)}}$$
$$= -\frac{4ia^2\sqrt{\cot(c+dx)}}{d} - \frac{2a^2\cot^{\frac{3}{2}}(c+dx)}{3d} - \frac{(8a^4)\operatorname{Subst}\left(\int \frac{1}{2ia^2-2a}\right.}{}$$
$$= -\frac{4(-1)^{3/4}a^2\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d} - \frac{4ia^2\sqrt{\cot(c+d}}{d}$$

Mathematica [A]
time = 1.21, size = 125, normalized size = 1.76

$$-\frac{2a^2e^{-2ic}\sqrt{\cot(c+dx)}\,(\cos(2(c+dx))+i\sin(2(c+dx)))\left(6i+\cot(c+dx)-6i\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\right)\sqrt{i\tan(c+dx)}\right)}{3d(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (-2*a^2*Sqrt[Cot[c + d*x]]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)])*(6*I + C
ot[c + d*x] - (6*I)*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(
c + d*x)))]]*Sqrt[I*Tan[c + d*x]]))/(3*d*E^((2*I)*c)*(Cos[d*x] + I*Sin[d*x]
)^2)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 14.17, size = 790, normalized size = 11.13

method	result
default	$-\frac{a^2\left(-6i\sqrt{-\frac{\cos(dx+c)-1-\sin(dx+c)}{\sin(dx+c)}}\,\sqrt{\frac{\cos(dx+c)-1+\sin(dx+c)}{\sin(dx+c)}}\,\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\ \mathrm{EllipticPi}\left(\sqrt{-\frac{\cos(dx+c)-1-\sin(dx+c)}{\sin(dx+c)}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a^2/d*(-6*I*cos(d*x+c)*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+
c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2
+1/2*I,1/2*2^(1/2))-6*I*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^
(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2
*I,1/2*2^(1/2))+6*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c
)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellipt
icPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*c
os(d*x+c)*sin(d*x+c)-6*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(
d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*E
llipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x
+c)*sin(d*x+c)+6*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)
-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellipti
cPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*si
n(d*x+c)-6*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin
(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(
cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*sin(d*x+c)+6*I*2^(1
/2)*cos(d*x+c)*sin(d*x+c)+2^(1/2)*cos(d*x+c)^2)*(cos(d*x+c)/sin(d*x+c))^(5/
2)*sin(d*x+c)/cos(d*x+c)^3*2^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(57) = 114$.
time = 0.53, size = 145, normalized size = 2.04

$$\frac{3\left((2i+2)\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+(2i+2)\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+(i-1)\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-(i-1)\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)\right)a^2-\frac{24i\,a^2}{\sqrt{\tan(dx+c)}}-\frac{4\,a^2}{\tan(dx+c)^{\frac{3}{2}}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*((2*I + 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)
))) + (2*I + 2)*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c))
)) + (I - 1)*sqrt(2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) -
 (I - 1)*sqrt(2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a^2
 - 24*I*a^2/sqrt(tan(d*x + c)) - 4*a^2/tan(d*x + c)^(3/2))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 297 vs. 2(57) = 114.
time = 1.21, size = 297, normalized size = 4.18

$$-\frac{3\sqrt{-\frac{16i\,a^4}{d^2}}\left(de^{(2i\,dx+2i\,c)}-d\right)\log\left(\frac{\left(4i\,a^2e^{(2i\,dx+2i\,c)}+\sqrt{-\frac{16i\,a^4}{d^2}}\left(i\,de^{(2i\,dx+2i\,c)}-i\,d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\right)e^{(-2i\,dx-2i\,c)}}{2\,a^2}\right)-3\sqrt{-\frac{16i\,a^4}{d^2}}\left(de^{(2i\,dx+2i\,c)}-d\right)\log\left(\frac{\left(4i\,a^2e^{(2i\,dx+2i\,c)}+\sqrt{-\frac{16i\,a^4}{d^2}}\left(-i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\right)e^{(-2i\,dx-2i\,c)}}{2\,a^2}\right)+8\left(7i\,a^2e^{(2i\,dx+2i\,c)}-5i\,a^2\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}}{12\left(de^{(2i\,dx+2i\,c)}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/12*(3*sqrt(-16*I*a^4/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*log(1/2*(4*I*a^2*e
^(2*I*d*x + 2*I*c) + sqrt(-16*I*a^4/d^2)*(I*d*e^(2*I*d*x + 2*I*c) - I*d)*sq
rt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*
I*c)/a^2) - 3*sqrt(-16*I*a^4/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*log(1/2*(4*I*
a^2*e^(2*I*d*x + 2*I*c) + sqrt(-16*I*a^4/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) + I
*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*
x - 2*I*c)/a^2) + 8*(7*I*a^2*e^(2*I*d*x + 2*I*c) - 5*I*a^2)*sqrt((I*e^(2*I*
d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))/(d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(5/2)*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^2*cot(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^{5/2}\,\left(a+a\,\tan(c+d\,x)\,1{}\mathrm{i}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] int(cot(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^2, x)
```

3.726 $\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=49

$$-\frac{4\sqrt[4]{-1}\,a^2\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}-\frac{2a^2\sqrt{\cot(c+dx)}}{d}$$

```
[Out] -4*(-1)^(1/4)*a^2*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/d-2*a^2*cot(d*x+c)^(
1/2)/d
```

Rubi [A]
time = 0.08, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3754, 3624, 3614, 214}

$$-\frac{2a^2\sqrt{\cot(c+dx)}}{d}-\frac{4\sqrt[4]{-1}\,a^2\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (-4*(-1)^(1/4)*a^2*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/d - (2*a^2*Sqrt[
Cot[c + d*x]])/d
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
```

```
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2\,dx = \int \frac{(ia+a\cot(c+dx))^2}{\sqrt{\cot(c+dx)}}\,dx$$
$$= -\frac{2a^2\sqrt{\cot(c+dx)}}{d} + \int \frac{-2a^2+2ia^2\cot(c+dx)}{\sqrt{\cot(c+dx)}}\,dx$$
$$= -\frac{2a^2\sqrt{\cot(c+dx)}}{d} + \frac{(8a^4)\,\text{Subst}\left(\int \frac{1}{2a^2+2ia^2x^2}\,dx, x, \sqrt{\cot(c+d}\right.}{d}$$
$$= -\frac{4\sqrt[4]{-1}\,a^2\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d} - \frac{2a^2\sqrt{\cot(c+dx)}}{d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.16, size = 70, normalized size = 1.43

$$\frac{2a^2\sqrt{\cot(c+dx)}\left(-1+2\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\right)\sqrt{i\tan(c+dx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (2*a^2*Sqrt[Cot[c + d*x]]*(-1 + 2*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(
1 + E^((2*I)*(c + d*x)))]]*Sqrt[I*Tan[c + d*x]]))/d
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 14.09, size = 736, normalized size = 15.02

method	result
default	$-\frac{a^2\left(2i\cos(dx+c)\sqrt{-\frac{\cos(dx+c)-1-\sin(dx+c)}{\sin(dx+c)}}\sqrt{\frac{\cos(dx+c)-1+\sin(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\text{EllipticF}\left(\sqrt{-\frac{\cos(dx+c)-}{\sin(}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^2/d*(2*I*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+si
n(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-
```

```
(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)-2*I*(-(
cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-
sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)+2*I*(-(cos(
d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c
))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(cos(d*x+c)-1-sin(d
*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-2*I*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d
*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),
1/2+1/2*I,1/2*2^(1/2))-2*cos(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))
^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/
2*I,1/2*2^(1/2))-2*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+
c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellip
ticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+
cos(d*x+c)*2^(1/2))*(cos(d*x+c)/sin(d*x+c))^(3/2)*sin(d*x+c)/cos(d*x+c)^2*2
^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.49, size = 131, normalized size = 2.67

$$\frac{\left(-(2i-2)\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-(2i-2)\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+(i+1)\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-(i+1)\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)\right)a^2-\frac{4a^2}{\sqrt{\tan(dx+c)}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*((-(2*I - 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c))
)) - (2*I - 2)*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))
) + (I + 1)*sqrt(2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) -
(I + 1)*sqrt(2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a^2
- 4*a^2/sqrt(tan(d*x + c)))/d
```

Fricas [C] Result contains complex when optimal does not.
time = 0.98, size = 236, normalized size = 4.82

$$-\frac{8a^2\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}-\sqrt{\frac{16ia^4}{d^2}}d\log\left(\frac{\left(4ia^2e^{(2idx+2ic)}+\sqrt{\frac{16ia^4}{d^2}}\left(de^{(2idx+2ic)}-d\right)\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\right)e^{(-2idx-2ic)}}{2a^2}\right)+\sqrt{\frac{16ia^4}{d^2}}d\log\left(\frac{\left(4ia^2e^{(2idx+2ic)}-\sqrt{\frac{16ia^4}{d^2}}\left(de^{(2idx+2ic)}-d\right)\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\right)e^{(-2idx-2ic)}}{2a^2}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(8*a^2*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)) - s
qrt(16*I*a^4/d^2)*d*log(1/2*(4*I*a^2*e^(2*I*d*x + 2*I*c) + sqrt(16*I*a^4/d^
2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x
```

```
 + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)/a^2) + sqrt(16*I*a^4/d^2)*d*log(1/2*(
4*I*a^2*e^(2*I*d*x + 2*I*c) - sqrt(16*I*a^4/d^2)*(d*e^(2*I*d*x + 2*I*c) - d
)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x
- 2*I*c)/a^2))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int \tan^2(c+dx)\cot^{\frac{3}{2}}(c+dx)\,dx + \int \left(-2i\tan(c+dx)\cot^{\frac{3}{2}}(c+dx)\right)dx + \int \left(-\cot^{\frac{3}{2}}(c+dx)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(3/2)*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -a**2*(Integral(tan(c + d*x)**2*cot(c + d*x)**(3/2), x) + Integral(-2*I*tan
(c + d*x)*cot(c + d*x)**(3/2), x) + Integral(-cot(c + d*x)**(3/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^2*cot(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cot(c+d\,x)^{3/2}\,(a+a\tan(c+d\,x)\;1\mathrm{i})^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] int(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^2, x)
```

3.727 $\int \sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=49

$$\frac{4(-1)^{3/4}a^2\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}-\frac{2a^2}{d\sqrt{\cot(c+dx)}}$$

```
[Out] 4*(-1)^(3/4)*a^2*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/d-2*a^2/d/cot(d*x+c)^
(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3754, 3623, 3614, 214}

$$\frac{4(-1)^{3/4}a^2\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}-\frac{2a^2}{d\sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (4*(-1)^(3/4)*a^2*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/d - (2*a^2)/(d*Sq
rt[Cot[c + d*x]])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^2\,dx &= \int \frac{(ia+a\cot(c+dx))^2}{\cot^{\frac{3}{2}}(c+dx)}\,dx\\ &= -\frac{2a^2}{d\sqrt{\cot(c+dx)}}+\int \frac{2ia^2+2a^2\cot(c+dx)}{\sqrt{\cot(c+dx)}}\,dx\\ &= -\frac{2a^2}{d\sqrt{\cot(c+dx)}}-\frac{(8a^4)\,\mathrm{Subst}\left(\int \frac{1}{-2ia^2+2a^2x^2}\,dx,x,\sqrt{\cot(c+\right.}{d}\\ &= \frac{4(-1)^{3/4}a^2\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}-\frac{2a^2}{d\sqrt{\cot(c+dx)}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.30, size = 83, normalized size = 1.69

$$\frac{2a^2\cot^{\frac{3}{2}}(c+dx)\left(-2\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\right)+\sqrt{i\tan(c+dx)}\right)(i\tan(c+dx))^{3/2}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (2*a^2*Cot[c + d*x]^(3/2)*(-2*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 +
E^((2*I)*(c + d*x)))]] + Sqrt[I*Tan[c + d*x]])*(I*Tan[c + d*x])^(3/2))/d
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 25.82, size = 426, normalized size = 8.69

method	result
default	$\frac{a^2\sqrt{\frac{\cos(dx+c)}{\sin(dx+c)}}\,(\cos(dx+c)+1)^2(-1+\cos(dx+c))\left(2i\sqrt{-\frac{\cos(dx+c)-1-\sin(dx+c)}{\sin(dx+c)}}\sqrt{\frac{\cos(dx+c)-1+\sin(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-1+\cos(d}{\sin(dx+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/d*(cos(d*x+c)/sin(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))*(2*I*s
in(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin
(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(
cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-2*(-(cos(
d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c
))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(
d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)+2*(-(cos(d*x+c)
-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/
2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))
/sin(d*x+c))^(1/2),1/2*2^(1/2))*sin(d*x+c)-cos(d*x+c)*2^(1/2)+2^(1/2))/cos(
d*x+c)/sin(d*x+c)^3*2^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.49, size = 131, normalized size = 2.67

$$-\frac{\left((2i+2)\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+(2i+2)\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+(i-1)\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-(i-1)\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)\right)a^2+4a^2\sqrt{\tan(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(((2*I + 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c))
)) + (2*I + 2)*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))
) + (I - 1)*sqrt(2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) -
(I - 1)*sqrt(2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a^2
+ 4*a^2*sqrt(tan(d*x + c)))/d
```

Fricas [C] Result contains complex when optimal does not.
time = 0.92, size = 290, normalized size = 5.92

$$\frac{\sqrt{-\frac{16i a^4}{d^2}}\left(de^{(2i dx+2i c)}+d\right)\log\left(\frac{\left(4i a^2 e^{(2i dx+2i c)}+\sqrt{-\frac{16i a^4}{d^2}}\left(i de^{(2i dx+2i c)}-i d\right)\sqrt{\frac{i e^{(2i dx+2i c)}+i}{e^{(2i dx+2i c)}-1}}\right)e^{(-2i dx-2i c)}}{2a^2}\right)-\sqrt{-\frac{16i a^4}{d^2}}\left(de^{(2i dx+2i c)}+d\right)\log\left(\frac{\left(4i a^2 e^{(2i dx+2i c)}+\sqrt{-\frac{16i a^4}{d^2}}\left(-i de^{(2i dx+2i c)}+i d\right)\sqrt{\frac{i e^{(2i dx+2i c)}+i}{e^{(2i dx+2i c)}-1}}\right)e^{(-2i dx-2i c)}}{2a^2}\right)-8\left(-i a^2 e^{(2i dx+2i c)}+i a^2\right)\sqrt{\frac{i e^{(2i dx+2i c)}+i}{e^{(2i dx+2i c)}-1}}}{4\left(de^{(2i dx+2i c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(-16*I*a^4/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(1/2*(4*I*a^2*e^(2*
I*d*x + 2*I*c) + sqrt(-16*I*a^4/d^2)*(I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt((
I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)
/a^2) - sqrt(-16*I*a^4/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(1/2*(4*I*a^2*e^
(2*I*d*x + 2*I*c) + sqrt(-16*I*a^4/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) + I*d)*sq
rt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*
I*c)/a^2) - 8*(-I*a^2*e^(2*I*d*x + 2*I*c) + I*a^2)*sqrt((I*e^(2*I*d*x + 2*I
*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int\tan^2(c+dx)\sqrt{\cot(c+dx)}\,dx+\int\left(-2i\tan(c+dx)\sqrt{\cot(c+dx)}\right)dx+\int\left(-\sqrt{\cot(c+dx)}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(1/2)*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -a**2*(Integral(tan(c + d*x)**2*sqrt(cot(c + d*x)), x) + Integral(-2*I*tan(
c + d*x)*sqrt(cot(c + d*x)), x) + Integral(-sqrt(cot(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^2*sqrt(cot(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\cot\left(c+d\,x\right)}\,\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] int(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^2, x)
```

3.728 $\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{\cot(c+dx)}}\,dx$

Optimal. Leaf size=71

$$\frac{4\sqrt[4]{-1}\,a^2\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}-\frac{2a^2}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{4ia^2}{d\sqrt{\cot(c+dx)}}$$

```
[Out] 4*(-1)^(1/4)*a^2*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/d-2/3*a^2/d/cot(d*x+c
)^(3/2)+4*I*a^2/d/cot(d*x+c)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3754, 3623, 3610, 3614, 214}

$$-\frac{2a^2}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{4ia^2}{d\sqrt{\cot(c+dx)}}+\frac{4\sqrt[4]{-1}\,a^2\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^2/Sqrt[Cot[c + d*x]],x]
```

```
[Out] (4*(-1)^(1/4)*a^2*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/d - (2*a^2)/(3*d*
Cot[c + d*x]^(3/2)) + ((4*I)*a^2)/(d*Sqrt[Cot[c + d*x]])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{\cot(c+dx)}}\,dx &= \int \frac{(ia+a\cot(c+dx))^2}{\cot^{\frac{5}{2}}(c+dx)}\,dx \\
&= -\frac{2a^2}{3d\cot^{\frac{3}{2}}(c+dx)}+\int \frac{2ia^2+2a^2\cot(c+dx)}{\cot^{\frac{3}{2}}(c+dx)}\,dx \\
&= -\frac{2a^2}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{4ia^2}{d\sqrt{\cot(c+dx)}}+\int \frac{2a^2-2ia^2\cot(c+dx)}{\sqrt{\cot(c+dx)}}\,dx \\
&= -\frac{2a^2}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{4ia^2}{d\sqrt{\cot(c+dx)}}+\frac{(8a^4)\,\mathrm{Subst}\left(\int \frac{1}{-2a^2-2ia^2x^2}\,dx,x,\sqrt{\cot
\end{aligned}$$

$$= \frac{4\sqrt[4]{-1}\,a^2\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}-\frac{2a^2}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{4ia^2}{d\sqrt{\cot(c+d}}$$

Mathematica [A]
time = 1.43, size = 90, normalized size = 1.27

$$-\frac{2a^2\left(1-6i\cot(c+dx)+6\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\right)\cot^2(c+dx)\sqrt{i\tan(c+dx)}\right)}{3d\cot^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^2/Sqrt[Cot[c + d*x]],x]
```

```
[Out] (-2*a^2*(1 - (6*I)*Cot[c + d*x] + 6*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))
/(1 + E^((2*I)*(c + d*x)))]]*Cot[c + d*x]^2*Sqrt[I*Tan[c + d*x]]))/(3*d*Cot
[c + d*x]^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 26.90, size = 485, normalized size = 6.83

method	result
default	$\frac{a^2(\cos(dx+c)+1)^2(-1+\cos(dx+c))\left(6i\sqrt{-\frac{\cos(dx+c)-1-\sin(dx+c)}{\sin(dx+c)}}\sqrt{\frac{\cos(dx+c)-1+\sin(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\ \mathrm{Ellipti}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^2/cot(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a^2/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))*(6*I*((-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*
x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticPi((-(cos(d*x+c)-1-si
n(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-6*I*((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1
-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticF((-(cos(d*x+c
)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+6*(-(cos(d*x+c)-1-sin(d*x+c)
)/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))
^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)*sin(d*x+c)+6*I*2^(1/2)*cos(d*x+c)^
2-6*I*2^(1/2)*cos(d*x+c)-2^(1/2)*cos(d*x+c)*sin(d*x+c)+2^(1/2)*sin(d*x+c))/
cos(d*x+c)/sin(d*x+c)^4/(cos(d*x+c)/sin(d*x+c))^(1/2)*2^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(57) = 114$.
time = 0.49, size = 146, normalized size = 2.06

$$-\frac{3\left(-(2i-2)\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-(2i-2)\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+(i+1)\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-(i+1)\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)\right)a^2+4\left(a^2-\frac{6ia^2}{\tan(dx+c)}\right)\tan(dx+c)^{\frac{3}{2}}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/cot(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*(3*(-(2*I - 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x +
c)))) - (2*I - 2)*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c
)))) + (I + 1)*sqrt(2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1)
 - (I + 1)*sqrt(2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a
^2 + 4*(a^2 - 6*I*a^2/tan(d*x + c))*tan(d*x + c)^(3/2))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(57) = 114$.

time = 0.68, size = 340, normalized size = 4.79

$$-\frac{3\sqrt{\frac{16i a^4}{d^2}}\left(de^{(4i dx+4i c)}+2de^{(2i dx+2i c)}+d\right)\log\left(\frac{\left(4i a^2e^{(2i dx+2i c)}+\sqrt{\frac{16i a^4}{d^2}}\left(de^{(2i dx+2i c)}-d\right)\sqrt{\frac{i e^{(2i dx+2i c)}+i}{e^{(2i dx+2i c)}-1}}\right)e^{(-2i dx-2i c)}}{2a^2}\right)-3\sqrt{\frac{16i a^4}{d^2}}\left(de^{(4i dx+4i c)}+2de^{(2i dx+2i c)}+d\right)\log\left(\frac{\left(4i a^2e^{(2i dx+2i c)}-\sqrt{\frac{16i a^4}{d^2}}\left(de^{(2i dx+2i c)}-d\right)\sqrt{\frac{i e^{(2i dx+2i c)}+i}{e^{(2i dx+2i c)}-1}}\right)e^{(-2i dx-2i c)}}{2a^2}\right)-8\left(7a^2e^{(4i dx+4i c)}-2a^2e^{(2i dx+2i c)}-5a^2\right)\sqrt{\frac{i e^{(2i dx+2i c)}+i}{e^{(2i dx+2i c)}-1}}}{12\left(de^{(4i dx+4i c)}+2de^{(2i dx+2i c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/cot(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/12*(3*sqrt(16*I*a^4/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c
) + d)*log(1/2*(4*I*a^2*e^(2*I*d*x + 2*I*c) + sqrt(16*I*a^4/d^2)*(d*e^(2*I*
d*x + 2*I*c) - d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1
)))*e^(-2*I*d*x - 2*I*c)/a^2) - 3*sqrt(16*I*a^4/d^2)*(d*e^(4*I*d*x + 4*I*c)
 + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(1/2*(4*I*a^2*e^(2*I*d*x + 2*I*c) - sqrt
(16*I*a^4/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)
/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)/a^2) - 8*(7*a^2*e^(4*I*d*
x + 4*I*c) - 2*a^2*e^(2*I*d*x + 2*I*c) - 5*a^2)*sqrt((I*e^(2*I*d*x + 2*I*c)
 + I)/(e^(2*I*d*x + 2*I*c) - 1)))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x +
 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int\frac{\tan^2(c+dx)}{\sqrt{\cot(c+dx)}}\,dx+\int\left(-\frac{2i\tan(c+dx)}{\sqrt{\cot(c+dx)}}\right)dx+\int\left(-\frac{1}{\sqrt{\cot(c+dx)}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**2/cot(d*x+c)**(1/2),x)
```

```
[Out] -a**2*(Integral(tan(c + d*x)**2/sqrt(cot(c + d*x)), x) + Integral(-2*I*tan(
c + d*x)/sqrt(cot(c + d*x)), x) + Integral(-1/sqrt(cot(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/cot(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^2/sqrt(cot(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2}{\sqrt{\mathrm{cot}\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^2/cot(c + d*x)^(1/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^2/cot(c + d*x)^(1/2), x)
```

3.729 $\int \frac{(a+ia\tan(c+dx))^2}{\cot^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=91

$$-\frac{4(-1)^{3/4}a^2\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}-\frac{2a^2}{5d\cot^{\frac{5}{2}}(c+dx)}+\frac{4ia^2}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{4a^2}{d\sqrt{\cot(c+dx)}}$$

```
[Out] -4*(-1)^(3/4)*a^2*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/d-2/5*a^2/d/cot(d*x+
c)^(5/2)+4/3*I*a^2/d/cot(d*x+c)^(3/2)+4*a^2/d/cot(d*x+c)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3754, 3623, 3610, 3614, 214}

$$\frac{4ia^2}{3d\cot^{\frac{3}{2}}(c+dx)}-\frac{2a^2}{5d\cot^{\frac{5}{2}}(c+dx)}+\frac{4a^2}{d\sqrt{\cot(c+dx)}}-\frac{4(-1)^{3/4}a^2\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^2/Cot[c + d*x]^(3/2),x]
```

```
[Out] (-4*(-1)^(3/4)*a^2*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/d - (2*a^2)/(5*d
*Cot[c + d*x]^(5/2)) + (((4*I)/3)*a^2)/(d*Cot[c + d*x]^(3/2)) + (4*a^2)/(d*
Sqrt[Cot[c + d*x]])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+ia\tan(c+dx))^2}{\cot^{\frac{3}{2}}(c+dx)}\,dx &= \int \frac{(ia+a\cot(c+dx))^2}{\cot^{\frac{7}{2}}(c+dx)}\,dx \\
&= -\frac{2a^2}{5d\cot^{\frac{5}{2}}(c+dx)} + \int \frac{2ia^2+2a^2\cot(c+dx)}{\cot^{\frac{5}{2}}(c+dx)}\,dx \\
&= -\frac{2a^2}{5d\cot^{\frac{5}{2}}(c+dx)} + \frac{4ia^2}{3d\cot^{\frac{3}{2}}(c+dx)} + \int \frac{2a^2-2ia^2\cot(c+dx)}{\cot^{\frac{3}{2}}(c+dx)}\,dx \\
&= -\frac{2a^2}{5d\cot^{\frac{5}{2}}(c+dx)} + \frac{4ia^2}{3d\cot^{\frac{3}{2}}(c+dx)} + \frac{4a^2}{d\sqrt{\cot(c+dx)}} + \int \frac{-2ia^2-2a^2\text{co}}{\sqrt{\cot(c+}} \\
&= -\frac{2a^2}{5d\cot^{\frac{5}{2}}(c+dx)} + \frac{4ia^2}{3d\cot^{\frac{3}{2}}(c+dx)} + \frac{4a^2}{d\sqrt{\cot(c+dx)}} - \frac{(8a^4)\,\text{Subst}\Big(\int \frac{}{2ia}}{} \\
&= -\frac{4(-1)^{3/4}a^2\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d} - \frac{2a^2}{5d\cot^{\frac{5}{2}}(c+dx)} + \frac{4ia}{3d\cot^{\frac{3}{2}}(}
\end{aligned}$$

Mathematica [A]

time = 3.35, size = 152, normalized size = 1.67

$$\frac{a^2e^{-2ic}(-i\cos(2(c+dx))+\sin(2(c+dx)))\left(2i\sec^2(c+dx)(27+33\cos(2(c+dx))+10i\sin(2(c+dx)))-\frac{120i\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\right)}{\sqrt{i\tan(c+dx)}}\right)}{30d\sqrt{\cot(c+dx)}\,(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^2/Cot[c + d*x]^(3/2),x]
```

```
[Out] (a^2*((-I)*Cos[2*(c + d*x)] + Sin[2*(c + d*x)])*((2*I)*Sec[c + d*x]^2*(27 +
 33*Cos[2*(c + d*x)] + (10*I)*Sin[2*(c + d*x)]) - ((120*I)*ArcTanh[Sqrt[(-1
 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])/Sqrt[I*Tan[c + d*x]]))
/(30*d*E^((2*I)*c)*Sqrt[Cot[c + d*x]]*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 27.30, size = 517, normalized size = 5.68

method	result
default	$\frac{a^2(-1+\cos(dx+c))\left(-30i\sin(dx+c)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\sqrt{\frac{\cos(dx+c)-1+\sin(dx+c)}{\sin(dx+c)}}\sqrt{-\frac{\cos(dx+c)-1-\sin(dx+c)}{\sin(dx+c)}}\,(\cos^2(dx+c$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^2/cot(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*a^2/d*(-1+cos(d*x+c))*(-30*I*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c
))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/si
n(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+30*sin(d*x+c)*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-s
in(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-(cos(d*x+c)-1-sin(d*
x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-30*sin(d*x+c)*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*
x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticF((-(cos(d*x+c)-1
-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+10*I*cos(d*x+c)^2*sin(d*x+c)*2^
(1/2)+33*cos(d*x+c)^3*2^(1/2)-10*I*cos(d*x+c)*sin(d*x+c)*2^(1/2)-33*2^(1/2)
*cos(d*x+c)^2-3*cos(d*x+c)*2^(1/2)+3*2^(1/2))*(cos(d*x+c)+1)^2/(cos(d*x+c)/
sin(d*x+c))^(3/2)/sin(d*x+c)^5/cos(d*x+c)*2^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(73) = 146.
time = 0.50, size = 161, normalized size = 1.77

$$-\frac{4\left(3a^2-\frac{10ia^2}{\tan(dx+c)}-\frac{30a^2}{\tan(dx+c)^2}\right)\tan(dx+c)^{\frac{5}{2}}-15\left((2i+2)\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+(2i+2)\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+(i-1)\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-(i-1)\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)\right)a^2}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/cot(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/30*(4*(3*a^2 - 10*I*a^2/tan(d*x + c) - 30*a^2/tan(d*x + c)^2)*tan(d*x +
c)^(5/2) - 15*((2*I + 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d
*x + c)))) + (2*I + 2)*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*
```

```
x + c)))) + (I - 1)*sqrt(2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c)
 + 1) - (I - 1)*sqrt(2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) +
1))*a^2)/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(73) = 146$.
time = 0.78, size = 391, normalized size = 4.30

$$-\frac{15 \sqrt{-\frac{16 i a^4}{d^2}} \left(d e^{(6 i d x+6 i c)}+3 d e^{(4 i d x+4 i c)}+3 d e^{(2 i d x+2 i c)}+d\right) \log \left(\frac{\left(4 i a^2 e^{(2 i d x+2 i c)}+\sqrt{-\frac{16 i a^4}{d^2}} \left(i d e^{(2 i d x+2 i c)}-i d\right) \sqrt{\frac{i e^{(2 i d x+2 i c)}+i}{e^{(2 i d x+2 i c)}-1}}\right) e^{(-2 i d x-2 i c)}}{2 a^2}\right)-15 \sqrt{-\frac{16 i a^4}{d^2}} \left(d e^{(6 i d x+6 i c)}+3 d e^{(4 i d x+4 i c)}+3 d e^{(2 i d x+2 i c)}+d\right) \log \left(\frac{\left(4 i a^2 e^{(2 i d x+2 i c)}+\sqrt{-\frac{16 i a^4}{d^2}} \left(-i d e^{(2 i d x+2 i c)}+i d\right) \sqrt{\frac{i e^{(2 i d x+2 i c)}+i}{e^{(2 i d x+2 i c)}-1}}\right) e^{(-2 i d x-2 i c)}}{2 a^2}\right)+8 \left(43 i a^2 e^{(6 i d x+6 i c)}+11 i a^2 e^{(4 i d x+4 i c)}-31 i a^2 e^{(2 i d x+2 i c)}-23 i a^2\right) \sqrt{\frac{i e^{(2 i d x+2 i c)}+i}{e^{(2 i d x+2 i c)}-1}}}{60 \left(d e^{(6 i d x+6 i c)}+3 d e^{(4 i d x+4 i c)}+3 d e^{(2 i d x+2 i c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/cot(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/60*(15*sqrt(-16*I*a^4/d^2)*(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I
*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)*log(1/2*(4*I*a^2*e^(2*I*d*x + 2*I*c) + s
qrt(-16*I*a^4/d^2)*(I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt((I*e^(2*I*d*x + 2*I
*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)/a^2) - 15*sqrt(-1
6*I*a^4/d^2)*(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*
d*x + 2*I*c) + d)*log(1/2*(4*I*a^2*e^(2*I*d*x + 2*I*c) + sqrt(-16*I*a^4/d^2
)*(-I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I
*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)/a^2) + 8*(43*I*a^2*e^(6*I*d*x + 6
*I*c) + 11*I*a^2*e^(4*I*d*x + 4*I*c) - 31*I*a^2*e^(2*I*d*x + 2*I*c) - 23*I*
a^2)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))/(d*e^(6*I
*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2 \left(\int \frac{\tan^2(c+dx)}{\cot^{\frac{3}{2}}(c+dx)}\, dx + \int \left(-\frac{2i \tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)}\right) dx + \int \left(-\frac{1}{\cot^{\frac{3}{2}}(c+dx)}\right) dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**2/cot(d*x+c)**(3/2),x)
```

```
[Out] -a**2*(Integral(tan(c + d*x)**2/cot(c + d*x)**(3/2), x) + Integral(-2*I*tan
(c + d*x)/cot(c + d*x)**(3/2), x) + Integral(-1/cot(c + d*x)**(3/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/cot(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^2/cot(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\tan(c+d\,x)\;1\mathrm{i})^2}{\cot\left(c+d\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^2/cot(c + d*x)^(3/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^2/cot(c + d*x)^(3/2), x)
```

3.730 $\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=106

$$\frac{8\sqrt[4]{-1}\,a^3\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}+\frac{8a^3\sqrt{\cot(c+dx)}}{d}-\frac{8ia^3\cot^{\frac{3}{2}}(c+dx)}{5d}-\frac{2\cot^{\frac{3}{2}}(c+dx)\,(ia^3+}{5d}$$

```
[Out] 8*(-1)^(1/4)*a^3*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/d-8/5*I*a^3*cot(d*x+c
)^(3/2)/d-2/5*cot(d*x+c)^(3/2)*(I*a^3+a^3*cot(d*x+c))/d+8*a^3*cot(d*x+c)^(1
/2)/d
```

Rubi [A]
time = 0.15, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3754, 3637, 3673, 3609, 3614, 214}

$$-\frac{8ia^3\cot^{\frac{3}{2}}(c+dx)}{5d}-\frac{2\cot^{\frac{3}{2}}(c+dx)\,(a^3\cot(c+dx)+ia^3)}{5d}+\frac{8a^3\sqrt{\cot(c+dx)}}{d}+\frac{8\sqrt[4]{-1}\,a^3\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(7/2)*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (8*(-1)^(1/4)*a^3*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/d + (8*a^3*Sqrt[C
ot[c + d*x]])/d - (((8*I)/5)*a^3*Cot[c + d*x]^(3/2))/d - (2*Cot[c + d*x]^(3
/2)*(I*a^3 + a^3*Cot[c + d*x]))/(5*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^3\,dx = \int \sqrt{\cot(c+dx)}\,(ia+a\cot(c+dx))^3\,dx$$

$$= -\frac{2\cot^{\frac{3}{2}}(c+dx)\left(ia^3+a^3\cot(c+dx)\right)}{5d} - \frac{1}{5}(2ia)\int \sqrt{\cot(c+dx)}$$

$$= -\frac{8ia^3\cot^{\frac{3}{2}}(c+dx)}{5d} - \frac{2\cot^{\frac{3}{2}}(c+dx)\left(ia^3+a^3\cot(c+dx)\right)}{5d} - \frac{1}{5}(2$$

$$= \frac{8a^3\sqrt{\cot(c+dx)}}{d} - \frac{8ia^3\cot^{\frac{3}{2}}(c+dx)}{5d} - \frac{2\cot^{\frac{3}{2}}(c+dx)\left(ia^3+a^3\right.}{5d}$$

$$= \frac{8a^3\sqrt{\cot(c+dx)}}{d} - \frac{8ia^3\cot^{\frac{3}{2}}(c+dx)}{5d} - \frac{2\cot^{\frac{3}{2}}(c+dx)\left(ia^3+a^3\right.}{5d}$$

$$= \frac{8\sqrt[4]{-1}\,a^3\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d} + \frac{8a^3\sqrt{\cot(c+dx)}}{d} -$$

Mathematica [A]

time = 2.21, size = 147, normalized size = 1.39

$$-\frac{a^3e^{-3ic}\sqrt{\cot(c+dx)}\,(\cos(3(c+dx))+i\sin(3(c+dx)))\left(\csc^2(c+dx)(-19+21\cos(2(c+dx))+5i\sin(2(c+dx)))+40\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\right)\sqrt{i\tan(c+dx)}\right)}{5d(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(7/2)*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] -1/5*(a^3*Sqrt[Cot[c + d*x]]*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)])*(Csc[c
 + d*x]^2*(-19 + 21*Cos[2*(c + d*x)] + (5*I)*Sin[2*(c + d*x)]) + 40*ArcTanh
[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]]*Sqrt[I*Tan[c +
 d*x]]))/(d*E^((3*I)*c)*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 14.35, size = 1482, normalized size = 13.98

method	result	size
default	Expression too large to display	1482

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*a^3/d*(-20*I*cos(d*x+c)^3*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2
)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))
^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1
/2*2^(1/2))+20*I*cos(d*x+c)^3*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)
*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^
(1/2)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-
20*cos(d*x+c)^3*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-
1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Elliptic
Pi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-20*
I*cos(d*x+c)^2*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1
+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticP
i((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+20*I
*cos(d*x+c)^2*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+
sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(
(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-20*cos(d*x+c)^2*
(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/si
n(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)
-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+20*I*cos(d*x+c)*(-(
cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-
sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-20*I*cos(d*x+c)*(-(cos
(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(cos(d*x+c)-1-sin(
```

```
d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+5*I*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+
20*cos(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+
sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi
((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+20*I*
(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/si
n(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)
-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-20*I*(-(cos(d*x+c)-
1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2
)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/
sin(d*x+c))^(1/2),1/2*2^(1/2))+21*cos(d*x+c)^3*2^(1/2)+20*(-(cos(d*x+c)-1-s
in(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/si
n(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-20*cos(d*x+c)*2^(1/2))*sin(d*x+c)*(c
os(d*x+c)/sin(d*x+c))^(7/2)/cos(d*x+c)^4*2^(1/2)
```

Maxima [**A**]
time = 0.49, size = 158, normalized size = 1.49

$$\frac{5\left((2i-2)\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+(2i-2)\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-(i+1)\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+(i+1)\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)\right)a^3+\frac{40a^3}{\sqrt{\tan(dx+c)}}-\frac{10ia^3}{\tan(dx+c)^{\frac{3}{2}}}-\frac{2a^3}{\tan(dx+c)^{\frac{5}{2}}}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/5*(5*((2*I - 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)
))) + (2*I - 2)*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c))
)) - (I + 1)*sqrt(2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) +
 (I + 1)*sqrt(2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a^3
 + 40*a^3/sqrt(tan(d*x + c)) - 10*I*a^3/tan(d*x + c)^(3/2) - 2*a^3/tan(d*x
+ c)^(5/2))/d
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(86) = 172$.
time = 0.65, size = 340, normalized size = 3.21

$$-\frac{5\sqrt{\frac{64ia^6}{d^2}}\left(de^{(4idx+4ic)}-2de^{(2idx+2ic)}+d\right)\log\left(\frac{\left(8ia^3e^{(2idx+2ic)}+\sqrt{\frac{64ia^6}{d^2}}\left(de^{(2idx+2ic)}-d\right)\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\right)e^{(-2idx-2ic)}}{4a^3}\right)-5\sqrt{\frac{64ia^6}{d^2}}\left(de^{(4idx+4ic)}-2de^{(2idx+2ic)}+d\right)\log\left(\frac{\left(8ia^3e^{(2idx+2ic)}-\sqrt{\frac{64ia^6}{d^2}}\left(de^{(2idx+2ic)}-d\right)\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\right)e^{(-2idx-2ic)}}{4a^3}\right)-16\left(13a^3e^{(4idx+4ic)}-19a^3e^{(2idx+2ic)}+8a^3\right)\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}}{20\left(de^{(4idx+4ic)}-2de^{(2idx+2ic)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/20*(5*sqrt(64*I*a^6/d^2)*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c
) + d)*log(1/4*(8*I*a^3*e^(2*I*d*x + 2*I*c) + sqrt(64*I*a^6/d^2)*(d*e^(2*I*
d*x + 2*I*c) - d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1
)))*e^(-2*I*d*x - 2*I*c)/a^3) - 5*sqrt(64*I*a^6/d^2)*(d*e^(4*I*d*x + 4*I*c)
 - 2*d*e^(2*I*d*x + 2*I*c) + d)*log(1/4*(8*I*a^3*e^(2*I*d*x + 2*I*c) - sqrt
```

```
(64*I*a^6/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)
/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)/a^3) - 16*(13*a^3*e^(4*I*
d*x + 4*I*c) - 19*a^3*e^(2*I*d*x + 2*I*c) + 8*a^3)*sqrt((I*e^(2*I*d*x + 2*I
*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))/(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*
x + 2*I*c) + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(7/2)*(a+I*a*tan(d*x+c))**3,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5987 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^3*cot(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^{7/2}\,(a+a\tan(c+d\,x)\ 1\mathrm{i})^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(7/2)*(a + a*tan(c + d*x)*1i)^3,x)
[Out] int(cot(c + d*x)^(7/2)*(a + a*tan(c + d*x)*1i)^3, x)
```

3.731 $\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=88

$$-\frac{8(-1)^{3/4}a^3\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}-\frac{16ia^3\sqrt{\cot(c+dx)}}{3d}-\frac{2\sqrt{\cot(c+dx)}\,(ia^3+a^3\cot(c+dx))}{3d}$$

```
[Out] -8*(-1)^(3/4)*a^3*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/d-16/3*I*a^3*cot(d*x
+c)^(1/2)/d-2/3*(I*a^3+a^3*cot(d*x+c))*cot(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.12, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3754, 3637, 3673, 3614, 214}

$$-\frac{16ia^3\sqrt{\cot(c+dx)}}{3d}-\frac{2\sqrt{\cot(c+dx)}\,(a^3\cot(c+dx)+ia^3)}{3d}-\frac{8(-1)^{3/4}a^3\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (-8*(-1)^(3/4)*a^3*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/d - (((16*I)/3)*
a^3*Sqrt[Cot[c + d*x]])/d - (2*Sqrt[Cot[c + d*x]]*(I*a^3 + a^3*Cot[c + d*x]
))/(3*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^3\,dx &= \int \frac{(ia+a\cot(c+dx))^3}{\sqrt{\cot(c+dx)}}\,dx\\ &= -\frac{2\sqrt{\cot(c+dx)}\,(ia^3+a^3\cot(c+dx))}{3d}-\frac{1}{3}(2ia)\int \frac{(-2ia-4a\\ &= -\frac{16ia^3\sqrt{\cot(c+dx)}}{3d}-\frac{2\sqrt{\cot(c+dx)}\,(ia^3+a^3\cot(c+dx))}{3d}\\ &= -\frac{16ia^3\sqrt{\cot(c+dx)}}{3d}-\frac{2\sqrt{\cot(c+dx)}\,(ia^3+a^3\cot(c+dx))}{3d}\\ &= -\frac{8(-1)^{3/4}a^3\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}-\frac{16ia^3\sqrt{\cot(c+}{3d}\end{aligned}$$

Mathematica [A]
time = 1.80, size = 125, normalized size = 1.42

$$-\frac{2a^3e^{-3ic}\sqrt{\cot(c+dx)}\,(\cos(3(c+dx))+i\sin(3(c+dx)))\left(9i+\cot(c+dx)-12i\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\right)\sqrt{i\tan(c+dx)}\right)}{3d(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (-2*a^3*Sqrt[Cot[c + d*x]]*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)])*(9*I + C
ot[c + d*x] - (12*I)*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*
```

```
(c + d*x))]]*Sqrt[I*Tan[c + d*x]]))/(3*d*E^((3*I)*c)*(Cos[d*x] + I*Sin[d*x
])^3)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 14.08, size = 790, normalized size = 8.98

method	result
default	$-\frac{a^3\left(-12i\sqrt{-\frac{\cos(dx+c)-1-\sin(dx+c)}{\sin(dx+c)}}\,\sqrt{\frac{\cos(dx+c)-1+\sin(dx+c)}{\sin(dx+c)}}\,\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{\cos(dx+c)-1-\sin}{\sin(dx+c)}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a^3/d*(-12*I*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c
)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellipt
icPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*c
os(d*x+c)*sin(d*x+c)-12*I*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((c
os(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^
(1/2))*sin(d*x+c)+12*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*
x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ell
ipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2)
)*cos(d*x+c)*sin(d*x+c)-12*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((
cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos
(d*x+c)*sin(d*x+c)+12*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d
*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*El
lipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2
))*sin(d*x+c)-12*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)
-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellipti
cF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*sin(d*x+c)+9*
I*2^(1/2)*cos(d*x+c)*sin(d*x+c)+2^(1/2)*cos(d*x+c)^2)*(cos(d*x+c)/sin(d*x+c
))^(5/2)*sin(d*x+c)/cos(d*x+c)^3*2^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(70) = 140.
time = 0.48, size = 145, normalized size = 1.65

$$\frac{3\left((2i+2)\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+(2i+2)\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+(i-1)\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-(i-1)\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)\right)a^3-\frac{18ia^3}{\sqrt{\tan(dx+c)}}-\frac{2a^3}{\tan(dx+c)^{\frac{3}{2}}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*(3*((2*I + 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)
))) + (2*I + 2)*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c))
```

```
)) + (I - 1)*sqrt(2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) -
 (I - 1)*sqrt(2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a^3
 - 18*I*a^3/sqrt(tan(d*x + c)) - 2*a^3/tan(d*x + c)^(3/2))/d
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 297 vs. 2(70) = 140.
time = 0.61, size = 297, normalized size = 3.38

$$-\frac{3\sqrt{-\frac{64i\,a^{6}}{d^{2}}}\left(de^{(2i\,dx+2i\,c)}-d\right)\log\left(\frac{\left(8i\,a^{3}e^{(2i\,dx+2i\,c)}+\sqrt{-\frac{64i\,a^{6}}{d^{2}}}\left(i\,de^{(2i\,dx+2i\,c)}-i\,d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\right)e^{(-2i\,dx-2i\,c)}}{4\,a^{3}}\right)-3\sqrt{-\frac{64i\,a^{6}}{d^{2}}}\left(de^{(2i\,dx+2i\,c)}-d\right)\log\left(\frac{\left(8i\,a^{3}e^{(2i\,dx+2i\,c)}+\sqrt{-\frac{64i\,a^{6}}{d^{2}}}\left(-i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\right)e^{(-2i\,dx-2i\,c)}}{4\,a^{3}}\right)+16\left(5i\,a^{3}e^{(2i\,dx+2i\,c)}-4i\,a^{3}\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}}{12\left(de^{(2i\,dx+2i\,c)}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/12*(3*sqrt(-64*I*a^6/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*log(1/4*(8*I*a^3*e
^(2*I*d*x + 2*I*c) + sqrt(-64*I*a^6/d^2)*(I*d*e^(2*I*d*x + 2*I*c) - I*d)*sq
rt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*
I*c)/a^3) - 3*sqrt(-64*I*a^6/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*log(1/4*(8*I*
a^3*e^(2*I*d*x + 2*I*c) + sqrt(-64*I*a^6/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) + I
*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*
x - 2*I*c)/a^3) + 16*(5*I*a^3*e^(2*I*d*x + 2*I*c) - 4*I*a^3)*sqrt((I*e^(2*I
*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))/(d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(5/2)*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^3*cot(d*x + c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^{5/2}\,\left(a+a\,\tan(c+d\,x)\,1{}\mathrm{i}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^3,x)
[Out] int(cot(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^3, x)
```

3.732 $\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=64

$$-\frac{8\sqrt[4]{-1}\,a^3\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}-\frac{2(ia^3+a^3\cot(c+dx))}{d\sqrt{\cot(c+dx)}}$$

```
[Out] -8*(-1)^(1/4)*a^3*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/d-2*(I*a^3+a^3*cot(d
*x+c))/d/cot(d*x+c)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3754, 3634, 12, 3614, 214}

$$-\frac{8\sqrt[4]{-1}\,a^3\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}-\frac{2(a^3\cot(c+dx)+ia^3)}{d\sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (-8*(-1)^(1/4)*a^3*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/d - (2*(I*a^3 +
a^3*Cot[c + d*x]))/(d*Sqrt[Cot[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3634

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-a^2)*(b*c - a*d)*(a + b*Tan[e + f*x]
)^(m - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] + Di
```

```
st[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[b*(b*c*(m - 2) - a*d*(m - 2*n - 4)) + (a*b*c*(m - 2)
+ b^2*d*(n + 1) - a^2*d*(m + n - 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3\,dx &= \int \frac{(ia+a\cot(c+dx))^3}{\cot^{\frac{3}{2}}(c+dx)}\,dx\\ &= -\frac{2(ia^3+a^3\cot(c+dx))}{d\sqrt{\cot(c+dx)}} - 2\int -\frac{2ia^2(ia+a\cot(c+dx))}{\sqrt{\cot(c+dx)}}\,dx\\ &= -\frac{2(ia^3+a^3\cot(c+dx))}{d\sqrt{\cot(c+dx)}} + (4ia^2)\int \frac{ia+a\cot(c+dx)}{\sqrt{\cot(c+dx)}}\,dx\\ &= -\frac{2(ia^3+a^3\cot(c+dx))}{d\sqrt{\cot(c+dx)}} - \frac{(8ia^4)\,\mathrm{Subst}\left(\int \frac{1}{-ia+ax^2}\,dx, x, \sqrt{\cot(c}\right.}{d}\\ &= -\frac{8\sqrt[4]{-1}\,a^3\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d} - \frac{2(ia^3+a^3\cot(c+}{d\sqrt{\cot(c+dx}}\end{aligned}$$

Mathematica [A]
time = 1.89, size = 125, normalized size = 1.95

$$\frac{2a^3e^{-3ic}(\cos(3(c+dx))+i\sin(3(c+dx)))\left(-i-\cot(c+dx)+\frac{4i\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\right)}{\sqrt{i\tan(c+dx)}}\right)}{d\sqrt{\cot(c+dx)}\,(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (2*a^3*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)])*(-I - Cot[c + d*x] + ((4*I)*
ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])/Sqrt[I
*Tan[c + d*x]]))/(d*E^((3*I)*c)*Sqrt[Cot[c + d*x]]*(Cos[d*x] + I*Sin[d*x])^
3)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 30.18, size = 748, normalized size = 11.69

method	result
default	$-\frac{a^{3}\left(4i\cos(dx+c)\sqrt{-\frac{\cos(dx+c)-1-\sin(dx+c)}{\sin(dx+c)}}\sqrt{\frac{\cos(dx+c)-1+\sin(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\ \text{EllipticF}\left(\sqrt{-\frac{\cos(dx+c}{\sin}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -a^3/d*(4*I*cos(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(
d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*E
llipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-4*I*cos
(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*
x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(co
s(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-4*cos(d*x+c
)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/
sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+
c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+4*I*(-(cos(d*x+c)
-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/
2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))
/sin(d*x+c))^(1/2),1/2*2^(1/2))-4*I*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))
^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/
2*I,1/2*2^(1/2))-4*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+
c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellip
ticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+
I*sin(d*x+c)*2^(1/2)+cos(d*x+c)*2^(1/2))*sin(d*x+c)*(cos(d*x+c)/sin(d*x+c))
^(3/2)/cos(d*x+c)^2*2^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(54) = 108$.
time = 0.49, size = 144, normalized size = 2.25

$$-\frac{\left(\left(2i-2\right)\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+\left(2i-2\right)\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)-\left(i+1\right)\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)+\left(i+1\right)\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)\right)a^{3}+2i\,a^{3}\sqrt{\tan\left(dx+c\right)}+\frac{2a^{3}}{\sqrt{\tan\left(dx+c\right)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -(((2*I - 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) +
 (2*I - 2)*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) -
(I + 1)*sqrt(2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + (I +
 1)*sqrt(2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a^3 + 2*
I*a^3*sqrt(tan(d*x + c)) + 2*a^3/sqrt(tan(d*x + c)))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(54) = 108$.
time = 0.52, size = 281, normalized size = 4.39

$$-\frac{16\,a^{3}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\,e^{(2i\,dx+2i\,c)}-\sqrt{\frac{64i\,a^{6}}{d^{2}}}\left(de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{\left(8i\,a^{3}e^{(2i\,dx+2i\,c)}+\sqrt{\frac{64i\,a^{6}}{d^{2}}}\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\right)e^{(-2i\,dx-2i\,c)}}{4\,a^{3}}\right)+\sqrt{\frac{64i\,a^{6}}{d^{2}}}\left(de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{\left(8i\,a^{3}e^{(2i\,dx+2i\,c)}-\sqrt{\frac{64i\,a^{6}}{d^{2}}}\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\right)e^{(-2i\,dx-2i\,c)}}{4\,a^{3}}\right)}{4\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(16*a^3*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*e^
(2*I*d*x + 2*I*c) - sqrt(64*I*a^6/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(1/4*
(8*I*a^3*e^(2*I*d*x + 2*I*c) + sqrt(64*I*a^6/d^2)*(d*e^(2*I*d*x + 2*I*c) -
d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x
 - 2*I*c)/a^3) + sqrt(64*I*a^6/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(1/4*(8*
I*a^3*e^(2*I*d*x + 2*I*c) - sqrt(64*I*a^6/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*
sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x -
2*I*c)/a^3))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia^{3}\left(\int i\cot^{\frac{3}{2}}(c+dx)\,dx+\int\left(-3\tan(c+dx)\cot^{\frac{3}{2}}(c+dx)\right)dx+\int\tan^{3}(c+dx)\cot^{\frac{3}{2}}(c+dx)\,dx+\int\left(-3i\tan^{2}(c+dx)\cot^{\frac{3}{2}}(c+dx)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(3/2)*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] -I*a**3*(Integral(I*cot(c + d*x)**(3/2), x) + Integral(-3*tan(c + d*x)*cot(
c + d*x)**(3/2), x) + Integral(tan(c + d*x)**3*cot(c + d*x)**(3/2), x) + In
tegral(-3*I*tan(c + d*x)**2*cot(c + d*x)**(3/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^3*cot(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cot(c+d\,x)^{3/2}\,\left(a+a\tan(c+d\,x)\;1\mathrm{i}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^3,x)
[Out] int(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^3, x)
```

3.733 $\int \sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=86

$$\frac{8(-1)^{3/4}a^3\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}-\frac{16a^3}{3d\sqrt{\cot(c+dx)}}-\frac{2(ia^3+a^3\cot(c+dx))}{3d\cot^{\frac{3}{2}}(c+dx)}$$

```
[Out] 8*(-1)^(3/4)*a^3*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/d-2/3*(I*a^3+a^3*cot(
d*x+c))/d/cot(d*x+c)^(3/2)-16/3*a^3/d/cot(d*x+c)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3754, 3634, 3672, 3614, 214}

$$-\frac{2(a^3\cot(c+dx)+ia^3)}{3d\cot^{\frac{3}{2}}(c+dx)}-\frac{16a^3}{3d\sqrt{\cot(c+dx)}}+\frac{8(-1)^{3/4}a^3\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (8*(-1)^(3/4)*a^3*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/d - (16*a^3)/(3*d
*Sqrt[Cot[c + d*x]]) - (2*(I*a^3 + a^3*Cot[c + d*x]))/(3*d*Cot[c + d*x]^(3/
2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3634

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-a^2)*(b*c - a*d)*(a + b*Tan[e + f*x]
)^(m - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] + Di
st[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[b*(b*c*(m - 2) - a*d*(m - 2*n - 4)) + (a*b*c*(m - 2)
+ b^2*d*(n + 1) - a^2*d*(m + n - 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
```

```
^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3672

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
 b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
 -1] && NeQ[a^2 + b^2, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^3\,dx &= \int \frac{(ia+a\cot(c+dx))^3}{\cot^{\frac{5}{2}}(c+dx)}\,dx \\
&= -\frac{2(ia^3+a^3\cot(c+dx))}{3d\cot^{\frac{3}{2}}(c+dx)} - \frac{2}{3}\int \frac{(ia+a\cot(c+dx))\,(-4ia^2 -}{\cot^{\frac{3}{2}}(c+dx)} \\
&= -\frac{16a^3}{3d\sqrt{\cot(c+dx)}} - \frac{2(ia^3+a^3\cot(c+dx))}{3d\cot^{\frac{3}{2}}(c+dx)} - \frac{2}{3}\int \frac{-6ia^3 -}{\sqrt{c}} \\
&= -\frac{16a^3}{3d\sqrt{\cot(c+dx)}} - \frac{2(ia^3+a^3\cot(c+dx))}{3d\cot^{\frac{3}{2}}(c+dx)} + \frac{(48a^6)\,\mathrm{Subst}(}{} \\
&= \frac{8(-1)^{3/4}a^3\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d} - \frac{16a^3}{3d\sqrt{\cot(c+d}}
\end{aligned}
$$

Mathematica [A]
time = 2.03, size = 147, normalized size = 1.71

$$
\frac{ia^3e^{-3ic}\sqrt{\cot(c+dx)}\,(\cos(3(c+dx))+i\sin(3(c+dx)))\left(\sec^2(c+dx)(-1+\cos(2(c+dx))+9i\sin(2(c+dx)))-24\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\right)\sqrt{i\tan(c+dx)}\right)}{3d(\cos(dx)+i\sin(dx))^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((I/3)*a^3*Sqrt[Cot[c + d*x]]*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)])*(Sec[
c + d*x]^2*(-1 + Cos[2*(c + d*x)] + (9*I)*Sin[2*(c + d*x)]) - 24*ArcTanh[Sq
rt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]]*Sqrt[I*Tan[c + d*
x]]))/(d*E^((3*I)*c)*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 28.40, size = 485, normalized size = 5.64

method	result
default	$\frac{a^3(-1+\cos(dx+c))\left(12i\sqrt{-\frac{\cos(dx+c)-1-\sin(dx+c)}{\sin(dx+c)}}\sqrt{\frac{\cos(dx+c)-1+\sin(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\ \mathrm{EllipticPi}\left(\sqrt{-\frac{\cos(dx}{}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a^3/d*(-1+cos(d*x+c))*(12*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*
x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)
)^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*
x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-12*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c
))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+
1/2*I,1/2*2^(1/2))*cos(d*x+c)*sin(d*x+c)+12*(-(cos(d*x+c)-1-sin(d*x+c))/sin
(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)
,1/2*2^(1/2))*cos(d*x+c)*sin(d*x+c)-I*2^(1/2)*cos(d*x+c)*sin(d*x+c)+I*sin(d
*x+c)*2^(1/2)-9*2^(1/2)*cos(d*x+c)^2+9*cos(d*x+c)*2^(1/2))*(cos(d*x+c)+1)^2
*(cos(d*x+c)/sin(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)^3*2^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(70) = 140$.
time = 0.50, size = 148, normalized size = 1.72

$$-\frac{3\left(\left(2i+2\right)\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+\left(2i+2\right)\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+\left(i-1\right)\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)-\left(i-1\right)\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)\right)a^3-2\left(-i\,a^3-\frac{9\,a^3}{\tan\left(dx+c\right)}\right)\tan\left(dx+c\right)^{\frac{3}{2}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/3*(3*((2*I + 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c
)))) + (2*I + 2)*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)
))) + (I - 1)*sqrt(2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1)
- (I - 1)*sqrt(2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a^
3 - 2*(-I*a^3 - 9*a^3/tan(d*x + c))*tan(d*x + c)^(3/2))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(70) = 140$.
time = 0.49, size = 341, normalized size = 3.97

$$\frac{3\sqrt{-\frac{64i\,a^6}{d^2}}\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{\left(8i\,a^3e^{(2i\,dx+2i\,c)}+\sqrt{-\frac{64i\,a^6}{d^2}}\left(i\,de^{(2i\,dx+2i\,c)}-i\,d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\right)e^{(-2i\,dx-2i\,c)}}{4\,a^3}\right)-3\sqrt{-\frac{64i\,a^6}{d^2}}\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{\left(8i\,a^3e^{(2i\,dx+2i\,c)}+\sqrt{-\frac{64i\,a^6}{d^2}}\left(-i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\right)e^{(-2i\,dx-2i\,c)}}{4\,a^3}\right)-16\left(-5i\,a^3e^{(4i\,dx+4i\,c)}+i\,a^3e^{(2i\,dx+2i\,c)}+4i\,a^3\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}}{12\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*sqrt(-64*I*a^6/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c
) + d)*log(1/4*(8*I*a^3*e^(2*I*d*x + 2*I*c) + sqrt(-64*I*a^6/d^2)*(I*d*e^(2
*I*d*x + 2*I*c) - I*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c
) - 1)))*e^(-2*I*d*x - 2*I*c)/a^3) - 3*sqrt(-64*I*a^6/d^2)*(d*e^(4*I*d*x +
4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(1/4*(8*I*a^3*e^(2*I*d*x + 2*I*c)
+ sqrt(-64*I*a^6/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt((I*e^(2*I*d*x +
 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)/a^3) - 16*(-5
*I*a^3*e^(4*I*d*x + 4*I*c) + I*a^3*e^(2*I*d*x + 2*I*c) + 4*I*a^3)*sqrt((I*e
^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))/(d*e^(4*I*d*x + 4*I*c)
+ 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia^3\left(\int i\sqrt{\cot(c+dx)}\,dx+\int\left(-3\tan(c+dx)\sqrt{\cot(c+dx)}\right)dx+\int\tan^3(c+dx)\sqrt{\cot(c+dx)}\,dx+\int\left(-3i\tan^2(c+dx)\sqrt{\cot(c+dx)}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(1/2)*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] -I*a**3*(Integral(I*sqrt(cot(c + d*x)), x) + Integral(-3*tan(c + d*x)*sqrt(
cot(c + d*x)), x) + Integral(tan(c + d*x)**3*sqrt(cot(c + d*x)), x) + Integ
ral(-3*I*tan(c + d*x)**2*sqrt(cot(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^3*sqrt(cot(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cot\left(c+d\,x\right)}\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^3,x)
[Out] int(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^3, x)
```

3.734 $\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{\cot(c+dx)}}\,dx$

Optimal. Leaf size=106

$$\frac{8\sqrt[4]{-1}\,a^3\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}-\frac{8a^3}{5d\cot^{\frac{3}{2}}(c+dx)}+\frac{8ia^3}{d\sqrt{\cot(c+dx)}}-\frac{2(ia^3+a^3\cot(c+dx))}{5d\cot^{\frac{5}{2}}(c+dx)}$$

```
[Out] 8*(-1)^(1/4)*a^3*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/d-8/5*a^3/d/cot(d*x+c
)^(3/2)-2/5*(I*a^3+a^3*cot(d*x+c))/d/cot(d*x+c)^(5/2)+8*I*a^3/d/cot(d*x+c)^
(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3754, 3634, 3672, 3610, 3614, 214}

$$-\frac{8a^3}{5d\cot^{\frac{3}{2}}(c+dx)}-\frac{2(a^3\cot(c+dx)+ia^3)}{5d\cot^{\frac{5}{2}}(c+dx)}+\frac{8ia^3}{d\sqrt{\cot(c+dx)}}+\frac{8\sqrt[4]{-1}\,a^3\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^3/Sqrt[Cot[c + d*x]],x]
```

```
[Out] (8*(-1)^(1/4)*a^3*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/d - (8*a^3)/(5*d*
Cot[c + d*x]^(3/2)) + ((8*I)*a^3)/(d*Sqrt[Cot[c + d*x]]) - (2*(I*a^3 + a^3*
Cot[c + d*x]))/(5*d*Cot[c + d*x]^(5/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3634

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-a^2)*(b*c - a*d)*(a + b*Tan[e + f*x]
)^(m - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] + Di
st[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[b*(b*c*(m - 2) - a*d*(m - 2*n - 4)) + (a*b*c*(m - 2)
+ b^2*d*(n + 1) - a^2*d*(m + n - 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3672

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
 b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
 -1] && NeQ[a^2 + b^2, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(ia+a\cot(c+dx))^3}{\cot^{\frac{7}{2}}(c+dx)}\,dx$$
$$= -\frac{2(ia^3+a^3\cot(c+dx))}{5d\cot^{\frac{5}{2}}(c+dx)} - \frac{2}{5}\int \frac{(ia+a\cot(c+dx))\left(-6ia^2-4a^2\cot(c+d\right.}{\cot^{\frac{5}{2}}(c+dx)}$$
$$= -\frac{8a^3}{5d\cot^{\frac{3}{2}}(c+dx)} - \frac{2(ia^3+a^3\cot(c+dx))}{5d\cot^{\frac{5}{2}}(c+dx)} - \frac{2}{5}\int \frac{-10ia^3-10a^3\cot(c+d}{\cot^{\frac{3}{2}}(c+dx)}$$
$$= -\frac{8a^3}{5d\cot^{\frac{3}{2}}(c+dx)} + \frac{8ia^3}{d\sqrt{\cot(c+dx)}} - \frac{2(ia^3+a^3\cot(c+dx))}{5d\cot^{\frac{5}{2}}(c+dx)} - \frac{2}{5}\int \frac{-10c}{}$$
$$= -\frac{8a^3}{5d\cot^{\frac{3}{2}}(c+dx)} + \frac{8ia^3}{d\sqrt{\cot(c+dx)}} - \frac{2(ia^3+a^3\cot(c+dx))}{5d\cot^{\frac{5}{2}}(c+dx)} - \frac{(80a^6)\,\mathrm{Sul}}{}$$
$$= \frac{8\sqrt[4]{-1}\,a^3\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d} - \frac{8a^3}{5d\cot^{\frac{3}{2}}(c+dx)} + \frac{8ia^3}{d\sqrt{\cot(c+}}$$

Mathematica [A]
time = 2.53, size = 164, normalized size = 1.55

$$\frac{a^3e^{-3ic}\sqrt{\cot(c+dx)}\,(\cos(3(c+dx))+i\sin(3(c+dx)))\left(\sec^3(c+dx)(-5\cos(c+dx)+5\cos(3(c+dx))+17i\sin(c+dx)+21i\sin(3(c+dx)))-80\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\right)\sqrt{i\tan(c+dx)}\right)}{10d(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^3/Sqrt[Cot[c + d*x]],x]
```

```
[Out] (a^3*Sqrt[Cot[c + d*x]]*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)])*(Sec[c + d*
x]^3*(-5*Cos[c + d*x] + 5*Cos[3*(c + d*x)] + (17*I)*Sin[c + d*x] + (21*I)*S
in[3*(c + d*x)]) - 80*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)
*(c + d*x)))]]*Sqrt[I*Tan[c + d*x]]))/(10*d*E^((3*I)*c)*(Cos[d*x] + I*Sin[d
*x])^3)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 29.32, size = 520, normalized size = 4.91

method	result
default	$\frac{a^3(-1+\cos(dx+c))\left(20i\sin(dx+c)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\sqrt{\frac{\cos(dx+c)-1+\sin(dx+c)}{\sin(dx+c)}}\sqrt{-\frac{\cos(dx+c)-1-\sin(dx+c)}{\sin(dx+c)}}\,(\cos^2(dx+c\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^3/cot(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a^3/d*(-1+cos(d*x+c))*(20*I*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(
d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((c
os(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d
*x+c))^(1/2)*cos(d*x+c)^2*sin(d*x+c)-20*I*EllipticF((-(cos(d*x+c)-1-sin(d*x
+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((co
s(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*
x+c))^(1/2)*cos(d*x+c)^2*sin(d*x+c)+20*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c
))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c)
)/sin(d*x+c))^(1/2)*cos(d*x+c)^2*sin(d*x+c)+21*I*2^(1/2)*cos(d*x+c)^3-21*I*
2^(1/2)*cos(d*x+c)^2-5*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)-I*2^(1/2)*cos(d*x+c)
+5*2^(1/2)*cos(d*x+c)*sin(d*x+c)+I*2^(1/2))*(cos(d*x+c)+1)^2/(cos(d*x+c)/si
n(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)^4*2^(1/2)
```

Maxima [A]
time = 0.50, size = 161, normalized size = 1.52

$$\frac{5\left(\left(2i-2\right)\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+\left(2i-2\right)\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)-\left(i+1\right)\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)+\left(i+1\right)\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)\right)a^3-2\left(i\,a^3+\frac{5\,a^3}{\tan(dx+c)}-\frac{20i\,a^3}{\tan(dx+c)^2}\right)\tan\left(dx+c\right)^{\frac{5}{2}}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/cot(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*(5*((2*I - 2)*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)
))) + (2*I - 2)*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c))
)) - (I + 1)*sqrt(2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) +
 (I + 1)*sqrt(2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a^3
 - 2*(I*a^3 + 5*a^3/tan(d*x + c) - 20*I*a^3/tan(d*x + c)^2)*tan(d*x + c)^(5
/2))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 390 vs. 2(86) = 172.
time = 0.47, size = 390, normalized size = 3.68

$$-\frac{5\sqrt{\frac{64i\,a^6}{d^2}}\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{\left(8i\,a^3e^{(2i\,dx+2i\,c)}+\sqrt{\frac{64i\,a^6}{d^2}}\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\right)e^{(-2i\,dx-2i\,c)}}{4\,a^3}\right)-5\sqrt{\frac{64i\,a^6}{d^2}}\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{\left(8i\,a^3e^{(2i\,dx+2i\,c)}-\sqrt{\frac{64i\,a^6}{d^2}}\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\right)e^{(-2i\,dx-2i\,c)}}{4\,a^3}\right)-16\left(13\,a^3e^{(6i\,dx+6i\,c)}+6\,a^3e^{(4i\,dx+4i\,c)}-11\,a^3e^{(2i\,dx+2i\,c)}-8\,a^3\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}}{20\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/cot(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/20*(5*sqrt(64*I*a^6/d^2)*(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c
) + 3*d*e^(2*I*d*x + 2*I*c) + d)*log(1/4*(8*I*a^3*e^(2*I*d*x + 2*I*c) + sqr
t(64*I*a^6/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I
)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)/a^3) - 5*sqrt(64*I*a^6/d
^2)*(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I
*c) + d)*log(1/4*(8*I*a^3*e^(2*I*d*x + 2*I*c) - sqrt(64*I*a^6/d^2)*(d*e^(2*
I*d*x + 2*I*c) - d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) -
```

```
 1)))*e^(-2*I*d*x - 2*I*c)/a^3) - 16*(13*a^3*e^(6*I*d*x + 6*I*c) + 6*a^3*e^
(4*I*d*x + 4*I*c) - 11*a^3*e^(2*I*d*x + 2*I*c) - 8*a^3)*sqrt((I*e^(2*I*d*x
+ 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4
*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia^3\left(\int \frac{i}{\sqrt{\cot(c+dx)}}\,dx + \int \left(-\frac{3\tan(c+dx)}{\sqrt{\cot(c+dx)}}\right)dx + \int \frac{\tan^3(c+dx)}{\sqrt{\cot(c+dx)}}\,dx + \int \left(-\frac{3i\tan^2(c+dx)}{\sqrt{\cot(c+dx)}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**3/cot(d*x+c)**(1/2),x)
```

```
[Out] -I*a**3*(Integral(I/sqrt(cot(c + d*x)), x) + Integral(-3*tan(c + d*x)/sqrt(
cot(c + d*x)), x) + Integral(tan(c + d*x)**3/sqrt(cot(c + d*x)), x) + Integ
ral(-3*I*tan(c + d*x)**2/sqrt(cot(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/cot(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^3/sqrt(cot(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\tan(c+d\,x)\;1\mathrm{i})^3}{\sqrt{\cot(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^3/cot(c + d*x)^(1/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^3/cot(c + d*x)^(1/2), x)
```

3.735 $\int \frac{\cot^{\frac{3}{2}}(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=220

$$-\frac{\left(\frac{5}{4}+\frac{3i}{4}\right)\operatorname{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,ad}+\frac{\left(\frac{5}{4}+\frac{3i}{4}\right)\operatorname{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,ad}-\frac{5\sqrt{\cot(c+dx)}}{2ad}-$$

```
[Out] 1/2*cot(d*x+c)^(3/2)/d/(I*a+a*cot(d*x+c))+(5/8+3/8*I)*arctan(-1+2^(1/2)*cot
(d*x+c)^(1/2))/a/d*2^(1/2)+(5/8+3/8*I)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/a
/d*2^(1/2)+(-5/16+3/16*I)*ln(1+cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2))/a/d*2^(
1/2)+(5/16-3/16*I)*ln(1+cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2))/a/d*2^(1/2)-5/
2*cot(d*x+c)^(1/2)/a/d
```

Rubi [A]
time = 0.18, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3754, 3631, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\left(\frac{5}{4}+\frac{3i}{4}\right)\operatorname{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,ad}+\frac{\left(\frac{5}{4}+\frac{3i}{4}\right)\operatorname{ArcTan}\left(\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,ad}+\frac{\cot^{\frac{3}{2}}(c+dx)}{2d(a\cot(c+dx)+ia)}-\frac{5\sqrt{\cot(c+dx)}}{2ad}-\frac{\left(\frac{5}{8}-\frac{3i}{8}\right)\log\left(\cot(c+dx)-\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,ad}+\frac{\left(\frac{5}{8}-\frac{3i}{8}\right)\log\left(\cot(c+dx)+\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(3/2)/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-5/4 - (3*I)/4)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*a*d) + (
(5/4 + (3*I)/4)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*a*d) - (5*
Sqrt[Cot[c + d*x]])/(2*a*d) + Cot[c + d*x]^(3/2)/(2*d*(I*a + a*Cot[c + d*x]
)) - ((5/8 - (3*I)/8)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(
Sqrt[2]*a*d) + ((5/8 - (3*I)/8)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c
+ d*x]])/(Sqrt[2]*a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3631

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((c + d*Tan[e + f*x])^(n - 1)/(
2*a*f*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a^2), Int[(c + d*Tan[e + f*x])
```

```
^(n - 2)*Simp[a*c^2 + a*d^2*(n - 1) - b*c*d*n - d*(a*c*(n - 2) + b*d*n)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)}{a+ia\tan(c+dx)}\,dx = \int \frac{\cot^{\frac{5}{2}}(c+dx)}{ia+a\cot(c+dx)}\,dx$$

$$= \frac{\cot^{\frac{3}{2}}(c+dx)}{2d(ia+a\cot(c+dx))} - \frac{\int \sqrt{\cot(c+dx)}\left(\frac{3ia}{2}-\frac{5}{2}a\cot(c+dx)\right)\,dx}{2a^2}$$

$$= -\frac{5\sqrt{\cot(c+dx)}}{2ad} + \frac{\cot^{\frac{3}{2}}(c+dx)}{2d(ia+a\cot(c+dx))} - \frac{\int \frac{\frac{5a}{2}+\frac{3}{2}ia\cot(c+dx)}{\sqrt{\cot(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{5\sqrt{\cot(c+dx)}}{2ad} + \frac{\cot^{\frac{3}{2}}(c+dx)}{2d(ia+a\cot(c+dx))} - \frac{\text{Subst}\left(\int \frac{-\frac{5a}{2}-\frac{3}{2}iax^2}{1+x^4}\,dx, x, \sqrt{\cot(c+\right.}{a^2 d}$$

$$= -\frac{5\sqrt{\cot(c+dx)}}{2ad} + \frac{\cot^{\frac{3}{2}}(c+dx)}{2d(ia+a\cot(c+dx))} - -\frac{\left(\frac{5}{4}+\frac{3i}{4}\right)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{c\right.}{ad}$$

$$= -\frac{5\sqrt{\cot(c+dx)}}{2ad} + \frac{\cot^{\frac{3}{2}}(c+dx)}{2d(ia+a\cot(c+dx))} - -\frac{\left(\frac{5}{8}+\frac{3i}{8}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx\right.}{ad}$$

$$= -\frac{5\sqrt{\cot(c+dx)}}{2ad} + \frac{\cot^{\frac{3}{2}}(c+dx)}{2d(ia+a\cot(c+dx))} - \frac{\left(\frac{5}{8}-\frac{3i}{8}\right)\log\left(1-\sqrt{2}\,\sqrt{\cot(c+d\right.}{\sqrt{2}\,ad}$$

$$= -\frac{\left(\frac{5}{4}+\frac{3i}{4}\right)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,ad} + \frac{\left(\frac{5}{4}+\frac{3i}{4}\right)\tan^{-1}\left(1+\sqrt{2}\,\sqrt{\cot(c+\right.}{\sqrt{2}\,ad}$$

Mathematica [A]
time = 1.15, size = 213, normalized size = 0.97

$$-\frac{\sqrt{\cot(c+dx)}\csc(c+dx)\sec(c+dx)\left(8+8\cos(2(c+dx))-(5-3i)\cos(c+dx)\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\sqrt{\sin(2(c+dx))}-(5+3i)\text{ArcSin}(\cos(c+dx)-\sin(c+dx))(\cos(c+dx)+i\sin(c+dx))\sqrt{\sin(2(c+dx))}-(3+5i)\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\sin(c+dx)\sqrt{\sin(2(c+dx))}+10i\sin(2(c+dx))\right)}{8ad(i+\cot(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(3/2)/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] -1/8*(Sqrt[Cot[c + d*x]]*Csc[c + d*x]*Sec[c + d*x]*(8 + 8*Cos[2*(c + d*x)]
- (5 - 3*I)*Cos[c + d*x]*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c +
d*x)]]]*Sqrt[Sin[2*(c + d*x)]] - (5 + 3*I)*ArcSin[Cos[c + d*x] - Sin[c + d*
x]]*(Cos[c + d*x] + I*Sin[c + d*x])*Sqrt[Sin[2*(c + d*x)]] - (3 + 5*I)*Log[
Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sin[c + d*x]*Sqrt[Sin
[2*(c + d*x)]] + (10*I)*Sin[2*(c + d*x)]))/(a*d*(I + Cot[c + d*x]))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 14.22, size = 1225, normalized size = 5.57

method	result	size
default	Expression too large to display	1225

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a/d*(I*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(
d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((cos(d*x+c)-
1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+
c)+3*I*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x
+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(cos(
d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)-4*I*(-(cos(d
*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c)
)^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d
*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)+I*(-(cos(d*x+c)-
1-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(
d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))
^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+3*I*(-(cos(d*x+c)-1-sin(d*x+c))/s
in(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/
2),1/2*2^(1/2))-4*I*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x
+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Elli
pticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))
-I*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)+cos(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/s
in(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1
/2),1/2+1/2*I,1/2*2^(1/2))+4*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*
((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2
*2^(1/2))*cos(d*x+c)+(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*
x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ell
ipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2)
)+4*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c)
)/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*
```

```
x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+cos(d*x+c)^3*2^
(1/2)-5*cos(d*x+c)*2^(1/2))*(cos(d*x+c)/sin(d*x+c))^(3/2)*sin(d*x+c)/cos(d*
x+c)^2*2^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 469 vs. 2(161) = 322.
time = 0.55, size = 469, normalized size = 2.13

$$\frac{ad\sqrt{\frac{i}{4a^2d^2}}e^{(2idx+2ic)}\log\left(2\left(2\left(ade^{(2idx+2ic)}-ad\right)\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{\frac{i}{4a^2d^2}}+ie^{(2idx+2ic)}\right)e^{(-2idx-2ic)}\right)-ad\sqrt{\frac{i}{4a^2d^2}}e^{(2idx+2ic)}\log\left(-2\left(2\left(ade^{(2idx+2ic)}-ad\right)\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{\frac{i}{4a^2d^2}}-ie^{(2idx+2ic)}\right)e^{(-2idx-2ic)}\right)+ad\sqrt{-\frac{4i}{a^2d^2}}e^{(2idx+2ic)}\log\left(\frac{\left(\left(ade^{(2idx+2ic)}-ad\right)\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{-\frac{4i}{a^2d^2}}+2i\right)e^{(-2idx-2ic)}}{ad}\right)-ad\sqrt{-\frac{4i}{a^2d^2}}e^{(2idx+2ic)}\log\left(-\frac{\left(\left(ade^{(2idx+2ic)}-ad\right)\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{-\frac{4i}{a^2d^2}}-2i\right)e^{(-2idx-2ic)}}{ad}\right)-\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\left(9e^{(2idx+2ic)}-1\right)e^{(-2idx-2ic)}}{4ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(a*d*sqrt(1/4*I/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(2*(2*(a*d*e^(2*I*d*x
 + 2*I*c) - a*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)
)*sqrt(1/4*I/(a^2*d^2)) + I*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)) - a*
d*sqrt(1/4*I/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(-2*(2*(a*d*e^(2*I*d*x + 2*I
*c) - a*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt
(1/4*I/(a^2*d^2)) - I*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)) + a*d*sqrt
(-4*I/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(((a*d*e^(2*I*d*x + 2*I*c) - a*d)*s
qrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-4*I/(a^2*d
^2)) + 2*I)*e^(-2*I*d*x - 2*I*c)/(a*d)) - a*d*sqrt(-4*I/(a^2*d^2))*e^(2*I*d
*x + 2*I*c)*log(-((a*d*e^(2*I*d*x + 2*I*c) - a*d)*sqrt((I*e^(2*I*d*x + 2*I*
c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-4*I/(a^2*d^2)) - 2*I)*e^(-2*I*d*x
- 2*I*c)/(a*d)) - sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1
))*(9*e^(2*I*d*x + 2*I*c) - 1))*e^(-2*I*d*x - 2*I*c)/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{\cot^{\frac{3}{2}}(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(3/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I*Integral(cot(c + d*x)**(3/2)/(tan(c + d*x) - I), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)^(3/2)/(I*a*tan(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^{3/2}}{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(3/2)/(a + a*tan(c + d*x)*1i),x)
```

```
[Out] int(cot(c + d*x)^(3/2)/(a + a*tan(c + d*x)*1i), x)
```

3.736 $\int \frac{\sqrt{\cot(c+dx)}}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=200

$$\frac{\left(\frac{3}{4}-\frac{i}{4}\right)\operatorname{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,ad}-\frac{\left(\frac{3}{4}-\frac{i}{4}\right)\operatorname{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,ad}+\frac{\sqrt{\cot(c+dx)}}{2d(ia+a\cot(c+dx$$

```
[Out] (-3/8+1/8*I)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/a/d*2^(1/2)+(-3/8+1/8*I)*a
rctan(1+2^(1/2)*cot(d*x+c)^(1/2))/a/d*2^(1/2)-(3/16+1/16*I)*ln(1+cot(d*x+c)
-2^(1/2)*cot(d*x+c)^(1/2))/a/d*2^(1/2)+(3/16+1/16*I)*ln(1+cot(d*x+c)+2^(1/2
)*cot(d*x+c)^(1/2))/a/d*2^(1/2)+1/2*cot(d*x+c)^(1/2)/d/(I*a+a*cot(d*x+c))
```

Rubi [A]
time = 0.14, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3754, 3631, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\left(\frac{3}{4}-\frac{i}{4}\right)\operatorname{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,ad}-\frac{\left(\frac{3}{4}-\frac{i}{4}\right)\operatorname{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,ad}+\frac{\sqrt{\cot(c+dx)}}{2d(a\cot(c+dx)+ia)}-\frac{\left(\frac{3}{8}+\frac{i}{8}\right)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,ad}+\frac{\left(\frac{3}{8}+\frac{i}{8}\right)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cot[c + d*x]]/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((3/4 - I/4)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*a*d) - ((3/4
- I/4)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*a*d) + Sqrt[Cot[c +
 d*x]]/(2*d*(I*a + a*Cot[c + d*x])) - ((3/8 + I/8)*Log[1 - Sqrt[2]*Sqrt[Cot
[c + d*x]] + Cot[c + d*x]])/(Sqrt[2]*a*d) + ((3/8 + I/8)*Log[1 + Sqrt[2]*Sq
rt[Cot[c + d*x]] + Cot[c + d*x]])/(Sqrt[2]*a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3631

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((c + d*Tan[e + f*x])^(n - 1)/(
2*a*f*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a^2), Int[(c + d*Tan[e + f*x])
^(n - 2)*Simp[a*c^2 + a*d^2*(n - 1) - b*c*d*n - d*(a*c*(n - 2) + b*d*n)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{\cot(c+dx)}}{a+ia\tan(c+dx)}\,dx &= \int \frac{\cot^{\frac{3}{2}}(c+dx)}{ia+a\cot(c+dx)}\,dx \\
&= \frac{\sqrt{\cot(c+dx)}}{2d(ia+a\cot(c+dx))} - \frac{\int \frac{\frac{ia}{2}-\frac{3}{2}a\cot(c+dx)}{\sqrt{\cot(c+dx)}}\,dx}{2a^2} \\
&= \frac{\sqrt{\cot(c+dx)}}{2d(ia+a\cot(c+dx))} - \frac{\text{Subst}\left(\int \frac{-\frac{ia}{2}+\frac{3ax^2}{2}}{1+x^4}\,dx, x, \sqrt{\cot(c+dx)}\right)}{a^2d} \\
&= \frac{\sqrt{\cot(c+dx)}}{2d(ia+a\cot(c+dx))} - -\frac{\left(\frac{3}{4}+\frac{i}{4}\right)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\cot(c+dx)}\right)}{ad} - \frac{\left(\frac{3}{4}-\right.}{} \\
&= \frac{\sqrt{\cot(c+dx)}}{2d(ia+a\cot(c+dx))} - \frac{\left(\frac{3}{8}-\frac{i}{8}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{\cot(c+dx)}\right)}{ad} - \\
&= \frac{\sqrt{\cot(c+dx)}}{2d(ia+a\cot(c+dx))} - \frac{\left(\frac{3}{8}+\frac{i}{8}\right)\log\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)}{\sqrt{2}\,ad} + \\
&= \frac{\left(\frac{3}{4}-\frac{i}{4}\right)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,ad} - \frac{\left(\frac{3}{4}-\frac{i}{4}\right)\tan^{-1}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx\right.}}{\sqrt{2}\,ad}
\end{aligned}
$$

Mathematica [A]
time = 1.01, size = 174, normalized size = 0.87

$$
\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\sqrt{\cot(c+dx)}\left((2-i)\csc(c+dx)\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)-(1-2i)\text{ArcSin}(\cos(c+dx)-\sin(c+dx))(\csc(c+dx)+i\sec(c+dx))+(1+2i)\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\sec(c+dx)+\frac{2-2i}{\sqrt{\sin(2(c+dx))}}\right)\sqrt{\sin(2(c+dx))}}{ad(i+\cot(c+dx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cot[c + d*x]]/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((1/8 + I/8)*Sqrt[Cot[c + d*x]]*((2 - I)*Csc[c + d*x]*Log[Cos[c + d*x] + Si
n[c + d*x] + Sqrt[Sin[2*(c + d*x)]]] - (1 - 2*I)*ArcSin[Cos[c + d*x] - Sin[
c + d*x]]*(Csc[c + d*x] + I*Sec[c + d*x]) + (1 + 2*I)*Log[Cos[c + d*x] + Si
n[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sec[c + d*x] + (2 - 2*I)/Sqrt[Sin[2*(c
 + d*x)]])*Sqrt[Sin[2*(c + d*x)]])/(a*d*(I + Cot[c + d*x]))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 14.07, size = 710, normalized size = 3.55

method	result

default	$-\frac{\left(-i\sqrt{-\frac{\cos(dx+c)-1-\sin(dx+c)}{\sin(dx+c)}}\,\sqrt{\frac{\cos(dx+c)-1+\sin(dx+c)}{\sin(dx+c)}}\,\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\ \mathrm{EllipticPi}\left(\sqrt{-\frac{\cos(dx+c)-1-\sin(dx}{\sin(dx+c)}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a/d*(-I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/
sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*
EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1
/2))+2*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(
d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*Elli
pticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))
+I*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^
(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2
*I,1/2*2^(1/2))*sin(d*x+c)+2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c
)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(
1/2)*sin(d*x+c)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/
2-1/2*I,1/2*2^(1/2))-3*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(
d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*E
llipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*sin(d*x
+c)-I*2^(1/2)*cos(d*x+c)*sin(d*x+c)-cos(d*x+c)^3*2^(1/2)+2^(1/2)*cos(d*x+c)
^2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))*(cos(d*x+c)/sin(d*x+c))^(1/2)/cos(d*x+
c)/sin(d*x+c)^3*2^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 471 vs. 2(145) = 290.
time = 0.49, size = 471, normalized size = 2.36

$$-\frac{\left(ad\sqrt{-\frac{i}{4a^2d^2}}e^{(2idx+2ic)}\log\left(-2\left(2\left(iade^{(2idx+2ic)}-iad\right)\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{-\frac{i}{4a^2d^2}}-ie^{(2idx+2ic)}\right)e^{(-2idx-2ic)}\right)-ad\sqrt{-\frac{i}{4a^2d^2}}e^{(2idx+2ic)}\log\left(-2\left(2\left(-iade^{(2idx+2ic)}+iad\right)\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{-\frac{i}{4a^2d^2}}-ie^{(2idx+2ic)}\right)e^{(-2idx-2ic)}\right)+ad\sqrt{\frac{i}{a^2d^2}}e^{(2idx+2ic)}\log\left(-\frac{\left(\left(ade^{(2idx+2ic)}-ad\right)\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{\frac{i}{a^2d^2}}+1\right)e^{(-2idx-2ic)}}{ad}\right)-ad\sqrt{\frac{i}{a^2d^2}}e^{(2idx+2ic)}\log\left(\frac{\left(\left(ade^{(2idx+2ic)}-ad\right)\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{\frac{i}{a^2d^2}}-1\right)e^{(-2idx-2ic)}}{ad}\right)-\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\left(-ie^{(2idx+2ic)}+i\right)\right)e^{(-2idx-2ic)}}{4ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(a*d*sqrt(-1/4*I/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(-2*(2*(I*a*d*e^(2*
I*d*x + 2*I*c) - I*a*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*
c) - 1))*sqrt(-1/4*I/(a^2*d^2)) - I*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*
c)) - a*d*sqrt(-1/4*I/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(-2*(2*(-I*a*d*e^(2
*I*d*x + 2*I*c) + I*a*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I
*c) - 1))*sqrt(-1/4*I/(a^2*d^2)) - I*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I
*c)) + a*d*sqrt(I/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(-((a*d*e^(2*I*d*x + 2*
I*c) - a*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqr
t(I/(a^2*d^2)) + 1)*e^(-2*I*d*x - 2*I*c)/(a*d)) - a*d*sqrt(I/(a^2*d^2))*e^(
2*I*d*x + 2*I*c)*log(((a*d*e^(2*I*d*x + 2*I*c) - a*d)*sqrt((I*e^(2*I*d*x +
2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(I/(a^2*d^2)) - 1)*e^(-2*I*d*x -
 2*I*c)/(a*d)) - sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)
)*(-I*e^(2*I*d*x + 2*I*c) + I))*e^(-2*I*d*x - 2*I*c)/(a*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{\sqrt{\cot{\left(c+dx \right)}}}{\tan{\left(c+dx \right)}-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(1/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I*Integral(sqrt(cot(c + d*x))/(tan(c + d*x) - I), x)/a
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cot(d*x + c))/(I*a*tan(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\sqrt{\mathrm{cot}\left(c+d\,x\right)}}{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(1/2)/(a + a*tan(c + d*x)*1i),x)
```

```
[Out] int(cot(c + d*x)^(1/2)/(a + a*tan(c + d*x)*1i), x)
```

3.737 $\int \frac{1}{\sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt[4]{-1}\,\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{2ad}+\frac{i\sqrt{\cot(c+dx)}}{2d(ia+a\cot(c+dx))}$$

```
[Out] 1/2*(-1)^(1/4)*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/a/d+1/2*I*cot(d*x+c)^(1
/2)/d/(I*a+a*cot(d*x+c))
```

Rubi [A]
time = 0.09, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3754, 3630, 3614, 214}

$$\frac{\sqrt[4]{-1}\,\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{2ad}+\frac{i\sqrt{\cot(c+dx)}}{2d(a\cot(c+dx)+ia)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])),x]
```

```
[Out] ((-1)^(1/4)*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/(2*a*d) + ((I/2)*Sqrt[C
ot[c + d*x]])/(d*(I*a + a*Cot[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3630

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(a*c + b*d))*((c + d*Tan[e + f*x])^n/(2*(
b*c - a*d)*f*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c
+ d*Tan[e + f*x])^(n - 1)*Simp[a*c*d*(n - 1) + b*c^2 + b*d^2*n - d*(b*c - a
*d)*(n - 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[0, n, 1]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))}\,dx &= \int \frac{\sqrt{\cot(c+dx)}}{ia+a\cot(c+dx)}\,dx\\ &= \frac{i\sqrt{\cot(c+dx)}}{2d(ia+a\cot(c+dx))}-\frac{\int \frac{-\frac{a}{2}+\frac{1}{2}ia\cot(c+dx)}{\sqrt{\cot(c+dx)}}\,dx}{2a^2}\\ &= \frac{i\sqrt{\cot(c+dx)}}{2d(ia+a\cot(c+dx))}-\frac{\text{Subst}\left(\int \frac{1}{\frac{a}{2}+\frac{1}{2}iax^2}\,dx,x,\sqrt{\cot(c+dx)}\right)}{4d}\\ &= \frac{\sqrt[4]{-1}\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{2ad}+\frac{i\sqrt{\cot(c+dx)}}{2d(ia+a\cot(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.94, size = 126, normalized size = 1.85

$$\frac{(i\cos(c+dx)+\sin(c+dx))\left(-\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\right)(\cos(c+dx)+i\sin(c+dx))+\cos(c+dx)\sqrt{i\tan(c+dx)}\right)}{2ad\sqrt{\cot(c+dx)}\sqrt{i\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])),x]
```

```
[Out] ((I*Cos[c + d*x] + Sin[c + d*x])*(-(ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))
/(1 + E^((2*I)*(c + d*x)))]]*(Cos[c + d*x] + I*Sin[c + d*x])) + Cos[c + d*x
]*Sqrt[I*Tan[c + d*x]]))/(2*a*d*Sqrt[Cot[c + d*x]]*Sqrt[I*Tan[c + d*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 13.60, size = 2687, normalized size = 39.51

method	result	size
default	Expression too large to display	2687

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)

[Out] -1/4/a/d*(-1+cos(d*x+c))^2*(-2^(1/2)*cos(d*x+c)*sin(d*x+c)+I*cos(d*x+c)^2*2
^(1/2)+I*cos(d*x+c)*2^(1/2)+(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*(
(cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*
2^(1/2))+2*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+3*cos(d*x+c)*(-(cos(d*x+c)-1-sin
(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(
d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-3*cos(d*x+c)*EllipticF((-(cos(d*x+c)-1
-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c)
)/sin(d*x+c))^(1/2)-I*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1
/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-
1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/
2)-I*sin(d*x+c)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2
*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin
(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)+4*cos(d*x+c)^3
*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*
(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)+2*cos(d*x+c)^2*EllipticF((-(c
os(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1
-sin(d*x+c))/sin(d*x+c))^(1/2)+(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2
)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))
^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1
/2*2^(1/2))*sin(d*x+c)-EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1
/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+
c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)-4*cos(d
*x+c)^3*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*
x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(co
s(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-2*cos(d*x+c
)^2*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c)
)/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*
x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-4*EllipticPi((-
(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos
(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(
cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*sin(d*x+c)+4*I*cos(
d*x+c)^2*sin(d*x+c)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)
,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))
/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)-4*I*cos(d*
x+c)^2*sin(d*x+c)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),
1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+si
n(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)+2
*I*cos(d*x+c)*sin(d*x+c)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^
```

```
(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*
x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)-2*I*c
os(d*x+c)*sin(d*x+c)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/
2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1
+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2
)-2*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c)
)/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*
x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)*sin(
d*x+c)+4*I*cos(d*x+c)^3*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^
(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c
)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(
1/2)+I*sin(d*x+c)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),
1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+si
n(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)+2
*I*cos(d*x+c)^2*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/
2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(
d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)-3*I
*cos(d*x+c)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/
2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+
c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)-2*I*cos
(d*x+c)^3*2^(1/2))*(cos(d*x+c)+1)^2/(4*I*cos(d*x+c)^2*sin(d*x+c)-2*I*cos(d*
x+c)*sin(d*x+c)+4*cos(d*x+c)^3-I*sin(d*x+c)-2*cos(d*x+c)^2-3*cos(d*x+c)+1)/
(cos(d*x+c)/sin(d*x+c))^(1/2)/sin(d*x+c)^4*2^(1/2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(52) = 104$.
time = 0.49, size = 269, normalized size = 3.96

$$-\frac{\left(ad\sqrt{\frac{i}{4\,a^2d^2}}\,e^{(2i\,dx+2i\,c)}\log\left(2\left(2\left(ade^{(2i\,dx+2i\,c)}-ad\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{i}{4\,a^2d^2}}+i\,e^{(2i\,dx+2i\,c)}\right)e^{(-2i\,dx-2i\,c)}\right)-ad\sqrt{\frac{i}{4\,a^2d^2}}\,e^{(2i\,dx+2i\,c)}\log\left(-2\left(2\left(ade^{(2i\,dx+2i\,c)}-ad\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{i}{4\,a^2d^2}}-i\,e^{(2i\,dx+2i\,c)}\right)e^{(-2i\,dx-2i\,c)}\right)-\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\left(e^{(2i\,dx+2i\,c)}-1\right)\right)e^{(-2i\,dx-2i\,c)}}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(a*d*sqrt(1/4*I/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(2*(2*(a*d*e^(2*I*d*
x + 2*I*c) - a*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1
```

```
))*sqrt(1/4*I/(a^2*d^2)) + I*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)) - a
*d*sqrt(1/4*I/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(-2*(2*(a*d*e^(2*I*d*x + 2*
I*c) - a*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqr
t(1/4*I/(a^2*d^2)) - I*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)) - sqrt((I
*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*(e^(2*I*d*x + 2*I*c) -
 1))*e^(-2*I*d*x - 2*I*c)/(a*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{1}{\tan\left(c+dx\right)\sqrt{\cot\left(c+dx\right)}-i\sqrt{\cot\left(c+dx\right)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(1/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I*Integral(1/(tan(c + d*x)*sqrt(cot(c + d*x)) - I*sqrt(cot(c + d*x))), x)/
a
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*tan(d*x + c) + a)*sqrt(cot(d*x + c))), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\sqrt{\cot\left(c+d\,x\right)}\,\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)),x)
```

```
[Out] int(1/(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)), x)
```

3.738 $\int \frac{1}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx$

Optimal. Leaf size=200

$$\frac{\left(\frac{1}{4}-\frac{3i}{4}\right)\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,ad}-\frac{\left(\frac{1}{4}-\frac{3i}{4}\right)\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,ad}-\frac{\sqrt{\cot(c+dx)}}{2d(ia+a\cot(c+a$$

```
[Out] (-1/8+3/8*I)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/a/d*2^(1/2)+(-1/8+3/8*I)*a
rctan(1+2^(1/2)*cot(d*x+c)^(1/2))/a/d*2^(1/2)-(1/16+3/16*I)*ln(1+cot(d*x+c)
-2^(1/2)*cot(d*x+c)^(1/2))/a/d*2^(1/2)+(1/16+3/16*I)*ln(1+cot(d*x+c)+2^(1/2
)*cot(d*x+c)^(1/2))/a/d*2^(1/2)-1/2*cot(d*x+c)^(1/2)/d/(I*a+a*cot(d*x+c))
```

Rubi [A]
time = 0.15, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3754, 3633, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\left(\frac{1}{4}-\frac{3i}{4}\right)\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,ad}-\frac{\left(\frac{1}{4}-\frac{3i}{4}\right)\mathrm{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,ad}-\frac{\sqrt{\cot(c+dx)}}{2d(a\cot(c+dx)+ia)}-\frac{\left(\frac{1}{8}+\frac{3i}{8}\right)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,ad}+\frac{\left(\frac{1}{8}+\frac{3i}{8}\right)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])),x]
```

```
[Out] ((1/4 - (3*I)/4)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*a*d) - ((
1/4 - (3*I)/4)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*a*d) - Sqrt
[Cot[c + d*x]]/(2*d*(I*a + a*Cot[c + d*x])) - ((1/8 + (3*I)/8)*Log[1 - Sqrt
[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(Sqrt[2]*a*d) + ((1/8 + (3*I)/8)*Lo
g[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(Sqrt[2]*a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3633

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-a)*((c + d*Tan[e + f*x])^(n + 1)/(2*f*(b*
c - a*d)*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*
Tan[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &
& NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx = \int \frac{1}{\sqrt{\cot(c+dx)}\,(ia+a\cot(c+dx))}\,dx$$

$$= -\frac{\sqrt{\cot(c+dx)}}{2d(ia+a\cot(c+dx))} - \frac{\int \frac{\frac{3ia}{2}-\frac{1}{2}a\cot(c+dx)}{\sqrt{\cot(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\sqrt{\cot(c+dx)}}{2d(ia+a\cot(c+dx))} - \frac{\text{Subst}\left(\int \frac{-\frac{3ia}{2}+\frac{ax^2}{2}}{1+x^4}\,dx, x, \sqrt{\cot(c+dx)}\right.}{a^2d}$$

$$= -\frac{\sqrt{\cot(c+dx)}}{2d(ia+a\cot(c+dx))} - -\frac{\left(\frac{1}{4}+\frac{3i}{4}\right)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\cot(c\right.}{ad}$$

$$= -\frac{\sqrt{\cot(c+dx)}}{2d(ia+a\cot(c+dx))} - \frac{\left(\frac{1}{8}-\frac{3i}{8}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{c\right.}{ad}$$

$$= -\frac{\sqrt{\cot(c+dx)}}{2d(ia+a\cot(c+dx))} - \frac{\left(\frac{1}{8}+\frac{3i}{8}\right)\log\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}+\right.}{\sqrt{2}\,ad}$$

$$= \frac{\left(\frac{1}{4}-\frac{3i}{4}\right)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,ad} - \frac{\left(\frac{1}{4}-\frac{3i}{4}\right)\tan^{-1}\left(1+\sqrt{\right.}{\sqrt{2}\,a}$$

Mathematica [A]
time = 1.14, size = 174, normalized size = 0.87

$$\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\sqrt{\cot(c+dx)}\left((2+i)\csc(c+dx)\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)+(1+2i)\text{ArcSin}(\cos(c+dx)-\sin(c+dx))(\csc(c+dx)+i\sec(c+dx))-(1-2i)\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\sec(c+dx)-\frac{2-2i}{\sqrt{\sin(2(c+dx))}}\right)\sqrt{\sin(2(c+dx))}}{ad(i+\cot(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])),x]
```

```
[Out] ((1/8 + I/8)*Sqrt[Cot[c + d*x]]*((2 + I)*Csc[c + d*x]*Log[Cos[c + d*x] + Si
n[c + d*x] + Sqrt[Sin[2*(c + d*x)]]] + (1 + 2*I)*ArcSin[Cos[c + d*x] - Sin[
c + d*x]]*(Csc[c + d*x] + I*Sec[c + d*x]) - (1 - 2*I)*Log[Cos[c + d*x] + Si
n[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sec[c + d*x] - (2 - 2*I)/Sqrt[Sin[2*(c
 + d*x)]])*Sqrt[Sin[2*(c + d*x)]])/(a*d*(I + Cot[c + d*x]))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 13.29, size = 4362, normalized size = 21.81

method	result	size

default	Expression too large to display	4362

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)

[Out] -1/4/a/d*(-1+cos(d*x+c))^2*(8*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+4*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-2*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-2*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-2^(1/2)*cos(d*x+c)^2-3*cos(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-6*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)-I*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)+2*cos(d*x+c)^3*2^(1/2)+(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)+(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*sin(d*x+c)-cos(d*x+c)*2^(1/2)+2*I*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)-2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+4*cos(d*x+c)^3*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2

```
)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^
(1/2))+2*cos(d*x+c)^2*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d
*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*El
lipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2
))-4*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2
*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin
(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*s
in(d*x+c)-2*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+si
n(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((
-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x
+c)*sin(d*x+c)-2*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)
-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellipti
cF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)*si
n(d*x+c)+4*I*cos(d*x+c)^3*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x
+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))
^(1/2)+2*I*cos(d*x+c)^2*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^
(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c
)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(
1/2)-3*I*cos(d*x+c)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2
),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+
sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)
+8*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c)
)^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*sin(d*x+
c)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2
^(1/2))+4*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin
(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*sin
(d*x+c)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,
1/2*2^(1/2))-8*I*((-1+cos(d*x+c))/sin(d*x+c))^(...
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 470 vs. 2(145) = 290.
time = 0.51, size = 470, normalized size = 2.35

$$\frac{\left(ad\sqrt{-\frac{i}{4a^2d^2}}e^{(2idx+2ic)}\log\left(-2\left(2\left(iade^{(2idx+2ic)}-iad\right)\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{-\frac{i}{4a^2d^2}}-ie^{(2idx+2ic)}\right)e^{(-2idx-2ic)}\right)-ad\sqrt{-\frac{i}{4a^2d^2}}e^{(2idx+2ic)}\log\left(-2\left(2\left(-iade^{(2idx+2ic)}+iad\right)\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{-\frac{i}{4a^2d^2}}-ie^{(2idx+2ic)}\right)e^{(-2idx-2ic)}\right)-ad\sqrt{\frac{i}{a^2d^2}}e^{(2idx+2ic)}\log\left(-\frac{\left(\left(ade^{(2idx+2ic)}-ad\right)\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{\frac{i}{a^2d^2}}+1\right)e^{(-2idx-2ic)}}{ad}\right)+ad\sqrt{\frac{i}{a^2d^2}}e^{(2idx+2ic)}\log\left(\frac{\left(\left(ade^{(2idx+2ic)}-ad\right)\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{\frac{i}{a^2d^2}}-1\right)e^{(-2idx-2ic)}}{ad}\right)+\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\left(ie^{(2idx+2ic)}-i\right)\right)e^{(-2idx-2ic)}}{4ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(a*d*sqrt(-1/4*I/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(-2*(2*(I*a*d*e^(2*I
*d*x + 2*I*c) - I*a*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c
) - 1))*sqrt(-1/4*I/(a^2*d^2)) - I*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c
)) - a*d*sqrt(-1/4*I/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(-2*(2*(-I*a*d*e^(2*
I*d*x + 2*I*c) + I*a*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*
c) - 1))*sqrt(-1/4*I/(a^2*d^2)) - I*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*
c)) - a*d*sqrt(I/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(-((a*d*e^(2*I*d*x + 2*I
*c) - a*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt
(I/(a^2*d^2)) + 1)*e^(-2*I*d*x - 2*I*c)/(a*d)) + a*d*sqrt(I/(a^2*d^2))*e^(2
*I*d*x + 2*I*c)*log(((a*d*e^(2*I*d*x + 2*I*c) - a*d)*sqrt((I*e^(2*I*d*x + 2
*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(I/(a^2*d^2)) - 1)*e^(-2*I*d*x -
2*I*c)/(a*d)) + sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))
*(I*e^(2*I*d*x + 2*I*c) - I))*e^(-2*I*d*x - 2*I*c)/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int \frac{1}{\tan{\left(c+dx \right)}\cot^{\frac{3}{2}}{\left(c+dx \right)}-i\cot^{\frac{3}{2}}{\left(c+dx \right)}}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(3/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I*Integral(1/(tan(c + d*x)*cot(c + d*x)**(3/2) - I*cot(c + d*x)**(3/2)), x
)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*tan(d*x + c) + a)*cot(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\mathrm{cot}\left(c+d\,x\right)}^{3/2}\,\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)),x)
```

```
[Out] int(1/(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)), x)
```

3.739 $\int \frac{1}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx$

Optimal. Leaf size=222

$$\frac{\left(\frac{3}{4}+\frac{5i}{4}\right)\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,ad}-\frac{\left(\frac{3}{4}+\frac{5i}{4}\right)\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,ad}-\frac{5i}{2ad\sqrt{\cot(c+dx)}}$$

```
[Out] (-3/8-5/8*I)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/a/d*2^(1/2)-(3/8+5/8*I)*ar
ctan(1+2^(1/2)*cot(d*x+c)^(1/2))/a/d*2^(1/2)+(3/16-5/16*I)*ln(1+cot(d*x+c)-
2^(1/2)*cot(d*x+c)^(1/2))/a/d*2^(1/2)+(-3/16+5/16*I)*ln(1+cot(d*x+c)+2^(1/2
)*cot(d*x+c)^(1/2))/a/d*2^(1/2)-5/2*I/a/d/cot(d*x+c)^(1/2)-1/2/d/(I*a+a*cot
(d*x+c))/cot(d*x+c)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3754, 3633, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\left(\frac{3}{4}+\frac{5i}{4}\right)\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,ad}-\frac{\left(\frac{3}{4}+\frac{5i}{4}\right)\mathrm{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,ad}-\frac{5i}{2ad\sqrt{\cot(c+dx)}}-\frac{1}{2d\sqrt{\cot(c+dx)}\,(a\cot(c+dx)+ia)}+\frac{\left(\frac{3}{8}-\frac{5i}{8}\right)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,ad}-\frac{\left(\frac{3}{8}-\frac{5i}{8}\right)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])),x]
```

```
[Out] ((3/4 + (5*I)/4)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*a*d) - ((
3/4 + (5*I)/4)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*a*d) - ((5*
I)/2)/(a*d*Sqrt[Cot[c + d*x]]) - 1/(2*d*Sqrt[Cot[c + d*x]]*(I*a + a*Cot[c +
 d*x])) + ((3/8 - (5*I)/8)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x
]])/(Sqrt[2]*a*d) - ((3/8 - (5*I)/8)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + C
ot[c + d*x]])/(Sqrt[2]*a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3633

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-a)*((c + d*Tan[e + f*x])^(n + 1)/(2*f*(b*
c - a*d)*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*
```

```
Tan[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &
& NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx = \int \frac{1}{\cot^{\frac{3}{2}}(c+dx)(ia+a\cot(c+dx))}\,dx$$

$$= -\frac{1}{2d\sqrt{\cot(c+dx)}\,(ia+a\cot(c+dx))} - \frac{\int \frac{\frac{5ia}{2}-\frac{3}{2}a\cot(c+dx)}{\cot^{\frac{3}{2}}(c+dx)}\,dx}{2a^2}$$

$$= -\frac{5i}{2ad\sqrt{\cot(c+dx)}} - \frac{1}{2d\sqrt{\cot(c+dx)}\,(ia+a\cot(c+dx))} - \ldots$$

$$= -\frac{5i}{2ad\sqrt{\cot(c+dx)}} - \frac{1}{2d\sqrt{\cot(c+dx)}\,(ia+a\cot(c+dx))} - \ldots$$

$$= -\frac{5i}{2ad\sqrt{\cot(c+dx)}} - \frac{1}{2d\sqrt{\cot(c+dx)}\,(ia+a\cot(c+dx))} - \ldots$$

$$= -\frac{5i}{2ad\sqrt{\cot(c+dx)}} - \frac{1}{2d\sqrt{\cot(c+dx)}\,(ia+a\cot(c+dx))} - \ldots$$

$$= -\frac{5i}{2ad\sqrt{\cot(c+dx)}} - \frac{1}{2d\sqrt{\cot(c+dx)}\,(ia+a\cot(c+dx))} + \ldots$$

$$= \frac{\left(\frac{3}{4}+\frac{5i}{4}\right)\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,ad} - \frac{\left(\frac{3}{4}+\frac{5i}{4}\right)\tan^{-1}\left(1+\ldots\right.}{\sqrt{2}\,\ldots}$$

Mathematica [A]
time = 1.16, size = 213, normalized size = 0.96

$$-\frac{\sqrt{\cot(c+dx)}\csc(c+dx)\sec(c+dx)\left(-8+8\cos(2(c+dx))+(3-5i)\cos(c+dx)\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\sqrt{\sin(2(c+dx))}+(3+5i)\mathrm{ArcSin}(\cos(c+dx)-\sin(c+dx))(\cos(c+dx)+i\sin(c+dx))\sqrt{\sin(2(c+dx))}+(5+3i)\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\sin(c+dx)\sqrt{\sin(2(c+dx))}+10i\sin(2(c+dx))\right)}{8ad(i+\cot(c+dx))}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])),x]

[Out] -1/8*(Sqrt[Cot[c + d*x]]*Csc[c + d*x]*Sec[c + d*x]*(-8 + 8*Cos[2*(c + d*x)] + (3 - 5*I)*Cos[c + d*x]*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d*x)]] + (3 + 5*I)*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*(Cos[c + d*x] + I*Sin[c + d*x])*Sqrt[Sin[2*(c + d*x)]] + (5 + 3*I)*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sin[c + d*x]*Sqrt[Sin[2*(c + d*x)]] + (10*I)*Sin[2*(c + d*x)]))/(a*d*(I + Cot[c + d*x]))

Maple [**C**] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 27.55, size = 4381, normalized size = 19.73

method	result	size
default	Expression too large to display	4381

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/4/a/d*(-1+cos(d*x+c))^2*(-9*2^(1/2)*cos(d*x+c)*sin(d*x+c)-16*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-8*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-4*2^(1/2)*sin(d*x+c)+(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+4*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+18*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+3*cos(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+12*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)-15*cos(d*x+c)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)-I*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)+20*cos(d*x+c)^3*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(

```
(cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin
(d*x+c))^(1/2)+10*cos(d*x+c)^2*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*
x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+
sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)
+(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/s
in(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c
)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)-4*I*2^(
1/2)-5*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))
*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^
(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)-4*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-
sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi((-(cos(d*x+c)-1-sin(d*x
+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-4*cos(d*x+c)^3*(-(cos(d*x+c)-
1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2
)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))
/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-2*cos(d*x+c)^2*(-(cos(d*x+c)-1-si
n(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin
(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-4*EllipticPi((-(cos(d*x+c)-1-sin(d*x+
c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c
))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*sin(d*x+c)-16*I*sin(d*x+c)*EllipticPi((-(
cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(cos(d*
x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)+20*I*sin(d*x+c)*Elli
pticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-(cos(d*x
+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)-8*I*cos(d*x+c)*sin(d*
x+c)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2
*2^(1/2))*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)+10*I*cos(d*x+c
)*sin(d*x+c)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^
(1/2))*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)-4*I*cos(d*x+c)^2*
sin(d*x+c)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2
*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c
))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)-2*I*cos(
d*x+c)*sin(d*x+c)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),
1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+si
n(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)-2
*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*...
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 548 vs. 2(161) = 322.
time = 0.47, size = 548, normalized size = 2.47

$$\frac{\left(ade^{(4i\,dx+4i\,c)}+ade^{(2i\,dx+2i\,c)}\right)\sqrt{\frac{i}{4\,a^2d^2}}\log\left(2\left(2\left(ade^{(2i\,dx+2i\,c)}-ad\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{i}{4\,a^2d^2}}+i\,e^{(2i\,dx+2i\,c)}\right)e^{(-2i\,dx-2i\,c)}\right)-\left(ade^{(4i\,dx+4i\,c)}+ade^{(2i\,dx+2i\,c)}\right)\sqrt{\frac{i}{4\,a^2d^2}}\log\left(-2\left(2\left(ade^{(2i\,dx+2i\,c)}-ad\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{i}{4\,a^2d^2}}-i\,e^{(2i\,dx+2i\,c)}\right)e^{(-2i\,dx-2i\,c)}\right)-\left(ade^{(4i\,dx+4i\,c)}+ade^{(2i\,dx+2i\,c)}\right)\sqrt{-\frac{4i}{a^2d^2}}\log\left(-\frac{\left(\left(ade^{(2i\,dx+2i\,c)}-ad\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{-\frac{4i}{a^2d^2}}+2i\right)e^{(-2i\,dx-2i\,c)}}{ad}\right)+\left(ade^{(4i\,dx+4i\,c)}+ade^{(2i\,dx+2i\,c)}\right)\sqrt{-\frac{4i}{a^2d^2}}\log\left(\frac{\left(\left(ade^{(2i\,dx+2i\,c)}-ad\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{-\frac{4i}{a^2d^2}}-2i\right)e^{(-2i\,dx-2i\,c)}}{ad}\right)-\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\left(9\,e^{(4i\,dx+4i\,c)}-8\,e^{(2i\,dx+2i\,c)}-1\right)}{4\left(ade^{(4i\,dx+4i\,c)}+ade^{(2i\,dx+2i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*((a*d*e^(4*I*d*x + 4*I*c) + a*d*e^(2*I*d*x + 2*I*c))*sqrt(1/4*I/(a^2*d^
2))*log(2*(2*(a*d*e^(2*I*d*x + 2*I*c) - a*d)*sqrt((I*e^(2*I*d*x + 2*I*c) +
I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(1/4*I/(a^2*d^2)) + I*e^(2*I*d*x + 2*I*c)
)*e^(-2*I*d*x - 2*I*c)) - (a*d*e^(4*I*d*x + 4*I*c) + a*d*e^(2*I*d*x + 2*I*c
))*sqrt(1/4*I/(a^2*d^2))*log(-2*(2*(a*d*e^(2*I*d*x + 2*I*c) - a*d)*sqrt((I*
e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(1/4*I/(a^2*d^2)) -
 I*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)) - (a*d*e^(4*I*d*x + 4*I*c) +
a*d*e^(2*I*d*x + 2*I*c))*sqrt(-4*I/(a^2*d^2))*log(-((a*d*e^(2*I*d*x + 2*I*c
) - a*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-
4*I/(a^2*d^2)) + 2*I)*e^(-2*I*d*x - 2*I*c)/(a*d)) + (a*d*e^(4*I*d*x + 4*I*c
) + a*d*e^(2*I*d*x + 2*I*c))*sqrt(-4*I/(a^2*d^2))*log(((a*d*e^(2*I*d*x + 2*
I*c) - a*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqr
t(-4*I/(a^2*d^2)) - 2*I)*e^(-2*I*d*x - 2*I*c)/(a*d)) - sqrt((I*e^(2*I*d*x +
 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*(9*e^(4*I*d*x + 4*I*c) - 8*e^(2*I*d
*x + 2*I*c) - 1))/(a*d*e^(4*I*d*x + 4*I*c) + a*d*e^(2*I*d*x + 2*I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{1}{\tan{\left(c+dx\right)}\cot^{\frac{5}{2}}{\left(c+dx\right)}-i\cot^{\frac{5}{2}}{\left(c+dx\right)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(5/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I*Integral(1/(tan(c + d*x)*cot(c + d*x)**(5/2) - I*cot(c + d*x)**(5/2)), x
)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*tan(d*x + c) + a)*cot(d*x + c)^(5/2)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cot\left(c+d\,x\right)^{5/2}\,\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)),x)
```

```
[Out] int(1/(cot(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)), x)
```

3.740 $\int \frac{\cot^{\frac{3}{2}}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=252

$$-\frac{\left(\frac{25}{16}+\frac{21i}{16}\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,a^2d}+\frac{\left(\frac{25}{16}+\frac{21i}{16}\right)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,a^2d}-\frac{25\sqrt{\cot(c+}}{8a^2d}$$

```
[Out] 7/8*cot(d*x+c)^(3/2)/a^2/d/(I+cot(d*x+c))+1/4*cot(d*x+c)^(5/2)/d/(I*a+a*cot
(d*x+c))^2+(25/32+21/32*I)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/a^2/d*2^(1/2
)+(25/32+21/32*I)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/a^2/d*2^(1/2)+(-25/64+
21/64*I)*ln(1+cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2))/a^2/d*2^(1/2)+(25/64-21/
64*I)*ln(1+cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2))/a^2/d*2^(1/2)-25/8*cot(d*x+
c)^(1/2)/a^2/d
```

Rubi [A]
time = 0.25, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {3754, 3639, 3676, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\left(\frac{25}{16}+\frac{21i}{16}\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,a^2d}+\frac{\left(\frac{25}{16}+\frac{21i}{16}\right)\text{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,a^2d}+\frac{7\cot^{\frac{3}{2}}(c+dx)}{8a^2d(\cot(c+dx)+i)}-\frac{25\sqrt{\cot(c+dx)}}{8a^2d}-\frac{\left(\frac{25}{32}-\frac{21i}{32}\right)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,a^2d}+\frac{\left(\frac{25}{32}-\frac{21i}{32}\right)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,a^2d}+\frac{\cot^{\frac{5}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(3/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((-25/16 - (21*I)/16)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*a^2*
d) + ((25/16 + (21*I)/16)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*
a^2*d) - (25*Sqrt[Cot[c + d*x]])/(8*a^2*d) + (7*Cot[c + d*x]^(3/2))/(8*a^2*
d*(I + Cot[c + d*x])) + Cot[c + d*x]^(5/2)/(4*d*(I*a + a*Cot[c + d*x])^2) -
 ((25/32 - (21*I)/32)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(
Sqrt[2]*a^2*d) + ((25/32 - (21*I)/32)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] +
Cot[c + d*x]])/(Sqrt[2]*a^2*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
```

```
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\cot^{\frac{7}{2}}(c+dx)}{(ia+a\cot(c+dx))^2}\,dx$$

$$= \frac{\cot^{\frac{5}{2}}(c+dx)}{4d(ia+a\cot(c+dx))^2} + \frac{\int \frac{\cot^{\frac{3}{2}}(c+dx)\left(-\frac{5ia}{2}+\frac{9}{2}a\cot(c+dx)\right)}{ia+a\cot(c+dx)}\,dx}{4a^2}$$

$$= \frac{7\cot^{\frac{3}{2}}(c+dx)}{8a^2d(i+\cot(c+dx))} + \frac{\cot^{\frac{5}{2}}(c+dx)}{4d(ia+a\cot(c+dx))^2} + \frac{\int \sqrt{\cot(c+dx)}\left(-\frac{21ia^2}{2}+\right.}{8a^4}$$

$$= -\frac{25\sqrt{\cot(c+dx)}}{8a^2d} + \frac{7\cot^{\frac{3}{2}}(c+dx)}{8a^2d(i+\cot(c+dx))} + \frac{\cot^{\frac{5}{2}}(c+dx)}{4d(ia+a\cot(c+dx))^2} + \frac{\int -}{}$$

$$= -\frac{25\sqrt{\cot(c+dx)}}{8a^2d} + \frac{7\cot^{\frac{3}{2}}(c+dx)}{8a^2d(i+\cot(c+dx))} + \frac{\cot^{\frac{5}{2}}(c+dx)}{4d(ia+a\cot(c+dx))^2} + \frac{\text{Sub}}{}$$

$$= -\frac{25\sqrt{\cot(c+dx)}}{8a^2d} + \frac{7\cot^{\frac{3}{2}}(c+dx)}{8a^2d(i+\cot(c+dx))} + \frac{\cot^{\frac{5}{2}}(c+dx)}{4d(ia+a\cot(c+dx))^2} + \frac{\left(\frac{25}{16}\right.}{}$$

$$= -\frac{25\sqrt{\cot(c+dx)}}{8a^2d} + \frac{7\cot^{\frac{3}{2}}(c+dx)}{8a^2d(i+\cot(c+dx))} + \frac{\cot^{\frac{5}{2}}(c+dx)}{4d(ia+a\cot(c+dx))^2} + \frac{\left(\frac{25}{32}\right.}{}$$

$$= -\frac{25\sqrt{\cot(c+dx)}}{8a^2d} + \frac{7\cot^{\frac{3}{2}}(c+dx)}{8a^2d(i+\cot(c+dx))} + \frac{\cot^{\frac{5}{2}}(c+dx)}{4d(ia+a\cot(c+dx))^2} - \frac{\left(\frac{25}{32}\right.}{}$$

$$= -\frac{\left(\frac{25}{16}+\frac{21i}{16}\right)\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,a^2d} + \frac{\left(\frac{25}{16}+\frac{21i}{16}\right)\tan^{-1}\left(1+\sqrt{2}\sqrt{\mathrm{c}\right.}{\sqrt{2}\,a^2d}$$

Mathematica [A]
time = 1.25, size = 232, normalized size = 0.92

$$-\frac{\cot^{\frac{3}{2}}(c+dx)\csc(c+dx)\sec^2(c+dx)\left(23\cos(c+dx)+41\cos(3(c+dx))+43i\sin(c+dx)-(25-21i)\cos(2(c+dx))\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\sqrt{\sin(2(c+dx))}-(21+25i)\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\sin^{\frac{3}{2}}(2(c+dx))+(21-25i)\text{ArcSin}(\cos(c+dx)-\sin(c+dx))\sqrt{\sin(2(c+dx))}\left(-i\cos(2(c+dx))+\sin(2(c+dx))\right)+43i\sin(3(c+dx))\right)}{32a^2d(i+\cot(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(3/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -1/32*(Cot[c + d*x]^(3/2)*Csc[c + d*x]*Sec[c + d*x]^2*(23*Cos[c + d*x] + 41
*Cos[3*(c + d*x)] + (43*I)*Sin[c + d*x] - (25 - 21*I)*Cos[2*(c + d*x)]*Log[
Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d*x)]
] - (21 + 25*I)*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*S
in[2*(c + d*x)]^(3/2) + (21 - 25*I)*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Sqr
t[Sin[2*(c + d*x)]]*((-I)*Cos[2*(c + d*x)] + Sin[2*(c + d*x)]) + (43*I)*Sin
[3*(c + d*x)]))/(a^2*d*(I + Cot[c + d*x])^2)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 13.89, size = 1261, normalized size = 5.00

method	result	size
default	Expression too large to display	1261

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/a^2/d*(4*I*2^(1/2)*cos(d*x+c)^4*sin(d*x+c)+23*I*EllipticPi((-(cos(d*x
+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((cos(d*x+c)-1+s
in(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+
c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)-2*I*((cos(d*x+c)-1+sin(d*x+c)
)/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*
x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-(cos(d*x+c)-1
-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)-21*I*((cos(d*x+c)-1+sin(d*x+c))/s
in(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(cos(d*x+c)
-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-(cos(d*x+c)-1-sin(d*x+c))/s
in(d*x+c))^(1/2)*cos(d*x+c)-4*2^(1/2)*cos(d*x+c)^5+23*I*EllipticPi((-(cos(d
*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((cos(d*x+c)-1
+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*
x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)-23*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x
+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/
2-1/2*I,1/2*2^(1/2))*cos(d*x+c)-2*I*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^
(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x
+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-(cos(d*x+c)-1-sin(d*x+c))/s
in(d*x+c))^(1/2)-2*cos(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)
*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^
(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/
2*2^(1/2))-21*I*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2
),1/2*2^(1/2))*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)+7*I*2^(1/2)*co
s(d*x+c)^2*sin(d*x+c)-23*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((co
s(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(
1/2))-2*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*
x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(co
s(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-5*cos(d*x+c
)^3*2^(1/2)+25*cos(d*x+c)*2^(1/2))*(cos(d*x+c)/sin(d*x+c))^(3/2)*sin(d*x+c)
/cos(d*x+c)^2*2^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 509 vs. 2(187) = 374.
time = 0.53, size = 509, normalized size = 2.02

$$\frac{4\,a^2d\sqrt{\frac{i}{16\,a^4d^2}}\,e^{(4i\,dx+4i\,c)}\log\left(2\left(4\left(a^2de^{(2i\,dx+2i\,c)}-a^2d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{i}{16\,a^4d^2}}+i\,e^{(2i\,dx+2i\,c)}\right)e^{(-2i\,dx-2i\,c)}\right)-4\,a^2d\sqrt{\frac{i}{16\,a^4d^2}}\,e^{(4i\,dx+4i\,c)}\log\left(-2\left(4\left(a^2de^{(2i\,dx+2i\,c)}-a^2d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{i}{16\,a^4d^2}}-i\,e^{(2i\,dx+2i\,c)}\right)e^{(-2i\,dx-2i\,c)}\right)+4\,a^2d\sqrt{-\frac{529i}{64\,a^4d^2}}\,e^{(4i\,dx+4i\,c)}\log\left(\frac{\left(8\left(a^2de^{(2i\,dx+2i\,c)}-a^2d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{-\frac{529i}{64\,a^4d^2}}+23i\right)e^{(-2i\,dx-2i\,c)}}{8\,a^2d}\right)-4\,a^2d\sqrt{-\frac{529i}{64\,a^4d^2}}\,e^{(4i\,dx+4i\,c)}\log\left(-\frac{\left(8\left(a^2de^{(2i\,dx+2i\,c)}-a^2d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{-\frac{529i}{64\,a^4d^2}}-23i\right)e^{(-2i\,dx-2i\,c)}}{8\,a^2d}\right)-\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\left(42\,e^{(4i\,dx+4i\,c)}-9\,e^{(2i\,dx+2i\,c)}-1\right)e^{(-4i\,dx-4i\,c)}}{16\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*a^2*d*sqrt(1/16*I/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(2*(4*(a^2*d*e^
(2*I*d*x + 2*I*c) - a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2
*I*c) - 1))*sqrt(1/16*I/(a^4*d^2)) + I*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2
*I*c)) - 4*a^2*d*sqrt(1/16*I/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(-2*(4*(a^2*
d*e^(2*I*d*x + 2*I*c) - a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x
 + 2*I*c) - 1))*sqrt(1/16*I/(a^4*d^2)) - I*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x
 - 2*I*c)) + 4*a^2*d*sqrt(-529/64*I/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(1/8*
(8*(a^2*d*e^(2*I*d*x + 2*I*c) - a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^
(2*I*d*x + 2*I*c) - 1))*sqrt(-529/64*I/(a^4*d^2)) + 23*I)*e^(-2*I*d*x - 2*I
*c)/(a^2*d)) - 4*a^2*d*sqrt(-529/64*I/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(-1
/8*(8*(a^2*d*e^(2*I*d*x + 2*I*c) - a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/
(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-529/64*I/(a^4*d^2)) - 23*I)*e^(-2*I*d*x -
2*I*c)/(a^2*d)) - sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1
))*(42*e^(4*I*d*x + 4*I*c) - 9*e^(2*I*d*x + 2*I*c) - 1))*e^(-4*I*d*x - 4*I*
c)/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\cot^{\frac{3}{2}}(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(3/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -Integral(cot(c + d*x)**(3/2)/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/
a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] integrate(cot(d*x + c)^(3/2)/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^{3/2}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(3/2)/(a + a*tan(c + d*x)*1i)^2,x)
[Out] int(cot(c + d*x)^(3/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.741 $\int \frac{\sqrt{\cot(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=232

$$\frac{\left(\frac{9}{16}-\frac{5i}{16}\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,a^2d}-\frac{\left(\frac{9}{16}-\frac{5i}{16}\right)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,a^2d}+\frac{5\sqrt{\cot(c+dx)}}{8a^2d(i+\cot(c+$$

```
[Out] 1/4*cot(d*x+c)^(3/2)/d/(I*a+a*cot(d*x+c))^2+(-9/32+5/32*I)*arctan(-1+2^(1/2
)*cot(d*x+c)^(1/2))/a^2/d*2^(1/2)+(-9/32+5/32*I)*arctan(1+2^(1/2)*cot(d*x+c
)^(1/2))/a^2/d*2^(1/2)-(9/64+5/64*I)*ln(1+cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/
2))/a^2/d*2^(1/2)+(9/64+5/64*I)*ln(1+cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2))/a
^2/d*2^(1/2)+5/8*cot(d*x+c)^(1/2)/a^2/d/(I+cot(d*x+c))
```

Rubi [A]
time = 0.23, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3754, 3639, 3676, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\left(\frac{9}{16}-\frac{5i}{16}\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,a^2d}-\frac{\left(\frac{9}{16}-\frac{5i}{16}\right)\text{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,a^2d}+\frac{5\sqrt{\cot(c+dx)}}{8a^2d(\cot(c+dx)+i)}-\frac{\left(\frac{9}{32}+\frac{5i}{32}\right)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,a^2d}+\frac{\left(\frac{9}{32}+\frac{5i}{32}\right)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,a^2d}+\frac{\cot^{\frac{3}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cot[c + d*x]]/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((9/16 - (5*I)/16)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*a^2*d)
- ((9/16 - (5*I)/16)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*a^2*d
) + (5*Sqrt[Cot[c + d*x]])/(8*a^2*d*(I + Cot[c + d*x])) + Cot[c + d*x]^(3/2
)/(4*d*(I*a + a*Cot[c + d*x])^2) - ((9/32 + (5*I)/32)*Log[1 - Sqrt[2]*Sqrt[
Cot[c + d*x]] + Cot[c + d*x]])/(Sqrt[2]*a^2*d) + ((9/32 + (5*I)/32)*Log[1 +
 Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(Sqrt[2]*a^2*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{\sqrt{\cot(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\cot^{\frac{5}{2}}(c+dx)}{(ia+a\cot(c+dx))^2}\,dx$$

$$= \frac{\cot^{\frac{3}{2}}(c+dx)}{4d(ia+a\cot(c+dx))^2} + \frac{\int \frac{\sqrt{\cot(c+dx)}\left(-\frac{3ia}{2}+\frac{7}{2}a\cot(c+dx)\right)}{ia+a\cot(c+dx)}\,dx}{4a^2}$$

$$= \frac{5\sqrt{\cot(c+dx)}}{8a^2d(i+\cot(c+dx))} + \frac{\cot^{\frac{3}{2}}(c+dx)}{4d(ia+a\cot(c+dx))^2} + \frac{\int \frac{-\frac{5ia^2}{2}+\frac{9}{2}a^2\cot(c+dx)}{\sqrt{\cot(c+dx)}}\,dx}{8a^4}$$

$$= \frac{5\sqrt{\cot(c+dx)}}{8a^2d(i+\cot(c+dx))} + \frac{\cot^{\frac{3}{2}}(c+dx)}{4d(ia+a\cot(c+dx))^2} + \frac{\operatorname{Subst}\left(\int \frac{\frac{5ia^2}{2}-\frac{9a^2x^2}{2}}{1+x^4}\,dx, x, \right.}{4a^4d}$$

$$= \frac{5\sqrt{\cot(c+dx)}}{8a^2d(i+\cot(c+dx))} + \frac{\cot^{\frac{3}{2}}(c+dx)}{4d(ia+a\cot(c+dx))^2} + -\frac{\left(\frac{9}{16}-\frac{5i}{16}\right)\operatorname{Subst}\left(\int \frac{1+x^2}{1+x^4}\right.}{a^2d}$$

$$= \frac{5\sqrt{\cot(c+dx)}}{8a^2d(i+\cot(c+dx))} + \frac{\cot^{\frac{3}{2}}(c+dx)}{4d(ia+a\cot(c+dx))^2} + -\frac{\left(\frac{9}{32}-\frac{5i}{32}\right)\operatorname{Subst}\left(\int \frac{}{1-\sqrt{2}}\right.}{}$$

$$= \frac{5\sqrt{\cot(c+dx)}}{8a^2d(i+\cot(c+dx))} + \frac{\cot^{\frac{3}{2}}(c+dx)}{4d(ia+a\cot(c+dx))^2} - \frac{\left(\frac{9}{32}+\frac{5i}{32}\right)\log\left(1-\sqrt{2}\sqrt{c}\right.}{\sqrt{2}}$$

$$= \frac{\left(\frac{9}{16}-\frac{5i}{16}\right)\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,a^2d} - \frac{\left(\frac{9}{16}-\frac{5i}{16}\right)\tan^{-1}\left(1+\sqrt{2}\sqrt{\cot(\right.}{\sqrt{2}\,a^2d}$$

Mathematica [A]

time = 0.94, size = 232, normalized size = 1.00

$$\frac{\cot^{\frac{3}{2}}(c+dx)\csc(c+dx)\sec^2(c+dx)\left(5i\cos(c+dx)-5i\cos(3(c+dx))+7\sin(c+dx)+(9+5i)\cos(2(c+dx))\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\sqrt{\sin(2(c+dx))}-(5-9i)\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\sin^{\frac{3}{2}}(2(c+dx))-(5+9i)\mathrm{ArcSin}(\cos(c+dx)-\sin(c+dx))\sqrt{\sin(2(c+dx))}\,(-i\cos(2(c+dx))+\sin(2(c+dx)))+7\sin(3(c+dx))\right)}{32a^2d(i+\cot(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cot[c + d*x]]/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (Cot[c + d*x]^(3/2)*Csc[c + d*x]*Sec[c + d*x]^2*((5*I)*Cos[c + d*x] - (5*I)
*Cos[3*(c + d*x)] + 7*Sin[c + d*x] + (9 + 5*I)*Cos[2*(c + d*x)]*Log[Cos[c +
 d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d*x)]] - (5
- 9*I)*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sin[2*(c +
 d*x)]^(3/2) - (5 + 9*I)*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Sqrt[Sin[2*(c
+ d*x)]]*((-I)*Cos[2*(c + d*x)] + Sin[2*(c + d*x)]) + 7*Sin[3*(c + d*x)]))/
(32*a^2*d*(I + Cot[c + d*x])^2)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 12.46, size = 778, normalized size = 3.35

method	result
default	$\left(-4i\sqrt{2}\,\left(\cos^4(dx+c)\right)\sin(dx+c)+2i\sqrt{-\frac{\cos(dx+c)-1-\sin(dx+c)}{\sin(dx+c)}}\,\sqrt{\frac{\cos(dx+c)-1+\sin(dx+c)}{\sin(dx+c)}}\,\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\mathrm{Ellipti}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/a^2/d*(-4*I*cos(d*x+c)^4*sin(d*x+c)*2^(1/2)+2*I*sin(d*x+c)*((-1+cos(d*
x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos
(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+
c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-7*I*sin(d*x+c)*((-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x
+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/
sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+4*I*cos(d*x+c)^3*sin(d*x+c)*2^(1/2
)+4*2^(1/2)*cos(d*x+c)^5-5*I*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)-2*(-(cos(d*x+c
)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1
/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c
))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)-7*((-1+cos(d*x+c))/s
in(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)
-1-sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi((-(cos(d*x+c)-1-sin(
d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+9*(-(cos(d*x+c)-1-sin(d*x+
c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos
(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2*2^(1/2))*sin(d*x+c)-4*cos(d*x+c)^4*2^(1/2)+5*I*cos(d*x+c)*sin(d
*x+c)*2^(1/2)+3*cos(d*x+c)^3*2^(1/2)-3*2^(1/2)*cos(d*x+c)^2)*(cos(d*x+c)+1)
```

```
^2*(-1+cos(d*x+c))*(cos(d*x+c)/sin(d*x+c))^(1/2)/cos(d*x+c)/sin(d*x+c)^3*2^
(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(171) = 342$.
time = 0.53, size = 511, normalized size = 2.20

$$-\frac{4\,a^2d\sqrt{-\frac{i}{16\,a^4d^2}}\,e^{(4i\,dx+4i\,c)}\log\left(-2\left(4\left(i\,a^2de^{(2i\,dx+2i\,c)}-i\,a^2d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{-\frac{i}{16\,a^4d^2}}-i\,e^{(2i\,dx+2i\,c)}\right)e^{(-2i\,dx-2i\,c)}\right)-4\,a^2d\sqrt{-\frac{i}{16\,a^4d^2}}\,e^{(4i\,dx+4i\,c)}\log\left(-2\left(4\left(-i\,a^2de^{(2i\,dx+2i\,c)}+i\,a^2d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{-\frac{i}{16\,a^4d^2}}-i\,e^{(2i\,dx+2i\,c)}\right)e^{(-2i\,dx-2i\,c)}\right)+4\,a^2d\sqrt{\frac{49i}{64\,a^4d^2}}\,e^{(4i\,dx+4i\,c)}\log\left(-\frac{\left(8\left(a^2de^{(2i\,dx+2i\,c)}-a^2d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{49i}{64\,a^4d^2}}+7\right)e^{(-2i\,dx-2i\,c)}}{8\,a^2d}\right)-4\,a^2d\sqrt{\frac{49i}{64\,a^4d^2}}\,e^{(4i\,dx+4i\,c)}\log\left(\frac{\left(8\left(a^2de^{(2i\,dx+2i\,c)}-a^2d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{49i}{64\,a^4d^2}}-7\right)e^{(-2i\,dx-2i\,c)}}{8\,a^2d}\right)-\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\left(-6i\,e^{(4i\,dx+4i\,c)}+5i\,e^{(2i\,dx+2i\,c)}+i\right)}{16\,a^2d}e^{(-4i\,dx-4i\,c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*a^2*d*sqrt(-1/16*I/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(-2*(4*(I*a^2
*d*e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*
d*x + 2*I*c) - 1))*sqrt(-1/16*I/(a^4*d^2)) - I*e^(2*I*d*x + 2*I*c))*e^(-2*I
*d*x - 2*I*c)) - 4*a^2*d*sqrt(-1/16*I/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(-2
*(4*(-I*a^2*d*e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c) +
I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-1/16*I/(a^4*d^2)) - I*e^(2*I*d*x + 2*I*
c))*e^(-2*I*d*x - 2*I*c)) + 4*a^2*d*sqrt(49/64*I/(a^4*d^2))*e^(4*I*d*x + 4*
I*c)*log(-1/8*(8*(a^2*d*e^(2*I*d*x + 2*I*c) - a^2*d)*sqrt((I*e^(2*I*d*x + 2
*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(49/64*I/(a^4*d^2)) + 7)*e^(-2*I*
d*x - 2*I*c)/(a^2*d)) - 4*a^2*d*sqrt(49/64*I/(a^4*d^2))*e^(4*I*d*x + 4*I*c)
*log(1/8*(8*(a^2*d*e^(2*I*d*x + 2*I*c) - a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c)
 + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(49/64*I/(a^4*d^2)) - 7)*e^(-2*I*d*x -
 2*I*c)/(a^2*d)) - sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) -
1))*(-6*I*e^(4*I*d*x + 4*I*c) + 5*I*e^(2*I*d*x + 2*I*c) + I))*e^(-4*I*d*x -
 4*I*c)/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt{\cot{\left(c + dx \right)}}}{\tan^{2}{\left(c + dx \right)} - 2 i \tan{\left(c + dx \right)} - 1}\, dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(1/2)/(a+I*a*tan(d*x+c))**2,x)
[Out] -Integral(sqrt(cot(c + d*x))/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/a
**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] integrate(sqrt(cot(d*x + c))/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cot\left(c+d\,x\right)}}{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(1/2)/(a + a*tan(c + d*x)*1i)^2,x)
[Out] int(cot(c + d*x)^(1/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.742 $\int \frac{1}{\sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=234

$$\frac{\left(\frac{1}{16}-\frac{3i}{16}\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,a^2d}-\frac{\left(\frac{1}{16}-\frac{3i}{16}\right)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,a^2d}+\frac{3i\sqrt{\cot(c+dx)}}{8a^2d(i+\cot(c+$$

```
[Out] (-1/32+3/32*I)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/a^2/d*2^(1/2)+(-1/32+3/3
2*I)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/a^2/d*2^(1/2)+(1/64+3/64*I)*ln(1+co
t(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2))/a^2/d*2^(1/2)-(1/64+3/64*I)*ln(1+cot(d*x
+c)+2^(1/2)*cot(d*x+c)^(1/2))/a^2/d*2^(1/2)+3/8*I*cot(d*x+c)^(1/2)/a^2/d/(I
+cot(d*x+c))+1/4*cot(d*x+c)^(1/2)/d/(I*a+a*cot(d*x+c))^2
```

Rubi [A]
time = 0.22, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3754, 3639, 3677, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\left(\frac{1}{16}-\frac{3i}{16}\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,a^2d}-\frac{\left(\frac{1}{16}-\frac{3i}{16}\right)\text{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,a^2d}+\frac{3i\sqrt{\cot(c+dx)}}{8a^2d(\cot(c+dx)+i)}+\frac{\left(\frac{1}{32}+\frac{3i}{32}\right)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,a^2d}-\frac{\left(\frac{1}{32}+\frac{3i}{32}\right)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,a^2d}+\frac{\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] ((1/16 - (3*I)/16)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*a^2*d)
 - ((1/16 - (3*I)/16)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*a^2*d
) + (((3*I)/8)*Sqrt[Cot[c + d*x]])/(a^2*d*(I + Cot[c + d*x])) + Sqrt[Cot[c
+ d*x]]/(4*d*(I*a + a*Cot[c + d*x])^2) + ((1/32 + (3*I)/32)*Log[1 - Sqrt[2]
*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(Sqrt[2]*a^2*d) - ((1/32 + (3*I)/32)*L
og[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(Sqrt[2]*a^2*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^2}\,dx = \int \frac{\cot^{\frac{3}{2}}(c+dx)}{(ia+a\cot(c+dx))^2}\,dx$$

$$= \frac{\sqrt{\cot(c+dx)}}{4d(ia+a\cot(c+dx))^2} + \frac{\int \frac{-\frac{ia}{2}+\frac{5}{2}a\cot(c+dx)}{\sqrt{\cot(c+dx)}\,(ia+a\cot(c+dx))}\,dx}{4a^2}$$

$$= \frac{3i\sqrt{\cot(c+dx)}}{8a^2d(i+\cot(c+dx))} + \frac{\sqrt{\cot(c+dx)}}{4d(ia+a\cot(c+dx))^2} + \frac{\int \frac{\frac{a^2}{2}-\frac{3}{2}ia^2\cot}{\sqrt{\cot(c+}}}{8a^4}$$

$$= \frac{3i\sqrt{\cot(c+dx)}}{8a^2d(i+\cot(c+dx))} + \frac{\sqrt{\cot(c+dx)}}{4d(ia+a\cot(c+dx))^2} + \frac{\text{Subst}\left(\int -\right.}{}$$

$$= \frac{3i\sqrt{\cot(c+dx)}}{8a^2d(i+\cot(c+dx))} + \frac{\sqrt{\cot(c+dx)}}{4d(ia+a\cot(c+dx))^2} + -\frac{\left(\frac{1}{16}+\frac{3i}{16}\right)}{}$$

$$= \frac{3i\sqrt{\cot(c+dx)}}{8a^2d(i+\cot(c+dx))} + \frac{\sqrt{\cot(c+dx)}}{4d(ia+a\cot(c+dx))^2} + -\frac{\left(\frac{1}{32}-\frac{3i}{32}\right)}{}$$

$$= \frac{3i\sqrt{\cot(c+dx)}}{8a^2d(i+\cot(c+dx))} + \frac{\sqrt{\cot(c+dx)}}{4d(ia+a\cot(c+dx))^2} + \frac{\left(\frac{1}{32}+\frac{3i}{32}\right)\text{lo}}{}$$

$$= \frac{\left(\frac{1}{16}-\frac{3i}{16}\right)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,a^2d} - \frac{\left(\frac{1}{16}-\frac{3i}{16}\right)\tan^{-1}\left(\right.}{}$$

Mathematica [A]
time = 0.92, size = 224, normalized size = 0.96

$$\frac{\csc^3(c+dx)\left(-\cos(c+dx)+\cos(3(c+dx))+3i\sin(c+dx)-(1+3i)\cos(2(c+dx))\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\sqrt{\sin(2(c+dx))}+(3-i)\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\sin^{\frac{3}{2}}(2(c+dx))-(3+i)\text{ArcSin}(\cos(c+dx)-\sin(c+dx))\sqrt{\sin(2(c+dx))}\,(-i\cos(2(c+dx))+\sin(2(c+dx)))+3i\sin(3(c+dx))\right)}{32a^2d\sqrt{\cot(c+dx)}\,(i+\cot(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (Csc[c + d*x]^3*(-Cos[c + d*x] + Cos[3*(c + d*x)] + (3*I)*Sin[c + d*x] - (1
 + 3*I)*Cos[2*(c + d*x)]*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c +
d*x)]]]*Sqrt[Sin[2*(c + d*x)]] + (3 - I)*Log[Cos[c + d*x] + Sin[c + d*x] +
Sqrt[Sin[2*(c + d*x)]]]*Sin[2*(c + d*x)]^(3/2) - (3 + I)*ArcSin[Cos[c + d*x
] - Sin[c + d*x]]*Sqrt[Sin[2*(c + d*x)]]*((-I)*Cos[2*(c + d*x)] + Sin[2*(c
+ d*x)]) + (3*I)*Sin[3*(c + d*x)]))/(32*a^2*d*Sqrt[Cot[c + d*x]]*(I + Cot[c
 + d*x])^2)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 13.38, size = 6915, normalized size = 29.55

method	result	size
default	Expression too large to display	6915

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(171) = 342$.
time = 0.45, size = 509, normalized size = 2.18

$$-\frac{\left(4a^2d\sqrt{\frac{i}{16a^4d^2}}e^{(4idx+4ic)}\log\left(2\left(4\left(a^2de^{(2idx+2ic)}-a^2d\right)\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{\frac{i}{16a^4d^2}}+ie^{(2idx+2ic)}\right)e^{(-2idx-2ic)}\right)-4a^2d\sqrt{\frac{i}{16a^4d^2}}e^{(4idx+4ic)}\log\left(-2\left(4\left(a^2de^{(2idx+2ic)}-a^2d\right)\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{\frac{i}{16a^4d^2}}-ie^{(2idx+2ic)}\right)e^{(-2idx-2ic)}\right)-4a^2d\sqrt{-\frac{i}{64a^4d^2}}e^{(4idx+4ic)}\log\left(\frac{\left(8\left(a^2de^{(2idx+2ic)}-a^2d\right)\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{-\frac{i}{64a^4d^2}}+i\right)e^{(-2idx-2ic)}}{8a^2d}\right)+4a^2d\sqrt{-\frac{i}{64a^4d^2}}e^{(4idx+4ic)}\log\left(-\frac{\left(8\left(a^2de^{(2idx+2ic)}-a^2d\right)\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{-\frac{i}{64a^4d^2}}-i\right)e^{(-2idx-2ic)}}{8a^2d}\right)-\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\left(2e^{(4idx+4ic)}-e^{(2idx+2ic)}-1\right)\right)e^{(-4idx-4ic)}}{16a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*a^2*d*sqrt(1/16*I/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(2*(4*(a^2*d*e
^(2*I*d*x + 2*I*c) - a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
2*I*c) - 1))*sqrt(1/16*I/(a^4*d^2)) + I*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x -
2*I*c)) - 4*a^2*d*sqrt(1/16*I/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(-2*(4*(a^2
*d*e^(2*I*d*x + 2*I*c) - a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*
x + 2*I*c) - 1))*sqrt(1/16*I/(a^4*d^2)) - I*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*
x - 2*I*c)) - 4*a^2*d*sqrt(-1/64*I/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(1/8*(
8*(a^2*d*e^(2*I*d*x + 2*I*c) - a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(
2*I*d*x + 2*I*c) - 1))*sqrt(-1/64*I/(a^4*d^2)) + I)*e^(-2*I*d*x - 2*I*c)/(a
^2*d)) + 4*a^2*d*sqrt(-1/64*I/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(-1/8*(8*(a
^2*d*e^(2*I*d*x + 2*I*c) - a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*
d*x + 2*I*c) - 1))*sqrt(-1/64*I/(a^4*d^2)) - I)*e^(-2*I*d*x - 2*I*c)/(a^2*d
)) - sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*(2*e^(4*I*
d*x + 4*I*c) - e^(2*I*d*x + 2*I*c) - 1))*e^(-4*I*d*x - 4*I*c)/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{\tan^2(c+dx)\sqrt{\cot(c+dx)}-2i\tan(c+dx)\sqrt{\cot(c+dx)}-\sqrt{\cot(c+dx)}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(1/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -Integral(1/(tan(c + d*x)**2*sqrt(cot(c + d*x)) - 2*I*tan(c + d*x)*sqrt(cot
(c + d*x)) - sqrt(cot(c + d*x))), x)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*tan(d*x + c) + a)^2*sqrt(cot(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{\cot(c+d\,x)}\,{\left(a+a\tan(c+d\,x)\,1{}\mathrm{i}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

```
[Out] int(1/(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

3.743 $\int \frac{1}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=234

$$-\frac{\left(\frac{1}{16}+\frac{3i}{16}\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}a^2d}+\frac{\left(\frac{1}{16}+\frac{3i}{16}\right)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}a^2d}+\frac{\sqrt{\cot(c+}}{8a^2d(i+\cot(}$$

```
[Out] (1/32+3/32*I)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/a^2/d*2^(1/2)+(1/32+3/32*
I)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/a^2/d*2^(1/2)+(1/64-3/64*I)*ln(1+cot(
d*x+c)-2^(1/2)*cot(d*x+c)^(1/2))/a^2/d*2^(1/2)+(-1/64+3/64*I)*ln(1+cot(d*x+
c)+2^(1/2)*cot(d*x+c)^(1/2))/a^2/d*2^(1/2)+1/8*cot(d*x+c)^(1/2)/a^2/d/(I+co
t(d*x+c))+1/4*I*cot(d*x+c)^(1/2)/d/(I*a+a*cot(d*x+c))^2
```

Rubi [A]
time = 0.21, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3754, 3638, 3677, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\left(\frac{1}{16}+\frac{3i}{16}\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}a^2d}+\frac{\left(\frac{1}{16}+\frac{3i}{16}\right)\text{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}a^2d}+\frac{\sqrt{\cot(c+dx)}}{8a^2d(\cot(c+dx)+i)}+\frac{\left(\frac{1}{32}-\frac{3i}{32}\right)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}a^2d}-\frac{\left(\frac{1}{32}-\frac{3i}{32}\right)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}a^2d}+\frac{i\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] ((-1/16 - (3*I)/16)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*a^2*d)
 + ((1/16 + (3*I)/16)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*a^2*
d) + Sqrt[Cot[c + d*x]]/(8*a^2*d*(I + Cot[c + d*x])) + ((I/4)*Sqrt[Cot[c +
d*x]])/(d*(I*a + a*Cot[c + d*x])^2) + ((1/32 - (3*I)/32)*Log[1 - Sqrt[2]*Sq
rt[Cot[c + d*x]] + Cot[c + d*x]])/(Sqrt[2]*a^2*d) - ((1/32 - (3*I)/32)*Log[
1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(Sqrt[2]*a^2*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3638

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*Sqrt[(c_.) + (d_.)*tan[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-b)*(a + b*Tan[e + f*x])^m*(Sqrt[c + d
*Tan[e + f*x]]/(2*a*f*m)), x] + Dist[1/(4*a^2*m), Int[(a + b*Tan[e + f*x])^
(m + 1)*(Simp[2*a*c*m + b*d + a*d*(2*m + 1)*Tan[e + f*x], x]/Sqrt[c + d*Tan
[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && IntegersQ[2*m]
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx = \int \frac{\sqrt{\cot(c+dx)}}{(ia+a\cot(c+dx))^2}\,dx$$

$$= \frac{i\sqrt{\cot(c+dx)}}{4d(ia+a\cot(c+dx))^2} + \frac{\int \frac{a-3ia\cot(c+dx)}{\sqrt{\cot(c+dx)}\,(ia+a\cot(c+dx))}\,dx}{8a^2}$$

$$= \frac{\sqrt{\cot(c+dx)}}{8a^2d(i+\cot(c+dx))} + \frac{i\sqrt{\cot(c+dx)}}{4d(ia+a\cot(c+dx))^2} + \frac{\int \frac{-3ia^2-a^2\cot\cdots}{\sqrt{\cot(c+\cdots}}}{16a^{\cdots}}$$

$$= \frac{\sqrt{\cot(c+dx)}}{8a^2d(i+\cot(c+dx))} + \frac{i\sqrt{\cot(c+dx)}}{4d(ia+a\cot(c+dx))^2} + \frac{\text{Subst}\Big(\int \frac{3ia^{\cdots}}{1\cdots}}{\cdots}$$

$$= \frac{\sqrt{\cot(c+dx)}}{8a^2d(i+\cot(c+dx))} + \frac{i\sqrt{\cot(c+dx)}}{4d(ia+a\cot(c+dx))^2} + -\frac{\left(\frac{1}{16}-\frac{3i}{16}\right)\cdots}{\cdots}$$

$$= \frac{\sqrt{\cot(c+dx)}}{8a^2d(i+\cot(c+dx))} + \frac{i\sqrt{\cot(c+dx)}}{4d(ia+a\cot(c+dx))^2} + \frac{\left(\frac{1}{32}+\frac{3i}{32}\right)\text{Su}\cdots}{\cdots}$$

$$= \frac{\sqrt{\cot(c+dx)}}{8a^2d(i+\cot(c+dx))} + \frac{i\sqrt{\cot(c+dx)}}{4d(ia+a\cot(c+dx))^2} + \frac{\left(\frac{1}{32}-\frac{3i}{32}\right)\log\cdots}{\cdots}$$

$$= -\frac{\left(\frac{1}{16}+\frac{3i}{16}\right)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,a^2d} + \frac{\left(\frac{1}{16}+\frac{3i}{16}\right)\tan^{-1}\left(\cdots\right.}{\cdots}$$

Mathematica [A]
time = 0.79, size = 222, normalized size = 0.95

$$\frac{\csc^3(c+dx)\left(3i\cos(c+dx)-3i\cos(3(c+dx))+\sin(c+dx)-(1-3i)\cos(2(c+dx))\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\sqrt{\sin(2(c+dx))}+(1+3i)\text{ArcSin}(\cos(c+dx)-\sin(c+dx))(\cos(2(c+dx))+i\sin(2(c+dx)))\sqrt{\sin(2(c+dx))}-(3+i)\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\sin^{\frac{3}{2}}(2(c+dx))+\sin(3(c+dx))\right)}{32a^2d\sqrt{\cot(c+dx)}\,(i+\cot(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (Csc[c + d*x]^3*((3*I)*Cos[c + d*x] - (3*I)*Cos[3*(c + d*x)] + Sin[c + d*x]
 - (1 - 3*I)*Cos[2*(c + d*x)]*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*
(c + d*x)]]]*Sqrt[Sin[2*(c + d*x)]] + (1 + 3*I)*ArcSin[Cos[c + d*x] - Sin[c
 + d*x]]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)])*Sqrt[Sin[2*(c + d*x)]] - (
3 + I)*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sin[2*(c +
 d*x)]^(3/2) + Sin[3*(c + d*x)]))/(32*a^2*d*Sqrt[Cot[c + d*x]]*(I + Cot[c +
 d*x])^2)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 14.19, size = 6919, normalized size = 29.57

method	result	size
default	Expression too large to display	6919

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 510 vs. $2(171) = 342$.
time = 0.61, size = 510, normalized size = 2.18

$$\frac{4 a^2 d\sqrt{-\frac{i}{16 a^4 d^2}} e^{(4 i d x+4 i c)} \log\left(-2 \left(4 \left(i a^2 d e^{(2 i d x+2 i c)} - i a^2 d\right)\sqrt{\frac{i e^{(2 i d x+2 i c)}+i}{e^{(2 i d x+2 i c)}-1}}\sqrt{-\frac{i}{16 a^4 d^2}} - i e^{(2 i d x+2 i c)}\right) e^{(-2 i d x-2 i c)}\right) - 4 a^2 d\sqrt{-\frac{i}{16 a^4 d^2}} e^{(4 i d x+4 i c)} \log\left(-2 \left(4 \left(-i a^2 d e^{(2 i d x+2 i c)} + i a^2 d\right)\sqrt{\frac{i e^{(2 i d x+2 i c)}+i}{e^{(2 i d x+2 i c)}-1}}\sqrt{-\frac{i}{16 a^4 d^2}} - i e^{(2 i d x+2 i c)}\right) e^{(-2 i d x-2 i c)}\right) - 4 a^2 d\sqrt{\frac{i}{64 a^4 d^2}} e^{(4 i d x+4 i c)} \log\left(-\frac{\left(8 \left(a^2 d e^{(2 i d x+2 i c)} - a^2 d\right)\sqrt{\frac{i e^{(2 i d x+2 i c)}+i}{e^{(2 i d x+2 i c)}-1}}\sqrt{\frac{i}{64 a^4 d^2}}+1\right) e^{(-2 i d x-2 i c)}}{8 a^2 d}\right) + 4 a^2 d\sqrt{\frac{i}{64 a^4 d^2}} e^{(4 i d x+4 i c)} \log\left(\frac{\left(8 \left(a^2 d e^{(2 i d x+2 i c)} - a^2 d\right)\sqrt{\frac{i e^{(2 i d x+2 i c)}+i}{e^{(2 i d x+2 i c)}-1}}\sqrt{\frac{i}{64 a^4 d^2}}-1\right) e^{(-2 i d x-2 i c)}}{8 a^2 d}\right) + \sqrt{\frac{i e^{(2 i d x+2 i c)}+i}{e^{(2 i d x+2 i c)}-1}}\left(-2 i e^{(4 i d x+4 i c)} + 3 i e^{(2 i d x+2 i c)} - i\right)}{16 a^2 d} e^{(-4 i d x-4 i c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*a^2*d*sqrt(-1/16*I/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(-2*(4*(I*a^2*
d*e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d
*x + 2*I*c) - 1))*sqrt(-1/16*I/(a^4*d^2)) - I*e^(2*I*d*x + 2*I*c))*e^(-2*I*
```

```
d*x - 2*I*c)) - 4*a^2*d*sqrt(-1/16*I/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(-2*
(4*(-I*a^2*d*e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I
)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-1/16*I/(a^4*d^2)) - I*e^(2*I*d*x + 2*I*c
))*e^(-2*I*d*x - 2*I*c)) - 4*a^2*d*sqrt(1/64*I/(a^4*d^2))*e^(4*I*d*x + 4*I*
c)*log(-1/8*(8*(a^2*d*e^(2*I*d*x + 2*I*c) - a^2*d)*sqrt((I*e^(2*I*d*x + 2*I
*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(1/64*I/(a^4*d^2)) + 1)*e^(-2*I*d*x
 - 2*I*c)/(a^2*d)) + 4*a^2*d*sqrt(1/64*I/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log
(1/8*(8*(a^2*d*e^(2*I*d*x + 2*I*c) - a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I
)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(1/64*I/(a^4*d^2)) - 1)*e^(-2*I*d*x - 2*I*
c)/(a^2*d)) + sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*(
-2*I*e^(4*I*d*x + 4*I*c) + 3*I*e^(2*I*d*x + 2*I*c) - I))*e^(-4*I*d*x - 4*I*
c)/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{\tan^2(c+dx)\cot^{\frac{3}{2}}(c+dx)-2i\tan(c+dx)\cot^{\frac{3}{2}}(c+dx)-\cot^{\frac{3}{2}}(c+dx)}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(3/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -Integral(1/(tan(c + d*x)**2*cot(c + d*x)**(3/2) - 2*I*tan(c + d*x)*cot(c +
 d*x)**(3/2) - cot(c + d*x)**(3/2)), x)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*tan(d*x + c) + a)^2*cot(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cot(c+d\,x)^{3/2}\,{\left(a+a\,\tan(c+d\,x)\,1{}\mathrm{i}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

```
[Out] int(1/(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

3.744 $\int \frac{1}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=234

$$-\frac{\left(\frac{9}{16}+\frac{5i}{16}\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,a^2d}+\frac{\left(\frac{9}{16}+\frac{5i}{16}\right)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,a^2d}+\frac{5i\sqrt{\cot(c+}}{8a^2d(i+\cot(c}$$

```
[Out] (9/32+5/32*I)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/a^2/d*2^(1/2)+(9/32+5/32*
I)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/a^2/d*2^(1/2)+(-9/64+5/64*I)*ln(1+cot
(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2))/a^2/d*2^(1/2)+(9/64-5/64*I)*ln(1+cot(d*x+
c)+2^(1/2)*cot(d*x+c)^(1/2))/a^2/d*2^(1/2)+5/8*I*cot(d*x+c)^(1/2)/a^2/d/(I+
cot(d*x+c))-1/4*cot(d*x+c)^(1/2)/d/(I*a+a*cot(d*x+c))^2
```

Rubi [A]
time = 0.23, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3754, 3640, 3677, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\left(\frac{9}{16}+\frac{5i}{16}\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,a^2d}+\frac{\left(\frac{9}{16}+\frac{5i}{16}\right)\text{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,a^2d}+\frac{5i\sqrt{\cot(c+dx)}}{8a^2d(\cot(c+dx)+i)}-\frac{\left(\frac{9}{32}-\frac{5i}{32}\right)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,a^2d}+\frac{\left(\frac{9}{32}-\frac{5i}{32}\right)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,a^2d}-\frac{\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] ((-9/16 - (5*I)/16)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*a^2*d)
 + ((9/16 + (5*I)/16)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*a^2*
d) + (((5*I)/8)*Sqrt[Cot[c + d*x]])/(a^2*d*(I + Cot[c + d*x])) - Sqrt[Cot[c
 + d*x]]/(4*d*(I*a + a*Cot[c + d*x])^2) - ((9/32 - (5*I)/32)*Log[1 - Sqrt[2
]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(Sqrt[2]*a^2*d) + ((9/32 - (5*I)/32)*
Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(Sqrt[2]*a^2*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{\sqrt{\cot(c+dx)}\,(ia+a\cot(c+dx))^2}\,dx$$

$$= -\frac{\sqrt{\cot(c+dx)}}{4d(ia+a\cot(c+dx))^2} + \frac{\int \frac{-\frac{7ia}{2}+\frac{3}{2}a\cot(c+dx)}{\sqrt{\cot(c+dx)}\,(ia+a\cot(c+dx))}\,dx}{4a^2}$$

$$= \frac{5i\sqrt{\cot(c+dx)}}{8a^2d(i+\cot(c+dx))} - \frac{\sqrt{\cot(c+dx)}}{4d(ia+a\cot(c+dx))^2} + \frac{\int \frac{-\frac{9a^2}{2}-\frac{5}{2}ia^2\cot}{\sqrt{\cot(c+}}}{8a^4}$$

$$= \frac{5i\sqrt{\cot(c+dx)}}{8a^2d(i+\cot(c+dx))} - \frac{\sqrt{\cot(c+dx)}}{4d(ia+a\cot(c+dx))^2} + \frac{\text{Subst}\left(\int \frac{\frac{9a^2}{2}+}{1}\right.}{}$$

$$= \frac{5i\sqrt{\cot(c+dx)}}{8a^2d(i+\cot(c+dx))} - \frac{\sqrt{\cot(c+dx)}}{4d(ia+a\cot(c+dx))^2} + \frac{\left(\frac{9}{16}-\frac{5i}{16}\right)\text{Sub}}{}$$

$$= \frac{5i\sqrt{\cot(c+dx)}}{8a^2d(i+\cot(c+dx))} - \frac{\sqrt{\cot(c+dx)}}{4d(ia+a\cot(c+dx))^2} + \frac{\left(\frac{9}{32}+\frac{5i}{32}\right)\text{Sub}}{}$$

$$= \frac{5i\sqrt{\cot(c+dx)}}{8a^2d(i+\cot(c+dx))} - \frac{\sqrt{\cot(c+dx)}}{4d(ia+a\cot(c+dx))^2} - \frac{\left(\frac{9}{32}-\frac{5i}{32}\right)\log}{}$$

$$= -\frac{\left(\frac{9}{16}+\frac{5i}{16}\right)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,a^2d} + \frac{\left(\frac{9}{16}+\frac{5i}{16}\right)\tan^{-1}\left(\right.}{}$$

Mathematica [A]

time = 1.21, size = 232, normalized size = 0.99

$$\frac{\cot^{\frac{3}{2}}(c+dx)\csc(c+dx)\sec^2(c+dx)\left(-7\cos(c+dx)+7\cos(3(c+dx))+5i\sin(c+dx)+(9-5i)\cos(2(c+dx))\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\sqrt{\sin(2(c+dx))}+(9+5i)\mathrm{ArcSin}(\cos(c+dx)-\sin(c+dx))(\cos(2(c+dx))+i\sin(2(c+dx)))\sqrt{\sin(2(c+dx))}+(5+9i)\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\sin^{\frac{3}{2}}(2(c+dx))+5i\sin(3(c+dx))\right)}{32a^2d(i+\cot(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (Cot[c + d*x]^(3/2)*Csc[c + d*x]*Sec[c + d*x]^2*(-7*Cos[c + d*x] + 7*Cos[3*
(c + d*x)] + (5*I)*Sin[c + d*x] + (9 - 5*I)*Cos[2*(c + d*x)]*Log[Cos[c + d*
x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d*x)]] + (9 + 5
*I)*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*
x)])*Sqrt[Sin[2*(c + d*x)]] + (5 + 9*I)*Log[Cos[c + d*x] + Sin[c + d*x] + S
qrt[Sin[2*(c + d*x)]]]*Sin[2*(c + d*x)]^(3/2) + (5*I)*Sin[3*(c + d*x)]))/(3
2*a^2*d*(I + Cot[c + d*x])^2)
```

Maple [**C**] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 13.56, size = 6924, normalized size = 29.59

method	result	size
default	Expression too large to display	6924

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 508 vs. 2(171) = 342.
time = 1.00, size = 508, normalized size = 2.17

$$\frac{\left(4\,a^2d\sqrt{\frac{i}{16\,a^4d^2}}\,e^{(4i\,dx+4i\,c)}\log\left(2\left(4\left(a^2de^{(2i\,dx+2i\,c)}-a^2d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{i}{16\,a^4d^2}}+i\,e^{(2i\,dx+2i\,c)}\right)e^{(-2i\,dx-2i\,c)}\right)-4\,a^2d\sqrt{\frac{i}{16\,a^4d^2}}\,e^{(4i\,dx+4i\,c)}\log\left(-2\left(4\left(a^2de^{(2i\,dx+2i\,c)}-a^2d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{i}{16\,a^4d^2}}-i\,e^{(2i\,dx+2i\,c)}\right)e^{(-2i\,dx-2i\,c)}\right)+4\,a^2d\sqrt{-\frac{49i}{64\,a^4d^2}}\,e^{(4i\,dx+4i\,c)}\log\left(\frac{\left(8\left(a^2de^{(2i\,dx+2i\,c)}-a^2d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{-\frac{49i}{64\,a^4d^2}}+7i\right)e^{(-2i\,dx-2i\,c)}}{8\,a^2d}\right)-4\,a^2d\sqrt{-\frac{49i}{64\,a^4d^2}}\,e^{(4i\,dx+4i\,c)}\log\left(-\frac{\left(8\left(a^2de^{(2i\,dx+2i\,c)}-a^2d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{-\frac{49i}{64\,a^4d^2}}-7i\right)e^{(-2i\,dx-2i\,c)}}{8\,a^2d}\right)+\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\left(6\,e^{(4i\,dx+4i\,c)}-7\,e^{(2i\,dx+2i\,c)}+1\right)\right)e^{(-4i\,dx-4i\,c)}}{16\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*a^2*d*sqrt(1/16*I/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(2*(4*(a^2*d*e^
(2*I*d*x + 2*I*c) - a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2
*I*c) - 1))*sqrt(1/16*I/(a^4*d^2)) + I*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2
*I*c)) - 4*a^2*d*sqrt(1/16*I/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(-2*(4*(a^2*
d*e^(2*I*d*x + 2*I*c) - a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x
 + 2*I*c) - 1))*sqrt(1/16*I/(a^4*d^2)) - I*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x
 - 2*I*c)) + 4*a^2*d*sqrt(-49/64*I/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(1/8*(
8*(a^2*d*e^(2*I*d*x + 2*I*c) - a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(
2*I*d*x + 2*I*c) - 1))*sqrt(-49/64*I/(a^4*d^2)) + 7*I)*e^(-2*I*d*x - 2*I*c)
/(a^2*d)) - 4*a^2*d*sqrt(-49/64*I/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(-1/8*(
8*(a^2*d*e^(2*I*d*x + 2*I*c) - a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(
2*I*d*x + 2*I*c) - 1))*sqrt(-49/64*I/(a^4*d^2)) - 7*I)*e^(-2*I*d*x - 2*I*c)
/(a^2*d)) + sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*(6*
e^(4*I*d*x + 4*I*c) - 7*e^(2*I*d*x + 2*I*c) + 1))*e^(-4*I*d*x - 4*I*c)/(a^2
*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(5/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*tan(d*x + c) + a)^2*cot(d*x + c)^(5/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cot\left(c+d\,x\right)^{5/2}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

```
[Out] int(1/(cot(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

3.745 $\int \frac{1}{\cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=254

$$\frac{\left(\frac{25}{16}-\frac{21i}{16}\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,a^2d}-\frac{\left(\frac{25}{16}-\frac{21i}{16}\right)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,a^2d}-\frac{25}{8a^2d\sqrt{\cot(c+}$$

```
[Out] (-25/32+21/32*I)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/a^2/d*2^(1/2)+(-25/32+
21/32*I)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/a^2/d*2^(1/2)-(25/64+21/64*I)*l
n(1+cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2))/a^2/d*2^(1/2)+(25/64+21/64*I)*ln(1
+cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2))/a^2/d*2^(1/2)-25/8/a^2/d/cot(d*x+c)^(
1/2)+7/8*I/a^2/d/(I+cot(d*x+c))/cot(d*x+c)^(1/2)-1/4/d/(I*a+a*cot(d*x+c))^2
/cot(d*x+c)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {3754, 3640, 3677, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\left(\frac{25}{16}-\frac{21i}{16}\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,a^2d}-\frac{\left(\frac{25}{16}-\frac{21i}{16}\right)\text{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,a^2d}-\frac{25}{8a^2d\sqrt{\cot(c+dx)}}+\frac{7i}{8a^2d\sqrt{\cot(c+dx)}\,(\cot(c+dx)+i)}-\frac{\left(\frac{25}{32}+\frac{21i}{32}\right)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,a^2d}+\frac{\left(\frac{25}{32}+\frac{21i}{32}\right)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,a^2d}-\frac{1}{4d\sqrt{\cot(c+dx)}\,(a\cot(c+dx)+ia)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cot[c + d*x]^(7/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] ((25/16 - (21*I)/16)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*a^2*d
) - ((25/16 - (21*I)/16)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*a
^2*d) - 25/(8*a^2*d*Sqrt[Cot[c + d*x]]) + ((7*I)/8)/(a^2*d*Sqrt[Cot[c + d*x
]]*(I + Cot[c + d*x])) - 1/(4*d*Sqrt[Cot[c + d*x]]*(I*a + a*Cot[c + d*x])^2
) - ((25/32 + (21*I)/32)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]
)/(Sqrt[2]*a^2*d) + ((25/32 + (21*I)/32)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]
 + Cot[c + d*x]])/(Sqrt[2]*a^2*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
```

```
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{\cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{\cot^{\frac{3}{2}}(c+dx)(ia+a\cot(c+dx))^2}\,dx$$

$$= -\frac{1}{4d\sqrt{\cot(c+dx)}\,(ia+a\cot(c+dx))^2} + \frac{\int \frac{-\frac{9ia}{2}+\frac{5}{2}a\cot(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(ia+a\cot(c+d}}{4a^2}$$

$$= \frac{7i}{8a^2d\sqrt{\cot(c+dx)}\,(i+\cot(c+dx))} - \frac{1}{4d\sqrt{\cot(c+dx)}\,(ia+a\,c}$$

$$= -\frac{25}{8a^2d\sqrt{\cot(c+dx)}} + \frac{7i}{8a^2d\sqrt{\cot(c+dx)}\,(i+\cot(c+dx))} - $$

$$= -\frac{25}{8a^2d\sqrt{\cot(c+dx)}} + \frac{7i}{8a^2d\sqrt{\cot(c+dx)}\,(i+\cot(c+dx))} - $$

$$= -\frac{25}{8a^2d\sqrt{\cot(c+dx)}} + \frac{7i}{8a^2d\sqrt{\cot(c+dx)}\,(i+\cot(c+dx))} - $$

$$= -\frac{25}{8a^2d\sqrt{\cot(c+dx)}} + \frac{7i}{8a^2d\sqrt{\cot(c+dx)}\,(i+\cot(c+dx))} - $$

$$= -\frac{25}{8a^2d\sqrt{\cot(c+dx)}} + \frac{7i}{8a^2d\sqrt{\cot(c+dx)}\,(i+\cot(c+dx))} - $$

$$= \frac{\left(\frac{25}{16}-\frac{21i}{16}\right)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,a^2d} - \frac{\left(\frac{25}{16}-\frac{21i}{16}\right)\tan^{-1}\left(1\right.}{\sqrt{}}$$

Mathematica [A]
time = 1.66, size = 232, normalized size = 0.91

$$\frac{\cot^{\frac{3}{2}}(c+dx)\csc(c+dx)\sec^2(c+dx)\left(-43i\cos(c+dx)+43i\cos(3(c+dx))+23\sin(c+dx)+(25+21i)\cos(2(c+dx))\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\sqrt{\sin(2(c+dx))}-(21-25i)\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\sin^{\frac{3}{2}}(2(c+dx))-(21+25i)\text{ArcSin}(\cos(c+dx)-\sin(c+dx))\sqrt{\sin(2(c+dx))}\,(-i\cos(2(c+dx))+\sin(2(c+dx)))-41\sin(3(c+dx))\right)}{32a^2d(i+\cot(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cot[c + d*x]^(7/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (Cot[c + d*x]^(3/2)*Csc[c + d*x]*Sec[c + d*x]^2*((-43*I)*Cos[c + d*x] + (43
*I)*Cos[3*(c + d*x)] + 23*Sin[c + d*x] + (25 + 21*I)*Cos[2*(c + d*x)]*Log[C
os[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d*x)]]
 - (21 - 25*I)*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*Si
```

```
n[2*(c + d*x)]^(3/2) - (21 + 25*I)*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Sqrt
[Sin[2*(c + d*x)]]*((-I)*Cos[2*(c + d*x)] + Sin[2*(c + d*x)]) - 41*Sin[3*(c
 + d*x)]))/(32*a^2*d*(I + Cot[c + d*x])^2)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 28.62, size = 6941, normalized size = 27.33

method	result	size
default	Expression too large to display	6941

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 602 vs. 2(187) = 374.
time = 1.52, size = 602, normalized size = 2.37

$$-\frac{4\left(a^{2}de^{(6i\,dx+6i\,c)}+a^{2}de^{(4i\,dx+4i\,c)}\right)\sqrt{-\frac{i}{16\,a^{4}d^{2}}}\log\left(-2\left(4\left(i\,a^{2}de^{(2i\,dx+2i\,c)}-i\,a^{2}d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{-\frac{i}{16\,a^{4}d^{2}}}-i\,e^{(2i\,dx+2i\,c)}\right)e^{(-2i\,dx-2i\,c)}\right)-4\left(a^{2}de^{(6i\,dx+6i\,c)}+a^{2}de^{(4i\,dx+4i\,c)}\right)\sqrt{-\frac{i}{16\,a^{4}d^{2}}}\log\left(-2\left(4\left(-i\,a^{2}de^{(2i\,dx+2i\,c)}+i\,a^{2}d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{-\frac{i}{16\,a^{4}d^{2}}}-i\,e^{(2i\,dx+2i\,c)}\right)e^{(-2i\,dx-2i\,c)}\right)+4\left(a^{2}de^{(6i\,dx+6i\,c)}+a^{2}de^{(4i\,dx+4i\,c)}\right)\sqrt{\frac{529i}{64\,a^{4}d^{2}}}\log\left(-\frac{\left(8\left(a^{2}de^{(2i\,dx+2i\,c)}-a^{2}d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{529i}{64\,a^{4}d^{2}}}+23\right)e^{(-2i\,dx-2i\,c)}}{8\,a^{2}d}\right)-4\left(a^{2}de^{(6i\,dx+6i\,c)}+a^{2}de^{(4i\,dx+4i\,c)}\right)\sqrt{\frac{529i}{64\,a^{4}d^{2}}}\log\left(\frac{\left(8\left(a^{2}de^{(2i\,dx+2i\,c)}-a^{2}d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{529i}{64\,a^{4}d^{2}}}-23\right)e^{(-2i\,dx-2i\,c)}}{8\,a^{2}d}\right)-\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\left(42i\,e^{(6i\,dx+6i\,c)}-33i\,e^{(4i\,dx+4i\,c)}-10i\,e^{(2i\,dx+2i\,c)}+i\right)}{16\left(a^{2}de^{(6i\,dx+6i\,c)}+a^{2}de^{(4i\,dx+4i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*(a^2*d*e^(6*I*d*x + 6*I*c) + a^2*d*e^(4*I*d*x + 4*I*c))*sqrt(-1/16
*I/(a^4*d^2))*log(-2*(4*(I*a^2*d*e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt((I*e^(
2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-1/16*I/(a^4*d^2)) -
I*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)) - 4*(a^2*d*e^(6*I*d*x + 6*I*c)
 + a^2*d*e^(4*I*d*x + 4*I*c))*sqrt(-1/16*I/(a^4*d^2))*log(-2*(4*(-I*a^2*d*e
^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x
+ 2*I*c) - 1))*sqrt(-1/16*I/(a^4*d^2)) - I*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x
 - 2*I*c)) + 4*(a^2*d*e^(6*I*d*x + 6*I*c) + a^2*d*e^(4*I*d*x + 4*I*c))*sqrt
(529/64*I/(a^4*d^2))*log(-1/8*(8*(a^2*d*e^(2*I*d*x + 2*I*c) - a^2*d)*sqrt((
I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(529/64*I/(a^4*d^
```

```
2)) + 23)*e^(-2*I*d*x - 2*I*c)/(a^2*d)) - 4*(a^2*d*e^(6*I*d*x + 6*I*c) + a^
2*d*e^(4*I*d*x + 4*I*c))*sqrt(529/64*I/(a^4*d^2))*log(1/8*(8*(a^2*d*e^(2*I*
d*x + 2*I*c) - a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c)
 - 1))*sqrt(529/64*I/(a^4*d^2)) - 23)*e^(-2*I*d*x - 2*I*c)/(a^2*d)) - sqrt(
(I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*(42*I*e^(6*I*d*x + 6
*I*c) - 33*I*e^(4*I*d*x + 4*I*c) - 10*I*e^(2*I*d*x + 2*I*c) + I))/(a^2*d*e^
(6*I*d*x + 6*I*c) + a^2*d*e^(4*I*d*x + 4*I*c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(7/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*tan(d*x + c) + a)^2*cot(d*x + c)^(7/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cot\left(c+d\,x\right)^{7/2}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(7/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

```
[Out] int(1/(cot(c + d*x)^(7/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

3.746 $\int \frac{\sqrt{\cot(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=273

$$\frac{\left(\frac{7}{16}-\frac{5i}{16}\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,a^3d}-\frac{\left(\frac{7}{16}-\frac{5i}{16}\right)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,a^3d}+\frac{\cot^{\frac{5}{2}}(c+}{6d(ia+a\cot(c}$$

```
[Out] 1/6*cot(d*x+c)^(5/2)/d/(I*a+a*cot(d*x+c))^3+1/3*cot(d*x+c)^(3/2)/a/d/(I*a+a
*cot(d*x+c))^2+(-7/32+5/32*I)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/a^3/d*2^(
1/2)+(-7/32+5/32*I)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/a^3/d*2^(1/2)-(7/64+
5/64*I)*ln(1+cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2))/a^3/d*2^(1/2)+(7/64+5/64*
I)*ln(1+cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2))/a^3/d*2^(1/2)+5/8*cot(d*x+c)^(
1/2)/d/(I*a^3+a^3*cot(d*x+c))
```

Rubi [A]

time = 0.32, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3754, 3639, 3676, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\left(\frac{7}{16}-\frac{5i}{16}\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,a^3d}-\frac{\left(\frac{7}{16}-\frac{5i}{16}\right)\text{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,a^3d}+\frac{5\sqrt{\cot(c+dx)}}{8d\left(a^3\cot(c+dx)+ia^3\right)}-\frac{\left(\frac{7}{32}+\frac{5i}{32}\right)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,a^3d}+\frac{\left(\frac{7}{32}+\frac{5i}{32}\right)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,a^3d}+\frac{\cot^{\frac{5}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}+\frac{\cot^{\frac{3}{2}}(c+dx)}{3ad(a\cot(c+dx)+ia)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cot[c + d*x]]/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((7/16 - (5*I)/16)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*a^3*d)
- ((7/16 - (5*I)/16)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*a^3*d
) + Cot[c + d*x]^(5/2)/(6*d*(I*a + a*Cot[c + d*x])^3) + Cot[c + d*x]^(3/2)/
(3*a*d*(I*a + a*Cot[c + d*x])^2) + (5*Sqrt[Cot[c + d*x]])/(8*d*(I*a^3 + a^3
*Cot[c + d*x])) - ((7/32 + (5*I)/32)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + C
ot[c + d*x]])/(Sqrt[2]*a^3*d) + ((7/32 + (5*I)/32)*Log[1 + Sqrt[2]*Sqrt[Cot
[c + d*x]] + Cot[c + d*x]])/(Sqrt[2]*a^3*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{\sqrt{\cot(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{\cot^{\frac{7}{2}}(c+dx)}{(ia+a\cot(c+dx))^3}\,dx$$

$$= \frac{\cot^{\frac{5}{2}}(c+dx)}{6d(ia+a\cot(c+dx))^3} + \frac{\int \frac{\cot^{\frac{3}{2}}(c+dx)\left(-\frac{5ia}{2}+\frac{11}{2}a\cot(c+dx)\right)}{(ia+a\cot(c+dx))^2}\,dx}{6a^2}$$

$$= \frac{\cot^{\frac{5}{2}}(c+dx)}{6d(ia+a\cot(c+dx))^3} + \frac{\cot^{\frac{3}{2}}(c+dx)}{3ad(ia+a\cot(c+dx))^2} + \frac{\int \frac{\sqrt{\cot(c+dx)}\,(-12ia^2}{ia+a\cot(c+d}}{24a^4}$$

$$= \frac{\cot^{\frac{5}{2}}(c+dx)}{6d(ia+a\cot(c+dx))^3} + \frac{\cot^{\frac{3}{2}}(c+dx)}{3ad(ia+a\cot(c+dx))^2} + \frac{5\sqrt{\cot(c+dx)}}{8d\,(ia^3+a^3\cot(c+dx)}$$

$$= \frac{\cot^{\frac{5}{2}}(c+dx)}{6d(ia+a\cot(c+dx))^3} + \frac{\cot^{\frac{3}{2}}(c+dx)}{3ad(ia+a\cot(c+dx))^2} + \frac{5\sqrt{\cot(c+dx)}}{8d\,(ia^3+a^3\cot(c+dx)}$$

$$= \frac{\cot^{\frac{5}{2}}(c+dx)}{6d(ia+a\cot(c+dx))^3} + \frac{\cot^{\frac{3}{2}}(c+dx)}{3ad(ia+a\cot(c+dx))^2} + \frac{5\sqrt{\cot(c+dx)}}{8d\,(ia^3+a^3\cot(c+dx)}$$

$$= \frac{\cot^{\frac{5}{2}}(c+dx)}{6d(ia+a\cot(c+dx))^3} + \frac{\cot^{\frac{3}{2}}(c+dx)}{3ad(ia+a\cot(c+dx))^2} + \frac{5\sqrt{\cot(c+dx)}}{8d\,(ia^3+a^3\cot(c+dx)}$$

$$= \frac{\cot^{\frac{5}{2}}(c+dx)}{6d(ia+a\cot(c+dx))^3} + \frac{\cot^{\frac{3}{2}}(c+dx)}{3ad(ia+a\cot(c+dx))^2} + \frac{5\sqrt{\cot(c+dx)}}{8d\,(ia^3+a^3\cot(c+dx)}$$

$$= \frac{\left(\frac{7}{16}-\frac{5i}{16}\right)\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,a^3d} - \frac{\left(\frac{7}{16}-\frac{5i}{16}\right)\tan^{-1}\left(1+\sqrt{2}\sqrt{\cot(}\right.}{\sqrt{2}\,a^3d}$$

Mathematica [A]
time = 1.39, size = 235, normalized size = 0.86

$$\frac{\cot^{\frac{5}{2}}(c+dx)\csc(c+dx)\sec^3(c+dx)\left(19i-19i\cos(4(c+dx))+(21+15i)\cos(3(c+dx))\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\sqrt{\sin(2(c+dx))}+12\sin(2(c+dx))-(21-15i)\text{ArcSin}(\cos(c+dx)-\sin(c+dx))\sqrt{\sin(2(c+dx))}(\cos(3(c+dx))+i\sin(3(c+dx)))-(15-21i)\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\sqrt{\sin(2(c+dx))}\sin(3(c+dx))+21\sin(4(c+dx))\right)}{96a^3d(i+\cot(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cot[c + d*x]]/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Cot[c + d*x]^(5/2)*Csc[c + d*x]*Sec[c + d*x]^3*(19*I - (19*I)*Cos[4*(c + d
*x)] + (21 + 15*I)*Cos[3*(c + d*x)]*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[
Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d*x)]] + 12*Sin[2*(c + d*x)] - (21 - 15*
I)*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Sqrt[Sin[2*(c + d*x)]]*(Cos[3*(c + d
*x)] + I*Sin[3*(c + d*x)]) - (15 - 21*I)*Log[Cos[c + d*x] + Sin[c + d*x] +
Sqrt[Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d*x)]]*Sin[3*(c + d*x)] + 21*Sin[4*
(c + d*x)]))/(96*a^3*d*(I + Cot[c + d*x])^3)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 13.24, size = 844, normalized size = 3.09

method	result
default	$-\frac{\left(-16i\sqrt{2}\,(\cos^5(dx+c))\sin(dx+c)-15i\sqrt{2}\,\cos(dx+c)\sin(dx+c)-16\sqrt{2}\,(\cos^7(dx+c))-3i\sqrt{-\frac{\cos(dx+c)-1-\sin(dx+c)}{\sin(dx+c)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48/a^3/d*(-16*I*2^(1/2)*cos(d*x+c)^5*sin(d*x+c)-15*I*2^(1/2)*cos(d*x+c)*
sin(d*x+c)-16*2^(1/2)*cos(d*x+c)^7+15*I*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+16*
2^(1/2)*cos(d*x+c)^6+18*I*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((c
os(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d
*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1
/2*I,1/2*2^(1/2))+16*I*2^(1/2)*cos(d*x+c)^6*sin(d*x+c)+12*I*2^(1/2)*cos(d*x
+c)^4*sin(d*x+c)-4*2^(1/2)*cos(d*x+c)^5-3*I*sin(d*x+c)*((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1
-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d
*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+4*cos(d*x+c)^4*2^(1/2)+3*(-(cos(d*x+c)-
1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2
)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))
/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)+18*((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-
1-sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi((-(cos(d*x+c)-1-sin(d
*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-21*(-(cos(d*x+c)-1-sin(d*x+
c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos
(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2*2^(1/2))*sin(d*x+c)-12*I*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)-7*cos(
d*x+c)^3*2^(1/2)+7*2^(1/2)*cos(d*x+c)^2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))*(
cos(d*x+c)/sin(d*x+c))^(1/2)/cos(d*x+c)/sin(d*x+c)^3*2^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 522 vs. 2(206) = 412.

time = 1.11, size = 522, normalized size = 1.91

$$-\frac{\left(12\,a^{3}d\sqrt{-\frac{i}{64\,a^{6}d^{2}}}\,e^{(6i\,dx+6i\,c)}\log\left(-2\left(8\left(i\,a^{3}de^{(2i\,dx+2i\,c)}-i\,a^{3}d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{-\frac{i}{64\,a^{6}d^{2}}}-i\,e^{(2i\,dx+2i\,c)}\right)e^{(-2i\,dx-2i\,c)}\right)-12\,a^{3}d\sqrt{-\frac{i}{64\,a^{6}d^{2}}}\,e^{(6i\,dx+6i\,c)}\log\left(-2\left(8\left(-i\,a^{3}de^{(2i\,dx+2i\,c)}+i\,a^{3}d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{-\frac{i}{64\,a^{6}d^{2}}}-i\,e^{(2i\,dx+2i\,c)}\right)e^{(-2i\,dx-2i\,c)}\right)+12\,a^{3}d\sqrt{\frac{9i}{16\,a^{6}d^{2}}}\,e^{(6i\,dx+6i\,c)}\log\left(-\frac{\left(4\left(a^{3}de^{(2i\,dx+2i\,c)}-a^{3}d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{9i}{16\,a^{6}d^{2}}}+3\right)e^{(-2i\,dx-2i\,c)}}{4\,a^{3}d}\right)-12\,a^{3}d\sqrt{\frac{9i}{16\,a^{6}d^{2}}}\,e^{(6i\,dx+6i\,c)}\log\left(\frac{\left(4\left(a^{3}de^{(2i\,dx+2i\,c)}-a^{3}d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{9i}{16\,a^{6}d^{2}}}-3\right)e^{(-2i\,dx-2i\,c)}}{4\,a^{3}d}\right)-\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\left(-20i\,e^{(6i\,dx+6i\,c)}+14i\,e^{(4i\,dx+4i\,c)}+5i\,e^{(2i\,dx+2i\,c)}+i\right)\right)e^{(-6i\,dx-6i\,c)}}{48\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/48*(12*a^3*d*sqrt(-1/64*I/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(-2*(8*(I*a^
3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I
*d*x + 2*I*c) - 1))*sqrt(-1/64*I/(a^6*d^2)) - I*e^(2*I*d*x + 2*I*c))*e^(-2*
I*d*x - 2*I*c)) - 12*a^3*d*sqrt(-1/64*I/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(
-2*(8*(-I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c)
+ I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-1/64*I/(a^6*d^2)) - I*e^(2*I*d*x + 2*
I*c))*e^(-2*I*d*x - 2*I*c)) + 12*a^3*d*sqrt(9/16*I/(a^6*d^2))*e^(6*I*d*x +
6*I*c)*log(-1/4*(4*(a^3*d*e^(2*I*d*x + 2*I*c) - a^3*d)*sqrt((I*e^(2*I*d*x +
 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(9/16*I/(a^6*d^2)) + 3)*e^(-2*I
*d*x - 2*I*c)/(a^3*d)) - 12*a^3*d*sqrt(9/16*I/(a^6*d^2))*e^(6*I*d*x + 6*I*c
)*log(1/4*(4*(a^3*d*e^(2*I*d*x + 2*I*c) - a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c
) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(9/16*I/(a^6*d^2)) - 3)*e^(-2*I*d*x -
 2*I*c)/(a^3*d)) - sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) -
1))*(-20*I*e^(6*I*d*x + 6*I*c) + 14*I*e^(4*I*d*x + 4*I*c) + 5*I*e^(2*I*d*x
+ 2*I*c) + I))*e^(-6*I*d*x - 6*I*c)/(a^3*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int\frac{\sqrt{\cot\left(c+dx\right)}}{\tan^{3}\left(c+dx\right)-3i\tan^{2}\left(c+dx\right)-3\tan\left(c+dx\right)+i}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(1/2)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] I*Integral(sqrt(cot(c + d*x))/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*ta
n(c + d*x) + I), x)/a**3
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cot(d*x + c))/(I*a*tan(d*x + c) + a)^3, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cot (c+d\,x)}}{{\left(a+a\,\tan (c+d\,x)\,1{}\mathrm{i}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(1/2)/(a + a*tan(c + d*x)*1i)^3,x)
[Out] int(cot(c + d*x)^(1/2)/(a + a*tan(c + d*x)*1i)^3, x)
```

3.747 $\int \frac{1}{\sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=267

$$-\frac{i\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{8\sqrt{2}\,a^3d}+\frac{i\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{8\sqrt{2}\,a^3d}+\frac{\cot^{\frac{3}{2}}(c+dx)}{6d(ia+a\cot(c+dx))^3}+\frac{\sqrt{}}{4ad(ia}$$

```
[Out] 1/6*cot(d*x+c)^(3/2)/d/(I*a+a*cot(d*x+c))^3+1/16*I*arctan(-1+2^(1/2)*cot(d*
x+c)^(1/2))/a^3/d*2^(1/2)+1/16*I*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/a^3/d*2
^(1/2)+1/32*I*ln(1+cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2))/a^3/d*2^(1/2)-1/32*
I*ln(1+cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2))/a^3/d*2^(1/2)+1/4*cot(d*x+c)^(1
/2)/a/d/(I*a+a*cot(d*x+c))^2+1/4*I*cot(d*x+c)^(1/2)/d/(I*a^3+a^3*cot(d*x+c)
)
```

Rubi [A]
time = 0.31, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3754, 3639, 3676, 3677, 12, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{i\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{8\sqrt{2}\,a^3d}+\frac{i\text{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{8\sqrt{2}\,a^3d}+\frac{i\sqrt{\cot(c+dx)}}{4d\left(a^3\cot(c+dx)+ia^3\right)}+\frac{i\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{16\sqrt{2}\,a^3d}-\frac{i\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{16\sqrt{2}\,a^3d}+\frac{\cot^{\frac{3}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}+\frac{\sqrt{\cot(c+dx)}}{4ad(a\cot(c+dx)+ia)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^3),x]
```

```
[Out] ((-1/8*I)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*a^3*d) + ((I/8)*
ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*a^3*d) + Cot[c + d*x]^(3/2
)/(6*d*(I*a + a*Cot[c + d*x])^3) + Sqrt[Cot[c + d*x]]/(4*a*d*(I*a + a*Cot[c
 + d*x])^2) + ((I/4)*Sqrt[Cot[c + d*x]])/(d*(I*a^3 + a^3*Cot[c + d*x])) + (
(I/16)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(Sqrt[2]*a^3*d)
- ((I/16)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(Sqrt[2]*a^3*
d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^3}\,dx = \int \frac{\cot^{\frac{5}{2}}(c+dx)}{(ia+a\cot(c+dx))^3}\,dx$$

$$= \frac{\cot^{\frac{3}{2}}(c+dx)}{6d(ia+a\cot(c+dx))^3} + \frac{\int \frac{\sqrt{\cot(c+dx)}\left(-\frac{3ia}{2}+\frac{9}{2}a\cot(c+dx)\right)}{(ia+a\cot(c+dx))^2}\,d}{6a^2}$$

$$= \frac{\cot^{\frac{3}{2}}(c+dx)}{6d(ia+a\cot(c+dx))^3} + \frac{\sqrt{\cot(c+dx)}}{4ad(ia+a\cot(c+dx))^2} + \frac{\int \frac{}{\sqrt{\mathrm{co}}}}{}$$

$$= \frac{\cot^{\frac{3}{2}}(c+dx)}{6d(ia+a\cot(c+dx))^3} + \frac{\sqrt{\cot(c+dx)}}{4ad(ia+a\cot(c+dx))^2} + \frac{i\sqrt{}}{4d\,(ia^3}$$

$$= \frac{\cot^{\frac{3}{2}}(c+dx)}{6d(ia+a\cot(c+dx))^3} + \frac{\sqrt{\cot(c+dx)}}{4ad(ia+a\cot(c+dx))^2} + \frac{i\sqrt{}}{4d\,(ia^3}$$

$$= \frac{\cot^{\frac{3}{2}}(c+dx)}{6d(ia+a\cot(c+dx))^3} + \frac{\sqrt{\cot(c+dx)}}{4ad(ia+a\cot(c+dx))^2} + \frac{i\sqrt{}}{4d\,(ia^3}$$

$$= \frac{\cot^{\frac{3}{2}}(c+dx)}{6d(ia+a\cot(c+dx))^3} + \frac{\sqrt{\cot(c+dx)}}{4ad(ia+a\cot(c+dx))^2} + \frac{i\sqrt{}}{4d\,(ia^3}$$

$$= \frac{\cot^{\frac{3}{2}}(c+dx)}{6d(ia+a\cot(c+dx))^3} + \frac{\sqrt{\cot(c+dx)}}{4ad(ia+a\cot(c+dx))^2} + \frac{i\sqrt{}}{4d\,(ia^3}$$

$$= \frac{\cot^{\frac{3}{2}}(c+dx)}{6d(ia+a\cot(c+dx))^3} + \frac{\sqrt{\cot(c+dx)}}{4ad(ia+a\cot(c+dx))^2} + \frac{i\sqrt{}}{4d\,(ia^3}$$

$$= \frac{\cot^{\frac{3}{2}}(c+dx)}{6d(ia+a\cot(c+dx))^3} + \frac{\sqrt{\cot(c+dx)}}{4ad(ia+a\cot(c+dx))^2} + \frac{i\sqrt{}}{4d\,(ia^3}$$

$$= -\frac{i\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{8\sqrt{2}\,a^3d} + \frac{i\tan^{-1}\left(1+\sqrt{2}\sqrt{\cot(}\right.}{8\sqrt{2}\,a^3d}$$

Mathematica [A]
time = 1.03, size = 231, normalized size = 0.87

$$\frac{\sqrt{\cot(c+dx)}\csc^3(c+dx)\sec(c+dx)\left(-1+\cos(4(c+dx))-6i\cos(3(c+dx))\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\sqrt{\sin(2(c+dx))}+6i\sin(2(c+dx))+6i\mathrm{ArcSin}(\cos(c+dx)-\sin(c+dx))\sqrt{\sin(2(c+dx))}(\cos(3(c+dx))+i\sin(3(c+dx)))+6\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\sqrt{\sin(2(c+dx))}\sin(3(c+dx))+3i\sin(4(c+dx))\right)}{96a^3d(i+\cot(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^3),x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*Csc[c + d*x]^3*Sec[c + d*x]*(-1 + Cos[4*(c + d*x)] - (6
*I)*Cos[3*(c + d*x)]*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)
```

```
]]]*Sqrt[Sin[2*(c + d*x)]] + (6*I)*Sin[2*(c + d*x)] + (6*I)*ArcSin[Cos[c +
d*x] - Sin[c + d*x]]*Sqrt[Sin[2*(c + d*x)]]*(Cos[3*(c + d*x)] + I*Sin[3*(c
+ d*x)]) + 6*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sqrt
[Sin[2*(c + d*x)]]*Sin[3*(c + d*x)] + (3*I)*Sin[4*(c + d*x)]))/(96*a^3*d*(I
 + Cot[c + d*x])^3)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 14.03, size = 9445, normalized size = 35.37

method	result	size
default	Expression too large to display	9445

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 518 vs. 2(206) = 412.
time = 1.26, size = 518, normalized size = 1.94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/48*(12*a^3*d*sqrt(1/64*I/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(2*(8*(a^3*d*
e^(2*I*d*x + 2*I*c) - a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
 2*I*c) - 1))*sqrt(1/64*I/(a^6*d^2)) + I*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x -
 2*I*c)) - 12*a^3*d*sqrt(1/64*I/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(-2*(8*(a
^3*d*e^(2*I*d*x + 2*I*c) - a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*
d*x + 2*I*c) - 1))*sqrt(1/64*I/(a^6*d^2)) - I*e^(2*I*d*x + 2*I*c))*e^(-2*I*
d*x - 2*I*c)) - 12*a^3*d*sqrt(-1/64*I/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(1/
8*(8*(a^3*d*e^(2*I*d*x + 2*I*c) - a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(
```

```
e^(2*I*d*x + 2*I*c) - 1))*sqrt(-1/64*I/(a^6*d^2)) + I)*e^(-2*I*d*x - 2*I*c)
/(a^3*d)) + 12*a^3*d*sqrt(-1/64*I/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(-1/8*(
8*(a^3*d*e^(2*I*d*x + 2*I*c) - a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(
2*I*d*x + 2*I*c) - 1))*sqrt(-1/64*I/(a^6*d^2)) - I)*e^(-2*I*d*x - 2*I*c)/(a
^3*d)) - sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*(2*e^(
6*I*d*x + 6*I*c) + e^(4*I*d*x + 4*I*c) - 2*e^(2*I*d*x + 2*I*c) - 1))*e^(-6*
I*d*x - 6*I*c)/(a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int\frac{1}{\tan^3(c+dx)\sqrt{\cot(c+dx)}-3i\tan^2(c+dx)\sqrt{\cot(c+dx)}-3\tan(c+dx)\sqrt{\cot(c+dx)}+i\sqrt{\cot(c+dx)}}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(1/2)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] I*Integral(1/(tan(c + d*x)**3*sqrt(cot(c + d*x)) - 3*I*tan(c + d*x)**2*sqrt
(cot(c + d*x)) - 3*tan(c + d*x)*sqrt(cot(c + d*x)) + I*sqrt(cot(c + d*x))),
 x)/a**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*tan(d*x + c) + a)^3*sqrt(cot(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{1}{\sqrt{\cot(c+d\,x)}\,{\left(a+a\,\tan(c+d\,x)\,1{}\mathrm{i}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^3),x)
```

```
[Out] int(1/(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^3), x)
```

3.748 $\int \frac{1}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=141

$$-\frac{(-1)^{3/4}\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{8a^3d}+\frac{\sqrt{\cot(c+dx)}}{6d(ia+a\cot(c+dx))^3}+\frac{i\sqrt{\cot(c+dx)}}{6ad(ia+a\cot(c+dx))^2}+\frac{\sqrt{\cot(}}{8d\,(ia^3+a^3}$$

```
[Out] -1/8*(-1)^(3/4)*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/a^3/d+1/6*cot(d*x+c)^(
1/2)/d/(I*a+a*cot(d*x+c))^3+1/6*I*cot(d*x+c)^(1/2)/a/d/(I*a+a*cot(d*x+c))^2
+1/8*cot(d*x+c)^(1/2)/d/(I*a^3+a^3*cot(d*x+c))
```

Rubi [A]
time = 0.20, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {3754, 3639, 3677, 12, 3630, 3614, 214}

$$\frac{\sqrt{\cot(c+dx)}}{8d\,(a^3\cot(c+dx)+ia^3)}-\frac{(-1)^{3/4}\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{8a^3d}+\frac{i\sqrt{\cot(c+dx)}}{6ad(a\cot(c+dx)+ia)^2}+\frac{\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^3),x]
```

```
[Out] -1/8*((-1)^(3/4)*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/(a^3*d) + Sqrt[Cot
[c + d*x]]/(6*d*(I*a + a*Cot[c + d*x])^3) + ((I/6)*Sqrt[Cot[c + d*x]])/(a*d
*(I*a + a*Cot[c + d*x])^2) + Sqrt[Cot[c + d*x]]/(8*d*(I*a^3 + a^3*Cot[c + d
*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3614

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2*(c^2/f), Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b*
Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

Rule 3630

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(a*c + b*d))*((c + d*Tan[e + f*x])^n/(2*(
b*c - a*d)*f*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c
+ d*Tan[e + f*x])^(n - 1)*Simp[a*c*d*(n - 1) + b*c^2 + b*d^2*n - d*(b*c - a
*d)*(n - 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[0, n, 1]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx = \int \frac{\cot^{\frac{3}{2}}(c+dx)}{(ia+a\cot(c+dx))^3}\,dx$$

$$= \frac{\sqrt{\cot(c+dx)}}{6d(ia+a\cot(c+dx))^3} + \frac{\int \frac{-\frac{ia}{2}+\frac{7}{2}a\cot(c+dx)}{\sqrt{\cot(c+dx)}\,(ia+a\cot(c+dx))^2}\,dx}{6a^2}$$

$$= \frac{\sqrt{\cot(c+dx)}}{6d(ia+a\cot(c+dx))^3} + \frac{i\sqrt{\cot(c+dx)}}{6ad(ia+a\cot(c+dx))^2} + \frac{\int -\frac{6ia^2\sqrt{c}}{ia+a}}{2}$$

$$= \frac{\sqrt{\cot(c+dx)}}{6d(ia+a\cot(c+dx))^3} + \frac{i\sqrt{\cot(c+dx)}}{6ad(ia+a\cot(c+dx))^2} - \frac{i\int \frac{\sqrt{\cot(c}}{ia+a\,\mathrm{co}}}{4a}$$

$$= \frac{\sqrt{\cot(c+dx)}}{6d(ia+a\cot(c+dx))^3} + \frac{i\sqrt{\cot(c+dx)}}{6ad(ia+a\cot(c+dx))^2} + \frac{\sqrt{\cot}}{8d\,(ia^3+a}$$

$$= \frac{\sqrt{\cot(c+dx)}}{6d(ia+a\cot(c+dx))^3} + \frac{i\sqrt{\cot(c+dx)}}{6ad(ia+a\cot(c+dx))^2} + \frac{\sqrt{\cot}}{8d\,(ia^3+a}$$

$$= -\frac{(-1)^{3/4}\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{8a^3d} + \frac{\sqrt{\cot(c+dx)}}{6d(ia+a\cot(c+d}$$

Mathematica [A]
time = 1.67, size = 154, normalized size = 1.09

$$\frac{i\sqrt{\cot(c+dx)}\,\csc^3(c+dx)\left(5\cos(c+dx)-5\cos(3(c+dx))+3i\sin(c+dx)-3i\sin(3(c+dx))+6\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\right)(\cos(3(c+dx))+i\sin(3(c+dx)))\sqrt{i\tan(c+dx)}\right)}{48a^3d(i+\cot(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^3),x]
```

```
[Out] ((I/48)*Sqrt[Cot[c + d*x]]*Csc[c + d*x]^3*(5*Cos[c + d*x] - 5*Cos[3*(c + d*
x)] + (3*I)*Sin[c + d*x] - (3*I)*Sin[3*(c + d*x)] + 6*ArcTanh[Sqrt[(-1 + E^
((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]]*(Cos[3*(c + d*x)] + I*Sin[3*
(c + d*x)])*Sqrt[I*Tan[c + d*x]]))/(a^3*d*(I + Cot[c + d*x])^3)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 15.08, size = 5851, normalized size = 41.50

method	result	size
default	Expression too large to display	5851

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 307 vs. 2(113) = 226.
time = 1.68, size = 307, normalized size = 2.18

$$\frac{\left(12\,a^3d\sqrt{-\frac{i}{64\,a^6d^2}}\,e^{(6i\,dx+6i\,c)}\log\left(-2\left(8\left(i\,a^3de^{(2i\,dx+2i\,c)}-i\,a^3d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{-\frac{i}{64\,a^6d^2}}-i\,e^{(2i\,dx+2i\,c)}\right)e^{(-2i\,dx-2i\,c)}\right)-12\,a^3d\sqrt{-\frac{i}{64\,a^6d^2}}\,e^{(6i\,dx+6i\,c)}\log\left(-2\left(8\left(-i\,a^3de^{(2i\,dx+2i\,c)}+i\,a^3d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{-\frac{i}{64\,a^6d^2}}-i\,e^{(2i\,dx+2i\,c)}\right)e^{(-2i\,dx-2i\,c)}\right)+\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\left(-4i\,e^{(6i\,dx+6i\,c)}+4i\,e^{(4i\,dx+4i\,c)}+i\,e^{(2i\,dx+2i\,c)}-i\right)\right)e^{(-6i\,dx-6i\,c)}}{48\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/48*(12*a^3*d*sqrt(-1/64*I/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(-2*(8*(I*a^3
*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*
d*x + 2*I*c) - 1))*sqrt(-1/64*I/(a^6*d^2)) - I*e^(2*I*d*x + 2*I*c))*e^(-2*I
*d*x - 2*I*c)) - 12*a^3*d*sqrt(-1/64*I/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(-
2*(8*(-I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) +
 I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-1/64*I/(a^6*d^2)) - I*e^(2*I*d*x + 2*I
*c))*e^(-2*I*d*x - 2*I*c)) + sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
 2*I*c) - 1))*(-4*I*e^(6*I*d*x + 6*I*c) + 4*I*e^(4*I*d*x + 4*I*c) + I*e^(2*
I*d*x + 2*I*c) - I))*e^(-6*I*d*x - 6*I*c)/(a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int\frac{1}{\tan^3(c+dx)\cot^{\frac{3}{2}}(c+dx)-3i\tan^2(c+dx)\cot^{\frac{3}{2}}(c+dx)-3\tan(c+dx)\cot^{\frac{3}{2}}(c+dx)+i\cot^{\frac{3}{2}}(c+dx)}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(3/2)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] I*Integral(1/(tan(c + d*x)**3*cot(c + d*x)**(3/2) - 3*I*tan(c + d*x)**2*cot
(c + d*x)**(3/2) - 3*tan(c + d*x)*cot(c + d*x)**(3/2) + I*cot(c + d*x)**(3/
2)), x)/a**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
[Out] integrate(1/((I*a*tan(d*x + c) + a)^3*cot(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cot\left(c+d\,x\right)^{3/2}\,\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^3),x)
[Out] int(1/(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^3), x)
```

3.749 $\int \frac{1}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=222

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{8\sqrt{2}\,a^3d}+\frac{\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{8\sqrt{2}\,a^3d}+\frac{i\sqrt{\cot(c+dx)}}{6d(ia+a\cot(c+dx))^3}+\frac{\sqrt{}}{12ad(ia}$$

```
[Out] 1/16*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/a^3/d*2^(1/2)+1/16*arctan(1+2^(1/2
)*cot(d*x+c)^(1/2))/a^3/d*2^(1/2)-1/32*ln(1+cot(d*x+c)-2^(1/2)*cot(d*x+c)^(
1/2))/a^3/d*2^(1/2)+1/32*ln(1+cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2))/a^3/d*2^
(1/2)+1/6*I*cot(d*x+c)^(1/2)/d/(I*a+a*cot(d*x+c))^3+1/12*cot(d*x+c)^(1/2)/a
/d/(I*a+a*cot(d*x+c))^2
```

Rubi [A]
time = 0.21, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3754, 3638, 3677, 21, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{8\sqrt{2}\,a^3d}+\frac{\text{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{8\sqrt{2}\,a^3d}-\frac{\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{16\sqrt{2}\,a^3d}+\frac{\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{16\sqrt{2}\,a^3d}+\frac{\sqrt{\cot(c+dx)}}{12ad(a\cot(c+dx)+ia)^2}+\frac{i\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^3),x]
```

```
[Out] -1/8*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/(Sqrt[2]*a^3*d) + ArcTan[1 + Sq
rt[2]*Sqrt[Cot[c + d*x]]]/(8*Sqrt[2]*a^3*d) + ((I/6)*Sqrt[Cot[c + d*x]])/(d
*(I*a + a*Cot[c + d*x])^3) + Sqrt[Cot[c + d*x]]/(12*a*d*(I*a + a*Cot[c + d*
x])^2) - Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/(16*Sqrt[2]*a^3
*d) + Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/(16*Sqrt[2]*a^3*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3638

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*Sqrt[(c_.) + (d_.)*tan[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-b)*(a + b*Tan[e + f*x])^m*(Sqrt[c + d
*Tan[e + f*x]]/(2*a*f*m)), x] + Dist[1/(4*a^2*m), Int[(a + b*Tan[e + f*x])^
(m + 1)*(Simp[2*a*c*m + b*d + a*d*(2*m + 1)*Tan[e + f*x], x]/Sqrt[c + d*Tan
[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && IntegersQ[2*m]
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx &= \int \frac{\sqrt{\cot(c+dx)}}{(ia+a\cot(c+dx))^3}\,dx \\
&= \frac{i\sqrt{\cot(c+dx)}}{6d(ia+a\cot(c+dx))^3} + \frac{\int \frac{a-5ia\cot(c+dx)}{\sqrt{\cot(c+dx)}\,(ia+a\cot(c+dx))^2}\,dx}{12a^2} \\
&= \frac{i\sqrt{\cot(c+dx)}}{6d(ia+a\cot(c+dx))^3} + \frac{\sqrt{\cot(c+dx)}}{12ad(ia+a\cot(c+dx))^2} + \frac{\int \frac{-\dots}{\sqrt{\cot(c\dots}}}{\dots} \\
&= \frac{i\sqrt{\cot(c+dx)}}{6d(ia+a\cot(c+dx))^3} + \frac{\sqrt{\cot(c+dx)}}{12ad(ia+a\cot(c+dx))^2} - \frac{\int \frac{\dots}{\sqrt{\cot(c\dots}}}{8\dots} \\
&= \frac{i\sqrt{\cot(c+dx)}}{6d(ia+a\cot(c+dx))^3} + \frac{\sqrt{\cot(c+dx)}}{12ad(ia+a\cot(c+dx))^2} + \frac{\mathrm{Subst}\left(\int \dots\right.}{\dots} \\
&= \frac{i\sqrt{\cot(c+dx)}}{6d(ia+a\cot(c+dx))^3} + \frac{\sqrt{\cot(c+dx)}}{12ad(ia+a\cot(c+dx))^2} + \frac{\mathrm{Subst}\left(\int \dots\right.}{\dots} \\
&= \frac{i\sqrt{\cot(c+dx)}}{6d(ia+a\cot(c+dx))^3} + \frac{\sqrt{\cot(c+dx)}}{12ad(ia+a\cot(c+dx))^2} + \frac{\mathrm{Subst}\left(\int \dots\right.}{\dots} \\
&= \frac{i\sqrt{\cot(c+dx)}}{6d(ia+a\cot(c+dx))^3} + \frac{\sqrt{\cot(c+dx)}}{12ad(ia+a\cot(c+dx))^2} + \frac{\mathrm{Subst}\left(\int \dots\right.}{\dots} \\
&= \frac{i\sqrt{\cot(c+dx)}}{6d(ia+a\cot(c+dx))^3} + \frac{\sqrt{\cot(c+dx)}}{12ad(ia+a\cot(c+dx))^2} - \frac{\log\left(1-\dots\right.}{\dots} \\
&= -\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{8\sqrt{2}\,a^3d} + \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{\cot(c+d\dots}\right.}{8\sqrt{2}\,a^3d}
\end{aligned}
$$

Mathematica [A]
time = 2.06, size = 224, normalized size = 1.01

$$
\frac{(\cos(3(c+dx))-i\sin(3(c+dx)))\left(6i\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\right)(\cos(3(c+dx))+i\sin(3(c+dx)))+6\mathrm{ArcTan}\left(\sqrt{\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\right)(-i\cos(3(c+dx))+\sin(3(c+dx)))+(3i\cos(c+dx)-3i\cos(3(c+dx))+\sin(c+dx)+\sin(3(c+dx)))\sqrt{i\tan(c+dx)}\right)}{48a^3d\sqrt{\cot(c+dx)}\sqrt{i\tan(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^3),x]
```

```
[Out] ((Cos[3*(c + d*x)] - I*Sin[3*(c + d*x)])*((6*I)*ArcTanh[Sqrt[(-1 + E^((2*I)
*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]]*(Cos[3*(c + d*x)] + I*Sin[3*(c + d
*x)]) + 6*ArcTan[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]
```

```
]*((-I)*Cos[3*(c + d*x)] + Sin[3*(c + d*x)]) + ((3*I)*Cos[c + d*x] - (3*I)*
Cos[3*(c + d*x)] + Sin[c + d*x] + Sin[3*(c + d*x)])*Sqrt[I*Tan[c + d*x]]))/
(48*a^3*d*Sqrt[Cot[c + d*x]]*Sqrt[I*Tan[c + d*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 14.42, size = 7710, normalized size = 34.73

method	result	size
default	Expression too large to display	7710

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 520 vs. 2(175) = 350.
time = 1.13, size = 520, normalized size = 2.34

$$\frac{12\,a^3d\sqrt{\frac{i}{64\,a^6d^2}}\,e^{(6i\,dx+6i\,c)}\log\left(2\left(8\left(a^3de^{(2i\,dx+2i\,c)}-a^3d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{i}{64\,a^6d^2}}+i\,e^{(2i\,dx+2i\,c)}\right)e^{(-2i\,dx-2i\,c)}\right)-12\,a^3d\sqrt{\frac{i}{64\,a^6d^2}}\,e^{(6i\,dx+6i\,c)}\log\left(-2\left(8\left(a^3de^{(2i\,dx+2i\,c)}-a^3d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{i}{64\,a^6d^2}}-i\,e^{(2i\,dx+2i\,c)}\right)e^{(-2i\,dx-2i\,c)}\right)+12\,a^3d\sqrt{-\frac{i}{64\,a^6d^2}}\,e^{(6i\,dx+6i\,c)}\log\left(\frac{\left(8\left(a^3de^{(2i\,dx+2i\,c)}-a^3d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{-\frac{i}{64\,a^6d^2}}+i\right)e^{(-2i\,dx-2i\,c)}}{8\,a^3d}\right)-12\,a^3d\sqrt{-\frac{i}{64\,a^6d^2}}\,e^{(6i\,dx+6i\,c)}\log\left(-\frac{\left(8\left(a^3de^{(2i\,dx+2i\,c)}-a^3d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{-\frac{i}{64\,a^6d^2}}-i\right)e^{(-2i\,dx-2i\,c)}}{8\,a^3d}\right)-\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\left(2\,e^{(6i\,dx+6i\,c)}-5\,e^{(4i\,dx+4i\,c)}+4\,e^{(2i\,dx+2i\,c)}-1\right)}{48\,a^3d}e^{(-6i\,dx-6i\,c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/48*(12*a^3*d*sqrt(1/64*I/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(2*(8*(a^3*d*e
^(2*I*d*x + 2*I*c) - a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
2*I*c) - 1))*sqrt(1/64*I/(a^6*d^2)) + I*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x -
2*I*c)) - 12*a^3*d*sqrt(1/64*I/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(-2*(8*(a^
3*d*e^(2*I*d*x + 2*I*c) - a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d
*x + 2*I*c) - 1))*sqrt(1/64*I/(a^6*d^2)) - I*e^(2*I*d*x + 2*I*c))*e^(-2*I*d
*x - 2*I*c)) + 12*a^3*d*sqrt(-1/64*I/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(1/8
*(8*(a^3*d*e^(2*I*d*x + 2*I*c) - a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e
^(2*I*d*x + 2*I*c) - 1))*sqrt(-1/64*I/(a^6*d^2)) + I)*e^(-2*I*d*x - 2*I*c)/
(a^3*d)) - 12*a^3*d*sqrt(-1/64*I/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(-1/8*(8
```

```
*(a^3*d*e^(2*I*d*x + 2*I*c) - a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2
*I*d*x + 2*I*c) - 1))*sqrt(-1/64*I/(a^6*d^2)) - I)*e^(-2*I*d*x - 2*I*c)/(a^
3*d)) - sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*(2*e^(6
*I*d*x + 6*I*c) - 5*e^(4*I*d*x + 4*I*c) + 4*e^(2*I*d*x + 2*I*c) - 1))*e^(-6
*I*d*x - 6*I*c)/(a^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(5/2)/(a+I*a*tan(d*x+c))**3,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
[Out] integrate(1/((I*a*tan(d*x + c) + a)^3*cot(d*x + c)^(5/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\cot\left(c+d\,x\right)}^{5/2}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^3),x)
[Out] int(1/(cot(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^3), x)
```

3.750 $\int \frac{1}{\cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=275

$$-\frac{\left(\frac{5}{16}-\frac{7i}{16}\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,a^3d}+\frac{\left(\frac{5}{16}-\frac{7i}{16}\right)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,a^3d}-\frac{\sqrt{\cot(c-}}{6d(ia+a\cot}$$

```
[Out] (5/32-7/32*I)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/a^3/d*2^(1/2)+(5/32-7/32*
I)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/a^3/d*2^(1/2)+(5/64+7/64*I)*ln(1+cot(
d*x+c)-2^(1/2)*cot(d*x+c)^(1/2))/a^3/d*2^(1/2)-(5/64+7/64*I)*ln(1+cot(d*x+c
)+2^(1/2)*cot(d*x+c)^(1/2))/a^3/d*2^(1/2)-1/6*cot(d*x+c)^(1/2)/d/(I*a+a*cot
(d*x+c))^3+1/3*I*cot(d*x+c)^(1/2)/a/d/(I*a+a*cot(d*x+c))^2+5/8*cot(d*x+c)^(
1/2)/d/(I*a^3+a^3*cot(d*x+c))
```

Rubi [A]

time = 0.32, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3754, 3640, 3677, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\left(\frac{5}{16}-\frac{7i}{16}\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,a^3d}+\frac{\left(\frac{5}{16}-\frac{7i}{16}\right)\text{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,a^3d}+\frac{5\sqrt{\cot(c+dx)}}{8d\left(a^3\cot(c+dx)+ia^3\right)}+\frac{\left(\frac{5}{32}+\frac{7i}{32}\right)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,a^3d}-\frac{\left(\frac{5}{32}+\frac{7i}{32}\right)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,a^3d}+\frac{i\sqrt{\cot(c+dx)}}{3ad(a\cot(c+dx)+ia)^2}-\frac{\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cot[c + d*x]^(7/2)*(a + I*a*Tan[c + d*x])^3),x]
```

```
[Out] ((-5/16 + (7*I)/16)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*a^3*d)
 + ((5/16 - (7*I)/16)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*a^3*
d) - Sqrt[Cot[c + d*x]]/(6*d*(I*a + a*Cot[c + d*x])^3) + ((I/3)*Sqrt[Cot[c
+ d*x]])/(a*d*(I*a + a*Cot[c + d*x])^2) + (5*Sqrt[Cot[c + d*x]])/(8*d*(I*a^
3 + a^3*Cot[c + d*x])) + ((5/32 + (7*I)/32)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*
x]] + Cot[c + d*x]])/(Sqrt[2]*a^3*d) - ((5/32 + (7*I)/32)*Log[1 + Sqrt[2]*S
qrt[Cot[c + d*x]] + Cot[c + d*x]])/(Sqrt[2]*a^3*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{\cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx = \int \frac{1}{\sqrt{\cot(c+dx)}\,(ia+a\cot(c+dx))^3}\,dx$$

$$= -\frac{\sqrt{\cot(c+dx)}}{6d(ia+a\cot(c+dx))^3} + \frac{\int \frac{-\frac{11ia}{2}+\frac{5}{2}a\cot(c+dx)}{\sqrt{\cot(c+dx)}\,(ia+a\cot(c+dx))^2}\,dx}{6a^2}$$

$$= -\frac{\sqrt{\cot(c+dx)}}{6d(ia+a\cot(c+dx))^3} + \frac{i\sqrt{\cot(c+dx)}}{3ad(ia+a\cot(c+dx))^2} + \frac{\int \frac{-1}{\sqrt{\cot(}}}{}$$

$$= -\frac{\sqrt{\cot(c+dx)}}{6d(ia+a\cot(c+dx))^3} + \frac{i\sqrt{\cot(c+dx)}}{3ad(ia+a\cot(c+dx))^2} + \frac{5\sqrt{c}}{8d\,(ia^3+}$$

$$= -\frac{\sqrt{\cot(c+dx)}}{6d(ia+a\cot(c+dx))^3} + \frac{i\sqrt{\cot(c+dx)}}{3ad(ia+a\cot(c+dx))^2} + \frac{5\sqrt{c}}{8d\,(ia^3+}$$

$$= -\frac{\sqrt{\cot(c+dx)}}{6d(ia+a\cot(c+dx))^3} + \frac{i\sqrt{\cot(c+dx)}}{3ad(ia+a\cot(c+dx))^2} + \frac{5\sqrt{c}}{8d\,(ia^3+}$$

$$= -\frac{\sqrt{\cot(c+dx)}}{6d(ia+a\cot(c+dx))^3} + \frac{i\sqrt{\cot(c+dx)}}{3ad(ia+a\cot(c+dx))^2} + \frac{5\sqrt{c}}{8d\,(ia^3+}$$

$$= -\frac{\sqrt{\cot(c+dx)}}{6d(ia+a\cot(c+dx))^3} + \frac{i\sqrt{\cot(c+dx)}}{3ad(ia+a\cot(c+dx))^2} + \frac{5\sqrt{c}}{8d\,(ia^3+}$$

$$= -\frac{\left(\frac{5}{16}-\frac{7i}{16}\right)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,a^3d} + \frac{\left(\frac{5}{16}-\frac{7i}{16}\right)\tan^{-1}\left(\right.}{}$$

Mathematica [A]
time = 2.08, size = 235, normalized size = 0.85

$$\frac{\cot^{\frac{5}{2}}(c+dx)\csc(c+dx)\sec^3(c+dx)\left(19i-19i\cos(4(c+dx))-(15+21i)\cos(3(c+dx))\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\sqrt{\sin(2(c+dx))}-12\sin(2(c+dx))+(21-15i)\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\sqrt{\sin(2(c+dx))}\sin(3(c+dx))+(21+15i)\mathrm{ArcSin}(\cos(c+dx)-\sin(c+dx))\sqrt{\sin(2(c+dx))}\,(-i\cos(3(c+dx))+\sin(3(c+dx)))+21\sin(4(c+dx))\right)}{96a^3d(i+\cot(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cot[c + d*x]^(7/2)*(a + I*a*Tan[c + d*x])^3),x]
```

```
[Out] (Cot[c + d*x]^(5/2)*Csc[c + d*x]*Sec[c + d*x]^3*(19*I - (19*I)*Cos[4*(c + d
*x)] - (15 + 21*I)*Cos[3*(c + d*x)]*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[
Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d*x)]] - 12*Sin[2*(c + d*x)] + (21 - 15*
I)*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c
+ d*x)]]*Sin[3*(c + d*x)] + (21 + 15*I)*ArcSin[Cos[c + d*x] - Sin[c + d*x]]
*Sqrt[Sin[2*(c + d*x)]]*((-I)*Cos[3*(c + d*x)] + Sin[3*(c + d*x)]) + 21*Sin
[4*(c + d*x)]))/(96*a^3*d*(I + Cot[c + d*x])^3)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 13.63, size = 9482, normalized size = 34.48

method	result	size
default	Expression too large to display	9482

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 522 vs. 2(206) = 412.
time = 1.04, size = 522, normalized size = 1.90

$$-\frac{12\,a^3 d\sqrt{-\frac{i}{64\,a^6d^2}}\,e^{(6i\,dx+6i\,c)}\log\left(-2\left(8\left(i\,a^3de^{(2i\,dx+2i\,c)}-i\,a^3d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{-\frac{i}{64\,a^6d^2}}-i\,e^{(2i\,dx+2i\,c)}\right)e^{(-2i\,dx-2i\,c)}\right)-12\,a^3d\sqrt{-\frac{i}{64\,a^6d^2}}\,e^{(6i\,dx+6i\,c)}\log\left(-2\left(8\left(-i\,a^3de^{(2i\,dx+2i\,c)}+i\,a^3d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{-\frac{i}{64\,a^6d^2}}-i\,e^{(2i\,dx+2i\,c)}\right)e^{(-2i\,dx-2i\,c)}\right)-12\,a^3d\sqrt{\frac{9i}{16\,a^6d^2}}\,e^{(6i\,dx+6i\,c)}\log\left(\frac{\left(4\left(a^3de^{(2i\,dx+2i\,c)}-a^3d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{9i}{16\,a^6d^2}}+3\right)e^{(-2i\,dx-2i\,c)}}{4\,a^3d}\right)+12\,a^3d\sqrt{\frac{9i}{16\,a^6d^2}}\,e^{(6i\,dx+6i\,c)}\log\left(-\frac{\left(4\left(a^3de^{(2i\,dx+2i\,c)}-a^3d\right)\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{9i}{16\,a^6d^2}}-3\right)e^{(-2i\,dx-2i\,c)}}{4\,a^3d}\right)-\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\left(-20i\,e^{(6i\,dx+6i\,c)}+26i\,e^{(4i\,dx+4i\,c)}-7i\,e^{(2i\,dx+2i\,c)}+i\right)}{48\,a^3d}\,e^{(-6i\,dx-6i\,c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/48*(12*a^3*d*sqrt(-1/64*I/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(-2*(8*(I*a^
3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I
*d*x + 2*I*c) - 1))*sqrt(-1/64*I/(a^6*d^2)) - I*e^(2*I*d*x + 2*I*c))*e^(-2*
I*d*x - 2*I*c)) - 12*a^3*d*sqrt(-1/64*I/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(
-2*(8*(-I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c)
+ I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-1/64*I/(a^6*d^2)) - I*e^(2*I*d*x + 2*
I*c))*e^(-2*I*d*x - 2*I*c)) - 12*a^3*d*sqrt(9/16*I/(a^6*d^2))*e^(6*I*d*x +
6*I*c)*log(1/4*(4*(a^3*d*e^(2*I*d*x + 2*I*c) - a^3*d)*sqrt((I*e^(2*I*d*x +
2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(9/16*I/(a^6*d^2)) + 3)*e^(-2*I*
d*x - 2*I*c)/(a^3*d)) + 12*a^3*d*sqrt(9/16*I/(a^6*d^2))*e^(6*I*d*x + 6*I*c)
*log(-1/4*(4*(a^3*d*e^(2*I*d*x + 2*I*c) - a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c
) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(9/16*I/(a^6*d^2)) - 3)*e^(-2*I*d*x -
 2*I*c)/(a^3*d)) - sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) -
```

```
1))*(-20*I*e^(6*I*d*x + 6*I*c) + 26*I*e^(4*I*d*x + 4*I*c) - 7*I*e^(2*I*d*x
+ 2*I*c) + I))*e^(-6*I*d*x - 6*I*c)/(a^3*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(7/2)/(a+I*a*tan(d*x+c))**3,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
[Out] integrate(1/((I*a*tan(d*x + c) + a)^3*cot(d*x + c)^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cot\left(c+d\,x\right)^{7/2}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(7/2)*(a + a*tan(c + d*x)*1i)^3),x)
[Out] int(1/(cot(c + d*x)^(7/2)*(a + a*tan(c + d*x)*1i)^3), x)
```

3.751 $\int \cot^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=174

$$-\frac{(1+i)\sqrt{a}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d}+\frac{26\sqrt{\cot(c+dx)}\,\sqrt{a+ia}}{15d}$$

```
[Out] (-1-I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*a^(
1/2)*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-2/15*I*cot(d*x+c)^(3/2)*(a+I*a*tan
(d*x+c))^(1/2)/d-2/5*cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(1/2)/d+26/15*cot(
d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.31, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4326, 3642, 3679, 12, 3625, 211}

$$-\frac{2\cot^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{2i\cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{15d}+\frac{26\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{15d}-\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(7/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-1 - I)*Sqrt[a]*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a
*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d + (26*Sqrt[Cot[c +
 d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(15*d) - (((2*I)/15)*Cot[c + d*x]^(3/2)*
Sqrt[a + I*a*Tan[c + d*x]])/d - (2*Cot[c + d*x]^(5/2)*Sqrt[a + I*a*Tan[c +
d*x]])/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
```

```
Q[c^2 + d^2, 0]
```

Rule 3642

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[d*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(a*(c^2 + d^2)*(n +
1)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[b*d*m - a
*c*(n + 1) + a*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
 e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0
] && LtQ[n, -1] && (IntegerQ[n] || IntegersQ[2*m, 2*n])
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \cot^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{\sqrt{a+ia\tan(c+dx)}}{\tan^{\frac{7}{2}}(c+dx)}\,dx$$

$$= -\frac{2\cot^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d} + \frac{\left(2\sqrt{\cot(c+dx)}\sqrt{\mathrm{ta}\right.}{}$$

$$= -\frac{2i\cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{15d} - \frac{2\cot^{\frac{5}{2}}(c+dx)\sqrt{a+i}}{5d}$$

$$= \frac{26\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{15d} - \frac{2i\cot^{\frac{3}{2}}(c+dx)\sqrt{a+}}{15d}$$

$$= \frac{26\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{15d} - \frac{2i\cot^{\frac{3}{2}}(c+dx)\sqrt{a+}}{15d}$$

$$= \frac{26\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{15d} - \frac{2i\cot^{\frac{3}{2}}(c+dx)\sqrt{a+}}{15d}$$

$$= -\frac{(1+i)\sqrt{a}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}}{d}$$

Mathematica [A]
time = 1.38, size = 149, normalized size = 0.86

$$\frac{e^{-i(c+dx)}\left(30e^{i(c+dx)} - 40e^{3i(c+dx)} + 34e^{5i(c+dx)} - 15\left(-1+e^{2i(c+dx)}\right)^{5/2}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{15d\left(-1+e^{2i(c+dx)}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(7/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((30*E^(I*(c + d*x)) - 40*E^((3*I)*(c + d*x)) + 34*E^((5*I)*(c + d*x)) - 15
*(-1 + E^((2*I)*(c + d*x)))^(5/2)*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I
)*(c + d*x))]])*Sqrt[Cot[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(15*d*E^(I*(
c + d*x))*(-1 + E^((2*I)*(c + d*x)))^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1145 vs. 2(139) = 278.
time = 41.19, size = 1146, normalized size = 6.59

method	result	size

default	Expression too large to display	1146

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/30/d*(-2*I*sin(d*x+c)*cos(d*x+c)*2^(1/2)-30*I*sin(d*x+c)*arctan(((-1+cos
(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+30
*I*sin(d*x+c)*cos(d*x+c)^2*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2
)-1)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-15*I*sin(d*x+c)*ln(-(((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1)/(((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1))*((-1
+cos(d*x+c))/sin(d*x+c))^(1/2)-30*sin(d*x+c)*cos(d*x+c)^2*arctan(((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-30*s
in(d*x+c)*cos(d*x+c)^2*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)
*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-15*sin(d*x+c)*cos(d*x+c)^2*ln(-(((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1)/(((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1
))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+30*I*sin(d*x+c)*cos(d*x+c)^2*arctan((
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)-30*I*sin(d*x+c)*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)*(
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+34*I*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)+15*I
*sin(d*x+c)*cos(d*x+c)^2*ln(-(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*si
n(d*x+c)-sin(d*x+c)-cos(d*x+c)+1)/(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/
2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+
34*cos(d*x+c)^3*2^(1/2)-26*I*sin(d*x+c)*2^(1/2)+30*sin(d*x+c)*arctan(((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+
30*sin(d*x+c)*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*((-1+cos
(d*x+c))/sin(d*x+c))^(1/2)+15*sin(d*x+c)*ln(-(((-1+cos(d*x+c))/sin(d*x+c))^
(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1)/(((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1))*((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)-32*2^(1/2)*cos(d*x+c)^2-28*cos(d*x+c)*2^(1/2)+26*2^(1/2))*s
in(d*x+c)*(cos(d*x+c)/sin(d*x+c))^(7/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*
x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)^3*2^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1134 vs. $2(130) = 260$.
time = 0.63, size = 1134, normalized size = 6.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/30*(3*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c)
+ 1)*((-(10*I + 10)*cos(3*d*x + 3*c) + (13*I + 13)*cos(d*x + c) - (10*I - 1
```

```
0)*sin(3*d*x + 3*c) + (13*I - 13)*sin(d*x + c))*cos(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) - 1)) + ((10*I - 10)*cos(3*d*x + 3*c) - (13*I - 13)
*cos(d*x + c) - (10*I + 10)*sin(3*d*x + 3*c) + (13*I + 13)*sin(d*x + c))*si
n(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)))*sqrt(a) + 15*(2*((I
 - 1)*cos(2*d*x + 2*c)^2 + (I - 1)*sin(2*d*x + 2*c)^2 - (2*I - 2)*cos(2*d*x
 + 2*c) + I - 1)*arctan2(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos
(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 - 1)) + 2*sin(d*x + c), 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos
(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 - 1)) + 2*cos(d*x + c)) + ((I + 1)*cos(2*d*x + 2*c)^2 + (I + 1)*sin(2*d*x
+ 2*c)^2 - (2*I + 2)*cos(2*d*x + 2*c) + I + 1)*log(4*cos(d*x + c)^2 + 4*sin
(d*x + c)^2 + 4*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x
+ 2*c) + 1)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1))^2 + s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1))^2) + 8*(cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + sin(d*x + c)*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)))))*(cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + ((-(30*I +
 30)*cos(5*d*x + 5*c) - (5*I + 5)*cos(3*d*x + 3*c) - (13*I + 13)*cos(d*x +
c) - (30*I - 30)*sin(5*d*x + 5*c) - (5*I - 5)*sin(3*d*x + 3*c) - (13*I - 13
)*sin(d*x + c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) +
8*((-(I + 1)*cos(d*x + c) - (I - 1)*sin(d*x + c))*cos(2*d*x + 2*c)^2 + (-(I
 + 1)*cos(d*x + c) - (I - 1)*sin(d*x + c))*sin(2*d*x + 2*c)^2 + 2*((I + 1)*
cos(d*x + c) + (I - 1)*sin(d*x + c))*cos(2*d*x + 2*c) - (I + 1)*cos(d*x + c
) - (I - 1)*sin(d*x + c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) - 1)) + ((30*I - 30)*cos(5*d*x + 5*c) + (5*I - 5)*cos(3*d*x + 3*c) + (13*
I - 13)*cos(d*x + c) - (30*I + 30)*sin(5*d*x + 5*c) - (5*I + 5)*sin(3*d*x +
 3*c) - (13*I + 13)*sin(d*x + c))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) - 1)) + 8*(((I - 1)*cos(d*x + c) - (I + 1)*sin(d*x + c))*cos(2*d*
x + 2*c)^2 + ((I - 1)*cos(d*x + c) - (I + 1)*sin(d*x + c))*sin(2*d*x + 2*c)
^2 + 2*(-(I - 1)*cos(d*x + c) + (I + 1)*sin(d*x + c))*cos(2*d*x + 2*c) + (I
 - 1)*cos(d*x + c) - (I + 1)*sin(d*x + c))*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) - 1)))*sqrt(a))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 - 2*cos(2*d*x + 2*c) + 1)^(5/4)*d)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(130) = 260$.
time = 0.92, size = 380, normalized size = 2.18

$$\frac{8\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\left(17\,e^{(5i\,dx+5i\,c)}-20\,e^{(3i\,dx+3i\,c)}+15\,e^{(i\,dx+i\,c)}\right)-15\left(de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{8i\,a}{d^2}}\log\left(\left(\sqrt{2}\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{8i\,a}{d^2}}+4i\,ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+15\left(de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{8i\,a}{d^2}}\log\left(-\left(\sqrt{2}\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{8i\,a}{d^2}}-4i\,ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)}{60\left(de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/60*(8*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*
c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*(17*e^(5*I*d*x + 5*I*c) - 20*e^(3*I*d*x
```

```
+ 3*I*c) + 15*e^(I*d*x + I*c)) - 15*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x
 + 2*I*c) + d)*sqrt(8*I*a/d^2)*log((sqrt(2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x
 + 2*I*c) - 1))*sqrt(8*I*a/d^2) + 4*I*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c))
+ 15*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*sqrt(8*I*a/d^2)*
log(-(sqrt(2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(8*I*a/d^2
) - 4*I*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)))/(d*e^(4*I*d*x + 4*I*c) - 2*d*
e^(2*I*d*x + 2*I*c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(7/2)*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*cot(d*x + c)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^{7/2}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(7/2)*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] int(cot(c + d*x)^(7/2)*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.752 $\int \cot^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=140

$$-\frac{(1-i)\sqrt{a}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d}-\frac{2i\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{3d}$$

```
[Out] (-1+I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*a^(
1/2)*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-2/3*cot(d*x+c)^(3/2)*(a+I*a*tan(d*
x+c))^(1/2)/d-2/3*I*cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.22, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4326, 3642, 3679, 12, 3625, 211}

$$-\frac{2\cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}-\frac{2i\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{3d}-\frac{(1-i)\sqrt{a}\,\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(5/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-1 + I)*Sqrt[a]*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a
*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d - (((2*I)/3)*Sqrt[
Cot[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d - (2*Cot[c + d*x]^(3/2)*Sqrt[a
+ I*a*Tan[c + d*x]])/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3642

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[d*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(a*(c^2 + d^2)*(n +
1)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[b*d*m - a
*c*(n + 1) + a*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
 e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0
] && LtQ[n, -1] && (IntegerQ[n] || IntegersQ[2*m, 2*n])
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \cot^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{\sqrt{a+ia\tan(c+dx)}}{\tan^{\frac{5}{2}}(c+dx)}dx$$

$$= -\frac{2\cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d} + \frac{\left(2\sqrt{\cot(c+dx)}\,\sqrt{\tan\right.}{}$$

$$= -\frac{2i\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{3d} - \frac{2\cot^{\frac{3}{2}}(c+dx)\sqrt{a+}}{3d}$$

$$= -\frac{2i\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{3d} - \frac{2\cot^{\frac{3}{2}}(c+dx)\sqrt{a+}}{3d}$$

$$= -\frac{2i\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{3d} - \frac{2\cot^{\frac{3}{2}}(c+dx)\sqrt{a+}}{3d}$$

$$= -\frac{(1-i)\sqrt{a}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}}{d}$$

Mathematica [A]
time = 1.06, size = 125, normalized size = 0.89

$$-\frac{ie^{-i(c+dx)}\left(4e^{3i(c+dx)}-3\left(-1+e^{2i(c+dx)}\right)^{3/2}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{3d\left(-1+e^{2i(c+dx)}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(5/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-1/3*I)*(4*E^((3*I)*(c + d*x)) - 3*(-1 + E^((2*I)*(c + d*x)))^(3/2)*ArcTa
nh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]])*Sqrt[Cot[c + d*x]]*Sqrt
[a + I*a*Tan[c + d*x]])/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*(c + d*x))))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1040 vs. 2(112) = 224.
time = 43.42, size = 1041, normalized size = 7.44

method	result	size
default	Expression too large to display	1041

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/d*(4*I*sin(d*x+c)*cos(d*x+c)*2^(1/2)-6*I*arctan(((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*2^(1/2)+1)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+6*I*arctan(((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*cos(d*x+c)^2+6*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)*((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+6*arctan(((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*2^(1/2)-1)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+3*ln(
-(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+
c)+1)/(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos
(d*x+c)-1))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+6*I*arctan(((-1
+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*cos(d*x+c)^2-3*I*ln(-(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+
c)+sin(d*x+c)+cos(d*x+c)-1)/(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin
(d*x+c)-sin(d*x+c)-cos(d*x+c)+1))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+3*I*ln
(-(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x
+c)-1)/(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-co
s(d*x+c)+1))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-2*I*sin(d*x+c)
*2^(1/2)-6*I*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)+4*2^(1/2)*cos(d*x+c)^2-6*arctan(((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*2^(1/2)+1)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-6*arctan(((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*((-1+cos(d*x+c))/sin(d*x+c))^(1
/2)-3*ln(-(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)
-cos(d*x+c)+1)/(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d
*x+c)+cos(d*x+c)-1))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-2*cos(d*x+c)*2^(1/2
)-2*2^(1/2))*sin(d*x+c)*(cos(d*x+c)/sin(d*x+c))^(5/2)*(a*(I*sin(d*x+c)+cos(
d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)^2*2^(1/2)
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 961 vs. $2(104) = 208$.
time = 0.59, size = 961, normalized size = 6.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(2*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) +
1)*((-(3*I - 3)*cos(3*d*x + 3*c) - (I - 1)*cos(d*x + c) + (3*I + 3)*sin(3*d
*x + 3*c) + (I + 1)*sin(d*x + c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) - 1)) + (-(3*I + 3)*cos(3*d*x + 3*c) - (I + 1)*cos(d*x + c) - (3*
I - 3)*sin(3*d*x + 3*c) - (I - 1)*sin(d*x + c))*sin(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) - 1)))*sqrt(a) + 3*(2*(-(I + 1)*cos(2*d*x + 2*c)^2
- (I + 1)*sin(2*d*x + 2*c)^2 + (2*I + 2)*cos(2*d*x + 2*c) - I - 1)*arctan2(
2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + 2*sin(d*x + c),
2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*
```

```
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + 2*cos(d*x + c))
+ ((I - 1)*cos(2*d*x + 2*c)^2 + (I - 1)*sin(2*d*x + 2*c)^2 - (2*I - 2)*cos(
2*d*x + 2*c) + I - 1)*log(4*cos(d*x + c)^2 + 4*sin(d*x + c)^2 + 4*sqrt(cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)*(cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1))^2 + sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) - 1))^2) + 8*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) - 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) - 1)))))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 -
2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 2*(((-(I - 1)*cos(d*x + c) + (I + 1
)*sin(d*x + c))*cos(2*d*x + 2*c)^2 + (-(I - 1)*cos(d*x + c) + (I + 1)*sin(d
*x + c))*sin(2*d*x + 2*c)^2 + 2*((I - 1)*cos(d*x + c) - (I + 1)*sin(d*x + c
))*cos(2*d*x + 2*c) - (I - 1)*cos(d*x + c) + (I + 1)*sin(d*x + c))*cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + ((-(I + 1)*cos(d*x + c)
- (I - 1)*sin(d*x + c))*cos(2*d*x + 2*c)^2 + (-(I + 1)*cos(d*x + c) - (I -
1)*sin(d*x + c))*sin(2*d*x + 2*c)^2 + 2*((I + 1)*cos(d*x + c) + (I - 1)*sin
(d*x + c))*cos(2*d*x + 2*c) - (I + 1)*cos(d*x + c) - (I - 1)*sin(d*x + c))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)))*sqrt(a))/((cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(5/4)*d)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(104) = 208$.
time = 0.98, size = 326, normalized size = 2.33

$$\frac{-16i\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\,e^{(3i\,dx+3i\,c)}-3\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{-\frac{8i\,a}{d^2}}\log\left(\left(\sqrt{2}\left(i\,de^{(2i\,dx+2i\,c)}-i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{-\frac{8i\,a}{d^2}}+4i\,ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+3\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{-\frac{8i\,a}{d^2}}\log\left(\left(\sqrt{2}\left(-i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{-\frac{8i\,a}{d^2}}+4i\,ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)}{12\left(de^{(2i\,dx+2i\,c)}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(-16*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x +
2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*e^(3*I*d*x + 3*I*c) - 3*(d*e^(2*I*d*
x + 2*I*c) - d)*sqrt(-8*I*a/d^2)*log((sqrt(2)*(I*d*e^(2*I*d*x + 2*I*c) - I*
d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2
*I*d*x + 2*I*c) - 1))*sqrt(-8*I*a/d^2) + 4*I*a*e^(I*d*x + I*c))*e^(-I*d*x -
 I*c)) + 3*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(-8*I*a/d^2)*log((sqrt(2)*(-I*d*
e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I
*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-8*I*a/d^2) + 4*I*a*e^(I
*d*x + I*c))*e^(-I*d*x - I*c)))/(d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(5/2)*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*cot(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^{5/2}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] int(cot(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.753 $\int \cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=102

$$\frac{(1+i)\sqrt{a}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d}-\frac{2\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{d}$$

```
[Out] (1+I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*a^(1
/2)*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-2*cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c
))^(1/2)/d
```

Rubi [A]
time = 0.13, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4326, 3629, 3625, 211}

$$\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(3/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((1 + I)*Sqrt[a]*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*
Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d - (2*Sqrt[Cot[c + d
*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3629

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-d)*(a + b*Tan[e + f*x])^m*((c + d*Ta
n[e + f*x])^(n + 1)/(f*m*(c^2 + d^2))), x] + Dist[a/(a*c - b*d), Int[(a + b
*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d,
```

```
 e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -1]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{\sqrt{a+ia\tan(c+dx)}}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

$$= -\frac{2\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d} + \left(i\sqrt{\cot(c+dx)}\sqrt{\tan\right.$$

$$= -\frac{2\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d} + \frac{\left(2a^2\sqrt{\cot(c+dx)}\sqrt{\right.}{}$$

$$= \frac{(1+i)\sqrt{a}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan}}{d}$$

Mathematica [A]
time = 0.88, size = 104, normalized size = 1.02

$$\frac{e^{-i(c+dx)}\left(-2e^{i(c+dx)}+\sqrt{-1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(3/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-2*E^(I*(c + d*x)) + Sqrt[-1 + E^((2*I)*(c + d*x))]*ArcTanh[E^(I*(c + d*x
))/Sqrt[-1 + E^((2*I)*(c + d*x))]])*Sqrt[Cot[c + d*x]]*Sqrt[a + I*a*Tan[c +
 d*x]])/(d*E^(I*(c + d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 574 vs. 2(84) = 168.
time = 43.48, size = 575, normalized size = 5.64

method	result
default	$-\frac{\left(2i\sin(dx+c)\arctan\left(\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\sqrt{2}+1\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}+2i\sin(dx+c)\arctan\left(\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\sqrt{2}-1\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(2*I*sin(d*x+c)*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)
*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+2*I*sin(d*x+c)*arctan(((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*2^(1/2)-1)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+I*sin(d*x+c
)*ln(-(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos
(d*x+c)+1)/(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c
)+cos(d*x+c)-1))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-2*sin(d*x+c)*arctan(((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)*((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)-2*sin(d*x+c)*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)-sin(d*x+c)*ln(-(((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1)/(((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1))*((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)+2*I*sin(d*x+c)*2^(1/2)+2*cos(d*x+c)*2^(1/2)-2*2^(1/2))*(cos(
d*x+c)/sin(d*x+c))^(3/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*sin
(d*x+c)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)*2^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(78) = 156$.
time = 0.58, size = 540, normalized size = 5.29

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/
4)*sqrt(a)*((2*I - 2)*arctan2(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 -
2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) - 1)) + 2*sin(d*x + c), 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 -
2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) - 1)) + 2*cos(d*x + c)) + (I + 1)*log(4*cos(d*x + c)^2 + 4*sin(d*x +
c)^2 + 4*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c)
+ 1)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1))^2 + sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1))^2) + 8*(cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + sin(d*x + c)*sin(1/2*ar
```

```
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1))))) - 4*(((I + 1)*cos(d*x + c
) + (I - 1)*sin(d*x + c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) - 1)) + (-(I - 1)*cos(d*x + c) + (I + 1)*sin(d*x + c))*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)))*sqrt(a))/((cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(78) = 156$.
time = 0.78, size = 282, normalized size = 2.76

$$-\frac{8\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\,e^{(i\,dx+i\,c)}-d\sqrt{\frac{8i\,a}{d^2}}\log\left(\left(\sqrt{2}\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{8i\,a}{d^2}}+4i\,ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+d\sqrt{\frac{8i\,a}{d^2}}\log\left(-\left(\sqrt{2}\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{8i\,a}{d^2}}-4i\,ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(8*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*
c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*e^(I*d*x + I*c) - d*sqrt(8*I*a/d^2)*log(
(sqrt(2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt
((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(8*I*a/d^2) + 4
*I*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + d*sqrt(8*I*a/d^2)*log(-(sqrt(2)*(
d*e^(2*I*d*x + 2*I*c) - d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I
*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(8*I*a/d^2) - 4*I*a*e^(I*
d*x + I*c))*e^(-I*d*x - I*c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,\cot^{\frac{3}{2}}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(3/2)*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*cot(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*cot(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^{3/2}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.754 $\int \sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=69

$$\frac{(1-i)\sqrt{a}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d}$$

```
[Out] (1-I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*a^(1
/2)*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.09, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {4326, 3625, 211}

$$\frac{(1-i)\sqrt{a}\,\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cot[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((1 - I)*Sqrt[a]*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*
Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx = \left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{\tan(c+dx)}}\,dx$$

$$= -\frac{\left(2ia^2\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\operatorname{Subst}\left(\int \frac{1}{-ia-2a^2x^2}\,dx,\right.}{d}$$

$$= \frac{(1-i)\sqrt{a}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}}{d}$$

Mathematica [A]
time = 0.77, size = 118, normalized size = 1.71

$$-\frac{ie^{-i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\sqrt{\frac{i\left(1+e^{2i(c+dx)}\right)}{-1+e^{2i(c+dx)}}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\sqrt{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cot[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-I)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[(I*(1 + E^((2*I)*(c + d*x))))/(-1
 + E^((2*I)*(c + d*x)))]*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*
x))]]*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^(I*(c + d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 399 vs. 2(56) = 112.
time = 46.09, size = 400, normalized size = 5.80

method	result
default	$-\frac{\sqrt{2}\sqrt{\frac{\cos(dx+c)}{\sin(dx+c)}}(-1+\cos(dx+c))\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(i\ln\left(\frac{\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\sqrt{2}\sin(dx+c)+\sin(dx+c)+\mathrm{co}}{-\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\sqrt{2}\sin(dx+c)+\cos(dx+c)+\mathrm{s}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*2^(1/2)*(cos(d*x+c)/sin(d*x+c))^(1/2)*(-1+cos(d*x+c))*(a*(I*sin(d*x+
c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(I*ln((((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
```

```
2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1)/(-((-1+cos(d*x+c))/sin(d*x+c))^
(1/2)*2^(1/2)*sin(d*x+c)+cos(d*x+c)+sin(d*x+c)-1))+2*I*arctan(((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)*2^(1/2)+1)+2*I*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*2^(1/2)-1)+2*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)+2*arc
tan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)+ln((-((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+cos(d*x+c)+sin(d*x+c)-1)/(((-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1)))/(I*sin(d*
x+c)+cos(d*x+c)-1)/((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(51) = 102$.
time = 0.58, size = 374, normalized size = 5.42

$$-\frac{\sqrt{a}\left(-(2i+2)\arctan\left(2\left(\cos(2dx+2c)^2+\sin(2dx+2c)^2-2\cos(2dx+2c)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\arctan(\sin(2dx+2c),\cos(2dx+2c)-1)\right)+2\sin(dx+c),2\left(\cos(2dx+2c)^2+\sin(2dx+2c)^2-2\cos(2dx+2c)+1\right)^{\frac{1}{4}}\cos\left(\frac{1}{2}\arctan(\sin(2dx+2c),\cos(2dx+2c)-1)\right)+2\cos(dx+c)\right)+(i-1)\log\left(4\cos(dx+c)^2+4\sin(dx+c)^2+4\sqrt{\cos(2dx+2c)^2+\sin(2dx+2c)^2-2\cos(2dx+2c)+1}\left(\cos\left(\frac{1}{2}\arctan(\sin(2dx+2c),\cos(2dx+2c)-1)\right)^2+\sin\left(\frac{1}{2}\arctan(\sin(2dx+2c),\cos(2dx+2c)-1)\right)^2\right)+8\left(\cos(2dx+2c)^2+\sin(2dx+2c)^2-2\cos(2dx+2c)+1\right)^{\frac{1}{4}}\left(\cos(dx+c)\cos\left(\frac{1}{2}\arctan(\sin(2dx+2c),\cos(2dx+2c)-1)\right)+\sin(dx+c)\sin\left(\frac{1}{2}\arctan(\sin(2dx+2c),\cos(2dx+2c)-1)\right)\right)\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(a)*(-(2*I + 2)*arctan2(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) - 1)) + 2*sin(d*x + c), 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) - 1)) + 2*cos(d*x + c)) + (I - 1)*log(4*cos(d*x + c)^2 + 4*sin(d*x
 + c)^2 + 4*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*
c) + 1)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1))^2 + sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1))^2) + 8*(cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + sin(d*x + c)*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)))))/d
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(51) = 102$.
time = 1.17, size = 217, normalized size = 3.14

$$\frac{1}{4}\sqrt{-\frac{8ia}{d^2}}\log\left(\left(\sqrt{2}\left(ide^{(2idx+2ic)}-id\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{-\frac{8ia}{d^2}}+4iae^{(idx+ic)}\right)e^{(-idx-ic)}\right)-\frac{1}{4}\sqrt{-\frac{8ia}{d^2}}\log\left(\left(\sqrt{2}\left(-ide^{(2idx+2ic)}+id\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{-\frac{8ia}{d^2}}+4iae^{(idx+ic)}\right)e^{(-idx-ic)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-8*I*a/d^2)*log((sqrt(2)*(I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(a/(e
^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I
*c) - 1))*sqrt(-8*I*a/d^2) + 4*I*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 1/4
*sqrt(-8*I*a/d^2)*log((sqrt(2)*(-I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(
2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c
) - 1))*sqrt(-8*I*a/d^2) + 4*I*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,\sqrt{\cot\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(1/2)*(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*sqrt(cot(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*sqrt(cot(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cot\left(c+d\,x\right)}\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.755 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{\cot(c+dx)}}\,dx$

Optimal. Leaf size=144

$$-\frac{2(-1)^{3/4}\sqrt{a}\,\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d}-\frac{(1+i)\sqrt{a}\,\tanh^{-1}\left(\frac{(1+i}{\sqrt{}}\right.}{}$$

[Out] -2*(-1)^(3/4)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*a^(1/2)*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-(1+I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*a^(1/2)*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d

Rubi [A]
time = 0.23, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4326, 3644, 3625, 211, 3680, 65, 223, 209}

$$-\frac{2(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[Cot[c + d*x]],x]

[Out] (-2*(-1)^(3/4)*Sqrt[a]*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d - ((1 + I)*Sqrt[a]*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3644

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(a*c - b*d)/a, Int[Sqrt[a + b*Tan[e + f*
x]]/Sqrt[c + d*Tan[e + f*x]], x], x] + Dist[d/a, Int[Sqrt[a + b*Tan[e + f*x
]]*((b + a*Tan[e + f*x])/Sqrt[c + d*Tan[e + f*x]]), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{\cot(c+dx)}}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \sqrt{\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}\,dx$$

$$= -\left(\left(i\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{\tan(c+dx)}}\,dx\right) + \frac{\left(\sqrt{\cot\right.}{}$$

$$= \frac{\left(ia\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\,\sqrt{a+iax}}\,dx, x, \tan(c+dx\right.}{d}$$

$$= -\frac{(1+i)\sqrt{a}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d}$$

$$= -\frac{(1+i)\sqrt{a}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d}$$

$$= -\frac{2(-1)^{3/4}\sqrt{a}\,\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+a}}{d}$$

Mathematica [F]
time = 43.24, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{\cot(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[Cot[c + d*x]],x]
[Out] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[Cot[c + d*x]], x]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 575 vs. 2(114) = 228.
time = 41.44, size = 576, normalized size = 4.00

method	result
default	$\frac{\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,\cos(dx+c)\left(i\sqrt{2}\,\ln\left(\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}+1\right)-2i\sqrt{2}\,\arctan\left(\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\right)-i\sqrt{2}\,\ln\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(1/2)/cot(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)*(I*2^(1/2)*
ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)-2*I*2^(1/2)*arctan(((-1+cos(d*x+c)
)/sin(d*x+c))^(1/2))-I*2^(1/2)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)+2^(
1/2)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)+2*2^(1/2)*arctan(((-1+cos(d*x
+c))/sin(d*x+c))^(1/2))-2^(1/2)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)+2*
I*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)+2*I*arctan(((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)+I*ln((-((-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*2^(1/2)*sin(d*x+c)+cos(d*x+c)+sin(d*x+c)-1)/(((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1))-2*arctan(((-1+cos(d*x
+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)-2*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)*2^(1/2)-1)-ln((((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin
(d*x+c)+cos(d*x+c)-1)/(-((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+
c)+cos(d*x+c)+sin(d*x+c)-1)))*(-1+cos(d*x+c))/sin(d*x+c)/(cos(d*x+c)/sin(d*
x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
2^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/cot(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)/sqrt(cot(d*x + c)), x)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 470 vs. 2(108) = 216.
time = 0.76, size = 470, normalized size = 3.26

$$-\frac{1}{4}\sqrt{\frac{8ia}{d^2}}\log\left(\left(\sqrt{2}\left(de^{(2idx+2ic)}-d\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{\frac{8ia}{d^2}}+4iae^{(idx+ic)}\right)e^{(-idx-ic)}\right)+\frac{1}{4}\sqrt{\frac{8ia}{d^2}}\log\left(-\left(\sqrt{2}\left(de^{(2idx+2ic)}-d\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{\frac{8ia}{d^2}}-4iae^{(idx+ic)}\right)e^{(-idx-ic)}\right)-\frac{1}{4}\sqrt{\frac{4ia}{d^2}}\log\left(-16\left(\sqrt{2}\left(iade^{(3idx+3ic)}-iade^{(idx+ic)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{\frac{4ia}{d^2}}+3a^2e^{(2idx+2ic)}-a^2\right)e^{(-2idx-2ic)}\right)+\frac{1}{4}\sqrt{\frac{4ia}{d^2}}\log\left(-16\left(\sqrt{2}\left(-iade^{(3idx+3ic)}+iade^{(idx+ic)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{\frac{4ia}{d^2}}+3a^2e^{(2idx+2ic)}-a^2\right)e^{(-2idx-2ic)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/cot(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(8*I*a/d^2)*log((sqrt(2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(a/(e^(2*
I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c)
- 1))*sqrt(8*I*a/d^2) + 4*I*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 1/4*sqrt
(8*I*a/d^2)*log(-(sqrt(2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sq
```

```
rt(8*I*a/d^2) - 4*I*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 1/4*sqrt(4*I*a/d
^2)*log(-16*(sqrt(2)*(I*a*d*e^(3*I*d*x + 3*I*c) - I*a*d*e^(I*d*x + I*c))*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*
x + 2*I*c) - 1))*sqrt(4*I*a/d^2) + 3*a^2*e^(2*I*d*x + 2*I*c) - a^2)*e^(-2*I
*d*x - 2*I*c)) + 1/4*sqrt(4*I*a/d^2)*log(-16*(sqrt(2)*(-I*a*d*e^(3*I*d*x +
3*I*c) + I*a*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e
^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(4*I*a/d^2) + 3*a^2*
e^(2*I*d*x + 2*I*c) - a^2)*e^(-2*I*d*x - 2*I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}{\sqrt{\cot\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)/cot(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))/sqrt(cot(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/cot(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)/sqrt(cot(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{\sqrt{\cot\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/cot(c + d*x)^(1/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^(1/2)/cot(c + d*x)^(1/2), x)
```

3.756 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{\cot^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=175

$$-\frac{\sqrt[4]{-1}\,\sqrt{a}\,\mathrm{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d}-\frac{(1-i)\sqrt{a}\,\tanh^{-1}\left(\frac{(1+i)\cdots}{\sqrt{a\cdots}}\right.}{}$$

```
[Out] -(-1)^(1/4)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(
1/2))*a^(1/2)*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+(-1+I)*arctanh((1+I)*a^(1
/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*a^(1/2)*cot(d*x+c)^(1/2)*tan
(d*x+c)^(1/2)/d+(a+I*a*tan(d*x+c))^(1/2)/d/cot(d*x+c)^(1/2)
```

Rubi [A]

time = 0.27, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4326, 3641, 21, 3636, 3625, 211, 3680, 65, 223, 209}

$$-\frac{\sqrt[4]{-1}\,\sqrt{a}\,\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\mathrm{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\cot(c+dx)}}-\frac{(1-i)\sqrt{a}\,\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[c + d*x]]/Cot[c + d*x]^(3/2),x]
```

```
[Out] -(((-1)^(1/4)*Sqrt[a]*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a
 + I*a*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d) - ((1 - I)*
Sqrt[a]*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d
*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d + Sqrt[a + I*a*Tan[c + d*x]]
/(d*Sqrt[Cot[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3636

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(c_.) + (d_.)*tan[(e_
.) + (f_.)*(x_)]], x_Symbol] :> Dist[2*a, Int[Sqrt[a + b*Tan[e + f*x]]/Sqrt
[c + d*Tan[e + f*x]], x], x] + Dist[b/a, Int[(b + a*Tan[e + f*x])*(Sqrt[a +
 b*Tan[e + f*x]]/Sqrt[c + d*Tan[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3641

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[d*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n - 1)/(f*(m + n - 1))), x] - Dist[1/(a*(m + n - 1)), Int[(a + b*
Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 2)*Simp[d*(b*c*m + a*d*(-1 + n))
- a*c^2*(m + n - 1) + d*(b*d*m - a*c*(m + 2*n - 2))*Tan[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1] && NeQ[m + n - 1, 0] && (IntegerQ[n]
|| IntegersQ[2*m, 2*n])
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
```

```
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

$$= \frac{\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\cot(c+dx)}} - \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{\left(\frac{a}{2}+\frac{1}{2}ia\tan(c+dx)\right)}{\sqrt{\mathrm{ta}}}}{a}$$

$$= \frac{\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\cot(c+dx)}} - \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{\tan(c+dx)}}}{2a}$$

$$= \frac{\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\cot(c+dx)}} - \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{\sqrt{a+ia\tan(c}}{\sqrt{\tan(c+}}$$

$$= \frac{\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\cot(c+dx)}} + \frac{\left(a\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\mathrm{Subst}\left(\int \frac{}{\sqrt{x}\,\sqrt{}}\right.}{2d}$$

$$= -\frac{(1-i)\sqrt{a}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx}}{d}$$

$$= -\frac{(1-i)\sqrt{a}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx}}{d}$$

$$= -\frac{\sqrt[4]{-1}\,\sqrt{a}\,\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx}}{d}$$

Mathematica [A]

time = 2.91, size = 223, normalized size = 1.27

$$\frac{\left(8-\frac{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\left(8\log\left(e^{i(c+dx)}+\sqrt{-1+e^{2i(c+dx)}}\right)+\sqrt{2}\left(-\log\left(1-3e^{2i(c+dx)}-2\sqrt{2}\,e^{i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\right)+\log\left(1-3e^{2i(c+dx)}+2\sqrt{2}\,e^{i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\right)\right)\right)}{\sqrt{-1+e^{2i(c+dx)}}}\right)\sqrt{a+ia\tan(c+dx)}}{8d\sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/Cot[c + d*x]^(3/2),x]
```

```
[Out] ((8 - ((1 + E^((2*I)*(c + d*x)))*(8*Log[E^(I*(c + d*x)) + Sqrt[-1 + E^((2*I
)*(c + d*x))]] + Sqrt[2]*(-Log[1 - 3*E^((2*I)*(c + d*x)) - 2*Sqrt[2]*E^(I*(
c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]] + Log[1 - 3*E^((2*I)*(c + d*x)) +
 2*Sqrt[2]*E^(I*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]])))/(E^(I*(c + d*
x))*Sqrt[-1 + E^((2*I)*(c + d*x))]))*Sqrt[a + I*a*Tan[c + d*x]])/(8*d*Sqrt[
Cot[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2092 vs. 2(140) = 280.
time = 41.99, size = 2093, normalized size = 11.96

method	result	size
default	Expression too large to display	2093

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(1/2)/cot(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*(2*I*ln(-(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(
d*x+c)+cos(d*x+c)-1)/(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)
-sin(d*x+c)-cos(d*x+c)+1))*cos(d*x+c)+4*arctan(((-1+cos(d*x+c))/sin(d*x+c))
^(1/2)*2^(1/2)+1)*sin(d*x+c)*cos(d*x+c)+2^(1/2)*ln(((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)+1)*cos(d*x+c)-2^(1/2)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)*c
os(d*x+c)+2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-4*I*arcta
n(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)*cos(d*x+c)^2-4*I*arctan(((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*cos(d*x+c)^2-2*I*ln(-(((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1)/(((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1))*
cos(d*x+c)^2-2*2^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-2^(1
/2)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)*cos(d*x+c)^2+2^(1/2)*ln(((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)-1)*cos(d*x+c)^2-2*2^(1/2)*arctan(((-1+cos(d*x+
c))/sin(d*x+c))^(1/2))*cos(d*x+c)+4*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)*2^(1/2)-1)*sin(d*x+c)*cos(d*x+c)+2*ln(-(((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1)/(((-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1))*sin(d*x+c)*cos(d*x+c)+
2*2^(1/2)*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2))*cos(d*x+c)^2-2*I*2^(1/
2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+4*I*arctan(((-1+cos(d*x+c))/sin(d*x+c
```

```
))^(1/2)*2^(1/2)+1)*cos(d*x+c)+4*I*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*2^(1/2)-1)*cos(d*x+c)-I*2^(1/2)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)*
cos(d*x+c)-2*I*2^(1/2)*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2))*cos(d*x+c
)-2*I*2^(1/2)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+4*I*arctan(((-1
+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)*sin(d*x+c)*cos(d*x+c)+4*I*arctan(
((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*sin(d*x+c)*cos(d*x+c)+2*I*ln(
-(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+
c)-1)/(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos
(d*x+c)+1))*sin(d*x+c)*cos(d*x+c)+I*2^(1/2)*ln(((-1+cos(d*x+c))/sin(d*x+c))
^(1/2)-1)*cos(d*x+c)-2^(1/2)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)*sin(d
*x+c)*cos(d*x+c)+2^(1/2)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)*sin(d*x+c
)*cos(d*x+c)-2*2^(1/2)*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2))*sin(d*x+c
)*cos(d*x+c)-4*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)*cos(d*x
+c)^2-4*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*cos(d*x+c)^2-2
*ln(-(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(
d*x+c)+1)/(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)
+cos(d*x+c)-1))*cos(d*x+c)^2+4*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^
(1/2)+1)*cos(d*x+c)+4*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*
cos(d*x+c)+2*ln(-(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin
(d*x+c)-cos(d*x+c)+1)/(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c
)+sin(d*x+c)+cos(d*x+c)-1))*cos(d*x+c)-I*2^(1/2)*ln(((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)+1)*sin(d*x+c)*cos(d*x+c)-2*I*2^(1/2)*arctan(((-1+cos(d*x+c))/si
n(d*x+c))^(1/2))*sin(d*x+c)*cos(d*x+c)-2*I*2^(1/2)*sin(d*x+c)*((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)*cos(d*x+c)+I*2^(1/2)*ln(((-1+cos(d*x+c))/sin(d*x+c))^
(1/2)-1)*sin(d*x+c)*cos(d*x+c)-2*2^(1/2)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)*cos(d*x+c)+I*2^(1/2)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)*c
os(d*x+c)^2-I*2^(1/2)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)*cos(d*x+c)^2
+2*I*2^(1/2)*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2))*cos(d*x+c)^2+2*I*2^
(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+2*((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*2^(1/2))*cos(d*x+c)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c)
)^(1/2)/(I*sin(d*x+c)+I*cos(d*x+c)-1+I-sin(d*x+c)+cos(d*x+c))/(cos(d*x+c)/s
in(d*x+c))^(3/2)/sin(d*x+c)^2/((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/cot(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)/cot(d*x + c)^(3/2), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 610 vs. 2(133) = 266.

time = 0.99, size = 610, normalized size = 3.49

$$-\frac{4\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\left(ie^{(3idx+3ic)}-ie^{(idx+ic)}\right)+\left(de^{(2idx+2ic)}+d\right)\sqrt{-\frac{8ia}{d^2}}\log\left(\left(\sqrt{2}\left(ide^{(2idx+2ic)}-id\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{-\frac{8ia}{d^2}}+4iae^{(idx+ic)}\right)e^{(-idx-ic)}\right)-\left(de^{(2idx+2ic)}+d\right)\sqrt{-\frac{8ia}{d^2}}\log\left(\left(\sqrt{2}\left(-ide^{(2idx+2ic)}+id\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{-\frac{8ia}{d^2}}+4iae^{(idx+ic)}\right)e^{(-idx-ic)}\right)-\left(de^{(2idx+2ic)}+d\right)\sqrt{-\frac{ia}{d^2}}\log\left(-16\left(2\sqrt{2}\left(ade^{(3idx+3ic)}-ade^{(idx+ic)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{-\frac{ia}{d^2}}+3a^2e^{(2idx+2ic)}-a^2\right)e^{(-2idx-2ic)}\right)+\left(de^{(2idx+2ic)}+d\right)\sqrt{-\frac{ia}{d^2}}\log\left(16\left(2\sqrt{2}\left(ade^{(3idx+3ic)}-ade^{(idx+ic)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{-\frac{ia}{d^2}}-3a^2e^{(2idx+2ic)}+a^2\right)e^{(-2idx-2ic)}\right)}{4\left(de^{(2idx+2ic)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/cot(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*
c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*(I*e^(3*I*d*x + 3*I*c) - I*e^(I*d*x + I*
c)) + (d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-8*I*a/d^2)*log((sqrt(2)*(I*d*e^(2*I
*d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x +
 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-8*I*a/d^2) + 4*I*a*e^(I*d*x +
 I*c))*e^(-I*d*x - I*c)) - (d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-8*I*a/d^2)*log
((sqrt(2)*(-I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-8*I*a/d
^2) + 4*I*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - (d*e^(2*I*d*x + 2*I*c) + d
)*sqrt(-I*a/d^2)*log(-16*(2*sqrt(2)*(a*d*e^(3*I*d*x + 3*I*c) - a*d*e^(I*d*x
 + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)
/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-I*a/d^2) + 3*a^2*e^(2*I*d*x + 2*I*c) - a^
2)*e^(-2*I*d*x - 2*I*c)) + (d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-I*a/d^2)*log(1
6*(2*sqrt(2)*(a*d*e^(3*I*d*x + 3*I*c) - a*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I
*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) -
 1))*sqrt(-I*a/d^2) - 3*a^2*e^(2*I*d*x + 2*I*c) + a^2)*e^(-2*I*d*x - 2*I*c)
))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}{\cot^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)/cot(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))/cot(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/cot(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)/cot(d*x + c)^(3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{{\cot\left(c+d\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/cot(c + d*x)^(3/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^(1/2)/cot(c + d*x)^(3/2), x)
```

3.757 $\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=218

$$-\frac{(2+2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d}-\frac{2ia^2\cot^{\frac{3}{2}}(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}-$$

```
[Out] (-2-2*I)*a^(3/2)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^
(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-2/5*I*a^2*cot(d*x+c)^(3/2)/d/(a+
I*a*tan(d*x+c))^(1/2)-2/5*a^2*cot(d*x+c)^(5/2)/d/(a+I*a*tan(d*x+c))^(1/2)-4
/5*I*a*cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(1/2)/d+12/5*a*cot(d*x+c)^(1/2)*
(a+I*a*tan(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.43, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {4326, 3634, 3677, 3679, 12, 3625, 211}

$$-\frac{(2+2i)a^{3/2}\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^2\cot^{\frac{5}{2}}(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}-\frac{2ia^2\cot^{\frac{3}{2}}(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}-\frac{4ia\cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}+\frac{12a\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(7/2)*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-2 - 2*I)*a^(3/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I
*a*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d - (((2*I)/5)*a^2
*Cot[c + d*x]^(3/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (2*a^2*Cot[c + d*x]^(
5/2))/(5*d*Sqrt[a + I*a*Tan[c + d*x]]) + (12*a*Sqrt[Cot[c + d*x]]*Sqrt[a +
I*a*Tan[c + d*x]])/(5*d) - (((4*I)/5)*a*Cot[c + d*x]^(3/2)*Sqrt[a + I*a*Tan
[c + d*x]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
```

```
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3634

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-a^2)*(b*c - a*d)*(a + b*Tan[e + f*x]
)^(m - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] + Di
st[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[b*(b*c*(m - 2) - a*d*(m - 2*n - 4)) + (a*b*c*(m - 2)
+ b^2*d*(n + 1) - a^2*d*(m + n - 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{(a+ia\tan(c+dx))^{3/2}}{\tan^{\frac{7}{2}}(c+dx)}\,dx$$

$$= -\frac{2a^2\cot^{\frac{5}{2}}(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}} - \frac{1}{5}\Big(2\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}$$

$$= -\frac{2ia^2\cot^{\frac{3}{2}}(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}} - \frac{2a^2\cot^{\frac{5}{2}}(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}} - \frac{\Big(2\sqrt{\mathrm{co}}}{}$$

$$= -\frac{2ia^2\cot^{\frac{3}{2}}(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}} - \frac{2a^2\cot^{\frac{5}{2}}(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}} - \frac{4ia\cot}{}$$

$$= -\frac{2ia^2\cot^{\frac{3}{2}}(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}} - \frac{2a^2\cot^{\frac{5}{2}}(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}} + \frac{12a\sqrt{\mathrm{c}}}{}$$

$$= -\frac{2ia^2\cot^{\frac{3}{2}}(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}} - \frac{2a^2\cot^{\frac{5}{2}}(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}} + \frac{12a\sqrt{\mathrm{c}}}{}$$

$$= -\frac{2ia^2\cot^{\frac{3}{2}}(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}} - \frac{2a^2\cot^{\frac{5}{2}}(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}} + \frac{12a\sqrt{\mathrm{c}}}{}$$

$$= -\frac{(2+2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}}{d}$$

<u>Mathematica</u> [A]
time = 1.92, size = 161, normalized size = 0.74

$$\frac{4a\left(e^{i(c+dx)}\left(5-10e^{2i(c+dx)}+9e^{4i(c+dx)}\right)-5\left(-1+e^{2i(c+dx)}\right)^{5/2}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\cos(c+dx)\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{5d\left(-1+e^{2i(c+dx)}\right)^2\left(1+e^{2i(c+dx)}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(7/2)*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (4*a*(E^(I*(c + d*x))*(5 - 10*E^((2*I)*(c + d*x)) + 9*E^((4*I)*(c + d*x)))
- 5*(-1 + E^((2*I)*(c + d*x)))^(5/2)*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((
2*I)*(c + d*x))]])*Cos[c + d*x]*Sqrt[Cot[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x
]])/(5*d*(-1 + E^((2*I)*(c + d*x)))^2*(1 + E^((2*I)*(c + d*x))))
```

<u>Maple</u> [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1146 vs. 2(175) = 350.
time = 47.15, size = 1147, normalized size = 5.26

method	result	size
default	Expression too large to display	1147

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/5/d*(-5*I*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*ln(-(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1)/(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1))-2*I*sin(d*x+c)*cos(d*x+c)*2^(1/2)-10*I*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)+10*I*cos(d*x+c)^2*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)-10*sin(d*x+c)*cos(d*x+c)^2*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-5*sin(d*x+c)*cos(d*x+c)^2*ln(-(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1)/(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-10*sin(d*x+c)*cos(d*x+c)^2*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+10*I*cos(d*x+c)^2*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)-10*I*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)+9*I*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+5*I*cos(d*x+c)^2*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*ln(-(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1)/(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1))+9*cos(d*x+c)^3*2^(1/2)-6*I*2^(1/2)*sin(d*x+c)-7*2^(1/2)*cos(d*x+c)^2+10*sin(d*x+c)*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+5*sin(d*x+c)*ln(-(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1)/(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+10*sin(d*x+c)*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-8*cos(d*x+c)*2^(1/2)+6*2^(1/2))*(cos(d*x+c)/sin(d*x+c))^(7/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*sin(d*x+c)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)^3*2^(1/2)*a

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1181 vs. $2(164) = 328$.
time = 0.65, size = 1181, normalized size = 5.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")

```
[Out] -1/15*(2*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c)
+ 1)*((-(15*I + 15)*a*cos(3*d*x + 3*c) + (16*I + 16)*a*cos(d*x + c) - (15*I
 - 15)*a*sin(3*d*x + 3*c) + (16*I - 16)*a*sin(d*x + c))*cos(3/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + ((15*I - 15)*a*cos(3*d*x + 3*c) - (
16*I - 16)*a*cos(d*x + c) - (15*I + 15)*a*sin(3*d*x + 3*c) + (16*I + 16)*a*
sin(d*x + c))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)))*sqr
t(a) + 15*(2*((I - 1)*a*cos(2*d*x + 2*c)^2 + (I - 1)*a*sin(2*d*x + 2*c)^2 -
 (2*I - 2)*a*cos(2*d*x + 2*c) + (I - 1)*a)*arctan2(2*(cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) - 1)) + 2*sin(d*x + c), 2*(cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) - 1)) + 2*cos(d*x + c)) + ((I + 1)*a*cos(2*d*x +
 2*c)^2 + (I + 1)*a*sin(2*d*x + 2*c)^2 - (2*I + 2)*a*cos(2*d*x + 2*c) + (I
+ 1)*a)*log(4*cos(d*x + c)^2 + 4*sin(d*x + c)^2 + 4*sqrt(cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)*(cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) - 1))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) - 1))^2) + 8*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*
d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) - 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) - 1)))))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x +
2*c) + 1)^(1/4)*sqrt(a) + 2*((-(15*I + 15)*a*cos(5*d*x + 5*c) + (5*I + 5)*a
*cos(3*d*x + 3*c) - (2*I + 2)*a*cos(d*x + c) - (15*I - 15)*a*sin(5*d*x + 5*
c) + (5*I - 5)*a*sin(3*d*x + 3*c) - (2*I - 2)*a*sin(d*x + c))*cos(5/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + 3*(((I + 1)*a*cos(d*x + c) +
(I - 1)*a*sin(d*x + c))*cos(2*d*x + 2*c)^2 + ((I + 1)*a*cos(d*x + c) + (I -
 1)*a*sin(d*x + c))*sin(2*d*x + 2*c)^2 + 2*(-(I + 1)*a*cos(d*x + c) - (I -
1)*a*sin(d*x + c))*cos(2*d*x + 2*c) + (I + 1)*a*cos(d*x + c) + (I - 1)*a*si
n(d*x + c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + ((15
*I - 15)*a*cos(5*d*x + 5*c) - (5*I - 5)*a*cos(3*d*x + 3*c) + (2*I - 2)*a*co
s(d*x + c) - (15*I + 15)*a*sin(5*d*x + 5*c) + (5*I + 5)*a*sin(3*d*x + 3*c)
- (2*I + 2)*a*sin(d*x + c))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) - 1)) + 3*((-(I - 1)*a*cos(d*x + c) + (I + 1)*a*sin(d*x + c))*cos(2*d*x
 + 2*c)^2 + (-(I - 1)*a*cos(d*x + c) + (I + 1)*a*sin(d*x + c))*sin(2*d*x +
2*c)^2 + 2*((I - 1)*a*cos(d*x + c) - (I + 1)*a*sin(d*x + c))*cos(2*d*x + 2*
c) - (I - 1)*a*cos(d*x + c) + (I + 1)*a*sin(d*x + c))*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) - 1)))*sqrt(a))/((cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(5/4)*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(164) = 328$.
time = 0.74, size = 402, normalized size = 1.84

$$\frac{8\sqrt{2}\left(9\,ae^{(5i\,dx+5i\,c)}-10\,ae^{(3i\,dx+3i\,c)}+5\,ae^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}-5\left(de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{32i\,a^3}{d^2}}\log\left(\frac{\left(\sqrt{2}\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{\frac{32i\,a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}+8i\,a^2e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{2\,a}\right)+5\left(de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{32i\,a^3}{d^2}}\log\left(-\frac{\left(\sqrt{2}\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{\frac{32i\,a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}-8i\,a^2e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{2\,a}\right)}{20\left(de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/20*(8*sqrt(2)*(9*a*e^(5*I*d*x + 5*I*c) - 10*a*e^(3*I*d*x + 3*I*c) + 5*a*e
^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*
c) + I)/(e^(2*I*d*x + 2*I*c) - 1)) - 5*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*
d*x + 2*I*c) + d)*sqrt(32*I*a^3/d^2)*log(1/2*(sqrt(2)*(d*e^(2*I*d*x + 2*I*c
) - d)*sqrt(32*I*a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*
d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)) + 8*I*a^2*e^(I*d*x + I*c))*e^(
-I*d*x - I*c)/a) + 5*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*
sqrt(32*I*a^3/d^2)*log(-1/2*(sqrt(2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(32*I*
a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)
/(e^(2*I*d*x + 2*I*c) - 1)) - 8*I*a^2*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a))
/(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(7/2)*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c+d\,x)^{7/2}\,{\left(a+a\,\tan(c+d\,x)\,1{}\mathrm{i}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(7/2)*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] int(cot(c + d*x)^(7/2)*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.758 $\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=139

$$-\frac{(2-2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d}-\frac{2ia\sqrt{\cot(c+dx)}\sqrt{a+ia}}{d}$$

```
[Out] (-2+2*I)*a^(3/2)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^
(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-2*I*a*cot(d*x+c)^(1/2)*(a+I*a*ta
n(d*x+c))^(1/2)/d-2/3*cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(3/2)/d
```

Rubi [A]
time = 0.19, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {4326, 3629, 3626, 3625, 211}

$$-\frac{(2-2i)a^{3/2}\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}-\frac{2ia\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-2 + 2*I)*a^(3/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I
*a*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d - ((2*I)*a*Sqrt[
Cot[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d - (2*Cot[c + d*x]^(3/2)*(a + I*
a*Tan[c + d*x])^(3/2))/(3*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3626

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*b*(a + b*Tan[e + f*x])^(m - 1)*((c +
 d*Tan[e + f*x])^(n + 1)/(f*(m - 1)*(a*c - b*d))), x] + Dist[2*(a^2/(a*c -
b*d)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2]
```

Rule 3629

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-d)*(a + b*Tan[e + f*x])^m*((c + d*Ta
n[e + f*x])^(n + 1)/(f*m*(c^2 + d^2))), x] + Dist[a/(a*c - b*d), Int[(a + b
*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -1]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{(a+ia\tan(c+dx))^{3/2}}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$

$$= -\frac{2\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d} + \left(i\sqrt{\cot(c+dx)}\,\sqrt{t\right.$$

$$= -\frac{2ia\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{d} - \frac{2\cot^{\frac{3}{2}}(c+dx)(a}{}$$

$$= -\frac{2ia\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{d} - \frac{2\cot^{\frac{3}{2}}(c+dx)(a}{}$$

$$= -\frac{(2-2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx}}{d}$$

Mathematica [A]
time = 1.26, size = 135, normalized size = 0.97

$$-\frac{4ia\left(e^{i(c+dx)}\left(-3+5e^{2i(c+dx)}\right)-3\left(-1+e^{2i(c+dx)}\right)^{3/2}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\cos(c+dx)\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{3d\left(-1+e^{4i(c+dx)}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((-4*I)/3)*a*(E^(I*(c + d*x))*(-3 + 5*E^((2*I)*(c + d*x))) - 3*(-1 + E^((2
*I)*(c + d*x)))^(3/2)*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))
]])*Cos[c + d*x]*Sqrt[Cot[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(d*(-1 + E^
((4*I)*(c + d*x))))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1041 vs. 2(113) = 226.
time = 41.65, size = 1042, normalized size = 7.50

method	result	size
default	Expression too large to display	1042

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/d*(5*I*2^(1/2)*cos(d*x+c)*sin(d*x+c)-6*I*arctan(((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*2^(1/2)+1)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+6*I*arctan(((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*cos(d*x+c)^2+6*I*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)*((-1
+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+6*arctan(((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*2^(1/2)+1)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+6*a
rctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)*cos(d*x+c)^2+3*ln(-(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*
sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1)/(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(
1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*cos(d*x+c)^2+5*2^(1/2)*cos(d*x+c)^2-4*I*2^(1/2)*sin(d*x+c)-3*I*ln(-(((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1)/(
((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)
+1))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+3*I*ln(-(((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1)/(((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1))*((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*cos(d*x+c)^2-6*I*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)*2^(1/2)-1)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-cos(d*x+c)*2^(1/2)-6*arcta
n(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)*((-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)-6*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*((-1+cos(d*x
+c))/sin(d*x+c))^(1/2)-3*ln(-(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*si
n(d*x+c)-sin(d*x+c)-cos(d*x+c)+1)/(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/
2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-
4*2^(1/2))*sin(d*x+c)*(cos(d*x+c)/sin(d*x+c))^(5/2)*(a*(I*sin(d*x+c)+cos(d*
x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)^2*2^(1/2)*a
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 995 vs. 2(105) = 210.

time = 0.60, size = 995, normalized size = 7.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) +
1)*((-(3*I - 3)*a*cos(3*d*x + 3*c) + (I - 1)*a*cos(d*x + c) + (3*I + 3)*a*s
in(3*d*x + 3*c) - (I + 1)*a*sin(d*x + c))*cos(3/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) - 1)) + (-(3*I + 3)*a*cos(3*d*x + 3*c) + (I + 1)*a*cos(d*
x + c) - (3*I - 3)*a*sin(3*d*x + 3*c) + (I - 1)*a*sin(d*x + c))*sin(3/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)))*sqrt(a) + 3*(2*(-(I + 1)*a*c
os(2*d*x + 2*c)^2 - (I + 1)*a*sin(2*d*x + 2*c)^2 + (2*I + 2)*a*cos(2*d*x +
2*c) - (I + 1)*a)*arctan2(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*co
s(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) - 1)) + 2*sin(d*x + c), 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*co
s(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) - 1)) + 2*cos(d*x + c)) + ((I - 1)*a*cos(2*d*x + 2*c)^2 + (I - 1)*a*sin(2
*d*x + 2*c)^2 - (2*I - 2)*a*cos(2*d*x + 2*c) + (I - 1)*a)*log(4*cos(d*x + c
)^2 + 4*sin(d*x + c)^2 + 4*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2
*cos(2*d*x + 2*c) + 1)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 - 1))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1))^2) + 8*(
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*(co
s(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + sin(d
*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)))))*(cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) +
 4*(((-(I - 1)*a*cos(d*x + c) + (I + 1)*a*sin(d*x + c))*cos(2*d*x + 2*c)^2
+ (-(I - 1)*a*cos(d*x + c) + (I + 1)*a*sin(d*x + c))*sin(2*d*x + 2*c)^2 + 2
*((I - 1)*a*cos(d*x + c) - (I + 1)*a*sin(d*x + c))*cos(2*d*x + 2*c) - (I -
1)*a*cos(d*x + c) + (I + 1)*a*sin(d*x + c))*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) - 1)) + ((-(I + 1)*a*cos(d*x + c) - (I - 1)*a*sin(d*x +
 c))*cos(2*d*x + 2*c)^2 + (-(I + 1)*a*cos(d*x + c) - (I - 1)*a*sin(d*x + c)
)*sin(2*d*x + 2*c)^2 + 2*((I + 1)*a*cos(d*x + c) + (I - 1)*a*sin(d*x + c))*
cos(2*d*x + 2*c) - (I + 1)*a*cos(d*x + c) - (I - 1)*a*sin(d*x + c))*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)))*sqrt(a))/((cos(2*d*x + 2
*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(5/4)*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(105) = 210$.
time = 0.99, size = 362, normalized size = 2.60

$$-\frac{8\sqrt{2}\left(5i\,ae^{(3i\,dx+3i\,c)}-3i\,ae^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}+3\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{-\frac{32i\,a^3}{d^2}}\log\left(\frac{\left(\sqrt{2}\left(i\,de^{(2i\,dx+2i\,c)}-i\,d\right)\sqrt{-\frac{32i\,a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}+8i\,a^2e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{2\,a}\right)-3\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{-\frac{32i\,a^3}{d^2}}\log\left(\frac{\left(\sqrt{2}\left(-i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{-\frac{32i\,a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}+8i\,a^2e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{2\,a}\right)}{12\left(de^{(2i\,dx+2i\,c)}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/12*(8*sqrt(2)*(5*I*a*e^(3*I*d*x + 3*I*c) - 3*I*a*e^(I*d*x + I*c))*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
2*I*c) - 1)) + 3*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(-32*I*a^3/d^2)*log(1/2*(s
qrt(2)*(I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(-32*I*a^3/d^2)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1
)) + 8*I*a^2*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a) - 3*(d*e^(2*I*d*x + 2*I*c
) - d)*sqrt(-32*I*a^3/d^2)*log(1/2*(sqrt(2)*(-I*d*e^(2*I*d*x + 2*I*c) + I*d
)*sqrt(-32*I*a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x
+ 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)) + 8*I*a^2*e^(I*d*x + I*c))*e^(-I*d
*x - I*c)/a))/(d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(5/2)*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^{5/2}\,{\left(a+a\,\tan(c+d\,x)\,1{}\mathrm{i}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] int(cot(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.759 $\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=103

$$\frac{(2+2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d}-\frac{2a\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}$$

```
[Out] (2+2*I)*a^(3/2)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(
1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-2*a*cot(d*x+c)^(1/2)*(a+I*a*tan(d
*x+c))^(1/2)/d
```

Rubi [A]
time = 0.14, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4326, 3626, 3625, 211}

$$\frac{(2+2i)a^{3/2}\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((2 + 2*I)*a^(3/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*
a*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d - (2*a*Sqrt[Cot[c
 + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3626

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*b*(a + b*Tan[e + f*x])^(m - 1)*((c +
 d*Tan[e + f*x])^(n + 1)/(f*(m - 1)*(a*c - b*d))), x] + Dist[2*(a^2/(a*c -
b*d)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{(a+ia\tan(c+dx))^{3/2}}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

$$= -\frac{2a\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d} + \left(2ia\sqrt{\cot(c+dx)}\right.$$

$$= -\frac{2a\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d} + \frac{\left(4a^3\sqrt{\cot(c+dx)}\right.}{}$$

$$= \frac{(2+2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{}}{d}$$

Mathematica [A]
time = 1.40, size = 105, normalized size = 1.02

$$-\frac{2ae^{-i(c+dx)}\left(e^{i(c+dx)}-\sqrt{-1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (-2*a*(E^(I*(c + d*x)) - Sqrt[-1 + E^((2*I)*(c + d*x))]*ArcTanh[E^(I*(c + d
*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]])*Sqrt[Cot[c + d*x]]*Sqrt[a + I*a*Tan[c
 + d*x]])/(d*E^(I*(c + d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 574 vs. 2(85) = 170.
time = 45.72, size = 575, normalized size = 5.58

method	result
default	$-\frac{\left(2i\sin(dx+c)\arctan\left(\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\sqrt{2}+1\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}+2i\sin(dx+c)\arctan\left(\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\sqrt{2}-1\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(2*I*sin(d*x+c)*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)*(
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+2*I*sin(d*x+c)*arctan(((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*2^(1/2)-1)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+I*sin(d*x+c)*
ln(-(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d
*x+c)+1)/(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+
cos(d*x+c)-1))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-2*sin(d*x+c)*arctan(((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
-2*sin(d*x+c)*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*((-1+cos
(d*x+c))/sin(d*x+c))^(1/2)-sin(d*x+c)*ln(-(((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1)/(((-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1))*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)+I*sin(d*x+c)*2^(1/2)+cos(d*x+c)*2^(1/2)-2^(1/2))*sin(d*x+c)*(c
os(d*x+c)/sin(d*x+c))^(3/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/
(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)*2^(1/2)*a
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 545 vs. $2(79) = 158$.
time = 0.60, size = 545, normalized size = 5.29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] (((2*I - 2)*a*arctan2(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*
d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) -
1)) + 2*sin(d*x + c), 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*
d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) -
1)) + 2*cos(d*x + c)) + (I + 1)*a*log(4*cos(d*x + c)^2 + 4*sin(d*x + c)^2 +
 4*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)*(
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1))^2 + sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1))^2) + 8*(cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + sin(d*x + c)*sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)))))*(cos(2*d*x + 2*c)^2 + sin(2*d*x
```

```
 + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) - 2*(((I + 1)*a*cos(d*x +
 c) + (I - 1)*a*sin(d*x + c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) - 1)) + (-(I - 1)*a*cos(d*x + c) + (I + 1)*a*sin(d*x + c))*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)))*sqrt(a))/((cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(79) = 158$.
time = 0.79, size = 302, normalized size = 2.93

$$-\frac{8\sqrt{2}a\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}e^{(i\,dx+i\,c)}-\sqrt{\frac{32i\,a^3}{d^2}}d\log\left(\frac{\left(\sqrt{2}\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{\frac{32i\,a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}+8i\,a^2e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{2\,a}\right)+\sqrt{\frac{32i\,a^3}{d^2}}d\log\left(-\frac{\left(\sqrt{2}\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{\frac{32i\,a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}-8i\,a^2e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{2\,a}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(8*sqrt(2)*a*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*
I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*e^(I*d*x + I*c) - sqrt(32*I*a^3/d^2)*d
*log(1/2*(sqrt(2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(32*I*a^3/d^2)*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*
c) - 1)) + 8*I*a^2*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a) + sqrt(32*I*a^3/d^2
)*d*log(-1/2*(sqrt(2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(32*I*a^3/d^2)*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
2*I*c) - 1)) - 8*I*a^2*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(3/2)*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+dx)^{3/2} (a+a\tan(c+dx)\ 1\mathrm{i})^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.760 $\int \sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=144

$$\frac{2\sqrt[4]{-1}\,a^{3/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d}+\frac{(2-2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{}}{\sqrt{a+}}\right.}{}$$

```
[Out] 2*(-1)^(1/4)*a^(3/2)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(
d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+(2-2*I)*a^(3/2)*arctanh(
(1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*t
an(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.23, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4326, 3636, 3625, 211, 3680, 65, 223, 209}

$$\frac{2\sqrt[4]{-1}\,a^{3/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{(2-2i)a^{3/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (2*(-1)^(1/4)*a^(3/2)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a
 + I*a*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d + ((2 - 2*I)
*a^(3/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c +
d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3636

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(c_.) + (d_.)*tan[(e_
.) + (f_.)*(x_)]], x_Symbol] :> Dist[2*a, Int[Sqrt[a + b*Tan[e + f*x]]/Sqrt
[c + d*Tan[e + f*x]], x], x] + Dist[b/a, Int[(b + a*Tan[e + f*x])*(Sqrt[a +
 b*Tan[e + f*x]]/Sqrt[c + d*Tan[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^{3/2}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{\tan(c+dx)}}\,dx$$

$$= \left(i\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{\sqrt{a+ia\tan(c+dx)}\,(ia}{\sqrt{\tan(c+a}}$$

$$= -\frac{\left(a^2\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\,\sqrt{a+iax}}\right.}{d}$$

$$= \frac{(2-2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}}{d}$$

$$= \frac{(2-2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}}{d}$$

$$= \frac{2\sqrt[4]{-1}\,a^{3/2}\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}}{d}$$

Mathematica [A]
time = 1.63, size = 255, normalized size = 1.77

$$-\frac{iae^{-i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{\frac{i\left(1+e^{2i(c+dx)}\right)}{-1+e^{2i(c+dx)}}}\left(8\log\left(e^{i(c+dx)}+\sqrt{-1+e^{2i(c+dx)}}\right)+\sqrt{2}\left(-\log\left(1-3e^{2i(c+dx)}-2\sqrt{2}\,e^{i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\right)+\log\left(1-3e^{2i(c+dx)}+2\sqrt{2}\,e^{i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\right)\right)\right)}{2\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-1/2*I)*a*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[(a*E^((2*I)*(c + d*x)))/(1
+ E^((2*I)*(c + d*x)))]*Sqrt[(I*(1 + E^((2*I)*(c + d*x))))/(-1 + E^((2*I)*(
c + d*x)))]*(8*Log[E^(I*(c + d*x)) + Sqrt[-1 + E^((2*I)*(c + d*x))]] + Sqrt
[2]*(-Log[1 - 3*E^((2*I)*(c + d*x)) - 2*Sqrt[2]*E^(I*(c + d*x))*Sqrt[-1 + E
^((2*I)*(c + d*x))]] + Log[1 - 3*E^((2*I)*(c + d*x)) + 2*Sqrt[2]*E^(I*(c +
d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]])))/(Sqrt[2]*d*E^(I*(c + d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 570 vs. 2(114) = 228.
time = 44.90, size = 571, normalized size = 3.97

method	result

default	$-\frac{\sqrt{\frac{\cos(dx+c)}{\sin(dx+c)}}\,\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))\left(i\sqrt{2}\,\ln\left(\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}-1\right)-2i\sqrt{2}\,\arctan\left(\sqrt{\frac{-1}{s}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/2/d*(cos(d*x+c)/sin(d*x+c))^(1/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(I*2^(1/2)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)-2*I*2^(1/2)*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2))-I*2^(1/2)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)-2^(1/2)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)-2*2^(1/2)*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2))+2^(1/2)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)+2*I*ln(-(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1)/(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1))+4*I*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)+4*I*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)+4*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)+4*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)+2*ln(-(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1)/(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1)))/(I*sin(d*x+c)+cos(d*x+c)-1)/((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*a

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*sqrt(cot(d*x + c)), x)

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(108) = 216$.
time = 0.94, size = 491, normalized size = 3.41

$$-\frac{1}{4}\sqrt{-\frac{4i a^3}{d^2}}\log\left(-16\left(\sqrt{2}\left(de^{(3i dx+3i c)}-de^{(i dx+i c)}\right)\sqrt{-\frac{4i a^3}{d^2}}\sqrt{\frac{a}{e^{(2i dx+2i c)}+1}}\sqrt{\frac{i e^{(2i dx+2i c)}+i}{e^{(2i dx+2i c)}-1}}+3a^2e^{(2i dx+2i c)}-a^2\right)e^{(-2i dx-2i c)}\right)+\frac{1}{4}\sqrt{-\frac{4i a^3}{d^2}}\log\left(16\left(\sqrt{2}\left(de^{(3i dx+3i c)}-de^{(i dx+i c)}\right)\sqrt{-\frac{4i a^3}{d^2}}\sqrt{\frac{a}{e^{(2i dx+2i c)}+1}}\sqrt{\frac{i e^{(2i dx+2i c)}+i}{e^{(2i dx+2i c)}-1}}-3a^2e^{(2i dx+2i c)}+a^2\right)e^{(-2i dx-2i c)}\right)+\frac{1}{4}\sqrt{-\frac{32i a^3}{d^2}}\log\left(\frac{\left(\sqrt{2}\left(i de^{(2i dx+2i c)}-i d\right)\sqrt{-\frac{32i a^3}{d^2}}\sqrt{\frac{a}{e^{(2i dx+2i c)}+1}}\sqrt{\frac{i e^{(2i dx+2i c)}+i}{e^{(2i dx+2i c)}-1}}+8i a^2e^{(i dx+i c)}\right)e^{(-i dx-i c)}}{2a}\right)-\frac{1}{4}\sqrt{-\frac{32i a^3}{d^2}}\log\left(\frac{\left(\sqrt{2}\left(-i de^{(2i dx+2i c)}+i d\right)\sqrt{-\frac{32i a^3}{d^2}}\sqrt{\frac{a}{e^{(2i dx+2i c)}+1}}\sqrt{\frac{i e^{(2i dx+2i c)}+i}{e^{(2i dx+2i c)}-1}}+8i a^2e^{(i dx+i c)}\right)e^{(-i dx-i c)}}{2a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")

[Out] -1/4*sqrt(-4*I*a^3/d^2)*log(-16*(sqrt(2)*(d*e^(3*I*d*x + 3*I*c) - d*e^(I*d*x + I*c))*sqrt(-4*I*a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2

```
*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)) + 3*a^2*e^(2*I*d*x + 2*I*c)
 - a^2)*e^(-2*I*d*x - 2*I*c)) + 1/4*sqrt(-4*I*a^3/d^2)*log(16*(sqrt(2)*(d*e
^(3*I*d*x + 3*I*c) - d*e^(I*d*x + I*c))*sqrt(-4*I*a^3/d^2)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1
)) - 3*a^2*e^(2*I*d*x + 2*I*c) + a^2)*e^(-2*I*d*x - 2*I*c)) + 1/4*sqrt(-32*
I*a^3/d^2)*log(1/2*(sqrt(2)*(I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(-32*I*a^3/
d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^
(2*I*d*x + 2*I*c) - 1)) + 8*I*a^2*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a) - 1/
4*sqrt(-32*I*a^3/d^2)*log(1/2*(sqrt(2)*(-I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqr
t(-32*I*a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I
*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)) + 8*I*a^2*e^(I*d*x + I*c))*e^(-I*d*x -
I*c)/a)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c+dx)-i))^{\frac{3}{2}}\sqrt{\cot(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(1/2)*(a+I*a*tan(d*x+c))**(3/2),x)
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)*sqrt(cot(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*sqrt(cot(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cot(c+d\,x)}\,\left(a+a\,\tan(c+d\,x)\,1\mathrm{i}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.761 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{\cot(c+dx)}}\,dx$

Optimal. Leaf size=216

$$-\frac{3(-1)^{3/4}a^{3/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d}-\frac{(2+2i)a^{3/2}\tanh^{-1}\left(\frac{(1}{}\right.}{}$$

```
[Out] -3*(-1)^(3/4)*a^(3/2)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan
(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-(2+2*I)*a^(3/2)*arctanh
((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*
tan(d*x+c)^(1/2)/d-a^2/d/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(1/2)+I*a^2/d/
cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4326, 3637, 3676, 3682, 3625, 211, 3680, 65, 223, 209}

$$-\frac{3(-1)^{3/4}a^{3/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{(2+2i)a^{3/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{a^2}{d\cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}+\frac{ia^2}{d\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(3/2)/Sqrt[Cot[c + d*x]],x]
```

```
[Out] (-3*(-1)^(3/4)*a^(3/2)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[
a + I*a*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d - ((2 + 2*I
)*a^(3/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c +
 d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d - a^2/(d*Cot[c + d*x]^(3/2
)*Sqrt[a + I*a*Tan[c + d*x]]) + (I*a^2)/(d*Sqrt[Cot[c + d*x]]*Sqrt[a + I*a*
Tan[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{\cot(c+dx)}}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^{3/2}\,d$$

$$= -\frac{a^2}{d\cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} + \left(a\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)$$

$$= -\frac{a^2}{d\cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} + \frac{ia^2}{d\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+}}$$

$$= -\frac{a^2}{d\cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} + \frac{ia^2}{d\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+}}$$

$$= -\frac{a^2}{d\cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} + \frac{ia^2}{d\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+}}$$

$$= -\frac{(2+2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+}}{d}$$

$$= -\frac{(2+2i)a^{3/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+}}{d}$$

$$= -\frac{3(-1)^{3/4}a^{3/2}\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+}}{d}$$

Mathematica [A]
time = 2.55, size = 228, normalized size = 1.06

$$\frac{ae^{-i(c+dx)}\left(\sqrt{2}\,e^{i(c+dx)}\left(-1+e^{2i(c+dx)}\right)-2\sqrt{2}\,\sqrt{-1+e^{2i(c+dx)}}\,\left(1+e^{2i(c+dx)}\right)\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)+3\sqrt{-1+e^{2i(c+dx)}}\,\left(1+e^{2i(c+dx)}\right)\tanh^{-1}\left(\frac{\sqrt{2}\,e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,d\left(1+e^{2i(c+dx)}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(3/2)/Sqrt[Cot[c + d*x]],x]
```

```
[Out] (a*(Sqrt[2]*E^(I*(c + d*x))*(-1 + E^((2*I)*(c + d*x))) - 2*Sqrt[2]*Sqrt[-1
+ E^((2*I)*(c + d*x))]*(1 + E^((2*I)*(c + d*x)))*ArcTanh[E^(I*(c + d*x))/Sq
rt[-1 + E^((2*I)*(c + d*x))]] + 3*Sqrt[-1 + E^((2*I)*(c + d*x))]*(1 + E^((2
*I)*(c + d*x)))*ArcTanh[(Sqrt[2]*E^(I*(c + d*x)))/Sqrt[-1 + E^((2*I)*(c + d
*x))]])*Sqrt[Cot[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[2]*d*E^(I*(c +
 d*x))*(1 + E^((2*I)*(c + d*x))))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2090 vs. 2(175) = 350.
time = 48.27, size = 2091, normalized size = 9.68

method	result	size
default	Expression too large to display	2091

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(3/2)/cot(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*(-3*I*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)*cos(d*x+c)^2*2^(1/2)-6
*I*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2))*cos(d*x+c)^2*2^(1/2)-8*I*arct
an(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)*cos(d*x+c)*sin(d*x+c)-8*I*
arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*cos(d*x+c)*sin(d*x+c)-
3*I*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)*cos(d*x+c)*2^(1/2)+3*I*ln(((-1
+cos(d*x+c))/sin(d*x+c))^(1/2)-1)*cos(d*x+c)*2^(1/2)+6*I*arctan(((-1+cos(d*
x+c))/sin(d*x+c))^(1/2))*cos(d*x+c)*2^(1/2)-4*I*ln(-(((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1)/(((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1))*cos(d*x+c)*s
in(d*x+c)+3*I*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)*cos(d*x+c)^2*2^(1/2)
+8*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)*sin(d*x+c)*cos(d*x+
c)-3*2^(1/2)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)*cos(d*x+c)+3*2^(1/2)*
ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)*cos(d*x+c)+8*I*arctan(((-1+cos(d*x
+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)*cos(d*x+c)^2+8*I*arctan(((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*2^(1/2)-1)*cos(d*x+c)^2+4*I*ln(-(((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1)/(((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1))*cos(d*x+c)^2-8
*I*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)*cos(d*x+c)-8*I*arct
an(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*cos(d*x+c)-4*I*ln(-(((-1+c
```

```
os(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1)/((
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-
1))*cos(d*x+c)+4*ln(-(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)
+sin(d*x+c)+cos(d*x+c)-1)/(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d
*x+c)-sin(d*x+c)-cos(d*x+c)+1))*cos(d*x+c)*sin(d*x+c)-2*((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+2*2^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*cos(d*x+c)^2+3*2^(1/2)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)*cos(d*
x+c)^2-3*2^(1/2)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)*cos(d*x+c)^2-6*2^
(1/2)*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2))*cos(d*x+c)+8*arctan(((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*sin(d*x+c)*cos(d*x+c)+6*2^(1/2)*arc
tan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2))*cos(d*x+c)^2-2*I*2^(1/2)*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)+6*I*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2))*cos
(d*x+c)*2^(1/2)*sin(d*x+c)-2*I*2^(1/2)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)+3*2^(1/2)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)*sin(d*x+c)*cos
(d*x+c)-3*2^(1/2)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)*sin(d*x+c)*cos(d
*x+c)-6*2^(1/2)*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2))*sin(d*x+c)*cos(d
*x+c)-8*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)*cos(d*x+c)^2-8
*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*cos(d*x+c)^2+8*arctan
(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)*cos(d*x+c)+8*arctan(((-1+cos
(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*cos(d*x+c)-2*2^(1/2)*sin(d*x+c)*((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)+2*I*2^(1/2)*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*cos(d*x+c)^2-2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-4*ln
(-(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x
+c)-1)/(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-co
s(d*x+c)+1))*cos(d*x+c)^2+4*ln(-(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)
*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1)/(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^
(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1))*cos(d*x+c)+2*I*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*cos(d*x+c)*2^(1/2)*sin(d*x+c)-3*I*ln(((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)+1)*cos(d*x+c)*2^(1/2)*sin(d*x+c)+3*I*ln(((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)-1)*cos(d*x+c)*2^(1/2)*sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c
))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+I*cos(d*x+c)-1+I-sin(d*x+c)+cos(d*x+c))/
(cos(d*x+c)/sin(d*x+c))^(1/2)/((-1+cos(d*x+c))/sin(d*x+c))^(1/2)/sin(d*x+c)
*2^(1/2)*a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/cot(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)/sqrt(cot(d*x + c)), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(166) = 332$.

time = 0.98, size = 637, normalized size = 2.95

$$\frac{4\sqrt{2}\left(ae^{(3idx+3ic)} - ae^{(idx+ic)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}} - \left(de^{(2idx+2ic)}+d\right)\sqrt{\frac{9ia^3}{d^2}}\log\left(-\frac{16}{3}\left(2\sqrt{2}\left(ide^{(3idx+3ic)} - ide^{(idx+ic)}\right)\sqrt{\frac{9ia^3}{d^2}}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}} + 9a^2e^{(2idx+2ic)} - 3a^2\right)e^{(-2idx-2ic)}\right) + \left(de^{(2idx+2ic)}+d\right)\sqrt{\frac{9ia^3}{d^2}}\log\left(-\frac{16}{3}\left(2\sqrt{2}\left(-ide^{(3idx+3ic)} + ide^{(idx+ic)}\right)\sqrt{\frac{9ia^3}{d^2}}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}} + 9a^2e^{(2idx+2ic)} - 3a^2\right)e^{(-2idx-2ic)}\right) - \left(de^{(2idx+2ic)}+d\right)\sqrt{\frac{32ia^3}{d^2}}\log\left(\frac{\left(\sqrt{2}\left(de^{(2idx+2ic)}-d\right)\sqrt{\frac{32ia^3}{d^2}}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}} + 8ia^2e^{(idx+ic)}\right)e^{(-idx-ic)}}{2a}\right) + \left(de^{(2idx+2ic)}+d\right)\sqrt{\frac{32ia^3}{d^2}}\log\left(-\frac{\left(\sqrt{2}\left(de^{(2idx+2ic)}-d\right)\sqrt{\frac{32ia^3}{d^2}}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}} - 8ia^2e^{(idx+ic)}\right)e^{(-idx-ic)}}{2a}\right)}{4\left(de^{(2idx+2ic)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/cot(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*(a*e^(3*I*d*x + 3*I*c) - a*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1
)) - (d*e^(2*I*d*x + 2*I*c) + d)*sqrt(9*I*a^3/d^2)*log(-16/3*(2*sqrt(2)*(I*
d*e^(3*I*d*x + 3*I*c) - I*d*e^(I*d*x + I*c))*sqrt(9*I*a^3/d^2)*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c)
 - 1)) + 9*a^2*e^(2*I*d*x + 2*I*c) - 3*a^2)*e^(-2*I*d*x - 2*I*c)) + (d*e^(2
*I*d*x + 2*I*c) + d)*sqrt(9*I*a^3/d^2)*log(-16/3*(2*sqrt(2)*(-I*d*e^(3*I*d*
x + 3*I*c) + I*d*e^(I*d*x + I*c))*sqrt(9*I*a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*
I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)) + 9*
a^2*e^(2*I*d*x + 2*I*c) - 3*a^2)*e^(-2*I*d*x - 2*I*c)) - (d*e^(2*I*d*x + 2*
I*c) + d)*sqrt(32*I*a^3/d^2)*log(1/2*(sqrt(2)*(d*e^(2*I*d*x + 2*I*c) - d)*s
qrt(32*I*a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*
I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)) + 8*I*a^2*e^(I*d*x + I*c))*e^(-I*d*x -
 I*c)/a) + (d*e^(2*I*d*x + 2*I*c) + d)*sqrt(32*I*a^3/d^2)*log(-1/2*(sqrt(2)
*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(32*I*a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c)
 + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)) - 8*I*a^
2*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{3}{2}}}{\sqrt{\cot\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(3/2)/cot(d*x+c)**(1/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)/sqrt(cot(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/cot(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)/sqrt(cot(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + a\tan(c + d\,x)\ 1\mathrm{i})^{3/2}}{\sqrt{\cot\left(c + d\,x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(3/2)/cot(c + d*x)^(1/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^(3/2)/cot(c + d*x)^(1/2), x)
```

3.762 $\int \cot^{\frac{9}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=222

$$\frac{(4-4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d}+\frac{104ia^2\sqrt{\cot(c+dx)}\sqrt{a+i}}{21d}$$

```
[Out] (4-4*I)*a^(5/2)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(
1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+32/21*a^2*cot(d*x+c)^(3/2)*(a+I*a
*tan(d*x+c))^(1/2)/d-6/7*I*a^2*cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(1/2)/d-
2/7*a^2*cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(1/2)/d+104/21*I*a^2*cot(d*x+c)
^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.43, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4326, 3634, 3679, 12, 3625, 211}

$$\frac{(4-4i)a^{5/2}\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^2\cot^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}-\frac{6ia^2\cot^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}+\frac{32a^2\cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{21d}+\frac{104ia^2\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(9/2)*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((4 - 4*I)*a^(5/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*
a*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d + (((104*I)/21)*a
^2*Sqrt[Cot[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d + (32*a^2*Cot[c + d*x]^
(3/2)*Sqrt[a + I*a*Tan[c + d*x]])/(21*d) - (((6*I)/7)*a^2*Cot[c + d*x]^(5/2
)*Sqrt[a + I*a*Tan[c + d*x]])/d - (2*a^2*Cot[c + d*x]^(7/2)*Sqrt[a + I*a*Ta
n[c + d*x]])/(7*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
```

```
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3634

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-a^2)*(b*c - a*d)*(a + b*Tan[e + f*x]
)^(m - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] + Di
st[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[b*(b*c*(m - 2) - a*d*(m - 2*n - 4)) + (a*b*c*(m - 2)
+ b^2*d*(n + 1) - a^2*d*(m + n - 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \cot^{\frac{9}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{(a+ia\tan(c+dx))^{5/2}}{\tan^{\frac{9}{2}}(c+dx)}\,dx$$

$$= -\frac{2a^2\cot^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d} - \frac{1}{7}\left(2\sqrt{\cot(c+dx)}\ \sqrt{\right.$$

$$= -\frac{6ia^2\cot^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d} - \frac{2a^2\cot^{\frac{7}{2}}(c+dx)\sqrt{a}}{7d}$$

$$= \frac{32a^2\cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{21d} - \frac{6ia^2\cot^{\frac{5}{2}}(c+dx)\sqrt{a}}{7d}$$

$$= \frac{104ia^2\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{21d} + \frac{32a^2\cot^{\frac{3}{2}}(c+dx)}{}$$

$$= \frac{104ia^2\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{21d} + \frac{32a^2\cot^{\frac{3}{2}}(c+dx)}{}$$

$$= \frac{104ia^2\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{21d} + \frac{32a^2\cot^{\frac{3}{2}}(c+dx)}{}$$

$$= \frac{(4-4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\ \sqrt{}}{d}$$

Mathematica [A]
time = 2.68, size = 167, normalized size = 0.75

$$\frac{4ia^2e^{-i(c+dx)}\left(-21e^{i(c+dx)}+70e^{3i(c+dx)}-77e^{5i(c+dx)}+40e^{7i(c+dx)}-21\left(-1+e^{2i(c+dx)}\right)^{7/2}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{21d\left(-1+e^{2i(c+dx)}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(9/2)*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((4*I)/21)*a^2*(-21*E^(I*(c + d*x)) + 70*E^((3*I)*(c + d*x)) - 77*E^((5*I)
*(c + d*x)) + 40*E^((7*I)*(c + d*x)) - 21*(-1 + E^((2*I)*(c + d*x)))^(7/2)*
ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]])*Sqrt[Cot[c + d*x]]
*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*(c + d*x)))^
3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1556 vs. 2(179) = 358.
time = 46.27, size = 1557, normalized size = 7.01

method	result	size
default	Expression too large to display	1557

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(9/2)*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/21/d*(84*I*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)*((-1+cos
(d*x+c))/sin(d*x+c))^(1/2)-129*2^(1/2)*cos(d*x+c)^2+84*arctan(((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)*2^(1/2)+1)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+84*arct
an(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)-168*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)*((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-168*arctan(((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*2^(1/2)-1)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-19*cos
(d*x+c)^3*2^(1/2)+84*I*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)
*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+42*I*ln((((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1)/(-((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*2^(1/2)*sin(d*x+c)+cos(d*x+c)+sin(d*x+c)-1))*((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)+52*I*sin(d*x+c)*2^(1/2)+84*arctan(((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*2^(1/2)+1)*cos(d*x+c)^4*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+84*arc
tan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*cos(d*x+c)^4*((-1+cos(d*x
+c))/sin(d*x+c))^(1/2)+42*ln((-((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*s
in(d*x+c)+cos(d*x+c)+sin(d*x+c)-1)/(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1
/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1))*cos(d*x+c)^4*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)-84*ln((-((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)
+cos(d*x+c)+sin(d*x+c)-1)/(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d
*x+c)+sin(d*x+c)+cos(d*x+c)-1))*cos(d*x+c)^2*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)+16*cos(d*x+c)*2^(1/2)+42*ln((-((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/
2)*sin(d*x+c)+cos(d*x+c)+sin(d*x+c)-1)/(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1))*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)+80*cos(d*x+c)^4*2^(1/2)+84*I*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*2^(1/2)+1)*cos(d*x+c)^4*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+84*I*arctan(((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*cos(d*x+c)^4*((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)+42*I*ln((((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x
+c)+sin(d*x+c)+cos(d*x+c)-1)/(-((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*s
in(d*x+c)+cos(d*x+c)+sin(d*x+c)-1))*cos(d*x+c)^4*((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)+80*I*cos(d*x+c)^3*sin(d*x+c)*2^(1/2)-168*I*arctan(((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*2^(1/2)+1)*cos(d*x+c)^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)-168*I*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*cos(d*x+c)^2*
((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-84*I*ln((((-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1)/(-((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*2^(1/2)*sin(d*x+c)+cos(d*x+c)+sin(d*x+c)-1))*cos(d*x+c)^2*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)-61*I*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)-68*I*cos(d
*x+c)*sin(d*x+c)*2^(1/2)+52*2^(1/2))*sin(d*x+c)*(cos(d*x+c)/sin(d*x+c))^(9/
```

```
2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-
1)/cos(d*x+c)^4*2^(1/2)*a^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3165 vs. $2(168) = 336$.
time = 0.79, size = 3165, normalized size = 14.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(9/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
[Out] -2/105*(2*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c)
 + 1)*(3*(-(35*I - 35)*a^2*cos(7*d*x + 7*c) + (35*I - 35)*a^2*cos(5*d*x + 5
*c) - (21*I - 21)*a^2*cos(3*d*x + 3*c) + (I - 1)*a^2*cos(d*x + c) + (35*I +
 35)*a^2*sin(7*d*x + 7*c) - (35*I + 35)*a^2*sin(5*d*x + 5*c) + (21*I + 21)*
a^2*sin(3*d*x + 3*c) - (I + 1)*a^2*sin(d*x + c))*cos(7/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) - 1)) + 5*(13*((I - 1)*a^2*cos(d*x + c) - (I + 1)*
a^2*sin(d*x + c))*cos(2*d*x + 2*c)^2 + (13*I - 13)*a^2*cos(d*x + c) + 13*((
I - 1)*a^2*cos(d*x + c) - (I + 1)*a^2*sin(d*x + c))*sin(2*d*x + 2*c)^2 - (1
3*I + 13)*a^2*sin(d*x + c) + 21*(-(I - 1)*a^2*cos(2*d*x + 2*c)^2 - (I - 1)*
a^2*sin(2*d*x + 2*c)^2 + (2*I - 2)*a^2*cos(2*d*x + 2*c) - (I - 1)*a^2)*cos(
3*d*x + 3*c) + 26*(-(I - 1)*a^2*cos(d*x + c) + (I + 1)*a^2*sin(d*x + c))*co
s(2*d*x + 2*c) + 21*((I + 1)*a^2*cos(2*d*x + 2*c)^2 + (I + 1)*a^2*sin(2*d*x
 + 2*c)^2 - (2*I + 2)*a^2*cos(2*d*x + 2*c) + (I + 1)*a^2)*sin(3*d*x + 3*c))
*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + 3*(-(35*I + 35)
*a^2*cos(7*d*x + 7*c) + (35*I + 35)*a^2*cos(5*d*x + 5*c) - (21*I + 21)*a^2*
cos(3*d*x + 3*c) + (I + 1)*a^2*cos(d*x + c) - (35*I - 35)*a^2*sin(7*d*x + 7
*c) + (35*I - 35)*a^2*sin(5*d*x + 5*c) - (21*I - 21)*a^2*sin(3*d*x + 3*c) +
 (I - 1)*a^2*sin(d*x + c))*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) - 1)) + 5*(13*((I + 1)*a^2*cos(d*x + c) + (I - 1)*a^2*sin(d*x + c))*cos(
2*d*x + 2*c)^2 + (13*I + 13)*a^2*cos(d*x + c) + 13*((I + 1)*a^2*cos(d*x + c
) + (I - 1)*a^2*sin(d*x + c))*sin(2*d*x + 2*c)^2 + (13*I - 13)*a^2*sin(d*x
+ c) + 21*(-(I + 1)*a^2*cos(2*d*x + 2*c)^2 - (I + 1)*a^2*sin(2*d*x + 2*c)^2
 + (2*I + 2)*a^2*cos(2*d*x + 2*c) - (I + 1)*a^2)*cos(3*d*x + 3*c) + 26*(-(I
 + 1)*a^2*cos(d*x + c) - (I - 1)*a^2*sin(d*x + c))*cos(2*d*x + 2*c) + 21*(-
(I - 1)*a^2*cos(2*d*x + 2*c)^2 - (I - 1)*a^2*sin(2*d*x + 2*c)^2 + (2*I - 2)
*a^2*cos(2*d*x + 2*c) - (I - 1)*a^2)*sin(3*d*x + 3*c))*sin(3/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) - 1)))*sqrt(a) + 105*(2*(-(I + 1)*a^2*cos(2*
d*x + 2*c)^4 - (I + 1)*a^2*sin(2*d*x + 2*c)^4 + (4*I + 4)*a^2*cos(2*d*x + 2
*c)^3 - (6*I + 6)*a^2*cos(2*d*x + 2*c)^2 + (4*I + 4)*a^2*cos(2*d*x + 2*c) +
 2*(-(I + 1)*a^2*cos(2*d*x + 2*c)^2 + (2*I + 2)*a^2*cos(2*d*x + 2*c) - (I +
 1)*a^2)*sin(2*d*x + 2*c)^2 - (I + 1)*a^2)*arctan2(2*(cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) - 1)) + 2*sin(d*x + c), 2*(cos(2*d*x + 2*c)^2 +
```

```
sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) - 1)) + 2*cos(d*x + c)) + ((I - 1)*a^2*cos(2*d*x
 + 2*c)^4 + (I - 1)*a^2*sin(2*d*x + 2*c)^4 - (4*I - 4)*a^2*cos(2*d*x + 2*c)
^3 + (6*I - 6)*a^2*cos(2*d*x + 2*c)^2 - (4*I - 4)*a^2*cos(2*d*x + 2*c) + 2*
((I - 1)*a^2*cos(2*d*x + 2*c)^2 - (2*I - 2)*a^2*cos(2*d*x + 2*c) + (I - 1)*
a^2)*sin(2*d*x + 2*c)^2 + (I - 1)*a^2)*log(4*cos(d*x + c)^2 + 4*sin(d*x + c
)^2 + 4*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) +
 1)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1))^2 + sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1))^2) + 8*(cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + sin(d*x + c)*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)))))*(cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 2*((152*(-(I - 1)*
a^2*cos(d*x + c) + (I + 1)*a^2*sin(d*x + c))*cos(2*d*x + 2*c)^2 - (152*I -
152)*a^2*cos(d*x + c) + 152*(-(I - 1)*a^2*cos(d*x + c) + (I + 1)*a^2*sin(d*
x + c))*sin(2*d*x + 2*c)^2 + (152*I + 152)*a^2*sin(d*x + c) + 105*(-(I - 1)
*a^2*cos(2*d*x + 2*c)^2 - (I - 1)*a^2*sin(2*d*x + 2*c)^2 + (2*I - 2)*a^2*co
s(2*d*x + 2*c) - (I - 1)*a^2)*cos(5*d*x + 5*c) + 245*((I - 1)*a^2*cos(2*d*x
 + 2*c)^2 + (I - 1)*a^2*sin(2*d*x + 2*c)^2 - (2*I - 2)*a^2*cos(2*d*x + 2*c)
 + (I - 1)*a^2)*cos(3*d*x + 3*c) + 304*((I - 1)*a^2*cos(d*x + c) - (I + 1)*
a^2*sin(d*x + c))*cos(2*d*x + 2*c) + 105*((I + 1)*a^2*cos(2*d*x + 2*c)^2 +
(I + 1)*a^2*sin(2*d*x + 2*c)^2 - (2*I + 2)*a^2*cos(2*d*x + 2*c) + (I + 1)*a
^2)*sin(5*d*x + 5*c) + 245*(-(I + 1)*a^2*cos(2*d*x + 2*c)^2 - (I + 1)*a^2*s
in(2*d*x + 2*c)^2 + (2*I + 2)*a^2*cos(2*d*x + 2*c) - (I + 1)*a^2)*sin(3*d*x
 + 3*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + 115*(((
I - 1)*a^2*cos(d*x + c) - (I + 1)*a^2*sin(d*x + c))*cos(2*d*x + 2*c)^4 + ((
I - 1)*a^2*cos(d*x + c) - (I + 1)*a^2*sin(d*x + c))*sin(2*d*x + 2*c)^4 + 4*
(-(I - 1)*a^2*cos(d*x + c) + (I + 1)*a^2*sin(d*x + c))*cos(2*d*x + 2*c)^3 +
 6*((I - 1)*a^2*cos(d*x + c) - (I + 1)*a^2*sin(d*x + c))*cos(2*d*x + 2*c)^2
 + (I - 1)*a^2*cos(d*x + c) + 2*(((I - 1)*a^2*cos(d*x + c) - (I + 1)*a^2*si
n(d*x + c))*cos(2*d*x + 2*c)^2 + (I - 1)*a^2*cos(d*x + c) - (I + 1)*a^2*sin
(d*x + c) + 2*(-(I - 1)*a^2*cos(d*x + c) + (I + 1)*a^2*sin(d*x + c))*cos(2*
d*x + 2*c))*sin(2*d*x + 2*c)^2 - (I + 1)*a^2*sin(d*x + c) + 4*(-(I - 1)*a^2
*cos(d*x + c) + (I + 1)*a^2*sin(d*x + c))*cos(2...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(168) = 336$.
time = 1.08, size = 466, normalized size = 2.10

$$-\frac{16\sqrt{2}\left(-40i\,a^2 e^{(7i\,dx+7i\,c)}+77i\,a^2 e^{(5i\,dx+5i\,c)}-70i\,a^2 e^{(3i\,dx+3i\,c)}+21i\,a^2 e^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}-21\sqrt{-\frac{128i\,a^5}{d^2}}\left(de^{(6i\,dx+6i\,c)}-3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}-d\right)\log\left(\frac{\left(16i\,a^3e^{(i\,dx+i\,c)}+\sqrt{2}\sqrt{-\frac{128i\,a^5}{d^2}}\left(i\,de^{(2i\,dx+2i\,c)}-i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\right)e^{(-i\,dx-i\,c)}}{4\,a^2}\right)+21\sqrt{-\frac{128i\,a^5}{d^2}}\left(de^{(6i\,dx+6i\,c)}-3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}-d\right)\log\left(\frac{\left(16i\,a^3e^{(i\,dx+i\,c)}+\sqrt{2}\sqrt{-\frac{128i\,a^5}{d^2}}\left(-i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\right)e^{(-i\,dx-i\,c)}}{4\,a^2}\right)}{84\left(de^{(6i\,dx+6i\,c)}-3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(9/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/84*(16*sqrt(2)*(-40*I*a^2*e^(7*I*d*x + 7*I*c) + 77*I*a^2*e^(5*I*d*x + 5*
I*c) - 70*I*a^2*e^(3*I*d*x + 3*I*c) + 21*I*a^2*e^(I*d*x + I*c))*sqrt(a/(e^(
```

```
2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c
) - 1)) - 21*sqrt(-128*I*a^5/d^2)*(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x +
 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)*log(1/4*(16*I*a^3*e^(I*d*x + I*c) +
sqrt(2)*sqrt(-128*I*a^5/d^2)*(I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I
*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) -
 1)))*e^(-I*d*x - I*c)/a^2) + 21*sqrt(-128*I*a^5/d^2)*(d*e^(6*I*d*x + 6*I*c
) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)*log(1/4*(16*I*a^
3*e^(I*d*x + I*c) + sqrt(2)*sqrt(-128*I*a^5/d^2)*(-I*d*e^(2*I*d*x + 2*I*c)
+ I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(
e^(2*I*d*x + 2*I*c) - 1)))*e^(-I*d*x - I*c)/a^2))/(d*e^(6*I*d*x + 6*I*c) -
3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(9/2)*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(9/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^(9/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c+d\,x)^{9/2}\,{\left(a+a\,\tan(c+d\,x)\,1{}\mathrm{i}\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(9/2)*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] int(cot(c + d*x)^(9/2)*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.763 $\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=176

$$-\frac{(4+4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d}+\frac{4a^2\sqrt{\cot(c+dx)}\sqrt{a+i\cdots}}{d}$$

```
[Out] (-4-4*I)*a^(5/2)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^
(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+4*a^2*cot(d*x+c)^(1/2)*(a+I*a*ta
n(d*x+c))^(1/2)/d-2/3*I*a*cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(3/2)/d-2/5*c
ot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(5/2)/d
```

Rubi [A]
time = 0.24, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {4326, 3629, 3626, 3625, 211}

$$-\frac{(4+4i)a^{5/2}\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{4a^2\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2ia\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}-\frac{2\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(7/2)*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-4 - 4*I)*a^(5/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I
*a*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d + (4*a^2*Sqrt[Co
t[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d - (((2*I)/3)*a*Cot[c + d*x]^(3/2)
*(a + I*a*Tan[c + d*x])^(3/2))/d - (2*Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d
*x])^(5/2))/(5*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3626

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*b*(a + b*Tan[e + f*x])^(m - 1)*((c +
```

```
 d*Tan[e + f*x])^(n + 1)/(f*(m - 1)*(a*c - b*d))), x] + Dist[2*(a^2/(a*c -
b*d)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2]
```

Rule 3629

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-d)*(a + b*Tan[e + f*x])^m*((c + d*Ta
n[e + f*x])^(n + 1)/(f*m*(c^2 + d^2))), x] + Dist[a/(a*c - b*d), Int[(a + b
*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -1]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{(a+ia\tan(c+dx))^{5/2}}{\tan^{\frac{7}{2}}(c+dx)}\,dx$$

$$= -\frac{2\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d} + \left(i\sqrt{\cot(c+dx)}\,\sqrt{\text{ta}\right.$$

$$= -\frac{2ia\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d} - \frac{2\cot^{\frac{5}{2}}(c+dx)(a+}{5d}$$

$$= \frac{4a^2\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{d} - \frac{2ia\cot^{\frac{3}{2}}(c+dx)(a+}{3d}$$

$$= \frac{4a^2\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{d} - \frac{2ia\cot^{\frac{3}{2}}(c+dx)(a+}{3d}$$

$$= -\frac{(4+4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}}{d}$$

Mathematica [A]

time = 1.98, size = 161, normalized size = 0.91

$$\frac{16a^2e^{i(c+dx)}\left(e^{i(c+dx)}\left(15-35e^{2i(c+dx)}+26e^{4i(c+dx)}\right)-15\left(-1+e^{2i(c+dx)}\right)^{5/2}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\cos^2(c+dx)\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{15d\left(-1+e^{4i(c+dx)}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(7/2)*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (16*a^2*E^(I*(c + d*x))*(E^(I*(c + d*x))*(15 - 35*E^((2*I)*(c + d*x)) + 26*
E^((4*I)*(c + d*x))) - 15*(-1 + E^((2*I)*(c + d*x)))^(5/2)*ArcTanh[E^(I*(c
+ d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]])*Cos[c + d*x]^2*Sqrt[Cot[c + d*x]]*
Sqrt[a + I*a*Tan[c + d*x]])/(15*d*(-1 + E^((4*I)*(c + d*x)))^2)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1148 vs. 2(143) = 286.
time = 47.78, size = 1149, normalized size = 6.53

method	result	size
default	Expression too large to display	1149

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15/d*(-30*I*sin(d*x+c)*ln(-(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*s
in(d*x+c)-sin(d*x+c)-cos(d*x+c)+1)/(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1
/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
-11*I*sin(d*x+c)*cos(d*x+c)*2^(1/2)-60*I*sin(d*x+c)*arctan(((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*2^(1/2)-1)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+60*I*sin(d
*x+c)*cos(d*x+c)^2*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)-60*sin(d*x+c)*cos(d*x+c)^2*arctan(((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-60*
sin(d*x+c)*cos(d*x+c)^2*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1
)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-30*sin(d*x+c)*cos(d*x+c)^2*ln(-(((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1)/((
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+
1))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+60*I*sin(d*x+c)*cos(d*x+c)^2*arctan(
((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)*((-1+cos(d*x+c))/sin(d*x+c))^
(1/2)+30*I*sin(d*x+c)*cos(d*x+c)^2*ln(-(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1)/(((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1))*((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)-60*I*sin(d*x+c)*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)
+1)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+52*I*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)
+52*cos(d*x+c)^3*2^(1/2)-38*I*sin(d*x+c)*2^(1/2)+60*sin(d*x+c)*arctan(((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
+60*sin(d*x+c)*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*((-1+co
```

```
s(d*x+c))/sin(d*x+c))^(1/2)+30*sin(d*x+c)*ln(-(((-1+cos(d*x+c))/sin(d*x+c))
^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1)/(((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1))*((-1+cos(d*x+c))/s
in(d*x+c))^(1/2)-41*2^(1/2)*cos(d*x+c)^2-49*cos(d*x+c)*2^(1/2)+38*2^(1/2))*
sin(d*x+c)*(cos(d*x+c)/sin(d*x+c))^(7/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d
*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)^3*2^(1/2)*a^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1267 vs. $2(134) = 268$.
time = 0.63, size = 1267, normalized size = 7.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/15*(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) +
1)*((-(30*I + 30)*a^2*cos(3*d*x + 3*c) + (31*I + 31)*a^2*cos(d*x + c) - (30
*I - 30)*a^2*sin(3*d*x + 3*c) + (31*I - 31)*a^2*sin(d*x + c))*cos(3/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + ((30*I - 30)*a^2*cos(3*d*x +
3*c) - (31*I - 31)*a^2*cos(d*x + c) - (30*I + 30)*a^2*sin(3*d*x + 3*c) + (3
1*I + 31)*a^2*sin(d*x + c))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) - 1)))*sqrt(a) + 15*(2*((I - 1)*a^2*cos(2*d*x + 2*c)^2 + (I - 1)*a^2*si
n(2*d*x + 2*c)^2 - (2*I - 2)*a^2*cos(2*d*x + 2*c) + (I - 1)*a^2)*arctan2(2*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + 2*sin(d*x + c), 2*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + 2*cos(d*x + c)) +
((I + 1)*a^2*cos(2*d*x + 2*c)^2 + (I + 1)*a^2*sin(2*d*x + 2*c)^2 - (2*I + 2
)*a^2*cos(2*d*x + 2*c) + (I + 1)*a^2)*log(4*cos(d*x + c)^2 + 4*sin(d*x + c)
^2 + 4*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) +
1)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1))^2 + sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1))^2) + 8*(cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + sin(d*x + c)*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)))))*(cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + ((-(30*I + 30)*a^2*
cos(5*d*x + 5*c) + (25*I + 25)*a^2*cos(3*d*x + 3*c) - (7*I + 7)*a^2*cos(d*x
 + c) - (30*I - 30)*a^2*sin(5*d*x + 5*c) + (25*I - 25)*a^2*sin(3*d*x + 3*c)
 - (7*I - 7)*a^2*sin(d*x + c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) - 1)) + 8*(((I + 1)*a^2*cos(d*x + c) + (I - 1)*a^2*sin(d*x + c))*cos
(2*d*x + 2*c)^2 + (I + 1)*a^2*cos(d*x + c) + ((I + 1)*a^2*cos(d*x + c) + (I
 - 1)*a^2*sin(d*x + c))*sin(2*d*x + 2*c)^2 + (I - 1)*a^2*sin(d*x + c) + 2*(
-(I + 1)*a^2*cos(d*x + c) - (I - 1)*a^2*sin(d*x + c))*cos(2*d*x + 2*c))*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + ((30*I - 30)*a^2*co
```

```
s(5*d*x + 5*c) - (25*I - 25)*a^2*cos(3*d*x + 3*c) + (7*I - 7)*a^2*cos(d*x +
 c) - (30*I + 30)*a^2*sin(5*d*x + 5*c) + (25*I + 25)*a^2*sin(3*d*x + 3*c) -
 (7*I + 7)*a^2*sin(d*x + c))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) - 1)) + 8*((-(I - 1)*a^2*cos(d*x + c) + (I + 1)*a^2*sin(d*x + c))*cos(
2*d*x + 2*c)^2 - (I - 1)*a^2*cos(d*x + c) + (-(I - 1)*a^2*cos(d*x + c) + (I
 + 1)*a^2*sin(d*x + c))*sin(2*d*x + 2*c)^2 + (I + 1)*a^2*sin(d*x + c) + 2*(
(I - 1)*a^2*cos(d*x + c) - (I + 1)*a^2*sin(d*x + c))*cos(2*d*x + 2*c))*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)))*sqrt(a))/((cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(5/4)*d)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 409 vs. $2(134) = 268$.
time = 0.80, size = 409, normalized size = 2.32

$$\frac{16\sqrt{2}\left(26a^2e^{(5idx+5ic)}-35a^2e^{(3idx+3ic)}+15a^2e^{(idx+ic)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}-15\sqrt{\frac{128ia^5}{d^2}}\left(de^{(4idx+4ic)}-2de^{(2idx+2ic)}+d\right)\log\left(\frac{\left(16ia^3e^{(idx+ic)}+\sqrt{2}\sqrt{\frac{128ia^5}{d^2}}\left(de^{(2idx+2ic)}-d\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\right)e^{(-idx-ic)}}{4a^2}\right)+15\sqrt{\frac{128ia^5}{d^2}}\left(de^{(4idx+4ic)}-2de^{(2idx+2ic)}+d\right)\log\left(\frac{\left(16ia^3e^{(idx+ic)}-\sqrt{2}\sqrt{\frac{128ia^5}{d^2}}\left(de^{(2idx+2ic)}-d\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\right)e^{(-idx-ic)}}{4a^2}\right)}{60\left(de^{(4idx+4ic)}-2de^{(2idx+2ic)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/60*(16*sqrt(2)*(26*a^2*e^(5*I*d*x + 5*I*c) - 35*a^2*e^(3*I*d*x + 3*I*c) +
 15*a^2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d
*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)) - 15*sqrt(128*I*a^5/d^2)*(d*e^(
4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*log(1/4*(16*I*a^3*e^(I*d*x
+ I*c) + sqrt(2)*sqrt(128*I*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*
c) - 1)))*e^(-I*d*x - I*c)/a^2) + 15*sqrt(128*I*a^5/d^2)*(d*e^(4*I*d*x + 4*
I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*log(1/4*(16*I*a^3*e^(I*d*x + I*c) - sqr
t(2)*sqrt(128*I*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^
(-I*d*x - I*c)/a^2))/(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(7/2)*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^(7/2), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^{7/2}\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d*x)^(7/2)*(a + a*tan(c + d*x)*1i)^(5/2),x)

[Out] int(cot(c + d*x)^(7/2)*(a + a*tan(c + d*x)*1i)^(5/2), x)

3.764 $\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=142

$$-\frac{(4-4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d}-\frac{4ia^2\sqrt{\cot(c+dx)}\sqrt{a+}}{d}$$

```
[Out] (-4+4*I)*a^(5/2)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^
(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-4*I*a^2*cot(d*x+c)^(1/2)*(a+I*a*
tan(d*x+c))^(1/2)/d-2/3*a*cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(3/2)/d
```

Rubi [A]
time = 0.19, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4326, 3626, 3625, 211}

$$-\frac{(4-4i)a^{5/2}\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{4ia^2\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2a\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-4 + 4*I)*a^(5/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I
*a*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d - ((4*I)*a^2*Sqr
t[Cot[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d - (2*a*Cot[c + d*x]^(3/2)*(a
+ I*a*Tan[c + d*x])^(3/2))/(3*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3626

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*b*(a + b*Tan[e + f*x])^(m - 1)*((c +
 d*Tan[e + f*x])^(n + 1)/(f*(m - 1)*(a*c - b*d))), x] + Dist[2*(a^2/(a*c -
b*d)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{(a+ia\tan(c+dx))^{5/2}}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$

$$= -\frac{2a\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d} + \left(2ia\sqrt{\cot(c+dx)}\right.$$

$$= -\frac{4ia^2\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d} - \frac{2a\cot^{\frac{3}{2}}(c+dx)(a}{}$$

$$= -\frac{4ia^2\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d} - \frac{2a\cot^{\frac{3}{2}}(c+dx)(a}{}$$

$$= -\frac{(4-4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}}{d}$$

Mathematica [A]
time = 1.55, size = 142, normalized size = 1.00

$$-\frac{4ia^2e^{-i(c+dx)}\left(e^{i(c+dx)}\left(-3+4e^{2i(c+dx)}\right)-3\left(-1+e^{2i(c+dx)}\right)^{3/2}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{3d\left(-1+e^{2i(c+dx)}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((-4*I)/3)*a^2*(E^(I*(c + d*x))*(-3 + 4*E^((2*I)*(c + d*x))) - 3*(-1 + E^(
(2*I)*(c + d*x)))^(3/2)*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x
))]])*Sqrt[Cot[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^(I*(c + d*x))*(-1
 + E^((2*I)*(c + d*x))))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1043 vs. 2(116) = 232.
time = 46.20, size = 1044, normalized size = 7.35

method	result	size
default	Expression too large to display	1044

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/d*(8*I*sin(d*x+c)*2^(1/2)*cos(d*x+c)-12*I*arctan(((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*2^(1/2)+1)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-6*I*ln(-(((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1)/((
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+
1))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+12*arctan(((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*2^(1/2)-1)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+12*arct
an(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)*((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*cos(d*x+c)^2+6*ln(-(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin
(d*x+c)-sin(d*x+c)-cos(d*x+c)+1)/(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2
)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*c
os(d*x+c)^2+12*I*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+12*I*arctan(((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)*2^(1/2)+1)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-7
*I*sin(d*x+c)*2^(1/2)+6*I*ln(-(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*s
in(d*x+c)+sin(d*x+c)+cos(d*x+c)-1)/(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1
/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*cos(d*x+c)^2-12*I*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)+8*2^(1/2)*cos(d*x+c)^2-12*arctan(((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-12*a
rctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)*((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)-6*ln(-(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-si
n(d*x+c)-cos(d*x+c)+1)/(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+
c)+sin(d*x+c)+cos(d*x+c)-1))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-cos(d*x+c)*
2^(1/2)-7*2^(1/2))*sin(d*x+c)*(cos(d*x+c)/sin(d*x+c))^(5/2)*(a*(I*sin(d*x+c
)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)^2*2^
(1/2)*a^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1059 vs. $2(108) = 216$.
time = 0.60, size = 1059, normalized size = 7.46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(2*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) +
1)*((-(3*I - 3)*a^2*cos(3*d*x + 3*c) + (2*I - 2)*a^2*cos(d*x + c) + (3*I +
```

```
3)*a^2*sin(3*d*x + 3*c) - (2*I + 2)*a^2*sin(d*x + c))*cos(3/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + (-(3*I + 3)*a^2*cos(3*d*x + 3*c) + (2
*I + 2)*a^2*cos(d*x + c) - (3*I - 3)*a^2*sin(3*d*x + 3*c) + (2*I - 2)*a^2*s
in(d*x + c))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)))*sqrt
(a) + 3*(2*(-(I + 1)*a^2*cos(2*d*x + 2*c)^2 - (I + 1)*a^2*sin(2*d*x + 2*c)^
2 + (2*I + 2)*a^2*cos(2*d*x + 2*c) - (I + 1)*a^2)*arctan2(2*(cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + 2*sin(d*x + c), 2*(cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + 2*cos(d*x + c)) + ((I - 1)*a^2*co
s(2*d*x + 2*c)^2 + (I - 1)*a^2*sin(2*d*x + 2*c)^2 - (2*I - 2)*a^2*cos(2*d*x
 + 2*c) + (I - 1)*a^2)*log(4*cos(d*x + c)^2 + 4*sin(d*x + c)^2 + 4*sqrt(cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)*(cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1))^2 + sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) - 1))^2) + 8*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) - 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) - 1)))))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 -
 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 2*(((-(I - 1)*a^2*cos(d*x + c) + (
I + 1)*a^2*sin(d*x + c))*cos(2*d*x + 2*c)^2 - (I - 1)*a^2*cos(d*x + c) + (-
(I - 1)*a^2*cos(d*x + c) + (I + 1)*a^2*sin(d*x + c))*sin(2*d*x + 2*c)^2 + (
I + 1)*a^2*sin(d*x + c) + 2*((I - 1)*a^2*cos(d*x + c) - (I + 1)*a^2*sin(d*x
 + c))*cos(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 - 1)) + ((-(I + 1)*a^2*cos(d*x + c) - (I - 1)*a^2*sin(d*x + c))*cos(2*d*x
+ 2*c)^2 - (I + 1)*a^2*cos(d*x + c) + (-(I + 1)*a^2*cos(d*x + c) - (I - 1)*
a^2*sin(d*x + c))*sin(2*d*x + 2*c)^2 - (I - 1)*a^2*sin(d*x + c) + 2*((I + 1
)*a^2*cos(d*x + c) + (I - 1)*a^2*sin(d*x + c))*cos(2*d*x + 2*c))*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)))*sqrt(a))/((cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(5/4)*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(108) = 216$.
time = 0.95, size = 366, normalized size = 2.58

$$-\frac{16\sqrt{2}\left(4ia^2e^{(3idx+3ic)}-3ia^2e^{(idx+ic)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}+3\sqrt{-\frac{128ia^5}{d^2}}\left(de^{(2idx+2ic)}-d\right)\log\left(\frac{\left(16ia^3e^{(idx+ic)}+\sqrt{2}\sqrt{-\frac{128ia^5}{d^2}}\left(ide^{(2idx+2ic)}-id\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\right)e^{(-idx-ic)}}{4a^2}\right)-3\sqrt{-\frac{128ia^5}{d^2}}\left(de^{(2idx+2ic)}-d\right)\log\left(\frac{\left(16ia^3e^{(idx+ic)}+\sqrt{2}\sqrt{-\frac{128ia^5}{d^2}}\left(-ide^{(2idx+2ic)}+id\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\right)e^{(-idx-ic)}}{4a^2}\right)}{12\left(de^{(2idx+2ic)}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(16*sqrt(2)*(4*I*a^2*e^(3*I*d*x + 3*I*c) - 3*I*a^2*e^(I*d*x + I*c))*s
qrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d
*x + 2*I*c) - 1)) + 3*sqrt(-128*I*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*log(
1/4*(16*I*a^3*e^(I*d*x + I*c) + sqrt(2)*sqrt(-128*I*a^5/d^2)*(I*d*e^(2*I*d*
x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*
```

```
I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-I*d*x - I*c)/a^2) - 3*sqrt(-128*I
*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*log(1/4*(16*I*a^3*e^(I*d*x + I*c) + s
qrt(2)*sqrt(-128*I*a^5/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I
*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) -
 1)))*e^(-I*d*x - I*c)/a^2))/(d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(5/2)*(a+I*a*tan(d*x+c))**(5/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^{5/2}\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(5/2),x)
[Out] int(cot(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.765 $\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=179

$$\frac{2(-1)^{3/4}a^{5/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d}+\frac{(4+4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)}{\sqrt{a}}\right.}{}$$

```
[Out] 2*(-1)^(3/4)*a^(5/2)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(
d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+(4+4*I)*a^(5/2)*arctanh(
(1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*t
an(d*x+c)^(1/2)/d-2*a^2*cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.32, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {4326, 3634, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{2(-1)^{3/4}a^{5/2}\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{(4+4i)a^{5/2}\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^2\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (2*(-1)^(3/4)*a^(5/2)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a
 + I*a*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d + ((4 + 4*I)
*a^(5/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c +
d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d - (2*a^2*Sqrt[Cot[c + d*x]]
*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3634

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-a^2)*(b*c - a*d)*(a + b*Tan[e + f*x]
)^(m - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] + Di
st[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[b*(b*c*(m - 2) - a*d*(m - 2*n - 4)) + (a*b*c*(m - 2)
+ b^2*d*(n + 1) - a^2*d*(m + n - 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{(a+ia\tan(c+dx))^{5/2}}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

$$= -\frac{2a^2\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{d} - \left(2\sqrt{\cot(c+dx)}\,\sqrt{\right.$$

$$= -\frac{2a^2\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{d} - \left(ia\sqrt{\cot(c+dx)}\right.$$

$$= -\frac{2a^2\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{d} - \frac{\left(ia^3\sqrt{\cot(c+dx)}\right.}{}$$

$$= \frac{(4+4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{}}{d}$$

$$= \frac{(4+4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{}}{d}$$

$$= \frac{2(-1)^{3/4}a^{5/2}\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}}{d}$$

Mathematica [A]
time = 2.58, size = 176, normalized size = 0.98

$$-\frac{\sqrt{2}\,a^2e^{-i(c+dx)}\left(\sqrt{2}\,e^{i(c+dx)}-2\sqrt{2}\,\sqrt{-1+e^{2i(c+dx)}}\,\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)+\sqrt{-1+e^{2i(c+dx)}}\,\tanh^{-1}\left(\frac{\sqrt{2}\,e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] -((Sqrt[2]*a^2*(Sqrt[2]*E^(I*(c + d*x)) - 2*Sqrt[2]*Sqrt[-1 + E^((2*I)*(c +
 d*x))]*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]] + Sqrt[-1 +
 E^((2*I)*(c + d*x))]*ArcTanh[(Sqrt[2]*E^(I*(c + d*x)))/Sqrt[-1 + E^((2*I)*
(c + d*x))]])*Sqrt[Cot[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^(I*(c + d
*x))))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 887 vs. $2(144) = 288$.
time = 49.08, size = 888, normalized size = 4.96

method	result
default	$-\frac{\left(i \ln\left(\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}+1\right)\sqrt{2}\,\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\sin(dx+c)-i \ln\left(\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}-1\right)\sqrt{2}\,\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(I*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)*2^(1/2)*((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*sin(d*x+c)-I*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)*2^
(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-2*I*arctan(((-1+cos(d*x
+c))/sin(d*x+c))^(1/2))*2^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+
c)+8*I*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)*((-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*sin(d*x+c)+8*I*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*2^(1/2)-1)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+4*I*ln(-(((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1)/(((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1
))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+ln(((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)+1)*2^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-ln(((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)*2^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)*sin(d*x+c)+2*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2))*2^(1/2)*((-1+cos
(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-8*sin(d*x+c)*arctan(((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*2^(1/2)+1)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-8*sin(d*x+c
)*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)-4*sin(d*x+c)*ln(-(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)
*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1)/(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^
(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1))*((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)+2*I*sin(d*x+c)*2^(1/2)+2*cos(d*x+c)*2^(1/2)-2*2^(1/2))*sin(d*x+c)*(cos(d
*x+c)/sin(d*x+c))^(3/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*s
in(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)*2^(1/2)*a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^(3/2), x)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(137) = 274$.
time = 0.84, size = 570, normalized size = 3.18

$$-\frac{8\sqrt{2}a^{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}e^{(idx+ic)}-\sqrt{\frac{4ia^{5}}{d^{2}}}d\log\left(-\frac{16\left(3a^{3}e^{(2idx+2ic)}-a^{3}+\sqrt{2}\sqrt{\frac{4ia^{5}}{d^{2}}}\left(ide^{(3idx+3ic)}-ide^{(idx+ic)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\right)e^{(-2idx-2ic)}}{a}\right)+\sqrt{\frac{4ia^{5}}{d^{2}}}d\log\left(-\frac{16\left(3a^{3}e^{(2idx+2ic)}-a^{3}+\sqrt{2}\sqrt{\frac{4ia^{5}}{d^{2}}}\left(-ide^{(3idx+3ic)}+ide^{(idx+ic)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\right)e^{(-2idx-2ic)}}{a}\right)-\sqrt{\frac{128ia^{5}}{d^{2}}}d\log\left(\frac{\left(16ia^{3}e^{(idx+ic)}+\sqrt{2}\sqrt{\frac{128ia^{5}}{d^{2}}}\left(de^{(2idx+2ic)}-d\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\right)e^{(-idx-ic)}}{4a^{2}}\right)+\sqrt{\frac{128ia^{5}}{d^{2}}}d\log\left(\frac{\left(16ia^{3}e^{(idx+ic)}-\sqrt{2}\sqrt{\frac{128ia^{5}}{d^{2}}}\left(de^{(2idx+2ic)}-d\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\right)e^{(-idx-ic)}}{4a^{2}}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/4*(8*sqrt(2)*a^2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x +
2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*e^(I*d*x + I*c) - sqrt(4*I*a^5/d^2)*
d*log(-16*(3*a^3*e^(2*I*d*x + 2*I*c) - a^3 + sqrt(2)*sqrt(4*I*a^5/d^2)*(I*d
*e^(3*I*d*x + 3*I*c) - I*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x
 - 2*I*c)/a) + sqrt(4*I*a^5/d^2)*d*log(-16*(3*a^3*e^(2*I*d*x + 2*I*c) - a^3
 + sqrt(2)*sqrt(4*I*a^5/d^2)*(-I*d*e^(3*I*d*x + 3*I*c) + I*d*e^(I*d*x + I*c
))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2
*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)/a) - sqrt(128*I*a^5/d^2)*d*log(
1/4*(16*I*a^3*e^(I*d*x + I*c) + sqrt(2)*sqrt(128*I*a^5/d^2)*(d*e^(2*I*d*x +
 2*I*c) - d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c)
+ I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-I*d*x - I*c)/a^2) + sqrt(128*I*a^5/d^2
)*d*log(1/4*(16*I*a^3*e^(I*d*x + I*c) - sqrt(2)*sqrt(128*I*a^5/d^2)*(d*e^(2
*I*d*x + 2*I*c) - d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x +
 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-I*d*x - I*c)/a^2))/d
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(3/2)*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^{3/2}\,(a+a\tan(c+d\,x)\;1\mathrm{i})^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(5/2),x)
[Out] int(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.766 $\int \sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=179

$$\frac{5\sqrt[4]{-1}\,a^{5/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d}+\frac{(4-4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{\cdots}}{\sqrt{a+\cdots}}\right.}{}$$

```
[Out] 5*(-1)^(1/4)*a^(5/2)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(
d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+(4-4*I)*a^(5/2)*arctanh(
(1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*t
an(d*x+c)^(1/2)/d-a^2*(a+I*a*tan(d*x+c))^(1/2)/d/cot(d*x+c)^(1/2)
```

Rubi [A]

time = 0.31, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {4326, 3637, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{5\sqrt[4]{-1}\,a^{5/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{(4-4i)a^{5/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{a^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (5*(-1)^(1/4)*a^(5/2)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a
 + I*a*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d + ((4 - 4*I)
*a^(5/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c +
d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d - (a^2*Sqrt[a + I*a*Tan[c +
 d*x]])/(d*Sqrt[Cot[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^{5/2}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{\tan(c+dx)}}\,dx$$

$$= -\frac{a^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\cot(c+dx)}} + \left(a\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right.$$

$$= -\frac{a^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\cot(c+dx)}} - \frac{1}{2}\left(5a\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+d\right.$$

$$= -\frac{a^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\cot(c+dx)}} - \frac{\left(5a^3\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx\right.}{}$$

$$= \frac{(4-4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}}{d}$$

$$= \frac{(4-4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}}{d}$$

$$= \frac{5\sqrt[4]{-1}\,a^{5/2}\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}}{d}$$

Mathematica [A]
time = 2.41, size = 241, normalized size = 1.35

$$\frac{2i\sqrt{2}\,a^2e^{i(c+dx)}\left(\sqrt{2}\,e^{i(c+dx)}\left(-1+e^{2i(c+dx)}\right)-4\sqrt{2}\,\sqrt{-1+e^{2i(c+dx)}}\left(1+e^{2i(c+dx)}\right)\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)+5\sqrt{-1+e^{2i(c+dx)}}\left(1+e^{2i(c+dx)}\right)\tanh^{-1}\left(\frac{\sqrt{2}\,e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\cos^2(c+dx)\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{d\left(1+e^{2i(c+dx)}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((2*I)*Sqrt[2]*a^2*E^(I*(c + d*x))*(Sqrt[2]*E^(I*(c + d*x))*(-1 + E^((2*I)*
(c + d*x))) - 4*Sqrt[2]*Sqrt[-1 + E^((2*I)*(c + d*x))]*(1 + E^((2*I)*(c + d
*x)))*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]] + 5*Sqrt[-1 +
 E^((2*I)*(c + d*x))]*(1 + E^((2*I)*(c + d*x)))*ArcTanh[(Sqrt[2]*E^(I*(c +
```

```
d*x)))/Sqrt[-1 + E^((2*I)*(c + d*x))]])*Cos[c + d*x]^2*Sqrt[Cot[c + d*x]]*S
qrt[a + I*a*Tan[c + d*x]])/(d*(1 + E^((2*I)*(c + d*x)))^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 739 vs. 2(144) = 288.
time = 44.74, size = 740, normalized size = 4.13

method	result
default	$-\frac{(-1+\cos(dx+c))\left(5i\sqrt{2}\,\ln\left(\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}-1\right)\cos(dx+c)-10i\sqrt{2}\,\arctan\left(\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\right)\cos(dx+c)-5i\ln\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))*(5*I*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)*cos(d*
x+c)*2^(1/2)-10*I*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2))*cos(d*x+c)*2^(
1/2)-5*I*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)*cos(d*x+c)*2^(1/2)-2*I*((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*2^(1/2)+2*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+16*I*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*2^(1/2)+1)*cos(d*x+c)+16*I*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2
^(1/2)-1)*cos(d*x+c)-5*2^(1/2)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)*cos
(d*x+c)-10*2^(1/2)*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2))*cos(d*x+c)+5*
2^(1/2)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)*cos(d*x+c)+8*I*ln(-(((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1)/((
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+
1))*cos(d*x+c)-2*I*2^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+16*arctan(((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)*cos(d*x+c)+16*arctan(((-1+cos(d*
x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*cos(d*x+c)+8*ln(-(((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1)/(((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1))*cos(d*x+c))*
(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)/sin(d*x+c))^(1/2
)/(I*sin(d*x+c)+cos(d*x+c)-1)/((-1+cos(d*x+c))/sin(d*x+c))^(1/2)/cos(d*x+c)
*2^(1/2)*a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*sqrt(cot(d*x + c)), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 648 vs. $2(137) = 274$.
time = 0.77, size = 648, normalized size = 3.62

$$-\frac{4\sqrt{2}\left(-ia^2e^{(3idx+3ic)}+ia^2e^{(idx+ic)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}+\sqrt{-\frac{25ia^5}{d^2}}\left(de^{(2idx+2ic)}+d\right)\log\left(-\frac{16\left(15a^3e^{(2idx+2ic)}-5a^3+2\sqrt{2}\sqrt{-\frac{25ia^5}{d^2}}\left(de^{(3idx+3ic)}-de^{(idx+ic)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\right)e^{(-2idx-2ic)}}{5a}\right)-\sqrt{-\frac{25ia^5}{d^2}}\left(de^{(2idx+2ic)}+d\right)\log\left(-\frac{16\left(15a^3e^{(2idx+2ic)}-5a^3-2\sqrt{2}\sqrt{-\frac{25ia^5}{d^2}}\left(de^{(3idx+3ic)}-de^{(idx+ic)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\right)e^{(-2idx-2ic)}}{5a}\right)-\sqrt{-\frac{128ia^5}{d^2}}\left(de^{(2idx+2ic)}+d\right)\log\left(\frac{\left(16ia^3e^{(idx+ic)}+\sqrt{2}\sqrt{-\frac{128ia^5}{d^2}}\left(ide^{(2idx+2ic)}-id\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\right)e^{(-idx-ic)}}{4a^2}\right)+\sqrt{-\frac{128ia^5}{d^2}}\left(de^{(2idx+2ic)}+d\right)\log\left(\frac{\left(16ia^3e^{(idx+ic)}+\sqrt{2}\sqrt{-\frac{128ia^5}{d^2}}\left(-ide^{(2idx+2ic)}+id\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\right)e^{(-idx-ic)}}{4a^2}\right)}{4\left(de^{(2idx+2ic)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*(-I*a^2*e^(3*I*d*x + 3*I*c) + I*a^2*e^(I*d*x + I*c))*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
2*I*c) - 1)) + sqrt(-25*I*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(-16/5*(1
5*a^3*e^(2*I*d*x + 2*I*c) - 5*a^3 + 2*sqrt(2)*sqrt(-25*I*a^5/d^2)*(d*e^(3*I
*d*x + 3*I*c) - d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((
I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)
/a) - sqrt(-25*I*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(-16/5*(15*a^3*e^(
2*I*d*x + 2*I*c) - 5*a^3 - 2*sqrt(2)*sqrt(-25*I*a^5/d^2)*(d*e^(3*I*d*x + 3*
I*c) - d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*
d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)/a) - sqr
t(-128*I*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(1/4*(16*I*a^3*e^(I*d*x +
I*c) + sqrt(2)*sqrt(-128*I*a^5/d^2)*(I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(a/
(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2
*I*c) - 1)))*e^(-I*d*x - I*c)/a^2) + sqrt(-128*I*a^5/d^2)*(d*e^(2*I*d*x + 2
*I*c) + d)*log(1/4*(16*I*a^3*e^(I*d*x + I*c) + sqrt(2)*sqrt(-128*I*a^5/d^2)
*(-I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I
*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-I*d*x - I*c)/a^2)
)/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(1/2)*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*sqrt(cot(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cot\left(c+d\,x\right)}\,\left(a+a\tan(c+d\,x)\;1\mathrm{i}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] int(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.767 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{\cot(c+dx)}}\,dx$

Optimal. Leaf size=222

$$-\frac{23(-1)^{3/4}a^{5/2}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{4d}-\frac{(4+4i)a^{5/2}\tanh^{-1}\left(\frac{(1\cdots}{\cdots}\right.}{}$$

```
[Out] -23/4*(-1)^(3/4)*a^(5/2)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*
tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-(4+4*I)*a^(5/2)*arct
anh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/
2)*tan(d*x+c)^(1/2)/d-1/2*a^2*(a+I*a*tan(d*x+c))^(1/2)/d/cot(d*x+c)^(3/2)+9
/4*I*a^2*(a+I*a*tan(d*x+c))^(1/2)/d/cot(d*x+c)^(1/2)
```

Rubi [A]

time = 0.39, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4326, 3637, 3678, 3682, 3625, 211, 3680, 65, 223, 209}

$$-\frac{23(-1)^{3/4}a^{5/2}\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{4d}-\frac{(4+4i)a^{5/2}\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{a^2\sqrt{a+ia\tan(c+dx)}}{2d\cot^{\frac{3}{2}}(c+dx)}+\frac{9ia^2\sqrt{a+ia\tan(c+dx)}}{4d\sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(5/2)/Sqrt[Cot[c + d*x]],x]
```

```
[Out] (-23*(-1)^(3/4)*a^(5/2)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt
[a + I*a*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(4*d) - ((4
+ 4*I)*a^(5/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Ta
n[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d - (a^2*Sqrt[a + I*a*T
an[c + d*x]])/(2*d*Cot[c + d*x]^(3/2)) + (((9*I)/4)*a^2*Sqrt[a + I*a*Tan[c
+ d*x]])/(d*Sqrt[Cot[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3678

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Dist[
1/(a*(m + n)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Simp
[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{\cot(c+dx)}}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^{5/2}\,dx$$

$$= -\frac{a^2\sqrt{a+ia\tan(c+dx)}}{2d\cot^{\frac{3}{2}}(c+dx)} + \frac{1}{2}\left(a\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \sqrt{\tan(c$$

$$= -\frac{a^2\sqrt{a+ia\tan(c+dx)}}{2d\cot^{\frac{3}{2}}(c+dx)} + \frac{9ia^2\sqrt{a+ia\tan(c+dx)}}{4d\sqrt{\cot(c+dx)}} + \frac{1}{2}\left(\sqrt{\cot(c+dx)}\right.$$

$$= -\frac{a^2\sqrt{a+ia\tan(c+dx)}}{2d\cot^{\frac{3}{2}}(c+dx)} + \frac{9ia^2\sqrt{a+ia\tan(c+dx)}}{4d\sqrt{\cot(c+dx)}} + \frac{1}{8}\left(23ia\sqrt{\cot(c+\right.$$

$$= -\frac{a^2\sqrt{a+ia\tan(c+dx)}}{2d\cot^{\frac{3}{2}}(c+dx)} + \frac{9ia^2\sqrt{a+ia\tan(c+dx)}}{4d\sqrt{\cot(c+dx)}} + \frac{\left(23ia^3\sqrt{\cot(c+\right.}{}$$

$$= -\frac{(4+4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+d}{d}$$

$$= -\frac{(4+4i)a^{5/2}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+d}{d}$$

$$= -\frac{23(-1)^{3/4}a^{5/2}\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+}{4d}$$

Mathematica [A]
time = 7.72, size = 318, normalized size = 1.43

$$-\frac{\left(\sqrt{2}\,e^{-i(3c+dx)}\sqrt{e^{idx}}\,\sqrt{-1+e^{2i(c+dx)}}\,\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\,\sqrt{\frac{i\left(1+e^{2i(c+dx)}\right)}{-1+e^{2i(c+dx)}}}\left(32\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)-23\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)-\sqrt{\cot(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)(\cos(2c)-i\sin(2c))\sqrt{\cos(dx)+i\sin(dx)}\,(-2+2\cos(2(c+dx))+9i\sin(2(c+dx)))\right)(a+ia\tan(c+dx))^{5/2}}{8d\sec^{\frac{5}{2}}(c+dx)(\cos(dx)+i\sin(dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(5/2)/Sqrt[Cot[c + d*x]],x]
```

```
[Out] -1/8*(((Sqrt[2]*Sqrt[E^(I*d*x)]*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[E^(I*(c
 + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[(I*(1 + E^((2*I)*(c + d*x))))/(-1
+ E^((2*I)*(c + d*x)))]*(32*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c +
 d*x))]] - 23*Sqrt[2]*ArcTanh[(Sqrt[2]*E^(I*(c + d*x)))/Sqrt[-1 + E^((2*I)*
(c + d*x))]]))/E^(I*(3*c + d*x)) - Sqrt[Cot[c + d*x]]*Sec[c + d*x]^(5/2)*(C
os[2*c] - I*Sin[2*c])*Sqrt[Cos[d*x] + I*Sin[d*x]]*(-2 + 2*Cos[2*(c + d*x)]
+ (9*I)*Sin[2*(c + d*x)]))*(a + I*a*Tan[c + d*x])^(5/2))/(d*Sec[c + d*x]^(5
/2)*(Cos[d*x] + I*Sin[d*x])^(5/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2351 vs. 2(175) = 350.
time = 51.21, size = 2352, normalized size = 10.59

method	result	size
default	Expression too large to display	2352

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(5/2)/cot(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d*(-64*I*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*cos(d*x+
c)^2+4*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-64*I*arctan(((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)*cos(d*x+c)^2-32*I*ln(-(((-1+cos
(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1)/(((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1)
)*cos(d*x+c)^2-4*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+64*I*arctan((
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*cos(d*x+c)^3+64*I*arctan(((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)*cos(d*x+c)^3+32*I*ln(-(((-1+cos(d*
x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1)/(((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1))*c
os(d*x+c)^3-23*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)*2^(1/2)*cos(d*x+c)^
3+46*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2))*2^(1/2)*cos(d*x+c)^3+23*ln(
((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)*2^(1/2)*cos(d*x+c)^3+64*arctan(((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*sin(d*x+c)*cos(d*x+c)^2+32*ln(-(((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1)
/(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+
c)+1))*sin(d*x+c)*cos(d*x+c)^2+64*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
```

```
*2^(1/2)+1)*sin(d*x+c)*cos(d*x+c)^2+22*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2
^(1/2)*cos(d*x+c)^3-22*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*cos(d*x+c
)-4*2^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-23*2^(1/2)*ln((
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)*cos(d*x+c)^2+23*2^(1/2)*ln(((-1+cos(d*
x+c))/sin(d*x+c))^(1/2)-1)*cos(d*x+c)^2-46*2^(1/2)*arctan(((-1+cos(d*x+c))/
sin(d*x+c))^(1/2))*cos(d*x+c)^2+64*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*2^(1/2)+1)*cos(d*x+c)^2+64*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1
/2)-1)*cos(d*x+c)^2-64*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)
*cos(d*x+c)^3-32*ln(-(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)
+sin(d*x+c)+cos(d*x+c)-1)/(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d
*x+c)-sin(d*x+c)-cos(d*x+c)+1))*cos(d*x+c)^3-64*arctan(((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)*2^(1/2)+1)*cos(d*x+c)^3-18*2^(1/2)*sin(d*x+c)*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*cos(d*x+c)+23*I*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-
1)*2^(1/2)*cos(d*x+c)^2+23*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)*2^(1/2)
*sin(d*x+c)*cos(d*x+c)^2-46*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2))*2^(1
/2)*sin(d*x+c)*cos(d*x+c)^2-23*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)*2^(
1/2)*sin(d*x+c)*cos(d*x+c)^2-22*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*
sin(d*x+c)*cos(d*x+c)^2+46*I*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2))*2^(
1/2)*cos(d*x+c)^2-23*I*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)*2^(1/2)*cos
(d*x+c)^2-64*I*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*sin(d*x
+c)*cos(d*x+c)^2+4*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+32*ln(-(((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1)/(
((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)
+1))*cos(d*x+c)^2-23*I*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)*2^(1/2)*sin
(d*x+c)*cos(d*x+c)^2+22*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*
x+c)*cos(d*x+c)^2-64*I*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)
*sin(d*x+c)*cos(d*x+c)^2-32*I*ln(-(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/
2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1)/(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1))*sin(d*x+c)*cos(d*x+c)^2-4*I*((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+4*I*((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*2^(1/2)*cos(d*x+c)^2-22*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*2^(1/2)*cos(d*x+c)+22*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*cos(d*
x+c)^3-23*I*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)*2^(1/2)*cos(d*x+c)^3-4
6*I*arctan(((-1+cos(d*x+c))/sin(d*x+c))^(1/2))*2^(1/2)*cos(d*x+c)^3+23*I*ln
(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)*2^(1/2)*cos(d*x+c)^3-18*I*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)*cos(d*x+c)+23*I*ln(((-1+cos(d*x
+c))/sin(d*x+c))^(1/2)-1)*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2+46*I*arctan(((-1+
cos(d*x+c))/sin(d*x+c))^(1/2))*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2)*(a*(I*sin(d
*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+I*cos(d*x+c)-1+I-sin(d*x+
c)+cos(d*x+c))/cos(d*x+c)/sin(d*x+c)/((-1+cos(d*x+c))/sin(d*x+c))^(1/2)/(co
s(d*x+c)/sin(d*x+c))^(1/2)*2^(1/2)*a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/cot(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)/sqrt(cot(d*x + c)), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 722 vs. $2(166) = 332$.
time = 0.94, size = 722, normalized size = 3.25

$$\frac{\sqrt{2}\left(11 a^{2} e^{(5 i d x+5 i c)}-4 a^{2} e^{(3 i d x+3 i c)}-7 a^{2} e^{(i d x+i c)}\right) \sqrt{\frac{a}{e^{(2 i d x+2 i c)}+1}} \sqrt{\frac{i e^{(2 i d x+2 i c)}+i}{e^{(2 i d x+2 i c)}-1}}-\sqrt{\frac{529 i a^{5}}{16 d^{2}}}\left(d e^{(4 i d x+4 i c)}+2 d e^{(2 i d x+2 i c)}+d\right) \log \left(-\frac{16\left(69 a^{3} e^{(2 i d x+2 i c)}-23 a^{3}+8 \sqrt{2} \sqrt{\frac{529 i a^{5}}{16 d^{2}}}\left(i d e^{(3 i d x+3 i c)}-i d e^{(i d x+i c)}\right) \sqrt{\frac{a}{e^{(2 i d x+2 i c)}+1}} \sqrt{\frac{i e^{(2 i d x+2 i c)}+i}{e^{(2 i d x+2 i c)}-1}}\right) e^{(-2 i d x-2 i c)}}{23 a}\right)+\sqrt{\frac{529 i a^{5}}{16 d^{2}}}\left(d e^{(4 i d x+4 i c)}+2 d e^{(2 i d x+2 i c)}+d\right) \log \left(-\frac{16\left(69 a^{3} e^{(2 i d x+2 i c)}-23 a^{3}+8 \sqrt{2} \sqrt{\frac{529 i a^{5}}{16 d^{2}}}\left(-i d e^{(3 i d x+3 i c)}+i d e^{(i d x+i c)}\right) \sqrt{\frac{a}{e^{(2 i d x+2 i c)}+1}} \sqrt{\frac{i e^{(2 i d x+2 i c)}+i}{e^{(2 i d x+2 i c)}-1}}\right) e^{(-2 i d x-2 i c)}}{23 a}\right)-\sqrt{\frac{128 i a^{5}}{d^{2}}}\left(d e^{(4 i d x+4 i c)}+2 d e^{(2 i d x+2 i c)}+d\right) \log \left(\frac{\left(16 i a^{3} e^{(i d x+i c)}+\sqrt{2} \sqrt{\frac{128 i a^{5}}{d^{2}}}\left(d e^{(2 i d x+2 i c)}-d\right) \sqrt{\frac{a}{e^{(2 i d x+2 i c)}+1}} \sqrt{\frac{i e^{(2 i d x+2 i c)}+i}{e^{(2 i d x+2 i c)}-1}}\right) e^{(-i d x-i c)}}{4 a^{2}}\right)+\sqrt{\frac{128 i a^{5}}{d^{2}}}\left(d e^{(4 i d x+4 i c)}+2 d e^{(2 i d x+2 i c)}+d\right) \log \left(\frac{\left(16 i a^{3} e^{(i d x+i c)}-\sqrt{2} \sqrt{\frac{128 i a^{5}}{d^{2}}}\left(d e^{(2 i d x+2 i c)}-d\right) \sqrt{\frac{a}{e^{(2 i d x+2 i c)}+1}} \sqrt{\frac{i e^{(2 i d x+2 i c)}+i}{e^{(2 i d x+2 i c)}-1}}\right) e^{(-i d x-i c)}}{4 a^{2}}\right)}{4\left(d e^{(4 i d x+4 i c)}+2 d e^{(2 i d x+2 i c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/cot(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*(11*a^2*e^(5*I*d*x + 5*I*c) - 4*a^2*e^(3*I*d*x + 3*I*c) - 7*a^
2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2
*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)) - sqrt(529/16*I*a^5/d^2)*(d*e^(4*I*d*
x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(-16/23*(69*a^3*e^(2*I*d*x + 2
*I*c) - 23*a^3 + 8*sqrt(2)*sqrt(529/16*I*a^5/d^2)*(I*d*e^(3*I*d*x + 3*I*c)
- I*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x
 + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)/a) + sqrt(5
29/16*I*a^5/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(
-16/23*(69*a^3*e^(2*I*d*x + 2*I*c) - 23*a^3 + 8*sqrt(2)*sqrt(529/16*I*a^5/d
^2)*(-I*d*e^(3*I*d*x + 3*I*c) + I*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^
(-2*I*d*x - 2*I*c)/a) - sqrt(128*I*a^5/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^
(2*I*d*x + 2*I*c) + d)*log(1/4*(16*I*a^3*e^(I*d*x + I*c) + sqrt(2)*sqrt(128
*I*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*s
qrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-I*d*x - I*c
)/a^2) + sqrt(128*I*a^5/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*
c) + d)*log(1/4*(16*I*a^3*e^(I*d*x + I*c) - sqrt(2)*sqrt(128*I*a^5/d^2)*(d*
e^(2*I*d*x + 2*I*c) - d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d
*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-I*d*x - I*c)/a^2))/(d*e^(4
*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(5/2)/cot(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/cot(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)/sqrt(cot(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + a\tan(c + d\,x)\ 1\mathrm{i})^{5/2}}{\sqrt{\cot\left(c + d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(5/2)/cot(c + d*x)^(1/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^(5/2)/cot(c + d*x)^(1/2), x)
```

3.768 $\int \frac{\cot^{\frac{5}{2}}(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=181

$$-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{a}\,d}+\frac{\cot^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}+\frac{7i\sqrt{}$$

```
[Out] (-1/2+1/2*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2
))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d/a^(1/2)+cot(d*x+c)^(3/2)/d/(a+I*a*ta
n(d*x+c))^(1/2)-5/3*cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(1/2)/a/d+7/3*I*cot
(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a/d
```

Rubi [A]

time = 0.33, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4326, 3640, 3679, 12, 3625, 211}

$$-\frac{5\cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{3ad}+\frac{\cot^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}+\frac{7i\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{3ad}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(5/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-1/2 + I/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan
[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[a]*d) + Cot[c + d*
x]^(3/2)/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((7*I)/3)*Sqrt[Cot[c + d*x]]*Sqr
t[a + I*a*Tan[c + d*x]])/(a*d) - (5*Cot[c + d*x]^(3/2)*Sqrt[a + I*a*Tan[c +
 d*x]])/(3*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
```

```
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx$$

$$= \frac{\cot^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} + \frac{\left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{\sqrt{a+ia\tan(c}{}}{a^2}$$

$$= \frac{\cot^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{5\cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{3ad} + \frac{\left(2\sqrt{\cot(c}\right.}{}$$

$$= \frac{\cot^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} + \frac{7i\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{3ad} - \frac{5\cot^{\frac{3}{2}}(c}{}$$

$$= \frac{\cot^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} + \frac{7i\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{3ad} - \frac{5\cot^{\frac{3}{2}}(c}{}$$

$$= \frac{\cot^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} + \frac{7i\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{3ad} - \frac{5\cot^{\frac{3}{2}}(c}{}$$

$$= -\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{a}\,d}$$

Mathematica [A]
time = 1.65, size = 159, normalized size = 0.88

$$\frac{i\left(3-18e^{2i(c+dx)}+7e^{4i(c+dx)}+3e^{i(c+dx)}\left(-1+e^{2i(c+dx)}\right)^{3/2}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\cot(c+dx)}}{3\sqrt{2}\,d\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(-1+e^{4i(c+dx)}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(5/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((I/3)*(3 - 18*E^((2*I)*(c + d*x)) + 7*E^((4*I)*(c + d*x)) + 3*E^(I*(c + d*
x))*(-1 + E^((2*I)*(c + d*x)))^(3/2)*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((
2*I)*(c + d*x))]])*Sqrt[Cot[c + d*x]])/(Sqrt[2]*d*Sqrt[(a*E^((2*I)*(c + d*x
)))/(1 + E^((2*I)*(c + d*x)))]*(-1 + E^((4*I)*(c + d*x))))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 373 vs. 2(144) = 288.
time = 43.98, size = 374, normalized size = 2.07

method	result
default	$\frac{\left(-\frac{1}{6}-\frac{i}{6}\right)\left(3i\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\sin(dx+c)\cos(dx+c)\arctan\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\sqrt{2}\right)\sqrt{2}+3i\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\sin(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/6-1/6*I)/d*(3*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c
)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))*2^(1/2)+3*
I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*arctan((1/2+1/2*I)*((-1+cos
(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))*2^(1/2)+3*((-1+cos(d*x+c))/sin(d*x+c))^
(1/2)*cos(d*x+c)^2*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^
(1/2))*2^(1/2)-3*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*arctan((1/2+1/2*I)*((-1
+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))*2^(1/2)-2*I*cos(d*x+c)*sin(d*x+c)+5
*I*cos(d*x+c)^2-2*sin(d*x+c)*cos(d*x+c)-5*cos(d*x+c)^2+7-7*I)*sin(d*x+c)*(c
os(d*x+c)/sin(d*x+c))^(5/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/
(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c)^2/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^(5/2)/sqrt(I*a*tan(d*x + c) + a), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 388 vs. 2(135) = 270.
time = 1.19, size = 388, normalized size = 2.14

$$-\frac{2\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\left(-7i\,e^{(4i\,dx+4i\,c)}+18i\,e^{(2i\,dx+2i\,c)}-3i\right)-3\left(ade^{(3i\,dx+3i\,c)}-ade^{(i\,dx+i\,c)}\right)\sqrt{-\frac{2i}{ad^2}}\log\left(-2\left(\sqrt{2}\left(i\,ade^{(2i\,dx+2i\,c)}-i\,ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{-\frac{2i}{ad^2}}-2i\,ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+3\left(ade^{(3i\,dx+3i\,c)}-ade^{(i\,dx+i\,c)}\right)\sqrt{-\frac{2i}{ad^2}}\log\left(-2\left(\sqrt{2}\left(-i\,ade^{(2i\,dx+2i\,c)}+i\,ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{-\frac{2i}{ad^2}}-2i\,ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)}{12\left(ade^{(3i\,dx+3i\,c)}-ade^{(i\,dx+i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/12*(2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I
*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*(-7*I*e^(4*I*d*x + 4*I*c) + 18*I*e^(2*I
*d*x + 2*I*c) - 3*I) - 3*(a*d*e^(3*I*d*x + 3*I*c) - a*d*e^(I*d*x + I*c))*sq
rt(-2*I/(a*d^2))*log(-2*(sqrt(2)*(I*a*d*e^(2*I*d*x + 2*I*c) - I*a*d)*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
2*I*c) - 1))*sqrt(-2*I/(a*d^2)) - 2*I*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c))
```

```
+ 3*(a*d*e^(3*I*d*x + 3*I*c) - a*d*e^(I*d*x + I*c))*sqrt(-2*I/(a*d^2))*log(
-2*(sqrt(2)*(-I*a*d*e^(2*I*d*x + 2*I*c) + I*a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-2
*I/(a*d^2)) - 2*I*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)))/(a*d*e^(3*I*d*x + 3
*I*c) - a*d*e^(I*d*x + I*c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(5/2)/(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(cot(d*x + c)^(5/2)/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(c+d\,x)^{5/2}}{\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(5/2)/(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int(cot(c + d*x)^(5/2)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.769 $\int \frac{\cot^{\frac{3}{2}}(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=140

$$\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{a}\,d}+\frac{\sqrt{\cot(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{3\sqrt{\cot(}}{}$$

```
[Out] (1/2+1/2*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
)*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d/a^(1/2)+cot(d*x+c)^(1/2)/d/(a+I*a*tan
(d*x+c))^(1/2)-3*cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a/d
```

Rubi [A]

time = 0.22, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4326, 3640, 3679, 12, 3625, 211}

$$-\frac{3\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{ad}+\frac{\sqrt{\cot(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}+\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(3/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((1/2 + I/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[
c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[a]*d) + Sqrt[Cot[c
+ d*x]]/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (3*Sqrt[Cot[c + d*x]]*Sqrt[a + I*a
*Tan[c + d*x]])/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
```

```
Q[c^2 + d^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx$$

$$= \frac{\sqrt{\cot(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}} + \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{\left(\frac{3a}{2}-ia\tan(c+dx)\right)\sqrt{\ }}{\tan}}{a^2}$$

$$= \frac{\sqrt{\cot(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}} - \frac{3\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{ad} + \frac{\left(2\sqrt{\cot(c}\right.}{}$$

$$= \frac{\sqrt{\cot(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}} - \frac{3\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{ad} + \frac{\left(i\sqrt{\cot(c}\right.}{}$$

$$= \frac{\sqrt{\cot(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}} - \frac{3\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{ad} + \frac{\left(a\sqrt{\cot(c}\right.}{}$$

$$= \frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{\sqrt{a}\,d} + \frac{}{a}$$

Mathematica [A]
time = 1.34, size = 139, normalized size = 0.99

$$\frac{e^{-2i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(1-5e^{2i(c+dx)}+e^{i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\cot(c+dx)}}{\sqrt{2}\,ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(3/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(1 - 5*E^((2*I)*(c
 + d*x)) + E^(I*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]*ArcTanh[E^(I*(c +
 d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]])*Sqrt[Cot[c + d*x]])/(Sqrt[2]*a*d*E^
((2*I)*(c + d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(113) = 226.
time = 43.38, size = 287, normalized size = 2.05

method	result

default	$\frac{\left(-\frac{1}{2}-\frac{i}{2}\right)\left(i\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\sin(dx+c)\arctan\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\sqrt{2}\right)\sqrt{2}+\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\cos(dx+c)\sqrt{2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2-1/2*I)/d*(I*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*arc
tan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))+((-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*cos(d*x+c)*2^(1/2)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*2^(1/2))+((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*arctan((1/2+1
/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))*2^(1/2)+3*I*sin(d*x+c)-2*
I*cos(d*x+c)+3*sin(d*x+c)+2*cos(d*x+c))*sin(d*x+c)*(cos(d*x+c)/sin(d*x+c))^
(3/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+
c))/cos(d*x+c)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^(3/2)/sqrt(I*a*tan(d*x + c) + a), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(106) = 212.
time = 1.06, size = 331, normalized size = 2.36

$$\frac{\left(ad\sqrt{\frac{2i}{ad^2}}e^{(idx+ic)}\log\left(2\left(\sqrt{2}\left(ade^{(2idx+2ic)}-ad\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{\frac{2i}{ad^2}}+2iae^{(idx+ic)}\right)e^{(-idx-ic)}\right)-ad\sqrt{\frac{2i}{ad^2}}e^{(idx+ic)}\log\left(-2\left(\sqrt{2}\left(ade^{(2idx+2ic)}-ad\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{\frac{2i}{ad^2}}-2iae^{(idx+ic)}\right)e^{(-idx-ic)}\right)-2\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\left(5e^{(2idx+2ic)}-1\right)\right)e^{(-idx-ic)}}{4ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(a*d*sqrt(2*I/(a*d^2))*e^(I*d*x + I*c)*log(2*(sqrt(2)*(a*d*e^(2*I*d*x +
 2*I*c) - a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c
) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(2*I/(a*d^2)) + 2*I*a*e^(I*d*x + I*c)
)*e^(-I*d*x - I*c)) - a*d*sqrt(2*I/(a*d^2))*e^(I*d*x + I*c)*log(-2*(sqrt(2)
*(a*d*e^(2*I*d*x + 2*I*c) - a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*
e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(2*I/(a*d^2)) - 2*I
*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*(5*e^(
2*I*d*x + 2*I*c) - 1))*e^(-I*d*x - I*c)/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{\frac{3}{2}}(c + dx)}{\sqrt{ia(\tan(c + dx) - i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(3/2)/(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral(cot(c + d*x)**(3/2)/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(cot(d*x + c)^(3/2)/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(c + d\,x)^{3/2}}{\sqrt{a + a\tan(c + d\,x)\ 1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(3/2)/(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int(cot(c + d*x)^(3/2)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.770 $\int \frac{\sqrt{\cot(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=105

$$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{a}\,d}+\frac{1}{d\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] (1/2-1/2*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
)*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d/a^(1/2)+1/d/cot(d*x+c)^(1/2)/(a+I*a*t
an(d*x+c))^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {4326, 3629, 3627, 3625, 211}

$$\frac{1}{d\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cot[c + d*x]]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((1/2 - I/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[
c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[a]*d) + 1/(d*Sqrt[C
ot[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3627

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
```

```
 + f*x])^n/(2*b*f*m)), x] - Dist[(a*c - b*d)/(2*b^2), Int[(a + b*Tan[e + f*
x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && EqQ
[m + n, 0] && LeQ[m, -2^(-1)]
```

Rule 3629

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-d)*(a + b*Tan[e + f*x])^m*((c + d*Ta
n[e + f*x])^(n + 1)/(f*m*(c^2 + d^2))), x] + Dist[a/(a*c - b*d), Int[(a + b
*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -1]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{\cot(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx &= \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{1}{\sqrt{\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}\,dx \\
&= \frac{2}{d\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}} + \left(i\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \\
&= \frac{1}{d\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}} + \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int}{2a} \\
&= \frac{1}{d\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}} - \frac{\left(ia\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)}{} \\
&= \frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{\sqrt{a}\,d} +
\end{aligned}
$$

Mathematica [A]

time = 1.00, size = 140, normalized size = 1.33

$$-\frac{ie^{-2i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(-1+e^{2i(c+dx)}+e^{i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\cot(c+dx)}}{\sqrt{2}\,ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cot[c + d*x]]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-I)*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(-1 + E^((2*I
)*(c + d*x)) + E^(I*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]*ArcTanh[E^(I*
(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]])*Sqrt[Cot[c + d*x]])/(Sqrt[2]*a*
d*E^((2*I)*(c + d*x)))
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 256 vs. 2(83) = 166.
time = 43.82, size = 257, normalized size = 2.45

method	result
default	$\frac{\left(-\frac{1}{2}-\frac{i}{2}\right)\left(i\sin(dx+c)\sqrt{2}\arctan\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\sqrt{2}\right)+i\sin(dx+c)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}+\cos(dx+c)\sqrt{2}\arctan}{d(i\,\mathrm{si}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2-1/2*I)/d*(I*sin(d*x+c)*2^(1/2)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*2^(1/2))+I*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+co
s(d*x+c)*2^(1/2)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1
/2))-sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-2^(1/2)*arctan((1/2+1/2*
I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)))*(cos(d*x+c)/sin(d*x+c))^(1/
2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c))
/((-1+cos(d*x+c))/sin(d*x+c))^(1/2)/a
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cot(d*x + c))/sqrt(I*a*tan(d*x + c) + a), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(77) = 154$.
time = 0.79, size = 333, normalized size = 3.17

$$-\frac{\left(ad\sqrt{-\frac{2i}{ad^2}}e^{(idx+ic)}\log\left(-2\left(\sqrt{2}\left(iade^{(2idx+2ic)}-iad\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{-\frac{2i}{ad^2}}-2iae^{(idx+ic)}\right)e^{(-idx-ic)}\right)-ad\sqrt{-\frac{2i}{ad^2}}e^{(idx+ic)}\log\left(-2\left(\sqrt{2}\left(-iade^{(2idx+2ic)}+iad\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{-\frac{2i}{ad^2}}-2iae^{(idx+ic)}\right)e^{(-idx-ic)}\right)+2\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\left(ie^{(2idx+2ic)}-i\right)\right)e^{(-idx-ic)}}{4ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(a*d*sqrt(-2*I/(a*d^2))*e^(I*d*x + I*c)*log(-2*(sqrt(2)*(I*a*d*e^(2*I*
d*x + 2*I*c) - I*a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x
+ 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-2*I/(a*d^2)) - 2*I*a*e^(I*d*
x + I*c))*e^(-I*d*x - I*c)) - a*d*sqrt(-2*I/(a*d^2))*e^(I*d*x + I*c)*log(-2
*(sqrt(2)*(-I*a*d*e^(2*I*d*x + 2*I*c) + I*a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-2*I
/(a*d^2)) - 2*I*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 2*sqrt(2)*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*
c) - 1))*(I*e^(2*I*d*x + 2*I*c) - I))*e^(-I*d*x - I*c)/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cot(c+dx)}}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(1/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(cot(c + d*x))/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cot(d*x + c))/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cot(c+d\,x)}}{\sqrt{a+a\tan(c+d\,x)\,1i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(1/2)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] int(cot(c + d*x)^(1/2)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.771 $\int \frac{1}{\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=108

$$-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{a}\,d}+\frac{i}{d\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(}}$$

```
[Out] (-1/2-1/2*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2
))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d/a^(1/2)+I/d/cot(d*x+c)^(1/2)/(a+I*a*
tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4326, 3627, 3625, 211}

$$\frac{i}{d\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cot[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] ((-1/2 - I/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan
[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[a]*d) + I/(d*Sqrt[
Cot[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3627

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
```

```
 + f*x])^n/(2*b*f*m)), x] - Dist[(a*c - b*d)/(2*b^2), Int[(a + b*Tan[e + f*
x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && EqQ
[m + n, 0] && LeQ[m, -2^(-1)]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx$$

$$= \frac{i}{d\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{\left(i\sqrt{\cot(c+dx)}\sqrt{\tan\right.}{}$$

$$= \frac{i}{d\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{\left(a\sqrt{\cot(c+dx)}\sqrt{\tan\right.}{}$$

$$= -\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{}}{\sqrt{a}\,d}$$

Mathematica [A]
time = 1.10, size = 138, normalized size = 1.28

$$\frac{e^{-2i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(-1+e^{2i(c+dx)}-e^{i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\cot(c+dx)}}{\sqrt{2}\,ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cot[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] (Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(-1 + E^((2*I)*(c
+ d*x)) - E^(I*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]*ArcTanh[E^(I*(c +
d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]])*Sqrt[Cot[c + d*x]])/(Sqrt[2]*a*d*E^(
(2*I)*(c + d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(85) = 170$.
time = 50.53, size = 270, normalized size = 2.50

method	result
default	$\frac{\left(-\frac{1}{2}+\frac{i}{2}\right)\left(i\sin(dx+c)\sqrt{2}\,\arctan\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\sqrt{2}\right)-i\sin(dx+c)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}+\cos(dx+c)\sqrt{2}\,\arctan}{d(i\sin(dx+c)+\cos(d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2+1/2*I)/d*(I*sin(d*x+c)*2^(1/2)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*2^(1/2))-I*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+co
s(d*x+c)*2^(1/2)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1
/2))-2^(1/2)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))
+sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2))*cos(d*x+c)*(a*(I*sin(d*x+c)
+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c))/sin(d*x+c)/((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)/(cos(d*x+c)/sin(d*x+c))^(1/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(I*a*tan(d*x + c) + a)*sqrt(cot(d*x + c))), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(78) = 156$.
time = 0.74, size = 329, normalized size = 3.05

$$-\frac{\left(ad\sqrt{\frac{2i}{ad^2}}\,e^{(i\,dx+i\,c)}\log\left(2\left(\sqrt{2}\left(ade^{(2i\,dx+2i\,c)}-ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{2i}{ad^2}}+2i\,ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-ad\sqrt{\frac{2i}{ad^2}}\,e^{(i\,dx+i\,c)}\log\left(-2\left(\sqrt{2}\left(ade^{(2i\,dx+2i\,c)}-ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{2i}{ad^2}}-2i\,ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-2\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\left(e^{(2i\,dx+2i\,c)}-1\right)\right)e^{(-i\,dx-i\,c)}}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] -1/4*(a*d*sqrt(2*I/(a*d^2))*e^(I*d*x + I*c)*log(2*(sqrt(2)*(a*d*e^(2*I*d*x
+ 2*I*c) - a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*
c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(2*I/(a*d^2)) + 2*I*a*e^(I*d*x + I*c
```

```
))*e^(-I*d*x - I*c)) - a*d*sqrt(2*I/(a*d^2))*e^(I*d*x + I*c)*log(-2*(sqrt(2
)*(a*d*e^(2*I*d*x + 2*I*c) - a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I
*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(2*I/(a*d^2)) - 2*
I*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*(e^(2
*I*d*x + 2*I*c) - 1))*e^(-I*d*x - I*c)/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,\sqrt{\cot\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(1/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(I*a*(tan(c + d*x) - I))*sqrt(cot(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(I*a*tan(d*x + c) + a)*sqrt(cot(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cot\left(c+d\,x\right)}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

```
[Out] int(1/(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

3.772 $\int \frac{1}{\cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=180

$$-\frac{2\sqrt[4]{-1}\,\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{a}\,d}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\ldots}{\ldots}$$

```
[Out] -2*(-1)^(1/4)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))
^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d/a^(1/2)+(-1/2+1/2*I)*arctanh((1
+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan
(d*x+c)^(1/2)/d/a^(1/2)-1/d/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]

time = 0.31, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {4326, 3639, 3682, 3625, 211, 3680, 65, 223, 209}

$$-\frac{2\sqrt[4]{-1}\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{1}{d\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cot[c + d*x]^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] (-2*(-1)^(1/4)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*
Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[a]*d) - ((1/2 -
 I/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x
]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[a]*d) - 1/(d*Sqrt[Cot[c +
d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{1}{\cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{\tan^{\frac{3}{2}}(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$

$$= -\frac{1}{d\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}} - \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\text{ta}\right.}{}$$

$$= -\frac{1}{d\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}} + \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\text{ta}\right.}{}$$

$$= -\frac{1}{d\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}} + \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\text{ta}\right.}{}$$

$$= -\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\text{t}}}{\sqrt{a}\,d}$$

$$= -\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\text{t}}}{\sqrt{a}\,d}$$

$$= -\frac{2\sqrt[4]{-1}\,\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\text{t}}}{\sqrt{a}\,d}$$

Mathematica [A]
time = 1.91, size = 210, normalized size = 1.17

$$\frac{ie^{-2i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(-1+e^{2i(c+dx)}+e^{i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\,\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)-2\sqrt{2}\,e^{i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\,\tanh^{-1}\left(\frac{\sqrt{2}\,e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\cot(c+dx)}}{\sqrt{2}\,ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cot[c + d*x]^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
[Out] (I*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(-1 + E^((2*I)*(
c + d*x)) + E^(I*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]*ArcTanh[E^(I*(c
```

```
+ d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]] - 2*Sqrt[2]*E^(I*(c + d*x))*Sqrt[-1
 + E^((2*I)*(c + d*x))]*ArcTanh[(Sqrt[2]*E^(I*(c + d*x)))/Sqrt[-1 + E^((2*I
)*(c + d*x))]])*Sqrt[Cot[c + d*x]])/(Sqrt[2]*a*d*E^((2*I)*(c + d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 620 vs. $2(141) = 282$.
time = 48.80, size = 621, normalized size = 3.45

method	result
default	$\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\cos(dx+c)\ln\left(\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}+1\right)-i\ln\left(\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}-i\right)+i\sin(dx+c)\sqrt{2}\,\arctan\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/2+1/2*I)/d*(-I*cos(d*x+c)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)-I*ln(
((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-I)+I*sin(d*x+c)*2^(1/2)*arctan((1/2+1/2*
I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))+I*ln(((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)+1)+I*cos(d*x+c)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-I)-I*ln((
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)+cos(d*x+c)*2^(1/2)*arctan((1/2+1/2*I)*
((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))-I*cos(d*x+c)*ln(((-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)+I)+I*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+I*co
s(d*x+c)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)+I*ln(((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)+I)-sin(d*x+c)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)+sin(d
*x+c)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+I)+sin(d*x+c)*ln(((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)+1)-sin(d*x+c)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-I)
-sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-2^(1/2)*arctan((1/2+1/2*I)*(
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)))*cos(d*x+c)^2*(a*(I*sin(d*x+c)+c
os(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c))/sin(d*x+c)^2/(cos(d*
x+c)/sin(d*x+c))^(3/2)/((-1+cos(d*x+c))/sin(d*x+c))^(1/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(I*a*tan(d*x + c) + a)*cot(d*x + c)^(3/2)), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 620 vs. $2(134) = 268$.

time = 1.07, size = 620, normalized size = 3.44

$$\frac{\left(ad\sqrt{-\frac{2i}{ad^2}}\,e^{(idx+ic)}\log\left(-2\left(\sqrt{2}\left(iade^{(2idx+2ic)}-iad\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{-\frac{2i}{ad^2}}-2iae^{(idx+ic)}\right)e^{(-idx-ic)}\right)-ad\sqrt{-\frac{2i}{ad^2}}\,e^{(idx+ic)}\log\left(-2\left(\sqrt{2}\left(-iade^{(2idx+2ic)}+iad\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{-\frac{2i}{ad^2}}-2iae^{(idx+ic)}\right)e^{(-idx-ic)}\right)+ad\sqrt{-\frac{4i}{ad^2}}\,e^{(idx+ic)}\log\left(-16\left(\sqrt{2}\left(a^2de^{(3idx+3ic)}-a^2de^{(idx+ic)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{-\frac{4i}{ad^2}}+3a^2e^{(2idx+2ic)}-a^2\right)e^{(-2idx-2ic)}\right)-ad\sqrt{-\frac{4i}{ad^2}}\,e^{(idx+ic)}\log\left(16\left(\sqrt{2}\left(a^2de^{(3idx+3ic)}-a^2de^{(idx+ic)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{-\frac{4i}{ad^2}}-3a^2e^{(2idx+2ic)}+a^2\right)e^{(-2idx-2ic)}\right)-2\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\left(-ie^{(2idx+2ic)}+i\right)\right)e^{(-idx-ic)}}{4ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/4*(a*d*sqrt(-2*I/(a*d^2))*e^(I*d*x + I*c)*log(-2*(sqrt(2)*(I*a*d*e^(2*I*d
*x + 2*I*c) - I*a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x +
 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-2*I/(a*d^2)) - 2*I*a*e^(I*d*x
 + I*c))*e^(-I*d*x - I*c)) - a*d*sqrt(-2*I/(a*d^2))*e^(I*d*x + I*c)*log(-2*
(sqrt(2)*(-I*a*d*e^(2*I*d*x + 2*I*c) + I*a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-2*I/
(a*d^2)) - 2*I*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + a*d*sqrt(-4*I/(a*d^2)
)*e^(I*d*x + I*c)*log(-16*(sqrt(2)*(a^2*d*e^(3*I*d*x + 3*I*c) - a^2*d*e^(I*
d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) +
 I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-4*I/(a*d^2)) + 3*a^2*e^(2*I*d*x + 2*I*
c) - a^2)*e^(-2*I*d*x - 2*I*c)) - a*d*sqrt(-4*I/(a*d^2))*e^(I*d*x + I*c)*lo
g(16*(sqrt(2)*(a^2*d*e^(3*I*d*x + 3*I*c) - a^2*d*e^(I*d*x + I*c))*sqrt(a/(e
^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I
*c) - 1))*sqrt(-4*I/(a*d^2)) - 3*a^2*e^(2*I*d*x + 2*I*c) + a^2)*e^(-2*I*d*x
 - 2*I*c)) - 2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x
 + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*(-I*e^(2*I*d*x + 2*I*c) + I))*e^(
-I*d*x - I*c)/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,\cot^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(3/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(I*a*(tan(c + d*x) - I))*cot(c + d*x)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(I*a*tan(d*x + c) + a)*cot(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cot\left(c+d\,x\right)^{3/2}\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
[Out] int(1/(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

3.773 $\int \frac{1}{\cot^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=217

$$-\frac{(-1)^{3/4}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{a}\,d}+\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\ldots}}{\sqrt{a+ia\,\mathrm{t}\ldots}}\right.}{}$$

```
[Out] -(-1)^(3/4)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(
1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d/a^(1/2)+(1/2+1/2*I)*arctanh((1+I)
*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*
x+c)^(1/2)/d/a^(1/2)-1/d/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(1/2)-2*I*(a+I
*a*tan(d*x+c))^(1/2)/a/d/cot(d*x+c)^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4326, 3639, 3678, 3682, 3625, 211, 3680, 65, 223, 209}

$$-\frac{(-1)^{3/4}\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{1}{d\cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{ad\sqrt{\cot(c+dx)}}+\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cot[c + d*x]^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] -(((-1)^(3/4)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*T
an[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[a]*d)) + ((1/2 +
 I/2)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x
]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[a]*d) - 1/(d*Cot[c + d*x]^
(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) - ((2*I)*Sqrt[a + I*a*Tan[c + d*x]])/(a*d
*Sqrt[Cot[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3678

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Dist[
1/(a*(m + n)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Simp
[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{1}{\cot^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{\tan^{\frac{5}{2}}(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$

$$= -\frac{1}{d\cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} - \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(\right.}{}$$

$$= -\frac{1}{d\cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} - \frac{2i\sqrt{a+ia\tan(c+dx)}}{ad\sqrt{\cot(c+dx)}}$$

$$= -\frac{1}{d\cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} - \frac{2i\sqrt{a+ia\tan(c+dx)}}{ad\sqrt{\cot(c+dx)}}$$

$$= -\frac{1}{d\cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} - \frac{2i\sqrt{a+ia\tan(c+dx)}}{ad\sqrt{\cot(c+dx)}}$$

$$= \frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(}{\sqrt{a}\,d}$$

$$= \frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(}{\sqrt{a}\,d}$$

$$= -\frac{(-1)^{3/4}\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{ta}{\sqrt{a}\,d}$$

Mathematica [A]
time = 2.35, size = 248, normalized size = 1.14

$$\frac{\left(1+2e^{2i(c+dx)}-3e^{4i(c+dx)}+e^{i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\left(1+e^{2i(c+dx)}\right)\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)+\sqrt{2}\,e^{i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\left(1+e^{2i(c+dx)}\right)\tanh^{-1}\left(\frac{\sqrt{2}\,e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\cot(c+dx)}}{\sqrt{2}\,d\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(1+e^{2i(c+dx)}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cot[c + d*x]^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] ((1 + 2*E^((2*I)*(c + d*x)) - 3*E^((4*I)*(c + d*x)) + E^(I*(c + d*x))*Sqrt[
-1 + E^((2*I)*(c + d*x))]*(1 + E^((2*I)*(c + d*x)))*ArcTanh[E^(I*(c + d*x))
/Sqrt[-1 + E^((2*I)*(c + d*x))]] + Sqrt[2]*E^(I*(c + d*x))*Sqrt[-1 + E^((2*
I)*(c + d*x))]*(1 + E^((2*I)*(c + d*x)))*ArcTanh[(Sqrt[2]*E^(I*(c + d*x)))/
```

```
Sqrt[-1 + E^((2*I)*(c + d*x))]])*Sqrt[Cot[c + d*x]])/(Sqrt[2]*d*Sqrt[(a*E^(
(2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 800 vs. $2(172) = 344$.
time = 48.24, size = 801, normalized size = 3.69

method	result
default	$\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\left(-2i\arctan\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\sqrt{2}\right)\sqrt{2}\,(\cos^2(dx+c))+i\ln\left(\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}+1\right)\cos(dx+c)\sin(dx+c)+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/4+1/4*I)/d*(-2*I*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2
^(1/2))*2^(1/2)*cos(d*x+c)^2+I*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)*cos
(d*x+c)*sin(d*x+c)+I*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+I)*cos(d*x+c)*si
n(d*x+c)+2*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))*c
os(d*x+c)*sin(d*x+c)*2^(1/2)-I*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-I)*cos
(d*x+c)*sin(d*x+c)+2*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-I*ln
(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)*cos(d*x+c)*sin(d*x+c)+2*I*arctan((1/
2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))*cos(d*x+c)*2^(1/2)-2*I
*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*co
s(d*x+c)^2-4*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*sin(d*x+c)-ln(((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)-I)*cos(d*x+c)^2+ln(((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)+I)*cos(d*x+c)^2+ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)*cos(d*
x+c)^2-ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)*cos(d*x+c)^2-4*I*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*sin(d*x+c)+ln(((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)-I)*cos(d*x+c)-ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+I)*cos(d*x+c)
-ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)*cos(d*x+c)+ln(((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)-1)*cos(d*x+c)+2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2))*cos(d*x
+c)^2*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+
c))/(cos(d*x+c)/sin(d*x+c))^(5/2)/((-1+cos(d*x+c))/sin(d*x+c))^(1/2)/sin(d*
x+c)^3/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima"
)
```

[Out] integrate(1/(sqrt(I*a*tan(d*x + c) + a)*cot(d*x + c)^(5/2)), x)

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 703 vs. $2(163) = 326$.
time = 1.39, size = 703, normalized size = 3.24

$$-\frac{2\sqrt{2}\sqrt{\frac{a}{e^{(2id x+2ic)}+1}}\sqrt{\frac{ie^{(2id x+2ic)}+i}{e^{(2id x+2ic)}-1}}\left(3e^{(4id x+4ic)}-2e^{(2id x+2ic)}-1\right)-\left(ade^{(3id x+3ic)}+ade^{(id x+ic)}\right)\sqrt{\frac{2i}{ad^2}}\log\left(2\left(\sqrt{2}\left(ade^{(2id x+2ic)}-ad\right)\sqrt{\frac{a}{e^{(2id x+2ic)}+1}}\sqrt{\frac{ie^{(2id x+2ic)}+i}{e^{(2id x+2ic)}-1}}\sqrt{\frac{2i}{ad^2}}+2iae^{(id x+ic)}\right)e^{(-id x-ic)}\right)+\left(ade^{(3id x+3ic)}+ade^{(id x+ic)}\right)\sqrt{\frac{2i}{ad^2}}\log\left(-2\left(\sqrt{2}\left(ade^{(2id x+2ic)}-ad\right)\sqrt{\frac{a}{e^{(2id x+2ic)}+1}}\sqrt{\frac{ie^{(2id x+2ic)}+i}{e^{(2id x+2ic)}-1}}\sqrt{\frac{2i}{ad^2}}-2iae^{(id x+ic)}\right)e^{(-id x-ic)}\right)+\left(ade^{(3id x+3ic)}+ade^{(id x+ic)}\right)\sqrt{\frac{i}{ad^2}}\log\left(-16\left(2\sqrt{2}\left(ia^2de^{(3id x+3ic)}-ia^2de^{(id x+ic)}\right)\sqrt{\frac{a}{e^{(2id x+2ic)}+1}}\sqrt{\frac{ie^{(2id x+2ic)}+i}{e^{(2id x+2ic)}-1}}\sqrt{\frac{i}{ad^2}}+3a^2e^{(2id x+2ic)}-a^2\right)e^{(-2id x-2ic)}\right)-\left(ade^{(3id x+3ic)}+ade^{(id x+ic)}\right)\sqrt{\frac{i}{ad^2}}\log\left(-16\left(2\sqrt{2}\left(-ia^2de^{(3id x+3ic)}+ia^2de^{(id x+ic)}\right)\sqrt{\frac{a}{e^{(2id x+2ic)}+1}}\sqrt{\frac{ie^{(2id x+2ic)}+i}{e^{(2id x+2ic)}-1}}\sqrt{\frac{i}{ad^2}}+3a^2e^{(2id x+2ic)}-a^2\right)e^{(-2id x-2ic)}\right)}{4\left(ade^{(3id x+3ic)}+ade^{(id x+ic)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")

[Out] -1/4*(2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*(3*e^(4*I*d*x + 4*I*c) - 2*e^(2*I*d*x + 2*I*c) - 1) - (a*d*e^(3*I*d*x + 3*I*c) + a*d*e^(I*d*x + I*c))*sqrt(2*I/(a*d^2))*log(2*(sqrt(2)*(a*d*e^(2*I*d*x + 2*I*c) - a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(2*I/(a*d^2)) + 2*I*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + (a*d*e^(3*I*d*x + 3*I*c) + a*d*e^(I*d*x + I*c))*sqrt(2*I/(a*d^2))*log(-2*(sqrt(2)*(a*d*e^(2*I*d*x + 2*I*c) - a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(2*I/(a*d^2)) - 2*I*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + (a*d*e^(3*I*d*x + 3*I*c) + a*d*e^(I*d*x + I*c))*sqrt(I/(a*d^2))*log(-16*(2*sqrt(2)*(I*a^2*d*e^(3*I*d*x + 3*I*c) - I*a^2*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(I/(a*d^2)) + 3*a^2*e^(2*I*d*x + 2*I*c) - a^2)*e^(-2*I*d*x - 2*I*c)) - (a*d*e^(3*I*d*x + 3*I*c) + a*d*e^(I*d*x + I*c))*sqrt(I/(a*d^2))*log(-16*(2*sqrt(2)*(-I*a^2*d*e^(3*I*d*x + 3*I*c) + I*a^2*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(I/(a*d^2)) + 3*a^2*e^(2*I*d*x + 2*I*c) - a^2)*e^(-2*I*d*x - 2*I*c)))/(a*d*e^(3*I*d*x + 3*I*c) + a*d*e^(I*d*x + I*c))

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\cot^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cot(d*x+c)**(5/2)/(a+I*a*tan(d*x+c))**(1/2),x)

[Out] Integral(1/(sqrt(I*a*(tan(c + d*x) - I))*cot(c + d*x)**(5/2)), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(I*a*tan(d*x + c) + a)*cot(d*x + c)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cot\left(c+d\,x\right)^{5/2}\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

```
[Out] int(1/(cot(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

3.774 $\int \frac{\cot^{\frac{5}{2}}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=221

$$-\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{a^{3/2}d}+\frac{\cot^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}+\frac{}{2ad}$$

```
[Out] (-1/4+1/4*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2
))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/a^(3/2)/d+5/2*cot(d*x+c)^(3/2)/a/d/(a+
I*a*tan(d*x+c))^(1/2)-7/2*cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(1/2)/a^2/d+1
3/2*I*cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^2/d+1/3*cot(d*x+c)^(3/2)/
d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.43, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4326, 3640, 3677, 3679, 12, 3625, 211}

$$-\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}-\frac{7\cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2a^2d}+\frac{13i\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{2a^2d}+\frac{5\cot^{\frac{3}{2}}(c+dx)}{2ad\sqrt{a+ia\tan(c+dx)}}+\frac{\cot^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(5/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-1/4 + I/4)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan
[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(a^(3/2)*d) + Cot[c + d*
x]^(3/2)/(3*d*(a + I*a*Tan[c + d*x])^(3/2)) + (5*Cot[c + d*x]^(3/2))/(2*a*d
*Sqrt[a + I*a*Tan[c + d*x]]) + (((13*I)/2)*Sqrt[Cot[c + d*x]]*Sqrt[a + I*a*
Tan[c + d*x]])/(a^2*d) - (7*Cot[c + d*x]^(3/2)*Sqrt[a + I*a*Tan[c + d*x]])/
(2*a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
```

```
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx$$

$$= \frac{\cot^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{\frac{9a}{2}-3}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a}}}{3a^2}$$

$$= \frac{\cot^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{5\cot^{\frac{3}{2}}(c+dx)}{2ad\sqrt{a+ia\tan(c+dx)}} + \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{}\right.}{}$$

$$= \frac{\cot^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{5\cot^{\frac{3}{2}}(c+dx)}{2ad\sqrt{a+ia\tan(c+dx)}} - \frac{7\cot^{\frac{3}{2}}(c+dx)\sqrt{a}}{2a^2}$$

$$= \frac{\cot^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{5\cot^{\frac{3}{2}}(c+dx)}{2ad\sqrt{a+ia\tan(c+dx)}} + \frac{13i\sqrt{\cot(c+dx)}}{2}$$

$$= \frac{\cot^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{5\cot^{\frac{3}{2}}(c+dx)}{2ad\sqrt{a+ia\tan(c+dx)}} + \frac{13i\sqrt{\cot(c+dx)}}{2}$$

$$= \frac{\cot^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{5\cot^{\frac{3}{2}}(c+dx)}{2ad\sqrt{a+ia\tan(c+dx)}} + \frac{13i\sqrt{\cot(c+dx)}}{2}$$

$$= -\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{a^{3/2}d}$$

Mathematica [A]
time = 1.80, size = 186, normalized size = 0.84

$$\frac{ie^{-4i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(1+18e^{2i(c+dx)}-87e^{4i(c+dx)}+52e^{6i(c+dx)}+3e^{3i(c+dx)}\left(-1+e^{2i(c+dx)}\right)^{3/2}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\cot(c+dx)}}{6\sqrt{2}\,a^2d\left(-1+e^{2i(c+dx)}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(5/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((I/6)*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(1 + 18*E^((
2*I)*(c + d*x)) - 87*E^((4*I)*(c + d*x)) + 52*E^((6*I)*(c + d*x)) + 3*E^((3
*I)*(c + d*x))*(-1 + E^((2*I)*(c + d*x)))^(3/2)*ArcTanh[E^(I*(c + d*x))/Sqr
t[-1 + E^((2*I)*(c + d*x))]])*Sqrt[Cot[c + d*x]])/(Sqrt[2]*a^2*d*E^((4*I)*(
c + d*x))*(-1 + E^((2*I)*(c + d*x))))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(175) = 350$.
time = 48.07, size = 445, normalized size = 2.01

method	result
default	$\frac{\left(-\frac{1}{12}-\frac{i}{12}\right)\left(\frac{\cos(dx+c)}{\sin(dx+c)}\right)^{\frac{5}{2}}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\ \sin(dx+c)\left(4i(\cos^4(dx+c))\sin(dx+c)+13i(\cos^3(dx+c))-39i\sin(dx+c)-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/12-1/12*I)/d*(cos(d*x+c)/sin(d*x+c))^(5/2)*(a*(I*sin(d*x+c)+cos(d*x+c))
/cos(d*x+c))^(1/2)*sin(d*x+c)*(4*I*cos(d*x+c)^4*sin(d*x+c)+13*I*cos(d*x+c)^
3-39*I*sin(d*x+c)-4*cos(d*x+c)^5+4*sin(d*x+c)*cos(d*x+c)^4-3*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*2^(1/2))*2^(1/2)-21*I*cos(d*x+c)+3*I*((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*sin(d*x+c)*cos(d*x+c)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*2^(1/2))*2^(1/2)+15*I*cos(d*x+c)^2*sin(d*x+c)-13*cos(d*x+c)^3+15
*sin(d*x+c)*cos(d*x+c)^2+3*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*arctan((1/2+1
/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))*2^(1/2)+4*I*cos(d*x+c)^5+
3*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*arctan((1/2+1/2*I)*((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))*2^(1/2)+21*cos(d*x+c)-39*sin(d*x+c))/
cos(d*x+c)^2/a^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(165) = 330$.
time = 1.33, size = 418, normalized size = 1.89

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\left(52i\,e^{(6i\,dx+6i\,c)}-87i\,e^{(4i\,dx+4i\,c)}+18i\,e^{(2i\,dx+2i\,c)}+i\right)+3\left(a^2de^{(5i\,dx+5i\,c)}-a^2de^{(3i\,dx+3i\,c)}\right)\sqrt{-\frac{i}{2a^3d^2}}\log\left(-4\left(\sqrt{2}\left(i\,a^2de^{(2i\,dx+2i\,c)}-i\,a^2d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{-\frac{i}{2a^3d^2}}-i\,ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-3\left(a^2de^{(5i\,dx+5i\,c)}-a^2de^{(3i\,dx+3i\,c)}\right)\sqrt{-\frac{i}{2a^3d^2}}\log\left(-4\left(\sqrt{2}\left(-i\,a^2de^{(2i\,dx+2i\,c)}+i\,a^2d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{-\frac{i}{2a^3d^2}}-i\,ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)}{12\left(a^2de^{(5i\,dx+5i\,c)}-a^2de^{(3i\,dx+3i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*(sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c)
 + I)/(e^(2*I*d*x + 2*I*c) - 1))*(52*I*e^(6*I*d*x + 6*I*c) - 87*I*e^(4*I*d*
x + 4*I*c) + 18*I*e^(2*I*d*x + 2*I*c) + I) + 3*(a^2*d*e^(5*I*d*x + 5*I*c) -
 a^2*d*e^(3*I*d*x + 3*I*c))*sqrt(-1/2*I/(a^3*d^2))*log(-4*(sqrt(2)*(I*a^2*d
*e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e
^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-1/2*I/(a^3*d^2)) -
 I*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 3*(a^2*d*e^(5*I*d*x + 5*I*c) - a^
2*d*e^(3*I*d*x + 3*I*c))*sqrt(-1/2*I/(a^3*d^2))*log(-4*(sqrt(2)*(-I*a^2*d*e
^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(
2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-1/2*I/(a^3*d^2)) - I
*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)))/(a^2*d*e^(5*I*d*x + 5*I*c) - a^2*d*e
^(3*I*d*x + 3*I*c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(5/2)/(a+I*a*tan(d*x+c))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(cot(d*x + c)^(5/2)/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cot(c+d\,x)}^{5/2}}{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(5/2)/(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int(cot(c + d*x)^(5/2)/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.775 $\int \frac{\cot^{\frac{3}{2}}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=182

$$\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{a^{3/2}d}+\frac{\sqrt{\cot(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}+\frac{}{6ad}$$

```
[Out] (1/4+1/4*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
)*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/a^(3/2)/d+11/6*cot(d*x+c)^(1/2)/a/d/(a+
I*a*tan(d*x+c))^(1/2)-25/6*cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^2/d+
1/3*cot(d*x+c)^(1/2)/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.34, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4326, 3640, 3677, 3679, 12, 3625, 211}

$$\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}-\frac{25\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{6a^2d}+\frac{11\sqrt{\cot(c+dx)}}{6ad\sqrt{a+ia\tan(c+dx)}}+\frac{\sqrt{\cot(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(3/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((1/4 + I/4)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[
c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(a^(3/2)*d) + Sqrt[Cot[c
+ d*x]]/(3*d*(a + I*a*Tan[c + d*x])^(3/2)) + (11*Sqrt[Cot[c + d*x]])/(6*a*d
*Sqrt[a + I*a*Tan[c + d*x]]) - (25*Sqrt[Cot[c + d*x]]*Sqrt[a + I*a*Tan[c +
d*x]])/(6*a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
```

```
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx$$

$$= \frac{\sqrt{\cot(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{\left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{\frac{7a}{2}}{\tan^{\frac{3}{2}}(c+dx)\sqrt{}}}{3a^2}$$

$$= \frac{\sqrt{\cot(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{11\sqrt{\cot(c+dx)}}{6ad\sqrt{a+ia\tan(c+dx)}} + \frac{\left(\sqrt{\cot(c+dx)}\right.}{}$$

$$= \frac{\sqrt{\cot(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{11\sqrt{\cot(c+dx)}}{6ad\sqrt{a+ia\tan(c+dx)}} - \frac{25\sqrt{\cot(c+dx)}}{}$$

$$= \frac{\sqrt{\cot(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{11\sqrt{\cot(c+dx)}}{6ad\sqrt{a+ia\tan(c+dx)}} - \frac{25\sqrt{\cot(c+dx)}}{}$$

$$= \frac{\sqrt{\cot(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{11\sqrt{\cot(c+dx)}}{6ad\sqrt{a+ia\tan(c+dx)}} - \frac{25\sqrt{\cot(c+dx)}}{}$$

$$= \frac{\left(\frac{1}{4}+\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{a^{3/2}d}$$

Mathematica [A]
time = 1.53, size = 156, normalized size = 0.86

$$\frac{e^{-4i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(1+13e^{2i(c+dx)}-38e^{4i(c+dx)}+3e^{3i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\cot(c+dx)}}{6\sqrt{2}\,a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(3/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(1 + 13*E^((2*I)*(
c + d*x)) - 38*E^((4*I)*(c + d*x)) + 3*E^((3*I)*(c + d*x))*Sqrt[-1 + E^((2*
I)*(c + d*x))]*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]])*Sqr
t[Cot[c + d*x]])/(6*Sqrt[2]*a^2*d*E^((4*I)*(c + d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(144) = 288.
time = 46.77, size = 344, normalized size = 1.89

method	result
default	$\frac{\left(\frac{1}{12}+\frac{i}{12}\right)\sin(dx+c)\left(\frac{\cos(dx+c)}{\sin(dx+c)}\right)^{\frac{3}{2}}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(3i\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\sin(dx+c)\arctan\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/12+1/12*I)/d*sin(d*x+c)*(cos(d*x+c)/sin(d*x+c))^(3/2)*(a*(I*sin(d*x+c)+c
os(d*x+c))/cos(d*x+c))^(1/2)*(3*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*
x+c)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))*2^(1/2)
-4*I*sin(d*x+c)*cos(d*x+c)^3-4*I*cos(d*x+c)^4-3*((-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*cos(d*x+c)*2^(1/2)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^
(1/2)*2^(1/2))-4*sin(d*x+c)*cos(d*x+c)^3+4*cos(d*x+c)^4-3*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(
1/2))*2^(1/2)-11*I*sin(d*x+c)*cos(d*x+c)-9*I*cos(d*x+c)^2-11*sin(d*x+c)*cos
(d*x+c)+9*cos(d*x+c)^2-25+25*I)/cos(d*x+c)/a^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(136) = 272.
time = 1.00, size = 355, normalized size = 1.95

$$\frac{\left(3\,a^2d\sqrt{\frac{i}{2\,a^3d^2}}\,e^{(3i\,dx+3i\,c)}\log\left(4\left(\sqrt{2}\left(a^2de^{(2i\,dx+2i\,c)}-a^2d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{i}{2\,a^3d^2}}+i\,ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-3\,a^2d\sqrt{\frac{i}{2\,a^3d^2}}\,e^{(3i\,dx+3i\,c)}\log\left(-4\left(\sqrt{2}\left(a^2de^{(2i\,dx+2i\,c)}-a^2d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{i}{2\,a^3d^2}}-i\,ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\left(38\,e^{(4i\,dx+4i\,c)}-13\,e^{(2i\,dx+2i\,c)}-1\right)\right)e^{(-3i\,dx-3i\,c)}}{12\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^2*d*sqrt(1/2*I/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(4*(sqrt(2)*(a^2
*d*e^(2*I*d*x + 2*I*c) - a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e
^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(1/2*I/(a^3*d^2)) +
I*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 3*a^2*d*sqrt(1/2*I/(a^3*d^2))*e^(3
*I*d*x + 3*I*c)*log(-4*(sqrt(2)*(a^2*d*e^(2*I*d*x + 2*I*c) - a^2*d)*sqrt(a/
(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2
```

```
*I*c) - 1))*sqrt(1/2*I/(a^3*d^2)) - I*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c))
- sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I
)/(e^(2*I*d*x + 2*I*c) - 1))*(38*e^(4*I*d*x + 4*I*c) - 13*e^(2*I*d*x + 2*I*
c) - 1))*e^(-3*I*d*x - 3*I*c)/(a^2*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(3/2)/(a+I*a*tan(d*x+c))**(3/2),x)
[Out] Integral(cot(c + d*x)**(3/2)/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(cot(d*x + c)^(3/2)/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(c+d\,x)^{3/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(3/2)/(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int(cot(c + d*x)^(3/2)/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.776 $\int \frac{\sqrt{\cot(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=145

$$\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{a^{3/2}d}+\frac{1}{3d\sqrt{\cot(c+dx)}\,(a+ia\tan(c+$$

[Out] (1/4-1/4*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
)*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/a^(3/2)/d+7/6/a/d/cot(d*x+c)^(1/2)/(a+I
*a*tan(d*x+c))^(1/2)+1/3/d/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2)

Rubi [A]
time = 0.23, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4326, 3640, 3677, 12, 3625, 211}

$$\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}+\frac{7}{6ad\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{1}{3d\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Cot[c + d*x]]/(a + I*a*Tan[c + d*x])^(3/2),x]

[Out] ((1/4 - I/4)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[
c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(a^(3/2)*d) + 1/(3*d*Sqrt
[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)) + 7/(6*a*d*Sqrt[Cot[c + d*x]]*
Sqrt[a + I*a*Tan[c + d*x]])

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
```

Q[c^2 + d^2, 0]

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{\sqrt{\cot(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{1}{\sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^{3/2}}\,dx$$

$$= \frac{1}{3d\sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^{3/2}} + \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)}{}$$

$$= \frac{1}{3d\sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^{3/2}} + \frac{7}{6ad\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c}}$$

$$= \frac{1}{3d\sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^{3/2}} + \frac{7}{6ad\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c}}$$

$$= \frac{1}{3d\sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^{3/2}} + \frac{7}{6ad\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c}}$$

$$= \frac{\left(\frac{1}{4}-\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{a^{3/2}d} +$$

Mathematica [A]
time = 1.32, size = 158, normalized size = 1.09

$$-\frac{ie^{-4i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(-1-7e^{2i(c+dx)}+8e^{4i(c+dx)}+3e^{3i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\cot(c+dx)}}{6\sqrt{2}\,a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cot[c + d*x]]/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-1/6*I)*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(-1 - 7*E
^((2*I)*(c + d*x)) + 8*E^((4*I)*(c + d*x)) + 3*E^((3*I)*(c + d*x))*Sqrt[-1
+ E^((2*I)*(c + d*x))]*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x)
)]])*Sqrt[Cot[c + d*x]])/(Sqrt[2]*a^2*d*E^((4*I)*(c + d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 467 vs. 2(114) = 228.
time = 45.21, size = 468, normalized size = 3.23

method	result

default	$\frac{\left(-\frac{1}{12}-\frac{i}{12}\right)\left(6i\sin(dx+c)\cos(dx+c)\sqrt{2}\,\arctan\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\sqrt{2}\right)+9i\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\cos(dx+c)\sin(dx+c}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/12-1/12*I)/d*(6*I*sin(d*x+c)*cos(d*x+c)*2^(1/2)*arctan((1/2+1/2*I)*((-1
+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))+9*I*((-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*sin(d*x+c)*cos(d*x+c)+7*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)
^2-3*I*sin(d*x+c)*2^(1/2)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*2^(1/2))+6*cos(d*x+c)^2*2^(1/2)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*2^(1/2))-9*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*si
n(d*x+c)+7*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-3*cos(d*x+c)*2^(
1/2)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))-7*I*((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)-3*2^(1/2)*arctan((1/2+1/2*I)*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*2^(1/2))-7*((-1+cos(d*x+c))/sin(d*x+c))^(1/2))*(cos(d*
x+c)/sin(d*x+c))^(1/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(2*I*
cos(d*x+c)*sin(d*x+c)+2*cos(d*x+c)^2-1)/((-1+cos(d*x+c))/sin(d*x+c))^(1/2)/
a^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 357 vs. 2(107) = 214.
time = 0.98, size = 357, normalized size = 2.46

$$-\frac{\left(3\,a^{2}d\sqrt{-\frac{i}{2\,a^{3}d^{2}}}\,e^{(3i\,dx+3i\,c)}\log\left(-4\left(\sqrt{2}\left(i\,a^{2}de^{(2i\,dx+2i\,c)}-i\,a^{2}d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{-\frac{i}{2\,a^{3}d^{2}}}-i\,ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-3\,a^{2}d\sqrt{-\frac{i}{2\,a^{3}d^{2}}}\,e^{(3i\,dx+3i\,c)}\log\left(-4\left(\sqrt{2}\left(-i\,a^{2}de^{(2i\,dx+2i\,c)}+i\,a^{2}d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{-\frac{i}{2\,a^{3}d^{2}}}-i\,ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\left(-8i\,e^{(4i\,dx+4i\,c)}+7i\,e^{(2i\,dx+2i\,c)}+i\right)\right)e^{(-3i\,dx-3i\,c)}}{12\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/12*(3*a^2*d*sqrt(-1/2*I/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(-4*(sqrt(2)*(
I*a^2*d*e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sq
```

```
rt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-1/2*I/(a^3*
d^2)) - I*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 3*a^2*d*sqrt(-1/2*I/(a^3*d
^2))*e^(3*I*d*x + 3*I*c)*log(-4*(sqrt(2)*(-I*a^2*d*e^(2*I*d*x + 2*I*c) + I*
a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(
e^(2*I*d*x + 2*I*c) - 1))*sqrt(-1/2*I/(a^3*d^2)) - I*a*e^(I*d*x + I*c))*e^(
-I*d*x - I*c)) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d
*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*(-8*I*e^(4*I*d*x + 4*I*c) + 7*I
*e^(2*I*d*x + 2*I*c) + I))*e^(-3*I*d*x - 3*I*c)/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cot{(c + dx)}}}{(ia(\tan{(c + dx)} - i))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(1/2)/(a+I*a*tan(d*x+c))**(3/2),x)
[Out] Integral(sqrt(cot(c + d*x))/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(cot(d*x + c))/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\mathrm{cot}\left(c+d\,x\right)}}{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(1/2)/(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int(cot(c + d*x)^(1/2)/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.777 $\int \frac{1}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=147

$$-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{a^{3/2}d}+\frac{1}{3d\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c-$$

```
[Out] (-1/4-1/4*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2
))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/a^(3/2)/d+1/2*I/a/d/cot(d*x+c)^(1/2)/(
a+I*a*tan(d*x+c))^(1/2)+1/3/d/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.18, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {4326, 3628, 3627, 3625, 211}

$$-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}+\frac{1}{3d\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}+\frac{i}{2ad\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

```
[Out] ((-1/4 - I/4)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan
[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(a^(3/2)*d) + 1/(3*d*Cot
[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)) + (I/2)/(a*d*Sqrt[Cot[c + d*x
]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3627

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
```

```
 + f*x])^n/(2*b*f*m)), x] - Dist[(a*c - b*d)/(2*b^2), Int[(a + b*Tan[e + f*
x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && EqQ
[m + n, 0] && LeQ[m, -2^(-1)]
```

Rule 3628

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a), Int[(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && EqQ[m
 + n + 1, 0] && LtQ[m, -1]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^{3/2}}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{\sqrt{\tan(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx$$

$$= \frac{1}{3d\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}} + \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{t\right.}}{}$$

$$= \frac{1}{3d\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}} + \frac{}{2ad\sqrt{\cot(c+dx)}\,\sqrt{}}$$

$$= \frac{1}{3d\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}} + \frac{}{2ad\sqrt{\cot(c+dx)}\,\sqrt{}}$$

$$= -\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{}}{a^{3/2}d}$$

Mathematica [A]

time = 1.64, size = 156, normalized size = 1.06

$$\frac{e^{-4i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(-1-e^{2i(c+dx)}+2e^{4i(c+dx)}-3e^{3i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\cot(c+dx)}}{6\sqrt{2}\,a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

```
[Out] (Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(-1 - E^((2*I)*(c
+ d*x)) + 2*E^((4*I)*(c + d*x)) - 3*E^((3*I)*(c + d*x))*Sqrt[-1 + E^((2*I)*
(c + d*x))]*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]])*Sqrt[C
ot[c + d*x]])/(6*Sqrt[2]*a^2*d*E^((4*I)*(c + d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 481 vs. 2(115) = 230.
time = 45.43, size = 482, normalized size = 3.28

method	result
default	$\frac{\left(\frac{1}{12}+\frac{i}{12}\right)\left(6i\arctan\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\sqrt{2}\right)\sqrt{2}\,(\cos^2(dx+c))-6\arctan\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\sqrt{2}\right)\cos(dx\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/12+1/12*I)/d*(6*I*2^(1/2)*cos(d*x+c)^2*arctan((1/2+1/2*I)*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*2^(1/2))-6*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*2^(1/2))*cos(d*x+c)*sin(d*x+c)*2^(1/2)-3*I*2^(1/2)*cos(d*x+c)*ar
ctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))-I*cos(d*x+c)^2
*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+3*I*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)+3*2^(1/2)*sin(d*x+c)*arctan((1/2+1/2*I)*((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)*2^(1/2))-3*I*2^(1/2)*arctan((1/2+1/2*I)*((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)*2^(1/2))+((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c
)^2+3*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*sin(d*x+c)+I*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)-((-1+cos(d*x+c))/sin(d*x+c))^(1/2))*cos(d*x+c)*(a*
(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(2*I*cos(d*x+c)*sin(d*x+c)+2*co
s(d*x+c)^2-1)/(cos(d*x+c)/sin(d*x+c))^(1/2)/sin(d*x+c)/((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)/a^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(107) = 214$.
time = 0.71, size = 355, normalized size = 2.41

$$-\frac{\left(3\,a^2 d\sqrt{\frac{i}{2\,a^3 d^2}}\,e^{(3i\,dx+3i\,c)}\log\left(4\left(\sqrt{2}\,\left(a^2 de^{(2i\,dx+2i\,c)}-a^2 d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\,\sqrt{\frac{i}{2\,a^3 d^2}}+i\,ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-3\,a^2 d\sqrt{\frac{i}{2\,a^3 d^2}}\,e^{(3i\,dx+3i\,c)}\log\left(-4\left(\sqrt{2}\,\left(a^2 de^{(2i\,dx+2i\,c)}-a^2 d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\,\sqrt{\frac{i}{2\,a^3 d^2}}-i\,ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-\sqrt{2}\,\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\,\left(2\,e^{(4i\,dx+4i\,c)}-e^{(2i\,dx+2i\,c)}-1\right)\right)e^{(-3i\,dx-3i\,c)}}{12\,a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] -1/12*(3*a^2*d*sqrt(1/2*I/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(4*(sqrt(2)*(a^
2*d*e^(2*I*d*x + 2*I*c) - a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*
e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(1/2*I/(a^3*d^2)) +
 I*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 3*a^2*d*sqrt(1/2*I/(a^3*d^2))*e^(
3*I*d*x + 3*I*c)*log(-4*(sqrt(2)*(a^2*d*e^(2*I*d*x + 2*I*c) - a^2*d)*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
2*I*c) - 1))*sqrt(1/2*I/(a^3*d^2)) - I*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c))
 - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) +
I)/(e^(2*I*d*x + 2*I*c) - 1))*(2*e^(4*I*d*x + 4*I*c) - e^(2*I*d*x + 2*I*c)
- 1))*e^(-3*I*d*x - 3*I*c)/(a^2*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{3}{2}}\sqrt{\cot\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(1/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/((I*a*(tan(c + d*x) - I))**(3/2)*sqrt(cot(c + d*x))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*tan(d*x + c) + a)^(3/2)*sqrt(cot(d*x + c))), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cot (c+d\,x)}\ (a+a\tan (c+d\,x)\ 1\mathrm{i})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(3/2)),x)
[Out] int(1/(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(3/2)), x)
```

3.778 $\int \frac{1}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=147

$$-\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{a^{3/2}d}+\frac{i}{3d\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+$$

```
[Out] (-1/4+1/4*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2
))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/a^(3/2)/d+1/2/a/d/cot(d*x+c)^(1/2)/(a+
I*a*tan(d*x+c))^(1/2)+1/3*I/d/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.19, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4326, 3627, 3625, 211}

$$-\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}+\frac{i}{3d\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}+\frac{1}{2ad\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

```
[Out] ((-1/4 + I/4)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan
[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(a^(3/2)*d) + (I/3)/(d*C
ot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)) + 1/(2*a*d*Sqrt[Cot[c + d*x
]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3627

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
```

```
 + f*x])^n/(2*b*f*m)), x] - Dist[(a*c - b*d)/(2*b^2), Int[(a + b*Tan[e + f*
x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && EqQ
[m + n, 0] && LeQ[m, -2^(-1)]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{1}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx = \left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{\tan^{\frac{3}{2}}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$$

$$= \frac{i}{3d\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}} - \frac{\left(i\sqrt{\cot(c+dx)}\sqrt{t\right.}}{}$$

$$= \frac{i}{3d\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}} + \frac{1}{2ad\sqrt{\cot(c+dx)}\sqrt{}}$$

$$= \frac{i}{3d\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}} + \frac{1}{2ad\sqrt{\cot(c+dx)}\sqrt{}}$$

$$= -\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{}}{a^{3/2}d}$$

Mathematica [A]
time = 0.91, size = 158, normalized size = 1.07

$$-\frac{ie^{-4i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(1-5e^{2i(c+dx)}+4e^{4i(c+dx)}-3e^{3i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\cot(c+dx)}}{6\sqrt{2}\,a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

```
[Out] ((-1/6*I)*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(1 - 5*E^
((2*I)*(c + d*x)) + 4*E^((4*I)*(c + d*x)) - 3*E^((3*I)*(c + d*x))*Sqrt[-1 +
 E^((2*I)*(c + d*x))]*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))
]])*Sqrt[Cot[c + d*x]])/(Sqrt[2]*a^2*d*E^((4*I)*(c + d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(115) = 230$.
time = 40.99, size = 484, normalized size = 3.29

method	result
default	$\frac{\left(\frac{1}{12}+\frac{i}{12}\right)\left(6i\sin(dx+c)\cos(dx+c)\sqrt{2}\,\arctan\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\sqrt{2}\right)-3i\sin(dx+c)\sqrt{2}\,\arctan\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\frac{-1+\cos(}{\sin(dx}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/12+1/12*I)/d*(6*I*sin(d*x+c)*cos(d*x+c)*2^(1/2)*arctan((1/2+1/2*I)*((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))-3*I*sin(d*x+c)*2^(1/2)*arctan((1/2+1
/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))-5*I*cos(d*x+c)^2*((-1+cos
(d*x+c))/sin(d*x+c))^(1/2)-3*I*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)+6*cos(d*x+c)^2*2^(1/2)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)*2^(1/2))-3*cos(d*x+c)*2^(1/2)*arctan((1/2+1/2*I)*((-1+cos(d*
x+c))/sin(d*x+c))^(1/2)*2^(1/2))-5*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d
*x+c)^2+3*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*sin(d*x+c)+5*I*((-1
+cos(d*x+c))/sin(d*x+c))^(1/2)-3*2^(1/2)*arctan((1/2+1/2*I)*((-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*2^(1/2))+5*((-1+cos(d*x+c))/sin(d*x+c))^(1/2))*cos(d*x+
c)^2*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(2*I*cos(d*x+c)*sin(d*x
+c)+2*cos(d*x+c)^2-1)/(cos(d*x+c)/sin(d*x+c))^(3/2)/((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)/sin(d*x+c)^2/a^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(107) = 214$.

time = 0.91, size = 356, normalized size = 2.42

$$\frac{\left(3\,a^{2}d\sqrt{-\frac{i}{2\,a^{3}d^{2}}}\,e^{(3i\,dx+3i\,c)}\log\left(-4\left(\sqrt{2}\left(i\,a^{2}de^{(2i\,dx+2i\,c)}-i\,a^{2}d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{-\frac{i}{2\,a^{3}d^{2}}}-i\,ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-3\,a^{2}d\sqrt{-\frac{i}{2\,a^{3}d^{2}}}\,e^{(3i\,dx+3i\,c)}\log\left(-4\left(\sqrt{2}\left(-i\,a^{2}de^{(2i\,dx+2i\,c)}+i\,a^{2}d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{-\frac{i}{2\,a^{3}d^{2}}}-i\,ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\left(-4i\,e^{(4i\,dx+4i\,c)}+5i\,e^{(2i\,dx+2i\,c)}-i\right)\right)e^{(-3i\,dx-3i\,c)}}{12\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/12*(3*a^2*d*sqrt(-1/2*I/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(-4*(sqrt(2)*(I
*a^2*d*e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqr
t((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-1/2*I/(a^3*d
^2)) - I*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 3*a^2*d*sqrt(-1/2*I/(a^3*d^
2))*e^(3*I*d*x + 3*I*c)*log(-4*(sqrt(2)*(-I*a^2*d*e^(2*I*d*x + 2*I*c) + I*a
^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e
^(2*I*d*x + 2*I*c) - 1))*sqrt(-1/2*I/(a^3*d^2)) - I*a*e^(I*d*x + I*c))*e^(-
I*d*x - I*c)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*
x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*(-4*I*e^(4*I*d*x + 4*I*c) + 5*I*
e^(2*I*d*x + 2*I*c) - I))*e^(-3*I*d*x - 3*I*c)/(a^2*d)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{3}{2}}\cot^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(3/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/((I*a*(tan(c + d*x) - I))**(3/2)*cot(c + d*x)**(3/2)), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^(3/2)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\mathrm{cot}\left(c+d\,x\right)}^{3/2}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

```
[Out] int(1/(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(3/2)), x)
```

3.779 $\int \frac{1}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=221

$$\frac{2(-1)^{3/4}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{a^{3/2}d}+\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right.}{}$$

```
[Out] 2*(-1)^(3/4)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^
(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/a^(3/2)/d+(1/4+1/4*I)*arctanh((1+I
)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d
*x+c)^(1/2)/a^(3/2)/d+3/2*I/a/d/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-1
/3/d/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.41, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4326, 3639, 3676, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{2(-1)^{3/4}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}+\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}-\frac{1}{3d\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}+\frac{3i}{2ad\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

```
[Out] (2*(-1)^(3/4)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*T
an[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(a^(3/2)*d) + ((1/4 +
I/4)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]
]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(a^(3/2)*d) - 1/(3*d*Cot[c + d*x]
^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)) + ((3*I)/2)/(a*d*Sqrt[Cot[c + d*x]]*Sq
rt[a + I*a*Tan[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{1}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{\tan^{\frac{5}{2}}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$$

$$= -\frac{1}{3d\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}} - \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\right.}}{}$$

$$= -\frac{1}{3d\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}} + \frac{}{2ad\sqrt{\cot(c+dx)}}$$

$$= -\frac{1}{3d\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}} + \frac{}{2ad\sqrt{\cot(c+dx)}}$$

$$= -\frac{1}{3d\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}} + \frac{}{2ad\sqrt{\cot(c+dx)}}$$

$$= \frac{\left(\frac{1}{4}+\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\mathrm{ta}}}{a^{3/2}d}$$

$$= \frac{\left(\frac{1}{4}+\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\mathrm{ta}}}{a^{3/2}d}$$

$$= \frac{2(-1)^{3/4}\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{}}{a^{3/2}d}$$

Mathematica [A]
time = 2.65, size = 226, normalized size = 1.02

$$\frac{e^{-4i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(1-11e^{2i(c+dx)}+10e^{4i(c+dx)}+3e^{3i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)-12\sqrt{2}\,e^{3i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{\sqrt{2}\,e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\cot(c+dx)}}{6\sqrt{2}\,a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

```
[Out] (Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(1 - 11*E^((2*I)*(
c + d*x)) + 10*E^((4*I)*(c + d*x)) + 3*E^((3*I)*(c + d*x))*Sqrt[-1 + E^((2*
I)*(c + d*x))]*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]] - 12
*Sqrt[2]*E^((3*I)*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]*ArcTanh[(Sqrt[2
```

```
]*E^(I*(c + d*x)))/Sqrt[-1 + E^((2*I)*(c + d*x))]])*Sqrt[Cot[c + d*x]])/(6*
Sqrt[2]*a^2*d*E^((4*I)*(c + d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1110 vs. $2(172) = 344$.
time = 48.09, size = 1111, normalized size = 5.03

method	result	size
default	Expression too large to display	1111

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/12+1/12*I)/d*(6*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+I)-11*((-1+cos(d*x
+c))/sin(d*x+c))^(1/2)-11*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2
-3*2^(1/2)*sin(d*x+c)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*2^(1/2))+9*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*sin(d*x+c)+6*ln((
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)-6*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)-1)+9*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c)+6*arctan(
(1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))*cos(d*x+c)*sin(d*x+
c)*2^(1/2)-6*I*2^(1/2)*cos(d*x+c)^2*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)*2^(1/2))+3*I*2^(1/2)*cos(d*x+c)*arctan((1/2+1/2*I)*((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))-12*I*sin(d*x+c)*cos(d*x+c)*ln(((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)+I)+12*I*sin(d*x+c)*cos(d*x+c)*ln(((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)-1)+12*I*sin(d*x+c)*cos(d*x+c)*ln(((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)-I)-12*I*sin(d*x+c)*cos(d*x+c)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1
/2)+1)+11*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+11*((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*cos(d*x+c)^2+12*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-I)*cos(d*x+
c)^2-12*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+I)*cos(d*x+c)^2-12*ln(((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)+1)*cos(d*x+c)^2+12*ln(((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)-1)*cos(d*x+c)^2-6*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-I)*cos(d*
x+c)+6*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+I)*cos(d*x+c)+6*ln(((-1+cos(d*
x+c))/sin(d*x+c))^(1/2)+1)*cos(d*x+c)-6*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)-1)*cos(d*x+c)+3*I*2^(1/2)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*2^(1/2))+6*I*sin(d*x+c)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+I)-6*
I*sin(d*x+c)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)-6*I*sin(d*x+c)*ln(((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)-I)+6*I*sin(d*x+c)*ln(((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)+1)-6*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-I))*cos(d*x+c)^3*(a
*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(2*I*cos(d*x+c)*sin(d*x+c)+2*c
os(d*x+c)^2-1)/((-1+cos(d*x+c))/sin(d*x+c))^(1/2)/sin(d*x+c)^3/(cos(d*x+c)/
sin(d*x+c))^(5/2)/a^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 650 vs. $2(163) = 326$.
time = 0.92, size = 650, normalized size = 2.94

$$\frac{\left(3a^{2}d\sqrt{\frac{i}{2a^{3}d^{2}}}e^{(3idx+3ic)}\log\left(4\left(\sqrt{2}\left(a^{2}de^{(2idx+2ic)}-a^{2}d\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{\frac{i}{2a^{3}d^{2}}}+iae^{(idx+ic)}\right)e^{(-idx-ic)}\right)-3a^{2}d\sqrt{\frac{i}{2a^{3}d^{2}}}e^{(3idx+3ic)}\log\left(-4\left(\sqrt{2}\left(a^{2}de^{(2idx+2ic)}-a^{2}d\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{\frac{i}{2a^{3}d^{2}}}-iae^{(idx+ic)}\right)e^{(-idx-ic)}\right)+3a^{2}d\sqrt{\frac{4i}{a^{3}d^{2}}}e^{(3idx+3ic)}\log\left(-16\left(\sqrt{2}\left(ia^{3}de^{(3idx+3ic)}-ia^{3}de^{(idx+ic)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{\frac{4i}{a^{3}d^{2}}}+3a^{2}e^{(2idx+2ic)}-a^{2}\right)e^{(-2idx-2ic)}\right)-3a^{2}d\sqrt{\frac{4i}{a^{3}d^{2}}}e^{(3idx+3ic)}\log\left(-16\left(\sqrt{2}\left(-ia^{3}de^{(3idx+3ic)}+ia^{3}de^{(idx+ic)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{\frac{4i}{a^{3}d^{2}}}+3a^{2}e^{(2idx+2ic)}-a^{2}\right)e^{(-2idx-2ic)}\right)+\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\left(10e^{(4idx+4ic)}-11e^{(2idx+2ic)}+1\right)\right)e^{(-3idx-3ic)}}{12a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/12*(3*a^2*d*sqrt(1/2*I/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(4*(sqrt(2)*(a^2
*d*e^(2*I*d*x + 2*I*c) - a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e
^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(1/2*I/(a^3*d^2)) +
I*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 3*a^2*d*sqrt(1/2*I/(a^3*d^2))*e^(3
*I*d*x + 3*I*c)*log(-4*(sqrt(2)*(a^2*d*e^(2*I*d*x + 2*I*c) - a^2*d)*sqrt(a/
(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2
*I*c) - 1))*sqrt(1/2*I/(a^3*d^2)) - I*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c))
+ 3*a^2*d*sqrt(4*I/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(-16*(sqrt(2)*(I*a^3*d
*e^(3*I*d*x + 3*I*c) - I*a^3*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c)
 + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(4*I
/(a^3*d^2)) + 3*a^2*e^(2*I*d*x + 2*I*c) - a^2)*e^(-2*I*d*x - 2*I*c)) - 3*a^
2*d*sqrt(4*I/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(-16*(sqrt(2)*(-I*a^3*d*e^(3
*I*d*x + 3*I*c) + I*a^3*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(4*I/(a^3
*d^2)) + 3*a^2*e^(2*I*d*x + 2*I*c) - a^2)*e^(-2*I*d*x - 2*I*c)) + sqrt(2)*s
qrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d
*x + 2*I*c) - 1))*(10*e^(4*I*d*x + 4*I*c) - 11*e^(2*I*d*x + 2*I*c) + 1))*e^
(-3*I*d*x - 3*I*c)/(a^2*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(5/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(1/((I*a*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cot\left(c+d\,x\right)^{5/2}\,\left(a+a\,\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(3/2)),x)
[Out] int(1/(cot(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(3/2)), x)
```

3.780 $\int \frac{1}{\cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=258

$$-\frac{3\sqrt[4]{-1}\,\mathrm{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{a^{3/2}d}+\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\mathrm{t}}}{\sqrt{a+ia\,\mathrm{t}}}\right.}{}$$

```
[Out] -3*(-1)^(1/4)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))
^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/a^(3/2)/d+(1/4-1/4*I)*arctanh((1+
I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(
d*x+c)^(1/2)/a^(3/2)/d+13/6*I/a/d/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(1/2)
-7/2*(a+I*a*tan(d*x+c))^(1/2)/a^2/d/cot(d*x+c)^(1/2)-1/3/d/cot(d*x+c)^(5/2)
/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.50, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {4326, 3639, 3676, 3678, 3682, 3625, 211, 3680, 65, 223, 209}

$$-\frac{3\sqrt[4]{-1}\,\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\mathrm{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}+\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}-\frac{7\sqrt{a+ia\tan(c+dx)}}{2a^2d\sqrt{\cot(c+dx)}}+\frac{13i}{6ad\cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}-\frac{1}{3d\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cot[c + d*x]^(7/2)*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

```
[Out] (-3*(-1)^(1/4)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*
Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(a^(3/2)*d) + ((1/4 -
 I/4)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x
]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(a^(3/2)*d) - 1/(3*d*Cot[c + d*x
]^(5/2)*(a + I*a*Tan[c + d*x])^(3/2)) + ((13*I)/6)/(a*d*Cot[c + d*x]^(3/2)*
Sqrt[a + I*a*Tan[c + d*x]]) - (7*Sqrt[a + I*a*Tan[c + d*x]])/(2*a^2*d*Sqrt[
Cot[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rule 3678

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Dist[
```

```
1/(a*(m + n)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Simp
[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{1}{\cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{\tan^{\frac{7}{2}}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$$

$$= -\frac{1}{3d\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}} - \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\mathrm{t}\right.}{}$$

$$= -\frac{1}{3d\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}} + \frac{13}{6ad\cot^{\frac{3}{2}}(c+dx)\sqrt{}}$$

$$= -\frac{1}{3d\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}} + \frac{13}{6ad\cot^{\frac{3}{2}}(c+dx)\sqrt{}}$$

$$= -\frac{1}{3d\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}} + \frac{13}{6ad\cot^{\frac{3}{2}}(c+dx)\sqrt{}}$$

$$= -\frac{1}{3d\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}} + \frac{13}{6ad\cot^{\frac{3}{2}}(c+dx)\sqrt{}}$$

$$= \frac{\left(\frac{1}{4}-\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan}}{a^{3/2}d}$$

$$= \frac{\left(\frac{1}{4}-\frac{i}{4}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan}}{a^{3/2}d}$$

$$= -\frac{3\sqrt[4]{-1}\,\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\mathrm{t}}}{a^{3/2}d}$$

Mathematica [A]
time = 2.94, size = 268, normalized size = 1.04

$$-\frac{i\left(-1+16e^{2i(c+dx)}+13e^{4i(c+dx)}-28e^{6i(c+dx)}+3e^{3i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\left(1+e^{2i(c+dx)}\right)\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)+18\sqrt{2}\,e^{3i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\left(1+e^{2i(c+dx)}\right)\tanh^{-1}\left(\frac{\sqrt{2}\,e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\cot(c+dx)}}{6\sqrt{2}\,d\left(\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{3/2}\left(1+e^{2i(c+dx)}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cot[c + d*x]^(7/2)*(a + I*a*Tan[c + d*x])^(3/2)),x]
[Out] ((-1/6*I)*(-1 + 16*E^((2*I)*(c + d*x)) + 13*E^((4*I)*(c + d*x)) - 28*E^((6*
I)*(c + d*x)) + 3*E^((3*I)*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]*(1 + E
```

```
^((2*I)*(c + d*x)))*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]]
 + 18*Sqrt[2]*E^((3*I)*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]*(1 + E^((2
*I)*(c + d*x)))*ArcTanh[(Sqrt[2]*E^(I*(c + d*x)))/Sqrt[-1 + E^((2*I)*(c + d
*x))]])*Sqrt[Cot[c + d*x]])/(Sqrt[2]*d*((a*E^((2*I)*(c + d*x)))/(1 + E^((2*
I)*(c + d*x))))^(3/2)*(1 + E^((2*I)*(c + d*x)))^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1264 vs. 2(202) = 404.
time = 45.04, size = 1265, normalized size = 4.90

method	result	size
default	Expression too large to display	1265

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/12+1/12*I)/d*(3*cos(d*x+c)*2^(1/2)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/s
in(d*x+c))^(1/2)*2^(1/2))+3*I*cos(d*x+c)*arctan((1/2+1/2*I)*((-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*2^(1/2))*2^(1/2)*sin(d*x+c)-6*I*arctan((1/2+1/2*I)*((-1
+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+3*c
os(d*x+c)^2*2^(1/2)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2
^(1/2))-6*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))*2^
(1/2)*cos(d*x+c)^3-18*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-I)*cos(d*x+c)^2
*sin(d*x+c)+18*I*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-I)*cos(d*x+c)^3+18*I
*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)*cos(d*x+c)^3-18*I*ln(((-1+cos(d*x
+c))/sin(d*x+c))^(1/2)+I)*cos(d*x+c)^3-18*I*ln(((-1+cos(d*x+c))/sin(d*x+c))
^(1/2)+1)*cos(d*x+c)^3+29*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3
-9*I*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-I)*cos(d*x+c)^2-9*I*ln(((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)-1)*cos(d*x+c)^2+9*I*ln(((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)+I)*cos(d*x+c)^2+9*I*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)*cos(d
*x+c)^2-18*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)*cos(d*x+c)^2*sin(d*x+c)
+18*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+I)*cos(d*x+c)^2*sin(d*x+c)+18*ln(
((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)*cos(d*x+c)^2*sin(d*x+c)-27*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*sin(d*x+c)+9*cos(d*x+c)*ln(((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)-I)*sin(d*x+c)+9*cos(d*x+c)*ln(((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)-1)*sin(d*x+c)-9*cos(d*x+c)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)+I)*sin(d*x+c)-9*cos(d*x+c)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)*si
n(d*x+c)-9*I*cos(d*x+c)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-I)-9*I*cos(d*
x+c)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)+9*I*cos(d*x+c)*ln(((-1+cos(d*
x+c))/sin(d*x+c))^(1/2)+I)+9*I*cos(d*x+c)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)+1)-29*I*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-6*I*((-1+cos(d*x
+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+6*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))
^(1/2)-29*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+29*((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*cos(d*x+c)^3+27*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos
(d*x+c)^2*sin(d*x+c))*cos(d*x+c)^3*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))
```

```
^(1/2)/(2*I*cos(d*x+c)*sin(d*x+c)+2*cos(d*x+c)^2-1)/((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)/(cos(d*x+c)/sin(d*x+c))^(7/2)/sin(d*x+c)^4/a^2
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 743 vs. 2(192) = 384.
time = 1.00, size = 743, normalized size = 2.88

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/12*(sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c)
 + I)/(e^(2*I*d*x + 2*I*c) - 1))*(28*I*e^(6*I*d*x + 6*I*c) - 13*I*e^(4*I*d*
x + 4*I*c) - 16*I*e^(2*I*d*x + 2*I*c) + I) - 3*(a^2*d*e^(5*I*d*x + 5*I*c) +
 a^2*d*e^(3*I*d*x + 3*I*c))*sqrt(-1/2*I/(a^3*d^2))*log(-4*(sqrt(2)*(I*a^2*d
*e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e
^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-1/2*I/(a^3*d^2)) -
 I*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 3*(a^2*d*e^(5*I*d*x + 5*I*c) + a^
2*d*e^(3*I*d*x + 3*I*c))*sqrt(-1/2*I/(a^3*d^2))*log(-4*(sqrt(2)*(-I*a^2*d*e
^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(
2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-1/2*I/(a^3*d^2)) - I
*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 3*(a^2*d*e^(5*I*d*x + 5*I*c) + a^2*
d*e^(3*I*d*x + 3*I*c))*sqrt(-9*I/(a^3*d^2))*log(-16/3*(2*sqrt(2)*(a^3*d*e^(
3*I*d*x + 3*I*c) - a^3*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-9*I/(a^3
*d^2)) + 9*a^2*e^(2*I*d*x + 2*I*c) - 3*a^2)*e^(-2*I*d*x - 2*I*c)) - 3*(a^2*
d*e^(5*I*d*x + 5*I*c) + a^2*d*e^(3*I*d*x + 3*I*c))*sqrt(-9*I/(a^3*d^2))*log
(16/3*(2*sqrt(2)*(a^3*d*e^(3*I*d*x + 3*I*c) - a^3*d*e^(I*d*x + I*c))*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
2*I*c) - 1))*sqrt(-9*I/(a^3*d^2)) - 9*a^2*e^(2*I*d*x + 2*I*c) + 3*a^2)*e^(-
2*I*d*x - 2*I*c)))/(a^2*d*e^(5*I*d*x + 5*I*c) + a^2*d*e^(3*I*d*x + 3*I*c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(7/2)/(a+I*a*tan(d*x+c))**(3/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(1/((I*a*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^(7/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cot\left(c+d\,x\right)^{7/2}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(7/2)*(a + a*tan(c + d*x)*1i)^(3/2)),x)
[Out] int(1/(cot(c + d*x)^(7/2)*(a + a*tan(c + d*x)*1i)^(3/2)), x)
```

3.781 $\int \frac{\cot^{\frac{5}{2}}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=258

$$-\frac{\left(\frac{1}{8}-\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{a^{5/2}d}+\frac{\cot^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}+\frac{}{10a}$$

```
[Out] (-1/8+1/8*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2
))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/a^(5/2)/d+89/20*cot(d*x+c)^(3/2)/a^2/d
/(a+I*a*tan(d*x+c))^(1/2)-361/60*cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(1/2)/
a^3/d+707/60*I*cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^3/d+1/5*cot(d*x+
c)^(3/2)/d/(a+I*a*tan(d*x+c))^(5/2)+7/10*cot(d*x+c)^(3/2)/a/d/(a+I*a*tan(d*
x+c))^(3/2)
```

Rubi [A]
time = 0.54, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4326, 3640, 3677, 3679, 12, 3625, 211}

$$-\frac{\left(\frac{1}{8}-\frac{i}{8}\right)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d}-\frac{361\cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{60a^3d}+\frac{707i\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{60a^3d}+\frac{89\cot^{\frac{3}{2}}(c+dx)}{20a^2d\sqrt{a+ia\tan(c+dx)}}+\frac{7\cot^{\frac{3}{2}}(c+dx)}{10ad(a+ia\tan(c+dx))^{3/2}}+\frac{\cot^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(5/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-1/8 + I/8)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan
[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(a^(5/2)*d) + Cot[c + d*
x]^(3/2)/(5*d*(a + I*a*Tan[c + d*x])^(5/2)) + (7*Cot[c + d*x]^(3/2))/(10*a*
d*(a + I*a*Tan[c + d*x])^(3/2)) + (89*Cot[c + d*x]^(3/2))/(20*a^2*d*Sqrt[a
+ I*a*Tan[c + d*x]]) + (((707*I)/60)*Sqrt[Cot[c + d*x]]*Sqrt[a + I*a*Tan[c
+ d*x]])/(a^3*d) - (361*Cot[c + d*x]^(3/2)*Sqrt[a + I*a*Tan[c + d*x]])/(60*
a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx$$

$$= \frac{\cot^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{\left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{\frac{13a}{2}-4ia\,\mathrm{t}}{\tan^{\frac{5}{2}}(c+dx)(a+}}{5a^2}$$

$$= \frac{\cot^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{7\cot^{\frac{3}{2}}(c+dx)}{10ad(a+ia\tan(c+dx))^{3/2}} + \frac{\left(\sqrt{\cot(c+dx)}\right.}{}$$

$$= \frac{\cot^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{7\cot^{\frac{3}{2}}(c+dx)}{10ad(a+ia\tan(c+dx))^{3/2}} + \frac{89\cot^{\frac{3}{2}}(c+}{20a^2d\sqrt{a+ia\,\mathrm{ta}}}$$

$$= \frac{\cot^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{7\cot^{\frac{3}{2}}(c+dx)}{10ad(a+ia\tan(c+dx))^{3/2}} + \frac{89\cot^{\frac{3}{2}}(c+}{20a^2d\sqrt{a+ia\,\mathrm{ta}}}$$

$$= \frac{\cot^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{7\cot^{\frac{3}{2}}(c+dx)}{10ad(a+ia\tan(c+dx))^{3/2}} + \frac{89\cot^{\frac{3}{2}}(c+}{20a^2d\sqrt{a+ia\,\mathrm{ta}}}$$

$$= \frac{\cot^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{7\cot^{\frac{3}{2}}(c+dx)}{10ad(a+ia\tan(c+dx))^{3/2}} + \frac{89\cot^{\frac{3}{2}}(c+}{20a^2d\sqrt{a+ia\,\mathrm{ta}}}$$

$$= \frac{\cot^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{7\cot^{\frac{3}{2}}(c+dx)}{10ad(a+ia\tan(c+dx))^{3/2}} + \frac{89\cot^{\frac{3}{2}}(c+}{20a^2d\sqrt{a+ia\,\mathrm{ta}}}$$

$$= -\frac{\left(\frac{1}{8}-\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{a^{5/2}d}$$

Mathematica [A]
time = 2.10, size = 199, normalized size = 0.77

$$\frac{ie^{-6i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3+33e^{2i(c+dx)}+348e^{4i(c+dx)}-1527e^{6i(c+dx)}+983e^{8i(c+dx)}+15e^{5i(c+dx)}\left(-1+e^{2i(c+dx)}\right)^{3/2}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\cot(c+dx)}}{60\sqrt{2}\,a^3d\left(-1+e^{2i(c+dx)}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(5/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((I/60)*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(3 + 33*E^(
(2*I)*(c + d*x)) + 348*E^((4*I)*(c + d*x)) - 1527*E^((6*I)*(c + d*x)) + 983
```

```
*E^((8*I)*(c + d*x)) + 15*E^((5*I)*(c + d*x))*(-1 + E^((2*I)*(c + d*x)))^(3
/2)*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]])*Sqrt[Cot[c + d
*x]])/(Sqrt[2]*a^3*d*E^((6*I)*(c + d*x))*(-1 + E^((2*I)*(c + d*x))))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 498 vs. 2(205) = 410.
time = 46.74, size = 499, normalized size = 1.93

method	result
default	$\left(-\frac{1}{120}-\frac{i}{120}\right)\sin(dx+c)\left(\frac{\cos(dx+c)}{\sin(dx+c)}\right)^{\frac{5}{2}}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(15i\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\sin(dx+c)\arctan\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\frac{-1+c}{\sin}}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/120-1/120*I)/d*sin(d*x+c)*(cos(d*x+c)/sin(d*x+c))^(5/2)*(a*(I*sin(d*x+c
)+cos(d*x+c))/cos(d*x+c))^(1/2)*(15*I*2^(1/2)*arctan((1/2+1/2*I)*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*
x+c)+225*I*cos(d*x+c)^3+48*sin(d*x+c)*cos(d*x+c)^6-48*cos(d*x+c)^7-707*I*si
n(d*x+c)-361*I*cos(d*x+c)+48*I*cos(d*x+c)^5+72*I*sin(d*x+c)*cos(d*x+c)^4-15
*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*arctan((1/2+1/2*I)*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))*2^(1/2)+72*sin(d*x+c)*cos(d*x+c)^4-48*
cos(d*x+c)^5+48*I*sin(d*x+c)*cos(d*x+c)^6+48*I*cos(d*x+c)^7+15*((-1+cos(d*x
+c))/sin(d*x+c))^(1/2)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*2^(1/2))*2^(1/2)+267*sin(d*x+c)*cos(d*x+c)^2-225*cos(d*x+c)^3+267*I*sin(d
*x+c)*cos(d*x+c)^2+15*I*2^(1/2)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c
)-707*sin(d*x+c)+361*cos(d*x+c))/cos(d*x+c)^2/a^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 431 vs. 2(194) = 388.
time = 0.64, size = 431, normalized size = 1.67

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\left(983i\,e^{(8i\,dx+8i\,c)}-1527i\,e^{(6i\,dx+6i\,c)}+348i\,e^{(4i\,dx+4i\,c)}+33i\,e^{(2i\,dx+2i\,c)}+3i\right)+30\left(a^3de^{(7i\,dx+7i\,c)}-a^3de^{(5i\,dx+5i\,c)}\right)\sqrt{-\frac{i}{8\,a^5d^2}}\log\left(-4\left(2\sqrt{2}\left(i\,a^3de^{(2i\,dx+2i\,c)}-i\,a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{-\frac{i}{8\,a^5d^2}}-i\,ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-30\left(a^3de^{(7i\,dx+7i\,c)}-a^3de^{(5i\,dx+5i\,c)}\right)\sqrt{-\frac{i}{8\,a^5d^2}}\log\left(-4\left(2\sqrt{2}\left(-i\,a^3de^{(2i\,dx+2i\,c)}+i\,a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{-\frac{i}{8\,a^5d^2}}-i\,ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)}{120\left(a^3de^{(7i\,dx+7i\,c)}-a^3de^{(5i\,dx+5i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")

[Out] 1/120*(sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*(983*I*e^(8*I*d*x + 8*I*c) - 1527*I*e^(6*I*d*x + 6*I*c) + 348*I*e^(4*I*d*x + 4*I*c) + 33*I*e^(2*I*d*x + 2*I*c) + 3*I) + 30*(a^3*d*e^(7*I*d*x + 7*I*c) - a^3*d*e^(5*I*d*x + 5*I*c))*sqrt(-1/8*I/(a^5*d^2))*log(-4*(2*sqrt(2)*(I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-1/8*I/(a^5*d^2)) - I*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 30*(a^3*d*e^(7*I*d*x + 7*I*c) - a^3*d*e^(5*I*d*x + 5*I*c))*sqrt(-1/8*I/(a^5*d^2))*log(-4*(2*sqrt(2)*(-I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-1/8*I/(a^5*d^2)) - I*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)))/(a^3*d*e^(7*I*d*x + 7*I*c) - a^3*d*e^(5*I*d*x + 5*I*c))

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**(5/2)/(a+I*a*tan(d*x+c))**(5/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(cot(d*x + c)^(5/2)/(I*a*tan(d*x + c) + a)^(5/2), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^{5/2}}{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d*x)^(5/2)/(a + a*tan(c + d*x)*1i)^(5/2),x)

[Out] int(cot(c + d*x)^(5/2)/(a + a*tan(c + d*x)*1i)^(5/2), x)

3.782 $\int \frac{\cot^{\frac{3}{2}}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=219

$$\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{a^{5/2}d}+\frac{\sqrt{\cot(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}+\frac{}{30ad}$$

```
[Out] (1/8+1/8*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
)*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/a^(5/2)/d+151/60*cot(d*x+c)^(1/2)/a^2/d
/(a+I*a*tan(d*x+c))^(1/2)-317/60*cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/
a^3/d+1/5*cot(d*x+c)^(1/2)/d/(a+I*a*tan(d*x+c))^(5/2)+17/30*cot(d*x+c)^(1/2
)/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.45, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4326, 3640, 3677, 3679, 12, 3625, 211}

$$\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d}-\frac{317\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{60a^3d}+\frac{151\sqrt{\cot(c+dx)}}{60a^2d\sqrt{a+ia\tan(c+dx)}}+\frac{17\sqrt{\cot(c+dx)}}{30ad(a+ia\tan(c+dx))^{3/2}}+\frac{\sqrt{\cot(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(3/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((1/8 + I/8)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[
c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(a^(5/2)*d) + Sqrt[Cot[c
+ d*x]]/(5*d*(a + I*a*Tan[c + d*x])^(5/2)) + (17*Sqrt[Cot[c + d*x]])/(30*a*
d*(a + I*a*Tan[c + d*x])^(3/2)) + (151*Sqrt[Cot[c + d*x]])/(60*a^2*d*Sqrt[a
 + I*a*Tan[c + d*x]]) - (317*Sqrt[Cot[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
/(60*a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
```

```
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx$$

$$= \frac{\sqrt{\cot(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{\frac{11a}{2}-3i}{\tan^{\frac{3}{2}}(c+dx)(a}}{5a^2}$$

$$= \frac{\sqrt{\cot(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{17\sqrt{\cot(c+dx)}}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{\left(\sqrt{\cot(c+dx)}\right.}{}$$

$$= \frac{\sqrt{\cot(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{17\sqrt{\cot(c+dx)}}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{151\sqrt{\cot(c}}{60a^2d\sqrt{a+ia}}$$

$$= \frac{\sqrt{\cot(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{17\sqrt{\cot(c+dx)}}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{151\sqrt{\cot(c}}{60a^2d\sqrt{a+ia}}$$

$$= \frac{\sqrt{\cot(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{17\sqrt{\cot(c+dx)}}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{151\sqrt{\cot(c}}{60a^2d\sqrt{a+ia}}$$

$$= \frac{\sqrt{\cot(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{17\sqrt{\cot(c+dx)}}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{151\sqrt{\cot(c}}{60a^2d\sqrt{a+ia}}$$

$$= \frac{\left(\frac{1}{8}+\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{a^{5/2}d} -$$

Mathematica [A]

time = 3.03, size = 165, normalized size = 0.75

$$\frac{\cot^{\frac{3}{2}}(c+dx)\sec(c+dx)\left((340-460\cos(2(c+dx)))\csc(c+dx)+15e^{2i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\csc(2(c+dx))-i(149+466\cos(2(c+dx)))\sec(c+dx)\right)}{60a^2d(i+\cot(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(3/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (Cot[c + d*x]^(3/2)*Sec[c + d*x]*((340 - 460*Cos[2*(c + d*x)])*Csc[c + d*x]
 + 15*E^((2*I)*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]*ArcTanh[E^(I*(c +
d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]]*Csc[2*(c + d*x)] - I*(149 + 466*Cos[2
*(c + d*x)])*Sec[c + d*x]))/(60*a^2*d*(I + Cot[c + d*x])^2*Sqrt[a + I*a*Tan
[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(174) = 348$.
time = 47.44, size = 398, normalized size = 1.82

method	result
default	$\frac{\left(\frac{1}{120}+\frac{i}{120}\right)\sin(dx+c)\left(\frac{\cos(dx+c)}{\sin(dx+c)}\right)^{\frac{3}{2}}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(-48i\left(\cos^{6}(dx+c)\right)-117i\left(\cos^{2}(dx+c)\right)-48\sin(dx+c)\left(\cos^{5}(dx+\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/120+1/120*I)/d*sin(d*x+c)*(cos(d*x+c)/sin(d*x+c))^(3/2)*(a*(I*sin(d*x+c)
+cos(d*x+c))/cos(d*x+c))^(1/2)*(-48*I*cos(d*x+c)^6-117*I*cos(d*x+c)^2-48*si
n(d*x+c)*cos(d*x+c)^5+48*cos(d*x+c)^6+15*I*2^(1/2)*arctan((1/2+1/2*I)*((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*s
in(d*x+c)-48*I*sin(d*x+c)*cos(d*x+c)^5-32*I*cos(d*x+c)^4-56*sin(d*x+c)*cos(
d*x+c)^3-15*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*2^(1/2)*arctan((1
/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))+32*cos(d*x+c)^4-151*I
*sin(d*x+c)*cos(d*x+c)-56*I*sin(d*x+c)*cos(d*x+c)^3-15*((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2
))*2^(1/2)-151*sin(d*x+c)*cos(d*x+c)+117*cos(d*x+c)^2-317+317*I)/cos(d*x+c)
/a^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(165) = 330$.
time = 0.58, size = 368, normalized size = 1.68

$$\frac{\left(30\,a^{3}d\sqrt{\frac{i}{8\,a^{5}d^{2}}}\,e^{(5i\,dx+5i\,c)}\log\left(4\left(2\sqrt{2}\left(a^{3}de^{(2i\,dx+2i\,c)}-a^{3}d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{i}{8\,a^{5}d^{2}}}+i\,ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-30\,a^{3}d\sqrt{\frac{i}{8\,a^{5}d^{2}}}\,e^{(5i\,dx+5i\,c)}\log\left(-4\left(2\sqrt{2}\left(a^{3}de^{(2i\,dx+2i\,c)}-a^{3}d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{i}{8\,a^{5}d^{2}}}-i\,ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\left(463\,e^{(6i\,dx+6i\,c)}-194\,e^{(4i\,dx+4i\,c)}-26\,e^{(2i\,dx+2i\,c)}-3\right)\right)e^{(-5i\,dx-5i\,c)}}{120\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/120*(30*a^3*d*sqrt(1/8*I/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(4*(2*sqrt(2)*
(a^3*d*e^(2*I*d*x + 2*I*c) - a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(
```

```
(I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(1/8*I/(a^5*d^2)
) + I*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 30*a^3*d*sqrt(1/8*I/(a^5*d^2))
*e^(5*I*d*x + 5*I*c)*log(-4*(2*sqrt(2)*(a^3*d*e^(2*I*d*x + 2*I*c) - a^3*d)*
sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*
d*x + 2*I*c) - 1))*sqrt(1/8*I/(a^5*d^2)) - I*a*e^(I*d*x + I*c))*e^(-I*d*x -
 I*c)) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I
*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*(463*e^(6*I*d*x + 6*I*c) - 194*e^(4*I*d
*x + 4*I*c) - 26*e^(2*I*d*x + 2*I*c) - 3))*e^(-5*I*d*x - 5*I*c)/(a^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(3/2)/(a+I*a*tan(d*x+c))**(5/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(cot(d*x + c)^(3/2)/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^{3/2}}{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(3/2)/(a + a*tan(c + d*x)*1i)^(5/2),x)
[Out] int(cot(c + d*x)^(3/2)/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.783 $\int \frac{\sqrt{\cot(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=182

$$\frac{\left(\frac{1}{8}-\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{a^{5/2}d}+\frac{1}{5d\sqrt{\cot(c+dx)}\,(a+ia\tan(c+$$

```
[Out] (1/8-1/8*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
)*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/a^(5/2)/d+67/60/a^2/d/cot(d*x+c)^(1/2)/
(a+I*a*tan(d*x+c))^(1/2)+1/5/d/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2)+13
/30/a/d/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.33, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4326, 3640, 3677, 12, 3625, 211}

$$\frac{\left(\frac{1}{8}-\frac{i}{8}\right)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d}+\frac{67}{60a^2d\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{13}{30ad\sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^{3/2}}+\frac{1}{5d\sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cot[c + d*x]]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((1/8 - I/8)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[
c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(a^(5/2)*d) + 1/(5*d*Sqrt
[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2)) + 13/(30*a*d*Sqrt[Cot[c + d*x]
]*(a + I*a*Tan[c + d*x])^(3/2)) + 67/(60*a^2*d*Sqrt[Cot[c + d*x]]*Sqrt[a +
I*a*Tan[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
```

```
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{\sqrt{\cot(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{1}{\sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^{5/2}}\,dx$$

$$= \frac{1}{5d\sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^{5/2}} + \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)}{}$$

$$= \frac{1}{5d\sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^{5/2}} + \frac{13}{30ad\sqrt{\cot(c+dx)}\,(a+ia\tan(c}$$

$$= \frac{1}{5d\sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^{5/2}} + \frac{13}{30ad\sqrt{\cot(c+dx)}\,(a+ia\tan(c}$$

$$= \frac{1}{5d\sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^{5/2}} + \frac{13}{30ad\sqrt{\cot(c+dx)}\,(a+ia\tan(c}$$

$$= \frac{1}{5d\sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^{5/2}} + \frac{13}{30ad\sqrt{\cot(c+dx)}\,(a+ia\tan(c}$$

$$= \frac{\left(\frac{1}{8}-\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{a^{5/2}d} +$$

Mathematica [A]
time = 1.86, size = 171, normalized size = 0.94

$$-\frac{ie^{-6i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(-3-16e^{2i(c+dx)}-64e^{4i(c+dx)}+83e^{6i(c+dx)}+15e^{5i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\cot(c+dx)}}{60\sqrt{2}\,a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cot[c + d*x]]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-1/60*I)*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(-3 - 16
*E^((2*I)*(c + d*x)) - 64*E^((4*I)*(c + d*x)) + 83*E^((6*I)*(c + d*x)) + 15
*E^((5*I)*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]*ArcTanh[E^(I*(c + d*x))
/Sqrt[-1 + E^((2*I)*(c + d*x))]])*Sqrt[Cot[c + d*x]])/(Sqrt[2]*a^3*d*E^((6*
I)*(c + d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 643 vs. 2(144) = 288.
time = 43.18, size = 644, normalized size = 3.54

method	result
default	$\frac{\left(-\frac{1}{120}-\frac{i}{120}\right)\left(60 i \arctan\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\sqrt{2}\right)\sqrt{2}\,\left(\cos^2(dx+c)\right)\sin(dx+c)-30 i \sin(dx+c)\cos(dx+c)\sqrt{2}\,\operatorname{arctan}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/120-1/120*I)/d*(60*I*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*2^(1/2))*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)-30*I*arctan((1/2+1/2*I)*((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))*2^(1/2)*cos(d*x+c)*sin(d*x+c)+60*arct
an((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))*2^(1/2)*cos(d*x+
c)^3+160*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3+172*I*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*sin(d*x+c)-15*I*arctan((1/2+1/2*I)*((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))*2^(1/2)*sin(d*x+c)-30*cos(d*x+c)^
2*2^(1/2)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))+16
0*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3-172*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*cos(d*x+c)^2*sin(d*x+c)-45*cos(d*x+c)*2^(1/2)*arctan((1/2+1/2
*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))-160*I*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*cos(d*x+c)-67*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)
+15*2^(1/2)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))-
160*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+67*sin(d*x+c)*((-1+cos(d*
x+c))/sin(d*x+c))^(1/2))*(cos(d*x+c)/sin(d*x+c))^(1/2)*(a*(I*sin(d*x+c)+cos
(d*x+c))/cos(d*x+c))^(1/2)/(4*I*cos(d*x+c)^2*sin(d*x+c)+4*cos(d*x+c)^3-I*si
n(d*x+c)-3*cos(d*x+c))/((-1+cos(d*x+c))/sin(d*x+c))^(1/2)/a^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(136) = 272.
time = 0.66, size = 370, normalized size = 2.03

$$-\frac{\left(30\,a^3 d\sqrt{-\frac{i}{8\,a^5 d^2}}\,e^{(5i\,dx+5i\,c)}\log\left(-4\left(2\sqrt{2}\left(i\,a^3 d e^{(2i\,dx+2i\,c)}-i\,a^3 d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{-\frac{i}{8\,a^5 d^2}}-i\,a e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-30\,a^3 d\sqrt{-\frac{i}{8\,a^5 d^2}}\,e^{(5i\,dx+5i\,c)}\log\left(-4\left(2\sqrt{2}\left(-i\,a^3 d e^{(2i\,dx+2i\,c)}+i\,a^3 d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{-\frac{i}{8\,a^5 d^2}}-i\,a e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\left(-83i\,e^{(6i\,dx+6i\,c)}+64i\,e^{(4i\,dx+4i\,c)}+16i\,e^{(2i\,dx+2i\,c)}+3i\right)\right)e^{(-5i\,dx-5i\,c)}}{120\,a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/120*(30*a^3*d*sqrt(-1/8*I/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(-4*(2*sqrt(
2)*(I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-1/8*I/(
a^5*d^2)) - I*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 30*a^3*d*sqrt(-1/8*I/(
a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(-4*(2*sqrt(2)*(-I*a^3*d*e^(2*I*d*x + 2*I*
c) + I*a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c)
 + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-1/8*I/(a^5*d^2)) - I*a*e^(I*d*x + I*
c))*e^(-I*d*x - I*c)) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e
^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*(-83*I*e^(6*I*d*x + 6*I*
c) + 64*I*e^(4*I*d*x + 4*I*c) + 16*I*e^(2*I*d*x + 2*I*c) + 3*I))*e^(-5*I*d*
x - 5*I*c)/(a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cot{(c+dx)}}}{(ia\,(\tan{(c+dx)}-i))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(1/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(sqrt(cot(c + d*x))/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cot(d*x + c))/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\mathrm{cot}\left(c+d\,x\right)}}{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(1/2)/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] int(cot(c + d*x)^(1/2)/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.784 $\int \frac{1}{\sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=188

$$-\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{a^{5/2}d}+\frac{i}{5d\sqrt{\cot(c+dx)}\,(a+ia\tan(c\ldots}$$

```
[Out] (-1/8-1/8*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2
))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/a^(5/2)/d-1/20*I/a^2/d/cot(d*x+c)^(1/2
)/(a+I*a*tan(d*x+c))^(1/2)+1/5*I/d/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2
)+1/10*I/a/d/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.33, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4326, 3638, 3677, 12, 3625, 211}

$$-\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d}-\frac{i}{20a^2d\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}+\frac{i}{10ad\sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^{3/2}}+\frac{i}{5d\sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2)),x]
```

```
[Out] ((-1/8 - I/8)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan
[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(a^(5/2)*d) + (I/5)/(d*S
qrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2)) + (I/10)/(a*d*Sqrt[Cot[c +
d*x]]*(a + I*a*Tan[c + d*x])^(3/2)) - (I/20)/(a^2*d*Sqrt[Cot[c + d*x]]*Sqrt
[a + I*a*Tan[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
```

```
Q[c^2 + d^2, 0]
```

Rule 3638

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*Sqrt[(c_.) + (d_.)*tan[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-b)*(a + b*Tan[e + f*x])^m*(Sqrt[c + d
*Tan[e + f*x]]/(2*a*f*m)), x] + Dist[1/(4*a^2*m), Int[(a + b*Tan[e + f*x])^
(m + 1)*(Simp[2*a*c*m + b*d + a*d*(2*m + 1)*Tan[e + f*x], x]/Sqrt[c + d*Tan
[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && IntegersQ[2*m]
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^{5/2}}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{\sqrt{\tan(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}\,d$$

$$= \frac{i}{5d\sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^{5/2}} - \frac{\left(\sqrt{\cot(c+dx)}\right.}{}$$

$$= \frac{i}{5d\sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^{5/2}} + \frac{}{10ad\sqrt{\cot(c+d}}$$

$$= \frac{i}{5d\sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^{5/2}} + \frac{}{10ad\sqrt{\cot(c+d}}$$

$$= \frac{i}{5d\sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^{5/2}} + \frac{}{10ad\sqrt{\cot(c+d}}$$

$$= \frac{i}{5d\sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^{5/2}} + \frac{}{10ad\sqrt{\cot(c+d}}$$

$$= -\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}}{a^{5/2}d}$$

Mathematica [A]
time = 1.77, size = 167, normalized size = 0.89

$$\frac{e^{-6i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(-1-2e^{2i(c+dx)}+2e^{4i(c+dx)}+e^{6i(c+dx)}-5e^{5i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\cot(c+dx)}}{20\sqrt{2}\,a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2)),x]
```

```
[Out] (Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(-1 - 2*E^((2*I)*(
c + d*x)) + 2*E^((4*I)*(c + d*x)) + E^((6*I)*(c + d*x)) - 5*E^((5*I)*(c + d
*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2
*I)*(c + d*x))]])*Sqrt[Cot[c + d*x]])/(20*Sqrt[2]*a^3*d*E^((6*I)*(c + d*x))
)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 542 vs. 2(147) = 294.
time = 47.20, size = 543, normalized size = 2.89

method	result
default	$\frac{\left(-\frac{1}{40}+\frac{i}{40}\right)\left(20 i \arctan\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\sqrt{2}\right)\sqrt{2}\,\left(\cos^{2}(dx+c)\right)\sin(dx+c)-10 i \sin(dx+c)\cos(dx+c)\sqrt{2}\,\arctan\left(\left(\frac{1}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/40+1/40*I)/d*(20*I*sin(d*x+c)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*2^(1/2))*2^(1/2)*cos(d*x+c)^2-10*I*sin(d*x+c)*arctan((1/2+1/2*
I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))*2^(1/2)*cos(d*x+c)-4*I*sin(d
*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+20*arctan((1/2+1/2*I)
*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))*2^(1/2)*cos(d*x+c)^3-5*I*sin(d
*x+c)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))*2^(1/2
)+4*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*sin(d*x+c)-10*cos(d*x+c
)^2*2^(1/2)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))-
I*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-15*cos(d*x+c)*2^(1/2)*arcta
n((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))+sin(d*x+c)*((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)+5*2^(1/2)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*2^(1/2)))*cos(d*x+c)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x
+c))^(1/2)/(4*I*cos(d*x+c)^2*sin(d*x+c)+4*cos(d*x+c)^3-I*sin(d*x+c)-3*cos(d
*x+c))/(cos(d*x+c)/sin(d*x+c))^(1/2)/sin(d*x+c)/((-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)/a^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 366 vs. 2(136) = 272.
time = 0.69, size = 366, normalized size = 1.95

$$-\frac{\left(10\,a^{3}d\sqrt{\frac{i}{8\,a^{5}d^{2}}}\,e^{(5i\,dx+5i\,c)}\log\left(4\left(2\sqrt{2}\left(a^{3}de^{(2i\,dx+2i\,c)}-a^{3}d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{i}{8\,a^{5}d^{2}}}+i\,ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-10\,a^{3}d\sqrt{\frac{i}{8\,a^{5}d^{2}}}\,e^{(5i\,dx+5i\,c)}\log\left(-4\left(2\sqrt{2}\left(a^{3}de^{(2i\,dx+2i\,c)}-a^{3}d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{i}{8\,a^{5}d^{2}}}-i\,ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\left(e^{(6i\,dx+6i\,c)}+2\,e^{(4i\,dx+4i\,c)}-2\,e^{(2i\,dx+2i\,c)}-1\right)\right)e^{(-5i\,dx-5i\,c)}}{40\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas"
)
```

```
[Out] -1/40*(10*a^3*d*sqrt(1/8*I/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(4*(2*sqrt(2)*
(a^3*d*e^(2*I*d*x + 2*I*c) - a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(
(I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(1/8*I/(a^5*d^2)
) + I*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 10*a^3*d*sqrt(1/8*I/(a^5*d^2))
*e^(5*I*d*x + 5*I*c)*log(-4*(2*sqrt(2)*(a^3*d*e^(2*I*d*x + 2*I*c) - a^3*d)*
sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*
d*x + 2*I*c) - 1))*sqrt(1/8*I/(a^5*d^2)) - I*a*e^(I*d*x + I*c))*e^(-I*d*x -
 I*c)) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I
*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*(e^(6*I*d*x + 6*I*c) + 2*e^(4*I*d*x + 4
*I*c) - 2*e^(2*I*d*x + 2*I*c) - 1))*e^(-5*I*d*x - 5*I*c)/(a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}\sqrt{\cot(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(1/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(1/((I*a*(tan(c + d*x) - I))**(5/2)*sqrt(cot(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*tan(d*x + c) + a)^(5/2)*sqrt(cot(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cot(c+d\,x)}\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

```
[Out] int(1/(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(5/2)), x)
```

3.785 $\int \frac{1}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=184

$$-\frac{\left(\frac{1}{8}-\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{a^{5/2}d}+\frac{1}{5d\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+$$

```
[Out] (-1/8+1/8*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2
))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/a^(5/2)/d+1/4/a^2/d/cot(d*x+c)^(1/2)/(
a+I*a*tan(d*x+c))^(1/2)+1/5/d/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(5/2)+1/6
*I/a/d/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.24, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {4326, 3628, 3627, 3625, 211}

$$-\frac{\left(\frac{1}{8}-\frac{i}{8}\right)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d}+\frac{1}{4a^2d\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{i}{6ad\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}+\frac{1}{5d\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(5/2)),x]
```

```
[Out] ((-1/8 + I/8)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan
[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(a^(5/2)*d) + 1/(5*d*Cot
[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^(5/2)) + (I/6)/(a*d*Cot[c + d*x]^(3/
2)*(a + I*a*Tan[c + d*x])^(3/2)) + 1/(4*a^2*d*Sqrt[Cot[c + d*x]]*Sqrt[a + I
*a*Tan[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3627

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
```

```
 + f*x])^n/(2*b*f*m)), x] - Dist[(a*c - b*d)/(2*b^2), Int[(a + b*Tan[e + f*
x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && EqQ
[m + n, 0] && LeQ[m, -2^(-1)]
```

Rule 3628

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a), Int[(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && EqQ[m
 + n + 1, 0] && LtQ[m, -1]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{1}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{\tan^{\frac{3}{2}}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$$

$$= \frac{1}{5d\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}} + \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\mathrm{ta}\right.}{}$$

$$= \frac{1}{5d\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}} + \frac{i}{6ad\cot^{\frac{3}{2}}(c+dx)(a-}$$

$$= \frac{1}{5d\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}} + \frac{i}{6ad\cot^{\frac{3}{2}}(c+dx)(a-}$$

$$= \frac{1}{5d\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}} + \frac{i}{6ad\cot^{\frac{3}{2}}(c+dx)(a-}$$

$$= -\frac{\left(\frac{1}{8}-\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{}}{a^{5/2}d}$$

Mathematica [A]
time = 1.29, size = 171, normalized size = 0.93

$$-\frac{ie^{-6i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3-4e^{2i(c+dx)}-16e^{4i(c+dx)}+17e^{6i(c+dx)}-15e^{5i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\cot(c+dx)}}{60\sqrt{2}\,a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(5/2)),x]
```

```
[Out] ((-1/60*I)*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(3 - 4*E
^((2*I)*(c + d*x)) - 16*E^((4*I)*(c + d*x)) + 17*E^((6*I)*(c + d*x)) - 15*E
^((5*I)*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]*ArcTanh[E^(I*(c + d*x))/S
qrt[-1 + E^((2*I)*(c + d*x))]])*Sqrt[Cot[c + d*x]])/(Sqrt[2]*a^3*d*E^((6*I)
*(c + d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 659 vs. 2(145) = 290.
time = 48.04, size = 660, normalized size = 3.59

method	result
default	$\frac{\left(\frac{1}{120}+\frac{i}{120}\right)\left(60i\arctan\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\sqrt{2}\right)\sqrt{2}\,\left(\cos^2(dx+c)\right)\sin(dx+c)-30i\sin(dx+c)\cos(dx+c)\sqrt{2}\,\arctan\left(\left(\frac{1}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/120+1/120*I)/d*(60*I*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)*2^(1/2))*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)-30*I*arctan((1/2+1/2*I)*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))*2^(1/2)*cos(d*x+c)*sin(d*x+c)+60*arcta
n((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))*2^(1/2)*cos(d*x+c
)^3-40*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3-28*I*((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*sin(d*x+c)-15*I*arctan((1/2+1/2*I)*((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))*2^(1/2)*sin(d*x+c)-30*cos(d*x+c)^2*2
^(1/2)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))-40*((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3+28*((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*cos(d*x+c)^2*sin(d*x+c)-45*cos(d*x+c)*2^(1/2)*arctan((1/2+1/2*I)*(
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))+40*I*((-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*cos(d*x+c)+13*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+15*2^
(1/2)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))+40*cos
(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-13*sin(d*x+c)*((-1+cos(d*x+c))/s
in(d*x+c))^(1/2))*cos(d*x+c)^2*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/
2)/(4*I*cos(d*x+c)^2*sin(d*x+c)+4*cos(d*x+c)^3-I*sin(d*x+c)-3*cos(d*x+c))/(
```

```
cos(d*x+c)/sin(d*x+c))^(3/2)/((-1+cos(d*x+c))/sin(d*x+c))^(1/2)/sin(d*x+c)^
2/a^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(136) = 272$.
time = 1.24, size = 369, normalized size = 2.01

$$\frac{\left(30\,a^3d\sqrt{-\frac{i}{8\,a^5d^2}}\,e^{(5i\,dx+5i\,c)}\log\left(-4\left(2\sqrt{2}\left(i\,a^3de^{(2i\,dx+2i\,c)}-i\,a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{-\frac{i}{8\,a^5d^2}}-i\,ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-30\,a^3d\sqrt{-\frac{i}{8\,a^5d^2}}\,e^{(5i\,dx+5i\,c)}\log\left(-4\left(2\sqrt{2}\left(-i\,a^3de^{(2i\,dx+2i\,c)}+i\,a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{-\frac{i}{8\,a^5d^2}}-i\,ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\left(-17i\,e^{(6i\,dx+6i\,c)}+16i\,e^{(4i\,dx+4i\,c)}+4i\,e^{(2i\,dx+2i\,c)}-3i\right)\right)e^{(-5i\,dx-5i\,c)}}{120\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/120*(30*a^3*d*sqrt(-1/8*I/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(-4*(2*sqrt(2
)*(I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-1/8*I/(a
^5*d^2)) - I*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 30*a^3*d*sqrt(-1/8*I/(a
^5*d^2))*e^(5*I*d*x + 5*I*c)*log(-4*(2*sqrt(2)*(-I*a^3*d*e^(2*I*d*x + 2*I*c
) + I*a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c)
+ I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-1/8*I/(a^5*d^2)) - I*a*e^(I*d*x + I*c
))*e^(-I*d*x - I*c)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^
(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*(-17*I*e^(6*I*d*x + 6*I*c
) + 16*I*e^(4*I*d*x + 4*I*c) + 4*I*e^(2*I*d*x + 2*I*c) - 3*I))*e^(-5*I*d*x
- 5*I*c)/(a^3*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(3/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(1/((I*a*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cot\left(c+d\,x\right)^{3/2}\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(5/2)),x)
[Out] int(1/(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(5/2)), x)
```

3.786 $\int \frac{1}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=186

$$\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{a^{5/2}d}+\frac{i}{5d\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+\ldots}$$

```
[Out] (1/8+1/8*I)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
)*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/a^(5/2)/d-1/4*I/a^2/d/cot(d*x+c)^(1/2)/
(a+I*a*tan(d*x+c))^(1/2)+1/5*I/d/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(5/2)+
1/6/a/d/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.24, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4326, 3627, 3625, 211}

$$\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d}-\frac{i}{4a^2d\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}+\frac{1}{6ad\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}+\frac{i}{5d\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^(5/2)),x]
```

```
[Out] ((1/8 + I/8)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[
c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(a^(5/2)*d) + (I/5)/(d*Co
t[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^(5/2)) + 1/(6*a*d*Cot[c + d*x]^(3/2
)*(a + I*a*Tan[c + d*x])^(3/2)) - (I/4)/(a^2*d*Sqrt[Cot[c + d*x]]*Sqrt[a +
I*a*Tan[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3627

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
```

```
 + f*x])^n/(2*b*f*m)), x] - Dist[(a*c - b*d)/(2*b^2), Int[(a + b*Tan[e + f*
x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && EqQ
[m + n, 0] && LeQ[m, -2^(-1)]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{1}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{\tan^{\frac{5}{2}}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$$

$$= \frac{i}{5d\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}} - \frac{\left(i\sqrt{\cot(c+dx)}\,\sqrt{\text{ta}\right.}{}$$

$$= \frac{i}{5d\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}} + \frac{1}{6ad\cot^{\frac{3}{2}}(c+dx)(a+}$$

$$= \frac{i}{5d\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}} + \frac{1}{6ad\cot^{\frac{3}{2}}(c+dx)(a+}$$

$$= \frac{i}{5d\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}} + \frac{1}{6ad\cot^{\frac{3}{2}}(c+dx)(a+}$$

$$= \frac{\left(\frac{1}{8}+\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\text{tan}}}{a^{5/2}d}$$

Mathematica [A]

time = 2.75, size = 158, normalized size = 0.85

$$\frac{\cot^{\frac{3}{2}}(c+dx)\sec(c+dx)\left(30e^{2i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\csc(2(c+dx))+2\sec(c+dx)(11i-26i\cos(2(c+dx))+20\sin(2(c+dx)))\right)}{120a^2d(i+\cot(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^(5/2)),x]
```

```
[Out] (Cot[c + d*x]^(3/2)*Sec[c + d*x]*(30*E^((2*I)*(c + d*x))*Sqrt[-1 + E^((2*I)
*(c + d*x))]*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]]*Csc[2*
(c + d*x)] + 2*Sec[c + d*x]*(11*I - (26*I)*Cos[2*(c + d*x)] + 20*Sin[2*(c +
 d*x)])))/(120*a^2*d*(I + Cot[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 660 vs. $2(146) = 292$.
time = 48.94, size = 661, normalized size = 3.55

method	result
default	$\frac{\left(-\frac{1}{120}-\frac{i}{120}\right)\left(60 i \arctan\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\sqrt{2}\right)\sqrt{2}\,(\cos^3(dx+c))-30 i \arctan\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\sqrt{2}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/120-1/120*I)/d*(60*I*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*2^(1/2))*2^(1/2)*cos(d*x+c)^3-30*I*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*2^(1/2))*2^(1/2)*cos(d*x+c)^2-60*arctan((1/2+1/2*I)*((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))*sin(d*x+c)*2^(1/2)*cos(d*x+c)^2+52*I
*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-40*I*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3-45*I*arctan((1/2+1/2*I)*((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)*2^(1/2))*2^(1/2)*cos(d*x+c)+30*arctan((1/2+1/2*I)*((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))*cos(d*x+c)*sin(d*x+c)*2^(1/2)+52*(
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*sin(d*x+c)+40*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*cos(d*x+c)^3+15*I*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*2^(1/2))*2^(1/2)+15*2^(1/2)*sin(d*x+c)*arctan((1/2+1/2*I)
*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))-37*I*sin(d*x+c)*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)+40*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)-37*
sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-40*cos(d*x+c)*((-1+cos(d*x+c)
)/sin(d*x+c))^(1/2))*cos(d*x+c)^3*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^
(1/2)/(4*I*cos(d*x+c)^2*sin(d*x+c)+4*cos(d*x+c)^3-I*sin(d*x+c)-3*cos(d*x+c)
)/((-1+cos(d*x+c))/sin(d*x+c))^(1/2)/sin(d*x+c)^3/(cos(d*x+c)/sin(d*x+c))^(
5/2)/a^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(136) = 272$.
time = 1.29, size = 368, normalized size = 1.98

$$\frac{30\,a^{3}d\sqrt{\frac{i}{8\,a^{5}d^{2}}}\,e^{(5i\,dx+5i\,c)}\log\left(4\left(2\sqrt{2}\left(a^{3}de^{(2i\,dx+2i\,c)}-a^{3}d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{i}{8\,a^{5}d^{2}}}+i\,ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-30\,a^{3}d\sqrt{\frac{i}{8\,a^{5}d^{2}}}\,e^{(5i\,dx+5i\,c)}\log\left(-4\left(2\sqrt{2}\left(a^{3}de^{(2i\,dx+2i\,c)}-a^{3}d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{i}{8\,a^{5}d^{2}}}-i\,ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\left(23\,e^{(6i\,dx+6i\,c)}-34\,e^{(4i\,dx+4i\,c)}+14\,e^{(2i\,dx+2i\,c)}-3\right)e^{(-5i\,dx-5i\,c)}}{120\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/120*(30*a^3*d*sqrt(1/8*I/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(4*(2*sqrt(2)*
(a^3*d*e^(2*I*d*x + 2*I*c) - a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(
(I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(1/8*I/(a^5*d^2)
) + I*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 30*a^3*d*sqrt(1/8*I/(a^5*d^2))
*e^(5*I*d*x + 5*I*c)*log(-4*(2*sqrt(2)*(a^3*d*e^(2*I*d*x + 2*I*c) - a^3*d)*
sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*
d*x + 2*I*c) - 1))*sqrt(1/8*I/(a^5*d^2)) - I*a*e^(I*d*x + I*c))*e^(-I*d*x -
 I*c)) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I
*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*(23*e^(6*I*d*x + 6*I*c) - 34*e^(4*I*d*x
 + 4*I*c) + 14*e^(2*I*d*x + 2*I*c) - 3))*e^(-5*I*d*x - 5*I*c)/(a^3*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(5/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6191 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cot\left(c+d\,x\right)^{5/2}\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(5/2)),x)
[Out] int(1/(cot(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(5/2)), x)
```

3.787 $\int \frac{1}{\cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=258

$$\frac{2\sqrt[4]{-1}\,\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{a^{5/2}d}+\frac{\left(\frac{1}{8}-\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan}}{\sqrt{a+ia\tan(}}\right.}{}$$

```
[Out] 2*(-1)^(1/4)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^
(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/a^(5/2)/d+(1/8-1/8*I)*arctanh((1+I
)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d
*x+c)^(1/2)/a^(5/2)/d+7/4/a^2/d/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-1
/5/d/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(5/2)+1/2*I/a/d/cot(d*x+c)^(3/2)/(
a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.52, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4326, 3639, 3676, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{2\sqrt[4]{-1}\,\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d}+\frac{\left(\frac{1}{8}-\frac{i}{8}\right)\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d}+\frac{7}{4a^2d\sqrt{\cot(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}+\frac{i}{2ad\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}-\frac{1}{5d\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cot[c + d*x]^(7/2)*(a + I*a*Tan[c + d*x])^(5/2)),x]
```

```
[Out] (2*(-1)^(1/4)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*T
an[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(a^(5/2)*d) + ((1/8 -
I/8)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]
]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(a^(5/2)*d) - 1/(5*d*Cot[c + d*x]
^(5/2)*(a + I*a*Tan[c + d*x])^(5/2)) + (I/2)/(a*d*Cot[c + d*x]^(3/2)*(a + I
*a*Tan[c + d*x])^(3/2)) + 7/(4*a^2*d*Sqrt[Cot[c + d*x]]*Sqrt[a + I*a*Tan[c
+ d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
```

```
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{1}{\cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{\tan^{\frac{7}{2}}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$$

$$= -\frac{1}{5d\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}} - \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\right.}{}}{}$$

$$= -\frac{1}{5d\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}} + \frac{}{2ad\cot^{\frac{3}{2}}(c+dx)(a}$$

$$= -\frac{1}{5d\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}} + \frac{}{2ad\cot^{\frac{3}{2}}(c+dx)(a}$$

$$= -\frac{1}{5d\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}} + \frac{}{2ad\cot^{\frac{3}{2}}(c+dx)(a}$$

$$= -\frac{1}{5d\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}} + \frac{}{2ad\cot^{\frac{3}{2}}(c+dx)(a}$$

$$= \frac{\left(\frac{1}{8}-\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\mathrm{ta}}}{a^{5/2}d}$$

$$= \frac{\left(\frac{1}{8}-\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\mathrm{ta}}}{a^{5/2}d}$$

$$= \frac{2\sqrt[4]{-1}\,\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a}\,\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\mathrm{ta}}}{a^{5/2}d}$$

Mathematica [A]
time = 3.82, size = 241, normalized size = 0.93

$$\frac{ie^{-6i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(1-8e^{2i(c+dx)}+48e^{4i(c+dx)}-41e^{6i(c+dx)}-5e^{5i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)+40\sqrt{2}\,e^{5i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{\sqrt{2}\,e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\cot(c+dx)}}{20\sqrt{2}\,a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cot[c + d*x]^(7/2)*(a + I*a*Tan[c + d*x])^(5/2)),x]
```

```
[Out] ((I/20)*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(1 - 8*E^((
2*I)*(c + d*x)) + 48*E^((4*I)*(c + d*x)) - 41*E^((6*I)*(c + d*x)) - 5*E^((5
```

```
*I)*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]*ArcTanh[E^(I*(c + d*x))/Sqrt[
-1 + E^((2*I)*(c + d*x))]] + 40*Sqrt[2]*E^((5*I)*(c + d*x))*Sqrt[-1 + E^((2
*I)*(c + d*x))]*ArcTanh[(Sqrt[2]*E^(I*(c + d*x)))/Sqrt[-1 + E^((2*I)*(c + d
*x))]])*Sqrt[Cot[c + d*x]])/(Sqrt[2]*a^3*d*E^((6*I)*(c + d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1577 vs. 2(202) = 404.
time = 46.56, size = 1578, normalized size = 6.12

method	result	size
default	Expression too large to display	1578

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/40-1/40*I)/d*(-15*cos(d*x+c)*2^(1/2)*arctan((1/2+1/2*I)*((-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*2^(1/2))+80*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*
x+c)^3+80*I*cos(d*x+c)^3*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-I)-80*I*cos(
d*x+c)^3*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)+80*I*cos(d*x+c)^3*ln(((-1
+cos(d*x+c))/sin(d*x+c))^(1/2)-1)-80*I*cos(d*x+c)^3*ln(((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)+I)-40*I*cos(d*x+c)^2*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-I
)+40*I*cos(d*x+c)^2*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)-40*I*cos(d*x+c
)^2*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)+40*I*cos(d*x+c)^2*ln(((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)+I)-60*I*cos(d*x+c)*ln(((-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)-I)+60*I*cos(d*x+c)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)-60*I*co
s(d*x+c)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)+60*I*cos(d*x+c)*ln(((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)+I)-49*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin
(d*x+c)-80*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)-10*cos(d*x+c)^2*
2^(1/2)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))+20*I
*sin(d*x+c)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))*
2^(1/2)*cos(d*x+c)^2-10*I*sin(d*x+c)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*2^(1/2))*2^(1/2)*cos(d*x+c)+20*arctan((1/2+1/2*I)*((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))*2^(1/2)*cos(d*x+c)^3-80*ln(((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)-I)*cos(d*x+c)^2*sin(d*x+c)-80*ln(((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)-1)*cos(d*x+c)^2*sin(d*x+c)+80*ln(((-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)+I)*cos(d*x+c)^2*sin(d*x+c)+80*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
+1)*cos(d*x+c)^2*sin(d*x+c)-84*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c
)^2*sin(d*x+c)+40*cos(d*x+c)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-I)*sin(d
*x+c)+40*cos(d*x+c)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)*sin(d*x+c)-40*
cos(d*x+c)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+I)*sin(d*x+c)-40*cos(d*x+c
)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)*sin(d*x+c)-5*I*sin(d*x+c)*arctan
((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))*2^(1/2)+84*I*((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*sin(d*x+c)+49*sin(d*x+c)*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)+5*2^(1/2)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/s
in(d*x+c))^(1/2)*2^(1/2))-80*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+
```

```
80*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3+20*I*ln(((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)-I)-20*I*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)+20*I*ln
(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)-20*I*ln(((-1+cos(d*x+c))/sin(d*x+c))
^(1/2)+I)+20*sin(d*x+c)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-I)-20*sin(d*x
+c)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)+20*sin(d*x+c)*ln(((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)-1)-20*sin(d*x+c)*ln(((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)+I))*cos(d*x+c)^4*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(4*I*cos(
d*x+c)^2*sin(d*x+c)+4*cos(d*x+c)^3-I*sin(d*x+c)-3*cos(d*x+c))/((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)/sin(d*x+c)^4/(cos(d*x+c)/sin(d*x+c))^(7/2)/a^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 662 vs. 2(192) = 384.
time = 0.97, size = 662, normalized size = 2.57

$$-\frac{\left(10a^3d\sqrt{-\frac{i}{8a^5d^2}}e^{(5idx+5ic)}\log\left(-4\left(2\sqrt{2}\left(ia^3de^{(2idx+2ic)}-ia^3d\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{-\frac{i}{8a^5d^2}}-iae^{(idx+ic)}\right)e^{(-idx-ic)}\right)-\ldots\text{[illegible]}\right)}{40a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas"
)
```

```
[Out] -1/40*(10*a^3*d*sqrt(-1/8*I/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(-4*(2*sqrt(2
)*(I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-1/8*I/(a
^5*d^2)) - I*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 10*a^3*d*sqrt(-1/8*I/(a
^5*d^2))*e^(5*I*d*x + 5*I*c)*log(-4*(2*sqrt(2)*(-I*a^3*d*e^(2*I*d*x + 2*I*c
) + I*a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c)
+ I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-1/8*I/(a^5*d^2)) - I*a*e^(I*d*x + I*c
))*e^(-I*d*x - I*c)) + 10*a^3*d*sqrt(-4*I/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*lo
g(-16*(sqrt(2)*(a^4*d*e^(3*I*d*x + 3*I*c) - a^4*d*e^(I*d*x + I*c))*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*
I*c) - 1))*sqrt(-4*I/(a^5*d^2)) + 3*a^2*e^(2*I*d*x + 2*I*c) - a^2)*e^(-2*I*
d*x - 2*I*c)) - 10*a^3*d*sqrt(-4*I/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(16*(s
qrt(2)*(a^4*d*e^(3*I*d*x + 3*I*c) - a^4*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1
```

```
))*sqrt(-4*I/(a^5*d^2)) - 3*a^2*e^(2*I*d*x + 2*I*c) + a^2)*e^(-2*I*d*x - 2*
I*c)) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*
c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*(-41*I*e^(6*I*d*x + 6*I*c) + 48*I*e^(4*I
*d*x + 4*I*c) - 8*I*e^(2*I*d*x + 2*I*c) + I))*e^(-5*I*d*x - 5*I*c)/(a^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(7/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cot(c+d\,x)^{7/2}\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(7/2)*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

```
[Out] int(1/(cot(c + d*x)^(7/2)*(a + a*tan(c + d*x)*1i)^(5/2)), x)
```

3.788 $\int (d\cot(e+fx))^n(a+ia\tan(e+fx))^3\,dx$

Optimal. Leaf size=139

$$\frac{ia^3d^2(1-2n)(d\cot(e+fx))^{-2+n}}{f(1-n)(2-n)}+\frac{d^2(d\cot(e+fx))^{-2+n}\left(ia^3+a^3\cot(e+fx)\right)}{f(1-n)}-\frac{4ia^3d^2(d\cot(e+fx))^{-2+}}{}$$

```
[Out] I*a^3*d^2*(1-2*n)*(d*cot(f*x+e))^(-2+n)/f/(n^2-3*n+2)+d^2*(d*cot(f*x+e))^(-
2+n)*(I*a^3+a^3*cot(f*x+e))/f/(1-n)-4*I*a^3*d^2*(d*cot(f*x+e))^(-2+n)*hyper
geom([1, -2+n],[-1+n],-I*cot(f*x+e))/f/(2-n)
```

Rubi [A]
time = 0.27, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3754, 3637, 3673, 3618, 12, 66}

$$-\frac{4ia^3d^2(d\cot(e+fx))^{n-2}\,{}_2F_1(1,n-2;n-1;-i\cot(e+fx))}{f(2-n)}+\frac{d^2\left(a^3\cot(e+fx)+ia^3\right)(d\cot(e+fx))^{n-2}}{f(1-n)}+\frac{ia^3d^2(1-2n)(d\cot(e+fx))^{n-2}}{f(1-n)(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^n*(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] (I*a^3*d^2*(1 - 2*n)*(d*Cot[e + f*x])^(-2 + n))/(f*(1 - n)*(2 - n)) + (d^2*
(d*Cot[e + f*x])^(-2 + n)*(I*a^3 + a^3*Cot[e + f*x]))/(f*(1 - n)) - ((4*I)*
a^3*d^2*(d*Cot[e + f*x])^(-2 + n)*Hypergeometric2F1[1, -2 + n, -1 + n, (-I)
*Cot[e + f*x]])/(f*(2 - n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int (d\cot(e+fx))^n(a+ia\tan(e+fx))^3\,dx = d^3\int (d\cot(e+fx))^{-3+n}(ia+a\cot(e+fx))^3\,dx$$

$$= \frac{d^2(d\cot(e+fx))^{-2+n}\left(ia^3+a^3\cot(e+fx)\right)}{f(1-n)} + \frac{(iad^2)\int (d\cot(}{}$$

$$= \frac{ia^3d^2(1-2n)(d\cot(e+fx))^{-2+n}}{f(1-n)(2-n)} + \frac{d^2(d\cot(e+fx))^{-2+n}\,(ia}{f(1-n}$$

$$= \frac{ia^3d^2(1-2n)(d\cot(e+fx))^{-2+n}}{f(1-n)(2-n)} + \frac{d^2(d\cot(e+fx))^{-2+n}\,(ia}{f(1-n}$$

$$= \frac{ia^3d^2(1-2n)(d\cot(e+fx))^{-2+n}}{f(1-n)(2-n)} + \frac{d^2(d\cot(e+fx))^{-2+n}\,(ia}{f(1-n}$$

$$= \frac{ia^3d^2(1-2n)(d\cot(e+fx))^{-2+n}}{f(1-n)(2-n)} + \frac{d^2(d\cot(e+fx))^{-2+n}\,(ia}{f(1-n}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 925 vs. $2(139) = 278$.
time = 8.36, size = 925, normalized size = 6.65

$$-\frac{4i\left(1+e^{2i(e+fx)}\right)^{-n}\left(\frac{i\left(1+e^{2i(e+fx)}\right)}{-1+e^{2i(e+fx)}}\right)^{n}\cos^3(e+fx)\cot^{-n}(e+fx)(d\cot(e+fx))^n\left(\left(1+e^{2i(e+fx)}\right)^n {}_2F_1\left(1,-n;1-n;-\frac{-1+e^{2i(e+fx)}}{1+e^{2i(e+fx)}}\right)-2^n {}_2F_1\left(-n,-n;1-n;\frac{1}{2}\left(1-e^{2i(e+fx)}\right)\right)\right)(a+ia\tan(e+fx))^3}{\left(e^{ie}+e^{3ie}\right)fn(\cos(fx)+i\sin(fx))^3} - \text{[illegible]}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cot[e + f*x])^n*(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] ((-4*I)*((I*(1 + E^((2*I)*(e + f*x))))/(-1 + E^((2*I)*(e + f*x))))^n*Cos[e
+ f*x]^3*(d*Cot[e + f*x])^n*((1 + E^((2*I)*(e + f*x)))^n*Hypergeometric2F1[
1, -n, 1 - n, -((-1 + E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f*x))))] - 2^
n*Hypergeometric2F1[-n, -n, 1 - n, (1 - E^((2*I)*(e + f*x)))/2])*(a + I*a*T
an[e + f*x])^3)/((E^(I*e) + E^((3*I)*e))*(1 + E^((2*I)*(e + f*x)))^n*f*n*Co
t[e + f*x]^n*(Cos[f*x] + I*Sin[f*x])^3) - ((4*I)*((-1 + E^((2*I)*(e + f*x))
)/(1 + E^((2*I)*(e + f*x))))^n*((I*(1 + E^((2*I)*(e + f*x))))/(-1 + E^((2*I
)*(e + f*x))))^n*Cos[e + f*x]^3*(d*Cot[e + f*x])^n*(((1 + E^((2*I)*e))*(-1
+ E^((2*I)*(e + f*x)))*(1 + E^((2*I)*(e + f*x)))^(-1 + n)*Hypergeometric2F1
[1, 1 - n, 2 - n, (1 - E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f*x)))])/(-1
 + n) + ((1 + E^((2*I)*(e + f*x)))^n*Hypergeometric2F1[1, -n, 1 - n, (1 - E
^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f*x)))])/n - (2^n*Hypergeometric2F1[
-n, -n, 1 - n, (1 - E^((2*I)*(e + f*x)))/2])/n)*(a + I*a*Tan[e + f*x])^3)/(
E^((3*I)*e)*(1 + E^((2*I)*e))*(-1 + E^((2*I)*(e + f*x)))^n*f*Cot[e + f*x]^n
*(Cos[f*x] + I*Sin[f*x])^3) + (Cos[e + f*x]^3*(d*Cot[e + f*x])^n*(((-3 + 2*
n + Cos[2*e])*Sec[e]^2*((-1/2*I)*Cos[3*e] - Sin[3*e]/2))/((-2 + n)*(-1 + n)
) + ((-Cos[e - f*x] + Cos[e + f*x])*Sec[e]^2*Sec[e + f*x]*((I/2)*Cos[3*e] +
 Sin[3*e]/2))/(-1 + n) + (Sec[e + f*x]^2*(I*Cos[3*e] + Sin[3*e]))/(-2 + n))
*(a + I*a*Tan[e + f*x])^3)/(f*(Cos[f*x] + I*Sin[f*x])^3) + (Cos[e + f*x]^3*
(d*Cot[e + f*x])^n*((Sec[e]^2*(-1 + Cos[2*e] + (3*I)*Sin[2*e])*((-1/2*I)*Co
s[3*e] - Sin[3*e]/2))/(-1 + n) + (Sec[e]^2*Sec[e + f*x]*((-1/2*I)*Cos[3*e]
- Sin[3*e]/2)*(-Cos[e - f*x] + Cos[e + f*x] - (3*I)*Sin[e - f*x] + (3*I)*Si
n[e + f*x]))/(-1 + n))*(a + I*a*Tan[e + f*x])^3)/(f*(Cos[f*x] + I*Sin[f*x])
^3)
```

Maple [F]
time = 0.78, size = 0, normalized size = 0.00

$$\int (d\cot(fx+e))^n (a+ia\tan(fx+e))^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^n*(a+I*a*tan(f*x+e))^3,x)
```

```
[Out] int((d*cot(f*x+e))^n*(a+I*a*tan(f*x+e))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*(a+I*a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^3*(d*cot(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*(a+I*a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(8*a^3*((I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) - 1))^
n*e^(6*I*f*x + 6*I*e)/(e^(6*I*f*x + 6*I*e) + 3*e^(4*I*f*x + 4*I*e) + 3*e^(2
*I*f*x + 2*I*e) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia^3\left(\int i(d\cot(e+fx))^n\,dx + \int\left(-3(d\cot(e+fx))^n\tan(e+fx)\right)dx + \int(d\cot(e+fx))^n\tan^3(e+fx)\,dx + \int\left(-3i(d\cot(e+fx))^n\tan^2(e+fx)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**n*(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] -I*a**3*(Integral(I*(d*cot(e + f*x))**n, x) + Integral(-3*(d*cot(e + f*x))*
*n*tan(e + f*x), x) + Integral((d*cot(e + f*x))**n*tan(e + f*x)**3, x) + In
tegral(-3*I*(d*cot(e + f*x))**n*tan(e + f*x)**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^3*(d*cot(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\cot(e+f\,x))^n\,\left(a+a\tan(e+f\,x)\,1\mathrm{i}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(e + f*x))^n*(a + a*tan(e + f*x)*1i)^3,x)
[Out] int((d*cot(e + f*x))^n*(a + a*tan(e + f*x)*1i)^3, x)
```

3.789 $\int (d\cot(e+fx))^n(a+ia\tan(e+fx))^2\,dx$

Optimal. Leaf size=72

$$\frac{a^2d(d\cot(e+fx))^{-1+n}}{f(1-n)}-\frac{2a^2d(d\cot(e+fx))^{-1+n}\,{}_2F_1(1,-1+n;n;-i\cot(e+fx))}{f(1-n)}$$

```
[Out] a^2*d*(d*cot(f*x+e))^(-1+n)/f/(1-n)-2*a^2*d*(d*cot(f*x+e))^(-1+n)*hypergeom
([1, -1+n],[n],-I*cot(f*x+e))/f/(1-n)
```

Rubi [A]
time = 0.13, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3754, 3624, 3618, 12, 66}

$$\frac{a^2d(d\cot(e+fx))^{n-1}}{f(1-n)}-\frac{2a^2d(d\cot(e+fx))^{n-1}\,{}_2F_1(1,n-1;n;-i\cot(e+fx))}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^n*(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (a^2*d*(d*Cot[e + f*x])^(-1 + n))/(f*(1 - n)) - (2*a^2*d*(d*Cot[e + f*x])^(
-1 + n)*Hypergeometric2F1[1, -1 + n, n, (-I)*Cot[e + f*x]])/(f*(1 - n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
```

```
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int (d\cot(e+fx))^n(a+ia\tan(e+fx))^2\,dx = d^2\int (d\cot(e+fx))^{-2+n}(ia+a\cot(e+fx))^2\,dx$$

$$= \frac{a^2d(d\cot(e+fx))^{-1+n}}{f(1-n)} + d^2\int (d\cot(e+fx))^{-2+n}\left(-2a^2+\right.$$

$$= \frac{a^2d(d\cot(e+fx))^{-1+n}}{f(1-n)} + \frac{(4ia^4d^2)\,\text{Subst}\left(\int \frac{2^{2-n}\left(-\frac{idx}{a^2}\right)^{-2+n}}{-4a^4-2a^2x}\,d\right.}{f}$$

$$= \frac{a^2d(d\cot(e+fx))^{-1+n}}{f(1-n)} + \frac{(i2^{4-n}a^4d^2)\,\text{Subst}\left(\int \frac{\left(-\frac{idx}{a^2}\right)^{-2+n}}{-4a^4-2a^2x}\,dx\right.}{f}$$

$$= \frac{a^2d(d\cot(e+fx))^{-1+n}}{f(1-n)} - \frac{2a^2d(d\cot(e+fx))^{-1+n}\,{}_2F_1(1,-1}{f(1-n)}$$

Mathematica [F]
time = 2.64, size = 0, normalized size = 0.00

$$\int (d\cot(e+fx))^n(a+ia\tan(e+fx))^2\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Cot[e + f*x])^n*(a + I*a*Tan[e + f*x])^2,x]
[Out] Integrate[(d*Cot[e + f*x])^n*(a + I*a*Tan[e + f*x])^2, x]
```

Maple [F]
time = 0.66, size = 0, normalized size = 0.00

$$\int (d\cot(fx+e))^n\,(a+ia\tan(fx+e))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^n*(a+I*a*tan(f*x+e))^2,x)
```

```
[Out] int((d*cot(f*x+e))^n*(a+I*a*tan(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^2*(d*cot(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(4*a^2*((I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) - 1))^
n*e^(4*I*f*x + 4*I*e)/(e^(4*I*f*x + 4*I*e) + 2*e^(2*I*f*x + 2*I*e) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^{2}\left(\int\left(-(d\cot(e+fx))^{n}\right)dx+\int(d\cot(e+fx))^{n}\tan^{2}(e+fx)\,dx+\int\left(-2i(d\cot(e+fx))^{n}\tan(e+fx)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**n*(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -a**2*(Integral(-(d*cot(e + f*x))**n, x) + Integral((d*cot(e + f*x))**n*tan
(e + f*x)**2, x) + Integral(-2*I*(d*cot(e + f*x))**n*tan(e + f*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^2*(d*cot(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\cot(e+fx))^n\,(a+a\tan(e+fx)\ 1\mathrm{i})^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(e + f*x))^n*(a + a*tan(e + f*x)*1i)^2,x)
[Out] int((d*cot(e + f*x))^n*(a + a*tan(e + f*x)*1i)^2, x)
```

3.790 $\int (d\cot(e+fx))^n(a+ia\tan(e+fx))\,dx$

Optimal. Leaf size=37

$$-\frac{ia(d\cot(e+fx))^n\,{}_2F_1(1,n;1+n;-i\cot(e+fx))}{fn}$$

```
[Out] -I*a*(d*cot(f*x+e))^n*hypergeom([1, n],[1+n],-I*cot(f*x+e))/f/n
```

Rubi [A]
time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3754, 3618, 66}

$$-\frac{ia(d\cot(e+fx))^n\,{}_2F_1(1,n;n+1;-i\cot(e+fx))}{fn}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^n*(a + I*a*Tan[e + f*x]),x]
```

```
[Out] ((-I)*a*(d*Cot[e + f*x])^n*Hypergeometric2F1[1, n, 1 + n, (-I)*Cot[e + f*x]
])/(f*n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int (d\cot(e+fx))^n(a+ia\tan(e+fx))\,dx = d\int (d\cot(e+fx))^{-1+n}(ia+a\cot(e+fx))\,dx$$
$$= -\frac{(ia^2d)\,\text{Subst}\left(\int \frac{\left(\frac{dx}{a}\right)^{-1+n}}{a^2+iax}\,dx, x, a\cot(e+fx)\right)}{f}$$
$$= -\frac{ia(d\cot(e+fx))^n\,{}_2F_1(1,n;1+n;-i\cot(e+fx))}{fn}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 166 vs. 2(37) = 74.
time = 0.75, size = 166, normalized size = 4.49

$$-\frac{2^{-1+n}e^{-ie}\left(1+e^{2i(e+fx)}\right)^{1-n}\left(\frac{i(1+e^{2i(e+fx)})}{-1+e^{2i(e+fx)}}\right)^{-1+n}\cos(e+fx)\cot^{-n}(e+fx)(d\cot(e+fx))^n\,{}_2F_1\left(1-n,1-n;2-n;\frac{1}{2}\left(1-e^{2i(e+fx)}\right)\right)(a+ia\tan(e+fx))}{f(-1+n)(\cos(fx)+i\sin(fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cot[e + f*x])^n*(a + I*a*Tan[e + f*x]),x]
```

```
[Out] -((2^(-1 + n)*(1 + E^((2*I)*(e + f*x)))^(1 - n)*((I*(1 + E^((2*I)*(e + f*x)
)))/(-1 + E^((2*I)*(e + f*x))))^(-1 + n)*Cos[e + f*x]*(d*Cot[e + f*x])^n*Hy
pergeometric2F1[1 - n, 1 - n, 2 - n, (1 - E^((2*I)*(e + f*x)))/2]*(a + I*a*
Tan[e + f*x]))/(E^(I*e)*f*(-1 + n)*Cot[e + f*x]^n*(Cos[f*x] + I*Sin[f*x])))
```

Maple [F]
time = 0.55, size = 0, normalized size = 0.00

$$\int (d\cot(fx+e))^n\,(a+ia\tan(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^n*(a+I*a*tan(f*x+e)),x)
```

```
[Out] int((d*cot(f*x+e))^n*(a+I*a*tan(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)*(d*cot(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(2*a*((I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) - 1))^n*
e^(2*I*f*x + 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-i(d\cot\left(e+fx\right))^{n}\right)\,dx+\int\left(d\cot\left(e+fx\right)\right)^{n}\tan\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**n*(a+I*a*tan(f*x+e)),x)
```

```
[Out] I*a*(Integral(-I*(d*cot(e + f*x))**n, x) + Integral((d*cot(e + f*x))**n*tan
(e + f*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)*(d*cot(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int{\left(d\,\mathrm{cot}\left(e+f\,x\right)\right)}^n\,\left(a+a\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(e + f*x))^n*(a + a*tan(e + f*x)*1i),x)
```

```
[Out] int((d*cot(e + f*x))^n*(a + a*tan(e + f*x)*1i), x)
```

3.791 $\int \frac{(d\cot(e+fx))^n}{a+ia\tan(e+fx)}\,dx$

Optimal. Leaf size=157

$$-\frac{(d\cot(e+fx))^{2+n}}{2d^2f(ia+a\cot(e+fx))}-\frac{in(d\cot(e+fx))^{2+n}\,{}_2F_1\left(1,\frac{2+n}{2};\frac{4+n}{2};-\cot^2(e+fx)\right)}{2ad^2f(2+n)}+\frac{(1+n)(d\cot(e+fx))}{}$$

```
[Out] -1/2*(d*cot(f*x+e))^(2+n)/d^2/f/(I*a+a*cot(f*x+e))-1/2*I*n*(d*cot(f*x+e))^(
2+n)*hypergeom([1, 1+1/2*n],[2+1/2*n],-cot(f*x+e)^2)/a/d^2/f/(2+n)+1/2*(1+n
)*(d*cot(f*x+e))^(3+n)*hypergeom([1, 3/2+1/2*n],[5/2+1/2*n],-cot(f*x+e)^2)/
a/d^3/f/(3+n)
```

Rubi [A]
time = 0.20, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3754, 3633, 3619, 3557, 371}

$$\frac{(n+1)(d\cot(e+fx))^{n+3}\,{}_2F_1\left(1,\frac{n+3}{2};\frac{n+5}{2};-\cot^2(e+fx)\right)}{2ad^3f(n+3)}-\frac{in(d\cot(e+fx))^{n+2}\,{}_2F_1\left(1,\frac{n+2}{2};\frac{n+4}{2};-\cot^2(e+fx)\right)}{2ad^2f(n+2)}-\frac{(d\cot(e+fx))^{n+2}}{2d^2f(a\cot(e+fx)+ia)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^n/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] -1/2*(d*Cot[e + f*x])^(2 + n)/(d^2*f*(I*a + a*Cot[e + f*x])) - ((I/2)*n*(d*
Cot[e + f*x])^(2 + n)*Hypergeometric2F1[1, (2 + n)/2, (4 + n)/2, -Cot[e + f
*x]^2])/(a*d^2*f*(2 + n)) + ((1 + n)*(d*Cot[e + f*x])^(3 + n)*Hypergeometri
c2F1[1, (3 + n)/2, (5 + n)/2, -Cot[e + f*x]^2])/(2*a*d^3*f*(3 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
```

```
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3633

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-a)*((c + d*Tan[e + f*x])^(n + 1)/(2*f*(b*
c - a*d)*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*
Tan[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &
& NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d\cot(e+fx))^n}{a+ia\tan(e+fx)}\,dx &= \frac{\int \frac{(d\cot(e+fx))^{1+n}}{ia+a\cot(e+fx)}\,dx}{d}\\
&= -\frac{(d\cot(e+fx))^{2+n}}{2d^2f(ia+a\cot(e+fx))} - \frac{\int (d\cot(e+fx))^{1+n}(-iadn+ad(1+n)\cot(e+f}{2a^2d^2}\\
&= -\frac{(d\cot(e+fx))^{2+n}}{2d^2f(ia+a\cot(e+fx))} + \frac{(in)\int (d\cot(e+fx))^{1+n}\,dx}{2ad} - \frac{(1+n)\int (d\cot(e+}{2ad^2}\\
&= -\frac{(d\cot(e+fx))^{2+n}}{2d^2f(ia+a\cot(e+fx))} - \frac{(in)\text{Subst}\left(\int \frac{x^{1+n}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{2af} + \frac{(1+n)}{}\\
&= -\frac{(d\cot(e+fx))^{2+n}}{2d^2f(ia+a\cot(e+fx))} - \frac{in(d\cot(e+fx))^{2+n}\,{}_2F_1\left(1,\frac{2+n}{2};\frac{4+n}{2};-\cot^2(e+f\right.}{2ad^2f(2+n)}
\end{aligned}$$

Mathematica [F]
time = 2.80, size = 0, normalized size = 0.00

$$\int \frac{(d\cot(e+fx))^n}{a+ia\tan(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Cot[e + f*x])^n/(a + I*a*Tan[e + f*x]),x]
```

[Out] Integrate[(d*Cot[e + f*x])^n/(a + I*a*Tan[e + f*x]), x]

Maple [F]
time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{(d\cot (fx+e))^{n}}{a+ia\tan (fx+e)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*cot(f*x+e))^n/(a+I*a*tan(f*x+e)),x)

[Out] int((d*cot(f*x+e))^n/(a+I*a*tan(f*x+e)),x)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*cot(f*x+e))^n/(a+I*a*tan(f*x+e)),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*cot(f*x+e))^n/(a+I*a*tan(f*x+e)),x, algorithm="fricas")

[Out] integral(1/2*((I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) - 1))^n*
(e^(2*I*f*x + 2*I*e) + 1)*e^(-2*I*f*x - 2*I*e)/a, x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int \frac{(d\cot (e+fx))^{n}}{\tan (e+fx)-i}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*cot(f*x+e))**n/(a+I*a*tan(f*x+e)),x)

[Out] -I*Integral((d*cot(e + f*x))**n/(tan(e + f*x) - I), x)/a

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n/(a+I*a*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((d*cot(f*x + e))^n/(I*a*tan(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\cot(e+f\,x))^n}{a+a\tan\left(e+f\,x\right)\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(e + f*x))^n/(a + a*tan(e + f*x)*1i),x)
[Out] int((d*cot(e + f*x))^n/(a + a*tan(e + f*x)*1i), x)
```

3.792 $\int \frac{(d\cot(e+fx))^n}{(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=202

$$-\frac{in(d\cot(e+fx))^{3+n}}{4a^2d^3f(i+\cot(e+fx))}-\frac{(d\cot(e+fx))^{3+n}}{4d^3f(ia+a\cot(e+fx))^2}+\frac{(1+n)^2(d\cot(e+fx))^{3+n}\,{}_2F_1\left(1,\frac{3+n}{2};\frac{5+n}{2};-\cot^2(\right.}{4a^2d^3f(3+n)}$$

```
[Out] -1/4*I*n*(d*cot(f*x+e))^(3+n)/a^2/d^3/f/(I+cot(f*x+e))-1/4*(d*cot(f*x+e))^(
3+n)/d^3/f/(I*a+a*cot(f*x+e))^2+1/4*(1+n)^2*(d*cot(f*x+e))^(3+n)*hypergeom(
[1, 3/2+1/2*n],[5/2+1/2*n],-cot(f*x+e)^2)/a^2/d^3/f/(3+n)+1/4*I*n*(2+n)*(d*
cot(f*x+e))^(4+n)*hypergeom([1, 2+1/2*n],[3+1/2*n],-cot(f*x+e)^2)/a^2/d^4/f
/(4+n)
```

Rubi [A]

time = 0.35, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3754, 3640, 3677, 3619, 3557, 371}

$$\frac{in(n+2)(d\cot(e+fx))^{n+4}\,{}_2F_1\left(1,\frac{n+4}{2};\frac{n+6}{2};-\cot^2(e+fx)\right)}{4a^2d^4f(n+4)}+\frac{(n+1)^2(d\cot(e+fx))^{n+3}\,{}_2F_1\left(1,\frac{n+3}{2};\frac{n+5}{2};-\cot^2(e+fx)\right)}{4a^2d^3f(n+3)}-\frac{in(d\cot(e+fx))^{n+3}}{4a^2d^3f(\cot(e+fx)+i)}-\frac{(d\cot(e+fx))^{n+3}}{4d^3f(a\cot(e+fx)+ia)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^n/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] ((-1/4*I)*n*(d*Cot[e + f*x])^(3 + n))/(a^2*d^3*f*(I + Cot[e + f*x])) - (d*C
ot[e + f*x])^(3 + n)/(4*d^3*f*(I*a + a*Cot[e + f*x])^2) + ((1 + n)^2*(d*Cot
[e + f*x])^(3 + n)*Hypergeometric2F1[1, (3 + n)/2, (5 + n)/2, -Cot[e + f*x]
^2])/(4*a^2*d^3*f*(3 + n)) + ((I/4)*n*(2 + n)*(d*Cot[e + f*x])^(4 + n)*Hype
rgeometric2F1[1, (4 + n)/2, (6 + n)/2, -Cot[e + f*x]^2])/(a^2*d^4*f*(4 + n)
)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{(d\cot(e+fx))^n}{(a+ia\tan(e+fx))^2}\,dx = \frac{\int \frac{(d\cot(e+fx))^{2+n}}{(ia+a\cot(e+fx))^2}\,dx}{d^2}$$
$$= -\frac{(d\cot(e+fx))^{3+n}}{4d^3f(ia+a\cot(e+fx))^2} + \frac{\int \frac{(d\cot(e+fx))^{2+n}(-iad(1-n)-ad(1+n)\cot(e+fx))}{ia+a\cot(e+fx)}\,dx}{4a^2d^3}$$
$$= -\frac{in(d\cot(e+fx))^{3+n}}{4a^2d^3f(i+\cot(e+fx))} - \frac{(d\cot(e+fx))^{3+n}}{4d^3f(ia+a\cot(e+fx))^2} + \frac{\int(d\cot(e+fx))^2}{}$$
$$= -\frac{in(d\cot(e+fx))^{3+n}}{4a^2d^3f(i+\cot(e+fx))} - \frac{(d\cot(e+fx))^{3+n}}{4d^3f(ia+a\cot(e+fx))^2} - \frac{(1+n)^2\int(d\cot(e}{4a^2d}$$
$$= -\frac{in(d\cot(e+fx))^{3+n}}{4a^2d^3f(i+\cot(e+fx))} - \frac{(d\cot(e+fx))^{3+n}}{4d^3f(ia+a\cot(e+fx))^2} + \frac{(1+n)^2\mathrm{Subst}\Big(\int}{}$$
$$= -\frac{in(d\cot(e+fx))^{3+n}}{4a^2d^3f(i+\cot(e+fx))} - \frac{(d\cot(e+fx))^{3+n}}{4d^3f(ia+a\cot(e+fx))^2} + \frac{(1+n)^2(d\cot(e+}{}$$

Mathematica [F]
time = 16.14, size = 0, normalized size = 0.00

$$\int \frac{(d\cot(e+fx))^n}{(a+ia\tan(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Cot[e + f*x])^n/(a + I*a*Tan[e + f*x])^2,x]
[Out] Integrate[(d*Cot[e + f*x])^n/(a + I*a*Tan[e + f*x])^2, x]
```

Maple [F]
time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{(d\cot(fx+e))^n}{(a+ia\tan(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^n/(a+I*a*tan(f*x+e))^2,x)
[Out] int((d*cot(f*x+e))^n/(a+I*a*tan(f*x+e))^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/4*((I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) - 1))^n*
(e^(4*I*f*x + 4*I*e) + 2*e^(2*I*f*x + 2*I*e) + 1)*e^(-4*I*f*x - 4*I*e)/a^2,
 x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{(d\cot(e+fx))^n}{\tan^2(e+fx)-2i\tan(e+fx)-1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**n/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -Integral((d*cot(e + f*x))**n/(tan(e + f*x)**2 - 2*I*tan(e + f*x) - 1), x)/
a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*cot(f*x + e))^n/(I*a*tan(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\cot(e+f\,x))^n}{(a+a\tan(e+f\,x)\,1\mathrm{i})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(e + f*x))^n/(a + a*tan(e + f*x)*1i)^2,x)
```

```
[Out] int((d*cot(e + f*x))^n/(a + a*tan(e + f*x)*1i)^2, x)
```

3.793 $\int (d\cot(e+fx))^n(a+ia\tan(e+fx))^m\,dx$

Optimal. Leaf size=95

$$\frac{F_1(1-n;1-m,1;2-n;-i\tan(e+fx),i\tan(e+fx))(d\cot(e+fx))^n(1+i\tan(e+fx))^{-m}\tan(e+fx)}{f(1-n)}$$

```
[Out] AppellF1(1-n,1-m,1,2-n,-I*tan(f*x+e),I*tan(f*x+e))*(d*cot(f*x+e))^n*tan(f*x
+e)*(a+I*a*tan(f*x+e))^m/f/(1-n)/((1+I*tan(f*x+e))^m)
```

Rubi [A]
time = 0.12, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4326, 3645, 140, 138}

$$\frac{\tan(e+fx)(1+i\tan(e+fx))^{-m}(a+ia\tan(e+fx))^m(d\cot(e+fx))^nF_1(1-n;1-m,1;2-n;-i\tan(e+fx),i\tan(e+fx))}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^n*(a + I*a*Tan[e + f*x])^m,x]
```

```
[Out] (AppellF1[1 - n, 1 - m, 1, 2 - n, (-I)*Tan[e + f*x], I*Tan[e + f*x]]*(d*Cot
[e + f*x])^n*Tan[e + f*x]*(a + I*a*Tan[e + f*x])^m)/(f*(1 - n)*(1 + I*Tan[e
 + f*x])^m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int (d\cot(e+fx))^n(a+ia\tan(e+fx))^m\,dx = ((d\cot(e+fx))^n(d\tan(e+fx))^n)\int (d\tan(e+fx))^{-n}(a+ic$$

$$= \frac{(ia^2(d\cot(e+fx))^n(d\tan(e+fx))^n)\,\mathrm{Subst}\left(\int \frac{\left(-\frac{idx}{a}\right)^{-n}(a+x)^{-}}{-a^2+ax}\right.}{f}$$

$$= \frac{(ia(d\cot(e+fx))^n(1+i\tan(e+fx))^{-m}(d\tan(e+fx))^n(a+}{}$$

$$= \frac{F_1(1-n;1-m,1;2-n;-i\tan(e+fx),i\tan(e+fx))(d\cot}{f}$$

Mathematica [F]
time = 5.62, size = 0, normalized size = 0.00

$$\int (d\cot(e+fx))^n(a+ia\tan(e+fx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Cot[e + f*x])^n*(a + I*a*Tan[e + f*x])^m,x]
[Out] Integrate[(d*Cot[e + f*x])^n*(a + I*a*Tan[e + f*x])^m, x]
```

Maple [F]
time = 0.68, size = 0, normalized size = 0.00

$$\int \left(d\cot\left(fx+e\right)\right)^{n}\left(a+ia\tan\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^n*(a+I*a*tan(f*x+e))^m,x)
[Out] int((d*cot(f*x+e))^n*(a+I*a*tan(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*(a+I*a*tan(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((d*cot(f*x + e))^n*(I*a*tan(f*x + e) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*(a+I*a*tan(f*x+e))^m,x, algorithm="fricas")
[Out] integral((2*a*e^(2*I*f*x + 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1))^m*((I*d*e^(2*I
*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) - 1))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cot(e+fx))^n\,(ia(\tan(e+fx)-i))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**n*(a+I*a*tan(f*x+e))**m,x)
[Out] Integral((d*cot(e + f*x))**n*(I*a*(tan(e + f*x) - I))**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*(a+I*a*tan(f*x+e))^m,x, algorithm="giac")
[Out] integrate((d*cot(f*x + e))^n*(I*a*tan(f*x + e) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\cot(e+f\,x))^n\,(a+a\tan(e+f\,x)\;1\mathrm{i})^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(e + f*x))^n*(a + a*tan(e + f*x)*1i)^m,x)
[Out] int((d*cot(e + f*x))^n*(a + a*tan(e + f*x)*1i)^m, x)
```

3.794 $\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=79

$$-\frac{2F_1\left(-\frac{1}{2};1-n,1;\frac{1}{2};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{\cot(c+dx)}\,(1+i\tan(c+dx))^{-n}(a+ia\tan(c+dx))^n}{d}$$

```
[Out] -2*AppellF1(-1/2,1-n,1,1/2,-I*tan(d*x+c),I*tan(d*x+c))*cot(d*x+c)^(1/2)*(a+
I*a*tan(d*x+c))^n/d/((1+I*tan(d*x+c))^n)
```

Rubi [A]
time = 0.12, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {4326, 3645, 129, 525, 524}

$$-\frac{2\sqrt{\cot(c+dx)}\,(1+i\tan(c+dx))^{-n}(a+ia\tan(c+dx))^n F_1\left(-\frac{1}{2};1-n,1;\frac{1}{2};-i\tan(c+dx),i\tan(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (-2*AppellF1[-1/2, 1 - n, 1, 1/2, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*Sqrt[C
ot[c + d*x]]*(a + I*a*Tan[c + d*x])^n)/(d*(1 + I*Tan[c + d*x])^n)
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{(a+ia\tan(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{\left(ia^2\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{(a+x)^{-1+n}}{\left(-\frac{ix}{a}\right)^{3/2}(-a^2+ax)}\,dx\right.}{d}$$

$$= -\frac{\left(2a^3\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{(a+iax^2)^{-1+n}}{x^2(-a^2+ia^2x^2)}\,dx,\right.}{d}$$

$$= -\frac{\left(2a^2\sqrt{\cot(c+dx)}\,(1+i\tan(c+dx))^{-n}\sqrt{\tan(c+dx)}\,(a+i\right.}{d}$$

$$= -\frac{2F_1\left(-\frac{1}{2};1-n,1;\frac{1}{2};-i\tan(c+dx),i\tan(c+dx)\right)\sqrt{\cot(c+d}}{d}$$

Mathematica [F]
time = 11.32, size = 0, normalized size = 0.00

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^n,x]
[Out] Integrate[Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^n, x]
```

Maple [F]
time = 0.75, size = 0, normalized size = 0.00

$$\int \left(\cot^{\frac{3}{2}}(dx+c)\right)(a+ia\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*cot(d*x + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*sqrt((I*e^(2
*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*(I*e^(2*I*d*x + 2*I*c) + I)
/(e^(2*I*d*x + 2*I*c) - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan (c + dx) - i))^n \cot^{\frac{3}{2}} (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(3/2)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**n*cot(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*cot(d*x + c)^(3/2), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^{3/2}\,\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^n,x)
[Out] int(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^n, x)
```

3.795 $\int \sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=79

$$\frac{2F_1\left(\frac{1}{2};1-n,1;\frac{3}{2};-i\tan(c+dx),i\tan(c+dx)\right)(1+i\tan(c+dx))^{-n}(a+ia\tan(c+dx))^n}{d\sqrt{\cot(c+dx)}}$$

```
[Out] 2*AppellF1(1/2,1-n,1,3/2,-I*tan(d*x+c),I*tan(d*x+c))*(a+I*a*tan(d*x+c))^n/d
/cot(d*x+c)^(1/2)/((1+I*tan(d*x+c))^n)
```

Rubi [A]
time = 0.11, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {4326, 3645, 129, 441, 440}

$$\frac{2(1+i\tan(c+dx))^{-n}(a+ia\tan(c+dx))^n F_1\left(\frac{1}{2};1-n,1;\frac{3}{2};-i\tan(c+dx),i\tan(c+dx)\right)}{d\sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (2*AppellF1[1/2, 1 - n, 1, 3/2, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*(a + I*a
*Tan[c + d*x])^n)/(d*Sqrt[Cot[c + d*x]]*(1 + I*Tan[c + d*x])^n)
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^n\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$$

$$= \frac{\left(ia^2\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{(a+x)^{-1+n}}{\sqrt{-\frac{ix}{a}}\,(-a^2+ax)}$$

$$d$$

$$= -\frac{\left(2a^3\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{(a+iax^2)^{-1+n}}{-a^2+ia^2x^2}\,dx\right.}{d}$$

$$= -\frac{\left(2a^2\sqrt{\cot(c+dx)}\,(1+i\tan(c+dx))^{-n}\sqrt{\tan(c+dx)}\,(a+\right.}{}$$

$$= \frac{2F_1\left(\tfrac{1}{2};1-n,1;\tfrac{3}{2};-i\tan(c+dx),i\tan(c+dx)\right)(1+i\tan(c+}{d\sqrt{\cot(c+dx)}}$$

Mathematica [F]
time = 5.64, size = 0, normalized size = 0.00

$$\int \sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^n,x]
[Out] Integrate[Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^n, x]
```

Maple [F]
time = 0.67, size = 0, normalized size = 0.00

$$\int \left(\sqrt{\cot}\,(dx+c)\right)(a+ia\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n,x)
[Out] int(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^n*sqrt(cot(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
[Out] integral((2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*sqrt((I*e^(2
*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c+dx)-i))^n\sqrt{\cot(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(1/2)*(a+I*a*tan(d*x+c))**n,x)
[Out] Integral((I*a*(tan(c + d*x) - I))**n*sqrt(cot(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^n*sqrt(cot(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cot\left(c+d\,x\right)}\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^n,x)
[Out] int(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^n, x)
```

3.796 $\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{\cot(c+dx)}}\,dx$

Optimal. Leaf size=81

$$\frac{2F_1\left(\frac{3}{2};1-n,1;\frac{5}{2};-i\tan(c+dx),i\tan(c+dx)\right)(1+i\tan(c+dx))^{-n}(a+ia\tan(c+dx))^n}{3d\cot^{\frac{3}{2}}(c+dx)}$$

```
[Out] 2/3*AppellF1(3/2,1-n,1,5/2,-I*tan(d*x+c),I*tan(d*x+c))*(a+I*a*tan(d*x+c))^n
/d/cot(d*x+c)^(3/2)/((1+I*tan(d*x+c))^n)
```

Rubi [A]
time = 0.12, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {4326, 3645, 129, 525, 524}

$$\frac{2(1+i\tan(c+dx))^{-n}(a+ia\tan(c+dx))^n F_1\left(\frac{3}{2};1-n,1;\frac{5}{2};-i\tan(c+dx),i\tan(c+dx)\right)}{3d\cot^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^n/Sqrt[Cot[c + d*x]],x]
```

```
[Out] (2*AppellF1[3/2, 1 - n, 1, 5/2, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*(a + I*a
*Tan[c + d*x])^n)/(3*d*Cot[c + d*x]^(3/2)*(1 + I*Tan[c + d*x])^n)
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{\cot(c+dx)}}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))^n\,dx$$

$$= \frac{\left(ia^2\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\mathrm{Subst}\left(\int \frac{\sqrt{-\frac{ix}{a}}\,(a+x)^{-1+n}}{-a^2+ax}\,dx, x, ia\tan(\right.}{d}$$

$$= -\frac{\left(2a^3\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\mathrm{Subst}\left(\int \frac{x^2(a+iax^2)^{-1+n}}{-a^2+ia^2x^2}\,dx, x, \sqrt{\tan(c\right.}{d}$$

$$= -\frac{\left(2a^2\sqrt{\cot(c+dx)}\,(1+i\tan(c+dx))^{-n}\sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx\right.}{d}$$

$$= \frac{2F_1\left(\frac{3}{2};1-n,1;\frac{5}{2};-i\tan(c+dx),i\tan(c+dx)\right)(1+i\tan(c+dx))^{-n}(a+ia}{3d\cot^{\frac{3}{2}}(c+dx)}$$

Mathematica [F]
time = 6.54, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{\cot(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^n/Sqrt[Cot[c + d*x]],x]
[Out] Integrate[(a + I*a*Tan[c + d*x])^n/Sqrt[Cot[c + d*x]], x]
```

Maple [F]
time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{\left(a+ia\tan \left(dx+c\right)\right)^{n}}{\sqrt{\cot \left(dx+c\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^n/cot(d*x+c)^(1/2),x)
[Out] int((a+I*a*tan(d*x+c))^n/cot(d*x+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/cot(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^n/sqrt(cot(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/cot(d*x+c)^(1/2),x, algorithm="fricas")
[Out] integral((2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*sqrt((I*e^(2
*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*(-I*e^(2*I*d*x + 2*I*c) + I
)/(e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ia \left(\tan \left(c+dx\right)-i\right)\right)^{n}}{\sqrt{\cot \left(c+dx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**n/cot(d*x+c)**(1/2),x)
[Out] Integral((I*a*(tan(c + d*x) - I))**n/sqrt(cot(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/cot(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^n/sqrt(cot(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\tan(c + d\,x)\ 1\mathrm{i})^n}{\sqrt{\cot\left(c + d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^n/cot(c + d*x)^(1/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^n/cot(c + d*x)^(1/2), x)
```

3.797 $\int \frac{(a+ia\tan(c+dx))^n}{\cot^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=81

$$\frac{2F_1\left(\frac{5}{2};1-n,1;\frac{7}{2};-i\tan(c+dx),i\tan(c+dx)\right)(1+i\tan(c+dx))^{-n}(a+ia\tan(c+dx))^n}{5d\cot^{\frac{5}{2}}(c+dx)}$$

```
[Out] 2/5*AppellF1(5/2,1-n,1,7/2,-I*tan(d*x+c),I*tan(d*x+c))*(a+I*a*tan(d*x+c))^n
/d/cot(d*x+c)^(5/2)/((1+I*tan(d*x+c))^n)
```

Rubi [A]
time = 0.12, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {4326, 3645, 129, 525, 524}

$$\frac{2(1+i\tan(c+dx))^{-n}(a+ia\tan(c+dx))^nF_1\left(\frac{5}{2};1-n,1;\frac{7}{2};-i\tan(c+dx),i\tan(c+dx)\right)}{5d\cot^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^n/Cot[c + d*x]^(3/2),x]
```

```
[Out] (2*AppellF1[5/2, 1 - n, 1, 7/2, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*(a + I*a
*Tan[c + d*x])^n)/(5*d*Cot[c + d*x]^(5/2)*(1 + I*Tan[c + d*x])^n)
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^n}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n\,dx$$

$$= \frac{\left(ia^2\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\mathrm{Subst}\left(\int \frac{\left(-\frac{ix}{a}\right)^{3/2}(a+x)^{-1+n}}{-a^2+ax}\,dx, x, ia\tan(c}{d}$$

$$= -\frac{\left(2a^3\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\mathrm{Subst}\left(\int \frac{x^4\left(a+iax^2\right)^{-1+n}}{-a^2+ia^2x^2}\,dx, x, \sqrt{\tan(c}}{d}$$

$$= -\frac{\left(2a^2\sqrt{\cot(c+dx)}\,(1+i\tan(c+dx))^{-n}\sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx}{d}$$

$$= \frac{2F_1\left(\frac{5}{2};1-n,1;\frac{7}{2};-i\tan(c+dx),i\tan(c+dx)\right)(1+i\tan(c+dx))^{-n}(a+ia}{5d\cot^{\frac{5}{2}}(c+dx)}$$

Mathematica [F]
time = 5.22, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(c+dx))^n}{\cot^{\frac{3}{2}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^n/Cot[c + d*x]^(3/2),x]
```

```
[Out] Integrate[(a + I*a*Tan[c + d*x])^n/Cot[c + d*x]^(3/2), x]
```

Maple [F]
time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{(a + ia\tan(dx + c))^n}{\cot(dx + c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^n/cot(d*x+c)^(3/2),x)
[Out] int((a+I*a*tan(d*x+c))^n/cot(d*x+c)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/cot(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^n/cot(d*x + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/cot(d*x+c)^(3/2),x, algorithm="fricas")
[Out] integral(-(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*sqrt((I*e^(
2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*(e^(4*I*d*x + 4*I*c) - 2*e
^(2*I*d*x + 2*I*c) + 1)/(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(c + dx) - i))^n}{\cot^{\frac{3}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**n/cot(d*x+c)**(3/2),x)
[Out] Integral((I*a*(tan(c + d*x) - I))**n/cot(c + d*x)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/cot(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n/cot(d*x + c)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^n}{\cot\left(c+d\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^n/cot(c + d*x)^(3/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^n/cot(c + d*x)^(3/2), x)
```

3.798 $\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=202

$$\frac{(a-b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a-b)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}+\frac{2a\sqrt{\cot(c+dx)}}{d}-\frac{2b}{}$$

[Out] -2/3*b*cot(d*x+c)^(3/2)/d-2/5*a*cot(d*x+c)^(5/2)/d-1/2*(a-b)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)-1/2*(a-b)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)+1/4*(a+b)*ln(1+cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)-1/4*(a+b)*ln(1+cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)+2*a*cot(d*x+c)^(1/2)/d

Rubi [A]
time = 0.15, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3754, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a-b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a-b)\text{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d}+\frac{(a+b)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{(a+b)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{2a\cot^{\frac{5}{2}}(c+dx)}{5d}+\frac{2a\sqrt{\cot(c+dx)}}{d}-\frac{2b\cot^{\frac{3}{2}}(c+dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^(7/2)*(a + b*Tan[c + d*x]),x]

[Out] ((a - b)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*d) - ((a - b)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*d) + (2*a*Sqrt[Cot[c + d*x]])/d - (2*b*Cot[c + d*x]^(3/2))/(3*d) - (2*a*Cot[c + d*x]^(5/2))/(5*d) + ((a + b)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*d) - ((a + b)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*d)

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))\,dx = \int \cot^{\frac{5}{2}}(c+dx)(b+a\cot(c+dx))\,dx$$
$$= -\frac{2a\cot^{\frac{5}{2}}(c+dx)}{5d} + \int \cot^{\frac{3}{2}}(c+dx)(-a+b\cot(c+dx))\,dx$$
$$= -\frac{2b\cot^{\frac{3}{2}}(c+dx)}{3d} - \frac{2a\cot^{\frac{5}{2}}(c+dx)}{5d} + \int \sqrt{\cot(c+dx)}\,(-b-a\,\mathrm{co}$$
$$= \frac{2a\sqrt{\cot(c+dx)}}{d} - \frac{2b\cot^{\frac{3}{2}}(c+dx)}{3d} - \frac{2a\cot^{\frac{5}{2}}(c+dx)}{5d} + \int \frac{a-b\,\mathrm{c}}{\sqrt{\mathrm{cot}}}$$
$$= \frac{2a\sqrt{\cot(c+dx)}}{d} - \frac{2b\cot^{\frac{3}{2}}(c+dx)}{3d} - \frac{2a\cot^{\frac{5}{2}}(c+dx)}{5d} + \frac{2\mathrm{Subst}\Big(\int}{}$$
$$= \frac{2a\sqrt{\cot(c+dx)}}{d} - \frac{2b\cot^{\frac{3}{2}}(c+dx)}{3d} - \frac{2a\cot^{\frac{5}{2}}(c+dx)}{5d} - \frac{(a-b)\mathrm{Su}}{}$$
$$= \frac{2a\sqrt{\cot(c+dx)}}{d} - \frac{2b\cot^{\frac{3}{2}}(c+dx)}{3d} - \frac{2a\cot^{\frac{5}{2}}(c+dx)}{5d} - \frac{(a-b)\mathrm{Su}}{}$$
$$= \frac{2a\sqrt{\cot(c+dx)}}{d} - \frac{2b\cot^{\frac{3}{2}}(c+dx)}{3d} - \frac{2a\cot^{\frac{5}{2}}(c+dx)}{5d} + \frac{(a+b)\,\mathrm{lo}}{}$$
$$= \frac{(a-b)\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d} - \frac{(a-b)\tan^{-1}\left(1+\sqrt{2}\right.}{\sqrt{2}\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.24, size = 66, normalized size = 0.33

$$-\frac{2\cot^{\frac{3}{2}}(c+dx)\left(3a\cot(c+dx)\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\tan^2(c+dx)\right)+5b\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\tan^2(c+dx)\right)\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(7/2)*(a + b*Tan[c + d*x]),x]
```

```
[Out] (-2*Cot[c + d*x]^(3/2)*(3*a*Cot[c + d*x]*Hypergeometric2F1[-5/4, 1, -1/4, -
Tan[c + d*x]^2] + 5*b*Hypergeometric2F1[-3/4, 1, 1/4, -Tan[c + d*x]^2]))/(1
5*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 19.13, size = 4321, normalized size = 21.39

method	result	size
default	Expression too large to display	4321

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/30/d*(30*cos(d*x+c)*2^(1/2)*a-36*cos(d*x+c)^3*2^(1/2)*a-10*cos(d*x+c)^2*s
in(d*x+c)*2^(1/2)*b-15*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*Ellipt
icPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(
(cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*a-15*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*
x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((cos(d*x+c)-1+
sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*b-15*(-(co
s(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x
+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-si
n(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a-15*(-(cos(d*x+c)-1-sin
(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(
d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b+30*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d
x+c))^(1/2)((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1
/2*2^(1/2))*b+15*I*cos(d*x+c)^3*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/
2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2
^(1/2))*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*a-15*I*cos(d*x+c)^3*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1
/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*
2^(1/2))*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*b-15*I*cos(d*x+c)^3*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(
1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*
I,1/2*2^(1/2))*a+15*I*cos(d*x+c)^3*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^
(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2
*I,1/2*2^(1/2))*b+15*I*cos(d*x+c)^2*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))
^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1
/2*2^(1/2))*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/s
in(d*x+c))^(1/2)*a-15*I*cos(d*x+c)^2*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)
)^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,
1/2*2^(1/2))*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*b-15*I*cos(d*x+c)^2*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c
))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-
1/2*I,1/2*2^(1/2))*a+15*I*cos(d*x+c)^2*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+

```
c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2
-1/2*I,1/2*2^(1/2))*b-15*I*cos(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c
))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I
,1/2*2^(1/2))*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*a+15*I*cos(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)
)^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,
1/2*2^(1/2))*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*b+15*I*cos(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))
^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/
2*I,1/2*2^(1/2))*a-15*I*cos(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^
(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2
*I,1/2*2^(1/2))*b+15*cos(d*x+c)^3*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(
1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2
*2^(1/2))*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)*b+15*cos(d*x+c)^3*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1
/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I
,1/2*2^(1/2))*a+15*cos(d*x+c)^3*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/
2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,
1/2*2^(1/2))*b-30*cos(d*x+c)^3*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2
)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))
^(1/2)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))
*b+15*cos(d*x+c)^2*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi
((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((cos
(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
a+15*cos(d*x+c)^2*(-(cos(d*x+c)-1-sin(d*x+c))/s...
```

Maxima [A]
time = 0.52, size = 162, normalized size = 0.80

$$-\frac{30\sqrt{2}\,(a-b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+30\sqrt{2}\,(a-b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+15\sqrt{2}\,(a+b)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-15\sqrt{2}\,(a+b)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\frac{120\,a}{\sqrt{\tan(dx+c)}}+\frac{40\,b}{\tan(dx+c)^{\frac{3}{2}}}+\frac{24\,a}{\tan(dx+c)^{\frac{5}{2}}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(30*sqrt(2)*(a - b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)
))) + 30*sqrt(2)*(a - b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)
))) + 15*sqrt(2)*(a + b)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) +
1) - 15*sqrt(2)*(a + b)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) +
1) - 120*a/sqrt(tan(d*x + c)) + 40*b/tan(d*x + c)^(3/2) + 24*a/tan(d*x + c)
^(5/2))/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(7/2)*(a+b*tan(d*x+c)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c)),x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)*cot(d*x + c)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c+d\,x)^{7/2}\,(a+b\tan(c+d\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(7/2)*(a + b*tan(c + d*x)),x)
[Out] int(cot(c + d*x)^(7/2)*(a + b*tan(c + d*x)), x)
```

3.799 $\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=184

$$-\frac{(a+b)\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a+b)\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}-\frac{2b\sqrt{\cot(c+dx)}}{d}-\frac{2}{}$$

```
[Out] -2/3*a*cot(d*x+c)^(3/2)/d+1/2*(a+b)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/d*2
^(1/2)+1/2*(a+b)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)+1/4*(a-b)*ln(
1+cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)-1/4*(a-b)*ln(1+cot(d*x+c)+
2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)-2*b*cot(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.12, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3754, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{(a+b)\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a+b)\mathrm{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d}+\frac{(a-b)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{(a-b)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{2a\cot^{\frac{3}{2}}(c+dx)}{3d}-\frac{2b\sqrt{\cot(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x]),x]
```

```
[Out] -(((a + b)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*d)) + ((a + b)*
ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*d) - (2*b*Sqrt[Cot[c + d*x
]])/d - (2*a*Cot[c + d*x]^(3/2))/(3*d) + ((a - b)*Log[1 - Sqrt[2]*Sqrt[Cot[
c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*d) - ((a - b)*Log[1 + Sqrt[2]*Sqrt[Co
t[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))\,dx = \int \cot^{\frac{3}{2}}(c+dx)(b+a\cot(c+dx))\,dx$$

$$= -\frac{2a\cot^{\frac{3}{2}}(c+dx)}{3d} + \int \sqrt{\cot(c+dx)}\,(-a+b\cot(c+dx))\,dx$$

$$= -\frac{2b\sqrt{\cot(c+dx)}}{d} - \frac{2a\cot^{\frac{3}{2}}(c+dx)}{3d} + \int \frac{-b-a\cot(c+dx)}{\sqrt{\cot(c+dx)}}\,dx$$

$$= -\frac{2b\sqrt{\cot(c+dx)}}{d} - \frac{2a\cot^{\frac{3}{2}}(c+dx)}{3d} + \frac{2\text{Subst}\left(\int \frac{b+ax^2}{1+x^4}\,dx, x, \sqrt{\text{co}\ldots}\right.}{d}$$

$$= -\frac{2b\sqrt{\cot(c+dx)}}{d} - \frac{2a\cot^{\frac{3}{2}}(c+dx)}{3d} - \frac{(a-b)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x\ldots\right.}{d}$$

$$= -\frac{2b\sqrt{\cot(c+dx)}}{d} - \frac{2a\cot^{\frac{3}{2}}(c+dx)}{3d} + \frac{(a-b)\text{Subst}\left(\int \frac{\sqrt{2}+2x\ldots}{-1-\sqrt{2}\,x-\ldots}\right.}{2\sqrt{2}\ldots}$$

$$= -\frac{2b\sqrt{\cot(c+dx)}}{d} - \frac{2a\cot^{\frac{3}{2}}(c+dx)}{3d} + \frac{(a-b)\log\left(1-\sqrt{2}\,\sqrt{\cot\ldots}\right.}{2\sqrt{2}\ldots}$$

$$= -\frac{(a+b)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d} + \frac{(a+b)\tan^{-1}\left(1+\sqrt{2}\ldots\right.}{\sqrt{2}\,d\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.17, size = 65, normalized size = 0.35

$$-\frac{2\sqrt{\cot(c+dx)}\left(a\cot(c+dx)\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\tan^2(c+dx)\right)+3b\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\tan^2(c+dx)\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x]),x]
```

```
[Out] (-2*Sqrt[Cot[c + d*x]]*(a*Cot[c + d*x]*Hypergeometric2F1[-3/4, 1, 1/4, -Tan
[c + d*x]^2] + 3*b*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[c + d*x]^2]))/(3*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 18.58, size = 2253, normalized size = 12.24

method	result	size
default	Expression too large to display	2253

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)

[Out] -1/6/d*(3*I*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(d*x+c)*b-3*I*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)*a-3*I*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)*b-3*I*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)*sin(d*x+c)*b+3*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)*sin(d*x+c)*a-3*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)*sin(d*x+c)*b+3*I*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)*sin(d*x+c)*a+3*I*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)*sin(d*x+c)*b+3*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)*sin(d*x+c)*a-3*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)*sin(d*x+c)*b-3*I*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)*sin(d*x+c)*a+3*I*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(d*x+c)*a-6*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)*sin(d*x+c)*a+3*(-(cos(d*x+c)

```
-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/
2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c)
)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(d*x+c)*a-3*(-(cos(d*x+c)-1-s
in(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/si
n(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(d*x+c)*b+3*(-(cos(d*x+c)-1-sin(d
*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*
x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)*a-3*(-(cos(d*x+c)-1-sin(d*x+c
))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)*b-6*(-(cos(d*x+c)-1-sin(d*x+c))/s
in(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/
2),1/2*2^(1/2))*sin(d*x+c)*a+2*2^(1/2)*cos(d*x+c)^2*a+6*2^(1/2)*cos(d*x+c)*
sin(d*x+c)*b)*sin(d*x+c)*(cos(d*x+c)/sin(d*x+c))^(5/2)/cos(d*x+c)^3*2^(1/2)
```

Maxima [A]
time = 0.51, size = 151, normalized size = 0.82

$$\frac{6\sqrt{2}(a+b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+6\sqrt{2}(a+b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-3\sqrt{2}(a-b)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+3\sqrt{2}(a-b)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\frac{24b}{\sqrt{\tan(dx+c)}}-\frac{8a}{\tan(dx+c)^{\frac{3}{2}}}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(6*sqrt(2)*(a + b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))
) + 6*sqrt(2)*(a + b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c))))
 - 3*sqrt(2)*(a - b)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) +
 3*sqrt(2)*(a - b)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) -
24*b/sqrt(tan(d*x + c)) - 8*a/tan(d*x + c)^(3/2))/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))\cot^{\frac{5}{2}}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(5/2)*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*cot(c + d*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)*cot(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^{5/2}\,(a+b\tan(c+d\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x)),x)
```

```
[Out] int(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x)), x)
```

3.800 $\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=166

$$-\frac{(a-b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a-b)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}-\frac{2a\sqrt{\cot(c+dx)}}{d}-\frac{(}{}$$

```
[Out] 1/2*(a-b)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)+1/2*(a-b)*arctan(1+
2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)-1/4*(a+b)*ln(1+cot(d*x+c)-2^(1/2)*cot(d
*x+c)^(1/2))/d*2^(1/2)+1/4*(a+b)*ln(1+cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2))/
d*2^(1/2)-2*a*cot(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.10, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3754, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{(a-b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a-b)\text{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d}-\frac{(a+b)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{(a+b)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{2a\sqrt{\cot(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x]),x]
```

```
[Out] -(((a - b)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*d)) + ((a - b)*
ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*d) - (2*a*Sqrt[Cot[c + d*x
]])/d - ((a + b)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqr
t[2]*d) + ((a + b)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*S
qrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$
\begin{aligned}
\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))\,dx &= \int \sqrt{\cot(c+dx)}\,(b+a\cot(c+dx))\,dx \\
&= -\frac{2a\sqrt{\cot(c+dx)}}{d} + \int \frac{-a+b\cot(c+dx)}{\sqrt{\cot(c+dx)}}\,dx \\
&= -\frac{2a\sqrt{\cot(c+dx)}}{d} + \frac{2\text{Subst}\left(\int \frac{a-bx^2}{1+x^4}\,dx, x, \sqrt{\cot(c+dx)}\right)}{d} \\
&= -\frac{2a\sqrt{\cot(c+dx)}}{d} + \frac{(a-b)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\cot(c+dx)}\right)}{d} + \\
&= -\frac{2a\sqrt{\cot(c+dx)}}{d} + \frac{(a-b)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{\cot(c+a}\right.}{2d} \\
&= -\frac{2a\sqrt{\cot(c+dx)}}{d} - \frac{(a+b)\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+\right.}{2\sqrt{2}\,d} \\
&= -\frac{(a-b)\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d} + \frac{(a-b)\tan^{-1}\left(1+\sqrt{2}\right.}{\sqrt{2}\,d}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.32, size = 153, normalized size = 0.92

$$
-\frac{\sqrt{\cot(c+dx)}\left(8a\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\tan^2(c+dx)\right)+\sqrt{2}\,b\left(2\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-2\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)+\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)-\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\right)\sqrt{\tan(c+dx)}\right)}{4d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x]),x]
```

```
[Out] -1/4*(Sqrt[Cot[c + d*x]]*(8*a*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[c + d*x]
^2] + Sqrt[2]*b*(2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 2*ArcTan[1 + Sq
rt[2]*Sqrt[Tan[c + d*x]]] + Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*
x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])*Sqrt[Tan[c + d*x]
]))/d
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 16.99, size = 2125, normalized size = 12.80

method	result	size
default	Expression too large to display	2125

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(I*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(
d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(
cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*b+I*(-(co
s(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x
+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-si
n(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a-I*(-(cos(d*x+c)-1-sin(
d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1
+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d
*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b-I*cos(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+
c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos
(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c
))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a-cos(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/si
n(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/
2),1/2-1/2*I,1/2*2^(1/2))*a-cos(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+
c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2
-1/2*I,1/2*2^(1/2))*b-cos(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1
/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*
2^(1/2))*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*a-cos(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*El
lipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2
))*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*b+2*cos(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(
d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*E
llipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*b+I*cos
(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*
x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(co
s(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a-I*cos(d*x
+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c)
)/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*
x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b+I*cos(d*x+c)*
(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/si
n(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)
-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*b-I*(-(cos(d*x+c)-1
-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/
sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a-(-(cos(d*x+c)-1-sin(d*x+c))/sin(
d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)
```

```
,1/2-1/2*I,1/2*2^(1/2))*a-(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((c
os(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^
(1/2))*b-(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*
x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((cos(d*x+c)-1+
sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*a-(-(cos(d
*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c)
)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((cos(d*x+c)-1+sin(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*b+2*(-(cos(d*x+c)-1-sin(d*x
+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c
))^(1/2),1/2*2^(1/2))*b+2*cos(d*x+c)*2^(1/2)*a)*sin(d*x+c)*(cos(d*x+c)/sin(
d*x+c))^(3/2)/cos(d*x+c)^2*2^(1/2)
```

Maxima [A]
time = 0.51, size = 139, normalized size = 0.84

$$\frac{2\sqrt{2}(a-b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}(a-b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+\sqrt{2}(a+b)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}(a+b)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\frac{8a}{\sqrt{\tan(dx+c)}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*(a - b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c))))
 + 2*sqrt(2)*(a - b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c))))
+ sqrt(2)*(a + b)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sq
rt(2)*(a + b)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - 8*a/s
qrt(tan(d*x + c)))/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))\cot^{\frac{3}{2}}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(3/2)*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*cot(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c)),x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)*cot(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^{3/2}\,(a+b\tan(c+d\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x)),x)
[Out] int(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x)), x)
```

3.801 $\int \sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=150

$$\frac{(a+b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a+b)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a-b)\log\left(1-\sqrt{2}\,\sqrt{\right.}$$

```
[Out] -1/2*(a+b)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)-1/2*(a+b)*arctan(1
+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)-1/4*(a-b)*ln(1+cot(d*x+c)-2^(1/2)*cot(
d*x+c)^(1/2))/d*2^(1/2)+1/4*(a-b)*ln(1+cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2))
/d*2^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3754, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a+b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a+b)\text{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d}-\frac{(a-b)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{(a-b)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x]),x]
```

```
[Out] ((a + b)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*d) - ((a + b)*Arc
Tan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*d) - ((a - b)*Log[1 - Sqrt[2]
*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*d) + ((a - b)*Log[1 + Sqrt[
2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))\,dx &= \int \frac{b+a\cot(c+dx)}{\sqrt{\cot(c+dx)}}\,dx \\
&= \frac{2\text{Subst}\left(\int \frac{-b-ax^2}{1+x^4}\,dx, x, \sqrt{\cot(c+dx)}\right)}{d} \\
&= \frac{(a-b)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\cot(c+dx)}\right)}{d} - \frac{(a+b)\text{Subst}\left(\int \frac{1+}{1+}\right.}{} \\
&= -\frac{(a-b)\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \sqrt{\cot(c+dx)}\right)}{2\sqrt{2}d} - \frac{(a-b)\text{S}}{} \\
&= -\frac{(a-b)\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)}{2\sqrt{2}d} + \frac{(a-b)\,\text{lo}}{} \\
&= \frac{(a+b)\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} - \frac{(a+b)\tan^{-1}\left(1+\sqrt{2}\right.}{\sqrt{2}d}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.24, size = 164, normalized size = 1.09

$$
\frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(3\sqrt{2}a\left(-2\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)+2\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)-\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)+\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\right)+8b\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)\right)}{12d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x]),x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(3*Sqrt[2]*a*(-2*ArcTan[1 - Sqrt[2]*
Sqrt[Tan[c + d*x]]] + 2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] - Log[1 - Sq
rt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x
]] + Tan[c + d*x]]) + 8*b*Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d*x]^2]*T
an[c + d*x]^(3/2)))/(12*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 14.32, size = 492, normalized size = 3.28

method	result
default	$-\frac{\sqrt{\frac{\cos(dx+c)}{\sin(dx+c)}}\,(-1+\cos(dx+c))\sqrt{-\frac{\cos(dx+c)-1-\sin(dx+c)}{\sin(dx+c)}}\sqrt{\frac{\cos(dx+c)-1+\sin(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\left(i\,\text{EllipticPi}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(cos(d*x+c)/sin(d*x+c))^(1/2)*(-1+cos(d*x+c))*(-(cos(d*x+c)-1-sin(d*
x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*(I*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(
d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a+I*EllipticPi((-(cos(d*x+c)-1-sin(d*x
+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b-I*EllipticPi((-(cos(d*x+c)-
1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a-I*EllipticPi((-(co
s(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*b-2*a*Ellip
ticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+EllipticPi(
(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a-Elli
pticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))
*b+EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2
^(1/2))*a-EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*
I,1/2*2^(1/2))*b)/sin(d*x+c)^2/cos(d*x+c)*(cos(d*x+c)+1)^2*2^(1/2)
```

Maxima [A]
time = 0.53, size = 128, normalized size = 0.85

$$-\frac{2\sqrt{2}(a+b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}(a+b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-\sqrt{2}(a-b)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}(a-b)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*(a + b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))
) + 2*sqrt(2)*(a + b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c))))
 - sqrt(2)*(a - b)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + s
qrt(2)*(a - b)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))\sqrt{\cot(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(1/2)*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*sqrt(cot(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)*sqrt(cot(d*x + c)), x)
```

Mupad [**B**]
time = 5.45, size = 86, normalized size = 0.57

$$\frac{(-1)^{1/4}\,a\,\mathrm{atanh}\left((-1)^{1/4}\,\sqrt{\cot\left(c+d\,x\right)}\right)}{d}-\frac{(-1)^{1/4}\,a\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{\cot\left(c+d\,x\right)}\right)}{d}+\frac{(-1)^{1/4}\,b\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{\cot\left(c+d\,x\right)}\right)\,1\mathrm{i}}{d}+\frac{(-1)^{1/4}\,b\,\mathrm{atanh}\left((-1)^{1/4}\,\sqrt{\cot\left(c+d\,x\right)}\right)\,1\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x)),x)
```

```
[Out] ((-1)^(1/4)*a*atanh((-1)^(1/4)*cot(c + d*x)^(1/2)))/d - ((-1)^(1/4)*a*atan(
(-1)^(1/4)*cot(c + d*x)^(1/2)))/d + ((-1)^(1/4)*b*atan((-1)^(1/4)*cot(c + d
*x)^(1/2))*1i)/d + ((-1)^(1/4)*b*atanh((-1)^(1/4)*cot(c + d*x)^(1/2))*1i)/d
```

3.802 $\int \frac{a+b\tan(c+dx)}{\sqrt{\cot(c+dx)}}\,dx$

Optimal. Leaf size=166

$$\frac{(a-b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a-b)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}+\frac{2b}{d\sqrt{\cot(c+dx)}}+\frac{(a$$

```
[Out] -1/2*(a-b)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)-1/2*(a-b)*arctan(1
+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)+1/4*(a+b)*ln(1+cot(d*x+c)-2^(1/2)*cot(
d*x+c)^(1/2))/d*2^(1/2)-1/4*(a+b)*ln(1+cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2))
/d*2^(1/2)+2*b/d/cot(d*x+c)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3754, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a-b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a-b)\text{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d}+\frac{(a+b)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{(a+b)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{2b}{d\sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])/Sqrt[Cot[c + d*x]],x]
```

```
[Out] ((a - b)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*d) - ((a - b)*Arc
Tan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*d) + (2*b)/(d*Sqrt[Cot[c + d*
x]]) + ((a + b)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt
[2]*d) - ((a + b)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sq
rt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\tan(c+dx)}{\sqrt{\cot(c+dx)}}\,dx &= \int \frac{b+a\cot(c+dx)}{\cot^{\frac{3}{2}}(c+dx)}\,dx \\
&= \frac{2b}{d\sqrt{\cot(c+dx)}} + \int \frac{a-b\cot(c+dx)}{\sqrt{\cot(c+dx)}}\,dx \\
&= \frac{2b}{d\sqrt{\cot(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{-a+bx^2}{1+x^4}\,dx, x, \sqrt{\cot(c+dx)}\right)}{d} \\
&= \frac{2b}{d\sqrt{\cot(c+dx)}} - \frac{(a-b)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\cot(c+dx)}\right)}{d} - \frac{(a+b)\text{Subst}\left(\right.}{} \\
&= \frac{2b}{d\sqrt{\cot(c+dx)}} - \frac{(a-b)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{\cot(c+dx)}\right)}{2d} - \frac{(a-b)}{} \\
&= \frac{2b}{d\sqrt{\cot(c+dx)}} + \frac{(a+b)\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)}{2\sqrt{2}\,d} - \frac{(a+b}{} \\
&= \frac{(a-b)\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d} - \frac{(a-b)\tan^{-1}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right.}{\sqrt{2}\,d}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.35, size = 194, normalized size = 1.17

$$
\frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(3b\left(2\left(\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\right)+\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)-\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)+8\sqrt{\tan(c+dx)}\right)+8a\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)\right)}{12d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])/Sqrt[Cot[c + d*x]],x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(3*b*(2*(Sqrt[2]*ArcTan[1 - Sqrt[2]*
Sqrt[Tan[c + d*x]]] - Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]) + Sqr
t[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Sqrt[2]*Log[1 + S
qrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + 8*Sqrt[Tan[c + d*x]]) + 8*a*Hyp
ergeometric2F1[3/4, 1, 7/4, -Tan[c + d*x]^2]*Tan[c + d*x]^(3/2)))/(12*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 48.65, size = 1112, normalized size = 6.70

method	result	size

default	Expression too large to display	1112

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*tan(d*x+c))/cot(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))*(I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)*a-I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)*b-I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(d*x+c)*a+I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(d*x+c)*b+((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)*a+((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)*b+((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(d*x+c)*a+((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(d*x+c)*b-2*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*sin(d*x+c)*b+2*cos(d*x+c)*2^(1/2)*b-2*2^(1/2)*b)/sin(d*x+c)^4/(cos(d*x+c)/sin(d*x+c))^(1/2)*2^(1/2)

Maxima [A]
time = 0.51, size = 139, normalized size = 0.84

$$-\frac{2\sqrt{2}(a-b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}(a-b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+\sqrt{2}(a+b)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}(a+b)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-8b\sqrt{\tan(dx+c)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(d*x+c))/cot(d*x+c)^(1/2),x, algorithm="maxima")

[Out] -1/4*(2*sqrt(2)*(a - b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*(a - b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c))))

```
 + sqrt(2)*(a + b)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - s
qrt(2)*(a + b)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - 8*b*
sqrt(tan(d*x + c)))/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/cot(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\tan{\left(c + dx \right)}}{\sqrt{\cot{\left(c + dx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/cot(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))/sqrt(cot(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/cot(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)/sqrt(cot(d*x + c)), x)
```

Mupad [**B**]
time = 5.73, size = 99, normalized size = 0.60

$$\frac{2\,b}{d\,\sqrt{\mathrm{cot}\left(c+d\,x\right)}}+\frac{{\left(-1\right)}^{1/4}\,a\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,\sqrt{\mathrm{cot}\left(c+d\,x\right)}\right)\,1{}\mathrm{i}}{d}+\frac{{\left(-1\right)}^{1/4}\,a\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,\sqrt{\mathrm{cot}\left(c+d\,x\right)}\right)\,1{}\mathrm{i}}{d}+\frac{{\left(-1\right)}^{1/4}\,b\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}}{\sqrt{\mathrm{cot}\left(c+d\,x\right)}}\right)\,1{}\mathrm{i}}{d}+\frac{{\left(-1\right)}^{1/4}\,b\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}}{\sqrt{\mathrm{cot}\left(c+d\,x\right)}}\right)\,1{}\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))/cot(c + d*x)^(1/2),x)
```

```
[Out] (2*b)/(d*cot(c + d*x)^(1/2)) + ((-1)^(1/4)*a*atan((-1)^(1/4)*cot(c + d*x)^(
1/2))*1i)/d + ((-1)^(1/4)*a*atanh((-1)^(1/4)*cot(c + d*x)^(1/2))*1i)/d + ((
-1)^(1/4)*b*atan((-1)^(1/4)/cot(c + d*x)^(1/2))*1i)/d + ((-1)^(1/4)*b*atanh
((-1)^(1/4)/cot(c + d*x)^(1/2))*1i)/d
```

3.803 $\int \frac{a+b\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=184

$$-\frac{(a+b)\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a+b)\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}+\frac{2b}{3d\cot^{\frac{3}{2}}(c+dx)}+ d\sqrt{}$$

```
[Out] 2/3*b/d/cot(d*x+c)^(3/2)+1/2*(a+b)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/d*2^
(1/2)+1/2*(a+b)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)+1/4*(a-b)*ln(1
+cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)-1/4*(a-b)*ln(1+cot(d*x+c)+2
^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)+2*a/d/cot(d*x+c)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3754, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{(a+b)\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a+b)\text{ArcTan}\left(\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d}+\frac{(a-b)\log\left(\cot(c+dx)-\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{(a-b)\log\left(\cot(c+dx)+\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{2a}{d\sqrt{\cot(c+dx)}}+\frac{2b}{3d\cot^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])/Cot[c + d*x]^(3/2),x]
```

```
[Out] -(((a + b)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*d)) + ((a + b)*
ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*d) + (2*b)/(3*d*Cot[c + d*
x]^(3/2)) + (2*a)/(d*Sqrt[Cot[c + d*x]]) + ((a - b)*Log[1 - Sqrt[2]*Sqrt[Co
t[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*d) - ((a - b)*Log[1 + Sqrt[2]*Sqrt[
Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)}\,dx &= \int \frac{b+a\cot(c+dx)}{\cot^{\frac{5}{2}}(c+dx)}\,dx \\
&= \frac{2b}{3d\cot^{\frac{3}{2}}(c+dx)} + \int \frac{a-b\cot(c+dx)}{\cot^{\frac{3}{2}}(c+dx)}\,dx \\
&= \frac{2b}{3d\cot^{\frac{3}{2}}(c+dx)} + \frac{2a}{d\sqrt{\cot(c+dx)}} + \int \frac{-b-a\cot(c+dx)}{\sqrt{\cot(c+dx)}}\,dx \\
&= \frac{2b}{3d\cot^{\frac{3}{2}}(c+dx)} + \frac{2a}{d\sqrt{\cot(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{b+ax^2}{1+x^4}\,dx, x, \sqrt{\cot(c+dx)}\right)}{d} \\
&= \frac{2b}{3d\cot^{\frac{3}{2}}(c+dx)} + \frac{2a}{d\sqrt{\cot(c+dx)}} - \frac{(a-b)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\cot(c+dx)}\right)}{d} \\
&= \frac{2b}{3d\cot^{\frac{3}{2}}(c+dx)} + \frac{2a}{d\sqrt{\cot(c+dx)}} + \frac{(a-b)\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}\,x-x^2}\,dx, x, \sqrt{\cot(c
}{2\sqrt{2}\,d} \\
&= \frac{2b}{3d\cot^{\frac{3}{2}}(c+dx)} + \frac{2a}{d\sqrt{\cot(c+dx)}} + \frac{(a-b)\log\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c
}{2\sqrt{2}\,d} \\
&= -\frac{(a+b)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d} + \frac{(a+b)\tan^{-1}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)
}{\sqrt{2}\,d}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.58, size = 194, normalized size = 1.05

$$\frac{\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\left(3a\left(2\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)-2\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}\right)+\sqrt{2}\,\log\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}+\tan(c+dx)\right)-\sqrt{2}\,\log\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}+\tan(c+dx)\right)+8\sqrt{\tan(c+dx)}\right)-8b\left(-1+{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(c+dx)\right)\right)\tan^{\frac{3}{2}}(c+dx)\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])/Cot[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(3*a*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*S
qrt[Tan[c + d*x]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + Sqr
t[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Sqrt[2]*Log[1 + S
qrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + 8*Sqrt[Tan[c + d*x]]) - 8*b*(-1
 + Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d*x]^2])*Tan[c + d*x]^(3/2)))/(1
2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 49.22, size = 1208, normalized size = 6.57

method	result	size
default	Expression too large to display	1208

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))/cot(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/d*(-1+cos(d*x+c))*(3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))
^(1/2)*sin(d*x+c)*cos(d*x+c)*a+3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(
d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c)*b-3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^
(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c
))/sin(d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c)*a-3*I*EllipticPi(((1-cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+si
n(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c)*b-3*EllipticPi(((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)
-1+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c)*a+3*EllipticPi(((1-c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x
+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d
*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c)*b-3*EllipticPi(
((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((
cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c)*a+3*Ellipt
icPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c)*b+6*E
llipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos
(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((c
os(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c)*a-2*2^(1/2)
*cos(d*x+c)*sin(d*x+c)*b-6*2^(1/2)*cos(d*x+c)^2*a+2*2^(1/2)*sin(d*x+c)*b+6*
cos(d*x+c)*2^(1/2)*a)*(cos(d*x+c)+1)^2/(cos(d*x+c)/sin(d*x+c))^(3/2)/sin(d*
x+c)^5*2^(1/2)
```

Maxima [A]

time = 0.54, size = 152, normalized size = 0.83

$$\frac{6\sqrt{2}(a+b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+6\sqrt{2}(a+b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-3\sqrt{2}(a-b)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+3\sqrt{2}(a-b)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+8\left(b+\frac{3a}{\tan(dx+c)}\right)\tan(dx+c)^{\frac{3}{2}}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/cot(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(6*sqrt(2)*(a + b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))
) + 6*sqrt(2)*(a + b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c))))
 - 3*sqrt(2)*(a - b)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) +
 3*sqrt(2)*(a - b)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) +
8*(b + 3*a/tan(d*x + c))*tan(d*x + c)^(3/2))/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/cot(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\tan\left(c + dx\right)}{\cot^{\frac{3}{2}}\left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/cot(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))/cot(c + d*x)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/cot(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)/cot(d*x + c)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\tan(c + d\,x)}{{\cot\left(c + d\,x\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))/cot(c + d*x)^(3/2),x)
```

```
[Out] int((a + b*tan(c + d*x))/cot(c + d*x)^(3/2), x)
```

3.804 $\int \frac{a+b\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=202

$$-\frac{(a-b)\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a-b)\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}+\frac{2b}{5d\cot^{\frac{5}{2}}(c+dx)}+\frac{}{3}$$

```
[Out] 2/5*b/d/cot(d*x+c)^(5/2)+2/3*a/d/cot(d*x+c)^(3/2)+1/2*(a-b)*arctan(-1+2^(1/
2)*cot(d*x+c)^(1/2))/d*2^(1/2)+1/2*(a-b)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))
/d*2^(1/2)-1/4*(a+b)*ln(1+cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)+1/
4*(a+b)*ln(1+cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)-2*b/d/cot(d*x+c
)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3754, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{(a-b)\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a-b)\mathrm{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d}-\frac{(a+b)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{(a+b)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{2a}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{2b}{5d\cot^{\frac{5}{2}}(c+dx)}-\frac{2b}{d\sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])/Cot[c + d*x]^(5/2),x]
```

```
[Out] -(((a - b)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*d)) + ((a - b)*
ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*d) + (2*b)/(5*d*Cot[c + d*
x]^(5/2)) + (2*a)/(3*d*Cot[c + d*x]^(3/2)) - (2*b)/(d*Sqrt[Cot[c + d*x]]) -
 ((a + b)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*d)
 + ((a + b)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*
d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
```

```
!IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)}\,dx &= \int \frac{b+a\cot(c+dx)}{\cot^{\frac{7}{2}}(c+dx)}\,dx \\
&= \frac{2b}{5d\cot^{\frac{5}{2}}(c+dx)} + \int \frac{a-b\cot(c+dx)}{\cot^{\frac{5}{2}}(c+dx)}\,dx \\
&= \frac{2b}{5d\cot^{\frac{5}{2}}(c+dx)} + \frac{2a}{3d\cot^{\frac{3}{2}}(c+dx)} + \int \frac{-b-a\cot(c+dx)}{\cot^{\frac{3}{2}}(c+dx)}\,dx \\
&= \frac{2b}{5d\cot^{\frac{5}{2}}(c+dx)} + \frac{2a}{3d\cot^{\frac{3}{2}}(c+dx)} - \frac{2b}{d\sqrt{\cot(c+dx)}} + \int \frac{-a+b\cot(c+dx)}{\sqrt{\cot(c+dx)}}\, d \\
&= \frac{2b}{5d\cot^{\frac{5}{2}}(c+dx)} + \frac{2a}{3d\cot^{\frac{3}{2}}(c+dx)} - \frac{2b}{d\sqrt{\cot(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{a-bx^2}{1+x^4}\,dx, x, \sqrt{\right.}{d} \\
&= \frac{2b}{5d\cot^{\frac{5}{2}}(c+dx)} + \frac{2a}{3d\cot^{\frac{3}{2}}(c+dx)} - \frac{2b}{d\sqrt{\cot(c+dx)}} + \frac{(a-b)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx\right.}{d} \\
&= \frac{2b}{5d\cot^{\frac{5}{2}}(c+dx)} + \frac{2a}{3d\cot^{\frac{3}{2}}(c+dx)} - \frac{2b}{d\sqrt{\cot(c+dx)}} + \frac{(a-b)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}}\right.}{} \\
&= \frac{2b}{5d\cot^{\frac{5}{2}}(c+dx)} + \frac{2a}{3d\cot^{\frac{3}{2}}(c+dx)} - \frac{2b}{d\sqrt{\cot(c+dx)}} - \frac{(a+b)\log\left(1-\sqrt{2}\,\sqrt{\right.}{2} \\
&= -\frac{(a-b)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d} + \frac{(a-b)\tan^{-1}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}\right.}{\sqrt{2}\,d}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.89, size = 207, normalized size = 1.02

$$\frac{\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\,\left(-40a\left(-1+{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(c+dx)\right)\right)\tan^{\frac{3}{2}}(c+dx)+3b\left(-10\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)+10\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}\right)-5\sqrt{2}\,\log\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}+\tan(c+dx)\right)+5\sqrt{2}\,\log\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}+\tan(c+dx)\right)-40\sqrt{\tan(c+dx)}+8\tan^{\frac{5}{2}}(c+dx)\right)\right)}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])/Cot[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-40*a*(-1 + Hypergeometric2F1[3/4,
1, 7/4, -Tan[c + d*x]^2])*Tan[c + d*x]^(3/2) + 3*b*(-10*Sqrt[2]*ArcTan[1 -
Sqrt[2]*Sqrt[Tan[c + d*x]]] + 10*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*
```

```
x]]] - 5*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + 5*Sqr
t[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - 40*Sqrt[Tan[c + d
*x]] + 8*Tan[c + d*x]^(5/2))))/(60*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 53.51, size = 1272, normalized size = 6.30

method	result	size
default	Expression too large to display	1272

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))/cot(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/30/d*(-1+cos(d*x+c))*(-15*I*sin(d*x+c)*EllipticPi((-(cos(d*x+c)-1-sin(d*
x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))
^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x
+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*a+15*I*sin(d*x+c)*EllipticPi((-(cos(d*x
+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+
c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*b+15*I*sin(d*x+c)*EllipticP
i((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1
+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)
*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*a-15*I*sin(d*x+
c)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2
^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d
*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*b+1
5*sin(d*x+c)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1
/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x
+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*
x+c)^2*a+15*sin(d*x+c)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(
1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)
-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1
/2)*cos(d*x+c)^2*b-30*sin(d*x+c)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(
d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-
1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/
2)*cos(d*x+c)^2*b+15*sin(d*x+c)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(
d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((c
os(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d
*x+c))^(1/2)*cos(d*x+c)^2*a+15*sin(d*x+c)*EllipticPi((-(cos(d*x+c)-1-sin(d*
x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))
^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x
+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*b+36*2^(1/2)*cos(d*x+c)^3*b-10*sin(d*x+
c)*2^(1/2)*cos(d*x+c)^2*a-36*2^(1/2)*cos(d*x+c)^2*b+10*cos(d*x+c)*sin(d*x+c
)*2^(1/2)*a-6*cos(d*x+c)*2^(1/2)*b+6*2^(1/2)*b)*(cos(d*x+c)+1)^2/(cos(d*x+c
)/sin(d*x+c))^(5/2)/sin(d*x+c)^6*2^(1/2)
```

Maxima [A]
time = 0.56, size = 165, normalized size = 0.82

$$\frac{8\left(3\,b+\frac{5\,a}{\tan(dx+c)}-\frac{15\,b}{\tan(dx+c)^2}\right)\tan\left(dx+c\right)^{\frac{5}{2}}+30\,\sqrt{2}\,(a-b)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+30\,\sqrt{2}\,(a-b)\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+15\,\sqrt{2}\,(a+b)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)-15\,\sqrt{2}\,(a+b)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/cot(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/60*(8*(3*b + 5*a/tan(d*x + c) - 15*b/tan(d*x + c)^2)*tan(d*x + c)^(5/2) +
 30*sqrt(2)*(a - b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) +
30*sqrt(2)*(a - b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) +
15*sqrt(2)*(a + b)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - 1
5*sqrt(2)*(a + b)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/cot(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\tan\left(c+dx\right)}{\cot^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/cot(d*x+c)**(5/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))/cot(c + d*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/cot(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)/cot(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\tan(c+d\,x)}{{\cot\left(c+d\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))/cot(c + d*x)^(5/2),x)
[Out] int((a + b*tan(c + d*x))/cot(c + d*x)^(5/2), x)
```

3.805 $\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=268

$$\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}+\frac{4ab}{}$$

```
[Out] 2/3*(a^2-b^2)*cot(d*x+c)^(3/2)/d-4/5*a*b*cot(d*x+c)^(5/2)/d-2/7*a^2*cot(d*x
+c)^(7/2)/d-1/2*(a^2+2*a*b-b^2)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/
2)-1/2*(a^2+2*a*b-b^2)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)-1/4*(a^
2-2*a*b-b^2)*ln(1+cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)+1/4*(a^2-2
*a*b-b^2)*ln(1+cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)+4*a*b*cot(d*x
+c)^(1/2)/d
```

Rubi [A]
time = 0.21, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3754, 3624, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d}+\frac{2(a^2-b^2)\cot^{\frac{3}{2}}(c+dx)}{3d}-\frac{(a^2-2ab-b^2)\log\left(\cot(c+dx)-\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{(a^2-2ab-b^2)\log\left(\cot(c+dx)+\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{2a^2\cot^{\frac{7}{2}}(c+dx)}{7d}-\frac{4ab\cot^{\frac{5}{2}}(c+dx)}{5d}+\frac{4ab\sqrt{\cot(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(9/2)*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((a^2 + 2*a*b - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*d) -
((a^2 + 2*a*b - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*d) +
(4*a*b*Sqrt[Cot[c + d*x]])/d + (2*(a^2 - b^2)*Cot[c + d*x]^(3/2))/(3*d) - (
4*a*b*Cot[c + d*x]^(5/2))/(5*d) - (2*a^2*Cot[c + d*x]^(7/2))/(7*d) - ((a^2
- 2*a*b - b^2)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[
2]*d) + ((a^2 - 2*a*b - b^2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d
*x]])/(2*Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
```

```
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^2\,dx = \int \cot^{\frac{5}{2}}(c+dx)(b+a\cot(c+dx))^2\,dx$$

$$= -\frac{2a^2\cot^{\frac{7}{2}}(c+dx)}{7d} + \int \cot^{\frac{5}{2}}(c+dx)\left(-a^2+b^2+2ab\cot(c+dx\right)$$

$$= -\frac{4ab\cot^{\frac{5}{2}}(c+dx)}{5d} - \frac{2a^2\cot^{\frac{7}{2}}(c+dx)}{7d} + \int \cot^{\frac{3}{2}}(c+dx)\left(-2ab\right.$$

$$= \frac{2(a^2-b^2)\cot^{\frac{3}{2}}(c+dx)}{3d} - \frac{4ab\cot^{\frac{5}{2}}(c+dx)}{5d} - \frac{2a^2\cot^{\frac{7}{2}}(c+dx)}{7d} +$$

$$= \frac{4ab\sqrt{\cot(c+dx)}}{d} + \frac{2(a^2-b^2)\cot^{\frac{3}{2}}(c+dx)}{3d} - \frac{4ab\cot^{\frac{5}{2}}(c+dx)}{5d}$$

$$= \frac{4ab\sqrt{\cot(c+dx)}}{d} + \frac{2(a^2-b^2)\cot^{\frac{3}{2}}(c+dx)}{3d} - \frac{4ab\cot^{\frac{5}{2}}(c+dx)}{5d}$$

$$= \frac{4ab\sqrt{\cot(c+dx)}}{d} + \frac{2(a^2-b^2)\cot^{\frac{3}{2}}(c+dx)}{3d} - \frac{4ab\cot^{\frac{5}{2}}(c+dx)}{5d}$$

$$= \frac{4ab\sqrt{\cot(c+dx)}}{d} + \frac{2(a^2-b^2)\cot^{\frac{3}{2}}(c+dx)}{3d} - \frac{4ab\cot^{\frac{5}{2}}(c+dx)}{5d}$$

$$= \frac{4ab\sqrt{\cot(c+dx)}}{d} + \frac{2(a^2-b^2)\cot^{\frac{3}{2}}(c+dx)}{3d} - \frac{4ab\cot^{\frac{5}{2}}(c+dx)}{5d}$$

$$= \frac{(a^2+2ab-b^2)\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d} - \frac{(a^2+2ab-b^2}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 1.57, size = 215, normalized size = 0.80

$$-\frac{\frac{2}{7}a^2\cot^{\frac{7}{2}}(c+dx)+\frac{2}{3}(a^2-b^2)\cot^{\frac{3}{2}}(c+dx)\left(-1+{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)\right)+\frac{1}{10}ab\left(-10\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)+10\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}\right)-40\sqrt{\cot(c+dx)}+8\cot^{\frac{5}{2}}(c+dx)-5\sqrt{2}\,\log\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)+5\sqrt{2}\,\log\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(9/2)*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((2*a^2*Cot[c + d*x]^(7/2))/7 + (2*(a^2 - b^2)*Cot[c + d*x]^(3/2)*(-1 + H
ypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2]))/3 + (a*b*(-10*Sqrt[2]*ArcT
an[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] + 10*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot
[c + d*x]]] - 40*Sqrt[Cot[c + d*x]] + 8*Cot[c + d*x]^(5/2) - 5*Sqrt[2]*Log[
1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] + 5*Sqrt[2]*Log[1 + Sqrt[2]*
Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/10)/d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 18.40, size = 7157, normalized size = 26.71

method	result	size
default	Expression too large to display	7157

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.51, size = 221, normalized size = 0.82

$$-\frac{210\sqrt{2}\,(a^2+2\,ab-b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+210\sqrt{2}\,(a^2+2\,ab-b^2)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-105\sqrt{2}\,(a^2-2\,ab-b^2)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+105\sqrt{2}\,(a^2-2\,ab-b^2)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\frac{1680\,ab}{\sqrt{\tan(dx+c)}}+\frac{336\,ab}{\tan(dx+c)^{\frac{5}{2}}}-\frac{280\,(a^2-b^2)}{\tan(dx+c)^{\frac{3}{2}}}+\frac{120\,a^2}{\tan(dx+c)^{\frac{7}{2}}}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/420*(210*sqrt(2)*(a^2 + 2*a*b - b^2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqr
t(tan(d*x + c)))) + 210*sqrt(2)*(a^2 + 2*a*b - b^2)*arctan(-1/2*sqrt(2)*(sq
rt(2) - 2/sqrt(tan(d*x + c)))) - 105*sqrt(2)*(a^2 - 2*a*b - b^2)*log(sqrt(2
)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + 105*sqrt(2)*(a^2 - 2*a*b - b^2
)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - 1680*a*b/sqrt(tan
(d*x + c)) + 336*a*b/tan(d*x + c)^(5/2) - 280*(a^2 - b^2)/tan(d*x + c)^(3/2
) + 120*a^2/tan(d*x + c)^(7/2))/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(9/2)*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^2*cot(d*x + c)^(9/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c+d\,x)^{9/2}\,\left(a+b\tan(c+d\,x)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(9/2)*(a + b*tan(c + d*x))^2,x)
```

```
[Out] int(cot(c + d*x)^(9/2)*(a + b*tan(c + d*x))^2, x)
```

3.806 $\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=249

$$\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}+\frac{2(a^2}{}$$

```
[Out] -4/3*a*b*cot(d*x+c)^(3/2)/d-2/5*a^2*cot(d*x+c)^(5/2)/d-1/2*(a^2-2*a*b-b^2)*
arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)-1/2*(a^2-2*a*b-b^2)*arctan(1+
2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)+1/4*(a^2+2*a*b-b^2)*ln(1+cot(d*x+c)-2^(
1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)-1/4*(a^2+2*a*b-b^2)*ln(1+cot(d*x+c)+2^(1/2
)*cot(d*x+c)^(1/2))/d*2^(1/2)+2*(a^2-b^2)*cot(d*x+c)^(1/2)/d
```

Rubi [A]

time = 0.19, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3754, 3624, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d}+\frac{2(a^2-b^2)\sqrt{\cot(c+dx)}}{d}+\frac{(a^2+2ab-b^2)\log\left(\cot(c+dx)-\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{(a^2+2ab-b^2)\log\left(\cot(c+dx)+\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{2a^2\cot^{\frac{5}{2}}(c+dx)}{5d}-\frac{4ab\cot^{\frac{3}{2}}(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(7/2)*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((a^2 - 2*a*b - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*d) -
((a^2 - 2*a*b - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*d) +
(2*(a^2 - b^2)*Sqrt[Cot[c + d*x]])/d - (4*a*b*Cot[c + d*x]^(3/2))/(3*d) - (
2*a^2*Cot[c + d*x]^(5/2))/(5*d) + ((a^2 + 2*a*b - b^2)*Log[1 - Sqrt[2]*Sqrt
[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*d) - ((a^2 + 2*a*b - b^2)*Log[1
+ Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^2\,dx = \int \cot^{\frac{3}{2}}(c+dx)(b+a\cot(c+dx))^2\,dx$$

$$= -\frac{2a^2\cot^{\frac{5}{2}}(c+dx)}{5d} + \int \cot^{\frac{3}{2}}(c+dx)\left(-a^2+b^2+2ab\cot(c+dx)\right)$$

$$= -\frac{4ab\cot^{\frac{3}{2}}(c+dx)}{3d} - \frac{2a^2\cot^{\frac{5}{2}}(c+dx)}{5d} + \int \sqrt{\cot(c+dx)}\,(-2ab$$

$$= \frac{2(a^2-b^2)\sqrt{\cot(c+dx)}}{d} - \frac{4ab\cot^{\frac{3}{2}}(c+dx)}{3d} - \frac{2a^2\cot^{\frac{5}{2}}(c+dx)}{5d}$$

$$= \frac{2(a^2-b^2)\sqrt{\cot(c+dx)}}{d} - \frac{4ab\cot^{\frac{3}{2}}(c+dx)}{3d} - \frac{2a^2\cot^{\frac{5}{2}}(c+dx)}{5d}$$

$$= \frac{2(a^2-b^2)\sqrt{\cot(c+dx)}}{d} - \frac{4ab\cot^{\frac{3}{2}}(c+dx)}{3d} - \frac{2a^2\cot^{\frac{5}{2}}(c+dx)}{5d}$$

$$= \frac{2(a^2-b^2)\sqrt{\cot(c+dx)}}{d} - \frac{4ab\cot^{\frac{3}{2}}(c+dx)}{3d} - \frac{2a^2\cot^{\frac{5}{2}}(c+dx)}{5d}$$

$$= \frac{2(a^2-b^2)\sqrt{\cot(c+dx)}}{d} - \frac{4ab\cot^{\frac{3}{2}}(c+dx)}{3d} - \frac{2a^2\cot^{\frac{5}{2}}(c+dx)}{5d}$$

$$= \frac{(a^2-2ab-b^2)\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d} - \frac{(a^2-2ab-b^2)}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.45, size = 202, normalized size = 0.81

$$-\frac{\frac{2}{5}a^2\cot^{\frac{5}{2}}(c+dx) - \frac{4}{3}ab\cot^{\frac{3}{2}}(c+dx)\left(-1+{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)\right) - \frac{1}{4}(a^2-b^2)\left(2\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right) - 2\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right) + 8\sqrt{\cot(c+dx)} + \sqrt{2}\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right) - \sqrt{2}\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(7/2)*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((2*a^2*Cot[c + d*x]^(5/2))/5 - (4*a*b*Cot[c + d*x]^(3/2)*(-1 + Hypergeom
etric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2]))/3 - ((a^2 - b^2)*(2*Sqrt[2]*ArcTan
[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c
+ d*x]]] + 8*Sqrt[Cot[c + d*x]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]
] + Cot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*
x]]))/4)/d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 18.64, size = 6328, normalized size = 25.41

method	result	size
default	Expression too large to display	6328

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.50, size = 209, normalized size = 0.84

$$-\frac{30\sqrt{2}\,(a^2-2\,ab-b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+30\sqrt{2}\,(a^2-2\,ab-b^2)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+15\sqrt{2}\,(a^2+2\,ab-b^2)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)-15\sqrt{2}\,(a^2+2\,ab-b^2)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)+\frac{80\,ab}{\tan(dx+c)^{\frac{3}{2}}}-\frac{120\,(a^2-b^2)}{\sqrt{\tan\left(dx+c\right)}}+\frac{24\,a^2}{\tan(dx+c)^{\frac{5}{2}}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/60*(30*sqrt(2)*(a^2 - 2*a*b - b^2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(
tan(d*x + c)))) + 30*sqrt(2)*(a^2 - 2*a*b - b^2)*arctan(-1/2*sqrt(2)*(sqrt(
2) - 2/sqrt(tan(d*x + c)))) + 15*sqrt(2)*(a^2 + 2*a*b - b^2)*log(sqrt(2)/sq
rt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - 15*sqrt(2)*(a^2 + 2*a*b - b^2)*log
(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + 80*a*b/tan(d*x + c)^(3
/2) - 120*(a^2 - b^2)/sqrt(tan(d*x + c)) + 24*a^2/tan(d*x + c)^(5/2))/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(7/2)*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4847 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^2*cot(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c+d\,x)^{7/2}\,(a+b\tan(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(7/2)*(a + b*tan(c + d*x))^2,x)
```

```
[Out] int(cot(c + d*x)^(7/2)*(a + b*tan(c + d*x))^2, x)
```

3.807 $\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=223

$$-\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}-\frac{4a}{}$$

```
[Out] -2/3*a^2*cot(d*x+c)^(3/2)/d+1/2*(a^2+2*a*b-b^2)*arctan(-1+2^(1/2)*cot(d*x+c
)^(1/2))/d*2^(1/2)+1/2*(a^2+2*a*b-b^2)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/d
*2^(1/2)+1/4*(a^2-2*a*b-b^2)*ln(1+cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2))/d*2^
(1/2)-1/4*(a^2-2*a*b-b^2)*ln(1+cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/
2)-4*a*b*cot(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.16, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3754, 3624, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d}+\frac{(a^2-2ab-b^2)\log\left(\cot(c+dx)-\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{(a^2-2ab-b^2)\log\left(\cot(c+dx)+\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{2a^2\cot^{\frac{3}{2}}(c+dx)}{3d}-\frac{4ab\sqrt{\cot(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((a^2 + 2*a*b - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*d))
 + ((a^2 + 2*a*b - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*d)
 - (4*a*b*Sqrt[Cot[c + d*x]])/d - (2*a^2*Cot[c + d*x]^(3/2))/(3*d) + ((a^2
- 2*a*b - b^2)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[
2]*d) - ((a^2 - 2*a*b - b^2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d
*x]])/(2*Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2\,dx = \int \sqrt{\cot(c+dx)}\,(b+a\cot(c+dx))^2\,dx$$

$$= -\frac{2a^2\cot^{\frac{3}{2}}(c+dx)}{3d} + \int \sqrt{\cot(c+dx)}\,\left(-a^2+b^2+2ab\cot(c+d\right.$$

$$= -\frac{4ab\sqrt{\cot(c+dx)}}{d} - \frac{2a^2\cot^{\frac{3}{2}}(c+dx)}{3d} + \int \frac{-2ab-(a^2-b^2)\,\mathrm{c}}{\sqrt{\cot(c+d}}$$

$$= -\frac{4ab\sqrt{\cot(c+dx)}}{d} - \frac{2a^2\cot^{\frac{3}{2}}(c+dx)}{3d} + \frac{2\mathrm{Subst}\left(\int \frac{2ab+(a^2-b^2)x}{1+x^4}\right.}{}$$

$$= -\frac{4ab\sqrt{\cot(c+dx)}}{d} - \frac{2a^2\cot^{\frac{3}{2}}(c+dx)}{3d} - \frac{(a^2-2ab-b^2)\,\mathrm{Subst}}{}$$

$$= -\frac{4ab\sqrt{\cot(c+dx)}}{d} - \frac{2a^2\cot^{\frac{3}{2}}(c+dx)}{3d} + \frac{(a^2-2ab-b^2)\,\mathrm{Subst}}{}$$

$$= -\frac{4ab\sqrt{\cot(c+dx)}}{d} - \frac{2a^2\cot^{\frac{3}{2}}(c+dx)}{3d} + \frac{(a^2-2ab-b^2)\log\left(1\right.}{}$$

$$= -\frac{(a^2+2ab-b^2)\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d} + \frac{(a^2+2ab-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.43, size = 199, normalized size = 0.89

$$\frac{4(a^2-b^2)\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)-a\left(6\sqrt{2}\,b\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-6\sqrt{2}\,b\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+24b\sqrt{\cot(c+dx)}+4a\cot^{\frac{3}{2}}(c+dx)+3\sqrt{2}\,b\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-3\sqrt{2}\,b\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (4*(a^2 - b^2)*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d
*x]^2] - a*(6*Sqrt[2]*b*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 6*Sqrt[2]*
```

```
b*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 24*b*Sqrt[Cot[c + d*x]] + 4*a*Co
t[c + d*x]^(3/2) + 3*Sqrt[2]*b*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c +
 d*x]] - 3*Sqrt[2]*b*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/(
6*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 18.97, size = 3512, normalized size = 15.75

method	result	size
default	Expression too large to display	3512

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/d*(-3*I*cos(d*x+c)*sin(d*x+c)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(
1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+
c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-(cos(d*x+c)-1-sin(d*x+c))/si
n(d*x+c))^(1/2)*a^2+3*I*cos(d*x+c)*sin(d*x+c)*((cos(d*x+c)-1+sin(d*x+c))/si
n(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)
-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-(cos(d*x+c)-1-sin
(d*x+c))/sin(d*x+c))^(1/2)*b^2+6*I*sin(d*x+c)*((cos(d*x+c)-1+sin(d*x+c))/si
n(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)
-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(cos(d*x+c)-1-sin
(d*x+c))/sin(d*x+c))^(1/2)*a*b-6*I*sin(d*x+c)*((cos(d*x+c)-1+sin(d*x+c))/si
n(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)
-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-(cos(d*x+c)-1-sin
(d*x+c))/sin(d*x+c))^(1/2)*a*b+3*I*cos(d*x+c)*sin(d*x+c)*((cos(d*x+c)-1+sin
(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-
(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(cos(d
*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*a^2-3*I*cos(d*x+c)*sin(d*x+c)*((cos(d
*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*El
lipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2
))*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*b^2+2*cos(d*x+c)^2*2^(1/2)
*a^2-6*cos(d*x+c)*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*
EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1
/2))*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*a*b-6*cos(d*x+c)*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c
))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-
1/2*I,1/2*2^(1/2))*a*b+12*cos(d*x+c)*sin(d*x+c)*2^(1/2)*a*b+3*sin(d*x+c)*(-
(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(
d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((cos(d*x+c)-1+sin(d*x+c))
/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*a^2-3*sin(d*x+c)*(-(c
os(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*
x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((cos(d*x+c)-1+sin(d*x+c))/s
```

```
in(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*b^2+3*sin(d*x+c)*(-(cos
(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin
(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a^2-3*sin(d*x+c)*(-(cos(d
*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c)
)^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d
*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b^2-6*sin(d*x+c)*(-(cos(d*x
+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^
(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(cos(d*x+c)-1-sin(d*x+
c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2+6*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x
+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c
))^(1/2),1/2*2^(1/2))*b^2+3*I*sin(d*x+c)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x
+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-si
n(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(cos(d*x+c)-1-sin(d*x+
c))/sin(d*x+c))^(1/2)*a^2-3*I*sin(d*x+c)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x
+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-si
n(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(cos(d*x+c)-1-sin(d*x+
c))/sin(d*x+c))^(1/2)*b^2-3*I*sin(d*x+c)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x
+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-si
n(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-(cos(d*x+c)-1-sin(d*x+
c))/sin(d*x+c))^(1/2)*a^2+3*I*sin(d*x+c)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x
+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-si
n(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-(cos(d*x+c)-1-sin(d*x+
c))/sin(d*x+c))^(1/2)*b^2+3*cos(d*x+c)*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c
))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/
2),1/2+1/2*I,1/2*2^(1/2))*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1
+cos(d*x+c))/sin(d*x+c))^(1/2)*a^2-3*cos(d*x+c)*sin(d*x+c)*(-(cos(d*x+c)-1-
sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*
x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(
1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*b^2+3*cos(d*x+c)*sin(d*x+c)*(-(cos(
d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c
))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(
d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a^2-3*cos(d*x+c)*sin(d*x+c
)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/
sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^...
```

Maxima [A]
time = 0.51, size = 190, normalized size = 0.85

$$\frac{6\sqrt{2}\,(a^2+2\,ab-b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+6\sqrt{2}\,(a^2+2\,ab-b^2)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-3\sqrt{2}\,(a^2-2\,ab-b^2)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+3\sqrt{2}\,(a^2-2\,ab-b^2)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\frac{48\,ab}{\sqrt{\tan(dx+c)}}-\frac{8\,a^2}{\tan(dx+c)^{\frac{3}{2}}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/12*(6*sqrt(2)*(a^2 + 2*a*b - b^2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(ta
n(d*x + c)))) + 6*sqrt(2)*(a^2 + 2*a*b - b^2)*arctan(-1/2*sqrt(2)*(sqrt(2)
- 2/sqrt(tan(d*x + c)))) - 3*sqrt(2)*(a^2 - 2*a*b - b^2)*log(sqrt(2)/sqrt(t
an(d*x + c)) + 1/tan(d*x + c) + 1) + 3*sqrt(2)*(a^2 - 2*a*b - b^2)*log(-sqr
t(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - 48*a*b/sqrt(tan(d*x + c)) -
 8*a^2/tan(d*x + c)^(3/2))/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(5/2)*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^2*cot(d*x + c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c+d\,x)^{5/2}\,{\left(a+b\tan(c+d\,x)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))^2,x)
```

```
[Out] int(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))^2, x)
```

3.808 $\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=204

$$-\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}-$$

```
[Out] 1/2*(a^2-2*a*b-b^2)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)+1/2*(a^2-
2*a*b-b^2)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)-1/4*(a^2+2*a*b-b^2)
*ln(1+cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)+1/4*(a^2+2*a*b-b^2)*ln
(1+cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)-2*a^2*cot(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.14, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3754, 3624, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d}-\frac{(a^2+2ab-b^2)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{(a^2+2ab-b^2)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{2a^2\sqrt{\cot(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((a^2 - 2*a*b - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*d))
 + ((a^2 - 2*a*b - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*d)
 - (2*a^2*Sqrt[Cot[c + d*x]])/d - ((a^2 + 2*a*b - b^2)*Log[1 - Sqrt[2]*Sqrt
[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*d) + ((a^2 + 2*a*b - b^2)*Log[1
+ Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$
\begin{aligned}
\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2\,dx &= \int \frac{(b+a\cot(c+dx))^2}{\sqrt{\cot(c+dx)}}\,dx \\
&= -\frac{2a^2\sqrt{\cot(c+dx)}}{d} + \int \frac{-a^2+b^2+2ab\cot(c+dx)}{\sqrt{\cot(c+dx)}}\,dx \\
&= -\frac{2a^2\sqrt{\cot(c+dx)}}{d} + \frac{2\text{Subst}\left(\int \frac{a^2-b^2-2abx^2}{1+x^4}\,dx, x, \sqrt{\cot(c+dx)}\right.}{d} \\
&= -\frac{2a^2\sqrt{\cot(c+dx)}}{d} + \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\cot(}\right.}{d} \\
&= -\frac{2a^2\sqrt{\cot(c+dx)}}{d} + \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x,\right.}{2d} \\
&= -\frac{2a^2\sqrt{\cot(c+dx)}}{d} - \frac{(a^2+2ab-b^2)\log\left(1-\sqrt{2}\,\sqrt{\cot(c+dx}\right.}{2\sqrt{2}\,d} \\
&= -\frac{(a^2-2ab-b^2)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d} + \frac{(a^2-2ab-}{}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.50, size = 170, normalized size = 0.83

$$
-\frac{2a^2\sqrt{\cot(c+dx)} + \frac{4}{3}ab\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right) + \frac{(a^2-b^2)\left(2\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)-2\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}\right)+\log\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\log\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)}{2\sqrt{2}}}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -((2*a^2*Sqrt[Cot[c + d*x]] + (4*a*b*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3
/4, 1, 7/4, -Cot[c + d*x]^2])/3 + ((a^2 - b^2)*(2*ArcTan[1 - Sqrt[2]*Sqrt[C
ot[c + d*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + Log[1 - Sqrt[2]*
Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + C
ot[c + d*x]]))/(2*Sqrt[2]))/d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 16.59, size = 3089, normalized size = 15.14

method	result	size

default	Expression too large to display	3089

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] -1/2/d*(2*cos(d*x+c)*2^(1/2)*a^2-((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*a^2+((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*b^2-((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*a^2+((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*b^2-I*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^2-I*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b^2+I*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*b^2+I*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a^2+((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*b^2-((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*a^2+((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*b^2-2*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*a*b-2*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*a*b+4*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(

```
1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c
))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(
1/2)*a*b-((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+
1/2*I,1/2*2^(1/2))*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)
*a^2-2*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/
2*I,1/2*2^(1/2))*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*a
*b-2*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*
I,1/2*2^(1/2))*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*a*b
+4*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2)
)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*a*b-I*cos(d*x+c)
*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/s
in(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c
)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^2-I*cos(d*x+c)*(
-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin
(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-
1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b^2+2*I*(-(cos(d*x+c
)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1
/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c
))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a*b-2*I*(-(cos(d*x+c)-1-sin(d*x
+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+
c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a*b+I*cos(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c
))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2+1/2*I,1/2*2^(1/2))*b^2+I*cos(d*x+c)...
```

<u>Maxima</u> [A]
time = 0.54, size = 177, normalized size = 0.87

$$\frac{2\sqrt{2}\left(a^2-2ab-b^2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+2\sqrt{2}\left(a^2-2ab-b^2\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+\sqrt{2}\left(a^2+2ab-b^2\right)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}\left(a^2+2ab-b^2\right)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)-\frac{8a^2}{\sqrt{\tan\left(dx+c\right)}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*(a^2 - 2*a*b - b^2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan
(d*x + c)))) + 2*sqrt(2)*(a^2 - 2*a*b - b^2)*arctan(-1/2*sqrt(2)*(sqrt(2) -
 2/sqrt(tan(d*x + c)))) + sqrt(2)*(a^2 + 2*a*b - b^2)*log(sqrt(2)/sqrt(tan(
d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*(a^2 + 2*a*b - b^2)*log(-sqrt(2)/
sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - 8*a^2/sqrt(tan(d*x + c)))/d
```

<u>Fricas</u> [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan{(c+dx)})^2\cot^{\frac{3}{2}}{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(3/2)*(a+b*tan(d*x+c))**2,x)
[Out] Integral((a + b*tan(c + d*x))**2*cot(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^2,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^2*cot(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c+d\,x)^{3/2}\,(a+b\tan(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^2,x)
[Out] int(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^2, x)
```

3.809 $\int \sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=204

$$\frac{(a^2+2ab-b^2)\operatorname{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a^2+2ab-b^2)\operatorname{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}+\frac{}{d\sqrt{}}$$

```
[Out] -1/2*(a^2+2*a*b-b^2)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)-1/2*(a^2
+2*a*b-b^2)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)-1/4*(a^2-2*a*b-b^2
)*ln(1+cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)+1/4*(a^2-2*a*b-b^2)*l
n(1+cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)+2*b^2/d/cot(d*x+c)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3754, 3623, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a^2+2ab-b^2)\operatorname{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a^2+2ab-b^2)\operatorname{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d}-\frac{(a^2-2ab-b^2)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{(a^2-2ab-b^2)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{2b^2}{d\sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((a^2 + 2*a*b - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*d) -
((a^2 + 2*a*b - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*d) +
(2*b^2)/(d*Sqrt[Cot[c + d*x]]) - ((a^2 - 2*a*b - b^2)*Log[1 - Sqrt[2]*Sqrt[
Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*d) + ((a^2 - 2*a*b - b^2)*Log[1 +
 Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^2\,dx &= \int \frac{(b+a\cot(c+dx))^2}{\cot^{\frac{3}{2}}(c+dx)}\,dx\\
&= \frac{2b^2}{d\sqrt{\cot(c+dx)}} + \int \frac{2ab+(a^2-b^2)\cot(c+dx)}{\sqrt{\cot(c+dx)}}\,dx\\
&= \frac{2b^2}{d\sqrt{\cot(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{-2ab+(-a^2+b^2)x^2}{1+x^4}\,dx, x, \sqrt{\cot(c+d}\right.}{d}\\
&= \frac{2b^2}{d\sqrt{\cot(c+dx)}} + \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\cot(c}\right.}{d}\\
&= \frac{2b^2}{d\sqrt{\cot(c+dx)}} - \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}\,x-x^2}\,dx, x,\right.}{2\sqrt{2}\,d}\\
&= \frac{2b^2}{d\sqrt{\cot(c+dx)}} - \frac{(a^2-2ab-b^2)\log\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right.}{2\sqrt{2}\,d}\\
&= \frac{(a^2+2ab-b^2)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d} - \frac{(a^2+2ab-}{}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.77, size = 173, normalized size = 0.85

$$-\frac{-\sqrt{2}\,ab\left(\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)-\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}\right)\right)-\frac{2a^2}{\sqrt{\cot(c+dx)}}+\frac{2(a^2-b^2)\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\cot^2(c+dx)\right)}{\sqrt{\cot(c+dx)}}+\frac{ab\left(-\log\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)+\log\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)}{\sqrt{2}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -((-(Sqrt[2]*a*b*(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - ArcTan[1 + Sqrt[
2]*Sqrt[Cot[c + d*x]]])) - (2*a^2)/Sqrt[Cot[c + d*x]] + (2*(a^2 - b^2)*Hype
rgeometric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2])/Sqrt[Cot[c + d*x]] + (a*b*(-L
og[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] + Log[1 + Sqrt[2]*Sqrt[Co
t[c + d*x]] + Cot[c + d*x]]))/Sqrt[2])/d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 52.49, size = 1757, normalized size = 8.61

method	result	size
default	Expression too large to display	1757

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(cos(d*x+c)/sin(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))*(-2*I*
sin(d*x+c)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2
*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c
))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*a*b+I*si
n(d*x+c)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I
,1/2*2^(1/2))*b^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x
+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)-I*sin(
d*x+c)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1
/2*2^(1/2))*a^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c
))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)+I*sin(d*
x+c)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2
*2^(1/2))*a^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))
/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)+2*I*sin(d*
x+c)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2
*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin
(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*a*b-I*sin(d*x+
c)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2
^(1/2))*b^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/s
in(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)-sin(d*x+c)*(
-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin
(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((cos(d*x+c)-1+sin(d*x+c)
)/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*a^2+2*sin(d*x+c)*(-(
cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d
*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((cos(d*x+c)-1+sin(d*x+c))/
sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*a*b+sin(d*x+c)*(-(cos(
d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c
))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((cos(d*x+c)-1+sin(d*x+c))/sin(
d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*b^2-sin(d*x+c)*(-(cos(d*x+
c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(
1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+
c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a^2+2*sin(d*x+c)*(-(cos(d*x+c)
-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/
2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c)
)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a*b+sin(d*x+c)*(-(cos(d*x+c)-1-s
in(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/si
n(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b^2+2*sin(d*x+c)*(-(cos(d*x+c)-1-sin
```

```
(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d
*x+c))^(1/2),1/2*2^(1/2))*a^2-2*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(
d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),
1/2*2^(1/2))*b^2+2*cos(d*x+c)*2^(1/2)*b^2-2*2^(1/2)*b^2)/sin(d*x+c)^3/cos(d
*x+c)*2^(1/2)
```

Maxima [A]
time = 0.50, size = 177, normalized size = 0.87

$$-\frac{2\sqrt{2}\left(a^2+2ab-b^2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+2\sqrt{2}\left(a^2+2ab-b^2\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)-\sqrt{2}\left(a^2-2ab-b^2\right)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}\left(a^2-2ab-b^2\right)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)-8b^2\sqrt{\tan\left(dx+c\right)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*(a^2 + 2*a*b - b^2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(ta
n(d*x + c)))) + 2*sqrt(2)*(a^2 + 2*a*b - b^2)*arctan(-1/2*sqrt(2)*(sqrt(2)
- 2/sqrt(tan(d*x + c)))) - sqrt(2)*(a^2 - 2*a*b - b^2)*log(sqrt(2)/sqrt(tan
(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*(a^2 - 2*a*b - b^2)*log(-sqrt(2)
/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - 8*b^2*sqrt(tan(d*x + c)))/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan\left(c+dx\right)\right)^2\sqrt{\cot\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(1/2)*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sqrt(cot(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^2*sqrt(cot(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cot(c+d\,x)}\,\left(a+b\tan(c+d\,x)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^2,x)
```

```
[Out] int(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^2, x)
```

3.810 $\int \frac{(a+b\tan(c+dx))^2}{\sqrt{\cot(c+dx)}}\,dx$

Optimal. Leaf size=223

$$\frac{(a^2-2ab-b^2)\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a^2-2ab-b^2)\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}+\frac{}{3d\,\mathrm{c}}$$

```
[Out] 2/3*b^2/d/cot(d*x+c)^(3/2)-1/2*(a^2-2*a*b-b^2)*arctan(-1+2^(1/2)*cot(d*x+c)
^(1/2))/d*2^(1/2)-1/2*(a^2-2*a*b-b^2)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/d*
2^(1/2)+1/4*(a^2+2*a*b-b^2)*ln(1+cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2))/d*2^(
1/2)-1/4*(a^2+2*a*b-b^2)*ln(1+cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2
)+4*a*b/d/cot(d*x+c)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3754, 3623, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a^2-2ab-b^2)\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a^2-2ab-b^2)\,\mathrm{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d}+\frac{(a^2+2ab-b^2)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{(a^2+2ab-b^2)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{4ab}{d\sqrt{\cot(c+dx)}}+\frac{2b^2}{3d\cot^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^2/Sqrt[Cot[c + d*x]],x]
```

```
[Out] ((a^2 - 2*a*b - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*d) -
((a^2 - 2*a*b - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*d) +
(2*b^2)/(3*d*Cot[c + d*x]^(3/2)) + (4*a*b)/(d*Sqrt[Cot[c + d*x]]) + ((a^2 +
 2*a*b - b^2)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2
]*d) - ((a^2 + 2*a*b - b^2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*
x]])/(2*Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
```

```
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{(a+b\tan(c+dx))^2}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(b+a\cot(c+dx))^2}{\cot^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2b^2}{3d\cot^{\frac{3}{2}}(c+dx)} + \int \frac{2ab+(a^2-b^2)\cot(c+dx)}{\cot^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2b^2}{3d\cot^{\frac{3}{2}}(c+dx)} + \frac{4ab}{d\sqrt{\cot(c+dx)}} + \int \frac{a^2-b^2-2ab\cot(c+dx)}{\sqrt{\cot(c+dx)}}\,dx$$
$$= \frac{2b^2}{3d\cot^{\frac{3}{2}}(c+dx)} + \frac{4ab}{d\sqrt{\cot(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{-a^2+b^2+2abx^2}{1+x^4}\,dx, x, \sqrt{\cot(c+\right.}{d}$$
$$= \frac{2b^2}{3d\cot^{\frac{3}{2}}(c+dx)} + \frac{4ab}{d\sqrt{\cot(c+dx)}} - \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{c\right.}{d}$$
$$= \frac{2b^2}{3d\cot^{\frac{3}{2}}(c+dx)} + \frac{4ab}{d\sqrt{\cot(c+dx)}} - \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx\right.}{2d}$$
$$= \frac{2b^2}{3d\cot^{\frac{3}{2}}(c+dx)} + \frac{4ab}{d\sqrt{\cot(c+dx)}} + \frac{(a^2+2ab-b^2)\log\left(1-\sqrt{2}\,\sqrt{\cot(c+\right.}{2\sqrt{2}\,d}$$
$$= \frac{(a^2-2ab-b^2)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d} - \frac{(a^2-2ab-b^2)\tan^{-1}\left(1+\right.}{\sqrt{2}\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.27, size = 77, normalized size = 0.35

$$\frac{2\left((-a^2+b^2)\;{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\cot^2(c+dx)\right)+a\left(a+6b\cot(c+dx)\;{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\cot^2(c+dx)\right)\right)\right)}{3d\cot^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^2/Sqrt[Cot[c + d*x]],x]
```

```
[Out] (2*((-a^2 + b^2)*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[c + d*x]^2] + a*(a +
6*b*Cot[c + d*x]*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2])))/(3*d*C
ot[c + d*x]^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 56.15, size = 1733, normalized size = 7.77

method	result	size
default	Expression too large to display	1733

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^2/cot(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))*(3*I*sin(d*x+c)*EllipticPi(((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b^2*((-1+cos(d*x
+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d
*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)+3*I*sin(d*x+c)*EllipticPi(
((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^2*((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)-6*I*sin(d*x+c)*El
lipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2)
)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*a*b-3*I*sin(
d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/
2*2^(1/2))*b^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)
+6*I*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2
-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d
*x+c)*a*b-3*I*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2),1/2-1/2*I,1/2*2^(1/2))*a^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((1-cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c)
)^(1/2)*cos(d*x+c)+3*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin
(d*x+c))^(1/2)*cos(d*x+c)+6*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/
sin(d*x+c))^(1/2)*cos(d*x+c)*a*b-3*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*b^2*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+s
in(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)+3*sin(d*x+c)*EllipticPi(((1-cos(d*x
```

```
+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a^2*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x
+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)+6*sin(d*x+c)*EllipticPi(((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*
x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(
d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*a*b-3*sin(d*x+c)*Elliptic
Pi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b^2*
((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)-12*sin(d*x+c)*
EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((
cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*a*b+2*sin(d*x+c)*2^(1
/2)*b^2*cos(d*x+c)+12*2^(1/2)*cos(d*x+c)^2*a*b-2*sin(d*x+c)*2^(1/2)*b^2-12*
2^(1/2)*cos(d*x+c)*a*b)/sin(d*x+c)^4/cos(d*x+c)/(cos(d*x+c)/sin(d*x+c))^(1/
2)*2^(1/2)
```

Maxima [A]
time = 0.50, size = 191, normalized size = 0.86

$$-\frac{6\sqrt{2}\,(a^2-2\,ab-b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+6\sqrt{2}\,(a^2-2\,ab-b^2)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+3\sqrt{2}\,(a^2+2\,ab-b^2)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-3\sqrt{2}\,(a^2+2\,ab-b^2)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-8\left(b^2+\frac{6\,ab}{\tan(dx+c)}\right)\tan(dx+c)^{\frac{3}{2}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^2/cot(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*(6*sqrt(2)*(a^2 - 2*a*b - b^2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(t
an(d*x + c)))) + 6*sqrt(2)*(a^2 - 2*a*b - b^2)*arctan(-1/2*sqrt(2)*(sqrt(2)
 - 2/sqrt(tan(d*x + c)))) + 3*sqrt(2)*(a^2 + 2*a*b - b^2)*log(sqrt(2)/sqrt(
tan(d*x + c)) + 1/tan(d*x + c) + 1) - 3*sqrt(2)*(a^2 + 2*a*b - b^2)*log(-sq
rt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - 8*(b^2 + 6*a*b/tan(d*x + c
))*tan(d*x + c)^(3/2))/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^2/cot(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(c+dx))^2}{\sqrt{\cot(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**2/cot(d*x+c)**(1/2),x)
[Out] Integral((a + b*tan(c + d*x))**2/sqrt(cot(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^2/cot(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^2/sqrt(cot(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\tan\left(c+d\,x\right)\right)}^2}{\sqrt{\mathrm{cot}\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^2/cot(c + d*x)^(1/2),x)
[Out] int((a + b*tan(c + d*x))^2/cot(c + d*x)^(1/2), x)
```

3.811 $\int \frac{(a+b\tan(c+dx))^2}{\cot^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=249

$$-\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}+\frac{}{5c}$$

```
[Out] 2/5*b^2/d/cot(d*x+c)^(5/2)+4/3*a*b/d/cot(d*x+c)^(3/2)+1/2*(a^2+2*a*b-b^2)*a
rctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)+1/2*(a^2+2*a*b-b^2)*arctan(1+2
^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)+1/4*(a^2-2*a*b-b^2)*ln(1+cot(d*x+c)-2^(1
/2)*cot(d*x+c)^(1/2))/d*2^(1/2)-1/4*(a^2-2*a*b-b^2)*ln(1+cot(d*x+c)+2^(1/2)
*cot(d*x+c)^(1/2))/d*2^(1/2)+2*(a^2-b^2)/d/cot(d*x+c)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3754, 3623, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d}+\frac{2(a^2-b^2)}{d\sqrt{\cot(c+dx)}}+\frac{(a^2-2ab-b^2)\log\left(\cot(c+dx)-\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{(a^2-2ab-b^2)\log\left(\cot(c+dx)+\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{4ab}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{2b^2}{5d\cot^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^2/Cot[c + d*x]^(3/2),x]
```

```
[Out] -(((a^2 + 2*a*b - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*d))
 + ((a^2 + 2*a*b - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*d)
 + (2*b^2)/(5*d*Cot[c + d*x]^(5/2)) + (4*a*b)/(3*d*Cot[c + d*x]^(3/2)) + (2
*(a^2 - b^2))/(d*Sqrt[Cot[c + d*x]]) + ((a^2 - 2*a*b - b^2)*Log[1 - Sqrt[2]
*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*d) - ((a^2 - 2*a*b - b^2)*L
og[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
```

```
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{(a+b\tan(c+dx))^2}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b+a\cot(c+dx))^2}{\cot^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2b^2}{5d\cot^{\frac{5}{2}}(c+dx)} + \int \frac{2ab+(a^2-b^2)\cot(c+dx)}{\cot^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2b^2}{5d\cot^{\frac{5}{2}}(c+dx)} + \frac{4ab}{3d\cot^{\frac{3}{2}}(c+dx)} + \int \frac{a^2-b^2-2ab\cot(c+dx)}{\cot^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2b^2}{5d\cot^{\frac{5}{2}}(c+dx)} + \frac{4ab}{3d\cot^{\frac{3}{2}}(c+dx)} + \frac{2(a^2-b^2)}{d\sqrt{\cot(c+dx)}} + \int \frac{-2ab-(a^2-b^2)}{\sqrt{\cot(c+}}$$

$$= \frac{2b^2}{5d\cot^{\frac{5}{2}}(c+dx)} + \frac{4ab}{3d\cot^{\frac{3}{2}}(c+dx)} + \frac{2(a^2-b^2)}{d\sqrt{\cot(c+dx)}} + \frac{2\mathrm{Subst}\left(\int \frac{2ab+(a^2-b^2)}{1+x^4}\right.}{}$$

$$= \frac{2b^2}{5d\cot^{\frac{5}{2}}(c+dx)} + \frac{4ab}{3d\cot^{\frac{3}{2}}(c+dx)} + \frac{2(a^2-b^2)}{d\sqrt{\cot(c+dx)}} - \frac{(a^2-2ab-b^2)\,\mathrm{Subs}}{}$$

$$= \frac{2b^2}{5d\cot^{\frac{5}{2}}(c+dx)} + \frac{4ab}{3d\cot^{\frac{3}{2}}(c+dx)} + \frac{2(a^2-b^2)}{d\sqrt{\cot(c+dx)}} + \frac{(a^2-2ab-b^2)\,\mathrm{Subs}}{}$$

$$= \frac{2b^2}{5d\cot^{\frac{5}{2}}(c+dx)} + \frac{4ab}{3d\cot^{\frac{3}{2}}(c+dx)} + \frac{2(a^2-b^2)}{d\sqrt{\cot(c+dx)}} + \frac{(a^2-2ab-b^2)\log\left(\right.}{}$$

$$= -\frac{(a^2+2ab-b^2)\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d} + \frac{(a^2+2ab-b^2)\tan^{-1}\left(1\right.}{\sqrt{2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.29, size = 81, normalized size = 0.33

$$\frac{-6(a^2-b^2)\ {}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\cot^2(c+dx)\right)+2a\left(3a+10b\cot(c+dx)\ {}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\cot^2(c+dx)\right)\right)}{15d\cot^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^2/Cot[c + d*x]^(3/2),x]
```

```
[Out] (-6*(a^2 - b^2)*Hypergeometric2F1[-5/4, 1, -1/4, -Cot[c + d*x]^2] + 2*a*(3*
a + 10*b*Cot[c + d*x]*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[c + d*x]^2]))/(1
5*d*Cot[c + d*x]^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 59.00, size = 1975, normalized size = 7.93

method	result	size
default	Expression too large to display	1975

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^2/cot(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30/d*(-1+cos(d*x+c))*(-15*I*sin(d*x+c)*a^2*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c
))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/si
n(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-30*I*sin(d*x+c)*((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1
-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-(cos(d*x+c)-1-sin(
d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a*b+15*I*sin(d*x+c)*b^2*((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/
2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-
(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+30*I*sin
(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d
*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*Ell
ipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2)
)*a*b+15*I*sin(d*x+c)*a^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1
+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2
)*cos(d*x+c)^2*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2
-1/2*I,1/2*2^(1/2))-15*I*sin(d*x+c)*b^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/si
n(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x
+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-30*cos(d*x+c)^2*sin(d*x+c)*(-(cos(d*x+c)-
1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2
)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/
sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2+30*cos(d*x+c)^2*sin(d*x+c)*(-(cos(d*x+c)
-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/
```

```
2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))
/sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2+15*cos(d*x+c)^2*sin(d*x+c)*(-(cos(d*x+c
)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1
/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c
))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a^2-30*cos(d*x+c)^2*sin(d*x+c)*
(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/si
n(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)
-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a*b-15*cos(d*x+c)^2
*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+si
n(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((
-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b^2+15*
cos(d*x+c)^2*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*Ellip
ticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*
((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*a^2-30*cos(d*x+c)^2*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))
^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1
/2*2^(1/2))*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/s
in(d*x+c))^(1/2)*a*b-15*cos(d*x+c)^2*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))
/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)
,1/2+1/2*I,1/2*2^(1/2))*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*b^2+30*cos(d*x+c)^3*2^(1/2)*a^2-36*cos(d*x+c)^
3*2^(1/2)*b^2+20*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)*a*b-30*cos(d*x+c)^2*2^(1/2
)*a^2+36*cos(d*x+c)^2*2^(1/2)*b^2-20*cos(d*x+c)*sin(d*x+c)*2^(1/2)*a*b+6*co
s(d*x+c)*2^(1/2)*b^2-6*2^(1/2)*b^2)*(cos(d*x+c)+1)^2/(cos(d*x+c)/sin(d*x+c)
)^(3/2)/cos(d*x+c)/sin(d*x+c)^5*2^(1/2)
```

Maxima [A]
time = 0.54, size = 212, normalized size = 0.85

$$\frac{8\left(3b^2+\frac{10ab}{\tan(dx+c)}+\frac{15(a^2-b^2)}{\tan(dx+c)^2}\right)\tan(dx+c)^{\frac{5}{2}}+30\sqrt{2}(a^2+2ab-b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+30\sqrt{2}(a^2+2ab-b^2)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-15\sqrt{2}(a^2-2ab-b^2)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+15\sqrt{2}(a^2-2ab-b^2)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^2/cot(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/60*(8*(3*b^2 + 10*a*b/tan(d*x + c) + 15*(a^2 - b^2)/tan(d*x + c)^2)*tan(d
*x + c)^(5/2) + 30*sqrt(2)*(a^2 + 2*a*b - b^2)*arctan(1/2*sqrt(2)*(sqrt(2)
+ 2/sqrt(tan(d*x + c)))) + 30*sqrt(2)*(a^2 + 2*a*b - b^2)*arctan(-1/2*sqrt(
2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - 15*sqrt(2)*(a^2 - 2*a*b - b^2)*log(s
qrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + 15*sqrt(2)*(a^2 - 2*a*b -
 b^2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(d*x+c))^2/cot(d*x+c)^(3/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(c+dx))^2}{\cot^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(d*x+c))**2/cot(d*x+c)**(3/2),x)

[Out] Integral((a + b*tan(c + d*x))**2/cot(c + d*x)**(3/2), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(d*x+c))^2/cot(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((b*tan(d*x + c) + a)^2/cot(d*x + c)^(3/2), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\tan(c+d\,x))^2}{\cot(c+d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tan(c + d*x))^2/cot(c + d*x)^(3/2),x)

[Out] int((a + b*tan(c + d*x))^2/cot(c + d*x)^(3/2), x)

3.812 $\int \frac{(a+b\tan(c+dx))^2}{\cot^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=268

$$-\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}+\frac{}{7c}$$

```
[Out] 2/7*b^2/d/cot(d*x+c)^(7/2)+4/5*a*b/d/cot(d*x+c)^(5/2)+2/3*(a^2-b^2)/d/cot(d
*x+c)^(3/2)+1/2*(a^2-2*a*b-b^2)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/
2)+1/2*(a^2-2*a*b-b^2)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)-1/4*(a^
2+2*a*b-b^2)*ln(1+cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)+1/4*(a^2+2
*a*b-b^2)*ln(1+cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)-4*a*b/d/cot(d
*x+c)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3754, 3623, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d}+\frac{2(a^2-b^2)}{3d\cot^{\frac{3}{2}}(c+dx)}-\frac{(a^2+2ab-b^2)\log\left(\cot(c+dx)-\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{(a^2+2ab-b^2)\log\left(\cot(c+dx)+\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{4ab}{5d\cot^{\frac{5}{2}}(c+dx)}-\frac{4ab}{d\sqrt{\cot(c+dx)}}+\frac{2b^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^2/Cot[c + d*x]^(5/2),x]
```

```
[Out] -(((a^2 - 2*a*b - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*d))
 + ((a^2 - 2*a*b - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*d)
 + (2*b^2)/(7*d*Cot[c + d*x]^(7/2)) + (4*a*b)/(5*d*Cot[c + d*x]^(5/2)) + (2
*(a^2 - b^2))/(3*d*Cot[c + d*x]^(3/2)) - (4*a*b)/(d*Sqrt[Cot[c + d*x]]) - (
(a^2 + 2*a*b - b^2)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*
Sqrt[2]*d) + ((a^2 + 2*a*b - b^2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[
c + d*x]])/(2*Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
```

```
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{(a+b\tan(c+dx))^2}{\cot^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b+a\cot(c+dx))^2}{\cot^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2b^2}{7d\cot^{\frac{7}{2}}(c+dx)} + \int \frac{2ab+(a^2-b^2)\cot(c+dx)}{\cot^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2b^2}{7d\cot^{\frac{7}{2}}(c+dx)} + \frac{4ab}{5d\cot^{\frac{5}{2}}(c+dx)} + \int \frac{a^2-b^2-2ab\cot(c+dx)}{\cot^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2b^2}{7d\cot^{\frac{7}{2}}(c+dx)} + \frac{4ab}{5d\cot^{\frac{5}{2}}(c+dx)} + \frac{2(a^2-b^2)}{3d\cot^{\frac{3}{2}}(c+dx)} + \int \frac{-2ab-(a^2-b^2)}{\cot^{\frac{3}{2}}(c+}$$

$$= \frac{2b^2}{7d\cot^{\frac{7}{2}}(c+dx)} + \frac{4ab}{5d\cot^{\frac{5}{2}}(c+dx)} + \frac{2(a^2-b^2)}{3d\cot^{\frac{3}{2}}(c+dx)} - \frac{4ab}{d\sqrt{\cot(c+dx)}} + \int$$

$$= \frac{2b^2}{7d\cot^{\frac{7}{2}}(c+dx)} + \frac{4ab}{5d\cot^{\frac{5}{2}}(c+dx)} + \frac{2(a^2-b^2)}{3d\cot^{\frac{3}{2}}(c+dx)} - \frac{4ab}{d\sqrt{\cot(c+dx)}} + \frac{2}{}$$

$$= \frac{2b^2}{7d\cot^{\frac{7}{2}}(c+dx)} + \frac{4ab}{5d\cot^{\frac{5}{2}}(c+dx)} + \frac{2(a^2-b^2)}{3d\cot^{\frac{3}{2}}(c+dx)} - \frac{4ab}{d\sqrt{\cot(c+dx)}} + \frac{(}{}$$

$$= \frac{2b^2}{7d\cot^{\frac{7}{2}}(c+dx)} + \frac{4ab}{5d\cot^{\frac{5}{2}}(c+dx)} + \frac{2(a^2-b^2)}{3d\cot^{\frac{3}{2}}(c+dx)} - \frac{4ab}{d\sqrt{\cot(c+dx)}} + \frac{(}{}$$

$$= \frac{2b^2}{7d\cot^{\frac{7}{2}}(c+dx)} + \frac{4ab}{5d\cot^{\frac{5}{2}}(c+dx)} + \frac{2(a^2-b^2)}{3d\cot^{\frac{3}{2}}(c+dx)} - \frac{4ab}{d\sqrt{\cot(c+dx)}} - \frac{(}{}$$

$$= -\frac{(a^2-2ab-b^2)\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d} + \frac{(a^2-2ab-b^2)\tan^{-1}\left(1\right.}{\sqrt{2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order

3 in optimal.
time = 0.40, size = 80, normalized size = 0.30

$$\frac{2\left(-5(a^2-b^2)\ {}_2F_1\left(-\frac{7}{4},1;-\frac{3}{4};-\cot^2(c+dx)\right)+a\left(5a+14b\cot(c+dx)\ {}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\cot^2(c+dx)\right)\right)\right)}{35d\cot^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^2/Cot[c + d*x]^(5/2),x]
```

```
[Out] (2*(-5*(a^2 - b^2)*Hypergeometric2F1[-7/4, 1, -3/4, -Cot[c + d*x]^2] + a*(5
*a + 14*b*Cot[c + d*x]*Hypergeometric2F1[-5/4, 1, -1/4, -Cot[c + d*x]^2])))
/(35*d*Cot[c + d*x]^(7/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 55.30, size = 1903, normalized size = 7.10

method	result	size
default	Expression too large to display	1903

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^2/cot(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/210/d*(-1+cos(d*x+c))*(210*I*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(
d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*
x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3*a*b+105*I*EllipticPi((-(cos(d*x+c)-1-s
in(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)*a^2*((-1+cos
(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(
cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3+105*I*EllipticPi((-
(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(d*x+
c)*b^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*
x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3-210*
I*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^
(1/2))*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x
+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*
x+c)^3*a*b-105*I*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1
/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)*b^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((c
os(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d
*x+c))^(1/2)*cos(d*x+c)^3-105*I*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(
d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(d*x+c)*a^2*((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-si
n(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3+105*EllipticPi((-(cos(d*x+c)-1-sin
(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)*a^2*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(co
s(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3+210*EllipticPi((-(cos
```

```
(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)*(
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1
/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3*a*b-105*Elli
pticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))
*sin(d*x+c)*b^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c
))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+
c)^3-420*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2
))*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))
/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)
^3*a*b+105*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2
*I,1/2*2^(1/2))*sin(d*x+c)*a^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x
+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))
^(1/2)*cos(d*x+c)^3+210*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^
(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/si
n(d*x+c))^(1/2)*cos(d*x+c)^3*a*b-105*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))
/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(d*x+c)*b^2*((-1+cos(d*x+c))/s
in(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)
-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3+504*2^(1/2)*cos(d*x+c)^4*a*b-
70*sin(d*x+c)*2^(1/2)*a^2*cos(d*x+c)^3+100*sin(d*x+c)*2^(1/2)*b^2*cos(d*x+c
)^3-504*2^(1/2)*cos(d*x+c)^3*a*b+70*sin(d*x+c)*2^(1/2)*a^2*cos(d*x+c)^2-100
*sin(d*x+c)*2^(1/2)*b^2*cos(d*x+c)^2-84*2^(1/2)*cos(d*x+c)^2*a*b-30*sin(d*x
+c)*2^(1/2)*b^2*cos(d*x+c)+84*2^(1/2)*cos(d*x+c)*a*b+30*sin(d*x+c)*2^(1/2)*
b^2)*(cos(d*x+c)+1)^2/(cos(d*x+c)/sin(d*x+c))^(5/2)/cos(d*x+c)/sin(d*x+c)^6
*2^(1/2)
```

Maxima [A]
time = 0.54, size = 224, normalized size = 0.84

$$\frac{8\left(15\,b^2+\frac{42\,ab}{\tan(dx+c)}-\frac{210\,ab}{\tan(dx+c)^3}+\frac{35\,(a^2-b^2)}{\tan(dx+c)^2}\right)\tan\left(dx+c\right)^{\frac{7}{2}}+210\,\sqrt{2}\,(a^2-2\,ab-b^2)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+210\,\sqrt{2}\,(a^2-2\,ab-b^2)\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+105\,\sqrt{2}\,(a^2+2\,ab-b^2)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)-105\,\sqrt{2}\,(a^2+2\,ab-b^2)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^2/cot(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/420*(8*(15*b^2 + 42*a*b/tan(d*x + c) - 210*a*b/tan(d*x + c)^3 + 35*(a^2 -
 b^2)/tan(d*x + c)^2)*tan(d*x + c)^(7/2) + 210*sqrt(2)*(a^2 - 2*a*b - b^2)*
arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 210*sqrt(2)*(a^2 - 2
*a*b - b^2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + 105*sqr
t(2)*(a^2 + 2*a*b - b^2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) +
1) - 105*sqrt(2)*(a^2 + 2*a*b - b^2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/ta
n(d*x + c) + 1))/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^2/cot(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan{(c+dx)})^2}{\cot^{\frac{5}{2}}{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**2/cot(d*x+c)**(5/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2/cot(c + d*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^2/cot(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^2/cot(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\tan(c+d\,x))^2}{\cot(c+d\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^2/cot(c + d*x)^(5/2),x)
```

```
[Out] int((a + b*tan(c + d*x))^2/cot(c + d*x)^(5/2), x)
```

3.813 $\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=299

$$\frac{(a-b)\left(a^2+4ab+b^2\right)\operatorname{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a-b)\left(a^2+4ab+b^2\right)\operatorname{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c$$

```
[Out] 2/3*a*(a^2-3*b^2)*cot(d*x+c)^(3/2)/d-32/35*a^2*b*cot(d*x+c)^(5/2)/d-2/7*a^2
*cot(d*x+c)^(5/2)*(b+a*cot(d*x+c))/d-1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(-1+2^
(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)-1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(1+2^(1/2
)*cot(d*x+c)^(1/2))/d*2^(1/2)-1/4*(a+b)*(a^2-4*a*b+b^2)*ln(1+cot(d*x+c)-2^(
1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)+1/4*(a+b)*(a^2-4*a*b+b^2)*ln(1+cot(d*x+c)+
2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)+2*b*(3*a^2-b^2)*cot(d*x+c)^(1/2)/d
```

Rubi [A]

time = 0.32, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3754, 3647, 3711, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a-b)\left(a^2+4ab+b^2\right)\operatorname{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a-b)\left(a^2+4ab+b^2\right)\operatorname{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d}+\frac{2a\left(a^2-3b^2\right)\cot^{\frac{3}{2}}(c+dx)}{3d}+\frac{2b\left(3a^2-b^2\right)\sqrt{\cot(c+dx)}}{d}-\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{2a^2\cot^{\frac{5}{2}}(c+dx)(a\cot(c+dx)+b)}{7d}-\frac{32a^2b\cot^{\frac{5}{2}}(c+dx)}{35d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(9/2)*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[
2]*d) - ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]
)/(Sqrt[2]*d) + (2*b*(3*a^2 - b^2)*Sqrt[Cot[c + d*x]])/d + (2*a*(a^2 - 3*b^
2)*Cot[c + d*x]^(3/2))/(3*d) - (32*a^2*b*Cot[c + d*x]^(5/2))/(35*d) - (2*a^
2*Cot[c + d*x]^(5/2)*(b + a*Cot[c + d*x]))/(7*d) - ((a + b)*(a^2 - 4*a*b +
b^2)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*d) + ((
a + b)*(a^2 - 4*a*b + b^2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x
]])/(2*Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
```

```
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^3\,dx = \int \cot^{\frac{3}{2}}(c+dx)(b+a\cot(c+dx))^3\,dx$$

$$= -\frac{2a^2\cot^{\frac{5}{2}}(c+dx)(b+a\cot(c+dx))}{7d} - \frac{2}{7}\int \cot^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}b(5a\right.$$

$$= -\frac{32a^2b\cot^{\frac{5}{2}}(c+dx)}{35d} - \frac{2a^2\cot^{\frac{5}{2}}(c+dx)(b+a\cot(c+dx))}{7d} - \frac{2}{7}\int$$

$$= \frac{2a(a^2-3b^2)\cot^{\frac{3}{2}}(c+dx)}{3d} - \frac{32a^2b\cot^{\frac{5}{2}}(c+dx)}{35d} - \frac{2a^2\cot^{\frac{5}{2}}(c+dx}{}$$

$$= \frac{2b(3a^2-b^2)\sqrt{\cot(c+dx)}}{d} + \frac{2a(a^2-3b^2)\cot^{\frac{3}{2}}(c+dx)}{3d} - \frac{32a^2b\,c}{}$$

$$= \frac{2b(3a^2-b^2)\sqrt{\cot(c+dx)}}{d} + \frac{2a(a^2-3b^2)\cot^{\frac{3}{2}}(c+dx)}{3d} - \frac{32a^2b\,c}{}$$

$$= \frac{2b(3a^2-b^2)\sqrt{\cot(c+dx)}}{d} + \frac{2a(a^2-3b^2)\cot^{\frac{3}{2}}(c+dx)}{3d} - \frac{32a^2b\,c}{}$$

$$= \frac{2b(3a^2-b^2)\sqrt{\cot(c+dx)}}{d} + \frac{2a(a^2-3b^2)\cot^{\frac{3}{2}}(c+dx)}{3d} - \frac{32a^2b\,c}{}$$

$$= \frac{2b(3a^2-b^2)\sqrt{\cot(c+dx)}}{d} + \frac{2a(a^2-3b^2)\cot^{\frac{3}{2}}(c+dx)}{3d} - \frac{32a^2b\,c}{}$$

$$= \frac{(a-b)(a^2+4ab+b^2)\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d} - \frac{(a-b)(}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 2.21, size = 229, normalized size = 0.77

$$-\frac{\frac{6}{5}a^2b\cot^{\frac{5}{2}}(c+dx)+\frac{2}{7}a^3\cot^{\frac{7}{2}}(c+dx)+\frac{2}{3}a(a^2-3b^2)\cot^{\frac{3}{2}}(c+dx)\left(-1+{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)\right)+\frac{1}{4}b(-3a^2+b^2)\left(2\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-2\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+8\sqrt{\cot(c+dx)}+\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(9/2)*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -(((6*a^2*b*Cot[c + d*x]^(5/2))/5 + (2*a^3*Cot[c + d*x]^(7/2))/7 + (2*a*(a^
2 - 3*b^2)*Cot[c + d*x]^(3/2)*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Cot[c +
 d*x]^2]))/3 + (b*(-3*a^2 + b^2)*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c +
 d*x]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 8*Sqrt[Cot[c +
 d*x]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[
2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/4)/d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 17.66, size = 9273, normalized size = 31.01

method	result	size
default	Expression too large to display	9273

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.51, size = 262, normalized size = 0.88

$$-\frac{210\,\sqrt{2}\,(a^3+3\,a^2b-3\,ab^2-b^3)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+210\,\sqrt{2}\,(a^3+3\,a^2b-3\,ab^2-b^3)\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)-105\,\sqrt{2}\,(a^3-3\,a^2b-3\,ab^2+b^3)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)+105\,\sqrt{2}\,(a^3-3\,a^2b-3\,ab^2+b^3)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)+\frac{504\,a^2b}{\tan(dx+c)^{\frac{5}{2}}}-\frac{840\,(3\,a^2b-b^3)}{\sqrt{\tan\left(dx+c\right)}}+\frac{120\,a^3}{\tan(dx+c)^{\frac{7}{2}}}-\frac{280\,(a^3-3\,ab^2)}{\tan(dx+c)^{\frac{3}{2}}}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/420*(210*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*arctan(1/2*sqrt(2)*(sqr
t(2) + 2/sqrt(tan(d*x + c)))) + 210*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)
*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - 105*sqrt(2)*(a^3 -
 3*a^2*b - 3*a*b^2 + b^3)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) +
 1) + 105*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*log(-sqrt(2)/sqrt(tan(d*x
 + c)) + 1/tan(d*x + c) + 1) + 504*a^2*b/tan(d*x + c)^(5/2) - 840*(3*a^2*b
- b^3)/sqrt(tan(d*x + c)) + 120*a^3/tan(d*x + c)^(7/2) - 280*(a^3 - 3*a*b^2
)/tan(d*x + c)^(3/2))/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(9/2)*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^3,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^3*cot(d*x + c)^(9/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c+d\,x)^{9/2}\,(a+b\tan(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(9/2)*(a + b*tan(c + d*x))^3,x)
[Out] int(cot(c + d*x)^(9/2)*(a + b*tan(c + d*x))^3, x)
```

3.814 $\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=270

$$\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(c$$

```
[Out] -8/5*a^2*b*cot(d*x+c)^(3/2)/d-2/5*a^2*cot(d*x+c)^(3/2)*(b+a*cot(d*x+c))/d-1
/2*(a+b)*(a^2-4*a*b+b^2)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)-1/2*
(a+b)*(a^2-4*a*b+b^2)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)+1/4*(a-b
)*(a^2+4*a*b+b^2)*ln(1+cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)-1/4*(
a-b)*(a^2+4*a*b+b^2)*ln(1+cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)+2*
a*(a^2-3*b^2)*cot(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.27, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3754, 3647, 3711, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d}+\frac{2a\left(a^2-3b^2\right)\sqrt{\cot(c+dx)}}{d}+\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\cot(c+dx)-\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\cot(c+dx)+\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{8a^2b\cot^{\frac{3}{2}}(c+dx)}{5d}-\frac{2a^2\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(7/2)*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[
2]*d) - ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]
)/(Sqrt[2]*d) + (2*a*(a^2 - 3*b^2)*Sqrt[Cot[c + d*x]])/d - (8*a^2*b*Cot[c +
 d*x]^(3/2))/(5*d) - (2*a^2*Cot[c + d*x]^(3/2)*(b + a*Cot[c + d*x]))/(5*d)
+ ((a - b)*(a^2 + 4*a*b + b^2)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c +
 d*x]])/(2*Sqrt[2]*d) - ((a - b)*(a^2 + 4*a*b + b^2)*Log[1 + Sqrt[2]*Sqrt[C
ot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
```

```
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^3\,dx = \int \sqrt{\cot(c+dx)}\,(b+a\cot(c+dx))^3\,dx$$
$$= -\frac{2a^2\cot^{\frac{3}{2}}(c+dx)(b+a\cot(c+dx))}{5d} - \frac{2}{5}\int \sqrt{\cot(c+dx)}\left(\frac{1}{2}b(3\right.$$
$$= -\frac{8a^2b\cot^{\frac{3}{2}}(c+dx)}{5d} - \frac{2a^2\cot^{\frac{3}{2}}(c+dx)(b+a\cot(c+dx))}{5d} - \frac{2}{5}\int$$
$$= \frac{2a(a^2-3b^2)\sqrt{\cot(c+dx)}}{d} - \frac{8a^2b\cot^{\frac{3}{2}}(c+dx)}{5d} - \frac{2a^2\cot^{\frac{3}{2}}(c+d}{}$$
$$= \frac{2a(a^2-3b^2)\sqrt{\cot(c+dx)}}{d} - \frac{8a^2b\cot^{\frac{3}{2}}(c+dx)}{5d} - \frac{2a^2\cot^{\frac{3}{2}}(c+d}{}$$
$$= \frac{2a(a^2-3b^2)\sqrt{\cot(c+dx)}}{d} - \frac{8a^2b\cot^{\frac{3}{2}}(c+dx)}{5d} - \frac{2a^2\cot^{\frac{3}{2}}(c+d}{}$$
$$= \frac{2a(a^2-3b^2)\sqrt{\cot(c+dx)}}{d} - \frac{8a^2b\cot^{\frac{3}{2}}(c+dx)}{5d} - \frac{2a^2\cot^{\frac{3}{2}}(c+d}{}$$
$$= \frac{2a(a^2-3b^2)\sqrt{\cot(c+dx)}}{d} - \frac{8a^2b\cot^{\frac{3}{2}}(c+dx)}{5d} - \frac{2a^2\cot^{\frac{3}{2}}(c+d}{}$$
$$= \frac{(a+b)\left(a^2-4ab+b^2\right)\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d} - \frac{(a+b)\,(}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 2.18, size = 225, normalized size = 0.83

$$-\frac{2a^2b\cot^{\frac{3}{2}}(c+dx)+\frac{2}{5}a^3\cot^{\frac{5}{2}}(c+dx)+\frac{2}{3}b(-3a^2+b^2)\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)-\frac{1}{4}a(a^2-3b^2)\left(2\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-2\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+8\sqrt{\cot(c+dx)}+\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(7/2)*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -((2*a^2*b*Cot[c + d*x]^(3/2) + (2*a^3*Cot[c + d*x]^(5/2))/5 + (2*b*(-3*a^2
 + b^2)*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2])
/3 - (a*(a^2 - 3*b^2)*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2
*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 8*Sqrt[Cot[c + d*x]] + Sq
rt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*Log[1 +
Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/4)/d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 18.48, size = 8774, normalized size = 32.50

method	result	size
default	Expression too large to display	8774

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.50, size = 240, normalized size = 0.89

$$-\frac{10\sqrt{2}\left(a^3-3a^2b-3ab^2+b^3\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+10\sqrt{2}\left(a^3-3a^2b-3ab^2+b^3\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+5\sqrt{2}\left(a^3+3a^2b-3ab^2-b^3\right)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)-5\sqrt{2}\left(a^3+3a^2b-3ab^2-b^3\right)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)+\frac{40a^2b}{\tan\left(dx+c\right)^{\frac{3}{2}}}+\frac{8a^3}{\tan\left(dx+c\right)^{\frac{5}{2}}}-\frac{40\left(a^3-3ab^2\right)}{\sqrt{\tan\left(dx+c\right)}}}{20d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/20*(10*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*arctan(1/2*sqrt(2)*(sqrt(
2) + 2/sqrt(tan(d*x + c)))) + 10*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*ar
ctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + 5*sqrt(2)*(a^3 + 3*a^
2*b - 3*a*b^2 - b^3)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) -
 5*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*log(-sqrt(2)/sqrt(tan(d*x + c))
+ 1/tan(d*x + c) + 1) + 40*a^2*b/tan(d*x + c)^(3/2) + 8*a^3/tan(d*x + c)^(5
/2) - 40*(a^3 - 3*a*b^2)/sqrt(tan(d*x + c)))/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(7/2)*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5987 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^3*cot(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c+d\,x)^{7/2}\,(a+b\tan(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(7/2)*(a + b*tan(c + d*x))^3,x)
```

```
[Out] int(cot(c + d*x)^(7/2)*(a + b*tan(c + d*x))^3, x)
```

3.815 $\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=245

$$-\frac{(a-b)\left(a^2+4ab+b^2\right)\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a-b)\left(a^2+4ab+b^2\right)\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}$$

```
[Out] 1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)+1/2
*(a-b)*(a^2+4*a*b+b^2)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)+1/4*(a+
b)*(a^2-4*a*b+b^2)*ln(1+cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)-1/4*
(a+b)*(a^2-4*a*b+b^2)*ln(1+cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)-1
6/3*a^2*b*cot(d*x+c)^(1/2)/d-2/3*a^2*(b+a*cot(d*x+c))*cot(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.24, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3754, 3647, 3711, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{(a-b)\left(a^2+4ab+b^2\right)\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a-b)\left(a^2+4ab+b^2\right)\mathrm{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d}+\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{2a^2\sqrt{\cot(c+dx)}\,(a\cot(c+dx)+b)}{3d}-\frac{16a^2b\sqrt{\cot(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -(((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqr
t[2]*d)) + ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x
]]])/(Sqrt[2]*d) - (16*a^2*b*Sqrt[Cot[c + d*x]])/(3*d) - (2*a^2*Sqrt[Cot[c
+ d*x]]*(b + a*Cot[c + d*x]))/(3*d) + ((a + b)*(a^2 - 4*a*b + b^2)*Log[1 -
Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*d) - ((a + b)*(a^2 -
 4*a*b + b^2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2
]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^3\,dx = \int \frac{(b+a\cot(c+dx))^3}{\sqrt{\cot(c+dx)}}\,dx$$

$$= -\frac{2a^2\sqrt{\cot(c+dx)}\,(b+a\cot(c+dx))}{3d} - \frac{2}{3}\int \frac{\frac{1}{2}b(a^2-3b^2)+\frac{3}{2}a$$

$$= -\frac{16a^2b\sqrt{\cot(c+dx)}}{3d} - \frac{2a^2\sqrt{\cot(c+dx)}\,(b+a\cot(c+dx))}{3d} -$$

$$= -\frac{16a^2b\sqrt{\cot(c+dx)}}{3d} - \frac{2a^2\sqrt{\cot(c+dx)}\,(b+a\cot(c+dx))}{3d} -$$

$$= -\frac{16a^2b\sqrt{\cot(c+dx)}}{3d} - \frac{2a^2\sqrt{\cot(c+dx)}\,(b+a\cot(c+dx))}{3d} -$$

$$= -\frac{16a^2b\sqrt{\cot(c+dx)}}{3d} - \frac{2a^2\sqrt{\cot(c+dx)}\,(b+a\cot(c+dx))}{3d} +$$

$$= -\frac{16a^2b\sqrt{\cot(c+dx)}}{3d} - \frac{2a^2\sqrt{\cot(c+dx)}\,(b+a\cot(c+dx))}{3d} +$$

$$= -\frac{(a-b)\,(a^2+4ab+b^2)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d} + \frac{(a-$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.03, size = 194, normalized size = 0.79

$$-\frac{2\left(9a^2b\sqrt{\cot(c+dx)}+a^3\cot^{\frac{3}{2}}(c+dx)-a(a^2-3b^2)\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)-\frac{3b(-3a^2+b^2)\left(2\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-2\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)}{4\sqrt{2}}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-2*(9*a^2*b*Sqrt[Cot[c + d*x]] + a^3*Cot[c + d*x]^(3/2) - a*(a^2 - 3*b^2)*
Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2] - (3*b*(
-3*a^2 + b^2)*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*ArcTan[1 + Sqrt
[2]*Sqrt[Cot[c + d*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]
] - Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/(4*Sqrt[2])))/(3*d
)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 17.94, size = 4528, normalized size = 18.48

method	result	size
default	Expression too large to display	4528

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/d*(-3*I*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+s
in(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(
(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*
x+c)*sin(d*x+c)*b^3-3*I*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos
(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1
/2))*cos(d*x+c)*sin(d*x+c)*a^3+3*I*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^
(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2
*I,1/2*2^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3+9*I*(-(cos(d*x+c)-1-sin(d*x+c))/s
in(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1
/2),1/2-1/2*I,1/2*2^(1/2))*sin(d*x+c)*a^2*b-9*I*(-(cos(d*x+c)-1-sin(d*x+c))
/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*
x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^
(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(d*x+c)*a*b^2-9*I*(-(cos(d*x+c)-1-sin(d*x+c
))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)*a^2*b+9*I*(-(cos(d*x+c)-1-sin(d*x
+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+
c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)*a*b^2+3*I*(-(cos(d*x+c)-1-sin(d
*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*
x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3+2*cos(d*x+c)^2
*2^(1/2)*a^3+18*cos(d*x+c)*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c
```

```
))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2
^(1/2))*a*b^2-9*cos(d*x+c)*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c
))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-
1/2*I,1/2*2^(1/2))*a^2*b-9*cos(d*x+c)*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c)
)/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))
^(1/2),1/2-1/2*I,1/2*2^(1/2))*a*b^2-9*cos(d*x+c)*sin(d*x+c)*(-(cos(d*x+c)-1
-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/
sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^2*b-9*cos(d*x+c)*sin(d*x+c)*(-(c
os(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*
x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d*x+c)-1-s
in(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a*b^2+18*cos(d*x+c)*sin
(d*x+c)*2^(1/2)*a^2*b-6*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^
(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1
/2))*a^3+3*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d
*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*El
lipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2
))*a^3+3*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x
+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Elli
pticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))
*b^3+3*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c
)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellipt
icPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a
^3+3*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-
1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Elliptic
Pi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*b^3
+3*I*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c
))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d
*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(d*x+c)*a^3
-3*I*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c
))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d
*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(d*x+c)*b^3
-3*I*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c
))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d
*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)*a^3
+3*I*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c
))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(cos(d
*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)*a^3
+3*I*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c
))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c...
```

Maxima [A]

time = 0.52, size = 220, normalized size = 0.90

$$\frac{6\sqrt{2}\left(a^3+3a^2b-3ab^2-b^3\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+6\sqrt{2}\left(a^3+3a^2b-3ab^2-b^3\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)-3\sqrt{2}\left(a^3-3a^2b-3ab^2+b^3\right)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)+3\sqrt{2}\left(a^3-3a^2b-3ab^2+b^3\right)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)-\frac{72a^2b}{\sqrt{\tan\left(dx+c\right)}}-\frac{8a^3}{\tan\left(dx+c\right)^{\frac{3}{2}}}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/12*(6*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*arctan(1/2*sqrt(2)*(sqrt(2)
 + 2/sqrt(tan(d*x + c)))) + 6*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - 3*sqrt(2)*(a^3 - 3*a^2*b
 - 3*a*b^2 + b^3)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + 3*
sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1
/tan(d*x + c) + 1) - 72*a^2*b/sqrt(tan(d*x + c)) - 8*a^3/tan(d*x + c)^(3/2)
)/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(5/2)*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^3*cot(d*x + c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c+d\,x)^{5/2}\,{\left(a+b\tan(c+d\,x)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))^3,x)
```

```
[Out] int(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))^3, x)
```

3.816 $\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=245

$$-\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot$$

```
[Out] 1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)+1/2
*(a+b)*(a^2-4*a*b+b^2)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)-1/4*(a-
b)*(a^2+4*a*b+b^2)*ln(1+cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)+1/4*
(a-b)*(a^2+4*a*b+b^2)*ln(1+cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)+2
*b^2*(b+a*cot(d*x+c))/d/cot(d*x+c)^(1/2)-2*a*(a^2+b^2)*cot(d*x+c)^(1/2)/d
```

Rubi [A]

time = 0.24, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3754, 3646, 3711, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}+\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d}-\frac{2a\left(a^2+b^2\right)\sqrt{\cot(c+dx)}}{d}-\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{2b^2(a\cot(c+dx)+b)}{d\sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -(((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqr
t[2]*d)) + ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x
]]])/(Sqrt[2]*d) - (2*a*(a^2 + b^2)*Sqrt[Cot[c + d*x]])/d + (2*b^2*(b + a*C
ot[c + d*x]))/(d*Sqrt[Cot[c + d*x]]) - ((a - b)*(a^2 + 4*a*b + b^2)*Log[1 -
 Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*d) + ((a - b)*(a^2
+ 4*a*b + b^2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[
2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3\,dx = \int \frac{(b+a\cot(c+dx))^3}{\cot^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2b^2(b+a\cot(c+dx))}{d\sqrt{\cot(c+dx)}} - 2\int \frac{-2ab^2 - \frac{1}{2}b(3a^2-b^2)\cot(c+dx) -}{\sqrt{\cot(c+dx}}$$

$$= -\frac{2a(a^2+b^2)\sqrt{\cot(c+dx)}}{d} + \frac{2b^2(b+a\cot(c+dx))}{d\sqrt{\cot(c+dx)}} - 2\int \frac{1}{2}a($$

$$= -\frac{2a(a^2+b^2)\sqrt{\cot(c+dx)}}{d} + \frac{2b^2(b+a\cot(c+dx))}{d\sqrt{\cot(c+dx)}} - \frac{4\text{Subst}\big(}{}$$

$$= -\frac{2a(a^2+b^2)\sqrt{\cot(c+dx)}}{d} + \frac{2b^2(b+a\cot(c+dx))}{d\sqrt{\cot(c+dx)}} + \frac{((a+b)}{}$$

$$= -\frac{2a(a^2+b^2)\sqrt{\cot(c+dx)}}{d} + \frac{2b^2(b+a\cot(c+dx))}{d\sqrt{\cot(c+dx)}} + \frac{((a+b)}{}$$

$$= -\frac{2a(a^2+b^2)\sqrt{\cot(c+dx)}}{d} + \frac{2b^2(b+a\cot(c+dx))}{d\sqrt{\cot(c+dx)}} - \frac{(a-b)\,(}{}$$

$$= -\frac{(a+b)\,(a^2-4ab+b^2)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d} + \frac{(a+}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 3.20, size = 188, normalized size = 0.77

$$-\frac{2\left(-3a^2b+a^3\cot(c+dx)-b(-3a^2+b^2)\ {}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\cot^2(c+dx)\right)+\frac{a(a^2-3b^2)\sqrt{\cot(c+dx)}\left(2\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-2\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)}{4\sqrt{2}}\right)}{d\sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-2*(-3*a^2*b + a^3*Cot[c + d*x] - b*(-3*a^2 + b^2)*Hypergeometric2F1[-1/4,
 1, 3/4, -Cot[c + d*x]^2] + (a*(a^2 - 3*b^2)*Sqrt[Cot[c + d*x]]*(2*ArcTan[1
 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] +
 Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[
Cot[c + d*x]] + Cot[c + d*x]]))/(4*Sqrt[2])))/(d*Sqrt[Cot[c + d*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 52.28, size = 4227, normalized size = 17.25

method	result	size
default	Expression too large to display	4227

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(-3*I*cos(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2),1/2+1/2*I,1/2*2^(1/2))*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
)*a^2*b-2*cos(d*x+c)*2^(1/2)*a^3-3*I*cos(d*x+c)*EllipticPi(((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((cos(d*x+c)-1+sin(d*x+
c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2)*a*b^2+3*I*cos(d*x+c)*((cos(d*x+c)-1+sin(d*x+c))/
sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2)*a^2*b+3*I*cos(d*x+c)*((cos(d*x+c)-1+sin(d*x+c))/
sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2)*a*b^2+EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))
^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*
x+c))^(1/2)*a^3-EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2
+1/2*I,1/2*2^(1/2))*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*b^3+(
(cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2
^(1/2))*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*a^3-((cos(d*x+c)-1+sin
(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((
```

1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*b^3+2*((cos(d*x+c)-1+sin(d*x+c))/sin(d*
x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*b^3-I*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
),1/2-1/2*I,1/2*2^(1/2))*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*a^3-I
*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^
(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2
2^(1/2))((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*b^3+I*EllipticPi(((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((cos(d*x+
c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((1-c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*a^3+I*EllipticPi(((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((cos(d*x+c)-1+sin(d*x+c)
)/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2)*b^3+cos(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((cos(d*x+c)-1+sin(d*x+c))/sin(
d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)*a^3-cos(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c)
)^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2)*b^3+cos(d*x+c)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2)*a^3-cos(d*x+c)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos
(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2-1/2*I,1/2*2^(1/2))*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
*b^3+2*cos(d*x+c)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x
+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2),1/2*2^(1/2))*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*b^3+3*Elliptic
Pi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((co
s(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*a^2*b-3*EllipticPi(((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((cos(d*x+c)-1+si
n(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*a*b^2+3*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x
+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((1-cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2)*a^2*b-3*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2)*a*b^2-6*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),
1/2*2^(1/2))*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*a^2*b-3*I*Ellipti
cPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2...

Maxima [A]
time = 0.50, size = 218, normalized size = 0.89

$$\frac{8\,b^3\sqrt{\tan\left(dx+c\right)}+2\,\sqrt{2}\,(a^3-3\,a^2b-3\,ab^2+b^3)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+2\,\sqrt{2}\,(a^3-3\,a^2b-3\,ab^2+b^3)\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+\sqrt{2}\,(a^3+3\,a^2b-3\,ab^2-b^3)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}\,(a^3+3\,a^2b-3\,ab^2-b^3)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)-\frac{8\,a^3}{\sqrt{\tan\left(dx+c\right)}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(8*b^3*sqrt(tan(d*x + c)) + 2*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*a
rctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*(a^3 - 3*a^
2*b - 3*a*b^2 + b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c))))
+ sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*log(sqrt(2)/sqrt(tan(d*x + c)) +
1/tan(d*x + c) + 1) - sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*log(-sqrt(2)/
sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - 8*a^3/sqrt(tan(d*x + c)))/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan\left(c+dx\right)\right)^{3}\cot^{\frac{3}{2}}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(3/2)*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*cot(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^3*cot(d*x + c)^(3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c+d\,x)^{3/2}\,\left(a+b\tan(c+d\,x)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^3,x)
[Out] int(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^3, x)
```

3.817 $\int \sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=245

$$\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(c-$$

```
[Out] 2/3*b^2*(b+a*cot(d*x+c))/d/cot(d*x+c)^(3/2)-1/2*(a-b)*(a^2+4*a*b+b^2)*arcta
n(-1+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)-1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(1
+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)-1/4*(a+b)*(a^2-4*a*b+b^2)*ln(1+cot(d*x
+c)-2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)+1/4*(a+b)*(a^2-4*a*b+b^2)*ln(1+cot(
d*x+c)+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)+16/3*a*b^2/d/cot(d*x+c)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3754, 3646, 3709, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d}-\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\cot(c+dx)-\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\cot(c+dx)+\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{2b^2(a\cot(c+dx)+b)}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{16ab^2}{3d\sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[
2]*d) - ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]
)/(Sqrt[2]*d) + (16*a*b^2)/(3*d*Sqrt[Cot[c + d*x]]) + (2*b^2*(b + a*Cot[c +
 d*x]))/(3*d*Cot[c + d*x]^(3/2)) - ((a + b)*(a^2 - 4*a*b + b^2)*Log[1 - Sqr
t[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*d) + ((a + b)*(a^2 - 4*
a*b + b^2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*d
)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^3\,dx = \int \frac{(b+a\cot(c+dx))^3}{\cot^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2b^2(b+a\cot(c+dx))}{3d\cot^{\frac{3}{2}}(c+dx)} - \frac{2}{3}\int \frac{-4ab^2-\frac{3}{2}b(3a^2-b^2)\cot(c+dx)\cdots}{\cot^{\frac{3}{2}}(c+\cdots}$$
$$= \frac{16ab^2}{3d\sqrt{\cot(c+dx)}} + \frac{2b^2(b+a\cot(c+dx))}{3d\cot^{\frac{3}{2}}(c+dx)} - \frac{2}{3}\int \frac{-\frac{3}{2}b(3a^2-b^2)\cdots}{\cdots}$$
$$= \frac{16ab^2}{3d\sqrt{\cot(c+dx)}} + \frac{2b^2(b+a\cot(c+dx))}{3d\cot^{\frac{3}{2}}(c+dx)} - \frac{4\text{Subst}\left(\int \frac{\frac{3}{2}b(3a^2-b^2\cdots}{\cdots}\right.}{\cdots}$$
$$= \frac{16ab^2}{3d\sqrt{\cot(c+dx)}} + \frac{2b^2(b+a\cot(c+dx))}{3d\cot^{\frac{3}{2}}(c+dx)} + \frac{((a+b)(a^2-4ab\cdots}{\cdots}$$
$$= \frac{16ab^2}{3d\sqrt{\cot(c+dx)}} + \frac{2b^2(b+a\cot(c+dx))}{3d\cot^{\frac{3}{2}}(c+dx)} - \frac{((a+b)(a^2-4ab\cdots}{\cdots}$$
$$= \frac{16ab^2}{3d\sqrt{\cot(c+dx)}} + \frac{2b^2(b+a\cot(c+dx))}{3d\cot^{\frac{3}{2}}(c+dx)} - \frac{(a+b)(a^2-4ab+\cdots}{\cdots}$$
$$= \frac{(a-b)(a^2+4ab+b^2)\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d} - \frac{(a-b\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.35, size = 100, normalized size = 0.41

$$\frac{(-6a^2b+2b^3)\ {}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\cot^2(c+dx)\right)+6a\left(a(b+a\cot(c+dx))-(a^2-3b^2)\cot(c+dx)\ {}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\cot^2(c+dx)\right)\right)}{3d\cot^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((-6*a^2*b + 2*b^3)*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[c + d*x]^2] + 6*a*
(a*(b + a*Cot[c + d*x]) - (a^2 - 3*b^2)*Cot[c + d*x]*Hypergeometric2F1[-1/4
, 1, 3/4, -Cot[c + d*x]^2]))/(3*d*Cot[c + d*x]^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 50.57, size = 2370, normalized size = 9.67

method	result	size
default	Expression too large to display	2370

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/d*(-1+cos(d*x+c))*(-3*I*b^3*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*
x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2)*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1
/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)+9*I*a^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)*b-9*I*b^2*((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)*a+3*I*a^3*((-1+cos(d*x
+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)+3*I*b^3*((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2
)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi(((1-co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)-9*
I*a^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x
+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*Elliptic
Pi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(
d*x+c)*b+9*I*b^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+
c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+
c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^
(1/2))*cos(d*x+c)*a-3*I*a^3*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)
-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
```

```
2)*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1
/2*I,1/2*2^(1/2))*cos(d*x+c)-3*a^3*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos
(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)+9*a^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)*b+9*b^2*((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)*a-3*b^3*((-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)-3*a^3*((-1+cos
(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi(((1-cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)+9*a^2*(
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1
/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi(((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)*
b+9*b^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d
*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*Ellipt
icPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*co
s(d*x+c)*a-3*b^3*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+
c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+
c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^
(1/2))*cos(d*x+c)+6*a^3*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+s
in(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*s
in(d*x+c)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2
))*cos(d*x+c)-18*b^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(
d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(
d*x+c)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*
cos(d*x+c)*a+2*cos(d*x+c)*sin(d*x+c)*2^(1/2)*b^3+18*2^(1/2)*b^2*cos(d*x+c)^
2*a-2*2^(1/2)*sin(d*x+c)*b^3-18*cos(d*x+c)*2^(1/2)*a*b^2)*(cos(d*x+c)+1)^2*
(cos(d*x+c)/sin(d*x+c))^(1/2)/sin(d*x+c)^3/cos(d*x+c)^2*2^(1/2)
```

<u>Maxima</u> [A]
time = 0.58, size = 221, normalized size = 0.90

$$-\frac{6\sqrt{2}\left(a^3+3a^2b-3ab^2-b^3\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+6\sqrt{2}\left(a^3+3a^2b-3ab^2-b^3\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)-3\sqrt{2}\left(a^3-3a^2b-3ab^2+b^3\right)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)+3\sqrt{2}\left(a^3-3a^2b-3ab^2+b^3\right)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)-8\left(b^3+\frac{9ab^2}{\tan\left(dx+c\right)}\right)\tan\left(dx+c\right)^{\frac{3}{2}}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/12*(6*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*arctan(1/2*sqrt(2)*(sqrt(2
) + 2/sqrt(tan(d*x + c)))) + 6*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*arct
```

```
an(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - 3*sqrt(2)*(a^3 - 3*a^2*
b - 3*a*b^2 + b^3)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + 3
*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*log(-sqrt(2)/sqrt(tan(d*x + c)) +
1/tan(d*x + c) + 1) - 8*(b^3 + 9*a*b^2/tan(d*x + c))*tan(d*x + c)^(3/2))/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))^3\sqrt{\cot(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(1/2)*(a+b*tan(d*x+c))**3,x)
[Out] Integral((a + b*tan(c + d*x))**3*sqrt(cot(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^3,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^3*sqrt(cot(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cot(c+d\,x)}\,(a+b\tan(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^3,x)
[Out] int(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^3, x)
```

3.818 $\int \frac{(a+b\tan(c+dx))^3}{\sqrt{\cot(c+dx)}}\,dx$

Optimal. Leaf size=272

$$\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c-$$

```
[Out] 8/5*a*b^2/d/cot(d*x+c)^(3/2)+2/5*b^2*(b+a*cot(d*x+c))/d/cot(d*x+c)^(5/2)-1/
2*(a+b)*(a^2-4*a*b+b^2)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)-1/2*(
a+b)*(a^2-4*a*b+b^2)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)+1/4*(a-b)
*(a^2+4*a*b+b^2)*ln(1+cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)-1/4*(a
-b)*(a^2+4*a*b+b^2)*ln(1+cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)+2*b
*(3*a^2-b^2)/d/cot(d*x+c)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3754, 3646, 3709, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d}-\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d}+\frac{2b\left(3a^2-b^2\right)}{d\sqrt{\cot(c+dx)}}+\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{2b^2(a\cot(c+dx)+b)}{5d\cot^{\frac{5}{2}}(c+dx)}+\frac{8ab^2}{5d\cot^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^3/Sqrt[Cot[c + d*x]],x]
```

```
[Out] ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[
2]*d) - ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]
)/(Sqrt[2]*d) + (8*a*b^2)/(5*d*Cot[c + d*x]^(3/2)) + (2*b*(3*a^2 - b^2))/(d
*Sqrt[Cot[c + d*x]]) + (2*b^2*(b + a*Cot[c + d*x]))/(5*d*Cot[c + d*x]^(5/2)
) + ((a - b)*(a^2 + 4*a*b + b^2)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c
 + d*x]])/(2*Sqrt[2]*d) - ((a - b)*(a^2 + 4*a*b + b^2)*Log[1 + Sqrt[2]*Sqrt
[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
```

```
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{(a+b\tan(c+dx))^3}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(b+a\cot(c+dx))^3}{\cot^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2b^2(b+a\cot(c+dx))}{5d\cot^{\frac{5}{2}}(c+dx)} - \frac{2}{5}\int \frac{-6ab^2 - \frac{5}{2}b(3a^2-b^2)\cot(c+dx) - \frac{1}{2}a(5a^2-3}{\cot^{\frac{5}{2}}(c+dx)}$$

$$= \frac{8ab^2}{5d\cot^{\frac{3}{2}}(c+dx)} + \frac{2b^2(b+a\cot(c+dx))}{5d\cot^{\frac{5}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5}{2}b(3a^2-b^2) - \frac{5}{2}a(a^2-3b}{\cot^{\frac{3}{2}}(c+dx)}$$

$$= \frac{8ab^2}{5d\cot^{\frac{3}{2}}(c+dx)} + \frac{2b(3a^2-b^2)}{d\sqrt{\cot(c+dx)}} + \frac{2b^2(b+a\cot(c+dx))}{5d\cot^{\frac{5}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5}{2}a(a^2-}{}$$

$$= \frac{8ab^2}{5d\cot^{\frac{3}{2}}(c+dx)} + \frac{2b(3a^2-b^2)}{d\sqrt{\cot(c+dx)}} + \frac{2b^2(b+a\cot(c+dx))}{5d\cot^{\frac{5}{2}}(c+dx)} - \frac{4\text{Subst}\left(\int \frac{\frac{5}{2}a(}{}\right.}{}$$

$$= \frac{8ab^2}{5d\cot^{\frac{3}{2}}(c+dx)} + \frac{2b(3a^2-b^2)}{d\sqrt{\cot(c+dx)}} + \frac{2b^2(b+a\cot(c+dx))}{5d\cot^{\frac{5}{2}}(c+dx)} - \frac{((a+b)(a^2-}{}$$

$$= \frac{8ab^2}{5d\cot^{\frac{3}{2}}(c+dx)} + \frac{2b(3a^2-b^2)}{d\sqrt{\cot(c+dx)}} + \frac{2b^2(b+a\cot(c+dx))}{5d\cot^{\frac{5}{2}}(c+dx)} - \frac{((a+b)(a^2-}{}$$

$$= \frac{8ab^2}{5d\cot^{\frac{3}{2}}(c+dx)} + \frac{2b(3a^2-b^2)}{d\sqrt{\cot(c+dx)}} + \frac{2b^2(b+a\cot(c+dx))}{5d\cot^{\frac{5}{2}}(c+dx)} + \frac{(a-b)(a^2+}{}$$

$$= \frac{(a+b)(a^2-4ab+b^2)\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d} - \frac{(a+b)(a^2-4ab+}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.53, size = 102, normalized size = 0.38

$$\frac{2\left((-9a^2b+3b^3)\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\cot^2(c+dx)\right)+a\left(a(9b+5a\cot(c+dx))-5(a^2-3b^2)\cot(c+dx)\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\cot^2(c+dx)\right)\right)\right)}{15d\cot^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^3/Sqrt[Cot[c + d*x]],x]
```

```
[Out] (2*((-9*a^2*b + 3*b^3)*Hypergeometric2F1[-5/4, 1, -1/4, -Cot[c + d*x]^2] +
a*(a*(9*b + 5*a*Cot[c + d*x]) - 5*(a^2 - 3*b^2)*Cot[c + d*x]*Hypergeometric
2F1[-3/4, 1, 1/4, -Cot[c + d*x]^2])))/(15*d*Cot[c + d*x]^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 64.47, size = 2472, normalized size = 9.09

method	result	size
default	Expression too large to display	2472

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^3/cot(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10/d*(-1+cos(d*x+c))*(-12*2^(1/2)*b^3*cos(d*x+c)^3+12*2^(1/2)*b^3*cos(d*x
+c)^2+2*2^(1/2)*b^3*cos(d*x+c)+5*I*b^3*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(
(cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-5*I*a^3*((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-5*I*b^3*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticPi(((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+15*a^2*((-1
+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)
*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*Ellip
ticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*b
-15*b^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d
*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*sin(
d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/
2*2^(1/2))*a-30*a^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d
*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d
*x+c)^2*sin(d*x+c)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1
/2*2^(1/2))*b+15*a^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(
d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(
d*x+c)^2*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
,1/2-1/2*I,1/2*2^(1/2))*b-15*b^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d
*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a+5*I*a^3*((-1+cos(d*x+c))/sin(d*x+c))^
(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+30*2^(1/2)*a^2*cos(d*x+c)^
3*b-30*2^(1/2)*a^2*cos(d*x+c)^2*b-2*b^3*2^(1/2)-15*I*a^2*((-1+cos(d*x+c))/s
in(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticPi(((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*b-15*I*b^2*((-1
+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)
*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*Ellip
ticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a
```

```
+15*I*a^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin
(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*si
n(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,
1/2*2^(1/2))*b+15*I*b^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+s
in(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*c
os(d*x+c)^2*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2),1/2-1/2*I,1/2*2^(1/2))*a+5*a^3*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos
(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-5*b^3*((-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+10*b^3*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticF(((1-cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+5*a^3*((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticPi(((1-cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-5*b^3*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticPi(((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+10*2^(1/2)*
b^2*cos(d*x+c)^2*sin(d*x+c)*a-10*2^(1/2)*b^2*sin(d*x+c)*cos(d*x+c)*a)*(cos(
d*x+c)+1)^2/sin(d*x+c)^4/(cos(d*x+c)/sin(d*x+c))^(1/2)/cos(d*x+c)^2*2^(1/2)
```

Maxima [A]
time = 0.51, size = 243, normalized size = 0.89

$$\frac{8\left(b^3+\frac{5\,ab^2}{\tan(dx+c)}+\frac{5\,(3\,a^2b-b^3)}{\tan(dx+c)^2}\right)\tan\left(dx+c\right)^{\frac{5}{2}}-10\,\sqrt{2}\,(a^3-3\,a^2b-3\,ab^2+b^3)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)-10\,\sqrt{2}\,(a^3-3\,a^2b-3\,ab^2+b^3)\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)-5\,\sqrt{2}\,(a^3+3\,a^2b-3\,ab^2-b^3)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)+5\,\sqrt{2}\,(a^3+3\,a^2b-3\,ab^2-b^3)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^3/cot(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/20*(8*(b^3 + 5*a*b^2/tan(d*x + c) + 5*(3*a^2*b - b^3)/tan(d*x + c)^2)*tan
(d*x + c)^(5/2) - 10*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*arctan(1/2*sqr
t(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) - 10*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^
2 + b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - 5*sqrt(2)*
(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x
+ c) + 1) + 5*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*log(-sqrt(2)/sqrt(tan
(d*x + c)) + 1/tan(d*x + c) + 1))/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^3/cot(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(c+dx))^3}{\sqrt{\cot(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**3/cot(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3/sqrt(cot(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^3/cot(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^3/sqrt(cot(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\tan(c+d\,x))^3}{\sqrt{\cot(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^3/cot(c + d*x)^(1/2),x)
```

```
[Out] int((a + b*tan(c + d*x))^3/cot(c + d*x)^(1/2), x)
```

3.819 $\int \frac{(a+b\tan(c+dx))^3}{\cot^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=299

$$-\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d}+\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot\right.}$$

```
[Out] 32/35*a*b^2/d/cot(d*x+c)^(5/2)+2/3*b*(3*a^2-b^2)/d/cot(d*x+c)^(3/2)+2/7*b^2
*(b+a*cot(d*x+c))/d/cot(d*x+c)^(7/2)+1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(-1+2^
(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)+1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(1+2^(1/2
)*cot(d*x+c)^(1/2))/d*2^(1/2)+1/4*(a+b)*(a^2-4*a*b+b^2)*ln(1+cot(d*x+c)-2^(
1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)-1/4*(a+b)*(a^2-4*a*b+b^2)*ln(1+cot(d*x+c)+
2^(1/2)*cot(d*x+c)^(1/2))/d*2^(1/2)+2*a*(a^2-3*b^2)/d/cot(d*x+c)^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3754, 3646, 3709, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d}+\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}d}+\frac{2b\left(3a^2-b^2\right)}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{2a\left(a^2-3b^2\right)}{d\sqrt{\cot(c+dx)}}+\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}d}-\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}d}+\frac{2b^2(a\cot(c+dx)+b)}{7d\cot^{\frac{7}{2}}(c+dx)}+\frac{32ab^2}{35d\cot^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^3/Cot[c + d*x]^(3/2),x]
```

```
[Out] -(((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqr
t[2]*d)) + ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x
]]])/(Sqrt[2]*d) + (32*a*b^2)/(35*d*Cot[c + d*x]^(5/2)) + (2*b*(3*a^2 - b^2
))/(3*d*Cot[c + d*x]^(3/2)) + (2*a*(a^2 - 3*b^2))/(d*Sqrt[Cot[c + d*x]]) +
(2*b^2*(b + a*Cot[c + d*x]))/(7*d*Cot[c + d*x]^(7/2)) + ((a + b)*(a^2 - 4*a
*b + b^2)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*d)
 - ((a + b)*(a^2 - 4*a*b + b^2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c
+ d*x]])/(2*Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{(a+b\tan(c+dx))^3}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b+a\cot(c+dx))^3}{\cot^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2b^2(b+a\cot(c+dx))}{7d\cot^{\frac{7}{2}}(c+dx)} - \frac{2}{7}\int \frac{-8ab^2 - \frac{7}{2}b(3a^2-b^2)\cot(c+dx) - \frac{1}{2}a(7a^2-5b^2 \ldots}{\cot^{\frac{7}{2}}(c+dx)}$$

$$= \frac{32ab^2}{35d\cot^{\frac{5}{2}}(c+dx)} + \frac{2b^2(b+a\cot(c+dx))}{7d\cot^{\frac{7}{2}}(c+dx)} - \frac{2}{7}\int \frac{-\frac{7}{2}b(3a^2-b^2) - \frac{7}{2}a(a^2-3b^2 \ldots}{\cot^{\frac{5}{2}}(c+dx)}$$

$$= \frac{32ab^2}{35d\cot^{\frac{5}{2}}(c+dx)} + \frac{2b(3a^2-b^2)}{3d\cot^{\frac{3}{2}}(c+dx)} + \frac{2b^2(b+a\cot(c+dx))}{7d\cot^{\frac{7}{2}}(c+dx)} - \frac{2}{7}\int \frac{-\frac{7}{2}a(a^2- \ldots}{\ldots}$$

$$= \frac{32ab^2}{35d\cot^{\frac{5}{2}}(c+dx)} + \frac{2b(3a^2-b^2)}{3d\cot^{\frac{3}{2}}(c+dx)} + \frac{2a(a^2-3b^2)}{d\sqrt{\cot(c+dx)}} + \frac{2b^2(b+a\cot(c+dx \ldots}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$$= \frac{32ab^2}{35d\cot^{\frac{5}{2}}(c+dx)} + \frac{2b(3a^2-b^2)}{3d\cot^{\frac{3}{2}}(c+dx)} + \frac{2a(a^2-3b^2)}{d\sqrt{\cot(c+dx)}} + \frac{2b^2(b+a\cot(c+dx \ldots}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$$= \frac{32ab^2}{35d\cot^{\frac{5}{2}}(c+dx)} + \frac{2b(3a^2-b^2)}{3d\cot^{\frac{3}{2}}(c+dx)} + \frac{2a(a^2-3b^2)}{d\sqrt{\cot(c+dx)}} + \frac{2b^2(b+a\cot(c+dx \ldots}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$$= \frac{32ab^2}{35d\cot^{\frac{5}{2}}(c+dx)} + \frac{2b(3a^2-b^2)}{3d\cot^{\frac{3}{2}}(c+dx)} + \frac{2a(a^2-3b^2)}{d\sqrt{\cot(c+dx)}} + \frac{2b^2(b+a\cot(c+dx \ldots}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$$= \frac{32ab^2}{35d\cot^{\frac{5}{2}}(c+dx)} + \frac{2b(3a^2-b^2)}{3d\cot^{\frac{3}{2}}(c+dx)} + \frac{2a(a^2-3b^2)}{d\sqrt{\cot(c+dx)}} + \frac{2b^2(b+a\cot(c+dx \ldots}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$$= -\frac{(a-b)(a^2+4ab+b^2)\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d} + \frac{(a-b)(a^2+4ab+ \ldots}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.62, size = 101, normalized size = 0.34

$$\frac{2\left(5b(-3a^2+b^2)\,{}_2F_1\left(-\frac{7}{4},1;-\frac{3}{4};-\cot^2(c+dx)\right) + a\left(a(15b+7a\cot(c+dx)) - 7(a^2-3b^2)\cot(c+dx)\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\cot^2(c+dx)\right)\right)\right)}{35d\cot^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^3/Cot[c + d*x]^(3/2),x]
```

```
[Out] (2*(5*b*(-3*a^2 + b^2)*Hypergeometric2F1[-7/4, 1, -3/4, -Cot[c + d*x]^2] +
a*(a*(15*b + 7*a*Cot[c + d*x]) - 7*(a^2 - 3*b^2)*Cot[c + d*x]*Hypergeometri
c2F1[-5/4, 1, -1/4, -Cot[c + d*x]^2])))/(35*d*Cot[c + d*x]^(7/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 54.52, size = 2598, normalized size = 8.69

method	result	size
default	Expression too large to display	2598

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^3/cot(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/210/d*(-1+cos(d*x+c))*(210*cos(d*x+c)^3*2^(1/2)*a^3-210*2^(1/2)*cos(d*x+
c)^4*a^3-126*2^(1/2)*b^2*cos(d*x+c)^2*a-756*cos(d*x+c)^3*2^(1/2)*a*b^2+756*
2^(1/2)*cos(d*x+c)^4*a*b^2+100*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)*b^3-100*2^(1
/2)*cos(d*x+c)^2*sin(d*x+c)*b^3+210*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)*a^2*b-3
0*cos(d*x+c)*sin(d*x+c)*2^(1/2)*b^3-630*cos(d*x+c)^3*sin(d*x+c)*EllipticF((
-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((cos(d*x+c)-1+si
n(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c
)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*a*b^2+315*cos(d*x+c)^3*sin(d*x+c)*Ellipti
cPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((
cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*a^2*b+315*cos(d*x+c)^3*sin
(d*x+c)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,
1/2*2^(1/2))*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*a*b^2+315*c
os(d*x+c)^3*sin(d*x+c)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(
1/2),1/2-1/2*I,1/2*2^(1/2))*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1
/2)*a^2*b+315*cos(d*x+c)^3*sin(d*x+c)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c)
)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((cos(d*x+c)-1+sin(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))
/sin(d*x+c))^(1/2)*a*b^2+105*I*cos(d*x+c)^3*sin(d*x+c)*EllipticPi((-(cos(d*
x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((cos(d*x+c)-1+
sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x
+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*a^3-105*I*cos(d*x+c)^3*sin(d*x+c)*Ellip
ticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*
((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*b^3-105*I*cos(d*x+c)^3*s
in(d*x+c)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*
I,1/2*2^(1/2))*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*a^3+105*I
*cos(d*x+c)^3*sin(d*x+c)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))
^(1/2),1/2-1/2*I,1/2*2^(1/2))*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*
((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^
(1/2)*b^3+315*I*cos(d*x+c)^3*sin(d*x+c)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+
c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((cos(d*x+c)-1+sin(d*x+c))/sin
```

```
(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c
))/sin(d*x+c))^(1/2)*a^2*b-315*I*cos(d*x+c)^3*sin(d*x+c)*EllipticPi((-(cos(
d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((cos(d*x+c)-
1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d
*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*a*b^2-315*I*cos(d*x+c)^3*sin(d*x+c)*E
llipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/
2))*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*a^2*b+315*I*cos(d*x+
c)^3*sin(d*x+c)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/
2-1/2*I,1/2*2^(1/2))*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*a*b
^2+210*cos(d*x+c)^3*sin(d*x+c)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*
x+c))^(1/2),1/2*2^(1/2))*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)
*a^3-105*cos(d*x+c)^3*sin(d*x+c)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin
(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c)
)^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(
d*x+c))^(1/2)*a^3-105*cos(d*x+c)^3*sin(d*x+c)*EllipticPi((-(cos(d*x+c)-1-si
n(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((cos(d*x+c)-1+sin(d*x+c
))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin
(d*x+c))/sin(d*x+c))^(1/2)*b^3-105*cos(d*x+c)^3*sin(d*x+c)*EllipticPi((-(co
s(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((cos(d*x+c
)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(cos
(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*a^3-105*cos(d*x+c)^3*sin(d*x+c)*Ell
ipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2)
)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))
^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*b^3-210*2^(1/2)*cos(d*
x+c)^3*sin(d*x+c)*a^2*b+126*cos(d*x+c)*2^(1/2)*a*b^2+30*2^(1/2)*sin(d*x+c)*
b^3)*(cos(d*x+c)+1)^2/cos(d*x+c)^2/sin(d*x+c)^5/(cos(d*x+c)/sin(d*x+c))^(3/
2)*2^(1/2)
```

Maxima [A]
time = 0.52, size = 265, normalized size = 0.89

$$\frac{8\left(15\,b^3+\frac{63\,ab^2}{\tan(dx+c)}+\frac{35\,(3\,a^2b-b^3)}{\tan(dx+c)^2}+\frac{105\,(a^3-3\,ab^2)}{\tan(dx+c)^3}\right)\tan\left(dx+c\right)^{\frac{7}{2}}+210\,\sqrt{2}\,(a^3+3\,a^2b-3\,ab^2-b^3)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+210\,\sqrt{2}\,(a^3+3\,a^2b-3\,ab^2-b^3)\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)-105\,\sqrt{2}\,(a^3-3\,a^2b-3\,ab^2+b^3)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)+105\,\sqrt{2}\,(a^3-3\,a^2b-3\,ab^2+b^3)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^3/cot(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/420*(8*(15*b^3 + 63*a*b^2/tan(d*x + c) + 35*(3*a^2*b - b^3)/tan(d*x + c)^
2 + 105*(a^3 - 3*a*b^2)/tan(d*x + c)^3)*tan(d*x + c)^(7/2) + 210*sqrt(2)*(a
^3 + 3*a^2*b - 3*a*b^2 - b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x
+ c)))) + 210*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*arctan(-1/2*sqrt(2)*(
sqrt(2) - 2/sqrt(tan(d*x + c)))) - 105*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b
^3)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + 105*sqrt(2)*(a^3
```

```
 - 3*a^2*b - 3*a*b^2 + b^3)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c
) + 1))/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^3/cot(d*x+c)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan(c + dx))^3}{\cot^{\frac{3}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**3/cot(d*x+c)**(3/2),x)
[Out] Integral((a + b*tan(c + d*x))**3/cot(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^3/cot(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^3/cot(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\tan(c + d\,x))^3}{\cot(c + d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^3/cot(c + d*x)^(3/2),x)
[Out] int((a + b*tan(c + d*x))^3/cot(c + d*x)^(3/2), x)
```

3.820 $\int \frac{\cot^{\frac{5}{2}}(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=271

$$-\frac{(a-b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}+\frac{(a-b)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}-\frac{2b^{7/2}\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{}}{}\right.}{a^{5/2}\,(a^2+}$$

[Out] -2*b^(7/2)*arctan(a^(1/2)*cot(d*x+c)^(1/2)/b^(1/2))/a^(5/2)/(a^2+b^2)/d-2/3*cot(d*x+c)^(3/2)/a/d+1/2*(a-b)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)+1/2*(a-b)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)+1/4*(a+b)*ln(1+cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)-1/4*(a+b)*ln(1+cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)+2*b*cot(d*x+c)^(1/2)/a^2/d

Rubi [A]
time = 0.46, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$,
Rules used = {3754, 3647, 3728, 3735, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{(a-b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d\,(a^2+b^2)}+\frac{(a-b)\text{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d\,(a^2+b^2)}+\frac{(a+b)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)}-\frac{(a+b)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)}+\frac{2b\sqrt{\cot(c+dx)}}{a^2d}-\frac{2b^{7/2}\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{a^{5/2}d\,(a^2+b^2)}-\frac{2\cot^{\frac{3}{2}}(c+dx)}{3ad}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^(5/2)/(a + b*Tan[c + d*x]),x]

[Out] -(((a - b)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)*d)) + ((a - b)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)*d) - (2*b^(7/2)*ArcTan[(Sqrt[a]*Sqrt[Cot[c + d*x]])/Sqrt[b]])/(a^(5/2)*(a^2 + b^2)*d) + (2*b*Sqrt[Cot[c + d*x]])/(a^2*d) - (2*Cot[c + d*x]^(3/2))/(3*a*d) + ((a + b)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)*d) - ((a + b)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)*d)

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
```

```
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3735

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dis
t[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*Simp[a*(A - C) - (A*b - b*C)*Ta
n[e + f*x], x], x], x] + Dist[(A*b^2 + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e
 + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a,
 b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0] &&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\cot^{\frac{7}{2}}(c+dx)}{b+a\cot(c+dx)}\,dx$$

$$= -\frac{2\cot^{\frac{3}{2}}(c+dx)}{3ad} - \frac{2\int \frac{\sqrt{\cot(c+dx)}\left(\frac{3b}{2}+\frac{3}{2}a\cot(c+dx)+\frac{3}{2}b\cot^2(c+dx)\right)}{b+a\cot(c+dx)}\,dx}{3a}$$

$$= \frac{2b\sqrt{\cot(c+dx)}}{a^2d} - \frac{2\cot^{\frac{3}{2}}(c+dx)}{3ad} + \frac{4\int \frac{\frac{3b^2}{4}-\frac{3}{4}(a^2-b^2)\cot^2(c+dx)}{\sqrt{\cot(c+dx)}\,(b+a\cot(c+dx))}\,dx}{3a^2}$$

$$= \frac{2b\sqrt{\cot(c+dx)}}{a^2d} - \frac{2\cot^{\frac{3}{2}}(c+dx)}{3ad} + \frac{4\int \frac{\frac{3a^2b}{4}-\frac{3}{4}a^3\cot(c+dx)}{\sqrt{\cot(c+dx)}}\,dx}{3a^2(a^2+b^2)} + \frac{b^4\int \frac{1+\text{co}}{\sqrt{\cot(c+d}}}{a^2\,(a}$$

$$= \frac{2b\sqrt{\cot(c+dx)}}{a^2d} - \frac{2\cot^{\frac{3}{2}}(c+dx)}{3ad} + \frac{8\text{Subst}\left(\int \frac{-\frac{3a^2b}{4}+\frac{3a^3x^2}{4}}{1+x^4}\,dx, x, \sqrt{\cot(c+dx)}\right.}{3a^2(a^2+b^2)d}$$

$$= \frac{2b\sqrt{\cot(c+dx)}}{a^2d} - \frac{2\cot^{\frac{3}{2}}(c+dx)}{3ad} + \frac{(a-b)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\cot(c+dx)}\right.}{(a^2+b^2)d}$$

$$= -\frac{2b^{7/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{a^{5/2}(a^2+b^2)d} + \frac{2b\sqrt{\cot(c+dx)}}{a^2d} - \frac{2\cot^{\frac{3}{2}}(c+dx)}{3ad} + \frac{(a}{}$$

$$= -\frac{2b^{7/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{a^{5/2}(a^2+b^2)d} + \frac{2b\sqrt{\cot(c+dx)}}{a^2d} - \frac{2\cot^{\frac{3}{2}}(c+dx)}{3ad} + \frac{(a}{}$$

$$= -\frac{(a-b)\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}(a^2+b^2)d} + \frac{(a-b)\tan^{-1}\left(1+\sqrt{2}\sqrt{\cot(c+dx}\right.}{\sqrt{2}(a^2+b^2)d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.39, size = 303, normalized size = 1.12

$$\frac{6\sqrt{2}a^{5/2}b\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-6\sqrt{2}a^{5/2}b\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)-24b^{7/2}\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)+24a^{5/2}b\sqrt{\cot(c+dx)}+24\sqrt{a}b^3\sqrt{\cot(c+dx)}-8a^{7/2}\cot^{\frac{3}{2}}(c+dx)-8a^{3/2}b^2\cot^{\frac{3}{2}}(c+dx)+8a^{7/2}\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)+3\sqrt{2}a^{5/2}b\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-3\sqrt{2}a^{5/2}b\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)}{12a^{5/2}(a^2+b^2)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(5/2)/(a + b*Tan[c + d*x]),x]
```

```
[Out] (6*Sqrt[2]*a^(5/2)*b*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 6*Sqrt[2]*a^(
5/2)*b*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] - 24*b^(7/2)*ArcTan[(Sqrt[a]*
Sqrt[Cot[c + d*x]])/Sqrt[b]] + 24*a^(5/2)*b*Sqrt[Cot[c + d*x]] + 24*Sqrt[a]
*b^3*Sqrt[Cot[c + d*x]] - 8*a^(7/2)*Cot[c + d*x]^(3/2) - 8*a^(3/2)*b^2*Cot[
c + d*x]^(3/2) + 8*a^(7/2)*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1, 7/4
, -Cot[c + d*x]^2] + 3*Sqrt[2]*a^(5/2)*b*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]
 + Cot[c + d*x]] - 3*Sqrt[2]*a^(5/2)*b*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] +
 Cot[c + d*x]])/(12*a^(5/2)*(a^2 + b^2)*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 19.34, size = 11313, normalized size = 41.75

method	result	size
default	Expression too large to display	11313

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.52, size = 207, normalized size = 0.76

$$-\frac{\frac{24\,b^{4}\arctan\left(\frac{a}{\sqrt{ab}\,\sqrt{\tan\left(dx+c\right)}}\right)}{\left(a^{4}+a^{2}b^{2}\right)\sqrt{ab}}-\frac{3\left(2\,\sqrt{2}\,(a-b)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+2\,\sqrt{2}\,(a-b)\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)-\sqrt{2}\,(a+b)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)+\sqrt{2}\,(a+b)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)\right)}{a^{2}+b^{2}}-\frac{8\left(\frac{3\,b}{\sqrt{\tan\left(dx+c\right)}}-\frac{a}{\tan\left(dx+c\right)^{\frac{3}{2}}}\right)}{a^{2}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(24*b^4*arctan(a/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^4 + a^2*b^2)*sqr
t(a*b)) - 3*(2*sqrt(2)*(a - b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x
 + c)))) + 2*sqrt(2)*(a - b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x
+ c)))) - sqrt(2)*(a + b)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) +
 1) + sqrt(2)*(a + b)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1)
)/(a^2 + b^2) - 8*(3*b/sqrt(tan(d*x + c)) - a/tan(d*x + c)^(3/2))/a^2)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(5/2)/(a+b*tan(d*x+c)),x)
[Out] Integral(cot(c + d*x)**(5/2)/(a + b*tan(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x, algorithm="giac")
[Out] integrate(cot(d*x + c)^(5/2)/(b*tan(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cot(c+d\,x)}^{5/2}}{a+b\tan(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(5/2)/(a + b*tan(c + d*x)),x)
[Out] int(cot(c + d*x)^(5/2)/(a + b*tan(c + d*x)), x)
```

3.821 $\int \frac{\cot^{\frac{3}{2}}(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=250

$$-\frac{(a+b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)d}+\frac{(a+b)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)d}+\frac{2b^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{}}{}\right.}{a^{3/2}\left(a^2+\right.}$$

[Out] 2*b^(5/2)*arctan(a^(1/2)*cot(d*x+c)^(1/2)/b^(1/2))/a^(3/2)/(a^2+b^2)/d+1/2*(a+b)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)+1/2*(a+b)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)-1/4*(a-b)*ln(1+cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)+1/4*(a-b)*ln(1+cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)-2*cot(d*x+c)^(1/2)/a/d

Rubi [A]
time = 0.33, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3754, 3647, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{(a+b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d\left(a^2+b^2\right)}+\frac{(a+b)\text{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}d\left(a^2+b^2\right)}-\frac{(a-b)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}d\left(a^2+b^2\right)}+\frac{(a-b)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}d\left(a^2+b^2\right)}+\frac{2b^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{a^{3/2}d\left(a^2+b^2\right)}-\frac{2\sqrt{\cot(c+dx)}}{ad}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^(3/2)/(a + b*Tan[c + d*x]),x]

[Out] -(((a + b)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)*d)) + ((a + b)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)*d) + (2*b^(5/2)*ArcTan[(Sqrt[a]*Sqrt[Cot[c + d*x]])/Sqrt[b]])/(a^(3/2)*(a^2 + b^2)*d) - (2*Sqrt[Cot[c + d*x]])/(a*d) - ((a - b)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)*d) + ((a - b)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)*d)

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\cot^{\frac{5}{2}}(c+dx)}{b+a\cot(c+dx)}\,dx$$

$$= -\frac{2\sqrt{\cot(c+dx)}}{ad} - \frac{2\int \frac{\frac{b}{2}+\frac{1}{2}a\cot(c+dx)+\frac{1}{2}b\cot^2(c+dx)}{\sqrt{\cot(c+dx)}\,(b+a\cot(c+dx))}\,dx}{a}$$

$$= -\frac{2\sqrt{\cot(c+dx)}}{ad} - \frac{2\int \frac{\frac{a^2}{2}+\frac{1}{2}ab\cot(c+dx)}{\sqrt{\cot(c+dx)}}\,dx}{a\,(a^2+b^2)} - \frac{b^3\int \frac{1+\cot^2(c+dx)}{\sqrt{\cot(c+dx)}\,(b+a\cot(c+dx))}\,dx}{a\,(a^2+b^2)}$$

$$= -\frac{2\sqrt{\cot(c+dx)}}{ad} - \frac{4\text{Subst}\left(\int \frac{-\frac{a^2}{2}-\frac{1}{2}abx^2}{1+x^4}\,dx, x, \sqrt{\cot(c+dx)}\right)}{a\,(a^2+b^2)\,d} - \frac{b^3\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{2\sqrt{\cot(c+dx)}}{ad} + \frac{(a-b)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\cot(c+dx)}\right)}{(a^2+b^2)\,d} + \frac{(2b^3)\,\text{Subst}\left(\cdots\right.}{\cdots}$$

$$= \frac{2b^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{a^{3/2}\,(a^2+b^2)\,d} - \frac{2\sqrt{\cot(c+dx)}}{ad} - \frac{(a-b)\text{Subst}\left(\int \frac{\sqrt{2}\cdots}{-1-\sqrt{2}\cdots}\right.}{2\sqrt{2}\cdots}$$

$$= \frac{2b^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{a^{3/2}\,(a^2+b^2)\,d} - \frac{2\sqrt{\cot(c+dx)}}{ad} - \frac{(a-b)\log\left(1-\sqrt{2}\,\sqrt{\cdots}\right.}{2\sqrt{2}\cdots}$$

$$= -\frac{(a+b)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} + \frac{(a+b)\tan^{-1}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}\cdots\right.}{\sqrt{2}\,(a^2+b^2)\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.43, size = 264, normalized size = 1.06

$$\frac{8a^{3/2}b\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right) - 3\left(2\sqrt{2}\,a^{5/2}\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right) - 2\sqrt{2}\,a^{5/2}\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}\right) - 8b^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right) + 8a^{5/2}\sqrt{\cot(c+dx)} + 8\sqrt{a}\,b^2\sqrt{\cot(c+dx)} + \sqrt{2}\,a^{5/2}\log\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right) - \sqrt{2}\,a^{5/2}\log\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)}{12a^{3/2}\,(a^2+b^2)\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(3/2)/(a + b*Tan[c + d*x]),x]
```

```
[Out] (8*a^(3/2)*b*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x
]^2] - 3*(2*Sqrt[2]*a^(5/2)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*Sqrt
[2]*a^(5/2)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] - 8*b^(5/2)*ArcTan[(Sqrt
[a]*Sqrt[Cot[c + d*x]])/Sqrt[b]] + 8*a^(5/2)*Sqrt[Cot[c + d*x]] + 8*Sqrt[a]
*b^2*Sqrt[Cot[c + d*x]] + Sqrt[2]*a^(5/2)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]
] + Cot[c + d*x]] - Sqrt[2]*a^(5/2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Co
t[c + d*x]]))/(12*a^(3/2)*(a^2 + b^2)*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 20.34, size = 10343, normalized size = 41.37

method	result	size
default	Expression too large to display	10343

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.49, size = 189, normalized size = 0.76

$$\frac{\frac{8 b^{3} \arctan\left(\frac{a}{\sqrt{ab}\,\sqrt{\tan\left(dx+c\right)}}\right)}{\left(a^{3}+ab^{2}\right)\sqrt{ab}}+\frac{2\sqrt{2}\,(a+b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+2\sqrt{2}\,(a+b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+\sqrt{2}\,(a-b)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)-\sqrt{2}\,(a-b)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)}{a^{2}+b^{2}}-\frac{8}{a\sqrt{\tan\left(dx+c\right)}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(8*b^3*arctan(a/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^3 + a*b^2)*sqrt(a*b
)) + (2*sqrt(2)*(a + b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))
) + 2*sqrt(2)*(a + b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c))))
 + sqrt(2)*(a - b)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - s
qrt(2)*(a - b)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^2
+ b^2) - 8/(a*sqrt(tan(d*x + c))))/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{\frac{3}{2}}\left(c+dx\right)}{a+b\tan\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(3/2)/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**(3/2)/(a + b*tan(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x, algorithm="giac")
[Out] integrate(cot(d*x + c)^(3/2)/(b*tan(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^{3/2}}{a+b\tan\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(3/2)/(a + b*tan(c + d*x)),x)
[Out] int(cot(c + d*x)^(3/2)/(a + b*tan(c + d*x)), x)
```

3.822 $\int \frac{\sqrt{\cot(c+dx)}}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=232

$$\frac{(a-b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)d}-\frac{(a-b)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)d}-\frac{2b^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{c}}{\ldots}\right.}{\sqrt{a}\left(a^2+b^2\ldots\right.}$$

```
[Out] -1/2*(a-b)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)-1/2*(a-b
)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)-1/4*(a+b)*ln(1+cot
(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)+1/4*(a+b)*ln(1+cot(d*
x+c)+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)-2*b^(3/2)*arctan(a^(1/2)
*cot(d*x+c)^(1/2)/b^(1/2))/(a^2+b^2)/d/a^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3754, 3654, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(a-b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d\left(a^2+b^2\right)}-\frac{(a-b)\text{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d\left(a^2+b^2\right)}-\frac{2b^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{\sqrt{a}\,d\left(a^2+b^2\right)}-\frac{(a+b)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)}+\frac{(a+b)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cot[c + d*x]]/(a + b*Tan[c + d*x]),x]
```

```
[Out] ((a - b)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)*d) -
((a - b)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)*d) -
(2*b^(3/2)*ArcTan[(Sqrt[a]*Sqrt[Cot[c + d*x]])/Sqrt[b]])/(Sqrt[a]*(a^2 + b^
2)*d) - ((a + b)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqr
t[2]*(a^2 + b^2)*d) + ((a + b)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c +
 d*x]])/(2*Sqrt[2]*(a^2 + b^2)*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3654

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[Simp[a^2*c - b^2*c + 2
*a*b*d + (2*a*b*c - a^2*d + b^2*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x]
], x], x] + Dist[(b*c - a*d)^2/(c^2 + d^2), Int[(1 + Tan[e + f*x]^2)/(Sqrt[
a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{\sqrt{\cot(c+dx)}}{a+b\tan(c+dx)}\,dx = \int \frac{\cot^{\frac{3}{2}}(c+dx)}{b+a\cot(c+dx)}\,dx$$

$$= \frac{\int \frac{-b+a\cot(c+dx)}{\sqrt{\cot(c+dx)}}\,dx}{a^2+b^2} + \frac{b^2\int \frac{1+\cot^2(c+dx)}{\sqrt{\cot(c+dx)}\,(b+a\cot(c+dx))}\,dx}{a^2+b^2}$$

$$= \frac{2\text{Subst}\left(\int \frac{b-ax^2}{1+x^4}\,dx, x, \sqrt{\cot(c+dx)}\right)}{(a^2+b^2)\,d} + \frac{b^2\text{Subst}\left(\int \frac{1}{\sqrt{-x}\,(b-ax)}\,dx, x, -\cot(c+\right.}{(a^2+b^2)\,d}$$

$$= -\frac{(a-b)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\cot(c+dx)}\right)}{(a^2+b^2)\,d} - \frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \sqrt{\cot\right.}{(a^2+b^2)\,d}$$

$$= -\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{\sqrt{a}\,(a^2+b^2)\,d} - \frac{(a-b)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{\cot(\right.}{2\,(a^2+b^2)\,d}$$

$$= -\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{\sqrt{a}\,(a^2+b^2)\,d} - \frac{(a+b)\log\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot\right.}{2\sqrt{2}\,(a^2+b^2)\,d}$$

$$= \frac{(a-b)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} - \frac{(a-b)\tan^{-1}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}\right.}{\sqrt{2}\,(a^2+b^2)\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.23, size = 227, normalized size = 0.98

$$\frac{-8a^{3/2}\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)-3b\left(2\sqrt{2}\,\sqrt{a}\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)-2\sqrt{2}\,\sqrt{a}\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}\right)+8\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)+\sqrt{2}\,\sqrt{a}\,\log\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\sqrt{2}\,\sqrt{a}\,\log\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)}{12\sqrt{a}\,(a^2+b^2)\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cot[c + d*x]]/(a + b*Tan[c + d*x]),x]
```

```
[Out] (-8*a^(3/2)*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]
^2] - 3*b*(2*Sqrt[2]*Sqrt[a]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*Sqr
t[2]*Sqrt[a]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 8*Sqrt[b]*ArcTan[(Sqr
t[a]*Sqrt[Cot[c + d*x]])/Sqrt[b]] + Sqrt[2]*Sqrt[a]*Log[1 - Sqrt[2]*Sqrt[Co
t[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*Sqrt[a]*Log[1 + Sqrt[2]*Sqrt[Cot[c +
d*x]] + Cot[c + d*x]]))/(12*Sqrt[a]*(a^2 + b^2)*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 16.05, size = 2278, normalized size = 9.82

method	result	size
default	Expression too large to display	2278

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(cos(d*x+c)/sin(d*x+c))^(1/2)*(-1+cos(d*x+c))*(-(cos(d*x+c)-1-sin(d*
x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)*EllipticPi((-(cos(d*x+c)-1-
sin(d*x+c))/sin(d*x+c))^(1/2),a/(b+(a^2+b^2)^(1/2)+a),1/2*2^(1/2))*a^2*b^2-
2*(a^2+b^2)^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),
a/(b+(a^2+b^2)^(1/2)+a),1/2*2^(1/2))*a*b^3+2*(a^2+b^2)^(1/2)*EllipticPi((-(
cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),-a/(-b+(a^2+b^2)^(1/2)-a),1/2*2^
(1/2))*a^2*b^2-2*(a^2+b^2)^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin
(d*x+c))^(1/2),-a/(-b+(a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b^3-3*(a^2+b^2)^(1/
2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2
^(1/2))*a^3*b-I*(a^2+b^2)^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(
d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a^4+I*(a^2+b^2)^(3/2)*EllipticPi((-(co
s(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a^2+I*(a^2+
b^2)^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2
*I,1/2*2^(1/2))*a^4-I*(a^2+b^2)^(3/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c)
)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^2-3*(a^2+b^2)^(1/2)*EllipticPi
((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^3*b
-3*(a^2+b^2)^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)
,1/2+1/2*I,1/2*2^(1/2))*a^2*b^2-(a^2+b^2)^(1/2)*EllipticPi((-(cos(d*x+c)-1-
sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a*b^3+(a^2+b^2)^(3/2)*
EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1
/2))*a*b+4*(a^2+b^2)^(1/2)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2*2^(1/2))*a^3*b+4*(a^2+b^2)^(1/2)*EllipticF((-(cos(d*x+c)-1-sin(
d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2*b^2+4*(a^2+b^2)^(1/2)*EllipticF(
(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*a*b^3-I*(a^2+b^2
)^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,
1/2*2^(1/2))*a*b^3-2*(a^2+b^2)^(3/2)*EllipticF((-(cos(d*x+c)-1-sin(d*x+c))/
sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-2*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))
/sin(d*x+c))^(1/2),a/(b+(a^2+b^2)^(1/2)+a),1/2*2^(1/2))*a*b^4+2*EllipticPi(
(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),-a/(-b+(a^2+b^2)^(1/2)-a),1/2
*2^(1/2))*a^3*b^2+2*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2
),-a/(-b+(a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b^4-2*(a^2+b^2)^(3/2)*EllipticF(
(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2+(a^2+b^2)^(3
/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*
2^(1/2))*a^2+(a^2+b^2)^(3/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x
+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^2+2*(a^2+b^2)^(1/2)*EllipticF((-(cos(d*
x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*a^4+2*(a^2+b^2)^(1/2)*Ell
```

```
ipticF((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*b^4-(a^2+
b^2)^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2
*I,1/2*2^(1/2))*a^4-(a^2+b^2)^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/
sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^4-2*EllipticPi((-(cos(d*x+c)-1-s
in(d*x+c))/sin(d*x+c))^(1/2),a/(b+(a^2+b^2)^(1/2)+a),1/2*2^(1/2))*a^3*b^2-3
*(a^2+b^2)^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1
/2-1/2*I,1/2*2^(1/2))*a^2*b^2-(a^2+b^2)^(1/2)*EllipticPi((-(cos(d*x+c)-1-si
n(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a*b^3+(a^2+b^2)^(3/2)*El
lipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2
))*a*b+I*(a^2+b^2)^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))
^(1/2),1/2-1/2*I,1/2*2^(1/2))*a^2*b^2+I*(a^2+b^2)^(1/2)*EllipticPi((-(cos(d
*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a*b^3+I*(a^2+b
^2)^(3/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*
I,1/2*2^(1/2))*a*b+I*(a^2+b^2)^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))
/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^3*b-I*(a^2+b^2)^(3/2)*EllipticP
i((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a*b-
I*(a^2+b^2)^(1/2)*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),
1/2-1/2*I,1/2*2^(1/2))*a^3*b-I*(a^2+b^2)^(1/2)*EllipticPi((-(cos(d*x+c)-1-s
in(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^2*b^2)*(cos(d*x+c)+1)
^2/cos(d*x+c)/sin(d*x+c)^2*2^(1/2)/a/(a^2+b^2)^(3/2)/(b+(a^2+b^2)^(1/2)+a)/
(-b+(a^2+b^2)^(1/2)-a)
```

Maxima [A]
time = 0.49, size = 174, normalized size = 0.75

$$-\frac{\frac{8\,b^2\arctan\left(\frac{a}{\sqrt{ab}\,\sqrt{\tan\left(dx+c\right)}}\right)}{(a^2+b^2)\sqrt{ab}}+\frac{2\,\sqrt{2}\,(a-b)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+2\,\sqrt{2}\,(a-b)\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)-\sqrt{2}\,(a+b)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}\,(a+b)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^2+b^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(8*b^2*arctan(a/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^2 + b^2)*sqrt(a*b)
) + (2*sqrt(2)*(a - b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c))))
 + 2*sqrt(2)*(a - b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c))))
- sqrt(2)*(a + b)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sq
rt(2)*(a + b)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^2 +
 b^2))/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cot (c + dx)}}{a + b \tan (c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(1/2)/(a+b*tan(d*x+c)),x)
[Out] Integral(sqrt(cot(c + d*x))/(a + b*tan(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x, algorithm="giac")
[Out] integrate(sqrt(cot(d*x + c))/(b*tan(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cot (c + d\,x)}}{a + b \tan (c + d\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(1/2)/(a + b*tan(c + d*x)),x)
[Out] int(cot(c + d*x)^(1/2)/(a + b*tan(c + d*x)), x)
```

3.823 $\int \frac{1}{\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))}\,dx$

Optimal. Leaf size=232

$$\frac{(a+b)\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}-\frac{(a+b)\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}+\frac{2\sqrt{a}\,\sqrt{b}\,\text{ArcTan}\left(\sqrt{\ }\right.}{(a^2+}$$

```
[Out] -1/2*(a+b)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)-1/2*(a+b
)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)+1/4*(a-b)*ln(1+cot
(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)-1/4*(a-b)*ln(1+cot(d*
x+c)+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)+2*arctan(a^(1/2)*cot(d*x
+c)^(1/2)/b^(1/2))*a^(1/2)*b^(1/2)/(a^2+b^2)/d
```

Rubi [A]
time = 0.20, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3754, 3653, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(a+b)\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d\,(a^2+b^2)}-\frac{(a+b)\text{ArcTan}\left(\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d\,(a^2+b^2)}+\frac{2\sqrt{a}\,\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{d\,(a^2+b^2)}+\frac{(a-b)\log\left(\cot(c+dx)-\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)}-\frac{(a-b)\log\left(\cot(c+dx)+\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])),x]
```

```
[Out] ((a + b)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)*d) -
((a + b)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)*d) +
(2*Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[a]*Sqrt[Cot[c + d*x]])/Sqrt[b]])/((a^2 + b^
2)*d) + ((a - b)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqr
t[2]*(a^2 + b^2)*d) - ((a - b)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c +
 d*x]])/(2*Sqrt[2]*(a^2 + b^2)*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3653

```
Int[Sqrt[(a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[Simp[a*c + b*d + (b*c -
a*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x]], x], x] - Dist[d*((b*c - a*d
)/(c^2 + d^2)), Int[(1 + Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]]*(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))}\,dx = \int \frac{\sqrt{\cot(c+dx)}}{b+a\cot(c+dx)}\,dx$$

$$= \frac{\int \frac{a+b\cot(c+dx)}{\sqrt{\cot(c+dx)}}\,dx}{a^2+b^2} - \frac{(ab)\int \frac{1+\cot^2(c+dx)}{\sqrt{\cot(c+dx)}\,(b+a\cot(c+dx))}\,dx}{a^2+b^2}$$

$$= \frac{2\text{Subst}\left(\int \frac{-a-bx^2}{1+x^4}\,dx, x, \sqrt{\cot(c+dx)}\right)}{(a^2+b^2)\,d} - \frac{(ab)\text{Subst}\left(\int \frac{1}{\sqrt{-x}\,(b-}\right.}{(a^2}$$

$$= -\frac{(a-b)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\cot(c+dx)}\right)}{(a^2+b^2)\,d} + \frac{(2ab)\text{Subst}\left(\int \frac{}{b+}\right.}{(}$$

$$= \frac{2\sqrt{a}\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{(a^2+b^2)\,d} + \frac{(a-b)\text{Subst}\left(\int \frac{\sqrt{2}}{-1-\sqrt{2}}\right.}{2\sqrt{2}}$$

$$= \frac{2\sqrt{a}\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{(a^2+b^2)\,d} + \frac{(a-b)\log\left(1-\sqrt{2}\sqrt{\right.}{2\sqrt{2}}$$

$$= \frac{(a+b)\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} - \frac{(a+b)\tan^{-1}\left(1+\sqrt{2}\right.}{\sqrt{2}\,(a^2+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.15, size = 204, normalized size = 0.88

$$\frac{6\sqrt{2}\,a\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-6\sqrt{2}\,a\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+24\sqrt{a}\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)-8b\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)+3\sqrt{2}\,a\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-3\sqrt{2}\,a\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)}{12(a^2+b^2)\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])),x]
```

```
[Out] (6*Sqrt[2]*a*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 6*Sqrt[2]*a*ArcTan[1
+ Sqrt[2]*Sqrt[Cot[c + d*x]]] + 24*Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[a]*Sqrt[Cot
[c + d*x]])/Sqrt[b]] - 8*b*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1, 7/4
, -Cot[c + d*x]^2] + 3*Sqrt[2]*a*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c
 + d*x]] - 3*Sqrt[2]*a*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/
(12*(a^2 + b^2)*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 17.46, size = 1892, normalized size = 8.16

method	result	size
default	Expression too large to display	1892

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*
x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*
(-1+cos(d*x+c))*(-3*I*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1
/2),1/2-1/2*I,1/2*2^(1/2))*(a^2+b^2)^(1/2)*a*b^2+3*I*EllipticPi((-(cos(d*x+
c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2+b^2)^(1/2)*a
^2*b+3*I*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I
,1/2*2^(1/2))*(a^2+b^2)^(1/2)*a*b^2-EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/
sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2+b^2)^(3/2)*a+EllipticPi((-(co
s(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2+b^2)^(
3/2)*b+EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1
/2*2^(1/2))*(a^2+b^2)^(1/2)*a^3-EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(
d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2+b^2)^(1/2)*b^3-EllipticPi((-(cos(
d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2+b^2)^(3/
2)*a+EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2
*2^(1/2))*(a^2+b^2)^(3/2)*b+EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+
c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2+b^2)^(1/2)*a^3-EllipticPi((-(cos(d*x+
c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2+b^2)^(1/2)*b
^3+2*a^3*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),a/(b+(a^2
+b^2)^(1/2)+a),1/2*2^(1/2))*b+2*a*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/si
n(d*x+c))^(1/2),a/(b+(a^2+b^2)^(1/2)+a),1/2*2^(1/2))*b^3-2*a^3*EllipticPi((
-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),-a/(-b+(a^2+b^2)^(1/2)-a),1/2*
2^(1/2))*b-2*a*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),-a/
(-b+(a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^3+I*EllipticPi((-(cos(d*x+c)-1-sin(d*
x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2+b^2)^(3/2)*a+I*Elliptic
Pi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^
2+b^2)^(3/2)*b+I*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1
/2+1/2*I,1/2*2^(1/2))*(a^2+b^2)^(1/2)*a^3+I*EllipticPi((-(cos(d*x+c)-1-sin(
d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2+b^2)^(1/2)*b^3-2*a^2*
EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),a/(b+(a^2+b^2)^(1/
2)+a),1/2*2^(1/2))*b*(a^2+b^2)^(1/2)+2*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c
))/sin(d*x+c))^(1/2),a/(b+(a^2+b^2)^(1/2)+a),1/2*2^(1/2))*b^2*(a^2+b^2)^(1/
2)*a-2*a^2*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),-a/(-b+
(a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b*(a^2+b^2)^(1/2)+2*EllipticPi((-(cos(d*x+c
)-1-sin(d*x+c))/sin(d*x+c))^(1/2),-a/(-b+(a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^
2*(a^2+b^2)^(1/2)*a+EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2
),1/2-1/2*I,1/2*2^(1/2))*(a^2+b^2)^(1/2)*a^2*b-EllipticPi((-(cos(d*x+c)-1-s
in(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2+b^2)^(1/2)*a*b^2+E
```

```
llipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/
2))*(a^2+b^2)^(1/2)*a^2*b-EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2+b^2)^(1/2)*a*b^2-I*EllipticPi((-(cos(d*
x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2+b^2)^(1/2)
*a^3-I*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1
/2*2^(1/2))*(a^2+b^2)^(1/2)*b^3-I*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/si
n(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2+b^2)^(3/2)*a-I*EllipticPi((-(co
s(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2+b^2)^(
3/2)*b-3*I*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2
*I,1/2*2^(1/2))*(a^2+b^2)^(1/2)*a^2*b)/sin(d*x+c)^3/(cos(d*x+c)/sin(d*x+c))
^(1/2)*2^(1/2)/(a^2+b^2)^(3/2)/(b+(a^2+b^2)^(1/2)+a)/(-b+(a^2+b^2)^(1/2)-a)
```

Maxima [A]
time = 0.50, size = 174, normalized size = 0.75

$$\frac{\frac{8\,ab\arctan\left(\frac{a}{\sqrt{ab}\,\sqrt{\tan\left(dx+c\right)}}\right)}{\left(a^{2}+b^{2}\right)\sqrt{ab}}-\frac{2\,\sqrt{2}\,(a+b)\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+2\,\sqrt{2}\,(a+b)\arctan\left(-\frac{1}{2}\,\sqrt{2}\,\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+\sqrt{2}\,(a-b)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}\,(a-b)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^{2}+b^{2}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(8*a*b*arctan(a/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^2 + b^2)*sqrt(a*b))
 - (2*sqrt(2)*(a + b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c))))
+ 2*sqrt(2)*(a + b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) +
 sqrt(2)*(a - b)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqr
t(2)*(a - b)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^2 +
b^2))/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\tan\left(c+dx\right)\right)\sqrt{\cot\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(1/2)/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(1/((a + b*tan(c + d*x))*sqrt(cot(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*tan(d*x + c) + a)*sqrt(cot(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{\cot\left(c+d\,x\right)}\,\left(a+b\tan\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))),x)
```

```
[Out] int(1/(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))), x)
```

3.824 $\int \frac{1}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx$

Optimal. Leaf size=232

$$-\frac{(a-b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}+\frac{(a-b)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}-\frac{2a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{}}{}\right.}{\sqrt{b}\,(a^2+}$$

```
[Out] 1/2*(a-b)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)+1/2*(a-b)
*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)+1/4*(a+b)*ln(1+cot(
d*x+c)-2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)-1/4*(a+b)*ln(1+cot(d*x
+c)+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)-2*a^(3/2)*arctan(a^(1/2)*
cot(d*x+c)^(1/2)/b^(1/2))/(a^2+b^2)/d/b^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3754, 3655, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{(a-b)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d\,(a^2+b^2)}+\frac{(a-b)\text{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d\,(a^2+b^2)}+\frac{(a+b)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)}-\frac{(a+b)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)}-\frac{2a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}\,d\,(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])),x]
```

```
[Out] -(((a - b)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)*d))
 + ((a - b)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)*d)
 - (2*a^(3/2)*ArcTan[(Sqrt[a]*Sqrt[Cot[c + d*x]])/Sqrt[b]])/(Sqrt[b]*(a^2 +
 b^2)*d) + ((a + b)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*
Sqrt[2]*(a^2 + b^2)*d) - ((a + b)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[
c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)/((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[(a + b*Tan[e + f*x])^m*
(c - d*Tan[e + f*x]), x], x] + Dist[d^2/(c^2 + d^2), Int[(a + b*Tan[e + f*x
])^m*((1 + Tan[e + f*x]^2)/(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = \int \frac{1}{\sqrt{\cot(c+dx)}\,(b+a\cot(c+dx))}\,dx$$

$$= \frac{\int \frac{b-a\cot(c+dx)}{\sqrt{\cot(c+dx)}}\,dx}{a^2+b^2} + \frac{a^2\int \frac{1+\cot^2(c+dx)}{\sqrt{\cot(c+dx)}\,(b+a\cot(c+dx))}\,dx}{a^2+b^2}$$

$$= \frac{2\text{Subst}\left(\int \frac{-b+ax^2}{1+x^4}\,dx, x, \sqrt{\cot(c+dx)}\right)}{(a^2+b^2)\,d} + \frac{a^2\text{Subst}\left(\int \frac{1}{\sqrt{-x}\,(b-ax}\right.}{(a^2+}$$

$$= -\frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \sqrt{\cot(c+dx)}\right)}{(a^2+b^2)\,d} + \frac{(a-b)\text{Subst}\left(\int \frac{1-}{1-}\right.}{(}$$

$$= -\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}\,(a^2+b^2)\,d} + \frac{(a-b)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+a}\right.}{2\,(a^2+}$$

$$= -\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}\,(a^2+b^2)\,d} + \frac{(a+b)\log\left(1-\sqrt{2}\,\sqrt{\cot}\right.}{2\sqrt{2}\,(a^2}$$

$$= -\frac{(a-b)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} + \frac{(a-b)\tan^{-1}\left(1+\sqrt{\right.}{\sqrt{2}\,(a^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.17, size = 226, normalized size = 0.97

$$-\frac{\frac{2a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}\,(a^2+b^2)} - \frac{2a\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)}{3(a^2+b^2)} - \frac{b\left(4\left(\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)-\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}\right)\right)+2\sqrt{2}\,\log\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-2\sqrt{2}\,\log\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)}{8(a^2+b^2)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])),x]
```

```
[Out] -(((2*a^(3/2)*ArcTan[(Sqrt[a]*Sqrt[Cot[c + d*x]])/Sqrt[b]])/(Sqrt[b]*(a^2 +
 b^2)) - (2*a*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*
x]^2])/(3*(a^2 + b^2)) - (b*(4*(Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x
]]] - Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]) + 2*Sqrt[2]*Log[1 - S
qrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - 2*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[
Cot[c + d*x]] + Cot[c + d*x]]))/(8*(a^2 + b^2)))/d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 16.66, size = 1900, normalized size = 8.19

method	result	size
default	Expression too large to display	1900

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(cos(d*x+c)+1)^2*(-EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2+b^2)^(3/2)*a-EllipticPi((-(cos(d*x+c)-1
-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2+b^2)^(3/2)*b+Ell
ipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2)
)*(a^2+b^2)^(1/2)*a^3+EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1
/2),1/2-1/2*I,1/2*2^(1/2))*(a^2+b^2)^(1/2)*b^3-EllipticPi((-(cos(d*x+c)-1-s
in(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2+b^2)^(3/2)*a-Ellip
ticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*
(a^2+b^2)^(3/2)*b+EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),
1/2+1/2*I,1/2*2^(1/2))*(a^2+b^2)^(1/2)*a^3+EllipticPi((-(cos(d*x+c)-1-sin(d
*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2+b^2)^(1/2)*b^3-2*Ellip
ticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),a/(b+(a^2+b^2)^(1/2)+a)
,1/2*2^(1/2))*a^4-I*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2
),1/2+1/2*I,1/2*2^(1/2))*(a^2+b^2)^(1/2)*a^3+I*EllipticPi((-(cos(d*x+c)-1-s
in(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2+b^2)^(3/2)*a+I*Ell
ipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2)
)*(a^2+b^2)^(1/2)*a^3-I*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^
(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2+b^2)^(3/2)*a-2*EllipticPi((-(cos(d*x+c)-1
-sin(d*x+c))/sin(d*x+c))^(1/2),a/(b+(a^2+b^2)^(1/2)+a),1/2*2^(1/2))*a^2*b^2
+2*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),-a/(-b+(a^2+b^2
)^(1/2)-a),1/2*2^(1/2))*(a^2+b^2)^(1/2)*a^3+2*EllipticPi((-(cos(d*x+c)-1-si
n(d*x+c))/sin(d*x+c))^(1/2),-a/(-b+(a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^2*b^2+
2*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),a/(b+(a^2+b^2)^(
1/2)+a),1/2*2^(1/2))*(a^2+b^2)^(1/2)*a^3+2*EllipticPi((-(cos(d*x+c)-1-sin(d
*x+c))/sin(d*x+c))^(1/2),-a/(-b+(a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^4+I*Ellip
ticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*
(a^2+b^2)^(3/2)*b+I*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2
),1/2+1/2*I,1/2*2^(1/2))*(a^2+b^2)^(1/2)*b^3-2*a^2*EllipticPi((-(cos(d*x+c)
-1-sin(d*x+c))/sin(d*x+c))^(1/2),a/(b+(a^2+b^2)^(1/2)+a),1/2*2^(1/2))*b*(a^
2+b^2)^(1/2)-2*a^2*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)
,-a/(-b+(a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b*(a^2+b^2)^(1/2)+3*EllipticPi((-(c
os(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2+b^2)^
(1/2)*a^2*b+3*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-
1/2*I,1/2*2^(1/2))*(a^2+b^2)^(1/2)*a*b^2+3*EllipticPi((-(cos(d*x+c)-1-sin(d
*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2+b^2)^(1/2)*a^2*b+3*Ell
ipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2)
)*(a^2+b^2)^(1/2)*a*b^2-I*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)
```

```
)^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2+b^2)^(1/2)*b^3-I*EllipticPi((-(cos(d*x+
c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2+b^2)^(3/2)*b
+I*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2
^(1/2))*(a^2+b^2)^(1/2)*a*b^2+I*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(
d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2+b^2)^(1/2)*a^2*b-I*EllipticPi((-(
cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2+b^2)
^(1/2)*a^2*b-I*EllipticPi((-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2
-1/2*I,1/2*2^(1/2))*(a^2+b^2)^(1/2)*a*b^2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))
/sin(d*x+c))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))/(cos(d*x+c)/sin(d*x+c))^(3/2)
/sin(d*x+c)^4*2^(1/2)/(a^2+b^2)^(3/2)/(b+(a^2+b^2)^(1/2)+a)/(-b+(a^2+b^2)^(
1/2)-a)
```

Maxima [**A**]
time = 0.55, size = 175, normalized size = 0.75

$$-\frac{\frac{8\,a^2\arctan\left(\frac{a}{\sqrt{ab}\,\sqrt{\tan\left(dx+c\right)}}\right)}{\left(a^2+b^2\right)\sqrt{ab}}-\frac{2\,\sqrt{2}\,(a-b)\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+2\,\sqrt{2}\,(a-b)\arctan\left(-\frac{1}{2}\,\sqrt{2}\,\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)-\sqrt{2}\,(a+b)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}\,(a+b)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^2+b^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(8*a^2*arctan(a/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^2 + b^2)*sqrt(a*b)
) - (2*sqrt(2)*(a - b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c))))
 + 2*sqrt(2)*(a - b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c))))
- sqrt(2)*(a + b)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sq
rt(2)*(a + b)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^2 +
 b^2))/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\tan\left(c+dx\right)\right)\cot^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(3/2)/(a+b*tan(d*x+c)),x)
[Out] Integral(1/((a + b*tan(c + d*x))*cot(c + d*x)**(3/2)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x, algorithm="giac")
[Out] Exception raised: AttributeError >> type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cot\left(c+d\,x\right)^{3/2}\,\left(a+b\,\tan\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))),x)
[Out] int(1/(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))), x)
```

3.825 $\int \frac{1}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx$

Optimal. Leaf size=250

$$-\frac{(a+b)\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}+\frac{(a+b)\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}+\frac{2a^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt{a}}{\ldots}\right.}{b^{3/2}\,(a^2\ldots}$$

```
[Out] 2*a^(5/2)*arctan(a^(1/2)*cot(d*x+c)^(1/2)/b^(1/2))/b^(3/2)/(a^2+b^2)/d+1/2*
(a+b)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)+1/2*(a+b)*arc
tan(1+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)-1/4*(a-b)*ln(1+cot(d*x+
c)-2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)+1/4*(a-b)*ln(1+cot(d*x+c)+
2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)/d*2^(1/2)+2/b/d/cot(d*x+c)^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3754, 3650, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{(a+b)\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d\,(a^2+b^2)}+\frac{(a+b)\mathrm{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d\,(a^2+b^2)}-\frac{(a-b)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)}+\frac{(a-b)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)}+\frac{2a^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d\,(a^2+b^2)}+\frac{2}{bd\sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])),x]
```

```
[Out] -(((a + b)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)*d))
 + ((a + b)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)*d)
 + (2*a^(5/2)*ArcTan[(Sqrt[a]*Sqrt[Cot[c + d*x]])/Sqrt[b]])/(b^(3/2)*(a^2 +
 b^2)*d) + 2/(b*d*Sqrt[Cot[c + d*x]]) - ((a - b)*Log[1 - Sqrt[2]*Sqrt[Cot[c
 + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)*d) + ((a - b)*Log[1 + Sqrt
[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = \int \frac{1}{\cot^{\frac{3}{2}}(c+dx)(b+a\cot(c+dx))}\,dx$$

$$= \frac{2}{bd\sqrt{\cot(c+dx)}} + \frac{2\int \frac{-\frac{a}{2}-\frac{1}{2}b\cot(c+dx)-\frac{1}{2}a\cot^2(c+dx)}{\sqrt{\cot(c+dx)}\,(b+a\cot(c+dx))}\,dx}{b}$$

$$= \frac{2}{bd\sqrt{\cot(c+dx)}} + \frac{2\int \frac{-\frac{ab}{2}-\frac{1}{2}b^2\cot(c+dx)}{\sqrt{\cot(c+dx)}}\,dx}{b\,(a^2+b^2)} - \frac{a^3\int \frac{1+\cot^2(c+}{\sqrt{\cot(c+dx)}\,(b}}{b\,(a^2+b^2}$$

$$= \frac{2}{bd\sqrt{\cot(c+dx)}} + \frac{4\text{Subst}\left(\int \frac{\frac{ab}{2}+\frac{b^2x^2}{2}}{1+x^4}\,dx, x, \sqrt{\cot(c+dx)}\right)}{b\,(a^2+b^2)\,d} - \frac{a^3}{}$$

$$= \frac{2}{bd\sqrt{\cot(c+dx)}} + \frac{(a-b)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\cot(c+dx)}\right)}{(a^2+b^2)\,d} +$$

$$= \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}\,(a^2+b^2)\,d} + \frac{2}{bd\sqrt{\cot(c+dx)}} - \frac{(a-b)\text{Su}}{}$$

$$= \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}\,(a^2+b^2)\,d} + \frac{2}{bd\sqrt{\cot(c+dx)}} - \frac{(a-b)\,\text{lo}}{}$$

$$= -\frac{(a+b)\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} + \frac{(a+b)\tan^{-1}\left(1+\sqrt{2}\right.}{\sqrt{2}\,(a^2+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.41, size = 193, normalized size = 0.77

$$\frac{8a^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{a\cot(c+dx)}{b}\right)+b\left(8b\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\cot^2(c+dx)\right)+\sqrt{2}\,a\sqrt{\cot(c+dx)}\left(-2\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)+2\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)-\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)+\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\right)}{4b\,(a^2+b^2)\,d\sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])),x]
```

```
[Out] (8*a^2*Hypergeometric2F1[-1/2, 1, 1/2, -((a*Cot[c + d*x])/b)] + b*(8*b*Hype
rgeometric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2] + Sqrt[2]*a*Sqrt[Cot[c + d*x]]
*(-2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] + 2*ArcTan[1 + Sqrt[2]*Sqrt[Cot
[c + d*x]]] - Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] + Log[1 +
Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])))/(4*b*(a^2 + b^2)*d*Sqrt[Cot[c
 + d*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 55.60, size = 5450, normalized size = 21.80

method	result	size
default	Expression too large to display	5450

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.53, size = 189, normalized size = 0.76

$$\frac{\frac{8\,a^3\arctan\left(\frac{a}{\sqrt{ab}\,\sqrt{\tan\left(dx+c\right)}}\right)}{\left(a^2b+b^3\right)\sqrt{ab}}+\frac{2\,\sqrt{2}\,(a+b)\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+2\,\sqrt{2}\,(a+b)\arctan\left(-\frac{1}{2}\,\sqrt{2}\,\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+\sqrt{2}\,(a-b)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)-\sqrt{2}\,(a-b)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)}{a^2+b^2}+\frac{8\,\sqrt{\tan\left(dx+c\right)}}{b}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(8*a^3*arctan(a/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^2*b + b^3)*sqrt(a*b
)) + (2*sqrt(2)*(a + b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))
) + 2*sqrt(2)*(a + b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c))))
 + sqrt(2)*(a - b)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - s
qrt(2)*(a - b)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^2
+ b^2) + 8*sqrt(tan(d*x + c))/b)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\tan\left(c+dx\right)\right)\cot^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(5/2)/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(1/((a + b*tan(c + d*x))*cot(c + d*x)**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x, algorithm="giac")
[Out] integrate(1/((b*tan(d*x + c) + a)*cot(d*x + c)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cot\left(c+d\,x\right)^{5/2}\,\left(a+b\,\tan\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))),x)
[Out] int(1/(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))), x)
```

3.826 $\int \frac{\cot^{\frac{5}{2}}(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=398

$$-\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}+\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}-\cdots$$

```
[Out] -b^(7/2)*(9*a^2+5*b^2)*arctan(a^(1/2)*cot(d*x+c)^(1/2)/b^(1/2))/a^(7/2)/(a^
2+b^2)^2/d-1/3*(2*a^2+5*b^2)*cot(d*x+c)^(3/2)/a^2/(a^2+b^2)/d+b^2*cot(d*x+c
)^(5/2)/a/(a^2+b^2)/d/(b+a*cot(d*x+c))+1/2*(a^2-2*a*b-b^2)*arctan(-1+2^(1/2
)*cot(d*x+c)^(1/2))/(a^2+b^2)^2/d*2^(1/2)+1/2*(a^2-2*a*b-b^2)*arctan(1+2^(1
/2)*cot(d*x+c)^(1/2))/(a^2+b^2)^2/d*2^(1/2)+1/4*(a^2+2*a*b-b^2)*ln(1+cot(d*
x+c)-2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)^2/d*2^(1/2)-1/4*(a^2+2*a*b-b^2)*ln
(1+cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)^2/d*2^(1/2)+b*(4*a^2+5*b^
2)*cot(d*x+c)^(1/2)/a^3/(a^2+b^2)/d
```

Rubi [A]
time = 0.76, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3754, 3646, 3728, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{b^2\cot^{\frac{5}{2}}(c+dx)}{ad\,(a^2+b^2)\,(a\cot(c+dx)+b)}-\frac{(2a^2+5b^2)\cot^{\frac{3}{2}}(c+dx)}{3a^2d\,(a^2+b^2)}+\frac{(a^2+2ab-b^2)\log\left(\cot(c+dx)-\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}-\frac{(a^2+2ab-b^2)\log\left(\cot(c+dx)+\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}-\frac{b^{7/2}(9a^2+5b^2)\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{a^{7/2}d\,(a^2+b^2)^2}+\frac{b(4a^2+5b^2)\sqrt{\cot(c+dx)}}{a^3d\,(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(5/2)/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((a^2 - 2*a*b - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*(a^
2 + b^2)^2*d)) + ((a^2 - 2*a*b - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]
])/(Sqrt[2]*(a^2 + b^2)^2*d) - (b^(7/2)*(9*a^2 + 5*b^2)*ArcTan[(Sqrt[a]*Sqr
t[Cot[c + d*x]])/Sqrt[b]])/(a^(7/2)*(a^2 + b^2)^2*d) + (b*(4*a^2 + 5*b^2)*S
qrt[Cot[c + d*x]])/(a^3*(a^2 + b^2)*d) - ((2*a^2 + 5*b^2)*Cot[c + d*x]^(3/2
))/(3*a^2*(a^2 + b^2)*d) + (b^2*Cot[c + d*x]^(5/2))/(a*(a^2 + b^2)*d*(b + a
*Cot[c + d*x])) + ((a^2 + 2*a*b - b^2)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] +
 Cot[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^2*d) - ((a^2 + 2*a*b - b^2)*Log[1 +
Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^2*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
```

```
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\cot^{\frac{9}{2}}(c+dx)}{(b+a\cot(c+dx))^2}\,dx$$

$$= \frac{b^2\cot^{\frac{5}{2}}(c+dx)}{a\left(a^2+b^2\right)d(b+a\cot(c+dx))} - \frac{\int \frac{\cot^{\frac{3}{2}}(c+dx)\left(-\frac{5b^2}{2}+ab\cot(c+dx)-\frac{1}{2}\left(2a^2+5b^2\right)\cot^2(c+\right.}{b+a\cot(c+dx)}}{a\left(a^2+b^2\right)}$$

$$= -\frac{\left(2a^2+5b^2\right)\cot^{\frac{3}{2}}(c+dx)}{3a^2\left(a^2+b^2\right)d} + \frac{b^2\cot^{\frac{5}{2}}(c+dx)}{a\left(a^2+b^2\right)d(b+a\cot(c+dx))} + \frac{2\int \sqrt{\cot(c+}}{}$$

$$= \frac{b(4a^2+5b^2)\sqrt{\cot(c+dx)}}{a^3\left(a^2+b^2\right)d} - \frac{\left(2a^2+5b^2\right)\cot^{\frac{3}{2}}(c+dx)}{3a^2\left(a^2+b^2\right)d} + \frac{b^2\cot^{\frac{5}{2}}(c+}{a\left(a^2+b^2\right)d(b+a}$$

$$= \frac{b(4a^2+5b^2)\sqrt{\cot(c+dx)}}{a^3\left(a^2+b^2\right)d} - \frac{\left(2a^2+5b^2\right)\cot^{\frac{3}{2}}(c+dx)}{3a^2\left(a^2+b^2\right)d} + \frac{b^2\cot^{\frac{5}{2}}(c+}{a\left(a^2+b^2\right)d(b+a}$$

$$= \frac{b(4a^2+5b^2)\sqrt{\cot(c+dx)}}{a^3\left(a^2+b^2\right)d} - \frac{\left(2a^2+5b^2\right)\cot^{\frac{3}{2}}(c+dx)}{3a^2\left(a^2+b^2\right)d} + \frac{b^2\cot^{\frac{5}{2}}(c+}{a\left(a^2+b^2\right)d(b+a}$$

$$= \frac{b(4a^2+5b^2)\sqrt{\cot(c+dx)}}{a^3\left(a^2+b^2\right)d} - \frac{\left(2a^2+5b^2\right)\cot^{\frac{3}{2}}(c+dx)}{3a^2\left(a^2+b^2\right)d} + \frac{b^2\cot^{\frac{5}{2}}(c+}{a\left(a^2+b^2\right)d(b+a}$$

$$= -\frac{b^{7/2}(9a^2+5b^2)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{a^{7/2}\left(a^2+b^2\right)^2 d} + \frac{b(4a^2+5b^2)\sqrt{\cot(c+dx)}}{a^3\left(a^2+b^2\right)d}$$

$$= -\frac{b^{7/2}(9a^2+5b^2)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{a^{7/2}\left(a^2+b^2\right)^2 d} + \frac{b(4a^2+5b^2)\sqrt{\cot(c+dx)}}{a^3\left(a^2+b^2\right)d}$$

$$= -\frac{(a^2-2ab-b^2)\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)^2 d} + \frac{(a^2-2ab-b^2)\tan^{-1}\left(1\right.}{\sqrt{2}\left(a^2\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.18, size = 455, normalized size = 1.14

$$\frac{-\frac{4ab\cot^{\frac{9}{2}}(c+dx)}{9(a^2+b^2)^2}-\frac{4b^2\left(15\cot^{\frac{7}{2}}(c+dx)-7b\left(\frac{3\cot^{\frac{5}{2}}(c+dx)}{a}-\frac{5b\left(-\frac{3b\left(-\frac{\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{a^{3/2}}+\frac{\sqrt{\cot(c+dx)}}{a}\right)}{a}+\frac{\cot^{\frac{3}{2}}(c+dx)}{a}\right)}{a}\right)\right)}{105(a^2+b^2)^2}+\frac{2(a^2-b^2)\left(7\cot^{\frac{3}{2}}(c+dx)-3\cot^{\frac{7}{2}}(c+dx)-7\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)\right)}{21(a^2+b^2)^2}+\frac{2a^2\cot^{\frac{11}{2}}(c+dx)\,{}_2F_1\left(2,\frac{11}{2};\frac{13}{2};-\frac{a\cot(c+dx)}{b}\right)}{11b^2(a^2+b^2)}-\frac{ab\left(360\sqrt{\cot(c+dx)}-72\cot^{\frac{5}{2}}(c+dx)+40\cot^{\frac{9}{2}}(c+dx)+45\left(2\left(\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)\right)+\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\right)}{90(a^2+b^2)^2}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(5/2)/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((4*a*b*Cot[c + d*x]^(9/2))/(9*(a^2 + b^2)^2) - (4*b^2*(15*Cot[c + d*x]^(
7/2) - 7*b*((3*Cot[c + d*x]^(5/2))/a - (5*b*((-3*b*(-((Sqrt[b]*ArcTan[(Sqrt
[a]*Sqrt[Cot[c + d*x]])/Sqrt[b]])/a^(3/2)) + Sqrt[Cot[c + d*x]]/a))/a + Cot
[c + d*x]^(3/2)/a))/a)))/(105*(a^2 + b^2)^2) + (2*(a^2 - b^2)*(7*Cot[c + d*
x]^(3/2) - 3*Cot[c + d*x]^(7/2) - 7*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/
4, 1, 7/4, -Cot[c + d*x]^2]))/(21*(a^2 + b^2)^2) + (2*a^2*Cot[c + d*x]^(11/
2)*Hypergeometric2F1[2, 11/2, 13/2, -((a*Cot[c + d*x])/b)])/(11*b^2*(a^2 +
b^2)) - (a*b*(360*Sqrt[Cot[c + d*x]] - 72*Cot[c + d*x]^(5/2) + 40*Cot[c + d
*x]^(9/2) + 45*(2*(Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - Sqrt[2]
*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]) + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot
[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + C
ot[c + d*x]])))/(90*(a^2 + b^2)^2))/d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 22.50, size = 37365, normalized size = 93.88

method	result	size
default	Expression too large to display	37365

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.54, size = 318, normalized size = 0.80

$$\frac{\frac{12\,b^4}{\left(a^5 b+a^3 b^3+\frac{a^6+a^4 b^2}{\tan\left(dx+c\right)}\right)\sqrt{\tan\left(dx+c\right)}}-\frac{12\,\left(9\,a^2 b^4+5\,b^6\right)\arctan\left(\frac{a}{\sqrt{ab}\,\sqrt{\tan\left(dx+c\right)}}\right)}{\left(a^7+2\,a^5 b^2+a^3 b^4\right)\sqrt{ab}}+\frac{3\,\left(2\,\sqrt{2}\,\left(a^2-2\,ab-b^2\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+2\,\sqrt{2}\,\left(a^2-2\,ab-b^2\right)\arctan\left(-\frac{1}{2}\,\sqrt{2}\,\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)-\sqrt{2}\,\left(a^2+2\,ab-b^2\right)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)+\sqrt{2}\,\left(a^2+2\,ab-b^2\right)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)\right)}{a^4+2\,a^2 b^2+b^4}+\frac{8\,\left(\frac{6\,b}{\sqrt{\tan\left(dx+c\right)}}-\frac{a}{\tan\left(dx+c\right)^{\frac{3}{2}}}\right)}{a^3}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/12*(12*b^4/((a^5*b + a^3*b^3 + (a^6 + a^4*b^2)/tan(d*x + c))*sqrt(tan(d*x
 + c))) - 12*(9*a^2*b^4 + 5*b^6)*arctan(a/(sqrt(a*b)*sqrt(tan(d*x + c))))/(
(a^7 + 2*a^5*b^2 + a^3*b^4)*sqrt(a*b)) + 3*(2*sqrt(2)*(a^2 - 2*a*b - b^2)*a
rctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*(a^2 - 2*a*
b - b^2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*(a
^2 + 2*a*b - b^2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sq
rt(2)*(a^2 + 2*a*b - b^2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c)
+ 1))/(a^4 + 2*a^2*b^2 + b^4) + 8*(6*b/sqrt(tan(d*x + c)) - a/tan(d*x + c)^
(3/2))/a^3)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(5/2)/(a+b*tan(d*x+c))**2,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^2,x, algorithm="giac")
[Out] integrate(cot(d*x + c)^(5/2)/(b*tan(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^{5/2}}{\left(a+b\tan\left(c+d\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(5/2)/(a + b*tan(c + d*x))^2,x)
[Out] int(cot(c + d*x)^(5/2)/(a + b*tan(c + d*x))^2, x)
```

3.827 $\int \frac{\cot^{\frac{3}{2}}(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=357

$$-\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}+\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}+\frac{b^{5/2}}{}$$

```
[Out] b^(5/2)*(7*a^2+3*b^2)*arctan(a^(1/2)*cot(d*x+c)^(1/2)/b^(1/2))/a^(5/2)/(a^2
+b^2)^2/d+b^2*cot(d*x+c)^(3/2)/a/(a^2+b^2)/d/(b+a*cot(d*x+c))+1/2*(a^2+2*a*
b-b^2)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)^2/d*2^(1/2)+1/2*(a^2+2
*a*b-b^2)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)^2/d*2^(1/2)-1/4*(a^2
-2*a*b-b^2)*ln(1+cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)^2/d*2^(1/2)
+1/4*(a^2-2*a*b-b^2)*ln(1+cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)^2/
d*2^(1/2)-(2*a^2+3*b^2)*cot(d*x+c)^(1/2)/a^2/(a^2+b^2)/d
```

Rubi [A]
time = 0.56, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3754, 3646, 3728, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{b^2\cot^{\frac{3}{2}}(c+dx)}{ad\,(a^2+b^2)\,(a\cot(c+dx)+b)}-\frac{(2a^2+3b^2)\sqrt{\cot(c+dx)}}{a^2d\,(a^2+b^2)}-\frac{(a^2-2ab-b^2)\log\left(\cot(c+dx)-\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{(a^2-2ab-b^2)\log\left(\cot(c+dx)+\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{b^{5/2}(7a^2+3b^2)\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{a^{5/2}d\,(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(3/2)/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((a^2 + 2*a*b - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*(a^
2 + b^2)^2*d)) + ((a^2 + 2*a*b - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]
])/(Sqrt[2]*(a^2 + b^2)^2*d) + (b^(5/2)*(7*a^2 + 3*b^2)*ArcTan[(Sqrt[a]*Sqr
t[Cot[c + d*x]])/Sqrt[b]])/(a^(5/2)*(a^2 + b^2)^2*d) - ((2*a^2 + 3*b^2)*Sqr
t[Cot[c + d*x]])/(a^2*(a^2 + b^2)*d) + (b^2*Cot[c + d*x]^(3/2))/(a*(a^2 + b
^2)*d*(b + a*Cot[c + d*x])) - ((a^2 - 2*a*b - b^2)*Log[1 - Sqrt[2]*Sqrt[Cot
[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^2*d) + ((a^2 - 2*a*b - b
^2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*(a^2 + b
^2)^2*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
```

```
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\cot^{\frac{7}{2}}(c+dx)}{(b+a\cot(c+dx))^2}\,dx$$

$$= \frac{b^2\cot^{\frac{3}{2}}(c+dx)}{a\,(a^2+b^2)\,d(b+a\cot(c+dx))} - \frac{\int \frac{\sqrt{\cot(c+dx)}\,\left(-\frac{3b^2}{2}+ab\cot(c+dx)-\frac{1}{2}(2a^2+3b^2)\right.}{b+a\cot(c+dx)}}{a\,(a^2+b^2)}$$

$$= -\frac{(2a^2+3b^2)\sqrt{\cot(c+dx)}}{a^2\,(a^2+b^2)\,d} + \frac{b^2\cot^{\frac{3}{2}}(c+dx)}{a\,(a^2+b^2)\,d(b+a\cot(c+dx))} + \frac{2\int \frac{-\frac{1}{4}b(2a^2+}{}}{}$$

$$= -\frac{(2a^2+3b^2)\sqrt{\cot(c+dx)}}{a^2\,(a^2+b^2)\,d} + \frac{b^2\cot^{\frac{3}{2}}(c+dx)}{a\,(a^2+b^2)\,d(b+a\cot(c+dx))} + \frac{2\int \frac{-\frac{1}{2}a^2(a^2-}{\sqrt{\mathrm{co}}}}{a^2(}$$

$$= -\frac{(2a^2+3b^2)\sqrt{\cot(c+dx)}}{a^2\,(a^2+b^2)\,d} + \frac{b^2\cot^{\frac{3}{2}}(c+dx)}{a\,(a^2+b^2)\,d(b+a\cot(c+dx))} + \frac{4\mathrm{Subst}\left(\int \frac{1}{2}\right.}{}$$

$$= -\frac{(2a^2+3b^2)\sqrt{\cot(c+dx)}}{a^2\,(a^2+b^2)\,d} + \frac{b^2\cot^{\frac{3}{2}}(c+dx)}{a\,(a^2+b^2)\,d(b+a\cot(c+dx))} + \frac{(a^2-2ab-}{}$$

$$= \frac{b^{5/2}(7a^2+3b^2)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{a^{5/2}\,(a^2+b^2)^2\,d} - \frac{(2a^2+3b^2)\sqrt{\cot(c+dx)}}{a^2\,(a^2+b^2)\,d} +$$

$$= \frac{b^{5/2}(7a^2+3b^2)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{a^{5/2}\,(a^2+b^2)^2\,d} - \frac{(2a^2+3b^2)\sqrt{\cot(c+dx)}}{a^2\,(a^2+b^2)\,d} +$$

$$= -\frac{(a^2+2ab-b^2)\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} + \frac{(a^2+2ab-b^2)\tan^{-1}\left(1\right.}{\sqrt{2}\,(a^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order

3 in optimal.
time = 6.12, size = 424, normalized size = 1.19

$$-\frac{\frac{4ab\cot^{\frac{7}{2}}(c+dx)}{7(a^2+b^2)^2}-\frac{4b^2\left(3\cot^{\frac{5}{2}}(c+dx)-5b\left(-\frac{3b\left(-\frac{\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{a^{3/2}}+\frac{\sqrt{\cot(c+dx)}}{a}\right)}{a}+\frac{\cot^{\frac{3}{2}}(c+dx)}{a}\right)\right)}{15(a^2+b^2)^2}+\frac{4ab\left(7\cot^{\frac{3}{2}}(c+dx)-3\cot^{\frac{7}{2}}(c+dx)-7\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)\right)}{21(a^2+b^2)^2}+\frac{2a^2\cot^{\frac{9}{2}}(c+dx)\,{}_2F_1\left(2,\frac{9}{2};\frac{11}{2};-\frac{a\cot(c+dx)}{b}\right)}{9b^2(a^2+b^2)}+\frac{(a^2-b^2)\left(40\sqrt{\cot(c+dx)}-8\cot^{\frac{5}{2}}(c+dx)+\frac{5}{2}\left(4\left(\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)-\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}\right)\right)+2\sqrt{2}\log\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-2\sqrt{2}\log\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\right)}{20(a^2+b^2)^2}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(3/2)/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((4*a*b*Cot[c + d*x]^(7/2))/(7*(a^2 + b^2)^2) - (4*b^2*(3*Cot[c + d*x]^(5
/2) - 5*b*((-3*b*(-((Sqrt[b]*ArcTan[(Sqrt[a]*Sqrt[Cot[c + d*x]])/Sqrt[b]])/
a^(3/2)) + Sqrt[Cot[c + d*x]]/a))/a + Cot[c + d*x]^(3/2)/a)))/(15*(a^2 + b^
2)^2) + (4*a*b*(7*Cot[c + d*x]^(3/2) - 3*Cot[c + d*x]^(7/2) - 7*Cot[c + d*x
]^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2]))/(21*(a^2 + b^2)^2
) + (2*a^2*Cot[c + d*x]^(9/2)*Hypergeometric2F1[2, 9/2, 11/2, -((a*Cot[c +
d*x])/b)])/(9*b^2*(a^2 + b^2)) + ((a^2 - b^2)*(40*Sqrt[Cot[c + d*x]] - 8*Co
t[c + d*x]^(5/2) + (5*(4*(Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] -
Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]) + 2*Sqrt[2]*Log[1 - Sqrt[2]
*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - 2*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c
+ d*x]] + Cot[c + d*x]]))/2))/(20*(a^2 + b^2)^2))/d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 23.15, size = 28980, normalized size = 81.18

method	result	size
default	Expression too large to display	28980

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.52, size = 303, normalized size = 0.85

$$-\frac{\frac{4b^3}{\left(a^4b+a^2b^3+\frac{a^5+a^3b^2}{\tan(dx+c)}\right)\sqrt{\tan(dx+c)}}-\frac{4(7a^2b^3+3b^5)\arctan\left(\frac{a}{\sqrt{ab}\,\sqrt{\tan(dx+c)}}\right)}{(a^6+2a^4b^2+a^2b^4)\sqrt{ab}}-\frac{2\sqrt{2}(a^2+2ab-b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}(a^2+2ab-b^2)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+\sqrt{2}(a^2-2ab-b^2)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}(a^2-2ab-b^2)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^4+2a^2b^2+b^4}+\frac{8}{a^2\sqrt{\tan(dx+c)}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(4*b^3/((a^4*b + a^2*b^3 + (a^5 + a^3*b^2)/tan(d*x + c))*sqrt(tan(d*x
+ c))) - 4*(7*a^2*b^3 + 3*b^5)*arctan(a/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a
^6 + 2*a^4*b^2 + a^2*b^4)*sqrt(a*b)) - (2*sqrt(2)*(a^2 + 2*a*b - b^2)*arcta
n(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*(a^2 + 2*a*b -
```

```
b^2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*(a^2 -
 2*a*b - b^2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2
)*(a^2 - 2*a*b - b^2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1)
)/(a^4 + 2*a^2*b^2 + b^4) + 8/(a^2*sqrt(tan(d*x + c))))/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(3/2)/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**(3/2)/(a + b*tan(c + d*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)^(3/2)/(b*tan(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cot(c+d\,x)}^{3/2}}{{(a+b\tan(c+d\,x))}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(3/2)/(a + b*tan(c + d*x))^2,x)
```

```
[Out] int(cot(c + d*x)^(3/2)/(a + b*tan(c + d*x))^2, x)
```

3.828 $\int \frac{\sqrt{\cot(c+dx)}}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=318

$$\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}-\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}-\frac{b^{3/2}(\cdots}{}$$

```
[Out] -b^(3/2)*(5*a^2+b^2)*arctan(a^(1/2)*cot(d*x+c)^(1/2)/b^(1/2))/a^(3/2)/(a^2+
b^2)^2/d-1/2*(a^2-2*a*b-b^2)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)^
2/d*2^(1/2)-1/2*(a^2-2*a*b-b^2)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2
)^2/d*2^(1/2)-1/4*(a^2+2*a*b-b^2)*ln(1+cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2))
/(a^2+b^2)^2/d*2^(1/2)+1/4*(a^2+2*a*b-b^2)*ln(1+cot(d*x+c)+2^(1/2)*cot(d*x+
c)^(1/2))/(a^2+b^2)^2/d*2^(1/2)+b^2*cot(d*x+c)^(1/2)/a/(a^2+b^2)/d/(b+a*cot
(d*x+c))
```

Rubi [A]
time = 0.38, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3754, 3646, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}-\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{b^2\sqrt{\cot(c+dx)}}{ad\,(a^2+b^2)\,(a\cot(c+dx)+b)}-\frac{(a^2+2ab-b^2)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{(a^2+2ab-b^2)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}-\frac{b^{3/2}(5a^2+b^2)\,\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{a^{3/2}d\,(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cot[c + d*x]]/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((a^2 - 2*a*b - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*(a^2
+ b^2)^2*d) - ((a^2 - 2*a*b - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/
(Sqrt[2]*(a^2 + b^2)^2*d) - (b^(3/2)*(5*a^2 + b^2)*ArcTan[(Sqrt[a]*Sqrt[Cot
[c + d*x]])/Sqrt[b]])/(a^(3/2)*(a^2 + b^2)^2*d) + (b^2*Sqrt[Cot[c + d*x]])/
(a*(a^2 + b^2)*d*(b + a*Cot[c + d*x])) - ((a^2 + 2*a*b - b^2)*Log[1 - Sqrt[
2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^2*d) + ((a^2
+ 2*a*b - b^2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[
2]*(a^2 + b^2)^2*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{\sqrt{\cot(c+dx)}}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\cot^{\frac{5}{2}}(c+dx)}{(b+a\cot(c+dx))^2}\,dx$$

$$= \frac{b^2\sqrt{\cot(c+dx)}}{a(a^2+b^2)\,d(b+a\cot(c+dx))} - \frac{\int \frac{-\frac{b^2}{2}+ab\cot(c+dx)-\frac{1}{2}(2a^2+b^2)\cot^2(c+dx)}{\sqrt{\cot(c+dx)}\,(b+a\cot(c+dx))}\,dx}{a(a^2+b^2)}$$

$$= \frac{b^2\sqrt{\cot(c+dx)}}{a(a^2+b^2)\,d(b+a\cot(c+dx))} - \frac{\int \frac{2a^2b-a(a^2-b^2)\cot(c+dx)}{\sqrt{\cot(c+dx)}}\,dx}{a(a^2+b^2)^2} + \frac{(b^2(5a^2+b^2))\ \int}{}$$

$$= \frac{b^2\sqrt{\cot(c+dx)}}{a(a^2+b^2)\,d(b+a\cot(c+dx))} - \frac{2\text{Subst}\left(\int \frac{-2a^2b+a(a^2-b^2)x^2}{1+x^4}\,dx, x, \sqrt{\cot(c+}\right.}{a(a^2+b^2)^2\,d}$$

$$= \frac{b^2\sqrt{\cot(c+dx)}}{a(a^2+b^2)\,d(b+a\cot(c+dx))} - \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\cot(}\right.}{(a^2+b^2)^2\,d}$$

$$= -\frac{b^{3/2}(5a^2+b^2)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{a^{3/2}(a^2+b^2)^2\,d} + \frac{b^2\sqrt{\cot(c+dx)}}{a(a^2+b^2)\,d(b+a\cot(c+dx}$$

$$= -\frac{b^{3/2}(5a^2+b^2)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{a^{3/2}(a^2+b^2)^2\,d} + \frac{b^2\sqrt{\cot(c+dx)}}{a(a^2+b^2)\,d(b+a\cot(c+dx}$$

$$= \frac{(a^2-2ab-b^2)\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} - \frac{(a^2-2ab-b^2)\tan^{-1}\left(1+\right.}{\sqrt{2}\,(a^2+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.72, size = 368, normalized size = 1.16

$$\frac{-28a^{3/2}b^2(a^2-b^2)\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)-12a^{7/2}(a^2+b^2)\cot^{\frac{7}{2}}(c+dx)\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};-\frac{a\cot(c+dx)}{b}\right)+7b^2\left(-6\sqrt{2}a^{5/2}b\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)+6\sqrt{2}a^{5/2}b\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+24b^{7/2}\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)-24a^{5/2}b\sqrt{\cot(c+dx)}-24\sqrt{a}b^3\sqrt{\cot(c+dx)}+4a^{7/2}\cot^{\frac{3}{2}}(c+dx)+4a^{3/2}b^2\cot^{\frac{3}{2}}(c+dx)-3\sqrt{2}a^{5/2}b\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)+3\sqrt{2}a^{5/2}b\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)}{42a^{3/2}b^2(a^2+b^2)^2\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cot[c + d*x]]/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-28*a^(3/2)*b^2*(a^2 - b^2)*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1, 7
/4, -Cot[c + d*x]^2] - 12*a^(7/2)*(a^2 + b^2)*Cot[c + d*x]^(7/2)*Hypergeome
tric2F1[2, 7/2, 9/2, -((a*Cot[c + d*x])/b)] + 7*b^2*(-6*Sqrt[2]*a^(5/2)*b*A
rcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] + 6*Sqrt[2]*a^(5/2)*b*ArcTan[1 + Sqrt
[2]*Sqrt[Cot[c + d*x]]] + 24*b^(7/2)*ArcTan[(Sqrt[a]*Sqrt[Cot[c + d*x]])/Sq
rt[b]] - 24*a^(5/2)*b*Sqrt[Cot[c + d*x]] - 24*Sqrt[a]*b^3*Sqrt[Cot[c + d*x]
```

```
] + 4*a^(7/2)*Cot[c + d*x]^(3/2) + 4*a^(3/2)*b^2*Cot[c + d*x]^(3/2) - 3*Sqr
t[2]*a^(5/2)*b*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] + 3*Sqrt[
2]*a^(5/2)*b*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/(42*a^(3/
2)*b^2*(a^2 + b^2)^2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 20.42, size = 18333, normalized size = 57.65

method	result	size
default	Expression too large to display	18333

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.50, size = 283, normalized size = 0.89

$$-\frac{\frac{4\,(5\,a^2b^2+b^4)\arctan\left(\frac{a}{\sqrt{ab}\,\sqrt{\tan\left(dx+c\right)}}\right)}{(a^5+2\,a^3b^2+ab^4)\sqrt{ab}}-\frac{4\,b^2}{\left(a^3b+ab^3+\frac{a^4+a^2b^2}{\tan(dx+c)}\right)\sqrt{\tan\left(dx+c\right)}}+\frac{2\,\sqrt{2}\,(a^2-2\,ab-b^2)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+2\,\sqrt{2}\,(a^2-2\,ab-b^2)\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)-\sqrt{2}\,(a^2+2\,ab-b^2)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}\,(a^2+2\,ab-b^2)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^4+2\,a^2b^2+b^4}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(4*(5*a^2*b^2 + b^4)*arctan(a/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^5 +
2*a^3*b^2 + a*b^4)*sqrt(a*b)) - 4*b^2/((a^3*b + a*b^3 + (a^4 + a^2*b^2)/tan
(d*x + c))*sqrt(tan(d*x + c))) + (2*sqrt(2)*(a^2 - 2*a*b - b^2)*arctan(1/2*
sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*(a^2 - 2*a*b - b^2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*(a^2 + 2*a*b
 - b^2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*(a^2
 + 2*a*b - b^2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^4
 + 2*a^2*b^2 + b^4))/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cot\left(c+dx\right)}}{\left(a+b\tan\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(1/2)/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(sqrt(cot(c + d*x))/(a + b*tan(c + d*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cot(d*x + c))/(b*tan(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cot\left(c+d\,x\right)}}{{\left(a+b\,\tan\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(1/2)/(a + b*tan(c + d*x))^2,x)
```

```
[Out] int(cot(c + d*x)^(1/2)/(a + b*tan(c + d*x))^2, x)
```

3.829 $\int \frac{1}{\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=315

$$\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}-\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}+\frac{\sqrt{b}\,(\cdots}{\cdots}$$

```
[Out] -1/2*(a^2+2*a*b-b^2)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)^2/d*2^(1
/2)-1/2*(a^2+2*a*b-b^2)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)^2/d*2^
(1/2)+1/4*(a^2-2*a*b-b^2)*ln(1+cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^
2)^2/d*2^(1/2)-1/4*(a^2-2*a*b-b^2)*ln(1+cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)
)/(a^2+b^2)^2/d*2^(1/2)+(3*a^2-b^2)*arctan(a^(1/2)*cot(d*x+c)^(1/2)/b^(1/2)
)*b^(1/2)/(a^2+b^2)^2/d/a^(1/2)-b*cot(d*x+c)^(1/2)/(a^2+b^2)/d/(b+a*cot(d*x
+c))
```

Rubi [A]
time = 0.35, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3754, 3648, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{\sqrt{b}\,(3a^2-b^2)\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{\sqrt{a}\,d\,(a^2+b^2)^2}+\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}-\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}-\frac{b\sqrt{\cot(c+dx)}}{d\,(a^2+b^2)\,(a\cot(c+dx)+b)}+\frac{(a^2-2ab-b^2)\log\left(\cot(c+dx)-\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}-\frac{(a^2-2ab-b^2)\log\left(\cot(c+dx)+\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^2),x]
```

```
[Out] ((a^2 + 2*a*b - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*(a^2
+ b^2)^2*d) - ((a^2 + 2*a*b - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/
(Sqrt[2]*(a^2 + b^2)^2*d) + (Sqrt[b]*(3*a^2 - b^2)*ArcTan[(Sqrt[a]*Sqrt[Cot
[c + d*x]])/Sqrt[b]])/(Sqrt[a]*(a^2 + b^2)^2*d) - (b*Sqrt[Cot[c + d*x]])/((
a^2 + b^2)*d*(b + a*Cot[c + d*x])) + ((a^2 - 2*a*b - b^2)*Log[1 - Sqrt[2]*S
qrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^2*d) - ((a^2 - 2*
a*b - b^2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*(
a^2 + b^2)^2*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^2}\,dx = \int \frac{\cot^{\frac{3}{2}}(c+dx)}{(b+a\cot(c+dx))^2}\,dx$$

$$= -\frac{b\sqrt{\cot(c+dx)}}{(a^2+b^2)\,d(b+a\cot(c+dx))} - \frac{\int \frac{\frac{b}{2}-a\cot(c+dx)-\frac{1}{2}b\cot^2(c+dx)}{\sqrt{\cot(c+dx)}\,(b+a\cot(c+dx)}}{a^2+b^2}$$

$$= -\frac{b\sqrt{\cot(c+dx)}}{(a^2+b^2)\,d(b+a\cot(c+dx))} - \frac{\int \frac{-a^2+b^2-2ab\cot(c+dx)}{\sqrt{\cot(c+dx)}}\,dx}{(a^2+b^2)^2} - \frac{(b}{}$$

$$= -\frac{b\sqrt{\cot(c+dx)}}{(a^2+b^2)\,d(b+a\cot(c+dx))} - \frac{2\mathrm{Subst}\left(\int \frac{a^2-b^2+2abx^2}{1+x^4}\,dx, x,\right.}{(a^2+b^2)^2\,d}$$

$$= -\frac{b\sqrt{\cot(c+dx)}}{(a^2+b^2)\,d(b+a\cot(c+dx))} + \frac{(b(3a^2-b^2))\,\mathrm{Subst}\left(\int \frac{1}{b+ax^2}\right.}{(a^2+b^2)}$$

$$= \frac{\sqrt{b}\,(3a^2-b^2)\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{\sqrt{a}\,(a^2+b^2)^2\,d} - \frac{b\sqrt{\cot(c}}{(a^2+b^2)\,d(b+}$$

$$= \frac{\sqrt{b}\,(3a^2-b^2)\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{\sqrt{a}\,(a^2+b^2)^2\,d} - \frac{b\sqrt{\cot(c}}{(a^2+b^2)\,d(b+}$$

$$= \frac{(a^2+2ab-b^2)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} - \frac{(a^2+2ab-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 4.78, size = 301, normalized size = 0.96

$$-\frac{-240b^2\left(-\frac{\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{\sqrt{a}}+\sqrt{\cot(c+dx)}\right)+80ab\cot^{\frac{3}{2}}(c+dx)+80ab\cot^{\frac{3}{2}}(c+dx)\left(-1+{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)\right)+\frac{24a^2(a^2+b^2)\cot^{\frac{5}{2}}(c+dx)\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};-\frac{a\cot(c+dx)}{b}\right)}{b^2}-15(a^2-b^2)\left(2\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)-2\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}\right)+8\sqrt{\cot(c+dx)}+\sqrt{2}\log\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\sqrt{2}\log\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)}{60\,(a^2+b^2)^2\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^2),x]
```

```
[Out] -1/60*(-240*b^2*(-((Sqrt[b]*ArcTan[(Sqrt[a]*Sqrt[Cot[c + d*x]])/Sqrt[b]])/S
qrt[a]) + Sqrt[Cot[c + d*x]]) + 80*a*b*Cot[c + d*x]^(3/2) + 80*a*b*Cot[c +
d*x]^(3/2)*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2]) + (24*a^2
*(a^2 + b^2)*Cot[c + d*x]^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, -((a*Cot[c +
 d*x])/b)])/b^2 - 15*(a^2 - b^2)*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c +
 d*x]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 8*Sqrt[Cot[c +
```

```
d*x]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[
2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/((a^2 + b^2)^2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 17.35, size = 20286, normalized size = 64.40

method	result	size
default	Expression too large to display	20286

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.52, size = 276, normalized size = 0.88

$$\frac{\frac{4\,(3\,a^2b-b^3)\arctan\left(\frac{a}{\sqrt{ab}\,\sqrt{\tan\left(dx+c\right)}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{ab}}-\frac{2\,\sqrt{2}\,(a^2+2\,ab-b^2)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+2\,\sqrt{2}\,(a^2+2\,ab-b^2)\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+\sqrt{2}\,(a^2-2\,ab-b^2)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}\,(a^2-2\,ab-b^2)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{4\,b}{\left(a^2b+b^3+\frac{a^3+ab^2}{\tan(dx+c)}\right)\sqrt{\tan\left(dx+c\right)}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(4*(3*a^2*b - b^3)*arctan(a/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^4 + 2*a
^2*b^2 + b^4)*sqrt(a*b)) - (2*sqrt(2)*(a^2 + 2*a*b - b^2)*arctan(1/2*sqrt(2
)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*(a^2 + 2*a*b - b^2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*(a^2 - 2*a*b - b^2
)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*(a^2 - 2*a
*b - b^2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^4 + 2*a
^2*b^2 + b^4) - 4*b/((a^2*b + b^3 + (a^3 + a*b^2)/tan(d*x + c))*sqrt(tan(d*
x + c))))/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\tan\left(c+dx\right)\right)^2\sqrt{\cot\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(1/2)/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(1/((a + b*tan(c + d*x))**2*sqrt(cot(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*tan(d*x + c) + a)^2*sqrt(cot(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{\cot\left(c+d\,x\right)}\,{\left(a+b\,\tan\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^2),x)
```

```
[Out] int(1/(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^2), x)
```

3.830 $\int \frac{1}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=313

$$-\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}+\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}-\frac{\sqrt{a}}{}$$

```
[Out] 1/2*(a^2-2*a*b-b^2)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)^2/d*2^(1/
2)+1/2*(a^2-2*a*b-b^2)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)^2/d*2^(
1/2)+1/4*(a^2+2*a*b-b^2)*ln(1+cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2
)^2/d*2^(1/2)-1/4*(a^2+2*a*b-b^2)*ln(1+cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2))
/(a^2+b^2)^2/d*2^(1/2)-(a^2-3*b^2)*arctan(a^(1/2)*cot(d*x+c)^(1/2)/b^(1/2))
*a^(1/2)/(a^2+b^2)^2/d/b^(1/2)+a*cot(d*x+c)^(1/2)/(a^2+b^2)/d/(b+a*cot(d*x+
c))
```

Rubi [A]
time = 0.34, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3754, 3649, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{\sqrt{a}\,(a^2-3b^2)\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}\,d\,(a^2+b^2)^2}-\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{a\sqrt{\cot(c+dx)}}{d\,(a^2+b^2)\,(a\cot(c+dx)+b)}+\frac{(a^2+2ab-b^2)\log\left(\cot(c+dx)-\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}-\frac{(a^2+2ab-b^2)\log\left(\cot(c+dx)+\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^2),x]
```

```
[Out] -(((a^2 - 2*a*b - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*(a^
2 + b^2)^2*d)) + ((a^2 - 2*a*b - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]
])/(Sqrt[2]*(a^2 + b^2)^2*d) - (Sqrt[a]*(a^2 - 3*b^2)*ArcTan[(Sqrt[a]*Sqrt[
Cot[c + d*x]])/Sqrt[b]])/(Sqrt[b]*(a^2 + b^2)^2*d) + (a*Sqrt[Cot[c + d*x]])
/((a^2 + b^2)*d*(b + a*Cot[c + d*x])) + ((a^2 + 2*a*b - b^2)*Log[1 - Sqrt[2
]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^2*d) - ((a^2 +
 2*a*b - b^2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2
]*(a^2 + b^2)^2*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx = \int \frac{\sqrt{\cot(c+dx)}}{(b+a\cot(c+dx))^2}\,dx$$

$$= \frac{a\sqrt{\cot(c+dx)}}{(a^2+b^2)\,d(b+a\cot(c+dx))} - \frac{\int \frac{-\frac{a}{2}-b\cot(c+dx)+\frac{1}{2}a\cot^2(c+dx)}{\sqrt{\cot(c+dx)}\,(b+a\cot(c+dx))}\,dx}{a^2+b^2}$$

$$= \frac{a\sqrt{\cot(c+dx)}}{(a^2+b^2)\,d(b+a\cot(c+dx))} - \frac{\int \frac{-2ab+(a^2-b^2)\cot(c+dx)}{\sqrt{\cot(c+dx)}}\,dx}{(a^2+b^2)^2} + \frac{(a(a}{}$$

$$= \frac{a\sqrt{\cot(c+dx)}}{(a^2+b^2)\,d(b+a\cot(c+dx))} - \frac{2\text{Subst}\left(\int \frac{2ab+(-a^2+b^2)x^2}{1+x^4}\,dx, x, \sqrt{}\right.}{(a^2+b^2)^2\,d}$$

$$= \frac{a\sqrt{\cot(c+dx)}}{(a^2+b^2)\,d(b+a\cot(c+dx))} - \frac{(a(a^2-3b^2))\,\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx,\right.}{(a^2+b^2)^2\,d}$$

$$= -\frac{\sqrt{a}\,(a^2-3b^2)\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}\,(a^2+b^2)^2\,d} + \frac{a\sqrt{\cot(c}}{(a^2+b^2)\,d(b+a}$$

$$= -\frac{\sqrt{a}\,(a^2-3b^2)\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}\,(a^2+b^2)^2\,d} + \frac{a\sqrt{\cot(c}}{(a^2+b^2)\,d(b+a}$$

$$= -\frac{(a^2-2ab-b^2)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} + \frac{(a^2-2ab-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.10, size = 380, normalized size = 1.21

$$-\frac{-\frac{4\sqrt{a}\,b^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{(a^2+b^2)^2}+\frac{4ab\sqrt{\cot(c+dx)}}{(a^2+b^2)^2}-\frac{\sqrt{a}\left(-b\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)+\sqrt{a}\,\sqrt{b}\,\sqrt{\cot(c+dx)}-a\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)\cot(c+dx)\right)}{\sqrt{b}\,(a^2+b^2)(b+a\cot(c+dx))}-\frac{2(a^2-b^2)\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)}{3(a^2+b^2)^2}-\frac{ab\left(2\left(\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)-\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}\right)\right)+8\sqrt{\cot(c+dx)}+\sqrt{2}\log\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\sqrt{2}\log\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)}{2(a^2+b^2)^2}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^2),x]
```

```
[Out] -(((-4*Sqrt[a]*b^(3/2)*ArcTan[(Sqrt[a]*Sqrt[Cot[c + d*x]])/Sqrt[b]])/(a^2 +
 b^2)^2 + (4*a*b*Sqrt[Cot[c + d*x]])/(a^2 + b^2)^2 - (Sqrt[a]*(-(b*ArcTan[(
Sqrt[a]*Sqrt[Cot[c + d*x]])/Sqrt[b]]) + Sqrt[a]*Sqrt[b]*Sqrt[Cot[c + d*x]]
- a*ArcTan[(Sqrt[a]*Sqrt[Cot[c + d*x]])/Sqrt[b]]*Cot[c + d*x]))/(Sqrt[b]*(a
^2 + b^2)*(b + a*Cot[c + d*x])) - (2*(a^2 - b^2)*Cot[c + d*x]^(3/2)*Hyperge
ometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2])/(3*(a^2 + b^2)^2) - (a*b*(2*(Sqrt
```

```
[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqr
t[Cot[c + d*x]]]) + 8*Sqrt[Cot[c + d*x]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot
[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + C
ot[c + d*x]]))/(2*(a^2 + b^2)^2))/d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 17.25, size = 20172, normalized size = 64.45

method	result	size
default	Expression too large to display	20172

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.51, size = 274, normalized size = 0.88

$$-\frac{\frac{4\,(a^3-3\,ab^2)\arctan\left(\frac{a}{\sqrt{ab}\,\sqrt{\tan\left(dx+c\right)}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{ab}}-\frac{2\sqrt{2}\,(a^2-2\,ab-b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+2\sqrt{2}\,(a^2-2\,ab-b^2)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)-\sqrt{2}\,(a^2+2\,ab-b^2)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}\,(a^2+2\,ab-b^2)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{4\,a}{\left(a^2b+b^3+\frac{a^3+ab^2}{\tan(dx+c)}\right)\sqrt{\tan\left(dx+c\right)}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(4*(a^3 - 3*a*b^2)*arctan(a/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^4 + 2*
a^2*b^2 + b^4)*sqrt(a*b)) - (2*sqrt(2)*(a^2 - 2*a*b - b^2)*arctan(1/2*sqrt(
2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*(a^2 - 2*a*b - b^2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*(a^2 + 2*a*b - b^
2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*(a^2 + 2*
a*b - b^2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^4 + 2*
a^2*b^2 + b^4) - 4*a/((a^2*b + b^3 + (a^3 + a*b^2)/tan(d*x + c))*sqrt(tan(d
*x + c))))/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\tan\left(c+dx\right)\right)^{2}\cot^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(3/2)/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(1/((a + b*tan(c + d*x))**2*cot(c + d*x)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*tan(d*x + c) + a)^2*cot(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cot\left(c+d\,x\right)^{3/2}\,{\left(a+b\,\tan\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^2),x)
```

```
[Out] int(1/(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^2), x)
```

3.831 $\int \frac{1}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=319

$$-\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}+\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}-\frac{a^{3/2}}{}$$

```
[Out] -a^(3/2)*(a^2+5*b^2)*arctan(a^(1/2)*cot(d*x+c)^(1/2)/b^(1/2))/b^(3/2)/(a^2+
b^2)^2/d+1/2*(a^2+2*a*b-b^2)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)^
2/d*2^(1/2)+1/2*(a^2+2*a*b-b^2)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2
)^2/d*2^(1/2)-1/4*(a^2-2*a*b-b^2)*ln(1+cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2))
/(a^2+b^2)^2/d*2^(1/2)+1/4*(a^2-2*a*b-b^2)*ln(1+cot(d*x+c)+2^(1/2)*cot(d*x+
c)^(1/2))/(a^2+b^2)^2/d*2^(1/2)-a^2*cot(d*x+c)^(1/2)/b/(a^2+b^2)/d/(b+a*cot
(d*x+c))
```

Rubi [A]

time = 0.40, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3754, 3650, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{(a^2+2ab-b^2)\,\text{ArcTan}\left(\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}-\frac{a^2\sqrt{\cot(c+dx)}}{bd\,(a^2+b^2)\,(a\cot(c+dx)+b)}-\frac{(a^2-2ab-b^2)\log\left(\cot(c+dx)-\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{(a^2-2ab-b^2)\log\left(\cot(c+dx)+\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}-\frac{a^{3/2}(a^2+5b^2)\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d\,(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^2),x]
```

```
[Out] -(((a^2 + 2*a*b - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*(a^
2 + b^2)^2*d)) + ((a^2 + 2*a*b - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]
])/(Sqrt[2]*(a^2 + b^2)^2*d) - (a^(3/2)*(a^2 + 5*b^2)*ArcTan[(Sqrt[a]*Sqrt[
Cot[c + d*x]])/Sqrt[b]])/(b^(3/2)*(a^2 + b^2)^2*d) - (a^2*Sqrt[Cot[c + d*x]
])/(b*(a^2 + b^2)*d*(b + a*Cot[c + d*x])) - ((a^2 - 2*a*b - b^2)*Log[1 - Sq
rt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^2*d) + ((a
^2 - 2*a*b - b^2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sq
rt[2]*(a^2 + b^2)^2*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx = \int \frac{1}{\sqrt{\cot(c+dx)}\,(b+a\cot(c+dx))^2}\,dx$$

$$= -\frac{a^2\sqrt{\cot(c+dx)}}{b\,(a^2+b^2)\,d(b+a\cot(c+dx))} - \frac{\int \frac{\frac{1}{2}(-a^2-2b^2)+ab\cot(c+dx)-\frac{1}{2}a^2\text{co}}{\sqrt{\cot(c+dx)}\,(b+a\cot(c+}}{b\,(a^2+b^2)}$$

$$= -\frac{a^2\sqrt{\cot(c+dx)}}{b\,(a^2+b^2)\,d(b+a\cot(c+dx))} - \frac{\int \frac{b(a^2-b^2)+2ab^2\cot(c+dx)}{\sqrt{\cot(c+dx)}}\,dx}{b\,(a^2+b^2)^2} + \cdots$$

$$= -\frac{a^2\sqrt{\cot(c+dx)}}{b\,(a^2+b^2)\,d(b+a\cot(c+dx))} - \frac{2\text{Subst}\left(\int \frac{-b(a^2-b^2)-2ab^2x^2}{1+x^4}\,dx \cdots\right.}{b\,(a^2+b^2)^2}$$

$$= -\frac{a^2\sqrt{\cot(c+dx)}}{b\,(a^2+b^2)\,d(b+a\cot(c+dx))} + \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4} \cdots\right.}{(a^2+b^2) \cdots}$$

$$= -\frac{a^{3/2}(a^2+5b^2)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}\,(a^2+b^2)^2\,d} - \frac{a^2\sqrt{\cot(\cdots}}{b\,(a^2+b^2)\,d(b+\cdots}$$

$$= -\frac{a^{3/2}(a^2+5b^2)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}\,(a^2+b^2)^2\,d} - \frac{a^2\sqrt{\cot(\cdots}}{b\,(a^2+b^2)\,d(b+\cdots}$$

$$= -\frac{(a^2+2ab-b^2)\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} + \frac{(a^2+2ab-\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 2.80, size = 279, normalized size = 0.87

$$-\frac{96a^{3/2}\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right) + \frac{24a^2(a^2+b^2)\sqrt{\cot(c+dx)}\left(\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{\cot(c+dx)}}+\frac{b}{b+a\cot(c+dx)}\right)}{b^2} - 32ab\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right) + 6\sqrt{2}\,(a^2-b^2)\left(2\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-2\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)}{24\,(a^2+b^2)^2\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^2),x]
```

```
[Out] -1/24*(96*a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[a]*Sqrt[Cot[c + d*x]])/Sqrt[b]] + (2
4*a^2*(a^2 + b^2)*Sqrt[Cot[c + d*x]]*((Sqrt[b]*ArcTan[(Sqrt[a]*Sqrt[Cot[c +
 d*x]])/Sqrt[b]])/(Sqrt[a]*Sqrt[Cot[c + d*x]]) + b/(b + a*Cot[c + d*x])))/b
^2 - 32*a*b*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]
^2] + 6*Sqrt[2]*(a^2 - b^2)*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*A
```

```
rcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]
+ Cot[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/((a^
2 + b^2)^2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 16.30, size = 20282, normalized size = 63.58

method	result	size
default	Expression too large to display	20282

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.52, size = 284, normalized size = 0.89

$$-\frac{\frac{4\,(a^4+5\,a^2b^2)\arctan\left(\frac{a}{\sqrt{ab}\,\sqrt{\tan\left(dx+c\right)}}\right)}{(a^4b+2\,a^2b^3+b^5)\sqrt{ab}}+\frac{4\,a^2}{\left(a^2b^2+b^4+\frac{a^3b+ab^3}{\tan(dx+c)}\right)\sqrt{\tan\left(dx+c\right)}}-\frac{2\,\sqrt{2}\,(a^2+2\,ab-b^2)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+2\,\sqrt{2}\,(a^2+2\,ab-b^2)\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+\sqrt{2}\,(a^2-2\,ab-b^2)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}\,(a^2-2\,ab-b^2)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^4+2\,a^2b^2+b^4}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(4*(a^4 + 5*a^2*b^2)*arctan(a/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^4*b
+ 2*a^2*b^3 + b^5)*sqrt(a*b)) + 4*a^2/((a^2*b^2 + b^4 + (a^3*b + a*b^3)/tan
(d*x + c))*sqrt(tan(d*x + c))) - (2*sqrt(2)*(a^2 + 2*a*b - b^2)*arctan(1/2*
sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*(a^2 + 2*a*b - b^2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*(a^2 - 2*a*b
 - b^2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*(a^2
 - 2*a*b - b^2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^4
 + 2*a^2*b^2 + b^4))/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\tan\left(c+dx\right)\right)^2\cot^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(5/2)/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(1/((a + b*tan(c + d*x))**2*cot(c + d*x)**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*tan(d*x + c) + a)^2*cot(d*x + c)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cot\left(c+d\,x\right)^{5/2}\,{\left(a+b\,\tan\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))^2),x)
```

```
[Out] int(1/(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))^2), x)
```

3.832 $\int \frac{1}{\cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=357

$$\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}-\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}+\frac{a^{5/2}(\cdots}{\cdots}$$

```
[Out] a^(5/2)*(3*a^2+7*b^2)*arctan(a^(1/2)*cot(d*x+c)^(1/2)/b^(1/2))/b^(5/2)/(a^2
+b^2)^2/d-1/2*(a^2-2*a*b-b^2)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)
^2/d*2^(1/2)-1/2*(a^2-2*a*b-b^2)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^
2)^2/d*2^(1/2)-1/4*(a^2+2*a*b-b^2)*ln(1+cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)
)/(a^2+b^2)^2/d*2^(1/2)+1/4*(a^2+2*a*b-b^2)*ln(1+cot(d*x+c)+2^(1/2)*cot(d*x
+c)^(1/2))/(a^2+b^2)^2/d*2^(1/2)+(3*a^2+2*b^2)/b^2/(a^2+b^2)/d/cot(d*x+c)^(
1/2)-a^2/b/(a^2+b^2)/d/(b+a*cot(d*x+c))/cot(d*x+c)^(1/2)
```

Rubi [A]
time = 0.58, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3754, 3650, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}-\frac{(a^2-2ab-b^2)\,\text{ArcTan}\left(\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}-\frac{a^2}{bd\,(a^2+b^2)\,\sqrt{\cot(c+dx)}\,(a\cot(c+dx)+b)}+\frac{3a^2+2b^2}{b^2d\,(a^2+b^2)\,\sqrt{\cot(c+dx)}}-\frac{(a^2+2ab-b^2)\log\left(\cot(c+dx)-\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{(a^2+2ab-b^2)\log\left(\cot(c+dx)+\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{a^{5/2}(3a^2+7b^2)\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d\,(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cot[c + d*x]^(7/2)*(a + b*Tan[c + d*x])^2),x]
```

```
[Out] ((a^2 - 2*a*b - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[2]*(a^2
+ b^2)^2*d) - ((a^2 - 2*a*b - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])/
(Sqrt[2]*(a^2 + b^2)^2*d) + (a^(5/2)*(3*a^2 + 7*b^2)*ArcTan[(Sqrt[a]*Sqrt[C
ot[c + d*x]])/Sqrt[b]])/(b^(5/2)*(a^2 + b^2)^2*d) + (3*a^2 + 2*b^2)/(b^2*(a
^2 + b^2)*d*Sqrt[Cot[c + d*x]]) - a^2/(b*(a^2 + b^2)*d*Sqrt[Cot[c + d*x]]*(
b + a*Cot[c + d*x])) - ((a^2 + 2*a*b - b^2)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*
x]] + Cot[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^2*d) + ((a^2 + 2*a*b - b^2)*Log
[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^2*d
)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
```

```
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] && !LeQ[n, -1]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{\cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx = \int \frac{1}{\cot^{\frac{3}{2}}(c+dx)(b+a\cot(c+dx))^2}\,dx$$

$$= -\frac{a^2}{b\left(a^2+b^2\right)d\sqrt{\cot(c+dx)}\,(b+a\cot(c+dx))} - \frac{\int \frac{\frac{1}{2}\left(-3a^2-2b^2\right)+}{\cot^{\frac{3}{2}}(c}}{}$$

$$= \frac{3a^2+2b^2}{b^2\left(a^2+b^2\right)d\sqrt{\cot(c+dx)}} - \frac{a^2}{b\left(a^2+b^2\right)d\sqrt{\cot(c+dx)}\,(b+a\,c}$$

$$= \frac{3a^2+2b^2}{b^2\left(a^2+b^2\right)d\sqrt{\cot(c+dx)}} - \frac{a^2}{b\left(a^2+b^2\right)d\sqrt{\cot(c+dx)}\,(b+a\,c}$$

$$= \frac{3a^2+2b^2}{b^2\left(a^2+b^2\right)d\sqrt{\cot(c+dx)}} - \frac{a^2}{b\left(a^2+b^2\right)d\sqrt{\cot(c+dx)}\,(b+a\,c}$$

$$= \frac{3a^2+2b^2}{b^2\left(a^2+b^2\right)d\sqrt{\cot(c+dx)}} - \frac{a^2}{b\left(a^2+b^2\right)d\sqrt{\cot(c+dx)}\,(b+a\,c}$$

$$= \frac{a^{5/2}\left(3a^2+7b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}\left(a^2+b^2\right)^2 d} + \frac{3a^2+2l}{b^2\left(a^2+b^2\right)d\sqrt{co}}$$

$$= \frac{a^{5/2}\left(3a^2+7b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}\left(a^2+b^2\right)^2 d} + \frac{3a^2+2l}{b^2\left(a^2+b^2\right)d\sqrt{co}}$$

$$= \frac{\left(a^2-2ab-b^2\right)\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)^2 d} - \frac{\left(a^2-2ab-b\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order

3 in optimal.
time = 0.61, size = 239, normalized size = 0.67

$$\frac{8a^2b^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{a\cot(c+dx)}{b}\right)+4a^2(a^2+b^2)\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};-\frac{a\cot(c+dx)}{b}\right)+b^2\left(-4(a^2-b^2)\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\cot^2(c+dx)\right)+\sqrt{2}\,ab\sqrt{\cot(c+dx)}\left(-2\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)+2\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)-\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)+\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\right)}{2b^2\left(a^2+b^2\right)^2d\sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cot[c + d*x]^(7/2)*(a + b*Tan[c + d*x])^2),x]
```

```
[Out] (8*a^2*b^2*Hypergeometric2F1[-1/2, 1, 1/2, -((a*Cot[c + d*x])/b)] + 4*a^2*(
a^2 + b^2)*Hypergeometric2F1[-1/2, 2, 1/2, -((a*Cot[c + d*x])/b)] + b^2*(-4
*(a^2 - b^2)*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2] + Sqrt[2]*a*b
*Sqrt[Cot[c + d*x]]*(-2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] + 2*ArcTan[1
 + Sqrt[2]*Sqrt[Cot[c + d*x]]] - Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c
 + d*x]] + Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])))/(2*b^2*(a^
2 + b^2)^2*d*Sqrt[Cot[c + d*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 59.64, size = 21870, normalized size = 61.26

method	result	size
default	Expression too large to display	21870

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(7/2)/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.52, size = 322, normalized size = 0.90

$$\frac{\frac{4\left(3a^5+7a^3b^2\right)\arctan\left(\frac{a}{\sqrt{ab}\sqrt{\tan(dx+c)}}\right)}{\left(a^4b^2+2a^2b^4+b^6\right)\sqrt{ab}}-\frac{2\sqrt{2}\left(a^2-2ab-b^2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}\left(a^2-2ab-b^2\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-\sqrt{2}\left(a^2+2ab-b^2\right)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}\left(a^2+2ab-b^2\right)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^4+2a^2b^2+b^4}+\frac{4\left(2a^2b+2b^3+\frac{3a^3+2ab^2}{\tan(dx+c)}\right)}{\frac{a^2b^3+b^5}{\sqrt{\tan(dx+c)}}+\frac{a^3b^2+ab^4}{\tan(dx+c)^{\frac{3}{2}}}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(7/2)/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(4*(3*a^5 + 7*a^3*b^2)*arctan(a/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^4*b
^2 + 2*a^2*b^4 + b^6)*sqrt(a*b)) - (2*sqrt(2)*(a^2 - 2*a*b - b^2)*arctan(1/
2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*(a^2 - 2*a*b - b^2)
*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*(a^2 + 2*a
*b - b^2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*(a
^2 + 2*a*b - b^2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a
^4 + 2*a^2*b^2 + b^4) + 4*(2*a^2*b + 2*b^3 + (3*a^3 + 2*a*b^2)/tan(d*x + c)
)/((a^2*b^3 + b^5)/sqrt(tan(d*x + c)) + (a^3*b^2 + a*b^4)/tan(d*x + c)^(3/2
)))/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(7/2)/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(7/2)/(a+b*tan(d*x+c))**2,x)
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(7/2)/(a+b*tan(d*x+c))^2,x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cot(c+d\,x)^{7/2}\,\left(a+b\tan(c+d\,x)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(7/2)*(a + b*tan(c + d*x))^2),x)
[Out] int(1/(cot(c + d*x)^(7/2)*(a + b*tan(c + d*x))^2), x)
```

3.833 $\int \frac{\cot^{\frac{5}{2}}(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=493

$$-\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 d}+\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c$$

```
[Out] -1/4*b^(7/2)*(99*a^4+102*a^2*b^2+35*b^4)*arctan(a^(1/2)*cot(d*x+c)^(1/2)/b^
(1/2))/a^(9/2)/(a^2+b^2)^3/d-1/12*(8*a^4+67*a^2*b^2+35*b^4)*cot(d*x+c)^(3/2
)/a^3/(a^2+b^2)^2/d+1/2*b^2*cot(d*x+c)^(7/2)/a/(a^2+b^2)/d/(b+a*cot(d*x+c))
^2+1/4*b^2*(15*a^2+7*b^2)*cot(d*x+c)^(5/2)/a^2/(a^2+b^2)^2/d/(b+a*cot(d*x+c
))+1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)^
3/d*2^(1/2)+1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/(a
^2+b^2)^3/d*2^(1/2)+1/4*(a-b)*(a^2+4*a*b+b^2)*ln(1+cot(d*x+c)-2^(1/2)*cot(d
*x+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/4*(a-b)*(a^2+4*a*b+b^2)*ln(1+cot(d*x+c
)+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)+1/4*b*(24*a^4+67*a^2*b^2+
35*b^4)*cot(d*x+c)^(1/2)/a^4/(a^2+b^2)^2/d
```

Rubi [A]
time = 1.08, antiderivative size = 493, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3754, 3646, 3726, 3728, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d\left(a^2+b^2\right)^3}+\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}d\left(a^2+b^2\right)^3}+\frac{b^2(15a^2+7b^2)\cot^{\frac{5}{2}}(c+dx)}{4a^2d\left(a^2+b^2\right)^2(a\cot(c+dx)+b)}+\frac{b^2\cot^{\frac{7}{2}}(c+dx)}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}+\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}d\left(a^2+b^2\right)^3}-\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}d\left(a^2+b^2\right)^3}+\frac{b(24a^4+67a^2b^2+35b^4)\sqrt{\cot(c+dx)}}{4a^4d\left(a^2+b^2\right)^2}-\frac{b^{7/2}(99a^4+102a^2b^2+35b^4)\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{4a^{9/2}d\left(a^2+b^2\right)^3}-\frac{(8a^4+67a^2b^2+35b^4)\cot^{\frac{3}{2}}(c+dx)}{12a^3d\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(5/2)/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -(((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqr
t[2]*(a^2 + b^2)^3*d)) + ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 + Sqrt[2]*Sq
rt[Cot[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)^3*d) - (b^(7/2)*(99*a^4 + 102*a^2*b
^2 + 35*b^4)*ArcTan[(Sqrt[a]*Sqrt[Cot[c + d*x]])/Sqrt[b]])/(4*a^(9/2)*(a^2
+ b^2)^3*d) + (b*(24*a^4 + 67*a^2*b^2 + 35*b^4)*Sqrt[Cot[c + d*x]])/(4*a^4*
(a^2 + b^2)^2*d) - ((8*a^4 + 67*a^2*b^2 + 35*b^4)*Cot[c + d*x]^(3/2))/(12*a
^3*(a^2 + b^2)^2*d) + (b^2*Cot[c + d*x]^(7/2))/(2*a*(a^2 + b^2)*d*(b + a*Co
t[c + d*x])^2) + (b^2*(15*a^2 + 7*b^2)*Cot[c + d*x]^(5/2))/(4*a^2*(a^2 + b^
2)^2*d*(b + a*Cot[c + d*x])) + ((a - b)*(a^2 + 4*a*b + b^2)*Log[1 - Sqrt[2]
*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^3*d) - ((a - b)
*(a^2 + 4*a*b + b^2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2
*Sqrt[2]*(a^2 + b^2)^3*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\cot^{\frac{11}{2}}(c+dx)}{(b+a\cot(c+dx))^3}\,dx$$

$$= \frac{b^2\cot^{\frac{7}{2}}(c+dx)}{2a\,(a^2+b^2)\,d(b+a\cot(c+dx))^2} - \frac{\int \frac{\cot^{\frac{5}{2}}(c+dx)\left(-\frac{7b^2}{2}+2ab\cot(c+dx)-\frac{1}{2}(4a^2+7b^2)\cot^2(c}{(b+a\cot(c+dx))^2}}{2a\,(a^2+b^2)}$$

$$= \frac{b^2\cot^{\frac{7}{2}}(c+dx)}{2a\,(a^2+b^2)\,d(b+a\cot(c+dx))^2} + \frac{b^2(15a^2+7b^2)\cot^{\frac{5}{2}}(c+dx)}{4a^2\,(a^2+b^2)^2\,d(b+a\cot(c+dx))} + \frac{\int \cot^{\frac{3}{}}}{}$$

$$= -\frac{(8a^4+67a^2b^2+35b^4)\cot^{\frac{3}{2}}(c+dx)}{12a^3\,(a^2+b^2)^2\,d} + \frac{b^2\cot^{\frac{7}{2}}(c+dx)}{2a\,(a^2+b^2)\,d(b+a\cot(c+dx))^2} + \frac{}{4a}$$

$$= \frac{b(24a^4+67a^2b^2+35b^4)\sqrt{\cot(c+dx)}}{4a^4\,(a^2+b^2)^2\,d} - \frac{(8a^4+67a^2b^2+35b^4)\cot^{\frac{3}{2}}(c+dx)}{12a^3\,(a^2+b^2)^2\,d}$$

$$= \frac{b(24a^4+67a^2b^2+35b^4)\sqrt{\cot(c+dx)}}{4a^4\,(a^2+b^2)^2\,d} - \frac{(8a^4+67a^2b^2+35b^4)\cot^{\frac{3}{2}}(c+dx)}{12a^3\,(a^2+b^2)^2\,d}$$

$$= \frac{b(24a^4+67a^2b^2+35b^4)\sqrt{\cot(c+dx)}}{4a^4\,(a^2+b^2)^2\,d} - \frac{(8a^4+67a^2b^2+35b^4)\cot^{\frac{3}{2}}(c+dx)}{12a^3\,(a^2+b^2)^2\,d}$$

$$= \frac{b(24a^4+67a^2b^2+35b^4)\sqrt{\cot(c+dx)}}{4a^4\,(a^2+b^2)^2\,d} - \frac{(8a^4+67a^2b^2+35b^4)\cot^{\frac{3}{2}}(c+dx)}{12a^3\,(a^2+b^2)^2\,d}$$

$$= -\frac{b^{7/2}(99a^4+102a^2b^2+35b^4)\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{4a^{9/2}\,(a^2+b^2)^3\,d} + \frac{b(24a^4+67a^2b^2}{4a^4\,(}$$

$$= -\frac{b^{7/2}(99a^4+102a^2b^2+35b^4)\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{4a^{9/2}\,(a^2+b^2)^3\,d} + \frac{b(24a^4+67a^2b^2}{4a^4\,(}$$

$$= -\frac{(a+b)\,(a^2-4ab+b^2)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d} + \frac{(a+b)\,(a^2-4ab+}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 6.26, size = 564, normalized size = 1.14

$$-\frac{-\frac{2a(a^2-3b^2)\cot^{\frac{11}{2}}(c+dx)}{11(a^2+b^2)^3}+\frac{2b(a^2-3b^2)\left(35\cot^{\frac{9}{2}}(c+dx)-3b\left(\frac{15\cot^{\frac{7}{2}}(c+dx)}{a}-\frac{7b\left(\frac{3\cot^{\frac{5}{2}}(c+dx)}{a}-\frac{5b\left(\frac{-3b\left(-\frac{\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{a^{3/2}}+\frac{\sqrt{\cot(c+dx)}}{a}\right)}{a}+\frac{\cot^{\frac{3}{2}}(c+dx)}{a}\right)}{a}\right)}{a}\right)\right)}{315(a^2+b^2)^3}+\frac{2a(a^2-3b^2)\left(77\cot^{\frac{3}{2}}(c+dx)-33\cot^{\frac{7}{2}}(c+dx)+21\cot^{\frac{11}{2}}(c+dx)-77\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)\right)}{231(a^2+b^2)^3}+\frac{4a^2\cot^{\frac{13}{2}}(c+dx)\,{}_2F_1\left(2,\frac{13}{2};\frac{15}{2};-\frac{a\cot(c+dx)}{b}\right)}{13b(a^2+b^2)^2}+\frac{2a^2\cot^{\frac{13}{2}}(c+dx)\,{}_2F_1\left(3,\frac{13}{2};\frac{15}{2};-\frac{a\cot(c+dx)}{b}\right)}{13b^3(a^2+b^2)}-\frac{b(3a^2-b^2)\left(360\sqrt{\cot(c+dx)}-72\cot^{\frac{5}{2}}(c+dx)+40\cot^{\frac{9}{2}}(c+dx)+45\left(2\left(\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)\right)+\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\right)}{180(a^2+b^2)^3}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(5/2)/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -((((-2*a*(a^2 - 3*b^2)*Cot[c + d*x]^(11/2))/(11*(a^2 + b^2)^3) + (2*b*(a^2
- 3*b^2)*(35*Cot[c + d*x]^(9/2) - 3*b*((15*Cot[c + d*x]^(7/2))/a - (7*b*((3
*Cot[c + d*x]^(5/2))/a - (5*b*((-3*b*(-((Sqrt[b]*ArcTan[(Sqrt[a]*Sqrt[Cot[c
 + d*x]])/Sqrt[b]])/a^(3/2)) + Sqrt[Cot[c + d*x]]/a))/a + Cot[c + d*x]^(3/2
)/a))/a))/a)))/(315*(a^2 + b^2)^3) + (2*a*(a^2 - 3*b^2)*(77*Cot[c + d*x]^(3
/2) - 33*Cot[c + d*x]^(7/2) + 21*Cot[c + d*x]^(11/2) - 77*Cot[c + d*x]^(3/2
)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2]))/(231*(a^2 + b^2)^3) + (
4*a^2*Cot[c + d*x]^(13/2)*Hypergeometric2F1[2, 13/2, 15/2, -((a*Cot[c + d*x
])/b)])/(13*b*(a^2 + b^2)^2) + (2*a^2*Cot[c + d*x]^(13/2)*Hypergeometric2F1
[3, 13/2, 15/2, -((a*Cot[c + d*x])/b)])/(13*b^3*(a^2 + b^2)) - (b*(3*a^2 -
b^2)*(360*Sqrt[Cot[c + d*x]] - 72*Cot[c + d*x]^(5/2) + 40*Cot[c + d*x]^(9/2
) + 45*(2*(Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - Sqrt[2]*ArcTan[
1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]) + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x
]] + Cot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d
*x]])))/(180*(a^2 + b^2)^3))/d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 28.12, size = 101140, normalized size = 205.15

method	result	size
default	Expression too large to display	101140

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.53, size = 456, normalized size = 0.92

$$-\frac{\frac{3\left(99a^4b^4+102a^2b^6+35b^8\right)\arctan\left(\frac{a}{\sqrt{ab}\sqrt{\tan(dx+c)}}\right)}{\left(a^{10}+3a^8b^2+3a^6b^4+a^4b^6\right)\sqrt{ab}}-\frac{3\left(2\sqrt{2}\left(a^3-3a^2b-3ab^2+b^3\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}\left(a^3-3a^2b-3ab^2+b^3\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-\sqrt{2}\left(a^3+3a^2b-3ab^2-b^3\right)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}\left(a^3+3a^2b-3ab^2-b^3\right)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)\right)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{3\left(\frac{19a^2b^5+11b^7}{\sqrt{\tan(dx+c)}}+\frac{21a^3b^4+13ab^6}{\tan(dx+c)^{\frac{3}{2}}}\right)}{a^8b^2+2a^6b^4+a^4b^6+\frac{2\left(a^9b+2a^7b^3+a^5b^5\right)}{\tan(dx+c)}+\frac{a^{10}+2a^8b^2+a^6b^4}{\tan(dx+c)^2}}-\frac{8\left(\frac{9b}{\sqrt{\tan(dx+c)}}-\frac{a}{\tan(dx+c)^{\frac{3}{2}}}\right)}{a^4}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/12*(3*(99*a^4*b^4 + 102*a^2*b^6 + 35*b^8)*arctan(a/(sqrt(a*b)*sqrt(tan(d
*x + c))))/((a^10 + 3*a^8*b^2 + 3*a^6*b^4 + a^4*b^6)*sqrt(a*b)) - 3*(2*sqrt
(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(ta
n(d*x + c)))) + 2*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*arctan(-1/2*sqrt(
2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b
^3)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*(a^3 + 3
*a^2*b - 3*a*b^2 - b^3)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) +
1))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 3*((19*a^2*b^5 + 11*b^7)/sqrt(tan
(d*x + c)) + (21*a^3*b^4 + 13*a*b^6)/tan(d*x + c)^(3/2))/(a^8*b^2 + 2*a^6*b
^4 + a^4*b^6 + 2*(a^9*b + 2*a^7*b^3 + a^5*b^5)/tan(d*x + c) + (a^10 + 2*a^8
*b^2 + a^6*b^4)/tan(d*x + c)^2) - 8*(9*b/sqrt(tan(d*x + c)) - a/tan(d*x + c
)^(3/2))/a^4)/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(5/2)/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)^(5/2)/(b*tan(d*x + c) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^{5/2}}{\left(a+b\tan\left(c+d\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(5/2)/(a + b*tan(c + d*x))^3,x)
[Out] int(cot(c + d*x)^(5/2)/(a + b*tan(c + d*x))^3, x)
```

3.834 $\int \frac{\cot^{\frac{3}{2}}(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=444

$$-\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 d}+\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c}\right.}{\sqrt{2}\left(a^2+b^2\right)^3 d}$$

```
[Out] 1/4*b^(5/2)*(63*a^4+46*a^2*b^2+15*b^4)*arctan(a^(1/2)*cot(d*x+c)^(1/2)/b^(1
/2))/a^(7/2)/(a^2+b^2)^3/d+1/2*b^2*cot(d*x+c)^(5/2)/a/(a^2+b^2)/d/(b+a*cot(
d*x+c))^2+1/4*b^2*(13*a^2+5*b^2)*cot(d*x+c)^(3/2)/a^2/(a^2+b^2)^2/d/(b+a*co
t(d*x+c))+1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/(a^
2+b^2)^3/d*2^(1/2)+1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(1+2^(1/2)*cot(d*x+c)^(1
/2))/(a^2+b^2)^3/d*2^(1/2)-1/4*(a+b)*(a^2-4*a*b+b^2)*ln(1+cot(d*x+c)-2^(1/2
)*cot(d*x+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)+1/4*(a+b)*(a^2-4*a*b+b^2)*ln(1+co
t(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/4*(8*a^4+31*a^2*
b^2+15*b^4)*cot(d*x+c)^(1/2)/a^3/(a^2+b^2)^2/d
```

Rubi [A]
time = 0.84, antiderivative size = 444, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3754, 3646, 3726, 3728, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d\left(a^2+b^2\right)^3}+\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d\left(a^2+b^2\right)^3}+\frac{b^2\left(13a^2+5b^2\right)\cot^{\frac{3}{2}}(c+dx)}{4a^2d\left(a^2+b^2\right)^2\left(a\cot(c+dx)+b\right)}+\frac{b^2\cot^{\frac{5}{2}}(c+dx)}{2ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}-\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)^3}+\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)^3}+\frac{b^{5/2}\left(63a^4+46a^2b^2+15b^4\right)\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{4a^{7/2}d\left(a^2+b^2\right)^3}-\frac{\left(8a^4+31a^2b^2+15b^4\right)\sqrt{\cot(c+dx)}}{4a^3d\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(3/2)/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -(((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqr
t[2]*(a^2 + b^2)^3*d)) + ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 + Sqrt[2]*Sq
rt[Cot[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)^3*d) + (b^(5/2)*(63*a^4 + 46*a^2*b^
2 + 15*b^4)*ArcTan[(Sqrt[a]*Sqrt[Cot[c + d*x]])/Sqrt[b]])/(4*a^(7/2)*(a^2 +
 b^2)^3*d) - ((8*a^4 + 31*a^2*b^2 + 15*b^4)*Sqrt[Cot[c + d*x]])/(4*a^3*(a^2
 + b^2)^2*d) + (b^2*Cot[c + d*x]^(5/2))/(2*a*(a^2 + b^2)*d*(b + a*Cot[c + d
*x])^2) + (b^2*(13*a^2 + 5*b^2)*Cot[c + d*x]^(3/2))/(4*a^2*(a^2 + b^2)^2*d*
(b + a*Cot[c + d*x])) - ((a + b)*(a^2 - 4*a*b + b^2)*Log[1 - Sqrt[2]*Sqrt[C
ot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^3*d) + ((a + b)*(a^2 -
 4*a*b + b^2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2
]*(a^2 + b^2)^3*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
```

```
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\cot^{\frac{9}{2}}(c+dx)}{(b+a\cot(c+dx))^3}\,dx$$

$$= \frac{b^2\cot^{\frac{5}{2}}(c+dx)}{2a\,(a^2+b^2)\,d(b+a\cot(c+dx))^2} - \frac{\int \frac{\cot^{\frac{3}{2}}(c+dx)\left(-\frac{5b^2}{2}+2ab\cot(c+dx)-\frac{1}{2}\left(4a^2+5b^2\right)\cot^2(\cdots}{(b+a\cot(c+dx))^2}}{2a\,(a^2+b^2)}$$

$$= \frac{b^2\cot^{\frac{5}{2}}(c+dx)}{2a\,(a^2+b^2)\,d(b+a\cot(c+dx))^2} + \frac{b^2(13a^2+5b^2)\cot^{\frac{3}{2}}(c+dx)}{4a^2\,(a^2+b^2)^2\,d(b+a\cot(c+dx))} + \frac{\int \sqrt{\cdots}}{\cdots}$$

$$= -\frac{(8a^4+31a^2b^2+15b^4)\sqrt{\cot(c+dx)}}{4a^3\,(a^2+b^2)^2\,d} + \frac{b^2\cot^{\frac{5}{2}}(c+dx)}{2a\,(a^2+b^2)\,d(b+a\cot(c+dx))^2} + \cdots$$

$$= -\frac{(8a^4+31a^2b^2+15b^4)\sqrt{\cot(c+dx)}}{4a^3\,(a^2+b^2)^2\,d} + \frac{b^2\cot^{\frac{5}{2}}(c+dx)}{2a\,(a^2+b^2)\,d(b+a\cot(c+dx))^2} + \cdots$$

$$= -\frac{(8a^4+31a^2b^2+15b^4)\sqrt{\cot(c+dx)}}{4a^3\,(a^2+b^2)^2\,d} + \frac{b^2\cot^{\frac{5}{2}}(c+dx)}{2a\,(a^2+b^2)\,d(b+a\cot(c+dx))^2} + \cdots$$

$$= -\frac{(8a^4+31a^2b^2+15b^4)\sqrt{\cot(c+dx)}}{4a^3\,(a^2+b^2)^2\,d} + \frac{b^2\cot^{\frac{5}{2}}(c+dx)}{2a\,(a^2+b^2)\,d(b+a\cot(c+dx))^2} + \cdots$$

$$= \frac{b^{5/2}(63a^4+46a^2b^2+15b^4)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{4a^{7/2}\,(a^2+b^2)^3\,d} - \frac{(8a^4+31a^2b^2+15b\cdots}{4a^3\,(a^2+\cdots}$$

$$= \frac{b^{5/2}(63a^4+46a^2b^2+15b^4)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{4a^{7/2}\,(a^2+b^2)^3\,d} - \frac{(8a^4+31a^2b^2+15b\cdots}{4a^3\,(a^2+\cdots}$$

$$= -\frac{(a-b)\,(a^2+4ab+b^2)\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d} + \frac{(a-b)\,(a^2+4ab+\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 6.18, size = 530, normalized size = 1.19

$$\frac{-\frac{2a(a^2-3b^2)\cot^{\frac{9}{2}}(c+dx)}{9(a^2+b^2)^3} + \frac{2b(a^2-3b^2)\left(15\cot^{\frac{7}{2}}(c+dx)-7b\left(\frac{3\cot^{\frac{5}{2}}(c+dx)}{a}-\frac{5b\left(-\frac{3b\left(-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{a^{3/2}}+\frac{\sqrt{\cot(c+dx)}}{a}\right)}{a}+\frac{\cot^{\frac{3}{2}}(c+dx)}{a}\right)}{a}\right)\right)}{105(a^2+b^2)^3} + \frac{2b(3a^2-b^2)\left(7\cot^{\frac{3}{2}}(c+dx)-3\cot^{\frac{7}{2}}(c+dx)-7\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)\right)}{21(a^2+b^2)^3} + \frac{4a^2\cot^{\frac{11}{2}}(c+dx)\,{}_2F_1\left(2,\frac{11}{2};\frac{13}{2};-\frac{a\cot(c+dx)}{b}\right)}{11b(a^2+b^2)^2} + \frac{2a^2\cot^{\frac{11}{2}}(c+dx)\,{}_2F_1\left(3,\frac{11}{2};\frac{13}{2};-\frac{a\cot(c+dx)}{b}\right)}{11b^3(a^2+b^2)} + \frac{a(a^2-3b^2)\left(360\sqrt{\cot(c+dx)}-72\cot^{\frac{5}{2}}(c+dx)+40\cot^{\frac{9}{2}}(c+dx)+45\left(2\left(\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)\right)+\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\right)}{180(a^2+b^2)^3}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(3/2)/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -(((-2*a*(a^2 - 3*b^2)*Cot[c + d*x]^(9/2))/(9*(a^2 + b^2)^3) + (2*b*(a^2 -
3*b^2)*(15*Cot[c + d*x]^(7/2) - 7*b*((3*Cot[c + d*x]^(5/2))/a - (5*b*((-3*b
*(-((Sqrt[b]*ArcTan[(Sqrt[a]*Sqrt[Cot[c + d*x]])/Sqrt[b]])/a^(3/2)) + Sqrt[
Cot[c + d*x]]/a))/a + Cot[c + d*x]^(3/2)/a))/a)))/(105*(a^2 + b^2)^3) + (2*
b*(3*a^2 - b^2)*(7*Cot[c + d*x]^(3/2) - 3*Cot[c + d*x]^(7/2) - 7*Cot[c + d*
x]^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2]))/(21*(a^2 + b^2)^
3) + (4*a^2*Cot[c + d*x]^(11/2)*Hypergeometric2F1[2, 11/2, 13/2, -((a*Cot[c
 + d*x])/b)])/(11*b*(a^2 + b^2)^2) + (2*a^2*Cot[c + d*x]^(11/2)*Hypergeomet
ric2F1[3, 11/2, 13/2, -((a*Cot[c + d*x])/b)])/(11*b^3*(a^2 + b^2)) + (a*(a^
2 - 3*b^2)*(360*Sqrt[Cot[c + d*x]] - 72*Cot[c + d*x]^(5/2) + 40*Cot[c + d*x
]^(9/2) + 45*(2*(Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - Sqrt[2]*A
rcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]) + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c
 + d*x]] + Cot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot
[c + d*x]]))))/(180*(a^2 + b^2)^3))/d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 32.52, size = 79208, normalized size = 178.40

method	result	size
default	Expression too large to display	79208

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.53, size = 439, normalized size = 0.99

$$\frac{\frac{(63\,a^4b^3+46\,a^2b^5+15\,b^7)\arctan\left(\frac{a}{\sqrt{ab}\,\sqrt{\tan\left(dx+c\right)}}\right)}{(a^9+3\,a^7b^2+3\,a^5b^4+a^3b^6)\sqrt{ab}}+\frac{2\sqrt{2}\,(a^3+3\,a^2b-3\,ab^2-b^3)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+2\sqrt{2}\,(a^3+3\,a^2b-3\,ab^2-b^3)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+\sqrt{2}\,(a^3-3\,a^2b-3\,ab^2+b^3)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)-\sqrt{2}\,(a^3-3\,a^2b-3\,ab^2+b^3)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{\frac{15\,a^2b^4+7\,b^6}{\sqrt{\tan\left(dx+c\right)}}+\frac{17\,a^3b^3+9\,ab^5}{\tan\left(dx+c\right)^{\frac{3}{2}}}}{a^7b^2+2\,a^5b^4+a^3b^6+\frac{2\,(a^8b+2\,a^6b^3+a^4b^5)}{\tan\left(dx+c\right)}+\frac{a^9+2\,a^7b^2+a^5b^4}{\tan\left(dx+c\right)^2}}-\frac{8}{a^3\sqrt{\tan\left(dx+c\right)}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*((63*a^4*b^3 + 46*a^2*b^5 + 15*b^7)*arctan(a/(sqrt(a*b)*sqrt(tan(d*x +
c))))/((a^9 + 3*a^7*b^2 + 3*a^5*b^4 + a^3*b^6)*sqrt(a*b)) + (2*sqrt(2)*(a^3
 + 3*a^2*b - 3*a*b^2 - b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x +
c)))) + 2*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*arctan(-1/2*sqrt(2)*(sqrt
(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*log(
sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*(a^3 - 3*a^2*b -
 3*a*b^2 + b^3)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - ((15*a^2*b^4 + 7*b^6)/sqrt(tan(d*x + c))
+ (17*a^3*b^3 + 9*a*b^5)/tan(d*x + c)^(3/2))/(a^7*b^2 + 2*a^5*b^4 + a^3*b^6
```

```
 + 2*(a^8*b + 2*a^6*b^3 + a^4*b^5)/tan(d*x + c) + (a^9 + 2*a^7*b^2 + a^5*b^
4)/tan(d*x + c)^2) - 8/(a^3*sqrt(tan(d*x + c))))/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(3/2)/(a+b*tan(d*x+c))**3,x)
[Out] Integral(cot(c + d*x)**(3/2)/(a + b*tan(c + d*x))**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^3,x, algorithm="giac")
[Out] integrate(cot(d*x + c)^(3/2)/(b*tan(d*x + c) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^{3/2}}{(a+b\tan(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(3/2)/(a + b*tan(c + d*x))^3,x)
[Out] int(cot(c + d*x)^(3/2)/(a + b*tan(c + d*x))^3, x)
```

3.835 $\int \frac{\sqrt{\cot(c+dx)}}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=396

$$\frac{(a+b)\left(a^2-4ab+b^2\right)\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 d}-\frac{(a+b)\left(a^2-4ab+b^2\right)\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 d}$$

```
[Out] -1/4*b^(3/2)*(35*a^4+6*a^2*b^2+3*b^4)*arctan(a^(1/2)*cot(d*x+c)^(1/2)/b^(1/
2))/a^(5/2)/(a^2+b^2)^3/d+1/2*b^2*cot(d*x+c)^(3/2)/a/(a^2+b^2)/d/(b+a*cot(d
*x+c))^2-1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/(a^2
+b^2)^3/d*2^(1/2)-1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(1+2^(1/2)*cot(d*x+c)^(1/
2))/(a^2+b^2)^3/d*2^(1/2)-1/4*(a-b)*(a^2+4*a*b+b^2)*ln(1+cot(d*x+c)-2^(1/2)
*cot(d*x+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)+1/4*(a-b)*(a^2+4*a*b+b^2)*ln(1+cot
(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)+1/4*b^2*(11*a^2+3*b
^2)*cot(d*x+c)^(1/2)/a^2/(a^2+b^2)^2/d/(b+a*cot(d*x+c))
```

Rubi [A]
time = 0.63, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3754, 3646, 3726, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(a+b)\left(a^2-4ab+b^2\right)\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d\left(a^2+b^2\right)^3}-\frac{(a+b)\left(a^2-4ab+b^2\right)\mathrm{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d\left(a^2+b^2\right)^3}+\frac{b^2\cot^{\frac{3}{2}}(c+dx)}{2ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}+\frac{b^2\left(11a^2+3b^2\right)\sqrt{\cot(c+dx)}}{4a^2d\left(a^2+b^2\right)^2\left(a\cot(c+dx)+b\right)}-\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)^3}+\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)^3}-\frac{b^{3/2}\left(35a^4+6a^2b^2+3b^4\right)\mathrm{ArcTan}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{4a^{5/2}d\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cot[c + d*x]]/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[
2]*(a^2 + b^2)^3*d) - ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 + Sqrt[2]*Sqrt[
Cot[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)^3*d) - (b^(3/2)*(35*a^4 + 6*a^2*b^2 +
3*b^4)*ArcTan[(Sqrt[a]*Sqrt[Cot[c + d*x]])/Sqrt[b]])/(4*a^(5/2)*(a^2 + b^2)
^3*d) + (b^2*Cot[c + d*x]^(3/2))/(2*a*(a^2 + b^2)*d*(b + a*Cot[c + d*x])^2)
 + (b^2*(11*a^2 + 3*b^2)*Sqrt[Cot[c + d*x]])/(4*a^2*(a^2 + b^2)^2*d*(b + a*
Cot[c + d*x])) - ((a - b)*(a^2 + 4*a*b + b^2)*Log[1 - Sqrt[2]*Sqrt[Cot[c +
d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^3*d) + ((a - b)*(a^2 + 4*a*b
+ b^2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*(a^2
+ b^2)^3*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
```

```
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
```

```
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{\sqrt{\cot(c+dx)}}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\cot^{\frac{7}{2}}(c+dx)}{(b+a\cot(c+dx))^3}\,dx$$

$$= \frac{b^2\cot^{\frac{3}{2}}(c+dx)}{2a\,(a^2+b^2)\,d(b+a\cot(c+dx))^2} - \frac{\int \frac{\sqrt{\cot(c+dx)}\left(-\frac{3b^2}{2}+2ab\cot(c+dx)-\frac{1}{2}(4a^2+\right.}{(b+a\cot(c+dx))^2}}{2a\,(a^2+b^2)}$$

$$= \frac{b^2\cot^{\frac{3}{2}}(c+dx)}{2a\,(a^2+b^2)\,d(b+a\cot(c+dx))^2} + \frac{b^2(11a^2+3b^2)\sqrt{\cot(c+dx)}}{4a^2\,(a^2+b^2)^2\,d(b+a\cot(c+dx))} + \frac{\int \frac{1}{4}}{}$$

$$= \frac{b^2\cot^{\frac{3}{2}}(c+dx)}{2a\,(a^2+b^2)\,d(b+a\cot(c+dx))^2} + \frac{b^2(11a^2+3b^2)\sqrt{\cot(c+dx)}}{4a^2\,(a^2+b^2)^2\,d(b+a\cot(c+dx))} + \frac{\int -}{}$$

$$= \frac{b^2\cot^{\frac{3}{2}}(c+dx)}{2a\,(a^2+b^2)\,d(b+a\cot(c+dx))^2} + \frac{b^2(11a^2+3b^2)\sqrt{\cot(c+dx)}}{4a^2\,(a^2+b^2)^2\,d(b+a\cot(c+dx))} + \frac{\text{Sub}}{}$$

$$= \frac{b^2\cot^{\frac{3}{2}}(c+dx)}{2a\,(a^2+b^2)\,d(b+a\cot(c+dx))^2} + \frac{b^2(11a^2+3b^2)\sqrt{\cot(c+dx)}}{4a^2\,(a^2+b^2)^2\,d(b+a\cot(c+dx))} - \frac{((a}{}$$

$$= -\frac{b^{3/2}(35a^4+6a^2b^2+3b^4)\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{4a^{5/2}\,(a^2+b^2)^3\,d} + \frac{b^2\cot^{\frac{3}{2}}(c}{2a\,(a^2+b^2)\,d(b+a}$$

$$= -\frac{b^{3/2}(35a^4+6a^2b^2+3b^4)\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{4a^{5/2}\,(a^2+b^2)^3\,d} + \frac{b^2\cot^{\frac{3}{2}}(c}{2a\,(a^2+b^2)\,d(b+a}$$

$$= \frac{(a+b)\,(a^2-4ab+b^2)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d} - \frac{(a+b)\,(a^2-4ab+}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.17, size = 499, normalized size = 1.26

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cot[c + d*x]]/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -((((-2*a*(a^2 - 3*b^2)*Cot[c + d*x]^(7/2))/(7*(a^2 + b^2)^3) + (2*b*(a^2 -
3*b^2)*(3*Cot[c + d*x]^(5/2) - 5*b*((-3*b*(-((Sqrt[b]*ArcTan[(Sqrt[a]*Sqrt[
```

```
Cot[c + d*x]])/Sqrt[b]])/a^(3/2)) + Sqrt[Cot[c + d*x]]/a))/a + Cot[c + d*x]
^(3/2)/a)))/(15*(a^2 + b^2)^3) - (2*a*(a^2 - 3*b^2)*(7*Cot[c + d*x]^(3/2) -
 3*Cot[c + d*x]^(7/2) - 7*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1, 7/4,
 -Cot[c + d*x]^2]))/(21*(a^2 + b^2)^3) + (4*a^2*Cot[c + d*x]^(9/2)*Hypergeo
metric2F1[2, 9/2, 11/2, -((a*Cot[c + d*x])/b)])/(9*b*(a^2 + b^2)^2) + (2*a^
2*Cot[c + d*x]^(9/2)*Hypergeometric2F1[3, 9/2, 11/2, -((a*Cot[c + d*x])/b)]
)/(9*b^3*(a^2 + b^2)) + (b*(3*a^2 - b^2)*(40*Sqrt[Cot[c + d*x]] - 8*Cot[c +
 d*x]^(5/2) + (5*(4*(Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - Sqrt[
2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]) + 2*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt
[Cot[c + d*x]] + Cot[c + d*x]] - 2*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x
]] + Cot[c + d*x]]))/2))/(20*(a^2 + b^2)^3))/d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 29.17, size = 50668, normalized size = 127.95

method	result	size
default	Expression too large to display	50668

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.52, size = 426, normalized size = 1.08

$$-\frac{\frac{(35a^4b^2+6a^2b^4+3b^6)\arctan\left(\frac{a}{\sqrt{ab}\sqrt{\tan(dx+c)}}\right)}{(a^8+3a^6b^2+3a^4b^4+a^2b^6)\sqrt{ab}}+\frac{2\sqrt{2}(a^3-3a^2b-3ab^2+b^3)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}(a^3-3a^2b-3ab^2+b^3)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-\sqrt{2}(a^3+3a^2b-3ab^2-b^3)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}(a^3+3a^2b-3ab^2-b^3)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{\frac{11a^2b^3+3b^5}{\sqrt{\tan(dx+c)}}+\frac{13a^3b^2+5ab^4}{\tan(dx+c)^{\frac{3}{2}}}}{a^6b^2+2a^4b^4+a^2b^6+\frac{2(a^7b+2a^5b^3+a^3b^5)}{\tan(dx+c)}+\frac{a^8+2a^6b^2+a^4b^4}{\tan(dx+c)^2}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*((35*a^4*b^2 + 6*a^2*b^4 + 3*b^6)*arctan(a/(sqrt(a*b)*sqrt(tan(d*x + c
))))/((a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*sqrt(a*b)) + (2*sqrt(2)*(a^3
- 3*a^2*b - 3*a*b^2 + b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c
)))) + 2*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*arctan(-1/2*sqrt(2)*(sqrt(
2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*log(s
qrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*(a^3 + 3*a^2*b -
3*a*b^2 - b^3)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^6
+ 3*a^4*b^2 + 3*a^2*b^4 + b^6) - ((11*a^2*b^3 + 3*b^5)/sqrt(tan(d*x + c)) +
 (13*a^3*b^2 + 5*a*b^4)/tan(d*x + c)^(3/2))/(a^6*b^2 + 2*a^4*b^4 + a^2*b^6
+ 2*(a^7*b + 2*a^5*b^3 + a^3*b^5)/tan(d*x + c) + (a^8 + 2*a^6*b^2 + a^4*b^4
)/tan(d*x + c)^2))/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cot(c+dx)}}{(a+b\tan(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(1/2)/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(sqrt(cot(c + d*x))/(a + b*tan(c + d*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cot(d*x + c))/(b*tan(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cot(c+d\,x)}}{(a+b\tan(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(1/2)/(a + b*tan(c + d*x))^3,x)
```

```
[Out] int(cot(c + d*x)^(1/2)/(a + b*tan(c + d*x))^3, x)
```

3.836 $\int \frac{1}{\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=392

$$\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 d}-\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c-$$

```
[Out] -1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)^3/
d*2^(1/2)-1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/(a^2
+b^2)^3/d*2^(1/2)+1/4*(a+b)*(a^2-4*a*b+b^2)*ln(1+cot(d*x+c)-2^(1/2)*cot(d*x
+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/4*(a+b)*(a^2-4*a*b+b^2)*ln(1+cot(d*x+c)+
2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)+1/4*(15*a^4-18*a^2*b^2-b^4)
*arctan(a^(1/2)*cot(d*x+c)^(1/2)/b^(1/2))*b^(1/2)/a^(3/2)/(a^2+b^2)^3/d+1/2
*b^2*cot(d*x+c)^(1/2)/a/(a^2+b^2)/d/(b+a*cot(d*x+c))^2-1/4*b*(9*a^2+b^2)*co
t(d*x+c)^(1/2)/a/(a^2+b^2)^2/d/(b+a*cot(d*x+c))
```

Rubi [A]
time = 0.66, antiderivative size = 392, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3754, 3646, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d\left(a^2+b^2\right)^3}-\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d\left(a^2+b^2\right)^3}+\frac{b^2\sqrt{\cot(c+dx)}}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}-\frac{b\left(9a^2+b^2\right)\sqrt{\cot(c+dx)}}{4ad\left(a^2+b^2\right)^2(a\cot(c+dx)+b)}+\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)^3}-\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)^3}+\frac{\sqrt{b}\left(15a^4-18a^2b^2-b^4\right)\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{4a^{3/2}d\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^3),x]
```

```
[Out] ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[
2]*(a^2 + b^2)^3*d) - ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 + Sqrt[2]*Sqrt[
Cot[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)^3*d) + (Sqrt[b]*(15*a^4 - 18*a^2*b^2 -
 b^4)*ArcTan[(Sqrt[a]*Sqrt[Cot[c + d*x]])/Sqrt[b]])/(4*a^(3/2)*(a^2 + b^2)^
3*d) + (b^2*Sqrt[Cot[c + d*x]])/(2*a*(a^2 + b^2)*d*(b + a*Cot[c + d*x])^2)
- (b*(9*a^2 + b^2)*Sqrt[Cot[c + d*x]])/(4*a*(a^2 + b^2)^2*d*(b + a*Cot[c +
d*x])) + ((a + b)*(a^2 - 4*a*b + b^2)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] +
Cot[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^3*d) - ((a + b)*(a^2 - 4*a*b + b^2)*L
og[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^3
*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
```

```
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
```

```
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^3}\,dx = \int \frac{\cot^{\frac{5}{2}}(c+dx)}{(b+a\cot(c+dx))^3}\,dx$$

$$= \frac{b^2\sqrt{\cot(c+dx)}}{2a\,(a^2+b^2)\,d(b+a\cot(c+dx))^2} - \frac{\int \frac{-\frac{b^2}{2}+2ab\cot(c+dx)-\frac{1}{2}(4a^2+b^2)\,\text{c}}{\sqrt{\cot(c+dx)}\,(b+a\cot(c\text{-}}}{2a\,(a^2+b^2)}$$

$$= \frac{b^2\sqrt{\cot(c+dx)}}{2a\,(a^2+b^2)\,d(b+a\cot(c+dx))^2} - \frac{b(9a^2+b^2)\sqrt{\cot(c+dx}}{4a\,(a^2+b^2)^2\,d(b+a\cot(c+}$$

$$= \frac{b^2\sqrt{\cot(c+dx)}}{2a\,(a^2+b^2)\,d(b+a\cot(c+dx))^2} - \frac{b(9a^2+b^2)\sqrt{\cot(c+dx}}{4a\,(a^2+b^2)^2\,d(b+a\cot(c+}$$

$$= \frac{b^2\sqrt{\cot(c+dx)}}{2a\,(a^2+b^2)\,d(b+a\cot(c+dx))^2} - \frac{b(9a^2+b^2)\sqrt{\cot(c+dx}}{4a\,(a^2+b^2)^2\,d(b+a\cot(c+}$$

$$= \frac{b^2\sqrt{\cot(c+dx)}}{2a\,(a^2+b^2)\,d(b+a\cot(c+dx))^2} - \frac{b(9a^2+b^2)\sqrt{\cot(c+dx}}{4a\,(a^2+b^2)^2\,d(b+a\cot(c+}$$

$$= \frac{\sqrt{b}\,(15a^4-18a^2b^2-b^4)\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{4a^{3/2}\,(a^2+b^2)^3\,d} + \frac{}{2a\,(a^2}$$

$$= \frac{\sqrt{b}\,(15a^4-18a^2b^2-b^4)\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{4a^{3/2}\,(a^2+b^2)^3\,d} + \frac{}{2a\,(a^2}$$

$$= \frac{(a-b)\,(a^2+4ab+b^2)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d} - \frac{(a-b}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.15, size = 462, normalized size = 1.18

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^3),x]
```

```
[Out] -(((-2*a*(a^2 - 3*b^2)*Cot[c + d*x]^(5/2))/(5*(a^2 + b^2)^3) + (2*b*(a^2 -
3*b^2)*(-3*b*(-((Sqrt[b]*ArcTan[(Sqrt[a]*Sqrt[Cot[c + d*x]])/Sqrt[b]])/a^(3
```

```
/2)) + Sqrt[Cot[c + d*x]]/a) + Cot[c + d*x]^(3/2)))/(3*(a^2 + b^2)^3) - (2*
b*(3*a^2 - b^2)*(Cot[c + d*x]^(3/2) - Cot[c + d*x]^(3/2)*Hypergeometric2F1[
3/4, 1, 7/4, -Cot[c + d*x]^2]))/(3*(a^2 + b^2)^3) + (4*a^2*Cot[c + d*x]^(7/
2)*Hypergeometric2F1[2, 7/2, 9/2, -((a*Cot[c + d*x])/b)])/(7*b*(a^2 + b^2)^
2) + (2*a^2*Cot[c + d*x]^(7/2)*Hypergeometric2F1[3, 7/2, 9/2, -((a*Cot[c +
d*x])/b)])/(7*b^3*(a^2 + b^2)) - (a*(a^2 - 3*b^2)*(40*Sqrt[Cot[c + d*x]] -
8*Cot[c + d*x]^(5/2) + (5*(4*(Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]
] - Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]) + 2*Sqrt[2]*Log[1 - Sqr
t[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - 2*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Co
t[c + d*x]] + Cot[c + d*x]]))/2))/(20*(a^2 + b^2)^3))/d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 19.07, size = 50771, normalized size = 129.52

method	result	size
default	Expression too large to display	50771

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.54, size = 418, normalized size = 1.07

$$\frac{\frac{(15a^4b-18a^2b^3-b^5)\arctan\left(\frac{a}{\sqrt{ab}\sqrt{\tan(dx+c)}}\right)}{(a^7+3a^5b^2+3a^3b^4+ab^6)\sqrt{ab}}-\frac{2\sqrt{2}(a^3+3a^2b-3ab^2-b^3)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}(a^3+3a^2b-3ab^2-b^3)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+\sqrt{2}(a^3-3a^2b-3ab^2+b^3)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}(a^3-3a^2b-3ab^2+b^3)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{\frac{7a^2b^2-b^4}{\sqrt{\tan(dx+c)}}+\frac{9a^3b+ab^3}{\tan(dx+c)^{\frac{3}{2}}}}{a^5b^2+2a^3b^4+ab^6+\frac{2(a^6b+2a^4b^3+a^2b^5)}{\tan(dx+c)}+\frac{a^7+2a^5b^2+a^3b^4}{\tan(dx+c)^2}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*((15*a^4*b - 18*a^2*b^3 - b^5)*arctan(a/(sqrt(a*b)*sqrt(tan(d*x + c))))
/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*sqrt(a*b)) - (2*sqrt(2)*(a^3 + 3*a^
2*b - 3*a*b^2 - b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) +
 2*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2
/sqrt(tan(d*x + c)))) + sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*log(sqrt(2)
/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^
2 + b^3)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^6 + 3*a^
4*b^2 + 3*a^2*b^4 + b^6) - ((7*a^2*b^2 - b^4)/sqrt(tan(d*x + c)) + (9*a^3*b
 + a*b^3)/tan(d*x + c)^(3/2))/(a^5*b^2 + 2*a^3*b^4 + a*b^6 + 2*(a^6*b + 2*a
^4*b^3 + a^2*b^5)/tan(d*x + c) + (a^7 + 2*a^5*b^2 + a^3*b^4)/tan(d*x + c)^2
))/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\tan(c+dx))^3\sqrt{\cot(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(1/2)/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(1/((a + b*tan(c + d*x))**3*sqrt(cot(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*tan(d*x + c) + a)^3*sqrt(cot(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{\cot(c+d\,x)}\,(a+b\tan(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^3),x)
```

```
[Out] int(1/(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^3), x)
```

3.837 $\int \frac{1}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=385

$$-\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,\left(a^2+b^2\right)^3 d}+\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot}\right.}{\sqrt{2}\,\left(a^2+b^2\right)^3 d}$$

```
[Out] 1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)^3/d
*2^(1/2)+1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+
b^2)^3/d*2^(1/2)+1/4*(a-b)*(a^2+4*a*b+b^2)*ln(1+cot(d*x+c)-2^(1/2)*cot(d*x+
c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/4*(a-b)*(a^2+4*a*b+b^2)*ln(1+cot(d*x+c)+2
^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/4*(3*a^4-26*a^2*b^2+3*b^4)
*arctan(a^(1/2)*cot(d*x+c)^(1/2)/b^(1/2))/(a^2+b^2)^3/d/a^(1/2)/b^(1/2)-1/2
*b*cot(d*x+c)^(1/2)/(a^2+b^2)/d/(b+a*cot(d*x+c))^2+1/4*(5*a^2-3*b^2)*cot(d*
x+c)^(1/2)/(a^2+b^2)^2/d/(b+a*cot(d*x+c))
```

Rubi [A]

time = 0.61, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3754, 3648, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d\left(a^2+b^2\right)^3}+\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d\left(a^2+b^2\right)^3}-\frac{b\sqrt{\cot(c+dx)}}{2d\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}+\frac{\left(5a^2-3b^2\right)\sqrt{\cot(c+dx)}}{4d\left(a^2+b^2\right)^2(a\cot(c+dx)+b)}+\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\cot(c+dx)-\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)^3}-\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\cot(c+dx)+\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)^3}-\frac{\left(3a^4-26a^2b^2+3b^4\right)\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{a}\,\sqrt{b}\,d\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^3),x]
```

```
[Out] -(((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqr
t[2]*(a^2 + b^2)^3*d)) + ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 + Sqrt[2]*Sq
rt[Cot[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)^3*d) - ((3*a^4 - 26*a^2*b^2 + 3*b^4
)*ArcTan[(Sqrt[a]*Sqrt[Cot[c + d*x]])/Sqrt[b]])/(4*Sqrt[a]*Sqrt[b]*(a^2 + b
^2)^3*d) - (b*Sqrt[Cot[c + d*x]])/(2*(a^2 + b^2)*d*(b + a*Cot[c + d*x])^2)
+ ((5*a^2 - 3*b^2)*Sqrt[Cot[c + d*x]])/(4*(a^2 + b^2)^2*d*(b + a*Cot[c + d*
x])) + ((a - b)*(a^2 + 4*a*b + b^2)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Co
t[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^3*d) - ((a - b)*(a^2 + 4*a*b + b^2)*Log
[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^3*d
)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
```

```
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx=\int \frac{\cot^{\frac{3}{2}}(c+dx)}{(b+a\cot(c+dx))^3}\,dx$$

$$=-\frac{b\sqrt{\cot(c+dx)}}{2\left(a^2+b^2\right)d(b+a\cot(c+dx))^2}-\frac{\int \frac{\frac{b}{2}-2a\cot(c+dx)-\frac{3}{2}b\cot^2(c+dx)}{\sqrt{\cot(c+dx)}\,(b+a\cot(c+dx))}}{2\left(a^2+b^2\right)}$$

$$=-\frac{b\sqrt{\cot(c+dx)}}{2\left(a^2+b^2\right)d(b+a\cot(c+dx))^2}+\frac{\left(5a^2-3b^2\right)\sqrt{\cot(c+dx)}}{4\left(a^2+b^2\right)^2d(b+a\cot(c+d}$$

$$=-\frac{b\sqrt{\cot(c+dx)}}{2\left(a^2+b^2\right)d(b+a\cot(c+dx))^2}+\frac{\left(5a^2-3b^2\right)\sqrt{\cot(c+dx)}}{4\left(a^2+b^2\right)^2d(b+a\cot(c+d}$$

$$=-\frac{b\sqrt{\cot(c+dx)}}{2\left(a^2+b^2\right)d(b+a\cot(c+dx))^2}+\frac{\left(5a^2-3b^2\right)\sqrt{\cot(c+dx)}}{4\left(a^2+b^2\right)^2d(b+a\cot(c+d}$$

$$=-\frac{b\sqrt{\cot(c+dx)}}{2\left(a^2+b^2\right)d(b+a\cot(c+dx))^2}+\frac{\left(5a^2-3b^2\right)\sqrt{\cot(c+dx)}}{4\left(a^2+b^2\right)^2d(b+a\cot(c+d}$$

$$=-\frac{\left(3a^4-26a^2b^2+3b^4\right)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{a}\sqrt{b}\left(a^2+b^2\right)^3d}-\frac{b\sqrt{}}{2\left(a^2+b^2\right)}$$

$$=-\frac{\left(3a^4-26a^2b^2+3b^4\right)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{a}\sqrt{b}\left(a^2+b^2\right)^3d}-\frac{b\sqrt{}}{2\left(a^2+b^2\right)}$$

$$=-\frac{(a+b)\left(a^2-4ab+b^2\right)\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)^3d}+\frac{(a+b}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.15, size = 492, normalized size = 1.28

$$-\frac{\frac{2b(a^2-3b^2)\left(-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{\sqrt{a}}+\sqrt{\cot(c+dx)}\right)}{(a^2+b^2)^3}-\frac{2a(a^2-3b^2)\cot^{\frac{3}{2}}(c+dx)}{3(a^2+b^2)^3}-\frac{-\frac{3\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)\sqrt{\cot(c+dx)}}{\sqrt{b}}+\frac{2a^2\cot^2(c+dx)}{(b+a\cot(c+dx))^2}+\frac{3a\cot(c+dx)}{b+a\cot(c+dx)}}{4a(a^2+b^2)\sqrt{\cot(c+dx)}}+\frac{2a(a^2-3b^2)\left(\cot^{\frac{3}{2}}(c+dx)-\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)\right)}{3(a^2+b^2)^3}+\frac{4a^2\cot^{\frac{5}{2}}(c+dx)\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};-\frac{a\cot(c+dx)}{b}\right)}{5b(a^2+b^2)^2}-\frac{b(3a^2-b^2)\left(2\left(\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)\right)+8\sqrt{\cot(c+dx)}+\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)}{4(a^2+b^2)^3}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^3),x]
```

```
[Out] -((((2*b*(a^2 - 3*b^2)*(-((Sqrt[b]*ArcTan[(Sqrt[a]*Sqrt[Cot[c + d*x]])/Sqrt[
b]])/Sqrt[a]) + Sqrt[Cot[c + d*x]]))/(a^2 + b^2)^3 - (2*a*(a^2 - 3*b^2)*Cot
[c + d*x]^(3/2))/(3*(a^2 + b^2)^3) - ((-3*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[Cot[
c + d*x]])/Sqrt[b]]*Sqrt[Cot[c + d*x]])/Sqrt[b] + (2*a^2*Cot[c + d*x]^2)/(b
 + a*Cot[c + d*x])^2 + (3*a*Cot[c + d*x])/(b + a*Cot[c + d*x]))/(4*a*(a^2 +
 b^2)*Sqrt[Cot[c + d*x]]) + (2*a*(a^2 - 3*b^2)*(Cot[c + d*x]^(3/2) - Cot[c
+ d*x]^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2]))/(3*(a^2 + b^
2)^3) + (4*a^2*Cot[c + d*x]^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, -((a*Cot[c
 + d*x])/b)])/(5*b*(a^2 + b^2)^2) - (b*(3*a^2 - b^2)*(2*(Sqrt[2]*ArcTan[1 -
 Sqrt[2]*Sqrt[Cot[c + d*x]]] - Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]
]]) + 8*Sqrt[Cot[c + d*x]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + C
ot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))
/(4*(a^2 + b^2)^3))/d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 20.92, size = 50714, normalized size = 131.72

method	result	size
default	Expression too large to display	50714

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.52, size = 409, normalized size = 1.06

$$-\frac{\frac{(3a^4-26a^2b^2+3b^4)\arctan\left(\frac{a}{\sqrt{ab}\sqrt{\tan(dx+c)}}\right)}{(a^6+3a^4b^2+3a^2b^4+b^6)\sqrt{ab}}-\frac{2\sqrt{2}(a^3-3a^2b-3ab^2+b^3)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}(a^3-3a^2b-3ab^2+b^3)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-\sqrt{2}(a^3+3a^2b-3ab^2-b^3)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}(a^3+3a^2b-3ab^2-b^3)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{\frac{3a^2b-5b^3}{\sqrt{\tan(dx+c)}}+\frac{5a^3-3ab^2}{\tan(dx+c)^{\frac{3}{2}}}}{a^4b^2+2a^2b^4+b^6+\frac{2(a^5b+2a^3b^3+ab^5)}{\tan(dx+c)}+\frac{a^6+2a^4b^2+a^2b^4}{\tan(dx+c)^2}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*((3*a^4 - 26*a^2*b^2 + 3*b^4)*arctan(a/(sqrt(a*b)*sqrt(tan(d*x + c))))
/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a*b)) - (2*sqrt(2)*(a^3 - 3*a^2*
```

```
b - 3*a*b^2 + b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2
*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/s
qrt(tan(d*x + c)))) - sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*log(sqrt(2)/s
qrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2
- b^3)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^6 + 3*a^4*
b^2 + 3*a^2*b^4 + b^6) - ((3*a^2*b - 5*b^3)/sqrt(tan(d*x + c)) + (5*a^3 - 3
*a*b^2)/tan(d*x + c)^(3/2))/(a^4*b^2 + 2*a^2*b^4 + b^6 + 2*(a^5*b + 2*a^3*b
^3 + a*b^5)/tan(d*x + c) + (a^6 + 2*a^4*b^2 + a^2*b^4)/tan(d*x + c)^2))/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\tan{(c+dx)})^3\cot^{\frac{3}{2}}{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(3/2)/(a+b*tan(d*x+c))**3,x)
[Out] Integral(1/((a + b*tan(c + d*x))**3*cot(c + d*x)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^3,x, algorithm="giac")
[Out] integrate(1/((b*tan(d*x + c) + a)^3*cot(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\mathrm{cot}\left(c+d\,x\right)}^{3/2}\,{\left(a+b\,\mathrm{tan}\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^3),x)
[Out] int(1/(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^3), x)
```

3.838 $\int \frac{1}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=385

$$-\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,\left(a^2+b^2\right)^3 d}+\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot}\right.}{\sqrt{2}\,\left(a^2+b^2\right)^3 d}$$

```
[Out] 1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)^3/d
*2^(1/2)+1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+
b^2)^3/d*2^(1/2)-1/4*(a+b)*(a^2-4*a*b+b^2)*ln(1+cot(d*x+c)-2^(1/2)*cot(d*x+
c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)+1/4*(a+b)*(a^2-4*a*b+b^2)*ln(1+cot(d*x+c)+2
^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/4*(a^4+18*a^2*b^2-15*b^4)*
arctan(a^(1/2)*cot(d*x+c)^(1/2)/b^(1/2))*a^(1/2)/b^(3/2)/(a^2+b^2)^3/d+1/2*
a*cot(d*x+c)^(1/2)/(a^2+b^2)/d/(b+a*cot(d*x+c))^2-1/4*a*(a^2-7*b^2)*cot(d*x
+c)^(1/2)/b/(a^2+b^2)^2/d/(b+a*cot(d*x+c))
```

Rubi [A]

time = 0.59, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3754, 3649, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d\left(a^2+b^2\right)^3}+\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d\left(a^2+b^2\right)^3}-\frac{a\left(a^2-7b^2\right)\sqrt{\cot(c+dx)}}{4bd\left(a^2+b^2\right)^2(a\cot(c+dx)+b)}+\frac{a\sqrt{\cot(c+dx)}}{2d\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}-\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\cot(c+dx)-\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)^3}+\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\cot(c+dx)+\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)^3}-\frac{\sqrt{a}\left(a^4+18a^2b^2-15b^4\right)\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^3),x]
```

```
[Out] -(((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqr
t[2]*(a^2 + b^2)^3*d)) + ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 + Sqrt[2]*Sq
rt[Cot[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)^3*d) - (Sqrt[a]*(a^4 + 18*a^2*b^2 -
 15*b^4)*ArcTan[(Sqrt[a]*Sqrt[Cot[c + d*x]])/Sqrt[b]])/(4*b^(3/2)*(a^2 + b^
2)^3*d) + (a*Sqrt[Cot[c + d*x]])/(2*(a^2 + b^2)*d*(b + a*Cot[c + d*x])^2) -
 (a*(a^2 - 7*b^2)*Sqrt[Cot[c + d*x]])/(4*b*(a^2 + b^2)^2*d*(b + a*Cot[c + d
*x])) - ((a + b)*(a^2 - 4*a*b + b^2)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + C
ot[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^3*d) + ((a + b)*(a^2 - 4*a*b + b^2)*Lo
g[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^3*
d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
```

```
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx = \int \frac{\sqrt{\cot(c+dx)}}{(b+a\cot(c+dx))^3}\,dx$$

$$= \frac{a\sqrt{\cot(c+dx)}}{2(a^2+b^2)\,d(b+a\cot(c+dx))^2} - \frac{\int \frac{-\frac{a}{2}-2b\cot(c+dx)+\frac{3}{2}a\cot^2(c+dx)}{\sqrt{\cot(c+dx)}\,(b+a\cot(c+dx))^2}\,d}{2(a^2+b^2)}$$

$$= \frac{a\sqrt{\cot(c+dx)}}{2(a^2+b^2)\,d(b+a\cot(c+dx))^2} - \frac{a(a^2-7b^2)\sqrt{\cot(c+dx)}}{4b(a^2+b^2)^2\,d(b+a\cot(c+dx}$$

$$= \frac{a\sqrt{\cot(c+dx)}}{2(a^2+b^2)\,d(b+a\cot(c+dx))^2} - \frac{a(a^2-7b^2)\sqrt{\cot(c+dx)}}{4b(a^2+b^2)^2\,d(b+a\cot(c+dx}$$

$$= \frac{a\sqrt{\cot(c+dx)}}{2(a^2+b^2)\,d(b+a\cot(c+dx))^2} - \frac{a(a^2-7b^2)\sqrt{\cot(c+dx)}}{4b(a^2+b^2)^2\,d(b+a\cot(c+dx}$$

$$= \frac{a\sqrt{\cot(c+dx)}}{2(a^2+b^2)\,d(b+a\cot(c+dx))^2} - \frac{a(a^2-7b^2)\sqrt{\cot(c+dx)}}{4b(a^2+b^2)^2\,d(b+a\cot(c+dx}$$

$$= -\frac{\sqrt{a}\,(a^4+18a^2b^2-15b^4)\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}(a^2+b^2)^3\,d} + \frac{}{2(a^2+}$$

$$= -\frac{\sqrt{a}\,(a^4+18a^2b^2-15b^4)\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}(a^2+b^2)^3\,d} + \frac{}{2(a^2+}$$

$$= -\frac{(a-b)(a^2+4ab+b^2)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d} + \frac{(a-b}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 2.65, size = 398, normalized size = 1.03

$$-\frac{24\sqrt{a}\sqrt{b}\,(a^2-3b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)-24a(a^2-3b^2)\sqrt{\cot(c+dx)}+\frac{24\sqrt{a}\sqrt{b}\,(a^2+b^2)\left(-\sqrt{a}\sqrt{b}\sqrt{\cot(c+dx)}+\mathrm{ArcTan}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)(b+a\cot(c+dx))\right)}{b+a\cot(c+dx)}+8b(-3a^2+b^2)\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)+\frac{8a^2(a^2+b^2)^2\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};-\frac{a\cot(c+dx)}{b}\right)}{b^3}+3a(a^2-3b^2)\left(2\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-2\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+8\sqrt{\cot(c+dx)}+\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)}{12(a^2+b^2)^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^3),x]
```

```
[Out] -1/12*(24*Sqrt[a]*Sqrt[b]*(a^2 - 3*b^2)*ArcTan[(Sqrt[a]*Sqrt[Cot[c + d*x]])
/Sqrt[b]] - 24*a*(a^2 - 3*b^2)*Sqrt[Cot[c + d*x]] + (24*Sqrt[a]*Sqrt[b]*(a^
2 + b^2)*(-(Sqrt[a]*Sqrt[b]*Sqrt[Cot[c + d*x]]) + ArcTan[(Sqrt[a]*Sqrt[Cot[
c + d*x]])/Sqrt[b]]*(b + a*Cot[c + d*x])))/(b + a*Cot[c + d*x]) + 8*b*(-3*a
^2 + b^2)*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2
] + (8*a^2*(a^2 + b^2)^2*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/2, 3, 5/2,
-((a*Cot[c + d*x])/b)])/b^3 + 3*a*(a^2 - 3*b^2)*(2*Sqrt[2]*ArcTan[1 - Sqrt[
2]*Sqrt[Cot[c + d*x]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] +
 8*Sqrt[Cot[c + d*x]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c
+ d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/((a^
2 + b^2)^3*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 22.54, size = 50673, normalized size = 131.62

method	result	size
default	Expression too large to display	50673

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.51, size = 415, normalized size = 1.08

$$-\frac{\frac{(a^5+18a^3b^2-15ab^4)\arctan\left(\frac{a}{\sqrt{ab}\sqrt{\tan(dx+c)}}\right)}{(a^6b+3a^4b^3+3a^2b^5+b^7)\sqrt{ab}}-\frac{2\sqrt{2}(a^3+3a^2b-3ab^2-b^3)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}(a^3+3a^2b-3ab^2-b^3)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+\sqrt{2}(a^3-3a^2b-3ab^2+b^3)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}(a^3-3a^2b-3ab^2+b^3)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{\frac{a^3b+9ab^3}{\sqrt{\tan(dx+c)}}-\frac{a^4-7a^2b^2}{\tan(dx+c)^{\frac{3}{2}}}}{a^4b^3+2a^2b^5+b^7+\frac{2(a^5b^2+2a^3b^4+ab^6)}{\tan(dx+c)}+\frac{a^6b+2a^4b^3+a^2b^5}{\tan(dx+c)^2}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*((a^5 + 18*a^3*b^2 - 15*a*b^4)*arctan(a/(sqrt(a*b)*sqrt(tan(d*x + c)))
)/((a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sqrt(a*b)) - (2*sqrt(2)*(a^3 + 3*a
^2*b - 3*a*b^2 - b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c))))
+ 2*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) -
2/sqrt(tan(d*x + c)))) + sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*log(sqrt(2
```

```
)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*(a^3 - 3*a^2*b - 3*a*b
^2 + b^3)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^6 + 3*a
^4*b^2 + 3*a^2*b^4 + b^6) - ((a^3*b + 9*a*b^3)/sqrt(tan(d*x + c)) - (a^4 -
7*a^2*b^2)/tan(d*x + c)^(3/2))/(a^4*b^3 + 2*a^2*b^5 + b^7 + 2*(a^5*b^2 + 2*
a^3*b^4 + a*b^6)/tan(d*x + c) + (a^6*b + 2*a^4*b^3 + a^2*b^5)/tan(d*x + c)^
2))/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(5/2)/(a+b*tan(d*x+c))**3,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^3,x, algorithm="giac")
[Out] integrate(1/((b*tan(d*x + c) + a)^3*cot(d*x + c)^(5/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cot(c+d\,x)^{5/2}\,(a+b\tan(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))^3),x)
[Out] int(1/(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))^3), x)
```

3.839 $\int \frac{1}{\cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=396

$$\frac{(a+b)\left(a^2-4ab+b^2\right)\operatorname{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,\left(a^2+b^2\right)^3 d}-\frac{(a+b)\left(a^2-4ab+b^2\right)\operatorname{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(c$$

```
[Out] -1/4*a^(3/2)*(3*a^4+6*a^2*b^2+35*b^4)*arctan(a^(1/2)*cot(d*x+c)^(1/2)/b^(1/
2))/b^(5/2)/(a^2+b^2)^3/d-1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(-1+2^(1/2)*cot(d
*x+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(1+2^(1/
2)*cot(d*x+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/4*(a-b)*(a^2+4*a*b+b^2)*ln(1+c
ot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)+1/4*(a-b)*(a^2+4*
a*b+b^2)*ln(1+cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/
2*a^2*cot(d*x+c)^(1/2)/b/(a^2+b^2)/d/(b+a*cot(d*x+c))^2-1/4*a^2*(3*a^2+11*b
^2)*cot(d*x+c)^(1/2)/b^2/(a^2+b^2)^2/d/(b+a*cot(d*x+c))
```

Rubi [A]
time = 0.65, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3754, 3650, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(a+b)\left(a^2-4ab+b^2\right)\operatorname{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,d\left(a^2+b^2\right)^3}-\frac{(a+b)\left(a^2-4ab+b^2\right)\operatorname{ArcTan}\left(\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}\,d\left(a^2+b^2\right)^3}-\frac{a^2\left(3a^2+11b^2\right)\sqrt{\cot(c+dx)}}{4b^2d\left(a^2+b^2\right)^2\left(a\cot(c+dx)+b\right)}-\frac{a^2\sqrt{\cot(c+dx)}}{2bd\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}-\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\cot(c+dx)-\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)^3}+\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\cot(c+dx)+\sqrt{2}\,\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)^3}-\frac{a^{3/2}\left(3a^4+6a^2b^2+35b^4\right)\operatorname{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{4b^{5/2}d\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cot[c + d*x]^(7/2)*(a + b*Tan[c + d*x])^3),x]
```

```
[Out] ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]])/(Sqrt[
2]*(a^2 + b^2)^3*d) - ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 + Sqrt[2]*Sqrt[
Cot[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)^3*d) - (a^(3/2)*(3*a^4 + 6*a^2*b^2 + 3
5*b^4)*ArcTan[(Sqrt[a]*Sqrt[Cot[c + d*x]])/Sqrt[b]])/(4*b^(5/2)*(a^2 + b^2)
^3*d) - (a^2*Sqrt[Cot[c + d*x]])/(2*b*(a^2 + b^2)*d*(b + a*Cot[c + d*x])^2)
 - (a^2*(3*a^2 + 11*b^2)*Sqrt[Cot[c + d*x]])/(4*b^2*(a^2 + b^2)^2*d*(b + a*
Cot[c + d*x])) - ((a - b)*(a^2 + 4*a*b + b^2)*Log[1 - Sqrt[2]*Sqrt[Cot[c +
d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^3*d) + ((a - b)*(a^2 + 4*a*b
+ b^2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*(a^2
+ b^2)^3*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
```

```
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{\cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx = \int \frac{1}{\sqrt{\cot(c+dx)}\,(b+a\cot(c+dx))^3}\,dx$$

$$= -\frac{a^2\sqrt{\cot(c+dx)}}{2b\,(a^2+b^2)\,d(b+a\cot(c+dx))^2} - \frac{\int \frac{\frac{1}{2}(-3a^2-4b^2)+2ab\cot(c+dx)-\frac{3}{2}a^2}{\sqrt{\cot(c+dx)}\,(b+a\cot(c}}{2b\,(a^2+b^2)}$$

$$= -\frac{a^2\sqrt{\cot(c+dx)}}{2b\,(a^2+b^2)\,d(b+a\cot(c+dx))^2} - \frac{a^2(3a^2+11b^2)\sqrt{\cot(c+}}{4b^2\,(a^2+b^2)^2\,d(b+a\cot(c+}$$

$$= -\frac{a^2\sqrt{\cot(c+dx)}}{2b\,(a^2+b^2)\,d(b+a\cot(c+dx))^2} - \frac{a^2(3a^2+11b^2)\sqrt{\cot(c+}}{4b^2\,(a^2+b^2)^2\,d(b+a\cot(c+}$$

$$= -\frac{a^2\sqrt{\cot(c+dx)}}{2b\,(a^2+b^2)\,d(b+a\cot(c+dx))^2} - \frac{a^2(3a^2+11b^2)\sqrt{\cot(c+}}{4b^2\,(a^2+b^2)^2\,d(b+a\cot(c+}$$

$$= -\frac{a^2\sqrt{\cot(c+dx)}}{2b\,(a^2+b^2)\,d(b+a\cot(c+dx))^2} - \frac{a^2(3a^2+11b^2)\sqrt{\cot(c+}}{4b^2\,(a^2+b^2)^2\,d(b+a\cot(c+}$$

$$= -\frac{a^{3/2}(3a^4+6a^2b^2+35b^4)\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{4b^{5/2}\,(a^2+b^2)^3\,d} - \frac{}{2b\,(a^2}$$

$$= -\frac{a^{3/2}(3a^4+6a^2b^2+35b^4)\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{4b^{5/2}\,(a^2+b^2)^3\,d} - \frac{}{2b\,(a^2}$$

$$= \frac{(a+b)\,(a^2-4ab+b^2)\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d} - \frac{(a+b)\,(}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.13, size = 387, normalized size = 0.98

$$-\frac{-\frac{2a^{3/2}(a^2-3b^2)\operatorname{ArcTan}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}(a^2+b^2)^3}+\frac{2a^2\sqrt{\cot(c+dx)}\left(\frac{\sqrt{b}\operatorname{ArcTan}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{\cot(c+dx)}}+\frac{b}{b+a\cot(c+dx)}\right)}{b(a^2+b^2)^2}+\frac{2a^2\sqrt{\cot(c+dx)}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};-\frac{a\cot(c+dx)}{b}\right)}{b^3(a^2+b^2)}+\frac{2a(a^2-3b^2)\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)}{3(a^2+b^2)^3}+\frac{b(3a^2-b^2)\left(4\left(\sqrt{2}\operatorname{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-\sqrt{2}\operatorname{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)\right)+2\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-2\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)}{8(a^2+b^2)^3}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cot[c + d*x]^(7/2)*(a + b*Tan[c + d*x])^3),x]
```

```
[Out] -(((-2*a^(3/2)*(a^2 - 3*b^2)*ArcTan[(Sqrt[a]*Sqrt[Cot[c + d*x]])/Sqrt[b]])/
(Sqrt[b]*(a^2 + b^2)^3) + (2*a^2*Sqrt[Cot[c + d*x]]*((Sqrt[b]*ArcTan[(Sqrt[
a]*Sqrt[Cot[c + d*x]])/Sqrt[b]])/(Sqrt[a]*Sqrt[Cot[c + d*x]]) + b/(b + a*Co
t[c + d*x])))/(b*(a^2 + b^2)^2) + (2*a^2*Sqrt[Cot[c + d*x]]*Hypergeometric2
F1[1/2, 3, 3/2, -((a*Cot[c + d*x])/b)])/(b^3*(a^2 + b^2)) + (2*a*(a^2 - 3*b
^2)*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2])/(3*
(a^2 + b^2)^3) + (b*(3*a^2 - b^2)*(4*(Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c
 + d*x]]] - Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]) + 2*Sqrt[2]*Log
[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - 2*Sqrt[2]*Log[1 + Sqrt[2]
*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/(8*(a^2 + b^2)^3))/d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 20.98, size = 50764, normalized size = 128.19

method	result	size
default	Expression too large to display	50764

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(7/2)/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.53, size = 425, normalized size = 1.07

$$-\frac{\frac{(3a^6+6a^4b^2+35a^2b^4)\arctan\left(\frac{a}{\sqrt{ab}\sqrt{\tan(dx+c)}}\right)}{(a^6b^2+3a^4b^4+3a^2b^6+b^8)\sqrt{ab}}+\frac{2\sqrt{2}(a^3-3a^2b-3ab^2+b^3)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}(a^3-3a^2b-3ab^2+b^3)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-\sqrt{2}(a^3+3a^2b-3ab^2-b^3)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}(a^3+3a^2b-3ab^2-b^3)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{\frac{5a^4b+13a^2b^3}{\sqrt{\tan(dx+c)}}+\frac{3a^5+11a^3b^2}{\tan(dx+c)^{\frac{3}{2}}}}{a^4b^4+2a^2b^6+b^8+\frac{2(a^5b^3+2a^3b^5+ab^7)}{\tan(dx+c)}+\frac{a^6b^2+2a^4b^4+a^2b^6}{\tan(dx+c)^2}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(7/2)/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*((3*a^6 + 6*a^4*b^2 + 35*a^2*b^4)*arctan(a/(sqrt(a*b)*sqrt(tan(d*x + c
))))/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*sqrt(a*b)) + (2*sqrt(2)*(a^3
- 3*a^2*b - 3*a*b^2 + b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c
)))) + 2*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*arctan(-1/2*sqrt(2)*(sqrt(
2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*log(s
```

```
qrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*(a^3 + 3*a^2*b -
3*a*b^2 - b^3)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^6
+ 3*a^4*b^2 + 3*a^2*b^4 + b^6) + ((5*a^4*b + 13*a^2*b^3)/sqrt(tan(d*x + c))
 + (3*a^5 + 11*a^3*b^2)/tan(d*x + c)^(3/2))/(a^4*b^4 + 2*a^2*b^6 + b^8 + 2*
(a^5*b^3 + 2*a^3*b^5 + a*b^7)/tan(d*x + c) + (a^6*b^2 + 2*a^4*b^4 + a^2*b^6
)/tan(d*x + c)^2))/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(7/2)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(7/2)/(a+b*tan(d*x+c))**3,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3064 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(7/2)/(a+b*tan(d*x+c))^3,x, algorithm="giac")
[Out] integrate(1/((b*tan(d*x + c) + a)^3*cot(d*x + c)^(7/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cot(c+d\,x)^{7/2}\,(a+b\tan(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(7/2)*(a + b*tan(c + d*x))^3),x)
[Out] int(1/(cot(c + d*x)^(7/2)*(a + b*tan(c + d*x))^3), x)
```

3.840 $\int \cot^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=261

$$\frac{\sqrt{ia-b}\,\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d}-\frac{\sqrt{ia+b}\,\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{}}{\sqrt{a+b\,\mathrm{t}}}\right.}{}$$

```
[Out] arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*(I*a-b)^(1/2)
*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)
/(a+b*tan(d*x+c))^(1/2))*(I*a+b)^(1/2)*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-
2/15*b*cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2)/a/d-2/5*cot(d*x+c)^(5/2)*(a+
b*tan(d*x+c))^(1/2)/d+2/15*(15*a^2+2*b^2)*cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))
^(1/2)/a^2/d
```

Rubi [A]
time = 0.61, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4326, 3649, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(15a^2+2b^2)\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{15a^2d}+\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{2\cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{5d}-\frac{2b\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{15ad}-\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(7/2)*Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] (Sqrt[I*a - b]*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c +
 d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d - (Sqrt[I*a + b]*ArcTanh[(
Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*
x]]*Sqrt[Tan[c + d*x]])/d + (2*(15*a^2 + 2*b^2)*Sqrt[Cot[c + d*x]]*Sqrt[a +
 b*Tan[c + d*x]])/(15*a^2*d) - (2*b*Cot[c + d*x]^(3/2)*Sqrt[a + b*Tan[c + d
*x]])/(15*a*d) - (2*Cot[c + d*x]^(5/2)*Sqrt[a + b*Tan[c + d*x]])/(5*d)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
```

```
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \cot^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{\sqrt{a+b\tan(c+dx)}}{\tan^{\frac{7}{2}}(c+dx)}\,dx$$

$$= -\frac{2\cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{5d} - \frac{1}{5}\Big(2\sqrt{\cot(c+dx)}\,\sqrt{\tan$$

$$= -\frac{2b\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{15ad} - \frac{2\cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\,\mathrm{t}}}{5d}$$

$$= \frac{2(15a^2+2b^2)\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{15a^2d} - \frac{2b\cot^{\frac{3}{2}}(c+}{}$$

$$= \frac{2(15a^2+2b^2)\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{15a^2d} - \frac{2b\cot^{\frac{3}{2}}(c+}{}$$

$$= \frac{2(15a^2+2b^2)\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{15a^2d} - \frac{2b\cot^{\frac{3}{2}}(c+}{}$$

$$= \frac{2(15a^2+2b^2)\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{15a^2d} - \frac{2b\cot^{\frac{3}{2}}(c+}{}$$

$$= \frac{\sqrt{ia-b}\,\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan}}{d}$$

Mathematica [A]

time = 2.30, size = 220, normalized size = 0.84

$$\frac{\sqrt{\cot(c+dx)}\left(15\sqrt[4]{-1}\,a^2\sqrt{-a+ib}\,\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\tan(c+dx)}+15\sqrt[4]{-1}\,a^2\sqrt{a+ib}\,\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\tan(c+dx)}-2(-15a^2-2b^2+ab\cot(c+dx)+3a^2\cot^2(c+dx))\sqrt{a+b\tan(c+dx)}\right)}{15a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(7/2)*Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*(15*(-1)^(1/4)*a^2*Sqrt[-a + I*b]*ArcTan[((-1)^(1/4)*Sq
rt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Tan[c + d*x
]] + 15*(-1)^(1/4)*a^2*Sqrt[a + I*b]*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[
Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Tan[c + d*x]] - 2*(-15*a^2 -
2*b^2 + a*b*Cot[c + d*x] + 3*a^2*Cot[c + d*x]^2)*Sqrt[a + b*Tan[c + d*x]]))
/(15*a^2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 36.90, size = 21305, normalized size = 81.63

method	result	size
default	Expression too large to display	21305

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(d*x + c) + a)*cot(d*x + c)^(7/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(7/2)*(a+b*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

$$\text{Exception raised: TypeError}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c+d\,x)^{7/2}\,\sqrt{a+b\tan\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(7/2)*(a + b*tan(c + d*x))^(1/2),x)
```

```
[Out] int(cot(c + d*x)^(7/2)*(a + b*tan(c + d*x))^(1/2), x)
```

3.841 $\int \cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=221

$$\frac{i\sqrt{ia-b}\,\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d}+\frac{i\sqrt{ia+b}\,\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\cdots}{\cdots}$$

```
[Out] I*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*(I*a-b)^(1/
2)*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+I*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(
1/2)/(a+b*tan(d*x+c))^(1/2))*(I*a+b)^(1/2)*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2
)/d-2/3*cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2)/d-2/3*b*cot(d*x+c)^(1/2)*(a
+b*tan(d*x+c))^(1/2)/a/d
```

Rubi [A]
time = 0.30, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {4326, 3649, 3730, 21, 3656, 924, 95, 211, 214}

$$\frac{i\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{2\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}-\frac{2b\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{3ad}+\frac{i\sqrt{b+ia}\,\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(5/2)*Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] (I*Sqrt[I*a - b]*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
 + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d + (I*Sqrt[I*a + b]*ArcTa
nh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c
+ d*x]]*Sqrt[Tan[c + d*x]])/d - (2*b*Sqrt[Cot[c + d*x]]*Sqrt[a + b*Tan[c +
d*x]])/(3*a*d) - (2*Cot[c + d*x]^(3/2)*Sqrt[a + b*Tan[c + d*x]])/(3*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
```

```
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}\,dx = \left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{\sqrt{a+b\tan(c+dx)}}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$

$$= -\frac{2\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{3d} - \frac{1}{3}\left(2\sqrt{\cot(c+dx)}\sqrt{\tan(\right.$$

$$= -\frac{2b\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{3ad} - \frac{2\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\,\mathrm{t}}}{3d}$$

$$= -\frac{2b\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{3ad} - \frac{2\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\,\mathrm{t}}}{3d}$$

$$= -\frac{2b\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{3ad} - \frac{2\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\,\mathrm{t}}}{3d}$$

$$= -\frac{2b\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{3ad} - \frac{2\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\,\mathrm{t}}}{3d}$$

$$= -\frac{2b\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{3ad} - \frac{2\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\,\mathrm{t}}}{3d}$$

$$= -\frac{2b\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{3ad} - \frac{2\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\,\mathrm{t}}}{3d}$$

$$= \frac{i\sqrt{ia-b}\,\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(}}{d}$$

Mathematica [A]
time = 0.78, size = 185, normalized size = 0.84

$$\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(-\frac{(-1)^{3/4}\sqrt{-a+ib}\,\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{(-1)^{3/4}\sqrt{a+ib}\,\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{2(a+b\tan(c+dx))^{3/2}}{3ad\tan^{\frac{3}{2}}(c+dx)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(5/2)*Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-(((-1)^(3/4)*Sqrt[-a + I*b]*ArcTan[
((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/d
) + ((-1)^(3/4)*Sqrt[a + I*b]*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c +
 d*x]])/Sqrt[a + b*Tan[c + d*x]]])/d - (2*(a + b*Tan[c + d*x])^(3/2))/(3*a*
d*Tan[c + d*x]^(3/2)))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 29.77, size = 10794, normalized size = 48.84

method	result	size
default	Expression too large to display	10794

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(d*x + c) + a)*cot(d*x + c)^(5/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(5/2)*(a+b*tan(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c+d\,x)^{5/2}\,\sqrt{a+b\tan\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))^(1/2),x)
```

```
[Out] int(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))^(1/2), x)
```

3.842 $\int \cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=179

$$-\frac{\sqrt{ia-b}\,\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d}+\frac{\sqrt{ia+b}\,\tanh^{-1}\left(\frac{\sqrt{ia+b}}{\sqrt{a+b}}\right.}{}$$

```
[Out] -arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*(I*a-b)^(1/2
)*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2
)/(a+b*tan(d*x+c))^(1/2))*(I*a+b)^(1/2)*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d
-2*cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.33, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4326, 3649, 3697, 3696, 95, 209, 212}

$$-\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{2\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{d}+\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(3/2)*Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] -((Sqrt[I*a - b]*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
 + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d) + (Sqrt[I*a + b]*ArcTan
h[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c +
 d*x]]*Sqrt[Tan[c + d*x]])/d - (2*Sqrt[Cot[c + d*x]]*Sqrt[a + b*Tan[c + d*x
]])/d
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{\sqrt{a+b\tan(c+dx)}}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$
$$= -\frac{2\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{d} - \left(2\sqrt{\cot(c+dx)}\,\sqrt{\tan\right.$$
$$= -\frac{2\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{d} - \frac{1}{2}\left((-ia-b)\sqrt{\cot(c+\right.$$
$$= -\frac{2\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{d} - \frac{\left((-ia-b)\sqrt{\cot(c+d\right.}{}$$
$$= -\frac{2\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{d} - \frac{\left((-ia-b)\sqrt{\cot(c+d\right.}{}$$
$$= -\frac{\sqrt{ia-b}\,\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\mathrm{ta}}}{d}$$

Mathematica [A]
time = 0.16, size = 174, normalized size = 0.97

$$-\frac{\sqrt{\cot(c+dx)}\left(\sqrt[4]{-1}\,\sqrt{-a+ib}\,\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\tan(c+dx)}+\sqrt[4]{-1}\,\sqrt{a+ib}\,\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\tan(c+dx)}+2\sqrt{a+b\tan(c+dx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(3/2)*Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] -((Sqrt[Cot[c + d*x]]*((-1)^(1/4)*Sqrt[-a + I*b]*ArcTan[((-1)^(1/4)*Sqrt[-a
 + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Tan[c + d*x]] +
(-1)^(1/4)*Sqrt[a + I*b]*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]
])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Tan[c + d*x]] + 2*Sqrt[a + b*Tan[c + d*x]
]))/d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 37.43, size = 10498, normalized size = 58.65

method	result	size
default	Expression too large to display	10498

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*tan(d*x + c) + a)*cot(d*x + c)^(3/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan\left(c+dx\right)}\,\cot^{\frac{3}{2}}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(3/2)*(a+b*tan(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*tan(c + d*x))*cot(c + d*x)**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^{3/2}\,\sqrt{a+b\tan\left(c+d\,x\right)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(1/2),x)
[Out] int(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(1/2), x)
```

3.843 $\int \sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=155

$$-\frac{i\sqrt{ia-b}\,\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d}-\frac{i\sqrt{ia+b}\,\tanh^{-1}\left(\frac{\sqrt{ia+b}}{\sqrt{a+b}}\right.}{}$$

```
[Out] -I*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*(I*a-b)^(1
/2)*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-I*arctanh((I*a+b)^(1/2)*tan(d*x+c)^
(1/2)/(a+b*tan(d*x+c))^(1/2))*(I*a+b)^(1/2)*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/
2)/d
```

Rubi [A]
time = 0.13, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4326, 3656, 924, 95, 211, 214}

$$-\frac{i\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{i\sqrt{b+ia}\,\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cot[c + d*x]]*Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] ((-I)*Sqrt[I*a - b]*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Ta
n[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d - (I*Sqrt[I*a + b]*Ar
cTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot
[c + d*x]]*Sqrt[Tan[c + d*x]])/d
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}\,dx = \left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{\tan(c+dx)}}\,dx$$

$$= \frac{\left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{\sqrt{x}\,(1+x^2)}\,dx, x, \tan$$

$$= \frac{\left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \left(\frac{ia-b}{2(i-x)\sqrt{x}\sqrt{a+bx}}$$

$$= \frac{\left((ia-b)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{1}{(i-x)\sqrt{x}\sqrt{a}}$$

$$= \frac{\left((ia-b)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{1}{i-(a+ib)x^2}\,dx$$

$$= -\frac{i\sqrt{ia-b}\,\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}}{d}$$

Mathematica [A]
time = 0.16, size = 143, normalized size = 0.92

$$\frac{(-1)^{3/4}\left(\sqrt{-a+ib}\,\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)-\sqrt{a+ib}\,\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cot[c + d*x]]*Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] ((-1)^(3/4)*(Sqrt[-a + I*b]*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c +
d*x]])/Sqrt[a + b*Tan[c + d*x]]] - Sqrt[a + I*b]*ArcTan[((-1)^(1/4)*Sqrt[a
+ I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])*Sqrt[Cot[c + d*x]]*Sq
rt[Tan[c + d*x]])/d
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 38.51, size = 2597, normalized size = 16.75

method	result	size
default	Expression too large to display	2597

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*2^(1/2)*(cos(d*x+c)/sin(d*x+c))^(1/2)*((a*cos(d*x+c)+b*sin(d*x+c))/cos
(d*x+c))^(1/2)*(((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-
b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*(((a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x
+c)+a*cos(d*x+c)-a)/(a^2+b^2)^(1/2)/sin(d*x+c))^(1/2)*(a*(-1+cos(d*x+c))/(-
b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*(I*EllipticPi((((a^2+b^2)^(1/2)*sin(d*
x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),-(
-b+(a^2+b^2)^(1/2))/(I*a-(a^2+b^2)^(1/2)+b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2
))/(a^2+b^2)^(1/2))^(1/2))*a*b^2-I*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-
b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^
2+b^2)^(1/2))/(I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^
2+b^2)^(1/2))^(1/2))*a*b^2-I*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(
d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)
^(1/2))/(I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)
^(1/2))^(1/2))*a^3+I*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a
*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),-(-b+(a^2+b^2)^(1/2))
/(I*a-(a^2+b^2)^(1/2)+b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))
^(1/2))*a^3-I*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*
x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),-(-b+(a^2+b^2)^(1/2))/(I*a-(
a^2+b^2)^(1/2)+b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))
*a*b*(a^2+b^2)^(1/2)+I*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)
-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2)
)/(I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2)
```

```
)^(1/2))*a*b*(a^2+b^2)^(1/2)+2*EllipticF((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin
(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),1/2*2^(1/2)*
((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*(a^2+b^2)^(1/2)*a^2+4*Ellipti
cF((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^
(1/2))/sin(d*x+c))^(1/2),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))
^(1/2))*(a^2+b^2)^(1/2)*b^2-4*EllipticF((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(
d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),1/2*2^(1/2)*(
(-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a^2*b-4*EllipticF((((a^2+b^2)^
(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+
c))^(1/2),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*b^3-Ell
ipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+
b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^2+b^2)^(1/2)-b),
1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a^2*(a^2+b^2)^(1/
2)-2*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(
-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^2+b^2)^(
1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*b^2*(a^2+
b^2)^(1/2)+2*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x
+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^
2+b^2)^(1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a
^2*b+2*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)
/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^2+b^2)
^(1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*b^3-Ell
ipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+
b^2)^(1/2))/sin(d*x+c))^(1/2),-(-b+(a^2+b^2)^(1/2))/(I*a-(a^2+b^2)^(1/2)+b)
,1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a^2*(a^2+b^2)^(1
/2)-2*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/
(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),-(-b+(a^2+b^2)^(1/2))/(I*a-(a^2+b^2)
^(1/2)+b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*b^2*(a^
2+b^2)^(1/2)+2*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d
*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),-(-b+(a^2+b^2)^(1/2))/(I*a-
(a^2+b^2)^(1/2)+b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2)
)*a^2*b+2*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)
+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),-(-b+(a^2+b^2)^(1/2))/(I*a-(a^2+
b^2)^(1/2)+b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*b^3
)*sin(d*x+c)^2/(-1+cos(d*x+c))/(a*cos(d*x+c)+b*sin(d*x+c))/(I*a+(a^2+b^2)^(
1/2)-b)/(I*a-(a^2+b^2)^(1/2)+b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*tan(d*x + c) + a)*sqrt(cot(d*x + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\tan{(c + dx)}}\,\sqrt{\cot{(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(1/2)*(a+b*tan(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*tan(c + d*x))*sqrt(cot(c + d*x)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cot\left(c+d\,x\right)}\,\sqrt{a+b\,\tan\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(1/2),x)
[Out] int(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(1/2), x)
```

3.844 $\int \frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{\cot(c+dx)}}\,dx$

Optimal. Leaf size=211

$$\frac{\sqrt{ia-b}\ \mathrm{ArcTan}\left(\frac{\sqrt{ia-b}\ \sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\ \sqrt{\tan(c+dx)}}{d}+\frac{2\sqrt{b}\ \tanh^{-1}\left(\frac{\sqrt{b}\ \sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\cdots}{\cdots}$$

```
[Out] arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*(I*a-b)^(1/2)
*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+2*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+
b*tan(d*x+c))^(1/2))*b^(1/2)*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-arctanh((I
*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*(I*a+b)^(1/2)*cot(d*x+
c)^(1/2)*tan(d*x+c)^(1/2)/d
```

Rubi [A]

time = 0.44, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4326, 3656, 920, 65, 223, 212, 6857, 95, 211, 214}

$$\frac{\sqrt{-b+ia}\ \sqrt{\tan(c+dx)}\ \sqrt{\cot(c+dx)}\ \mathrm{ArcTan}\left(\frac{\sqrt{-b+ia}\ \sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{2\sqrt{b}\ \sqrt{\tan(c+dx)}\ \sqrt{\cot(c+dx)}\ \tanh^{-1}\left(\frac{\sqrt{b}\ \sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{\sqrt{b+ia}\ \sqrt{\tan(c+dx)}\ \sqrt{\cot(c+dx)}\ \tanh^{-1}\left(\frac{\sqrt{b+ia}\ \sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Tan[c + d*x]]/Sqrt[Cot[c + d*x]],x]
```

```
[Out] (Sqrt[I*a - b]*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c +
 d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d + (2*Sqrt[b]*ArcTanh[(Sqrt
[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[T
an[c + d*x]])/d - (Sqrt[I*a + b]*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])
/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 920

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Dist[e*(g/c), Int[(d + e*x)^(m - 1)*(f + g*x)^(n - 1), x
], x] + Dist[1/c, Int[Simp[c*d*f - a*e*g + (c*e*f + c*d*g)*x, x]*(d + e*x)^
(m - 1)*((f + g*x)^(n - 1)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[m, 0
] && GtQ[n, 0]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{\cot(c+dx)}}\,dx &= \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}\,dx \\
&= \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\mathrm{Subst}\left(\int \frac{\sqrt{x}\,\sqrt{a+bx}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d} \\
&= \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\mathrm{Subst}\left(\int \frac{-b+ax}{\sqrt{x}\,\sqrt{a+bx}\,(1+x^2)}\,dx, x, \tan(c+\right.}{d} \\
&= \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\mathrm{Subst}\left(\int \left(\frac{-a-ib}{2(i-x)\sqrt{x}\,\sqrt{a+bx}} + \frac{a-}{2\sqrt{x}\,(i+x)}\right.\right.}{d} \\
&= \frac{\left((-a-ib)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\mathrm{Subst}\left(\int \frac{1}{(i-x)\sqrt{x}\,\sqrt{a+bx}}\,dx, x,\right.}{2d} \\
&= \frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d} + \frac{((-a}{} \\
&= \frac{\sqrt{ia-b}\,\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d} +
\end{aligned}
$$

Mathematica [A]

time = 0.86, size = 209, normalized size = 0.99

$$
\frac{\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\left(\sqrt[4]{-1}\left(\sqrt{-a+ib}\,\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+\sqrt{a+ib}\,\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\right)+\frac{2\sqrt{a}\,\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{1+\frac{b\tan(c+dx)}{a}}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Tan[c + d*x]]/Sqrt[Cot[c + d*x]],x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-1)^(1/4)*(Sqrt[-a + I*b]*ArcTan[(
(-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + S
```

```
qrt[a + I*b]*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a +
b*Tan[c + d*x]]]) + (2*Sqrt[a]*Sqrt[b]*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])
/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a])/Sqrt[a + b*Tan[c + d*x]]))/d
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 35.28, size = 5190, normalized size = 24.60

method	result	size
default	Expression too large to display	5190

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(1/2)/cot(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)/cot(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*tan(d*x + c) + a)/sqrt(cot(d*x + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)/cot(d*x+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\tan\left(c + dx\right)}}{\sqrt{\cot\left(c + dx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(1/2)/cot(d*x+c)**(1/2),x)
[Out] Integral(sqrt(a + b*tan(c + d*x))/sqrt(cot(c + d*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)/cot(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{\cot(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^(1/2)/cot(c + d*x)^(1/2),x)
```

```
[Out] int((a + b*tan(c + d*x))^(1/2)/cot(c + d*x)^(1/2), x)
```

3.845 $\int \frac{\sqrt{a+b\tan(c+dx)}}{\cot^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=244

$$\frac{i\sqrt{ia-b}\,\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right.}{}$$

```
[Out] I*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*(I*a-b)^(1/
2)*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+a*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(
a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d/b^(1/2)+I*arctan
h((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*(I*a+b)^(1/2)*cot(
d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+(a+b*tan(d*x+c))^(1/2)/d/cot(d*x+c)^(1/2)
```

Rubi [A]
time = 0.52, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {4326, 3651, 3736, 6857, 65, 223, 212, 924, 95, 211, 214}

$$\frac{i\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{\sqrt{a+b\tan(c+dx)}}{d\sqrt{\cot(c+dx)}}+\frac{a\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{b}\,d}+\frac{i\sqrt{b+ia}\,\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Tan[c + d*x]]/Cot[c + d*x]^(3/2),x]
```

```
[Out] (I*Sqrt[I*a - b]*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
 + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d + (a*ArcTanh[(Sqrt[b]*Sq
rt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c +
 d*x]])/(Sqrt[b]*d) + (I*Sqrt[I*a + b]*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c +
d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d +
 Sqrt[a + b*Tan[c + d*x]]/(d*Sqrt[Cot[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
```

```
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 3651

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m - 1)*((c +
d*Tan[e + f*x])^n/(f*(m + n - 1))), x] + Dist[1/(m + n - 1), Int[(a + b*Tan
[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a^2*c*(m + n - 1) - b*
(b*c*(m - 1) + a*d*n) + (2*a*b*c + a^2*d - b^2*d)*(m + n - 1)*Tan[e + f*x]
+ b*(b*c*n + a*d*(2*m + n - 2))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
 0] && GtQ[m, 1] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\tan(c+dx)}}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}\,dx$$

$$= \frac{\sqrt{a+b\tan(c+dx)}}{d\sqrt{\cot(c+dx)}} + \left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{-\frac{a}{2}-b\tan(c+d\ldots}{\sqrt{\tan(c+dx)}\ \ldots}$$

$$= \frac{\sqrt{a+b\tan(c+dx)}}{d\sqrt{\cot(c+dx)}} + \frac{\left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{-\frac{a}{2}-bx\ldots}{\sqrt{x}\sqrt{a+\ldots}}\right.}{d}$$

$$= \frac{\sqrt{a+b\tan(c+dx)}}{d\sqrt{\cot(c+dx)}} + \frac{\left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int\left(\frac{a}{2\sqrt{x}\sqrt{a\ldots}}\right.\right.}{d}$$

$$= \frac{\sqrt{a+b\tan(c+dx)}}{d\sqrt{\cot(c+dx)}} - \frac{\left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{\sqrt{x}\,(1+x^2)}\ldots\right.}{d}$$

$$= \frac{\sqrt{a+b\tan(c+dx)}}{d\sqrt{\cot(c+dx)}} - \frac{\left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int\left(\frac{i}{2(i-x)\sqrt{x}\ldots}\right.\right.}{\ldots}$$

$$= \frac{\sqrt{a+b\tan(c+dx)}}{d\sqrt{\cot(c+dx)}} + \frac{\left(a\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,\ldots\right.}{d}$$

$$= \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{b}\,d} + \frac{\sqrt{a+b\tan\ldots}}{d\sqrt{\cot\ldots}}$$

$$= \frac{i\sqrt{ia-b}\,\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} + \ldots$$

Mathematica [A]
time = 0.79, size = 261, normalized size = 1.07

$$\frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{a+b\tan(c+dx)}+\sqrt{b}\sqrt{1+\frac{b\tan(c+dx)}{a}}\left(-(-1)^{3/4}\sqrt{-a+ib}\,\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+(-1)^{3/4}\sqrt{a+ib}\,\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}\right)\right)}{\sqrt{b}\,d\sqrt{1+\frac{b\tan(c+dx)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Tan[c + d*x]]/Cot[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(Sqrt[a]*ArcSinh[(Sqrt[b]*Sqrt[Tan[c
 + d*x]])/Sqrt[a]]*Sqrt[a + b*Tan[c + d*x]] + Sqrt[b]*Sqrt[1 + (b*Tan[c + d
```

```
*x])/a]*(-((-1)^(3/4)*Sqrt[-a + I*b]*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt
[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]) + (-1)^(3/4)*Sqrt[a + I*b]*ArcTa
n[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] +
 Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]])))/(Sqrt[b]*d*Sqrt[1 + (b*Tan[
c + d*x])/a])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 37.31, size = 12857, normalized size = 52.69

method	result	size
default	Expression too large to display	12857

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(1/2)/cot(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)/cot(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(d*x + c) + a)/cot(d*x + c)^(3/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)/cot(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \tan{\left(c + dx \right)}}}{\cot^{\frac{3}{2}}{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(1/2)/cot(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(c + d*x))/cot(c + d*x)**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)/cot(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\tan\left(c+d\,x\right)}}{{\cot\left(c+d\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^(1/2)/cot(c + d*x)^(3/2),x)
```

```
[Out] int((a + b*tan(c + d*x))^(1/2)/cot(c + d*x)^(3/2), x)
```

3.846 $\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=306

$$-\frac{(ia-b)^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d}-\frac{(ia+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ia+b}}{\sqrt{a+}}\right.}{}$$

```
[Out] -(I*a-b)^(3/2)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2)
)*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-(I*a+b)^(3/2)*arctanh((I*a+b)^(1/2)*t
an(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d
+2/105*(35*a^2-3*b^2)*cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2)/a/d-16/35*b*c
ot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(1/2)/d-2/7*a*cot(d*x+c)^(7/2)*(a+b*tan(d*
x+c))^(1/2)/d+4/105*b*(70*a^2+3*b^2)*cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2
)/a^2/d
```

Rubi [A]
time = 0.85, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4326, 3648, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(35a^2-3b^2)\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{105ad}+\frac{4b(70a^2+3b^2)\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{105a^2d}-\frac{(-b+ia)^{3/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\mathrm{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{2a\cot^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{7d}-\frac{16b\cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{35d}-\frac{(b+ia)^{3/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(9/2)*(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] -(((I*a - b)^(3/2)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan
[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d) - ((I*a + b)^(3/2)*Ar
cTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot
[c + d*x]]*Sqrt[Tan[c + d*x]])/d + (4*b*(70*a^2 + 3*b^2)*Sqrt[Cot[c + d*x]]
*Sqrt[a + b*Tan[c + d*x]])/(105*a^2*d) + (2*(35*a^2 - 3*b^2)*Cot[c + d*x]^(
3/2)*Sqrt[a + b*Tan[c + d*x]])/(105*a*d) - (16*b*Cot[c + d*x]^(5/2)*Sqrt[a
+ b*Tan[c + d*x]])/(35*d) - (2*a*Cot[c + d*x]^(7/2)*Sqrt[a + b*Tan[c + d*x]
])/(7*d)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
```

```
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{(a+b\tan(c+dx))^{3/2}}{\tan^{\frac{9}{2}}(c+dx)}\,dx$$

$$= -\frac{2a\cot^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{7d} - \frac{1}{7}\Big(2\sqrt{\cot(c+dx)}\,\sqrt{\text{ta}}$$

$$= -\frac{16b\cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{35d} - \frac{2a\cot^{\frac{7}{2}}(c+dx)\sqrt{a+}}{7d}$$

$$= \frac{2(35a^2-3b^2)\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{105ad} - \frac{16b\cot^{\frac{5}{2}}(c+d}{}$$

$$= \frac{4b(70a^2+3b^2)\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{105a^2d} + \frac{2(35a^2-3}{}$$

$$= \frac{4b(70a^2+3b^2)\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{105a^2d} + \frac{2(35a^2-3}{}$$

$$= \frac{4b(70a^2+3b^2)\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{105a^2d} + \frac{2(35a^2-3}{}$$

$$= \frac{4b(70a^2+3b^2)\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{105a^2d} + \frac{2(35a^2-3}{}$$

$$= -\frac{(ia-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\text{t}}}{d}$$

Mathematica [A]

time = 3.30, size = 255, normalized size = 0.83

$$\frac{\cot^{\frac{7}{2}}(c+dx)\left(105\sqrt[4]{-1}\,a^2\sqrt{-a+ib}\,(ia+b)\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\tan^{\frac{7}{2}}(c+dx)-105(-1)^{3/4}a^2(a+ib)^{3/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\tan^{\frac{7}{2}}(c+dx)+2\sqrt{a+b\tan(c+dx)}\,(-15a^3-24a^2b\tan(c+dx)+a(35a^2-3b^2)\tan^2(c+dx)+2b(70a^2+3b^2)\tan^3(c+dx))\right)}{105a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(9/2)*(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] (Cot[c + d*x]^(7/2)*(105*(-1)^(1/4)*a^2*Sqrt[-a + I*b]*(I*a + b)*ArcTan[((-
1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Tan[c
 + d*x]^(7/2) - 105*(-1)^(3/4)*a^2*(a + I*b)^(3/2)*ArcTan[((-1)^(1/4)*Sqrt[
a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Tan[c + d*x]^(7/2) +
 2*Sqrt[a + b*Tan[c + d*x]]*(-15*a^3 - 24*a^2*b*Tan[c + d*x] + a*(35*a^2 -
3*b^2)*Tan[c + d*x]^2 + 2*b*(70*a^2 + 3*b^2)*Tan[c + d*x]^3)))/(105*a^2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 35.28, size = 25460, normalized size = 83.20

method	result	size
default	Expression too large to display	25460

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^(9/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(9/2)*(a+b*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^(9/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c+d\,x)^{9/2}\,{\left(a+b\,\tan(c+d\,x)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(9/2)*(a + b*tan(c + d*x))^(3/2),x)
```

```
[Out] int(cot(c + d*x)^(9/2)*(a + b*tan(c + d*x))^(3/2), x)
```

3.847 $\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=264

$$-\frac{i(ia-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d}+\frac{i(ia+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ia}}{\sqrt{}}\right.}{}$$

```
[Out] -I*(I*a-b)^(3/2)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/
2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+I*(I*a+b)^(3/2)*arctanh((I*a+b)^(1/
2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/
2)/d-4/5*b*cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2)/d-2/5*a*cot(d*x+c)^(5/2)
*(a+b*tan(d*x+c))^(1/2)/d+2/5*(5*a^2-b^2)*cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))
^(1/2)/a/d
```

Rubi [A]
time = 0.66, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4326, 3648, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(5a^2-b^2)\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{5ad}-\frac{i(-b+ia)^{3/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{2a\cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{5d}-\frac{4b\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{5d}+\frac{i(b+ia)^{3/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(7/2)*(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-I)*(I*a - b)^(3/2)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*
Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d + (I*(I*a + b)^(3/2
)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt
[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d + (2*(5*a^2 - b^2)*Sqrt[Cot[c + d*x]]*
Sqrt[a + b*Tan[c + d*x]])/(5*a*d) - (4*b*Cot[c + d*x]^(3/2)*Sqrt[a + b*Tan[
c + d*x]])/(5*d) - (2*a*Cot[c + d*x]^(5/2)*Sqrt[a + b*Tan[c + d*x]])/(5*d)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
```

```
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{(a+b\tan(c+dx))^{3/2}}{\tan^{\frac{7}{2}}(c+dx)}\,dx$$

$$= -\frac{2a\cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{5d} - \frac{1}{5}\Big(2\sqrt{\cot(c+dx)}\,\sqrt{t$$

$$= -\frac{4b\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{5d} - \frac{2a\cot^{\frac{5}{2}}(c+dx)\sqrt{a+}}{5d}$$

$$= \frac{2(5a^2-b^2)\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{5ad} - \frac{4b\cot^{\frac{3}{2}}(c+d}{}$$

$$= \frac{2(5a^2-b^2)\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{5ad} - \frac{4b\cot^{\frac{3}{2}}(c+d}{}$$

$$= \frac{2(5a^2-b^2)\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{5ad} - \frac{4b\cot^{\frac{3}{2}}(c+d}{}$$

$$= \frac{2(5a^2-b^2)\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{5ad} - \frac{4b\cot^{\frac{3}{2}}(c+d}{}$$

$$= -\frac{i(ia-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,+}{d}$$

Mathematica [A]
time = 1.58, size = 219, normalized size = 0.83

$$\frac{\cot^{\frac{5}{2}}(c+dx)\left(-5\sqrt[4]{-1}\,a(-a+ib)^{3/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\tan^{\frac{5}{2}}(c+dx)+5\sqrt[4]{-1}\,a(a+ib)^{3/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\tan^{\frac{5}{2}}(c+dx)+2\sqrt{a+b\tan(c+dx)}\,(-a^2-2ab\tan(c+dx)+(5a^2-b^2)\tan^2(c+dx))\right)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(7/2)*(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] (Cot[c + d*x]^(5/2)*(-5*(-1)^(1/4)*a*(-a + I*b)^(3/2)*ArcTan[((-1)^(1/4)*Sq
rt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Tan[c + d*x]^(5/
2) + 5*(-1)^(1/4)*a*(a + I*b)^(3/2)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[T
an[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Tan[c + d*x]^(5/2) + 2*Sqrt[a + b*T
an[c + d*x]]*(-a^2 - 2*a*b*Tan[c + d*x] + (5*a^2 - b^2)*Tan[c + d*x]^2)))/(
5*a*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 35.01, size = 23629, normalized size = 89.50

method	result	size
default	Expression too large to display	23629

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^(7/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(7/2)*(a+b*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c+d\,x)^{7/2}\,\left(a+b\tan(c+d\,x)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(7/2)*(a + b*tan(c + d*x))^(3/2),x)
```

```
[Out] int(cot(c + d*x)^(7/2)*(a + b*tan(c + d*x))^(3/2), x)
```

3.848 $\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=213

$$\frac{(ia-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d}+\frac{(ia+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ia+b}}{\sqrt{a+b}}\right.}{}$$

```
[Out] (I*a-b)^(3/2)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))
*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+(I*a+b)^(3/2)*arctanh((I*a+b)^(1/2)*ta
n(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-
2/3*a*cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2)/d-8/3*b*cot(d*x+c)^(1/2)*(a+b
*tan(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.51, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4326, 3648, 3730, 3697, 3696, 95, 209, 212}

$$\frac{(-b+ia)^{3/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{2a\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}-\frac{8b\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{3d}+\frac{(b+ia)^{3/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((I*a - b)^(3/2)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
 + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d + ((I*a + b)^(3/2)*ArcTa
nh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c
+ d*x]]*Sqrt[Tan[c + d*x]])/d - (8*b*Sqrt[Cot[c + d*x]]*Sqrt[a + b*Tan[c +
d*x]])/(3*d) - (2*a*Cot[c + d*x]^(3/2)*Sqrt[a + b*Tan[c + d*x]])/(3*d)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
```

```
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$
\begin{aligned}
\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}\,dx &= \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{(a+b\tan(c+dx))^{3/2}}{\tan^{\frac{5}{2}}(c+dx)}\,dx \\
&= -\frac{2a\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}-\frac{1}{3}\Big(2\sqrt{\cot(c+dx)}\,\sqrt{\mathrm{ta}} \\
&= -\frac{8b\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{3d}-\frac{2a\cot^{\frac{3}{2}}(c+dx)\sqrt{a+}}{3d} \\
&= -\frac{8b\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{3d}-\frac{2a\cot^{\frac{3}{2}}(c+dx)\sqrt{a+}}{3d} \\
&= -\frac{8b\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{3d}-\frac{2a\cot^{\frac{3}{2}}(c+dx)\sqrt{a+}}{3d} \\
&= -\frac{8b\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{3d}-\frac{2a\cot^{\frac{3}{2}}(c+dx)\sqrt{a+}}{3d} \\
&= \frac{(ia-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\mathrm{ta}}}{d}
\end{aligned}
$$

Mathematica [A]
time = 1.27, size = 196, normalized size = 0.92

$$
\frac{\cot^{\frac{3}{2}}(c+dx)\left(-3\sqrt[4]{-1}\,\sqrt{-a+ib}\,(ia+b)\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\tan^{\frac{3}{2}}(c+dx)+3(-1)^{3/4}(a+ib)^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\tan^{\frac{3}{2}}(c+dx)-2\sqrt{a+b\tan(c+dx)}\,(a+4b\tan(c+dx))\right)}{3d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] (Cot[c + d*x]^(3/2)*(-3*(-1)^(1/4)*Sqrt[-a + I*b]*(I*a + b)*ArcTan[((-1)^(1
/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Tan[c + d*
```

```
x]^(3/2) + 3*(-1)^(3/4)*(a + I*b)^(3/2)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sq
rt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Tan[c + d*x]^(3/2) - 2*Sqrt[a +
 b*Tan[c + d*x]]*(a + 4*b*Tan[c + d*x])))/(3*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 33.80, size = 12725, normalized size = 59.74

method	result	size
default	Expression too large to display	12725

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^(5/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(5/2)*(a+b*tan(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c+d\,x)^{5/2}\,{\left(a+b\tan(c+d\,x)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))^(3/2),x)
[Out] int(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))^(3/2), x)
```

3.849 $\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=185

$$\frac{i(ia-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d}-\frac{i(ia+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ia+}}{\sqrt{a-}}\right.}{}$$

```
[Out] I*(I*a-b)^(3/2)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2
))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-I*(I*a+b)^(3/2)*arctanh((I*a+b)^(1/2
)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2
)/d-2*a*cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.38, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4326, 3648, 3697, 3696, 95, 209, 212}

$$\frac{i(-b+ia)^{3/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{2a\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{d}-\frac{i(b+ia)^{3/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] (I*(I*a - b)^(3/2)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan
[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d - (I*(I*a + b)^(3/2)*A
rcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Co
t[c + d*x]]*Sqrt[Tan[c + d*x]])/d - (2*a*Sqrt[Cot[c + d*x]]*Sqrt[a + b*Tan[
c + d*x]])/d
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}\,dx = \left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{(a+b\tan(c+dx))^{3/2}}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

$$= -\frac{2a\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{d} - \left(2\sqrt{\cot(c+dx)}\sqrt{t\right.$$

$$= -\frac{2a\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{d} + \frac{1}{2}\left(i(a-ib)^2\sqrt{\cot(c\right.$$

$$= -\frac{2a\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{d} + \frac{\left(i(a-ib)^2\sqrt{\cot(c+\right.}{}$$

$$= -\frac{2a\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{d} + \frac{\left(i(a-ib)^2\sqrt{\cot(c+\right.}{}$$

$$= \frac{i(ia-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{}}{d}$$

Mathematica [A]
time = 0.50, size = 175, normalized size = 0.95

$$\frac{\sqrt{\cot(c+dx)}\left(\sqrt[4]{-1}(-a+ib)^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\tan(c+dx)}-\sqrt[4]{-1}(a+ib)^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\tan(c+dx)}-2a\sqrt{a+b\tan(c+dx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*((-1)^(1/4)*(-a + I*b)^(3/2)*ArcTan[((-1)^(1/4)*Sqrt[-a
 + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Tan[c + d*x]] -
(-1)^(1/4)*(a + I*b)^(3/2)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*
x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Tan[c + d*x]] - 2*a*Sqrt[a + b*Tan[c +
d*x]]))/d
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 32.69, size = 11638, normalized size = 62.91

method	result	size
default	Expression too large to display	11638

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^(3/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(3/2)*(a+b*tan(d*x+c))**(3/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^{3/2}\,{\left(a+b\tan(c+d\,x)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(3/2),x)
[Out] int(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(3/2), x)
```

3.850 $\int \sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=212

$$-\frac{(ia-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d}+\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(}}{\sqrt{a+b\tan}}\right.}{}$$

```
[Out] -(I*a-b)^(3/2)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2)
)*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+2*b^(3/2)*arctanh(b^(1/2)*tan(d*x+c)^
(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-(I*a+b)^(
3/2)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x
+c)^(1/2)*tan(d*x+c)^(1/2)/d
```

Rubi [A]

time = 0.56, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4326, 3656, 924, 65, 223, 212, 6857, 95, 211, 214}

$$-\frac{(-b+ia)^{3/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{2b^{3/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{(b+ia)^{3/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] -(((I*a - b)^(3/2)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan
[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d) + (2*b^(3/2)*ArcTanh[
(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*S
qrt[Tan[c + d*x]])/d - ((I*a + b)^(3/2)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c +
 d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

[n, 0]

Rubi steps

$$\int \sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{(a+b\tan(c+dx))^{3/2}}{\sqrt{\tan(c+dx)}}\,dx$$

$$= \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\mathrm{Subst}\left(\int \frac{(a+bx)^{3/2}}{\sqrt{x}\,(1+x^2)}\,dx, x, \mathrm{ta}\right.}{d}$$

$$= \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\mathrm{Subst}\left(\int \left(\frac{b^2}{\sqrt{x}\,\sqrt{a+bx}} + \right.\right.}{d}$$

$$= \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\mathrm{Subst}\left(\int \frac{a^2-b^2+2abx}{\sqrt{x}\,\sqrt{a+bx}\,(1+x^2}\right.}{d}$$

$$= \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\mathrm{Subst}\left(\int \left(\frac{-2ab+i(a^2-b^2)}{2(i-x)\sqrt{x}\,\sqrt{a+b}}\right.\right.}{d}$$

$$= \frac{\left(i(a-ib)^2\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\mathrm{Subst}\left(\int \frac{}{\sqrt{x}\,(i+x)}\right.}{2d}$$

$$= \frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c}}{d}$$

$$= -\frac{(ia-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}}{d}$$

Mathematica [A]
time = 0.60, size = 237, normalized size = 1.12

$$\frac{\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\left(\sqrt[4]{-1}\left(\sqrt{-a+ib}\,(ia+b)\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+\sqrt{a+ib}\,(-ia+b)\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\right)\sqrt{a+b\tan(c+dx)}+2\sqrt{a}\,b^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{1+\frac{b\tan(c+dx)}{a}}\right)}{d\sqrt{a+b\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-1)^(1/4)*(Sqrt[-a + I*b]*(I*a + b
)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c +
d*x]]] + Sqrt[a + I*b]*((-I)*a + b)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[T
an[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])*Sqrt[a + b*Tan[c + d*x]] + 2*Sqrt[
```

```
a]*b^(3/2)*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 + (b*Tan[c
+ d*x])/a]))/(d*Sqrt[a + b*Tan[c + d*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 34.23, size = 6349, normalized size = 29.95

method	result	size
default	Expression too large to display	6349

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^(3/2)*sqrt(cot(d*x + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx))^{\frac{3}{2}} \sqrt{\cot(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(1/2)*(a+b*tan(d*x+c))**(3/2),x)
[Out] Integral((a + b*tan(c + d*x))**(3/2)*sqrt(cot(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(3/2)*sqrt(cot(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cot (c+d\,x)}\ (a+b\tan(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(3/2),x)
```

```
[Out] int(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(3/2), x)
```

3.851 $\int \frac{(a+b\tan(c+dx))^{3/2}}{\sqrt{\cot(c+dx)}}\,dx$

Optimal. Leaf size=246

$$-\frac{i(ia-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d}+\frac{3a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan}}{\sqrt{a+b\tan}}\right.}{}$$

```
[Out] -I*(I*a-b)^(3/2)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/
2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+I*(I*a+b)^(3/2)*arctanh((I*a+b)^(1/
2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/
2)/d+3*a*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*b^(1/2)*c
ot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+b*(a+b*tan(d*x+c))^(1/2)/d/cot(d*x+c)^(1
/2)
```

Rubi [A]
time = 0.92, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4326, 3651, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$-\frac{i(-b+ia)^{3/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{b\sqrt{a+b\tan(c+dx)}}{d\sqrt{\cot(c+dx)}}+\frac{3a\sqrt{b}\,\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{i(b+ia)^{3/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^(3/2)/Sqrt[Cot[c + d*x]],x]
```

```
[Out] ((-I)*(I*a - b)^(3/2)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*
Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d + (3*a*Sqrt[b]*ArcT
anh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x
]]*Sqrt[Tan[c + d*x]])/d + (I*(I*a + b)^(3/2)*ArcTanh[(Sqrt[I*a + b]*Sqrt[T
an[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x
]])/d + (b*Sqrt[a + b*Tan[c + d*x]])/(d*Sqrt[Cot[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3651

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m - 1)*((c +
d*Tan[e + f*x])^n/(f*(m + n - 1))), x] + Dist[1/(m + n - 1), Int[(a + b*Tan
[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a^2*c*(m + n - 1) - b*
(b*c*(m - 1) + a*d*n) + (2*a*b*c + a^2*d - b^2*d)*(m + n - 1)*Tan[e + f*x]
+ b*(b*c*n + a*d*(2*m + n - 2))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
 0] && GtQ[m, 1] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(c+dx))^{3/2}}{\sqrt{\cot(c+dx)}}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{3/2}\,dx$$

$$= \frac{b\sqrt{a+b\tan(c+dx)}}{d\sqrt{\cot(c+dx)}} + \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{-\frac{ab}{2}+(a^2-b^2}{\sqrt{\tan(c+}}$$

$$= \frac{b\sqrt{a+b\tan(c+dx)}}{d\sqrt{\cot(c+dx)}} + \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\mathrm{Subst}\left(\int \frac{-\frac{ab}{2}+(a^2}{\sqrt{x}\,\sqrt{a}}\right.}{d}$$

$$= \frac{b\sqrt{a+b\tan(c+dx)}}{d\sqrt{\cot(c+dx)}} + \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\mathrm{Subst}\left(\int \left(\frac{}{2\sqrt{x}\,}\right.\right.}{}$$

$$= \frac{b\sqrt{a+b\tan(c+dx)}}{d\sqrt{\cot(c+dx)}} - \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\mathrm{Subst}\left(\int \frac{2ab-}{\sqrt{x}\,\sqrt{a}}\right.}{d}$$

$$= \frac{b\sqrt{a+b\tan(c+dx)}}{d\sqrt{\cot(c+dx)}} - \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\mathrm{Subst}\left(\int \left(\frac{a}{2(i-x)}\right.\right.}{}$$

$$= \frac{b\sqrt{a+b\tan(c+dx)}}{d\sqrt{\cot(c+dx)}} + \frac{\left((a-ib)^2\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\mathrm{Subst}\left(\int\right.}{2d}$$

$$= \frac{3a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d} + \frac{b\sqrt{}}{d}$$

$$= -\frac{i(ia-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx}}{d}$$

Mathematica [A]
time = 2.10, size = 292, normalized size = 1.19

$$\frac{\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\left(3\sqrt{a}\,\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)(a+b\tan(c+dx))+\sqrt{1+\frac{b\tan(c+dx)}{a}}\left(-\sqrt[4]{-1}\,(-a+ib)^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{a+b\tan(c+dx)}+\sqrt[4]{-1}\,(a+ib)^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{a+b\tan(c+dx)}+b\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))\right)\right)}{d\sqrt{a+b\tan(c+dx)}\,\sqrt{1+\frac{b\tan(c+dx)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^(3/2)/Sqrt[Cot[c + d*x]],x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(3*Sqrt[a]*Sqrt[b]*ArcSinh[(Sqrt[b]*
Sqrt[Tan[c + d*x]])/Sqrt[a]]*(a + b*Tan[c + d*x]) + Sqrt[1 + (b*Tan[c + d*x
])/a]*(-((-1)^(1/4)*(-a + I*b)^(3/2)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt
```

```
[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]]) + (-1)^
(1/4)*(a + I*b)^(3/2)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/
Sqrt[a + b*Tan[c + d*x]]]*Sqrt[a + b*Tan[c + d*x]] + b*Sqrt[Tan[c + d*x]]*(
a + b*Tan[c + d*x]])))/(d*Sqrt[a + b*Tan[c + d*x]]*Sqrt[1 + (b*Tan[c + d*x]
)/a])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 38.46, size = 13888, normalized size = 56.46

method	result	size
default	Expression too large to display	13888

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(3/2)/cot(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(3/2)/cot(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(3/2)/sqrt(cot(d*x + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(3/2)/cot(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan{(c + dx)})^{\frac{3}{2}}}{\sqrt{\cot{(c + dx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(3/2)/cot(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**(3/2)/sqrt(cot(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(3/2)/cot(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\tan(c+d\,x))^{3/2}}{\sqrt{\cot\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^(3/2)/cot(c + d*x)^(1/2),x)
```

```
[Out] int((a + b*tan(c + d*x))^(3/2)/cot(c + d*x)^(1/2), x)
```

3.852 $\int \frac{(a+b\tan(c+dx))^{3/2}}{\cot^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=286

$$\frac{(ia-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d}+\frac{(3a^2-8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right.}{}$$

```
[Out] (I*a-b)^(3/2)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))
*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+(I*a+b)^(3/2)*arctanh((I*a+b)^(1/2)*ta
n(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+
1/4*(3*a^2-8*b^2)*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*
cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d/b^(1/2)+3/4*a*(a+b*tan(d*x+c))^(1/2)/d/
cot(d*x+c)^(1/2)+1/2*(a+b*tan(d*x+c))^(3/2)/d/cot(d*x+c)^(1/2)
```

Rubi [A]
time = 1.02, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {4326, 3651, 3728, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{(3a^2-8b^2)\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{4\sqrt{b}\,d}+\frac{(-b+ia)^{3/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{(a+b\tan(c+dx))^{3/2}}{2d\sqrt{\cot(c+dx)}}+\frac{3a\sqrt{a+b\tan(c+dx)}}{4d\sqrt{\cot(c+dx)}}+\frac{(b+ia)^{3/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^(3/2)/Cot[c + d*x]^(3/2),x]
```

```
[Out] ((I*a - b)^(3/2)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
 + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d + ((3*a^2 - 8*b^2)*ArcTa
nh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]
]*Sqrt[Tan[c + d*x]])/(4*Sqrt[b]*d) + ((I*a + b)^(3/2)*ArcTanh[(Sqrt[I*a +
b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Ta
n[c + d*x]])/d + (3*a*Sqrt[a + b*Tan[c + d*x]])/(4*d*Sqrt[Cot[c + d*x]]) +
(a + b*Tan[c + d*x])^(3/2)/(2*d*Sqrt[Cot[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3651

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m - 1)*((c +
d*Tan[e + f*x])^n/(f*(m + n - 1))), x] + Dist[1/(m + n - 1), Int[(a + b*Tan
[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a^2*c*(m + n - 1) - b*
(b*c*(m - 1) + a*d*n) + (2*a*b*c + a^2*d - b^2*d)*(m + n - 1)*Tan[e + f*x]
+ b*(b*c*n + a*d*(2*m + n - 2))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
 0] && GtQ[m, 1] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
```

```
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(c+dx))^{3/2}}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}\,dx$$

$$= \frac{(a+b\tan(c+dx))^{3/2}}{2d\sqrt{\cot(c+dx)}} + \frac{1}{2}\left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{\sqrt{a+b\tan(c}}{}$$

$$= \frac{3a\sqrt{a+b\tan(c+dx)}}{4d\sqrt{\cot(c+dx)}} + \frac{(a+b\tan(c+dx))^{3/2}}{2d\sqrt{\cot(c+dx)}} + \frac{1}{2}\left(\sqrt{\cot(c+dx)}\sqrt{\tan(\right.$$

$$= \frac{3a\sqrt{a+b\tan(c+dx)}}{4d\sqrt{\cot(c+dx)}} + \frac{(a+b\tan(c+dx))^{3/2}}{2d\sqrt{\cot(c+dx)}} + \frac{\left(\sqrt{\cot(c+dx)}\sqrt{\tan(c\right.}{}$$

$$= \frac{3a\sqrt{a+b\tan(c+dx)}}{4d\sqrt{\cot(c+dx)}} + \frac{(a+b\tan(c+dx))^{3/2}}{2d\sqrt{\cot(c+dx)}} + \frac{\left(\sqrt{\cot(c+dx)}\sqrt{\tan(c\right.}{}$$

$$= \frac{3a\sqrt{a+b\tan(c+dx)}}{4d\sqrt{\cot(c+dx)}} + \frac{(a+b\tan(c+dx))^{3/2}}{2d\sqrt{\cot(c+dx)}} - \frac{\left(\sqrt{\cot(c+dx)}\sqrt{\tan(c\right.}{}$$

$$= \frac{3a\sqrt{a+b\tan(c+dx)}}{4d\sqrt{\cot(c+dx)}} + \frac{(a+b\tan(c+dx))^{3/2}}{2d\sqrt{\cot(c+dx)}} - \frac{\left(\sqrt{\cot(c+dx)}\sqrt{\tan(c\right.}{}$$

$$= \frac{3a\sqrt{a+b\tan(c+dx)}}{4d\sqrt{\cot(c+dx)}} + \frac{(a+b\tan(c+dx))^{3/2}}{2d\sqrt{\cot(c+dx)}} - \frac{\left(i(a-ib)^2\sqrt{\cot(c+dx\right.}{}$$

$$= \frac{(3a^2-8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{4\sqrt{b}\,d} +$$

$$= \frac{(ia-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d}$$

Mathematica [A]
time = 6.00, size = 312, normalized size = 1.09

$$\frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(-4\sqrt[4]{-1}\sqrt{-a+ib}\,(ia+b)\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{a+b\tan(c+dx)}+4(-1)^{3/4}(a+ib)^{3/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{a+b\tan(c+dx)}+\frac{(3a^2-8b^2)\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)(a+b\tan(c+dx))}{\sqrt{a}\sqrt{b}\sqrt{1+\frac{b\tan(c+dx)}{a}}}+\sqrt{\tan(c+dx)}\left(5a^2+7ab\tan(c+dx)+2b^2\tan^2(c+dx)\right)\right)}{4d\sqrt{a+b\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^(3/2)/Cot[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-4*(-1)^(1/4)*Sqrt[-a + I*b]*(I*a +
 b)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
+ d*x]]]*Sqrt[a + b*Tan[c + d*x]] + 4*(-1)^(3/4)*(a + I*b)^(3/2)*ArcTan[((-
1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[a
 + b*Tan[c + d*x]] + ((3*a^2 - 8*b^2)*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/
Sqrt[a]]*(a + b*Tan[c + d*x]))/(Sqrt[a]*Sqrt[b]*Sqrt[1 + (b*Tan[c + d*x])/a
]) + Sqrt[Tan[c + d*x]]*(5*a^2 + 7*a*b*Tan[c + d*x] + 2*b^2*Tan[c + d*x]^2)
))/(4*d*Sqrt[a + b*Tan[c + d*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 35.70, size = 16676, normalized size = 58.31

method	result	size
default	Expression too large to display	16676

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(3/2)/cot(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(3/2)/cot(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(3/2)/cot(d*x + c)^(3/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(3/2)/cot(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan(c + dx))^{\frac{3}{2}}}{\cot^{\frac{3}{2}}(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(3/2)/cot(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x))**(3/2)/cot(c + d*x)**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(3/2)/cot(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\tan(c+d\,x))^{3/2}}{\cot\left(c+d\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^(3/2)/cot(c + d*x)^(3/2),x)
```

```
[Out] int((a + b*tan(c + d*x))^(3/2)/cot(c + d*x)^(3/2), x)
```

3.853 $\int \cot^{\frac{11}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=358

$$\frac{(ia-b)^{5/2}\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d}-\frac{(ia+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{ia+b}}{\sqrt{a+b}}\right.}{}$$

```
[Out] (I*a-b)^(5/2)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))
*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-(I*a+b)^(5/2)*arctanh((I*a+b)^(1/2)*ta
n(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+
2/315*b*(231*a^2-5*b^2)*cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2)/a/d+2/105*(
21*a^2-25*b^2)*cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(1/2)/d-38/63*a*b*cot(d*x+
c)^(7/2)*(a+b*tan(d*x+c))^(1/2)/d-2/9*a^2*cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))
^(1/2)/d-2/315*(315*a^4-483*a^2*b^2-10*b^4)*cot(d*x+c)^(1/2)*(a+b*tan(d*x+c
))^(1/2)/a^2/d
```

Rubi [A]

time = 1.13, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4326, 3646, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(21a^2-25b^2)\cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{105d}+\frac{2b(231a^2-5b^2)\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{315ad}-\frac{2a^2\cot^{\frac{9}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{9d}-\frac{2(315a^4-483a^2b^2-10b^4)\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{315a^2d}+\frac{(-b+ia)^{5/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{38ab\cot^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{63d}-\frac{(b+ia)^{5/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(11/2)*(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((I*a - b)^(5/2)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
 + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d - ((I*a + b)^(5/2)*ArcTa
nh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c
+ d*x]]*Sqrt[Tan[c + d*x]])/d - (2*(315*a^4 - 483*a^2*b^2 - 10*b^4)*Sqrt[Co
t[c + d*x]]*Sqrt[a + b*Tan[c + d*x]])/(315*a^2*d) + (2*b*(231*a^2 - 5*b^2)*
Cot[c + d*x]^(3/2)*Sqrt[a + b*Tan[c + d*x]])/(315*a*d) + (2*(21*a^2 - 25*b^
2)*Cot[c + d*x]^(5/2)*Sqrt[a + b*Tan[c + d*x]])/(105*d) - (38*a*b*Cot[c + d
*x]^(7/2)*Sqrt[a + b*Tan[c + d*x]])/(63*d) - (2*a^2*Cot[c + d*x]^(9/2)*Sqrt
[a + b*Tan[c + d*x]])/(9*d)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
```

```
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \cot^{\frac{11}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{(a+b\tan(c+dx))^{5/2}}{\tan^{\frac{11}{2}}(c+dx)}\,dx$$

$$= -\frac{2a^2\cot^{\frac{9}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{9d} + \frac{1}{9}\Big(2\sqrt{\cot(c+dx)}\ \sqrt{}$$

$$= -\frac{38ab\cot^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{63d} - \frac{2a^2\cot^{\frac{9}{2}}(c+dx)\sqrt{}}{9d}$$

$$= \frac{2(21a^2-25b^2)\cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{105d} - \frac{38ab\cot^{\frac{7}{2}}(}{}$$

$$= \frac{2b(231a^2-5b^2)\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{315ad} + \frac{2(21a^2-}{}$$

$$= -\frac{2(315a^4-483a^2b^2-10b^4)\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx}}{315a^2d}$$

$$= -\frac{2(315a^4-483a^2b^2-10b^4)\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx}}{315a^2d}$$

$$= -\frac{2(315a^4-483a^2b^2-10b^4)\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx}}{315a^2d}$$

$$= -\frac{2(315a^4-483a^2b^2-10b^4)\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx}}{315a^2d}$$

$$= \frac{(ia-b)^{5/2}\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\ \sqrt{}}{d}$$

Mathematica [A]

time = 5.49, size = 329, normalized size = 0.92

$$\frac{\cot^{\frac{9}{2}}(c+dx)\left(-315\sqrt[4]{-1}\,a^2(-a+ib)^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\tan^{\frac{9}{2}}(c+dx)-315\sqrt[4]{-1}\,a^2(a+ib)^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\tan^{\frac{9}{2}}(c+dx)+\frac{1}{4}\sec^4(c+dx)\left(-987a^4+1374a^2b^2+30b^4+4(280a^4-483a^2b^2-10b^4)\cos(2(c+dx))+(-413a^4+558a^2b^2+10b^4)\cos(4(c+dx))+272a^3b\sin(2(c+dx))-10ab^3\sin(2(c+dx))-326a^3b\sin(4(c+dx))+5ab^3\sin(4(c+dx))\right)\sqrt{a+b\tan(c+dx)}\right)}{315a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(11/2)*(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] (Cot[c + d*x]^(9/2)*(-315*(-1)^(1/4)*a^2*(-a + I*b)^(5/2)*ArcTan[((-1)^(1/4
)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Tan[c + d*x]
^(9/2) - 315*(-1)^(1/4)*a^2*(a + I*b)^(5/2)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b
]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Tan[c + d*x]^(9/2) + (Sec[c
```

```
 + d*x]^4*(-987*a^4 + 1374*a^2*b^2 + 30*b^4 + 4*(280*a^4 - 483*a^2*b^2 - 10
*b^4)*Cos[2*(c + d*x)] + (-413*a^4 + 558*a^2*b^2 + 10*b^4)*Cos[4*(c + d*x)]
 + 272*a^3*b*Sin[2*(c + d*x)] - 10*a*b^3*Sin[2*(c + d*x)] - 326*a^3*b*Sin[4
*(c + d*x)] + 5*a*b^3*Sin[4*(c + d*x)])*Sqrt[a + b*Tan[c + d*x]])/4))/(315*
a^2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 35.86, size = 49804, normalized size = 139.12

method	result	size
default	Expression too large to display	49804

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(11/2)*(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(11/2)*(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^(11/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(11/2)*(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(11/2)*(a+b*tan(d*x+c))**(5/2),x)
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(11/2)*(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c+d\,x)^{11/2}\,(a+b\tan(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(11/2)*(a + b*tan(c + d*x))^(5/2),x)
[Out] int(cot(c + d*x)^(11/2)*(a + b*tan(c + d*x))^(5/2), x)
```

3.854 $\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=310

$$\frac{i(ia-b)^{5/2}\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d}+\frac{i(ia+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{ia+b}}{\sqrt{a+}}\right.}{}$$

```
[Out] I*(I*a-b)^(5/2)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2
))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+I*(I*a+b)^(5/2)*arctanh((I*a+b)^(1/2
)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2
)/d+2/21*(7*a^2-9*b^2)*cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2)/d-6/7*a*b*co
t(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(1/2)/d-2/7*a^2*cot(d*x+c)^(7/2)*(a+b*tan(d
*x+c))^(1/2)/d+2/21*b*(49*a^2-3*b^2)*cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2
)/a/d
```

Rubi [A]
time = 0.94, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4326, 3646, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(7a^2-9b^2)\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{21d}+\frac{2b(49a^2-3b^2)\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{21ad}-\frac{2a^2\cot^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{7d}+\frac{i(-b+ia)^{5/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{6ab\cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{7d}+\frac{i(b+ia)^{5/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(9/2)*(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] (I*(I*a - b)^(5/2)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan
[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d + (I*(I*a + b)^(5/2)*A
rcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Co
t[c + d*x]]*Sqrt[Tan[c + d*x]])/d + (2*b*(49*a^2 - 3*b^2)*Sqrt[Cot[c + d*x]
]*Sqrt[a + b*Tan[c + d*x]])/(21*a*d) + (2*(7*a^2 - 9*b^2)*Cot[c + d*x]^(3/2
)*Sqrt[a + b*Tan[c + d*x]])/(21*d) - (6*a*b*Cot[c + d*x]^(5/2)*Sqrt[a + b*T
an[c + d*x]])/(7*d) - (2*a^2*Cot[c + d*x]^(7/2)*Sqrt[a + b*Tan[c + d*x]])/(
7*d)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
```

```
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}\,dx = \left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{(a+b\tan(c+dx))^{5/2}}{\tan^{\frac{9}{2}}(c+dx)}\,dx$$

$$= -\frac{2a^2\cot^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{7d} + \frac{1}{7}\Big(2\sqrt{\cot(c+dx)}\sqrt{\mathrm{t}$$

$$= -\frac{6ab\cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{7d} - \frac{2a^2\cot^{\frac{7}{2}}(c+dx)\sqrt{a+}}{7d}$$

$$= \frac{2(7a^2-9b^2)\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{21d} - \frac{6ab\cot^{\frac{5}{2}}(c+dx}{}$$

$$= \frac{2b(49a^2-3b^2)\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{21ad} + \frac{2(7a^2-9b}{}$$

$$= \frac{2b(49a^2-3b^2)\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{21ad} + \frac{2(7a^2-9b}{}$$

$$= \frac{2b(49a^2-3b^2)\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{21ad} + \frac{2(7a^2-9b}{}$$

$$= \frac{2b(49a^2-3b^2)\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{21ad} + \frac{2(7a^2-9b}{}$$

$$= \frac{i(ia-b)^{5/2}\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\mathrm{ta}}}{d}$$

Mathematica [A]

time = 3.57, size = 274, normalized size = 0.88

$$\frac{\cot^{\frac{7}{2}}(c+dx)\left(21(-1)^{3/4}a(-a+ib)^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\tan^{\frac{7}{2}}(c+dx)-21(-1)^{3/4}a(a+ib)^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\tan^{\frac{7}{2}}(c+dx)-\frac{1}{2}\sec^3(c+dx)\left(a(2a^2+9b^2)\cos(c+dx)+(10a^3-9ab^2)\cos(3(c+dx))+2b(-40a^2+3b^2+(58a^2-3b^2)\cos(2(c+dx)))\sin(c+dx)\right)\sqrt{a+b\tan(c+dx)}\right)}{21ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(9/2)*(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] (Cot[c + d*x]^(7/2)*(21*(-1)^(3/4)*a*(-a + I*b)^(5/2)*ArcTan[((-1)^(1/4)*Sq
rt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Tan[c + d*x]^(7/
2) - 21*(-1)^(3/4)*a*(a + I*b)^(5/2)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[
Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Tan[c + d*x]^(7/2) - (Sec[c + d*x]
^3*(a*(2*a^2 + 9*b^2)*Cos[c + d*x] + (10*a^3 - 9*a*b^2)*Cos[3*(c + d*x)] +
2*b*(-40*a^2 + 3*b^2 + (58*a^2 - 3*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])*Sqr
t[a + b*Tan[c + d*x]])/2))/(21*a*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 35.89, size = 33758, normalized size = 108.90

method	result	size
default	Expression too large to display	33758

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^(9/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(9/2)*(a+b*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c+d\,x)^{9/2}\,\left(a+b\tan(c+d\,x)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(9/2)*(a + b*tan(c + d*x))^(5/2),x)
```

```
[Out] int(cot(c + d*x)^(9/2)*(a + b*tan(c + d*x))^(5/2), x)
```

3.855 $\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=259

$$-\frac{(ia-b)^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d}+\frac{(ia+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{ia+}}{\sqrt{a}}\right.}{}$$

```
[Out] -(I*a-b)^(5/2)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2)
)*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+(I*a+b)^(5/2)*arctanh((I*a+b)^(1/2)*t
an(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d
-22/15*a*b*cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2)/d-2/5*a^2*cot(d*x+c)^(5/
2)*(a+b*tan(d*x+c))^(1/2)/d+2/15*(15*a^2-23*b^2)*cot(d*x+c)^(1/2)*(a+b*tan(
d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.73, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4326, 3646, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(15a^2-23b^2)\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{15d}-\frac{2a^2\cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{5d}-\frac{(-b+ia)^{5/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\mathrm{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{22ab\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{15d}+\frac{(b+ia)^{5/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(7/2)*(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] -(((I*a - b)^(5/2)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan
[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d) + ((I*a + b)^(5/2)*Ar
cTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot
[c + d*x]]*Sqrt[Tan[c + d*x]])/d + (2*(15*a^2 - 23*b^2)*Sqrt[Cot[c + d*x]]*
Sqrt[a + b*Tan[c + d*x]])/(15*d) - (22*a*b*Cot[c + d*x]^(3/2)*Sqrt[a + b*Ta
n[c + d*x]])/(15*d) - (2*a^2*Cot[c + d*x]^(5/2)*Sqrt[a + b*Tan[c + d*x]])/(
5*d)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
```

```
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{(a+b\tan(c+dx))^{5/2}}{\tan^{\frac{7}{2}}(c+dx)}\,dx$$

$$= -\frac{2a^2\cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{5d} + \frac{1}{5}\Big(2\sqrt{\cot(c+dx)}\,\sqrt{}$$

$$= -\frac{22ab\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{15d} - \frac{2a^2\cot^{\frac{5}{2}}(c+dx)\sqrt{a}}{5d}$$

$$= \frac{2(15a^2-23b^2)\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{15d} - \frac{22ab\cot^{\frac{3}{2}}}{}$$

$$= \frac{2(15a^2-23b^2)\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{15d} - \frac{22ab\cot^{\frac{3}{2}}}{}$$

$$= \frac{2(15a^2-23b^2)\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{15d} - \frac{22ab\cot^{\frac{3}{2}}}{}$$

$$= \frac{2(15a^2-23b^2)\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{15d} - \frac{22ab\cot^{\frac{3}{2}}}{}$$

$$= -\frac{(ia-b)^{5/2}\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{}}{d}$$

Mathematica [A]
time = 1.80, size = 214, normalized size = 0.83

$$\frac{\cot^{\frac{5}{2}}(c+dx)\left(15\sqrt[4]{-1}\,(-a+ib)^{5/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\tan^{\frac{5}{2}}(c+dx)+15\sqrt[4]{-1}\,(a+ib)^{5/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\tan^{\frac{5}{2}}(c+dx)+2\sqrt{a+b\tan(c+dx)}\left(-3a^2-11ab\tan(c+dx)+(15a^2-23b^2)\tan^2(c+dx)\right)\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(7/2)*(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] (Cot[c + d*x]^(5/2)*(15*(-1)^(1/4)*(-a + I*b)^(5/2)*ArcTan[((-1)^(1/4)*Sqrt
[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Tan[c + d*x]^(5/2)
 + 15*(-1)^(1/4)*(a + I*b)^(5/2)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[
c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Tan[c + d*x]^(5/2) + 2*Sqrt[a + b*Tan[
c + d*x]]*(-3*a^2 - 11*a*b*Tan[c + d*x] + (15*a^2 - 23*b^2)*Tan[c + d*x]^2)
))/(15*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 35.14, size = 33357, normalized size = 128.79

method	result	size
default	Expression too large to display	33357

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^(7/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(7/2)*(a+b*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c+d\,x)^{7/2}\,\left(a+b\tan(c+d\,x)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(7/2)*(a + b*tan(c + d*x))^(5/2),x)
```

```
[Out] int(cot(c + d*x)^(7/2)*(a + b*tan(c + d*x))^(5/2), x)
```

3.856 $\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=222

$$-\frac{i(ia-b)^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d}-\frac{i(ia+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{ia+}}{\sqrt{a}}\right.}{}$$

```
[Out] -I*(I*a-b)^(5/2)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/
2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-I*(I*a+b)^(5/2)*arctanh((I*a+b)^(1/
2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/
2)/d-2/3*a^2*cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2)/d-14/3*a*b*cot(d*x+c)^
(1/2)*(a+b*tan(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.54, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4326, 3646, 3730, 3697, 3696, 95, 209, 212}

$$-\frac{2a^2\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}-\frac{i(-b+ia)^{5/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\mathrm{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{14ab\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{3d}-\frac{i(b+ia)^{5/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-I)*(I*a - b)^(5/2)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*
Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d - (I*(I*a + b)^(5/2
)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt
[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d - (14*a*b*Sqrt[Cot[c + d*x]]*Sqrt[a +
b*Tan[c + d*x]])/(3*d) - (2*a^2*Cot[c + d*x]^(3/2)*Sqrt[a + b*Tan[c + d*x]]
)/(3*d)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
```

```
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{(a+b\tan(c+dx))^{5/2}}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$

$$= -\frac{2a^2\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{3d} + \frac{1}{3}\Big(2\sqrt{\cot(c+dx)}\,\sqrt{t\ldots}$$

$$= -\frac{14ab\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{3d} - \frac{2a^2\cot^{\frac{3}{2}}(c+dx)\sqrt{\ldots}}{3d\ldots}$$

$$= -\frac{14ab\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{3d} - \frac{2a^2\cot^{\frac{3}{2}}(c+dx)\sqrt{\ldots}}{3d\ldots}$$

$$= -\frac{14ab\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{3d} - \frac{2a^2\cot^{\frac{3}{2}}(c+dx)\sqrt{\ldots}}{3d\ldots}$$

$$= -\frac{14ab\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{3d} - \frac{2a^2\cot^{\frac{3}{2}}(c+dx)\sqrt{\ldots}}{3d\ldots}$$

$$= -\frac{i(ia-b)^{5/2}\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\ldots}}{d}$$

Mathematica [A]
time = 1.13, size = 190, normalized size = 0.86

$$\frac{\cot^{\frac{3}{2}}(c+dx)\left(-3(-1)^{3/4}(-a+ib)^{5/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\tan^{\frac{3}{2}}(c+dx)+3(-1)^{3/4}(a+ib)^{5/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\tan^{\frac{3}{2}}(c+dx)-2a\sqrt{a+b\tan(c+dx)}\,(a+7b\tan(c+dx))\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] (Cot[c + d*x]^(3/2)*(-3*(-1)^(3/4)*(-a + I*b)^(5/2)*ArcTan[((-1)^(1/4)*Sqrt
[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Tan[c + d*x]^(3/2)
```

```
 + 3*(-1)^(3/4)*(a + I*b)^(5/2)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c
 + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Tan[c + d*x]^(3/2) - 2*a*Sqrt[a + b*Tan
[c + d*x]]*(a + 7*b*Tan[c + d*x])))/(3*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 32.75, size = 16676, normalized size = 75.12

method	result	size
default	Expression too large to display	16676

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^(5/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(5/2)*(a+b*tan(d*x+c))**(5/2),x)
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c+d\,x)^{5/2}\,(a+b\tan(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))^(5/2),x)
```

```
[Out] int(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))^(5/2), x)
```

3.857 $\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=243

$$\frac{(ia-b)^{5/2}\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d}+\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\cdots}{\cdots}$$

```
[Out] (I*a-b)^(5/2)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))
*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+2*b^(5/2)*arctanh(b^(1/2)*tan(d*x+c)^(
1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-(I*a+b)^(5
/2)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+
c)^(1/2)*tan(d*x+c)^(1/2)/d-2*a^2*cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)/d
```

Rubi [A]

time = 1.09, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4326, 3646, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$-\frac{2a^2\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{d}+\frac{(-b+ia)^{5/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{2b^{5/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{(b+ia)^{5/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((I*a - b)^(5/2)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
 + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d + (2*b^(5/2)*ArcTanh[(Sq
rt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt
[Tan[c + d*x]])/d - ((I*a + b)^(5/2)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*
x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d - (
2*a^2*Sqrt[Cot[c + d*x]]*Sqrt[a + b*Tan[c + d*x]])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{(a+b\tan(c+dx))^{5/2}}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

$$= -\frac{2a^2\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{d} + \left(2\sqrt{\cot(c+dx)}\,\sqrt{\right.$$

$$= -\frac{2a^2\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{d} + \frac{\left(2\sqrt{\cot(c+dx)}\,\sqrt{\right.}{}$$

$$= -\frac{2a^2\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{d} + \frac{\left(2\sqrt{\cot(c+dx)}\,\sqrt{\right.}{}$$

$$= -\frac{2a^2\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{d} + \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{t\right.}{}$$

$$= -\frac{2a^2\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{d} + \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{t\right.}{}$$

$$= -\frac{2a^2\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{d} - \frac{\left((a-ib)^3\sqrt{\cot(c+\right.}{}$$

$$= \frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+}}{d}$$

$$= \frac{(ia-b)^{5/2}\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{t}}{d}$$

Mathematica [A]

time = 2.09, size = 253, normalized size = 1.04

$$\frac{\sqrt{\cot(c+dx)}\left(-\sqrt[4]{-1}\,(-a+ib)^{5/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\tan(c+dx)}-\sqrt[4]{-1}\,(a+ib)^{5/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\tan(c+dx)}-2a^2\sqrt{a+b\tan(c+dx)}+\frac{2\sqrt{a}\,b^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{\tan(c+dx)}\,\sqrt{1+\frac{b\tan(c+dx)}{a}}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*(-((-1)^(1/4)*(-a + I*b)^(5/2)*ArcTan[((-1)^(1/4)*Sqrt[
-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Tan[c + d*x]])
 - (-1)^(1/4)*(a + I*b)^(5/2)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c +
 d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Tan[c + d*x]] - 2*a^2*Sqrt[a + b*Tan
[c + d*x]] + (2*Sqrt[a]*b^(5/2)*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a
]]*Sqrt[Tan[c + d*x]]*Sqrt[1 + (b*Tan[c + d*x])/a])/Sqrt[a + b*Tan[c + d*x]
]))/d
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 36.01, size = 27482, normalized size = 113.09

method	result	size
default	Expression too large to display	27482

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^(3/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(3/2)*(a+b*tan(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c+d\,x)^{3/2}\,\left(a+b\tan(c+d\,x)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(5/2),x)
[Out] int(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(5/2), x)
```

3.858 $\int \sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=248

$$\frac{i(ia-b)^{5/2}\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d}+\frac{5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c}}{\sqrt{a+b\tan(c}}\right.}{}$$

```
[Out] I*(I*a-b)^(5/2)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2
))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+5*a*b^(3/2)*arctanh(b^(1/2)*tan(d*x+
c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+I*(I*a
+b)^(5/2)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*co
t(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+b^2*(a+b*tan(d*x+c))^(1/2)/d/cot(d*x+c)^(
1/2)
```

Rubi [A]
time = 1.04, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4326, 3647, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{i(-b+ia)^{5/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{5ab^{3/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{b^2\sqrt{a+b\tan(c+dx)}}{d\sqrt{\cot(c+dx)}}+\frac{i(b+ia)^{5/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] (I*(I*a - b)^(5/2)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan
[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d + (5*a*b^(3/2)*ArcTanh
[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*
Sqrt[Tan[c + d*x]])/d + (I*(I*a + b)^(5/2)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[
c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])
/d + (b^2*Sqrt[a + b*Tan[c + d*x]])/(d*Sqrt[Cot[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{5/2}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{(a+b\tan(c+dx))^{5/2}}{\sqrt{\tan(c+dx)}}\,dx$$

$$= \frac{b^2\sqrt{a+b\tan(c+dx)}}{d\sqrt{\cot(c+dx)}} + \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{1}{2}$$

$$= \frac{b^2\sqrt{a+b\tan(c+dx)}}{d\sqrt{\cot(c+dx)}} + \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\text{Sub}}{}$$

$$= \frac{b^2\sqrt{a+b\tan(c+dx)}}{d\sqrt{\cot(c+dx)}} + \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\text{Sub}}{}$$

$$= \frac{b^2\sqrt{a+b\tan(c+dx)}}{d\sqrt{\cot(c+dx)}} + \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\text{Sub}}{}$$

$$= \frac{b^2\sqrt{a+b\tan(c+dx)}}{d\sqrt{\cot(c+dx)}} + \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\text{Sub}}{}$$

$$= \frac{b^2\sqrt{a+b\tan(c+dx)}}{d\sqrt{\cot(c+dx)}} - \frac{\left((ia-b)^3\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+d}\right.}{}$$

$$= \frac{5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c-}}{d}$$

$$= \frac{i(ia-b)^{5/2}\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{}}{d}$$

Mathematica [**A**]
time = 1.12, size = 241, normalized size = 0.97

$$\frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left((-1)^{3/4}(-a+ib)^{5/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)-(-1)^{3/4}(a+ib)^{5/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+b^2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}+\frac{5a^{3/2}b^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{1+\frac{b\tan(c+dx)}{a}}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-1)^(3/4)*(-a + I*b)^(5/2)*ArcTan[
((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] -
(-1)^(3/4)*(a + I*b)^(5/2)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*
x]])/Sqrt[a + b*Tan[c + d*x]]] + b^2*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c +
d*x]] + (5*a^(3/2)*b^(3/2)*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sq
rt[1 + (b*Tan[c + d*x])/a])/Sqrt[a + b*Tan[c + d*x]]))/d
```

Maple [**C**] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 35.21, size = 15065, normalized size = 60.75

method	result	size
default	Expression too large to display	15065

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(5/2)*sqrt(cot(d*x + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(1/2)*(a+b*tan(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cot\left(c+d\,x\right)}\ \left(a+b\tan(c+d\,x)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(5/2),x)
[Out] int(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(5/2), x)
```

3.859 $\int \frac{(a+b\tan(c+dx))^{5/2}}{\sqrt{\cot(c+dx)}}\,dx$

Optimal. Leaf size=291

$$-\frac{(ia-b)^{5/2}\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d}+\frac{\sqrt{b}\,(15a^2-8b^2)\tanh^{-1}\left(\right.}{}$$

```
[Out] -(I*a-b)^(5/2)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2)
)*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+(I*a+b)^(5/2)*arctanh((I*a+b)^(1/2)*t
an(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d
+1/4*(15*a^2-8*b^2)*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2)
)*b^(1/2)*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+1/2*b^2*(a+b*tan(d*x+c))^(1/2
)/d/cot(d*x+c)^(3/2)+9/4*a*b*(a+b*tan(d*x+c))^(1/2)/d/cot(d*x+c)^(1/2)
```

Rubi [A]
time = 1.58, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {4326, 3647, 3728, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{\sqrt{b}\,(15a^2-8b^2)\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{4d}-\frac{(-b+ia)^{5/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{b^2\sqrt{a+b\tan(c+dx)}}{2d\cot^{\frac{3}{2}}(c+dx)}+\frac{9ab\sqrt{a+b\tan(c+dx)}}{4d\sqrt{\cot(c+dx)}}+\frac{(b+ia)^{5/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^(5/2)/Sqrt[Cot[c + d*x]],x]
```

```
[Out] -(((I*a - b)^(5/2)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan
[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d) + (Sqrt[b]*(15*a^2 -
8*b^2)*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[
Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(4*d) + ((I*a + b)^(5/2)*ArcTanh[(Sqrt[I*
a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqr
t[Tan[c + d*x]])/d + (b^2*Sqrt[a + b*Tan[c + d*x]])/(2*d*Cot[c + d*x]^(3/2)
) + (9*a*b*Sqrt[a + b*Tan[c + d*x]])/(4*d*Sqrt[Cot[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
```

```
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(c+dx))^{5/2}}{\sqrt{\cot(c+dx)}}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{5/2}\,dx$$

$$= \frac{b^2\sqrt{a+b\tan(c+dx)}}{2d\cot^{\frac{3}{2}}(c+dx)} + \frac{1}{2}\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{\sqrt{\tan(c+dx}}{}$$

$$= \frac{b^2\sqrt{a+b\tan(c+dx)}}{2d\cot^{\frac{3}{2}}(c+dx)} + \frac{9ab\sqrt{a+b\tan(c+dx)}}{4d\sqrt{\cot(c+dx)}} + \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c\right.}}{}$$

$$= \frac{b^2\sqrt{a+b\tan(c+dx)}}{2d\cot^{\frac{3}{2}}(c+dx)} + \frac{9ab\sqrt{a+b\tan(c+dx)}}{4d\sqrt{\cot(c+dx)}} + \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c\right.}}{}$$

$$= \frac{b^2\sqrt{a+b\tan(c+dx)}}{2d\cot^{\frac{3}{2}}(c+dx)} + \frac{9ab\sqrt{a+b\tan(c+dx)}}{4d\sqrt{\cot(c+dx)}} + \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c\right.}}{}$$

$$= \frac{b^2\sqrt{a+b\tan(c+dx)}}{2d\cot^{\frac{3}{2}}(c+dx)} + \frac{9ab\sqrt{a+b\tan(c+dx)}}{4d\sqrt{\cot(c+dx)}} - \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c\right.}}{}$$

$$= \frac{b^2\sqrt{a+b\tan(c+dx)}}{2d\cot^{\frac{3}{2}}(c+dx)} + \frac{9ab\sqrt{a+b\tan(c+dx)}}{4d\sqrt{\cot(c+dx)}} - \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c\right.}}{}$$

$$= \frac{b^2\sqrt{a+b\tan(c+dx)}}{2d\cot^{\frac{3}{2}}(c+dx)} + \frac{9ab\sqrt{a+b\tan(c+dx)}}{4d\sqrt{\cot(c+dx)}} + \frac{\left((a-ib)^3\sqrt{\cot(c+dx)}\right.}{}$$

$$= \frac{\sqrt{b}\,(15a^2-8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx}}{4d}$$

$$= -\frac{(ia-b)^{5/2}\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d}$$

Mathematica [A]
time = 2.52, size = 284, normalized size = 0.98

$$\frac{\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\left(4\sqrt[4]{-1}\,(-a+ib)^{5/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+4\sqrt[4]{-1}\,(a+ib)^{5/2}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+9ab\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}+2b^2\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}+\frac{\sqrt{a}\,\sqrt{b}\,(15a^2-8b^2)\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{1+\frac{b\tan(c+dx)}{a}}}{\sqrt{a+b\tan(c+dx)}}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^(5/2)/Sqrt[Cot[c + d*x]],x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(4*(-1)^(1/4)*(-a + I*b)^(5/2)*ArcTa
n[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]
+ 4*(-1)^(1/4)*(a + I*b)^(5/2)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c
+ d*x]])/Sqrt[a + b*Tan[c + d*x]]] + 9*a*b*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Ta
n[c + d*x]] + 2*b^2*Tan[c + d*x]^(3/2)*Sqrt[a + b*Tan[c + d*x]] + (Sqrt[a]*
Sqrt[b]*(15*a^2 - 8*b^2)*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt
[1 + (b*Tan[c + d*x])/a])/Sqrt[a + b*Tan[c + d*x]]))/(4*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 40.20, size = 17291, normalized size = 59.42

method	result	size
default	Expression too large to display	17291

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(5/2)/cot(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/2)/cot(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^(5/2)/sqrt(cot(d*x + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/2)/cot(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(5/2)/cot(d*x+c)**(1/2),x)
```

[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/2)/cot(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\tan(c+d\,x))^{5/2}}{\sqrt{\cot\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^(5/2)/cot(c + d*x)^(1/2),x)
```

```
[Out] int((a + b*tan(c + d*x))^(5/2)/cot(c + d*x)^(1/2), x)
```

3.860 $\int \frac{(a+b\tan(c+dx))^{5/2}}{\cot^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=337

$$-\frac{i(ia-b)^{5/2}\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{d}+\frac{5a(a^2-8b^2)\tanh^{-1}\left(\frac{\sqrt{\cdot}}{\sqrt{a}}\right.}{}$$

```
[Out] -I*(I*a-b)^(5/2)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/
2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-I*(I*a+b)^(5/2)*arctanh((I*a+b)^(1/
2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/
2)/d+5/8*a*(a^2-8*b^2)*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1
/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d/b^(1/2)+1/3*b^2*(a+b*tan(d*x+c))^(
1/2)/d/cot(d*x+c)^(5/2)+13/12*a*b*(a+b*tan(d*x+c))^(1/2)/d/cot(d*x+c)^(3/2)
+1/8*(11*a^2-8*b^2)*(a+b*tan(d*x+c))^(1/2)/d/cot(d*x+c)^(1/2)
```

Rubi [A]
time = 1.59, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {4326, 3647, 3728, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{(11a^2-8b^2)\sqrt{a+b\tan(c+dx)}}{8d\sqrt{\cot(c+dx)}}+\frac{5a(a^2-8b^2)\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{8\sqrt{b}\,d}-\frac{i(-b+ia)^{5/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{b^2\sqrt{a+b\tan(c+dx)}}{3d\cot^{\frac{5}{2}}(c+dx)}+\frac{13ab\sqrt{a+b\tan(c+dx)}}{12d\cot^{\frac{3}{2}}(c+dx)}-\frac{i(b+ia)^{5/2}\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^(5/2)/Cot[c + d*x]^(3/2),x]
```

```
[Out] ((-I)*(I*a - b)^(5/2)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*
Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d + (5*a*(a^2 - 8*b^2
)*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c
 + d*x]]*Sqrt[Tan[c + d*x]])/(8*Sqrt[b]*d) - (I*(I*a + b)^(5/2)*ArcTanh[(Sq
rt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]
]*Sqrt[Tan[c + d*x]])/d + (b^2*Sqrt[a + b*Tan[c + d*x]])/(3*d*Cot[c + d*x]^
(5/2)) + (13*a*b*Sqrt[a + b*Tan[c + d*x]])/(12*d*Cot[c + d*x]^(3/2)) + ((11
*a^2 - 8*b^2)*Sqrt[a + b*Tan[c + d*x]])/(8*d*Sqrt[Cot[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
```

```
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(c+dx))^{5/2}}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}\,dx$$

$$= \frac{b^2\sqrt{a+b\tan(c+dx)}}{3d\cot^{\frac{5}{2}}(c+dx)} + \frac{1}{3}\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{\tan^{\frac{3}{2}}(c+dx)\cdots}{\cdots}$$

$$= \frac{b^2\sqrt{a+b\tan(c+dx)}}{3d\cot^{\frac{5}{2}}(c+dx)} + \frac{13ab\sqrt{a+b\tan(c+dx)}}{12d\cot^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(\cdots}{\cdots}$$

$$= \frac{b^2\sqrt{a+b\tan(c+dx)}}{3d\cot^{\frac{5}{2}}(c+dx)} + \frac{13ab\sqrt{a+b\tan(c+dx)}}{12d\cot^{\frac{3}{2}}(c+dx)} + \frac{(11a^2-8b^2)\sqrt{a+b\,\mathrm{ta}\cdots}}{8d\sqrt{\cot(c+d\cdots}}$$

$$= \frac{b^2\sqrt{a+b\tan(c+dx)}}{3d\cot^{\frac{5}{2}}(c+dx)} + \frac{13ab\sqrt{a+b\tan(c+dx)}}{12d\cot^{\frac{3}{2}}(c+dx)} + \frac{(11a^2-8b^2)\sqrt{a+b\,\mathrm{ta}\cdots}}{8d\sqrt{\cot(c+d\cdots}}$$

$$= \frac{b^2\sqrt{a+b\tan(c+dx)}}{3d\cot^{\frac{5}{2}}(c+dx)} + \frac{13ab\sqrt{a+b\tan(c+dx)}}{12d\cot^{\frac{3}{2}}(c+dx)} + \frac{(11a^2-8b^2)\sqrt{a+b\,\mathrm{ta}\cdots}}{8d\sqrt{\cot(c+d\cdots}}$$

$$= \frac{b^2\sqrt{a+b\tan(c+dx)}}{3d\cot^{\frac{5}{2}}(c+dx)} + \frac{13ab\sqrt{a+b\tan(c+dx)}}{12d\cot^{\frac{3}{2}}(c+dx)} + \frac{(11a^2-8b^2)\sqrt{a+b\,\mathrm{ta}\cdots}}{8d\sqrt{\cot(c+d\cdots}}$$

$$= \frac{b^2\sqrt{a+b\tan(c+dx)}}{3d\cot^{\frac{5}{2}}(c+dx)} + \frac{13ab\sqrt{a+b\tan(c+dx)}}{12d\cot^{\frac{3}{2}}(c+dx)} + \frac{(11a^2-8b^2)\sqrt{a+b\,\mathrm{ta}\cdots}}{8d\sqrt{\cot(c+d\cdots}}$$

$$= \frac{b^2\sqrt{a+b\tan(c+dx)}}{3d\cot^{\frac{5}{2}}(c+dx)} + \frac{13ab\sqrt{a+b\tan(c+dx)}}{12d\cot^{\frac{3}{2}}(c+dx)} + \frac{(11a^2-8b^2)\sqrt{a+b\,\mathrm{ta}\cdots}}{8d\sqrt{\cot(c+d\cdots}}$$

$$= \frac{5a(a^2-8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{8\sqrt{b}\,d} + \cdots$$

$$= -\frac{i(ia-b)^{5/2}\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\cdots}{d}$$

Mathematica [A]
time = 4.00, size = 320, normalized size = 0.95

$$\frac{\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\left(-24(-1)^{3/4}(-a+ib)^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+24(-1)^{3/4}(a+ib)^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+3(11a^2-8b^2)\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}+26ab\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}+8b^2\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}+\frac{15a^{3/2}(a^2-8b^2)\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{1+\frac{b\tan(c+dx)}{a}}}{\sqrt{b}\,\sqrt{a+b\tan(c+dx)}}\right)}{24d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Tan[c + d*x])^(5/2)/Cot[c + d*x]^(3/2),x]

[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-24*(-1)^(3/4)*(-a + I*b)^(5/2)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + 24*(-1)^(3/4)*(a + I*b)^(5/2)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + 3*(11*a^2 - 8*b^2)*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]] + 26*a*b*Tan[c + d*x]^(3/2)*Sqrt[a + b*Tan[c + d*x]] + 8*b^2*Tan[c + d*x]^(5/2)*Sqrt[a + b*Tan[c + d*x]] + (15*a^(3/2)*(a^2 - 8*b^2)*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a])/(Sqrt[b]*Sqrt[a + b*Tan[c + d*x]])))/(24*d)

Maple [**C**] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 43.03, size = 18203, normalized size = 54.01

method	result	size
default	Expression too large to display	18203

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*tan(d*x+c))^(5/2)/cot(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] result too large to display

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(d*x+c))^(5/2)/cot(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((b*tan(d*x + c) + a)^(5/2)/cot(d*x + c)^(3/2), x)

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(d*x+c))^(5/2)/cot(d*x+c)^(3/2),x, algorithm="fricas")

[Out] Timed out

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(5/2)/cot(d*x+c)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/2)/cot(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\tan(c+d\,x))^{5/2}}{\cot\left(c+d\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^(5/2)/cot(c + d*x)^(3/2),x)
```

```
[Out] int((a + b*tan(c + d*x))^(5/2)/cot(c + d*x)^(3/2), x)
```

3.861 $\int \frac{\cot^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal. Leaf size=220

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{\sqrt{ia-b}\,d}-\frac{\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\ }}{\sqrt{ia+b}}$$

```
[Out] -arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(
1/2)*tan(d*x+c)^(1/2)/d/(I*a-b)^(1/2)-arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2
)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d/(I*a+b)^(1/2)
-2/3*cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2)/a/d+4/3*b*cot(d*x+c)^(1/2)*(a+
b*tan(d*x+c))^(1/2)/a^2/d
```

Rubi [A]
time = 0.32, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4326, 3650, 3730, 12, 3656, 926, 95, 211, 214}

$$\frac{4b\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{3a^2d}-\frac{\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}-\frac{2\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{3ad}-\frac{\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(5/2)/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] -((ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt
[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[I*a - b]*d)) - (ArcTanh[(Sqrt[I*a
+ b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[
Tan[c + d*x]])/(Sqrt[I*a + b]*d) + (4*b*Sqrt[Cot[c + d*x]]*Sqrt[a + b*Tan[c
 + d*x]])/(3*a^2*d) - (2*Cot[c + d*x]^(3/2)*Sqrt[a + b*Tan[c + d*x]])/(3*a*
d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
```

```
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx$$

$$= -\frac{2\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{3ad} - \frac{\left(2\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int}{3a}$$

$$= \frac{4b\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{3a^2d} - \frac{2\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{3ad}$$

$$= \frac{4b\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{3a^2d} - \frac{2\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{3ad}$$

$$= \frac{4b\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{3a^2d} - \frac{2\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{3ad}$$

$$= \frac{4b\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{3a^2d} - \frac{2\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{3ad}$$

$$= \frac{4b\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{3a^2d} - \frac{2\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{3ad}$$

$$= \frac{4b\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{3a^2d} - \frac{2\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{3ad}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{\sqrt{ia-b}\,d} - \frac{\tanh^{-1}}{}$$

Mathematica [A]

time = 3.55, size = 193, normalized size = 0.88

$$\frac{\sqrt{\cot(c+dx)}\left(\frac{3(-1)^{3/4}\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\tan(c+dx)}}{\sqrt{-a+ib}}+\frac{3(-1)^{3/4}\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\tan(c+dx)}}{\sqrt{a+ib}}-\frac{2(-2b+a\cot(c+dx))\sqrt{a+b\tan(c+dx)}}{a^2}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(5/2)/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*((3*(-1)^(3/4)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[T
an[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Tan[c + d*x]])/Sqrt[-a + I*b]
+ (3*(-1)^(3/4)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a
 + b*Tan[c + d*x]]]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*b] - (2*(-2*b + a*Cot[c
+ d*x])*Sqrt[a + b*Tan[c + d*x]])/a^2))/(3*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 32.69, size = 8425, normalized size = 38.30

method	result	size
default	Expression too large to display	8425

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^(5/2)/sqrt(b*tan(d*x + c) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(5/2)/(a+b*tan(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^{5/2}}{\sqrt{a+b\tan\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(5/2)/(a + b*tan(c + d*x))^(1/2),x)
```

```
[Out] int(cot(c + d*x)^(5/2)/(a + b*tan(c + d*x))^(1/2), x)
```

3.862 $\int \frac{\cot^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal. Leaf size=187

$$-\frac{i\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{\sqrt{ia-b}\,d}+\frac{i\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia+b}}$$

```
[Out] -I*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)
^(1/2)*tan(d*x+c)^(1/2)/d/(I*a-b)^(1/2)+I*arctanh((I*a+b)^(1/2)*tan(d*x+c)^
(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d/(I*a+b)^(
1/2)-2*cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)/a/d
```

Rubi [A]

time = 0.20, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4326, 3650, 12, 3656, 924, 95, 211, 214}

$$-\frac{i\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}-\frac{2\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}{ad}+\frac{i\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(3/2)/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] ((-I)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*S
qrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[I*a - b]*d) + (I*ArcTanh[(Sqrt[
I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*S
qrt[Tan[c + d*x]])/(Sqrt[I*a + b]*d) - (2*Sqrt[Cot[c + d*x]]*Sqrt[a + b*Tan
[c + d*x]])/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx = \left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx$$

$$= -\frac{2\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{ad} - \frac{\left(2\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int}{a}$$

$$= -\frac{2\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{ad} - \left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int$$

$$= -\frac{2\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{ad} - \frac{\left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Sul}}{}$$

$$= -\frac{2\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{ad} - \frac{\left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Sul}}{}$$

$$= -\frac{2\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{ad} + \frac{\left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Sul}}{}$$

$$= -\frac{2\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{ad} - \frac{\left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Sul}}{}$$

$$= -\frac{i\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{ia-b}\,d} + \frac{i\tanh^{-}}{}$$

Mathematica [A]
time = 0.66, size = 177, normalized size = 0.95

$$\frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(\frac{\sqrt[4]{-1}\,\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}} - \frac{\sqrt[4]{-1}\,\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{a+ib}} - \frac{2\sqrt{a+b\tan(c+dx)}}{a\sqrt{\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(3/2)/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(((-1)^(1/4)*ArcTan[((-1)^(1/4)*Sqrt
[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[-a + I*b] -
((-1)^(1/4)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b
*Tan[c + d*x]]])/Sqrt[a + I*b] - (2*Sqrt[a + b*Tan[c + d*x]])/(a*Sqrt[Tan[c
 + d*x]])))/d
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 38.72, size = 6056, normalized size = 32.39

method	result	size
default	Expression too large to display	6056

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^(3/2)/sqrt(b*tan(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(3/2)/(a+b*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)**(3/2)/sqrt(a + b*tan(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)^(3/2)/sqrt(b*tan(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(c+d\,x)^{3/2}}{\sqrt{a+b\tan\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(3/2)/(a + b*tan(c + d*x))^(1/2),x)
```

```
[Out] int(cot(c + d*x)^(3/2)/(a + b*tan(c + d*x))^(1/2), x)
```

3.863 $\int \frac{\sqrt{\cot(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal. Leaf size=149

$$\frac{\mathrm{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{\sqrt{ia-b}\,d}+\frac{\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{c}}{\sqrt{ia+b}\,d}$$

```
[Out] arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1
/2)*tan(d*x+c)^(1/2)/d/(I*a-b)^(1/2)+arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)
/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d/(I*a+b)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4326, 3656, 926, 95, 211, 214}

$$\frac{\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\mathrm{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}+\frac{\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cot[c + d*x]]/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] (ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[C
ot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[I*a - b]*d) + (ArcTanh[(Sqrt[I*a + b
]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan
[c + d*x]])/(Sqrt[I*a + b]*d)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{\sqrt{\cot(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{1}{\sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)}}\,dx$$

$$= \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}\,(1+x^2)}\,dx, x, \tan(c+d$$

$$= \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \left(\frac{i}{2(i-x)\sqrt{x}\,\sqrt{a+bx}} + \frac{i}{2\sqrt{x}\,(i+x)\sqrt{}}\right.\right.}{d}$$

$$= \frac{\left(i\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{1}{(i-x)\sqrt{x}\,\sqrt{a+bx}}\,dx, x, \tan(c+d}{2d}$$

$$= \frac{\left(i\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{1}{i-(-a+ib)x^2}\,dx, x, \frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+d}}\right.}{d}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{\sqrt{ia-b}\,d} + \frac{\tanh^{-1}\left(\frac{\sqrt{}}{}\right.}{}$$

Mathematica [A]
time = 0.17, size = 144, normalized size = 0.97

$$\frac{(-1)^{3/4}\left(-\frac{\operatorname{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}}-\frac{\operatorname{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{a+ib}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cot[c + d*x]]/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] ((-1)^(3/4)*(-(ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a
 + b*Tan[c + d*x]]]/Sqrt[-a + I*b]) - ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt
[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]/Sqrt[a + I*b])*Sqrt[Cot[c + d*x]]
*Sqrt[Tan[c + d*x]])/d
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 35.23, size = 2048, normalized size = 13.74

method	result	size
default	Expression too large to display	2048

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*2^(1/2)*(cos(d*x+c)/sin(d*x+c))^(1/2)*((a*cos(d*x+c)+b*sin(d*x+c))/cos(
d*x+c))^(1/2)*(2*I*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*c
os(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I
*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1
/2))*a*b*(a^2+b^2)^(1/2)-2*I*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(
d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)
^(1/2))/(-I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2
)^(1/2))^(1/2))*a*b*(a^2+b^2)^(1/2)-I*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+
c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+
(a^2+b^2)^(1/2))/(I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/
(a^2+b^2)^(1/2))^(1/2))*a^3-2*I*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*s
in(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b
^2)^(1/2))/(I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b
^2)^(1/2))^(1/2))*a*b^2+I*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x
+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1
/2))/(-I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(
1/2))^(1/2))*a^3+2*I*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a
*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/
(-I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))
^(1/2))*a*b^2+2*(a^2+b^2)^(1/2)*EllipticF((((a^2+b^2)^(1/2)*sin(d*x+c)-b*si
```

```
n(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),1/2*2^(1/2)
*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a^2+4*(a^2+b^2)^(1/2)*Ellipt
icF((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)
^(1/2))/sin(d*x+c))^(1/2),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2)
)^(1/2))*b^2-EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x
+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^
2+b^2)^(1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a
^2*(a^2+b^2)^(1/2)-EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*c
os(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(-
I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(
1/2))*a^2*(a^2+b^2)^(1/2)-4*EllipticF((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*
x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),1/2*2^(1/2)*((-
b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a^2*b-4*EllipticF((((a^2+b^2)^(1
/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c)
)^(1/2),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*b^3+Ellip
ticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^
2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^2+b^2)^(1/2)-b),1/
2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a^2*b+EllipticPi(((
(a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2)
)/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(-I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/
2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a^2*b)*(a*(-1+cos(d*x+c))/
(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*(((a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d
*x+c)+a*cos(d*x+c)-a)/(a^2+b^2)^(1/2)/sin(d*x+c))^(1/2)*(((a^2+b^2)^(1/2)*s
in(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/
2)*sin(d*x+c)^2/(-1+cos(d*x+c))/(a*cos(d*x+c)+b*sin(d*x+c))/(-I*a+(a^2+b^2)
^(1/2)-b)/(I*a+(a^2+b^2)^(1/2)-b)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(cot(d*x + c))/sqrt(b*tan(d*x + c) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cot(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(1/2)/(a+b*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(cot(c + d*x))/sqrt(a + b*tan(c + d*x)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cot(c+d\,x)}}{\sqrt{a+b\tan(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(1/2)/(a + b*tan(c + d*x))^(1/2),x)
```

```
[Out] int(cot(c + d*x)^(1/2)/(a + b*tan(c + d*x))^(1/2), x)
```

3.864 $\int \frac{1}{\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal. Leaf size=155

$$\frac{i\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{ia-b}\,d}-\frac{i\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{}}{\sqrt{ia+b}\,d}$$

```
[Out] I*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^
(1/2)*tan(d*x+c)^(1/2)/d/(I*a-b)^(1/2)-I*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(
1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d/(I*a+b)^(1
/2)
```

Rubi [A]
time = 0.14, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4326, 3656, 924, 95, 211, 214}

$$\frac{i\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}-\frac{i\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cot[c + d*x]]*Sqrt[a + b*Tan[c + d*x]]),x]
```

```
[Out] (I*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt
[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[I*a - b]*d) - (I*ArcTanh[(Sqrt[I*a
 + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt
[Tan[c + d*x]])/(Sqrt[I*a + b]*d)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}\,dx = \left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\,dx$$

$$= \frac{\left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{a+bx}\,(1+x^2)}\,dx,\right.}{d}$$

$$= \frac{\left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \left(-\frac{1}{2(i-x)\sqrt{x}\sqrt{a+}}\right.\right.}{d}$$

$$= -\frac{\left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{1}{(i-x)\sqrt{x}\sqrt{a+bx}}\right.}{2d}$$

$$= \frac{\left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{1}{i-(-a+ib)x^2}\,dx, x, \right.}{d}$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+}}{\sqrt{ia-b}\,d}$$

Mathematica [A]
time = 0.18, size = 143, normalized size = 0.92

$$\frac{\sqrt[4]{-1}\left(-\frac{\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}}+\frac{\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{a+ib}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cot[c + d*x]]*Sqrt[a + b*Tan[c + d*x]]),x]
```

```
[Out] ((-1)^(1/4)*(-(ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a
 + b*Tan[c + d*x]]]/Sqrt[-a + I*b]) + ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt
[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]/Sqrt[a + I*b])*Sqrt[Cot[c + d*x]]
*Sqrt[Tan[c + d*x]])/d
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 33.72, size = 1631, normalized size = 10.52

method	result	size
default	Expression too large to display	1631

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(I*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)
/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^2+b^2)
^(1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a*(a^2+
b^2)^(1/2)-I*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x
+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^
2+b^2)^(1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a
*b-I*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(
-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),-(-b+(a^2+b^2)^(1/2))/(I*a-(a^2+b^2)^
(1/2)+b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a*(a^2+b
^2)^(1/2)+I*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+
c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),-(-b+(a^2+b^2)^(1/2))/(I*a-(a^
2+b^2)^(1/2)+b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a
*b+2*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(
-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^2+b^2)^(
1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*b*(a^2+b^
2)^(1/2)-EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+
a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^2+b^
2)^(1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a^2-2
*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(
a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^2+b^2)^(1/2)
```

```
-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*b^2+2*Ellipti
cPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)
^(1/2))/sin(d*x+c))^(1/2),-(-b+(a^2+b^2)^(1/2))/(I*a-(a^2+b^2)^(1/2)+b),1/2
*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*b*(a^2+b^2)^(1/2)-El
lipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2
+b^2)^(1/2))/sin(d*x+c))^(1/2),-(-b+(a^2+b^2)^(1/2))/(I*a-(a^2+b^2)^(1/2)+b
),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a^2-2*EllipticP
i((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(
1/2))/sin(d*x+c))^(1/2),-(-b+(a^2+b^2)^(1/2))/(I*a-(a^2+b^2)^(1/2)+b),1/2*2
^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*b^2)*cos(d*x+c)*(a*(-1
+cos(d*x+c))/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*(((a^2+b^2)^(1/2)*sin(d
*x+c)+b*sin(d*x+c)+a*cos(d*x+c)-a)/(a^2+b^2)^(1/2)/sin(d*x+c))^(1/2)*(((a^2
+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/si
n(d*x+c))^(1/2)*sin(d*x+c)*((a*cos(d*x+c)+b*sin(d*x+c))/cos(d*x+c))^(1/2)*2
^(1/2)/(a*cos(d*x+c)+b*sin(d*x+c))/(-1+cos(d*x+c))/(cos(d*x+c)/sin(d*x+c))^
(1/2)/(I*a-(a^2+b^2)^(1/2)+b)/(I*a+(a^2+b^2)^(1/2)-b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*tan(d*x + c) + a)*sqrt(cot(d*x + c))), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\tan\left(c+dx\right)}\,\sqrt{\cot\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(1/2)/(a+b*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*tan(c + d*x))*sqrt(cot(c + d*x))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cot\left(c+d\,x\right)}\,\sqrt{a+b\tan\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(1/2)), x)
```

3.865 $\int \frac{1}{\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal. Leaf size=212

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{ia-b}\,d}+\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{c}}{\sqrt{b}\,d}$$

```
[Out] -arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(
1/2)*tan(d*x+c)^(1/2)/d/(I*a-b)^(1/2)+2*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a
+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d/b^(1/2)-arctanh((
I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(
d*x+c)^(1/2)/d/(I*a+b)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4326, 3656, 924, 65, 223, 212, 926, 95, 211, 214}

$$-\frac{\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}+\frac{2\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{b}\,d}-\frac{\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cot[c + d*x]^(3/2)*Sqrt[a + b*Tan[c + d*x]]),x]
```

```
[Out] -((ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt
[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[I*a - b]*d)) + (2*ArcTanh[(Sqrt[b]
*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[
c + d*x]])/(Sqrt[b]*d) - (ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a
 + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[I*a + b]*d
)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{1}{\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx = \left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{\tan^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$$

$$= \frac{\left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{x^{3/2}}{\sqrt{a+bx}\,(1+x^2)}\,dx, x, \right.}{d}$$

$$= \frac{\left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt{x}\sqrt{a+bx}} - \right.\right.}{d}$$

$$= \frac{\left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx, x, \right.}{d}$$

$$= -\frac{\left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \left(\frac{i}{2(i-x)\sqrt{x}\sqrt{a+bx}} \right.\right.}{d}$$

$$= -\frac{\left(i\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{1}{(i-x)\sqrt{x}\sqrt{a+bx}} \right.}{2d}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{b}\,d}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{ia-b}\,d}$$

Mathematica [A]

time = 1.18, size = 209, normalized size = 0.99

$$\frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left((-1)^{3/4}\left(\frac{\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}}+\frac{\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{a+ib}}\right)+\frac{2\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{1+\frac{b\tan(c+dx)}{a}}}{\sqrt{b}\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cot[c + d*x]^(3/2)*Sqrt[a + b*Tan[c + d*x]]),x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-1)^(3/4)*(ArcTan[((-1)^(1/4)*Sqrt
[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]/Sqrt[-a + I*b] + A
rcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]
]]/Sqrt[a + I*b]) + (2*Sqrt[a]*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]
]*Sqrt[1 + (b*Tan[c + d*x])/a])/(Sqrt[b]*Sqrt[a + b*Tan[c + d*x]])))/d
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 36.24, size = 4705, normalized size = 22.19

method	result	size
default	Expression too large to display	4705

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*2^(1/2)*(-(a*cos(d*x+c)-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-b
+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*(((a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+
c)+a*cos(d*x+c)-a)/(a^2+b^2)^(1/2)/sin(d*x+c))^(1/2)*(a*(-1+cos(d*x+c))/(-b
+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*(-4*I*EllipticPi((-(a*cos(d*x+c)-(a^2+b
^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)
,(-b+(a^2+b^2)^(1/2))/(I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1
/2))/(a^2+b^2)^(1/2))^(1/2))*b^3*(a^2+b^2)^(1/2)+3*I*EllipticPi((-(a*cos(d*
x+c)-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^(1/2))/sin(d*
x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/2)*((-b+(
a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a^2*b^2+I*EllipticPi((-(a*cos(d*x+c
)-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c
))^(1/2),-(-b+(a^2+b^2)^(1/2))/(I*a-(a^2+b^2)^(1/2)+b),1/2*2^(1/2)*((-b+(a^
2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a^2*b*(a^2+b^2)^(1/2)-I*EllipticPi((-
(a*cos(d*x+c)-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^(1/2
))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/
2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a^2*b*(a^2+b^2)^(1/2)+4*I*
EllipticPi((-(a*cos(d*x+c)-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(
a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),-(-b+(a^2+b^2)^(1/2))/(I*a-(a^2+b^2)^(1/2
)+b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*b^3*(a^2+b^2
)^(1/2)-3*I*EllipticPi((-(a*cos(d*x+c)-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x
+c)-a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),-(-b+(a^2+b^2)^(1/2))/(I*a-(a
^2+b^2)^(1/2)+b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*
a^2*b^2-3*EllipticPi((-(a*cos(d*x+c)-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c
)-a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(-b+(a^2+b
^2)^(1/2)-a),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a^2*
b*(a^2+b^2)^(1/2)+2*EllipticPi((-(a*cos(d*x+c)-(a^2+b^2)^(1/2)*sin(d*x+c)+b
*sin(d*x+c)-a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/
(-b+(a^2+b^2)^(1/2)-a),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(
1/2))*a*b^2*(a^2+b^2)^(1/2)+2*EllipticPi((-(a*cos(d*x+c)-(a^2+b^2)^(1/2)*si
n(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),-(-b+(a^2+b
```

```
^2)^(1/2))/(I*a-(a^2+b^2)^(1/2)+b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b
^2)^(1/2))^(1/2))*a*b^2*(a^2+b^2)^(1/2)+3*EllipticPi((-(a*cos(d*x+c)-(a^2+b
^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)
,(-b+(a^2+b^2)^(1/2))/(a-b+(a^2+b^2)^(1/2)),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2
))/(a^2+b^2)^(1/2))^(1/2))*a^2*b*(a^2+b^2)^(1/2)+2*EllipticPi((-(a*cos(d*x+
c)-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+
c))^(1/2),(-b+(a^2+b^2)^(1/2))/(a-b+(a^2+b^2)^(1/2)),1/2*2^(1/2)*((-b+(a^2+
b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a*b^2*(a^2+b^2)^(1/2)+2*EllipticPi((-(a
*cos(d*x+c)-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^(1/2))
/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/2)
*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a*b^2*(a^2+b^2)^(1/2)-Ellipt
icPi((-(a*cos(d*x+c)-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+b^
2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(a-b+(a^2+b^2)^(1/2)),1/2*
2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a^4-4*EllipticPi((-(a
*cos(d*x+c)-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^(1/2))
/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(a-b+(a^2+b^2)^(1/2)),1/2*2^(1/2)*(
(-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*b^4+EllipticPi((-(a*cos(d*x+c)
-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c)
)^(1/2),(-b+(a^2+b^2)^(1/2))/(-b+(a^2+b^2)^(1/2)-a),1/2*2^(1/2)*((-b+(a^2+b
^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a^4+4*EllipticPi((-(a*cos(d*x+c)-(a^2+b^
2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),
(-b+(a^2+b^2)^(1/2))/(-b+(a^2+b^2)^(1/2)-a),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2
))/(a^2+b^2)^(1/2))^(1/2))*b^4+4*I*EllipticPi((-(a*cos(d*x+c)-(a^2+b^2)^(1/
2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a
^2+b^2)^(1/2))/(I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a
^2+b^2)^(1/2))^(1/2))*b^4-4*I*EllipticPi((-(a*cos(d*x+c)-(a^2+b^2)^(1/2)*si
n(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),-(-b+(a^2+b
^2)^(1/2))/(I*a-(a^2+b^2)^(1/2)+b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b
^2)^(1/2))^(1/2))*b^4+4*EllipticPi((-(a*cos(d*x+c)-(a^2+b^2)^(1/2)*sin(d*x+
c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/
2))/(a-b+(a^2+b^2)^(1/2)),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2)
)^(1/2))*b^3*(a^2+b^2)^(1/2)-EllipticPi((-(a*cos(d*x+c)-(a^2+b^2)^(1/2)*sin
(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2
)^(1/2))/(I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2
)^(1/2))^(1/2))*a^3*b-2*EllipticPi((-(a*cos(d*x+c)-(a^2+b^2)^(1/2)*sin(d*x+
c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/
2))/(I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/2)*((-b+(a...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*tan(d*x + c) + a)*cot(d*x + c)^(3/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\tan(c+dx)}\,\cot^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(3/2)/(a+b*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*tan(c + d*x))*cot(c + d*x)**(3/2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\mathrm{cot}\left(c+d\,x\right)}^{3/2}\,\sqrt{a+b\,\mathrm{tan}\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(1/2)), x)
```

3.866 $\int \frac{1}{\cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal. Leaf size=248

$$-\frac{i\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{ia-b}\,d}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{}}{b^{3/2}d}$$

```
[Out] -a*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2
)*tan(d*x+c)^(1/2)/b^(3/2)/d-I*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*t
an(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d/(I*a-b)^(1/2)+I*arcta
nh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*
tan(d*x+c)^(1/2)/d/(I*a+b)^(1/2)+(a+b*tan(d*x+c))^(1/2)/b/d/cot(d*x+c)^(1/2
)
```

Rubi [A]

time = 0.55, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {4326, 3647, 3736, 6857, 65, 223, 212, 924, 95, 211, 214}

$$-\frac{i\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}-\frac{a\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{b^{3/2}d}+\frac{\sqrt{a+b\tan(c+dx)}}{bd\sqrt{\cot(c+dx)}}+\frac{i\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cot[c + d*x]^(5/2)*Sqrt[a + b*Tan[c + d*x]]),x]
```

```
[Out] ((-I)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*S
qrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[I*a - b]*d) - (a*ArcTanh[(Sqrt[
b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Ta
n[c + d*x]])/(b^(3/2)*d) + (I*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sq
rt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[I*a +
b]*d) + Sqrt[a + b*Tan[c + d*x]]/(b*d*Sqrt[Cot[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
```

```
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{\cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{\tan^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$$

$$= \frac{\sqrt{a+b\tan(c+dx)}}{bd\sqrt{\cot(c+dx)}} + \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{-}{\sqrt{\tan}}}{b}$$

$$= \frac{\sqrt{a+b\tan(c+dx)}}{bd\sqrt{\cot(c+dx)}} + \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\text{Subst}\Big(\int}{bd}$$

$$= \frac{\sqrt{a+b\tan(c+dx)}}{bd\sqrt{\cot(c+dx)}} + \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\text{Subst}\Big(\int}{}$$

$$= \frac{\sqrt{a+b\tan(c+dx)}}{bd\sqrt{\cot(c+dx)}} - \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\text{Subst}\Big(\int}{d}$$

$$= \frac{\sqrt{a+b\tan(c+dx)}}{bd\sqrt{\cot(c+dx)}} - \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\text{Subst}\Big(\int}{}$$

$$= \frac{\sqrt{a+b\tan(c+dx)}}{bd\sqrt{\cot(c+dx)}} + \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\text{Subst}\Big(\int}{2d}$$

$$= -\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{b^{3/2}d}$$

$$= -\frac{i\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx}}{\sqrt{ia-b}\,d}$$

Mathematica [A]
time = 2.47, size = 243, normalized size = 0.98

$$\frac{\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\left(\frac{\sqrt[4]{-1}\,b\,\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}} - \frac{\sqrt[4]{-1}\,b\,\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{a+ib}} + \sqrt{\tan(c+dx)}\,\sqrt{a+b\tan(c+dx)} - \frac{a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{1+\frac{b\tan(c+dx)}{a}}}{\sqrt{b}\,\sqrt{a+b\tan(c+dx)}}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cot[c + d*x]^(5/2)*Sqrt[a + b*Tan[c + d*x]]),x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(((-1)^(1/4)*b*ArcTan[((-1)^(1/4)*Sq
rt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[-a + I*b]
```

```
- ((-1)^(1/4)*b*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a
 + b*Tan[c + d*x]]])/Sqrt[a + I*b] + Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c +
d*x]] - (a^(3/2)*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 + (b*
Tan[c + d*x])/a])/(Sqrt[b]*Sqrt[a + b*Tan[c + d*x]])))/(b*d)
```

Maple [**C**] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 35.54, size = 11480, normalized size = 46.29

method	result	size
default	Expression too large to display	11480

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*tan(d*x + c) + a)*cot(d*x + c)^(5/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\tan\left(c + dx\right)}\,\cot^{\frac{5}{2}}\left(c + dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(5/2)/(a+b*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*tan(c + d*x))*cot(c + d*x)**(5/2)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cot(c+d\,x)^{5/2}\,\sqrt{a+b\tan(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))^(1/2)),x)
[Out] int(1/(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))^(1/2)), x)
```

3.867 $\int \frac{\cot^{\frac{5}{2}}(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=281

$$-\frac{i\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{(ia-b)^{3/2}d}-\frac{i\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{3}}$$

```
[Out] -I*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)
^(1/2)*tan(d*x+c)^(1/2)/(I*a-b)^(3/2)/d-I*arctanh((I*a+b)^(1/2)*tan(d*x+c)^
(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a+b)^(3/
2)/d-2/3*cot(d*x+c)^(3/2)/a/d/(a+b*tan(d*x+c))^(1/2)+2/3*b^2*(5*a^2+8*b^2)/
a^3/(a^2+b^2)/d/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2)+8/3*b*cot(d*x+c)^(1
/2)/a^2/d/(a+b*tan(d*x+c))^(1/2)
```

Rubi [A]

time = 0.65, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4326, 3650, 3730, 3731, 3697, 3696, 95, 209, 212}

$$\frac{8b\sqrt{\cot(c+dx)}}{3a^2d\sqrt{a+b\tan(c+dx)}}+\frac{2b^2(5a^2+8b^2)}{3a^3d(a^2+b^2)\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}-\frac{i\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{3/2}}-\frac{2\cot^{\frac{3}{2}}(c+dx)}{3ad\sqrt{a+b\tan(c+dx)}}-\frac{i\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(5/2)/(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-I)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*S
qrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/((I*a - b)^(3/2)*d) - (I*ArcTanh[(Sqr
t[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]
*Sqrt[Tan[c + d*x]])/((I*a + b)^(3/2)*d) + (2*b^2*(5*a^2 + 8*b^2))/(3*a^3*(
a^2 + b^2)*d*Sqrt[Cot[c + d*x]]*Sqrt[a + b*Tan[c + d*x]]) + (8*b*Sqrt[Cot[c
 + d*x]])/(3*a^2*d*Sqrt[a + b*Tan[c + d*x]]) - (2*Cot[c + d*x]^(3/2))/(3*a*
d*Sqrt[a + b*Tan[c + d*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
```

```
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3731

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)
*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp
[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - a*C*(b*c*(m + 1) + a*d*(n
+ 1)) - (m + 1)*(b*c - a*d)*(A*b - b*C)*Tan[e + f*x] - d*(A*b^2 + a^2*C)*(m
 + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m,
 -1] &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx = \left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx$$

$$= -\frac{2\cot^{\frac{3}{2}}(c+dx)}{3ad\sqrt{a+b\tan(c+dx)}} - \frac{\left(2\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{2b+\frac{3}{2}a\tan(c+}{\tan^{\frac{3}{2}}(c+dx)(a}}{3a}$$

$$= \frac{8b\sqrt{\cot(c+dx)}}{3a^2d\sqrt{a+b\tan(c+dx)}} - \frac{2\cot^{\frac{3}{2}}(c+dx)}{3ad\sqrt{a+b\tan(c+dx)}} + \frac{\left(4\sqrt{\cot(c+dx)}\sqrt{t\right.}}{}$$

$$= \frac{2b^2(5a^2+8b^2)}{3a^3(a^2+b^2)\,d\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}} + \frac{8b\sqrt{\cot(c+dx)}}{3a^2d\sqrt{a+b\tan(c+dx)}}$$

$$= \frac{2b^2(5a^2+8b^2)}{3a^3(a^2+b^2)\,d\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}} + \frac{8b\sqrt{\cot(c+dx)}}{3a^2d\sqrt{a+b\tan(c+dx)}}$$

$$= \frac{2b^2(5a^2+8b^2)}{3a^3(a^2+b^2)\,d\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}} + \frac{8b\sqrt{\cot(c+dx)}}{3a^2d\sqrt{a+b\tan(c+dx)}}$$

$$= \frac{2b^2(5a^2+8b^2)}{3a^3(a^2+b^2)\,d\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}} + \frac{8b\sqrt{\cot(c+dx)}}{3a^2d\sqrt{a+b\tan(c+dx)}}$$

$$= -\frac{i\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{(ia-b)^{3/2}d} - \frac{i\tanh}{}$$

Mathematica [A]
time = 5.78, size = 241, normalized size = 0.86

$$\frac{\sqrt{\cot(c+dx)}\left(-\frac{3(-1)^{3/4}\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\tan(c+dx)}}{(-a+ib)^{3/2}}+\frac{3(-1)^{3/4}\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\tan(c+dx)}}{(a+ib)^{3/2}}+\frac{-2a^2(a^2+b^2)\cot(c+dx)+2b(4a(a^2+b^2)+b(5a^2+8b^2)\tan(c+dx))}{a^3(a^2+b^2)\sqrt{a+b\tan(c+dx)}}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(5/2)/(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*((-3*(-1)^(3/4)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[
Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Tan[c + d*x]])/(-a + I*b)^(3/
2) + (3*(-1)^(3/4)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqr
t[a + b*Tan[c + d*x]]]*Sqrt[Tan[c + d*x]])/(a + I*b)^(3/2) + (-2*a^2*(a^2 +
```

```
b^2)*Cot[c + d*x] + 2*b*(4*a*(a^2 + b^2) + b*(5*a^2 + 8*b^2)*Tan[c + d*x])
)/(a^3*(a^2 + b^2)*Sqrt[a + b*Tan[c + d*x]])))/(3*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 37.94, size = 9931, normalized size = 35.34

method	result	size
default	Expression too large to display	9931

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(cot(d*x + c)^(5/2)/(b*tan(d*x + c) + a)^(3/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(5/2)/(a+b*tan(d*x+c))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^{5/2}}{\left(a+b\tan\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(5/2)/(a + b*tan(c + d*x))^(3/2),x)
```

```
[Out] int(cot(c + d*x)^(5/2)/(a + b*tan(c + d*x))^(3/2), x)
```

3.868 $\int \frac{\cot^{\frac{3}{2}}(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=233

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{(ia-b)^{3/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{c}}{(ia+b)^{3/2}d}$$

```
[Out] arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1
/2)*tan(d*x+c)^(1/2)/(I*a-b)^(3/2)/d-arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)
/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a+b)^(3/2)/d-
2*b*(a^2+2*b^2)/a^2/(a^2+b^2)/d/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2)-2*c
ot(d*x+c)^(1/2)/a/d/(a+b*tan(d*x+c))^(1/2)
```

Rubi [A]

time = 0.50, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4326, 3650, 3730, 3697, 3696, 95, 209, 212}

$$-\frac{2b(a^2+2b^2)}{a^2d(a^2+b^2)\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}+\frac{\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\operatorname{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{3/2}}-\frac{2\sqrt{\cot(c+dx)}}{ad\sqrt{a+b\tan(c+dx)}}-\frac{\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(3/2)/(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] (ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[C
ot[c + d*x]]*Sqrt[Tan[c + d*x]])/((I*a - b)^(3/2)*d) - (ArcTanh[(Sqrt[I*a +
 b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[T
an[c + d*x]])/((I*a + b)^(3/2)*d) - (2*b*(a^2 + 2*b^2))/(a^2*(a^2 + b^2)*d*
Sqrt[Cot[c + d*x]]*Sqrt[a + b*Tan[c + d*x]]) - (2*Sqrt[Cot[c + d*x]])/(a*d*
Sqrt[a + b*Tan[c + d*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
```

```
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx = \left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx$$

$$= -\frac{2\sqrt{\cot(c+dx)}}{ad\sqrt{a+b\tan(c+dx)}} - \frac{\left(2\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{b+\frac{1}{2}a\tan(}{\sqrt{\tan(c+}}}{a}$$

$$= -\frac{2b(a^2+2b^2)}{a^2(a^2+b^2)\,d\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}} - \frac{2\sqrt{\cot(c+dx)}}{ad\sqrt{a+b\tan(c+dx)}}$$

$$= -\frac{2b(a^2+2b^2)}{a^2(a^2+b^2)\,d\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}} - \frac{2\sqrt{\cot(c+dx)}}{ad\sqrt{a+b\tan(c+dx)}}$$

$$= -\frac{2b(a^2+2b^2)}{a^2(a^2+b^2)\,d\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}} - \frac{2\sqrt{\cot(c+dx)}}{ad\sqrt{a+b\tan(c+dx)}}$$

$$= -\frac{2b(a^2+2b^2)}{a^2(a^2+b^2)\,d\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}} - \frac{2\sqrt{\cot(c+dx)}}{ad\sqrt{a+b\tan(c+dx)}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{(ia-b)^{3/2}d} - \frac{\tanh^{-1}(}{}$$

Mathematica [A]
time = 5.92, size = 229, normalized size = 0.98

$$-\frac{\sqrt{\cot(c+dx)}\left(\frac{\sqrt[4]{-1}\,a\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\tan(c+dx)}}{(-a+ib)^{3/2}}+\frac{\frac{\sqrt[4]{-1}\,a^2(a-ib)\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\tan(c+dx)}}{\sqrt{a+ib}}+\frac{2\left(a\left(a^2+b^2\right)+b\left(a^2+2b^2\right)\tan(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}}{a(a^2+b^2)}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(3/2)/(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] -((Sqrt[Cot[c + d*x]]*(((-1)^(1/4)*a*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt
[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Tan[c + d*x]])/(-a + I*b)^(3
/2) + (((-1)^(1/4)*a^2*(a - I*b)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[
c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*b] + (2
*(a*(a^2 + b^2) + b*(a^2 + 2*b^2)*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x]])/
(a*(a^2 + b^2))))/(a*d))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 38.64, size = 9450, normalized size = 40.56

method	result	size
default	Expression too large to display	9450

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^(3/2)/(b*tan(d*x + c) + a)^(3/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)}{(a+b\tan(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(3/2)/(a+b*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(cot(c + d*x)**(3/2)/(a + b*tan(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)^(3/2)/(b*tan(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^{3/2}}{\left(a+b\tan\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(3/2)/(a + b*tan(c + d*x))^(3/2),x)
```

```
[Out] int(cot(c + d*x)^(3/2)/(a + b*tan(c + d*x))^(3/2), x)
```

3.869 $\int \frac{\sqrt{\cot(c+dx)}}{(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=199

$$\frac{i\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{(ia-b)^{3/2}d}+\frac{i\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{}}{(ia+b)^{3/2}d}$$

```
[Out] I*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^
(1/2)*tan(d*x+c)^(1/2)/(I*a-b)^(3/2)/d+I*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(
1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a+b)^(3/2
)/d+2*b^2/a/(a^2+b^2)/d/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4326, 3650, 3697, 3696, 95, 209, 212}

$$\frac{2b^2}{ad(a^2+b^2)\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}+\frac{i\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{3/2}}+\frac{i\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cot[c + d*x]]/(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] (I*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt
[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/((I*a - b)^(3/2)*d) + (I*ArcTanh[(Sqrt[I
*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sq
rt[Tan[c + d*x]])/((I*a + b)^(3/2)*d) + (2*b^2)/(a*(a^2 + b^2)*d*Sqrt[Cot[c
 + d*x]]*Sqrt[a + b*Tan[c + d*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{\sqrt{\cot(c+dx)}}{(a+b\tan(c+dx))^{3/2}}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{1}{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{3/2}}\,dx$$

$$= \frac{2b^2}{a\,(a^2+b^2)\,d\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}} + \frac{\left(2\sqrt{\cot(c+dx)}\,\sqrt{\tan(c\cdots\right.}{\cdots}$$

$$= \frac{2b^2}{a\,(a^2+b^2)\,d\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}} + \frac{\left((a-ib)\sqrt{\cot(c+dx)}\,\sqrt{\cdots\right.}{\cdots}$$

$$= \frac{2b^2}{a\,(a^2+b^2)\,d\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}} + \frac{\left((a-ib)\sqrt{\cot(c+dx)}\,\sqrt{\cdots\right.}{\cdots}$$

$$= \frac{2b^2}{a\,(a^2+b^2)\,d\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}} + \frac{\left((a-ib)\sqrt{\cot(c+dx)}\,\sqrt{\cdots\right.}{\cdots}$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{(ia-b)^{3/2}d} + \frac{i\tanh^{-1}\cdots}{\cdots}$$

Mathematica [A]
time = 1.04, size = 203, normalized size = 1.02

$$-\frac{\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\left(\frac{(-1)^{3/4}(a+ib)\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}} + \frac{\sqrt[4]{-1}\,(ia+b)\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{a+ib}} - \frac{2b^2\sqrt{\tan(c+dx)}}{a\sqrt{a+b\tan(c+dx)}}\right)}{(a^2+b^2)\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cot[c + d*x]]/(a + b*Tan[c + d*x])^(3/2),x]
```

```
[Out] -((Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(((-1)^(3/4)*(a + I*b)*ArcTan[((-1
)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[
-a + I*b] + ((-1)^(1/4)*(I*a + b)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan
[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[a + I*b] - (2*b^2*Sqrt[Tan[c +
d*x]])/(a*Sqrt[a + b*Tan[c + d*x]])))/((a^2 + b^2)*d))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 38.35, size = 4830, normalized size = 24.27

method	result	size

default	Expression too large to display	4830

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/d*(cos(d*x+c)/sin(d*x+c))^(1/2)*((a*cos(d*x+c)+b*sin(d*x+c))/cos(d*x+c))^(1/2)*sin(d*x+c)*(-I*(((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*(((a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)+a*cos(d*x+c)-a)/(a^2+b^2)^(1/2)/sin(d*x+c))^(1/2)*(a*(-1+cos(d*x+c))/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*sin(d*x+c)*a^4+3*I*(a^2+b^2)^(1/2)*(((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*(((a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)+a*cos(d*x+c)-a)/(a^2+b^2)^(1/2)/sin(d*x+c))^(1/2)*(a*(-1+cos(d*x+c))/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*sin(d*x+c)*a^2*b-3*I*(((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*(((a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)+a*cos(d*x+c)-a)/(a^2+b^2)^(1/2)/sin(d*x+c))^(1/2)*(a*(-1+cos(d*x+c))/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*sin(d*x+c)*a^2*b^2+3*I*(((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*(((a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)+a*cos(d*x+c)-a)/(a^2+b^2)^(1/2)/sin(d*x+c))^(1/2)*(a*(-1+cos(d*x+c))/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(-I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*sin(d*x+c)*a^2*b^2+I*(((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*(((a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)+a*cos(d*x+c)-a)/(a^2+b^2)^(1/2)/sin(d*x+c))^(1/2)*(a*(-1+cos(d*x+c))/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(-I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*sin(d*x+c)*a^4-3*I*(a^2+b^2)^(1/2)*(((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*(((a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)+a*cos(d*x+c)-a)/(a^2+b^2)^(1/2)/sin(d*x+c))^(1/2)*(a*(-1+cos(d*x+c))/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^

```
2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(-I*a+(a^2+b^2)^(1/2)-b),1
/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*sin(d*x+c)*a^2*b-(
a^2+b^2)^(1/2)*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d
*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(
a^2+b^2)^(1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))
*(((a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)+a*cos(d*x+c)-a)/(a^2+b^2)^(1/2)/
sin(d*x+c))^(1/2)*(a*(-1+cos(d*x+c))/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)
*sin(d*x+c)*(((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(
a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*a^3+2*(a^2+b^2)^(1/2)*(((a^2+b^2)^(1/2)*s
in(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/
2)*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b
+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^2+b^2)^(1/
2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*(((a^2+b^2)
^(1/2)*sin(d*x+c)+b*sin(d*x+c)+a*cos(d*x+c)-a)/(a^2+b^2)^(1/2)/sin(d*x+c))^
(1/2)*(a*(-1+cos(d*x+c))/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*sin(d*x+c)*
a*b^2-(a^2+b^2)^(1/2)*(((a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)+a*cos(d*x+c
)-a)/(a^2+b^2)^(1/2)/sin(d*x+c))^(1/2)*(a*(-1+cos(d*x+c))/(-b+(a^2+b^2)^(1/
2))/sin(d*x+c))^(1/2)*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-
a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))
/(-I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2)
)^(1/2))*sin(d*x+c)*(((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+
a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*a^3+2*(a^2+b^2)^(1/2)*(((a^2+b^2)
^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x
+c))^(1/2)*(((a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)+a*cos(d*x+c)-a)/(a^2+b
^2)^(1/2)/sin(d*x+c))^(1/2)*(a*(-1+cos(d*x+c))/(-b+(a^2+b^2)^(1/2))/sin(d*x
+c))^(1/2)*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c
)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(-I*a+(a^2
+b^2)^(1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*si
n(d*x+c)*a*b^2+2*(a^2+b^2)^(1/2)*(((a^2+b^2)^(1...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(cot(d*x + c))/(b*tan(d*x + c) + a)^(3/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cot{\left(c+dx \right)}}}{\left(a+b\tan{\left(c+dx \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(1/2)/(a+b*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(cot(c + d*x))/(a + b*tan(c + d*x))**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\mathrm{cot}\left(c+d\,x\right)}}{{\left(a+b\,\mathrm{tan}\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(1/2)/(a + b*tan(c + d*x))^(3/2),x)
```

```
[Out] int(cot(c + d*x)^(1/2)/(a + b*tan(c + d*x))^(3/2), x)
```

3.870 $\int \frac{1}{\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=189

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{(ia-b)^{3/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{}}{(ia+b)^{3/2}}$$

```
[Out] -arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(
1/2)*tan(d*x+c)^(1/2)/(I*a-b)^(3/2)/d+arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2
)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a+b)^(3/2)/d
-2*b/(a^2+b^2)/d/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2)
```

Rubi [A]

time = 0.36, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4326, 3649, 3697, 3696, 95, 209, 212}

$$-\frac{2b}{d(a^2+b^2)\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}-\frac{\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{3/2}}+\frac{\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^(3/2)),x]
```

```
[Out] -((ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt
[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/((I*a - b)^(3/2)*d)) + (ArcTanh[(Sqrt[I*
a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqr
t[Tan[c + d*x]])/((I*a + b)^(3/2)*d) - (2*b)/((a^2 + b^2)*d*Sqrt[Cot[c + d*
x]]*Sqrt[a + b*Tan[c + d*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{\sqrt{\tan(c+dx)}}{(a+b\tan(c+dx))^{3/2}}\,dx$$

$$= -\frac{2b}{(a^2+b^2)\,d\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}} - \frac{\left(2\sqrt{\cot(c\cdots\right.}{\cdots}$$

$$= -\frac{2b}{(a^2+b^2)\,d\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}} - \frac{\left((-ia-b)\cdots\right.}{\cdots}$$

$$= -\frac{2b}{(a^2+b^2)\,d\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}} - \frac{\left((-ia-b)\cdots\right.}{\cdots}$$

$$= -\frac{2b}{(a^2+b^2)\,d\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}} - \frac{\left((-ia-b)\cdots\right.}{\cdots}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+\cdots}{(ia-b)^{3/2}d}$$

Mathematica [A]
time = 2.05, size = 183, normalized size = 0.97

$$\frac{\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\left(\frac{\sqrt[4]{-1}\,\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(-a+ib)^{3/2}}+\frac{\sqrt[4]{-1}\,\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(a+ib)^{3/2}}-\frac{2b\sqrt{\tan(c+dx)}}{(a^2+b^2)\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^(3/2)),x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(((-1)^(1/4)*ArcTan[((-1)^(1/4)*Sqrt
[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(-a + I*b)^(3/2)
+ ((-1)^(1/4)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a +
 b*Tan[c + d*x]]])/(a + I*b)^(3/2) - (2*b*Sqrt[Tan[c + d*x]])/((a^2 + b^2)*
Sqrt[a + b*Tan[c + d*x]])))/d
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 39.41, size = 4836, normalized size = 25.59

method	result	size

default	Expression too large to display	4836

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/d*(2*I*sin(d*x+c)*(((a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)+a*cos(d*x+c)-a)/(a^2+b^2)^(1/2)/sin(d*x+c))^(1/2)*(a*(-1+cos(d*x+c))/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*(((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*(a^2+b^2)^(1/2)*a*b^2+2*I*sin(d*x+c)*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),-(-b+(a^2+b^2)^(1/2))/(I*a-(a^2+b^2)^(1/2)+b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*(((a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)+a*cos(d*x+c)-a)/(a^2+b^2)^(1/2)/sin(d*x+c))^(1/2)*(a*(-1+cos(d*x+c))/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*(((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*a*b^3-2*I*sin(d*x+c)*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),-(-b+(a^2+b^2)^(1/2))/(I*a-(a^2+b^2)^(1/2)+b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*(((a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)+a*cos(d*x+c)-a)/(a^2+b^2)^(1/2)/sin(d*x+c))^(1/2)*(a*(-1+cos(d*x+c))/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*(((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*(a^2+b^2)^(1/2)*a*b^2-2*I*sin(d*x+c)*(((a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)+a*cos(d*x+c)-a)/(a^2+b^2)^(1/2)/sin(d*x+c))^(1/2)*(a*(-1+cos(d*x+c))/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*(((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*a*b^3+I*sin(d*x+c)*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),-(-b+(a^2+b^2)^(1/2))/(I*a-(a^2+b^2)^(1/2)+b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*(((a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)+a*cos(d*x+c)-a)/(a^2+b^2)^(1/2)/sin(d*x+c))^(1/2)*(a*(-1+cos(d*x+c))/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*(((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*(a^2+b^2)^(1/2)*a^3-I*sin(d*x+c)*(((a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)+a*cos(d*x+c)-a)/(a^2+b^2)^(1/2)/sin(d*x+c))^(1/2)*(a*(-1+cos(d*x+c))/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*(((a^2+b^2)^(1/2)*sin(d*x+c)

```
-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*(a^2+b
^2)^(1/2)*a^3-3*sin(d*x+c)*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*
x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),-(-b+(a^2+b^2)^
(1/2))/(I*a-(a^2+b^2)^(1/2)+b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^
(1/2))^(1/2))*(((a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)+a*cos(d*x+c)-a)/(a^
2+b^2)^(1/2)/sin(d*x+c))^(1/2)*(a*(-1+cos(d*x+c))/(-b+(a^2+b^2)^(1/2))/sin(
d*x+c))^(1/2)*(((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b
+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*(a^2+b^2)^(1/2)*a^2*b+sin(d*x+c)*Ellipt
icPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2
)^(1/2))/sin(d*x+c))^(1/2),-(-b+(a^2+b^2)^(1/2))/(I*a-(a^2+b^2)^(1/2)+b),1/
2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*(((a^2+b^2)^(1/2)*s
in(d*x+c)+b*sin(d*x+c)+a*cos(d*x+c)-a)/(a^2+b^2)^(1/2)/sin(d*x+c))^(1/2)*(a
*(-1+cos(d*x+c))/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*(((a^2+b^2)^(1/2)*s
in(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/
2)*a^4+3*sin(d*x+c)*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*
cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),-(-b+(a^2+b^2)^(1/2))/
(I*a-(a^2+b^2)^(1/2)+b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^
(1/2))*(((a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)+a*cos(d*x+c)-a)/(a^2+b^2)^
(1/2)/sin(d*x+c))^(1/2)*(a*(-1+cos(d*x+c))/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))
^(1/2)*(((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b
^2)^(1/2))/sin(d*x+c))^(1/2)*a^2*b^2-3*sin(d*x+c)*(((a^2+b^2)^(1/2)*sin(d*x
+c)+b*sin(d*x+c)+a*cos(d*x+c)-a)/(a^2+b^2)^(1/2)/sin(d*x+c))^(1/2)*(a*(-1+c
os(d*x+c))/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*EllipticPi((((a^2+b^2)^(1
/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c)
)^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/2)*((-b+(a^2+
b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*(((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+
c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*(a^2+b^2)^(1/2)*a
^2*b+sin(d*x+c)*(((a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)+a*cos(d*x+c)-a)/(
a^2+b^2)^(1/2)/sin(d*x+c))^(1/2)*(a*(-1+cos(d*x...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*tan(d*x + c) + a)^(3/2)*sqrt(cot(d*x + c))), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\tan{(c+dx)})^{\frac{3}{2}}\sqrt{\cot{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(1/2)/(a+b*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/((a + b*tan(c + d*x))**(3/2)*sqrt(cot(c + d*x))), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\mathrm{cot}\left(c+d\,x\right)}\,{\left(a+b\,\mathrm{tan}\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(3/2)), x)
```

3.871 $\int \frac{1}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=194

$$-\frac{i\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{(ia-b)^{3/2}d}-\frac{i\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{3/2}}$$

```
[Out] -I*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)
^(1/2)*tan(d*x+c)^(1/2)/(I*a-b)^(3/2)/d-I*arctanh((I*a+b)^(1/2)*tan(d*x+c)^
(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a+b)^(3/
2)/d+2*a/(a^2+b^2)/d/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2)
```

Rubi [A]

time = 0.35, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4326, 3648, 3697, 3696, 95, 209, 212}

$$\frac{2a}{d(a^2+b^2)\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}-\frac{i\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{3/2}}-\frac{i\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(3/2)),x]
```

```
[Out] ((-I)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*S
qrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/((I*a - b)^(3/2)*d) - (I*ArcTanh[(Sqr
t[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]
*Sqrt[Tan[c + d*x]])/((I*a + b)^(3/2)*d) + (2*a)/((a^2 + b^2)*d*Sqrt[Cot[c
+ d*x]]*Sqrt[a + b*Tan[c + d*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{1}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{\tan^{\frac{3}{2}}(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx$$

$$= \frac{2a}{(a^2+b^2)\,d\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}} - \frac{\left(2\sqrt{\cot(c+dx\right.}{}$$

$$= \frac{2a}{(a^2+b^2)\,d\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}} - \frac{\left((a-ib)\sqrt{\cot\right.}{}$$

$$= \frac{2a}{(a^2+b^2)\,d\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}} - \frac{\left((a-ib)\sqrt{\cot\right.}{}$$

$$= \frac{2a}{(a^2+b^2)\,d\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}} - \frac{\left((a-ib)\sqrt{\cot\right.}{}$$

$$= -\frac{i\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+d}}{(ia-b)^{3/2}d}$$

Mathematica [A]
time = 1.84, size = 202, normalized size = 1.04

$$\frac{\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\left(-\frac{(-1)^{3/4}\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(-a+ib)^{3/2}}+\frac{\frac{\sqrt[4]{-1}\,(ia+b)\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(a+ib)^{3/2}}+\frac{2a\sqrt{\tan(c+dx)}}{(a+ib)\sqrt{a+b\tan(c+dx)}}}{a-ib}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(3/2)),x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-(((-1)^(3/4)*ArcTan[((-1)^(1/4)*Sq
rt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(-a + I*b)^(3/2
)) + (((-1)^(1/4)*(I*a + b)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d
*x]])/Sqrt[a + b*Tan[c + d*x]]])/(a + I*b)^(3/2) + (2*a*Sqrt[Tan[c + d*x]])
/((a + I*b)*Sqrt[a + b*Tan[c + d*x]]))/(a - I*b)))/d
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 34.00, size = 4822, normalized size = 24.86

method	result	size

default	Expression too large to display	4822

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/d*(-3*I*b^2*(((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*(((a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)+a*cos(d*x+c)-a)/(a^2+b^2)^(1/2)/sin(d*x+c))^(1/2)*(a*(-1+cos(d*x+c))/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),-(-b+(a^2+b^2)^(1/2))/(I*a-(a^2+b^2)^(1/2)+b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*sin(d*x+c)*a-3*I*(((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*(((a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)+a*cos(d*x+c)-a)/(a^2+b^2)^(1/2)/sin(d*x+c))^(1/2)*(a*(-1+cos(d*x+c))/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*(a^2+b^2)^(1/2)*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*sin(d*x+c)*a*b-I*a^3*(((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*(((a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)+a*cos(d*x+c)-a)/(a^2+b^2)^(1/2)/sin(d*x+c))^(1/2)*(a*(-1+cos(d*x+c))/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),-(-b+(a^2+b^2)^(1/2))/(I*a-(a^2+b^2)^(1/2)+b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*sin(d*x+c)+3*I*b^2*(((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*(((a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)+a*cos(d*x+c)-a)/(a^2+b^2)^(1/2)/sin(d*x+c))^(1/2)*(a*(-1+cos(d*x+c))/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*sin(d*x+c)*a+3*I*(((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*(((a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)+a*cos(d*x+c)-a)/(a^2+b^2)^(1/2)/sin(d*x+c))^(1/2)*(a*(-1+cos(d*x+c))/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*(a^2+b^2)^(1/2)*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),-(-b+(a^2+b^2)^(1/2))/(I*a-(a^2+b^2)^(1/2)+b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*sin(d*x+c)*a*b+I*a^3*(((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*(((a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)+a*cos(d*x+c)-a)/(a^2+b^2)^(1/2)/sin(d*x+c))^(1/2)*(a*(-1+cos(d*x+c))/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/2)*((-

```
b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*sin(d*x+c)+a^2*(((a^2+b^2)^(1/2)
*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(
1/2)*(((a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)+a*cos(d*x+c)-a)/(a^2+b^2)^(1
/2)/sin(d*x+c))^(1/2)*(a*(-1+cos(d*x+c))/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(
1/2)*(a^2+b^2)^(1/2)*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a
*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),-(-b+(a^2+b^2)^(1/2))
/(I*a-(a^2+b^2)^(1/2)+b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))
^(1/2))*sin(d*x+c)-2*b^2*(((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*
x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*(((a^2+b^2)^(1/2)*sin(d*x+c)
+b*sin(d*x+c)+a*cos(d*x+c)-a)/(a^2+b^2)^(1/2)/sin(d*x+c))^(1/2)*(a*(-1+cos(
d*x+c))/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*(a^2+b^2)^(1/2)*EllipticPi((
((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2
))/sin(d*x+c))^(1/2),-(-b+(a^2+b^2)^(1/2))/(I*a-(a^2+b^2)^(1/2)+b),1/2*2^(1
/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*sin(d*x+c)+a^2*(((a^2+b^2
)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*
x+c))^(1/2)*(((a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)+a*cos(d*x+c)-a)/(a^2+
b^2)^(1/2)/sin(d*x+c))^(1/2)*(a*(-1+cos(d*x+c))/(-b+(a^2+b^2)^(1/2))/sin(d*
x+c))^(1/2)*(a^2+b^2)^(1/2)*EllipticPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d
*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^
(1/2))/(I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^
(1/2))^(1/2))*sin(d*x+c)-2*b^2*(((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*
cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*(((a^2+b^2)^(1/2)*sin(
d*x+c)+b*sin(d*x+c)+a*cos(d*x+c)-a)/(a^2+b^2)^(1/2)/sin(d*x+c))^(1/2)*(a*(-
1+cos(d*x+c))/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*(a^2+b^2)^(1/2)*Ellipt
icPi((((a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2
)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^2+b^2)^(1/2)-b),1/2
*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*sin(d*x+c)-2*a^2*(((
a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)-a*cos(d*x+c)+a)/(-b+(a^2+b^2)^(1/2))
/sin(d*x+c))^(1/2)*(((a^2+b^2)^(1/2)*sin(d*x+c)...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^(3/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\tan(c+dx))^{\frac{3}{2}}\cot^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(3/2)/(a+b*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/((a + b*tan(c + d*x))**(3/2)*cot(c + d*x)**(3/2)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\cot\left(c+d\,x\right)}^{3/2}\,{\left(a+b\,\tan\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(3/2)), x)
```

3.872 $\int \frac{1}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=255

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{(ia-b)^{3/2}d}+\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(}}{b^{3/2}d}$$

```
[Out] arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1
/2)*tan(d*x+c)^(1/2)/(I*a-b)^(3/2)/d+2*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+
b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/b^(3/2)/d-arctanh((I
*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d
*x+c)^(1/2)/(I*a+b)^(3/2)/d-2*a^2/b/(a^2+b^2)/d/cot(d*x+c)^(1/2)/(a+b*tan(d
*x+c))^(1/2)
```

Rubi [A]
time = 0.91, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4326, 3646, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$-\frac{2a^2}{bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}+\frac{\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\operatorname{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{3/2}}+\frac{2\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{b^{3/2}d}-\frac{\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^(3/2)),x]
```

```
[Out] (ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[C
ot[c + d*x]]*Sqrt[Tan[c + d*x]])/((I*a - b)^(3/2)*d) + (2*ArcTanh[(Sqrt[b]*
Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c
 + d*x]])/(b^(3/2)*d) - (ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a
+ b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/((I*a + b)^(3/2)*
d) - (2*a^2)/(b*(a^2 + b^2)*d*Sqrt[Cot[c + d*x]]*Sqrt[a + b*Tan[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
```

```
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{\tan^{\frac{5}{2}}(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx$$

$$= -\frac{2a^2}{b\,(a^2+b^2)\,d\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}} + \frac{\left(2\sqrt{\cot(c \cdots\right.}{\cdots}$$

$$= -\frac{2a^2}{b\,(a^2+b^2)\,d\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}} + \frac{\left(2\sqrt{\cot(c \cdots\right.}{\cdots}$$

$$= -\frac{2a^2}{b\,(a^2+b^2)\,d\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}} + \frac{\left(2\sqrt{\cot(c \cdots\right.}{\cdots}$$

$$= -\frac{2a^2}{b\,(a^2+b^2)\,d\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}} + \frac{\left(\sqrt{\cot(c - \cdots\right.}{\cdots}$$

$$= -\frac{2a^2}{b\,(a^2+b^2)\,d\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}} + \frac{\left(2\sqrt{\cot(c \cdots\right.}{\cdots}$$

$$= -\frac{2a^2}{b\,(a^2+b^2)\,d\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}} + \frac{\left(2\sqrt{\cot(c \cdots\right.}{\cdots}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)} \cdots}{b^{3/2}d}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)} \cdots}{(ia-b)^{3/2}d}$$

Mathematica [A]
time = 2.06, size = 341, normalized size = 1.34

$$-\frac{\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\left(2\sqrt{a}\,\sqrt{-a+ib}\,\sqrt{a+ib}\,(a^2+b^2)\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{1+\frac{b\tan(c+dx)}{a}}+\sqrt{b}\left(\sqrt[4]{-1}\,(a+ib)^{3/2}b\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{a+b\tan(c+dx)}+\sqrt{-a+ib}\left(-2a^2\sqrt{a+ib}\,\sqrt{\tan(c+dx)}-\sqrt[4]{-1}\,(a-ib)b\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{a+b\tan(c+dx)}\right)\right)\right)}{(-a+ib)^{3/2}(a+ib)^{3/2}b^{3/2}d\sqrt{a+b\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^(3/2)),x]
```

```
[Out] -((Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(2*Sqrt[a]*Sqrt[-a + I*b]*Sqrt[a +
 I*b]*(a^2 + b^2)*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 + (b
```

```
*Tan[c + d*x])/a] + Sqrt[b]*((-1)^(1/4)*(a + I*b)^(3/2)*b*ArcTan[((-1)^(1/4
)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[a + b*T
an[c + d*x]] + Sqrt[-a + I*b]*(-2*a^2*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]] - (-
1)^(1/4)*(a - I*b)*b*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/S
qrt[a + b*Tan[c + d*x]]]*Sqrt[a + b*Tan[c + d*x]])))/((-a + I*b)^(3/2)*(a
+ I*b)^(3/2)*b^(3/2)*d*Sqrt[a + b*Tan[c + d*x]]))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 36.48, size = 14605, normalized size = 57.27

method	result	size
default	Expression too large to display	14605

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^(5/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(5/2)/(a+b*tan(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cot\left(c+d\,x\right)^{5/2}\,\left(a+b\tan\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))^(3/2)), x)
```

3.873 $\int \frac{1}{\cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=310

$$\frac{i\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{(ia-b)^{3/2}d}-\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\mathrm{co}}}{b^{5/2}d}$$

```
[Out] I*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^
(1/2)*tan(d*x+c)^(1/2)/(I*a-b)^(3/2)/d-3*a*arctanh(b^(1/2)*tan(d*x+c)^(1/2)
/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/b^(5/2)/d+I*arct
anh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)
*tan(d*x+c)^(1/2)/(I*a+b)^(3/2)/d-2*a^2/b/(a^2+b^2)/d/cot(d*x+c)^(3/2)/(a+b
*tan(d*x+c))^(1/2)+(3*a^2+b^2)*(a+b*tan(d*x+c))^(1/2)/b^2/(a^2+b^2)/d/cot(d
*x+c)^(1/2)
```

Rubi [A]

time = 1.16, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {4326, 3646, 3728, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$-\frac{2a^2}{bd\left(a^2+b^2\right)\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}+\frac{\left(3a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}{b^2d\left(a^2+b^2\right)\sqrt{\cot(c+dx)}}+\frac{i\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{3/2}}-\frac{3a\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{b^{5/2}d}+\frac{i\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cot[c + d*x]^(7/2)*(a + b*Tan[c + d*x])^(3/2)),x]
```

```
[Out] (I*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt
[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/((I*a - b)^(3/2)*d) - (3*a*ArcTanh[(Sqrt
[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[T
an[c + d*x]])/(b^(5/2)*d) + (I*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/S
qrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/((I*a + b)^
(3/2)*d) - (2*a^2)/(b*(a^2 + b^2)*d*Cot[c + d*x]^(3/2)*Sqrt[a + b*Tan[c + d
*x]]) + ((3*a^2 + b^2)*Sqrt[a + b*Tan[c + d*x]])/(b^2*(a^2 + b^2)*d*Sqrt[Co
t[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
```

```
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{\cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx = \left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{\tan^{\frac{7}{2}}(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx$$

$$= -\frac{2a^2}{b(a^2+b^2)\,d\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}} + \frac{\left(2\sqrt{\cot(c+\right.}{}$$

$$= -\frac{2a^2}{b(a^2+b^2)\,d\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}} + \frac{(3a^2+b^2)\,\sqrt{}}{b^2\,(a^2+b^2}$$

$$= -\frac{2a^2}{b(a^2+b^2)\,d\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}} + \frac{(3a^2+b^2)\,\sqrt{}}{b^2\,(a^2+b^2}$$

$$= -\frac{2a^2}{b(a^2+b^2)\,d\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}} + \frac{(3a^2+b^2)\,\sqrt{}}{b^2\,(a^2+b^2}$$

$$= -\frac{2a^2}{b(a^2+b^2)\,d\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}} + \frac{(3a^2+b^2)\,\sqrt{}}{b^2\,(a^2+b^2}$$

$$= -\frac{2a^2}{b(a^2+b^2)\,d\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}} + \frac{(3a^2+b^2)\,\sqrt{}}{b^2\,(a^2+b^2}$$

$$= -\frac{2a^2}{b(a^2+b^2)\,d\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}} + \frac{(3a^2+b^2)\,\sqrt{}}{b^2\,(a^2+b^2}$$

$$= -\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+}}{b^{5/2}d}$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+d}}{(ia-b)^{3/2}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.22, size = 319, normalized size = 1.03

$$\frac{(-1)^{3/4}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{(-a+ib)^{3/2}d} - \frac{(-1)^{3/4}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{(a+ib)^{3/2}d} - \frac{1}{(a-ib)d\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}} - \frac{1}{(a+ib)d\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}} + \frac{2\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};-\frac{b\tan(c+dx)}{a}\right)\sqrt{1+\frac{b\tan(c+dx)}{a}}}{5ad\cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cot[c + d*x]^(7/2)*(a + b*Tan[c + d*x])^(3/2)),x]
```

```
[Out] ((-1)^(3/4)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a +
b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/((-a + I*b)^(3/2)*d
) - ((-1)^(3/4)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a
 + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/((a + I*b)^(3/2)
*d) - 1/((a - I*b)*d*Sqrt[Cot[c + d*x]]*Sqrt[a + b*Tan[c + d*x]]) - 1/((a +
 I*b)*d*Sqrt[Cot[c + d*x]]*Sqrt[a + b*Tan[c + d*x]]) + (2*Hypergeometric2F1
[3/2, 5/2, 7/2, -((b*Tan[c + d*x])/a)]*Sqrt[1 + (b*Tan[c + d*x])/a])/(5*a*d
*Cot[c + d*x]^(5/2)*Sqrt[a + b*Tan[c + d*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 36.79, size = 15849, normalized size = 51.13

method	result	size
default	Expression too large to display	15849

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(7/2)/(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(7/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^(7/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(7/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(7/2)/(a+b*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(7/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cot\left(c+d\,x\right)^{7/2}\,\left(a+b\tan\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(7/2)*(a + b*tan(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cot(c + d*x)^(7/2)*(a + b*tan(c + d*x))^(3/2)), x)
```

3.874 $\int \frac{\cot^{\frac{5}{2}}(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=338

$$\frac{\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{(ia-b)^{5/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\text{co}}}{(ia+b)^{5/2}d}$$

```
[Out] arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1
/2)*tan(d*x+c)^(1/2)/(I*a-b)^(5/2)/d+arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)
/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a+b)^(5/2)/d+
4/3*b^2*(4*a^4+15*a^2*b^2+8*b^4)/a^4/(a^2+b^2)^2/d/cot(d*x+c)^(1/2)/(a+b*ta
n(d*x+c))^(1/2)-2/3*cot(d*x+c)^(3/2)/a/d/(a+b*tan(d*x+c))^(3/2)+2/3*b^2*(7*
a^2+8*b^2)/a^3/(a^2+b^2)/d/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2)+4*b*cot(
d*x+c)^(1/2)/a^2/d/(a+b*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.88, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4326, 3650, 3730, 3731, 3697, 3696, 95, 209, 212}

$$\frac{4b\sqrt{\cot(c+dx)}}{a^2d(a+b\tan(c+dx))^{3/2}}+\frac{4b^2(4a^4+15a^2b^2+8b^4)}{3a^4d(a^2+b^2)^2\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}+\frac{2b^2(7a^2+8b^2)}{3a^3d(a^2+b^2)\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}}+\frac{\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{5/2}}-\frac{2\cot^{\frac{3}{2}}(c+dx)}{3ad(a+b\tan(c+dx))^{3/2}}+\frac{\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(5/2)/(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] (ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[C
ot[c + d*x]]*Sqrt[Tan[c + d*x]])/((I*a - b)^(5/2)*d) + (ArcTanh[(Sqrt[I*a +
 b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[T
an[c + d*x]])/((I*a + b)^(5/2)*d) + (2*b^2*(7*a^2 + 8*b^2))/(3*a^3*(a^2 + b
^2)*d*Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^(3/2)) + (4*b*Sqrt[Cot[c + d*
x]])/(a^2*d*(a + b*Tan[c + d*x])^(3/2)) - (2*Cot[c + d*x]^(3/2))/(3*a*d*(a
+ b*Tan[c + d*x])^(3/2)) + (4*b^2*(4*a^4 + 15*a^2*b^2 + 8*b^4))/(3*a^4*(a^2
 + b^2)^2*d*Sqrt[Cot[c + d*x]]*Sqrt[a + b*Tan[c + d*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
```

```
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3731

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)
*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp
[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - a*C*(b*c*(m + 1) + a*d*(n
+ 1)) - (m + 1)*(b*c - a*d)*(A*b - b*C)*Tan[e + f*x] - d*(A*b^2 + a^2*C)*(m
 + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m,
 -1] &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx$$

$$= -\frac{2\cot^{\frac{3}{2}}(c+dx)}{3ad(a+b\tan(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{3b+\frac{3}{2}a\tan(}{\tan^{\frac{3}{2}}(c+dx}}{3a}$$

$$= \frac{4b\sqrt{\cot(c+dx)}}{a^2d(a+b\tan(c+dx))^{3/2}} - \frac{2\cot^{\frac{3}{2}}(c+dx)}{3ad(a+b\tan(c+dx))^{3/2}} + \frac{\left(4\sqrt{\cot(c+dx)}\right.}{}$$

$$= \frac{2b^2(7a^2+8b^2)}{3a^3(a^2+b^2)\,d\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}} + \frac{4b\sqrt{\cot(c+dx)}}{a^2d(a+b\tan(c+dx)}$$

$$= \frac{2b^2(7a^2+8b^2)}{3a^3(a^2+b^2)\,d\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}} + \frac{4b\sqrt{\cot(c+dx)}}{a^2d(a+b\tan(c+dx)}$$

$$= \frac{2b^2(7a^2+8b^2)}{3a^3(a^2+b^2)\,d\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}} + \frac{4b\sqrt{\cot(c+dx)}}{a^2d(a+b\tan(c+dx)}$$

$$= \frac{2b^2(7a^2+8b^2)}{3a^3(a^2+b^2)\,d\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}} + \frac{4b\sqrt{\cot(c+dx)}}{a^2d(a+b\tan(c+dx)}$$

$$= \frac{2b^2(7a^2+8b^2)}{3a^3(a^2+b^2)\,d\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}} + \frac{4b\sqrt{\cot(c+dx)}}{a^2d(a+b\tan(c+dx)}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{(ia-b)^{5/2}d} + \frac{\tanh^{-1}\left(\right.}{}$$

Mathematica [A]
time = 6.30, size = 504, normalized size = 1.49

$$\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\left(-\frac{2}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}} - \frac{2\left(-\frac{6b}{ad\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{3/2}} - \frac{3\left(\frac{ab\sqrt{\tan(c+dx)}}{3(ia-b)(a+b\tan(c+dx))^{3/2}} + \frac{16b^2\sqrt{\tan(c+dx)}}{3a(a+b\tan(c+dx))^{3/2}} - \frac{ab\sqrt{\tan(c+dx)}}{3(ia+b)(a+b\tan(c+dx))^{3/2}} + \frac{32b^2\sqrt{\tan(c+dx)}}{3a^2\sqrt{a+b\tan(c+dx)}} - \frac{-\frac{3\sqrt[4]{-1}\,a^2\operatorname{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(-a+ib)^{3/2}} + \frac{(5a-2ib)b\sqrt{\tan(c+dx)}}{(a-ib)\sqrt{a+b\tan(c+dx)}}}{3(ia+b)} + \frac{-\frac{3\sqrt[4]{-1}\,a^2\operatorname{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(a+ib)^{3/2}} + \frac{(5a+2ib)b\sqrt{\tan(c+dx)}}{(a+ib)\sqrt{a+b\tan(c+dx)}}}{3(ia-b)}\right)}{2ad}\right)}{3a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(5/2)/(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-2/(3*a*d*Tan[c + d*x]^(3/2)*(a + b*
Tan[c + d*x])^(3/2)) - (2*((-6*b)/(a*d*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*
x])^(3/2)) - (3*((a*b*Sqrt[Tan[c + d*x]])/(3*(I*a - b)*(a + b*Tan[c + d*x])
^(3/2)) + (16*b^2*Sqrt[Tan[c + d*x]])/(3*a*(a + b*Tan[c + d*x])^(3/2)) - (a
*b*Sqrt[Tan[c + d*x]])/(3*(I*a + b)*(a + b*Tan[c + d*x])^(3/2)) + (32*b^2*S
qrt[Tan[c + d*x]])/(3*a^2*Sqrt[a + b*Tan[c + d*x]]) - ((-3*(-1)^(1/4)*a^2*A
rcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x
]]])/(-a + I*b)^(3/2) + ((5*a - (2*I)*b)*b*Sqrt[Tan[c + d*x]])/((a - I*b)*S
qrt[a + b*Tan[c + d*x]]))/(3*(I*a + b)) + ((-3*(-1)^(1/4)*a^2*ArcTan[((-1)^
(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(a + I*b
)^(3/2) + ((5*a + (2*I)*b)*b*Sqrt[Tan[c + d*x]])/((a + I*b)*Sqrt[a + b*Tan[
c + d*x]]))/(3*(I*a - b))))/(2*a*d)))/(3*a))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 37.55, size = 41735, normalized size = 123.48

method	result	size
default	Expression too large to display	41735

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^(5/2)/(b*tan(d*x + c) + a)^(5/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(5/2)/(a+b*tan(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^{5/2}}{\left(a+b\tan\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(5/2)/(a + b*tan(c + d*x))^(5/2),x)
```

```
[Out] int(cot(c + d*x)^(5/2)/(a + b*tan(c + d*x))^(5/2), x)
```

3.875 $\int \frac{\cot^{\frac{3}{2}}(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=305

$$\frac{i\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{(ia-b)^{5/2}d}-\frac{i\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{}}{(ia+b)^{5/2}d}$$

```
[Out] I*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^
(1/2)*tan(d*x+c)^(1/2)/(I*a-b)^(5/2)/d-I*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(
1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a+b)^(5/2
)/d-2/3*b*(3*a^4+17*a^2*b^2+8*b^4)/a^3/(a^2+b^2)^2/d/cot(d*x+c)^(1/2)/(a+b*
tan(d*x+c))^(1/2)-2/3*b*(3*a^2+4*b^2)/a^2/(a^2+b^2)/d/cot(d*x+c)^(1/2)/(a+b
*tan(d*x+c))^(3/2)-2*cot(d*x+c)^(1/2)/a/d/(a+b*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.72, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4326, 3650, 3730, 3697, 3696, 95, 209, 212}

$$-\frac{2b(3a^2+4b^2)}{3a^2d(a^2+b^2)\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}}-\frac{2b(3a^4+17a^2b^2+8b^4)}{3a^3d(a^2+b^2)^2\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}+\frac{i\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{5/2}}-\frac{2\sqrt{\cot(c+dx)}}{ad(a+b\tan(c+dx))^{3/2}}-\frac{i\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(3/2)/(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] (I*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt
[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/((I*a - b)^(5/2)*d) - (I*ArcTanh[(Sqrt[I
*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sq
rt[Tan[c + d*x]])/((I*a + b)^(5/2)*d) - (2*b*(3*a^2 + 4*b^2))/(3*a^2*(a^2 +
 b^2)*d*Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^(3/2)) - (2*Sqrt[Cot[c + d*
x]])/(a*d*(a + b*Tan[c + d*x])^(3/2)) - (2*b*(3*a^4 + 17*a^2*b^2 + 8*b^4))/
(3*a^3*(a^2 + b^2)^2*d*Sqrt[Cot[c + d*x]]*Sqrt[a + b*Tan[c + d*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
```

```
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx$$

$$= -\frac{2\sqrt{\cot(c+dx)}}{ad(a+b\tan(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{2b+\frac{1}{2}a\tan(\ldots}{\sqrt{\tan(c+\ldots}}}{a}$$

$$= -\frac{2b(3a^2+4b^2)}{3a^2(a^2+b^2)\,d\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}} - \frac{2\sqrt{\cot(c+dx)}}{ad(a+b\tan(c+dx))\ldots}$$

$$= -\frac{2b(3a^2+4b^2)}{3a^2(a^2+b^2)\,d\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}} - \frac{2\sqrt{\cot(c+dx)}}{ad(a+b\tan(c+dx))\ldots}$$

$$= -\frac{2b(3a^2+4b^2)}{3a^2(a^2+b^2)\,d\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}} - \frac{2\sqrt{\cot(c+dx)}}{ad(a+b\tan(c+dx))\ldots}$$

$$= -\frac{2b(3a^2+4b^2)}{3a^2(a^2+b^2)\,d\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}} - \frac{2\sqrt{\cot(c+dx)}}{ad(a+b\tan(c+dx))\ldots}$$

$$= -\frac{2b(3a^2+4b^2)}{3a^2(a^2+b^2)\,d\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}} - \frac{2\sqrt{\cot(c+dx)}}{ad(a+b\tan(c+dx))\ldots}$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{(ia-b)^{5/2}d} - \frac{i\tanh^{-1}\ldots}{\ldots}$$

Mathematica [A]

time = 4.10, size = 296, normalized size = 0.97

$$-\frac{\sqrt{\cot(c+dx)}\left(-\frac{3\sqrt[4]{-1}\,a\left(\frac{(a+ib)^2\operatorname{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}}-\frac{(a-ib)^2\operatorname{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{a+ib}}\right)\sqrt{\tan(c+dx)}}{(a^2+b^2)^2}+\frac{6}{(a+b\tan(c+dx))^{3/2}}+\frac{2b(3a^2+4b^2)\tan(c+dx)}{a(a^2+b^2)(a+b\tan(c+dx))^{3/2}}+\frac{2b(3a^4+17a^2b^2+8b^4)\tan(c+dx)}{a^2(a^2+b^2)^2\sqrt{a+b\tan(c+dx)}}\right)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(3/2)/(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] -1/3*(Sqrt[Cot[c + d*x]]*((-3*(-1)^(1/4)*a*(((a + I*b)^2*ArcTan[((-1)^(1/4)
*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[-a + I*
b] - ((a - I*b)^2*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt
[a + b*Tan[c + d*x]]])/Sqrt[a + I*b])*Sqrt[Tan[c + d*x]])/(a^2 + b^2)^2 + 6
/(a + b*Tan[c + d*x])^(3/2) + (2*b*(3*a^2 + 4*b^2)*Tan[c + d*x])/(a*(a^2 +
b^2)*(a + b*Tan[c + d*x])^(3/2)) + (2*b*(3*a^4 + 17*a^2*b^2 + 8*b^4)*Tan[c
+ d*x])/(a^2*(a^2 + b^2)^2*Sqrt[a + b*Tan[c + d*x]])))/(a*d)
```

Maple [**C**] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 38.19, size = 26450, normalized size = 86.72

method	result	size
default	Expression too large to display	26450

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^(3/2)/(b*tan(d*x + c) + a)^(5/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)}{(a+b\tan(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(3/2)/(a+b*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(cot(c + d*x)**(3/2)/(a + b*tan(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)^(3/2)/(b*tan(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^{3/2}}{(a+b\tan(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(3/2)/(a + b*tan(c + d*x))^(5/2),x)
```

```
[Out] int(cot(c + d*x)^(3/2)/(a + b*tan(c + d*x))^(5/2), x)
```

3.876 $\int \frac{\sqrt{\cot(c+dx)}}{(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=252

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{(ia-b)^{5/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{}}{(ia+b)^{5/2}}$$

```
[Out] -arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(
1/2)*tan(d*x+c)^(1/2)/(I*a-b)^(5/2)/d-arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2
)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a+b)^(5/2)/d
+4/3*b^2*(4*a^2+b^2)/a^2/(a^2+b^2)^2/d/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1
/2)+2/3*b^2/a/(a^2+b^2)/d/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.55, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4326, 3650, 3730, 3697, 3696, 95, 209, 212}

$$\frac{4b^2(4a^2+b^2)}{3a^2d\,(a^2+b^2)^2\,\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}+\frac{2b^2}{3ad\,(a^2+b^2)\,\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}}-\frac{\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{5/2}}-\frac{\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cot[c + d*x]]/(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] -((ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt
[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/((I*a - b)^(5/2)*d)) - (ArcTanh[(Sqrt[I*
a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqr
t[Tan[c + d*x]])/((I*a + b)^(5/2)*d) + (2*b^2)/(3*a*(a^2 + b^2)*d*Sqrt[Cot[
c + d*x]]*(a + b*Tan[c + d*x])^(3/2)) + (4*b^2*(4*a^2 + b^2))/(3*a^2*(a^2 +
 b^2)^2*d*Sqrt[Cot[c + d*x]]*Sqrt[a + b*Tan[c + d*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
```

```
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{\sqrt{\cot(c+dx)}}{(a+b\tan(c+dx))^{5/2}}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{1}{\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^{5/2}}\,dx$$

$$= \frac{2b^2}{3a\left(a^2+b^2\right)d\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}} + \frac{\left(2\sqrt{\cot(c+dx)}\,\sqrt{\tan\right.}{}$$

$$= \frac{2b^2}{3a\left(a^2+b^2\right)d\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}} + \frac{4b^2(}{3a^2\left(a^2+b^2\right)^2 d\sqrt{\cot(c}}$$

$$= \frac{2b^2}{3a\left(a^2+b^2\right)d\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}} + \frac{4b^2(}{3a^2\left(a^2+b^2\right)^2 d\sqrt{\cot(c}}$$

$$= \frac{2b^2}{3a\left(a^2+b^2\right)d\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}} + \frac{4b^2(}{3a^2\left(a^2+b^2\right)^2 d\sqrt{\cot(c}}$$

$$= \frac{2b^2}{3a\left(a^2+b^2\right)d\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}} + \frac{4b^2(}{3a^2\left(a^2+b^2\right)^2 d\sqrt{\cot(c}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{(ia-b)^{5/2}d} - \frac{\tanh^{-}}{}$$

Mathematica [A]
time = 2.08, size = 255, normalized size = 1.01

$$\frac{\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\left(-3(-1)^{3/4}\left(\frac{(a+ib)^2\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}}+\frac{(a-ib)^2\text{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{a+ib}}\right)+\frac{2b^2\left(a^2+b^2\right)\sqrt{\tan(c+dx)}}{a(a+b\tan(c+dx))^{3/2}}+\frac{4b^2\left(4a^2+b^2\right)\sqrt{\tan(c+dx)}}{a^2\sqrt{a+b\tan(c+dx)}}\right)}{3\left(a^2+b^2\right)^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cot[c + d*x]]/(a + b*Tan[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-3*(-1)^(3/4)*(((a + I*b)^2*ArcTan[
((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/S
qrt[-a + I*b] + ((a - I*b)^2*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c +
d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[a + I*b]) + (2*b^2*(a^2 + b^2)*Sqrt[
Tan[c + d*x]])/(a*(a + b*Tan[c + d*x])^(3/2)) + (4*b^2*(4*a^2 + b^2)*Sqrt[T
an[c + d*x]])/(a^2*Sqrt[a + b*Tan[c + d*x]])))/(3*(a^2 + b^2)^2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 27.44, size = 20642, normalized size = 81.91

method	result	size
default	Expression too large to display	20642

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cot(d*x + c))/(b*tan(d*x + c) + a)^(5/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cot{\left(c + dx \right)}}}{\left(a + b \tan{\left(c + dx \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(1/2)/(a+b*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(sqrt(cot(c + d*x))/(a + b*tan(c + d*x))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cot\left(c+d\,x\right)}}{{\left(a+b\,\tan\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(1/2)/(a + b*tan(c + d*x))^(5/2),x)
```

```
[Out] int(cot(c + d*x)^(1/2)/(a + b*tan(c + d*x))^(5/2), x)
```

3.877 $\int \frac{1}{\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=251

$$-\frac{i\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{(ia-b)^{5/2}d}+\frac{i\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\cdots}{(ia+b)^{5/2}\cdots}$$

```
[Out] -I*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)
^(1/2)*tan(d*x+c)^(1/2)/(I*a-b)^(5/2)/d+I*arctanh((I*a+b)^(1/2)*tan(d*x+c)^
(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a+b)^(5/
2)/d-2/3*b*(5*a^2-b^2)/a/(a^2+b^2)^2/d/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1
/2)-2/3*b/(a^2+b^2)/d/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.51, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4326, 3649, 3730, 3697, 3696, 95, 209, 212}

$$-\frac{2b(5a^2-b^2)}{3ad\,(a^2+b^2)^2\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}-\frac{2b}{3d\,(a^2+b^2)\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}}-\frac{i\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{5/2}}+\frac{i\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^(5/2)),x]
```

```
[Out] ((-I)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*S
qrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/((I*a - b)^(5/2)*d) + (I*ArcTanh[(Sqr
t[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]
*Sqrt[Tan[c + d*x]])/((I*a + b)^(5/2)*d) - (2*b)/(3*(a^2 + b^2)*d*Sqrt[Cot[
c + d*x]]*(a + b*Tan[c + d*x])^(3/2)) - (2*b*(5*a^2 - b^2))/(3*a*(a^2 + b^2
)^2*d*Sqrt[Cot[c + d*x]]*Sqrt[a + b*Tan[c + d*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
```

```
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{5/2}}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{\sqrt{\tan(c+dx)}}{(a+b\tan(c+dx))^{5/2}}\,dx$$

$$= -\frac{2b}{3\left(a^2+b^2\right)d\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cot\right.}{}$$

$$= -\frac{2b}{3\left(a^2+b^2\right)d\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}} - \frac{}{3a\left(a^2+\right.}$$

$$= -\frac{2b}{3\left(a^2+b^2\right)d\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}} - \frac{}{3a\left(a^2+\right.}$$

$$= -\frac{2b}{3\left(a^2+b^2\right)d\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}} - \frac{}{3a\left(a^2+\right.}$$

$$= -\frac{2b}{3\left(a^2+b^2\right)d\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}} - \frac{}{3a\left(a^2+\right.}$$

$$= -\frac{i\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+\right.}{(ia-b)^{5/2}d}$$

Mathematica [A]
time = 5.24, size = 214, normalized size = 0.85

$$\frac{\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\left(-\frac{3\sqrt[4]{-1}\,\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(-a+ib)^{5/2}}+\frac{3\sqrt[4]{-1}\,\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(a+ib)^{5/2}}+\frac{2b\sqrt{\tan(c+dx)}\,\left(-6a^3+\left(-5a^2b+b^3\right)\tan(c+dx)\right)}{a\left(a^2+b^2\right)^2(a+b\tan(c+dx))^{3/2}}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^(5/2)),x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-3*(-1)^(1/4)*ArcTan[((-1)^(1/4)*S
qrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(-a + I*b)^(5/
2) + (3*(-1)^(1/4)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqr
t[a + b*Tan[c + d*x]]])/(a + I*b)^(5/2) + (2*b*Sqrt[Tan[c + d*x]]*(-6*a^3 +
 (-5*a^2*b + b^3)*Tan[c + d*x]))/(a*(a^2 + b^2)^2*(a + b*Tan[c + d*x])^(3/2
))))/(3*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 38.88, size = 19740, normalized size = 78.65

method	result	size
default	Expression too large to display	19740

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*tan(d*x + c) + a)^(5/2)*sqrt(cot(d*x + c))), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\tan(c+dx))^{\frac{5}{2}}\sqrt{\cot(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(1/2)/(a+b*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(1/((a + b*tan(c + d*x))**(5/2)*sqrt(cot(c + d*x))), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{\cot\left(c+d\,x\right)}\,{\left(a+b\,\tan\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(5/2)), x)
```

3.878 $\int \frac{1}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=239

$$\frac{\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{(ia-b)^{5/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{c}}{(ia+b)^{5/2}d}$$

```
[Out] arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1
/2)*tan(d*x+c)^(1/2)/(I*a-b)^(5/2)/d+arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)
/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a+b)^(5/2)/d+
4/3*(a^2-2*b^2)/(a^2+b^2)^2/d/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2)+2/3*a
/(a^2+b^2)/d/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.51, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4326, 3648, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2a}{3d(a^2+b^2)\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}}+\frac{4(a^2-2b^2)}{3d(a^2+b^2)^2\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}+\frac{\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{5/2}}+\frac{\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(5/2)),x]
```

```
[Out] (ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[C
ot[c + d*x]]*Sqrt[Tan[c + d*x]])/((I*a - b)^(5/2)*d) + (ArcTanh[(Sqrt[I*a +
 b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[T
an[c + d*x]])/((I*a + b)^(5/2)*d) + (2*a)/(3*(a^2 + b^2)*d*Sqrt[Cot[c + d*x
]]*(a + b*Tan[c + d*x])^(3/2)) + (4*(a^2 - 2*b^2))/(3*(a^2 + b^2)^2*d*Sqrt[
Cot[c + d*x]]*Sqrt[a + b*Tan[c + d*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
```

```
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{1}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{\tan^{\frac{3}{2}}(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx$$

$$= \frac{2a}{3(a^2+b^2)\,d\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cot(c\right.}}{}$$

$$= \frac{2a}{3(a^2+b^2)\,d\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}} + \frac{}{3(a^2+b^2)^2}$$

$$= \frac{2a}{3(a^2+b^2)\,d\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}} + \frac{}{3(a^2+b^2)^2}$$

$$= \frac{2a}{3(a^2+b^2)\,d\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}} + \frac{}{3(a^2+b^2)^2}$$

$$= \frac{2a}{3(a^2+b^2)\,d\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}} + \frac{}{3(a^2+b^2)^2}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx}}{(ia-b)^{5/2}d}$$

Mathematica [A]

time = 4.16, size = 218, normalized size = 0.91

$$\frac{\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\left(\frac{3(-1)^{3/4}\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{-a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(-a+ib)^{5/2}} + \frac{3(-1)^{3/4}\mathrm{ArcTan}\left(\frac{\sqrt[4]{-1}\,\sqrt{a+ib}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(a+ib)^{5/2}} + \frac{2\sqrt{\tan(c+dx)}\,(3a(a^2-b^2)+2b(a^2-2b^2)\tan(c+dx))}{(a^2+b^2)^2(a+b\tan(c+dx))^{3/2}}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(5/2)),x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((3*(-1)^(3/4)*ArcTan[((-1)^(1/4)*Sq
rt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(-a + I*b)^(5/2
) + (3*(-1)^(3/4)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt
[a + b*Tan[c + d*x]]])/(a + I*b)^(5/2) + (2*Sqrt[Tan[c + d*x]]*(3*a*(a^2 -
b^2) + 2*b*(a^2 - 2*b^2)*Tan[c + d*x]))/((a^2 + b^2)^2*(a + b*Tan[c + d*x])
^(3/2))))/(3*d)
```

Maple [**C**] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 37.08, size = 20598, normalized size = 86.18

method	result	size
default	Expression too large to display	20598

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^(3/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(3/2)/(a+b*tan(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cot(c+d\,x)^{3/2}\,(a+b\tan(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(5/2)), x)
```

3.879 $\int \frac{1}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=254

$$\frac{i\text{ArcTan}\left(\frac{\sqrt{ia-b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}}{(ia-b)^{5/2}d}-\frac{i\tanh^{-1}\left(\frac{\sqrt{ia+b}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{}}{(ia+b)^{5/2}d}$$

```
[Out] I*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^
(1/2)*tan(d*x+c)^(1/2)/(I*a-b)^(5/2)/d-I*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(
1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a+b)^(5/2
)/d+2/3*a*(a^2+7*b^2)/b/(a^2+b^2)^2/d/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/
2)-2/3*a^2/b/(a^2+b^2)/d/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.57, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4326, 3646, 3730, 3697, 3696, 95, 209, 212}

$$-\frac{2a^2}{3bd(a^2+b^2)\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}}+\frac{2a(a^2+7b^2)}{3bd(a^2+b^2)^2\sqrt{\cot(c+dx)}\,\sqrt{a+b\tan(c+dx)}}+\frac{i\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\text{ArcTan}\left(\frac{\sqrt{-b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{5/2}}-\frac{i\sqrt{\tan(c+dx)}\,\sqrt{\cot(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{b+ia}\,\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^(5/2)),x]
```

```
[Out] (I*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt
[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/((I*a - b)^(5/2)*d) - (I*ArcTanh[(Sqrt[I
*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sq
rt[Tan[c + d*x]])/((I*a + b)^(5/2)*d) - (2*a^2)/(3*b*(a^2 + b^2)*d*Sqrt[Cot
[c + d*x]]*(a + b*Tan[c + d*x])^(3/2)) + (2*a*(a^2 + 7*b^2))/(3*b*(a^2 + b^
2)^2*d*Sqrt[Cot[c + d*x]]*Sqrt[a + b*Tan[c + d*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
```

```
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{1}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx = \left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{\tan^{\frac{5}{2}}(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx$$

$$= -\frac{2a^2}{3b\left(a^2+b^2\right)d\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}} + \frac{\left(2\sqrt{\cot(\right.}{}$$

$$= -\frac{2a^2}{3b\left(a^2+b^2\right)d\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}} + \frac{}{3b\left(a^2+b\right.}$$

$$= -\frac{2a^2}{3b\left(a^2+b^2\right)d\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}} + \frac{}{3b\left(a^2+b\right.}$$

$$= -\frac{2a^2}{3b\left(a^2+b^2\right)d\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}} + \frac{}{3b\left(a^2+b\right.}$$

$$= -\frac{2a^2}{3b\left(a^2+b^2\right)d\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^{3/2}} + \frac{}{3b\left(a^2+b\right.}$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{(ia-b)^{5/2}d}$$

Mathematica [A]
time = 4.30, size = 209, normalized size = 0.82

$$\frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(\frac{3\sqrt[4]{-1}\operatorname{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(-a+ib)^{5/2}} - \frac{3\sqrt[4]{-1}\operatorname{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(a+ib)^{5/2}} + \frac{2a\sqrt{\tan(c+dx)}\left(6ab+\left(a^2+7b^2\right)\tan(c+dx)\right)}{\left(a^2+b^2\right)^2(a+b\tan(c+dx))^{3/2}}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^(5/2)),x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((3*(-1)^(1/4)*ArcTan[((-1)^(1/4)*Sq
rt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(-a + I*b)^(5/2
) - (3*(-1)^(1/4)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt
[a + b*Tan[c + d*x]]])/(a + I*b)^(5/2) + (2*a*Sqrt[Tan[c + d*x]]*(6*a*b + (
a^2 + 7*b^2)*Tan[c + d*x]))/((a^2 + b^2)^2*(a + b*Tan[c + d*x])^(3/2))))/(3
*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 37.44, size = 19720, normalized size = 77.64

method	result	size
default	Expression too large to display	19720

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^(5/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)**(5/2)/(a+b*tan(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6191 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cot\left(c+d\,x\right)^{5/2}\,\left(a+b\,\tan\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))^(5/2)), x)
```

3.880 $\int (d\cot(e+fx))^n (a+b\tan(e+fx))^3\,dx$

Optimal. Leaf size=206

$$\frac{a^2bd^2(1-2n)(d\cot(e+fx))^{-2+n}}{f(1-n)(2-n)}+\frac{a^2d^2(d\cot(e+fx))^{-2+n}(b+a\cot(e+fx))}{f(1-n)}-\frac{b(3a^2-b^2)\,d^2(d\cot(e+f$$

```
[Out] a^2*b*d^2*(1-2*n)*(d*cot(f*x+e))^(-2+n)/f/(n^2-3*n+2)+a^2*d^2*(d*cot(f*x+e)
)^(-2+n)*(b+a*cot(f*x+e))/f/(1-n)-b*(3*a^2-b^2)*d^2*(d*cot(f*x+e))^(-2+n)*h
ypergeom([1, -1+1/2*n],[1/2*n],-cot(f*x+e)^2)/f/(2-n)-a*(a^2-3*b^2)*d*(d*co
t(f*x+e))^(-1+n)*hypergeom([1, -1/2+1/2*n],[1/2+1/2*n],-cot(f*x+e)^2)/f/(1-
n)
```

Rubi [A]
time = 0.30, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3754, 3647, 3711, 3619, 3557, 371}

$$-\frac{bd^2(3a^2-b^2)\,(d\cot(e+fx))^{n-2}\,{}_2F_1\left(1,\frac{n-2}{2};\frac{n}{2};-\cot^2(e+fx)\right)}{f(2-n)}-\frac{ad(a^2-3b^2)\,(d\cot(e+fx))^{n-1}\,{}_2F_1\left(1,\frac{n-1}{2};\frac{n+1}{2};-\cot^2(e+fx)\right)}{f(1-n)}+\frac{a^2d^2(a\cot(e+fx)+b)(d\cot(e+fx))^{n-2}}{f(1-n)}+\frac{a^2bd^2(1-2n)(d\cot(e+fx))^{n-2}}{f(1-n)(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^n*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (a^2*b*d^2*(1 - 2*n)*(d*Cot[e + f*x])^(-2 + n))/(f*(1 - n)*(2 - n)) + (a^2*
d^2*(d*Cot[e + f*x])^(-2 + n)*(b + a*Cot[e + f*x]))/(f*(1 - n)) - (b*(3*a^2
 - b^2)*d^2*(d*Cot[e + f*x])^(-2 + n)*Hypergeometric2F1[1, (-2 + n)/2, n/2,
 -Cot[e + f*x]^2])/(f*(2 - n)) - (a*(a^2 - 3*b^2)*d*(d*Cot[e + f*x])^(-1 +
n)*Hypergeometric2F1[1, (-1 + n)/2, (1 + n)/2, -Cot[e + f*x]^2])/(f*(1 - n)
)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int (d\cot(e+fx))^n(a+b\tan(e+fx))^3\,dx = d^3\int (d\cot(e+fx))^{-3+n}(b+a\cot(e+fx))^3\,dx$$
$$= \frac{a^2d^2(d\cot(e+fx))^{-2+n}(b+a\cot(e+fx))}{f(1-n)} + \frac{d^2\int(d\cot(e+ \ldots}{}$$
$$= \frac{a^2bd^2(1-2n)(d\cot(e+fx))^{-2+n}}{f(1-n)(2-n)} + \frac{a^2d^2(d\cot(e+fx))^{-2+n}(\ldots}{f(1-n \ldots}$$
$$= \frac{a^2bd^2(1-2n)(d\cot(e+fx))^{-2+n}}{f(1-n)(2-n)} + \frac{a^2d^2(d\cot(e+fx))^{-2+n}(\ldots}{f(1-n \ldots}$$
$$= \frac{a^2bd^2(1-2n)(d\cot(e+fx))^{-2+n}}{f(1-n)(2-n)} + \frac{a^2d^2(d\cot(e+fx))^{-2+n}(\ldots}{f(1-n \ldots}$$
$$= \frac{a^2bd^2(1-2n)(d\cot(e+fx))^{-2+n}}{f(1-n)(2-n)} + \frac{a^2d^2(d\cot(e+fx))^{-2+n}(\ldots}{f(1-n \ldots}$$

Mathematica [A]
time = 0.95, size = 141, normalized size = 0.68

$$\frac{(d\cot(e+fx))^n\left(-b(-3a^2+b^2)(-1+n)\,{}_2F_1\left(1,\frac{1}{2}(-2+n);\frac{n}{2};-\cot^2(e+fx)\right)+a\left(a(-3b(-1+n)-a(-2+n)\cot(e+fx))+(a^2-3b^2)(-2+n)\cot(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(-1+n);\frac{1+n}{2};-\cot^2(e+fx)\right)\right)\right)\tan^2(e+fx)}{f(-2+n)(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cot[e + f*x])^n*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] ((d*Cot[e + f*x])^n*(-(b*(-3*a^2 + b^2)*(-1 + n)*Hypergeometric2F1[1, (-2 +
 n)/2, n/2, -Cot[e + f*x]^2]) + a*(a*(-3*b*(-1 + n) - a*(-2 + n)*Cot[e + f*
x]) + (a^2 - 3*b^2)*(-2 + n)*Cot[e + f*x]*Hypergeometric2F1[1, (-1 + n)/2,
(1 + n)/2, -Cot[e + f*x]^2]))*Tan[e + f*x]^2)/(f*(-2 + n)*(-1 + n))
```

Maple [F]
time = 0.54, size = 0, normalized size = 0.00

$$\int (d\cot(fx+e))^n\,(a+b\tan(fx+e))^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^n*(a+b*tan(f*x+e))^3,x)
```

```
[Out] int((d*cot(f*x+e))^n*(a+b*tan(f*x+e))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^3*(d*cot(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*tan(f*x + e)^3 + 3*a*b^2*tan(f*x + e)^2 + 3*a^2*b*tan(f*x + e
) + a^3)*(d*cot(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cot(e+fx))^n (a+b\tan(e+fx))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**n*(a+b*tan(f*x+e))**3,x)
```

```
[Out] Integral((d*cot(e + f*x))**n*(a + b*tan(e + f*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)^3*(d*cot(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (d\cot(e+f\,x))^n\,(a+b\tan(e+f\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(e + f*x))^n*(a + b*tan(e + f*x))^3,x)
```

```
[Out] int((d*cot(e + f*x))^n*(a + b*tan(e + f*x))^3, x)
```

3.881 $\int (d\cot(e+fx))^n(a+b\tan(e+fx))^2\,dx$

Optimal. Leaf size=132

$$\frac{a^2d(d\cot(e+fx))^{-1+n}}{f(1-n)}-\frac{(a^2-b^2)\,d(d\cot(e+fx))^{-1+n}\,{}_2F_1\left(1,\frac{1}{2}(-1+n);\frac{1+n}{2};-\cot^2(e+fx)\right)}{f(1-n)}-\frac{2ab(d\cot}{}$$

```
[Out] a^2*d*(d*cot(f*x+e))^(-1+n)/f/(1-n)-(a^2-b^2)*d*(d*cot(f*x+e))^(-1+n)*hyper
geom([1, -1/2+1/2*n],[1/2+1/2*n],-cot(f*x+e)^2)/f/(1-n)-2*a*b*(d*cot(f*x+e)
)^n*hypergeom([1, 1/2*n],[1+1/2*n],-cot(f*x+e)^2)/f/n
```

Rubi [A]
time = 0.16, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3754, 3624, 3619, 3557, 371}

$$-\frac{d(a^2-b^2)\,(d\cot(e+fx))^{n-1}\,{}_2F_1\left(1,\frac{n-1}{2};\frac{n+1}{2};-\cot^2(e+fx)\right)}{f(1-n)}+\frac{a^2d(d\cot(e+fx))^{n-1}}{f(1-n)}-\frac{2ab(d\cot(e+fx))^n\,{}_2F_1\left(1,\frac{n}{2};\frac{n+2}{2};-\cot^2(e+fx)\right)}{fn}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^n*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (a^2*d*(d*Cot[e + f*x])^(-1 + n))/(f*(1 - n)) - ((a^2 - b^2)*d*(d*Cot[e + f
*x])^(-1 + n)*Hypergeometric2F1[1, (-1 + n)/2, (1 + n)/2, -Cot[e + f*x]^2])
/(f*(1 - n)) - (2*a*b*(d*Cot[e + f*x])^n*Hypergeometric2F1[1, n/2, (2 + n)/
2, -Cot[e + f*x]^2])/(f*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int (d\cot(e+fx))^n(a+b\tan(e+fx))^2\,dx = d^2\int (d\cot(e+fx))^{-2+n}(b+a\cot(e+fx))^2\,dx$$

$$= \frac{a^2d(d\cot(e+fx))^{-1+n}}{f(1-n)} + d^2\int (d\cot(e+fx))^{-2+n}\left(-a^2+b^2+\right.$$

$$= \frac{a^2d(d\cot(e+fx))^{-1+n}}{f(1-n)} + (2abd)\int (d\cot(e+fx))^{-1+n}\,dx - ((($$

$$= \frac{a^2d(d\cot(e+fx))^{-1+n}}{f(1-n)} - \frac{(2abd^2)\,\mathrm{Subst}\left(\int \frac{x^{-1+n}}{d^2+x^2}\,dx, x, d\cot(e\right.}{f}$$

$$= \frac{a^2d(d\cot(e+fx))^{-1+n}}{f(1-n)} - \frac{(a^2-b^2)\,d(d\cot(e+fx))^{-1+n}\,{}_2F_1(1}{f(1-}$$

Mathematica [A]
time = 0.50, size = 107, normalized size = 0.81

$$-\frac{d(d\cot(e+fx))^{-1+n}\left(-\left((a^2-b^2)\,n\,{}_2F_1\left(1,\tfrac{1}{2}(-1+n);\tfrac{1+n}{2};-\cot^2(e+fx)\right)\right)+a\left(an+2b(-1+n)\cot(e+fx)\,{}_2F_1\left(1,\tfrac{n}{2};\tfrac{2+n}{2};-\cot^2(e+fx)\right)\right)\right)}{f(-1+n)n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cot[e + f*x])^n*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] -((d*(d*Cot[e + f*x])^(-1 + n)*(-((a^2 - b^2)*n*Hypergeometric2F1[1, (-1 +
n)/2, (1 + n)/2, -Cot[e + f*x]^2]) + a*(a*n + 2*b*(-1 + n)*Cot[e + f*x]*Hyp
ergeometric2F1[1, n/2, (2 + n)/2, -Cot[e + f*x]^2])))/(f*(-1 + n)*n))
```

Maple [F]
time = 0.42, size = 0, normalized size = 0.00

$$\int \left(d\cot\left(fx+e\right)\right)^n\left(a+b\tan\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^n*(a+b*tan(f*x+e))^2,x)
```

```
[Out] int((d*cot(f*x+e))^n*(a+b*tan(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2*(d*cot(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)*(d*cot(f*x + e))^n
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cot(e+fx))^n (a+b\tan(e+fx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**n*(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral((d*cot(e + f*x))**n*(a + b*tan(e + f*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2*(d*cot(f*x + e))^n, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (d\cot(e+f\,x))^n\,(a+b\tan(e+f\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(e + f*x))^n*(a + b*tan(e + f*x))^2,x)
[Out] int((d*cot(e + f*x))^n*(a + b*tan(e + f*x))^2, x)
```

3.882 $\int (d\cot(e+fx))^n(a+b\tan(e+fx))\,dx$

Optimal. Leaf size=96

$$-\frac{b(d\cot(e+fx))^n\,{}_2F_1\left(1,\frac{n}{2};\frac{2+n}{2};-\cot^2(e+fx)\right)}{fn}-\frac{a(d\cot(e+fx))^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\cot^2(e+fx)\right)}{df(1+n)}$$

```
[Out] -b*(d*cot(f*x+e))^n*hypergeom([1, 1/2*n],[1+1/2*n],-cot(f*x+e)^2)/f/n-a*(d*
cot(f*x+e))^(1+n)*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],-cot(f*x+e)^2)/d/f/(
1+n)
```

Rubi [A]
time = 0.09, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3754, 3619, 3557, 371}

$$-\frac{a(d\cot(e+fx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\cot^2(e+fx)\right)}{df(n+1)}-\frac{b(d\cot(e+fx))^n\,{}_2F_1\left(1,\frac{n}{2};\frac{n+2}{2};-\cot^2(e+fx)\right)}{fn}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^n*(a + b*Tan[e + f*x]),x]
```

```
[Out] -((b*(d*Cot[e + f*x])^n*Hypergeometric2F1[1, n/2, (2 + n)/2, -Cot[e + f*x]^
2])/(f*n)) - (a*(d*Cot[e + f*x])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3
 + n)/2, -Cot[e + f*x]^2])/(d*f*(1 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int (d\cot(e+fx))^n(a+b\tan(e+fx))\,dx &= d\int (d\cot(e+fx))^{-1+n}(b+a\cot(e+fx))\,dx\\ &= a\int (d\cot(e+fx))^n\,dx+(bd)\int (d\cot(e+fx))^{-1+n}\,dx\\ &= -\frac{(ad)\mathrm{Subst}\left(\int \frac{x^n}{d^2+x^2}\,dx,x,d\cot(e+fx)\right)}{f}-\frac{(bd^2)\,\mathrm{Subst}\left(\int \frac{x^{-1+n}}{d^2+x^2}\right.}{}\\ &= -\frac{b(d\cot(e+fx))^n\,{}_2F_1\left(1,\frac{n}{2};\frac{2+n}{2};-\cot^2(e+fx)\right)}{fn}-\frac{a(d\cot(e+}{}\end{aligned}$$

Mathematica [A]
time = 0.22, size = 88, normalized size = 0.92

$$-\frac{(d\cot(e+fx))^n\left(b(1+n)\,{}_2F_1\left(1,\frac{n}{2};\frac{2+n}{2};-\cot^2(e+fx)\right)+an\cot(e+fx)\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\cot^2(e+fx)\right)\right)}{fn(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cot[e + f*x])^n*(a + b*Tan[e + f*x]),x]
[Out] -(((d*Cot[e + f*x])^n*(b*(1 + n)*Hypergeometric2F1[1, n/2, (2 + n)/2, -Cot[
e + f*x]^2] + a*n*Cot[e + f*x]*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -
Cot[e + f*x]^2]))/(f*n*(1 + n)))
```

Maple [F]
time = 0.31, size = 0, normalized size = 0.00

$$\int \left(d\cot\left(fx+e\right)\right)^n\left(a+b\tan\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^n*(a+b*tan(f*x+e)),x)
[Out] int((d*cot(f*x+e))^n*(a+b*tan(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)*(d*cot(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e) + a)*(d*cot(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cot(e+fx))^n (a+b\tan(e+fx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**n*(a+b*tan(f*x+e)),x)
```

```
[Out] Integral((d*cot(e + f*x))**n*(a + b*tan(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)*(d*cot(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\cot(e+f\,x))^n\,(a+b\tan(e+f\,x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(e + f*x))^n*(a + b*tan(e + f*x)),x)
```

```
[Out] int((d*cot(e + f*x))^n*(a + b*tan(e + f*x)), x)
```

3.883 $\int \frac{(d\cot(e+fx))^n}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=182

$$-\frac{b(d\cot(e+fx))^{2+n}\,{}_2F_1\left(1,\frac{2+n}{2};\frac{4+n}{2};-\cot^2(e+fx)\right)}{(a^2+b^2)\,d^2f(2+n)}-\frac{a^2(d\cot(e+fx))^{2+n}\,{}_2F_1\left(1,2+n;3+n;-\frac{a\cot(e+fx)}{b}\right.}{b\,(a^2+b^2)\,d^2f(2+n)}$$

```
[Out] -b*(d*cot(f*x+e))^(2+n)*hypergeom([1, 1+1/2*n],[2+1/2*n],-cot(f*x+e)^2)/(a^
2+b^2)/d^2/f/(2+n)-a^2*(d*cot(f*x+e))^(2+n)*hypergeom([1, 2+n],[3+n],-a*cot
(f*x+e)/b)/b/(a^2+b^2)/d^2/f/(2+n)+a*(d*cot(f*x+e))^(3+n)*hypergeom([1, 3/2
+1/2*n],[5/2+1/2*n],-cot(f*x+e)^2)/(a^2+b^2)/d^3/f/(3+n)
```

Rubi [A]
time = 0.28, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3754, 3655, 3619, 3557, 371, 3715, 66}

$$\frac{a(d\cot(e+fx))^{n+3}\,{}_2F_1\left(1,\frac{n+3}{2};\frac{n+5}{2};-\cot^2(e+fx)\right)}{d^3f(n+3)\,(a^2+b^2)}-\frac{b(d\cot(e+fx))^{n+2}\,{}_2F_1\left(1,\frac{n+2}{2};\frac{n+4}{2};-\cot^2(e+fx)\right)}{d^2f(n+2)\,(a^2+b^2)}-\frac{a^2(d\cot(e+fx))^{n+2}\,{}_2F_1\left(1,n+2;n+3;-\frac{a\cot(e+fx)}{b}\right)}{bd^2f(n+2)\,(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^n/(a + b*Tan[e + f*x]),x]
```

```
[Out] -((b*(d*Cot[e + f*x])^(2 + n)*Hypergeometric2F1[1, (2 + n)/2, (4 + n)/2, -C
ot[e + f*x]^2])/((a^2 + b^2)*d^2*f*(2 + n))) - (a^2*(d*Cot[e + f*x])^(2 + n
)*Hypergeometric2F1[1, 2 + n, 3 + n, -((a*Cot[e + f*x])/b)])/(b*(a^2 + b^2)
*d^2*f*(2 + n)) + (a*(d*Cot[e + f*x])^(3 + n)*Hypergeometric2F1[1, (3 + n)/
2, (5 + n)/2, -Cot[e + f*x]^2])/((a^2 + b^2)*d^3*f*(3 + n))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)/((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[(a + b*Tan[e + f*x])^m*
(c - d*Tan[e + f*x]), x], x] + Dist[d^2/(c^2 + d^2), Int[(a + b*Tan[e + f*x
])^m*((1 + Tan[e + f*x]^2)/(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{(d\cot(e+fx))^n}{a+b\tan(e+fx)}\,dx = \frac{\int \frac{(d\cot(e+fx))^{1+n}}{b+a\cot(e+fx)}\,dx}{d}$$

$$= \frac{\int (d\cot(e+fx))^{1+n}(b-a\cot(e+fx))\,dx}{(a^2+b^2)\,d} + \frac{a^2\int \frac{(d\cot(e+fx))^{1+n}(1+\cot^2(e+fx))}{b+a\cot(e+fx)}\,dx}{(a^2+b^2)\,d}$$

$$= -\frac{a\int (d\cot(e+fx))^{2+n}\,dx}{(a^2+b^2)\,d^2} + \frac{b\int (d\cot(e+fx))^{1+n}\,dx}{(a^2+b^2)\,d} + \frac{a^2\text{Subst}\left(\int \frac{(-dx)^{1+n}}{b-ax}\,dx, \right.}{(a^2+b^2)}$$

$$= -\frac{a^2(d\cot(e+fx))^{2+n}\,{}_2F_1\left(1,2+n;3+n;-\frac{a\cot(e+fx)}{b}\right)}{b\,(a^2+b^2)\,d^2f(2+n)} - \frac{b\text{Subst}\left(\int \frac{x^{1+n}}{d^2+x^2}\,dx, x, \right.}{(a^2+b^2)}$$

$$= -\frac{b(d\cot(e+fx))^{2+n}\,{}_2F_1\left(1,\frac{2+n}{2};\frac{4+n}{2};-\cot^2(e+fx)\right)}{(a^2+b^2)\,d^2f(2+n)} - \frac{a^2(d\cot(e+fx))^{2+n}\,{}_2F_1}{b\,(a^2+}$$

Mathematica [A]
time = 0.64, size = 145, normalized size = 0.80

$$-\frac{\cot^2(e+fx)(d\cot(e+fx))^n\left(b^2(3+n)\,{}_2F_1\left(1,\frac{2+n}{2};\frac{4+n}{2};-\cot^2(e+fx)\right)+a\left(a(3+n)\,{}_2F_1\left(1,2+n;3+n;-\frac{a\cot(e+fx)}{b}\right)-b(2+n)\cot(e+fx)\,{}_2F_1\left(1,\frac{3+n}{2};\frac{5+n}{2};-\cot^2(e+fx)\right)\right)\right)}{b\,(a^2+b^2)\,f(2+n)(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cot[e + f*x])^n/(a + b*Tan[e + f*x]),x]
```

```
[Out] -((Cot[e + f*x]^2*(d*Cot[e + f*x])^n*(b^2*(3 + n)*Hypergeometric2F1[1, (2 +
 n)/2, (4 + n)/2, -Cot[e + f*x]^2] + a*(a*(3 + n)*Hypergeometric2F1[1, 2 +
n, 3 + n, -((a*Cot[e + f*x])/b)] - b*(2 + n)*Cot[e + f*x]*Hypergeometric2F1
[1, (3 + n)/2, (5 + n)/2, -Cot[e + f*x]^2])))/(b*(a^2 + b^2)*f*(2 + n)*(3 +
 n)))
```

Maple [F]
time = 2.32, size = 0, normalized size = 0.00

$$\int \frac{(d\cot(fx+e))^n}{a+b\tan(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^n/(a+b*tan(f*x+e)),x)
```

```
[Out] int((d*cot(f*x+e))^n/(a+b*tan(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*cot(f*x + e))^n/(b*tan(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((d*cot(f*x + e))^n/(b*tan(f*x + e) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\cot(e+fx))^n}{a+b\tan(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**n/(a+b*tan(f*x+e)),x)
```

```
[Out] Integral((d*cot(e + f*x))**n/(a + b*tan(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n/(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*cot(f*x + e))^n/(b*tan(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\cot(e+f\,x))^n}{a+b\tan(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(e + f*x))^n/(a + b*tan(e + f*x)),x)
```

```
[Out] int((d*cot(e + f*x))^n/(a + b*tan(e + f*x)), x)
```

3.884 $\int \frac{(d\cot(e+fx))^n}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=250

$$-\frac{a^2(d\cot(e+fx))^{3+n}}{b\left(a^2+b^2\right)d^3f(b+a\cot(e+fx))}+\frac{\left(a^2-b^2\right)(d\cot(e+fx))^{3+n}\,{}_2F_1\left(1,\frac{3+n}{2};\frac{5+n}{2};-\cot^2(e+fx)\right)}{\left(a^2+b^2\right)^2d^3f(3+n)}+\frac{a^2(b^2n+$$

```
[Out] -a^2*(d*cot(f*x+e))^(3+n)/b/(a^2+b^2)/d^3/f/(b+a*cot(f*x+e))+(a^2-b^2)*(d*c
ot(f*x+e))^(3+n)*hypergeom([1, 3/2+1/2*n],[5/2+1/2*n],-cot(f*x+e)^2)/(a^2+b
^2)^2/d^3/f/(3+n)+a^2*(b^2*n+a^2*(2+n))*(d*cot(f*x+e))^(3+n)*hypergeom([1,
3+n],[4+n],-a*cot(f*x+e)/b)/b^2/(a^2+b^2)^2/d^3/f/(3+n)+2*a*b*(d*cot(f*x+e)
)^(4+n)*hypergeom([1, 2+1/2*n],[3+1/2*n],-cot(f*x+e)^2)/(a^2+b^2)^2/d^4/f/(
4+n)
```

Rubi [A]
time = 0.55, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3754, 3650, 3734, 3619, 3557, 371, 3715, 66}

$$\frac{2ab(d\cot(e+fx))^{n+4}\,{}_2F_1\left(1,\frac{n+4}{2};\frac{n+6}{2};-\cot^2(e+fx)\right)}{d^4f(n+4)\left(a^2+b^2\right)^2}+\frac{\left(a^2-b^2\right)(d\cot(e+fx))^{n+3}\,{}_2F_1\left(1,\frac{n+3}{2};\frac{n+5}{2};-\cot^2(e+fx)\right)}{d^3f(n+3)\left(a^2+b^2\right)^2}+\frac{a^2\left(a^2(n+2)+b^2n\right)(d\cot(e+fx))^{n+3}\,{}_2F_1\left(1,n+3;n+4;-\frac{a\cot(e+fx)}{b}\right)}{b^2d^3f(n+3)\left(a^2+b^2\right)^2}-\frac{a^2(d\cot(e+fx))^{n+3}}{bd^3f\left(a^2+b^2\right)(a\cot(e+fx)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^n/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] -((a^2*(d*Cot[e + f*x])^(3 + n))/(b*(a^2 + b^2)*d^3*f*(b + a*Cot[e + f*x]))
) + ((a^2 - b^2)*(d*Cot[e + f*x])^(3 + n)*Hypergeometric2F1[1, (3 + n)/2, (
5 + n)/2, -Cot[e + f*x]^2])/((a^2 + b^2)^2*d^3*f*(3 + n)) + (a^2*(b^2*n + a
^2*(2 + n))*(d*Cot[e + f*x])^(3 + n)*Hypergeometric2F1[1, 3 + n, 4 + n, -((
a*Cot[e + f*x])/b)])/(b^2*(a^2 + b^2)^2*d^3*f*(3 + n)) + (2*a*b*(d*Cot[e +
f*x])^(4 + n)*Hypergeometric2F1[1, (4 + n)/2, (6 + n)/2, -Cot[e + f*x]^2])/
((a^2 + b^2)^2*d^4*f*(4 + n))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
```

```
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{(d\cot(e+fx))^n}{(a+b\tan(e+fx))^2}\,dx = \frac{\int \frac{(d\cot(e+fx))^{2+n}}{(b+a\cot(e+fx))^2}\,dx}{d^2}$$

$$= -\frac{a^2(d\cot(e+fx))^{3+n}}{b\left(a^2+b^2\right)d^3f(b+a\cot(e+fx))} - \frac{\int \frac{(d\cot(e+fx))^{2+n}\left(-d\left(b^2-a^2(2+n)\right)+abd\cot(e+fx\right.}{b+a\cot(e+fx)}}{b\left(a^2+b^2\right)d^3}$$

$$= -\frac{a^2(d\cot(e+fx))^{3+n}}{b\left(a^2+b^2\right)d^3f(b+a\cot(e+fx))} - \frac{\int (d\cot(e+fx))^{2+n}\left(b\left(a^2-b^2\right)d+2ab^2\right.}{b\left(a^2+b^2\right)^2d^3}$$

$$= -\frac{a^2(d\cot(e+fx))^{3+n}}{b\left(a^2+b^2\right)d^3f(b+a\cot(e+fx))} - \frac{(2ab)\int (d\cot(e+fx))^{3+n}\,dx}{\left(a^2+b^2\right)^2d^3} - \frac{\left(a^2-b^2\right.}{}$$

$$= -\frac{a^2(d\cot(e+fx))^{3+n}}{b\left(a^2+b^2\right)d^3f(b+a\cot(e+fx))} + \frac{a^2\left(b^2n+a^2(2+n)\right)(d\cot(e+fx))^{3+n}\,{}_2}{b^2\left(a^2+b^2\right)^2d^3f}$$

$$= -\frac{a^2(d\cot(e+fx))^{3+n}}{b\left(a^2+b^2\right)d^3f(b+a\cot(e+fx))} + \frac{\left(a^2-b^2\right)(d\cot(e+fx))^{3+n}\,{}_2F_1\left(1,\frac{3+n}{2};\right.}{\left(a^2+b^2\right)^2d^3f(3+n}$$

Mathematica [A]
time = 0.89, size = 192, normalized size = 0.77

$$-\frac{\cot^3(e+fx)(d\cot(e+fx))^n\left(b^2\left(-a^2+b^2\right)(4+n)\,{}_2F_1\left(1,\frac{3+n}{2};\frac{5+n}{2};-\cot^2(e+fx)\right)+a\left(2ab^2(4+n)\,{}_2F_1\left(1,3+n;4+n;-\frac{a\cot(e+fx)}{b}\right)-2b^3(3+n)\cot(e+fx)\,{}_2F_1\left(1,\frac{4+n}{2};\frac{6+n}{2};-\cot^2(e+fx)\right)+a\left(a^2+b^2\right)(4+n)\,{}_2F_1\left(2,3+n;4+n;-\frac{a\cot(e+fx)}{b}\right)\right)\right)}{b^2\left(a^2+b^2\right)^2f(3+n)(4+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cot[e + f*x])^n/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] -((Cot[e + f*x]^3*(d*Cot[e + f*x])^n*(b^2*(-a^2 + b^2)*(4 + n)*Hypergeometr
ic2F1[1, (3 + n)/2, (5 + n)/2, -Cot[e + f*x]^2] + a*(2*a*b^2*(4 + n)*Hyperg
eometric2F1[1, 3 + n, 4 + n, -((a*Cot[e + f*x])/b)] - 2*b^3*(3 + n)*Cot[e +
 f*x]*Hypergeometric2F1[1, (4 + n)/2, (6 + n)/2, -Cot[e + f*x]^2] + a*(a^2
+ b^2)*(4 + n)*Hypergeometric2F1[2, 3 + n, 4 + n, -((a*Cot[e + f*x])/b)]))
)/(b^2*(a^2 + b^2)^2*f*(3 + n)*(4 + n)))
```

Maple [F]
time = 2.60, size = 0, normalized size = 0.00

$$\int \frac{(d\cot(fx+e))^n}{(a+b\tan(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^n/(a+b*tan(f*x+e))^2,x)
```

```
[Out] int((d*cot(f*x+e))^n/(a+b*tan(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*cot(f*x + e))^n/(b*tan(f*x + e) + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((d*cot(f*x + e))^n/(b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\cot(e+fx))^n}{(a+b\tan(e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**n/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral((d*cot(e + f*x))**n/(a + b*tan(e + f*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*cot(f*x + e))^n/(b*tan(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\cot(e+f\,x))^n}{(a+b\tan(e+f\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(e + f*x))^n/(a + b*tan(e + f*x))^2,x)
[Out] int((d*cot(e + f*x))^n/(a + b*tan(e + f*x))^2, x)
```

3.885 $\int (d\cot(e+fx))^n(a+b\tan(e+fx))^m\,dx$

Optimal. Leaf size=193

$$\frac{F_1\left(1-n;-m,1;2-n;-\frac{b\tan(e+fx)}{a},-i\tan(e+fx)\right)(d\cot(e+fx))^n\tan(e+fx)(a+b\tan(e+fx))^m\left(1\right.}{2f(1-n)}$$

```
[Out] 1/2*AppellF1(1-n,1,-m,2-n,-I*tan(f*x+e),-b*tan(f*x+e)/a)*(d*cot(f*x+e))^n*t
an(f*x+e)*(a+b*tan(f*x+e))^m/f/(1-n)/((1+b*tan(f*x+e)/a)^m)+1/2*AppellF1(1-
n,1,-m,2-n,I*tan(f*x+e),-b*tan(f*x+e)/a)*(d*cot(f*x+e))^n*tan(f*x+e)*(a+b*t
an(f*x+e))^m/f/(1-n)/((1+b*tan(f*x+e)/a)^m)
```

Rubi [A]
time = 0.19, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4326, 3656, 926, 140, 138}

$$\frac{\tan(e+fx)(d\cot(e+fx))^n(a+b\tan(e+fx))^m\left(\frac{b\tan(e+fx)}{a}+1\right)^{-m}F_1\left(1-n;-m,1;2-n;-\frac{b\tan(e+fx)}{a},-i\tan(e+fx)\right)}{2f(1-n)}+\frac{\tan(e+fx)(d\cot(e+fx))^n(a+b\tan(e+fx))^m\left(\frac{b\tan(e+fx)}{a}+1\right)^{-m}F_1\left(1-n;-m,1;2-n;-\frac{b\tan(e+fx)}{a},i\tan(e+fx)\right)}{2f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^n*(a + b*Tan[e + f*x])^m,x]
```

```
[Out] (AppellF1[1 - n, -m, 1, 2 - n, -((b*Tan[e + f*x])/a), (-I)*Tan[e + f*x]]*(d
*Cot[e + f*x])^n*Tan[e + f*x]*(a + b*Tan[e + f*x])^m)/(2*f*(1 - n)*(1 + (b*
Tan[e + f*x])/a)^m) + (AppellF1[1 - n, -m, 1, 2 - n, -((b*Tan[e + f*x])/a),
 I*Tan[e + f*x]]*(d*Cot[e + f*x])^n*Tan[e + f*x]*(a + b*Tan[e + f*x])^m)/(2
*f*(1 - n)*(1 + (b*Tan[e + f*x])/a)^m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
```

```
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int (d\cot(e+fx))^n(a+b\tan(e+fx))^m\,dx = ((d\cot(e+fx))^n(d\tan(e+fx))^n)\int (d\tan(e+fx))^{-n}(a+b\,\mathrm{t}$$

$$= \frac{((d\cot(e+fx))^n(d\tan(e+fx))^n)\,\mathrm{Subst}\left(\int \frac{(dx)^{-n}(a+bx)^m}{1+x^2}\,dx, x,\right.}{f}$$

$$= \frac{((d\cot(e+fx))^n(d\tan(e+fx))^n)\,\mathrm{Subst}\left(\int \left(\frac{i(dx)^{-n}(a+bx)^m}{2(i-x)}+\frac{i}{}\right.\right.}{f}$$

$$= \frac{(i(d\cot(e+fx))^n(d\tan(e+fx))^n)\,\mathrm{Subst}\left(\int \frac{(dx)^{-n}(a+bx)^m}{i-x}\,dx, x\right.}{2f}$$

$$= \frac{\left(i(d\cot(e+fx))^n(d\tan(e+fx))^n(a+b\tan(e+fx))^m\left(1+\right.\right.}{2f}$$

$$= \frac{F_1\left(1-n;-m,1;2-n;-\frac{b\tan(e+fx)}{a},-i\tan(e+fx)\right)(d\cot(e+}{2f(1-}$$

Mathematica [F]
time = 3.41, size = 0, normalized size = 0.00

$$\int (d\cot(e+fx))^n(a+b\tan(e+fx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Cot[e + f*x])^n*(a + b*Tan[e + f*x])^m,x]
```

```
[Out] Integrate[(d*Cot[e + f*x])^n*(a + b*Tan[e + f*x])^m, x]
```

Maple [F]
time = 0.38, size = 0, normalized size = 0.00

$$\int (d\cot (fx+e))^n (a+b\tan (fx+e))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^n*(a+b*tan(f*x+e))^m,x)
```

```
[Out] int((d*cot(f*x+e))^n*(a+b*tan(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*(a+b*tan(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((d*cot(f*x + e))^n*(b*tan(f*x + e) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*(a+b*tan(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((d*cot(f*x + e))^n*(b*tan(f*x + e) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cot (e+fx))^n (a+b\tan (e+fx))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**n*(a+b*tan(f*x+e))**m,x)
```

```
[Out] Integral((d*cot(e + f*x))**n*(a + b*tan(e + f*x))**m, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*(a+b*tan(f*x+e))^m,x, algorithm="giac")
[Out] integrate((d*cot(f*x + e))^n*(b*tan(f*x + e) + a)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\cot(e+f\,x))^n\,(a+b\tan(e+f\,x))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(e + f*x))^n*(a + b*tan(e + f*x))^m,x)
[Out] int((d*cot(e + f*x))^n*(a + b*tan(e + f*x))^m, x)
```

3.886 $\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=155

$$-\frac{F_1\left(-\frac{1}{2};1,-n;\frac{1}{2};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^n\left(1+\frac{b\tan(c+dx)}{a}\right)^{-n}}{d}-\frac{F_1}{}$$

```
[Out] -AppellF1(-1/2,1,-n,1/2,-I*tan(d*x+c),-b*tan(d*x+c)/a)*cot(d*x+c)^(1/2)*(a+
b*tan(d*x+c))^n/d/((1+b*tan(d*x+c)/a)^n)-AppellF1(-1/2,1,-n,1/2,I*tan(d*x+c
),-b*tan(d*x+c)/a)*cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^n/d/((1+b*tan(d*x+c)/a
)^n)
```

Rubi [A]
time = 0.20, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4326, 3656, 926, 129, 525, 524}

$$-\frac{\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}F_1\left(-\frac{1}{2};1,-n;\frac{1}{2};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)}{d}-\frac{\sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}F_1\left(-\frac{1}{2};1,-n;\frac{1}{2};i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] -((AppellF1[-1/2, 1, -n, 1/2, (-I)*Tan[c + d*x], -((b*Tan[c + d*x])/a)]*Sqr
t[Cot[c + d*x]]*(a + b*Tan[c + d*x])^n)/(d*(1 + (b*Tan[c + d*x])/a)^n)) - (
AppellF1[-1/2, 1, -n, 1/2, I*Tan[c + d*x], -((b*Tan[c + d*x])/a)]*Sqrt[Cot[
c + d*x]]*(a + b*Tan[c + d*x])^n)/(d*(1 + (b*Tan[c + d*x])/a)^n)
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
```

```
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n\,dx = \left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{(a+b\tan(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{\left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{(a+bx)^n}{x^{3/2}(1+x^2)}\,dx, x, \tan(c+\right.}{d}$$

$$= \frac{\left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \left(\frac{i(a+bx)^n}{2(i-x)x^{3/2}} + \frac{i(a+bx)^n}{2x^{3/2}(i+x)}\right)\right.}{d}$$

$$= \frac{\left(i\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{(a+bx)^n}{(i-x)x^{3/2}}\,dx, x, \tan(c+\right.}{2d}$$

$$= \frac{\left(i\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{(a+bx^2)^n}{x^2(i-x^2)}\,dx, x, \sqrt{\tan(c\right.}{d}$$

$$= \frac{\left(i\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^n\left(1+\frac{b\tan(c+}{a}\right.\right.}{d}$$

$$= -\frac{F_1\left(-\frac{1}{2};1,-n;\frac{1}{2};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)\sqrt{\cot(c+dx)}\,(a}{d}$$

Mathematica [F]
time = 3.92, size = 0, normalized size = 0.00

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^n,x]
[Out] Integrate[Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^n, x]
```

Maple [F]
time = 0.36, size = 0, normalized size = 0.00

$$\int \left(\cot^{\frac{3}{2}}(dx+c)\right)(a+b\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^n,x)
[Out] int(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*cot(d*x + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n*cot(d*x + c)^(3/2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(3/2)*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*cot(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^{3/2}\,{\left(a+b\tan(c+d\,x)\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^n,x)
```

```
[Out] int(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^n, x)
```

3.887 $\int \sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=153

$$\frac{F_1\left(\frac{1}{2};1,-n;\frac{3}{2};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)(a+b\tan(c+dx))^n\left(1+\frac{b\tan(c+dx)}{a}\right)^{-n}}{d\sqrt{\cot(c+dx)}}+\frac{F_1\left(\frac{1}{2};1,-n;\frac{3}{2};i\tan(c\right.}{}$$

```
[Out] AppellF1(1/2,1,-n,3/2,-I*tan(d*x+c),-b*tan(d*x+c)/a)*(a+b*tan(d*x+c))^n/d/c
ot(d*x+c)^(1/2)/((1+b*tan(d*x+c)/a)^n)+AppellF1(1/2,1,-n,3/2,I*tan(d*x+c),-
b*tan(d*x+c)/a)*(a+b*tan(d*x+c))^n/d/cot(d*x+c)^(1/2)/((1+b*tan(d*x+c)/a)^n
)
```

Rubi [A]
time = 0.18, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4326, 3656, 926, 129, 441, 440}

$$\frac{(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}F_1\left(\frac{1}{2};1,-n;\frac{3}{2};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)}{d\sqrt{\cot(c+dx)}}+\frac{(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}F_1\left(\frac{1}{2};1,-n;\frac{3}{2};i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)}{d\sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (AppellF1[1/2, 1, -n, 3/2, (-I)*Tan[c + d*x], -((b*Tan[c + d*x])/a)]*(a + b
*Tan[c + d*x])^n)/(d*Sqrt[Cot[c + d*x]]*(1 + (b*Tan[c + d*x])/a)^n) + (Appe
llF1[1/2, 1, -n, 3/2, I*Tan[c + d*x], -((b*Tan[c + d*x])/a)]*(a + b*Tan[c +
 d*x])^n)/(d*Sqrt[Cot[c + d*x]]*(1 + (b*Tan[c + d*x])/a)^n)
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
```

```
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^n\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{(a+b\tan(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$$

$$= \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{(a+bx)^n}{\sqrt{x}\,(1+x^2)}\,dx, x, \tan(\right.}{d}$$

$$= \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \left(\frac{i(a+bx)^n}{2(i-x)\sqrt{x}} + \frac{i(a+b}{2\sqrt{x}\,(}\right.\right.}{d}$$

$$= \frac{\left(i\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{(a+bx)^n}{(i-x)\sqrt{x}}\,dx, x, \tan(\right.}{2d}$$

$$= \frac{\left(i\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{(a+bx^2)^n}{i-x^2}\,dx, x, \sqrt{\tan}\right.}{d}$$

$$= \frac{\left(i\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^n\left(1+\frac{b\tan(}{}\right.\right.}{d}$$

$$= \frac{F_1\left(\frac{1}{2};1,-n;\frac{3}{2};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)(a+b\tan(c+dx))}{d\sqrt{\cot(c+dx)}}$$

Mathematica [F]
time = 4.63, size = 0, normalized size = 0.00

$$\int \sqrt{\cot(c+dx)}\,(a+b\tan(c+dx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^n,x]
[Out] Integrate[Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^n, x]
```

Maple [F]
time = 0.36, size = 0, normalized size = 0.00

$$\int \left(\sqrt{\cot}\,(dx+c)\right)(a+b\tan(dx+c))^n\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^n,x)
[Out] int(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n*sqrt(cot(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*tan(d*x + c) + a)^n*sqrt(cot(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx))^n \sqrt{\cot(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(1/2)*(a+b*tan(d*x+c))**n,x)
[Out] Integral((a + b*tan(c + d*x))**n*sqrt(cot(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^n*sqrt(cot(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cot(c + d\,x)}\, (a + b\tan(c + d\,x))^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^n,x)
[Out] int(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^n, x)
```

3.888 $\int \frac{(a+b\tan(c+dx))^n}{\sqrt{\cot(c+dx)}}\,dx$

Optimal. Leaf size=159

$$\frac{F_1\left(\frac{3}{2};1,-n;\frac{5}{2};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)(a+b\tan(c+dx))^n\left(1+\frac{b\tan(c+dx)}{a}\right)^{-n}}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{F_1\left(\frac{3}{2};1,-n;\frac{5}{2};i\tan(c\right.}{}$$

```
[Out] 1/3*AppellF1(3/2,1,-n,5/2,-I*tan(d*x+c),-b*tan(d*x+c)/a)*(a+b*tan(d*x+c))^n
/d/cot(d*x+c)^(3/2)/((1+b*tan(d*x+c)/a)^n)+1/3*AppellF1(3/2,1,-n,5/2,I*tan(
d*x+c),-b*tan(d*x+c)/a)*(a+b*tan(d*x+c))^n/d/cot(d*x+c)^(3/2)/((1+b*tan(d*x
+c)/a)^n)
```

Rubi [A]
time = 0.19, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4326, 3656, 926, 129, 525, 524}

$$\frac{(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}F_1\left(\frac{3}{2};1,-n;\frac{5}{2};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}F_1\left(\frac{3}{2};1,-n;\frac{5}{2};i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)}{3d\cot^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^n/Sqrt[Cot[c + d*x]],x]
```

```
[Out] (AppellF1[3/2, 1, -n, 5/2, (-I)*Tan[c + d*x], -((b*Tan[c + d*x])/a)]*(a + b
*Tan[c + d*x])^n)/(3*d*Cot[c + d*x]^(3/2)*(1 + (b*Tan[c + d*x])/a)^n) + (Ap
pellF1[3/2, 1, -n, 5/2, I*Tan[c + d*x], -((b*Tan[c + d*x])/a)]*(a + b*Tan[c
 + d*x])^n)/(3*d*Cot[c + d*x]^(3/2)*(1 + (b*Tan[c + d*x])/a)^n)
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{(a+b\tan(c+dx))^n}{\sqrt{\cot(c+dx)}}\,dx = \left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^n\,dx$$

$$= \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{\sqrt{x}\,(a+bx)^n}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\left(\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \left(\frac{i\sqrt{x}\,(a+bx)^n}{2(i-x)} + \frac{i\sqrt{x}\,(a+bx)^n}{2(i+x)}\right)\,dx, x\right.}{d}$$

$$= \frac{\left(i\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{\sqrt{x}\,(a+bx)^n}{i-x}\,dx, x, \tan(c+dx)\right)}{2d} +$$

$$= \frac{\left(i\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{x^2(a+bx^2)^n}{i-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d} -$$

$$= \frac{\left(i\sqrt{\cot(c+dx)}\,\sqrt{\tan(c+dx)}\,(a+b\tan(c+dx))^n\left(1+\frac{b\tan(c+dx)}{a}\right)^{-n}\right)\text{Sub}}{d}$$

$$= \frac{F_1\left(\frac{3}{2}; 1, -n; \frac{5}{2}; -i\tan(c+dx), -\frac{b\tan(c+dx)}{a}\right)(a+b\tan(c+dx))^n\left(1+\frac{b\tan(c+d}{a}\right.}{3d\cot^{\frac{3}{2}}(c+dx)}$$

Mathematica [F]
time = 5.83, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(c+dx))^n}{\sqrt{\cot(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Tan[c + d*x])^n/Sqrt[Cot[c + d*x]],x]
[Out] Integrate[(a + b*Tan[c + d*x])^n/Sqrt[Cot[c + d*x]], x]
```

Maple [F]
time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(dx+c))^n}{\sqrt{\cot(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^n/cot(d*x+c)^(1/2),x)
[Out] int((a+b*tan(d*x+c))^n/cot(d*x+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^n/cot(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n/sqrt(cot(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^n/cot(d*x+c)^(1/2),x, algorithm="fricas")
[Out] integral((b*tan(d*x + c) + a)^n/sqrt(cot(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan(c + dx))^n}{\sqrt{\cot(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**n/cot(d*x+c)**(1/2),x)
[Out] Integral((a + b*tan(c + d*x))**n/sqrt(cot(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^n/cot(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^n/sqrt(cot(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\tan(c + d\,x))^n}{\sqrt{\cot(c + d\,x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^n/cot(c + d*x)^(1/2),x)
[Out] int((a + b*tan(c + d*x))^n/cot(c + d*x)^(1/2), x)
```

3.889 $\int \frac{(a+b\tan(c+dx))^n}{\cot^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=159

$$\frac{F_1\left(\frac{5}{2};1,-n;\frac{7}{2};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)(a+b\tan(c+dx))^n\left(1+\frac{b\tan(c+dx)}{a}\right)^{-n}}{5d\cot^{\frac{5}{2}}(c+dx)}+\frac{F_1\left(\frac{5}{2};1,-n;\frac{7}{2};i\tan(c\right.}{}$$

```
[Out] 1/5*AppellF1(5/2,1,-n,7/2,-I*tan(d*x+c),-b*tan(d*x+c)/a)*(a+b*tan(d*x+c))^n
/d/cot(d*x+c)^(5/2)/((1+b*tan(d*x+c)/a)^n)+1/5*AppellF1(5/2,1,-n,7/2,I*tan(
d*x+c),-b*tan(d*x+c)/a)*(a+b*tan(d*x+c))^n/d/cot(d*x+c)^(5/2)/((1+b*tan(d*x
+c)/a)^n)
```

Rubi [A]
time = 0.19, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4326, 3656, 926, 129, 525, 524}

$$\frac{(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}F_1\left(\frac{5}{2};1,-n;\frac{7}{2};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)}{5d\cot^{\frac{5}{2}}(c+dx)}+\frac{(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}F_1\left(\frac{5}{2};1,-n;\frac{7}{2};i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)}{5d\cot^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^n/Cot[c + d*x]^(3/2),x]
```

```
[Out] (AppellF1[5/2, 1, -n, 7/2, (-I)*Tan[c + d*x], -((b*Tan[c + d*x])/a)]*(a + b
*Tan[c + d*x])^n)/(5*d*Cot[c + d*x]^(5/2)*(1 + (b*Tan[c + d*x])/a)^n) + (Ap
pellF1[5/2, 1, -n, 7/2, I*Tan[c + d*x], -((b*Tan[c + d*x])/a)]*(a + b*Tan[c
 + d*x])^n)/(5*d*Cot[c + d*x]^(5/2)*(1 + (b*Tan[c + d*x])/a)^n)
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \frac{(a+b\tan(c+dx))^n}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n\,dx$$
$$= \frac{\left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{x^{3/2}(a+bx)^n}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}$$
$$= \frac{\left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \left(\frac{ix^{3/2}(a+bx)^n}{2(i-x)}+\frac{ix^{3/2}(a+bx)^n}{2(i+x)}\right)\,dx,x,\text{t}\right.}{d}$$
$$= \frac{\left(i\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{x^{3/2}(a+bx)^n}{i-x}\,dx,x,\tan(c+dx)\right)}{2d}+\frac{(}{}$$
$$= \frac{\left(i\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{x^4(a+bx^2)^n}{i-x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{d}-$$
$$= \frac{\left(i\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^n\left(1+\frac{b\tan(c+dx)}{a}\right)^{-n}\right)\text{Sub}}{d}$$
$$= \frac{F_1\left(\frac{5}{2};1,-n;\frac{7}{2};-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)(a+b\tan(c+dx))^n\left(1+\frac{b\tan(c+d}{a}\right.}{5d\cot^{\frac{5}{2}}(c+dx)}$$

Mathematica [F]
time = 5.83, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(c+dx))^n}{\cot^{\frac{3}{2}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Tan[c + d*x])^n/Cot[c + d*x]^(3/2),x]
[Out] Integrate[(a + b*Tan[c + d*x])^n/Cot[c + d*x]^(3/2), x]
```

Maple [F]
time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(dx+c))^n}{\cot(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^n/cot(d*x+c)^(3/2),x)
[Out] int((a+b*tan(d*x+c))^n/cot(d*x+c)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^n/cot(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n/cot(d*x + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^n/cot(d*x+c)^(3/2),x, algorithm="fricas")
[Out] integral((b*tan(d*x + c) + a)^n/cot(d*x + c)^(3/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(c+dx))^n}{\cot^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**n/cot(d*x+c)**(3/2),x)
[Out] Integral((a + b*tan(c + d*x))**n/cot(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^n/cot(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^n/cot(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\tan(c+d\,x))^n}{\cot(c+d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^n/cot(c + d*x)^(3/2),x)
[Out] int((a + b*tan(c + d*x))^n/cot(c + d*x)^(3/2), x)
```

3.890 $\int (a+ia\tan(e+fx))^3(c-ic\tan(e+fx))\,dx$

Optimal. Leaf size=25

$$-\frac{ic(a+ia\tan(e+fx))^3}{3f}$$

```
[Out] -1/3*I*c*(a+I*a*tan(f*x+e))^3/f
```

Rubi [A]

time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {3603, 3568, 32}

$$-\frac{ic(a+ia\tan(e+fx))^3}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x]),x]
```

```
[Out] ((-1/3*I)*c*(a + I*a*Tan[e + f*x])^3)/f
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int (a+ia\tan(e+fx))^3(c-ic\tan(e+fx))\,dx = (ac)\int \sec^2(e+fx)(a+ia\tan(e+fx))^2\,dx$$
$$= -\frac{(ic)\text{Subst}\left(\int (a+x)^2\,dx, x, ia\tan(e+fx)\right)}{f}$$
$$= -\frac{ic(a+ia\tan(e+fx))^3}{3f}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 55 vs. $2(25) = 50$.
time = 0.21, size = 55, normalized size = 2.20

$$\frac{a^3c(3fx - 3\text{ArcTan}(\tan(e+fx)) + 3\tan(e+fx) + 3i\tan^2(e+fx) - \tan^3(e+fx))}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x]),x]
```

```
[Out] (a^3*c*(3*f*x - 3*ArcTan[Tan[e + f*x]] + 3*Tan[e + f*x] + (3*I)*Tan[e + f*x
]^2 - Tan[e + f*x]^3))/(3*f)
```

Maple [A]
time = 0.07, size = 23, normalized size = 0.92

method	result	size
derivativedivides	$\frac{a^3c(-\tan(fx+e)+i)^3}{3f}$	23
default	$\frac{a^3c(-\tan(fx+e)+i)^3}{3f}$	23
risch	$\frac{8ia^3c(3\,e^{4i(fx+e)}+3\,e^{2i(fx+e)}+1)}{3f(e^{2i(fx+e)}+1)^3}$	48
norman	$\frac{a^3c\tan(fx+e)}{f}+\frac{ia^3c(\tan^2(fx+e))}{f}-\frac{a^3c(\tan^3(fx+e))}{3f}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3*(c-I*c*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/f*a^3*c*(-tan(f*x+e)+I)^3
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(20) = 40$.
time = 0.50, size = 48, normalized size = 1.92

$$-\frac{a^3c\tan(fx+e)^3 - 3i\,a^3c\tan(fx+e)^2 - 3\,a^3c\tan(fx+e)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c-I*c*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/3*(a^3*c*tan(f*x + e)^3 - 3*I*a^3*c*tan(f*x + e)^2 - 3*a^3*c*tan(f*x + e
))/f
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(20) = 40$.
time = 1.10, size = 83, normalized size = 3.32

$$-\frac{8\left(-3i\,a^3ce^{(4i\,fx+4i\,e)}-3i\,a^3ce^{(2i\,fx+2i\,e)}-i\,a^3c\right)}{3\left(fe^{(6i\,fx+6i\,e)}+3\,fe^{(4i\,fx+4i\,e)}+3\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c-I*c*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] -8/3*(-3*I*a^3*c*e^(4*I*f*x + 4*I*e) - 3*I*a^3*c*e^(2*I*f*x + 2*I*e) - I*a^
3*c)/(f*e^(6*I*f*x + 6*I*e) + 3*f*e^(4*I*f*x + 4*I*e) + 3*f*e^(2*I*f*x + 2*
I*e) + f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(20) = 40$.
time = 0.18, size = 114, normalized size = 4.56

$$\frac{24ia^3ce^{4ie}e^{4ifx}+24ia^3ce^{2ie}e^{2ifx}+8ia^3c}{3fe^{6ie}e^{6ifx}+9fe^{4ie}e^{4ifx}+9fe^{2ie}e^{2ifx}+3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3*(c-I*c*tan(f*x+e)),x)
```

```
[Out] (24*I*a**3*c*exp(4*I*e)*exp(4*I*f*x) + 24*I*a**3*c*exp(2*I*e)*exp(2*I*f*x)
+ 8*I*a**3*c)/(3*f*exp(6*I*e)*exp(6*I*f*x) + 9*f*exp(4*I*e)*exp(4*I*f*x) +
9*f*exp(2*I*e)*exp(2*I*f*x) + 3*f)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(20) = 40$.
time = 0.56, size = 83, normalized size = 3.32

$$-\frac{8\left(-3i\,a^3ce^{(4i\,fx+4i\,e)}-3i\,a^3ce^{(2i\,fx+2i\,e)}-i\,a^3c\right)}{3\left(fe^{(6i\,fx+6i\,e)}+3\,fe^{(4i\,fx+4i\,e)}+3\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c-I*c*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] -8/3*(-3*I*a^3*c*e^(4*I*f*x + 4*I*e) - 3*I*a^3*c*e^(2*I*f*x + 2*I*e) - I*a^
3*c)/(f*e^(6*I*f*x + 6*I*e) + 3*f*e^(4*I*f*x + 4*I*e) + 3*f*e^(2*I*f*x + 2*
I*e) + f)
```

Mupad [B]
time = 4.78, size = 36, normalized size = 1.44

$$\frac{a^3\,c\tan(e+f\,x)\,\left(-\tan(e+f\,x)^2+\tan(e+f\,x)\,3\mathrm{i}+3\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i),x)
[Out] (a^3*c*tan(e + f*x)*(tan(e + f*x)*3i - tan(e + f*x)^2 + 3))/(3*f)
```

3.891 $\int (a+ia\tan(e+fx))^2(c-ic\tan(e+fx))\,dx$

Optimal. Leaf size=25

$$-\frac{ic(a+ia\tan(e+fx))^2}{2f}$$

```
[Out] -1/2*I*c*(a+I*a*tan(f*x+e))^2/f
```

Rubi [A]
time = 0.05, antiderivative size = 36, normalized size of antiderivative = 1.44, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {3603, 3567, 3852, 8}

$$\frac{a^2c\tan(e+fx)}{f}+\frac{ia^2c\sec^2(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x]),x]
```

```
[Out] ((I/2)*a^2*c*Sec[e + f*x]^2)/f + (a^2*c*Tan[e + f*x])/f
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+ia\tan(e+fx))^2(c-ic\tan(e+fx))\,dx &= (ac)\int \sec^2(e+fx)(a+ia\tan(e+fx))\,dx\\ &= \frac{ia^2c\sec^2(e+fx)}{2f}+\left(a^2c\right)\int \sec^2(e+fx)\,dx\\ &= \frac{ia^2c\sec^2(e+fx)}{2f}-\frac{(a^2c)\,\mathrm{Subst}(\int 1\,dx,x,-\tan(e+fx))}{f}\\ &= \frac{ia^2c\sec^2(e+fx)}{2f}+\frac{a^2c\tan(e+fx)}{f}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 45, normalized size = 1.80

$$\frac{a^2c(2fx-2\mathrm{ArcTan}(\tan(e+fx))+2\tan(e+fx)+i\tan^2(e+fx))}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x]),x]
[Out] (a^2*c*(2*f*x - 2*ArcTan[Tan[e + f*x]] + 2*Tan[e + f*x] + I*Tan[e + f*x]^2)
)/(2*f)
```

Maple [A]
time = 0.07, size = 31, normalized size = 1.24

method	result	size
derivativedivides	$-\frac{ia^2c\left(-\frac{\left(\tan^2(fx+e)\right)}{2}+i\tan(fx+e)\right)}{f}$	31
default	$-\frac{ia^2c\left(-\frac{\left(\tan^2(fx+e)\right)}{2}+i\tan(fx+e)\right)}{f}$	31
norman	$\frac{a^2c\tan(fx+e)}{f}+\frac{ia^2c\left(\tan^2(fx+e)\right)}{2f}$	34
risch	$\frac{2ia^2c\left(2\,\mathrm{e}^{2i(fx+e)}+1\right)}{f\left(\mathrm{e}^{2i(fx+e)}+1\right)^2}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2*(c-I*c*tan(f*x+e)),x,method=_RETURNVERBOSE)
[Out] -I/f*a^2*c*(-1/2*tan(f*x+e)^2+I*tan(f*x+e))
```

Maxima [A]
time = 0.54, size = 34, normalized size = 1.36

$$-\frac{-i\,a^2c\tan\left(fx+e\right)^2-2\,a^2c\tan\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c-I*c*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/2*(-I*a^2*c*tan(f*x + e)^2 - 2*a^2*c*tan(f*x + e))/f
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(20) = 40$.
time = 0.99, size = 54, normalized size = 2.16

$$-\frac{2\left(-2i\,a^2ce^{(2i\,fx+2i\,e)}-i\,a^2c\right)}{fe^{(4i\,fx+4i\,e)}+2\,fe^{(2i\,fx+2i\,e)}+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c-I*c*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] -2*(-2*I*a^2*c*e^(2*I*f*x + 2*I*e) - I*a^2*c)/(f*e^(4*I*f*x + 4*I*e) + 2*f*
e^(2*I*f*x + 2*I*e) + f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(20) = 40$.
time = 0.17, size = 68, normalized size = 2.72

$$\frac{4ia^2ce^{2ie}e^{2ifx}+2ia^2c}{fe^{4ie}e^{4ifx}+2fe^{2ie}e^{2ifx}+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2*(c-I*c*tan(f*x+e)),x)
```

```
[Out] (4*I*a**2*c*exp(2*I*e)*exp(2*I*f*x) + 2*I*a**2*c)/(f*exp(4*I*e)*exp(4*I*f*x
) + 2*f*exp(2*I*e)*exp(2*I*f*x) + f)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(20) = 40$.
time = 0.48, size = 54, normalized size = 2.16

$$-\frac{2\left(-2i\,a^2ce^{(2i\,fx+2i\,e)}-i\,a^2c\right)}{fe^{(4i\,fx+4i\,e)}+2\,fe^{(2i\,fx+2i\,e)}+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c-I*c*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] -2*(-2*I*a^2*c*e^(2*I*f*x + 2*I*e) - I*a^2*c)/(f*e^(4*I*f*x + 4*I*e) + 2*f*
e^(2*I*f*x + 2*I*e) + f)
```

Mupad [B]
time = 4.73, size = 26, normalized size = 1.04

$$\frac{a^2\,c\tan(e+f\,x)\,\left(2+\tan(e+f\,x)\,1\mathrm{i}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i),x)
[Out] (a^2*c*tan(e + f*x)*(tan(e + f*x)*1i + 2))/(2*f)
```

3.892 $\int (a+ia\tan(e+fx))(c-ic\tan(e+fx))\,dx$

Optimal. Leaf size=12

$$\frac{ac\tan(e+fx)}{f}$$

[Out] a*c*tan(f*x+e)/f

Rubi [A]

time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3603, 3852, 8}

$$\frac{ac\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

[In] Int[(a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*x]),x]

[Out] (a*c*Tan[e + f*x])/f

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int (a+ia\tan(e+fx))(c-ic\tan(e+fx))\,dx &= (ac)\int \sec^2(e+fx)\,dx\\
&= -\frac{(ac)\text{Subst}\left(\int 1\,dx, x, -\tan(e+fx)\right)}{f}\\
&= \frac{ac\tan(e+fx)}{f}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.00

$$\frac{ac\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*x]),x]
```

```
[Out] (a*c*Tan[e + f*x])/f
```

Maple [A]
time = 0.02, size = 13, normalized size = 1.08

method	result	size
derivativedivides	$\frac{ac\tan(fx+e)}{f}$	13
default	$\frac{ac\tan(fx+e)}{f}$	13
norman	$\frac{ac\tan(fx+e)}{f}$	13
risch	$\frac{2iac}{f(e^{2i(fx+e)}+1)}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] a*c*tan(f*x+e)/f
```

Maxima [A]
time = 0.52, size = 13, normalized size = 1.08

$$\frac{ac\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] a*c*tan(f*x + e)/f
```

Fricas [C] Result contains complex when optimal does not.
time = 1.12, size = 20, normalized size = 1.67

$$\frac{2i\,ac}{fe^{(2i\,fx+2i\,e)}+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 2*I*a*c/(f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.08, size = 24, normalized size = 2.00

$$\frac{2iac}{fe^{2ie}e^{2ifx} + f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e)),x)
```

```
[Out] 2*I*a*c/(f*exp(2*I*e)*exp(2*I*f*x) + f)
```

Giac [A]
time = 0.43, size = 13, normalized size = 1.08

$$\frac{ac\tan(fx + e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] a*c*tan(f*x + e)/f
```

Mupad [B]
time = 4.69, size = 12, normalized size = 1.00

$$\frac{a\,c\tan(e + f\,x)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i),x)
```

```
[Out] (a*c*tan(e + f*x))/f
```

3.893 $\int \frac{c-ic\tan(e+fx)}{a+ia\tan(e+fx)}\,dx$

Optimal. Leaf size=23

$$\frac{ic}{f(a+ia\tan(e+fx))}$$

```
[Out] I*c/f/(a+I*a*tan(f*x+e))
```

Rubi [A]
time = 0.05, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {3603, 3568, 32}

$$\frac{ic}{f(a+ia\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (I*c)/(f*(a + I*a*Tan[e + f*x]))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{c - ic\tan(e+fx)}{a+ia\tan(e+fx)}\,dx = (ac)\int \frac{\sec^2(e+fx)}{(a+ia\tan(e+fx))^2}\,dx$$
$$= -\frac{(ic)\text{Subst}\left(\int \frac{1}{(a+x)^2}\,dx, x, ia\tan(e+fx)\right)}{f}$$
$$= \frac{ic}{f(a+ia\tan(e+fx))}$$

Mathematica [A]
time = 0.12, size = 32, normalized size = 1.39

$$\frac{c(i\cos(2(e+fx))+\sin(2(e+fx)))}{2af}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - I*c*Tan[e + f*x])/(a + I*a*Tan[e + f*x]),x]
[Out] (c*(I*Cos[2*(e + f*x)] + Sin[2*(e + f*x)]))/(2*a*f)
```

Maple [A]
time = 0.12, size = 20, normalized size = 0.87

method	result	size
derivativedivides	$\frac{c}{fa(\tan(fx+e)-i)}$	20
default	$\frac{c}{fa(\tan(fx+e)-i)}$	20
risch	$\frac{ic\,\mathrm{e}^{-2i(fx+e)}}{2af}$	20
norman	$\frac{\frac{ic}{af}+\frac{c\tan(fx+e)}{af}}{1+\tan^2(fx+e)}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
[Out] 1/f*c/a/(tan(f*x+e)-I)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.15, size = 19, normalized size = 0.83

$$\frac{i\,ce^{(-2i\,fx-2i\,e)}}{2\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
[Out] 1/2*I*c*e^(-2*I*f*x - 2*I*e)/(a*f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(15) = 30$.
time = 0.08, size = 42, normalized size = 1.83

$$\begin{cases} \frac{ice^{-2ie}e^{-2ifx}}{2af} & \text{for}\ afe^{2ie} \neq 0 \\ \frac{cxe^{-2ie}}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e)),x)
[Out] Piecewise((I*c*exp(-2*I*e)*exp(-2*I*f*x)/(2*a*f), Ne(a*f*exp(2*I*e), 0)), (
c*x*exp(-2*I*e)/a, True))
```

Giac [A]
time = 0.45, size = 33, normalized size = 1.43

$$-\frac{2\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{af\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-i\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e)),x, algorithm="giac")
[Out] -2*c*tan(1/2*f*x + 1/2*e)/(a*f*(tan(1/2*f*x + 1/2*e) - I)^2)
```

Mupad [B]
time = 4.78, size = 23, normalized size = 1.00

$$\frac{c\,1{}\mathrm{i}}{a\,f\,\left(1+\tan\left(e+f\,x\right)\,1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)/(a + a*tan(e + f*x)*1i),x)
[Out] (c*1i)/(a*f*(tan(e + f*x)*1i + 1))
```

3.894 $\int \frac{c-ic\tan(e+fx)}{(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=25

$$\frac{ic}{2f(a+ia\tan(e+fx))^2}$$

```
[Out] 1/2*I*c/f/(a+I*a*tan(f*x+e))^2
```

Rubi [A]
time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {3603, 3568, 32}

$$\frac{ic}{2f(a+ia\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] ((I/2)*c)/(f*(a + I*a*Tan[e + f*x])^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{c-ic\tan(e+fx)}{(a+ia\tan(e+fx))^2}\,dx=(ac)\int \frac{\sec^2(e+fx)}{(a+ia\tan(e+fx))^3}\,dx$$
$$=-\frac{(ic)\text{Subst}\left(\int \frac{1}{(a+x)^3}\,dx,x,ia\tan(e+fx)\right)}{f}$$
$$=\frac{ic}{2f(a+ia\tan(e+fx))^2}$$

Mathematica [A]
time = 0.56, size = 45, normalized size = 1.80

$$\frac{(-3i+\tan(e+fx))(c-ic\tan(e+fx))}{8a^2f(-i+\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - I*c*Tan[e + f*x])/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] ((-3*I + Tan[e + f*x])*(c - I*c*Tan[e + f*x]))/(8*a^2*f*(-I + Tan[e + f*x])
^2)
```

Maple [A]
time = 0.15, size = 22, normalized size = 0.88

method	result	size
derivativedivides	$-\frac{ic}{2f\,a^2(\tan(fx+e)-i)^2}$	22
default	$-\frac{ic}{2f\,a^2(\tan(fx+e)-i)^2}$	22
risch	$\frac{ic\,\mathrm{e}^{-2i(fx+e)}}{4a^2f}+\frac{ic\,\mathrm{e}^{-4i(fx+e)}}{8a^2f}$	40
norman	$\frac{\frac{c\tan(fx+e)}{af}+\frac{ic}{2af}-\frac{ic\left(\tan^2(fx+e)\right)}{2af}}{a\left(1+\tan^2(fx+e)\right)^2}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/f*c/a^2/(tan(f*x+e)-I)^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.88, size = 35, normalized size = 1.40

$$\frac{\left(2i\,ce^{(2i\,fx+2i\,e)}+i\,c\right)e^{(-4i\,fx-4i\,e)}}{8\,a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*I*c*e^(2*I*f*x + 2*I*e) + I*c)*e^(-4*I*f*x - 4*I*e)/(a^2*f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(19) = 38$.
time = 0.16, size = 100, normalized size = 4.00

$$\begin{cases} \frac{\left(8ia^2cfe^{4ie}e^{-2ifx}+4ia^2cfe^{2ie}e^{-4ifx}\right)e^{-6ie}}{32a^4f^2} & \text{for } a^4f^2e^{6ie}\neq 0 \\ \frac{x\left(ce^{2ie}+c\right)e^{-4ie}}{2a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] Piecewise(((8*I*a**2*c*f*exp(4*I*e)*exp(-2*I*f*x) + 4*I*a**2*c*f*exp(2*I*e)
*exp(-4*I*f*x))*exp(-6*I*e)/(32*a**4*f**2), Ne(a**4*f**2*exp(6*I*e), 0)), (
x*(c*exp(2*I*e) + c)*exp(-4*I*e)/(2*a**2), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(20) = 40$.
time = 0.53, size = 65, normalized size = 2.60

$$-\frac{2\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-i\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^2f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-i\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -2*(c*tan(1/2*f*x + 1/2*e)^3 - I*c*tan(1/2*f*x + 1/2*e)^2 - c*tan(1/2*f*x +
 1/2*e))/(a^2*f*(tan(1/2*f*x + 1/2*e) - I)^4)
```

Mupad [B]
time = 4.81, size = 21, normalized size = 0.84

$$-\frac{c\,1\mathrm{i}}{2\,a^2\,f\,\left(\mathrm{tan}\left(e+f\,x\right)-\mathrm{i}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)/(a + a*tan(e + f*x)*1i)^2,x)
[Out] -(c*1i)/(2*a^2*f*(tan(e + f*x) - 1i)^2)
```

3.895 $\int \frac{c-ic\tan(e+fx)}{(a+ia\tan(e+fx))^3} \, dx$

Optimal. Leaf size=25

$$\frac{ic}{3f(a+ia\tan(e+fx))^3}$$

```
[Out] 1/3*I*c/f/(a+I*a*tan(f*x+e))^3
```

Rubi [A]
time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {3603, 3568, 32}

$$\frac{ic}{3f(a+ia\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] ((I/3)*c)/(f*(a + I*a*Tan[e + f*x])^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\begin{aligned}\int \frac{c-ic\tan(e+fx)}{(a+ia\tan(e+fx))^3}\,dx &= (ac)\int \frac{\sec^2(e+fx)}{(a+ia\tan(e+fx))^4}\,dx \\ &= -\frac{(ic)\mathrm{Subst}\left(\int \frac{1}{(a+x)^4}\,dx, x, ia\tan(e+fx)\right)}{f} \\ &= \frac{ic}{3f(a+ia\tan(e+fx))^3}\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 56 vs. $2(25) = 50$.
time = 0.47, size = 56, normalized size = 2.24

$$\frac{c(3+4\cos(2(e+fx))+2i\sin(2(e+fx)))(i\cos(4(e+fx))+\sin(4(e+fx)))}{24a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - I*c*Tan[e + f*x])/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] (c*(3 + 4*Cos[2*(e + f*x)] + (2*I)*Sin[2*(e + f*x)])*(I*Cos[4*(e + f*x)] +
Sin[4*(e + f*x)]))/(24*a^3*f)
```

Maple [**A**]
time = 0.18, size = 21, normalized size = 0.84

method	result	size
derivativedivides	$-\frac{c}{3f\,a^3(\tan(fx+e)-i)^3}$	21
default	$-\frac{c}{3f\,a^3(\tan(fx+e)-i)^3}$	21
risch	$\frac{ic\,\mathrm{e}^{-2i(fx+e)}}{8a^3f}+\frac{ic\,\mathrm{e}^{-4i(fx+e)}}{8a^3f}+\frac{ic\,\mathrm{e}^{-6i(fx+e)}}{24a^3f}$	59
norman	$\frac{\frac{c\tan(fx+e)}{af}-\frac{ic\left(\tan^2(fx+e)\right)}{af}+\frac{ic}{3af}-\frac{c\left(\tan^3(fx+e)\right)}{3af}}{a^2(1+\tan^2(fx+e))^3}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*c/a^3/(tan(f*x+e)-I)^3
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(20) = 40$.
time = 1.22, size = 48, normalized size = 1.92

$$\frac{\left(3i\,ce^{(4i\,fx+4i\,e)}+3i\,ce^{(2i\,fx+2i\,e)}+i\,c\right)e^{(-6i\,fx-6i\,e)}}{24\,a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*I*c*e^(4*I*f*x + 4*I*e) + 3*I*c*e^(2*I*f*x + 2*I*e) + I*c)*e^(-6*I*
f*x - 6*I*e)/(a^3*f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(19) = 38$.
time = 0.19, size = 141, normalized size = 5.64

$$\begin{cases}\frac{(192ia^6cf^2e^{10ie}e^{-2ifx}+192ia^6cf^2e^{8ie}e^{-4ifx}+64ia^6cf^2e^{6ie}e^{-6ifx})e^{-12ie}}{1536a^9f^3} & \text{for } a^9f^3e^{12ie}\neq 0\\ \frac{x(ce^{4ie}+2ce^{2ie}+c)e^{-6ie}}{4a^3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] Piecewise(((192*I*a**6*c*f**2*exp(10*I*e)*exp(-2*I*f*x) + 192*I*a**6*c*f**2
*exp(8*I*e)*exp(-4*I*f*x) + 64*I*a**6*c*f**2*exp(6*I*e)*exp(-6*I*f*x))*exp(
-12*I*e)/(1536*a**9*f**3), Ne(a**9*f**3*exp(12*I*e), 0)), (x*(c*exp(4*I*e)
+ 2*c*exp(2*I*e) + c)*exp(-6*I*e)/(4*a**3), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(20) = 40$.
time = 0.63, size = 96, normalized size = 3.84

$$-\frac{2\left(3\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-6i\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-10\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+6i\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{3\,a^3f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-i\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/3*(3*c*tan(1/2*f*x + 1/2*e)^5 - 6*I*c*tan(1/2*f*x + 1/2*e)^4 - 10*c*tan(
1/2*f*x + 1/2*e)^3 + 6*I*c*tan(1/2*f*x + 1/2*e)^2 + 3*c*tan(1/2*f*x + 1/2*e
))/(a^3*f*(tan(1/2*f*x + 1/2*e) - I)^6)
```

Mupad [B]
time = 4.79, size = 20, normalized size = 0.80

$$-\frac{c}{3\,a^3\,f\,{\left(\mathrm{tan}\left(e+f\,x\right)-\mathrm{i}\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)/(a + a*tan(e + f*x)*1i)^3,x)
```

```
[Out] -c/(3*a^3*f*(tan(e + f*x) - 1i)^3)
```

3.896 $\int (a+ia\tan(e+fx))^4(c-ic\tan(e+fx))^2\,dx$

Optimal. Leaf size=58

$$-\frac{ic^2(a+ia\tan(e+fx))^4}{2f}+\frac{ic^2(a+ia\tan(e+fx))^5}{5af}$$

```
[Out] -1/2*I*c^2*(a+I*a*tan(f*x+e))^4/f+1/5*I*c^2*(a+I*a*tan(f*x+e))^5/a/f
```

Rubi [A]
time = 0.07, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 45}

$$\frac{ic^2(a+ia\tan(e+fx))^5}{5af}-\frac{ic^2(a+ia\tan(e+fx))^4}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^4*(c - I*c*Tan[e + f*x])^2,x]
```

```
[Out] ((-1/2*I)*c^2*(a + I*a*Tan[e + f*x])^4)/f + ((I/5)*c^2*(a + I*a*Tan[e + f*x
])^5)/(a*f)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int (a+ia\tan(e+fx))^4(c-ic\tan(e+fx))^2\,dx = (a^2c^2)\int \sec^4(e+fx)(a+ia\tan(e+fx))^2\,dx$$
$$= -\frac{(ic^2)\operatorname{Subst}\left(\int (a-x)(a+x)^3\,dx, x, ia\tan(e+fx)\right)}{af}$$
$$= -\frac{(ic^2)\operatorname{Subst}\left(\int \left(2a(a+x)^3-(a+x)^4\right)\,dx, x, ia\tan(e+$$
$$af$$
$$= -\frac{ic^2(a+ia\tan(e+fx))^4}{2f}+\frac{ic^2(a+ia\tan(e+fx))^5}{5af}$$

Mathematica [A]
time = 1.13, size = 80, normalized size = 1.38

$$\frac{a^4c^2\sec(e)\sec^5(e+fx)(5i\cos(fx)+5i\cos(2e+fx)+5\sin(fx)-5\sin(2e+fx)+5\sin(2e+3fx)+\sin(4e+5fx))}{20f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^4*(c - I*c*Tan[e + f*x])^2,x]
[Out] (a^4*c^2*Sec[e]*Sec[e + f*x]^5*((5*I)*Cos[f*x] + (5*I)*Cos[2*e + f*x] + 5*S
in[f*x] - 5*Sin[2*e + f*x] + 5*Sin[2*e + 3*f*x] + Sin[4*e + 5*f*x]))/(20*f)
```

Maple [A]
time = 0.08, size = 50, normalized size = 0.86

method	result	size
derivativedivides	$\frac{a^4c^2\left(\tan(fx+e)-\frac{\left(\tan^5(fx+e)\right)}{5}+\frac{i\left(\tan^4(fx+e)\right)}{2}+i\left(\tan^2(fx+e)\right)\right)}{f}$	50
default	$\frac{a^4c^2\left(\tan(fx+e)-\frac{\left(\tan^5(fx+e)\right)}{5}+\frac{i\left(\tan^4(fx+e)\right)}{2}+i\left(\tan^2(fx+e)\right)\right)}{f}$	50
risch	$\frac{8ia^4c^2\left(10\,e^{6i(fx+e)}+10\,e^{4i(fx+e)}+5\,e^{2i(fx+e)}+1\right)}{5f\left(e^{2i(fx+e)}+1\right)^5}$	61
norman	$\frac{a^4c^2\tan(fx+e)}{f}+\frac{ia^4c^2\left(\tan^2(fx+e)\right)}{f}-\frac{a^4c^2\left(\tan^5(fx+e)\right)}{5f}+\frac{ia^4c^2\left(\tan^4(fx+e)\right)}{2f}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^4*(c-I*c*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
[Out] 1/f*a^4*c^2*(tan(f*x+e)-1/5*tan(f*x+e)^5+1/2*I*tan(f*x+e)^4+I*tan(f*x+e)^2)
```

Maxima [A]
time = 0.49, size = 72, normalized size = 1.24

$$-\frac{2\,a^4c^2\tan\left(fx+e\right)^5-5i\,a^4c^2\tan\left(fx+e\right)^4-10i\,a^4c^2\tan\left(fx+e\right)^2-10\,a^4c^2\tan\left(fx+e\right)}{10\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^4*(c-I*c*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/10*(2*a^4*c^2*tan(f*x + e)^5 - 5*I*a^4*c^2*tan(f*x + e)^4 - 10*I*a^4*c^2
*tan(f*x + e)^2 - 10*a^4*c^2*tan(f*x + e))/f
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(48) = 96$.
time = 1.19, size = 133, normalized size = 2.29

$$-\frac{8\left(-10i\,a^4c^2e^{(6i\,fx+6i\,e)}-10i\,a^4c^2e^{(4i\,fx+4i\,e)}-5i\,a^4c^2e^{(2i\,fx+2i\,e)}-i\,a^4c^2\right)}{5\left(fe^{(10i\,fx+10i\,e)}+5\,fe^{(8i\,fx+8i\,e)}+10\,fe^{(6i\,fx+6i\,e)}+10\,fe^{(4i\,fx+4i\,e)}+5\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^4*(c-I*c*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -8/5*(-10*I*a^4*c^2*e^(6*I*f*x + 6*I*e) - 10*I*a^4*c^2*e^(4*I*f*x + 4*I*e)
- 5*I*a^4*c^2*e^(2*I*f*x + 2*I*e) - I*a^4*c^2)/(f*e^(10*I*f*x + 10*I*e) + 5
*f*e^(8*I*f*x + 8*I*e) + 10*f*e^(6*I*f*x + 6*I*e) + 10*f*e^(4*I*f*x + 4*I*e
) + 5*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(44) = 88$.
time = 0.32, size = 182, normalized size = 3.14

$$\frac{80ia^4c^2e^{6ie}e^{6ifx}+80ia^4c^2e^{4ie}e^{4ifx}+40ia^4c^2e^{2ie}e^{2ifx}+8ia^4c^2}{5fe^{10ie}e^{10ifx}+25fe^{8ie}e^{8ifx}+50fe^{6ie}e^{6ifx}+50fe^{4ie}e^{4ifx}+25fe^{2ie}e^{2ifx}+5f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**4*(c-I*c*tan(f*x+e))**2,x)
```

```
[Out] (80*I*a**4*c**2*exp(6*I*e)*exp(6*I*f*x) + 80*I*a**4*c**2*exp(4*I*e)*exp(4*I
*f*x) + 40*I*a**4*c**2*exp(2*I*e)*exp(2*I*f*x) + 8*I*a**4*c**2)/(5*f*exp(10
*I*e)*exp(10*I*f*x) + 25*f*exp(8*I*e)*exp(8*I*f*x) + 50*f*exp(6*I*e)*exp(6*
I*f*x) + 50*f*exp(4*I*e)*exp(4*I*f*x) + 25*f*exp(2*I*e)*exp(2*I*f*x) + 5*f)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(48) = 96$.
time = 0.71, size = 133, normalized size = 2.29

$$-\frac{8\left(-10i\,a^4c^2e^{(6i\,fx+6i\,e)}-10i\,a^4c^2e^{(4i\,fx+4i\,e)}-5i\,a^4c^2e^{(2i\,fx+2i\,e)}-i\,a^4c^2\right)}{5\left(fe^{(10i\,fx+10i\,e)}+5\,fe^{(8i\,fx+8i\,e)}+10\,fe^{(6i\,fx+6i\,e)}+10\,fe^{(4i\,fx+4i\,e)}+5\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^4*(c-I*c*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -8/5*(-10*I*a^4*c^2*e^(6*I*f*x + 6*I*e) - 10*I*a^4*c^2*e^(4*I*f*x + 4*I*e)
- 5*I*a^4*c^2*e^(2*I*f*x + 2*I*e) - I*a^4*c^2)/(f*e^(10*I*f*x + 10*I*e) + 5
*f*e^(8*I*f*x + 8*I*e) + 10*f*e^(6*I*f*x + 6*I*e) + 10*f*e^(4*I*f*x + 4*I*e
) + 5*f*e^(2*I*f*x + 2*I*e) + f)
```

Mupad [B]
time = 4.80, size = 80, normalized size = 1.38

$$\frac{a^4\,c^2\,\sin\left(e+f\,x\right)\,\left(10\cos\left(e+f\,x\right)^4+\cos\left(e+f\,x\right)^3\,\sin\left(e+f\,x\right)\,10{}\mathrm{i}+\cos\left(e+f\,x\right)\,\sin\left(e+f\,x\right)^3\,5{}\mathrm{i}-2\sin\left(e+f\,x\right)^4\right)}{10\,f\,\cos\left(e+f\,x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^4*(c - c*tan(e + f*x)*1i)^2,x)
```

```
[Out] (a^4*c^2*sin(e + f*x)*(cos(e + f*x)*sin(e + f*x)^3*5i + cos(e + f*x)^3*sin(
e + f*x)*10i + 10*cos(e + f*x)^4 - 2*sin(e + f*x)^4))/(10*f*cos(e + f*x)^5)
```

3.897 $\int (a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^2\,dx$

Optimal. Leaf size=61

$$\frac{ia^3c^2\sec^4(e+fx)}{4f}+\frac{a^3c^2\tan(e+fx)}{f}+\frac{a^3c^2\tan^3(e+fx)}{3f}$$

```
[Out] 1/4*I*a^3*c^2*sec(f*x+e)^4/f+a^3*c^2*tan(f*x+e)/f+1/3*a^3*c^2*tan(f*x+e)^3/
f
```

Rubi [A]
time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}=0.097$, Rules used = {3603, 3567, 3852}

$$\frac{a^3c^2\tan^3(e+fx)}{3f}+\frac{a^3c^2\tan(e+fx)}{f}+\frac{ia^3c^2\sec^4(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])^2,x]
```

```
[Out] ((I/4)*a^3*c^2*Sec[e + f*x]^4)/f + (a^3*c^2*Tan[e + f*x])/f + (a^3*c^2*Tan[
e + f*x]^3)/(3*f)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^2\,dx &= (a^2c^2)\int \sec^4(e+fx)(a+ia\tan(e+fx))\,dx\\ &= \frac{ia^3c^2\sec^4(e+fx)}{4f}+(a^3c^2)\int \sec^4(e+fx)\,dx\\ &= \frac{ia^3c^2\sec^4(e+fx)}{4f}-\frac{(a^3c^2)\,\mathrm{Subst}\left(\int (1+x^2)\,dx,x,-\tan\right.}{f}\\ &= \frac{ia^3c^2\sec^4(e+fx)}{4f}+\frac{a^3c^2\tan(e+fx)}{f}+\frac{a^3c^2\tan^3(e+fx}{3f}\end{aligned}$$

Mathematica [A]
time = 0.99, size = 52, normalized size = 0.85

$$\frac{a^3c^2\sec(e)\sec^4(e+fx)(3i\cos(e)-3\sin(e)+4\sin(e+2fx)+\sin(3e+4fx))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])^2,x]
[Out] (a^3*c^2*Sec[e]*Sec[e + f*x]^4*((3*I)*Cos[e] - 3*Sin[e] + 4*Sin[e + 2*f*x]
+ Sin[3*e + 4*f*x]))/(12*f)
```

Maple [A]
time = 0.07, size = 54, normalized size = 0.89

method	result	size
risch	$\frac{4ia^3c^2\left(6\,\mathrm{e}^{4i(fx+e)}+4\,\mathrm{e}^{2i(fx+e)}+1\right)}{3f\left(\mathrm{e}^{2i(fx+e)}+1\right)^4}$	50
derivativedivides	$\frac{ia^3c^2\left(\frac{\left(\tan^4(fx+e)\right)}{4}+\frac{\left(\tan^2(fx+e)\right)}{2}-\frac{i\left(\tan^3(fx+e)\right)}{3}-i\tan(fx+e)\right)}{f}$	54
default	$\frac{ia^3c^2\left(\frac{\left(\tan^4(fx+e)\right)}{4}+\frac{\left(\tan^2(fx+e)\right)}{2}-\frac{i\left(\tan^3(fx+e)\right)}{3}-i\tan(fx+e)\right)}{f}$	54
norman	$\frac{a^3c^2\tan(fx+e)}{f}+\frac{a^3c^2\left(\tan^3(fx+e)\right)}{3f}+\frac{ia^3c^2\left(\tan^2(fx+e)\right)}{2f}+\frac{ia^3c^2\left(\tan^4(fx+e)\right)}{4f}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3*(c-I*c*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
[Out] I/f*a^3*c^2*(1/4*tan(f*x+e)^4+1/2*tan(f*x+e)^2-1/3*I*tan(f*x+e)^3-I*tan(f*x
+e))
```

Maxima [A]
time = 0.50, size = 72, normalized size = 1.18

$$\frac{3i\,a^{3}c^{2}\tan\left(fx+e\right)^{4}+4\,a^{3}c^{2}\tan\left(fx+e\right)^{3}+6i\,a^{3}c^{2}\tan\left(fx+e\right)^{2}+12\,a^{3}c^{2}\tan\left(fx+e\right)}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c-I*c*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/12*(3*I*a^3*c^2*tan(f*x + e)^4 + 4*a^3*c^2*tan(f*x + e)^3 + 6*I*a^3*c^2*t
an(f*x + e)^2 + 12*a^3*c^2*tan(f*x + e))/f
```

Fricas [A]
time = 1.27, size = 102, normalized size = 1.67

$$-\frac{4\left(-6i\,a^{3}c^{2}e^{(4i\,fx+4i\,e)}-4i\,a^{3}c^{2}e^{(2i\,fx+2i\,e)}-i\,a^{3}c^{2}\right)}{3\left(fe^{(8i\,fx+8i\,e)}+4\,fe^{(6i\,fx+6i\,e)}+6\,fe^{(4i\,fx+4i\,e)}+4\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c-I*c*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -4/3*(-6*I*a^3*c^2*e^(4*I*f*x + 4*I*e) - 4*I*a^3*c^2*e^(2*I*f*x + 2*I*e) -
I*a^3*c^2)/(f*e^(8*I*f*x + 8*I*e) + 4*f*e^(6*I*f*x + 6*I*e) + 6*f*e^(4*I*f*
x + 4*I*e) + 4*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(53) = 106.
time = 0.26, size = 138, normalized size = 2.26

$$\frac{24ia^{3}c^{2}e^{4ie}e^{4ifx}+16ia^{3}c^{2}e^{2ie}e^{2ifx}+4ia^{3}c^{2}}{3fe^{8ie}e^{8ifx}+12fe^{6ie}e^{6ifx}+18fe^{4ie}e^{4ifx}+12fe^{2ie}e^{2ifx}+3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3*(c-I*c*tan(f*x+e))**2,x)
```

```
[Out] (24*I*a**3*c**2*exp(4*I*e)*exp(4*I*f*x) + 16*I*a**3*c**2*exp(2*I*e)*exp(2*I
*f*x) + 4*I*a**3*c**2)/(3*f*exp(8*I*e)*exp(8*I*f*x) + 12*f*exp(6*I*e)*exp(6
*I*f*x) + 18*f*exp(4*I*e)*exp(4*I*f*x) + 12*f*exp(2*I*e)*exp(2*I*f*x) + 3*f
)
```

Giac [A]
time = 0.62, size = 102, normalized size = 1.67

$$-\frac{4\left(-6i\,a^{3}c^{2}e^{(4i\,fx+4i\,e)}-4i\,a^{3}c^{2}e^{(2i\,fx+2i\,e)}-i\,a^{3}c^{2}\right)}{3\left(fe^{(8i\,fx+8i\,e)}+4\,fe^{(6i\,fx+6i\,e)}+6\,fe^{(4i\,fx+4i\,e)}+4\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c-I*c*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -4/3*(-6*I*a^3*c^2*e^(4*I*f*x + 4*I*e) - 4*I*a^3*c^2*e^(2*I*f*x + 2*I*e) -
I*a^3*c^2)/(f*e^(8*I*f*x + 8*I*e) + 4*f*e^(6*I*f*x + 6*I*e) + 6*f*e^(4*I*f*
x + 4*I*e) + 4*f*e^(2*I*f*x + 2*I*e) + f)
```

Mupad [B]
time = 4.72, size = 80, normalized size = 1.31

$$\frac{a^3\,c^2\,\sin\left(e+f\,x\right)\,\left(12\cos\left(e+f\,x\right)^3+\cos\left(e+f\,x\right)^2\,\sin\left(e+f\,x\right)\,6{}\mathrm{i}+4\,\cos\left(e+f\,x\right)\,\sin\left(e+f\,x\right)^2+\sin\left(e+f\,x\right)^3\,3{}\mathrm{i}\right)}{12\,f\,\cos\left(e+f\,x\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i)^2,x)
```

```
[Out] (a^3*c^2*sin(e + f*x)*(4*cos(e + f*x)*sin(e + f*x)^2 + cos(e + f*x)^2*sin(e
 + f*x)*6i + 12*cos(e + f*x)^3 + sin(e + f*x)^3*3i))/(12*f*cos(e + f*x)^4)
```

3.898 $\int (a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^2\,dx$

Optimal. Leaf size=38

$$\frac{a^2c^2\tan(e+fx)}{f}+\frac{a^2c^2\tan^3(e+fx)}{3f}$$

```
[Out] a^2*c^2*tan(f*x+e)/f+1/3*a^2*c^2*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {3603, 3852}

$$\frac{a^2c^2\tan^3(e+fx)}{3f}+\frac{a^2c^2\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^2,x]
```

```
[Out] (a^2*c^2*Tan[e + f*x])/f + (a^2*c^2*Tan[e + f*x]^3)/(3*f)
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^2\,dx &= \left(a^2c^2\right)\int \sec^4(e+fx)\,dx\\ &= -\frac{\left(a^2c^2\right)\operatorname{Subst}\left(\int \left(1+x^2\right)\,dx,x,-\tan(e+fx)\right)}{f}\\ &= \frac{a^2c^2\tan(e+fx)}{f}+\frac{a^2c^2\tan^3(e+fx)}{3f}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 29, normalized size = 0.76

$$\frac{a^2c^2\left(\tan(e+fx)+\frac{1}{3}\tan^3(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^2,x]
[Out] (a^2*c^2*(Tan[e + f*x] + Tan[e + f*x]^3/3))/f
```

Maple [A]
time = 0.04, size = 28, normalized size = 0.74

method	result	size
derivativedivides	$\frac{a^{2} c^{2} \left(\frac{\left(\tan^{3}\left(f x +e \right)\right)}{3}+\tan \left(f x +e \right)\right)}{f}$	28
default	$\frac{a^{2} c^{2} \left(\frac{\left(\tan^{3}\left(f x +e \right)\right)}{3}+\tan \left(f x +e \right)\right)}{f}$	28
norman	$\frac{a^{2} c^{2} \tan \left(f x +e \right)}{f}+\frac{a^{2} c^{2} \left(\tan^{3}\left(f x +e \right)\right)}{3 f}$	37
risch	$\frac{4 i a^{2} c^{2} \left(3 \,{\mathrm e}^{2 i \left(f x +e \right)}+1\right)}{3 f \left({\mathrm e}^{2 i \left(f x +e \right)}+1\right)^{3}}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2*(c-I*c*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
[Out] 1/f*a^2*c^2*(1/3*tan(f*x+e)^3+tan(f*x+e))
```

Maxima [A]
time = 0.50, size = 37, normalized size = 0.97

$$\frac{a^2c^2\tan\left(fx+e\right)^3+3\,a^2c^2\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c-I*c*tan(f*x+e))^2,x, algorithm="maxima")
[Out] 1/3*(a^2*c^2*tan(f*x + e)^3 + 3*a^2*c^2*tan(f*x + e))/f
```

Fricas [C] Result contains complex when optimal does not.
time = 1.17, size = 71, normalized size = 1.87

$$-\frac{4\left(-3i\,a^2c^2e^{(2i\,fx+2i\,e)}-i\,a^2c^2\right)}{3\left(fe^{(6i\,fx+6i\,e)}+3\,fe^{(4i\,fx+4i\,e)}+3\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c-I*c*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -4/3*(-3*I*a^2*c^2*e^(2*I*f*x + 2*I*e) - I*a^2*c^2)/(f*e^(6*I*f*x + 6*I*e)
+ 3*f*e^(4*I*f*x + 4*I*e) + 3*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.17, size = 94, normalized size = 2.47

$$\frac{12ia^2c^2e^{2ie}e^{2ifx} + 4ia^2c^2}{3fe^{6ie}e^{6ifx} + 9fe^{4ie}e^{4ifx} + 9fe^{2ie}e^{2ifx} + 3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2*(c-I*c*tan(f*x+e))**2,x)
```

```
[Out] (12*I*a**2*c**2*exp(2*I*e)*exp(2*I*f*x) + 4*I*a**2*c**2)/(3*f*exp(6*I*e)*ex
p(6*I*f*x) + 9*f*exp(4*I*e)*exp(4*I*f*x) + 9*f*exp(2*I*e)*exp(2*I*f*x) + 3*
f)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(38) = 76.
time = 0.56, size = 166, normalized size = 4.37

$$-\frac{3\,a^2c^2\tan\left(fx\right)^3\tan\left(e\right)^2 + 3\,a^2c^2\tan\left(fx\right)^2\tan\left(e\right)^3 + a^2c^2\tan\left(fx\right)^3 - 3\,a^2c^2\tan\left(fx\right)^2\tan\left(e\right) - 3\,a^2c^2\tan\left(fx\right)\tan\left(e\right)^2 + a^2c^2\tan\left(e\right)^3 + 3\,a^2c^2\tan\left(fx\right) + 3\,a^2c^2\tan\left(e\right)}{3\,\left(f\tan\left(fx\right)^3\tan\left(e\right)^3 - 3\,f\tan\left(fx\right)^2\tan\left(e\right)^2 + 3\,f\tan\left(fx\right)\tan\left(e\right) - f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c-I*c*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*a^2*c^2*tan(f*x)^3*tan(e)^2 + 3*a^2*c^2*tan(f*x)^2*tan(e)^3 + a^2*c
^2*tan(f*x)^3 - 3*a^2*c^2*tan(f*x)^2*tan(e) - 3*a^2*c^2*tan(f*x)*tan(e)^2 +
 a^2*c^2*tan(e)^3 + 3*a^2*c^2*tan(f*x) + 3*a^2*c^2*tan(e))/(f*tan(f*x)^3*ta
n(e)^3 - 3*f*tan(f*x)^2*tan(e)^2 + 3*f*tan(f*x)*tan(e) - f)
```

Mupad [B]
time = 4.64, size = 27, normalized size = 0.71

$$\frac{a^2\,c^2\,\tan(e+f\,x)\,\left(\tan(e+f\,x)^2+3\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i)^2,x)
```

```
[Out] (a^2*c^2*tan(e + f*x)*(tan(e + f*x)^2 + 3))/(3*f)
```

3.899 $\int (a+ia\tan(e+fx))(c-ic\tan(e+fx))^2\,dx$

Optimal. Leaf size=25

$$\frac{ia(c-ic\tan(e+fx))^2}{2f}$$

```
[Out] 1/2*I*a*(c-I*c*tan(f*x+e))^2/f
```

Rubi [A]
time = 0.05, antiderivative size = 36, normalized size of antiderivative = 1.44, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {3603, 3567, 3852, 8}

$$\frac{ac^2\tan(e+fx)}{f}-\frac{iac^2\sec^2(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^2,x]
```

```
[Out] ((-1/2*I)*a*c^2*Sec[e + f*x]^2)/f + (a*c^2*Tan[e + f*x])/f
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int (a+ia\tan(e+fx))(c-ic\tan(e+fx))^2\,dx = (ac)\int \sec^2(e+fx)(c-ic\tan(e+fx))\,dx$$
$$= -\frac{iac^2\sec^2(e+fx)}{2f} + (ac^2)\int \sec^2(e+fx)\,dx$$
$$= -\frac{iac^2\sec^2(e+fx)}{2f} - \frac{(ac^2)\,\mathrm{Subst}(\int 1\,dx, x, -\tan(e+fx$$
$$= -\frac{iac^2\sec^2(e+fx)}{2f} + \frac{ac^2\tan(e+fx)}{f}$$

Mathematica [A]
time = 0.16, size = 45, normalized size = 1.80

$$\frac{ac^2(2fx - 2\mathrm{ArcTan}(\tan(e+fx)) + 2\tan(e+fx) - i\tan^2(e+fx))}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^2,x]
[Out] (a*c^2*(2*f*x - 2*ArcTan[Tan[e + f*x]] + 2*Tan[e + f*x] - I*Tan[e + f*x]^2)
)/(2*f)
```

Maple [A]
time = 0.07, size = 31, normalized size = 1.24

method	result	size
risch	$\frac{2ia\,c^2}{f\left(e^{2i(fx+e)}+1\right)^2}$	24
derivativedivides	$-\frac{ia\,c^2\left(\frac{\left(\tan^2(fx+e)\right)}{2}+i\tan(fx+e)\right)}{f}$	31
default	$-\frac{ia\,c^2\left(\frac{\left(\tan^2(fx+e)\right)}{2}+i\tan(fx+e)\right)}{f}$	31
norman	$\frac{a\,c^2\tan(fx+e)}{f} - \frac{ia\,c^2\left(\tan^2(fx+e)\right)}{2f}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
[Out] -I/f*a*c^2*(1/2*tan(f*x+e)^2+I*tan(f*x+e))
```

Maxima [A]
time = 0.55, size = 34, normalized size = 1.36

$$\frac{-i\,ac^2\tan\left(fx+e\right)^2 + 2\,ac^2\tan\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e))^2,x, algorithm="maxima")
[Out] 1/2*(-I*a*c^2*tan(f*x + e)^2 + 2*a*c^2*tan(f*x + e))/f
```

Fricas [A]
time = 1.48, size = 35, normalized size = 1.40

$$\frac{2i\,ac^2}{fe^{(4i\,fx+4i\,e)}+2\,fe^{(2i\,fx+2i\,e)}+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e))^2,x, algorithm="fricas")
[Out] 2*I*a*c^2/(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(19) = 38.
time = 0.12, size = 44, normalized size = 1.76

$$\frac{2iac^2}{fe^{4ie}e^{4ifx}+2fe^{2ie}e^{2ifx}+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e))**2,x)
[Out] 2*I*a*c**2/(f*exp(4*I*e)*exp(4*I*f*x) + 2*f*exp(2*I*e)*exp(2*I*f*x) + f)
```

Giac [A]
time = 0.50, size = 35, normalized size = 1.40

$$\frac{2i\,ac^2}{fe^{(4i\,fx+4i\,e)}+2\,fe^{(2i\,fx+2i\,e)}+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e))^2,x, algorithm="giac")
[Out] 2*I*a*c^2/(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*e) + f)
```

Mupad [B]
time = 4.54, size = 26, normalized size = 1.04

$$-\frac{a\,c^2\tan(e+f\,x)\,\left(-2+\tan(e+f\,x)\,1\mathrm{i}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i)^2,x)
[Out] -(a*c^2*tan(e + f*x)*(tan(e + f*x)*1i - 2))/(2*f)
```

3.900 $\int \frac{(c-ic\tan(e+fx))^2}{a+ia\tan(e+fx)}\,dx$

Optimal. Leaf size=55

$$-\frac{c^2x}{a}-\frac{ic^2\log(\cos(e+fx))}{af}+\frac{2ic^2}{f(a+ia\tan(e+fx))}$$

```
[Out] -c^2*x/a-I*c^2*ln(cos(f*x+e))/a/f+2*I*c^2/f/(a+I*a*tan(f*x+e))
```

Rubi [A]
time = 0.08, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {3603, 3568, 45}

$$\frac{2ic^2}{f(a+ia\tan(e+fx))}-\frac{ic^2\log(\cos(e+fx))}{af}-\frac{c^2x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])^2/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] -((c^2*x)/a) - (I*c^2*Log[Cos[e + f*x]])/(a*f) + ((2*I)*c^2)/(f*(a + I*a*Ta
n[e + f*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\begin{aligned}\int \frac{(c-ic\tan(e+fx))^2}{a+ia\tan(e+fx)}\,dx &= (a^2c^2)\int \frac{\sec^4(e+fx)}{(a+ia\tan(e+fx))^3}\,dx\\ &= -\frac{(ic^2)\,\text{Subst}\left(\int \frac{a-x}{(a+x)^2}\,dx, x, ia\tan(e+fx)\right)}{af}\\ &= -\frac{(ic^2)\,\text{Subst}\left(\int \left(\frac{1}{-a-x}+\frac{2a}{(a+x)^2}\right)dx, x, ia\tan(e+fx)\right)}{af}\\ &= -\frac{c^2x}{a}-\frac{ic^2\log(\cos(e+fx))}{af}+\frac{2ic^2}{f(a+ia\tan(e+fx))}\end{aligned}$$

Mathematica [A]
time = 0.76, size = 74, normalized size = 1.35

$$-\frac{c^2(-2+\log(\cos^2(e+fx))+i(2+\log(\cos^2(e+fx)))\tan(e+fx)+2\text{ArcTan}(\tan(fx))(-i+\tan(e+fx)))}{2af(-i+\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - I*c*Tan[e + f*x])^2/(a + I*a*Tan[e + f*x]),x]
[Out] -1/2*(c^2*(-2 + Log[Cos[e + f*x]^2] + I*(2 + Log[Cos[e + f*x]^2])*Tan[e + f
*x] + 2*ArcTan[Tan[f*x]]*(-I + Tan[e + f*x])))/(a*f*(-I + Tan[e + f*x]))
```

Maple [A]
time = 0.17, size = 38, normalized size = 0.69

method	result	size
derivativedivides	$\frac{c^2\left(\frac{2}{\tan(fx+e)-i}+i\ln(\tan(fx+e)-i)\right)}{fa}$	38
default	$\frac{c^2\left(\frac{2}{\tan(fx+e)-i}+i\ln(\tan(fx+e)-i)\right)}{fa}$	38
risch	$\frac{ic^2e^{-2i(fx+e)}}{af}-\frac{2c^2x}{a}-\frac{2c^2e}{af}-\frac{ic^2\ln(e^{2i(fx+e)}+1)}{af}$	68
norman	$\frac{\frac{2ic^2}{af}-\frac{c^2x}{a}+\frac{2c^2\tan(fx+e)}{af}-\frac{c^2x\left(\tan^2(fx+e)\right)}{a}}{1+\tan^2(fx+e)}+\frac{ic^2\ln(1+\tan^2(fx+e))}{2af}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
[Out] 1/f*c^2/a*(2/(tan(f*x+e)-I)+I*ln(tan(f*x+e)-I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.09, size = 69, normalized size = 1.25

$$-\frac{\left(2\,c^2 f x e^{(2i\,fx+2i\,e)}+i\,c^2 e^{(2i\,fx+2i\,e)}\log\left(e^{(2i\,fx+2i\,e)}+1\right)-i\,c^2\right)e^{(-2i\,fx-2i\,e)}}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(2*c^2*f*x*e^(2*I*f*x + 2*I*e) + I*c^2*e^(2*I*f*x + 2*I*e)*log(e^(2*I*f*x
+ 2*I*e) + 1) - I*c^2)*e^(-2*I*f*x - 2*I*e)/(a*f)
```

Sympy [A]
time = 0.16, size = 100, normalized size = 1.82

$$\begin{cases}\frac{ic^2e^{-2ie}e^{-2ifx}}{af} & \text{for}\ afe^{2ie}\neq 0\\ x\left(\frac{2c^2}{a}+\frac{(-2c^2e^{2ie}+2c^2)e^{-2ie}}{a}\right) & \text{otherwise}\end{cases}-\frac{2c^2x}{a}-\frac{ic^2\log\left(e^{2ifx}+e^{-2ie}\right)}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**2/(a+I*a*tan(f*x+e)),x)
```

```
[Out] Piecewise((I*c**2*exp(-2*I*e)*exp(-2*I*f*x)/(a*f), Ne(a*f*exp(2*I*e), 0)),
(x*(2*c**2/a + (-2*c**2*exp(2*I*e) + 2*c**2)*exp(-2*I*e)/a), True)) - 2*c**
2*x/a - I*c**2*log(exp(2*I*f*x) + exp(-2*I*e))/(a*f)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(51) = 102$.
time = 0.54, size = 125, normalized size = 2.27

$$-\frac{\frac{i\,c^2\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}{a}-\frac{2i\,c^2\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-i\right)}{a}+\frac{i\,c^2\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}{a}+\frac{3i\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+10\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-3i\,c^2}{a\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-i\right)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] -(I*c^2*log(tan(1/2*f*x + 1/2*e) + 1)/a - 2*I*c^2*log(tan(1/2*f*x + 1/2*e)
- I)/a + I*c^2*log(tan(1/2*f*x + 1/2*e) - 1)/a + (3*I*c^2*tan(1/2*f*x + 1/2
*e)^2 + 10*c^2*tan(1/2*f*x + 1/2*e) - 3*I*c^2)/(a*(tan(1/2*f*x + 1/2*e) - I
)^2))/f
```

Mupad [B]
time = 4.62, size = 48, normalized size = 0.87

$$\frac{c^2\,2\mathrm{i}}{a\,f\,\left(1+\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}+\frac{c^2\,\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,1\mathrm{i}}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^2/(a + a*tan(e + f*x)*1i),x)
```

```
[Out] (c^2*2i)/(a*f*(tan(e + f*x)*1i + 1)) + (c^2*log(tan(e + f*x) - 1i)*1i)/(a*f
)
```

3.901 $\int \frac{(c-ic\tan(e+fx))^2}{(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=28

$$\frac{c^2\tan(e+fx)}{f(a+ia\tan(e+fx))^2}$$

[Out] c^2*tan(f*x+e)/f/(a+I*a*tan(f*x+e))^2

Rubi [A]

time = 0.07, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 34}

$$\frac{c^2\tan(e+fx)}{f(a+ia\tan(e+fx))^2}$$

Antiderivative was successfully verified.

[In] Int[(c - I*c*Tan[e + f*x])^2/(a + I*a*Tan[e + f*x])^2,x]

[Out] (c^2*Tan[e + f*x])/(f*(a + I*a*Tan[e + f*x])^2)

Rule 34

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[d*x*((
a + b*x)^(m + 1)/(b*(m + 2))), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c-ic\tan(e+fx))^2}{(a+ia\tan(e+fx))^2}\,dx &= \left(a^2c^2\right)\int \frac{\sec^4(e+fx)}{(a+ia\tan(e+fx))^4}\,dx\\
&= -\frac{(ic^2)\,\text{Subst}\left(\int \frac{a-x}{(a+x)^3}\,dx,x,ia\tan(e+fx)\right)}{af}\\
&= \frac{c^2\tan(e+fx)}{f(a+ia\tan(e+fx))^2}
\end{aligned}
$$

Mathematica [A]
time = 0.26, size = 34, normalized size = 1.21

$$\frac{c^2(i\cos(4(e+fx))+\sin(4(e+fx)))}{4a^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - I*c*Tan[e + f*x])^2/(a + I*a*Tan[e + f*x])^2,x]
[Out] (c^2*(I*Cos[4*(e + f*x)] + Sin[4*(e + f*x)]))/(4*a^2*f)
```

Maple [A]
time = 0.18, size = 39, normalized size = 1.39

method	result	size
risch	$\frac{ic^2 \mathrm{e}^{-4i(fx+e)}}{4f\,a^2}$	22
derivativedivides	$\frac{c^2\left(-\frac{1}{\tan(fx+e)-i}-\frac{i}{(\tan(fx+e)-i)^2}\right)}{f\,a^2}$	39
default	$\frac{c^2\left(-\frac{1}{\tan(fx+e)-i}-\frac{i}{(\tan(fx+e)-i)^2}\right)}{f\,a^2}$	39
norman	$\frac{\frac{c^2\tan(fx+e)}{af}-\frac{c^2\left(\tan^3(fx+e)\right)}{af}-\frac{2ic^2\left(\tan^2(fx+e)\right)}{af}}{a\left(1+\tan^2(fx+e)\right)^2}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
[Out] 1/f*c^2/a^2*(-1/(tan(f*x+e)-I)-I/(tan(f*x+e)-I)^2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.03, size = 21, normalized size = 0.75

$$\frac{i\,c^2 e^{(-4i\,fx-4i\,e)}}{4\,a^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/4*I*c^2*e^(-4*I*f*x - 4*I*e)/(a^2*f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(24) = 48.
time = 0.13, size = 51, normalized size = 1.82

$$\begin{cases} \frac{ic^2e^{-4ie}e^{-4ifx}}{4a^2f} & \text{for}\ a^2fe^{4ie}\neq 0 \\ \frac{c^2xe^{-4ie}}{a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**2/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] Piecewise((I*c**2*exp(-4*I*e)*exp(-4*I*f*x)/(4*a**2*f), Ne(a**2*f*exp(4*I*e
), 0)), (c**2*x*exp(-4*I*e)/a**2, True))
```

Giac [A]
time = 0.59, size = 54, normalized size = 1.93

$$-\frac{2\left(c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^2f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-i\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -2*(c^2*tan(1/2*f*x + 1/2*e)^3 - c^2*tan(1/2*f*x + 1/2*e))/(a^2*f*(tan(1/2*
f*x + 1/2*e) - I)^4)
```

Mupad [B]
time = 4.60, size = 28, normalized size = 1.00

$$-\frac{c^2\tan(e+f\,x)}{a^2\,f\,\left(\tan\left(e+f\,x\right)-\mathrm{i}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^2/(a + a*tan(e + f*x)*1i)^2,x)
```

```
[Out] -(c^2*tan(e + f*x))/(a^2*f*(tan(e + f*x) - 1i)^2)
```

3.902 $\int \frac{(c-ic\tan(e+fx))^2}{(a+ia\tan(e+fx))^3}\,dx$

Optimal. Leaf size=58

$$\frac{2ic^2}{3f(a+ia\tan(e+fx))^3}-\frac{ic^2}{2af(a+ia\tan(e+fx))^2}$$

```
[Out] 2/3*I*c^2/f/(a+I*a*tan(f*x+e))^3-1/2*I*c^2/a/f/(a+I*a*tan(f*x+e))^2
```

Rubi [A]
time = 0.08, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}=0.097$, Rules used = {3603, 3568, 45}

$$\frac{2ic^2}{3f(a+ia\tan(e+fx))^3}-\frac{ic^2}{2af(a+ia\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])^2/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] (((2*I)/3)*c^2)/(f*(a + I*a*Tan[e + f*x])^3) - ((I/2)*c^2)/(a*f*(a + I*a*Ta
n[e + f*x])^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{(c-ic\tan(e+fx))^2}{(a+ia\tan(e+fx))^3}\,dx = (a^2c^2)\int \frac{\sec^4(e+fx)}{(a+ia\tan(e+fx))^5}\,dx$$
$$= -\frac{(ic^2)\,\text{Subst}\left(\int \frac{a-x}{(a+x)^4}\,dx, x, ia\tan(e+fx)\right)}{af}$$
$$= -\frac{(ic^2)\,\text{Subst}\left(\int \left(\frac{2a}{(a+x)^4}-\frac{1}{(a+x)^3}\right)\,dx, x, ia\tan(e+fx)\right)}{af}$$
$$= \frac{2ic^2}{3f(a+ia\tan(e+fx))^3}-\frac{ic^2}{2af(a+ia\tan(e+fx))^2}$$

Mathematica [A]
time = 0.97, size = 53, normalized size = 0.91

$$\frac{c^2(5\cos(e+fx)+i\sin(e+fx))(i\cos(5(e+fx))+\sin(5(e+fx)))}{24a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - I*c*Tan[e + f*x])^2/(a + I*a*Tan[e + f*x])^3,x]
[Out] (c^2*(5*Cos[e + f*x] + I*Sin[e + f*x])*(I*Cos[5*(e + f*x)] + Sin[5*(e + f*x
)]))/(24*a^3*f)
```

Maple [A]
time = 0.19, size = 39, normalized size = 0.67

method	result	size
derivativedivides	$\frac{c^2\left(\frac{i}{2(\tan(fx+e)-i)^2}-\frac{2}{3(\tan(fx+e)-i)^3}\right)}{f\,a^3}$	39
default	$\frac{c^2\left(\frac{i}{2(\tan(fx+e)-i)^2}-\frac{2}{3(\tan(fx+e)-i)^3}\right)}{f\,a^3}$	39
risch	$\frac{ic^2\mathrm{e}^{-4i(fx+e)}}{8a^3f}+\frac{ic^2\mathrm{e}^{-6i(fx+e)}}{12a^3f}$	44
norman	$\frac{\frac{c^2\tan(fx+e)}{af}-\frac{2ic^2\left(\tan^2(fx+e)\right)}{af}+\frac{ic^2}{6af}-\frac{5c^2\left(\tan^3(fx+e)\right)}{3af}+\frac{ic^2\left(\tan^4(fx+e)\right)}{2af}}{a^2\left(1+\tan^2(fx+e)\right)^3}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
[Out] 1/f*c^2/a^3*(1/2*I/(tan(f*x+e)-I)^2-2/3/(tan(f*x+e)-I)^3)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.89, size = 39, normalized size = 0.67

$$\frac{\left(3i\,c^2e^{(2i\,fx+2i\,e)}+2i\,c^2\right)e^{(-6i\,fx-6i\,e)}}{24\,a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*I*c^2*e^(2*I*f*x + 2*I*e) + 2*I*c^2)*e^(-6*I*f*x - 6*I*e)/(a^3*f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(46) = 92.
time = 0.23, size = 107, normalized size = 1.84

$$\begin{cases} \frac{\left(12ia^3c^2fe^{6ie}e^{-4ifx}+8ia^3c^2fe^{4ie}e^{-6ifx}\right)e^{-10ie}}{96a^6f^2} & \text{for}\ a^6f^2e^{10ie}\neq 0 \\ \frac{x\left(c^2e^{2ie}+c^2\right)e^{-6ie}}{2a^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**2/(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] Piecewise((((12*I*a**3*c**2*f*exp(6*I*e)*exp(-4*I*f*x) + 8*I*a**3*c**2*f*exp
(4*I*e)*exp(-6*I*f*x))*exp(-10*I*e)/(96*a**6*f**2), Ne(a**6*f**2*exp(10*I*e
), 0)), (x*(c**2*exp(2*I*e) + c**2)*exp(-6*I*e)/(2*a**3), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(48) = 96.
time = 0.69, size = 106, normalized size = 1.83

$$-\frac{2\left(3\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-3i\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-8\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+3i\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{3\,a^3f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-i\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/3*(3*c^2*tan(1/2*f*x + 1/2*e)^5 - 3*I*c^2*tan(1/2*f*x + 1/2*e)^4 - 8*c^2
*tan(1/2*f*x + 1/2*e)^3 + 3*I*c^2*tan(1/2*f*x + 1/2*e)^2 + 3*c^2*tan(1/2*f*
x + 1/2*e))/(a^3*f*(tan(1/2*f*x + 1/2*e) - I)^6)
```

Mupad [B]
time = 4.62, size = 56, normalized size = 0.97

$$\frac{c^2\,\left(3\tan(e+f\,x)+1{}\mathrm{i}\right)}{6\,a^3\,f\,\left(-{\tan\left(e+f\,x\right)}^3\,1{}\mathrm{i}-3\,{\tan\left(e+f\,x\right)}^2+\tan\left(e+f\,x\right)\,3{}\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^2/(a + a*tan(e + f*x)*1i)^3,x)
```

```
[Out] (c^2*(3*tan(e + f*x) + 1i))/(6*a^3*f*(tan(e + f*x)*3i - 3*tan(e + f*x)^2 -
tan(e + f*x)^3*1i + 1))
```

3.903 $\int \frac{(c-ic\tan(e+fx))^2}{(a+ia\tan(e+fx))^4}\,dx$

Optimal. Leaf size=62

$$\frac{ic^2}{2f(a+ia\tan(e+fx))^4}-\frac{ia^2c^2}{3f\left(a^2+ia^2\tan(e+fx)\right)^3}$$

```
[Out] 1/2*I*c^2/f/(a+I*a*tan(f*x+e))^4-1/3*I*a^2*c^2/f/(a^2+I*a^2*tan(f*x+e))^3
```

Rubi [A]
time = 0.08, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 45}

$$\frac{ic^2}{2f(a+ia\tan(e+fx))^4}-\frac{ia^2c^2}{3f\left(a^2+ia^2\tan(e+fx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])^2/(a + I*a*Tan[e + f*x])^4,x]
```

```
[Out] ((I/2)*c^2)/(f*(a + I*a*Tan[e + f*x])^4) - ((I/3)*a^2*c^2)/(f*(a^2 + I*a^2*
Tan[e + f*x])^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c-ic\tan(e+fx))^2}{(a+ia\tan(e+fx))^4}\,dx &= (a^2c^2)\int \frac{\sec^4(e+fx)}{(a+ia\tan(e+fx))^6}\,dx\\
&= -\frac{(ic^2)\,\text{Subst}\left(\int \frac{a-x}{(a+x)^5}\,dx, x, ia\tan(e+fx)\right)}{af}\\
&= -\frac{(ic^2)\,\text{Subst}\left(\int \left(\frac{2a}{(a+x)^5}-\frac{1}{(a+x)^4}\right)\,dx, x, ia\tan(e+fx)\right)}{af}\\
&= \frac{ic^2}{2f(a+ia\tan(e+fx))^4}-\frac{ic^2}{3af(a+ia\tan(e+fx))^3}
\end{aligned}
$$

Mathematica [A]
time = 1.07, size = 58, normalized size = 0.94

$$
\frac{c^2(8+9\cos(2(e+fx))+3i\sin(2(e+fx)))(i\cos(6(e+fx))+\sin(6(e+fx)))}{96a^4f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - I*c*Tan[e + f*x])^2/(a + I*a*Tan[e + f*x])^4,x]
```

```
[Out] (c^2*(8 + 9*Cos[2*(e + f*x)] + (3*I)*Sin[2*(e + f*x)])*(I*Cos[6*(e + f*x)]
+ Sin[6*(e + f*x)]))/(96*a^4*f)
```

Maple [A]
time = 0.22, size = 39, normalized size = 0.63

method	result	size
derivativedivides	$\frac{c^2\left(\frac{i}{2(\tan(fx+e)-i)^4}+\frac{1}{3(\tan(fx+e)-i)^3}\right)}{f\,a^4}$	39
default	$\frac{c^2\left(\frac{i}{2(\tan(fx+e)-i)^4}+\frac{1}{3(\tan(fx+e)-i)^3}\right)}{f\,a^4}$	39
risch	$\frac{ic^2\mathrm{e}^{-4i(fx+e)}}{16a^4f}+\frac{ic^2\mathrm{e}^{-6i(fx+e)}}{12a^4f}+\frac{ic^2\mathrm{e}^{-8i(fx+e)}}{32a^4f}$	65
norman	$\frac{\frac{c^2\tan(fx+e)}{af}+\frac{ic^2}{6af}-\frac{8c^2\left(\tan^3(fx+e)\right)}{3af}+\frac{c^2\left(\tan^5(fx+e)\right)}{3af}-\frac{7ic^2\left(\tan^2(fx+e)\right)}{3af}+\frac{3ic^2\left(\tan^4(fx+e)\right)}{2af}}{(1+\tan^2(fx+e))^4a^3}$	124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*c^2/a^4*(1/2*I/(tan(f*x+e)-I)^4+1/3/(tan(f*x+e)-I)^3)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.27, size = 54, normalized size = 0.87

$$\frac{\left(6i\,c^2e^{(4i\,fx+4i\,e)}+8i\,c^2e^{(2i\,fx+2i\,e)}+3i\,c^2\right)e^{(-8i\,fx-8i\,e)}}{96\,a^4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/96*(6*I*c^2*e^(4*I*f*x + 4*I*e) + 8*I*c^2*e^(2*I*f*x + 2*I*e) + 3*I*c^2)*
e^(-8*I*f*x - 8*I*e)/(a^4*f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(49) = 98.
time = 0.26, size = 151, normalized size = 2.44

$$\begin{cases}\frac{\left(384ia^8c^2f^2e^{14ie}e^{-4ifx}+512ia^8c^2f^2e^{12ie}e^{-6ifx}+192ia^8c^2f^2e^{10ie}e^{-8ifx}\right)e^{-18ie}}{6144a^{12}f^3} & \text{for } a^{12}f^3e^{18ie}\neq 0\\ \frac{x\left(c^2e^{4ie}+2c^2e^{2ie}+c^2\right)e^{-8ie}}{4a^4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**2/(a+I*a*tan(f*x+e))**4,x)
```

```
[Out] Piecewise(((384*I*a**8*c**2*f**2*exp(14*I*e)*exp(-4*I*f*x) + 512*I*a**8*c**
2*f**2*exp(12*I*e)*exp(-6*I*f*x) + 192*I*a**8*c**2*f**2*exp(10*I*e)*exp(-8*
I*f*x))*exp(-18*I*e)/(6144*a**12*f**3), Ne(a**12*f**3*exp(18*I*e), 0)), (x*
(c**2*exp(4*I*e) + 2*c**2*exp(2*I*e) + c**2)*exp(-8*I*e)/(4*a**4), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(52) = 104.
time = 0.81, size = 140, normalized size = 2.26

$$-\frac{2\left(3\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7-6i\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-17\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+16i\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+17\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-6i\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{3\,a^4f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-i\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -2/3*(3*c^2*tan(1/2*f*x + 1/2*e)^7 - 6*I*c^2*tan(1/2*f*x + 1/2*e)^6 - 17*c^
2*tan(1/2*f*x + 1/2*e)^5 + 16*I*c^2*tan(1/2*f*x + 1/2*e)^4 + 17*c^2*tan(1/2
*f*x + 1/2*e)^3 - 6*I*c^2*tan(1/2*f*x + 1/2*e)^2 - 3*c^2*tan(1/2*f*x + 1/2*
e))/(a^4*f*(tan(1/2*f*x + 1/2*e) - I)^8)
```

Mupad [B]
time = 4.72, size = 67, normalized size = 1.08

$$\frac{c^2\left(-1+\tan(e+f\,x)\ 2\mathrm{i}\right)}{6\,a^4\,f\,\left(\tan\left(e+f\,x\right)^4\,1\mathrm{i}+4\tan\left(e+f\,x\right)^3-\tan\left(e+f\,x\right)^2\,6\mathrm{i}-4\tan\left(e+f\,x\right)+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^2/(a + a*tan(e + f*x)*1i)^4,x)
```

```
[Out] (c^2*(tan(e + f*x)*2i - 1))/(6*a^4*f*(4*tan(e + f*x)^3 - tan(e + f*x)^2*6i
- 4*tan(e + f*x) + tan(e + f*x)^4*1i + 1i))
```

3.904 $\int (a+ia\tan(e+fx))^5(c-ic\tan(e+fx))^3\,dx$

Optimal. Leaf size=88

$$-\frac{4ic^3(a+ia\tan(e+fx))^5}{5f}+\frac{2ic^3(a+ia\tan(e+fx))^6}{3af}-\frac{ic^3(a+ia\tan(e+fx))^7}{7a^2f}$$

```
[Out] -4/5*I*c^3*(a+I*a*tan(f*x+e))^5/f+2/3*I*c^3*(a+I*a*tan(f*x+e))^6/a/f-1/7*I*
c^3*(a+I*a*tan(f*x+e))^7/a^2/f
```

Rubi [A]
time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 45}

$$-\frac{ic^3(a+ia\tan(e+fx))^7}{7a^2f}+\frac{2ic^3(a+ia\tan(e+fx))^6}{3af}-\frac{4ic^3(a+ia\tan(e+fx))^5}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^5*(c - I*c*Tan[e + f*x])^3,x]
```

```
[Out] (((-4*I)/5)*c^3*(a + I*a*Tan[e + f*x])^5)/f + (((2*I)/3)*c^3*(a + I*a*Tan[e
 + f*x])^6)/(a*f) - ((I/7)*c^3*(a + I*a*Tan[e + f*x])^7)/(a^2*f)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int (a+ia\tan(e+fx))^5(c-ic\tan(e+fx))^3\,dx = (a^3c^3)\int \sec^6(e+fx)(a+ia\tan(e+fx))^2\,dx$$
$$= -\frac{(ic^3)\,\mathrm{Subst}\left(\int (a-x)^2(a+x)^4\,dx, x, ia\tan(e+fx)\right)}{a^2f}$$
$$= -\frac{(ic^3)\,\mathrm{Subst}\left(\int \left(4a^2(a+x)^4-4a(a+x)^5+(a+x)^6\right)\,d$$
$$= -\frac{4ic^3(a+ia\tan(e+fx))^5}{5f}+\frac{2ic^3(a+ia\tan(e+fx))^6}{3af}$$

Mathematica [A]
time = 1.34, size = 93, normalized size = 1.06

$$\frac{a^5c^3\sec(e)\sec^7(e+fx)(35i\cos(fx)+35i\cos(2e+fx)+35\sin(fx)-35\sin(2e+fx)+42\sin(2e+3fx)+14\sin(4e+5fx)+2\sin(6e+7fx))}{210f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^5*(c - I*c*Tan[e + f*x])^3,x]
```

```
[Out] (a^5*c^3*Sec[e]*Sec[e + f*x]^7*((35*I)*Cos[f*x] + (35*I)*Cos[2*e + f*x] + 3
5*Sin[f*x] - 35*Sin[2*e + f*x] + 42*Sin[2*e + 3*f*x] + 14*Sin[4*e + 5*f*x]
+ 2*Sin[6*e + 7*f*x]))/(210*f)
```

Maple [A]
time = 0.09, size = 81, normalized size = 0.92

method	result
risch	$\frac{128ia^5c^3\left(35\,\mathrm{e}^{8i(fx+e)}+35\,\mathrm{e}^{6i(fx+e)}+21\,\mathrm{e}^{4i(fx+e)}+7\,\mathrm{e}^{2i(fx+e)}+1\right)}{105f\left(\mathrm{e}^{2i(fx+e)}+1\right)^7}$
derivativedivides	$\frac{a^5c^3\left(\tan(fx+e)-\frac{\left(\tan^7(fx+e)\right)}{7}+\frac{i\left(\tan^6(fx+e)\right)}{3}-\frac{\left(\tan^5(fx+e)\right)}{5}+i\left(\tan^4(fx+e)\right)+\frac{\left(\tan^3(fx+e)\right)}{3}+i\left(\tan^2(fx+e)\right)\right)}{f}$
default	$\frac{a^5c^3\left(\tan(fx+e)-\frac{\left(\tan^7(fx+e)\right)}{7}+\frac{i\left(\tan^6(fx+e)\right)}{3}-\frac{\left(\tan^5(fx+e)\right)}{5}+i\left(\tan^4(fx+e)\right)+\frac{\left(\tan^3(fx+e)\right)}{3}+i\left(\tan^2(fx+e)\right)\right)}{f}$
norman	$\frac{a^5c^3\tan(fx+e)}{f}+\frac{ia^5c^3\left(\tan^2(fx+e)\right)}{f}+\frac{ia^5c^3\left(\tan^4(fx+e)\right)}{f}+\frac{a^5c^3\left(\tan^3(fx+e)\right)}{3f}-\frac{a^5c^3\left(\tan^5(fx+e)\right)}{5f}-\frac{a^5c}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^5*(c-I*c*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a^5*c^3*(tan(f*x+e)-1/7*tan(f*x+e)^7+1/3*I*tan(f*x+e)^6-1/5*tan(f*x+e)^
5+I*tan(f*x+e)^4+1/3*tan(f*x+e)^3+I*tan(f*x+e)^2)
```

Maxima [A]
time = 0.49, size = 123, normalized size = 1.40

$$-\frac{15\,a^5c^3\tan(fx+e)^7-35i\,a^5c^3\tan(fx+e)^6+21\,a^5c^3\tan(fx+e)^5-105i\,a^5c^3\tan(fx+e)^4-35\,a^5c^3\tan(fx+e)^3-105i\,a^5c^3\tan(fx+e)^2-105\,a^5c^3\tan(fx+e)}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^5*(c-I*c*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/105*(15*a^5*c^3*tan(f*x + e)^7 - 35*I*a^5*c^3*tan(f*x + e)^6 + 21*a^5*c^
3*tan(f*x + e)^5 - 105*I*a^5*c^3*tan(f*x + e)^4 - 35*a^5*c^3*tan(f*x + e)^3
 - 105*I*a^5*c^3*tan(f*x + e)^2 - 105*a^5*c^3*tan(f*x + e))/f
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(73) = 146.
time = 0.92, size = 177, normalized size = 2.01

$$-\frac{128\left(-35i\,a^5c^3e^{(8i\,fx+8i\,e)}-35i\,a^5c^3e^{(6i\,fx+6i\,e)}-21i\,a^5c^3e^{(4i\,fx+4i\,e)}-7i\,a^5c^3e^{(2i\,fx+2i\,e)}-i\,a^5c^3\right)}{105\left(fe^{(14i\,fx+14i\,e)}+7\,fe^{(12i\,fx+12i\,e)}+21\,fe^{(10i\,fx+10i\,e)}+35\,fe^{(8i\,fx+8i\,e)}+35\,fe^{(6i\,fx+6i\,e)}+21\,fe^{(4i\,fx+4i\,e)}+7\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^5*(c-I*c*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -128/105*(-35*I*a^5*c^3*e^(8*I*f*x + 8*I*e) - 35*I*a^5*c^3*e^(6*I*f*x + 6*I
*e) - 21*I*a^5*c^3*e^(4*I*f*x + 4*I*e) - 7*I*a^5*c^3*e^(2*I*f*x + 2*I*e) -
I*a^5*c^3)/(f*e^(14*I*f*x + 14*I*e) + 7*f*e^(12*I*f*x + 12*I*e) + 21*f*e^(1
0*I*f*x + 10*I*e) + 35*f*e^(8*I*f*x + 8*I*e) + 35*f*e^(6*I*f*x + 6*I*e) + 2
1*f*e^(4*I*f*x + 4*I*e) + 7*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(73) = 146.
time = 0.52, size = 245, normalized size = 2.78

$$\frac{4480ia^5c^3e^{8ie}e^{8ifx}+4480ia^5c^3e^{6ie}e^{6ifx}+2688ia^5c^3e^{4ie}e^{4ifx}+896ia^5c^3e^{2ie}e^{2ifx}+128ia^5c^3}{105fe^{14ie}e^{14ifx}+735fe^{12ie}e^{12ifx}+2205fe^{10ie}e^{10ifx}+3675fe^{8ie}e^{8ifx}+3675fe^{6ie}e^{6ifx}+2205fe^{4ie}e^{4ifx}+735fe^{2ie}e^{2ifx}+105f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**5*(c-I*c*tan(f*x+e))**3,x)
```

```
[Out] (4480*I*a**5*c**3*exp(8*I*e)*exp(8*I*f*x) + 4480*I*a**5*c**3*exp(6*I*e)*exp
(6*I*f*x) + 2688*I*a**5*c**3*exp(4*I*e)*exp(4*I*f*x) + 896*I*a**5*c**3*exp(
2*I*e)*exp(2*I*f*x) + 128*I*a**5*c**3)/(105*f*exp(14*I*e)*exp(14*I*f*x) + 7
35*f*exp(12*I*e)*exp(12*I*f*x) + 2205*f*exp(10*I*e)*exp(10*I*f*x) + 3675*f*
exp(8*I*e)*exp(8*I*f*x) + 3675*f*exp(6*I*e)*exp(6*I*f*x) + 2205*f*exp(4*I*e
)*exp(4*I*f*x) + 735*f*exp(2*I*e)*exp(2*I*f*x) + 105*f)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(73) = 146.
time = 0.89, size = 177, normalized size = 2.01

$$-\frac{128\left(-35i\,a^5c^3e^{(8i\,fx+8i\,e)}-35i\,a^5c^3e^{(6i\,fx+6i\,e)}-21i\,a^5c^3e^{(4i\,fx+4i\,e)}-7i\,a^5c^3e^{(2i\,fx+2i\,e)}-i\,a^5c^3\right)}{105\left(fe^{(14i\,fx+14i\,e)}+7\,fe^{(12i\,fx+12i\,e)}+21\,fe^{(10i\,fx+10i\,e)}+35\,fe^{(8i\,fx+8i\,e)}+35\,fe^{(6i\,fx+6i\,e)}+21\,fe^{(4i\,fx+4i\,e)}+7\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I*a*tan(f*x+e))^5*(c-I*c*tan(f*x+e))^3,x, algorithm="giac")

[Out] -128/105*(-35*I*a^5*c^3*e^(8*I*f*x + 8*I*e) - 35*I*a^5*c^3*e^(6*I*f*x + 6*I*e) - 21*I*a^5*c^3*e^(4*I*f*x + 4*I*e) - 7*I*a^5*c^3*e^(2*I*f*x + 2*I*e) - I*a^5*c^3)/(f*e^(14*I*f*x + 14*I*e) + 7*f*e^(12*I*f*x + 12*I*e) + 21*f*e^(10*I*f*x + 10*I*e) + 35*f*e^(8*I*f*x + 8*I*e) + 35*f*e^(6*I*f*x + 6*I*e) + 21*f*e^(4*I*f*x + 4*I*e) + 7*f*e^(2*I*f*x + 2*I*e) + f)

Mupad [B]
time = 4.90, size = 96, normalized size = 1.09

$$\frac{a^5\,c^3\,\left(-\cos\left(e+f\,x\right)^7\,35{}\mathrm{i}+64\,\sin\left(e+f\,x\right)\,\cos\left(e+f\,x\right)^6+32\,\sin\left(e+f\,x\right)\,\cos\left(e+f\,x\right)^4+24\,\sin\left(e+f\,x\right)\,\cos\left(e+f\,x\right)^2+\cos\left(e+f\,x\right)\,35{}\mathrm{i}-15\,\sin\left(e+f\,x\right)\right)}{105\,f\,\cos\left(e+f\,x\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*tan(e + f*x)*1i)^5*(c - c*tan(e + f*x)*1i)^3,x)

[Out] (a^5*c^3*(cos(e + f*x)*35i - 15*sin(e + f*x) + 24*cos(e + f*x)^2*sin(e + f*x) + 32*cos(e + f*x)^4*sin(e + f*x) + 64*cos(e + f*x)^6*sin(e + f*x) - cos(e + f*x)^7*35i))/(105*f*cos(e + f*x)^7)

3.905 $\int (a+ia\tan(e+fx))^4(c-ic\tan(e+fx))^3\,dx$

Optimal. Leaf size=82

$$\frac{ia^4c^3\sec^6(e+fx)}{6f}+\frac{a^4c^3\tan(e+fx)}{f}+\frac{2a^4c^3\tan^3(e+fx)}{3f}+\frac{a^4c^3\tan^5(e+fx)}{5f}$$

```
[Out] 1/6*I*a^4*c^3*sec(f*x+e)^6/f+a^4*c^3*tan(f*x+e)/f+2/3*a^4*c^3*tan(f*x+e)^3/
f+1/5*a^4*c^3*tan(f*x+e)^5/f
```

Rubi [A]
time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}=0.097$, Rules used = {3603, 3567, 3852}

$$\frac{a^4c^3\tan^5(e+fx)}{5f}+\frac{2a^4c^3\tan^3(e+fx)}{3f}+\frac{a^4c^3\tan(e+fx)}{f}+\frac{ia^4c^3\sec^6(e+fx)}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^4*(c - I*c*Tan[e + f*x])^3,x]
```

```
[Out] ((I/6)*a^4*c^3*Sec[e + f*x]^6)/f + (a^4*c^3*Tan[e + f*x])/f + (2*a^4*c^3*Ta
n[e + f*x]^3)/(3*f) + (a^4*c^3*Tan[e + f*x]^5)/(5*f)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int (a+ia\tan(e+fx))^4(c-ic\tan(e+fx))^3\,dx = (a^3c^3)\int \sec^6(e+fx)(a+ia\tan(e+fx))\,dx$$
$$= \frac{ia^4c^3\sec^6(e+fx)}{6f} + (a^4c^3)\int \sec^6(e+fx)\,dx$$
$$= \frac{ia^4c^3\sec^6(e+fx)}{6f} - \frac{(a^4c^3)\,\mathrm{Subst}\left(\int (1+2x^2+x^4)\,dx, x\right.}{f}$$
$$= \frac{ia^4c^3\sec^6(e+fx)}{6f} + \frac{a^4c^3\tan(e+fx)}{f} + \frac{2a^4c^3\tan^3(e+}{3f}$$

Mathematica [A]
time = 1.21, size = 63, normalized size = 0.77

$$\frac{a^4c^3\sec(e)\sec^6(e+fx)(10i\cos(e)-10\sin(e)+15\sin(e+2fx)+6\sin(3e+4fx)+\sin(5e+6fx))}{60f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^4*(c - I*c*Tan[e + f*x])^3,x]
```

```
[Out] (a^4*c^3*Sec[e]*Sec[e + f*x]^6*((10*I)*Cos[e] - 10*Sin[e] + 15*Sin[e + 2*f*
x] + 6*Sin[3*e + 4*f*x] + Sin[5*e + 6*f*x]))/(60*f)
```

Maple [A]
time = 0.08, size = 75, normalized size = 0.91

method	result
risch	$\frac{16ia^4c^3\left(20\,\mathrm{e}^{6i(fx+e)}+15\,\mathrm{e}^{4i(fx+e)}+6\,\mathrm{e}^{2i(fx+e)}+1\right)}{15f\left(\mathrm{e}^{2i(fx+e)}+1\right)^6}$
derivativedivides	$-\frac{ia^4c^3\left(i\tan(fx+e)-\frac{\left(\tan^6(fx+e)\right)}{6}+\frac{i\left(\tan^5(fx+e)\right)}{5}-\frac{\left(\tan^4(fx+e)\right)}{2}+\frac{2i\left(\tan^3(fx+e)\right)}{3}-\frac{\left(\tan^2(fx+e)\right)}{2}\right)}{f}$
default	$-\frac{ia^4c^3\left(i\tan(fx+e)-\frac{\left(\tan^6(fx+e)\right)}{6}+\frac{i\left(\tan^5(fx+e)\right)}{5}-\frac{\left(\tan^4(fx+e)\right)}{2}+\frac{2i\left(\tan^3(fx+e)\right)}{3}-\frac{\left(\tan^2(fx+e)\right)}{2}\right)}{f}$
norman	$\frac{a^4c^3\tan(fx+e)}{f}+\frac{2a^4c^3\left(\tan^3(fx+e)\right)}{3f}+\frac{a^4c^3\left(\tan^5(fx+e)\right)}{5f}+\frac{ia^4c^3\left(\tan^2(fx+e)\right)}{2f}+\frac{ia^4c^3\left(\tan^4(fx+e)\right)}{2f}+ ia$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^4*(c-I*c*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -I/f*a^4*c^3*(I*tan(f*x+e)-1/6*tan(f*x+e)^6+1/5*I*tan(f*x+e)^5-1/2*tan(f*x+
e)^4+2/3*I*tan(f*x+e)^3-1/2*tan(f*x+e)^2)
```

Maxima [A]
time = 0.52, size = 106, normalized size = 1.29

$$\frac{5i\,a^4c^3\tan\left(fx+e\right)^6+6\,a^4c^3\tan\left(fx+e\right)^5+15i\,a^4c^3\tan\left(fx+e\right)^4+20\,a^4c^3\tan\left(fx+e\right)^3+15i\,a^4c^3\tan\left(fx+e\right)^2+30\,a^4c^3\tan\left(fx+e\right)}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^4*(c-I*c*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/30*(5*I*a^4*c^3*tan(f*x + e)^6 + 6*a^4*c^3*tan(f*x + e)^5 + 15*I*a^4*c^3*
tan(f*x + e)^4 + 20*a^4*c^3*tan(f*x + e)^3 + 15*I*a^4*c^3*tan(f*x + e)^2 +
30*a^4*c^3*tan(f*x + e))/f
```

Fricas [A]
time = 1.09, size = 146, normalized size = 1.78

$$-\frac{16\left(-20i\,a^4c^3e^{(6i\,fx+6i\,e)}-15i\,a^4c^3e^{(4i\,fx+4i\,e)}-6i\,a^4c^3e^{(2i\,fx+2i\,e)}-i\,a^4c^3\right)}{15\left(fe^{(12i\,fx+12i\,e)}+6\,fe^{(10i\,fx+10i\,e)}+15\,fe^{(8i\,fx+8i\,e)}+20\,fe^{(6i\,fx+6i\,e)}+15\,fe^{(4i\,fx+4i\,e)}+6\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^4*(c-I*c*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -16/15*(-20*I*a^4*c^3*e^(6*I*f*x + 6*I*e) - 15*I*a^4*c^3*e^(4*I*f*x + 4*I*e
) - 6*I*a^4*c^3*e^(2*I*f*x + 2*I*e) - I*a^4*c^3)/(f*e^(12*I*f*x + 12*I*e) +
 6*f*e^(10*I*f*x + 10*I*e) + 15*f*e^(8*I*f*x + 8*I*e) + 20*f*e^(6*I*f*x + 6
*I*e) + 15*f*e^(4*I*f*x + 4*I*e) + 6*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(73) = 146.
time = 0.43, size = 201, normalized size = 2.45

$$\frac{320ia^4c^3e^{6ie}e^{6ifx}+240ia^4c^3e^{4ie}e^{4ifx}+96ia^4c^3e^{2ie}e^{2ifx}+16ia^4c^3}{15fe^{12ie}e^{12ifx}+90fe^{10ie}e^{10ifx}+225fe^{8ie}e^{8ifx}+300fe^{6ie}e^{6ifx}+225fe^{4ie}e^{4ifx}+90fe^{2ie}e^{2ifx}+15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**4*(c-I*c*tan(f*x+e))**3,x)
```

```
[Out] (320*I*a**4*c**3*exp(6*I*e)*exp(6*I*f*x) + 240*I*a**4*c**3*exp(4*I*e)*exp(4
*I*f*x) + 96*I*a**4*c**3*exp(2*I*e)*exp(2*I*f*x) + 16*I*a**4*c**3)/(15*f*ex
p(12*I*e)*exp(12*I*f*x) + 90*f*exp(10*I*e)*exp(10*I*f*x) + 225*f*exp(8*I*e)
*exp(8*I*f*x) + 300*f*exp(6*I*e)*exp(6*I*f*x) + 225*f*exp(4*I*e)*exp(4*I*f*
x) + 90*f*exp(2*I*e)*exp(2*I*f*x) + 15*f)
```

Giac [A]
time = 0.78, size = 146, normalized size = 1.78

$$-\frac{16\left(-20i\,a^4c^3e^{(6i\,fx+6i\,e)}-15i\,a^4c^3e^{(4i\,fx+4i\,e)}-6i\,a^4c^3e^{(2i\,fx+2i\,e)}-i\,a^4c^3\right)}{15\left(fe^{(12i\,fx+12i\,e)}+6\,fe^{(10i\,fx+10i\,e)}+15\,fe^{(8i\,fx+8i\,e)}+20\,fe^{(6i\,fx+6i\,e)}+15\,fe^{(4i\,fx+4i\,e)}+6\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^4*(c-I*c*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -16/15*(-20*I*a^4*c^3*e^(6*I*f*x + 6*I*e) - 15*I*a^4*c^3*e^(4*I*f*x + 4*I*e
) - 6*I*a^4*c^3*e^(2*I*f*x + 2*I*e) - I*a^4*c^3)/(f*e^(12*I*f*x + 12*I*e) +
 6*f*e^(10*I*f*x + 10*I*e) + 15*f*e^(8*I*f*x + 8*I*e) + 20*f*e^(6*I*f*x + 6
*I*e) + 15*f*e^(4*I*f*x + 4*I*e) + 6*f*e^(2*I*f*x + 2*I*e) + f)
```

Mupad [B]
time = 4.71, size = 117, normalized size = 1.43

$$\frac{a^4\,c^3\,\sin\left(e+f\,x\right)\,\left(30\cos\left(e+f\,x\right)^5+\cos\left(e+f\,x\right)^4\,\sin\left(e+f\,x\right)\,15{}\mathrm{i}+20\cos\left(e+f\,x\right)^3\sin\left(e+f\,x\right)^2+\cos\left(e+f\,x\right)^2\sin\left(e+f\,x\right)^3\,15{}\mathrm{i}+6\,\cos\left(e+f\,x\right)\,\sin\left(e+f\,x\right)^4+\sin\left(e+f\,x\right)^5\,5{}\mathrm{i}\right)}{30\,f\,\cos\left(e+f\,x\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^4*(c - c*tan(e + f*x)*1i)^3,x)
```

```
[Out] (a^4*c^3*sin(e + f*x)*(6*cos(e + f*x)*sin(e + f*x)^4 + cos(e + f*x)^4*sin(e
 + f*x)*15i + 30*cos(e + f*x)^5 + sin(e + f*x)^5*5i + cos(e + f*x)^2*sin(e
+ f*x)^3*15i + 20*cos(e + f*x)^3*sin(e + f*x)^2))/(30*f*cos(e + f*x)^6)
```

3.906 $\int (a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^3\,dx$

Optimal. Leaf size=59

$$\frac{a^3c^3\tan(e+fx)}{f}+\frac{2a^3c^3\tan^3(e+fx)}{3f}+\frac{a^3c^3\tan^5(e+fx)}{5f}$$

```
[Out] a^3*c^3*tan(f*x+e)/f+2/3*a^3*c^3*tan(f*x+e)^3/f+1/5*a^3*c^3*tan(f*x+e)^5/f
```

Rubi [A]
time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}=0.065$, Rules used = {3603, 3852}

$$\frac{a^3c^3\tan^5(e+fx)}{5f}+\frac{2a^3c^3\tan^3(e+fx)}{3f}+\frac{a^3c^3\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])^3,x]
```

```
[Out] (a^3*c^3*Tan[e + f*x])/f + (2*a^3*c^3*Tan[e + f*x]^3)/(3*f) + (a^3*c^3*Tan[
e + f*x]^5)/(5*f)
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^3\,dx &= \left(a^3c^3\right)\int \sec^6(e+fx)\,dx\\ &= -\frac{\left(a^3c^3\right)\operatorname{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx,x,-\tan(e+fx)\right)}{f}\\ &= \frac{a^3c^3\tan(e+fx)}{f}+\frac{2a^3c^3\tan^3(e+fx)}{3f}+\frac{a^3c^3\tan^5(e+f}{5f}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 41, normalized size = 0.69

$$\frac{a^3c^3\left(\tan(e+fx)+\frac{2}{3}\tan^3(e+fx)+\frac{1}{5}\tan^5(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])^3,x]
[Out] (a^3*c^3*(Tan[e + f*x] + (2*Tan[e + f*x]^3)/3 + Tan[e + f*x]^5/5))/f
```

Maple [A]
time = 0.04, size = 38, normalized size = 0.64

method	result	size
derivativedivides	$\frac{a^3c^3\left(\frac{\left(\tan^5(fx+e)\right)}{5}+\frac{2\left(\tan^3(fx+e)\right)}{3}+\tan(fx+e)\right)}{f}$	38
default	$\frac{a^3c^3\left(\frac{\left(\tan^5(fx+e)\right)}{5}+\frac{2\left(\tan^3(fx+e)\right)}{3}+\tan(fx+e)\right)}{f}$	38
risch	$\frac{16ia^3c^3\left(10\,\mathrm{e}^{4i(fx+e)}+5\,\mathrm{e}^{2i(fx+e)}+1\right)}{15f\left(\mathrm{e}^{2i(fx+e)}+1\right)^5}$	50
norman	$\frac{a^3c^3\tan(fx+e)}{f}+\frac{2a^3c^3\left(\tan^3(fx+e)\right)}{3f}+\frac{a^3c^3\left(\tan^5(fx+e)\right)}{5f}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3*(c-I*c*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
[Out] 1/f*a^3*c^3*(1/5*tan(f*x+e)^5+2/3*tan(f*x+e)^3+tan(f*x+e))
```

Maxima [A]
time = 0.52, size = 55, normalized size = 0.93

$$\frac{3\,a^3c^3\tan\left(fx+e\right)^5+10\,a^3c^3\tan\left(fx+e\right)^3+15\,a^3c^3\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c-I*c*tan(f*x+e))^3,x, algorithm="maxima")
[Out] 1/15*(3*a^3*c^3*tan(f*x + e)^5 + 10*a^3*c^3*tan(f*x + e)^3 + 15*a^3*c^3*tan
(f*x + e))/f
```

Fricas [C] Result contains complex when optimal does not.
time = 0.94, size = 115, normalized size = 1.95

$$-\frac{16\left(-10i\,a^3c^3e^{(4i\,fx+4i\,e)}-5i\,a^3c^3e^{(2i\,fx+2i\,e)}-i\,a^3c^3\right)}{15\left(fe^{(10i\,fx+10i\,e)}+5\,fe^{(8i\,fx+8i\,e)}+10\,fe^{(6i\,fx+6i\,e)}+10\,fe^{(4i\,fx+4i\,e)}+5\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c-I*c*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -16/15*(-10*I*a^3*c^3*e^(4*I*f*x + 4*I*e) - 5*I*a^3*c^3*e^(2*I*f*x + 2*I*e)
 - I*a^3*c^3)/(f*e^(10*I*f*x + 10*I*e) + 5*f*e^(8*I*f*x + 8*I*e) + 10*f*e^(
6*I*f*x + 6*I*e) + 10*f*e^(4*I*f*x + 4*I*e) + 5*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [**C**] Result contains complex when optimal does not.
time = 0.32, size = 156, normalized size = 2.64

$$\frac{160ia^3c^3e^{4ie}e^{4ifx}+80ia^3c^3e^{2ie}e^{2ifx}+16ia^3c^3}{15fe^{10ie}e^{10ifx}+75fe^{8ie}e^{8ifx}+150fe^{6ie}e^{6ifx}+150fe^{4ie}e^{4ifx}+75fe^{2ie}e^{2ifx}+15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3*(c-I*c*tan(f*x+e))**3,x)
```

```
[Out] (160*I*a**3*c**3*exp(4*I*e)*exp(4*I*f*x) + 80*I*a**3*c**3*exp(2*I*e)*exp(2*
I*f*x) + 16*I*a**3*c**3)/(15*f*exp(10*I*e)*exp(10*I*f*x) + 75*f*exp(8*I*e)*
exp(8*I*f*x) + 150*f*exp(6*I*e)*exp(6*I*f*x) + 150*f*exp(4*I*e)*exp(4*I*f*x
) + 75*f*exp(2*I*e)*exp(2*I*f*x) + 15*f)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 371 vs. 2(58) = 116.
time = 0.72, size = 371, normalized size = 6.29

$$-\frac{15\,a^3c^3\tan(fx)^5\tan(e)^4+15\,a^3c^3\tan(fx)^4\tan(e)^5+10\,a^3c^3\tan(fx)^5\tan(e)^2-30\,a^3c^3\tan(fx)^4\tan(e)^3-30\,a^3c^3\tan(fx)^3\tan(e)^4+10\,a^3c^3\tan(fx)^2\tan(e)^5+3\,a^3c^3\tan(fx)^5-5\,a^3c^3\tan(fx)^4\tan(e)+60\,a^3c^3\tan(fx)^3\tan(e)^2+60\,a^3c^3\tan(fx)^2\tan(e)^3-5\,a^3c^3\tan(fx)\tan(e)^4+3\,a^3c^3\tan(e)^5+10\,a^3c^3\tan(fx)^3-30\,a^3c^3\tan(fx)^2\tan(e)-30\,a^3c^3\tan(fx)\tan(e)^2+10\,a^3c^3\tan(e)^3+15\,a^3c^3\tan(fx)+15\,a^3c^3\tan(e)}{15\left(f\tan(fx)^5\tan(e)^5-5\,f\tan(fx)^4\tan(e)^4+10\,f\tan(fx)^3\tan(e)^3-10\,f\tan(fx)^2\tan(e)^2+5\,f\tan(fx)\tan(e)-f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c-I*c*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/15*(15*a^3*c^3*tan(f*x)^5*tan(e)^4 + 15*a^3*c^3*tan(f*x)^4*tan(e)^5 + 10
*a^3*c^3*tan(f*x)^5*tan(e)^2 - 30*a^3*c^3*tan(f*x)^4*tan(e)^3 - 30*a^3*c^3*
tan(f*x)^3*tan(e)^4 + 10*a^3*c^3*tan(f*x)^2*tan(e)^5 + 3*a^3*c^3*tan(f*x)^5
 - 5*a^3*c^3*tan(f*x)^4*tan(e) + 60*a^3*c^3*tan(f*x)^3*tan(e)^2 + 60*a^3*c^
3*tan(f*x)^2*tan(e)^3 - 5*a^3*c^3*tan(f*x)*tan(e)^4 + 3*a^3*c^3*tan(e)^5 +
10*a^3*c^3*tan(f*x)^3 - 30*a^3*c^3*tan(f*x)^2*tan(e) - 30*a^3*c^3*tan(f*x)*
tan(e)^2 + 10*a^3*c^3*tan(e)^3 + 15*a^3*c^3*tan(f*x) + 15*a^3*c^3*tan(e))/(
f*tan(f*x)^5*tan(e)^5 - 5*f*tan(f*x)^4*tan(e)^4 + 10*f*tan(f*x)^3*tan(e)^3
- 10*f*tan(f*x)^2*tan(e)^2 + 5*f*tan(f*x)*tan(e) - f)
```

Mupad [**B**]
time = 4.73, size = 39, normalized size = 0.66

$$\frac{a^3\,c^3\,\tan(e+f\,x)\,\left(3\,{\tan(e+f\,x)}^4+10\,{\tan(e+f\,x)}^2+15\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i)^3,x)
```

```
[Out] (a^3*c^3*tan(e + f*x)*(10*tan(e + f*x)^2 + 3*tan(e + f*x)^4 + 15))/(15*f)
```

3.907 $\int (a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^3\,dx$

Optimal. Leaf size=61

$$-\frac{ia^2c^3\sec^4(e+fx)}{4f}+\frac{a^2c^3\tan(e+fx)}{f}+\frac{a^2c^3\tan^3(e+fx)}{3f}$$

```
[Out] -1/4*I*a^2*c^3*sec(f*x+e)^4/f+a^2*c^3*tan(f*x+e)/f+1/3*a^2*c^3*tan(f*x+e)^3
/f
```

Rubi [A]
time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3567, 3852}

$$\frac{a^2c^3\tan^3(e+fx)}{3f}+\frac{a^2c^3\tan(e+fx)}{f}-\frac{ia^2c^3\sec^4(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^3,x]
```

```
[Out] ((-1/4*I)*a^2*c^3*Sec[e + f*x]^4)/f + (a^2*c^3*Tan[e + f*x])/f + (a^2*c^3*T
an[e + f*x]^3)/(3*f)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int (a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^3\,dx = \left(a^2c^2\right)\int \sec^4(e+fx)(c-ic\tan(e+fx))\,dx$$
$$= -\frac{ia^2c^3\sec^4(e+fx)}{4f} + \left(a^2c^3\right)\int \sec^4(e+fx)\,dx$$
$$= -\frac{ia^2c^3\sec^4(e+fx)}{4f} - \frac{\left(a^2c^3\right)\mathrm{Subst}\left(\int \left(1+x^2\right)\,dx, x, -\mathrm{ta}\right.}{f}$$
$$= -\frac{ia^2c^3\sec^4(e+fx)}{4f} + \frac{a^2c^3\tan(e+fx)}{f} + \frac{a^2c^3\tan^3(e+}{3f}$$

Mathematica [A]
time = 1.03, size = 52, normalized size = 0.85

$$\frac{a^2c^3\sec(e)\sec^4(e+fx)(-3i\cos(e)-3\sin(e)+4\sin(e+2fx)+\sin(3e+4fx))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^3,x]
```

```
[Out] (a^2*c^3*Sec[e]*Sec[e + f*x]^4*((-3*I)*Cos[e] - 3*Sin[e] + 4*Sin[e + 2*f*x]
 + Sin[3*e + 4*f*x]))/(12*f)
```

Maple [A]
time = 0.07, size = 54, normalized size = 0.89

method	result	size
risch	$\frac{4ia^2c^3\left(4\,\mathrm{e}^{2i(fx+e)}+1\right)}{3f\left(\mathrm{e}^{2i(fx+e)}+1\right)^4}$	39
derivativedivides	$\frac{ia^2c^3\left(-i\tan(fx+e)-\frac{\left(\tan^4(fx+e)\right)}{4}-\frac{i\left(\tan^3(fx+e)\right)}{3}-\frac{\left(\tan^2(fx+e)\right)}{2}\right)}{f}$	54
default	$\frac{ia^2c^3\left(-i\tan(fx+e)-\frac{\left(\tan^4(fx+e)\right)}{4}-\frac{i\left(\tan^3(fx+e)\right)}{3}-\frac{\left(\tan^2(fx+e)\right)}{2}\right)}{f}$	54
norman	$\frac{a^2c^3\tan(fx+e)}{f}+\frac{a^2c^3\left(\tan^3(fx+e)\right)}{3f}-\frac{ia^2c^3\left(\tan^2(fx+e)\right)}{2f}-\frac{ia^2c^3\left(\tan^4(fx+e)\right)}{4f}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2*(c-I*c*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] I/f*a^2*c^3*(-I*tan(f*x+e)-1/4*tan(f*x+e)^4-1/3*I*tan(f*x+e)^3-1/2*tan(f*x+
e)^2)
```

Maxima [A]
time = 0.53, size = 72, normalized size = 1.18

$$-\frac{3i\,a^2c^3\tan\left(fx+e\right)^4-4\,a^2c^3\tan\left(fx+e\right)^3+6i\,a^2c^3\tan\left(fx+e\right)^2-12\,a^2c^3\tan\left(fx+e\right)}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c-I*c*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/12*(3*I*a^2*c^3*tan(f*x + e)^4 - 4*a^2*c^3*tan(f*x + e)^3 + 6*I*a^2*c^3*
tan(f*x + e)^2 - 12*a^2*c^3*tan(f*x + e))/f
```

Fricas [A]
time = 1.20, size = 84, normalized size = 1.38

$$-\frac{4\left(-4i\,a^2c^3e^{(2i\,fx+2i\,e)}-i\,a^2c^3\right)}{3\left(fe^{(8i\,fx+8i\,e)}+4\,fe^{(6i\,fx+6i\,e)}+6\,fe^{(4i\,fx+4i\,e)}+4\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c-I*c*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -4/3*(-4*I*a^2*c^3*e^(2*I*f*x + 2*I*e) - I*a^2*c^3)/(f*e^(8*I*f*x + 8*I*e)
+ 4*f*e^(6*I*f*x + 6*I*e) + 6*f*e^(4*I*f*x + 4*I*e) + 4*f*e^(2*I*f*x + 2*I*
e) + f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(53) = 106.
time = 0.28, size = 112, normalized size = 1.84

$$\frac{16ia^2c^3e^{2ie}e^{2ifx}+4ia^2c^3}{3fe^{8ie}e^{8ifx}+12fe^{6ie}e^{6ifx}+18fe^{4ie}e^{4ifx}+12fe^{2ie}e^{2ifx}+3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2*(c-I*c*tan(f*x+e))**3,x)
```

```
[Out] (16*I*a**2*c**3*exp(2*I*e)*exp(2*I*f*x) + 4*I*a**2*c**3)/(3*f*exp(8*I*e)*ex
p(8*I*f*x) + 12*f*exp(6*I*e)*exp(6*I*f*x) + 18*f*exp(4*I*e)*exp(4*I*f*x) +
12*f*exp(2*I*e)*exp(2*I*f*x) + 3*f)
```

Giac [A]
time = 0.62, size = 84, normalized size = 1.38

$$-\frac{4\left(-4i\,a^2c^3e^{(2i\,fx+2i\,e)}-i\,a^2c^3\right)}{3\left(fe^{(8i\,fx+8i\,e)}+4\,fe^{(6i\,fx+6i\,e)}+6\,fe^{(4i\,fx+4i\,e)}+4\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c-I*c*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -4/3*(-4*I*a^2*c^3*e^(2*I*f*x + 2*I*e) - I*a^2*c^3)/(f*e^(8*I*f*x + 8*I*e)
+ 4*f*e^(6*I*f*x + 6*I*e) + 6*f*e^(4*I*f*x + 4*I*e) + 4*f*e^(2*I*f*x + 2*I*
e) + f)
```

Mupad [B]
time = 4.75, size = 80, normalized size = 1.31

$$\frac{a^2\,c^3\,\sin\left(e+f\,x\right)\,\left(12\cos\left(e+f\,x\right)^3-\cos\left(e+f\,x\right)^2\,\sin\left(e+f\,x\right)\,6\mathrm{i}+4\,\cos\left(e+f\,x\right)\,\sin\left(e+f\,x\right)^2-\sin\left(e+f\,x\right)^3\,3\mathrm{i}\right)}{12\,f\,\cos\left(e+f\,x\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i)^3,x)
```

```
[Out] (a^2*c^3*sin(e + f*x)*(4*cos(e + f*x)*sin(e + f*x)^2 - cos(e + f*x)^2*sin(e
 + f*x)*6i + 12*cos(e + f*x)^3 - sin(e + f*x)^3*3i))/(12*f*cos(e + f*x)^4)
```

3.908 $\int (a+ia\tan(e+fx))(c-ic\tan(e+fx))^3\,dx$

Optimal. Leaf size=25

$$\frac{ia(c-ic\tan(e+fx))^3}{3f}$$

```
[Out] 1/3*I*a*(c-I*c*tan(f*x+e))^3/f
```

Rubi [A]

time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {3603, 3568, 32}

$$\frac{ia(c-ic\tan(e+fx))^3}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^3,x]
```

```
[Out] ((I/3)*a*(c - I*c*Tan[e + f*x])^3)/f
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int (a+ia\tan(e+fx))(c-ic\tan(e+fx))^3\,dx = (ac)\int \sec^2(e+fx)(c-ic\tan(e+fx))^2\,dx$$
$$= \frac{(ia)\text{Subst}\left(\int (c+x)^2\,dx, x, -ic\tan(e+fx)\right)}{f}$$
$$= \frac{ia(c-ic\tan(e+fx))^3}{3f}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 55 vs. $2(25) = 50$.
time = 0.20, size = 55, normalized size = 2.20

$$\frac{ac^3(3fx - 3\text{ArcTan}(\tan(e+fx)) + 3\tan(e+fx) - 3i\tan^2(e+fx) - \tan^3(e+fx))}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^3,x]
[Out] (a*c^3*(3*f*x - 3*ArcTan[Tan[e + f*x]] + 3*Tan[e + f*x] - (3*I)*Tan[e + f*x
]^2 - Tan[e + f*x]^3))/(3*f)
```

Maple [A]
time = 0.07, size = 21, normalized size = 0.84

method	result	size
derivativedivides	$-\frac{a\,c^3(\tan(fx+e)+i)^3}{3f}$	21
default	$-\frac{a\,c^3(\tan(fx+e)+i)^3}{3f}$	21
risch	$\frac{8ia\,c^3}{3f\left(e^{2i(fx+e)}+1\right)^3}$	24
norman	$\frac{a\,c^3\tan(fx+e)}{f} - \frac{a\,c^3(\tan^3(fx+e))}{3f} - \frac{ia\,c^3(\tan^2(fx+e))}{f}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
[Out] -1/3/f*a*c^3*(tan(f*x+e)+I)^3
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(20) = 40$.
time = 0.53, size = 48, normalized size = 1.92

$$-\frac{ac^3\tan(fx+e)^3 + 3i\,ac^3\tan(fx+e)^2 - 3\,ac^3\tan(fx+e)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/3*(a*c^3*tan(f*x + e)^3 + 3*I*a*c^3*tan(f*x + e)^2 - 3*a*c^3*tan(f*x + e
))/f
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(20) = 40$.
time = 1.55, size = 48, normalized size = 1.92

$$\frac{8i\,ac^3}{3\left(fe^{(6i\,fx+6i\,e)}+3\,fe^{(4i\,fx+4i\,e)}+3\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 8/3*I*a*c^3/(f*e^(6*I*f*x + 6*I*e) + 3*f*e^(4*I*f*x + 4*I*e) + 3*f*e^(2*I*f
*x + 2*I*e) + f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(19) = 38$.
time = 0.15, size = 66, normalized size = 2.64

$$\frac{8iac^3}{3fe^{6ie}e^{6ifx}+9fe^{4ie}e^{4ifx}+9fe^{2ie}e^{2ifx}+3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e))**3,x)
```

```
[Out] 8*I*a*c**3/(3*f*exp(6*I*e)*exp(6*I*f*x) + 9*f*exp(4*I*e)*exp(4*I*f*x) + 9*f
*exp(2*I*e)*exp(2*I*f*x) + 3*f)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(20) = 40$.
time = 0.57, size = 48, normalized size = 1.92

$$\frac{8i\,ac^3}{3\left(fe^{(6i\,fx+6i\,e)}+3\,fe^{(4i\,fx+4i\,e)}+3\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 8/3*I*a*c^3/(f*e^(6*I*f*x + 6*I*e) + 3*f*e^(4*I*f*x + 4*I*e) + 3*f*e^(2*I*f
*x + 2*I*e) + f)
```

Mupad [B]
time = 4.73, size = 34, normalized size = 1.36

$$-\frac{a\,c^3\tan(e+f\,x)\,\left(\tan(e+f\,x)^2+\tan(e+f\,x)\,3\mathrm{i}-3\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i)^3,x)
[Out] -(a*c^3*tan(e + f*x)*(tan(e + f*x)*3i + tan(e + f*x)^2 - 3))/(3*f)
```

3.909 $\int \frac{(c-ic\tan(e+fx))^3}{a+ia\tan(e+fx)}\,dx$

Optimal. Leaf size=71

$$-\frac{4c^3x}{a}-\frac{4ic^3\log(\cos(e+fx))}{af}+\frac{c^3\tan(e+fx)}{af}+\frac{4ic^3}{f(a+ia\tan(e+fx))}$$

```
[Out] -4*c^3*x/a-4*I*c^3*ln(cos(f*x+e))/a/f+c^3*tan(f*x+e)/a/f+4*I*c^3/f/(a+I*a*t
an(f*x+e))
```

Rubi [A]
time = 0.08, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}=0.097$, Rules used = {3603, 3568, 45}

$$\frac{c^3\tan(e+fx)}{af}+\frac{4ic^3}{f(a+ia\tan(e+fx))}-\frac{4ic^3\log(\cos(e+fx))}{af}-\frac{4c^3x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])^3/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (-4*c^3*x)/a - ((4*I)*c^3*Log[Cos[e + f*x]])/(a*f) + (c^3*Tan[e + f*x])/(a*
f) + ((4*I)*c^3)/(f*(a + I*a*Tan[e + f*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c-ic\tan(e+fx))^3}{a+ia\tan(e+fx)}\,dx &= \left(a^3c^3\right)\int \frac{\sec^6(e+fx)}{(a+ia\tan(e+fx))^4}\,dx \\
&= -\frac{\left(ic^3\right)\text{Subst}\left(\int \frac{(a-x)^2}{(a+x)^2}\,dx, x, ia\tan(e+fx)\right)}{a^2f} \\
&= -\frac{\left(ic^3\right)\text{Subst}\left(\int \left(1+\frac{4a^2}{(a+x)^2}-\frac{4a}{a+x}\right)\,dx, x, ia\tan(e+fx)\right)}{a^2f} \\
&= -\frac{4c^3x}{a}-\frac{4ic^3\log(\cos(e+fx))}{af}+\frac{c^3\tan(e+fx)}{af}+\frac{4ic^3}{f(a+ia\tan(e+fx))}
\end{aligned}
$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 234 vs. 2(71) = 142.
time = 1.01, size = 234, normalized size = 3.30

$$
\frac{ic^3\sec^2(e+fx)\left(-i\cos(3e+2fx)+i\cos(e+2fx)\log\left(\cos^2(e+fx)\right)+i\cos(3e+2fx)\log\left(\cos^2(e+fx)\right)+i\cos(e)\left(-3+2\log\left(\cos^2(e+fx)\right)\right)+\sin(e)+8\text{ArcTan}(\tan(fx))\cos(e)\cos(e+fx)(\cos(e+fx)+i\sin(e+fx))-2\sin(e+2fx)-\log\left(\cos^2(e+fx)\right)\sin(e+2fx)-\sin(3e+2fx)-\log\left(\cos^2(e+fx)\right)\sin(3e+2fx)\right)}{2af\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)(-i+\tan(e+fx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - I*c*Tan[e + f*x])^3/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] ((I/2)*c^3*Sec[e + f*x]^2*((-I)*Cos[3*e + 2*f*x] + I*Cos[e + 2*f*x]*Log[Cos
[e + f*x]^2] + I*Cos[3*e + 2*f*x]*Log[Cos[e + f*x]^2] + I*Cos[e]*(-3 + 2*Lo
g[Cos[e + f*x]^2]) + Sin[e] + 8*ArcTan[Tan[f*x]]*Cos[e]*Cos[e + f*x]*(Cos[e
 + f*x] + I*Sin[e + f*x]) - 2*Sin[e + 2*f*x] - Log[Cos[e + f*x]^2]*Sin[e +
2*f*x] - Sin[3*e + 2*f*x] - Log[Cos[e + f*x]^2]*Sin[3*e + 2*f*x]))/(a*f*(Co
s[e/2] - Sin[e/2])*(Cos[e/2] + Sin[e/2])*(-I + Tan[e + f*x]))
```

Maple [A]
time = 0.17, size = 44, normalized size = 0.62

method	result	size
derivativedivides	$\frac{c^3\left(\tan(fx+e)+4i\ln(\tan(fx+e)-i)+\frac{4}{\tan(fx+e)-i}\right)}{fa}$	44
default	$\frac{c^3\left(\tan(fx+e)+4i\ln(\tan(fx+e)-i)+\frac{4}{\tan(fx+e)-i}\right)}{fa}$	44
risch	$\frac{2ic^3\mathrm{e}^{-2i(fx+e)}}{af}-\frac{8c^3x}{a}-\frac{8c^3e}{af}+\frac{2ic^3}{fa\left(\mathrm{e}^{2i(fx+e)}+1\right)}-\frac{4ic^3\ln\left(\mathrm{e}^{2i(fx+e)}+1\right)}{af}$	93
norman	$\frac{\frac{4ic^3}{af}+\frac{c^3\left(\tan^3(fx+e)\right)}{af}-\frac{4c^3x}{a}-\frac{4c^3x\left(\tan^2(fx+e)\right)}{a}+\frac{5c^3\tan(fx+e)}{af}}{1+\tan^2(fx+e)}+\frac{2ic^3\ln\left(1+\tan^2(fx+e)\right)}{af}$	112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*c^3/a*(tan(f*x+e)+4*I*ln(tan(f*x+e)-I)+4/(tan(f*x+e)-I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.01, size = 125, normalized size = 1.76

$$-\frac{2\left(4\,c^{3}fxe^{(4i\,fx+4i\,e)}-i\,c^{3}+2\left(2\,c^{3}fx-i\,c^{3}\right)e^{(2i\,fx+2i\,e)}+2\left(i\,c^{3}e^{(4i\,fx+4i\,e)}+i\,c^{3}e^{(2i\,fx+2i\,e)}\right)\log\left(e^{(2i\,fx+2i\,e)}+1\right)\right)}{afe^{(4i\,fx+4i\,e)}+afe^{(2i\,fx+2i\,e)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] -2*(4*c^3*f*x*e^(4*I*f*x + 4*I*e) - I*c^3 + 2*(2*c^3*f*x - I*c^3)*e^(2*I*f*
x + 2*I*e) + 2*(I*c^3*e^(4*I*f*x + 4*I*e) + I*c^3*e^(2*I*f*x + 2*I*e))*log(
e^(2*I*f*x + 2*I*e) + 1))/(a*f*e^(4*I*f*x + 4*I*e) + a*f*e^(2*I*f*x + 2*I*e
))
```

Sympy [A]
time = 0.22, size = 133, normalized size = 1.87

$$\frac{2ic^{3}}{afe^{2ie}e^{2ifx}+af}+\begin{cases}\frac{2ic^{3}e^{-2ie}e^{-2ifx}}{af} & \text{for}\ afe^{2ie}\neq 0\\ x\left(\frac{8c^{3}}{a}+\frac{\left(-8c^{3}e^{2ie}+4c^{3}\right)e^{-2ie}}{a}\right) & \text{otherwise}\end{cases}-\frac{8c^{3}x}{a}-\frac{4ic^{3}\log\left(e^{2ifx}+e^{-2ie}\right)}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**3/(a+I*a*tan(f*x+e)),x)
```

```
[Out] 2*I*c**3/(a*f*exp(2*I*e)*exp(2*I*f*x) + a*f) + Piecewise((2*I*c**3*exp(-2*I
*e)*exp(-2*I*f*x)/(a*f), Ne(a*f*exp(2*I*e), 0)), (x*(8*c**3/a + (-8*c**3*ex
p(2*I*e) + 4*c**3)*exp(-2*I*e)/a), True)) - 8*c**3*x/a - 4*I*c**3*log(exp(2
*I*f*x) + exp(-2*I*e))/(a*f)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(68) = 136$.
time = 0.57, size = 184, normalized size = 2.59

$$\frac{2\left(-\frac{2i\,c^3\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}{a}+\frac{4i\,c^3\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-i\right)}{a}-\frac{2i\,c^3\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}{a}+\frac{2i\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2i\,c^3}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)a}-\frac{2\left(3i\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+8\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-3i\,c^3\right)}{a\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-i\right)^2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] 2*(-2*I*c^3*log(tan(1/2*f*x + 1/2*e) + 1)/a + 4*I*c^3*log(tan(1/2*f*x + 1/2
*e) - I)/a - 2*I*c^3*log(tan(1/2*f*x + 1/2*e) - 1)/a + (2*I*c^3*tan(1/2*f*x
 + 1/2*e)^2 - c^3*tan(1/2*f*x + 1/2*e) - 2*I*c^3)/((tan(1/2*f*x + 1/2*e)^2
- 1)*a) - 2*(3*I*c^3*tan(1/2*f*x + 1/2*e)^2 + 8*c^3*tan(1/2*f*x + 1/2*e) -
3*I*c^3)/(a*(tan(1/2*f*x + 1/2*e) - I)^2))/f
```

Mupad [B]
time = 4.78, size = 64, normalized size = 0.90

$$\frac{c^3\tan(e+f\,x)}{a\,f}+\frac{c^3\,4\mathrm{i}}{a\,f\,\left(1+\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}+\frac{c^3\,\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,4\mathrm{i}}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^3/(a + a*tan(e + f*x)*1i),x)
```

```
[Out] (c^3*tan(e + f*x))/(a*f) + (c^3*4i)/(a*f*(tan(e + f*x)*1i + 1)) + (c^3*log(
tan(e + f*x) - 1i)*4i)/(a*f)
```

3.910 $\int \frac{(c-ic\tan(e+fx))^3}{(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=83

$$\frac{c^3x}{a^2}+\frac{ic^3\log(\cos(e+fx))}{a^2f}+\frac{2ic^3}{f(a+ia\tan(e+fx))^2}-\frac{4ic^3}{f\left(a^2+ia^2\tan(e+fx)\right)}$$

```
[Out] c^3*x/a^2+I*c^3*ln(cos(f*x+e))/a^2/f+2*I*c^3/f/(a+I*a*tan(f*x+e))^2-4*I*c^3
/f/(a^2+I*a^2*tan(f*x+e))
```

Rubi [A]
time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {3603, 3568, 45}

$$-\frac{4ic^3}{f\left(a^2+ia^2\tan(e+fx)\right)}+\frac{ic^3\log(\cos(e+fx))}{a^2f}+\frac{c^3x}{a^2}+\frac{2ic^3}{f(a+ia\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])^3/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (c^3*x)/a^2 + (I*c^3*Log[Cos[e + f*x]])/(a^2*f) + ((2*I)*c^3)/(f*(a + I*a*T
an[e + f*x])^2) - ((4*I)*c^3)/(f*(a^2 + I*a^2*Tan[e + f*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c-ic\tan(e+fx))^3}{(a+ia\tan(e+fx))^2}\,dx &= \left(a^3c^3\right)\int \frac{\sec^6(e+fx)}{(a+ia\tan(e+fx))^5}\,dx \\
&= -\frac{\left(ic^3\right)\mathrm{Subst}\left(\int \frac{(a-x)^2}{(a+x)^3}\,dx, x, ia\tan(e+fx)\right)}{a^2f} \\
&= -\frac{\left(ic^3\right)\mathrm{Subst}\left(\int \left(\frac{4a^2}{(a+x)^3}-\frac{4a}{(a+x)^2}+\frac{1}{a+x}\right)dx, x, ia\tan(e+fx)\right)}{a^2f} \\
&= \frac{c^3x}{a^2}+\frac{ic^3\log(\cos(e+fx))}{a^2f}+\frac{2ic^3}{f(a+ia\tan(e+fx))^2}-\frac{4ic^3}{f\left(a^2+ia^2\tan(e+fx)\right)}
\end{aligned}
$$

Mathematica [A]
time = 1.12, size = 115, normalized size = 1.39

$$
-\frac{c^3\sec^2(e+fx)\left(-2i+i\cos(2(e+fx))\left(1+\log\left(\cos^2(e+fx)\right)\right)+2\mathrm{ArcTan}(\tan(fx))(\cos(2(e+fx))+i\sin(2(e+fx)))+\sin(2(e+fx))-\log\left(\cos^2(e+fx)\right)\sin(2(e+fx))\right)}{2a^2f(-i+\tan(e+fx))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - I*c*Tan[e + f*x])^3/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] -1/2*(c^3*Sec[e + f*x]^2*(-2*I + I*Cos[2*(e + f*x)]*(1 + Log[Cos[e + f*x]^2
]) + 2*ArcTan[Tan[f*x]]*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)]) + Sin[2*(e
+ f*x)] - Log[Cos[e + f*x]^2]*Sin[2*(e + f*x)]))/(a^2*f*(-I + Tan[e + f*x])
^2)
```

Maple [A]
time = 0.19, size = 52, normalized size = 0.63

method	result	size
derivativedivides	$\frac{c^3\left(-\frac{2i}{(\tan(fx+e)-i)^2}-i\ln(\tan(fx+e)-i)-\frac{4}{\tan(fx+e)-i}\right)}{f\,a^2}$	52
default	$\frac{c^3\left(-\frac{2i}{(\tan(fx+e)-i)^2}-i\ln(\tan(fx+e)-i)-\frac{4}{\tan(fx+e)-i}\right)}{f\,a^2}$	52
risch	$-\frac{ic^3\mathrm{e}^{-2i(fx+e)}}{a^2f}+\frac{ic^3\mathrm{e}^{-4i(fx+e)}}{2a^2f}+\frac{2c^3x}{a^2}+\frac{2c^3e}{a^2f}+\frac{ic^3\ln\left(\mathrm{e}^{2i(fx+e)}+1\right)}{a^2f}$	89
norman	$\frac{\frac{c^3x}{a}-\frac{2ic^3}{af}+\frac{c^3x\left(\tan^4(fx+e)\right)}{a}-\frac{4c^3\left(\tan^3(fx+e)\right)}{af}+\frac{2c^3x\left(\tan^2(fx+e)\right)}{a}-\frac{6ic^3\left(\tan^2(fx+e)\right)}{af}}{a\left(1+\tan^2(fx+e)\right)^2}-\frac{ic^3\ln\left(1+\tan^2(fx+e)\right)}{2a^2f}$	134

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*c^3/a^2*(-2*I/(tan(f*x+e)-I)^2-I*ln(tan(f*x+e)-I)-4/(tan(f*x+e)-I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.10, size = 84, normalized size = 1.01

$$\frac{\left(4\,c^{3}fxe^{(4i\,fx+4i\,e)}+2i\,c^{3}e^{(4i\,fx+4i\,e)}\log\left(e^{(2i\,fx+2i\,e)}+1\right)-2i\,c^{3}e^{(2i\,fx+2i\,e)}+i\,c^{3}\right)e^{(-4i\,fx-4i\,e)}}{2\,a^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*c^3*f*x*e^(4*I*f*x + 4*I*e) + 2*I*c^3*e^(4*I*f*x + 4*I*e)*log(e^(2*I
*f*x + 2*I*e) + 1) - 2*I*c^3*e^(2*I*f*x + 2*I*e) + I*c^3)*e^(-4*I*f*x - 4*I
*e)/(a^2*f)
```

Sympy [A]
time = 0.30, size = 167, normalized size = 2.01

$$\begin{cases}\frac{\left(-2ia^{2}c^{3}fe^{4ie}e^{-2ifx}+ia^{2}c^{3}fe^{2ie}e^{-4ifx}\right)e^{-6ie}}{2a^{4}f^{2}} & \text{for}\: a^{4}f^{2}e^{6ie}\neq 0\\x\left(-\frac{2c^{3}}{a^{2}}+\frac{\left(2c^{3}e^{4ie}-2c^{3}e^{2ie}+2c^{3}\right)e^{-4ie}}{a^{2}}\right) & \text{otherwise}\end{cases}+\frac{2c^{3}x}{a^{2}}+\frac{ic^{3}\log\left(e^{2ifx}+e^{-2ie}\right)}{a^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**3/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] Piecewise(((-2*I*a**2*c**3*f*exp(4*I*e)*exp(-2*I*f*x) + I*a**2*c**3*f*exp(2
*I*e)*exp(-4*I*f*x))*exp(-6*I*e)/(2*a**4*f**2), Ne(a**4*f**2*exp(6*I*e), 0)
), (x*(-2*c**3/a**2 + (2*c**3*exp(4*I*e) - 2*c**3*exp(2*I*e) + 2*c**3)*exp(
-4*I*e)/a**2), True)) + 2*c**3*x/a**2 + I*c**3*log(exp(2*I*f*x) + exp(-2*I*
e))/(a**2*f)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(76) = 152$.
time = 0.64, size = 159, normalized size = 1.92

$$-\frac{-\frac{6i\,c^{3}\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}{a^{2}}+\frac{12i\,c^{3}\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-i\right)}{a^{2}}-\frac{6i\,c^{3}\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}{a^{2}}+\frac{-25i\,c^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-100\,c^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+198i\,c^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+100\,c^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-25i\,c^{3}}{a^{2}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-i\right)^{4}}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/6*(-6*I*c^3*log(tan(1/2*f*x + 1/2*e) + 1)/a^2 + 12*I*c^3*log(tan(1/2*f*x
 + 1/2*e) - I)/a^2 - 6*I*c^3*log(tan(1/2*f*x + 1/2*e) - 1)/a^2 + (-25*I*c^3
*tan(1/2*f*x + 1/2*e)^4 - 100*c^3*tan(1/2*f*x + 1/2*e)^3 + 198*I*c^3*tan(1/
2*f*x + 1/2*e)^2 + 100*c^3*tan(1/2*f*x + 1/2*e) - 25*I*c^3)/(a^2*(tan(1/2*f
*x + 1/2*e) - I)^4))/f
```

Mupad [B]
time = 4.88, size = 76, normalized size = 0.92

$$-\frac{\frac{2\,c^3}{a^2}+\frac{c^3\,\tan(e+f\,x)\,4{}\mathrm{i}}{a^2}}{f\,\left({\tan\left(e+f\,x\right)}^2\,1{}\mathrm{i}+2\,\tan\left(e+f\,x\right)-\mathrm{i}\right)}-\frac{c^3\,\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,1{}\mathrm{i}}{a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^3/(a + a*tan(e + f*x)*1i)^2,x)
```

```
[Out] - ((2*c^3)/a^2 + (c^3*tan(e + f*x)*4i)/a^2)/(f*(2*tan(e + f*x) + tan(e + f*
x)^2*1i - 1i)) - (c^3*log(tan(e + f*x) - 1i)*1i)/(a^2*f)
```

3.911 $\int \frac{(c-ic\tan(e+fx))^3}{(a+ia\tan(e+fx))^3}\,dx$

Optimal. Leaf size=50

$$\frac{ic^3(a^2-ia^2\tan(e+fx))^3}{6f\,(a^3+ia^3\tan(e+fx))^3}$$

```
[Out] 1/6*I*c^3*(a^2-I*a^2*tan(f*x+e))^3/f/(a^3+I*a^3*tan(f*x+e))^3
```

Rubi [A]
time = 0.08, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 37}

$$\frac{ic^3(a^2-ia^2\tan(e+fx))^3}{6f\,(a^3+ia^3\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])^3/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] ((I/6)*c^3*(a^2 - I*a^2*Tan[e + f*x])^3)/(f*(a^3 + I*a^3*Tan[e + f*x])^3)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{(c-ic\tan(e+fx))^3}{(a+ia\tan(e+fx))^3}\,dx = \left(a^3c^3\right)\int \frac{\sec^6(e+fx)}{(a+ia\tan(e+fx))^6}\,dx$$
$$= -\frac{\left(ic^3\right)\text{Subst}\left(\int \frac{(a-x)^2}{(a+x)^4}\,dx, x, ia\tan(e+fx)\right)}{a^2f}$$
$$= \frac{ic^3(1-i\tan(e+fx))^3}{6f(a+ia\tan(e+fx))^3}$$

Mathematica [A]
time = 0.28, size = 34, normalized size = 0.68

$$\frac{c^3(i\cos(6(e+fx))+\sin(6(e+fx)))}{6a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - I*c*Tan[e + f*x])^3/(a + I*a*Tan[e + f*x])^3,x]
[Out] (c^3*(I*Cos[6*(e + f*x)] + Sin[6*(e + f*x)]))/(6*a^3*f)
```

Maple [A]
time = 0.19, size = 50, normalized size = 1.00

method	result	size
risch	$\frac{ic^3 e^{-6i(fx+e)}}{6a^3f}$	22
derivativedivides	$\frac{c^3\left(\frac{2i}{(\tan(fx+e)-i)^2}-\frac{4}{3(\tan(fx+e)-i)^3}+\frac{1}{\tan(fx+e)-i}\right)}{f\,a^3}$	50
default	$\frac{c^3\left(\frac{2i}{(\tan(fx+e)-i)^2}-\frac{4}{3(\tan(fx+e)-i)^3}+\frac{1}{\tan(fx+e)-i}\right)}{f\,a^3}$	50
norman	$\frac{\frac{c^3\left(\tan^5(fx+e)\right)}{af}+\frac{c^3\tan(fx+e)}{af}-\frac{2ic^3\left(\tan^2(fx+e)\right)}{af}+\frac{ic^3}{3af}-\frac{10c^3\left(\tan^3(fx+e)\right)}{3af}+\frac{3ic^3\left(\tan^4(fx+e)\right)}{af}}{a^2\left(1+\tan^2(fx+e)\right)^3}$	123

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
[Out] 1/f*c^3/a^3*(2*I/(tan(f*x+e)-I)^2-4/3/(tan(f*x+e)-I)^3+1/(tan(f*x+e)-I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.30, size = 21, normalized size = 0.42

$$\frac{i\,c^3 e^{(-6i\,fx-6i\,e)}}{6\,a^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/6*I*c^3*e^(-6*I*f*x - 6*I*e)/(a^3*f)
```

Sympy [A]
time = 0.19, size = 51, normalized size = 1.02

$$\begin{cases} \frac{ic^3e^{-6ie}e^{-6ifx}}{6a^3f} & \text{for } a^3fe^{6ie}\neq 0 \\ \frac{c^3xe^{-6ie}}{a^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**3/(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] Piecewise((I*c**3*exp(-6*I*e)*exp(-6*I*f*x)/(6*a**3*f), Ne(a**3*f*exp(6*I*e
), 0)), (c**3*x*exp(-6*I*e)/a**3, True))
```

Giac [A]
time = 0.81, size = 72, normalized size = 1.44

$$-\frac{2\left(3\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-10\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+3\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{3\,a^3f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-i\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/3*(3*c^3*tan(1/2*f*x + 1/2*e)^5 - 10*c^3*tan(1/2*f*x + 1/2*e)^3 + 3*c^3*
tan(1/2*f*x + 1/2*e))/(a^3*f*(tan(1/2*f*x + 1/2*e) - I)^6)
```

Mupad [B]
time = 4.82, size = 59, normalized size = 1.18

$$-\frac{c^3\,\left(\tan(e+f\,x)^2\,1\mathrm{i}-\frac{1}{3}\mathrm{i}\right)}{a^3\,f\,\left(-\tan\left(e+f\,x\right)^3\,1\mathrm{i}-3\tan\left(e+f\,x\right)^2+\tan\left(e+f\,x\right)\,3\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^3/(a + a*tan(e + f*x)*1i)^3,x)
```

```
[Out] -(c^3*(tan(e + f*x)^2*1i - 1i/3))/(a^3*f*(tan(e + f*x)*3i - 3*tan(e + f*x)^
2 - tan(e + f*x)^3*1i + 1))
```

3.912 $\int \frac{(c-ic\tan(e+fx))^3}{(a+ia\tan(e+fx))^4} dx$

Optimal. Leaf size=87

$$\frac{ic^3}{f(a+ia\tan(e+fx))^4}-\frac{4ic^3}{3af(a+ia\tan(e+fx))^3}+\frac{ic^3}{2f\left(a^2+ia^2\tan(e+fx)\right)^2}$$

```
[Out] I*c^3/f/(a+I*a*tan(f*x+e))^4-4/3*I*c^3/a/f/(a+I*a*tan(f*x+e))^3+1/2*I*c^3/f
/(a^2+I*a^2*tan(f*x+e))^2
```

Rubi [A]
time = 0.09, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 45}

$$\frac{ic^3}{2f\left(a^2+ia^2\tan(e+fx)\right)^2}-\frac{4ic^3}{3af(a+ia\tan(e+fx))^3}+\frac{ic^3}{f(a+ia\tan(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])^3/(a + I*a*Tan[e + f*x])^4,x]
```

```
[Out] (I*c^3)/(f*(a + I*a*Tan[e + f*x])^4) - (((4*I)/3)*c^3)/(a*f*(a + I*a*Tan[e
+ f*x])^3) + ((I/2)*c^3)/(f*(a^2 + I*a^2*Tan[e + f*x])^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\begin{aligned}\int \frac{(c-ic\tan(e+fx))^3}{(a+ia\tan(e+fx))^4}\,dx &= \left(a^3c^3\right)\int \frac{\sec^6(e+fx)}{(a+ia\tan(e+fx))^7}\,dx\\ &= -\frac{\left(ic^3\right)\mathrm{Subst}\left(\int \frac{(a-x)^2}{(a+x)^5}\,dx, x, ia\tan(e+fx)\right)}{a^2f}\\ &= -\frac{\left(ic^3\right)\mathrm{Subst}\left(\int \left(\frac{4a^2}{(a+x)^5}-\frac{4a}{(a+x)^4}+\frac{1}{(a+x)^3}\right)\,dx, x, ia\tan(e+fx)\right)}{a^2f}\\ &= \frac{ic^3}{f(a+ia\tan(e+fx))^4}-\frac{4ic^3}{3af(a+ia\tan(e+fx))^3}+\frac{ic^3}{2f\left(a^2+ia^2\tan(e+fx\right.}\end{aligned}$$

Mathematica [A]
time = 1.16, size = 64, normalized size = 0.74

$$-\frac{c^3\sec^3(e+fx)(\cos(3(e+fx))-i\sin(3(e+fx)))(-7i+\tan(e+fx))}{48a^4f(-i+\tan(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - I*c*Tan[e + f*x])^3/(a + I*a*Tan[e + f*x])^4,x]
```

```
[Out] -1/48*(c^3*Sec[e + f*x]^3*(Cos[3*(e + f*x)] - I*Sin[3*(e + f*x)])*(-7*I + T
an[e + f*x]))/(a^4*f*(-I + Tan[e + f*x])^4)
```

Maple [A]
time = 0.23, size = 53, normalized size = 0.61

method	result	siz
risch	$\frac{ic^3 e^{-6i(fx+e)}}{12a^4f}+\frac{ic^3 e^{-8i(fx+e)}}{16a^4f}$	44
derivativedivides	$\frac{c^3\left(-\frac{i}{2(\tan(fx+e)-i)^2}+\frac{i}{(\tan(fx+e)-i)^4}+\frac{4}{3(\tan(fx+e)-i)^3}\right)}{f\,a^4}$	53
default	$\frac{c^3\left(-\frac{i}{2(\tan(fx+e)-i)^2}+\frac{i}{(\tan(fx+e)-i)^4}+\frac{4}{3(\tan(fx+e)-i)^3}\right)}{f\,a^4}$	53
norman	$\frac{\frac{c^3\tan(fx+e)}{af}+\frac{ic^3}{6af}-\frac{14c^3\left(\tan^3(fx+e)\right)}{3af}+\frac{7c^3\left(\tan^5(fx+e)\right)}{3af}-\frac{17ic^3\left(\tan^2(fx+e)\right)}{6af}-\frac{ic^3\left(\tan^6(fx+e)\right)}{2af}+\frac{9ic^3\left(\tan^4(fx+e)\right)}{2af}}{\left(1+\tan^2(fx+e)\right)^4a^3}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*c^3/a^4*(-1/2*I/(tan(f*x+e)-I)^2+I/(tan(f*x+e)-I)^4+4/3/(tan(f*x+e)-I)^
3)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.16, size = 39, normalized size = 0.45

$$\frac{\left(4i\,c^3e^{(2i\,fx+2i\,e)}+3i\,c^3\right)e^{(-8i\,fx-8i\,e)}}{48\,a^4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/48*(4*I*c^3*e^(2*I*f*x + 2*I*e) + 3*I*c^3)*e^(-8*I*f*x - 8*I*e)/(a^4*f)
```

Sympy [A]
time = 0.28, size = 107, normalized size = 1.23

$$\begin{cases} \frac{\left(16ia^4c^3fe^{8ie}e^{-6ifx}+12ia^4c^3fe^{6ie}e^{-8ifx}\right)e^{-14ie}}{192a^8f^2} & \text{for } a^8f^2e^{14ie}\neq 0\\ \frac{x\left(c^3e^{2ie}+c^3\right)e^{-8ie}}{2a^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**3/(a+I*a*tan(f*x+e))**4,x)
```

```
[Out] Piecewise(((16*I*a**4*c**3*f*exp(8*I*e)*exp(-6*I*f*x) + 12*I*a**4*c**3*f*ex
p(6*I*e)*exp(-8*I*f*x))*exp(-14*I*e)/(192*a**8*f**2), Ne(a**8*f**2*exp(14*I
*e), 0)), (x*(c**3*exp(2*I*e) + c**3)*exp(-8*I*e)/(2*a**4), True))
```

Giac [A]
time = 0.88, size = 140, normalized size = 1.61

$$-\frac{2\left(3\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7-3i\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-17\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+10i\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+17\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-3i\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{3\,a^4f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-i\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -2/3*(3*c^3*tan(1/2*f*x + 1/2*e)^7 - 3*I*c^3*tan(1/2*f*x + 1/2*e)^6 - 17*c^
3*tan(1/2*f*x + 1/2*e)^5 + 10*I*c^3*tan(1/2*f*x + 1/2*e)^4 + 17*c^3*tan(1/2
```

```
*f*x + 1/2*e)^3 - 3*I*c^3*tan(1/2*f*x + 1/2*e)^2 - 3*c^3*tan(1/2*f*x + 1/2*
e))/(a^4*f*(tan(1/2*f*x + 1/2*e) - I)^8)
```

Mupad [B]
time = 4.85, size = 77, normalized size = 0.89

$$\frac{c^3\,\left(3\tan(e+f\,x)^2+\tan(e+f\,x)\;2\mathrm{i}-1\right)}{6\,a^4\,f\,\left(\tan\left(e+f\,x\right)^4\,1\mathrm{i}+4\tan\left(e+f\,x\right)^3-\tan\left(e+f\,x\right)^2\,6\mathrm{i}-4\tan\left(e+f\,x\right)+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^3/(a + a*tan(e + f*x)*1i)^4,x)
[Out] (c^3*(tan(e + f*x)*2i + 3*tan(e + f*x)^2 - 1))/(6*a^4*f*(4*tan(e + f*x)^3 -
 tan(e + f*x)^2*6i - 4*tan(e + f*x) + tan(e + f*x)^4*1i + 1i))
```

3.913 $\int \frac{(c-ic\tan(e+fx))^3}{(a+ia\tan(e+fx))^5} dx$

Optimal. Leaf size=90

$$\frac{4ic^3}{5f(a+ia\tan(e+fx))^5}+\frac{ic^3}{3a^2f(a+ia\tan(e+fx))^3}-\frac{ia^3c^3}{f\left(a^2+ia^2\tan(e+fx)\right)^4}$$

```
[Out] 4/5*I*c^3/f/(a+I*a*tan(f*x+e))^5+1/3*I*c^3/a^2/f/(a+I*a*tan(f*x+e))^3-I*a^3
*c^3/f/(a^2+I*a^2*tan(f*x+e))^4
```

Rubi [A]
time = 0.09, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 45}

$$\frac{ic^3}{3a^2f(a+ia\tan(e+fx))^3}-\frac{ia^3c^3}{f\left(a^2+ia^2\tan(e+fx)\right)^4}+\frac{4ic^3}{5f(a+ia\tan(e+fx))^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])^3/(a + I*a*Tan[e + f*x])^5,x]
```

```
[Out] (((4*I)/5)*c^3)/(f*(a + I*a*Tan[e + f*x])^5) + ((I/3)*c^3)/(a^2*f*(a + I*a*
Tan[e + f*x])^3) - (I*a^3*c^3)/(f*(a^2 + I*a^2*Tan[e + f*x])^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{(c-ic\tan(e+fx))^3}{(a+ia\tan(e+fx))^5}\,dx = \left(a^3c^3\right)\int \frac{\sec^6(e+fx)}{(a+ia\tan(e+fx))^8}\,dx$$
$$= -\frac{\left(ic^3\right)\text{Subst}\left(\int \frac{(a-x)^2}{(a+x)^6}\,dx, x, ia\tan(e+fx)\right)}{a^2f}$$
$$= -\frac{\left(ic^3\right)\text{Subst}\left(\int \left(\frac{4a^2}{(a+x)^6}-\frac{4a}{(a+x)^5}+\frac{1}{(a+x)^4}\right)\,dx, x, ia\tan(e+fx)\right)}{a^2f}$$
$$= \frac{4ic^3}{5f(a+ia\tan(e+fx))^5}-\frac{ic^3}{af(a+ia\tan(e+fx))^4}+\frac{ic^3}{3a^2f(a+ia\tan(e+fx$$

Mathematica [A]
time = 1.34, size = 58, normalized size = 0.64

$$\frac{c^3(15+16\cos(2(e+fx))+4i\sin(2(e+fx)))(i\cos(8(e+fx))+\sin(8(e+fx)))}{240a^5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - I*c*Tan[e + f*x])^3/(a + I*a*Tan[e + f*x])^5,x]
[Out] (c^3*(15 + 16*Cos[2*(e + f*x)] + (4*I)*Sin[2*(e + f*x)])*(I*Cos[8*(e + f*x)
] + Sin[8*(e + f*x)]))/(240*a^5*f)
```

Maple [A]
time = 0.29, size = 52, normalized size = 0.58

method	result
derivativedivides	$\frac{c^3\left(-\frac{1}{3(\tan(fx+e)-i)^3}+\frac{4}{5(\tan(fx+e)-i)^5}-\frac{i}{(\tan(fx+e)-i)^4}\right)}{f\,a^5}$
default	$\frac{c^3\left(-\frac{1}{3(\tan(fx+e)-i)^3}+\frac{4}{5(\tan(fx+e)-i)^5}-\frac{i}{(\tan(fx+e)-i)^4}\right)}{f\,a^5}$
risch	$\frac{ic^3e^{-6i(fx+e)}}{24a^5f}+\frac{ic^3e^{-8i(fx+e)}}{16a^5f}+\frac{ic^3e^{-10i(fx+e)}}{40a^5f}$
norman	$\frac{\frac{c^3\tan(fx+e)}{af}+\frac{2ic^3}{15af}-\frac{19c^3\left(\tan^3(fx+e)\right)}{3af}+\frac{77c^3\left(\tan^5(fx+e)\right)}{15af}-\frac{c^3\left(\tan^7(fx+e)\right)}{3af}-\frac{10ic^3\left(\tan^2(fx+e)\right)}{3af}-\frac{2ic^3\left(\tan^6(fx+e)\right)}{af}+\frac{22ic}{}}{\left(1+\tan^2(fx+e)\right)^5a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^5,x,method=_RETURNVERBOSE)
[Out] 1/f*c^3/a^5*(-1/3/(tan(f*x+e)-I)^3+4/5/(tan(f*x+e)-I)^5-I/(tan(f*x+e)-I)^4)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 1.53, size = 54, normalized size = 0.60

$$\frac{\left(10i\,c^3e^{(4i\,fx+4i\,e)}+15i\,c^3e^{(2i\,fx+2i\,e)}+6i\,c^3\right)e^{(-10i\,fx-10i\,e)}}{240\,a^5f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] 1/240*(10*I*c^3*e^(4*I*f*x + 4*I*e) + 15*I*c^3*e^(2*I*f*x + 2*I*e) + 6*I*c^
3)*e^(-10*I*f*x - 10*I*e)/(a^5*f)
```

Sympy [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(75) = 150$.
time = 0.36, size = 151, normalized size = 1.68

$$\begin{cases} \frac{\left(640ia^{10}c^3f^2e^{18ie}e^{-6ifx}+960ia^{10}c^3f^2e^{16ie}e^{-8ifx}+384ia^{10}c^3f^2e^{14ie}e^{-10ifx}\right)e^{-24ie}}{15360a^{15}f^3} & \text{for } a^{15}f^3e^{24ie}\neq 0 \\ \frac{x\left(c^3e^{4ie}+2c^3e^{2ie}+c^3\right)e^{-10ie}}{4a^5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**3/(a+I*a*tan(f*x+e))**5,x)
```

```
[Out] Piecewise((((640*I*a**10*c**3*f**2*exp(18*I*e)*exp(-6*I*f*x) + 960*I*a**10*c
**3*f**2*exp(16*I*e)*exp(-8*I*f*x) + 384*I*a**10*c**3*f**2*exp(14*I*e)*exp(
-10*I*f*x))*exp(-24*I*e)/(15360*a**15*f**3), Ne(a**15*f**3*exp(24*I*e), 0))
, (x*(c**3*exp(4*I*e) + 2*c**3*exp(2*I*e) + c**3)*exp(-10*I*e)/(4*a**5), Tr
ue))
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(77) = 154$.
time = 1.09, size = 174, normalized size = 1.93

$$-\frac{2\left(15\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9-30i\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8-140\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+170i\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+282\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-170i\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-140\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+30i\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+15\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{15\,a^5f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-i\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^5,x, algorithm="giac")
```

```
[Out] -2/15*(15*c^3*tan(1/2*f*x + 1/2*e)^9 - 30*I*c^3*tan(1/2*f*x + 1/2*e)^8 - 14
0*c^3*tan(1/2*f*x + 1/2*e)^7 + 170*I*c^3*tan(1/2*f*x + 1/2*e)^6 + 282*c^3*t
an(1/2*f*x + 1/2*e)^5 - 170*I*c^3*tan(1/2*f*x + 1/2*e)^4 - 140*c^3*tan(1/2*
f*x + 1/2*e)^3 + 30*I*c^3*tan(1/2*f*x + 1/2*e)^2 + 15*c^3*tan(1/2*f*x + 1/2
*e))/(a^5*f*(tan(1/2*f*x + 1/2*e) - I)^10)
```

Mupad [B]
time = 4.86, size = 88, normalized size = 0.98

$$\frac{c^3\left(-\tan(e+f\,x)^2\,5\mathrm{i}+5\tan(e+f\,x)+2\mathrm{i}\right)}{15\,a^5\,f\,\left(\tan\left(e+f\,x\right)^5\,1\mathrm{i}+5\tan\left(e+f\,x\right)^4-\tan\left(e+f\,x\right)^3\,10\mathrm{i}-10\tan\left(e+f\,x\right)^2+\tan\left(e+f\,x\right)\,5\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^3/(a + a*tan(e + f*x)*1i)^5,x)
```

```
[Out] (c^3*(5*tan(e + f*x) - tan(e + f*x)^2*5i + 2i))/(15*a^5*f*(tan(e + f*x)*5i
- 10*tan(e + f*x)^2 - tan(e + f*x)^3*10i + 5*tan(e + f*x)^4 + tan(e + f*x)^
5*1i + 1))
```

3.914 $\int (a+ia\tan(e+fx))^5(c-ic\tan(e+fx))^4\,dx$

Optimal. Leaf size=100

$$\frac{ia^5c^4\sec^8(e+fx)}{8f}+\frac{a^5c^4\tan(e+fx)}{f}+\frac{a^5c^4\tan^3(e+fx)}{f}+\frac{3a^5c^4\tan^5(e+fx)}{5f}+\frac{a^5c^4\tan^7(e+fx)}{7f}$$

```
[Out] 1/8*I*a^5*c^4*sec(f*x+e)^8/f+a^5*c^4*tan(f*x+e)/f+a^5*c^4*tan(f*x+e)^3/f+3/
5*a^5*c^4*tan(f*x+e)^5/f+1/7*a^5*c^4*tan(f*x+e)^7/f
```

Rubi [A]
time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3567, 3852}

$$\frac{a^5c^4\tan^7(e+fx)}{7f}+\frac{3a^5c^4\tan^5(e+fx)}{5f}+\frac{a^5c^4\tan^3(e+fx)}{f}+\frac{a^5c^4\tan(e+fx)}{f}+\frac{ia^5c^4\sec^8(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^5*(c - I*c*Tan[e + f*x])^4,x]
```

```
[Out] ((I/8)*a^5*c^4*Sec[e + f*x]^8)/f + (a^5*c^4*Tan[e + f*x])/f + (a^5*c^4*Tan[
e + f*x]^3)/f + (3*a^5*c^4*Tan[e + f*x]^5)/(5*f) + (a^5*c^4*Tan[e + f*x]^7)
/(7*f)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int (a+ia\tan(e+fx))^5(c-ic\tan(e+fx))^4\,dx = (a^4c^4)\int \sec^8(e+fx)(a+ia\tan(e+fx))\,dx$$
$$= \frac{ia^5c^4\sec^8(e+fx)}{8f} + (a^5c^4)\int \sec^8(e+fx)\,dx$$
$$= \frac{ia^5c^4\sec^8(e+fx)}{8f} - \frac{(a^5c^4)\,\text{Subst}\left(\int (1+3x^2+3x^4+x^6)\right.}{f}$$
$$= \frac{ia^5c^4\sec^8(e+fx)}{8f} + \frac{a^5c^4\tan(e+fx)}{f} + \frac{a^5c^4\tan^3(e+fx}{f}$$

Mathematica [A]

time = 1.22, size = 74, normalized size = 0.74

$$\frac{a^5c^4\sec(e)\sec^8(e+fx)(35i\cos(e)-35\sin(e)+56\sin(e+2fx)+28\sin(3e+4fx)+8\sin(5e+6fx)+\sin(7e+8fx))}{280f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^5*(c - I*c*Tan[e + f*x])^4,x]
```

```
[Out] (a^5*c^4*Sec[e]*Sec[e + f*x]^8*((35*I)*Cos[e] - 35*Sin[e] + 56*Sin[e + 2*f*
x] + 28*Sin[3*e + 4*f*x] + 8*Sin[5*e + 6*f*x] + Sin[7*e + 8*f*x]))/(280*f)
```

Maple [A]

time = 0.09, size = 96, normalized size = 0.96

method	result
risch	$\frac{32ia^5c^4\left(70\,e^{8i(fx+e)}+56\,e^{6i(fx+e)}+28\,e^{4i(fx+e)}+8\,e^{2i(fx+e)}+1\right)}{35f\left(e^{2i(fx+e)}+1\right)^8}$
derivativedivides	$\frac{ia^5c^4\left(\frac{(\tan^8(fx+e))}{8}+\frac{(\tan^6(fx+e))}{2}-\frac{i(\tan^7(fx+e))}{7}+\frac{3(\tan^4(fx+e))}{4}-\frac{3i(\tan^5(fx+e))}{5}+\frac{(\tan^2(fx+e))}{2}-i(\tan^3(fx+e))-i\right.}{f}$
default	$\frac{ia^5c^4\left(\frac{(\tan^8(fx+e))}{8}+\frac{(\tan^6(fx+e))}{2}-\frac{i(\tan^7(fx+e))}{7}+\frac{3(\tan^4(fx+e))}{4}-\frac{3i(\tan^5(fx+e))}{5}+\frac{(\tan^2(fx+e))}{2}-i(\tan^3(fx+e))-i\right.}{f}$
norman	$\frac{a^5c^4\tan(fx+e)}{f}+\frac{a^5c^4(\tan^3(fx+e))}{f}+\frac{3a^5c^4(\tan^5(fx+e))}{5f}+\frac{a^5c^4(\tan^7(fx+e))}{7f}+\frac{ia^5c^4(\tan^2(fx+e))}{2f}+\frac{3ia^5}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^5*(c-I*c*tan(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] I/f*a^5*c^4*(1/8*tan(f*x+e)^8+1/2*tan(f*x+e)^6-1/7*I*tan(f*x+e)^7+3/4*tan(f
*x+e)^4-3/5*I*tan(f*x+e)^5+1/2*tan(f*x+e)^2-I*tan(f*x+e)^3-I*tan(f*x+e))
```

Maxima [A]
time = 0.50, size = 140, normalized size = 1.40

$$-\frac{-35i\,a^5c^4\tan(fx+e)^8-40\,a^5c^4\tan(fx+e)^7-140i\,a^5c^4\tan(fx+e)^6-168\,a^5c^4\tan(fx+e)^5-210i\,a^5c^4\tan(fx+e)^4-280\,a^5c^4\tan(fx+e)^3-140i\,a^5c^4\tan(fx+e)^2-280\,a^5c^4\tan(fx+e)}{280\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^5*(c-I*c*tan(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] -1/280*(-35*I*a^5*c^4*tan(f*x + e)^8 - 40*a^5*c^4*tan(f*x + e)^7 - 140*I*a^
5*c^4*tan(f*x + e)^6 - 168*a^5*c^4*tan(f*x + e)^5 - 210*I*a^5*c^4*tan(f*x +
 e)^4 - 280*a^5*c^4*tan(f*x + e)^3 - 140*I*a^5*c^4*tan(f*x + e)^2 - 280*a^5
*c^4*tan(f*x + e))/f
```

Fricas [A]
time = 1.19, size = 190, normalized size = 1.90

$$-\frac{32\left(-70i\,a^5c^4e^{(8i\,fx+8i\,e)}-56i\,a^5c^4e^{(6i\,fx+6i\,e)}-28i\,a^5c^4e^{(4i\,fx+4i\,e)}-8i\,a^5c^4e^{(2i\,fx+2i\,e)}-i\,a^5c^4\right)}{35\left(fe^{(16i\,fx+16i\,e)}+8\,fe^{(14i\,fx+14i\,e)}+28\,fe^{(12i\,fx+12i\,e)}+56\,fe^{(10i\,fx+10i\,e)}+70\,fe^{(8i\,fx+8i\,e)}+56\,fe^{(6i\,fx+6i\,e)}+28\,fe^{(4i\,fx+4i\,e)}+8\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^5*(c-I*c*tan(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] -32/35*(-70*I*a^5*c^4*e^(8*I*f*x + 8*I*e) - 56*I*a^5*c^4*e^(6*I*f*x + 6*I*e
) - 28*I*a^5*c^4*e^(4*I*f*x + 4*I*e) - 8*I*a^5*c^4*e^(2*I*f*x + 2*I*e) - I*
a^5*c^4)/(f*e^(16*I*f*x + 16*I*e) + 8*f*e^(14*I*f*x + 14*I*e) + 28*f*e^(12*
I*f*x + 12*I*e) + 56*f*e^(10*I*f*x + 10*I*e) + 70*f*e^(8*I*f*x + 8*I*e) + 5
6*f*e^(6*I*f*x + 6*I*e) + 28*f*e^(4*I*f*x + 4*I*e) + 8*f*e^(2*I*f*x + 2*I*e
) + f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(90) = 180.
time = 0.69, size = 264, normalized size = 2.64

$$\frac{2240ia^5c^4e^{8ie}e^{8ifx}+1792ia^5c^4e^{6ie}e^{6ifx}+896ia^5c^4e^{4ie}e^{4ifx}+256ia^5c^4e^{2ie}e^{2ifx}+32ia^5c^4}{35fe^{16ie}e^{16ifx}+280fe^{14ie}e^{14ifx}+980fe^{12ie}e^{12ifx}+1960fe^{10ie}e^{10ifx}+2450fe^{8ie}e^{8ifx}+1960fe^{6ie}e^{6ifx}+980fe^{4ie}e^{4ifx}+280fe^{2ie}e^{2ifx}+35f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**5*(c-I*c*tan(f*x+e))**4,x)
```

```
[Out] (2240*I*a**5*c**4*exp(8*I*e)*exp(8*I*f*x) + 1792*I*a**5*c**4*exp(6*I*e)*exp
(6*I*f*x) + 896*I*a**5*c**4*exp(4*I*e)*exp(4*I*f*x) + 256*I*a**5*c**4*exp(2
*I*e)*exp(2*I*f*x) + 32*I*a**5*c**4)/(35*f*exp(16*I*e)*exp(16*I*f*x) + 280*
f*exp(14*I*e)*exp(14*I*f*x) + 980*f*exp(12*I*e)*exp(12*I*f*x) + 1960*f*exp(
10*I*e)*exp(10*I*f*x) + 2450*f*exp(8*I*e)*exp(8*I*f*x) + 1960*f*exp(6*I*e)*
exp(6*I*f*x) + 980*f*exp(4*I*e)*exp(4*I*f*x) + 280*f*exp(2*I*e)*exp(2*I*f*x
) + 35*f)
```

Giac [A]
time = 0.99, size = 190, normalized size = 1.90

$$-\frac{32 \left(-70 i a^{5} c^{4} e^{\left(8 i f x+8 i e\right)}-56 i a^{5} c^{4} e^{\left(6 i f x+6 i e\right)}-28 i a^{5} c^{4} e^{\left(4 i f x+4 i e\right)}-8 i a^{5} c^{4} e^{\left(2 i f x+2 i e\right)}-i a^{5} c^{4}\right)}{35 \left(f e^{\left(16 i f x+16 i e\right)}+8 f e^{\left(14 i f x+14 i e\right)}+28 f e^{\left(12 i f x+12 i e\right)}+56 f e^{\left(10 i f x+10 i e\right)}+70 f e^{\left(8 i f x+8 i e\right)}+56 f e^{\left(6 i f x+6 i e\right)}+28 f e^{\left(4 i f x+4 i e\right)}+8 f e^{\left(2 i f x+2 i e\right)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^5*(c-I*c*tan(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -32/35*(-70*I*a^5*c^4*e^(8*I*f*x + 8*I*e) - 56*I*a^5*c^4*e^(6*I*f*x + 6*I*e
) - 28*I*a^5*c^4*e^(4*I*f*x + 4*I*e) - 8*I*a^5*c^4*e^(2*I*f*x + 2*I*e) - I*
a^5*c^4)/(f*e^(16*I*f*x + 16*I*e) + 8*f*e^(14*I*f*x + 14*I*e) + 28*f*e^(12*
I*f*x + 12*I*e) + 56*f*e^(10*I*f*x + 10*I*e) + 70*f*e^(8*I*f*x + 8*I*e) + 5
6*f*e^(6*I*f*x + 6*I*e) + 28*f*e^(4*I*f*x + 4*I*e) + 8*f*e^(2*I*f*x + 2*I*e
) + f)
```

Mupad [B]
time = 4.74, size = 95, normalized size = 0.95

$$\frac{a^{5} c^{4} \left(-\cos\left(e+f x\right)^{8} 35\mathrm{i}+128 \sin\left(e+f x\right) \cos\left(e+f x\right)^{7}+64 \sin\left(e+f x\right) \cos\left(e+f x\right)^{5}+48 \sin\left(e+f x\right) \cos\left(e+f x\right)^{3}+40 \sin\left(e+f x\right) \cos\left(e+f x\right)+35\mathrm{i}\right)}{280 f \cos\left(e+f x\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^5*(c - c*tan(e + f*x)*1i)^4,x)
```

```
[Out] (a^5*c^4*(40*cos(e + f*x)*sin(e + f*x) + 48*cos(e + f*x)^3*sin(e + f*x) + 6
4*cos(e + f*x)^5*sin(e + f*x) + 128*cos(e + f*x)^7*sin(e + f*x) - cos(e + f
*x)^8*35i + 35i))/(280*f*cos(e + f*x)^8)
```

3.915 $\int (a+ia\tan(e+fx))^4(c-ic\tan(e+fx))^4\,dx$

Optimal. Leaf size=77

$$\frac{a^4c^4\tan(e+fx)}{f}+\frac{a^4c^4\tan^3(e+fx)}{f}+\frac{3a^4c^4\tan^5(e+fx)}{5f}+\frac{a^4c^4\tan^7(e+fx)}{7f}$$

```
[Out] a^4*c^4*tan(f*x+e)/f+a^4*c^4*tan(f*x+e)^3/f+3/5*a^4*c^4*tan(f*x+e)^5/f+1/7*
a^4*c^4*tan(f*x+e)^7/f
```

Rubi [A]
time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {3603, 3852}

$$\frac{a^4c^4\tan^7(e+fx)}{7f}+\frac{3a^4c^4\tan^5(e+fx)}{5f}+\frac{a^4c^4\tan^3(e+fx)}{f}+\frac{a^4c^4\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^4*(c - I*c*Tan[e + f*x])^4,x]
```

```
[Out] (a^4*c^4*Tan[e + f*x])/f + (a^4*c^4*Tan[e + f*x]^3)/f + (3*a^4*c^4*Tan[e +
f*x]^5)/(5*f) + (a^4*c^4*Tan[e + f*x]^7)/(7*f)
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+ia\tan(e+fx))^4(c-ic\tan(e+fx))^4\,dx &= \left(a^4c^4\right)\int \sec^8(e+fx)\,dx\\ &= -\frac{\left(a^4c^4\right)\text{Subst}\left(\int \left(1+3x^2+3x^4+x^6\right)\,dx,x,-\tan(e+\\ }{f}\\ &= \frac{a^4c^4\tan(e+fx)}{f}+\frac{a^4c^4\tan^3(e+fx)}{f}+\frac{3a^4c^4\tan^5(e+}{5f}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 49, normalized size = 0.64

$$\frac{a^4c^4\left(\tan(e+fx)+\tan^3(e+fx)+\frac{3}{5}\tan^5(e+fx)+\frac{1}{7}\tan^7(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^4*(c - I*c*Tan[e + f*x])^4,x]
```

```
[Out] (a^4*c^4*(Tan[e + f*x] + Tan[e + f*x]^3 + (3*Tan[e + f*x]^5)/5 + Tan[e + f*
x]^7/7))/f
```

Maple [A]
time = 0.06, size = 46, normalized size = 0.60

method	result	size
derivativedivides	$\frac{a^4c^4\left(\frac{\left(\tan^7(fx+e)\right)}{7}+\frac{3\left(\tan^5(fx+e)\right)}{5}+\tan^3(fx+e)+\tan(fx+e)\right)}{f}$	46
default	$\frac{a^4c^4\left(\frac{\left(\tan^7(fx+e)\right)}{7}+\frac{3\left(\tan^5(fx+e)\right)}{5}+\tan^3(fx+e)+\tan(fx+e)\right)}{f}$	46
risch	$\frac{32ia^4c^4\left(35\,e^{6i(fx+e)}+21\,e^{4i(fx+e)}+7\,e^{2i(fx+e)}+1\right)}{35f\left(e^{2i(fx+e)}+1\right)^7}$	61
norman	$\frac{a^4c^4\tan(fx+e)}{f}+\frac{a^4c^4\left(\tan^3(fx+e)\right)}{f}+\frac{3a^4c^4\left(\tan^5(fx+e)\right)}{5f}+\frac{a^4c^4\left(\tan^7(fx+e)\right)}{7f}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^4*(c-I*c*tan(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a^4*c^4*(1/7*tan(f*x+e)^7+3/5*tan(f*x+e)^5+tan(f*x+e)^3+tan(f*x+e))
```

Maxima [A]
time = 0.50, size = 72, normalized size = 0.94

$$\frac{5\,a^4c^4\tan\left(fx+e\right)^7+21\,a^4c^4\tan\left(fx+e\right)^5+35\,a^4c^4\tan\left(fx+e\right)^3+35\,a^4c^4\tan\left(fx+e\right)}{35\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^4*(c-I*c*tan(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 1/35*(5*a^4*c^4*tan(f*x + e)^7 + 21*a^4*c^4*tan(f*x + e)^5 + 35*a^4*c^4*tan
(f*x + e)^3 + 35*a^4*c^4*tan(f*x + e))/f
```

Fricas [C] Result contains complex when optimal does not.
time = 0.83, size = 159, normalized size = 2.06

$$-\frac{32\left(-35i\,a^4c^4e^{(6i\,fx+6i\,e)}-21i\,a^4c^4e^{(4i\,fx+4i\,e)}-7i\,a^4c^4e^{(2i\,fx+2i\,e)}-i\,a^4c^4\right)}{35\left(fe^{(14i\,fx+14i\,e)}+7\,fe^{(12i\,fx+12i\,e)}+21\,fe^{(10i\,fx+10i\,e)}+35\,fe^{(8i\,fx+8i\,e)}+35\,fe^{(6i\,fx+6i\,e)}+21\,fe^{(4i\,fx+4i\,e)}+7\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^4*(c-I*c*tan(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] -32/35*(-35*I*a^4*c^4*e^(6*I*f*x + 6*I*e) - 21*I*a^4*c^4*e^(4*I*f*x + 4*I*e
) - 7*I*a^4*c^4*e^(2*I*f*x + 2*I*e) - I*a^4*c^4)/(f*e^(14*I*f*x + 14*I*e) +
 7*f*e^(12*I*f*x + 12*I*e) + 21*f*e^(10*I*f*x + 10*I*e) + 35*f*e^(8*I*f*x +
 8*I*e) + 35*f*e^(6*I*f*x + 6*I*e) + 21*f*e^(4*I*f*x + 4*I*e) + 7*f*e^(2*I*
f*x + 2*I*e) + f)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.46, size = 219, normalized size = 2.84

$$\frac{1120ia^4c^4e^{6ie}e^{6ifx}+672ia^4c^4e^{4ie}e^{4ifx}+224ia^4c^4e^{2ie}e^{2ifx}+32ia^4c^4}{35fe^{14ie}e^{14ifx}+245fe^{12ie}e^{12ifx}+735fe^{10ie}e^{10ifx}+1225fe^{8ie}e^{8ifx}+1225fe^{6ie}e^{6ifx}+735fe^{4ie}e^{4ifx}+245fe^{2ie}e^{2ifx}+35f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**4*(c-I*c*tan(f*x+e))**4,x)
```

```
[Out] (1120*I*a**4*c**4*exp(6*I*e)*exp(6*I*f*x) + 672*I*a**4*c**4*exp(4*I*e)*exp(
4*I*f*x) + 224*I*a**4*c**4*exp(2*I*e)*exp(2*I*f*x) + 32*I*a**4*c**4)/(35*f*
exp(14*I*e)*exp(14*I*f*x) + 245*f*exp(12*I*e)*exp(12*I*f*x) + 735*f*exp(10*
I*e)*exp(10*I*f*x) + 1225*f*exp(8*I*e)*exp(8*I*f*x) + 1225*f*exp(6*I*e)*exp
(6*I*f*x) + 735*f*exp(4*I*e)*exp(4*I*f*x) + 245*f*exp(2*I*e)*exp(2*I*f*x) +
 35*f)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 650 vs. 2(77) = 154.
time = 1.23, size = 650, normalized size = 8.44

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^4*(c-I*c*tan(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -1/35*(35*a^4*c^4*tan(f*x)^7*tan(e)^6 + 35*a^4*c^4*tan(f*x)^6*tan(e)^7 + 35
*a^4*c^4*tan(f*x)^7*tan(e)^4 - 105*a^4*c^4*tan(f*x)^6*tan(e)^5 - 105*a^4*c^
4*tan(f*x)^5*tan(e)^6 + 35*a^4*c^4*tan(f*x)^4*tan(e)^7 + 21*a^4*c^4*tan(f*x
)^7*tan(e)^2 - 35*a^4*c^4*tan(f*x)^6*tan(e)^3 + 315*a^4*c^4*tan(f*x)^5*tan(
e)^4 + 315*a^4*c^4*tan(f*x)^4*tan(e)^5 - 35*a^4*c^4*tan(f*x)^3*tan(e)^6 + 2
1*a^4*c^4*tan(f*x)^2*tan(e)^7 + 5*a^4*c^4*tan(f*x)^7 - 7*a^4*c^4*tan(f*x)^6
*tan(e) + 105*a^4*c^4*tan(f*x)^5*tan(e)^2 - 315*a^4*c^4*tan(f*x)^4*tan(e)^3
 - 315*a^4*c^4*tan(f*x)^3*tan(e)^4 + 105*a^4*c^4*tan(f*x)^2*tan(e)^5 - 7*a^
4*c^4*tan(f*x)*tan(e)^6 + 5*a^4*c^4*tan(e)^7 + 21*a^4*c^4*tan(f*x)^5 - 35*a
^4*c^4*tan(f*x)^4*tan(e) + 315*a^4*c^4*tan(f*x)^3*tan(e)^2 + 315*a^4*c^4*ta
n(f*x)^2*tan(e)^3 - 35*a^4*c^4*tan(f*x)*tan(e)^4 + 21*a^4*c^4*tan(e)^5 + 35
*a^4*c^4*tan(f*x)^3 - 105*a^4*c^4*tan(f*x)^2*tan(e) - 105*a^4*c^4*tan(f*x)*
```

```
tan(e)^2 + 35*a^4*c^4*tan(e)^3 + 35*a^4*c^4*tan(f*x) + 35*a^4*c^4*tan(e))/(
f*tan(f*x)^7*tan(e)^7 - 7*f*tan(f*x)^6*tan(e)^6 + 21*f*tan(f*x)^5*tan(e)^5
- 35*f*tan(f*x)^4*tan(e)^4 + 35*f*tan(f*x)^3*tan(e)^3 - 21*f*tan(f*x)^2*tan
(e)^2 + 7*f*tan(f*x)*tan(e) - f)
```

Mupad [B]
time = 4.58, size = 82, normalized size = 1.06

$$\frac{a^4\,c^4\,\sin\left(e+f\,x\right)\,\left(35\,{\cos\left(e+f\,x\right)}^6+35\,{\cos\left(e+f\,x\right)}^4\,{\sin\left(e+f\,x\right)}^2+21\,{\cos\left(e+f\,x\right)}^2\,{\sin\left(e+f\,x\right)}^4+5\,{\sin\left(e+f\,x\right)}^6\right)}{35\,f\,{\cos\left(e+f\,x\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^4*(c - c*tan(e + f*x)*1i)^4,x)
```

```
[Out] (a^4*c^4*sin(e + f*x)*(35*cos(e + f*x)^6 + 5*sin(e + f*x)^6 + 21*cos(e + f*
x)^2*sin(e + f*x)^4 + 35*cos(e + f*x)^4*sin(e + f*x)^2))/(35*f*cos(e + f*x)
^7)
```

3.916 $\int (a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^4\,dx$

Optimal. Leaf size=82

$$-\frac{ia^3c^4\sec^6(e+fx)}{6f}+\frac{a^3c^4\tan(e+fx)}{f}+\frac{2a^3c^4\tan^3(e+fx)}{3f}+\frac{a^3c^4\tan^5(e+fx)}{5f}$$

```
[Out] -1/6*I*a^3*c^4*sec(f*x+e)^6/f+a^3*c^4*tan(f*x+e)/f+2/3*a^3*c^4*tan(f*x+e)^3
/f+1/5*a^3*c^4*tan(f*x+e)^5/f
```

Rubi [A]
time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {3603, 3567, 3852}

$$\frac{a^3c^4\tan^5(e+fx)}{5f}+\frac{2a^3c^4\tan^3(e+fx)}{3f}+\frac{a^3c^4\tan(e+fx)}{f}-\frac{ia^3c^4\sec^6(e+fx)}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])^4,x]
```

```
[Out] ((-1/6*I)*a^3*c^4*Sec[e + f*x]^6)/f + (a^3*c^4*Tan[e + f*x])/f + (2*a^3*c^4
*Tan[e + f*x]^3)/(3*f) + (a^3*c^4*Tan[e + f*x]^5)/(5*f)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int (a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^4\,dx = (a^3c^3)\int \sec^6(e+fx)(c-ic\tan(e+fx))\,dx$$
$$= -\frac{ia^3c^4\sec^6(e+fx)}{6f} + (a^3c^4)\int \sec^6(e+fx)\,dx$$
$$= -\frac{ia^3c^4\sec^6(e+fx)}{6f} - \frac{(a^3c^4)\,\mathrm{Subst}\left(\int (1+2x^2+x^4)\,dx,\right.}{f}$$
$$= -\frac{ia^3c^4\sec^6(e+fx)}{6f} + \frac{a^3c^4\tan(e+fx)}{f} + \frac{2a^3c^4\tan^3(e+}{3f}$$

Mathematica [A]
time = 1.04, size = 63, normalized size = 0.77

$$\frac{a^3c^4\sec(e)\sec^6(e+fx)(-10i\cos(e)-10\sin(e)+15\sin(e+2fx)+6\sin(3e+4fx)+\sin(5e+6fx))}{60f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])^4,x]
[Out] (a^3*c^4*Sec[e]*Sec[e + f*x]^6*((-10*I)*Cos[e] - 10*Sin[e] + 15*Sin[e + 2*f
*x] + 6*Sin[3*e + 4*f*x] + Sin[5*e + 6*f*x]))/(60*f)
```

Maple [A]
time = 0.08, size = 75, normalized size = 0.91

method	result
risch	$\frac{16ia^3c^4\left(15\,e^{4i(fx+e)}+6\,e^{2i(fx+e)}+1\right)}{15f\left(e^{2i(fx+e)}+1\right)^6}$
derivativedivides	$-\frac{ia^3c^4\left(\frac{\left(\tan^6(fx+e)\right)}{6}+\frac{\left(\tan^4(fx+e)\right)}{2}+\frac{i\left(\tan^5(fx+e)\right)}{5}+\frac{\left(\tan^2(fx+e)\right)}{2}+\frac{2i\left(\tan^3(fx+e)\right)}{3}+i\tan(fx+e)\right)}{f}$
default	$-\frac{ia^3c^4\left(\frac{\left(\tan^6(fx+e)\right)}{6}+\frac{\left(\tan^4(fx+e)\right)}{2}+\frac{i\left(\tan^5(fx+e)\right)}{5}+\frac{\left(\tan^2(fx+e)\right)}{2}+\frac{2i\left(\tan^3(fx+e)\right)}{3}+i\tan(fx+e)\right)}{f}$
norman	$\frac{a^3c^4\tan(fx+e)}{f}+\frac{2a^3c^4\left(\tan^3(fx+e)\right)}{3f}+\frac{a^3c^4\left(\tan^5(fx+e)\right)}{5f}-\frac{ia^3c^4\left(\tan^2(fx+e)\right)}{2f}-\frac{ia^3c^4\left(\tan^4(fx+e)\right)}{2f}-\frac{ia^3c}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3*(c-I*c*tan(f*x+e))^4,x,method=_RETURNVERBOSE)
[Out] -I/f*a^3*c^4*(1/6*tan(f*x+e)^6+1/2*tan(f*x+e)^4+1/5*I*tan(f*x+e)^5+1/2*tan(
f*x+e)^2+2/3*I*tan(f*x+e)^3+I*tan(f*x+e))
```

Maxima [A]
time = 0.51, size = 106, normalized size = 1.29

$$-\frac{5i\,a^3c^4\tan\left(fx+e\right)^6-6\,a^3c^4\tan\left(fx+e\right)^5+15i\,a^3c^4\tan\left(fx+e\right)^4-20\,a^3c^4\tan\left(fx+e\right)^3+15i\,a^3c^4\tan\left(fx+e\right)^2-30\,a^3c^4\tan\left(fx+e\right)}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c-I*c*tan(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] -1/30*(5*I*a^3*c^4*tan(f*x + e)^6 - 6*a^3*c^4*tan(f*x + e)^5 + 15*I*a^3*c^4
*tan(f*x + e)^4 - 20*a^3*c^4*tan(f*x + e)^3 + 15*I*a^3*c^4*tan(f*x + e)^2 -
 30*a^3*c^4*tan(f*x + e))/f
```

Fricas [A]
time = 1.02, size = 128, normalized size = 1.56

$$-\frac{16\left(-15i\,a^3c^4e^{(4i\,fx+4i\,e)}-6i\,a^3c^4e^{(2i\,fx+2i\,e)}-i\,a^3c^4\right)}{15\left(fe^{(12i\,fx+12i\,e)}+6\,fe^{(10i\,fx+10i\,e)}+15\,fe^{(8i\,fx+8i\,e)}+20\,fe^{(6i\,fx+6i\,e)}+15\,fe^{(4i\,fx+4i\,e)}+6\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c-I*c*tan(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] -16/15*(-15*I*a^3*c^4*e^(4*I*f*x + 4*I*e) - 6*I*a^3*c^4*e^(2*I*f*x + 2*I*e)
 - I*a^3*c^4)/(f*e^(12*I*f*x + 12*I*e) + 6*f*e^(10*I*f*x + 10*I*e) + 15*f*e
^(8*I*f*x + 8*I*e) + 20*f*e^(6*I*f*x + 6*I*e) + 15*f*e^(4*I*f*x + 4*I*e) +
6*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(73) = 146$.
time = 0.42, size = 175, normalized size = 2.13

$$\frac{240ia^3c^4e^{4ie}e^{4ifx}+96ia^3c^4e^{2ie}e^{2ifx}+16ia^3c^4}{15fe^{12ie}e^{12ifx}+90fe^{10ie}e^{10ifx}+225fe^{8ie}e^{8ifx}+300fe^{6ie}e^{6ifx}+225fe^{4ie}e^{4ifx}+90fe^{2ie}e^{2ifx}+15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3*(c-I*c*tan(f*x+e))**4,x)
```

```
[Out] (240*I*a**3*c**4*exp(4*I*e)*exp(4*I*f*x) + 96*I*a**3*c**4*exp(2*I*e)*exp(2*
I*f*x) + 16*I*a**3*c**4)/(15*f*exp(12*I*e)*exp(12*I*f*x) + 90*f*exp(10*I*e)
*exp(10*I*f*x) + 225*f*exp(8*I*e)*exp(8*I*f*x) + 300*f*exp(6*I*e)*exp(6*I*f
*x) + 225*f*exp(4*I*e)*exp(4*I*f*x) + 90*f*exp(2*I*e)*exp(2*I*f*x) + 15*f)
```

Giac [A]
time = 0.78, size = 128, normalized size = 1.56

$$-\frac{16\left(-15i\,a^3c^4e^{(4i\,fx+4i\,e)}-6i\,a^3c^4e^{(2i\,fx+2i\,e)}-i\,a^3c^4\right)}{15\left(fe^{(12i\,fx+12i\,e)}+6\,fe^{(10i\,fx+10i\,e)}+15\,fe^{(8i\,fx+8i\,e)}+20\,fe^{(6i\,fx+6i\,e)}+15\,fe^{(4i\,fx+4i\,e)}+6\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c-I*c*tan(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -16/15*(-15*I*a^3*c^4*e^(4*I*f*x + 4*I*e) - 6*I*a^3*c^4*e^(2*I*f*x + 2*I*e)
 - I*a^3*c^4)/(f*e^(12*I*f*x + 12*I*e) + 6*f*e^(10*I*f*x + 10*I*e) + 15*f*e
^(8*I*f*x + 8*I*e) + 20*f*e^(6*I*f*x + 6*I*e) + 15*f*e^(4*I*f*x + 4*I*e) +
6*f*e^(2*I*f*x + 2*I*e) + f)
```

Mupad [B]
time = 4.55, size = 117, normalized size = 1.43

$$\frac{a^3\,c^4\,\sin\left(e+f\,x\right)\,\left(30\cos\left(e+f\,x\right)^5-\cos\left(e+f\,x\right)^4\,\sin\left(e+f\,x\right)\,15\mathrm{i}+20\cos\left(e+f\,x\right)^3\sin\left(e+f\,x\right)^2-\cos\left(e+f\,x\right)^2\sin\left(e+f\,x\right)^3\,15\mathrm{i}+6\,\cos\left(e+f\,x\right)\,\sin\left(e+f\,x\right)^4-\sin\left(e+f\,x\right)^5\,5\mathrm{i}\right)}{30\,f\,\cos\left(e+f\,x\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i)^4,x)
```

```
[Out] (a^3*c^4*sin(e + f*x)*(6*cos(e + f*x)*sin(e + f*x)^4 - cos(e + f*x)^4*sin(e
 + f*x)*15i + 30*cos(e + f*x)^5 - sin(e + f*x)^5*5i - cos(e + f*x)^2*sin(e
+ f*x)^3*15i + 20*cos(e + f*x)^3*sin(e + f*x)^2))/(30*f*cos(e + f*x)^6)
```

3.917 $\int (a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^4\,dx$

Optimal. Leaf size=58

$$\frac{ia^2(c-ic\tan(e+fx))^4}{2f}-\frac{ia^2(c-ic\tan(e+fx))^5}{5cf}$$

```
[Out] 1/2*I*a^2*(c-I*c*tan(f*x+e))^4/f-1/5*I*a^2*(c-I*c*tan(f*x+e))^5/c/f
```

Rubi [A]
time = 0.07, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 45}

$$\frac{ia^2(c-ic\tan(e+fx))^4}{2f}-\frac{ia^2(c-ic\tan(e+fx))^5}{5cf}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^4,x]
```

```
[Out] ((I/2)*a^2*(c - I*c*Tan[e + f*x])^4)/f - ((I/5)*a^2*(c - I*c*Tan[e + f*x])^
5)/(c*f)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int (a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^4\,dx = (a^2c^2)\int \sec^4(e+fx)(c-ic\tan(e+fx))^2\,dx$$
$$= \frac{(ia^2)\,\text{Subst}\left(\int (c-x)(c+x)^3\,dx, x, -ic\tan(e+fx)\right)}{cf}$$
$$= \frac{(ia^2)\,\text{Subst}\left(\int \left(2c(c+x)^3-(c+x)^4\right)\,dx, x, -ic\tan(e+f\right.}{cf}$$
$$= \frac{ia^2(c-ic\tan(e+fx))^4}{2f} - \frac{ia^2(c-ic\tan(e+fx))^5}{5cf}$$

Mathematica [A]
time = 0.95, size = 80, normalized size = 1.38

$$\frac{a^2c^4\sec(e)\sec^5(e+fx)(-5i\cos(fx)-5i\cos(2e+fx)+5\sin(fx)-5\sin(2e+fx)+5\sin(2e+3fx)+\sin(4e+5fx))}{20f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^4,x]
```

```
[Out] (a^2*c^4*Sec[e]*Sec[e + f*x]^5*((-5*I)*Cos[f*x] - (5*I)*Cos[2*e + f*x] + 5*
Sin[f*x] - 5*Sin[2*e + f*x] + 5*Sin[2*e + 3*f*x] + Sin[4*e + 5*f*x]))/(20*f
)
```

Maple [A]
time = 0.07, size = 50, normalized size = 0.86

method	result	size
risch	$\frac{8ia^2c^4\left(5\,e^{2i(fx+e)}+1\right)}{5f\left(e^{2i(fx+e)}+1\right)^5}$	39
derivativedivides	$\frac{a^2c^4\left(\tan(fx+e)-\frac{\left(\tan^5(fx+e)\right)}{5}-\frac{i\left(\tan^4(fx+e)\right)}{2}-i\left(\tan^2(fx+e)\right)\right)}{f}$	50
default	$\frac{a^2c^4\left(\tan(fx+e)-\frac{\left(\tan^5(fx+e)\right)}{5}-\frac{i\left(\tan^4(fx+e)\right)}{2}-i\left(\tan^2(fx+e)\right)\right)}{f}$	50
norman	$\frac{a^2c^4\tan(fx+e)}{f}-\frac{a^2c^4\left(\tan^5(fx+e)\right)}{5f}-\frac{ia^2c^4\left(\tan^2(fx+e)\right)}{f}-\frac{ia^2c^4\left(\tan^4(fx+e)\right)}{2f}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2*(c-I*c*tan(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a^2*c^4*(tan(f*x+e)-1/5*tan(f*x+e)^5-1/2*I*tan(f*x+e)^4-I*tan(f*x+e)^2)
```

Maxima [A]
time = 0.53, size = 72, normalized size = 1.24

$$-\frac{2\,a^2c^4\tan\left(fx+e\right)^5+5i\,a^2c^4\tan\left(fx+e\right)^4+10i\,a^2c^4\tan\left(fx+e\right)^2-10\,a^2c^4\tan\left(fx+e\right)}{10\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c-I*c*tan(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] -1/10*(2*a^2*c^4*tan(f*x + e)^5 + 5*I*a^2*c^4*tan(f*x + e)^4 + 10*I*a^2*c^4
*tan(f*x + e)^2 - 10*a^2*c^4*tan(f*x + e))/f
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(48) = 96.
time = 1.12, size = 97, normalized size = 1.67

$$-\frac{8\left(-5i\,a^2c^4e^{(2i\,fx+2i\,e)}-i\,a^2c^4\right)}{5\left(fe^{(10i\,fx+10i\,e)}+5\,fe^{(8i\,fx+8i\,e)}+10\,fe^{(6i\,fx+6i\,e)}+10\,fe^{(4i\,fx+4i\,e)}+5\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c-I*c*tan(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] -8/5*(-5*I*a^2*c^4*e^(2*I*f*x + 2*I*e) - I*a^2*c^4)/(f*e^(10*I*f*x + 10*I*e
) + 5*f*e^(8*I*f*x + 8*I*e) + 10*f*e^(6*I*f*x + 6*I*e) + 10*f*e^(4*I*f*x +
4*I*e) + 5*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(44) = 88.
time = 0.30, size = 131, normalized size = 2.26

$$\frac{40ia^2c^4e^{2ie}e^{2ifx}+8ia^2c^4}{5fe^{10ie}e^{10ifx}+25fe^{8ie}e^{8ifx}+50fe^{6ie}e^{6ifx}+50fe^{4ie}e^{4ifx}+25fe^{2ie}e^{2ifx}+5f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2*(c-I*c*tan(f*x+e))**4,x)
```

```
[Out] (40*I*a**2*c**4*exp(2*I*e)*exp(2*I*f*x) + 8*I*a**2*c**4)/(5*f*exp(10*I*e)*e
xp(10*I*f*x) + 25*f*exp(8*I*e)*exp(8*I*f*x) + 50*f*exp(6*I*e)*exp(6*I*f*x)
+ 50*f*exp(4*I*e)*exp(4*I*f*x) + 25*f*exp(2*I*e)*exp(2*I*f*x) + 5*f)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(48) = 96.
time = 0.73, size = 97, normalized size = 1.67

$$-\frac{8\left(-5i\,a^2c^4e^{(2i\,fx+2i\,e)}-i\,a^2c^4\right)}{5\left(fe^{(10i\,fx+10i\,e)}+5\,fe^{(8i\,fx+8i\,e)}+10\,fe^{(6i\,fx+6i\,e)}+10\,fe^{(4i\,fx+4i\,e)}+5\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c-I*c*tan(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -8/5*(-5*I*a^2*c^4*e^(2*I*f*x + 2*I*e) - I*a^2*c^4)/(f*e^(10*I*f*x + 10*I*e
) + 5*f*e^(8*I*f*x + 8*I*e) + 10*f*e^(6*I*f*x + 6*I*e) + 10*f*e^(4*I*f*x +
4*I*e) + 5*f*e^(2*I*f*x + 2*I*e) + f)
```

Mupad [B]
time = 4.60, size = 80, normalized size = 1.38

$$-\frac{a^2\,c^4\,\sin\left(e+f\,x\right)\,\left(-10\,{\cos\left(e+f\,x\right)}^4+{\cos\left(e+f\,x\right)}^3\,\sin\left(e+f\,x\right)\,10{}\mathrm{i}+\cos\left(e+f\,x\right)\,{\sin\left(e+f\,x\right)}^3\,5{}\mathrm{i}+2\,{\sin\left(e+f\,x\right)}^4\right)}{10\,f\,{\cos\left(e+f\,x\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i)^4,x)
```

```
[Out] -(a^2*c^4*sin(e + f*x)*(cos(e + f*x)*sin(e + f*x)^3*5i + cos(e + f*x)^3*sin
(e + f*x)*10i - 10*cos(e + f*x)^4 + 2*sin(e + f*x)^4))/(10*f*cos(e + f*x)^5
)
```

3.918 $\int (a+ia\tan(e+fx))(c-ic\tan(e+fx))^4\,dx$

Optimal. Leaf size=25

$$\frac{ia(c-ic\tan(e+fx))^4}{4f}$$

[Out] 1/4*I*a*(c-I*c*tan(f*x+e))^4/f

Rubi [A]
time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3603, 3568, 32}

$$\frac{ia(c-ic\tan(e+fx))^4}{4f}$$

Antiderivative was successfully verified.

[In] Int[(a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^4,x]

[Out] ((I/4)*a*(c - I*c*Tan[e + f*x])^4)/f

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int (a+ia\tan(e+fx))(c-ic\tan(e+fx))^4\,dx = (ac)\int \sec^2(e+fx)(c-ic\tan(e+fx))^3\,dx$$
$$= \frac{(ia)\text{Subst}\left(\int (c+x)^3\,dx, x, -ic\tan(e+fx)\right)}{f}$$
$$= \frac{ia(c-ic\tan(e+fx))^4}{4f}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 85 vs. $2(25) = 50$.
time = 0.60, size = 85, normalized size = 3.40

$$\frac{ac^4\sec(e)\sec^4(e+fx)(-3i\cos(e)-2i\cos(e+2fx)-2i\cos(3e+2fx)-3\sin(e)+2\sin(e+2fx)-2\sin(3e+2fx)+\sin(3e+4fx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^4,x]
```

```
[Out] (a*c^4*Sec[e]*Sec[e + f*x]^4*((-3*I)*Cos[e] - (2*I)*Cos[e + 2*f*x] - (2*I)*
Cos[3*e + 2*f*x] - 3*Sin[e] + 2*Sin[e + 2*f*x] - 2*Sin[3*e + 2*f*x] + Sin[3
*e + 4*f*x]))/(4*f)
```

Maple [A]
time = 0.07, size = 22, normalized size = 0.88

method	result	size
derivativedivides	$\frac{ia\,c^4(\tan(fx+e)+i)^4}{4f}$	22
default	$\frac{ia\,c^4(\tan(fx+e)+i)^4}{4f}$	22
risch	$\frac{4ia\,c^4}{f\left(e^{2i(fx+e)}+1\right)^4}$	24
norman	$\frac{a\,c^4\tan(fx+e)}{f}-\frac{a\,c^4(\tan^3(fx+e))}{f}-\frac{3ia\,c^4(\tan^2(fx+e))}{2f}+\frac{ia\,c^4(\tan^4(fx+e))}{4f}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I/f*a*c^4*(tan(f*x+e)+I)^4
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(20) = 40$.
time = 0.50, size = 64, normalized size = 2.56

$$\frac{i\,ac^4\tan(fx+e)^4 - 4\,ac^4\tan(fx+e)^3 - 6i\,ac^4\tan(fx+e)^2 + 4\,ac^4\tan(fx+e)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 1/4*(I*a*c^4*tan(f*x + e)^4 - 4*a*c^4*tan(f*x + e)^3 - 6*I*a*c^4*tan(f*x +
e)^2 + 4*a*c^4*tan(f*x + e))/f
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(20) = 40$.
time = 1.16, size = 61, normalized size = 2.44

$$\frac{4i\,ac^4}{fe^{(8i\,fx+8i\,e)} + 4\,fe^{(6i\,fx+6i\,e)} + 6\,fe^{(4i\,fx+4i\,e)} + 4\,fe^{(2i\,fx+2i\,e)} + f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 4*I*a*c^4/(f*e^(8*I*f*x + 8*I*e) + 4*f*e^(6*I*f*x + 6*I*e) + 6*f*e^(4*I*f*x
 + 4*I*e) + 4*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(19) = 38$.
time = 0.20, size = 82, normalized size = 3.28

$$\frac{4iac^4}{fe^{8ie}e^{8ifx} + 4fe^{6ie}e^{6ifx} + 6fe^{4ie}e^{4ifx} + 4fe^{2ie}e^{2ifx} + f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e))**4,x)
```

```
[Out] 4*I*a*c**4/(f*exp(8*I*e)*exp(8*I*f*x) + 4*f*exp(6*I*e)*exp(6*I*f*x) + 6*f*e
xp(4*I*e)*exp(4*I*f*x) + 4*f*exp(2*I*e)*exp(2*I*f*x) + f)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(20) = 40$.
time = 0.63, size = 61, normalized size = 2.44

$$\frac{4i\,ac^4}{fe^{(8i\,fx+8i\,e)} + 4\,fe^{(6i\,fx+6i\,e)} + 6\,fe^{(4i\,fx+4i\,e)} + 4\,fe^{(2i\,fx+2i\,e)} + f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 4*I*a*c^4/(f*e^(8*I*f*x + 8*I*e) + 4*f*e^(6*I*f*x + 6*I*e) + 6*f*e^(4*I*f*x
 + 4*I*e) + 4*f*e^(2*I*f*x + 2*I*e) + f)
```

Mupad [B]
time = 4.59, size = 78, normalized size = 3.12

$$-\frac{a\,c^4\,\sin\left(e+f\,x\right)\,\left(-4\cos\left(e+f\,x\right)^3+\cos\left(e+f\,x\right)^2\,\sin\left(e+f\,x\right)\,6{}\mathrm{i}+4\,\cos\left(e+f\,x\right)\,\sin\left(e+f\,x\right)^2-\sin\left(e+f\,x\right)^3\,1{}\mathrm{i}\right)}{4\,f\,\cos\left(e+f\,x\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i)^4,x)
```

```
[Out] -(a*c^4*sin(e + f*x)*(4*cos(e + f*x)*sin(e + f*x)^2 + cos(e + f*x)^2*sin(e
+ f*x)*6i - 4*cos(e + f*x)^3 - sin(e + f*x)^3*1i))/(4*f*cos(e + f*x)^4)
```

3.919 $\int \frac{(c-ic\tan(e+fx))^4}{a+ia\tan(e+fx)}\,dx$

Optimal. Leaf size=95

$$-\frac{12c^4x}{a}-\frac{12ic^4\log(\cos(e+fx))}{af}+\frac{5c^4\tan(e+fx)}{af}-\frac{ic^4\tan^2(e+fx)}{2af}+\frac{8ic^4}{f(a+ia\tan(e+fx))}$$

```
[Out] -12*c^4*x/a-12*I*c^4*ln(cos(f*x+e))/a/f+5*c^4*tan(f*x+e)/a/f-1/2*I*c^4*tan(
f*x+e)^2/a/f+8*I*c^4/f/(a+I*a*tan(f*x+e))
```

Rubi [A]
time = 0.09, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 45}

$$-\frac{ic^4\tan^2(e+fx)}{2af}+\frac{5c^4\tan(e+fx)}{af}+\frac{8ic^4}{f(a+ia\tan(e+fx))}-\frac{12ic^4\log(\cos(e+fx))}{af}-\frac{12c^4x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])^4/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (-12*c^4*x)/a - ((12*I)*c^4*Log[Cos[e + f*x]])/(a*f) + (5*c^4*Tan[e + f*x])
/(a*f) - ((I/2)*c^4*Tan[e + f*x]^2)/(a*f) + ((8*I)*c^4)/(f*(a + I*a*Tan[e +
 f*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\begin{aligned}\int \frac{(c-ic\tan(e+fx))^4}{a+ia\tan(e+fx)}\,dx &= (a^4c^4)\int \frac{\sec^8(e+fx)}{(a+ia\tan(e+fx))^5}\,dx\\ &= -\frac{(ic^4)\,\text{Subst}\left(\int \frac{(a-x)^3}{(a+x)^2}\,dx, x, ia\tan(e+fx)\right)}{a^3 f}\\ &= -\frac{(ic^4)\,\text{Subst}\left(\int \left(5a-x+\frac{8a^3}{(a+x)^2}-\frac{12a^2}{a+x}\right)\,dx, x, ia\tan(e+fx)\right)}{a^3 f}\\ &= -\frac{12c^4x}{a}-\frac{12ic^4\log(\cos(e+fx))}{af}+\frac{5c^4\tan(e+fx)}{af}-\frac{ic^4\tan^2(e+fx)}{2af}+\frac{}{f(a\ldots}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 194 vs. $2(95)=190$.
time = 2.08, size = 194, normalized size = 2.04

$$\frac{c^4\cos(e)\sec(e+fx)(\cos(fx)+i\sin(fx))\left(-24fx-12i\log\left(\cos^2(e+fx)\right)+24fx\sec^2(e)-i\sec^2(e+fx)+10\sec(e)\sec(e+fx)\sin(fx)+8\sin(2fx)+12\log\left(\cos^2(e+fx)\right)\tan(e)+\sec^2(e+fx)\tan(e)+10i\sec(e)\sec(e+fx)\sin(fx)\tan(e)-8i\sin(2fx)\tan(e)-24fx\tan^2(e)-24i\text{ArcTan}(\tan(fx))(-i+\tan(e))+8\cos(2fx)(i+\tan(e))\right)}{2f(a+ia\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - I*c*Tan[e + f*x])^4/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (c^4*Cos[e]*Sec[e + f*x]*(Cos[f*x] + I*Sin[f*x])*(-24*f*x - (12*I)*Log[Cos[
e + f*x]^2] + 24*f*x*Sec[e]^2 - I*Sec[e + f*x]^2 + 10*Sec[e]*Sec[e + f*x]*S
in[f*x] + 8*Sin[2*f*x] + 12*Log[Cos[e + f*x]^2]*Tan[e] + Sec[e + f*x]^2*Tan
[e] + (10*I)*Sec[e]*Sec[e + f*x]*Sin[f*x]*Tan[e] - (8*I)*Sin[2*f*x]*Tan[e]
- 24*f*x*Tan[e]^2 - (24*I)*ArcTan[Tan[f*x]]*(-I + Tan[e]) + 8*Cos[2*f*x]*(I
 + Tan[e])))/(2*f*(a + I*a*Tan[e + f*x]))
```

Maple [A]
time = 0.16, size = 57, normalized size = 0.60

method	result	si
derivativedivides	$\frac{c^4\left(5\tan(fx+e)-\frac{i\left(\tan^2(fx+e)\right)}{2}+\frac{8}{\tan(fx+e)-i}+12i\ln(\tan(fx+e)-i)\right)}{fa}$	5
default	$\frac{c^4\left(5\tan(fx+e)-\frac{i\left(\tan^2(fx+e)\right)}{2}+\frac{8}{\tan(fx+e)-i}+12i\ln(\tan(fx+e)-i)\right)}{fa}$	5
risch	$\frac{4ic^4\mathrm{e}^{-2i(fx+e)}}{af}-\frac{24c^4x}{a}-\frac{24c^4e}{af}+\frac{2ic^4\left(4\,\mathrm{e}^{2i(fx+e)}+5\right)}{fa\left(\mathrm{e}^{2i(fx+e)}+1\right)^2}-\frac{12ic^4\ln\left(\mathrm{e}^{2i(fx+e)}+1\right)}{af}$	1

norman	$\frac{\frac{17ic^4}{2af}-\frac{12c^4x}{a}+\frac{5c^4\left(\tan^3(fx+e)\right)}{af}-\frac{12c^4x\left(\tan^2(fx+e)\right)}{a}+\frac{13c^4\tan(fx+e)}{af}-\frac{ic^4\left(\tan^4(fx+e)\right)}{2af}}{1+\tan^2(fx+e)}+\frac{6ic^4\ln(1+\tan^2(fx+e))}{af}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*c^4/a*(5*tan(f*x+e)-1/2*I*tan(f*x+e)^2+8/(tan(f*x+e)-I)+12*I*ln(tan(f*x
+e)-I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(89) = 178.
time = 1.01, size = 179, normalized size = 1.88

$$-\frac{2\left(12\,c^4fxe^{(6i\,fx+6i\,e)}-2i\,c^4+6\left(4\,c^4fx-i\,c^4\right)e^{(4i\,fx+4i\,e)}+3\left(4\,c^4fx-3i\,c^4\right)e^{(2i\,fx+2i\,e)}+6\left(i\,c^4e^{(6i\,fx+6i\,e)}+2i\,c^4e^{(4i\,fx+4i\,e)}+i\,c^4e^{(2i\,fx+2i\,e)}\right)\log\left(e^{(2i\,fx+2i\,e)}+1\right)\right)}{afe^{(6i\,fx+6i\,e)}+2\,afe^{(4i\,fx+4i\,e)}+afe^{(2i\,fx+2i\,e)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] -2*(12*c^4*f*x*e^(6*I*f*x + 6*I*e) - 2*I*c^4 + 6*(4*c^4*f*x - I*c^4)*e^(4*I
*f*x + 4*I*e) + 3*(4*c^4*f*x - 3*I*c^4)*e^(2*I*f*x + 2*I*e) + 6*(I*c^4*e^(6
*I*f*x + 6*I*e) + 2*I*c^4*e^(4*I*f*x + 4*I*e) + I*c^4*e^(2*I*f*x + 2*I*e))*
log(e^(2*I*f*x + 2*I*e) + 1))/(a*f*e^(6*I*f*x + 6*I*e) + 2*a*f*e^(4*I*f*x +
 4*I*e) + a*f*e^(2*I*f*x + 2*I*e))
```

Sympy [A]
time = 0.28, size = 175, normalized size = 1.84

$$\frac{8ic^4e^{2ie}e^{2ifx}+10ic^4}{afe^{4ie}e^{4ifx}+2afe^{2ie}e^{2ifx}+af}+\begin{cases}\frac{4ic^4e^{-2ie}e^{-2ifx}}{af} & \text{for } afe^{2ie}\neq 0\\ x\left(\frac{24c^4}{a}+\frac{\left(-24c^4e^{2ie}+8c^4\right)e^{-2ie}}{a}\right) & \text{otherwise}\end{cases}-\frac{24c^4x}{a}-\frac{12ic^4\log\left(e^{2ifx}+e^{-2ie}\right)}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**4/(a+I*a*tan(f*x+e)),x)
```

```
[Out] (8*I*c**4*exp(2*I*e)*exp(2*I*f*x) + 10*I*c**4)/(a*f*exp(4*I*e)*exp(4*I*f*x)
 + 2*a*f*exp(2*I*e)*exp(2*I*f*x) + a*f) + Piecewise((4*I*c**4*exp(-2*I*e)*e
xp(-2*I*f*x)/(a*f), Ne(a*f*exp(2*I*e), 0)), (x*(24*c**4/a + (-24*c**4*exp(2
*I*e) + 8*c**4)*exp(-2*I*e)/a), True)) - 24*c**4*x/a - 12*I*c**4*log(exp(2*
I*f*x) + exp(-2*I*e))/(a*f)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(89) = 178$.
time = 0.66, size = 200, normalized size = 2.11

$$\frac{2\left(-\frac{6i\,c^4\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}{a}+\frac{12i\,c^4\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-i\right)}{a}-\frac{6i\,c^4\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}{a}-\frac{13\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-9i\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-24\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+9i\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+13\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-i\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+i\right)^2a}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] 2*(-6*I*c^4*log(tan(1/2*f*x + 1/2*e) + 1)/a + 12*I*c^4*log(tan(1/2*f*x + 1/
2*e) - I)/a - 6*I*c^4*log(tan(1/2*f*x + 1/2*e) - 1)/a - (13*c^4*tan(1/2*f*x
 + 1/2*e)^5 - 9*I*c^4*tan(1/2*f*x + 1/2*e)^4 - 24*c^4*tan(1/2*f*x + 1/2*e)^
3 + 9*I*c^4*tan(1/2*f*x + 1/2*e)^2 + 13*c^4*tan(1/2*f*x + 1/2*e))/((tan(1/2
*f*x + 1/2*e)^3 - I*tan(1/2*f*x + 1/2*e)^2 - tan(1/2*f*x + 1/2*e) + I)^2*a)
)/f
```

Mupad [B]
time = 4.68, size = 85, normalized size = 0.89

$$\frac{5\,c^4\tan(e+f\,x)}{a\,f}+\frac{c^4\,8\mathrm{i}}{a\,f\,\left(1+\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}-\frac{c^4\tan(e+f\,x)^2\,1\mathrm{i}}{2\,a\,f}+\frac{c^4\,\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,12\mathrm{i}}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^4/(a + a*tan(e + f*x)*1i),x)
```

```
[Out] (5*c^4*tan(e + f*x))/(a*f) + (c^4*8i)/(a*f*(tan(e + f*x)*1i + 1)) - (c^4*ta
n(e + f*x)^2*1i)/(2*a*f) + (c^4*log(tan(e + f*x) - 1i)*12i)/(a*f)
```

3.920 $\int \frac{(c-ic\tan(e+fx))^4}{(a+ia\tan(e+fx))^2} dx$

Optimal. Leaf size=101

$$\frac{6c^4x}{a^2}+\frac{6ic^4\log(\cos(e+fx))}{a^2f}-\frac{c^4\tan(e+fx)}{a^2f}+\frac{4ic^4}{f(a+ia\tan(e+fx))^2}-\frac{12ic^4}{f\left(a^2+ia^2\tan(e+fx)\right)}$$

```
[Out] 6*c^4*x/a^2+6*I*c^4*ln(cos(f*x+e))/a^2/f-c^4*tan(f*x+e)/a^2/f+4*I*c^4/f/(a+
I*a*tan(f*x+e))^2-12*I*c^4/f/(a^2+I*a^2*tan(f*x+e))
```

Rubi [A]
time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 45}

$$-\frac{c^4\tan(e+fx)}{a^2f}-\frac{12ic^4}{f\left(a^2+ia^2\tan(e+fx)\right)}+\frac{6ic^4\log(\cos(e+fx))}{a^2f}+\frac{6c^4x}{a^2}+\frac{4ic^4}{f(a+ia\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])^4/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (6*c^4*x)/a^2 + ((6*I)*c^4*Log[Cos[e + f*x]])/(a^2*f) - (c^4*Tan[e + f*x])/
(a^2*f) + ((4*I)*c^4)/(f*(a + I*a*Tan[e + f*x])^2) - ((12*I)*c^4)/(f*(a^2 +
 I*a^2*Tan[e + f*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c-ic\tan(e+fx))^4}{(a+ia\tan(e+fx))^2}\,dx &= \left(a^4c^4\right)\int \frac{\sec^8(e+fx)}{(a+ia\tan(e+fx))^6}\,dx\\
&= -\frac{\left(ic^4\right)\operatorname{Subst}\left(\int \frac{(a-x)^3}{(a+x)^3}\,dx, x, ia\tan(e+fx)\right)}{a^3f}\\
&= -\frac{\left(ic^4\right)\operatorname{Subst}\left(\int \left(-1+\frac{8a^3}{(a+x)^3}-\frac{12a^2}{(a+x)^2}+\frac{6a}{a+x}\right)dx, x, ia\tan(e+fx)\right)}{a^3f}\\
&= \frac{6c^4x}{a^2}+\frac{6ic^4\log(\cos(e+fx))}{a^2f}-\frac{c^4\tan(e+fx)}{a^2f}+\frac{4ic^4}{f(a+ia\tan(e+fx))^2}-\frac{}{f}
\end{aligned}
$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 279 vs. 2(101) = 202.
time = 1.74, size = 279, normalized size = 2.76

$$
\frac{c^4\sec^2(e+fx)(\cos(fx)+i\sin(fx))^2\left(12fx+8i\cos(2fx)+i\cos(2e-fx)\sec(e)\sec(e+fx)-i\cos(2e+fx)\sec(e)\sec(e+fx)-24fx\sin^2(e)-12\text{ArcTan}(\tan(fx))(\cos(2e)+i\sin(2e))-12ifx\sin(2e)-2\cos(4fx)\sin(2e)+6\log\left(\cos^2(e+fx)\right)\sin(2e)+8\sin(2fx)+2i\sin(2e)\sin(4fx)-\sec(e)\sec(e+fx)\sin(2e-fx)+\sec(e)\sec(e+fx)\sin(2e+fx)+12ifx\tan(e)+2i\cos(2e)\left(6ifx-\cos(4fx)-3\log\left(\cos^2(e+fx)\right)+i\sin(4fx)+6fx\tan(e)\right)\right)}{2a^2f(-i+\tan(e+fx))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - I*c*Tan[e + f*x])^4/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (c^4*Sec[e + f*x]^2*(Cos[f*x] + I*Sin[f*x])^2*(12*f*x + (8*I)*Cos[2*f*x] +
I*Cos[2*e - f*x]*Sec[e]*Sec[e + f*x] - I*Cos[2*e + f*x]*Sec[e]*Sec[e + f*x]
 - 24*f*x*Sin[e]^2 - 12*ArcTan[Tan[f*x]]*(Cos[2*e] + I*Sin[2*e]) - (12*I)*f
*x*Sin[2*e] - 2*Cos[4*f*x]*Sin[2*e] + 6*Log[Cos[e + f*x]^2]*Sin[2*e] + 8*Si
n[2*f*x] + (2*I)*Sin[2*e]*Sin[4*f*x] - Sec[e]*Sec[e + f*x]*Sin[2*e - f*x] +
 Sec[e]*Sec[e + f*x]*Sin[2*e + f*x] + (12*I)*f*x*Tan[e] + (2*I)*Cos[2*e]*((
6*I)*f*x - Cos[4*f*x] - 3*Log[Cos[e + f*x]^2] + I*Sin[4*f*x] + 6*f*x*Tan[e]
)))/(2*a^2*f*(-I + Tan[e + f*x])^2)
```

Maple [A]
time = 0.19, size = 60, normalized size = 0.59

method	result
derivativedivides	$\frac{c^4\left(-\tan(fx+e)-\frac{12}{\tan(fx+e)-i}-\frac{4i}{(\tan(fx+e)-i)^2}-6i\ln(\tan(fx+e)-i)\right)}{f\,a^2}$
default	$\frac{c^4\left(-\tan(fx+e)-\frac{12}{\tan(fx+e)-i}-\frac{4i}{(\tan(fx+e)-i)^2}-6i\ln(\tan(fx+e)-i)\right)}{f\,a^2}$
risch	$-\frac{4ic^4e^{-2i(fx+e)}}{a^2f}+\frac{ic^4e^{-4i(fx+e)}}{a^2f}+\frac{12c^4x}{a^2}+\frac{12c^4e}{a^2f}-\frac{2ic^4}{f\,a^2\left(e^{2i(fx+e)}+1\right)}+\frac{6ic^4\ln\left(e^{2i(fx+e)}+1\right)}{a^2f}$

norman	$\frac{-\frac{8ic^4}{af}+\frac{6c^4x}{a}-\frac{14c^4\left(\tan^3(fx+e)\right)}{af}-\frac{c^4\left(\tan^5(fx+e)\right)}{af}+\frac{12c^4x\left(\tan^2(fx+e)\right)}{a}+\frac{6c^4x\left(\tan^4(fx+e)\right)}{a}-\frac{5c^4\tan(fx+e)}{af}-\frac{16ic^4\left(\tan^2(\right.}{af}}{a(1+\tan^2(fx+e))^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*c^4/a^2*(-tan(f*x+e)-12/(tan(f*x+e)-I)-4*I/(tan(f*x+e)-I)^2-6*I*ln(tan(
f*x+e)-I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.02, size = 143, normalized size = 1.42

$$\frac{12\,c^4fxe^{(6i\,fx+6i\,e)}-3i\,c^4e^{(2i\,fx+2i\,e)}+i\,c^4+6\,(2\,c^4fx-i\,c^4)e^{(4i\,fx+4i\,e)}-6\left(-i\,c^4e^{(6i\,fx+6i\,e)}-i\,c^4e^{(4i\,fx+4i\,e)}\right)\log\left(e^{(2i\,fx+2i\,e)}+1\right)}{a^2fe^{(6i\,fx+6i\,e)}+a^2fe^{(4i\,fx+4i\,e)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] (12*c^4*f*x*e^(6*I*f*x + 6*I*e) - 3*I*c^4*e^(2*I*f*x + 2*I*e) + I*c^4 + 6*(
2*c^4*f*x - I*c^4)*e^(4*I*f*x + 4*I*e) - 6*(-I*c^4*e^(6*I*f*x + 6*I*e) - I*
c^4*e^(4*I*f*x + 4*I*e))*log(e^(2*I*f*x + 2*I*e) + 1))/(a^2*f*e^(6*I*f*x +
6*I*e) + a^2*f*e^(4*I*f*x + 4*I*e))
```

Sympy [A]
time = 0.31, size = 199, normalized size = 1.97

$$-\frac{2ic^4}{a^2fe^{2ie}e^{2ifx}+a^2f}+\begin{cases}\frac{\left(-4ia^2c^4fe^{4ie}e^{-2ifx}+ia^2c^4fe^{2ie}e^{-4ifx}\right)e^{-6ie}}{a^4f^2} & \text{for } a^4f^2e^{6ie}\neq 0\\ x\left(-\frac{12c^4}{a^2}+\frac{\left(12c^4e^{4ie}-8c^4e^{2ie}+4c^4\right)e^{-4ie}}{a^2}\right) & \text{otherwise}\end{cases}+\frac{12c^4x}{a^2}+\frac{6ic^4\log\left(e^{2ifx}+e^{-2ie}\right)}{a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**4/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -2*I*c**4/(a**2*f*exp(2*I*e)*exp(2*I*f*x) + a**2*f) + Piecewise(((-4*I*a**2
*c**4*f*exp(4*I*e)*exp(-2*I*f*x) + I*a**2*c**4*f*exp(2*I*e)*exp(-4*I*f*x))*
```

```
exp(-6*I*e)/(a**4*f**2), Ne(a**4*f**2*exp(6*I*e), 0)), (x*(-12*c**4/a**2 +
(12*c**4*exp(4*I*e) - 8*c**4*exp(2*I*e) + 4*c**4)*exp(-4*I*e)/a**2), True))
 + 12*c**4*x/a**2 + 6*I*c**4*log(exp(2*I*f*x) + exp(-2*I*e))/(a**2*f)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(95) = 190.
time = 0.73, size = 217, normalized size = 2.15

$$-\frac{-\frac{6i\,c^4\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}{a^2}+\frac{12i\,c^4\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-i\right)}{a^2}-\frac{6i\,c^4\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}{a^2}-\frac{2\left(-3i\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+3i\,c^4\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)a^2}+\frac{-25i\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-108\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+182i\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+108\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-25i\,c^4}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-i\right)^4}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -(-6*I*c^4*log(tan(1/2*f*x + 1/2*e) + 1)/a^2 + 12*I*c^4*log(tan(1/2*f*x + 1
/2*e) - I)/a^2 - 6*I*c^4*log(tan(1/2*f*x + 1/2*e) - 1)/a^2 - 2*(-3*I*c^4*ta
n(1/2*f*x + 1/2*e)^2 + c^4*tan(1/2*f*x + 1/2*e) + 3*I*c^4)/((tan(1/2*f*x +
1/2*e)^2 - 1)*a^2) + (-25*I*c^4*tan(1/2*f*x + 1/2*e)^4 - 108*c^4*tan(1/2*f*
x + 1/2*e)^3 + 182*I*c^4*tan(1/2*f*x + 1/2*e)^2 + 108*c^4*tan(1/2*f*x + 1/2
*e) - 25*I*c^4)/(a^2*(tan(1/2*f*x + 1/2*e) - I)^4))/f
```

Mupad [B]
time = 4.84, size = 93, normalized size = 0.92

$$-\frac{\frac{8\,c^4}{a^2}+\frac{c^4\,\tan(e+f\,x)\,12\mathrm{i}}{a^2}}{f\,\left(\tan\left(e+f\,x\right)^2\,1\mathrm{i}+2\,\tan\left(e+f\,x\right)-\mathrm{i}\right)}-\frac{c^4\,\tan(e+f\,x)}{a^2\,f}-\frac{c^4\,\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,6\mathrm{i}}{a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^4/(a + a*tan(e + f*x)*1i)^2,x)
```

```
[Out] - ((8*c^4)/a^2 + (c^4*tan(e + f*x)*12i)/a^2)/(f*(2*tan(e + f*x) + tan(e + f
*x)^2*1i - 1i)) - (c^4*tan(e + f*x))/(a^2*f) - (c^4*log(tan(e + f*x) - 1i)*
6i)/(a^2*f)
```

3.921 $\int \frac{(c-ic\tan(e+fx))^4}{(a+ia\tan(e+fx))^3}\,dx$

Optimal. Leaf size=114

$$-\frac{c^4x}{a^3}-\frac{ic^4\log(\cos(e+fx))}{a^3f}+\frac{8ic^4}{3f(a+ia\tan(e+fx))^3}-\frac{6ic^4}{af(a+ia\tan(e+fx))^2}+\frac{6ic^4}{f\left(a^3+ia^3\tan(e+fx)\right)}$$

```
[Out] -c^4*x/a^3-I*c^4*ln(cos(f*x+e))/a^3/f+8/3*I*c^4/f/(a+I*a*tan(f*x+e))^3-6*I*
c^4/a/f/(a+I*a*tan(f*x+e))^2+6*I*c^4/f/(a^3+I*a^3*tan(f*x+e))
```

Rubi [A]
time = 0.09, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 45}

$$\frac{6ic^4}{f\left(a^3+ia^3\tan(e+fx)\right)}-\frac{ic^4\log(\cos(e+fx))}{a^3f}-\frac{c^4x}{a^3}-\frac{6ic^4}{af(a+ia\tan(e+fx))^2}+\frac{8ic^4}{3f(a+ia\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])^4/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] -((c^4*x)/a^3) - (I*c^4*Log[Cos[e + f*x]])/(a^3*f) + (((8*I)/3)*c^4)/(f*(a
+ I*a*Tan[e + f*x])^3) - ((6*I)*c^4)/(a*f*(a + I*a*Tan[e + f*x])^2) + ((6*I
)*c^4)/(f*(a^3 + I*a^3*Tan[e + f*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{(c-ic\tan(e+fx))^4}{(a+ia\tan(e+fx))^3}\,dx = \left(a^4c^4\right)\int \frac{\sec^8(e+fx)}{(a+ia\tan(e+fx))^7}\,dx$$
$$= -\frac{(ic^4)\operatorname{Subst}\left(\int \frac{(a-x)^3}{(a+x)^4}\,dx, x, ia\tan(e+fx)\right)}{a^3f}$$
$$= -\frac{(ic^4)\operatorname{Subst}\left(\int \left(\frac{1}{-a-x}+\frac{8a^3}{(a+x)^4}-\frac{12a^2}{(a+x)^3}+\frac{6a}{(a+x)^2}\right)dx, x, ia\tan(e+fx)\right)}{a^3f}$$
$$= -\frac{c^4x}{a^3}-\frac{ic^4\log(\cos(e+fx))}{a^3f}+\frac{8ic^4}{3f(a+ia\tan(e+fx))^3}-\frac{6ic^4}{af(a+ia\tan(e+$$

Mathematica [A]
time = 1.05, size = 121, normalized size = 1.06

$$\frac{c^4\sec^3(e+fx)(-3\cos(e+fx)+\cos(3(e+fx))(-2-6ifx+6\log(\cos(e+fx)))-9i\sin(e+fx)+2i\sin(3(e+fx))+6fx\sin(3(e+fx))+6i\log(\cos(e+fx))\sin(3(e+fx)))}{6a^3f(-i+\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - I*c*Tan[e + f*x])^4/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] (c^4*Sec[e + f*x]^3*(-3*Cos[e + f*x] + Cos[3*(e + f*x)]*(-2 - (6*I)*f*x + 6
*Log[Cos[e + f*x]]) - (9*I)*Sin[e + f*x] + (2*I)*Sin[3*(e + f*x)] + 6*f*x*S
in[3*(e + f*x)] + (6*I)*Log[Cos[e + f*x]]*Sin[3*(e + f*x)]))/(6*a^3*f*(-I +
 Tan[e + f*x])^3)
```

Maple [A]
time = 0.24, size = 65, normalized size = 0.57

method	result
derivativedivides	$\frac{c^4\left(\frac{6i}{(\tan(fx+e)-i)^2}-\frac{8}{3(\tan(fx+e)-i)^3}+i\ln(\tan(fx+e)-i)+\frac{6}{\tan(fx+e)-i}\right)}{f\,a^3}$
default	$\frac{c^4\left(\frac{6i}{(\tan(fx+e)-i)^2}-\frac{8}{3(\tan(fx+e)-i)^3}+i\ln(\tan(fx+e)-i)+\frac{6}{\tan(fx+e)-i}\right)}{f\,a^3}$
risch	$\frac{ic^4 e^{-2i(fx+e)}}{a^3f}-\frac{ic^4 e^{-4i(fx+e)}}{2a^3f}+\frac{ic^4 e^{-6i(fx+e)}}{3a^3f}-\frac{2c^4x}{a^3}-\frac{2c^4e}{a^3f}-\frac{ic^4\ln\left(e^{2i(fx+e)}+1\right)}{a^3f}$
norman	$\frac{\frac{4ic^4\left(\tan^2(fx+e)\right)}{af}-\frac{c^4x}{a}+\frac{8ic^4}{3af}-\frac{8c^4\left(\tan^3(fx+e)\right)}{3af}+\frac{6c^4\left(\tan^5(fx+e)\right)}{af}-\frac{3c^4x\left(\tan^2(fx+e)\right)}{a}-\frac{3c^4x\left(\tan^4(fx+e)\right)}{a}-\frac{c^4x\left(\tan^6(fx+\right.}{a}}{a^2(1+\tan^2(fx+e))^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*c^4/a^3*(6*I/(tan(f*x+e)-I)^2-8/3/(tan(f*x+e)-I)^3+I*ln(tan(f*x+e)-I)+6
/(tan(f*x+e)-I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.22, size = 99, normalized size = 0.87

$$-\frac{\left(12\,c^4 fxe^{(6i\,fx+6i\,e)}+6i\,c^4e^{(6i\,fx+6i\,e)}\log\left(e^{(2i\,fx+2i\,e)}+1\right)-6i\,c^4e^{(4i\,fx+4i\,e)}+3i\,c^4e^{(2i\,fx+2i\,e)}-2i\,c^4\right)e^{(-6i\,fx-6i\,e)}}{6\,a^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(12*c^4*f*x*e^(6*I*f*x + 6*I*e) + 6*I*c^4*e^(6*I*f*x + 6*I*e)*log(e^(2
*I*f*x + 2*I*e) + 1) - 6*I*c^4*e^(4*I*f*x + 4*I*e) + 3*I*c^4*e^(2*I*f*x + 2
*I*e) - 2*I*c^4)*e^(-6*I*f*x - 6*I*e)/(a^3*f)
```

Sympy [A]
time = 0.36, size = 212, normalized size = 1.86

$$\begin{cases}\frac{\left(6ia^6c^4f^2e^{10ie}e^{-2ifx}-3ia^6c^4f^2e^{8ie}e^{-4ifx}+2ia^6c^4f^2e^{6ie}e^{-6ifx}\right)e^{-12ie}}{6a^9f^3} & \text{for}\ a^9f^3e^{12ie}\neq 0\\ x\left(\frac{2c^4}{a^3}+\frac{\left(-2c^4e^{6ie}+2c^4e^{4ie}-2c^4e^{2ie}+2c^4\right)e^{-6ie}}{a^3}\right) & \text{otherwise}\end{cases}-\frac{2c^4x}{a^3}-\frac{ic^4\log\left(e^{2ifx}+e^{-2ie}\right)}{a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**4/(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] Piecewise((((6*I*a**6*c**4*f**2*exp(10*I*e)*exp(-2*I*f*x) - 3*I*a**6*c**4*f*
*2*exp(8*I*e)*exp(-4*I*f*x) + 2*I*a**6*c**4*f**2*exp(6*I*e)*exp(-6*I*f*x))*
exp(-12*I*e)/(6*a**9*f**3), Ne(a**9*f**3*exp(12*I*e), 0)), (x*(2*c**4/a**3
+ (-2*c**4*exp(6*I*e) + 2*c**4*exp(4*I*e) - 2*c**4*exp(2*I*e) + 2*c**4)*exp
(-6*I*e)/a**3), True)) - 2*c**4*x/a**3 - I*c**4*log(exp(2*I*f*x) + exp(-2*I
*e))/(a**3*f)
```

Giac [A]
time = 0.85, size = 193, normalized size = 1.69

$$-\frac{\frac{30i\,c^4\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}{a^3}-\frac{60i\,c^4\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-i\right)}{a^3}+\frac{30i\,c^4\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}{a^3}+\frac{147i\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+1002\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-2445i\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-3820\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+2445i\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1002\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-147i\,c^4}{a^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-i\right)^6}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/30*(30*I*c^4*log(tan(1/2*f*x + 1/2*e) + 1)/a^3 - 60*I*c^4*log(tan(1/2*f*
x + 1/2*e) - I)/a^3 + 30*I*c^4*log(tan(1/2*f*x + 1/2*e) - 1)/a^3 + (147*I*c
^4*tan(1/2*f*x + 1/2*e)^6 + 1002*c^4*tan(1/2*f*x + 1/2*e)^5 - 2445*I*c^4*ta
n(1/2*f*x + 1/2*e)^4 - 3820*c^4*tan(1/2*f*x + 1/2*e)^3 + 2445*I*c^4*tan(1/2
*f*x + 1/2*e)^2 + 1002*c^4*tan(1/2*f*x + 1/2*e) - 147*I*c^4)/(a^3*(tan(1/2*
f*x + 1/2*e) - I)^6))/f
```

Mupad [B]
time = 4.77, size = 103, normalized size = 0.90

$$-\frac{\frac{6\,c^4\,\tan(e+f\,x)}{a^3}-\frac{c^4\,8\mathrm{i}}{3\,a^3}+\frac{c^4\,{\tan(e+f\,x)}^2\,6\mathrm{i}}{a^3}}{f\,\left(-{\tan\left(e+f\,x\right)}^3\,1\mathrm{i}-3\,{\tan\left(e+f\,x\right)}^2+\tan\left(e+f\,x\right)\,3\mathrm{i}+1\right)}+\frac{c^4\,\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,1\mathrm{i}}{a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^4/(a + a*tan(e + f*x)*1i)^3,x)
```

```
[Out] (c^4*log(tan(e + f*x) - 1i)*1i)/(a^3*f) - ((6*c^4*tan(e + f*x))/a^3 - (c^4*
8i)/(3*a^3) + (c^4*tan(e + f*x)^2*6i)/a^3)/(f*(tan(e + f*x)*3i - 3*tan(e +
f*x)^2 - tan(e + f*x)^3*1i + 1))
```

3.922 $\int \frac{(c-ic\tan(e+fx))^4}{(a+ia\tan(e+fx))^4}\,dx$

Optimal. Leaf size=50

$$\frac{ic^4(a^2 - ia^2\tan(e + fx))^4}{8f\,(a^3 + ia^3\tan(e + fx))^4}$$

[Out] 1/8*I*c^4*(a^2-I*a^2*tan(f*x+e))^4/f/(a^3+I*a^3*tan(f*x+e))^4

Rubi [A]
time = 0.07, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 37}

$$\frac{ic^4(a^2 - ia^2\tan(e + fx))^4}{8f\,(a^3 + ia^3\tan(e + fx))^4}$$

Antiderivative was successfully verified.

[In] Int[(c - I*c*Tan[e + f*x])^4/(a + I*a*Tan[e + f*x])^4,x]

[Out] ((I/8)*c^4*(a^2 - I*a^2*Tan[e + f*x])^4)/(f*(a^3 + I*a^3*Tan[e + f*x])^4)

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{(c-ic\tan(e+fx))^4}{(a+ia\tan(e+fx))^4}\,dx = \left(a^4c^4\right)\int \frac{\sec^8(e+fx)}{(a+ia\tan(e+fx))^8}\,dx$$
$$= -\frac{(ic^4)\,\text{Subst}\left(\int \frac{(a-x)^3}{(a+x)^5}\,dx, x, ia\tan(e+fx)\right)}{a^3f}$$
$$= \frac{ic^4(1-i\tan(e+fx))^4}{8f(a+ia\tan(e+fx))^4}$$

Mathematica [A]
time = 0.28, size = 34, normalized size = 0.68

$$\frac{c^4(i\cos(8(e+fx))+\sin(8(e+fx)))}{8a^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - I*c*Tan[e + f*x])^4/(a + I*a*Tan[e + f*x])^4,x]
```

```
[Out] (c^4*(I*Cos[8*(e + f*x)] + Sin[8*(e + f*x)]))/(8*a^4*f)
```

Maple [A]
time = 0.23, size = 66, normalized size = 1.32

method	result
risch	$\frac{ic^4 e^{-8i(fx+e)}}{8a^4f}$
derivativedivides	$\frac{c^4\left(-\frac{1}{\tan(fx+e)-i}+\frac{2i}{(\tan(fx+e)-i)^4}-\frac{3i}{(\tan(fx+e)-i)^2}+\frac{4}{(\tan(fx+e)-i)^3}\right)}{f\,a^4}$
default	$\frac{c^4\left(-\frac{1}{\tan(fx+e)-i}+\frac{2i}{(\tan(fx+e)-i)^4}-\frac{3i}{(\tan(fx+e)-i)^2}+\frac{4}{(\tan(fx+e)-i)^3}\right)}{f\,a^4}$
norman	$\frac{\frac{c^4\tan(fx+e)}{af}-\frac{7c^4\left(\tan^3(fx+e)\right)}{af}+\frac{7c^4\left(\tan^5(fx+e)\right)}{af}-\frac{c^4\left(\tan^7(fx+e)\right)}{af}-\frac{4ic^4\left(\tan^2(fx+e)\right)}{af}-\frac{4ic^4\left(\tan^6(fx+e)\right)}{af}+\frac{8ic^4\left(\tan^4(fx+\right.}{af}}{\left(1+\tan^2(fx+e)\right)^4a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*c^4/a^4*(-1/(tan(f*x+e)-I)+2*I/(tan(f*x+e)-I)^4-3*I/(tan(f*x+e)-I)^2+4/
(tan(f*x+e)-I)^3)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.76, size = 21, normalized size = 0.42

$$\frac{i\,c^4e^{(-8i\,fx-8i\,e)}}{8\,a^4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/8*I*c^4*e^(-8*I*f*x - 8*I*e)/(a^4*f)
```

Sympy [A]
time = 0.26, size = 51, normalized size = 1.02

$$\begin{cases} \frac{ic^4e^{-8ie}e^{-8ifx}}{8a^4f} & \text{for}\ a^4fe^{8ie}\neq 0 \\ \frac{c^4xe^{-8ie}}{a^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**4/(a+I*a*tan(f*x+e))**4,x)
```

```
[Out] Piecewise((I*c**4*exp(-8*I*e)*exp(-8*I*f*x)/(8*a**4*f), Ne(a**4*f*exp(8*I*e
), 0)), (c**4*x*exp(-8*I*e)/a**4, True))
```

Giac [A]
time = 0.96, size = 88, normalized size = 1.76

$$-\frac{2\left(c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7-7\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+7\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^4f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-i\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -2*(c^4*tan(1/2*f*x + 1/2*e)^7 - 7*c^4*tan(1/2*f*x + 1/2*e)^5 + 7*c^4*tan(1
/2*f*x + 1/2*e)^3 - c^4*tan(1/2*f*x + 1/2*e))/(a^4*f*(tan(1/2*f*x + 1/2*e)
- I)^8)
```

Mupad [B]
time = 4.72, size = 76, normalized size = 1.52

$$-\frac{c^4\tan(e+f\,x)\,\left(\tan(e+f\,x)^2\,1\mathrm{i}-\mathrm{i}\right)}{a^4\,f\,\left(\tan\left(e+f\,x\right)^4\,1\mathrm{i}+4\tan\left(e+f\,x\right)^3-\tan\left(e+f\,x\right)^2\,6\mathrm{i}-4\tan\left(e+f\,x\right)+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - c*tan(e + f*x)*1i)^4/(a + a*tan(e + f*x)*1i)^4,x)

[Out] -(c^4*tan(e + f*x)*(tan(e + f*x)^2*1i - 1i))/(a^4*f*(4*tan(e + f*x)^3 - tan(e + f*x)^2*6i - 4*tan(e + f*x) + tan(e + f*x)^4*1i + 1i))

3.923 $\int \frac{(c-ic\tan(e+fx))^4}{(a+ia\tan(e+fx))^5}\,dx$

Optimal. Leaf size=87

$$\frac{ic^4(1-i\tan(e+fx))^4}{10f(a+ia\tan(e+fx))^5}+\frac{ic^4(a-ia\tan(e+fx))^4}{80a^5f(a+ia\tan(e+fx))^4}$$

```
[Out] 1/10*I*c^4*(1-I*tan(f*x+e))^4/f/(a+I*a*tan(f*x+e))^5+1/80*I*c^4*(a-I*a*tan(
f*x+e))^4/a^5/f/(a+I*a*tan(f*x+e))^4
```

Rubi [A]
time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {3603, 3568, 47, 37}

$$\frac{ic^4(a-ia\tan(e+fx))^4}{80a^5f(a+ia\tan(e+fx))^4}+\frac{ic^4(1-i\tan(e+fx))^4}{10f(a+ia\tan(e+fx))^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])^4/(a + I*a*Tan[e + f*x])^5,x]
```

```
[Out] ((I/10)*c^4*(1 - I*Tan[e + f*x])^4)/(f*(a + I*a*Tan[e + f*x])^5) + ((I/80)*
c^4*(a - I*a*Tan[e + f*x])^4)/(a^5*f*(a + I*a*Tan[e + f*x])^4)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
```

```
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] && !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{(c-ic\tan(e+fx))^4}{(a+ia\tan(e+fx))^5}\,dx = \left(a^4c^4\right)\int \frac{\sec^8(e+fx)}{(a+ia\tan(e+fx))^9}\,dx$$
$$= -\frac{(ic^4)\,\mathrm{Subst}\left(\int \frac{(a-x)^3}{(a+x)^6}\,dx, x, ia\tan(e+fx)\right)}{a^3f}$$
$$= \frac{ic^4(1-i\tan(e+fx))^4}{10f(a+ia\tan(e+fx))^5} - \frac{(ic^4)\,\mathrm{Subst}\left(\int \frac{(a-x)^3}{(a+x)^5}\,dx, x, ia\tan(e+fx)\right)}{10a^4f}$$
$$= \frac{ic^4(1-i\tan(e+fx))^4}{10f(a+ia\tan(e+fx))^5} + \frac{ic^4(a-ia\tan(e+fx))^4}{80a^5f(a+ia\tan(e+fx))^4}$$

Mathematica [A]
time = 1.01, size = 53, normalized size = 0.61

$$\frac{c^4(9\cos(e+fx)+i\sin(e+fx))(i\cos(9(e+fx))+\sin(9(e+fx)))}{80a^5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - I*c*Tan[e + f*x])^4/(a + I*a*Tan[e + f*x])^5,x]
[Out] (c^4*(9*Cos[e + f*x] + I*Sin[e + f*x])*(I*Cos[9*(e + f*x)] + Sin[9*(e + f*x
)]))/(80*a^5*f)
```

Maple [A]
time = 0.28, size = 66, normalized size = 0.76

method	result
risch	$\frac{ic^4 e^{-8i(fx+e)}}{16a^5f} + \frac{ic^4 e^{-10i(fx+e)}}{20a^5f}$
derivativedivides	$\frac{c^4\left(-\frac{3i}{(\tan(fx+e)-i)^4}-\frac{2}{(\tan(fx+e)-i)^3}+\frac{i}{2(\tan(fx+e)-i)^2}+\frac{8}{5(\tan(fx+e)-i)^5}\right)}{f\,a^5}$

default	$\frac{c^4\left(-\frac{3i}{(\tan(fx+e)-i)^4}-\frac{2}{(\tan(fx+e)-i)^3}+\frac{i}{2(\tan(fx+e)-i)^2}+\frac{8}{5(\tan(fx+e)-i)^5}\right)}{f\,a^5}$
norman	$\frac{\frac{c^4\tan(fx+e)}{af}-\frac{4ic^4\left(\tan^2(fx+e)\right)}{af}+\frac{13ic^4\left(\tan^4(fx+e)\right)}{af}+\frac{ic^4}{10af}-\frac{9c^4\left(\tan^3(fx+e)\right)}{af}+\frac{63c^4\left(\tan^5(fx+e)\right)}{5af}-\frac{3c^4\left(\tan^7(fx+e)\right)}{af}+\frac{ic}{}}{(1+\tan^2(fx+e))^5a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*c^4/a^5*(-3*I/(tan(f*x+e)-I)^4-2/(tan(f*x+e)-I)^3+1/2*I/(tan(f*x+e)-I)^
2+8/5/(tan(f*x+e)-I)^5)
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]

time = 1.14, size = 39, normalized size = 0.45

$$\frac{\left(5i\,c^4e^{(2i\,fx+2i\,e)}+4i\,c^4\right)e^{(-10i\,fx-10i\,e)}}{80\,a^5f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] 1/80*(5*I*c^4*e^(2*I*f*x + 2*I*e) + 4*I*c^4)*e^(-10*I*f*x - 10*I*e)/(a^5*f)
```

Sympy [A]

time = 0.36, size = 107, normalized size = 1.23

$$\begin{cases}\frac{\left(20ia^5c^4fe^{10ie}e^{-8ifx}+16ia^5c^4fe^{8ie}e^{-10ifx}\right)e^{-18ie}}{320a^{10}f^2} & \text{for } a^{10}f^2e^{18ie}\neq 0\\ \frac{x\left(c^4e^{2ie}+c^4\right)e^{-10ie}}{2a^5} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**4/(a+I*a*tan(f*x+e))**5,x)
```

```
[Out] Piecewise(((20*I*a**5*c**4*f*exp(10*I*e)*exp(-8*I*f*x) + 16*I*a**5*c**4*f*e
xp(8*I*e)*exp(-10*I*f*x))*exp(-18*I*e)/(320*a**10*f**2), Ne(a**10*f**2*exp(
18*I*e), 0)), (x*(c**4*exp(2*I*e) + c**4)*exp(-10*I*e)/(2*a**5), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(75) = 150.
time = 1.20, size = 174, normalized size = 2.00

$$-\frac{2\left(5\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9-5i\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8-50\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+35i\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+98\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-35i\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-50\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+5i\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+5\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{5\,a^5f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-i\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e))^5,x, algorithm="giac")
```

```
[Out] -2/5*(5*c^4*tan(1/2*f*x + 1/2*e)^9 - 5*I*c^4*tan(1/2*f*x + 1/2*e)^8 - 50*c^
4*tan(1/2*f*x + 1/2*e)^7 + 35*I*c^4*tan(1/2*f*x + 1/2*e)^6 + 98*c^4*tan(1/2
*f*x + 1/2*e)^5 - 35*I*c^4*tan(1/2*f*x + 1/2*e)^4 - 50*c^4*tan(1/2*f*x + 1/
2*e)^3 + 5*I*c^4*tan(1/2*f*x + 1/2*e)^2 + 5*c^4*tan(1/2*f*x + 1/2*e))/(a^5*
f*(tan(1/2*f*x + 1/2*e) - I)^10)
```

Mupad [B]
time = 4.87, size = 98, normalized size = 1.13

$$\frac{c^4\left(-5\tan(e+f\,x)^3-\tan(e+f\,x)^2\,5\mathrm{i}+5\tan(e+f\,x)+1\mathrm{i}\right)}{10\,a^5\,f\,\left(\tan\left(e+f\,x\right)^5\,1\mathrm{i}+5\tan\left(e+f\,x\right)^4-\tan\left(e+f\,x\right)^3\,10\mathrm{i}-10\tan\left(e+f\,x\right)^2+\tan\left(e+f\,x\right)\,5\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^4/(a + a*tan(e + f*x)*1i)^5,x)
```

```
[Out] (c^4*(5*tan(e + f*x) - tan(e + f*x)^2*5i - 5*tan(e + f*x)^3 + 1i))/(10*a^5*
f*(tan(e + f*x)*5i - 10*tan(e + f*x)^2 - tan(e + f*x)^3*10i + 5*tan(e + f*x
)^4 + tan(e + f*x)^5*1i + 1))
```

3.924 $\int \frac{(a+ia\tan(e+fx))^4}{c-ic\tan(e+fx)}\,dx$

Optimal. Leaf size=95

$$-\frac{12a^4x}{c}+\frac{12ia^4\log(\cos(e+fx))}{cf}+\frac{5a^4\tan(e+fx)}{cf}+\frac{ia^4\tan^2(e+fx)}{2cf}-\frac{8ia^4}{f(c-ic\tan(e+fx))}$$

```
[Out] -12*a^4*x/c+12*I*a^4*ln(cos(f*x+e))/c/f+5*a^4*tan(f*x+e)/c/f+1/2*I*a^4*tan(
f*x+e)^2/c/f-8*I*a^4/f/(c-I*c*tan(f*x+e))
```

Rubi [A]
time = 0.10, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 45}

$$\frac{ia^4\tan^2(e+fx)}{2cf}+\frac{5a^4\tan(e+fx)}{cf}-\frac{8ia^4}{f(c-ic\tan(e+fx))}+\frac{12ia^4\log(\cos(e+fx))}{cf}-\frac{12a^4x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^4/(c - I*c*Tan[e + f*x]),x]
```

```
[Out] (-12*a^4*x)/c + ((12*I)*a^4*Log[Cos[e + f*x]])/(c*f) + (5*a^4*Tan[e + f*x])
/(c*f) + ((I/2)*a^4*Tan[e + f*x]^2)/(c*f) - ((8*I)*a^4)/(f*(c - I*c*Tan[e +
 f*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+ia\tan(e+fx))^4}{c-ic\tan(e+fx)}\,dx &= \left(a^4c^4\right)\int \frac{\sec^8(e+fx)}{(c-ic\tan(e+fx))^5}\,dx\\
&= \frac{\left(ia^4\right)\operatorname{Subst}\left(\int \frac{(c-x)^3}{(c+x)^2}\,dx, x, -ic\tan(e+fx)\right)}{c^3f}\\
&= \frac{\left(ia^4\right)\operatorname{Subst}\left(\int \left(5c-x+\frac{8c^3}{(c+x)^2}-\frac{12c^2}{c+x}\right)\,dx, x, -ic\tan(e+fx)\right)}{c^3f}\\
&= -\frac{12a^4x}{c}+\frac{12ia^4\log(\cos(e+fx))}{cf}+\frac{5a^4\tan(e+fx)}{cf}+\frac{ia^4\tan^2(e+fx)}{2cf}-\frac{}{f(}
\end{aligned}
$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 376 vs. 2(95) = 190.
time = 1.60, size = 376, normalized size = 3.96

$$
-\frac{a^4\sec(e)\sec^2(e+fx)\left(-3i\cos(2e+3fx)+6fx\cos(2e+3fx)+2i\cos(4e+3fx)+6fx\cos(4e+3fx)+\cos(fx)\left(5i+18fx-9i\log\left(\cos^2(e+fx)\right)\right)+\cos(2e+fx)\left(10i+18fx-9i\log\left(\cos^2(e+fx)\right)\right)-3i\cos(2e+3fx)\log\left(\cos^2(e+fx)\right)-3i\cos(4e+3fx)\log\left(\cos^2(e+fx)\right)-13\sin(fx)-6ifx\sin(fx)-3\log\left(\cos^2(e+fx)\right)\sin(fx)+2\sin(2e+fx)-6ifx\sin(2e+fx)-3\log\left(\cos^2(e+fx)\right)\sin(2e+fx)-7\sin(2e+3fx)-6ifx\sin(2e+3fx)-3\log\left(\cos^2(e+fx)\right)\sin(2e+3fx)-2\sin(4e+3fx)-6ifx\sin(4e+3fx)-3\log\left(\cos^2(e+fx)\right)\sin(4e+3fx)\right)\left(\cos(e+5fx)+i\sin(e+5fx)\right)}{4cf(\cos(fx)+i\sin(fx))^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^4/(c - I*c*Tan[e + f*x]),x]
```

```
[Out] -1/4*(a^4*Sec[e]*Sec[e + f*x]^2*((-3*I)*Cos[2*e + 3*f*x] + 6*f*x*Cos[2*e +
3*f*x] + (2*I)*Cos[4*e + 3*f*x] + 6*f*x*Cos[4*e + 3*f*x] + Cos[f*x]*(5*I +
18*f*x - (9*I)*Log[Cos[e + f*x]^2]) + Cos[2*e + f*x]*(10*I + 18*f*x - (9*I)
*Log[Cos[e + f*x]^2]) - (3*I)*Cos[2*e + 3*f*x]*Log[Cos[e + f*x]^2] - (3*I)*
Cos[4*e + 3*f*x]*Log[Cos[e + f*x]^2] - 13*Sin[f*x] - (6*I)*f*x*Sin[f*x] - 3
*Log[Cos[e + f*x]^2]*Sin[f*x] + 2*Sin[2*e + f*x] - (6*I)*f*x*Sin[2*e + f*x]
 - 3*Log[Cos[e + f*x]^2]*Sin[2*e + f*x] - 7*Sin[2*e + 3*f*x] - (6*I)*f*x*Si
n[2*e + 3*f*x] - 3*Log[Cos[e + f*x]^2]*Sin[2*e + 3*f*x] - 2*Sin[4*e + 3*f*x
] - (6*I)*f*x*Sin[4*e + 3*f*x] - 3*Log[Cos[e + f*x]^2]*Sin[4*e + 3*f*x])*(C
os[e + 5*f*x] + I*Sin[e + 5*f*x]))/(c*f*(Cos[f*x] + I*Sin[f*x])^4)
```

Maple [**A**]
time = 0.19, size = 57, normalized size = 0.60

method	result
derivativedivides	$\frac{a^4\left(5\tan(fx+e)+\frac{i\left(\tan^2(fx+e)\right)}{2}+\frac{8}{\tan(fx+e)+i}-12i\ln(\tan(fx+e)+i)\right)}{fc}$
default	$\frac{a^4\left(5\tan(fx+e)+\frac{i\left(\tan^2(fx+e)\right)}{2}+\frac{8}{\tan(fx+e)+i}-12i\ln(\tan(fx+e)+i)\right)}{fc}$

risch	$-\frac{4ia^4 e^{2i(fx+e)}}{cf}+\frac{24a^4 e}{cf}+\frac{2ia^4\left(6\,e^{2i(fx+e)}+5\right)}{fc\left(e^{2i(fx+e)}+1\right)^2}+\frac{12ia^4\ln\left(e^{2i(fx+e)}+1\right)}{fc}$
norman	$\frac{-\frac{12a^4x}{c}-\frac{17ia^4}{2cf}-\frac{12a^4x\left(\tan^2(fx+e)\right)}{c}+\frac{13a^4\tan(fx+e)}{cf}+\frac{5a^4\left(\tan^3(fx+e)\right)}{cf}+\frac{ia^4\left(\tan^4(fx+e)\right)}{2cf}}{1+\tan^2(fx+e)}-\frac{6ia^4\ln\left(1+\tan^2(fx+e)\right)}{cf}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^4/(c-I*c*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a^4/c*(5*tan(f*x+e)+1/2*I*tan(f*x+e)^2+8/(tan(f*x+e)+I)-12*I*ln(tan(f*x
+e)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^4/(c-I*c*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.19, size = 137, normalized size = 1.44

$$-\frac{2\left(2i\,a^4e^{(6i\,fx+6i\,e)}+4i\,a^4e^{(4i\,fx+4i\,e)}-4i\,a^4e^{(2i\,fx+2i\,e)}-5i\,a^4+6\left(-i\,a^4e^{(4i\,fx+4i\,e)}-2i\,a^4e^{(2i\,fx+2i\,e)}-i\,a^4\right)\log\left(e^{(2i\,fx+2i\,e)}+1\right)\right)}{cfe^{(4i\,fx+4i\,e)}+2\,cfe^{(2i\,fx+2i\,e)}+cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^4/(c-I*c*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] -2*(2*I*a^4*e^(6*I*f*x + 6*I*e) + 4*I*a^4*e^(4*I*f*x + 4*I*e) - 4*I*a^4*e^(
2*I*f*x + 2*I*e) - 5*I*a^4 + 6*(-I*a^4*e^(4*I*f*x + 4*I*e) - 2*I*a^4*e^(2*I
*f*x + 2*I*e) - I*a^4)*log(e^(2*I*f*x + 2*I*e) + 1))/(c*f*e^(4*I*f*x + 4*I*
e) + 2*c*f*e^(2*I*f*x + 2*I*e) + c*f)
```

Sympy [A]
time = 0.25, size = 143, normalized size = 1.51

$$\frac{12ia^4\log\left(e^{2ifx}+e^{-2ie}\right)}{cf}+\frac{12ia^4e^{2ie}e^{2ifx}+10ia^4}{cfe^{4ie}e^{4ifx}+2cfe^{2ie}e^{2ifx}+cf}+\begin{cases}-\frac{4ia^4e^{2ie}e^{2ifx}}{cf} & \text{for } cf\neq 0\\ \frac{8a^4xe^{2ie}}{c} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**4/(c-I*c*tan(f*x+e)),x)
```

```
[Out] 12*I*a**4*log(exp(2*I*f*x) + exp(-2*I*e))/(c*f) + (12*I*a**4*exp(2*I*e)*exp
(2*I*f*x) + 10*I*a**4)/(c*f*exp(4*I*e)*exp(4*I*f*x) + 2*c*f*exp(2*I*e)*exp(
2*I*f*x) + c*f) + Piecewise((-4*I*a**4*exp(2*I*e)*exp(2*I*f*x)/(c*f), Ne(c*
f, 0)), (8*a**4*x*exp(2*I*e)/c, True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(89) = 178.
time = 0.65, size = 200, normalized size = 2.11

$$\frac{2\left(\frac{6i\,a^4\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}{c}-\frac{12i\,a^4\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+i\right)}{c}+\frac{6i\,a^4\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}{c}-\frac{13\,a^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+9i\,a^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-24\,a^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-9i\,a^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+13\,a^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+i\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-i\right)^2c}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^4/(c-I*c*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] 2*(6*I*a^4*log(tan(1/2*f*x + 1/2*e) + 1)/c - 12*I*a^4*log(tan(1/2*f*x + 1/2
*e) + I)/c + 6*I*a^4*log(tan(1/2*f*x + 1/2*e) - 1)/c - (13*a^4*tan(1/2*f*x
+ 1/2*e)^5 + 9*I*a^4*tan(1/2*f*x + 1/2*e)^4 - 24*a^4*tan(1/2*f*x + 1/2*e)^3
 - 9*I*a^4*tan(1/2*f*x + 1/2*e)^2 + 13*a^4*tan(1/2*f*x + 1/2*e))/((tan(1/2*
f*x + 1/2*e)^3 + I*tan(1/2*f*x + 1/2*e)^2 - tan(1/2*f*x + 1/2*e) - I)^2*c))
/f
```

Mupad [B]
time = 4.72, size = 82, normalized size = 0.86

$$\frac{5\,a^4\tan(e+f\,x)}{c\,f}+\frac{a^4\tan(e+f\,x)^2\,1\mathrm{i}}{2\,c\,f}+\frac{8\,a^4}{c\,f\,\left(\tan\left(e+f\,x\right)+1\mathrm{i}\right)}-\frac{a^4\,\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,12\mathrm{i}}{c\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^4/(c - c*tan(e + f*x)*1i),x)
```

```
[Out] (5*a^4*tan(e + f*x))/(c*f) + (a^4*tan(e + f*x)^2*1i)/(2*c*f) + (8*a^4)/(c*f
*(tan(e + f*x) + 1i)) - (a^4*log(tan(e + f*x) + 1i)*12i)/(c*f)
```

3.925 $\int \frac{(a+ia\tan(e+fx))^3}{c-ic\tan(e+fx)}\,dx$

Optimal. Leaf size=71

$$-\frac{4a^3x}{c}+\frac{4ia^3\log(\cos(e+fx))}{cf}+\frac{a^3\tan(e+fx)}{cf}-\frac{4ia^3}{f(c-ic\tan(e+fx))}$$

```
[Out] -4*a^3*x/c+4*I*a^3*ln(cos(f*x+e))/c/f+a^3*tan(f*x+e)/c/f-4*I*a^3/f/(c-I*c*t
an(f*x+e))
```

Rubi [A]
time = 0.09, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {3603, 3568, 45}

$$\frac{a^3\tan(e+fx)}{cf}-\frac{4ia^3}{f(c-ic\tan(e+fx))}+\frac{4ia^3\log(\cos(e+fx))}{cf}-\frac{4a^3x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^3/(c - I*c*Tan[e + f*x]),x]
```

```
[Out] (-4*a^3*x)/c + ((4*I)*a^3*Log[Cos[e + f*x]])/(c*f) + (a^3*Tan[e + f*x])/(c*
f) - ((4*I)*a^3)/(f*(c - I*c*Tan[e + f*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+ia\tan(e+fx))^3}{c-ic\tan(e+fx)}\,dx &= \left(a^3c^3\right)\int \frac{\sec^6(e+fx)}{(c-ic\tan(e+fx))^4}\,dx \\
&= \frac{\left(ia^3\right)\operatorname{Subst}\left(\int \frac{(c-x)^2}{(c+x)^2}\,dx, x, -ic\tan(e+fx)\right)}{c^2f} \\
&= \frac{\left(ia^3\right)\operatorname{Subst}\left(\int \left(1+\frac{4c^2}{(c+x)^2}-\frac{4c}{c+x}\right)\,dx, x, -ic\tan(e+fx)\right)}{c^2f} \\
&= -\frac{4a^3x}{c}+\frac{4ia^3\log(\cos(e+fx))}{cf}+\frac{a^3\tan(e+fx)}{cf}-\frac{4ia^3}{f(c-ic\tan(e+fx))}
\end{aligned}
$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 214 vs. $2(71) = 142$.
time = 1.43, size = 214, normalized size = 3.01

$$
\frac{a^3\sec(e)\left(\cos(3e+2fx)-2ifx\cos(3e+2fx)+\cos(e)\left(3-4ifx-2\log\left(\cos^2(e+fx)\right)\right)+\cos(e+2fx)\left(-2ifx-\log\left(\cos^2(e+fx)\right)\right)-\cos(3e+2fx)\log\left(\cos^2(e+fx)\right)-i\sin(e)+2i\sin(e+2fx)-2fx\sin(e+2fx)+i\log\left(\cos^2(e+fx)\right)\sin(e+2fx)+i\sin(3e+2fx)-2fx\sin(3e+2fx)+i\log\left(\cos^2(e+fx)\right)\sin(3e+2fx)\right)(-i+\tan(e+fx))}{2cf}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3/(c - I*c*Tan[e + f*x]),x]
```

```
[Out] (a^3*Sec[e]*(Cos[3*e + 2*f*x] - (2*I)*f*x*Cos[3*e + 2*f*x] + Cos[e]*(3 - (4
*I)*f*x - 2*Log[Cos[e + f*x]^2]) + Cos[e + 2*f*x]*((-2*I)*f*x - Log[Cos[e +
 f*x]^2]) - Cos[3*e + 2*f*x]*Log[Cos[e + f*x]^2] - I*Sin[e] + (2*I)*Sin[e +
 2*f*x] - 2*f*x*Sin[e + 2*f*x] + I*Log[Cos[e + f*x]^2]*Sin[e + 2*f*x] + I*S
in[3*e + 2*f*x] - 2*f*x*Sin[3*e + 2*f*x] + I*Log[Cos[e + f*x]^2]*Sin[3*e +
2*f*x])*(-I + Tan[e + f*x]))/(2*c*f)
```

Maple [**A**]
time = 0.17, size = 44, normalized size = 0.62

method	result	size
derivativedivides	$\frac{a^3\left(\tan(fx+e)-4i\ln(\tan(fx+e)+i)+\frac{4}{\tan(fx+e)+i}\right)}{fc}$	44
default	$\frac{a^3\left(\tan(fx+e)-4i\ln(\tan(fx+e)+i)+\frac{4}{\tan(fx+e)+i}\right)}{fc}$	44
risch	$-\frac{2ia^3e^{2i(fx+e)}}{cf}+\frac{8a^3e}{cf}+\frac{2ia^3}{fc\left(e^{2i(fx+e)}+1\right)}+\frac{4ia^3\ln\left(e^{2i(fx+e)}+1\right)}{fc}$	84
norman	$\frac{-\frac{4ia^3}{cf}+\frac{a^3\left(\tan^3(fx+e)\right)}{cf}-\frac{4a^3x}{c}-\frac{4a^3x\left(\tan^2(fx+e)\right)}{c}+\frac{5a^3\tan(fx+e)}{cf}}{1+\tan^2(fx+e)}-\frac{2ia^3\ln\left(1+\tan^2(fx+e)\right)}{cf}$	112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a^3/c*(tan(f*x+e)-4*I*ln(tan(f*x+e)+I)+4/(tan(f*x+e)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.46, size = 93, normalized size = 1.31

$$-\frac{2\left(i\,a^{3}e^{(4i\,fx+4i\,e)}+i\,a^{3}e^{(2i\,fx+2i\,e)}-i\,a^{3}+2\left(-i\,a^{3}e^{(2i\,fx+2i\,e)}-i\,a^{3}\right)\log\left(e^{(2i\,fx+2i\,e)}+1\right)\right)}{cfe^{(2i\,fx+2i\,e)}+cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] -2*(I*a^3*e^(4*I*f*x + 4*I*e) + I*a^3*e^(2*I*f*x + 2*I*e) - I*a^3 + 2*(-I*a
^3*e^(2*I*f*x + 2*I*e) - I*a^3)*log(e^(2*I*f*x + 2*I*e) + 1))/(c*f*e^(2*I*f
*x + 2*I*e) + c*f)
```

Sympy [A]
time = 0.21, size = 100, normalized size = 1.41

$$\frac{2ia^{3}}{cfe^{2ie}e^{2ifx}+cf}+\frac{4ia^{3}\log\left(e^{2ifx}+e^{-2ie}\right)}{cf}+\begin{cases}-\frac{2ia^{3}e^{2ie}e^{2ifx}}{cf} & \text{for}\ cf\neq 0\\ \frac{4a^{3}xe^{2ie}}{c} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3/(c-I*c*tan(f*x+e)),x)
```

```
[Out] 2*I*a**3/(c*f*exp(2*I*e)*exp(2*I*f*x) + c*f) + 4*I*a**3*log(exp(2*I*f*x) +
exp(-2*I*e))/(c*f) + Piecewise((-2*I*a**3*exp(2*I*e)*exp(2*I*f*x)/(c*f), Ne
(c*f, 0)), (4*a**3*x*exp(2*I*e)/c, True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(68) = 136$.

time = 0.60, size = 184, normalized size = 2.59

$$\frac{2\left(\frac{2i a^3 \log\left(\tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)+1\right)}{c} - \frac{4i a^3 \log\left(\tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)+i\right)}{c} + \frac{2i a^3 \log\left(\tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)-1\right)}{c} + \frac{-2i a^3 \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^2 - a^3 \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)+2i a^3}{\left(\tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^2-1\right)c} - \frac{2\left(-3i a^3 \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^2+8 a^3 \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)+3i a^3\right)}{c\left(\tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)+i\right)^2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] 2*(2*I*a^3*log(tan(1/2*f*x + 1/2*e) + 1)/c - 4*I*a^3*log(tan(1/2*f*x + 1/2*
e) + I)/c + 2*I*a^3*log(tan(1/2*f*x + 1/2*e) - 1)/c + (-2*I*a^3*tan(1/2*f*x
 + 1/2*e)^2 - a^3*tan(1/2*f*x + 1/2*e) + 2*I*a^3)/((tan(1/2*f*x + 1/2*e)^2
- 1)*c) - 2*(-3*I*a^3*tan(1/2*f*x + 1/2*e)^2 + 8*a^3*tan(1/2*f*x + 1/2*e) +
 3*I*a^3)/(c*(tan(1/2*f*x + 1/2*e) + I)^2))/f
```

Mupad [B]

time = 4.62, size = 61, normalized size = 0.86

$$\frac{a^3 \tan(e + f\,x)}{c\,f} + \frac{4\,a^3}{c\,f\,\left(\tan\left(e + f\,x\right) + 1\mathrm{i}\right)} - \frac{a^3\,\ln\left(\tan(e + f\,x) + 1\mathrm{i}\right)\,4\mathrm{i}}{c\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^3/(c - c*tan(e + f*x)*1i),x)
```

```
[Out] (a^3*tan(e + f*x))/(c*f) + (4*a^3)/(c*f*(tan(e + f*x) + 1i)) - (a^3*log(tan
(e + f*x) + 1i)*4i)/(c*f)
```

3.926 $\int \frac{(a+ia\tan(e+fx))^2}{c-ic\tan(e+fx)}\,dx$

Optimal. Leaf size=55

$$-\frac{a^2x}{c}+\frac{ia^2\log(\cos(e+fx))}{cf}-\frac{2ia^2}{f(c-ic\tan(e+fx))}$$

```
[Out] -a^2*x/c+I*a^2*ln(cos(f*x+e))/c/f-2*I*a^2/f/(c-I*c*tan(f*x+e))
```

Rubi [A]
time = 0.08, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}=0.097$, Rules used = {3603, 3568, 45}

$$-\frac{2ia^2}{f(c-ic\tan(e+fx))}+\frac{ia^2\log(\cos(e+fx))}{cf}-\frac{a^2x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2/(c - I*c*Tan[e + f*x]),x]
```

```
[Out] -((a^2*x)/c) + (I*a^2*Log[Cos[e + f*x]])/(c*f) - ((2*I)*a^2)/(f*(c - I*c*Ta
n[e + f*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^2}{c-ic\tan(e+fx)}\,dx=\left(a^2c^2\right)\int \frac{\sec^4(e+fx)}{(c-ic\tan(e+fx))^3}\,dx$$
$$=\frac{(ia^2)\,\mathrm{Subst}\left(\int \frac{c-x}{(c+x)^2}\,dx,x,-ic\tan(e+fx)\right)}{cf}$$
$$=\frac{(ia^2)\,\mathrm{Subst}\left(\int \left(\frac{1}{-c-x}+\frac{2c}{(c+x)^2}\right)\,dx,x,-ic\tan(e+fx)\right)}{cf}$$
$$=-\frac{a^2x}{c}+\frac{ia^2\log(\cos(e+fx))}{cf}-\frac{2ia^2}{f(c-ic\tan(e+fx))}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 130 vs. $2(55) = 110$.
time = 0.85, size = 130, normalized size = 2.36

$$-\frac{a^2(\cos(e+fx)\left(2i+4fx-i\log\left(\cos^2(e+fx)\right)\right)-2\mathrm{ArcTan}(\tan(3e+fx))(\cos(e+fx)-i\sin(e+fx))+\left(-2-4ifx-\log\left(\cos^2(e+fx)\right)\right)\sin(e+fx))\left(\cos(e+3fx)+i\sin(e+3fx)\right)}{2cf(\cos(fx)+i\sin(fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2/(c - I*c*Tan[e + f*x]),x]
```

```
[Out] -1/2*(a^2*(Cos[e + f*x]*(2*I + 4*f*x - I*Log[Cos[e + f*x]^2]) - 2*ArcTan[Ta
n[3*e + f*x]]*(Cos[e + f*x] - I*Sin[e + f*x]) + (-2 - (4*I)*f*x - Log[Cos[e
 + f*x]^2])*Sin[e + f*x])*(Cos[e + 3*f*x] + I*Sin[e + 3*f*x]))/(c*f*(Cos[f*
x] + I*Sin[f*x])^2)
```

Maple [A]
time = 0.16, size = 38, normalized size = 0.69

method	result	size
derivativedivides	$\frac{a^2\left(\frac{2}{\tan(fx+e)+i}-i\ln(\tan(fx+e)+i)\right)}{fc}$	38
default	$\frac{a^2\left(\frac{2}{\tan(fx+e)+i}-i\ln(\tan(fx+e)+i)\right)}{fc}$	38
risch	$-\frac{ia^2 e^{2i(fx+e)}}{cf}+\frac{2a^2e}{cf}+\frac{ia^2\ln\left(e^{2i(fx+e)}+1\right)}{cf}$	59
norman	$\frac{-\frac{2ia^2}{cf}-\frac{a^2x}{c}-\frac{a^2x\left(\tan^2(fx+e)\right)}{c}+\frac{2a^2\tan(fx+e)}{cf}}{1+\tan^2(fx+e)}-\frac{ia^2\ln\left(1+\tan^2(fx+e)\right)}{2cf}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a^2/c*(2/(tan(f*x+e)+I)-I*ln(tan(f*x+e)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.23, size = 41, normalized size = 0.75

$$\frac{-i\,a^2e^{(2i\,fx+2i\,e)}+i\,a^2\log\left(e^{(2i\,fx+2i\,e)}+1\right)}{cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] (-I*a^2*e^(2*I*f*x + 2*I*e) + I*a^2*log(e^(2*I*f*x + 2*I*e) + 1))/(c*f)
```

Sympy [A]
time = 0.19, size = 68, normalized size = 1.24

$$\frac{ia^2\log\left(e^{2ifx}+e^{-2ie}\right)}{cf}+\begin{cases}-\frac{ia^2e^{2ie}e^{2ifx}}{cf} & \text{for } cf\neq 0\\ \frac{2a^2xe^{2ie}}{c} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2/(c-I*c*tan(f*x+e)),x)
```

```
[Out] I*a**2*log(exp(2*I*f*x) + exp(-2*I*e))/(c*f) + Piecewise((-I*a**2*exp(2*I*e
)*exp(2*I*f*x)/(c*f), Ne(c*f, 0)), (2*a**2*x*exp(2*I*e)/c, True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(51) = 102.
time = 0.52, size = 125, normalized size = 2.27

$$-\frac{-\frac{i\,a^2\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}{c}+\frac{2i\,a^2\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+i\right)}{c}-\frac{i\,a^2\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}{c}+\frac{-3i\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+10\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+3i\,a^2}{c\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+i\right)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] -(-I*a^2*log(tan(1/2*f*x + 1/2*e) + 1)/c + 2*I*a^2*log(tan(1/2*f*x + 1/2*e)
 + I)/c - I*a^2*log(tan(1/2*f*x + 1/2*e) - 1)/c + (-3*I*a^2*tan(1/2*f*x + 1
/2*e)^2 + 10*a^2*tan(1/2*f*x + 1/2*e) + 3*I*a^2)/(c*(tan(1/2*f*x + 1/2*e) +
 I)^2))/f
```

Mupad [B]
time = 4.68, size = 45, normalized size = 0.82

$$\frac{2\,a^2}{c\,f\,\left(\tan\left(e+f\,x\right)+1{}\mathrm{i}\right)}-\frac{a^2\,\ln\left(\tan(e+f\,x)+1{}\mathrm{i}\right)\,1{}\mathrm{i}}{c\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^2/(c - c*tan(e + f*x)*1i),x)
```

```
[Out] (2*a^2)/(c*f*(tan(e + f*x) + 1i)) - (a^2*log(tan(e + f*x) + 1i)*1i)/(c*f)
```

3.927 $\int \frac{a+ia\tan(e+fx)}{c-ic\tan(e+fx)}\,dx$

Optimal. Leaf size=23

$$-\frac{ia}{f(c-ic\tan(e+fx))}$$

[Out] -I*a/f/(c-I*c*tan(f*x+e))

Rubi [A]
time = 0.06, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {3603, 3568, 32}

$$-\frac{ia}{f(c-ic\tan(e+fx))}$$

Antiderivative was successfully verified.

[In] Int[(a + I*a*Tan[e + f*x])/(c - I*c*Tan[e + f*x]),x]

[Out] ((-I)*a)/(f*(c - I*c*Tan[e + f*x]))

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{a+ia\tan(e+fx)}{c-ic\tan(e+fx)}\,dx = (ac)\int \frac{\sec^2(e+fx)}{(c-ic\tan(e+fx))^2}\,dx$$
$$= \frac{(ia)\mathrm{Subst}\left(\int \frac{1}{(c+x)^2}\,dx, x, -ic\tan(e+fx)\right)}{f}$$
$$= -\frac{ia}{f(c-ic\tan(e+fx))}$$

Mathematica [A]
time = 0.14, size = 32, normalized size = 1.39

$$\frac{a(-i\cos(2(e+fx))+\sin(2(e+fx)))}{2cf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])/(c - I*c*Tan[e + f*x]),x]
[Out] (a*((-I)*Cos[2*(e + f*x)] + Sin[2*(e + f*x)]))/(2*c*f)
```

Maple [A]
time = 0.15, size = 20, normalized size = 0.87

method	result	size
derivativedivides	$\frac{a}{fc(\tan(fx+e)+i)}$	20
default	$\frac{a}{fc(\tan(fx+e)+i)}$	20
risch	$-\frac{ia\,e^{2i(fx+e)}}{2cf}$	20
norman	$\frac{-\frac{ia}{cf}+\frac{a\tan(fx+e)}{cf}}{1+\tan^2(fx+e)}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e)),x,method=_RETURNVERBOSE)
[Out] 1/f*a/c/(tan(f*x+e)+I)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.05, size = 19, normalized size = 0.83

$$-\frac{i\,ae^{(2i\,fx+2i\,e)}}{2\,cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/2*I*a*e^(2*I*f*x + 2*I*e)/(c*f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(17) = 34$.
time = 0.08, size = 37, normalized size = 1.61

$$\begin{cases} -\frac{iae^{2ie}e^{2ifx}}{2cf} & \text{for } cf \neq 0 \\ \frac{axe^{2ie}}{c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e)),x)
```

```
[Out] Piecewise((-I*a*exp(2*I*e)*exp(2*I*f*x)/(2*c*f), Ne(c*f, 0)), (a*x*exp(2*I*
e)/c, True))
```

Giac [A]
time = 0.45, size = 33, normalized size = 1.43

$$-\frac{2\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{cf\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+i\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] -2*a*tan(1/2*f*x + 1/2*e)/(c*f*(tan(1/2*f*x + 1/2*e) + I)^2)
```

Mupad [B]
time = 4.70, size = 19, normalized size = 0.83

$$\frac{a}{c\,f\,\left(\tan\left(e+f\,x\right)+1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)/(c - c*tan(e + f*x)*1i),x)
```

```
[Out] a/(c*f*(tan(e + f*x) + 1i))
```

3.928 $\int \frac{1}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))}\,dx$

Optimal. Leaf size=37

$$\frac{x}{2ac}+\frac{\cos(e+fx)\sin(e+fx)}{2acf}$$

[Out] 1/2*x/a/c+1/2*cos(f*x+e)*sin(f*x+e)/a/c/f

Rubi [A]
time = 0.05, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 2715, 8}

$$\frac{\sin(e+fx)\cos(e+fx)}{2acf}+\frac{x}{2ac}$$

Antiderivative was successfully verified.

[In] Int[1/((a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*x])),x]

[Out] x/(2*a*c) + (Cos[e + f*x]*Sin[e + f*x])/(2*a*c*f)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))}\,dx = \frac{\int \cos^2(e+fx)\,dx}{ac}$$
$$= \frac{\cos(e+fx)\sin(e+fx)}{2acf} + \frac{\int 1\,dx}{2ac}$$
$$= \frac{x}{2ac} + \frac{\cos(e+fx)\sin(e+fx)}{2acf}$$

Mathematica [A]
time = 0.03, size = 29, normalized size = 0.78

$$\frac{2(e+fx)+\sin(2(e+fx))}{4acf}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*x])),x]
[Out] (2*(e + f*x) + Sin[2*(e + f*x)])/(4*a*c*f)
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 64, normalized size = 1.73

method	result	size
risch	$\frac{x}{2ac}+\frac{\sin(2fx+2e)}{4acf}$	31
norman	$\frac{\frac{x}{2ac}+\frac{\tan(fx+e)}{2acf}+\frac{x\left(\tan^2(fx+e)\right)}{2ac}}{1+\tan^2(fx+e)}$	58
derivativedivides	$\frac{\frac{i\ln(\tan(fx+e)+i)}{4}+\frac{1}{4\tan(fx+e)+4i}-\frac{i\ln(\tan(fx+e)-i)}{4}+\frac{1}{4\tan(fx+e)-4i}}{fac}$	64
default	$\frac{\frac{i\ln(\tan(fx+e)+i)}{4}+\frac{1}{4\tan(fx+e)+4i}-\frac{i\ln(\tan(fx+e)-i)}{4}+\frac{1}{4\tan(fx+e)-4i}}{fac}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e)),x,method=_RETURNVERBOSE)
[Out] 1/f/a/c*(1/4*I*ln(tan(f*x+e)+I)+1/4/(tan(f*x+e)+I)-1/4*I*ln(tan(f*x+e)-I)+1
/4/(tan(f*x+e)-I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [C] Result contains complex when optimal does not.
time = 1.42, size = 49, normalized size = 1.32

$$\frac{\left(4\,fxe^{(2i\,fx+2i\,e)}-i\,e^{(4i\,fx+4i\,e)}+i\right)e^{(-2i\,fx-2i\,e)}}{8\,acf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*f*x*e^(2*I*f*x + 2*I*e) - I*e^(4*I*f*x + 4*I*e) + I)*e^(-2*I*f*x - 2
*I*e)/(a*c*f)
```

Sympy [A]
time = 0.14, size = 117, normalized size = 3.16

$$\begin{cases} \frac{\left(-8iacfe^{4ie}e^{2ifx}+8iacfe^{-2ifx}\right)e^{-2ie}}{64a^2c^2f^2} & \text{for } a^2c^2f^2e^{2ie}\neq 0 \\ x\left(\frac{\left(e^{4ie}+2e^{2ie}+1\right)e^{-2ie}}{4ac}-\frac{1}{2ac}\right) & \text{otherwise} \end{cases}+\frac{x}{2ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e)),x)
```

```
[Out] Piecewise((((-8*I*a*c*f*exp(4*I*e)*exp(2*I*f*x) + 8*I*a*c*f*exp(-2*I*f*x))*e
xp(-2*I*e)/(64*a**2*c**2*f**2), Ne(a**2*c**2*f**2*exp(2*I*e), 0)), (x*((exp
(4*I*e) + 2*exp(2*I*e) + 1)*exp(-2*I*e)/(4*a*c) - 1/(2*a*c)), True)) + x/(2
*a*c)
```

Giac [A]
time = 0.48, size = 46, normalized size = 1.24

$$\frac{\frac{fx+e}{ac}+\frac{\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)ac}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*((f*x + e)/(a*c) + tan(f*x + e)/((tan(f*x + e)^2 + 1)*a*c))/f
```

Mupad [B]
time = 4.59, size = 32, normalized size = 0.86

$$\frac{\frac{\sin(2\,e+2\,f\,x)}{4\,a\,c}+\frac{f\,x}{2\,a\,c}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i)),x)
[Out] (sin(2*e + 2*f*x)/(4*a*c) + (f*x)/(2*a*c))/f
```

3.929 $\int \frac{1}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))}\,dx$

Optimal. Leaf size=87

$$\frac{3x}{8a^2c}+\frac{i\cos^4(e+fx)}{4a^2cf}+\frac{3\cos(e+fx)\sin(e+fx)}{8a^2cf}+\frac{\cos^3(e+fx)\sin(e+fx)}{4a^2cf}$$

```
[Out] 3/8*x/a^2/c+1/4*I*cos(f*x+e)^4/a^2/c/f+3/8*cos(f*x+e)*sin(f*x+e)/a^2/c/f+1/
4*cos(f*x+e)^3*sin(f*x+e)/a^2/c/f
```

Rubi [A]
time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {3603, 3567, 2715, 8}

$$\frac{i\cos^4(e+fx)}{4a^2cf}+\frac{\sin(e+fx)\cos^3(e+fx)}{4a^2cf}+\frac{3\sin(e+fx)\cos(e+fx)}{8a^2cf}+\frac{3x}{8a^2c}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])),x]
```

```
[Out] (3*x)/(8*a^2*c) + ((I/4)*Cos[e + f*x]^4)/(a^2*c*f) + (3*Cos[e + f*x]*Sin[e
+ f*x])/(8*a^2*c*f) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a^2*c*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
```

```
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))}\,dx = \frac{\int \cos^4(e+fx)(c-ic\tan(e+fx))\,dx}{a^2c^2}$$
$$= \frac{i\cos^4(e+fx)}{4a^2cf} + \frac{\int \cos^4(e+fx)\,dx}{a^2c}$$
$$= \frac{i\cos^4(e+fx)}{4a^2cf} + \frac{\cos^3(e+fx)\sin(e+fx)}{4a^2cf} + \frac{3\int \cos^2(e-$$
$$= \frac{i\cos^4(e+fx)}{4a^2cf} + \frac{3\cos(e+fx)\sin(e+fx)}{8a^2cf} + \frac{\cos^3(e+f}{4}$$
$$= \frac{3x}{8a^2c} + \frac{i\cos^4(e+fx)}{4a^2cf} + \frac{3\cos(e+fx)\sin(e+fx)}{8a^2cf} + \cos$$

Mathematica [A]
time = 0.77, size = 81, normalized size = 0.93

$$-\frac{-7+12ifx+2\cos(2(e+fx))+3i\sec(e+fx)\sin(3(e+fx))+6i\tan(e+fx)-12fx\tan(e+fx)}{32a^2cf(-i+\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])),x]
[Out] -1/32*(-7 + (12*I)*f*x + 2*Cos[2*(e + f*x)] + (3*I)*Sec[e + f*x]*Sin[3*(e +
 f*x)] + (6*I)*Tan[e + f*x] - 12*f*x*Tan[e + f*x])/(a^2*c*f*(-I + Tan[e + f
*x]))
```

Maple [A]
time = 0.19, size = 78, normalized size = 0.90

method	result	size
risch	$\frac{3x}{8a^2c}+\frac{ie^{-4i(fx+e)}}{32a^2cf}+\frac{i\cos(2fx+2e)}{8a^2cf}+\frac{\sin(2fx+2e)}{4a^2cf}$	73
derivativedivides	$\frac{-\frac{3i\ln(\tan(fx+e)-i)}{16}-\frac{i}{8(\tan(fx+e)-i)^2}+\frac{1}{4\tan(fx+e)-4i}+\frac{3i\ln(\tan(fx+e)+i)}{16}+\frac{1}{8\tan(fx+e)+8i}}{f\,a^2c}$	78
default	$\frac{-\frac{3i\ln(\tan(fx+e)-i)}{16}-\frac{i}{8(\tan(fx+e)-i)^2}+\frac{1}{4\tan(fx+e)-4i}+\frac{3i\ln(\tan(fx+e)+i)}{16}+\frac{1}{8\tan(fx+e)+8i}}{f\,a^2c}$	78
norman	$\frac{\frac{3x}{8ac}+\frac{5\tan(fx+e)}{8acf}+\frac{3\left(\tan^3(fx+e)\right)}{8acf}+\frac{3x\left(\tan^2(fx+e)\right)}{4ac}+\frac{3x\left(\tan^4(fx+e)\right)}{8ac}+\frac{i}{4acf}}{a\left(1+\tan^2(fx+e)\right)^2}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a^2/c*(-3/16*I*ln(tan(f*x+e)-I)-1/8*I/(tan(f*x+e)-I)^2+1/4/(tan(f*x+e)-
I)+3/16*I*ln(tan(f*x+e)+I)+1/8/(tan(f*x+e)+I))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 1.46, size = 61, normalized size = 0.70

$$\frac{\left(12\,fxe^{(4i\,fx+4i\,e)}-2i\,e^{(6i\,fx+6i\,e)}+6i\,e^{(2i\,fx+2i\,e)}+i\right)e^{(-4i\,fx-4i\,e)}}{32\,a^2cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/32*(12*f*x*e^(4*I*f*x + 4*I*e) - 2*I*e^(6*I*f*x + 6*I*e) + 6*I*e^(2*I*f*x
 + 2*I*e) + I)*e^(-4*I*f*x - 4*I*e)/(a^2*c*f)
```

Sympy [**A**]
time = 0.20, size = 178, normalized size = 2.05

$$\begin{cases}\frac{\left(-512ia^4c^2f^2e^{8ie}e^{2ifx}+1536ia^4c^2f^2e^{4ie}e^{-2ifx}+256ia^4c^2f^2e^{2ie}e^{-4ifx}\right)e^{-6ie}}{8192a^6c^3f^3} & \text{for}\: a^6c^3f^3e^{6ie}\neq 0\\x\left(\frac{\left(e^{6ie}+3e^{4ie}+3e^{2ie}+1\right)e^{-4ie}}{8a^2c}-\frac{3}{8a^2c}\right) & \text{otherwise}\end{cases}+\frac{3x}{8a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**2/(c-I*c*tan(f*x+e)),x)
```

```
[Out] Piecewise(((-512*I*a**4*c**2*f**2*exp(8*I*e)*exp(2*I*f*x) + 1536*I*a**4*c**
2*f**2*exp(4*I*e)*exp(-2*I*f*x) + 256*I*a**4*c**2*f**2*exp(2*I*e)*exp(-4*I*
f*x))*exp(-6*I*e)/(8192*a**6*c**3*f**3), Ne(a**6*c**3*f**3*exp(6*I*e), 0)),
 (x*((exp(6*I*e) + 3*exp(4*I*e) + 3*exp(2*I*e) + 1)*exp(-4*I*e)/(8*a**2*c)
- 3/(8*a**2*c)), True)) + 3*x/(8*a**2*c)
```

Giac [A]
time = 0.55, size = 118, normalized size = 1.36

$$-\frac{\frac{6i\log(i\tan(fx+e)+1)}{a^2c}-\frac{6i\log(i\tan(fx+e)-1)}{a^2c}+\frac{2(3\tan(fx+e)+5i)}{a^2c(-i\tan(fx+e)+1)}+\frac{-9i\tan(fx+e)^2-26\tan(fx+e)+21i}{a^2c(\tan(fx+e)-i)^2}}{32f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/32*(6*I*log(I*tan(f*x + e) + 1)/(a^2*c) - 6*I*log(I*tan(f*x + e) - 1)/(a
^2*c) + 2*(3*tan(f*x + e) + 5*I)/(a^2*c*(-I*tan(f*x + e) + 1)) + (-9*I*tan(
f*x + e)^2 - 26*tan(f*x + e) + 21*I)/(a^2*c*(tan(f*x + e) - I)^2))/f
```

Mupad [B]
time = 4.75, size = 66, normalized size = 0.76

$$\frac{3x}{8a^2c}-\frac{\frac{3\tan(e+fx)^2}{8}-\frac{\tan(e+fx)\,3\mathrm{i}}{8}+\frac{1}{4}}{a^2\,c\,f\,(1+\tan(e+f\,x)\,1\mathrm{i})^2\,(\tan(e+f\,x)+1\mathrm{i})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i)),x)
```

```
[Out] (3*x)/(8*a^2*c) - ((3*tan(e + f*x)^2)/8 - (tan(e + f*x)*3i)/8 + 1/4)/(a^2*c
*f*(tan(e + f*x)*1i + 1)^2*(tan(e + f*x) + 1i))
```

3.930 $\int \frac{1}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))}\,dx$

Optimal. Leaf size=124

$$\frac{x}{4a^3c}-\frac{i}{16a^3f(c-ic\tan(e+fx))}+\frac{ic^2}{12a^3f(c+ic\tan(e+fx))^3}+\frac{ic}{8a^3f(c+ic\tan(e+fx))^2}+\frac{3i}{16a^3f(c+ic\tan$$

```
[Out] 1/4*x/a^3/c-1/16*I/a^3/f/(c-I*c*tan(f*x+e))+1/12*I*c^2/a^3/f/(c+I*c*tan(f*x
+e))^3+1/8*I*c/a^3/f/(c+I*c*tan(f*x+e))^2+3/16*I/a^3/f/(c+I*c*tan(f*x+e))
```

Rubi [A]
time = 0.11, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {3603, 3568, 46, 212}

$$\frac{ic^2}{12a^3f(c+ic\tan(e+fx))^3}+\frac{ic}{8a^3f(c+ic\tan(e+fx))^2}-\frac{i}{16a^3f(c-ic\tan(e+fx))}+\frac{3i}{16a^3f(c+ic\tan(e+fx))}+\frac{x}{4a^3c}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])),x]
```

```
[Out] x/(4*a^3*c) - (I/16)/(a^3*f*(c - I*c*Tan[e + f*x])) + ((I/12)*c^2)/(a^3*f*(
c + I*c*Tan[e + f*x])^3) + ((I/8)*c)/(a^3*f*(c + I*c*Tan[e + f*x])^2) + ((3
*I)/16)/(a^3*f*(c + I*c*Tan[e + f*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] && !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))}\,dx = \frac{\int \cos^6(e+fx)(c-ic\tan(e+fx))^2\,dx}{a^3c^3}$$
$$= \frac{(ic^4)\,\text{Subst}\left(\int \frac{1}{(c-x)^4(c+x)^2}\,dx, x, -ic\tan(e+fx)\right)}{a^3f}$$
$$= \frac{(ic^4)\,\text{Subst}\left(\int \left(\frac{1}{4c^2(c-x)^4} + \frac{1}{4c^3(c-x)^3} + \frac{3}{16c^4(c-x)^2} + \frac{1}{16c^4(c+x}\right.\right.}{a^3f}$$
$$= -\frac{i}{16a^3f(c-ic\tan(e+fx))} + \frac{ic^2}{12a^3f(c+ic\tan(e+fx)}$$
$$= \frac{x}{4a^3c} - \frac{i}{16a^3f(c-ic\tan(e+fx))} + \frac{ic^2}{12a^3f(c+ic\tan(e}$$

Mathematica [A]
time = 0.81, size = 101, normalized size = 0.81

$$-\frac{\sec^2(e+fx)(9i+3(i+4fx)\cos(2(e+fx))-i\cos(4(e+fx))+3\sin(2(e+fx))+12ifx\sin(2(e+fx))+2\sin(4(e+fx)))}{48a^3cf(-i+\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])),x]
```

```
[Out] -1/48*(Sec[e + f*x]^2*(9*I + 3*(I + 4*f*x)*Cos[2*(e + f*x)] - I*Cos[4*(e +
f*x)] + 3*Sin[2*(e + f*x)] + (12*I)*f*x*Sin[2*(e + f*x)] + 2*Sin[4*(e + f*x
)]))/(a^3*c*f*(-I + Tan[e + f*x])^2)
```

Maple [A]
time = 0.21, size = 91, normalized size = 0.73

method	result	size
derivativedivides	$\frac{-\frac{i\ln(\tan(fx+e)-i)}{8}-\frac{i}{8(\tan(fx+e)-i)^2}-\frac{1}{12(\tan(fx+e)-i)^3}+\frac{3}{16(\tan(fx+e)-i)}+\frac{i\ln(\tan(fx+e)+i)}{8}+\frac{1}{16\tan(fx+e)+16i}}{f\,a^3c}$	91
default	$\frac{-\frac{i\ln(\tan(fx+e)-i)}{8}-\frac{i}{8(\tan(fx+e)-i)^2}-\frac{1}{12(\tan(fx+e)-i)^3}+\frac{3}{16(\tan(fx+e)-i)}+\frac{i\ln(\tan(fx+e)+i)}{8}+\frac{1}{16\tan(fx+e)+16i}}{f\,a^3c}$	91

risch	$\frac{x}{4a^3c}+\frac{ie^{-4i(fx+e)}}{16a^3cf}+\frac{ie^{-6i(fx+e)}}{96a^3cf}+\frac{5i\cos(2fx+2e)}{32a^3cf}+\frac{7\sin(2fx+2e)}{32a^3cf}$	94
norman	$\frac{\frac{x}{4ac}+\frac{3\tan(fx+e)}{4acf}+\frac{2\left(\tan^3(fx+e)\right)}{3acf}+\frac{\tan^5(fx+e)}{4acf}+\frac{3x\left(\tan^2(fx+e)\right)}{4ac}+\frac{3x\left(\tan^4(fx+e)\right)}{4ac}+\frac{x\left(\tan^6(fx+e)\right)}{4ac}+\frac{i}{3acf}}{(1+\tan^2(fx+e))^3a^2}$	145

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a^3/c*(-1/8*I*ln(tan(f*x+e)-I)-1/8*I/(tan(f*x+e)-I)^2-1/12/(tan(f*x+e)-
I)^3+3/16/(tan(f*x+e)-I)+1/8*I*ln(tan(f*x+e)+I)+1/16/(tan(f*x+e)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.17, size = 73, normalized size = 0.59

$$\frac{\left(24\,fxe^{(6i\,fx+6i\,e)}-3i\,e^{(8i\,fx+8i\,e)}+18i\,e^{(4i\,fx+4i\,e)}+6i\,e^{(2i\,fx+2i\,e)}+i\right)e^{(-6i\,fx-6i\,e)}}{96\,a^3cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/96*(24*f*x*e^(6*I*f*x + 6*I*e) - 3*I*e^(8*I*f*x + 8*I*e) + 18*I*e^(4*I*f*
x + 4*I*e) + 6*I*e^(2*I*f*x + 2*I*e) + I)*e^(-6*I*f*x - 6*I*e)/(a^3*c*f)
```

Sympy [A]
time = 0.24, size = 214, normalized size = 1.73

$$\begin{cases} \frac{\left(-24576ia^9c^3f^3e^{14ie}e^{2ifx}+147456ia^9c^3f^3e^{10ie}e^{-2ifx}+49152ia^9c^3f^3e^{8ie}e^{-4ifx}+8192ia^9c^3f^3e^{6ie}e^{-6ifx}\right)e^{-12ie}}{786432a^{12}c^4f^4} & \text{for } a^{12}c^4f^4e^{12ie}\neq 0 \\ x\left(\frac{\left(e^{8ie}+4e^{6ie}+6e^{4ie}+4e^{2ie}+1\right)e^{-6ie}}{16a^3c}-\frac{1}{4a^3c}\right) & \text{otherwise} \end{cases}+\frac{x}{4a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**3/(c-I*c*tan(f*x+e)),x)
```

```
[Out] Piecewise(((-24576*I*a**9*c**3*f**3*exp(14*I*e)*exp(2*I*f*x) + 147456*I*a**
9*c**3*f**3*exp(10*I*e)*exp(-2*I*f*x) + 49152*I*a**9*c**3*f**3*exp(8*I*e)*e
```

```
xp(-4*I*f*x) + 8192*I*a**9*c**3*f**3*exp(6*I*e)*exp(-6*I*f*x))*exp(-12*I*e)
/(786432*a**12*c**4*f**4), Ne(a**12*c**4*f**4*exp(12*I*e), 0)), (x*((exp(8*
I*e) + 4*exp(6*I*e) + 6*exp(4*I*e) + 4*exp(2*I*e) + 1)*exp(-6*I*e)/(16*a**3
*c) - 1/(4*a**3*c)), True)) + x/(4*a**3*c)
```

Giac [A]
time = 0.61, size = 123, normalized size = 0.99

$$-\frac{-\frac{6i\log(\tan(fx+e)+i)}{a^3c}+\frac{6i\log(\tan(fx+e)-i)}{a^3c}+\frac{3(2i\tan(fx+e)-3)}{a^3c(\tan(fx+e)+i)}+\frac{-11i\tan(fx+e)^3-42\tan(fx+e)^2+57i\tan(fx+e)+30}{a^3c(\tan(fx+e)-i)^3}}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/48*(-6*I*log(tan(f*x + e) + I)/(a^3*c) + 6*I*log(tan(f*x + e) - I)/(a^3*
c) + 3*(2*I*tan(f*x + e) - 3)/(a^3*c*(tan(f*x + e) + I)) + (-11*I*tan(f*x +
 e)^3 - 42*tan(f*x + e)^2 + 57*I*tan(f*x + e) + 30)/(a^3*c*(tan(f*x + e) -
I)^3))/f
```

Mupad [B]
time = 4.90, size = 77, normalized size = 0.62

$$\frac{x}{4\,a^3\,c}-\frac{\frac{\tan(e+f\,x)^3\,1\mathrm{i}}{4}+\frac{\tan(e+f\,x)^2}{2}-\frac{\tan(e+f\,x)\,1\mathrm{i}}{12}+\frac{1}{3}}{a^3\,c\,f\,\left(1+\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^3\,\left(\tan\left(e+f\,x\right)+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i)),x)
```

```
[Out] x/(4*a^3*c) - (tan(e + f*x)^2/2 - (tan(e + f*x)*1i)/12 + (tan(e + f*x)^3*1i
)/4 + 1/3)/(a^3*c*f*(tan(e + f*x)*1i + 1)^3*(tan(e + f*x) + 1i))
```

3.931 $\int \frac{(a+ia\tan(e+fx))^4}{(c-ic\tan(e+fx))^2}\,dx$

Optimal. Leaf size=101

$$\frac{6a^4x}{c^2}-\frac{6ia^4\log(\cos(e+fx))}{c^2f}-\frac{a^4\tan(e+fx)}{c^2f}-\frac{4ia^4}{f(c-ic\tan(e+fx))^2}+\frac{12ia^4}{f\left(c^2-ic^2\tan(e+fx)\right)}$$

```
[Out] 6*a^4*x/c^2-6*I*a^4*ln(cos(f*x+e))/c^2/f-a^4*tan(f*x+e)/c^2/f-4*I*a^4/f/(c-
I*c*tan(f*x+e))^2+12*I*a^4/f/(c^2-I*c^2*tan(f*x+e))
```

Rubi [A]
time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 45}

$$-\frac{a^4\tan(e+fx)}{c^2f}+\frac{12ia^4}{f\left(c^2-ic^2\tan(e+fx)\right)}-\frac{6ia^4\log(\cos(e+fx))}{c^2f}+\frac{6a^4x}{c^2}-\frac{4ia^4}{f(c-ic\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^4/(c - I*c*Tan[e + f*x])^2,x]
```

```
[Out] (6*a^4*x)/c^2 - ((6*I)*a^4*Log[Cos[e + f*x]])/(c^2*f) - (a^4*Tan[e + f*x])/
(c^2*f) - ((4*I)*a^4)/(f*(c - I*c*Tan[e + f*x])^2) + ((12*I)*a^4)/(f*(c^2 -
 I*c^2*Tan[e + f*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+ia\tan(e+fx))^4}{(c-ic\tan(e+fx))^2}\,dx &= \left(a^4c^4\right)\int \frac{\sec^8(e+fx)}{(c-ic\tan(e+fx))^6}\,dx\\
&= \frac{(ia^4)\,\mathrm{Subst}\left(\int \frac{(c-x)^3}{(c+x)^3}\,dx,x,-ic\tan(e+fx)\right)}{c^3f}\\
&= \frac{(ia^4)\,\mathrm{Subst}\left(\int \left(-1+\frac{8c^3}{(c+x)^3}-\frac{12c^2}{(c+x)^2}+\frac{6c}{c+x}\right)dx,x,-ic\tan(e+fx)\right)}{c^3f}\\
&= \frac{6a^4x}{c^2}-\frac{6ia^4\log(\cos(e+fx))}{c^2f}-\frac{a^4\tan(e+fx)}{c^2f}-\frac{4ia^4}{f(c-ic\tan(e+fx))^2}+
\end{aligned}
$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 374 vs. $2(101) = 202$.
time = 1.55, size = 374, normalized size = 3.70

$$
\frac{a^4\sec(e)\sec(e+fx)(\cos(2(e+3fx))+i\sin(2(e+3fx)))\left(-3i\cos(2e+3fx)+6fx\cos(2e+3fx)-i\cos(4e+3fx)+6fx\cos(4e+3fx)+\cos(fx)\left(7i+6fx-3i\log\left(\cos^2(e+fx)\right)\right)+\cos(2e+fx)\left(9i+6fx-3i\log\left(\cos^2(e+fx)\right)\right)-3i\cos(2e+3fx)\log\left(\cos^2(e+fx)\right)-3i\cos(4e+3fx)\log\left(\cos^2(e+fx)\right)+\sin(fx)-6ifx\sin(fx)-3\log\left(\cos^2(e+fx)\right)\sin(fx)+3\sin(2e+fx)-6ifx\sin(2e+fx)-3\log\left(\cos^2(e+fx)\right)\sin(2e+fx)-\sin(2e+3fx)-6ifx\sin(2e+3fx)-3\log\left(\cos^2(e+fx)\right)\sin(2e+3fx)+\sin(4e+3fx)-6ifx\sin(4e+3fx)-3\log\left(\cos^2(e+fx)\right)\sin(4e+3fx)\right)}{4c^2f(\cos(fx)+i\sin(fx))^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^4/(c - I*c*Tan[e + f*x])^2,x]
```

```
[Out] (a^4*Sec[e]*Sec[e + f*x]*(Cos[2*(e + 3*f*x)] + I*Sin[2*(e + 3*f*x)])*((-3*I
)*Cos[2*e + 3*f*x] + 6*f*x*Cos[2*e + 3*f*x] - I*Cos[4*e + 3*f*x] + 6*f*x*Co
s[4*e + 3*f*x] + Cos[f*x]*(7*I + 6*f*x - (3*I)*Log[Cos[e + f*x]^2]) + Cos[2
*e + f*x]*(9*I + 6*f*x - (3*I)*Log[Cos[e + f*x]^2]) - (3*I)*Cos[2*e + 3*f*x
]*Log[Cos[e + f*x]^2] - (3*I)*Cos[4*e + 3*f*x]*Log[Cos[e + f*x]^2] + Sin[f*
x] - (6*I)*f*x*Sin[f*x] - 3*Log[Cos[e + f*x]^2]*Sin[f*x] + 3*Sin[2*e + f*x]
 - (6*I)*f*x*Sin[2*e + f*x] - 3*Log[Cos[e + f*x]^2]*Sin[2*e + f*x] - Sin[2*
e + 3*f*x] - (6*I)*f*x*Sin[2*e + 3*f*x] - 3*Log[Cos[e + f*x]^2]*Sin[2*e + 3
*f*x] + Sin[4*e + 3*f*x] - (6*I)*f*x*Sin[4*e + 3*f*x] - 3*Log[Cos[e + f*x]^
2]*Sin[4*e + 3*f*x]))/(4*c^2*f*(Cos[f*x] + I*Sin[f*x])^4)
```

Maple [A]
time = 0.21, size = 60, normalized size = 0.59

method	result
derivativedivides	$\frac{a^4\left(-\tan(fx+e)+\frac{4i}{(\tan(fx+e)+i)^2}+6i\ln(\tan(fx+e)+i)-\frac{12}{\tan(fx+e)+i}\right)}{f\,c^2}$
default	$\frac{a^4\left(-\tan(fx+e)+\frac{4i}{(\tan(fx+e)+i)^2}+6i\ln(\tan(fx+e)+i)-\frac{12}{\tan(fx+e)+i}\right)}{f\,c^2}$
risch	$-\frac{ia^4e^{4i(fx+e)}}{c^2f}+\frac{4ia^4e^{2i(fx+e)}}{c^2f}-\frac{12a^4e}{f\,c^2}-\frac{2ia^4}{f\,c^2\left(e^{2i(fx+e)}+1\right)}-\frac{6ia^4\ln\left(e^{2i(fx+e)}+1\right)}{f\,c^2}$

norman	$\frac{\frac{8ia^4}{cf}+\frac{6a^4x}{c}+\frac{12a^4x\left(\tan^2(fx+e)\right)}{c}+\frac{6a^4x\left(\tan^4(fx+e)\right)}{c}-\frac{5a^4\tan(fx+e)}{cf}-\frac{14a^4\left(\tan^3(fx+e)\right)}{cf}-\frac{a^4\left(\tan^5(fx+e)\right)}{cf}+\frac{16ia^4\left(\tan^2(fx\right.}{cf}}{c(1+\tan^2(fx+e))^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^4/(c-I*c*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a^4/c^2*(-tan(f*x+e)+4*I/(tan(f*x+e)+I)^2+6*I*ln(tan(f*x+e)+I)-12/(tan(
f*x+e)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^4/(c-I*c*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.03, size = 111, normalized size = 1.10

$$\frac{-i\,a^4e^{(6i\,fx+6i\,e)}+3i\,a^4e^{(4i\,fx+4i\,e)}+4i\,a^4e^{(2i\,fx+2i\,e)}-2i\,a^4-6\left(i\,a^4e^{(2i\,fx+2i\,e)}+i\,a^4\right)\log\left(e^{(2i\,fx+2i\,e)}+1\right)}{c^2fe^{(2i\,fx+2i\,e)}+c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^4/(c-I*c*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] (-I*a^4*e^(6*I*f*x + 6*I*e) + 3*I*a^4*e^(4*I*f*x + 4*I*e) + 4*I*a^4*e^(2*I*
f*x + 2*I*e) - 2*I*a^4 - 6*(I*a^4*e^(2*I*f*x + 2*I*e) + I*a^4)*log(e^(2*I*f
*x + 2*I*e) + 1))/(c^2*f*e^(2*I*f*x + 2*I*e) + c^2*f)
```

Sympy [A]
time = 0.33, size = 155, normalized size = 1.53

$$-\frac{2ia^4}{c^2fe^{2ie}e^{2ifx}+c^2f}-\frac{6ia^4\log\left(e^{2ifx}+e^{-2ie}\right)}{c^2f}+\begin{cases}\frac{-ia^4c^2fe^{4ie}e^{4ifx}+4ia^4c^2fe^{2ie}e^{2ifx}}{c^4f^2} & \text{for } c^4f^2\neq 0\\ \frac{x\left(4a^4e^{4ie}-8a^4e^{2ie}\right)}{c^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**4/(c-I*c*tan(f*x+e))**2,x)
```

```
[Out] -2*I*a**4/(c**2*f*exp(2*I*e)*exp(2*I*f*x) + c**2*f) - 6*I*a**4*log(exp(2*I*
f*x) + exp(-2*I*e))/(c**2*f) + Piecewise(((-I*a**4*c**2*f*exp(4*I*e)*exp(4*
```

```
I*f*x) + 4*I*a**4*c**2*f*exp(2*I*e)*exp(2*I*f*x))/(c**4*f**2), Ne(c**4*f**2
, 0)), (x*(4*a**4*exp(4*I*e) - 8*a**4*exp(2*I*e))/c**2, True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(95) = 190$.
time = 0.71, size = 217, normalized size = 2.15

$$-\frac{\frac{6i\,a^4\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}{c^2}-\frac{12i\,a^4\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+i\right)}{c^2}+\frac{6i\,a^4\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}{c^2}-\frac{2\left(3i\,a^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-3i\,a^4\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)c^2}+\frac{25i\,a^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-108\,a^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-182i\,a^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+108\,a^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+25i\,a^4}{c^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+i\right)^4}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^4/(c-I*c*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -(6*I*a^4*log(tan(1/2*f*x + 1/2*e) + 1)/c^2 - 12*I*a^4*log(tan(1/2*f*x + 1/
2*e) + I)/c^2 + 6*I*a^4*log(tan(1/2*f*x + 1/2*e) - 1)/c^2 - 2*(3*I*a^4*tan(
1/2*f*x + 1/2*e)^2 + a^4*tan(1/2*f*x + 1/2*e) - 3*I*a^4)/((tan(1/2*f*x + 1/
2*e)^2 - 1)*c^2) + (25*I*a^4*tan(1/2*f*x + 1/2*e)^4 - 108*a^4*tan(1/2*f*x +
 1/2*e)^3 - 182*I*a^4*tan(1/2*f*x + 1/2*e)^2 + 108*a^4*tan(1/2*f*x + 1/2*e)
 + 25*I*a^4)/(c^2*(tan(1/2*f*x + 1/2*e) + I)^4))/f
```

Mupad [B]
time = 4.68, size = 90, normalized size = 0.89

$$-\frac{\frac{12\,a^4\,\tan(e+f\,x)}{c^2}+\frac{a^4\,8\mathrm{i}}{c^2}}{f\,\left({\tan\left(e+f\,x\right)}^2+\tan\left(e+f\,x\right)\,2\mathrm{i}-1\right)}-\frac{a^4\,\tan(e+f\,x)}{c^2\,f}+\frac{a^4\,\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,6\mathrm{i}}{c^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^4/(c - c*tan(e + f*x)*1i)^2,x)
```

```
[Out] (a^4*log(tan(e + f*x) + 1i)*6i)/(c^2*f) - (a^4*tan(e + f*x))/(c^2*f) - ((a^
4*8i)/c^2 + (12*a^4*tan(e + f*x))/c^2)/(f*(tan(e + f*x)*2i + tan(e + f*x)^2
 - 1))
```

3.932 $\int \frac{(a+ia\tan(e+fx))^3}{(c-ic\tan(e+fx))^2}\,dx$

Optimal. Leaf size=83

$$\frac{a^3x}{c^2}-\frac{ia^3\log(\cos(e+fx))}{c^2f}-\frac{2ia^3}{f(c-ic\tan(e+fx))^2}+\frac{4ia^3}{f\left(c^2-ic^2\tan(e+fx)\right)}$$

```
[Out] a^3*x/c^2-I*a^3*ln(cos(f*x+e))/c^2/f-2*I*a^3/f/(c-I*c*tan(f*x+e))^2+4*I*a^3
/f/(c^2-I*c^2*tan(f*x+e))
```

Rubi [A]
time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {3603, 3568, 45}

$$\frac{4ia^3}{f\left(c^2-ic^2\tan(e+fx)\right)}-\frac{ia^3\log(\cos(e+fx))}{c^2f}+\frac{a^3x}{c^2}-\frac{2ia^3}{f(c-ic\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^3/(c - I*c*Tan[e + f*x])^2,x]
```

```
[Out] (a^3*x)/c^2 - (I*a^3*Log[Cos[e + f*x]])/(c^2*f) - ((2*I)*a^3)/(f*(c - I*c*T
an[e + f*x])^2) + ((4*I)*a^3)/(f*(c^2 - I*c^2*Tan[e + f*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^3}{(c-ic\tan(e+fx))^2}\,dx = (a^3c^3)\int \frac{\sec^6(e+fx)}{(c-ic\tan(e+fx))^5}\,dx$$
$$= \frac{(ia^3)\,\text{Subst}\left(\int \frac{(c-x)^2}{(c+x)^3}\,dx, x, -ic\tan(e+fx)\right)}{c^2f}$$
$$= \frac{(ia^3)\,\text{Subst}\left(\int \left(\frac{4c^2}{(c+x)^3}-\frac{4c}{(c+x)^2}+\frac{1}{c+x}\right)\,dx, x, -ic\tan(e+fx)\right)}{c^2f}$$
$$= \frac{a^3x}{c^2}-\frac{ia^3\log(\cos(e+fx))}{c^2f}-\frac{2ia^3}{f(c-ic\tan(e+fx))^2}+\frac{4ia^3}{f\,(c^2-ic^2\tan(e+f$$

Mathematica [A]
time = 1.04, size = 113, normalized size = 1.36

$$\frac{a^3(2i+\cos(2(e+fx))\left(-i+2fx-i\log\left(\cos^2(e+fx)\right)\right)+\left(1-2ifx-\log\left(\cos^2(e+fx)\right)\right)\sin(2(e+fx)))\left(\cos(2e+5fx)+i\sin(2e+5fx)\right)}{2c^2f(\cos(fx)+i\sin(fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3/(c - I*c*Tan[e + f*x])^2,x]
[Out] (a^3*(2*I + Cos[2*(e + f*x)]*(-I + 2*f*x - I*Log[Cos[e + f*x]^2]) + (1 - (2
*I)*f*x - Log[Cos[e + f*x]^2])*Sin[2*(e + f*x)])*(Cos[2*e + 5*f*x] + I*Sin[
2*e + 5*f*x]))/(2*c^2*f*(Cos[f*x] + I*Sin[f*x])^3)
```

Maple [A]
time = 0.18, size = 52, normalized size = 0.63

method	result
derivativedivides	$\frac{a^3\left(\frac{2i}{(\tan(fx+e)+i)^2}+i\ln(\tan(fx+e)+i)-\frac{4}{\tan(fx+e)+i}\right)}{f\,c^2}$
default	$\frac{a^3\left(\frac{2i}{(\tan(fx+e)+i)^2}+i\ln(\tan(fx+e)+i)-\frac{4}{\tan(fx+e)+i}\right)}{f\,c^2}$
risch	$-\frac{ia^3e^{4i(fx+e)}}{2c^2f}+\frac{ia^3e^{2i(fx+e)}}{c^2f}-\frac{2a^3e}{c^2f}-\frac{ia^3\ln\left(e^{2i(fx+e)}+1\right)}{c^2f}$
norman	$\frac{\frac{a^3x}{c}+\frac{2ia^3}{cf}+\frac{a^3x\left(\tan^4(fx+e)\right)}{c}+\frac{2a^3x\left(\tan^2(fx+e)\right)}{c}-\frac{4a^3\left(\tan^3(fx+e)\right)}{cf}+\frac{6ia^3\left(\tan^2(fx+e)\right)}{cf}}{c(1+\tan^2(fx+e))^2}+\frac{ia^3\ln(1+\tan^2(fx+e))}{2c^2f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
[Out] 1/f*a^3/c^2*(2*I/(tan(f*x+e)+I)^2+I*ln(tan(f*x+e)+I)-4/(tan(f*x+e)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.99, size = 57, normalized size = 0.69

$$\frac{-i\,a^3 e^{(4i\,fx+4i\,e)} + 2i\,a^3 e^{(2i\,fx+2i\,e)} - 2i\,a^3 \log\left(e^{(2i\,fx+2i\,e)} + 1\right)}{2\,c^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(-I*a^3*e^(4*I*f*x + 4*I*e) + 2*I*a^3*e^(2*I*f*x + 2*I*e) - 2*I*a^3*log
(e^(2*I*f*x + 2*I*e) + 1))/(c^2*f)
```

Sympy [A]
time = 0.25, size = 122, normalized size = 1.47

$$-\frac{ia^3 \log\left(e^{2ifx} + e^{-2ie}\right)}{c^2 f} + \begin{cases} \frac{-ia^3c^2fe^{4ie}e^{4ifx} + 2ia^3c^2fe^{2ie}e^{2ifx}}{2c^4f^2} & \text{for } c^4f^2 \neq 0 \\ \frac{x\left(2a^3e^{4ie} - 2a^3e^{2ie}\right)}{c^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3/(c-I*c*tan(f*x+e))**2,x)
```

```
[Out] -I*a**3*log(exp(2*I*f*x) + exp(-2*I*e))/(c**2*f) + Piecewise(((-I*a**3*c**2
*f*exp(4*I*e)*exp(4*I*f*x) + 2*I*a**3*c**2*f*exp(2*I*e)*exp(2*I*f*x))/(2*c*
*4*f**2), Ne(c**4*f**2, 0)), (x*(2*a**3*exp(4*I*e) - 2*a**3*exp(2*I*e))/c**
2, True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(76) = 152.
time = 0.60, size = 159, normalized size = 1.92

$$-\frac{\frac{6i\,a^3 \log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}{c^2} - \frac{12i\,a^3 \log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+i\right)}{c^2} + \frac{6i\,a^3 \log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}{c^2} + \frac{25i\,a^3 \tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 - 100\,a^3 \tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 - 198i\,a^3 \tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 + 100\,a^3 \tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) + 25i\,a^3}{c^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+i\right)^4}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/6*(6*I*a^3*log(tan(1/2*f*x + 1/2*e) + 1)/c^2 - 12*I*a^3*log(tan(1/2*f*x
+ 1/2*e) + I)/c^2 + 6*I*a^3*log(tan(1/2*f*x + 1/2*e) - 1)/c^2 + (25*I*a^3*t
an(1/2*f*x + 1/2*e)^4 - 100*a^3*tan(1/2*f*x + 1/2*e)^3 - 198*I*a^3*tan(1/2*
f*x + 1/2*e)^2 + 100*a^3*tan(1/2*f*x + 1/2*e) + 25*I*a^3)/(c^2*(tan(1/2*f*x
 + 1/2*e) + I)^4))/f
```

Mupad [B]
time = 4.71, size = 73, normalized size = 0.88

$$-\frac{\frac{4\,a^3\,\tan(e+f\,x)}{c^2}+\frac{a^3\,2\mathrm{i}}{c^2}}{f\,\left({\tan\left(e+f\,x\right)}^2+\tan\left(e+f\,x\right)\,2\mathrm{i}-1\right)}+\frac{a^3\,\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{c^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^3/(c - c*tan(e + f*x)*1i)^2,x)
```

```
[Out] (a^3*log(tan(e + f*x) + 1i)*1i)/(c^2*f) - ((a^3*2i)/c^2 + (4*a^3*tan(e + f*
x))/c^2)/(f*(tan(e + f*x)*2i + tan(e + f*x)^2 - 1))
```

3.933 $\int \frac{(a+ia\tan(e+fx))^2}{(c-ic\tan(e+fx))^2}\,dx$

Optimal. Leaf size=28

$$\frac{a^2\tan(e+fx)}{f(c-ic\tan(e+fx))^2}$$

```
[Out] a^2*tan(f*x+e)/f/(c-I*c*tan(f*x+e))^2
```

Rubi [A]
time = 0.07, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 34}

$$\frac{a^2\tan(e+fx)}{f(c-ic\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2/(c - I*c*Tan[e + f*x])^2,x]
```

```
[Out] (a^2*Tan[e + f*x])/(f*(c - I*c*Tan[e + f*x])^2)
```

Rule 34

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[d*x*((
a + b*x)^(m + 1)/(b*(m + 2))), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^2}{(c-ic\tan(e+fx))^2}\,dx = (a^2c^2)\int \frac{\sec^4(e+fx)}{(c-ic\tan(e+fx))^4}\,dx$$
$$= \frac{(ia^2)\,\text{Subst}\left(\int \frac{c-x}{(c+x)^3}\,dx, x, -ic\tan(e+fx)\right)}{cf}$$
$$= \frac{a^2\tan(e+fx)}{f(c-ic\tan(e+fx))^2}$$

Mathematica [A]
time = 0.28, size = 34, normalized size = 1.21

$$\frac{a^2(-i\cos(4(e+fx))+\sin(4(e+fx)))}{4c^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2/(c - I*c*Tan[e + f*x])^2,x]
[Out] (a^2*((-I)*Cos[4*(e + f*x)] + Sin[4*(e + f*x)]))/(4*c^2*f)
```

Maple [A]
time = 0.18, size = 39, normalized size = 1.39

method	result	size
risch	$-\frac{ia^2 e^{4i(fx+e)}}{4c^2f}$	22
derivativedivides	$\frac{a^2\left(\frac{i}{(\tan(fx+e)+i)^2}-\frac{1}{\tan(fx+e)+i}\right)}{f\,c^2}$	39
default	$\frac{a^2\left(\frac{i}{(\tan(fx+e)+i)^2}-\frac{1}{\tan(fx+e)+i}\right)}{f\,c^2}$	39
norman	$\frac{\frac{a^2\tan(fx+e)}{cf}-\frac{a^2\left(\tan^3(fx+e)\right)}{cf}+\frac{2ia^2\left(\tan^2(fx+e)\right)}{cf}}{c\left(1+\tan^2(fx+e)\right)^2}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
[Out] 1/f*a^2/c^2*(I/(tan(f*x+e)+I)^2-1/(tan(f*x+e)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^2,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

Fricas [A]
time = 0.92, size = 21, normalized size = 0.75

$$-\frac{i\,a^2e^{(4i\,fx+4i\,e)}}{4\,c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^2,x, algorithm="fricas")

[Out] -1/4*I*a^2*e^(4*I*f*x + 4*I*e)/(c^2*f)

Sympy [A]
time = 0.13, size = 46, normalized size = 1.64

$$\begin{cases} -\frac{ia^2e^{4ie}e^{4ifx}}{4c^2f} & \text{for}\ c^2f \neq 0 \\ \frac{a^2xe^{4ie}}{c^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I*a*tan(f*x+e))**2/(c-I*c*tan(f*x+e))**2,x)

[Out] Piecewise((-I*a**2*exp(4*I*e)*exp(4*I*f*x)/(4*c**2*f), Ne(c**2*f, 0)), (a**2*x*exp(4*I*e)/c**2, True))

Giac [A]
time = 0.59, size = 54, normalized size = 1.93

$$-\frac{2\left(a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{c^2f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+i\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^2,x, algorithm="giac")

[Out] -2*(a^2*tan(1/2*f*x + 1/2*e)^3 - a^2*tan(1/2*f*x + 1/2*e))/(c^2*f*(tan(1/2*f*x + 1/2*e) + I)^4)

Mupad [B]
time = 4.74, size = 28, normalized size = 1.00

$$-\frac{a^2\tan(e+f\,x)}{c^2\,f\,\left(\tan\left(e+f\,x\right)+1\mathrm{i}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*tan(e + f*x)*1i)^2/(c - c*tan(e + f*x)*1i)^2,x)

[Out] -(a^2*tan(e + f*x))/(c^2*f*(tan(e + f*x) + 1i)^2)

3.934 $\int \frac{a+ia\tan(e+fx)}{(c-ic\tan(e+fx))^2}\,dx$

Optimal. Leaf size=25

$$-\frac{ia}{2f(c-ic\tan(e+fx))^2}$$

```
[Out] -1/2*I*a/f/(c-I*c*tan(f*x+e))^2
```

Rubi [A]
time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {3603, 3568, 32}

$$-\frac{ia}{2f(c-ic\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])/(c - I*c*Tan[e + f*x])^2,x]
```

```
[Out] ((-1/2*I)*a)/(f*(c - I*c*Tan[e + f*x])^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{a+ia\tan(e+fx)}{(c-ic\tan(e+fx))^2}\,dx = (ac)\int \frac{\sec^2(e+fx)}{(c-ic\tan(e+fx))^3}\,dx$$
$$= \frac{(ia)\text{Subst}\left(\int \frac{1}{(c+x)^3}\,dx, x, -ic\tan(e+fx)\right)}{f}$$
$$= -\frac{ia}{2f(c-ic\tan(e+fx))^2}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 51 vs. $2(25) = 50$.
time = 0.40, size = 51, normalized size = 2.04

$$\frac{a(3\cos(e+fx)-i\sin(e+fx))(-i\cos(3(e+fx))+\sin(3(e+fx)))}{8c^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])/(c - I*c*Tan[e + f*x])^2,x]
```

```
[Out] (a*(3*Cos[e + f*x] - I*Sin[e + f*x])*((-I)*Cos[3*(e + f*x)] + Sin[3*(e + f*
x)]))/(8*c^2*f)
```

Maple [**A**]
time = 0.17, size = 22, normalized size = 0.88

method	result	size
derivativedivides	$\frac{ia}{2f\,c^2(\tan(fx+e)+i)^2}$	22
default	$\frac{ia}{2f\,c^2(\tan(fx+e)+i)^2}$	22
risch	$-\frac{ia\,e^{4i(fx+e)}}{8c^2f}-\frac{ia\,e^{2i(fx+e)}}{4c^2f}$	40
norman	$\frac{\frac{a\tan(fx+e)}{cf}-\frac{ia}{2cf}+\frac{ia\left(\tan^2(fx+e)\right)}{2cf}}{c(1+\tan^2(fx+e))^2}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I/f*a/c^2/(tan(f*x+e)+I)^2
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.04, size = 35, normalized size = 1.40

$$\frac{-i\,ae^{(4i\,fx+4i\,e)}-2i\,ae^{(2i\,fx+2i\,e)}}{8\,c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(-I*a*e^(4*I*f*x + 4*I*e) - 2*I*a*e^(2*I*f*x + 2*I*e))/(c^2*f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(20) = 40.
time = 0.14, size = 88, normalized size = 3.52

$$\begin{cases}\frac{-4iac^2fe^{4ie}e^{4ifx}-8iac^2fe^{2ie}e^{2ifx}}{32c^4f^2} & \text{for } c^4f^2\neq 0\\ \frac{x\left(ae^{4ie}+ae^{2ie}\right)}{2c^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))**2,x)
```

```
[Out] Piecewise(((-4*I*a*c**2*f*exp(4*I*e)*exp(4*I*f*x) - 8*I*a*c**2*f*exp(2*I*e)
*exp(2*I*f*x))/(32*c**4*f**2), Ne(c**4*f**2, 0)), (x*(a*exp(4*I*e) + a*exp(
2*I*e))/(2*c**2), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(20) = 40.
time = 0.51, size = 65, normalized size = 2.60

$$-\frac{2\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+i\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{c^2f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+i\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -2*(a*tan(1/2*f*x + 1/2*e)^3 + I*a*tan(1/2*f*x + 1/2*e)^2 - a*tan(1/2*f*x +
 1/2*e))/(c^2*f*(tan(1/2*f*x + 1/2*e) + I)^4)
```

Mupad [B]
time = 4.61, size = 21, normalized size = 0.84

$$\frac{a\,1\mathrm{i}}{2\,c^2\,f\,{\left(\tan\left(e+f\,x\right)+1\mathrm{i}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)/(c - c*tan(e + f*x)*1i)^2,x)
[Out] (a*1i)/(2*c^2*f*(tan(e + f*x) + 1i)^2)
```

3.935 $\int \frac{1}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^2}\,dx$

Optimal. Leaf size=101

$$\frac{3x}{8ac^2}-\frac{i}{8af(c-ic\tan(e+fx))^2}-\frac{i}{4af\left(c^2-ic^2\tan(e+fx)\right)}+\frac{i}{8af\left(c^2+ic^2\tan(e+fx)\right)}$$

```
[Out] 3/8*x/a/c^2-1/8*I/a/f/(c-I*c*tan(f*x+e))^2-1/4*I/a/f/(c^2-I*c^2*tan(f*x+e))
+1/8*I/a/f/(c^2+I*c^2*tan(f*x+e))
```

Rubi [A]
time = 0.11, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {3603, 3568, 46, 212}

$$-\frac{i}{4af\left(c^2-ic^2\tan(e+fx)\right)}+\frac{i}{8af\left(c^2+ic^2\tan(e+fx)\right)}+\frac{3x}{8ac^2}-\frac{i}{8af(c-ic\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^2),x]
```

```
[Out] (3*x)/(8*a*c^2) - (I/8)/(a*f*(c - I*c*Tan[e + f*x])^2) - (I/4)/(a*f*(c^2 -
I*c^2*Tan[e + f*x])) + (I/8)/(a*f*(c^2 + I*c^2*Tan[e + f*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
```

```
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^2}\,dx &= \frac{\int \frac{\cos^2(e+fx)}{c-ic\tan(e+fx)}\,dx}{ac}\\
&= \frac{(ic^2)\,\text{Subst}\left(\int \frac{1}{(c-x)^2(c+x)^3}\,dx, x, -ic\tan(e+fx)\right)}{af}\\
&= \frac{(ic^2)\,\text{Subst}\left(\int \left(\frac{1}{8c^3(c-x)^2}+\frac{1}{4c^2(c+x)^3}+\frac{1}{4c^3(c+x)^2}+\frac{3}{8c^3(c^2-x^2)}\right.\right.}{af}\\
&= -\frac{i}{8af(c-ic\tan(e+fx))^2}-\frac{i}{4af\,(c^2-ic^2\tan(e+fx))}\\
&= \frac{3x}{8ac^2}-\frac{i}{8af(c-ic\tan(e+fx))^2}-\frac{i}{4af\,(c^2-ic^2\tan(e+
\end{aligned}$$

Mathematica [A]
time = 0.66, size = 102, normalized size = 1.01

$$-\frac{(\cos(2(e+fx))+i\sin(2(e+fx)))(7+12ifx-2\cos(2(e+fx))+3i\sec(e+fx)\sin(3(e+fx))+6i\tan(e+fx)+12fx\tan(e+fx))}{32ac^2f(-i+\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^2),x]
```

```
[Out] -1/32*((Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)])*(7 + (12*I)*f*x - 2*Cos[2*(e
 + f*x)] + (3*I)*Sec[e + f*x]*Sin[3*(e + f*x)] + (6*I)*Tan[e + f*x] + 12*f*
x*Tan[e + f*x]))/(a*c^2*f*(-I + Tan[e + f*x]))
```

Maple [A]
time = 0.17, size = 78, normalized size = 0.77

method	result	size
risch	$\frac{3x}{8a\,c^2}-\frac{i e^{4i(fx+e)}}{32c^2af}-\frac{i\cos(2fx+2e)}{8c^2af}+\frac{\sin(2fx+2e)}{4c^2af}$	73
derivativedivides	$\frac{-\frac{3i\ln(\tan(fx+e)-i)}{16}+\frac{1}{8\tan(fx+e)-8i}+\frac{i}{8(\tan(fx+e)+i)^2}+\frac{3i\ln(\tan(fx+e)+i)}{16}+\frac{1}{4\tan(fx+e)+4i}}{fa\,c^2}$	78
default	$\frac{-\frac{3i\ln(\tan(fx+e)-i)}{16}+\frac{1}{8\tan(fx+e)-8i}+\frac{i}{8(\tan(fx+e)+i)^2}+\frac{3i\ln(\tan(fx+e)+i)}{16}+\frac{1}{4\tan(fx+e)+4i}}{fa\,c^2}$	78

norman	$\frac{\frac{3x}{8ac}+\frac{5\tan(fx+e)}{8acf}+\frac{3\left(\tan^3(fx+e)\right)}{8acf}+\frac{3x\left(\tan^2(fx+e)\right)}{4ac}+\frac{3x\left(\tan^4(fx+e)\right)}{8ac}-\frac{i}{4acf}}{\left(1+\tan^2(fx+e)\right)^2 c}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a/c^2*(-3/16*I*ln(tan(f*x+e)-I)+1/8/(tan(f*x+e)-I)+1/8*I/(tan(f*x+e)+I)
^2+3/16*I*ln(tan(f*x+e)+I)+1/4/(tan(f*x+e)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.05, size = 61, normalized size = 0.60

$$\frac{\left(12\,fxe^{(2i\,fx+2i\,e)}-i\,e^{(6i\,fx+6i\,e)}-6i\,e^{(4i\,fx+4i\,e)}+2i\right)e^{(-2i\,fx-2i\,e)}}{32\,ac^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/32*(12*f*x*e^(2*I*f*x + 2*I*e) - I*e^(6*I*f*x + 6*I*e) - 6*I*e^(4*I*f*x +
 4*I*e) + 2*I)*e^(-2*I*f*x - 2*I*e)/(a*c^2*f)
```

Sympy [A]
time = 0.19, size = 172, normalized size = 1.70

$$\begin{cases}\frac{\left(-256ia^2c^4f^2e^{6ie}e^{4ifx}-1536ia^2c^4f^2e^{4ie}e^{2ifx}+512ia^2c^4f^2e^{-2ifx}\right)e^{-2ie}}{8192a^3c^6f^3} & \text{for } a^3c^6f^3e^{2ie}\neq 0\\ x\left(\frac{\left(e^{6ie}+3e^{4ie}+3e^{2ie}+1\right)e^{-2ie}}{8ac^2}-\frac{3}{8ac^2}\right) & \text{otherwise}\end{cases}+\frac{3x}{8ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))**2,x)
```

```
[Out] Piecewise(((-256*I*a**2*c**4*f**2*exp(6*I*e)*exp(4*I*f*x) - 1536*I*a**2*c**
4*f**2*exp(4*I*e)*exp(2*I*f*x) + 512*I*a**2*c**4*f**2*exp(-2*I*f*x))*exp(-2
*I*e)/(8192*a**3*c**6*f**3), Ne(a**3*c**6*f**3*exp(2*I*e), 0)), (x*((exp(6*
```

```
I*e) + 3*exp(4*I*e) + 3*exp(2*I*e) + 1)*exp(-2*I*e)/(8*a*c**2) - 3/(8*a*c**
2)), True)) + 3*x/(8*a*c**2)
```

Giac [A]
time = 0.54, size = 118, normalized size = 1.17

$$-\frac{-\frac{6i\log(-i\tan(fx+e)+1)}{ac^2}+\frac{6i\log(-i\tan(fx+e)-1)}{ac^2}+\frac{2(3\tan(fx+e)-5i)}{ac^2(i\tan(fx+e)+1)}+\frac{9i\tan(fx+e)^2-26\tan(fx+e)-21i}{ac^2(\tan(fx+e)+i)^2}}{32f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/32*(-6*I*log(-I*tan(f*x + e) + 1)/(a*c^2) + 6*I*log(-I*tan(f*x + e) - 1)
/(a*c^2) + 2*(3*tan(f*x + e) - 5*I)/(a*c^2*(I*tan(f*x + e) + 1)) + (9*I*tan
(f*x + e)^2 - 26*tan(f*x + e) - 21*I)/(a*c^2*(tan(f*x + e) + I)^2))/f
```

Mupad [B]
time = 4.88, size = 66, normalized size = 0.65

$$\frac{3x}{8ac^2}+\frac{\frac{\tan(e+fx)^2\,3\mathrm{i}}{8}-\frac{3\tan(e+fx)}{8}+\frac{1}{4}\mathrm{i}}{a\,c^2\,f\,\left(1+\tan\left(e+f\,x\right)\,1\mathrm{i}\right)\,{\left(\tan\left(e+f\,x\right)+1\mathrm{i}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i)^2),x)
```

```
[Out] (3*x)/(8*a*c^2) + ((tan(e + f*x)^2*3i)/8 - (3*tan(e + f*x))/8 + 1i/4)/(a*c^
2*f*(tan(e + f*x)*1i + 1)*(tan(e + f*x) + 1i)^2)
```

3.936 $\int \frac{1}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^2}\,dx$

Optimal. Leaf size=64

$$\frac{3x}{8a^2c^2}+\frac{3\cos(e+fx)\sin(e+fx)}{8a^2c^2f}+\frac{\cos^3(e+fx)\sin(e+fx)}{4a^2c^2f}$$

```
[Out] 3/8*x/a^2/c^2+3/8*cos(f*x+e)*sin(f*x+e)/a^2/c^2/f+1/4*cos(f*x+e)^3*sin(f*x+
e)/a^2/c^2/f
```

Rubi [A]
time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {3603, 2715, 8}

$$\frac{\sin(e+fx)\cos^3(e+fx)}{4a^2c^2f}+\frac{3\sin(e+fx)\cos(e+fx)}{8a^2c^2f}+\frac{3x}{8a^2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^2),x]
```

```
[Out] (3*x)/(8*a^2*c^2) + (3*Cos[e + f*x]*Sin[e + f*x])/(8*a^2*c^2*f) + (Cos[e +
f*x]^3*Sin[e + f*x])/(4*a^2*c^2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^2}\,dx = \frac{\int \cos^4(e+fx)\,dx}{a^2c^2}$$
$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4a^2c^2f} + \frac{3\int \cos^2(e+fx)\,dx}{4a^2c^2}$$
$$= \frac{3\cos(e+fx)\sin(e+fx)}{8a^2c^2f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4a^2c^2f} + \cdots$$
$$= \frac{3x}{8a^2c^2} + \frac{3\cos(e+fx)\sin(e+fx)}{8a^2c^2f} + \frac{\cos^3(e+fx)\sin(e+\cdots}{4a^2c^2f}$$

Mathematica [A]
time = 0.06, size = 39, normalized size = 0.61

$$\frac{12(e+fx)+8\sin(2(e+fx))+\sin(4(e+fx))}{32a^2c^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^2),x]
[Out] (12*(e + f*x) + 8*Sin[2*(e + f*x)] + Sin[4*(e + f*x)])/(32*a^2*c^2*f)
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 92, normalized size = 1.44

method	result	size
risch	$\frac{3x}{8a^2c^2}+\frac{\sin(4fx+4e)}{32a^2c^2f}+\frac{\sin(2fx+2e)}{4a^2c^2f}$	51
derivativedivides	$\frac{\frac{i}{16(\tan(fx+e)+i)^2}+\frac{3i\ln(\tan(fx+e)+i)}{16}+\frac{3}{16(\tan(fx+e)+i)}-\frac{3i\ln(\tan(fx+e)-i)}{16}-\frac{i}{16(\tan(fx+e)-i)^2}+\frac{3}{16(\tan(fx+e)-i)}}{f\,a^2c^2}$	92
default	$\frac{\frac{i}{16(\tan(fx+e)+i)^2}+\frac{3i\ln(\tan(fx+e)+i)}{16}+\frac{3}{16(\tan(fx+e)+i)}-\frac{3i\ln(\tan(fx+e)-i)}{16}-\frac{i}{16(\tan(fx+e)-i)^2}+\frac{3}{16(\tan(fx+e)-i)}}{f\,a^2c^2}$	92
norman	$\frac{\frac{3x}{8ac}+\frac{5\tan(fx+e)}{8acf}+\frac{3\left(\tan^3(fx+e)\right)}{8acf}+\frac{3x\left(\tan^2(fx+e)\right)}{4ac}+\frac{3x\left(\tan^4(fx+e)\right)}{8ac}}{(1+\tan^2(fx+e))^2ac}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
[Out] 1/f/a^2/c^2*(1/16*I/(tan(f*x+e)+I)^2+3/16*I*ln(tan(f*x+e)+I)+3/16/(tan(f*x+
e)+I)-3/16*I*ln(tan(f*x+e)-I)-1/16*I/(tan(f*x+e)-I)^2+3/16/(tan(f*x+e)-I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [C] Result contains complex when optimal does not.
time = 1.29, size = 73, normalized size = 1.14

$$\frac{\left(24\,fxe^{(4i\,fx+4i\,e)}-i\,e^{(8i\,fx+8i\,e)}-8i\,e^{(6i\,fx+6i\,e)}+8i\,e^{(2i\,fx+2i\,e)}+i\right)e^{(-4i\,fx-4i\,e)}}{64\,a^2c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] 1/64*(24*f*x*e^(4*I*f*x + 4*I*e) - I*e^(8*I*f*x + 8*I*e) - 8*I*e^(6*I*f*x +
 6*I*e) + 8*I*e^(2*I*f*x + 2*I*e) + I)*e^(-4*I*f*x - 4*I*e)/(a^2*c^2*f)
```

Sympy [A]
time = 0.25, size = 221, normalized size = 3.45

$$\begin{cases}\frac{\left(-4096ia^6c^6f^3e^{10ie}e^{4ifx}-32768ia^6c^6f^3e^{8ie}e^{2ifx}+32768ia^6c^6f^3e^{4ie}e^{-2ifx}+4096ia^6c^6f^3e^{2ie}e^{-4ifx}\right)e^{-6ie}}{262144a^8c^8f^4} & \text{for } a^8c^8f^4e^{6ie}\neq 0\\ x\left(\frac{\left(e^{8ie}+4e^{6ie}+6e^{4ie}+4e^{2ie}+1\right)e^{-4ie}}{16a^2c^2}-\frac{3}{8a^2c^2}\right) & \text{otherwise}\end{cases}+\frac{3x}{8a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**2/(c-I*c*tan(f*x+e))**2,x)
```

```
[Out] Piecewise(((-4096*I*a**6*c**6*f**3*exp(10*I*e)*exp(4*I*f*x) - 32768*I*a**6*
c**6*f**3*exp(8*I*e)*exp(2*I*f*x) + 32768*I*a**6*c**6*f**3*exp(4*I*e)*exp(-
2*I*f*x) + 4096*I*a**6*c**6*f**3*exp(2*I*e)*exp(-4*I*f*x))*exp(-6*I*e)/(262
144*a**8*c**8*f**4), Ne(a**8*c**8*f**4*exp(6*I*e), 0)), (x*((exp(8*I*e) + 4
*exp(6*I*e) + 6*exp(4*I*e) + 4*exp(2*I*e) + 1)*exp(-4*I*e)/(16*a**2*c**2) -
 3/(8*a**2*c**2)), True)) + 3*x/(8*a**2*c**2)
```

Giac [A]
time = 0.57, size = 61, normalized size = 0.95

$$\frac{\frac{3\,(fx+e)}{a^2c^2}+\frac{3\,\tan(fx+e)^3+5\,\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)^2a^2c^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*(f*x + e)/(a^2*c^2) + (3*tan(f*x + e)^3 + 5*tan(f*x + e))/((tan(f*x
+ e)^2 + 1)^2*a^2*c^2))/f
```

Mupad [B]
time = 4.73, size = 38, normalized size = 0.59

$$\frac{2\sin\left(2e+2fx\right)+\frac{\sin\left(4e+4fx\right)}{4}+3fx}{8a^2c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i)^2),x)
```

```
[Out] (2*sin(2*e + 2*f*x) + sin(4*e + 4*f*x)/4 + 3*f*x)/(8*a^2*c^2*f)
```

3.937 $\int \frac{1}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^2}\,dx$

Optimal. Leaf size=114

$$\frac{5x}{16a^3c^2}+\frac{i\cos^6(e+fx)}{6a^3c^2f}+\frac{5\cos(e+fx)\sin(e+fx)}{16a^3c^2f}+\frac{5\cos^3(e+fx)\sin(e+fx)}{24a^3c^2f}+\frac{\cos^5(e+fx)\sin(e+fx)}{6a^3c^2f}$$

```
[Out] 5/16*x/a^3/c^2+1/6*I*cos(f*x+e)^6/a^3/c^2/f+5/16*cos(f*x+e)*sin(f*x+e)/a^3/
c^2/f+5/24*cos(f*x+e)^3*sin(f*x+e)/a^3/c^2/f+1/6*cos(f*x+e)^5*sin(f*x+e)/a^
3/c^2/f
```

Rubi [A]
time = 0.09, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {3603, 3567, 2715, 8}

$$\frac{i\cos^6(e+fx)}{6a^3c^2f}+\frac{\sin(e+fx)\cos^5(e+fx)}{6a^3c^2f}+\frac{5\sin(e+fx)\cos^3(e+fx)}{24a^3c^2f}+\frac{5\sin(e+fx)\cos(e+fx)}{16a^3c^2f}+\frac{5x}{16a^3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])^2),x]
```

```
[Out] (5*x)/(16*a^3*c^2) + ((I/6)*Cos[e + f*x]^6)/(a^3*c^2*f) + (5*Cos[e + f*x]*S
in[e + f*x])/(16*a^3*c^2*f) + (5*Cos[e + f*x]^3*Sin[e + f*x])/(24*a^3*c^2*f
) + (Cos[e + f*x]^5*Sin[e + f*x])/(6*a^3*c^2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
```

```
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^2}\,dx = \frac{\int \cos^6(e+fx)(c-ic\tan(e+fx))\,dx}{a^3c^3}$$
$$= \frac{i\cos^6(e+fx)}{6a^3c^2f} + \frac{\int \cos^6(e+fx)\,dx}{a^3c^2}$$
$$= \frac{i\cos^6(e+fx)}{6a^3c^2f} + \frac{\cos^5(e+fx)\sin(e+fx)}{6a^3c^2f} + \frac{5\int \cos^4(e+}{6a^3c}$$
$$= \frac{i\cos^6(e+fx)}{6a^3c^2f} + \frac{5\cos^3(e+fx)\sin(e+fx)}{24a^3c^2f} + \frac{\cos^5(e+f}{6}$$
$$= \frac{i\cos^6(e+fx)}{6a^3c^2f} + \frac{5\cos(e+fx)\sin(e+fx)}{16a^3c^2f} + \frac{5\cos^3(e+}{24}$$
$$= \frac{5x}{16a^3c^2} + \frac{i\cos^6(e+fx)}{6a^3c^2f} + \frac{5\cos(e+fx)\sin(e+fx)}{16a^3c^2f} + \frac{5}{}$$

Mathematica [A]

time = 0.97, size = 135, normalized size = 1.18

$$\frac{\sec^3(e+fx)(\cos(2(e+fx))+i\sin(2(e+fx)))(60i(i+2fx)\cos(e+fx)+15\cos(3(e+fx))+\cos(5(e+fx))+60i\sin(e+fx)-120fx\sin(e+fx)+45i\sin(3(e+fx))+5i\sin(5(e+fx)))}{384a^3c^2f(-i+\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])^2),x]
```

```
[Out] (Sec[e + f*x]^3*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)])*((60*I)*(I + 2*f*x)
*Cos[e + f*x] + 15*Cos[3*(e + f*x)] + Cos[5*(e + f*x)] + (60*I)*Sin[e + f*x
] - 120*f*x*Sin[e + f*x] + (45*I)*Sin[3*(e + f*x)] + (5*I)*Sin[5*(e + f*x)]
))/(384*a^3*c^2*f*(-I + Tan[e + f*x])^3)
```

Maple [A]

time = 0.21, size = 105, normalized size = 0.92

method	result
derivativedivides	$\frac{\frac{i}{32(\tan(fx+e)+i)^2}+\frac{5i\ln(\tan(fx+e)+i)}{32}+\frac{1}{8\tan(fx+e)+8i}-\frac{5i\ln(\tan(fx+e)-i)}{32}-\frac{3i}{32(\tan(fx+e)-i)^2}-\frac{1}{24(\tan(fx+e)-i)^3}+\frac{3}{16(\tan(f}}{f\,a^3c^2}$
default	$\frac{\frac{i}{32(\tan(fx+e)+i)^2}+\frac{5i\ln(\tan(fx+e)+i)}{32}+\frac{1}{8\tan(fx+e)+8i}-\frac{5i\ln(\tan(fx+e)-i)}{32}-\frac{3i}{32(\tan(fx+e)-i)^2}-\frac{1}{24(\tan(fx+e)-i)^3}+\frac{3}{16(\tan(f}}{f\,a^3c^2}$

risch	$\frac{5x}{16a^3c^2} + \frac{ie^{-6i(fx+e)}}{192a^3c^2f} + \frac{i\cos(4fx+4e)}{32a^3c^2f} + \frac{3\sin(4fx+4e)}{64a^3c^2f} + \frac{5i\cos(2fx+2e)}{64a^3c^2f} + \frac{15\sin(2fx+2e)}{64a^3c^2f}$
norman	$\frac{\frac{5x}{16ac}+\frac{i}{6acf}+\frac{11\tan(fx+e)}{16acf}+\frac{5\left(\tan^3(fx+e)\right)}{6acf}+\frac{5\left(\tan^5(fx+e)\right)}{16acf}+\frac{15x\left(\tan^2(fx+e)\right)}{16ac}+\frac{15x\left(\tan^4(fx+e)\right)}{16ac}+\frac{5x\left(\tan^6(fx+e)\right)}{16ac}}{(1+\tan^2(fx+e))^3a^2c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a^3/c^2*(1/32*I/(tan(f*x+e)+I)^2+5/32*I*ln(tan(f*x+e)+I)+1/8/(tan(f*x+e
)+I)-5/32*I*ln(tan(f*x+e)-I)-3/32*I/(tan(f*x+e)-I)^2-1/24/(tan(f*x+e)-I)^3+
3/16/(tan(f*x+e)-I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.21, size = 85, normalized size = 0.75

$$\frac{\left(120\,fxe^{(6i\,fx+6i\,e)} - 3i\,e^{(10i\,fx+10i\,e)} - 30i\,e^{(8i\,fx+8i\,e)} + 60i\,e^{(4i\,fx+4i\,e)} + 15i\,e^{(2i\,fx+2i\,e)} + 2i\right)e^{(-6i\,fx-6i\,e)}}{384\,a^3c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] 1/384*(120*f*x*e^(6*I*f*x + 6*I*e) - 3*I*e^(10*I*f*x + 10*I*e) - 30*I*e^(8*
I*f*x + 8*I*e) + 60*I*e^(4*I*f*x + 4*I*e) + 15*I*e^(2*I*f*x + 2*I*e) + 2*I)
*e^(-6*I*f*x - 6*I*e)/(a^3*c^2*f)
```

Sympy [A]
time = 0.35, size = 258, normalized size = 2.26

$$\begin{cases} \frac{\left(-50331648ia^{12}c^8f^4e^{16ie}e^{4ifx} - 503316480ia^{12}c^8f^4e^{14ie}e^{2ifx} + 1006632960ia^{12}c^8f^4e^{10ie}e^{-2ifx} + 251658240ia^{12}c^8f^4e^{8ie}e^{-4ifx} + 33554432ia^{12}c^8f^4e^{6ie}e^{-6ifx}\right)e^{-12ie}}{6442450944a^{15}c^{10}f^5} & \text{for } a^{15}c^{10}f^5e^{12ie} \neq 0 \\ x\left(\frac{\left(e^{10ie}+5e^{8ie}+10e^{6ie}+10e^{4ie}+5e^{2ie}+1\right)e^{-6ie}}{32a^3c^2} - \frac{5}{16a^3c^2}\right) & \text{otherwise} \end{cases} + \frac{5x}{16a^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**3/(c-I*c*tan(f*x+e))**2,x)
```

```
[Out] Piecewise(((-50331648*I*a**12*c**8*f**4*exp(16*I*e)*exp(4*I*f*x) - 50331648
0*I*a**12*c**8*f**4*exp(14*I*e)*exp(2*I*f*x) + 1006632960*I*a**12*c**8*f**4
*exp(10*I*e)*exp(-2*I*f*x) + 251658240*I*a**12*c**8*f**4*exp(8*I*e)*exp(-4*
I*f*x) + 33554432*I*a**12*c**8*f**4*exp(6*I*e)*exp(-6*I*f*x))*exp(-12*I*e)/
(6442450944*a**15*c**10*f**5), Ne(a**15*c**10*f**5*exp(12*I*e), 0)), (x*((e
xp(10*I*e) + 5*exp(8*I*e) + 10*exp(6*I*e) + 10*exp(4*I*e) + 5*exp(2*I*e) +
1)*exp(-6*I*e)/(32*a**3*c**2) - 5/(16*a**3*c**2)), True)) + 5*x/(16*a**3*c*
*2)
```

Giac [A]
time = 0.67, size = 137, normalized size = 1.20

$$-\frac{-\frac{30i\log(\tan(fx+e)+i)}{a^3c^2}+\frac{30i\log(\tan(fx+e)-i)}{a^3c^2}+\frac{3\left(-15i\tan(fx+e)^2+38\tan(fx+e)+25i\right)}{a^3c^2(-i\tan(fx+e)+1)^2}-\frac{55i\tan(fx+e)^3+201\tan(fx+e)^2-255i\tan(fx+e)-117}{a^3c^2(\tan(fx+e)-i)^3}}{192f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/192*(-30*I*log(tan(f*x + e) + I)/(a^3*c^2) + 30*I*log(tan(f*x + e) - I)/
(a^3*c^2) + 3*(-15*I*tan(f*x + e)^2 + 38*tan(f*x + e) + 25*I)/(a^3*c^2*(-I*
tan(f*x + e) + 1)^2) - (55*I*tan(f*x + e)^3 + 201*tan(f*x + e)^2 - 255*I*ta
n(f*x + e) - 117)/(a^3*c^2*(tan(f*x + e) - I)^3))/f
```

Mupad [B]
time = 5.22, size = 88, normalized size = 0.77

$$\frac{5\,x}{16\,a^3\,c^2}-\frac{\frac{\mathrm{tan}\left(e+f\,x\right)^4\,5{}\mathrm{i}}{16}+\frac{5\,\mathrm{tan}\left(e+f\,x\right)^3}{16}+\frac{\mathrm{tan}\left(e+f\,x\right)^2\,25{}\mathrm{i}}{48}+\frac{25\,\mathrm{tan}\left(e+f\,x\right)}{48}+\frac{1}{6}{}\mathrm{i}}{a^3\,c^2\,f\,{\left(1+\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^3\,{\left(\mathrm{tan}\left(e+f\,x\right)+1{}\mathrm{i}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i)^2),x)
```

```
[Out] (5*x)/(16*a^3*c^2) - ((25*tan(e + f*x))/48 + (tan(e + f*x)^2*25i)/48 + (5*t
an(e + f*x)^3)/16 + (tan(e + f*x)^4*5i)/16 + 1i/6)/(a^3*c^2*f*(tan(e + f*x)
*1i + 1)^3*(tan(e + f*x) + 1i)^2)
```

3.938 $\int \frac{(a+ia\tan(e+fx))^6}{(c-ic\tan(e+fx))^3}\,dx$

Optimal. Leaf size=154

$$-\frac{40a^6x}{c^3}+\frac{40ia^6\log(\cos(e+fx))}{c^3f}+\frac{9a^6\tan(e+fx)}{c^3f}+\frac{ia^6\tan^2(e+fx)}{2c^3f}-\frac{32ia^6}{3f(c-ic\tan(e+fx))^3}+\frac{4}{cf(c-ic}$$

```
[Out] -40*a^6*x/c^3+40*I*a^6*ln(cos(f*x+e))/c^3/f+9*a^6*tan(f*x+e)/c^3/f+1/2*I*a^
6*tan(f*x+e)^2/c^3/f-32/3*I*a^6/f/(c-I*c*tan(f*x+e))^3+40*I*a^6/c/f/(c-I*c*
tan(f*x+e))^2-80*I*a^6/f/(c^3-I*c^3*tan(f*x+e))
```

Rubi [A]
time = 0.12, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 45}

$$\frac{ia^6\tan^2(e+fx)}{2c^3f}+\frac{9a^6\tan(e+fx)}{c^3f}-\frac{80ia^6}{f\left(c^3-ic^3\tan(e+fx)\right)}+\frac{40ia^6\log(\cos(e+fx))}{c^3f}-\frac{40a^6x}{c^3}+\frac{40ia^6}{cf(c-ic\tan(e+fx))^2}-\frac{32ia^6}{3f(c-ic\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^6/(c - I*c*Tan[e + f*x])^3,x]
```

```
[Out] (-40*a^6*x)/c^3 + ((40*I)*a^6*Log[Cos[e + f*x]])/(c^3*f) + (9*a^6*Tan[e + f
*x])/(c^3*f) + ((I/2)*a^6*Tan[e + f*x]^2)/(c^3*f) - (((32*I)/3)*a^6)/(f*(c
- I*c*Tan[e + f*x])^3) + ((40*I)*a^6)/(c*f*(c - I*c*Tan[e + f*x])^2) - ((80
*I)*a^6)/(f*(c^3 - I*c^3*Tan[e + f*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
```

```
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^6}{(c-ic\tan(e+fx))^3}\,dx = (a^6c^6)\int \frac{\sec^{12}(e+fx)}{(c-ic\tan(e+fx))^9}\,dx$$
$$= \frac{(ia^6)\operatorname{Subst}\left(\int \frac{(c-x)^5}{(c+x)^4}\,dx, x, -ic\tan(e+fx)\right)}{c^5f}$$
$$= \frac{(ia^6)\operatorname{Subst}\left(\int \left(9c-x+\frac{32c^5}{(c+x)^4}-\frac{80c^4}{(c+x)^3}+\frac{80c^3}{(c+x)^2}-\frac{40c^2}{c+x}\right)dx, x, -ic\tan(e+fx\right.}{c^5f}$$
$$= -\frac{40a^6x}{c^3}+\frac{40ia^6\log(\cos(e+fx))}{c^3f}+\frac{9a^6\tan(e+fx)}{c^3f}+\frac{ia^6\tan^2(e+fx)}{2c^3f}-\frac{}{3}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 569 vs. 2(154) = 308.
time = 3.62, size = 569, normalized size = 3.69

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^6/(c - I*c*Tan[e + f*x])^3,x]
```

```
[Out] -1/12*(a^6*Sec[e]*Sec[e + f*x]^2*(Cos[3*(e + 3*f*x)] + I*Sin[3*(e + 3*f*x)]
)*(I*Cos[2*e + 3*f*x] + 120*f*x*Cos[2*e + 3*f*x] + (55*I)*Cos[4*e + 3*f*x]
+ 120*f*x*Cos[4*e + 3*f*x] - (25*I)*Cos[4*e + 5*f*x] + 60*f*x*Cos[4*e + 5*f
*x] + (2*I)*Cos[6*e + 5*f*x] + 60*f*x*Cos[6*e + 5*f*x] + 10*Cos[2*e + f*x]*
(11*I + 6*f*x - (3*I)*Log[Cos[e + f*x]^2]) + Cos[f*x]*(83*I + 60*f*x - (30*
I)*Log[Cos[e + f*x]^2]) - (60*I)*Cos[2*e + 3*f*x]*Log[Cos[e + f*x]^2] - (60
*I)*Cos[4*e + 3*f*x]*Log[Cos[e + f*x]^2] - (30*I)*Cos[4*e + 5*f*x]*Log[Cos[
e + f*x]^2] - (30*I)*Cos[6*e + 5*f*x]*Log[Cos[e + f*x]^2] + 43*Sin[f*x] - (
60*I)*f*x*Sin[f*x] - 30*Log[Cos[e + f*x]^2]*Sin[f*x] + 70*Sin[2*e + f*x] -
(60*I)*f*x*Sin[2*e + f*x] - 30*Log[Cos[e + f*x]^2]*Sin[2*e + f*x] + 11*Sin[
2*e + 3*f*x] - (120*I)*f*x*Sin[2*e + 3*f*x] - 60*Log[Cos[e + f*x]^2]*Sin[2*
e + 3*f*x] + 65*Sin[4*e + 3*f*x] - (120*I)*f*x*Sin[4*e + 3*f*x] - 60*Log[Co
s[e + f*x]^2]*Sin[4*e + 3*f*x] - 29*Sin[4*e + 5*f*x] - (60*I)*f*x*Sin[4*e +
 5*f*x] - 30*Log[Cos[e + f*x]^2]*Sin[4*e + 5*f*x] - 2*Sin[6*e + 5*f*x] - (6
0*I)*f*x*Sin[6*e + 5*f*x] - 30*Log[Cos[e + f*x]^2]*Sin[6*e + 5*f*x]))/(c^3*
f*(Cos[f*x] + I*Sin[f*x])^6)
```

Maple [A]
time = 0.27, size = 84, normalized size = 0.55

method	result
derivativedivides	$\frac{a^6\left(9\tan(fx+e)+\frac{i\left(\tan^2(fx+e)\right)}{2}-\frac{32}{3(\tan(fx+e)+i)^3}+\frac{80}{\tan(fx+e)+i}-\frac{40i}{(\tan(fx+e)+i)^2}-40i\ln(\tan(fx+e)+i)\right)}{f\,c^3}$
default	$\frac{a^6\left(9\tan(fx+e)+\frac{i\left(\tan^2(fx+e)\right)}{2}-\frac{32}{3(\tan(fx+e)+i)^3}+\frac{80}{\tan(fx+e)+i}-\frac{40i}{(\tan(fx+e)+i)^2}-40i\ln(\tan(fx+e)+i)\right)}{f\,c^3}$
risch	$-\frac{4ia^6e^{6i(fx+e)}}{3c^3f}+\frac{6ia^6e^{4i(fx+e)}}{c^3f}-\frac{24ia^6e^{2i(fx+e)}}{c^3f}+\frac{80a^6e}{f\,c^3}+\frac{2ia^6\left(10\,e^{2i(fx+e)}+9\right)}{f\,c^3\left(e^{2i(fx+e)}+1\right)^2}+\frac{40ia^6\ln\left(e^{2i(fx+e)}+1\right)}{f\,c^3}$
norman	$\frac{-\frac{40a^6x}{c}-\frac{313ia^6}{6cf}-\frac{120a^6x\left(\tan^2(fx+e)\right)}{c}-\frac{120a^6x\left(\tan^4(fx+e)\right)}{c}-\frac{40a^6x\left(\tan^6(fx+e)\right)}{c}+\frac{41a^6\tan(fx+e)}{cf}+\frac{289a^6\left(\tan^3(fx+e)\right)}{3cf}}{c^2(1+\tan^2(fx+e))^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^6/(c-I*c*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a^6/c^3*(9*tan(f*x+e)+1/2*I*tan(f*x+e)^2-32/3/(tan(f*x+e)+I)^3+80/(tan(
f*x+e)+I)-40*I/(tan(f*x+e)+I)^2-40*I*ln(tan(f*x+e)+I))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^6/(c-I*c*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]

time = 1.02, size = 173, normalized size = 1.12

$$-\frac{2\left(2i\,a^6e^{(10i\,fx+10i\,e)}-5i\,a^6e^{(8i\,fx+8i\,e)}+20i\,a^6e^{(6i\,fx+6i\,e)}+63i\,a^6e^{(4i\,fx+4i\,e)}+6i\,a^6e^{(2i\,fx+2i\,e)}-27i\,a^6+60\left(-i\,a^6e^{(4i\,fx+4i\,e)}-2i\,a^6e^{(2i\,fx+2i\,e)}-i\,a^6\right)\log\left(e^{(2i\,fx+2i\,e)}+1\right)\right)}{3\left(c^3fe^{(4i\,fx+4i\,e)}+2\,c^3fe^{(2i\,fx+2i\,e)}+c^3f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^6/(c-I*c*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -2/3*(2*I*a^6*e^(10*I*f*x + 10*I*e) - 5*I*a^6*e^(8*I*f*x + 8*I*e) + 20*I*a^
6*e^(6*I*f*x + 6*I*e) + 63*I*a^6*e^(4*I*f*x + 4*I*e) + 6*I*a^6*e^(2*I*f*x +
 2*I*e) - 27*I*a^6 + 60*(-I*a^6*e^(4*I*f*x + 4*I*e) - 2*I*a^6*e^(2*I*f*x +
2*I*e) - I*a^6)*log(e^(2*I*f*x + 2*I*e) + 1))/(c^3*f*e^(4*I*f*x + 4*I*e) +
2*c^3*f*e^(2*I*f*x + 2*I*e) + c^3*f)
```

Sympy [A]
time = 0.45, size = 246, normalized size = 1.60

$$\frac{40ia^6\log\left(e^{2ifx}+e^{-2ie}\right)}{c^3f}+\frac{20ia^6e^{2ie}e^{2ifx}+18ia^6}{c^3fe^{4ie}e^{4ifx}+2c^3fe^{2ie}e^{2ifx}+c^3f}+\begin{cases}\frac{-4ia^6c^6f^2e^{6ie}e^{6ifx}+18ia^6c^6f^2e^{4ie}e^{4ifx}-72ia^6c^6f^2e^{2ie}e^{2ifx}}{3c^9f^3} & \text{for } c^9f^3\neq 0\\ \frac{x\left(8a^6e^{6ie}-24a^6e^{4ie}+48a^6e^{2ie}\right)}{c^3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**6/(c-I*c*tan(f*x+e))**3,x)
```

```
[Out] 40*I*a**6*log(exp(2*I*f*x) + exp(-2*I*e))/(c**3*f) + (20*I*a**6*exp(2*I*e)*
exp(2*I*f*x) + 18*I*a**6)/(c**3*f*exp(4*I*e)*exp(4*I*f*x) + 2*c**3*f*exp(2*
I*e)*exp(2*I*f*x) + c**3*f) + Piecewise(((-4*I*a**6*c**6*f**2*exp(6*I*e)*ex
p(6*I*f*x) + 18*I*a**6*c**6*f**2*exp(4*I*e)*exp(4*I*f*x) - 72*I*a**6*c**6*f
**2*exp(2*I*e)*exp(2*I*f*x))/(3*c**9*f**3), Ne(c**9*f**3, 0)), (x*(8*a**6*e
xp(6*I*e) - 24*a**6*exp(4*I*e) + 48*a**6*exp(2*I*e))/c**3, True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(140) = 280.
time = 1.09, size = 287, normalized size = 1.86

$$-\frac{2\left(-\frac{60ia^6\log\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{c^3}+\frac{120ia^6\log\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+i\right)}{c^3}-\frac{60ia^6\log\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{c^3}-\frac{3\left(-30ia^6\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-9a^6\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+61ia^6\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+9a^6\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-30ia^6\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^2c^3}+\frac{2\left(-147ia^6\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6+930a^6\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+2421ia^6\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-3340a^6\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2421ia^6\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+930a^6\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+147ia^6\right)}{c^3\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+i\right)^6}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^6/(c-I*c*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/3*(-60*I*a^6*log(tan(1/2*f*x + 1/2*e) + 1)/c^3 + 120*I*a^6*log(tan(1/2*f
*x + 1/2*e) + I)/c^3 - 60*I*a^6*log(tan(1/2*f*x + 1/2*e) - 1)/c^3 - 3*(-30*
I*a^6*tan(1/2*f*x + 1/2*e)^4 - 9*a^6*tan(1/2*f*x + 1/2*e)^3 + 61*I*a^6*tan(
1/2*f*x + 1/2*e)^2 + 9*a^6*tan(1/2*f*x + 1/2*e) - 30*I*a^6)/((tan(1/2*f*x +
 1/2*e)^2 - 1)^2*c^3) + 2*(-147*I*a^6*tan(1/2*f*x + 1/2*e)^6 + 930*a^6*tan(
1/2*f*x + 1/2*e)^5 + 2421*I*a^6*tan(1/2*f*x + 1/2*e)^4 - 3340*a^6*tan(1/2*f
*x + 1/2*e)^3 - 2421*I*a^6*tan(1/2*f*x + 1/2*e)^2 + 930*a^6*tan(1/2*f*x + 1
/2*e) + 147*I*a^6)/(c^3*(tan(1/2*f*x + 1/2*e) + I)^6))/f
```

Mupad [B]
time = 4.75, size = 139, normalized size = 0.90

$$\frac{9\,a^6\tan(e+f\,x)}{c^3\,f}-\frac{\frac{80\,a^6\tan(e+f\,x)^2}{c^3}-\frac{152\,a^6}{3\,c^3}+\frac{a^6\tan(e+f\,x)\,120\mathrm{i}}{c^3}}{f\,\left(-{\tan\left(e+f\,x\right)}^3-{\tan\left(e+f\,x\right)}^2\,3\mathrm{i}+3\tan\left(e+f\,x\right)+1\mathrm{i}\right)}+\frac{a^6\tan(e+f\,x)^2\,1\mathrm{i}}{2\,c^3\,f}-\frac{a^6\,\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,40\mathrm{i}}{c^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^6/(c - c*tan(e + f*x)*1i)^3,x)
```

```
[Out] (9*a^6*tan(e + f*x))/(c^3*f) - ((a^6*tan(e + f*x)*120i)/c^3 - (152*a^6)/(3*
c^3) + (80*a^6*tan(e + f*x)^2)/c^3)/(f*(3*tan(e + f*x) - tan(e + f*x)^2*3i
- tan(e + f*x)^3 + 1i)) + (a^6*tan(e + f*x)^2*1i)/(2*c^3*f) - (a^6*log(tan(
e + f*x) + 1i)*40i)/(c^3*f)
```

3.939 $\int \frac{(a+ia\tan(e+fx))^5}{(c-ic\tan(e+fx))^3}\,dx$

Optimal. Leaf size=134

$$-\frac{8a^5x}{c^3}+\frac{8ia^5\log(\cos(e+fx))}{c^3f}+\frac{a^5\tan(e+fx)}{c^3f}-\frac{16ia^5}{3f(c-ic\tan(e+fx))^3}-\frac{24ia^5}{f\left(c^3-ic^3\tan(e+fx)\right)}+\frac{}{f\left(c^4\right.}$$

```
[Out] -8*a^5*x/c^3+8*I*a^5*ln(cos(f*x+e))/c^3/f+a^5*tan(f*x+e)/c^3/f-16/3*I*a^5/f
/(c-I*c*tan(f*x+e))^3-24*I*a^5/f/(c^3-I*c^3*tan(f*x+e))+16*I*a^5*c^5/f/(c^4
-I*c^4*tan(f*x+e))^2
```

Rubi [A]

time = 0.10, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 45}

$$\frac{a^5\tan(e+fx)}{c^3f}-\frac{24ia^5}{f\left(c^3-ic^3\tan(e+fx)\right)}+\frac{8ia^5\log(\cos(e+fx))}{c^3f}-\frac{8a^5x}{c^3}+\frac{16ia^5c^5}{f\left(c^4-ic^4\tan(e+fx)\right)^2}-\frac{16ia^5}{3f(c-ic\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^5/(c - I*c*Tan[e + f*x])^3,x]
```

```
[Out] (-8*a^5*x)/c^3 + ((8*I)*a^5*Log[Cos[e + f*x]])/(c^3*f) + (a^5*Tan[e + f*x])
/(c^3*f) - (((16*I)/3)*a^5)/(f*(c - I*c*Tan[e + f*x])^3) - ((24*I)*a^5)/(f*
(c^3 - I*c^3*Tan[e + f*x])) + ((16*I)*a^5*c^5)/(f*(c^4 - I*c^4*Tan[e + f*x]
)^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
```

```
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+ia\tan(e+fx))^5}{(c-ic\tan(e+fx))^3}\,dx &= \left(a^5c^5\right)\int \frac{\sec^{10}(e+fx)}{(c-ic\tan(e+fx))^8}\,dx \\
&= \frac{\left(ia^5\right)\operatorname{Subst}\left(\int \frac{(c-x)^4}{(c+x)^4}\,dx, x, -ic\tan(e+fx)\right)}{c^4f} \\
&= \frac{\left(ia^5\right)\operatorname{Subst}\left(\int \left(1+\frac{16c^4}{(c+x)^4}-\frac{32c^3}{(c+x)^3}+\frac{24c^2}{(c+x)^2}-\frac{8c}{c+x}\right)dx, x, -ic\tan(e+fx)\right)}{c^4f} \\
&= -\frac{8a^5x}{c^3}+\frac{8ia^5\log(\cos(e+fx))}{c^3f}+\frac{a^5\tan(e+fx)}{c^3f}-\frac{16ia^5}{3f(c-ic\tan(e+fx))^3} -
\end{aligned}
$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 923 vs. $2(134) = 268$.
time = 6.91, size = 923, normalized size = 6.89

$$
-\frac{8x\cos(5e)\cos^5(e+fx)(a+ia\tan(e+fx))^5}{c^3(\cos(fx)+i\sin(fx))^5}+\frac{4i\cos(5e)\cos^5(e+fx)\log\left(\cos^2(e+fx)\right)(a+ia\tan(e+fx))^5}{c^3f(\cos(fx)+i\sin(fx))^5}+\frac{\cos(6fx)\cos^5(e+fx)\left(-\frac{2i\cos(e)}{3c^3}+\frac{2\sin(e)}{3c^3}\right)(a+ia\tan(e+fx))^5}{f(\cos(fx)+i\sin(fx))^5}+\frac{\cos(4fx)\cos^5(e+fx)\left(\frac{2i\cos(e)}{c^3}+\frac{2\sin(e)}{c^3}\right)(a+ia\tan(e+fx))^5}{f(\cos(fx)+i\sin(fx))^5}+\frac{\cos(2fx)\cos^5(e+fx)\left(-\frac{6i\cos(3e)}{c^3}-\frac{6\sin(3e)}{c^3}\right)(a+ia\tan(e+fx))^5}{f(\cos(fx)+i\sin(fx))^5}+\frac{8ix\cos^5(e+fx)\sin(5e)(a+ia\tan(e+fx))^5}{c^3(\cos(fx)+i\sin(fx))^5}+\frac{4\cos^5(e+fx)\log\left(\cos^2(e+fx)\right)\sin(5e)(a+ia\tan(e+fx))^5}{c^3f(\cos(fx)+i\sin(fx))^5}+\frac{\cos^4(e+fx)\left(\frac{\cos(5e)}{c^3}-\frac{i\sin(5e)}{c^3}\right)\sin(fx)(a+ia\tan(e+\cdots}{f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)(\cos(fx)+i\sin\cdots}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^5/(c - I*c*Tan[e + f*x])^3,x]
```

```
[Out] (-8*x*Cos[5*e]*Cos[e + f*x]^5*(a + I*a*Tan[e + f*x])^5)/(c^3*(Cos[f*x] + I*
Sin[f*x])^5) + ((4*I)*Cos[5*e]*Cos[e + f*x]^5*Log[Cos[e + f*x]^2]*(a + I*a*
Tan[e + f*x])^5)/(c^3*f*(Cos[f*x] + I*Sin[f*x])^5) + (Cos[6*f*x]*Cos[e + f*
x]^5*((((-2*I)/3)*Cos[e])/c^3 + (2*Sin[e])/(3*c^3))*(a + I*a*Tan[e + f*x])^
5)/(f*(Cos[f*x] + I*Sin[f*x])^5) + (Cos[4*f*x]*Cos[e + f*x]^5*(((2*I)*Cos[e
])/c^3 + (2*Sin[e])/c^3)*(a + I*a*Tan[e + f*x])^5)/(f*(Cos[f*x] + I*Sin[f*x
])^5) + (Cos[2*f*x]*Cos[e + f*x]^5*(((-6*I)*Cos[3*e])/c^3 - (6*Sin[3*e])/c^
3)*(a + I*a*Tan[e + f*x])^5)/(f*(Cos[f*x] + I*Sin[f*x])^5) + ((8*I)*x*Cos[e
 + f*x]^5*Sin[5*e]*(a + I*a*Tan[e + f*x])^5)/(c^3*(Cos[f*x] + I*Sin[f*x])^5
) + (4*Cos[e + f*x]^5*Log[Cos[e + f*x]^2]*Sin[5*e]*(a + I*a*Tan[e + f*x])^5
)/(c^3*f*(Cos[f*x] + I*Sin[f*x])^5) + (Cos[e + f*x]^4*(Cos[5*e]/c^3 - (I*Si
n[5*e])/c^3)*Sin[f*x]*(a + I*a*Tan[e + f*x])^5)/(f*(Cos[e/2] - Sin[e/2])*(C
os[e/2] + Sin[e/2])*(Cos[f*x] + I*Sin[f*x])^5) + (Cos[e + f*x]^5*((6*Cos[3*
e])/c^3 - ((6*I)*Sin[3*e])/c^3)*Sin[2*f*x]*(a + I*a*Tan[e + f*x])^5)/(f*(Co
s[f*x] + I*Sin[f*x])^5) + (Cos[e + f*x]^5*((-2*Cos[e])/c^3 + ((2*I)*Sin[e])
/c^3)*Sin[4*f*x]*(a + I*a*Tan[e + f*x])^5)/(f*(Cos[f*x] + I*Sin[f*x])^5) +
(Cos[e + f*x]^5*((2*Cos[e])/(3*c^3) + (((2*I)/3)*Sin[e])/c^3)*Sin[6*f*x]*(a
```

```
 + I*a*Tan[e + f*x])^5)/(f*(Cos[f*x] + I*Sin[f*x])^5) + (x*Cos[e + f*x]^5*(
(4*Cos[e]^3)/c^3 - (4*Cos[e]^5)/c^3 - ((16*I)*Cos[e]^2*Sin[e])/c^3 + ((24*I
)*Cos[e]^4*Sin[e])/c^3 - (24*Cos[e]*Sin[e]^2)/c^3 + (60*Cos[e]^3*Sin[e]^2)/
c^3 + ((16*I)*Sin[e]^3)/c^3 - ((80*I)*Cos[e]^2*Sin[e]^3)/c^3 - (60*Cos[e]*S
in[e]^4)/c^3 + ((24*I)*Sin[e]^5)/c^3 + (4*Sin[e]^3*Tan[e])/c^3 + (4*Sin[e]^
5*Tan[e])/c^3 - I*((8*Cos[5*e])/c^3 - ((8*I)*Sin[5*e])/c^3)*Tan[e])*(a + I*
a*Tan[e + f*x])^5)/(Cos[f*x] + I*Sin[f*x])^5
```

Maple [A]
time = 0.24, size = 71, normalized size = 0.53

method	result
derivativedivides	$\frac{a^{5}\left(\tan(fx+e)+\frac{24}{\tan(fx+e)+i}-\frac{16}{3(\tan(fx+e)+i)^{3}}-\frac{16i}{(\tan(fx+e)+i)^{2}}-8i\ln(\tan(fx+e)+i)\right)}{f\,c^{3}}$
default	$\frac{a^{5}\left(\tan(fx+e)+\frac{24}{\tan(fx+e)+i}-\frac{16}{3(\tan(fx+e)+i)^{3}}-\frac{16i}{(\tan(fx+e)+i)^{2}}-8i\ln(\tan(fx+e)+i)\right)}{f\,c^{3}}$
risch	$-\frac{2ia^{5}e^{6i(fx+e)}}{3c^{3}f}+\frac{2ia^{5}e^{4i(fx+e)}}{c^{3}f}-\frac{6ia^{5}e^{2i(fx+e)}}{c^{3}f}+\frac{16a^{5}e}{f\,c^{3}}+\frac{2ia^{5}}{f\,c^{3}\left(e^{2i(fx+e)}+1\right)}+\frac{8ia^{5}\ln\left(e^{2i(fx+e)}+1\right)}{f\,c^{3}}$
norman	$\frac{\frac{a^{5}\left(\tan^{7}(fx+e)\right)}{cf}-\frac{40ia^{5}\left(\tan^{4}(fx+e)\right)}{cf}-\frac{32ia^{5}\left(\tan^{2}(fx+e)\right)}{cf}-\frac{8a^{5}x}{c}-\frac{40ia^{5}}{3cf}-\frac{24a^{5}x\left(\tan^{2}(fx+e)\right)}{c}-\frac{24a^{5}x\left(\tan^{4}(fx+e)\right)}{c}-\frac{8a^{5}x \cdots}{}}{c^{2}\left(1+\tan^{2}(fx+e)\right)^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^5/(c-I*c*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a^5/c^3*(tan(f*x+e)+24/(tan(f*x+e)+I)-16/3/(tan(f*x+e)+I)^3-16*I/(tan(f
*x+e)+I)^2-8*I*ln(tan(f*x+e)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^5/(c-I*c*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.53, size = 127, normalized size = 0.95

$$-\frac{2\left(i\,a^{5}e^{(8i\,fx+8i\,e)}-2i\,a^{5}e^{(6i\,fx+6i\,e)}+6i\,a^{5}e^{(4i\,fx+4i\,e)}+9i\,a^{5}e^{(2i\,fx+2i\,e)}-3i\,a^{5}+12\left(-i\,a^{5}e^{(2i\,fx+2i\,e)}-i\,a^{5}\right)\log\left(e^{(2i\,fx+2i\,e)}+1\right)\right)}{3\left(c^{3}fe^{(2i\,fx+2i\,e)}+c^{3}f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^5/(c-I*c*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -2/3*(I*a^5*e^(8*I*f*x + 8*I*e) - 2*I*a^5*e^(6*I*f*x + 6*I*e) + 6*I*a^5*e^(
4*I*f*x + 4*I*e) + 9*I*a^5*e^(2*I*f*x + 2*I*e) - 3*I*a^5 + 12*(-I*a^5*e^(2*
I*f*x + 2*I*e) - I*a^5)*log(e^(2*I*f*x + 2*I*e) + 1))/(c^3*f*e^(2*I*f*x + 2
*I*e) + c^3*f)
```

Sympy [A]
time = 0.39, size = 202, normalized size = 1.51

$$\frac{2ia^5}{c^3fe^{2ie}e^{2ifx}+c^3f}+\frac{8ia^5\log\left(e^{2ifx}+e^{-2ie}\right)}{c^3f}+\begin{cases}\frac{-2ia^5c^6f^2e^{6ie}e^{6ifx}+6ia^5c^6f^2e^{4ie}e^{4ifx}-18ia^5c^6f^2e^{2ie}e^{2ifx}}{3c^9f^3} & \text{for } c^9f^3\neq 0\\ \frac{x\left(4a^5e^{6ie}-8a^5e^{4ie}+12a^5e^{2ie}\right)}{c^3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**5/(c-I*c*tan(f*x+e))**3,x)
```

```
[Out] 2*I*a**5/(c**3*f*exp(2*I*e)*exp(2*I*f*x) + c**3*f) + 8*I*a**5*log(exp(2*I*f
*x) + exp(-2*I*e))/(c**3*f) + Piecewise(((-2*I*a**5*c**6*f**2*exp(6*I*e)*ex
p(6*I*f*x) + 6*I*a**5*c**6*f**2*exp(4*I*e)*exp(4*I*f*x) - 18*I*a**5*c**6*f*
*2*exp(2*I*e)*exp(2*I*f*x))/(3*c**9*f**3), Ne(c**9*f**3, 0)), (x*(4*a**5*ex
p(6*I*e) - 8*a**5*exp(4*I*e) + 12*a**5*exp(2*I*e))/c**3, True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(123) = 246.
time = 1.03, size = 253, normalized size = 1.89

$$-\frac{2\left(-\frac{60ia^5\log\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{c^3}+\frac{120ia^5\log\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+i\right)}{c^3}-\frac{60ia^5\log\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{c^3}-\frac{15\left(-4ia^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+4ia^5\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)c^3}+\frac{2\left(-147ia^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6+942a^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+2445ia^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-3460a^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2445ia^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+942a^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+147ia^5\right)}{c^3\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+i\right)^6}\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^5/(c-I*c*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/15*(-60*I*a^5*log(tan(1/2*f*x + 1/2*e) + 1)/c^3 + 120*I*a^5*log(tan(1/2*
f*x + 1/2*e) + I)/c^3 - 60*I*a^5*log(tan(1/2*f*x + 1/2*e) - 1)/c^3 - 15*(-4
*I*a^5*tan(1/2*f*x + 1/2*e)^2 - a^5*tan(1/2*f*x + 1/2*e) + 4*I*a^5)/((tan(1
/2*f*x + 1/2*e)^2 - 1)*c^3) + 2*(-147*I*a^5*tan(1/2*f*x + 1/2*e)^6 + 942*a^
5*tan(1/2*f*x + 1/2*e)^5 + 2445*I*a^5*tan(1/2*f*x + 1/2*e)^4 - 3460*a^5*tan
(1/2*f*x + 1/2*e)^3 - 2445*I*a^5*tan(1/2*f*x + 1/2*e)^2 + 942*a^5*tan(1/2*f
*x + 1/2*e) + 147*I*a^5)/(c^3*(tan(1/2*f*x + 1/2*e) + I)^6))/f
```

Mupad [B]
time = 5.69, size = 138, normalized size = 1.03

$$\frac{a^5\left(\ln\left(\tan(e+f\,x)+1{}\mathrm{i}\right)\,8{}\mathrm{i}+31\,\tan(e+f\,x)+24\,\ln\left(\tan(e+f\,x)+1{}\mathrm{i}\right)\,\tan(e+f\,x)-\ln\left(\tan(e+f\,x)+1{}\mathrm{i}\right)\,{\tan(e+f\,x)}^2\,24{}\mathrm{i}-8\,\ln\left(\tan(e+f\,x)+1{}\mathrm{i}\right)\,{\tan(e+f\,x)}^3-{\tan(e+f\,x)}^2\,21{}\mathrm{i}+3\,{\tan(e+f\,x)}^3-{\tan(e+f\,x)}^4\,1{}\mathrm{i}+\frac{40}{3}{}\mathrm{i}\right)}{c^3\,f\,{\left(-1+\tan\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^5/(c - c*tan(e + f*x)*1i)^3,x)
```

```
[Out] (a^5*(log(tan(e + f*x) + 1i)*8i + 31*tan(e + f*x) + 24*log(tan(e + f*x) + 1
i)*tan(e + f*x) - log(tan(e + f*x) + 1i)*tan(e + f*x)^2*24i - 8*log(tan(e +
 f*x) + 1i)*tan(e + f*x)^3 - tan(e + f*x)^2*21i + 3*tan(e + f*x)^3 - tan(e
+ f*x)^4*1i + 40i/3))/(c^3*f*(tan(e + f*x)*1i - 1)^3)
```

3.940 $\int \frac{(a+ia\tan(e+fx))^4}{(c-ic\tan(e+fx))^3}\,dx$

Optimal. Leaf size=114

$$-\frac{a^4x}{c^3}+\frac{ia^4\log(\cos(e+fx))}{c^3f}-\frac{8ia^4}{3f(c-ic\tan(e+fx))^3}+\frac{6ia^4}{cf(c-ic\tan(e+fx))^2}-\frac{6ia^4}{f\left(c^3-ic^3\tan(e+fx)\right)}$$

```
[Out] -a^4*x/c^3+I*a^4*ln(cos(f*x+e))/c^3/f-8/3*I*a^4/f/(c-I*c*tan(f*x+e))^3+6*I*
a^4/c/f/(c-I*c*tan(f*x+e))^2-6*I*a^4/f/(c^3-I*c^3*tan(f*x+e))
```

Rubi [A]
time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 45}

$$-\frac{6ia^4}{f\left(c^3-ic^3\tan(e+fx)\right)}+\frac{ia^4\log(\cos(e+fx))}{c^3f}-\frac{a^4x}{c^3}+\frac{6ia^4}{cf(c-ic\tan(e+fx))^2}-\frac{8ia^4}{3f(c-ic\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^4/(c - I*c*Tan[e + f*x])^3,x]
```

```
[Out] -((a^4*x)/c^3) + (I*a^4*Log[Cos[e + f*x]])/(c^3*f) - (((8*I)/3)*a^4)/(f*(c
- I*c*Tan[e + f*x])^3) + ((6*I)*a^4)/(c*f*(c - I*c*Tan[e + f*x])^2) - ((6*I
)*a^4)/(f*(c^3 - I*c^3*Tan[e + f*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+ia\tan(e+fx))^4}{(c-ic\tan(e+fx))^3}\,dx &= \left(a^4c^4\right)\int \frac{\sec^8(e+fx)}{(c-ic\tan(e+fx))^7}\,dx\\
&= \frac{\left(ia^4\right)\operatorname{Subst}\left(\int \frac{(c-x)^3}{(c+x)^4}\,dx,x,-ic\tan(e+fx)\right)}{c^3f}\\
&= \frac{\left(ia^4\right)\operatorname{Subst}\left(\int \left(\frac{1}{-c-x}+\frac{8c^3}{(c+x)^4}-\frac{12c^2}{(c+x)^3}+\frac{6c}{(c+x)^2}\right)dx,x,-ic\tan(e+fx)\right)}{c^3f}\\
&= -\frac{a^4x}{c^3}+\frac{ia^4\log(\cos(e+fx))}{c^3f}-\frac{8ia^4}{3f(c-ic\tan(e+fx))^3}+\frac{6ia^4}{cf(c-ic\tan(e+\ldots}
\end{aligned}$$

Mathematica [A]
time = 1.17, size = 143, normalized size = 1.25

$$\frac{a^4\left(-3i\cos(e+fx)+\cos(3(e+fx))\left(-2i-6fx+3i\log\left(\cos^2(e+fx)\right)\right)-9\sin(e+fx)+2\sin(3(e+fx))+6ifx\sin(3(e+fx))+3\log\left(\cos^2(e+fx)\right)\sin(3(e+fx))\right)(\cos(3e+7fx)+i\sin(3e+7fx))}{6c^3f(\cos(fx)+i\sin(fx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^4/(c - I*c*Tan[e + f*x])^3,x]
```

```
[Out] (a^4*((-3*I)*Cos[e + f*x] + Cos[3*(e + f*x)]*(-2*I - 6*f*x + (3*I)*Log[Cos[
e + f*x]^2]) - 9*Sin[e + f*x] + 2*Sin[3*(e + f*x)] + (6*I)*f*x*Sin[3*(e + f
*x)] + 3*Log[Cos[e + f*x]^2]*Sin[3*(e + f*x)])*(Cos[3*e + 7*f*x] + I*Sin[3*
e + 7*f*x]))/(6*c^3*f*(Cos[f*x] + I*Sin[f*x])^4)
```

Maple [A]
time = 0.22, size = 65, normalized size = 0.57

method	result
derivativedivides	$\frac{a^4\left(-\frac{6i}{(\tan(fx+e)+i)^2}+\frac{6}{\tan(fx+e)+i}-i\ln(\tan(fx+e)+i)-\frac{8}{3(\tan(fx+e)+i)^3}\right)}{f\,c^3}$
default	$\frac{a^4\left(-\frac{6i}{(\tan(fx+e)+i)^2}+\frac{6}{\tan(fx+e)+i}-i\ln(\tan(fx+e)+i)-\frac{8}{3(\tan(fx+e)+i)^3}\right)}{f\,c^3}$
risch	$-\frac{ia^4e^{6i(fx+e)}}{3c^3f}+\frac{ia^4e^{4i(fx+e)}}{2c^3f}-\frac{ia^4e^{2i(fx+e)}}{c^3f}+\frac{2a^4e}{c^3f}+\frac{ia^4\ln\left(e^{2i(fx+e)}+1\right)}{c^3f}$
norman	$\frac{-\frac{4ia^4\left(\tan^2(fx+e)\right)}{cf}-\frac{a^4x}{c}-\frac{8ia^4}{3cf}-\frac{3a^4x\left(\tan^2(fx+e)\right)}{c}-\frac{3a^4x\left(\tan^4(fx+e)\right)}{c}-\frac{a^4x\left(\tan^6(fx+e)\right)}{c}+\frac{2a^4\tan(fx+e)}{cf}-\frac{8a^4\left(\tan^3(fx+\ldots\right.}{3cf}}{c^2\left(1+\tan^2(fx+e)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^4/(c-I*c*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a^4/c^3*(-6*I/(tan(f*x+e)+I)^2+6/(tan(f*x+e)+I)-I*ln(tan(f*x+e)+I)-8/3/
(tan(f*x+e)+I)^3)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^4/(c-I*c*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.28, size = 72, normalized size = 0.63

$$\frac{-2i\,a^4e^{(6i\,fx+6i\,e)}+3i\,a^4e^{(4i\,fx+4i\,e)}-6i\,a^4e^{(2i\,fx+2i\,e)}+6i\,a^4\log\left(e^{(2i\,fx+2i\,e)}+1\right)}{6\,c^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^4/(c-I*c*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(-2*I*a^4*e^(6*I*f*x + 6*I*e) + 3*I*a^4*e^(4*I*f*x + 4*I*e) - 6*I*a^4*e
^(2*I*f*x + 2*I*e) + 6*I*a^4*log(e^(2*I*f*x + 2*I*e) + 1))/(c^3*f)
```

Sympy [A]
time = 0.34, size = 168, normalized size = 1.47

$$\frac{ia^4\log\left(e^{2ifx}+e^{-2ie}\right)}{c^3f}+\begin{cases}\frac{-2ia^4c^6f^2e^{6ie}e^{6ifx}+3ia^4c^6f^2e^{4ie}e^{4ifx}-6ia^4c^6f^2e^{2ie}e^{2ifx}}{6c^9f^3} & \text{for } c^9f^3\neq 0\\ \frac{x\left(2a^4e^{6ie}-2a^4e^{4ie}+2a^4e^{2ie}\right)}{c^3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**4/(c-I*c*tan(f*x+e))**3,x)
```

```
[Out] I*a**4*log(exp(2*I*f*x) + exp(-2*I*e))/(c**3*f) + Piecewise(((-2*I*a**4*c**
6*f**2*exp(6*I*e)*exp(6*I*f*x) + 3*I*a**4*c**6*f**2*exp(4*I*e)*exp(4*I*f*x)
 - 6*I*a**4*c**6*f**2*exp(2*I*e)*exp(2*I*f*x))/(6*c**9*f**3), Ne(c**9*f**3,
 0)), (x*(2*a**4*exp(6*I*e) - 2*a**4*exp(4*I*e) + 2*a**4*exp(2*I*e))/c**3,
True))
```

Giac [A]
time = 0.85, size = 193, normalized size = 1.69

$$-\frac{-\frac{30i\,a^4\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}{c^3}+\frac{60i\,a^4\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+i\right)}{c^3}-\frac{30i\,a^4\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}{c^3}+\frac{-147i\,a^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+1002\,a^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+2445i\,a^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-3820\,a^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2445i\,a^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1002\,a^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+147i\,a^4}{c^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+i\right)^6}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^4/(c-I*c*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/30*(-30*I*a^4*log(tan(1/2*f*x + 1/2*e) + 1)/c^3 + 60*I*a^4*log(tan(1/2*f
*x + 1/2*e) + I)/c^3 - 30*I*a^4*log(tan(1/2*f*x + 1/2*e) - 1)/c^3 + (-147*I
*a^4*tan(1/2*f*x + 1/2*e)^6 + 1002*a^4*tan(1/2*f*x + 1/2*e)^5 + 2445*I*a^4*
tan(1/2*f*x + 1/2*e)^4 - 3820*a^4*tan(1/2*f*x + 1/2*e)^3 - 2445*I*a^4*tan(1
/2*f*x + 1/2*e)^2 + 1002*a^4*tan(1/2*f*x + 1/2*e) + 147*I*a^4)/(c^3*(tan(1/
2*f*x + 1/2*e) + I)^6))/f
```

Mupad [B]
time = 4.73, size = 102, normalized size = 0.89

$$-\frac{\frac{6\,a^4\,\tan(e+f\,x)^2}{c^3}-\frac{8\,a^4}{3\,c^3}+\frac{a^4\,\tan(e+f\,x)\,6\mathrm{i}}{c^3}}{f\,\left(-{\tan\left(e+f\,x\right)}^3-{\tan\left(e+f\,x\right)}^2\,3\mathrm{i}+3\,\tan\left(e+f\,x\right)+1\mathrm{i}\right)}-\frac{a^4\,\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{c^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^4/(c - c*tan(e + f*x)*1i)^3,x)
```

```
[Out] - ((a^4*tan(e + f*x)*6i)/c^3 - (8*a^4)/(3*c^3) + (6*a^4*tan(e + f*x)^2)/c^3
)/(f*(3*tan(e + f*x) - tan(e + f*x)^2*3i - tan(e + f*x)^3 + 1i)) - (a^4*log
(tan(e + f*x) + 1i)*1i)/(c^3*f)
```

3.941 $\int \frac{(a+ia\tan(e+fx))^3}{(c-ic\tan(e+fx))^3}\,dx$

Optimal. Leaf size=50

$$-\frac{ia^3(c^2+ic^2\tan(e+fx))^3}{6f\,(c^3-ic^3\tan(e+fx))^3}$$

```
[Out] -1/6*I*a^3*(c^2+I*c^2*tan(f*x+e))^3/f/(c^3-I*c^3*tan(f*x+e))^3
```

Rubi [A]
time = 0.07, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {3603, 3568, 37}

$$-\frac{ia^3(c^2+ic^2\tan(e+fx))^3}{6f\,(c^3-ic^3\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^3/(c - I*c*Tan[e + f*x])^3,x]
```

```
[Out] ((-1/6*I)*a^3*(c^2 + I*c^2*Tan[e + f*x])^3)/(f*(c^3 - I*c^3*Tan[e + f*x])^3
)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^3}{(c-ic\tan(e+fx))^3}\,dx = (a^3c^3)\int \frac{\sec^6(e+fx)}{(c-ic\tan(e+fx))^6}\,dx$$
$$= \frac{(ia^3)\,\text{Subst}\left(\int \frac{(c-x)^2}{(c+x)^4}\,dx, x, -ic\tan(e+fx)\right)}{c^2 f}$$
$$= -\frac{ia^3(c+ic\tan(e+fx))^3}{6f\,(c^2-ic^2\tan(e+fx))^3}$$

Mathematica [A]
time = 0.28, size = 34, normalized size = 0.68

$$\frac{a^3(-i\cos(6(e+fx))+\sin(6(e+fx)))}{6c^3 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3/(c - I*c*Tan[e + f*x])^3,x]
[Out] (a^3*((-I)*Cos[6*(e + f*x)] + Sin[6*(e + f*x)]))/(6*c^3*f)
```

Maple [A]
time = 0.21, size = 50, normalized size = 1.00

method	result	size
risch	$-\frac{ia^3 e^{6i(fx+e)}}{6c^3 f}$	22
derivativedivides	$\frac{a^3\left(-\frac{2i}{(\tan(fx+e)+i)^2}+\frac{1}{\tan(fx+e)+i}-\frac{4}{3(\tan(fx+e)+i)^3}\right)}{f\,c^3}$	50
default	$\frac{a^3\left(-\frac{2i}{(\tan(fx+e)+i)^2}+\frac{1}{\tan(fx+e)+i}-\frac{4}{3(\tan(fx+e)+i)^3}\right)}{f\,c^3}$	50
norman	$\frac{\frac{a^3\tan(fx+e)}{cf}+\frac{a^3\left(\tan^5(fx+e)\right)}{cf}+\frac{2ia^3\left(\tan^2(fx+e)\right)}{cf}-\frac{ia^3}{3cf}-\frac{10a^3\left(\tan^3(fx+e)\right)}{3cf}-\frac{3ia^3\left(\tan^4(fx+e)\right)}{cf}}{c^2\left(1+\tan^2(fx+e)\right)^3}$	123

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
[Out] 1/f*a^3/c^3*(-2*I/(tan(f*x+e)+I)^2+1/(tan(f*x+e)+I)-4/3/(tan(f*x+e)+I)^3)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.61, size = 21, normalized size = 0.42

$$-\frac{i a^3 e^{(6i fx+6i e)}}{6 c^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/6*I*a^3*e^(6*I*f*x + 6*I*e)/(c^3*f)
```

Sympy [A]
time = 0.20, size = 46, normalized size = 0.92

$$\begin{cases} -\frac{ia^3e^{6ie}e^{6ifx}}{6c^3f} & \text{for } c^3f \neq 0 \\ \frac{a^3xe^{6ie}}{c^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3/(c-I*c*tan(f*x+e))**3,x)
```

```
[Out] Piecewise((-I*a**3*exp(6*I*e)*exp(6*I*f*x)/(6*c**3*f), Ne(c**3*f, 0)), (a**
3*x*exp(6*I*e)/c**3, True))
```

Giac [A]
time = 0.79, size = 72, normalized size = 1.44

$$-\frac{2 \left(3 a^3 \tan \left(\frac{1}{2} fx + \frac{1}{2} e\right)^5 - 10 a^3 \tan \left(\frac{1}{2} fx + \frac{1}{2} e\right)^3 + 3 a^3 \tan \left(\frac{1}{2} fx + \frac{1}{2} e\right)\right)}{3 c^3 f \left(\tan \left(\frac{1}{2} fx + \frac{1}{2} e\right) + i\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/3*(3*a^3*tan(1/2*f*x + 1/2*e)^5 - 10*a^3*tan(1/2*f*x + 1/2*e)^3 + 3*a^3*
tan(1/2*f*x + 1/2*e))/(c^3*f*(tan(1/2*f*x + 1/2*e) + I)^6)
```

Mupad [B]
time = 4.71, size = 55, normalized size = 1.10

$$-\frac{a^3 \left(\tan(e+f\,x)^2 - \frac{1}{3}\right)}{c^3\,f\,\left(-\tan\left(e+f\,x\right)^3 - \tan\left(e+f\,x\right)^2\,3\mathrm{i} + 3\tan\left(e+f\,x\right) + 1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^3/(c - c*tan(e + f*x)*1i)^3,x)
```

```
[Out] -(a^3*(tan(e + f*x)^2 - 1/3))/(c^3*f*(3*tan(e + f*x) - tan(e + f*x)^2*3i -
tan(e + f*x)^3 + 1i))
```

3.942 $\int \frac{(a+ia\tan(e+fx))^2}{(c-ic\tan(e+fx))^3}\,dx$

Optimal. Leaf size=58

$$-\frac{2ia^2}{3f(c-ic\tan(e+fx))^3}+\frac{ia^2}{2cf(c-ic\tan(e+fx))^2}$$

```
[Out] -2/3*I*a^2/f/(c-I*c*tan(f*x+e))^3+1/2*I*a^2/c/f/(c-I*c*tan(f*x+e))^2
```

Rubi [A]
time = 0.08, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {3603, 3568, 45}

$$\frac{ia^2}{2cf(c-ic\tan(e+fx))^2}-\frac{2ia^2}{3f(c-ic\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2/(c - I*c*Tan[e + f*x])^3,x]
```

```
[Out] (((-2*I)/3)*a^2)/(f*(c - I*c*Tan[e + f*x])^3) + ((I/2)*a^2)/(c*f*(c - I*c*T
an[e + f*x])^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^2}{(c-ic\tan(e+fx))^3}\,dx = (a^2c^2)\int \frac{\sec^4(e+fx)}{(c-ic\tan(e+fx))^5}\,dx$$
$$= \frac{(ia^2)\,\text{Subst}\left(\int \frac{c-x}{(c+x)^4}\,dx, x, -ic\tan(e+fx)\right)}{cf}$$
$$= \frac{(ia^2)\,\text{Subst}\left(\int \left(\frac{2c}{(c+x)^4}-\frac{1}{(c+x)^3}\right)dx, x, -ic\tan(e+fx)\right)}{cf}$$
$$= -\frac{2ia^2}{3f(c-ic\tan(e+fx))^3}+\frac{ia^2}{2cf(c-ic\tan(e+fx))^2}$$

Mathematica [A]
time = 0.67, size = 53, normalized size = 0.91

$$\frac{a^2(5\cos(e+fx)-i\sin(e+fx))(-i\cos(5(e+fx))+\sin(5(e+fx)))}{24c^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2/(c - I*c*Tan[e + f*x])^3,x]
[Out] (a^2*(5*Cos[e + f*x] - I*Sin[e + f*x])*((-I)*Cos[5*(e + f*x)] + Sin[5*(e +
f*x)]))/(24*c^3*f)
```

Maple [A]
time = 0.20, size = 39, normalized size = 0.67

method	result	size
derivativedivides	$\frac{a^2\left(-\frac{2}{3(\tan(fx+e)+i)^3}-\frac{i}{2(\tan(fx+e)+i)^2}\right)}{f\,c^3}$	39
default	$\frac{a^2\left(-\frac{2}{3(\tan(fx+e)+i)^3}-\frac{i}{2(\tan(fx+e)+i)^2}\right)}{f\,c^3}$	39
risch	$-\frac{ia^2 e^{6i(fx+e)}}{12c^3f}-\frac{ia^2 e^{4i(fx+e)}}{8c^3f}$	44
norman	$\frac{\frac{a^2\tan(fx+e)}{cf}+\frac{2ia^2\left(\tan^2(fx+e)\right)}{cf}-\frac{ia^2}{6cf}-\frac{5a^2\left(\tan^3(fx+e)\right)}{3cf}-\frac{ia^2\left(\tan^4(fx+e)\right)}{2cf}}{c^2\left(1+\tan^2(fx+e)\right)^3}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
[Out] 1/f*a^2/c^3*(-2/3/(tan(f*x+e)+I)^3-1/2*I/(tan(f*x+e)+I)^2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.21, size = 39, normalized size = 0.67

$$\frac{-2i\,a^2e^{(6i\,fx+6i\,e)}-3i\,a^2e^{(4i\,fx+4i\,e)}}{24\,c^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/24*(-2*I*a^2*e^(6*I*f*x + 6*I*e) - 3*I*a^2*e^(4*I*f*x + 4*I*e))/(c^3*f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(46) = 92$.
time = 0.20, size = 95, normalized size = 1.64

$$\begin{cases} \frac{-8ia^2c^3fe^{6ie}e^{6ifx}-12ia^2c^3fe^{4ie}e^{4ifx}}{96c^6f^2} & \text{for } c^6f^2\neq 0 \\ \frac{x\left(a^2e^{6ie}+a^2e^{4ie}\right)}{2c^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2/(c-I*c*tan(f*x+e))**3,x)
```

```
[Out] Piecewise(((-8*I*a**2*c**3*f*exp(6*I*e)*exp(6*I*f*x) - 12*I*a**2*c**3*f*exp
(4*I*e)*exp(4*I*f*x))/(96*c**6*f**2), Ne(c**6*f**2, 0)), (x*(a**2*exp(6*I*e
) + a**2*exp(4*I*e))/(2*c**3), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(48) = 96$.
time = 0.74, size = 106, normalized size = 1.83

$$-\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+3i\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-8\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-3i\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{3\,c^3f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+i\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/3*(3*a^2*tan(1/2*f*x + 1/2*e)^5 + 3*I*a^2*tan(1/2*f*x + 1/2*e)^4 - 8*a^2
*tan(1/2*f*x + 1/2*e)^3 - 3*I*a^2*tan(1/2*f*x + 1/2*e)^2 + 3*a^2*tan(1/2*f*
x + 1/2*e))/(c^3*f*(tan(1/2*f*x + 1/2*e) + I)^6)
```

Mupad [B]
time = 4.74, size = 56, normalized size = 0.97

$$\frac{a^2\,\left(1+\tan(e+f\,x)\;3{}\mathrm{i}\right)}{6\,c^3\,f\,\left(-{\tan\left(e+f\,x\right)}^3-{\tan\left(e+f\,x\right)}^2\,3{}\mathrm{i}+3\,\tan\left(e+f\,x\right)+1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^2/(c - c*tan(e + f*x)*1i)^3,x)
```

```
[Out] (a^2*(tan(e + f*x)*3i + 1))/(6*c^3*f*(3*tan(e + f*x) - tan(e + f*x)^2*3i -
tan(e + f*x)^3 + 1i))
```

3.943 $\int \frac{a+ia\tan(e+fx)}{(c-ic\tan(e+fx))^3}\,dx$

Optimal. Leaf size=25

$$-\frac{ia}{3f(c-ic\tan(e+fx))^3}$$

[Out] -1/3*I*a/f/(c-I*c*tan(f*x+e))^3

Rubi [A]
time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {3603, 3568, 32}

$$-\frac{ia}{3f(c-ic\tan(e+fx))^3}$$

Antiderivative was successfully verified.

[In] Int[(a + I*a*Tan[e + f*x])/(c - I*c*Tan[e + f*x])^3,x]

[Out] ((-1/3*I)*a)/(f*(c - I*c*Tan[e + f*x])^3)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{a+ia\tan(e+fx)}{(c-ic\tan(e+fx))^3}\,dx = (ac)\int \frac{\sec^2(e+fx)}{(c-ic\tan(e+fx))^4}\,dx$$
$$= \frac{(ia)\text{Subst}\left(\int \frac{1}{(c+x)^4}\,dx, x, -ic\tan(e+fx)\right)}{f}$$
$$= -\frac{ia}{3f(c-ic\tan(e+fx))^3}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 56 vs. 2(25) = 50.
time = 0.36, size = 56, normalized size = 2.24

$$\frac{a(3+4\cos(2(e+fx))-2i\sin(2(e+fx)))(-i\cos(4(e+fx))+\sin(4(e+fx)))}{24c^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])/(c - I*c*Tan[e + f*x])^3,x]
```

```
[Out] (a*(3 + 4*Cos[2*(e + f*x)] - (2*I)*Sin[2*(e + f*x)])*((-I)*Cos[4*(e + f*x)]
 + Sin[4*(e + f*x)]))/(24*c^3*f)
```

Maple [A]
time = 0.18, size = 21, normalized size = 0.84

method	result	size
derivativedivides	$-\frac{a}{3f\,c^3(\tan(fx+e)+i)^3}$	21
default	$-\frac{a}{3f\,c^3(\tan(fx+e)+i)^3}$	21
risch	$-\frac{ia\,e^{6i(fx+e)}}{24c^3f}-\frac{ia\,e^{4i(fx+e)}}{8c^3f}-\frac{ia\,e^{2i(fx+e)}}{8c^3f}$	59
norman	$\frac{\frac{a\tan(fx+e)}{cf}+\frac{ia\left(\tan^2(fx+e)\right)}{cf}-\frac{ia}{3cf}-\frac{a\left(\tan^3(fx+e)\right)}{3cf}}{c^2(1+\tan^2(fx+e))^3}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*a/c^3/(tan(f*x+e)+I)^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(20) = 40$.
time = 1.03, size = 48, normalized size = 1.92

$$\frac{-i\,ae^{(6i\,fx+6i\,e)} - 3i\,ae^{(4i\,fx+4i\,e)} - 3i\,ae^{(2i\,fx+2i\,e)}}{24\,c^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/24*(-I*a*e^(6*I*f*x + 6*I*e) - 3*I*a*e^(4*I*f*x + 4*I*e) - 3*I*a*e^(2*I*f
*x + 2*I*e))/(c^3*f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(20) = 40$.
time = 0.18, size = 129, normalized size = 5.16

$$\begin{cases} \frac{-64iac^6f^2e^{6ie}e^{6ifx}-192iac^6f^2e^{4ie}e^{4ifx}-192iac^6f^2e^{2ie}e^{2ifx}}{1536c^9f^3} & \text{for } c^9f^3 \neq 0 \\ \frac{x\left(ae^{6ie}+2ae^{4ie}+ae^{2ie}\right)}{4c^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))**3,x)
```

```
[Out] Piecewise(((-64*I*a*c**6*f**2*exp(6*I*e)*exp(6*I*f*x) - 192*I*a*c**6*f**2*e
xp(4*I*e)*exp(4*I*f*x) - 192*I*a*c**6*f**2*exp(2*I*e)*exp(2*I*f*x))/(1536*c
**9*f**3), Ne(c**9*f**3, 0)), (x*(a*exp(6*I*e) + 2*a*exp(4*I*e) + a*exp(2*I
*e))/(4*c**3), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(20) = 40$.
time = 0.68, size = 96, normalized size = 3.84

$$-\frac{2\left(3\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+6i\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-10\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-6i\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{3\,c^3f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+i\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/3*(3*a*tan(1/2*f*x + 1/2*e)^5 + 6*I*a*tan(1/2*f*x + 1/2*e)^4 - 10*a*tan(
1/2*f*x + 1/2*e)^3 - 6*I*a*tan(1/2*f*x + 1/2*e)^2 + 3*a*tan(1/2*f*x + 1/2*e
))/(c^3*f*(tan(1/2*f*x + 1/2*e) + I)^6)
```

Mupad [B]
time = 4.64, size = 20, normalized size = 0.80

$$-\frac{a}{3\,c^3\,f\,{\left(\mathrm{tan}\left(e+f\,x\right)+1{}\mathrm{i}\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)/(c - c*tan(e + f*x)*1i)^3,x)
```

```
[Out] -a/(3*c^3*f*(tan(e + f*x) + 1i)^3)
```

3.944 $\int \frac{1}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^3}\,dx$

Optimal. Leaf size=131

$$\frac{x}{4ac^3}-\frac{i}{12af(c-ic\tan(e+fx))^3}-\frac{i}{8acf(c-ic\tan(e+fx))^2}-\frac{3i}{16af\left(c^3-ic^3\tan(e+fx)\right)}+\frac{i}{16af\left(c^3+ic^3\right.}$$

```
[Out] 1/4*x/a/c^3-1/12*I/a/f/(c-I*c*tan(f*x+e))^3-1/8*I/a/c/f/(c-I*c*tan(f*x+e))^
2-3/16*I/a/f/(c^3-I*c^3*tan(f*x+e))+1/16*I/a/f/(c^3+I*c^3*tan(f*x+e))
```

Rubi [A]
time = 0.11, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {3603, 3568, 46, 212}

$$-\frac{3i}{16af\left(c^3-ic^3\tan(e+fx)\right)}+\frac{i}{16af\left(c^3+ic^3\tan(e+fx)\right)}+\frac{x}{4ac^3}-\frac{i}{8acf(c-ic\tan(e+fx))^2}-\frac{i}{12af(c-ic\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^3),x]
```

```
[Out] x/(4*a*c^3) - (I/12)/(a*f*(c - I*c*Tan[e + f*x])^3) - (I/8)/(a*c*f*(c - I*c
*Tan[e + f*x])^2) - ((3*I)/16)/(a*f*(c^3 - I*c^3*Tan[e + f*x])) + (I/16)/(a
*f*(c^3 + I*c^3*Tan[e + f*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^3}\,dx = \frac{\int \frac{\cos^2(e+fx)}{(c-ic\tan(e+fx))^2}\,dx}{ac}$$
$$= \frac{(ic^2)\,\text{Subst}\left(\int \frac{1}{(c-x)^2(c+x)^4}\,dx, x, -ic\tan(e+fx)\right)}{af}$$
$$= \frac{(ic^2)\,\text{Subst}\left(\int \left(\frac{1}{16c^4(c-x)^2} + \frac{1}{4c^2(c+x)^4} + \frac{1}{4c^3(c+x)^3} + \frac{3}{16c^4(c+x)^2}\right.\right.}{af}$$
$$= -\frac{i}{12af(c-ic\tan(e+fx))^3} - \frac{i}{8acf(c-ic\tan(e+fx))^2}$$
$$= \frac{x}{4ac^3} - \frac{i}{12af(c-ic\tan(e+fx))^3} - \frac{i}{8acf(c-ic\tan(e+$$

Mathematica [A]

time = 0.67, size = 115, normalized size = 0.88

$$\frac{\sec(e+fx)(\cos(3(e+fx))+i\sin(3(e+fx)))(-9+(-3-12ifx)\cos(2(e+fx))+\cos(4(e+fx))-3i\sin(2(e+fx))-12fx\sin(2(e+fx))-2i\sin(4(e+fx)))}{48ac^3f(-i+\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^3),x]
```

```
[Out] (Sec[e + f*x]*(Cos[3*(e + f*x)] + I*Sin[3*(e + f*x)])*(-9 + (-3 - (12*I)*f*
x)*Cos[2*(e + f*x)] + Cos[4*(e + f*x)] - (3*I)*Sin[2*(e + f*x)] - 12*f*x*Si
n[2*(e + f*x)] - (2*I)*Sin[4*(e + f*x)]))/(48*a*c^3*f*(-I + Tan[e + f*x]))
```

Maple [A]

time = 0.24, size = 91, normalized size = 0.69

method	result	size
derivativedivides	$\frac{\frac{i}{8(\tan(fx+e)+i)^2}+\frac{i\ln(\tan(fx+e)+i)}{8}-\frac{1}{12(\tan(fx+e)+i)^3}+\frac{3}{16(\tan(fx+e)+i)}-\frac{i\ln(\tan(fx+e)-i)}{8}+\frac{1}{16\tan(fx+e)-16i}}{f\,a\,c^3}$	91
default	$\frac{\frac{i}{8(\tan(fx+e)+i)^2}+\frac{i\ln(\tan(fx+e)+i)}{8}-\frac{1}{12(\tan(fx+e)+i)^3}+\frac{3}{16(\tan(fx+e)+i)}-\frac{i\ln(\tan(fx+e)-i)}{8}+\frac{1}{16\tan(fx+e)-16i}}{f\,a\,c^3}$	91

risch	$\frac{x}{4a\,c^3}-\frac{i\mathrm{e}^{6i(fx+e)}}{96a\,c^3f}-\frac{i\mathrm{e}^{4i(fx+e)}}{16a\,c^3f}-\frac{5i\cos(2fx+2e)}{32a\,c^3f}+\frac{7\sin(2fx+2e)}{32a\,c^3f}$	94
norman	$\frac{\frac{x}{4ac}+\frac{3\tan(fx+e)}{4acf}+\frac{2\left(\tan^3(fx+e)\right)}{3acf}+\frac{\tan^5(fx+e)}{4acf}+\frac{3x\left(\tan^2(fx+e)\right)}{4ac}+\frac{3x\left(\tan^4(fx+e)\right)}{4ac}+\frac{x\left(\tan^6(fx+e)\right)}{4ac}-\frac{i}{3acf}}{(1+\tan^2(fx+e))^3c^2}$	145

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a/c^3*(1/8*I/(tan(f*x+e)+I)^2+1/8*I*ln(tan(f*x+e)+I)-1/12/(tan(f*x+e)+I
)^3+3/16/(tan(f*x+e)+I)-1/8*I*ln(tan(f*x+e)-I)+1/16/(tan(f*x+e)-I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.18, size = 73, normalized size = 0.56

$$\frac{\left(24\,fxe^{(2i\,fx+2i\,e)}-i\,e^{(8i\,fx+8i\,e)}-6i\,e^{(6i\,fx+6i\,e)}-18i\,e^{(4i\,fx+4i\,e)}+3i\right)e^{(-2i\,fx-2i\,e)}}{96\,ac^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/96*(24*f*x*e^(2*I*f*x + 2*I*e) - I*e^(8*I*f*x + 8*I*e) - 6*I*e^(6*I*f*x +
 6*I*e) - 18*I*e^(4*I*f*x + 4*I*e) + 3*I)*e^(-2*I*f*x - 2*I*e)/(a*c^3*f)
```

Sympy [A]
time = 0.22, size = 207, normalized size = 1.58

$$\begin{cases}\frac{\left(-8192ia^3c^9f^3e^{8ie}e^{6ifx}-49152ia^3c^9f^3e^{6ie}e^{4ifx}-147456ia^3c^9f^3e^{4ie}e^{2ifx}+24576ia^3c^9f^3e^{-2ifx}\right)e^{-2ie}}{786432a^4c^{12}f^4} & \text{for}\;a^4c^{12}f^4e^{2ie}\neq 0\\ x\left(\frac{\left(e^{8ie}+4e^{6ie}+6e^{4ie}+4e^{2ie}+1\right)e^{-2ie}}{16ac^3}-\frac{1}{4ac^3}\right) & \text{otherwise}\end{cases}+\frac{x}{4ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))**3,x)
```

```
[Out] Piecewise(((-8192*I*a**3*c**9*f**3*exp(8*I*e)*exp(6*I*f*x) - 49152*I*a**3*c
**9*f**3*exp(6*I*e)*exp(4*I*f*x) - 147456*I*a**3*c**9*f**3*exp(4*I*e)*exp(2
```

```
*I*f*x) + 24576*I*a**3*c**9*f**3*exp(-2*I*f*x))*exp(-2*I*e)/(786432*a**4*c*
*12*f**4), Ne(a**4*c**12*f**4*exp(2*I*e), 0)), (x*((exp(8*I*e) + 4*exp(6*I*
e) + 6*exp(4*I*e) + 4*exp(2*I*e) + 1)*exp(-2*I*e)/(16*a*c**3) - 1/(4*a*c**3
)), True)) + x/(4*a*c**3)
```

Giac [A]

time = 0.59, size = 123, normalized size = 0.94

$$-\frac{-\frac{6i\log(\tan(fx+e)+i)}{ac^3}+\frac{6i\log(\tan(fx+e)-i)}{ac^3}+\frac{3(-2i\tan(fx+e)-3)}{ac^3(\tan(fx+e)-i)}+\frac{11i\tan(fx+e)^3-42\tan(fx+e)^2-57i\tan(fx+e)+30}{ac^3(\tan(fx+e)+i)^3}}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/48*(-6*I*log(tan(f*x + e) + I)/(a*c^3) + 6*I*log(tan(f*x + e) - I)/(a*c^
3) + 3*(-2*I*tan(f*x + e) - 3)/(a*c^3*(tan(f*x + e) - I)) + (11*I*tan(f*x +
 e)^3 - 42*tan(f*x + e)^2 - 57*I*tan(f*x + e) + 30)/(a*c^3*(tan(f*x + e) +
I)^3))/f
```

Mupad [B]

time = 4.83, size = 77, normalized size = 0.59

$$\frac{x}{4\,a\,c^3}-\frac{-\frac{\tan(e+f\,x)^3\,1\mathrm{i}}{4}+\frac{\tan(e+f\,x)^2}{2}+\frac{\tan(e+f\,x)\,1\mathrm{i}}{12}+\frac{1}{3}}{a\,c^3\,f\,\left(1+\tan\left(e+f\,x\right)\,1\mathrm{i}\right)\,{\left(\tan\left(e+f\,x\right)+1\mathrm{i}\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i)^3),x)
```

```
[Out] x/(4*a*c^3) - ((tan(e + f*x)*1i)/12 + tan(e + f*x)^2/2 - (tan(e + f*x)^3*1i
)/4 + 1/3)/(a*c^3*f*(tan(e + f*x)*1i + 1)*(tan(e + f*x) + 1i)^3)
```

3.945 $\int \frac{1}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^3}\,dx$

Optimal. Leaf size=161

$$\frac{5x}{16a^2c^3}-\frac{i}{24a^2f(c-ic\tan(e+fx))^3}-\frac{3i}{32a^2cf(c-ic\tan(e+fx))^2}+\frac{i}{32a^2cf(c+ic\tan(e+fx))^2}-\frac{}{16a^2f\,(c}$$

```
[Out] 5/16*x/a^2/c^3-1/24*I/a^2/f/(c-I*c*tan(f*x+e))^3-3/32*I/a^2/c/f/(c-I*c*tan(
f*x+e))^2+1/32*I/a^2/c/f/(c+I*c*tan(f*x+e))^2-3/16*I/a^2/f/(c^3-I*c^3*tan(f
*x+e))+1/8*I/a^2/f/(c^3+I*c^3*tan(f*x+e))
```

Rubi [A]
time = 0.12, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {3603, 3568, 46, 212}

$$-\frac{3i}{16a^2f\,(c^3-ic^3\tan(e+fx))}+\frac{i}{8a^2f\,(c^3+ic^3\tan(e+fx))}+\frac{5x}{16a^2c^3}-\frac{3i}{32a^2cf(c-ic\tan(e+fx))^2}+\frac{i}{32a^2cf(c+ic\tan(e+fx))^2}-\frac{i}{24a^2f(c-ic\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^3),x]
```

```
[Out] (5*x)/(16*a^2*c^3) - (I/24)/(a^2*f*(c - I*c*Tan[e + f*x])^3) - ((3*I)/32)/(
a^2*c*f*(c - I*c*Tan[e + f*x])^2) + (I/32)/(a^2*c*f*(c + I*c*Tan[e + f*x])^
2) - ((3*I)/16)/(a^2*f*(c^3 - I*c^3*Tan[e + f*x])) + (I/8)/(a^2*f*(c^3 + I*
c^3*Tan[e + f*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^3}\,dx &= \frac{\int \frac{\cos^4(e+fx)}{c-ic\tan(e+fx)}\,dx}{a^2c^2}\\
&= \frac{(ic^3)\,\text{Subst}\left(\int \frac{1}{(c-x)^3(c+x)^4}\,dx, x, -ic\tan(e+fx)\right)}{a^2f}\\
&= \frac{(ic^3)\,\text{Subst}\left(\int \left(\frac{1}{16c^4(c-x)^3}+\frac{1}{8c^5(c-x)^2}+\frac{1}{8c^3(c+x)^4}+\frac{3}{16c^4(c+x}\right.\right.}{a^2f}\\
&= -\frac{i}{24a^2f(c-ic\tan(e+fx))^3}-\frac{3i}{32a^2cf(c-ic\tan(e+fx}\\
&= \frac{5x}{16a^2c^3}-\frac{i}{24a^2f(c-ic\tan(e+fx))^3}-\frac{3i}{32a^2cf(c-ic\tan}
\end{aligned}$$

Mathematica [A]
time = 0.97, size = 111, normalized size = 0.69

$$\frac{(\cos(e+fx)+i\sin(e+fx))(60(-i+2fx)\cos(e+fx)+15i\cos(3(e+fx))+i\cos(5(e+fx))+60\sin(e+fx)-120ifx\sin(e+fx)+45\sin(3(e+fx))+5\sin(5(e+fx)))}{384a^2c^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^3),x]
[Out] ((Cos[e + f*x] + I*Sin[e + f*x])*(60*(-I + 2*f*x)*Cos[e + f*x] + (15*I)*Cos
[3*(e + f*x)] + I*Cos[5*(e + f*x)] + 60*Sin[e + f*x] - (120*I)*f*x*Sin[e +
f*x] + 45*Sin[3*(e + f*x)] + 5*Sin[5*(e + f*x)]))/(384*a^2*c^3*f)
```

Maple [A]
time = 0.18, size = 105, normalized size = 0.65

method	result
derivativedivides	$\frac{\frac{3i}{32(\tan(fx+e)+i)^2}+\frac{5i\ln(\tan(fx+e)+i)}{32}-\frac{1}{24(\tan(fx+e)+i)^3}+\frac{3}{16(\tan(fx+e)+i)}-\frac{5i\ln(\tan(fx+e)-i)}{32}-\frac{i}{32(\tan(fx+e)-i)^2}+\frac{}{8\tan(f}}{f\,a^2c^3}$

default	$\frac{\frac{3i}{32(\tan(fx+e)+i)^2}+\frac{5i\ln(\tan(fx+e)+i)}{32}-\frac{1}{24(\tan(fx+e)+i)^3}+\frac{3}{16(\tan(fx+e)+i)}-\frac{5i\ln(\tan(fx+e)-i)}{32}-\frac{i}{32(\tan(fx+e)-i)^2}+\frac{}{8\tan}}{f\,a^2c^3}$
risch	$\frac{5x}{16a^2c^3}-\frac{i e^{6i(fx+e)}}{192a^2c^3f}-\frac{i\cos(4fx+4e)}{32a^2c^3f}+\frac{3\sin(4fx+4e)}{64a^2c^3f}-\frac{5i\cos(2fx+2e)}{64a^2c^3f}+\frac{15\sin(2fx+2e)}{64a^2c^3f}$
norman	$\frac{\frac{5x}{16ac}-\frac{i}{6acf}+\frac{11\tan(fx+e)}{16acf}+\frac{5\left(\tan^3(fx+e)\right)}{6acf}+\frac{5\left(\tan^5(fx+e)\right)}{16acf}+\frac{15x\left(\tan^2(fx+e)\right)}{16ac}+\frac{15x\left(\tan^4(fx+e)\right)}{16ac}+\frac{5x\left(\tan^6(fx+e)\right)}{16ac}}{\left(1+\tan^2(fx+e)\right)^3a\,c^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a^2/c^3*(3/32*I/(tan(f*x+e)+I)^2+5/32*I*ln(tan(f*x+e)+I)-1/24/(tan(f*x+
e)+I)^3+3/16/(tan(f*x+e)+I)-5/32*I*ln(tan(f*x+e)-I)-1/32*I/(tan(f*x+e)-I)^2
+1/8/(tan(f*x+e)-I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.85, size = 85, normalized size = 0.53

$$\frac{\left(120\,fxe^{(4i\,fx+4i\,e)}-2i\,e^{(10i\,fx+10i\,e)}-15i\,e^{(8i\,fx+8i\,e)}-60i\,e^{(6i\,fx+6i\,e)}+30i\,e^{(2i\,fx+2i\,e)}+3i\right)e^{(-4i\,fx-4i\,e)}}{384\,a^2c^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^3,x, algorithm="fricas"
)
```

```
[Out] 1/384*(120*f*x*e^(4*I*f*x + 4*I*e) - 2*I*e^(10*I*f*x + 10*I*e) - 15*I*e^(8*
I*f*x + 8*I*e) - 60*I*e^(6*I*f*x + 6*I*e) + 30*I*e^(2*I*f*x + 2*I*e) + 3*I)
*e^(-4*I*f*x - 4*I*e)/(a^2*c^3*f)
```

Sympy [A]
time = 0.34, size = 258, normalized size = 1.60

$$\begin{cases}\frac{\left(-33554432ia^8c^{12}f^4e^{12ie}e^{6ifx}-251658240ia^8c^{12}f^4e^{10ie}e^{4ifx}-1006632960ia^8c^{12}f^4e^{8ie}e^{2ifx}+503316480ia^8c^{12}f^4e^{4ie}e^{-2ifx}+50331648ia^8c^{12}f^4e^{2ie}e^{-4ifx}\right)e^{-6ie}}{6442450944a^{10}c^{15}f^5} & \text{for } a^{10}c^{15}f^5e^{6ie}\neq 0\\ x\left(\frac{\left(e^{10ie}+5e^{8ie}+10e^{6ie}+10e^{4ie}+5e^{2ie}+1\right)e^{-4ie}}{32a^2c^3}-\frac{5}{16a^2c^3}\right) & \text{otherwise}\end{cases}+\frac{5x}{16a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**2/(c-I*c*tan(f*x+e))**3,x)
```

```
[Out] Piecewise(((-33554432*I*a**8*c**12*f**4*exp(12*I*e)*exp(6*I*f*x) - 25165824
0*I*a**8*c**12*f**4*exp(10*I*e)*exp(4*I*f*x) - 1006632960*I*a**8*c**12*f**4
*exp(8*I*e)*exp(2*I*f*x) + 503316480*I*a**8*c**12*f**4*exp(4*I*e)*exp(-2*I*
f*x) + 50331648*I*a**8*c**12*f**4*exp(2*I*e)*exp(-4*I*f*x))*exp(-6*I*e)/(64
42450944*a**10*c**15*f**5), Ne(a**10*c**15*f**5*exp(6*I*e), 0)), (x*((exp(1
0*I*e) + 5*exp(8*I*e) + 10*exp(6*I*e) + 10*exp(4*I*e) + 5*exp(2*I*e) + 1)*e
xp(-4*I*e)/(32*a**2*c**3) - 5/(16*a**2*c**3)), True)) + 5*x/(16*a**2*c**3)
```

Giac [A]
time = 0.67, size = 137, normalized size = 0.85

$$-\frac{-\frac{30i\log\left(\tan\left(fx+e\right)+i\right)}{a^{2}c^{3}}+\frac{30i\log\left(\tan\left(fx+e\right)-i\right)}{a^{2}c^{3}}+\frac{3\left(15i\tan\left(fx+e\right)^{2}+38\tan\left(fx+e\right)-25i\right)}{a^{2}c^{3}\left(i\tan\left(fx+e\right)+1\right)^{2}}-\frac{-55i\tan\left(fx+e\right)^{3}+201\tan\left(fx+e\right)^{2}+255i\tan\left(fx+e\right)-117}{a^{2}c^{3}\left(\tan\left(fx+e\right)+i\right)^{3}}}{192f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/192*(-30*I*log(tan(f*x + e) + I)/(a^2*c^3) + 30*I*log(tan(f*x + e) - I)/
(a^2*c^3) + 3*(15*I*tan(f*x + e)^2 + 38*tan(f*x + e) - 25*I)/(a^2*c^3*(I*ta
n(f*x + e) + 1)^2) - (-55*I*tan(f*x + e)^3 + 201*tan(f*x + e)^2 + 255*I*tan
(f*x + e) - 117)/(a^2*c^3*(tan(f*x + e) + I)^3))/f
```

Mupad [B]
time = 5.25, size = 87, normalized size = 0.54

$$\frac{5\,x}{16\,a^2\,c^3}-\frac{\frac{5\,{\mathrm{tan}\left(e+f\,x\right)}^4}{16}+\frac{{\mathrm{tan}\left(e+f\,x\right)}^3\,5{}\mathrm{i}}{16}+\frac{25\,{\mathrm{tan}\left(e+f\,x\right)}^2}{48}+\frac{\mathrm{tan}\left(e+f\,x\right)\,25{}\mathrm{i}}{48}+\frac{1}{6}}{a^2\,c^3\,f\,{\left(1+\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^2\,{\left(\mathrm{tan}\left(e+f\,x\right)+1{}\mathrm{i}\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i)^3),x)
```

```
[Out] (5*x)/(16*a^2*c^3) - ((tan(e + f*x)*25i)/48 + (25*tan(e + f*x)^2)/48 + (tan
(e + f*x)^3*5i)/16 + (5*tan(e + f*x)^4)/16 + 1/6)/(a^2*c^3*f*(tan(e + f*x)*
1i + 1)^2*(tan(e + f*x) + 1i)^3)
```

3.946 $\int \frac{1}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^3}\,dx$

Optimal. Leaf size=91

$$\frac{5x}{16a^3c^3}+\frac{5\cos(e+fx)\sin(e+fx)}{16a^3c^3f}+\frac{5\cos^3(e+fx)\sin(e+fx)}{24a^3c^3f}+\frac{\cos^5(e+fx)\sin(e+fx)}{6a^3c^3f}$$

```
[Out] 5/16*x/a^3/c^3+5/16*cos(f*x+e)*sin(f*x+e)/a^3/c^3/f+5/24*cos(f*x+e)^3*sin(f
*x+e)/a^3/c^3/f+1/6*cos(f*x+e)^5*sin(f*x+e)/a^3/c^3/f
```

Rubi [A]
time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 2715, 8}

$$\frac{\sin(e+fx)\cos^5(e+fx)}{6a^3c^3f}+\frac{5\sin(e+fx)\cos^3(e+fx)}{24a^3c^3f}+\frac{5\sin(e+fx)\cos(e+fx)}{16a^3c^3f}+\frac{5x}{16a^3c^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])^3),x]
```

```
[Out] (5*x)/(16*a^3*c^3) + (5*Cos[e + f*x]*Sin[e + f*x])/(16*a^3*c^3*f) + (5*Cos[
e + f*x]^3*Sin[e + f*x])/(24*a^3*c^3*f) + (Cos[e + f*x]^5*Sin[e + f*x])/(6*
a^3*c^3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^3}\,dx = \frac{\int \cos^6(e+fx)\,dx}{a^3c^3}$$
$$= \frac{\cos^5(e+fx)\sin(e+fx)}{6a^3c^3f} + \frac{5\int \cos^4(e+fx)\,dx}{6a^3c^3}$$
$$= \frac{5\cos^3(e+fx)\sin(e+fx)}{24a^3c^3f} + \frac{\cos^5(e+fx)\sin(e+fx)}{6a^3c^3f} +$$
$$= \frac{5\cos(e+fx)\sin(e+fx)}{16a^3c^3f} + \frac{5\cos^3(e+fx)\sin(e+fx)}{24a^3c^3f} +$$
$$= \frac{5x}{16a^3c^3} + \frac{5\cos(e+fx)\sin(e+fx)}{16a^3c^3f} + \frac{5\cos^3(e+fx)\sin(}{24a^3c^3f}$$

Mathematica [A]
time = 0.06, size = 49, normalized size = 0.54

$$\frac{60e+60fx+45\sin(2(e+fx))+9\sin(4(e+fx))+\sin(6(e+fx))}{192a^3c^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])^3),x]
```

```
[Out] (60*e + 60*f*x + 45*Sin[2*(e + f*x)] + 9*Sin[4*(e + f*x)] + Sin[6*(e + f*x)
])/(192*a^3*c^3*f)
```

Maple [C] Result contains complex when optimal does not.
time = 0.18, size = 118, normalized size = 1.30

method	result
risch	$\frac{5x}{16a^3c^3}+\frac{\sin(6fx+6e)}{192a^3c^3f}+\frac{3\sin(4fx+4e)}{64a^3c^3f}+\frac{15\sin(2fx+2e)}{64a^3c^3f}$
derivativedivides	$\frac{-\frac{5i\ln(\tan(fx+e)-i)}{32}-\frac{i}{16(\tan(fx+e)-i)^2}-\frac{1}{48(\tan(fx+e)-i)^3}+\frac{5}{32(\tan(fx+e)-i)}+\frac{i}{16(\tan(fx+e)+i)^2}+\frac{5i\ln(\tan(fx+e)+i)}{32}-\frac{}{48(\mathrm{ta}}}{f\,a^3c^3}$
default	$\frac{-\frac{5i\ln(\tan(fx+e)-i)}{32}-\frac{i}{16(\tan(fx+e)-i)^2}-\frac{1}{48(\tan(fx+e)-i)^3}+\frac{5}{32(\tan(fx+e)-i)}+\frac{i}{16(\tan(fx+e)+i)^2}+\frac{5i\ln(\tan(fx+e)+i)}{32}-\frac{}{48(\mathrm{ta}}}{f\,a^3c^3}$
norman	$\frac{\frac{5x}{16ac}+\frac{11\tan(fx+e)}{16acf}+\frac{5\left(\tan^3(fx+e)\right)}{6acf}+\frac{5\left(\tan^5(fx+e)\right)}{16acf}+\frac{15x\left(\tan^2(fx+e)\right)}{16ac}+\frac{15x\left(\tan^4(fx+e)\right)}{16ac}+\frac{5x\left(\tan^6(fx+e)\right)}{16ac}}{\left(1+\tan^2(fx+e)\right)^3a^2c^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a^3/c^3*(-5/32*I*ln(tan(f*x+e)-I)-1/16*I/(tan(f*x+e)-I)^2-1/48/(tan(f*x
+e)-I)^3+5/32/(tan(f*x+e)-I)+1/16*I/(tan(f*x+e)+I)^2+5/32*I*ln(tan(f*x+e)+I
)-1/48/(tan(f*x+e)+I)^3+5/32/(tan(f*x+e)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [C] Result contains complex when optimal does not.
time = 1.34, size = 97, normalized size = 1.07

$$\frac{\left(120\, fxe^{(6i\, fx+6i\, e)} - i\, e^{(12i\, fx+12i\, e)} - 9i\, e^{(10i\, fx+10i\, e)} - 45i\, e^{(8i\, fx+8i\, e)} + 45i\, e^{(4i\, fx+4i\, e)} + 9i\, e^{(2i\, fx+2i\, e)} + i\right)e^{(-6i\, fx-6i\, e)}}{384\, a^3c^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^3,x, algorithm="fricas"
)
```

```
[Out] 1/384*(120*f*x*e^(6*I*f*x + 6*I*e) - I*e^(12*I*f*x + 12*I*e) - 9*I*e^(10*I*
f*x + 10*I*e) - 45*I*e^(8*I*f*x + 8*I*e) + 45*I*e^(4*I*f*x + 4*I*e) + 9*I*e
^(2*I*f*x + 2*I*e) + I)*e^(-6*I*f*x - 6*I*e)/(a^3*c^3*f)
```

Sympy [A]
time = 0.37, size = 296, normalized size = 3.25

$$\begin{cases} \frac{\left(-103079215104ia^{15}c^{15}f^5e^{18ie}e^{6ifx} - 927712935936ia^{15}c^{15}f^5e^{16ie}e^{4ifx} - 4638564679680ia^{15}c^{15}f^5e^{14ie}e^{2ifx} + 4638564679680ia^{15}c^{15}f^5e^{10ie}e^{-2ifx} + 927712935936ia^{15}c^{15}f^5e^{8ie}e^{-4ifx} + 103079215104ia^{15}c^{15}f^5e^{6ie}e^{-6ifx}\right)e^{-12ie}}{39582418599936a^{18}c^{18}f^6} & \text{for}\ a^{18}c^{18}f^6e^{12ie} \neq 0 \\ x\left(\frac{\left(e^{12ie}+6e^{10ie}+15e^{8ie}+20e^{6ie}+15e^{4ie}+6e^{2ie}+1\right)e^{-6ie}}{64a^3c^3} - \frac{5}{16a^3c^3}\right) & \text{otherwise} \end{cases} + \frac{5x}{16a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**3/(c-I*c*tan(f*x+e))**3,x)
```

```
[Out] Piecewise(((-103079215104*I*a**15*c**15*f**5*exp(18*I*e)*exp(6*I*f*x) - 927
712935936*I*a**15*c**15*f**5*exp(16*I*e)*exp(4*I*f*x) - 4638564679680*I*a**
15*c**15*f**5*exp(14*I*e)*exp(2*I*f*x) + 4638564679680*I*a**15*c**15*f**5*e
xp(10*I*e)*exp(-2*I*f*x) + 927712935936*I*a**15*c**15*f**5*exp(8*I*e)*exp(-
4*I*f*x) + 103079215104*I*a**15*c**15*f**5*exp(6*I*e)*exp(-6*I*f*x))*exp(-1
2*I*e)/(39582418599936*a**18*c**18*f**6), Ne(a**18*c**18*f**6*exp(12*I*e),
0)), (x*((exp(12*I*e) + 6*exp(10*I*e) + 15*exp(8*I*e) + 20*exp(6*I*e) + 15*
exp(4*I*e) + 6*exp(2*I*e) + 1)*exp(-6*I*e)/(64*a**3*c**3) - 5/(16*a**3*c**3
)), True)) + 5*x/(16*a**3*c**3)
```

Giac [A]
time = 0.69, size = 72, normalized size = 0.79

$$\frac{\frac{15\,(fx+e)}{a^3c^3} + \frac{15\,\tan(fx+e)^5+40\,\tan(fx+e)^3+33\,\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)^3 a^3c^3}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/48*(15*(f*x + e)/(a^3*c^3) + (15*tan(f*x + e)^5 + 40*tan(f*x + e)^3 + 33*
tan(f*x + e))/((tan(f*x + e)^2 + 1)^3*a^3*c^3))/f
```

Mupad [B]
time = 4.72, size = 57, normalized size = 0.63

$$\frac{5\,x}{16\,a^3\,c^3} + \frac{\cos\left(e+f\,x\right)^6\,\left(\frac{5\tan(e+f\,x)^5}{16} + \frac{5\tan(e+f\,x)^3}{6} + \frac{11\tan(e+f\,x)}{16}\right)}{a^3\,c^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i)^3),x)
```

```
[Out] (5*x)/(16*a^3*c^3) + (cos(e + f*x)^6*((11*tan(e + f*x))/16 + (5*tan(e + f*x
)^3)/6 + (5*tan(e + f*x)^5)/16))/(a^3*c^3*f)
```

3.947 $\int \frac{(a+ia\tan(e+fx))^6}{(c-ic\tan(e+fx))^4}\,dx$

Optimal. Leaf size=160

$$\frac{10a^6x}{c^4}-\frac{10ia^6\log(\cos(e+fx))}{c^4f}-\frac{a^6\tan(e+fx)}{c^4f}-\frac{8ia^6}{f(c-ic\tan(e+fx))^4}+\frac{80ia^6}{3cf(c-ic\tan(e+fx))^3}-\frac{}{f\,(c^2}$$

```
[Out] 10*a^6*x/c^4-10*I*a^6*ln(cos(f*x+e))/c^4/f-a^6*tan(f*x+e)/c^4/f-8*I*a^6/f/(
c-I*c*tan(f*x+e))^4+80/3*I*a^6/c/f/(c-I*c*tan(f*x+e))^3-40*I*a^6/f/(c^2-I*c
^2*tan(f*x+e))^2+40*I*a^6/f/(c^4-I*c^4*tan(f*x+e))
```

Rubi [A]
time = 0.12, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 45}

$$-\frac{a^6\tan(e+fx)}{c^4f}+\frac{40ia^6}{f\,(c^4-ic^4\tan(e+fx))}-\frac{10ia^6\log(\cos(e+fx))}{c^4f}+\frac{10a^6x}{c^4}-\frac{40ia^6}{f\,(c^2-ic^2\tan(e+fx))^2}+\frac{80ia^6}{3cf(c-ic\tan(e+fx))^3}-\frac{8ia^6}{f(c-ic\tan(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^6/(c - I*c*Tan[e + f*x])^4,x]
```

```
[Out] (10*a^6*x)/c^4 - ((10*I)*a^6*Log[Cos[e + f*x]])/(c^4*f) - (a^6*Tan[e + f*x]
)/(c^4*f) - ((8*I)*a^6)/(f*(c - I*c*Tan[e + f*x])^4) + (((80*I)/3)*a^6)/(c*
f*(c - I*c*Tan[e + f*x])^3) - ((40*I)*a^6)/(f*(c^2 - I*c^2*Tan[e + f*x])^2)
 + ((40*I)*a^6)/(f*(c^4 - I*c^4*Tan[e + f*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
```

```
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^6}{(c-ic\tan(e+fx))^4}\,dx = \left(a^6c^6\right)\int \frac{\sec^{12}(e+fx)}{(c-ic\tan(e+fx))^{10}}\,dx$$
$$= \frac{\left(ia^6\right)\text{Subst}\left(\int \frac{(c-x)^5}{(c+x)^5}\,dx, x, -ic\tan(e+fx)\right)}{c^5f}$$
$$= \frac{\left(ia^6\right)\text{Subst}\left(\int \left(-1+\frac{32c^5}{(c+x)^5}-\frac{80c^4}{(c+x)^4}+\frac{80c^3}{(c+x)^3}-\frac{40c^2}{(c+x)^2}+\frac{10c}{c+x}\right)\,dx, x, -ic\tan(e}{c^5f}$$
$$= \frac{10a^6x}{c^4}-\frac{10ia^6\log(\cos(e+fx))}{c^4f}-\frac{a^6\tan(e+fx)}{c^4f}-\frac{8ia^6}{f(c-ic\tan(e+fx))^4}+$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 455 vs. $2(160) = 320$.
time = 3.86, size = 455, normalized size = 2.84

$$\frac{a^6\sec(e)\sec(e+fx)(\cos(4(e+fx))+i\sin(4(e+fx)))\left(20i\cos(fx)+20i\cos(2e+fx)+53i\cos(2e+3fx)+60fx\cos(2e+3fx)+65i\cos(4e+3fx)+60fx\cos(4e+3fx)-15i\cos(4e+5fx)+60fx\cos(4e+5fx)-3i\cos(6e+5fx)+60fx\cos(6e+5fx)-30i\cos(2e+3fx)\log\left(\cos^2(e+fx)\right)-30i\cos(4e+3fx)\log\left(\cos^2(e+fx)\right)-30i\cos(4e+5fx)\log\left(\cos^2(e+fx)\right)-30i\cos(6e+5fx)\log\left(\cos^2(e+fx)\right)+40\sin(fx)+40\sin(2e+fx)+43\sin(2e+3fx)-60ifx\sin(2e+3fx)-30\log\left(\cos^2(e+fx)\right)\sin(2e+3fx)+55\sin(4e+3fx)-60ifx\sin(4e+3fx)-30\log\left(\cos^2(e+fx)\right)\sin(4e+3fx)-9\sin(4e+5fx)-60ifx\sin(4e+5fx)-30\log\left(\cos^2(e+fx)\right)\sin(4e+5fx)+3\sin(6e+5fx)-60ifx\sin(6e+5fx)-30\log\left(\cos^2(e+fx)\right)\sin(6e+5fx)\right)}{24c^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^6/(c - I*c*Tan[e + f*x])^4,x]
```

```
[Out] (a^6*Sec[e]*Sec[e + f*x]*(Cos[4*(e + f*x)] + I*Sin[4*(e + f*x)])*((20*I)*Co
s[f*x] + (20*I)*Cos[2*e + f*x] + (53*I)*Cos[2*e + 3*f*x] + 60*f*x*Cos[2*e +
 3*f*x] + (65*I)*Cos[4*e + 3*f*x] + 60*f*x*Cos[4*e + 3*f*x] - (15*I)*Cos[4*
e + 5*f*x] + 60*f*x*Cos[4*e + 5*f*x] - (3*I)*Cos[6*e + 5*f*x] + 60*f*x*Cos[
6*e + 5*f*x] - (30*I)*Cos[2*e + 3*f*x]*Log[Cos[e + f*x]^2] - (30*I)*Cos[4*e
 + 3*f*x]*Log[Cos[e + f*x]^2] - (30*I)*Cos[4*e + 5*f*x]*Log[Cos[e + f*x]^2]
 - (30*I)*Cos[6*e + 5*f*x]*Log[Cos[e + f*x]^2] + 40*Sin[f*x] + 40*Sin[2*e +
 f*x] + 43*Sin[2*e + 3*f*x] - (60*I)*f*x*Sin[2*e + 3*f*x] - 30*Log[Cos[e +
f*x]^2]*Sin[2*e + 3*f*x] + 55*Sin[4*e + 3*f*x] - (60*I)*f*x*Sin[4*e + 3*f*x
] - 30*Log[Cos[e + f*x]^2]*Sin[4*e + 3*f*x] - 9*Sin[4*e + 5*f*x] - (60*I)*f
*x*Sin[4*e + 5*f*x] - 30*Log[Cos[e + f*x]^2]*Sin[4*e + 5*f*x] + 3*Sin[6*e +
 5*f*x] - (60*I)*f*x*Sin[6*e + 5*f*x] - 30*Log[Cos[e + f*x]^2]*Sin[6*e + 5*
f*x]))/(24*c^4*f)
```

Maple [A]
time = 0.29, size = 87, normalized size = 0.54

method	result

derivativedivides	$\frac{a^6\left(-\tan(fx+e)-\frac{8i}{(\tan(fx+e)+i)^4}+\frac{40i}{(\tan(fx+e)+i)^2}+\frac{80}{3(\tan(fx+e)+i)^3}-\frac{40}{\tan(fx+e)+i}+10i\ln(\tan(fx+e)+i)\right)}{f\,c^4}$
default	$\frac{a^6\left(-\tan(fx+e)-\frac{8i}{(\tan(fx+e)+i)^4}+\frac{40i}{(\tan(fx+e)+i)^2}+\frac{80}{3(\tan(fx+e)+i)^3}-\frac{40}{\tan(fx+e)+i}+10i\ln(\tan(fx+e)+i)\right)}{f\,c^4}$
risch	$-\frac{ia^6 e^{8i(fx+e)}}{2c^4 f}+\frac{4ia^6 e^{6i(fx+e)}}{3c^4 f}-\frac{3ia^6 e^{4i(fx+e)}}{c^4 f}+\frac{8ia^6 e^{2i(fx+e)}}{c^4 f}-\frac{20a^6 e}{f\,c^4}-\frac{2ia^6}{f\,c^4\left(e^{2i(fx+e)}+1\right)}-\frac{10ia^6\ln(e$
norman	$\frac{\frac{10a^6x}{c}+\frac{56ia^6}{3cf}+\frac{40a^6x\left(\tan^2(fx+e)\right)}{c}+\frac{60a^6x\left(\tan^4(fx+e)\right)}{c}+\frac{40a^6x\left(\tan^6(fx+e)\right)}{c}+\frac{10a^6x\left(\tan^8(fx+e)\right)}{c}-\frac{9a^6\tan(fx+e)}{cf}-\frac{148a}{}}{(1-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^6/(c-I*c*tan(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a^6/c^4*(-tan(f*x+e)-8*I/(tan(f*x+e)+I)^4+40*I/(tan(f*x+e)+I)^2+80/3/(t
an(f*x+e)+I)^3-40/(tan(f*x+e)+I)+10*I*ln(tan(f*x+e)+I))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^6/(c-I*c*tan(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]

time = 1.15, size = 142, normalized size = 0.89

$$\frac{-3i\,a^6e^{(10i\,fx+10i\,e)}+5i\,a^6e^{(8i\,fx+8i\,e)}-10i\,a^6e^{(6i\,fx+6i\,e)}+30i\,a^6e^{(4i\,fx+4i\,e)}+48i\,a^6e^{(2i\,fx+2i\,e)}-12i\,a^6-60\left(i\,a^6e^{(2i\,fx+2i\,e)}+i\,a^6\right)\log\left(e^{(2i\,fx+2i\,e)}+1\right)}{6\left(c^4fe^{(2i\,fx+2i\,e)}+c^4f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^6/(c-I*c*tan(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/6*(-3*I*a^6*e^(10*I*f*x + 10*I*e) + 5*I*a^6*e^(8*I*f*x + 8*I*e) - 10*I*a^
6*e^(6*I*f*x + 6*I*e) + 30*I*a^6*e^(4*I*f*x + 4*I*e) + 48*I*a^6*e^(2*I*f*x
+ 2*I*e) - 12*I*a^6 - 60*(I*a^6*e^(2*I*f*x + 2*I*e) + I*a^6)*log(e^(2*I*f*x
 + 2*I*e) + 1))/(c^4*f*e^(2*I*f*x + 2*I*e) + c^4*f)
```

Sympy [A]

time = 0.53, size = 243, normalized size = 1.52

$$-\frac{2ia^6}{c^4fe^{2ie}e^{2ifx}+c^4f}-\frac{10ia^6\log\left(e^{2ifx}+e^{-2ie}\right)}{c^4f}+\begin{cases}\frac{-3ia^6c^{12}f^3e^{8ie}e^{8ifx}+8ia^6c^{12}f^3e^{6ie}e^{6ifx}-18ia^6c^{12}f^3e^{4ie}e^{4ifx}+48ia^6c^{12}f^3e^{2ie}e^{2ifx}}{6c^{16}f^4} & \text{for } c^{16}f^4\neq 0\\ \frac{x\left(4a^6e^{8ie}-8a^6e^{6ie}+12a^6e^{4ie}-16a^6e^{2ie}\right)}{c^4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**6/(c-I*c*tan(f*x+e))**4,x)
```

```
[Out] -2*I*a**6/(c**4*f*exp(2*I*e)*exp(2*I*f*x) + c**4*f) - 10*I*a**6*log(exp(2*I
*f*x) + exp(-2*I*e))/(c**4*f) + Piecewise(((-3*I*a**6*c**12*f**3*exp(8*I*e)
*exp(8*I*f*x) + 8*I*a**6*c**12*f**3*exp(6*I*e)*exp(6*I*f*x) - 18*I*a**6*c**
12*f**3*exp(4*I*e)*exp(4*I*f*x) + 48*I*a**6*c**12*f**3*exp(2*I*e)*exp(2*I*f
*x))/(6*c**16*f**4), Ne(c**16*f**4, 0)), (x*(4*a**6*exp(8*I*e) - 8*a**6*exp
(6*I*e) + 12*a**6*exp(4*I*e) - 16*a**6*exp(2*I*e))/c**4, True))
```

Giac [A]
time = 1.25, size = 285, normalized size = 1.78

$$-\frac{\frac{420i\,a^6\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}{c^4}-\frac{840i\,a^6\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+i\right)}{c^4}+\frac{420i\,a^6\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}{c^4}-\frac{84\left(5i\,a^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-5i\,a^6\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)c^4}+\frac{2283i\,a^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8-18936\,a^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7-69300i\,a^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+141512\,a^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+183106i\,a^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-141512\,a^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-69300i\,a^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+18936\,a^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2283i\,a^6}{c^4\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+i\right)^8}}{42\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^6/(c-I*c*tan(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -1/42*(420*I*a^6*log(tan(1/2*f*x + 1/2*e) + 1)/c^4 - 840*I*a^6*log(tan(1/2*
f*x + 1/2*e) + I)/c^4 + 420*I*a^6*log(tan(1/2*f*x + 1/2*e) - 1)/c^4 - 84*(5
*I*a^6*tan(1/2*f*x + 1/2*e)^2 + a^6*tan(1/2*f*x + 1/2*e) - 5*I*a^6)/((tan(1
/2*f*x + 1/2*e)^2 - 1)*c^4) + (2283*I*a^6*tan(1/2*f*x + 1/2*e)^8 - 18936*a^
6*tan(1/2*f*x + 1/2*e)^7 - 69300*I*a^6*tan(1/2*f*x + 1/2*e)^6 + 141512*a^6*
tan(1/2*f*x + 1/2*e)^5 + 183106*I*a^6*tan(1/2*f*x + 1/2*e)^4 - 141512*a^6*t
an(1/2*f*x + 1/2*e)^3 - 69300*I*a^6*tan(1/2*f*x + 1/2*e)^2 + 18936*a^6*tan(
1/2*f*x + 1/2*e) + 2283*I*a^6)/(c^4*(tan(1/2*f*x + 1/2*e) + I)^8))/f
```

Mupad [B]
time = 6.95, size = 170, normalized size = 1.06

$$\frac{a^6\left(10\,\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)-60\,\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\tan(e+f\,x)^2+10\,\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\tan(e+f\,x)^4-76\,\tan(e+f\,x)^2-4\,\tan(e+f\,x)^4+\frac{56}{3}-\frac{\tan(e+f\,x)\,197\mathrm{i}}{3}-\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\tan(e+f\,x)\,40\mathrm{i}+\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\tan(e+f\,x)^3\,40\mathrm{i}+\tan(e+f\,x)^3\,34\mathrm{i}+\tan(e+f\,x)^5\,1\mathrm{i}\right)\,1\mathrm{i}}{c^4\,f\,\left(-1+\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^6/(c - c*tan(e + f*x)*1i)^4,x)
```

```
[Out] (a^6*(10*log(tan(e + f*x) + 1i) - (tan(e + f*x)*197i)/3 - log(tan(e + f*x)
+ 1i)*tan(e + f*x)*40i - 60*log(tan(e + f*x) + 1i)*tan(e + f*x)^2 + log(tan
(e + f*x) + 1i)*tan(e + f*x)^3*40i + 10*log(tan(e + f*x) + 1i)*tan(e + f*x)
^4 - 76*tan(e + f*x)^2 + tan(e + f*x)^3*34i - 4*tan(e + f*x)^4 + tan(e + f*
x)^5*1i + 56/3)*1i)/(c^4*f*(tan(e + f*x)*1i - 1)^4)
```

3.948 $\int \frac{(a+ia\tan(e+fx))^5}{(c-ic\tan(e+fx))^4}\,dx$

Optimal. Leaf size=146

$$\frac{a^5x}{c^4}-\frac{ia^5\log(\cos(e+fx))}{c^4f}-\frac{4ia^5}{f(c-ic\tan(e+fx))^4}-\frac{12ia^5}{f\left(c^2-ic^2\tan(e+fx)\right)^2}+\frac{32ia^5c^5}{3f\left(c^3-ic^3\tan(e+fx)\right)^3}+$$

```
[Out] a^5*x/c^4-I*a^5*ln(cos(f*x+e))/c^4/f-4*I*a^5/f/(c-I*c*tan(f*x+e))^4-12*I*a^
5/f/(c^2-I*c^2*tan(f*x+e))^2+32/3*I*a^5*c^5/f/(c^3-I*c^3*tan(f*x+e))^3+8*I*
a^5/f/(c^4-I*c^4*tan(f*x+e))
```

Rubi [A]
time = 0.10, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 45}

$$\frac{8ia^5}{f\left(c^4-ic^4\tan(e+fx)\right)}-\frac{ia^5\log(\cos(e+fx))}{c^4f}+\frac{a^5x}{c^4}-\frac{12ia^5}{f\left(c^2-ic^2\tan(e+fx)\right)^2}+\frac{32ia^5c^5}{3f\left(c^3-ic^3\tan(e+fx)\right)^3}-\frac{4ia^5}{f(c-ic\tan(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^5/(c - I*c*Tan[e + f*x])^4,x]
```

```
[Out] (a^5*x)/c^4 - (I*a^5*Log[Cos[e + f*x]])/(c^4*f) - ((4*I)*a^5)/(f*(c - I*c*T
an[e + f*x])^4) - ((12*I)*a^5)/(f*(c^2 - I*c^2*Tan[e + f*x])^2) + (((32*I)/
3)*a^5*c^5)/(f*(c^3 - I*c^3*Tan[e + f*x])^3) + ((8*I)*a^5)/(f*(c^4 - I*c^4*
Tan[e + f*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
```

```
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\begin{aligned}\int \frac{(a+ia\tan(e+fx))^5}{(c-ic\tan(e+fx))^4}\,dx &= \left(a^5c^5\right)\int \frac{\sec^{10}(e+fx)}{(c-ic\tan(e+fx))^9}\,dx\\ &= \frac{(ia^5)\,\text{Subst}\left(\int \frac{(c-x)^4}{(c+x)^5}\,dx, x, -ic\tan(e+fx)\right)}{c^4 f}\\ &= \frac{(ia^5)\,\text{Subst}\left(\int \left(\frac{16c^4}{(c+x)^5}-\frac{32c^3}{(c+x)^4}+\frac{24c^2}{(c+x)^3}-\frac{8c}{(c+x)^2}+\frac{1}{c+x}\right)dx, x, -ic\tan(e+fx\right.}{c^4 f}\\ &= \frac{a^5x}{c^4}-\frac{ia^5\log(\cos(e+fx))}{c^4 f}-\frac{4ia^5}{f(c-ic\tan(e+fx))^4}+\frac{32ia^5}{3cf(c-ic\tan(e+fx}\end{aligned}$$

Mathematica [A]

time = 1.48, size = 151, normalized size = 1.03

$$\frac{a^5(-6i+16i\cos(2(e+fx))+3\cos(4(e+fx))\left(-i+4fx-2i\log\left(\cos^2(e+fx)\right)\right)+8\sin(2(e+fx))+3\sin(4(e+fx))-12ifx\sin(4(e+fx))-6\log\left(\cos^2(e+fx)\right)\sin(4(e+fx)))\left(\cos(4e+9fx)+i\sin(4e+9fx)\right)}{12c^4f(\cos(fx)+i\sin(fx))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^5/(c - I*c*Tan[e + f*x])^4,x]
```

```
[Out] (a^5*(-6*I + (16*I)*Cos[2*(e + f*x)] + 3*Cos[4*(e + f*x)]*(-I + 4*f*x - (2*
I)*Log[Cos[e + f*x]^2]) + 8*Sin[2*(e + f*x)] + 3*Sin[4*(e + f*x)] - (12*I)*
f*x*Sin[4*(e + f*x)] - 6*Log[Cos[e + f*x]^2]*Sin[4*(e + f*x)])*(Cos[4*e + 9
*f*x] + I*Sin[4*e + 9*f*x]))/(12*c^4*f*(Cos[f*x] + I*Sin[f*x])^5)
```

Maple [A]

time = 0.25, size = 79, normalized size = 0.54

method	result
derivativedivides	$\frac{a^5\left(-\frac{4i}{(\tan(fx+e)+i)^4}-\frac{8}{\tan(fx+e)+i}+\frac{32}{3(\tan(fx+e)+i)^3}+i\ln(\tan(fx+e)+i)+\frac{12i}{(\tan(fx+e)+i)^2}\right)}{f\,c^4}$
default	$\frac{a^5\left(-\frac{4i}{(\tan(fx+e)+i)^4}-\frac{8}{\tan(fx+e)+i}+\frac{32}{3(\tan(fx+e)+i)^3}+i\ln(\tan(fx+e)+i)+\frac{12i}{(\tan(fx+e)+i)^2}\right)}{f\,c^4}$
risch	$-\frac{ia^5e^{8i(fx+e)}}{4c^4f}+\frac{ia^5e^{6i(fx+e)}}{3c^4f}-\frac{ia^5e^{4i(fx+e)}}{2c^4f}+\frac{ia^5e^{2i(fx+e)}}{c^4f}-\frac{2a^5e}{c^4f}-\frac{ia^5\ln\left(e^{2i(fx+e)}+1\right)}{c^4f}$
norman	$\frac{\frac{a^5x}{c}+\frac{a^5x\left(\tan^8(fx+e)\right)}{c}+\frac{8ia^5}{3cf}+\frac{4a^5x\left(\tan^2(fx+e)\right)}{c}+\frac{6a^5x\left(\tan^4(fx+e)\right)}{c}+\frac{4a^5x\left(\tan^6(fx+e)\right)}{c}-\frac{40a^5\left(\tan^3(fx+e)\right)}{3cf}+\frac{32a^5\left(\tan^5(\right.}{3cf}}{(1+\tan^2(fx+e))^4c^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^5/(c-I*c*tan(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a^5/c^4*(-4*I/(tan(f*x+e)+I)^4-8/(tan(f*x+e)+I)+32/3/(tan(f*x+e)+I)^3+I
*ln(tan(f*x+e)+I)+12*I/(tan(f*x+e)+I)^2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^5/(c-I*c*tan(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.61, size = 87, normalized size = 0.60

$$\frac{-3i\,a^5e^{(8i\,fx+8i\,e)}+4i\,a^5e^{(6i\,fx+6i\,e)}-6i\,a^5e^{(4i\,fx+4i\,e)}+12i\,a^5e^{(2i\,fx+2i\,e)}-12i\,a^5\log\left(e^{(2i\,fx+2i\,e)}+1\right)}{12\,c^4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^5/(c-I*c*tan(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/12*(-3*I*a^5*e^(8*I*f*x + 8*I*e) + 4*I*a^5*e^(6*I*f*x + 6*I*e) - 6*I*a^5*
e^(4*I*f*x + 4*I*e) + 12*I*a^5*e^(2*I*f*x + 2*I*e) - 12*I*a^5*log(e^(2*I*f*
x + 2*I*e) + 1))/(c^4*f)
```

Sympy [A]
time = 0.45, size = 209, normalized size = 1.43

$$-\frac{ia^5\log\left(e^{2ifx}+e^{-2ie}\right)}{c^4f}+\begin{cases}\frac{-6ia^5c^{12}f^3e^{8ie}e^{8ifx}+8ia^5c^{12}f^3e^{6ie}e^{6ifx}-12ia^5c^{12}f^3e^{4ie}e^{4ifx}+24ia^5c^{12}f^3e^{2ie}e^{2ifx}}{24c^{16}f^4} & \text{for } c^{16}f^4\neq 0\\ \frac{x\left(2a^5e^{8ie}-2a^5e^{6ie}+2a^5e^{4ie}-2a^5e^{2ie}\right)}{c^4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**5/(c-I*c*tan(f*x+e))**4,x)
```

```
[Out] -I*a**5*log(exp(2*I*f*x) + exp(-2*I*e))/(c**4*f) + Piecewise(((-6*I*a**5*c*
*12*f**3*exp(8*I*e)*exp(8*I*f*x) + 8*I*a**5*c**12*f**3*exp(6*I*e)*exp(6*I*f
*x) - 12*I*a**5*c**12*f**3*exp(4*I*e)*exp(4*I*f*x) + 24*I*a**5*c**12*f**3*e
xp(2*I*e)*exp(2*I*f*x))/(24*c**16*f**4), Ne(c**16*f**4, 0)), (x*(2*a**5*exp
```

```
(8*I*e) - 2*a**5*exp(6*I*e) + 2*a**5*exp(4*I*e) - 2*a**5*exp(2*I*e))/c**4,
True))
```

Giac [A]
time = 1.12, size = 227, normalized size = 1.55

$$-\frac{\frac{420i\,a^5\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}{c^4}-\frac{840i\,a^5\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+i\right)}{c^4}+\frac{420i\,a^5\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}{c^4}+\frac{2283i\,a^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8-18264\,a^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7-70644i\,a^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+136808\,a^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+191170i\,a^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-136808\,a^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-70644i\,a^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+18264\,a^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2283i\,a^5}{c^4\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+i\right)^8}}{420\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^5/(c-I*c*tan(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -1/420*(420*I*a^5*log(tan(1/2*f*x + 1/2*e) + 1)/c^4 - 840*I*a^5*log(tan(1/2
*f*x + 1/2*e) + I)/c^4 + 420*I*a^5*log(tan(1/2*f*x + 1/2*e) - 1)/c^4 + (228
3*I*a^5*tan(1/2*f*x + 1/2*e)^8 - 18264*a^5*tan(1/2*f*x + 1/2*e)^7 - 70644*I
*a^5*tan(1/2*f*x + 1/2*e)^6 + 136808*a^5*tan(1/2*f*x + 1/2*e)^5 + 191170*I*
a^5*tan(1/2*f*x + 1/2*e)^4 - 136808*a^5*tan(1/2*f*x + 1/2*e)^3 - 70644*I*a^
5*tan(1/2*f*x + 1/2*e)^2 + 18264*a^5*tan(1/2*f*x + 1/2*e) + 2283*I*a^5)/(c^
4*(tan(1/2*f*x + 1/2*e) + I)^8))/f
```

Mupad [B]
time = 6.07, size = 146, normalized size = 1.00

$$\frac{a^5\left(\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)-6\,\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\tan(e+f\,x)^2+\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\tan(e+f\,x)^4-12\tan(e+f\,x)^2+\frac{8}{3}-\frac{\tan(e+f\,x)\,32\mathrm{i}}{3}-\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\tan(e+f\,x)\,4\mathrm{i}+\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\tan(e+f\,x)^3\,4\mathrm{i}+\tan(e+f\,x)^3\,8\mathrm{i}\right)\,1\mathrm{i}}{c^4\,f\,\left(-1+\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^5/(c - c*tan(e + f*x)*1i)^4,x)
```

```
[Out] (a^5*(log(tan(e + f*x) + 1i) - (tan(e + f*x)*32i)/3 - log(tan(e + f*x) + 1i
)*tan(e + f*x)*4i - 6*log(tan(e + f*x) + 1i)*tan(e + f*x)^2 + log(tan(e + f
*x) + 1i)*tan(e + f*x)^3*4i + log(tan(e + f*x) + 1i)*tan(e + f*x)^4 - 12*ta
n(e + f*x)^2 + tan(e + f*x)^3*8i + 8/3)*1i)/(c^4*f*(tan(e + f*x)*1i - 1)^4)
```

3.949 $\int \frac{(a+ia\tan(e+fx))^4}{(c-ic\tan(e+fx))^4}\,dx$

Optimal. Leaf size=50

$$-\frac{ia^4(c^2+ic^2\tan(e+fx))^4}{8f\,(c^3-ic^3\tan(e+fx))^4}$$

```
[Out] -1/8*I*a^4*(c^2+I*c^2*tan(f*x+e))^4/f/(c^3-I*c^3*tan(f*x+e))^4
```

Rubi [A]
time = 0.07, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 37}

$$-\frac{ia^4(c^2+ic^2\tan(e+fx))^4}{8f\,(c^3-ic^3\tan(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^4/(c - I*c*Tan[e + f*x])^4,x]
```

```
[Out] ((-1/8*I)*a^4*(c^2 + I*c^2*Tan[e + f*x])^4)/(f*(c^3 - I*c^3*Tan[e + f*x])^4
)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^4}{(c-ic\tan(e+fx))^4}\,dx = \left(a^4c^4\right)\int \frac{\sec^8(e+fx)}{(c-ic\tan(e+fx))^8}\,dx$$
$$= \frac{(ia^4)\,\mathrm{Subst}\left(\int \frac{(c-x)^3}{(c+x)^5}\,dx, x, -ic\tan(e+fx)\right)}{c^3f}$$
$$= -\frac{ia^4(c+ic\tan(e+fx))^4}{8f\left(c^2-ic^2\tan(e+fx)\right)^4}$$

Mathematica [A]
time = 0.39, size = 34, normalized size = 0.68

$$\frac{a^4(-i\cos(8(e+fx))+\sin(8(e+fx)))}{8c^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^4/(c - I*c*Tan[e + f*x])^4,x]
```

```
[Out] (a^4*((-I)*Cos[8*(e + f*x)] + Sin[8*(e + f*x)]))/(8*c^4*f)
```

Maple [A]
time = 0.20, size = 66, normalized size = 1.32

method	result
risch	$-\frac{ia^4 e^{8i(fx+e)}}{8c^4f}$
derivativedivides	$\frac{a^4\left(-\frac{1}{\tan(fx+e)+i}+\frac{4}{(\tan(fx+e)+i)^3}+\frac{3i}{(\tan(fx+e)+i)^2}-\frac{2i}{(\tan(fx+e)+i)^4}\right)}{f\,c^4}$
default	$\frac{a^4\left(-\frac{1}{\tan(fx+e)+i}+\frac{4}{(\tan(fx+e)+i)^3}+\frac{3i}{(\tan(fx+e)+i)^2}-\frac{2i}{(\tan(fx+e)+i)^4}\right)}{f\,c^4}$
norman	$\frac{\frac{a^4\tan(fx+e)}{cf}-\frac{7a^4\left(\tan^3(fx+e)\right)}{cf}+\frac{7a^4\left(\tan^5(fx+e)\right)}{cf}-\frac{a^4\left(\tan^7(fx+e)\right)}{cf}-\frac{8ia^4\left(\tan^4(fx+e)\right)}{cf}+\frac{4ia^4\left(\tan^2(fx+e)\right)}{cf}+\frac{4ia^4\left(\tan^6(fx\right.}{cf}}{\left(1+\tan^2(fx+e)\right)^4c^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^4/(c-I*c*tan(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a^4/c^4*(-1/(tan(f*x+e)+I)+4/(tan(f*x+e)+I)^3+3*I/(tan(f*x+e)+I)^2-2*I/
(tan(f*x+e)+I)^4)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^4/(c-I*c*tan(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.18, size = 21, normalized size = 0.42

$$-\frac{i\,a^4 e^{(8i\,fx+8i\,e)}}{8\,c^4 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^4/(c-I*c*tan(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] -1/8*I*a^4*e^(8*I*f*x + 8*I*e)/(c^4*f)
```

Sympy [A]
time = 0.26, size = 46, normalized size = 0.92

$$\begin{cases} -\frac{ia^4e^{8ie}e^{8ifx}}{8c^4f} & \text{for } c^4f \neq 0 \\ \frac{a^4xe^{8ie}}{c^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**4/(c-I*c*tan(f*x+e))**4,x)
```

```
[Out] Piecewise((-I*a**4*exp(8*I*e)*exp(8*I*f*x)/(8*c**4*f), Ne(c**4*f, 0)), (a**
4*x*exp(8*I*e)/c**4, True))
```

Giac [A]
time = 0.98, size = 88, normalized size = 1.76

$$-\frac{2\left(a^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7-7\,a^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+7\,a^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-a^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{c^4f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+i\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^4/(c-I*c*tan(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -2*(a^4*tan(1/2*f*x + 1/2*e)^7 - 7*a^4*tan(1/2*f*x + 1/2*e)^5 + 7*a^4*tan(1
/2*f*x + 1/2*e)^3 - a^4*tan(1/2*f*x + 1/2*e))/(c^4*f*(tan(1/2*f*x + 1/2*e)
+ I)^8)
```

Mupad [B]
time = 4.72, size = 69, normalized size = 1.38

$$-\frac{a^4\tan(e+f\,x)\,\left(\tan(e+f\,x)^2-1\right)}{c^4\,f\,\left(\tan\left(e+f\,x\right)^4+\tan\left(e+f\,x\right)^3\,4\mathrm{i}-6\tan\left(e+f\,x\right)^2-\tan\left(e+f\,x\right)\,4\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^4/(c - c*tan(e + f*x)*1i)^4,x)
```

```
[Out] -(a^4*tan(e + f*x)*(tan(e + f*x)^2 - 1))/(c^4*f*(tan(e + f*x)^3*4i - 6*tan(
e + f*x)^2 - tan(e + f*x)*4i + tan(e + f*x)^4 + 1))
```

3.950 $\int \frac{(a+ia\tan(e+fx))^3}{(c-ic\tan(e+fx))^4}\,dx$

Optimal. Leaf size=87

$$-\frac{ia^3}{f(c-ic\tan(e+fx))^4}+\frac{4ia^3}{3cf(c-ic\tan(e+fx))^3}-\frac{ia^3}{2f\left(c^2-ic^2\tan(e+fx)\right)^2}$$

```
[Out] -I*a^3/f/(c-I*c*tan(f*x+e))^4+4/3*I*a^3/c/f/(c-I*c*tan(f*x+e))^3-1/2*I*a^3/
f/(c^2-I*c^2*tan(f*x+e))^2
```

Rubi [A]
time = 0.09, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}=0.097$, Rules used = {3603, 3568, 45}

$$-\frac{ia^3}{2f\left(c^2-ic^2\tan(e+fx)\right)^2}+\frac{4ia^3}{3cf(c-ic\tan(e+fx))^3}-\frac{ia^3}{f(c-ic\tan(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^3/(c - I*c*Tan[e + f*x])^4,x]
```

```
[Out] ((-I)*a^3)/(f*(c - I*c*Tan[e + f*x])^4) + (((4*I)/3)*a^3)/(c*f*(c - I*c*Tan
[e + f*x])^3) - ((I/2)*a^3)/(f*(c^2 - I*c^2*Tan[e + f*x])^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^3}{(c-ic\tan(e+fx))^4}\,dx = \left(a^3c^3\right)\int \frac{\sec^6(e+fx)}{(c-ic\tan(e+fx))^7}\,dx$$

$$= \frac{\left(ia^3\right)\text{Subst}\left(\int \frac{(c-x)^2}{(c+x)^5}\,dx, x, -ic\tan(e+fx)\right)}{c^2f}$$

$$= \frac{\left(ia^3\right)\text{Subst}\left(\int \left(\frac{4c^2}{(c+x)^5}-\frac{4c}{(c+x)^4}+\frac{1}{(c+x)^3}\right)\,dx, x, -ic\tan(e+fx)\right)}{c^2f}$$

$$= -\frac{ia^3}{f(c-ic\tan(e+fx))^4}+\frac{4ia^3}{3cf(c-ic\tan(e+fx))^3}-\frac{ia^3}{2f\left(c^2-ic^2\tan(e+f\right.}$$

Mathematica [A]
time = 0.97, size = 53, normalized size = 0.61

$$\frac{a^3(7\cos(e+fx)-i\sin(e+fx))(-i\cos(7(e+fx))+\sin(7(e+fx)))}{48c^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3/(c - I*c*Tan[e + f*x])^4,x]
```

```
[Out] (a^3*(7*Cos[e + f*x] - I*Sin[e + f*x])*((-I)*Cos[7*(e + f*x)] + Sin[7*(e + f*x)]))/(48*c^4*f)
```

Maple [A]
time = 0.23, size = 53, normalized size = 0.61

method	result	si
risch	$-\frac{ia^3e^{8i(fx+e)}}{16c^4f}-\frac{ia^3e^{6i(fx+e)}}{12c^4f}$	44
derivativedivides	$\frac{a^3\left(\frac{i}{2(\tan(fx+e)+i)^2}-\frac{i}{(\tan(fx+e)+i)^4}+\frac{4}{3(\tan(fx+e)+i)^3}\right)}{f\,c^4}$	53
default	$\frac{a^3\left(\frac{i}{2(\tan(fx+e)+i)^2}-\frac{i}{(\tan(fx+e)+i)^4}+\frac{4}{3(\tan(fx+e)+i)^3}\right)}{f\,c^4}$	53
norman	$\frac{\frac{a^3\tan(fx+e)}{cf}-\frac{ia^3}{6cf}-\frac{14a^3\left(\tan^3(fx+e)\right)}{3cf}+\frac{7a^3\left(\tan^5(fx+e)\right)}{3cf}+\frac{ia^3\left(\tan^6(fx+e)\right)}{2cf}+\frac{17ia^3\left(\tan^2(fx+e)\right)}{6cf}-\frac{9ia^3\left(\tan^4(fx+e)\right)}{2cf}}{\left(1+\tan^2(fx+e)\right)^4c^3}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a^3/c^4*(1/2*I/(tan(f*x+e)+I)^2-I/(tan(f*x+e)+I)^4+4/3/(tan(f*x+e)+I)^3
)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^4,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.40, size = 39, normalized size = 0.45

$$\frac{-3i\,a^3e^{(8i\,fx+8i\,e)}-4i\,a^3e^{(6i\,fx+6i\,e)}}{48\,c^4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^4,x, algorithm="fricas")
[Out] 1/48*(-3*I*a^3*e^(8*I*f*x + 8*I*e) - 4*I*a^3*e^(6*I*f*x + 6*I*e))/(c^4*f)
```

Sympy [A]
time = 0.27, size = 95, normalized size = 1.09

$$\begin{cases} \frac{-12ia^3c^4fe^{8ie}e^{8ifx}-16ia^3c^4fe^{6ie}e^{6ifx}}{192c^8f^2} & \text{for } c^8f^2\neq 0 \\ \frac{x\left(a^3e^{8ie}+a^3e^{6ie}\right)}{2c^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3/(c-I*c*tan(f*x+e))**4,x)
[Out] Piecewise(((-12*I*a**3*c**4*f*exp(8*I*e)*exp(8*I*f*x) - 16*I*a**3*c**4*f*ex
p(6*I*e)*exp(6*I*f*x))/(192*c**8*f**2), Ne(c**8*f**2, 0)), (x*(a**3*exp(8*I
*e) + a**3*exp(6*I*e))/(2*c**4), True))
```

Giac [A]
time = 0.88, size = 140, normalized size = 1.61

$$-\frac{2\left(3\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+3i\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-17\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-10i\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+17\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+3i\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{3\,c^4f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+i\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^4,x, algorithm="giac")
[Out] -2/3*(3*a^3*tan(1/2*f*x + 1/2*e)^7 + 3*I*a^3*tan(1/2*f*x + 1/2*e)^6 - 17*a^
3*tan(1/2*f*x + 1/2*e)^5 - 10*I*a^3*tan(1/2*f*x + 1/2*e)^4 + 17*a^3*tan(1/2
```

```
*f*x + 1/2*e)^3 + 3*I*a^3*tan(1/2*f*x + 1/2*e)^2 - 3*a^3*tan(1/2*f*x + 1/2*
e))/(c^4*f*(tan(1/2*f*x + 1/2*e) + I)^8)
```

Mupad [B]
time = 4.77, size = 75, normalized size = 0.86

$$\frac{a^3 \left(\tan(e+f\,x)^2\,3\mathrm{i}+2\tan(e+f\,x)-\mathrm{i}\right)}{6\,c^4\,f\,\left(\tan\left(e+f\,x\right)^4+\tan\left(e+f\,x\right)^3\,4\mathrm{i}-6\tan\left(e+f\,x\right)^2-\tan\left(e+f\,x\right)\,4\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^3/(c - c*tan(e + f*x)*1i)^4,x)
[Out] (a^3*(2*tan(e + f*x) + tan(e + f*x)^2*3i - 1i))/(6*c^4*f*(tan(e + f*x)^3*4i
 - 6*tan(e + f*x)^2 - tan(e + f*x)*4i + tan(e + f*x)^4 + 1))
```

3.951 $\int \frac{(a+ia\tan(e+fx))^2}{(c-ic\tan(e+fx))^4}\,dx$

Optimal. Leaf size=62

$$-\frac{ia^2}{2f(c-ic\tan(e+fx))^4}+\frac{ia^2c^2}{3f\left(c^2-ic^2\tan(e+fx)\right)^3}$$

```
[Out] -1/2*I*a^2/f/(c-I*c*tan(f*x+e))^4+1/3*I*a^2*c^2/f/(c^2-I*c^2*tan(f*x+e))^3
```

Rubi [A]
time = 0.08, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {3603, 3568, 45}

$$\frac{ia^2c^2}{3f\left(c^2-ic^2\tan(e+fx)\right)^3}-\frac{ia^2}{2f(c-ic\tan(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2/(c - I*c*Tan[e + f*x])^4,x]
```

```
[Out] ((-1/2*I)*a^2)/(f*(c - I*c*Tan[e + f*x])^4) + ((I/3)*a^2*c^2)/(f*(c^2 - I*c
^2*Tan[e + f*x])^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^2}{(c-ic\tan(e+fx))^4}\,dx = (a^2c^2)\int \frac{\sec^4(e+fx)}{(c-ic\tan(e+fx))^6}\,dx$$
$$= \frac{(ia^2)\,\text{Subst}\left(\int \frac{c-x}{(c+x)^5}\,dx, x, -ic\tan(e+fx)\right)}{cf}$$
$$= \frac{(ia^2)\,\text{Subst}\left(\int \left(\frac{2c}{(c+x)^5}-\frac{1}{(c+x)^4}\right)dx, x, -ic\tan(e+fx)\right)}{cf}$$
$$= -\frac{ia^2}{2f(c-ic\tan(e+fx))^4}+\frac{ia^2}{3cf(c-ic\tan(e+fx))^3}$$

Mathematica [A]
time = 0.75, size = 75, normalized size = 1.21

$$\frac{a^2(8+9\cos(2(e+fx))-3i\sin(2(e+fx)))(-i\cos(6e+8fx)+\sin(6e+8fx))}{96c^4f(\cos(fx)+i\sin(fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2/(c - I*c*Tan[e + f*x])^4,x]
[Out] (a^2*(8 + 9*Cos[2*(e + f*x)] - (3*I)*Sin[2*(e + f*x)])*((-I)*Cos[6*e + 8*f*
x] + Sin[6*e + 8*f*x]))/(96*c^4*f*(Cos[f*x] + I*Sin[f*x])^2)
```

Maple [A]
time = 0.23, size = 39, normalized size = 0.63

method	result	size
derivativedivides	$\frac{a^2\left(\frac{1}{3(\tan(fx+e)+i)^3}-\frac{i}{2(\tan(fx+e)+i)^4}\right)}{f\,c^4}$	39
default	$\frac{a^2\left(\frac{1}{3(\tan(fx+e)+i)^3}-\frac{i}{2(\tan(fx+e)+i)^4}\right)}{f\,c^4}$	39
risch	$-\frac{ia^2 e^{8i(fx+e)}}{32c^4f}-\frac{ia^2 e^{6i(fx+e)}}{12c^4f}-\frac{ia^2 e^{4i(fx+e)}}{16c^4f}$	65
norman	$\frac{\frac{a^2\tan(fx+e)}{cf}-\frac{ia^2}{6cf}-\frac{8a^2\left(\tan^3(fx+e)\right)}{3cf}+\frac{a^2\left(\tan^5(fx+e)\right)}{3cf}-\frac{3ia^2\left(\tan^4(fx+e)\right)}{2cf}+\frac{7ia^2\left(\tan^2(fx+e)\right)}{3cf}}{\left(1+\tan^2(fx+e)\right)^4c^3}$	124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^4,x,method=_RETURNVERBOSE)
[Out] 1/f*a^2/c^4*(1/3/(tan(f*x+e)+I)^3-1/2*I/(tan(f*x+e)+I)^4)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.40, size = 54, normalized size = 0.87

$$\frac{-3i\,a^2e^{(8i\,fx+8i\,e)}-8i\,a^2e^{(6i\,fx+6i\,e)}-6i\,a^2e^{(4i\,fx+4i\,e)}}{96\,c^4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/96*(-3*I*a^2*e^(8*I*f*x + 8*I*e) - 8*I*a^2*e^(6*I*f*x + 6*I*e) - 6*I*a^2*
e^(4*I*f*x + 4*I*e))/(c^4*f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(49) = 98$.
time = 0.23, size = 139, normalized size = 2.24

$$\begin{cases} \frac{-192ia^2c^8f^2e^{8ie}e^{8ifx}-512ia^2c^8f^2e^{6ie}e^{6ifx}-384ia^2c^8f^2e^{4ie}e^{4ifx}}{6144c^{12}f^3} & \text{for } c^{12}f^3\neq 0 \\ \frac{x\left(a^2e^{8ie}+2a^2e^{6ie}+a^2e^{4ie}\right)}{4c^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2/(c-I*c*tan(f*x+e))**4,x)
```

```
[Out] Piecewise(((-192*I*a**2*c**8*f**2*exp(8*I*e)*exp(8*I*f*x) - 512*I*a**2*c**8
*f**2*exp(6*I*e)*exp(6*I*f*x) - 384*I*a**2*c**8*f**2*exp(4*I*e)*exp(4*I*f*x
))/(6144*c**12*f**3), Ne(c**12*f**3, 0)), (x*(a**2*exp(8*I*e) + 2*a**2*exp(
6*I*e) + a**2*exp(4*I*e))/(4*c**4), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(52) = 104$.
time = 0.78, size = 140, normalized size = 2.26

$$-\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+6i\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-17\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-16i\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+17\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+6i\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{3\,c^4f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+i\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -2/3*(3*a^2*tan(1/2*f*x + 1/2*e)^7 + 6*I*a^2*tan(1/2*f*x + 1/2*e)^6 - 17*a^
2*tan(1/2*f*x + 1/2*e)^5 - 16*I*a^2*tan(1/2*f*x + 1/2*e)^4 + 17*a^2*tan(1/2
*f*x + 1/2*e)^3 + 6*I*a^2*tan(1/2*f*x + 1/2*e)^2 - 3*a^2*tan(1/2*f*x + 1/2*
e))/(c^4*f*(tan(1/2*f*x + 1/2*e) + I)^8)
```

Mupad [B]
time = 4.71, size = 64, normalized size = 1.03

$$\frac{a^2\left(2\tan(e+f\,x)-\mathrm{i}\right)}{6\,c^4\,f\,\left({\tan\left(e+f\,x\right)}^4+{\tan\left(e+f\,x\right)}^3\,4{}\mathrm{i}-6\,{\tan\left(e+f\,x\right)}^2-\tan\left(e+f\,x\right)\,4{}\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^2/(c - c*tan(e + f*x)*1i)^4,x)
```

```
[Out] (a^2*(2*tan(e + f*x) - 1i))/(6*c^4*f*(tan(e + f*x)^3*4i - 6*tan(e + f*x)^2
- tan(e + f*x)*4i + tan(e + f*x)^4 + 1))
```

3.952 $\int \frac{a+ia\tan(e+fx)}{(c-ic\tan(e+fx))^4}\,dx$

Optimal. Leaf size=25

$$-\frac{ia}{4f(c-ic\tan(e+fx))^4}$$

[Out] -1/4*I*a/f/(c-I*c*tan(f*x+e))^4

Rubi [A]
time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {3603, 3568, 32}

$$-\frac{ia}{4f(c-ic\tan(e+fx))^4}$$

Antiderivative was successfully verified.

[In] Int[(a + I*a*Tan[e + f*x])/(c - I*c*Tan[e + f*x])^4,x]

[Out] ((-1/4*I)*a)/(f*(c - I*c*Tan[e + f*x])^4)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{a+ia\tan(e+fx)}{(c-ic\tan(e+fx))^4}\,dx = (ac)\int \frac{\sec^2(e+fx)}{(c-ic\tan(e+fx))^5}\,dx$$
$$= \frac{(ia)\text{Subst}\left(\int \frac{1}{(c+x)^5}\,dx, x, -ic\tan(e+fx)\right)}{f}$$
$$= -\frac{ia}{4f(c-ic\tan(e+fx))^4}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 74 vs. 2(25) = 50.
time = 0.45, size = 74, normalized size = 2.96

$$\frac{a(10\cos(e+fx)+5\cos(3(e+fx))-i(2\sin(e+fx)+3\sin(3(e+fx))))(-i\cos(5(e+fx))+\sin(5(e+fx)))}{64c^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])/(c - I*c*Tan[e + f*x])^4,x]
```

```
[Out] (a*(10*Cos[e + f*x] + 5*Cos[3*(e + f*x)] - I*(2*Sin[e + f*x] + 3*Sin[3*(e +
 f*x)]))*((-I)*Cos[5*(e + f*x)] + Sin[5*(e + f*x)]))/(64*c^4*f)
```

Maple [A]
time = 0.24, size = 22, normalized size = 0.88

method	result	size
derivativedivides	$-\frac{ia}{4f\,c^4(\tan(fx+e)+i)^4}$	22
default	$-\frac{ia}{4f\,c^4(\tan(fx+e)+i)^4}$	22
risch	$-\frac{ia\,e^{8i(fx+e)}}{64c^4f}-\frac{ia\,e^{6i(fx+e)}}{16c^4f}-\frac{3ia\,e^{4i(fx+e)}}{32c^4f}-\frac{ia\,e^{2i(fx+e)}}{16c^4f}$	78
norman	$\frac{\frac{a\tan(fx+e)}{cf}-\frac{ia}{4cf}-\frac{a\left(\tan^3(fx+e)\right)}{cf}-\frac{ia\left(\tan^4(fx+e)\right)}{4cf}+\frac{3ia\left(\tan^2(fx+e)\right)}{2cf}}{\left(1+\tan^2(fx+e)\right)^4c^3}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I/f*a/c^4/(tan(f*x+e)+I)^4
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(20) = 40$.
time = 1.69, size = 61, normalized size = 2.44

$$\frac{-i\,ae^{(8i\,fx+8i\,e)}-4i\,ae^{(6i\,fx+6i\,e)}-6i\,ae^{(4i\,fx+4i\,e)}-4i\,ae^{(2i\,fx+2i\,e)}}{64\,c^4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/64*(-I*a*e^(8*I*f*x + 8*I*e) - 4*I*a*e^(6*I*f*x + 6*I*e) - 6*I*a*e^(4*I*f
*x + 4*I*e) - 4*I*a*e^(2*I*f*x + 2*I*e))/(c^4*f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(20) = 40$.
time = 0.24, size = 167, normalized size = 6.68

$$\begin{cases}\frac{-8192iac^{12}f^3e^{8ie}e^{8ifx}-32768iac^{12}f^3e^{6ie}e^{6ifx}-49152iac^{12}f^3e^{4ie}e^{4ifx}-32768iac^{12}f^3e^{2ie}e^{2ifx}}{524288c^{16}f^4} & \text{for } c^{16}f^4\neq 0\\ \frac{x(ae^{8ie}+3ae^{6ie}+3ae^{4ie}+ae^{2ie})}{8c^4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))**4,x)
```

```
[Out] Piecewise(((-8192*I*a*c**12*f**3*exp(8*I*e)*exp(8*I*f*x) - 32768*I*a*c**12*
f**3*exp(6*I*e)*exp(6*I*f*x) - 49152*I*a*c**12*f**3*exp(4*I*e)*exp(4*I*f*x)
 - 32768*I*a*c**12*f**3*exp(2*I*e)*exp(2*I*f*x))/(524288*c**16*f**4), Ne(c*
*16*f**4, 0)), (x*(a*exp(8*I*e) + 3*a*exp(6*I*e) + 3*a*exp(4*I*e) + a*exp(2
*I*e))/(8*c**4), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(20) = 40$.
time = 0.72, size = 125, normalized size = 5.00

$$-\frac{2\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+3i\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-7\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-8i\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+7\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+3i\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{c^4f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+i\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -2*(a*tan(1/2*f*x + 1/2*e)^7 + 3*I*a*tan(1/2*f*x + 1/2*e)^6 - 7*a*tan(1/2*f
*x + 1/2*e)^5 - 8*I*a*tan(1/2*f*x + 1/2*e)^4 + 7*a*tan(1/2*f*x + 1/2*e)^3 +
 3*I*a*tan(1/2*f*x + 1/2*e)^2 - a*tan(1/2*f*x + 1/2*e))/(c^4*f*(tan(1/2*f*x
 + 1/2*e) + I)^8)
```

Mupad [B]
time = 4.65, size = 21, normalized size = 0.84

$$-\frac{a\,1\mathrm{i}}{4\,c^4\,f\,{\left(\mathrm{tan}\left(e+f\,x\right)+1\mathrm{i}\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)/(c - c*tan(e + f*x)*1i)^4,x)
```

```
[Out] -(a*1i)/(4*c^4*f*(tan(e + f*x) + 1i)^4)
```

3.953 $\int \frac{1}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^4}\,dx$

Optimal. Leaf size=162

$$\frac{5x}{32ac^4}-\frac{i}{16af(c-ic\tan(e+fx))^4}-\frac{i}{12acf(c-ic\tan(e+fx))^3}-\frac{3i}{32af\left(c^2-ic^2\tan(e+fx)\right)^2}-\frac{i}{8af\left(c^4-i\right.}$$

```
[Out] 5/32*x/a/c^4-1/16*I/a/f/(c-I*c*tan(f*x+e))^4-1/12*I/a/c/f/(c-I*c*tan(f*x+e)
)^3-3/32*I/a/f/(c^2-I*c^2*tan(f*x+e))^2-1/8*I/a/f/(c^4-I*c^4*tan(f*x+e))+1/
32*I/a/f/(c^4+I*c^4*tan(f*x+e))
```

Rubi [A]
time = 0.12, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {3603, 3568, 46, 212}

$$-\frac{i}{8af\left(c^4-ic^4\tan(e+fx)\right)}+\frac{i}{32af\left(c^4+ic^4\tan(e+fx)\right)}+\frac{5x}{32ac^4}-\frac{3i}{32af\left(c^2-ic^2\tan(e+fx)\right)^2}-\frac{i}{12acf(c-ic\tan(e+fx))^3}-\frac{i}{16af(c-ic\tan(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^4),x]
```

```
[Out] (5*x)/(32*a*c^4) - (I/16)/(a*f*(c - I*c*Tan[e + f*x])^4) - (I/12)/(a*c*f*(c
 - I*c*Tan[e + f*x])^3) - ((3*I)/32)/(a*f*(c^2 - I*c^2*Tan[e + f*x])^2) - (
I/8)/(a*f*(c^4 - I*c^4*Tan[e + f*x])) + (I/32)/(a*f*(c^4 + I*c^4*Tan[e + f*
x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^4}\,dx = \frac{\int \frac{\cos^2(e+fx)}{(c-ic\tan(e+fx))^3}\,dx}{ac}$$

$$= \frac{(ic^2)\,\mathrm{Subst}\left(\int \frac{1}{(c-x)^2(c+x)^5}\,dx, x, -ic\tan(e+fx)\right)}{af}$$

$$= \frac{(ic^2)\,\mathrm{Subst}\left(\int \left(\frac{1}{32c^5(c-x)^2} + \frac{1}{4c^2(c+x)^5} + \frac{1}{4c^3(c+x)^4} + \frac{3}{16c^4(c+x)^{3}}\right.\right.}{af}$$

$$= -\frac{i}{16af(c-ic\tan(e+fx))^4} - \frac{i}{12acf(c-ic\tan(e+fx))^3}$$

$$= \frac{5x}{32ac^4} - \frac{i}{16af(c-ic\tan(e+fx))^4} - \frac{i}{12acf(c-ic\tan(e$$

Mathematica [A]
time = 0.78, size = 134, normalized size = 0.83

$$\frac{\sec(e+fx)(\cos(4(e+fx))+i\sin(4(e+fx)))(-180\cos(e+fx)+(-20-120ifx)\cos(3(e+fx))+9\cos(5(e+fx))+60i\sin(e+fx)-20i\sin(3(e+fx))-120fx\sin(3(e+fx))-15i\sin(5(e+fx)))}{768ac^4f(-i+\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^4),x]
```

```
[Out] (Sec[e + f*x]*(Cos[4*(e + f*x)] + I*Sin[4*(e + f*x)])*(-180*Cos[e + f*x] +
(-20 - (120*I)*f*x)*Cos[3*(e + f*x)] + 9*Cos[5*(e + f*x)] + (60*I)*Sin[e +
f*x] - (20*I)*Sin[3*(e + f*x)] - 120*f*x*Sin[3*(e + f*x)] - (15*I)*Sin[5*(e
 + f*x)]))/(768*a*c^4*f*(-I + Tan[e + f*x]))
```

Maple [A]
time = 0.22, size = 105, normalized size = 0.65

method	result
derivativedivides	$\frac{\frac{3i}{32(\tan(fx+e)+i)^2}-\frac{i}{16(\tan(fx+e)+i)^4}+\frac{5i\ln(\tan(fx+e)+i)}{64}-\frac{1}{12(\tan(fx+e)+i)^3}+\frac{1}{8\tan(fx+e)+8i}-\frac{5i\ln(\tan(fx+e)-i)}{64}+\frac{}{32\tan(f}}{fac^4}$

default	$\frac{\frac{3i}{32(\tan(fx+e)+i)^2}-\frac{i}{16(\tan(fx+e)+i)^4}+\frac{5i\ln(\tan(fx+e)+i)}{64}-\frac{1}{12(\tan(fx+e)+i)^3}+\frac{1}{8\tan(fx+e)+8i}-\frac{5i\ln(\tan(fx+e)-i)}{64}+\frac{}{32\tan(}}{f a c^4}$
risch	$\frac{5x}{32a\,c^4}-\frac{i\mathrm{e}^{8i(fx+e)}}{256a\,c^4f}-\frac{5i\mathrm{e}^{6i(fx+e)}}{192a\,c^4f}-\frac{5i\mathrm{e}^{4i(fx+e)}}{64a\,c^4f}-\frac{9i\cos(2fx+2e)}{64a\,c^4f}+\frac{11\sin(2fx+2e)}{64a\,c^4f}$
norman	$\frac{\frac{5x}{32ac}+\frac{27\tan(fx+e)}{32acf}+\frac{73\left(\tan^3(fx+e)\right)}{96acf}+\frac{55\left(\tan^5(fx+e)\right)}{96acf}+\frac{5\left(\tan^7(fx+e)\right)}{32acf}+\frac{5x\left(\tan^2(fx+e)\right)}{8ac}+\frac{15x\left(\tan^4(fx+e)\right)}{16ac}+\frac{5x\left(\tan^6(fx\right.}{8ac}}{\left(1+\tan^2(fx+e)\right)^4c^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a/c^4*(3/32*I/(tan(f*x+e)+I)^2-1/16*I/(tan(f*x+e)+I)^4+5/64*I*ln(tan(f*
x+e)+I)-1/12/(tan(f*x+e)+I)^3+1/8/(tan(f*x+e)+I)-5/64*I*ln(tan(f*x+e)-I)+1/
32/(tan(f*x+e)-I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.63, size = 85, normalized size = 0.52

$$\frac{\left(120\,fxe^{(2i\,fx+2i\,e)}-3i\,e^{(10i\,fx+10i\,e)}-20i\,e^{(8i\,fx+8i\,e)}-60i\,e^{(6i\,fx+6i\,e)}-120i\,e^{(4i\,fx+4i\,e)}+12i\right)e^{(-2i\,fx-2i\,e)}}{768\,ac^4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/768*(120*f*x*e^(2*I*f*x + 2*I*e) - 3*I*e^(10*I*f*x + 10*I*e) - 20*I*e^(8*
I*f*x + 8*I*e) - 60*I*e^(6*I*f*x + 6*I*e) - 120*I*e^(4*I*f*x + 4*I*e) + 12*
I)*e^(-2*I*f*x - 2*I*e)/(a*c^4*f)
```

Sympy [A]
time = 0.28, size = 246, normalized size = 1.52

$$\begin{cases}\frac{\left(-25165824ia^4c^{16}f^4e^{10ie}e^{8ifx}-167772160ia^4c^{16}f^4e^{8ie}e^{6ifx}-503316480ia^4c^{16}f^4e^{6ie}e^{4ifx}-1006632960ia^4c^{16}f^4e^{4ie}e^{2ifx}+100663296ia^4c^{16}f^4e^{-2ifx}\right)e^{-2ie}}{6442450944a^5c^{20}f^5} & \text{for } a^5c^{20}f^5e^{2ie}\neq 0\\ x\left(\frac{\left(e^{10ie}+5e^{8ie}+10e^{6ie}+10e^{4ie}+5e^{2ie}+1\right)e^{-2ie}}{32ac^4}-\frac{5}{32ac^4}\right) & \text{otherwise}\end{cases}+\frac{5x}{32ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))**4,x)
```

```
[Out] Piecewise(((-25165824*I*a**4*c**16*f**4*exp(10*I*e)*exp(8*I*f*x) - 16777216
0*I*a**4*c**16*f**4*exp(8*I*e)*exp(6*I*f*x) - 503316480*I*a**4*c**16*f**4*e
xp(6*I*e)*exp(4*I*f*x) - 1006632960*I*a**4*c**16*f**4*exp(4*I*e)*exp(2*I*f*
x) + 100663296*I*a**4*c**16*f**4*exp(-2*I*f*x))*exp(-2*I*e)/(6442450944*a**
5*c**20*f**5), Ne(a**5*c**20*f**5*exp(2*I*e), 0)), (x*((exp(10*I*e) + 5*exp
(8*I*e) + 10*exp(6*I*e) + 10*exp(4*I*e) + 5*exp(2*I*e) + 1)*exp(-2*I*e)/(32
*a*c**4) - 5/(32*a*c**4)), True)) + 5*x/(32*a*c**4)
```

Giac [A]
time = 0.64, size = 140, normalized size = 0.86

$$-\frac{\frac{60i\log(i\tan(fx+e)+1)}{ac^4}-\frac{60i\log(i\tan(fx+e)-1)}{ac^4}-\frac{12\,(5\tan(fx+e)-7i)}{ac^4(-i\tan(fx+e)-1)}+\frac{125i\tan(fx+e)^4-596\tan(fx+e)^3-1110i\tan(fx+e)^2+996\tan(fx+e)+405i}{ac^4(\tan(fx+e)+i)^4}}{768\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -1/768*(60*I*log(I*tan(f*x + e) + 1)/(a*c^4) - 60*I*log(I*tan(f*x + e) - 1)
/(a*c^4) - 12*(5*tan(f*x + e) - 7*I)/(a*c^4*(-I*tan(f*x + e) - 1)) + (125*I
*tan(f*x + e)^4 - 596*tan(f*x + e)^3 - 1110*I*tan(f*x + e)^2 + 996*tan(f*x
+ e) + 405*I)/(a*c^4*(tan(f*x + e) + I)^4))/f
```

Mupad [B]
time = 5.17, size = 88, normalized size = 0.54

$$\frac{5\,x}{32\,a\,c^4}-\frac{-\frac{\tan(e+f\,x)^4\,5\mathrm{i}}{32}+\frac{15\tan(e+f\,x)^3}{32}+\frac{\tan(e+f\,x)^2\,35\mathrm{i}}{96}+\frac{5\tan(e+f\,x)}{32}+\frac{1}{3}\mathrm{i}}{a\,c^4\,f\,\left(1+\tan\left(e+f\,x\right)\,1\mathrm{i}\right)\,{\left(\tan\left(e+f\,x\right)+1\mathrm{i}\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i)^4),x)
```

```
[Out] (5*x)/(32*a*c^4) - ((5*tan(e + f*x))/32 + (tan(e + f*x)^2*35i)/96 + (15*tan
(e + f*x)^3)/32 - (tan(e + f*x)^4*5i)/32 + 1i/3)/(a*c^4*f*(tan(e + f*x)*1i
+ 1)*(tan(e + f*x) + 1i)^4)
```

3.954 $\int \frac{1}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^4}\,dx$

Optimal. Leaf size=193

$$\frac{15x}{64a^2c^4}-\frac{i}{32a^2f(c-ic\tan(e+fx))^4}-\frac{i}{16a^2cf(c-ic\tan(e+fx))^3}-\frac{3i}{32a^2f\left(c^2-ic^2\tan(e+fx)\right)^2}+\frac{}{64a^2f\,(}$$

```
[Out] 15/64*x/a^2/c^4-1/32*I/a^2/f/(c-I*c*tan(f*x+e))^4-1/16*I/a^2/c/f/(c-I*c*tan
(f*x+e))^3-3/32*I/a^2/f/(c^2-I*c^2*tan(f*x+e))^2+1/64*I/a^2/f/(c^2+I*c^2*ta
n(f*x+e))^2-5/32*I/a^2/f/(c^4-I*c^4*tan(f*x+e))+5/64*I/a^2/f/(c^4+I*c^4*tan
(f*x+e))
```

Rubi [A]
time = 0.14, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {3603, 3568, 46, 212}

$$-\frac{5i}{32a^2f\left(c^4-ic^4\tan(e+fx)\right)}+\frac{5i}{64a^2f\left(c^4+ic^4\tan(e+fx)\right)}+\frac{15x}{64a^2c^4}-\frac{3i}{32a^2f\left(c^2-ic^2\tan(e+fx)\right)^2}+\frac{i}{64a^2f\left(c^2+ic^2\tan(e+fx)\right)^2}-\frac{i}{16a^2cf(c-ic\tan(e+fx))^3}-\frac{i}{32a^2f(c-ic\tan(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^4),x]
```

```
[Out] (15*x)/(64*a^2*c^4) - (I/32)/(a^2*f*(c - I*c*Tan[e + f*x])^4) - (I/16)/(a^2
*c*f*(c - I*c*Tan[e + f*x])^3) - ((3*I)/32)/(a^2*f*(c^2 - I*c^2*Tan[e + f*x
])^2) + (I/64)/(a^2*f*(c^2 + I*c^2*Tan[e + f*x])^2) - ((5*I)/32)/(a^2*f*(c^
4 - I*c^4*Tan[e + f*x])) + ((5*I)/64)/(a^2*f*(c^4 + I*c^4*Tan[e + f*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^4}\,dx = \frac{\int \frac{\cos^4(e+fx)}{(c-ic\tan(e+fx))^2}\,dx}{a^2c^2}$$
$$= \frac{(ic^3)\,\text{Subst}\left(\int \frac{1}{(c-x)^3(c+x)^5}\,dx, x, -ic\tan(e+fx)\right)}{a^2f}$$
$$= \frac{(ic^3)\,\text{Subst}\left(\int \left(\frac{1}{32c^5(c-x)^3}+\frac{5}{64c^6(c-x)^2}+\frac{1}{8c^3(c+x)^5}+\frac{3}{16c^4(c+$$
$$= -\frac{i}{32a^2f(c-ic\tan(e+fx))^4}-\frac{i}{16a^2cf(c-ic\tan(e+fx$$
$$= \frac{15x}{64a^2c^4}-\frac{i}{32a^2f(c-ic\tan(e+fx))^4}-\frac{i}{16a^2cf(c-ic\tan$$

Mathematica [A]
time = 0.95, size = 139, normalized size = 0.72

$$\frac{\sec^2(e+fx)(-i\cos(4(e+fx))+\sin(4(e+fx)))(-80+(-30-120ifx)\cos(2(e+fx))+16\cos(4(e+fx))+\cos(6(e+fx))-30i\sin(2(e+fx))-120fx\sin(2(e+fx))-32i\sin(4(e+fx))-3i\sin(6(e+fx)))}{512a^2c^4f(-i+\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^4),x]
```

```
[Out] (Sec[e + f*x]^2*((-I)*Cos[4*(e + f*x)] + Sin[4*(e + f*x)])*(-80 + (-30 - (1
20*I)*f*x)*Cos[2*(e + f*x)] + 16*Cos[4*(e + f*x)] + Cos[6*(e + f*x)] - (30*
I)*Sin[2*(e + f*x)] - 120*f*x*Sin[2*(e + f*x)] - (32*I)*Sin[4*(e + f*x)] -
(3*I)*Sin[6*(e + f*x)]))/(512*a^2*c^4*f*(-I + Tan[e + f*x])^2)
```

Maple [A]
time = 0.28, size = 119, normalized size = 0.62

method	result
derivativedivides	$\frac{-\frac{15i\ln(\tan(fx+e)-i)}{128}-\frac{i}{64(\tan(fx+e)-i)^2}+\frac{5}{64(\tan(fx+e)-i)}+\frac{3i}{32(\tan(fx+e)+i)^2}-\frac{i}{32(\tan(fx+e)+i)^4}+\frac{15i\ln(\tan(fx+e)+i)}{128}-\frac{}{16(}}{f\,a^2c^4}$

default	$\frac{-\frac{15i\ln(\tan(fx+e)-i)}{128}-\frac{i}{64(\tan(fx+e)-i)^2}+\frac{5}{64(\tan(fx+e)-i)}+\frac{3i}{32(\tan(fx+e)+i)^2}-\frac{i}{32(\tan(fx+e)+i)^4}+\frac{15i\ln(\tan(fx+e)+i)}{128}-\ldots}{f\,a^2c^4}$
risch	$\frac{15x}{64a^2c^4}-\frac{ie^{8i(fx+e)}}{512a^2c^4f}-\frac{ie^{6i(fx+e)}}{64a^2c^4f}-\frac{7i\cos(4fx+4e)}{128a^2c^4f}+\frac{\sin(4fx+4e)}{16a^2c^4f}-\frac{7i\cos(2fx+2e)}{64a^2c^4f}+\frac{13\sin(2fx+2e)}{64a^2c^4f}$
norman	$\frac{\frac{15x}{64ac}+\frac{49\tan(fx+e)}{64acf}+\frac{73\left(\tan^3(fx+e)\right)}{64acf}+\frac{55\left(\tan^5(fx+e)\right)}{64acf}+\frac{15\left(\tan^7(fx+e)\right)}{64acf}+\frac{15x\left(\tan^2(fx+e)\right)}{16ac}+\frac{45x\left(\tan^4(fx+e)\right)}{32ac}+\frac{15x\left(\tan^6\ldots\right)}{16\ldots}}{\left(1+\tan^2(fx+e)\right)^4a\,c^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a^2/c^4*(-15/128*I*ln(tan(f*x+e)-I)-1/64*I/(tan(f*x+e)-I)^2+5/64/(tan(f
*x+e)-I)+3/32*I/(tan(f*x+e)+I)^2-1/32*I/(tan(f*x+e)+I)^4+15/128*I*ln(tan(f*
x+e)+I)-1/16/(tan(f*x+e)+I)^3+5/32/(tan(f*x+e)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^4,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.56, size = 97, normalized size = 0.50

$$\frac{\left(120\,fxe^{(4i\,fx+4i\,e)}-i\,e^{(12i\,fx+12i\,e)}-8i\,e^{(10i\,fx+10i\,e)}-30i\,e^{(8i\,fx+8i\,e)}-80i\,e^{(6i\,fx+6i\,e)}+24i\,e^{(2i\,fx+2i\,e)}+2i\right)e^{(-4i\,fx-4i\,e)}}{512\,a^2c^4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^4,x, algorithm="fricas"
)
```

```
[Out] 1/512*(120*f*x*e^(4*I*f*x + 4*I*e) - I*e^(12*I*f*x + 12*I*e) - 8*I*e^(10*I*
f*x + 10*I*e) - 30*I*e^(8*I*f*x + 8*I*e) - 80*I*e^(6*I*f*x + 6*I*e) + 24*I*
e^(2*I*f*x + 2*I*e) + 2*I)*e^(-4*I*f*x - 4*I*e)/(a^2*c^4*f)
```

Sympy [A]
time = 0.39, size = 296, normalized size = 1.53

$$\begin{cases}\frac{\left(-8589934592ia^{10}c^{20}f^5e^{14ie}e^{8ifx}-68719476736ia^{10}c^{20}f^5e^{12ie}e^{6ifx}-257698037760ia^{10}c^{20}f^5e^{10ie}e^{4ifx}-687194767360ia^{10}c^{20}f^5e^{8ie}e^{2ifx}+206158430208ia^{10}c^{20}f^5e^{4ie}e^{-2ifx}+17179869184ia^{10}c^{20}f^5e^{2ie}e^{-4ifx}\right)e^{-6ie}}{4398046511104a^{12}c^{24}f^6} & \text{for } a^{12}c^{24}f^6e^{6ie}\neq 0\\ x\left(\frac{\left(e^{12ie}+6e^{10ie}+15e^{8ie}+20e^{6ie}+15e^{4ie}+6e^{2ie}+1\right)e^{-4ie}}{64a^2c^4}-\frac{15}{64a^2c^4}\right) & \text{otherwise}\end{cases}+\frac{15x}{64a^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**2/(c-I*c*tan(f*x+e))**4,x)
```

```
[Out] Piecewise(((-8589934592*I*a**10*c**20*f**5*exp(14*I*e)*exp(8*I*f*x) - 68719
476736*I*a**10*c**20*f**5*exp(12*I*e)*exp(6*I*f*x) - 257698037760*I*a**10*c
**20*f**5*exp(10*I*e)*exp(4*I*f*x) - 687194767360*I*a**10*c**20*f**5*exp(8*
I*e)*exp(2*I*f*x) + 206158430208*I*a**10*c**20*f**5*exp(4*I*e)*exp(-2*I*f*x
) + 17179869184*I*a**10*c**20*f**5*exp(2*I*e)*exp(-4*I*f*x))*exp(-6*I*e)/(4
398046511104*a**12*c**24*f**6), Ne(a**12*c**24*f**6*exp(6*I*e), 0)), (x*((e
xp(12*I*e) + 6*exp(10*I*e) + 15*exp(8*I*e) + 20*exp(6*I*e) + 15*exp(4*I*e)
+ 6*exp(2*I*e) + 1)*exp(-4*I*e)/(64*a**2*c**4) - 15/(64*a**2*c**4)), True))
 + 15*x/(64*a**2*c**4)
```

Giac [A]
time = 0.72, size = 149, normalized size = 0.77

$$-\frac{-\frac{60i\log\left(-i\tan\left(fx+e\right)+1\right)}{a^2c^4}+\frac{60i\log\left(-i\tan\left(fx+e\right)-1\right)}{a^2c^4}+\frac{2\left(-45i\tan\left(fx+e\right)^2-110\tan\left(fx+e\right)+69i\right)}{a^2c^4\left(\tan\left(fx+e\right)-i\right)^2}+\frac{125i\tan\left(fx+e\right)^4-580\tan\left(fx+e\right)^3-1038i\tan\left(fx+e\right)^2+868\tan\left(fx+e\right)+301i}{a^2c^4\left(\tan\left(fx+e\right)+i\right)^4}}{512f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -1/512*(-60*I*log(-I*tan(f*x + e) + 1)/(a^2*c^4) + 60*I*log(-I*tan(f*x + e)
 - 1)/(a^2*c^4) + 2*(-45*I*tan(f*x + e)^2 - 110*tan(f*x + e) + 69*I)/(a^2*c
^4*(tan(f*x + e) - I)^2) + (125*I*tan(f*x + e)^4 - 580*tan(f*x + e)^3 - 103
8*I*tan(f*x + e)^2 + 868*tan(f*x + e) + 301*I)/(a^2*c^4*(tan(f*x + e) + I)^
4))/f
```

Mupad [B]
time = 6.05, size = 98, normalized size = 0.51

$$\frac{15\,x}{64\,a^2\,c^4}-\frac{\frac{15\,{\mathrm{tan}\left(e+f\,x\right)}^5}{64}+\frac{{\mathrm{tan}\left(e+f\,x\right)}^4\,15{}\mathrm{i}}{32}+\frac{5\,{\mathrm{tan}\left(e+f\,x\right)}^3}{32}+\frac{{\mathrm{tan}\left(e+f\,x\right)}^2\,25{}\mathrm{i}}{32}-\frac{17\,\mathrm{tan}\left(e+f\,x\right)}{64}+\frac{1}{4}{}\mathrm{i}}{a^2\,c^4\,f\,{\left(1+\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^2\,{\left(\mathrm{tan}\left(e+f\,x\right)+1{}\mathrm{i}\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i)^4),x)
```

```
[Out] (15*x)/(64*a^2*c^4) - ((tan(e + f*x)^2*25i)/32 - (17*tan(e + f*x))/64 + (5*
tan(e + f*x)^3)/32 + (tan(e + f*x)^4*15i)/32 + (15*tan(e + f*x)^5)/64 + 1i/
4)/(a^2*c^4*f*(tan(e + f*x)*1i + 1)^2*(tan(e + f*x) + 1i)^4)
```

3.955 $\int \frac{1}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^4}\,dx$

Optimal. Leaf size=223

$$\frac{35x}{128a^3c^4}-\frac{i}{64a^3f(c-ic\tan(e+fx))^4}-\frac{i}{24a^3cf(c-ic\tan(e+fx))^3}+\frac{i}{96a^3cf(c+ic\tan(e+fx))^3}-\frac{}{64a^3f(}$$

```
[Out] 35/128*x/a^3/c^4-1/64*I/a^3/f/(c-I*c*tan(f*x+e))^4-1/24*I/a^3/c/f/(c-I*c*ta
n(f*x+e))^3+1/96*I/a^3/c/f/(c+I*c*tan(f*x+e))^3-5/64*I/a^3/f/(c^2-I*c^2*tan
(f*x+e))^2+5/128*I/a^3/f/(c^2+I*c^2*tan(f*x+e))^2-5/32*I/a^3/f/(c^4-I*c^4*t
an(f*x+e))+15/128*I/a^3/f/(c^4+I*c^4*tan(f*x+e))
```

Rubi [A]
time = 0.15, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {3603, 3568, 46, 212}

$$-\frac{5i}{32a^3f(c^4-ic^4\tan(e+fx))}+\frac{15i}{128a^3f(c^4+ic^4\tan(e+fx))}+\frac{35x}{128a^3c^4}-\frac{5i}{64a^3f(c^2-ic^2\tan(e+fx))^2}+\frac{5i}{128a^3f(c^2+ic^2\tan(e+fx))^2}-\frac{i}{24a^3cf(c-ic\tan(e+fx))^3}+\frac{i}{96a^3cf(c+ic\tan(e+fx))^3}-\frac{i}{64a^3f(c-ic\tan(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])^4),x]
```

```
[Out] (35*x)/(128*a^3*c^4) - (I/64)/(a^3*f*(c - I*c*Tan[e + f*x])^4) - (I/24)/(a^
3*c*f*(c - I*c*Tan[e + f*x])^3) + (I/96)/(a^3*c*f*(c + I*c*Tan[e + f*x])^3)
 - ((5*I)/64)/(a^3*f*(c^2 - I*c^2*Tan[e + f*x])^2) + ((5*I)/128)/(a^3*f*(c^
2 + I*c^2*Tan[e + f*x])^2) - ((5*I)/32)/(a^3*f*(c^4 - I*c^4*Tan[e + f*x]))
+ ((15*I)/128)/(a^3*f*(c^4 + I*c^4*Tan[e + f*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
```

```
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^4}\,dx &= \frac{\int \frac{\cos^6(e+fx)}{c-ic\tan(e+fx)}\,dx}{a^3c^3} \\
&= \frac{(ic^4)\,\mathrm{Subst}\left(\int \frac{1}{(c-x)^4(c+x)^5}\,dx, x, -ic\tan(e+fx)\right)}{a^3f} \\
&= \frac{(ic^4)\,\mathrm{Subst}\left(\int \left(\frac{1}{32c^5(c-x)^4}+\frac{5}{64c^6(c-x)^3}+\frac{15}{128c^7(c-x)^2}+\frac{1}{16c^4(c\ldots}\right.\right.}{} \\
&= -\frac{i}{64a^3f(c-ic\tan(e+fx))^4}-\frac{i}{24a^3cf(c-ic\tan(e+f\ldots} \\
&= \frac{35x}{128a^3c^4}-\frac{i}{64a^3f(c-ic\tan(e+fx))^4}-\frac{i}{24a^3cf(c-ic\,\mathrm{t}\ldots}
\end{aligned}$$

Mathematica [A]
time = 1.43, size = 133, normalized size = 0.60

$$\frac{(\cos(e+fx)+i\sin(e+fx))(420(-i+2fx)\cos(e+fx)+126i\cos(3(e+fx))+14i\cos(5(e+fx))+i\cos(7(e+fx))+420\sin(e+fx)-840ifx\sin(e+fx)+378\sin(3(e+fx))+70\sin(5(e+fx))+7\sin(7(e+fx)))}{3072a^3c^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])^4),x]
```

```
[Out] ((Cos[e + f*x] + I*Sin[e + f*x])*(420*(-I + 2*f*x)*Cos[e + f*x] + (126*I)*C
os[3*(e + f*x)] + (14*I)*Cos[5*(e + f*x)] + I*Cos[7*(e + f*x)] + 420*Sin[e
+ f*x] - (840*I)*f*x*Sin[e + f*x] + 378*Sin[3*(e + f*x)] + 70*Sin[5*(e + f*
x)] + 7*Sin[7*(e + f*x)]))/(3072*a^3*c^4*f)
```

Maple [A]
time = 0.24, size = 132, normalized size = 0.59

method	result
derivativedivides	$\frac{-\frac{35i\ln(\tan(fx+e)-i)}{256}-\frac{5i}{128(\tan(fx+e)-i)^2}-\frac{1}{96(\tan(fx+e)-i)^3}+\frac{15}{128(\tan(fx+e)-i)}+\frac{5i}{64(\tan(fx+e)+i)^2}-\frac{i}{64(\tan(fx+e)+i)^4}+}{f\,a^3c^4}$
default	$\frac{-\frac{35i\ln(\tan(fx+e)-i)}{256}-\frac{5i}{128(\tan(fx+e)-i)^2}-\frac{1}{96(\tan(fx+e)-i)^3}+\frac{15}{128(\tan(fx+e)-i)}+\frac{5i}{64(\tan(fx+e)+i)^2}-\frac{i}{64(\tan(fx+e)+i)^4}+}{f\,a^3c^4}$
risch	$\frac{35x}{128a^3c^4}-\frac{ie^{8i(fx+e)}}{1024a^3c^4f}-\frac{i\cos(6fx+6e)}{128a^3c^4f}+\frac{\sin(6fx+6e)}{96a^3c^4f}-\frac{7i\cos(4fx+4e)}{256a^3c^4f}+\frac{7\sin(4fx+4e)}{128a^3c^4f}-\frac{7i\cos(2fx+2e)}{128a^3c^4f}$
norman	$\frac{\frac{35x}{128ac}-\frac{i}{8acf}+\frac{93\tan(fx+e)}{128acf}+\frac{511\left(\tan^3(fx+e)\right)}{384acf}+\frac{385\left(\tan^5(fx+e)\right)}{384acf}+\frac{35\left(\tan^7(fx+e)\right)}{128acf}+\frac{35x\left(\tan^2(fx+e)\right)}{32ac}+\frac{105x\left(\tan^4(fx+e)\right)}{64ac}}{\left(1+\tan^2(fx+e)\right)^4a^2c^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a^3/c^4*(-35/256*I*ln(tan(f*x+e)-I)-5/128*I/(tan(f*x+e)-I)^2-1/96/(tan(
f*x+e)-I)^3+15/128/(tan(f*x+e)-I)+5/64*I/(tan(f*x+e)+I)^2-1/64*I/(tan(f*x+e
)+I)^4+35/256*I*ln(tan(f*x+e)+I)-1/24/(tan(f*x+e)+I)^3+5/32/(tan(f*x+e)+I))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^4,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]

time = 1.84, size = 109, normalized size = 0.49

$$\frac{\left(840\,fxe^{(6i\,fx+6i\,e)}-3i\,e^{(14i\,fx+14i\,e)}-28i\,e^{(12i\,fx+12i\,e)}-126i\,e^{(10i\,fx+10i\,e)}-420i\,e^{(8i\,fx+8i\,e)}+252i\,e^{(4i\,fx+4i\,e)}+42i\,e^{(2i\,fx+2i\,e)}+4i\right)e^{(-6i\,fx-6i\,e)}}{3072\,a^3c^4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^4,x, algorithm="fricas"
)
```

```
[Out] 1/3072*(840*f*x*e^(6*I*f*x + 6*I*e) - 3*I*e^(14*I*f*x + 14*I*e) - 28*I*e^(1
2*I*f*x + 12*I*e) - 126*I*e^(10*I*f*x + 10*I*e) - 420*I*e^(8*I*f*x + 8*I*e)
 + 252*I*e^(4*I*f*x + 4*I*e) + 42*I*e^(2*I*f*x + 2*I*e) + 4*I)*e^(-6*I*f*x
- 6*I*e)/(a^3*c^4*f)
```

Sympy [A]
time = 0.48, size = 333, normalized size = 1.49

$$\begin{cases} \frac{\left(-10133099161583616 i a^{18} c^{24} f^{6} e^{20 i e} e^{8 i f x} - 94575592174780416 i a^{18} c^{24} f^{6} e^{18 i e} e^{6 i f x} - 425590164786511872 i a^{18} c^{24} f^{6} e^{16 i e} e^{4 i f x} - 1418633882621706240 i a^{18} c^{24} f^{6} e^{14 i e} e^{2 i f x} + 851180329573023744 i a^{18} c^{24} f^{6} e^{10 i e} e^{- 2 i f x} + 141863388262170624 i a^{18} c^{24} f^{6} e^{8 i e} e^{- 4 i f x} + 13510798882111488 i a^{18} c^{24} f^{6} e^{6 i e} e^{- 6 i f x}\right) e^{- 12 i e}}{10376293541461622784 a^{21} c^{28} f^{7}} & \text{for}\: a^{21} c^{28} f^{7} e^{12 i e} \neq 0 \\ x \left(\frac{\left(e^{14 i e} + 7 e^{12 i e} + 21 e^{10 i e} + 35 e^{8 i e} + 35 e^{6 i e} + 21 e^{4 i e} + 7 e^{2 i e} + 1\right) e^{- 6 i e}}{128 a^{3} c^{4}} - \frac{35}{128 a^{3} c^{4}}\right) & \text{otherwise} \end{cases} + \frac{35 x}{128 a^{3} c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**3/(c-I*c*tan(f*x+e))**4,x)
```

```
[Out] Piecewise(((-10133099161583616*I*a**18*c**24*f**6*exp(20*I*e)*exp(8*I*f*x)
- 94575592174780416*I*a**18*c**24*f**6*exp(18*I*e)*exp(6*I*f*x) - 425590164
786511872*I*a**18*c**24*f**6*exp(16*I*e)*exp(4*I*f*x) - 1418633882621706240
*I*a**18*c**24*f**6*exp(14*I*e)*exp(2*I*f*x) + 851180329573023744*I*a**18*c
**24*f**6*exp(10*I*e)*exp(-2*I*f*x) + 141863388262170624*I*a**18*c**24*f**6
*exp(8*I*e)*exp(-4*I*f*x) + 13510798882111488*I*a**18*c**24*f**6*exp(6*I*e)
*exp(-6*I*f*x))*exp(-12*I*e)/(10376293541461622784*a**21*c**28*f**7), Ne(a*
*21*c**28*f**7*exp(12*I*e), 0)), (x*((exp(14*I*e) + 7*exp(12*I*e) + 21*exp(
10*I*e) + 35*exp(8*I*e) + 35*exp(6*I*e) + 21*exp(4*I*e) + 7*exp(2*I*e) + 1)
*exp(-6*I*e)/(128*a**3*c**4) - 35/(128*a**3*c**4)), True)) + 35*x/(128*a**3
*c**4)
```

Giac [A]
time = 0.82, size = 160, normalized size = 0.72

$$-\frac{\frac{420 i \log(\tan(fx+e)-i)}{a^3c^4} - \frac{420 i \log(i\tan(fx+e)-1)}{a^3c^4} - \frac{2\left(385\tan(fx+e)^3 - 1335 i\tan(fx+e)^2 - 1575\tan(fx+e) + 641 i\right)}{a^3c^4(i\tan(fx+e)+1)^3} + \frac{875 i\tan(fx+e)^4 - 3980\tan(fx+e)^3 - 6930 i\tan(fx+e)^2 + 5548\tan(fx+e) + 1771 i}{a^3c^4(\tan(fx+e)+i)^4}}{3072 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -1/3072*(420*I*log(tan(f*x + e) - I)/(a^3*c^4) - 420*I*log(I*tan(f*x + e) -
 1)/(a^3*c^4) - 2*(385*tan(f*x + e)^3 - 1335*I*tan(f*x + e)^2 - 1575*tan(f*
x + e) + 641*I)/(a^3*c^4*(I*tan(f*x + e) + 1)^3) + (875*I*tan(f*x + e)^4 -
3980*tan(f*x + e)^3 - 6930*I*tan(f*x + e)^2 + 5548*tan(f*x + e) + 1771*I)/(
a^3*c^4*(tan(f*x + e) + I)^4))/f
```

Mupad [B]
time = 7.10, size = 109, normalized size = 0.49

$$\frac{35\,x}{128\,a^3\,c^4}-\frac{\frac{{\mathrm{tan}\left(e+f\,x\right)}^6\,35{}\mathrm{i}}{128}-\frac{35\,{\mathrm{tan}\left(e+f\,x\right)}^5}{128}+\frac{{\mathrm{tan}\left(e+f\,x\right)}^4\,35{}\mathrm{i}}{48}-\frac{35\,{\mathrm{tan}\left(e+f\,x\right)}^3}{48}+\frac{{\mathrm{tan}\left(e+f\,x\right)}^2\,77{}\mathrm{i}}{128}-\frac{77\,\mathrm{tan}\left(e+f\,x\right)}{128}+\frac{1}{8}{}\mathrm{i}}{a^3\,c^4\,f\,{\left(1+\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^3\,{\left(\mathrm{tan}\left(e+f\,x\right)+1{}\mathrm{i}\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i)^4),x)
```

```
[Out] (35*x)/(128*a^3*c^4) - ((tan(e + f*x)^2*77i)/128 - (77*tan(e + f*x))/128 -
(35*tan(e + f*x)^3)/48 + (tan(e + f*x)^4*35i)/48 - (35*tan(e + f*x)^5)/128
+ (tan(e + f*x)^6*35i)/128 + 1i/8)/(a^3*c^4*f*(tan(e + f*x)*1i + 1)^3*(tan(
e + f*x) + 1i)^4)
```

3.956 $\int (a+ia\tan(e+fx))^3\sqrt{c-ic\tan(e+fx)}\,dx$

Optimal. Leaf size=92

$$\frac{8ia^3\sqrt{c-ic\tan(e+fx)}}{f}-\frac{8ia^3(c-ic\tan(e+fx))^{3/2}}{3cf}+\frac{2ia^3(c-ic\tan(e+fx))^{5/2}}{5c^2f}$$

```
[Out] 8*I*a^3*(c-I*c*tan(f*x+e))^(1/2)/f-8/3*I*a^3*(c-I*c*tan(f*x+e))^(3/2)/c/f+2
/5*I*a^3*(c-I*c*tan(f*x+e))^(5/2)/c^2/f
```

Rubi [A]
time = 0.12, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3603, 3568, 45}

$$\frac{2ia^3(c-ic\tan(e+fx))^{5/2}}{5c^2f}-\frac{8ia^3(c-ic\tan(e+fx))^{3/2}}{3cf}+\frac{8ia^3\sqrt{c-ic\tan(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^3*Sqrt[c - I*c*Tan[e + f*x]],x]
```

```
[Out] ((8*I)*a^3*Sqrt[c - I*c*Tan[e + f*x]])/f - (((8*I)/3)*a^3*(c - I*c*Tan[e +
f*x])^(3/2))/(c*f) + (((2*I)/5)*a^3*(c - I*c*Tan[e + f*x])^(5/2))/(c^2*f)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int (a+ia\tan(e+fx))^3\sqrt{c-ic\tan(e+fx)}\,dx = \left(a^3c^3\right)\int \frac{\sec^6(e+fx)}{(c-ic\tan(e+fx))^{5/2}}\,dx$$
$$= \frac{(ia^3)\,\text{Subst}\left(\int \frac{(c-x)^2}{\sqrt{c+x}}\,dx, x, -ic\tan(e+fx)\right)}{c^2f}$$
$$= \frac{(ia^3)\,\text{Subst}\left(\int \left(\frac{4c^2}{\sqrt{c+x}}-4c\sqrt{c+x}+(c+x)^{3/2}\right)dx,\right.}{c^2f}$$
$$= \frac{8ia^3\sqrt{c-ic\tan(e+fx)}}{f}-\frac{8ia^3(c-ic\tan(e+fx))^{3/2}}{3cf}$$

Mathematica [A]
time = 1.32, size = 61, normalized size = 0.66

$$\frac{2ia^3\sec^2(e+fx)(20+23\cos(2(e+fx))+7i\sin(2(e+fx)))\sqrt{c-ic\tan(e+fx)}}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3*Sqrt[c - I*c*Tan[e + f*x]],x]
```

```
[Out] (((2*I)/15)*a^3*Sec[e + f*x]^2*(20 + 23*Cos[2*(e + f*x)] + (7*I)*Sin[2*(e +
 f*x)])*Sqrt[c - I*c*Tan[e + f*x]])/f
```

Maple [A]
time = 0.36, size = 66, normalized size = 0.72

method	result	size
derivativedivides	$\frac{2ia^3\left(\frac{(c-ic\tan(fx+e))^{\frac{5}{2}}}{5}-\frac{4c(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}+4c^2\sqrt{c-ic\tan(fx+e)}\right)}{f\,c^2}$	66
default	$\frac{2ia^3\left(\frac{(c-ic\tan(fx+e))^{\frac{5}{2}}}{5}-\frac{4c(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}+4c^2\sqrt{c-ic\tan(fx+e)}\right)}{f\,c^2}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/f*a^3/c^2*(1/5*(c-I*c*tan(f*x+e))^(5/2)-4/3*c*(c-I*c*tan(f*x+e))^(3/2)+
4*c^2*(c-I*c*tan(f*x+e))^(1/2))
```

Maxima [A]
time = 0.28, size = 70, normalized size = 0.76

$$\frac{2i\left(3\left(-i\,c\tan\left(fx+e\right)+c\right)^{\frac{5}{2}}a^{3}-20\left(-i\,c\tan\left(fx+e\right)+c\right)^{\frac{3}{2}}a^{3}c+60\,\sqrt{-i\,c\tan\left(fx+e\right)+c}\,a^{3}c^{2}\right)}{15\,c^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^3,x, algorithm="maxim
a")
```

```
[Out] 2/15*I*(3*(-I*c*tan(f*x + e) + c)^(5/2)*a^3 - 20*(-I*c*tan(f*x + e) + c)^(3
/2)*a^3*c + 60*sqrt(-I*c*tan(f*x + e) + c)*a^3*c^2)/(c^2*f)
```

Fricas [A]
time = 1.22, size = 88, normalized size = 0.96

$$-\frac{8\,\sqrt{2}\,\left(-15i\,a^{3}e^{(4i\,fx+4i\,e)}-20i\,a^{3}e^{(2i\,fx+2i\,e)}-8i\,a^{3}\right)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{15\left(fe^{(4i\,fx+4i\,e)}+2\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^3,x, algorithm="frica
s")
```

```
[Out] -8/15*sqrt(2)*(-15*I*a^3*e^(4*I*f*x + 4*I*e) - 20*I*a^3*e^(2*I*f*x + 2*I*e)
 - 8*I*a^3)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(f*e^(4*I*f*x + 4*I*e) + 2*f*
e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia^{3}\left(\int i\sqrt{-ic\tan\left(e+fx\right)+c}\,dx+\int\left(-3\sqrt{-ic\tan\left(e+fx\right)+c}\tan\left(e+fx\right)\right)dx+\int\sqrt{-ic\tan\left(e+fx\right)+c}\tan^{3}\left(e+fx\right)dx+\int\left(-3i\sqrt{-ic\tan\left(e+fx\right)+c}\tan^{2}\left(e+fx\right)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**(1/2)*(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] -I*a**3*(Integral(I*sqrt(-I*c*tan(e + f*x) + c), x) + Integral(-3*sqrt(-I*c
*tan(e + f*x) + c)*tan(e + f*x), x) + Integral(sqrt(-I*c*tan(e + f*x) + c)*
tan(e + f*x)**3, x) + Integral(-3*I*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x
)**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^3,x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^3*sqrt(-I*c*tan(f*x + e) + c), x)
```

Mupad [B]
time = 6.21, size = 155, normalized size = 1.68

$$\frac{4\,a^3\,\sqrt{\frac{c\,\left(\cos\left(2\,e+2\,f\,x\right)+1-\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\left(\cos\left(2\,e+2\,f\,x\right)\,321{}\mathrm{i}+\cos\left(4\,e+4\,f\,x\right)\,132{}\mathrm{i}+\cos\left(6\,e+6\,f\,x\right)\,23{}\mathrm{i}-35\,\sin\left(2\,e+2\,f\,x\right)-28\,\sin\left(4\,e+4\,f\,x\right)-7\,\sin\left(6\,e+6\,f\,x\right)+212{}\mathrm{i}\right)}{15\,f\,\left(15\,\cos\left(2\,e+2\,f\,x\right)+6\,\cos\left(4\,e+4\,f\,x\right)+\cos\left(6\,e+6\,f\,x\right)+10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i)^(1/2),x)
```

```
[Out] (4*a^3*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x)
+ 1))^(1/2)*(cos(2*e + 2*f*x)*321i + cos(4*e + 4*f*x)*132i + cos(6*e + 6*f*
x)*23i - 35*sin(2*e + 2*f*x) - 28*sin(4*e + 4*f*x) - 7*sin(6*e + 6*f*x) + 2
12i))/(15*f*(15*cos(2*e + 2*f*x) + 6*cos(4*e + 4*f*x) + cos(6*e + 6*f*x) +
10))
```

3.957 $\int (a+ia\tan(e+fx))^2\sqrt{c-ic\tan(e+fx)}\,dx$

Optimal. Leaf size=60

$$\frac{4ia^2\sqrt{c-ic\tan(e+fx)}}{f}-\frac{2ia^2(c-ic\tan(e+fx))^{3/2}}{3cf}$$

```
[Out] 4*I*a^2*(c-I*c*tan(f*x+e))^(1/2)/f-2/3*I*a^2*(c-I*c*tan(f*x+e))^(3/2)/c/f
```

Rubi [A]
time = 0.10, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3603, 3568, 45}

$$\frac{4ia^2\sqrt{c-ic\tan(e+fx)}}{f}-\frac{2ia^2(c-ic\tan(e+fx))^{3/2}}{3cf}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2*Sqrt[c - I*c*Tan[e + f*x]],x]
```

```
[Out] ((4*I)*a^2*Sqrt[c - I*c*Tan[e + f*x]])/f - (((2*I)/3)*a^2*(c - I*c*Tan[e +
f*x])^(3/2))/(c*f)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int (a+ia\tan(e+fx))^2\sqrt{c-ic\tan(e+fx)}\,dx = (a^2c^2)\int \frac{\sec^4(e+fx)}{(c-ic\tan(e+fx))^{3/2}}\,dx$$
$$= \frac{(ia^2)\,\text{Subst}\left(\int \frac{c-x}{\sqrt{c+x}}\,dx, x, -ic\tan(e+fx)\right)}{cf}$$
$$= \frac{(ia^2)\,\text{Subst}\left(\int \left(\frac{2c}{\sqrt{c+x}}-\sqrt{c+x}\right)\,dx, x, -ic\tan(e+$$
$$= \frac{4ia^2\sqrt{c-ic\tan(e+fx)}}{f} - \frac{2ia^2(c-ic\tan(e+fx))^{3/2}}{3cf}$$

Mathematica [A]
time = 0.92, size = 37, normalized size = 0.62

$$-\frac{2a^2(-5i+\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2*Sqrt[c - I*c*Tan[e + f*x]],x]
[Out] (-2*a^2*(-5*I + Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(3*f)
```

Maple [A]
time = 0.23, size = 47, normalized size = 0.78

method	result	size
derivativedivides	$-\frac{2ia^2\left(\frac{(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}-2c\sqrt{c-ic\tan(fx+e)}\right)}{fc}$	47
default	$-\frac{2ia^2\left(\frac{(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}-2c\sqrt{c-ic\tan(fx+e)}\right)}{fc}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
[Out] -2*I/f*a^2/c*(1/3*(c-I*c*tan(f*x+e))^(3/2)-2*c*(c-I*c*tan(f*x+e))^(1/2))
```

Maxima [A]
time = 0.28, size = 47, normalized size = 0.78

$$-\frac{2i\left((-ic\tan(fx+e)+c)^{\frac{3}{2}}a^2-6\sqrt{-ic\tan(fx+e)+c}\,a^2c\right)}{3cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^2,x, algorithm="maxim
a")
```

```
[Out] -2/3*I*((-I*c*tan(f*x + e) + c)^(3/2)*a^2 - 6*sqrt(-I*c*tan(f*x + e) + c)*a
^2*c)/(c*f)
```

Fricas [A]
time = 1.36, size = 60, normalized size = 1.00

$$-\frac{4\sqrt{2}\left(-3i\,a^{2}e^{(2i\,fx+2i\,e)}-2i\,a^{2}\right)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{3\left(fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^2,x, algorithm="frica
s")
```

```
[Out] -4/3*sqrt(2)*(-3*I*a^2*e^(2*I*f*x + 2*I*e) - 2*I*a^2)*sqrt(c/(e^(2*I*f*x +
2*I*e) + 1))/(f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^{2}\left(\int\sqrt{-ic\tan{(e+fx)}+c}\,\tan^{2}{(e+fx)}\,dx+\int\left(-2i\sqrt{-ic\tan{(e+fx)}+c}\,\tan{(e+fx)}\right)dx+\int\left(-\sqrt{-ic\tan{(e+fx)}+c}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**(1/2)*(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -a**2*(Integral(sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2, x) + Integral(
-2*I*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x), x) + Integral(-sqrt(-I*c*tan
(e + f*x) + c), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^2,x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^2*sqrt(-I*c*tan(f*x + e) + c), x)
```

Mupad [B]
time = 4.79, size = 87, normalized size = 1.45

$$\frac{2\,a^2\,\sqrt{\frac{c\,\left(\cos\left(2\,e+2\,f\,x\right)+1-\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\left(\cos\left(2\,e+2\,f\,x\right)\,5{}\mathrm{i}-\sin\left(2\,e+2\,f\,x\right)+5{}\mathrm{i}\right)}{3\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i)^(1/2),x)
```

```
[Out] (2*a^2*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x)
+ 1))^(1/2)*(cos(2*e + 2*f*x)*5i - sin(2*e + 2*f*x) + 5i))/(3*f*(cos(2*e +
2*f*x) + 1))
```

3.958 $\int (a+ia\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx$

Optimal. Leaf size=25

$$\frac{2ia\sqrt{c-ic\tan(e+fx)}}{f}$$

[Out] 2*I*a*(c-I*c*tan(f*x+e))^(1/2)/f

Rubi [A]
time = 0.07, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 32}

$$\frac{2ia\sqrt{c-ic\tan(e+fx)}}{f}$$

Antiderivative was successfully verified.

[In] Int[(a + I*a*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]],x]

[Out] ((2*I)*a*Sqrt[c - I*c*Tan[e + f*x]])/f

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int (a + ia\tan(e+fx))\sqrt{c - ic\tan(e+fx)}\,dx = (ac)\int \frac{\sec^2(e+fx)}{\sqrt{c - ic\tan(e+fx)}}\,dx$$
$$= \frac{(ia)\text{Subst}\left(\int \frac{1}{\sqrt{c+x}}\,dx, x, -ic\tan(e+fx)\right)}{f}$$
$$= \frac{2ia\sqrt{c - ic\tan(e+fx)}}{f}$$

Mathematica [A]
time = 0.42, size = 25, normalized size = 1.00

$$\frac{2ia\sqrt{c - ic\tan(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]],x]
[Out] ((2*I)*a*Sqrt[c - I*c*Tan[e + f*x]])/f
```

Maple [A]
time = 0.19, size = 22, normalized size = 0.88

method	result	size
derivativedivides	$\frac{2ia\sqrt{c - ic\tan(fx+e)}}{f}$	22
default	$\frac{2ia\sqrt{c - ic\tan(fx+e)}}{f}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
[Out] 2*I*a*(c-I*c*tan(f*x+e))^(1/2)/f
```

Maxima [A]
time = 0.29, size = 20, normalized size = 0.80

$$\frac{2i\sqrt{-ic\tan(fx+e)+c}\,a}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e)),x, algorithm="maxima"
)
```

[Out] 2*I*sqrt(-I*c*tan(f*x + e) + c)*a/f

Fricas [A]
time = 0.92, size = 27, normalized size = 1.08

$$\frac{2i\sqrt{2}\,a\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e)),x, algorithm="fricas")

[Out] 2*I*sqrt(2)*a*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/f

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(20) = 40.
time = 1.69, size = 42, normalized size = 1.68

$$\begin{cases} \frac{2ia\sqrt{-ic\tan{(e+fx)}+c}}{f} & \text{for } f\neq 0\\ x(ia\tan{(e)}+a)\sqrt{-ic\tan{(e)}+c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I*c*tan(f*x+e))**(1/2)*(a+I*a*tan(f*x+e)),x)

[Out] Piecewise((2*I*a*sqrt(-I*c*tan(e + f*x) + c)/f, Ne(f, 0)), (x*(I*a*tan(e) + a)*sqrt(-I*c*tan(e) + c), True))

Giac [A]
time = 1.15, size = 20, normalized size = 0.80

$$\frac{2i\sqrt{-i\,c\tan{(fx+e)}+c}\,a}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e)),x, algorithm="giac")

[Out] 2*I*sqrt(-I*c*tan(f*x + e) + c)*a/f

Mupad [B]
time = 0.46, size = 29, normalized size = 1.16

$$\frac{\sqrt{2}\,a\sqrt{\frac{c}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,2\mathrm{i}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i)^(1/2),x)

[Out] (2^(1/2)*a*(c/(exp(e*2i + f*x*2i) + 1))^(1/2)*2i)/f

3.959 $\int \frac{\sqrt{c-ic\tan(e+fx)}}{a+ia\tan(e+fx)}\,dx$

Optimal. Leaf size=95

$$\frac{i\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}\,af}+\frac{i\sqrt{c-ic\tan(e+fx)}}{2af(1+i\tan(e+fx))}$$

```
[Out] 1/4*I*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^(1/2)/a/f*2^(
1/2)+1/2*I*(c-I*c*tan(f*x+e))^(1/2)/a/f/(1+I*tan(f*x+e))
```

Rubi [A]
time = 0.12, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3603, 3568, 44, 65, 212}

$$\frac{i\sqrt{c-ic\tan(e+fx)}}{2af(1+i\tan(e+fx))}+\frac{i\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}\,af}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - I*c*Tan[e + f*x]]/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] ((I/2)*Sqrt[c]*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])])/(Sqrt
[2]*a*f) + ((I/2)*Sqrt[c - I*c*Tan[e + f*x]])/(a*f*(1 + I*Tan[e + f*x]))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{c-ic\tan(e+fx)}}{a+ia\tan(e+fx)}\,dx &= \frac{\int \cos^2(e+fx)(c-ic\tan(e+fx))^{3/2}\,dx}{ac}\\
&= \frac{(ic^2)\,\text{Subst}\left(\int \frac{1}{(c-x)^2\sqrt{c+x}}\,dx, x, -ic\tan(e+fx)\right)}{af}\\
&= \frac{i\sqrt{c-ic\tan(e+fx)}}{2af(1+i\tan(e+fx))} + \frac{(ic)\text{Subst}\left(\int \frac{1}{(c-x)\sqrt{c+x}}\,dx, x, -ic\tan(e+fx)\right)}{4af}\\
&= \frac{i\sqrt{c-ic\tan(e+fx)}}{2af(1+i\tan(e+fx))} + \frac{(ic)\text{Subst}\left(\int \frac{1}{2c-x^2}\,dx, x, \sqrt{c-ic\tan(e+fx)}\right)}{2af}\\
&= \frac{i\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}\,af} + \frac{i\sqrt{c-ic\tan(e+fx)}}{2af(1+i\tan(e+fx))}
\end{aligned}
$$

Mathematica [A]
time = 0.75, size = 110, normalized size = 1.16

$$
\frac{(i\cos(e+fx)+\sin(e+fx))\left(\sqrt{2}\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)(\cos(e+fx)+i\sin(e+fx))+2\cos(e+fx)\sqrt{c-ic\tan(e+fx)}\right)}{4af}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - I*c*Tan[e + f*x]]/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] ((I*Cos[e + f*x] + Sin[e + f*x])*(Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - I*c*Tan[
e + f*x]]/(Sqrt[2]*Sqrt[c])]*(Cos[e + f*x] + I*Sin[e + f*x]) + 2*Cos[e + f*
x]*Sqrt[c - I*c*Tan[e + f*x]]))/(4*a*f)
```

Maple [A]
time = 0.36, size = 78, normalized size = 0.82

method	result	size
derivativedivides	$\frac{2ic^2\left(\frac{\sqrt{c-ic\tan(fx+e)}}{4c(c+ic\tan(fx+e))}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)}{8c^{\frac{3}{2}}}\right)}{fa}$	78
default	$\frac{2ic^2\left(\frac{\sqrt{c-ic\tan(fx+e)}}{4c(c+ic\tan(fx+e))}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)}{8c^{\frac{3}{2}}}\right)}{fa}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/f/a*c^2*(1/4*(c-I*c*tan(f*x+e))^(1/2)/c/(c+I*c*tan(f*x+e))+1/8/c^(3/2)*
2^(1/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [A]
time = 0.52, size = 110, normalized size = 1.16

$$-\frac{i\left(\frac{\sqrt{2}\,c^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{c}-\sqrt{-i\,c\tan(fx+e)+c}}{\sqrt{2}\,\sqrt{c}+\sqrt{-i\,c\tan(fx+e)+c}}\right)}{a}+\frac{4\sqrt{-i\,c\tan(fx+e)+c}\,c^2}{(-i\,c\tan(fx+e)+c)a-2\,ac}\right)}{8\,cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e)),x, algorithm="maxima"
)
```

```
[Out] -1/8*I*(sqrt(2)*c^(3/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*c*tan(f*x + e) + c)
)/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/a + 4*sqrt(-I*c*tan(f*x
+ e) + c)*c^2/((-I*c*tan(f*x + e) + c)*a - 2*a*c))/(c*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(73) = 146.

time = 1.07, size = 261, normalized size = 2.75

$$\frac{\left(\sqrt{\frac{1}{2}}\, af\sqrt{-\frac{c}{a^2f^2}}\, e^{(2i\,fx+2i\,e)}\log\left(\frac{\left(\sqrt{2}\,\sqrt{\frac{1}{2}}\,(afe^{(2i\,fx+2i\,e)}+af)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\,\sqrt{-\frac{c}{a^2f^2}}+ic\right)e^{(-i\,fx-i\,e)}}{af}\right)-\sqrt{\frac{1}{2}}\, af\sqrt{-\frac{c}{a^2f^2}}\, e^{(2i\,fx+2i\,e)}\log\left(-\frac{\left(\sqrt{2}\,\sqrt{\frac{1}{2}}\,(afe^{(2i\,fx+2i\,e)}+af)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\,\sqrt{-\frac{c}{a^2f^2}}-ic\right)e^{(-i\,fx-i\,e)}}{af}\right)+\sqrt{2}\,\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\,\left(i\,e^{(2i\,fx+2i\,e)}+i\right)\right)e^{(-2i\,fx-2i\,e)}}{4\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e)),x, algorithm="fricas"
)
```

```
[Out] 1/4*(sqrt(1/2)*a*f*sqrt(-c/(a^2*f^2))*e^(2*I*f*x + 2*I*e)*log((sqrt(2)*sqrt
(1/2)*(a*f*e^(2*I*f*x + 2*I*e) + a*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqr
t(-c/(a^2*f^2)) + I*c)*e^(-I*f*x - I*e)/(a*f)) - sqrt(1/2)*a*f*sqrt(-c/(a^2
*f^2))*e^(2*I*f*x + 2*I*e)*log(-(sqrt(2)*sqrt(1/2)*(a*f*e^(2*I*f*x + 2*I*e)
 + a*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-c/(a^2*f^2)) - I*c)*e^(-I*f
*x - I*e)/(a*f)) + sqrt(2)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*(I*e^(2*I*f*x
+ 2*I*e) + I))*e^(-2*I*f*x - 2*I*e)/(a*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{\sqrt{-ic\tan\left(e+fx\right)+c}}{\tan\left(e+fx\right)-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] -I*Integral(sqrt(-I*c*tan(e + f*x) + c)/(tan(e + f*x) - I), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-I*c*tan(f*x + e) + c)/(I*a*tan(f*x + e) + a), x)
```

Mupad [B]
time = 0.30, size = 81, normalized size = 0.85

$$\frac{\sqrt{2}\,\sqrt{-c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}}}{2\,\sqrt{-c}}\right)\,1{}\mathrm{i}}{4\,a\,f}+\frac{c\,\sqrt{c-c\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2\,a\,f\,\left(c+c\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^(1/2)/(a + a*tan(e + f*x)*1i),x)
```

```
[Out] (2^(1/2)*(-c)^(1/2)*atan((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*(-c)^(1
/2)))*1i)/(4*a*f) + (c*(c - c*tan(e + f*x)*1i)^(1/2)*1i)/(2*a*f*(c + c*tan(
e + f*x)*1i))
```

3.960 $\int \frac{\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=138

$$\frac{3i\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{16\sqrt{2}\,a^2f}+\frac{i\sqrt{c-ic\tan(e+fx)}}{4a^2f(1+i\tan(e+fx))^2}+\frac{3i\sqrt{c-ic\tan(e+fx)}}{16a^2f(1+i\tan(e+fx))}$$

```
[Out] 3/32*I*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^(1/2)/a^2/f*
2^(1/2)+1/4*I*(c-I*c*tan(f*x+e))^(1/2)/a^2/f/(1+I*tan(f*x+e))^2+3/16*I*(c-I
*c*tan(f*x+e))^(1/2)/a^2/f/(1+I*tan(f*x+e))
```

Rubi [A]
time = 0.13, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3603, 3568, 44, 65, 212}

$$\frac{3i\sqrt{c-ic\tan(e+fx)}}{16a^2f(1+i\tan(e+fx))}+\frac{i\sqrt{c-ic\tan(e+fx)}}{4a^2f(1+i\tan(e+fx))^2}+\frac{3i\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{16\sqrt{2}\,a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - I*c*Tan[e + f*x]]/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (((3*I)/16)*Sqrt[c]*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])])/
(Sqrt[2]*a^2*f) + ((I/4)*Sqrt[c - I*c*Tan[e + f*x]])/(a^2*f*(1 + I*Tan[e +
f*x])^2) + (((3*I)/16)*Sqrt[c - I*c*Tan[e + f*x]])/(a^2*f*(1 + I*Tan[e + f*
x]))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx = \frac{\int \cos^4(e+fx)(c-ic\tan(e+fx))^{5/2}\,dx}{a^2c^2}$$

$$= \frac{(ic^3)\,\text{Subst}\left(\int \frac{1}{(c-x)^3\sqrt{c+x}}\,dx, x, -ic\tan(e+fx)\right)}{a^2f}$$

$$= \frac{i\sqrt{c-ic\tan(e+fx)}}{4a^2f(1+i\tan(e+fx))^2} + \frac{(3ic^2)\,\text{Subst}\left(\int \frac{1}{(c-x)^2\sqrt{c+x}}\,dx, x, -ic\tan(e+ f\right.}{8a^2f}$$

$$= \frac{i\sqrt{c-ic\tan(e+fx)}}{4a^2f(1+i\tan(e+fx))^2} + \frac{3i\sqrt{c-ic\tan(e+fx)}}{16a^2f(1+i\tan(e+fx))} + \frac{(3ic)\text{Subst}\left(\int \frac{}{(c-x)\sqrt{}}\right.}{}$$

$$= \frac{i\sqrt{c-ic\tan(e+fx)}}{4a^2f(1+i\tan(e+fx))^2} + \frac{3i\sqrt{c-ic\tan(e+fx)}}{16a^2f(1+i\tan(e+fx))} + \frac{(3ic)\text{Subst}\left(\int \frac{1}{2c-x^2}\,d\right.}{}$$

$$= \frac{3i\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{16\sqrt{2}\,a^2f} + \frac{i\sqrt{c-ic\tan(e+fx)}}{4a^2f(1+i\tan(e+fx))^2} + \frac{3i\sqrt{c}}{16a^2f(}$$

Mathematica [A]

time = 1.10, size = 136, normalized size = 0.99

$$\frac{(i\cos(2(e+fx))+\sin(2(e+fx)))\left(3\sqrt{2}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\,\sqrt{c}}\right)(\cos(2(e+fx))+i\sin(2(e+fx)))+(7+7\cos(2(e+fx))+3i\sin(2(e+fx)))\sqrt{c-ic\tan(e+fx)}\right)}{32a^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - I*c*Tan[e + f*x]]/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] ((I*Cos[2*(e + f*x)] + Sin[2*(e + f*x)])*(3*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c
- I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])]*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x
)]) + (7 + 7*Cos[2*(e + f*x)] + (3*I)*Sin[2*(e + f*x)])*Sqrt[c - I*c*Tan[e
+ f*x]]))/(32*a^2*f)
```

Maple [A]
time = 0.32, size = 117, normalized size = 0.85

method	result
derivativedivides	$-\frac{2ic^3\left(-\frac{\sqrt{c-ic\tan(fx+e)}}{8c(c+ic\tan(fx+e))^2}-\frac{3\left(\frac{\sqrt{c-ic\tan(fx+e)}}{4c(c+ic\tan(fx+e))}+\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+}}{2\sqrt{c}}\right.}{8c^{\frac{3}{2}}}\right.}{8c}\right.}{f\,a^2}$
default	$-\frac{2ic^3\left(-\frac{\sqrt{c-ic\tan(fx+e)}}{8c(c+ic\tan(fx+e))^2}-\frac{3\left(\frac{\sqrt{c-ic\tan(fx+e)}}{4c(c+ic\tan(fx+e))}+\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+}}{2\sqrt{c}}\right.}{8c^{\frac{3}{2}}}\right.}{8c}\right.}{f\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/f/a^2*c^3*(-1/8*(c-I*c*tan(f*x+e))^(1/2)/c/(c+I*c*tan(f*x+e))^2-3/8/c*
(1/4*(c-I*c*tan(f*x+e))^(1/2)/c/(c+I*c*tan(f*x+e))+1/8/c^(3/2)*2^(1/2)*arct
anh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))))
```

Maxima [A]

time = 0.51, size = 159, normalized size = 1.15

$$-\frac{i\left(\frac{3\sqrt{2}\,c^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{c}-\sqrt{-i\,c\tan\left(fx+e\right)+c}}{\sqrt{2}\,\sqrt{c}+\sqrt{-i\,c\tan\left(fx+e\right)+c}}\right)}{a^{2}}+\frac{4\left(3\left(-i\,c\tan(fx+e)+c\right)^{\frac{3}{2}}c^{2}-10\,\sqrt{-i\,c\tan\left(fx+e\right)+c}\,c^{3}\right)}{\left(-i\,c\tan(fx+e)+c\right)^{2}a^{2}-4\left(-i\,c\tan(fx+e)+c\right)a^{2}c+4\,a^{2}c^{2}}\right)}{64\,cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxim
a")
```

```
[Out] -1/64*I*(3*sqrt(2)*c^(3/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*c*tan(f*x + e) +
 c))/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/a^2 + 4*(3*(-I*c*tan(
f*x + e) + c)^(3/2)*c^2 - 10*sqrt(-I*c*tan(f*x + e) + c)*c^3)/((-I*c*tan(f*
x + e) + c)^2*a^2 - 4*(-I*c*tan(f*x + e) + c)*a^2*c + 4*a^2*c^2))/(c*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(108) = 216.
time = 1.27, size = 287, normalized size = 2.08

$$\frac{\left(3\sqrt{\frac{1}{2}}\,a^{2}f\sqrt{-\frac{c}{a^{4}f^{2}}}\,e^{(4i\,fx+4i\,e)}\log\left(\frac{3\left(\sqrt{2}\,\sqrt{\frac{1}{2}}\,(a^{2}fe^{(2i\,fx+2i\,e)}+a^{2}f)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\,\sqrt{-\frac{c}{a^{4}f^{2}}}+i\,c\right)e^{(-i\,fx-i\,e)}}{8\,a^{2}f}\right)-3\sqrt{\frac{1}{2}}\,a^{2}f\sqrt{-\frac{c}{a^{4}f^{2}}}\,e^{(4i\,fx+4i\,e)}\log\left(-\frac{3\left(\sqrt{2}\,\sqrt{\frac{1}{2}}\,(a^{2}fe^{(2i\,fx+2i\,e)}+a^{2}f)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\,\sqrt{-\frac{c}{a^{4}f^{2}}}-i\,c\right)e^{(-i\,fx-i\,e)}}{8\,a^{2}f}\right)+\sqrt{2}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\left(5i\,e^{(4i\,fx+4i\,e)}+7i\,e^{(2i\,fx+2i\,e)}+2i\right)\right)e^{(-4i\,fx-4i\,e)}}{32\,a^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="frica
s")
```

```
[Out] 1/32*(3*sqrt(1/2)*a^2*f*sqrt(-c/(a^4*f^2))*e^(4*I*f*x + 4*I*e)*log(3/8*(sqr
t(2)*sqrt(1/2)*(a^2*f*e^(2*I*f*x + 2*I*e) + a^2*f)*sqrt(c/(e^(2*I*f*x + 2*I
*e) + 1))*sqrt(-c/(a^4*f^2)) + I*c)*e^(-I*f*x - I*e)/(a^2*f)) - 3*sqrt(1/2)
*a^2*f*sqrt(-c/(a^4*f^2))*e^(4*I*f*x + 4*I*e)*log(-3/8*(sqrt(2)*sqrt(1/2)*(
a^2*f*e^(2*I*f*x + 2*I*e) + a^2*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-
c/(a^4*f^2)) - I*c)*e^(-I*f*x - I*e)/(a^2*f)) + sqrt(2)*sqrt(c/(e^(2*I*f*x
+ 2*I*e) + 1))*(5*I*e^(4*I*f*x + 4*I*e) + 7*I*e^(2*I*f*x + 2*I*e) + 2*I))*e
^(-4*I*f*x - 4*I*e)/(a^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt{-ic\tan\left(e+fx\right)+c}}{\tan^{2}\left(e+fx\right)-2i\tan\left(e+fx\right)-1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -Integral(sqrt(-I*c*tan(e + f*x) + c)/(tan(e + f*x)**2 - 2*I*tan(e + f*x) -
 1), x)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(-I*c*tan(f*x + e) + c)/(I*a*tan(f*x + e) + a)^2, x)
```

Mupad [B]
time = 4.85, size = 132, normalized size = 0.96

$$\frac{\frac{c^2\,\sqrt{c-c\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}}\,5{}\mathrm{i}}{8\,a^2\,f}-\frac{c\,{\left(c-c\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{3/2}\,3{}\mathrm{i}}{16\,a^2\,f}}{{\left(c-c\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^2-4\,c\,\left(c-c\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}\right)+4\,c^2}+\frac{\sqrt{2}\,\sqrt{-c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}}}{2\,\sqrt{-c}}\right)\,3{}\mathrm{i}}{32\,a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^(1/2)/(a + a*tan(e + f*x)*1i)^2,x)
```

```
[Out] ((c^2*(c - c*tan(e + f*x)*1i)^(1/2)*5i)/(8*a^2*f) - (c*(c - c*tan(e + f*x)*
1i)^(3/2)*3i)/(16*a^2*f))/((c - c*tan(e + f*x)*1i)^2 - 4*c*(c - c*tan(e + f
*x)*1i) + 4*c^2) + (2^(1/2)*(-c)^(1/2)*atan((2^(1/2)*(c - c*tan(e + f*x)*1i
)^(1/2))/(2*(-c)^(1/2)))*3i)/(32*a^2*f)
```

3.961 $\int \frac{\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^3}\,dx$

Optimal. Leaf size=181

$$\frac{5i\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{64\sqrt{2}a^3f}+\frac{i\sqrt{c-ic\tan(e+fx)}}{6a^3f(1+i\tan(e+fx))^3}+\frac{5i\sqrt{c-ic\tan(e+fx)}}{48a^3f(1+i\tan(e+fx))^2}+\frac{5i\sqrt{c-ic}}{64a^3f(1+}$$

```
[Out] 5/128*I*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^(1/2)/a^3/f
*2^(1/2)+1/6*I*(c-I*c*tan(f*x+e))^(1/2)/a^3/f/(1+I*tan(f*x+e))^3+5/48*I*(c-
I*c*tan(f*x+e))^(1/2)/a^3/f/(1+I*tan(f*x+e))^2+5/64*I*(c-I*c*tan(f*x+e))^(1
/2)/a^3/f/(1+I*tan(f*x+e))
```

Rubi [A]
time = 0.14, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3603, 3568, 44, 65, 212}

$$\frac{5i\sqrt{c-ic\tan(e+fx)}}{64a^3f(1+i\tan(e+fx))}+\frac{5i\sqrt{c-ic\tan(e+fx)}}{48a^3f(1+i\tan(e+fx))^2}+\frac{i\sqrt{c-ic\tan(e+fx)}}{6a^3f(1+i\tan(e+fx))^3}+\frac{5i\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{64\sqrt{2}a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - I*c*Tan[e + f*x]]/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] (((5*I)/64)*Sqrt[c]*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])])/
(Sqrt[2]*a^3*f) + ((I/6)*Sqrt[c - I*c*Tan[e + f*x]])/(a^3*f*(1 + I*Tan[e +
f*x])^3) + (((5*I)/48)*Sqrt[c - I*c*Tan[e + f*x]])/(a^3*f*(1 + I*Tan[e + f*
x])^2) + (((5*I)/64)*Sqrt[c - I*c*Tan[e + f*x]])/(a^3*f*(1 + I*Tan[e + f*x]
))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^3}\,dx = \frac{\int \cos^6(e+fx)(c-ic\tan(e+fx))^{7/2}\,dx}{a^3c^3}$$

$$= \frac{(ic^4)\,\text{Subst}\left(\int \frac{1}{(c-x)^4\sqrt{c+x}}\,dx, x, -ic\tan(e+fx)\right)}{a^3f}$$

$$= \frac{i\sqrt{c-ic\tan(e+fx)}}{6a^3f(1+i\tan(e+fx))^3} + \frac{(5ic^3)\,\text{Subst}\left(\int \frac{1}{(c-x)^3\sqrt{c+x}}\,dx, x, -ic\tan(e+f\right.}{12a^3f}$$

$$= \frac{i\sqrt{c-ic\tan(e+fx)}}{6a^3f(1+i\tan(e+fx))^3} + \frac{5i\sqrt{c-ic\tan(e+fx)}}{48a^3f(1+i\tan(e+fx))^2} + \frac{(5ic^2)\,\text{Subst}\left(\int \frac{}{(c-x}\right.}{}$$

$$= \frac{i\sqrt{c-ic\tan(e+fx)}}{6a^3f(1+i\tan(e+fx))^3} + \frac{5i\sqrt{c-ic\tan(e+fx)}}{48a^3f(1+i\tan(e+fx))^2} + \frac{5i\sqrt{c-ic\tan(e+}}{64a^3f(1+i\tan(e+}$$

$$= \frac{i\sqrt{c-ic\tan(e+fx)}}{6a^3f(1+i\tan(e+fx))^3} + \frac{5i\sqrt{c-ic\tan(e+fx)}}{48a^3f(1+i\tan(e+fx))^2} + \frac{5i\sqrt{c-ic\tan(e+}}{64a^3f(1+i\tan(e+}$$

$$= \frac{5i\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{64\sqrt{2}\,a^3f} + \frac{i\sqrt{c-ic\tan(e+fx)}}{6a^3f(1+i\tan(e+fx))^3} + \frac{5i\sqrt{c}}{48a^3f(}$$

Mathematica [A]
time = 1.49, size = 149, normalized size = 0.82

$$\frac{(i\cos(3(e+fx))+\sin(3(e+fx)))\left(15\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)(\cos(3(e+fx))+i\sin(3(e+fx)))+(93\cos(e+fx)+41\cos(3(e+fx))+100i\cos^2(e+fx)\sin(e+fx))\sqrt{c-ic\tan(e+fx)}\right)}{384a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - I*c*Tan[e + f*x]]/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] ((I*Cos[3*(e + f*x)] + Sin[3*(e + f*x)])*(15*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c
 - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])]*(Cos[3*(e + f*x)] + I*Sin[3*(e + f*
x)]) + (93*Cos[e + f*x] + 41*Cos[3*(e + f*x)] + (100*I)*Cos[e + f*x]^2*Sin[
e + f*x])*Sqrt[c - I*c*Tan[e + f*x]]))/(384*a^3*f)
```

Maple [A]
time = 0.34, size = 156, normalized size = 0.86

method	result

derivativedivides	$\frac{2ic^4\left(\frac{\sqrt{c-ic\tan(fx+e)}}{12c(c+ic\tan(fx+e))^3}+\frac{\frac{5\sqrt{c-ic\tan(fx+e)}}{96c(c+ic\tan(fx+e))^2}+\frac{5\left(\frac{3\sqrt{c-ic\tan(fx+e)}}{32c(c+ic\tan(fx+e))}+\frac{3\sqrt{2}\,\mathrm{arc}}{}\right)}{1}}{c}\right)}{f\,a^3}$
default	$\frac{2ic^4\left(\frac{\sqrt{c-ic\tan(fx+e)}}{12c(c+ic\tan(fx+e))^3}+\frac{\frac{5\sqrt{c-ic\tan(fx+e)}}{96c(c+ic\tan(fx+e))^2}+\frac{5\left(\frac{3\sqrt{c-ic\tan(fx+e)}}{32c(c+ic\tan(fx+e))}+\frac{3\sqrt{2}\,\mathrm{arc}}{}\right)}{1}}{c}\right)}{f\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/f/a^3*c^4*(1/12*(c-I*c*tan(f*x+e))^(1/2)/c/(c+I*c*tan(f*x+e))^3+5/12/c*
(1/8*(c-I*c*tan(f*x+e))^(1/2)/c/(c+I*c*tan(f*x+e))^2+3/8/c*(1/4*(c-I*c*tan(
f*x+e))^(1/2)/c/(c+I*c*tan(f*x+e))+1/8/c^(3/2)*2^(1/2)*arctanh(1/2*(c-I*c*t
an(f*x+e))^(1/2)*2^(1/2)/c^(1/2)))))
```

Maxima [A]
time = 0.52, size = 200, normalized size = 1.10

$$-\frac{i\left(\frac{4\left(15\,(-ic\tan(fx+e)+c)^{\frac{5}{2}}c^2-80\,(-ic\tan(fx+e)+c)^{\frac{3}{2}}c^3+132\,\sqrt{-ic\tan(fx+e)+c}\,c^4\right)}{(-ic\tan(fx+e)+c)^3a^3-6\,(-ic\tan(fx+e)+c)^2a^3c+12\,(-ic\tan(fx+e)+c)a^3c^2-8\,a^3c^3}+\frac{15\,\sqrt{2}\,c^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{c}-\sqrt{-ic\tan(fx+e)+c}}{\sqrt{2}\,\sqrt{c}+\sqrt{-ic\tan(fx+e)+c}}\right)}{a^3}\right)}{768\,cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="maxim
a")
```

```
[Out] -1/768*I*(4*(15*(-I*c*tan(f*x + e) + c)^(5/2)*c^2 - 80*(-I*c*tan(f*x + e) +
 c)^(3/2)*c^3 + 132*sqrt(-I*c*tan(f*x + e) + c)*c^4)/((-I*c*tan(f*x + e) +
```

```
c)^3*a^3 - 6*(-I*c*tan(f*x + e) + c)^2*a^3*c + 12*(-I*c*tan(f*x + e) + c)*a
^3*c^2 - 8*a^3*c^3) + 15*sqrt(2)*c^(3/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*c*
tan(f*x + e) + c))/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/a^3)/(c
*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(143) = 286$.
time = 1.01, size = 299, normalized size = 1.65

$$\frac{\left(15\sqrt{\frac{1}{2}}\,a^3f\sqrt{-\frac{c}{a^6f^2}}\,e^{(6i\,fx+6i\,e)}\log\left(\frac{5\left(\sqrt{2}\sqrt{\frac{1}{2}}\,(a^3fe^{(2i\,fx+2i\,e)}+a^3f)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{-\frac{c}{a^6f^2}}+i\,c\right)e^{(-i\,fx-i\,e)}}{32\,a^3f}\right)-15\sqrt{\frac{1}{2}}\,a^3f\sqrt{-\frac{c}{a^6f^2}}\,e^{(6i\,fx+6i\,e)}\log\left(-\frac{5\left(\sqrt{2}\sqrt{\frac{1}{2}}\,(a^3fe^{(2i\,fx+2i\,e)}+a^3f)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{-\frac{c}{a^6f^2}}-i\,c\right)e^{(-i\,fx-i\,e)}}{32\,a^3f}\right)+\sqrt{2}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\left(33i\,e^{(6i\,fx+6i\,e)}+59i\,e^{(4i\,fx+4i\,e)}+34i\,e^{(2i\,fx+2i\,e)}+8i\right)\right)e^{(-6i\,fx-6i\,e)}}{384\,a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="frica
s")
```

```
[Out] 1/384*(15*sqrt(1/2)*a^3*f*sqrt(-c/(a^6*f^2))*e^(6*I*f*x + 6*I*e)*log(5/32*(
sqrt(2)*sqrt(1/2)*(a^3*f*e^(2*I*f*x + 2*I*e) + a^3*f)*sqrt(c/(e^(2*I*f*x +
2*I*e) + 1))*sqrt(-c/(a^6*f^2)) + I*c)*e^(-I*f*x - I*e)/(a^3*f)) - 15*sqrt(
1/2)*a^3*f*sqrt(-c/(a^6*f^2))*e^(6*I*f*x + 6*I*e)*log(-5/32*(sqrt(2)*sqrt(1
/2)*(a^3*f*e^(2*I*f*x + 2*I*e) + a^3*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*s
qrt(-c/(a^6*f^2)) - I*c)*e^(-I*f*x - I*e)/(a^3*f)) + sqrt(2)*sqrt(c/(e^(2*I
*f*x + 2*I*e) + 1))*(33*I*e^(6*I*f*x + 6*I*e) + 59*I*e^(4*I*f*x + 4*I*e) +
34*I*e^(2*I*f*x + 2*I*e) + 8*I))*e^(-6*I*f*x - 6*I*e)/(a^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int\frac{\sqrt{-ic\tan\left(e+fx\right)+c}}{\tan^3\left(e+fx\right)-3i\tan^2\left(e+fx\right)-3\tan\left(e+fx\right)+i}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] I*Integral(sqrt(-I*c*tan(e + f*x) + c)/(tan(e + f*x)**3 - 3*I*tan(e + f*x)*
*2 - 3*tan(e + f*x) + I), x)/a**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(-I*c*tan(f*x + e) + c)/(I*a*tan(f*x + e) + a)^3, x)
```

Mupad [B]
time = 4.98, size = 179, normalized size = 0.99

$$\frac{\frac{c^3\,\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}\,11\mathrm{i}}{16\,a^3\,f}-\frac{c^2\,{\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}^{3/2}\,5\mathrm{i}}{12\,a^3\,f}+\frac{c\,{\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}^{5/2}\,5\mathrm{i}}{64\,a^3\,f}}{6\,c\,{\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}^2-12\,c^2\,\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)-{\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}^3+8\,c^3}+\frac{\sqrt{2}\,\sqrt{-c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}}{2\,\sqrt{-c}}\right)\,5\mathrm{i}}{128\,a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^(1/2)/(a + a*tan(e + f*x)*1i)^3,x)
```

```
[Out] ((c^3*(c - c*tan(e + f*x)*1i)^(1/2)*11i)/(16*a^3*f) - (c^2*(c - c*tan(e + f
*x)*1i)^(3/2)*5i)/(12*a^3*f) + (c*(c - c*tan(e + f*x)*1i)^(5/2)*5i)/(64*a^3
*f))/(6*c*(c - c*tan(e + f*x)*1i)^2 - 12*c^2*(c - c*tan(e + f*x)*1i) - (c -
 c*tan(e + f*x)*1i)^3 + 8*c^3) + (2^(1/2)*(-c)^(1/2)*atan((2^(1/2)*(c - c*t
an(e + f*x)*1i)^(1/2))/(2*(-c)^(1/2)))*5i)/(128*a^3*f)
```

3.962 $\int (a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=94

$$\frac{8ia^3(c-ic\tan(e+fx))^{3/2}}{3f}-\frac{8ia^3(c-ic\tan(e+fx))^{5/2}}{5cf}+\frac{2ia^3(c-ic\tan(e+fx))^{7/2}}{7c^2f}$$

```
[Out] 8/3*I*a^3*(c-I*c*tan(f*x+e))^(3/2)/f-8/5*I*a^3*(c-I*c*tan(f*x+e))^(5/2)/c/f
+2/7*I*a^3*(c-I*c*tan(f*x+e))^(7/2)/c^2/f
```

Rubi [A]
time = 0.12, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3603, 3568, 45}

$$\frac{2ia^3(c-ic\tan(e+fx))^{7/2}}{7c^2f}-\frac{8ia^3(c-ic\tan(e+fx))^{5/2}}{5cf}+\frac{8ia^3(c-ic\tan(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])^(3/2),x]
```

```
[Out] (((8*I)/3)*a^3*(c - I*c*Tan[e + f*x])^(3/2))/f - (((8*I)/5)*a^3*(c - I*c*Ta
n[e + f*x])^(5/2))/(c*f) + (((2*I)/7)*a^3*(c - I*c*Tan[e + f*x])^(7/2))/(c^
2*f)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int (a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^{3/2}\,dx = (a^3c^3)\int \frac{\sec^6(e+fx)}{(c-ic\tan(e+fx))^{3/2}}\,dx$$
$$= \frac{(ia^3)\,\mathrm{Subst}\left(\int (c-x)^2\sqrt{c+x}\,dx, x, -ic\tan(e+fx)\right)}{c^2f}$$
$$= \frac{(ia^3)\,\mathrm{Subst}\left(\int \left(4c^2\sqrt{c+x}-4c(c+x)^{3/2}+(c+x)^{5/2}\right.\right.}{c^2f}$$
$$= \frac{8ia^3(c-ic\tan(e+fx))^{3/2}}{3f} - \frac{8ia^3(c-ic\tan(e+fx))}{5cf}$$

Mathematica [A]
time = 1.37, size = 94, normalized size = 1.00

$$\frac{2a^3c\sec^3(e+fx)(i\cos(e-2fx)+\sin(e-2fx))(28+43\cos(2(e+fx))+27i\sin(2(e+fx)))\sqrt{c-ic\tan(e+fx)}}{105f(\cos(fx)+i\sin(fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])^(3/2),x]
[Out] (2*a^3*c*Sec[e + f*x]^3*(I*Cos[e - 2*f*x] + Sin[e - 2*f*x])*(28 + 43*Cos[2*
(e + f*x)] + (27*I)*Sin[2*(e + f*x)])*Sqrt[c - I*c*Tan[e + f*x]])/(105*f*(C
os[f*x] + I*Sin[f*x])^3)
```

Maple [A]
time = 0.32, size = 66, normalized size = 0.70

method	result	size
derivativedivides	$\frac{2ia^3\left(\frac{(c-ic\tan(fx+e))^{\frac{7}{2}}}{7}-\frac{4c(c-ic\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{4c^2(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}\right)}{f\,c^2}$	66
default	$\frac{2ia^3\left(\frac{(c-ic\tan(fx+e))^{\frac{7}{2}}}{7}-\frac{4c(c-ic\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{4c^2(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}\right)}{f\,c^2}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3*(c-I*c*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2*I/f*a^3/c^2*(1/7*(c-I*c*tan(f*x+e))^(7/2)-4/5*c*(c-I*c*tan(f*x+e))^(5/2)+
4/3*c^2*(c-I*c*tan(f*x+e))^(3/2))
```

Maxima [A]
time = 0.30, size = 70, normalized size = 0.74

$$\frac{2i\left(15\,(-i\,c\tan(fx+e)+c)^{\frac{7}{2}}a^3-84\,(-i\,c\tan(fx+e)+c)^{\frac{5}{2}}a^3c+140\,(-i\,c\tan(fx+e)+c)^{\frac{3}{2}}a^3c^2\right)}{105\,c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c-I*c*tan(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] 2/105*I*(15*(-I*c*tan(f*x + e) + c)^(7/2)*a^3 - 84*(-I*c*tan(f*x + e) + c)^
(5/2)*a^3*c + 140*(-I*c*tan(f*x + e) + c)^(3/2)*a^3*c^2)/(c^2*f)
```

Fricas [A]
time = 1.15, size = 104, normalized size = 1.11

$$-\frac{16\sqrt{2}\left(-35i\,a^{3}ce^{(4i\,fx+4i\,e)}-28i\,a^{3}ce^{(2i\,fx+2i\,e)}-8i\,a^{3}c\right)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{105\left(fe^{(6i\,fx+6i\,e)}+3\,fe^{(4i\,fx+4i\,e)}+3\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c-I*c*tan(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] -16/105*sqrt(2)*(-35*I*a^3*c*e^(4*I*f*x + 4*I*e) - 28*I*a^3*c*e^(2*I*f*x +
2*I*e) - 8*I*a^3*c)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(f*e^(6*I*f*x + 6*I*e
) + 3*f*e^(4*I*f*x + 4*I*e) + 3*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia^{3}\left(\int ic\sqrt{-ic\tan{\left(e+fx\right)}+c}\,dx+\int\left(-2c\sqrt{-ic\tan{\left(e+fx\right)}+c}\tan{\left(e+fx\right)}\right)dx+\int\left(-2c\sqrt{-ic\tan{\left(e+fx\right)}+c}\tan^{3}{\left(e+fx\right)}\right)dx+\int\left(-ic\sqrt{-ic\tan{\left(e+fx\right)}+c}\tan^{4}{\left(e+fx\right)}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3*(c-I*c*tan(f*x+e))**(3/2),x)
```

```
[Out] -I*a**3*(Integral(I*c*sqrt(-I*c*tan(e + f*x) + c), x) + Integral(-2*c*sqrt(
-I*c*tan(e + f*x) + c)*tan(e + f*x), x) + Integral(-2*c*sqrt(-I*c*tan(e + f
*x) + c)*tan(e + f*x)**3, x) + Integral(-I*c*sqrt(-I*c*tan(e + f*x) + c)*ta
n(e + f*x)**4, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c-I*c*tan(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^3*(-I*c*tan(f*x + e) + c)^(3/2), x)
```

Mupad [B]
time = 8.62, size = 95, normalized size = 1.01

$$\frac{16\,a^3\,c\,\sqrt{c+\frac{c\,\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1}}\,\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}\,28{}\mathrm{i}+{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,35{}\mathrm{i}+8{}\mathrm{i}\right)}{105\,f\,{\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i)^(3/2),x)
```

```
[Out] (16*a^3*c*(c + (c*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1)
)^(1/2)*(exp(e*2i + f*x*2i)*28i + exp(e*4i + f*x*4i)*35i + 8i))/(105*f*(exp
(e*2i + f*x*2i) + 1)^3)
```

3.963 $\int (a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=62

$$\frac{4ia^2(c-ic\tan(e+fx))^{3/2}}{3f}-\frac{2ia^2(c-ic\tan(e+fx))^{5/2}}{5cf}$$

```
[Out] 4/3*I*a^2*(c-I*c*tan(f*x+e))^(3/2)/f-2/5*I*a^2*(c-I*c*tan(f*x+e))^(5/2)/c/f
```

Rubi [A]
time = 0.11, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3603, 3568, 45}

$$\frac{4ia^2(c-ic\tan(e+fx))^{3/2}}{3f}-\frac{2ia^2(c-ic\tan(e+fx))^{5/2}}{5cf}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^(3/2),x]
```

```
[Out] (((4*I)/3)*a^2*(c - I*c*Tan[e + f*x])^(3/2))/f - (((2*I)/5)*a^2*(c - I*c*Ta
n[e + f*x])^(5/2))/(c*f)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$
\begin{aligned}
\int (a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{3/2}\,dx &= (a^2c^2)\int \frac{\sec^4(e+fx)}{\sqrt{c-ic\tan(e+fx)}}\,dx \\
&= \frac{(ia^2)\,\text{Subst}\left(\int (c-x)\sqrt{c+x}\,dx, x, -ic\tan(e+fx)\right)}{cf} \\
&= \frac{(ia^2)\,\text{Subst}\left(\int \left(2c\sqrt{c+x}-(c+x)^{3/2}\right)dx, x, -ic\tan(\right.}{cf} \\
&= \frac{4ia^2(c-ic\tan(e+fx))^{3/2}}{3f} - \frac{2ia^2(c-ic\tan(e+fx))}{5cf}
\end{aligned}
$$

Mathematica [A]
time = 1.03, size = 80, normalized size = 1.29

$$
-\frac{2a^2c\sec(e+fx)(\cos(e-fx)-i\sin(e-fx))(-7i+3\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{15f(\cos(fx)+i\sin(fx))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^(3/2),x]
[Out] (-2*a^2*c*Sec[e + f*x]*(Cos[e - f*x] - I*Sin[e - f*x])*(-7*I + 3*Tan[e + f*
x])*Sqrt[c - I*c*Tan[e + f*x]])/(15*f*(Cos[f*x] + I*Sin[f*x])^2)
```

Maple [A]
time = 0.30, size = 47, normalized size = 0.76

method	result	size
derivativedivides	$-\frac{2ia^2\left(\frac{(c-ic\tan(fx+e))^{\frac{5}{2}}}{5}-\frac{2c(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}\right)}{fc}$	47
default	$-\frac{2ia^2\left(\frac{(c-ic\tan(fx+e))^{\frac{5}{2}}}{5}-\frac{2c(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}\right)}{fc}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2*(c-I*c*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2*I/f*a^2/c*(1/5*(c-I*c*tan(f*x+e))^(5/2)-2/3*c*(c-I*c*tan(f*x+e))^(3/2))
```

Maxima [A]
time = 0.29, size = 48, normalized size = 0.77

$$
-\frac{2i\left(3\left(-i\,c\tan\left(fx+e\right)+c\right)^{\frac{5}{2}}a^2-10\left(-i\,c\tan\left(fx+e\right)+c\right)^{\frac{3}{2}}a^2c\right)}{15\,cf}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c-I*c*tan(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -2/15*I*(3*(-I*c*tan(f*x + e) + c)^(5/2)*a^2 - 10*(-I*c*tan(f*x + e) + c)^(
3/2)*a^2*c)/(c*f)
```

Fricas [A]
time = 1.54, size = 75, normalized size = 1.21

$$-\frac{8\sqrt{2}\left(-5i\,a^2ce^{(2i\,fx+2i\,e)}-2i\,a^2c\right)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{15\left(fe^{(4i\,fx+4i\,e)}+2\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c-I*c*tan(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] -8/15*sqrt(2)*(-5*I*a^2*c*e^(2*I*f*x + 2*I*e) - 2*I*a^2*c)*sqrt(c/(e^(2*I*f
*x + 2*I*e) + 1))/(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int\left(-c\sqrt{-ic\tan(e+fx)+c}\right)dx+\int\left(-c\sqrt{-ic\tan(e+fx)+c}\tan^2(e+fx)\right)dx+\int\left(-ic\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)\right)dx+\int\left(-ic\sqrt{-ic\tan(e+fx)+c}\tan^3(e+fx)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2*(c-I*c*tan(f*x+e))**(3/2),x)
```

```
[Out] -a**2*(Integral(-c*sqrt(-I*c*tan(e + f*x) + c), x) + Integral(-c*sqrt(-I*c*
tan(e + f*x) + c)*tan(e + f*x)**2, x) + Integral(-I*c*sqrt(-I*c*tan(e + f*x
) + c)*tan(e + f*x), x) + Integral(-I*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e +
 f*x)**3, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c-I*c*tan(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^2*(-I*c*tan(f*x + e) + c)^(3/2), x)
```

Mupad [B]
time = 6.79, size = 168, normalized size = 2.71

$$\frac{8\,a^2\,c\,\sqrt{\frac{2\,c}{{\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1}}\,\left({\mathrm{e}}^{-e\,2{}\mathrm{i}-f\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}+\frac{{\mathrm{e}}^{-e\,4{}\mathrm{i}-f\,x\,4{}\mathrm{i}}\,1{}\mathrm{i}}{2}+\frac{1}{2}{}\mathrm{i}\right)\,\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}\,7{}\mathrm{i}+{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,5{}\mathrm{i}+2{}\mathrm{i}\right)}{15\,f\,\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}+1{}\mathrm{i}\right)\,\left(2\,{\mathrm{e}}^{-e\,2{}\mathrm{i}-f\,x\,2{}\mathrm{i}}+2\,{\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+\frac{{\mathrm{e}}^{-e\,4{}\mathrm{i}-f\,x\,4{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}}{2}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i)^(3/2),x)
```

```
[Out] (8*a^2*c*((2*c)/(exp(e*2i + f*x*2i) + 1))^(1/2)*(exp(- e*2i - f*x*2i)*1i +
(exp(- e*4i - f*x*4i)*1i)/2 + 1i/2)*(exp(e*2i + f*x*2i)*7i + exp(e*4i + f*x
*4i)*5i + 2i))/(15*f*(exp(e*2i + f*x*2i)*1i + 1i)*(2*exp(- e*2i - f*x*2i) +
 2*exp(e*2i + f*x*2i) + exp(- e*4i - f*x*4i)/2 + exp(e*4i + f*x*4i)/2 + 3))
```

3.964 $\int (a+ia\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=27

$$\frac{2ia(c-ic\tan(e+fx))^{3/2}}{3f}$$

[Out] 2/3*I*a*(c-I*c*tan(f*x+e))^(3/2)/f

Rubi [A]
time = 0.07, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 32}

$$\frac{2ia(c-ic\tan(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

[In] Int[(a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(3/2),x]

[Out] (((2*I)/3)*a*(c - I*c*Tan[e + f*x])^(3/2))/f

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int (a+ia\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = (ac)\int \sec^2(e+fx)\sqrt{c-ic\tan(e+fx)}\,dx$$
$$= \frac{(ia)\mathrm{Subst}\left(\int \sqrt{c+x}\,dx, x, -ic\tan(e+fx)\right)}{f}$$
$$= \frac{2ia(c-ic\tan(e+fx))^{3/2}}{3f}$$

Mathematica [A]
time = 0.55, size = 54, normalized size = 2.00

$$\frac{2ac\sec(e+fx)(i\cos(e)+\sin(e))(\cos(fx)-i\sin(fx))\sqrt{c-ic\tan(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*a*c*Sec[e + f*x]*(I*Cos[e] + Sin[e])*(Cos[f*x] - I*Sin[f*x])*Sqrt[c - I*
c*Tan[e + f*x]])/(3*f)
```

Maple [A]
time = 0.16, size = 22, normalized size = 0.81

method	result	size
derivativedivides	$\frac{2ia(c-ic\tan(fx+e))^{\frac{3}{2}}}{3f}$	22
default	$\frac{2ia(c-ic\tan(fx+e))^{\frac{3}{2}}}{3f}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*a*(c-I*c*tan(f*x+e))^(3/2)/f
```

Maxima [A]
time = 0.28, size = 20, normalized size = 0.74

$$\frac{2i\,(-i\,c\tan(fx+e)+c)^{\frac{3}{2}}a}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] 2/3*I*(-I*c*tan(f*x + e) + c)^(3/2)*a/f
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(20) = 40.
time = 0.88, size = 41, normalized size = 1.52

$$\frac{4i\sqrt{2}\,ac\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{3\,(fe^{(2i\,fx+2i\,e)}+f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 4/3*I*sqrt(2)*a*c*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(f*e^(2*I*f*x + 2*I*e)
+ f)
```

Sympy [A]
time = 1.75, size = 44, normalized size = 1.63

$$\begin{cases} \frac{2ia(-ic\tan{(e+fx)}+c)^{\frac{3}{2}}}{3f} & \text{for}\ f\neq 0 \\ x(ia\tan{(e)}+a)\,(-ic\tan{(e)}+c)^{\frac{3}{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e))**(3/2),x)
```

```
[Out] Piecewise((2*I*a*(-I*c*tan(e + f*x) + c)**(3/2)/(3*f), Ne(f, 0)), (x*(I*a*t
an(e) + a)*(-I*c*tan(e) + c)**(3/2), True))
```

Giac [A]
time = 0.46, size = 20, normalized size = 0.74

$$\frac{2i\,(-i\,c\tan{(fx+e)}+c)^{\frac{3}{2}}a}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*I*(-I*c*tan(f*x + e) + c)^(3/2)*a/f
```

Mupad [B]
time = 0.20, size = 47, normalized size = 1.74

$$\frac{\sqrt{2}\,a\,c\,\sqrt{\frac{c}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,4\mathrm{i}}{3\,\left(f+f\,\mathrm{e}^{e\,2\mathrm{i}}\,\mathrm{e}^{f\,x\,2\mathrm{i}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i)^(3/2),x)
```

```
[Out] (2^(1/2)*a*c*(c/(exp(e*2i + f*x*2i) + 1))^(1/2)*4i)/(3*(f + f*exp(e*2i)*exp
(f*x*2i)))
```

3.965 $\int \frac{(c-ic\tan(e+fx))^{3/2}}{a+ia\tan(e+fx)}\,dx$

Optimal. Leaf size=95

$$-\frac{ic^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}\,af}+\frac{ic^2\sqrt{c-ic\tan(e+fx)}}{af(c+ic\tan(e+fx))}$$

```
[Out] -1/2*I*c^(3/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))/a/f*2^
(1/2)+I*c^2*(c-I*c*tan(f*x+e))^(1/2)/a/f/(c+I*c*tan(f*x+e))
```

Rubi [A]
time = 0.12, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3603, 3568, 43, 65, 212}

$$\frac{ic^2\sqrt{c-ic\tan(e+fx)}}{af(c+ic\tan(e+fx))}-\frac{ic^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}\,af}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])^(3/2)/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] ((-I)*c^(3/2)*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])])/(Sqrt[
2]*a*f) + (I*c^2*Sqrt[c - I*c*Tan[e + f*x]])/(a*f*(c + I*c*Tan[e + f*x]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{(c-ic\tan(e+fx))^{3/2}}{a+ia\tan(e+fx)}\,dx = \frac{\int \cos^2(e+fx)(c-ic\tan(e+fx))^{5/2}\,dx}{ac}$$
$$= \frac{(ic^2)\,\text{Subst}\left(\int \frac{\sqrt{c+x}}{(c-x)^2}\,dx, x, -ic\tan(e+fx)\right)}{af}$$
$$= \frac{ic^2\sqrt{c-ic\tan(e+fx)}}{af(c+ic\tan(e+fx))} - \frac{(ic^2)\,\text{Subst}\left(\int \frac{1}{(c-x)\sqrt{c+x}}\,dx, x, -ic\tan(e+f$$
$$= \frac{ic^2\sqrt{c-ic\tan(e+fx)}}{af(c+ic\tan(e+fx))} - \frac{(ic^2)\,\text{Subst}\left(\int \frac{1}{2c-x^2}\,dx, x, \sqrt{c-ic\tan(e+fx)}$$
$$= -\frac{ic^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}\,af} + \frac{ic^2\sqrt{c-ic\tan(e+fx)}}{af(c+ic\tan(e+fx))}$$

Mathematica [A]
time = 0.91, size = 114, normalized size = 1.20

$$\frac{(-ic\cos(e+fx)-c\sin(e+fx))\left(\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)(\cos(e+fx)+i\sin(e+fx))-2\cos(e+fx)\sqrt{c-ic\tan(e+fx)}\right)}{2af}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - I*c*Tan[e + f*x])^(3/2)/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (((-I)*c*Cos[e + f*x] - c*Sin[e + f*x])*(Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - I
*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])]*(Cos[e + f*x] + I*Sin[e + f*x]) - 2*Cos
[e + f*x]*Sqrt[c - I*c*Tan[e + f*x]]))/(2*a*f)
```

Maple [A]
time = 0.32, size = 77, normalized size = 0.81

method	result	size
derivativedivides	$\frac{2ic^2\left(\frac{\sqrt{c-ic\tan(fx+e)}}{2c+2ic\tan(fx+e)}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)}{4\sqrt{c}}\right)}{fa}$	77
default	$\frac{2ic^2\left(\frac{\sqrt{c-ic\tan(fx+e)}}{2c+2ic\tan(fx+e)}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)}{4\sqrt{c}}\right)}{fa}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/f/a*c^2*(1/4*(c-I*c*tan(f*x+e))^(1/2)/(1/2*c+1/2*I*c*tan(f*x+e))-1/4*2^
(1/2)/c^(1/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [A]
time = 0.51, size = 110, normalized size = 1.16

$$\frac{i\left(\frac{\sqrt{2}\,c^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{c}-\sqrt{-i\,c\tan(fx+e)+c}}{\sqrt{2}\,\sqrt{c}+\sqrt{-i\,c\tan(fx+e)+c}}\right)}{a}-\frac{4\,\sqrt{-i\,c\tan(fx+e)+c}\,c^3}{(-i\,c\tan(fx+e)+c)a-2\,ac}\right)}{4\,cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e)),x, algorithm="maxima"
)
```

```
[Out] 1/4*I*(sqrt(2)*c^(5/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*c*tan(f*x + e) + c))
/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/a - 4*sqrt(-I*c*tan(f*x +
 e) + c)*c^3/((-I*c*tan(f*x + e) + c)*a - 2*a*c))/(c*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(77) = 154$.

time = 0.71, size = 267, normalized size = 2.81

$$-\frac{\left(\sqrt{2}\,af\sqrt{-\frac{c^3}{a^2f^2}}\,e^{(2i\,fx+2i\,e)}\log\left(-\frac{2\left(i\,c^2+\left(afe^{(2i\,fx+2i\,e)}+af\right)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\,\sqrt{-\frac{c^3}{a^2f^2}}\right)e^{(-i\,fx-i\,e)}}{af}\right)-\sqrt{2}\,af\sqrt{-\frac{c^3}{a^2f^2}}\,e^{(2i\,fx+2i\,e)}\log\left(-\frac{2\left(i\,c^2-\left(afe^{(2i\,fx+2i\,e)}+af\right)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\,\sqrt{-\frac{c^3}{a^2f^2}}\right)e^{(-i\,fx-i\,e)}}{af}\right)+2\sqrt{2}\left(-i\,ce^{(2i\,fx+2i\,e)}-i\,c\right)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-2i\,fx-2i\,e)}}{4\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e)),x, algorithm="fricas"
)
```

```
[Out] -1/4*(sqrt(2)*a*f*sqrt(-c^3/(a^2*f^2))*e^(2*I*f*x + 2*I*e)*log(-2*(I*c^2 +
(a*f*e^(2*I*f*x + 2*I*e) + a*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-c^3
/(a^2*f^2)))*e^(-I*f*x - I*e)/(a*f)) - sqrt(2)*a*f*sqrt(-c^3/(a^2*f^2))*e^(
2*I*f*x + 2*I*e)*log(-2*(I*c^2 - (a*f*e^(2*I*f*x + 2*I*e) + a*f)*sqrt(c/(e^
(2*I*f*x + 2*I*e) + 1))*sqrt(-c^3/(a^2*f^2)))*e^(-I*f*x - I*e)/(a*f)) + 2*s
qrt(2)*(-I*c*e^(2*I*f*x + 2*I*e) - I*c)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))*
e^(-2*I*f*x - 2*I*e)/(a*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\left(\int\frac{c\sqrt{-ic\tan\left(e+fx\right)+c}}{\tan\left(e+fx\right)-i}\,dx+\int\left(-\frac{ic\sqrt{-ic\tan\left(e+fx\right)+c}\,\tan\left(e+fx\right)}{\tan\left(e+fx\right)-i}\right)dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**(3/2)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] -I*(Integral(c*sqrt(-I*c*tan(e + f*x) + c)/(tan(e + f*x) - I), x) + Integra
l(-I*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)/(tan(e + f*x) - I), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((-I*c*tan(f*x + e) + c)^(3/2)/(I*a*tan(f*x + e) + a), x)
```

Mupad [B]
time = 0.29, size = 83, normalized size = 0.87

$$\frac{\sqrt{2}\,{\left(-c\right)}^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}}}{2\,\sqrt{-c}}\right)\,1{}\mathrm{i}}{2\,a\,f}+\frac{c^2\,\sqrt{c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}}\,1{}\mathrm{i}}{a\,f\,\left(c+c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^(3/2)/(a + a*tan(e + f*x)*1i),x)
```

```
[Out] (2^(1/2)*(-c)^(3/2)*atan((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*(-c)^(1
/2)))*1i)/(2*a*f) + (c^2*(c - c*tan(e + f*x)*1i)^(1/2)*1i)/(a*f*(c + c*tan(
e + f*x)*1i))
```

3.966 $\int \frac{(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=146

$$-\frac{ic^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{8\sqrt{2}\,a^2f}+\frac{ic^3\sqrt{c-ic\tan(e+fx)}}{2a^2f(c+ic\tan(e+fx))^2}-\frac{ic^2\sqrt{c-ic\tan(e+fx)}}{8a^2f(c+ic\tan(e+fx))}$$

```
[Out] -1/16*I*c^(3/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))/a^2/f
*2^(1/2)+1/2*I*c^3*(c-I*c*tan(f*x+e))^(1/2)/a^2/f/(c+I*c*tan(f*x+e))^2-1/8*
I*c^2*(c-I*c*tan(f*x+e))^(1/2)/a^2/f/(c+I*c*tan(f*x+e))
```

Rubi [A]
time = 0.13, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3603, 3568, 43, 44, 65, 212}

$$-\frac{ic^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{8\sqrt{2}\,a^2f}+\frac{ic^3\sqrt{c-ic\tan(e+fx)}}{2a^2f(c+ic\tan(e+fx))^2}-\frac{ic^2\sqrt{c-ic\tan(e+fx)}}{8a^2f(c+ic\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])^(3/2)/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] ((-1/8*I)*c^(3/2)*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])])/(S
qrt[2]*a^2*f) + ((I/2)*c^3*Sqrt[c - I*c*Tan[e + f*x]])/(a^2*f*(c + I*c*Tan[
e + f*x])^2) - ((I/8)*c^2*Sqrt[c - I*c*Tan[e + f*x]])/(a^2*f*(c + I*c*Tan[e
 + f*x]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^2}\,dx = \frac{\int \cos^4(e+fx)(c-ic\tan(e+fx))^{7/2}\,dx}{a^2c^2}$$

$$= \frac{(ic^3)\,\text{Subst}\left(\int \frac{\sqrt{c+x}}{(c-x)^3}\,dx, x, -ic\tan(e+fx)\right)}{a^2f}$$

$$= \frac{ic^3\sqrt{c-ic\tan(e+fx)}}{2a^2f(c+ic\tan(e+fx))^2} - \frac{(ic^3)\,\text{Subst}\left(\int \frac{1}{(c-x)^2\sqrt{c+x}}\,dx, x, -ic\tan(e+\right.}{4a^2f}$$

$$= \frac{ic^3\sqrt{c-ic\tan(e+fx)}}{2a^2f(c+ic\tan(e+fx))^2} - \frac{ic^2\sqrt{c-ic\tan(e+fx)}}{8a^2f(c+ic\tan(e+fx))} - \frac{(ic^2)\,\text{Subst}\left(\int \frac{}{(c-}\right.}{}$$

$$= \frac{ic^3\sqrt{c-ic\tan(e+fx)}}{2a^2f(c+ic\tan(e+fx))^2} - \frac{ic^2\sqrt{c-ic\tan(e+fx)}}{8a^2f(c+ic\tan(e+fx))} - \frac{(ic^2)\,\text{Subst}\left(\int \frac{1}{2c-}\right.}{}$$

$$= -\frac{ic^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{8\sqrt{2}\,a^2f} + \frac{ic^3\sqrt{c-ic\tan(e+fx)}}{2a^2f(c+ic\tan(e+fx))^2} - \frac{ic^2}{8a^2}$$

Mathematica [A]

time = 1.41, size = 136, normalized size = 0.93

$$\frac{c(\cos(2(e+fx))-i\sin(2(e+fx)))\left(\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)(-i\cos(2(e+fx))+\sin(2(e+fx)))+(3i+3i\cos(2(e+fx))+\sin(2(e+fx)))\sqrt{c-ic\tan(e+fx)}\right)}{16a^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - I*c*Tan[e + f*x])^(3/2)/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (c*(Cos[2*(e + f*x)] - I*Sin[2*(e + f*x)])*(Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c
- I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])]*((-I)*Cos[2*(e + f*x)] + Sin[2*(e +
f*x)]) + (3*I + (3*I)*Cos[2*(e + f*x)] + Sin[2*(e + f*x)])*Sqrt[c - I*c*Tan
[e + f*x]]))/(16*a^2*f)
```

Maple [A]

time = 0.32, size = 97, normalized size = 0.66

method	result
derivativedivides	$-\frac{2ic^3\left(-\frac{4\left(\frac{(c-ic\tan(fx+e))^{\frac{3}{2}}}{64c}+\frac{\sqrt{c-ic\tan(fx+e)}}{32}\right)}{(c+ic\tan(fx+e))^2}+\frac{\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)}{32c^{\frac{3}{2}}}\right.}{fa^2}$

default	$-\frac{2ic^{3}\left(-\frac{4\left(\frac{(c-ic\tan(fx+e))^{\frac{3}{2}}}{64c}+\frac{\sqrt{c-ic\tan(fx+e)}}{32}\right)}{(c+ic\tan(fx+e))^{2}}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)}{32c^{\frac{3}{2}}}\right.}{f\,a^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/f/a^2*c^3*(-4*(1/64/c*(c-I*c*tan(f*x+e))^(3/2)+1/32*(c-I*c*tan(f*x+e))
^(1/2))/(c+I*c*tan(f*x+e))^2+1/32/c^(3/2)*2^(1/2)*arctanh(1/2*(c-I*c*tan(f*
x+e))^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [A]
time = 0.54, size = 157, normalized size = 1.08

$$\frac{i\left(\frac{\sqrt{2}\,c^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{c}-\sqrt{-i\,c\tan(fx+e)+c}}{\sqrt{2}\,\sqrt{c}+\sqrt{-i\,c\tan(fx+e)+c}}\right)}{a^{2}}+\frac{4\left((-i\,c\tan(fx+e)+c)^{\frac{3}{2}}c^{3}+2\,\sqrt{-i\,c\tan(fx+e)+c}\,c^{4}\right)}{(-i\,c\tan(fx+e)+c)^{2}a^{2}-4\,(-i\,c\tan(fx+e)+c)a^{2}c+4\,a^{2}c^{2}}\right)}{32\,cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxim
a")
```

```
[Out] 1/32*I*(sqrt(2)*c^(5/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*c*tan(f*x + e) + c)
)/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/a^2 + 4*((-I*c*tan(f*x +
 e) + c)^(3/2)*c^3 + 2*sqrt(-I*c*tan(f*x + e) + c)*c^4)/((-I*c*tan(f*x + e)
 + c)^2*a^2 - 4*(-I*c*tan(f*x + e) + c)*a^2*c + 4*a^2*c^2))/(c*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(116) = 232.
time = 0.76, size = 303, normalized size = 2.08

$$-\frac{\left(\sqrt{\frac{1}{2}}\,a^{2}f\sqrt{-\frac{c^{3}}{a^{4}f^{2}}}\,e^{(4i\,fx+4i\,e)}\log\left(-\frac{\left(\sqrt{2}\,\sqrt{\frac{1}{2}}\,(a^{2}fe^{(2i\,fx+2i\,e)}+a^{2}f)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\,\sqrt{-\frac{c^{3}}{a^{4}f^{2}}}+i\,c^{2}\right)e^{(-i\,fx-i\,e)}}{4\,a^{2}f}\right)-\sqrt{\frac{1}{2}}\,a^{2}f\sqrt{-\frac{c^{3}}{a^{4}f^{2}}}\,e^{(4i\,fx+4i\,e)}\log\left(\frac{\left(\sqrt{2}\,\sqrt{\frac{1}{2}}\,(a^{2}fe^{(2i\,fx+2i\,e)}+a^{2}f)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\,\sqrt{-\frac{c^{3}}{a^{4}f^{2}}}-i\,c^{2}\right)e^{(-i\,fx-i\,e)}}{4\,a^{2}f}\right)-\sqrt{2}\,(i\,ce^{(4i\,fx+4i\,e)}+3i\,ce^{(2i\,fx+2i\,e)}+2i\,c)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-4i\,fx-4i\,e)}}{16\,a^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="frica
s")
```

```
[Out] -1/16*(sqrt(1/2)*a^2*f*sqrt(-c^3/(a^4*f^2))*e^(4*I*f*x + 4*I*e)*log(-1/4*(s
qrt(2)*sqrt(1/2)*(a^2*f*e^(2*I*f*x + 2*I*e) + a^2*f)*sqrt(c/(e^(2*I*f*x + 2
*I*e) + 1))*sqrt(-c^3/(a^4*f^2)) + I*c^2)*e^(-I*f*x - I*e)/(a^2*f)) - sqrt(
```

```
1/2)*a^2*f*sqrt(-c^3/(a^4*f^2))*e^(4*I*f*x + 4*I*e)*log(1/4*(sqrt(2)*sqrt(1
/2)*(a^2*f*e^(2*I*f*x + 2*I*e) + a^2*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*s
qrt(-c^3/(a^4*f^2)) - I*c^2)*e^(-I*f*x - I*e)/(a^2*f)) - sqrt(2)*(I*c*e^(4*
I*f*x + 4*I*e) + 3*I*c*e^(2*I*f*x + 2*I*e) + 2*I*c)*sqrt(c/(e^(2*I*f*x + 2*
I*e) + 1)))*e^(-4*I*f*x - 4*I*e)/(a^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{c\sqrt{-ic\tan(e+fx)+c}}{\tan^2(e+fx)-2i\tan(e+fx)-1}\,dx+\int\left(-\frac{ic\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)}{\tan^2(e+fx)-2i\tan(e+fx)-1}\right)dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**(3/2)/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -(Integral(c*sqrt(-I*c*tan(e + f*x) + c)/(tan(e + f*x)**2 - 2*I*tan(e + f*x
) - 1), x) + Integral(-I*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)/(tan(e
+ f*x)**2 - 2*I*tan(e + f*x) - 1), x))/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac"
)
```

```
[Out] integrate((-I*c*tan(f*x + e) + c)^(3/2)/(I*a*tan(f*x + e) + a)^2, x)
```

Mupad [B]
time = 4.89, size = 134, normalized size = 0.92

$$\frac{\frac{c^3\sqrt{c-c\tan(e+fx)\,1\mathrm{i}}\,1\mathrm{i}}{4a^2f}+\frac{c^2(c-c\tan(e+fx)\,1\mathrm{i})^{3/2}\,1\mathrm{i}}{8a^2f}}{(c-c\tan(e+fx)\,1\mathrm{i})^2-4c\,(c-c\tan(e+fx)\,1\mathrm{i})+4c^2}+\frac{\sqrt{2}\,(-c)^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+fx)\,1\mathrm{i}}}{2\sqrt{-c}}\right)\,1\mathrm{i}}{16a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^(3/2)/(a + a*tan(e + f*x)*1i)^2,x)
```

```
[Out] ((c^3*(c - c*tan(e + f*x)*1i)^(1/2)*1i)/(4*a^2*f) + (c^2*(c - c*tan(e + f*x
)*1i)^(3/2)*1i)/(8*a^2*f))/((c - c*tan(e + f*x)*1i)^2 - 4*c*(c - c*tan(e +
f*x)*1i) + 4*c^2) + (2^(1/2)*(-c)^(3/2)*atan((2^(1/2)*(c - c*tan(e + f*x)*1
i)^(1/2))/(2*(-c)^(1/2)))*1i)/(16*a^2*f)
```

3.967 $\int \frac{(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^3}\,dx$

Optimal. Leaf size=193

$$-\frac{ic^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{32\sqrt{2}\,a^3f}+\frac{ic^4\sqrt{c-ic\tan(e+fx)}}{3a^3f(c+ic\tan(e+fx))^3}-\frac{ic^3\sqrt{c-ic\tan(e+fx)}}{24a^3f(c+ic\tan(e+fx))^2}-\frac{ic^2\sqrt{c-}}{32a^3f(c-}$$

```
[Out] -1/64*I*c^(3/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))/a^3/f
*2^(1/2)+1/3*I*c^4*(c-I*c*tan(f*x+e))^(1/2)/a^3/f/(c+I*c*tan(f*x+e))^3-1/24
*I*c^3*(c-I*c*tan(f*x+e))^(1/2)/a^3/f/(c+I*c*tan(f*x+e))^2-1/32*I*c^2*(c-I*
c*tan(f*x+e))^(1/2)/a^3/f/(c+I*c*tan(f*x+e))
```

Rubi [A]
time = 0.15, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3603, 3568, 43, 44, 65, 212}

$$-\frac{ic^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{32\sqrt{2}\,a^3f}+\frac{ic^4\sqrt{c-ic\tan(e+fx)}}{3a^3f(c+ic\tan(e+fx))^3}-\frac{ic^3\sqrt{c-ic\tan(e+fx)}}{24a^3f(c+ic\tan(e+fx))^2}-\frac{ic^2\sqrt{c-ic\tan(e+fx)}}{32a^3f(c+ic\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])^(3/2)/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] ((-1/32*I)*c^(3/2)*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])])/(
Sqrt[2]*a^3*f) + ((I/3)*c^4*Sqrt[c - I*c*Tan[e + f*x]])/(a^3*f*(c + I*c*Tan
[e + f*x])^3) - ((I/24)*c^3*Sqrt[c - I*c*Tan[e + f*x]])/(a^3*f*(c + I*c*Tan
[e + f*x])^2) - ((I/32)*c^2*Sqrt[c - I*c*Tan[e + f*x]])/(a^3*f*(c + I*c*Tan
[e + f*x]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^3}\,dx = \frac{\int \cos^6(e+fx)(c-ic\tan(e+fx))^{9/2}\,dx}{a^3c^3}$$

$$= \frac{(ic^4)\,\text{Subst}\left(\int \frac{\sqrt{c+x}}{(c-x)^4}\,dx, x, -ic\tan(e+fx)\right)}{a^3 f}$$

$$= \frac{ic^4\sqrt{c-ic\tan(e+fx)}}{3a^3f(c+ic\tan(e+fx))^3} - \frac{(ic^4)\,\text{Subst}\left(\int \frac{1}{(c-x)^3\sqrt{c+x}}\,dx, x, -ic\tan(e+\right.}{6a^3f}$$

$$= \frac{ic^4\sqrt{c-ic\tan(e+fx)}}{3a^3f(c+ic\tan(e+fx))^3} - \frac{ic^3\sqrt{c-ic\tan(e+fx)}}{24a^3f(c+ic\tan(e+fx))^2} - \frac{(ic^3)\,\text{Subst}\left(\int \frac{}{(c-}\right.}{}$$

$$= \frac{ic^4\sqrt{c-ic\tan(e+fx)}}{3a^3f(c+ic\tan(e+fx))^3} - \frac{ic^3\sqrt{c-ic\tan(e+fx)}}{24a^3f(c+ic\tan(e+fx))^2} - \frac{ic^2\sqrt{c-ic\tan(}}{32a^3f(c+ic\tan(}$$

$$= \frac{ic^4\sqrt{c-ic\tan(e+fx)}}{3a^3f(c+ic\tan(e+fx))^3} - \frac{ic^3\sqrt{c-ic\tan(e+fx)}}{24a^3f(c+ic\tan(e+fx))^2} - \frac{ic^2\sqrt{c-ic\tan(}}{32a^3f(c+ic\tan(}$$

$$= -\frac{ic^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{32\sqrt{2}\,a^3f} + \frac{ic^4\sqrt{c-ic\tan(e+fx)}}{3a^3f(c+ic\tan(e+fx))^3} - \frac{ic^3}{24a^3}$$

Mathematica [A]
time = 1.88, size = 152, normalized size = 0.79

$$\frac{c(\cos(3(e+fx))-i\sin(3(e+fx)))\left(3\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)(-i\cos(3(e+fx))+\sin(3(e+fx)))+(39i\cos(e+fx)+11i\cos(3(e+fx))+20\cos^2(e+fx)\sin(e+fx))\sqrt{c-ic\tan(e+fx)}\right)}{192a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - I*c*Tan[e + f*x])^(3/2)/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] (c*(Cos[3*(e + f*x)] - I*Sin[3*(e + f*x)])*(3*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[
c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])]*((-I)*Cos[3*(e + f*x)] + Sin[3*(e
+ f*x)]) + ((39*I)*Cos[e + f*x] + (11*I)*Cos[3*(e + f*x)] + 20*Cos[e + f*x]
^2*Sin[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]]))/(192*a^3*f)
```

Maple [A]
time = 0.33, size = 116, normalized size = 0.60

method	result

derivativedivides	$\frac{2ic^4\left(\frac{-\frac{(c-ic\tan(fx+e))^{\frac{5}{2}}}{64c^2}+\frac{(c-ic\tan(fx+e))^{\frac{3}{2}}}{12c}+\frac{\sqrt{c-ic\tan(fx+e)}}{16}}{(c+ic\tan(fx+e))^3}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx}{2\sqrt{c}}\right.}{128c^{\frac{5}{2}}}\right.}{f\,a^3}$
default	$\frac{2ic^4\left(\frac{-\frac{(c-ic\tan(fx+e))^{\frac{5}{2}}}{64c^2}+\frac{(c-ic\tan(fx+e))^{\frac{3}{2}}}{12c}+\frac{\sqrt{c-ic\tan(fx+e)}}{16}}{(c+ic\tan(fx+e))^3}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx}{2\sqrt{c}}\right.}{128c^{\frac{5}{2}}}\right.}{f\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/f/a^3*c^4*(8*(-1/512/c^2*(c-I*c*tan(f*x+e))^(5/2)+1/96/c*(c-I*c*tan(f*x
+e))^(3/2)+1/128*(c-I*c*tan(f*x+e))^(1/2))/(c+I*c*tan(f*x+e))^3-1/128/c^(5/
2)*2^(1/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [A]
time = 0.50, size = 200, normalized size = 1.04

$$\frac{i\left(\frac{3\sqrt{2}\,c^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{c}-\sqrt{-i\,c\tan\left(fx+e\right)+c}}{\sqrt{2}\,\sqrt{c}+\sqrt{-i\,c\tan\left(fx+e\right)+c}}\right)}{a^3}+\frac{4\left(3\left(-i\,c\tan(fx+e)+c\right)^{\frac{5}{2}}c^3-16\left(-i\,c\tan(fx+e)+c\right)^{\frac{3}{2}}c^4-12\sqrt{-i\,c\tan\left(fx+e\right)+c}\,c^5\right)}{\left(-i\,c\tan(fx+e)+c\right)^3a^3-6\left(-i\,c\tan(fx+e)+c\right)^2a^3c+12\left(-i\,c\tan(fx+e)+c\right)a^3c^2-8\,a^3c^3}\right)}{384\,cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="maxim
a")
```

```
[Out] 1/384*I*(3*sqrt(2)*c^(5/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*c*tan(f*x + e) +
 c))/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/a^3 + 4*(3*(-I*c*tan(
f*x + e) + c)^(5/2)*c^3 - 16*(-I*c*tan(f*x + e) + c)^(3/2)*c^4 - 12*sqrt(-I
*c*tan(f*x + e) + c)*c^5)/((-I*c*tan(f*x + e) + c)^3*a^3 - 6*(-I*c*tan(f*x
+ e) + c)^2*a^3*c + 12*(-I*c*tan(f*x + e) + c)*a^3*c^2 - 8*a^3*c^3))/(c*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(155) = 310.
time = 0.86, size = 317, normalized size = 1.64

$$-\frac{\left(3\sqrt{\frac{1}{2}}\,a^3f\sqrt{-\frac{c^3}{a^6f^2}}\,e^{(6i\,fx+6i\,e)}\log\left(-\frac{\left(\sqrt{2}\,\sqrt{\frac{1}{2}}\,(a^3fe^{(2i\,fx+2i\,e)}+a^3f)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\,\sqrt{-\frac{c^3}{a^6f^2}}+i\,c^2\right)e^{(-i\,fx-i\,e)}}{16\,a^3f}\right)-3\sqrt{\frac{1}{2}}\,a^3f\sqrt{-\frac{c^3}{a^6f^2}}\,e^{(6i\,fx+6i\,e)}\log\left(\frac{\left(\sqrt{2}\,\sqrt{\frac{1}{2}}\,(a^3fe^{(2i\,fx+2i\,e)}+a^3f)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\,\sqrt{-\frac{c^3}{a^6f^2}}-i\,c^2\right)e^{(-i\,fx-i\,e)}}{16\,a^3f}\right)-\sqrt{2}\left(3i\,ce^{(6i\,fx+6i\,e)}+17i\,ce^{(4i\,fx+4i\,e)}+22i\,ce^{(2i\,fx+2i\,e)}+8i\,c\right)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-6i\,fx-6i\,e)}}{192\,a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="frica
s")
```

```
[Out] -1/192*(3*sqrt(1/2)*a^3*f*sqrt(-c^3/(a^6*f^2))*e^(6*I*f*x + 6*I*e)*log(-1/1
6*(sqrt(2)*sqrt(1/2)*(a^3*f*e^(2*I*f*x + 2*I*e) + a^3*f)*sqrt(c/(e^(2*I*f*x
 + 2*I*e) + 1))*sqrt(-c^3/(a^6*f^2)) + I*c^2)*e^(-I*f*x - I*e)/(a^3*f)) - 3
*sqrt(1/2)*a^3*f*sqrt(-c^3/(a^6*f^2))*e^(6*I*f*x + 6*I*e)*log(1/16*(sqrt(2)
*sqrt(1/2)*(a^3*f*e^(2*I*f*x + 2*I*e) + a^3*f)*sqrt(c/(e^(2*I*f*x + 2*I*e)
+ 1))*sqrt(-c^3/(a^6*f^2)) - I*c^2)*e^(-I*f*x - I*e)/(a^3*f)) - sqrt(2)*(3*
I*c*e^(6*I*f*x + 6*I*e) + 17*I*c*e^(4*I*f*x + 4*I*e) + 22*I*c*e^(2*I*f*x +
2*I*e) + 8*I*c)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-6*I*f*x - 6*I*e)/(a^
3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\left(\int \frac{c\sqrt{-ic\tan\left(e+fx\right)+c}}{\tan^{3}\left(e+fx\right)-3i\tan^{2}\left(e+fx\right)-3\tan\left(e+fx\right)+i}\,dx+\int\left(-\frac{ic\sqrt{-ic\tan\left(e+fx\right)+c}\tan\left(e+fx\right)}{\tan^{3}\left(e+fx\right)-3i\tan^{2}\left(e+fx\right)-3\tan\left(e+fx\right)+i}\right)dx\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**(3/2)/(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] I*(Integral(c*sqrt(-I*c*tan(e + f*x) + c)/(tan(e + f*x)**3 - 3*I*tan(e + f*
x)**2 - 3*tan(e + f*x) + I), x) + Integral(-I*c*sqrt(-I*c*tan(e + f*x) + c)
*tan(e + f*x)/(tan(e + f*x)**3 - 3*I*tan(e + f*x)**2 - 3*tan(e + f*x) + I),
 x))/a**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="giac"
)
```

```
[Out] integrate((-I*c*tan(f*x + e) + c)^(3/2)/(I*a*tan(f*x + e) + a)^3, x)
```

Mupad [B]
time = 4.94, size = 181, normalized size = 0.94

$$\frac{\frac{c^4\,\sqrt{c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}}\,1{}\mathrm{i}}{8\,a^3\,f}+\frac{c^3\,{\left(c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{3/2}\,1{}\mathrm{i}}{6\,a^3\,f}-\frac{c^2\,{\left(c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{5/2}\,1{}\mathrm{i}}{32\,a^3\,f}}{6\,c\,{\left(c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^2-12\,c^2\,\left(c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)-{\left(c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^3+8\,c^3}+\frac{\sqrt{2}\,{\left(-c\right)}^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}}}{2\,\sqrt{-c}}\right)\,1{}\mathrm{i}}{64\,a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^(3/2)/(a + a*tan(e + f*x)*1i)^3,x)
```

```
[Out] ((c^4*(c - c*tan(e + f*x)*1i)^(1/2)*1i)/(8*a^3*f) + (c^3*(c - c*tan(e + f*x
)*1i)^(3/2)*1i)/(6*a^3*f) - (c^2*(c - c*tan(e + f*x)*1i)^(5/2)*1i)/(32*a^3*
f))/(6*c*(c - c*tan(e + f*x)*1i)^2 - 12*c^2*(c - c*tan(e + f*x)*1i) - (c -
c*tan(e + f*x)*1i)^3 + 8*c^3) + (2^(1/2)*(-c)^(3/2)*atan((2^(1/2)*(c - c*ta
n(e + f*x)*1i)^(1/2))/(2*(-c)^(1/2)))*1i)/(64*a^3*f)
```

3.968 $\int (a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=94

$$\frac{8ia^3(c-ic\tan(e+fx))^{5/2}}{5f}-\frac{8ia^3(c-ic\tan(e+fx))^{7/2}}{7cf}+\frac{2ia^3(c-ic\tan(e+fx))^{9/2}}{9c^2f}$$

```
[Out] 8/5*I*a^3*(c-I*c*tan(f*x+e))^(5/2)/f-8/7*I*a^3*(c-I*c*tan(f*x+e))^(7/2)/c/f
+2/9*I*a^3*(c-I*c*tan(f*x+e))^(9/2)/c^2/f
```

Rubi [A]
time = 0.10, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3603, 3568, 45}

$$\frac{2ia^3(c-ic\tan(e+fx))^{9/2}}{9c^2f}-\frac{8ia^3(c-ic\tan(e+fx))^{7/2}}{7cf}+\frac{8ia^3(c-ic\tan(e+fx))^{5/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])^(5/2),x]
```

```
[Out] (((8*I)/5)*a^3*(c - I*c*Tan[e + f*x])^(5/2))/f - (((8*I)/7)*a^3*(c - I*c*Ta
n[e + f*x])^(7/2))/(c*f) + (((2*I)/9)*a^3*(c - I*c*Tan[e + f*x])^(9/2))/(c^
2*f)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int (a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^{5/2}\,dx = \left(a^3c^3\right)\int \frac{\sec^6(e+fx)}{\sqrt{c-ic\tan(e+fx)}}\,dx$$
$$= \frac{\left(ia^3\right)\operatorname{Subst}\left(\int (c-x)^2(c+x)^{3/2}\,dx, x, -ic\tan(e+fx)\right)}{c^2 f}$$
$$= \frac{\left(ia^3\right)\operatorname{Subst}\left(\int \left(4c^2(c+x)^{3/2}-4c(c+x)^{5/2}+(c+x)^{7/}\right.\right.}{c^2 f}$$
$$= \frac{8ia^3(c-ic\tan(e+fx))^{5/2}}{5f} - \frac{8ia^3(c-ic\tan(e+fx))}{7cf}$$

Mathematica [A]
time = 1.91, size = 100, normalized size = 1.06

$$\frac{2a^3c^2\sec^4(e+fx)(i\cos(2e-fx)+\sin(2e-fx))(36+71\cos(2(e+fx))+55i\sin(2(e+fx)))\sqrt{c-ic\tan(e+fx)}}{315f(\cos(fx)+i\sin(fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])^(5/2),x]
```

```
[Out] (2*a^3*c^2*Sec[e + f*x]^4*(I*Cos[2*e - f*x] + Sin[2*e - f*x])*(36 + 71*Cos[
2*(e + f*x)] + (55*I)*Sin[2*(e + f*x)])*Sqrt[c - I*c*Tan[e + f*x]])/(315*f*
(Cos[f*x] + I*Sin[f*x])^3)
```

Maple [A]
time = 0.32, size = 66, normalized size = 0.70

method	result	size
derivativedivides	$\frac{2ia^3\left(\frac{(c-ic\tan(fx+e))^{\frac{9}{2}}}{9}-\frac{4c(c-ic\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{4c^2(c-ic\tan(fx+e))^{\frac{5}{2}}}{5}\right)}{f\,c^2}$	66
default	$\frac{2ia^3\left(\frac{(c-ic\tan(fx+e))^{\frac{9}{2}}}{9}-\frac{4c(c-ic\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{4c^2(c-ic\tan(fx+e))^{\frac{5}{2}}}{5}\right)}{f\,c^2}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3*(c-I*c*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/f*a^3/c^2*(1/9*(c-I*c*tan(f*x+e))^(9/2)-4/7*c*(c-I*c*tan(f*x+e))^(7/2)+
4/5*c^2*(c-I*c*tan(f*x+e))^(5/2))
```

Maxima [A]
time = 0.28, size = 70, normalized size = 0.74

$$\frac{2i\left(35\left(-ic\tan\left(fx+e\right)+c\right)^{\frac{9}{2}}a^3-180\left(-ic\tan\left(fx+e\right)+c\right)^{\frac{7}{2}}a^3c+252\left(-ic\tan\left(fx+e\right)+c\right)^{\frac{5}{2}}a^3c^2\right)}{315\,c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c-I*c*tan(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] 2/315*I*(35*(-I*c*tan(f*x + e) + c)^(9/2)*a^3 - 180*(-I*c*tan(f*x + e) + c)
^(7/2)*a^3*c + 252*(-I*c*tan(f*x + e) + c)^(5/2)*a^3*c^2)/(c^2*f)
```

Fricas [A]
time = 0.79, size = 123, normalized size = 1.31

$$-\frac{32\sqrt{2}\left(-63i\,a^3c^2e^{(4i\,fx+4i\,e)}-36i\,a^3c^2e^{(2i\,fx+2i\,e)}-8i\,a^3c^2\right)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{315\left(fe^{(8i\,fx+8i\,e)}+4\,fe^{(6i\,fx+6i\,e)}+6\,fe^{(4i\,fx+4i\,e)}+4\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c-I*c*tan(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] -32/315*sqrt(2)*(-63*I*a^3*c^2*e^(4*I*f*x + 4*I*e) - 36*I*a^3*c^2*e^(2*I*f*
x + 2*I*e) - 8*I*a^3*c^2)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(f*e^(8*I*f*x +
 8*I*e) + 4*f*e^(6*I*f*x + 6*I*e) + 6*f*e^(4*I*f*x + 4*I*e) + 4*f*e^(2*I*f*
x + 2*I*e) + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia^3\left(\int ic^2\sqrt{-ic\tan\left(e+fx\right)+c}\,dx+\int\left(-c^2\sqrt{-ic\tan\left(e+fx\right)+c}\tan\left(e+fx\right)\right)dx+\int\left(-2c^2\sqrt{-ic\tan\left(e+fx\right)+c}\tan^3\left(e+fx\right)\right)dx+\int\left(-c^2\sqrt{-ic\tan\left(e+fx\right)+c}\tan^5\left(e+fx\right)\right)dx+\int 2ic^2\sqrt{-ic\tan\left(e+fx\right)+c}\tan^2\left(e+fx\right)dx+\int ic^2\sqrt{-ic\tan\left(e+fx\right)+c}\tan^4\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3*(c-I*c*tan(f*x+e))**(5/2),x)
```

```
[Out] -I*a**3*(Integral(I*c**2*sqrt(-I*c*tan(e + f*x) + c), x) + Integral(-c**2*s
qrt(-I*c*tan(e + f*x) + c)*tan(e + f*x), x) + Integral(-2*c**2*sqrt(-I*c*ta
n(e + f*x) + c)*tan(e + f*x)**3, x) + Integral(-c**2*sqrt(-I*c*tan(e + f*x)
 + c)*tan(e + f*x)**5, x) + Integral(2*I*c**2*sqrt(-I*c*tan(e + f*x) + c)*t
an(e + f*x)**2, x) + Integral(I*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*
x)**4, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c-I*c*tan(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^3*(-I*c*tan(f*x + e) + c)^(5/2), x)
```

Mupad [B]
time = 8.23, size = 97, normalized size = 1.03

$$\frac{32\,a^3\,c^2\,\sqrt{c+\frac{c\,\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{{\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,36\mathrm{i}+{\mathrm{e}}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,63\mathrm{i}+8\mathrm{i}\right)}{315\,f\,{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i)^(5/2),x)
```

```
[Out] (32*a^3*c^2*(c + (c*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) +
1))^(1/2)*(exp(e*2i + f*x*2i)*36i + exp(e*4i + f*x*4i)*63i + 8i))/(315*f*(e
xp(e*2i + f*x*2i) + 1)^4)
```

3.969 $\int (a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=62

$$\frac{4ia^2(c-ic\tan(e+fx))^{5/2}}{5f}-\frac{2ia^2(c-ic\tan(e+fx))^{7/2}}{7cf}$$

```
[Out] 4/5*I*a^2*(c-I*c*tan(f*x+e))^(5/2)/f-2/7*I*a^2*(c-I*c*tan(f*x+e))^(7/2)/c/f
```

Rubi [A]
time = 0.10, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3603, 3568, 45}

$$\frac{4ia^2(c-ic\tan(e+fx))^{5/2}}{5f}-\frac{2ia^2(c-ic\tan(e+fx))^{7/2}}{7cf}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^(5/2),x]
```

```
[Out] (((4*I)/5)*a^2*(c - I*c*Tan[e + f*x])^(5/2))/f - (((2*I)/7)*a^2*(c - I*c*Ta
n[e + f*x])^(7/2))/(c*f)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int (a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{5/2}\,dx = (a^2c^2)\int \sec^4(e+fx)\sqrt{c-ic\tan(e+fx)}\,dx$$
$$= \frac{(ia^2)\,\text{Subst}\left(\int (c-x)(c+x)^{3/2}\,dx, x, -ic\tan(e+fx)\right)}{cf}$$
$$= \frac{(ia^2)\,\text{Subst}\left(\int \left(2c(c+x)^{3/2}-(c+x)^{5/2}\right)\,dx, x, -ic\tan\right.}{cf}$$
$$= \frac{4ia^2(c-ic\tan(e+fx))^{5/2}}{5f} - \frac{2ia^2(c-ic\tan(e+fx))}{7cf}$$

Mathematica [A]
time = 1.37, size = 78, normalized size = 1.26

$$-\frac{2a^2c^2\sec^2(e+fx)(\cos(2e)-i\sin(2e))(-9i+5\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{35f(\cos(fx)+i\sin(fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^(5/2),x]
[Out] (-2*a^2*c^2*Sec[e + f*x]^2*(Cos[2*e] - I*Sin[2*e])*(-9*I + 5*Tan[e + f*x])*
Sqrt[c - I*c*Tan[e + f*x]])/(35*f*(Cos[f*x] + I*Sin[f*x])^2)
```

Maple [A]
time = 0.26, size = 47, normalized size = 0.76

method	result	size
derivativedivides	$-\frac{2ia^2\left(\frac{(c-ic\tan(fx+e))^{\frac{7}{2}}}{7}-\frac{2c(c-ic\tan(fx+e))^{\frac{5}{2}}}{5}\right)}{fc}$	47
default	$-\frac{2ia^2\left(\frac{(c-ic\tan(fx+e))^{\frac{7}{2}}}{7}-\frac{2c(c-ic\tan(fx+e))^{\frac{5}{2}}}{5}\right)}{fc}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2*(c-I*c*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
[Out] -2*I/f*a^2/c*(1/7*(c-I*c*tan(f*x+e))^(7/2)-2/5*c*(c-I*c*tan(f*x+e))^(5/2))
```

Maxima [A]
time = 0.29, size = 48, normalized size = 0.77

$$-\frac{2i\left(5\left(-i\,c\tan\left(fx+e\right)+c\right)^{\frac{7}{2}}a^2-14\left(-i\,c\tan\left(fx+e\right)+c\right)^{\frac{5}{2}}a^2c\right)}{35\,cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c-I*c*tan(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -2/35*I*(5*(-I*c*tan(f*x + e) + c)^(7/2)*a^2 - 14*(-I*c*tan(f*x + e) + c)^(
5/2)*a^2*c)/(c*f)
```

Fricas [A]
time = 0.97, size = 92, normalized size = 1.48

$$-\frac{16\sqrt{2}\left(-7i\,a^2c^2e^{(2i\,fx+2i\,e)}-2i\,a^2c^2\right)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{35\left(fe^{(6i\,fx+6i\,e)}+3\,fe^{(4i\,fx+4i\,e)}+3\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c-I*c*tan(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] -16/35*sqrt(2)*(-7*I*a^2*c^2*e^(2*I*f*x + 2*I*e) - 2*I*a^2*c^2)*sqrt(c/(e^(
2*I*f*x + 2*I*e) + 1))/(f*e^(6*I*f*x + 6*I*e) + 3*f*e^(4*I*f*x + 4*I*e) + 3
*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int\left(-c^2\sqrt{-ic\tan{(e+fx)}+c}\right)dx+\int\left(-2c^2\sqrt{-ic\tan{(e+fx)}+c}\tan^2{(e+fx)}\right)dx+\int\left(-c^2\sqrt{-ic\tan{(e+fx)}+c}\tan^4{(e+fx)}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2*(c-I*c*tan(f*x+e))**(5/2),x)
```

```
[Out] -a**2*(Integral(-c**2*sqrt(-I*c*tan(e + f*x) + c), x) + Integral(-2*c**2*sq
rt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2, x) + Integral(-c**2*sqrt(-I*c*ta
n(e + f*x) + c)*tan(e + f*x)**4, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c-I*c*tan(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^2*(-I*c*tan(f*x + e) + c)^(5/2), x)
```

Mupad [B]
time = 9.95, size = 83, normalized size = 1.34

$$\frac{16\,a^2\,c^2\,\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}\,7{}\mathrm{i}+2{}\mathrm{i}\right)\,\sqrt{c+\frac{c\,\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1}}}{35\,f\,{\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i)^(5/2),x)
```

```
[Out] (16*a^2*c^2*(exp(e*2i + f*x*2i)*7i + 2i)*(c + (c*(exp(e*2i + f*x*2i)*1i - 1
i)*1i)/(exp(e*2i + f*x*2i) + 1))^(1/2))/(35*f*(exp(e*2i + f*x*2i) + 1)^3)
```

3.970 $\int (a+ia\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=27

$$\frac{2ia(c-ic\tan(e+fx))^{5/2}}{5f}$$

```
[Out] 2/5*I*a*(c-I*c*tan(f*x+e))^(5/2)/f
```

Rubi [A]
time = 0.08, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 32}

$$\frac{2ia(c-ic\tan(e+fx))^{5/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(5/2),x]
```

```
[Out] (((2*I)/5)*a*(c - I*c*Tan[e + f*x])^(5/2))/f
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int (a+ia\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx = (ac)\int \sec^2(e+fx)(c-ic\tan(e+fx))^{3/2}\,dx$$
$$= \frac{(ia)\text{Subst}\left(\int (c+x)^{3/2}\,dx, x, -ic\tan(e+fx)\right)}{f}$$
$$= \frac{2ia(c-ic\tan(e+fx))^{5/2}}{5f}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 70 vs. $2(27) = 54$.
time = 0.68, size = 70, normalized size = 2.59

$$\frac{2ac^2\sec^2(e+fx)(\cos(fx)-i\sin(fx))(i\cos(2e+fx)+\sin(2e+fx))\sqrt{c-ic\tan(e+fx)}}{5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(5/2),x]
```

```
[Out] (2*a*c^2*Sec[e + f*x]^2*(Cos[f*x] - I*Sin[f*x])*(I*Cos[2*e + f*x] + Sin[2*e
 + f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(5*f)
```

Maple [**A**]
time = 0.22, size = 22, normalized size = 0.81

method	result	size
derivativedivides	$\frac{2ia(c-ic\tan(fx+e))^{\frac{5}{2}}}{5f}$	22
default	$\frac{2ia(c-ic\tan(fx+e))^{\frac{5}{2}}}{5f}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*I*a*(c-I*c*tan(f*x+e))^(5/2)/f
```

Maxima [**A**]
time = 0.31, size = 20, normalized size = 0.74

$$\frac{2i\left(-i\,c\tan\left(fx+e\right)+c\right)^{\frac{5}{2}}a}{5\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x, algorithm="maxima"
)
```

```
[Out] 2/5*I*(-I*c*tan(f*x + e) + c)^(5/2)*a/f
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(20) = 40$.
time = 0.94, size = 56, normalized size = 2.07

$$\frac{8i\sqrt{2}\,ac^2\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{5\left(fe^{(4i\,fx+4i\,e)}+2\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x, algorithm="fricas"
)
```

```
[Out] 8/5*I*sqrt(2)*a*c^2*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(f*e^(4*I*f*x + 4*I*e
) + 2*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [A]
time = 4.43, size = 44, normalized size = 1.63

$$\begin{cases} \frac{2ia(-ic\tan{(e+fx)}+c)^{\frac{5}{2}}}{5f} & \text{for}\ f\neq 0 \\ x(ia\tan{(e)}+a)\left(-ic\tan{(e)}+c\right)^{\frac{5}{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e))**(5/2),x)
```

```
[Out] Piecewise((2*I*a*(-I*c*tan(e + f*x) + c)**(5/2)/(5*f), Ne(f, 0)), (x*(I*a*t
an(e) + a)*(-I*c*tan(e) + c)**(5/2), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)*(-I*c*tan(f*x + e) + c)^(5/2), x)
```

Mupad [B]
time = 0.31, size = 120, normalized size = 4.44

$$\frac{a\,c^2\,\sqrt{\frac{c\,\left(\cos\left(2\,e+2\,f\,x\right)+1-\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\left(2\,\cos\left(2\,e+2\,f\,x\right)+\cos\left(4\,e+4\,f\,x\right)+1-\sin\left(2\,e+2\,f\,x\right)\,2{}\mathrm{i}-\sin\left(4\,e+4\,f\,x\right)\,1{}\mathrm{i}\right)\,4{}\mathrm{i}}{5\,f\,\left(4\,\cos\left(2\,e+2\,f\,x\right)+\cos\left(4\,e+4\,f\,x\right)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i)^(5/2),x)
```

```
[Out] (a*c^2*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x)
+ 1))^(1/2)*(2*cos(2*e + 2*f*x) + cos(4*e + 4*f*x) - sin(2*e + 2*f*x)*2i -
sin(4*e + 4*f*x)*1i + 1)*4i)/(5*f*(4*cos(2*e + 2*f*x) + cos(4*e + 4*f*x) +
3))
```

3.971 $\int \frac{(c-ic\tan(e+fx))^{5/2}}{a+ia\tan(e+fx)}\,dx$

Optimal. Leaf size=125

$$-\frac{3i\sqrt{2}\,c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\,\sqrt{c}}\right)}{af}+\frac{3ic^2\sqrt{c-ic\tan(e+fx)}}{af}+\frac{ic^2(c-ic\tan(e+fx))^{3/2}}{af(c+ic\tan(e+fx))}$$

```
[Out] -3*I*c^(5/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/
a/f+3*I*c^2*(c-I*c*tan(f*x+e))^(1/2)/a/f+I*c^2*(c-I*c*tan(f*x+e))^(3/2)/a/f
/(c+I*c*tan(f*x+e))
```

Rubi [A]
time = 0.13, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3603, 3568, 43, 52, 65, 212}

$$-\frac{3i\sqrt{2}\,c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\,\sqrt{c}}\right)}{af}+\frac{3ic^2\sqrt{c-ic\tan(e+fx)}}{af}+\frac{ic^2(c-ic\tan(e+fx))^{3/2}}{af(c+ic\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])^(5/2)/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] ((-3*I)*Sqrt[2]*c^(5/2)*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c]
)])/(a*f) + ((3*I)*c^2*Sqrt[c - I*c*Tan[e + f*x]])/(a*f) + (I*c^2*(c - I*c*
Tan[e + f*x])^(3/2))/(a*f*(c + I*c*Tan[e + f*x]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{(c-ic\tan(e+fx))^{5/2}}{a+ia\tan(e+fx)}\,dx = \frac{\int \cos^2(e+fx)(c-ic\tan(e+fx))^{7/2}\,dx}{ac}$$
$$= \frac{(ic^2)\,\text{Subst}\left(\int \frac{(c+x)^{3/2}}{(c-x)^2}\,dx, x, -ic\tan(e+fx)\right)}{af}$$
$$= \frac{ic^2(c-ic\tan(e+fx))^{3/2}}{af(c+ic\tan(e+fx))} - \frac{(3ic^2)\,\text{Subst}\left(\int \frac{\sqrt{c+x}}{c-x}\,dx, x, -ic\tan(e+fx)\right)}{2af}$$
$$= \frac{3ic^2\sqrt{c-ic\tan(e+fx)}}{af} + \frac{ic^2(c-ic\tan(e+fx))^{3/2}}{af(c+ic\tan(e+fx))} - \frac{(3ic^3)\,\text{Subst}\left(\int \frac{}{(c-}\right.}{}$$
$$= \frac{3ic^2\sqrt{c-ic\tan(e+fx)}}{af} + \frac{ic^2(c-ic\tan(e+fx))^{3/2}}{af(c+ic\tan(e+fx))} - \frac{(6ic^3)\,\text{Subst}\left(\int \frac{1}{2c-}\right.}{}$$
$$= -\frac{3i\sqrt{2}c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{af} + \frac{3ic^2\sqrt{c-ic\tan(e+fx)}}{af} +$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(c - I*c*Tan[e + f*x])^(5/2)/(a + I*a*Tan[e + f*x]),x]
```
```
[Out] $Aborted
```

Maple [A]
time = 0.30, size = 95, normalized size = 0.76

method	result
derivativedivides	$\frac{2ic^2\left(\sqrt{c-ic\tan(fx+e)}-4c\left(-\frac{\sqrt{c-ic\tan(fx+e)}}{8\left(\frac{c}{2}+\frac{ic\tan(fx+e)}{2}\right)}+\frac{3\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e}}{2\sqrt{c}}\right.}{8\sqrt{c}}\right.\right.}{fa}$

default	$\frac{2ic^2\left(\sqrt{c-ic\tan\left(fx+e\right)}-4c\left(-\frac{\sqrt{c-ic\tan\left(fx+e\right)}}{8\left(\frac{c}{2}+\frac{ic\tan\left(fx+e\right)}{2}\right)}+\frac{3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan\left(fx+\right.}}{2\sqrt{c}}\right.}{8\sqrt{c}}\right.\right.}{fa}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/f/a*c^2*((c-I*c*tan(f*x+e))^(1/2)-4*c*(-1/8*(c-I*c*tan(f*x+e))^(1/2)/(1
/2*c+1/2*I*c*tan(f*x+e))+3/8*2^(1/2)/c^(1/2)*arctanh(1/2*(c-I*c*tan(f*x+e))
^(1/2)*2^(1/2)/c^(1/2))))
```

Maxima [A]
time = 0.50, size = 133, normalized size = 1.06

$$\frac{i\left(\frac{3\sqrt{2}\,c^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-i\,c\tan\left(fx+e\right)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-i\,c\tan\left(fx+e\right)+c}}\right)}{a}-\frac{4\sqrt{-i\,c\tan\left(fx+e\right)+c}\,c^4}{\left(-i\,c\tan\left(fx+e\right)+c\right)a-2\,ac}+\frac{4\sqrt{-i\,c\tan\left(fx+e\right)+c}\,c^3}{a}\right)}{2\,cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e)),x, algorithm="maxima"
)
```

```
[Out] 1/2*I*(3*sqrt(2)*c^(7/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*c*tan(f*x + e) + c
))/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/a - 4*sqrt(-I*c*tan(f*x
 + e) + c)*c^4/((-I*c*tan(f*x + e) + c)*a - 2*a*c) + 4*sqrt(-I*c*tan(f*x +
e) + c)*c^3/a)/(c*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(102) = 204.
time = 1.50, size = 272, normalized size = 2.18

$$-\frac{\left(3\sqrt{2}\,a\sqrt{-\frac{c^5}{a^2f^2}}\,fe^{\left(2i\,fx+2i\,e\right)}\log\left(-\frac{12\left(i\,c^3+\left(afe^{\left(2i\,fx+2i\,e\right)}+af\right)\sqrt{-\frac{c^5}{a^2f^2}}\sqrt{\frac{c}{e^{\left(2i\,fx+2i\,e\right)}+1}}\right)e^{\left(-i\,fx-i\,e\right)}}{af}\right)-3\sqrt{2}\,a\sqrt{-\frac{c^5}{a^2f^2}}\,fe^{\left(2i\,fx+2i\,e\right)}\log\left(-\frac{12\left(i\,c^3-\left(afe^{\left(2i\,fx+2i\,e\right)}+af\right)\sqrt{-\frac{c^5}{a^2f^2}}\sqrt{\frac{c}{e^{\left(2i\,fx+2i\,e\right)}+1}}\right)e^{\left(-i\,fx-i\,e\right)}}{af}\right)+2\sqrt{2}\left(-3i\,c^2e^{\left(2i\,fx+2i\,e\right)}-i\,c^2\right)\sqrt{\frac{c}{e^{\left(2i\,fx+2i\,e\right)}+1}}\right)e^{\left(-2i\,fx-2i\,e\right)}}{2\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e)),x, algorithm="fricas"
)
```

```
[Out] -1/2*(3*sqrt(2)*a*sqrt(-c^5/(a^2*f^2))*f*e^(2*I*f*x + 2*I*e)*log(-12*(I*c^3
 + (a*f*e^(2*I*f*x + 2*I*e) + a*f)*sqrt(-c^5/(a^2*f^2))*sqrt(c/(e^(2*I*f*x
+ 2*I*e) + 1)))*e^(-I*f*x - I*e)/(a*f)) - 3*sqrt(2)*a*sqrt(-c^5/(a^2*f^2))*
```

```
f*e^(2*I*f*x + 2*I*e)*log(-12*(I*c^3 - (a*f*e^(2*I*f*x + 2*I*e) + a*f)*sqrt
(-c^5/(a^2*f^2))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/(a*f))
 + 2*sqrt(2)*(-3*I*c^2*e^(2*I*f*x + 2*I*e) - I*c^2)*sqrt(c/(e^(2*I*f*x + 2*
I*e) + 1)))*e^(-2*I*f*x - 2*I*e)/(a*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\left(\int \frac{c^2\sqrt{-ic\tan(e+fx)+c}}{\tan(e+fx)-i}\,dx+\int\left(-\frac{c^2\sqrt{-ic\tan(e+fx)+c}\tan^2(e+fx)}{\tan(e+fx)-i}\right)dx+\int\left(-\frac{2ic^2\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)}{\tan(e+fx)-i}\right)dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**(5/2)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] -I*(Integral(c**2*sqrt(-I*c*tan(e + f*x) + c)/(tan(e + f*x) - I), x) + Inte
gral(-c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2/(tan(e + f*x) - I),
x) + Integral(-2*I*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)/(tan(e + f
*x) - I), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((-I*c*tan(f*x + e) + c)^(5/2)/(I*a*tan(f*x + e) + a), x)
```

Mupad [B]
time = 0.35, size = 109, normalized size = 0.87

$$\frac{c^2\sqrt{c-c\tan(e+fx)\,1\mathrm{i}}\;2\mathrm{i}}{af}-\frac{\sqrt{2}\,(-c)^{5/2}\operatorname{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+fx)\,1\mathrm{i}}}{2\sqrt{-c}}\right)3\mathrm{i}}{af}+\frac{c^3\sqrt{c-c\tan(e+fx)\,1\mathrm{i}}\;2\mathrm{i}}{af\,(c+c\tan(e+fx)\,1\mathrm{i})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^(5/2)/(a + a*tan(e + f*x)*1i),x)
```

```
[Out] (c^2*(c - c*tan(e + f*x)*1i)^(1/2)*2i)/(a*f) - (2^(1/2)*(-c)^(5/2)*atan((2^
(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*(-c)^(1/2)))*3i)/(a*f) + (c^3*(c -
c*tan(e + f*x)*1i)^(1/2)*2i)/(a*f*(c + c*tan(e + f*x)*1i))
```

3.972 $\int \frac{(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=146

$$\frac{3ic^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}\,a^2f}+\frac{ic^3(c-ic\tan(e+fx))^{3/2}}{2a^2f(c+ic\tan(e+fx))^2}-\frac{3ic^3\sqrt{c-ic\tan(e+fx)}}{4a^2f(c+ic\tan(e+fx))}$$

```
[Out] 3/8*I*c^(5/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))/a^2/f*2
^(1/2)+1/2*I*c^3*(c-I*c*tan(f*x+e))^(3/2)/a^2/f/(c+I*c*tan(f*x+e))^2-3/4*I*
c^3*(c-I*c*tan(f*x+e))^(1/2)/a^2/f/(c+I*c*tan(f*x+e))
```

Rubi [A]
time = 0.13, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3603, 3568, 43, 65, 212}

$$\frac{3ic^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}\,a^2f}-\frac{3ic^3\sqrt{c-ic\tan(e+fx)}}{4a^2f(c+ic\tan(e+fx))}+\frac{ic^3(c-ic\tan(e+fx))^{3/2}}{2a^2f(c+ic\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])^(5/2)/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (((3*I)/4)*c^(5/2)*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])])/(
Sqrt[2]*a^2*f) + ((I/2)*c^3*(c - I*c*Tan[e + f*x])^(3/2))/(a^2*f*(c + I*c*T
an[e + f*x])^2) - (((3*I)/4)*c^3*Sqrt[c - I*c*Tan[e + f*x]])/(a^2*f*(c + I*
c*Tan[e + f*x]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^2}\,dx &= \frac{\int \cos^4(e+fx)(c-ic\tan(e+fx))^{9/2}\,dx}{a^2c^2} \\
&= \frac{(ic^3)\operatorname{Subst}\left(\int \frac{(c+x)^{3/2}}{(c-x)^3}\,dx, x, -ic\tan(e+fx)\right)}{a^2 f} \\
&= \frac{ic^3(c-ic\tan(e+fx))^{3/2}}{2a^2f(c+ic\tan(e+fx))^2} - \frac{(3ic^3)\operatorname{Subst}\left(\int \frac{\sqrt{c+x}}{(c-x)^2}\,dx, x, -ic\tan(e+fx)\right.}{4a^2 f} \\
&= \frac{ic^3(c-ic\tan(e+fx))^{3/2}}{2a^2f(c+ic\tan(e+fx))^2} - \frac{3ic^3\sqrt{c-ic\tan(e+fx)}}{4a^2f(c+ic\tan(e+fx))} + \frac{(3ic^3)\operatorname{Subst}\left(\int \frac{}{(c-}\right.}{} \\
&= \frac{ic^3(c-ic\tan(e+fx))^{3/2}}{2a^2f(c+ic\tan(e+fx))^2} - \frac{3ic^3\sqrt{c-ic\tan(e+fx)}}{4a^2f(c+ic\tan(e+fx))} + \frac{(3ic^3)\operatorname{Subst}\left(\int \frac{1}{2c-}\right.}{} \\
&= \frac{3ic^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}\,a^2f} + \frac{ic^3(c-ic\tan(e+fx))^{3/2}}{2a^2f(c+ic\tan(e+fx))^2} - \frac{3ic^3\sqrt{}}{4a^2f}
\end{aligned}
$$

Mathematica [A]

time = 1.55, size = 138, normalized size = 0.95

$$
\frac{c^2(i\cos(2(e+fx))+\sin(2(e+fx)))\left(3\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)(\cos(2(e+fx))+i\sin(2(e+fx)))-(1+\cos(2(e+fx))+5i\sin(2(e+fx)))\sqrt{c-ic\tan(e+fx)}\right)}{8a^2f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - I*c*Tan[e + f*x])^(5/2)/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (c^2*(I*Cos[2*(e + f*x)] + Sin[2*(e + f*x)])*(3*Sqrt[2]*Sqrt[c]*ArcTanh[Sqr
t[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])]*(Cos[2*(e + f*x)] + I*Sin[2*(e +
 f*x)]) - (1 + Cos[2*(e + f*x)] + (5*I)*Sin[2*(e + f*x)])*Sqrt[c - I*c*Tan[
e + f*x]]))/(8*a^2*f)
```

Maple [A]
time = 0.29, size = 95, normalized size = 0.65

method	result
derivativedivides	$-\frac{2ic^3\left(-\frac{4\left(\frac{5(c-ic\tan(fx+e))^{\frac{3}{2}}}{32}-\frac{3c\sqrt{c-ic\tan(fx+e)}}{16}\right)}{(c+ic\tan(fx+e))^2}-\frac{3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{}}{2\sqrt{c}}\right.}{16\sqrt{c}}\right.}{f a^2}$
default	$-\frac{2ic^3\left(-\frac{4\left(\frac{5(c-ic\tan(fx+e))^{\frac{3}{2}}}{32}-\frac{3c\sqrt{c-ic\tan(fx+e)}}{16}\right)}{(c+ic\tan(fx+e))^2}-\frac{3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{}}{2\sqrt{c}}\right.}{16\sqrt{c}}\right.}{f a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/f/a^2*c^3*(-4*(5/32*(c-I*c*tan(f*x+e))^(3/2)-3/16*c*(c-I*c*tan(f*x+e))
^(1/2))/(c+I*c*tan(f*x+e))^2-3/16*2^(1/2)/c^(1/2)*arctanh(1/2*(c-I*c*tan(f*
x+e))^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [A]
time = 0.52, size = 159, normalized size = 1.09

$$-\frac{i\left(\frac{3\sqrt{2}\,c^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-ic\tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic\tan(fx+e)+c}}\right)}{a^2}-\frac{4\left(5\,(-ic\tan(fx+e)+c)^{\frac{3}{2}}c^4-6\sqrt{-ic\tan(fx+e)+c}\,c^5\right)}{(-ic\tan(fx+e)+c)^2a^2-4\,(-ic\tan(fx+e)+c)a^2c+4\,a^2c^2}\right)}{16\,cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxim
a")
```

```
[Out] -1/16*I*(3*sqrt(2)*c^(7/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*c*tan(f*x + e) +
 c))/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/a^2 - 4*(5*(-I*c*tan(
```

```
f*x + e) + c)^(3/2)*c^4 - 6*sqrt(-I*c*tan(f*x + e) + c)*c^5)/((-I*c*tan(f*x
 + e) + c)^2*a^2 - 4*(-I*c*tan(f*x + e) + c)*a^2*c + 4*a^2*c^2))/(c*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(116) = 232$.
time = 1.79, size = 311, normalized size = 2.13

$$-\frac{\left(3\sqrt{\frac{1}{2}}a^2f\sqrt{-\frac{c^5}{a^4f^2}}e^{(4ifx+4ie)}\log\left(-\frac{3\left(-ic^3+\sqrt{2}\sqrt{\frac{1}{2}}\left(a^2fe^{(2ifx+2ie)}+a^2f\right)\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}\sqrt{-\frac{c^5}{a^4f^2}}\right)e^{(-ifx-ie)}}{2a^2f}\right)-3\sqrt{\frac{1}{2}}a^2f\sqrt{-\frac{c^5}{a^4f^2}}e^{(4ifx+4ie)}\log\left(-\frac{3\left(-ic^3-\sqrt{2}\sqrt{\frac{1}{2}}\left(a^2fe^{(2ifx+2ie)}+a^2f\right)\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}\sqrt{-\frac{c^5}{a^4f^2}}\right)e^{(-ifx-ie)}}{2a^2f}\right)-\sqrt{2}\left(-3ic^2e^{(4ifx+4ie)}-ic^2e^{(2ifx+2ie)}+2ic^2\right)\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}\right)e^{(-4ifx-4ie)}}{8a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="frica
s")
```

```
[Out] -1/8*(3*sqrt(1/2)*a^2*f*sqrt(-c^5/(a^4*f^2))*e^(4*I*f*x + 4*I*e)*log(-3/2*(
-I*c^3 + sqrt(2)*sqrt(1/2)*(a^2*f*e^(2*I*f*x + 2*I*e) + a^2*f)*sqrt(c/(e^(2
*I*f*x + 2*I*e) + 1))*sqrt(-c^5/(a^4*f^2)))*e^(-I*f*x - I*e)/(a^2*f)) - 3*s
qrt(1/2)*a^2*f*sqrt(-c^5/(a^4*f^2))*e^(4*I*f*x + 4*I*e)*log(-3/2*(-I*c^3 -
sqrt(2)*sqrt(1/2)*(a^2*f*e^(2*I*f*x + 2*I*e) + a^2*f)*sqrt(c/(e^(2*I*f*x +
2*I*e) + 1))*sqrt(-c^5/(a^4*f^2)))*e^(-I*f*x - I*e)/(a^2*f)) - sqrt(2)*(-3*
I*c^2*e^(4*I*f*x + 4*I*e) - I*c^2*e^(2*I*f*x + 2*I*e) + 2*I*c^2)*sqrt(c/(e^
(2*I*f*x + 2*I*e) + 1)))*e^(-4*I*f*x - 4*I*e)/(a^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{c^2\sqrt{-ic\tan(e+fx)+c}}{\tan^2(e+fx)-2i\tan(e+fx)-1}\,dx+\int\left(-\frac{c^2\sqrt{-ic\tan(e+fx)+c}\tan^2(e+fx)}{\tan^2(e+fx)-2i\tan(e+fx)-1}\right)dx+\int\left(-\frac{2ic^2\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)}{\tan^2(e+fx)-2i\tan(e+fx)-1}\right)dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**(5/2)/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -(Integral(c**2*sqrt(-I*c*tan(e + f*x) + c)/(tan(e + f*x)**2 - 2*I*tan(e +
f*x) - 1), x) + Integral(-c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2/
(tan(e + f*x)**2 - 2*I*tan(e + f*x) - 1), x) + Integral(-2*I*c**2*sqrt(-I*c
*tan(e + f*x) + c)*tan(e + f*x)/(tan(e + f*x)**2 - 2*I*tan(e + f*x) - 1), x
))/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac"
)
```

```
[Out] integrate((-I*c*tan(f*x + e) + c)^(5/2)/(I*a*tan(f*x + e) + a)^2, x)
```

Mupad [B]
time = 0.28, size = 135, normalized size = 0.92

$$-\frac{\frac{c^4\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}\,3\mathrm{i}}{2\,a^2\,f}-\frac{c^3\,{\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}^{3/2}\,5\mathrm{i}}{4\,a^2\,f}}{{\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}^2-4\,c\,\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)+4\,c^2}+\frac{\sqrt{2}\,{\left(-c\right)}^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}}{2\,\sqrt{-c}}\right)\,3\mathrm{i}}{8\,a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^(5/2)/(a + a*tan(e + f*x)*1i)^2,x)
```

```
[Out] (2^(1/2)*(-c)^(5/2)*atan((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*(-c)^(1
/2)))*3i)/(8*a^2*f) - ((c^4*(c - c*tan(e + f*x)*1i)^(1/2)*3i)/(2*a^2*f) - (
c^3*(c - c*tan(e + f*x)*1i)^(3/2)*5i)/(4*a^2*f))/((c - c*tan(e + f*x)*1i)^2
 - 4*c*(c - c*tan(e + f*x)*1i) + 4*c^2)
```

3.973 $\int \frac{(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^3}\,dx$

Optimal. Leaf size=193

$$\frac{ic^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{16\sqrt{2}\,a^3f}+\frac{ic^4(c-ic\tan(e+fx))^{3/2}}{3a^3f(c+ic\tan(e+fx))^3}-\frac{ic^4\sqrt{c-ic\tan(e+fx)}}{4a^3f(c+ic\tan(e+fx))^2}+\frac{ic^3\sqrt{c-ic}}{16a^3f(c+ic}$$

```
[Out] 1/32*I*c^(5/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))/a^3/f*
2^(1/2)+1/3*I*c^4*(c-I*c*tan(f*x+e))^(3/2)/a^3/f/(c+I*c*tan(f*x+e))^3-1/4*I
*c^4*(c-I*c*tan(f*x+e))^(1/2)/a^3/f/(c+I*c*tan(f*x+e))^2+1/16*I*c^3*(c-I*c*
tan(f*x+e))^(1/2)/a^3/f/(c+I*c*tan(f*x+e))
```

Rubi [A]
time = 0.15, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3603, 3568, 43, 44, 65, 212}

$$\frac{ic^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{16\sqrt{2}\,a^3f}-\frac{ic^4\sqrt{c-ic\tan(e+fx)}}{4a^3f(c+ic\tan(e+fx))^2}+\frac{ic^4(c-ic\tan(e+fx))^{3/2}}{3a^3f(c+ic\tan(e+fx))^3}+\frac{ic^3\sqrt{c-ic\tan(e+fx)}}{16a^3f(c+ic\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])^(5/2)/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] ((I/16)*c^(5/2)*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])])/(Sqr
t[2]*a^3*f) + ((I/3)*c^4*(c - I*c*Tan[e + f*x])^(3/2))/(a^3*f*(c + I*c*Tan[
e + f*x])^3) - ((I/4)*c^4*Sqrt[c - I*c*Tan[e + f*x]])/(a^3*f*(c + I*c*Tan[e
 + f*x])^2) + ((I/16)*c^3*Sqrt[c - I*c*Tan[e + f*x]])/(a^3*f*(c + I*c*Tan[e
 + f*x]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^3}\,dx = \frac{\int \cos^6(e+fx)(c-ic\tan(e+fx))^{11/2}\,dx}{a^3c^3}$$

$$= \frac{(ic^4)\,\text{Subst}\left(\int \frac{(c+x)^{3/2}}{(c-x)^4}\,dx, x, -ic\tan(e+fx)\right)}{a^3f}$$

$$= \frac{ic^4(c-ic\tan(e+fx))^{3/2}}{3a^3f(c+ic\tan(e+fx))^3} - \frac{(ic^4)\,\text{Subst}\left(\int \frac{\sqrt{c+x}}{(c-x)^3}\,dx, x, -ic\tan(e+fx)\right)}{2a^3f}$$

$$= \frac{ic^4(c-ic\tan(e+fx))^{3/2}}{3a^3f(c+ic\tan(e+fx))^3} - \frac{ic^4\sqrt{c-ic\tan(e+fx)}}{4a^3f(c+ic\tan(e+fx))^2} + \frac{(ic^4)\,\text{Subst}\left(\int \frac{}{(c-x}\right.}{}$$

$$= \frac{ic^4(c-ic\tan(e+fx))^{3/2}}{3a^3f(c+ic\tan(e+fx))^3} - \frac{ic^4\sqrt{c-ic\tan(e+fx)}}{4a^3f(c+ic\tan(e+fx))^2} + \frac{ic^3\sqrt{c-ic\tan(e}}{16a^3f(c+ic\tan(e}$$

$$= \frac{ic^4(c-ic\tan(e+fx))^{3/2}}{3a^3f(c+ic\tan(e+fx))^3} - \frac{ic^4\sqrt{c-ic\tan(e+fx)}}{4a^3f(c+ic\tan(e+fx))^2} + \frac{ic^3\sqrt{c-ic\tan(e}}{16a^3f(c+ic\tan(e}$$

$$= \frac{ic^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{16\sqrt{2}\,a^3f} + \frac{ic^4(c-ic\tan(e+fx))^{3/2}}{3a^3f(c+ic\tan(e+fx))^3} - \frac{ic^4\sqrt{}}{4a^3f(}$$

Mathematica [A]
time = 2.44, size = 152, normalized size = 0.79

$$\frac{c^2(i\cos(3(e+fx))+\sin(3(e+fx)))\left(3\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)(\cos(3(e+fx))+i\sin(3(e+fx)))+(9\cos(e+fx)+5\cos(3(e+fx))-44i\cos^2(e+fx)\sin(e+fx))\sqrt{c-ic\tan(e+fx)}\right)}{96a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - I*c*Tan[e + f*x])^(5/2)/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] (c^2*(I*Cos[3*(e + f*x)] + Sin[3*(e + f*x)])*(3*Sqrt[2]*Sqrt[c]*ArcTanh[Sqr
t[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])]*(Cos[3*(e + f*x)] + I*Sin[3*(e +
 f*x)]) + (9*Cos[e + f*x] + 5*Cos[3*(e + f*x)] - (44*I)*Cos[e + f*x]^2*Sin[
e + f*x])*Sqrt[c - I*c*Tan[e + f*x]]))/(96*a^3*f)
```

Maple [A]
time = 0.34, size = 114, normalized size = 0.59

method	result

derivativedivides	$\frac{2ic^4\left(\frac{\frac{(c-ic\tan(fx+e))^{\frac{5}{2}}}{32c}+\frac{(c-ic\tan(fx+e))^{\frac{3}{2}}}{6}-\frac{c\sqrt{c-ic\tan(fx+e)}}{8}}{(c+ic\tan(fx+e))^3}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx}}{2\sqrt{c}}\right.}{64c^{\frac{3}{2}}}\right.}{f\,a^3}$
default	$\frac{2ic^4\left(\frac{\frac{(c-ic\tan(fx+e))^{\frac{5}{2}}}{32c}+\frac{(c-ic\tan(fx+e))^{\frac{3}{2}}}{6}-\frac{c\sqrt{c-ic\tan(fx+e)}}{8}}{(c+ic\tan(fx+e))^3}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx}}{2\sqrt{c}}\right.}{64c^{\frac{3}{2}}}\right.}{f\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/f/a^3*c^4*(8*(1/256/c*(c-I*c*tan(f*x+e))^(5/2)+1/48*(c-I*c*tan(f*x+e))^
(3/2)-1/64*c*(c-I*c*tan(f*x+e))^(1/2))/(c+I*c*tan(f*x+e))^3+1/64/c^(3/2)*2^
(1/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [A]
time = 0.51, size = 200, normalized size = 1.04

$$-\frac{i\left(\frac{3\sqrt{2}\,c^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{c}-\sqrt{-i\,c\tan\left(fx+e\right)+c}}{\sqrt{2}\,\sqrt{c}+\sqrt{-i\,c\tan\left(fx+e\right)+c}}\right)}{a^3}+\frac{4\left(3\left(-i\,c\tan(fx+e)+c\right)^{\frac{5}{2}}c^4+16\left(-i\,c\tan(fx+e)+c\right)^{\frac{3}{2}}c^5-12\sqrt{-i\,c\tan\left(fx+e\right)+c}\,c^6\right)}{\left(-i\,c\tan(fx+e)+c\right)^3a^3-6\left(-i\,c\tan(fx+e)+c\right)^2a^3c+12\left(-i\,c\tan(fx+e)+c\right)a^3c^2-8\,a^3c^3}\right)}{192\,cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="maxim
a")
```

```
[Out] -1/192*I*(3*sqrt(2)*c^(7/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*c*tan(f*x + e)
+ c))/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/a^3 + 4*(3*(-I*c*tan
(f*x + e) + c)^(5/2)*c^4 + 16*(-I*c*tan(f*x + e) + c)^(3/2)*c^5 - 12*sqrt(-
I*c*tan(f*x + e) + c)*c^6)/((-I*c*tan(f*x + e) + c)^3*a^3 - 6*(-I*c*tan(f*x
 + e) + c)^2*a^3*c + 12*(-I*c*tan(f*x + e) + c)*a^3*c^2 - 8*a^3*c^3))/(c*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(155) = 310$.
time = 1.41, size = 325, normalized size = 1.68

$$\frac{\left(3\sqrt{\frac{1}{2}}\,a^3f\sqrt{-\frac{c^5}{a^6f^2}}\,e^{(6i\,fx+6i\,e)}\log\left(\frac{\left(i\,c^3+\sqrt{2}\,\sqrt{\frac{1}{2}}\,(a^3fe^{(2i\,fx+2i\,e)}+a^3f)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\,\sqrt{-\frac{c^5}{a^6f^2}}\right)e^{(-i\,fx-i\,e)}}{8\,a^3f}\right)-3\sqrt{\frac{1}{2}}\,a^3f\sqrt{-\frac{c^5}{a^6f^2}}\,e^{(6i\,fx+6i\,e)}\log\left(\frac{\left(i\,c^3-\sqrt{2}\,\sqrt{\frac{1}{2}}\,(a^3fe^{(2i\,fx+2i\,e)}+a^3f)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\,\sqrt{-\frac{c^5}{a^6f^2}}\right)e^{(-i\,fx-i\,e)}}{8\,a^3f}\right)+\sqrt{2}\left(-3i\,c^2e^{(6i\,fx+6i\,e)}-i\,c^2e^{(4i\,fx+4i\,e)}+10i\,c^2e^{(2i\,fx+2i\,e)}+8i\,c^2\right)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-6i\,fx-6i\,e)}}{96\,a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="frica
s")
```

```
[Out] 1/96*(3*sqrt(1/2)*a^3*f*sqrt(-c^5/(a^6*f^2))*e^(6*I*f*x + 6*I*e)*log(1/8*(I
*c^3 + sqrt(2)*sqrt(1/2)*(a^3*f*e^(2*I*f*x + 2*I*e) + a^3*f)*sqrt(c/(e^(2*I
*f*x + 2*I*e) + 1))*sqrt(-c^5/(a^6*f^2)))*e^(-I*f*x - I*e)/(a^3*f)) - 3*sqr
t(1/2)*a^3*f*sqrt(-c^5/(a^6*f^2))*e^(6*I*f*x + 6*I*e)*log(1/8*(I*c^3 - sqrt
(2)*sqrt(1/2)*(a^3*f*e^(2*I*f*x + 2*I*e) + a^3*f)*sqrt(c/(e^(2*I*f*x + 2*I*
e) + 1))*sqrt(-c^5/(a^6*f^2)))*e^(-I*f*x - I*e)/(a^3*f)) + sqrt(2)*(-3*I*c^
2*e^(6*I*f*x + 6*I*e) - I*c^2*e^(4*I*f*x + 4*I*e) + 10*I*c^2*e^(2*I*f*x + 2
*I*e) + 8*I*c^2)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-6*I*f*x - 6*I*e)/(a
^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\left(\int \frac{c^2\sqrt{-ic\tan(e+fx)+c}}{\tan^3(e+fx)-3i\tan^2(e+fx)-3\tan(e+fx)+i}\,dx+\int\left(-\frac{c^2\sqrt{-ic\tan(e+fx)+c}\,\tan^2(e+fx)}{\tan^3(e+fx)-3i\tan^2(e+fx)-3\tan(e+fx)+i}\right)dx+\int\left(-\frac{2ic^2\sqrt{-ic\tan(e+fx)+c}\,\tan(e+fx)}{\tan^3(e+fx)-3i\tan^2(e+fx)-3\tan(e+fx)+i}\right)dx\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**(5/2)/(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] I*(Integral(c**2*sqrt(-I*c*tan(e + f*x) + c)/(tan(e + f*x)**3 - 3*I*tan(e +
 f*x)**2 - 3*tan(e + f*x) + I), x) + Integral(-c**2*sqrt(-I*c*tan(e + f*x)
+ c)*tan(e + f*x)**2/(tan(e + f*x)**3 - 3*I*tan(e + f*x)**2 - 3*tan(e + f*x
) + I), x) + Integral(-2*I*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)/(t
an(e + f*x)**3 - 3*I*tan(e + f*x)**2 - 3*tan(e + f*x) + I), x))/a**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="giac"
)
```

```
[Out] integrate((-I*c*tan(f*x + e) + c)^(5/2)/(I*a*tan(f*x + e) + a)^3, x)
```

Mupad [B]
time = 4.93, size = 181, normalized size = 0.94

$$\frac{-\frac{c^5\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}\,1\mathrm{i}}{4\,a^3\,f}+\frac{c^4\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{3/2}\,1\mathrm{i}}{3\,a^3\,f}+\frac{c^3\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{5/2}\,1\mathrm{i}}{16\,a^3\,f}}{6\,c\,(c-c\tan(e+f\,x)\,1\mathrm{i})^2-12\,c^2\,(c-c\tan(e+f\,x)\,1\mathrm{i})-(c-c\tan(e+f\,x)\,1\mathrm{i})^3+8\,c^3}+\frac{\sqrt{2}\,(-c)^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}}{2\,\sqrt{-c}}\right)\,1\mathrm{i}}{32\,a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^(5/2)/(a + a*tan(e + f*x)*1i)^3,x)
```

```
[Out] ((c^4*(c - c*tan(e + f*x)*1i)^(3/2)*1i)/(3*a^3*f) - (c^5*(c - c*tan(e + f*x
)*1i)^(1/2)*1i)/(4*a^3*f) + (c^3*(c - c*tan(e + f*x)*1i)^(5/2)*1i)/(16*a^3*
f))/(6*c*(c - c*tan(e + f*x)*1i)^2 - 12*c^2*(c - c*tan(e + f*x)*1i) - (c -
c*tan(e + f*x)*1i)^3 + 8*c^3) + (2^(1/2)*(-c)^(5/2)*atan((2^(1/2)*(c - c*ta
n(e + f*x)*1i)^(1/2))/(2*(-c)^(1/2)))*1i)/(32*a^3*f)
```

3.974 $\int \frac{(a+ia\tan(e+fx))^3}{\sqrt{c-ic\tan(e+fx)}}\,dx$

Optimal. Leaf size=90

$$-\frac{8ia^3}{f\sqrt{c-ic\tan(e+fx)}}-\frac{8ia^3\sqrt{c-ic\tan(e+fx)}}{cf}+\frac{2ia^3(c-ic\tan(e+fx))^{3/2}}{3c^2f}$$

```
[Out] -8*I*a^3/f/(c-I*c*tan(f*x+e))^(1/2)-8*I*a^3*(c-I*c*tan(f*x+e))^(1/2)/c/f+2/
3*I*a^3*(c-I*c*tan(f*x+e))^(3/2)/c^2/f
```

Rubi [A]
time = 0.11, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3603, 3568, 45}

$$\frac{2ia^3(c-ic\tan(e+fx))^{3/2}}{3c^2f}-\frac{8ia^3\sqrt{c-ic\tan(e+fx)}}{cf}-\frac{8ia^3}{f\sqrt{c-ic\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^3/Sqrt[c - I*c*Tan[e + f*x]],x]
```

```
[Out] ((-8*I)*a^3)/(f*Sqrt[c - I*c*Tan[e + f*x]]) - ((8*I)*a^3*Sqrt[c - I*c*Tan[e
 + f*x]])/(c*f) + (((2*I)/3)*a^3*(c - I*c*Tan[e + f*x])^(3/2))/(c^2*f)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
```

```
, 0] || GtQ[m, n]))
```

Rubi steps

$$\begin{aligned}\int \frac{(a+ia\tan(e+fx))^3}{\sqrt{c-ic\tan(e+fx)}}\,dx &= \left(a^3c^3\right)\int \frac{\sec^6(e+fx)}{(c-ic\tan(e+fx))^{7/2}}\,dx\\ &= \frac{\left(ia^3\right)\operatorname{Subst}\left(\int \frac{(c-x)^2}{(c+x)^{3/2}}\,dx,x,-ic\tan(e+fx)\right)}{c^2f}\\ &= \frac{\left(ia^3\right)\operatorname{Subst}\left(\int \left(\frac{4c^2}{(c+x)^{3/2}}-\frac{4c}{\sqrt{c+x}}+\sqrt{c+x}\right)dx,x,-ic\tan(e+fx)\right)}{c^2f}\\ &= -\frac{8ia^3}{f\sqrt{c-ic\tan(e+fx)}}-\frac{8ia^3\sqrt{c-ic\tan(e+fx)}}{cf}+\frac{2ia^3(c-ic\tan(e+}{3c^2f}\end{aligned}$$

Mathematica [A]
time = 1.29, size = 94, normalized size = 1.04

$$\frac{2a^3\sec(e+fx)(12+11\cos(2(e+fx))-5i\sin(2(e+fx)))(-i\cos(e+4fx)+\sin(e+4fx))\sqrt{c-ic\tan(e+fx)}}{3cf(\cos(fx)+i\sin(fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3/Sqrt[c - I*c*Tan[e + f*x]],x]
[Out] (2*a^3*Sec[e + f*x]*(12 + 11*Cos[2*(e + f*x)] - (5*I)*Sin[2*(e + f*x)])*((-
I)*Cos[e + 4*f*x] + Sin[e + 4*f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(3*c*f*(Cos
[f*x] + I*Sin[f*x])^3)
```

Maple [A]
time = 0.25, size = 66, normalized size = 0.73

method	result	size
derivativedivides	$\frac{2ia^3\left(\frac{(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}-4c\sqrt{c-ic\tan(fx+e)}-\frac{4c^2}{\sqrt{c-ic\tan(fx+e)}}\right)}{f\,c^2}$	66
default	$\frac{2ia^3\left(\frac{(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}-4c\sqrt{c-ic\tan(fx+e)}-\frac{4c^2}{\sqrt{c-ic\tan(fx+e)}}\right)}{f\,c^2}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/f*a^3/c^2*(1/3*(c-I*c*tan(f*x+e))^(3/2)-4*c*(c-I*c*tan(f*x+e))^(1/2)-4*
c^2/(c-I*c*tan(f*x+e))^(1/2))
```

Maxima [A]
time = 0.29, size = 73, normalized size = 0.81

$$-\frac{2i\left(\frac{12\,a^3c}{\sqrt{-i\,c\tan\left(fx+e\right)+c}}-\frac{(-i\,c\tan(fx+e)+c)^{\frac{3}{2}}a^3-12\,\sqrt{-i\,c\tan\left(fx+e\right)+c}\,a^3c}{c}\right)}{3\,cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -2/3*I*(12*a^3*c/sqrt(-I*c*tan(f*x + e) + c) - ((-I*c*tan(f*x + e) + c)^(3/
2)*a^3 - 12*sqrt(-I*c*tan(f*x + e) + c)*a^3*c)/c)/(c*f)
```

Fricas [A]
time = 1.44, size = 78, normalized size = 0.87

$$-\frac{4\,\sqrt{2}\,\left(3i\,a^3e^{(4i\,fx+4i\,e)}+12i\,a^3e^{(2i\,fx+2i\,e)}+8i\,a^3\right)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{3\,\left(cfe^{(2i\,fx+2i\,e)}+cf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] -4/3*sqrt(2)*(3*I*a^3*e^(4*I*f*x + 4*I*e) + 12*I*a^3*e^(2*I*f*x + 2*I*e) +
8*I*a^3)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(c*f*e^(2*I*f*x + 2*I*e) + c*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia^3\left(\int\frac{i}{\sqrt{-ic\tan\left(e+fx\right)+c}}\,dx+\int\left(-\frac{3\tan\left(e+fx\right)}{\sqrt{-ic\tan\left(e+fx\right)+c}}\right)dx+\int\frac{\tan^3\left(e+fx\right)}{\sqrt{-ic\tan\left(e+fx\right)+c}}\,dx+\int\left(-\frac{3i\tan^2\left(e+fx\right)}{\sqrt{-ic\tan\left(e+fx\right)+c}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3/(c-I*c*tan(f*x+e))**(1/2),x)
```

```
[Out] -I*a**3*(Integral(I/sqrt(-I*c*tan(e + f*x) + c), x) + Integral(-3*tan(e + f
*x)/sqrt(-I*c*tan(e + f*x) + c), x) + Integral(tan(e + f*x)**3/sqrt(-I*c*ta
n(e + f*x) + c), x) + Integral(-3*I*tan(e + f*x)**2/sqrt(-I*c*tan(e + f*x)
+ c), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^3/sqrt(-I*c*tan(f*x + e) + c), x)
```

Mupad [B]
time = 5.28, size = 113, normalized size = 1.26

$$-\frac{2\,a^3\,\sqrt{\frac{c\,\left(\cos\left(2\,e+2\,f\,x\right)+1-\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\left(\cos\left(2\,e+2\,f\,x\right)\,23{}\mathrm{i}+\cos\left(4\,e+4\,f\,x\right)\,3{}\mathrm{i}-7\,\sin\left(2\,e+2\,f\,x\right)-3\,\sin\left(4\,e+4\,f\,x\right)+20{}\mathrm{i}\right)}{3\,c\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^3/(c - c*tan(e + f*x)*1i)^(1/2),x)
```

```
[Out] -(2*a^3*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x)
 + 1))^(1/2)*(cos(2*e + 2*f*x)*23i + cos(4*e + 4*f*x)*3i - 7*sin(2*e + 2*f*
x) - 3*sin(4*e + 4*f*x) + 20i))/(3*c*f*(cos(2*e + 2*f*x) + 1))
```

3.975 $\int \frac{(a+ia\tan(e+fx))^2}{\sqrt{c-ic\tan(e+fx)}}\,dx$

Optimal. Leaf size=58

$$-\frac{4ia^2}{f\sqrt{c-ic\tan(e+fx)}}-\frac{2ia^2\sqrt{c-ic\tan(e+fx)}}{cf}$$

```
[Out] -4*I*a^2/f/(c-I*c*tan(f*x+e))^(1/2)-2*I*a^2*(c-I*c*tan(f*x+e))^(1/2)/c/f
```

Rubi [A]
time = 0.10, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3603, 3568, 45}

$$-\frac{2ia^2\sqrt{c-ic\tan(e+fx)}}{cf}-\frac{4ia^2}{f\sqrt{c-ic\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2/Sqrt[c - I*c*Tan[e + f*x]],x]
```

```
[Out] ((-4*I)*a^2)/(f*Sqrt[c - I*c*Tan[e + f*x]]) - ((2*I)*a^2*Sqrt[c - I*c*Tan[e
 + f*x]])/(c*f)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+ia\tan(e+fx))^2}{\sqrt{c-ic\tan(e+fx)}}\,dx &= (a^2c^2)\int \frac{\sec^4(e+fx)}{(c-ic\tan(e+fx))^{5/2}}\,dx \\
&= \frac{(ia^2)\,\mathrm{Subst}\left(\int \frac{c-x}{(c+x)^{3/2}}\,dx, x, -ic\tan(e+fx)\right)}{cf} \\
&= \frac{(ia^2)\,\mathrm{Subst}\left(\int \left(\frac{2c}{(c+x)^{3/2}}-\frac{1}{\sqrt{c+x}}\right)dx, x, -ic\tan(e+fx)\right)}{cf} \\
&= -\frac{4ia^2}{f\sqrt{c-ic\tan(e+fx)}}-\frac{2ia^2\sqrt{c-ic\tan(e+fx)}}{cf}
\end{aligned}$$

Mathematica [A]
time = 0.99, size = 91, normalized size = 1.57

$$\frac{2a^2(-2i\cos(2e)-i\cos(2fx)-2\sin(2e)+\sin(2fx))(\cos(e+fx)+i\sin(e+fx))^2\sqrt{c-ic\tan(e+fx)}}{cf(\cos(fx)+i\sin(fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2/Sqrt[c - I*c*Tan[e + f*x]],x]
```

```
[Out] (2*a^2*((-2*I)*Cos[2*e] - I*Cos[2*f*x] - 2*Sin[2*e] + Sin[2*f*x])*(Cos[e +
f*x] + I*Sin[e + f*x])^2*Sqrt[c - I*c*Tan[e + f*x]])/(c*f*(Cos[f*x] + I*Sin
[f*x])^2)
```

Maple [A]
time = 0.26, size = 45, normalized size = 0.78

method	result	size
derivativedivides	$-\frac{2ia^2\left(\sqrt{c-ic\tan(fx+e)}+\frac{2c}{\sqrt{c-ic\tan(fx+e)}}\right)}{fc}$	45
default	$-\frac{2ia^2\left(\sqrt{c-ic\tan(fx+e)}+\frac{2c}{\sqrt{c-ic\tan(fx+e)}}\right)}{fc}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/f*a^2/c*((c-I*c*tan(f*x+e))^(1/2)+2*c/(c-I*c*tan(f*x+e))^(1/2))
```

Maxima [A]
time = 0.30, size = 47, normalized size = 0.81

$$-\frac{2i\left(\sqrt{-i\,c\tan\left(fx+e\right)+c}\,a^2+\frac{2\,a^2c}{\sqrt{-i\,c\tan\left(fx+e\right)+c}}\right)}{cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -2*I*(sqrt(-I*c*tan(f*x + e) + c)*a^2 + 2*a^2*c/sqrt(-I*c*tan(f*x + e) + c)
)/(c*f)
```

Fricas [A]
time = 1.40, size = 50, normalized size = 0.86

$$-\frac{2\sqrt{2}\left(i\,a^2e^{(2i\,fx+2i\,e)}+2i\,a^2\right)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] -2*sqrt(2)*(I*a^2*e^(2*I*f*x + 2*I*e) + 2*I*a^2)*sqrt(c/(e^(2*I*f*x + 2*I*e
) + 1))/(c*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int\frac{\tan^2\left(e+fx\right)}{\sqrt{-ic\tan\left(e+fx\right)+c}}\,dx+\int\left(-\frac{2i\tan\left(e+fx\right)}{\sqrt{-ic\tan\left(e+fx\right)+c}}\right)dx+\int\left(-\frac{1}{\sqrt{-ic\tan\left(e+fx\right)+c}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2/(c-I*c*tan(f*x+e))**(1/2),x)
```

```
[Out] -a**2*(Integral(tan(e + f*x)**2/sqrt(-I*c*tan(e + f*x) + c), x) + Integral(
-2*I*tan(e + f*x)/sqrt(-I*c*tan(e + f*x) + c), x) + Integral(-1/sqrt(-I*c*t
an(e + f*x) + c), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^2/sqrt(-I*c*tan(f*x + e) + c), x)
```

Mupad [B]
time = 4.86, size = 77, normalized size = 1.33

$$-\frac{2\,a^2\,\sqrt{\frac{c\,\left(\cos\left(2\,e+2\,f\,x\right)+1-\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\left(\cos\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}-\sin\left(2\,e+2\,f\,x\right)+2{}\mathrm{i}\right)}{c\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^2/(c - c*tan(e + f*x)*1i)^(1/2),x)
```

```
[Out] -(2*a^2*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x)
 + 1))^(1/2)*(cos(2*e + 2*f*x)*1i - sin(2*e + 2*f*x) + 2i))/(c*f)
```

3.976 $\int \frac{a+ia\tan(e+fx)}{\sqrt{c-ic\tan(e+fx)}}\,dx$

Optimal. Leaf size=25

$$-\frac{2ia}{f\sqrt{c-ic\tan(e+fx)}}$$

```
[Out] -2*I*a/f/(c-I*c*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 32}

$$-\frac{2ia}{f\sqrt{c-ic\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])/Sqrt[c - I*c*Tan[e + f*x]],x]
```

```
[Out] ((-2*I)*a)/(f*Sqrt[c - I*c*Tan[e + f*x]])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{a+ia\tan(e+fx)}{\sqrt{c-ic\tan(e+fx)}}\,dx = (ac)\int \frac{\sec^2(e+fx)}{(c-ic\tan(e+fx))^{3/2}}\,dx$$
$$= \frac{(ia)\text{Subst}\left(\int \frac{1}{(c+x)^{3/2}}\,dx, x, -ic\tan(e+fx)\right)}{f}$$
$$= -\frac{2ia}{f\sqrt{c-ic\tan(e+fx)}}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 64 vs. $2(25) = 50$.
time = 0.48, size = 64, normalized size = 2.56

$$\frac{2a\cos(e+fx)(\cos(fx)-i\sin(fx))(-i\cos(e+2fx)+\sin(e+2fx))\sqrt{c-ic\tan(e+fx)}}{cf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])/Sqrt[c - I*c*Tan[e + f*x]],x]
```

```
[Out] (2*a*Cos[e + f*x]*(Cos[f*x] - I*Sin[f*x])*((-I)*Cos[e + 2*f*x] + Sin[e + 2*
f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(c*f)
```

Maple [A]
time = 0.20, size = 22, normalized size = 0.88

method	result	size
derivativedivides	$-\frac{2ia}{f\sqrt{c-ic\tan(fx+e)}}$	22
default	$-\frac{2ia}{f\sqrt{c-ic\tan(fx+e)}}$	22
risch	$-\frac{ia\sqrt{2}}{\sqrt{\frac{c}{e^{2i(fx+e)}+1}}\,f}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I*a/f/(c-I*c*tan(f*x+e))^(1/2)
```

Maxima [A]
time = 0.29, size = 20, normalized size = 0.80

$$-\frac{2i\,a}{\sqrt{-i\,c\tan(fx+e)+c}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] -2*I*a/(sqrt(-I*c*tan(f*x + e) + c)*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(20) = 40$.
time = 1.55, size = 45, normalized size = 1.80

$$\frac{\sqrt{2}\left(-i\,ae^{(2i\,fx+2i\,e)}-i\,a\right)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] sqrt(2)*(-I*a*e^(2*I*f*x + 2*I*e) - I*a)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/
(c*f)
```

Sympy [A]
time = 1.46, size = 44, normalized size = 1.76

$$\begin{cases} -\frac{2ia}{f\sqrt{-ic\tan{(e+fx)}+c}} & \text{for}\: f\neq 0 \\ \frac{x(ia\tan{(e)}+a)}{\sqrt{-ic\tan{(e)}+c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))**(1/2),x)
```

```
[Out] Piecewise((-2*I*a/(f*sqrt(-I*c*tan(e + f*x) + c)), Ne(f, 0)), (x*(I*a*tan(e
) + a)/sqrt(-I*c*tan(e) + c), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)/sqrt(-I*c*tan(f*x + e) + c), x)
```

Mupad [B]
time = 4.80, size = 65, normalized size = 2.60

$$\frac{a\left(\sin\left(2\,e+2\,f\,x\right)-{\cos\left(e+f\,x\right)}^2\,2{}\mathrm{i}\right)\,\sqrt{-\frac{c\,\left(-2\,{\cos\left(e+f\,x\right)}^2+\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{2\,{\cos\left(e+f\,x\right)}^2}}}{c\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)/(c - c*tan(e + f*x)*1i)^(1/2),x)
```

```
[Out] (a*(sin(2*e + 2*f*x) - cos(e + f*x)^2*2i)*(-(c*(sin(2*e + 2*f*x)*1i - 2*cos
(e + f*x)^2))/(2*cos(e + f*x)^2))^(1/2))/(c*f)
```

3.977 $\int \frac{1}{(a+ia\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}\,dx$

Optimal. Leaf size=124

$$\frac{3i\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}\,a\sqrt{c}\,f}-\frac{3i}{4af\sqrt{c-ic\tan(e+fx)}}+\frac{i}{2af(1+i\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}$$

```
[Out] 3/8*I*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))/a/f*2^(1/2)/c^(
1/2)-3/4*I/a/f/(c-I*c*tan(f*x+e))^(1/2)+1/2*I/a/f/(c-I*c*tan(f*x+e))^(1/2)/
(1+I*tan(f*x+e))
```

Rubi [A]
time = 0.13, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3603, 3568, 44, 53, 65, 212}

$$-\frac{3i}{4af\sqrt{c-ic\tan(e+fx)}}+\frac{i}{2af(1+i\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}+\frac{3i\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}\,a\sqrt{c}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]]),x]
```

```
[Out] (((3*I)/4)*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])])/(Sqrt[2]*
a*Sqrt[c]*f) - ((3*I)/4)/(a*f*Sqrt[c - I*c*Tan[e + f*x]]) + (I/2)/(a*f*(1 +
 I*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}\,dx = \frac{\int \cos^2(e+fx)\sqrt{c-ic\tan(e+fx)}\,dx}{ac}$$
$$= \frac{(ic^2)\operatorname{Subst}\left(\int \frac{1}{(c-x)^2(c+x)^{3/2}}\,dx, x, -ic\tan(e+fx)\right)}{af}$$
$$= \frac{i}{2af(1+i\tan(e+fx))\sqrt{c-ic\tan(e+fx)}} + \frac{(3ic)\mathrm{Subs}}{}$$
$$= -\frac{3i}{4af\sqrt{c-ic\tan(e+fx)}} + \frac{i}{2af(1+i\tan(e+fx))\sqrt{c}}$$
$$= -\frac{3i}{4af\sqrt{c-ic\tan(e+fx)}} + \frac{i}{2af(1+i\tan(e+fx))\sqrt{c}}$$
$$= \frac{3i\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}\,a\sqrt{c}\,f} - \frac{3i}{4af\sqrt{c-ic\tan(e}}$$

Mathematica [A]
time = 1.26, size = 117, normalized size = 0.94

$$-\frac{ie^{-2i(e+fx)}\left(-1+e^{2i(e+fx)}+2e^{4i(e+fx)}-3e^{2i(e+fx)}\sqrt{1+e^{2i(e+fx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(e+fx)}}\right)\right)\sqrt{c-ic\tan(e+fx)}}{8acf}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]]),x]
[Out] ((-1/8*I)*(-1 + E^((2*I)*(e + f*x)) + 2*E^((4*I)*(e + f*x)) - 3*E^((2*I)*(e
 + f*x))*Sqrt[1 + E^((2*I)*(e + f*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(e + f*x))
]])*Sqrt[c - I*c*Tan[e + f*x]])/(a*c*E^((2*I)*(e + f*x))*f)
```

Maple [A]
time = 0.27, size = 102, normalized size = 0.82

method	result

derivativedivides	$\frac{2ic^2\left(\frac{\frac{\sqrt{c-ic\tan(fx+e)}}{2c+2ic\tan(fx+e)}+\frac{3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)}{4\sqrt{c}}}{4c^2}-\frac{1}{4c^2\sqrt{c-ic\tan(}}\right.}{fa}$
default	$\frac{2ic^2\left(\frac{\frac{\sqrt{c-ic\tan(fx+e)}}{2c+2ic\tan(fx+e)}+\frac{3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)}{4\sqrt{c}}}{4c^2}-\frac{1}{4c^2\sqrt{c-ic\tan(}}\right.}{fa}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/f/a*c^2*(1/4/c^2*(1/4*(c-I*c*tan(f*x+e))^(1/2)/(1/2*c+1/2*I*c*tan(f*x+e
))+3/4*2^(1/2)/c^(1/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2)
))-1/4/c^2/(c-I*c*tan(f*x+e))^(1/2))
```

Maxima [A]
time = 0.50, size = 131, normalized size = 1.06

$$-\frac{i\left(\frac{3\sqrt{2}\sqrt{c}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-ic\tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic\tan(fx+e)+c}}\right)}{a}+\frac{4\left(3\left(-ic\tan(fx+e)+c\right)c-4c^2\right)}{\left(-ic\tan(fx+e)+c\right)^{\frac{3}{2}}a-2\sqrt{-ic\tan(fx+e)+c}\,ac}\right)}{16\,cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e)),x, algorithm="maxim
a")
```

```
[Out] -1/16*I*(3*sqrt(2)*sqrt(c)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*c*tan(f*x + e) +
 c))/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/a + 4*(3*(-I*c*tan(f*
x + e) + c)*c - 4*c^2)/((-I*c*tan(f*x + e) + c)^(3/2)*a - 2*sqrt(-I*c*tan(f
*x + e) + c)*a*c))/(c*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(95) = 190.
time = 1.66, size = 284, normalized size = 2.29

$$\frac{\left(-3i\sqrt{\frac{1}{2}}\,acf\sqrt{\frac{1}{a^2cf^2}}\,e^{(2ifx+2ie)}\log\left(-\frac{3\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(iafe^{(2ifx+2ie)}+iaf\right)\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{1}{a^2cf^2}}-i\right)e^{(-ifx-ie)}}{2af}\right)+3i\sqrt{\frac{1}{2}}\,acf\sqrt{\frac{1}{a^2cf^2}}\,e^{(2ifx+2ie)}\log\left(-\frac{3\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(-iafe^{(2ifx+2ie)}-iaf\right)\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{1}{a^2cf^2}}-i\right)e^{(-ifx-ie)}}{2af}\right)+\sqrt{2}\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}\left(-2ie^{(4ifx+4ie)}-ie^{(2ifx+2ie)}+i\right)\right)e^{(-2ifx-2ie)}}{8\,acf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e)),x, algorithm="frica
s")
```

```
[Out] 1/8*(-3*I*sqrt(1/2)*a*c*f*sqrt(1/(a^2*c*f^2))*e^(2*I*f*x + 2*I*e)*log(-3/2*
(sqrt(2)*sqrt(1/2)*(I*a*f*e^(2*I*f*x + 2*I*e) + I*a*f)*sqrt(c/(e^(2*I*f*x +
 2*I*e) + 1))*sqrt(1/(a^2*c*f^2)) - I)*e^(-I*f*x - I*e)/(a*f)) + 3*I*sqrt(1
/2)*a*c*f*sqrt(1/(a^2*c*f^2))*e^(2*I*f*x + 2*I*e)*log(-3/2*(sqrt(2)*sqrt(1/
2)*(-I*a*f*e^(2*I*f*x + 2*I*e) - I*a*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*s
qrt(1/(a^2*c*f^2)) - I)*e^(-I*f*x - I*e)/(a*f)) + sqrt(2)*sqrt(c/(e^(2*I*f*
x + 2*I*e) + 1))*(-2*I*e^(4*I*f*x + 4*I*e) - I*e^(2*I*f*x + 2*I*e) + I))*e^
(-2*I*f*x - 2*I*e)/(a*c*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int \frac{1}{\sqrt{-ic\tan{(e+fx)}+c}\tan{(e+fx)}-i\sqrt{-ic\tan{(e+fx)}+c}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-I*c*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] -I*Integral(1/(sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) - I*sqrt(-I*c*tan(e
 + f*x) + c)), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e)),x, algorithm="giac"
)
```

```
[Out] integrate(1/((I*a*tan(f*x + e) + a)*sqrt(-I*c*tan(f*x + e) + c)), x)
```

Mupad [B]
time = 5.03, size = 113, normalized size = 0.91

$$-\frac{\frac{c\,1\mathrm{i}}{a\,f}-\frac{(c-c\tan(e+f\,x)\,1\mathrm{i})\,3\mathrm{i}}{4\,a\,f}}{2\,c\,\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}-\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^{3/2}}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}}{2\,\sqrt{-c}}\right)\,3\mathrm{i}}{8\,a\,\sqrt{-c}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i)^(1/2)),x)
```

```
[Out] - ((c*1i)/(a*f) - ((c - c*tan(e + f*x)*1i)*3i)/(4*a*f))/(2*c*(c - c*tan(e +
 f*x)*1i)^(1/2) - (c - c*tan(e + f*x)*1i)^(3/2)) - (2^(1/2)*atan((2^(1/2)*(
c - c*tan(e + f*x)*1i)^(1/2))/(2*(-c)^(1/2)))*3i)/(8*a*(-c)^(1/2)*f)
```

3.978 $\int \frac{1}{(a+ia\tan(e+fx))^2\sqrt{c-ic\tan(e+fx)}}\,dx$

Optimal. Leaf size=167

$$\frac{15i\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{32\sqrt{2}\,a^2\sqrt{c}\,f}-\frac{15i}{32a^2f\sqrt{c-ic\tan(e+fx)}}+\frac{i}{4a^2f(1+i\tan(e+fx))^2\sqrt{c-ic\tan(e+fx)}}$$

```
[Out] 15/64*I*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))/a^2/f*2^(1/2)
/c^(1/2)-15/32*I/a^2/f/(c-I*c*tan(f*x+e))^(1/2)+1/4*I/a^2/f/(c-I*c*tan(f*x+
e))^(1/2)/(1+I*tan(f*x+e))^2+5/16*I/a^2/f/(c-I*c*tan(f*x+e))^(1/2)/(1+I*tan
(f*x+e))
```

Rubi [A]

time = 0.14, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3603, 3568, 44, 53, 65, 212}

$$-\frac{15i}{32a^2f\sqrt{c-ic\tan(e+fx)}}+\frac{5i}{16a^2f(1+i\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}+\frac{i}{4a^2f(1+i\tan(e+fx))^2\sqrt{c-ic\tan(e+fx)}}+\frac{15i\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{32\sqrt{2}\,a^2\sqrt{c}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^2*Sqrt[c - I*c*Tan[e + f*x]]),x]
```

```
[Out] (((15*I)/32)*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])])/(Sqrt[2
]*a^2*Sqrt[c]*f) - ((15*I)/32)/(a^2*f*Sqrt[c - I*c*Tan[e + f*x]]) + (I/4)/(
a^2*f*(1 + I*Tan[e + f*x])^2*Sqrt[c - I*c*Tan[e + f*x]]) + ((5*I)/16)/(a^2*
f*(1 + I*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^2\sqrt{c-ic\tan(e+fx)}}\,dx = \frac{\int \cos^4(e+fx)(c-ic\tan(e+fx))^{3/2}\,dx}{a^2c^2}$$

$$= \frac{(ic^3)\,\text{Subst}\left(\int \frac{1}{(c-x)^3(c+x)^{3/2}}\,dx, x, -ic\tan(e+fx)\right)}{a^2f}$$

$$= \frac{i}{4a^2f(1+i\tan(e+fx))^2\sqrt{c-ic\tan(e+fx)}} + \frac{(5ic^2}{}$$

$$= \frac{i}{4a^2f(1+i\tan(e+fx))^2\sqrt{c-ic\tan(e+fx)}} + \frac{}{16a^2}$$

$$= -\frac{15i}{32a^2f\sqrt{c-ic\tan(e+fx)}} + \frac{}{4a^2f(1+i\tan(e+fx}$$

$$= -\frac{15i}{32a^2f\sqrt{c-ic\tan(e+fx)}} + \frac{}{4a^2f(1+i\tan(e+fx}$$

$$= \frac{15i\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{32\sqrt{2}\,a^2\sqrt{c}\,f} - \frac{15}{32a^2f\sqrt{c-ic}}$$

Mathematica [A]
time = 1.66, size = 141, normalized size = 0.84

$$-\frac{ie^{-4i(e+fx)}\sqrt{\frac{c}{1+e^{2i(e+fx)}}}\left(-2-11e^{2i(e+fx)}-e^{4i(e+fx)}+8e^{6i(e+fx)}-15e^{4i(e+fx)}\sqrt{1+e^{2i(e+fx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(e+fx)}}\right)\right)}{32\sqrt{2}\,a^2cf}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^2*Sqrt[c - I*c*Tan[e + f*x]]),x]
```

```
[Out] ((-1/32*I)*Sqrt[c/(1 + E^((2*I)*(e + f*x)))]*(-2 - 11*E^((2*I)*(e + f*x)) -
 E^((4*I)*(e + f*x)) + 8*E^((6*I)*(e + f*x)) - 15*E^((4*I)*(e + f*x))*Sqrt[
1 + E^((2*I)*(e + f*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(e + f*x))]]))/(Sqrt[2]*
a^2*c*E^((4*I)*(e + f*x))*f)
```

Maple [A]
time = 0.34, size = 120, normalized size = 0.72

method	result

derivativedivides	$-\frac{2ic^3\left(\frac{1}{8c^3\sqrt{c-ic\tan(fx+e)}}-\frac{\frac{-\frac{7(c-ic\tan(fx+e))^{\frac{3}{2}}}{8}+\frac{9c\sqrt{c-ic\tan(fx+e)}}{4}}{(c+ic\tan(fx+e))^2}+\frac{15\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{}}{}\right.}{}}{8c^3}\right.}{f\,a^2}$
default	$-\frac{2ic^3\left(\frac{1}{8c^3\sqrt{c-ic\tan(fx+e)}}-\frac{\frac{-\frac{7(c-ic\tan(fx+e))^{\frac{3}{2}}}{8}+\frac{9c\sqrt{c-ic\tan(fx+e)}}{4}}{(c+ic\tan(fx+e))^2}+\frac{15\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{}}{}\right.}{}}{8c^3}\right.}{f\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE
)
```

```
[Out] -2*I/f/a^2*c^3*(1/8/c^3/(c-I*c*tan(f*x+e))^(1/2)-1/8/c^3*(4*(-7/32*(c-I*c*t
an(f*x+e))^(3/2)+9/16*c*(c-I*c*tan(f*x+e))^(1/2))/(c+I*c*tan(f*x+e))^2+15/1
6*2^(1/2)/c^(1/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))))
```

Maxima [A]
time = 0.52, size = 176, normalized size = 1.05

$$-\frac{i\left(\frac{4\left(15\left(-ic\tan(fx+e)+c\right)^2c-50\left(-ic\tan(fx+e)+c\right)c^2+32c^3\right)}{\left(-ic\tan(fx+e)+c\right)^{\frac{5}{2}}a^2-4\left(-ic\tan(fx+e)+c\right)^{\frac{3}{2}}a^2c+4\sqrt{-ic\tan(fx+e)+c}\,a^2c^2}+\frac{15\sqrt{2}\sqrt{c}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-ic\tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic\tan(fx+e)+c}}\right)}{a^2}\right)}{128\,cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="max
ima")
```

```
[Out] -1/128*I*(4*(15*(-I*c*tan(f*x + e) + c)^2*c - 50*(-I*c*tan(f*x + e) + c)*c^
2 + 32*c^3)/((-I*c*tan(f*x + e) + c)^(5/2)*a^2 - 4*(-I*c*tan(f*x + e) + c)^
(3/2)*a^2*c + 4*sqrt(-I*c*tan(f*x + e) + c)*a^2*c^2) + 15*sqrt(2)*sqrt(c)*l
og(-(sqrt(2)*sqrt(c) - sqrt(-I*c*tan(f*x + e) + c))/(sqrt(2)*sqrt(c) + sqrt
(-I*c*tan(f*x + e) + c)))/a^2)/(c*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(130) = 260$.
time = 1.26, size = 308, normalized size = 1.84

$$\frac{\left(-15i\sqrt{\frac{1}{2}}\,a^2cf\sqrt{\frac{1}{a^4cf^2}}\,e^{(4ifx+4ie)}\log\left(-\frac{15\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(ia^2fe^{(2ifx+2ie)}+ia^2f\right)\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{1}{a^4cf^2}}-i\right)e^{(-ifx-ie)}}{16a^2f}\right)+15i\sqrt{\frac{1}{2}}\,a^2cf\sqrt{\frac{1}{a^4cf^2}}\,e^{(4ifx+4ie)}\log\left(-\frac{15\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(-ia^2fe^{(2ifx+2ie)}-ia^2f\right)\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{1}{a^4cf^2}}-i\right)e^{(-ifx-ie)}}{16a^2f}\right)+\sqrt{2}\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}\left(-8ie^{(6ifx+6ie)}+ie^{(4ifx+4ie)}+11ie^{(2ifx+2ie)}+2i\right)\right)e^{(-4ifx-4ie)}}{64\,a^2cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="fri
cas")
```

```
[Out] 1/64*(-15*I*sqrt(1/2)*a^2*c*f*sqrt(1/(a^4*c*f^2))*e^(4*I*f*x + 4*I*e)*log(-
15/16*(sqrt(2)*sqrt(1/2)*(I*a^2*f*e^(2*I*f*x + 2*I*e) + I*a^2*f)*sqrt(c/(e^
(2*I*f*x + 2*I*e) + 1))*sqrt(1/(a^4*c*f^2)) - I)*e^(-I*f*x - I*e)/(a^2*f))
+ 15*I*sqrt(1/2)*a^2*c*f*sqrt(1/(a^4*c*f^2))*e^(4*I*f*x + 4*I*e)*log(-15/16
*(sqrt(2)*sqrt(1/2)*(-I*a^2*f*e^(2*I*f*x + 2*I*e) - I*a^2*f)*sqrt(c/(e^(2*I
*f*x + 2*I*e) + 1))*sqrt(1/(a^4*c*f^2)) - I)*e^(-I*f*x - I*e)/(a^2*f)) + sq
rt(2)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*(-8*I*e^(6*I*f*x + 6*I*e) + I*e^(4*
I*f*x + 4*I*e) + 11*I*e^(2*I*f*x + 2*I*e) + 2*I))*e^(-4*I*f*x - 4*I*e)/(a^2
*c*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{\sqrt{-ic\tan\left(e+fx\right)+c}\tan^{2}\left(e+fx\right)-2i\sqrt{-ic\tan\left(e+fx\right)+c}\tan\left(e+fx\right)-\sqrt{-ic\tan\left(e+fx\right)+c}}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-I*c*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -Integral(1/(sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 2*I*sqrt(-I*c*ta
n(e + f*x) + c)*tan(e + f*x) - sqrt(-I*c*tan(e + f*x) + c)), x)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="gia
c")
```

```
[Out] integrate(1/((I*a*tan(f*x + e) + a)^2*sqrt(-I*c*tan(f*x + e) + c)), x)
```

Mupad [B]
time = 5.00, size = 156, normalized size = 0.93

$$-\frac{\frac{{\left(c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^2\,15{}\mathrm{i}}{32\,a^2\,f}+\frac{c^2\,1{}\mathrm{i}}{a^2\,f}-\frac{c\,\left(c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)\,25{}\mathrm{i}}{16\,a^2\,f}}{-4\,c\,{\left(c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{3/2}+{\left(c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{5/2}+4\,c^2\,\sqrt{c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}}}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}}}{2\,\sqrt{-c}}\right)\,15{}\mathrm{i}}{64\,a^2\,\sqrt{-c}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i)^(1/2)),x)
```

```
[Out] - ((((c - c*tan(e + f*x)*1i)^2*15i)/(32*a^2*f) + (c^2*1i)/(a^2*f) - (c*(c -
c*tan(e + f*x)*1i)*25i)/(16*a^2*f))/((c - c*tan(e + f*x)*1i)^(5/2) - 4*c*(c
 - c*tan(e + f*x)*1i)^(3/2) + 4*c^2*(c - c*tan(e + f*x)*1i)^(1/2)) - (2^(1/
2)*atan((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*(-c)^(1/2)))*15i)/(64*a^
2*(-c)^(1/2)*f)
```

3.979 $\int \frac{1}{(a+ia\tan(e+fx))^3\sqrt{c-ic\tan(e+fx)}}\,dx$

Optimal. Leaf size=210

$$\frac{35i\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{128\sqrt{2}\,a^3\sqrt{c}\,f}-\frac{35i}{128a^3f\sqrt{c-ic\tan(e+fx)}}+\frac{i}{6a^3f(1+i\tan(e+fx))^3\sqrt{c-ic\tan}}$$

```
[Out] 35/256*I*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))/a^3/f*2^(1/2
)/c^(1/2)-35/128*I/a^3/f/(c-I*c*tan(f*x+e))^(1/2)+1/6*I/a^3/f/(c-I*c*tan(f*
x+e))^(1/2)/(1+I*tan(f*x+e))^3+7/48*I/a^3/f/(c-I*c*tan(f*x+e))^(1/2)/(1+I*t
an(f*x+e))^2+35/192*I/a^3/f/(c-I*c*tan(f*x+e))^(1/2)/(1+I*tan(f*x+e))
```

Rubi [A]
time = 0.15, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3603, 3568, 44, 53, 65, 212}

$$-\frac{35i}{128a^3f\sqrt{c-ic\tan(e+fx)}}+\frac{35i}{192a^3f(1+i\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}+\frac{7i}{48a^3f(1+i\tan(e+fx))^2\sqrt{c-ic\tan(e+fx)}}+\frac{i}{6a^3f(1+i\tan(e+fx))^3\sqrt{c-ic\tan(e+fx)}}+\frac{35i\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{128\sqrt{2}\,a^3\sqrt{c}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^3*Sqrt[c - I*c*Tan[e + f*x]]),x]
```

```
[Out] (((35*I)/128)*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])])/(Sqrt[
2]*a^3*Sqrt[c]*f) - ((35*I)/128)/(a^3*f*Sqrt[c - I*c*Tan[e + f*x]]) + (I/6)
/(a^3*f*(1 + I*Tan[e + f*x])^3*Sqrt[c - I*c*Tan[e + f*x]]) + ((7*I)/48)/(a^
3*f*(1 + I*Tan[e + f*x])^2*Sqrt[c - I*c*Tan[e + f*x]]) + ((35*I)/192)/(a^3*
f*(1 + I*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+ia\tan(e+fx))^3\sqrt{c-ic\tan(e+fx)}}\,dx &= \frac{\int \cos^6(e+fx)(c-ic\tan(e+fx))^{5/2}\,dx}{a^3c^3} \\
&= \frac{(ic^4)\,\text{Subst}\left(\int \frac{1}{(c-x)^4(c+x)^{3/2}}\,dx, x, -ic\tan(e+fx)\right)}{a^3 f} \\
&= \frac{i}{6a^3f(1+i\tan(e+fx))^3\sqrt{c-ic\tan(e+fx)}} + \frac{(7ic^3 \\
&= \frac{i}{6a^3f(1+i\tan(e+fx))^3\sqrt{c-ic\tan(e+fx)}} + \frac{}{48a^3 \\
&= \frac{i}{6a^3f(1+i\tan(e+fx))^3\sqrt{c-ic\tan(e+fx)}} + \frac{}{48a^3 \\
&= -\frac{35i}{128a^3f\sqrt{c-ic\tan(e+fx)}} + \frac{}{6a^3f(1+i\tan(e+f \\
&= -\frac{35i}{128a^3f\sqrt{c-ic\tan(e+fx)}} + \frac{}{6a^3f(1+i\tan(e+f \\
&= \frac{35i\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{128\sqrt{2}\,a^3\sqrt{c}\,f} - \frac{35}{128a^3f\sqrt{c-ic}
\end{aligned}
$$

Mathematica [A]
time = 2.07, size = 146, normalized size = 0.70

$$
-\frac{i\sec^2(e+fx)\left(125+105e^{2i(e+fx)}\sqrt{1+e^{2i(e+fx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(e+fx)}}\right)+85\cos(2(e+fx))-40\cos(4(e+fx))-7i\sin(2(e+fx))-56i\sin(4(e+fx))\right)\sqrt{c-ic\tan(e+fx)}}{768a^3cf(-i+\tan(e+fx))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^3*Sqrt[c - I*c*Tan[e + f*x]]),x]
```

```
[Out] ((-1/768*I)*Sec[e + f*x]^2*(125 + 105*E^((2*I)*(e + f*x))*Sqrt[1 + E^((2*I)
*(e + f*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(e + f*x))]] + 85*Cos[2*(e + f*x)] -
 40*Cos[4*(e + f*x)] - (7*I)*Sin[2*(e + f*x)] - (56*I)*Sin[4*(e + f*x)])*Sq
rt[c - I*c*Tan[e + f*x]])/(a^3*c*f*(-I + Tan[e + f*x])^2)
```

Maple [A]
time = 0.38, size = 139, normalized size = 0.66

method	result
derivativedivides	$\frac{2ic^4\left(-\frac{1}{16c^4\sqrt{c-ic\tan(fx+e)}}+\frac{\frac{\frac{19(c-ic\tan(fx+e))^{\frac{5}{2}}}{16}-\frac{17c(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}+\frac{29c^2\sqrt{c-ic\tan(fx+}}{4}}{(c+ic\tan(fx+e))^3}}{16c^4}\right.}{f\,a^3}$
default	$\frac{2ic^4\left(-\frac{1}{16c^4\sqrt{c-ic\tan(fx+e)}}+\frac{\frac{\frac{19(c-ic\tan(fx+e))^{\frac{5}{2}}}{16}-\frac{17c(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}+\frac{29c^2\sqrt{c-ic\tan(fx+}}{4}}{(c+ic\tan(fx+e))^3}}{16c^4}\right.}{f\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE
)
```

```
[Out] 2*I/f/a^3*c^4*(-1/16/c^4/(c-I*c*tan(f*x+e))^(1/2)+1/16/c^4*(8*(19/128*(c-I*
c*tan(f*x+e))^(5/2)-17/24*c*(c-I*c*tan(f*x+e))^(3/2)+29/32*c^2*(c-I*c*tan(f
*x+e))^(1/2))/(c+I*c*tan(f*x+e))^3+35/32*2^(1/2)/c^(1/2)*arctanh(1/2*(c-I*c
*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))))
```

Maxima [A]

time = 0.52, size = 217, normalized size = 1.03

$$-\frac{i\left(\frac{4\left(105(-ic\tan(fx+e)+c)^3c-560(-ic\tan(fx+e)+c)^2c^2+924(-ic\tan(fx+e)+c)c^3-384c^4\right)}{(-ic\tan(fx+e)+c)^{\frac{7}{2}}a^3-6(-ic\tan(fx+e)+c)^{\frac{5}{2}}a^3c+12(-ic\tan(fx+e)+c)^{\frac{3}{2}}a^3c^2-8\sqrt{-ic\tan(fx+e)+c}\,a^3c^3}+\frac{105\sqrt{2}\sqrt{c}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-ic\tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic\tan(fx+e)+c}}\right)}{a^3}\right)}{1536\,cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="max
ima")
```

```
[Out] -1/1536*I*(4*(105*(-I*c*tan(f*x + e) + c)^3*c - 560*(-I*c*tan(f*x + e) + c)
^2*c^2 + 924*(-I*c*tan(f*x + e) + c)*c^3 - 384*c^4)/((-I*c*tan(f*x + e) + c
)^(7/2)*a^3 - 6*(-I*c*tan(f*x + e) + c)^(5/2)*a^3*c + 12*(-I*c*tan(f*x + e)
 + c)^(3/2)*a^3*c^2 - 8*sqrt(-I*c*tan(f*x + e) + c)*a^3*c^3) + 105*sqrt(2)*
sqrt(c)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*c*tan(f*x + e) + c))/(sqrt(2)*sqrt(
c) + sqrt(-I*c*tan(f*x + e) + c)))/a^3)/(c*f)
```

Fricas [A]

time = 1.61, size = 320, normalized size = 1.52

$$\frac{\left(-105i\sqrt{\frac{1}{2}}\,a^3cf\sqrt{\frac{1}{a^6cf^2}}\,e^{(6i\,fx+6i\,e)}\log\left(-\frac{35\left(\sqrt{2}\sqrt{\frac{1}{2}}\,(i\,a^3fe^{(2i\,fx+2i\,e)}+i\,a^3f)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{1}{a^6cf^2}}-i\right)e^{(-i\,fx-i\,e)}}{64\,a^3f}\right)+105i\sqrt{\frac{1}{2}}\,a^3cf\sqrt{\frac{1}{a^6cf^2}}\,e^{(6i\,fx+6i\,e)}\log\left(-\frac{35\left(\sqrt{2}\sqrt{\frac{1}{2}}\,(-i\,a^3fe^{(2i\,fx+2i\,e)}-i\,a^3f)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{1}{a^6cf^2}}-i\right)e^{(-i\,fx-i\,e)}}{64\,a^3f}\right)+\sqrt{2}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\,(-48i\,e^{(8i\,fx+8i\,e)}+39i\,e^{(6i\,fx+6i\,e)}+125i\,e^{(4i\,fx+4i\,e)}+46i\,e^{(2i\,fx+2i\,e)}+8i)\right)e^{(-6i\,fx-6i\,e)}}{768\,a^3cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="fri
cas")
```

```
[Out] 1/768*(-105*I*sqrt(1/2)*a^3*c*f*sqrt(1/(a^6*c*f^2))*e^(6*I*f*x + 6*I*e)*log
(-35/64*(sqrt(2)*sqrt(1/2)*(I*a^3*f*e^(2*I*f*x + 2*I*e) + I*a^3*f)*sqrt(c/(
e^(2*I*f*x + 2*I*e) + 1))*sqrt(1/(a^6*c*f^2)) - I)*e^(-I*f*x - I*e)/(a^3*f)
) + 105*I*sqrt(1/2)*a^3*c*f*sqrt(1/(a^6*c*f^2))*e^(6*I*f*x + 6*I*e)*log(-35
/64*(sqrt(2)*sqrt(1/2)*(-I*a^3*f*e^(2*I*f*x + 2*I*e) - I*a^3*f)*sqrt(c/(e^(
2*I*f*x + 2*I*e) + 1))*sqrt(1/(a^6*c*f^2)) - I)*e^(-I*f*x - I*e)/(a^3*f)) +
 sqrt(2)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*(-48*I*e^(8*I*f*x + 8*I*e) + 39*
I*e^(6*I*f*x + 6*I*e) + 125*I*e^(4*I*f*x + 4*I*e) + 46*I*e^(2*I*f*x + 2*I*e
) + 8*I))*e^(-6*I*f*x - 6*I*e)/(a^3*c*f)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int\frac{1}{\sqrt{-ic\tan{(e+fx)}+c}\tan^3{(e+fx)}-3i\sqrt{-ic\tan{(e+fx)}+c}\tan^2{(e+fx)}-3\sqrt{-ic\tan{(e+fx)}+c}\tan{(e+fx)}+i\sqrt{-ic\tan{(e+fx)}+c}}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-I*c*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] I*Integral(1/(sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**3 - 3*I*sqrt(-I*c*t
an(e + f*x) + c)*tan(e + f*x)**2 - 3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*
x) + I*sqrt(-I*c*tan(e + f*x) + c)), x)/a**3
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="gia
c")
```

```
[Out] integrate(1/((I*a*tan(f*x + e) + a)^3*sqrt(-I*c*tan(f*x + e) + c)), x)
```

Mupad [B]

time = 5.01, size = 202, normalized size = 0.96

$$\frac{\frac{{\left(c-c\tan\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^3\,35{}\mathrm{i}}{128\,a^3\,f}-\frac{c^3\,1{}\mathrm{i}}{a^3\,f}-\frac{c\,{\left(c-c\tan\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^2\,35{}\mathrm{i}}{24\,a^3\,f}+\frac{c^2\,\left(c-c\tan\left(e+f\,x\right)\,1{}\mathrm{i}\right)\,77{}\mathrm{i}}{32\,a^3\,f}}{6\,c\,{\left(c-c\tan\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{5/2}-{\left(c-c\tan\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{7/2}+8\,c^3\,\sqrt{c-c\tan\left(e+f\,x\right)\,1{}\mathrm{i}}-12\,c^2\,{\left(c-c\tan\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan\left(e+f\,x\right)\,1{}\mathrm{i}}}{2\,\sqrt{-c}}\right)\,35{}\mathrm{i}}{256\,a^3\,\sqrt{-c}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i)^(1/2)),x)
```

```
[Out] (((c - c*tan(e + f*x)*1i)^3*35i)/(128*a^3*f) - (c^3*1i)/(a^3*f) - (c*(c - c
*tan(e + f*x)*1i)^2*35i)/(24*a^3*f) + (c^2*(c - c*tan(e + f*x)*1i)*77i)/(32
*a^3*f))/(6*c*(c - c*tan(e + f*x)*1i)^(5/2) - (c - c*tan(e + f*x)*1i)^(7/2)
 + 8*c^3*(c - c*tan(e + f*x)*1i)^(1/2) - 12*c^2*(c - c*tan(e + f*x)*1i)^(3/
2)) - (2^(1/2)*atan((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*(-c)^(1/2)))
*35i)/(256*a^3*(-c)^(1/2)*f)
```

3.980 $\int \frac{(a+ia\tan(e+fx))^3}{(c-ic\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=90

$$-\frac{8ia^3}{3f(c-ic\tan(e+fx))^{3/2}}+\frac{8ia^3}{cf\sqrt{c-ic\tan(e+fx)}}+\frac{2ia^3\sqrt{c-ic\tan(e+fx)}}{c^2f}$$

```
[Out] 8*I*a^3/c/f/(c-I*c*tan(f*x+e))^(1/2)+2*I*a^3*(c-I*c*tan(f*x+e))^(1/2)/c^2/f
-8/3*I*a^3/f/(c-I*c*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.12, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {3603, 3568, 45}

$$\frac{2ia^3\sqrt{c-ic\tan(e+fx)}}{c^2f}+\frac{8ia^3}{cf\sqrt{c-ic\tan(e+fx)}}-\frac{8ia^3}{3f(c-ic\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^3/(c - I*c*Tan[e + f*x])^(3/2),x]
```

```
[Out] (((-8*I)/3)*a^3)/(f*(c - I*c*Tan[e + f*x])^(3/2)) + ((8*I)*a^3)/(c*f*Sqrt[c
 - I*c*Tan[e + f*x]]) + ((2*I)*a^3*Sqrt[c - I*c*Tan[e + f*x]])/(c^2*f)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^3}{(c-ic\tan(e+fx))^{3/2}}\,dx = (a^3c^3)\int \frac{\sec^6(e+fx)}{(c-ic\tan(e+fx))^{9/2}}\,dx$$
$$= \frac{(ia^3)\,\mathrm{Subst}\left(\int \frac{(c-x)^2}{(c+x)^{5/2}}\,dx, x, -ic\tan(e+fx)\right)}{c^2 f}$$
$$= \frac{(ia^3)\,\mathrm{Subst}\left(\int \left(\frac{4c^2}{(c+x)^{5/2}} - \frac{4c}{(c+x)^{3/2}} + \frac{1}{\sqrt{c+x}}\right)dx, x, -ic\tan(e+fx)\right)}{c^2 f}$$
$$= -\frac{8ia^3}{3f(c-ic\tan(e+fx))^{3/2}} + \frac{8ia^3}{cf\sqrt{c-ic\tan(e+fx)}} + \frac{2ia^3\sqrt{c-ic\tan(e}{c^2 f}$$

Mathematica [A]
time = 2.01, size = 94, normalized size = 1.04

$$\frac{2a^3(4i+7i\cos(2(e+fx))+9\sin(2(e+fx)))(\cos(2e+5fx)+i\sin(2e+5fx))\sqrt{c-ic\tan(e+fx)}}{3c^2f(\cos(fx)+i\sin(fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3/(c - I*c*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*a^3*(4*I + (7*I)*Cos[2*(e + f*x)] + 9*Sin[2*(e + f*x)])*(Cos[2*e + 5*f*x
] + I*Sin[2*e + 5*f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(3*c^2*f*(Cos[f*x] + I*
Sin[f*x])^3)
```

Maple [A]
time = 0.26, size = 64, normalized size = 0.71

method	result	size
derivativedivides	$\frac{2ia^3\left(\sqrt{c-ic\tan(fx+e)}+\frac{4c}{\sqrt{c-ic\tan(fx+e)}}-\frac{4c^2}{3(c-ic\tan(fx+e))^{\frac{3}{2}}}\right)}{f\,c^2}$	64
default	$\frac{2ia^3\left(\sqrt{c-ic\tan(fx+e)}+\frac{4c}{\sqrt{c-ic\tan(fx+e)}}-\frac{4c^2}{3(c-ic\tan(fx+e))^{\frac{3}{2}}}\right)}{f\,c^2}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/f*a^3/c^2*((c-I*c*tan(f*x+e))^(1/2)+4*c/(c-I*c*tan(f*x+e))^(1/2)-4/3*c^
2/(c-I*c*tan(f*x+e))^(3/2))
```

Maxima [A]
time = 0.28, size = 71, normalized size = 0.79

$$\frac{2i\left(\frac{3\sqrt{-ic\tan\left(fx+e\right)+c}\,a^3}{c}+\frac{4\left(3\left(-ic\tan(fx+e)+c\right)a^3-a^3c\right)}{\left(-ic\tan(fx+e)+c\right)^{\frac{3}{2}}}\right)}{3\,cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] 2/3*I*(3*sqrt(-I*c*tan(f*x + e) + c)*a^3/c + 4*(3*(-I*c*tan(f*x + e) + c)*a
^3 - a^3*c)/(-I*c*tan(f*x + e) + c)^(3/2))/(c*f)
```

Fricas [A]
time = 1.17, size = 65, normalized size = 0.72

$$-\frac{2\sqrt{2}\left(i\,a^3e^{(4i\,fx+4i\,e)}-4i\,a^3e^{(2i\,fx+2i\,e)}-8i\,a^3\right)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{3\,c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] -2/3*sqrt(2)*(I*a^3*e^(4*I*f*x + 4*I*e) - 4*I*a^3*e^(2*I*f*x + 2*I*e) - 8*I
*a^3)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(c^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia^3\left(\int\frac{i}{-ic\sqrt{-ic\tan\left(e+fx\right)+c}\,\tan\left(e+fx\right)+c\sqrt{-ic\tan\left(e+fx\right)+c}}\,dx+\int\left(-\frac{3\tan\left(e+fx\right)}{-ic\sqrt{-ic\tan\left(e+fx\right)+c}\,\tan\left(e+fx\right)+c\sqrt{-ic\tan\left(e+fx\right)+c}}\right)dx+\int\frac{\tan^3\left(e+fx\right)}{-ic\sqrt{-ic\tan\left(e+fx\right)+c}\,\tan\left(e+fx\right)+c\sqrt{-ic\tan\left(e+fx\right)+c}}\,dx+\int\left(-\frac{3i\tan^2\left(e+fx\right)}{-ic\sqrt{-ic\tan\left(e+fx\right)+c}\,\tan\left(e+fx\right)+c\sqrt{-ic\tan\left(e+fx\right)+c}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3/(c-I*c*tan(f*x+e))**(3/2),x)
```

```
[Out] -I*a**3*(Integral(I/(-I*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c*sqrt
(-I*c*tan(e + f*x) + c)), x) + Integral(-3*tan(e + f*x)/(-I*c*sqrt(-I*c*tan
(e + f*x) + c)*tan(e + f*x) + c*sqrt(-I*c*tan(e + f*x) + c)), x) + Integral
(tan(e + f*x)**3/(-I*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c*sqrt(-I
*c*tan(e + f*x) + c)), x) + Integral(-3*I*tan(e + f*x)**2/(-I*c*sqrt(-I*c*t
an(e + f*x) + c)*tan(e + f*x) + c*sqrt(-I*c*tan(e + f*x) + c)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^3/(-I*c*tan(f*x + e) + c)^(3/2), x)
```

Mupad [B]
time = 5.15, size = 98, normalized size = 1.09

$$\frac{2\,a^3\,\sqrt{\frac{c\,\left(\cos\left(2\,e+2\,f\,x\right)+1-\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\left(\cos\left(2\,e+2\,f\,x\right)\,4{}\mathrm{i}-\cos\left(4\,e+4\,f\,x\right)\,1{}\mathrm{i}-4\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(4\,e+4\,f\,x\right)+8{}\mathrm{i}\right)}{3\,c^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^3/(c - c*tan(e + f*x)*1i)^(3/2),x)
```

```
[Out] (2*a^3*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x)
+ 1))^(1/2)*(cos(2*e + 2*f*x)*4i - cos(4*e + 4*f*x)*1i - 4*sin(2*e + 2*f*x)
 + sin(4*e + 4*f*x) + 8i))/(3*c^2*f)
```

3.981 $\int \frac{(a+ia\tan(e+fx))^2}{(c-ic\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=60

$$-\frac{4ia^2}{3f(c-ic\tan(e+fx))^{3/2}}+\frac{2ia^2}{cf\sqrt{c-ic\tan(e+fx)}}$$

```
[Out] 2*I*a^2/c/f/(c-I*c*tan(f*x+e))^(1/2)-4/3*I*a^2/f/(c-I*c*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.11, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3603, 3568, 45}

$$\frac{2ia^2}{cf\sqrt{c-ic\tan(e+fx)}}-\frac{4ia^2}{3f(c-ic\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2/(c - I*c*Tan[e + f*x])^(3/2),x]
```

```
[Out] (((-4*I)/3)*a^2)/(f*(c - I*c*Tan[e + f*x])^(3/2)) + ((2*I)*a^2)/(c*f*Sqrt[c
 - I*c*Tan[e + f*x]])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\begin{aligned}\int \frac{(a+ia\tan(e+fx))^2}{(c-ic\tan(e+fx))^{3/2}}\,dx &= \left(a^2c^2\right)\int \frac{\sec^4(e+fx)}{(c-ic\tan(e+fx))^{7/2}}\,dx\\ &= \frac{\left(ia^2\right)\text{Subst}\left(\int \frac{c-x}{(c+x)^{5/2}}\,dx, x, -ic\tan(e+fx)\right)}{cf}\\ &= \frac{\left(ia^2\right)\text{Subst}\left(\int \left(\frac{2c}{(c+x)^{5/2}}-\frac{1}{(c+x)^{3/2}}\right)dx, x, -ic\tan(e+fx)\right)}{cf}\\ &= -\frac{4ia^2}{3f(c-ic\tan(e+fx))^{3/2}}+\frac{2ia^2}{cf\sqrt{c-ic\tan(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 1.40, size = 93, normalized size = 1.55

$$\frac{2a^2\cos(e+fx)(i\cos(e+fx)+3\sin(e+fx))(\cos(2(e+2fx))+i\sin(2(e+2fx)))\sqrt{c-ic\tan(e+fx)}}{3c^2f(\cos(fx)+i\sin(fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2/(c - I*c*Tan[e + f*x])^(3/2),x]
[Out] (2*a^2*Cos[e + f*x]*(I*Cos[e + f*x] + 3*Sin[e + f*x])*(Cos[2*(e + 2*f*x)] +
 I*Sin[2*(e + 2*f*x)])*Sqrt[c - I*c*Tan[e + f*x]])/(3*c^2*f*(Cos[f*x] + I*S
in[f*x])^2)
```

Maple [A]
time = 0.24, size = 47, normalized size = 0.78

method	result	size
risch	$-\frac{i a^2\left(e^{2i(fx+e)}-2\right)\sqrt{2}}{3c\sqrt{\frac{c}{e^{2i(fx+e)}+1}}\,f}$	44
derivativedivides	$-\frac{2ia^2\left(\frac{2c}{3(c-ic\tan(fx+e))^{\frac{3}{2}}}-\frac{1}{\sqrt{c-ic\tan(fx+e)}}\right)}{fc}$	47
default	$-\frac{2ia^2\left(\frac{2c}{3(c-ic\tan(fx+e))^{\frac{3}{2}}}-\frac{1}{\sqrt{c-ic\tan(fx+e)}}\right)}{fc}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/f*a^2/c*(2/3*c/(c-I*c*tan(f*x+e))^(3/2)-1/(c-I*c*tan(f*x+e))^(1/2))
```

Maxima [A]
time = 0.28, size = 46, normalized size = 0.77

$$\frac{2i\left(3\left(-i\,c\tan\left(fx+e\right)+c\right)a^2-2\,a^2c\right)}{3\left(-i\,c\tan\left(fx+e\right)+c\right)^{\frac{3}{2}}cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] 2/3*I*(3*(-I*c*tan(f*x + e) + c)*a^2 - 2*a^2*c)/((-I*c*tan(f*x + e) + c)^(3
/2)*c*f)
```

Fricas [A]
time = 1.09, size = 65, normalized size = 1.08

$$\frac{\sqrt{2}\left(-i\,a^2e^{(4i\,fx+4i\,e)}+i\,a^2e^{(2i\,fx+2i\,e)}+2i\,a^2\right)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{3\,c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/3*sqrt(2)*(-I*a^2*e^(4*I*f*x + 4*I*e) + I*a^2*e^(2*I*f*x + 2*I*e) + 2*I*a
^2)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(c^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int\frac{\tan^2\left(e+fx\right)}{-ic\sqrt{-ic\tan\left(e+fx\right)+c}\,\tan\left(e+fx\right)+c\sqrt{-ic\tan\left(e+fx\right)+c}}\,dx+\int\left(-\frac{2i\tan\left(e+fx\right)}{-ic\sqrt{-ic\tan\left(e+fx\right)+c}\,\tan\left(e+fx\right)+c\sqrt{-ic\tan\left(e+fx\right)+c}}\right)dx+\int\left(-\frac{1}{-ic\sqrt{-ic\tan\left(e+fx\right)+c}\,\tan\left(e+fx\right)+c\sqrt{-ic\tan\left(e+fx\right)+c}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2/(c-I*c*tan(f*x+e))**(3/2),x)
```

```
[Out] -a**2*(Integral(tan(e + f*x)**2/(-I*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f
*x) + c*sqrt(-I*c*tan(e + f*x) + c)), x) + Integral(-2*I*tan(e + f*x)/(-I*c
*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c*sqrt(-I*c*tan(e + f*x) + c)),
 x) + Integral(-1/(-I*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c*sqrt(-
I*c*tan(e + f*x) + c)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^2/(-I*c*tan(f*x + e) + c)^(3/2), x)
```

Mupad [B]
time = 5.03, size = 98, normalized size = 1.63

$$\frac{a^2\sqrt{\frac{c\,\left(\cos\left(2\,e+2\,f\,x\right)+1-\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\left(\cos\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}-\cos\left(4\,e+4\,f\,x\right)\,1{}\mathrm{i}-\sin\left(2\,e+2\,f\,x\right)+\sin\left(4\,e+4\,f\,x\right)+2{}\mathrm{i}\right)}{3\,c^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^2/(c - c*tan(e + f*x)*1i)^(3/2),x)
```

```
[Out] (a^2*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) +
1))^(1/2)*(cos(2*e + 2*f*x)*1i - cos(4*e + 4*f*x)*1i - sin(2*e + 2*f*x) + s
in(4*e + 4*f*x) + 2i))/(3*c^2*f)
```

3.982 $\int \frac{a+ia\tan(e+fx)}{(c-ic\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=27

$$-\frac{2ia}{3f(c-ic\tan(e+fx))^{3/2}}$$

```
[Out] -2/3*I*a/f/(c-I*c*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 32}

$$-\frac{2ia}{3f(c-ic\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])/(c - I*c*Tan[e + f*x])^(3/2),x]
```

```
[Out] (((-2*I)/3)*a)/(f*(c - I*c*Tan[e + f*x])^(3/2))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{a+ia\tan(e+fx)}{(c-ic\tan(e+fx))^{3/2}}\,dx = (ac)\int \frac{\sec^2(e+fx)}{(c-ic\tan(e+fx))^{5/2}}\,dx$$
$$= \frac{(ia)\text{Subst}\left(\int \frac{1}{(c+x)^{5/2}}\,dx, x, -ic\tan(e+fx)\right)}{f}$$
$$= -\frac{2ia}{3f(c-ic\tan(e+fx))^{3/2}}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 72 vs. 2(27) = 54.
time = 0.80, size = 72, normalized size = 2.67

$$\frac{2a\cos^2(e+fx)(\cos(fx)-i\sin(fx))(-i\cos(2e+3fx)+\sin(2e+3fx))\sqrt{c-ic\tan(e+fx)}}{3c^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])/(c - I*c*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*a*Cos[e + f*x]^2*(Cos[f*x] - I*Sin[f*x])*((-I)*Cos[2*e + 3*f*x] + Sin[2*e + 3*f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(3*c^2*f)
```

Maple [A]
time = 0.20, size = 22, normalized size = 0.81

method	result	size
derivativedivides	$-\frac{2ia}{3f(c-ic\tan(fx+e))^{\frac{3}{2}}}$	22
default	$-\frac{2ia}{3f(c-ic\tan(fx+e))^{\frac{3}{2}}}$	22
risch	$-\frac{ia\left(e^{2i(fx+e)}+1\right)\sqrt{2}}{6c\sqrt{\frac{c}{e^{2i(fx+e)}+1}}\,f}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*a/f/(c-I*c*tan(f*x+e))^(3/2)
```

Maxima [A]
time = 0.30, size = 20, normalized size = 0.74

$$-\frac{2i\,a}{3\left(-i\,c\tan\left(fx+e\right)+c\right)^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] -2/3*I*a/((-I*c*tan(f*x + e) + c)^(3/2)*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(20) = 40.
time = 1.55, size = 59, normalized size = 2.19

$$\frac{\sqrt{2}\left(-i\,ae^{(4i\,fx+4i\,e)}-2i\,ae^{(2i\,fx+2i\,e)}-i\,a\right)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{6\,c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/6*sqrt(2)*(-I*a*e^(4*I*f*x + 4*I*e) - 2*I*a*e^(2*I*f*x + 2*I*e) - I*a)*sq
rt(c/(e^(2*I*f*x + 2*I*e) + 1))/(c^2*f)
```

Sympy [A]
time = 7.32, size = 46, normalized size = 1.70

$$\begin{cases} -\frac{2ia}{3f(-ic\tan(e+fx)+c)^{\frac{3}{2}}} & \text{for } f \neq 0 \\ \frac{x(ia\tan(e)+a)}{(-ic\tan(e)+c)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))**(3/2),x)
```

```
[Out] Piecewise((-2*I*a/(3*f*(-I*c*tan(e + f*x) + c)**(3/2)), Ne(f, 0)), (x*(I*a*
tan(e) + a)/(-I*c*tan(e) + c)**(3/2), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)/(-I*c*tan(f*x + e) + c)^(3/2), x)
```

Mupad [B]
time = 5.09, size = 93, normalized size = 3.44

$$\frac{a\,\sqrt{-\frac{c\,\left(-2\,{\cos\left(e+f\,x\right)}^2+\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{2\,{\cos\left(e+f\,x\right)}^2}}\,\left(-{\cos\left(e+f\,x\right)}^2\,4{}\mathrm{i}-{\cos\left(2\,e+2\,f\,x\right)}^2\,2{}\mathrm{i}+2\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(4\,e+4\,f\,x\right)+2{}\mathrm{i}\right)}{6\,c^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)/(c - c*tan(e + f*x)*1i)^(3/2),x)
```

```
[Out] (a*(-(c*(sin(2*e + 2*f*x)*1i - 2*cos(e + f*x)^2))/(2*cos(e + f*x)^2))^(1/2)
*(2*sin(2*e + 2*f*x) + sin(4*e + 4*f*x) - cos(2*e + 2*f*x)^2*2i - cos(e + f
*x)^2*4i + 2i))/(6*c^2*f)
```

3.983 $\int \frac{1}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=156

$$\frac{5i\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{8\sqrt{2}\,ac^{3/2}f}-\frac{5i}{12af(c-ic\tan(e+fx))^{3/2}}+\frac{i}{2af(1+i\tan(e+fx))(c-ic\tan(e+f}$$

```
[Out] 5/16*I*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))/a/c^(3/2)/f*2^
(1/2)-5/8*I/a/c/f/(c-I*c*tan(f*x+e))^(1/2)-5/12*I/a/f/(c-I*c*tan(f*x+e))^(3
/2)+1/2*I/a/f/(1+I*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2)
```

Rubi [A]

time = 0.14, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3603, 3568, 44, 53, 65, 212}

$$\frac{5i\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{8\sqrt{2}\,ac^{3/2}f}-\frac{5i}{8acf\sqrt{c-ic\tan(e+fx)}}-\frac{5i}{12af(c-ic\tan(e+fx))^{3/2}}+\frac{i}{2af(1+i\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(3/2)),x]
```

```
[Out] (((5*I)/8)*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])])/(Sqrt[2]*
a*c^(3/2)*f) - ((5*I)/12)/(a*f*(c - I*c*Tan[e + f*x])^(3/2)) + (I/2)/(a*f*(
1 + I*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(3/2)) - ((5*I)/8)/(a*c*f*Sqrt[c
 - I*c*Tan[e + f*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{\int \frac{\cos^2(e+fx)}{\sqrt{c-ic\tan(e+fx)}}\,dx}{ac}$$

$$= \frac{(ic^2)\operatorname{Subst}\left(\int \frac{1}{(c-x)^2(c+x)^{5/2}}\,dx, x, -ic\tan(e+fx)\right)}{af}$$

$$= \frac{i}{2af(1+i\tan(e+fx))(c-ic\tan(e+fx))^{3/2}} + \frac{(5ic)\mathrm{S}}{}$$

$$= -\frac{5i}{12af(c-ic\tan(e+fx))^{3/2}} + \frac{}{2af(1+i\tan(e+fx)}$$

$$= -\frac{5i}{12af(c-ic\tan(e+fx))^{3/2}} + \frac{}{2af(1+i\tan(e+fx)}$$

$$= -\frac{5i}{12af(c-ic\tan(e+fx))^{3/2}} + \frac{}{2af(1+i\tan(e+fx)}$$

$$= \frac{5i\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{8\sqrt{2}\,ac^{3/2}f} - \frac{5i}{12af(c-ic\tan(}$$

Mathematica [A]
time = 2.06, size = 138, normalized size = 0.88

$$\frac{(\cos(e+fx)+i\sin(e+fx))\left(15ie^{-i(e+fx)}\sqrt{1+e^{2i(e+fx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(e+fx)}}\right)-27i\cos(e+fx)+i\cos(3(e+fx))+5\sin(e+fx)+5\sin(3(e+fx))\right)\sqrt{c-ic\tan(e+fx)}}{48ac^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(3/2)),x]
```

```
[Out] ((Cos[e + f*x] + I*Sin[e + f*x])*(((15*I)*Sqrt[1 + E^((2*I)*(e + f*x))]*Arc
Tanh[Sqrt[1 + E^((2*I)*(e + f*x))]])/E^(I*(e + f*x)) - (27*I)*Cos[e + f*x]
+ I*Cos[3*(e + f*x)] + 5*Sin[e + f*x] + 5*Sin[3*(e + f*x)])*Sqrt[c - I*c*Ta
n[e + f*x]])/(48*a*c^2*f)
```

Maple [A]
time = 0.34, size = 121, normalized size = 0.78

method	result

derivativedivides	$\frac{2ic^2\left(\frac{\frac{\sqrt{c-ic\tan(fx+e)}}{4c+4ic\tan(fx+e)}+\frac{5\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)}{8\sqrt{c}}}{4c^3}-\frac{1}{4c^3\sqrt{c-ic\tan(fx}}\right.}{fa}$
default	$\frac{2ic^2\left(\frac{\frac{\sqrt{c-ic\tan(fx+e)}}{4c+4ic\tan(fx+e)}+\frac{5\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)}{8\sqrt{c}}}{4c^3}-\frac{1}{4c^3\sqrt{c-ic\tan(fx}}\right.}{fa}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/f/a*c^2*(1/4/c^3*(1/8*(c-I*c*tan(f*x+e))^(1/2)/(1/2*c+1/2*I*c*tan(f*x+e
))+5/8*2^(1/2)/c^(1/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2)
))-1/4/c^3/(c-I*c*tan(f*x+e))^(1/2)-1/12/c^2/(c-I*c*tan(f*x+e))^(3/2))
```

Maxima [A]
time = 0.50, size = 147, normalized size = 0.94

$$-\frac{i\left(\frac{4\left(15\left(-ic\tan(fx+e)+c\right)^2-20\left(-ic\tan(fx+e)+c\right)c-8c^2\right)}{\left(-ic\tan(fx+e)+c\right)^{\frac{5}{2}}a-2\left(-ic\tan(fx+e)+c\right)^{\frac{3}{2}}ac}+\frac{15\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-ic\tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic\tan(fx+e)+c}}\right)}{a\sqrt{c}}\right)}{96\,cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -1/96*I*(4*(15*(-I*c*tan(f*x + e) + c)^2 - 20*(-I*c*tan(f*x + e) + c)*c - 8
*c^2)/((-I*c*tan(f*x + e) + c)^(5/2)*a - 2*(-I*c*tan(f*x + e) + c)^(3/2)*a*
c) + 15*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*c*tan(f*x + e) + c))/(sqrt(
2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/(a*sqrt(c)))/(c*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(120) = 240$.
time = 1.34, size = 310, normalized size = 1.99

$$\frac{\left(-15i\sqrt{\frac{1}{2}}\,ac^2f\sqrt{\frac{1}{a^2c^3f^2}}\,e^{(2i\,fx+2i\,e)}\log\left(-\frac{5\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(iacfe^{(2i\,fx+2i\,e)}+iacf\right)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{1}{a^2c^3f^2}}-i\right)e^{(-i\,fx-i\,e)}}{4\,acf}\right)+15i\sqrt{\frac{1}{2}}\,ac^2f\sqrt{\frac{1}{a^2c^3f^2}}\,e^{(2i\,fx+2i\,e)}\log\left(-\frac{5\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(-iacfe^{(2i\,fx+2i\,e)}-iacf\right)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{1}{a^2c^3f^2}}-i\right)e^{(-i\,fx-i\,e)}}{4\,acf}\right)+\sqrt{2}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\left(-2i\,e^{(6i\,fx+6i\,e)}-16i\,e^{(4i\,fx+4i\,e)}-11i\,e^{(2i\,fx+2i\,e)}+3i\right)\right)e^{(-2i\,fx-2i\,e)}}{48\,ac^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/48*(-15*I*sqrt(1/2)*a*c^2*f*sqrt(1/(a^2*c^3*f^2))*e^(2*I*f*x + 2*I*e)*log
(-5/4*(sqrt(2)*sqrt(1/2)*(I*a*c*f*e^(2*I*f*x + 2*I*e) + I*a*c*f)*sqrt(c/(e^
(2*I*f*x + 2*I*e) + 1))*sqrt(1/(a^2*c^3*f^2)) - I)*e^(-I*f*x - I*e)/(a*c*f)
) + 15*I*sqrt(1/2)*a*c^2*f*sqrt(1/(a^2*c^3*f^2))*e^(2*I*f*x + 2*I*e)*log(-5
/4*(sqrt(2)*sqrt(1/2)*(-I*a*c*f*e^(2*I*f*x + 2*I*e) - I*a*c*f)*sqrt(c/(e^(2
*I*f*x + 2*I*e) + 1))*sqrt(1/(a^2*c^3*f^2)) - I)*e^(-I*f*x - I*e)/(a*c*f))
+ sqrt(2)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*(-2*I*e^(6*I*f*x + 6*I*e) - 16*
I*e^(4*I*f*x + 4*I*e) - 11*I*e^(2*I*f*x + 2*I*e) + 3*I))*e^(-2*I*f*x - 2*I*
e)/(a*c^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{1}{-ic\sqrt{-ic\tan\left(e+fx\right)+c}\tan^{2}\left(e+fx\right)-ic\sqrt{-ic\tan\left(e+fx\right)+c}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))**(3/2),x)
```

```
[Out] -I*Integral(1/(-I*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - I*c*sqrt(
-I*c*tan(e + f*x) + c)), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((I*a*tan(f*x + e) + a)*(-I*c*tan(f*x + e) + c)^(3/2)), x)
```

Mupad [B]
time = 0.49, size = 139, normalized size = 0.89

$$-\frac{\frac{c\,1\mathrm{i}}{3\,a\,f}+\frac{\left(c-c\,\tan\left(e+f\,x\right)\,1\mathrm{i}\right)\,5\mathrm{i}}{6\,a\,f}-\frac{{\left(c-c\,\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}^2\,5\mathrm{i}}{8\,a\,c\,f}}{2\,c\,{\left(c-c\,\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}^{3/2}-{\left(c-c\,\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}^{5/2}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\,\tan\left(e+f\,x\right)\,1\mathrm{i}}}{2\,\sqrt{-c}}\right)\,5\mathrm{i}}{16\,a\,{\left(-c\right)}^{3/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i)^(3/2)),x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*(-c)^(1/2)))*5i)/(
16*a*(-c)^(3/2)*f) - ((c*1i)/(3*a*f) + ((c - c*tan(e + f*x)*1i)*5i)/(6*a*f)
 - ((c - c*tan(e + f*x)*1i)^2*5i)/(8*a*c*f))/(2*c*(c - c*tan(e + f*x)*1i)^(
3/2) - (c - c*tan(e + f*x)*1i)^(5/2))
```

3.984 $\int \frac{1}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=199

$$\frac{35i\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{64\sqrt{2}\,a^2c^{3/2}f}-\frac{35i}{96a^2f(c-ic\tan(e+fx))^{3/2}}+\frac{i}{4a^2f(1+i\tan(e+fx))^2(c-ic\tan(e$$

```
[Out] 35/128*I*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))/a^2/c^(3/2)/
f*2^(1/2)-35/64*I/a^2/c/f/(c-I*c*tan(f*x+e))^(1/2)-35/96*I/a^2/f/(c-I*c*tan
(f*x+e))^(3/2)+1/4*I/a^2/f/(1+I*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(3/2)+7/16
*I/a^2/f/(1+I*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2)
```

Rubi [A]

time = 0.16, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3603, 3568, 44, 53, 65, 212}

$$\frac{35i\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{64\sqrt{2}\,a^2c^{3/2}f}-\frac{35i}{64a^2cf\sqrt{c-ic\tan(e+fx)}}-\frac{35i}{96a^2f(c-ic\tan(e+fx))^{3/2}}+\frac{7i}{16a^2f(1+i\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}+\frac{i}{4a^2f(1+i\tan(e+fx))^2(c-ic\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^(3/2)),x]
```

```
[Out] (((35*I)/64)*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])])/(Sqrt[2
]*a^2*c^(3/2)*f) - ((35*I)/96)/(a^2*f*(c - I*c*Tan[e + f*x])^(3/2)) + (I/4)
/(a^2*f*(1 + I*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^(3/2)) + ((7*I)/16)/(
a^2*f*(1 + I*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(3/2)) - ((35*I)/64)/(a^2
*c*f*Sqrt[c - I*c*Tan[e + f*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{\int \cos^4(e+fx)\sqrt{c-ic\tan(e+fx)}\,dx}{a^2c^2}$$

$$= \frac{(ic^3)\,\text{Subst}\Big(\int \frac{1}{(c-x)^3(c+x)^{5/2}}\,dx, x, -ic\tan(e+fx)\Big)}{a^2f}$$

$$= \frac{i}{4a^2f(1+i\tan(e+fx))^2(c-ic\tan(e+fx))^{3/2}} + \frac{(7i}{}$$

$$= \frac{i}{4a^2f(1+i\tan(e+fx))^2(c-ic\tan(e+fx))^{3/2}} + \frac{}{16}$$

$$= -\frac{35i}{96a^2f(c-ic\tan(e+fx))^{3/2}} + \frac{}{4a^2f(1+i\tan(e+}$$

$$= -\frac{35i}{96a^2f(c-ic\tan(e+fx))^{3/2}} + \frac{}{4a^2f(1+i\tan(e+}$$

$$= -\frac{35i}{96a^2f(c-ic\tan(e+fx))^{3/2}} + \frac{}{4a^2f(1+i\tan(e+}$$

$$= \frac{35i\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{64\sqrt{2}\,a^2c^{3/2}f} - \frac{3}{96a^2f(c-ic\mathrm{t}}$$

Mathematica [A]
time = 1.98, size = 145, normalized size = 0.73

$$-\frac{ie^{-4i(e+fx)}\left(-6-45e^{2i(e+fx)}+41e^{4i(e+fx)}+88e^{6i(e+fx)}+8e^{8i(e+fx)}-105e^{4i(e+fx)}\sqrt{1+e^{2i(e+fx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(e+fx)}}\right)\right)\sqrt{c-ic\tan(e+fx)}}{384a^2c^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^(3/2)),x]
```

```
[Out] ((-1/384*I)*(-6 - 45*E^((2*I)*(e + f*x)) + 41*E^((4*I)*(e + f*x)) + 88*E^((
6*I)*(e + f*x)) + 8*E^((8*I)*(e + f*x)) - 105*E^((4*I)*(e + f*x))*Sqrt[1 +
E^((2*I)*(e + f*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(e + f*x))]])*Sqrt[c - I*c*T
an[e + f*x]])/(a^2*c^2*E^((4*I)*(e + f*x))*f)
```

Maple [A]
time = 0.33, size = 139, normalized size = 0.70

method	result
derivativedivides	$-\frac{2ic^3\left(\frac{3}{16c^4\sqrt{c-ic\tan(fx+e)}}+\frac{1}{24c^3(c-ic\tan(fx+e))^{\frac{3}{2}}}-\frac{\frac{-\frac{11(c-ic\tan(fx+e))^{\frac{3}{2}}}{8}+\frac{13c\sqrt{c-ic\tan(fx}}{4}}{(c+ic\tan(fx+e))^2}}{}\right.}{f a^2}$
default	$-\frac{2ic^3\left(\frac{3}{16c^4\sqrt{c-ic\tan(fx+e)}}+\frac{1}{24c^3(c-ic\tan(fx+e))^{\frac{3}{2}}}-\frac{\frac{-\frac{11(c-ic\tan(fx+e))^{\frac{3}{2}}}{8}+\frac{13c\sqrt{c-ic\tan(fx}}{4}}{(c+ic\tan(fx+e))^2}}{}\right.}{f a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -2*I/f/a^2*c^3*(3/16/c^4/(c-I*c*tan(f*x+e))^(1/2)+1/24/c^3/(c-I*c*tan(f*x+e
))^(3/2)-1/16/c^4*(4*(-11/32*(c-I*c*tan(f*x+e))^(3/2)+13/16*c*(c-I*c*tan(f*
x+e))^(1/2))/(c+I*c*tan(f*x+e))^2+35/16*2^(1/2)/c^(1/2)*arctanh(1/2*(c-I*c*
tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))))
```

Maxima [A]
time = 0.52, size = 192, normalized size = 0.96

$$-\frac{i\left(\frac{4\left(105\left(-ic\tan(fx+e)+c\right)^3-350\left(-ic\tan(fx+e)+c\right)^2c+224\left(-ic\tan(fx+e)+c\right)c^2+64c^3\right)}{\left(-ic\tan(fx+e)+c\right)^{\frac{7}{2}}a^2-4\left(-ic\tan(fx+e)+c\right)^{\frac{5}{2}}a^2c+4\left(-ic\tan(fx+e)+c\right)^{\frac{3}{2}}a^2c^2}+\frac{105\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-ic\tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic\tan(fx+e)+c}}\right)}{a^2\sqrt{c}}\right)}{768\,cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] -1/768*I*(4*(105*(-I*c*tan(f*x + e) + c)^3 - 350*(-I*c*tan(f*x + e) + c)^2*
c + 224*(-I*c*tan(f*x + e) + c)*c^2 + 64*c^3)/((-I*c*tan(f*x + e) + c)^(7/2
)*a^2 - 4*(-I*c*tan(f*x + e) + c)^(5/2)*a^2*c + 4*(-I*c*tan(f*x + e) + c)^(
3/2)*a^2*c^2) + 105*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*c*tan(f*x + e)
+ c))/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/(a^2*sqrt(c)))/(c*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(155) = 310$.

time = 1.43, size = 334, normalized size = 1.68

$$\frac{\left(-105i\sqrt{\frac{1}{2}}\,a^2c^2f\sqrt{\frac{1}{a^4c^3f^2}}\,e^{(4i\,fx+4i\,e)}\log\left(-\frac{35\left(\sqrt{2}\sqrt{\frac{1}{2}}\,(i\,a^2cfe^{(2i\,fx+2i\,e)}+i\,a^2cf)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{1}{a^4c^3f^2}}-i\right)e^{(-i\,fx-i\,e)}}{32\,a^2cf}\right)+105i\sqrt{\frac{1}{2}}\,a^2c^2f\sqrt{\frac{1}{a^4c^3f^2}}\,e^{(4i\,fx+4i\,e)}\log\left(-\frac{35\left(\sqrt{2}\sqrt{\frac{1}{2}}\,(-i\,a^2cfe^{(2i\,fx+2i\,e)}-i\,a^2cf)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{1}{a^4c^3f^2}}-i\right)e^{(-i\,fx-i\,e)}}{32\,a^2cf}\right)+\sqrt{2}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\left(-8i\,e^{(8i\,fx+8i\,e)}-88i\,e^{(6i\,fx+6i\,e)}-41i\,e^{(4i\,fx+4i\,e)}+45i\,e^{(2i\,fx+2i\,e)}+6i\right)\right)e^{(-4i\,fx-4i\,e)}}{384\,a^2c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/384*(-105*I*sqrt(1/2)*a^2*c^2*f*sqrt(1/(a^4*c^3*f^2))*e^(4*I*f*x + 4*I*e)
*log(-35/32*(sqrt(2)*sqrt(1/2)*(I*a^2*c*f*e^(2*I*f*x + 2*I*e) + I*a^2*c*f)*
sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(1/(a^4*c^3*f^2)) - I)*e^(-I*f*x - I*
e)/(a^2*c*f)) + 105*I*sqrt(1/2)*a^2*c^2*f*sqrt(1/(a^4*c^3*f^2))*e^(4*I*f*x
+ 4*I*e)*log(-35/32*(sqrt(2)*sqrt(1/2)*(-I*a^2*c*f*e^(2*I*f*x + 2*I*e) - I*
a^2*c*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(1/(a^4*c^3*f^2)) - I)*e^(-I
*f*x - I*e)/(a^2*c*f)) + sqrt(2)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*(-8*I*e^
(8*I*f*x + 8*I*e) - 88*I*e^(6*I*f*x + 6*I*e) - 41*I*e^(4*I*f*x + 4*I*e) + 4
5*I*e^(2*I*f*x + 2*I*e) + 6*I))*e^(-4*I*f*x - 4*I*e)/(a^2*c^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{-ic\sqrt{-ic\tan\left(e+fx\right)+c}\,\tan^3\left(e+fx\right)-c\sqrt{-ic\tan\left(e+fx\right)+c}\,\tan^2\left(e+fx\right)-ic\sqrt{-ic\tan\left(e+fx\right)+c}\,\tan\left(e+fx\right)-c\sqrt{-ic\tan\left(e+fx\right)+c}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**2/(c-I*c*tan(f*x+e))**(3/2),x)
```

```
[Out] -Integral(1/(-I*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**3 - c*sqrt(-I*c
*tan(e + f*x) + c)*tan(e + f*x)**2 - I*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e
+ f*x) - c*sqrt(-I*c*tan(e + f*x) + c)), x)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((I*a*tan(f*x + e) + a)^2*(-I*c*tan(f*x + e) + c)^(3/2)), x)
```

Mupad [B]
time = 5.19, size = 182, normalized size = 0.91

$$-\frac{-\frac{(c-c\tan(e+f\,x)\,1\mathrm{i})^2\,175\mathrm{i}}{96\,a^2\,f}+\frac{c^2\,1\mathrm{i}}{3\,a^2\,f}+\frac{(c-c\tan(e+f\,x)\,1\mathrm{i})^3\,35\mathrm{i}}{64\,a^2\,c\,f}+\frac{c\,(c-c\tan(e+f\,x)\,1\mathrm{i})\,7\mathrm{i}}{6\,a^2\,f}}{-4\,c\,\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^{5/2}+\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^{7/2}+4\,c^2\,\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^{3/2}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}}{2\,\sqrt{-c}}\right)\,35\mathrm{i}}{128\,a^2\,{\left(-c\right)}^{3/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i)^(3/2)),x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*(-c)^(1/2)))*35i)/
(128*a^2*(-c)^(3/2)*f) - ((c^2*1i)/(3*a^2*f) - ((c - c*tan(e + f*x)*1i)^2*1
75i)/(96*a^2*f) + ((c - c*tan(e + f*x)*1i)^3*35i)/(64*a^2*c*f) + (c*(c - c*
tan(e + f*x)*1i)*7i)/(6*a^2*f))/((c - c*tan(e + f*x)*1i)^(7/2) - 4*c*(c - c
*tan(e + f*x)*1i)^(5/2) + 4*c^2*(c - c*tan(e + f*x)*1i)^(3/2))
```

3.985 $\int \frac{1}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=242

$$\frac{105i\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{256\sqrt{2}\,a^3c^{3/2}f}-\frac{35i}{128a^3f(c-ic\tan(e+fx))^{3/2}}+\frac{i}{6a^3f(1+i\tan(e+fx))^3(c-ic\tan}$$

```
[Out] 105/512*I*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))/a^3/c^(3/2)
/f*2^(1/2)-105/256*I/a^3/c/f/(c-I*c*tan(f*x+e))^(1/2)-35/128*I/a^3/f/(c-I*c
*tan(f*x+e))^(3/2)+1/6*I/a^3/f/(1+I*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(3/2)+
3/16*I/a^3/f/(1+I*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(3/2)+21/64*I/a^3/f/(1+I
*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2)
```

Rubi [A]

time = 0.17, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = $\{3603, 3568, 44, 53, 65, 212\}$

$$\frac{105i\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{256\sqrt{2}\,a^3c^{3/2}f}-\frac{105i}{256a^3cf\sqrt{c-ic\tan(e+fx)}}-\frac{35i}{128a^3f(c-ic\tan(e+fx))^{3/2}}+\frac{21i}{64a^3f(1+i\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}+\frac{3i}{16a^3f(1+i\tan(e+fx))^2(c-ic\tan(e+fx))^{3/2}}+\frac{i}{6a^3f(1+i\tan(e+fx))^3(c-ic\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])^(3/2)),x]
```

```
[Out] (((105*I)/256)*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])])/(Sqrt
[2]*a^3*c^(3/2)*f) - ((35*I)/128)/(a^3*f*(c - I*c*Tan[e + f*x])^(3/2)) + (I
/6)/(a^3*f*(1 + I*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])^(3/2)) + ((3*I)/16
)/(a^3*f*(1 + I*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^(3/2)) + ((21*I)/64)
/(a^3*f*(1 + I*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(3/2)) - ((105*I)/256)/
(a^3*c*f*Sqrt[c - I*c*Tan[e + f*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
```

```
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{\int \cos^6(e+fx)(c-ic\tan(e+fx))^{3/2}\,dx}{a^3c^3}$$

$$= \frac{(ic^4)\,\mathrm{Subst}\left(\int \frac{1}{(c-x)^4(c+x)^{5/2}}\,dx, x, -ic\tan(e+fx)\right)}{a^3 f}$$

$$= \frac{i}{6a^3f(1+i\tan(e+fx))^3(c-ic\tan(e+fx))^{3/2}} + \frac{(3i}{\ }$$

$$= \frac{i}{6a^3f(1+i\tan(e+fx))^3(c-ic\tan(e+fx))^{3/2}} + \frac{\ }{16}$$

$$= \frac{i}{6a^3f(1+i\tan(e+fx))^3(c-ic\tan(e+fx))^{3/2}} + \frac{\ }{16}$$

$$= -\frac{35i}{128a^3f(c-ic\tan(e+fx))^{3/2}} + \frac{\ }{6a^3f(1+i\tan(e+}$$

$$= -\frac{35i}{128a^3f(c-ic\tan(e+fx))^{3/2}} + \frac{\ }{6a^3f(1+i\tan(e+}$$

$$= -\frac{35i}{128a^3f(c-ic\tan(e+fx))^{3/2}} + \frac{\ }{6a^3f(1+i\tan(e+}$$

$$= \frac{105i\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{256\sqrt{2}\,a^3c^{3/2}f} - \frac{\ }{128a^3f(c-i}$$

Mathematica [A]
time = 2.67, size = 160, normalized size = 0.66

$$\frac{(\cos(e+fx)-i\sin(e+fx))\left(315ie^{i(e+fx)}\sqrt{1+e^{2i(e+fx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(e+fx)}}\right)+172i\cos(e+fx)-166i\cos(3(e+fx))-8i\cos(5(e+fx))+258\sin(e+fx)+282\sin(3(e+fx))+24\sin(5(e+fx))\right)\sqrt{c-ic\tan(e+fx)}}{1536a^3c^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])^(3/2)),x]
```

```
[Out] ((Cos[e + f*x] - I*Sin[e + f*x])*((315*I)*E^(I*(e + f*x))*Sqrt[1 + E^((2*I)
*(e + f*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(e + f*x))]] + (172*I)*Cos[e + f*x]
- (166*I)*Cos[3*(e + f*x)] - (8*I)*Cos[5*(e + f*x)] + 258*Sin[e + f*x] + 28
2*Sin[3*(e + f*x)] + 24*Sin[5*(e + f*x)])*Sqrt[c - I*c*Tan[e + f*x]])/(1536
*a^3*c^2*f)
```

Maple [A]
time = 0.38, size = 158, normalized size = 0.65

method	result
derivativedivides	$\frac{2ic^4\left(-\frac{1}{8c^5\sqrt{c-ic\tan(fx+e)}}-\frac{1}{48c^4(c-ic\tan(fx+e))^{\frac{3}{2}}}+\frac{\frac{\frac{41(c-ic\tan(fx+e))^{\frac{5}{2}}}{32}-\frac{35c(c-ic\tan(fx+e))^{\frac{3}{2}}}{6}+\frac{55c^2\sqrt{\cdots}}{}}{(c+ic\tan(fx+e))^3}}{}\right.}{f\,a^3}$
default	$\frac{2ic^4\left(-\frac{1}{8c^5\sqrt{c-ic\tan(fx+e)}}-\frac{1}{48c^4(c-ic\tan(fx+e))^{\frac{3}{2}}}+\frac{\frac{\frac{41(c-ic\tan(fx+e))^{\frac{5}{2}}}{32}-\frac{35c(c-ic\tan(fx+e))^{\frac{3}{2}}}{6}+\frac{55c^2\sqrt{\cdots}}{}}{(c+ic\tan(fx+e))^3}}{}\right.}{f\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 2*I/f/a^3*c^4*(-1/8/c^5/(c-I*c*tan(f*x+e))^(1/2)-1/48/c^4/(c-I*c*tan(f*x+e)
)^(3/2)+1/16/c^5*(8*(41/256*(c-I*c*tan(f*x+e))^(5/2)-35/48*c*(c-I*c*tan(f*x
+e))^(3/2)+55/64*c^2*(c-I*c*tan(f*x+e))^(1/2))/(c+I*c*tan(f*x+e))^3+105/64*
2^(1/2)/c^(1/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))))
```

Maxima [A]

time = 0.54, size = 233, normalized size = 0.96

$$-\frac{i\left(\frac{4\left(315\left(-ic\tan(fx+e)+c\right)^4-1680\left(-ic\tan(fx+e)+c\right)^3c+2772\left(-ic\tan(fx+e)+c\right)^2c^2-1152\left(-ic\tan(fx+e)+c\right)c^3-256c^4\right)}{\left(-ic\tan(fx+e)+c\right)^{\frac{9}{2}}a^3-6\left(-ic\tan(fx+e)+c\right)^{\frac{7}{2}}a^3c+12\left(-ic\tan(fx+e)+c\right)^{\frac{5}{2}}a^3c^2-8\left(-ic\tan(fx+e)+c\right)^{\frac{3}{2}}a^3c^3}+\frac{315\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-ic\tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic\tan(fx+e)+c}}\right)}{a^3\sqrt{c}}\right)}{3072\,cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] -1/3072*I*(4*(315*(-I*c*tan(f*x + e) + c)^4 - 1680*(-I*c*tan(f*x + e) + c)^
3*c + 2772*(-I*c*tan(f*x + e) + c)^2*c^2 - 1152*(-I*c*tan(f*x + e) + c)*c^3
 - 256*c^4)/((-I*c*tan(f*x + e) + c)^(9/2)*a^3 - 6*(-I*c*tan(f*x + e) + c)^
(7/2)*a^3*c + 12*(-I*c*tan(f*x + e) + c)^(5/2)*a^3*c^2 - 8*(-I*c*tan(f*x +
e) + c)^(3/2)*a^3*c^3) + 315*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*c*tan(
f*x + e) + c))/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/(a^3*sqrt(c
)))/(c*f)
```

Fricas [A]
time = 1.52, size = 346, normalized size = 1.43

$$\frac{\left(-315i\sqrt{\frac{1}{2}}a^3c^2f\sqrt{\frac{1}{a^6c^3f^2}}e^{(6ifx+6ie)}\log\left(-\frac{105\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(ia^3cfe^{(2ifx+2ie)}+ia^3cf\right)\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{1}{a^6c^3f^2}}-i\right)e^{(-ifx-ie)}}{128a^3cf}\right)+315i\sqrt{\frac{1}{2}}a^3c^2f\sqrt{\frac{1}{a^6c^3f^2}}e^{(6ifx+6ie)}\log\left(-\frac{105\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(-ia^3cfe^{(2ifx+2ie)}-ia^3cf\right)\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{1}{a^6c^3f^2}}-i\right)e^{(-ifx-ie)}}{128a^3cf}\right)+\sqrt{2}\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}\left(-16ie^{(10ifx+10ie)}-224ie^{(8ifx+8ie)}-43ie^{(6ifx+6ie)}+215ie^{(4ifx+4ie)}+58ie^{(2ifx+2ie)}+8i\right)\right)e^{(-6ifx-6ie)}}{1536a^3c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/1536*(-315*I*sqrt(1/2)*a^3*c^2*f*sqrt(1/(a^6*c^3*f^2))*e^(6*I*f*x + 6*I*e
)*log(-105/128*(sqrt(2)*sqrt(1/2)*(I*a^3*c*f*e^(2*I*f*x + 2*I*e) + I*a^3*c*
f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(1/(a^6*c^3*f^2)) - I)*e^(-I*f*x -
 I*e)/(a^3*c*f)) + 315*I*sqrt(1/2)*a^3*c^2*f*sqrt(1/(a^6*c^3*f^2))*e^(6*I*f
*x + 6*I*e)*log(-105/128*(sqrt(2)*sqrt(1/2)*(-I*a^3*c*f*e^(2*I*f*x + 2*I*e)
 - I*a^3*c*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(1/(a^6*c^3*f^2)) - I)*
e^(-I*f*x - I*e)/(a^3*c*f)) + sqrt(2)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*(-1
6*I*e^(10*I*f*x + 10*I*e) - 224*I*e^(8*I*f*x + 8*I*e) - 43*I*e^(6*I*f*x + 6
*I*e) + 215*I*e^(4*I*f*x + 4*I*e) + 58*I*e^(2*I*f*x + 2*I*e) + 8*I))*e^(-6*
I*f*x - 6*I*e)/(a^3*c^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int\frac{1}{-ic\sqrt{-ic\tan(e+fx)+c}\tan^4(e+fx)-2c\sqrt{-ic\tan(e+fx)+c}\tan^3(e+fx)-2c\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)+ic\sqrt{-ic\tan(e+fx)+c}}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**3/(c-I*c*tan(f*x+e))**(3/2),x)
```

```
[Out] I*Integral(1/(-I*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**4 - 2*c*sqrt(-
I*c*tan(e + f*x) + c)*tan(e + f*x)**3 - 2*c*sqrt(-I*c*tan(e + f*x) + c)*tan
(e + f*x) + I*c*sqrt(-I*c*tan(e + f*x) + c)), x)/a**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((I*a*tan(f*x + e) + a)^3*(-I*c*tan(f*x + e) + c)^(3/2)), x)
```

Mupad [B]
time = 5.07, size = 229, normalized size = 0.95

$$-\frac{\frac{(c-c\tan(e+fx)\,1\mathrm{i})^3\,35\mathrm{i}}{16\,a^3\,f}+\frac{c^3\,1\mathrm{i}}{3\,a^3\,f}-\frac{(c-c\tan(e+fx)\,1\mathrm{i})^4\,105\mathrm{i}}{256\,a^3\,c\,f}-\frac{c\,(c-c\tan(e+fx)\,1\mathrm{i})^2\,231\mathrm{i}}{64\,a^3\,f}+\frac{c^2\,(c-c\tan(e+fx)\,1\mathrm{i})\,3\mathrm{i}}{2\,a^3\,f}}{6\,c\,(c-c\tan(e+fx)\,1\mathrm{i})^{7/2}-(c-c\tan(e+fx)\,1\mathrm{i})^{9/2}+8\,c^3\,(c-c\tan(e+fx)\,1\mathrm{i})^{3/2}-12\,c^2\,(c-c\tan(e+fx)\,1\mathrm{i})^{5/2}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+fx)\,1\mathrm{i}}}{2\,\sqrt{-c}}\right)\,105\mathrm{i}}{512\,a^3\,(-c)^{3/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i)^(3/2)),x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*(-c)^(1/2)))*105i)
/(512*a^3*(-c)^(3/2)*f) - (((c - c*tan(e + f*x)*1i)^3*35i)/(16*a^3*f) + (c^
3*1i)/(3*a^3*f) - ((c - c*tan(e + f*x)*1i)^4*105i)/(256*a^3*c*f) - (c*(c -
c*tan(e + f*x)*1i)^2*231i)/(64*a^3*f) + (c^2*(c - c*tan(e + f*x)*1i)*3i)/(2
*a^3*f))/(6*c*(c - c*tan(e + f*x)*1i)^(7/2) - (c - c*tan(e + f*x)*1i)^(9/2)
 + 8*c^3*(c - c*tan(e + f*x)*1i)^(3/2) - 12*c^2*(c - c*tan(e + f*x)*1i)^(5/
2))
```

3.986 $\int \frac{(a+ia\tan(e+fx))^3}{(c-ic\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=92

$$-\frac{8ia^3}{5f(c-ic\tan(e+fx))^{5/2}}+\frac{8ia^3}{3cf(c-ic\tan(e+fx))^{3/2}}-\frac{2ia^3}{c^2f\sqrt{c-ic\tan(e+fx)}}$$

```
[Out] -2*I*a^3/c^2/f/(c-I*c*tan(f*x+e))^(1/2)-8/5*I*a^3/f/(c-I*c*tan(f*x+e))^(5/2
)+8/3*I*a^3/c/f/(c-I*c*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.12, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3603, 3568, 45}

$$-\frac{2ia^3}{c^2f\sqrt{c-ic\tan(e+fx)}}+\frac{8ia^3}{3cf(c-ic\tan(e+fx))^{3/2}}-\frac{8ia^3}{5f(c-ic\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^3/(c - I*c*Tan[e + f*x])^(5/2),x]
```

```
[Out] (((-8*I)/5)*a^3)/(f*(c - I*c*Tan[e + f*x])^(5/2)) + (((8*I)/3)*a^3)/(c*f*(c
 - I*c*Tan[e + f*x])^(3/2)) - ((2*I)*a^3)/(c^2*f*Sqrt[c - I*c*Tan[e + f*x]]
)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
```

```
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^3}{(c-ic\tan(e+fx))^{5/2}}\,dx = (a^3c^3)\int \frac{\sec^6(e+fx)}{(c-ic\tan(e+fx))^{11/2}}\,dx$$
$$= \frac{(ia^3)\,\text{Subst}\left(\int \frac{(c-x)^2}{(c+x)^{7/2}}\,dx, x, -ic\tan(e+fx)\right)}{c^2f}$$
$$= \frac{(ia^3)\,\text{Subst}\left(\int \left(\frac{4c^2}{(c+x)^{7/2}}-\frac{4c}{(c+x)^{5/2}}+\frac{1}{(c+x)^{3/2}}\right)\,dx, x, -ic\tan(e+fx)\right)}{c^2f}$$
$$= -\frac{8ia^3}{5f(c-ic\tan(e+fx))^{5/2}}+\frac{8ia^3}{3cf(c-ic\tan(e+fx))^{3/2}}-\frac{2ia^3}{c^2f\sqrt{c-ic\tan(}}$$

Mathematica [A]
time = 3.00, size = 98, normalized size = 1.07

$$\frac{2a^3\cos(e+fx)(-4+11\cos(2(e+fx))-5i\sin(2(e+fx)))(-i\cos(3(e+2fx))+\sin(3(e+2fx)))\sqrt{c-ic\tan(e+fx)}}{15c^3f(\cos(fx)+i\sin(fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3/(c - I*c*Tan[e + f*x])^(5/2),x]
[Out] (2*a^3*Cos[e + f*x]*(-4 + 11*Cos[2*(e + f*x)] - (5*I)*Sin[2*(e + f*x)])*((-
I)*Cos[3*(e + 2*f*x)] + Sin[3*(e + 2*f*x)])*Sqrt[c - I*c*Tan[e + f*x]])/(15
*c^3*f*(Cos[f*x] + I*Sin[f*x])^3)
```

Maple [A]
time = 0.27, size = 66, normalized size = 0.72

method	result	size
risch	$-\frac{i a^3\left(3\,{\rm e}^{4i(fx+e)}-4\,{\rm e}^{2i(fx+e)}+8\right)\sqrt{2}}{15c^2\sqrt{\frac{c}{{\rm e}^{2i(fx+e)}+1}}\,f}$	57
derivativedivides	$\frac{2ia^3\left(-\frac{4c^2}{5(c-ic\tan(fx+e))^{\frac{5}{2}}}+\frac{4c}{3(c-ic\tan(fx+e))^{\frac{3}{2}}}-\frac{1}{\sqrt{c-ic\tan(fx+e)}}\right)}{f\,c^2}$	66
default	$\frac{2ia^3\left(-\frac{4c^2}{5(c-ic\tan(fx+e))^{\frac{5}{2}}}+\frac{4c}{3(c-ic\tan(fx+e))^{\frac{3}{2}}}-\frac{1}{\sqrt{c-ic\tan(fx+e)}}\right)}{f\,c^2}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/f*a^3/c^2*(-4/5*c^2/(c-I*c*tan(f*x+e))^(5/2)+4/3*c/(c-I*c*tan(f*x+e))^(
3/2)-1/(c-I*c*tan(f*x+e))^(1/2))
```

Maxima [**A**]
time = 0.29, size = 68, normalized size = 0.74

$$-\frac{2i\left(15\left(-i\,c\tan\left(fx+e\right)+c\right)^{2}a^{3}-20\left(-i\,c\tan\left(fx+e\right)+c\right)a^{3}c+12\,a^{3}c^{2}\right)}{15\left(-i\,c\tan\left(fx+e\right)+c\right)^{\frac{5}{2}}c^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -2/15*I*(15*(-I*c*tan(f*x + e) + c)^2*a^3 - 20*(-I*c*tan(f*x + e) + c)*a^3*
c + 12*a^3*c^2)/((-I*c*tan(f*x + e) + c)^(5/2)*c^2*f)
```

Fricas [**A**]
time = 1.91, size = 80, normalized size = 0.87

$$\frac{\sqrt{2}\left(-3i\,a^{3}e^{(6i\,fx+6i\,e)}+i\,a^{3}e^{(4i\,fx+4i\,e)}-4i\,a^{3}e^{(2i\,fx+2i\,e)}-8i\,a^{3}\right)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{15\,c^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/15*sqrt(2)*(-3*I*a^3*e^(6*I*f*x + 6*I*e) + I*a^3*e^(4*I*f*x + 4*I*e) - 4*
I*a^3*e^(2*I*f*x + 2*I*e) - 8*I*a^3)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(c^3
*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-ia^{3}\left(\int\frac{i}{-c^{2}\sqrt{-ic\tan\left(e+fx\right)+c}\tan^{2}\left(e+fx\right)-2ic^{2}\sqrt{-ic\tan\left(e+fx\right)+c}\tan\left(e+fx\right)+c^{2}\sqrt{-ic\tan\left(e+fx\right)+c}}\,dx+\int\left(-\frac{3\tan\left(e+fx\right)}{-c^{2}\sqrt{-ic\tan\left(e+fx\right)+c}\tan^{2}\left(e+fx\right)-2ic^{2}\sqrt{-ic\tan\left(e+fx\right)+c}\tan\left(e+fx\right)+c^{2}\sqrt{-ic\tan\left(e+fx\right)+c}}\right)dx+\int\frac{\tan^{3}\left(e+fx\right)}{-c^{2}\sqrt{-ic\tan\left(e+fx\right)+c}\tan^{2}\left(e+fx\right)-2ic^{2}\sqrt{-ic\tan\left(e+fx\right)+c}\tan\left(e+fx\right)+c^{2}\sqrt{-ic\tan\left(e+fx\right)+c}}\,dx+\int\left(-\frac{3i\tan^{2}\left(e+fx\right)}{-c^{2}\sqrt{-ic\tan\left(e+fx\right)+c}\tan^{2}\left(e+fx\right)-2ic^{2}\sqrt{-ic\tan\left(e+fx\right)+c}\tan\left(e+fx\right)+c^{2}\sqrt{-ic\tan\left(e+fx\right)+c}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3/(c-I*c*tan(f*x+e))**(5/2),x)
```

```
[Out] -I*a**3*(Integral(I/(-c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 2*
I*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c**2*sqrt(-I*c*tan(e + f*
```

```
x) + c)), x) + Integral(-3*tan(e + f*x)/(-c**2*sqrt(-I*c*tan(e + f*x) + c)*
tan(e + f*x)**2 - 2*I*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c**2*
sqrt(-I*c*tan(e + f*x) + c)), x) + Integral(tan(e + f*x)**3/(-c**2*sqrt(-I*
c*tan(e + f*x) + c)*tan(e + f*x)**2 - 2*I*c**2*sqrt(-I*c*tan(e + f*x) + c)*
tan(e + f*x) + c**2*sqrt(-I*c*tan(e + f*x) + c)), x) + Integral(-3*I*tan(e
+ f*x)**2/(-c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 2*I*c**2*sqr
t(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c**2*sqrt(-I*c*tan(e + f*x) + c)),
x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^3/(-I*c*tan(f*x + e) + c)^(5/2), x)
```

Mupad [**B**]
time = 5.46, size = 121, normalized size = 1.32

$$-\frac{a^3\,\sqrt{\frac{c\,\left(\cos\left(2\,e+2\,f\,x\right)+1-\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\left(\cos\left(2\,e+2\,f\,x\right)\,4{}\mathrm{i}-\cos\left(4\,e+4\,f\,x\right)\,1{}\mathrm{i}+\cos\left(6\,e+6\,f\,x\right)\,3{}\mathrm{i}-4\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(4\,e+4\,f\,x\right)-3\,\sin\left(6\,e+6\,f\,x\right)+8{}\mathrm{i}\right)}{15\,c^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^3/(c - c*tan(e + f*x)*1i)^(5/2),x)
```

```
[Out] -(a^3*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) +
 1))^(1/2)*(cos(2*e + 2*f*x)*4i - cos(4*e + 4*f*x)*1i + cos(6*e + 6*f*x)*3i
 - 4*sin(2*e + 2*f*x) + sin(4*e + 4*f*x) - 3*sin(6*e + 6*f*x) + 8i))/(15*c^
3*f)
```

3.987 $\int \frac{(a+ia\tan(e+fx))^2}{(c-ic\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{4ia^2}{5f(c-ic\tan(e+fx))^{5/2}}+\frac{2ia^2}{3cf(c-ic\tan(e+fx))^{3/2}}$$

```
[Out] -4/5*I*a^2/f/(c-I*c*tan(f*x+e))^(5/2)+2/3*I*a^2/c/f/(c-I*c*tan(f*x+e))^(3/2
)
```

Rubi [A]
time = 0.11, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3603, 3568, 45}

$$\frac{2ia^2}{3cf(c-ic\tan(e+fx))^{3/2}}-\frac{4ia^2}{5f(c-ic\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2/(c - I*c*Tan[e + f*x])^(5/2),x]
```

```
[Out] (((-4*I)/5)*a^2)/(f*(c - I*c*Tan[e + f*x])^(5/2)) + (((2*I)/3)*a^2)/(c*f*(c
 - I*c*Tan[e + f*x])^(3/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\begin{aligned}\int \frac{(a+ia\tan(e+fx))^2}{(c-ic\tan(e+fx))^{5/2}}\,dx &= \left(a^2c^2\right)\int \frac{\sec^4(e+fx)}{(c-ic\tan(e+fx))^{9/2}}\,dx\\ &= \frac{\left(ia^2\right)\text{Subst}\left(\int \frac{c-x}{(c+x)^{7/2}}\,dx, x, -ic\tan(e+fx)\right)}{cf}\\ &= \frac{\left(ia^2\right)\text{Subst}\left(\int \left(\frac{2c}{(c+x)^{7/2}}-\frac{1}{(c+x)^{5/2}}\right)\,dx, x, -ic\tan(e+fx)\right)}{cf}\\ &= -\frac{4ia^2}{5f(c-ic\tan(e+fx))^{5/2}}+\frac{2ia^2}{3cf(c-ic\tan(e+fx))^{3/2}}\end{aligned}$$

Mathematica [A]
time = 2.10, size = 95, normalized size = 1.53

$$\frac{2a^2\cos^2(e+fx)(-i\cos(e+fx)+5\sin(e+fx))(\cos(3e+5fx)+i\sin(3e+5fx))\sqrt{c-ic\tan(e+fx)}}{15c^3f(\cos(fx)+i\sin(fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2/(c - I*c*Tan[e + f*x])^(5/2),x]
```

```
[Out] (2*a^2*Cos[e + f*x]^2*((-I)*Cos[e + f*x] + 5*Sin[e + f*x])*(Cos[3*e + 5*f*x
] + I*Sin[3*e + 5*f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(15*c^3*f*(Cos[f*x] + I
*Sin[f*x])^2)
```

Maple [A]
time = 0.26, size = 47, normalized size = 0.76

method	result	size
derivativedivides	$-\frac{2ia^2\left(\frac{2c}{5(c-ic\tan(fx+e))^{\frac{5}{2}}}-\frac{1}{3(c-ic\tan(fx+e))^{\frac{3}{2}}}\right)}{fc}$	47
default	$-\frac{2ia^2\left(\frac{2c}{5(c-ic\tan(fx+e))^{\frac{5}{2}}}-\frac{1}{3(c-ic\tan(fx+e))^{\frac{3}{2}}}\right)}{fc}$	47
risch	$-\frac{ia^2\left(3\,\mathrm{e}^{4i(fx+e)}+\mathrm{e}^{2i(fx+e)}-2\right)\sqrt{2}}{30c^2\sqrt{\frac{c}{\mathrm{e}^{2i(fx+e)}+1}}\,f}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/f*a^2/c*(2/5*c/(c-I*c*tan(f*x+e))^(5/2)-1/3/(c-I*c*tan(f*x+e))^(3/2))
```

Maxima [A]
time = 0.29, size = 46, normalized size = 0.74

$$\frac{2i\,(5\,(-i\,c\tan\left(fx+e\right)+c)a^2-6\,a^2c)}{15\,(-i\,c\tan\left(fx+e\right)+c)^{\frac{5}{2}}cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] 2/15*I*(5*(-I*c*tan(f*x + e) + c)*a^2 - 6*a^2*c)/((-I*c*tan(f*x + e) + c)^(
5/2)*c*f)
```

Fricas [A]
time = 1.79, size = 80, normalized size = 1.29

$$\frac{\sqrt{2}\,\left(-3i\,a^2e^{(6i\,fx+6i\,e)}-4i\,a^2e^{(4i\,fx+4i\,e)}+i\,a^2e^{(2i\,fx+2i\,e)}+2i\,a^2\right)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{30\,c^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/30*sqrt(2)*(-3*I*a^2*e^(6*I*f*x + 6*I*e) - 4*I*a^2*e^(4*I*f*x + 4*I*e) +
I*a^2*e^(2*I*f*x + 2*I*e) + 2*I*a^2)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(c^3
*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int\frac{\tan^2\left(e+fx\right)}{-c^2\sqrt{-ic\tan\left(e+fx\right)+c}\,\tan^2\left(e+fx\right)-2ic^2\sqrt{-ic\tan\left(e+fx\right)+c}\,\tan\left(e+fx\right)+c^2\sqrt{-ic\tan\left(e+fx\right)+c}}\,dx+\int\left(-\frac{2i\tan\left(e+fx\right)}{-c^2\sqrt{-ic\tan\left(e+fx\right)+c}\,\tan^2\left(e+fx\right)-2ic^2\sqrt{-ic\tan\left(e+fx\right)+c}\,\tan\left(e+fx\right)+c^2\sqrt{-ic\tan\left(e+fx\right)+c}}\right)dx+\int\left(-\frac{1}{-c^2\sqrt{-ic\tan\left(e+fx\right)+c}\,\tan^2\left(e+fx\right)-2ic^2\sqrt{-ic\tan\left(e+fx\right)+c}\,\tan\left(e+fx\right)+c^2\sqrt{-ic\tan\left(e+fx\right)+c}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2/(c-I*c*tan(f*x+e))**(5/2),x)
```

```
[Out] -a**2*(Integral(tan(e + f*x)**2/(-c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e +
f*x)**2 - 2*I*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c**2*sqrt(-I*
c*tan(e + f*x) + c)), x) + Integral(-2*I*tan(e + f*x)/(-c**2*sqrt(-I*c*tan(
e + f*x) + c)*tan(e + f*x)**2 - 2*I*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e
+ f*x) + c**2*sqrt(-I*c*tan(e + f*x) + c)), x) + Integral(-1/(-c**2*sqrt(-I
*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 2*I*c**2*sqrt(-I*c*tan(e + f*x) + c)
*tan(e + f*x) + c**2*sqrt(-I*c*tan(e + f*x) + c)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^2/(-I*c*tan(f*x + e) + c)^(5/2), x)
```

Mupad [B]
time = 5.55, size = 123, normalized size = 1.98

$$\frac{a^2\,\sqrt{\frac{c\,\left(\cos\left(2\,e+2\,f\,x\right)+1-\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\left(\cos\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}-\cos\left(4\,e+4\,f\,x\right)\,4{}\mathrm{i}-\cos\left(6\,e+6\,f\,x\right)\,3{}\mathrm{i}-\sin\left(2\,e+2\,f\,x\right)+4\,\sin\left(4\,e+4\,f\,x\right)+3\,\sin\left(6\,e+6\,f\,x\right)+2{}\mathrm{i}\right)}{30\,c^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^2/(c - c*tan(e + f*x)*1i)^(5/2),x)
```

```
[Out] (a^2*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) +
1))^(1/2)*(cos(2*e + 2*f*x)*1i - cos(4*e + 4*f*x)*4i - cos(6*e + 6*f*x)*3i
- sin(2*e + 2*f*x) + 4*sin(4*e + 4*f*x) + 3*sin(6*e + 6*f*x) + 2i))/(30*c^3
*f)
```

3.988 $\int \frac{a+ia\tan(e+fx)}{(c-ic\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=27

$$-\frac{2ia}{5f(c-ic\tan(e+fx))^{5/2}}$$

```
[Out] -2/5*I*a/f/(c-I*c*tan(f*x+e))^(5/2)
```

Rubi [A]
time = 0.07, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 32}

$$-\frac{2ia}{5f(c-ic\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])/(c - I*c*Tan[e + f*x])^(5/2),x]
```

```
[Out] (((-2*I)/5)*a)/(f*(c - I*c*Tan[e + f*x])^(5/2))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{a+ia\tan(e+fx)}{(c-ic\tan(e+fx))^{5/2}}\,dx = (ac)\int \frac{\sec^2(e+fx)}{(c-ic\tan(e+fx))^{7/2}}\,dx$$
$$= \frac{(ia)\text{Subst}\left(\int \frac{1}{(c+x)^{7/2}}\,dx, x, -ic\tan(e+fx)\right)}{f}$$
$$= -\frac{2ia}{5f(c-ic\tan(e+fx))^{5/2}}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 72 vs. 2(27) = 54.
time = 1.07, size = 72, normalized size = 2.67

$$\frac{2a\cos^3(e+fx)(\cos(fx)-i\sin(fx))(-i\cos(3e+4fx)+\sin(3e+4fx))\sqrt{c-ic\tan(e+fx)}}{5c^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])/(c - I*c*Tan[e + f*x])^(5/2),x]
```

```
[Out] (2*a*Cos[e + f*x]^3*(Cos[f*x] - I*Sin[f*x])*((-I)*Cos[3*e + 4*f*x] + Sin[3*
e + 4*f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(5*c^3*f)
```

Maple [**A**]
time = 0.18, size = 22, normalized size = 0.81

method	result	size
derivativedivides	$-\frac{2ia}{5f(c-ic\tan(fx+e))^{\frac{5}{2}}}$	22
default	$-\frac{2ia}{5f(c-ic\tan(fx+e))^{\frac{5}{2}}}$	22
risch	$-\frac{ia\left(e^{4i(fx+e)}+2\,e^{2i(fx+e)}+1\right)\sqrt{2}}{20c^2\sqrt{\frac{c}{e^{2i(fx+e)}+1}}\,f}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*I*a/f/(c-I*c*tan(f*x+e))^(5/2)
```

Maxima [**A**]
time = 0.28, size = 20, normalized size = 0.74

$$-\frac{2i\,a}{5\,(-i\,c\tan{(fx+e)}+c)^{\frac{5}{2}}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="maxima"
)
```

```
[Out] -2/5*I*a/((-I*c*tan(f*x + e) + c)^(5/2)*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(20) = 40$.
time = 1.51, size = 72, normalized size = 2.67

$$\frac{\sqrt{2}\left(-i\,ae^{(6i\,fx+6i\,e)}-3i\,ae^{(4i\,fx+4i\,e)}-3i\,ae^{(2i\,fx+2i\,e)}-i\,a\right)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{20\,c^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/20*sqrt(2)*(-I*a*e^(6*I*f*x + 6*I*e) - 3*I*a*e^(4*I*f*x + 4*I*e) - 3*I*a*
e^(2*I*f*x + 2*I*e) - I*a)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(c^3*f)
```

Sympy [A]
time = 9.30, size = 46, normalized size = 1.70

$$\begin{cases} -\frac{2ia}{5f(-ic\tan{(e+fx)}+c)^{\frac{5}{2}}} & \text{for } f \neq 0 \\ \frac{x(ia\tan{(e)}+a)}{(-ic\tan{(e)}+c)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))**(5/2),x)
```

```
[Out] Piecewise((-2*I*a/(5*f*(-I*c*tan(e + f*x) + c)**(5/2)), Ne(f, 0)), (x*(I*a*
tan(e) + a)/(-I*c*tan(e) + c)**(5/2), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)/(-I*c*tan(f*x + e) + c)^(5/2), x)
```

Mupad [B]
time = 5.49, size = 118, normalized size = 4.37

$$\frac{a\sqrt{-\frac{c\left(-2\cos\left(e+f\,x\right)^2+\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{2\cos\left(e+f\,x\right)^2}}\,\left(-\cos\left(e+f\,x\right)^2\,6{}\mathrm{i}-\cos\left(2\,e+2\,f\,x\right)^2\,6{}\mathrm{i}-\cos\left(3\,e+3\,f\,x\right)^2\,2{}\mathrm{i}+3\,\sin\left(2\,e+2\,f\,x\right)+3\,\sin\left(4\,e+4\,f\,x\right)+\sin\left(6\,e+6\,f\,x\right)+6{}\mathrm{i}\right)}{20\,c^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)/(c - c*tan(e + f*x)*1i)^(5/2),x)
```

```
[Out] (a*(-(c*(sin(2*e + 2*f*x)*1i - 2*cos(e + f*x)^2))/(2*cos(e + f*x)^2))^(1/2)
*(3*sin(2*e + 2*f*x) + 3*sin(4*e + 4*f*x) + sin(6*e + 6*f*x) - cos(2*e + 2*
f*x)^2*6i - cos(3*e + 3*f*x)^2*2i - cos(e + f*x)^2*6i + 6i))/(20*c^3*f)
```

3.989 $\int \frac{1}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=188

$$\frac{7i\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{16\sqrt{2}\,ac^{5/2}f}-\frac{7i}{20af(c-ic\tan(e+fx))^{5/2}}+\frac{i}{2af(1+i\tan(e+fx))(c-ic\tan(e+f}$$

```
[Out] 7/32*I*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))/a/c^(5/2)/f*2^
(1/2)-7/16*I/a/c^2/f/(c-I*c*tan(f*x+e))^(1/2)-7/20*I/a/f/(c-I*c*tan(f*x+e))
^(5/2)+1/2*I/a/f/(1+I*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2)-7/24*I/a/c/f/(c-
I*c*tan(f*x+e))^(3/2)
```

Rubi [A]

time = 0.16, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3603, 3568, 44, 53, 65, 212}

$$\frac{7i\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{16\sqrt{2}\,ac^{5/2}f}-\frac{7i}{16ac^2f\sqrt{c-ic\tan(e+fx)}}-\frac{7i}{24acf(c-ic\tan(e+fx))^{3/2}}-\frac{7i}{20af(c-ic\tan(e+fx))^{5/2}}+\frac{i}{2af(1+i\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(5/2)),x]
```

```
[Out] (((7*I)/16)*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])])/(Sqrt[2]
*a*c^(5/2)*f) - ((7*I)/20)/(a*f*(c - I*c*Tan[e + f*x])^(5/2)) + (I/2)/(a*f*
(1 + I*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(5/2)) - ((7*I)/24)/(a*c*f*(c -
 I*c*Tan[e + f*x])^(3/2)) - ((7*I)/16)/(a*c^2*f*Sqrt[c - I*c*Tan[e + f*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{\int \frac{\cos^2(e+fx)}{(c-ic\tan(e+fx))^{3/2}}\,dx}{ac}$$

$$= \frac{(ic^2)\,\text{Subst}\left(\int \frac{1}{(c-x)^2(c+x)^{7/2}}\,dx, x, -ic\tan(e+fx)\right)}{af}$$

$$= \frac{i}{2af(1+i\tan(e+fx))(c-ic\tan(e+fx))^{5/2}} + \frac{(7ic)\text{S}}{}$$

$$= -\frac{7i}{20af(c-ic\tan(e+fx))^{5/2}} + \frac{}{2af(1+i\tan(e+fx)}$$

$$= -\frac{7i}{20af(c-ic\tan(e+fx))^{5/2}} + \frac{}{2af(1+i\tan(e+fx)}$$

$$= -\frac{7i}{20af(c-ic\tan(e+fx))^{5/2}} + \frac{}{2af(1+i\tan(e+fx)}$$

$$= -\frac{7i}{20af(c-ic\tan(e+fx))^{5/2}} + \frac{}{2af(1+i\tan(e+fx)}$$

$$= \frac{7i\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{16\sqrt{2}\,ac^{5/2}f} - \frac{7i}{20af(c-ic\tan}$$

Mathematica [A]

time = 2.52, size = 149, normalized size = 0.79

$$\frac{(\cos(2(e+fx))+i\sin(2(e+fx)))\left(-148i+105ie^{-2i(e+fx)}\sqrt{1+e^{2i(e+fx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(e+fx)}}\right)-139i\cos(2(e+fx))+9i\cos(4(e+fx))-63\sin(2(e+fx))+21\sin(4(e+fx))\right)\sqrt{c-ic\tan(e+fx)}}{480ac^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(5/2)),x]
```

```
[Out] ((Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)])*(-148*I + ((105*I)*Sqrt[1 + E^((2*
I)*(e + f*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(e + f*x))]])/E^((2*I)*(e + f*x))
- (139*I)*Cos[2*(e + f*x)] + (9*I)*Cos[4*(e + f*x)] - 63*Sin[2*(e + f*x)] +
 21*Sin[4*(e + f*x)])*Sqrt[c - I*c*Tan[e + f*x]])/(480*a*c^3*f)
```

Maple [A]

time = 0.32, size = 140, normalized size = 0.74

method	result

derivativedivides	$\frac{2ic^2\left(-\frac{3}{16c^4\sqrt{c-ic\tan(fx+e)}}-\frac{1}{12c^3(c-ic\tan(fx+e))^{\frac{3}{2}}}-\frac{1}{20c^2(c-ic\tan(fx+e))^{\frac{5}{2}}}+\frac{\frac{\sqrt{c-ic\tan(fx+e}}{2c+2ic\tan(fx+e)}}{}\right.}{fa}$
default	$\frac{2ic^2\left(-\frac{3}{16c^4\sqrt{c-ic\tan(fx+e)}}-\frac{1}{12c^3(c-ic\tan(fx+e))^{\frac{3}{2}}}-\frac{1}{20c^2(c-ic\tan(fx+e))^{\frac{5}{2}}}+\frac{\frac{\sqrt{c-ic\tan(fx+e}}{2c+2ic\tan(fx+e)}}{}\right.}{fa}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/f/a*c^2*(-3/16/c^4/(c-I*c*tan(f*x+e))^(1/2)-1/12/c^3/(c-I*c*tan(f*x+e))
^(3/2)-1/20/c^2/(c-I*c*tan(f*x+e))^(5/2)+1/16/c^4*(1/4*(c-I*c*tan(f*x+e))^(
1/2)/(1/2*c+1/2*I*c*tan(f*x+e))+7/4*2^(1/2)/c^(1/2)*arctanh(1/2*(c-I*c*tan(
f*x+e))^(1/2)*2^(1/2)/c^(1/2))))
```

Maxima [A]
time = 0.49, size = 169, normalized size = 0.90

$$-\frac{i\left(\frac{4\left(105(-ic\tan(fx+e)+c)^3-140(-ic\tan(fx+e)+c)^2c-56(-ic\tan(fx+e)+c)c^2-48c^3\right)}{(-ic\tan(fx+e)+c)^{\frac{7}{2}}ac-2(-ic\tan(fx+e)+c)^{\frac{5}{2}}ac^2}+\frac{105\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-ic\tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic\tan(fx+e)+c}}\right)}{ac^{\frac{3}{2}}}\right)}{960\,cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -1/960*I*(4*(105*(-I*c*tan(f*x + e) + c)^3 - 140*(-I*c*tan(f*x + e) + c)^2*
c - 56*(-I*c*tan(f*x + e) + c)*c^2 - 48*c^3)/((-I*c*tan(f*x + e) + c)^(7/2)
*a*c - 2*(-I*c*tan(f*x + e) + c)^(5/2)*a*c^2) + 105*sqrt(2)*log(-(sqrt(2)*s
qrt(c) - sqrt(-I*c*tan(f*x + e) + c))/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x
+ e) + c)))/(a*c^(3/2)))/(c*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(145) = 290$.
time = 1.25, size = 330, normalized size = 1.76

$$\frac{\left(-105i\sqrt{\frac{1}{2}}ac^3f\sqrt{\frac{1}{a^2c^5f^2}}e^{(2ifx+2ie)}\log\left(-\frac{7\left(\sqrt{2}\sqrt{\frac{1}{2}}(iac^2fe^{(2ifx+2ie)}+iac^2f)\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{1}{a^2c^5f^2}}-i\right)e^{(-ifx-ie)}}{8ac^2f}\right)+105i\sqrt{\frac{1}{2}}ac^3f\sqrt{\frac{1}{a^2c^5f^2}}e^{(2ifx+2ie)}\log\left(-\frac{7\left(\sqrt{2}\sqrt{\frac{1}{2}}(-iac^2fe^{(2ifx+2ie)}-iac^2f)\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{1}{a^2c^5f^2}}-i\right)e^{(-ifx-ie)}}{8ac^2f}\right)+\sqrt{2}\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}\left(-6ie^{(8ifx+8ie)}-38ie^{(6ifx+6ie)}-148ie^{(4ifx+4ie)}-101ie^{(2ifx+2ie)}+15i\right)\right)e^{(-2ifx-2ie)}}{480\,ac^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/480*(-105*I*sqrt(1/2)*a*c^3*f*sqrt(1/(a^2*c^5*f^2))*e^(2*I*f*x + 2*I*e)*l
og(-7/8*(sqrt(2)*sqrt(1/2)*(I*a*c^2*f*e^(2*I*f*x + 2*I*e) + I*a*c^2*f)*sqrt
(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(1/(a^2*c^5*f^2)) - I)*e^(-I*f*x - I*e)/(
a*c^2*f)) + 105*I*sqrt(1/2)*a*c^3*f*sqrt(1/(a^2*c^5*f^2))*e^(2*I*f*x + 2*I*
e)*log(-7/8*(sqrt(2)*sqrt(1/2)*(-I*a*c^2*f*e^(2*I*f*x + 2*I*e) - I*a*c^2*f)
*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(1/(a^2*c^5*f^2)) - I)*e^(-I*f*x - I
*e)/(a*c^2*f)) + sqrt(2)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*(-6*I*e^(8*I*f*x
 + 8*I*e) - 38*I*e^(6*I*f*x + 6*I*e) - 148*I*e^(4*I*f*x + 4*I*e) - 101*I*e^
(2*I*f*x + 2*I*e) + 15*I))*e^(-2*I*f*x - 2*I*e)/(a*c^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int \frac{1}{-c^{2}\sqrt{-ic\tan\left(e+fx\right)+c}\tan^{3}\left(e+fx\right)-ic^{2}\sqrt{-ic\tan\left(e+fx\right)+c}\tan^{2}\left(e+fx\right)-c^{2}\sqrt{-ic\tan\left(e+fx\right)+c}\tan\left(e+fx\right)-ic^{2}\sqrt{-ic\tan\left(e+fx\right)+c}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))**(5/2),x)
```

```
[Out] -I*Integral(1/(-c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**3 - I*c**2*s
qrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - c**2*sqrt(-I*c*tan(e + f*x) +
c)*tan(e + f*x) - I*c**2*sqrt(-I*c*tan(e + f*x) + c)), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((I*a*tan(f*x + e) + a)*(-I*c*tan(f*x + e) + c)^(5/2)), x)
```

Mupad [B]
time = 0.64, size = 165, normalized size = 0.88

$$-\frac{\frac{c\,1\mathrm{i}}{5\,a\,f}+\frac{\left(c-c\,\tan(e+f\,x)\,1\mathrm{i}\right)\,7\mathrm{i}}{30\,a\,f}+\frac{{\left(c-c\,\tan(e+f\,x)\,1\mathrm{i}\right)}^2\,7\mathrm{i}}{12\,a\,c\,f}-\frac{{\left(c-c\,\tan(e+f\,x)\,1\mathrm{i}\right)}^3\,7\mathrm{i}}{16\,a\,c^2\,f}}{2\,c\,{\left(c-c\,\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}^{5/2}-{\left(c-c\,\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}^{7/2}}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\,\tan\left(e+f\,x\right)\,1\mathrm{i}}}{2\,\sqrt{-c}}\right)\,7\mathrm{i}}{32\,a\,{\left(-c\right)}^{5/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i)^(5/2)),x)
```

```
[Out] - ((c*1i)/(5*a*f) + ((c - c*tan(e + f*x)*1i)*7i)/(30*a*f) + ((c - c*tan(e +
 f*x)*1i)^2*7i)/(12*a*c*f) - ((c - c*tan(e + f*x)*1i)^3*7i)/(16*a*c^2*f))/(
2*c*(c - c*tan(e + f*x)*1i)^(5/2) - (c - c*tan(e + f*x)*1i)^(7/2)) - (2^(1/
2)*atan((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*(-c)^(1/2)))*7i)/(32*a*(
-c)^(5/2)*f)
```

3.990 $\int \frac{1}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=231

$$\frac{63i\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{128\sqrt{2}\,a^2c^{5/2}f}-\frac{63i}{160a^2f(c-ic\tan(e+fx))^{5/2}}+\frac{i}{4a^2f(1+i\tan(e+fx))^2(c-ic\tan(}$$

```
[Out] 63/256*I*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))/a^2/c^(5/2)/
f*2^(1/2)-63/128*I/a^2/c^2/f/(c-I*c*tan(f*x+e))^(1/2)-63/160*I/a^2/f/(c-I*c
*tan(f*x+e))^(5/2)+1/4*I/a^2/f/(1+I*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(5/2)+
9/16*I/a^2/f/(1+I*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2)-21/64*I/a^2/c/f/(c-I
*c*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.17, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3603, 3568, 44, 53, 65, 212}

$$\frac{63i\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{128\sqrt{2}\,a^2c^{5/2}f}-\frac{63i}{128a^2c^2f\sqrt{c-ic\tan(e+fx)}}-\frac{21i}{64a^2cf(c-ic\tan(e+fx))^{3/2}}-\frac{63i}{160a^2f(c-ic\tan(e+fx))^{5/2}}+\frac{9i}{16a^2f(1+i\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}+\frac{i}{4a^2f(1+i\tan(e+fx))^2(c-ic\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^(5/2)),x]
```

```
[Out] (((63*I)/128)*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])])/(Sqrt[
2]*a^2*c^(5/2)*f) - ((63*I)/160)/(a^2*f*(c - I*c*Tan[e + f*x])^(5/2)) + (I/
4)/(a^2*f*(1 + I*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^(5/2)) + ((9*I)/16)
/(a^2*f*(1 + I*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(5/2)) - ((21*I)/64)/(a
^2*c*f*(c - I*c*Tan[e + f*x])^(3/2)) - ((63*I)/128)/(a^2*c^2*f*Sqrt[c - I*c
*Tan[e + f*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
```

```
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{\int \frac{\cos^4(e+fx)}{\sqrt{c-ic\tan(e+fx)}}\,dx}{a^2c^2}$$

$$= \frac{(ic^3)\,\text{Subst}\left(\int \frac{1}{(c-x)^3(c+x)^{7/2}}\,dx, x, -ic\tan(e+fx)\right)}{a^2f}$$

$$= \frac{i}{4a^2f(1+i\tan(e+fx))^2(c-ic\tan(e+fx))^{5/2}} + \frac{(9i}{}$$

$$= \frac{i}{4a^2f(1+i\tan(e+fx))^2(c-ic\tan(e+fx))^{5/2}} + \frac{}{16}$$

$$= -\frac{63i}{160a^2f(c-ic\tan(e+fx))^{5/2}} + \frac{}{4a^2f(1+i\tan(e+}$$

$$= -\frac{63i}{160a^2f(c-ic\tan(e+fx))^{5/2}} + \frac{}{4a^2f(1+i\tan(e+}$$

$$= -\frac{63i}{160a^2f(c-ic\tan(e+fx))^{5/2}} + \frac{}{4a^2f(1+i\tan(e+}$$

$$= -\frac{63i}{160a^2f(c-ic\tan(e+fx))^{5/2}} + \frac{}{4a^2f(1+i\tan(e+}$$

$$= \frac{63i\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{128\sqrt{2}\,a^2c^{5/2}f} - \frac{6}{160a^2f(c-ic}$$

Mathematica [A]
time = 3.66, size = 182, normalized size = 0.79

$$\frac{\sec^2(e+fx)(-i\cos(3(e+fx))+\sin(3(e+fx)))\left(315e^{-i(e+fx)}\sqrt{1+e^{2i(e+fx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(e+fx)}}\right)-547\cos(e+fx)+31\cos(3(e+fx))+2\cos(5(e+fx))-141i\sin(e+fx)-159i\sin(3(e+fx))-18i\sin(5(e+fx))\right)\sqrt{c-ic\tan(e+fx)}}{1280a^2c^3f(-i+\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^(5/2)),x]
```

```
[Out] (Sec[e + f*x]^2*((-I)*Cos[3*(e + f*x)] + Sin[3*(e + f*x)])*((315*Sqrt[1 + E
^((2*I)*(e + f*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(e + f*x))]])/E^(I*(e + f*x))
 - 547*Cos[e + f*x] + 31*Cos[3*(e + f*x)] + 2*Cos[5*(e + f*x)] - (141*I)*Si
n[e + f*x] - (159*I)*Sin[3*(e + f*x)] - (18*I)*Sin[5*(e + f*x)])*Sqrt[c - I
*c*Tan[e + f*x]])/(1280*a^2*c^3*f*(-I + Tan[e + f*x])^2)
```

Maple [A]
time = 0.34, size = 158, normalized size = 0.68

method	result
derivativedivides	$-\frac{2ic^3\left(-\frac{\frac{-\frac{15(c-ic\tan(fx+e))^{\frac{3}{2}}}{16}+\frac{17c\sqrt{c-ic\tan(fx+e)}}{8}}{(c+ic\tan(fx+e))^2}+\frac{63\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{\cdots}}{2\sqrt{c}}\right)}{32\sqrt{c}}}{16c^5}\cdots\right)}{f\,a^2}$
default	$-\frac{2ic^3\left(-\frac{\frac{-\frac{15(c-ic\tan(fx+e))^{\frac{3}{2}}}{16}+\frac{17c\sqrt{c-ic\tan(fx+e)}}{8}}{(c+ic\tan(fx+e))^2}+\frac{63\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{\cdots}}{2\sqrt{c}}\right)}{32\sqrt{c}}}{16c^5}\cdots\right)}{f\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -2*I/f/a^2*c^3*(-1/16/c^5*(4*(-15/64*(c-I*c*tan(f*x+e))^(3/2)+17/32*c*(c-I*
c*tan(f*x+e))^(1/2))/(c+I*c*tan(f*x+e))^2+63/32*2^(1/2)/c^(1/2)*arctanh(1/2
*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2)))+3/16/c^5/(c-I*c*tan(f*x+e))^(1/
2)+1/16/c^4/(c-I*c*tan(f*x+e))^(3/2)+1/40/c^3/(c-I*c*tan(f*x+e))^(5/2))
```

Maxima [A]
time = 0.50, size = 214, normalized size = 0.93

$$-\frac{i\left(\frac{4\left(315\,(-ic\tan(fx+e)+c)^4-1050\,(-ic\tan(fx+e)+c)^3c+672\,(-ic\tan(fx+e)+c)^2c^2+192\,(-ic\tan(fx+e)+c)c^3+128\,c^4\right)}{(-ic\tan(fx+e)+c)^{\frac{9}{2}}a^2c-4\,(-ic\tan(fx+e)+c)^{\frac{7}{2}}a^2c^2+4\,(-ic\tan(fx+e)+c)^{\frac{5}{2}}a^2c^3}+\frac{315\sqrt{2}\,\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-ic\tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic\tan(fx+e)+c}}\right)}{a^2c^{\frac{3}{2}}}\right)}{2560\,cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] -1/2560*I*(4*(315*(-I*c*tan(f*x + e) + c)^4 - 1050*(-I*c*tan(f*x + e) + c)^
3*c + 672*(-I*c*tan(f*x + e) + c)^2*c^2 + 192*(-I*c*tan(f*x + e) + c)*c^3 +
 128*c^4)/((-I*c*tan(f*x + e) + c)^(9/2)*a^2*c - 4*(-I*c*tan(f*x + e) + c)^
(7/2)*a^2*c^2 + 4*(-I*c*tan(f*x + e) + c)^(5/2)*a^2*c^3) + 315*sqrt(2)*log(
-(sqrt(2)*sqrt(c) - sqrt(-I*c*tan(f*x + e) + c))/(sqrt(2)*sqrt(c) + sqrt(-I
*c*tan(f*x + e) + c)))/(a^2*c^(3/2)))/(c*f)
```

Fricas [A]
time = 1.50, size = 354, normalized size = 1.53

$$\frac{\left(-315i\sqrt{\frac{1}{2}}a^2c^3f\sqrt{\frac{1}{a^4c^5f^2}}e^{(4ifx+4ie)}\log\left(-\frac{63\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(ia^2c^2fe^{(2ifx+2ie)}+ia^2c^2f\right)\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{1}{a^4c^5f^2}}-i\right)e^{(-ifx-ie)}}{64a^2c^2f}\right)+315i\sqrt{\frac{1}{2}}a^2c^3f\sqrt{\frac{1}{a^4c^5f^2}}e^{(4ifx+4ie)}\log\left(-\frac{63\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(-ia^2c^2fe^{(2ifx+2ie)}-ia^2c^2f\right)\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{1}{a^4c^5f^2}}-i\right)e^{(-ifx-ie)}}{64a^2c^2f}\right)+\sqrt{2}\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}\left(-8ie^{(10ifx+10ie)}-64ie^{(8ifx+8ie)}-344ie^{(6ifx+6ie)}-203ie^{(4ifx+4ie)}+95ie^{(2ifx+2ie)}+10i\right)\right)e^{(-4ifx-4ie)}}{1280a^2c^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] 1/1280*(-315*I*sqrt(1/2)*a^2*c^3*f*sqrt(1/(a^4*c^5*f^2))*e^(4*I*f*x + 4*I*e
)*log(-63/64*(sqrt(2)*sqrt(1/2)*(I*a^2*c^2*f*e^(2*I*f*x + 2*I*e) + I*a^2*c^
2*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(1/(a^4*c^5*f^2)) - I)*e^(-I*f*x
 - I*e)/(a^2*c^2*f)) + 315*I*sqrt(1/2)*a^2*c^3*f*sqrt(1/(a^4*c^5*f^2))*e^(4
*I*f*x + 4*I*e)*log(-63/64*(sqrt(2)*sqrt(1/2)*(-I*a^2*c^2*f*e^(2*I*f*x + 2*
I*e) - I*a^2*c^2*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(1/(a^4*c^5*f^2))
 - I)*e^(-I*f*x - I*e)/(a^2*c^2*f)) + sqrt(2)*sqrt(c/(e^(2*I*f*x + 2*I*e) +
 1))*(-8*I*e^(10*I*f*x + 10*I*e) - 64*I*e^(8*I*f*x + 8*I*e) - 344*I*e^(6*I*
f*x + 6*I*e) - 203*I*e^(4*I*f*x + 4*I*e) + 95*I*e^(2*I*f*x + 2*I*e) + 10*I)
)*e^(-4*I*f*x - 4*I*e)/(a^2*c^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{-c^2\sqrt{-ic\tan(e+fx)+c}\tan^4(e+fx)-2c^2\sqrt{-ic\tan(e+fx)+c}\tan^2(e+fx)-c^2\sqrt{-ic\tan(e+fx)+c}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**2/(c-I*c*tan(f*x+e))**(5/2),x)
```

```
[Out] -Integral(1/(-c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**4 - 2*c**2*sqr
t(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - c**2*sqrt(-I*c*tan(e + f*x) + c)
), x)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((I*a*tan(f*x + e) + a)^2*(-I*c*tan(f*x + e) + c)^(5/2)), x)
```

Mupad [B]
time = 5.64, size = 208, normalized size = 0.90

$$-\frac{\frac{(c-c\tan(e+f\,x)\,1\mathrm{i})^2\,21\mathrm{i}}{20\,a^2\,f}+\frac{c^2\,1\mathrm{i}}{5\,a^2\,f}-\frac{(c-c\tan(e+f\,x)\,1\mathrm{i})^3\,105\mathrm{i}}{64\,a^2\,c\,f}+\frac{(c-c\tan(e+f\,x)\,1\mathrm{i})^4\,63\mathrm{i}}{128\,a^2\,c^2\,f}+\frac{c\,(c-c\tan(e+f\,x)\,1\mathrm{i})\,3\mathrm{i}}{10\,a^2\,f}}{-4\,c\,(c-c\tan\left(e+f\,x\right)\,1\mathrm{i})^{7/2}+(c-c\tan\left(e+f\,x\right)\,1\mathrm{i})^{9/2}+4\,c^2\,(c-c\tan\left(e+f\,x\right)\,1\mathrm{i})^{5/2}}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}}{2\,\sqrt{-c}}\right)\,63\mathrm{i}}{256\,a^2\,(-c)^{5/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i)^(5/2)),x)
```

```
[Out] - (((c - c*tan(e + f*x)*1i)^2*21i)/(20*a^2*f) + (c^2*1i)/(5*a^2*f) - ((c -
c*tan(e + f*x)*1i)^3*105i)/(64*a^2*c*f) + ((c - c*tan(e + f*x)*1i)^4*63i)/(
128*a^2*c^2*f) + (c*(c - c*tan(e + f*x)*1i)*3i)/(10*a^2*f))/((c - c*tan(e +
 f*x)*1i)^(9/2) - 4*c*(c - c*tan(e + f*x)*1i)^(7/2) + 4*c^2*(c - c*tan(e +
f*x)*1i)^(5/2)) - (2^(1/2)*atan((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*
(-c)^(1/2)))*63i)/(256*a^2*(-c)^(5/2)*f)
```

3.991 $\int \frac{1}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=274

$$\frac{231i\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{512\sqrt{2}\,a^3c^{5/2}f}-\frac{231i}{640a^3f(c-ic\tan(e+fx))^{5/2}}+\frac{i}{6a^3f(1+i\tan(e+fx))^3(c-ic\tan}$$

```
[Out] 231/1024*I*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))/a^3/c^(5/2
)/f*2^(1/2)-231/512*I/a^3/c^2/f/(c-I*c*tan(f*x+e))^(1/2)-231/640*I/a^3/f/(c
-I*c*tan(f*x+e))^(5/2)+1/6*I/a^3/f/(1+I*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(5
/2)+11/48*I/a^3/f/(1+I*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(5/2)+33/64*I/a^3/f
/(1+I*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2)-77/256*I/a^3/c/f/(c-I*c*tan(f*x+
e))^(3/2)
```

Rubi [A]

time = 0.18, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3603, 3568, 44, 53, 65, 212}

$$\frac{231i\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{512\sqrt{2}\,a^3c^{5/2}f}-\frac{231i}{512a^3c^2f\sqrt{c-ic\tan(e+fx)}}-\frac{77i}{256a^3cf(c-ic\tan(e+fx))^{3/2}}-\frac{231i}{640a^3f(c-ic\tan(e+fx))^{5/2}}+\frac{33i}{64a^3f(1+i\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}+\frac{11i}{48a^3f(1+i\tan(e+fx))^2(c-ic\tan(e+fx))^{5/2}}+\frac{i}{6a^3f(1+i\tan(e+fx))^3(c-ic\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])^(5/2)),x]
```

```
[Out] (((231*I)/512)*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])])/(Sqrt
[2]*a^3*c^(5/2)*f) - ((231*I)/640)/(a^3*f*(c - I*c*Tan[e + f*x])^(5/2)) + (
I/6)/(a^3*f*(1 + I*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])^(5/2)) + ((11*I)/
48)/(a^3*f*(1 + I*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^(5/2)) + ((33*I)/6
4)/(a^3*f*(1 + I*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(5/2)) - ((77*I)/256)
/(a^3*c*f*(c - I*c*Tan[e + f*x])^(3/2)) - ((231*I)/512)/(a^3*c^2*f*Sqrt[c -
 I*c*Tan[e + f*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{\int \cos^6(e+fx)\sqrt{c-ic\tan(e+fx)}\,dx}{a^3c^3}$$

$$= \frac{(ic^4)\operatorname{Subst}\left(\int \frac{1}{(c-x)^4(c+x)^{7/2}}\,dx, x, -ic\tan(e+fx)\right)}{a^3 f}$$

$$= \frac{i}{6a^3f(1+i\tan(e+fx))^3(c-ic\tan(e+fx))^{5/2}} + \frac{(11}{}$$

$$= \frac{i}{6a^3f(1+i\tan(e+fx))^3(c-ic\tan(e+fx))^{5/2}} + \frac{}{48}$$

$$= \frac{i}{6a^3f(1+i\tan(e+fx))^3(c-ic\tan(e+fx))^{5/2}} + \frac{}{48}$$

$$= -\frac{231i}{640a^3f(c-ic\tan(e+fx))^{5/2}} + \frac{}{6a^3f(1+i\tan(e+}$$

$$= -\frac{231i}{640a^3f(c-ic\tan(e+fx))^{5/2}} + \frac{}{6a^3f(1+i\tan(e+}$$

$$= -\frac{231i}{640a^3f(c-ic\tan(e+fx))^{5/2}} + \frac{}{6a^3f(1+i\tan(e+}$$

$$= -\frac{231i}{640a^3f(c-ic\tan(e+fx))^{5/2}} + \frac{}{6a^3f(1+i\tan(e+}$$

$$= \frac{231i\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{512\sqrt{2}\,a^3c^{5/2}f} - \frac{}{640a^3f(c-i}$$

Mathematica [A]
time = 3.30, size = 171, normalized size = 0.62

$$-\frac{ie^{-6i(e+fx)}\left(-40-350e^{2i(e+fx)}-1645e^{4i(e+fx)}+1433e^{6i(e+fx)}+3184e^{8i(e+fx)}+464e^{10i(e+fx)}+48e^{12i(e+fx)}-3465e^{6i(e+fx)}\sqrt{1+e^{2i(e+fx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(e+fx)}}\right)\right)\sqrt{c-ic\tan(e+fx)}}{15360a^3c^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])^(5/2)),x]
```

```
[Out] ((-1/15360*I)*(-40 - 350*E^((2*I)*(e + f*x)) - 1645*E^((4*I)*(e + f*x)) + 1
433*E^((6*I)*(e + f*x)) + 3184*E^((8*I)*(e + f*x)) + 464*E^((10*I)*(e + f*x
)) + 48*E^((12*I)*(e + f*x)) - 3465*E^((6*I)*(e + f*x))*Sqrt[1 + E^((2*I)*(
```

```
e + f*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(e + f*x))]])*Sqrt[c - I*c*Tan[e + f*x
]])/(a^3*c^3*E^((6*I)*(e + f*x))*f)
```

Maple [A]
time = 0.30, size = 177, normalized size = 0.65

method	result
derivativedivides	$\frac{2ic^4\left(-\frac{5}{32c^6\sqrt{c-ic\tan(fx+e)}}-\frac{1}{24c^5(c-ic\tan(fx+e))^{\frac{3}{2}}}-\frac{1}{80c^4(c-ic\tan(fx+e))^{\frac{5}{2}}}+\frac{\frac{71(c-ic\tan(fx+e))^{\frac{5}{2}}}{32}-59c(c-}{}\right.}{f a^3}$
default	$\frac{2ic^4\left(-\frac{5}{32c^6\sqrt{c-ic\tan(fx+e)}}-\frac{1}{24c^5(c-ic\tan(fx+e))^{\frac{3}{2}}}-\frac{1}{80c^4(c-ic\tan(fx+e))^{\frac{5}{2}}}+\frac{\frac{71(c-ic\tan(fx+e))^{\frac{5}{2}}}{32}-59c(c-}{}\right.}{f a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 2*I/f/a^3*c^4*(-5/32/c^6/(c-I*c*tan(f*x+e))^(1/2)-1/24/c^5/(c-I*c*tan(f*x+e
))^(3/2)-1/80/c^4/(c-I*c*tan(f*x+e))^(5/2)+1/32/c^6*(8*(71/256*(c-I*c*tan(f
*x+e))^(5/2)-59/48*c*(c-I*c*tan(f*x+e))^(3/2)+89/64*c^2*(c-I*c*tan(f*x+e))^
(1/2))/(c+I*c*tan(f*x+e))^3+231/64*2^(1/2)/c^(1/2)*arctanh(1/2*(c-I*c*tan(f
*x+e))^(1/2)*2^(1/2)/c^(1/2))))
```

Maxima [A]
time = 0.49, size = 255, normalized size = 0.93

$$-\frac{i\left(\frac{4\left(3465\,(-i\,c\tan(fx+e)+c)^5-18480\,(-i\,c\tan(fx+e)+c)^4c+30492\,(-i\,c\tan(fx+e)+c)^3c^2-12672\,(-i\,c\tan(fx+e)+c)^2c^3-2816\,(-i\,c\tan(fx+e)+c)c^4-1536\,c^5\right)}{(-i\,c\tan(fx+e)+c)^{\frac{11}{2}}a^3c-6\,(-i\,c\tan(fx+e)+c)^{\frac{9}{2}}a^3c^2+12\,(-i\,c\tan(fx+e)+c)^{\frac{7}{2}}a^3c^3-8\,(-i\,c\tan(fx+e)+c)^{\frac{5}{2}}a^3c^4}+\frac{3465\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-i\,c\tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-i\,c\tan(fx+e)+c}}\right)}{a^3c^{\frac{3}{2}}}\right)}{30720\,cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] -1/30720*I*(4*(3465*(-I*c*tan(f*x + e) + c)^5 - 18480*(-I*c*tan(f*x + e) +
c)^4*c + 30492*(-I*c*tan(f*x + e) + c)^3*c^2 - 12672*(-I*c*tan(f*x + e) + c
)^2*c^3 - 2816*(-I*c*tan(f*x + e) + c)*c^4 - 1536*c^5)/((-I*c*tan(f*x + e)
```

```
+ c)^(11/2)*a^3*c - 6*(-I*c*tan(f*x + e) + c)^(9/2)*a^3*c^2 + 12*(-I*c*tan(
f*x + e) + c)^(7/2)*a^3*c^3 - 8*(-I*c*tan(f*x + e) + c)^(5/2)*a^3*c^4) + 34
65*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*c*tan(f*x + e) + c))/(sqrt(2)*sq
rt(c) + sqrt(-I*c*tan(f*x + e) + c)))/(a^3*c^(3/2)))/(c*f)
```

Fricas [A]
time = 1.11, size = 366, normalized size = 1.34

$$\frac{\left(-3465i\sqrt{\frac{1}{2}}\,a^3c^3f\sqrt{\frac{1}{a^6c^5f^2}}\,e^{(6i\,fx+6i\,e)}\log\left(-\frac{231\left(\sqrt{2}\sqrt{\frac{1}{2}}\,(i\,a^3c^2fe^{(2i\,fx+2i\,e)}+i\,a^3c^2f)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{1}{a^6c^5f^2}}-i\right)e^{(-i\,fx-i\,e)}}{256\,a^3c^2f}\right)+3465i\sqrt{\frac{1}{2}}\,a^3c^3f\sqrt{\frac{1}{a^6c^5f^2}}\,e^{(6i\,fx+6i\,e)}\log\left(-\frac{231\left(\sqrt{2}\sqrt{\frac{1}{2}}\,(-i\,a^3c^2fe^{(2i\,fx+2i\,e)}-i\,a^3c^2f)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{1}{a^6c^5f^2}}-i\right)e^{(-i\,fx-i\,e)}}{256\,a^3c^2f}\right)+\sqrt{2}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\left(-48i\,e^{(12i\,fx+12i\,e)}-464i\,e^{(10i\,fx+10i\,e)}-3184i\,e^{(8i\,fx+8i\,e)}-1433i\,e^{(6i\,fx+6i\,e)}+1645i\,e^{(4i\,fx+4i\,e)}+350i\,e^{(2i\,fx+2i\,e)}+40i\right)\right)e^{(-6i\,fx-6i\,e)}}{15360\,a^3c^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] 1/15360*(-3465*I*sqrt(1/2)*a^3*c^3*f*sqrt(1/(a^6*c^5*f^2))*e^(6*I*f*x + 6*I
*e)*log(-231/256*(sqrt(2)*sqrt(1/2)*(I*a^3*c^2*f*e^(2*I*f*x + 2*I*e) + I*a^
3*c^2*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(1/(a^6*c^5*f^2)) - I)*e^(-I
*f*x - I*e)/(a^3*c^2*f)) + 3465*I*sqrt(1/2)*a^3*c^3*f*sqrt(1/(a^6*c^5*f^2))
*e^(6*I*f*x + 6*I*e)*log(-231/256*(sqrt(2)*sqrt(1/2)*(-I*a^3*c^2*f*e^(2*I*f
*x + 2*I*e) - I*a^3*c^2*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(1/(a^6*c^
5*f^2)) - I)*e^(-I*f*x - I*e)/(a^3*c^2*f)) + sqrt(2)*sqrt(c/(e^(2*I*f*x + 2
*I*e) + 1))*(-48*I*e^(12*I*f*x + 12*I*e) - 464*I*e^(10*I*f*x + 10*I*e) - 31
84*I*e^(8*I*f*x + 8*I*e) - 1433*I*e^(6*I*f*x + 6*I*e) + 1645*I*e^(4*I*f*x +
 4*I*e) + 350*I*e^(2*I*f*x + 2*I*e) + 40*I))*e^(-6*I*f*x - 6*I*e)/(a^3*c^3*
f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int\frac{1}{-c^2\sqrt{-ic\tan(e+fx)+c}\,\tan^5(e+fx)+ic^2\sqrt{-ic\tan(e+fx)+c}\,\tan^4(e+fx)-2c^2\sqrt{-ic\tan(e+fx)+c}\,\tan^3(e+fx)+2ic^2\sqrt{-ic\tan(e+fx)+c}\,\tan^2(e+fx)-c^2\sqrt{-ic\tan(e+fx)+c}\,\tan(e+fx)+ic^2\sqrt{-ic\tan(e+fx)+c}}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**3/(c-I*c*tan(f*x+e))**(5/2),x)
```

```
[Out] I*Integral(1/(-c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**5 + I*c**2*sq
rt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**4 - 2*c**2*sqrt(-I*c*tan(e + f*x) +
 c)*tan(e + f*x)**3 + 2*I*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2
- c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + I*c**2*sqrt(-I*c*tan(e +
f*x) + c)), x)/a**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((I*a*tan(f*x + e) + a)^3*(-I*c*tan(f*x + e) + c)^(5/2)), x)
```

Mupad [B]
time = 5.14, size = 255, normalized size = 0.93

$$-\frac{-\frac{(c-c\tan(e+f\,x)\,1\mathrm{i})^3\,2541\mathrm{i}}{640\,a^3\,f}+\frac{c^3\,1\mathrm{i}}{5\,a^3\,f}+\frac{(c-c\tan(e+f\,x)\,1\mathrm{i})^4\,77\mathrm{i}}{32\,a^3\,c\,f}-\frac{(c-c\tan(e+f\,x)\,1\mathrm{i})^5\,231\mathrm{i}}{512\,a^3\,c^2\,f}+\frac{c\,(c-c\tan(e+f\,x)\,1\mathrm{i})^2\,33\mathrm{i}}{20\,a^3\,f}+\frac{c^2\,(c-c\tan(e+f\,x)\,1\mathrm{i})\,11\mathrm{i}}{30\,a^3\,f}}{6\,c\,(c-c\tan\left(e+f\,x\right)\,1\mathrm{i})^{9/2}-(c-c\tan\left(e+f\,x\right)\,1\mathrm{i})^{11/2}+8\,c^3\,(c-c\tan\left(e+f\,x\right)\,1\mathrm{i})^{5/2}-12\,c^2\,(c-c\tan\left(e+f\,x\right)\,1\mathrm{i})^{7/2}}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}}{2\,\sqrt{-c}}\right)\,231\mathrm{i}}{1024\,a^3\,(-c)^{5/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i)^(5/2)),x)
```

```
[Out] - ((c^3*1i)/(5*a^3*f) - ((c - c*tan(e + f*x)*1i)^3*2541i)/(640*a^3*f) + ((c
 - c*tan(e + f*x)*1i)^4*77i)/(32*a^3*c*f) - ((c - c*tan(e + f*x)*1i)^5*231i
)/(512*a^3*c^2*f) + (c*(c - c*tan(e + f*x)*1i)^2*33i)/(20*a^3*f) + (c^2*(c
- c*tan(e + f*x)*1i)*11i)/(30*a^3*f))/(6*c*(c - c*tan(e + f*x)*1i)^(9/2) -
(c - c*tan(e + f*x)*1i)^(11/2) + 8*c^3*(c - c*tan(e + f*x)*1i)^(5/2) - 12*c
^2*(c - c*tan(e + f*x)*1i)^(7/2)) - (2^(1/2)*atan((2^(1/2)*(c - c*tan(e + f
*x)*1i)^(1/2))/(2*(-c)^(1/2)))*231i)/(1024*a^3*(-c)^(5/2)*f)
```

3.992 $\int (a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}\,dx$

Optimal. Leaf size=154

$$-\frac{3ia^{5/2}\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{f}+\frac{3ia^2\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{2f}+\frac{ia(a-$$

```
[Out] -3*I*a^(5/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*c*tan(f*x
+e))^(1/2))*c^(1/2)/f+3/2*I*a^2*(a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))
^(1/2)/f+1/2*I*a*(c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^(3/2)/f
```

Rubi [A]

time = 0.12, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3604, 52, 65, 223, 209}

$$-\frac{3ia^{5/2}\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{f}+\frac{3ia^2\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{2f}+\frac{ia(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^(5/2)*Sqrt[c - I*c*Tan[e + f*x]],x]
```

```
[Out] ((-3*I)*a^(5/2)*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a
]*Sqrt[c - I*c*Tan[e + f*x]])])/f + (((3*I)/2)*a^2*Sqrt[a + I*a*Tan[e + f*x
]]*Sqrt[c - I*c*Tan[e + f*x]])/f + ((I/2)*a*(a + I*a*Tan[e + f*x])^(3/2)*Sq
rt[c - I*c*Tan[e + f*x]])/f
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{(a+iax)^{3/2}}{\sqrt{c-icx}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{ia(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}{2f} + \frac{(3a^2c}{}$$

$$= \frac{3ia^2\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{2f} + \frac{ia(a}{}$$

$$= \frac{3ia^2\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{2f} + \frac{ia(a}{}$$

$$= \frac{3ia^2\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{2f} + \frac{ia(a}{}$$

$$= -\frac{3ia^{5/2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{f} + \frac{3ia^2}{}$$

Mathematica [A]

time = 2.87, size = 87, normalized size = 0.56

$$\frac{a^2c\left(4-6\text{ArcTan}\left(e^{i(e+fx)}\right)\cos(e+fx)+i\tan(e+fx)\right)(i+\tan(e+fx))\sqrt{a+ia\tan(e+fx)}}{2f\sqrt{c-ic\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(5/2)*Sqrt[c - I*c*Tan[e + f*x]],x]
```

```
[Out] (a^2*c*(4 - 6*ArcTan[E^(I*(e + f*x))]*Cos[e + f*x] + I*Tan[e + f*x])*(I + T
an[e + f*x])*Sqrt[a + I*a*Tan[e + f*x]])/(2*f*Sqrt[c - I*c*Tan[e + f*x]])
```

Maple [A]
time = 0.44, size = 154, normalized size = 1.00

method	result
derivativedivides	$\frac{\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,a^{2}\left(4i\sqrt{ac\left(1+\tan^{2}\left(fx+e\right)\right)}\,\sqrt{ac}\right.}{2f\sqrt{ac\,(1+\mathrm{ta}}}$
default	$\frac{\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,a^{2}\left(4i\sqrt{ac\left(1+\tan^{2}\left(fx+e\right)\right)}\,\sqrt{ac}\right.}{2f\sqrt{ac\,(1+\mathrm{ta}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^(5/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/2/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)*a^2*(4*I*(a*c*
(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)-tan(f*x+e)*(a*c*(1+tan(f*x+e)^2))^(1/2)
*(a*c)^(1/2)+3*a*c*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1
/2))/(a*c)^(1/2)))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(a*c)^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 710 vs. 2(118) = 236.
time = 0.60, size = 710, normalized size = 4.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^(5/2),x, algorithm="m
axima")
```

```
[Out] (20*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 12*a^2*cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 20*I*a^2*sin(3/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 12*I*a^2*sin(1/2*arctan2(sin(2*f*x +
```

```
2*e), cos(2*f*x + 2*e))) - 6*(a^2*cos(4*f*x + 4*e) + 2*a^2*cos(2*f*x + 2*e
) + I*a^2*sin(4*f*x + 4*e) + 2*I*a^2*sin(2*f*x + 2*e) + a^2)*arctan2(cos(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), sin(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))) + 1) - 6*(a^2*cos(4*f*x + 4*e) + 2*a^2*cos(2*f*x
+ 2*e) + I*a^2*sin(4*f*x + 4*e) + 2*I*a^2*sin(2*f*x + 2*e) + a^2)*arctan2(c
os(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), -sin(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) + 1) - 3*(I*a^2*cos(4*f*x + 4*e) + 2*I*a^2*
cos(2*f*x + 2*e) - a^2*sin(4*f*x + 4*e) - 2*a^2*sin(2*f*x + 2*e) + I*a^2)*l
og(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) + 1) - 3*(-I*a^2*cos(4*f*x + 4*e) - 2*I*a^2*cos(2*f*x
 + 2*e) + a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f*x + 2*e) - I*a^2)*log(cos(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) + 1))*sqrt(a)*sqrt(c)/(f*(-4*I*cos(4*f*x + 4*e) - 8*I*cos(2*f*
x + 2*e) + 4*sin(4*f*x + 4*e) + 8*sin(2*f*x + 2*e) - 4*I))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(118) = 236$.
time = 1.07, size = 380, normalized size = 2.47

$$\frac{3\sqrt{\frac{a^5c}{f^2}}\left(fe^{(2i fx+2i e)}+f\right)\log\left(\frac{4\left(2\left(a^2e^{(3i fx+3i e)}+a^2e^{(i fx+i e)}\right)\sqrt{\frac{a}{e^{(2i fx+2i e)}+1}}\sqrt{\frac{c}{e^{(2i fx+2i e)}+1}}-\sqrt{\frac{a^5c}{f^2}}\left(i fe^{(2i fx+2i e)}-i f\right)\right)}{a^2e^{(2i fx+2i e)}+a^2}\right)-3\sqrt{\frac{a^5c}{f^2}}\left(fe^{(2i fx+2i e)}+f\right)\log\left(\frac{4\left(2\left(a^2e^{(3i fx+3i e)}+a^2e^{(i fx+i e)}\right)\sqrt{\frac{a}{e^{(2i fx+2i e)}+1}}\sqrt{\frac{c}{e^{(2i fx+2i e)}+1}}-\sqrt{\frac{a^5c}{f^2}}\left(-i fe^{(2i fx+2i e)}+i f\right)\right)}{a^2e^{(2i fx+2i e)}+a^2}\right)-4\left(-5i a^2e^{(3i fx+3i e)}-3i a^2e^{(i fx+i e)}\right)\sqrt{\frac{a}{e^{(2i fx+2i e)}+1}}\sqrt{\frac{c}{e^{(2i fx+2i e)}+1}}}{4\left(fe^{(2i fx+2i e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^(5/2),x, algorithm="f
ricas")
```

```
[Out] 1/4*(3*sqrt(a^5*c/f^2)*(f*e^(2*I*f*x + 2*I*e) + f)*log(4*(2*(a^2*e^(3*I*f*x
 + 3*I*e) + a^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(
e^(2*I*f*x + 2*I*e) + 1)) - sqrt(a^5*c/f^2)*(I*f*e^(2*I*f*x + 2*I*e) - I*f)
)/(a^2*e^(2*I*f*x + 2*I*e) + a^2)) - 3*sqrt(a^5*c/f^2)*(f*e^(2*I*f*x + 2*I*
e) + f)*log(4*(2*(a^2*e^(3*I*f*x + 3*I*e) + a^2*e^(I*f*x + I*e))*sqrt(a/(e^
(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - sqrt(a^5*c/f^2)
*(-I*f*e^(2*I*f*x + 2*I*e) + I*f))/(a^2*e^(2*I*f*x + 2*I*e) + a^2)) - 4*(-5
*I*a^2*e^(3*I*f*x + 3*I*e) - 3*I*a^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x +
2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(e+fx)-i))^{\frac{5}{2}}\sqrt{-ic(\tan(e+fx)+i)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**(1/2)*(a+I*a*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral((I*a*(tan(e + f*x) - I))**(5/2)*sqrt(-I*c*(tan(e + f*x) + I)), x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(118) = 236$.
time = 1.45, size = 325, normalized size = 2.11

$$\frac{15\left(a^{3}c-a^{2}c\right)\sqrt{-ac}\,e^{\left(9i\,fx+9i\,e\right)}+74\left(a^{3}c-a^{2}c\right)\sqrt{-ac}\,e^{\left(7i\,fx+7i\,e\right)}+132\left(a^{3}c-a^{2}c\right)\sqrt{-ac}\,e^{\left(5i\,fx+5i\,e\right)}+102\left(a^{3}c-a^{2}c\right)\sqrt{-ac}\,e^{\left(3i\,fx+3i\,e\right)}+29\left(a^{3}c-a^{2}c\right)\sqrt{-ac}\,e^{\left(i\,fx+i\,e\right)}}{4\left((a-1)cfe^{\left(10i\,fx+10i\,e\right)}+5\,(a-1)cfe^{\left(8i\,fx+8i\,e\right)}+10\,(a-1)cfe^{\left(6i\,fx+6i\,e\right)}+10\,(a-1)cfe^{\left(4i\,fx+4i\,e\right)}+5\,(a-1)cfe^{\left(2i\,fx+2i\,e\right)}+(a-1)cf\right)}-\frac{12i\,a^{\frac{5}{2}}\sqrt{c}\,\arctan\left(e^{\left(i\,fx+i\,e\right)}\right)-\frac{i\left(5\,a^{\frac{5}{2}}\sqrt{c}\,e^{\left(3i\,fx+3i\,e\right)}+7\,a^{\frac{5}{2}}\sqrt{c}\,e^{\left(i\,fx+i\,e\right)}\right)}{\left(e^{\left(2i\,fx+2i\,e\right)}+1\right)^{2}}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^(5/2),x, algorithm="g
iac")
```

```
[Out] 1/4*(15*(a^3*c - a^2*c)*sqrt(-a*c)*e^(9*I*f*x + 9*I*e) + 74*(a^3*c - a^2*c)
*sqrt(-a*c)*e^(7*I*f*x + 7*I*e) + 132*(a^3*c - a^2*c)*sqrt(-a*c)*e^(5*I*f*x
 + 5*I*e) + 102*(a^3*c - a^2*c)*sqrt(-a*c)*e^(3*I*f*x + 3*I*e) + 29*(a^3*c
- a^2*c)*sqrt(-a*c)*e^(I*f*x + I*e))/((a - 1)*c*f*e^(10*I*f*x + 10*I*e) + 5
*(a - 1)*c*f*e^(8*I*f*x + 8*I*e) + 10*(a - 1)*c*f*e^(6*I*f*x + 6*I*e) + 10*
(a - 1)*c*f*e^(4*I*f*x + 4*I*e) + 5*(a - 1)*c*f*e^(2*I*f*x + 2*I*e) + (a -
1)*c*f) - 1/4*(12*I*a^(5/2)*sqrt(c)*arctan(e^(I*f*x + I*e)) - I*(5*a^(5/2)*
sqrt(c)*e^(3*I*f*x + 3*I*e) + 7*a^(5/2)*sqrt(c)*e^(I*f*x + I*e))/(e^(2*I*f*
x + 2*I*e) + 1)^2)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\tan(e+f\,x)\;1\mathrm{i})^{5/2}\,\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(5/2)*(c - c*tan(e + f*x)*1i)^(1/2),x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^(5/2)*(c - c*tan(e + f*x)*1i)^(1/2), x)
```

3.993 $\int (a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}\,dx$

Optimal. Leaf size=106

$$-\frac{2ia^{3/2}\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{f}+\frac{ia\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{f}$$

```
[Out] -2*I*a^(3/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*c*tan(f*x
+e))^(1/2))*c^(1/2)/f+I*a*(a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))^(1/2)
/f
```

Rubi [A]
time = 0.09, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3604, 52, 65, 223, 209}

$$\frac{ia\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{f}-\frac{2ia^{3/2}\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^(3/2)*Sqrt[c - I*c*Tan[e + f*x]],x]
```

```
[Out] ((-2*I)*a^(3/2)*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a
]*Sqrt[c - I*c*Tan[e + f*x]])])/f + (I*a*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c
- I*c*Tan[e + f*x]])/f
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}\,dx=\frac{(ac)\text{Subst}\left(\int\frac{\sqrt{a+iax}}{\sqrt{c-icx}}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{ia\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{f}+\frac{(a^2c)}{\quad}$$

$$(2iac$$

$$=\frac{ia\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{f}-\frac{\quad}{\quad}$$

$$=\frac{ia\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{f}-\frac{(2iac}{\quad}$$

$$=-\frac{2ia^{3/2}\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{f}+\frac{ia}{\quad}$$

Mathematica [A]
time = 2.00, size = 129, normalized size = 1.22

$$\frac{ace^{-\frac{1}{2}i(4e+fx)}\left(2\text{ArcTan}\left(e^{i(e+fx)}\right)-\sec(e+fx)\right)\left(-i\cos\left(\frac{3e}{2}\right)+\sin\left(\frac{3e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-i\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a+ia\tan(e+fx)}}{\sqrt{2}\sqrt{\frac{c}{1+e^{2i(e+fx)}}}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(3/2)*Sqrt[c - I*c*Tan[e + f*x]],x]
```

```
[Out] (a*c*(2*ArcTan[E^(I*(e + f*x))] - Sec[e + f*x])*((-I)*Cos[(3*e)/2] + Sin[(3
*e)/2])*(Cos[(e + f*x)/2] - I*Sin[(e + f*x)/2])*Sqrt[a + I*a*Tan[e + f*x]])
/(Sqrt[2]*E^((I/2)*(4*e + f*x))*Sqrt[c/(1 + E^((2*I)*(e + f*x)))]*f)
```

Maple [A]
time = 0.35, size = 122, normalized size = 1.15

method	result
derivativedivides	$\frac{\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,a\left(i\sqrt{ac\left(1+\tan^{2}\left(fx+e\right)\right)}\,\sqrt{ac}+ac\right.}{f\sqrt{ac\left(1+\tan^{2}\left(fx+e\right)\right)}\,\sqrt{ac}}$
default	$\frac{\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,a\left(i\sqrt{ac\left(1+\tan^{2}\left(fx+e\right)\right)}\,\sqrt{ac}+ac\right.}{f\sqrt{ac\left(1+\tan^{2}\left(fx+e\right)\right)}\,\sqrt{ac}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^(3/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)*a*(I*(a*c*(1+tan
(f*x+e)^2))^(1/2)*(a*c)^(1/2)+a*c*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))
^(1/2)*(a*c)^(1/2))/(a*c)^(1/2)))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(a*c)^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 481 vs. 2(82) = 164.
time = 0.60, size = 481, normalized size = 4.54

$$-\frac{\left(2\left(a\cos\left(2fx+2e\right)+ia\sin\left(2fx+2e\right)+a\right)\arctan\left(\cos\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right),\sin\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)+1\right)+2\left(a\cos\left(2fx+2e\right)+ia\sin\left(2fx+2e\right)+a\right)\arctan\left(\cos\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right),-\sin\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)+1\right)-4a\cos\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)-\left(-ia\cos\left(2fx+2e\right)+a\sin\left(2fx+2e\right)-ia\right)\log\left(\cos\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)^{2}+\sin\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)^{2}+2\sin\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)+1\right)-\left(ia\cos\left(2fx+2e\right)-a\sin\left(2fx+2e\right)+ia\right)\log\left(\cos\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)^{2}+\sin\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)^{2}-2\sin\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)+1\right)-4ia\sin\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)\right)\sqrt{a}\sqrt{c}}{-2f\left(i\cos\left(2fx+2e\right)-\sin\left(2fx+2e\right)+i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^(3/2),x, algorithm="m
axima")
```

```
[Out] -(2*(a*cos(2*f*x + 2*e) + I*a*sin(2*f*x + 2*e) + a)*arctan2(cos(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), sin(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e))) + 1) + 2*(a*cos(2*f*x + 2*e) + I*a*sin(2*f*x + 2*e) + a)*a
rctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), -sin(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) - 4*a*cos(1/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))) - (-I*a*cos(2*f*x + 2*e) + a*sin(2*f*x + 2*e)
- I*a)*log(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/2*arctan2(sin(2*f
```

```
*x + 2*e), cos(2*f*x + 2*e))) + 1) - (I*a*cos(2*f*x + 2*e) - a*sin(2*f*x +
2*e) + I*a)*log(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + si
n(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sin(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) - 4*I*a*sin(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/(f*(-2*I*cos(2*f*x + 2*e) + 2*si
n(2*f*x + 2*e) - 2*I))
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(82) = 164$.
time = 1.52, size = 304, normalized size = 2.87

$$-\frac{-4i\,a\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}e^{(i\,fx+i\,e)} - \sqrt{\frac{a^3c}{f^2}}f\log\left(\frac{4\left(2\left(ae^{(3i\,fx+3i\,e)}+ae^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}} - \sqrt{\frac{a^3c}{f^2}}\left(i\,fe^{(2i\,fx+2i\,e)}-i\,f\right)\right)}{ae^{(2i\,fx+2i\,e)}+a}\right) + \sqrt{\frac{a^3c}{f^2}}f\log\left(\frac{4\left(2\left(ae^{(3i\,fx+3i\,e)}+ae^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}} - \sqrt{\frac{a^3c}{f^2}}\left(-i\,fe^{(2i\,fx+2i\,e)}+i\,f\right)\right)}{ae^{(2i\,fx+2i\,e)}+a}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^(3/2),x, algorithm="f
ricas")
```

```
[Out] -1/2*(-4*I*a*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e)
+ 1))*e^(I*f*x + I*e) - sqrt(a^3*c/f^2)*f*log(4*(2*(a*e^(3*I*f*x + 3*I*e) +
 a*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x +
2*I*e) + 1)) - sqrt(a^3*c/f^2)*(I*f*e^(2*I*f*x + 2*I*e) - I*f))/(a*e^(2*I*f
*x + 2*I*e) + a)) + sqrt(a^3*c/f^2)*f*log(4*(2*(a*e^(3*I*f*x + 3*I*e) + a*e
^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*
e) + 1)) - sqrt(a^3*c/f^2)*(-I*f*e^(2*I*f*x + 2*I*e) + I*f))/(a*e^(2*I*f*x
+ 2*I*e) + a)))/f
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan{(e + fx)} - i))^{\frac{3}{2}}\sqrt{-ic\,(\tan{(e + fx)} + i)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**(1/2)*(a+I*a*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((I*a*(tan(e + f*x) - I))**(3/2)*sqrt(-I*c*(tan(e + f*x) + I)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^(3/2)*sqrt(-I*c*tan(f*x + e) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a\tan(e + f\,x)\ 1\mathrm{i})^{3/2}\,\sqrt{c - c\tan\left(e + f\,x\right)\,1\mathrm{i}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(3/2)*(c - c*tan(e + f*x)*1i)^(1/2),x)
[Out] int((a + a*tan(e + f*x)*1i)^(3/2)*(c - c*tan(e + f*x)*1i)^(1/2), x)
```

3.994 $\int \sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}\,dx$

Optimal. Leaf size=63

$$-\frac{2i\sqrt{a}\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{f}$$

```
[Out] -2*I*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*c*tan(f*x+e))^(1/
2))*a^(1/2)*c^(1/2)/f
```

Rubi [A]
time = 0.08, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3604, 65, 223, 209}

$$-\frac{2i\sqrt{a}\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]],x]
```

```
[Out] ((-2*I)*Sqrt[a]*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a
]*Sqrt[c - I*c*Tan[e + f*x]])])/f
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{1}{\sqrt{a+iax}\,\sqrt{c-icx}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(2ic)\text{Subst}\left(\int \frac{1}{\sqrt{2c-\frac{cx^2}{a}}}\,dx, x, \sqrt{a+ia\tan(e+fx}\right.}{f}$$

$$= -\frac{(2ic)\text{Subst}\left(\int \frac{1}{1+\frac{cx^2}{a}}\,dx, x, \frac{\sqrt{a+ia\tan(e+fx)}}{\sqrt{c-ic\tan(e+fx)}}\right)}{f}$$

$$= -\frac{2i\sqrt{a}\,\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{f}$$

Mathematica [A]
time = 0.94, size = 74, normalized size = 1.17

$$-\frac{i\sqrt{2}\,ce^{-i(e+fx)}\text{ArcTan}\left(e^{i(e+fx)}\right)\sqrt{a+ia\tan(e+fx)}}{\sqrt{\frac{c}{1+e^{2i(e+fx)}}}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]],x]
```

```
[Out] ((-I)*Sqrt[2]*c*ArcTan[E^(I*(e + f*x))]*Sqrt[a + I*a*Tan[e + f*x]])/(E^(I*(
e + f*x))*Sqrt[c/(1 + E^((2*I)*(e + f*x)))]*f)
```

Maple [A]
time = 0.33, size = 96, normalized size = 1.52

method	result

derivativedivides	$\frac{\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,ac\ln\left(\frac{ca\tan(fx+e)+\sqrt{ac\left(1+\tan^2\left(fx+\right.\right.}}{\sqrt{ac}}\right.}{f\sqrt{ac\left(1+\tan^2\left(fx+e\right)\right)}\,\sqrt{ac}}$
default	$\frac{\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,ac\ln\left(\frac{ca\tan(fx+e)+\sqrt{ac\left(1+\tan^2\left(fx+\right.\right.}}{\sqrt{ac}}\right.}{f\sqrt{ac\left(1+\tan^2\left(fx+e\right)\right)}\,\sqrt{ac}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)/(a*c*(1+tan(f*x+
e)^2))^(1/2)*a*c*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2
))/(a*c)^(1/2))/(a*c)^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(47) = 94$.
time = 0.61, size = 114, normalized size = 1.81

$$\frac{\sqrt{a}\,\sqrt{c}\,\left(-2i\arctan\left(\cos\left(fx+e\right),\sin\left(fx+e\right)+1\right)-2i\arctan\left(\cos\left(fx+e\right),-\sin\left(fx+e\right)+1\right)+\log\left(\cos\left(fx+e\right)^2+\sin\left(fx+e\right)^2+2\sin\left(fx+e\right)+1\right)-\log\left(\cos\left(fx+e\right)^2+\sin\left(fx+e\right)^2-2\sin\left(fx+e\right)+1\right)\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^(1/2),x, algorithm="m
axima")
```

```
[Out] 1/2*sqrt(a)*sqrt(c)*(-2*I*arctan2(cos(f*x + e), sin(f*x + e) + 1) - 2*I*arc
tan2(cos(f*x + e), -sin(f*x + e) + 1) + log(cos(f*x + e)^2 + sin(f*x + e)^2
 + 2*sin(f*x + e) + 1) - log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x +
e) + 1))/f
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(47) = 94$.
time = 1.29, size = 229, normalized size = 3.63

$$\frac{1}{2}\sqrt{\frac{ac}{f^2}}\log\left(\frac{4\left(2\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}\left(e^{(3ifx+3ie)}+e^{(ifx+ie)}\right)-\left(ife^{(2ifx+2ie)}-if\right)\sqrt{\frac{ac}{f^2}}\right)}{e^{(2ifx+2ie)}+1}\right)-\frac{1}{2}\sqrt{\frac{ac}{f^2}}\log\left(\frac{4\left(2\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}\left(e^{(3ifx+3ie)}+e^{(ifx+ie)}\right)-\left(-ife^{(2ifx+2ie)}+if\right)\sqrt{\frac{ac}{f^2}}\right)}{e^{(2ifx+2ie)}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/2*sqrt(a*c/f^2)*log(4*(2*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I
*f*x + 2*I*e) + 1))*(e^(3*I*f*x + 3*I*e) + e^(I*f*x + I*e)) - (I*f*e^(2*I*f
```

```
*x + 2*I*e) - I*f)*sqrt(a*c/f^2))/(e^(2*I*f*x + 2*I*e) + 1)) - 1/2*sqrt(a*c
/f^2)*log(4*(2*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e
) + 1))*(e^(3*I*f*x + 3*I*e) + e^(I*f*x + I*e)) - (-I*f*e^(2*I*f*x + 2*I*e)
 + I*f)*sqrt(a*c/f^2))/(e^(2*I*f*x + 2*I*e) + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\left(\tan\left(e+fx\right)-i\right)}\,\sqrt{-ic\left(\tan\left(e+fx\right)+i\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**(1/2)*(a+I*a*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(e + f*x) - I))*sqrt(-I*c*(tan(e + f*x) + I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(I*a*tan(f*x + e) + a)*sqrt(-I*c*tan(f*x + e) + c), x)
```

Mupad [B]
time = 5.77, size = 62, normalized size = 0.98

$$-\frac{\sqrt{a}\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,\left(\sqrt{a+a\tan\left(e+f\,x\right)\,1\mathrm{i}}-\sqrt{a}\right)}{\sqrt{a}\,\left(\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}-\sqrt{c}\right)}\right)\,4\mathrm{i}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2),x)
```

```
[Out] -(a^(1/2)*c^(1/2)*atan((c^(1/2)*((a + a*tan(e + f*x)*1i)^(1/2) - a^(1/2)))/
(a^(1/2)*((c - c*tan(e + f*x)*1i)^(1/2) - c^(1/2))))*4i)/f
```

3.995 $\int \frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{a+ia\tan(e+fx)}}\,dx$

Optimal. Leaf size=41

$$\frac{i\sqrt{c-ic\tan(e+fx)}}{f\sqrt{a+ia\tan(e+fx)}}$$

```
[Out] I*(c-I*c*tan(f*x+e))^(1/2)/f/(a+I*a*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {3604, 37}

$$\frac{i\sqrt{c-ic\tan(e+fx)}}{f\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - I*c*Tan[e + f*x]]/Sqrt[a + I*a*Tan[e + f*x]],x]
```

```
[Out] (I*Sqrt[c - I*c*Tan[e + f*x]])/(f*Sqrt[a + I*a*Tan[e + f*x]])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{a+ia\tan(e+fx)}}\,dx &= \frac{(ac)\text{Subst}\left(\int \frac{1}{(a+iax)^{3/2}\sqrt{c-icx}}\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{i\sqrt{c-ic\tan(e+fx)}}{f\sqrt{a+ia\tan(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 0.69, size = 41, normalized size = 1.00

$$\frac{i\sqrt{c - ic\tan(e + fx)}}{f\sqrt{a + ia\tan(e + fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - I*c*Tan[e + f*x]]/Sqrt[a + I*a*Tan[e + f*x]],x]
```

```
[Out] (I*Sqrt[c - I*c*Tan[e + f*x]])/(f*Sqrt[a + I*a*Tan[e + f*x]])
```

Maple [A]
time = 0.33, size = 65, normalized size = 1.59

method	result	size
risch	$\frac{i\sqrt{\frac{c}{e^{2i(fx+e)}+1}}}{\sqrt{\frac{a\,e^{2i(fx+e)}}{e^{2i(fx+e)}+1}}\,f}$	50
derivativedivides	$-\frac{i\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,(1+i\tan(fx+e))}{fa(-\tan(fx+e)+i)^2}$	65
default	$-\frac{i\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,(1+i\tan(fx+e))}{fa(-\tan(fx+e)+i)^2}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] -I/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)/a*(1+I*tan(f*x+
e))/(-tan(f*x+e)+I)^2
```

Maxima [A]
time = 0.54, size = 37, normalized size = 0.90

$$\frac{i\sqrt{c}\,\sqrt{-i\,\tan\left(fx+e\right)+1}}{\sqrt{a}\,f\sqrt{i\,\tan\left(fx+e\right)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(1/2),x, algorithm="m
axima")
```

```
[Out] I*sqrt(c)*sqrt(-I*tan(f*x + e) + 1)/(sqrt(a)*f*sqrt(I*tan(f*x + e) + 1))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(33) = 66$.

time = 1.67, size = 67, normalized size = 1.63

$$\frac{\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\,\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\,\left(i\,e^{(2i\,fx+2i\,e)}+i\right)e^{(-i\,fx-i\,e)}}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(1/2),x, algorithm="f
ricas")
```

```
[Out] sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*(I*e^(2
*I*f*x + 2*I*e) + I)*e^(-I*f*x - I*e)/(a*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-ic\left(\tan\left(e+fx\right)+i\right)}}{\sqrt{ia\left(\tan\left(e+fx\right)-i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(-I*c*(tan(e + f*x) + I))/sqrt(I*a*(tan(e + f*x) - I)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-I*c*tan(f*x + e) + c)/sqrt(I*a*tan(f*x + e) + a), x)
```

Mupad [**B**]
time = 0.71, size = 34, normalized size = 0.83

$$\frac{\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}\;1\mathrm{i}}{f\,\sqrt{a+a\tan\left(e+f\,x\right)\,1\mathrm{i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^(1/2)/(a + a*tan(e + f*x)*1i)^(1/2),x)
```

```
[Out] ((c - c*tan(e + f*x)*1i)^(1/2)*1i)/(f*(a + a*tan(e + f*x)*1i)^(1/2))
```

3.996 $\int \frac{\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=90

$$\frac{i\sqrt{c-ic\tan(e+fx)}}{3f(a+ia\tan(e+fx))^{3/2}}+\frac{i\sqrt{c-ic\tan(e+fx)}}{3af\sqrt{a+ia\tan(e+fx)}}$$

```
[Out] 1/3*I*(c-I*c*tan(f*x+e))^(1/2)/a/f/(a+I*a*tan(f*x+e))^(1/2)+1/3*I*(c-I*c*ta
n(f*x+e))^(1/2)/f/(a+I*a*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}=0.086$, Rules used = {3604, 47, 37}

$$\frac{i\sqrt{c-ic\tan(e+fx)}}{3af\sqrt{a+ia\tan(e+fx)}}+\frac{i\sqrt{c-ic\tan(e+fx)}}{3f(a+ia\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - I*c*Tan[e + f*x]]/(a + I*a*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((I/3)*Sqrt[c - I*c*Tan[e + f*x]])/(f*(a + I*a*Tan[e + f*x])^(3/2)) + ((I/3
)*Sqrt[c - I*c*Tan[e + f*x]])/(a*f*Sqrt[a + I*a*Tan[e + f*x]])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
```

```
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^{3/2}}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{1}{(a+iax)^{5/2}\sqrt{c-icx}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{i\sqrt{c-ic\tan(e+fx)}}{3f(a+ia\tan(e+fx))^{3/2}} + \frac{c\text{Subst}\left(\int \frac{1}{(a+iax)^{3/2}\sqrt{c-icx}}\,dx, x, \tan(e+f\right.}{3f}$$

$$= \frac{i\sqrt{c-ic\tan(e+fx)}}{3f(a+ia\tan(e+fx))^{3/2}} + \frac{i\sqrt{c-ic\tan(e+fx)}}{3af\sqrt{a+ia\tan(e+fx)}}$$

Mathematica [A]
time = 0.87, size = 68, normalized size = 0.76

$$\frac{(2+i\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{3af(-i+\tan(e+fx))\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - I*c*Tan[e + f*x]]/(a + I*a*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((2 + I*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(3*a*f*(-I + Tan[e + f*x]
)*Sqrt[a + I*a*Tan[e + f*x]])
```

Maple [A]
time = 0.32, size = 74, normalized size = 0.82

method	result	size
derivativedivides	$\frac{\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\left(3i\tan(fx+e)-\left(\tan^2(fx+e)\right)+2\right)}{3f\,a^2\left(-\tan(fx+e)+i\right)^3}$	74
default	$\frac{\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\left(3i\tan(fx+e)-\left(\tan^2(fx+e)\right)+2\right)}{3f\,a^2\left(-\tan(fx+e)+i\right)^3}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(3/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/3/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)/a^2*(3*I*tan(f
*x+e)-tan(f*x+e)^2+2)/(-tan(f*x+e)+I)^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(3/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 1.34, size = 80, normalized size = 0.89

$$\frac{\sqrt{\frac{a}{e^{(2i fx+2ie)}+1}}\sqrt{\frac{c}{e^{(2i fx+2ie)}+1}}\left(3i e^{(4i fx+4ie)}+4i e^{(2i fx+2ie)}+i\right)e^{(-3i fx-3ie)}}{6a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/6*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*(3*
I*e^(4*I*f*x + 4*I*e) + 4*I*e^(2*I*f*x + 2*I*e) + I)*e^(-3*I*f*x - 3*I*e)/(
a^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-ic\left(\tan\left(e+fx\right)+i\right)}}{\left(ia\left(\tan\left(e+fx\right)-i\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(-I*c*(tan(e + f*x) + I))/(I*a*(tan(e + f*x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-I*c*tan(f*x + e) + c)/(I*a*tan(f*x + e) + a)^(3/2), x)
```

Mupad [B]
time = 6.31, size = 135, normalized size = 1.50

$$\frac{\sqrt{\frac{a\,\left(\cos\left(2\,e+2\,f\,x\right)+1+\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\sqrt{\frac{c\,\left(\cos\left(2\,e+2\,f\,x\right)+1-\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\left(\cos\left(2\,e+2\,f\,x\right)\,4{}\mathrm{i}+\cos\left(4\,e+4\,f\,x\right)\,1{}\mathrm{i}+4\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(4\,e+4\,f\,x\right)+3{}\mathrm{i}\right)}{12\,a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^(1/2)/(a + a*tan(e + f*x)*1i)^(3/2),x)
```

```
[Out] (((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^
(1/2)*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) +
 1))^(1/2)*(cos(2*e + 2*f*x)*4i + cos(4*e + 4*f*x)*1i + 4*sin(2*e + 2*f*x)
+ sin(4*e + 4*f*x) + 3i))/(12*a^2*f)
```

3.997 $\int \frac{\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=136

$$\frac{i\sqrt{c-ic\tan(e+fx)}}{5f(a+ia\tan(e+fx))^{5/2}}+\frac{2i\sqrt{c-ic\tan(e+fx)}}{15af(a+ia\tan(e+fx))^{3/2}}+\frac{2i\sqrt{c-ic\tan(e+fx)}}{15a^2f\sqrt{a+ia\tan(e+fx)}}$$

```
[Out] 2/15*I*(c-I*c*tan(f*x+e))^(1/2)/a^2/f/(a+I*a*tan(f*x+e))^(1/2)+1/5*I*(c-I*c
*tan(f*x+e))^(1/2)/f/(a+I*a*tan(f*x+e))^(5/2)+2/15*I*(c-I*c*tan(f*x+e))^(1/
2)/a/f/(a+I*a*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.09, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {3604, 47, 37}

$$\frac{2i\sqrt{c-ic\tan(e+fx)}}{15a^2f\sqrt{a+ia\tan(e+fx)}}+\frac{2i\sqrt{c-ic\tan(e+fx)}}{15af(a+ia\tan(e+fx))^{3/2}}+\frac{i\sqrt{c-ic\tan(e+fx)}}{5f(a+ia\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - I*c*Tan[e + f*x]]/(a + I*a*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((I/5)*Sqrt[c - I*c*Tan[e + f*x]])/(f*(a + I*a*Tan[e + f*x])^(5/2)) + (((2*
I)/15)*Sqrt[c - I*c*Tan[e + f*x]])/(a*f*(a + I*a*Tan[e + f*x])^(3/2)) + (((
2*I)/15)*Sqrt[c - I*c*Tan[e + f*x]])/(a^2*f*Sqrt[a + I*a*Tan[e + f*x]])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^{5/2}}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{1}{(a+iax)^{7/2}\sqrt{c-icx}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{i\sqrt{c-ic\tan(e+fx)}}{5f(a+ia\tan(e+fx))^{5/2}} + \frac{(2c)\text{Subst}\left(\int \frac{1}{(a+iax)^{5/2}\sqrt{c-icx}}\,dx, x, \tan(e-\right.}{5f}$$

$$= \frac{i\sqrt{c-ic\tan(e+fx)}}{5f(a+ia\tan(e+fx))^{5/2}} + \frac{2i\sqrt{c-ic\tan(e+fx)}}{15af(a+ia\tan(e+fx))^{3/2}} + \frac{(2c)\text{Subst}\left(\int\right.}{}$$

$$= \frac{i\sqrt{c-ic\tan(e+fx)}}{5f(a+ia\tan(e+fx))^{5/2}} + \frac{2i\sqrt{c-ic\tan(e+fx)}}{15af(a+ia\tan(e+fx))^{3/2}} + \frac{2i\sqrt{c-ic\,\mathrm{t}}}{15a^2f\sqrt{a+ia}}$$

Mathematica [A]
time = 1.27, size = 90, normalized size = 0.66

$$-\frac{i\sec^2(e+fx)(5+9\cos(2(e+fx))+6i\sin(2(e+fx)))\sqrt{c-ic\tan(e+fx)}}{30a^2f(-i+\tan(e+fx))^2\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - I*c*Tan[e + f*x]]/(a + I*a*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-1/30*I)*Sec[e + f*x]^2*(5 + 9*Cos[2*(e + f*x)] + (6*I)*Sin[2*(e + f*x)])
*Sqrt[c - I*c*Tan[e + f*x]])/(a^2*f*(-I + Tan[e + f*x])^2*Sqrt[a + I*a*Tan[
e + f*x]])
```

Maple [A]
time = 0.37, size = 85, normalized size = 0.62

method	result
derivativedivides	$-\frac{\sqrt{-c\,(i\tan(fx+e)-1)}\,\sqrt{a\,(1+i\tan(fx+e))}\,(8i(\tan^2(fx+e))-2(\tan^3(fx+e))-7i+13\tan}{15f\,a^3(-\tan(fx+e)+i)^4}$
default	$-\frac{\sqrt{-c\,(i\tan(fx+e)-1)}\,\sqrt{a\,(1+i\tan(fx+e))}\,(8i(\tan^2(fx+e))-2(\tan^3(fx+e))-7i+13\tan}{15f\,a^3(-\tan(fx+e)+i)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(5/2),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/15/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)/a^3*(8*I*tan
(f*x+e)^2-2*tan(f*x+e)^3-7*I+13*tan(f*x+e))/(-tan(f*x+e)+I)^4
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(5/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 1.40, size = 92, normalized size = 0.68

$$\frac{\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\,\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\,\left(15i\,e^{(6i\,fx+6i\,e)}+25i\,e^{(4i\,fx+4i\,e)}+13i\,e^{(2i\,fx+2i\,e)}+3i\right)e^{(-5i\,fx-5i\,e)}}{60\,a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(5/2),x, algorithm="f
ricas")
```

```
[Out] 1/60*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*(1
5*I*e^(6*I*f*x + 6*I*e) + 25*I*e^(4*I*f*x + 4*I*e) + 13*I*e^(2*I*f*x + 2*I*
e) + 3*I)*e^(-5*I*f*x - 5*I*e)/(a^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-ic\left(\tan\left(e+fx\right)+i\right)}}{\left(ia\left(\tan\left(e+fx\right)-i\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(-I*c*(tan(e + f*x) + I))/(I*a*(tan(e + f*x) - I))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(5/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-I*c*tan(f*x + e) + c)/(I*a*tan(f*x + e) + a)^(5/2), x)
```

Mupad [B]
time = 6.15, size = 160, normalized size = 1.18

$$\frac{\sqrt{\frac{a\,\left(\cos\left(2\,e+2\,f\,x\right)+1+\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\sqrt{\frac{c\,\left(\cos\left(2\,e+2\,f\,x\right)+1-\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\left(\cos\left(2\,e+2\,f\,x\right)\,25{}\mathrm{i}+\cos\left(4\,e+4\,f\,x\right)\,13{}\mathrm{i}+\cos\left(6\,e+6\,f\,x\right)\,3{}\mathrm{i}+25\,\sin\left(2\,e+2\,f\,x\right)+13\,\sin\left(4\,e+4\,f\,x\right)+3\,\sin\left(6\,e+6\,f\,x\right)+15{}\mathrm{i}\right)}{120\,a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^(1/2)/(a + a*tan(e + f*x)*1i)^(5/2),x)
```

```
[Out] (((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^
(1/2)*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) +
 1))^(1/2)*(cos(2*e + 2*f*x)*25i + cos(4*e + 4*f*x)*13i + cos(6*e + 6*f*x)*
3i + 25*sin(2*e + 2*f*x) + 13*sin(4*e + 4*f*x) + 3*sin(6*e + 6*f*x) + 15i))
/(120*a^3*f)
```

3.998 $\int \frac{\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=182

$$\frac{i\sqrt{c-ic\tan(e+fx)}}{7f(a+ia\tan(e+fx))^{7/2}}+\frac{3i\sqrt{c-ic\tan(e+fx)}}{35af(a+ia\tan(e+fx))^{5/2}}+\frac{2i\sqrt{c-ic\tan(e+fx)}}{35a^2f(a+ia\tan(e+fx))^{3/2}}+\frac{2i\sqrt{c-ic\tan(e+fx)}}{35a^3f\sqrt{a+ia\tan(e+fx)}}$$

```
[Out] 2/35*I*(c-I*c*tan(f*x+e))^(1/2)/a^3/f/(a+I*a*tan(f*x+e))^(1/2)+1/7*I*(c-I*c
*tan(f*x+e))^(1/2)/f/(a+I*a*tan(f*x+e))^(7/2)+3/35*I*(c-I*c*tan(f*x+e))^(1/
2)/a/f/(a+I*a*tan(f*x+e))^(5/2)+2/35*I*(c-I*c*tan(f*x+e))^(1/2)/a^2/f/(a+I*
a*tan(f*x+e))^(3/2)
```

Rubi [A]

time = 0.11, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {3604, 47, 37}

$$\frac{2i\sqrt{c-ic\tan(e+fx)}}{35a^3f\sqrt{a+ia\tan(e+fx)}}+\frac{2i\sqrt{c-ic\tan(e+fx)}}{35a^2f(a+ia\tan(e+fx))^{3/2}}+\frac{3i\sqrt{c-ic\tan(e+fx)}}{35af(a+ia\tan(e+fx))^{5/2}}+\frac{i\sqrt{c-ic\tan(e+fx)}}{7f(a+ia\tan(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - I*c*Tan[e + f*x]]/(a + I*a*Tan[e + f*x])^(7/2),x]
```

```
[Out] ((I/7)*Sqrt[c - I*c*Tan[e + f*x]])/(f*(a + I*a*Tan[e + f*x])^(7/2)) + (((3*
I)/35)*Sqrt[c - I*c*Tan[e + f*x]])/(a*f*(a + I*a*Tan[e + f*x])^(5/2)) + (((
2*I)/35)*Sqrt[c - I*c*Tan[e + f*x]])/(a^2*f*(a + I*a*Tan[e + f*x])^(3/2)) +
 (((2*I)/35)*Sqrt[c - I*c*Tan[e + f*x]])/(a^3*f*Sqrt[a + I*a*Tan[e + f*x]])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^{7/2}}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{1}{(a+iax)^{9/2}\sqrt{c-icx}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{i\sqrt{c-ic\tan(e+fx)}}{7f(a+ia\tan(e+fx))^{7/2}} + \frac{(3c)\text{Subst}\left(\int \frac{1}{(a+iax)^{7/2}\sqrt{c-icx}}\,dx, x, \tan(e \cdots\right.}{7f}$$

$$= \frac{i\sqrt{c-ic\tan(e+fx)}}{7f(a+ia\tan(e+fx))^{7/2}} + \frac{3i\sqrt{c-ic\tan(e+fx)}}{35af(a+ia\tan(e+fx))^{5/2}} + \frac{(6c)\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{i\sqrt{c-ic\tan(e+fx)}}{7f(a+ia\tan(e+fx))^{7/2}} + \frac{3i\sqrt{c-ic\tan(e+fx)}}{35af(a+ia\tan(e+fx))^{5/2}} + \frac{2i\sqrt{c-ic\,\mathrm{t}\cdots}}{35a^2f(a+ia\,\mathrm{t}\cdots}$$

$$= \frac{i\sqrt{c-ic\tan(e+fx)}}{7f(a+ia\tan(e+fx))^{7/2}} + \frac{3i\sqrt{c-ic\tan(e+fx)}}{35af(a+ia\tan(e+fx))^{5/2}} + \frac{2i\sqrt{c-ic\,\mathrm{t}\cdots}}{35a^2f(a+ia\,\mathrm{t}\cdots}$$

Mathematica [**A**]
time = 1.70, size = 105, normalized size = 0.58

$$-\frac{\sec^3(e+fx)(28\cos(e+fx)+20\cos(3(e+fx))+7i\sin(e+fx)+15i\sin(3(e+fx)))\sqrt{c-ic\tan(e+fx)}}{140a^3f(-i+\tan(e+fx))^3\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - I*c*Tan[e + f*x]]/(a + I*a*Tan[e + f*x])^(7/2),x]
```

```
[Out] -1/140*(Sec[e + f*x]^3*(28*Cos[e + f*x] + 20*Cos[3*(e + f*x)] + (7*I)*Sin[e
 + f*x] + (15*I)*Sin[3*(e + f*x)])*Sqrt[c - I*c*Tan[e + f*x]])/(a^3*f*(-I +
 Tan[e + f*x])^3*Sqrt[a + I*a*Tan[e + f*x]])
```

Maple [**A**]
time = 0.38, size = 95, normalized size = 0.52

method	result

derivativedivides	$\frac{\sqrt{-c\,(i\tan(fx+e)-1)}\,\sqrt{a\,(1+i\tan(fx+e))}\,(10i(\tan^3(fx+e))-2(\tan^4(fx+e))-25i\tan(fx+e}{35f\,a^4(-\tan(fx+e)+i)^5}$
default	$\frac{\sqrt{-c\,(i\tan(fx+e)-1)}\,\sqrt{a\,(1+i\tan(fx+e))}\,(10i(\tan^3(fx+e))-2(\tan^4(fx+e))-25i\tan(fx+e}{35f\,a^4(-\tan(fx+e)+i)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(7/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/35/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)/a^4*(10*I*tan
(f*x+e)^3-2*tan(f*x+e)^4-25*I*tan(f*x+e)+21*tan(f*x+e)^2-12)/(-tan(f*x+e)+I
)^5
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(7/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 2.26, size = 104, normalized size = 0.57

$$\frac{\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\,\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\,\left(35i\,e^{(8i\,fx+8i\,e)}+70i\,e^{(6i\,fx+6i\,e)}+56i\,e^{(4i\,fx+4i\,e)}+26i\,e^{(2i\,fx+2i\,e)}+5i\right)e^{(-7i\,fx-7i\,e)}}{280\,a^4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(7/2),x, algorithm="f
ricas")
```

```
[Out] 1/280*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*(
35*I*e^(8*I*f*x + 8*I*e) + 70*I*e^(6*I*f*x + 6*I*e) + 56*I*e^(4*I*f*x + 4*I
*e) + 26*I*e^(2*I*f*x + 2*I*e) + 5*I)*e^(-7*I*f*x - 7*I*e)/(a^4*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-ic\,(\tan(e+fx)+i)}}{(ia\,(\tan(e+fx)-i))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e))**(7/2),x)
```

```
[Out] Integral(sqrt(-I*c*(tan(e + f*x) + I))/(I*a*(tan(e + f*x) - I))**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(7/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-I*c*tan(f*x + e) + c)/(I*a*tan(f*x + e) + a)^(7/2), x)
```

Mupad [B]
time = 6.73, size = 183, normalized size = 1.01

$$\frac{\sqrt{\frac{a\,\left(\cos\left(2\,e+2\,f\,x\right)+1+\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\sqrt{\frac{c\,\left(\cos\left(2\,e+2\,f\,x\right)+1-\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\left(\cos\left(2\,e+2\,f\,x\right)\,70{}\mathrm{i}+\cos\left(4\,e+4\,f\,x\right)\,56{}\mathrm{i}+\cos\left(6\,e+6\,f\,x\right)\,26{}\mathrm{i}+\cos\left(8\,e+8\,f\,x\right)\,5{}\mathrm{i}+70\,\sin\left(2\,e+2\,f\,x\right)+56\,\sin\left(4\,e+4\,f\,x\right)+26\,\sin\left(6\,e+6\,f\,x\right)+5\,\sin\left(8\,e+8\,f\,x\right)+35{}\mathrm{i}\right)}{560\,a^4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^(1/2)/(a + a*tan(e + f*x)*1i)^(7/2),x)
```

```
[Out] (((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^
(1/2)*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) +
 1))^(1/2)*(cos(2*e + 2*f*x)*70i + cos(4*e + 4*f*x)*56i + cos(6*e + 6*f*x)*
26i + cos(8*e + 8*f*x)*5i + 70*sin(2*e + 2*f*x) + 56*sin(4*e + 4*f*x) + 26*
sin(6*e + 6*f*x) + 5*sin(8*e + 8*f*x) + 35i))/(560*a^4*f)
```

3.999 $\int (a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=159

$$-\frac{ia^{5/2}c^{3/2}\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{f}+\frac{a^2c\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{2f}$$

```
[Out] -I*a^(5/2)*c^(3/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*c*t
an(f*x+e))^(1/2))/f+1/2*a^2*c*(a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))^(
1/2)*tan(f*x+e)/f+1/3*I*a*(a+I*a*tan(f*x+e))^(3/2)*(c-I*c*tan(f*x+e))^(3/2)
/f
```

Rubi [A]
time = 0.11, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3604, 51, 38, 65, 223, 209}

$$-\frac{ia^{5/2}c^{3/2}\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{f}+\frac{a^2c\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{2f}+\frac{ia(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^(5/2)*(c - I*c*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-I)*a^(5/2)*c^(3/2)*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a]*
Sqrt[c - I*c*Tan[e + f*x]])])/f + (a^2*c*Tan[e + f*x]*Sqrt[a + I*a*Tan[e +
f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/(2*f) + ((I/3)*a*(a + I*a*Tan[e + f*x])^(
3/2)*(c - I*c*Tan[e + f*x])^(3/2))/f
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 51

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[2*c*(n/(m + n + 1)
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{3/2}\,dx = \frac{(ac)\text{Subst}\left(\int (a+iax)^{3/2}\sqrt{c-icx}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{ia(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}{3f} + \frac{(a^2}{}$$

$$= \frac{a^2c\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+}}{2f}$$

$$= \frac{a^2c\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+}}{2f}$$

$$= \frac{a^2c\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+}}{2f}$$

$$= -\frac{ia^{5/2}c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{f} + \frac{a^2c\,t}{}$$

Mathematica [A]
time = 3.43, size = 101, normalized size = 0.64

$$-\frac{a^2c^2\sec^2(e+fx)\left(-4+12\text{ArcTan}\left(e^{i(e+fx)}\right)\cos^3(e+fx)+3i\sin(2(e+fx))\right)(i+\tan(e+fx))\sqrt{a+ia\tan(e+fx)}}{12f\sqrt{c-ic\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(5/2)*(c - I*c*Tan[e + f*x])^(3/2),x]
```

```
[Out] -1/12*(a^2*c^2*Sec[e + f*x]^2*(-4 + 12*ArcTan[E^(I*(e + f*x))]*Cos[e + f*x]
^3 + (3*I)*Sin[2*(e + f*x)])*(I + Tan[e + f*x])*Sqrt[a + I*a*Tan[e + f*x]])
/(f*Sqrt[c - I*c*Tan[e + f*x]])
```

Maple [A]
time = 0.33, size = 186, normalized size = 1.17

method	result
derivativedivides	$\frac{\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,a^2c\left(2i\sqrt{ac\left(1+\tan^2\left(fx+e\right)\right)}\,\sqrt{ac}\right.}{}$

default	$\frac{\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,a^{2}c\left(2i\sqrt{ac\left(1+\tan^{2}\left(fx+e\right)\right)}\,\sqrt{ac}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(5/2)*(c-I*c*tan(f*x+e))^(3/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/6/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^2*c*(2*I*(a*
c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^2+3*a*c*ln((c*a*tan(f*x+e)
+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))+2*I*(a*c*(1+tan(f*x
+e)^2))^(1/2)*(a*c)^(1/2)+3*tan(f*x+e)*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(
1/2))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(a*c)^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 949 vs. 2(126) = 252.
time = 0.62, size = 949, normalized size = 5.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)*(c-I*c*tan(f*x+e))^(3/2),x, algorithm="m
axima")
```

```
[Out] -(12*a^2*c*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 32*a^2*c*
cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 12*a^2*c*cos(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 12*I*a^2*c*sin(5/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) - 32*I*a^2*c*sin(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 12*I*a^2*c*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) + 6*(a^2*c*cos(6*f*x + 6*e) + 3*a^2*c*cos(4*f*x + 4*e) + 3*a
^2*c*cos(2*f*x + 2*e) + I*a^2*c*sin(6*f*x + 6*e) + 3*I*a^2*c*sin(4*f*x + 4*
e) + 3*I*a^2*c*sin(2*f*x + 2*e) + a^2*c)*arctan2(cos(1/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))), sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + 1) + 6*(a^2*c*cos(6*f*x + 6*e) + 3*a^2*c*cos(4*f*x + 4*e) + 3*a^2*c
*cos(2*f*x + 2*e) + I*a^2*c*sin(6*f*x + 6*e) + 3*I*a^2*c*sin(4*f*x + 4*e) +
 3*I*a^2*c*sin(2*f*x + 2*e) + a^2*c)*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))), -sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) + 1) + 3*(I*a^2*c*cos(6*f*x + 6*e) + 3*I*a^2*c*cos(4*f*x + 4*e) + 3*I*a^
2*c*cos(2*f*x + 2*e) - a^2*c*sin(6*f*x + 6*e) - 3*a^2*c*sin(4*f*x + 4*e) -
3*a^2*c*sin(2*f*x + 2*e) + I*a^2*c)*log(cos(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^
2 + 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 3*(-I*a^2
*c*cos(6*f*x + 6*e) - 3*I*a^2*c*cos(4*f*x + 4*e) - 3*I*a^2*c*cos(2*f*x + 2*
e) + a^2*c*sin(6*f*x + 6*e) + 3*a^2*c*sin(4*f*x + 4*e) + 3*a^2*c*sin(2*f*x
```

```
+ 2*e) - I*a^2*c)*log(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^
2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sin(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1))*sqrt(a)*sqrt(c)/(f*(-12*I*co
s(6*f*x + 6*e) - 36*I*cos(4*f*x + 4*e) - 36*I*cos(2*f*x + 2*e) + 12*sin(6*f
*x + 6*e) + 36*sin(4*f*x + 4*e) + 36*sin(2*f*x + 2*e) - 12*I))
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 455 vs. $2(126) = 252$.
time = 1.50, size = 455, normalized size = 2.86

$$\frac{3\sqrt{\frac{a^5c^3}{f^2}}\left(fe^{(4i\,fx+4i\,e)}+2\,fe^{(2i\,fx+2i\,e)}+f\right)\log\left(\frac{4\left(2\left(a^2ce^{(3i\,fx+3i\,e)}+a^2ce^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}-\sqrt{\frac{a^5c^3}{f^2}}\left(i\,fe^{(2i\,fx+2i\,e)}-i\,f\right)\right)}{a^2ce^{(2i\,fx+2i\,e)}+a^2c}\right)-3\sqrt{\frac{a^5c^3}{f^2}}\left(fe^{(4i\,fx+4i\,e)}+2\,fe^{(2i\,fx+2i\,e)}+f\right)\log\left(\frac{4\left(2\left(a^2ce^{(3i\,fx+3i\,e)}+a^2ce^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}-\sqrt{\frac{a^5c^3}{f^2}}\left(-i\,fe^{(2i\,fx+2i\,e)}+i\,f\right)\right)}{a^2ce^{(2i\,fx+2i\,e)}+a^2c}\right)-4\left(3i\,a^2ce^{(5i\,fx+5i\,e)}-8i\,a^2ce^{(3i\,fx+3i\,e)}-3i\,a^2ce^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{12\left(fe^{(4i\,fx+4i\,e)}+2\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)*(c-I*c*tan(f*x+e))^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/12*(3*sqrt(a^5*c^3/f^2)*(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*e)
+ f)*log(4*(2*(a^2*c*e^(3*I*f*x + 3*I*e) + a^2*c*e^(I*f*x + I*e))*sqrt(a/(e
^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - sqrt(a^5*c^3/f
^2)*(I*f*e^(2*I*f*x + 2*I*e) - I*f))/(a^2*c*e^(2*I*f*x + 2*I*e) + a^2*c)) -
 3*sqrt(a^5*c^3/f^2)*(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*e) + f)*
log(4*(2*(a^2*c*e^(3*I*f*x + 3*I*e) + a^2*c*e^(I*f*x + I*e))*sqrt(a/(e^(2*I
*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - sqrt(a^5*c^3/f^2)*(
-I*f*e^(2*I*f*x + 2*I*e) + I*f))/(a^2*c*e^(2*I*f*x + 2*I*e) + a^2*c)) - 4*(
3*I*a^2*c*e^(5*I*f*x + 5*I*e) - 8*I*a^2*c*e^(3*I*f*x + 3*I*e) - 3*I*a^2*c*e
^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*
e) + 1)))/(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(5/2)*(c-I*c*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)*(c-I*c*tan(f*x+e))^(3/2),x, algorithm="g
iac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a\tan(e + f\,x)\ 1\mathrm{i})^{5/2}\,(c - c\tan(e + f\,x)\ 1\mathrm{i})^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(5/2)*(c - c*tan(e + f*x)*1i)^(3/2),x)
[Out] int((a + a*tan(e + f*x)*1i)^(5/2)*(c - c*tan(e + f*x)*1i)^(3/2), x)
```

3.1000 $\int (a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=113

$$-\frac{ia^{3/2}c^{3/2}\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{f}+\frac{ac\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{2f}$$

```
[Out] -I*a^(3/2)*c^(3/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*c*t
an(f*x+e))^(1/2))/f+1/2*a*c*(a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))^(1/
2)*tan(f*x+e)/f
```

Rubi [A]
time = 0.10, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3604, 38, 65, 223, 209}

$$\frac{ac\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{2f}-\frac{ia^{3/2}c^{3/2}\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^(3/2)*(c - I*c*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-I)*a^(3/2)*c^(3/2)*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a]*
Sqrt[c - I*c*Tan[e + f*x]])])/f + (a*c*Tan[e + f*x]*Sqrt[a + I*a*Tan[e + f*
x]]*Sqrt[c - I*c*Tan[e + f*x]])/(2*f)
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}\,dx = \frac{(ac)\text{Subst}\left(\int \sqrt{a+iax}\,\sqrt{c-icx}\,dx,x,\tan(e+fx)\right.}{f}$$

$$= \frac{ac\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+}}{2f}$$

$$= \frac{ac\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+}}{2f}$$

$$= \frac{ac\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+}}{2f}$$

$$= -\frac{ia^{3/2}c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{f}+\frac{ac}{}$$

Mathematica [A]
time = 2.85, size = 133, normalized size = 1.18

$$\frac{ac^2e^{-2ie}\left(\cos\left(\frac{3e}{2}\right)+i\sin\left(\frac{3e}{2}\right)\right)\left(\cos\left(\frac{e}{2}+fx\right)-i\sin\left(\frac{e}{2}+fx\right)\right)\sqrt{a+ia\tan(e+fx)}\left(-2i\text{ArcTan}\left(e^{i(e+fx)}\right)+\sec(e+fx)\tan(e+fx)\right)}{2\sqrt{2}\sqrt{\frac{c}{1+e^{2i(e+fx)}}}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(3/2)*(c - I*c*Tan[e + f*x])^(3/2),x]
```

```
[Out] (a*c^2*(Cos[(3*e)/2] + I*Sin[(3*e)/2])*(Cos[e/2 + f*x] - I*Sin[e/2 + f*x])*
Sqrt[a + I*a*Tan[e + f*x]]*((-2*I)*ArcTan[E^(I*(e + f*x))] + Sec[e + f*x]*T
an[e + f*x]))/(2*Sqrt[2]*E^((2*I)*e)*Sqrt[c/(1 + E^((2*I)*(e + f*x)))]*f)
```

Maple [A]
time = 0.34, size = 128, normalized size = 1.13

method	result
derivativedivides	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,ac\left(\tan(fx+e)\sqrt{ac(1+\tan^2(fx+e)}\right.}{2f\sqrt{ac(1+\tan^2(fx+e))}\,\sqrt{a}}$
default	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,ac\left(\tan(fx+e)\sqrt{ac(1+\tan^2(fx+e)}\right.}{2f\sqrt{ac(1+\tan^2(fx+e))}\,\sqrt{a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(3/2)*(c-I*c*tan(f*x+e))^(3/2),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/2/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a*c*(tan(f*x+
e)*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)+a*c*ln((c*a*tan(f*x+e)+(a*c*(1+
tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2)))/(a*c*(1+tan(f*x+e)^2))^(1/2
)/(a*c)^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 689 vs. 2(90) = 180.
time = 0.59, size = 689, normalized size = 6.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)*(c-I*c*tan(f*x+e))^(3/2),x, algorithm="m
axima")
```

```
[Out] -(4*a*c*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*a*c*cos(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*I*a*c*sin(3/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*I*a*c*sin(1/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))) + 2*(a*c*cos(4*f*x + 4*e) + 2*a*c*cos(2*f*x + 2*e) +
 I*a*c*sin(4*f*x + 4*e) + 2*I*a*c*sin(2*f*x + 2*e) + a*c)*arctan2(cos(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), sin(1/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))) + 1) + 2*(a*c*cos(4*f*x + 4*e) + 2*a*c*cos(2*f*x + 2
*e) + I*a*c*sin(4*f*x + 4*e) + 2*I*a*c*sin(2*f*x + 2*e) + a*c)*arctan2(cos(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), -sin(1/2*arctan2(sin(2*f*
```

```
x + 2*e), cos(2*f*x + 2*e))) + 1) - (-I*a*c*cos(4*f*x + 4*e) - 2*I*a*c*cos(
2*f*x + 2*e) + a*c*sin(4*f*x + 4*e) + 2*a*c*sin(2*f*x + 2*e) - I*a*c)*log(c
os(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + 1) - (I*a*c*cos(4*f*x + 4*e) + 2*I*a*c*cos(2*f*x + 2*e)
 - a*c*sin(4*f*x + 4*e) - 2*a*c*sin(2*f*x + 2*e) + I*a*c)*log(cos(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e)))^2 - 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + 1))*sqrt(a)*sqrt(c)/(f*(-4*I*cos(4*f*x + 4*e) - 8*I*cos(2*f*x + 2*e
) + 4*sin(4*f*x + 4*e) + 8*sin(2*f*x + 2*e) - 4*I))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(90) = 180$.
time = 1.82, size = 379, normalized size = 3.35

$$\frac{\sqrt{\frac{a^3c^3}{f^2}}\left(fe^{(2i\,fx+2i\,e)}+f\right)\log\left(\frac{4\left(2\left(ace^{(3i\,fx+3i\,e)}+ace^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}-\sqrt{\frac{a^3c^3}{f^2}}\left(i\,fe^{(2i\,fx+2i\,e)}-i\,f\right)\right)}{ace^{(2i\,fx+2i\,e)}+ac}\right)-\sqrt{\frac{a^3c^3}{f^2}}\left(fe^{(2i\,fx+2i\,e)}+f\right)\log\left(\frac{4\left(2\left(ace^{(3i\,fx+3i\,e)}+ace^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}-\sqrt{\frac{a^3c^3}{f^2}}\left(-i\,fe^{(2i\,fx+2i\,e)}+i\,f\right)\right)}{ace^{(2i\,fx+2i\,e)}+ac}\right)-4\left(i\,ace^{(3i\,fx+3i\,e)}-i\,ace^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{4\left(fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)*(c-I*c*tan(f*x+e))^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/4*(sqrt(a^3*c^3/f^2)*(f*e^(2*I*f*x + 2*I*e) + f)*log(4*(2*(a*c*e^(3*I*f*x
 + 3*I*e) + a*c*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(
e^(2*I*f*x + 2*I*e) + 1)) - sqrt(a^3*c^3/f^2)*(I*f*e^(2*I*f*x + 2*I*e) - I*
f))/(a*c*e^(2*I*f*x + 2*I*e) + a*c)) - sqrt(a^3*c^3/f^2)*(f*e^(2*I*f*x + 2*
I*e) + f)*log(4*(2*(a*c*e^(3*I*f*x + 3*I*e) + a*c*e^(I*f*x + I*e))*sqrt(a/(
e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - sqrt(a^3*c^3/
f^2)*(-I*f*e^(2*I*f*x + 2*I*e) + I*f))/(a*c*e^(2*I*f*x + 2*I*e) + a*c)) - 4
*(I*a*c*e^(3*I*f*x + 3*I*e) - I*a*c*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2
*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(e+fx)-i))^{\frac{3}{2}}\,(-ic(\tan(e+fx)+i))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(3/2)*(c-I*c*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((I*a*(tan(e + f*x) - I))**(3/2)*(-I*c*(tan(e + f*x) + I))**(3/2),
x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(90) = 180$.
time = 1.35, size = 307, normalized size = 2.72

$$-\frac{7\left(\left(a^2c-ac\right)\sqrt{-ac}\,e^{(9i\,fx+9i\,e)}+6\left(a^2c-ac\right)\sqrt{-ac}\,e^{(7i\,fx+7i\,e)}+12\left(a^2c-ac\right)\sqrt{-ac}\,e^{(5i\,fx+5i\,e)}+10\left(a^2c-ac\right)\sqrt{-ac}\,e^{(3i\,fx+3i\,e)}+3\left(a^2c-ac\right)\sqrt{-ac}\,e^{(i\,fx+i\,e)}\right)}{4\left((a-1)fe^{(10i\,fx+10i\,e)}+5\,(a-1)fe^{(8i\,fx+8i\,e)}+10\,(a-1)fe^{(6i\,fx+6i\,e)}+10\,(a-1)fe^{(4i\,fx+4i\,e)}+5\,(a-1)fe^{(2i\,fx+2i\,e)}+(a-1)f\right)}-\frac{4i\,a^{\frac{3}{2}}c^{\frac{3}{2}}\arctan\left(e^{(i\,fx+i\,e)}\right)-\frac{i\left(3\,a^{\frac{3}{2}}c^{\frac{3}{2}}e^{(3i\,fx+3i\,e)}+a^{\frac{3}{2}}c^{\frac{3}{2}}e^{(i\,fx+i\,e)}\right)}{\left(e^{(2i\,fx+2i\,e)}+1\right)^2}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)*(c-I*c*tan(f*x+e))^(3/2),x, algorithm="g
iac")
```

```
[Out] -7/4*((a^2*c - a*c)*sqrt(-a*c)*e^(9*I*f*x + 9*I*e) + 6*(a^2*c - a*c)*sqrt(-
a*c)*e^(7*I*f*x + 7*I*e) + 12*(a^2*c - a*c)*sqrt(-a*c)*e^(5*I*f*x + 5*I*e)
+ 10*(a^2*c - a*c)*sqrt(-a*c)*e^(3*I*f*x + 3*I*e) + 3*(a^2*c - a*c)*sqrt(-a
*c)*e^(I*f*x + I*e))/((a - 1)*f*e^(10*I*f*x + 10*I*e) + 5*(a - 1)*f*e^(8*I*
f*x + 8*I*e) + 10*(a - 1)*f*e^(6*I*f*x + 6*I*e) + 10*(a - 1)*f*e^(4*I*f*x +
 4*I*e) + 5*(a - 1)*f*e^(2*I*f*x + 2*I*e) + (a - 1)*f) - 1/4*(4*I*a^(3/2)*c
^(3/2)*arctan(e^(I*f*x + I*e)) - I*(3*a^(3/2)*c^(3/2)*e^(3*I*f*x + 3*I*e) +
 a^(3/2)*c^(3/2)*e^(I*f*x + I*e))/(e^(2*I*f*x + 2*I*e) + 1)^2)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a\tan(e + f\,x)\;1\mathrm{i})^{3/2}\,(c - c\tan(e + f\,x)\;1\mathrm{i})^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(3/2)*(c - c*tan(e + f*x)*1i)^(3/2),x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^(3/2)*(c - c*tan(e + f*x)*1i)^(3/2), x)
```

3.1001 $\int \sqrt{a+ia\tan(e+fx)}\,(c-ic\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=106

$$-\frac{2i\sqrt{a}\,c^{3/2}\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{f}-\frac{ic\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{f}$$

```
[Out] -2*I*c^(3/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*c*tan(f*x
+e))^(1/2))*a^(1/2)/f-I*c*(a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))^(1/2)
/f
```

Rubi [A]
time = 0.10, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3604, 52, 65, 223, 209}

$$-\frac{2i\sqrt{a}\,c^{3/2}\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{f}-\frac{ic\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[e + f*x]]*(c - I*c*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-2*I)*Sqrt[a]*c^(3/2)*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a
]*Sqrt[c - I*c*Tan[e + f*x]])])/f - (I*c*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c
- I*c*Tan[e + f*x]])/f
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \sqrt{a+ia\tan(e+fx)}\,(c-ic\tan(e+fx))^{3/2}\,dx=\frac{(ac)\mathrm{Subst}\left(\int \frac{\sqrt{c-icx}}{\sqrt{a+iax}}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=-\frac{ic\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{f}+\frac{(ac^2}{}$$

$$\frac{(2ic}{}$$

$$=-\frac{ic\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{f}-$$

$$=-\frac{ic\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{f}-\frac{(2ic}{}$$

$$=-\frac{2i\sqrt{a}\,c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{f}-\frac{ic\sqrt{}}{}$$

Mathematica [A]
time = 1.51, size = 100, normalized size = 0.94

$$-\frac{i\sqrt{2}\,ce^{-i(e+fx)}\sqrt{\frac{c}{1+e^{2i(e+fx)}}}\,\left(e^{i(e+fx)}+\left(1+e^{2i(e+fx)}\right)\mathrm{ArcTan}\left(e^{i(e+fx)}\right)\right)\sqrt{a+ia\tan(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Tan[e + f*x]]*(c - I*c*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-I)*Sqrt[2]*c*Sqrt[c/(1 + E^((2*I)*(e + f*x)))]*(E^(I*(e + f*x)) + (1 + E
^((2*I)*(e + f*x)))*ArcTan[E^(I*(e + f*x))])*Sqrt[a + I*a*Tan[e + f*x]])/(E
^(I*(e + f*x))*f)
```

Maple [A]
time = 0.34, size = 122, normalized size = 1.15

method	result
derivativedivides	$\frac{\left(-i\sqrt{ac\left(1+\tan^2\left(fx+e\right)\right)}\,\sqrt{ac}+ac\ln\left(\frac{ca\tan(fx+e)+\sqrt{ac\left(1+\tan^2\left(fx+e\right)\right)}\,\sqrt{ac}}{\sqrt{ac}}\right)\right)\sqrt{}}{f\sqrt{ac\left(1+\tan^2\left(fx+e\right)\right)}\,\sqrt{ac}}$
default	$\frac{\left(-i\sqrt{ac\left(1+\tan^2\left(fx+e\right)\right)}\,\sqrt{ac}+ac\ln\left(\frac{ca\tan(fx+e)+\sqrt{ac\left(1+\tan^2\left(fx+e\right)\right)}\,\sqrt{ac}}{\sqrt{ac}}\right)\right)\sqrt{}}{f\sqrt{ac\left(1+\tan^2\left(fx+e\right)\right)}\,\sqrt{ac}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))^(3/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/f*(-I*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)+a*c*ln((c*a*tan(f*x+e)+(a*
c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2)))*(a*(1+I*tan(f*x+e)))^(
1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*c/(a*c*(1+tan(f*x+e)^2))^(1/2)/(a*c)^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(82) = 164$.
time = 0.57, size = 277, normalized size = 2.61

$$-\frac{\left(2\left(c\cos\left(2fx+2e\right)+ic\sin\left(2fx+2e\right)+c\right)\arctan\left(\cos\left(fx+e\right),\sin\left(fx+e\right)+1\right)+2\left(c\cos\left(2fx+2e\right)+ic\sin\left(2fx+2e\right)+c\right)\arctan\left(\cos\left(fx+e\right),-\sin\left(fx+e\right)+1\right)+4c\cos\left(fx+e\right)-\left(-ic\cos\left(2fx+2e\right)+c\sin\left(2fx+2e\right)-ic\right)\log\left(\cos\left(fx+e\right)^2+\sin\left(fx+e\right)^2+2\sin\left(fx+e\right)+1\right)-\left(ic\cos\left(2fx+2e\right)-c\sin\left(2fx+2e\right)+ic\right)\log\left(\cos\left(fx+e\right)^2+\sin\left(fx+e\right)^2-2\sin\left(fx+e\right)+1\right)+4ic\sin\left(fx+e\right)\right)\sqrt{a}\sqrt{c}}{-2f\left(i\cos\left(2fx+2e\right)-\sin\left(2fx+2e\right)+i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))^(3/2),x, algorithm="m
axima")
```

```
[Out] -(2*(c*cos(2*f*x + 2*e) + I*c*sin(2*f*x + 2*e) + c)*arctan2(cos(f*x + e), s
in(f*x + e) + 1) + 2*(c*cos(2*f*x + 2*e) + I*c*sin(2*f*x + 2*e) + c)*arctan
2(cos(f*x + e), -sin(f*x + e) + 1) + 4*c*cos(f*x + e) - (-I*c*cos(2*f*x + 2
*e) + c*sin(2*f*x + 2*e) - I*c)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin
(f*x + e) + 1) - (I*c*cos(2*f*x + 2*e) - c*sin(2*f*x + 2*e) + I*c)*log(cos(
f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1) + 4*I*c*sin(f*x + e))*sqr
t(a)*sqrt(c)/(f*(-2*I*cos(2*f*x + 2*e) + 2*sin(2*f*x + 2*e) - 2*I))
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(82) = 164$.
time = 2.06, size = 304, normalized size = 2.87

$$-\frac{4i\,c\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\,e^{(i\,fx+i\,e)}-\sqrt{\frac{ac^3}{f^2}}\,f\log\left(\frac{4\left(2\left(ce^{(3i\,fx+3i\,e)}+ce^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}-\sqrt{\frac{ac^3}{f^2}}\left(i\,fe^{(2i\,fx+2i\,e)}-i\,f\right)\right)}{ce^{(2i\,fx+2i\,e)}+c}\right)+\sqrt{\frac{ac^3}{f^2}}\,f\log\left(\frac{4\left(2\left(ce^{(3i\,fx+3i\,e)}+ce^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}-\sqrt{\frac{ac^3}{f^2}}\left(-i\,fe^{(2i\,fx+2i\,e)}+i\,f\right)\right)}{ce^{(2i\,fx+2i\,e)}+c}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))^(3/2),x, algorithm="f
ricas")
```

```
[Out] -1/2*(4*I*c*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) +
 1))*e^(I*f*x + I*e) - sqrt(a*c^3/f^2)*f*log(4*(2*(c*e^(3*I*f*x + 3*I*e) +
c*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2
*I*e) + 1)) - sqrt(a*c^3/f^2)*(I*f*e^(2*I*f*x + 2*I*e) - I*f))/(c*e^(2*I*f*
x + 2*I*e) + c)) + sqrt(a*c^3/f^2)*f*log(4*(2*(c*e^(3*I*f*x + 3*I*e) + c*e^
(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e
) + 1)) - sqrt(a*c^3/f^2)*(-I*f*e^(2*I*f*x + 2*I*e) + I*f))/(c*e^(2*I*f*x +
 2*I*e) + c)))/f
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\left(\tan\left(e+fx\right)-i\right)}\left(-ic(\tan\left(e+fx\right)+i)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(1/2)*(c-I*c*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(e + f*x) - I))*(-I*c*(tan(e + f*x) + I))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(I*a*tan(f*x + e) + a)*(-I*c*tan(f*x + e) + c)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+a\tan\left(e+f\,x\right)\,1\mathrm{i}}\,\left(c-c\tan(e+f\,x)\,1\mathrm{i}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(1/2)*(c - c*tan(e + f*x)*1i)^(3/2),x)
[Out] int((a + a*tan(e + f*x)*1i)^(1/2)*(c - c*tan(e + f*x)*1i)^(3/2), x)
```

3.1002 $\int \frac{(c-ic\tan(e+fx))^{3/2}}{\sqrt{a+ia\tan(e+fx)}}\,dx$

Optimal. Leaf size=106

$$\frac{2ic^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{a}\,f}+\frac{2ic\sqrt{c-ic\tan(e+fx)}}{f\sqrt{a+ia\tan(e+fx)}}$$

```
[Out] 2*I*c^(3/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*c*tan(f*x+
e))^(1/2))/f/a^(1/2)+2*I*c*(c-I*c*tan(f*x+e))^(1/2)/f/(a+I*a*tan(f*x+e))^(1
/2)
```

Rubi [A]
time = 0.09, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3604, 49, 65, 223, 209}

$$\frac{2ic^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{a}\,f}+\frac{2ic\sqrt{c-ic\tan(e+fx)}}{f\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])^(3/2)/Sqrt[a + I*a*Tan[e + f*x]],x]
```

```
[Out] ((2*I)*c^(3/2)*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c
 - I*c*Tan[e + f*x]])])/(Sqrt[a]*f) + ((2*I)*c*Sqrt[c - I*c*Tan[e + f*x]])/(
f*Sqrt[a + I*a*Tan[e + f*x]])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(c-ic\tan(e+fx))^{3/2}}{\sqrt{a+ia\tan(e+fx)}}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{\sqrt{c-icx}}{(a+iax)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{2ic\sqrt{c-ic\tan(e+fx)}}{f\sqrt{a+ia\tan(e+fx)}} - \frac{c^2\text{Subst}\left(\int \frac{1}{\sqrt{a+iax}\sqrt{c-icx}}\,dx, x, \tan(e+\right.}{f}$$

$$= \frac{2ic\sqrt{c-ic\tan(e+fx)}}{f\sqrt{a+ia\tan(e+fx)}} + \frac{(2ic^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{2c-\frac{cx^2}{a}}}\,dx, x, \sqrt{a+ia\tan\right.}{af}$$

$$= \frac{2ic\sqrt{c-ic\tan(e+fx)}}{f\sqrt{a+ia\tan(e+fx)}} + \frac{(2ic^2)\,\text{Subst}\left(\int \frac{1}{1+\frac{cx^2}{a}}\,dx, x, \frac{\sqrt{a+ia\tan(e+f}}{\sqrt{c-ic\tan(e+f}}\right.}{af}$$

$$= \frac{2ic^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{a}\,f} + \frac{2ic\sqrt{c-ic\tan(e+fx)}}{f\sqrt{a+ia\tan(e+fx)}}$$

Mathematica [A]

time = 1.93, size = 119, normalized size = 1.12

$$\frac{c^2(\cos(fx)+i\sin(fx))(i\cos(fx)+\sin(fx))(1+\text{ArcTan}(\cos(e+fx)+i\sin(e+fx))\sec(e+fx)-i\tan(e+fx))}{f\sqrt{\frac{c}{2+2\cos(2(e+fx))+2i\sin(2(e+fx))}}\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - I*c*Tan[e + f*x])^(3/2)/Sqrt[a + I*a*Tan[e + f*x]],x]
```

```
[Out] (c^2*(Cos[f*x] + I*Sin[f*x])*(I*Cos[f*x] + Sin[f*x])*(1 + ArcTan[Cos[e + f*
x] + I*Sin[e + f*x]]*Sec[e + f*x] - I*Tan[e + f*x]))/(f*Sqrt[c/(2 + 2*Cos[2
*(e + f*x)] + (2*I)*Sin[2*(e + f*x)])]*Sqrt[a + I*a*Tan[e + f*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(84) = 168.
time = 0.38, size = 269, normalized size = 2.54

method	result
derivativedivides	$\frac{i\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,c\left(i\ln\left(\frac{ca\tan(fx+e)+\sqrt{ac\left(1+\tan^2\left(fx-\right.\right.}}{\sqrt{ac}}\right.\right.}{}$
default	$\frac{i\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,c\left(i\ln\left(\frac{ca\tan(fx+e)+\sqrt{ac\left(1+\tan^2\left(fx-\right.\right.}}{\sqrt{ac}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] I/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)/a*c*(I*ln((c*a*t
an(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))*a*c*tan(f*
x+e)^2-I*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)
^(1/2))*a*c-2*I*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)+2*ln((c
*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))*a*c*ta
n(f*x+e)-2*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c*(1+tan(f*x+e)^2))
^(1/2)/(-tan(f*x+e)+I)^2/(a*c)^(1/2)
```

Maxima [A]
time = 0.62, size = 139, normalized size = 1.31

$$-\frac{\left(2ic\arctan\left(\cos\left(fx+e\right),\sin\left(fx+e\right)+1\right)+2ic\arctan\left(\cos\left(fx+e\right),-\sin\left(fx+e\right)+1\right)-4ic\cos\left(fx+e\right)+c\log\left(\cos\left(fx+e\right)^2+\sin\left(fx+e\right)^2+2\sin\left(fx+e\right)+1\right)-c\log\left(\cos\left(fx+e\right)^2+\sin\left(fx+e\right)^2-2\sin\left(fx+e\right)+1\right)-4c\sin\left(fx+e\right)\right)\sqrt{c}}{2\sqrt{a}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(1/2),x, algorithm="m
axima")
```

```
[Out] -1/2*(2*I*c*arctan2(cos(f*x + e), sin(f*x + e) + 1) + 2*I*c*arctan2(cos(f*x
 + e), -sin(f*x + e) + 1) - 4*I*c*cos(f*x + e) + c*log(cos(f*x + e)^2 + sin
(f*x + e)^2 + 2*sin(f*x + e) + 1) - c*log(cos(f*x + e)^2 + sin(f*x + e)^2 -
 2*sin(f*x + e) + 1) - 4*c*sin(f*x + e))*sqrt(c)/(sqrt(a)*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(82) = 164$.
time = 1.85, size = 357, normalized size = 3.37

$$-\frac{\left(af\sqrt{\frac{c^3}{af^2}}e^{(ifx+ie)}\log\left(\frac{4\left(2\left(ce^{(3ifx+3ie)}+ce^{(ifx+ie)}\right)\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}-\left(iafe^{(2ifx+2ie)}-iaf\right)\sqrt{\frac{c^3}{af^2}}\right)}{ce^{(2ifx+2ie)}+c}\right)-af\sqrt{\frac{c^3}{af^2}}e^{(ifx+ie)}\log\left(\frac{4\left(2\left(ce^{(3ifx+3ie)}+ce^{(ifx+ie)}\right)\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}-\left(-iafe^{(2ifx+2ie)}+iaf\right)\sqrt{\frac{c^3}{af^2}}\right)}{ce^{(2ifx+2ie)}+c}\right)+4\left(-ice^{(2ifx+2ie)}-ic\right)\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}\right)e^{(-ifx-ie)}}{2af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(1/2),x, algorithm="f
ricas")
```

```
[Out] -1/2*(a*f*sqrt(c^3/(a*f^2))*e^(I*f*x + I*e)*log(4*(2*(c*e^(3*I*f*x + 3*I*e)
 + c*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x
+ 2*I*e) + 1)) - (I*a*f*e^(2*I*f*x + 2*I*e) - I*a*f)*sqrt(c^3/(a*f^2)))/(c*
e^(2*I*f*x + 2*I*e) + c)) - a*f*sqrt(c^3/(a*f^2))*e^(I*f*x + I*e)*log(4*(2*
(c*e^(3*I*f*x + 3*I*e) + c*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1
))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - (-I*a*f*e^(2*I*f*x + 2*I*e) + I*a*f)
*sqrt(c^3/(a*f^2)))/(c*e^(2*I*f*x + 2*I*e) + c)) + 4*(-I*c*e^(2*I*f*x + 2*I
*e) - I*c)*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) +
1)))*e^(-I*f*x - I*e)/(a*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-ic\left(\tan\left(e+fx\right)+i\right)\right)^{\frac{3}{2}}}{\sqrt{ia\left(\tan\left(e+fx\right)-i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**(3/2)/(a+I*a*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral((-I*c*(tan(e + f*x) + I))**(3/2)/sqrt(I*a*(tan(e + f*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((-I*c*tan(f*x + e) + c)^(3/2)/sqrt(I*a*tan(f*x + e) + a), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c - c\tan(e + f\,x)\ 1\mathrm{i})^{3/2}}{\sqrt{a + a\tan\left(e + f\,x\right)\ 1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^(3/2)/(a + a*tan(e + f*x)*1i)^(1/2),x)
[Out] int((c - c*tan(e + f*x)*1i)^(3/2)/(a + a*tan(e + f*x)*1i)^(1/2), x)
```

3.1003 $\int \frac{(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=43

$$\frac{i(c-ic\tan(e+fx))^{3/2}}{3f(a+ia\tan(e+fx))^{3/2}}$$

```
[Out] 1/3*I*(c-I*c*tan(f*x+e))^(3/2)/f/(a+I*a*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {3604, 37}

$$\frac{i(c-ic\tan(e+fx))^{3/2}}{3f(a+ia\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])^(3/2)/(a + I*a*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((I/3)*(c - I*c*Tan[e + f*x])^(3/2))/(f*(a + I*a*Tan[e + f*x])^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^{3/2}}\,dx &= \frac{(ac)\text{Subst}\left(\int \frac{\sqrt{c-icx}}{(a+iax)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{i(c-ic\tan(e+fx))^{3/2}}{3f(a+ia\tan(e+fx))^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 1.20, size = 69, normalized size = 1.60

$$\frac{c(1 - i\tan(e + fx))\sqrt{c - ic\tan(e + fx)}}{3af(-i + \tan(e + fx))\sqrt{a + ia\tan(e + fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - I*c*Tan[e + f*x])^(3/2)/(a + I*a*Tan[e + f*x])^(3/2),x]
```

```
[Out] (c*(1 - I*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(3*a*f*(-I + Tan[e + f*
x])*Sqrt[a + I*a*Tan[e + f*x]])
```

Maple [A]
time = 0.45, size = 64, normalized size = 1.49

method	result	size
derivativedivides	$\frac{\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\,c(1+\tan^2(fx+e))}{3f\,a^2(-\tan(fx+e)+i)^3}$	64
default	$\frac{\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\,c(1+\tan^2(fx+e))}{3f\,a^2(-\tan(fx+e)+i)^3}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(3/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/3/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)/a^2*c*(1+tan(f
*x+e)^2)/(-tan(f*x+e)+I)^3
```

Maxima [A]
time = 0.54, size = 37, normalized size = 0.86

$$\frac{(i\,c\cos(3\,fx + 3\,e) + c\sin(3\,fx + 3\,e))\sqrt{c}}{3\,a^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(3/2),x, algorithm="m
axima")
```

```
[Out] 1/3*(I*c*cos(3*f*x + 3*e) + c*sin(3*f*x + 3*e))*sqrt(c)/(a^(3/2)*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(33) = 66.
time = 1.55, size = 71, normalized size = 1.65

$$\frac{\left(i\,ce^{(2i\,fx+2i\,e)} + i\,c\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\,\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\,e^{(-3i\,fx-3i\,e)}}{3\,a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/3*(I*c*e^(2*I*f*x + 2*I*e) + I*c)*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(
c/(e^(2*I*f*x + 2*I*e) + 1))*e^(-3*I*f*x - 3*I*e)/(a^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ic(\tan{(e+fx)}+i))^{\frac{3}{2}}}{(ia\,(\tan{(e+fx)}-i))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**(3/2)/(a+I*a*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((-I*c*(tan(e + f*x) + I))**(3/2)/(I*a*(tan(e + f*x) - I))**(3/2),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((-I*c*tan(f*x + e) + c)^(3/2)/(I*a*tan(f*x + e) + a)^(3/2), x)
```

Mupad [B]
time = 5.52, size = 62, normalized size = 1.44

$$-\frac{c\,(-1+\tan(e+f\,x)\;1\mathrm{i})\;\sqrt{-c\;(-1+\tan{(e+f\,x)}\;1\mathrm{i})}}{3\,a\,f\;\,(\tan{(e+f\,x)}-\mathrm{i})\;\sqrt{a\;(1+\tan{(e+f\,x)}\;1\mathrm{i})}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^(3/2)/(a + a*tan(e + f*x)*1i)^(3/2),x)
```

```
[Out] -(c*(tan(e + f*x)*1i - 1)*(-c*(tan(e + f*x)*1i - 1))^(1/2))/(3*a*f*(tan(e +
 f*x) - 1i)*(a*(tan(e + f*x)*1i + 1))^(1/2))
```

3.1004 $\int \frac{(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=90

$$\frac{i(c-ic\tan(e+fx))^{3/2}}{5f(a+ia\tan(e+fx))^{5/2}}+\frac{i(c-ic\tan(e+fx))^{3/2}}{15af(a+ia\tan(e+fx))^{3/2}}$$

```
[Out] 1/5*I*(c-I*c*tan(f*x+e))^(3/2)/f/(a+I*a*tan(f*x+e))^(5/2)+1/15*I*(c-I*c*tan
(f*x+e))^(3/2)/a/f/(a+I*a*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.09, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {3604, 47, 37}

$$\frac{i(c-ic\tan(e+fx))^{3/2}}{15af(a+ia\tan(e+fx))^{3/2}}+\frac{i(c-ic\tan(e+fx))^{3/2}}{5f(a+ia\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])^(3/2)/(a + I*a*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((I/5)*(c - I*c*Tan[e + f*x])^(3/2))/(f*(a + I*a*Tan[e + f*x])^(5/2)) + ((I
/15)*(c - I*c*Tan[e + f*x])^(3/2))/(a*f*(a + I*a*Tan[e + f*x])^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
```

```
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{\sqrt{c-icx}}{(a+iax)^{7/2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{i(c-ic\tan(e+fx))^{3/2}}{5f(a+ia\tan(e+fx))^{5/2}} + \frac{c\text{Subst}\left(\int \frac{\sqrt{c-icx}}{(a+iax)^{5/2}}\,dx, x, \tan(e+fx)\right)}{5f}$$
$$= \frac{i(c-ic\tan(e+fx))^{3/2}}{5f(a+ia\tan(e+fx))^{5/2}} + \frac{i(c-ic\tan(e+fx))^{3/2}}{15af(a+ia\tan(e+fx))^{3/2}}$$

Mathematica [A]
time = 1.65, size = 79, normalized size = 0.88

$$\frac{c(1-i\tan(e+fx))(-4i+\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{15a^2f(-i+\tan(e+fx))^2\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - I*c*Tan[e + f*x])^(3/2)/(a + I*a*Tan[e + f*x])^(5/2),x]
[Out] (c*(1 - I*Tan[e + f*x])*(-4*I + Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(
15*a^2*f*(-I + Tan[e + f*x])^2*Sqrt[a + I*a*Tan[e + f*x]])
```

Maple [A]
time = 0.35, size = 75, normalized size = 0.83

method	result	size
derivativedivides	$\frac{\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,c(1+\tan^2(fx+e))(4i-\tan(fx+e))}{15f\,a^3(-\tan(fx+e)+i)^4}$	75
default	$\frac{\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,c(1+\tan^2(fx+e))(4i-\tan(fx+e))}{15f\,a^3(-\tan(fx+e)+i)^4}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(5/2),x,method=_RETURNVERBO
SE)
[Out] 1/15/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)/a^3*c*(1+tan(
f*x+e)^2)*(4*I-tan(f*x+e))/(-tan(f*x+e)+I)^4
```

Maxima [A]
time = 0.61, size = 92, normalized size = 1.02

$$\frac{\left(3i\,c\cos\left(5\,fx+5\,e\right)+5i\,c\cos\left(\frac{3}{5}\,\arctan\left(\sin\left(5\,fx+5\,e\right),\cos\left(5\,fx+5\,e\right)\right)\right)+3\,c\sin\left(5\,fx+5\,e\right)+5\,c\sin\left(\frac{3}{5}\,\arctan\left(\sin\left(5\,fx+5\,e\right),\cos\left(5\,fx+5\,e\right)\right)\right)\right)\sqrt{c}}{30\,a^{\frac{5}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(5/2),x, algorithm="m
axima")
```

```
[Out] 1/30*(3*I*c*cos(5*f*x + 5*e) + 5*I*c*cos(3/5*arctan2(sin(5*f*x + 5*e), cos(
5*f*x + 5*e))) + 3*c*sin(5*f*x + 5*e) + 5*c*sin(3/5*arctan2(sin(5*f*x + 5*e
), cos(5*f*x + 5*e))))*sqrt(c)/(a^(5/2)*f)
```

Fricas [A]
time = 1.13, size = 84, normalized size = 0.93

$$\frac{\left(5i\,ce^{\left(4i\,fx+4i\,e\right)}+8i\,ce^{\left(2i\,fx+2i\,e\right)}+3i\,c\right)\sqrt{\frac{a}{e^{\left(2i\,fx+2i\,e\right)}+1}}\sqrt{\frac{c}{e^{\left(2i\,fx+2i\,e\right)}+1}}\,e^{\left(-5i\,fx-5i\,e\right)}}{30\,a^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(5/2),x, algorithm="f
ricas")
```

```
[Out] 1/30*(5*I*c*e^(4*I*f*x + 4*I*e) + 8*I*c*e^(2*I*f*x + 2*I*e) + 3*I*c)*sqrt(a
/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*e^(-5*I*f*x -
 5*I*e)/(a^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-ic(\tan\left(e+fx\right)+i)\right)^{\frac{3}{2}}}{\left(ia\left(\tan\left(e+fx\right)-i\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**(3/2)/(a+I*a*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral((-I*c*(tan(e + f*x) + I))**(3/2)/(I*a*(tan(e + f*x) - I))**(5/2),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(5/2),x, algorithm="giac")

[Out] integrate((-I*c*tan(f*x + e) + c)^(3/2)/(I*a*tan(f*x + e) + a)^(5/2), x)

Mupad [B]
time = 6.10, size = 159, normalized size = 1.77

$$\frac{c\,\sqrt{\frac{a\,\left(\cos\left(2\,e+2\,f\,x\right)+1+\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\sqrt{\frac{c\,\left(\cos\left(2\,e+2\,f\,x\right)+1-\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\left(5\,\sin\left(2\,e+2\,f\,x\right)+8\,\sin\left(4\,e+4\,f\,x\right)+3\,\sin\left(6\,e+6\,f\,x\right)+\cos\left(2\,e+2\,f\,x\right)\,5{}\mathrm{i}+\cos\left(4\,e+4\,f\,x\right)\,8{}\mathrm{i}+\cos\left(6\,e+6\,f\,x\right)\,3{}\mathrm{i}\right)}{60\,a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - c*tan(e + f*x)*1i)^(3/2)/(a + a*tan(e + f*x)*1i)^(5/2),x)

[Out] (c*((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2)*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2)*(cos(2*e + 2*f*x)*5i + cos(4*e + 4*f*x)*8i + cos(6*e + 6*f*x)*3i + 5*sin(2*e + 2*f*x) + 8*sin(4*e + 4*f*x) + 3*sin(6*e + 6*f*x)))/(60*a^3*f)

3.1005 $\int \frac{(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=136

$$\frac{i(c-ic\tan(e+fx))^{3/2}}{7f(a+ia\tan(e+fx))^{7/2}}+\frac{2i(c-ic\tan(e+fx))^{3/2}}{35af(a+ia\tan(e+fx))^{5/2}}+\frac{2i(c-ic\tan(e+fx))^{3/2}}{105a^2f(a+ia\tan(e+fx))^{3/2}}$$

```
[Out] 1/7*I*(c-I*c*tan(f*x+e))^(3/2)/f/(a+I*a*tan(f*x+e))^(7/2)+2/35*I*(c-I*c*tan
(f*x+e))^(3/2)/a/f/(a+I*a*tan(f*x+e))^(5/2)+2/105*I*(c-I*c*tan(f*x+e))^(3/2
)/a^2/f/(a+I*a*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.10, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {3604, 47, 37}

$$\frac{2i(c-ic\tan(e+fx))^{3/2}}{105a^2f(a+ia\tan(e+fx))^{3/2}}+\frac{2i(c-ic\tan(e+fx))^{3/2}}{35af(a+ia\tan(e+fx))^{5/2}}+\frac{i(c-ic\tan(e+fx))^{3/2}}{7f(a+ia\tan(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])^(3/2)/(a + I*a*Tan[e + f*x])^(7/2),x]
```

```
[Out] ((I/7)*(c - I*c*Tan[e + f*x])^(3/2))/(f*(a + I*a*Tan[e + f*x])^(7/2)) + (((
2*I)/35)*(c - I*c*Tan[e + f*x])^(3/2))/(a*f*(a + I*a*Tan[e + f*x])^(5/2)) +
 (((2*I)/105)*(c - I*c*Tan[e + f*x])^(3/2))/(a^2*f*(a + I*a*Tan[e + f*x])^(
3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^{7/2}}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{\sqrt{c-icx}}{(a+iax)^{9/2}}\,dx,x,\tan(e+fx)\right)}{f}$$
$$= \frac{i(c-ic\tan(e+fx))^{3/2}}{7f(a+ia\tan(e+fx))^{7/2}} + \frac{(2c)\text{Subst}\left(\int \frac{\sqrt{c-icx}}{(a+iax)^{7/2}}\,dx,x,\tan(e+fx)\right)}{7f}$$
$$= \frac{i(c-ic\tan(e+fx))^{3/2}}{7f(a+ia\tan(e+fx))^{7/2}} + \frac{2i(c-ic\tan(e+fx))^{3/2}}{35af(a+ia\tan(e+fx))^{5/2}} + \frac{(2c)\text{Subst}\left(\int \right.}{}$$
$$= \frac{i(c-ic\tan(e+fx))^{3/2}}{7f(a+ia\tan(e+fx))^{7/2}} + \frac{2i(c-ic\tan(e+fx))^{3/2}}{35af(a+ia\tan(e+fx))^{5/2}} + \frac{2i(c-ic\,\text{ta}}{105a^2f(a+ia}$$

Mathematica [A]
time = 2.17, size = 101, normalized size = 0.74

$$\frac{ic\sec^2(e+fx)(21+25\cos(2(e+fx))+10i\sin(2(e+fx)))(i+\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{210a^3f(-i+\tan(e+fx))^3\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - I*c*Tan[e + f*x])^(3/2)/(a + I*a*Tan[e + f*x])^(7/2),x]
```

```
[Out] ((I/210)*c*Sec[e + f*x]^2*(21 + 25*Cos[2*(e + f*x)] + (10*I)*Sin[2*(e + f*x
)])*(I + Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(a^3*f*(-I + Tan[e + f*x
])^3*Sqrt[a + I*a*Tan[e + f*x]])
```

Maple [A]
time = 0.37, size = 85, normalized size = 0.62

method	result
derivativedivides	$-\frac{\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,c(1+\tan^2(fx+e))\left(10i\tan(fx+e)-2(\tan^2(fx\right.}{105f\,a^4(-\tan(fx+e)+i)^5}$
default	$-\frac{\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,c(1+\tan^2(fx+e))\left(10i\tan(fx+e)-2(\tan^2(fx\right.}{105f\,a^4(-\tan(fx+e)+i)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(7/2),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/105/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)/a^4*c*(1+ta
n(f*x+e)^2)*(10*I*tan(f*x+e)-2*tan(f*x+e)^2+23)/(-tan(f*x+e)+I)^5
```

Maxima [A]
time = 0.63, size = 146, normalized size = 1.07

$$\frac{\left(15 i c \cos\left(7 f x + 7 e\right) + 42 i c \cos\left(\frac{5}{7} \arctan\left(\sin\left(7 f x + 7 e\right), \cos\left(7 f x + 7 e\right)\right)\right) + 35 i c \cos\left(\frac{3}{7} \arctan\left(\sin\left(7 f x + 7 e\right), \cos\left(7 f x + 7 e\right)\right)\right) + 15 c \sin\left(7 f x + 7 e\right) + 42 c \sin\left(\frac{5}{7} \arctan\left(\sin\left(7 f x + 7 e\right), \cos\left(7 f x + 7 e\right)\right)\right) + 35 c \sin\left(\frac{3}{7} \arctan\left(\sin\left(7 f x + 7 e\right), \cos\left(7 f x + 7 e\right)\right)\right)\right) \sqrt{c}}{420 a^{\frac{7}{2}} f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(7/2),x, algorithm="m
axima")
```

```
[Out] 1/420*(15*I*c*cos(7*f*x + 7*e) + 42*I*c*cos(5/7*arctan2(sin(7*f*x + 7*e), c
os(7*f*x + 7*e))) + 35*I*c*cos(3/7*arctan2(sin(7*f*x + 7*e), cos(7*f*x + 7*
e))) + 15*c*sin(7*f*x + 7*e) + 42*c*sin(5/7*arctan2(sin(7*f*x + 7*e), cos(7
*f*x + 7*e))) + 35*c*sin(3/7*arctan2(sin(7*f*x + 7*e), cos(7*f*x + 7*e))))*
sqrt(c)/(a^(7/2)*f)
```

Fricas [A]
time = 1.43, size = 97, normalized size = 0.71

$$\frac{\left(35 i c e^{\left(6 i f x + 6 i e\right)} + 77 i c e^{\left(4 i f x + 4 i e\right)} + 57 i c e^{\left(2 i f x + 2 i e\right)} + 15 i c\right) \sqrt{\frac{a}{e^{\left(2 i f x + 2 i e\right)} + 1}} \sqrt{\frac{c}{e^{\left(2 i f x + 2 i e\right)} + 1}} e^{\left(-7 i f x - 7 i e\right)}}{420 a^{4} f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(7/2),x, algorithm="f
ricas")
```

```
[Out] 1/420*(35*I*c*e^(6*I*f*x + 6*I*e) + 77*I*c*e^(4*I*f*x + 4*I*e) + 57*I*c*e^(
2*I*f*x + 2*I*e) + 15*I*c)*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I
*f*x + 2*I*e) + 1))*e^(-7*I*f*x - 7*I*e)/(a^4*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(- i c \left(\tan\left(e + f x\right) + i\right)\right)^{\frac{3}{2}}}{\left(i a \left(\tan\left(e + f x\right) - i\right)\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**(3/2)/(a+I*a*tan(f*x+e))**(7/2),x)
```

```
[Out] Integral((-I*c*(tan(e + f*x) + I))**(3/2)/(I*a*(tan(e + f*x) - I))**(7/2),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(7/2),x, algorithm="g
iac")
```

```
[Out] integrate((-I*c*tan(f*x + e) + c)^(3/2)/(I*a*tan(f*x + e) + a)^(7/2), x)
```

Mupad [B]
time = 6.51, size = 182, normalized size = 1.34

$$\frac{c\,\sqrt{\frac{a\,\left(\cos\left(2\,e+2\,f\,x\right)+1+\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\sqrt{\frac{c\,\left(\cos\left(2\,e+2\,f\,x\right)+1-\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\left(35\,\sin\left(2\,e+2\,f\,x\right)+77\,\sin\left(4\,e+4\,f\,x\right)+57\,\sin\left(6\,e+6\,f\,x\right)+15\,\sin\left(8\,e+8\,f\,x\right)+\cos\left(2\,e+2\,f\,x\right)\,35{}\mathrm{i}+\cos\left(4\,e+4\,f\,x\right)\,77{}\mathrm{i}+\cos\left(6\,e+6\,f\,x\right)\,57{}\mathrm{i}+\cos\left(8\,e+8\,f\,x\right)\,15{}\mathrm{i}\right)}{840\,a^4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^(3/2)/(a + a*tan(e + f*x)*1i)^(7/2),x)
```

```
[Out] (c*((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1)
)^(1/2)*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x)
 + 1))^(1/2)*(cos(2*e + 2*f*x)*35i + cos(4*e + 4*f*x)*77i + cos(6*e + 6*f*x
)*57i + cos(8*e + 8*f*x)*15i + 35*sin(2*e + 2*f*x) + 77*sin(4*e + 4*f*x) +
57*sin(6*e + 6*f*x) + 15*sin(8*e + 8*f*x)))/(840*a^4*f)
```

3.1006 $\int \frac{(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=182

$$\frac{i(c-ic\tan(e+fx))^{3/2}}{9f(a+ia\tan(e+fx))^{9/2}}+\frac{i(c-ic\tan(e+fx))^{3/2}}{21af(a+ia\tan(e+fx))^{7/2}}+\frac{2i(c-ic\tan(e+fx))^{3/2}}{105a^2f(a+ia\tan(e+fx))^{5/2}}+\frac{2i(c-ic\tan(e}{315a^3f(a+ia\tan}$$

```
[Out] 1/9*I*(c-I*c*tan(f*x+e))^(3/2)/f/(a+I*a*tan(f*x+e))^(9/2)+1/21*I*(c-I*c*tan
(f*x+e))^(3/2)/a/f/(a+I*a*tan(f*x+e))^(7/2)+2/105*I*(c-I*c*tan(f*x+e))^(3/2
)/a^2/f/(a+I*a*tan(f*x+e))^(5/2)+2/315*I*(c-I*c*tan(f*x+e))^(3/2)/a^3/f/(a+
I*a*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.11, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {3604, 47, 37}

$$\frac{2i(c-ic\tan(e+fx))^{3/2}}{315a^3f(a+ia\tan(e+fx))^{3/2}}+\frac{2i(c-ic\tan(e+fx))^{3/2}}{105a^2f(a+ia\tan(e+fx))^{5/2}}+\frac{i(c-ic\tan(e+fx))^{3/2}}{21af(a+ia\tan(e+fx))^{7/2}}+\frac{i(c-ic\tan(e+fx))^{3/2}}{9f(a+ia\tan(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])^(3/2)/(a + I*a*Tan[e + f*x])^(9/2),x]
```

```
[Out] ((I/9)*(c - I*c*Tan[e + f*x])^(3/2))/(f*(a + I*a*Tan[e + f*x])^(9/2)) + ((I
/21)*(c - I*c*Tan[e + f*x])^(3/2))/(a*f*(a + I*a*Tan[e + f*x])^(7/2)) + (((
2*I)/105)*(c - I*c*Tan[e + f*x])^(3/2))/(a^2*f*(a + I*a*Tan[e + f*x])^(5/2)
) + (((2*I)/315)*(c - I*c*Tan[e + f*x])^(3/2))/(a^3*f*(a + I*a*Tan[e + f*x]
)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^{9/2}}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{\sqrt{c-icx}}{(a+iax)^{11/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{i(c-ic\tan(e+fx))^{3/2}}{9f(a+ia\tan(e+fx))^{9/2}} + \frac{c\text{Subst}\left(\int \frac{\sqrt{c-icx}}{(a+iax)^{9/2}}\,dx, x, \tan(e+fx)\right)}{3f}$$

$$= \frac{i(c-ic\tan(e+fx))^{3/2}}{9f(a+ia\tan(e+fx))^{9/2}} + \frac{i(c-ic\tan(e+fx))^{3/2}}{21af(a+ia\tan(e+fx))^{7/2}} + \frac{(2c)\text{Subst}\left(\int \cdot\right.}{}$$

$$= \frac{i(c-ic\tan(e+fx))^{3/2}}{9f(a+ia\tan(e+fx))^{9/2}} + \frac{i(c-ic\tan(e+fx))^{3/2}}{21af(a+ia\tan(e+fx))^{7/2}} + \frac{2i(c-ic\,\text{ta}}{105a^2f(a+ia}$$

$$= \frac{i(c-ic\tan(e+fx))^{3/2}}{9f(a+ia\tan(e+fx))^{9/2}} + \frac{i(c-ic\tan(e+fx))^{3/2}}{21af(a+ia\tan(e+fx))^{7/2}} + \frac{2i(c-ic\,\text{ta}}{105a^2f(a+ia}$$

Mathematica [A]
time = 2.81, size = 115, normalized size = 0.63

$$\frac{c\sec^2(e+fx)(92+140\cos(2(e+fx))+35i\sec(e+fx)\sin(3(e+fx))+27i\tan(e+fx))(i+\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{1260a^4f(-i+\tan(e+fx))^4\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - I*c*Tan[e + f*x])^(3/2)/(a + I*a*Tan[e + f*x])^(9/2),x]
```

```
[Out] (c*Sec[e + f*x]^2*(92 + 140*Cos[2*(e + f*x)] + (35*I)*Sec[e + f*x]*Sin[3*(e
 + f*x)] + (27*I)*Tan[e + f*x])*(I + Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x
]])/(1260*a^4*f*(-I + Tan[e + f*x])^4*Sqrt[a + I*a*Tan[e + f*x]])
```

Maple [A]
time = 0.40, size = 97, normalized size = 0.53

method	result

derivativedivides	$\frac{i\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,c(1+\tan^2(fx+e))(2i(\tan^3(fx+e))-33i\tan(fx+}{315f\,a^5(-\tan(fx+e)+i)^6}$
default	$\frac{i\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,c(1+\tan^2(fx+e))(2i(\tan^3(fx+e))-33i\tan(fx+}{315f\,a^5(-\tan(fx+e)+i)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(9/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/315*I/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)/a^5*c*(1+t
an(f*x+e)^2)*(2*I*tan(f*x+e)^3-33*I*tan(f*x+e)+12*tan(f*x+e)^2-58)/(-tan(f*
x+e)+I)^6
```

Maxima [A]

time = 0.61, size = 200, normalized size = 1.10

$$\frac{\left(35i\,c\cos\left(9\,fx+9\,e\right)+135i\,c\cos\left(\frac{7}{9}\arctan\left(\sin\left(9\,fx+9\,e\right),\cos\left(9\,fx+9\,e\right)\right)\right)+189i\,c\cos\left(\frac{5}{9}\arctan\left(\sin\left(9\,fx+9\,e\right),\cos\left(9\,fx+9\,e\right)\right)\right)+105i\,c\cos\left(\frac{1}{3}\arctan\left(\sin\left(9\,fx+9\,e\right),\cos\left(9\,fx+9\,e\right)\right)\right)+35\,c\sin\left(9\,fx+9\,e\right)+135\,c\sin\left(\frac{7}{9}\arctan\left(\sin\left(9\,fx+9\,e\right),\cos\left(9\,fx+9\,e\right)\right)\right)+189\,c\sin\left(\frac{5}{9}\arctan\left(\sin\left(9\,fx+9\,e\right),\cos\left(9\,fx+9\,e\right)\right)\right)+105\,c\sin\left(\frac{1}{3}\arctan\left(\sin\left(9\,fx+9\,e\right),\cos\left(9\,fx+9\,e\right)\right)\right)\right)\sqrt{c}}{2520\,a^{\frac{9}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(9/2),x, algorithm="m
axima")
```

```
[Out] 1/2520*(35*I*c*cos(9*f*x + 9*e) + 135*I*c*cos(7/9*arctan2(sin(9*f*x + 9*e),
 cos(9*f*x + 9*e))) + 189*I*c*cos(5/9*arctan2(sin(9*f*x + 9*e), cos(9*f*x +
 9*e))) + 105*I*c*cos(1/3*arctan2(sin(9*f*x + 9*e), cos(9*f*x + 9*e))) + 35
*c*sin(9*f*x + 9*e) + 135*c*sin(7/9*arctan2(sin(9*f*x + 9*e), cos(9*f*x + 9
*e))) + 189*c*sin(5/9*arctan2(sin(9*f*x + 9*e), cos(9*f*x + 9*e))) + 105*c*
sin(1/3*arctan2(sin(9*f*x + 9*e), cos(9*f*x + 9*e))))*sqrt(c)/(a^(9/2)*f)
```

Fricas [A]

time = 1.13, size = 110, normalized size = 0.60

$$\frac{\left(105i\,ce^{\left(8i\,fx+8i\,e\right)}+294i\,ce^{\left(6i\,fx+6i\,e\right)}+324i\,ce^{\left(4i\,fx+4i\,e\right)}+170i\,ce^{\left(2i\,fx+2i\,e\right)}+35i\,c\right)\sqrt{\frac{a}{e^{\left(2i\,fx+2i\,e\right)}+1}}\,\sqrt{\frac{c}{e^{\left(2i\,fx+2i\,e\right)}+1}}\,e^{\left(-9i\,fx-9i\,e\right)}}{2520\,a^5f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(9/2),x, algorithm="f
ricas")
```

```
[Out] 1/2520*(105*I*c*e^(8*I*f*x + 8*I*e) + 294*I*c*e^(6*I*f*x + 6*I*e) + 324*I*c
*e^(4*I*f*x + 4*I*e) + 170*I*c*e^(2*I*f*x + 2*I*e) + 35*I*c)*sqrt(a/(e^(2*I
*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*e^(-9*I*f*x - 9*I*e)/
(a^5*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**(3/2)/(a+I*a*tan(f*x+e))**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(9/2),x, algorithm="g
iac")
```

```
[Out] integrate((-I*c*tan(f*x + e) + c)^(3/2)/(I*a*tan(f*x + e) + a)^(9/2), x)
```

Mupad [B]
time = 7.30, size = 205, normalized size = 1.13

$$\frac{c\,\sqrt{\frac{a\,\left(\cos\left(2\,e+2\,f\,x\right)+1+\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\sqrt{\frac{c\,\left(\cos\left(2\,e+2\,f\,x\right)+1-\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\left(105\,\sin\left(2\,e+2\,f\,x\right)+294\,\sin\left(4\,e+4\,f\,x\right)+324\,\sin\left(6\,e+6\,f\,x\right)+170\,\sin\left(8\,e+8\,f\,x\right)+35\,\sin\left(10\,e+10\,f\,x\right)+\cos\left(2\,e+2\,f\,x\right)\,105{}\mathrm{i}+\cos\left(4\,e+4\,f\,x\right)\,294{}\mathrm{i}+\cos\left(6\,e+6\,f\,x\right)\,324{}\mathrm{i}+\cos\left(8\,e+8\,f\,x\right)\,170{}\mathrm{i}+\cos\left(10\,e+10\,f\,x\right)\,35{}\mathrm{i}\right)}{5040\,a^5\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^(3/2)/(a + a*tan(e + f*x)*1i)^(9/2),x)
```

```
[Out] (c*((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1)
)^(1/2)*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x)
 + 1))^(1/2)*(cos(2*e + 2*f*x)*105i + cos(4*e + 4*f*x)*294i + cos(6*e + 6*f
*x)*324i + cos(8*e + 8*f*x)*170i + cos(10*e + 10*f*x)*35i + 105*sin(2*e + 2
*f*x) + 294*sin(4*e + 4*f*x) + 324*sin(6*e + 6*f*x) + 170*sin(8*e + 8*f*x)
+ 35*sin(10*e + 10*f*x)))/(5040*a^5*f)
```

3.1007 $\int (a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=168

$$-\frac{3ia^{5/2}c^{5/2}\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{4f}+\frac{3a^2c^2\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{8f}$$

[Out] -3/4*I*a^(5/2)*c^(5/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*c*tan(f*x+e))^(1/2))/f+3/8*a^2*c^2*(a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))^(1/2)*tan(f*x+e)/f+1/4*a*c*tan(f*x+e)*(a+I*a*tan(f*x+e))^(3/2)*(c-I*c*tan(f*x+e))^(3/2)/f

Rubi [A]

time = 0.12, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3604, 38, 65, 223, 209}

$$-\frac{3ia^{5/2}c^{5/2}\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{4f}+\frac{3a^2c^2\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{8f}+\frac{ac\tan(e+fx)(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}{4f}$$

Antiderivative was successfully verified.

[In] Int[(a + I*a*Tan[e + f*x])^(5/2)*(c - I*c*Tan[e + f*x])^(5/2),x]

[Out] (((-3*I)/4)*a^(5/2)*c^(5/2)*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])])/f + (3*a^2*c^2*Tan[e + f*x]*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/(8*f) + (a*c*Tan[e + f*x]*(a + I*a*Tan[e + f*x])^(3/2)*(c - I*c*Tan[e + f*x])^(3/2))/(4*f)

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{5/2}\,dx = \frac{(ac)\text{Subst}\left(\int (a+iax)^{3/2}(c-icx)^{3/2}\,dx,x,\tan(e+f\right.}{f}$$

$$= \frac{ac\tan(e+fx)(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+}{4f}$$

$$= \frac{3a^2c^2\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(}{8f}$$

$$= \frac{3a^2c^2\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(}{8f}$$

$$= \frac{3a^2c^2\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(}{8f}$$

$$= -\frac{3ia^{5/2}c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{4f} + \frac{3a}{}$$

Mathematica [A]
time = 4.35, size = 110, normalized size = 0.65

$$-\frac{a^2c^3\sec^3(e+fx)\left(24\text{ArcTan}\left(e^{i(e+fx)}\right)\cos^4(e+fx)+11i\sin(e+fx)+3i\sin(3(e+fx))\right)(i+\tan(e+fx))\sqrt{a+ia\tan(e+fx)}}{32f\sqrt{c-ic\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(5/2)*(c - I*c*Tan[e + f*x])^(5/2),x]
```

```
[Out] -1/32*(a^2*c^3*Sec[e + f*x]^3*(24*ArcTan[E^(I*(e + f*x))]*Cos[e + f*x]^4 +
(11*I)*Sin[e + f*x] + (3*I)*Sin[3*(e + f*x)])*(I + Tan[e + f*x])*Sqrt[a + I
*a*Tan[e + f*x]])/(f*Sqrt[c - I*c*Tan[e + f*x]])
```

Maple [A]
time = 0.34, size = 164, normalized size = 0.98

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))}\ \sqrt{-c(i\tan(fx+e)-1)}\ a^2c^2\left(2\sqrt{ac}\ \sqrt{ac(1+\tan^2(fx+e))}\right.}{8f\sqrt{ac}\ \sqrt{}}$
default	$\frac{\sqrt{a(1+i\tan(fx+e))}\ \sqrt{-c(i\tan(fx+e)-1)}\ a^2c^2\left(2\sqrt{ac}\ \sqrt{ac(1+\tan^2(fx+e))}\right.}{8f\sqrt{ac}\ \sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(5/2)*(c-I*c*tan(f*x+e))^(5/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/8/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^2*c^2*(2*(a*
c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^3+3*a*c*ln((c*a*tan(f*x+e)
+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))+5*tan(f*x+e)*(a*c*(
1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2)/(a*c*(1+tan(f*x+e)^2))^(1/2
)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1249 vs. 2(136) = 272.
time = 0.79, size = 1249, normalized size = 7.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)*(c-I*c*tan(f*x+e))^(5/2),x, algorithm="m
axima")
```

```
[Out] -(12*a^2*c^2*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 44*a^2*
c^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 44*a^2*c^2*cos(3
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 12*a^2*c^2*cos(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 12*I*a^2*c^2*sin(7/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) + 44*I*a^2*c^2*sin(5/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) - 44*I*a^2*c^2*sin(3/2*arctan2(sin(2*f*x + 2*e), c
```

```
os(2*f*x + 2*e))) - 12*I*a^2*c^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) + 6*(a^2*c^2*cos(8*f*x + 8*e) + 4*a^2*c^2*cos(6*f*x + 6*e) + 6*a
^2*c^2*cos(4*f*x + 4*e) + 4*a^2*c^2*cos(2*f*x + 2*e) + I*a^2*c^2*sin(8*f*x
+ 8*e) + 4*I*a^2*c^2*sin(6*f*x + 6*e) + 6*I*a^2*c^2*sin(4*f*x + 4*e) + 4*I*
a^2*c^2*sin(2*f*x + 2*e) + a^2*c^2)*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))), sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 + 1) + 6*(a^2*c^2*cos(8*f*x + 8*e) + 4*a^2*c^2*cos(6*f*x + 6*e) + 6*a^2*c^
2*cos(4*f*x + 4*e) + 4*a^2*c^2*cos(2*f*x + 2*e) + I*a^2*c^2*sin(8*f*x + 8*e
) + 4*I*a^2*c^2*sin(6*f*x + 6*e) + 6*I*a^2*c^2*sin(4*f*x + 4*e) + 4*I*a^2*c
^2*sin(2*f*x + 2*e) + a^2*c^2)*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e))), -sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1
) + 3*(I*a^2*c^2*cos(8*f*x + 8*e) + 4*I*a^2*c^2*cos(6*f*x + 6*e) + 6*I*a^2*
c^2*cos(4*f*x + 4*e) + 4*I*a^2*c^2*cos(2*f*x + 2*e) - a^2*c^2*sin(8*f*x + 8
*e) - 4*a^2*c^2*sin(6*f*x + 6*e) - 6*a^2*c^2*sin(4*f*x + 4*e) - 4*a^2*c^2*s
in(2*f*x + 2*e) + I*a^2*c^2)*log(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*s
in(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 3*(-I*a^2*c^2*co
s(8*f*x + 8*e) - 4*I*a^2*c^2*cos(6*f*x + 6*e) - 6*I*a^2*c^2*cos(4*f*x + 4*e
) - 4*I*a^2*c^2*cos(2*f*x + 2*e) + a^2*c^2*sin(8*f*x + 8*e) + 4*a^2*c^2*sin
(6*f*x + 6*e) + 6*a^2*c^2*sin(4*f*x + 4*e) + 4*a^2*c^2*sin(2*f*x + 2*e) - I
*a^2*c^2)*log(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sin(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) + 1))*sqrt(a)*sqrt(c)/(f*(-16*I*cos(8*f*x
+ 8*e) - 64*I*cos(6*f*x + 6*e) - 96*I*cos(4*f*x + 4*e) - 64*I*cos(2*f*x + 2
*e) + 16*sin(8*f*x + 8*e) + 64*sin(6*f*x + 6*e) + 96*sin(4*f*x + 4*e) + 64*
sin(2*f*x + 2*e) - 16*I))
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(136) = 272$.
time = 1.24, size = 534, normalized size = 3.18

$$\frac{3\sqrt{\frac{a^5c^5}{f^2}}\left(fe^{(6ifx+6ie)}+3fe^{(4ifx+4ie)}+3fe^{(2ifx+2ie)}+f\right)\log\left(\frac{4\left(2\left(a^2c^2e^{(3ifx+3ie)}+a^2c^2e^{(ifx+ie)}\right)\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}-\sqrt{\frac{a^5c^5}{f^2}}\left(ife^{(2ifx+2ie)}-if\right)\right)}{a^2c^2e^{(2ifx+2ie)}+a^2c^2}\right)-3\sqrt{\frac{a^5c^5}{f^2}}\left(fe^{(6ifx+6ie)}+3fe^{(4ifx+4ie)}+3fe^{(2ifx+2ie)}+f\right)\log\left(\frac{4\left(2\left(a^2c^2e^{(3ifx+3ie)}+a^2c^2e^{(ifx+ie)}\right)\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}-\sqrt{\frac{a^5c^5}{f^2}}\left(-ife^{(2ifx+2ie)}+if\right)\right)}{a^2c^2e^{(2ifx+2ie)}+a^2c^2}\right)+4\left(-3ia^2c^2e^{(7ifx+7ie)}-11ia^2c^2e^{(5ifx+5ie)}+11ia^2c^2e^{(3ifx+3ie)}+3ia^2c^2e^{(ifx+ie)}\right)\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}}{16\left(fe^{(6ifx+6ie)}+3fe^{(4ifx+4ie)}+3fe^{(2ifx+2ie)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)*(c-I*c*tan(f*x+e))^(5/2),x, algorithm="f
ricas")
```

```
[Out] 1/16*(3*sqrt(a^5*c^5/f^2)*(f*e^(6*I*f*x + 6*I*e) + 3*f*e^(4*I*f*x + 4*I*e)
+ 3*f*e^(2*I*f*x + 2*I*e) + f)*log(4*(2*(a^2*c^2*e^(3*I*f*x + 3*I*e) + a^2*
c^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x +
 2*I*e) + 1)) - sqrt(a^5*c^5/f^2)*(I*f*e^(2*I*f*x + 2*I*e) - I*f))/(a^2*c^2
*e^(2*I*f*x + 2*I*e) + a^2*c^2)) - 3*sqrt(a^5*c^5/f^2)*(f*e^(6*I*f*x + 6*I*
e) + 3*f*e^(4*I*f*x + 4*I*e) + 3*f*e^(2*I*f*x + 2*I*e) + f)*log(4*(2*(a^2*c
^2*e^(3*I*f*x + 3*I*e) + a^2*c^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*
e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - sqrt(a^5*c^5/f^2)*(-I*f*e^(2*I
```

```
*f*x + 2*I*e) + I*f))/(a^2*c^2*e^(2*I*f*x + 2*I*e) + a^2*c^2)) + 4*(-3*I*a^
2*c^2*e^(7*I*f*x + 7*I*e) - 11*I*a^2*c^2*e^(5*I*f*x + 5*I*e) + 11*I*a^2*c^2
*e^(3*I*f*x + 3*I*e) + 3*I*a^2*c^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*
I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(f*e^(6*I*f*x + 6*I*e) + 3*f*
e^(4*I*f*x + 4*I*e) + 3*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(5/2)*(c-I*c*tan(f*x+e))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)*(c-I*c*tan(f*x+e))^(5/2),x, algorithm="g
iac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a\tan(e + f\,x)\ 1\mathrm{i})^{5/2}\,(c - c\tan(e + f\,x)\ 1\mathrm{i})^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(5/2)*(c - c*tan(e + f*x)*1i)^(5/2),x)
[Out] int((a + a*tan(e + f*x)*1i)^(5/2)*(c - c*tan(e + f*x)*1i)^(5/2), x)
```

3.1008 $\int (a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=159

$$-\frac{ia^{3/2}c^{5/2}\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{f}+\frac{ac^2\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{2f}$$

```
[Out] -I*a^(3/2)*c^(5/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*c*t
an(f*x+e))^(1/2))/f+1/2*a*c^2*(a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))^(
1/2)*tan(f*x+e)/f-1/3*I*c*(a+I*a*tan(f*x+e))^(3/2)*(c-I*c*tan(f*x+e))^(3/2)
/f
```

Rubi [A]

time = 0.11, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3604, 51, 38, 65, 223, 209}

$$-\frac{ia^{3/2}c^{5/2}\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{f}+\frac{ac^2\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{2f}-\frac{ic(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^(3/2)*(c - I*c*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-I)*a^(3/2)*c^(5/2)*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a]*
Sqrt[c - I*c*Tan[e + f*x]])])/f + (a*c^2*Tan[e + f*x]*Sqrt[a + I*a*Tan[e +
f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/(2*f) - ((I/3)*c*(a + I*a*Tan[e + f*x])^(
3/2)*(c - I*c*Tan[e + f*x])^(3/2))/f
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 51

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[2*c*(n/(m + n + 1)
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{5/2}\,dx = \frac{(ac)\text{Subst}\left(\int \sqrt{a+iax}\,(c-icx)^{3/2}\,dx, x, \tan(e+fx\right.}{f}$$

$$= -\frac{ic(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}{3f} +$$

$$= \frac{ac^2\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e}}{2f}$$

$$= \frac{ac^2\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e}}{2f}$$

$$= \frac{ac^2\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e}}{2f}$$

$$= -\frac{ia^{3/2}c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{f} + \frac{ac^2}{}$$

Mathematica [A]
time = 3.12, size = 169, normalized size = 1.06

$$-\frac{ie^{-2i(e+fx)}\left(\frac{c}{1+e^{2i(e+fx)}}\right)^{5/2}\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\left(e^{i(e+fx)}\left(-3+8e^{2i(e+fx)}+3e^{4i(e+fx)}\right)+3\left(1+e^{2i(e+fx)}\right)^3\text{ArcTan}\left(e^{i(e+fx)}\right)\right)(a+ia\tan(e+fx))^{3/2}}{3f\sec^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(3/2)*(c - I*c*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-1/3*I)*(c/(1 + E^((2*I)*(e + f*x))))^(5/2)*Sqrt[E^(I*(e + f*x))/(1 + E^(
(2*I)*(e + f*x)))]*(E^(I*(e + f*x))*(-3 + 8*E^((2*I)*(e + f*x)) + 3*E^((4*I
)*(e + f*x))) + 3*(1 + E^((2*I)*(e + f*x)))^3*ArcTan[E^(I*(e + f*x))])*(a +
 I*a*Tan[e + f*x])^(3/2))/(E^((2*I)*(e + f*x))*f*Sec[e + f*x]^(3/2))
```

Maple [A]
time = 0.33, size = 186, normalized size = 1.17

method	result

derivativedivides	$\frac{\sqrt{a\,(1+i\tan\,(fx+e))}\ \sqrt{-c\,(i\tan\,(fx+e)-1)}\ c^2a\left(-2i\sqrt{ac\,(1+\tan^2\,(fx+e))}\ \sqrt{ac}\right.}{}$
default	$\frac{\sqrt{a\,(1+i\tan\,(fx+e))}\ \sqrt{-c\,(i\tan\,(fx+e)-1)}\ c^2a\left(-2i\sqrt{ac\,(1+\tan^2\,(fx+e))}\ \sqrt{ac}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(3/2)*(c-I*c*tan(f*x+e))^(5/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/6/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*c^2*a*(-2*I*(a
*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^2+3*a*c*ln((c*a*tan(f*x+e
)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))-2*I*(a*c*(1+tan(f*
x+e)^2))^(1/2)*(a*c)^(1/2)+3*tan(f*x+e)*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^
(1/2))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(a*c)^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 949 vs. $2(126) = 252$.
time = 0.65, size = 949, normalized size = 5.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)*(c-I*c*tan(f*x+e))^(5/2),x, algorithm="m
axima")
```

```
[Out] -(12*a*c^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 32*a*c^2*
cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 12*a*c^2*cos(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 12*I*a*c^2*sin(5/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) + 32*I*a*c^2*sin(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 12*I*a*c^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) + 6*(a*c^2*cos(6*f*x + 6*e) + 3*a*c^2*cos(4*f*x + 4*e) + 3*a
*c^2*cos(2*f*x + 2*e) + I*a*c^2*sin(6*f*x + 6*e) + 3*I*a*c^2*sin(4*f*x + 4*
e) + 3*I*a*c^2*sin(2*f*x + 2*e) + a*c^2)*arctan2(cos(1/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))), sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + 1) + 6*(a*c^2*cos(6*f*x + 6*e) + 3*a*c^2*cos(4*f*x + 4*e) + 3*a*c^2
*cos(2*f*x + 2*e) + I*a*c^2*sin(6*f*x + 6*e) + 3*I*a*c^2*sin(4*f*x + 4*e) +
 3*I*a*c^2*sin(2*f*x + 2*e) + a*c^2)*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))), -sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) + 1) + 3*(I*a*c^2*cos(6*f*x + 6*e) + 3*I*a*c^2*cos(4*f*x + 4*e) + 3*I*a*
c^2*cos(2*f*x + 2*e) - a*c^2*sin(6*f*x + 6*e) - 3*a*c^2*sin(4*f*x + 4*e) -
3*a*c^2*sin(2*f*x + 2*e) + I*a*c^2)*log(cos(1/2*arctan2(sin(2*f*x + 2*e), c
```

```
os(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^
2 + 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 3*(-I*a*c
^2*cos(6*f*x + 6*e) - 3*I*a*c^2*cos(4*f*x + 4*e) - 3*I*a*c^2*cos(2*f*x + 2*
e) + a*c^2*sin(6*f*x + 6*e) + 3*a*c^2*sin(4*f*x + 4*e) + 3*a*c^2*sin(2*f*x
+ 2*e) - I*a*c^2)*log(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^
2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sin(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1))*sqrt(a)*sqrt(c)/(f*(-12*I*co
s(6*f*x + 6*e) - 36*I*cos(4*f*x + 4*e) - 36*I*cos(2*f*x + 2*e) + 12*sin(6*f
*x + 6*e) + 36*sin(4*f*x + 4*e) + 36*sin(2*f*x + 2*e) - 12*I))
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 455 vs. $2(126) = 252$.
time = 1.14, size = 455, normalized size = 2.86

$$\frac{3\sqrt{\frac{a^3c^5}{f^2}}\left(fe^{(4i\,fx+4i\,e)}+2\,fe^{(2i\,fx+2i\,e)}+f\right)\log\left(\frac{4\left(2\left(ac^2e^{(3i\,fx+3i\,e)}+ac^2e^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}-\sqrt{\frac{a^3c^5}{f^2}}\left(i\,fe^{(2i\,fx+2i\,e)}-i\,f\right)\right)}{ac^2e^{(2i\,fx+2i\,e)}+ac^2}\right)-3\sqrt{\frac{a^3c^5}{f^2}}\left(fe^{(4i\,fx+4i\,e)}+2\,fe^{(2i\,fx+2i\,e)}+f\right)\log\left(\frac{4\left(2\left(ac^2e^{(3i\,fx+3i\,e)}+ac^2e^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}-\sqrt{\frac{a^3c^5}{f^2}}\left(-i\,fe^{(2i\,fx+2i\,e)}+i\,f\right)\right)}{ac^2e^{(2i\,fx+2i\,e)}+ac^2}\right)-4\left(3i\,ac^2e^{(5i\,fx+5i\,e)}+8i\,ac^2e^{(3i\,fx+3i\,e)}-3i\,ac^2e^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{12\left(fe^{(4i\,fx+4i\,e)}+2\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)*(c-I*c*tan(f*x+e))^(5/2),x, algorithm="f
ricas")
```

```
[Out] 1/12*(3*sqrt(a^3*c^5/f^2)*(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*e)
+ f)*log(4*(2*(a*c^2*e^(3*I*f*x + 3*I*e) + a*c^2*e^(I*f*x + I*e))*sqrt(a/(e
^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - sqrt(a^3*c^5/f
^2)*(I*f*e^(2*I*f*x + 2*I*e) - I*f))/(a*c^2*e^(2*I*f*x + 2*I*e) + a*c^2)) -
 3*sqrt(a^3*c^5/f^2)*(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*e) + f)*
log(4*(2*(a*c^2*e^(3*I*f*x + 3*I*e) + a*c^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I
*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - sqrt(a^3*c^5/f^2)*(
-I*f*e^(2*I*f*x + 2*I*e) + I*f))/(a*c^2*e^(2*I*f*x + 2*I*e) + a*c^2)) - 4*(
3*I*a*c^2*e^(5*I*f*x + 5*I*e) + 8*I*a*c^2*e^(3*I*f*x + 3*I*e) - 3*I*a*c^2*e
^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*
e) + 1)))/(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(3/2)*(c-I*c*tan(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)*(c-I*c*tan(f*x+e))^(5/2),x, algorithm="g
iac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\tan(e+f\,x)\ 1\mathrm{i})^{3/2}\,(c-c\tan(e+f\,x)\ 1\mathrm{i})^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(3/2)*(c - c*tan(e + f*x)*1i)^(5/2),x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^(3/2)*(c - c*tan(e + f*x)*1i)^(5/2), x)
```

3.1009 $\int \sqrt{a+ia\tan(e+fx)}\,(c-ic\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=154

$$-\frac{3i\sqrt{a}\,c^{5/2}\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{f}-\frac{3ic^2\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{2f}-\frac{ic\sqrt{a}}{}$$

```
[Out] -3*I*c^(5/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*c*tan(f*x
+e))^(1/2))*a^(1/2)/f-3/2*I*c^2*(a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))
^(1/2)/f-1/2*I*c*(a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))^(3/2)/f
```

Rubi [A]
time = 0.11, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3604, 52, 65, 223, 209}

$$-\frac{3i\sqrt{a}\,c^{5/2}\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{f}-\frac{3ic^2\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{2f}-\frac{ic\sqrt{a+ia\tan(e+fx)}\,(c-ic\tan(e+fx))^{3/2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[e + f*x]]*(c - I*c*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-3*I)*Sqrt[a]*c^(5/2)*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a
]*Sqrt[c - I*c*Tan[e + f*x]])])/f - (((3*I)/2)*c^2*Sqrt[a + I*a*Tan[e + f*x
]]*Sqrt[c - I*c*Tan[e + f*x]])/f - ((I/2)*c*Sqrt[a + I*a*Tan[e + f*x]]*(c -
 I*c*Tan[e + f*x])^(3/2))/f
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \sqrt{a+ia\tan(e+fx)}\,(c-ic\tan(e+fx))^{5/2}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{(c-icx)^{3/2}}{\sqrt{a+iax}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{ic\sqrt{a+ia\tan(e+fx)}\,(c-ic\tan(e+fx))^{3/2}}{2f} + \frac{(3a}{}$$

$$= -\frac{3ic^2\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{2f} - \frac{ic}{}$$

$$= -\frac{3ic^2\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{2f} - \frac{ic}{}$$

$$= -\frac{3ic^2\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{2f} - \frac{ic}{}$$

$$= -\frac{3i\sqrt{a}\,c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{f} - \frac{3ic^2}{}$$

Mathematica [A]

time = 1.86, size = 155, normalized size = 1.01

$$-\frac{ice^{-i(e+fx)}\left(\frac{c}{1+e^{2i(e+fx)}}\right)^{3/2}\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\left(e^{i(e+fx)}\left(5+3e^{2i(e+fx)}\right)+3\left(1+e^{2i(e+fx)}\right)^2\text{ArcTan}\left(e^{i(e+fx)}\right)\right)\sqrt{a+ia\tan(e+fx)}}{f\sqrt{\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Tan[e + f*x]]*(c - I*c*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-I)*c*(c/(1 + E^((2*I)*(e + f*x))))^(3/2)*Sqrt[E^(I*(e + f*x))/(1 + E^((2
*I)*(e + f*x)))]*(E^(I*(e + f*x))*(5 + 3*E^((2*I)*(e + f*x))) + 3*(1 + E^((
2*I)*(e + f*x)))^2*ArcTan[E^(I*(e + f*x))])*Sqrt[a + I*a*Tan[e + f*x]])/(E^
(I*(e + f*x))*f*Sqrt[Sec[e + f*x]])
```

Maple [A]
time = 0.33, size = 153, normalized size = 0.99

method	result
derivativedivides	$-\frac{\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,c^{2}\left(4i\sqrt{ac\left(1+\tan^{2}\left(fx+e\right)\right)}\,\sqrt{a\right.}}{2f\sqrt{ac\left(1+\right.}}$
default	$-\frac{\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,c^{2}\left(4i\sqrt{ac\left(1+\tan^{2}\left(fx+e\right)\right)}\,\sqrt{a\right.}}{2f\sqrt{ac\left(1+\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))^(5/2),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/2/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*c^2*(4*I*(a*c
*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)+tan(f*x+e)*(a*c*(1+tan(f*x+e)^2))^(1/2
)*(a*c)^(1/2)-3*a*c*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(
1/2))/(a*c)^(1/2)))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(a*c)^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(118) = 236$.
time = 0.59, size = 479, normalized size = 3.11

$$-\frac{\left(12c^{2}\cos\left(3fx+3e\right)+20c^{2}\cos\left(fx+e\right)+12ic^{2}\sin\left(3fx+3e\right)+20ic^{2}\sin\left(fx+e\right)+6\left(c^{2}\cos\left(4fx+4e\right)+2c^{2}\cos\left(2fx+2e\right)+ic^{2}\sin\left(4fx+4e\right)+2ic^{2}\sin\left(2fx+2e\right)+c^{2}\right)\arctan\left(\cos\left(fx+e\right),\sin\left(fx+e\right)+1\right)+6\left(c^{2}\cos\left(4fx+4e\right)+2c^{2}\cos\left(2fx+2e\right)+ic^{2}\sin\left(4fx+4e\right)+2ic^{2}\sin\left(2fx+2e\right)+c^{2}\right)\arctan\left(\cos\left(fx+e\right),-\sin\left(fx+e\right)+1\right)+3\left(ic^{2}\cos\left(4fx+4e\right)+2ic^{2}\cos\left(2fx+2e\right)-c^{2}\sin\left(4fx+4e\right)-2c^{2}\sin\left(2fx+2e\right)+ic^{2}\right)\log\left(\cos\left(fx+e\right)^{2}+\sin\left(fx+e\right)^{2}+2\sin\left(fx+e\right)+1\right)+3\left(-ic^{2}\cos\left(4fx+4e\right)-2ic^{2}\cos\left(2fx+2e\right)+c^{2}\sin\left(4fx+4e\right)+2c^{2}\sin\left(2fx+2e\right)-ic^{2}\right)\log\left(\cos\left(fx+e\right)^{2}+\sin\left(fx+e\right)^{2}-2\sin\left(fx+e\right)+1\right)\right)\sqrt{a}\sqrt{c}}{-4f\left(i\cos\left(4fx+4e\right)+2i\cos\left(2fx+2e\right)-\sin\left(4fx+4e\right)-2\sin\left(2fx+2e\right)+i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))^(5/2),x, algorithm="m
axima")
```

```
[Out] -(12*c^2*cos(3*f*x + 3*e) + 20*c^2*cos(f*x + e) + 12*I*c^2*sin(3*f*x + 3*e)
 + 20*I*c^2*sin(f*x + e) + 6*(c^2*cos(4*f*x + 4*e) + 2*c^2*cos(2*f*x + 2*e)
 + I*c^2*sin(4*f*x + 4*e) + 2*I*c^2*sin(2*f*x + 2*e) + c^2)*arctan2(cos(f*x
 + e), sin(f*x + e) + 1) + 6*(c^2*cos(4*f*x + 4*e) + 2*c^2*cos(2*f*x + 2*e)
 + I*c^2*sin(4*f*x + 4*e) + 2*I*c^2*sin(2*f*x + 2*e) + c^2)*arctan2(cos(f*x
 + e), -sin(f*x + e) + 1) + 3*(I*c^2*cos(4*f*x + 4*e) + 2*I*c^2*cos(2*f*x +
 2*e) - c^2*sin(4*f*x + 4*e) - 2*c^2*sin(2*f*x + 2*e) + I*c^2)*log(cos(f*x
+ e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1) + 3*(-I*c^2*cos(4*f*x + 4*e)
- 2*I*c^2*cos(2*f*x + 2*e) + c^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f*x + 2*e)
- I*c^2)*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1))*sqrt(a)
*sqrt(c)/(f*(-4*I*cos(4*f*x + 4*e) - 8*I*cos(2*f*x + 2*e) + 4*sin(4*f*x + 4
*e) + 8*sin(2*f*x + 2*e) - 4*I))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(118) = 236$.
time = 0.89, size = 380, normalized size = 2.47

$$\frac{3\sqrt{\frac{ac^5}{f^2}}\left(fe^{(2i\,fx+2i\,e)}+f\right)\log\left(\frac{4\left(2\left(c^2e^{(3i\,fx+3i\,e)}+c^2e^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}-\sqrt{\frac{ac^5}{f^2}}\left(i\,fe^{(2i\,fx+2i\,e)}-i\,f\right)\right)}{c^2e^{(2i\,fx+2i\,e)}+c^2}\right)-3\sqrt{\frac{ac^5}{f^2}}\left(fe^{(2i\,fx+2i\,e)}+f\right)\log\left(\frac{4\left(2\left(c^2e^{(3i\,fx+3i\,e)}+c^2e^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}-\sqrt{\frac{ac^5}{f^2}}\left(-i\,fe^{(2i\,fx+2i\,e)}+i\,f\right)\right)}{c^2e^{(2i\,fx+2i\,e)}+c^2}\right)-4\left(3i\,c^2e^{(3i\,fx+3i\,e)}+5i\,c^2e^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{4\left(fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))^(5/2),x, algorithm="f
ricas")
```

```
[Out] 1/4*(3*sqrt(a*c^5/f^2)*(f*e^(2*I*f*x + 2*I*e) + f)*log(4*(2*(c^2*e^(3*I*f*x
 + 3*I*e) + c^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(
e^(2*I*f*x + 2*I*e) + 1)) - sqrt(a*c^5/f^2)*(I*f*e^(2*I*f*x + 2*I*e) - I*f)
)/(c^2*e^(2*I*f*x + 2*I*e) + c^2)) - 3*sqrt(a*c^5/f^2)*(f*e^(2*I*f*x + 2*I*
e) + f)*log(4*(2*(c^2*e^(3*I*f*x + 3*I*e) + c^2*e^(I*f*x + I*e))*sqrt(a/(e^
(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - sqrt(a*c^5/f^2)
*(-I*f*e^(2*I*f*x + 2*I*e) + I*f))/(c^2*e^(2*I*f*x + 2*I*e) + c^2)) - 4*(3*
I*c^2*e^(3*I*f*x + 3*I*e) + 5*I*c^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2
*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{ia\left(\tan\left(e+fx\right)-i\right)}\left(-ic(\tan\left(e+fx\right)+i)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(1/2)*(c-I*c*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(e + f*x) - I))*(-I*c*(tan(e + f*x) + I))**(5/2), x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(118) = 236$.

time = 1.67, size = 314, normalized size = 2.04

$$-\frac{11\,(ac^2-c^2)\sqrt{-ac}\,e^{(9i\,fx+9i\,e)}+50\,(ac^2-c^2)\sqrt{-ac}\,e^{(7i\,fx+7i\,e)}+84\,(ac^2-c^2)\sqrt{-ac}\,e^{(5i\,fx+5i\,e)}+62\,(ac^2-c^2)\sqrt{-ac}\,e^{(3i\,fx+3i\,e)}+17\,(ac^2-c^2)\sqrt{-ac}\,e^{(i\,fx+i\,e)}}{4\,((a-1)fe^{(10i\,fx+10i\,e)}+5\,(a-1)fe^{(8i\,fx+8i\,e)}+10\,(a-1)fe^{(6i\,fx+6i\,e)}+10\,(a-1)fe^{(4i\,fx+4i\,e)}+5\,(a-1)fe^{(2i\,fx+2i\,e)}+(a-1)f)}-\frac{32i\,\sqrt{a}\,c^{\frac{5}{2}}\arctan\left(e^{(i\,fx+i\,e)}\right)+\frac{i\left(11\,\sqrt{a}\,c^{\frac{5}{2}}e^{(3i\,fx+3i\,e)}+9\,\sqrt{a}\,c^{\frac{5}{2}}e^{(i\,fx+i\,e)}\right)}{\left(e^{(2i\,fx+2i\,e)}+1\right)^2}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))^(5/2),x, algorithm="g
iac")
```

```
[Out] -1/4*(11*(a*c^2 - c^2)*sqrt(-a*c)*e^(9*I*f*x + 9*I*e) + 50*(a*c^2 - c^2)*sq
rt(-a*c)*e^(7*I*f*x + 7*I*e) + 84*(a*c^2 - c^2)*sqrt(-a*c)*e^(5*I*f*x + 5*I
*e) + 62*(a*c^2 - c^2)*sqrt(-a*c)*e^(3*I*f*x + 3*I*e) + 17*(a*c^2 - c^2)*sq
rt(-a*c)*e^(I*f*x + I*e))/((a - 1)*f*e^(10*I*f*x + 10*I*e) + 5*(a - 1)*f*e^
(8*I*f*x + 8*I*e) + 10*(a - 1)*f*e^(6*I*f*x + 6*I*e) + 10*(a - 1)*f*e^(4*I*
f*x + 4*I*e) + 5*(a - 1)*f*e^(2*I*f*x + 2*I*e) + (a - 1)*f) - 1/4*(32*I*sqr
t(a)*c^(5/2)*arctan(e^(I*f*x + I*e)) + I*(11*sqrt(a)*c^(5/2)*e^(3*I*f*x + 3
*I*e) + 9*sqrt(a)*c^(5/2)*e^(I*f*x + I*e))/(e^(2*I*f*x + 2*I*e) + 1)^2)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+a\tan\left(e+f\,x\right)\,1\mathrm{i}}\,\left(c-c\tan(e+f\,x)\,1\mathrm{i}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(1/2)*(c - c*tan(e + f*x)*1i)^(5/2),x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^(1/2)*(c - c*tan(e + f*x)*1i)^(5/2), x)
```

3.1010 $\int \frac{(c-ic\tan(e+fx))^{5/2}}{\sqrt{a+ia\tan(e+fx)}}\,dx$

Optimal. Leaf size=153

$$\frac{6ic^{5/2}\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{a}\,f}+\frac{3ic^2\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{af}+\frac{2ic(c-ic\tan}{f\sqrt{a+ia\,\text{ta}}}$$

```
[Out] 6*I*c^(5/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*c*tan(f*x+
e))^(1/2))/f/a^(1/2)+3*I*c^2*(a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))^(1
/2)/a/f+2*I*c*(c-I*c*tan(f*x+e))^(3/2)/f/(a+I*a*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3604, 49, 52, 65, 223, 209}

$$\frac{6ic^{5/2}\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{a}\,f}+\frac{3ic^2\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{af}+\frac{2ic(c-ic\tan(e+fx))^{3/2}}{f\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])^(5/2)/Sqrt[a + I*a*Tan[e + f*x]],x]
```

```
[Out] ((6*I)*c^(5/2)*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c
- I*c*Tan[e + f*x]])])/(Sqrt[a]*f) + ((3*I)*c^2*Sqrt[a + I*a*Tan[e + f*x]]*
Sqrt[c - I*c*Tan[e + f*x]])/(a*f) + ((2*I)*c*(c - I*c*Tan[e + f*x])^(3/2))/
(f*Sqrt[a + I*a*Tan[e + f*x]])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(c-ic\tan(e+fx))^{5/2}}{\sqrt{a+ia\tan(e+fx)}}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{(c-icx)^{3/2}}{(a+iax)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{2ic(c-ic\tan(e+fx))^{3/2}}{f\sqrt{a+ia\tan(e+fx)}} - \frac{(3c^2)\,\text{Subst}\left(\int \frac{\sqrt{c-icx}}{\sqrt{a+iax}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{3ic^2\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{af} + \frac{2ic(c-ic\tan(e+fx))^{3/2}}{f\sqrt{a+ia\tan(e+fx)}}$$

$$= \frac{3ic^2\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{af} + \frac{2ic(c-ic\tan(e+fx))^{3/2}}{f\sqrt{a+ia\tan(e+fx)}}$$

$$= \frac{3ic^2\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{af} + \frac{2ic(c-ic\tan(e+fx))^{3/2}}{f\sqrt{a+ia\tan(e+fx)}}$$

$$= \frac{6ic^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{a}\,f} + \frac{3ic^2\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-}}{af}$$

Mathematica [A]
time = 2.44, size = 113, normalized size = 0.74

$$\frac{c^3(\cos(fx)+i\sin(fx))(i\cos(fx)+\sin(fx))\left(5+6\text{ArcTan}(\cos(e+fx)+i\sin(e+fx))\sec(e+fx)-4i\tan(e+fx)+\tan^2(e+fx)\right)}{f\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - I*c*Tan[e + f*x])^(5/2)/Sqrt[a + I*a*Tan[e + f*x]],x]
```

```
[Out] (c^3*(Cos[f*x] + I*Sin[f*x])*(I*Cos[f*x] + Sin[f*x])*(5 + 6*ArcTan[Cos[e +
f*x] + I*Sin[e + f*x]]*Sec[e + f*x] - (4*I)*Tan[e + f*x] + Tan[e + f*x]^2))
/(f*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(124) = 248.
time = 0.40, size = 300, normalized size = 1.96

method	result

derivativedivides	$\frac{i\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,c^{2}\left(3i\ln\left(\frac{ca\tan(fx+e)+\sqrt{ac\left(1+\tan^{2}(f\right.}}{\sqrt{ac}}\right.\right.}{}$
default	$\frac{i\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,c^{2}\left(3i\ln\left(\frac{ca\tan(fx+e)+\sqrt{ac\left(1+\tan^{2}(f\right.}}{\sqrt{ac}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] I/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)*c^2/a*(3*I*ln((c
*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))*a*c*ta
n(f*x+e)^2-3*I*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))
/(a*c)^(1/2))*a*c-6*I*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)+6
*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))*
a*c*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^2-5*(a*c
*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(-tan(f*
x+e)+I)^2/(a*c)^(1/2)
```

Maxima [A]
time = 0.59, size = 203, normalized size = 1.33

$$-\frac{\left(6i\,c^{2}\arctan\left(\cos\left(fx+e\right),\sin\left(fx+e\right)+1\right)\cos\left(fx+e\right)+6i\,c^{2}\arctan\left(\cos\left(fx+e\right),-\sin\left(fx+e\right)+1\right)\cos\left(fx+e\right)-8i\,c^{2}\cos\left(fx+e\right)^{2}+3\,c^{2}\cos\left(fx+e\right)\log\left(\cos\left(fx+e\right)^{2}+\sin\left(fx+e\right)^{2}+2\,\sin\left(fx+e\right)+1\right)-3\,c^{2}\cos\left(fx+e\right)\log\left(\cos\left(fx+e\right)^{2}+\sin\left(fx+e\right)^{2}-2\,\sin\left(fx+e\right)+1\right)-8\,c^{2}\cos\left(fx+e\right)\sin\left(fx+e\right)-2i\,c^{2}\right)\sqrt{c}}{2\,\sqrt{a}\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(1/2),x, algorithm="m
axima")
```

```
[Out] -1/2*(6*I*c^2*arctan2(cos(f*x + e), sin(f*x + e) + 1)*cos(f*x + e) + 6*I*c^
2*arctan2(cos(f*x + e), -sin(f*x + e) + 1)*cos(f*x + e) - 8*I*c^2*cos(f*x +
 e)^2 + 3*c^2*cos(f*x + e)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x
+ e) + 1) - 3*c^2*cos(f*x + e)*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(
f*x + e) + 1) - 8*c^2*cos(f*x + e)*sin(f*x + e) - 2*I*c^2)*sqrt(c)/(sqrt(a)
*f*cos(f*x + e))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(121) = 242$.
time = 1.33, size = 378, normalized size = 2.47

$$-\frac{\left(3\,\sqrt{\frac{c^{5}}{af^{2}}}\,afe^{\left(ifx+ie\right)}\log\left(\frac{4\left(2\left(c^{2}e^{\left(3ifx+3ie\right)}+c^{2}e^{\left(ifx+ie\right)}\right)\sqrt{\frac{a}{e^{\left(2ifx+2ie\right)}+1}}\,\sqrt{\frac{c}{e^{\left(2ifx+2ie\right)}+1}}-\left(iafe^{\left(2ifx+2ie\right)}-iaf\right)\sqrt{\frac{c^{5}}{af^{2}}}\right)}{c^{2}e^{\left(2ifx+2ie\right)}+c^{2}}\right)-3\,\sqrt{\frac{c^{5}}{af^{2}}}\,afe^{\left(ifx+ie\right)}\log\left(\frac{4\left(2\left(c^{2}e^{\left(3ifx+3ie\right)}+c^{2}e^{\left(ifx+ie\right)}\right)\sqrt{\frac{a}{e^{\left(2ifx+2ie\right)}+1}}\,\sqrt{\frac{c}{e^{\left(2ifx+2ie\right)}+1}}-\left(-iafe^{\left(2ifx+2ie\right)}+iaf\right)\sqrt{\frac{c^{5}}{af^{2}}}\right)}{c^{2}e^{\left(2ifx+2ie\right)}+c^{2}}\right)+4\left(-3i\,c^{2}e^{\left(2ifx+2ie\right)}-2i\,c^{2}\right)\sqrt{\frac{a}{e^{\left(2ifx+2ie\right)}+1}}\,\sqrt{\frac{c}{e^{\left(2ifx+2ie\right)}+1}}\right)e^{\left(-ifx-ie\right)}}{2\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(1/2),x, algorithm="f
ricas")
```

```
[Out] -1/2*(3*sqrt(c^5/(a*f^2))*a*f*e^(I*f*x + I*e)*log(4*(2*(c^2*e^(3*I*f*x + 3*
I*e) + c^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*
I*f*x + 2*I*e) + 1)) - (I*a*f*e^(2*I*f*x + 2*I*e) - I*a*f)*sqrt(c^5/(a*f^2)
))/(c^2*e^(2*I*f*x + 2*I*e) + c^2)) - 3*sqrt(c^5/(a*f^2))*a*f*e^(I*f*x + I*
e)*log(4*(2*(c^2*e^(3*I*f*x + 3*I*e) + c^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*
f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - (-I*a*f*e^(2*I*f*x +
 2*I*e) + I*a*f)*sqrt(c^5/(a*f^2)))/(c^2*e^(2*I*f*x + 2*I*e) + c^2)) + 4*(-
3*I*c^2*e^(2*I*f*x + 2*I*e) - 2*I*c^2)*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sq
rt(c/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/(a*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ic(\tan(e+fx)+i))^{\frac{5}{2}}}{\sqrt{ia(\tan(e+fx)-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**(5/2)/(a+I*a*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral((-I*c*(tan(e + f*x) + I))**(5/2)/sqrt(I*a*(tan(e + f*x) - I)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((-I*c*tan(f*x + e) + c)^(5/2)/sqrt(I*a*tan(f*x + e) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c-c\tan(e+f\,x)\ 1\mathrm{i})^{5/2}}{\sqrt{a+a\tan(e+f\,x)\ 1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^(5/2)/(a + a*tan(e + f*x)*1i)^(1/2),x)
```

```
[Out] int((c - c*tan(e + f*x)*1i)^(5/2)/(a + a*tan(e + f*x)*1i)^(1/2), x)
```

3.1011 $\int \frac{(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=155

$$-\frac{2ic^{5/2}\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{a^{3/2}f}-\frac{2ic^2\sqrt{c-ic\tan(e+fx)}}{af\sqrt{a+ia\tan(e+fx)}}+\frac{2ic(c-ic\tan(e+fx))^{3/2}}{3f(a+ia\tan(e+fx))^{3/2}}$$

```
[Out] -2*I*c^(5/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*c*tan(f*x
+e))^(1/2))/a^(3/2)/f-2*I*c^2*(c-I*c*tan(f*x+e))^(1/2)/a/f/(a+I*a*tan(f*x+e
))^(1/2)+2/3*I*c*(c-I*c*tan(f*x+e))^(3/2)/f/(a+I*a*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.12, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3604, 49, 65, 223, 209}

$$-\frac{2ic^{5/2}\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{a^{3/2}f}-\frac{2ic^2\sqrt{c-ic\tan(e+fx)}}{af\sqrt{a+ia\tan(e+fx)}}+\frac{2ic(c-ic\tan(e+fx))^{3/2}}{3f(a+ia\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])^(5/2)/(a + I*a*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-2*I)*c^(5/2)*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c
 - I*c*Tan[e + f*x]])])/(a^(3/2)*f) - ((2*I)*c^2*Sqrt[c - I*c*Tan[e + f*x]]
)/(a*f*Sqrt[a + I*a*Tan[e + f*x]]) + (((2*I)/3)*c*(c - I*c*Tan[e + f*x])^(3
/2))/(f*(a + I*a*Tan[e + f*x])^(3/2))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^{3/2}}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{(c-icx)^{3/2}}{(a+iax)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{2ic(c-ic\tan(e+fx))^{3/2}}{3f(a+ia\tan(e+fx))^{3/2}} - \frac{c^2\text{Subst}\left(\int \frac{\sqrt{c-icx}}{(a+iax)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{2ic^2\sqrt{c-ic\tan(e+fx)}}{af\sqrt{a+ia\tan(e+fx)}} + \frac{2ic(c-ic\tan(e+fx))^{3/2}}{3f(a+ia\tan(e+fx))^{3/2}} + \frac{c^3\text{Subst}\left(\int \frac{}{\sqrt{a-}}\right.}{}$$

$$= -\frac{2ic^2\sqrt{c-ic\tan(e+fx)}}{af\sqrt{a+ia\tan(e+fx)}} + \frac{2ic(c-ic\tan(e+fx))^{3/2}}{3f(a+ia\tan(e+fx))^{3/2}} - \frac{(2ic^3)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2ic^2\sqrt{c-ic\tan(e+fx)}}{af\sqrt{a+ia\tan(e+fx)}} + \frac{2ic(c-ic\tan(e+fx))^{3/2}}{3f(a+ia\tan(e+fx))^{3/2}} - \frac{(2ic^3)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2ic^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{a^{3/2}f} - \frac{2ic^2\sqrt{c-ic\tan(e+fx)}}{af\sqrt{a+ia\tan(e+fx)}} +$$

Mathematica [**A**]
time = 2.70, size = 109, normalized size = 0.70

$$-\frac{2i\sqrt{2}\,c^2e^{-2i(e+fx)}\sqrt{\frac{c}{1+e^{2i(e+fx)}}}\,\left(-1+3e^{2i(e+fx)}+3e^{3i(e+fx)}\mathrm{ArcTan}\left(e^{i(e+fx)}\right)\right)}{3af\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - I*c*Tan[e + f*x])^(5/2)/(a + I*a*Tan[e + f*x])^(3/2),x]
```

```
[Out] (((-2*I)/3)*Sqrt[2]*c^2*Sqrt[c/(1 + E^((2*I)*(e + f*x)))]*(-1 + 3*E^((2*I)*
(e + f*x)) + 3*E^((3*I)*(e + f*x))*ArcTan[E^(I*(e + f*x))]))/(a*E^((2*I)*(e
 + f*x))*f*Sqrt[a + I*a*Tan[e + f*x]])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(124) = 248.
time = 0.46, size = 350, normalized size = 2.26

method	result
derivativedivides	$\frac{\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,c^2\left(9i\ln\left(\frac{ca\tan(fx+e)+\sqrt{ac\left(1+\tan^2\left(f\right.\right.}}{\sqrt{ac}}\right.\right.}{}$
default	$\frac{\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,c^2\left(9i\ln\left(\frac{ca\tan(fx+e)+\sqrt{ac\left(1+\tan^2\left(f\right.\right.}}{\sqrt{ac}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(3/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/3/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)*c^2/a^2*(9*I*l
n((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))*a*
c*tan(f*x+e)^2-3*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2
))/(a*c)^(1/2))*a*c*tan(f*x+e)^3-3*I*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^
2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))*a*c+9*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*
x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)-12*I*(a*c*(1+tan(f*
x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)+8*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^
(1/2)*tan(f*x+e)^2-4*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c*(1+tan(
f*x+e)^2))^(1/2)/(-tan(f*x+e)+I)^3/(a*c)^(1/2)
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 440 vs. 2(121) = 242.
time = 0.65, size = 440, normalized size = 2.84

$$\frac{\left(-6i\,c^2\arctan\left(\cos\left(\frac{1}{3}\arctan\left(\sin\left(3fx+3e\right),\cos\left(3fx+3e\right)\right)\right),\sin\left(\frac{1}{3}\arctan\left(\sin\left(3fx+3e\right),\cos\left(3fx+3e\right)\right)\right)+1\right)-6i\,c^2\arctan\left(\cos\left(\frac{1}{3}\arctan\left(\sin\left(3fx+3e\right),\cos\left(3fx+3e\right)\right)\right),-\sin\left(\frac{1}{3}\arctan\left(\sin\left(3fx+3e\right),\cos\left(3fx+3e\right)\right)\right)+1\right)+4i\,c^2\cos\left(3fx+3e\right)+3c^2\log\left(\cos\left(\frac{1}{3}\arctan\left(\sin\left(3fx+3e\right),\cos\left(3fx+3e\right)\right)\right)^2+\sin\left(\frac{1}{3}\arctan\left(\sin\left(3fx+3e\right),\cos\left(3fx+3e\right)\right)\right)^2+2\sin\left(\frac{1}{3}\arctan\left(\sin\left(3fx+3e\right),\cos\left(3fx+3e\right)\right)\right)+1\right)-3c^2\log\left(\cos\left(\frac{1}{3}\arctan\left(\sin\left(3fx+3e\right),\cos\left(3fx+3e\right)\right)\right)^2+\sin\left(\frac{1}{3}\arctan\left(\sin\left(3fx+3e\right),\cos\left(3fx+3e\right)\right)\right)^2-2\sin\left(\frac{1}{3}\arctan\left(\sin\left(3fx+3e\right),\cos\left(3fx+3e\right)\right)\right)+1\right)+4c^2\sin\left(3fx+3e\right)-12\left(i\,c^2\cos\left(3fx+3e\right)+c^2\sin\left(3fx+3e\right)\right)\cos\left(\frac{2}{3}\arctan\left(\sin\left(3fx+3e\right),\cos\left(3fx+3e\right)\right)\right)+12\left(c^2\cos\left(3fx+3e\right)-i\,c^2\sin\left(3fx+3e\right)\right)\sin\left(\frac{2}{3}\arctan\left(\sin\left(3fx+3e\right),\cos\left(3fx+3e\right)\right)\right)\right)\sqrt{c}}{6a^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(3/2),x, algorithm="m
axima")
```

```
[Out] 1/6*(-6*I*c^2*arctan2(cos(1/3*arctan2(sin(3*f*x + 3*e), cos(3*f*x + 3*e))),
 sin(1/3*arctan2(sin(3*f*x + 3*e), cos(3*f*x + 3*e))) + 1) - 6*I*c^2*arctan
2(cos(1/3*arctan2(sin(3*f*x + 3*e), cos(3*f*x + 3*e))), -sin(1/3*arctan2(si
n(3*f*x + 3*e), cos(3*f*x + 3*e))) + 1) + 4*I*c^2*cos(3*f*x + 3*e) + 3*c^2*
log(cos(1/3*arctan2(sin(3*f*x + 3*e), cos(3*f*x + 3*e)))^2 + sin(1/3*arctan
2(sin(3*f*x + 3*e), cos(3*f*x + 3*e)))^2 + 2*sin(1/3*arctan2(sin(3*f*x + 3*
e), cos(3*f*x + 3*e))) + 1) - 3*c^2*log(cos(1/3*arctan2(sin(3*f*x + 3*e), c
os(3*f*x + 3*e)))^2 + sin(1/3*arctan2(sin(3*f*x + 3*e), cos(3*f*x + 3*e)))^
2 - 2*sin(1/3*arctan2(sin(3*f*x + 3*e), cos(3*f*x + 3*e))) + 1) + 4*c^2*sin
(3*f*x + 3*e) - 12*(I*c^2*cos(3*f*x + 3*e) + c^2*sin(3*f*x + 3*e))*cos(2/3*
arctan2(sin(3*f*x + 3*e), cos(3*f*x + 3*e))) + 12*(c^2*cos(3*f*x + 3*e) - I
*c^2*sin(3*f*x + 3*e))*sin(2/3*arctan2(sin(3*f*x + 3*e), cos(3*f*x + 3*e)))
)*sqrt(c)/(a^(3/2)*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(121) = 242$.
time = 1.17, size = 405, normalized size = 2.61

$$\frac{3\,a^2 f\sqrt{\frac{c^5}{a^3 f^2}}\,e^{(3i\,fx+3i\,e)}\log\left(\frac{4\left(2\left(c^2e^{(3i\,fx+3i\,e)}+c^2e^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}-\left(i\,a^2fe^{(2i\,fx+2i\,e)}-i\,a^2f\right)\sqrt{\frac{c^5}{a^3f^2}}\right)}{c^2e^{(2i\,fx+2i\,e)}+c^2}\right)-3\,a^2f\sqrt{\frac{c^5}{a^3f^2}}\,e^{(3i\,fx+3i\,e)}\log\left(\frac{4\left(2\left(c^2e^{(3i\,fx+3i\,e)}+c^2e^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}-\left(-i\,a^2fe^{(2i\,fx+2i\,e)}+i\,a^2f\right)\sqrt{\frac{c^5}{a^3f^2}}\right)}{c^2e^{(2i\,fx+2i\,e)}+c^2}\right)-4\left(3i\,c^2e^{(4i\,fx+4i\,e)}+2i\,c^2e^{(2i\,fx+2i\,e)}-i\,c^2\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-3i\,fx-3i\,e)}}{6\,a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/6*(3*a^2*f*sqrt(c^5/(a^3*f^2))*e^(3*I*f*x + 3*I*e)*log(4*(2*(c^2*e^(3*I*f
*x + 3*I*e) + c^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c
/(e^(2*I*f*x + 2*I*e) + 1)) - (I*a^2*f*e^(2*I*f*x + 2*I*e) - I*a^2*f)*sqrt(
c^5/(a^3*f^2)))/(c^2*e^(2*I*f*x + 2*I*e) + c^2)) - 3*a^2*f*sqrt(c^5/(a^3*f^
2))*e^(3*I*f*x + 3*I*e)*log(4*(2*(c^2*e^(3*I*f*x + 3*I*e) + c^2*e^(I*f*x +
I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) -
 (-I*a^2*f*e^(2*I*f*x + 2*I*e) + I*a^2*f)*sqrt(c^5/(a^3*f^2)))/(c^2*e^(2*I*
f*x + 2*I*e) + c^2)) - 4*(3*I*c^2*e^(4*I*f*x + 4*I*e) + 2*I*c^2*e^(2*I*f*x
+ 2*I*e) - I*c^2)*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*
I*e) + 1)))*e^(-3*I*f*x - 3*I*e)/(a^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-ic\left(\tan\left(e+fx\right)+i\right)\right)^{\frac{5}{2}}}{\left(ia\left(\tan\left(e+fx\right)-i\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**(5/2)/(a+I*a*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((-I*c*(tan(e + f*x) + I))**(5/2)/(I*a*(tan(e + f*x) - I))**(3/2),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((-I*c*tan(f*x + e) + c)^(5/2)/(I*a*tan(f*x + e) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{5/2}}{{\left(a+a\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^(5/2)/(a + a*tan(e + f*x)*1i)^(3/2),x)
```

```
[Out] int((c - c*tan(e + f*x)*1i)^(5/2)/(a + a*tan(e + f*x)*1i)^(3/2), x)
```

3.1012 $\int \frac{(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=43

$$\frac{i(c-ic\tan(e+fx))^{5/2}}{5f(a+ia\tan(e+fx))^{5/2}}$$

```
[Out] 1/5*I*(c-I*c*tan(f*x+e))^(5/2)/f/(a+I*a*tan(f*x+e))^(5/2)
```

Rubi [A]
time = 0.07, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {3604, 37}

$$\frac{i(c-ic\tan(e+fx))^{5/2}}{5f(a+ia\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])^(5/2)/(a + I*a*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((I/5)*(c - I*c*Tan[e + f*x])^(5/2))/(f*(a + I*a*Tan[e + f*x])^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx &= \frac{(ac)\text{Subst}\left(\int \frac{(c-icx)^{3/2}}{(a+iax)^{7/2}}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{i(c-ic\tan(e+fx))^{5/2}}{5f(a+ia\tan(e+fx))^{5/2}}
\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 90 vs. $2(43) = 86$.
time = 2.13, size = 90, normalized size = 2.09

$$-\frac{ic^2\sec^2(e+fx)(\cos(2(e+fx))-i\sin(2(e+fx)))\sqrt{c-ic\tan(e+fx)}}{5a^2f(-i+\tan(e+fx))^2\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - I*c*Tan[e + f*x])^(5/2)/(a + I*a*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-1/5*I)*c^2*Sec[e + f*x]^2*(Cos[2*(e + f*x)] - I*Sin[2*(e + f*x)])*Sqrt[c
 - I*c*Tan[e + f*x]])/(a^2*f*(-I + Tan[e + f*x])^2*Sqrt[a + I*a*Tan[e + f*x
]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(34) = 68$.
time = 0.37, size = 75, normalized size = 1.74

method	result	size
derivativedivides	$\frac{\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,c^{2}\left(1+\tan^{2}\left(fx+e\right)\right)\left(\tan\left(fx+e\right)+i\right)}{5f\,a^{3}\left(-\tan\left(fx+e\right)+i\right)^{4}}$	75
default	$\frac{\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,c^{2}\left(1+\tan^{2}\left(fx+e\right)\right)\left(\tan\left(fx+e\right)+i\right)}{5f\,a^{3}\left(-\tan\left(fx+e\right)+i\right)^{4}}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(5/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/5/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)*c^2/a^3*(1+tan
(f*x+e)^2)*(tan(f*x+e)+I)/(-tan(f*x+e)+I)^4
```

Maxima [A]
time = 0.63, size = 41, normalized size = 0.95

$$\frac{\left(i\,c^{2}\cos\left(5\,fx+5\,e\right)+c^{2}\sin\left(5\,fx+5\,e\right)\right)\sqrt{c}}{5\,a^{\frac{5}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(5/2),x, algorithm="m
axima")
```

```
[Out] 1/5*(I*c^2*cos(5*f*x + 5*e) + c^2*sin(5*f*x + 5*e))*sqrt(c)/(a^(5/2)*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(33) = 66$.

time = 0.81, size = 75, normalized size = 1.74

$$\frac{\left(i\,c^2e^{(2i\,fx+2i\,e)}+i\,c^2\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\,\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\,e^{(-5i\,fx-5i\,e)}}{5\,a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(5/2),x, algorithm="f
ricas")
```

```
[Out] 1/5*(I*c^2*e^(2*I*f*x + 2*I*e) + I*c^2)*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*s
qrt(c/(e^(2*I*f*x + 2*I*e) + 1))*e^(-5*I*f*x - 5*I*e)/(a^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(-ic(\tan{(e+fx)}+i))^{\frac{5}{2}}}{(ia\,(\tan{(e+fx)}-i))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**(5/2)/(a+I*a*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral((-I*c*(tan(e + f*x) + I))**(5/2)/(I*a*(tan(e + f*x) - I))**(5/2),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(5/2),x, algorithm="g
iac")
```

```
[Out] integrate((-I*c*tan(f*x + e) + c)^(5/2)/(I*a*tan(f*x + e) + a)^(5/2), x)
```

Mupad [B]
time = 7.76, size = 65, normalized size = 1.51

$$\frac{c^2\,(\tan(e+f\,x)+1\mathrm{i})^5\,\sqrt{a\,\left(1+\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}\,\sqrt{-c\,\left(-1+\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}}{5\,a^3\,f\,\left(\tan\left(e+f\,x\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^(5/2)/(a + a*tan(e + f*x)*1i)^(5/2),x)
```

```
[Out] (c^2*(tan(e + f*x) + 1i)^5*(a*(tan(e + f*x)*1i + 1))^(1/2)*(-c*(tan(e + f*x
)*1i - 1))^(1/2))/(5*a^3*f*(tan(e + f*x)^2 + 1)^3)
```

3.1013 $\int \frac{(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=90

$$\frac{i(c-ic\tan(e+fx))^{5/2}}{7f(a+ia\tan(e+fx))^{7/2}}+\frac{i(c-ic\tan(e+fx))^{5/2}}{35af(a+ia\tan(e+fx))^{5/2}}$$

```
[Out] 1/7*I*(c-I*c*tan(f*x+e))^(5/2)/f/(a+I*a*tan(f*x+e))^(7/2)+1/35*I*(c-I*c*tan
(f*x+e))^(5/2)/a/f/(a+I*a*tan(f*x+e))^(5/2)
```

Rubi [A]
time = 0.09, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {3604, 47, 37}

$$\frac{i(c-ic\tan(e+fx))^{5/2}}{35af(a+ia\tan(e+fx))^{5/2}}+\frac{i(c-ic\tan(e+fx))^{5/2}}{7f(a+ia\tan(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])^(5/2)/(a + I*a*Tan[e + f*x])^(7/2),x]
```

```
[Out] ((I/7)*(c - I*c*Tan[e + f*x])^(5/2))/(f*(a + I*a*Tan[e + f*x])^(7/2)) + ((I
/35)*(c - I*c*Tan[e + f*x])^(5/2))/(a*f*(a + I*a*Tan[e + f*x])^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
```

```
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^{7/2}}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{(c-icx)^{3/2}}{(a+iax)^{9/2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{i(c-ic\tan(e+fx))^{5/2}}{7f(a+ia\tan(e+fx))^{7/2}} + \frac{c\text{Subst}\left(\int \frac{(c-icx)^{3/2}}{(a+iax)^{7/2}}\,dx, x, \tan(e+fx)\right)}{7f}$$
$$= \frac{i(c-ic\tan(e+fx))^{5/2}}{7f(a+ia\tan(e+fx))^{7/2}} + \frac{i(c-ic\tan(e+fx))^{5/2}}{35af(a+ia\tan(e+fx))^{5/2}}$$

Mathematica [A]
time = 2.98, size = 100, normalized size = 1.11

$$-\frac{ic^2\sec^2(e+fx)(\cos(2(e+fx))-i\sin(2(e+fx)))(-6i+\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{35a^3f(-i+\tan(e+fx))^3\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - I*c*Tan[e + f*x])^(5/2)/(a + I*a*Tan[e + f*x])^(7/2),x]
```

```
[Out] ((-1/35*I)*c^2*Sec[e + f*x]^2*(Cos[2*(e + f*x)] - I*Sin[2*(e + f*x)])*(-6*I
 + Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(a^3*f*(-I + Tan[e + f*x])^3*S
qrt[a + I*a*Tan[e + f*x]])
```

Maple [A]
time = 0.39, size = 87, normalized size = 0.97

method	result
derivativedivides	$\frac{\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,c^2\left(1+\tan^2(fx+e)\right)\left(5i\tan(fx+e)-\left(\tan^2(fx+e)\right)-\right.}{35f\,a^4(-\tan(fx+e)+i)^5}$
default	$\frac{\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,c^2\left(1+\tan^2(fx+e)\right)\left(5i\tan(fx+e)-\left(\tan^2(fx+e)\right)-\right.}{35f\,a^4(-\tan(fx+e)+i)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(7/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/35/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)*c^2/a^4*(1+ta
n(f*x+e)^2)*(5*I*tan(f*x+e)-tan(f*x+e)^2-6)/(-tan(f*x+e)+I)^5
```

Maxima [A]
time = 0.61, size = 100, normalized size = 1.11

$$\frac{\left(5i\,c^2\cos\left(7\,fx+7\,e\right)+7i\,c^2\cos\left(\frac{5}{7}\,\arctan\left(\sin\left(7\,fx+7\,e\right),\cos\left(7\,fx+7\,e\right)\right)\right)+5\,c^2\sin\left(7\,fx+7\,e\right)+7\,c^2\sin\left(\frac{5}{7}\,\arctan\left(\sin\left(7\,fx+7\,e\right),\cos\left(7\,fx+7\,e\right)\right)\right)\right)\sqrt{c}}{70\,a^{\frac{7}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(7/2),x, algorithm="m
axima")
```

```
[Out] 1/70*(5*I*c^2*cos(7*f*x + 7*e) + 7*I*c^2*cos(5/7*arctan2(sin(7*f*x + 7*e),
cos(7*f*x + 7*e))) + 5*c^2*sin(7*f*x + 7*e) + 7*c^2*sin(5/7*arctan2(sin(7*f
*x + 7*e), cos(7*f*x + 7*e))))*sqrt(c)/(a^(7/2)*f)
```

Fricas [A]
time = 1.17, size = 90, normalized size = 1.00

$$\frac{\left(7i\,c^2e^{(4i\,fx+4i\,e)}+12i\,c^2e^{(2i\,fx+2i\,e)}+5i\,c^2\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\,\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\,e^{(-7i\,fx-7i\,e)}}{70\,a^4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(7/2),x, algorithm="f
ricas")
```

```
[Out] 1/70*(7*I*c^2*e^(4*I*f*x + 4*I*e) + 12*I*c^2*e^(2*I*f*x + 2*I*e) + 5*I*c^2)
*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*e^(-7*
I*f*x - 7*I*e)/(a^4*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-ic(\tan\left(e+fx\right)+i)\right)^{\frac{5}{2}}}{\left(ia\left(\tan\left(e+fx\right)-i\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**(5/2)/(a+I*a*tan(f*x+e))**(7/2),x)
```

```
[Out] Integral((-I*c*(tan(e + f*x) + I))**(5/2)/(I*a*(tan(e + f*x) - I))**(7/2),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(7/2),x, algorithm="g
iac")
```

```
[Out] integrate((-I*c*tan(f*x + e) + c)^(5/2)/(I*a*tan(f*x + e) + a)^(7/2), x)
```

Mupad [B]
time = 6.47, size = 161, normalized size = 1.79

$$\frac{c^2\sqrt{\frac{a\left(\cos\left(2e+2fx\right)+1+\sin\left(2e+2fx\right)\,1\mathrm{i}\right)}{\cos\left(2e+2fx\right)+1}}\,\sqrt{\frac{c\left(\cos\left(2e+2fx\right)+1-\sin\left(2e+2fx\right)\,1\mathrm{i}\right)}{\cos\left(2e+2fx\right)+1}}\,\left(7\sin\left(4e+4fx\right)+12\sin\left(6e+6fx\right)+5\sin\left(8e+8fx\right)+\cos\left(4e+4fx\right)\,7\mathrm{i}+\cos\left(6e+6fx\right)\,12\mathrm{i}+\cos\left(8e+8fx\right)\,5\mathrm{i}\right)}{140\,a^4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^(5/2)/(a + a*tan(e + f*x)*1i)^(7/2),x)
```

```
[Out] (c^2*((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) +
1))^(1/2)*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*
x) + 1))^(1/2)*(cos(4*e + 4*f*x)*7i + cos(6*e + 6*f*x)*12i + cos(8*e + 8*f*
x)*5i + 7*sin(4*e + 4*f*x) + 12*sin(6*e + 6*f*x) + 5*sin(8*e + 8*f*x)))/(14
0*a^4*f)
```

3.1014 $\int \frac{(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=136

$$\frac{i(c-ic\tan(e+fx))^{5/2}}{9f(a+ia\tan(e+fx))^{9/2}}+\frac{2i(c-ic\tan(e+fx))^{5/2}}{63af(a+ia\tan(e+fx))^{7/2}}+\frac{2i(c-ic\tan(e+fx))^{5/2}}{315a^2f(a+ia\tan(e+fx))^{5/2}}$$

```
[Out] 1/9*I*(c-I*c*tan(f*x+e))^(5/2)/f/(a+I*a*tan(f*x+e))^(9/2)+2/63*I*(c-I*c*tan
(f*x+e))^(5/2)/a/f/(a+I*a*tan(f*x+e))^(7/2)+2/315*I*(c-I*c*tan(f*x+e))^(5/2
)/a^2/f/(a+I*a*tan(f*x+e))^(5/2)
```

Rubi [A]

time = 0.10, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {3604, 47, 37}

$$\frac{2i(c-ic\tan(e+fx))^{5/2}}{315a^2f(a+ia\tan(e+fx))^{5/2}}+\frac{2i(c-ic\tan(e+fx))^{5/2}}{63af(a+ia\tan(e+fx))^{7/2}}+\frac{i(c-ic\tan(e+fx))^{5/2}}{9f(a+ia\tan(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])^(5/2)/(a + I*a*Tan[e + f*x])^(9/2),x]
```

```
[Out] ((I/9)*(c - I*c*Tan[e + f*x])^(5/2))/(f*(a + I*a*Tan[e + f*x])^(9/2)) + (((
2*I)/63)*(c - I*c*Tan[e + f*x])^(5/2))/(a*f*(a + I*a*Tan[e + f*x])^(7/2)) +
 (((2*I)/315)*(c - I*c*Tan[e + f*x])^(5/2))/(a^2*f*(a + I*a*Tan[e + f*x])^(
5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^{9/2}}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{(c-icx)^{3/2}}{(a+iax)^{11/2}}\,dx,x,\tan(e+fx)\right)}{f}$$
$$= \frac{i(c-ic\tan(e+fx))^{5/2}}{9f(a+ia\tan(e+fx))^{9/2}} + \frac{(2c)\text{Subst}\left(\int \frac{(c-icx)^{3/2}}{(a+iax)^{9/2}}\,dx,x,\tan(e+fx)\right)}{9f}$$
$$= \frac{i(c-ic\tan(e+fx))^{5/2}}{9f(a+ia\tan(e+fx))^{9/2}} + \frac{2i(c-ic\tan(e+fx))^{5/2}}{63af(a+ia\tan(e+fx))^{7/2}} + \frac{(2c)\text{Subst}\left(\int \frac{(c-}{(a-}\right.}{}$$
$$= \frac{i(c-ic\tan(e+fx))^{5/2}}{9f(a+ia\tan(e+fx))^{9/2}} + \frac{2i(c-ic\tan(e+fx))^{5/2}}{63af(a+ia\tan(e+fx))^{7/2}} + \frac{2i(c-ic\tan}{315a^2f(a+ia\tan}$$

Mathematica [A]
time = 3.62, size = 112, normalized size = 0.82

$$\frac{c^2\sec^4(e+fx)(45+49\cos(2(e+fx))+14i\sin(2(e+fx)))(i\cos(2(e+fx))+\sin(2(e+fx)))\sqrt{c-ic\tan(e+fx)}}{630a^4f(-i+\tan(e+fx))^4\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - I*c*Tan[e + f*x])^(5/2)/(a + I*a*Tan[e + f*x])^(9/2),x]
```

```
[Out] (c^2*Sec[e + f*x]^4*(45 + 49*Cos[2*(e + f*x)] + (14*I)*Sin[2*(e + f*x)])*(I
*Cos[2*(e + f*x)] + Sin[2*(e + f*x)])*Sqrt[c - I*c*Tan[e + f*x]])/(630*a^4*
f*(-I + Tan[e + f*x])^4*Sqrt[a + I*a*Tan[e + f*x]])
```

Maple [A]
time = 0.39, size = 99, normalized size = 0.73

method	result
derivativedivides	$-\frac{i\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,c^2(1+\tan^2(fx+e))(2i(\tan^3(fx+e))-33i\tan(}{315fa^5(-\tan(fx+e)+i)^6}$
default	$-\frac{i\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,c^2(1+\tan^2(fx+e))(2i(\tan^3(fx+e))-33i\tan(}{315fa^5(-\tan(fx+e)+i)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(9/2),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/315*I/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)*c^2/a^5*(
1+tan(f*x+e)^2)*(2*I*tan(f*x+e)^3-33*I*tan(f*x+e)+12*tan(f*x+e)^2+47)/(-tan
(f*x+e)+I)^6
```

Maxima [A]
time = 0.64, size = 158, normalized size = 1.16

$$\frac{\left(35i\,c^{2}\cos\left(9\,fx+9\,e\right)+90i\,c^{2}\cos\left(\frac{7}{9}\,\arctan\left(\sin\left(9\,fx+9\,e\right),\cos\left(9\,fx+9\,e\right)\right)\right)+63i\,c^{2}\cos\left(\frac{5}{9}\,\arctan\left(\sin\left(9\,fx+9\,e\right),\cos\left(9\,fx+9\,e\right)\right)\right)+35\,c^{2}\sin\left(9\,fx+9\,e\right)+90\,c^{2}\sin\left(\frac{7}{9}\,\arctan\left(\sin\left(9\,fx+9\,e\right),\cos\left(9\,fx+9\,e\right)\right)\right)+63\,c^{2}\sin\left(\frac{5}{9}\,\arctan\left(\sin\left(9\,fx+9\,e\right),\cos\left(9\,fx+9\,e\right)\right)\right)\right)\sqrt{c}}{1260\,a^{\frac{9}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(9/2),x, algorithm="m
axima")
```

```
[Out] 1/1260*(35*I*c^2*cos(9*f*x + 9*e) + 90*I*c^2*cos(7/9*arctan2(sin(9*f*x + 9*
e), cos(9*f*x + 9*e))) + 63*I*c^2*cos(5/9*arctan2(sin(9*f*x + 9*e), cos(9*f
*x + 9*e))) + 35*c^2*sin(9*f*x + 9*e) + 90*c^2*sin(7/9*arctan2(sin(9*f*x +
9*e), cos(9*f*x + 9*e))) + 63*c^2*sin(5/9*arctan2(sin(9*f*x + 9*e), cos(9*f
*x + 9*e))))*sqrt(c)/(a^(9/2)*f)
```

Fricas [A]
time = 1.11, size = 105, normalized size = 0.77

$$\frac{\left(63i\,c^{2}e^{\left(6i\,fx+6i\,e\right)}+153i\,c^{2}e^{\left(4i\,fx+4i\,e\right)}+125i\,c^{2}e^{\left(2i\,fx+2i\,e\right)}+35i\,c^{2}\right)\sqrt{\frac{a}{e^{\left(2i\,fx+2i\,e\right)}+1}}\sqrt{\frac{c}{e^{\left(2i\,fx+2i\,e\right)}+1}}\,e^{\left(-9i\,fx-9i\,e\right)}}{1260\,a^{5}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(9/2),x, algorithm="f
ricas")
```

```
[Out] 1/1260*(63*I*c^2*e^(6*I*f*x + 6*I*e) + 153*I*c^2*e^(4*I*f*x + 4*I*e) + 125*
I*c^2*e^(2*I*f*x + 2*I*e) + 35*I*c^2)*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqr
t(c/(e^(2*I*f*x + 2*I*e) + 1))*e^(-9*I*f*x - 9*I*e)/(a^5*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**(5/2)/(a+I*a*tan(f*x+e))**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(9/2),x, algorithm="g
iac")
```

```
[Out] integrate((-I*c*tan(f*x + e) + c)^(5/2)/(I*a*tan(f*x + e) + a)^(9/2), x)
```

Mupad [B]
time = 7.33, size = 184, normalized size = 1.35

$$\frac{c^2\,\sqrt{\frac{a\,\left(\cos\left(2\,e+2\,f\,x\right)+1+\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\sqrt{\frac{c\,\left(\cos\left(2\,e+2\,f\,x\right)+1-\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\left(63\,\sin\left(4\,e+4\,f\,x\right)+153\,\sin\left(6\,e+6\,f\,x\right)+125\,\sin\left(8\,e+8\,f\,x\right)+35\,\sin\left(10\,e+10\,f\,x\right)+\cos\left(4\,e+4\,f\,x\right)\,63{}\mathrm{i}+\cos\left(6\,e+6\,f\,x\right)\,153{}\mathrm{i}+\cos\left(8\,e+8\,f\,x\right)\,125{}\mathrm{i}+\cos\left(10\,e+10\,f\,x\right)\,35{}\mathrm{i}\right)}{2520\,a^5\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^(5/2)/(a + a*tan(e + f*x)*1i)^(9/2),x)
```

```
[Out] (c^2*((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) +
1))^(1/2)*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*
x) + 1))^(1/2)*(cos(4*e + 4*f*x)*63i + cos(6*e + 6*f*x)*153i + cos(8*e + 8*
f*x)*125i + cos(10*e + 10*f*x)*35i + 63*sin(4*e + 4*f*x) + 153*sin(6*e + 6*
f*x) + 125*sin(8*e + 8*f*x) + 35*sin(10*e + 10*f*x)))/(2520*a^5*f)
```

3.1015 $\int \frac{(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=182

$$\frac{i(c-ic\tan(e+fx))^{5/2}}{11f(a+ia\tan(e+fx))^{11/2}}+\frac{i(c-ic\tan(e+fx))^{5/2}}{33af(a+ia\tan(e+fx))^{9/2}}+\frac{2i(c-ic\tan(e+fx))^{5/2}}{231a^2f(a+ia\tan(e+fx))^{7/2}}+\frac{2i(c-ic\tan(e+fx))^{5/2}}{1155a^3f(a+ia\tan(e+fx))^{5/2}}$$

```
[Out] 1/11*I*(c-I*c*tan(f*x+e))^(5/2)/f/(a+I*a*tan(f*x+e))^(11/2)+1/33*I*(c-I*c*t
an(f*x+e))^(5/2)/a/f/(a+I*a*tan(f*x+e))^(9/2)+2/231*I*(c-I*c*tan(f*x+e))^(5
/2)/a^2/f/(a+I*a*tan(f*x+e))^(7/2)+2/1155*I*(c-I*c*tan(f*x+e))^(5/2)/a^3/f/
(a+I*a*tan(f*x+e))^(5/2)
```

Rubi [A]
time = 0.11, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {3604, 47, 37}

$$\frac{2i(c-ic\tan(e+fx))^{5/2}}{1155a^3f(a+ia\tan(e+fx))^{5/2}}+\frac{2i(c-ic\tan(e+fx))^{5/2}}{231a^2f(a+ia\tan(e+fx))^{7/2}}+\frac{i(c-ic\tan(e+fx))^{5/2}}{33af(a+ia\tan(e+fx))^{9/2}}+\frac{i(c-ic\tan(e+fx))^{5/2}}{11f(a+ia\tan(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])^(5/2)/(a + I*a*Tan[e + f*x])^(11/2),x]
```

```
[Out] ((I/11)*(c - I*c*Tan[e + f*x])^(5/2))/(f*(a + I*a*Tan[e + f*x])^(11/2)) + (
(I/33)*(c - I*c*Tan[e + f*x])^(5/2))/(a*f*(a + I*a*Tan[e + f*x])^(9/2)) + (
((2*I)/231)*(c - I*c*Tan[e + f*x])^(5/2))/(a^2*f*(a + I*a*Tan[e + f*x])^(7/
2)) + (((2*I)/1155)*(c - I*c*Tan[e + f*x])^(5/2))/(a^3*f*(a + I*a*Tan[e + f
*x])^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^{11/2}}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{(c-icx)^{3/2}}{(a+iax)^{13/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{i(c-ic\tan(e+fx))^{5/2}}{11f(a+ia\tan(e+fx))^{11/2}} + \frac{(3c)\text{Subst}\left(\int \frac{(c-icx)^{3/2}}{(a+iax)^{11/2}}\,dx, x, \tan(e+fx)\right)}{11f}$$

$$= \frac{i(c-ic\tan(e+fx))^{5/2}}{11f(a+ia\tan(e+fx))^{11/2}} + \frac{i(c-ic\tan(e+fx))^{5/2}}{33af(a+ia\tan(e+fx))^{9/2}} + \frac{(2c)\text{Subst}\left(\int\right.}{}$$

$$= \frac{i(c-ic\tan(e+fx))^{5/2}}{11f(a+ia\tan(e+fx))^{11/2}} + \frac{i(c-ic\tan(e+fx))^{5/2}}{33af(a+ia\tan(e+fx))^{9/2}} + \frac{2i(c-ic\,\mathrm{t}}{231a^2f(a+}$$

$$= \frac{i(c-ic\tan(e+fx))^{5/2}}{11f(a+ia\tan(e+fx))^{11/2}} + \frac{i(c-ic\tan(e+fx))^{5/2}}{33af(a+ia\tan(e+fx))^{9/2}} + \frac{2i(c-ic\,\mathrm{t}}{231a^2f(a+}$$

Mathematica [A]
time = 5.32, size = 128, normalized size = 0.70

$$\frac{c^2\sec^4(e+fx)(\cos(2(e+fx))-i\sin(2(e+fx)))(272+336\cos(2(e+fx))+63i\sec(e+fx)\sin(3(e+fx))+55i\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{4620a^5f(-i+\tan(e+fx))^5\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - I*c*Tan[e + f*x])^(5/2)/(a + I*a*Tan[e + f*x])^(11/2),x]
```

```
[Out] (c^2*Sec[e + f*x]^4*(Cos[2*(e + f*x)] - I*Sin[2*(e + f*x)])*(272 + 336*Cos[
2*(e + f*x)] + (63*I)*Sec[e + f*x]*Sin[3*(e + f*x)] + (55*I)*Tan[e + f*x])*
Sqrt[c - I*c*Tan[e + f*x]])/(4620*a^5*f*(-I + Tan[e + f*x])^5*Sqrt[a + I*a*
Tan[e + f*x]])
```

Maple [A]
time = 0.37, size = 110, normalized size = 0.60

method	result
derivativedivides	$\frac{i\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,c^2\left(1+\tan^2(fx+e)\right)\left(2i\left(\tan^4(fx+e)\right)-45i\left(\tan^2(f\right.\right.}{1155f\,a^6\left(-\tan(fx+e)+i\right)^7}$

default	$\frac{i\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,c^{2}\left(1+\tan^{2}\left(fx+e\right)\right)\left(2i\left(\tan^{4}\left(fx+e\right)\right)-45i\left(\tan^{2}\right.\right.}{1155f\,a^{6}\left(-\tan\left(fx+e\right)+i\right)^{7}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(11/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/1155*I/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)*c^2/a^6*(
1+tan(f*x+e)^2)*(2*I*tan(f*x+e)^4-45*I*tan(f*x+e)^2+14*tan(f*x+e)^3-152*I-9
1*tan(f*x+e))/(-tan(f*x+e)+I)^7
```

Maxima [A]
time = 0.57, size = 216, normalized size = 1.19

$$\frac{\left(105i\,c^{2}\cos\left(11\,fx+11\,e\right)+385i\,c^{2}\cos\left(\frac{9}{11}\arctan\left(\sin\left(11\,fx+11\,e\right),\cos\left(11\,fx+11\,e\right)\right)\right)+495i\,c^{2}\cos\left(\frac{7}{11}\arctan\left(\sin\left(11\,fx+11\,e\right),\cos\left(11\,fx+11\,e\right)\right)\right)+231i\,c^{2}\cos\left(\frac{5}{11}\arctan\left(\sin\left(11\,fx+11\,e\right),\cos\left(11\,fx+11\,e\right)\right)\right)+105\,c^{2}\sin\left(11\,fx+11\,e\right)+385\,c^{2}\sin\left(\frac{9}{11}\arctan\left(\sin\left(11\,fx+11\,e\right),\cos\left(11\,fx+11\,e\right)\right)\right)+495\,c^{2}\sin\left(\frac{7}{11}\arctan\left(\sin\left(11\,fx+11\,e\right),\cos\left(11\,fx+11\,e\right)\right)\right)+231\,c^{2}\sin\left(\frac{5}{11}\arctan\left(\sin\left(11\,fx+11\,e\right),\cos\left(11\,fx+11\,e\right)\right)\right)\right)\sqrt{c}}{9240\,a^{\frac{11}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(11/2),x, algorithm="
maxima")
```

```
[Out] 1/9240*(105*I*c^2*cos(11*f*x + 11*e) + 385*I*c^2*cos(9/11*arctan2(sin(11*f*
x + 11*e), cos(11*f*x + 11*e))) + 495*I*c^2*cos(7/11*arctan2(sin(11*f*x + 1
1*e), cos(11*f*x + 11*e))) + 231*I*c^2*cos(5/11*arctan2(sin(11*f*x + 11*e),
 cos(11*f*x + 11*e))) + 105*c^2*sin(11*f*x + 11*e) + 385*c^2*sin(9/11*arcta
n2(sin(11*f*x + 11*e), cos(11*f*x + 11*e))) + 495*c^2*sin(7/11*arctan2(sin(
11*f*x + 11*e), cos(11*f*x + 11*e))) + 231*c^2*sin(5/11*arctan2(sin(11*f*x
+ 11*e), cos(11*f*x + 11*e))))*sqrt(c)/(a^(11/2)*f)
```

Fricas [A]
time = 0.99, size = 120, normalized size = 0.66

$$\frac{\left(231i\,c^{2}e^{\left(8i\,fx+8i\,e\right)}+726i\,c^{2}e^{\left(6i\,fx+6i\,e\right)}+880i\,c^{2}e^{\left(4i\,fx+4i\,e\right)}+490i\,c^{2}e^{\left(2i\,fx+2i\,e\right)}+105i\,c^{2}\right)\sqrt{\frac{a}{e^{\left(2i\,fx+2i\,e\right)}+1}}\,\sqrt{\frac{c}{e^{\left(2i\,fx+2i\,e\right)}+1}}\,e^{\left(-11i\,fx-11i\,e\right)}}{9240\,a^{6}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(11/2),x, algorithm="
fricas")
```

```
[Out] 1/9240*(231*I*c^2*e^(8*I*f*x + 8*I*e) + 726*I*c^2*e^(6*I*f*x + 6*I*e) + 880
*I*c^2*e^(4*I*f*x + 4*I*e) + 490*I*c^2*e^(2*I*f*x + 2*I*e) + 105*I*c^2)*sqr
t(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*e^(-11*I*f
*x - 11*I*e)/(a^6*f)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**(5/2)/(a+I*a*tan(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(11/2),x, algorithm="
giac")
```

```
[Out] integrate((-I*c*tan(f*x + e) + c)^(5/2)/(I*a*tan(f*x + e) + a)^(11/2), x)
```

Mupad [**B**]
time = 8.15, size = 207, normalized size = 1.14

$$\frac{c^2\,\sqrt{\frac{a\,\left(\cos\left(2\,e+2\,f\,x\right)+1+\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\sqrt{\frac{c\,\left(\cos\left(2\,e+2\,f\,x\right)+1-\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\left(231\,\sin\left(4\,e+4\,f\,x\right)+726\,\sin\left(6\,e+6\,f\,x\right)+880\,\sin\left(8\,e+8\,f\,x\right)+490\,\sin\left(10\,e+10\,f\,x\right)+105\,\sin\left(12\,e+12\,f\,x\right)+\cos\left(4\,e+4\,f\,x\right)\,231{}\mathrm{i}+\cos\left(6\,e+6\,f\,x\right)\,726{}\mathrm{i}+\cos\left(8\,e+8\,f\,x\right)\,880{}\mathrm{i}+\cos\left(10\,e+10\,f\,x\right)\,490{}\mathrm{i}+\cos\left(12\,e+12\,f\,x\right)\,105{}\mathrm{i}\right)}{18480\,a^6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^(5/2)/(a + a*tan(e + f*x)*1i)^(11/2),x)
```

```
[Out] (c^2*((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) +
1))^(1/2)*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*
x) + 1))^(1/2)*(cos(4*e + 4*f*x)*231i + cos(6*e + 6*f*x)*726i + cos(8*e + 8
*f*x)*880i + cos(10*e + 10*f*x)*490i + cos(12*e + 12*f*x)*105i + 231*sin(4*
e + 4*f*x) + 726*sin(6*e + 6*f*x) + 880*sin(8*e + 8*f*x) + 490*sin(10*e + 1
0*f*x) + 105*sin(12*e + 12*f*x)))/(18480*a^6*f)
```

3.1016 $\int \frac{(a+ia\tan(e+fx))^{7/2}}{\sqrt{c-ic\tan(e+fx)}}\,dx$

Optimal. Leaf size=204

$$\frac{15ia^{7/2}\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{c}f}-\frac{2ia(a+ia\tan(e+fx))^{5/2}}{f\sqrt{c-ic\tan(e+fx)}}-\frac{15ia^3\sqrt{a+ia\tan(e+fx)}\sqrt{c}}{2cf}$$

```
[Out] 15*I*a^(7/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*c*tan(f*x
+e))^(1/2))/f/c^(1/2)-15/2*I*a^3*(a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e)
)^(1/2)/c/f-5/2*I*a^2*(c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^(3/2)/c/f
-2*I*a*(a+I*a*tan(f*x+e))^(5/2)/f/(c-I*c*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3604, 49, 52, 65, 223, 209}

$$\frac{15ia^{7/2}\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{c}f}-\frac{15ia^3\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{2cf}-\frac{5ia^2(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}{2cf}-\frac{2ia(a+ia\tan(e+fx))^{5/2}}{f\sqrt{c-ic\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^(7/2)/Sqrt[c - I*c*Tan[e + f*x]],x]
```

```
[Out] ((15*I)*a^(7/2)*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c
 - I*c*Tan[e + f*x]])])/(Sqrt[c]*f) - ((2*I)*a*(a + I*a*Tan[e + f*x])^(5/2)
)/(f*Sqrt[c - I*c*Tan[e + f*x]]) - (((15*I)/2)*a^3*Sqrt[a + I*a*Tan[e + f*x
]]*Sqrt[c - I*c*Tan[e + f*x]])/(c*f) - (((5*I)/2)*a^2*(a + I*a*Tan[e + f*x]
)^(3/2)*Sqrt[c - I*c*Tan[e + f*x]])/(c*f)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^{7/2}}{\sqrt{c-ic\tan(e+fx)}}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{(a+iax)^{5/2}}{(c-icx)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{2ia(a+ia\tan(e+fx))^{5/2}}{f\sqrt{c-ic\tan(e+fx)}} - \frac{(5a^2)\,\text{Subst}\left(\int \frac{(a+iax)^{3/2}}{\sqrt{c-icx}}\,dx, x, \tan(e+fx\right.}{f}$$

$$= -\frac{2ia(a+ia\tan(e+fx))^{5/2}}{f\sqrt{c-ic\tan(e+fx)}} - \frac{5ia^2(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+}}{2cf}$$

$$= -\frac{2ia(a+ia\tan(e+fx))^{5/2}}{f\sqrt{c-ic\tan(e+fx)}} - \frac{15ia^3\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+}}{2cf}$$

$$= -\frac{2ia(a+ia\tan(e+fx))^{5/2}}{f\sqrt{c-ic\tan(e+fx)}} - \frac{15ia^3\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+}}{2cf}$$

$$= -\frac{2ia(a+ia\tan(e+fx))^{5/2}}{f\sqrt{c-ic\tan(e+fx)}} - \frac{15ia^3\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+}}{2cf}$$

$$= \frac{15ia^{7/2}\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{c}\,f} - \frac{2ia(a+ia\tan(e+fx))^{5/2}}{f\sqrt{c-ic\tan(e+fx)}}$$

Mathematica [A]
time = 8.56, size = 340, normalized size = 1.67

$$\frac{15ie^{-i(4e+fx)}\sqrt{e^{ifx}}\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\,\text{ArcTan}\left(e^{i(e+fx)}\right)(a+ia\tan(e+fx))^{7/2}}{\sqrt{\frac{c}{1+e^{2i(e+fx)}}}\,f\sec^{\frac{7}{2}}(e+fx)(\cos(fx)+i\sin(fx))^{7/2}} + \frac{\cos^3(e+fx)\left(\cos(2fx)\left(-\frac{4i\cos(e)}{c}-\frac{4\sin(e)}{c}\right)+\sec(e)(16\cos(e)+i\sin(e))\left(-\frac{i\cos(3e)}{2c}-\frac{\sin(3e)}{2c}\right)+\sec(e)\sec(e+fx)\left(\frac{\cos(3e)}{2c}-\frac{i\sin(3e)}{2c}\right)\sin(fx)+\left(\frac{4\cos(e)}{c}-\frac{4i\sin(e)}{c}\right)\sin(2fx)\right)\sqrt{\sec(e+fx)(c\cos(e+fx)-ic\sin(e+fx))}\,(a+ia\tan(e+fx))^{7/2}}{f(\cos(fx)+i\sin(fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(7/2)/Sqrt[c - I*c*Tan[e + f*x]],x]
```

```
[Out] ((15*I)*Sqrt[E^(I*f*x)]*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*Arc
Tan[E^(I*(e + f*x))]*(a + I*a*Tan[e + f*x])^(7/2))/(E^(I*(4*e + f*x))*Sqrt[
c/(1 + E^((2*I)*(e + f*x)))]*f*Sec[e + f*x]^(7/2)*(Cos[f*x] + I*Sin[f*x])^(
7/2)) + (Cos[e + f*x]^3*(Cos[2*f*x]*(((-4*I)*Cos[e])/c - (4*Sin[e])/c) + Se
c[e]*(16*Cos[e] + I*Sin[e])*(((-1/2*I)*Cos[3*e])/c - Sin[3*e]/(2*c)) + Sec[
e]*Sec[e + f*x]*(Cos[3*e]/(2*c) - ((I/2)*Sin[3*e])/c)*Sin[f*x] + ((4*Cos[e]
```

```
)/c - ((4*I)*Sin[e])/c)*Sin[2*f*x])*Sqrt[Sec[e + f*x]*(c*Cos[e + f*x] - I*c
*Sin[e + f*x])]*(a + I*a*Tan[e + f*x])^(7/2))/(f*(Cos[f*x] + I*Sin[f*x])^3)
```

Maple [A]
time = 0.42, size = 328, normalized size = 1.61

method	result
derivativedivides	$-\frac{\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,a^{3}\left(30i\ln\left(\frac{ca\tan(fx+e)+\sqrt{ac\left(1+\tan^{2}\left(\right.\right.}}{\sqrt{ac}}\right.\right.}{}$
default	$-\frac{\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,a^{3}\left(30i\ln\left(\frac{ca\tan(fx+e)+\sqrt{ac\left(1+\tan^{2}\left(\right.\right.}}{\sqrt{ac}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(7/2)/(c-I*c*tan(f*x+e))^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/2/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^3/c*(30*I*l
n((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))*a*
c*tan(f*x+e)+6*I*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^2+15*l
n((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))*a*
c*tan(f*x+e)^2-(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^3-24*I*(
a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)-15*a*c*ln((c*a*tan(f*x+e)+(a*c*(1+t
an(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))-31*tan(f*x+e)*(a*c*(1+tan(f*x
+e)^2))^(1/2)*(a*c)^(1/2))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(a*c)^(1/2)/(tan(f*
x+e)+I)^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 843 vs. 2(160) = 320.
time = 0.59, size = 843, normalized size = 4.13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(7/2)/(c-I*c*tan(f*x+e))^(1/2),x, algorithm="m
axima")
```

```
[Out] -2*(36*a^3*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 36*I*a^3*
sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 30*(a^3*cos(4*f*x +
4*e) + 2*a^3*cos(2*f*x + 2*e) + I*a^3*sin(4*f*x + 4*e) + 2*I*a^3*sin(2*f*x
+ 2*e) + a^3)*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))),
 sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) - 30*(a^3*cos(4*
f*x + 4*e) + 2*a^3*cos(2*f*x + 2*e) + I*a^3*sin(4*f*x + 4*e) + 2*I*a^3*sin(
```

```
2*f*x + 2*e) + a^3)*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))), -sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 4*(8*a^
3*cos(4*f*x + 4*e) + 16*a^3*cos(2*f*x + 2*e) + 8*I*a^3*sin(4*f*x + 4*e) + 1
6*I*a^3*sin(2*f*x + 2*e) + 15*a^3)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) - 15*(I*a^3*cos(4*f*x + 4*e) + 2*I*a^3*cos(2*f*x + 2*e) - a^3*
sin(4*f*x + 4*e) - 2*a^3*sin(2*f*x + 2*e) + I*a^3)*log(cos(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e)))^2 + 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
 1) - 15*(-I*a^3*cos(4*f*x + 4*e) - 2*I*a^3*cos(2*f*x + 2*e) + a^3*sin(4*f*
x + 4*e) + 2*a^3*sin(2*f*x + 2*e) - I*a^3)*log(cos(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e)))^2 - 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) - 4*
(-8*I*a^3*cos(4*f*x + 4*e) - 16*I*a^3*cos(2*f*x + 2*e) + 8*a^3*sin(4*f*x +
4*e) + 16*a^3*sin(2*f*x + 2*e) - 15*I*a^3)*sin(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((-8*I*c*cos(4*f*x + 4*e) - 16*I*c*co
s(2*f*x + 2*e) + 8*c*sin(4*f*x + 4*e) + 16*c*sin(2*f*x + 2*e) - 8*I*c)*f)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(160) = 320$.
time = 1.01, size = 416, normalized size = 2.04

$$-\frac{15 \sqrt{\frac{a^7}{cf^2}}\left(cfe^{(2i fx+2i e)}+cf\right)\log\left(\frac{4\left(2\left(a^3e^{(3i fx+3i e)}+a^3e^{(i fx+i e)}\right)\sqrt{\frac{a}{e^{(2i fx+2i e)}+1}}\sqrt{\frac{c}{e^{(2i fx+2i e)}+1}}-\sqrt{\frac{a^7}{cf^2}}\left(icfe^{(2i fx+2i e)}-icf\right)\right)}{a^3e^{(2i fx+2i e)}+a^3}\right)-15\sqrt{\frac{a^7}{cf^2}}\left(cfe^{(2i fx+2i e)}+cf\right)\log\left(\frac{4\left(2\left(a^3e^{(3i fx+3i e)}+a^3e^{(i fx+i e)}\right)\sqrt{\frac{a}{e^{(2i fx+2i e)}+1}}\sqrt{\frac{c}{e^{(2i fx+2i e)}+1}}-\sqrt{\frac{a^7}{cf^2}}\left(-icfe^{(2i fx+2i e)}+icf\right)\right)}{a^3e^{(2i fx+2i e)}+a^3}\right)+4\left(8i a^3e^{(5i fx+5i e)}+25i a^3e^{(3i fx+3i e)}+15i a^3e^{(i fx+i e)}\right)\sqrt{\frac{a}{e^{(2i fx+2i e)}+1}}\sqrt{\frac{c}{e^{(2i fx+2i e)}+1}}}{4\left(cfe^{(2i fx+2i e)}+cf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(7/2)/(c-I*c*tan(f*x+e))^(1/2),x, algorithm="f
ricas")
```

```
[Out] -1/4*(15*sqrt(a^7/(c*f^2))*(c*f*e^(2*I*f*x + 2*I*e) + c*f)*log(4*(2*(a^3*e^
(3*I*f*x + 3*I*e) + a^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*
sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - sqrt(a^7/(c*f^2))*(I*c*f*e^(2*I*f*x + 2
*I*e) - I*c*f))/(a^3*e^(2*I*f*x + 2*I*e) + a^3)) - 15*sqrt(a^7/(c*f^2))*(c*
f*e^(2*I*f*x + 2*I*e) + c*f)*log(4*(2*(a^3*e^(3*I*f*x + 3*I*e) + a^3*e^(I*f
*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) +
1)) - sqrt(a^7/(c*f^2))*(-I*c*f*e^(2*I*f*x + 2*I*e) + I*c*f))/(a^3*e^(2*I*f
*x + 2*I*e) + a^3)) + 4*(8*I*a^3*e^(5*I*f*x + 5*I*e) + 25*I*a^3*e^(3*I*f*x
+ 3*I*e) + 15*I*a^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt
(c/(e^(2*I*f*x + 2*I*e) + 1)))/(c*f*e^(2*I*f*x + 2*I*e) + c*f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(7/2)/(c-I*c*tan(f*x+e))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(7/2)/(c-I*c*tan(f*x+e))^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^(7/2)/sqrt(-I*c*tan(f*x + e) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+a\tan(e+f\,x)\,1\mathrm{i}\right)^{7/2}}{\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(7/2)/(c - c*tan(e + f*x)*1i)^(1/2),x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^(7/2)/(c - c*tan(e + f*x)*1i)^(1/2), x)
```

3.1017 $\int \frac{(a+ia\tan(e+fx))^{5/2}}{\sqrt{c-ic\tan(e+fx)}}\,dx$

Optimal. Leaf size=153

$$\frac{6ia^{5/2}\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{c}\,f}-\frac{2ia(a+ia\tan(e+fx))^{3/2}}{f\sqrt{c-ic\tan(e+fx)}}-\frac{3ia^2\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-}}{cf}$$

```
[Out] 6*I*a^(5/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*c*tan(f*x+
e))^(1/2))/f/c^(1/2)-3*I*a^2*(a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))^(1
/2)/c/f-2*I*a*(a+I*a*tan(f*x+e))^(3/2)/f/(c-I*c*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3604, 49, 52, 65, 223, 209}

$$\frac{6ia^{5/2}\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{c}\,f}-\frac{3ia^2\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{cf}-\frac{2ia(a+ia\tan(e+fx))^{3/2}}{f\sqrt{c-ic\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^(5/2)/Sqrt[c - I*c*Tan[e + f*x]],x]
```

```
[Out] ((6*I)*a^(5/2)*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c
- I*c*Tan[e + f*x]])])/(Sqrt[c]*f) - ((2*I)*a*(a + I*a*Tan[e + f*x])^(3/2))
/(f*Sqrt[c - I*c*Tan[e + f*x]]) - ((3*I)*a^2*Sqrt[a + I*a*Tan[e + f*x]]*Sqr
t[c - I*c*Tan[e + f*x]])/(c*f)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^{5/2}}{\sqrt{c-ic\tan(e+fx)}}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{(a+iax)^{3/2}}{(c-icx)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{2ia(a+ia\tan(e+fx))^{3/2}}{f\sqrt{c-ic\tan(e+fx)}} - \frac{(3a^2)\,\text{Subst}\left(\int \frac{\sqrt{a+iax}}{\sqrt{c-icx}}\,dx, x, \tan(e+fx\right.}{f}$$

$$= -\frac{2ia(a+ia\tan(e+fx))^{3/2}}{f\sqrt{c-ic\tan(e+fx)}} - \frac{3ia^2\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+}}{cf}$$

$$= -\frac{2ia(a+ia\tan(e+fx))^{3/2}}{f\sqrt{c-ic\tan(e+fx)}} - \frac{3ia^2\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+}}{cf}$$

$$= -\frac{2ia(a+ia\tan(e+fx))^{3/2}}{f\sqrt{c-ic\tan(e+fx)}} - \frac{3ia^2\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+}}{cf}$$

$$= \frac{6ia^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{c}\,f} - \frac{2ia(a+ia\tan(e+fx))^{3/2}}{f\sqrt{c-ic\tan(e+fx)}} -$$

Mathematica [A]
time = 3.24, size = 155, normalized size = 1.01

$$-\frac{2ie^{-3i(e+fx)}\sqrt{\frac{c}{1+e^{2i(e+fx)}}}\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\left(e^{i(e+fx)}\left(3+2e^{2i(e+fx)}\right)-3\left(1+e^{2i(e+fx)}\right)\text{ArcTan}\left(e^{i(e+fx)}\right)\right)(a+ia\tan(e+fx))^{5/2}}{cf\sec^{\frac{5}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(5/2)/Sqrt[c - I*c*Tan[e + f*x]],x]
```

```
[Out] ((-2*I)*Sqrt[c/(1 + E^((2*I)*(e + f*x)))]*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I
)*(e + f*x)))]*(E^(I*(e + f*x))*(3 + 2*E^((2*I)*(e + f*x))) - 3*(1 + E^((2*
I)*(e + f*x)))*ArcTan[E^(I*(e + f*x))])*(a + I*a*Tan[e + f*x])^(5/2))/(c*E^
((3*I)*(e + f*x))*f*Sec[e + f*x]^(5/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(124) = 248.
time = 0.43, size = 299, normalized size = 1.95

method	result
derivativedivides	$\frac{i\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\ \sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\ a^{2}\left(3i\ln\left(\frac{ca\tan(fx+e)+\sqrt{ac\left(1+\tan^{2}\left(fx\right.\right.}}{\sqrt{ac}}\right.\right.}{}$
default	$\frac{i\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\ \sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\ a^{2}\left(3i\ln\left(\frac{ca\tan(fx+e)+\sqrt{ac\left(1+\tan^{2}\left(fx\right.\right.}}{\sqrt{ac}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] I/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^2/c*(3*I*ln((c
*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))*a*c*ta
n(f*x+e)^2-3*I*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))
/(a*c)^(1/2))*a*c-6*I*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)-6
*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))*
a*c*tan(f*x+e)-(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^2+5*(a*c
*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(a*c)^(1
/2)/(tan(f*x+e)+I)^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 578 vs. $2(121) = 242$.
time = 0.59, size = 578, normalized size = 3.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(1/2),x, algorithm="m
axima")
```

```
[Out] (6*(a^2*cos(2*f*x + 2*e) + I*a^2*sin(2*f*x + 2*e) + a^2)*arctan2(cos(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) + 1) + 6*(a^2*cos(2*f*x + 2*e) + I*a^2*sin(2*f*x + 2*
e) + a^2)*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), -si
n(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) - 4*(2*a^2*cos(2*f*
x + 2*e) + 2*I*a^2*sin(2*f*x + 2*e) + 3*a^2)*cos(1/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))) + 3*(I*a^2*cos(2*f*x + 2*e) - a^2*sin(2*f*x + 2*e) +
 I*a^2)*log(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 3*(-I*a^2*cos(2*f*x + 2*e) + a^2*sin(
2*f*x + 2*e) - I*a^2)*log(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sin(1/2*
```

```
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 4*(-2*I*a^2*cos(2*f*x +
 2*e) + 2*a^2*sin(2*f*x + 2*e) - 3*I*a^2)*sin(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((-2*I*c*cos(2*f*x + 2*e) + 2*c*sin(2*
f*x + 2*e) - 2*I*c)*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 358 vs. $2(121) = 242$.
time = 1.11, size = 358, normalized size = 2.34

$$-\frac{3\sqrt{\frac{a^5}{cf^2}}\,cf\log\left(\frac{4\left(2\left(a^2e^{(3i fx+3ie)}+a^2e^{(i fx+ie)}\right)\sqrt{\frac{a}{e^{(2i fx+2ie)}+1}}\sqrt{\frac{c}{e^{(2i fx+2ie)}+1}}-\left(icfe^{(2i fx+2ie)}-icf\right)\sqrt{\frac{a^5}{cf^2}}\right)}{a^2e^{(2i fx+2ie)}+a^2}\right)-3\sqrt{\frac{a^5}{cf^2}}\,cf\log\left(\frac{4\left(2\left(a^2e^{(3i fx+3ie)}+a^2e^{(i fx+ie)}\right)\sqrt{\frac{a}{e^{(2i fx+2ie)}+1}}\sqrt{\frac{c}{e^{(2i fx+2ie)}+1}}-\left(-icfe^{(2i fx+2ie)}+icf\right)\sqrt{\frac{a^5}{cf^2}}\right)}{a^2e^{(2i fx+2ie)}+a^2}\right)+4\left(2i\,a^2e^{(3i fx+3ie)}+3i\,a^2e^{(i fx+ie)}\right)\sqrt{\frac{a}{e^{(2i fx+2ie)}+1}}\sqrt{\frac{c}{e^{(2i fx+2ie)}+1}}}{2cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(1/2),x, algorithm="f
ricas")
```

```
[Out] -1/2*(3*sqrt(a^5/(c*f^2))*c*f*log(4*(2*(a^2*e^(3*I*f*x + 3*I*e) + a^2*e^(I*
f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) +
 1)) - (I*c*f*e^(2*I*f*x + 2*I*e) - I*c*f)*sqrt(a^5/(c*f^2)))/(a^2*e^(2*I*f
*x + 2*I*e) + a^2)) - 3*sqrt(a^5/(c*f^2))*c*f*log(4*(2*(a^2*e^(3*I*f*x + 3*
I*e) + a^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*
I*f*x + 2*I*e) + 1)) - (-I*c*f*e^(2*I*f*x + 2*I*e) + I*c*f)*sqrt(a^5/(c*f^2
)))/(a^2*e^(2*I*f*x + 2*I*e) + a^2)) + 4*(2*I*a^2*e^(3*I*f*x + 3*I*e) + 3*I
*a^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x
+ 2*I*e) + 1)))/(c*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan{(e+fx)}-i))^{\frac{5}{2}}}{\sqrt{-ic\,(\tan{(e+fx)}+i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(5/2)/(c-I*c*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral((I*a*(tan(e + f*x) - I))**(5/2)/sqrt(-I*c*(tan(e + f*x) + I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^(5/2)/sqrt(-I*c*tan(f*x + e) + c), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\tan(e+f\,x)\;1\mathrm{i})^{5/2}}{\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(5/2)/(c - c*tan(e + f*x)*1i)^(1/2),x)
[Out] int((a + a*tan(e + f*x)*1i)^(5/2)/(c - c*tan(e + f*x)*1i)^(1/2), x)
```

3.1018 $\int \frac{(a+ia\tan(e+fx))^{3/2}}{\sqrt{c-ic\tan(e+fx)}}\,dx$

Optimal. Leaf size=106

$$\frac{2ia^{3/2}\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{c}\,f}-\frac{2ia\sqrt{a+ia\tan(e+fx)}}{f\sqrt{c-ic\tan(e+fx)}}$$

```
[Out] 2*I*a^(3/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*c*tan(f*x+
e))^(1/2))/f/c^(1/2)-2*I*a*(a+I*a*tan(f*x+e))^(1/2)/f/(c-I*c*tan(f*x+e))^(1
/2)
```

Rubi [A]
time = 0.10, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3604, 49, 65, 223, 209}

$$\frac{2ia^{3/2}\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{c}\,f}-\frac{2ia\sqrt{a+ia\tan(e+fx)}}{f\sqrt{c-ic\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^(3/2)/Sqrt[c - I*c*Tan[e + f*x]],x]
```

```
[Out] ((2*I)*a^(3/2)*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c
 - I*c*Tan[e + f*x]])])/(Sqrt[c]*f) - ((2*I)*a*Sqrt[a + I*a*Tan[e + f*x]])/(
f*Sqrt[c - I*c*Tan[e + f*x]])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^{3/2}}{\sqrt{c-ic\tan(e+fx)}}\,dx=\frac{(ac)\text{Subst}\left(\int\frac{\sqrt{a+iax}}{(c-icx)^{3/2}}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=-\frac{2ia\sqrt{a+ia\tan(e+fx)}}{f\sqrt{c-ic\tan(e+fx)}}-\frac{a^2\text{Subst}\left(\int\frac{1}{\sqrt{a+iax}\sqrt{c-icx}}\,dx,x,\tan(e\right.}{f}$$

$$=-\frac{2ia\sqrt{a+ia\tan(e+fx)}}{f\sqrt{c-ic\tan(e+fx)}}+\frac{(2ia)\text{Subst}\left(\int\frac{1}{\sqrt{2c-\frac{cx^2}{a}}}\,dx,x,\sqrt{a+ia\tan\right.}{f}$$

$$=-\frac{2ia\sqrt{a+ia\tan(e+fx)}}{f\sqrt{c-ic\tan(e+fx)}}+\frac{(2ia)\text{Subst}\left(\int\frac{1}{1+\frac{cx^2}{a}}\,dx,x,\frac{\sqrt{a+ia\tan(e+f}}{\sqrt{c-ic\tan(e+f}}\right.}{f}$$

$$=\frac{2ia^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{c}f}-\frac{2ia\sqrt{a+ia\tan(e+fx)}}{f\sqrt{c-ic\tan(e+fx)}}$$

Mathematica [A]

time = 2.13, size = 123, normalized size = 1.16

$$-\frac{2ie^{-2i(e+fx)}\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\left(e^{i(e+fx)}-\operatorname{ArcTan}\left(e^{i(e+fx)}\right)\right)(a+ia\tan(e+fx))^{3/2}}{\sqrt{\frac{c}{1+e^{2i(e+fx)}}}\,f\sec^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(3/2)/Sqrt[c - I*c*Tan[e + f*x]],x]
```

```
[Out] ((-2*I)*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*(E^(I*(e + f*x)) -
ArcTan[E^(I*(e + f*x))])*(a + I*a*Tan[e + f*x])^(3/2))/(E^((2*I)*(e + f*x))
*Sqrt[c/(1 + E^((2*I)*(e + f*x)))]*f*Sec[e + f*x]^(3/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(84) = 168.
time = 0.39, size = 267, normalized size = 2.52

method	result
derivativedivides	$\frac{i\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,a\left(i\ln\left(\frac{ca\tan(fx+e)+\sqrt{ac\left(1+\tan^{2}\left(fx\right.\right.}}{\sqrt{ac}}\right.\right.}{}$
default	$\frac{i\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,a\left(i\ln\left(\frac{ca\tan(fx+e)+\sqrt{ac\left(1+\tan^{2}\left(fx\right.\right.}}{\sqrt{ac}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] I/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a/c*(I*ln((c*a*t
an(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))*a*c*tan(f*
x+e)^2-I*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)
^(1/2))*a*c-2*I*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)-2*ln((c
*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))*a*c*ta
n(f*x+e)+2*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c*(1+tan(f*x+e)^2))
^(1/2)/(tan(f*x+e)+I)^2/(a*c)^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(82) = 164.
time = 0.56, size = 343, normalized size = 3.24

$$-\frac{\left(-2i\,a\arctan\left(\cos\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right),\sin\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)+1\right)-2i\,a\arctan\left(\cos\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right),-\sin\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)+1\right)+4i\,a\cos\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)+a\log\left(\cos\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)^{2}+\sin\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)^{2}+2\sin\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)+1\right)-a\log\left(\cos\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)^{2}+\sin\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)^{2}-2\sin\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)+1\right)-4a\sin\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)\right)\sqrt{a}}{2\sqrt{c}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(1/2),x, algorithm="m
axima")
```

```
[Out] -1/2*(-2*I*a*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))),
sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) - 2*I*a*arctan2(c
os(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), -sin(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 4*I*a*cos(1/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))) + a*log(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin
(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) - a*log(cos(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e)))^2 - 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) + 1) - 4*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqr
t(a)/(sqrt(c)*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(82) = 164$.
time = 0.96, size = 337, normalized size = 3.18

$$-\frac{cf\sqrt{\frac{a^3}{cf^2}}\log\left(\frac{4\left(2\left(ae^{(3i\,fx+3i\,e)}+ae^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}-\left(i\,cfe^{(2i\,fx+2i\,e)}-i\,cf\right)\sqrt{\frac{a^3}{cf^2}}\right)}{ae^{(2i\,fx+2i\,e)}+a}\right)-cf\sqrt{\frac{a^3}{cf^2}}\log\left(\frac{4\left(2\left(ae^{(3i\,fx+3i\,e)}+ae^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}-\left(-i\,cfe^{(2i\,fx+2i\,e)}+i\,cf\right)\sqrt{\frac{a^3}{cf^2}}\right)}{ae^{(2i\,fx+2i\,e)}+a}\right)+4\left(i\,ae^{(3i\,fx+3i\,e)}+i\,ae^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{2\,cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(1/2),x, algorithm="f
ricas")
```

```
[Out] -1/2*(c*f*sqrt(a^3/(c*f^2))*log(4*(2*(a*e^(3*I*f*x + 3*I*e) + a*e^(I*f*x +
I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) -
 (I*c*f*e^(2*I*f*x + 2*I*e) - I*c*f)*sqrt(a^3/(c*f^2)))/(a*e^(2*I*f*x + 2*I
*e) + a)) - c*f*sqrt(a^3/(c*f^2))*log(4*(2*(a*e^(3*I*f*x + 3*I*e) + a*e^(I*
f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) +
 1)) - (-I*c*f*e^(2*I*f*x + 2*I*e) + I*c*f)*sqrt(a^3/(c*f^2)))/(a*e^(2*I*f*
x + 2*I*e) + a)) + 4*(I*a*e^(3*I*f*x + 3*I*e) + I*a*e^(I*f*x + I*e))*sqrt(a
/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(c*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan{(e+fx)}-i))^{\frac{3}{2}}}{\sqrt{-ic\,(\tan{(e+fx)}+i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(3/2)/(c-I*c*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral((I*a*(tan(e + f*x) - I))**(3/2)/sqrt(-I*c*(tan(e + f*x) + I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(1/2),x, algorithm="g
iac")
[Out] integrate((I*a*tan(f*x + e) + a)^(3/2)/sqrt(-I*c*tan(f*x + e) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\tan(e+f\,x)\ 1\mathrm{i})^{3/2}}{\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(3/2)/(c - c*tan(e + f*x)*1i)^(1/2),x)
[Out] int((a + a*tan(e + f*x)*1i)^(3/2)/(c - c*tan(e + f*x)*1i)^(1/2), x)
```

3.1019 $\int \frac{\sqrt{a+ia\tan(e+fx)}}{\sqrt{c-ic\tan(e+fx)}}\,dx$

Optimal. Leaf size=41

$$-\frac{i\sqrt{a+ia\tan(e+fx)}}{f\sqrt{c-ic\tan(e+fx)}}$$

```
[Out] -I*(a+I*a*tan(f*x+e))^(1/2)/f/(c-I*c*tan(f*x+e))^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {3604, 37}

$$-\frac{i\sqrt{a+ia\tan(e+fx)}}{f\sqrt{c-ic\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[e + f*x]]/Sqrt[c - I*c*Tan[e + f*x]],x]
```

```
[Out] ((-I)*Sqrt[a + I*a*Tan[e + f*x]])/(f*Sqrt[c - I*c*Tan[e + f*x]])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+ia\tan(e+fx)}}{\sqrt{c-ic\tan(e+fx)}}\,dx &= \frac{(ac)\text{Subst}\left(\int \frac{1}{\sqrt{a+iax}\,(c-icx)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}\\ &= -\frac{i\sqrt{a+ia\tan(e+fx)}}{f\sqrt{c-ic\tan(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 0.79, size = 64, normalized size = 1.56

$$\frac{\cos(e+fx)(-i\cos(e+fx)+\sin(e+fx))\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{cf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Tan[e + f*x]]/Sqrt[c - I*c*Tan[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*((-I)*Cos[e + f*x] + Sin[e + f*x])*Sqrt[a + I*a*Tan[e + f*x]]
*Sqrt[c - I*c*Tan[e + f*x]])/(c*f)
```

Maple [A]
time = 0.37, size = 63, normalized size = 1.54

method	result	size
risch	$-\frac{i\sqrt{\frac{a\,\mathrm{e}^{2i(fx+e)}}{\mathrm{e}^{2i(fx+e)}+1}}}{\sqrt{\frac{c}{\mathrm{e}^{2i(fx+e)}+1}}\,f}$	50
derivativedivides	$-\frac{i\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\left(i\tan(fx+e)-1\right)}{fc(\tan(fx+e)+i)^{2}}$	63
default	$-\frac{i\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\left(i\tan(fx+e)-1\right)}{fc(\tan(fx+e)+i)^{2}}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] -I/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)/c*(I*tan(f*x+e)
-1)/(tan(f*x+e)+I)^2
```

Maxima [A]
time = 0.55, size = 27, normalized size = 0.66

$$\frac{\sqrt{a}\left(-i\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)}{\sqrt{c}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(1/2),x, algorithm="m
axima")
```

```
[Out] sqrt(a)*(-I*cos(f*x + e) + sin(f*x + e))/(sqrt(c)*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(33) = 66$.
time = 1.10, size = 68, normalized size = 1.66

$$\frac{\sqrt{\frac{a}{e^{(2i fx+2i e)}+1}}\sqrt{\frac{c}{e^{(2i fx+2i e)}+1}}\left(-i e^{(3i fx+3i e)}-i e^{(i fx+i e)}\right)}{cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(1/2),x, algorithm="f
ricas")
```

```
[Out] sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*(-I*e^(
3*I*f*x + 3*I*e) - I*e^(I*f*x + I*e))/(c*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ia\left(\tan\left(e+fx\right)-i\right)}}{\sqrt{-ic\left(\tan\left(e+fx\right)+i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(1/2)/(c-I*c*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(e + f*x) - I))/sqrt(-I*c*(tan(e + f*x) + I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(I*a*tan(f*x + e) + a)/sqrt(-I*c*tan(f*x + e) + c), x)
```

Mupad [B]
time = 5.38, size = 49, normalized size = 1.20

$$\frac{\sqrt{a\,\left(1+\tan\left(e+f\,x\right)\,1{}\mathrm{i}\right)}\,\sqrt{-c\,\left(-1+\tan\left(e+f\,x\right)\,1{}\mathrm{i}\right)}}{c\,f\,\left(\tan\left(e+f\,x\right)+1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(1/2)/(c - c*tan(e + f*x)*1i)^(1/2),x)
```

```
[Out] ((a*(tan(e + f*x)*1i + 1))^(1/2)*(-c*(tan(e + f*x)*1i - 1))^(1/2))/(c*f*(ta
n(e + f*x) + 1i))
```

3.1020 $\int \frac{1}{\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}\,dx$

Optimal. Leaf size=44

$$\frac{\tan(e+fx)}{f\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}$$

```
[Out] tan(f*x+e)/f/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {3604, 39}

$$\frac{\tan(e+fx)}{f\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]]),x]
```

```
[Out] Tan[e + f*x]/(f*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}\,dx &= \frac{(ac)\text{Subst}\left(\int \frac{1}{(a+iax)^{3/2}(c-icx)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{\tan(e+fx)}{f\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 0.78, size = 64, normalized size = 1.45

$$\frac{(\cos(e+fx)+i\sin(e+fx))\sin(e+fx)\sqrt{c-ic\tan(e+fx)}}{cf\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]]),x]
```

```
[Out] ((Cos[e + f*x] + I*Sin[e + f*x])*Sin[e + f*x]*Sqrt[c - I*c*Tan[e + f*x]])/(
c*f*Sqrt[a + I*a*Tan[e + f*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(38) = 76.
time = 0.35, size = 82, normalized size = 1.86

method	result	size
risch	$-\frac{i\left(e^{2i(fx+e)}-1\right)}{2\sqrt{\frac{a\,e^{2i(fx+e)}}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{c}{e^{2i(fx+e)}+1}}\,f}$	74
derivativedivides	$\frac{\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\left(1+\tan^{2}(fx+e)\right)\tan(fx+e)}{fac(\tan(fx+e)+i)^{2}(-\tan(fx+e)+i)^{2}}$	82
default	$\frac{\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\left(1+\tan^{2}(fx+e)\right)\tan(fx+e)}{fac(\tan(fx+e)+i)^{2}(-\tan(fx+e)+i)^{2}}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)/a/c*(1+tan(f*x+e
)^2)*tan(f*x+e)/(tan(f*x+e)+I)^2/(-tan(f*x+e)+I)^2
```

Maxima [A]
time = 0.57, size = 17, normalized size = 0.39

$$\frac{\sin\left(fx+e\right)}{\sqrt{a}\,\sqrt{c}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(1/2),x, algorithm=
"maxima")
```

```
[Out] sin(f*x + e)/(sqrt(a)*sqrt(c)*f)
```

Fricas [A]
time = 1.34, size = 71, normalized size = 1.61

$$\frac{\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\,\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\,\left(-i\,e^{(4i\,fx+4i\,e)}+i\right)e^{(-i\,fx-i\,e)}}{2\,acf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(1/2),x, algorithm=
"fricas")
```

```
[Out] 1/2*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*(-I
*e^(4*I*f*x + 4*I*e) + I)*e^(-I*f*x - I*e)/(a*c*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{ia\,(\tan{(e+fx)}-i)}\,\sqrt{-ic\,(\tan{(e+fx)}+i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-I*c*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(I*a*(tan(e + f*x) - I))*sqrt(-I*c*(tan(e + f*x) + I))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate(1/(sqrt(I*a*tan(f*x + e) + a)*sqrt(-I*c*tan(f*x + e) + c)), x)
```

Mupad [B]
time = 5.48, size = 112, normalized size = 2.55

$$\frac{\left(\cos\left(2\,e+2\,f\,x\right)\,1\mathrm{i}+\sin\left(2\,e+2\,f\,x\right)-\mathrm{i}\right)\,\sqrt{\frac{a\,\left(\cos\left(2\,e+2\,f\,x\right)+1+\sin\left(2\,e+2\,f\,x\right)\,1\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}}{2\,a\,f\,\sqrt{\frac{c\,\left(\cos\left(2\,e+2\,f\,x\right)+1-\sin\left(2\,e+2\,f\,x\right)\,1\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2)),x)
```

```
[Out] ((cos(2*e + 2*f*x)*1i + sin(2*e + 2*f*x) - 1i)*((a*(cos(2*e + 2*f*x) + sin(
2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2))/(2*a*f*((c*(cos(2*e +
2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

3.1021 $\int \frac{1}{(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}\,dx$

Optimal. Leaf size=94

$$\frac{i}{3f(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}+\frac{2\tan(e+fx)}{3af\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}$$

```
[Out] 2/3*tan(f*x+e)/a/f/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(1/2)+1/3*I/
f/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {3604, 47, 39}

$$\frac{2\tan(e+fx)}{3af\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}+\frac{i}{3f(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^(3/2)*Sqrt[c - I*c*Tan[e + f*x]]),x]
```

```
[Out] (I/3)/(f*(a + I*a*Tan[e + f*x])^(3/2)*Sqrt[c - I*c*Tan[e + f*x]]) + (2*Tan[
e + f*x])/(3*a*f*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
```

```
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}\,dx = \frac{(ac)\mathrm{Subst}\left(\int \frac{1}{(a+iax)^{5/2}(c-icx)^{3/2}}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{i}{3f(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}} + \frac{(2\cdots}{\cdots}$$

$$= \frac{i}{3f(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}} + \frac{\cdots}{3a\cdots}$$

Mathematica [A]
time = 1.07, size = 73, normalized size = 0.78

$$\frac{(3i - i\cos(2(e+fx)) + 2\sin(2(e+fx)))\sqrt{c-ic\tan(e+fx)}}{6acf\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^(3/2)*Sqrt[c - I*c*Tan[e + f*x]]),x]
[Out] ((3*I - I*Cos[2*(e + f*x)] + 2*Sin[2*(e + f*x)])*Sqrt[c - I*c*Tan[e + f*x]]
)/(6*a*c*f*Sqrt[a + I*a*Tan[e + f*x]])
```

Maple [A]
time = 0.38, size = 109, normalized size = 1.16

method	result
derivativedivides	$\frac{\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\left(2i\left(\tan^3(fx+e)\right)-2\left(\tan^4(fx+e)\right)+2i\tan(fx+e\cdots\right.}{3f\,a^2c(\tan(fx+e)+i)^2(-\tan(fx+e)+i)^3}$
default	$\frac{\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\left(2i\left(\tan^3(fx+e)\right)-2\left(\tan^4(fx+e)\right)+2i\tan(fx+e\cdots\right.}{3f\,a^2c(\tan(fx+e)+i)^2(-\tan(fx+e)+i)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(3/2),x,method=_RETURNVER
BOSE)
[Out] 1/3/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)/a^2/c*(2*I*tan
(f*x+e)^3-2*tan(f*x+e)^4+2*I*tan(f*x+e)-3*tan(f*x+e)^2-1)/(tan(f*x+e)+I)^2/
(-tan(f*x+e)+I)^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(3/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 1.12, size = 119, normalized size = 1.27

$$\frac{\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\left(-3i\,e^{(6i\,fx+6i\,e)}-4i\,e^{(5i\,fx+5i\,e)}+3i\,e^{(4i\,fx+4i\,e)}-4i\,e^{(3i\,fx+3i\,e)}+7i\,e^{(2i\,fx+2i\,e)}+i\right)e^{(-3i\,fx-3i\,e)}}{12\,a^2cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(3/2),x, algorithm=
"fricas")
```

```
[Out] 1/12*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*(-
3*I*e^(6*I*f*x + 6*I*e) - 4*I*e^(5*I*f*x + 5*I*e) + 3*I*e^(4*I*f*x + 4*I*e)
 - 4*I*e^(3*I*f*x + 3*I*e) + 7*I*e^(2*I*f*x + 2*I*e) + I)*e^(-3*I*f*x - 3*I
*e)/(a^2*c*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(\tan\left(e+fx\right)-i\right)\right)^{\frac{3}{2}}\sqrt{-ic\left(\tan\left(e+fx\right)+i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-I*c*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/((I*a*(tan(e + f*x) - I))**(3/2)*sqrt(-I*c*(tan(e + f*x) + I))),
 x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate(1/((I*a*tan(f*x + e) + a)^(3/2)*sqrt(-I*c*tan(f*x + e) + c)), x)
```

Mupad [B]
time = 5.63, size = 135, normalized size = 1.44

$$\frac{\sqrt{\frac{a\,\left(\cos\left(2\,e+2\,f\,x\right)+1+\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\left(\cos\left(2\,e+2\,f\,x\right)\,6{}\mathrm{i}+\cos\left(4\,e+4\,f\,x\right)\,1{}\mathrm{i}+6\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(4\,e+4\,f\,x\right)-3{}\mathrm{i}\right)}{12\,a^2\,f\,\sqrt{\frac{c\,\left(\cos\left(2\,e+2\,f\,x\right)+1-\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^(3/2)*(c - c*tan(e + f*x)*1i)^(1/2)),x)
```

```
[Out] (((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^
(1/2)*(cos(2*e + 2*f*x)*6i + cos(4*e + 4*f*x)*1i + 6*sin(2*e + 2*f*x) + sin
(4*e + 4*f*x) - 3i))/(12*a^2*f*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i
+ 1))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

3.1022 $\int \frac{1}{(a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}}\,dx$

Optimal. Leaf size=140

$$\frac{i}{5f(a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}}+\frac{i}{5af(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}+\frac{}{5a^2f\sqrt{a}}$$

```
[Out] 2/5*tan(f*x+e)/a^2/f/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(1/2)+1/5*
I/f/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(5/2)+1/5*I/a/f/(c-I*c*tan(
f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(3/2)
```

Rubi [A]

time = 0.09, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {3604, 47, 39}

$$\frac{2\tan(e+fx)}{5a^2f\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}+\frac{i}{5af(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}+\frac{i}{5f(a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^(5/2)*Sqrt[c - I*c*Tan[e + f*x]]),x]
```

```
[Out] (I/5)/(f*(a + I*a*Tan[e + f*x])^(5/2)*Sqrt[c - I*c*Tan[e + f*x]]) + (I/5)/(
a*f*(a + I*a*Tan[e + f*x])^(3/2)*Sqrt[c - I*c*Tan[e + f*x]]) + (2*Tan[e + f
*x])/(5*a^2*f*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
```

```
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{1}{(a+iax)^{7/2}(c-icx)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{i}{5f(a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}} + \frac{(3}{}$$

$$= \frac{i}{5f(a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}} + \frac{}{5a}$$

$$= \frac{i}{5f(a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}} + \frac{}{5a}$$

Mathematica [A]
time = 1.66, size = 99, normalized size = 0.71

$$\frac{(12-4\cos(2(e+fx))-3i\sec(e+fx)\sin(3(e+fx))+5i\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{20a^2cf(-i+\tan(e+fx))\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^(5/2)*Sqrt[c - I*c*Tan[e + f*x]]),x]
[Out] ((12 - 4*Cos[2*(e + f*x)] - (3*I)*Sec[e + f*x]*Sin[3*(e + f*x)] + (5*I)*Tan
[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(20*a^2*c*f*(-I + Tan[e + f*x])*Sqrt
[a + I*a*Tan[e + f*x]])
```

Maple [A]
time = 0.40, size = 118, normalized size = 0.84

method	result
derivativedivides	$-\frac{\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\left(4i\left(\tan^4(fx+e)\right)-2\left(\tan^5(fx+e)\right)+6i\left(\tan^2(f\right.\right.}{5f\,a^3c(-\tan(fx+e)+i)^4(\tan(fx+e)+i)^2}$
default	$-\frac{\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\left(4i\left(\tan^4(fx+e)\right)-2\left(\tan^5(fx+e)\right)+6i\left(\tan^2(f\right.\right.}{5f\,a^3c(-\tan(fx+e)+i)^4(\tan(fx+e)+i)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(5/2),x,method=_RETURNVER
BOSE)
```

```
[Out] -1/5/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)/a^3/c*(4*I*ta
n(f*x+e)^4-2*tan(f*x+e)^5+6*I*tan(f*x+e)^2-tan(f*x+e)^3+2*I+tan(f*x+e))/(-t
an(f*x+e)+I)^4/(tan(f*x+e)+I)^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(5/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 0.95, size = 131, normalized size = 0.94

$$\frac{\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\,\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\,\left(-5i\,e^{(8i\,fx+8i\,e)}-16i\,e^{(7i\,fx+7i\,e)}+10i\,e^{(6i\,fx+6i\,e)}-16i\,e^{(5i\,fx+5i\,e)}+20i\,e^{(4i\,fx+4i\,e)}+6i\,e^{(2i\,fx+2i\,e)}+i\right)e^{(-5i\,fx-5i\,e)}}{40\,a^3cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(5/2),x, algorithm=
"fricas")
```

```
[Out] 1/40*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*(-
5*I*e^(8*I*f*x + 8*I*e) - 16*I*e^(7*I*f*x + 7*I*e) + 10*I*e^(6*I*f*x + 6*I*
e) - 16*I*e^(5*I*f*x + 5*I*e) + 20*I*e^(4*I*f*x + 4*I*e) + 6*I*e^(2*I*f*x +
 2*I*e) + I)*e^(-5*I*f*x - 5*I*e)/(a^3*c*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(\tan\left(e+fx\right)-i\right)\right)^{\frac{5}{2}}\sqrt{-ic\left(\tan\left(e+fx\right)+i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-I*c*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral(1/((I*a*(tan(e + f*x) - I))**(5/2)*sqrt(-I*c*(tan(e + f*x) + I))),
 x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(5/2),x, algorithm=
"giac")
```

```
[Out] integrate(1/((I*a*tan(f*x + e) + a)^(5/2)*sqrt(-I*c*tan(f*x + e) + c)), x)
```

Mupad [B]
time = 5.96, size = 158, normalized size = 1.13

$$\frac{\sqrt{\frac{a\,\left(\cos\left(2\,e+2\,f\,x\right)+1+\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\left(\cos\left(2\,e+2\,f\,x\right)\,15{}\mathrm{i}+\cos\left(4\,e+4\,f\,x\right)\,5{}\mathrm{i}+\cos\left(6\,e+6\,f\,x\right)\,1{}\mathrm{i}+15\,\sin\left(2\,e+2\,f\,x\right)+5\,\sin\left(4\,e+4\,f\,x\right)+\sin\left(6\,e+6\,f\,x\right)-5{}\mathrm{i}\right)}{40\,a^3\,f\,\sqrt{\frac{c\,\left(\cos\left(2\,e+2\,f\,x\right)+1-\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^(5/2)*(c - c*tan(e + f*x)*1i)^(1/2)),x)
```

```
[Out] (((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^
(1/2)*(cos(2*e + 2*f*x)*15i + cos(4*e + 4*f*x)*5i + cos(6*e + 6*f*x)*1i + 1
5*sin(2*e + 2*f*x) + 5*sin(4*e + 4*f*x) + sin(6*e + 6*f*x) - 5i))/(40*a^3*f
*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^
(1/2))
```

3.1023 $\int \frac{1}{(a+ia\tan(e+fx))^{7/2}\sqrt{c-ic\tan(e+fx)}}\,dx$

Optimal. Leaf size=186

$$\frac{i}{7f(a+ia\tan(e+fx))^{7/2}\sqrt{c-ic\tan(e+fx)}}+\frac{4i}{35af(a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}}+\frac{}{35a^2f(}$$

```
[Out] 8/35*tan(f*x+e)/a^3/f/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(1/2)+1/7
*I/f/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(7/2)+4/35*I/a/f/(c-I*c*ta
n(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(5/2)+4/35*I/a^2/f/(c-I*c*tan(f*x+e))^(1
/2)/(a+I*a*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.11, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {3604, 47, 39}

$$\frac{8\tan(e+fx)}{35a^3f\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}+\frac{4i}{35a^2f(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}+\frac{4i}{35af(a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}}+\frac{i}{7f(a+ia\tan(e+fx))^{7/2}\sqrt{c-ic\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^(7/2)*Sqrt[c - I*c*Tan[e + f*x]]),x]
```

```
[Out] (I/7)/(f*(a + I*a*Tan[e + f*x])^(7/2)*Sqrt[c - I*c*Tan[e + f*x]]) + ((4*I)/
35)/(a*f*(a + I*a*Tan[e + f*x])^(5/2)*Sqrt[c - I*c*Tan[e + f*x]]) + ((4*I)/
35)/(a^2*f*(a + I*a*Tan[e + f*x])^(3/2)*Sqrt[c - I*c*Tan[e + f*x]]) + (8*Ta
n[e + f*x])/(35*a^3*f*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]]
)
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^{7/2}\sqrt{c-ic\tan(e+fx)}}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{1}{(a+iax)^{9/2}(c-icx)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{i}{7f(a+ia\tan(e+fx))^{7/2}\sqrt{c-ic\tan(e+fx)}} + \frac{(4}{}$$

$$= \frac{i}{7f(a+ia\tan(e+fx))^{7/2}\sqrt{c-ic\tan(e+fx)}} + \frac{}{35}$$

$$= \frac{i}{7f(a+ia\tan(e+fx))^{7/2}\sqrt{c-ic\tan(e+fx)}} + \frac{}{35}$$

$$= \frac{i}{7f(a+ia\tan(e+fx))^{7/2}\sqrt{c-ic\tan(e+fx)}} + \frac{}{35}$$

Mathematica [A]
time = 2.23, size = 115, normalized size = 0.62

$$\frac{\sec^2(e+fx)(-35i-84i\cos(2(e+fx))+15i\cos(4(e+fx))+56\sin(2(e+fx))-20\sin(4(e+fx)))\sqrt{c-ic\tan(e+fx)}}{280a^3cf(-i+\tan(e+fx))^2\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^(7/2)*Sqrt[c - I*c*Tan[e + f*x]]),x]
```

```
[Out] (Sec[e + f*x]^2*(-35*I - (84*I)*Cos[2*(e + f*x)] + (15*I)*Cos[4*(e + f*x)]
+ 56*Sin[2*(e + f*x)] - 20*Sin[4*(e + f*x)])*Sqrt[c - I*c*Tan[e + f*x]])/(2
80*a^3*c*f*(-I + Tan[e + f*x])^2*Sqrt[a + I*a*Tan[e + f*x]])
```

Maple [A]
time = 0.42, size = 130, normalized size = 0.70

method	result
derivativedivides	$\frac{\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,(24i(\tan^5(fx+e))-8(\tan^6(fx+e))+28i(\tan^3(f}{35f\,a^4c(-\tan(fx+e)+i)^5(\tan(fx+e)+i)^2}$

default	$\frac{\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\left(24i\left(\tan^{5}\left(fx+e\right)\right)-8\left(\tan^{6}\left(fx+e\right)\right)+28i\left(\tan^{3}\left(fx\right.\right.\right.}{35f\,a^{4}c\left(-\tan\left(fx+e\right)+i\right)^{5}\left(\tan\left(fx+e\right)+i\right)^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(7/2),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/35/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)/a^4/c*(24*I*t
an(f*x+e)^5-8*tan(f*x+e)^6+28*I*tan(f*x+e)^3+12*tan(f*x+e)^4+4*I*tan(f*x+e)
+33*tan(f*x+e)^2+13)/(-tan(f*x+e)+I)^5/(tan(f*x+e)+I)^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(7/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 0.79, size = 143, normalized size = 0.77

$$\frac{\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}\,\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}\,\left(-35i\,e^{(10ifx+10ie)}-208i\,e^{(9ifx+9ie)}+105i\,e^{(8ifx+8ie)}-208i\,e^{(7ifx+7ie)}+210i\,e^{(6ifx+6ie)}+98i\,e^{(4ifx+4ie)}+33i\,e^{(2ifx+2ie)}+5i\right)e^{(-7ifx-7ie)}}{560\,a^{4}cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(7/2),x, algorithm=
"fricas")
```

```
[Out] 1/560*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*(
-35*I*e^(10*I*f*x + 10*I*e) - 208*I*e^(9*I*f*x + 9*I*e) + 105*I*e^(8*I*f*x
+ 8*I*e) - 208*I*e^(7*I*f*x + 7*I*e) + 210*I*e^(6*I*f*x + 6*I*e) + 98*I*e^(
4*I*f*x + 4*I*e) + 33*I*e^(2*I*f*x + 2*I*e) + 5*I)*e^(-7*I*f*x - 7*I*e)/(a^
4*c*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(ia\left(\tan\left(e+fx\right)-i\right)\right)^{\frac{7}{2}}\sqrt{-ic\left(\tan\left(e+fx\right)+i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-I*c*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e))**(7/2),x)
```

```
[Out] Integral(1/((I*a*(tan(e + f*x) - I))**(7/2)*sqrt(-I*c*(tan(e + f*x) + I))),
 x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(7/2),x, algorithm=
"giac")
```

```
[Out] integrate(1/((I*a*tan(f*x + e) + a)^(7/2)*sqrt(-I*c*tan(f*x + e) + c)), x)
```

Mupad [B]
time = 6.42, size = 183, normalized size = 0.98

$$\frac{\sqrt{\frac{a\,\left(\cos\left(2\,e+2\,f\,x\right)+1+\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\left(\cos\left(2\,e+2\,f\,x\right)\,140{}\mathrm{i}+\cos\left(4\,e+4\,f\,x\right)\,70{}\mathrm{i}+\cos\left(6\,e+6\,f\,x\right)\,28{}\mathrm{i}+\cos\left(8\,e+8\,f\,x\right)\,5{}\mathrm{i}+140\,\sin\left(2\,e+2\,f\,x\right)+70\,\sin\left(4\,e+4\,f\,x\right)+28\,\sin\left(6\,e+6\,f\,x\right)+5\,\sin\left(8\,e+8\,f\,x\right)-35{}\mathrm{i}\right)}{560\,a^4\,f\,\sqrt{\frac{c\,\left(\cos\left(2\,e+2\,f\,x\right)+1-\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^(7/2)*(c - c*tan(e + f*x)*1i)^(1/2)),x)
```

```
[Out] (((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^
(1/2)*(cos(2*e + 2*f*x)*140i + cos(4*e + 4*f*x)*70i + cos(6*e + 6*f*x)*28i
+ cos(8*e + 8*f*x)*5i + 140*sin(2*e + 2*f*x) + 70*sin(4*e + 4*f*x) + 28*sin
(6*e + 6*f*x) + 5*sin(8*e + 8*f*x) - 35i))/(560*a^4*f*((c*(cos(2*e + 2*f*x)
 - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

3.1024 $\int \frac{(a+ia\tan(e+fx))^{9/2}}{(c-ic\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=255

$$-\frac{35ia^{9/2}\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{c^{3/2}f}-\frac{2ia(a+ia\tan(e+fx))^{7/2}}{3f(c-ic\tan(e+fx))^{3/2}}+\frac{14ia^2(a+ia\tan(e+fx))^{5/2}}{3cf\sqrt{c-ic\tan(e+fx)}}+$$

```
[Out] -35*I*a^(9/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*c*tan(f*
x+e))^(1/2))/c^(3/2)/f+35/2*I*a^4*(a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e
))^(1/2)/c^2/f+35/6*I*a^3*(c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^(3/2)
/c^2/f+14/3*I*a^2*(a+I*a*tan(f*x+e))^(5/2)/c/f/(c-I*c*tan(f*x+e))^(1/2)-2/3
*I*a*(a+I*a*tan(f*x+e))^(7/2)/f/(c-I*c*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.15, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3604, 49, 52, 65, 223, 209}

$$-\frac{35ia^{9/2}\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{c^{3/2}f}+\frac{35ia^4\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{2c^2f}+\frac{35ia^3(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}{6c^2f}+\frac{14ia^2(a+ia\tan(e+fx))^{5/2}}{3cf\sqrt{c-ic\tan(e+fx)}}-\frac{2ia(a+ia\tan(e+fx))^{7/2}}{3f(c-ic\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^(9/2)/(c - I*c*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-35*I)*a^(9/2)*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[
c - I*c*Tan[e + f*x]])])/(c^(3/2)*f) - (((2*I)/3)*a*(a + I*a*Tan[e + f*x])^
(7/2))/(f*(c - I*c*Tan[e + f*x])^(3/2)) + (((14*I)/3)*a^2*(a + I*a*Tan[e +
f*x])^(5/2))/(c*f*Sqrt[c - I*c*Tan[e + f*x]]) + (((35*I)/2)*a^4*Sqrt[a + I*
a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/(c^2*f) + (((35*I)/6)*a^3*(a +
I*a*Tan[e + f*x])^(3/2)*Sqrt[c - I*c*Tan[e + f*x]])/(c^2*f)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^{9/2}}{(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{(a+iax)^{7/2}}{(c-icx)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{2ia(a+ia\tan(e+fx))^{7/2}}{3f(c-ic\tan(e+fx))^{3/2}} - \frac{(7a^2)\,\text{Subst}\left(\int \frac{(a+iax)^{5/2}}{(c-icx)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3f}$$

$$= -\frac{2ia(a+ia\tan(e+fx))^{7/2}}{3f(c-ic\tan(e+fx))^{3/2}} + \frac{14ia^2(a+ia\tan(e+fx))^{5/2}}{3cf\sqrt{c-ic\tan(e+fx)}} + \frac{(35a^3)\,\text{Subst}}{}$$

$$= -\frac{2ia(a+ia\tan(e+fx))^{7/2}}{3f(c-ic\tan(e+fx))^{3/2}} + \frac{14ia^2(a+ia\tan(e+fx))^{5/2}}{3cf\sqrt{c-ic\tan(e+fx)}} + \frac{35ia^3(a+ia}{}$$

$$= -\frac{2ia(a+ia\tan(e+fx))^{7/2}}{3f(c-ic\tan(e+fx))^{3/2}} + \frac{14ia^2(a+ia\tan(e+fx))^{5/2}}{3cf\sqrt{c-ic\tan(e+fx)}} + \frac{35ia^4\sqrt{a+i}}{}$$

$$= -\frac{2ia(a+ia\tan(e+fx))^{7/2}}{3f(c-ic\tan(e+fx))^{3/2}} + \frac{14ia^2(a+ia\tan(e+fx))^{5/2}}{3cf\sqrt{c-ic\tan(e+fx)}} + \frac{35ia^4\sqrt{a+i}}{}$$

$$= -\frac{2ia(a+ia\tan(e+fx))^{7/2}}{3f(c-ic\tan(e+fx))^{3/2}} + \frac{14ia^2(a+ia\tan(e+fx))^{5/2}}{3cf\sqrt{c-ic\tan(e+fx)}} + \frac{35ia^4\sqrt{a+i}}{}$$

$$= -\frac{35ia^{9/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{c^{3/2}f} - \frac{2ia(a+ia\tan(e+fx))^{7/2}}{3f(c-ic\tan(e+fx))^{3/2}}$$

Mathematica [A]
time = 9.79, size = 386, normalized size = 1.51

$$-\frac{35ie^{-i(5e+fx)}\sqrt{e^{ifx}}\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\,\text{ArcTan}\left(e^{i(e+fx)}\right)(a+ia\tan(e+fx))^{9/2}}{c\sqrt{\frac{c}{1+e^{2i(e+fx)}}}\,f\sec^{\frac{9}{2}}(e+fx)(\cos(fx)+i\sin(fx))^{9/2}} + \frac{\cos^4(e+fx)\left(-\frac{4i\cos(4fx)}{3c^2}+\cos(2fx)\left(\frac{32i\cos(2e)}{3c^2}+\frac{32\sin(2e)}{3c^2}\right)+\sec(e)(36\cos(e)+i\sin(e))\left(\frac{i\cos(4e)}{2c^2}+\frac{\sin(4e)}{2c^2}\right)-\sec(e)\sec(e+fx)\left(\frac{\cos(4e)}{2c^2}-\frac{i\sin(4e)}{2c^2}\right)\sin(fx)+\left(-\frac{32\cos(2e)}{3c^2}+\frac{32i\sin(2e)}{3c^2}\right)\sin(2fx)+\frac{4\sin(4fx)}{3c^2}\right)\sqrt{\sec(e+fx)(c\cos(e+fx)-ic\sin(e+fx))}\,(a+ia\tan(e+fx))^{9/2}}{f(\cos(fx)+i\sin(fx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(9/2)/(c - I*c*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-35*I)*Sqrt[E^(I*f*x)]*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*Ar
cTan[E^(I*(e + f*x))]*(a + I*a*Tan[e + f*x])^(9/2))/(c*E^(I*(5*e + f*x))*Sq
rt[c/(1 + E^((2*I)*(e + f*x)))]*f*Sec[e + f*x]^(9/2)*(Cos[f*x] + I*Sin[f*x]
)^(9/2)) + (Cos[e + f*x]^4*((((-4*I)/3)*Cos[4*f*x])/c^2 + Cos[2*f*x]*((((32
```

```
*I)/3)*Cos[2*e])/c^2 + (32*Sin[2*e])/(3*c^2)) + Sec[e]*(36*Cos[e] + I*Sin[e
])*(((I/2)*Cos[4*e])/c^2 + Sin[4*e]/(2*c^2)) - Sec[e]*Sec[e + f*x]*(Cos[4*e
]/(2*c^2) - ((I/2)*Sin[4*e])/c^2)*Sin[f*x] + ((-32*Cos[2*e])/(3*c^2) + (((3
2*I)/3)*Sin[2*e])/c^2)*Sin[2*f*x] + (4*Sin[4*f*x])/(3*c^2))*Sqrt[Sec[e + f*
x]*(c*Cos[e + f*x] - I*c*Sin[e + f*x])]*(a + I*a*Tan[e + f*x])^(9/2))/(f*(C
os[f*x] + I*Sin[f*x])^4)
```

Maple [A]
time = 0.39, size = 409, normalized size = 1.60

method	result
derivativedivides	$\frac{\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,a^{4}\left(315i\ln\left(\frac{ca\tan(fx+e)+\sqrt{ac\left(1+\tan^{2}(\right.}}{\sqrt{ac}}\right.\right.}{}$
default	$\frac{\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,a^{4}\left(315i\ln\left(\frac{ca\tan(fx+e)+\sqrt{ac\left(1+\tan^{2}(\right.}}{\sqrt{ac}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(9/2)/(c-I*c*tan(f*x+e))^(3/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/6/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^4/c^2*(315*I
*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))*
a*c*tan(f*x+e)^2+105*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^
(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^3+27*I*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c
)^(1/2)*tan(f*x+e)^3-3*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^
4-105*I*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^
(1/2))*a*c-315*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))
/(a*c)^(1/2))*a*c*tan(f*x+e)-393*I*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)
*tan(f*x+e)-259*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^2+164*(
a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(a*c)
^(1/2)/(tan(f*x+e)+I)^3
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 979 vs. 2(199) = 398.
time = 0.70, size = 979, normalized size = 3.84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(9/2)/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="m
axima")
```

[Out] -6*(210*(a^4*cos(4*f*x + 4*e) + 2*a^4*cos(2*f*x + 2*e) + I*a^4*sin(4*f*x + 4*e) + 2*I*a^4*sin(2*f*x + 2*e) + a^4)*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 210*(a^4*cos(4*f*x + 4*e) + 2*a^4*cos(2*f*x + 2*e) + I*a^4*sin(4*f*x + 4*e) + 2*I*a^4*sin(2*f*x + 2*e) + a^4)*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), -sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 4*(8*a^4*cos(4*f*x + 4*e) + 16*a^4*cos(2*f*x + 2*e) + 8*I*a^4*sin(4*f*x + 4*e) + 16*I*a^4*sin(2*f*x + 2*e) - 31*a^4)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 12*(24*a^4*cos(4*f*x + 4*e) + 48*a^4*cos(2*f*x + 2*e) + 24*I*a^4*sin(4*f*x + 4*e) + 48*I*a^4*sin(2*f*x + 2*e) + 35*a^4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 105*(I*a^4*cos(4*f*x + 4*e) + 2*I*a^4*cos(2*f*x + 2*e) - a^4*sin(4*f*x + 4*e) - 2*a^4*sin(2*f*x + 2*e) + I*a^4)*log(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 105*(-I*a^4*cos(4*f*x + 4*e) - 2*I*a^4*cos(2*f*x + 2*e) + a^4*sin(4*f*x + 4*e) + 2*a^4*sin(2*f*x + 2*e) - I*a^4)*log(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 4*(8*I*a^4*cos(4*f*x + 4*e) + 16*I*a^4*cos(2*f*x + 2*e) - 8*a^4*sin(4*f*x + 4*e) - 16*a^4*sin(2*f*x + 2*e) - 31*I*a^4)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 12*(-24*I*a^4*cos(4*f*x + 4*e) - 48*I*a^4*cos(2*f*x + 2*e) + 24*a^4*sin(4*f*x + 4*e) + 48*a^4*sin(2*f*x + 2*e) - 35*I*a^4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((-72*I*c^2*cos(4*f*x + 4*e) - 144*I*c^2*cos(2*f*x + 2*e) + 72*c^2*sin(4*f*x + 4*e) + 144*c^2*sin(2*f*x + 2*e) - 72*I*c^2)*f)

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(199) = 398$.
time = 0.99, size = 451, normalized size = 1.77

$$\frac{105 \sqrt{\frac{a^9}{c^3 f^2}} \left(c^2 f e^{(2i f x+2i e)} + c^2 f\right) \log\left(\frac{4\left(2\left(a^4 e^{(3i f x+3i e)}+a^4 e^{(i f x+i e)}\right)\sqrt{\frac{a}{e^{(2i f x+2i e)}+1}}\sqrt{\frac{c}{e^{(2i f x+2i e)}+1}}-\sqrt{\frac{a^9}{c^3 f^2}}\left(i c^2 f e^{(2i f x+2i e)}-i c^2 f\right)\right)}{a^4 e^{(2i f x+2i e)}+a^4}\right) - 105 \sqrt{\frac{a^9}{c^3 f^2}} \left(c^2 f e^{(2i f x+2i e)} + c^2 f\right) \log\left(\frac{4\left(2\left(a^4 e^{(3i f x+3i e)}+a^4 e^{(i f x+i e)}\right)\sqrt{\frac{a}{e^{(2i f x+2i e)}+1}}\sqrt{\frac{c}{e^{(2i f x+2i e)}+1}}-\sqrt{\frac{a^9}{c^3 f^2}}\left(-i c^2 f e^{(2i f x+2i e)}+i c^2 f\right)\right)}{a^4 e^{(2i f x+2i e)}+a^4}\right) - 4\left(8i a^4 e^{(7i f x+7i e)} - 56i a^4 e^{(5i f x+5i e)} - 175i a^4 e^{(3i f x+3i e)} - 105i a^4 e^{(i f x+i e)}\right)\sqrt{\frac{a}{e^{(2i f x+2i e)}+1}}\sqrt{\frac{c}{e^{(2i f x+2i e)}+1}}}{12\left(c^2 f e^{(2i f x+2i e)} + c^2 f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I*a*tan(f*x+e))^(9/2)/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="fricas")

[Out] 1/12*(105*sqrt(a^9/(c^3*f^2))*(c^2*f*e^(2*I*f*x + 2*I*e) + c^2*f)*log(4*(2*(a^4*e^(3*I*f*x + 3*I*e) + a^4*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - sqrt(a^9/(c^3*f^2))*(I*c^2*f*e^(2*I*f*x + 2*I*e) - I*c^2*f))/(a^4*e^(2*I*f*x + 2*I*e) + a^4)) - 105*sqrt(a^9/(c^3*f^2))*(c^2*f*e^(2*I*f*x + 2*I*e) + c^2*f)*log(4*(2*(a^4*e^(3*I*f*x + 3*I*e) + a^4*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - sqrt(a^9/(c^3*f^2))*(-I*c^2*f*e^(2*I*f*x + 2*I*e) + I*c^2*f))/(a^4*e^(2*I*f*x + 2*I*e) + a^4)) - 4*(8*I*a^4*e^(7*I*f*x + 7*I

```
*e) - 56*I*a^4*e^(5*I*f*x + 5*I*e) - 175*I*a^4*e^(3*I*f*x + 3*I*e) - 105*I*
a^4*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x +
 2*I*e) + 1)))/(c^2*f*e^(2*I*f*x + 2*I*e) + c^2*f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(9/2)/(c-I*c*tan(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7317 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(9/2)/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^(9/2)/(-I*c*tan(f*x + e) + c)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+a\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{9/2}}{{\left(c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(9/2)/(c - c*tan(e + f*x)*1i)^(3/2),x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^(9/2)/(c - c*tan(e + f*x)*1i)^(3/2), x)
```

3.1025 $\int \frac{(a+ia\tan(e+fx))^{7/2}}{(c-ic\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=204

$$-\frac{10ia^{7/2}\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{c^{3/2}f}-\frac{2ia(a+ia\tan(e+fx))^{5/2}}{3f(c-ic\tan(e+fx))^{3/2}}+\frac{10ia^2(a+ia\tan(e+fx))^{3/2}}{3cf\sqrt{c-ic\tan(e+fx)}}+$$

```
[Out] -10*I*a^(7/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*c*tan(f*
x+e))^(1/2))/c^(3/2)/f+5*I*a^3*(a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))^
(1/2)/c^2/f+10/3*I*a^2*(a+I*a*tan(f*x+e))^(3/2)/c/f/(c-I*c*tan(f*x+e))^(1/2
)-2/3*I*a*(a+I*a*tan(f*x+e))^(5/2)/f/(c-I*c*tan(f*x+e))^(3/2)
```

Rubi [A]

time = 0.13, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3604, 49, 52, 65, 223, 209}

$$-\frac{10ia^{7/2}\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{c^{3/2}f}+\frac{5ia^3\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{c^2f}+\frac{10ia^2(a+ia\tan(e+fx))^{3/2}}{3cf\sqrt{c-ic\tan(e+fx)}}-\frac{2ia(a+ia\tan(e+fx))^{5/2}}{3f(c-ic\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^(7/2)/(c - I*c*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-10*I)*a^(7/2)*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[
c - I*c*Tan[e + f*x]])])/(c^(3/2)*f) - (((2*I)/3)*a*(a + I*a*Tan[e + f*x])^
(5/2))/(f*(c - I*c*Tan[e + f*x])^(3/2)) + (((10*I)/3)*a^2*(a + I*a*Tan[e +
f*x])^(3/2))/(c*f*Sqrt[c - I*c*Tan[e + f*x]]) + ((5*I)*a^3*Sqrt[a + I*a*Tan
[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/(c^2*f)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^{7/2}}{(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{(a+iax)^{5/2}}{(c-icx)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{2ia(a+ia\tan(e+fx))^{5/2}}{3f(c-ic\tan(e+fx))^{3/2}} - \frac{(5a^2)\,\text{Subst}\left(\int \frac{(a+iax)^{3/2}}{(c-icx)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3f}$$

$$= -\frac{2ia(a+ia\tan(e+fx))^{5/2}}{3f(c-ic\tan(e+fx))^{3/2}} + \frac{10ia^2(a+ia\tan(e+fx))^{3/2}}{3cf\sqrt{c-ic\tan(e+fx)}} + \frac{(5a^3)\,\text{Subst}\Big(}{}$$

$$= -\frac{2ia(a+ia\tan(e+fx))^{5/2}}{3f(c-ic\tan(e+fx))^{3/2}} + \frac{10ia^2(a+ia\tan(e+fx))^{3/2}}{3cf\sqrt{c-ic\tan(e+fx)}} + \frac{5ia^3\sqrt{a+ia}}{}$$

$$= -\frac{2ia(a+ia\tan(e+fx))^{5/2}}{3f(c-ic\tan(e+fx))^{3/2}} + \frac{10ia^2(a+ia\tan(e+fx))^{3/2}}{3cf\sqrt{c-ic\tan(e+fx)}} + \frac{5ia^3\sqrt{a+ia}}{}$$

$$= -\frac{2ia(a+ia\tan(e+fx))^{5/2}}{3f(c-ic\tan(e+fx))^{3/2}} + \frac{10ia^2(a+ia\tan(e+fx))^{3/2}}{3cf\sqrt{c-ic\tan(e+fx)}} + \frac{5ia^3\sqrt{a+ia}}{}$$

$$= -\frac{10ia^{7/2}\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{c^{3/2}f} - \frac{2ia(a+ia\tan(e+fx))^{5/2}}{3f(c-ic\tan(e+fx))^{3/2}}$$

Mathematica [A]
time = 8.96, size = 348, normalized size = 1.71

$$-\frac{10ie^{-i(4e+fx)}\sqrt{e^{ifx}}\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\,\text{ArcTan}\left(e^{i(e+fx)}\right)(a+ia\tan(e+fx))^{7/2}}{c\sqrt{\frac{c}{1+e^{2i(e+fx)}}}\,f\sec^{\frac{7}{2}}(e+fx)(\cos(fx)+i\sin(fx))^{7/2}} + \frac{\cos^3(e+fx)\left(\frac{5i\cos(3e)}{c^2}+\cos(4fx)\left(-\frac{2i\cos(e)}{3c^2}+\frac{2\sin(e)}{3c^2}\right)+\cos(2fx)\left(\frac{10i\cos(e)}{3c^2}+\frac{10\sin(e)}{3c^2}\right)+\frac{5\sin(3e)}{c^2}+\left(-\frac{10\cos(e)}{3c^2}+\frac{10i\sin(e)}{3c^2}\right)\sin(2fx)+\left(\frac{2\cos(e)}{3c^2}+\frac{2i\sin(e)}{3c^2}\right)\sin(4fx)\right)\sqrt{\sec(e+fx)(c\cos(e+fx)-ic\sin(e+fx))}\,(a+ia\tan(e+fx))^{7/2}}{f(\cos(fx)+i\sin(fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(7/2)/(c - I*c*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-10*I)*Sqrt[E^(I*f*x)]*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*Ar
cTan[E^(I*(e + f*x))]*(a + I*a*Tan[e + f*x])^(7/2))/(c*E^(I*(4*e + f*x))*Sq
rt[c/(1 + E^((2*I)*(e + f*x)))]*f*Sec[e + f*x]^(7/2)*(Cos[f*x] + I*Sin[f*x]
)^(7/2)) + (Cos[e + f*x]^3*(((5*I)*Cos[3*e])/c^2 + Cos[4*f*x]*((((-2*I)/3)*
Cos[e])/c^2 + (2*Sin[e])/(3*c^2)) + Cos[2*f*x]*((((10*I)/3)*Cos[e])/c^2 + (
10*Sin[e])/(3*c^2)) + (5*Sin[3*e])/c^2 + ((-10*Cos[e])/(3*c^2) + (((10*I)/3
)*Sin[e])/c^2)*Sin[2*f*x] + ((2*Cos[e])/(3*c^2) + (((2*I)/3)*Sin[e])/c^2)*S
```

```
in[4*f*x])*Sqrt[Sec[e + f*x]*(c*Cos[e + f*x] - I*c*Sin[e + f*x])]*(a + I*a*
Tan[e + f*x])^(7/2))/(f*(Cos[f*x] + I*Sin[f*x])^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(164) = 328.
time = 0.36, size = 379, normalized size = 1.86

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^3\left(45i\ln\left(\frac{ca\tan(fx+e)+\sqrt{ac(1+\tan^2(}}{\sqrt{ac}}\right.\right.}{}$
default	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^3\left(45i\ln\left(\frac{ca\tan(fx+e)+\sqrt{ac(1+\tan^2(}}{\sqrt{ac}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(7/2)/(c-I*c*tan(f*x+e))^(3/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/3/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^3/c^2*(45*I*
ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))*a
*c*tan(f*x+e)^2+15*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1
/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^3+3*I*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(
1/2)*tan(f*x+e)^3-15*I*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c
)^(1/2))/(a*c)^(1/2))*a*c-45*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2
)*(a*c)^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)-57*I*(a*c*(1+tan(f*x+e)^2))^(1/2
)*(a*c)^(1/2)*tan(f*x+e)-37*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*
x+e)^2+23*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c*(1+tan(f*x+e)^2))^
(1/2)/(tan(f*x+e)+I)^3/(a*c)^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 706 vs. 2(160) = 320.
time = 0.58, size = 706, normalized size = 3.46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(7/2)/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="m
axima")
```

```
[Out] -3*(30*(a^3*cos(2*f*x + 2*e) + I*a^3*sin(2*f*x + 2*e) + a^3)*arctan2(cos(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), sin(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))) + 1) + 30*(a^3*cos(2*f*x + 2*e) + I*a^3*sin(2*f*x
 + 2*e) + a^3)*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
```

```
, -sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 8*(a^3*cos(2
*f*x + 2*e) + I*a^3*sin(2*f*x + 2*e) + a^3)*cos(3/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) - 12*(4*a^3*cos(2*f*x + 2*e) + 4*I*a^3*sin(2*f*x + 2*
e) + 5*a^3)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 15*(I*a^
3*cos(2*f*x + 2*e) - a^3*sin(2*f*x + 2*e) + I*a^3)*log(cos(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e)))^2 + 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
 1) + 15*(-I*a^3*cos(2*f*x + 2*e) + a^3*sin(2*f*x + 2*e) - I*a^3)*log(cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) + 1) + 8*(I*a^3*cos(2*f*x + 2*e) - a^3*sin(2*f*x + 2*e) + I*a
^3)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 12*(-4*I*a^3*cos
(2*f*x + 2*e) + 4*a^3*sin(2*f*x + 2*e) - 5*I*a^3)*sin(1/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((-18*I*c^2*cos(2*f*x + 2*e) +
 18*c^2*sin(2*f*x + 2*e) - 18*I*c^2)*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(160) = 320$.
time = 0.81, size = 385, normalized size = 1.89

$$\frac{15\sqrt{\frac{a^7}{c^3f^2}}\,c^2f\log\left(\frac{4\left(2\left(a^3e^{(3i\,fx+3i\,e)}+a^3e^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}-\left(i\,c^2fe^{(2i\,fx+2i\,e)}-i\,c^2f\right)\sqrt{\frac{a^7}{c^3f^2}}\right)}{a^3e^{(2i\,fx+2i\,e)}+a^3}\right)-15\sqrt{\frac{a^7}{c^3f^2}}\,c^2f\log\left(\frac{4\left(2\left(a^3e^{(3i\,fx+3i\,e)}+a^3e^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}-\left(-i\,c^2fe^{(2i\,fx+2i\,e)}+i\,c^2f\right)\sqrt{\frac{a^7}{c^3f^2}}\right)}{a^3e^{(2i\,fx+2i\,e)}+a^3}\right)-4\left(2i\,a^3e^{(5i\,fx+5i\,e)}-10i\,a^3e^{(3i\,fx+3i\,e)}-15i\,a^3e^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{6\,c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(7/2)/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/6*(15*sqrt(a^7/(c^3*f^2))*c^2*f*log(4*(2*(a^3*e^(3*I*f*x + 3*I*e) + a^3*e
^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*
e) + 1)) - (I*c^2*f*e^(2*I*f*x + 2*I*e) - I*c^2*f)*sqrt(a^7/(c^3*f^2)))/(a^
3*e^(2*I*f*x + 2*I*e) + a^3)) - 15*sqrt(a^7/(c^3*f^2))*c^2*f*log(4*(2*(a^3*
e^(3*I*f*x + 3*I*e) + a^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)
)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - (-I*c^2*f*e^(2*I*f*x + 2*I*e) + I*c^2
*f)*sqrt(a^7/(c^3*f^2)))/(a^3*e^(2*I*f*x + 2*I*e) + a^3)) - 4*(2*I*a^3*e^(5
*I*f*x + 5*I*e) - 10*I*a^3*e^(3*I*f*x + 3*I*e) - 15*I*a^3*e^(I*f*x + I*e))*
sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(c^2*f
)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(7/2)/(c-I*c*tan(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(7/2)/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^(7/2)/(-I*c*tan(f*x + e) + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + a\tan(e + f\,x)\;1\mathrm{i})^{7/2}}{(c - c\tan\left(e + f\,x\right)\;1\mathrm{i})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(7/2)/(c - c*tan(e + f*x)*1i)^(3/2),x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^(7/2)/(c - c*tan(e + f*x)*1i)^(3/2), x)
```

3.1026 $\int \frac{(a+ia\tan(e+fx))^{5/2}}{(c-ic\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=155

$$-\frac{2ia^{5/2}\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{c^{3/2}f}-\frac{2ia(a+ia\tan(e+fx))^{3/2}}{3f(c-ic\tan(e+fx))^{3/2}}+\frac{2ia^2\sqrt{a+ia\tan(e+fx)}}{cf\sqrt{c-ic\tan(e+fx)}}$$

```
[Out] -2*I*a^(5/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*c*tan(f*x
+e))^(1/2))/c^(3/2)/f+2*I*a^2*(a+I*a*tan(f*x+e))^(1/2)/c/f/(c-I*c*tan(f*x+e
))^(1/2)-2/3*I*a*(a+I*a*tan(f*x+e))^(3/2)/f/(c-I*c*tan(f*x+e))^(3/2)
```

Rubi [A]

time = 0.12, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3604, 49, 65, 223, 209}

$$-\frac{2ia^{5/2}\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{c^{3/2}f}+\frac{2ia^2\sqrt{a+ia\tan(e+fx)}}{cf\sqrt{c-ic\tan(e+fx)}}-\frac{2ia(a+ia\tan(e+fx))^{3/2}}{3f(c-ic\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^(5/2)/(c - I*c*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-2*I)*a^(5/2)*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c
 - I*c*Tan[e + f*x]])])/(c^(3/2)*f) - (((2*I)/3)*a*(a + I*a*Tan[e + f*x])^(
3/2))/(f*(c - I*c*Tan[e + f*x])^(3/2)) + ((2*I)*a^2*Sqrt[a + I*a*Tan[e + f*
x]])/(c*f*Sqrt[c - I*c*Tan[e + f*x]])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^{5/2}}{(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{(a+iax)^{3/2}}{(c-icx)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{2ia(a+ia\tan(e+fx))^{3/2}}{3f(c-ic\tan(e+fx))^{3/2}} - \frac{a^2\text{Subst}\left(\int \frac{\sqrt{a+iax}}{(c-icx)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{2ia(a+ia\tan(e+fx))^{3/2}}{3f(c-ic\tan(e+fx))^{3/2}} + \frac{2ia^2\sqrt{a+ia\tan(e+fx)}}{cf\sqrt{c-ic\tan(e+fx)}} + \frac{a^3\text{Subst}\left(\int \cdots\right.}{}$$

$$= -\frac{2ia(a+ia\tan(e+fx))^{3/2}}{3f(c-ic\tan(e+fx))^{3/2}} + \frac{2ia^2\sqrt{a+ia\tan(e+fx)}}{cf\sqrt{c-ic\tan(e+fx)}} - \frac{(2ia^2)\,\text{Subst}\left(\cdots\right.}{}$$

$$= -\frac{2ia(a+ia\tan(e+fx))^{3/2}}{3f(c-ic\tan(e+fx))^{3/2}} + \frac{2ia^2\sqrt{a+ia\tan(e+fx)}}{cf\sqrt{c-ic\tan(e+fx)}} - \frac{(2ia^2)\,\text{Subst}\left(\cdots\right.}{}$$

$$= -\frac{2ia^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{c^{3/2}f} - \frac{2ia(a+ia\tan(e+fx))^{3/2}}{3f(c-ic\tan(e+fx))^{3/2}}$$

Mathematica [A]
time = 5.47, size = 184, normalized size = 1.19

$$\frac{2a^2\cos(e+fx)\left(\cos\left(\frac{1}{2}(e-2fx)\right)-i\sin\left(\frac{1}{2}(e-2fx)\right)\right)\left(-1-\cos(2(e+fx))+3\text{ArcTan}\left(e^{i(e+fx)}\right)\cos(e+fx)(\cos(2(e+fx))-i\sin(2(e+fx)))+2i\sin(2(e+fx))\right)\left(i\cos\left(\frac{1}{2}(e+4fx)\right)+\sin\left(\frac{1}{2}(e+4fx)\right)\right)(-i+\tan(e+fx))^2\sqrt{a+ia\tan(e+fx)}}{3cf\sqrt{c-ic\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(5/2)/(c - I*c*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*a^2*Cos[e + f*x]*(Cos[(e - 2*f*x)/2] - I*Sin[(e - 2*f*x)/2])*(-1 - Cos[2
*(e + f*x)] + 3*ArcTan[E^(I*(e + f*x))]*Cos[e + f*x]*(Cos[2*(e + f*x)] - I*
Sin[2*(e + f*x)]) + (2*I)*Sin[2*(e + f*x)])*(I*Cos[(e + 4*f*x)/2] + Sin[(e
+ 4*f*x)/2])*(-I + Tan[e + f*x])^2*Sqrt[a + I*a*Tan[e + f*x]])/(3*c*f*Sqrt[
c - I*c*Tan[e + f*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(124) = 248.
time = 0.37, size = 348, normalized size = 2.25

method	result
derivativedivides	$\frac{\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,a^{2}\left(9i\ln\left(\frac{ca\tan(fx+e)+\sqrt{ac\left(1+\tan^{2}\left(fx\right.\right.}}{\sqrt{ac}}\right.\right.}{}$
default	$\frac{\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,a^{2}\left(9i\ln\left(\frac{ca\tan(fx+e)+\sqrt{ac\left(1+\tan^{2}\left(fx\right.\right.}}{\sqrt{ac}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(3/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/3/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^2/c^2*(9*I*l
n((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))*a*
c*tan(f*x+e)^2+3*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2
))/(a*c)^(1/2))*a*c*tan(f*x+e)^3-3*I*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^
2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))*a*c-9*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*
x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)-12*I*(a*c*(1+tan(f*
x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)-8*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^
(1/2)*tan(f*x+e)^2+4*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c*(1+tan(
f*x+e)^2))^(1/2)/(tan(f*x+e)+I)^3/(a*c)^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 414 vs. 2(121) = 242.
time = 0.61, size = 414, normalized size = 2.67

$$\frac{\left(-6i\,a^{2}\arctan\left(\cos\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right),\sin\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)+1\right)-6i\,a^{2}\arctan\left(\cos\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right),-\sin\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)+1\right)-4i\,a^{2}\cos\left(\frac{3}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)+12i\,a^{2}\cos\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)+3a^{2}\log\left(\cos\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)^{2}+\sin\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)^{2}+2\sin\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)+1\right)-3a^{2}\log\left(\cos\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)^{2}+\sin\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)^{2}-2\sin\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)+1\right)+4a^{2}\sin\left(\frac{3}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)-12a^{2}\sin\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)\right)\sqrt{a}}{6c^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="m
axima")
```

```
[Out] 1/6*(-6*I*a^2*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))),
 sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) - 6*I*a^2*arctan
2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), -sin(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) - 4*I*a^2*cos(3/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))) + 12*I*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e))) + 3*a^2*log(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) - 3*a^2*log(cos(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e)))^2 - 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) + 1) + 4*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
- 12*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)/(c^(
3/2)*f)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(121) = 242$.
time = 0.80, size = 385, normalized size = 2.48

$$\frac{3\,c^{2}f\sqrt{\frac{a^{5}}{c^{3}f^{2}}}\log\left(\frac{4\left(2\left(a^{2}e^{(3i\,fx+3i\,e)}+a^{2}e^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}-\left(i\,c^{2}fe^{(2i\,fx+2i\,e)}-i\,c^{2}f\right)\sqrt{\frac{a^{5}}{c^{3}f^{2}}}\right)}{a^{2}e^{(2i\,fx+2i\,e)}+a^{2}}\right)-3\,c^{2}f\sqrt{\frac{a^{5}}{c^{3}f^{2}}}\log\left(\frac{4\left(2\left(a^{2}e^{(3i\,fx+3i\,e)}+a^{2}e^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}-\left(-i\,c^{2}fe^{(2i\,fx+2i\,e)}+i\,c^{2}f\right)\sqrt{\frac{a^{5}}{c^{3}f^{2}}}\right)}{a^{2}e^{(2i\,fx+2i\,e)}+a^{2}}\right)-4\left(i\,a^{2}e^{(5i\,fx+5i\,e)}-2i\,a^{2}e^{(3i\,fx+3i\,e)}-3i\,a^{2}e^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{6\,c^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/6*(3*c^2*f*sqrt(a^5/(c^3*f^2))*log(4*(2*(a^2*e^(3*I*f*x + 3*I*e) + a^2*e^
(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e
) + 1)) - (I*c^2*f*e^(2*I*f*x + 2*I*e) - I*c^2*f)*sqrt(a^5/(c^3*f^2)))/(a^2
*e^(2*I*f*x + 2*I*e) + a^2)) - 3*c^2*f*sqrt(a^5/(c^3*f^2))*log(4*(2*(a^2*e^
(3*I*f*x + 3*I*e) + a^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*
sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - (-I*c^2*f*e^(2*I*f*x + 2*I*e) + I*c^2*f
)*sqrt(a^5/(c^3*f^2)))/(a^2*e^(2*I*f*x + 2*I*e) + a^2)) - 4*(I*a^2*e^(5*I*f
*x + 5*I*e) - 2*I*a^2*e^(3*I*f*x + 3*I*e) - 3*I*a^2*e^(I*f*x + I*e))*sqrt(a
/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(c^2*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(ia\left(\tan\left(e+fx\right)-i\right)\right)^{\frac{5}{2}}}{\left(-ic\left(\tan\left(e+fx\right)+i\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(5/2)/(c-I*c*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((I*a*(tan(e + f*x) - I))**(5/2)/(-I*c*(tan(e + f*x) + I))**(3/2),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^(5/2)/(-I*c*tan(f*x + e) + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\tan(e+f\,x)\;1\mathrm{i})^{5/2}}{(c-c\tan\left(e+f\,x\right)\;1\mathrm{i})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(5/2)/(c - c*tan(e + f*x)*1i)^(3/2),x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^(5/2)/(c - c*tan(e + f*x)*1i)^(3/2), x)
```

3.1027 $\int \frac{(a+ia\tan(e+fx))^{3/2}}{(c-ic\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=43

$$-\frac{i(a+ia\tan(e+fx))^{3/2}}{3f(c-ic\tan(e+fx))^{3/2}}$$

```
[Out] -1/3*I*(a+I*a*tan(f*x+e))^(3/2)/f/(c-I*c*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {3604, 37}

$$-\frac{i(a+ia\tan(e+fx))^{3/2}}{3f(c-ic\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^(3/2)/(c - I*c*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-1/3*I)*(a + I*a*Tan[e + f*x])^(3/2))/(f*(c - I*c*Tan[e + f*x])^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+ia\tan(e+fx))^{3/2}}{(c-ic\tan(e+fx))^{3/2}}\,dx &= \frac{(ac)\text{Subst}\left(\int \frac{\sqrt{a+iax}}{(c-icx)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}\\ &= -\frac{i(a+ia\tan(e+fx))^{3/2}}{3f(c-ic\tan(e+fx))^{3/2}}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 87 vs. 2(43) = 86.
time = 1.52, size = 87, normalized size = 2.02

$$\frac{a\cos(e+fx)(\cos(fx)-i\sin(fx))(-i\cos(3e+4fx)+\sin(3e+4fx))\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{3c^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(3/2)/(c - I*c*Tan[e + f*x])^(3/2),x]
```

```
[Out] (a*Cos[e + f*x]*(Cos[f*x] - I*Sin[f*x])*((-I)*Cos[3*e + 4*f*x] + Sin[3*e + 4*f*x])*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/(3*c^2*f)
```

Maple [A]
time = 0.34, size = 62, normalized size = 1.44

method	result	size
derivativedivides	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a(1+\tan^2(fx+e))}{3f\,c^2(\tan(fx+e)+i)^3}$	62
default	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a(1+\tan^2(fx+e))}{3f\,c^2(\tan(fx+e)+i)^3}$	62
risch	$-\frac{ia\sqrt{\frac{a\,e^{2i(fx+e)}}{e^{2i(fx+e)}+1}}\,e^{2i(fx+e)}}{3c\sqrt{\frac{c}{e^{2i(fx+e)}+1}}\,f}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a/c^2*(1+tan(f*x+e)^2)/(tan(f*x+e)+I)^3
```

Maxima [A]
time = 0.54, size = 37, normalized size = 0.86

$$\frac{(-i\,a\cos(3\,fx+3\,e)+a\sin(3\,fx+3\,e))\sqrt{a}}{3\,c^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*(-I*a*cos(3*f*x + 3*e) + a*sin(3*f*x + 3*e))*sqrt(a)/(c^(3/2)*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(33) = 66$.
time = 0.74, size = 71, normalized size = 1.65

$$\frac{\left(-i\,ae^{(5i\,fx+5i\,e)}-i\,ae^{(3i\,fx+3i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\,\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{3\,c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/3*(-I*a*e^(5*I*f*x + 5*I*e) - I*a*e^(3*I*f*x + 3*I*e))*sqrt(a/(e^(2*I*f*x
 + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(c^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ia(\tan(e+fx)-i))^{\frac{3}{2}}}{(-ic\,(\tan(e+fx)+i))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(3/2)/(c-I*c*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((I*a*(tan(e + f*x) - I))**(3/2)/(-I*c*(tan(e + f*x) + I))**(3/2),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^(3/2)/(-I*c*tan(f*x + e) + c)^(3/2), x)
```

Mupad [B]
time = 0.77, size = 108, normalized size = 2.51

$$-\frac{\sqrt{2}\,a\,(\cos(2\,f\,x)+\sin(2\,f\,x)\,1\mathrm{i})\,(\cos(2\,e)+\sin(2\,e)\,1\mathrm{i})\,\sqrt{\frac{a\,(\cos(2\,e+2\,f\,x)+1+\sin(2\,e+2\,f\,x)\,1\mathrm{i})}{\cos(2\,e+2\,f\,x)+1}}\,1\mathrm{i}}{6\,c\,f\,\sqrt{\frac{c}{\cos(2\,e+2\,f\,x)+1+\sin(2\,e+2\,f\,x)\,1\mathrm{i}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(3/2)/(c - c*tan(e + f*x)*1i)^(3/2),x)
```

```
[Out] -(2^(1/2)*a*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)*((a*(cos(
2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2)*1i)/
(6*c*f*(c/(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))^(1/2))
```

3.1028 $\int \frac{\sqrt{a+ia\tan(e+fx)}}{(c-ic\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=90

$$-\frac{i\sqrt{a+ia\tan(e+fx)}}{3f(c-ic\tan(e+fx))^{3/2}}-\frac{i\sqrt{a+ia\tan(e+fx)}}{3cf\sqrt{c-ic\tan(e+fx)}}$$

```
[Out] -1/3*I*(a+I*a*tan(f*x+e))^(1/2)/c/f/(c-I*c*tan(f*x+e))^(1/2)-1/3*I*(a+I*a*t
an(f*x+e))^(1/2)/f/(c-I*c*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {3604, 47, 37}

$$-\frac{i\sqrt{a+ia\tan(e+fx)}}{3cf\sqrt{c-ic\tan(e+fx)}}-\frac{i\sqrt{a+ia\tan(e+fx)}}{3f(c-ic\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[e + f*x]]/(c - I*c*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-1/3*I)*Sqrt[a + I*a*Tan[e + f*x]])/(f*(c - I*c*Tan[e + f*x])^(3/2)) - ((
I/3)*Sqrt[a + I*a*Tan[e + f*x]])/(c*f*Sqrt[c - I*c*Tan[e + f*x]])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
```

```
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+ia\tan(e+fx)}}{(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{(ac)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+iax}\,(c-icx)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{i\sqrt{a+ia\tan(e+fx)}}{3f(c-ic\tan(e+fx))^{3/2}} + \frac{a\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+iax}\,(c-icx)^{3/2}}\,dx, x, \tan(e+f\right.}{3f}$$

$$= -\frac{i\sqrt{a+ia\tan(e+fx)}}{3f(c-ic\tan(e+fx))^{3/2}} - \frac{i\sqrt{a+ia\tan(e+fx)}}{3cf\sqrt{c-ic\tan(e+fx)}}$$

Mathematica [A]
time = 1.19, size = 89, normalized size = 0.99

$$\frac{(2+2\cos(2(e+fx))-i\sin(2(e+fx)))(-i\cos(2(e+fx))+\sin(2(e+fx)))\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{6c^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Tan[e + f*x]]/(c - I*c*Tan[e + f*x])^(3/2),x]
[Out] ((2 + 2*Cos[2*(e + f*x)] - I*Sin[2*(e + f*x)])*((-I)*Cos[2*(e + f*x)] + Sin
[2*(e + f*x)])*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/(6*c^
2*f)
```

Maple [A]
time = 0.36, size = 70, normalized size = 0.78

method	result	size
risch	$-\frac{i\sqrt{\frac{a\,e^{2i(fx+e)}}{e^{2i(fx+e)}+1}}\,\left(e^{2i(fx+e)}+3\right)}{6c\sqrt{\frac{c}{e^{2i(fx+e)}+1}}\,f}$	64
derivativedivides	$\frac{\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\left(3i\tan(fx+e)+\tan^2(fx+e)-2\right)}{3f\,c^2(\tan(fx+e)+i)^3}$	70
default	$\frac{\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\left(3i\tan(fx+e)+\tan^2(fx+e)-2\right)}{3f\,c^2(\tan(fx+e)+i)^3}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(3/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/3/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)/c^2*(3*I*tan(f
*x+e)+tan(f*x+e)^2-2)/(tan(f*x+e)+I)^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 0.93, size = 81, normalized size = 0.90

$$\frac{\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\,\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\,\left(-i\,e^{(5i\,fx+5i\,e)}-4i\,e^{(3i\,fx+3i\,e)}-3i\,e^{(i\,fx+i\,e)}\right)}{6\,c^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/6*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*(-I
*e^(5*I*f*x + 5*I*e) - 4*I*e^(3*I*f*x + 3*I*e) - 3*I*e^(I*f*x + I*e))/(c^2*
f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ia\left(\tan\left(e+fx\right)-i\right)}}{\left(-ic\left(\tan\left(e+fx\right)+i\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(1/2)/(c-I*c*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(e + f*x) - I))/(-I*c*(tan(e + f*x) + I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(I*a*tan(f*x + e) + a)/(-I*c*tan(f*x + e) + c)^(3/2), x)
```

Mupad [B]
time = 5.05, size = 114, normalized size = 1.27

$$-\frac{\sqrt{\frac{a\,\left(\cos\left(2\,e+2\,f\,x\right)+1+\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\left(\cos\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}-\sin\left(2\,e+2\,f\,x\right)+3{}\mathrm{i}\right)}{6\,c\,f\,\sqrt{\frac{c\,\left(\cos\left(2\,e+2\,f\,x\right)+1-\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(1/2)/(c - c*tan(e + f*x)*1i)^(3/2),x)
```

```
[Out] -(((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))
^(1/2)*(cos(2*e + 2*f*x)*1i - sin(2*e + 2*f*x) + 3i))/(6*c*f*((c*(cos(2*e +
 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

3.1029 $\int \frac{1}{\sqrt{a+ia\tan(e+fx)}\,(c-ic\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=137

$$\frac{i}{f\sqrt{a+ia\tan(e+fx)}\,(c-ic\tan(e+fx))^{3/2}}-\frac{2i\sqrt{a+ia\tan(e+fx)}}{3af(c-ic\tan(e+fx))^{3/2}}-\frac{2i\sqrt{a+ia\tan(e+fx)}}{3acf\sqrt{c-ic\tan(e+fx)}}$$

```
[Out] -2/3*I*(a+I*a*tan(f*x+e))^(1/2)/a/c/f/(c-I*c*tan(f*x+e))^(1/2)+I/f/(a+I*a*t
an(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(3/2)-2/3*I*(a+I*a*tan(f*x+e))^(1/2)/a/
f/(c-I*c*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.10, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {3604, 47, 37}

$$-\frac{2i\sqrt{a+ia\tan(e+fx)}}{3acf\sqrt{c-ic\tan(e+fx)}}-\frac{2i\sqrt{a+ia\tan(e+fx)}}{3af(c-ic\tan(e+fx))^{3/2}}+\frac{i}{f\sqrt{a+ia\tan(e+fx)}\,(c-ic\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + I*a*Tan[e + f*x]]*(c - I*c*Tan[e + f*x])^(3/2)),x]
```

```
[Out] I/(f*Sqrt[a + I*a*Tan[e + f*x]]*(c - I*c*Tan[e + f*x])^(3/2)) - (((2*I)/3)*
Sqrt[a + I*a*Tan[e + f*x]])/(a*f*(c - I*c*Tan[e + f*x])^(3/2)) - (((2*I)/3)
*Sqrt[a + I*a*Tan[e + f*x]])/(a*c*f*Sqrt[c - I*c*Tan[e + f*x]])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+ia\tan(e+fx)}\,(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{(ac)\mathrm{Subst}\left(\int \frac{1}{(a+iax)^{3/2}(c-icx)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{i}{f\sqrt{a+ia\tan(e+fx)}\,(c-ic\tan(e+fx))^{3/2}} + \frac{(2c)\mathrm{S}}{}$$

$$= \frac{i}{f\sqrt{a+ia\tan(e+fx)}\,(c-ic\tan(e+fx))^{3/2}} - \frac{2i\sqrt{}}{3af(}$$

$$= \frac{i}{f\sqrt{a+ia\tan(e+fx)}\,(c-ic\tan(e+fx))^{3/2}} - \frac{2i\sqrt{}}{3af(}$$

Mathematica [A]
time = 1.35, size = 89, normalized size = 0.65

$$\frac{i(\cos(2(e+fx))+i\sin(2(e+fx)))(-3+\cos(2(e+fx))-2i\sin(2(e+fx)))\sqrt{c-ic\tan(e+fx)}}{6c^2f\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + I*a*Tan[e + f*x]]*(c - I*c*Tan[e + f*x])^(3/2)),x]
```

```
[Out] ((I/6)*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)])*(-3 + Cos[2*(e + f*x)] - (2*
I)*Sin[2*(e + f*x)])*Sqrt[c - I*c*Tan[e + f*x]])/(c^2*f*Sqrt[a + I*a*Tan[e
+ f*x]])
```

Maple [A]
time = 0.40, size = 109, normalized size = 0.80

method	result
risch	$-\frac{i\left(\mathrm{e}^{4i(fx+e)}+6\,\mathrm{e}^{2i(fx+e)}-3\right)}{12c\sqrt{\frac{a\,\mathrm{e}^{2i(fx+e)}}{\mathrm{e}^{2i(fx+e)}+1}}\,\left(\mathrm{e}^{2i(fx+e)}+1\right)\sqrt{\frac{c}{\mathrm{e}^{2i(fx+e)}+1}}\,f}$
derivativedivides	$\frac{\sqrt{a\,(1+i\tan(fx+e))}\,\sqrt{-c\,(i\tan(fx+e)-1)}\,\left(2i\left(\tan^3(fx+e)\right)+2\left(\tan^4(fx+e)\right)+2i\tan(fx+e)-\right.}{3fa\,c^2(\tan(fx+e)+i)^3(-\tan(fx+e)+i)^2}$

default	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,(2i(\tan^3(fx+e))+2(\tan^4(fx+e))+2i\tan(fx+e}{3fac^2(\tan(fx+e)+i)^3(-\tan(fx+e)+i)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(3/2),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/3/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)/a/c^2*(2*I*tan
(f*x+e)^3+2*tan(f*x+e)^4+2*I*tan(f*x+e)+3*tan(f*x+e)^2+1)/(tan(f*x+e)+I)^3/
(-tan(f*x+e)+I)^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(3/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 0.90, size = 119, normalized size = 0.87

$$\frac{\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}\left(-ie^{(6ifx+6ie)}-7ie^{(4ifx+4ie)}+4ie^{(3ifx+3ie)}-3ie^{(2ifx+2ie)}+4ie^{(ifx+ie)}+3i\right)e^{(-ifx-ie)}}{12\,ac^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(3/2),x, algorithm=
"fricas")
```

```
[Out] 1/12*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*(-
I*e^(6*I*f*x + 6*I*e) - 7*I*e^(4*I*f*x + 4*I*e) + 4*I*e^(3*I*f*x + 3*I*e) -
 3*I*e^(2*I*f*x + 2*I*e) + 4*I*e^(I*f*x + I*e) + 3*I)*e^(-I*f*x - I*e)/(a*c
^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{ia(\tan(e+fx)-i)}\,(-ic(\tan(e+fx)+i))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**(1/2)/(c-I*c*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/(sqrt(I*a*(tan(e + f*x) - I))*(-I*c*(tan(e + f*x) + I))**(3/2)),
 x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate(1/(sqrt(I*a*tan(f*x + e) + a)*(-I*c*tan(f*x + e) + c)^(3/2)), x)
```

Mupad [B]
time = 0.39, size = 117, normalized size = 0.85

$$\frac{\sqrt{\frac{a\,\left(\cos\left(2\,e+2\,f\,x\right)+1+\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\left(\cos\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}+2\,\sin\left(2\,e+2\,f\,x\right)-3{}\mathrm{i}\right)}{6\,a\,c\,f\,\sqrt{\frac{c\,\left(\cos\left(2\,e+2\,f\,x\right)+1-\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^(1/2)*(c - c*tan(e + f*x)*1i)^(3/2)),x)
```

```
[Out] (((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^
(1/2)*(cos(2*e + 2*f*x)*1i + 2*sin(2*e + 2*f*x) - 3i))/(6*a*c*f*((c*(cos(2*
e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

3.1030 $\int \frac{1}{(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=101

$$\frac{\tan(e+fx)}{3f(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}+\frac{2\tan(e+fx)}{3acf\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}$$

```
[Out] 2/3*tan(f*x+e)/a/c/f/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(1/2)+1/3*
tan(f*x+e)/f/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {3604, 40, 39}

$$\frac{2\tan(e+fx)}{3acf\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}+\frac{\tan(e+fx)}{3f(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^(3/2)*(c - I*c*Tan[e + f*x])^(3/2)),x]
```

```
[Out] Tan[e + f*x]/(3*f*(a + I*a*Tan[e + f*x])^(3/2)*(c - I*c*Tan[e + f*x])^(3/2)
) + (2*Tan[e + f*x])/(3*a*c*f*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e
 + f*x]])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Dist[(2*m +
3)/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{1}{(a+iax)^{5/2}(c-icx)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\tan(e+fx)}{3f(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}} + \frac{2S}{-}$$
$$= \frac{\tan(e+fx)}{3f(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}} + \frac{}{3a}$$

Mathematica [A]
time = 1.71, size = 103, normalized size = 1.02

$$\frac{\sec(e+fx)(-i\cos(2(e+fx))+\sin(2(e+fx)))(9\sin(e+fx)+\sin(3(e+fx)))\sqrt{c-ic\tan(e+fx)}}{12ac^2f(-i+\tan(e+fx))\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^(3/2)*(c - I*c*Tan[e + f*x])^(3/2)),x]
```

```
[Out] (Sec[e + f*x]*((-I)*Cos[2*(e + f*x)] + Sin[2*(e + f*x)])*(9*Sin[e + f*x] +
Sin[3*(e + f*x)])*Sqrt[c - I*c*Tan[e + f*x]])/(12*a*c^2*f*(-I + Tan[e + f*x
])*Sqrt[a + I*a*Tan[e + f*x]])
```

Maple [A]
time = 0.37, size = 95, normalized size = 0.94

method	result
derivativedivides	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,(1+\tan^2(fx+e))\tan(fx+e)(2(\tan^2(fx+e))+3}{3f\,a^2c^2(-\tan(fx+e)+i)^3(\tan(fx+e)+i)^3}$
default	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,(1+\tan^2(fx+e))\tan(fx+e)(2(\tan^2(fx+e))+3}{3f\,a^2c^2(-\tan(fx+e)+i)^3(\tan(fx+e)+i)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(3/2),x,method=_RETURNVER
BOSE)
```

```
[Out] -1/3/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)/a^2/c^2*(1+ta
n(f*x+e)^2)*tan(f*x+e)*(2*tan(f*x+e)^2+3)/(-tan(f*x+e)+I)^3/(tan(f*x+e)+I)^
3
```

Maxima [A]
time = 0.62, size = 48, normalized size = 0.48

$$\frac{\sin(3\,fx+3\,e)+9\,\sin\left(\frac{1}{3}\,\arctan(\sin(3\,fx+3\,e),\cos(3\,fx+3\,e))\right)}{12\,a^{\frac{3}{2}}c^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(3/2),x, algorithm=
"maxima")
```

```
[Out] 1/12*(sin(3*f*x + 3*e) + 9*sin(1/3*arctan2(sin(3*f*x + 3*e), cos(3*f*x + 3*
e))))/(a^(3/2)*c^(3/2)*f)
```

Fricas [A]
time = 0.82, size = 95, normalized size = 0.94

$$\frac{\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\left(-i\,e^{(8i\,fx+8i\,e)}-10i\,e^{(6i\,fx+6i\,e)}+10i\,e^{(2i\,fx+2i\,e)}+i\right)e^{(-3i\,fx-3i\,e)}}{24\,a^2c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(3/2),x, algorithm=
"fricas")
```

```
[Out] 1/24*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*(-
I*e^(8*I*f*x + 8*I*e) - 10*I*e^(6*I*f*x + 6*I*e) + 10*I*e^(2*I*f*x + 2*I*e)
 + I)*e^(-3*I*f*x - 3*I*e)/(a^2*c^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(\tan\left(e+fx\right)-i\right)\right)^{\frac{3}{2}}\left(-ic\left(\tan\left(e+fx\right)+i\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**(3/2)/(c-I*c*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/((I*a*(tan(e + f*x) - I))**(3/2)*(-I*c*(tan(e + f*x) + I))**(3/2
)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate(1/((I*a*tan(f*x + e) + a)^(3/2)*(-I*c*tan(f*x + e) + c)^(3/2)), x
)
```

Mupad [B]
time = 5.54, size = 138, normalized size = 1.37

$$\frac{\sqrt{\frac{a\,\left(\cos\left(2\,e+2\,f\,x\right)+1+\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\left(\cos\left(2\,e+2\,f\,x\right)\,8{}\mathrm{i}+\cos\left(4\,e+4\,f\,x\right)\,1{}\mathrm{i}+10\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(4\,e+4\,f\,x\right)-9{}\mathrm{i}\right)}{24\,a^2\,c\,f\,\sqrt{\frac{c\,\left(\cos\left(2\,e+2\,f\,x\right)+1-\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^(3/2)*(c - c*tan(e + f*x)*1i)^(3/2)),x)
```

```
[Out] (((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^
(1/2)*(cos(2*e + 2*f*x)*8i + cos(4*e + 4*f*x)*1i + 10*sin(2*e + 2*f*x) + si
n(4*e + 4*f*x) - 9i))/(24*a^2*c*f*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*
1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

3.1031 $\int \frac{1}{(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=147

$$\frac{i}{5f(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{3/2}}+\frac{4\tan(e+fx)}{15af(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}+\frac{}{15a}$$

```
[Out] 8/15*tan(f*x+e)/a^2/c/f/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(1/2)+1
/5*I/f/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(3/2)+4/15*tan(f*x+e)/a/
f/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(3/2)
```

Rubi [A]

time = 0.10, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3604, 47, 40, 39}

$$\frac{8\tan(e+fx)}{15a^2cf\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}+\frac{4\tan(e+fx)}{15af(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}+\frac{i}{5f(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^(5/2)*(c - I*c*Tan[e + f*x])^(3/2)),x]
```

```
[Out] (I/5)/(f*(a + I*a*Tan[e + f*x])^(5/2)*(c - I*c*Tan[e + f*x])^(3/2)) + (4*Ta
n[e + f*x])/(15*a*f*(a + I*a*Tan[e + f*x])^(3/2)*(c - I*c*Tan[e + f*x])^(3/
2)) + (8*Tan[e + f*x])/(15*a^2*c*f*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*
Tan[e + f*x]])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Dist[(2*m +
3)/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
```

```
(EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{1}{(a+iax)^{7/2}(c-icx)^{5/2}}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{i}{5f(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{3/2}} + \frac{(4$$

$$= \frac{i}{5f(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{3/2}} + \frac{}{15$$

$$= \frac{i}{5f(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{3/2}} + \frac{}{15$$

Mathematica [A]
time = 2.65, size = 93, normalized size = 0.63

$$-\frac{i(-45+20\cos(2(e+fx))+\cos(4(e+fx))+40i\sin(2(e+fx))+4i\sin(4(e+fx)))\sqrt{c-ic\tan(e+fx)}}{120a^2c^2f\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^(5/2)*(c - I*c*Tan[e + f*x])^(3/2)),x]
```

```
[Out] ((-1/120*I)*(-45 + 20*Cos[2*(e + f*x)] + Cos[4*(e + f*x)] + (40*I)*Sin[2*(e
 + f*x)] + (4*I)*Sin[4*(e + f*x)])*Sqrt[c - I*c*Tan[e + f*x]])/(a^2*c^2*f*S
qrt[a + I*a*Tan[e + f*x]])
```

Maple [A]
time = 0.38, size = 130, normalized size = 0.88

method	result

derivativedivides	$-\frac{\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\left(8i\left(\tan^5\left(fx+e\right)\right)-8\left(\tan^6\left(fx+e\right)\right)+20i\left(\tan^3\left(\right.\right.\right.}{15f\,a^3c^2\left(-\tan\left(fx+e\right)+i\right)^4\left(\tan\left(fx+e\right)+i\right)^3}$
default	$-\frac{\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\left(8i\left(\tan^5\left(fx+e\right)\right)-8\left(\tan^6\left(fx+e\right)\right)+20i\left(\tan^3\left(\right.\right.\right.}{15f\,a^3c^2\left(-\tan\left(fx+e\right)+i\right)^4\left(\tan\left(fx+e\right)+i\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(3/2),x,method=_RETURNVER
BOSE)
```

```
[Out] -1/15/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)/a^3/c^2*(8*I
*tan(f*x+e)^5-8*tan(f*x+e)^6+20*I*tan(f*x+e)^3-20*tan(f*x+e)^4+12*I*tan(f*x
+e)-15*tan(f*x+e)^2-3)/(-tan(f*x+e)+I)^4/(tan(f*x+e)+I)^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(3/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 1.06, size = 143, normalized size = 0.97

$$\frac{\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\,\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\,\left(-5i\,e^{(10i\,fx+10i\,e)}-65i\,e^{(8i\,fx+8i\,e)}-48i\,e^{(7i\,fx+7i\,e)}+30i\,e^{(6i\,fx+6i\,e)}-48i\,e^{(5i\,fx+5i\,e)}+110i\,e^{(4i\,fx+4i\,e)}+23i\,e^{(2i\,fx+2i\,e)}+3i\right)e^{(-5i\,fx-5i\,e)}}{240\,a^3c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(3/2),x, algorithm=
"fricas")
```

```
[Out] 1/240*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*(
-5*I*e^(10*I*f*x + 10*I*e) - 65*I*e^(8*I*f*x + 8*I*e) - 48*I*e^(7*I*f*x + 7
*I*e) + 30*I*e^(6*I*f*x + 6*I*e) - 48*I*e^(5*I*f*x + 5*I*e) + 110*I*e^(4*I*
f*x + 4*I*e) + 23*I*e^(2*I*f*x + 2*I*e) + 3*I)*e^(-5*I*f*x - 5*I*e)/(a^3*c^
2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(ia\left(\tan\left(e+fx\right)-i\right)\right)^{\frac{5}{2}}\left(-ic\left(\tan\left(e+fx\right)+i\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**(5/2)/(c-I*c*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/((I*a*(tan(e + f*x) - I))**(5/2)*(-I*c*(tan(e + f*x) + I))**(3/2
)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate(1/((I*a*tan(f*x + e) + a)^(5/2)*(-I*c*tan(f*x + e) + c)^(3/2)), x
)
```

Mupad [B]
time = 5.91, size = 163, normalized size = 1.11

$$\frac{\sqrt{\frac{a\,\left(\cos\left(2\,e+2\,f\,x\right)+1+\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\left(\cos\left(2\,e+2\,f\,x\right)\,85{}\mathrm{i}+\cos\left(4\,e+4\,f\,x\right)\,20{}\mathrm{i}+\cos\left(6\,e+6\,f\,x\right)\,3{}\mathrm{i}+95\,\sin\left(2\,e+2\,f\,x\right)+20\,\sin\left(4\,e+4\,f\,x\right)+3\,\sin\left(6\,e+6\,f\,x\right)-60{}\mathrm{i}\right)}{240\,a^3\,c\,f\,\sqrt{\frac{c\,\left(\cos\left(2\,e+2\,f\,x\right)+1-\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^(5/2)*(c - c*tan(e + f*x)*1i)^(3/2)),x)
```

```
[Out] (((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^
(1/2)*(cos(2*e + 2*f*x)*85i + cos(4*e + 4*f*x)*20i + cos(6*e + 6*f*x)*3i +
95*sin(2*e + 2*f*x) + 20*sin(4*e + 4*f*x) + 3*sin(6*e + 6*f*x) - 60i))/(240
*a^3*c*f*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x
) + 1))^(1/2))
```

3.1032 $\int \frac{1}{(a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=193

$$\frac{i}{7f(a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))^{3/2}}+\frac{i}{7af(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{3/2}}+\frac{}{21a^2}$$

```
[Out] 8/21*tan(f*x+e)/a^3/c/f/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(1/2)+1
/7*I/f/(a+I*a*tan(f*x+e))^(7/2)/(c-I*c*tan(f*x+e))^(3/2)+1/7*I/a/f/(a+I*a*t
an(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(3/2)+4/21*tan(f*x+e)/a^2/f/(a+I*a*tan(
f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.11, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3604, 47, 40, 39}

$$\frac{8\tan(e+fx)}{21a^3cf\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}+\frac{4\tan(e+fx)}{21a^2f(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}+\frac{i}{7af(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{3/2}}+\frac{i}{7f(a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^(7/2)*(c - I*c*Tan[e + f*x])^(3/2)),x]
```

```
[Out] (I/7)/(f*(a + I*a*Tan[e + f*x])^(7/2)*(c - I*c*Tan[e + f*x])^(3/2)) + (I/7)
/(a*f*(a + I*a*Tan[e + f*x])^(5/2)*(c - I*c*Tan[e + f*x])^(3/2)) + (4*Tan[e
 + f*x])/(21*a^2*f*(a + I*a*Tan[e + f*x])^(3/2)*(c - I*c*Tan[e + f*x])^(3/2
)) + (8*Tan[e + f*x])/(21*a^3*c*f*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*T
an[e + f*x]])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Dist[(2*m +
3)/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
```

```
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{1}{(a+iax)^{9/2}(c-icx)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{i}{7f(a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))^{3/2}} + \frac{(5}{\quad}$$

$$= \frac{i}{7f(a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))^{3/2}} + \frac{}{7c}$$

$$= \frac{i}{7f(a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))^{3/2}} + \frac{}{7c}$$

$$= \frac{i}{7f(a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))^{3/2}} + \frac{}{7c}$$

Mathematica [A]
time = 3.53, size = 151, normalized size = 0.78

$$\frac{\sec^3(e+fx)(\cos(2(e+fx))+i\sin(2(e+fx)))(-140\cos(e+fx)+42\cos(3(e+fx))+2\cos(5(e+fx))-70i\sin(e+fx)+63i\sin(3(e+fx))+5i\sin(5(e+fx)))\sqrt{c-ic\tan(e+fx)}}{336a^3c^2f(-i+\tan(e+fx))^3\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^(7/2)*(c - I*c*Tan[e + f*x])^(3/2)),x]
```

```
[Out] (Sec[e + f*x]^3*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)])*(-140*Cos[e + f*x]
+ 42*Cos[3*(e + f*x)] + 2*Cos[5*(e + f*x)] - (70*I)*Sin[e + f*x] + (63*I)*S
in[3*(e + f*x)] + (5*I)*Sin[5*(e + f*x)])*Sqrt[c - I*c*Tan[e + f*x]])/(336*
a^3*c^2*f*(-I + Tan[e + f*x])^3*Sqrt[a + I*a*Tan[e + f*x]])
```

Maple [A]
time = 0.39, size = 141, normalized size = 0.73

method	result
derivativedivides	$\frac{\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\left(16i(\tan^6(fx+e))-8(\tan^7(fx+e))+40i(\tan^4(f\right.}{21f\,a^4c^2(-\tan(fx+e)+i)^5(\tan(fx+e)+}$
default	$\frac{\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\left(16i(\tan^6(fx+e))-8(\tan^7(fx+e))+40i(\tan^4(f\right.}{21f\,a^4c^2(-\tan(fx+e)+i)^5(\tan(fx+e)+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))^(7/2)/(c-I*c*tan(f*x+e))^(3/2),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/21/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)/a^4/c^2*(16*I
*tan(f*x+e)^6-8*tan(f*x+e)^7+40*I*tan(f*x+e)^4-12*tan(f*x+e)^5+30*I*tan(f*x
+e)^2+5*tan(f*x+e)^3+6*I+9*tan(f*x+e))/(-tan(f*x+e)+I)^5/(tan(f*x+e)+I)^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(7/2)/(c-I*c*tan(f*x+e))^(3/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 1.52, size = 155, normalized size = 0.80

$$\frac{\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\,\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\,\left(-7i\,e^{(12i\,fx+12i\,e)}-112i\,e^{(10i\,fx+10i\,e)}-192i\,e^{(9i\,fx+9i\,e)}+105i\,e^{(8i\,fx+8i\,e)}-192i\,e^{(7i\,fx+7i\,e)}+280i\,e^{(6i\,fx+6i\,e)}+91i\,e^{(4i\,fx+4i\,e)}+24i\,e^{(2i\,fx+2i\,e)}+3i\right)e^{(-7i\,fx-7i\,e)}}{672\,a^4c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(7/2)/(c-I*c*tan(f*x+e))^(3/2),x, algorithm=
"fricas")
```

```
[Out] 1/672*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*(
-7*I*e^(12*I*f*x + 12*I*e) - 112*I*e^(10*I*f*x + 10*I*e) - 192*I*e^(9*I*f*x
 + 9*I*e) + 105*I*e^(8*I*f*x + 8*I*e) - 192*I*e^(7*I*f*x + 7*I*e) + 280*I*e
^(6*I*f*x + 6*I*e) + 91*I*e^(4*I*f*x + 4*I*e) + 24*I*e^(2*I*f*x + 2*I*e) +
3*I)*e^(-7*I*f*x - 7*I*e)/(a^4*c^2*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**(7/2)/(c-I*c*tan(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(7/2)/(c-I*c*tan(f*x+e))^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate(1/((I*a*tan(f*x + e) + a)^(7/2)*(-I*c*tan(f*x + e) + c)^(3/2)), x
)
```

Mupad [**B**]
time = 6.52, size = 186, normalized size = 0.96

$$\frac{\sqrt{\frac{a\,\left(\cos\left(2\,e+2\,f\,x\right)+1+\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\left(\cos\left(2\,e+2\,f\,x\right)\,203{}\mathrm{i}+\cos\left(4\,e+4\,f\,x\right)\,70{}\mathrm{i}+\cos\left(6\,e+6\,f\,x\right)\,21{}\mathrm{i}+\cos\left(8\,e+8\,f\,x\right)\,3{}\mathrm{i}+217\,\sin\left(2\,e+2\,f\,x\right)+70\,\sin\left(4\,e+4\,f\,x\right)+21\,\sin\left(6\,e+6\,f\,x\right)+3\,\sin\left(8\,e+8\,f\,x\right)-105{}\mathrm{i}\right)}{672\,a^4\,c\,f\,\sqrt{\frac{c\,\left(\cos\left(2\,e+2\,f\,x\right)+1-\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^(7/2)*(c - c*tan(e + f*x)*1i)^(3/2)),x)
```

```
[Out] (((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^
(1/2)*(cos(2*e + 2*f*x)*203i + cos(4*e + 4*f*x)*70i + cos(6*e + 6*f*x)*21i
+ cos(8*e + 8*f*x)*3i + 217*sin(2*e + 2*f*x) + 70*sin(4*e + 4*f*x) + 21*sin
(6*e + 6*f*x) + 3*sin(8*e + 8*f*x) - 105i))/(672*a^4*c*f*((c*(cos(2*e + 2*f
*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

3.1033 $\int \frac{(a+ia\tan(e+fx))^{11/2}}{(c-ic\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=304

$$\frac{63ia^{11/2}\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{c^{5/2}f}-\frac{2ia(a+ia\tan(e+fx))^{9/2}}{5f(c-ic\tan(e+fx))^{5/2}}+\frac{6ia^2(a+ia\tan(e+fx))^{7/2}}{5cf(c-ic\tan(e+fx))^{3/2}}-4$$

```
[Out] 63*I*a^(11/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*c*tan(f*
x+e))^(1/2))/c^(5/2)/f-63/2*I*a^5*(a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e
))^(1/2)/c^3/f-21/2*I*a^4*(c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^(3/2)
/c^3/f-42/5*I*a^3*(a+I*a*tan(f*x+e))^(5/2)/c^2/f/(c-I*c*tan(f*x+e))^(1/2)-2
/5*I*a*(a+I*a*tan(f*x+e))^(9/2)/f/(c-I*c*tan(f*x+e))^(5/2)+6/5*I*a^2*(a+I*a
*tan(f*x+e))^(7/2)/c/f/(c-I*c*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.17, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3604, 49, 52, 65, 223, 209}

$$\frac{63ia^{11/2}\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{c^{5/2}f}-\frac{63ia^5\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{2c^3f}-\frac{21ia^4(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}{2c^3f}-\frac{42ia^3(a+ia\tan(e+fx))^{5/2}}{5c^2f\sqrt{c-ic\tan(e+fx)}}+\frac{6ia^2(a+ia\tan(e+fx))^{7/2}}{5cf(c-ic\tan(e+fx))^{3/2}}-\frac{2ia(a+ia\tan(e+fx))^{9/2}}{5f(c-ic\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^(11/2)/(c - I*c*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((63*I)*a^(11/2)*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[
c - I*c*Tan[e + f*x]])])/(c^(5/2)*f) - (((2*I)/5)*a*(a + I*a*Tan[e + f*x])^
(9/2))/(f*(c - I*c*Tan[e + f*x])^(5/2)) + (((6*I)/5)*a^2*(a + I*a*Tan[e + f
*x])^(7/2))/(c*f*(c - I*c*Tan[e + f*x])^(3/2)) - (((42*I)/5)*a^3*(a + I*a*T
an[e + f*x])^(5/2))/(c^2*f*Sqrt[c - I*c*Tan[e + f*x]]) - (((63*I)/2)*a^5*Sq
rt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/(c^3*f) - (((21*I)/2)*
a^4*(a + I*a*Tan[e + f*x])^(3/2)*Sqrt[c - I*c*Tan[e + f*x]])/(c^3*f)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1 + 1))), x] + Dist[n*((b*c - a*d)/(
```

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^{11/2}}{(c-ic\tan(e+fx))^{5/2}}\,dx=\frac{(ac)\text{Subst}\left(\int \frac{(a+iax)^{9/2}}{(c-icx)^{7/2}}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=-\frac{2ia(a+ia\tan(e+fx))^{9/2}}{5f(c-ic\tan(e+fx))^{5/2}}-\frac{(9a^2)\,\text{Subst}\left(\int \frac{(a+iax)^{7/2}}{(c-icx)^{5/2}}\,dx,x,\tan(e+fx)\right)}{5f}$$

$$=-\frac{2ia(a+ia\tan(e+fx))^{9/2}}{5f(c-ic\tan(e+fx))^{5/2}}+\frac{6ia^2(a+ia\tan(e+fx))^{7/2}}{5cf(c-ic\tan(e+fx))^{3/2}}+\frac{(21a^3)\,\text{Subs}}{}$$

$$=-\frac{2ia(a+ia\tan(e+fx))^{9/2}}{5f(c-ic\tan(e+fx))^{5/2}}+\frac{6ia^2(a+ia\tan(e+fx))^{7/2}}{5cf(c-ic\tan(e+fx))^{3/2}}-\frac{42ia^3(a+i}{5c^2f\sqrt{c-}}$$

$$=-\frac{2ia(a+ia\tan(e+fx))^{9/2}}{5f(c-ic\tan(e+fx))^{5/2}}+\frac{6ia^2(a+ia\tan(e+fx))^{7/2}}{5cf(c-ic\tan(e+fx))^{3/2}}-\frac{42ia^3(a+i}{5c^2f\sqrt{c-}}$$

$$=-\frac{2ia(a+ia\tan(e+fx))^{9/2}}{5f(c-ic\tan(e+fx))^{5/2}}+\frac{6ia^2(a+ia\tan(e+fx))^{7/2}}{5cf(c-ic\tan(e+fx))^{3/2}}-\frac{42ia^3(a+i}{5c^2f\sqrt{c-}}$$

$$=-\frac{2ia(a+ia\tan(e+fx))^{9/2}}{5f(c-ic\tan(e+fx))^{5/2}}+\frac{6ia^2(a+ia\tan(e+fx))^{7/2}}{5cf(c-ic\tan(e+fx))^{3/2}}-\frac{42ia^3(a+i}{5c^2f\sqrt{c-}}$$

$$=-\frac{2ia(a+ia\tan(e+fx))^{9/2}}{5f(c-ic\tan(e+fx))^{5/2}}+\frac{6ia^2(a+ia\tan(e+fx))^{7/2}}{5cf(c-ic\tan(e+fx))^{3/2}}-\frac{42ia^3(a+i}{5c^2f\sqrt{c-}}$$

$$=\frac{63ia^{11/2}\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{c^{5/2}f}-\frac{2ia(a+ia\tan(e+fx))^{9/2}}{5f(c-ic\tan(e+fx))^{5/2}}$$

Mathematica [A]
time = 13.03, size = 459, normalized size = 1.51

$$\frac{63ie^{-i(6e+fx)}\sqrt{e^{ifx}}\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\,\text{ArcTan}\left(e^{i(e+fx)}\right)(a+ia\tan(e+fx))^{11/2}}{c^2\sqrt{\frac{c}{1+e^{2i(e+fx)}}}\,f\sec^{\frac{11}{2}}(e+fx)(\cos(fx)+i\sin(fx))^{11/2}}+\frac{\cos^5(e+fx)\left(\cos(6fx)\left(-\frac{4i\cos(e)}{5c^3}+\frac{4\sin(e)}{5c^3}\right)+\cos(4fx)\left(\frac{16i\cos(e)}{5c^3}+\frac{16\sin(e)}{5c^3}\right)+\cos(2fx)\left(-\frac{20i\cos(3e)}{c^3}-\frac{20\sin(3e)}{c^3}\right)+\sec(e)(64\cos(e)+i\sin(e))\left(-\frac{i\cos(5e)}{2c^3}-\frac{\sin(5e)}{2c^3}\right)+\sec(e)\sec(e+fx)\left(\frac{\cos(5e)}{2c^3}-\frac{i\sin(5e)}{2c^3}\right)\sin(fx)+\left(\frac{20\cos(3e)}{c^3}-\frac{20i\sin(3e)}{c^3}\right)\sin(2fx)+\left(-\frac{16\cos(e)}{5c^3}+\frac{16i\sin(e)}{5c^3}\right)\sin(4fx)+\left(\frac{4\cos(e)}{5c^3}+\frac{4i\sin(e)}{5c^3}\right)\sin(6fx)\right)\sqrt{\sec(e+fx)(c\cos(e+fx)-ic\sin(e+fx))}\,(a+ia\tan(e+fx))^{11/2}}{f(\cos(fx)+i\sin(fx))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(11/2)/(c - I*c*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((63*I)*Sqrt[E^(I*f*x)]*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*Arc
Tan[E^(I*(e + f*x))]*(a + I*a*Tan[e + f*x])^(11/2))/(c^2*E^(I*(6*e + f*x))*
```

```
Sqrt[c/(1 + E^((2*I)*(e + f*x)))]*f*Sec[e + f*x]^(11/2)*(Cos[f*x] + I*Sin[f
*x])^(11/2)) + (Cos[e + f*x]^5*(Cos[6*f*x]*((((-4*I)/5)*Cos[e])/c^3 + (4*Si
n[e])/(5*c^3)) + Cos[4*f*x]*((((16*I)/5)*Cos[e])/c^3 + (16*Sin[e])/(5*c^3))
 + Cos[2*f*x]*(((-20*I)*Cos[3*e])/c^3 - (20*Sin[3*e])/c^3) + Sec[e]*(64*Cos
[e] + I*Sin[e])*(((-1/2*I)*Cos[5*e])/c^3 - Sin[5*e]/(2*c^3)) + Sec[e]*Sec[e
 + f*x]*(Cos[5*e]/(2*c^3) - ((I/2)*Sin[5*e])/c^3)*Sin[f*x] + ((20*Cos[3*e])
/c^3 - ((20*I)*Sin[3*e])/c^3)*Sin[2*f*x] + ((-16*Cos[e])/(5*c^3) + (((16*I)
/5)*Sin[e])/c^3)*Sin[4*f*x] + ((4*Cos[e])/(5*c^3) + (((4*I)/5)*Sin[e])/c^3)
*Sin[6*f*x])*Sqrt[Sec[e + f*x]*(c*Cos[e + f*x] - I*c*Sin[e + f*x])]*(a + I*
a*Tan[e + f*x])^(11/2))/(f*(Cos[f*x] + I*Sin[f*x])^5)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(244) = 488$.
time = 0.34, size = 490, normalized size = 1.61

method	result
derivativedivides	$-\frac{\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,a^{5}\left(1260 i\ln\left(\frac{ca\tan(fx+e)+\sqrt{ac\left(1+\tan^{2}\right.}}{\sqrt{ac}}\right.\right.}{}$
default	$-\frac{\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,a^{5}\left(1260 i\ln\left(\frac{ca\tan(fx+e)+\sqrt{ac\left(1+\tan^{2}\right.}}{\sqrt{ac}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(11/2)/(c-I*c*tan(f*x+e))^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/10/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^5/c^3*(126
0*I*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2
))*a*c*tan(f*x+e)^3+315*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*
c)^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^4+60*I*(a*c*(1+tan(f*x+e)^2))^(1/2)*(
a*c)^(1/2)*tan(f*x+e)^4-5*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+
e)^5-1260*I*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a
*c)^(1/2))*a*c*tan(f*x+e)-1890*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1
/2)*(a*c)^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^2-1964*I*(a*c*(1+tan(f*x+e)^2)
)^(1/2)*(a*c)^(1/2)*tan(f*x+e)^2-866*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/
2)*tan(f*x+e)^3+315*a*c*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*
c)^(1/2))/(a*c)^(1/2))+496*I*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)+1659*
tan(f*x+e)*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c*(1+tan(f*x+e)^2))
^(1/2)/(a*c)^(1/2)/(tan(f*x+e)+I)^4
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1159 vs. $2(238) = 476$.
time = 0.61, size = 1159, normalized size = 3.81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(11/2)/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="
maxima")
```

```
[Out] 10*(630*(a^5*cos(4*f*x + 4*e) + 2*a^5*cos(2*f*x + 2*e) + I*a^5*sin(4*f*x +
4*e) + 2*I*a^5*sin(2*f*x + 2*e) + a^5)*arctan2(cos(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))), sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
))) + 1) + 630*(a^5*cos(4*f*x + 4*e) + 2*a^5*cos(2*f*x + 2*e) + I*a^5*sin(4
*f*x + 4*e) + 2*I*a^5*sin(2*f*x + 2*e) + a^5)*arctan2(cos(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))), -sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) + 1) - 32*(a^5*cos(4*f*x + 4*e) + 2*a^5*cos(2*f*x + 2*e) + I*a^
5*sin(4*f*x + 4*e) + 2*I*a^5*sin(2*f*x + 2*e) + a^5)*cos(5/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) + 20*(8*a^5*cos(4*f*x + 4*e) + 16*a^5*cos(2*
f*x + 2*e) + 8*I*a^5*sin(4*f*x + 4*e) + 16*I*a^5*sin(2*f*x + 2*e) - 9*a^5)*
cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 60*(16*a^5*cos(4*f*x
 + 4*e) + 32*a^5*cos(2*f*x + 2*e) + 16*I*a^5*sin(4*f*x + 4*e) + 32*I*a^5*si
n(2*f*x + 2*e) + 21*a^5)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) + 315*(I*a^5*cos(4*f*x + 4*e) + 2*I*a^5*cos(2*f*x + 2*e) - a^5*sin(4*f*x
 + 4*e) - 2*a^5*sin(2*f*x + 2*e) + I*a^5)*log(cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e)))^2 + 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 315
*(-I*a^5*cos(4*f*x + 4*e) - 2*I*a^5*cos(2*f*x + 2*e) + a^5*sin(4*f*x + 4*e)
 + 2*a^5*sin(2*f*x + 2*e) - I*a^5)*log(cos(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
 - 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 32*(-I*a^5
*cos(4*f*x + 4*e) - 2*I*a^5*cos(2*f*x + 2*e) + a^5*sin(4*f*x + 4*e) + 2*a^5
*sin(2*f*x + 2*e) - I*a^5)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) + 20*(8*I*a^5*cos(4*f*x + 4*e) + 16*I*a^5*cos(2*f*x + 2*e) - 8*a^5*sin
(4*f*x + 4*e) - 16*a^5*sin(2*f*x + 2*e) - 9*I*a^5)*sin(3/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))) + 60*(-16*I*a^5*cos(4*f*x + 4*e) - 32*I*a^5*co
s(2*f*x + 2*e) + 16*a^5*sin(4*f*x + 4*e) + 32*a^5*sin(2*f*x + 2*e) - 21*I*a
^5)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/(
(-200*I*c^3*cos(4*f*x + 4*e) - 400*I*c^3*cos(2*f*x + 2*e) + 200*c^3*sin(4*f
*x + 4*e) + 400*c^3*sin(2*f*x + 2*e) - 200*I*c^3)*f)
```

Fricas [A]
time = 1.26, size = 466, normalized size = 1.53

$$-\frac{315\,\left(c^3 f e^{(2i\,fx+2i\,e)} + c^3 f\right)\sqrt{\frac{a^{11}}{c^5 f^2}}\,\log\left(\frac{4\left(2\left(a^5 e^{(3i\,fx+3i\,e)} + a^5 e^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}} - \left(i\,c^3 f e^{(2i\,fx+2i\,e)} - i\,c^3 f\right)\sqrt{\frac{a^{11}}{c^5 f^2}}\right)}{a^5 e^{(2i\,fx+2i\,e)} + a^5}\right) - 315\,\left(c^3 f e^{(2i\,fx+2i\,e)} + c^3 f\right)\sqrt{\frac{a^{11}}{c^5 f^2}}\,\log\left(\frac{4\left(2\left(a^5 e^{(3i\,fx+3i\,e)} + a^5 e^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}} - \left(-i\,c^3 f e^{(2i\,fx+2i\,e)} + i\,c^3 f\right)\sqrt{\frac{a^{11}}{c^5 f^2}}\right)}{a^5 e^{(2i\,fx+2i\,e)} + a^5}\right) + 4\left(8i\,a^5 e^{(9i\,fx+9i\,e)} - 24i\,a^5 e^{(7i\,fx+7i\,e)} + 168i\,a^5 e^{(5i\,fx+5i\,e)} + 525i\,a^5 e^{(3i\,fx+3i\,e)} + 315i\,a^5 e^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{20\,\left(c^3 f e^{(2i\,fx+2i\,e)} + c^3 f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(11/2)/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="
fricas")
```

```
[Out] -1/20*(315*(c^3*f*e^(2*I*f*x + 2*I*e) + c^3*f)*sqrt(a^11/(c^5*f^2))*log(4*(
2*(a^5*e^(3*I*f*x + 3*I*e) + a^5*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*
e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - (I*c^3*f*e^(2*I*f*x + 2*I*e) -
 I*c^3*f)*sqrt(a^11/(c^5*f^2)))/(a^5*e^(2*I*f*x + 2*I*e) + a^5)) - 315*(c^3
*f*e^(2*I*f*x + 2*I*e) + c^3*f)*sqrt(a^11/(c^5*f^2))*log(4*(2*(a^5*e^(3*I*f
*x + 3*I*e) + a^5*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c
/(e^(2*I*f*x + 2*I*e) + 1)) - (-I*c^3*f*e^(2*I*f*x + 2*I*e) + I*c^3*f)*sqrt
(a^11/(c^5*f^2)))/(a^5*e^(2*I*f*x + 2*I*e) + a^5)) + 4*(8*I*a^5*e^(9*I*f*x
+ 9*I*e) - 24*I*a^5*e^(7*I*f*x + 7*I*e) + 168*I*a^5*e^(5*I*f*x + 5*I*e) + 5
25*I*a^5*e^(3*I*f*x + 3*I*e) + 315*I*a^5*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*
x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(c^3*f*e^(2*I*f*x + 2*I
*e) + c^3*f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(11/2)/(c-I*c*tan(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(11/2)/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^(11/2)/(-I*c*tan(f*x + e) + c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+a\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{11/2}}{{\left(c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(11/2)/(c - c*tan(e + f*x)*1i)^(5/2),x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^(11/2)/(c - c*tan(e + f*x)*1i)^(5/2), x)
```

3.1034 $\int \frac{(a+ia\tan(e+fx))^{9/2}}{(c-ic\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=253

$$\frac{14ia^{9/2}\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{c^{5/2}f}-\frac{2ia(a+ia\tan(e+fx))^{7/2}}{5f(c-ic\tan(e+fx))^{5/2}}+\frac{14ia^2(a+ia\tan(e+fx))^{5/2}}{15cf(c-ic\tan(e+fx))^{3/2}}-1$$

```
[Out] 14*I*a^(9/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*c*tan(f*x
+e))^(1/2))/c^(5/2)/f-7*I*a^4*(a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))^(
1/2)/c^3/f-14/3*I*a^3*(a+I*a*tan(f*x+e))^(3/2)/c^2/f/(c-I*c*tan(f*x+e))^(1/
2)-2/5*I*a*(a+I*a*tan(f*x+e))^(7/2)/f/(c-I*c*tan(f*x+e))^(5/2)+14/15*I*a^2*
(a+I*a*tan(f*x+e))^(5/2)/c/f/(c-I*c*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.15, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3604, 49, 52, 65, 223, 209}

$$\frac{14ia^{9/2}\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{c^{5/2}f}-\frac{7ia^4\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{c^3f}-\frac{14ia^3(a+ia\tan(e+fx))^{3/2}}{3c^2f\sqrt{c-ic\tan(e+fx)}}+\frac{14ia^2(a+ia\tan(e+fx))^{5/2}}{15cf(c-ic\tan(e+fx))^{3/2}}-\frac{2ia(a+ia\tan(e+fx))^{7/2}}{5f(c-ic\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^(9/2)/(c - I*c*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((14*I)*a^(9/2)*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c
 - I*c*Tan[e + f*x]])])/(c^(5/2)*f) - (((2*I)/5)*a*(a + I*a*Tan[e + f*x])^(
7/2))/(f*(c - I*c*Tan[e + f*x])^(5/2)) + (((14*I)/15)*a^2*(a + I*a*Tan[e +
f*x])^(5/2))/(c*f*(c - I*c*Tan[e + f*x])^(3/2)) - (((14*I)/3)*a^3*(a + I*a*
Tan[e + f*x])^(3/2))/(c^2*f*Sqrt[c - I*c*Tan[e + f*x]]) - ((7*I)*a^4*Sqrt[a
 + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/(c^3*f)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^{9/2}}{(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{(a+iax)^{7/2}}{(c-icx)^{7/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{2ia(a+ia\tan(e+fx))^{7/2}}{5f(c-ic\tan(e+fx))^{5/2}} - \frac{(7a^2)\,\text{Subst}\left(\int \frac{(a+iax)^{5/2}}{(c-icx)^{5/2}}\,dx, x, \tan(e+fx)\right)}{5f}$$

$$= -\frac{2ia(a+ia\tan(e+fx))^{7/2}}{5f(c-ic\tan(e+fx))^{5/2}} + \frac{14ia^2(a+ia\tan(e+fx))^{5/2}}{15cf(c-ic\tan(e+fx))^{3/2}} + \frac{(7a^3)\,\text{Subst}}{}$$

$$= -\frac{2ia(a+ia\tan(e+fx))^{7/2}}{5f(c-ic\tan(e+fx))^{5/2}} + \frac{14ia^2(a+ia\tan(e+fx))^{5/2}}{15cf(c-ic\tan(e+fx))^{3/2}} - \frac{14ia^3(a+i}{3c^2f\sqrt{c-}}$$

$$= -\frac{2ia(a+ia\tan(e+fx))^{7/2}}{5f(c-ic\tan(e+fx))^{5/2}} + \frac{14ia^2(a+ia\tan(e+fx))^{5/2}}{15cf(c-ic\tan(e+fx))^{3/2}} - \frac{14ia^3(a+i}{3c^2f\sqrt{c-}}$$

$$= -\frac{2ia(a+ia\tan(e+fx))^{7/2}}{5f(c-ic\tan(e+fx))^{5/2}} + \frac{14ia^2(a+ia\tan(e+fx))^{5/2}}{15cf(c-ic\tan(e+fx))^{3/2}} - \frac{14ia^3(a+i}{3c^2f\sqrt{c-}}$$

$$= -\frac{2ia(a+ia\tan(e+fx))^{7/2}}{5f(c-ic\tan(e+fx))^{5/2}} + \frac{14ia^2(a+ia\tan(e+fx))^{5/2}}{15cf(c-ic\tan(e+fx))^{3/2}} - \frac{14ia^3(a+i}{3c^2f\sqrt{c-}}$$

$$= \frac{14ia^{9/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{c^{5/2}f} - \frac{2ia(a+ia\tan(e+fx))^{7/2}}{5f(c-ic\tan(e+fx))^{5/2}}$$

Mathematica [A]
time = 11.51, size = 390, normalized size = 1.54

$$\frac{14ie^{-i(5e+fx)}\sqrt{e^{ifx}}\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\,\text{ArcTan}\left(e^{i(e+fx)}\right)(a+ia\tan(e+fx))^{9/2}}{c^2\sqrt{\frac{c}{1+e^{2i(e+fx)}}}\,f\sec^{\frac{9}{2}}(e+fx)(\cos(fx)+i\sin(fx))^{9/2}} + \frac{\cos^4(e+fx)\left(-\frac{7i\cos(4e)}{c^3}+\frac{14i\cos(4fx)}{15c^3}+\cos(2fx)\left(-\frac{14i\cos(2e)}{3c^3}-\frac{14\sin(2e)}{3c^3}\right)+\cos(6fx)\left(-\frac{2i\cos(2e)}{5c^3}+\frac{2\sin(2e)}{5c^3}\right)-\frac{7\sin(4e)}{c^3}+\left(\frac{14\cos(2e)}{3c^3}-\frac{14i\sin(2e)}{3c^3}\right)\sin(2fx)-\frac{14\sin(4fx)}{15c^3}+\left(\frac{2\cos(2e)}{5c^3}+\frac{2i\sin(2e)}{5c^3}\right)\sin(6fx)\right)\sqrt{\sec(e+fx)(c\cos(e+fx)-ic\sin(e+fx))}\,(a+ia\tan(e+fx))^{9/2}}{f(\cos(fx)+i\sin(fx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(9/2)/(c - I*c*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((14*I)*Sqrt[E^(I*f*x)]*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*Arc
Tan[E^(I*(e + f*x))]*(a + I*a*Tan[e + f*x])^(9/2))/(c^2*E^(I*(5*e + f*x))*S
qrt[c/(1 + E^((2*I)*(e + f*x)))]*f*Sec[e + f*x]^(9/2)*(Cos[f*x] + I*Sin[f*x
])^(9/2)) + (Cos[e + f*x]^4*(((-7*I)*Cos[4*e])/c^3 + (((14*I)/15)*Cos[4*f*x
```

```
])/c^3 + Cos[2*f*x]*((((-14*I)/3)*Cos[2*e])/c^3 - (14*Sin[2*e])/(3*c^3)) +
Cos[6*f*x]*((((-2*I)/5)*Cos[2*e])/c^3 + (2*Sin[2*e])/(5*c^3)) - (7*Sin[4*e]
)/c^3 + ((14*Cos[2*e])/(3*c^3) - (((14*I)/3)*Sin[2*e])/c^3)*Sin[2*f*x] - (1
4*Sin[4*f*x])/(15*c^3) + ((2*Cos[2*e])/(5*c^3) + (((2*I)/5)*Sin[2*e])/c^3)*
Sin[6*f*x])*Sqrt[Sec[e + f*x]*(c*Cos[e + f*x] - I*c*Sin[e + f*x])]*(a + I*a
*Tan[e + f*x])^(9/2))/(f*(Cos[f*x] + I*Sin[f*x])^4)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(204) = 408$.
time = 0.35, size = 460, normalized size = 1.82

method	result
derivativedivides	$-\frac{\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,a^{4}\left(420i\ln\left(\frac{ca\tan(fx+e)+\sqrt{ac\left(1+\tan^{2}\right.}}{\sqrt{ac}}\right.\right.}{}$
default	$-\frac{\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,a^{4}\left(420i\ln\left(\frac{ca\tan(fx+e)+\sqrt{ac\left(1+\tan^{2}\right.}}{\sqrt{ac}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(9/2)/(c-I*c*tan(f*x+e))^(5/2),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/15/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^4/c^3*(420
*I*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2)
)*a*c*tan(f*x+e)^3+105*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c
)^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^4+15*I*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a
*c)^(1/2)*tan(f*x+e)^4-420*I*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2
)*(a*c)^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)-630*ln((c*a*tan(f*x+e)+(a*c*(1+t
an(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^2-658*I*(a*c*(
1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^2-292*(a*c)^(1/2)*(a*c*(1+tan
(f*x+e)^2))^(1/2)*tan(f*x+e)^3+105*a*c*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e
)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))+167*I*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a
*c)^(1/2)+548*tan(f*x+e)*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c*(1+
tan(f*x+e)^2))^(1/2)/(tan(f*x+e)+I)^4/(a*c)^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 826 vs. $2(199) = 398$.
time = 0.61, size = 826, normalized size = 3.26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(9/2)/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="m
axima")
```

```
[Out] 15*(210*(a^4*cos(2*f*x + 2*e) + I*a^4*sin(2*f*x + 2*e) + a^4)*arctan2(cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), sin(1/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) + 1) + 210*(a^4*cos(2*f*x + 2*e) + I*a^4*sin(2*f
*x + 2*e) + a^4)*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)), -sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) - 24*(a^4*co
s(2*f*x + 2*e) + I*a^4*sin(2*f*x + 2*e) + a^4)*cos(5/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) + 80*(a^4*cos(2*f*x + 2*e) + I*a^4*sin(2*f*x + 2*e
) + a^4)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 60*(6*a^4*c
os(2*f*x + 2*e) + 6*I*a^4*sin(2*f*x + 2*e) + 7*a^4)*cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))) + 105*(I*a^4*cos(2*f*x + 2*e) - a^4*sin(2*f*x
 + 2*e) + I*a^4)*log(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 105*(-I*a^4*cos(2*f*x + 2*e)
 + a^4*sin(2*f*x + 2*e) - I*a^4)*log(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 -
 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 24*(-I*a^4*c
os(2*f*x + 2*e) + a^4*sin(2*f*x + 2*e) - I*a^4)*sin(5/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))) + 80*(I*a^4*cos(2*f*x + 2*e) - a^4*sin(2*f*x + 2*
e) + I*a^4)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 60*(-6*I
*a^4*cos(2*f*x + 2*e) + 6*a^4*sin(2*f*x + 2*e) - 7*I*a^4)*sin(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((-450*I*c^3*cos(2*f*x
 + 2*e) + 450*c^3*sin(2*f*x + 2*e) - 450*I*c^3)*f)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(199) = 398$.
time = 1.28, size = 400, normalized size = 1.58

$$-\frac{105\,\sqrt{\frac{a^9}{c^5 f^2}}\,c^3 f \log\left(\frac{4\left(2\,\left(a^4 e^{(3i\,fx+3i\,e)}+a^4 e^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}-\left(i\,c^3 f e^{(2i\,fx+2i\,e)}-i\,c^3 f\right)\sqrt{\frac{a^9}{c^5 f^2}}\right)}{a^4 e^{(2i\,fx+2i\,e)}+a^4}\right)-105\,\sqrt{\frac{a^9}{c^5 f^2}}\,c^3 f \log\left(\frac{4\left(2\,\left(a^4 e^{(3i\,fx+3i\,e)}+a^4 e^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}-\left(-i\,c^3 f e^{(2i\,fx+2i\,e)}+i\,c^3 f\right)\sqrt{\frac{a^9}{c^5 f^2}}\right)}{a^4 e^{(2i\,fx+2i\,e)}+a^4}\right)+4\,\left(6i\,a^4 e^{(7i\,fx+7i\,e)}-14i\,a^4 e^{(5i\,fx+5i\,e)}+70i\,a^4 e^{(3i\,fx+3i\,e)}+105i\,a^4 e^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{30\,c^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(9/2)/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="f
ricas")
```

```
[Out] -1/30*(105*sqrt(a^9/(c^5*f^2))*c^3*f*log(4*(2*(a^4*e^(3*I*f*x + 3*I*e) + a^
4*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2
*I*e) + 1)) - (I*c^3*f*e^(2*I*f*x + 2*I*e) - I*c^3*f)*sqrt(a^9/(c^5*f^2)))/
(a^4*e^(2*I*f*x + 2*I*e) + a^4)) - 105*sqrt(a^9/(c^5*f^2))*c^3*f*log(4*(2*(
a^4*e^(3*I*f*x + 3*I*e) + a^4*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e)
+ 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - (-I*c^3*f*e^(2*I*f*x + 2*I*e) + I
*c^3*f)*sqrt(a^9/(c^5*f^2)))/(a^4*e^(2*I*f*x + 2*I*e) + a^4)) + 4*(6*I*a^4*
e^(7*I*f*x + 7*I*e) - 14*I*a^4*e^(5*I*f*x + 5*I*e) + 70*I*a^4*e^(3*I*f*x +
3*I*e) + 105*I*a^4*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(
c/(e^(2*I*f*x + 2*I*e) + 1)))/(c^3*f)
```

Sympy [**F(-2)**]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(9/2)/(c-I*c*tan(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7317 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(9/2)/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="g
iac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^(9/2)/(-I*c*tan(f*x + e) + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+a\tan(e+f\,x)\,1\mathrm{i}\right)^{9/2}}{\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(9/2)/(c - c*tan(e + f*x)*1i)^(5/2),x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^(9/2)/(c - c*tan(e + f*x)*1i)^(5/2), x)
```

3.1035 $\int \frac{(a+ia\tan(e+fx))^{7/2}}{(c-ic\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=204

$$\frac{2ia^{7/2}\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{c^{5/2}f}-\frac{2ia(a+ia\tan(e+fx))^{5/2}}{5f(c-ic\tan(e+fx))^{5/2}}+\frac{2ia^2(a+ia\tan(e+fx))^{3/2}}{3cf(c-ic\tan(e+fx))^{3/2}}-\frac{2ia}{c^2}$$

```
[Out] 2*I*a^(7/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*c*tan(f*x+
e))^(1/2))/c^(5/2)/f-2*I*a^3*(a+I*a*tan(f*x+e))^(1/2)/c^2/f/(c-I*c*tan(f*x+
e))^(1/2)-2/5*I*a*(a+I*a*tan(f*x+e))^(5/2)/f/(c-I*c*tan(f*x+e))^(5/2)+2/3*I
*a^2*(a+I*a*tan(f*x+e))^(3/2)/c/f/(c-I*c*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.13, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3604, 49, 65, 223, 209}

$$\frac{2ia^{7/2}\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c-ic\tan(e+fx)}}\right)}{c^{5/2}f}-\frac{2ia^3\sqrt{a+ia\tan(e+fx)}}{c^2f\sqrt{c-ic\tan(e+fx)}}+\frac{2ia^2(a+ia\tan(e+fx))^{3/2}}{3cf(c-ic\tan(e+fx))^{3/2}}-\frac{2ia(a+ia\tan(e+fx))^{5/2}}{5f(c-ic\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^(7/2)/(c - I*c*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((2*I)*a^(7/2)*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c
- I*c*Tan[e + f*x]])])/(c^(5/2)*f) - (((2*I)/5)*a*(a + I*a*Tan[e + f*x])^(5
/2))/(f*(c - I*c*Tan[e + f*x])^(5/2)) + (((2*I)/3)*a^2*(a + I*a*Tan[e + f*x
])^(3/2))/(c*f*(c - I*c*Tan[e + f*x])^(3/2)) - ((2*I)*a^3*Sqrt[a + I*a*Tan[
e + f*x]])/(c^2*f*Sqrt[c - I*c*Tan[e + f*x]])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+ia\tan(e+fx))^{7/2}}{(c-ic\tan(e+fx))^{5/2}}\,dx &= \frac{(ac)\text{Subst}\left(\int \frac{(a+iax)^{5/2}}{(c-icx)^{7/2}}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{2ia(a+ia\tan(e+fx))^{5/2}}{5f(c-ic\tan(e+fx))^{5/2}} - \frac{a^2\text{Subst}\left(\int \frac{(a+iax)^{3/2}}{(c-icx)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{2ia(a+ia\tan(e+fx))^{5/2}}{5f(c-ic\tan(e+fx))^{5/2}} + \frac{2ia^2(a+ia\tan(e+fx))^{3/2}}{3cf(c-ic\tan(e+fx))^{3/2}} + \frac{a^3\text{Subst}\left(\int \cdots\right.}{\cdots} \\
&= -\frac{2ia(a+ia\tan(e+fx))^{5/2}}{5f(c-ic\tan(e+fx))^{5/2}} + \frac{2ia^2(a+ia\tan(e+fx))^{3/2}}{3cf(c-ic\tan(e+fx))^{3/2}} - \frac{2ia^3\sqrt{a+ic\cdots}}{c^2f\sqrt{c-ic\cdots}} \\
&= -\frac{2ia(a+ia\tan(e+fx))^{5/2}}{5f(c-ic\tan(e+fx))^{5/2}} + \frac{2ia^2(a+ia\tan(e+fx))^{3/2}}{3cf(c-ic\tan(e+fx))^{3/2}} - \frac{2ia^3\sqrt{a+ic\cdots}}{c^2f\sqrt{c-ic\cdots}} \\
&= -\frac{2ia(a+ia\tan(e+fx))^{5/2}}{5f(c-ic\tan(e+fx))^{5/2}} + \frac{2ia^2(a+ia\tan(e+fx))^{3/2}}{3cf(c-ic\tan(e+fx))^{3/2}} - \frac{2ia^3\sqrt{a+ic\cdots}}{c^2f\sqrt{c-ic\cdots}} \\
&= \frac{2ia^{7/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{c^{5/2}f} - \frac{2ia(a+ia\tan(e+fx))^{5/2}}{5f(c-ic\tan(e+fx))^{5/2}} + \cdots
\end{aligned}
$$

Mathematica [**A**]
time = 9.70, size = 205, normalized size = 1.00

$$
\frac{2a^3\cos^2(e+fx)\left(\cos\left(\frac{1}{2}(e-4fx)\right)-i\sin\left(\frac{1}{2}(e-4fx)\right)\right)\left(4i\cos(e+fx)+9i\cos(3(e+fx))+6\sin(e+fx)-15i\text{ArcTan}\left(e^{i(e+fx)}\right)\cos(e+fx)(\cos(3(e+fx))-i\sin(3(e+fx)))+6\sin(3(e+fx))\right)\left(i\cos\left(\frac{1}{2}(e+6fx)\right)+\sin\left(\frac{1}{2}(e+6fx)\right)\right)(-i+\tan(e+fx))^3\sqrt{a+ia\tan(e+fx)}}{15c^2f\sqrt{c-ic\tan(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(7/2)/(c - I*c*Tan[e + f*x])^(5/2),x]
```

```
[Out] (2*a^3*Cos[e + f*x]^2*(Cos[(e - 4*f*x)/2] - I*Sin[(e - 4*f*x)/2])*((4*I)*Co
s[e + f*x] + (9*I)*Cos[3*(e + f*x)] + 6*Sin[e + f*x] - (15*I)*ArcTan[E^(I*(
e + f*x))]*Cos[e + f*x]*(Cos[3*(e + f*x)] - I*Sin[3*(e + f*x)]) + 6*Sin[3*(
e + f*x)])*(I*Cos[(e + 6*f*x)/2] + Sin[(e + 6*f*x)/2])*(-I + Tan[e + f*x])^
3*Sqrt[a + I*a*Tan[e + f*x]])/(15*c^2*f*Sqrt[c - I*c*Tan[e + f*x]])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 428 vs. 2(164) = 328.
time = 0.35, size = 429, normalized size = 2.10

method	result
derivativedivides	$-\frac{\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\ \sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\ a^{3}\left(60i\ln\left(\frac{ca\tan(fx+e)+\sqrt{ac\left(1+\tan^{2}\left(\right.\right.}}{\sqrt{ac}}\right.\right.}{}$
default	$-\frac{\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\ \sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\ a^{3}\left(60i\ln\left(\frac{ca\tan(fx+e)+\sqrt{ac\left(1+\tan^{2}\left(\right.\right.}}{\sqrt{ac}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(7/2)/(c-I*c*tan(f*x+e))^(5/2),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/15/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^3/c^3*(60*
I*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))
*a*c*tan(f*x+e)^3+15*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^
(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^4-60*I*ln((c*a*tan(f*x+e)+(a*c*(1+tan(f*
x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)-90*ln((c*a*tan(f*x+
e)+(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^2-
94*I*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^2-46*(a*c)^(1/2)*(
a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^3+15*a*c*ln((c*a*tan(f*x+e)+(a*c*(1+
tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))+26*I*(a*c*(1+tan(f*x+e)^2))^
(1/2)*(a*c)^(1/2)+74*tan(f*x+e)*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(
a*c*(1+tan(f*x+e)^2))^(1/2)/(tan(f*x+e)+I)^4/(a*c)^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(160) = 320$.
time = 0.59, size = 472, normalized size = 2.31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(7/2)/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="m
axima")
```

```
[Out] -1/30*(-30*I*a^3*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)), sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) - 30*I*a^3*ar
ctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), -sin(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 12*I*a^3*cos(5/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) - 20*I*a^3*cos(3/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) + 60*I*a^3*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) + 15*a^3*log(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) - 15*a^3*log(cos(1/2*arctan
```

```
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e)))^2 - 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) + 1) - 12*a^3*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2
0*a^3*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 60*a^3*sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)/(c^(5/2)*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 400 vs. 2(160) = 320.
time = 1.22, size = 400, normalized size = 1.96

$$-\frac{15\,c^3 f\sqrt{\frac{a^7}{c^5 f^2}}\log\left(\frac{4\left(2\left(a^3 e^{(3i fx+3i e)}+a^3 e^{(i fx+i e)}\right)\sqrt{\frac{a}{e^{(2i fx+2i e)}+1}}\sqrt{\frac{c}{e^{(2i fx+2i e)}+1}}-\left(i c^3 f e^{(2i fx+2i e)}-i c^3 f\right)\sqrt{\frac{a^7}{c^5 f^2}}\right)}{a^3 e^{(2i fx+2i e)}+a^3}\right)-15\,c^3 f\sqrt{\frac{a^7}{c^5 f^2}}\log\left(\frac{4\left(2\left(a^3 e^{(3i fx+3i e)}+a^3 e^{(i fx+i e)}\right)\sqrt{\frac{a}{e^{(2i fx+2i e)}+1}}\sqrt{\frac{c}{e^{(2i fx+2i e)}+1}}-\left(-i c^3 f e^{(2i fx+2i e)}+i c^3 f\right)\sqrt{\frac{a^7}{c^5 f^2}}\right)}{a^3 e^{(2i fx+2i e)}+a^3}\right)+4\left(3i\,a^3 e^{(7i fx+7i e)}-2i\,a^3 e^{(5i fx+5i e)}+10i\,a^3 e^{(3i fx+3i e)}+15i\,a^3 e^{(i fx+i e)}\right)\sqrt{\frac{a}{e^{(2i fx+2i e)}+1}}\sqrt{\frac{c}{e^{(2i fx+2i e)}+1}}}{30\,c^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(7/2)/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="f
ricas")
```

```
[Out] -1/30*(15*c^3*f*sqrt(a^7/(c^5*f^2))*log(4*(2*(a^3*e^(3*I*f*x + 3*I*e) + a^3
*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*
I*e) + 1)) - (I*c^3*f*e^(2*I*f*x + 2*I*e) - I*c^3*f)*sqrt(a^7/(c^5*f^2)))/(
a^3*e^(2*I*f*x + 2*I*e) + a^3)) - 15*c^3*f*sqrt(a^7/(c^5*f^2))*log(4*(2*(a^
3*e^(3*I*f*x + 3*I*e) + a^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) +
1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - (-I*c^3*f*e^(2*I*f*x + 2*I*e) + I*c
^3*f)*sqrt(a^7/(c^5*f^2)))/(a^3*e^(2*I*f*x + 2*I*e) + a^3)) + 4*(3*I*a^3*e^
(7*I*f*x + 7*I*e) - 2*I*a^3*e^(5*I*f*x + 5*I*e) + 10*I*a^3*e^(3*I*f*x + 3*I
*e) + 15*I*a^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e
^(2*I*f*x + 2*I*e) + 1)))/(c^3*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(7/2)/(c-I*c*tan(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(7/2)/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="g
iac")
```

[Out] integrate((I*a*tan(f*x + e) + a)^(7/2)/(-I*c*tan(f*x + e) + c)^(5/2), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+a\tan(e+f\,x)\ 1\mathrm{i})^{7/2}}{(c-c\tan\left(e+f\,x\right)\ 1\mathrm{i})^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*tan(e + f*x)*1i)^(7/2)/(c - c*tan(e + f*x)*1i)^(5/2),x)

[Out] int((a + a*tan(e + f*x)*1i)^(7/2)/(c - c*tan(e + f*x)*1i)^(5/2), x)

3.1036 $\int \frac{(a+ia\tan(e+fx))^{5/2}}{(c-ic\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=43

$$-\frac{i(a+ia\tan(e+fx))^{5/2}}{5f(c-ic\tan(e+fx))^{5/2}}$$

```
[Out] -1/5*I*(a+I*a*tan(f*x+e))^(5/2)/f/(c-I*c*tan(f*x+e))^(5/2)
```

Rubi [A]
time = 0.08, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {3604, 37}

$$-\frac{i(a+ia\tan(e+fx))^{5/2}}{5f(c-ic\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^(5/2)/(c - I*c*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-1/5*I)*(a + I*a*Tan[e + f*x])^(5/2))/(f*(c - I*c*Tan[e + f*x])^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+ia\tan(e+fx))^{5/2}}{(c-ic\tan(e+fx))^{5/2}}\,dx &= \frac{(ac)\text{Subst}\left(\int \frac{(a+iax)^{3/2}}{(c-icx)^{7/2}}\,dx, x, \tan(e+fx)\right)}{f}\\
&= -\frac{i(a+ia\tan(e+fx))^{5/2}}{5f(c-ic\tan(e+fx))^{5/2}}
\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 91 vs. 2(43) = 86.
time = 2.61, size = 91, normalized size = 2.12

$$\frac{a^2\cos(e+fx)(-i\cos(5e+7fx)+\sin(5e+7fx))\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{5c^3f(\cos(fx)+i\sin(fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(5/2)/(c - I*c*Tan[e + f*x])^(5/2),x]
```

```
[Out] (a^2*Cos[e + f*x]*((-I)*Cos[5*e + 7*f*x] + Sin[5*e + 7*f*x])*Sqrt[a + I*a*T
an[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/(5*c^3*f*(Cos[f*x] + I*Sin[f*x])^2
)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(34) = 68.
time = 0.38, size = 75, normalized size = 1.74

method	result	size
risch	$-\frac{ia^2\sqrt{\frac{a\,e^{2i(fx+e)}}{e^{2i(fx+e)}+1}}\,e^{4i(fx+e)}}{5c^2\sqrt{\frac{c}{e^{2i(fx+e)}+1}}\,f}$	65
derivativedivides	$-\frac{\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,a^2\left(1+\tan^2\left(fx+e\right)\right)\left(-\tan\left(fx+e\right)+i\right)}{5f\,c^3\left(\tan\left(fx+e\right)+i\right)^4}$	75
default	$-\frac{\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,a^2\left(1+\tan^2\left(fx+e\right)\right)\left(-\tan\left(fx+e\right)+i\right)}{5f\,c^3\left(\tan\left(fx+e\right)+i\right)^4}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(5/2),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/5/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^2/c^3*(1+ta
n(f*x+e)^2)*(-tan(f*x+e)+I)/(tan(f*x+e)+I)^4
```

Maxima [A]
time = 0.56, size = 41, normalized size = 0.95

$$\frac{\left(-i\,a^2\cos\left(5\,fx+5\,e\right)+a^2\sin\left(5\,fx+5\,e\right)\right)\sqrt{a}}{5\,c^{\frac{5}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="m
axima")
```

```
[Out] 1/5*(-I*a^2*cos(5*f*x + 5*e) + a^2*sin(5*f*x + 5*e))*sqrt(a)/(c^(5/2)*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(33) = 66.
time = 0.97, size = 75, normalized size = 1.74

$$\frac{\left(-i\,a^2 e^{(7i\,fx+7i\,e)} - i\,a^2 e^{(5i\,fx+5i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\,\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{5\,c^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="f
ricas")
```

```
[Out] 1/5*(-I*a^2*e^(7*I*f*x + 7*I*e) - I*a^2*e^(5*I*f*x + 5*I*e))*sqrt(a/(e^(2*I
*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(c^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan{(e+fx)}-i))^{\frac{5}{2}}}{(-ic\,(\tan{(e+fx)}+i))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(5/2)/(c-I*c*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral((I*a*(tan(e + f*x) - I))**(5/2)/(-I*c*(tan(e + f*x) + I))**(5/2),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="g
iac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^(5/2)/(-I*c*tan(f*x + e) + c)^(5/2), x)
```

Mupad [B]
time = 5.29, size = 114, normalized size = 2.65

$$-\frac{a^2\left(\cos\left(4\,e+4\,f\,x\right)+\sin\left(4\,e+4\,f\,x\right)\,1{}\mathrm{i}\right)\,\sqrt{\frac{a\,\left(\cos\left(2\,e+2\,f\,x\right)+1+\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,1{}\mathrm{i}}{5\,c^2\,f\,\sqrt{\frac{c\,\left(\cos\left(2\,e+2\,f\,x\right)+1-\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(5/2)/(c - c*tan(e + f*x)*1i)^(5/2),x)
```

```
[Out] -(a^2*(cos(4*e + 4*f*x) + sin(4*e + 4*f*x)*1i)*((a*(cos(2*e + 2*f*x) + sin(
2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2)*1i)/(5*c^2*f*((c*(cos(2
*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

3.1037 $\int \frac{(a+ia\tan(e+fx))^{3/2}}{(c-ic\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=90

$$-\frac{i(a+ia\tan(e+fx))^{3/2}}{5f(c-ic\tan(e+fx))^{5/2}}-\frac{i(a+ia\tan(e+fx))^{3/2}}{15cf(c-ic\tan(e+fx))^{3/2}}$$

```
[Out] -1/5*I*(a+I*a*tan(f*x+e))^(3/2)/f/(c-I*c*tan(f*x+e))^(5/2)-1/15*I*(a+I*a*ta
n(f*x+e))^(3/2)/c/f/(c-I*c*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.09, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {3604, 47, 37}

$$-\frac{i(a+ia\tan(e+fx))^{3/2}}{15cf(c-ic\tan(e+fx))^{3/2}}-\frac{i(a+ia\tan(e+fx))^{3/2}}{5f(c-ic\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^(3/2)/(c - I*c*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-1/5*I)*(a + I*a*Tan[e + f*x])^(3/2))/(f*(c - I*c*Tan[e + f*x])^(5/2)) -
((I/15)*(a + I*a*Tan[e + f*x])^(3/2))/(c*f*(c - I*c*Tan[e + f*x])^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
```

```
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^{3/2}}{(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{\sqrt{a+iax}}{(c-icx)^{7/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{i(a+ia\tan(e+fx))^{3/2}}{5f(c-ic\tan(e+fx))^{5/2}} + \frac{a\text{Subst}\left(\int \frac{\sqrt{a+iax}}{(c-icx)^{5/2}}\,dx, x, \tan(e+fx)\right)}{5f}$$

$$= -\frac{i(a+ia\tan(e+fx))^{3/2}}{5f(c-ic\tan(e+fx))^{5/2}} - \frac{i(a+ia\tan(e+fx))^{3/2}}{15cf(c-ic\tan(e+fx))^{3/2}}$$

Mathematica [A]
time = 2.18, size = 106, normalized size = 1.18

$$\frac{a\cos(e+fx)(\cos(fx)-i\sin(fx))(4\cos(e+fx)-i\sin(e+fx))(-i\cos(4e+5fx)+\sin(4e+5fx))\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{15c^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(3/2)/(c - I*c*Tan[e + f*x])^(5/2),x]
[Out] (a*Cos[e + f*x]*(Cos[f*x] - I*Sin[f*x])*(4*Cos[e + f*x] - I*Sin[e + f*x])*(
(-I)*Cos[4*e + 5*f*x] + Sin[4*e + 5*f*x])*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c
 - I*c*Tan[e + f*x]])/(15*c^3*f)
```

Maple [A]
time = 0.39, size = 71, normalized size = 0.79

method	result	size
derivativedivides	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a(1+\tan^2(fx+e))(4i+\tan(fx+e))}{15f\,c^3(\tan(fx+e)+i)^4}$	71
default	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a(1+\tan^2(fx+e))(4i+\tan(fx+e))}{15f\,c^3(\tan(fx+e)+i)^4}$	71
risch	$-\frac{ia\sqrt{\frac{a\,e^{2i(fx+e)}}{e^{2i(fx+e)}+1}}\,(3\,e^{4i(fx+e)}+5\,e^{2i(fx+e)})}{30c^2\sqrt{\frac{c}{e^{2i(fx+e)}+1}}\,f}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(5/2),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/15/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a/c^3*(1+tan
(f*x+e)^2)*(4*I+tan(f*x+e))/(tan(f*x+e)+I)^4
```

Maxima [A]
time = 0.64, size = 120, normalized size = 1.33

$$\frac{\left(-3i\,a\cos\left(\frac{5}{2}\,\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)\right)\right)-5i\,a\cos\left(\frac{3}{2}\,\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)\right)\right)+3\,a\sin\left(\frac{5}{2}\,\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)\right)\right)+5\,a\sin\left(\frac{3}{2}\,\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)\right)\right)\right)\sqrt{a}}{30\,c^{\frac{5}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="m
axima")
```

```
[Out] 1/30*(-3*I*a*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 5*I*a*c
os(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 3*a*sin(5/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) + 5*a*sin(3/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))))*sqrt(a)/(c^(5/2)*f)
```

Fricas [A]
time = 1.19, size = 84, normalized size = 0.93

$$\frac{\left(-3i\,ae^{\left(7i\,fx+7i\,e\right)}-8i\,ae^{\left(5i\,fx+5i\,e\right)}-5i\,ae^{\left(3i\,fx+3i\,e\right)}\right)\sqrt{\frac{a}{e^{\left(2i\,fx+2i\,e\right)}+1}}\sqrt{\frac{c}{e^{\left(2i\,fx+2i\,e\right)}+1}}}{30\,c^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="f
ricas")
```

```
[Out] 1/30*(-3*I*a*e^(7*I*f*x + 7*I*e) - 8*I*a*e^(5*I*f*x + 5*I*e) - 5*I*a*e^(3*I
*f*x + 3*I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e
) + 1))/(c^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(ia\left(\tan\left(e+fx\right)-i\right)\right)^{\frac{3}{2}}}{\left(-ic\left(\tan\left(e+fx\right)+i\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(3/2)/(c-I*c*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral((I*a*(tan(e + f*x) - I))**(3/2)/(-I*c*(tan(e + f*x) + I))**(5/2),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="g
iac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^(3/2)/(-I*c*tan(f*x + e) + c)^(5/2), x)
```

Mupad [B]
time = 5.47, size = 136, normalized size = 1.51

$$-\frac{a\,\sqrt{\frac{a\,\left(\cos\left(2\,e+2\,f\,x\right)+1+\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\left(-5\,\sin\left(2\,e+2\,f\,x\right)-3\,\sin\left(4\,e+4\,f\,x\right)+\cos\left(2\,e+2\,f\,x\right)\,5{}\mathrm{i}+\cos\left(4\,e+4\,f\,x\right)\,3{}\mathrm{i}\right)}{30\,c^2\,f\,\sqrt{\frac{c\,\left(\cos\left(2\,e+2\,f\,x\right)+1-\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(3/2)/(c - c*tan(e + f*x)*1i)^(5/2),x)
```

```
[Out] -(a*((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1
))^(1/2)*(cos(2*e + 2*f*x)*5i + cos(4*e + 4*f*x)*3i - 5*sin(2*e + 2*f*x) -
3*sin(4*e + 4*f*x)))/(30*c^2*f*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i
+ 1))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

3.1038 $\int \frac{\sqrt{a+ia\tan(e+fx)}}{(c-ic\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=136

$$-\frac{i\sqrt{a+ia\tan(e+fx)}}{5f(c-ic\tan(e+fx))^{5/2}}-\frac{2i\sqrt{a+ia\tan(e+fx)}}{15cf(c-ic\tan(e+fx))^{3/2}}-\frac{2i\sqrt{a+ia\tan(e+fx)}}{15c^2f\sqrt{c-ic\tan(e+fx)}}$$

```
[Out] -2/15*I*(a+I*a*tan(f*x+e))^(1/2)/c^2/f/(c-I*c*tan(f*x+e))^(1/2)-1/5*I*(a+I*
a*tan(f*x+e))^(1/2)/f/(c-I*c*tan(f*x+e))^(5/2)-2/15*I*(a+I*a*tan(f*x+e))^(1
/2)/c/f/(c-I*c*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.09, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {3604, 47, 37}

$$-\frac{2i\sqrt{a+ia\tan(e+fx)}}{15c^2f\sqrt{c-ic\tan(e+fx)}}-\frac{2i\sqrt{a+ia\tan(e+fx)}}{15cf(c-ic\tan(e+fx))^{3/2}}-\frac{i\sqrt{a+ia\tan(e+fx)}}{5f(c-ic\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[e + f*x]]/(c - I*c*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-1/5*I)*Sqrt[a + I*a*Tan[e + f*x]])/(f*(c - I*c*Tan[e + f*x])^(5/2)) - ((
(2*I)/15)*Sqrt[a + I*a*Tan[e + f*x]])/(c*f*(c - I*c*Tan[e + f*x])^(3/2)) -
(((2*I)/15)*Sqrt[a + I*a*Tan[e + f*x]])/(c^2*f*Sqrt[c - I*c*Tan[e + f*x]])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+ia\tan(e+fx)}}{(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{1}{\sqrt{a+iax}\,(c-icx)^{7/2}}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= -\frac{i\sqrt{a+ia\tan(e+fx)}}{5f(c-ic\tan(e+fx))^{5/2}} + \frac{(2a)\text{Subst}\left(\int \frac{1}{\sqrt{a+iax}\,(c-icx)^{5/2}}\,dx,x,\tan(e-\right.}{5f}$$

$$= -\frac{i\sqrt{a+ia\tan(e+fx)}}{5f(c-ic\tan(e+fx))^{5/2}} - \frac{2i\sqrt{a+ia\tan(e+fx)}}{15cf(c-ic\tan(e+fx))^{3/2}} + \frac{(2a)\text{Subst}\left(\int -\right.}{}$$

$$= -\frac{i\sqrt{a+ia\tan(e+fx)}}{5f(c-ic\tan(e+fx))^{5/2}} - \frac{2i\sqrt{a+ia\tan(e+fx)}}{15cf(c-ic\tan(e+fx))^{3/2}} - \frac{2i\sqrt{a+ia\,\text{ta}}}{15c^2f\sqrt{c-ic\,\text{t}}}$$

Mathematica [A]
time = 1.90, size = 102, normalized size = 0.75

$$\frac{(19\cos(e+fx)+9\cos(3(e+fx))-24i\cos^2(e+fx)\sin(e+fx))(-i\cos(3(e+fx))+\sin(3(e+fx)))\sqrt{a+ia\tan(e+fx)}\,\sqrt{c-ic\tan(e+fx)}}{60c^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Tan[e + f*x]]/(c - I*c*Tan[e + f*x])^(5/2),x]
[Out] ((19*Cos[e + f*x] + 9*Cos[3*(e + f*x)] - (24*I)*Cos[e + f*x]^2*Sin[e + f*x]
)*((-I)*Cos[3*(e + f*x)] + Sin[3*(e + f*x)])*Sqrt[a + I*a*Tan[e + f*x]]*Sqr
t[c - I*c*Tan[e + f*x]])/(60*c^3*f)
```

Maple [A]
time = 0.39, size = 83, normalized size = 0.61

method	result
risch	$-\frac{i\sqrt{\frac{a\,e^{2i(fx+e)}}{e^{2i(fx+e)}+1}}\,(3\,e^{4i(fx+e)}+10\,e^{2i(fx+e)}+15)}{60c^2\sqrt{\frac{c}{e^{2i(fx+e)}+1}}\,f}$
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,(8i(\tan^2(fx+e))+2(\tan^3(fx+e))-7i-13\tan(fx-}{15fc^3(\tan(fx+e)+i)^4}$

default	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,(8i(\tan^2(fx+e))+2(\tan^3(fx+e))-7i-13\tan(f}{15fc^3(\tan(fx+e)+i)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(5/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/15/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)/c^3*(8*I*tan(
f*x+e)^2+2*tan(f*x+e)^3-7*I-13*tan(f*x+e))/(tan(f*x+e)+I)^4
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 1.08, size = 93, normalized size = 0.68

$$\frac{\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\,\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\left(-3i\,e^{(7i\,fx+7i\,e)}-13i\,e^{(5i\,fx+5i\,e)}-25i\,e^{(3i\,fx+3i\,e)}-15i\,e^{(i\,fx+i\,e)}\right)}{60\,c^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="f
ricas")
```

```
[Out] 1/60*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*(-
3*I*e^(7*I*f*x + 7*I*e) - 13*I*e^(5*I*f*x + 5*I*e) - 25*I*e^(3*I*f*x + 3*I*
e) - 15*I*e^(I*f*x + I*e))/(c^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{ia(\tan(e+fx)-i)}}{(-ic(\tan(e+fx)+i))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(1/2)/(c-I*c*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(e + f*x) - I))/(-I*c*(tan(e + f*x) + I))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(I*a*tan(f*x + e) + a)/(-I*c*tan(f*x + e) + c)^(5/2), x)
```

Mupad [B]
time = 5.48, size = 137, normalized size = 1.01

$$-\frac{\sqrt{\frac{a\,\left(\cos\left(2\,e+2\,f\,x\right)+1+\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\left(\cos\left(2\,e+2\,f\,x\right)\,10{}\mathrm{i}+\cos\left(4\,e+4\,f\,x\right)\,3{}\mathrm{i}-10\,\sin\left(2\,e+2\,f\,x\right)-3\,\sin\left(4\,e+4\,f\,x\right)+15{}\mathrm{i}\right)}{60\,c^2\,f\,\sqrt{\frac{c\,\left(\cos\left(2\,e+2\,f\,x\right)+1-\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(1/2)/(c - c*tan(e + f*x)*1i)^(5/2),x)
```

```
[Out] -(((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))
^(1/2)*(cos(2*e + 2*f*x)*10i + cos(4*e + 4*f*x)*3i - 10*sin(2*e + 2*f*x) -
3*sin(4*e + 4*f*x) + 15i))/(60*c^2*f*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*
x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

3.1039 $\int \frac{1}{\sqrt{a+ia\tan(e+fx)}\,(c-ic\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=186

$$\frac{i}{f\sqrt{a+ia\tan(e+fx)}\,(c-ic\tan(e+fx))^{5/2}}-\frac{3i\sqrt{a+ia\tan(e+fx)}}{5af(c-ic\tan(e+fx))^{5/2}}-\frac{2i\sqrt{a+ia\tan(e+fx)}}{5acf(c-ic\tan(e+fx))^{3/2}}-$$

```
[Out] -2/5*I*(a+I*a*tan(f*x+e))^(1/2)/a/c^2/f/(c-I*c*tan(f*x+e))^(1/2)+I/f/(a+I*a
*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(5/2)-3/5*I*(a+I*a*tan(f*x+e))^(1/2)/
a/f/(c-I*c*tan(f*x+e))^(5/2)-2/5*I*(a+I*a*tan(f*x+e))^(1/2)/a/c/f/(c-I*c*ta
n(f*x+e))^(3/2)
```

Rubi [A]
time = 0.11, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {3604, 47, 37}

$$-\frac{2i\sqrt{a+ia\tan(e+fx)}}{5ac^2f\sqrt{c-ic\tan(e+fx)}}-\frac{2i\sqrt{a+ia\tan(e+fx)}}{5acf(c-ic\tan(e+fx))^{3/2}}-\frac{3i\sqrt{a+ia\tan(e+fx)}}{5af(c-ic\tan(e+fx))^{5/2}}+\frac{i}{f\sqrt{a+ia\tan(e+fx)}\,(c-ic\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + I*a*Tan[e + f*x]]*(c - I*c*Tan[e + f*x])^(5/2)),x]
```

```
[Out] I/(f*Sqrt[a + I*a*Tan[e + f*x]]*(c - I*c*Tan[e + f*x])^(5/2)) - (((3*I)/5)*
Sqrt[a + I*a*Tan[e + f*x]])/(a*f*(c - I*c*Tan[e + f*x])^(5/2)) - (((2*I)/5)
*Sqrt[a + I*a*Tan[e + f*x]])/(a*c*f*(c - I*c*Tan[e + f*x])^(3/2)) - (((2*I)
/5)*Sqrt[a + I*a*Tan[e + f*x]])/(a*c^2*f*Sqrt[c - I*c*Tan[e + f*x]])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+ia\tan(e+fx)}\,(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{1}{(a+iax)^{3/2}(c-icx)^{7/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{i}{f\sqrt{a+ia\tan(e+fx)}\,(c-ic\tan(e+fx))^{5/2}} + \frac{(3c)\text{S}}{}$$

$$= \frac{i}{f\sqrt{a+ia\tan(e+fx)}\,(c-ic\tan(e+fx))^{5/2}} - \frac{3i\sqrt{}}{5af(}$$

$$= \frac{i}{f\sqrt{a+ia\tan(e+fx)}\,(c-ic\tan(e+fx))^{5/2}} - \frac{3i\sqrt{}}{5af(}$$

$$= \frac{i}{f\sqrt{a+ia\tan(e+fx)}\,(c-ic\tan(e+fx))^{5/2}} - \frac{3i\sqrt{}}{5af(}$$

Mathematica [A]
time = 2.09, size = 106, normalized size = 0.57

$$\frac{(\cos(3(e+fx))+i\sin(3(e+fx)))(-10i\cos(e+fx)+2i\cos(3(e+fx))-5\sin(e+fx)+3\sin(3(e+fx)))\sqrt{c-ic\tan(e+fx)}}{20c^3f\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + I*a*Tan[e + f*x]]*(c - I*c*Tan[e + f*x])^(5/2)),x]
```

```
[Out] ((Cos[3*(e + f*x)] + I*Sin[3*(e + f*x)])*((-10*I)*Cos[e + f*x] + (2*I)*Cos[
3*(e + f*x)] - 5*Sin[e + f*x] + 3*Sin[3*(e + f*x)])*Sqrt[c - I*c*Tan[e + f*
x]])/(20*c^3*f*Sqrt[a + I*a*Tan[e + f*x]])
```

Maple [A]
time = 0.40, size = 118, normalized size = 0.63

method	result

risch	$-\frac{i\left(\mathrm{e}^{6i(fx+e)}+5\,\mathrm{e}^{4i(fx+e)}+15\,\mathrm{e}^{2i(fx+e)}-5\right)}{40c^{2}\sqrt{\frac{a\,\mathrm{e}^{2i(fx+e)}}{\mathrm{e}^{2i(fx+e)}+1}}\,\sqrt{\frac{c}{\mathrm{e}^{2i(fx+e)}+1}}\,\left(\mathrm{e}^{2i(fx+e)}+1\right)f}$
derivativedivides	$\frac{\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\left(4i\left(\tan^{4}(fx+e)\right)+2\left(\tan^{5}(fx+e)\right)+6i\left(\tan^{2}(fx-\right.\right.}{5fa\,c^{3}(\tan(fx+e)+i)^{4}(-\tan(fx+e)+i)^{2}}$
default	$\frac{\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\left(4i\left(\tan^{4}(fx+e)\right)+2\left(\tan^{5}(fx+e)\right)+6i\left(\tan^{2}(fx-\right.\right.}{5fa\,c^{3}(\tan(fx+e)+i)^{4}(-\tan(fx+e)+i)^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(5/2),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/5/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)/a/c^3*(4*I*tan
(f*x+e)^4+2*tan(f*x+e)^5+6*I*tan(f*x+e)^2+tan(f*x+e)^3+2*I-tan(f*x+e))/(tan
(f*x+e)+I)^4/(-tan(f*x+e)+I)^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(5/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 1.18, size = 131, normalized size = 0.70

$$\frac{\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}\,\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}\,\left(-i\,e^{(8ifx+8ie)}-6i\,e^{(6ifx+6ie)}-20i\,e^{(4ifx+4ie)}+16i\,e^{(3ifx+3ie)}-10i\,e^{(2ifx+2ie)}+16i\,e^{(ifx+ie)}+5i\right)e^{(-ifx-ie)}}{40\,ac^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(5/2),x, algorithm=
"fricas")
```

```
[Out] 1/40*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*(-
I*e^(8*I*f*x + 8*I*e) - 6*I*e^(6*I*f*x + 6*I*e) - 20*I*e^(4*I*f*x + 4*I*e)
+ 16*I*e^(3*I*f*x + 3*I*e) - 10*I*e^(2*I*f*x + 2*I*e) + 16*I*e^(I*f*x + I*e
) + 5*I)*e^(-I*f*x - I*e)/(a*c^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ia\left(\tan\left(e+fx\right)-i\right)}\,\left(-ic\left(\tan\left(e+fx\right)+i\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**(1/2)/(c-I*c*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral(1/(sqrt(I*a*(tan(e + f*x) - I))*(-I*c*(tan(e + f*x) + I))**(5/2)),
 x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(5/2),x, algorithm=
"giac")
```

```
[Out] integrate(1/(sqrt(I*a*tan(f*x + e) + a)*(-I*c*tan(f*x + e) + c)^(5/2)), x)
```

Mupad [B]
time = 5.36, size = 128, normalized size = 0.69

$$-\frac{\sqrt{\frac{a\,\left(\cos\left(2\,e+2\,f\,x\right)+1+\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\left(\cos\left(4\,e+4\,f\,x\right)\,1{}\mathrm{i}-10\,\sin\left(2\,e+2\,f\,x\right)-\sin\left(4\,e+4\,f\,x\right)+15{}\mathrm{i}\right)}{40\,a\,c^2\,f\,\sqrt{\frac{c\,\left(\cos\left(2\,e+2\,f\,x\right)+1-\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^(1/2)*(c - c*tan(e + f*x)*1i)^(5/2)),x)
```

```
[Out] -(((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))
^(1/2)*(cos(4*e + 4*f*x)*1i - 10*sin(2*e + 2*f*x) - sin(4*e + 4*f*x) + 15i)
)/(40*a*c^2*f*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e +
2*f*x) + 1))^(1/2))
```

3.1040 $\int \frac{1}{(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=234

$$\frac{i}{3f(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{5/2}}+\frac{4i}{3af\sqrt{a+ia\tan(e+fx)}\,(c-ic\tan(e+fx))^{5/2}}-\frac{4i\sqrt{\ldots}}{5a^2f(\ldots}$$

```
[Out] -8/15*I*(a+I*a*tan(f*x+e))^(1/2)/a^2/c^2/f/(c-I*c*tan(f*x+e))^(1/2)+4/3*I/a
/f/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(5/2)-4/5*I*(a+I*a*tan(f*x+e
))^(1/2)/a^2/f/(c-I*c*tan(f*x+e))^(5/2)+1/3*I/f/(a+I*a*tan(f*x+e))^(3/2)/(c
-I*c*tan(f*x+e))^(5/2)-8/15*I*(a+I*a*tan(f*x+e))^(1/2)/a^2/c/f/(c-I*c*tan(f
*x+e))^(3/2)
```

Rubi [A]
time = 0.13, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {3604, 47, 37}

$$-\frac{8i\sqrt{a+ia\tan(e+fx)}}{15a^2c^2f\sqrt{c-ic\tan(e+fx)}}-\frac{8i\sqrt{a+ia\tan(e+fx)}}{15a^2cf(c-ic\tan(e+fx))^{3/2}}-\frac{4i\sqrt{a+ia\tan(e+fx)}}{5a^2f(c-ic\tan(e+fx))^{5/2}}+\frac{4i}{3af\sqrt{a+ia\tan(e+fx)}\,(c-ic\tan(e+fx))^{5/2}}+\frac{i}{3f(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^(3/2)*(c - I*c*Tan[e + f*x])^(5/2)),x]
```

```
[Out] (I/3)/(f*(a + I*a*Tan[e + f*x])^(3/2)*(c - I*c*Tan[e + f*x])^(5/2)) + ((4*I
)/3)/(a*f*Sqrt[a + I*a*Tan[e + f*x]]*(c - I*c*Tan[e + f*x])^(5/2)) - (((4*I
)/5)*Sqrt[a + I*a*Tan[e + f*x]])/(a^2*f*(c - I*c*Tan[e + f*x])^(5/2)) - (((
8*I)/15)*Sqrt[a + I*a*Tan[e + f*x]])/(a^2*c*f*(c - I*c*Tan[e + f*x])^(3/2))
 - (((8*I)/15)*Sqrt[a + I*a*Tan[e + f*x]])/(a^2*c^2*f*Sqrt[c - I*c*Tan[e +
f*x]])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
```

```
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{1}{(a+iax)^{5/2}(c-icx)^{7/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{i}{3f(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{5/2}} + \frac{(4}{}$$

$$= \frac{i}{3f(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{5/2}} + \frac{}{3a}$$

$$= \frac{i}{3f(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{5/2}} + \frac{}{3a}$$

$$= \frac{i}{3f(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{5/2}} + \frac{}{3a}$$

$$= \frac{i}{3f(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{5/2}} + \frac{}{3a}$$

Mathematica [A]
time = 2.90, size = 130, normalized size = 0.56

$$\frac{\sec(e+fx)(\cos(3(e+fx))+i\sin(3(e+fx)))(-45+20\cos(2(e+fx))+\cos(4(e+fx))-40i\sin(2(e+fx))-4i\sin(4(e+fx)))\sqrt{c-ic\tan(e+fx)}}{120ac^3f(-i+\tan(e+fx))\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^(3/2)*(c - I*c*Tan[e + f*x])^(5/2)),x]
```

```
[Out] (Sec[e + f*x]*(Cos[3*(e + f*x)] + I*Sin[3*(e + f*x)])*(-45 + 20*Cos[2*(e +
f*x)] + Cos[4*(e + f*x)] - (40*I)*Sin[2*(e + f*x)] - (4*I)*Sin[4*(e + f*x)]
)*Sqrt[c - I*c*Tan[e + f*x]])/(120*a*c^3*f*(-I + Tan[e + f*x])*Sqrt[a + I*a
*Tan[e + f*x]])
```

Maple [A]
time = 0.41, size = 130, normalized size = 0.56

method	result
derivativedivides	$-\frac{\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\left(8i\left(\tan^5\left(fx+e\right)\right)+8\left(\tan^6\left(fx+e\right)\right)+20i\left(\tan^3\left(\right.\right.\right.}{15f\,a^2c^3\left(-\tan\left(fx+e\right)+i\right)^3\left(\tan\left(fx+e\right)+i\right)^4}$
default	$-\frac{\sqrt{a\left(1+i\tan\left(fx+e\right)\right)}\,\sqrt{-c\left(i\tan\left(fx+e\right)-1\right)}\,\left(8i\left(\tan^5\left(fx+e\right)\right)+8\left(\tan^6\left(fx+e\right)\right)+20i\left(\tan^3\left(\right.\right.\right.}{15f\,a^2c^3\left(-\tan\left(fx+e\right)+i\right)^3\left(\tan\left(fx+e\right)+i\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(5/2),x,method=_RETURNVER
BOSE)
```

```
[Out] -1/15/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)/a^2/c^3*(8*I
*tan(f*x+e)^5+8*tan(f*x+e)^6+20*I*tan(f*x+e)^3+20*tan(f*x+e)^4+12*I*tan(f*x
+e)+15*tan(f*x+e)^2+3)/(-tan(f*x+e)+I)^3/(tan(f*x+e)+I)^4
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(5/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 1.36, size = 143, normalized size = 0.61

$$\frac{\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}\,\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}\,\left(-3i\,e^{(10ifx+10ie)}-23i\,e^{(8ifx+8ie)}-110i\,e^{(6ifx+6ie)}+48i\,e^{(5ifx+5ie)}-30i\,e^{(4ifx+4ie)}+48i\,e^{(3ifx+3ie)}+65i\,e^{(2ifx+2ie)}+5i\right)e^{(-3ifx-3ie)}}{240\,a^2c^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(5/2),x, algorithm=
"fricas")
```

```
[Out] 1/240*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*(
-3*I*e^(10*I*f*x + 10*I*e) - 23*I*e^(8*I*f*x + 8*I*e) - 110*I*e^(6*I*f*x +
6*I*e) + 48*I*e^(5*I*f*x + 5*I*e) - 30*I*e^(4*I*f*x + 4*I*e) + 48*I*e^(3*I*
f*x + 3*I*e) + 65*I*e^(2*I*f*x + 2*I*e) + 5*I)*e^(-3*I*f*x - 3*I*e)/(a^2*c^
3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia(\tan(e+fx)-i))^{\frac{3}{2}}(-ic(\tan(e+fx)+i))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**(3/2)/(c-I*c*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral(1/((I*a*(tan(e + f*x) - I))**(3/2)*(-I*c*(tan(e + f*x) + I))**(5/2
)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(5/2),x, algorithm=
"giac")
```

```
[Out] integrate(1/((I*a*tan(f*x + e) + a)^(3/2)*(-I*c*tan(f*x + e) + c)^(5/2)), x
)
```

Mupad [B]
time = 5.53, size = 140, normalized size = 0.60

$$\frac{\sqrt{\frac{a\,\left(\cos\left(2\,e+2\,f\,x\right)+1+\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\left(\cos\left(2\,e+2\,f\,x\right)\,20{}\mathrm{i}+\cos\left(4\,e+4\,f\,x\right)\,1{}\mathrm{i}+40\,\sin\left(2\,e+2\,f\,x\right)+4\,\sin\left(4\,e+4\,f\,x\right)-45{}\mathrm{i}\right)}{120\,a^2\,c^2\,f\,\sqrt{\frac{c\,\left(\cos\left(2\,e+2\,f\,x\right)+1-\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^(3/2)*(c - c*tan(e + f*x)*1i)^(5/2)),x)
```

```
[Out] (((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^
(1/2)*(cos(2*e + 2*f*x)*20i + cos(4*e + 4*f*x)*1i + 40*sin(2*e + 2*f*x) + 4
*sin(4*e + 4*f*x) - 45i))/(120*a^2*c^2*f*((c*(cos(2*e + 2*f*x) - sin(2*e +
2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

3.1041 $\int \frac{1}{(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=154

$$\frac{\tan(e+fx)}{5f(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{5/2}}+\frac{4\tan(e+fx)}{15acf(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}+\frac{}{15}$$

```
[Out] 8/15*tan(f*x+e)/a^2/c^2/f/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(1/2)
+1/5*tan(f*x+e)/f/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(5/2)+4/15*ta
n(f*x+e)/a/c/f/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.11, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {3604, 40, 39}

$$\frac{8\tan(e+fx)}{15a^2c^2f\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}+\frac{4\tan(e+fx)}{15acf(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}+\frac{\tan(e+fx)}{5f(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^(5/2)*(c - I*c*Tan[e + f*x])^(5/2)),x]
```

```
[Out] Tan[e + f*x]/(5*f*(a + I*a*Tan[e + f*x])^(5/2)*(c - I*c*Tan[e + f*x])^(5/2)
) + (4*Tan[e + f*x])/(15*a*c*f*(a + I*a*Tan[e + f*x])^(3/2)*(c - I*c*Tan[e
+ f*x])^(3/2)) + (8*Tan[e + f*x])/(15*a^2*c^2*f*Sqrt[a + I*a*Tan[e + f*x]]*
Sqrt[c - I*c*Tan[e + f*x]])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Dist[(2*m +
3)/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{1}{(a+iax)^{7/2}(c-icx)^{7/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{5f(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{5/2}} + \frac{4\ldots}{\ldots}$$

$$= \frac{\tan(e+fx)}{5f(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{5/2}} + \frac{\ldots}{15\ldots}$$

$$= \frac{\tan(e+fx)}{5f(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{5/2}} + \frac{\ldots}{15\ldots}$$

Mathematica [A]
time = 3.70, size = 117, normalized size = 0.76

$$-\frac{\sec^2(e+fx)(\cos(3(e+fx))+i\sin(3(e+fx)))(150\sin(e+fx)+25\sin(3(e+fx))+3\sin(5(e+fx)))\sqrt{c-ic\tan(e+fx)}}{240a^2c^3f(-i+\tan(e+fx))^2\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^(5/2)*(c - I*c*Tan[e + f*x])^(5/2)),x]
```

```
[Out] -1/240*(Sec[e + f*x]^2*(Cos[3*(e + f*x)] + I*Sin[3*(e + f*x)])*(150*Sin[e +
 f*x] + 25*Sin[3*(e + f*x)] + 3*Sin[5*(e + f*x)])*Sqrt[c - I*c*Tan[e + f*x]
])/(a^2*c^3*f*(-I + Tan[e + f*x])^2*Sqrt[a + I*a*Tan[e + f*x]])
```

Maple [A]
time = 0.35, size = 105, normalized size = 0.68

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,(1+\tan^2(fx+e))\tan(fx+e)(8(\tan^4(fx+e))+20(\ldots}{15f\,a^3c^3(\tan(fx+e)+i)^4(-\tan(fx+e)+i)^4}$
default	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,(1+\tan^2(fx+e))\tan(fx+e)(8(\tan^4(fx+e))+20(\ldots}{15f\,a^3c^3(\tan(fx+e)+i)^4(-\tan(fx+e)+i)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(5/2),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/15/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)/a^3/c^3*(1+ta
n(f*x+e)^2)*tan(f*x+e)*(8*tan(f*x+e)^4+20*tan(f*x+e)^2+15)/(tan(f*x+e)+I)^4
/(-tan(f*x+e)+I)^4
```

Maxima [A]
time = 0.63, size = 76, normalized size = 0.49

$$\frac{3 \sin (5 f x+5 e)+25 \sin \left(\frac{3}{5} \arctan (\sin (5 f x+5 e), \cos (5 f x+5 e))\right)+150 \sin \left(\frac{1}{5} \arctan (\sin (5 f x+5 e), \cos (5 f x+5 e))\right)}{240 a^{\frac{5}{2}} c^{\frac{5}{2}} f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(5/2),x, algorithm=
"maxima")
```

```
[Out] 1/240*(3*sin(5*f*x + 5*e) + 25*sin(3/5*arctan2(sin(5*f*x + 5*e), cos(5*f*x
 + 5*e))) + 150*sin(1/5*arctan2(sin(5*f*x + 5*e), cos(5*f*x + 5*e))))/(a^(5/
2)*c^(5/2)*f)
```

Fricas [A]
time = 1.08, size = 119, normalized size = 0.77

$$\frac{\sqrt{\frac{a}{e^{(2 i f x+2 i e)}+1}} \sqrt{\frac{c}{e^{(2 i f x+2 i e)}+1}}\left(-3 i e^{(12 i f x+12 i e)}-28 i e^{(10 i f x+10 i e)}-175 i e^{(8 i f x+8 i e)}+175 i e^{(4 i f x+4 i e)}+28 i e^{(2 i f x+2 i e)}+3 i\right) e^{(-5 i f x-5 i e)}}{480 a^{3} c^{3} f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(5/2),x, algorithm=
"fricas")
```

```
[Out] 1/480*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*(
-3*I*e^(12*I*f*x + 12*I*e) - 28*I*e^(10*I*f*x + 10*I*e) - 175*I*e^(8*I*f*x
+ 8*I*e) + 175*I*e^(4*I*f*x + 4*I*e) + 28*I*e^(2*I*f*x + 2*I*e) + 3*I)*e^(-
5*I*f*x - 5*I*e)/(a^3*c^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i a (\tan (e+f x)-i))^{\frac{5}{2}}(-i c (\tan (e+f x)+i))^{\frac{5}{2}}} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**(5/2)/(c-I*c*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral(1/((I*a*(tan(e + f*x) - I))**(5/2)*(-I*c*(tan(e + f*x) + I))**(5/2
)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(5/2),x, algorithm=
"giac")

[Out] integrate(1/((I*a*tan(f*x + e) + a)^(5/2)*(-I*c*tan(f*x + e) + c)^(5/2)), x
)
```

Mupad [B]
time = 6.13, size = 163, normalized size = 1.06

$$\frac{\sqrt{\frac{a\,\left(\cos\left(2\,e+2\,f\,x\right)+1+\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\left(\cos\left(2\,e+2\,f\,x\right)\,125{}\mathrm{i}+\cos\left(4\,e+4\,f\,x\right)\,22{}\mathrm{i}+\cos\left(6\,e+6\,f\,x\right)\,3{}\mathrm{i}+175\,\sin\left(2\,e+2\,f\,x\right)+28\,\sin\left(4\,e+4\,f\,x\right)+3\,\sin\left(6\,e+6\,f\,x\right)-150{}\mathrm{i}\right)}{480\,a^3\,c^2\,f\,\sqrt{\frac{c\,\left(\cos\left(2\,e+2\,f\,x\right)+1-\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^(5/2)*(c - c*tan(e + f*x)*1i)^(5/2)),x)

[Out] (((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^
(1/2)*(cos(2*e + 2*f*x)*125i + cos(4*e + 4*f*x)*22i + cos(6*e + 6*f*x)*3i +
 175*sin(2*e + 2*f*x) + 28*sin(4*e + 4*f*x) + 3*sin(6*e + 6*f*x) - 150i))/(
480*a^3*c^2*f*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e +
2*f*x) + 1))^(1/2))
```

3.1042 $\int \frac{1}{(a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=200

$$\frac{i}{7f(a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))^{5/2}}+\frac{6\tan(e+fx)}{35af(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{5/2}}+\frac{}{35a}$$

```
[Out] 16/35*tan(f*x+e)/a^3/c^2/f/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(1/2
)+1/7*I/f/(a+I*a*tan(f*x+e))^(7/2)/(c-I*c*tan(f*x+e))^(5/2)+6/35*tan(f*x+e)
/a/f/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(5/2)+8/35*tan(f*x+e)/a^2/
c/f/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.12, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3604, 47, 40, 39}

$$\frac{16\tan(e+fx)}{35a^3c^2f\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}+\frac{8\tan(e+fx)}{35a^2cf(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}+\frac{6\tan(e+fx)}{35af(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{5/2}}+\frac{i}{7f(a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^(7/2)*(c - I*c*Tan[e + f*x])^(5/2)),x]
```

```
[Out] (I/7)/(f*(a + I*a*Tan[e + f*x])^(7/2)*(c - I*c*Tan[e + f*x])^(5/2)) + (6*Ta
n[e + f*x])/(35*a*f*(a + I*a*Tan[e + f*x])^(5/2)*(c - I*c*Tan[e + f*x])^(5/
2)) + (8*Tan[e + f*x])/(35*a^2*c*f*(a + I*a*Tan[e + f*x])^(3/2)*(c - I*c*Ta
n[e + f*x])^(3/2)) + (16*Tan[e + f*x])/(35*a^3*c^2*f*Sqrt[a + I*a*Tan[e + f
*x]]*Sqrt[c - I*c*Tan[e + f*x]])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Dist[(2*m +
3)/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
```

```
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{1}{(a+iax)^{9/2}(c-icx)^{7/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{i}{7f(a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))^{5/2}} + \frac{(6}{}$$

$$= \frac{i}{7f(a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))^{5/2}} + \frac{}{35}$$

$$= \frac{i}{7f(a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))^{5/2}} + \frac{}{35}$$

$$= \frac{i}{7f(a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))^{5/2}} + \frac{}{35}$$

Mathematica [A]
time = 5.53, size = 115, normalized size = 0.58

$$-\frac{i(-350+175\cos(2(e+fx))+14\cos(4(e+fx))+\cos(6(e+fx))+350i\sin(2(e+fx))+56i\sin(4(e+fx))+6i\sin(6(e+fx)))\sqrt{c-ic\tan(e+fx)}}{1120a^3c^3f\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^(7/2)*(c - I*c*Tan[e + f*x])^(5/2)),x]
[Out] ((-1/1120*I)*(-350 + 175*Cos[2*(e + f*x)] + 14*Cos[4*(e + f*x)] + Cos[6*(e
+ f*x)] + (350*I)*Sin[2*(e + f*x)] + (56*I)*Sin[4*(e + f*x)] + (6*I)*Sin[6*
(e + f*x)])*Sqrt[c - I*c*Tan[e + f*x]])/(a^3*c^3*f*Sqrt[a + I*a*Tan[e + f*x
]])
```

Maple [A]
time = 0.37, size = 151, normalized size = 0.76

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,(16i(\tan^7(fx+e))-16(\tan^8(fx+e))+56i(\tan^5(}{35f\,a^4c^3(-\tan(fx+e)+i)^5(\mathrm{ta}}$
default	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,(16i(\tan^7(fx+e))-16(\tan^8(fx+e))+56i(\tan^5(}{35f\,a^4c^3(-\tan(fx+e)+i)^5(\mathrm{ta}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))^(7/2)/(c-I*c*tan(f*x+e))^(5/2),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/35/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)/a^4/c^3*(16*I
*tan(f*x+e)^7-16*tan(f*x+e)^8+56*I*tan(f*x+e)^5-56*tan(f*x+e)^6+70*I*tan(f*
x+e)^3-70*tan(f*x+e)^4+30*I*tan(f*x+e)-35*tan(f*x+e)^2-5)/(-tan(f*x+e)+I)^5
/(tan(f*x+e)+I)^4
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(7/2)/(c-I*c*tan(f*x+e))^(5/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]

time = 0.89, size = 167, normalized size = 0.84

$$\frac{\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\,\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\,\left(-7i\,e^{(14i\,fx+14i\,e)}-77i\,e^{(12i\,fx+12i\,e)}-595i\,e^{(10i\,fx+10i\,e)}-320i\,e^{(9i\,fx+9i\,e)}+175i\,e^{(8i\,fx+8i\,e)}-320i\,e^{(7i\,fx+7i\,e)}+875i\,e^{(6i\,fx+6i\,e)}+217i\,e^{(4i\,fx+4i\,e)}+47i\,e^{(2i\,fx+2i\,e)}+5i\right)e^{(-7i\,fx-7i\,e)}}{2240\,a^4c^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(7/2)/(c-I*c*tan(f*x+e))^(5/2),x, algorithm=
"fricas")
```

```
[Out] 1/2240*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*
(-7*I*e^(14*I*f*x + 14*I*e) - 77*I*e^(12*I*f*x + 12*I*e) - 595*I*e^(10*I*f*
x + 10*I*e) - 320*I*e^(9*I*f*x + 9*I*e) + 175*I*e^(8*I*f*x + 8*I*e) - 320*I
*e^(7*I*f*x + 7*I*e) + 875*I*e^(6*I*f*x + 6*I*e) + 217*I*e^(4*I*f*x + 4*I*e
) + 47*I*e^(2*I*f*x + 2*I*e) + 5*I)*e^(-7*I*f*x - 7*I*e)/(a^4*c^3*f)
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**(7/2)/(c-I*c*tan(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4848 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(7/2)/(c-I*c*tan(f*x+e))^(5/2),x, algorithm=
"giac")
```

```
[Out] integrate(1/((I*a*tan(f*x + e) + a)^(7/2)*(-I*c*tan(f*x + e) + c)^(5/2)), x
)
```

Mupad [B]
time = 6.70, size = 186, normalized size = 0.93

$$\frac{\sqrt{\frac{a\,(\cos(2\,e+2\,f\,x)+1+\sin(2\,e+2\,f\,x)\,1\mathrm{i})}{\cos(2\,e+2\,f\,x)+1}}\,(\cos(2\,e+2\,f\,x)\,630\mathrm{i}+\cos(4\,e+4\,f\,x)\,168\mathrm{i}+\cos(6\,e+6\,f\,x)\,42\mathrm{i}+\cos(8\,e+8\,f\,x)\,5\mathrm{i}+770\,\sin(2\,e+2\,f\,x)+182\,\sin(4\,e+4\,f\,x)+42\,\sin(6\,e+6\,f\,x)+5\,\sin(8\,e+8\,f\,x)-525\mathrm{i})}{2240\,a^4\,c^2\,f\,\sqrt{\frac{c\,(\cos(2\,e+2\,f\,x)+1-\sin(2\,e+2\,f\,x)\,1\mathrm{i})}{\cos(2\,e+2\,f\,x)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^(7/2)*(c - c*tan(e + f*x)*1i)^(5/2)),x)
```

```
[Out] (((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^
(1/2)*(cos(2*e + 2*f*x)*630i + cos(4*e + 4*f*x)*168i + cos(6*e + 6*f*x)*42i
 + cos(8*e + 8*f*x)*5i + 770*sin(2*e + 2*f*x) + 182*sin(4*e + 4*f*x) + 42*s
in(6*e + 6*f*x) + 5*sin(8*e + 8*f*x) - 525i))/(2240*a^4*c^2*f*((c*(cos(2*e
+ 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

3.1043 $\int (a+ia\tan(e+fx))^4(c-ic\tan(e+fx))^n\,dx$

Optimal. Leaf size=134

$$\frac{8ia^4(c-ic\tan(e+fx))^n}{fn}-\frac{12ia^4(c-ic\tan(e+fx))^{1+n}}{cf(1+n)}+\frac{6ia^4(c-ic\tan(e+fx))^{2+n}}{c^2f(2+n)}-\frac{ia^4(c-ic\tan(e+fx))^{3+n}}{c^3f(3+n)}$$

```
[Out] 8*I*a^4*(c-I*c*tan(f*x+e))^n/f/n-12*I*a^4*(c-I*c*tan(f*x+e))^(1+n)/c/f/(1+n
)+6*I*a^4*(c-I*c*tan(f*x+e))^(2+n)/c^2/f/(2+n)-I*a^4*(c-I*c*tan(f*x+e))^(3+
n)/c^3/f/(3+n)
```

Rubi [A]

time = 0.12, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 45}

$$-\frac{ia^4(c-ic\tan(e+fx))^{n+3}}{c^3f(n+3)}+\frac{6ia^4(c-ic\tan(e+fx))^{n+2}}{c^2f(n+2)}+\frac{8ia^4(c-ic\tan(e+fx))^n}{fn}-\frac{12ia^4(c-ic\tan(e+fx))^{n+1}}{cf(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^4*(c - I*c*Tan[e + f*x])^n,x]
```

```
[Out] ((8*I)*a^4*(c - I*c*Tan[e + f*x])^n)/(f*n) - ((12*I)*a^4*(c - I*c*Tan[e + f
*x])^(1 + n))/(c*f*(1 + n)) + ((6*I)*a^4*(c - I*c*Tan[e + f*x])^(2 + n))/(c
^2*f*(2 + n)) - (I*a^4*(c - I*c*Tan[e + f*x])^(3 + n))/(c^3*f*(3 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$
\begin{aligned}
\int (a+ia\tan(e+fx))^4(c-ic\tan(e+fx))^n\,dx &= \left(a^4c^4\right)\int \sec^8(e+fx)(c-ic\tan(e+fx))^{-4+n}\,dx\\
&= \frac{\left(ia^4\right)\operatorname{Subst}\left(\int (c-x)^3(c+x)^{-1+n}\,dx,x,-ic\tan(e+fx)\right)}{c^3f}\\
&= \frac{\left(ia^4\right)\operatorname{Subst}\left(\int \left(8c^3(c+x)^{-1+n}-12c^2(c+x)^n+6c(c+x)\right.\right.}{c^3f}\\
&= \frac{8ia^4(c-ic\tan(e+fx))^n}{fn}-\frac{12ia^4(c-ic\tan(e+fx))^{1+n}}{cf(1+n)}
\end{aligned}
$$

Mathematica [A]
time = 4.44, size = 180, normalized size = 1.34

$$
\frac{ia^4e^{n(-\log(c\sec(e+fx))+\log(c-ic\tan(e+fx)))}\sec^2(e+fx)(c\sec(e+fx))^n(\cos(4fx)+i\sin(4fx))\left(3(12+7n+n^2)+(12+11n+6n^2+n^3)(-1+2\cos(2(e+fx)))+in(20+9n+n^2+2(11+6n+n^2)\cos(2(e+fx)))\tan(e+fx)\right)}{fn(1+n)(2+n)(3+n)(\cos(fx)+i\sin(fx))^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^4*(c - I*c*Tan[e + f*x])^n,x]
```

```
[Out] (I*a^4*E^(n*(-Log[c*Sec[e + f*x]] + Log[c - I*c*Tan[e + f*x]]))*Sec[e + f*x
]^2*(c*Sec[e + f*x])^n*(Cos[4*f*x] + I*Sin[4*f*x])*(3*(12 + 7*n + n^2) + (1
2 + 11*n + 6*n^2 + n^3)*(-1 + 2*Cos[2*(e + f*x)]) + I*n*(20 + 9*n + n^2 + 2
*(11 + 6*n + n^2)*Cos[2*(e + f*x)])*Tan[e + f*x]))/(f*n*(1 + n)*(2 + n)*(3
+ n)*(Cos[f*x] + I*Sin[f*x])^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.08, size = 1784, normalized size = 13.31

method	result	size
risch	Expression too large to display	1784

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^4*(c-I*c*tan(f*x+e))^n,x,method=_RETURNVERBOSE)
```

```
[Out] 8*I*a^4/(1+n)/f/(exp(2*I*(f*x+e))+1)^3/(3+n)/n/(2+n)*(n^3*c^n*2^n/((exp(2*I
*(f*x+e))+1)^n)*exp(-1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^3*n)*exp(1/2*I
*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^2*csgn(I*c)*n)*exp(1/2*I*Pi*csgn(I*c/(ex
p(2*I*(f*x+e))+1))^2*csgn(I/(exp(2*I*(f*x+e))+1))*n)*exp(-1/2*I*Pi*csgn(I*c
/(exp(2*I*(f*x+e))+1))*csgn(I*c)*csgn(I/(exp(2*I*(f*x+e))+1))*n)*exp(6*I*f*
x)*exp(6*I*e)+6*n^2*c^n*2^n/((exp(2*I*(f*x+e))+1)^n)*exp(-1/2*I*Pi*csgn(I*c
/(exp(2*I*(f*x+e))+1))^3*n)*exp(1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^2*c
```

```
sgn(I*c)*n)*exp(1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^2*csgn(I/(exp(2*I*(
f*x+e))+1))*n)*exp(-1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))*csgn(I*c)*csgn(
I/(exp(2*I*(f*x+e))+1))*n)*exp(6*I*f*x)*exp(6*I*e)+11*n*c^n*2^n/((exp(2*I*(
f*x+e))+1)^n)*exp(-1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^3*n)*exp(1/2*I*P
i*csgn(I*c/(exp(2*I*(f*x+e))+1))^2*csgn(I*c)*n)*exp(1/2*I*Pi*csgn(I*c/(exp(
2*I*(f*x+e))+1))^2*csgn(I/(exp(2*I*(f*x+e))+1))*n)*exp(-1/2*I*Pi*csgn(I*c/(
exp(2*I*(f*x+e))+1))*csgn(I*c)*csgn(I/(exp(2*I*(f*x+e))+1))*n)*exp(6*I*f*x)
*exp(6*I*e)+3*n^2*c^n*2^n/((exp(2*I*(f*x+e))+1)^n)*exp(-1/2*I*Pi*csgn(I*c/(
exp(2*I*(f*x+e))+1))^3*n)*exp(1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^2*csg
n(I*c)*n)*exp(1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^2*csgn(I/(exp(2*I*(f*
x+e))+1))*n)*exp(-1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))*csgn(I*c)*csgn(I/
(exp(2*I*(f*x+e))+1))*n)*exp(4*I*f*x)*exp(4*I*e)+6*c^n*2^n/((exp(2*I*(f*x+e
))+1)^n)*exp(-1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^3*n)*exp(1/2*I*Pi*csg
n(I*c/(exp(2*I*(f*x+e))+1))^2*csgn(I*c)*n)*exp(1/2*I*Pi*csgn(I*c/(exp(2*I*(
f*x+e))+1))^2*csgn(I/(exp(2*I*(f*x+e))+1))*n)*exp(-1/2*I*Pi*csgn(I*c/(exp(2
*I*(f*x+e))+1))*csgn(I*c)*csgn(I/(exp(2*I*(f*x+e))+1))*n)*exp(6*I*f*x)*exp(
6*I*e)+15*n*c^n*2^n/((exp(2*I*(f*x+e))+1)^n)*exp(-1/2*I*Pi*csgn(I*c/(exp(2*
I*(f*x+e))+1))^3*n)*exp(1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^2*csgn(I*c)
*n)*exp(1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^2*csgn(I/(exp(2*I*(f*x+e))+
1))*n)*exp(-1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))*csgn(I*c)*csgn(I/(exp(2
*I*(f*x+e))+1))*n)*exp(4*I*f*x)*exp(4*I*e)+18*c^n*2^n/((exp(2*I*(f*x+e))+1)
^n)*exp(-1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^3*n)*exp(1/2*I*Pi*csgn(I*c
/(exp(2*I*(f*x+e))+1))^2*csgn(I*c)*n)*exp(1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e
))+1))^2*csgn(I/(exp(2*I*(f*x+e))+1))*n)*exp(-1/2*I*Pi*csgn(I*c/(exp(2*I*(f
*x+e))+1))*csgn(I*c)*csgn(I/(exp(2*I*(f*x+e))+1))*n)*exp(4*I*f*x)*exp(4*I*e
)+6*n*c^n*2^n/((exp(2*I*(f*x+e))+1)^n)*exp(-1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x
+e))+1))^3*n)*exp(1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^2*csgn(I*c)*n)*ex
p(1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^2*csgn(I/(exp(2*I*(f*x+e))+1))*n)
*exp(-1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))*csgn(I*c)*csgn(I/(exp(2*I*(f*
x+e))+1))*n)*exp(2*I*f*x)*exp(2*I*e)+18*c^n*2^n/((exp(2*I*(f*x+e))+1)^n)*ex
p(-1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^3*n)*exp(1/2*I*Pi*csgn(I*c/(exp(
2*I*(f*x+e))+1))^2*csgn(I*c)*n)*exp(1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))
^2*csgn(I/(exp(2*I*(f*x+e))+1))*n)*exp(-1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))
+1))*csgn(I*c)*csgn(I/(exp(2*I*(f*x+e))+1))*n)*exp(2*I*f*x)*exp(2*I*e)+6*c^
n*2^n/((exp(2*I*(f*x+e))+1)^n)*exp(1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))*
n*(csgn(I*c/(exp(2*I*(f*x+e))+1))-csgn(I/(exp(2*I*(f*x+e))+1)))*(-csgn(I*c/
(exp(2*I*(f*x+e))+1))+csgn(I*c))))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 905 vs. 2(122) = 244.
time = 0.59, size = 905, normalized size = 6.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^4*(c-I*c*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] (3*2^(n + 4)*a^4*c^n*cos(n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))
 - 3*I*2^(n + 4)*a^4*c^n*sin(n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
 1)) + 48*(a^4*c^n*n + 3*a^4*c^n)*2^n*cos(-2*f*x + n*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e) + 1) - 2*e) + 24*(a^4*c^n*n^2 + 5*a^4*c^n*n + 6*a^4*c^
n)*2^n*cos(-4*f*x + n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 4*e
) + 8*(a^4*c^n*n^3 + 6*a^4*c^n*n^2 + 11*a^4*c^n*n + 6*a^4*c^n)*2^n*cos(-6*f
*x + n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 6*e) + 48*(-I*a^4*
c^n*n - 3*I*a^4*c^n)*2^n*sin(-2*f*x + n*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e) + 1) - 2*e) + 24*(-I*a^4*c^n*n^2 - 5*I*a^4*c^n*n - 6*I*a^4*c^n)*2^n
*sin(-4*f*x + n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 4*e) + 8*
(-I*a^4*c^n*n^3 - 6*I*a^4*c^n*n^2 - 11*I*a^4*c^n*n - 6*I*a^4*c^n)*2^n*sin(-
6*f*x + n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 6*e))/(((-I*n^4
 - 6*I*n^3 - 11*I*n^2 - 6*I*n)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2
*cos(2*f*x + 2*e) + 1)^(1/2*n)*cos(6*f*x + 6*e) - 3*(I*n^4 + 6*I*n^3 + 11*I
*n^2 + 6*I*n)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
 + 1)^(1/2*n)*cos(4*f*x + 4*e) + (n^4 + 6*n^3 + 11*n^2 + 6*n)*(cos(2*f*x +
2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/2*n)*sin(6*f*x + 6
*e) + 3*(n^4 + 6*n^3 + 11*n^2 + 6*n)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)^(1/2*n)*sin(4*f*x + 4*e) + (-I*n^4 - 6*I*n^3 -
 11*I*n^2 - 3*(I*n^4 + 6*I*n^3 + 11*I*n^2 + 6*I*n)*cos(2*f*x + 2*e) + 3*(n^
4 + 6*n^3 + 11*n^2 + 6*n)*sin(2*f*x + 2*e) - 6*I*n)*(cos(2*f*x + 2*e)^2 + s
in(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/2*n))*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(122) = 244$.
time = 1.10, size = 247, normalized size = 1.84

$$-\frac{8\left(-6i\,a^4 + \left(-i\,a^4n^3 - 6i\,a^4n^2 - 11i\,a^4n - 6i\,a^4\right)e^{(6i\,fx+6i\,e)} + 3\left(-i\,a^4n^2 - 5i\,a^4n - 6i\,a^4\right)e^{(4i\,fx+4i\,e)} + 6\left(-i\,a^4n - 3i\,a^4\right)e^{(2i\,fx+2i\,e)}\right)\left(\frac{2c}{e^{(2i\,fx+2i\,e)}+1}\right)^n}{fn^4 + 6\,fn^3 + 11\,fn^2 + 6\,fn + \left(fn^4 + 6\,fn^3 + 11\,fn^2 + 6\,fn\right)e^{(6i\,fx+6i\,e)} + 3\left(fn^4 + 6\,fn^3 + 11\,fn^2 + 6\,fn\right)e^{(4i\,fx+4i\,e)} + 3\left(fn^4 + 6\,fn^3 + 11\,fn^2 + 6\,fn\right)e^{(2i\,fx+2i\,e)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^4*(c-I*c*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] -8*(-6*I*a^4 + (-I*a^4*n^3 - 6*I*a^4*n^2 - 11*I*a^4*n - 6*I*a^4)*e^(6*I*f*x
 + 6*I*e) + 3*(-I*a^4*n^2 - 5*I*a^4*n - 6*I*a^4)*e^(4*I*f*x + 4*I*e) + 6*(-
I*a^4*n - 3*I*a^4)*e^(2*I*f*x + 2*I*e))*(2*c/(e^(2*I*f*x + 2*I*e) + 1))^n/(
f*n^4 + 6*f*n^3 + 11*f*n^2 + 6*f*n + (f*n^4 + 6*f*n^3 + 11*f*n^2 + 6*f*n)*e
^(6*I*f*x + 6*I*e) + 3*(f*n^4 + 6*f*n^3 + 11*f*n^2 + 6*f*n)*e^(4*I*f*x + 4*
I*e) + 3*(f*n^4 + 6*f*n^3 + 11*f*n^2 + 6*f*n)*e^(2*I*f*x + 2*I*e))
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2225 vs. $2(110) = 220$.
time = 1.74, size = 2225, normalized size = 16.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**4*(c-I*c*tan(f*x+e))**n,x)
```

```
[Out] Piecewise((x*(I*a*tan(e) + a)**4*(-I*c*tan(e) + c)**n, Eq(f, 0)), (-6*a**4*
f*x*tan(e + f*x)**3/(6*c**3*f*tan(e + f*x)**3 + 18*I*c**3*f*tan(e + f*x)**2
 - 18*c**3*f*tan(e + f*x) - 6*I*c**3*f) - 18*I*a**4*f*x*tan(e + f*x)**2/(6*
c**3*f*tan(e + f*x)**3 + 18*I*c**3*f*tan(e + f*x)**2 - 18*c**3*f*tan(e + f*
x) - 6*I*c**3*f) + 18*a**4*f*x*tan(e + f*x)/(6*c**3*f*tan(e + f*x)**3 + 18*
I*c**3*f*tan(e + f*x)**2 - 18*c**3*f*tan(e + f*x) - 6*I*c**3*f) + 6*I*a**4*
f*x/(6*c**3*f*tan(e + f*x)**3 + 18*I*c**3*f*tan(e + f*x)**2 - 18*c**3*f*tan
(e + f*x) - 6*I*c**3*f) - 3*I*a**4*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**3
/(6*c**3*f*tan(e + f*x)**3 + 18*I*c**3*f*tan(e + f*x)**2 - 18*c**3*f*tan(e
+ f*x) - 6*I*c**3*f) + 9*a**4*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(6*c
**3*f*tan(e + f*x)**3 + 18*I*c**3*f*tan(e + f*x)**2 - 18*c**3*f*tan(e + f*x
) - 6*I*c**3*f) + 9*I*a**4*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(6*c**3*f*
tan(e + f*x)**3 + 18*I*c**3*f*tan(e + f*x)**2 - 18*c**3*f*tan(e + f*x) - 6*
I*c**3*f) - 3*a**4*log(tan(e + f*x)**2 + 1)/(6*c**3*f*tan(e + f*x)**3 + 18*
I*c**3*f*tan(e + f*x)**2 - 18*c**3*f*tan(e + f*x) - 6*I*c**3*f) + 36*a**4*t
an(e + f*x)**2/(6*c**3*f*tan(e + f*x)**3 + 18*I*c**3*f*tan(e + f*x)**2 - 18
*c**3*f*tan(e + f*x) - 6*I*c**3*f) + 36*I*a**4*tan(e + f*x)/(6*c**3*f*tan(e
 + f*x)**3 + 18*I*c**3*f*tan(e + f*x)**2 - 18*c**3*f*tan(e + f*x) - 6*I*c**
3*f) - 16*a**4/(6*c**3*f*tan(e + f*x)**3 + 18*I*c**3*f*tan(e + f*x)**2 - 18
*c**3*f*tan(e + f*x) - 6*I*c**3*f), Eq(n, -3)), (6*a**4*f*x*tan(e + f*x)**2
/(c**2*f*tan(e + f*x)**2 + 2*I*c**2*f*tan(e + f*x) - c**2*f) + 12*I*a**4*f*
x*tan(e + f*x)/(c**2*f*tan(e + f*x)**2 + 2*I*c**2*f*tan(e + f*x) - c**2*f)
- 6*a**4*f*x/(c**2*f*tan(e + f*x)**2 + 2*I*c**2*f*tan(e + f*x) - c**2*f) +
3*I*a**4*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(c**2*f*tan(e + f*x)**2 +
 2*I*c**2*f*tan(e + f*x) - c**2*f) - 6*a**4*log(tan(e + f*x)**2 + 1)*tan(e
+ f*x)/(c**2*f*tan(e + f*x)**2 + 2*I*c**2*f*tan(e + f*x) - c**2*f) - 3*I*a*
*4*log(tan(e + f*x)**2 + 1)/(c**2*f*tan(e + f*x)**2 + 2*I*c**2*f*tan(e + f*
x) - c**2*f) - a**4*tan(e + f*x)**3/(c**2*f*tan(e + f*x)**2 + 2*I*c**2*f*ta
n(e + f*x) - c**2*f) - 15*a**4*tan(e + f*x)/(c**2*f*tan(e + f*x)**2 + 2*I*c
**2*f*tan(e + f*x) - c**2*f) - 10*I*a**4/(c**2*f*tan(e + f*x)**2 + 2*I*c**2
*f*tan(e + f*x) - c**2*f), Eq(n, -2)), (-24*a**4*f*x*tan(e + f*x)/(2*c*f*ta
n(e + f*x) + 2*I*c*f) - 24*I*a**4*f*x/(2*c*f*tan(e + f*x) + 2*I*c*f) - 12*I
*a**4*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*c*f*tan(e + f*x) + 2*I*c*f)
+ 12*a**4*log(tan(e + f*x)**2 + 1)/(2*c*f*tan(e + f*x) + 2*I*c*f) + I*a**4*
tan(e + f*x)**3/(2*c*f*tan(e + f*x) + 2*I*c*f) + 9*a**4*tan(e + f*x)**2/(2*
c*f*tan(e + f*x) + 2*I*c*f) + 26*a**4/(2*c*f*tan(e + f*x) + 2*I*c*f), Eq(n,
 -1)), (8*a**4*x + 4*I*a**4*log(tan(e + f*x)**2 + 1)/f + a**4*tan(e + f*x)*
*3/(3*f) - 2*I*a**4*tan(e + f*x)**2/f - 7*a**4*tan(e + f*x)/f, Eq(n, 0)), (
a**4*n**3*(-I*c*tan(e + f*x) + c)**n*tan(e + f*x)**3/(f*n**4 + 6*f*n**3 + 1
1*f*n**2 + 6*f*n) - 3*I*a**4*n**3*(-I*c*tan(e + f*x) + c)**n*tan(e + f*x)**
2/(f*n**4 + 6*f*n**3 + 11*f*n**2 + 6*f*n) - 3*a**4*n**3*(-I*c*tan(e + f*x)
```

```
+ c)**n*tan(e + f*x)/(f*n**4 + 6*f*n**3 + 11*f*n**2 + 6*f*n) + I*a**4*n**3*
(-I*c*tan(e + f*x) + c)**n/(f*n**4 + 6*f*n**3 + 11*f*n**2 + 6*f*n) + 3*a**4
*n**2*(-I*c*tan(e + f*x) + c)**n*tan(e + f*x)**3/(f*n**4 + 6*f*n**3 + 11*f*
n**2 + 6*f*n) - 15*I*a**4*n**2*(-I*c*tan(e + f*x) + c)**n*tan(e + f*x)**2/(
f*n**4 + 6*f*n**3 + 11*f*n**2 + 6*f*n) - 21*a**4*n**2*(-I*c*tan(e + f*x) +
c)**n*tan(e + f*x)/(f*n**4 + 6*f*n**3 + 11*f*n**2 + 6*f*n) + 9*I*a**4*n**2*
(-I*c*tan(e + f*x) + c)**n/(f*n**4 + 6*f*n**3 + 11*f*n**2 + 6*f*n) + 2*a**4
*n*(-I*c*tan(e + f*x) + c)**n*tan(e + f*x)**3/(f*n**4 + 6*f*n**3 + 11*f*n**
2 + 6*f*n) - 12*I*a**4*n*(-I*c*tan(e + f*x) + c)**n*tan(e + f*x)**2/(f*n**4
 + 6*f*n**3 + 11*f*n**2 + 6*f*n) - 42*a**4*n*(-I*c*tan(e + f*x) + c)**n*tan
(e + f*x)/(f*n**4 + 6*f*n**3 + 11*f*n**2 + 6*f*n) + 32*I*a**4*n*(-I*c*tan(e
 + f*x) + c)**n/(f*n**4 + 6*f*n**3 + 11*f*n**2 + 6*f*n) + 48*I*a**4*(-I*c*t
an(e + f*x) + c)**n/(f*n**4 + 6*f*n**3 + 11*f*n**2 + 6*f*n), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^4*(c-I*c*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^4*(-I*c*tan(f*x + e) + c)^n, x)
```

Mupad [B]
time = 9.23, size = 229, normalized size = 1.71

$$-\frac{\mathrm{e}^{-e\,3\mathrm{i}-f\,x\,3\mathrm{i}}\,{\left(c-\frac{c\,\sin(e+f\,x)\,1\mathrm{i}}{\cos(e+f\,x)}\right)}^n\,\left(\frac{48\,a^4}{f\,n\,(n^3\,1\mathrm{i}+n^2\,6\mathrm{i}+n\,11\mathrm{i}+6\mathrm{i})}+\frac{48\,a^4\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,(n+3)}{f\,n\,(n^3\,1\mathrm{i}+n^2\,6\mathrm{i}+n\,11\mathrm{i}+6\mathrm{i})}+\frac{24\,a^4\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,(n^2+5\,n+6)}{f\,n\,(n^3\,1\mathrm{i}+n^2\,6\mathrm{i}+n\,11\mathrm{i}+6\mathrm{i})}+\frac{8\,a^4\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,(n^3+6\,n^2+11\,n+6)}{f\,n\,(n^3\,1\mathrm{i}+n^2\,6\mathrm{i}+n\,11\mathrm{i}+6\mathrm{i})}\right)}{8\,{\cos\left(e+f\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^4*(c - c*tan(e + f*x)*1i)^n,x)
```

```
[Out] -(exp(- e*3i - f*x*3i)*(c - (c*sin(e + f*x)*1i)/cos(e + f*x))^n*((48*a^4)/(
f*n*(n*11i + n^2*6i + n^3*1i + 6i)) + (48*a^4*exp(e*2i + f*x*2i)*(n + 3))/(
f*n*(n*11i + n^2*6i + n^3*1i + 6i)) + (24*a^4*exp(e*4i + f*x*4i)*(5*n + n^2
 + 6))/(f*n*(n*11i + n^2*6i + n^3*1i + 6i)) + (8*a^4*exp(e*6i + f*x*6i)*(11
*n + 6*n^2 + n^3 + 6))/(f*n*(n*11i + n^2*6i + n^3*1i + 6i))))/(8*cos(e + f*
x)^3)
```

3.1044 $\int (a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^n\,dx$

Optimal. Leaf size=99

$$\frac{4ia^3(c-ic\tan(e+fx))^n}{fn}-\frac{4ia^3(c-ic\tan(e+fx))^{1+n}}{cf(1+n)}+\frac{ia^3(c-ic\tan(e+fx))^{2+n}}{c^2f(2+n)}$$

```
[Out] 4*I*a^3*(c-I*c*tan(f*x+e))^n/f/n-4*I*a^3*(c-I*c*tan(f*x+e))^(1+n)/c/f/(1+n)
+I*a^3*(c-I*c*tan(f*x+e))^(2+n)/c^2/f/(2+n)
```

Rubi [A]
time = 0.11, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 45}

$$\frac{ia^3(c-ic\tan(e+fx))^{n+2}}{c^2f(n+2)}+\frac{4ia^3(c-ic\tan(e+fx))^n}{fn}-\frac{4ia^3(c-ic\tan(e+fx))^{n+1}}{cf(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])^n,x]
```

```
[Out] ((4*I)*a^3*(c - I*c*Tan[e + f*x])^n)/(f*n) - ((4*I)*a^3*(c - I*c*Tan[e + f*
x])^(1 + n))/(c*f*(1 + n)) + (I*a^3*(c - I*c*Tan[e + f*x])^(2 + n))/(c^2*f*
(2 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int (a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^n\,dx = \left(a^3c^3\right)\int \sec^6(e+fx)(c-ic\tan(e+fx))^{-3+n}\,dx$$
$$= \frac{\left(ia^3\right)\operatorname{Subst}\left(\int (c-x)^2(c+x)^{-1+n}\,dx, x, -ic\tan(e+fx)\right)}{c^2f}$$
$$= \frac{\left(ia^3\right)\operatorname{Subst}\left(\int \left(4c^2(c+x)^{-1+n} - 4c(c+x)^n + (c+x)^{1+n}\right)}{c^2f}$$
$$= \frac{4ia^3(c-ic\tan(e+fx))^n}{fn} - \frac{4ia^3(c-ic\tan(e+fx))^{1+n}}{cf(1+n)}$$

Mathematica [A]
time = 2.96, size = 110, normalized size = 1.11

$$\frac{ia^3e^{n(-\log(c\sec(e+fx))+\log(c-ic\tan(e+fx)))}\sec^2(e+fx)(c\sec(e+fx))^n\left(2(2+n)+\left(4+3n+n^2\right)\cos(2(e+fx))+in(3+n)\sin(2(e+fx))\right)}{fn(1+n)(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])^n,x]
```

```
[Out] (I*a^3*E^(n*(-Log[c*Sec[e + f*x]] + Log[c - I*c*Tan[e + f*x]]))*Sec[e + f*x
]^2*(c*Sec[e + f*x])^n*(2*(2 + n) + (4 + 3*n + n^2)*Cos[2*(e + f*x)] + I*n*
(3 + n)*Sin[2*(e + f*x)]))/(f*n*(1 + n)*(2 + n))
```

Maple [A]
time = 1.29, size = 129, normalized size = 1.30

method	result	size
norman	$\frac{ia^3(n^2+5n+8)e^{n\ln(c-ic\tan(fx+e))}}{(1+n)(2+n)fn} - \frac{ia^3(\tan^2(fx+e))e^{n\ln(c-ic\tan(fx+e))}}{f(2+n)} - \frac{2a^3(3+n)\tan(fx+e)e^{n\ln(c-ic\tan(fx+e))}}{(1+n)f(2+n)}$	129
risch	Expression too large to display	1052

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3*(c-I*c*tan(f*x+e))^n,x,method=_RETURNVERBOSE)
```

```
[Out] I*a^3*(n^2+5*n+8)/(1+n)/(2+n)/f/n*exp(n*ln(c-I*c*tan(f*x+e)))-I/f/(2+n)*a^3
*tan(f*x+e)^2*exp(n*ln(c-I*c*tan(f*x+e)))-2*a^3*(3+n)/(1+n)/f/(2+n)*tan(f*x
+e)*exp(n*ln(c-I*c*tan(f*x+e)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 576 vs. 2(90) = 180.
time = 0.57, size = 576, normalized size = 5.82

$$\frac{2^{n+3}a^3c^n\cos\left(n\arctan\left(\sin(2fx+2e),\cos(2fx+2e)+1\right)\right) - i\cdot 2^{n+3}a^3c^n\sin\left(n\arctan\left(\sin(2fx+2e),\cos(2fx+2e)+1\right)\right) + 8\left(a^3c^nn+2a^3c^n\right)2^n\cos\left(-2fx+n\arctan\left(\sin(2fx+2e),\cos(2fx+2e)+1\right)-2e\right) + 4\left(a^3c^nn^2+3a^3c^nn+2a^3c^n\right)2^n\cos\left(-4fx+n\arctan\left(\sin(2fx+2e),\cos(2fx+2e)+1\right)-4e\right) + 8\left(-ia^3c^nn-2ia^3c^n\right)2^n\sin\left(-2fx+n\arctan\left(\sin(2fx+2e),\cos(2fx+2e)+1\right)-2e\right) + 4\left(-ia^3c^nn^2-3ia^3c^nn-2ia^3c^n\right)2^n\sin\left(-4fx+n\arctan\left(\sin(2fx+2e),\cos(2fx+2e)+1\right)-4e\right)}{\left(\left(-in^3-3in^2-2in\right)\left(\cos(2fx+2e)^2+\sin(2fx+2e)^2+2\cos(2fx+2e)+1\right)^{\frac{1}{2}n}\cos(4fx+4e) + \left(n^3+3n^2+2n\right)\left(\cos(2fx+2e)^2+\sin(2fx+2e)^2+2\cos(2fx+2e)+1\right)^{\frac{1}{2}n}\sin(4fx+4e) + \left(-in^3-3in^2-2\left(in^3+3in^2+2in\right)\cos(2fx+2e)+2\left(n^3+3n^2+2n\right)\sin(2fx+2e)-2in\right)\left(\cos(2fx+2e)^2+\sin(2fx+2e)^2+2\cos(2fx+2e)+1\right)^{\frac{1}{2}n}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c-I*c*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] (2^(n + 3)*a^3*c^n*cos(n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) -
 I*2^(n + 3)*a^3*c^n*sin(n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))
 + 8*(a^3*c^n*n + 2*a^3*c^n)*2^n*cos(-2*f*x + n*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e) + 1) - 2*e) + 4*(a^3*c^n*n^2 + 3*a^3*c^n*n + 2*a^3*c^n)*2^n
*cos(-4*f*x + n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 4*e) + 8*
(-I*a^3*c^n*n - 2*I*a^3*c^n)*2^n*sin(-2*f*x + n*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e) + 1) - 2*e) + 4*(-I*a^3*c^n*n^2 - 3*I*a^3*c^n*n - 2*I*a^3*c
^n)*2^n*sin(-4*f*x + n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 4*
e))/(((-I*n^3 - 3*I*n^2 - 2*I*n)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
 2*cos(2*f*x + 2*e) + 1)^(1/2*n)*cos(4*f*x + 4*e) + (n^3 + 3*n^2 + 2*n)*(co
s(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/2*n)*sin
(4*f*x + 4*e) + (-I*n^3 - 3*I*n^2 - 2*(I*n^3 + 3*I*n^2 + 2*I*n)*cos(2*f*x +
 2*e) + 2*(n^3 + 3*n^2 + 2*n)*sin(2*f*x + 2*e) - 2*I*n)*(cos(2*f*x + 2*e)^2
 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/2*n))*f)
```

Fricas [A]
time = 1.15, size = 155, normalized size = 1.57

$$-\frac{4\left(-2i\,a^3+\left(-i\,a^3n^2-3i\,a^3n-2i\,a^3\right)e^{(4i\,fx+4i\,e)}+2\left(-i\,a^3n-2i\,a^3\right)e^{(2i\,fx+2i\,e)}\right)\left(\frac{2\,c}{e^{(2i\,fx+2i\,e)}+1}\right)^n}{fn^3+3\,fn^2+2\,fn+\left(fn^3+3\,fn^2+2\,fn\right)e^{(4i\,fx+4i\,e)}+2\left(fn^3+3\,fn^2+2\,fn\right)e^{(2i\,fx+2i\,e)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c-I*c*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] -4*(-2*I*a^3 + (-I*a^3*n^2 - 3*I*a^3*n - 2*I*a^3)*e^(4*I*f*x + 4*I*e) + 2*(
-I*a^3*n - 2*I*a^3)*e^(2*I*f*x + 2*I*e))*(2*c/(e^(2*I*f*x + 2*I*e) + 1))^n/
(f*n^3 + 3*f*n^2 + 2*f*n + (f*n^3 + 3*f*n^2 + 2*f*n)*e^(4*I*f*x + 4*I*e) +
2*(f*n^3 + 3*f*n^2 + 2*f*n)*e^(2*I*f*x + 2*I*e))
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 979 vs. 2(80) = 160.
time = 0.99, size = 979, normalized size = 9.89

$$\begin{cases} x\left(ia\tan\left(e\right)+a\right)^3\left(-ic\tan\left(e\right)+c\right)^n & \text{for}\: f=0 \\ \frac{2a^3fx\tan^2\left(e+fx\right)}{2c^2f\tan^2\left(e+fx\right)+4ic^2f\tan\left(e+fx\right)-2c^2f}+\frac{4ia^3fx\tan\left(e+fx\right)}{2c^2f\tan^2\left(e+fx\right)+4ic^2f\tan\left(e+fx\right)-2c^2f}-\frac{2a^3fx}{2c^2f\tan^2\left(e+fx\right)+4ic^2f\tan\left(e+fx\right)-2c^2f}+\frac{ia^3\log\left(\tan^2\left(e+fx\right)+1\right)\tan^2\left(e+fx\right)}{2c^2f\tan^2\left(e+fx\right)+4ic^2f\tan\left(e+fx\right)-2c^2f}-\frac{2a^3\log\left(\tan^2\left(e+fx\right)+1\right)\tan\left(e+fx\right)}{2c^2f\tan^2\left(e+fx\right)+4ic^2f\tan\left(e+fx\right)-2c^2f}-\frac{ia^3\log\left(\tan^2\left(e+fx\right)+1\right)}{2c^2f\tan^2\left(e+fx\right)+4ic^2f\tan\left(e+fx\right)-2c^2f}-\frac{8a^3\tan\left(e+fx\right)}{2c^2f\tan^2\left(e+fx\right)+4ic^2f\tan\left(e+fx\right)-2c^2f}-\frac{4ia^3}{2c^2f\tan^2\left(e+fx\right)+4ic^2f\tan\left(e+fx\right)-2c^2f} & \text{for}\: n=-2 \\ -\frac{4a^3fx\tan\left(e+fx\right)}{cf\tan\left(e+fx\right)+icf}-\frac{4ia^3fx}{cf\tan\left(e+fx\right)+icf}-\frac{2ia^3\log\left(\tan^2\left(e+fx\right)+1\right)\tan\left(e+fx\right)}{cf\tan\left(e+fx\right)+icf}+\frac{2a^3\log\left(\tan^2\left(e+fx\right)+1\right)}{cf\tan\left(e+fx\right)+icf}+\frac{a^3\tan^2\left(e+fx\right)}{cf\tan\left(e+fx\right)+icf}+\frac{5a^3}{cf\tan\left(e+fx\right)+icf} & \text{for}\: n=-1 \\ 4a^3x+\frac{2ia^3\log\left(\tan^2\left(e+fx\right)+1\right)}{f}-\frac{ia^3\tan^2\left(e+fx\right)}{2f}-\frac{3a^3\tan\left(e+fx\right)}{f} & \text{for}\: n=0 \\ -\frac{ia^3n^2\left(-ic\tan\left(e+fx\right)+c\right)^n\tan^2\left(e+fx\right)}{fn^3+3fn^2+2fn}-\frac{2a^3n^2\left(-ic\tan\left(e+fx\right)+c\right)^n\tan\left(e+fx\right)}{fn^3+3fn^2+2fn}+\frac{ia^3n^2\left(-ic\tan\left(e+fx\right)+c\right)^n}{fn^3+3fn^2+2fn}-\frac{ia^3n\left(-ic\tan\left(e+fx\right)+c\right)^n\tan^2\left(e+fx\right)}{fn^3+3fn^2+2fn}-\frac{6a^3n\left(-ic\tan\left(e+fx\right)+c\right)^n\tan\left(e+fx\right)}{fn^3+3fn^2+2fn}+\frac{5ia^3n\left(-ic\tan\left(e+fx\right)+c\right)^n}{fn^3+3fn^2+2fn}+\frac{8ia^3\left(-ic\tan\left(e+fx\right)+c\right)^n}{fn^3+3fn^2+2fn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3*(c-I*c*tan(f*x+e))**n,x)
```

```
[Out] Piecewise((x*(I*a*tan(e) + a)**3*(-I*c*tan(e) + c)**n, Eq(f, 0)), (2*a**3*f
*x*tan(e + f*x)**2/(2*c**2*f*tan(e + f*x)**2 + 4*I*c**2*f*tan(e + f*x) - 2*
```

```
c**2*f) + 4*I*a**3*f*x*tan(e + f*x)/(2*c**2*f*tan(e + f*x)**2 + 4*I*c**2*f*
tan(e + f*x) - 2*c**2*f) - 2*a**3*f*x/(2*c**2*f*tan(e + f*x)**2 + 4*I*c**2*
f*tan(e + f*x) - 2*c**2*f) + I*a**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**
2/(2*c**2*f*tan(e + f*x)**2 + 4*I*c**2*f*tan(e + f*x) - 2*c**2*f) - 2*a**3*
log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*c**2*f*tan(e + f*x)**2 + 4*I*c**2*
f*tan(e + f*x) - 2*c**2*f) - I*a**3*log(tan(e + f*x)**2 + 1)/(2*c**2*f*tan(
e + f*x)**2 + 4*I*c**2*f*tan(e + f*x) - 2*c**2*f) - 8*a**3*tan(e + f*x)/(2*
c**2*f*tan(e + f*x)**2 + 4*I*c**2*f*tan(e + f*x) - 2*c**2*f) - 4*I*a**3/(2*
c**2*f*tan(e + f*x)**2 + 4*I*c**2*f*tan(e + f*x) - 2*c**2*f), Eq(n, -2)), (
-4*a**3*f*x*tan(e + f*x)/(c*f*tan(e + f*x) + I*c*f) - 4*I*a**3*f*x/(c*f*tan
(e + f*x) + I*c*f) - 2*I*a**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(c*f*ta
n(e + f*x) + I*c*f) + 2*a**3*log(tan(e + f*x)**2 + 1)/(c*f*tan(e + f*x) + I
*c*f) + a**3*tan(e + f*x)**2/(c*f*tan(e + f*x) + I*c*f) + 5*a**3/(c*f*tan(e
 + f*x) + I*c*f), Eq(n, -1)), (4*a**3*x + 2*I*a**3*log(tan(e + f*x)**2 + 1)
/f - I*a**3*tan(e + f*x)**2/(2*f) - 3*a**3*tan(e + f*x)/f, Eq(n, 0)), (-I*a
**3*n**2*(-I*c*tan(e + f*x) + c)**n*tan(e + f*x)**2/(f*n**3 + 3*f*n**2 + 2*
f*n) - 2*a**3*n**2*(-I*c*tan(e + f*x) + c)**n*tan(e + f*x)/(f*n**3 + 3*f*n*
*2 + 2*f*n) + I*a**3*n**2*(-I*c*tan(e + f*x) + c)**n/(f*n**3 + 3*f*n**2 + 2
*f*n) - I*a**3*n*(-I*c*tan(e + f*x) + c)**n*tan(e + f*x)**2/(f*n**3 + 3*f*n
**2 + 2*f*n) - 6*a**3*n*(-I*c*tan(e + f*x) + c)**n*tan(e + f*x)/(f*n**3 + 3
*f*n**2 + 2*f*n) + 5*I*a**3*n*(-I*c*tan(e + f*x) + c)**n/(f*n**3 + 3*f*n**2
 + 2*f*n) + 8*I*a**3*(-I*c*tan(e + f*x) + c)**n/(f*n**3 + 3*f*n**2 + 2*f*n)
, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c-I*c*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((I*a*tan(f*x + e) + a)^3*(-I*c*tan(f*x + e) + c)^n, x)
```

Mupad [B]
time = 1.68, size = 230, normalized size = 2.32

$$\frac{2\,a^3\,{\left(\frac{c\,\left(\cos\left(2\,e+2\,f\,x\right)+1-\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}\right)}^n\,\left(n\,7{}\mathrm{i}+\cos\left(2\,e+2\,f\,x\right)\,16{}\mathrm{i}+\cos\left(4\,e+4\,f\,x\right)\,4{}\mathrm{i}-2\,n^2\,\sin\left(2\,e+2\,f\,x\right)-n^2\,\sin\left(4\,e+4\,f\,x\right)+n\,\cos\left(2\,e+2\,f\,x\right)\,10{}\mathrm{i}+n\,\cos\left(4\,e+4\,f\,x\right)\,3{}\mathrm{i}-6\,n\,\sin\left(2\,e+2\,f\,x\right)-3\,n\,\sin\left(4\,e+4\,f\,x\right)+n^2\,1{}\mathrm{i}+n^2\,\cos\left(2\,e+2\,f\,x\right)\,2{}\mathrm{i}+n^2\,\cos\left(4\,e+4\,f\,x\right)\,1{}\mathrm{i}+12{}\mathrm{i}\right)}{f\,n\,\left(4\,\cos\left(2\,e+2\,f\,x\right)+\cos\left(4\,e+4\,f\,x\right)+3\right)\,\left(n^2+3\,n+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i)^n,x)
[Out] (2*a^3*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x)
+ 1))^n*(n*7i + cos(2*e + 2*f*x)*16i + cos(4*e + 4*f*x)*4i - 2*n^2*sin(2*e
+ 2*f*x) - n^2*sin(4*e + 4*f*x) + n*cos(2*e + 2*f*x)*10i + n*cos(4*e + 4*f*
x)*3i - 6*n*sin(2*e + 2*f*x) - 3*n*sin(4*e + 4*f*x) + n^2*1i + n^2*cos(2*e
+ 2*f*x)*2i + n^2*cos(4*e + 4*f*x)*1i + 12i))/(f*n*(4*cos(2*e + 2*f*x) + co
s(4*e + 4*f*x) + 3)*(3*n + n^2 + 2))
```

3.1045 $\int (a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^n\,dx$

Optimal. Leaf size=64

$$\frac{2ia^2(c-ic\tan(e+fx))^n}{fn}-\frac{ia^2(c-ic\tan(e+fx))^{1+n}}{cf(1+n)}$$

```
[Out] 2*I*a^2*(c-I*c*tan(f*x+e))^n/f/n-I*a^2*(c-I*c*tan(f*x+e))^(1+n)/c/f/(1+n)
```

Rubi [A]
time = 0.09, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {3603, 3568, 45}

$$\frac{2ia^2(c-ic\tan(e+fx))^n}{fn}-\frac{ia^2(c-ic\tan(e+fx))^{n+1}}{cf(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^n,x]
```

```
[Out] ((2*I)*a^2*(c - I*c*Tan[e + f*x])^n)/(f*n) - (I*a^2*(c - I*c*Tan[e + f*x])^
(1 + n))/(c*f*(1 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$
\begin{aligned}
\int (a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^n\,dx &= \left(a^2c^2\right)\int \sec^4(e+fx)(c-ic\tan(e+fx))^{-2+n}\,dx \\
&= \frac{\left(ia^2\right)\mathrm{Subst}\left(\int (c-x)(c+x)^{-1+n}\,dx, x, -ic\tan(e+fx)\right)}{cf} \\
&= \frac{\left(ia^2\right)\mathrm{Subst}\left(\int \left(2c(c+x)^{-1+n}-(c+x)^n\right)\,dx, x, -ic\tan(e
}{cf} \\
&= \frac{2ia^2(c-ic\tan(e+fx))^n}{fn} - \frac{ia^2(c-ic\tan(e+fx))^{1+n}}{cf(1+n)}
\end{aligned}
$$

Mathematica [A]
time = 1.62, size = 72, normalized size = 1.12

$$
-\frac{a^2e^{n(-\log(c\sec(e+fx))+\log(c-ic\tan(e+fx)))}(c\sec(e+fx))^n(-i(2+n)+n\tan(e+fx))}{fn(1+n)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^n,x]
```

```
[Out] -((a^2*E^(n*(-Log[c*Sec[e + f*x]] + Log[c - I*c*Tan[e + f*x]]))*(c*Sec[e +
f*x])^n*((-I)*(2 + n) + n*Tan[e + f*x]))/(f*n*(1 + n)))
```

Maple [A]
time = 1.00, size = 78, normalized size = 1.22

method	result
norman	$\frac{i(a^2n+2a^2)e^{n\ln(c-ic\tan(fx+e))}}{fn(1+n)} - \frac{a^2\tan(fx+e)e^{n\ln(c-ic\tan(fx+e))}}{(1+n)f}$
risch	$\frac{2ia^2\left(n\,c^n2^n\left(e^{2i(fx+e)}+1\right)^{-n}e^{-\frac{i\pi\mathrm{csgn}\left(\frac{ic}{e^{2i(fx+e)}+1}\right)^3n}{2}}e^{\frac{i\pi\mathrm{csgn}\left(\frac{ic}{e^{2i(fx+e)}+1}\right)^2\mathrm{csgn}(ic)n}{2}}e^{\frac{i\pi\mathrm{csgn}\left(\frac{ic}{e^{2i(fx+e)}+1}\right)^2\mathrm{csgn}\left(\frac{i}{e^{2i(fx+e)}+1}\right)}{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2*(c-I*c*tan(f*x+e))^n,x,method=_RETURNVERBOSE)
```

```
[Out] I/f/n/(1+n)*(a^2*n+2*a^2)*exp(n*ln(c-I*c*tan(f*x+e)))-a^2/(1+n)/f*tan(f*x+e
)*exp(n*ln(c-I*c*tan(f*x+e)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(58) = 116.
time = 0.56, size = 287, normalized size = 4.48

$$
\frac{2^{n+1}a^2c^n\cos\left(n\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)+1\right)\right)-i\cdot 2^{n+1}a^2c^n\sin\left(n\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)+1\right)\right)+2\left(a^2c^nn+a^2c^n\right)2^n\cos\left(-2fx+n\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)+1\right)-2e\right)+2\left(-ia^2c^nn-ia^2c^n\right)2^n\sin\left(-2fx+n\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)+1\right)-2e\right)}{\left(-in^2+\left(-in^2-in\right)\cos\left(2fx+2e\right)+\left(n^2+n\right)\sin\left(2fx+2e\right)-in\right)\left(\cos\left(2fx+2e\right)^2+\sin\left(2fx+2e\right)^2+2\cos\left(2fx+2e\right)+1\right)^{\frac{1}{2}n}f}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c-I*c*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] (2^(n + 1)*a^2*c^n*cos(n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) -
 I*2^(n + 1)*a^2*c^n*sin(n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))
 + 2*(a^2*c^n*n + a^2*c^n)*2^n*cos(-2*f*x + n*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e) + 1) - 2*e) + 2*(-I*a^2*c^n*n - I*a^2*c^n)*2^n*sin(-2*f*x + n
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 2*e))/((-I*n^2 + (-I*n^2
 - I*n)*cos(2*f*x + 2*e) + (n^2 + n)*sin(2*f*x + 2*e) - I*n)*(cos(2*f*x + 2
*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/2*n)*f)
```

Fricas [A]
time = 0.93, size = 81, normalized size = 1.27

$$-\frac{2\left(-i\,a^{2}+(-i\,a^{2}n-i\,a^{2})e^{(2i\,fx+2i\,e)}\right)\left(\frac{2\,c}{e^{(2i\,fx+2i\,e)}+1}\right)^{n}}{fn^{2}+fn+(fn^{2}+fn)e^{(2i\,fx+2i\,e)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c-I*c*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] -2*(-I*a^2 + (-I*a^2*n - I*a^2)*e^(2*I*f*x + 2*I*e))*(2*c/(e^(2*I*f*x + 2*I
*e) + 1))^n/(f*n^2 + f*n + (f*n^2 + f*n)*e^(2*I*f*x + 2*I*e))
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(49) = 98.
time = 0.53, size = 311, normalized size = 4.86

$$\begin{cases} x(ia\tan(e)+a)^{2}(-ic\tan(e)+c)^{n} & \text{for}\ f=0\\ -\frac{2a^{2}fx\tan(e+fx)}{2cf\tan(e+fx)+2icf}-\frac{2ia^{2}fx}{2cf\tan(e+fx)+2icf}-\frac{ia^{2}\log(\tan^{2}(e+fx)+1)\tan(e+fx)}{2cf\tan(e+fx)+2icf}+\frac{a^{2}\log(\tan^{2}(e+fx)+1)}{2cf\tan(e+fx)+2icf}+\frac{4a^{2}}{2cf\tan(e+fx)+2icf} & \text{for}\ n=-1\\ 2a^{2}x+\frac{ia^{2}\log(\tan^{2}(e+fx)+1)}{f}-\frac{a^{2}\tan(e+fx)}{f} & \text{for}\ n=0\\ -\frac{a^{2}n(-ic\tan(e+fx)+c)^{n}\tan(e+fx)}{fn^{2}+fn}+\frac{ia^{2}n(-ic\tan(e+fx)+c)^{n}}{fn^{2}+fn}+\frac{2ia^{2}(-ic\tan(e+fx)+c)^{n}}{fn^{2}+fn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2*(c-I*c*tan(f*x+e))**n,x)
```

```
[Out] Piecewise((x*(I*a*tan(e) + a)**2*(-I*c*tan(e) + c)**n, Eq(f, 0)), (-2*a**2*
f*x*tan(e + f*x)/(2*c*f*tan(e + f*x) + 2*I*c*f) - 2*I*a**2*f*x/(2*c*f*tan(e
 + f*x) + 2*I*c*f) - I*a**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*c*f*ta
n(e + f*x) + 2*I*c*f) + a**2*log(tan(e + f*x)**2 + 1)/(2*c*f*tan(e + f*x) +
 2*I*c*f) + 4*a**2/(2*c*f*tan(e + f*x) + 2*I*c*f), Eq(n, -1)), (2*a**2*x +
I*a**2*log(tan(e + f*x)**2 + 1)/f - a**2*tan(e + f*x)/f, Eq(n, 0)), (-a**2*
n*(-I*c*tan(e + f*x) + c)**n*tan(e + f*x)/(f*n**2 + f*n) + I*a**2*n*(-I*c*t
an(e + f*x) + c)**n/(f*n**2 + f*n) + 2*I*a**2*(-I*c*tan(e + f*x) + c)**n/(f
*n**2 + f*n), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c-I*c*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((I*a*tan(f*x + e) + a)^2*(-I*c*tan(f*x + e) + c)^n, x)
```

Mupad [B]
time = 0.39, size = 112, normalized size = 1.75

$$\frac{a^2\left(\frac{c\,(\cos(2\,e+2\,f\,x)+1-\sin(2\,e+2\,f\,x)\,1\mathrm{i})}{\cos(2\,e+2\,f\,x)+1}\right)^n\,(n\,1\mathrm{i}+\cos\left(2\,e+2\,f\,x\right)\,2\mathrm{i}+n\,\cos\left(2\,e+2\,f\,x\right)\,1\mathrm{i}-n\,\sin\left(2\,e+2\,f\,x\right)+2\mathrm{i})}{f\,n\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)\,\left(n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i)^n,x)
[Out] (a^2*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) +
1))^n*(n*1i + cos(2*e + 2*f*x)*2i + n*cos(2*e + 2*f*x)*1i - n*sin(2*e + 2*f
*x) + 2i))/(f*n*(cos(2*e + 2*f*x) + 1)*(n + 1))
```

3.1046 $\int (a+ia\tan(e+fx))(c-ic\tan(e+fx))^n\,dx$

Optimal. Leaf size=26

$$\frac{ia(c-ic\tan(e+fx))^n}{fn}$$

```
[Out] I*a*(c-I*c*tan(f*x+e))^n/f/n
```

Rubi [A]
time = 0.06, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {3603, 3568, 32}

$$\frac{ia(c-ic\tan(e+fx))^n}{fn}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^n,x]
```

```
[Out] (I*a*(c - I*c*Tan[e + f*x])^n)/(f*n)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int (a+ia\tan(e+fx))(c-ic\tan(e+fx))^n\,dx = (ac)\int \sec^2(e+fx)(c-ic\tan(e+fx))^{-1+n}\,dx$$
$$= \frac{(ia)\text{Subst}\left(\int (c+x)^{-1+n}\,dx, x, -ic\tan(e+fx)\right)}{f}$$
$$= \frac{ia(c-ic\tan(e+fx))^n}{fn}$$

Mathematica [A]
time = 0.42, size = 51, normalized size = 1.96

$$\frac{iae^{n(-\log(c\sec(e+fx))+\log(c-ic\tan(e+fx)))}(c\sec(e+fx))^n}{fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^n,x]
```

```
[Out] (I*a*E^(n*(-Log[c*Sec[e + f*x]] + Log[c - I*c*Tan[e + f*x]]))*(c*Sec[e + f*
x])^n)/(f*n)
```

Maple [A]
time = 0.22, size = 25, normalized size = 0.96

method	result
derivativedivides	$\frac{ia(c-ic\tan(fx+e))^n}{fn}$
default	$\frac{ia(c-ic\tan(fx+e))^n}{fn}$
norman	$\frac{ia\,\mathrm{e}^{n\ln(c-ic\tan(fx+e))}}{fn}$
risch	$\frac{ia\,\mathrm{e}^{\frac{n\left(-i\pi\mathrm{csgn}\left(\frac{ic}{\mathrm{e}^{2i(fx+e)}+1}\right)^3+i\pi\mathrm{csgn}\left(\frac{ic}{\mathrm{e}^{2i(fx+e)}+1}\right)^2\mathrm{csgn}(ic)+i\pi\mathrm{csgn}\left(\frac{ic}{\mathrm{e}^{2i(fx+e)}+1}\right)^2\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2i(fx+e)}+1}\right)-i\pi\,\mathrm{csgn}\left(\frac{ic}{\mathrm{e}^{2i(fx+}\right.\cdots}{2}}}{fn}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e))^n,x,method=_RETURNVERBOSE)
```

```
[Out] I*a*(c-I*c*tan(f*x+e))^n/f/n
```

Maxima [A]
time = 0.52, size = 25, normalized size = 0.96

$$\frac{i\,ac^n(-i\,\tan(fx+e)+1)^n}{fn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] I*a*c^n*(-I*tan(f*x + e) + 1)^n/(f*n)
```

Fricas [A]
time = 1.17, size = 28, normalized size = 1.08

$$\frac{i\,a\left(\frac{2\,c}{e^{(2i\,fx+2i\,e)}+1}\right)^{n}}{fn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] I*a*(2*c/(e^(2*I*f*x + 2*I*e) + 1))^n/(f*n)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(19) = 38.
time = 0.21, size = 70, normalized size = 2.69

$$\begin{cases} x(ia\tan{(e)}+a) & \text{for } f=0 \wedge n=0 \\ ax+\frac{ia\log{(\tan^2{(e+fx)}+1)}}{2f} & \text{for } n=0 \\ x(ia\tan{(e)}+a)\left(-ic\tan{(e)}+c\right)^{n} & \text{for } f=0 \\ \frac{ia(-ic\tan{(e+fx)}+c)^{n}}{fn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e))**n,x)
```

```
[Out] Piecewise((x*(I*a*tan(e) + a), Eq(f, 0) & Eq(n, 0)), (a*x + I*a*log(tan(e +
 f*x)**2 + 1)/(2*f), Eq(n, 0)), (x*(I*a*tan(e) + a)*(-I*c*tan(e) + c)**n, E
q(f, 0)), (I*a*(-I*c*tan(e + f*x) + c)**n/(f*n), True))
```

Giac [A]
time = 1.22, size = 23, normalized size = 0.88

$$\frac{i\left(-i\,c\tan\left(fx+e\right)+c\right)^{n}a}{fn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] I*(-I*c*tan(f*x + e) + c)^n*a/(f*n)
```

Mupad [B]
time = 4.86, size = 40, normalized size = 1.54

$$\frac{a\left(\frac{2\,c}{\cos(2\,e+2\,f\,x)+1+\sin(2\,e+2\,f\,x)\,1\mathrm{i}}\right)^n\,1\mathrm{i}}{f\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i)^n,x)
[Out] (a*((2*c)/(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))^n*1i)/(f*n)
```

3.1047 $\int \frac{(c-ic\tan(e+fx))^n}{a+ia\tan(e+fx)}\,dx$

Optimal. Leaf size=52

$$\frac{i\,{}_2F_1\left(2,n;1+n;\frac{1}{2}(1-i\tan(e+fx))\right)(c-ic\tan(e+fx))^n}{4afn}$$

```
[Out] 1/4*I*hypergeom([2, n],[1+n],1/2-1/2*I*tan(f*x+e))*(c-I*c*tan(f*x+e))^n/a/f
/n
```

Rubi [A]
time = 0.09, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 70}

$$\frac{i(c-ic\tan(e+fx))^n\,{}_2F_1\left(2,n;n+1;\frac{1}{2}(1-i\tan(e+fx))\right)}{4afn}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])^n/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] ((I/4)*Hypergeometric2F1[2, n, 1 + n, (1 - I*Tan[e + f*x])/2]*(c - I*c*Tan[
e + f*x])^n)/(a*f*n)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{(c-ic\tan(e+fx))^n}{a+ia\tan(e+fx)}\,dx = \frac{\int \cos^2(e+fx)(c-ic\tan(e+fx))^{1+n}\,dx}{ac}$$
$$= \frac{(ic^2)\,\text{Subst}\left(\int \frac{(c+x)^{-1+n}}{(c-x)^2}\,dx, x, -ic\tan(e+fx)\right)}{af}$$
$$= \frac{i\,{}_2F_1\left(2,n;1+n;\frac{1}{2}(1-i\tan(e+fx))\right)(c-ic\tan(e+fx))^n}{4afn}$$

Mathematica [A]
time = 58.69, size = 79, normalized size = 1.52

$$\frac{i2^{-2+n}\left(\frac{c}{1+e^{2i(e+fx)}}\right)^n\left(1+e^{2i(e+fx)}\right)^2\,{}_2F_1\left(2,2-n;3-n;1+e^{2i(e+fx)}\right)}{af(-2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - I*c*Tan[e + f*x])^n/(a + I*a*Tan[e + f*x]),x]
[Out] (I*2^(-2 + n)*(c/(1 + E^((2*I)*(e + f*x))))^n*(1 + E^((2*I)*(e + f*x)))^2*H
ypergeometric2F1[2, 2 - n, 3 - n, 1 + E^((2*I)*(e + f*x))])/(a*f*(-2 + n))
```

Maple [F]
time = 2.90, size = 0, normalized size = 0.00

$$\int \frac{(c-ic\tan(fx+e))^n}{a+ia\tan(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e)),x)
[Out] int((c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
[Out] integral(1/2*(2*c/(e^(2*I*f*x + 2*I*e) + 1))^n*(e^(2*I*f*x + 2*I*e) + 1)*e^
(-2*I*f*x - 2*I*e)/a, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{(-ic\tan{(e+fx)}+c)^n}{\tan{(e+fx)}-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**n/(a+I*a*tan(f*x+e)),x)
[Out] -I*Integral((-I*c*tan(e + f*x) + c)**n/(tan(e + f*x) - I), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((-I*c*tan(f*x + e) + c)^n/(I*a*tan(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{(c-c\tan(e+f\,x)\ 1\mathrm{i})^n}{a+a\tan\left(e+f\,x\right)\ 1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^n/(a + a*tan(e + f*x)*1i),x)
[Out] int((c - c*tan(e + f*x)*1i)^n/(a + a*tan(e + f*x)*1i), x)
```

3.1048 $\int \frac{(c-ic\tan(e+fx))^n}{(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=52

$$\frac{i\,{}_2F_1\left(3,n;1+n;\frac{1}{2}(1-i\tan(e+fx))\right)(c-ic\tan(e+fx))^n}{8a^2fn}$$

```
[Out] 1/8*I*hypergeom([3, n],[1+n],1/2-1/2*I*tan(f*x+e))*(c-I*c*tan(f*x+e))^n/a^2
/f/n
```

Rubi [A]
time = 0.09, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 70}

$$\frac{i(c-ic\tan(e+fx))^n\,{}_2F_1\left(3,n;n+1;\frac{1}{2}(1-i\tan(e+fx))\right)}{8a^2fn}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])^n/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] ((I/8)*Hypergeometric2F1[3, n, 1 + n, (1 - I*Tan[e + f*x])/2]*(c - I*c*Tan[
e + f*x])^n)/(a^2*f*n)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\begin{aligned}\int \frac{(c-ic\tan(e+fx))^n}{(a+ia\tan(e+fx))^2}\,dx &= \frac{\int \cos^4(e+fx)(c-ic\tan(e+fx))^{2+n}\,dx}{a^2c^2}\\ &= \frac{(ic^3)\,\text{Subst}\left(\int \frac{(c+x)^{-1+n}}{(c-x)^3}\,dx, x, -ic\tan(e+fx)\right)}{a^2f}\\ &= \frac{i\,{}_2F_1\left(3,n;1+n;\frac{1}{2}(1-i\tan(e+fx))\right)(c-ic\tan(e+fx))^n}{8a^2fn}\end{aligned}$$

Mathematica [**F**]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(c - I*c*Tan[e + f*x])^n/(a + I*a*Tan[e + f*x])^2,x]
[Out] $Aborted
```

Maple [**F**]
time = 2.69, size = 0, normalized size = 0.00

$$\int \frac{(c-ic\tan(fx+e))^n}{(a+ia\tan(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^2,x)
[Out] int((c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^2,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/4*(2*c/(e^(2*I*f*x + 2*I*e) + 1))^n*(e^(4*I*f*x + 4*I*e) + 2*e^(
2*I*f*x + 2*I*e) + 1)*e^(-4*I*f*x - 4*I*e)/a^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{(-ic\tan{(e+fx)}+c)^n}{\tan^2{(e+fx)}-2i\tan{(e+fx)}-1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**n/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -Integral((-I*c*tan(e + f*x) + c)**n/(tan(e + f*x)**2 - 2*I*tan(e + f*x) -
1), x)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((-I*c*tan(f*x + e) + c)^n/(I*a*tan(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c-c\tan(e+f\,x)\,1\mathrm{i})^n}{(a+a\tan\left(e+f\,x\right)\,1\mathrm{i})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^n/(a + a*tan(e + f*x)*1i)^2,x)
```

```
[Out] int((c - c*tan(e + f*x)*1i)^n/(a + a*tan(e + f*x)*1i)^2, x)
```

3.1049 $\int \frac{(c-ic\tan(e+fx))^n}{(a+ia\tan(e+fx))^3}\,dx$

Optimal. Leaf size=52

$$\frac{i\,{}_2F_1\left(4,n;1+n;\frac{1}{2}(1-i\tan(e+fx))\right)(c-ic\tan(e+fx))^n}{16a^3fn}$$

```
[Out] 1/16*I*hypergeom([4, n],[1+n],1/2-1/2*I*tan(f*x+e))*(c-I*c*tan(f*x+e))^n/a^
3/f/n
```

Rubi [A]
time = 0.09, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 70}

$$\frac{i(c-ic\tan(e+fx))^n\,{}_2F_1\left(4,n;n+1;\frac{1}{2}(1-i\tan(e+fx))\right)}{16a^3fn}$$

Antiderivative was successfully verified.

```
[In] Int[(c - I*c*Tan[e + f*x])^n/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] ((I/16)*Hypergeometric2F1[4, n, 1 + n, (1 - I*Tan[e + f*x])/2]*(c - I*c*Tan
[e + f*x])^n)/(a^3*f*n)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{(c-ic\tan(e+fx))^n}{(a+ia\tan(e+fx))^3}\,dx = \frac{\int \cos^6(e+fx)(c-ic\tan(e+fx))^{3+n}\,dx}{a^3c^3}$$
$$= \frac{(ic^4)\,\text{Subst}\left(\int \frac{(c+x)^{-1+n}}{(c-x)^4}\,dx, x, -ic\tan(e+fx)\right)}{a^3f}$$
$$= \frac{i\,{}_2F_1\left(4,n;1+n;\frac{1}{2}(1-i\tan(e+fx))\right)(c-ic\tan(e+fx))^n}{16a^3fn}$$

Mathematica [F]
time = 124.57, size = 0, normalized size = 0.00

$$\int \frac{(c-ic\tan(e+fx))^n}{(a+ia\tan(e+fx))^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c - I*c*Tan[e + f*x])^n/(a + I*a*Tan[e + f*x])^3,x]
[Out] Integrate[(c - I*c*Tan[e + f*x])^n/(a + I*a*Tan[e + f*x])^3, x]
```

Maple [F]
time = 2.74, size = 0, normalized size = 0.00

$$\int \frac{(c-ic\tan(fx+e))^n}{(a+ia\tan(fx+e))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^3,x)
[Out] int((c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^3,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^3,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(1/8*(2*c/(e^(2*I*f*x + 2*I*e) + 1))^n*(e^(6*I*f*x + 6*I*e) + 3*e^(
4*I*f*x + 4*I*e) + 3*e^(2*I*f*x + 2*I*e) + 1)*e^(-6*I*f*x - 6*I*e)/a^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int \frac{(-ic\tan{(e+fx)}+c)^n}{\tan^3{(e+fx)}-3i\tan^2{(e+fx)}-3\tan{(e+fx)}+i}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))**n/(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] I*Integral((-I*c*tan(e + f*x) + c)**n/(tan(e + f*x)**3 - 3*I*tan(e + f*x)**
2 - 3*tan(e + f*x) + I), x)/a**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((-I*c*tan(f*x + e) + c)^n/(I*a*tan(f*x + e) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^n}{{\left(a+a\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*tan(e + f*x)*1i)^n/(a + a*tan(e + f*x)*1i)^3,x)
```

```
[Out] int((c - c*tan(e + f*x)*1i)^n/(a + a*tan(e + f*x)*1i)^3, x)
```

3.1050 $\int (a+ia\tan(e+fx))^m(c-ic\tan(e+fx))^n\,dx$

Optimal. Leaf size=66

$$\frac{i\,{}_2F_1\left(1,m+n;1+n;\frac{1}{2}(1-i\tan(e+fx))\right)(a+ia\tan(e+fx))^m(c-ic\tan(e+fx))^n}{2fn}$$

```
[Out] 1/2*I*hypergeom([1, n+m],[1+n],1/2-1/2*I*tan(f*x+e))*(a+I*a*tan(f*x+e))^m*(
c-I*c*tan(f*x+e))^n/f/n
```

Rubi [A]
time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.32, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3604, 72, 71}

$$-\frac{i2^{n-1}(1-i\tan(e+fx))^{-n}(a+ia\tan(e+fx))^m(c-ic\tan(e+fx))^n\,{}_2F_1\left(m,1-n;m+1;\frac{1}{2}(i\tan(e+fx)+1)\right)}{fm}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^m*(c - I*c*Tan[e + f*x])^n,x]
```

```
[Out] ((-I)*2^(-1 + n)*Hypergeometric2F1[m, 1 - n, 1 + m, (1 + I*Tan[e + f*x])/2]
*(a + I*a*Tan[e + f*x])^m*(c - I*c*Tan[e + f*x])^n)/(f*m*(1 - I*Tan[e + f*x
])^n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
```

```
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a+ia\tan(e+fx))^m(c-ic\tan(e+fx))^n\,dx = \frac{(ac)\text{Subst}\left(\int (a+iax)^{-1+m}(c-icx)^{-1+n}\,dx, x, \tan(e+\right.}{f}$$

$$= \frac{\left(2^{-1+n}a(c-ic\tan(e+fx))^n\left(\frac{c-ic\tan(e+fx)}{c}\right)^{-n}\right)\text{Subst}}{f}$$

$$= -\frac{i2^{-1+n}\,{}_2F_1\left(m,1-n;1+m;\frac{1}{2}(1+i\tan(e+fx))\right)(1}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 142 vs. $2(66)=132$.
time = 13.84, size = 142, normalized size = 2.15

$$-\frac{i2^{-1+m+n}c\left(e^{ifx}\right)^m\left(\frac{c}{1+e^{2i(e+fx)}}\right)^{-1+n}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^m{}_2F_1\left(1,1-n;1+m;-e^{2i(e+fx)}\right)\sec^{-m}(e+fx)(\cos(fx)+i\sin(fx))^{-m}(a+ia\tan(e+fx))^m}{fm}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^m*(c - I*c*Tan[e + f*x])^n,x]
```

```
[Out] ((-I)*2^(-1 + m + n)*c*(E^(I*f*x))^m*(c/(1 + E^((2*I)*(e + f*x))))^(-1 + n)
*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^m*Hypergeometric2F1[1, 1 - n,
1 + m, -E^((2*I)*(e + f*x))]*(a + I*a*Tan[e + f*x])^m)/(f*m*Sec[e + f*x]^m*
(Cos[f*x] + I*Sin[f*x])^m)
```

Maple [F]
time = 2.67, size = 0, normalized size = 0.00

$$\int (a+ia\tan(fx+e))^m(c-ic\tan(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e))^n,x)
```

```
[Out] int((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^m*(-I*c*tan(f*x + e) + c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((2*c/(e^(2*I*f*x + 2*I*e) + 1))^n*e^(2*I*f*m*x + 2*I*m*e + m*log(a
/c) + m*log(2*c/(e^(2*I*f*x + 2*I*e) + 1))), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan{(e+fx)}-i))^m\,(-ic(\tan{(e+fx)}+i))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**m*(c-I*c*tan(f*x+e))**n,x)
```

```
[Out] Integral((I*a*(tan(e + f*x) - I))**m*(-I*c*(tan(e + f*x) + I))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^m*(-I*c*tan(f*x + e) + c)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(a+a\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^m\,{\left(c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^m*(c - c*tan(e + f*x)*1i)^n,x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^m*(c - c*tan(e + f*x)*1i)^n, x)
```

3.1051 $\int (a+ia\tan(e+fx))^m(c-ic\tan(e+fx))^4\,dx$

Optimal. Leaf size=134

$$-\frac{8ic^4(a+ia\tan(e+fx))^m}{fm}+\frac{12ic^4(a+ia\tan(e+fx))^{1+m}}{af(1+m)}-\frac{6ic^4(a+ia\tan(e+fx))^{2+m}}{a^2f(2+m)}+\frac{ic^4(a+ia\tan(e+fx))^{3+m}}{a^3f(3+m)}$$

```
[Out] -8*I*c^4*(a+I*a*tan(f*x+e))^m/f/m+12*I*c^4*(a+I*a*tan(f*x+e))^(1+m)/a/f/(1+
m)-6*I*c^4*(a+I*a*tan(f*x+e))^(2+m)/a^2/f/(2+m)+I*c^4*(a+I*a*tan(f*x+e))^(3
+m)/a^3/f/(3+m)
```

Rubi [A]

time = 0.12, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 45}

$$\frac{ic^4(a+ia\tan(e+fx))^{m+3}}{a^3f(m+3)}-\frac{6ic^4(a+ia\tan(e+fx))^{m+2}}{a^2f(m+2)}-\frac{8ic^4(a+ia\tan(e+fx))^m}{fm}+\frac{12ic^4(a+ia\tan(e+fx))^{m+1}}{af(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^m*(c - I*c*Tan[e + f*x])^4,x]
```

```
[Out] ((-8*I)*c^4*(a + I*a*Tan[e + f*x])^m)/(f*m) + ((12*I)*c^4*(a + I*a*Tan[e +
f*x])^(1 + m))/(a*f*(1 + m)) - ((6*I)*c^4*(a + I*a*Tan[e + f*x])^(2 + m))/(
a^2*f*(2 + m)) + (I*c^4*(a + I*a*Tan[e + f*x])^(3 + m))/(a^3*f*(3 + m))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int (a+ia\tan(e+fx))^m(c-ic\tan(e+fx))^4\,dx = (a^4c^4)\int \sec^8(e+fx)(a+ia\tan(e+fx))^{-4+m}\,dx$$
$$= -\frac{(ic^4)\,\text{Subst}\left(\int (a-x)^3(a+x)^{-1+m}\,dx, x, ia\tan(e+fx)\right)}{a^3f}$$
$$= -\frac{(ic^4)\,\text{Subst}\left(\int (8a^3(a+x)^{-1+m} - 12a^2(a+x)^m + 6a(a}{a^3f}$$
$$= -\frac{8ic^4(a+ia\tan(e+fx))^m}{fm} + \frac{12ic^4(a+ia\tan(e+fx))}{af(1+m)}$$

Mathematica [F]
time = 61.69, size = 0, normalized size = 0.00

$$\int (a+ia\tan(e+fx))^m(c-ic\tan(e+fx))^4\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^m*(c - I*c*Tan[e + f*x])^4,x]
[Out] Integrate[(a + I*a*Tan[e + f*x])^m*(c - I*c*Tan[e + f*x])^4, x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.28, size = 5385, normalized size = 40.19

method	result	size
risch	Expression too large to display	5385

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e))^4,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e))^4,x, algorithm="maxima")
[Out] integrate((-I*c*tan(f*x + e) + c)^4*(I*a*tan(f*x + e) + a)^m, x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(122) = 244$.
time = 0.94, size = 255, normalized size = 1.90

$$-\frac{8\left(i c^4 m^3 + 6 i c^4 m^2 + 11 i c^4 m + 6 i c^4 e^{(6 i f x + 6 i e)} + 6 i c^4 + 6\left(i c^4 m + 3 i c^4\right) e^{(4 i f x + 4 i e)} + 3\left(i c^4 m^2 + 5 i c^4 m + 6 i c^4\right) e^{(2 i f x + 2 i e)}\right)\left(\frac{2 a e^{(2 i f x + 2 i e)}}{e^{(2 i f x + 2 i e)} + 1}\right)^m}{f m^4 + 6 f m^3 + 11 f m^2 + 6 f m + \left(f m^4 + 6 f m^3 + 11 f m^2 + 6 f m\right) e^{(6 i f x + 6 i e)} + 3\left(f m^4 + 6 f m^3 + 11 f m^2 + 6 f m\right) e^{(4 i f x + 4 i e)} + 3\left(f m^4 + 6 f m^3 + 11 f m^2 + 6 f m\right) e^{(2 i f x + 2 i e)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] -8*(I*c^4*m^3 + 6*I*c^4*m^2 + 11*I*c^4*m + 6*I*c^4*e^(6*I*f*x + 6*I*e) + 6*
I*c^4 + 6*(I*c^4*m + 3*I*c^4)*e^(4*I*f*x + 4*I*e) + 3*(I*c^4*m^2 + 5*I*c^4*
m + 6*I*c^4)*e^(2*I*f*x + 2*I*e))*(2*a*e^(2*I*f*x + 2*I*e)/(e^(2*I*f*x + 2*
I*e) + 1))^m/(f*m^4 + 6*f*m^3 + 11*f*m^2 + 6*f*m + (f*m^4 + 6*f*m^3 + 11*f*
m^2 + 6*f*m)*e^(6*I*f*x + 6*I*e) + 3*(f*m^4 + 6*f*m^3 + 11*f*m^2 + 6*f*m)*e
^(4*I*f*x + 4*I*e) + 3*(f*m^4 + 6*f*m^3 + 11*f*m^2 + 6*f*m)*e^(2*I*f*x + 2*
I*e))
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2225 vs. $2(110) = 220$.
time = 1.64, size = 2225, normalized size = 16.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**m*(c-I*c*tan(f*x+e))**4,x)
```

```
[Out] Piecewise((x*(I*a*tan(e) + a)**m*(-I*c*tan(e) + c)**4, Eq(f, 0)), (-6*c**4*
f*x*tan(e + f*x)**3/(6*a**3*f*tan(e + f*x)**3 - 18*I*a**3*f*tan(e + f*x)**2
 - 18*a**3*f*tan(e + f*x) + 6*I*a**3*f) + 18*I*c**4*f*x*tan(e + f*x)**2/(6*
a**3*f*tan(e + f*x)**3 - 18*I*a**3*f*tan(e + f*x)**2 - 18*a**3*f*tan(e + f*
x) + 6*I*a**3*f) + 18*c**4*f*x*tan(e + f*x)/(6*a**3*f*tan(e + f*x)**3 - 18*
I*a**3*f*tan(e + f*x)**2 - 18*a**3*f*tan(e + f*x) + 6*I*a**3*f) - 6*I*c**4*
f*x/(6*a**3*f*tan(e + f*x)**3 - 18*I*a**3*f*tan(e + f*x)**2 - 18*a**3*f*tan
(e + f*x) + 6*I*a**3*f) + 3*I*c**4*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**3
/(6*a**3*f*tan(e + f*x)**3 - 18*I*a**3*f*tan(e + f*x)**2 - 18*a**3*f*tan(e
 + f*x) + 6*I*a**3*f) + 9*c**4*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(6*a
**3*f*tan(e + f*x)**3 - 18*I*a**3*f*tan(e + f*x)**2 - 18*a**3*f*tan(e + f*x
) + 6*I*a**3*f) - 9*I*c**4*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(6*a**3*f*
tan(e + f*x)**3 - 18*I*a**3*f*tan(e + f*x)**2 - 18*a**3*f*tan(e + f*x) + 6*
I*a**3*f) - 3*c**4*log(tan(e + f*x)**2 + 1)/(6*a**3*f*tan(e + f*x)**3 - 18*
I*a**3*f*tan(e + f*x)**2 - 18*a**3*f*tan(e + f*x) + 6*I*a**3*f) + 36*c**4*t
an(e + f*x)**2/(6*a**3*f*tan(e + f*x)**3 - 18*I*a**3*f*tan(e + f*x)**2 - 18
*a**3*f*tan(e + f*x) + 6*I*a**3*f) - 36*I*c**4*tan(e + f*x)/(6*a**3*f*tan(e
 + f*x)**3 - 18*I*a**3*f*tan(e + f*x)**2 - 18*a**3*f*tan(e + f*x) + 6*I*a**
3*f) - 16*c**4/(6*a**3*f*tan(e + f*x)**3 - 18*I*a**3*f*tan(e + f*x)**2 - 18
```

```
*a**3*f*tan(e + f*x) + 6*I*a**3*f), Eq(m, -3)), (6*c**4*f*x*tan(e + f*x)**2
/(a**2*f*tan(e + f*x)**2 - 2*I*a**2*f*tan(e + f*x) - a**2*f) - 12*I*c**4*f*
x*tan(e + f*x)/(a**2*f*tan(e + f*x)**2 - 2*I*a**2*f*tan(e + f*x) - a**2*f)
- 6*c**4*f*x/(a**2*f*tan(e + f*x)**2 - 2*I*a**2*f*tan(e + f*x) - a**2*f) -
3*I*c**4*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(a**2*f*tan(e + f*x)**2 -
 2*I*a**2*f*tan(e + f*x) - a**2*f) - 6*c**4*log(tan(e + f*x)**2 + 1)*tan(e
+ f*x)/(a**2*f*tan(e + f*x)**2 - 2*I*a**2*f*tan(e + f*x) - a**2*f) + 3*I*c*
*4*log(tan(e + f*x)**2 + 1)/(a**2*f*tan(e + f*x)**2 - 2*I*a**2*f*tan(e + f*
x) - a**2*f) - c**4*tan(e + f*x)**3/(a**2*f*tan(e + f*x)**2 - 2*I*a**2*f*ta
n(e + f*x) - a**2*f) - 15*c**4*tan(e + f*x)/(a**2*f*tan(e + f*x)**2 - 2*I*a
**2*f*tan(e + f*x) - a**2*f) + 10*I*c**4/(a**2*f*tan(e + f*x)**2 - 2*I*a**2
*f*tan(e + f*x) - a**2*f), Eq(m, -2)), (-24*c**4*f*x*tan(e + f*x)/(2*a*f*ta
n(e + f*x) - 2*I*a*f) + 24*I*c**4*f*x/(2*a*f*tan(e + f*x) - 2*I*a*f) + 12*I
*c**4*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*a*f*tan(e + f*x) - 2*I*a*f)
+ 12*c**4*log(tan(e + f*x)**2 + 1)/(2*a*f*tan(e + f*x) - 2*I*a*f) - I*c**4*
tan(e + f*x)**3/(2*a*f*tan(e + f*x) - 2*I*a*f) + 9*c**4*tan(e + f*x)**2/(2*
a*f*tan(e + f*x) - 2*I*a*f) + 26*c**4/(2*a*f*tan(e + f*x) - 2*I*a*f), Eq(m,
 -1)), (8*c**4*x - 4*I*c**4*log(tan(e + f*x)**2 + 1)/f + c**4*tan(e + f*x)*
*3/(3*f) + 2*I*c**4*tan(e + f*x)**2/f - 7*c**4*tan(e + f*x)/f, Eq(m, 0)), (
c**4*m**3*(I*a*tan(e + f*x) + a)**m*tan(e + f*x)**3/(f*m**4 + 6*f*m**3 + 11
*f*m**2 + 6*f*m) + 3*I*c**4*m**3*(I*a*tan(e + f*x) + a)**m*tan(e + f*x)**2/
(f*m**4 + 6*f*m**3 + 11*f*m**2 + 6*f*m) - 3*c**4*m**3*(I*a*tan(e + f*x) + a
)**m*tan(e + f*x)/(f*m**4 + 6*f*m**3 + 11*f*m**2 + 6*f*m) - I*c**4*m**3*(I*
a*tan(e + f*x) + a)**m/(f*m**4 + 6*f*m**3 + 11*f*m**2 + 6*f*m) + 3*c**4*m**
2*(I*a*tan(e + f*x) + a)**m*tan(e + f*x)**3/(f*m**4 + 6*f*m**3 + 11*f*m**2
+ 6*f*m) + 15*I*c**4*m**2*(I*a*tan(e + f*x) + a)**m*tan(e + f*x)**2/(f*m**4
 + 6*f*m**3 + 11*f*m**2 + 6*f*m) - 21*c**4*m**2*(I*a*tan(e + f*x) + a)**m*t
an(e + f*x)/(f*m**4 + 6*f*m**3 + 11*f*m**2 + 6*f*m) - 9*I*c**4*m**2*(I*a*ta
n(e + f*x) + a)**m/(f*m**4 + 6*f*m**3 + 11*f*m**2 + 6*f*m) + 2*c**4*m*(I*a*
tan(e + f*x) + a)**m*tan(e + f*x)**3/(f*m**4 + 6*f*m**3 + 11*f*m**2 + 6*f*m
) + 12*I*c**4*m*(I*a*tan(e + f*x) + a)**m*tan(e + f*x)**2/(f*m**4 + 6*f*m**
3 + 11*f*m**2 + 6*f*m) - 42*c**4*m*(I*a*tan(e + f*x) + a)**m*tan(e + f*x)/(
f*m**4 + 6*f*m**3 + 11*f*m**2 + 6*f*m) - 32*I*c**4*m*(I*a*tan(e + f*x) + a)
**m/(f*m**4 + 6*f*m**3 + 11*f*m**2 + 6*f*m) - 48*I*c**4*(I*a*tan(e + f*x) +
 a)**m/(f*m**4 + 6*f*m**3 + 11*f*m**2 + 6*f*m), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e))^4,x, algorithm="giac")
[Out] integrate((-I*c*tan(f*x + e) + c)^4*(I*a*tan(f*x + e) + a)^m, x)
```

Mupad [B]
time = 9.87, size = 332, normalized size = 2.48

$$-\frac{4\,c^4\,{\left(\frac{a\,\left(\cos\left(2\,e+2\,f\,x\right)+1+\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}\right)}^m\,\left(\cos\left(2\,e+2\,f\,x\right)\,3{}\mathrm{i}+\cos\left(4\,e+4\,f\,x\right)\,3{}\mathrm{i}+\cos\left(6\,e+6\,f\,x\right)\,1{}\mathrm{i}+3\,\sin\left(2\,e+2\,f\,x\right)+3\,\sin\left(4\,e+4\,f\,x\right)+\sin\left(6\,e+6\,f\,x\right)+1{}\mathrm{i}\right)\,\left(11\,m+18\,\cos\left(2\,e+2\,f\,x\right)+18\,\cos\left(4\,e+4\,f\,x\right)+6\,\cos\left(6\,e+6\,f\,x\right)+15\,m\,\cos\left(2\,e+2\,f\,x\right)+6\,m\,\cos\left(4\,e+4\,f\,x\right)+6\,m^2+m^3+3\,m^2\,\cos\left(2\,e+2\,f\,x\right)+6+\sin\left(2\,e+2\,f\,x\right)\,18{}\mathrm{i}+\sin\left(4\,e+4\,f\,x\right)\,18{}\mathrm{i}+\sin\left(6\,e+6\,f\,x\right)\,6{}\mathrm{i}+m^2\,\sin\left(2\,e+2\,f\,x\right)\,3{}\mathrm{i}+m\,\sin\left(2\,e+2\,f\,x\right)\,15{}\mathrm{i}+m\,\sin\left(4\,e+4\,f\,x\right)\,6{}\mathrm{i}\right)}{f\,m\,\left(m^3+6\,m^2+11\,m+6\right)\,\left(15\,\cos\left(2\,e+2\,f\,x\right)+6\,\cos\left(4\,e+4\,f\,x\right)+\cos\left(6\,e+6\,f\,x\right)+10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^m*(c - c*tan(e + f*x)*1i)^4,x)
```

```
[Out] -(4*c^4*((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x)
 + 1))^m*(cos(2*e + 2*f*x)*3i + cos(4*e + 4*f*x)*3i + cos(6*e + 6*f*x)*1i +
 3*sin(2*e + 2*f*x) + 3*sin(4*e + 4*f*x) + sin(6*e + 6*f*x) + 1i)*(11*m + 1
8*cos(2*e + 2*f*x) + 18*cos(4*e + 4*f*x) + 6*cos(6*e + 6*f*x) + sin(2*e + 2
*f*x)*18i + sin(4*e + 4*f*x)*18i + sin(6*e + 6*f*x)*6i + m^2*sin(2*e + 2*f*
x)*3i + 15*m*cos(2*e + 2*f*x) + 6*m*cos(4*e + 4*f*x) + m*sin(2*e + 2*f*x)*1
5i + m*sin(4*e + 4*f*x)*6i + 6*m^2 + m^3 + 3*m^2*cos(2*e + 2*f*x) + 6))/(f*
m*(11*m + 6*m^2 + m^3 + 6)*(15*cos(2*e + 2*f*x) + 6*cos(4*e + 4*f*x) + cos(
6*e + 6*f*x) + 10))
```

3.1052 $\int (a+ia\tan(e+fx))^m(c-ic\tan(e+fx))^3\,dx$

Optimal. Leaf size=99

$$-\frac{4ic^3(a+ia\tan(e+fx))^m}{fm}+\frac{4ic^3(a+ia\tan(e+fx))^{1+m}}{af(1+m)}-\frac{ic^3(a+ia\tan(e+fx))^{2+m}}{a^2f(2+m)}$$

```
[Out] -4*I*c^3*(a+I*a*tan(f*x+e))^m/f/m+4*I*c^3*(a+I*a*tan(f*x+e))^(1+m)/a/f/(1+m
)-I*c^3*(a+I*a*tan(f*x+e))^(2+m)/a^2/f/(2+m)
```

Rubi [A]
time = 0.10, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 45}

$$-\frac{ic^3(a+ia\tan(e+fx))^{m+2}}{a^2f(m+2)}-\frac{4ic^3(a+ia\tan(e+fx))^m}{fm}+\frac{4ic^3(a+ia\tan(e+fx))^{m+1}}{af(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^m*(c - I*c*Tan[e + f*x])^3,x]
```

```
[Out] ((-4*I)*c^3*(a + I*a*Tan[e + f*x])^m)/(f*m) + ((4*I)*c^3*(a + I*a*Tan[e + f
*x])^(1 + m))/(a*f*(1 + m)) - (I*c^3*(a + I*a*Tan[e + f*x])^(2 + m))/(a^2*f
*(2 + m))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int (a+ia\tan(e+fx))^m(c-ic\tan(e+fx))^3\,dx = (a^3c^3)\int \sec^6(e+fx)(a+ia\tan(e+fx))^{-3+m}\,dx$$
$$= -\frac{(ic^3)\,\mathrm{Subst}\left(\int (a-x)^2(a+x)^{-1+m}\,dx, x, ia\tan(e+fx\right.}{a^2f}$$
$$= -\frac{(ic^3)\,\mathrm{Subst}\left(\int \left(4a^2(a+x)^{-1+m}-4a(a+x)^m+(a+x\right.\right.}{a^2f}$$
$$= -\frac{4ic^3(a+ia\tan(e+fx))^m}{fm}+\frac{4ic^3(a+ia\tan(e+fx))}{af(1+m)}$$

Mathematica [A]
time = 14.76, size = 161, normalized size = 1.63

$$-\frac{i2^{2+m}c^3\left(e^{ifx}\right)^m\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^m\left(2+2e^{4i(e+fx)}+3m+m^2+2e^{2i(e+fx)}(2+m)\right)\sec^{-m}(e+fx)(\cos(fx)+i\sin(fx))^{-m}(a+ia\tan(e+fx))^m}{\left(1+e^{2i(e+fx)}\right)^2fm(1+m)(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^m*(c - I*c*Tan[e + f*x])^3,x]
```

```
[Out] ((-I)*2^(2 + m)*c^3*(E^(I*f*x))^m*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))
))^m*(2 + 2*E^((4*I)*(e + f*x)) + 3*m + m^2 + 2*E^((2*I)*(e + f*x))*(2 + m)
)*(a + I*a*Tan[e + f*x])^m)/((1 + E^((2*I)*(e + f*x)))^2*f*m*(1 + m)*(2 + m
)*Sec[e + f*x]^m*(Cos[f*x] + I*Sin[f*x])^m)
```

Maple [A]
time = 1.21, size = 129, normalized size = 1.30

method	result	s
norman	$\frac{ic^3(\tan^2(fx+e))e^{m\ln(a+ia\tan(fx+e))}}{f(2+m)}-\frac{2c^3(3+m)\tan(fx+e)e^{m\ln(a+ia\tan(fx+e))}}{f(1+m)(2+m)}-\frac{ic^3(m^2+5m+8)e^{m\ln(a+ia\tan(fx+e))}}{fm(1+m)(2+m)}$	
risch	Expression too large to display	3

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] I/f/(2+m)*c^3*tan(f*x+e)^2*exp(m*ln(a+I*a*tan(f*x+e)))-2*c^3*(3+m)/f/(1+m)/
(2+m)*tan(f*x+e)*exp(m*ln(a+I*a*tan(f*x+e)))-I*c^3*(m^2+5*m+8)/f/m/(1+m)/(2
+m)*exp(m*ln(a+I*a*tan(f*x+e)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((-I*c*tan(f*x + e) + c)^3*(I*a*tan(f*x + e) + a)^m, x)
```

Fricas [A]
time = 1.03, size = 163, normalized size = 1.65

$$-\frac{4\left(i\,c^3m^2+3i\,c^3m+2i\,c^3e^{(4i\,fx+4i\,e)}+2i\,c^3+2\left(i\,c^3m+2i\,c^3\right)e^{(2i\,fx+2i\,e)}\right)\left(\frac{2\,ae^{(2i\,fx+2i\,e)}}{e^{(2i\,fx+2i\,e)}+1}\right)^m}{fm^3+3\,fm^2+2\,fm+(fm^3+3\,fm^2+2\,fm)e^{(4i\,fx+4i\,e)}+2\,(fm^3+3\,fm^2+2\,fm)e^{(2i\,fx+2i\,e)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -4*(I*c^3*m^2 + 3*I*c^3*m + 2*I*c^3*e^(4*I*f*x + 4*I*e) + 2*I*c^3 + 2*(I*c^
3*m + 2*I*c^3)*e^(2*I*f*x + 2*I*e))*(2*a*e^(2*I*f*x + 2*I*e)/(e^(2*I*f*x +
2*I*e) + 1))^m/(f*m^3 + 3*f*m^2 + 2*f*m + (f*m^3 + 3*f*m^2 + 2*f*m)*e^(4*I*
f*x + 4*I*e) + 2*(f*m^3 + 3*f*m^2 + 2*f*m)*e^(2*I*f*x + 2*I*e))
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 979 vs. 2(80) = 160.
time = 0.87, size = 979, normalized size = 9.89

$$\begin{cases} x(ia\tan(e)+a)^m(-ic\tan(e)+c)^3 & \text{for } f=0 \\ \frac{2c^3fx\tan^2(e+fx)}{2a^2f\tan^2(e+fx)-4ia^2f\tan(e+fx)-2a^2f}-\frac{4ic^3fx\tan(e+fx)}{2a^2f\tan^2(e+fx)-4ia^2f\tan(e+fx)-2a^2f}-\frac{2c^3fx}{2a^2f\tan^2(e+fx)-4ia^2f\tan(e+fx)-2a^2f}-\frac{ic^3\log(\tan^2(e+fx)+1)\tan^2(e+fx)}{2a^2f\tan^2(e+fx)-4ia^2f\tan(e+fx)-2a^2f}-\frac{2c^3\log(\tan^2(e+fx)+1)\tan(e+fx)}{2a^2f\tan^2(e+fx)-4ia^2f\tan(e+fx)-2a^2f}+\frac{ic^3\log(\tan^2(e+fx)+1)}{2a^2f\tan^2(e+fx)-4ia^2f\tan(e+fx)-2a^2f}-\frac{8c^3\tan(e+fx)}{2a^2f\tan^2(e+fx)-4ia^2f\tan(e+fx)-2a^2f}+\frac{4ic^3}{2a^2f\tan^2(e+fx)-4ia^2f\tan(e+fx)-2a^2f} & \text{for } m=-2 \\ -\frac{4c^3fx\tan(e+fx)}{af\tan(e+fx)-iaf}+\frac{4ic^3fx}{af\tan(e+fx)-iaf}+\frac{2ic^3\log(\tan^2(e+fx)+1)\tan(e+fx)}{af\tan(e+fx)-iaf}+\frac{2c^3\log(\tan^2(e+fx)+1)}{af\tan(e+fx)-iaf}+\frac{c^3\tan^2(e+fx)}{af\tan(e+fx)-iaf}+\frac{5c^3}{af\tan(e+fx)-iaf} & \text{for } m=-1 \\ 4c^3x-\frac{2ic^3\log(\tan^2(e+fx)+1)}{f}+\frac{ic^3\tan^2(e+fx)}{2f}-\frac{3c^3\tan(e+fx)}{f} & \text{for } m=0 \\ \frac{ic^3m^2(ia\tan(e+fx)+a)^m\tan^2(e+fx)}{fm^3+3fm^2+2fm}-\frac{2c^3m^2(ia\tan(e+fx)+a)^m\tan(e+fx)}{fm^3+3fm^2+2fm}-\frac{ic^3m^2(ia\tan(e+fx)+a)^m}{fm^3+3fm^2+2fm}+\frac{ic^3m(ia\tan(e+fx)+a)^m\tan^2(e+fx)}{fm^3+3fm^2+2fm}-\frac{6c^3m(ia\tan(e+fx)+a)^m\tan(e+fx)}{fm^3+3fm^2+2fm}-\frac{5ic^3m(ia\tan(e+fx)+a)^m}{fm^3+3fm^2+2fm}-\frac{8ic^3(ia\tan(e+fx)+a)^m}{fm^3+3fm^2+2fm} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**m*(c-I*c*tan(f*x+e))**3,x)
```

```
[Out] Piecewise((x*(I*a*tan(e) + a)**m*(-I*c*tan(e) + c)**3, Eq(f, 0)), (2*c**3*f
*x*tan(e + f*x)**2/(2*a**2*f*tan(e + f*x)**2 - 4*I*a**2*f*tan(e + f*x) - 2*
a**2*f) - 4*I*c**3*f*x*tan(e + f*x)/(2*a**2*f*tan(e + f*x)**2 - 4*I*a**2*f*
tan(e + f*x) - 2*a**2*f) - 2*c**3*f*x/(2*a**2*f*tan(e + f*x)**2 - 4*I*a**2*
f*tan(e + f*x) - 2*a**2*f) - I*c**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**
2/(2*a**2*f*tan(e + f*x)**2 - 4*I*a**2*f*tan(e + f*x) - 2*a**2*f) - 2*c**3*
log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*a**2*f*tan(e + f*x)**2 - 4*I*a**2*
f*tan(e + f*x) - 2*a**2*f) + I*c**3*log(tan(e + f*x)**2 + 1)/(2*a**2*f*tan(
e + f*x)**2 - 4*I*a**2*f*tan(e + f*x) - 2*a**2*f) - 8*c**3*tan(e + f*x)/(2*
a**2*f*tan(e + f*x)**2 - 4*I*a**2*f*tan(e + f*x) - 2*a**2*f) + 4*I*c**3/(2*
a**2*f*tan(e + f*x)**2 - 4*I*a**2*f*tan(e + f*x) - 2*a**2*f), Eq(m, -2)), (
-4*c**3*f*x*tan(e + f*x)/(a*f*tan(e + f*x) - I*a*f) + 4*I*c**3*f*x/(a*f*tan
(e + f*x) - I*a*f) + 2*I*c**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(a*f*ta
n(e + f*x) - I*a*f) + 2*c**3*log(tan(e + f*x)**2 + 1)/(a*f*tan(e + f*x) - I
*a*f) + c**3*tan(e + f*x)**2/(a*f*tan(e + f*x) - I*a*f) + 5*c**3/(a*f*tan(e
```

```
 + f*x) - I*a*f), Eq(m, -1)), (4*c**3*x - 2*I*c**3*log(tan(e + f*x)**2 + 1)
/f + I*c**3*tan(e + f*x)**2/(2*f) - 3*c**3*tan(e + f*x)/f, Eq(m, 0)), (I*c*
*3*m**2*(I*a*tan(e + f*x) + a)**m*tan(e + f*x)**2/(f*m**3 + 3*f*m**2 + 2*f*
m) - 2*c**3*m**2*(I*a*tan(e + f*x) + a)**m*tan(e + f*x)/(f*m**3 + 3*f*m**2
+ 2*f*m) - I*c**3*m**2*(I*a*tan(e + f*x) + a)**m/(f*m**3 + 3*f*m**2 + 2*f*m
) + I*c**3*m*(I*a*tan(e + f*x) + a)**m*tan(e + f*x)**2/(f*m**3 + 3*f*m**2 +
 2*f*m) - 6*c**3*m*(I*a*tan(e + f*x) + a)**m*tan(e + f*x)/(f*m**3 + 3*f*m**
2 + 2*f*m) - 5*I*c**3*m*(I*a*tan(e + f*x) + a)**m/(f*m**3 + 3*f*m**2 + 2*f*
m) - 8*I*c**3*(I*a*tan(e + f*x) + a)**m/(f*m**3 + 3*f*m**2 + 2*f*m), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((-I*c*tan(f*x + e) + c)^3*(I*a*tan(f*x + e) + a)^m, x)
```

Mupad [B]
time = 1.36, size = 229, normalized size = 2.31

$$-\frac{2\,c^3\,{\left(\frac{a\,\left(\cos(2\,e+2\,f\,x)+1+\sin(2\,e+2\,f\,x)\,1{}\mathrm{i}\right)}{\cos(2\,e+2\,f\,x)+1}\right)}^m\,\left(m\,7{}\mathrm{i}+\cos\left(2\,e+2\,f\,x\right)\,16{}\mathrm{i}+\cos\left(4\,e+4\,f\,x\right)\,4{}\mathrm{i}+2\,m^2\,\sin\left(2\,e+2\,f\,x\right)+m^2\,\sin\left(4\,e+4\,f\,x\right)+m\,\cos\left(2\,e+2\,f\,x\right)\,10{}\mathrm{i}+m\,\cos\left(4\,e+4\,f\,x\right)\,3{}\mathrm{i}+6\,m\,\sin\left(2\,e+2\,f\,x\right)+3\,m\,\sin\left(4\,e+4\,f\,x\right)+m^2\,1{}\mathrm{i}+m^2\,\cos\left(2\,e+2\,f\,x\right)\,2{}\mathrm{i}+m^2\,\cos\left(4\,e+4\,f\,x\right)\,1{}\mathrm{i}+12{}\mathrm{i}\right)}{f\,m\,\left(4\,\cos\left(2\,e+2\,f\,x\right)+\cos\left(4\,e+4\,f\,x\right)+3\right)\,\left(m^2+3\,m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^m*(c - c*tan(e + f*x)*1i)^3,x)
```

```
[Out] -(2*c^3*((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x)
 + 1))^m*(m*7i + cos(2*e + 2*f*x)*16i + cos(4*e + 4*f*x)*4i + 2*m^2*sin(2*e
 + 2*f*x) + m^2*sin(4*e + 4*f*x) + m*cos(2*e + 2*f*x)*10i + m*cos(4*e + 4*f
*x)*3i + 6*m*sin(2*e + 2*f*x) + 3*m*sin(4*e + 4*f*x) + m^2*1i + m^2*cos(2*e
 + 2*f*x)*2i + m^2*cos(4*e + 4*f*x)*1i + 12i))/(f*m*(4*cos(2*e + 2*f*x) + c
os(4*e + 4*f*x) + 3)*(3*m + m^2 + 2))
```

3.1053 $\int (a+ia\tan(e+fx))^m(c-ic\tan(e+fx))^2\,dx$

Optimal. Leaf size=64

$$-\frac{2ic^2(a+ia\tan(e+fx))^m}{fm}+\frac{ic^2(a+ia\tan(e+fx))^{1+m}}{af(1+m)}$$

```
[Out] -2*I*c^2*(a+I*a*tan(f*x+e))^m/f/m+I*c^2*(a+I*a*tan(f*x+e))^(1+m)/a/f/(1+m)
```

Rubi [A]
time = 0.09, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 45}

$$\frac{ic^2(a+ia\tan(e+fx))^{m+1}}{af(m+1)}-\frac{2ic^2(a+ia\tan(e+fx))^m}{fm}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^m*(c - I*c*Tan[e + f*x])^2,x]
```

```
[Out] ((-2*I)*c^2*(a + I*a*Tan[e + f*x])^m)/(f*m) + (I*c^2*(a + I*a*Tan[e + f*x])
^(1 + m))/(a*f*(1 + m))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int (a+ia\tan(e+fx))^m(c-ic\tan(e+fx))^2\,dx = \left(a^2c^2\right)\int \sec^4(e+fx)(a+ia\tan(e+fx))^{-2+m}\,dx$$
$$= -\frac{\left(ic^2\right)\operatorname{Subst}\left(\int (a-x)(a+x)^{-1+m}\,dx, x, ia\tan(e+fx\right.}{af}$$
$$= -\frac{\left(ic^2\right)\operatorname{Subst}\left(\int \left(2a(a+x)^{-1+m}-(a+x)^m\right)\,dx, x, ia\,\mathrm{ta}\right.}{af}$$
$$= -\frac{2ic^2(a+ia\tan(e+fx))^m}{fm}+\frac{ic^2(a+ia\tan(e+fx))^{1}}{af(1+m)}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 131 vs. $2(64) = 128$.
time = 41.63, size = 131, normalized size = 2.05

$$-\frac{i2^{1+m}c^2e^{-i(e+fx)}\left(e^{ifx}\right)^m\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{1+m}\left(1+e^{2i(e+fx)}+m\right)\sec^{-m}(e+fx)(\cos(fx)+i\sin(fx))^{-m}(a+ia\tan(e+fx))^m}{fm(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^m*(c - I*c*Tan[e + f*x])^2,x]
```

```
[Out] ((-I)*2^(1 + m)*c^2*(E^(I*f*x))^m*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))
))^(1 + m)*(1 + E^((2*I)*(e + f*x)) + m)*(a + I*a*Tan[e + f*x])^m)/(E^(I*(e
 + f*x))*f*m*(1 + m)*Sec[e + f*x]^m*(Cos[f*x] + I*Sin[f*x])^m)
```

Maple [**A**]
time = 0.94, size = 78, normalized size = 1.22

method	result	size
norman	$-\frac{c^2\tan(fx+e)\mathrm{e}^{m\ln(a+ia\tan(fx+e))}}{f(1+m)}-\frac{i\left(c^2m+2c^2\right)\mathrm{e}^{m\ln(a+ia\tan(fx+e))}}{fm(1+m)}$	78
risch	Expression too large to display	1604

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -c^2/f/(1+m)*tan(f*x+e)*exp(m*ln(a+I*a*tan(f*x+e)))-I/f/m/(1+m)*(c^2*m+2*c^
2)*exp(m*ln(a+I*a*tan(f*x+e)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((-I*c*tan(f*x + e) + c)^2*(I*a*tan(f*x + e) + a)^m, x)
```

Fricas [A]
time = 0.93, size = 89, normalized size = 1.39

$$-\frac{2\left(i\,c^2m+i\,c^2e^{(2i\,fx+2i\,e)}+i\,c^2\right)\left(\frac{2\,ae^{(2i\,fx+2i\,e)}}{e^{(2i\,fx+2i\,e)}+1}\right)^m}{fm^2+fm+(fm^2+fm)e^{(2i\,fx+2i\,e)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -2*(I*c^2*m + I*c^2*e^(2*I*f*x + 2*I*e) + I*c^2)*(2*a*e^(2*I*f*x + 2*I*e)/(
e^(2*I*f*x + 2*I*e) + 1))^m/(f*m^2 + f*m + (f*m^2 + f*m)*e^(2*I*f*x + 2*I*e
))
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(49) = 98.
time = 0.46, size = 313, normalized size = 4.89

$$\begin{cases} x(ia\tan(e)+a)^m(-ic\tan(e)+c)^2 & \text{for } f=0\\ -\frac{2c^2fx\tan(e+fx)}{2af\tan(e+fx)-2iaf}+\frac{2ic^2fx}{2af\tan(e+fx)-2iaf}+\frac{ic^2\log(\tan^2(e+fx)+1)\tan(e+fx)}{2af\tan(e+fx)-2iaf}+\frac{c^2\log(\tan^2(e+fx)+1)}{2af\tan(e+fx)-2iaf}+\frac{4c^2}{2af\tan(e+fx)-2iaf} & \text{for } m=-1\\ 2c^2x-\frac{ic^2\log(\tan^2(e+fx)+1)}{f}-\frac{c^2\tan(e+fx)}{f} & \text{for } m=0\\ -\frac{c^2m(ia\tan(e+fx)+a)^m\tan(e+fx)}{fm^2+fm}-\frac{ic^2m(ia\tan(e+fx)+a)^m}{fm^2+fm}-\frac{2ic^2(ia\tan(e+fx)+a)^m}{fm^2+fm} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**m*(c-I*c*tan(f*x+e))**2,x)
```

```
[Out] Piecewise((x*(I*a*tan(e) + a)**m*(-I*c*tan(e) + c)**2, Eq(f, 0)), (-2*c**2*
f*x*tan(e + f*x)/(2*a*f*tan(e + f*x) - 2*I*a*f) + 2*I*c**2*f*x/(2*a*f*tan(e
 + f*x) - 2*I*a*f) + I*c**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*a*f*ta
n(e + f*x) - 2*I*a*f) + c**2*log(tan(e + f*x)**2 + 1)/(2*a*f*tan(e + f*x) -
 2*I*a*f) + 4*c**2/(2*a*f*tan(e + f*x) - 2*I*a*f), Eq(m, -1)), (2*c**2*x -
I*c**2*log(tan(e + f*x)**2 + 1)/f - c**2*tan(e + f*x)/f, Eq(m, 0)), (-c**2*
m*(I*a*tan(e + f*x) + a)**m*tan(e + f*x)/(f*m**2 + f*m) - I*c**2*m*(I*a*tan
(e + f*x) + a)**m/(f*m**2 + f*m) - 2*I*c**2*(I*a*tan(e + f*x) + a)**m/(f*m*
*2 + f*m), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((-I*c*tan(f*x + e) + c)^2*(I*a*tan(f*x + e) + a)^m, x)
```

Mupad [B]
time = 0.37, size = 112, normalized size = 1.75

$$-\frac{c^2\left(\frac{a\,\left(\cos\left(2\,e+2\,f\,x\right)+1+\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}\right)^m\,\left(m\,1{}\mathrm{i}+\cos\left(2\,e+2\,f\,x\right)\,2{}\mathrm{i}+m\,\cos\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}+m\,\sin\left(2\,e+2\,f\,x\right)+2{}\mathrm{i}\right)}{f\,m\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)\,\left(m+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^m*(c - c*tan(e + f*x)*1i)^2,x)
```

```
[Out] -(c^2*((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) +
 1))^m*(m*1i + cos(2*e + 2*f*x)*2i + m*cos(2*e + 2*f*x)*1i + m*sin(2*e + 2*
f*x) + 2i))/(f*m*(cos(2*e + 2*f*x) + 1)*(m + 1))
```

3.1054 $\int (a+ia\tan(e+fx))^m(c-ic\tan(e+fx))\,dx$

Optimal. Leaf size=26

$$-\frac{ic(a+ia\tan(e+fx))^m}{fm}$$

[Out] -I*c*(a+I*a*tan(f*x+e))^m/f/m

Rubi [A]
time = 0.06, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {3603, 3568, 32}

$$-\frac{ic(a+ia\tan(e+fx))^m}{fm}$$

Antiderivative was successfully verified.

[In] Int[(a + I*a*Tan[e + f*x])^m*(c - I*c*Tan[e + f*x]),x]

[Out] ((-I)*c*(a + I*a*Tan[e + f*x])^m)/(f*m)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c + d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] && !(IGtQ[n, 0] && (LtQ[m, 0] || GtQ[m, n]))
```

Rubi steps

$$\begin{aligned}\int (a+ia\tan(e+fx))^m(c-ic\tan(e+fx))\,dx &= (ac)\int \sec^2(e+fx)(a+ia\tan(e+fx))^{-1+m}\,dx\\ &= -\frac{(ic)\mathrm{Subst}\left(\int (a+x)^{-1+m}\,dx,x,ia\tan(e+fx)\right)}{f}\\ &= -\frac{ic(a+ia\tan(e+fx))^m}{fm}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 95 vs. $2(26) = 52$.
time = 1.97, size = 95, normalized size = 3.65

$$-\frac{i2^m c\left(e^{ifx}\right)^m\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^m\sec^{-m}(e+fx)(\cos(fx)+i\sin(fx))^{-m}(a+ia\tan(e+fx))^m}{fm}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^m*(c - I*c*Tan[e + f*x]),x]
```

```
[Out] ((-I)*2^m*c*(E^(I*f*x))^m*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^m*(a
+ I*a*Tan[e + f*x])^m)/(f*m*Sec[e + f*x]^m*(Cos[f*x] + I*Sin[f*x])^m)
```

Maple [A]
time = 0.20, size = 25, normalized size = 0.96

method	result
derivativedivides	$-\frac{ic(a+ia\tan(fx+e))^m}{fm}$
default	$-\frac{ic(a+ia\tan(fx+e))^m}{fm}$
norman	$-\frac{ic\,\mathrm{e}^{m\ln(a+ia\tan(fx+e))}}{fm}$
risch	$-\frac{ic\,\mathrm{e}^{\frac{m\left(-i\pi\mathrm{csgn}\left(i\mathrm{e}^{2i(fx+e)}\right)^3+2i\pi\mathrm{csgn}\left(i\mathrm{e}^{2i(fx+e)}\right)^2\mathrm{csgn}\left(i\mathrm{e}^{i(fx+e)}\right)-i\pi\,\mathrm{csgn}\left(i\mathrm{e}^{2i(fx+e)}\right)\mathrm{csgn}\left(i\mathrm{e}^{i(fx+e)}\right)^2+i\pi\,\mathrm{csgn}\left(i\mathrm{e}^{2i(fx}\right.\right.}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -I*c*(a+I*a*tan(f*x+e))^m/f/m
```

Maxima [A]
time = 0.54, size = 25, normalized size = 0.96

$$-\frac{i\,a^m c(i\tan(fx+e)+1)^m}{fm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] -I*a^m*c*(I*tan(f*x + e) + 1)^m/(f*m)
```

Fricas [A]
time = 1.12, size = 38, normalized size = 1.46

$$-\frac{i\,c\left(\frac{2\,a e^{(2i\,fx+2i\,e)}}{e^{(2i\,fx+2i\,e)}+1}\right)^{m}}{fm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] -I*c*(2*a*e^(2*I*f*x + 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1))^m/(f*m)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(20) = 40.
time = 0.22, size = 71, normalized size = 2.73

$$\begin{cases} x(-ic\tan{(e)}+c) & \text{for } f=0\wedge m=0 \\ cx-\frac{ic\log{(\tan^2{(e+fx)}+1)}}{2f} & \text{for } m=0 \\ x(ia\tan{(e)}+a)^m\,(-ic\tan{(e)}+c) & \text{for } f=0 \\ -\frac{ic(ia\tan{(e+fx)}+a)^m}{fm} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**m*(c-I*c*tan(f*x+e)),x)
```

```
[Out] Piecewise((x*(-I*c*tan(e) + c), Eq(f, 0) & Eq(m, 0)), (c*x - I*c*log(tan(e
+ f*x)**2 + 1)/(2*f), Eq(m, 0)), (x*(I*a*tan(e) + a)**m*(-I*c*tan(e) + c),
Eq(f, 0)), (-I*c*(I*a*tan(e + f*x) + a)**m/(f*m), True))
```

Giac [A]
time = 1.17, size = 23, normalized size = 0.88

$$-\frac{i\,(i\,a\tan{(fx+e)}+a)^{m}c}{fm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] -I*(I*a*tan(f*x + e) + a)^m*c/(f*m)
```

<u>**Mupad**</u> [B]
time = 0.17, size = 46, normalized size = 1.77

$$-\frac{c\,{\left(\frac{a\,\left(2\,{\cos\left(e+f\,x\right)}^2+\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{2\,{\cos\left(e+f\,x\right)}^2}\right)}^m\,1{}\mathrm{i}}{f\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^m*(c - c*tan(e + f*x)*1i),x)
```

```
[Out] -(c*((a*(sin(2*e + 2*f*x)*1i + 2*cos(e + f*x)^2))/(2*cos(e + f*x)^2))^m*1i)
/(f*m)
```

3.1055 $\int \frac{(a+ia\tan(e+fx))^m}{c-ic\tan(e+fx)}\,dx$

Optimal. Leaf size=52

$$-\frac{i\,{}_2F_1\left(2,m;1+m;\frac{1}{2}(1+i\tan(e+fx))\right)(a+ia\tan(e+fx))^m}{4cfm}$$

```
[Out] -1/4*I*hypergeom([2, m],[1+m],1/2+1/2*I*tan(f*x+e))*(a+I*a*tan(f*x+e))^m/c/
f/m
```

Rubi [A]
time = 0.09, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 70}

$$-\frac{i(a+ia\tan(e+fx))^m\,{}_2F_1\left(2,m;m+1;\frac{1}{2}(i\tan(e+fx)+1)\right)}{4cfm}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^m/(c - I*c*Tan[e + f*x]),x]
```

```
[Out] ((-1/4*I)*Hypergeometric2F1[2, m, 1 + m, (1 + I*Tan[e + f*x])/2]*(a + I*a*T
an[e + f*x])^m)/(c*f*m)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^m}{c-ic\tan(e+fx)}\,dx = \frac{\int \cos^2(e+fx)(a+ia\tan(e+fx))^{1+m}\,dx}{ac}$$
$$= -\frac{(ia^2)\,\text{Subst}\left(\int \frac{(a+x)^{-1+m}}{(a-x)^2}\,dx, x, ia\tan(e+fx)\right)}{cf}$$
$$= -\frac{i\,{}_2F_1\left(2,m;1+m;\frac{1}{2}(1+i\tan(e+fx))\right)(a+ia\tan(e+fx))^m}{4cfm}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 133 vs. 2(52) = 104.
time = 16.05, size = 133, normalized size = 2.56

$$-\frac{i2^{-2+m}\left(e^{ifx}\right)^m\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^m\left(1+e^{2i(e+fx)}\right)^2\,{}_2F_1\left(1,2;1+m;-e^{2i(e+fx)}\right)\sec^{-m}(e+fx)(\cos(fx)+i\sin(fx))^{-m}(a+ia\tan(e+fx))^m}{cfm}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^m/(c - I*c*Tan[e + f*x]),x]
```

```
[Out] ((-I)*2^(-2 + m)*(E^(I*f*x))^m*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^
m*(1 + E^((2*I)*(e + f*x)))^2*Hypergeometric2F1[1, 2, 1 + m, -E^((2*I)*(e +
 f*x))]*(a + I*a*Tan[e + f*x])^m)/(c*f*m*Sec[e + f*x]^m*(Cos[f*x] + I*Sin[f
*x])^m)
```

Maple [F]
time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(fx+e))^m}{c-ic\tan(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e)),x)
```

```
[Out] int((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/2*(2*a*e^(2*I*f*x + 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1))^m*(e^(2*I*
f*x + 2*I*e) + 1)/c, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int\frac{(ia\tan{(e+fx)}+a)^m}{\tan{(e+fx)}+i}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**m/(c-I*c*tan(f*x+e)),x)
```

```
[Out] I*Integral((I*a*tan(e + f*x) + a)**m/(tan(e + f*x) + I), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^m/(-I*c*tan(f*x + e) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{(a+a\tan(e+f\,x)\,1\mathrm{i})^m}{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^m/(c - c*tan(e + f*x)*1i),x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^m/(c - c*tan(e + f*x)*1i), x)
```

3.1056 $\int \frac{(a+ia\tan(e+fx))^m}{(c-ic\tan(e+fx))^2}\,dx$

Optimal. Leaf size=52

$$-\frac{i\,{}_2F_1\left(3,m;1+m;\frac{1}{2}(1+i\tan(e+fx))\right)(a+ia\tan(e+fx))^m}{8c^2fm}$$

```
[Out] -1/8*I*hypergeom([3, m],[1+m],1/2+1/2*I*tan(f*x+e))*(a+I*a*tan(f*x+e))^m/c^
2/f/m
```

Rubi [A]
time = 0.09, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 70}

$$-\frac{i(a+ia\tan(e+fx))^m\,{}_2F_1\left(3,m;m+1;\frac{1}{2}(i\tan(e+fx)+1)\right)}{8c^2fm}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^m/(c - I*c*Tan[e + f*x])^2,x]
```

```
[Out] ((-1/8*I)*Hypergeometric2F1[3, m, 1 + m, (1 + I*Tan[e + f*x])/2]*(a + I*a*T
an[e + f*x])^m)/(c^2*f*m)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^m}{(c-ic\tan(e+fx))^2}\,dx = \frac{\int \cos^4(e+fx)(a+ia\tan(e+fx))^{2+m}\,dx}{a^2c^2}$$
$$= -\frac{(ia^3)\,\text{Subst}\left(\int \frac{(a+x)^{-1+m}}{(a-x)^3}\,dx, x, ia\tan(e+fx)\right)}{c^2f}$$
$$= -\frac{i\,{}_2F_1\left(3,m;1+m;\frac{1}{2}(1+i\tan(e+fx))\right)(a+ia\tan(e+fx))^m}{8c^2fm}$$

Mathematica [**F**]
time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^m/(c - I*c*Tan[e + f*x])^2,x]
[Out] $Aborted
```

Maple [**F**]
time = 2.58, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(fx+e))^m}{(c-ic\tan(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e))^2,x)
[Out] int((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e))^2,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e))^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/4*(2*a*e^(2*I*f*x + 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1))^m*(e^(4*I*
f*x + 4*I*e) + 2*e^(2*I*f*x + 2*I*e) + 1)/c^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{(ia\tan{(e+fx)}+a)^m}{\tan^2{(e+fx)}+2i\tan{(e+fx)}-1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**m/(c-I*c*tan(f*x+e))**2,x)
```

```
[Out] -Integral((I*a*tan(e + f*x) + a)**m/(tan(e + f*x)**2 + 2*I*tan(e + f*x) - 1
), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^m/(-I*c*tan(f*x + e) + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a+a\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^m}{{\left(c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^m/(c - c*tan(e + f*x)*1i)^2,x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^m/(c - c*tan(e + f*x)*1i)^2, x)
```

3.1057 $\int \frac{(a+ia\tan(e+fx))^m}{(c-ic\tan(e+fx))^3}\,dx$

Optimal. Leaf size=52

$$-\frac{i\,{}_2F_1\left(4,m;1+m;\frac{1}{2}(1+i\tan(e+fx))\right)(a+ia\tan(e+fx))^m}{16c^3fm}$$

```
[Out] -1/16*I*hypergeom([4, m],[1+m],1/2+1/2*I*tan(f*x+e))*(a+I*a*tan(f*x+e))^m/c
^3/f/m
```

Rubi [A]
time = 0.08, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 70}

$$-\frac{i(a+ia\tan(e+fx))^m\,{}_2F_1\left(4,m;m+1;\frac{1}{2}(i\tan(e+fx)+1)\right)}{16c^3fm}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^m/(c - I*c*Tan[e + f*x])^3,x]
```

```
[Out] ((-1/16*I)*Hypergeometric2F1[4, m, 1 + m, (1 + I*Tan[e + f*x])/2]*(a + I*a*
Tan[e + f*x])^m)/(c^3*f*m)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^m}{(c-ic\tan(e+fx))^3}\,dx = \frac{\int \cos^6(e+fx)(a+ia\tan(e+fx))^{3+m}\,dx}{a^3c^3}$$
$$= -\frac{(ia^4)\,\text{Subst}\left(\int \frac{(a+x)^{-1+m}}{(a-x)^4}\,dx, x, ia\tan(e+fx)\right)}{c^3f}$$
$$= -\frac{i\,{}_2F_1\left(4,m;1+m;\frac{1}{2}(1+i\tan(e+fx))\right)(a+ia\tan(e+fx))^m}{16c^3fm}$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^m/(c - I*c*Tan[e + f*x])^3,x]
[Out] $Aborted
```

Maple [F]
time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(fx+e))^m}{(c-ic\tan(fx+e))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e))^3,x)
[Out] int((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e))^3,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e))^3,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(1/8*(2*a*e^(2*I*f*x + 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1))^m*(e^(6*I*
f*x + 6*I*e) + 3*e^(4*I*f*x + 4*I*e) + 3*e^(2*I*f*x + 2*I*e) + 1)/c^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{(ia\tan{(e+fx)}+a)^m}{\tan^3{(e+fx)}+3i\tan^2{(e+fx)}-3\tan{(e+fx)}-i}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**m/(c-I*c*tan(f*x+e))**3,x)
```

```
[Out] -I*Integral((I*a*tan(e + f*x) + a)**m/(tan(e + f*x)**3 + 3*I*tan(e + f*x)**
2 - 3*tan(e + f*x) - I), x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^m/(-I*c*tan(f*x + e) + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{(a+a\tan(e+f\,x)\,1\mathrm{i})^m}{(c-c\tan(e+f\,x)\,1\mathrm{i})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^m/(c - c*tan(e + f*x)*1i)^3,x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^m/(c - c*tan(e + f*x)*1i)^3, x)
```

3.1058 $\int \frac{(a+ia\tan(e+fx))^m}{(c-ic\tan(e+fx))^4}\,dx$

Optimal. Leaf size=52

$$-\frac{i\,{}_2F_1\left(5,m;1+m;\frac{1}{2}(1+i\tan(e+fx))\right)(a+ia\tan(e+fx))^m}{32c^4fm}$$

```
[Out] -1/32*I*hypergeom([5, m],[1+m],1/2+1/2*I*tan(f*x+e))*(a+I*a*tan(f*x+e))^m/c
^4/f/m
```

Rubi [A]
time = 0.08, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3603, 3568, 70}

$$-\frac{i(a+ia\tan(e+fx))^m\,{}_2F_1\left(5,m;m+1;\frac{1}{2}(i\tan(e+fx)+1)\right)}{32c^4fm}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^m/(c - I*c*Tan[e + f*x])^4,x]
```

```
[Out] ((-1/32*I)*Hypergeometric2F1[5, m, 1 + m, (1 + I*Tan[e + f*x])/2]*(a + I*a*
Tan[e + f*x])^m)/(c^4*f*m)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+ia\tan(e+fx))^m}{(c-ic\tan(e+fx))^4}\,dx &= \frac{\int \cos^8(e+fx)(a+ia\tan(e+fx))^{4+m}\,dx}{a^4c^4} \\
&= -\frac{(ia^5)\,\mathrm{Subst}\left(\int \frac{(a+x)^{-1+m}}{(a-x)^5}\,dx, x, ia\tan(e+fx)\right)}{c^4f} \\
&= -\frac{i\,{}_2F_1\left(5,m;1+m;\frac{1}{2}(1+i\tan(e+fx))\right)(a+ia\tan(e+fx))^m}{32c^4fm}
\end{aligned}
$$

Mathematica [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\$\mathrm{Aborted}$$

Verification is not applicable to the result.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^m/(c - I*c*Tan[e + f*x])^4,x]
```

```
[Out] $Aborted
```

Maple [**F**]
time = 2.44, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(fx+e))^m}{(c-ic\tan(fx+e))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e))^4,x)
```

```
[Out] int((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e))^4,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] integral(1/16*(2*a*e^(2*I*f*x + 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1))^m*(e^(8*I
*f*x + 8*I*e) + 4*e^(6*I*f*x + 6*I*e) + 6*e^(4*I*f*x + 4*I*e) + 4*e^(2*I*f*
x + 2*I*e) + 1)/c^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(ia\tan{(e+fx)}+a)^m}{\tan^4{(e+fx)}+4i\tan^3{(e+fx)}-6\tan^2{(e+fx)}-4i\tan{(e+fx)}+1}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**m/(c-I*c*tan(f*x+e))**4,x)
```

```
[Out] Integral((I*a*tan(e + f*x) + a)**m/(tan(e + f*x)**4 + 4*I*tan(e + f*x)**3 -
 6*tan(e + f*x)**2 - 4*I*tan(e + f*x) + 1), x)/c**4
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e))^4,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^m/(-I*c*tan(f*x + e) + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a+a\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^m}{{\left(c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^m/(c - c*tan(e + f*x)*1i)^4,x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^m/(c - c*tan(e + f*x)*1i)^4, x)
```

3.1059 $\int (a+ia\tan(e+fx))^m(c-ic\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=67

$$\frac{i\,{}_2F_1\left(1,\frac{5}{2}+m;\frac{7}{2};\frac{1}{2}(1-i\tan(e+fx))\right)(a+ia\tan(e+fx))^m(c-ic\tan(e+fx))^{5/2}}{5f}$$

```
[Out] 1/5*I*hypergeom([1, 5/2+m],[7/2],1/2-1/2*I*tan(f*x+e))*(a+I*a*tan(f*x+e))^m
*(c-I*c*tan(f*x+e))^(5/2)/f
```

Rubi [A]
time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.31, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3604, 72, 71}

$$\frac{i2^m(c-ic\tan(e+fx))^{5/2}(1+i\tan(e+fx))^{-m}(a+ia\tan(e+fx))^m\,{}_2F_1\left(\frac{5}{2},1-m;\frac{7}{2};\frac{1}{2}(1-i\tan(e+fx))\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^m*(c - I*c*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((I/5)*2^m*Hypergeometric2F1[5/2, 1 - m, 7/2, (1 - I*Tan[e + f*x])/2]*(a +
I*a*Tan[e + f*x])^m*(c - I*c*Tan[e + f*x])^(5/2))/(f*(1 + I*Tan[e + f*x])^m
)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
```

```
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a+ia\tan(e+fx))^m(c-ic\tan(e+fx))^{5/2}\,dx = \frac{(ac)\text{Subst}\left(\int (a+iax)^{-1+m}(c-icx)^{3/2}\,dx, x, \tan(e+\right.}{f}$$
$$= \frac{\left(2^{-1+m}c(a+ia\tan(e+fx))^m\left(\frac{a+ia\tan(e+fx)}{a}\right)^{-m}\right)\text{Su}}{f}$$
$$= \frac{i2^m\,{}_2F_1\left(\frac{5}{2},1-m;\frac{7}{2};\frac{1}{2}(1-i\tan(e+fx))\right)(1+i\tan(e}{5}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 141 vs. 2(67) = 134.
time = 65.04, size = 141, normalized size = 2.10

$$-\frac{i2^{\frac{3}{2}+m}c\left(e^{ifx}\right)^m\left(\frac{c}{1+e^{2i(e+fx)}}\right)^{3/2}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^m{}_2F_1\left(-\frac{3}{2},1;1+m;-e^{2i(e+fx)}\right)\sec^{-m}(e+fx)(\cos(fx)+i\sin(fx))^{-m}(a+ia\tan(e+fx))^m}{fm}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^m*(c - I*c*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-I)*2^(3/2 + m)*c*(E^(I*f*x))^m*(c/(1 + E^((2*I)*(e + f*x))))^(3/2)*(E^(I
*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^m*Hypergeometric2F1[-3/2, 1, 1 + m,
-E^((2*I)*(e + f*x))]*(a + I*a*Tan[e + f*x])^m)/(f*m*Sec[e + f*x]^m*(Cos[f*
x] + I*Sin[f*x])^m)
```

Maple [F]
time = 3.55, size = 0, normalized size = 0.00

$$\int (a+ia\tan(fx+e))^m\,(c-ic\tan(fx+e))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e))^(5/2),x)
```

```
[Out] int((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((-I*c*tan(f*x + e) + c)^(5/2)*(I*a*tan(f*x + e) + a)^m, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(4*sqrt(2)*c^2*(2*a*e^(2*I*f*x + 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1))^
m*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(e^(4*I*f*x + 4*I*e) + 2*e^(2*I*f*x + 2
*I*e) + 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**m*(c-I*c*tan(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((-I*c*tan(f*x + e) + c)^(5/2)*(I*a*tan(f*x + e) + a)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\tan(e+f\,x)\;1\mathrm{i})^m\,(c-c\tan(e+f\,x)\;1\mathrm{i})^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^m*(c - c*tan(e + f*x)*1i)^(5/2),x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^m*(c - c*tan(e + f*x)*1i)^(5/2), x)
```

3.1060 $\int (a+ia\tan(e+fx))^m(c-ic\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=67

$$\frac{i\,{}_2F_1\left(1,\frac{3}{2}+m;\frac{5}{2};\frac{1}{2}(1-i\tan(e+fx))\right)(a+ia\tan(e+fx))^m(c-ic\tan(e+fx))^{3/2}}{3f}$$

```
[Out] 1/3*I*hypergeom([1, 3/2+m],[5/2],1/2-1/2*I*tan(f*x+e))*(a+I*a*tan(f*x+e))^m
*(c-I*c*tan(f*x+e))^(3/2)/f
```

Rubi [A]
time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.31, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3604, 72, 71}

$$\frac{i2^m(c-ic\tan(e+fx))^{3/2}(1+i\tan(e+fx))^{-m}(a+ia\tan(e+fx))^m\,{}_2F_1\left(\frac{3}{2},1-m;\frac{5}{2};\frac{1}{2}(1-i\tan(e+fx))\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^m*(c - I*c*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((I/3)*2^m*Hypergeometric2F1[3/2, 1 - m, 5/2, (1 - I*Tan[e + f*x])/2]*(a +
I*a*Tan[e + f*x])^m*(c - I*c*Tan[e + f*x])^(3/2))/(f*(1 + I*Tan[e + f*x])^m
)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
```

```
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+ia\tan(e+fx))^m(c-ic\tan(e+fx))^{3/2}\,dx &= \frac{(ac)\text{Subst}\left(\int (a+iax)^{-1+m}\sqrt{c-icx}\,dx,x,\tan(e+fx\right.}{f}\\ &= \frac{\left(2^{-1+m}c(a+ia\tan(e+fx))^m\left(\frac{a+ia\tan(e+fx)}{a}\right)^{-m}\right)\text{Sub}}{f}\\ &= \frac{i2^m\,{}_2F_1\left(\frac{3}{2},1-m;\frac{5}{2};\frac{1}{2}(1-i\tan(e+fx))\right)(1+i\tan(e}{3j}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 141 vs. 2(67) = 134.
time = 6.24, size = 141, normalized size = 2.10

$$-\frac{i2^{\frac{1}{2}+m}c\left(e^{ifx}\right)^m\sqrt{\frac{c}{1+e^{2i(e+fx)}}}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^m{}_2F_1\left(-\frac{1}{2},1;1+m;-e^{2i(e+fx)}\right)\sec^{-m}(e+fx)(\cos(fx)+i\sin(fx))^{-m}(a+ia\tan(e+fx))^m}{fm}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^m*(c - I*c*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-I)*2^(1/2 + m)*c*(E^(I*f*x))^m*Sqrt[c/(1 + E^((2*I)*(e + f*x)))]*(E^(I*(
e + f*x))/(1 + E^((2*I)*(e + f*x))))^m*Hypergeometric2F1[-1/2, 1, 1 + m, -E
^((2*I)*(e + f*x))]*(a + I*a*Tan[e + f*x])^m)/(f*m*Sec[e + f*x]^m*(Cos[f*x]
 + I*Sin[f*x])^m)
```

Maple [F]
time = 0.84, size = 0, normalized size = 0.00

$$\int (a+ia\tan(fx+e))^m(c-ic\tan(fx+e))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e))^(3/2),x)
```

```
[Out] int((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((-I*c*tan(f*x + e) + c)^(3/2)*(I*a*tan(f*x + e) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(2*sqrt(2)*c*(2*a*e^(2*I*f*x + 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1))^m*
sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(e^(2*I*f*x + 2*I*e) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(e+fx)-i))^m\,(-ic(\tan(e+fx)+i))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**m*(c-I*c*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((I*a*(tan(e + f*x) - I))**m*(-I*c*(tan(e + f*x) + I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((-I*c*tan(f*x + e) + c)^(3/2)*(I*a*tan(f*x + e) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\tan(e+f\,x)\,1\mathrm{i})^m\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^m*(c - c*tan(e + f*x)*1i)^(3/2),x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^m*(c - c*tan(e + f*x)*1i)^(3/2), x)
```

3.1061 $\int (a+ia\tan(e+fx))^m \sqrt{c-ic\tan(e+fx)}\,dx$

Optimal. Leaf size=65

$$\frac{i\,{}_2F_1\left(1,\frac{1}{2}+m;\frac{3}{2};\frac{1}{2}(1-i\tan(e+fx))\right)(a+ia\tan(e+fx))^m\sqrt{c-ic\tan(e+fx)}}{f}$$

```
[Out] I*hypergeom([1, 1/2+m],[3/2],1/2-1/2*I*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2)
*(a+I*a*tan(f*x+e))^m/f
```

Rubi [A]
time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.32, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3604, 72, 71}

$$\frac{i2^m\sqrt{c-ic\tan(e+fx)}\,(1+i\tan(e+fx))^{-m}(a+ia\tan(e+fx))^m\,{}_2F_1\left(\frac{1}{2},1-m;\frac{3}{2};\frac{1}{2}(1-i\tan(e+fx))\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^m*Sqrt[c - I*c*Tan[e + f*x]],x]
```

```
[Out] (I*2^m*Hypergeometric2F1[1/2, 1 - m, 3/2, (1 - I*Tan[e + f*x])/2]*(a + I*a*
Tan[e + f*x])^m*Sqrt[c - I*c*Tan[e + f*x]])/(f*(1 + I*Tan[e + f*x])^m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a+ia\tan(e+fx))^m\sqrt{c-ic\tan(e+fx)}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{(a+iax)^{-1+m}}{\sqrt{c-icx}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(2^{-1+m}c(a+ia\tan(e+fx))^m\left(\frac{a+ia\tan(e+fx)}{a}\right)^{-m}\right)\text{Sul}}{f}$$

$$= \frac{i2^m\,{}_2F_1\left(\frac{1}{2},1-m;\frac{3}{2};\frac{1}{2}(1-i\tan(e+fx))\right)(1+i\tan(e}{f}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 141 vs. 2(65) = 130.
time = 1.55, size = 141, normalized size = 2.17

$$-\frac{i2^{-\frac{1}{2}+m}c\left(e^{ifx}\right)^m\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^m{}_2F_1\left(\frac{1}{2},1;1+m;-e^{2i(e+fx)}\right)\sec^{-m}(e+fx)(\cos(fx)+i\sin(fx))^{-m}(a+ia\tan(e+fx))^m}{\sqrt{\frac{c}{1+e^{2i(e+fx)}}}\,fm}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^m*Sqrt[c - I*c*Tan[e + f*x]],x]
```

```
[Out] ((-I)*2^(-1/2 + m)*c*(E^(I*f*x))^m*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)
)))^m*Hypergeometric2F1[1/2, 1, 1 + m, -E^((2*I)*(e + f*x))]*(a + I*a*Tan[e
 + f*x])^m)/(Sqrt[c/(1 + E^((2*I)*(e + f*x)))]*f*m*Sec[e + f*x]^m*(Cos[f*x]
 + I*Sin[f*x])^m)
```

Maple [F]
time = 1.22, size = 0, normalized size = 0.00

$$\int (a+ia\tan(fx+e))^m\sqrt{c-ic\tan(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e))^(1/2),x)
```

```
[Out] int((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(-I*c*tan(f*x + e) + c)*(I*a*tan(f*x + e) + a)^m, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(2)*(2*a*e^(2*I*f*x + 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1))^m*sqrt
(c/(e^(2*I*f*x + 2*I*e) + 1)), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan{(e+fx)}-i))^m \sqrt{-ic(\tan{(e+fx)}+i)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**m*(c-I*c*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral((I*a*(tan(e + f*x) - I))**m*sqrt(-I*c*(tan(e + f*x) + I)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(-I*c*tan(f*x + e) + c)*(I*a*tan(f*x + e) + a)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(a+a\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^m\,\sqrt{c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^m*(c - c*tan(e + f*x)*1i)^(1/2),x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^m*(c - c*tan(e + f*x)*1i)^(1/2), x)
```

3.1062 $\int \frac{(a+ia\tan(e+fx))^m}{\sqrt{c-ic\tan(e+fx)}}\,dx$

Optimal. Leaf size=65

$$-\frac{i\,{}_2F_1\left(1,-\frac{1}{2}+m;\frac{1}{2};\frac{1}{2}(1-i\tan(e+fx))\right)(a+ia\tan(e+fx))^m}{f\sqrt{c-ic\tan(e+fx)}}$$

```
[Out] -I*hypergeom([1, -1/2+m],[1/2],1/2-1/2*I*tan(f*x+e))*(a+I*a*tan(f*x+e))^m/f
/(c-I*c*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.32, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3604, 72, 71}

$$-\frac{i2^m(1+i\tan(e+fx))^{-m}(a+ia\tan(e+fx))^m\,{}_2F_1\left(-\frac{1}{2},1-m;\frac{1}{2};\frac{1}{2}(1-i\tan(e+fx))\right)}{f\sqrt{c-ic\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^m/Sqrt[c - I*c*Tan[e + f*x]],x]
```

```
[Out] ((-I)*2^m*Hypergeometric2F1[-1/2, 1 - m, 1/2, (1 - I*Tan[e + f*x])/2]*(a +
I*a*Tan[e + f*x])^m)/(f*(1 + I*Tan[e + f*x])^m*Sqrt[c - I*c*Tan[e + f*x]])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
```

```
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^m}{\sqrt{c-ic\tan(e+fx)}}\,dx = \frac{(ac)\mathrm{Subst}\left(\int \frac{(a+iax)^{-1+m}}{(c-icx)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(2^{-1+m}c(a+ia\tan(e+fx))^m\left(\frac{a+ia\tan(e+fx)}{a}\right)^{-m}\right)\mathrm{Subst}\left(\int \frac{\left(\frac{1}{2}+\frac{ix}{2}\right)^{-1+m}}{(c-icx)^{3/2}}\,dx, x\right.}{f}$$

$$= -\frac{i2^m\,{}_2F_1\left(-\frac{1}{2},1-m;\frac{1}{2};\frac{1}{2}(1-i\tan(e+fx))\right)(1+i\tan(e+fx))^{-m}(a+ia\tan}{f\sqrt{c-ic\tan(e+fx)}}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 141 vs. $2(65) = 130$.
time = 6.03, size = 141, normalized size = 2.17

$$-\frac{i2^{-\frac{3}{2}+m}c\left(e^{ifx}\right)^m\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^m {}_2F_1\left(1,\frac{3}{2};1+m;-e^{2i(e+fx)}\right)\sec^{-m}(e+fx)(\cos(fx)+i\sin(fx))^{-m}(a+ia\tan(e+fx))^m}{\left(\frac{c}{1+e^{2i(e+fx)}}\right)^{3/2}fm}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^m/Sqrt[c - I*c*Tan[e + f*x]],x]
[Out] ((-I)*2^(-3/2 + m)*c*(E^(I*f*x))^m*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)
)))^m*Hypergeometric2F1[1, 3/2, 1 + m, -E^((2*I)*(e + f*x))]*(a + I*a*Tan[e
 + f*x])^m)/((c/(1 + E^((2*I)*(e + f*x))))^(3/2)*f*m*Sec[e + f*x]^m*(Cos[f*
x] + I*Sin[f*x])^m)
```

Maple [**F**]
time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(fx+e))^m}{\sqrt{c-ic\tan(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e))^(1/2),x)
[Out] int((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^m/sqrt(-I*c*tan(f*x + e) + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(1/2*sqrt(2)*(2*a*e^(2*I*f*x + 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1))^m*
sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*(e^(2*I*f*x + 2*I*e) + 1)/c, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan{(e+fx)}-i))^m}{\sqrt{-ic\,(\tan{(e+fx)}+i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**m/(c-I*c*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral((I*a*(tan(e + f*x) - I))**m/sqrt(-I*c*(tan(e + f*x) + I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^m/sqrt(-I*c*tan(f*x + e) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a+a\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^m}{\sqrt{c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^m/(c - c*tan(e + f*x)*1i)^(1/2),x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^m/(c - c*tan(e + f*x)*1i)^(1/2), x)
```

3.1063 $\int \frac{(a+ia\tan(e+fx))^m}{(c-ic\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{i\,{}_2F_1\left(1,-\frac{3}{2}+m;-\frac{1}{2};\frac{1}{2}(1-i\tan(e+fx))\right)(a+ia\tan(e+fx))^m}{3f(c-ic\tan(e+fx))^{3/2}}$$

```
[Out] -1/3*I*hypergeom([1, -3/2+m],[-1/2],1/2-1/2*I*tan(f*x+e))*(a+I*a*tan(f*x+e)
)^m/f/(c-I*c*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.31, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{3604, 72, 71\}$

$$-\frac{i2^m(1+i\tan(e+fx))^{-m}(a+ia\tan(e+fx))^m\,{}_2F_1\left(-\frac{3}{2},1-m;-\frac{1}{2};\frac{1}{2}(1-i\tan(e+fx))\right)}{3f(c-ic\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^m/(c - I*c*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-1/3*I)*2^m*Hypergeometric2F1[-3/2, 1 - m, -1/2, (1 - I*Tan[e + f*x])/2]*
(a + I*a*Tan[e + f*x])^m)/(f*(1 + I*Tan[e + f*x])^m*(c - I*c*Tan[e + f*x])^
(3/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
```

```
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^m}{(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{(a+iax)^{-1+m}}{(c-icx)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(2^{-1+m}c(a+ia\tan(e+fx))^m\left(\frac{a+ia\tan(e+fx)}{a}\right)^{-m}\right)\text{Subst}\left(\int \frac{\left(\frac{1}{2}+\frac{ix}{2}\right)^{-1+m}}{(c-icx)^{5/2}}\,dx\right.}{f}$$

$$= -\frac{i2^m\,{}_2F_1\left(-\frac{3}{2},1-m;-\frac{1}{2};\frac{1}{2}(1-i\tan(e+fx))\right)(1+i\tan(e+fx))^{-m}(a+}{3f(c-ic\tan(e+fx))^{3/2}}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 141 vs. 2(67) = 134.
time = 106.73, size = 141, normalized size = 2.10

$$-\frac{i2^{-\frac{5}{2}+m}c\left(e^{ifx}\right)^m\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^m {}_2F_1\left(1,\frac{5}{2};1+m;-e^{2i(e+fx)}\right)\sec^{-m}(e+fx)(\cos(fx)+i\sin(fx))^{-m}(a+ia\tan(e+fx))^m}{\left(\frac{c}{1+e^{2i(e+fx)}}\right)^{5/2}fm}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^m/(c - I*c*Tan[e + f*x])^(3/2),x]
[Out] ((-I)*2^(-5/2 + m)*c*(E^(I*f*x))^m*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)
)))^m*Hypergeometric2F1[1, 5/2, 1 + m, -E^((2*I)*(e + f*x))]*(a + I*a*Tan[e
 + f*x])^m)/((c/(1 + E^((2*I)*(e + f*x))))^(5/2)*f*m*Sec[e + f*x]^m*(Cos[f*
x] + I*Sin[f*x])^m)
```

Maple [**F**]
time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(fx+e))^m}{(c-ic\tan(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e))^(3/2),x)
[Out] int((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e))^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(1/4*sqrt(2)*(2*a*e^(2*I*f*x + 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1))^m*
sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*(e^(4*I*f*x + 4*I*e) + 2*e^(2*I*f*x + 2*I
*e) + 1)/c^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan{(e+fx)}-i))^m}{(-ic(\tan{(e+fx)}+i))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**m/(c-I*c*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((I*a*(tan(e + f*x) - I))**m/(-I*c*(tan(e + f*x) + I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^m/(-I*c*tan(f*x + e) + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\tan(e+f\,x)\,1\mathrm{i})^m}{(c-c\tan{(e+f\,x)}\,1\mathrm{i})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^m/(c - c*tan(e + f*x)*1i)^(3/2),x)
[Out] int((a + a*tan(e + f*x)*1i)^m/(c - c*tan(e + f*x)*1i)^(3/2), x)
```

3.1064 $\int \frac{(a+ia\tan(e+fx))^m}{(c-ic\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{i\,{}_2F_1\left(1,-\frac{5}{2}+m;-\frac{3}{2};\frac{1}{2}(1-i\tan(e+fx))\right)(a+ia\tan(e+fx))^m}{5f(c-ic\tan(e+fx))^{5/2}}$$

```
[Out] -1/5*I*hypergeom([1, -5/2+m],[-3/2],1/2-1/2*I*tan(f*x+e))*(a+I*a*tan(f*x+e)
)^m/f/(c-I*c*tan(f*x+e))^(5/2)
```

Rubi [A]
time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.31, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3604, 72, 71}

$$-\frac{i2^m(1+i\tan(e+fx))^{-m}(a+ia\tan(e+fx))^m\,{}_2F_1\left(-\frac{5}{2},1-m;-\frac{3}{2};\frac{1}{2}(1-i\tan(e+fx))\right)}{5f(c-ic\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^m/(c - I*c*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-1/5*I)*2^m*Hypergeometric2F1[-5/2, 1 - m, -3/2, (1 - I*Tan[e + f*x])/2]*
(a + I*a*Tan[e + f*x])^m)/(f*(1 + I*Tan[e + f*x])^m*(c - I*c*Tan[e + f*x])^
(5/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
```

```
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^m}{(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{(ac)\text{Subst}\left(\int \frac{(a+iax)^{-1+m}}{(c-icx)^{7/2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\left(2^{-1+m}c(a+ia\tan(e+fx))^m\left(\frac{a+ia\tan(e+fx)}{a}\right)^{-m}\right)\text{Subst}\left(\int \frac{\left(\frac{1}{2}+\frac{ix}{2}\right)^{-1+m}}{(c-icx)^{7/2}}\,dx\right.}{f}$$
$$= -\frac{i2^m\,{}_2F_1\left(-\frac{5}{2},1-m;-\frac{3}{2};\frac{1}{2}(1-i\tan(e+fx))\right)(1+i\tan(e+fx))^{-m}(a+}{5f(c-ic\tan(e+fx))^{5/2}}$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^m/(c - I*c*Tan[e + f*x])^(5/2),x]
[Out] $Aborted
```

Maple [F]
time = 3.09, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(fx+e))^m}{(c-ic\tan(fx+e))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e))^(5/2),x)
[Out] int((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e))^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(1/8*sqrt(2)*(2*a*e^(2*I*f*x + 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1))^m*
sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*(e^(6*I*f*x + 6*I*e) + 3*e^(4*I*f*x + 4*I
*e) + 3*e^(2*I*f*x + 2*I*e) + 1)/c^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(e+fx)-i))^m}{(-ic(\tan(e+fx)+i))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**m/(c-I*c*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral((I*a*(tan(e + f*x) - I))**m/(-I*c*(tan(e + f*x) + I))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^m/(-I*c*tan(f*x + e) + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\tan(e+f\,x)\;1\mathrm{i})^m}{(c-c\tan(e+f\,x)\;1\mathrm{i})^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^m/(c - c*tan(e + f*x)*1i)^(5/2),x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^m/(c - c*tan(e + f*x)*1i)^(5/2), x)
```

3.1065 $\int (a+ia\tan(e+fx))^3(c+d\tan(e+fx))\,dx$

Optimal. Leaf size=110

$$4a^3(c-id)x-\frac{4a^3(ic+d)\log(\cos(e+fx))}{f}-\frac{2a^3(c-id)\tan(e+fx)}{f}+\frac{a(ic+d)(a+ia\tan(e+fx))^2}{2f}+\frac{d(a\ -$$

```
[Out] 4*a^3*(c-I*d)*x-4*a^3*(I*c+d)*ln(cos(f*x+e))/f-2*a^3*(c-I*d)*tan(f*x+e)/f+1
/2*a*(I*c+d)*(a+I*a*tan(f*x+e))^2/f+1/3*d*(a+I*a*tan(f*x+e))^3/f
```

Rubi [A]
time = 0.06, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3608, 3559, 3558, 3556}

$$-\frac{2a^3(c-id)\tan(e+fx)}{f}-\frac{4a^3(d+ic)\log(\cos(e+fx))}{f}+4a^3x(c-id)+\frac{a(d+ic)(a+ia\tan(e+fx))^2}{2f}+\frac{d(a+ia\tan(e+fx))^3}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^3*(c + d*Tan[e + f*x]),x]
```

```
[Out] 4*a^3*(c - I*d)*x - (4*a^3*(I*c + d)*Log[Cos[e + f*x]])/f - (2*a^3*(c - I*d
)*Tan[e + f*x])/f + (a*(I*c + d)*(a + I*a*Tan[e + f*x])^2)/(2*f) + (d*(a +
I*a*Tan[e + f*x])^3)/(3*f)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
```

```
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (a+ia\tan(e+fx))^3(c+d\tan(e+fx))\,dx &= \frac{d(a+ia\tan(e+fx))^3}{3f}-(-c+id)\int (a+ia\tan(e+fx))^3\\ &= \frac{a(ic+d)(a+ia\tan(e+fx))^2}{2f}+\frac{d(a+ia\tan(e+fx))^3}{3f}+\\ &= 4a^3(c-id)x-\frac{2a^3(c-id)\tan(e+fx)}{f}+\frac{a(ic+d)(a+ia}{2f}\\ &= 4a^3(c-id)x-\frac{4a^3(ic+d)\log(\cos(e+fx))}{f}-\frac{2a^3(c-id)}{f}\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 331 vs. $2(110) = 220$.
time = 3.40, size = 331, normalized size = 3.01

$$\frac{a^3\sec(e)\sec^3(e+fx)\left(6cfx\cos(2e+3fx)-6idfx\cos(2e+3fx)+6cfx\cos(4e+3fx)-6idfx\cos(4e+3fx)-3ic\cos(2e+3fx)\log\left(\cos^2(e+fx)\right)-3d\cos(2e+3fx)\log\left(\cos^2(e+fx)\right)-3ic\cos(4e+3fx)\log\left(\cos^2(e+fx)\right)-3d\cos(4e+3fx)\log\left(\cos^2(e+fx)\right)+3\cos(fx)\left(-ic-3d+6cfx-6idfx+(-3ic-3d)\log\left(\cos^2(e+fx)\right)\right)+3\cos(2e+fx)\left(-ic-3d+6cfx-6idfx+(-3ic-3d)\log\left(\cos^2(e+fx)\right)\right)-18c\sin(fx)+24id\sin(fx)+9c\sin(2e+fx)-15id\sin(2e+fx)-9c\sin(2e+3fx)+13id\sin(2e+3fx)\right)}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3*(c + d*Tan[e + f*x]),x]
```

```
[Out] (a^3*Sec[e]*Sec[e + f*x]^3*(6*c*f*x*Cos[2*e + 3*f*x] - (6*I)*d*f*x*Cos[2*e
+ 3*f*x] + 6*c*f*x*Cos[4*e + 3*f*x] - (6*I)*d*f*x*Cos[4*e + 3*f*x] - (3*I)*
c*Cos[2*e + 3*f*x]*Log[Cos[e + f*x]^2] - 3*d*Cos[2*e + 3*f*x]*Log[Cos[e + f
*x]^2] - (3*I)*c*Cos[4*e + 3*f*x]*Log[Cos[e + f*x]^2] - 3*d*Cos[4*e + 3*f*x
]*Log[Cos[e + f*x]^2] + 3*Cos[f*x]*((-I)*c - 3*d + 6*c*f*x - (6*I)*d*f*x +
((-3*I)*c - 3*d)*Log[Cos[e + f*x]^2]) + 3*Cos[2*e + f*x]*((-I)*c - 3*d + 6*
c*f*x - (6*I)*d*f*x + ((-3*I)*c - 3*d)*Log[Cos[e + f*x]^2]) - 18*c*Sin[f*x]
 + (24*I)*d*Sin[f*x] + 9*c*Sin[2*e + f*x] - (15*I)*d*Sin[2*e + f*x] - 9*c*S
in[2*e + 3*f*x] + (13*I)*d*Sin[2*e + 3*f*x]))/(12*f)
```

Maple [**A**]
time = 0.12, size = 100, normalized size = 0.91

method	result
derivativedivides	$\frac{a^3\left(-\frac{id\left(\tan^3(fx+e)\right)}{3}-\frac{ic\left(\tan^2(fx+e)\right)}{2}+4i\tan(fx+e)d-\frac{3d\left(\tan^2(fx+e)\right)}{2}-3c\tan(fx+e)+\frac{(4ic+4d)\ln\left(1+\tan^2(fx+e)\right)}{2}+(-4\right.}{f}$

default	$\frac{a^3 (-\frac{id(\tan^3(fx+e))}{3} - \frac{ic(\tan^2(fx+e))}{2} + 4i\tan(fx+e)d - \frac{3d(\tan^2(fx+e))}{2} - 3c\tan(fx+e) + \frac{(4ic+4d)\ln(1+\tan^2(fx+e))}{2} + (-}{f}$
norman	$(-4ia^3d+4a^3c)\,x - \frac{(ia^3c+3a^3d)(\tan^2(fx+e))}{2f} - \frac{(-4ia^3d+3a^3c)\tan(fx+e)}{f} - \frac{ia^3d(\tan^3(fx+e))}{3f} + \frac{2(}{}$
risch	$\frac{8ia^3de}{f} - \frac{8a^3ce}{f} - \frac{2a^3(12ic\,e^{4i(fx+e)}+24d\,e^{4i(fx+e)}+21ic\,e^{2i(fx+e)}+33\,e^{2i(fx+e)}d+9ic+13d)}{3f(e^{2i(fx+e)}+1)^3} - \frac{4a^3\ln(e^{2i(fx+e)}}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3*(c+d*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a^3*(-1/3*I*d*tan(f*x+e)^3-1/2*I*c*tan(f*x+e)^2+4*I*tan(f*x+e)*d-3/2*d*
tan(f*x+e)^2-3*c*tan(f*x+e)+1/2*(4*I*c+4*d)*ln(1+tan(f*x+e)^2)+(-4*I*d+4*c)
*arctan(tan(f*x+e)))
```

Maxima [**A**]
time = 0.51, size = 114, normalized size = 1.04

$$-\frac{2i\,a^3d\tan(fx+e)^3+3\,(i\,a^3c+3\,a^3d)\tan(fx+e)^2-24\,(a^3c-i\,a^3d)(fx+e)+12\,(-i\,a^3c-a^3d)\log\left(\tan(fx+e)^2+1\right)+6\,(3\,a^3c-4i\,a^3d)\tan(fx+e)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c+d*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/6*(2*I*a^3*d*tan(f*x + e)^3 + 3*(I*a^3*c + 3*a^3*d)*tan(f*x + e)^2 - 24*
(a^3*c - I*a^3*d)*(f*x + e) + 12*(-I*a^3*c - a^3*d)*log(tan(f*x + e)^2 + 1)
 + 6*(3*a^3*c - 4*I*a^3*d)*tan(f*x + e))/f
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(98) = 196.
time = 0.96, size = 205, normalized size = 1.86

$$-\frac{2\left(9i\,a^3c+13\,a^3d+12\,(i\,a^3c+2\,a^3d)e^{(4i\,fx+4i\,e)}+3\,(7i\,a^3c+11\,a^3d)e^{(2i\,fx+2i\,e)}+6\left(i\,a^3c+a^3d+(i\,a^3c+a^3d)e^{(6i\,fx+6i\,e)}+3\,(i\,a^3c+a^3d)e^{(4i\,fx+4i\,e)}+3\,(i\,a^3c+a^3d)e^{(2i\,fx+2i\,e)}\right)\log\left(e^{(2i\,fx+2i\,e)}+1\right)\right)}{3\left(fe^{(6i\,fx+6i\,e)}+3\,fe^{(4i\,fx+4i\,e)}+3\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c+d*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] -2/3*(9*I*a^3*c + 13*a^3*d + 12*(I*a^3*c + 2*a^3*d)*e^(4*I*f*x + 4*I*e) + 3
*(7*I*a^3*c + 11*a^3*d)*e^(2*I*f*x + 2*I*e) + 6*(I*a^3*c + a^3*d + (I*a^3*c
 + a^3*d)*e^(6*I*f*x + 6*I*e) + 3*(I*a^3*c + a^3*d)*e^(4*I*f*x + 4*I*e) + 3
*(I*a^3*c + a^3*d)*e^(2*I*f*x + 2*I*e))*log(e^(2*I*f*x + 2*I*e) + 1))/(f*e^
(6*I*f*x + 6*I*e) + 3*f*e^(4*I*f*x + 4*I*e) + 3*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [**A**]
time = 0.39, size = 184, normalized size = 1.67

$$-\frac{4ia^3(c-id)\log\left(e^{2ifx}+e^{-2ie}\right)}{f}+\frac{-18ia^3c-26a^3d+\left(-42ia^3ce^{2ie}-66a^3de^{2ie}\right)e^{2ifx}+\left(-24ia^3ce^{4ie}-48a^3de^{4ie}\right)e^{4ifx}}{3fe^{6ie}e^{6ifx}+9fe^{4ie}e^{4ifx}+9fe^{2ie}e^{2ifx}+3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3*(c+d*tan(f*x+e)),x)
```

```
[Out] -4*I*a**3*(c - I*d)*log(exp(2*I*f*x) + exp(-2*I*e))/f + (-18*I*a**3*c - 26*
a**3*d + (-42*I*a**3*c*exp(2*I*e) - 66*a**3*d*exp(2*I*e))*exp(2*I*f*x) + (-
24*I*a**3*c*exp(4*I*e) - 48*a**3*d*exp(4*I*e))*exp(4*I*f*x))/(3*f*exp(6*I*e
)*exp(6*I*f*x) + 9*f*exp(4*I*e)*exp(4*I*f*x) + 9*f*exp(2*I*e)*exp(2*I*f*x)
+ 3*f)
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(98) = 196$.
time = 0.60, size = 333, normalized size = 3.03

$$-\frac{2\left(6i\,a^3ce^{(6i\,fx+6i\,e)}\log\left(e^{(2i\,fx+2i\,e)}+1\right)+6\,a^3de^{(6i\,fx+6i\,e)}\log\left(e^{(2i\,fx+2i\,e)}+1\right)+18i\,a^3ce^{(4i\,fx+4i\,e)}\log\left(e^{(2i\,fx+2i\,e)}+1\right)+18\,a^3de^{(4i\,fx+4i\,e)}\log\left(e^{(2i\,fx+2i\,e)}+1\right)+18i\,a^3ce^{(2i\,fx+2i\,e)}\log\left(e^{(2i\,fx+2i\,e)}+1\right)+18\,a^3de^{(2i\,fx+2i\,e)}\log\left(e^{(2i\,fx+2i\,e)}+1\right)+12i\,a^3ce^{(4i\,fx+4i\,e)}+24\,a^3de^{(4i\,fx+4i\,e)}+21i\,a^3ce^{(2i\,fx+2i\,e)}+33\,a^3de^{(2i\,fx+2i\,e)}+6i\,a^3c\log\left(e^{(2i\,fx+2i\,e)}+1\right)+6\,a^3d\log\left(e^{(2i\,fx+2i\,e)}+1\right)+9i\,a^3c+13\,a^3d\right)}{3\left(fe^{(6i\,fx+6i\,e)}+3\,fe^{(4i\,fx+4i\,e)}+3\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c+d*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] -2/3*(6*I*a^3*c*e^(6*I*f*x + 6*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 6*a^3*d*
e^(6*I*f*x + 6*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 18*I*a^3*c*e^(4*I*f*x +
4*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 18*a^3*d*e^(4*I*f*x + 4*I*e)*log(e^(2
*I*f*x + 2*I*e) + 1) + 18*I*a^3*c*e^(2*I*f*x + 2*I*e)*log(e^(2*I*f*x + 2*I*
e) + 1) + 18*a^3*d*e^(2*I*f*x + 2*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 12*I*
a^3*c*e^(4*I*f*x + 4*I*e) + 24*a^3*d*e^(4*I*f*x + 4*I*e) + 21*I*a^3*c*e^(2*
I*f*x + 2*I*e) + 33*a^3*d*e^(2*I*f*x + 2*I*e) + 6*I*a^3*c*log(e^(2*I*f*x +
2*I*e) + 1) + 6*a^3*d*log(e^(2*I*f*x + 2*I*e) + 1) + 9*I*a^3*c + 13*a^3*d)/
(f*e^(6*I*f*x + 6*I*e) + 3*f*e^(4*I*f*x + 4*I*e) + 3*f*e^(2*I*f*x + 2*I*e)
+ f)
```

Mupad [**B**]
time = 5.42, size = 125, normalized size = 1.14

$$\frac{\tan(e+f\,x)\,\left(-a^3\,(2\,c-d\,1\mathrm{i})+a^3\,(2\,d+c\,1\mathrm{i})\,1\mathrm{i}+a^3\,d\,1\mathrm{i}\right)}{f}+\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(4\,a^3\,d+a^3\,c\,4\mathrm{i}\right)}{f}-\frac{\tan(e+f\,x)^2\,\left(\frac{a^3\,(2\,d+c\,1\mathrm{i})}{2}+\frac{a^3\,d}{2}\right)}{f}-\frac{a^3\,d\,\tan(e+f\,x)^3\,1\mathrm{i}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^3*(c + d*tan(e + f*x)),x)
```

```
[Out] (tan(e + f*x)*(a^3*(c*1i + 2*d)*1i - a^3*(2*c - d*1i) + a^3*d*1i))/f + (log
(tan(e + f*x) + 1i)*(a^3*c*4i + 4*a^3*d))/f - (tan(e + f*x)^2*((a^3*(c*1i +
 2*d))/2 + (a^3*d)/2))/f - (a^3*d*tan(e + f*x)^3*1i)/(3*f)
```

3.1066 $\int (a+ia\tan(e+fx))^2(c+d\tan(e+fx))\,dx$

Optimal. Leaf size=80

$$2a^2(c-id)x-\frac{2a^2(ic+d)\log(\cos(e+fx))}{f}-\frac{a^2(c-id)\tan(e+fx)}{f}+\frac{d(a+ia\tan(e+fx))^2}{2f}$$

```
[Out] 2*a^2*(c-I*d)*x-2*a^2*(I*c+d)*ln(cos(f*x+e))/f-a^2*(c-I*d)*tan(f*x+e)/f+1/2
*d*(a+I*a*tan(f*x+e))^2/f
```

Rubi [A]
time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3608, 3558, 3556}

$$-\frac{a^2(c-id)\tan(e+fx)}{f}-\frac{2a^2(d+ic)\log(\cos(e+fx))}{f}+2a^2x(c-id)+\frac{d(a+ia\tan(e+fx))^2}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2*(c + d*Tan[e + f*x]),x]
```

```
[Out] 2*a^2*(c - I*d)*x - (2*a^2*(I*c + d)*Log[Cos[e + f*x]])/f - (a^2*(c - I*d)*
Tan[e + f*x])/f + (d*(a + I*a*Tan[e + f*x])^2)/(2*f)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && !LtQ[m, 0]
```

Rubi steps

$$\int (a+ia\tan(e+fx))^2(c+d\tan(e+fx))\,dx = \frac{d(a+ia\tan(e+fx))^2}{2f} - (-c+id)\int (a+ia\tan(e+fx))^2$$
$$= 2a^2(c-id)x - \frac{a^2(c-id)\tan(e+fx)}{f} + \frac{d(a+ia\tan(e+f}{2f}$$
$$= 2a^2(c-id)x - \frac{2a^2(ic+d)\log(\cos(e+fx))}{f} - \frac{a^2(c-id)\,\text{ta}}{f}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 263 vs. 2(80) = 160.
time = 1.86, size = 263, normalized size = 3.29

$$\frac{a^2\sec(e)\sec^2(e+fx)(\cos(2fx)+i\sin(2fx))\left(-8(c-id)\text{ArcTan}(\tan(3e+fx))\cos(e)\cos^2(e+fx)-i\left(4icfx\cos(3e+2fx)+4dfx\cos(3e+2fx)+(ic+d)\cos(e+2fx)\left(4fx-i\log\left(\cos^2(e+fx)\right)\right)+c\cos(3e+2fx)\log\left(\cos^2(e+fx)\right)-id\cos(3e+2fx)\log\left(\cos^2(e+fx)\right)+2\cos(e)\left(-id+4icfx+4dfx+(c-id)\log\left(\cos^2(e+fx)\right)\right)+2ic\sin(e)+4d\sin(e)-2ic\sin(e+2fx)-4d\sin(e+2fx)\right)\right)}{4f(\cos(fx)+i\sin(fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2*(c + d*Tan[e + f*x]),x]
```

```
[Out] (a^2*Sec[e]*Sec[e + f*x]^2*(Cos[2*f*x] + I*Sin[2*f*x])*(-8*(c - I*d)*ArcTan
[Tan[3*e + f*x]]*Cos[e]*Cos[e + f*x]^2 - I*((4*I)*c*f*x*Cos[3*e + 2*f*x] +
4*d*f*x*Cos[3*e + 2*f*x] + (I*c + d)*Cos[e + 2*f*x]*(4*f*x - I*Log[Cos[e +
f*x]^2]) + c*Cos[3*e + 2*f*x]*Log[Cos[e + f*x]^2] - I*d*Cos[3*e + 2*f*x]*Lo
g[Cos[e + f*x]^2] + 2*Cos[e]*((-I)*d + (4*I)*c*f*x + 4*d*f*x + (c - I*d)*Lo
g[Cos[e + f*x]^2]) + (2*I)*c*Sin[e] + 4*d*Sin[e] - (2*I)*c*Sin[e + 2*f*x] -
 4*d*Sin[e + 2*f*x])))/(4*f*(Cos[f*x] + I*Sin[f*x])^2)
```

Maple [A]
time = 0.11, size = 76, normalized size = 0.95

method	result	si
derivativedivides	$\frac{a^2\left(-\frac{d\left(\tan^2(fx+e)\right)}{2}-c\tan(fx+e)+2i\tan(fx+e)d+\frac{(2ic+2d)\ln\left(1+\tan^2(fx+e)\right)}{2}+(-2id+2c)\arctan(\tan(fx+e))\right)}{f}$	76
default	$\frac{a^2\left(-\frac{d\left(\tan^2(fx+e)\right)}{2}-c\tan(fx+e)+2i\tan(fx+e)d+\frac{(2ic+2d)\ln\left(1+\tan^2(fx+e)\right)}{2}+(-2id+2c)\arctan(\tan(fx+e))\right)}{f}$	76
norman	$(-2ia^2d+2a^2c)\,x-\frac{(-2ia^2d+a^2c)\tan(fx+e)}{f}-\frac{a^2d\left(\tan^2(fx+e)\right)}{2f}+\frac{(ia^2c+a^2d)\ln(1+\tan^2(fx+e))}{f}$	87
risch	$\frac{4ia^2de}{f}-\frac{4a^2ce}{f}-\frac{2a^2\left(ic\,e^{2i(fx+e)}+3\,e^{2i(fx+e)}d+ic+2d\right)}{f\left(e^{2i(fx+e)}+1\right)^2}-\frac{2a^2\ln\left(e^{2i(fx+e)}+1\right)d}{f}-\frac{2ia^2\ln\left(e^{2i(fx+e)}+1\right)c}{f}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2*(c+d*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a^2*(-1/2*d*tan(f*x+e)^2-c*tan(f*x+e)+2*I*d*tan(f*x+e)+1/2*(2*d+2*I*c)*
ln(1+tan(f*x+e)^2)+(2*c-2*I*d)*arctan(tan(f*x+e)))
```

Maxima [A]
time = 0.52, size = 87, normalized size = 1.09

$$-\frac{a^2d\tan\left(fx+e\right)^2-4\left(a^2c-i\,a^2d\right)(fx+e)-2\left(i\,a^2c+a^2d\right)\log\left(\tan\left(fx+e\right)^2+1\right)+2\left(a^2c-2i\,a^2d\right)\tan\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c+d*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/2*(a^2*d*tan(f*x + e)^2 - 4*(a^2*c - I*a^2*d)*(f*x + e) - 2*(I*a^2*c + a
^2*d)*log(tan(f*x + e)^2 + 1) + 2*(a^2*c - 2*I*a^2*d)*tan(f*x + e))/f
```

Fricas [A]
time = 1.05, size = 141, normalized size = 1.76

$$-\frac{2\left(i\,a^2c+2\,a^2d+\left(i\,a^2c+3\,a^2d\right)e^{(2i\,fx+2i\,e)}+\left(i\,a^2c+a^2d+\left(i\,a^2c+a^2d\right)e^{(4i\,fx+4i\,e)}+2\left(i\,a^2c+a^2d\right)e^{(2i\,fx+2i\,e)}\right)\log\left(e^{(2i\,fx+2i\,e)}+1\right)\right)}{fe^{(4i\,fx+4i\,e)}+2\,fe^{(2i\,fx+2i\,e)}+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c+d*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] -2*(I*a^2*c + 2*a^2*d + (I*a^2*c + 3*a^2*d)*e^(2*I*f*x + 2*I*e) + (I*a^2*c
+ a^2*d + (I*a^2*c + a^2*d)*e^(4*I*f*x + 4*I*e) + 2*(I*a^2*c + a^2*d)*e^(2*
I*f*x + 2*I*e))*log(e^(2*I*f*x + 2*I*e) + 1))/(f*e^(4*I*f*x + 4*I*e) + 2*f*
e^(2*I*f*x + 2*I*e) + f)
```

Sympy [A]
time = 0.32, size = 122, normalized size = 1.52

$$-\frac{2ia^2(c-id)\log\left(e^{2ifx}+e^{-2ie}\right)}{f}+\frac{-2ia^2c-4a^2d+\left(-2ia^2ce^{2ie}-6a^2de^{2ie}\right)e^{2ifx}}{fe^{4ie}e^{4ifx}+2fe^{2ie}e^{2ifx}+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2*(c+d*tan(f*x+e)),x)
```

```
[Out] -2*I*a**2*(c - I*d)*log(exp(2*I*f*x) + exp(-2*I*e))/f + (-2*I*a**2*c - 4*a*
*2*d + (-2*I*a**2*c*exp(2*I*e) - 6*a**2*d*exp(2*I*e))*exp(2*I*f*x))/(f*exp(
4*I*e)*exp(4*I*f*x) + 2*f*exp(2*I*e)*exp(2*I*f*x) + f)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(73) = 146.
time = 0.53, size = 228, normalized size = 2.85

$$-\frac{2\left(i\,a^2ce^{(4i\,fx+4i\,e)}\log\left(e^{(2i\,fx+2i\,e)}+1\right)+a^2de^{(4i\,fx+4i\,e)}\log\left(e^{(2i\,fx+2i\,e)}+1\right)+2i\,a^2ce^{(2i\,fx+2i\,e)}\log\left(e^{(2i\,fx+2i\,e)}+1\right)+2\,a^2de^{(2i\,fx+2i\,e)}\log\left(e^{(2i\,fx+2i\,e)}+1\right)+i\,a^2ce^{(2i\,fx+2i\,e)}+3\,a^2de^{(2i\,fx+2i\,e)}+i\,a^2c\log\left(e^{(2i\,fx+2i\,e)}+1\right)+a^2d\log\left(e^{(2i\,fx+2i\,e)}+1\right)+i\,a^2c+2\,a^2d\right)}{fe^{(4i\,fx+4i\,e)}+2\,fe^{(2i\,fx+2i\,e)}+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c+d*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] -2*(I*a^2*c*e^(4*I*f*x + 4*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + a^2*d*e^(4*I
*f*x + 4*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 2*I*a^2*c*e^(2*I*f*x + 2*I*e)*
log(e^(2*I*f*x + 2*I*e) + 1) + 2*a^2*d*e^(2*I*f*x + 2*I*e)*log(e^(2*I*f*x +
 2*I*e) + 1) + I*a^2*c*e^(2*I*f*x + 2*I*e) + 3*a^2*d*e^(2*I*f*x + 2*I*e) +
I*a^2*c*log(e^(2*I*f*x + 2*I*e) + 1) + a^2*d*log(e^(2*I*f*x + 2*I*e) + 1) +
 I*a^2*c + 2*a^2*d)/(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*e) + f)
```

Mupad [B]
time = 5.07, size = 76, normalized size = 0.95

$$\frac{\tan(e+f\,x)\,\left(a^2\,d\,1\mathrm{i}+a^2\,\left(d+c\,1\mathrm{i}\right)\,1\mathrm{i}\right)}{f}+\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(2\,a^2\,d+a^2\,c\,2\mathrm{i}\right)}{f}-\frac{a^2\,d\,{\tan(e+f\,x)}^2}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^2*(c + d*tan(e + f*x)),x)
```

```
[Out] (tan(e + f*x)*(a^2*d*1i + a^2*(c*1i + d)*1i))/f + (log(tan(e + f*x) + 1i)*(
a^2*c*2i + 2*a^2*d))/f - (a^2*d*tan(e + f*x)^2)/(2*f)
```

3.1067 $\int (a+ia\tan(e+fx))(c+d\tan(e+fx))\,dx$

Optimal. Leaf size=46

$$a(c-id)x-\frac{a(ic+d)\log(\cos(e+fx))}{f}+\frac{iad\tan(e+fx)}{f}$$

```
[Out] a*(c-I*d)*x-a*(I*c+d)*ln(cos(f*x+e))/f+I*a*d*tan(f*x+e)/f
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used = {3606, 3556}

$$-\frac{a(d+ic)\log(\cos(e+fx))}{f}+ax(c-id)+\frac{iad\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])*(c + d*Tan[e + f*x]),x]
```

```
[Out] a*(c - I*d)*x - (a*(I*c + d)*Log[Cos[e + f*x]])/f + (I*a*d*Tan[e + f*x])/f
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rubi steps

$$\begin{aligned}\int (a+ia\tan(e+fx))(c+d\tan(e+fx))\,dx &= a(c-id)x+\frac{iad\tan(e+fx)}{f}+(a(ic+d))\int \tan(e+fx)\,dx\\ &= a(c-id)x-\frac{a(ic+d)\log(\cos(e+fx))}{f}+\frac{iad\tan(e+fx)}{f}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 66, normalized size = 1.43

$$acx-\frac{iad\text{ArcTan}(\tan(e+fx))}{f}-\frac{iac\log(\cos(e+fx))}{f}-\frac{ad\log(\cos(e+fx))}{f}+\frac{iad\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])*(c + d*Tan[e + f*x]),x]
```

```
[Out] a*c*x - (I*a*d*ArcTan[Tan[e + f*x]])/f - (I*a*c*Log[Cos[e + f*x]])/f - (a*d
*Log[Cos[e + f*x]])/f + (I*a*d*Tan[e + f*x])/f
```

Maple [A]
time = 0.08, size = 50, normalized size = 1.09

method	result	size
derivativedivides	$\frac{a\left(i\tan(fx+e)d+\frac{(ic+d)\ln\left(1+\tan^2(fx+e)\right)}{2}+(-id+c)\arctan(\tan(fx+e))\right)}{f}$	50
default	$\frac{a\left(i\tan(fx+e)d+\frac{(ic+d)\ln\left(1+\tan^2(fx+e)\right)}{2}+(-id+c)\arctan(\tan(fx+e))\right)}{f}$	50
norman	$(-iad+ac)\,x+\frac{iad\tan(fx+e)}{f}+\frac{(iac+ad)\ln(1+\tan^2(fx+e))}{2f}$	52
risch	$\frac{2iade}{f}-\frac{2ace}{f}-\frac{2ad}{f(e^{2i(fx+e)}+1)}-\frac{a\ln(e^{2i(fx+e)}+1)d}{f}-\frac{ia\ln(e^{2i(fx+e)}+1)c}{f}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))*(c+d*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a*(I*tan(f*x+e)*d+1/2*(I*c+d)*ln(1+tan(f*x+e)^2)+(c-I*d)*arctan(tan(f*x
+e)))
```

Maxima [A]
time = 0.55, size = 55, normalized size = 1.20

$$-\frac{-2i\,ad\tan\left(fx+e\right)-2\left(ac-i\,ad\right)\left(fx+e\right)+\left(-i\,ac-ad\right)\log\left(\tan\left(fx+e\right)^{2}+1\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c+d*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/2*(-2*I*a*d*tan(f*x + e) - 2*(a*c - I*a*d)*(f*x + e) + (-I*a*c - a*d)*lo
g(tan(f*x + e)^2 + 1))/f
```

Fricas [A]
time = 0.93, size = 67, normalized size = 1.46

$$-\frac{2\,ad-\left(-i\,ac-ad+\left(-i\,ac-ad\right)e^{(2i\,fx+2i\,e)}\right)\log\left(e^{(2i\,fx+2i\,e)}+1\right)}{fe^{(2i\,fx+2i\,e)}+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c+d*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(2*a*d - (-I*a*c - a*d + (-I*a*c - a*d)*e^(2*I*f*x + 2*I*e))*log(e^(2*I*f*
x + 2*I*e) + 1))/(f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [A]
time = 0.25, size = 53, normalized size = 1.15

$$-\frac{2ad}{fe^{2ie}e^{2ifx}+f}-\frac{ia(c-id)\log\left(e^{2ifx}+e^{-2ie}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c+d*tan(f*x+e)),x)
```

```
[Out] -2*a*d/(f*exp(2*I*e)*exp(2*I*f*x) + f) - I*a*(c - I*d)*log(exp(2*I*f*x) + e
xp(-2*I*e))/f
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(42) = 84$.
time = 0.46, size = 110, normalized size = 2.39

$$\frac{-iace^{(2ifx+2ie)}\log\left(e^{(2ifx+2ie)}+1\right)-ade^{(2ifx+2ie)}\log\left(e^{(2ifx+2ie)}+1\right)-iac\log\left(e^{(2ifx+2ie)}+1\right)-ad\log\left(e^{(2ifx+2ie)}+1\right)-2ad}{fe^{(2ifx+2ie)}+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c+d*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] (-I*a*c*e^(2*I*f*x + 2*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) - a*d*e^(2*I*f*x +
 2*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) - I*a*c*log(e^(2*I*f*x + 2*I*e) + 1) -
 a*d*log(e^(2*I*f*x + 2*I*e) + 1) - 2*a*d)/(f*e^(2*I*f*x + 2*I*e) + f)
```

Mupad [B]
time = 4.88, size = 38, normalized size = 0.83

$$\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(a\,d+a\,c\,1\mathrm{i}\right)}{f}+\frac{a\,d\tan(e+f\,x)\,1\mathrm{i}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)*(c + d*tan(e + f*x)),x)
```

```
[Out] (log(tan(e + f*x) + 1i)*(a*c*1i + a*d))/f + (a*d*tan(e + f*x)*1i)/f
```

3.1068 $\int \frac{c+d\tan(e+fx)}{a+ia\tan(e+fx)}\,dx$

Optimal. Leaf size=47

$$\frac{(c-id)x}{2a}+\frac{ic-d}{2f(a+ia\tan(e+fx))}$$

```
[Out] 1/2*(c-I*d)*x/a+1/2*(I*c-d)/f/(a+I*a*tan(f*x+e))
```

Rubi [A]
time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {3607, 8}

$$\frac{-d+ic}{2f(a+ia\tan(e+fx))}+\frac{x(c-id)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] ((c - I*d)*x)/(2*a) + (I*c - d)/(2*f*(a + I*a*Tan[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{c+d\tan(e+fx)}{a+ia\tan(e+fx)}\,dx &= \frac{ic-d}{2f(a+ia\tan(e+fx))}+\frac{(c-id)\int 1\,dx}{2a}\\ &= \frac{(c-id)x}{2a}+\frac{ic-d}{2f(a+ia\tan(e+fx))}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 102 vs. 2(47) = 94.

time = 0.46, size = 102, normalized size = 2.17

$$\frac{\cos(e+fx)(c+d\tan(e+fx))(c-2icfx+d(i-2fx)+(d-2idfx+c(-i+2fx))\tan(e+fx))}{4af(c\cos(e+fx)+d\sin(e+fx))(-i+\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (Cos[e + f*x]*(c + d*Tan[e + f*x])*(c - (2*I)*c*f*x + d*(I - 2*f*x) + (d -
(2*I)*d*f*x + c*(-I + 2*f*x))*Tan[e + f*x]))/(4*a*f*(c*Cos[e + f*x] + d*Sin
[e + f*x])*(-I + Tan[e + f*x]))
```

Maple [A]
time = 0.17, size = 70, normalized size = 1.49

method	result	size
risch	$-\frac{ixd}{2a}+\frac{xc}{2a}-\frac{e^{-2i(fx+e)}d}{4af}+\frac{ie^{-2i(fx+e)}c}{4af}$	54
derivativedivides	$\frac{-\frac{i(id-c)\ln(\tan(fx+e)+i)}{4}+\left(-\frac{ic}{4}-\frac{d}{4}\right)\ln(\tan(fx+e)-i)-\frac{-\frac{c}{2}-\frac{id}{2}}{\tan(fx+e)-i}}{fa}$	70
default	$\frac{-\frac{i(id-c)\ln(\tan(fx+e)+i)}{4}+\left(-\frac{ic}{4}-\frac{d}{4}\right)\ln(\tan(fx+e)-i)-\frac{-\frac{c}{2}-\frac{id}{2}}{\tan(fx+e)-i}}{fa}$	70
norman	$\frac{\frac{(-id+c)x}{2a}-\frac{-ic+d}{2af}+\frac{(id+c)\tan(fx+e)}{2af}+\frac{(-id+c)x\left(\tan^2(fx+e)\right)}{2a}}{1+\tan^2(fx+e)}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))/(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a*(-1/4*I*(I*d-c)*ln(tan(f*x+e)+I)+(-1/4*I*c-1/4*d)*ln(tan(f*x+e)-I)-(-
1/2*c-1/2*I*d)/(tan(f*x+e)-I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.94, size = 44, normalized size = 0.94

$$\frac{\left(2\,(c-i\,d)fxe^{(2i\,fx+2i\,e)}+i\,c-d\right)e^{(-2i\,fx-2i\,e)}}{4\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(c - I*d)*f*x*e^(2*I*f*x + 2*I*e) + I*c - d)*e^(-2*I*f*x - 2*I*e)/(a
*f)
```

Sympy [A]
time = 0.13, size = 87, normalized size = 1.85

$$\begin{cases} \frac{(ic-d)e^{-2ie}e^{-2ifx}}{4af} & \text{for } afe^{2ie} \neq 0 \\ x\left(-\frac{c-id}{2a} + \frac{(ce^{2ie}+c-ide^{2ie}+id)e^{-2ie}}{2a}\right) & \text{otherwise} \end{cases} + \frac{x(c-id)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))/(a+I*a*tan(f*x+e)),x)
```

```
[Out] Piecewise((((I*c - d)*exp(-2*I*e)*exp(-2*I*f*x)/(4*a*f), Ne(a*f*exp(2*I*e),
0)), (x*(-(c - I*d)/(2*a) + (c*exp(2*I*e) + c - I*d*exp(2*I*e) + I*d)*exp(-
2*I*e)/(2*a)), True)) + x*(c - I*d)/(2*a)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(36) = 72$.
time = 0.45, size = 90, normalized size = 1.91

$$-\frac{\frac{(i\,c+d)\log(\tan(fx+e)-i)}{a} + \frac{(-i\,c-d)\log(-i\,\tan(fx+e)+1)}{a} + \frac{-i\,c\tan(fx+e)-d\tan(fx+e)-3\,c-i\,d}{a(\tan(fx+e)-i)}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/4*((I*c + d)*log(tan(f*x + e) - I)/a + (-I*c - d)*log(-I*tan(f*x + e) +
1)/a + (-I*c*tan(f*x + e) - d*tan(f*x + e) - 3*c - I*d)/(a*(tan(f*x + e) -
I)))/f
```

Mupad [B]
time = 5.01, size = 45, normalized size = 0.96

$$-\frac{x\,(d+c\,1\mathrm{i})\,1\mathrm{i}}{2\,a} + \frac{-\frac{d}{2\,a} + \frac{c\,1\mathrm{i}}{2\,a}}{f\,\left(1+\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))/(a + a*tan(e + f*x)*1i),x)
```

```
[Out] ((c*1i)/(2*a) - d/(2*a))/(f*(tan(e + f*x)*1i + 1)) - (x*(c*1i + d)*1i)/(2*a
)
```

3.1069 $\int \frac{c+d\tan(e+fx)}{(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=80

$$\frac{(c-id)x}{4a^2}+\frac{ic-d}{4f(a+ia\tan(e+fx))^2}+\frac{ic+d}{4f\left(a^2+ia^2\tan(e+fx)\right)}$$

```
[Out] 1/4*(c-I*d)*x/a^2+1/4*(I*c-d)/f/(a+I*a*tan(f*x+e))^2+1/4*(I*c+d)/f/(a^2+I*a
^2*tan(f*x+e))
```

Rubi [A]

time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.115$, Rules used = {3607, 3560, 8}

$$\frac{d+ic}{4f\left(a^2+ia^2\tan(e+fx)\right)}+\frac{x(c-id)}{4a^2}+\frac{-d+ic}{4f(a+ia\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] ((c - I*d)*x)/(4*a^2) + (I*c - d)/(4*f*(a + I*a*Tan[e + f*x])^2) + (I*c + d
)/(4*f*(a^2 + I*a^2*Tan[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rubi steps

$$\int \frac{c+d\tan(e+fx)}{(a+ia\tan(e+fx))^2}\,dx = \frac{ic-d}{4f(a+ia\tan(e+fx))^2}+\frac{(c-id)\int \frac{1}{a+ia\tan(e+fx)}\,dx}{2a}$$
$$= \frac{ic-d}{4f(a+ia\tan(e+fx))^2}+\frac{ic+d}{4f\,(a^2+ia^2\tan(e+fx))}+\frac{(c-id)\int 1\,dx}{4a^2}$$
$$= \frac{(c-id)x}{4a^2}+\frac{ic-d}{4f(a+ia\tan(e+fx))^2}+\frac{ic+d}{4f\,(a^2+ia^2\tan(e+fx))}$$

Mathematica [A]
time = 0.58, size = 94, normalized size = 1.18

$$-\frac{\sec^2(e+fx)(4ic+(d(-1-4ifx)+c(i+4fx))\cos(2(e+fx))+(c+id+4icfx+4dfx)\sin(2(e+fx)))}{16a^2f(-i+\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] -1/16*(Sec[e + f*x]^2*((4*I)*c + (d*(-1 - (4*I)*f*x) + c*(I + 4*f*x))*Cos[2
*(e + f*x)] + (c + I*d + (4*I)*c*f*x + 4*d*f*x)*Sin[2*(e + f*x)]))/(a^2*f*(
-I + Tan[e + f*x])^2)
```

Maple [A]
time = 0.20, size = 91, normalized size = 1.14

method	result	size
risch	$-\frac{ixd}{4a^2}+\frac{xc}{4a^2}+\frac{ic\,\mathrm{e}^{-2i(fx+e)}}{4a^2f}-\frac{\mathrm{e}^{-4i(fx+e)}d}{16f\,a^2}+\frac{i\mathrm{e}^{-4i(fx+e)}c}{16f\,a^2}$	73
derivativedivides	$\frac{-\frac{i(id-c)\ln(\tan(fx+e)+i)}{8}-\frac{-\frac{c}{4}+\frac{id}{4}}{\tan(fx+e)-i}+\left(-\frac{ic}{8}-\frac{d}{8}\right)\ln(\tan(fx+e)-i)-\frac{\frac{ic}{2}-\frac{d}{2}}{2(\tan(fx+e)-i)^2}}{f\,a^2}$	91
default	$\frac{-\frac{i(id-c)\ln(\tan(fx+e)+i)}{8}-\frac{-\frac{c}{4}+\frac{id}{4}}{\tan(fx+e)-i}+\left(-\frac{ic}{8}-\frac{d}{8}\right)\ln(\tan(fx+e)-i)-\frac{\frac{ic}{2}-\frac{d}{2}}{2(\tan(fx+e)-i)^2}}{f\,a^2}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))/(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a^2*(-1/8*I*(I*d-c)*ln(tan(f*x+e)+I)-(-1/4*c+1/4*I*d)/(tan(f*x+e)-I)+(-
1/8*I*c-1/8*d)*ln(tan(f*x+e)-I)-1/2*(1/2*I*c-1/2*d)/(tan(f*x+e)-I)^2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*tan(f*x+e))/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

Fricas [A]
time = 1.07, size = 57, normalized size = 0.71

$$\frac{\left(4\,(c-i\,d)fxe^{(4i\,fx+4i\,e)}+4i\,ce^{(2i\,fx+2i\,e)}+i\,c-d\right)e^{(-4i\,fx-4i\,e)}}{16\,a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*tan(f*x+e))/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")

[Out] 1/16*(4*(c - I*d)*f*x*e^(4*I*f*x + 4*I*e) + 4*I*c*e^(2*I*f*x + 2*I*e) + I*c - d)*e^(-4*I*f*x - 4*I*e)/(a^2*f)

Sympy [A]
time = 0.19, size = 162, normalized size = 2.02

$$\begin{cases}\frac{\left(16ia^2cfe^{4ie}e^{-2ifx}+\left(4ia^2cfe^{2ie}-4a^2dfe^{2ie}\right)e^{-4ifx}\right)e^{-6ie}}{64a^4f^2} & \text{for } a^4f^2e^{6ie}\neq 0\\ x\left(-\frac{c-id}{4a^2}+\frac{\left(ce^{4ie}+2ce^{2ie}+c-ide^{4ie}+id\right)e^{-4ie}}{4a^2}\right) & \text{otherwise}\end{cases}+\frac{x(c-id)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*tan(f*x+e))/(a+I*a*tan(f*x+e))**2,x)

[Out] Piecewise(((16*I*a**2*c*f*exp(4*I*e)*exp(-2*I*f*x) + (4*I*a**2*c*f*exp(2*I*e) - 4*a**2*d*f*exp(2*I*e))*exp(-4*I*f*x))*exp(-6*I*e)/(64*a**4*f**2), Ne(a**4*f**2*exp(6*I*e), 0)), (x*(-(c - I*d)/(4*a**2) + (c*exp(4*I*e) + 2*c*exp(2*I*e) + c - I*d*exp(4*I*e) + I*d)*exp(-4*I*e)/(4*a**2)), True)) + x*(c - I*d)/(4*a**2)

Giac [A]
time = 0.54, size = 117, normalized size = 1.46

$$-\frac{\frac{2\,(-i\,c-d)\log(\tan(fx+e)+i)}{a^2}-\frac{2\,(-i\,c-d)\log(\tan(fx+e)-i)}{a^2}-\frac{3i\,c\tan(fx+e)^2+3\,d\tan(fx+e)^2+10\,c\tan(fx+e)-10i\,d\tan(fx+e)-11i\,c-3\,d}{a^2(\tan(fx+e)-i)^2}}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*tan(f*x+e))/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")

[Out] -1/16*(2*(-I*c - d)*log(tan(f*x + e) + I)/a^2 - 2*(-I*c - d)*log(tan(f*x + e) - I)/a^2 - (3*I*c*tan(f*x + e)^2 + 3*d*tan(f*x + e)^2 + 10*c*tan(f*x + e) - 10*I*d*tan(f*x + e) - 11*I*c - 3*d)/(a^2*(tan(f*x + e) - I)^2))/f

Mupad [B]
time = 5.02, size = 70, normalized size = 0.88

$$\frac{\tan(e+f\,x)\,\left(\frac{d}{4\,a^2}+\frac{c\,1\mathrm{i}}{4\,a^2}\right)+\frac{c}{2\,a^2}}{f\,\left({\tan\left(e+f\,x\right)}^2\,1\mathrm{i}+2\tan\left(e+f\,x\right)-\mathrm{i}\right)}-\frac{x\,\left(d+c\,1\mathrm{i}\right)\,1\mathrm{i}}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))/(a + a*tan(e + f*x)*1i)^2,x)
```

```
[Out] (tan(e + f*x)*((c*1i)/(4*a^2) + d/(4*a^2)) + c/(2*a^2))/(f*(2*tan(e + f*x)
+ tan(e + f*x)^2*1i - 1i)) - (x*(c*1i + d)*1i)/(4*a^2)
```

3.1070 $\int \frac{c+d\tan(e+fx)}{(a+ia\tan(e+fx))^3}\,dx$

Optimal. Leaf size=112

$$\frac{(c-id)x}{8a^3}+\frac{ic-d}{6f(a+ia\tan(e+fx))^3}+\frac{ic+d}{8af(a+ia\tan(e+fx))^2}+\frac{ic+d}{8f\left(a^3+ia^3\tan(e+fx)\right)}$$

```
[Out] 1/8*(c-I*d)*x/a^3+1/6*(I*c-d)/f/(a+I*a*tan(f*x+e))^3+1/8*(I*c+d)/a/f/(a+I*a
*tan(f*x+e))^2+1/8*(I*c+d)/f/(a^3+I*a^3*tan(f*x+e))
```

Rubi [A]
time = 0.06, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3607, 3560, 8}

$$\frac{d+ic}{8f\left(a^3+ia^3\tan(e+fx)\right)}+\frac{x(c-id)}{8a^3}+\frac{-d+ic}{6f(a+ia\tan(e+fx))^3}+\frac{d+ic}{8af(a+ia\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] ((c - I*d)*x)/(8*a^3) + (I*c - d)/(6*f*(a + I*a*Tan[e + f*x])^3) + (I*c + d
)/(8*a*f*(a + I*a*Tan[e + f*x])^2) + (I*c + d)/(8*f*(a^3 + I*a^3*Tan[e + f*
x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rubi steps

$$\int \frac{c+d\tan(e+fx)}{(a+ia\tan(e+fx))^3}\,dx = \frac{ic-d}{6f(a+ia\tan(e+fx))^3} + \frac{(c-id)\int \frac{1}{(a+ia\tan(e+fx))^2}\,dx}{2a}$$

$$= \frac{ic-d}{6f(a+ia\tan(e+fx))^3} + \frac{ic+d}{8af(a+ia\tan(e+fx))^2} + \frac{(c-id)\int \frac{1}{a+ia\tan(e+fx}}{4a^2}$$

$$= \frac{ic-d}{6f(a+ia\tan(e+fx))^3} + \frac{ic+d}{8af(a+ia\tan(e+fx))^2} + \frac{ic+d}{8f\,(a^3+ia^3\tan(e+f}$$

$$= \frac{(c-id)x}{8a^3} + \frac{ic-d}{6f(a+ia\tan(e+fx))^3} + \frac{ic+d}{8af(a+ia\tan(e+fx))^2} + \frac{}{8f\,(a^3+}$$

Mathematica [A]

time = 0.75, size = 150, normalized size = 1.34

$$\frac{\sec^3(e+fx)((-27c+3id)\cos(e+fx)+2(-c-id+6icfx+6dfx)\cos(3(e+fx))-9ic\sin(e+fx)-9d\sin(e+fx)+2ic\sin(3(e+fx))-2d\sin(3(e+fx))-12cfx\sin(3(e+fx))+12idfx\sin(3(e+fx)))}{96a^3f(-i+\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] (Sec[e + f*x]^3*((-27*c + (3*I)*d)*Cos[e + f*x] + 2*(-c - I*d + (6*I)*c*f*x
 + 6*d*f*x)*Cos[3*(e + f*x)] - (9*I)*c*Sin[e + f*x] - 9*d*Sin[e + f*x] + (2
*I)*c*Sin[3*(e + f*x)] - 2*d*Sin[3*(e + f*x)] - 12*c*f*x*Sin[3*(e + f*x)] +
 (12*I)*d*f*x*Sin[3*(e + f*x)]))/(96*a^3*f*(-I + Tan[e + f*x])^3)
```

Maple [A]

time = 0.21, size = 112, normalized size = 1.00

method	result
derivativedivides	$\frac{-\frac{i(id-c)\ln(\tan(fx+e)+i)}{16}-\frac{-\frac{c}{8}+\frac{id}{8}}{\tan(fx+e)-i}-\frac{\frac{ic}{4}+\frac{d}{4}}{2(\tan(fx+e)-i)^2}+\left(-\frac{ic}{16}-\frac{d}{16}\right)\ln(\tan(fx+e)-i)-\frac{\frac{c}{2}+\frac{id}{2}}{3(\tan(fx+e)-i)^3}}{f\,a^3}$
default	$\frac{-\frac{i(id-c)\ln(\tan(fx+e)+i)}{16}-\frac{-\frac{c}{8}+\frac{id}{8}}{\tan(fx+e)-i}-\frac{\frac{ic}{4}+\frac{d}{4}}{2(\tan(fx+e)-i)^2}+\left(-\frac{ic}{16}-\frac{d}{16}\right)\ln(\tan(fx+e)-i)-\frac{\frac{c}{2}+\frac{id}{2}}{3(\tan(fx+e)-i)^3}}{f\,a^3}$
risch	$-\frac{ixd}{8a^3}+\frac{xc}{8a^3}+\frac{e^{-2i(fx+e)}d}{16a^3f}+\frac{3ie^{-2i(fx+e)}c}{16a^3f}-\frac{e^{-4i(fx+e)}d}{32a^3f}+\frac{3ic\,e^{-4i(fx+e)}}{32a^3f}-\frac{e^{-6i(fx+e)}d}{48a^3f}+\frac{ie^{-6i(fx+e)}c}{48a^3f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))/(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a^3*(-1/16*I*(I*d-c)*ln(tan(f*x+e)+I)-(-1/8*c+1/8*I*d)/(tan(f*x+e)-I)-1
/2*(1/4*I*c+1/4*d)/(tan(f*x+e)-I)^2+(-1/16*I*c-1/16*d)*ln(tan(f*x+e)-I)-1/3
*(1/2*c+1/2*I*d)/(tan(f*x+e)-I)^3)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))/(a+I*a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.29, size = 80, normalized size = 0.71

$$\frac{\left(12\,(c-i\,d)fxe^{(6i\,fx+6i\,e)}-6\,(-3i\,c-d)e^{(4i\,fx+4i\,e)}-3\,(-3i\,c+d)e^{(2i\,fx+2i\,e)}+2i\,c-2\,d\right)e^{(-6i\,fx-6i\,e)}}{96\,a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))/(a+I*a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/96*(12*(c - I*d)*f*x*e^(6*I*f*x + 6*I*e) - 6*(-3*I*c - d)*e^(4*I*f*x + 4*
I*e) - 3*(-3*I*c + d)*e^(2*I*f*x + 2*I*e) + 2*I*c - 2*d)*e^(-6*I*f*x - 6*I*
e)/(a^3*f)
```

Sympy [A]
time = 0.25, size = 258, normalized size = 2.30

$$\begin{cases}\frac{((512ia^6cf^2e^{6ie}-512a^6df^2e^{6ie})e^{-6ifx}+(2304ia^6cf^2e^{8ie}-768a^6df^2e^{8ie})e^{-4ifx}+(4608ia^6cf^2e^{10ie}+1536a^6df^2e^{10ie})e^{-2ifx})e^{-12ie}}{24576a^9f^3} & \text{for } a^9f^3e^{12ie}\neq 0\\ x\left(-\frac{c-id}{8a^3}+\frac{(ce^{6ie}+3ce^{4ie}+3ce^{2ie}+c-ide^{6ie}-ide^{4ie}+ide^{2ie}+id)e^{-6ie}}{8a^3}\right) & \text{otherwise}\end{cases}+\frac{x(c-id)}{8a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))/(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] Piecewise((((512*I*a**6*c*f**2*exp(6*I*e) - 512*a**6*d*f**2*exp(6*I*e))*exp
(-6*I*f*x) + (2304*I*a**6*c*f**2*exp(8*I*e) - 768*a**6*d*f**2*exp(8*I*e))*e
xp(-4*I*f*x) + (4608*I*a**6*c*f**2*exp(10*I*e) + 1536*a**6*d*f**2*exp(10*I*
e))*exp(-2*I*f*x))*exp(-12*I*e)/(24576*a**9*f**3), Ne(a**9*f**3*exp(12*I*e)
, 0)), (x*(-(c - I*d)/(8*a**3) + (c*exp(6*I*e) + 3*c*exp(4*I*e) + 3*c*exp(2
*I*e) + c - I*d*exp(6*I*e) - I*d*exp(4*I*e) + I*d*exp(2*I*e) + I*d)*exp(-6*
I*e)/(8*a**3)), True)) + x*(c - I*d)/(8*a**3)
```

Giac [A]
time = 0.67, size = 140, normalized size = 1.25

$$-\frac{\frac{6\,(i\,c+d)\log(\tan(fx+e)-i)}{a^3}+\frac{6\,(-i\,c-d)\log(i\,\tan(fx+e)-1)}{a^3}+\frac{-11i\,c\tan(fx+e)^3-11\,d\tan(fx+e)^3-45\,c\tan(fx+e)^2+45i\,d\tan(fx+e)^2+69i\,c\tan(fx+e)+69\,d\tan(fx+e)+51\,c-19i\,d}{a^3(\tan(fx+e)-i)^3}}{96\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))/(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/96*(6*(I*c + d)*log(tan(f*x + e) - I)/a^3 + 6*(-I*c - d)*log(I*tan(f*x +
 e) - 1)/a^3 + (-11*I*c*tan(f*x + e)^3 - 11*d*tan(f*x + e)^3 - 45*c*tan(f*x
 + e)^2 + 45*I*d*tan(f*x + e)^2 + 69*I*c*tan(f*x + e) + 69*d*tan(f*x + e) +
 51*c - 19*I*d)/(a^3*(tan(f*x + e) - I)^3))/f
```

Mupad [B]
time = 5.10, size = 111, normalized size = 0.99

$$-\frac{x\,\left(d+c\,1\mathrm{i}\right)\,1\mathrm{i}}{8\,a^3}-\frac{\mathrm{tan}\left(e+f\,x\right)\,\left(\frac{3\,c}{8\,a^3}-\frac{d\,3\mathrm{i}}{8\,a^3}\right)-\frac{c\,5\mathrm{i}}{12\,a^3}-\frac{d}{12\,a^3}+{\mathrm{tan}\left(e+f\,x\right)}^2\,\left(\frac{d}{8\,a^3}+\frac{c\,1\mathrm{i}}{8\,a^3}\right)}{f\,\left(-{\mathrm{tan}\left(e+f\,x\right)}^3\,1\mathrm{i}-3\,{\mathrm{tan}\left(e+f\,x\right)}^2+\mathrm{tan}\left(e+f\,x\right)\,3\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))/(a + a*tan(e + f*x)*1i)^3,x)
```

```
[Out] - (x*(c*1i + d)*1i)/(8*a^3) - (tan(e + f*x)*((3*c)/(8*a^3) - (d*3i)/(8*a^3)
) - (c*5i)/(12*a^3) - d/(12*a^3) + tan(e + f*x)^2*((c*1i)/(8*a^3) + d/(8*a^
3)))/(f*(tan(e + f*x)*3i - 3*tan(e + f*x)^2 - tan(e + f*x)^3*1i + 1))
```

3.1071 $\int (a+ia\tan(e+fx))^3(c+d\tan(e+fx))^2\,dx$

Optimal. Leaf size=153

$$4a^3(c-id)^2x-\frac{4ia^3(c-id)^2\log(\cos(e+fx))}{f}-\frac{2a^3(c-id)^2\tan(e+fx)}{f}+\frac{ia(c-id)^2(a+ia\tan(e+fx))^2}{2f}+$$

```
[Out] 4*a^3*(c-I*d)^2*x-4*I*a^3*(c-I*d)^2*ln(cos(f*x+e))/f-2*a^3*(c-I*d)^2*tan(f*
x+e)/f+1/2*I*a*(c-I*d)^2*(a+I*a*tan(f*x+e))^2/f+2/3*c*d*(a+I*a*tan(f*x+e))^
3/f-1/4*I*d^2*(a+I*a*tan(f*x+e))^4/a/f
```

Rubi [A]
time = 0.14, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3624, 3608, 3559, 3558, 3556}

$$-\frac{2a^3(c-id)^2\tan(e+fx)}{f}-\frac{4ia^3(c-id)^2\log(\cos(e+fx))}{f}+4a^3x(c-id)^2+\frac{2cd(a+ia\tan(e+fx))^3}{3f}+\frac{ia(c-id)^2(a+ia\tan(e+fx))^2}{2f}-\frac{id^2(a+ia\tan(e+fx))^4}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^2,x]
```

```
[Out] 4*a^3*(c - I*d)^2*x - ((4*I)*a^3*(c - I*d)^2*Log[Cos[e + f*x]])/f - (2*a^3*
(c - I*d)^2*Tan[e + f*x])/f + ((I/2)*a*(c - I*d)^2*(a + I*a*Tan[e + f*x])^2
)/f + (2*c*d*(a + I*a*Tan[e + f*x])^3)/(3*f) - ((I/4)*d^2*(a + I*a*Tan[e +
f*x])^4)/(a*f)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
```

```
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && !LtQ[m, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && !LeQ
[m, -1] && !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (a+ia\tan(e+fx))^3(c+d\tan(e+fx))^2\,dx &= -\frac{id^2(a+ia\tan(e+fx))^4}{4af}+\int (a+ia\tan(e+fx))^3\,(c^2 -\\ &= \frac{2cd(a+ia\tan(e+fx))^3}{3f}-\frac{id^2(a+ia\tan(e+fx))^4}{4af}+(c\\ &= \frac{ia(c-id)^2(a+ia\tan(e+fx))^2}{2f}+\frac{2cd(a+ia\tan(e+fx)}{3f}\\ &= 4a^3(c-id)^2x-\frac{2a^3(c-id)^2\tan(e+fx)}{f}+\frac{ia(c-id)^2(a}{}\\ &= 4a^3(c-id)^2x-\frac{4ia^3(c-id)^2\log(\cos(e+fx))}{f}-\frac{2a^3(c-}{}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 948 vs. $2(153) = 306$.
time = 7.27, size = 948, normalized size = 6.20

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^2,x]
```

```
[Out] (Cos[e + f*x]^3*(c^2*Cos[(3*e)/2] - (2*I)*c*d*Cos[(3*e)/2] - d^2*Cos[(3*e)/
2] - I*c^2*Sin[(3*e)/2] - 2*c*d*Sin[(3*e)/2] + I*d^2*Sin[(3*e)/2])*((-2*I)*
Cos[(3*e)/2]*Log[Cos[e + f*x]^2] - 2*Log[Cos[e + f*x]^2]*Sin[(3*e)/2])*(a +
 I*a*Tan[e + f*x])^3)/(f*(Cos[f*x] + I*Sin[f*x])^3) + (Cos[e + f*x]*(3*c^2*
Cos[e] - (18*I)*c*d*Cos[e] - 15*d^2*Cos[e] + 4*c*d*Sin[e] - (6*I)*d^2*Sin[e
])*((-1/6*I)*Cos[3*e] - Sin[3*e]/6)*(a + I*a*Tan[e + f*x])^3)/(f*(Cos[e/2]
- Sin[e/2])*(Cos[e/2] + Sin[e/2])*(Cos[f*x] + I*Sin[f*x])^3) + (Sec[e + f*x
]*((-1/4*I)*d^2*Cos[3*e] - (d^2*Sin[3*e])/4)*(a + I*a*Tan[e + f*x])^3)/(f*(
```

```
Cos[f*x] + I*Sin[f*x])^3) + ((c - I*d)^2*Cos[e + f*x]^3*(4*f*x*Cos[3*e] - (
4*I)*f*x*Sin[3*e])*(a + I*a*Tan[e + f*x])^3)/(f*(Cos[f*x] + I*Sin[f*x])^3)
+ ((Cos[3*e]/3 - (I/3)*Sin[3*e])*((-2*I)*c*d*Sin[f*x] - 3*d^2*Sin[f*x])*(a
+ I*a*Tan[e + f*x])^3)/(f*(Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[e/2])*(Cos[
f*x] + I*Sin[f*x])^3) + (Cos[e + f*x]^2*(Cos[3*e]/3 - (I/3)*Sin[3*e])*(-9*c
^2*Sin[f*x] + (26*I)*c*d*Sin[f*x] + 15*d^2*Sin[f*x])*(a + I*a*Tan[e + f*x])
^3)/(f*(Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[e/2])*(Cos[f*x] + I*Sin[f*x])^
3) + (x*Cos[e + f*x]^3*(-2*c^2*Cos[e] + (4*I)*c*d*Cos[e] + 2*d^2*Cos[e] + 2
*c^2*Cos[e]^3 - (4*I)*c*d*Cos[e]^3 - 2*d^2*Cos[e]^3 + (4*I)*c^2*Sin[e] + 8*
c*d*Sin[e] - (4*I)*d^2*Sin[e] - (8*I)*c^2*Cos[e]^2*Sin[e] - 16*c*d*Cos[e]^2
*Sin[e] + (8*I)*d^2*Cos[e]^2*Sin[e] - 12*c^2*Cos[e]*Sin[e]^2 + (24*I)*c*d*C
os[e]*Sin[e]^2 + 12*d^2*Cos[e]*Sin[e]^2 + (8*I)*c^2*Sin[e]^3 + 16*c*d*Sin[e
]^3 - (8*I)*d^2*Sin[e]^3 + 2*c^2*Sin[e]*Tan[e] - (4*I)*c*d*Sin[e]*Tan[e] -
2*d^2*Sin[e]*Tan[e] + 2*c^2*Sin[e]^3*Tan[e] - (4*I)*c*d*Sin[e]^3*Tan[e] - 2
*d^2*Sin[e]^3*Tan[e] + I*(c - I*d)^2*(4*Cos[3*e] - (4*I)*Sin[3*e])*Tan[e])*
(a + I*a*Tan[e + f*x])^3)/(Cos[f*x] + I*Sin[f*x])^3
```

Maple [A]

time = 0.13, size = 176, normalized size = 1.15

method	result
derivativedivides	$\frac{a^3\left(-\frac{id^2\left(\tan^4(fx+e)\right)}{4}-\frac{2icd\left(\tan^3(fx+e)\right)}{3}-\frac{ic^2\left(\tan^2(fx+e)\right)}{2}+2id^2\left(\tan^2(fx+e)\right)-d^2\left(\tan^3(fx+e)\right)+8icd\tan(fx+e)-3\right.}{}$
default	$\frac{a^3\left(-\frac{id^2\left(\tan^4(fx+e)\right)}{4}-\frac{2icd\left(\tan^3(fx+e)\right)}{3}-\frac{ic^2\left(\tan^2(fx+e)\right)}{2}+2id^2\left(\tan^2(fx+e)\right)-d^2\left(\tan^3(fx+e)\right)+8icd\tan(fx+e)-3\right.}{}$
norman	$\left(-8ia^3cd+4a^3c^2-4a^3d^2\right)x-\frac{\left(2ia^3cd+3a^3d^2\right)\left(\tan^3(fx+e)\right)}{3f}-\frac{\left(-8ia^3cd+3a^3c^2-4a^3d^2\right)\tan(fx+e)}{f}-$
risch	$\frac{16ia^3cde}{f}-\frac{8a^3c^2e}{f}+\frac{8a^3d^2e}{f}-\frac{2ia^3\left(12c^2e^{6i(fx+e)}-36d^2e^{6i(fx+e)}-48icd\,e^{6i(fx+e)}+33c^2e^{4i(fx+e)}-69d^2e^{4i(fx+\right.}{3f\left(e^2\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3*(c+d*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a^3*(-1/4*I*d^2*tan(f*x+e)^4-2/3*I*c*d*tan(f*x+e)^3-1/2*I*c^2*tan(f*x+e
)^2+2*I*d^2*tan(f*x+e)^2-d^2*tan(f*x+e)^3+8*I*c*d*tan(f*x+e)-3*c*d*tan(f*x+
e)^2-3*c^2*tan(f*x+e)+4*d^2*tan(f*x+e)+1/2*(-4*I*d^2+4*I*c^2+8*c*d)*ln(1+ta
n(f*x+e)^2)+(-8*I*c*d-4*d^2+4*c^2)*arctan(tan(f*x+e)))
```

Maxima [A]

time = 0.49, size = 187, normalized size = 1.22

$$-\frac{3i\,a^3d^2\tan(fx+e)^4+4\left(2i\,a^3cd+3\,a^3d^2\right)\tan(fx+e)^3+6\left(i\,a^3c^2+6\,a^3cd-4i\,a^3d^2\right)\tan(fx+e)^2-48\left(a^3c^2-2i\,a^3cd-a^3d^2\right)(fx+e)+24\left(-i\,a^3c^2-2\,a^3cd+i\,a^3d^2\right)\log\left(\tan(fx+e)^2+1\right)+12\left(3\,a^3c^2-8i\,a^3cd-4\,a^3d^2\right)\tan(fx+e)}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c+d*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/12*(3*I*a^3*d^2*tan(f*x + e)^4 + 4*(2*I*a^3*c*d + 3*a^3*d^2)*tan(f*x + e
)^3 + 6*(I*a^3*c^2 + 6*a^3*c*d - 4*I*a^3*d^2)*tan(f*x + e)^2 - 48*(a^3*c^2
- 2*I*a^3*c*d - a^3*d^2)*(f*x + e) + 24*(-I*a^3*c^2 - 2*a^3*c*d + I*a^3*d^2
)*log(tan(f*x + e)^2 + 1) + 12*(3*a^3*c^2 - 8*I*a^3*c*d - 4*a^3*d^2)*tan(f*
x + e))/f
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(132) = 264$.
time = 1.12, size = 371, normalized size = 2.42

$$-\frac{2\left(9i\,a^3c^2 + 26\,a^3cd - 15i\,a^3d^2 + 12\,(i\,a^3c^2 + 4\,a^3cd - 3i\,a^3d^2)e^{(6i\,fx+6i\,e)} + 3\,(11i\,a^3c^2 + 38\,a^3cd - 23i\,a^3d^2)e^{(4i\,fx+4i\,e)} + 2\,(15i\,a^3c^2 + 46\,a^3cd - 27i\,a^3d^2)e^{(2i\,fx+2i\,e)} + 6\left(i\,a^3c^2 + 2\,a^3cd - i\,a^3d^2 + (i\,a^3c^2 + 2\,a^3cd - i\,a^3d^2)e^{(8i\,fx+8i\,e)} + 4\,(i\,a^3c^2 + 2\,a^3cd - i\,a^3d^2)e^{(6i\,fx+6i\,e)} + 6\,(i\,a^3c^2 + 2\,a^3cd - i\,a^3d^2)e^{(4i\,fx+4i\,e)} + 4\,(i\,a^3c^2 + 2\,a^3cd - i\,a^3d^2)e^{(2i\,fx+2i\,e)}\right)\log\left(e^{(2i\,fx+2i\,e)} + 1\right)\right)}{3\left(fe^{(8i\,fx+8i\,e)} + 4\,fe^{(6i\,fx+6i\,e)} + 6\,fe^{(4i\,fx+4i\,e)} + 4\,fe^{(2i\,fx+2i\,e)} + f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c+d*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(9*I*a^3*c^2 + 26*a^3*c*d - 15*I*a^3*d^2 + 12*(I*a^3*c^2 + 4*a^3*c*d -
 3*I*a^3*d^2)*e^(6*I*f*x + 6*I*e) + 3*(11*I*a^3*c^2 + 38*a^3*c*d - 23*I*a^3
*d^2)*e^(4*I*f*x + 4*I*e) + 2*(15*I*a^3*c^2 + 46*a^3*c*d - 27*I*a^3*d^2)*e^
(2*I*f*x + 2*I*e) + 6*(I*a^3*c^2 + 2*a^3*c*d - I*a^3*d^2 + (I*a^3*c^2 + 2*a
^3*c*d - I*a^3*d^2)*e^(8*I*f*x + 8*I*e) + 4*(I*a^3*c^2 + 2*a^3*c*d - I*a^3*
d^2)*e^(6*I*f*x + 6*I*e) + 6*(I*a^3*c^2 + 2*a^3*c*d - I*a^3*d^2)*e^(4*I*f*x
 + 4*I*e) + 4*(I*a^3*c^2 + 2*a^3*c*d - I*a^3*d^2)*e^(2*I*f*x + 2*I*e))*log(
e^(2*I*f*x + 2*I*e) + 1))/(f*e^(8*I*f*x + 8*I*e) + 4*f*e^(6*I*f*x + 6*I*e)
+ 6*f*e^(4*I*f*x + 4*I*e) + 4*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(129) = 258$.
time = 0.61, size = 313, normalized size = 2.05

$$-\frac{4ia^3(c-id)^2\log\left(e^{2ifx} + e^{-2ie}\right)}{f} + \frac{-18ia^3c^2 - 52a^3cd + 30ia^3d^2 + \left(-60ia^3c^2e^{2ie} - 184a^3cde^{2ie} + 108ia^3d^2e^{2ie}\right)e^{2ifx} + \left(-66ia^3c^2e^{4ie} - 228a^3cde^{4ie} + 138ia^3d^2e^{4ie}\right)e^{4ifx} + \left(-24ia^3c^2e^{6ie} - 96a^3cde^{6ie} + 72ia^3d^2e^{6ie}\right)e^{6ifx}}{3fe^{8ie}e^{8ifx} + 12fe^{6ie}e^{6ifx} + 18fe^{4ie}e^{4ifx} + 12fe^{2ie}e^{2ifx} + 3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3*(c+d*tan(f*x+e))**2,x)
```

```
[Out] -4*I*a**3*(c - I*d)**2*log(exp(2*I*f*x) + exp(-2*I*e))/f + (-18*I*a**3*c**2
 - 52*a**3*c*d + 30*I*a**3*d**2 + (-60*I*a**3*c**2*exp(2*I*e) - 184*a**3*c*
d*exp(2*I*e) + 108*I*a**3*d**2*exp(2*I*e))*exp(2*I*f*x) + (-66*I*a**3*c**2*
exp(4*I*e) - 228*a**3*c*d*exp(4*I*e) + 138*I*a**3*d**2*exp(4*I*e))*exp(4*I*
f*x) + (-24*I*a**3*c**2*exp(6*I*e) - 96*a**3*c*d*exp(6*I*e) + 72*I*a**3*d**
2*exp(6*I*e))*exp(6*I*f*x))/(3*f*exp(8*I*e)*exp(8*I*f*x) + 12*f*exp(6*I*e)*
exp(6*I*f*x) + 18*f*exp(4*I*e)*exp(4*I*f*x) + 12*f*exp(2*I*e)*exp(2*I*f*x)
+ 3*f)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 670 vs. $2(132) = 264$.

time = 0.76, size = 670, normalized size = 4.38

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c+d*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -2/3*(6*I*a^3*c^2*e^(8*I*f*x + 8*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 12*a^3
*c*d*e^(8*I*f*x + 8*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) - 6*I*a^3*d^2*e^(8*I*
f*x + 8*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 24*I*a^3*c^2*e^(6*I*f*x + 6*I*e
)*log(e^(2*I*f*x + 2*I*e) + 1) + 48*a^3*c*d*e^(6*I*f*x + 6*I*e)*log(e^(2*I*
f*x + 2*I*e) + 1) - 24*I*a^3*d^2*e^(6*I*f*x + 6*I*e)*log(e^(2*I*f*x + 2*I*e
) + 1) + 36*I*a^3*c^2*e^(4*I*f*x + 4*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 72
*a^3*c*d*e^(4*I*f*x + 4*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) - 36*I*a^3*d^2*e^
(4*I*f*x + 4*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 24*I*a^3*c^2*e^(2*I*f*x +
2*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 48*a^3*c*d*e^(2*I*f*x + 2*I*e)*log(e^
(2*I*f*x + 2*I*e) + 1) - 24*I*a^3*d^2*e^(2*I*f*x + 2*I*e)*log(e^(2*I*f*x +
2*I*e) + 1) + 12*I*a^3*c^2*e^(6*I*f*x + 6*I*e) + 48*a^3*c*d*e^(6*I*f*x + 6*
I*e) - 36*I*a^3*d^2*e^(6*I*f*x + 6*I*e) + 33*I*a^3*c^2*e^(4*I*f*x + 4*I*e)
+ 114*a^3*c*d*e^(4*I*f*x + 4*I*e) - 69*I*a^3*d^2*e^(4*I*f*x + 4*I*e) + 30*I
*a^3*c^2*e^(2*I*f*x + 2*I*e) + 92*a^3*c*d*e^(2*I*f*x + 2*I*e) - 54*I*a^3*d^
2*e^(2*I*f*x + 2*I*e) + 6*I*a^3*c^2*log(e^(2*I*f*x + 2*I*e) + 1) + 12*a^3*c
*d*log(e^(2*I*f*x + 2*I*e) + 1) - 6*I*a^3*d^2*log(e^(2*I*f*x + 2*I*e) + 1)
+ 9*I*a^3*c^2 + 26*a^3*c*d - 15*I*a^3*d^2)/(f*e^(8*I*f*x + 8*I*e) + 4*f*e^(
6*I*f*x + 6*I*e) + 6*f*e^(4*I*f*x + 4*I*e) + 4*f*e^(2*I*f*x + 2*I*e) + f)
```

Mupad [B]
time = 4.92, size = 217, normalized size = 1.42

$$\frac{\tan(e+fx)^2\left(\frac{a^3\,d^2\,1\mathrm{i}}{2}-\frac{a^3\,(c^2\,1\mathrm{i}+4\,c\,d-d^2\,1\mathrm{i})}{2}+a^3\,d\,(d+c\,1\mathrm{i})\,1\mathrm{i}\right)}{f}-\frac{\tan(e+fx)^3\left(\frac{a^3\,d^2}{3}+\frac{2\,a^3\,d\,(d+c\,1\mathrm{i})}{3}\right)}{f}+\frac{\ln(\tan(e+fx)+1\mathrm{i})\,(a^3\,c^2\,4\mathrm{i}+8\,a^3\,c\,d-a^3\,d^2\,4\mathrm{i})}{f}+\frac{\tan(e+fx)\,(a^3\,d^2+a^3\,(c^2\,1\mathrm{i}+4\,c\,d-d^2\,1\mathrm{i})\,1\mathrm{i}-2\,a^3\,c\,(c-d\,1\mathrm{i})+2\,a^3\,d\,(d+c\,1\mathrm{i}))}{f}-\frac{a^3\,d^2\tan(e+fx)^4\,1\mathrm{i}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^3*(c + d*tan(e + f*x))^2,x)
```

```
[Out] (tan(e + f*x)^2*((a^3*d^2*1i)/2 - (a^3*(4*c*d + c^2*1i - d^2*1i))/2 + a^3*d
*(c*1i + d)*1i))/f - (tan(e + f*x)^3*((a^3*d^2)/3 + (2*a^3*d*(c*1i + d))/3)
)/f + (log(tan(e + f*x) + 1i)*(a^3*c^2*4i - a^3*d^2*4i + 8*a^3*c*d))/f + (t
an(e + f*x)*(a^3*d^2 + a^3*(4*c*d + c^2*1i - d^2*1i)*1i - 2*a^3*c*(c - d*1i
) + 2*a^3*d*(c*1i + d)))/f - (a^3*d^2*tan(e + f*x)^4*1i)/(4*f)
```

3.1072 $\int (a+ia\tan(e+fx))^2(c+d\tan(e+fx))^2\,dx$

Optimal. Leaf size=116

$$2a^2(c-id)^2x-\frac{2ia^2(c-id)^2\log(\cos(e+fx))}{f}-\frac{a^2(c-id)^2\tan(e+fx)}{f}+\frac{cd(a+ia\tan(e+fx))^2}{f}-\frac{id^2(a+ia$$

```
[Out] 2*a^2*(c-I*d)^2*x-2*I*a^2*(c-I*d)^2*ln(cos(f*x+e))/f-a^2*(c-I*d)^2*tan(f*x+
e)/f+c*d*(a+I*a*tan(f*x+e))^2/f-1/3*I*d^2*(a+I*a*tan(f*x+e))^3/a/f
```

Rubi [A]
time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3624, 3608, 3558, 3556}

$$-\frac{a^2(c-id)^2\tan(e+fx)}{f}-\frac{2ia^2(c-id)^2\log(\cos(e+fx))}{f}+2a^2x(c-id)^2+\frac{cd(a+ia\tan(e+fx))^2}{f}-\frac{id^2(a+ia\tan(e+fx))^3}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^2,x]
```

```
[Out] 2*a^2*(c - I*d)^2*x - ((2*I)*a^2*(c - I*d)^2*Log[Cos[e + f*x]])/f - (a^2*(c
 - I*d)^2*Tan[e + f*x])/f + (c*d*(a + I*a*Tan[e + f*x])^2)/f - ((I/3)*d^2*(
a + I*a*Tan[e + f*x])^3)/(a*f)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
```

```
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\int (a+ia\tan(e+fx))^2(c+d\tan(e+fx))^2\,dx &= -\frac{id^2(a+ia\tan(e+fx))^3}{3af} + \int (a+ia\tan(e+fx))^2\left(c^2\right. \\
&= \frac{cd(a+ia\tan(e+fx))^2}{f} - \frac{id^2(a+ia\tan(e+fx))^3}{3af} + (c \\
&= 2a^2(c-id)^2x - \frac{a^2(c-id)^2\tan(e+fx)}{f} + \frac{cd(a+ia\tan(}{f} \\
&= 2a^2(c-id)^2x - \frac{2ia^2(c-id)^2\log(\cos(e+fx))}{f} - \frac{a^2(c-}{}
\end{aligned}
$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 261 vs. 2(116) = 232.
time = 3.19, size = 261, normalized size = 2.25

$$
\frac{\left((c-id)^2\cos^2(e+fx)\log\left(\cos^2(e+fx)\right)(-i\cos(2e)-\sin(2e))+4(c-id)^2fx\cos^2(e+fx)(\cos(2e)-i\sin(2e))-2(c-id)^2\text{ArcTan}(\tan(3e+fx))\cos^2(e+fx)(\cos(2e)-i\sin(2e))-\frac{1}{3}(3c^2-12icd-7d^2)\cos(e+fx)\sec(e)(\cos(2e)-i\sin(2e))\sin(fx)-\frac{1}{3}d^2\sec(e)\sec(e+fx)(\cos(2e)-i\sin(2e))\sin(fx)-\frac{1}{3}d(\cos(2e)-i\sin(2e))(3c-3id+d\tan(e))\right)(a+ia\tan(e+fx))^2}{f(\cos(fx)+i\sin(fx))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^2,x]
```

```
[Out] (((c - I*d)^2*Cos[e + f*x]^2*Log[Cos[e + f*x]^2]*((-I)*Cos[2*e] - Sin[2*e])
 + 4*(c - I*d)^2*f*x*Cos[e + f*x]^2*(Cos[2*e] - I*Sin[2*e]) - 2*(c - I*d)^2
*ArcTan[Tan[3*e + f*x]]*Cos[e + f*x]^2*(Cos[2*e] - I*Sin[2*e]) - ((3*c^2 -
(12*I)*c*d - 7*d^2)*Cos[e + f*x]*Sec[e]*(Cos[2*e] - I*Sin[2*e])*Sin[f*x])/3
 - (d^2*Sec[e]*Sec[e + f*x]*(Cos[2*e] - I*Sin[2*e])*Sin[f*x])/3 - (d*(Cos[2
*e] - I*Sin[2*e])*(3*c - (3*I)*d + d*Tan[e]))/3)*(a + I*a*Tan[e + f*x])^2)/
(f*(Cos[f*x] + I*Sin[f*x])^2)
```

Maple [A]
time = 0.14, size = 135, normalized size = 1.16

method	result
derivativedivides	$\frac{a^2\left(id^2\left(\tan^2(fx+e)\right)-\frac{d^2\left(\tan^3(fx+e)\right)}{3}+4icd\tan(fx+e)-cd\left(\tan^2(fx+e)\right)-c^2\tan(fx+e)+2d^2\tan(fx+e)+\frac{\left(2ic^2-2id^2\right.}{}\right.}{f}$
default	$\frac{a^2\left(id^2\left(\tan^2(fx+e)\right)-\frac{d^2\left(\tan^3(fx+e)\right)}{3}+4icd\tan(fx+e)-cd\left(\tan^2(fx+e)\right)-c^2\tan(fx+e)+2d^2\tan(fx+e)+\frac{\left(2ic^2-2id^2\right.}{}\right.}{f}$

norman	$\left(-4ia^2cd+2a^2c^2-2a^2d^2\right)x-\frac{\left(-ia^2d^2+a^2cd\right)\left(\tan^2(fx+e)\right)}{f}-\frac{\left(-4ia^2cd+a^2c^2-2a^2d^2\right)\tan(fx+e)}{f}-$
risch	$\frac{8ia^2cde}{f}-\frac{4a^2c^2e}{f}+\frac{4a^2d^2e}{f}-\frac{2ia^2\left(3c^2e^{4i(fx+e)}-15d^2e^{4i(fx+e)}-18icd\,e^{4i(fx+e)}+6c^2e^{2i(fx+e)}-18d^2e^{2i(fx+e)}-3\right.}{3f\left(e^{2i(fx+e)}+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2*(c+d*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a^2*(I*d^2*tan(f*x+e)^2-1/3*d^2*tan(f*x+e)^3+4*I*c*d*tan(f*x+e)-c*d*tan
(f*x+e)^2-c^2*tan(f*x+e)+2*d^2*tan(f*x+e)+1/2*(4*c*d-2*I*d^2+2*I*c^2)*ln(1+
tan(f*x+e)^2)+(-2*d^2+2*c^2-4*I*c*d)*arctan(tan(f*x+e)))
```

Maxima [A]
time = 0.50, size = 149, normalized size = 1.28

$$-\frac{a^2d^2\tan(fx+e)^3+3\,(a^2cd-i\,a^2d^2)\tan(fx+e)^2-6\,(a^2c^2-2i\,a^2cd-a^2d^2)(fx+e)-3\,(i\,a^2c^2+2\,a^2cd-i\,a^2d^2)\log\left(\tan(fx+e)^2+1\right)+3\,(a^2c^2-4i\,a^2cd-2\,a^2d^2)\tan(fx+e)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c+d*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(a^2*d^2*tan(f*x + e)^3 + 3*(a^2*c*d - I*a^2*d^2)*tan(f*x + e)^2 - 6*(
a^2*c^2 - 2*I*a^2*c*d - a^2*d^2)*(f*x + e) - 3*(I*a^2*c^2 + 2*a^2*c*d - I*a
^2*d^2)*log(tan(f*x + e)^2 + 1) + 3*(a^2*c^2 - 4*I*a^2*c*d - 2*a^2*d^2)*tan
(f*x + e))/f
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(104) = 208.
time = 1.13, size = 286, normalized size = 2.47

$$-\frac{2\left(3i\,a^2c^2+12\,a^2cd-7i\,a^2d^2+3\,(i\,a^2c^2+6\,a^2cd-5i\,a^2d^2)e^{(4i\,fx+4i\,e)}+6\,(i\,a^2c^2+5\,a^2cd-3i\,a^2d^2)e^{(2i\,fx+2i\,e)}+3\left(i\,a^2c^2+2\,a^2cd-i\,a^2d^2+(i\,a^2c^2+2\,a^2cd-i\,a^2d^2)e^{(6i\,fx+6i\,e)}+3\,(i\,a^2c^2+2\,a^2cd-i\,a^2d^2)e^{(4i\,fx+4i\,e)}+3\,(i\,a^2c^2+2\,a^2cd-i\,a^2d^2)e^{(2i\,fx+2i\,e)}\right)\log\left(e^{(2i\,fx+2i\,e)}+1\right)\right)}{3\left(fe^{(6i\,fx+6i\,e)}+3\,fe^{(4i\,fx+4i\,e)}+3\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c+d*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(3*I*a^2*c^2 + 12*a^2*c*d - 7*I*a^2*d^2 + 3*(I*a^2*c^2 + 6*a^2*c*d - 5
*I*a^2*d^2)*e^(4*I*f*x + 4*I*e) + 6*(I*a^2*c^2 + 5*a^2*c*d - 3*I*a^2*d^2)*e
^(2*I*f*x + 2*I*e) + 3*(I*a^2*c^2 + 2*a^2*c*d - I*a^2*d^2 + (I*a^2*c^2 + 2*
a^2*c*d - I*a^2*d^2)*e^(6*I*f*x + 6*I*e) + 3*(I*a^2*c^2 + 2*a^2*c*d - I*a^2
*d^2)*e^(4*I*f*x + 4*I*e) + 3*(I*a^2*c^2 + 2*a^2*c*d - I*a^2*d^2)*e^(2*I*f*
x + 2*I*e))*log(e^(2*I*f*x + 2*I*e) + 1))/(f*e^(6*I*f*x + 6*I*e) + 3*f*e^(4
*I*f*x + 4*I*e) + 3*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(97) = 194.
time = 0.44, size = 236, normalized size = 2.03

$$-\frac{2ia^2(c-id)^2\log\left(e^{2ifx}+e^{-2ie}\right)}{f}+\frac{-6ia^2c^2-24a^2cd+14ia^2d^2+\left(-12ia^2c^2e^{2ie}-60a^2cde^{2ie}+36ia^2d^2e^{2ie}\right)e^{2ifx}+\left(-6ia^2c^2e^{4ie}-36a^2cde^{4ie}+30ia^2d^2e^{4ie}\right)e^{4ifx}}{3fe^{6ie}e^{6ifx}+9fe^{4ie}e^{4ifx}+9fe^{2ie}e^{2ifx}+3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2*(c+d*tan(f*x+e))**2,x)
```

```
[Out] -2*I*a**2*(c - I*d)**2*log(exp(2*I*f*x) + exp(-2*I*e))/f + (-6*I*a**2*c**2
- 24*a**2*c*d + 14*I*a**2*d**2 + (-12*I*a**2*c**2*exp(2*I*e) - 60*a**2*c*d*
exp(2*I*e) + 36*I*a**2*d**2*exp(2*I*e))*exp(2*I*f*x) + (-6*I*a**2*c**2*exp(
4*I*e) - 36*a**2*c*d*exp(4*I*e) + 30*I*a**2*d**2*exp(4*I*e))*exp(4*I*f*x))/
(3*f*exp(6*I*e)*exp(6*I*f*x) + 9*f*exp(4*I*e)*exp(4*I*f*x) + 9*f*exp(2*I*e)
*exp(2*I*f*x) + 3*f)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 512 vs. 2(104) = 208.
time = 0.66, size = 512, normalized size = 4.41

$$-\frac{2\left(3i\,a^2c^2e^{(6i\,fx+6i\,e)}\log\left(e^{(2i\,fx+2i\,e)}+1\right)+6\,a^2cde^{(6i\,fx+6i\,e)}\log\left(e^{(2i\,fx+2i\,e)}+1\right)-3i\,a^2d^2e^{(6i\,fx+6i\,e)}\log\left(e^{(2i\,fx+2i\,e)}+1\right)+9i\,a^2c^2e^{(4i\,fx+4i\,e)}\log\left(e^{(2i\,fx+2i\,e)}+1\right)+18\,a^2cde^{(4i\,fx+4i\,e)}\log\left(e^{(2i\,fx+2i\,e)}+1\right)-9i\,a^2d^2e^{(4i\,fx+4i\,e)}\log\left(e^{(2i\,fx+2i\,e)}+1\right)+9i\,a^2c^2e^{(2i\,fx+2i\,e)}\log\left(e^{(2i\,fx+2i\,e)}+1\right)+18\,a^2cde^{(2i\,fx+2i\,e)}\log\left(e^{(2i\,fx+2i\,e)}+1\right)-9i\,a^2d^2e^{(2i\,fx+2i\,e)}\log\left(e^{(2i\,fx+2i\,e)}+1\right)+3i\,a^2c^2e^{(4i\,fx+4i\,e)}+18\,a^2cde^{(4i\,fx+4i\,e)}-15i\,a^2d^2e^{(4i\,fx+4i\,e)}+6i\,a^2c^2e^{(2i\,fx+2i\,e)}+30\,a^2cde^{(2i\,fx+2i\,e)}-18i\,a^2d^2e^{(2i\,fx+2i\,e)}+3i\,a^2c^2\log\left(e^{(2i\,fx+2i\,e)}+1\right)+6\,a^2cd\log\left(e^{(2i\,fx+2i\,e)}+1\right)-3i\,a^2d^2\log\left(e^{(2i\,fx+2i\,e)}+1\right)+3i\,a^2c^2+12\,a^2cd-7i\,a^2d^2\right)}{3\left(fe^{(6i\,fx+6i\,e)}+3\,fe^{(4i\,fx+4i\,e)}+3\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c+d*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*I*a^2*c^2*e^(6*I*f*x + 6*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 6*a^2*
c*d*e^(6*I*f*x + 6*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) - 3*I*a^2*d^2*e^(6*I*f
*x + 6*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 9*I*a^2*c^2*e^(4*I*f*x + 4*I*e)*
log(e^(2*I*f*x + 2*I*e) + 1) + 18*a^2*c*d*e^(4*I*f*x + 4*I*e)*log(e^(2*I*f*
x + 2*I*e) + 1) - 9*I*a^2*d^2*e^(4*I*f*x + 4*I*e)*log(e^(2*I*f*x + 2*I*e) +
 1) + 9*I*a^2*c^2*e^(2*I*f*x + 2*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 18*a^2
*c*d*e^(2*I*f*x + 2*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) - 9*I*a^2*d^2*e^(2*I*
f*x + 2*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 3*I*a^2*c^2*e^(4*I*f*x + 4*I*e)
 + 18*a^2*c*d*e^(4*I*f*x + 4*I*e) - 15*I*a^2*d^2*e^(4*I*f*x + 4*I*e) + 6*I*
a^2*c^2*e^(2*I*f*x + 2*I*e) + 30*a^2*c*d*e^(2*I*f*x + 2*I*e) - 18*I*a^2*d^2
*e^(2*I*f*x + 2*I*e) + 3*I*a^2*c^2*log(e^(2*I*f*x + 2*I*e) + 1) + 6*a^2*c*d
*log(e^(2*I*f*x + 2*I*e) + 1) - 3*I*a^2*d^2*log(e^(2*I*f*x + 2*I*e) + 1) +
3*I*a^2*c^2 + 12*a^2*c*d - 7*I*a^2*d^2)/(f*e^(6*I*f*x + 6*I*e) + 3*f*e^(4*I
*f*x + 4*I*e) + 3*f*e^(2*I*f*x + 2*I*e) + f)
```

Mupad [B]
time = 4.92, size = 139, normalized size = 1.20

$$\frac{\tan(e+f\,x)^2\,\left(\frac{a^2\,d^2\,1\mathrm{i}}{2}+\frac{a^2\,d\,(d+c\,2\mathrm{i})\,1\mathrm{i}}{2}\right)}{f}+\frac{\tan(e+f\,x)\,\left(a^2\,d^2+a^2\,d\,(d+c\,2\mathrm{i})+a^2\,c\,(2\,d+c\,1\mathrm{i})\,1\mathrm{i}\right)}{f}+\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(a^2\,c^2\,2\mathrm{i}+4\,a^2\,c\,d-a^2\,d^2\,2\mathrm{i}\right)}{f}-\frac{a^2\,d^2\,\tan(e+f\,x)^3}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^2*(c + d*tan(e + f*x))^2,x)
```

```
[Out] (tan(e + f*x)^2*((a^2*d^2*1i)/2 + (a^2*d*(c*2i + d)*1i)/2))/f + (tan(e + f*
x)*(a^2*d^2 + a^2*d*(c*2i + d) + a^2*c*(c*1i + 2*d)*1i))/f + (log(tan(e + f
*x) + 1i)*(a^2*c^2*2i - a^2*d^2*2i + 4*a^2*c*d))/f - (a^2*d^2*tan(e + f*x)^
3)/(3*f)
```

3.1073 $\int (a+ia\tan(e+fx))(c+d\tan(e+fx))^2\,dx$

Optimal. Leaf size=78

$$a(c-id)^2x-\frac{ia(c-id)^2\log(\cos(e+fx))}{f}+\frac{ad(ic+d)\tan(e+fx)}{f}+\frac{ia(c+d\tan(e+fx))^2}{2f}$$

```
[Out] a*(c-I*d)^2*x-I*a*(c-I*d)^2*ln(cos(f*x+e))/f+a*d*(I*c+d)*tan(f*x+e)/f+1/2*I
*a*(c+d*tan(f*x+e))^2/f
```

Rubi [A]
time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3609, 3606, 3556}

$$\frac{ia(c+d\tan(e+fx))^2}{2f}+\frac{ad(d+ic)\tan(e+fx)}{f}-\frac{ia(c-id)^2\log(\cos(e+fx))}{f}+ax(c-id)^2$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])*(c + d*Tan[e + f*x])^2,x]
```

```
[Out] a*(c - I*d)^2*x - (I*a*(c - I*d)^2*Log[Cos[e + f*x]])/f + (a*d*(I*c + d)*Ta
n[e + f*x])/f + ((I/2)*a*(c + d*Tan[e + f*x])^2)/f
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\int (a+ia\tan(e+fx))(c+d\tan(e+fx))^2\,dx = \frac{ia(c+d\tan(e+fx))^2}{2f} + \int (c+d\tan(e+fx))(a(c-id)$$
$$= a(c-id)^2x + \frac{ad(ic+d)\tan(e+fx)}{f} + \frac{ia(c+d\tan(e+$$
$$= a(c-id)^2x - \frac{ia(c-id)^2\log(\cos(e+fx))}{f} + \frac{ad(ic+d)\text{ta}}{f}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 175 vs. $2(78) = 156$.
time = 1.48, size = 175, normalized size = 2.24

$$\frac{(\cos(fx)-i\sin(fx))\left(4(c-id)^2fx\cos(e+fx)(\cos(e)-i\sin(e))-2(c-id)^2\text{ArcTan}(\tan(2e+fx))\cos(e+fx)(\cos(e)-i\sin(e))-i(c-id)^2\cos(e+fx)\log\left(\cos^2(e+fx)\right)(\cos(e)-i\sin(e))+d^2\sec(e+fx)(i\cos(e)+\sin(e))+2(2c-id)d\sin(fx)(i+\tan(e))\right)(a+ia\tan(e+fx))}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])*(c + d*Tan[e + f*x])^2,x]
```

```
[Out] ((Cos[f*x] - I*Sin[f*x])*(4*(c - I*d)^2*f*x*Cos[e + f*x]*(Cos[e] - I*Sin[e]
) - 2*(c - I*d)^2*ArcTan[Tan[2*e + f*x]]*Cos[e + f*x]*(Cos[e] - I*Sin[e]) -
 I*(c - I*d)^2*Cos[e + f*x]*Log[Cos[e + f*x]^2]*(Cos[e] - I*Sin[e]) + d^2*S
ec[e + f*x]*(I*Cos[e] + Sin[e]) + 2*(2*c - I*d)*d*Sin[f*x]*(I + Tan[e]))*(a
 + I*a*Tan[e + f*x]))/(2*f)
```

Maple [A]
time = 0.11, size = 94, normalized size = 1.21

method	result
derivativedivides	$\frac{a\left(\frac{id^2\left(\tan^2(fx+e)\right)}{2}+2icd\tan(fx+e)+d^2\tan(fx+e)+\frac{\left(ic^2-id^2+2cd\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(-2icd+c^2-d^2\right)\arctan(\tan(f\right.}{f}$
default	$\frac{a\left(\frac{id^2\left(\tan^2(fx+e)\right)}{2}+2icd\tan(fx+e)+d^2\tan(fx+e)+\frac{\left(ic^2-id^2+2cd\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(-2icd+c^2-d^2\right)\arctan(\tan(f\right.}{f}$
norman	$\left(-2iacd+a\,c^2-a\,d^2\right)x+\frac{\left(2iacd+a\,d^2\right)\tan(fx+e)}{f}+\frac{ia\,d^2\left(\tan^2(fx+e)\right)}{2f}+\frac{ia\left(-2icd+c^2-d^2\right)\ln(1+\tan}{2f}$
risch	$\frac{4iacde}{f}-\frac{2a\,c^2e}{f}+\frac{2a\,d^2e}{f}-\frac{2ad\left(-2ie^{2i(fx+e)}d+2\,e^{2i(fx+e)}c-id+2c\right)}{f\left(e^{2i(fx+e)}+1\right)^2}-\frac{2a\ln\left(e^{2i(fx+e)}+1\right)cd}{f}-\frac{ia\ln\left(e^{2i(fx+\right.}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))*(c+d*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a*(1/2*I*d^2*tan(f*x+e)^2+2*I*c*d*tan(f*x+e)+d^2*tan(f*x+e)+1/2*(-I*d^2
+I*c^2+2*c*d)*ln(1+tan(f*x+e)^2)+(-2*I*c*d+c^2-d^2)*arctan(tan(f*x+e)))
```

Maxima [A]
time = 0.51, size = 98, normalized size = 1.26

$$-\frac{-i\,ad^2\tan\left(fx+e\right)^2-2\left(ac^2-2i\,acd-ad^2\right)\left(fx+e\right)+\left(-i\,ac^2-2\,acd+i\,ad^2\right)\log\left(\tan\left(fx+e\right)^2+1\right)+2\left(-2i\,acd-ad^2\right)\tan\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c+d*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(-I*a*d^2*tan(f*x + e)^2 - 2*(a*c^2 - 2*I*a*c*d - a*d^2)*(f*x + e) + (
-I*a*c^2 - 2*a*c*d + I*a*d^2)*log(tan(f*x + e)^2 + 1) + 2*(-2*I*a*c*d - a*d
^2)*tan(f*x + e))/f
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(69) = 138.
time = 1.09, size = 158, normalized size = 2.03

$$-\frac{4\,acd-2i\,ad^2+4\left(acd-i\,ad^2\right)e^{\left(2i\,fx+2i\,e\right)}-\left(-i\,ac^2-2\,acd+i\,ad^2+\left(-i\,ac^2-2\,acd+i\,ad^2\right)e^{\left(4i\,fx+4i\,e\right)}-2\left(i\,ac^2+2\,acd-i\,ad^2\right)e^{\left(2i\,fx+2i\,e\right)}\right)\log\left(e^{\left(2i\,fx+2i\,e\right)}+1\right)}{fe^{\left(4i\,fx+4i\,e\right)}+2\,fe^{\left(2i\,fx+2i\,e\right)}+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c+d*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -(4*a*c*d - 2*I*a*d^2 + 4*(a*c*d - I*a*d^2)*e^(2*I*f*x + 2*I*e) - (-I*a*c^2
 - 2*a*c*d + I*a*d^2 + (-I*a*c^2 - 2*a*c*d + I*a*d^2)*e^(4*I*f*x + 4*I*e) -
 2*(I*a*c^2 + 2*a*c*d - I*a*d^2)*e^(2*I*f*x + 2*I*e))*log(e^(2*I*f*x + 2*I*
e) + 1))/(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [A]
time = 0.36, size = 119, normalized size = 1.53

$$-\frac{ia(c-id)^2\log\left(e^{2ifx}+e^{-2ie}\right)}{f}+\frac{-4acd+2iad^2+\left(-4acde^{2ie}+4iad^2e^{2ie}\right)e^{2ifx}}{fe^{4ie}e^{4ifx}+2fe^{2ie}e^{2ifx}+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c+d*tan(f*x+e))**2,x)
```

```
[Out] -I*a*(c - I*d)**2*log(exp(2*I*f*x) + exp(-2*I*e))/f + (-4*a*c*d + 2*I*a*d**
2 + (-4*a*c*d*exp(2*I*e) + 4*I*a*d**2*exp(2*I*e))*exp(2*I*f*x))/(f*exp(4*I*
e)*exp(4*I*f*x) + 2*f*exp(2*I*e)*exp(2*I*f*x) + f)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(69) = 138.
time = 0.56, size = 301, normalized size = 3.86

$$\frac{-i\,ac^2e^{\left(4i\,fx+4i\,e\right)}\log\left(e^{\left(2i\,fx+2i\,e\right)}+1\right)-2\,acde^{\left(4i\,fx+4i\,e\right)}\log\left(e^{\left(2i\,fx+2i\,e\right)}+1\right)+i\,ad^2e^{\left(4i\,fx+4i\,e\right)}\log\left(e^{\left(2i\,fx+2i\,e\right)}+1\right)-2i\,ac^2e^{\left(2i\,fx+2i\,e\right)}\log\left(e^{\left(2i\,fx+2i\,e\right)}+1\right)-4\,acde^{\left(2i\,fx+2i\,e\right)}\log\left(e^{\left(2i\,fx+2i\,e\right)}+1\right)+2i\,ad^2e^{\left(2i\,fx+2i\,e\right)}\log\left(e^{\left(2i\,fx+2i\,e\right)}+1\right)-4\,acde^{\left(2i\,fx+2i\,e\right)}+4i\,ad^2e^{\left(2i\,fx+2i\,e\right)}-i\,ac^2\log\left(e^{\left(2i\,fx+2i\,e\right)}+1\right)-2\,acd\log\left(e^{\left(2i\,fx+2i\,e\right)}+1\right)+i\,ad^2\log\left(e^{\left(2i\,fx+2i\,e\right)}+1\right)-4\,acd+2i\,ad^2}{fe^{\left(4i\,fx+4i\,e\right)}+2\,fe^{\left(2i\,fx+2i\,e\right)}+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c+d*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] (-I*a*c^2*e^(4*I*f*x + 4*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) - 2*a*c*d*e^(4*I
*f*x + 4*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + I*a*d^2*e^(4*I*f*x + 4*I*e)*lo
g(e^(2*I*f*x + 2*I*e) + 1) - 2*I*a*c^2*e^(2*I*f*x + 2*I*e)*log(e^(2*I*f*x +
 2*I*e) + 1) - 4*a*c*d*e^(2*I*f*x + 2*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 2
*I*a*d^2*e^(2*I*f*x + 2*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) - 4*a*c*d*e^(2*I*
f*x + 2*I*e) + 4*I*a*d^2*e^(2*I*f*x + 2*I*e) - I*a*c^2*log(e^(2*I*f*x + 2*I
*e) + 1) - 2*a*c*d*log(e^(2*I*f*x + 2*I*e) + 1) + I*a*d^2*log(e^(2*I*f*x +
2*I*e) + 1) - 4*a*c*d + 2*I*a*d^2)/(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x
+ 2*I*e) + f)
```

Mupad [B]
time = 5.11, size = 75, normalized size = 0.96

$$\frac{\tan(e+f\,x)\,\left(a\,d^2+2\mathrm{i}\,a\,c\,d\right)}{f}+\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(1\mathrm{i}\,a\,c^2+2\,a\,c\,d-1\mathrm{i}\,a\,d^2\right)}{f}+\frac{a\,d^2\,{\tan(e+f\,x)}^2\,1\mathrm{i}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)*(c + d*tan(e + f*x))^2,x)
```

```
[Out] (tan(e + f*x)*(a*d^2 + a*c*d*2i))/f + (log(tan(e + f*x) + 1i)*(a*c^2*1i - a
*d^2*1i + 2*a*c*d))/f + (a*d^2*tan(e + f*x)^2*1i)/(2*f)
```

3.1074 $\int \frac{(c+d\tan(e+fx))^2}{a+ia\tan(e+fx)}\,dx$

Optimal. Leaf size=75

$$\frac{(c^2-2icd+d^2)\,x}{2a}+\frac{id^2\log(\cos(e+fx))}{af}+\frac{i(c+id)^2}{2f(a+ia\tan(e+fx))}$$

```
[Out] 1/2*(c^2-2*I*c*d+d^2)*x/a+I*d^2*ln(cos(f*x+e))/a/f+1/2*I*(c+I*d)^2/f/(a+I*a
*tan(f*x+e))
```

Rubi [A]

time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}=0.071$, Rules used = {3621, 3556}

$$\frac{x(c^2-2icd+d^2)}{2a}+\frac{i(c+id)^2}{2f(a+ia\tan(e+fx))}+\frac{id^2\log(\cos(e+fx))}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^2/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] ((c^2 - (2*I)*c*d + d^2)*x)/(2*a) + (I*d^2*Log[Cos[e + f*x]])/(a*f) + ((I/2
)*(c + I*d)^2)/(f*(a + I*a*Tan[e + f*x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3621

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(a*c + b*d)^2*((a + b*Tan[e + f*x])^
m/(2*a^3*f*m)), x] + Dist[1/(2*a^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[
a*c^2 - 2*b*c*d + a*d^2 - 2*b*d^2*Tan[e + f*x], x], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LeQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+d\tan(e+fx))^2}{a+ia\tan(e+fx)}\,dx &= \frac{i(c+id)^2}{2f(a+ia\tan(e+fx))}+\frac{\int \left(a(c^2-2icd+d^2)-2iad^2\tan(e+fx)\right)\,dx}{2a^2}\\
&= \frac{(c^2-2icd+d^2)\,x}{2a}+\frac{i(c+id)^2}{2f(a+ia\tan(e+fx))}-\frac{(id^2)\int \tan(e+fx)\,dx}{a}\\
&= \frac{(c^2-2icd+d^2)\,x}{2a}+\frac{id^2\log(\cos(e+fx))}{af}+\frac{i(c+id)^2}{2f(a+ia\tan(e+fx))}
\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 155 vs. $2(75) = 150$.
time = 1.01, size = 155, normalized size = 2.07

$$\frac{c^2 + 2icd - d^2 - 2ic^2fx - 4cdfx + 2id^2fx + 2d^2\log\left(\cos^2(e+fx)\right) + \left(d^2(i - 2fx) + c^2(-i + 2fx) + 2c(d - 2idfx) + 2id^2\log\left(\cos^2(e+fx)\right)\right)\tan(e+fx) + 4d^2\text{ArcTan}(\tan(fx))(-i + \tan(e+fx))}{4af(-i + \tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^2/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (c^2 + (2*I)*c*d - d^2 - (2*I)*c^2*f*x - 4*c*d*f*x + (2*I)*d^2*f*x + 2*d^2*
Log[Cos[e + f*x]^2] + (d^2*(I - 2*f*x) + c^2*(-I + 2*f*x) + 2*c*(d - (2*I)*
d*f*x) + (2*I)*d^2*Log[Cos[e + f*x]^2])*Tan[e + f*x] + 4*d^2*ArcTan[Tan[f*x
]]*(-I + Tan[e + f*x]))/(4*a*f*(-I + Tan[e + f*x]))
```

Maple [A]
time = 0.24, size = 93, normalized size = 1.24

method	result	s
derivativedivides	$\frac{\left(-\frac{1}{2}cd-\frac{1}{4}ic^2-\frac{3}{4}id^2\right)\ln(\tan(fx+e)-i)-\frac{-icd-\frac{1}{2}c^2+\frac{1}{2}d^2}{\tan(fx+e)-i}-\frac{i\left(2icd-c^2+d^2\right)\ln(\tan(fx+e)+i)}{4}}{fa}$	9
default	$\frac{\left(-\frac{1}{2}cd-\frac{1}{4}ic^2-\frac{3}{4}id^2\right)\ln(\tan(fx+e)-i)-\frac{-icd-\frac{1}{2}c^2+\frac{1}{2}d^2}{\tan(fx+e)-i}-\frac{i\left(2icd-c^2+d^2\right)\ln(\tan(fx+e)+i)}{4}}{fa}$	9
risch	$-\frac{ixcd}{a}+\frac{c^2x}{2a}+\frac{3x\,d^2}{2a}-\frac{\mathrm{e}^{-2i(fx+e)}cd}{2af}+\frac{i\mathrm{e}^{-2i(fx+e)}c^2}{4af}-\frac{i\mathrm{e}^{-2i(fx+e)}d^2}{4af}+\frac{2d^2e}{af}+\frac{id^2\ln\left(\mathrm{e}^{2i(fx+e)}+1\right)}{af}$	[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^2/(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a*((-1/2*c*d-1/4*I*c^2-3/4*I*d^2)*ln(tan(f*x+e)-I)-(-I*c*d-1/2*c^2+1/2*
d^2)/(tan(f*x+e)-I)-1/4*I*(2*I*c*d-c^2+d^2)*ln(tan(f*x+e)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.13, size = 88, normalized size = 1.17

$$\frac{\left(2\left(c^2 - 2i\,cd + 3\,d^2\right)fxe^{(2i\,fx+2i\,e)} + 4i\,d^2e^{(2i\,fx+2i\,e)}\log\left(e^{(2i\,fx+2i\,e)} + 1\right) + i\,c^2 - 2\,cd - i\,d^2\right)e^{(-2i\,fx-2i\,e)}}{4\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(c^2 - 2*I*c*d + 3*d^2)*f*x*e^(2*I*f*x + 2*I*e) + 4*I*d^2*e^(2*I*f*x
 + 2*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + I*c^2 - 2*c*d - I*d^2)*e^(-2*I*f*x
 - 2*I*e)/(a*f)
```

Sympy [A]
time = 0.26, size = 170, normalized size = 2.27

$$\begin{cases} \frac{(ic^2-2cd-id^2)e^{-2ie}e^{-2ifx}}{4af} & \text{for } afe^{2ie}\neq 0 \\ x\left(-\frac{c^2-2icd+3d^2}{2a}+\frac{(c^2e^{2ie}+c^2-2icde^{2ie}+2icd+3d^2e^{2ie}-d^2)e^{-2ie}}{2a}\right) & \text{otherwise} \end{cases} + \frac{id^2\log\left(e^{2ifx}+e^{-2ie}\right)}{af} + \frac{x(c^2-2icd+3d^2)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**2/(a+I*a*tan(f*x+e)),x)
```

```
[Out] Piecewise((((I*c**2 - 2*c*d - I*d**2)*exp(-2*I*e)*exp(-2*I*f*x)/(4*a*f), Ne(
a*f*exp(2*I*e), 0)), (x*(-(c**2 - 2*I*c*d + 3*d**2)/(2*a) + (c**2*exp(2*I*e
) + c**2 - 2*I*c*d*exp(2*I*e) + 2*I*c*d + 3*d**2*exp(2*I*e) - d**2)*exp(-2*
I*e)/(2*a)), True)) + I*d**2*log(exp(2*I*f*x) + exp(-2*I*e))/(a*f) + x*(c**
2 - 2*I*c*d + 3*d**2)/(2*a)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(63) = 126.
time = 0.56, size = 131, normalized size = 1.75

$$-\frac{\frac{(i\,c^2+2\,cd+3i\,d^2)\log(\tan(fx+e)-i)}{a}+\frac{(-i\,c^2-2\,cd+i\,d^2)\log(i\,\tan(fx+e)-1)}{a}+\frac{-i\,c^2\tan(fx+e)-2\,cd\tan(fx+e)-3i\,d^2\tan(fx+e)-3\,c^2-2i\,cd-d^2}{a(\tan(fx+e)-i)}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/4*((I*c^2 + 2*c*d + 3*I*d^2)*log(tan(f*x + e) - I)/a + (-I*c^2 - 2*c*d +
 I*d^2)*log(I*tan(f*x + e) - 1)/a + (-I*c^2*tan(f*x + e) - 2*c*d*tan(f*x +
e) - 3*I*d^2*tan(f*x + e) - 3*c^2 - 2*I*c*d - d^2)/(a*(tan(f*x + e) - I)))/
f
```

Mupad [B]
time = 5.62, size = 112, normalized size = 1.49

$$-\frac{\frac{c\,d}{a}-\frac{c^2\,1\mathrm{i}}{2\,a}+\frac{d^2\,1\mathrm{i}}{2\,a}}{f\,\left(1+\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}-\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(c^2-c\,d\,2\mathrm{i}+3\,d^2\right)\,1\mathrm{i}}{4\,a\,f}+\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(c^2\,1\mathrm{i}+2\,c\,d-d^2\,1\mathrm{i}\right)}{4\,a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^2/(a + a*tan(e + f*x)*1i),x)
```

```
[Out] (log(tan(e + f*x) + 1i)*(2*c*d + c^2*1i - d^2*1i))/(4*a*f) - (log(tan(e + f
*x) - 1i)*(c^2 - c*d*2i + 3*d^2)*1i)/(4*a*f) - ((d^2*1i)/(2*a) - (c^2*1i)/(
2*a) + (c*d)/a)/(f*(tan(e + f*x)*1i + 1))
```

3.1075 $\int \frac{(c+d\tan(e+fx))^2}{(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=91

$$\frac{(c-id)^2x}{4a^2}+\frac{(c+id)(ic+3d)}{4a^2f(1+i\tan(e+fx))}+\frac{i(c+id)^2}{4f(a+ia\tan(e+fx))^2}$$

```
[Out] 1/4*(c-I*d)^2*x/a^2+1/4*(c+I*d)*(I*c+3*d)/a^2/f/(1+I*tan(f*x+e))+1/4*I*(c+I
*d)^2/f/(a+I*a*tan(f*x+e))^2
```

Rubi [A]

time = 0.10, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}=0.107$, Rules used = {3621, 3607, 8}

$$\frac{(c+id)(3d+ic)}{4a^2f(1+i\tan(e+fx))}+\frac{x(c-id)^2}{4a^2}+\frac{i(c+id)^2}{4f(a+ia\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^2/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] ((c - I*d)^2*x)/(4*a^2) + ((c + I*d)*(I*c + 3*d))/(4*a^2*f*(1 + I*Tan[e + f
*x])) + ((I/4)*(c + I*d)^2)/(f*(a + I*a*Tan[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rule 3621

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(a*c + b*d)^2*((a + b*Tan[e + f*x])^
m/(2*a^3*f*m)), x] + Dist[1/(2*a^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[
a*c^2 - 2*b*c*d + a*d^2 - 2*b*d^2*Tan[e + f*x], x], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LeQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+d\tan(e+fx))^2}{(a+ia\tan(e+fx))^2}\,dx &= \frac{i(c+id)^2}{4f(a+ia\tan(e+fx))^2}+\frac{\int \frac{a(c^2-2icd+d^2)-2iad^2\tan(e+fx)}{a+ia\tan(e+fx)}\,dx}{2a^2}\\ &= \frac{(c+id)(ic+3d)}{4a^2f(1+i\tan(e+fx))}+\frac{i(c+id)^2}{4f(a+ia\tan(e+fx))^2}+\frac{(c-id)^2\int 1\,dx}{4a^2}\\ &= \frac{(c-id)^2x}{4a^2}+\frac{(c+id)(ic+3d)}{4a^2f(1+i\tan(e+fx))}+\frac{i(c+id)^2}{4f(a+ia\tan(e+fx))^2}\end{aligned}$$

Mathematica [A]
time = 1.07, size = 134, normalized size = 1.47

$$-\frac{\sec^2(e+fx)\,(4i(c^2+d^2)+(cd(-2-8ifx)+c^2(i+4fx)-d^2(i+4fx))\cos(2(e+fx))+(d^2(-1-4ifx)+c^2(1+4ifx)+2cd(i+4fx))\sin(2(e+fx)))}{16a^2f(-i+\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^2/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] -1/16*(Sec[e + f*x]^2*((4*I)*(c^2 + d^2) + (c*d*(-2 - (8*I)*f*x) + c^2*(I +
 4*f*x) - d^2*(I + 4*f*x))*Cos[2*(e + f*x)] + (d^2*(-1 - (4*I)*f*x) + c^2*(
1 + (4*I)*f*x) + 2*c*d*(I + 4*f*x))*Sin[2*(e + f*x)]))/(a^2*f*(-I + Tan[e +
 f*x])^2)
```

Maple [A]
time = 0.21, size = 123, normalized size = 1.35

method	result
derivativedivides	$\frac{\left(\frac{1}{8}id^2-\frac{1}{8}ic^2-\frac{1}{4}cd\right)\ln(\tan(fx+e)-i)-\frac{-cd+\frac{1}{2}ic^2-\frac{1}{2}id^2}{2(\tan(fx+e)-i)^2}-\frac{\frac{1}{2}icd-\frac{1}{4}c^2-\frac{3}{4}d^2}{\tan(fx+e)-i}-\frac{i\left(2icd-c^2+d^2\right)\ln(\tan(fx+e)+i)}{8}}{f\,a^2}$
default	$\frac{\left(\frac{1}{8}id^2-\frac{1}{8}ic^2-\frac{1}{4}cd\right)\ln(\tan(fx+e)-i)-\frac{-cd+\frac{1}{2}ic^2-\frac{1}{2}id^2}{2(\tan(fx+e)-i)^2}-\frac{\frac{1}{2}icd-\frac{1}{4}c^2-\frac{3}{4}d^2}{\tan(fx+e)-i}-\frac{i\left(2icd-c^2+d^2\right)\ln(\tan(fx+e)+i)}{8}}{f\,a^2}$
risch	$-\frac{ixcd}{2a^2}+\frac{x\,c^2}{4a^2}-\frac{x\,d^2}{4a^2}+\frac{i\mathrm{e}^{-2i(fx+e)}c^2}{4a^2f}+\frac{i\mathrm{e}^{-2i(fx+e)}d^2}{4a^2f}-\frac{\mathrm{e}^{-4i(fx+e)}cd}{8a^2f}+\frac{i\mathrm{e}^{-4i(fx+e)}c^2}{16a^2f}-\frac{i\mathrm{e}^{-4i(fx+e)}d^2}{16a^2f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a^2*((1/8*I*d^2-1/8*I*c^2-1/4*c*d)*ln(tan(f*x+e)-I)-1/2*(-c*d+1/2*I*c^2
-1/2*I*d^2)/(tan(f*x+e)-I)^2-(1/2*I*c*d-1/4*c^2-3/4*d^2)/(tan(f*x+e)-I)-1/8
*I*(2*I*c*d-c^2+d^2)*ln(tan(f*x+e)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.73, size = 83, normalized size = 0.91

$$\frac{\left(4\left(c^2-2i\,cd-d^2\right)fxe^{(4i\,fx+4i\,e)}+i\,c^2-2\,cd-i\,d^2-4\left(-i\,c^2-i\,d^2\right)e^{(2i\,fx+2i\,e)}\right)e^{(-4i\,fx-4i\,e)}}{16\,a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(c^2 - 2*I*c*d - d^2)*f*x*e^(4*I*f*x + 4*I*e) + I*c^2 - 2*c*d - I*d
^2 - 4*(-I*c^2 - I*d^2)*e^(2*I*f*x + 2*I*e))*e^(-4*I*f*x - 4*I*e)/(a^2*f)
```

Sympy [A]
time = 0.24, size = 258, normalized size = 2.84

$$\begin{cases}\frac{\left(\left(16ia^2c^2fe^{4ie}+16ia^2d^2fe^{4ie}\right)e^{-2ifx}+\left(4ia^2c^2fe^{2ie}-8a^2cdfe^{2ie}-4ia^2d^2fe^{2ie}\right)e^{-4ifx}\right)e^{-6ie}}{64a^4f^2} & \text{for}\ a^4f^2e^{6ie}\neq 0\\ x\left(-\frac{c^2-2icd-d^2}{4a^2}+\frac{\left(c^2e^{4ie}+2c^2e^{2ie}+c^2-2icde^{4ie}+2icd-d^2e^{4ie}+2d^2e^{2ie}-d^2\right)e^{-4ie}}{4a^2}\right) & \text{otherwise}\end{cases}+\frac{x(c^2-2icd-d^2)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**2/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] Piecewise((((16*I*a**2*c**2*f*exp(4*I*e) + 16*I*a**2*d**2*f*exp(4*I*e))*exp
(-2*I*f*x) + (4*I*a**2*c**2*f*exp(2*I*e) - 8*a**2*c*d*f*exp(2*I*e) - 4*I*a*
*2*d**2*f*exp(2*I*e))*exp(-4*I*f*x))*exp(-6*I*e)/(64*a**4*f**2), Ne(a**4*f*
*2*exp(6*I*e), 0)), (x*(-(c**2 - 2*I*c*d - d**2)/(4*a**2) + (c**2*exp(4*I*e
) + 2*c**2*exp(2*I*e) + c**2 - 2*I*c*d*exp(4*I*e) + 2*I*c*d - d**2*exp(4*I*
e) + 2*d**2*exp(2*I*e) - d**2)*exp(-4*I*e)/(4*a**2)), True)) + x*(c**2 - 2*
I*c*d - d**2)/(4*a**2)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(73) = 146$.
time = 0.64, size = 176, normalized size = 1.93

$$-\frac{\frac{2\left(-i\,c^2-2\,cd+i\,d^2\right)\log\left(-i\,\tan\left(fx+e\right)+1\right)}{a^2}+\frac{2\left(i\,c^2+2\,cd-i\,d^2\right)\log\left(-i\,\tan\left(fx+e\right)-1\right)}{a^2}+\frac{-3i\,c^2\tan\left(fx+e\right)^2-6\,cd\tan\left(fx+e\right)^2+3i\,d^2\tan\left(fx+e\right)^2-10\,c^2\tan\left(fx+e\right)+20i\,cd\tan\left(fx+e\right)-6\,d^2\tan\left(fx+e\right)+11i\,c^2+6\,cd+5i\,d^2}{a^2\left(\tan\left(fx+e\right)-i\right)^2}}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/16*(2*(-I*c^2 - 2*c*d + I*d^2)*log(-I*tan(f*x + e) + 1)/a^2 + 2*(I*c^2 +
 2*c*d - I*d^2)*log(-I*tan(f*x + e) - 1)/a^2 + (-3*I*c^2*tan(f*x + e)^2 - 6
*c*d*tan(f*x + e)^2 + 3*I*d^2*tan(f*x + e)^2 - 10*c^2*tan(f*x + e) + 20*I*c
*d*tan(f*x + e) - 6*d^2*tan(f*x + e) + 11*I*c^2 + 6*c*d + 5*I*d^2)/(a^2*(ta
n(f*x + e) - I)^2))/f
```

Mupad [B]
time = 5.24, size = 93, normalized size = 1.02

$$-\frac{x\,{\left(d+c\,1{}\mathrm{i}\right)}^2}{4\,a^2}+\frac{\mathrm{tan}\left(e+f\,x\right)\,\left(\frac{c\,d}{2\,a^2}+\frac{c^2\,1{}\mathrm{i}}{4\,a^2}+\frac{d^2\,3{}\mathrm{i}}{4\,a^2}\right)+\frac{c^2}{2\,a^2}+\frac{d^2}{2\,a^2}}{f\,\left({\mathrm{tan}\left(e+f\,x\right)}^2\,1{}\mathrm{i}+2\,\mathrm{tan}\left(e+f\,x\right)-\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^2/(a + a*tan(e + f*x)*1i)^2,x)
```

```
[Out] (tan(e + f*x)*((c^2*1i)/(4*a^2) + (d^2*3i)/(4*a^2) + (c*d)/(2*a^2)) + c^2/(
2*a^2) + d^2/(2*a^2))/(f*(2*tan(e + f*x) + tan(e + f*x)^2*1i - 1i)) - (x*(c
*1i + d)^2)/(4*a^2)
```

3.1076 $\int \frac{(c+d\tan(e+fx))^2}{(a+ia\tan(e+fx))^3}\,dx$

Optimal. Leaf size=129

$$\frac{(c-id)^2x}{8a^3}+\frac{i(c+id)^2}{6f(a+ia\tan(e+fx))^3}+\frac{(c+id)(ic+3d)}{8af(a+ia\tan(e+fx))^2}+\frac{i(c-id)^2}{8f\left(a^3+ia^3\tan(e+fx)\right)}$$

```
[Out] 1/8*(c-I*d)^2*x/a^3+1/6*I*(c+I*d)^2/f/(a+I*a*tan(f*x+e))^3+1/8*(c+I*d)*(I*c
+3*d)/a/f/(a+I*a*tan(f*x+e))^2+1/8*I*(c-I*d)^2/f/(a^3+I*a^3*tan(f*x+e))
```

Rubi [A]

time = 0.12, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3621, 3607, 3560, 8}

$$\frac{i(c-id)^2}{8f\left(a^3+ia^3\tan(e+fx)\right)}+\frac{x(c-id)^2}{8a^3}+\frac{(c+id)(3d+ic)}{8af(a+ia\tan(e+fx))^2}+\frac{i(c+id)^2}{6f(a+ia\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^2/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] ((c - I*d)^2*x)/(8*a^3) + ((I/6)*(c + I*d)^2)/(f*(a + I*a*Tan[e + f*x])^3)
+ ((c + I*d)*(I*c + 3*d))/(8*a*f*(a + I*a*Tan[e + f*x])^2) + ((I/8)*(c - I*
d)^2)/(f*(a^3 + I*a^3*Tan[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rule 3621

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(a*c + b*d)^2*((a + b*Tan[e + f*x])^
```

```
m/(2*a^3*f*m)), x] + Dist[1/(2*a^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[
a*c^2 - 2*b*c*d + a*d^2 - 2*b*d^2*Tan[e + f*x], x], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LeQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+d\tan(e+fx))^2}{(a+ia\tan(e+fx))^3}\,dx &= \frac{i(c+id)^2}{6f(a+ia\tan(e+fx))^3} + \frac{\int \frac{a(c^2-2icd+d^2)-2iad^2\tan(e+fx)}{(a+ia\tan(e+fx))^2}\,dx}{2a^2} \\
&= \frac{i(c+id)^2}{6f(a+ia\tan(e+fx))^3} + \frac{(c+id)(ic+3d)}{8af(a+ia\tan(e+fx))^2} + \frac{(c-id)^2 \int \frac{1}{a+ia\tan(e+f\cdots}}{4a^2} \\
&= \frac{i(c+id)^2}{6f(a+ia\tan(e+fx))^3} + \frac{(c+id)(ic+3d)}{8af(a+ia\tan(e+fx))^2} + \frac{i(c-id)^2}{8f\,(a^3+ia^3\tan(e+f\cdots} \\
&= \frac{(c-id)^2x}{8a^3} + \frac{i(c+id)^2}{6f(a+ia\tan(e+fx))^3} + \frac{(c+id)(ic+3d)}{8af(a+ia\tan(e+fx))^2} + \frac{\cdots}{8f\,(a^3+\cdots}
\end{aligned}
$$

Mathematica [A]

time = 1.07, size = 256, normalized size = 1.98

$$
\frac{\sec^3(e+fx)(\cos(fx)+i\sin(fx))^3\,(3(c+id)(3ic+d)\cos(4fx)(\cos(e)-i\sin(e))+6(3c+id)(ic+d)\cos(2fx)(\cos(e)+i\sin(e))+12(c-id)^2fx(\cos(3e)+i\sin(3e))+2(c+id)^2\cos(6fx)(i\cos(3e)+\sin(3e))+6(c-id)(3c+id)(\cos(e)+i\sin(e))\sin(2fx)+3(3c-id)(c+id)(\cos(e)-i\sin(e))\sin(4fx)+2(c+id)^2(\cos(3e)-i\sin(3e))\sin(6fx))}{96f(a+ia\tan(e+fx))^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^2/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] (Sec[e + f*x]^3*(Cos[f*x] + I*Sin[f*x])^3*(3*(c + I*d)*((3*I)*c + d)*Cos[4*
f*x]*(Cos[e] - I*Sin[e]) + 6*(3*c + I*d)*(I*c + d)*Cos[2*f*x]*(Cos[e] + I*S
in[e]) + 12*(c - I*d)^2*f*x*(Cos[3*e] + I*Sin[3*e]) + 2*(c + I*d)^2*Cos[6*f
*x]*(I*Cos[3*e] + Sin[3*e]) + 6*(c - I*d)*(3*c + I*d)*(Cos[e] + I*Sin[e])*S
in[2*f*x] + 3*(3*c - I*d)*(c + I*d)*(Cos[e] - I*Sin[e])*Sin[4*f*x] + 2*(c +
 I*d)^2*(Cos[3*e] - I*Sin[3*e])*Sin[6*f*x]))/(96*f*(a + I*a*Tan[e + f*x])^3
)
```

Maple [A]

time = 0.23, size = 152, normalized size = 1.18

method	result
derivativedivides	$\frac{-\frac{i\left(2icd-c^2+d^2\right)\ln(\tan(fx+e)+i)}{16}-\frac{\frac{1}{2}cd+\frac{1}{4}ic^2+\frac{3}{4}id^2}{2(\tan(fx+e)-i)^2}-\frac{icd+\frac{1}{2}c^2-\frac{1}{2}d^2}{3(\tan(fx+e)-i)^3}+\left(-\frac{1}{16}ic^2+\frac{1}{16}id^2-\frac{1}{8}cd\right)\ln(\tan(fx+e)-i)-\frac{\frac{1}{4}icd-\frac{1}{8}\cdots}{\tan(fx\cdots}}{f\,a^3}$
default	$\frac{-\frac{i\left(2icd-c^2+d^2\right)\ln(\tan(fx+e)+i)}{16}-\frac{\frac{1}{2}cd+\frac{1}{4}ic^2+\frac{3}{4}id^2}{2(\tan(fx+e)-i)^2}-\frac{icd+\frac{1}{2}c^2-\frac{1}{2}d^2}{3(\tan(fx+e)-i)^3}+\left(-\frac{1}{16}ic^2+\frac{1}{16}id^2-\frac{1}{8}cd\right)\ln(\tan(fx+e)-i)-\frac{\frac{1}{4}icd-\frac{1}{8}\cdots}{\tan(fx\cdots}}{f\,a^3}$

risch	$-\frac{ixcd}{4a^3}+\frac{x\,c^2}{8a^3}-\frac{x\,d^2}{8a^3}+\frac{\mathrm{e}^{-2i(fx+e)}cd}{8a^3f}+\frac{3i\mathrm{e}^{-2i(fx+e)}c^2}{16a^3f}+\frac{i\mathrm{e}^{-2i(fx+e)}d^2}{16a^3f}-\frac{\mathrm{e}^{-4i(fx+e)}cd}{16a^3f}+\frac{3i\mathrm{e}^{-4i(fx+e)}c^2}{32a^3f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a^3*(-1/16*I*(2*I*c*d-c^2+d^2)*ln(tan(f*x+e)+I)-1/2*(1/2*c*d+1/4*I*c^2+
3/4*I*d^2)/(tan(f*x+e)-I)^2-1/3*(I*c*d+1/2*c^2-1/2*d^2)/(tan(f*x+e)-I)^3+(-
1/16*I*c^2+1/16*I*d^2-1/8*c*d)*ln(tan(f*x+e)-I)-(1/4*I*c*d-1/8*c^2+1/8*d^2)
/(tan(f*x+e)-I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.74, size = 114, normalized size = 0.88

$$\frac{\left(12\,(c^2-2i\,cd-d^2)fxe^{(6i\,fx+6i\,e)}+2i\,c^2-4\,cd-2i\,d^2-6\,(-3i\,c^2-2\,cd-i\,d^2)e^{(4i\,fx+4i\,e)}-3\,(-3i\,c^2+2\,cd-i\,d^2)e^{(2i\,fx+2i\,e)}\right)e^{(-6i\,fx-6i\,e)}}{96\,a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/96*(12*(c^2 - 2*I*c*d - d^2)*f*x*e^(6*I*f*x + 6*I*e) + 2*I*c^2 - 4*c*d -
2*I*d^2 - 6*(-3*I*c^2 - 2*c*d - I*d^2)*e^(4*I*f*x + 4*I*e) - 3*(-3*I*c^2 +
2*c*d - I*d^2)*e^(2*I*f*x + 2*I*e))*e^(-6*I*f*x - 6*I*e)/(a^3*f)
```

Sympy [A]
time = 0.39, size = 401, normalized size = 3.11

$$\begin{cases}\frac{\left(\left(512ia^6c^2f^2e^{6ie}-1024a^6cdf^2e^{6ie}-512ia^6d^2f^2e^{6ie}\right)e^{-6ifx}+\left(2304ia^6c^2f^2e^{8ie}-1536a^6cdf^2e^{8ie}+768ia^6d^2f^2e^{8ie}\right)e^{-4ifx}+\left(4608ia^6c^2f^2e^{10ie}+3072a^6cdf^2e^{10ie}+1536ia^6d^2f^2e^{10ie}\right)e^{-2ifx}\right)e^{-12ie}}{24576a^9f^3} & \text{for } a^9f^3e^{12ie}\neq 0\\ x\left(-\frac{c^2-2icd-d^2}{8a^3}+\frac{\left(c^2e^{6ie}+3c^2e^{4ie}+3c^2e^{2ie}+c^2-2icde^{6ie}-2icde^{4ie}+2icde^{2ie}+2icd-d^2e^{6ie}+d^2e^{4ie}+d^2e^{2ie}-d^2\right)e^{-6ie}}{8a^3}\right) & \text{otherwise}\end{cases}+\frac{x(c^2-2icd-d^2)}{8a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**2/(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] Piecewise((((512*I*a**6*c**2*f**2*exp(6*I*e) - 1024*a**6*c*d*f**2*exp(6*I*e
) - 512*I*a**6*d**2*f**2*exp(6*I*e))*exp(-6*I*f*x) + (2304*I*a**6*c**2*f**2
*exp(8*I*e) - 1536*a**6*c*d*f**2*exp(8*I*e) + 768*I*a**6*d**2*f**2*exp(8*I*
```

```
e))*exp(-4*I*f*x) + (4608*I*a**6*c**2*f**2*exp(10*I*e) + 3072*a**6*c*d*f**2
*exp(10*I*e) + 1536*I*a**6*d**2*f**2*exp(10*I*e))*exp(-2*I*f*x))*exp(-12*I*
e)/(24576*a**9*f**3), Ne(a**9*f**3*exp(12*I*e), 0)), (x*(-(c**2 - 2*I*c*d -
 d**2)/(8*a**3) + (c**2*exp(6*I*e) + 3*c**2*exp(4*I*e) + 3*c**2*exp(2*I*e)
+ c**2 - 2*I*c*d*exp(6*I*e) - 2*I*c*d*exp(4*I*e) + 2*I*c*d*exp(2*I*e) + 2*I
*c*d - d**2*exp(6*I*e) + d**2*exp(4*I*e) + d**2*exp(2*I*e) - d**2)*exp(-6*I
*e)/(8*a**3)), True)) + x*(c**2 - 2*I*c*d - d**2)/(8*a**3)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(104) = 208$.
time = 0.78, size = 215, normalized size = 1.67

$$-\frac{\frac{6\left(i c^2+2 c d-i d^2\right) \log(\tan(fx+e)-i)}{a^3}+\frac{6\left(-i c^2-2 c d+i d^2\right) \log(i \tan(fx+e)-1)}{a^3}+\frac{-11 i c^2 \tan(fx+e)^3-22 c d \tan(fx+e)^3+11 i d^2 \tan(fx+e)^3-45 c^2 \tan(fx+e)^2+90 i c d \tan(fx+e)^2+45 d^2 \tan(fx+e)^2+69 i c^2 \tan(fx+e)+138 c d \tan(fx+e)-21 i d^2 \tan(fx+e)+51 c^2-38 i c d-3 d^2}{a^3(\tan(fx+e)-i)^3}}{96 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/96*(6*(I*c^2 + 2*c*d - I*d^2)*log(tan(f*x + e) - I)/a^3 + 6*(-I*c^2 - 2*
c*d + I*d^2)*log(I*tan(f*x + e) - 1)/a^3 + (-11*I*c^2*tan(f*x + e)^3 - 22*c
*d*tan(f*x + e)^3 + 11*I*d^2*tan(f*x + e)^3 - 45*c^2*tan(f*x + e)^2 + 90*I*
c*d*tan(f*x + e)^2 + 45*d^2*tan(f*x + e)^2 + 69*I*c^2*tan(f*x + e) + 138*c*
d*tan(f*x + e) - 21*I*d^2*tan(f*x + e) + 51*c^2 - 38*I*c*d - 3*d^2)/(a^3*(t
an(f*x + e) - I)^3))/f
```

Mupad [B]
time = 5.44, size = 148, normalized size = 1.15

$$\frac{\frac{c\,d}{6\,a^3}-\tan(e+f\,x)\,\left(\frac{3\,c^2}{8\,a^3}+\frac{d^2}{8\,a^3}-\frac{c\,d\,3\mathrm{i}}{4\,a^3}\right)-\tan(e+f\,x)^2\,\left(\frac{c\,d}{4\,a^3}+\frac{c^2\,1\mathrm{i}}{8\,a^3}-\frac{d^2\,1\mathrm{i}}{8\,a^3}\right)+\frac{c^2\,5\mathrm{i}}{12\,a^3}+\frac{d^2\,1\mathrm{i}}{12\,a^3}}{f\,\left(-{\tan\left(e+f\,x\right)}^3\,1\mathrm{i}-3\,{\tan\left(e+f\,x\right)}^2+\tan\left(e+f\,x\right)\,3\mathrm{i}+1\right)}-\frac{x\,{\left(d+c\,1\mathrm{i}\right)}^2}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^2/(a + a*tan(e + f*x)*1i)^3,x)
```

```
[Out] ((c^2*5i)/(12*a^3) - tan(e + f*x)*((3*c^2)/(8*a^3) + d^2/(8*a^3) - (c*d*3i)
/(4*a^3)) - tan(e + f*x)^2*((c^2*1i)/(8*a^3) - (d^2*1i)/(8*a^3) + (c*d)/(4*
a^3)) + (d^2*1i)/(12*a^3) + (c*d)/(6*a^3))/(f*(tan(e + f*x)*3i - 3*tan(e +
f*x)^2 - tan(e + f*x)^3*1i + 1)) - (x*(c*1i + d)^2)/(8*a^3)
```

3.1077 $\int (a+ia\tan(e+fx))^3(c+d\tan(e+fx))^3\,dx$

Optimal. Leaf size=190

$$4a^3(c-id)^3x+\frac{4a^3(ic+d)^3\log(\cos(e+fx))}{f}+\frac{4ia^3(c-id)^2d\tan(e+fx)}{f}+\frac{2a^3(ic+d)(c+d\tan(e+fx))^2}{f}-$$

```
[Out] 4*a^3*(c-I*d)^3*x+4*a^3*(I*c+d)^3*ln(cos(f*x+e))/f+4*I*a^3*(c-I*d)^2*d*tan(
f*x+e)/f+2*a^3*(I*c+d)*(c+d*tan(f*x+e))^2/f+4/3*I*a^3*(c+d*tan(f*x+e))^3/f+
1/20*a^3*(I*c-11*d)*(c+d*tan(f*x+e))^4/d^2/f-1/5*(a^3+I*a^3*tan(f*x+e))*(c+
d*tan(f*x+e))^4/d/f
```

Rubi [A]
time = 0.24, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3637, 3673, 3609, 3606, 3556}

$$\frac{a^3(-11d+ic)(c+d\tan(e+fx))^4}{20d^2f}-\frac{(a^3+ia^3\tan(e+fx))\,(c+d\tan(e+fx))^4}{5df}+\frac{4ia^3(c+d\tan(e+fx))^3}{3f}+\frac{2a^3(d+ic)(c+d\tan(e+fx))^2}{f}+\frac{4ia^3d(c-id)^2\tan(e+fx)}{f}+\frac{4a^3(d+ic)^3\log(\cos(e+fx))}{f}+4a^3x(c-id)^3$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^3,x]
```

```
[Out] 4*a^3*(c - I*d)^3*x + (4*a^3*(I*c + d)^3*Log[Cos[e + f*x]])/f + ((4*I)*a^3*
(c - I*d)^2*d*Tan[e + f*x])/f + (2*a^3*(I*c + d)*(c + d*Tan[e + f*x])^2)/f
+ (((4*I)/3)*a^3*(c + d*Tan[e + f*x])^3)/f + (a^3*(I*c - 11*d)*(c + d*Tan[e
 + f*x])^4)/(20*d^2*f) - ((a^3 + I*a^3*Tan[e + f*x])*(c + d*Tan[e + f*x])^4
)/(5*d*f)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
```

```
0] && GtQ[m, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (a+ia\tan(e+fx))^3(c+d\tan(e+fx))^3\,dx &= -\frac{(a^3+ia^3\tan(e+fx))\,(c+d\tan(e+fx))^4}{5df}+\frac{a\int(a+ia\\
&= \frac{a^3(ic-11d)(c+d\tan(e+fx))^4}{20d^2f}-\frac{(a^3+ia^3\tan(e+fx))}{5d}\\
&= \frac{4ia^3(c+d\tan(e+fx))^3}{3f}+\frac{a^3(ic-11d)(c+d\tan(e+fx)}{20d^2f}\\
&= \frac{2a^3(ic+d)(c+d\tan(e+fx))^2}{f}+\frac{4ia^3(c+d\tan(e+fx))}{3f}\\
&= 4a^3(c-id)^3x+\frac{4ia^3(c-id)^2d\tan(e+fx)}{f}+\frac{2a^3(ic+d)(}{}\\
&= 4a^3(c-id)^3x+\frac{4a^3(ic+d)^3\log(\cos(e+fx))}{f}+\frac{4ia^3(c-}{}
\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1564 vs. 2(190) = 380.
time = 8.15, size = 1564, normalized size = 8.23

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^3,x]
```

```
[Out] (Cos[e + f*x]^3*((-I)*c^3*Cos[(3*e)/2] - 3*c^2*d*Cos[(3*e)/2] + (3*I)*c*d^2
*Cos[(3*e)/2] + d^3*Cos[(3*e)/2] - c^3*Sin[(3*e)/2] + (3*I)*c^2*d*Sin[(3*e)
/2] + 3*c*d^2*Sin[(3*e)/2] - I*d^3*Sin[(3*e)/2])*(2*Cos[(3*e)/2]*Log[Cos[e
 + f*x]^2] - (2*I)*Log[Cos[e + f*x]^2]*Sin[(3*e)/2])*(a + I*a*Tan[e + f*x])^
3)/(f*(Cos[f*x] + I*Sin[f*x])^3) + (Sec[e]*Sec[e + f*x]^2*(Cos[3*e]/240 - (
I/240)*Sin[3*e])*((-45*I)*c^3*Cos[f*x] - 405*c^2*d*Cos[f*x] + (585*I)*c*d^2
*Cos[f*x] + 225*d^3*Cos[f*x] + 300*c^3*f*x*Cos[f*x] - (900*I)*c^2*d*f*x*Cos
[f*x] - 900*c*d^2*f*x*Cos[f*x] + (300*I)*d^3*f*x*Cos[f*x] - (45*I)*c^3*Cos[
2*e + f*x] - 405*c^2*d*Cos[2*e + f*x] + (585*I)*c*d^2*Cos[2*e + f*x] + 225*
d^3*Cos[2*e + f*x] + 300*c^3*f*x*Cos[2*e + f*x] - (900*I)*c^2*d*f*x*Cos[2*e
 + f*x] - 900*c*d^2*f*x*Cos[2*e + f*x] + (300*I)*d^3*f*x*Cos[2*e + f*x] - (
15*I)*c^3*Cos[2*e + 3*f*x] - 135*c^2*d*Cos[2*e + 3*f*x] + (225*I)*c*d^2*Cos
[2*e + 3*f*x] + 105*d^3*Cos[2*e + 3*f*x] + 150*c^3*f*x*Cos[2*e + 3*f*x] - (
450*I)*c^2*d*f*x*Cos[2*e + 3*f*x] - 450*c*d^2*f*x*Cos[2*e + 3*f*x] + (150*I
)*d^3*f*x*Cos[2*e + 3*f*x] - (15*I)*c^3*Cos[4*e + 3*f*x] - 135*c^2*d*Cos[4*
e + 3*f*x] + (225*I)*c*d^2*Cos[4*e + 3*f*x] + 105*d^3*Cos[4*e + 3*f*x] + 15
0*c^3*f*x*Cos[4*e + 3*f*x] - (450*I)*c^2*d*f*x*Cos[4*e + 3*f*x] - 450*c*d^2
*f*x*Cos[4*e + 3*f*x] + (150*I)*d^3*f*x*Cos[4*e + 3*f*x] + 30*c^3*f*x*Cos[4
*e + 5*f*x] - (90*I)*c^2*d*f*x*Cos[4*e + 5*f*x] - 90*c*d^2*f*x*Cos[4*e + 5*
f*x] + (30*I)*d^3*f*x*Cos[4*e + 5*f*x] + 30*c^3*f*x*Cos[6*e + 5*f*x] - (90*
I)*c^2*d*f*x*Cos[6*e + 5*f*x] - 90*c*d^2*f*x*Cos[6*e + 5*f*x] + (30*I)*d^3*
f*x*Cos[6*e + 5*f*x] - 270*c^3*Sin[f*x] + (1140*I)*c^2*d*Sin[f*x] + 1260*c*
d^2*Sin[f*x] - (470*I)*d^3*Sin[f*x] + 180*c^3*Sin[2*e + f*x] - (810*I)*c^2*
d*Sin[2*e + f*x] - 990*c*d^2*Sin[2*e + f*x] + (360*I)*d^3*Sin[2*e + f*x] -
180*c^3*Sin[2*e + 3*f*x] + (750*I)*c^2*d*Sin[2*e + 3*f*x] + 810*c*d^2*Sin[2
*e + 3*f*x] - (280*I)*d^3*Sin[2*e + 3*f*x] + 45*c^3*Sin[4*e + 3*f*x] - (225
*I)*c^2*d*Sin[4*e + 3*f*x] - 315*c*d^2*Sin[4*e + 3*f*x] + (135*I)*d^3*Sin[4
*e + 3*f*x] - 45*c^3*Sin[4*e + 5*f*x] + (195*I)*c^2*d*Sin[4*e + 5*f*x] + 22
5*c*d^2*Sin[4*e + 5*f*x] - (83*I)*d^3*Sin[4*e + 5*f*x])*(a + I*a*Tan[e + f*
x])^3)/(f*(Cos[f*x] + I*Sin[f*x])^3) + (x*Cos[e + f*x]^3*(-2*c^3*Cos[e] + (
6*I)*c^2*d*Cos[e] + 6*c*d^2*Cos[e] - (2*I)*d^3*Cos[e] + 2*c^3*Cos[e]^3 - (6
*I)*c^2*d*Cos[e]^3 - 6*c*d^2*Cos[e]^3 + (2*I)*d^3*Cos[e]^3 + (4*I)*c^3*Sin[
e] + 12*c^2*d*Sin[e] - (12*I)*c*d^2*Sin[e] - 4*d^3*Sin[e] - (8*I)*c^3*Cos[e
]^2*Sin[e] - 24*c^2*d*Cos[e]^2*Sin[e] + (24*I)*c*d^2*Cos[e]^2*Sin[e] + 8*d^
3*Cos[e]^2*Sin[e] - 12*c^3*Cos[e]*Sin[e]^2 + (36*I)*c^2*d*Cos[e]*Sin[e]^2 +
 36*c*d^2*Cos[e]*Sin[e]^2 - (12*I)*d^3*Cos[e]*Sin[e]^2 + (8*I)*c^3*Sin[e]^3
 + 24*c^2*d*Sin[e]^3 - (24*I)*c*d^2*Sin[e]^3 - 8*d^3*Sin[e]^3 + 2*c^3*Sin[e
]*Tan[e] - (6*I)*c^2*d*Sin[e]*Tan[e] - 6*c*d^2*Sin[e]*Tan[e] + (2*I)*d^3*Si
n[e]*Tan[e] + 2*c^3*Sin[e]^3*Tan[e] - (6*I)*c^2*d*Sin[e]^3*Tan[e] - 6*c*d^2
*Sin[e]^3*Tan[e] + (2*I)*d^3*Sin[e]^3*Tan[e] + ((-I)*c - d)^3*(4*Cos[3*e] -
 (4*I)*Sin[3*e])*Tan[e])*(a + I*a*Tan[e + f*x])^3)/(Cos[f*x] + I*Sin[f*x])^
```

3

Maple [A]
time = 0.19, size = 269, normalized size = 1.42

method	result
derivativedivides	$\frac{a^3\left(-\frac{ic^3\left(\tan^2(fx+e)\right)}{2}-4id^3\tan(fx+e)-\frac{id^3\left(\tan^5(fx+e)\right)}{5}+12ic^2d\tan(fx+e)-\frac{3d^3\left(\tan^4(fx+e)\right)}{4}+6icd^2\left(\tan^2(fx+e)\right)-i\right.}{}$
default	$\frac{a^3\left(-\frac{ic^3\left(\tan^2(fx+e)\right)}{2}-4id^3\tan(fx+e)-\frac{id^3\left(\tan^5(fx+e)\right)}{5}+12ic^2d\tan(fx+e)-\frac{3d^3\left(\tan^4(fx+e)\right)}{4}+6icd^2\left(\tan^2(fx+e)\right)-i\right.}{}$
norman	$(-12ia^3c^2d+4ia^3d^3+4a^3c^3-12a^3c\,d^2)\,x-\frac{3(ia^3c\,d^2+a^3d^3)\left(\tan^4(fx+e)\right)}{4f}-\frac{(-12ia^3c^2d+4ia^3d^3+}{}$
risch	$\frac{12ia^3\ln(e^{2i(fx+e)}+1)c\,d^2}{f}+\frac{24ia^3c^2de}{f}-\frac{8ia^3d^3e}{f}-\frac{4ia^3\ln(e^{2i(fx+e)}+1)c^3}{f}-\frac{8a^3c^3e}{f}+\frac{24a^3c\,d^2e}{f}+\frac{2a^3(-60}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3*(c+d*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a^3*(-1/2*I*c^3*tan(f*x+e)^2-4*I*d^3*tan(f*x+e)-1/5*I*d^3*tan(f*x+e)^5+
12*I*c^2*d*tan(f*x+e)-3/4*d^3*tan(f*x+e)^4+6*I*c*d^2*tan(f*x+e)^2-I*c^2*d*t
an(f*x+e)^3-3*c*d^2*tan(f*x+e)^3+4/3*I*d^3*tan(f*x+e)^3-3/4*I*c*d^2*tan(f*x
+e)^4-9/2*c^2*d*tan(f*x+e)^2+2*d^3*tan(f*x+e)^2-3*c^3*tan(f*x+e)+12*c*d^2*t
an(f*x+e)+1/2*(-12*I*c*d^2-4*d^3+4*I*c^3+12*c^2*d)*ln(1+tan(f*x+e)^2)+(4*I*
d^3-12*I*c^2*d-12*c*d^2+4*c^3)*arctan(tan(f*x+e)))
```

Maxima [A]
time = 0.50, size = 269, normalized size = 1.42

$$-\frac{12i\,a^3d^3\tan(fx+e)^5+45\,(i\,a^3cd^2+a^3d^3)\tan(fx+e)^4+20\,(3i\,a^3c^2d+9\,a^3cd^2-4i\,a^3d^3)\tan(fx+e)^3+30\,(i\,a^3c^3+9\,a^3c^2d-12i\,a^3cd^2-4\,a^3d^3)\tan(fx+e)^2-240\,(a^3c^3-3i\,a^3c^2d-3\,a^3cd^2+i\,a^3d^3)(fx+e)+120\,(-i\,a^3c^3-3\,a^3c^2d+3i\,a^3cd^2+a^3d^3)\log(\tan(fx+e)^2+1)+60\,(3\,a^3c^3-12i\,a^3c^2d-12\,a^3cd^2+4i\,a^3d^3)\tan(fx+e)}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c+d*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(12*I*a^3*d^3*tan(f*x + e)^5 + 45*(I*a^3*c*d^2 + a^3*d^3)*tan(f*x + e
)^4 + 20*(3*I*a^3*c^2*d + 9*a^3*c*d^2 - 4*I*a^3*d^3)*tan(f*x + e)^3 + 30*(I
*a^3*c^3 + 9*a^3*c^2*d - 12*I*a^3*c*d^2 - 4*a^3*d^3)*tan(f*x + e)^2 - 240*(
a^3*c^3 - 3*I*a^3*c^2*d - 3*a^3*c*d^2 + I*a^3*d^3)*(f*x + e) + 120*(-I*a^3*
c^3 - 3*a^3*c^2*d + 3*I*a^3*c*d^2 + a^3*d^3)*log(tan(f*x + e)^2 + 1) + 60*(
3*a^3*c^3 - 12*I*a^3*c^2*d - 12*a^3*c*d^2 + 4*I*a^3*d^3)*tan(f*x + e))/f
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 577 vs. 2(177) = 354.
time = 0.93, size = 577, normalized size = 3.04

$$-\frac{2\,(45i\,a^3c^3+195\,a^3c^2d-225i\,a^3cd^2-83\,a^3d^3+60\,(i\,a^3c^3+6\,a^3c^2d-9i\,a^3cd^2-4\,a^3d^3)e^{(8i\,fx+8i\,e)}+45\,(5i\,a^3c^3+27\,a^3c^2d-35i\,a^3cd^2-13\,a^3d^3)e^{(6i\,fx+6i\,e)}+5\,(63i\,a^3c^3+309\,a^3c^2d-369i\,a^3cd^2-139\,a^3d^3)e^{(4i\,fx+4i\,e)}+5\,(39i\,a^3c^3+177\,a^3c^2d-207i\,a^3cd^2-77\,a^3d^3)e^{(2i\,fx+2i\,e)}+30\,(i\,a^3c^3+3\,a^3c^2d-3i\,a^3cd^2-a^3d^3+(i\,a^3c^3+3\,a^3c^2d-3i\,a^3cd^2-a^3d^3)e^{(10i\,fx+10i\,e)}+5\,(i\,a^3c^3+3\,a^3c^2d-3i\,a^3cd^2-a^3d^3)e^{(8i\,fx+8i\,e)}+10\,(i\,a^3c^3+3\,a^3c^2d-3i\,a^3cd^2-a^3d^3)e^{(6i\,fx+6i\,e)}+10\,(i\,a^3c^3+3\,a^3c^2d-3i\,a^3cd^2-a^3d^3)e^{(4i\,fx+4i\,e)}+5\,(i\,a^3c^3+3\,a^3c^2d-3i\,a^3cd^2-a^3d^3)e^{(2i\,fx+2i\,e)})\log(e^{(2i\,fx+2i\,e)}+1))}{15\,(f e^{(10i\,fx+10i\,e)}+5\,f e^{(8i\,fx+8i\,e)}+10\,f e^{(6i\,fx+6i\,e)}+10\,f e^{(4i\,fx+4i\,e)}+5\,f e^{(2i\,fx+2i\,e)}+f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c+d*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -2/15*(45*I*a^3*c^3 + 195*a^3*c^2*d - 225*I*a^3*c*d^2 - 83*a^3*d^3 + 60*(I*
a^3*c^3 + 6*a^3*c^2*d - 9*I*a^3*c*d^2 - 4*a^3*d^3)*e^(8*I*f*x + 8*I*e) + 45
*(5*I*a^3*c^3 + 27*a^3*c^2*d - 35*I*a^3*c*d^2 - 13*a^3*d^3)*e^(6*I*f*x + 6*
I*e) + 5*(63*I*a^3*c^3 + 309*a^3*c^2*d - 369*I*a^3*c*d^2 - 139*a^3*d^3)*e^(
4*I*f*x + 4*I*e) + 5*(39*I*a^3*c^3 + 177*a^3*c^2*d - 207*I*a^3*c*d^2 - 77*a
^3*d^3)*e^(2*I*f*x + 2*I*e) + 30*(I*a^3*c^3 + 3*a^3*c^2*d - 3*I*a^3*c*d^2 -
 a^3*d^3 + (I*a^3*c^3 + 3*a^3*c^2*d - 3*I*a^3*c*d^2 - a^3*d^3)*e^(10*I*f*x
+ 10*I*e) + 5*(I*a^3*c^3 + 3*a^3*c^2*d - 3*I*a^3*c*d^2 - a^3*d^3)*e^(8*I*f*
x + 8*I*e) + 10*(I*a^3*c^3 + 3*a^3*c^2*d - 3*I*a^3*c*d^2 - a^3*d^3)*e^(6*I*
f*x + 6*I*e) + 10*(I*a^3*c^3 + 3*a^3*c^2*d - 3*I*a^3*c*d^2 - a^3*d^3)*e^(4*
I*f*x + 4*I*e) + 5*(I*a^3*c^3 + 3*a^3*c^2*d - 3*I*a^3*c*d^2 - a^3*d^3)*e^(2
*I*f*x + 2*I*e))*log(e^(2*I*f*x + 2*I*e) + 1))/(f*e^(10*I*f*x + 10*I*e) + 5
*f*e^(8*I*f*x + 8*I*e) + 10*f*e^(6*I*f*x + 6*I*e) + 10*f*e^(4*I*f*x + 4*I*e
) + 5*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(167) = 334$.
time = 0.81, size = 476, normalized size = 2.51

$$-\frac{4ia^3(c-id)^3\log\left(e^{2ifx}+e^{-2ie}\right)}{f}+\frac{-90ia^3c^3-390a^3c^2d+450ia^3cd^2+166a^3d^3+\left(-390ia^3c^3e^{2ie}-1770a^3c^2de^{2ie}+2070ia^3cd^2e^{2ie}+770a^3d^3e^{2ie}\right)e^{2ifx}+\left(-630ia^3c^3e^{4ie}-3090a^3c^2de^{4ie}+3690ia^3cd^2e^{4ie}+1390a^3d^3e^{4ie}\right)e^{4ifx}+\left(-450ia^3c^3e^{6ie}-2430a^3c^2de^{6ie}+3150ia^3cd^2e^{6ie}+1170a^3d^3e^{6ie}\right)e^{6ifx}+\left(-120ia^3c^3e^{8ie}-720a^3c^2de^{8ie}+1080ia^3cd^2e^{8ie}+480a^3d^3e^{8ie}\right)e^{8ifx}}{15fe^{10ie}e^{10ifx}+75fe^{8ie}e^{8ifx}+150fe^{6ie}e^{6ifx}+150fe^{4ie}e^{4ifx}+75fe^{2ie}e^{2ifx}+15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3*(c+d*tan(f*x+e))**3,x)
```

```
[Out] -4*I*a**3*(c - I*d)**3*log(exp(2*I*f*x) + exp(-2*I*e))/f + (-90*I*a**3*c**3
 - 390*a**3*c**2*d + 450*I*a**3*c*d**2 + 166*a**3*d**3 + (-390*I*a**3*c**3*
exp(2*I*e) - 1770*a**3*c**2*d*exp(2*I*e) + 2070*I*a**3*c*d**2*exp(2*I*e) +
770*a**3*d**3*exp(2*I*e))*exp(2*I*f*x) + (-630*I*a**3*c**3*exp(4*I*e) - 309
0*a**3*c**2*d*exp(4*I*e) + 3690*I*a**3*c*d**2*exp(4*I*e) + 1390*a**3*d**3*e
xp(4*I*e))*exp(4*I*f*x) + (-450*I*a**3*c**3*exp(6*I*e) - 2430*a**3*c**2*d*e
xp(6*I*e) + 3150*I*a**3*c*d**2*exp(6*I*e) + 1170*a**3*d**3*exp(6*I*e))*exp(
6*I*f*x) + (-120*I*a**3*c**3*exp(8*I*e) - 720*a**3*c**2*d*exp(8*I*e) + 1080
*I*a**3*c*d**2*exp(8*I*e) + 480*a**3*d**3*exp(8*I*e))*exp(8*I*f*x))/(15*f*e
xp(10*I*e)*exp(10*I*f*x) + 75*f*exp(8*I*e)*exp(8*I*f*x) + 150*f*exp(6*I*e)*
exp(6*I*f*x) + 150*f*exp(4*I*e)*exp(4*I*f*x) + 75*f*exp(2*I*e)*exp(2*I*f*x)
 + 15*f)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1117 vs. $2(177) = 354$.
time = 1.03, size = 1117, normalized size = 5.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c+d*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/15*(30*I*a^3*c^3*e^(10*I*f*x + 10*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 90
*a^3*c^2*d*e^(10*I*f*x + 10*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) - 90*I*a^3*c*
d^2*e^(10*I*f*x + 10*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) - 30*a^3*d^3*e^(10*I
*f*x + 10*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 150*I*a^3*c^3*e^(8*I*f*x + 8*
I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 450*a^3*c^2*d*e^(8*I*f*x + 8*I*e)*log(e
^(2*I*f*x + 2*I*e) + 1) - 450*I*a^3*c*d^2*e^(8*I*f*x + 8*I*e)*log(e^(2*I*f*
x + 2*I*e) + 1) - 150*a^3*d^3*e^(8*I*f*x + 8*I*e)*log(e^(2*I*f*x + 2*I*e) +
 1) + 300*I*a^3*c^3*e^(6*I*f*x + 6*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 900*
a^3*c^2*d*e^(6*I*f*x + 6*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) - 900*I*a^3*c*d^
2*e^(6*I*f*x + 6*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) - 300*a^3*d^3*e^(6*I*f*x
 + 6*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 300*I*a^3*c^3*e^(4*I*f*x + 4*I*e)*
log(e^(2*I*f*x + 2*I*e) + 1) + 900*a^3*c^2*d*e^(4*I*f*x + 4*I*e)*log(e^(2*I
*f*x + 2*I*e) + 1) - 900*I*a^3*c*d^2*e^(4*I*f*x + 4*I*e)*log(e^(2*I*f*x + 2
*I*e) + 1) - 300*a^3*d^3*e^(4*I*f*x + 4*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) +
 150*I*a^3*c^3*e^(2*I*f*x + 2*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 450*a^3*c
^2*d*e^(2*I*f*x + 2*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) - 450*I*a^3*c*d^2*e^(
2*I*f*x + 2*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) - 150*a^3*d^3*e^(2*I*f*x + 2*
I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 60*I*a^3*c^3*e^(8*I*f*x + 8*I*e) + 360*
a^3*c^2*d*e^(8*I*f*x + 8*I*e) - 540*I*a^3*c*d^2*e^(8*I*f*x + 8*I*e) - 240*a
^3*d^3*e^(8*I*f*x + 8*I*e) + 225*I*a^3*c^3*e^(6*I*f*x + 6*I*e) + 1215*a^3*c
^2*d*e^(6*I*f*x + 6*I*e) - 1575*I*a^3*c*d^2*e^(6*I*f*x + 6*I*e) - 585*a^3*d
^3*e^(6*I*f*x + 6*I*e) + 315*I*a^3*c^3*e^(4*I*f*x + 4*I*e) + 1545*a^3*c^2*d
*e^(4*I*f*x + 4*I*e) - 1845*I*a^3*c*d^2*e^(4*I*f*x + 4*I*e) - 695*a^3*d^3*e
^(4*I*f*x + 4*I*e) + 195*I*a^3*c^3*e^(2*I*f*x + 2*I*e) + 885*a^3*c^2*d*e^(2
*I*f*x + 2*I*e) - 1035*I*a^3*c*d^2*e^(2*I*f*x + 2*I*e) - 385*a^3*d^3*e^(2*I
*f*x + 2*I*e) + 30*I*a^3*c^3*log(e^(2*I*f*x + 2*I*e) + 1) + 90*a^3*c^2*d*lo
g(e^(2*I*f*x + 2*I*e) + 1) - 90*I*a^3*c*d^2*log(e^(2*I*f*x + 2*I*e) + 1) -
30*a^3*d^3*log(e^(2*I*f*x + 2*I*e) + 1) + 45*I*a^3*c^3 + 195*a^3*c^2*d - 22
5*I*a^3*c*d^2 - 83*a^3*d^3)/(f*e^(10*I*f*x + 10*I*e) + 5*f*e^(8*I*f*x + 8*I
*e) + 10*f*e^(6*I*f*x + 6*I*e) + 10*f*e^(4*I*f*x + 4*I*e) + 5*f*e^(2*I*f*x
+ 2*I*e) + f)
```

Mupad [B]
time = 5.19, size = 356, normalized size = 1.87

$$-\frac{\tan(e+f\,x)^4\,\left(\frac{a^3\,d^3}{4}+\frac{a^3\,d^2\,(2\,d+c\,3\mathrm{i})}{4}\right)}{f}-\frac{\tan(e+f\,x)\,\left(a^3\,d^3\,1\mathrm{i}-a^3\,c\,(c^2\,1\mathrm{i}+6\,c\,d-d^2\,3\mathrm{i})\,1\mathrm{i}-a^3\,d\,(c^2\,3\mathrm{i}+6\,c\,d-d^2\,1\mathrm{i})+a^3\,c^2\,(2\,c-d\,3\mathrm{i})+a^3\,d^2\,(2\,d+c\,3\mathrm{i})\,1\mathrm{i}\right)}{f}+\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(a^3\,c^3\,4\mathrm{i}+12\,a^3\,c^2\,d-a^3\,c\,d^2\,12\mathrm{i}-4\,a^3\,d^3\right)}{f}+\frac{\tan(e+f\,x)^3\,\left(\frac{a^3\,d^3\,1\mathrm{i}}{3}-\frac{a^3\,d\,(c^2\,3\mathrm{i}+6\,c\,d-d^2\,1\mathrm{i})}{3}+\frac{a^3\,d^2\,(2\,d+c\,3\mathrm{i})\,1\mathrm{i}}{3}\right)}{f}+\frac{\tan(e+f\,x)^2\,\left(\frac{a^3\,d^3}{2}-\frac{a^3\,c\,(c^2\,1\mathrm{i}+6\,c\,d-d^2\,3\mathrm{i})}{2}+\frac{a^3\,d\,(c^2\,3\mathrm{i}+6\,c\,d-d^2\,1\mathrm{i})\,1\mathrm{i}}{2}+\frac{a^3\,d^2\,(2\,d+c\,3\mathrm{i})}{2}\right)}{f}-\frac{a^3\,d^3\,\tan(e+f\,x)^5\,1\mathrm{i}}{5\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^3*(c + d*tan(e + f*x))^3,x)
```

```
[Out] (log(tan(e + f*x) + 1i)*(a^3*c^3*4i - 4*a^3*d^3 - a^3*c*d^2*12i + 12*a^3*c^
2*d))/f - (tan(e + f*x)*(a^3*d^3*1i - a^3*c*(6*c*d + c^2*1i - d^2*3i)*1i -
```

```
a^3*d*(6*c*d + c^2*3i - d^2*1i) + a^3*c^2*(2*c - d*3i) + a^3*d^2*(c*3i + 2*
d)*1i))/f - (tan(e + f*x)^4*((a^3*d^3)/4 + (a^3*d^2*(c*3i + 2*d))/4))/f + (
tan(e + f*x)^3*((a^3*d^3*1i)/3 - (a^3*d*(6*c*d + c^2*3i - d^2*1i))/3 + (a^3
*d^2*(c*3i + 2*d)*1i)/3))/f + (tan(e + f*x)^2*((a^3*d^3)/2 - (a^3*c*(6*c*d
+ c^2*1i - d^2*3i))/2 + (a^3*d*(6*c*d + c^2*3i - d^2*1i)*1i)/2 + (a^3*d^2*(
c*3i + 2*d))/2))/f - (a^3*d^3*tan(e + f*x)^5*1i)/(5*f)
```

3.1078 $\int (a+ia\tan(e+fx))^2(c+d\tan(e+fx))^3\,dx$

Optimal. Leaf size=141

$$2a^2(c-id)^3x+\frac{2a^2(ic+d)^3\log(\cos(e+fx))}{f}+\frac{2ia^2(c-id)^2d\tan(e+fx)}{f}+\frac{a^2(ic+d)(c+d\tan(e+fx))^2}{f}+$$

```
[Out] 2*a^2*(c-I*d)^3*x+2*a^2*(I*c+d)^3*ln(cos(f*x+e))/f+2*I*a^2*(c-I*d)^2*d*tan(
f*x+e)/f+a^2*(I*c+d)*(c+d*tan(f*x+e))^2/f+2/3*I*a^2*(c+d*tan(f*x+e))^3/f-1/
4*a^2*(c+d*tan(f*x+e))^4/d/f
```

Rubi [A]
time = 0.14, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3624, 3609, 3606, 3556}

$$-\frac{a^2(c+d\tan(e+fx))^4}{4df}+\frac{2ia^2(c+d\tan(e+fx))^3}{3f}+\frac{a^2(d+ic)(c+d\tan(e+fx))^2}{f}+\frac{2ia^2d(c-id)^2\tan(e+fx)}{f}+\frac{2a^2(d+ic)^3\log(\cos(e+fx))}{f}+2a^2x(c-id)^3$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^3,x]
```

```
[Out] 2*a^2*(c - I*d)^3*x + (2*a^2*(I*c + d)^3*Log[Cos[e + f*x]])/f + ((2*I)*a^2*
(c - I*d)^2*d*Tan[e + f*x])/f + (a^2*(I*c + d)*(c + d*Tan[e + f*x])^2)/f +
(((2*I)/3)*a^2*(c + d*Tan[e + f*x])^3)/f - (a^2*(c + d*Tan[e + f*x])^4)/(4*
d*f)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int (a+ia\tan(e+fx))^2(c+d\tan(e+fx))^3\,dx = -\frac{a^2(c+d\tan(e+fx))^4}{4df}+\int \left(2a^2+2ia^2\tan(e+fx)\right)($$
$$=\frac{2ia^2(c+d\tan(e+fx))^3}{3f}-\frac{a^2(c+d\tan(e+fx))^4}{4df}+\int($$
$$=\frac{a^2(ic+d)(c+d\tan(e+fx))^2}{f}+\frac{2ia^2(c+d\tan(e+fx))}{3f}$$
$$=2a^2(c-id)^3x+\frac{2ia^2(c-id)^2d\tan(e+fx)}{f}+\frac{a^2(ic+d)(}{}$$
$$=2a^2(c-id)^3x+\frac{2a^2(ic+d)^3\log(\cos(e+fx))}{f}+\frac{2ia^2(c-}{}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 733 vs. 2(141) = 282.
time = 6.39, size = 733, normalized size = 5.20

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^3,x]
```

```
[Out] (Sec[e + f*x]^2*((c - I*d)^3*Cos[e + f*x]^4*Log[Cos[e + f*x]^2]*((-I)*Cos[2
*e] - Sin[2*e]) + 2*(c - I*d)^3*f*x*Cos[e + f*x]^4*(Cos[2*e] - I*Sin[2*e])
- 2*(c - I*d)^3*ArcTan[Tan[3*e + f*x]]*Cos[e + f*x]^4*(Cos[2*e] - I*Sin[2*e
]) + (Sec[e]*(Cos[2*e] - I*Sin[2*e])*(6*(3*c^3*f*x + 3*c*d^2*(2*I - 3*f*x)
+ d^3*(2 + (3*I)*f*x) + c^2*d*(-3 - (9*I)*f*x))*Cos[e] + 3*(c - I*d)^2*(-3*
d + 4*c*f*x - (4*I)*d*f*x)*Cos[e + 2*f*x] - 9*c^2*d*Cos[3*e + 2*f*x] + (18*
I)*c*d^2*Cos[3*e + 2*f*x] + 9*d^3*Cos[3*e + 2*f*x] + 12*c^3*f*x*Cos[3*e + 2
*f*x] - (36*I)*c^2*d*f*x*Cos[3*e + 2*f*x] - 36*c*d^2*f*x*Cos[3*e + 2*f*x] +
 (12*I)*d^3*f*x*Cos[3*e + 2*f*x] + 3*c^3*f*x*Cos[3*e + 4*f*x] - (9*I)*c^2*d
*f*x*Cos[3*e + 4*f*x] - 9*c*d^2*f*x*Cos[3*e + 4*f*x] + (3*I)*d^3*f*x*Cos[3*
e + 4*f*x] + 3*c^3*f*x*Cos[5*e + 4*f*x] - (9*I)*c^2*d*f*x*Cos[5*e + 4*f*x]
- 9*c*d^2*f*x*Cos[5*e + 4*f*x] + (3*I)*d^3*f*x*Cos[5*e + 4*f*x] + 9*c^3*Sin
```

[e] - (54*I)*c^2*d*Sin[e] - 63*c*d^2*Sin[e] + (24*I)*d^3*Sin[e] - 9*c^3*Sin
[e + 2*f*x] + (54*I)*c^2*d*Sin[e + 2*f*x] + 57*c*d^2*Sin[e + 2*f*x] - (20*I
)*d^3*Sin[e + 2*f*x] + 3*c^3*Sin[3*e + 2*f*x] - (18*I)*c^2*d*Sin[3*e + 2*f*
x] - 27*c*d^2*Sin[3*e + 2*f*x] + (12*I)*d^3*Sin[3*e + 2*f*x] - 3*c^3*Sin[3*
e + 4*f*x] + (18*I)*c^2*d*Sin[3*e + 4*f*x] + 21*c*d^2*Sin[3*e + 4*f*x] - (8
*I)*d^3*Sin[3*e + 4*f*x]))/24)*(a + I*a*Tan[e + f*x])^2)/(f*(Cos[f*x] + I*S
in[f*x])^2)

Maple [A]
time = 0.16, size = 210, normalized size = 1.49

method	result
derivativedivides	$a^2\left(\frac{2id^3(\tan^3(fx+e))}{3}-\frac{d^3(\tan^4(fx+e))}{4}+3icd^2(\tan^2(fx+e))-cd^2(\tan^3(fx+e))+6ic^2d\tan(fx+e)-2id^3\tan(fx+e)-\frac{3c\ldots}{}\right.$ …
default	$a^2\left(\frac{2id^3(\tan^3(fx+e))}{3}-\frac{d^3(\tan^4(fx+e))}{4}+3icd^2(\tan^2(fx+e))-cd^2(\tan^3(fx+e))+6ic^2d\tan(fx+e)-2id^3\tan(fx+e)-\frac{3c\ldots}{}\right.$ …
norman	$(-6ia^2c^2d+2ia^2d^3+2a^2c^3-6a^2cd^2)x-\frac{(-2ia^2d^3+3a^2cd^2)(\tan^3(fx+e))}{3f}+\frac{(6ia^2cd^2-3a^2c^2d+2a\ldots}{2f}$ …
risch	$-\frac{2ia^2\ln(e^{2i(fx+e)}+1)c^3}{f}+\frac{12ia^2c^2de}{f}-\frac{4ia^2d^3e}{f}+\frac{6ia^2\ln(e^{2i(fx+e)}+1)cd^2}{f}-\frac{4a^2c^3e}{f}+\frac{12a^2cd^2e}{f}+\frac{2a^2(99\ldots}{}$ …

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I*a*tan(f*x+e))^2*(c+d*tan(f*x+e))^3,x,method=_RETURNVERBOSE)

[Out] 1/f*a^2*(2/3*I*d^3*tan(f*x+e)^3-1/4*d^3*tan(f*x+e)^4+3*I*c*d^2*tan(f*x+e)^2
-c*d^2*tan(f*x+e)^3+6*I*c^2*d*tan(f*x+e)-2*I*d^3*tan(f*x+e)-3/2*c^2*d*tan(f
*x+e)^2+d^3*tan(f*x+e)^2-c^3*tan(f*x+e)+6*c*d^2*tan(f*x+e)+1/2*(-2*d^3+6*c^
2*d-6*I*c*d^2+2*I*c^3)*ln(1+tan(f*x+e)^2)+(-6*c*d^2+2*I*d^3+2*c^3-6*I*c^2*d
)*arctan(tan(f*x+e)))

Maxima [A]
time = 0.50, size = 224, normalized size = 1.59

$$-\frac{3\,a^2d^3\tan(fx+e)^4+4\,(3\,a^2cd^2-2i\,a^2d^3)\tan(fx+e)^3+6\,(3\,a^2c^2d-6i\,a^2cd^2-2\,a^2d^3)\tan(fx+e)^2-24\,(a^2c^3-3i\,a^2c^2d-3\,a^2cd^2+i\,a^2d^3)(fx+e)-12\,(i\,a^2c^3+3\,a^2c^2d-3i\,a^2cd^2-a^2d^3)\log\left(\tan(fx+e)^2+1\right)+12\,(a^2c^3-6i\,a^2c^2d-6\,a^2cd^2+2i\,a^2d^3)\tan(fx+e)}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I*a*tan(f*x+e))^2*(c+d*tan(f*x+e))^3,x, algorithm="maxima")

[Out] -1/12*(3*a^2*d^3*tan(f*x + e)^4 + 4*(3*a^2*c*d^2 - 2*I*a^2*d^3)*tan(f*x + e
)^3 + 6*(3*a^2*c^2*d - 6*I*a^2*c*d^2 - 2*a^2*d^3)*tan(f*x + e)^2 - 24*(a^2*
c^3 - 3*I*a^2*c^2*d - 3*a^2*c*d^2 + I*a^2*d^3)*(f*x + e) - 12*(I*a^2*c^3 +
3*a^2*c^2*d - 3*I*a^2*c*d^2 - a^2*d^3)*log(tan(f*x + e)^2 + 1) + 12*(a^2*c^
3 - 6*I*a^2*c^2*d - 6*a^2*c*d^2 + 2*I*a^2*d^3)*tan(f*x + e))/f

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 469 vs. 2(130) = 260.
time = 0.90, size = 469, normalized size = 3.33

$$-\frac{2\left(3i a^2c^3+18a^2c^2d-21i a^2cd^2-8a^2d^3+3\left(i a^2c^3+9a^2c^2d-15i a^2cd^2-7a^2d^3\right)e^{(6i fx+6i e)}+9\left(i a^2c^3+8a^2c^2d-11i a^2cd^2-4a^2d^3\right)e^{(4i fx+4i e)}+\left(9i a^2c^3+63a^2c^2d-75i a^2cd^2-29a^2d^3\right)e^{(2i fx+2i e)}+3\left(i a^2c^3+3a^2c^2d-3i a^2cd^2-a^2d^3+\left(i a^2c^3+3a^2c^2d-3i a^2cd^2-a^2d^3\right)e^{(8i fx+8i e)}+4\left(i a^2c^3+3a^2c^2d-3i a^2cd^2-a^2d^3\right)e^{(6i fx+6i e)}+6\left(i a^2c^3+3a^2c^2d-3i a^2cd^2-a^2d^3\right)e^{(4i fx+4i e)}+4\left(i a^2c^3+3a^2c^2d-3i a^2cd^2-a^2d^3\right)e^{(2i fx+2i e)}\right)\log\left(e^{(2i fx+2i e)}+1\right)\right)}{3\left(fe^{(8i fx+8i e)}+4fe^{(6i fx+6i e)}+6fe^{(4i fx+4i e)}+4fe^{(2i fx+2i e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c+d*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -2/3*(3*I*a^2*c^3 + 18*a^2*c^2*d - 21*I*a^2*c*d^2 - 8*a^2*d^3 + 3*(I*a^2*c^
3 + 9*a^2*c^2*d - 15*I*a^2*c*d^2 - 7*a^2*d^3)*e^(6*I*f*x + 6*I*e) + 9*(I*a^
2*c^3 + 8*a^2*c^2*d - 11*I*a^2*c*d^2 - 4*a^2*d^3)*e^(4*I*f*x + 4*I*e) + (9*
I*a^2*c^3 + 63*a^2*c^2*d - 75*I*a^2*c*d^2 - 29*a^2*d^3)*e^(2*I*f*x + 2*I*e)
 + 3*(I*a^2*c^3 + 3*a^2*c^2*d - 3*I*a^2*c*d^2 - a^2*d^3 + (I*a^2*c^3 + 3*a^
2*c^2*d - 3*I*a^2*c*d^2 - a^2*d^3)*e^(8*I*f*x + 8*I*e) + 4*(I*a^2*c^3 + 3*a
^2*c^2*d - 3*I*a^2*c*d^2 - a^2*d^3)*e^(6*I*f*x + 6*I*e) + 6*(I*a^2*c^3 + 3*
a^2*c^2*d - 3*I*a^2*c*d^2 - a^2*d^3)*e^(4*I*f*x + 4*I*e) + 4*(I*a^2*c^3 + 3
*a^2*c^2*d - 3*I*a^2*c*d^2 - a^2*d^3)*e^(2*I*f*x + 2*I*e))*log(e^(2*I*f*x +
 2*I*e) + 1))/(f*e^(8*I*f*x + 8*I*e) + 4*f*e^(6*I*f*x + 6*I*e) + 6*f*e^(4*I
*f*x + 4*I*e) + 4*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 381 vs. 2(122) = 244.
time = 0.73, size = 381, normalized size = 2.70

$$-\frac{2ia^2(c-id)^3\log\left(e^{2ifx}+e^{-2ie}\right)}{f}+\frac{-6ia^2c^3-36a^2c^2d+42ia^2cd^2+16a^2d^3+\left(-18ia^2c^3e^{2ie}-126a^2c^2de^{2ie}+150ia^2cd^2e^{2ie}+58a^2d^3e^{2ie}\right)e^{2ifx}+\left(-18ia^2c^3e^{4ie}-144a^2c^2de^{4ie}+198ia^2cd^2e^{4ie}+72a^2d^3e^{4ie}\right)e^{4ifx}+\left(-6ia^2c^3e^{6ie}-54a^2c^2de^{6ie}+90ia^2cd^2e^{6ie}+42a^2d^3e^{6ie}\right)e^{6ifx}}{3fe^{8ie}e^{8ifx}+12fe^{6ie}e^{6ifx}+18fe^{4ie}e^{4ifx}+12fe^{2ie}e^{2ifx}+3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2*(c+d*tan(f*x+e))**3,x)
```

```
[Out] -2*I*a**2*(c - I*d)**3*log(exp(2*I*f*x) + exp(-2*I*e))/f + (-6*I*a**2*c**3
- 36*a**2*c**2*d + 42*I*a**2*c*d**2 + 16*a**2*d**3 + (-18*I*a**2*c**3*exp(2
*I*e) - 126*a**2*c**2*d*exp(2*I*e) + 150*I*a**2*c*d**2*exp(2*I*e) + 58*a**2
*d**3*exp(2*I*e))*exp(2*I*f*x) + (-18*I*a**2*c**3*exp(4*I*e) - 144*a**2*c**
2*d*exp(4*I*e) + 198*I*a**2*c*d**2*exp(4*I*e) + 72*a**2*d**3*exp(4*I*e))*ex
p(4*I*f*x) + (-6*I*a**2*c**3*exp(6*I*e) - 54*a**2*c**2*d*exp(6*I*e) + 90*I*
a**2*c*d**2*exp(6*I*e) + 42*a**2*d**3*exp(6*I*e))*exp(6*I*f*x))/(3*f*exp(8*
I*e)*exp(8*I*f*x) + 12*f*exp(6*I*e)*exp(6*I*f*x) + 18*f*exp(4*I*e)*exp(4*I*
f*x) + 12*f*exp(2*I*e)*exp(2*I*f*x) + 3*f)
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 904 vs. 2(130) = 260.
time = 0.94, size = 904, normalized size = 6.41

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I*a*tan(f*x+e))^2*(c+d*tan(f*x+e))^3,x, algorithm="giac")

[Out] -2/3*(3*I*a^2*c^3*e^(8*I*f*x + 8*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 9*a^2*c^2*d*e^(8*I*f*x + 8*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) - 9*I*a^2*c*d^2*e^(8*I*f*x + 8*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) - 3*a^2*d^3*e^(8*I*f*x + 8*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 12*I*a^2*c^3*e^(6*I*f*x + 6*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 36*a^2*c^2*d*e^(6*I*f*x + 6*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) - 36*I*a^2*c*d^2*e^(6*I*f*x + 6*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) - 12*a^2*d^3*e^(6*I*f*x + 6*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 18*I*a^2*c^3*e^(4*I*f*x + 4*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 54*a^2*c^2*d*e^(4*I*f*x + 4*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) - 54*I*a^2*c*d^2*e^(4*I*f*x + 4*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) - 18*a^2*d^3*e^(4*I*f*x + 4*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 12*I*a^2*c^3*e^(2*I*f*x + 2*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 36*a^2*c^2*d*e^(2*I*f*x + 2*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) - 36*I*a^2*c*d^2*e^(2*I*f*x + 2*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) - 12*a^2*d^3*e^(2*I*f*x + 2*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 3*I*a^2*c^3*e^(6*I*f*x + 6*I*e) + 27*a^2*c^2*d*e^(6*I*f*x + 6*I*e) - 45*I*a^2*c*d^2*e^(6*I*f*x + 6*I*e) - 21*a^2*d^3*e^(6*I*f*x + 6*I*e) + 9*I*a^2*c^3*e^(4*I*f*x + 4*I*e) + 72*a^2*c^2*d*e^(4*I*f*x + 4*I*e) - 99*I*a^2*c*d^2*e^(4*I*f*x + 4*I*e) - 36*a^2*d^3*e^(4*I*f*x + 4*I*e) + 9*I*a^2*c^3*e^(2*I*f*x + 2*I*e) + 63*a^2*c^2*d*e^(2*I*f*x + 2*I*e) - 75*I*a^2*c*d^2*e^(2*I*f*x + 2*I*e) - 29*a^2*d^3*e^(2*I*f*x + 2*I*e) + 3*I*a^2*c^3*log(e^(2*I*f*x + 2*I*e) + 1) + 9*a^2*c^2*d*log(e^(2*I*f*x + 2*I*e) + 1) - 9*I*a^2*c*d^2*log(e^(2*I*f*x + 2*I*e) + 1) - 3*a^2*d^3*log(e^(2*I*f*x + 2*I*e) + 1) + 3*I*a^2*c^3 + 18*a^2*c^2*d - 21*I*a^2*c*d^2 - 8*a^2*d^3)/(f*e^(8*I*f*x + 8*I*e) + 4*f*e^(6*I*f*x + 6*I*e) + 6*f*e^(4*I*f*x + 4*I*e) + 4*f*e^(2*I*f*x + 2*I*e) + f)

Mupad [B]
time = 5.06, size = 223, normalized size = 1.58

$$\frac{\tan(e+f\,x)^2\,\left(\frac{a^2\,d^3}{2}+\frac{a^2\,d^2\,(d+c\,3\mathrm{i})}{2}+\frac{a^2\,c\,d\,(d+c\,1\mathrm{i})\,3\mathrm{i}}{2}\right)}{f}+\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(a^2\,c^3\,2\mathrm{i}+6\,a^2\,c^2\,d-a^2\,c\,d^2\,6\mathrm{i}-2\,a^2\,d^3\right)}{f}-\frac{\tan(e+f\,x)\,\left(a^2\,d^3\,1\mathrm{i}+a^2\,d^2\,(d+c\,3\mathrm{i})\,1\mathrm{i}-a^2\,c^2\,(3\,d+c\,1\mathrm{i})\,1\mathrm{i}-3\,a^2\,c\,d\,(d+c\,1\mathrm{i})\right)}{f}+\frac{\tan(e+f\,x)^3\,\left(\frac{a^2\,d^3\,1\mathrm{i}}{3}+\frac{a^2\,d^2\,(d+c\,3\mathrm{i})\,1\mathrm{i}}{3}\right)}{f}-\frac{a^2\,d^3\,\tan(e+f\,x)^4}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*tan(e + f*x)*1i)^2*(c + d*tan(e + f*x))^3,x)

[Out] (tan(e + f*x)^2*((a^2*d^3)/2 + (a^2*d^2*(c*3i + d))/2 + (a^2*c*d*(c*1i + d)*3i)/2))/f + (log(tan(e + f*x) + 1i)*(a^2*c^3*2i - 2*a^2*d^3 - a^2*c*d^2*6i + 6*a^2*c^2*d))/f - (tan(e + f*x)*(a^2*d^3*1i + a^2*d^2*(c*3i + d)*1i - a^2*c^2*(c*1i + 3*d)*1i - 3*a^2*c*d*(c*1i + d)))/f + (tan(e + f*x)^3*((a^2*d^3*1i)/3 + (a^2*d^2*(c*3i + d)*1i)/3))/f - (a^2*d^3*tan(e + f*x)^4)/(4*f)

3.1079 $\int (a+ia\tan(e+fx))(c+d\tan(e+fx))^3\,dx$

Optimal. Leaf size=107

$$a(c-id)^3x+\frac{a(ic+d)^3\log(\cos(e+fx))}{f}+\frac{ia(c-id)^2d\tan(e+fx)}{f}+\frac{a(ic+d)(c+d\tan(e+fx))^2}{2f}+\frac{ia(c+\cdots}{}$$

```
[Out] a*(c-I*d)^3*x+a*(I*c+d)^3*ln(cos(f*x+e))/f+I*a*(c-I*d)^2*d*tan(f*x+e)/f+1/2
*a*(I*c+d)*(c+d*tan(f*x+e))^2/f+1/3*I*a*(c+d*tan(f*x+e))^3/f
```

Rubi [A]

time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3609, 3606, 3556}

$$\frac{iad(c-id)^2\tan(e+fx)}{f}+\frac{ia(c+d\tan(e+fx))^3}{3f}+\frac{a(d+ic)(c+d\tan(e+fx))^2}{2f}+\frac{a(d+ic)^3\log(\cos(e+fx))}{f}+ax(c-id)^3$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])*(c + d*Tan[e + f*x])^3,x]
```

```
[Out] a*(c - I*d)^3*x + (a*(I*c + d)^3*Log[Cos[e + f*x]])/f + (I*a*(c - I*d)^2*d*
Tan[e + f*x])/f + (a*(I*c + d)*(c + d*Tan[e + f*x])^2)/(2*f) + ((I/3)*a*(c
+ d*Tan[e + f*x])^3)/f
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\int (a+ia\tan(e+fx))(c+d\tan(e+fx))^3\,dx = \frac{ia(c+d\tan(e+fx))^3}{3f} + \int (c+d\tan(e+fx))^2(a(c-id)$$
$$= \frac{a(ic+d)(c+d\tan(e+fx))^2}{2f} + \frac{ia(c+d\tan(e+fx))^3}{3f} +$$
$$= a(c-id)^3x + \frac{ia(c-id)^2d\tan(e+fx)}{f} + \frac{a(ic+d)(c+d\,\mathrm{ta}}{2f}$$
$$= a(c-id)^3x + \frac{a(ic+d)^3\log(\cos(e+fx))}{f} + \frac{ia(c-id)^2d\,\mathrm{ta}}{f}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 219 vs. 2(107) = 214.
time = 3.47, size = 219, normalized size = 2.05

$$\frac{(\cos(fx)-i\sin(fx))\left(12(c-id)^3fx\cos(e+fx)(\cos(e)-i\sin(e))-6(c-id)^3\mathrm{ArcTan}(\tan(2e+fx))\cos(e+fx)(\cos(e)-i\sin(e))-3i(c-id)^3\cos(e+fx)\log\left(\cos^2(e+fx)\right)(\cos(e)-i\sin(e))-2d\left(-9c^2+9icd+4d^2\right)\sin(fx)(i+\tan(e))+2d^3\sec^2(e+fx)\sin(fx)(i+\tan(e))+d^2\cos(e)\sec(e+fx)(i+\tan(e))(9c-3id+2d\tan(e))\right)(a+ia\tan(e+fx))}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])*(c + d*Tan[e + f*x])^3,x]
```

```
[Out] ((Cos[f*x] - I*Sin[f*x])*(12*(c - I*d)^3*f*x*Cos[e + f*x]*(Cos[e] - I*Sin[e
]) - 6*(c - I*d)^3*ArcTan[Tan[2*e + f*x]]*Cos[e + f*x]*(Cos[e] - I*Sin[e])
- (3*I)*(c - I*d)^3*Cos[e + f*x]*Log[Cos[e + f*x]^2]*(Cos[e] - I*Sin[e]) -
2*d*(-9*c^2 + (9*I)*c*d + 4*d^2)*Sin[f*x]*(I + Tan[e]) + 2*d^3*Sec[e + f*x]
^2*Sin[f*x]*(I + Tan[e]) + d^2*Cos[e]*Sec[e + f*x]*(I + Tan[e])*(9*c - (3*I
)*d + 2*d*Tan[e]))*(a + I*a*Tan[e + f*x]))/(6*f)
```

Maple [A]
time = 0.10, size = 155, normalized size = 1.45

method	result
norman	$\left(-3ia\,c^2d+ia\,d^3+a\,c^3-3ac\,d^2\right)x+\frac{\left(3iac\,d^2+a\,d^3\right)\left(\tan^2(fx+e)\right)}{2f}+\frac{ia\,d^3\left(\tan^3(fx+e)\right)}{3f}-\frac{iad(3icd-}{}$
derivativedivides	$\frac{a\left(\frac{id^3\left(\tan^3(fx+e)\right)}{3}+\frac{3ic\,d^2\left(\tan^2(fx+e)\right)}{2}+3ic^2d\tan(fx+e)-id^3\tan(fx+e)+\frac{d^3\left(\tan^2(fx+e)\right)}{2}+3c\,d^2\tan(fx+e)+\frac{(ic^3-3ic}{}\right.}{f}$
default	$\frac{a\left(\frac{id^3\left(\tan^3(fx+e)\right)}{3}+\frac{3ic\,d^2\left(\tan^2(fx+e)\right)}{2}+3ic^2d\tan(fx+e)-id^3\tan(fx+e)+\frac{d^3\left(\tan^2(fx+e)\right)}{2}+3c\,d^2\tan(fx+e)+\frac{(ic^3-3ic}{}\right.}{f}$
risch	$-\frac{2ia\,d^3e}{f}-\frac{ia\ln\left(\mathrm{e}^{2i(fx+e)}+1\right)c^3}{f}+\frac{3ia\ln\left(\mathrm{e}^{2i(fx+e)}+1\right)c\,d^2}{f}+\frac{6ia\,c^2de}{f}-\frac{2a\,c^3e}{f}+\frac{6ac\,d^2e}{f}+\frac{2ad(18icd\,\mathrm{e}^{4i(fx}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))*(c+d*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a*(1/3*I*d^3*tan(f*x+e)^3+3/2*I*c*d^2*tan(f*x+e)^2+3*I*c^2*d*tan(f*x+e)
-I*d^3*tan(f*x+e)+1/2*d^3*tan(f*x+e)^2+3*c*d^2*tan(f*x+e)+1/2*(-3*I*c*d^2-d
^3+I*c^3+3*c^2*d)*ln(1+tan(f*x+e)^2)+(I*d^3-3*I*c^2*d-3*c*d^2+c^3)*arctan(t
an(f*x+e)))
```

Maxima [A]
time = 0.51, size = 150, normalized size = 1.40

$$-\frac{-2i\,ad^3\tan\left(fx+e\right)^3+3\left(-3i\,acd^2-ad^3\right)\tan\left(fx+e\right)^2-6\left(ac^3-3i\,ac^2d-3\,acd^2+i\,ad^3\right)(fx+e)+3\left(-i\,ac^3-3\,ac^2d+3i\,acd^2+ad^3\right)\log\left(\tan\left(fx+e\right)^2+1\right)+6\left(-3i\,ac^2d-3\,acd^2+i\,ad^3\right)\tan\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c+d*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/6*(-2*I*a*d^3*tan(f*x + e)^3 + 3*(-3*I*a*c*d^2 - a*d^3)*tan(f*x + e)^2 -
 6*(a*c^3 - 3*I*a*c^2*d - 3*a*c*d^2 + I*a*d^3)*(f*x + e) + 3*(-I*a*c^3 - 3*
a*c^2*d + 3*I*a*c*d^2 + a*d^3)*log(tan(f*x + e)^2 + 1) + 6*(-3*I*a*c^2*d -
3*a*c*d^2 + I*a*d^3)*tan(f*x + e))/f
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(95) = 190.
time = 1.08, size = 288, normalized size = 2.69

$$-\frac{18\,ac^2d-18i\,acd^2-8\,ad^3+18\left(ac^2d-2i\,acd^2-ad^3\right)e^{(4i\,fx+4i\,e)}+18\left(2\,ac^2d-3i\,acd^2-ad^3\right)e^{(2i\,fx+2i\,e)}+3\left(i\,ac^3+3\,ac^2d-3i\,acd^2-ad^3+\left(i\,ac^3+3\,ac^2d-3i\,acd^2-ad^3\right)e^{(6i\,fx+6i\,e)}+3\left(i\,ac^3+3\,ac^2d-3i\,acd^2-ad^3\right)e^{(4i\,fx+4i\,e)}+3\left(i\,ac^3+3\,ac^2d-3i\,acd^2-ad^3\right)e^{(2i\,fx+2i\,e)}\right)\log\left(e^{(2i\,fx+2i\,e)}+1\right)}{3\left(fe^{(6i\,fx+6i\,e)}+3\,fe^{(4i\,fx+4i\,e)}+3\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c+d*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/3*(18*a*c^2*d - 18*I*a*c*d^2 - 8*a*d^3 + 18*(a*c^2*d - 2*I*a*c*d^2 - a*d
^3)*e^(4*I*f*x + 4*I*e) + 18*(2*a*c^2*d - 3*I*a*c*d^2 - a*d^3)*e^(2*I*f*x +
 2*I*e) + 3*(I*a*c^3 + 3*a*c^2*d - 3*I*a*c*d^2 - a*d^3 + (I*a*c^3 + 3*a*c^2
*d - 3*I*a*c*d^2 - a*d^3)*e^(6*I*f*x + 6*I*e) + 3*(I*a*c^3 + 3*a*c^2*d - 3*
I*a*c*d^2 - a*d^3)*e^(4*I*f*x + 4*I*e) + 3*(I*a*c^3 + 3*a*c^2*d - 3*I*a*c*d
^2 - a*d^3)*e^(2*I*f*x + 2*I*e))*log(e^(2*I*f*x + 2*I*e) + 1))/(f*e^(6*I*f*
x + 6*I*e) + 3*f*e^(4*I*f*x + 4*I*e) + 3*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(88) = 176.
time = 0.50, size = 223, normalized size = 2.08

$$-\frac{ia(c-id)^3\log\left(e^{2ifx}+e^{-2ie}\right)}{f}+\frac{-18ac^2d+18iacd^2+8ad^3+\left(-36ac^2de^{2ie}+54iacd^2e^{2ie}+18ad^3e^{2ie}\right)e^{2ifx}+\left(-18ac^2de^{4ie}+36iacd^2e^{4ie}+18ad^3e^{4ie}\right)e^{4ifx}}{3fe^{6ie}e^{6ifx}+9fe^{4ie}e^{4ifx}+9fe^{2ie}e^{2ifx}+3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c+d*tan(f*x+e))**3,x)
```

```
[Out] -I*a*(c - I*d)**3*log(exp(2*I*f*x) + exp(-2*I*e))/f + (-18*a*c**2*d + 18*I*
a*c*d**2 + 8*a*d**3 + (-36*a*c**2*d*exp(2*I*e) + 54*I*a*c*d**2*exp(2*I*e) +
```

```
 18*a*d**3*exp(2*I*e))*exp(2*I*f*x) + (-18*a*c**2*d*exp(4*I*e) + 36*I*a*c*d
**2*exp(4*I*e) + 18*a*d**3*exp(4*I*e))*exp(4*I*f*x))/(3*f*exp(6*I*e)*exp(6*
I*f*x) + 9*f*exp(4*I*e)*exp(4*I*f*x) + 9*f*exp(2*I*e)*exp(2*I*f*x) + 3*f)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(95) = 190$.
time = 0.75, size = 597, normalized size = 5.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c+d*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/3*(-3*I*a*c^3*e^(6*I*f*x + 6*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) - 9*a*c^2*
d*e^(6*I*f*x + 6*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 9*I*a*c*d^2*e^(6*I*f*x
 + 6*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 3*a*d^3*e^(6*I*f*x + 6*I*e)*log(e^
(2*I*f*x + 2*I*e) + 1) - 9*I*a*c^3*e^(4*I*f*x + 4*I*e)*log(e^(2*I*f*x + 2*I
*e) + 1) - 27*a*c^2*d*e^(4*I*f*x + 4*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 27
*I*a*c*d^2*e^(4*I*f*x + 4*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 9*a*d^3*e^(4*
I*f*x + 4*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) - 9*I*a*c^3*e^(2*I*f*x + 2*I*e)
*log(e^(2*I*f*x + 2*I*e) + 1) - 27*a*c^2*d*e^(2*I*f*x + 2*I*e)*log(e^(2*I*f
*x + 2*I*e) + 1) + 27*I*a*c*d^2*e^(2*I*f*x + 2*I*e)*log(e^(2*I*f*x + 2*I*e)
 + 1) + 9*a*d^3*e^(2*I*f*x + 2*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) - 18*a*c^2
*d*e^(4*I*f*x + 4*I*e) + 36*I*a*c*d^2*e^(4*I*f*x + 4*I*e) + 18*a*d^3*e^(4*I
*f*x + 4*I*e) - 36*a*c^2*d*e^(2*I*f*x + 2*I*e) + 54*I*a*c*d^2*e^(2*I*f*x +
2*I*e) + 18*a*d^3*e^(2*I*f*x + 2*I*e) - 3*I*a*c^3*log(e^(2*I*f*x + 2*I*e) +
 1) - 9*a*c^2*d*log(e^(2*I*f*x + 2*I*e) + 1) + 9*I*a*c*d^2*log(e^(2*I*f*x +
 2*I*e) + 1) + 3*a*d^3*log(e^(2*I*f*x + 2*I*e) + 1) - 18*a*c^2*d + 18*I*a*c
*d^2 + 8*a*d^3)/(f*e^(6*I*f*x + 6*I*e) + 3*f*e^(4*I*f*x + 4*I*e) + 3*f*e^(2
*I*f*x + 2*I*e) + f)
```

Mupad [B]
time = 5.07, size = 122, normalized size = 1.14

$$\frac{\tan(e+f\,x)\,\left(3\mathrm{i}\,a\,c^2\,d+3\,a\,c\,d^2-1\mathrm{i}\,a\,d^3\right)}{f}+\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(1\mathrm{i}\,a\,c^3+3\,a\,c^2\,d-3\mathrm{i}\,a\,c\,d^2-a\,d^3\right)}{f}+\frac{\tan(e+f\,x)^2\,\left(\frac{a\,d^3}{2}+\frac{3\mathrm{i}\,a\,c\,d^2}{2}\right)}{f}+\frac{a\,d^3\,\tan(e+f\,x)^3\,1\mathrm{i}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)*(c + d*tan(e + f*x))^3,x)
```

```
[Out] (tan(e + f*x)*(3*a*c*d^2 - a*d^3*1i + a*c^2*d*3i))/f + (log(tan(e + f*x) +
1i)*(a*c^3*1i - a*d^3 - a*c*d^2*3i + 3*a*c^2*d))/f + (tan(e + f*x)^2*((a*d^
3)/2 + (a*c*d^2*3i)/2))/f + (a*d^3*tan(e + f*x)^3*1i)/(3*f)
```

3.1080 $\int \frac{(c+d\tan(e+fx))^3}{a+ia\tan(e+fx)}\,dx$

Optimal. Leaf size=129

$$\frac{(c^3-3ic^2d+3cd^2+3id^3)\,x}{2a}+\frac{(3ic-d)d^2\log(\cos(e+fx))}{af}-\frac{(c+3id)d^2\tan(e+fx)}{2af}+\frac{(ic-d)(c+d\tan(e$$

[Out] 1/2*(c^3-3*I*c^2*d+3*c*d^2+3*I*d^3)*x/a+(3*I*c-d)*d^2*ln(cos(f*x+e))/a/f-1/2*(c+3*I*d)*d^2*tan(f*x+e)/a/f+1/2*(I*c-d)*(c+d*tan(f*x+e))^2/f/(a+I*a*tan(f*x+e))

Rubi [A]
time = 0.11, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3631, 3606, 3556}

$$\frac{x(c^3-3ic^2d+3cd^2+3id^3)}{2a}-\frac{d^2(c+3id)\tan(e+fx)}{2af}+\frac{d^2(-d+3ic)\log(\cos(e+fx))}{af}+\frac{(-d+ic)(c+d\tan(e+fx))^2}{2f(a+ia\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^3/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] ((c^3 - (3*I)*c^2*d + 3*c*d^2 + (3*I)*d^3)*x)/(2*a) + (((3*I)*c - d)*d^2*Log[Cos[e + f*x]])/(a*f) - ((c + (3*I)*d)*d^2*Tan[e + f*x])/(2*a*f) + ((I*c - d)*(c + d*Tan[e + f*x])^2)/(2*f*(a + I*a*Tan[e + f*x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e + f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3631

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a^2), Int[(c + d*Tan[e + f*x])^(n - 2)*Simp[a*c^2 + a*d^2*(n - 1) - b*c*d*n - d*(a*c*(n - 2) + b*d*n)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^3}{a+ia\tan(e+fx)}\,dx = \frac{(ic-d)(c+d\tan(e+fx))^2}{2f(a+ia\tan(e+fx))} + \frac{\int (c+d\tan(e+fx))\left(a(c^2-3icd+2d^2)-a(c\right.}{2a^2}$$
$$= \frac{(c^3-3ic^2d+3cd^2+3id^3)\,x}{2a} - \frac{(c+3id)d^2\tan(e+fx)}{2af} + \frac{(ic-d)(c+d\tan(e}{2f(a+ia\tan(e+}$$
$$= \frac{(c^3-3ic^2d+3cd^2+3id^3)\,x}{2a} + \frac{(3ic-d)d^2\log(\cos(e+fx))}{af} - \frac{(c+3id)d^2\tan(}{2af}$$

Mathematica [A]
time = 2.04, size = 236, normalized size = 1.83

$$\frac{\sec(e+fx)(\cos(fx)+i\sin(fx))\left(-4(3c+id)d^2fx(\cos(e)+i\sin(e))+2(c^3-3ic^2d+3cd^2+3id^3)\,fx(\cos(e)+i\sin(e))+4(3c+id)d^2\mathrm{ArcTan}(\tan(fx))(\cos(e)+i\sin(e))+2i(3c+id)d^2\log\left(\cos^2(e+fx)\right)(\cos(e)+i\sin(e))+(c+id)^3\cos(2fx)(i\cos(e)+\sin(e))+(c+id)^3(\cos(e)-i\sin(e))\sin(2fx)+4d^3\sec(e+fx)\sin(fx)(-i+\tan(e))\right)}{4f(a+ia\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^3/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (Sec[e + f*x]*(Cos[f*x] + I*Sin[f*x])*(-4*(3*c + I*d)*d^2*f*x*(Cos[e] + I*S
in[e]) + 2*(c^3 - (3*I)*c^2*d + 3*c*d^2 + (3*I)*d^3)*f*x*(Cos[e] + I*Sin[e]
) + 4*(3*c + I*d)*d^2*ArcTan[Tan[f*x]]*(Cos[e] + I*Sin[e]) + (2*I)*(3*c + I
*d)*d^2*Log[Cos[e + f*x]^2]*(Cos[e] + I*Sin[e]) + (c + I*d)^3*Cos[2*f*x]*(I
*Cos[e] + Sin[e]) + (c + I*d)^3*(Cos[e] - I*Sin[e])*Sin[2*f*x] + 4*d^3*Sec[
e + f*x]*Sin[f*x]*(-I + Tan[e])))/(4*f*(a + I*a*Tan[e + f*x]))
```

Maple [A]
time = 0.24, size = 133, normalized size = 1.03

method	result
derivativedivides	$\frac{-id^3\tan(fx+e)+\left(-\frac{3}{4}c^2d+\frac{5}{4}d^3-\frac{1}{4}ic^3-\frac{9}{4}icd^2\right)\ln(\tan(fx+e)-i)-\frac{-\frac{3}{2}ic^2d+\frac{1}{2}id^3-\frac{1}{2}c^3+\frac{3}{2}cd^2}{\tan(fx+e)-i}-\frac{i\left(3ic^2d-id^3-c^3+3cd^2\right)\ln(\mathrm{t}}{4}}{fa}$
default	$\frac{-id^3\tan(fx+e)+\left(-\frac{3}{4}c^2d+\frac{5}{4}d^3-\frac{1}{4}ic^3-\frac{9}{4}icd^2\right)\ln(\tan(fx+e)-i)-\frac{-\frac{3}{2}ic^2d+\frac{1}{2}id^3-\frac{1}{2}c^3+\frac{3}{2}cd^2}{\tan(fx+e)-i}-\frac{i\left(3ic^2d-id^3-c^3+3cd^2\right)\ln(\mathrm{t}}{4}}{fa}$
risch	$-\frac{3ixc^2d}{2a}+\frac{5ixd^3}{2a}+\frac{c^3x}{2a}+\frac{9xcd^2}{2a}-\frac{3\,\mathrm{e}^{-2i(fx+e)}c^2d}{4af}+\frac{\mathrm{e}^{-2i(fx+e)}d^3}{4af}+\frac{i\mathrm{e}^{-2i(fx+e)}c^3}{4af}-\frac{3i\mathrm{e}^{-2i(fx+e)}cd^2}{4af}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^3/(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a*(-I*d^3*tan(f*x+e)+(-3/4*c^2*d+5/4*d^3-1/4*I*c^3-9/4*I*c*d^2)*ln(tan(
f*x+e)-I)-(-3/2*I*c^2*d+1/2*I*d^3-1/2*c^3+3/2*c*d^2)/(tan(f*x+e)-I)-1/4*I*(
3*I*c^2*d-I*d^3-c^3+3*c*d^2)*ln(tan(f*x+e)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.77, size = 204, normalized size = 1.58

$$\frac{2\,(c^3-3i\,c^2d+9\,cd^2+5i\,d^3)fxe^{(4i\,fx+4i\,e)}+i\,c^3-3\,c^2d-3i\,cd^2+d^3+(i\,c^3-3\,c^2d-3i\,cd^2+9\,d^3+2\,(c^3-3i\,c^2d+9\,cd^2+5i\,d^3)fx)e^{(2i\,fx+2i\,e)}-4\,((-3i\,cd^2+d^3)e^{(4i\,fx+4i\,e)}+(-3i\,cd^2+d^3)e^{(2i\,fx+2i\,e)})\log\left(e^{(2i\,fx+2i\,e)}+1\right)}{4\,(afe^{(4i\,fx+4i\,e)}+afe^{(2i\,fx+2i\,e)})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(c^3 - 3*I*c^2*d + 9*c*d^2 + 5*I*d^3)*f*x*e^(4*I*f*x + 4*I*e) + I*c^
3 - 3*c^2*d - 3*I*c*d^2 + d^3 + (I*c^3 - 3*c^2*d - 3*I*c*d^2 + 9*d^3 + 2*(c
^3 - 3*I*c^2*d + 9*c*d^2 + 5*I*d^3)*f*x)*e^(2*I*f*x + 2*I*e) - 4*((-3*I*c*d
^2 + d^3)*e^(4*I*f*x + 4*I*e) + (-3*I*c*d^2 + d^3)*e^(2*I*f*x + 2*I*e))*log
(e^(2*I*f*x + 2*I*e) + 1))/(a*f*e^(4*I*f*x + 4*I*e) + a*f*e^(2*I*f*x + 2*I*
e))
```

Sympy [A]
time = 0.44, size = 260, normalized size = 2.02

$$\frac{2d^3}{afe^{2ie}e^{2ifx}+af}+\begin{cases}\frac{(ic^3-3c^2d-3icd^2+d^3)e^{-2ie}e^{-2ifx}}{4af} & \text{for } afe^{2ie}\neq 0\\ x\left(-\frac{c^3-3ic^2d+9cd^2+5id^3}{2a}+\frac{(c^3e^{2ie}+c^3-3ic^2de^{2ie}+3ic^2d+9cd^2e^{2ie}-3cd^2+5id^3e^{2ie}-id^3)e^{-2ie}}{2a}\right) & \text{otherwise}\end{cases}+\frac{id^2\cdot(3c+id)\log\left(e^{2ifx}+e^{-2ie}\right)}{af}+\frac{x(c^3-3ic^2d+9cd^2+5id^3)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**3/(a+I*a*tan(f*x+e)),x)
```

```
[Out] 2*d**3/(a*f*exp(2*I*e)*exp(2*I*f*x) + a*f) + Piecewise((((I*c**3 - 3*c**2*d
 - 3*I*c*d**2 + d**3)*exp(-2*I*e)*exp(-2*I*f*x)/(4*a*f), Ne(a*f*exp(2*I*e),
0)), (x*(-(c**3 - 3*I*c**2*d + 9*c*d**2 + 5*I*d**3)/(2*a) + (c**3*exp(2*I*e
) + c**3 - 3*I*c**2*d*exp(2*I*e) + 3*I*c**2*d + 9*c*d**2*exp(2*I*e) - 3*c*d
**2 + 5*I*d**3*exp(2*I*e) - I*d**3)*exp(-2*I*e)/(2*a)), True)) + I*d**2*(3*
c + I*d)*log(exp(2*I*f*x) + exp(-2*I*e))/(a*f) + x*(c**3 - 3*I*c**2*d + 9*c
*d**2 + 5*I*d**3)/(2*a)
```

Giac [A]
time = 0.73, size = 186, normalized size = 1.44

$$-\frac{\frac{4i\,d^3\tan(fx+e)}{a}-\frac{(i\,c^3+3\,c^2d-3i\,cd^2-d^3)\log(\tan(fx+e)+i)}{a}+\frac{(i\,c^3+3\,c^2d+9i\,cd^2-5\,d^3)\log(-i\,\tan(fx+e)-1)}{a}+\frac{-i\,c^3\tan(fx+e)-3\,c^2d\tan(fx+e)-9i\,cd^2\tan(fx+e)+5\,d^3\tan(fx+e)-3\,c^3-3i\,c^2d-3\,cd^2-3i\,d^3}{a(\tan(fx+e)-i)}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/4*(4*I*d^3*tan(f*x + e)/a - (I*c^3 + 3*c^2*d - 3*I*c*d^2 - d^3)*log(tan(
f*x + e) + I)/a + (I*c^3 + 3*c^2*d + 9*I*c*d^2 - 5*d^3)*log(-I*tan(f*x + e)
 - 1)/a + (-I*c^3*tan(f*x + e) - 3*c^2*d*tan(f*x + e) - 9*I*c*d^2*tan(f*x +
 e) + 5*d^3*tan(f*x + e) - 3*c^3 - 3*I*c^2*d - 3*c*d^2 - 3*I*d^3)/(a*(tan(f
*x + e) - I)))/f
```

Mupad [B]
time = 5.66, size = 175, normalized size = 1.36

$$-\frac{\frac{3\,c^2\,d-d^3}{2\,a}+\frac{\left(d^3\,1{}\mathrm{i}+3\,c\,d^2\right)\,1{}\mathrm{i}}{2\,a}-\frac{-d^3+c^3\,1{}\mathrm{i}}{2\,a}}{f\,\left(1+\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}-\frac{d^3\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}}{a\,f}-\frac{\ln\left(\mathrm{tan}\left(e+f\,x\right)+1{}\mathrm{i}\right)\,\left(-c^3\,1{}\mathrm{i}-3\,c^2\,d+c\,d^2\,3{}\mathrm{i}+d^3\right)}{4\,a\,f}-\frac{\ln\left(\mathrm{tan}\left(e+f\,x\right)-\mathrm{i}\right)\,\left(c^3\,1{}\mathrm{i}+3\,c^2\,d+c\,d^2\,9{}\mathrm{i}-5\,d^3\right)}{4\,a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^3/(a + a*tan(e + f*x)*1i),x)
```

```
[Out] - ((3*c^2*d - d^3)/(2*a) + ((3*c*d^2 + d^3*1i)*1i)/(2*a) - (c^3*1i - d^3)/(
2*a))/(f*(tan(e + f*x)*1i + 1)) - (d^3*tan(e + f*x)*1i)/(a*f) - (log(tan(e
+ f*x) + 1i)*(c*d^2*3i - 3*c^2*d - c^3*1i + d^3))/(4*a*f) - (log(tan(e + f*
x) - 1i)*(c*d^2*9i + 3*c^2*d + c^3*1i - 5*d^3))/(4*a*f)
```

3.1081 $\int \frac{(c+d\tan(e+fx))^3}{(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=136

$$\frac{(c^3-3ic^2d-3cd^2-3id^3)\,x}{4a^2}+\frac{d^3\log(\cos(e+fx))}{a^2f}+\frac{(c+id)^2(ic+3d)}{4a^2f(1+i\tan(e+fx))}+\frac{(ic-d)(c+d\tan(e+fx))^2}{4f(a+ia\tan(e+fx))^2}$$

```
[Out] 1/4*(c^3-3*I*c^2*d-3*c*d^2-3*I*d^3)*x/a^2+d^3*ln(cos(f*x+e))/a^2/f+1/4*(c+I
*d)^2*(I*c+3*d)/a^2/f/(1+I*tan(f*x+e))+1/4*(I*c-d)*(c+d*tan(f*x+e))^2/f/(a+
I*a*tan(f*x+e))^2
```

Rubi [A]
time = 0.21, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}=0.179$, Rules used = {3639, 3670, 3556, 3607, 8}

$$\frac{x(c^3-3ic^2d-3cd^2-3id^3)}{4a^2}+\frac{(c+id)^2(3d+ic)}{4a^2f(1+i\tan(e+fx))}+\frac{d^3\log(\cos(e+fx))}{a^2f}+\frac{(-d+ic)(c+d\tan(e+fx))^2}{4f(a+ia\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^3/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] ((c^3 - (3*I)*c^2*d - 3*c*d^2 - (3*I)*d^3)*x)/(4*a^2) + (d^3*Log[Cos[e + f*
x]])/(a^2*f) + ((c + I*d)^2*(I*c + 3*d))/(4*a^2*f*(1 + I*Tan[e + f*x])) + (
(I*c - d)*(c + d*Tan[e + f*x])^2)/(4*f*(a + I*a*Tan[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
```

```
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3670

```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)]))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[B*(d/
b), Int[Tan[e + f*x], x], x] + Dist[1/b, Int[Simp[A*b*c + (A*b*d + B*(b*c -
 a*d))*Tan[e + f*x], x]/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d,
e, f, A, B}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^3}{(a+ia\tan(e+fx))^2}\,dx = \frac{(ic-d)(c+d\tan(e+fx))^2}{4f(a+ia\tan(e+fx))^2} - \frac{\int \frac{(c+d\tan(e+fx))\left(-2a\left(c^2-2icd+d^2\right)+4iad^2\tan(e+fx)\right)}{a+ia\tan(e+fx)}\,dx}{4a^2}$$

$$= \frac{(ic-d)(c+d\tan(e+fx))^2}{4f(a+ia\tan(e+fx))^2} + \frac{i\int \frac{-2a^2c\left(2cd+i\left(c^2+d^2\right)\right)-2a^2d\left(ic^2+4cd+3id^2\right)\tan(e+fx)}{a+ia\tan(e+fx)}}{4a^3}$$

$$= \frac{d^3\log(\cos(e+fx))}{a^2f} + \frac{(c+id)^2(ic+3d)}{4f\left(a^2+ia^2\tan(e+fx)\right)} + \frac{(ic-d)(c+d\tan(e+fx)}{4f(a+ia\tan(e+fx))^2}$$

$$= \frac{\left(c^3-3ic^2d-3cd^2-3id^3\right)x}{4a^2} + \frac{d^3\log(\cos(e+fx))}{a^2f} + \frac{(c+id)^2(ic+3d)}{4f\left(a^2+ia^2\tan(e+fx\right.}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 305 vs. 2(136) = 272.
time = 2.03, size = 305, normalized size = 2.24

$$-\frac{\sec^2(e+fx)\left(4ic^3+12icd^2-8d^3+\cos(2(e+fx))\left(3c^2d(-1-4ifx)+d^3(1+4ifx)+c^3(i+4fx)-3cd^2(i+4fx)+8d^3\log\left(\cos^2(e+fx)\right)\right)+c^3\sin(2(e+fx))+3ic^2d\sin(2(e+fx))-3cd^2\sin(2(e+fx))-id^3\sin(2(e+fx))+4ic^3fx\sin(2(e+fx))+12c^2dfx\sin(2(e+fx))-12icd^2fx\sin(2(e+fx))-4d^3fx\sin(2(e+fx))+8id^3\log\left(\cos^2(e+fx)\right)\sin(2(e+fx))+16d^3\mathrm{ArcTan}(\tan(fx))(-i\cos(2(e+fx))+\sin(2(e+fx)))\right)}{16a^2f(-i+\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^3/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] -1/16*(Sec[e + f*x]^2*((4*I)*c^3 + (12*I)*c*d^2 - 8*d^3 + Cos[2*(e + f*x)]*
(3*c^2*d*(-1 - (4*I)*f*x) + d^3*(1 + (4*I)*f*x) + c^3*(I + 4*f*x) - 3*c*d^2
*(I + 4*f*x) + 8*d^3*Log[Cos[e + f*x]^2]) + c^3*Sin[2*(e + f*x)] + (3*I)*c^
2*d*Sin[2*(e + f*x)] - 3*c*d^2*Sin[2*(e + f*x)] - I*d^3*Sin[2*(e + f*x)] +
(4*I)*c^3*f*x*Sin[2*(e + f*x)] + 12*c^2*d*f*x*Sin[2*(e + f*x)] - (12*I)*c*d
```

```
^2*f*x*Sin[2*(e + f*x)] - 4*d^3*f*x*Sin[2*(e + f*x)] + (8*I)*d^3*Log[Cos[e
+ f*x]^2]*Sin[2*(e + f*x)] + 16*d^3*ArcTan[Tan[f*x]]*((-I)*Cos[2*(e + f*x)]
 + Sin[2*(e + f*x)])))/(a^2*f*(-I + Tan[e + f*x])^2)
```

Maple [A]
time = 0.24, size = 159, normalized size = 1.17

method	result
derivativedivides	$\frac{\left(-\frac{1}{8}ic^{3}+\frac{3}{8}icd^{2}-\frac{3}{8}c^{2}d-\frac{7}{8}d^{3}\right)\ln(\tan(fx+e)-i)-\frac{-\frac{3}{2}c^{2}d+\frac{1}{2}d^{3}+\frac{1}{2}ic^{3}-\frac{3}{2}icd^{2}}{2(\tan(fx+e)-i)^{2}}-\frac{\frac{3}{4}ic^{2}d-\frac{5}{4}id^{3}-\frac{1}{4}c^{3}-\frac{9}{4}cd^{2}}{\tan(fx+e)-i}-\frac{i\left(3ic^{2}d-id^{3}-c^{3}\right.}{}}{f\,a^{2}}$
default	$\frac{\left(-\frac{1}{8}ic^{3}+\frac{3}{8}icd^{2}-\frac{3}{8}c^{2}d-\frac{7}{8}d^{3}\right)\ln(\tan(fx+e)-i)-\frac{-\frac{3}{2}c^{2}d+\frac{1}{2}d^{3}+\frac{1}{2}ic^{3}-\frac{3}{2}icd^{2}}{2(\tan(fx+e)-i)^{2}}-\frac{\frac{3}{4}ic^{2}d-\frac{5}{4}id^{3}-\frac{1}{4}c^{3}-\frac{9}{4}cd^{2}}{\tan(fx+e)-i}-\frac{i\left(3ic^{2}d-id^{3}-c^{3}\right.}{}}{f\,a^{2}}$
risch	$-\frac{3ixc^{2}d}{4a^{2}}-\frac{7ixd^{3}}{4a^{2}}+\frac{c^{3}x}{4a^{2}}-\frac{3xcd^{2}}{4a^{2}}-\frac{e^{-2i(fx+e)}d^{3}}{2a^{2}f}+\frac{ie^{-2i(fx+e)}c^{3}}{4a^{2}f}+\frac{3ie^{-2i(fx+e)}cd^{2}}{4a^{2}f}-\frac{3\,e^{-4i(fx+e)}c^{2}d}{16a^{2}f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a^2*((-1/8*I*c^3+3/8*I*c*d^2-3/8*c^2*d-7/8*d^3)*ln(tan(f*x+e)-I)-1/2*(-
3/2*c^2*d+1/2*d^3+1/2*I*c^3-3/2*I*c*d^2)/(tan(f*x+e)-I)^2-(3/4*I*c^2*d-5/4*
I*d^3-1/4*c^3-9/4*c*d^2)/(tan(f*x+e)-I)-1/8*I*(3*I*c^2*d-I*d^3-c^3+3*c*d^2)
*ln(tan(f*x+e)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.04, size = 131, normalized size = 0.96

$$\frac{\left(16\,d^{3}e^{(4i\,fx+4i\,e)}\log\left(e^{(2i\,fx+2i\,e)}+1\right)+4\left(c^{3}-3i\,c^{2}d-3\,cd^{2}-7i\,d^{3}\right)fxe^{(4i\,fx+4i\,e)}+i\,c^{3}-3\,c^{2}d-3i\,cd^{2}+d^{3}-4\left(-i\,c^{3}-3i\,cd^{2}+2\,d^{3}\right)e^{(2i\,fx+2i\,e)}\right)e^{(-4i\,fx-4i\,e)}}{16\,a^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(16*d^3*e^(4*I*f*x + 4*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + 4*(c^3 - 3*
I*c^2*d - 3*c*d^2 - 7*I*d^3)*f*x*e^(4*I*f*x + 4*I*e) + I*c^3 - 3*c^2*d - 3*
I*c*d^2 + d^3 - 4*(-I*c^3 - 3*I*c*d^2 + 2*d^3)*e^(2*I*f*x + 2*I*e))*e^(-4*I
*f*x - 4*I*e)/(a^2*f)
```

Sympy [A]

time = 0.49, size = 393, normalized size = 2.89

$$\begin{cases} \frac{\left(\left(16 i a^{2} c^{3} f e^{4 i e} + 48 i a^{2} c d^{2} f e^{4 i e} - 32 a^{2} d^{3} f e^{4 i e}\right) e^{- 2 i f x} + \left(4 i a^{2} c^{3} f e^{2 i e} - 12 a^{2} c^{2} d f e^{2 i e} - 12 i a^{2} c d^{2} f e^{2 i e} + 4 a^{2} d^{3} f e^{2 i e}\right) e^{- 4 i f x}\right) e^{- 6 i e}}{64 a^{4} f^{2}} & \text{for}\: a^{4} f^{2} e^{6 i e} \neq 0 \\ x \left(- \frac{c^{3} - 3 i c^{2} d - 3 c d^{2} - 7 i d^{3}}{4 a^{2}} + \frac{\left(c^{3} e^{4 i e} + 2 c^{3} e^{2 i e} + c^{3} - 3 i c^{2} d e^{4 i e} + 3 i c^{2} d - 3 c d^{2} e^{4 i e} + 6 c d^{2} e^{2 i e} - 3 c d^{2} - 7 i d^{3} e^{4 i e} + 4 i d^{3} e^{2 i e} - i d^{3}\right) e^{- 4 i e}}{4 a^{2}}\right) & \text{otherwise} \end{cases} + \frac{d^{3} \log{\left(e^{2 i f x} + e^{- 2 i e} \right)}}{a^{2} f} + \frac{x \left(c^{3} - 3 i c^{2} d - 3 c d^{2} - 7 i d^{3}\right)}{4 a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**3/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] Piecewise((((16*I*a**2*c**3*f*exp(4*I*e) + 48*I*a**2*c*d**2*f*exp(4*I*e) -
32*a**2*d**3*f*exp(4*I*e))*exp(-2*I*f*x) + (4*I*a**2*c**3*f*exp(2*I*e) - 12
*a**2*c**2*d*f*exp(2*I*e) - 12*I*a**2*c*d**2*f*exp(2*I*e) + 4*a**2*d**3*f*e
xp(2*I*e))*exp(-4*I*f*x))*exp(-6*I*e)/(64*a**4*f**2), Ne(a**4*f**2*exp(6*I*
e), 0)), (x*(-(c**3 - 3*I*c**2*d - 3*c*d**2 - 7*I*d**3)/(4*a**2) + (c**3*ex
p(4*I*e) + 2*c**3*exp(2*I*e) + c**3 - 3*I*c**2*d*exp(4*I*e) + 3*I*c**2*d -
3*c*d**2*exp(4*I*e) + 6*c*d**2*exp(2*I*e) - 3*c*d**2 - 7*I*d**3*exp(4*I*e)
+ 4*I*d**3*exp(2*I*e) - I*d**3)*exp(-4*I*e)/(4*a**2)), True)) + d**3*log(ex
p(2*I*f*x) + exp(-2*I*e))/(a**2*f) + x*(c**3 - 3*I*c**2*d - 3*c*d**2 - 7*I*
d**3)/(4*a**2)
```

Giac [A]

time = 0.82, size = 226, normalized size = 1.66

$$-\frac{\frac{2 \, {\left(-i \, c^{3} - 3 \, c^{2} d + 3 i \, c d^{2} + d^{3}\right)} \log\left(\tan\left(f x + e\right) + i\right)}{a^{2}} + \frac{2 \, {\left(i \, c^{3} + 3 \, c^{2} d - 3 i \, c d^{2} + 7 \, d^{3}\right)} \log\left(\tan\left(f x + e\right) - i\right)}{a^{2}} + \frac{-3 i \, c^{3} \tan\left(f x + e\right)^{2} - 9 \, c^{2} d \tan\left(f x + e\right)^{2} + 9 i \, c d^{2} \tan\left(f x + e\right)^{2} - 21 \, d^{3} \tan\left(f x + e\right)^{2} - 10 \, c^{3} \tan\left(f x + e\right) + 30 i \, c^{2} d \tan\left(f x + e\right) - 18 \, c d^{2} \tan\left(f x + e\right) + 22 i \, d^{3} \tan\left(f x + e\right) + 11 i \, c^{3} + 9 \, c^{2} d + 15 i \, c d^{2} + 5 \, d^{3}}{a^{2} {\left(\tan\left(f x + e\right) - i\right)}^{2}}}{16 \, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/16*(2*(-I*c^3 - 3*c^2*d + 3*I*c*d^2 + d^3)*log(tan(f*x + e) + I)/a^2 + 2
*(I*c^3 + 3*c^2*d - 3*I*c*d^2 + 7*d^3)*log(tan(f*x + e) - I)/a^2 + (-3*I*c^
3*tan(f*x + e)^2 - 9*c^2*d*tan(f*x + e)^2 + 9*I*c*d^2*tan(f*x + e)^2 - 21*d
^3*tan(f*x + e)^2 - 10*c^3*tan(f*x + e) + 30*I*c^2*d*tan(f*x + e) - 18*c*d^
2*tan(f*x + e) + 22*I*d^3*tan(f*x + e) + 11*I*c^3 + 9*c^2*d + 15*I*c*d^2 +
5*d^3)/(a^2*(tan(f*x + e) - I)^2))/f
```

Mupad [B]

time = 5.66, size = 184, normalized size = 1.35

$$\frac{\mathrm{tan}\left(e+f\,x\right)\,\left(\frac{3\,c^2\,d}{4\,a^2}-\frac{5\,d^3}{4\,a^2}+\frac{c^3\,1{}\mathrm{i}}{4\,a^2}+\frac{c\,d^2\,9{}\mathrm{i}}{4\,a^2}\right)+\frac{c^3}{2\,a^2}+\frac{3\,c\,d^2}{2\,a^2}+\frac{d^3\,1{}\mathrm{i}}{a^2}}{f\,\left({\mathrm{tan}\left(e+f\,x\right)}^2\,1{}\mathrm{i}+2\,\mathrm{tan}\left(e+f\,x\right)-\mathrm{i}\right)}-\frac{\ln\left(\mathrm{tan}\left(e+f\,x\right)+1{}\mathrm{i}\right)\,\left(-c^3\,1{}\mathrm{i}-3\,c^2\,d+c\,d^2\,3{}\mathrm{i}+d^3\right)}{8\,a^2\,f}-\frac{\ln\left(\mathrm{tan}\left(e+f\,x\right)-\mathrm{i}\right)\,\left(c^3\,1{}\mathrm{i}+3\,c^2\,d-c\,d^2\,3{}\mathrm{i}+7\,d^3\right)}{8\,a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^3/(a + a*tan(e + f*x)*1i)^2,x)
```

```
[Out] (tan(e + f*x)*((c^3*1i)/(4*a^2) - (5*d^3)/(4*a^2) + (c*d^2*9i)/(4*a^2) + (3
*c^2*d)/(4*a^2)) + c^3/(2*a^2) + (d^3*1i)/a^2 + (3*c*d^2)/(2*a^2))/(f*(2*ta
n(e + f*x) + tan(e + f*x)^2*1i - 1i)) - (log(tan(e + f*x) + 1i)*(c*d^2*3i -
 3*c^2*d - c^3*1i + d^3))/(8*a^2*f) - (log(tan(e + f*x) - 1i)*(3*c^2*d - c*
d^2*3i + c^3*1i + 7*d^3))/(8*a^2*f)
```

3.1082 $\int \frac{(c+d\tan(e+fx))^3}{(a+ia\tan(e+fx))^3}\,dx$

Optimal. Leaf size=140

$$\frac{(c-id)^3x}{8a^3}+\frac{(c+id)(c-3id)(ic+d)}{8a^3f(1+i\tan(e+fx))}+\frac{(c+id)^2(ic+d)}{8af(a+ia\tan(e+fx))^2}+\frac{i(c+d\tan(e+fx))^3}{6f(a+ia\tan(e+fx))^3}$$

```
[Out] 1/8*(c-I*d)^3*x/a^3+1/8*(c+I*d)*(c-3*I*d)*(I*c+d)/a^3/f/(1+I*tan(f*x+e))+1/
8*(c+I*d)^2*(I*c+d)/a/f/(a+I*a*tan(f*x+e))^2+1/6*I*(c+d*tan(f*x+e))^3/f/(a+
I*a*tan(f*x+e))^3
```

Rubi [A]
time = 0.15, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3627, 3621, 3607, 8}

$$\frac{(c+id)(c-3id)(d+ic)}{8a^3f(1+i\tan(e+fx))}+\frac{x(c-id)^3}{8a^3}+\frac{i(c+d\tan(e+fx))^3}{6f(a+ia\tan(e+fx))^3}+\frac{(c+id)^2(d+ic)}{8af(a+ia\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^3/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] ((c - I*d)^3*x)/(8*a^3) + ((c + I*d)*(c - (3*I)*d)*(I*c + d))/(8*a^3*f*(1 +
 I*Tan[e + f*x])) + ((c + I*d)^2*(I*c + d))/(8*a*f*(a + I*a*Tan[e + f*x])^2
) + ((I/6)*(c + d*Tan[e + f*x])^3)/(f*(a + I*a*Tan[e + f*x])^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rule 3621

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(a*c + b*d)^2*((a + b*Tan[e + f*x])^
m/(2*a^3*f*m)), x] + Dist[1/(2*a^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[
a*c^2 - 2*b*c*d + a*d^2 - 2*b*d^2*Tan[e + f*x], x], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LeQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Rule 3627

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^n/(2*b*f*m)), x] - Dist[(a*c - b*d)/(2*b^2), Int[(a + b*Tan[e + f*
x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && EqQ
[m + n, 0] && LeQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^3}{(a+ia\tan(e+fx))^3}\,dx = \frac{i(c+d\tan(e+fx))^3}{6f(a+ia\tan(e+fx))^3} + \frac{(c-id)\int \frac{(c+d\tan(e+fx))^2}{(a+ia\tan(e+fx))^2}\,dx}{2a}$$

$$= \frac{(c+id)^2(ic+d)}{8af(a+ia\tan(e+fx))^2} + \frac{i(c+d\tan(e+fx))^3}{6f(a+ia\tan(e+fx))^3} + \frac{(c-id)\int \frac{a(c^2-2icd+d^2)}{a+ia\,\mathrm{t}}}{4a^3}$$

$$= \frac{(c+id)(c-3id)(ic+d)}{8a^3f(1+i\tan(e+fx))} + \frac{(c+id)^2(ic+d)}{8af(a+ia\tan(e+fx))^2} + \frac{i(c+d\tan(e+fx))}{6f(a+ia\tan(e+fx}$$

$$= \frac{(c-id)^3x}{8a^3} + \frac{(c+id)(c-3id)(ic+d)}{8a^3f(1+i\tan(e+fx))} + \frac{(c+id)^2(ic+d)}{8af(a+ia\tan(e+fx))^2} + \frac{i(c+}{6f(a+}$$

Mathematica [A]
time = 1.61, size = 260, normalized size = 1.86

$$\frac{\sec^3(e+fx)(\cos(fx)+i\sin(fx))^3\,(9(c+id)^2(ic+d)\cos(4fx)(\cos(e)-i\sin(e))+18i(c-id)^2(c+id)\cos(2fx)(\cos(e)+i\sin(e))+12(c-id)^3fx(\cos(3e)+i\sin(3e))+2(c+id)^3\cos(6fx)(i\cos(3e)+\sin(3e))+18(c-id)^2(c+id)(\cos(e)+i\sin(e))\sin(2fx)+9(c-id)(c+id)^2(\cos(e)-i\sin(e))\sin(4fx)+2(c+id)^3(\cos(3e)-i\sin(3e))\sin(6fx))}{96f(a+ia\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^3/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] (Sec[e + f*x]^3*(Cos[f*x] + I*Sin[f*x])^3*(9*(c + I*d)^2*(I*c + d)*Cos[4*f*
x]*(Cos[e] - I*Sin[e]) + (18*I)*(c - I*d)^2*(c + I*d)*Cos[2*f*x]*(Cos[e] +
I*Sin[e]) + 12*(c - I*d)^3*f*x*(Cos[3*e] + I*Sin[3*e]) + 2*(c + I*d)^3*Cos[
6*f*x]*(I*Cos[3*e] + Sin[3*e]) + 18*(c - I*d)^2*(c + I*d)*(Cos[e] + I*Sin[e
])*Sin[2*f*x] + 9*(c - I*d)*(c + I*d)^2*(Cos[e] - I*Sin[e])*Sin[4*f*x] + 2*
(c + I*d)^3*(Cos[3*e] - I*Sin[3*e])*Sin[6*f*x]))/(96*f*(a + I*a*Tan[e + f*x
])^3)
```

Maple [A]
time = 0.28, size = 197, normalized size = 1.41

method	result

derivativedivides	$\frac{-\frac{i\left(3ic^2d-id^3-c^3+3c\,d^2\right)\ln(\tan(fx+e)+i)}{16}-\frac{-\frac{1}{8}c^3+\frac{3}{8}c\,d^2+\frac{3}{8}ic^2d+\frac{7}{8}id^3}{\tan(fx+e)-i}+\left(\frac{3}{16}ic\,d^2+\frac{1}{16}d^3-\frac{1}{16}ic^3-\frac{3}{16}c^2d\right)\ln(\tan(fx+e)-i)}{f\,a^3}$
default	$\frac{-\frac{i\left(3ic^2d-id^3-c^3+3c\,d^2\right)\ln(\tan(fx+e)+i)}{16}-\frac{-\frac{1}{8}c^3+\frac{3}{8}c\,d^2+\frac{3}{8}ic^2d+\frac{7}{8}id^3}{\tan(fx+e)-i}+\left(\frac{3}{16}ic\,d^2+\frac{1}{16}d^3-\frac{1}{16}ic^3-\frac{3}{16}c^2d\right)\ln(\tan(fx+e)-i)}{f\,a^3}$
risch	$\frac{3ie^{-2i(fx+e)}c^3}{16a^3f}+\frac{3ie^{-4i(fx+e)}c^3}{32a^3f}+\frac{x\,c^3}{8a^3}-\frac{3xc\,d^2}{8a^3}+\frac{3\,\mathrm{e}^{-2i(fx+e)}c^2d}{16a^3f}+\frac{3\,\mathrm{e}^{-2i(fx+e)}d^3}{16a^3f}+\frac{ie^{-6i(fx+e)}c^3}{48a^3f}+3$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a^3*(-1/16*I*(3*I*c^2*d-I*d^3-c^3+3*c*d^2)*ln(tan(f*x+e)+I)-(-1/8*c^3+3
/8*c*d^2+3/8*I*c^2*d+7/8*I*d^3)/(tan(f*x+e)-I)+(3/16*I*c*d^2+1/16*d^3-1/16*
I*c^3-3/16*c^2*d)*ln(tan(f*x+e)-I)-1/2*(3/4*c^2*d-5/4*d^3+1/4*I*c^3+9/4*I*c
*d^2)/(tan(f*x+e)-I)^2-1/3*(3/2*I*c^2*d-1/2*I*d^3+1/2*c^3-3/2*c*d^2)/(tan(f
*x+e)-I)^3)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.88, size = 143, normalized size = 1.02

$$\frac{\left(12\left(c^3-3i\,c^2d-3\,cd^2+i\,d^3\right)fxe^{(6i\,fx+6i\,e)}+2i\,c^3-6\,c^2d-6i\,cd^2+2\,d^3-18\left(-i\,c^3-c^2d-i\,cd^2-d^3\right)e^{(4i\,fx+4i\,e)}-9\left(-i\,c^3+c^2d-i\,cd^2+d^3\right)e^{(2i\,fx+2i\,e)}\right)e^{(-6i\,fx-6i\,e)}}{96\,a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/96*(12*(c^3 - 3*I*c^2*d - 3*c*d^2 + I*d^3)*f*x*e^(6*I*f*x + 6*I*e) + 2*I*
c^3 - 6*c^2*d - 6*I*c*d^2 + 2*d^3 - 18*(-I*c^3 - c^2*d - I*c*d^2 - d^3)*e^(
4*I*f*x + 4*I*e) - 9*(-I*c^3 + c^2*d - I*c*d^2 + d^3)*e^(2*I*f*x + 2*I*e))*
e^(-6*I*f*x - 6*I*e)/(a^3*f)
```

Sympy [A]
time = 0.46, size = 552, normalized size = 3.94

$$\begin{cases} \frac{\left(\left(512ia^6c^3f^2e^{6ie}-1536a^6c^2df^2e^{6ie}-1536ia^6cd^2f^2e^{6ie}+512a^6d^3f^2e^{6ie}\right)e^{-6ifx}+\left(2304ia^6c^3f^2e^{8ie}-2304a^6c^2df^2e^{8ie}+2304ia^6cd^2f^2e^{8ie}-2304a^6d^3f^2e^{8ie}\right)e^{-4ifx}+\left(4608ia^6c^3f^2e^{10ie}+4608a^6c^2df^2e^{10ie}+4608ia^6cd^2f^2e^{10ie}+4608a^6d^3f^2e^{10ie}\right)e^{-2ifx}\right)e^{-12ie}}{24576a^9f^3} & \text{for}\ a^9f^3e^{12ie}\neq 0 \\ x\left(-\frac{c^3-3ic^2d-3cd^2+id^3}{8a^3}+\frac{\left(c^3e^{6ie}+3c^3e^{4ie}+3c^3e^{2ie}+c^3-3ic^2de^{6ie}-3ic^2de^{4ie}+3ic^2de^{2ie}+3ic^2d-3cd^2e^{6ie}+3cd^2e^{4ie}+3cd^2e^{2ie}-3cd^2+id^3e^{6ie}-3id^3e^{4ie}+3id^3e^{2ie}-id^3\right)e^{-6ie}}{8a^3}\right) & \text{otherwise} \end{cases}+\frac{x\left(c^3-3ic^2d-3cd^2+id^3\right)}{8a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**3/(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] Piecewise((((512*I*a**6*c**3*f**2*exp(6*I*e) - 1536*a**6*c**2*d*f**2*exp(6*
I*e) - 1536*I*a**6*c*d**2*f**2*exp(6*I*e) + 512*a**6*d**3*f**2*exp(6*I*e))*
exp(-6*I*f*x) + (2304*I*a**6*c**3*f**2*exp(8*I*e) - 2304*a**6*c**2*d*f**2*e
xp(8*I*e) + 2304*I*a**6*c*d**2*f**2*exp(8*I*e) - 2304*a**6*d**3*f**2*exp(8*
I*e))*exp(-4*I*f*x) + (4608*I*a**6*c**3*f**2*exp(10*I*e) + 4608*a**6*c**2*d
*f**2*exp(10*I*e) + 4608*I*a**6*c*d**2*f**2*exp(10*I*e) + 4608*a**6*d**3*f*
*2*exp(10*I*e))*exp(-2*I*f*x))*exp(-12*I*e)/(24576*a**9*f**3), Ne(a**9*f**3
*exp(12*I*e), 0)), (x*(-(c**3 - 3*I*c**2*d - 3*c*d**2 + I*d**3)/(8*a**3) +
(c**3*exp(6*I*e) + 3*c**3*exp(4*I*e) + 3*c**3*exp(2*I*e) + c**3 - 3*I*c**2*
d*exp(6*I*e) - 3*I*c**2*d*exp(4*I*e) + 3*I*c**2*d*exp(2*I*e) + 3*I*c**2*d -
 3*c*d**2*exp(6*I*e) + 3*c*d**2*exp(4*I*e) + 3*c*d**2*exp(2*I*e) - 3*c*d**2
 + I*d**3*exp(6*I*e) - 3*I*d**3*exp(4*I*e) + 3*I*d**3*exp(2*I*e) - I*d**3)*
exp(-6*I*e)/(8*a**3)), True)) + x*(c**3 - 3*I*c**2*d - 3*c*d**2 + I*d**3)/(
8*a**3)
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(116) = 232$.
time = 0.95, size = 286, normalized size = 2.04

$$-\frac{\frac{6\,\left(i\,c^{3}+3\,c^{2}d-3i\,cd^{2}-d^{3}\right)\log\left(\tan\left(fx+e\right)-i\right)}{a^{3}}+\frac{6\,\left(-i\,c^{3}-3\,c^{2}d+3i\,cd^{2}+d^{3}\right)\log\left(i\,\tan\left(fx+e\right)-1\right)}{a^{3}}+\frac{-11i\,c^{3}\tan\left(fx+e\right)^{3}-33\,c^{2}d\tan\left(fx+e\right)^{3}+33i\,cd^{2}\tan\left(fx+e\right)^{3}+11\,d^{3}\tan\left(fx+e\right)^{3}-45\,c^{3}\tan\left(fx+e\right)^{2}+135i\,c^{2}d\tan\left(fx+e\right)^{2}+135\,cd^{2}\tan\left(fx+e\right)^{2}+51i\,d^{3}\tan\left(fx+e\right)^{2}+69i\,c^{3}\tan\left(fx+e\right)+207\,c^{2}d\tan\left(fx+e\right)-63i\,cd^{2}\tan\left(fx+e\right)+75\,d^{3}\tan\left(fx+e\right)+51\,c^{3}-57i\,c^{2}d-9\,cd^{2}-29i\,d^{3}}{a^{3}\left(\tan\left(fx+e\right)-i\right)^{3}}}{96\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/96*(6*(I*c^3 + 3*c^2*d - 3*I*c*d^2 - d^3)*log(tan(f*x + e) - I)/a^3 + 6*
(-I*c^3 - 3*c^2*d + 3*I*c*d^2 + d^3)*log(I*tan(f*x + e) - 1)/a^3 + (-11*I*c
^3*tan(f*x + e)^3 - 33*c^2*d*tan(f*x + e)^3 + 33*I*c*d^2*tan(f*x + e)^3 + 1
1*d^3*tan(f*x + e)^3 - 45*c^3*tan(f*x + e)^2 + 135*I*c^2*d*tan(f*x + e)^2 +
 135*c*d^2*tan(f*x + e)^2 + 51*I*d^3*tan(f*x + e)^2 + 69*I*c^3*tan(f*x + e)
 + 207*c^2*d*tan(f*x + e) - 63*I*c*d^2*tan(f*x + e) + 75*d^3*tan(f*x + e) +
 51*c^3 - 57*I*c^2*d - 9*c*d^2 - 29*I*d^3)/(a^3*(tan(f*x + e) - I)^3))/f
```

Mupad [**B**]
time = 5.61, size = 183, normalized size = 1.31

$$\frac{\frac{5\,d^{3}}{12\,a^{3}}-\tan(e+f\,x)\,\left(\frac{3\,c^{3}}{8\,a^{3}}-\frac{d^{3}\,9\mathrm{i}}{8\,a^{3}}+\frac{3\,c\,d^{2}}{8\,a^{3}}-\frac{c^{2}\,d\,9\mathrm{i}}{8\,a^{3}}\right)-\tan(e+f\,x)^{2}\,\left(\frac{7\,d^{3}}{8\,a^{3}}+\frac{3\,c^{2}\,d}{8\,a^{3}}+\frac{c^{3}\,1\mathrm{i}}{8\,a^{3}}-\frac{c\,d^{2}\,3\mathrm{i}}{8\,a^{3}}\right)+\frac{c^{2}\,d}{4\,a^{3}}+\frac{c^{3}\,5\mathrm{i}}{12\,a^{3}}+\frac{c\,d^{2}\,1\mathrm{i}}{4\,a^{3}}}{f\,\left(-{\tan\left(e+f\,x\right)}^{3}\,1\mathrm{i}-3\,{\tan\left(e+f\,x\right)}^{2}+\tan\left(e+f\,x\right)\,3\mathrm{i}+1\right)}+\frac{x\,{\left(d+c\,1\mathrm{i}\right)}^{3}\,1\mathrm{i}}{8\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^3/(a + a*tan(e + f*x)*1i)^3,x)
```

```
[Out] ((c^3*5i)/(12*a^3) - tan(e + f*x)*((3*c^3)/(8*a^3) - (d^3*9i)/(8*a^3) + (3*
c*d^2)/(8*a^3) - (c^2*d*9i)/(8*a^3)) + (5*d^3)/(12*a^3) - tan(e + f*x)^2*((
```

```
c^3*1i)/(8*a^3) + (7*d^3)/(8*a^3) - (c*d^2*3i)/(8*a^3) + (3*c^2*d)/(8*a^3))
 + (c*d^2*1i)/(4*a^3) + (c^2*d)/(4*a^3))/(f*(tan(e + f*x)*3i - 3*tan(e + f*
x)^2 - tan(e + f*x)^3*1i + 1)) + (x*(c*1i + d)^3*1i)/(8*a^3)
```

3.1083 $\int \frac{(a+ia\tan(e+fx))^3}{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=115

$$\frac{4a^3x}{c-id}-\frac{a^3(ic-3d)\log(\cos(e+fx))}{d^2f}-\frac{a^3(c+id)^2\log(c\cos(e+fx)+d\sin(e+fx))}{d^2(ic+d)f}-\frac{a^3+ia^3\tan(e+fx)}{df}$$

```
[Out] 4*a^3*x/(c-I*d)-a^3*(I*c-3*d)*ln(cos(f*x+e))/d^2/f-a^3*(c+I*d)^2*ln(c*cos(f
*x+e)+d*sin(f*x+e))/d^2/(I*c+d)/f+(-a^3-I*a^3*tan(f*x+e))/d/f
```

Rubi [A]
time = 0.25, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3637, 3670, 3556, 3612, 3611}

$$-\frac{a^3(-3d+ic)\log(\cos(e+fx))}{d^2f}-\frac{a^3(c+id)^2\log(c\cos(e+fx)+d\sin(e+fx))}{d^2f(d+ic)}+\frac{4a^3x}{c-id}-\frac{a^3+ia^3\tan(e+fx)}{df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^3/(c + d*Tan[e + f*x]),x]
```

```
[Out] (4*a^3*x)/(c - I*d) - (a^3*(I*c - 3*d)*Log[Cos[e + f*x]])/(d^2*f) - (a^3*(c
 + I*d)^2*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/(d^2*(I*c + d)*f) - (a^3 +
I*a^3*Tan[e + f*x])/(d*f)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3670

```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)]))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[B*(d/
b), Int[Tan[e + f*x], x], x] + Dist[1/b, Int[Simp[A*b*c + (A*b*d + B*(b*c -
 a*d))*Tan[e + f*x], x]/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d,
e, f, A, B}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^3}{c+d\tan(e+fx)}\,dx = -\frac{a^3+ia^3\tan(e+fx)}{df} + \frac{a\int \frac{(a+ia\tan(e+fx))(a(ic+d)+a(c+3id)\tan(e+fx))}{c+d\tan(e+fx)}\,dx}{d}$$

$$= -\frac{a^3+ia^3\tan(e+fx)}{df} + \frac{a\int \frac{a^2d(ic+d)-a^2(ic^2-3cd-4id^2)\tan(e+fx)}{c+d\tan(e+fx)}\,dx}{d^2} + \frac{(a^3(ic-3 \cdots}{\cdots}$$

$$= \frac{4a^3x}{c-id} - \frac{a^3(ic-3d)\log(\cos(e+fx))}{d^2f} - \frac{a^3+ia^3\tan(e+fx)}{df} - \frac{(a^3(c+id)^2 \cdots}{d \cdots}$$

$$= \frac{4a^3x}{c-id} - \frac{a^3(ic-3d)\log(\cos(e+fx))}{d^2f} - \frac{a^3(c+id)^2\log(c\cos(e+fx)+d\sin \cdots}{d^2(ic+d)f}$$

Mathematica [A]

time = 4.56, size = 229, normalized size = 1.99

$$\frac{a^3\sec(e+fx)\left(\cos(fx)\left(8d^2fx-i(c^2+2icd+3d^2)\log\left(\cos^2(e+fx)\right)+i(c+id)^2\log\left((c\cos(e+fx)+d\sin(e+fx))^2\right)\right)+\cos(2e+fx)\left(8d^2fx-i(c^2+2icd+3d^2)\log\left(\cos^2(e+fx)\right)+i(c+id)^2\log\left((c\cos(e+fx)+d\sin(e+fx))^2\right)\right)-4i(c-id)d\sin(fx)\right)}{4(c-id)d^2f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3/(c + d*Tan[e + f*x]),x]
```

```
[Out] (a^3*Sec[e + f*x]*(Cos[f*x]*(8*d^2*f*x - I*(c^2 + (2*I)*c*d + 3*d^2)*Log[Co
s[e + f*x]^2] + I*(c + I*d)^2*Log[(c*Cos[e + f*x] + d*Sin[e + f*x])^2]) + C
os[2*e + f*x]*(8*d^2*f*x - I*(c^2 + (2*I)*c*d + 3*d^2)*Log[Cos[e + f*x]^2]
+ I*(c + I*d)^2*Log[(c*Cos[e + f*x] + d*Sin[e + f*x])^2]) - (4*I)*(c - I*d)
```

```
*d*Sin[f*x]))/(4*(c - I*d)*d^2*f*(Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[e/2]
))
```

Maple [A]

time = 0.27, size = 116, normalized size = 1.01

method	result
norman	$\frac{4a^3x}{-id+c}-\frac{ia^3\tan(fx+e)}{df}+\frac{ia^3(2icd+c^2-d^2)\ln(c+d\tan(fx+e))}{d^2f(-id+c)}+\frac{2ia^3\ln(1+\tan^2(fx+e))}{f(-id+c)}$
derivativedivides	$\frac{a^3\left(-\frac{i\tan(fx+e)}{d}+\frac{\frac{(4ic-4d)\ln\left(1+\tan^2(fx+e)\right)}{2}+(4id+4c)\arctan(\tan(fx+e))}{c^2+d^2}+\frac{\left(ic^3-3ic\,d^2-3c^2d+d^3\right)\ln(c+d\tan(fx+e))}{d^2\left(c^2+d^2\right)}\right)}{f}$
default	$\frac{a^3\left(-\frac{i\tan(fx+e)}{d}+\frac{\frac{(4ic-4d)\ln\left(1+\tan^2(fx+e)\right)}{2}+(4id+4c)\arctan(\tan(fx+e))}{c^2+d^2}+\frac{\left(ic^3-3ic\,d^2-3c^2d+d^3\right)\ln(c+d\tan(fx+e))}{d^2\left(c^2+d^2\right)}\right)}{f}$
risch	$-\frac{8a^3x}{id-c}-\frac{4a^3cx}{d(ic+d)}-\frac{4a^3ce}{df(ic+d)}-\frac{2ia^3e}{f(ic+d)}-\frac{2ia^3x}{ic+d}+\frac{2ia^3c^2x}{d^2(ic+d)}-\frac{6ia^3e}{df}-\frac{ia^3\ln(\mathrm{e}^{2i(fx+e)}+1)c}{d^2f}+\frac{2ia^3c^2e}{d^2f(ic+d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a^3*(-I/d*tan(f*x+e)+1/(c^2+d^2)*(1/2*(4*I*c-4*d)*ln(1+tan(f*x+e)^2)+(4
*I*d+4*c)*arctan(tan(f*x+e)))+1/d^2*(I*c^3-3*I*c*d^2-3*c^2*d+d^3)/(c^2+d^2)
*ln(c+d*tan(f*x+e)))
```

Maxima [A]

time = 0.52, size = 144, normalized size = 1.25

$$\frac{-\frac{i\,a^3\tan(fx+e)}{d}+\frac{4\,(a^3c+i\,a^3d)(fx+e)}{c^2+d^2}+\frac{(i\,a^3c^3-3\,a^3c^2d-3i\,a^3cd^2+a^3d^3)\log(d\tan(fx+e)+c)}{c^2d^2+d^4}-\frac{2\,(-i\,a^3c+a^3d)\log\left(\tan(fx+e)^2+1\right)}{c^2+d^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] (-I*a^3*tan(f*x + e)/d + 4*(a^3*c + I*a^3*d)*(f*x + e)/(c^2 + d^2) + (I*a^3
*c^3 - 3*a^3*c^2*d - 3*I*a^3*c*d^2 + a^3*d^3)*log(d*tan(f*x + e) + c)/(c^2*
d^2 + d^4) - 2*(-I*a^3*c + a^3*d)*log(tan(f*x + e)^2 + 1)/(c^2 + d^2))/f
```

Fricas [A]

time = 0.91, size = 216, normalized size = 1.88

$$\frac{2i\,a^3cd+2\,a^3d^2-\left(a^3c^2+2i\,a^3cd-a^3d^2+(a^3c^2+2i\,a^3cd-a^3d^2)e^{(2i\,fx+2i\,e)}\right)\log\left(\frac{(i\,c+d)e^{(2i\,fx+2i\,e)}+i\,c-d}{i\,c+d}\right)+\left(a^3c^2+2i\,a^3cd+3\,a^3d^2+(a^3c^2+2i\,a^3cd+3\,a^3d^2)e^{(2i\,fx+2i\,e)}\right)\log\left(e^{(2i\,fx+2i\,e)}+1\right)}{(i\,cd^2+d^3)fe^{(2i\,fx+2i\,e)}+(i\,cd^2+d^3)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] (2*I*a^3*c*d + 2*a^3*d^2 - (a^3*c^2 + 2*I*a^3*c*d - a^3*d^2 + (a^3*c^2 + 2*
I*a^3*c*d - a^3*d^2)*e^(2*I*f*x + 2*I*e))*log(((I*c + d)*e^(2*I*f*x + 2*I*e
) + I*c - d)/(I*c + d)) + (a^3*c^2 + 2*I*a^3*c*d + 3*a^3*d^2 + (a^3*c^2 + 2
*I*a^3*c*d + 3*a^3*d^2)*e^(2*I*f*x + 2*I*e))*log(e^(2*I*f*x + 2*I*e) + 1))/
((I*c*d^2 + d^3)*f*e^(2*I*f*x + 2*I*e) + (I*c*d^2 + d^3)*f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(94) = 188$.
time = 10.04, size = 257, normalized size = 2.23

$$\frac{2a^3}{dfe^{2ie}e^{2ifx} + df} - \frac{ia^3(c+3id)\log\left(e^{2ifx} + \frac{a^3c^2+3ia^3cd-2a^3d^2-ia^3d(c+3id)}{a^3c^2e^{2ie}+2ia^3cde^{2ie}+a^3d^2e^{2ie}}\right)}{d^2f} + \frac{ia^3(c+id)^2\log\left(e^{2ifx} + \frac{a^3c^2+3ia^3cd-2a^3d^2+\frac{ia^3d(c+id)^2}{c-id}}{a^3c^2e^{2ie}+2ia^3cde^{2ie}+a^3d^2e^{2ie}}\right)}{d^2f\,(c-id)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3/(c+d*tan(f*x+e)),x)
```

```
[Out] 2*a**3/(d*f*exp(2*I*e)*exp(2*I*f*x) + d*f) - I*a**3*(c + 3*I*d)*log(exp(2*I
*f*x) + (a**3*c**2 + 3*I*a**3*c*d - 2*a**3*d**2 - I*a**3*d*(c + 3*I*d))/(a*
*3*c**2*exp(2*I*e) + 2*I*a**3*c*d*exp(2*I*e) + a**3*d**2*exp(2*I*e)))/(d**2
*f) + I*a**3*(c + I*d)**2*log(exp(2*I*f*x) + (a**3*c**2 + 3*I*a**3*c*d - 2*
a**3*d**2 + I*a**3*d*(c + I*d)**2/(c - I*d))/(a**3*c**2*exp(2*I*e) + 2*I*a*
*3*c*d*exp(2*I*e) + a**3*d**2*exp(2*I*e)))/(d**2*f*(c - I*d))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(110) = 220$.
time = 0.60, size = 247, normalized size = 2.15

$$-\frac{-\frac{8i\,a^3\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+i\right)}{c-i\,d} + \frac{\left(-i\,a^3c^2+2\,a^3cd+i\,a^3d^2\right)\log\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-2\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-c\right)}{cd^2-i\,d^3} - \frac{\left(-i\,a^3c+3\,a^3d\right)\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}{d^2} + \frac{\left(i\,a^3c-3\,a^3d\right)\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}{d^2} + \frac{-i\,a^3c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,a^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-2i\,a^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+i\,a^3c-3\,a^3d}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)d^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] -(-8*I*a^3*log(tan(1/2*f*x + 1/2*e) + I)/(c - I*d) + (-I*a^3*c^2 + 2*a^3*c*
d + I*a^3*d^2)*log(c*tan(1/2*f*x + 1/2*e)^2 - 2*d*tan(1/2*f*x + 1/2*e) - c)
/(c*d^2 - I*d^3) - (-I*a^3*c + 3*a^3*d)*log(tan(1/2*f*x + 1/2*e) + 1)/d^2 +
 (I*a^3*c - 3*a^3*d)*log(tan(1/2*f*x + 1/2*e) - 1)/d^2 + (-I*a^3*c*tan(1/2*
f*x + 1/2*e)^2 + 3*a^3*d*tan(1/2*f*x + 1/2*e)^2 - 2*I*a^3*d*tan(1/2*f*x + 1
/2*e) + I*a^3*c - 3*a^3*d)/((tan(1/2*f*x + 1/2*e)^2 - 1)*d^2))/f
```

Mupad [B]
time = 7.43, size = 99, normalized size = 0.86

$$\frac{a^3\,\ln\left(\mathrm{tan}\left(e+f\,x\right)+1{}\mathrm{i}\right)\,4{}\mathrm{i}}{f\,\left(c-d\,1{}\mathrm{i}\right)}-\frac{a^3\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}}{d\,f}-\frac{\ln\left(c+d\,\mathrm{tan}\left(e+f\,x\right)\right)\,\left(-a^3\,c^2\,1{}\mathrm{i}+2\,a^3\,c\,d+a^3\,d^2\,1{}\mathrm{i}\right)}{d^2\,f\,\left(c-d\,1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^3/(c + d*tan(e + f*x)),x)
```

```
[Out] (a^3*log(tan(e + f*x) + 1i)*4i)/(f*(c - d*1i)) - (a^3*tan(e + f*x)*1i)/(d*f
) - (log(c + d*tan(e + f*x))*(a^3*d^2*1i - a^3*c^2*1i + 2*a^3*c*d))/(d^2*f*
(c - d*1i))
```

3.1084 $\int \frac{(a+ia\tan(e+fx))^2}{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=106

$$-\frac{a^2c(c+id)x}{(c-id)d^2}+\frac{a^2(c+2id)x}{d^2}+\frac{a^2\log(\cos(e+fx))}{df}-\frac{a^2(ic-d)\log(c\cos(e+fx)+d\sin(e+fx))}{d(ic+d)f}$$

```
[Out] -a^2*c*(c+I*d)*x/(c-I*d)/d^2+a^2*(c+2*I*d)*x/d^2+a^2*ln(cos(f*x+e))/d/f-a^2
*(I*c-d)*ln(c*cos(f*x+e)+d*sin(f*x+e))/d/(I*c+d)/f
```

Rubi [A]
time = 0.09, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3622, 3556, 3565, 3611}

$$-\frac{a^2cx(c+id)}{d^2(c-id)}+\frac{a^2x(c+2id)}{d^2}-\frac{a^2(-d+ic)\log(c\cos(e+fx)+d\sin(e+fx))}{df(d+ic)}+\frac{a^2\log(\cos(e+fx))}{df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2/(c + d*Tan[e + f*x]),x]
```

```
[Out] -((a^2*c*(c + I*d)*x)/((c - I*d)*d^2)) + (a^2*(c + (2*I)*d)*x)/d^2 + (a^2*L
og[Cos[e + f*x]])/(d*f) - (a^2*(I*c - d)*Log[c*Cos[e + f*x] + d*Sin[e + f*x
]])/(d*(I*c + d)*f)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3565

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3622

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^2/((a_.) + (b_.)*tan[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[d*(2*b*c - a*d)*(x/b^2), x] + (Dist[d^2/b, In
```

```
t[Tan[e + f*x], x], x] + Dist[(b*c - a*d)^2/b^2, Int[1/(a + b*Tan[e + f*x])
, x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 +
 b^2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^2}{c+d\tan(e+fx)}\,dx = \frac{a^2(c+2id)x}{d^2} - \frac{a^2\int\tan(e+fx)\,dx}{d} + \frac{(-iac+ad)^2\int\frac{1}{c+d\tan(e+fx)}\,dx}{d^2}$$
$$= -\frac{a^2c(c+id)x}{(c-id)d^2} + \frac{a^2(c+2id)x}{d^2} + \frac{a^2\log(\cos(e+fx))}{df} + \frac{(-iac+ad)^2\int\frac{d-c\tan(}{c+d\tan(}}{d\left(c^2+d^2\right)}$$
$$= -\frac{a^2c(c+id)x}{(c-id)d^2} + \frac{a^2(c+2id)x}{d^2} + \frac{a^2\log(\cos(e+fx))}{df} - \frac{a^2(ic-d)\log(c\cos(e}{d(ic}$$

Mathematica [A]
time = 1.81, size = 176, normalized size = 1.66

$$\frac{a^2\left(8dfx+2(-ic+d)\text{ArcTan}\left(\frac{d\cos(3e+fx)-c\sin(3e+fx)}{c\cos(3e+fx)+d\sin(3e+fx)}\right)+(-2ic-2d)\text{ArcTan}(\tan(3e+fx))+c\log\left(\cos^2(e+fx)\right)-id\log\left(\cos^2(e+fx)\right)-c\log\left((c\cos(e+fx)+d\sin(e+fx))^2\right)-id\log\left((c\cos(e+fx)+d\sin(e+fx))^2\right)\right)}{2(c-id)df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2/(c + d*Tan[e + f*x]),x]
```

```
[Out] (a^2*(8*d*f*x + 2*((-I)*c + d)*ArcTan[(d*Cos[3*e + f*x] - c*Sin[3*e + f*x])
/(c*Cos[3*e + f*x] + d*Sin[3*e + f*x])] + ((-2*I)*c - 2*d)*ArcTan[Tan[3*e +
 f*x]] + c*Log[Cos[e + f*x]^2] - I*d*Log[Cos[e + f*x]^2] - c*Log[(c*Cos[e +
 f*x] + d*Sin[e + f*x])^2] - I*d*Log[(c*Cos[e + f*x] + d*Sin[e + f*x])^2]))
/(2*(c - I*d)*d*f)
```

Maple [A]
time = 0.21, size = 95, normalized size = 0.90

method	result
norman	$\frac{2a^2x}{-id+c}+\frac{ia^2\ln\left(1+\tan^2(fx+e)\right)}{f(-id+c)}-\frac{(ia^2d+a^2c)\ln(c+d\tan(fx+e))}{df(-id+c)}$
derivativedivides	$\frac{a^2\left(\frac{\left(-2icd-c^2+d^2\right)\ln(c+d\tan(fx+e))}{\left(c^2+d^2\right)d}+\frac{\frac{(2ic-2d)\ln\left(1+\tan^2(fx+e)\right)}{2}+(2id+2c)\arctan(\tan(fx+e))}{c^2+d^2}\right)}{f}$
default	$\frac{a^2\left(\frac{\left(-2icd-c^2+d^2\right)\ln(c+d\tan(fx+e))}{\left(c^2+d^2\right)d}+\frac{\frac{(2ic-2d)\ln\left(1+\tan^2(fx+e)\right)}{2}+(2id+2c)\arctan(\tan(fx+e))}{c^2+d^2}\right)}{f}$

risch	$-\frac{2a^2x}{id-c}-\frac{2ia^2x}{d}-\frac{2ia^2e}{df}+\frac{2a^2e}{f(id-c)}-\frac{2ia^2cx}{d(id-c)}-\frac{2ia^2ce}{df(id-c)}+\frac{a^2\ln\left(\mathrm{e}^{2i(fx+e)}+1\right)}{df}+\frac{ia^2\ln\left(\mathrm{e}^{2i(fx+e)}-\frac{id+c}{id-c}\right.}{f(id-c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a^2*((-2*I*c*d-c^2+d^2)/(c^2+d^2)/d*ln(c+d*tan(f*x+e))+1/(c^2+d^2)*(1/2
*(2*I*c-2*d)*ln(1+tan(f*x+e)^2)+(2*I*d+2*c)*arctan(tan(f*x+e))))
```

Maxima [A]
time = 0.49, size = 117, normalized size = 1.10

$$\frac{\frac{2\,(a^2c+i\,a^2d)(fx+e)}{c^2+d^2}-\frac{(a^2c^2+2i\,a^2cd-a^2d^2)\log(d\tan(fx+e)+c)}{c^2d+d^3}-\frac{(-i\,a^2c+a^2d)\log\left(\tan(fx+e)^2+1\right)}{c^2+d^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] (2*(a^2*c + I*a^2*d)*(f*x + e)/(c^2 + d^2) - (a^2*c^2 + 2*I*a^2*c*d - a^2*d
^2)*log(d*tan(f*x + e) + c)/(c^2*d + d^3) - (-I*a^2*c + a^2*d)*log(tan(f*x
+ e)^2 + 1)/(c^2 + d^2))/f
```

Fricas [A]
time = 1.04, size = 86, normalized size = 0.81

$$\frac{(-i\,a^2c+a^2d)\log\left(\frac{(i\,c+d)e^{(2i\,fx+2i\,e)}+i\,c-d}{i\,c+d}\right)+(i\,a^2c+a^2d)\log\left(e^{(2i\,fx+2i\,e)}+1\right)}{(i\,cd+d^2)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] ((-I*a^2*c + a^2*d)*log(((I*c + d)*e^(2*I*f*x + 2*I*e) + I*c - d)/(I*c + d)
) + (I*a^2*c + a^2*d)*log(e^(2*I*f*x + 2*I*e) + 1))/((I*c*d + d^2)*f)
```

Sympy [A]
time = 4.22, size = 92, normalized size = 0.87

$$\frac{a^2\log\left(e^{2ifx}+e^{-2ie}\right)}{df}-\frac{a^2(c+id)\log\left(e^{2ifx}+\frac{\left(a^2c+ia^2d+\frac{ia^2d(c+id)}{c-id}\right)e^{-2ie}}{a^2c}\right)}{df\,(c-id)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2/(c+d*tan(f*x+e)),x)
```

```
[Out] a**2*log(exp(2*I*f*x) + exp(-2*I*e))/(d*f) - a**2*(c + I*d)*log(exp(2*I*f*x
) + (a**2*c + I*a**2*d + I*a**2*d*(c + I*d)/(c - I*d))*exp(-2*I*e)/(a**2*c)
)/(d*f*(c - I*d))
```

Giac [A]
time = 0.54, size = 127, normalized size = 1.20

$$\frac{\frac{a^2\log\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{d}+\frac{4ia^2\log\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+i\right)}{c-id}+\frac{a^2\log\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{d}-\frac{(a^2c+ia^2d)\log\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-c\right)}{cd-id^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] (a^2*log(tan(1/2*f*x + 1/2*e) + 1)/d + 4*I*a^2*log(tan(1/2*f*x + 1/2*e) + I
)/(c - I*d) + a^2*log(tan(1/2*f*x + 1/2*e) - 1)/d - (a^2*c + I*a^2*d)*log(c
*tan(1/2*f*x + 1/2*e)^2 - 2*d*tan(1/2*f*x + 1/2*e) - c)/(c*d - I*d^2))/f
```

Mupad [B]
time = 5.75, size = 64, normalized size = 0.60

$$\frac{a^2\,\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,2\mathrm{i}}{f\,\left(c-d\,1\mathrm{i}\right)}-\frac{a^2\,\ln\left(c+d\tan(e+f\,x)\right)\,\left(c+d\,1\mathrm{i}\right)}{d\,f\,\left(c-d\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^2/(c + d*tan(e + f*x)),x)
```

```
[Out] (a^2*log(tan(e + f*x) + 1i)*2i)/(f*(c - d*1i)) - (a^2*log(c + d*tan(e + f*x
))*(c + d*1i))/(d*f*(c - d*1i))
```

3.1085 $\int \frac{a+ia\tan(e+fx)}{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=45

$$\frac{ax}{c-id}+\frac{a\log(c\cos(e+fx)+d\sin(e+fx))}{(ic+d)f}$$

```
[Out] a*x/(c-I*d)+a*ln(c*cos(f*x+e)+d*sin(f*x+e))/(I*c+d)/f
```

Rubi [A]
time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {3612, 3611}

$$\frac{a\log(c\cos(e+fx)+d\sin(e+fx))}{f(d+ic)}+\frac{ax}{c-id}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])/(c + d*Tan[e + f*x]),x]
```

```
[Out] (a*x)/(c - I*d) + (a*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((I*c + d)*f)
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+ia\tan(e+fx)}{c+d\tan(e+fx)}\,dx&=\frac{ax}{c-id}+\frac{a\int \frac{d-c\tan(e+fx)}{c+d\tan(e+fx)}\,dx}{ic+d}\\&=\frac{ax}{c-id}+\frac{a\log(c\cos(e+fx)+d\sin(e+fx))}{(ic+d)f}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 95 vs. 2(45) = 90.
time = 0.57, size = 95, normalized size = 2.11

$$\frac{4afx+2a\text{ArcTan}\left(\frac{d\cos(2e+fx)-c\sin(2e+fx)}{c\cos(2e+fx)+d\sin(2e+fx)}\right)-ia\log\left((c\cos(e+fx)+d\sin(e+fx))^2\right)}{2cf-2idf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])/(c + d*Tan[e + f*x]),x]
```

```
[Out] (4*a*f*x + 2*a*ArcTan[(d*Cos[2*e + f*x] - c*Sin[2*e + f*x])/(c*Cos[2*e + f*
x] + d*Sin[2*e + f*x])] - I*a*Log[(c*Cos[e + f*x] + d*Sin[e + f*x])^2])/(2*
c*f - (2*I)*d*f)
```

Maple [A]
time = 0.20, size = 83, normalized size = 1.84

method	result	size
norman	$\frac{ax}{-id+c}+\frac{ia\ln\left(1+\tan^2(fx+e)\right)}{2f(-id+c)}-\frac{ia\ln(c+d\tan(fx+e))}{f(-id+c)}$	65
derivativedivides	$\frac{a\left(\frac{\frac{(ic-d)\ln\left(1+\tan^2(fx+e)\right)}{2}+(id+c)\arctan(\tan(fx+e))}{c^2+d^2}-\frac{(ic-d)\ln(c+d\tan(fx+e))}{c^2+d^2}\right)}{f}$	83
default	$\frac{a\left(\frac{\frac{(ic-d)\ln\left(1+\tan^2(fx+e)\right)}{2}+(id+c)\arctan(\tan(fx+e))}{c^2+d^2}-\frac{(ic-d)\ln(c+d\tan(fx+e))}{c^2+d^2}\right)}{f}$	83
risch	$-\frac{2ax}{id-c}-\frac{2iax}{ic+d}-\frac{2iae}{f(ic+d)}+\frac{a\ln\left(e^{2i(fx+e)}-\frac{id+c}{id-c}\right)}{f(ic+d)}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))/(c+d*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a*(1/(c^2+d^2)*(1/2*(I*c-d)*ln(1+tan(f*x+e)^2)+(c+I*d)*arctan(tan(f*x+e
)))-(I*c-d)/(c^2+d^2)*ln(c+d*tan(f*x+e)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(43) = 86.
time = 0.49, size = 93, normalized size = 2.07

$$\frac{\frac{2\,(ac+i\,ad)(fx+e)}{c^2+d^2}+\frac{2\,(-i\,ac+ad)\log(d\tan(fx+e)+c)}{c^2+d^2}+\frac{(i\,ac-ad)\log\left(\tan(fx+e)^2+1\right)}{c^2+d^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c+d*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a*c + I*a*d)*(f*x + e)/(c^2 + d^2) + 2*(-I*a*c + a*d)*log(d*tan(f*x
 + e) + c)/(c^2 + d^2) + (I*a*c - a*d)*log(tan(f*x + e)^2 + 1)/(c^2 + d^2))
/f
```

Fricas [A]
time = 1.26, size = 44, normalized size = 0.98

$$\frac{a\log\left(\frac{(ic+d)e^{(2i\,fx+2i\,e)}+i\,c-d}{i\,c+d}\right)}{(i\,c+d)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c+d*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] a*log(((I*c + d)*e^(2*I*f*x + 2*I*e) + I*c - d)/(I*c + d))/((I*c + d)*f)
```

Sympy [A]
time = 0.83, size = 44, normalized size = 0.98

$$-\frac{ia\log\left(\frac{c+id}{ce^{2ie}-ide^{2ie}}+e^{2ifx}\right)}{f\,(c-id)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c+d*tan(f*x+e)),x)
```

```
[Out] -I*a*log((c + I*d)/(c*exp(2*I*e) - I*d*exp(2*I*e)) + exp(2*I*f*x))/(f*(c -
I*d))
```

Giac [A]
time = 0.46, size = 71, normalized size = 1.58

$$-\frac{\frac{i\,a\log\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-2\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-c\right)}{c-i\,d}-\frac{2i\,a\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+i\right)}{c-i\,d}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c+d*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] -(I*a*log(c*tan(1/2*f*x + 1/2*e)^2 - 2*d*tan(1/2*f*x + 1/2*e) - c)/(c - I*d
) - 2*I*a*log(tan(1/2*f*x + 1/2*e) + I)/(c - I*d))/f
```

Mupad [B]
time = 5.18, size = 43, normalized size = 0.96

$$-\frac{a\,\mathrm{atan}\left(\frac{c\,1\mathrm{i}-d+d\tan(e+f\,x)\,2\mathrm{i}}{c-d\,1\mathrm{i}}\right)\,2\mathrm{i}}{f\,\left(d+c\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)/(c + d*tan(e + f*x)),x)
```

```
[Out] -(a*atan((c*1i - d + d*tan(e + f*x)*2i)/(c - d*1i))*2i)/(f*(c*1i + d))
```

3.1086 $\int \frac{1}{(a+ia\tan(e+fx))(c+d\tan(e+fx))}\,dx$

Optimal. Leaf size=128

$$\frac{x}{2a(c+id)}-\frac{cdx}{a(ic-d)\left(c^2+d^2\right)}-\frac{d^2\log(c\cos(e+fx)+d\sin(e+fx))}{a(ic-d)\left(c^2+d^2\right)f}-\frac{1}{2(ic-d)f(a+ia\tan(e+fx))}$$

```
[Out] 1/2*x/a/(c+I*d)-c*d*x/a/(I*c-d)/(c^2+d^2)-d^2*ln(c*cos(f*x+e)+d*sin(f*x+e))
/a/(I*c-d)/(c^2+d^2)/f-1/2/(I*c-d)/f/(a+I*a*tan(f*x+e))
```

Rubi [A]
time = 0.10, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3632, 3560, 8, 3565, 3611}

$$-\frac{d^2\log(c\cos(e+fx)+d\sin(e+fx))}{af(-d+ic)\left(c^2+d^2\right)}-\frac{cdx}{a(-d+ic)\left(c^2+d^2\right)}-\frac{1}{2f(-d+ic)(a+ia\tan(e+fx))}+\frac{x}{2a(c+id)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])*(c + d*Tan[e + f*x])),x]
```

```
[Out] x/(2*a*(c + I*d)) - (c*d*x)/(a*(I*c - d)*(c^2 + d^2)) - (d^2*Log[c*Cos[e +
f*x] + d*Sin[e + f*x]])/(a*(I*c - d)*(c^2 + d^2)*f) - 1/(2*(I*c - d)*f*(a +
 I*a*Tan[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3565

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3632

```
Int[1/(((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Tan[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Tan[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))(c+d\tan(e+fx))}\,dx = \frac{\int \frac{1}{a+ia\tan(e+fx)}\,dx}{c+id} - \frac{d\int \frac{1}{c+d\tan(e+fx)}\,dx}{a(ic-d)}$$

$$= -\frac{cdx}{a(ic-d)\left(c^2+d^2\right)} - \frac{1}{2(ic-d)f(a+ia\tan(e+fx))} +$$

$$= \frac{x}{2a(c+id)} - \frac{cdx}{a(ic-d)\left(c^2+d^2\right)} - \frac{d^2\log(c\cos(e+fx)+}{a(ic-d)\left(c^2+\right.}$$

Mathematica [A]

time = 1.27, size = 206, normalized size = 1.61

$$\frac{c^2+d^2-2ic^2fx+4cdfx+2id^2fx+2d^2\log\left((c\cos(e+fx)+d\sin(e+fx))^2\right)+\left((c+id)(-ic-d+2cfx+2idfx)+2id^2\log\left((c\cos(e+fx)+d\sin(e+fx))^2\right)\right)\tan(e+fx)-4d^2\text{ArcTan}\left(\frac{d\cos(fx)+c\sin(fx)}{-c\cos(fx)+d\sin(fx)}\right)(-i+\tan(e+fx))}{4a(c-id)(c+id)^2f(-i+\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])*(c + d*Tan[e + f*x])),x]
```

```
[Out] (c^2 + d^2 - (2*I)*c^2*f*x + 4*c*d*f*x + (2*I)*d^2*f*x + 2*d^2*Log[(c*Cos[e
 + f*x] + d*Sin[e + f*x])^2] + ((c + I*d)*((-I)*c - d + 2*c*f*x + (2*I)*d*f
*x) + (2*I)*d^2*Log[(c*Cos[e + f*x] + d*Sin[e + f*x])^2])*Tan[e + f*x] - 4*
d^2*ArcTan[(d*Cos[f*x] + c*Sin[f*x])/(-(c*Cos[f*x]) + d*Sin[f*x])]*(-I + Ta
n[e + f*x]))/(4*a*(c - I*d)*(c + I*d)^2*f*(-I + Tan[e + f*x]))
```

Maple [A]

time = 0.30, size = 117, normalized size = 0.91

method	result
derivativedivides	$\frac{\frac{1}{(2id+2c)(\tan(fx+e)-i)}+\frac{(-ic+3d)\ln(\tan(fx+e)-i)}{4(id+c)^2}-\frac{id^2\ln(c+d\tan(fx+e))}{(id-c)(id+c)^2}-\frac{i\ln(\tan(fx+e)+i)}{4id-4c}}{fa}$
default	$\frac{\frac{1}{(2id+2c)(\tan(fx+e)-i)}+\frac{(-ic+3d)\ln(\tan(fx+e)-i)}{4(id+c)^2}-\frac{id^2\ln(c+d\tan(fx+e))}{(id-c)(id+c)^2}-\frac{i\ln(\tan(fx+e)+i)}{4id-4c}}{fa}$

risch	$-\frac{x}{2a(id-c)}+\frac{i\mathrm{e}^{-2i(fx+e)}}{4af(id+c)}+\frac{2d^2x}{a(ic^2d+id^3+c^3+c\,d^2)}+\frac{2d^2e}{af(ic^2d+id^3+c^3+c\,d^2)}+\frac{id^2\ln\left(\mathrm{e}^{2i(fx+e)}-\frac{id+c}{id-c}\right)}{af(ic^2d+id^3+c^3+c\,d^2)}$
norman	$\frac{\frac{\tan(fx+e)}{2af(id+c)}+\frac{i}{2af(id+c)}+\frac{\left(2icd+c^2+d^2\right)x}{2\left(c^2+d^2\right)a(id+c)}+\frac{\left(2icd+c^2+d^2\right)x\left(\tan^2(fx+e)\right)}{2\left(c^2+d^2\right)a(id+c)}}{1+\tan^2(fx+e)}+\frac{d^2\ln(c+d\tan(fx+e))}{af(-ic^3-ic\,d^2+c^2d+d^3)}-\frac{d^2\ln\left(1+\tan^2\right.}{2af(-ic^3-ic\,d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))/(c+d*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a*(1/(2*I*d+2*c)/(tan(f*x+e)-I)+1/4/(c+I*d)^2*(-I*c+3*d)*ln(tan(f*x+e)-
I)-I*d^2/(I*d-c)/(c+I*d)^2*ln(c+d*tan(f*x+e))-I/(4*I*d-4*c)*ln(tan(f*x+e)+I
))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c+d*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]

time = 1.16, size = 131, normalized size = 1.02

$$\frac{\left(2\,(i\,c^2-2\,cd+3i\,d^2)fxe^{(2i\,fx+2i\,e)}-4\,d^2e^{(2i\,fx+2i\,e)}\log\left(\frac{(i\,c+d)e^{(2i\,fx+2i\,e)}+i\,c-d}{i\,c+d}\right)-c^2-d^2\right)e^{(-2i\,fx-2i\,e)}}{4\,(i\,ac^3-ac^2d+i\,acd^2-ad^3)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c+d*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(I*c^2 - 2*c*d + 3*I*d^2)*f*x*e^(2*I*f*x + 2*I*e) - 4*d^2*e^(2*I*f*x
 + 2*I*e)*log(((I*c + d)*e^(2*I*f*x + 2*I*e) + I*c - d)/(I*c + d)) - c^2 -
d^2)*e^(-2*I*f*x - 2*I*e)/((I*a*c^3 - a*c^2*d + I*a*c*d^2 - a*d^3)*f)
```

Sympy [A]

time = 2.61, size = 248, normalized size = 1.94

$$\frac{x(c+3id)}{2ac^2+4iacd-2ad^2}+\begin{cases}\frac{ie^{-2ifx}}{4acfe^{2ie}+4iadfe^{2ie}} & \text{for } 4acfe^{2ie}+4iadfe^{2ie}\neq 0\\ x\left(-\frac{c+3id}{2ac^2+4iacd-2ad^2}+\frac{ce^{2ie}+c+3ide^{2ie}+id}{2ac^2e^{2ie}+4iacde^{2ie}-2ad^2e^{2ie}}\right) & \text{otherwise}\end{cases}+\frac{id^2\log\left(\frac{c+id}{ce^{2ie}-ide^{2ie}}+e^{2ifx}\right)}{af\,(c-id)\,(c+id)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c+d*tan(f*x+e)),x)
```

```
[Out] x*(c + 3*I*d)/(2*a*c**2 + 4*I*a*c*d - 2*a*d**2) + Piecewise((I*exp(-2*I*f*x
)/(4*a*c*f*exp(2*I*e) + 4*I*a*d*f*exp(2*I*e)), Ne(4*a*c*f*exp(2*I*e) + 4*I*
a*d*f*exp(2*I*e), 0)), (x*(-(c + 3*I*d)/(2*a*c**2 + 4*I*a*c*d - 2*a*d**2) +
 (c*exp(2*I*e) + c + 3*I*d*exp(2*I*e) + I*d)/(2*a*c**2*exp(2*I*e) + 4*I*a*c
*d*exp(2*I*e) - 2*a*d**2*exp(2*I*e))), True)) + I*d**2*log((c + I*d)/(c*exp
(2*I*e) - I*d*exp(2*I*e)) + exp(2*I*f*x))/(a*f*(c - I*d)*(c + I*d)**2)
```

Giac [A]
time = 0.51, size = 178, normalized size = 1.39

$$-\frac{-\frac{4i\,d^3\log(-i\,d\tan(fx+e)-i\,c)}{ac^3d+i\,ac^2d^2+acd^3+i\,ad^4}-\frac{(-i\,c+3\,d)\log(i\,\tan(fx+e)+1)}{ac^2+2i\,acd-ad^2}-\frac{8\,\log(\tan(fx+e)+i)}{-8i\,ac-8\,ad}-\frac{i\,c\tan(fx+e)-3\,d\tan(fx+e)+3\,c+5i\,d}{(ac^2+2i\,acd-ad^2)(\tan(fx+e)-i)}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c+d*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/4*(-4*I*d^3*log(-I*d*tan(f*x + e) - I*c)/(a*c^3*d + I*a*c^2*d^2 + a*c*d^
3 + I*a*d^4) - (-I*c + 3*d)*log(I*tan(f*x + e) + 1)/(a*c^2 + 2*I*a*c*d - a*
d^2) - 8*log(tan(f*x + e) + I)/(-8*I*a*c - 8*a*d) - (I*c*tan(f*x + e) - 3*d
*tan(f*x + e) + 3*c + 5*I*d)/((a*c^2 + 2*I*a*c*d - a*d^2)*(tan(f*x + e) - I
)))/f
```

Mupad [B]
time = 9.31, size = 2639, normalized size = 20.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)*(c + d*tan(e + f*x))),x)
```

```
[Out] symsum(log(-a*d^2*(c*1i + d)^2*(c*d + d^2*tan(e + f*x) + d^2*2i + 2*root(a^
3*c^3*d^3*e^3*64i + 16*a^3*c^4*d^2*e^3 - 16*a^3*c^2*d^4*e^3 + a^3*c^5*d*e^3
*32i + a^3*c*d^5*e^3*32i - 16*a^3*d^6*e^3 + 16*a^3*c^6*e^3 - 2*a*c^2*d^2*e
+ a*c^3*d*e*4i + a*c*d^3*e*4i + 13*a*d^4*e + a*c^4*e - c*d^2*1i + 3*d^3, e,
 k)*a*c^3 - root(a^3*c^3*d^3*e^3*64i + 16*a^3*c^4*d^2*e^3 - 16*a^3*c^2*d^4*
e^3 + a^3*c^5*d*e^3*32i + a^3*c*d^5*e^3*32i - 16*a^3*d^6*e^3 + 16*a^3*c^6*e
^3 - 2*a*c^2*d^2*e + a*c^3*d*e*4i + a*c*d^3*e*4i + 13*a*d^4*e + a*c^4*e - c
*d^2*1i + 3*d^3, e, k)*a*d^3*2i - 8*root(a^3*c^3*d^3*e^3*64i + 16*a^3*c^4*d
^2*e^3 - 16*a^3*c^2*d^4*e^3 + a^3*c^5*d*e^3*32i + a^3*c*d^5*e^3*32i - 16*a^
3*d^6*e^3 + 16*a^3*c^6*e^3 - 2*a*c^2*d^2*e + a*c^3*d*e*4i + a*c*d^3*e*4i +
13*a*d^4*e + a*c^4*e - c*d^2*1i + 3*d^3, e, k)^2*a^2*c^4*tan(e + f*x) - 24*
root(a^3*c^3*d^3*e^3*64i + 16*a^3*c^4*d^2*e^3 - 16*a^3*c^2*d^4*e^3 + a^3*c^
5*d*e^3*32i + a^3*c*d^5*e^3*32i - 16*a^3*d^6*e^3 + 16*a^3*c^6*e^3 - 2*a*c^2
*d^2*e + a*c^3*d*e*4i + a*c*d^3*e*4i + 13*a*d^4*e + a*c^4*e - c*d^2*1i + 3*
d^3, e, k)^2*a^2*d^4*tan(e + f*x) - 6*root(a^3*c^3*d^3*e^3*64i + 16*a^3*c^4
*d^2*e^3 - 16*a^3*c^2*d^4*e^3 + a^3*c^5*d*e^3*32i + a^3*c*d^5*e^3*32i - 16*
```

a^3*d^6*e^3 + 16*a^3*c^6*e^3 - 2*a*c^2*d^2*e + a*c^3*d*e*4i + a*c*d^3*e*4i + 13*a*d^4*e + a*c^4*e - c*d^2*1i + 3*d^3, e, k)*a*c*d^2 + root(a^3*c^3*d^3*e^3*64i + 16*a^3*c^4*d^2*e^3 - 16*a^3*c^2*d^4*e^3 + a^3*c^5*d*e^3*32i + a^3*c*d^5*e^3*32i - 16*a^3*d^6*e^3 + 16*a^3*c^6*e^3 - 2*a*c^2*d^2*e + a*c^3*d*e*4i + a*c*d^3*e*4i + 13*a*d^4*e + a*c^4*e - c*d^2*1i + 3*d^3, e, k)*a*c^2*d*6i - 12*root(a^3*c^3*d^3*e^3*64i + 16*a^3*c^4*d^2*e^3 - 16*a^3*c^2*d^4*e^3 + a^3*c^5*d*e^3*32i + a^3*c*d^5*e^3*32i - 16*a^3*d^6*e^3 + 16*a^3*c^6*e^3 - 2*a*c^2*d^2*e + a*c^3*d*e*4i + a*c*d^3*e*4i + 13*a*d^4*e + a*c^4*e - c*d^2*1i + 3*d^3, e, k)*a*d^3*tan(e + f*x) + root(a^3*c^3*d^3*e^3*64i + 16*a^3*c^4*d^2*e^3 - 16*a^3*c^2*d^4*e^3 + a^3*c^5*d*e^3*32i + a^3*c*d^5*e^3*32i - 16*a^3*d^6*e^3 + 16*a^3*c^6*e^3 - 2*a*c^2*d^2*e + a*c^3*d*e*4i + a*c*d^3*e*4i + 13*a*d^4*e + a*c^4*e - c*d^2*1i + 3*d^3, e, k)^2*a^2*c^2*d^2*64i - 32*root(a^3*c^3*d^3*e^3*64i + 16*a^3*c^4*d^2*e^3 - 16*a^3*c^2*d^4*e^3 + a^3*c^5*d*e^3*32i + a^3*c*d^5*e^3*32i - 16*a^3*d^6*e^3 + 16*a^3*c^6*e^3 - 2*a*c^2*d^2*e + a*c^3*d*e*4i + a*c*d^3*e*4i + 13*a*d^4*e + a*c^4*e - c*d^2*1i + 3*d^3, e, k)^2*a^2*c*d^3 + 32*root(a^3*c^3*d^3*e^3*64i + 16*a^3*c^4*d^2*e^3 - 16*a^3*c^2*d^4*e^3 + a^3*c^5*d*e^3*32i + a^3*c*d^5*e^3*32i - 16*a^3*d^6*e^3 + 16*a^3*c^6*e^3 - 2*a*c^2*d^2*e + a*c^3*d*e*4i + a*c*d^3*e*4i + 13*a*d^4*e + a*c^4*e - c*d^2*1i + 3*d^3, e, k)^2*a^2*c^3*d + root(a^3*c^3*d^3*e^3*64i + 16*a^3*c^4*d^2*e^3 - 16*a^3*c^2*d^4*e^3 + a^3*c^5*d*e^3*32i + a^3*c*d^5*e^3*32i - 16*a^3*d^6*e^3 + 16*a^3*c^6*e^3 - 2*a*c^2*d^2*e + a*c^3*d*e*4i + a*c*d^3*e*4i + 13*a*d^4*e + a*c^4*e - c*d^2*1i + 3*d^3, e, k)^2*a^2*c*d^3*tan(e + f*x)*48i - root(a^3*c^3*d^3*e^3*64i + 16*a^3*c^4*d^2*e^3 - 16*a^3*c^2*d^4*e^3 + a^3*c^5*d*e^3*32i + a^3*c*d^5*e^3*32i - 16*a^3*d^6*e^3 + 16*a^3*c^6*e^3 - 2*a*c^2*d^2*e + a*c^3*d*e*4i + a*c*d^3*e*4i + 13*a*d^4*e + a*c^4*e - c*d^2*1i + 3*d^3, e, k)^2*a^2*c^3*d*tan(e + f*x)*16i + root(a^3*c^3*d^3*e^3*64i + 16*a^3*c^4*d^2*e^3 - 16*a^3*c^2*d^4*e^3 + a^3*c^5*d*e^3*32i + a^3*c*d^5*e^3*32i - 16*a^3*d^6*e^3 + 16*a^3*c^6*e^3 - 2*a*c^2*d^2*e + a*c^3*d*e*4i + a*c*d^3*e*4i + 13*a*d^4*e + a*c^4*e - c*d^2*1i + 3*d^3, e, k)*a*c*d^2*tan(e + f*x)*16i + 4*root(a^3*c^3*d^3*e^3*64i + 16*a^3*c^4*d^2*e^3 - 16*a^3*c^2*d^4*e^3 + a^3*c^5*d*e^3*32i + a^3*c*d^5*e^3*32i - 16*a^3*d^6*e^3 + 16*a^3*c^6*e^3 - 2*a*c^2*d^2*e + a*c^3*d*e*4i + a*c*d^3*e*4i + 13*a*d^4*e + a*c^4*e - c*d^2*1i + 3*d^3, e, k)*a*c^2*d*tan(e + f*x) + 32*root(a^3*c^3*d^3*e^3*64i + 16*a^3*c^4*d^2*e^3 - 16*a^3*c^2*d^4*e^3 + a^3*c^5*d*e^3*32i + a^3*c*d^5*e^3*32i - 16*a^3*d^6*e^3 + 16*a^3*c^6*e^3 - 2*a*c^2*d^2*e + a*c^3*d*e*4i + a*c*d^3*e*4i + 13*a*d^4*e + a*c^4*e - c*d^2*1i + 3*d^3, e, k)^2*a^2*c^2*d^2*tan(e + f*x)))*root(a^3*c^3*d^3*e^3*64i + 16*a^3*c^4*d^2*e^3 - 16*a^3*c^2*d^4*e^3 + a^3*c^5*d*e^3*32i + a^3*c*d^5*e^3*32i - 16*a^3*d^6*e^3 + 16*a^3*c^6*e^3 - 2*a*c^2*d^2*e + a*c^3*d*e*4i + a*c*d^3*e*4i + 13*a*d^4*e + a*c^4*e - c*d^2*1i + 3*d^3, e, k), k, 1, 3)/f + 1/(2*f*(a*d - a*c*1i + a*c*tan(e + f*x) + a*d*tan(e + f*x)*1i))

3.1087 $\int \frac{1}{(a+ia\tan(e+fx))^2(c+d\tan(e+fx))}\,dx$

Optimal. Leaf size=174

$$\frac{(c^3+3ic^2d-3cd^2+3id^3)\,x}{4a^2(c-id)(c+id)^3}-\frac{d^3\log(c\cos(e+fx)+d\sin(e+fx))}{a^2(c-id)(c+id)^3f}+\frac{ic-3d}{4a^2(c+id)^2f(1+i\tan(e+fx))}-\frac{}{4(i}$$

```
[Out] 1/4*(c^3+3*I*c^2*d-3*c*d^2+3*I*d^3)*x/a^2/(c-I*d)/(c+I*d)^3-d^3*ln(c*cos(f*
x+e)+d*sin(f*x+e))/a^2/(c-I*d)/(c+I*d)^3/f+1/4*(I*c-3*d)/a^2/(c+I*d)^2/f/(1
+I*tan(f*x+e))-1/4/(I*c-d)/f/(a+I*a*tan(f*x+e))^2
```

Rubi [A]
time = 0.28, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3640, 3677, 3612, 3611}

$$\frac{x(c^3+3ic^2d-3cd^2+3id^3)}{4a^2(c-id)(c+id)^3}-\frac{d^3\log(c\cos(e+fx)+d\sin(e+fx))}{a^2f(c-id)(c+id)^3}+\frac{-3d+ic}{4a^2f(c+id)^2(1+i\tan(e+fx))}-\frac{1}{4f(-d+ic)(a+ia\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^2*(c + d*Tan[e + f*x])),x]
```

```
[Out] ((c^3 + (3*I)*c^2*d - 3*c*d^2 + (3*I)*d^3)*x)/(4*a^2*(c - I*d)*(c + I*d)^3)
 - (d^3*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/(a^2*(c - I*d)*(c + I*d)^3*f)
 + (I*c - 3*d)/(4*a^2*(c + I*d)^2*f*(1 + I*Tan[e + f*x])) - 1/(4*(I*c - d)*
f*(a + I*a*Tan[e + f*x])^2)
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
```

```
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^2(c+d\tan(e+fx))}\,dx = -\frac{1}{4(ic-d)f(a+ia\tan(e+fx))^2} - \frac{\int \frac{-2a(ic-2d)-2iad\tan(\cdots}{(a+ia\tan(e+fx))(c+d\tan\cdots}}{4a^2(ic-d)\cdots}$$

$$= \frac{ic-3d}{4a^2(c+id)^2f(1+i\tan(e+fx))} - \frac{1}{4(ic-d)f(a+ia\tan(\cdots}$$

$$= \frac{(c^3+3ic^2d-3cd^2+3id^3)\,x}{4a^2(c+id)^2\,(c^2+d^2)} + \frac{ic-3d}{4a^2(c+id)^2f(1+i\tan(e+\cdots}$$

$$= \frac{(c^3+3ic^2d-3cd^2+3id^3)\,x}{4a^2(c+id)^2\,(c^2+d^2)} - \frac{d^3\log(c\cos(e+fx)+d\sin\cdots}{a^2(c+id)^2\,(c^2+d^2\cdots}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 372 vs. 2(174) = 348.
time = 1.51, size = 372, normalized size = 2.14

$$-\frac{\sec^2(e+fx)\left(4ic^3-8c^2d+4icd^2-8d^3+\cos(2(e+fx))\left((c+id)^2(ic+d+4cfx+4idfx)-8d^3\log\left((c\cos(e+fx)+d\sin(e+fx))^2\right)\right)+c^3\sin(2(e+fx))+ic^2d\sin(2(e+fx))+cd^2\sin(2(e+fx))+id^3\sin(2(e+fx))+4ic^3fx\sin(2(e+fx))-12c^2dfx\sin(2(e+fx))-12icd^2fx\sin(2(e+fx))+4d^3fx\sin(2(e+fx))-8id^3\log\left((c\cos(e+fx)+d\sin(e+fx))^2\right)\sin(2(e+fx))+16d^3\text{ArcTan}\left(\frac{-2cd\cos(fx)+(-c^2+d^2)\sin(fx)}{(c^2-d^2)\cos(fx)-2cd\sin(fx)}\right)(-i\cos(2(e+fx))+\sin(2(e+fx)))\right)}{16a^2(c-id)(c+id)^3f(-i+\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^2*(c + d*Tan[e + f*x])),x]
```

```
[Out] -1/16*(Sec[e + f*x]^2*((4*I)*c^3 - 8*c^2*d + (4*I)*c*d^2 - 8*d^3 + Cos[2*(e
 + f*x)]*((c + I*d)^2*(I*c + d + 4*c*f*x + (4*I)*d*f*x) - 8*d^3*Log[(c*Cos[
e + f*x] + d*Sin[e + f*x])^2]) + c^3*Sin[2*(e + f*x)] + I*c^2*d*Sin[2*(e +
f*x)] + c*d^2*Sin[2*(e + f*x)] + I*d^3*Sin[2*(e + f*x)] + (4*I)*c^3*f*x*Sin
```

```
[2*(e + f*x)] - 12*c^2*d*f*x*Sin[2*(e + f*x)] - (12*I)*c*d^2*f*x*Sin[2*(e +
 f*x)] + 4*d^3*f*x*Sin[2*(e + f*x)] - (8*I)*d^3*Log[(c*Cos[e + f*x] + d*Sin
[e + f*x])^2]*Sin[2*(e + f*x)] + 16*d^3*ArcTan[(-2*c*d*Cos[f*x] + (-c^2 + d
^2)*Sin[f*x])/((c^2 - d^2)*Cos[f*x] - 2*c*d*Sin[f*x])]*((-I)*Cos[2*(e + f*x
)] + Sin[2*(e + f*x)])))/(a^2*(c - I*d)*(c + I*d)^3*f*(-I + Tan[e + f*x])^2
)
```

Maple [A]

time = 0.41, size = 177, normalized size = 1.02

method	result
derivativedivides	$\frac{-\frac{-4icd-c^2+3d^2}{4(id+c)^3(\tan(fx+e)-i)}+\frac{\left(-ic^2+7id^2+4cd\right)\ln(\tan(fx+e)-i)}{8(id+c)^3}-\frac{ic^2-id^2-2cd}{4(id+c)^3(\tan(fx+e)-i)^2}+\frac{d^3\ln(c+d\tan(fx+e))}{(id-c)(id+c)^3}-\frac{i\ln(\tan(fx+}{8id-8c}}{f\,a^2}$
default	$\frac{-\frac{-4icd-c^2+3d^2}{4(id+c)^3(\tan(fx+e)-i)}+\frac{\left(-ic^2+7id^2+4cd\right)\ln(\tan(fx+e)-i)}{8(id+c)^3}-\frac{ic^2-id^2-2cd}{4(id+c)^3(\tan(fx+e)-i)^2}+\frac{d^3\ln(c+d\tan(fx+e))}{(id-c)(id+c)^3}-\frac{i\ln(\tan(fx+}{8id-8c}}{f\,a^2}$
risch	$-\frac{x}{4a^2(id-c)}-\frac{e^{-2i(fx+e)}d}{2a^2(id+c)^2f}+\frac{ie^{-2i(fx+e)}c}{4a^2(id+c)^2f}+\frac{ie^{-4i(fx+e)}}{16a^2(id+c)f}+\frac{2id^3x}{a^2(2ic^3d+2ic\,d^3+c^4-d^4)}+\frac{2id^3e}{a^2f(2ic^3d+2ic\,d^3+}$
norman	$\frac{\frac{-ic+2d}{2af\left(-2icd-c^2+d^2\right)}+\frac{(3id+c)\left(\tan^3(fx+e)\right)}{4af\left(2icd+c^2-d^2\right)}+\frac{(5id+3c)\tan(fx+e)}{4af\left(2icd+c^2-d^2\right)}+\frac{\left(3ic^2d+3id^3+c^3-3c\,d^2\right)x}{4\left(c^2+d^2\right)a\left(2icd+c^2-d^2\right)}-\frac{d\left(\tan^2(fx+e)\right)}{2af\left(2icd+c^2-d^2\right)}+\frac{\left(3ic^2d+3i\right.}{2(c}}{a(1+\tan^2(fx+e))^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a^2*(-1/4*(-4*I*c*d-c^2+3*d^2)/(c+I*d)^3/(tan(f*x+e)-I)+1/8*(-I*c^2+7*I
*d^2+4*c*d)/(c+I*d)^3*ln(tan(f*x+e)-I)-1/4*(I*c^2-I*d^2-2*c*d)/(c+I*d)^3/(t
an(f*x+e)-I)^2+d^3/(I*d-c)/(c+I*d)^3*ln(c+d*tan(f*x+e))-I/(8*I*d-8*c)*ln(ta
n(f*x+e)+I))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]

time = 1.25, size = 188, normalized size = 1.08

$$-\frac{\left(16\,d^3e^{(4i\,fx+4i\,e)}\log\left(\frac{(ic+d)e^{(2i\,fx+2i\,e)}+ic-d}{ic+d}\right)-4\,(c^3+3i\,c^2d-3\,cd^2+7i\,d^3)fxe^{(4i\,fx+4i\,e)}-i\,c^3+c^2d-i\,cd^2+d^3-4\,(i\,c^3-2\,c^2d+i\,cd^2-2\,d^3)e^{(2i\,fx+2i\,e)}\right)e^{(-4i\,fx-4i\,e)}}{16\,(a^2c^4+2i\,a^2c^3d+2i\,a^2cd^3-a^2d^4)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/16*(16*d^3*e^(4*I*f*x + 4*I*e)*log(((I*c + d)*e^(2*I*f*x + 2*I*e) + I*c
- d)/(I*c + d)) - 4*(c^3 + 3*I*c^2*d - 3*c*d^2 + 7*I*d^3)*f*x*e^(4*I*f*x +
4*I*e) - I*c^3 + c^2*d - I*c*d^2 + d^3 - 4*(I*c^3 - 2*c^2*d + I*c*d^2 - 2*d
^3)*e^(2*I*f*x + 2*I*e))*e^(-4*I*f*x - 4*I*e)/((a^2*c^4 + 2*I*a^2*c^3*d + 2
*I*a^2*c*d^3 - a^2*d^4)*f)
```

Sympy [A]
time = 5.91, size = 610, normalized size = 3.51

$$\frac{x(c^2+4icd-7d^2)}{4a^2c^3+12ia^2c^2d-12a^2cd^2-4ia^2d^3}+\begin{cases}\frac{(4ia^2c^2fe^{2ie}-8a^2cdfe^{2ie}-4ia^2d^2fe^{2ie})e^{-4ifx}+(16ia^2c^2fe^{4ie}-48a^2cdfe^{4ie}-32ia^2d^2fe^{4ie})e^{-2ifx}}{64a^4c^3f^2e^{6ie}+192ia^4c^2df^2e^{6ie}-192a^4cd^2f^2e^{6ie}-64ia^4d^3f^2e^{6ie}} & \text{for } 64a^4c^3f^2e^{6ie}+192ia^4c^2df^2e^{6ie}-192a^4cd^2f^2e^{6ie}-64ia^4d^3f^2e^{6ie}\neq 0\\ x\left(-\frac{c^2+4icd-7d^2}{4a^2c^3+12ia^2c^2d-12a^2cd^2-4ia^2d^3}+\frac{c^2e^{4ie}+2c^2e^{2ie}+c^2+4icde^{4ie}+6icde^{2ie}+2icd-7d^2e^{4ie}-4d^2e^{2ie}-d^2}{4a^2c^3e^{4ie}+12ia^2c^2de^{4ie}-12a^2cd^2e^{4ie}-4ia^2d^3e^{4ie}}\right) & \text{otherwise}\end{cases}-\frac{d^3\log\left(\frac{c+id}{ce^{2ie}-ide^{2ie}}+e^{2ifx}\right)}{a^2f\,(c-id)\,(c+id)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**2/(c+d*tan(f*x+e)),x)
```

```
[Out] x*(c**2 + 4*I*c*d - 7*d**2)/(4*a**2*c**3 + 12*I*a**2*c**2*d - 12*a**2*c*d**
2 - 4*I*a**2*d**3) + Piecewise((((4*I*a**2*c**2*f*exp(2*I*e) - 8*a**2*c*d*f
*exp(2*I*e) - 4*I*a**2*d**2*f*exp(2*I*e))*exp(-4*I*f*x) + (16*I*a**2*c**2*f
*exp(4*I*e) - 48*a**2*c*d*f*exp(4*I*e) - 32*I*a**2*d**2*f*exp(4*I*e))*exp(-
2*I*f*x))/(64*a**4*c**3*f**2*exp(6*I*e) + 192*I*a**4*c**2*d*f**2*exp(6*I*e)
 - 192*a**4*c*d**2*f**2*exp(6*I*e) - 64*I*a**4*d**3*f**2*exp(6*I*e)), Ne(64
*a**4*c**3*f**2*exp(6*I*e) + 192*I*a**4*c**2*d*f**2*exp(6*I*e) - 192*a**4*c
*d**2*f**2*exp(6*I*e) - 64*I*a**4*d**3*f**2*exp(6*I*e), 0)), (x*(-(c**2 + 4
*I*c*d - 7*d**2)/(4*a**2*c**3 + 12*I*a**2*c**2*d - 12*a**2*c*d**2 - 4*I*a**
2*d**3) + (c**2*exp(4*I*e) + 2*c**2*exp(2*I*e) + c**2 + 4*I*c*d*exp(4*I*e)
+ 6*I*c*d*exp(2*I*e) + 2*I*c*d - 7*d**2*exp(4*I*e) - 4*d**2*exp(2*I*e) - d*
*2)/(4*a**2*c**3*exp(4*I*e) + 12*I*a**2*c**2*d*exp(4*I*e) - 12*a**2*c*d**2*
exp(4*I*e) - 4*I*a**2*d**3*exp(4*I*e))), True)) - d**3*log((c + I*d)/(c*exp
(2*I*e) - I*d*exp(2*I*e)) + exp(2*I*f*x))/(a**2*f*(c - I*d)*(c + I*d)**3)
```

Giac [A]
time = 0.55, size = 296, normalized size = 1.70

$$-\frac{\frac{d^4\log(-i\,d\tan(fx+e)-i\,c)}{a^2c^4d+2i\,a^2c^3d^2+2i\,a^2cd^4-a^2d^5}+\frac{2\,(c^2+4i\,cd-7\,d^2)\log(i\,\tan(fx+e)+1)}{-16i\,a^2c^3+48\,a^2c^2d+48i\,a^2cd^2-16\,a^2d^3}-\frac{2\log(\tan(fx+e)+i)}{-16i\,a^2c-16\,a^2d}-\frac{2\left(3\,c^2\tan(fx+e)^2+12i\,cd\tan(fx+e)^2-21\,d^2\tan(fx+e)^2-10i\,c^2\tan(fx+e)+40\,cd\tan(fx+e)+54i\,d^2\tan(fx+e)-11\,c^2-36i\,cd+37\,d^2\right)}{-32\,(i\,a^2c^3-3\,a^2c^2d-3i\,a^2cd^2+a^2d^3)(\tan(fx+e)-i)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] -(d^4*log(-I*d*tan(f*x + e) - I*c)/(a^2*c^4*d + 2*I*a^2*c^3*d^2 + 2*I*a^2*c
*d^4 - a^2*d^5) + 2*(c^2 + 4*I*c*d - 7*d^2)*log(I*tan(f*x + e) + 1)/(-16*I*
a^2*c^3 + 48*a^2*c^2*d + 48*I*a^2*c*d^2 - 16*a^2*d^3) - 2*log(tan(f*x + e)
+ I)/(-16*I*a^2*c - 16*a^2*d) - 2*(3*c^2*tan(f*x + e)^2 + 12*I*c*d*tan(f*x
+ e)^2 - 21*d^2*tan(f*x + e)^2 - 10*I*c^2*tan(f*x + e) + 40*c*d*tan(f*x + e
) + 54*I*d^2*tan(f*x + e) - 11*c^2 - 36*I*c*d + 37*d^2)/((-32*I*a^2*c^3 + 9
6*a^2*c^2*d + 96*I*a^2*c*d^2 - 32*a^2*d^3)*(tan(f*x + e) - I)^2))/f
```

Mupad [B]
time = 9.23, size = 1384, normalized size = 7.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^2*(c + d*tan(e + f*x))),x)
```

```
[Out] symsum(log(root(640*a^6*c^4*d^4*e^3 - a^6*c^5*d^3*e^3*256i + a^6*c^3*d^5*e^
3*256i + 256*a^6*c^6*d^2*e^3 + 256*a^6*c^2*d^6*e^3 - a^6*c^7*d*e^3*256i + a
^6*c*d^7*e^3*256i - 64*a^6*d^8*e^3 - 64*a^6*c^8*e^3 + a^2*c*d^5*e*18i - a^2
*c^5*d*e*6i + a^2*c^3*d^3*e*12i + 15*a^2*c^4*d^2*e + 9*a^2*c^2*d^4*e + 57*a
^2*d^6*e - a^2*c^6*e - c^2*d^3 - c*d^4*4i + 7*d^5, e, k)*(root(640*a^6*c^4*
d^4*e^3 - a^6*c^5*d^3*e^3*256i + a^6*c^3*d^5*e^3*256i + 256*a^6*c^6*d^2*e^3
 + 256*a^6*c^2*d^6*e^3 - a^6*c^7*d*e^3*256i + a^6*c*d^7*e^3*256i - 64*a^6*d
^8*e^3 - 64*a^6*c^8*e^3 + a^2*c*d^5*e*18i - a^2*c^5*d*e*6i + a^2*c^3*d^3*e*
12i + 15*a^2*c^4*d^2*e + 9*a^2*c^2*d^4*e + 57*a^2*d^6*e - a^2*c^6*e - c^2*d
^3 - c*d^4*4i + 7*d^5, e, k)*((a^2*d^6 + a^2*c*d^5*4i - 6*a^2*c^2*d^4 - a^2
*c^3*d^3*4i + a^2*c^4*d^2)*(128*a^4*c*d^5 + 128*a^4*c^5*d - a^4*c^2*d^4*512
i - 768*a^4*c^3*d^3 + a^4*c^4*d^2*512i) - tan(e + f*x)*(a^2*d^6 + a^2*c*d^5
*4i - 6*a^2*c^2*d^4 - a^2*c^3*d^3*4i + a^2*c^4*d^2)*(32*a^4*c^6 - 96*a^4*d^
6 + a^4*c*d^5*384i + a^4*c^5*d*128i + 608*a^4*c^2*d^4 - a^4*c^3*d^3*512i -
288*a^4*c^4*d^2)) + (a^2*d^6 + a^2*c*d^5*4i - 6*a^2*c^2*d^4 - a^2*c^3*d^3*4
i + a^2*c^4*d^2)*(4*a^2*c^5 + a^2*d^5*12i + 44*a^2*c*d^4 + a^2*c^4*d*20i -
a^2*c^2*d^3*64i - 48*a^2*c^3*d^2) + tan(e + f*x)*(48*a^2*d^5 - a^2*c*d^4*12
0i + 8*a^2*c^4*d - 104*a^2*c^2*d^3 + a^2*c^3*d^2*40i)*(a^2*d^6 + a^2*c*d^5*
4i - 6*a^2*c^2*d^4 - a^2*c^3*d^3*4i + a^2*c^4*d^2)) - (13*c*d^3 - c^3*d + d
^4*12i - c^2*d^2*6i)*(a^2*d^6 + a^2*c*d^5*4i - 6*a^2*c^2*d^4 - a^2*c^3*d^3*
4i + a^2*c^4*d^2) + tan(e + f*x)*(c*d^3*6i - 9*d^4 + c^2*d^2)*(a^2*d^6 + a^
2*c*d^5*4i - 6*a^2*c^2*d^4 - a^2*c^3*d^3*4i + a^2*c^4*d^2))*root(640*a^6*c^
4*d^4*e^3 - a^6*c^5*d^3*e^3*256i + a^6*c^3*d^5*e^3*256i + 256*a^6*c^6*d^2*e
^3 + 256*a^6*c^2*d^6*e^3 - a^6*c^7*d*e^3*256i + a^6*c*d^7*e^3*256i - 64*a^6
*d^8*e^3 - 64*a^6*c^8*e^3 + a^2*c*d^5*e*18i - a^2*c^5*d*e*6i + a^2*c^3*d^3*
e*12i + 15*a^2*c^4*d^2*e + 9*a^2*c^2*d^4*e + 57*a^2*d^6*e - a^2*c^6*e - c^2
*d^3 - c*d^4*4i + 7*d^5, e, k), k, 1, 3)/f + (((c + d*2i)*1i)/(2*a^2*(c*d*2
i + c^2 - d^2)) - (tan(e + f*x)*(c + d*3i))/(4*a^2*(c*d*2i + c^2 - d^2)))/(
f*(tan(e + f*x)*2i - tan(e + f*x)^2 + 1))
```

3.1088 $\int \frac{1}{(a+ia\tan(e+fx))^3(c+d\tan(e+fx))}\,dx$

Optimal. Leaf size=234

$$\frac{(c^4+4ic^3d-6c^2d^2-4icd^3-7d^4)\,x}{8a^3(c-id)(c+id)^4}+\frac{d^4\log(c\cos(e+fx)+d\sin(e+fx))}{a^3(c+id)^4(ic+d)f}-\frac{1}{6(ic-d)f(a+ia\tan(e+fx))}$$

```
[Out] 1/8*(c^4+4*I*c^3*d-6*c^2*d^2-4*I*c*d^3-7*d^4)*x/a^3/(c-I*d)/(c+I*d)^4+d^4*l
n(c*cos(f*x+e)+d*sin(f*x+e))/a^3/(c+I*d)^4/(I*c+d)/f-1/6/(I*c-d)/f/(a+I*a*t
an(f*x+e))^3+1/8*(I*c-3*d)/a/(c+I*d)^2/f/(a+I*a*tan(f*x+e))^2+1/8*(c^2+4*I*
c*d-7*d^2)/(I*c-d)^3/f/(a^3+I*a^3*tan(f*x+e))
```

Rubi [A]

time = 0.46, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{3640, 3677, 3612, 3611\}$

$$\frac{c^2+4icd-7d^2}{8f(-d+ic)^3\,(a^3+ia^3\tan(e+fx))}+\frac{x(c^4+4ic^3d-6c^2d^2-4icd^3-7d^4)}{8a^3(c-id)(c+id)^4}+\frac{d^4\log(c\cos(e+fx)+d\sin(e+fx))}{a^3f(c+id)^4(d+ic)}+\frac{-3d+ic}{8af(c+id)^2(a+ia\tan(e+fx))^2}-\frac{1}{6f(-d+ic)(a+ia\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^3*(c + d*Tan[e + f*x])),x]
```

```
[Out] ((c^4 + (4*I)*c^3*d - 6*c^2*d^2 - (4*I)*c*d^3 - 7*d^4)*x)/(8*a^3*(c - I*d)*
(c + I*d)^4) + (d^4*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/(a^3*(c + I*d)^4*
(I*c + d)*f) - 1/(6*(I*c - d)*f*(a + I*a*Tan[e + f*x])^3) + (I*c - 3*d)/(8*
a*(c + I*d)^2*f*(a + I*a*Tan[e + f*x])^2) + (c^2 + (4*I)*c*d - 7*d^2)/(8*(I
*c - d)^3*f*(a^3 + I*a^3*Tan[e + f*x]))
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
```

```
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^3(c+d\tan(e+fx))}\,dx = -\frac{1}{6(ic-d)f(a+ia\tan(e+fx))^3} - \frac{\int \frac{-3a(ic-2d)-3iad\,\mathrm{ta}}{(a+ia\tan(e+fx))^2(c+d}}{6a^2(ic-}$$

$$= -\frac{1}{6(ic-d)f(a+ia\tan(e+fx))^3} + \frac{ic-3}{8a(c+id)^2f(a+ia}$$

$$= -\frac{1}{6(ic-d)f(a+ia\tan(e+fx))^3} + \frac{ic-3}{8a(c+id)^2f(a+ia}$$

$$= \frac{(c^4+4ic^3d-6c^2d^2-4icd^3-7d^4)\,x}{8a^3(c-id)(c+id)^4} - \frac{1}{6(ic-d)f(a+ia}$$

$$= \frac{(c^4+4ic^3d-6c^2d^2-4icd^3-7d^4)\,x}{8a^3(c-id)(c+id)^4} + \frac{d^4\log(c\cos(e+f}{a^3(c+id}$$

Mathematica [A]
time = 2.02, size = 435, normalized size = 1.86

$$\frac{\sec^3(e+fx)\left(-3(9c^4+28ic^3d-18c^2d^2+28icd^3-27d^4)\cos(e+fx)+2\cos(3(e+fx))\left(-36ic^2d^2fx+c^4(-1+6ifx)+d^4(1-42ifx)+2cd^3(-i+12fx)-2c^3d(i+12fx)+24d^4\log\left((c\cos(e+fx)+d\sin(e+fx))^2\right)\right)-9ic^4\sin(e+fx)+36c^3d\sin(e+fx)+42ic^2d^2\sin(e+fx)+36cd^3\sin(e+fx)+51id^4\sin(e+fx)+2ic^4\sin(3(e+fx))-4c^3d\sin(3(e+fx))-4cd^3\sin(3(e+fx))-2id^4\sin(3(e+fx))-12c^4fx\sin(3(e+fx))-48ic^3dfx\sin(3(e+fx))+72c^2d^2fx\sin(3(e+fx))+48icd^3fx\sin(3(e+fx))+84d^4fx\sin(3(e+fx))+48id^4\log\left((c\cos(e+fx)+d\sin(e+fx))^2\right)\sin(3(e+fx))\right)}{96a^3(c-id)(c+id)^4f(-i+\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^3*(c + d*Tan[e + f*x])),x]
```

```
[Out] (Sec[e + f*x]^3*(-3*(9*c^4 + (28*I)*c^3*d - 18*c^2*d^2 + (28*I)*c*d^3 - 27*
d^4)*Cos[e + f*x] + 2*Cos[3*(e + f*x)]*((-36*I)*c^2*d^2*f*x + c^4*(-1 + (6*
```

```
I)*f*x) + d^4*(1 - (42*I)*f*x) + 2*c*d^3*(-I + 12*f*x) - 2*c^3*d*(I + 12*f*
x) + 24*d^4*Log[(c*Cos[e + f*x] + d*Sin[e + f*x])^2]) - (9*I)*c^4*Sin[e + f
*x] + 36*c^3*d*Sin[e + f*x] + (42*I)*c^2*d^2*Sin[e + f*x] + 36*c*d^3*Sin[e
+ f*x] + (51*I)*d^4*Sin[e + f*x] + (2*I)*c^4*Sin[3*(e + f*x)] - 4*c^3*d*Sin
[3*(e + f*x)] - 4*c*d^3*Sin[3*(e + f*x)] - (2*I)*d^4*Sin[3*(e + f*x)] - 12*
c^4*f*x*Sin[3*(e + f*x)] - (48*I)*c^3*d*f*x*Sin[3*(e + f*x)] + 72*c^2*d^2*f
*x*Sin[3*(e + f*x)] + (48*I)*c*d^3*f*x*Sin[3*(e + f*x)] + 84*d^4*f*x*Sin[3*
(e + f*x)] + (48*I)*d^4*Log[(c*Cos[e + f*x] + d*Sin[e + f*x])^2]*Sin[3*(e +
 f*x)]))/(96*a^3*(c - I*d)*(c + I*d)^4*f*(-I + Tan[e + f*x])^3)
```

Maple [A]
time = 0.56, size = 248, normalized size = 1.06

method	result
derivativedivides	$\frac{\frac{\left(-ic^3+11ic\,d^2+5c^2d-15d^3\right)\ln(\tan(fx+e)-i)}{16(id+c)^4}-\frac{-5ic^2d+7id^3-c^3+11c\,d^2}{8(id+c)^4(\tan(fx+e)-i)}-\frac{ic^3-7ic\,d^2-5c^2d+3d^3}{8(id+c)^4(\tan(fx+e)-i)^2}-\frac{3ic^2d-id^3+c^3-3c\,d^2}{6(id+c)^4(\tan(fx+e)-i)^3}+}{f\,a^3}$
default	$\frac{\frac{\left(-ic^3+11ic\,d^2+5c^2d-15d^3\right)\ln(\tan(fx+e)-i)}{16(id+c)^4}-\frac{-5ic^2d+7id^3-c^3+11c\,d^2}{8(id+c)^4(\tan(fx+e)-i)}-\frac{ic^3-7ic\,d^2-5c^2d+3d^3}{8(id+c)^4(\tan(fx+e)-i)^2}-\frac{3ic^2d-id^3+c^3-3c\,d^2}{6(id+c)^4(\tan(fx+e)-i)^3}+}{f\,a^3}$
risch	$-\frac{x}{8a^3(id-c)}-\frac{5\,\mathrm{e}^{-2i(fx+e)}cd}{8a^3(id+c)^3f}+\frac{3i\mathrm{e}^{-2i(fx+e)}c^2}{16a^3(id+c)^3f}-\frac{11i\mathrm{e}^{-2i(fx+e)}d^2}{16a^3(id+c)^3f}-\frac{5\,\mathrm{e}^{-4i(fx+e)}d}{32a^3(id+c)^2f}+\frac{3i\mathrm{e}^{-4i(fx+e)}c}{32a^3(id+c)^2f}+$
norman	$\frac{-16icd-5c^2+17d^2}{12af\left(ic^3-3ic\,d^2-3c^2d+d^3\right)}+\frac{\left(20icd+7c^2-17d^2\right)\tan(fx+e)}{8af\left(3ic^2d-id^3+c^3-3c\,d^2\right)}+\frac{\left(4icd+c^2-7d^2\right)\left(\tan^5(fx+e)\right)}{8af\left(3ic^2d-id^3+c^3-3c\,d^2\right)}+\frac{\left(5icd+c^2-7d^2\right)\left(\tan^3(fx+e)\right)}{3af\left(3ic^2d-id^3+c^3-3c\,d^2\right)}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a^3*(1/16/(c+I*d)^4*(-I*c^3+11*I*c*d^2+5*c^2*d-15*d^3)*ln(tan(f*x+e)-I)
-1/8*(-5*I*c^2*d+7*I*d^3-c^3+11*c*d^2)/(c+I*d)^4/(tan(f*x+e)-I)-1/8*(I*c^3-
7*I*c*d^2-5*c^2*d+3*d^3)/(c+I*d)^4/(tan(f*x+e)-I)^2-1/6*(3*I*c^2*d-I*d^3+c^
3-3*c*d^2)/(c+I*d)^4/(tan(f*x+e)-I)^3+I*d^4/(I*d-c)/(c+I*d)^4*ln(c+d*tan(f*
x+e))-I/(16*I*d-16*c)*ln(tan(f*x+e)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]

time = 1.35, size = 275, normalized size = 1.18

$$-\frac{\left(96\,d^4e^{(6i\,fx+6i\,e)}\log\left(\frac{(i\,c+d)e^{(2i\,fx+2i\,e)}+i\,c-d}{i\,c+d}\right)-2\,c^4-4i\,c^3d-4i\,cd^3+2\,d^4-12\,(-i\,c^4+4\,c^3d+6i\,c^2d^2-4\,cd^3+15i\,d^4)fxe^{(6i\,fx+6i\,e)}-6\,(3\,c^4+10i\,c^3d-8\,c^2d^2+10i\,cd^3-11\,d^4)e^{(4i\,fx+4i\,e)}-3\,(3\,c^4+8i\,c^3d-2\,c^2d^2+8i\,cd^3-5\,d^4)e^{(2i\,fx+2i\,e)}\right)e^{(-6i\,fx-6i\,e)}}{96\,(-i\,a^3c^5+3\,a^3c^4d+2i\,a^3c^3d^2+2\,a^3c^2d^3+3i\,a^3cd^4-a^3d^5)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/96*(96*d^4*e^(6*I*f*x + 6*I*e)*log(((I*c + d)*e^(2*I*f*x + 2*I*e) + I*c
- d)/(I*c + d)) - 2*c^4 - 4*I*c^3*d - 4*I*c*d^3 + 2*d^4 - 12*(-I*c^4 + 4*c^
3*d + 6*I*c^2*d^2 - 4*c*d^3 + 15*I*d^4)*f*x*e^(6*I*f*x + 6*I*e) - 6*(3*c^4
+ 10*I*c^3*d - 8*c^2*d^2 + 10*I*c*d^3 - 11*d^4)*e^(4*I*f*x + 4*I*e) - 3*(3*
c^4 + 8*I*c^3*d - 2*c^2*d^2 + 8*I*c*d^3 - 5*d^4)*e^(2*I*f*x + 2*I*e))*e^(-6
*I*f*x - 6*I*e)/((-I*a^3*c^5 + 3*a^3*c^4*d + 2*I*a^3*c^3*d^2 + 2*a^3*c^2*d^
3 + 3*I*a^3*c*d^4 - a^3*d^5)*f)
```

Sympy [A]

time = 14.34, size = 1192, normalized size = 5.09

$$\frac{x(c^3+5ic^2d-11cd^2-15id^3)}{8a^3c^4+32ia^3c^3d-48a^3c^2d^2-32ia^3cd^3+8a^3d^4}+\begin{cases}\frac{(512ia^6c^5f^2e^{6ie}-2560a^6c^4df^2e^{6ie}-5120ia^6c^3d^2f^2e^{6ie}+5120a^6c^2d^3f^2e^{6ie}+2560ia^6cd^4f^2e^{6ie}-512a^6d^5f^2e^{6ie})e^{-6ifx}+(2304ia^6c^5f^2e^{8ie}-13056a^6c^4df^2e^{8ie}-29184ia^6c^3d^2f^2e^{8ie}+32256a^6c^2d^3f^2e^{8ie}+17664ia^6cd^4f^2e^{8ie}-3840a^6d^5f^2e^{8ie})e^{-4ifx}+(4608ia^6c^5f^2e^{10ie}-29184a^6c^4df^2e^{10ie}-76800ia^6c^3d^2f^2e^{10ie}+101376a^6c^2d^3f^2e^{10ie}+66048ia^6cd^4f^2e^{10ie}-16896a^6d^5f^2e^{10ie})e^{-2ifx}}{24576a^9c^6f^3e^{12ie}+147456ia^9c^5df^3e^{12ie}-368640a^9c^4d^2f^3e^{12ie}-491520ia^9c^3d^3f^3e^{12ie}+368640a^9c^2d^4f^3e^{12ie}+147456ia^9cd^5f^3e^{12ie}-24576a^9d^6f^3e^{12ie}} & \text{for } 24576a^9c^6f^3e^{12ie}+147456ia^9c^5df^3e^{12ie}-368640a^9c^4d^2f^3e^{12ie}-491520ia^9c^3d^3f^3e^{12ie}+368640a^9c^2d^4f^3e^{12ie}+147456ia^9cd^5f^3e^{12ie}-24576a^9d^6f^3e^{12ie}\neq 0 \\ x\left(-\frac{c^3+5ic^2d-11cd^2-15id^3}{8a^3c^4+32ia^3c^3d-48a^3c^2d^2-32ia^3cd^3+8a^3d^4}+\frac{c^3e^{6ie}+3c^3e^{4ie}+3c^3e^{2ie}+c^3+5ic^2de^{6ie}+13ic^2de^{4ie}+11ic^2de^{2ie}+3ic^2d-11cd^2e^{6ie}-21cd^2e^{4ie}-13cd^2e^{2ie}-3cd^2-15id^3e^{6ie}-11id^3e^{4ie}-5id^3e^{2ie}-id^3}{8a^3c^4e^{6ie}+32ia^3c^3de^{6ie}-48a^3c^2d^2e^{6ie}-32ia^3cd^3e^{6ie}+8a^3d^4e^{6ie}}\right) & \text{otherwise}\end{cases}-\frac{id^4\log\left(\frac{c+id}{ce^{2ie}-ide^{2ie}}+e^{2ifx}\right)}{a^3f(c-id)(c+id)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**3/(c+d*tan(f*x+e)),x)
```

```
[Out] x*(c**3 + 5*I*c**2*d - 11*c*d**2 - 15*I*d**3)/(8*a**3*c**4 + 32*I*a**3*c**3
*d - 48*a**3*c**2*d**2 - 32*I*a**3*c*d**3 + 8*a**3*d**4) + Piecewise((((512
*I*a**6*c**5*f**2*exp(6*I*e) - 2560*a**6*c**4*d*f**2*exp(6*I*e) - 5120*I*a*
*6*c**3*d**2*f**2*exp(6*I*e) + 5120*a**6*c**2*d**3*f**2*exp(6*I*e) + 2560*I
*a**6*c*d**4*f**2*exp(6*I*e) - 512*a**6*d**5*f**2*exp(6*I*e))*exp(-6*I*f*x)
 + (2304*I*a**6*c**5*f**2*exp(8*I*e) - 13056*a**6*c**4*d*f**2*exp(8*I*e) -
29184*I*a**6*c**3*d**2*f**2*exp(8*I*e) + 32256*a**6*c**2*d**3*f**2*exp(8*I*
e) + 17664*I*a**6*c*d**4*f**2*exp(8*I*e) - 3840*a**6*d**5*f**2*exp(8*I*e))*
exp(-4*I*f*x) + (4608*I*a**6*c**5*f**2*exp(10*I*e) - 29184*a**6*c**4*d*f**2
*exp(10*I*e) - 76800*I*a**6*c**3*d**2*f**2*exp(10*I*e) + 101376*a**6*c**2*d
**3*f**2*exp(10*I*e) + 66048*I*a**6*c*d**4*f**2*exp(10*I*e) - 16896*a**6*d*
*5*f**2*exp(10*I*e))*exp(-2*I*f*x))/(24576*a**9*c**6*f**3*exp(12*I*e) + 147
456*I*a**9*c**5*d*f**3*exp(12*I*e) - 368640*a**9*c**4*d**2*f**3*exp(12*I*e)
 - 491520*I*a**9*c**3*d**3*f**3*exp(12*I*e) + 368640*a**9*c**2*d**4*f**3*ex
p(12*I*e) + 147456*I*a**9*c*d**5*f**3*exp(12*I*e) - 24576*a**9*d**6*f**3*ex
p(12*I*e)), Ne(24576*a**9*c**6*f**3*exp(12*I*e) + 147456*I*a**9*c**5*d*f**3
*exp(12*I*e) - 368640*a**9*c**4*d**2*f**3*exp(12*I*e) - 491520*I*a**9*c**3*
d**3*f**3*exp(12*I*e) + 368640*a**9*c**2*d**4*f**3*exp(12*I*e) + 147456*I*a
**9*c*d**5*f**3*exp(12*I*e) - 24576*a**9*d**6*f**3*exp(12*I*e), 0)), (x*(-(
c**3 + 5*I*c**2*d - 11*c*d**2 - 15*I*d**3)/(8*a**3*c**4 + 32*I*a**3*c**3*d
- 48*a**3*c**2*d**2 - 32*I*a**3*c*d**3 + 8*a**3*d**4) + (c**3*exp(6*I*e) +
```

```
3*c**3*exp(4*I*e) + 3*c**3*exp(2*I*e) + c**3 + 5*I*c**2*d*exp(6*I*e) + 13*I
*c**2*d*exp(4*I*e) + 11*I*c**2*d*exp(2*I*e) + 3*I*c**2*d - 11*c*d**2*exp(6*
I*e) - 21*c*d**2*exp(4*I*e) - 13*c*d**2*exp(2*I*e) - 3*c*d**2 - 15*I*d**3*e
xp(6*I*e) - 11*I*d**3*exp(4*I*e) - 5*I*d**3*exp(2*I*e) - I*d**3)/(8*a**3*c*
*4*exp(6*I*e) + 32*I*a**3*c**3*d*exp(6*I*e) - 48*a**3*c**2*d**2*exp(6*I*e)
- 32*I*a**3*c*d**3*exp(6*I*e) + 8*a**3*d**4*exp(6*I*e))), True)) - I*d**4*l
og((c + I*d)/(c*exp(2*I*e) - I*d*exp(2*I*e)) + exp(2*I*f*x))/(a**3*f*(c - I
*d)*(c + I*d)**4)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(203) = 406$.
time = 0.69, size = 445, normalized size = 1.90

$$-\frac{\frac{96i\,d^{5}\log\left(-i\,d\tan\left(fx+e\right)-i\,c\right)}{a^{3}c^{5}d+3i\,a^{3}c^{4}d^{2}-2\,a^{3}c^{3}d^{3}+2i\,a^{3}c^{2}d^{4}-3\,a^{3}cd^{5}-i\,a^{3}d^{6}}-\frac{6\left(-i\,c^{3}+5\,c^{2}d+11i\,cd^{2}-15\,d^{3}\right)\log\left(i\,\tan\left(fx+e\right)+1\right)}{a^{3}c^{4}+4i\,a^{3}c^{3}d-6\,a^{3}c^{2}d^{2}-4i\,a^{3}cd^{3}+a^{3}d^{4}}-\frac{192\,\log\left(\tan\left(fx+e\right)+i\right)}{-32i\,a^{3}c-32\,a^{3}d}-\frac{11i\,c^{3}\tan\left(fx+e\right)^{3}-55\,c^{2}d\tan\left(fx+e\right)^{3}-121i\,cd^{2}\tan\left(fx+e\right)^{3}+165\,d^{3}\tan\left(fx+e\right)^{3}+45\,c^{3}\tan\left(fx+e\right)^{2}+225i\,c^{2}d\tan\left(fx+e\right)^{2}-495\,cd^{2}\tan\left(fx+e\right)^{2}-579i\,d^{3}\tan\left(fx+e\right)^{2}-69i\,c^{3}\tan\left(fx+e\right)+345\,c^{2}d\tan\left(fx+e\right)+711i\,cd^{2}\tan\left(fx+e\right)-699\,d^{3}\tan\left(fx+e\right)-51\,c^{3}-223i\,c^{2}d+385\,cd^{2}+301i\,d^{3}}{\left(a^{3}c^{4}+4i\,a^{3}c^{3}d-6\,a^{3}c^{2}d^{2}-4i\,a^{3}cd^{3}+a^{3}d^{4}\right)\left(\tan\left(fx+e\right)-i\right)^{3}}}{96\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/96*(96*I*d^5*log(-I*d*tan(f*x + e) - I*c)/(a^3*c^5*d + 3*I*a^3*c^4*d^2 -
 2*a^3*c^3*d^3 + 2*I*a^3*c^2*d^4 - 3*a^3*c*d^5 - I*a^3*d^6) - 6*(-I*c^3 + 5
*c^2*d + 11*I*c*d^2 - 15*d^3)*log(I*tan(f*x + e) + 1)/(a^3*c^4 + 4*I*a^3*c^
3*d - 6*a^3*c^2*d^2 - 4*I*a^3*c*d^3 + a^3*d^4) - 192*log(tan(f*x + e) + I)/
(-32*I*a^3*c - 32*a^3*d) - (11*I*c^3*tan(f*x + e)^3 - 55*c^2*d*tan(f*x + e)
^3 - 121*I*c*d^2*tan(f*x + e)^3 + 165*d^3*tan(f*x + e)^3 + 45*c^3*tan(f*x +
 e)^2 + 225*I*c^2*d*tan(f*x + e)^2 - 495*c*d^2*tan(f*x + e)^2 - 579*I*d^3*t
an(f*x + e)^2 - 69*I*c^3*tan(f*x + e) + 345*c^2*d*tan(f*x + e) + 711*I*c*d^
2*tan(f*x + e) - 699*d^3*tan(f*x + e) - 51*c^3 - 223*I*c^2*d + 385*c*d^2 +
301*I*d^3)/((a^3*c^4 + 4*I*a^3*c^3*d - 6*a^3*c^2*d^2 - 4*I*a^3*c*d^3 + a^3*
d^4)*(tan(f*x + e) - I)^3))/f
```

Mupad [B]
time = 10.00, size = 1952, normalized size = 8.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^3*(c + d*tan(e + f*x))),x)
```

```
[Out] symsum(log(- (81*c*d^5 + c^5*d + d^6*56i - c^2*d^4*64i - 30*c^3*d^3 + c^4*d
^2*8i)*(a^3*d^8 + a^3*c*d^7*6i - 15*a^3*c^2*d^6 - a^3*c^3*d^5*20i + 15*a^3*
c^4*d^4 + a^3*c^5*d^3*6i - a^3*c^6*d^2) - root(a^9*c^5*d^5*e^3*7168i + 3584
*a^9*c^6*d^4*e^3 - 3584*a^9*c^4*d^6*e^3 + 3328*a^9*c^8*d^2*e^3 - 3328*a^9*c
^2*d^8*e^3 + a^9*c^7*d^3*e^3*2048i + a^9*c^3*d^7*e^3*2048i - a^9*c^9*d*e^3*
1536i - a^9*c*d^9*e^3*1536i + 256*a^9*d^10*e^3 - 256*a^9*c^10*e^3 - a^3*c*d
^7*e*56i - a^3*c^7*d*e*8i - 68*a^3*c^2*d^6*e + a^3*c^5*d^3*e*56i - 54*a^3*c
^4*d^4*e + 28*a^3*c^6*d^2*e + a^3*c^3*d^5*e*8i - 241*a^3*d^8*e - a^3*c^8*e
```

- c^3*d^4*1i + 5*c^2*d^5 + c*d^6*11i - 15*d^7, e, k)*((a^3*d^8 + a^3*c*d^7*6i - 15*a^3*c^2*d^6 - a^3*c^3*d^5*20i + 15*a^3*c^4*d^4 + a^3*c^5*d^3*6i - a^3*c^6*d^2)*(8*a^3*c^7 - a^3*d^7*56i - 264*a^3*c*d^6 + a^3*c^6*d*56i + a^3*c^2*d^5*520i + 568*a^3*c^3*d^4 - a^3*c^4*d^3*392i - 184*a^3*c^5*d^2) + root(a^9*c^5*d^5*e^3*7168i + 3584*a^9*c^6*d^4*e^3 - 3584*a^9*c^4*d^6*e^3 + 3328*a^9*c^8*d^2*e^3 - 3328*a^9*c^2*d^8*e^3 + a^9*c^7*d^3*e^3*2048i + a^9*c^3*d^7*e^3*2048i - a^9*c^9*d*e^3*1536i - a^9*c*d^9*e^3*1536i + 256*a^9*d^10*e^3 - 256*a^9*c^10*e^3 - a^3*c*d^7*e*56i - a^3*c^7*d*e*8i - 68*a^3*c^2*d^6*e + a^3*c^5*d^3*e*56i - 54*a^3*c^4*d^4*e + 28*a^3*c^6*d^2*e + a^3*c^3*d^5*e*8i - 241*a^3*d^8*e - a^3*c^8*e - c^3*d^4*1i + 5*c^2*d^5 + c*d^6*11i - 15*d^7, e, k)*((512*a^6*c^7*d - 512*a^6*c*d^7 + a^6*c^2*d^6*3072i + 7680*a^6*c^3*d^5 - a^6*c^4*d^4*10240i - 7680*a^6*c^5*d^3 + a^6*c^6*d^2*3072i)*(a^3*d^8 + a^3*c*d^7*6i - 15*a^3*c^2*d^6 - a^3*c^3*d^5*20i + 15*a^3*c^4*d^4 + a^3*c^5*d^3*6i - a^3*c^6*d^2) - tan(e + f*x)*(a^3*d^8 + a^3*c*d^7*6i - 15*a^3*c^2*d^6 - a^3*c^3*d^5*20i + 15*a^3*c^4*d^4 + a^3*c^5*d^3*6i - a^3*c^6*d^2)*(128*a^6*c^8 + 384*a^6*d^8 - a^6*c*d^7*2304i + a^6*c^7*d*768i - 5888*a^6*c^2*d^6 + a^6*c^3*d^5*8448i + 7680*a^6*c^4*d^4 - a^6*c^5*d^3*4864i - 2304*a^6*c^6*d^2)) + tan(e + f*x)*(a^3*c*d^6*688i - 192*a^3*d^7 + 16*a^3*c^6*d + 976*a^3*c^2*d^5 - a^3*c^3*d^4*736i - 352*a^3*c^4*d^3 + a^3*c^5*d^2*112i)*(a^3*d^8 + a^3*c*d^7*6i - 15*a^3*c^2*d^6 - a^3*c^3*d^5*20i + 15*a^3*c^4*d^4 + a^3*c^5*d^3*6i - a^3*c^6*d^2)) - tan(e + f*x)*(49*d^6 - c*d^5*56i - 30*c^2*d^4 + c^3*d^3*8i + c^4*d^2)*(a^3*d^8 + a^3*c*d^7*6i - 15*a^3*c^2*d^6 - a^3*c^3*d^5*20i + 15*a^3*c^4*d^4 + a^3*c^5*d^3*6i - a^3*c^6*d^2))*root(a^9*c^5*d^5*e^3*7168i + 3584*a^9*c^6*d^4*e^3 - 3584*a^9*c^4*d^6*e^3 + 3328*a^9*c^8*d^2*e^3 - 3328*a^9*c^2*d^8*e^3 + a^9*c^7*d^3*e^3*2048i + a^9*c^3*d^7*e^3*2048i - a^9*c^9*d*e^3*1536i - a^9*c*d^9*e^3*1536i + 256*a^9*d^10*e^3 - 256*a^9*c^10*e^3 - a^3*c*d^7*e*56i - a^3*c^7*d*e*8i - 68*a^3*c^2*d^6*e + a^3*c^5*d^3*e*56i - 54*a^3*c^4*d^4*e + 28*a^3*c^6*d^2*e + a^3*c^3*d^5*e*8i - 241*a^3*d^8*e - a^3*c^8*e - c^3*d^4*1i + 5*c^2*d^5 + c*d^6*11i - 15*d^7, e, k), k, 1, 3)/f - ((c*d*32i + 10*c^2 - 34*d^2)/(24*a^3*(3*c*d^2 - c^2*d*3i - c^3 + d^3*1i)) + (tan(e + f*x)*(c*d*12i + 3*c^2 - 17*d^2)*1i)/(8*a^3*(3*c*d^2 - c^2*d*3i - c^3 + d^3*1i)) - (tan(e + f*x)^2*(c*d*4i + c^2 - 7*d^2))/(8*a^3*(3*c*d^2 - c^2*d*3i - c^3 + d^3*1i)))/(f*(3*tan(e + f*x) + tan(e + f*x)^2*3i - tan(e + f*x)^3 - 1i))

3.1089 $\int \frac{(a+ia\tan(e+fx))^3}{(c+d\tan(e+fx))^2}\,dx$

Optimal. Leaf size=142

$$\frac{4a^3x}{(c-id)^2}+\frac{ia^3\log(\cos(e+fx))}{d^2f}-\frac{a^3(ic-d)(c-3id)\log(c\cos(e+fx)+d\sin(e+fx))}{(c-id)^2d^2f}+\frac{(c+id)\,(a^3+ia^3\tan(e+fx))}{(c-id)df(c+d\tan(e+fx))}$$

```
[Out] 4*a^3*x/(c-I*d)^2+I*a^3*ln(cos(f*x+e))/d^2/f-a^3*(I*c-d)*(c-3*I*d)*ln(c*cos
(f*x+e)+d*sin(f*x+e))/(c-I*d)^2/d^2/f+(c+I*d)*(a^3+I*a^3*tan(f*x+e))/(c-I*d
)/d/f/(c+d*tan(f*x+e))
```

Rubi [A]
time = 0.26, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3634, 3670, 3556, 3612, 3611}

$$-\frac{a^3(-d+ic)(c-3id)\log(c\cos(e+fx)+d\sin(e+fx))}{d^2f(c-id)^2}+\frac{(c+id)\,(a^3+ia^3\tan(e+fx))}{df(c-id)(c+d\tan(e+fx))}+\frac{4a^3x}{(c-id)^2}+\frac{ia^3\log(\cos(e+fx))}{d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^3/(c + d*Tan[e + f*x])^2,x]
```

```
[Out] (4*a^3*x)/(c - I*d)^2 + (I*a^3*Log[Cos[e + f*x]])/(d^2*f) - (a^3*(I*c - d)*
(c - (3*I)*d)*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((c - I*d)^2*d^2*f) + (
(c + I*d)*(a^3 + I*a^3*Tan[e + f*x]))/((c - I*d)*d*f*(c + d*Tan[e + f*x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3634

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-a^2)*(b*c - a*d)*(a + b*Tan[e + f*x]
)^(m - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] + Di
st[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[b*(b*c*(m - 2) - a*d*(m - 2*n - 4)) + (a*b*c*(m - 2)
+ b^2*d*(n + 1) - a^2*d*(m + n - 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3670

```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)]))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[B*(d/
b), Int[Tan[e + f*x], x], x] + Dist[1/b, Int[Simp[A*b*c + (A*b*d + B*(b*c -
 a*d))*Tan[e + f*x], x]/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d,
e, f, A, B}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^3}{(c+d\tan(e+fx))^2}\,dx = \frac{(c+id)\left(a^3+ia^3\tan(e+fx)\right)}{(c-id)df(c+d\tan(e+fx))} - \frac{\int \frac{(a+ia\tan(e+fx))\left(-a^2(c+3id)+a^2(ic+d)\tan(e+fx)\right)}{c+d\tan(e+fx)}}{d(ic+d)}$$

$$= \frac{(c+id)\left(a^3+ia^3\tan(e+fx)\right)}{(c-id)df(c+d\tan(e+fx))} - \frac{(ia^3)\int\tan(e+fx)\,dx}{d^2} - \frac{\int \frac{-a^3(c+3id)d+a^3(c}{c+d\,\mathrm{ta}}}{d^2(}$$

$$= -\frac{4a^3x}{(ic+d)^2} + \frac{ia^3\log(\cos(e+fx))}{d^2f} + \frac{(c+id)\left(a^3+ia^3\tan(e+fx)\right)}{(c-id)df(c+d\tan(e+fx))} + \frac{(a^3(c}{}$$

$$= -\frac{4a^3x}{(ic+d)^2} + \frac{ia^3\log(\cos(e+fx))}{d^2f} - \frac{a^3(ic-d)(c-3id)\log(c\cos(e+fx)+}{(c-id)^2d^2f}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1936 vs. 2(142) = 284.
time = 7.66, size = 1936, normalized size = 13.63

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3/(c + d*Tan[e + f*x])^2,x]
```

```
[Out] ((I/2)*Cos[3*e]*Cos[e + f*x]^3*Log[Cos[e + f*x]^2]*(a + I*a*Tan[e + f*x])^3
)/(d^2*f*(Cos[f*x] + I*Sin[f*x])^3) + (Cos[e + f*x]^3*(c^2*Cos[(3*e)/2] - (
```

2*I)*c*d*Cos[(3*e)/2] + 3*d^2*Cos[(3*e)/2] - I*c^2*Sin[(3*e)/2] - 2*c*d*Sin
[(3*e)/2] - (3*I)*d^2*Sin[(3*e)/2])*((ArcTan[(2*c*d*Cos[4*e + f*x] - c^2*Si
n[4*e + f*x] + d^2*Sin[4*e + f*x])/(c^2*Cos[4*e + f*x] - d^2*Cos[4*e + f*x]
 + 2*c*d*Sin[4*e + f*x])]*Cos[(3*e)/2])/d^2 - (I*ArcTan[(2*c*d*Cos[4*e + f*
x] - c^2*Sin[4*e + f*x] + d^2*Sin[4*e + f*x])/(c^2*Cos[4*e + f*x] - d^2*Cos
[4*e + f*x] + 2*c*d*Sin[4*e + f*x])]*Sin[(3*e)/2])/d^2)*(a + I*a*Tan[e + f*
x])^3)/((c - I*d)^2*f*(Cos[f*x] + I*Sin[f*x])^3) + (Cos[e + f*x]^3*(c^2*Cos
[(3*e)/2] - (2*I)*c*d*Cos[(3*e)/2] + 3*d^2*Cos[(3*e)/2] - I*c^2*Sin[(3*e)/2
] - 2*c*d*Sin[(3*e)/2] - (3*I)*d^2*Sin[(3*e)/2])*(((-1/2*I)*Cos[(3*e)/2]*Lo
g[(c*Cos[e + f*x] + d*Sin[e + f*x])^2])/d^2 - (Log[(c*Cos[e + f*x] + d*Sin[
e + f*x])^2]*Sin[(3*e)/2])/(2*d^2))*(a + I*a*Tan[e + f*x])^3)/((c - I*d)^2*
f*(Cos[f*x] + I*Sin[f*x])^3) + (Cos[e + f*x]^3*Log[Cos[e + f*x]^2]*Sin[3*e]
*(a + I*a*Tan[e + f*x])^3)/(2*d^2*f*(Cos[f*x] + I*Sin[f*x])^3) + (Cos[e + f
x]^3(4*f*x*Cos[3*e] - (4*I)*f*x*Sin[3*e])*(a + I*a*Tan[e + f*x])^3)/((c -
 I*d)^2*f*(Cos[f*x] + I*Sin[f*x])^3) + (Cos[e + f*x]^3*(Cos[3*e]/d - (I*Sin
[3*e])/d)*(I*c^2*Sin[f*x] - 2*c*d*Sin[f*x] - I*d^2*Sin[f*x])*(a + I*a*Tan[e
 + f*x])^3)/((c - I*d)*f*(c*Cos[e] + d*Sin[e])*(Cos[f*x] + I*Sin[f*x])^3*(c
*Cos[e + f*x] + d*Sin[e + f*x])) + (x*Cos[e + f*x]^3*(Cos[e]/(2*d^2) - Cos[
e]^3/(2*d^2) - (I*Sin[e])/d^2 + ((2*I)*Cos[e]^2*Sin[e])/d^2 + (3*Cos[e]*Sin
[e]^2)/d^2 - ((2*I)*Sin[e]^3)/d^2 + (5*c*Cos[e]^4)/((c - I*d)^2*(c*Cos[e] +
 d*Sin[e])) + (c^3*Cos[e]^4)/((c - I*d)^2*d^2*(c*Cos[e] + d*Sin[e])) - (I*c
^2*Cos[e]^4)/((c - I*d)^2*d*(c*Cos[e] + d*Sin[e])) + ((3*I)*d*Cos[e]^4)/((c
 - I*d)^2*(c*Cos[e] + d*Sin[e])) - ((20*I)*c*Cos[e]^3*Sin[e])/((c - I*d)^2*
(c*Cos[e] + d*Sin[e])) - ((4*I)*c^3*Cos[e]^3*Sin[e])/((c - I*d)^2*d^2*(c*Co
s[e] + d*Sin[e])) - (4*c^2*Cos[e]^3*Sin[e])/((c - I*d)^2*d*(c*Cos[e] + d*Si
n[e])) + (12*d*Cos[e]^3*Sin[e])/((c - I*d)^2*(c*Cos[e] + d*Sin[e])) - (30*c
*Cos[e]^2*Sin[e]^2)/((c - I*d)^2*(c*Cos[e] + d*Sin[e])) - (6*c^3*Cos[e]^2*S
in[e]^2)/((c - I*d)^2*d^2*(c*Cos[e] + d*Sin[e])) + ((6*I)*c^2*Cos[e]^2*Sin[
e]^2)/((c - I*d)^2*d*(c*Cos[e] + d*Sin[e])) - ((18*I)*d*Cos[e]^2*Sin[e]^2)/
((c - I*d)^2*(c*Cos[e] + d*Sin[e])) + ((20*I)*c*Cos[e]*Sin[e]^3)/((c - I*d)
^2*(c*Cos[e] + d*Sin[e])) + ((4*I)*c^3*Cos[e]*Sin[e]^3)/((c - I*d)^2*d^2*(c
*Cos[e] + d*Sin[e])) + (4*c^2*Cos[e]*Sin[e]^3)/((c - I*d)^2*d*(c*Cos[e] + d
*Sin[e])) - (12*d*Cos[e]*Sin[e]^3)/((c - I*d)^2*(c*Cos[e] + d*Sin[e])) + (5
*c*Sin[e]^4)/((c - I*d)^2*(c*Cos[e] + d*Sin[e])) + (c^3*Sin[e]^4)/((c - I*d
)^2*d^2*(c*Cos[e] + d*Sin[e])) - (I*c^2*Sin[e]^4)/((c - I*d)^2*d*(c*Cos[e]
+ d*Sin[e])) + ((3*I)*d*Sin[e]^4)/((c - I*d)^2*(c*Cos[e] + d*Sin[e])) + ((-
5*c - (3*I)*d + c*Cos[2*e] - (3*I)*d*Cos[2*e] + I*c*Sin[2*e] + 3*d*Sin[2*e]
)*(Cos[3*e] - I*Sin[3*e]))/((c - I*d)^2*(c + I*d + c*Cos[2*e] - I*d*Cos[2*e
] + I*c*Sin[2*e] + d*Sin[2*e])) + ((1 - Cos[2*e] - I*Sin[2*e])*(Cos[3*e]/d^
2 - (I*Sin[3*e])/d^2))/(1 + Cos[2*e] + I*Sin[2*e]) + ((-c^3 + c^3*Cos[2*e]
+ I*c^3*Sin[2*e])*(Cos[3*e]/d^2 - (I*Sin[3*e])/d^2))/((c - I*d)^2*(c + I*d
+ c*Cos[2*e] - I*d*Cos[2*e] + I*c*Sin[2*e] + d*Sin[2*e])) + ((-c^2 + 3*c^2*
Cos[2*e] + (3*I)*c^2*Sin[2*e])*(((-I)*Cos[3*e])/d - Sin[3*e]/d))/((c - I*d)
^2*(c + I*d + c*Cos[2*e] - I*d*Cos[2*e] + I*c*Sin[2*e] + d*Sin[2*e])) - (Si
n[e]*Tan[e])/(2*d^2) - (Sin[e]^3*Tan[e])/(2*d^2))*(a + I*a*Tan[e + f*x])^3)

```
/(Cos[f*x] + I*Sin[f*x])^3
```

Maple [A]
time = 0.28, size = 175, normalized size = 1.23

method	result
derivativedivides	$\frac{a^3\left(\frac{(-ic^4-6ic^2d^2+3id^4+8cd^3)\ln(c+d\tan(fx+e))}{(c^2+d^2)^2 d^2}-\frac{ic^3-3icd^2-3c^2d+d^3}{d^2(c^2+d^2)(c+d\tan(fx+e))}+\frac{\frac{(4ic^2-4id^2-8cd)\ln(1+\tan^2(fx+e))}{2}+(8icd \cdots}{(c^2+d^2)^2}\right.}{f}$
default	$\frac{a^3\left(\frac{(-ic^4-6ic^2d^2+3id^4+8cd^3)\ln(c+d\tan(fx+e))}{(c^2+d^2)^2 d^2}-\frac{ic^3-3icd^2-3c^2d+d^3}{d^2(c^2+d^2)(c+d\tan(fx+e))}+\frac{\frac{(4ic^2-4id^2-8cd)\ln(1+\tan^2(fx+e))}{2}+(8icd \cdots}{(c^2+d^2)^2}\right.}{f}$
norman	$\frac{\frac{i(2ia^3cd+a^3c^2-a^3d^2)\tan(fx+e)}{cf(-id+c)d}+\frac{4a^3cx}{-2icd+c^2-d^2}-\frac{4da^3x\tan(fx+e)}{2icd-c^2+d^2}}{c+d\tan(fx+e)}+\frac{2ia^3\ln(1+\tan^2(fx+e))}{f(-2icd+c^2-d^2)}-\frac{ia^3(-2icd+c^2+3d^2)\cdots}{d^2f(-2icd\cdots}$
risch	$-\frac{8a^3x}{2icd-c^2+d^2}-\frac{4a^3cx}{d(ic^2-id^2+2cd)}-\frac{4a^3ce}{df(ic^2-id^2+2cd)}-\frac{2ia^3c^2e}{d^2f(ic^2-id^2+2cd)}-\frac{6ia^3x}{ic^2-id^2+2cd}-\frac{6ia^3e}{f(ic^2-id^2+2cd\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a^3*(1/(c^2+d^2)^2*(-I*c^4-6*I*c^2*d^2+3*I*d^4+8*c*d^3)/d^2*ln(c+d*tan(
f*x+e))-(I*c^3-3*I*c*d^2-3*c^2*d+d^3)/d^2/(c^2+d^2)/(c+d*tan(f*x+e))+1/(c^2
+d^2)^2*(1/2*(4*I*c^2-4*I*d^2-8*c*d)*ln(1+tan(f*x+e)^2)+(8*I*c*d+4*c^2-4*d^
2)*arctan(tan(f*x+e))))
```

Maxima [A]
time = 0.48, size = 249, normalized size = 1.75

$$\frac{\frac{4\,(a^3c^2+2i\,a^3cd-a^3d^2)(fx+e)}{c^4+2\,c^2d^2+d^4}+\frac{(-i\,a^3c^4-6i\,a^3c^2d^2+8\,a^3cd^3+3i\,a^3d^4)\log(d\tan(fx+e)+c)}{c^4d^2+2\,c^2d^4+d^6}-\frac{2\,(-i\,a^3c^2+2\,a^3cd+i\,a^3d^2)\log\left(\tan(fx+e)^2+1\right)}{c^4+2\,c^2d^2+d^4}+\frac{-i\,a^3c^3+3\,a^3c^2d+3i\,a^3cd^2-a^3d^3}{c^3d^2+cd^4+(c^2d^3+d^5)\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] (4*(a^3*c^2 + 2*I*a^3*c*d - a^3*d^2)*(f*x + e)/(c^4 + 2*c^2*d^2 + d^4) + (-
I*a^3*c^4 - 6*I*a^3*c^2*d^2 + 8*a^3*c*d^3 + 3*I*a^3*d^4)*log(d*tan(f*x + e)
 + c)/(c^4*d^2 + 2*c^2*d^4 + d^6) - 2*(-I*a^3*c^2 + 2*a^3*c*d + I*a^3*d^2)*
log(tan(f*x + e)^2 + 1)/(c^4 + 2*c^2*d^2 + d^4) + (-I*a^3*c^3 + 3*a^3*c^2*d
 + 3*I*a^3*c*d^2 - a^3*d^3)/(c^3*d^2 + c*d^4 + (c^2*d^3 + d^5)*tan(f*x + e)
))/f
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(131) = 262$.
time = 1.26, size = 303, normalized size = 2.13

$$\frac{2i\,a^3c^2d-4\,a^3cd^2-2i\,a^3d^3-\left(a^3c^3-i\,a^3c^2d+5\,a^3cd^2+3i\,a^3d^3+(a^3c^3-3i\,a^3c^2d+a^3cd^2-3i\,a^3d^3)e^{(2i\,fx+2i\,e)}\right)\log\left(\frac{(i\,c+d)e^{(2i\,fx+2i\,e)}+i\,c-d}{i\,c+d}\right)+\left(a^3c^3-i\,a^3c^2d+a^3cd^2-i\,a^3d^3+(a^3c^3-3i\,a^3c^2d-3\,a^3cd^2+i\,a^3d^3)e^{(2i\,fx+2i\,e)}\right)\log\left(e^{(2i\,fx+2i\,e)}+1\right)}{(-i\,c^3d^2-3\,c^2d^3+3i\,cd^4+d^5)fe^{(2i\,fx+2i\,e)}+(-i\,c^3d^2-c^2d^3-i\,cd^4-d^5)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] (2*I*a^3*c^2*d - 4*a^3*c*d^2 - 2*I*a^3*d^3 - (a^3*c^3 - I*a^3*c^2*d + 5*a^3
*c*d^2 + 3*I*a^3*d^3 + (a^3*c^3 - 3*I*a^3*c^2*d + a^3*c*d^2 - 3*I*a^3*d^3)*
e^(2*I*f*x + 2*I*e))*log(((I*c + d)*e^(2*I*f*x + 2*I*e) + I*c - d)/(I*c + d
)) + (a^3*c^3 - I*a^3*c^2*d + a^3*c*d^2 - I*a^3*d^3 + (a^3*c^3 - 3*I*a^3*c^
2*d - 3*a^3*c*d^2 + I*a^3*d^3)*e^(2*I*f*x + 2*I*e))*log(e^(2*I*f*x + 2*I*e)
 + 1))/((-I*c^3*d^2 - 3*c^2*d^3 + 3*I*c*d^4 + d^5)*f*e^(2*I*f*x + 2*I*e) +
(-I*c^3*d^2 - c^2*d^3 - I*c*d^4 - d^5)*f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(116) = 232$.
time = 16.93, size = 374, normalized size = 2.63

$$-\frac{ia^3(c-3id)(c+id)\log\left(e^{2ifx}+\frac{a^3c^2+\frac{ia^3cd(c-3id)(c+id)}{(c-id)^2}-ia^3cd+\frac{a^3d^2(c-3id)(c+id)}{(c-id)^2}+2a^3d^2}{a^3c^2e^{2ie}-2ia^3cde^{2ie}+a^3d^2e^{2ie}}\right)}{d^2f(c-id)^2}+\frac{ia^3\log\left(\frac{a^3c^2-2ia^3cd+a^3d^2}{a^3c^2e^{2ie}-2ia^3cde^{2ie}+a^3d^2e^{2ie}}+e^{2ifx}\right)}{d^2f}+\frac{-2a^3c^2-4ia^3cd+2a^3d^2}{c^3df-ic^2d^2f+cd^3f-id^4f+(c^3dfe^{2ie}-3ic^2d^2fe^{2ie}-3cd^3fe^{2ie}+id^4fe^{2ie})e^{2ifx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3/(c+d*tan(f*x+e))**2,x)
```

```
[Out] -I*a**3*(c - 3*I*d)*(c + I*d)*log(exp(2*I*f*x) + (a**3*c**2 + I*a**3*c*d*(c
 - 3*I*d)*(c + I*d)/(c - I*d)**2 - I*a**3*c*d + a**3*d**2*(c - 3*I*d)*(c +
I*d)/(c - I*d)**2 + 2*a**3*d**2)/(a**3*c**2*exp(2*I*e) - 2*I*a**3*c*d*exp(2
*I*e) + a**3*d**2*exp(2*I*e)))/(d**2*f*(c - I*d)**2) + I*a**3*log((a**3*c**
2 - 2*I*a**3*c*d + a**3*d**2)/(a**3*c**2*exp(2*I*e) - 2*I*a**3*c*d*exp(2*I*
e) + a**3*d**2*exp(2*I*e)) + exp(2*I*f*x))/(d**2*f) + (-2*a**3*c**2 - 4*I*a
**3*c*d + 2*a**3*d**2)/(c**3*d*f - I*c**2*d**2*f + c*d**3*f - I*d**4*f + (c
**3*d*f*exp(2*I*e) - 3*I*c**2*d**2*f*exp(2*I*e) - 3*c*d**3*f*exp(2*I*e) + I
*d**4*f*exp(2*I*e))*exp(2*I*f*x))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(131) = 262$.
time = 0.64, size = 389, normalized size = 2.74

$$\frac{\frac{8a^3\log\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+i\right)}{-ic^2-2cd+id^2}+\frac{ia^3\log\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{d^2}+\frac{ia^3\log\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{d^2}+\frac{(-ia^3c^2-2a^3cd-3ia^3d^2)\log\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-c\right)}{c^2d^2-2icd^3-d^4}-\frac{-ia^3c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-2a^3c^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-3ia^3c^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4ia^3c^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2a^3c^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+8ia^3cd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-2a^3d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+ia^3c^4+2a^3c^3d+3ia^3c^2d^2}{(c^3d^2-2ic^2d^3-cd^4)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-c\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] (8*a^3*log(tan(1/2*f*x + 1/2*e) + I)/(-I*c^2 - 2*c*d + I*d^2) + I*a^3*log(t
an(1/2*f*x + 1/2*e) + 1)/d^2 + I*a^3*log(tan(1/2*f*x + 1/2*e) - 1)/d^2 + (-
I*a^3*c^2 - 2*a^3*c*d - 3*I*a^3*d^2)*log(c*tan(1/2*f*x + 1/2*e)^2 - 2*d*tan
(1/2*f*x + 1/2*e) - c)/(c^2*d^2 - 2*I*c*d^3 - d^4) - (-I*a^3*c^4*tan(1/2*f*
x + 1/2*e)^2 - 2*a^3*c^3*d*tan(1/2*f*x + 1/2*e)^2 - 3*I*a^3*c^2*d^2*tan(1/2
*f*x + 1/2*e)^2 + 4*I*a^3*c^3*d*tan(1/2*f*x + 1/2*e) + 2*a^3*c^2*d^2*tan(1/
```

```
2*f*x + 1/2*e) + 8*I*a^3*c*d^3*tan(1/2*f*x + 1/2*e) - 2*a^3*d^4*tan(1/2*f*x
 + 1/2*e) + I*a^3*c^4 + 2*a^3*c^3*d + 3*I*a^3*c^2*d^2)/((c^3*d^2 - 2*I*c^2*
d^3 - c*d^4)*(c*tan(1/2*f*x + 1/2*e)^2 - 2*d*tan(1/2*f*x + 1/2*e) - c)))/f
```

Mupad [B]
time = 6.82, size = 133, normalized size = 0.94

$$-\frac{4\,a^3\,\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)}{f\,\left(c^2\,1\mathrm{i}+2\,c\,d-d^2\,1\mathrm{i}\right)}+\frac{a^3\,\left(-c^2\,1\mathrm{i}+2\,c\,d+d^2\,1\mathrm{i}\right)}{d^3\,f\,\left(\tan\left(e+f\,x\right)+\frac{c}{d}\right)\,\left(c-d\,1\mathrm{i}\right)}+\frac{a^3\,\ln\left(c+d\tan(e+f\,x)\right)\,\left(c^2\,1\mathrm{i}+2\,c\,d+d^2\,3\mathrm{i}\right)}{d^2\,f\,{\left(d+c\,1\mathrm{i}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^3/(c + d*tan(e + f*x))^2,x)
```

```
[Out] (a^3*(2*c*d - c^2*1i + d^2*1i))/(d^3*f*(tan(e + f*x) + c/d)*(c - d*1i)) - (
4*a^3*log(tan(e + f*x) + 1i))/(f*(2*c*d + c^2*1i - d^2*1i)) + (a^3*log(c +
d*tan(e + f*x))*(2*c*d + c^2*1i + d^2*3i))/(d^2*f*(c*1i + d)^2)
```

3.1090 $\int \frac{(a+ia\tan(e+fx))^2}{(c+d\tan(e+fx))^2}\,dx$

Optimal. Leaf size=93

$$\frac{2a^2x}{(c-id)^2}-\frac{2ia^2\log(c\cos(e+fx)+d\sin(e+fx))}{(c-id)^2f}+\frac{a^2(ic-d)}{d(ic+d)f(c+d\tan(e+fx))}$$

```
[Out] 2*a^2*x/(c-I*d)^2-2*I*a^2*ln(c*cos(f*x+e)+d*sin(f*x+e))/(c-I*d)^2/f+a^2*(I*
c-d)/d/(I*c+d)/f/(c+d*tan(f*x+e))
```

Rubi [A]
time = 0.13, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3623, 3612, 3611}

$$\frac{a^2(-d+ic)}{df(d+ic)(c+d\tan(e+fx))}-\frac{2ia^2\log(c\cos(e+fx)+d\sin(e+fx))}{f(c-id)^2}+\frac{2a^2x}{(c-id)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2/(c + d*Tan[e + f*x])^2,x]
```

```
[Out] (2*a^2*x)/(c - I*d)^2 - ((2*I)*a^2*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((
c - I*d)^2*f) + (a^2*(I*c - d))/(d*(I*c + d)*f*(c + d*Tan[e + f*x]))
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

```
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^2}{(c+d\tan(e+fx))^2}\,dx = \frac{a^2(ic-d)}{d(ic+d)f(c+d\tan(e+fx))} + \frac{\int \frac{2a^2(c+id)+2a^2(ic-d)\tan(e+fx)}{c+d\tan(e+fx)}\,dx}{c^2+d^2}$$
$$= \frac{2a^2x}{(c-id)^2} + \frac{a^2(ic-d)}{d(ic+d)f(c+d\tan(e+fx))} - \frac{(2ia^2)\int \frac{d-c\tan(e+fx)}{c+d\tan(e+fx)}\,dx}{(c-id)^2}$$
$$= \frac{2a^2x}{(c-id)^2} - \frac{2ia^2\log(c\cos(e+fx)+d\sin(e+fx))}{(c-id)^2f} + \frac{a^2(ic-d)}{d(ic+d)f(c+d\tan(e}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 253 vs. 2(93) = 186.
time = 2.35, size = 253, normalized size = 2.72

$$\frac{a^2(\cos(e+fx)+i\sin(e+fx))^2\left(\frac{\log((c\cos(e+fx)+d\sin(e+fx))^2)(-i\cos(2e)-\sin(2e))}{f}+4x(\cos(2e)-i\sin(2e))+\frac{2\text{ArcTan}\left(\frac{2cd\cos(3e+fx)+(-c^2+d^2)\sin(3e+fx)}{(c^2-d^2)\cos(3e+fx)+2cd\sin(3e+fx)}\right)(\cos(2e)-i\sin(2e))}{f}-\frac{(c-id)(c+id)(\cos(2e)-i\sin(2e))\sin(fx)}{f(c\cos(e)+d\sin(e))(c\cos(e+fx)+d\sin(e+fx))}\right)}{(c-id)^2(\cos(fx)+i\sin(fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2/(c + d*Tan[e + f*x])^2,x]
```

```
[Out] (a^2*(Cos[e + f*x] + I*Sin[e + f*x])^2*((Log[(c*Cos[e + f*x] + d*Sin[e + f*
x])^2]*((-I)*Cos[2*e] - Sin[2*e]))/f + 4*x*(Cos[2*e] - I*Sin[2*e]) + (2*Arc
Tan[(2*c*d*Cos[3*e + f*x] + (-c^2 + d^2)*Sin[3*e + f*x])/((c^2 - d^2)*Cos[3
*e + f*x] + 2*c*d*Sin[3*e + f*x])]*(Cos[2*e] - I*Sin[2*e]))/f - ((c - I*d)*
(c + I*d)*(Cos[2*e] - I*Sin[2*e])*Sin[f*x])/(f*(c*Cos[e] + d*Sin[e])*(c*Cos
[e + f*x] + d*Sin[e + f*x]))))/((c - I*d)^2*(Cos[f*x] + I*Sin[f*x])^2)
```

Maple [A]
time = 0.22, size = 153, normalized size = 1.65

method	result
derivativedivides	$\frac{a^2\left(\frac{\frac{(2ic^2-2id^2-4cd)\ln(1+\tan^2(fx+e))}{2}+(4icd+2c^2-2d^2)\arctan(\tan(fx+e))}{(c^2+d^2)^2}-\frac{-2icd-c^2+d^2}{(c^2+d^2)d(c+d\tan(fx+e))}-\frac{2(ic^2-id^2-2cd)\,\mathrm{l}}{(c^2+}\right.}{f}$
default	$\frac{a^2\left(\frac{\frac{(2ic^2-2id^2-4cd)\ln(1+\tan^2(fx+e))}{2}+(4icd+2c^2-2d^2)\arctan(\tan(fx+e))}{(c^2+d^2)^2}-\frac{-2icd-c^2+d^2}{(c^2+d^2)d(c+d\tan(fx+e))}-\frac{2(ic^2-id^2-2cd)\,\mathrm{l}}{(c^2+}\right.}{f}$
norman	$\frac{\frac{2a^2cx}{(-id+c)^2}-\frac{2a^2dx\tan(fx+e)}{(ic+d)^2}-\frac{(ia^2d+a^2c)\tan(fx+e)}{f(-id+c)c}}{c+d\tan(fx+e)}+\frac{ia^2\ln(1+\tan^2(fx+e))}{f(-2icd+c^2-d^2)}-\frac{2ia^2\ln(c+d\tan(fx+e))}{f(-2icd+c^2-d^2)}$

risch	$-\frac{4a^2x}{2icd-c^2+d^2}-\frac{4ia^2x}{ic^2-id^2+2cd}-\frac{4ia^2e}{f(ic^2-id^2+2cd)}+\frac{2a^2d}{f(-id+c)^2\left(-\mathrm{i}\mathrm{e}^{2i(fx+e)}d+id+\mathrm{e}^{2i(fx+e)}c+c\right)}-\frac{}{f(-id+c)^2(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a^2*(1/(c^2+d^2)^2*(1/2*(2*I*c^2-2*I*d^2-4*c*d)*ln(1+tan(f*x+e)^2)+(4*I
*c*d+2*c^2-2*d^2)*arctan(tan(f*x+e)))-(-2*I*c*d-c^2+d^2)/(c^2+d^2)/d/(c+d*t
an(f*x+e))-2*(I*c^2-I*d^2-2*c*d)/(c^2+d^2)^2*ln(c+d*tan(f*x+e)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(86) = 172$.
time = 0.49, size = 218, normalized size = 2.34

$$\frac{\frac{2\,(a^2c^2+2i\,a^2cd-a^2d^2)(fx+e)}{c^4+2\,c^2d^2+d^4}-\frac{2\,(i\,a^2c^2-2\,a^2cd-i\,a^2d^2)\log(d\tan(fx+e)+c)}{c^4+2\,c^2d^2+d^4}+\frac{(i\,a^2c^2-2\,a^2cd-i\,a^2d^2)\log\left(\tan(fx+e)^2+1\right)}{c^4+2\,c^2d^2+d^4}+\frac{a^2c^2+2i\,a^2cd-a^2d^2}{c^3d+cd^3+(c^2d^2+d^4)\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] (2*(a^2*c^2 + 2*I*a^2*c*d - a^2*d^2)*(f*x + e)/(c^4 + 2*c^2*d^2 + d^4) - 2*
(I*a^2*c^2 - 2*a^2*c*d - I*a^2*d^2)*log(d*tan(f*x + e) + c)/(c^4 + 2*c^2*d^
2 + d^4) + (I*a^2*c^2 - 2*a^2*c*d - I*a^2*d^2)*log(tan(f*x + e)^2 + 1)/(c^4
 + 2*c^2*d^2 + d^4) + (a^2*c^2 + 2*I*a^2*c*d - a^2*d^2)/(c^3*d + c*d^3 + (c
^2*d^2 + d^4)*tan(f*x + e)))/f
```

Fricas [A]
time = 1.12, size = 143, normalized size = 1.54

$$-\frac{2\left(a^2c+i\,a^2d+\left(a^2c+i\,a^2d+(a^2c-i\,a^2d)e^{(2i\,fx+2i\,e)}\right)\log\left(\frac{(i\,c+d)e^{(2i\,fx+2i\,e)}+i\,c-d}{i\,c+d}\right)\right)}{(-i\,c^3-3\,c^2d+3i\,cd^2+d^3)fe^{(2i\,fx+2i\,e)}+(-i\,c^3-c^2d-i\,cd^2-d^3)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -2*(a^2*c + I*a^2*d + (a^2*c + I*a^2*d + (a^2*c - I*a^2*d)*e^(2*I*f*x + 2*I
*e))*log(((I*c + d)*e^(2*I*f*x + 2*I*e) + I*c - d)/(I*c + d)))/((-I*c^3 - 3
*c^2*d + 3*I*c*d^2 + d^3)*f*e^(2*I*f*x + 2*I*e) + (-I*c^3 - c^2*d - I*c*d^2
 - d^3)*f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(73) = 146$.
time = 1.75, size = 156, normalized size = 1.68

$$-\frac{2ia^2\log\left(\frac{c+id}{ce^{2ie}-ide^{2ie}}+e^{2ifx}\right)}{f\,(c-id)^2}+\frac{-2ia^2c+2a^2d}{c^3f-ic^2df+cd^2f-id^3f+(c^3fe^{2ie}-3ic^2dfe^{2ie}-3cd^2fe^{2ie}+id^3fe^{2ie})\,e^{2ifx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2/(c+d*tan(f*x+e))**2,x)
```

```
[Out] -2*I*a**2*log((c + I*d)/(c*exp(2*I*e) - I*d*exp(2*I*e)) + exp(2*I*f*x))/(f*
(c - I*d)**2) + (-2*I*a**2*c + 2*a**2*d)/(c**3*f - I*c**2*d*f + c*d**2*f -
I*d**3*f + (c**3*f*exp(2*I*e) - 3*I*c**2*d*f*exp(2*I*e) - 3*c*d**2*f*exp(2*
I*e) + I*d**3*f*exp(2*I*e))*exp(2*I*f*x))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(86) = 172$.
time = 0.59, size = 232, normalized size = 2.49

$$\frac{2\left(\frac{a^2\log\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-c\right)}{ic^2+2cd-id^2}+\frac{2a^2\log\left(-i\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{-ic^2-2cd+id^2}-\frac{a^2c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-ia^2c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-2a^2cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-ia^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-a^2c^2}{(ic^3+2c^2d-icd^2)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-c\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 2*(a^2*log(c*tan(1/2*f*x + 1/2*e)^2 - 2*d*tan(1/2*f*x + 1/2*e) - c)/(I*c^2
+ 2*c*d - I*d^2) + 2*a^2*log(-I*tan(1/2*f*x + 1/2*e) + 1)/(-I*c^2 - 2*c*d +
 I*d^2) - (a^2*c^2*tan(1/2*f*x + 1/2*e)^2 - I*a^2*c^2*tan(1/2*f*x + 1/2*e)
- 2*a^2*c*d*tan(1/2*f*x + 1/2*e) - I*a^2*d^2*tan(1/2*f*x + 1/2*e) - a^2*c^2
)/((I*c^3 + 2*c^2*d - I*c*d^2)*(c*tan(1/2*f*x + 1/2*e)^2 - 2*d*tan(1/2*f*x
+ 1/2*e) - c)))/f
```

Mupad [B]
time = 5.18, size = 139, normalized size = 1.49

$$-\frac{a^2\,\mathrm{atanh}\left(\frac{c^2+d^2}{(d+c\,1\mathrm{i})^2}+\frac{\tan(e+f\,x)\,\left(2\,c^4\,d^2+4\,c^2\,d^4+2\,d^6\right)}{(d+c\,1\mathrm{i})^2\,\left(c^3\,d+c^2\,d^2\,1\mathrm{i}+c\,d^3+d^4\,1\mathrm{i}\right)}\right)\,4\mathrm{i}}{f\,(d+c\,1\mathrm{i})^2}+\frac{a^2\,(c+d\,1\mathrm{i})}{d^2\,f\,\left(\tan\left(e+f\,x\right)+\frac{c}{d}\right)\,\left(c-d\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^2/(c + d*tan(e + f*x))^2,x)
```

```
[Out] (a^2*(c + d*1i))/(d^2*f*(tan(e + f*x) + c/d)*(c - d*1i)) - (a^2*atanh((c^2
+ d^2)/(c*1i + d)^2 + (tan(e + f*x)*(2*d^6 + 4*c^2*d^4 + 2*c^4*d^2))/((c*1i
 + d)^2*(c*d^3 + c^3*d + d^4*1i + c^2*d^2*1i)))*4i)/(f*(c*1i + d)^2)
```

3.1091 $\int \frac{a+ia\tan(e+fx)}{(c+d\tan(e+fx))^2}\,dx$

Optimal. Leaf size=75

$$\frac{ax}{(c-id)^2}-\frac{ia\log(c\cos(e+fx)+d\sin(e+fx))}{(c-id)^2f}-\frac{a}{(ic+d)f(c+d\tan(e+fx))}$$

```
[Out] a*x/(c-I*d)^2-I*a*ln(c*cos(f*x+e)+d*sin(f*x+e))/(c-I*d)^2/f-a/(I*c+d)/f/(c+
d*tan(f*x+e))
```

Rubi [A]
time = 0.11, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3610, 3612, 3611}

$$-\frac{a}{f(d+ic)(c+d\tan(e+fx))}-\frac{ia\log(c\cos(e+fx)+d\sin(e+fx))}{f(c-id)^2}+\frac{ax}{(c-id)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])/(c + d*Tan[e + f*x])^2,x]
```

```
[Out] (a*x)/(c - I*d)^2 - (I*a*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((c - I*d)^2
*f) - a/((I*c + d)*f*(c + d*Tan[e + f*x]))
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+ia\tan(e+fx)}{(c+d\tan(e+fx))^2}\,dx &= -\frac{a}{(ic+d)f(c+d\tan(e+fx))}+\frac{\int \frac{a(c+id)+a(ic-d)\tan(e+fx)}{c+d\tan(e+fx)}\,dx}{c^2+d^2}\\ &= \frac{ax}{(c-id)^2}-\frac{a}{(ic+d)f(c+d\tan(e+fx))}-\frac{(ia)\int \frac{d-c\tan(e+fx)}{c+d\tan(e+fx)}\,dx}{(c-id)^2}\\ &= \frac{ax}{(c-id)^2}-\frac{ia\log(c\cos(e+fx)+d\sin(e+fx))}{(c-id)^2f}-\frac{a}{(ic+d)f(c+d\tan(e+f}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 302 vs. $2(75) = 150$.
time = 2.48, size = 302, normalized size = 4.03

$$\frac{\cos(e+fx)(\cos(e)-i\sin(e))(\cos(fx)-i\sin(fx))\left(4\text{ArcTan}\left(\frac{2cd\cos(2e+fx)+(-c^2+d^2)\sin(2e+fx)}{(c^2-d^2)\cos(2e+fx)+2cd\sin(2e+fx)}\right)+\frac{(c^2+d^2)\cos(fx)(4fx-i\log((c\cos(e+fx)+d\sin(e+fx))^2))+(c^2-d^2)\cos(2e+fx)(4fx-i\log((c\cos(e+fx)+d\sin(e+fx))^2))-2d(2(ic+d)\sin(fx)+c(-4fx+i\log((c\cos(e+fx)+d\sin(e+fx))^2))\sin(2e+fx))}{(c\cos(e)+d\sin(e))(c\cos(e+fx)+d\sin(e+fx))}\right)(a+ia\tan(e+fx))}{4(c-id)^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])/(c + d*Tan[e + f*x])^2,x]
```

```
[Out] (Cos[e + f*x]*(Cos[e] - I*Sin[e])*(Cos[f*x] - I*Sin[f*x])*(4*ArcTan[(2*c*d*
Cos[2*e + f*x] + (-c^2 + d^2)*Sin[2*e + f*x])/((c^2 - d^2)*Cos[2*e + f*x] +
 2*c*d*Sin[2*e + f*x])] + ((c^2 + d^2)*Cos[f*x]*(4*f*x - I*Log[(c*Cos[e + f
*x] + d*Sin[e + f*x])^2]) + (c^2 - d^2)*Cos[2*e + f*x]*(4*f*x - I*Log[(c*Co
s[e + f*x] + d*Sin[e + f*x])^2]) - 2*d*(2*(I*c + d)*Sin[f*x] + c*(-4*f*x +
I*Log[(c*Cos[e + f*x] + d*Sin[e + f*x])^2])*Sin[2*e + f*x]))/((c*Cos[e] + d
*Sin[e])*(c*Cos[e + f*x] + d*Sin[e + f*x])))*(a + I*a*Tan[e + f*x]))/(4*(c
- I*d)^2*f)
```

Maple [A]
time = 0.24, size = 139, normalized size = 1.85

method	result
derivativedivides	$\frac{a\left(\frac{ic-d}{\left(c^2+d^2\right)(c+d\tan(fx+e))}-\frac{\left(ic^2-id^2-2cd\right)\ln(c+d\tan(fx+e))}{\left(c^2+d^2\right)^2}+\frac{\frac{\left(ic^2-id^2-2cd\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(2icd+c^2-d^2\right)\arctan(\text{ta}}{\left(c^2+d^2\right)^2}\right.}{f}$
default	$\frac{a\left(\frac{ic-d}{\left(c^2+d^2\right)(c+d\tan(fx+e))}-\frac{\left(ic^2-id^2-2cd\right)\ln(c+d\tan(fx+e))}{\left(c^2+d^2\right)^2}+\frac{\frac{\left(ic^2-id^2-2cd\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(2icd+c^2-d^2\right)\arctan(\text{ta}}{\left(c^2+d^2\right)^2}\right.}{f}$
norman	$\frac{\frac{acx}{(-id+c)^2}-\frac{iad\tan(fx+e)}{f(-id+c)c}-\frac{adx\tan(fx+e)}{(ic+d)^2}}{c+d\tan(fx+e)}+\frac{ia\ln\left(1+\tan^2(fx+e)\right)}{2f\left(-2icd+c^2-d^2\right)}-\frac{ia\ln(c+d\tan(fx+e))}{f\left(-2icd+c^2-d^2\right)}$

risch	$-\frac{2ax}{2icd-c^2+d^2}-\frac{2iax}{ic^2-id^2+2cd}-\frac{2iae}{f(ic^2-id^2+2cd)}-\frac{2iad}{f(ic+d)^2(e^{2i(fx+e)}d+ic\,e^{2i(fx+e)}-d+ic)}+\frac{a\ln\left(e^{2i(fx+e)}-\right.}{f(ic^2-id^2+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a*((I*c-d)/(c^2+d^2)/(c+d*tan(f*x+e))-(I*c^2-I*d^2-2*c*d)/(c^2+d^2)^2*l
n(c+d*tan(f*x+e))+1/(c^2+d^2)^2*(1/2*(I*c^2-I*d^2-2*c*d)*ln(1+tan(f*x+e)^2)
+(2*I*c*d+c^2-d^2)*arctan(tan(f*x+e))))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(70) = 140$.
time = 0.51, size = 184, normalized size = 2.45

$$\frac{\frac{2\,(ac^2+2i\,acd-ad^2)(fx+e)}{c^4+2\,c^2d^2+d^4}+\frac{2\,(-i\,ac^2+2\,acd+i\,ad^2)\log(d\tan(fx+e)+c)}{c^4+2\,c^2d^2+d^4}+\frac{(i\,ac^2-2\,acd-i\,ad^2)\log\left(\tan(fx+e)^2+1\right)}{c^4+2\,c^2d^2+d^4}+\frac{2\,(i\,ac-ad)}{c^3+cd^2+(c^2d+d^3)\tan(fx+e)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a*c^2 + 2*I*a*c*d - a*d^2)*(f*x + e)/(c^4 + 2*c^2*d^2 + d^4) + 2*(-
I*a*c^2 + 2*a*c*d + I*a*d^2)*log(d*tan(f*x + e) + c)/(c^4 + 2*c^2*d^2 + d^4
) + (I*a*c^2 - 2*a*c*d - I*a*d^2)*log(tan(f*x + e)^2 + 1)/(c^4 + 2*c^2*d^2
+ d^4) + 2*(I*a*c - a*d)/(c^3 + c*d^2 + (c^2*d + d^3)*tan(f*x + e)))/f
```

Fricas [A]
time = 0.86, size = 128, normalized size = 1.71

$$\frac{-2i\,ad-\left(ac+i\,ad+(ac-i\,ad)e^{(2i\,fx+2i\,e)}\right)\log\left(\frac{(i\,c+d)e^{(2i\,fx+2i\,e)}+i\,c-d}{i\,c+d}\right)}{(-i\,c^3-3\,c^2d+3i\,cd^2+d^3)fe^{(2i\,fx+2i\,e)}+(-i\,c^3-c^2d-i\,cd^2-d^3)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] (-2*I*a*d - (a*c + I*a*d + (a*c - I*a*d)*e^(2*I*f*x + 2*I*e))*log(((I*c + d
)*e^(2*I*f*x + 2*I*e) + I*c - d)/(I*c + d)))/((-I*c^3 - 3*c^2*d + 3*I*c*d^2
 + d^3)*f*e^(2*I*f*x + 2*I*e) + (-I*c^3 - c^2*d - I*c*d^2 - d^3)*f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(58) = 116$.
time = 1.67, size = 143, normalized size = 1.91

$$\frac{2ad}{c^3f-ic^2df+cd^2f-id^3f+(c^3fe^{2ie}-3ic^2dfe^{2ie}-3cd^2fe^{2ie}+id^3fe^{2ie})\,e^{2ifx}}-\frac{ia\log\left(\frac{c+id}{ce^{2ie}-ide^{2ie}}+e^{2ifx}\right)}{f\,(c-id)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))**2,x)
```

```
[Out] 2*a*d/(c**3*f - I*c**2*d*f + c*d**2*f - I*d**3*f + (c**3*f*exp(2*I*e) - 3*I
*c**2*d*f*exp(2*I*e) - 3*c*d**2*f*exp(2*I*e) + I*d**3*f*exp(2*I*e))*exp(2*I
*f*x)) - I*a*log((c + I*d)/(c*exp(2*I*e) - I*d*exp(2*I*e)) + exp(2*I*f*x))/
(f*(c - I*d)**2)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(70) = 140$.
time = 0.55, size = 186, normalized size = 2.48

$$\frac{2\left(\frac{a\log\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-c\right)}{2ic^{2}+4cd-2id^{2}}+\frac{a\log\left(-i\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{-ic^{2}-2cd+id^{2}}-\frac{ac^{2}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-2iad^{2}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-ac^{2}}{-2\left(-ic^{3}-2c^{2}d+icd^{2}\right)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-c\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 2*(a*log(c*tan(1/2*f*x + 1/2*e)^2 - 2*d*tan(1/2*f*x + 1/2*e) - c)/(2*I*c^2
+ 4*c*d - 2*I*d^2) + a*log(-I*tan(1/2*f*x + 1/2*e) + 1)/(-I*c^2 - 2*c*d + I
*d^2) - (a*c^2*tan(1/2*f*x + 1/2*e)^2 - 2*I*a*d^2*tan(1/2*f*x + 1/2*e) - a*
c^2)/((2*I*c^3 + 4*c^2*d - 2*I*c*d^2)*(c*tan(1/2*f*x + 1/2*e)^2 - 2*d*tan(1
/2*f*x + 1/2*e) - c)))/f
```

Mupad [B]
time = 5.13, size = 135, normalized size = 1.80

$$-\frac{2\,a\,\mathrm{atan}\left(\frac{\left(c^2+d^2\right)\,1{}\mathrm{i}}{{\left(d+c\,1{}\mathrm{i}\right)}^2}-\frac{\tan\left(e+f\,x\right)\,\left(2\,c^4\,d^2+4\,c^2\,d^4+2\,d^6\right)}{{\left(d+c\,1{}\mathrm{i}\right)}^2\,\left(c^3\,d\,1{}\mathrm{i}-c^2\,d^2+c\,d^3\,1{}\mathrm{i}-d^4\right)}\right)}{f\,{\left(d+c\,1{}\mathrm{i}\right)}^2}+\frac{a\,1{}\mathrm{i}}{d\,f\,\left(\tan\left(e+f\,x\right)+\frac{c}{d}\right)\,\left(c-d\,1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)/(c + d*tan(e + f*x))^2,x)
```

```
[Out] (a*1i)/(d*f*(tan(e + f*x) + c/d)*(c - d*1i)) - (2*a*atan(((c^2 + d^2)*1i)/(
c*1i + d)^2 - (tan(e + f*x)*(2*d^6 + 4*c^2*d^4 + 2*c^4*d^2))/((c*1i + d)^2*
(c*d^3*1i + c^3*d*1i - d^4 - c^2*d^2))))/(f*(c*1i + d)^2)
```

3.1092 $\int \frac{1}{(a+ia\tan(e+fx))(c+d\tan(e+fx))^2}\,dx$

Optimal. Leaf size=202

$$\frac{(c^3+3ic^2d+3cd^2-3id^3)\,x}{2a(c-id)^2(c+id)^3}+\frac{(3c-id)d^2\log(c\cos(e+fx)+d\sin(e+fx))}{a(ic-d)^3(c-id)^2f}+\frac{(c-3id)d}{2a(c-id)(c+id)^2f(c+d\tan}$$

```
[Out] 1/2*(c^3+3*I*c^2*d+3*c*d^2-3*I*d^3)*x/a/(c-I*d)^2/(c+I*d)^3+(3*c-I*d)*d^2*l
n(c*cos(f*x+e)+d*sin(f*x+e))/a/(I*c-d)^3/(c-I*d)^2/f+1/2*(c-3*I*d)*d/a/(c-I
*d)/(c+I*d)^2/f/(c+d*tan(f*x+e))-1/2/(I*c-d)/f/(a+I*a*tan(f*x+e))/(c+d*tan(
f*x+e))
```

Rubi [A]
time = 0.24, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3633, 3610, 3612, 3611}

$$\frac{x(c^3+3ic^2d+3cd^2-3id^3)}{2a(c-id)^2(c+id)^3}+\frac{d^2(3c-id)\log(c\cos(e+fx)+d\sin(e+fx))}{af(-d+ic)^3(c-id)^2}+\frac{d(c-3id)}{2af(c-id)(c+id)^2(c+d\tan(e+fx))}-\frac{1}{2f(-d+ic)(a+ia\tan(e+fx))(c+d\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])*(c + d*Tan[e + f*x])^2),x]
```

```
[Out] ((c^3 + (3*I)*c^2*d + 3*c*d^2 - (3*I)*d^3)*x)/(2*a*(c - I*d)^2*(c + I*d)^3)
 + ((3*c - I*d)*d^2*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/(a*(I*c - d)^3*(c
 - I*d)^2*f) + ((c - (3*I)*d)*d)/(2*a*(c - I*d)*(c + I*d)^2*f*(c + d*Tan[e
+ f*x])) - 1/(2*(I*c - d)*f*(a + I*a*Tan[e + f*x])*(c + d*Tan[e + f*x]))
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
```

```
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3633

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-a)*((c + d*Tan[e + f*x])^(n + 1)/(2*f*(b*
c - a*d)*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*
Tan[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &
& NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))(c+d\tan(e+fx))^2}\,dx = -\frac{1}{2(ic-d)f(a+ia\tan(e+fx))(c+d\tan(e+fx))} + \frac{\int}{}$$

$$= \frac{(c-3id)d}{2a(c-id)(c+id)^2f(c+d\tan(e+fx))} - \frac{}{2(ic-d)f(a}$$

$$= \frac{(c^3+3ic^2d+3cd^2-3id^3)\,x}{2a(c-id)^2(c+id)^3} + \frac{(c-3id)d}{2a(c-id)(c+id)^2f(c+}$$

$$= \frac{(c^3+3ic^2d+3cd^2-3id^3)\,x}{2a(c-id)^2(c+id)^3} + \frac{(3c-id)d^2\log(c\cos(e+}{a(ic-d)^3(c}$$

Mathematica [A]

time = 2.99, size = 385, normalized size = 1.91

$$\frac{\sec(e+fx)(\cos(fx)+i\sin(fx))\left(-\frac{4(3c-id)d^2\operatorname{ArcTan}\left(\frac{d\cos(fx)+c\sin(fx)}{-c\cos(fx)+d\sin(fx)}\right)\left(\cos\left(\frac{e}{2}\right)+i\sin\left(\frac{e}{2}\right)\right)^2}{(c-id)^2f}+\frac{2d^2(3ic+d)\log\left((c\cos(e+fx)+d\sin(e+fx))^2\right)\left(\cos\left(\frac{e}{2}\right)+i\sin\left(\frac{e}{2}\right)\right)^2}{(c-id)^2f}-\frac{4(3c-id)d^2x(\cos(e)+i\sin(e))}{(c-id)^2}+\frac{2(c^3+3ic^2d+3cd^2-3id^3)x(\cos(e)+i\sin(e))}{(c-id)^2}+\frac{(c+id)\cos(2fx)(i\cos(e)+\sin(e))}{f}+\frac{(c+id)(\cos(e)-i\sin(e))\sin(2fx)}{f}+\frac{4i(c+id)d^3(\cos(e)+i\sin(e))\sin(fx)}{(c-id)f(c\cos(e)+d\sin(e))(c\cos(e+fx)+d\sin(e+fx))}\right)}{4(c+id)^3(a+ia\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])*(c + d*Tan[e + f*x])^2),x]
```

```
[Out] (Sec[e + f*x]*(Cos[f*x] + I*Sin[f*x])*((-4*(3*c - I*d)*d^2*ArcTan[(d*Cos[f*
x] + c*Sin[f*x])/(-(c*Cos[f*x]) + d*Sin[f*x])]*(Cos[e/2] + I*Sin[e/2])^2)/(
(c - I*d)^2*f) + (2*d^2*((3*I)*c + d)*Log[(c*Cos[e + f*x] + d*Sin[e + f*x])
^2]*(Cos[e/2] + I*Sin[e/2])^2)/((c - I*d)^2*f) - (4*(3*c - I*d)*d^2*x*(Cos[
e] + I*Sin[e]))/(c - I*d)^2 + (2*(c^3 + (3*I)*c^2*d + 3*c*d^2 - (3*I)*d^3)*
x*(Cos[e] + I*Sin[e]))/(c - I*d)^2 + ((c + I*d)*Cos[2*f*x]*(I*Cos[e] + Sin[
e]))/f + ((c + I*d)*(Cos[e] - I*Sin[e])*Sin[2*f*x])/f + ((4*I)*(c + I*d)*d^
3*(Cos[e] + I*Sin[e])*Sin[f*x])/((c - I*d)*f*(c*Cos[e] + d*Sin[e])*(c*Cos[e
 + f*x] + d*Sin[e + f*x]))))/(4*(c + I*d)^3*(a + I*a*Tan[e + f*x]))
```

Maple [A]

time = 0.45, size = 163, normalized size = 0.81

method	result
derivativedivides	$\frac{\frac{d^{2}(3ic+d)\ln(c+d\tan(fx+e))}{(id-c)^{2}(id+c)^{3}}-\frac{id^{2}\left(c^{2}+d^{2}\right)}{(id-c)^{2}(id+c)^{3}(c+d\tan(fx+e))}+\frac{i\ln(\tan(fx+e)+i)}{4(id-c)^{2}}+\frac{1}{2(id+c)^{2}(\tan(fx+e)-i)}+\frac{(-ic+5d)\ln(\tan(fx-}{4(id+c)^{3}}}{fa}$
default	$\frac{\frac{d^{2}(3ic+d)\ln(c+d\tan(fx+e))}{(id-c)^{2}(id+c)^{3}}-\frac{id^{2}\left(c^{2}+d^{2}\right)}{(id-c)^{2}(id+c)^{3}(c+d\tan(fx+e))}+\frac{i\ln(\tan(fx+e)+i)}{4(id-c)^{2}}+\frac{1}{2(id+c)^{2}(\tan(fx+e)-i)}+\frac{(-ic+5d)\ln(\tan(fx-}{4(id+c)^{3}}}{fa}$
norman	$\frac{\frac{1}{2af(-ic+d)}+\frac{c\left(3ic^{2}d-3id^{3}+c^{3}+3c\,d^{2}\right)x}{2(-id+c)^{2}(id+c)^{3}a}+\frac{c\left(3ic^{2}d-3id^{3}+c^{3}+3c\,d^{2}\right)x\left(\tan^{2}(fx+e)\right)}{2(-id+c)^{2}(id+c)^{3}a}+\frac{d\left(3ic^{2}d-3id^{3}+c^{3}+3c\,d^{2}\right)x\tan(fx+e)}{2(-id+c)^{2}(id+c)^{3}a}+\frac{d(3}{}}{(1+\tan^{2}(fx+e))(c+d\tan(fx+e))}$
risch	$-\frac{x}{2a(2icd-c^{2}+d^{2})}+\frac{\mathrm{i}e^{-2i(fx+e)}}{4a(2icd+c^{2}-d^{2})f}+\frac{6d^{2}cx}{a(ic^{4}d+2ic^{2}d^{3}+id^{5}+c^{5}+2c^{3}d^{2}+c\,d^{4})}+\frac{6d^{2}ce}{af(ic^{4}d+2ic^{2}d^{3}+id^{5}+c^{5}+2c^{3}d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a*(d^2*(3*I*c+d)/(I*d-c)^2/(c+I*d)^3*ln(c+d*tan(f*x+e))-I*d^2*(c^2+d^2)
/(I*d-c)^2/(c+I*d)^3/(c+d*tan(f*x+e))+1/4*I/(I*d-c)^2*ln(tan(f*x+e)+I)+1/2/
(c+I*d)^2/(tan(f*x+e)-I)+1/4/(c+I*d)^3*(-I*c+5*d)*ln(tan(f*x+e)-I))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]

time = 1.35, size = 336, normalized size = 1.66

$$\frac{i\,c^{4}+2i\,c^{2}d^{2}+i\,d^{4}+2\left(c^{4}+2i\,c^{3}d+12\,c^{2}d^{2}-14i\,cd^{3}-5\,d^{4}\right)fxe^{(4i\,fx+4i\,e)}+\left(i\,c^{4}+2\,c^{3}d-6\,cd^{3}-9i\,d^{4}+2\left(c^{4}+4i\,c^{3}d+6\,c^{2}d^{2}+4i\,cd^{3}+5\,d^{4}\right)fx\right)e^{(2i\,fx+2i\,e)}-4\left(\left(-3i\,c^{2}d^{2}-4\,cd^{3}+i\,d^{4}\right)e^{(4i\,fx+4i\,e)}+\left(-3i\,c^{2}d^{2}+2\,cd^{3}-i\,d^{4}\right)e^{(2i\,fx+2i\,e)}\right)\log\left(\frac{(i\,c+d)e^{(2i\,fx+2i\,e)}+i\,c-d}{i\,c+d}\right)}{4\left(\left(ac^{6}+3\,ac^{4}d^{2}+3\,ac^{2}d^{4}+ad^{6}\right)fe^{(4i\,fx+4i\,e)}+\left(ac^{6}+2i\,ac^{5}d+ac^{4}d^{2}+4i\,ac^{3}d^{3}-ac^{2}d^{4}+2i\,acd^{5}-ad^{6}\right)fe^{(2i\,fx+2i\,e)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(I*c^4 + 2*I*c^2*d^2 + I*d^4 + 2*(c^4 + 2*I*c^3*d + 12*c^2*d^2 - 14*I*c
*d^3 - 5*d^4)*f*x*e^(4*I*f*x + 4*I*e) + (I*c^4 + 2*c^3*d - 6*c*d^3 - 9*I*d^
4 + 2*(c^4 + 4*I*c^3*d + 6*c^2*d^2 + 4*I*c*d^3 + 5*d^4)*f*x)*e^(2*I*f*x + 2
*I*e) - 4*((-3*I*c^2*d^2 - 4*c*d^3 + I*d^4)*e^(4*I*f*x + 4*I*e) + (-3*I*c^2
*d^2 + 2*c*d^3 - I*d^4)*e^(2*I*f*x + 2*I*e))*log(((I*c + d)*e^(2*I*f*x + 2*
```

```
I*e) + I*c - d)/(I*c + d)))/((a*c^6 + 3*a*c^4*d^2 + 3*a*c^2*d^4 + a*d^6)*f*
e^(4*I*f*x + 4*I*e) + (a*c^6 + 2*I*a*c^5*d + a*c^4*d^2 + 4*I*a*c^3*d^3 - a*
c^2*d^4 + 2*I*a*c*d^5 - a*d^6)*f*e^(2*I*f*x + 2*I*e))
```

Sympy [A]
time = 14.84, size = 510, normalized size = 2.52

$$-\frac{2d^3}{ac^5f + iac^4df + 2ac^3d^2f + 2iac^2d^3f + acd^4f + iad^5f + \left(ac^5fe^{2ie} - iac^4dfe^{2ie} + 2ac^3d^2fe^{2ie} - 2iac^2d^3fe^{2ie} + acd^4fe^{2ie} - iad^5fe^{2ie}\right)e^{2ifx}} + \frac{x(c+5id)}{2ac^3 + 6iac^2d - 6acd^2 - 2iad^3} + \begin{cases} \frac{ie^{-2ifx}}{4ac^2fe^{2ie}+8iacdfe^{2ie}-4ad^2fe^{2ie}} & \text{for}\ 4ac^2fe^{2ie} + 8iacdfe^{2ie} - 4ad^2fe^{2ie} \neq 0 \\ x\left(-\frac{c+5id}{2ac^3+6iac^2d-6acd^2-2iad^3} + \frac{ce^{2ie}+c+5ide^{2ie}+id}{2ac^3e^{2ie}+6iac^2de^{2ie}-6acd^2e^{2ie}-2iad^3e^{2ie}}\right) & \text{otherwise} \end{cases} + \frac{id^2\cdot(3c-id)\log\left(\frac{c+id}{ce^{2ie}-ide^{2ie}} + e^{2ifx}\right)}{af\left(c-id\right)^2\left(c+id\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))**2,x)
```

```
[Out] -2*d**3/(a*c**5*f + I*a*c**4*d*f + 2*a*c**3*d**2*f + 2*I*a*c**2*d**3*f + a*
c*d**4*f + I*a*d**5*f + (a*c**5*f*exp(2*I*e) - I*a*c**4*d*f*exp(2*I*e) + 2*
a*c**3*d**2*f*exp(2*I*e) - 2*I*a*c**2*d**3*f*exp(2*I*e) + a*c*d**4*f*exp(2*
I*e) - I*a*d**5*f*exp(2*I*e))*exp(2*I*f*x)) + x*(c + 5*I*d)/(2*a*c**3 + 6*I
*a*c**2*d - 6*a*c*d**2 - 2*I*a*d**3) + Piecewise((I*exp(-2*I*f*x)/(4*a*c**2
*f*exp(2*I*e) + 8*I*a*c*d*f*exp(2*I*e) - 4*a*d**2*f*exp(2*I*e)), Ne(4*a*c**
2*f*exp(2*I*e) + 8*I*a*c*d*f*exp(2*I*e) - 4*a*d**2*f*exp(2*I*e), 0)), (x*(-
(c + 5*I*d)/(2*a*c**3 + 6*I*a*c**2*d - 6*a*c*d**2 - 2*I*a*d**3) + (c*exp(2*
I*e) + c + 5*I*d*exp(2*I*e) + I*d)/(2*a*c**3*exp(2*I*e) + 6*I*a*c**2*d*exp(
2*I*e) - 6*a*c*d**2*exp(2*I*e) - 2*I*a*d**3*exp(2*I*e))), True)) + I*d**2*(
3*c - I*d)*log((c + I*d)/(c*exp(2*I*e) - I*d*exp(2*I*e)) + exp(2*I*f*x))/(a
*f*(c - I*d)**2*(c + I*d)**3)
```

Giac [A]
time = 0.59, size = 342, normalized size = 1.69

$$\frac{\frac{16\left(3cd^3-id^4\right)\log(id\tan(fx+e)+ic)}{-2iac^5d+2ac^4d^2-4iac^3d^3+4ac^2d^4-2iacd^5+2ad^6} - \frac{2(ic-5d)\log(\tan(fx+e)-i)}{ac^3+3iac^2d-3acd^2-iad^3} - \frac{16\log(-i\tan(fx+e)+1)}{8iac^2+16acd-8iad^2} + \frac{ic^2d\tan(fx+e)^2-2cd^2\tan(fx+e)^2-id^3\tan(fx+e)^2+ic^3\tan(fx+e)+3c^2d\tan(fx+e)-15icd^2\tan(fx+e)-13d^3\tan(fx+e)+5c^3-6ic^2d-13cd^2+8id^3}{\left(ac^4+2ac^2d^2+ad^4\right)\left(d\tan(fx+e)^2+c\tan(fx+e)-id\tan(fx+e)-ic\right)}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/8*(16*(3*c*d^3 - I*d^4)*log(I*d*tan(f*x + e) + I*c)/(-2*I*a*c^5*d + 2*a*c
^4*d^2 - 4*I*a*c^3*d^3 + 4*a*c^2*d^4 - 2*I*a*c*d^5 + 2*a*d^6) - 2*(I*c - 5*
d)*log(tan(f*x + e) - I)/(a*c^3 + 3*I*a*c^2*d - 3*a*c*d^2 - I*a*d^3) - 16*l
og(-I*tan(f*x + e) + 1)/(8*I*a*c^2 + 16*a*c*d - 8*I*a*d^2) + (I*c^2*d*tan(f
*x + e)^2 - 2*c*d^2*tan(f*x + e)^2 - I*d^3*tan(f*x + e)^2 + I*c^3*tan(f*x +
 e) + 3*c^2*d*tan(f*x + e) - 15*I*c*d^2*tan(f*x + e) - 13*d^3*tan(f*x + e)
+ 5*c^3 - 6*I*c^2*d - 13*c*d^2 + 8*I*d^3)/((a*c^4 + 2*a*c^2*d^2 + a*d^4)*(d
*tan(f*x + e)^2 + c*tan(f*x + e) - I*d*tan(f*x + e) - I*c)))/f
```

Mupad [B]
time = 8.77, size = 1334, normalized size = 6.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)*(c + d*tan(e + f*x))^2),x)
```

```
[Out] symsum(log(tan(e + f*x)*(a*d^4 - a*c^2*d^2 + a*c*d^3*2i)*(c*d^3*6i + 9*d^4
- c^2*d^2) - (a*d^4 - a*c^2*d^2 + a*c*d^3*2i)*(11*c*d^3 + c^3*d - d^4*6i) -
 root(a^3*c^5*d^5*e^3*192i + a^3*c^7*d^3*e^3*128i + a^3*c^3*d^7*e^3*128i +
48*a^3*c^8*d^2*e^3 - 48*a^3*c^2*d^8*e^3 + 32*a^3*c^6*d^4*e^3 - 32*a^3*c^4*d
^6*e^3 + a^3*c^9*d*e^3*32i + a^3*c*d^9*e^3*32i - 16*a^3*d^10*e^3 + 16*a^3*c
^10*e^3 + 135*a*c^2*d^4*e + a*c^3*d^3*e*12i - 3*a*c^4*d^2*e - a*c*d^5*e*90i
 + a*c^5*d*e*6i - 21*a*d^6*e + a*c^6*e - c^2*d^2*3i + 14*c*d^3 - d^4*5i, e,
 k)*((a*d^4 - a*c^2*d^2 + a*c*d^3*2i)*(2*a*c^6 - 6*a*d^6 - 10*a*c^2*d^4 + a
*c^3*d^3*16i - 2*a*c^4*d^2 + a*c*d^5*8i + a*c^5*d*8i) + root(a^3*c^5*d^5*e^
3*192i + a^3*c^7*d^3*e^3*128i + a^3*c^3*d^7*e^3*128i + 48*a^3*c^8*d^2*e^3 -
 48*a^3*c^2*d^8*e^3 + 32*a^3*c^6*d^4*e^3 - 32*a^3*c^4*d^6*e^3 + a^3*c^9*d*e
^3*32i + a^3*c*d^9*e^3*32i - 16*a^3*d^10*e^3 + 16*a^3*c^10*e^3 + 135*a*c^2*
d^4*e + a*c^3*d^3*e*12i - 3*a*c^4*d^2*e - a*c*d^5*e*90i + a*c^5*d*e*6i - 21
*a*d^6*e + a*c^6*e - c^2*d^2*3i + 14*c*d^3 - d^4*5i, e, k)*((a*d^4 - a*c^2*
d^2 + a*c*d^3*2i)*(32*a^2*c^7*d - 32*a^2*c*d^7 + a^2*c^2*d^6*64i - 32*a^2*c
^3*d^5 + a^2*c^4*d^4*128i + 32*a^2*c^5*d^3 + a^2*c^6*d^2*64i) + tan(e + f*x
)*(a*d^4 - a*c^2*d^2 + a*c*d^3*2i)*(a^2*c*d^7*48i - 24*a^2*d^8 - 8*a^2*c^8
- a^2*c^7*d*16i - 16*a^2*c^2*d^6 + a^2*c^3*d^5*80i + 32*a^2*c^4*d^4 + a^2*c
^5*d^3*16i + 16*a^2*c^6*d^2)) + tan(e + f*x)*(a*d^4 - a*c^2*d^2 + a*c*d^3*2
i)*(a*d^6*12i + a*c^2*d^4*40i - 8*a*c^3*d^3 + a*c^4*d^2*28i - 12*a*c*d^5 +
4*a*c^5*d)))*root(a^3*c^5*d^5*e^3*192i + a^3*c^7*d^3*e^3*128i + a^3*c^3*d^7
*e^3*128i + 48*a^3*c^8*d^2*e^3 - 48*a^3*c^2*d^8*e^3 + 32*a^3*c^6*d^4*e^3 -
32*a^3*c^4*d^6*e^3 + a^3*c^9*d*e^3*32i + a^3*c*d^9*e^3*32i - 16*a^3*d^10*e^
3 + 16*a^3*c^10*e^3 + 135*a*c^2*d^4*e + a*c^3*d^3*e*12i - 3*a*c^4*d^2*e - a
*c*d^5*e*90i + a*c^5*d*e*6i - 21*a*d^6*e + a*c^6*e - c^2*d^2*3i + 14*c*d^3
- d^4*5i, e, k), k, 1, 3)/f - ((c*d*2i - 2*c^2 + 4*d^2)/(4*a*d*(c*d^2 + c^2
*d*1i + c^3 + d^3*1i)) - (tan(e + f*x)*(c - d*3i))/(2*a*(c*d^2 + c^2*d*1i +
 c^3 + d^3*1i)))/(f*(tan(e + f*x)*(c/d - 1i) - (c*1i)/d + tan(e + f*x)^2))
```

3.1093 $\int \frac{1}{(a+ia\tan(e+fx))^2(c+d\tan(e+fx))^2}\,dx$

Optimal. Leaf size=271

$$\frac{(c^4+4ic^3d-6c^2d^2+12icd^3+9d^4)\,x}{4a^2(c-id)^2(c+id)^4}-\frac{2(2c-id)d^3\log(c\cos(e+fx)+d\sin(e+fx))}{a^2(c-id)^2(c+id)^4f}+\frac{d(c^2+}{4a^2(c-id)(c+id}$$

```
[Out] 1/4*(c^4+4*I*c^3*d-6*c^2*d^2+12*I*c*d^3+9*d^4)*x/a^2/(c-I*d)^2/(c+I*d)^4-2*
(2*c-I*d)*d^3*ln(c*cos(f*x+e)+d*sin(f*x+e))/a^2/(c-I*d)^2/(c+I*d)^4/f+1/4*d
*(c^2+4*I*c*d+9*d^2)/a^2/(c-I*d)/(c+I*d)^3/f/(c+d*tan(f*x+e))+1/4*(I*c-4*d)
/a^2/(c+I*d)^2/f/(1+I*tan(f*x+e))/(c+d*tan(f*x+e))-1/4/(I*c-d)/f/(a+I*a*tan
(f*x+e))^2/(c+d*tan(f*x+e))
```

Rubi [A]
time = 0.39, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3640, 3677, 3610, 3612, 3611}

$$\frac{d(c^2+4icd+9d^2)}{4a^2f(c-id)(c+id)^3(c+d\tan(e+fx))}+\frac{x(c^4+4ic^3d-6c^2d^2+12icd^3+9d^4)}{4a^2(c-id)^2(c+id)^4}-\frac{2d^3(2c-id)\log(c\cos(e+fx)+d\sin(e+fx))}{a^2f(c-id)^2(c+id)^4}+\frac{-4d+ic}{4a^2f(c+id)^2(1+i\tan(e+fx))(c+d\tan(e+fx))}-\frac{1}{4f(-d+ic)(a+ia\tan(e+fx))^2(c+d\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^2),x]
```

```
[Out] ((c^4 + (4*I)*c^3*d - 6*c^2*d^2 + (12*I)*c*d^3 + 9*d^4)*x)/(4*a^2*(c - I*d)
^2*(c + I*d)^4) - (2*(2*c - I*d)*d^3*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/
(a^2*(c - I*d)^2*(c + I*d)^4*f) + (d*(c^2 + (4*I)*c*d + 9*d^2))/(4*a^2*(c -
 I*d)*(c + I*d)^3*f*(c + d*Tan[e + f*x])) + (I*c - 4*d)/(4*a^2*(c + I*d)^2*
f*(1 + I*Tan[e + f*x])*(c + d*Tan[e + f*x])) - 1/(4*(I*c - d)*f*(a + I*a*Ta
n[e + f*x])^2*(c + d*Tan[e + f*x]))
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] && !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^2(c+d\tan(e+fx))^2}\,dx = -\frac{1}{4(ic-d)f(a+ia\tan(e+fx))^2(c+d\tan(e+fx))} - \cdots$$

$$= \frac{ic-4d}{4a^2(c+id)^2f(1+i\tan(e+fx))(c+d\tan(e+fx))} - \cdots$$

$$= \frac{d\left(c^2+4icd+9d^2\right)}{4a^2(c+id)^2\left(c^2+d^2\right)f(c+d\tan(e+fx))} + \cdots$$

$$= \frac{\left(c^4+4ic^3d-6c^2d^2+12icd^3+9d^4\right)x}{4a^2(c+id)^2\left(c^2+d^2\right)^2} + \cdots$$

$$= \frac{\left(c^4+4ic^3d-6c^2d^2+12icd^3+9d^4\right)x}{4a^2(c+id)^2\left(c^2+d^2\right)^2} - \frac{2(2c-id)d^3\log(\cdots}{a^2(c\cdots}$$

Mathematica [A]
time = 4.07, size = 476, normalized size = 1.76

$$\frac{\sec^2(e+fx)(\cos(fx)+i\sin(fx))^2\left(\frac{4i(c+id)(c+3id)\cos(2fx)}{f}-\frac{32i(2c-id)d^3\mathrm{ArcTan}\left(\frac{-2cd\cos(fx)+\left(-c^2+d^2\right)\sin(fx)}{\left(c^2-d^2\right)\cos(fx)-2cd\sin(fx)}\right)(\cos(e)+i\sin(e))^2}{(c-id)^2f}-\frac{16(2c-id)d^3\log\left((c\cos(e+fx)+d\sin(e+fx))^2\right)(\cos(e)+i\sin(e))^2}{(c-id)^2f}+\frac{4(c^4+4ic^3d-6c^2d^2+12icd^3+9d^4)x(\cos(2e)+i\sin(2e))}{(c-id)^2}+\frac{32(2c-id)d^3x(-i\cos(2e)+\sin(2e))}{(c-id)^2}+\frac{(c+id)^2\cos(4fx)(i\cos(2e)+\sin(2e))}{f}+\frac{4(c+id)(c+3id)\sin(2fx)}{f}+\frac{(c+id)^2(\cos(2e)-i\sin(2e))\sin(4fx)}{f}-\frac{16(c+id)d^4(\cos(2e)+i\sin(2e))\sin(fx)}{(c-id)f(c\cos(e)+d\sin(e))(c\cos(e+fx)+d\sin(e+fx))}\right)}{16(c+id)^4(a+ia\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^2),x]
```

```
[Out] (Sec[e + f*x]^2*(Cos[f*x] + I*Sin[f*x])^2*(((4*I)*(c + I*d)*(c + (3*I)*d)*C
os[2*f*x])/f - ((32*I)*(2*c - I*d)*d^3*ArcTan[(-2*c*d*Cos[f*x] + (-c^2 + d^
2)*Sin[f*x])/((c^2 - d^2)*Cos[f*x] - 2*c*d*Sin[f*x])]*(Cos[e] + I*Sin[e])^2
)/((c - I*d)^2*f) - (16*(2*c - I*d)*d^3*Log[(c*Cos[e + f*x] + d*Sin[e + f*x
])^2]*(Cos[e] + I*Sin[e])^2)/((c - I*d)^2*f) + (4*(c^4 + (4*I)*c^3*d - 6*c^
2*d^2 + (12*I)*c*d^3 + 9*d^4)*x*(Cos[2*e] + I*Sin[2*e]))/(c - I*d)^2 + (32*
(2*c - I*d)*d^3*x*((-I)*Cos[2*e] + Sin[2*e]))/(c - I*d)^2 + ((c + I*d)^2*Co
s[4*f*x]*(I*Cos[2*e] + Sin[2*e]))/f + (4*(c + I*d)*(c + (3*I)*d)*Sin[2*f*x]
)/f + ((c + I*d)^2*(Cos[2*e] - I*Sin[2*e])*Sin[4*f*x])/f - (16*(c + I*d)*d^
4*(Cos[2*e] + I*Sin[2*e])*Sin[f*x])/((c - I*d)*f*(c*Cos[e] + d*Sin[e])*(c*C
os[e + f*x] + d*Sin[e + f*x]))))/(16*(c + I*d)^4*(a + I*a*Tan[e + f*x])^2)
```

Maple [A]
time = 0.70, size = 227, normalized size = 0.84 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a^2*(1/8*I/(I*d-c)^2*ln(tan(f*x+e)+I)+2*d^3*(I*d-2*c)/(I*d-c)^2/(c+I*d)
^4*ln(c+d*tan(f*x+e))+d^3*(c^2+d^2)/(I*d-c)^2/(c+I*d)^4/(c+d*tan(f*x+e))-1/
4*(-6*I*c*d-c^2+5*d^2)/(c+I*d)^4/(tan(f*x+e)-I)+1/8*(-I*c^2+17*I*d^2+6*c*d)
/(c+I*d)^4*ln(tan(f*x+e)-I)-1/4*(I*c^2-I*d^2-2*c*d)/(c+I*d)^4/(tan(f*x+e)-I
)^2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 513 vs. 2(240) = 480.
time = 1.08, size = 513, normalized size = 1.89

$$-\frac{c^5+ic^4d+2c^3d^2+2ic^2d^3+cd^4+id^5-4\left(ic^5-3c^4d-2ic^3d^2-34c^2d^3+45icd^4+17d^5\right)fxe^{(6ifx+6ie)}+4\left(c^5+ic^4d+6c^3d^2-2ic^2d^3-3cd^4-11id^5-\left(ic^5-5c^4d-10ic^3d^2-22c^2d^3-11icd^4-17d^5\right)fx\right)e^{(4ifx+4ie)}+\left(5c^5+11ic^4d+10c^3d^2+22ic^2d^3+5cd^4+11id^5\right)e^{(2ifx+2ie)}-32\left(\left(-2ic^2d^3-3cd^4+id^5\right)e^{(6ifx+6ie)}+\left(-2ic^2d^3+cd^4-id^5\right)e^{(4ifx+4ie)}\right)\log\left(\frac{(ic+d)e^{(2ifx+2ie)}+ic-d}{ic+d}\right)}{16\left(\left(ia^2c^7-a^2c^6d+3ia^2c^5d^2-3a^2c^4d^3+3ia^2c^3d^4-3a^2c^2d^5+ia^2cd^6-a^2d^7\right)fe^{(6ifx+6ie)}+\left(ia^2c^7-3a^2c^6d-ia^2c^5d^2-5a^2c^4d^3-5ia^2c^3d^4-a^2c^2d^5-3ia^2cd^6+a^2d^7\right)fe^{(4ifx+4ie)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/16*(c^5 + I*c^4*d + 2*c^3*d^2 + 2*I*c^2*d^3 + c*d^4 + I*d^5 - 4*(I*c^5 -
 3*c^4*d - 2*I*c^3*d^2 - 34*c^2*d^3 + 45*I*c*d^4 + 17*d^5)*f*x*e^(6*I*f*x +
 6*I*e) + 4*(c^5 + I*c^4*d + 6*c^3*d^2 - 2*I*c^2*d^3 - 3*c*d^4 - 11*I*d^5 -
 (I*c^5 - 5*c^4*d - 10*I*c^3*d^2 - 22*c^2*d^3 - 11*I*c*d^4 - 17*d^5)*f*x)*e
^(4*I*f*x + 4*I*e) + (5*c^5 + 11*I*c^4*d + 10*c^3*d^2 + 22*I*c^2*d^3 + 5*c*
d^4 + 11*I*d^5)*e^(2*I*f*x + 2*I*e) - 32*((-2*I*c^2*d^3 - 3*c*d^4 + I*d^5)*
e^(6*I*f*x + 6*I*e) + (-2*I*c^2*d^3 + c*d^4 - I*d^5)*e^(4*I*f*x + 4*I*e))*l
og(((I*c + d)*e^(2*I*f*x + 2*I*e) + I*c - d)/(I*c + d)))/((I*a^2*c^7 - a^2*
c^6*d + 3*I*a^2*c^5*d^2 - 3*a^2*c^4*d^3 + 3*I*a^2*c^3*d^4 - 3*a^2*c^2*d^5 +
 I*a^2*c*d^6 - a^2*d^7)*f*e^(6*I*f*x + 6*I*e) + (I*a^2*c^7 - 3*a^2*c^6*d -
I*a^2*c^5*d^2 - 5*a^2*c^4*d^3 - 5*I*a^2*c^3*d^4 - a^2*c^2*d^5 - 3*I*a^2*c*d
^6 + a^2*d^7)*f*e^(4*I*f*x + 4*I*e))
```

Sympy [A]
time = 31.06, size = 966, normalized size = 3.56

$$-\frac{2id^4}{a^2c^6f+2ia^2c^5df+a^2c^4d^2f+4ia^2c^3d^3f-a^2c^2d^4f+2ia^2cd^5f-a^2d^6f+\left(a^2c^6fe^{2ie}+3a^2c^4d^2fe^{2ie}+3a^2c^2d^4fe^{2ie}+a^2d^6fe^{2ie}\right)e^{2ifx}}+\frac{x(c^2+6icd-17d^2)}{4a^2c^4+16ia^2c^3d-24a^2c^2d^2-16ia^2cd^3+4a^2d^4}+\begin{cases}\frac{\left(4ia^2c^3fe^{2ie}-12a^2c^2dfe^{2ie}-12ia^2cd^2fe^{2ie}+4a^2d^3fe^{2ie}\right)e^{-4ifx}+\left(16ia^2c^3fe^{4ie}-80a^2c^2dfe^{4ie}-112ia^2cd^2fe^{4ie}+48a^2d^3fe^{4ie}\right)e^{-2ifx}}{64a^4c^5f^2e^{6ie}+320ia^4c^4df^2e^{6ie}-640a^4c^3d^2f^2e^{6ie}-640ia^4c^2d^3f^2e^{6ie}+320a^4cd^4f^2e^{6ie}+64ia^4d^5f^2e^{6ie}} & \text{for } 64a^4c^5f^2e^{6ie}+320ia^4c^4df^2e^{6ie}-640a^4c^3d^2f^2e^{6ie}-640ia^4c^2d^3f^2e^{6ie}+320a^4cd^4f^2e^{6ie}+64ia^4d^5f^2e^{6ie}\neq 0\\ x\left(-\frac{c^2+6icd-17d^2}{4a^2c^4+16ia^2c^3d-24a^2c^2d^2-16ia^2cd^3+4a^2d^4}+\frac{c^2e^{4ie}+2c^2e^{2ie}+c^2+6icde^{4ie}+8icde^{2ie}+2icd-17d^2e^{4ie}-6d^2e^{2ie}-d^2}{4a^2c^4e^{4ie}+16ia^2c^3de^{4ie}-24a^2c^2d^2e^{4ie}-16ia^2cd^3e^{4ie}+4a^2d^4e^{4ie}}\right) & \text{otherwise}\end{cases}-\frac{2d^3\cdot(2c-id)\log\left(\frac{c+id}{ce^{2ie}-ide^{2ie}}+e^{2ifx}\right)}{a^2f(c-id)^2(c+id)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**2/(c+d*tan(f*x+e))**2,x)
```

```
[Out] -2*I*d**4/(a**2*c**6*f + 2*I*a**2*c**5*d*f + a**2*c**4*d**2*f + 4*I*a**2*c*
*3*d**3*f - a**2*c**2*d**4*f + 2*I*a**2*c*d**5*f - a**2*d**6*f + (a**2*c**6
*f*exp(2*I*e) + 3*a**2*c**4*d**2*f*exp(2*I*e) + 3*a**2*c**2*d**4*f*exp(2*I*
e) + a**2*d**6*f*exp(2*I*e))*exp(2*I*f*x)) + x*(c**2 + 6*I*c*d - 17*d**2)/(
4*a**2*c**4 + 16*I*a**2*c**3*d - 24*a**2*c**2*d**2 - 16*I*a**2*c*d**3 + 4*a
**2*d**4) + Piecewise((((4*I*a**2*c**3*f*exp(2*I*e) - 12*a**2*c**2*d*f*exp(
2*I*e) - 12*I*a**2*c*d**2*f*exp(2*I*e) + 4*a**2*d**3*f*exp(2*I*e))*exp(-4*I
*f*x) + (16*I*a**2*c**3*f*exp(4*I*e) - 80*a**2*c**2*d*f*exp(4*I*e) - 112*I*
a**2*c*d**2*f*exp(4*I*e) + 48*a**2*d**3*f*exp(4*I*e))*exp(-2*I*f*x))/(64*a*
*4*c**5*f**2*exp(6*I*e) + 320*I*a**4*c**4*d*f**2*exp(6*I*e) - 640*a**4*c**3
*d**2*f**2*exp(6*I*e) - 640*I*a**4*c**2*d**3*f**2*exp(6*I*e) + 320*a**4*c*d
**4*f**2*exp(6*I*e) + 64*I*a**4*d**5*f**2*exp(6*I*e)), Ne(64*a**4*c**5*f**2
*exp(6*I*e) + 320*I*a**4*c**4*d*f**2*exp(6*I*e) - 640*a**4*c**3*d**2*f**2*e
xp(6*I*e) - 640*I*a**4*c**2*d**3*f**2*exp(6*I*e) + 320*a**4*c*d**4*f**2*exp
(6*I*e) + 64*I*a**4*d**5*f**2*exp(6*I*e), 0)), (x*(-(c**2 + 6*I*c*d - 17*d*
*2)/(4*a**2*c**4 + 16*I*a**2*c**3*d - 24*a**2*c**2*d**2 - 16*I*a**2*c*d**3
+ 4*a**2*d**4) + (c**2*exp(4*I*e) + 2*c**2*exp(2*I*e) + c**2 + 6*I*c*d*exp(
4*I*e) + 8*I*c*d*exp(2*I*e) + 2*I*c*d - 17*d**2*exp(4*I*e) - 6*d**2*exp(2*I
*e) - d**2)/(4*a**2*c**4*exp(4*I*e) + 16*I*a**2*c**3*d*exp(4*I*e) - 24*a**2
*c**2*d**2*exp(4*I*e) - 16*I*a**2*c*d**3*exp(4*I*e) + 4*a**2*d**4*exp(4*I*e
))), True)) - 2*d**3*(2*c - I*d)*log((c + I*d)/(c*exp(2*I*e) - I*d*exp(2*I*
e)) + exp(2*I*f*x))/(a**2*f*(c - I*d)**2*(c + I*d)**4)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(240) = 480$.
time = 0.69, size = 496, normalized size = 1.83

$$-\frac{\frac{2\left(2\,cd^{4}-i\,d^{5}\right)\log\left(d\tan\left(fx+e\right)+c\right)}{a^{2}c^{6}d+2i\,a^{2}c^{5}d^{2}+a^{2}c^{4}d^{3}+4i\,a^{2}c^{3}d^{4}-a^{2}c^{2}d^{5}+2i\,a^{2}cd^{6}-a^{2}d^{7}}-\frac{2\left(c^{2}+6i\,cd-17\,d^{2}\right)\log\left(i\,\tan\left(fx+e\right)+1\right)}{16i\,a^{2}c^{4}-64\,a^{2}c^{3}d-96i\,a^{2}c^{2}d^{2}+64\,a^{2}cd^{3}+16i\,a^{2}d^{4}}+\frac{2\,\log\left(-i\,\tan\left(fx+e\right)+1\right)}{16i\,a^{2}c^{2}+32\,a^{2}cd-16i\,a^{2}d^{2}}-\frac{4\,cd^{4}\tan\left(fx+e\right)-2i\,d^{5}\tan\left(fx+e\right)+5\,c^{2}d^{3}-2i\,cd^{4}+d^{5}}{\left(a^{2}c^{6}+2i\,a^{2}c^{5}d+a^{2}c^{4}d^{2}+4i\,a^{2}c^{3}d^{3}-a^{2}c^{2}d^{4}+2i\,a^{2}cd^{5}-a^{2}d^{6}\right)\left(d\tan\left(fx+e\right)+c\right)}+\frac{2\left(3\,c^{2}\tan\left(fx+e\right)^{2}+18i\,cd\tan\left(fx+e\right)^{2}-51\,d^{2}\tan\left(fx+e\right)^{2}-10i\,c^{2}\tan\left(fx+e\right)+60\,cd\tan\left(fx+e\right)+122i\,d^{2}\tan\left(fx+e\right)-11\,c^{2}-50i\,cd+75\,d^{2}\right)}{-32\left(-i\,a^{2}c^{4}+4\,a^{2}c^{3}d+6i\,a^{2}c^{2}d^{2}-4\,a^{2}cd^{3}-i\,a^{2}d^{4}\right)\left(\tan\left(fx+e\right)-i\right)^{2}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -(2*(2*c*d^4 - I*d^5)*log(d*tan(f*x + e) + c)/(a^2*c^6*d + 2*I*a^2*c^5*d^2
+ a^2*c^4*d^3 + 4*I*a^2*c^3*d^4 - a^2*c^2*d^5 + 2*I*a^2*c*d^6 - a^2*d^7) -
2*(c^2 + 6*I*c*d - 17*d^2)*log(I*tan(f*x + e) + 1)/(16*I*a^2*c^4 - 64*a^2*c
^3*d - 96*I*a^2*c^2*d^2 + 64*a^2*c*d^3 + 16*I*a^2*d^4) + 2*log(-I*tan(f*x +
 e) + 1)/(16*I*a^2*c^2 + 32*a^2*c*d - 16*I*a^2*d^2) - (4*c*d^4*tan(f*x + e)
 - 2*I*d^5*tan(f*x + e) + 5*c^2*d^3 - 2*I*c*d^4 + d^5)/((a^2*c^6 + 2*I*a^2*
c^5*d + a^2*c^4*d^2 + 4*I*a^2*c^3*d^3 - a^2*c^2*d^4 + 2*I*a^2*c*d^5 - a^2*d
^6)*(d*tan(f*x + e) + c)) + 2*(3*c^2*tan(f*x + e)^2 + 18*I*c*d*tan(f*x + e)
^2 - 51*d^2*tan(f*x + e)^2 - 10*I*c^2*tan(f*x + e) + 60*c*d*tan(f*x + e) +
122*I*d^2*tan(f*x + e) - 11*c^2 - 50*I*c*d + 75*d^2)/((32*I*a^2*c^4 - 128*a
^2*c^3*d - 192*I*a^2*c^2*d^2 + 128*a^2*c*d^3 + 32*I*a^2*d^4)*(tan(f*x + e)
- I)^2))/f
```

Mupad [B]
time = 10.33, size = 1984, normalized size = 7.32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^2*(c + d*tan(e + f*x))^2),x)
```

```
[Out] symsum(log((a^2*d^6 + a^2*c*d^5*4i - 6*a^2*c^2*d^4 - a^2*c^3*d^3*4i + a^2*c
^4*d^2)*(c^5*d - 95*c*d^5 + d^6*72i + c^2*d^4*16i - 14*c^3*d^3 + c^4*d^2*8i
) - root(1792*a^6*c^6*d^6*e^3 + 1088*a^6*c^8*d^4*e^3 + 1088*a^6*c^4*d^8*e^3
 - a^6*c^9*d^3*e^3*768i + a^6*c^3*d^9*e^3*768i - a^6*c^7*d^5*e^3*512i + a^6
*c^5*d^7*e^3*512i + 128*a^6*c^10*d^2*e^3 + 128*a^6*c^2*d^10*e^3 - a^6*c^11*
d*e^3*256i + a^6*c*d^11*e^3*256i - 64*a^6*d^12*e^3 - 64*a^6*c^12*e^3 - a^2*
c*d^7*e*984i - a^2*c^7*d*e*8i + 1020*a^2*c^2*d^6*e + a^2*c^3*d^5*e*72i + 42
*a^2*c^4*d^4*e + a^2*c^5*d^3*e*24i + 28*a^2*c^6*d^2*e - 273*a^2*d^8*e - a^2
*c^8*e - c^2*d^4*22i - 4*c^3*d^3 + 56*c*d^5 - d^6*34i, e, k)*((a^2*d^6 + a^
2*c*d^5*4i - 6*a^2*c^2*d^4 - a^2*c^3*d^3*4i + a^2*c^4*d^2)*(a^2*c*d^7*88i -
 36*a^2*d^8 - 4*a^2*c^8 - a^2*c^7*d*24i + a^2*c^3*d^5*152i + 104*a^2*c^4*d^
4 + a^2*c^5*d^3*40i + 64*a^2*c^6*d^2) + root(1792*a^6*c^6*d^6*e^3 + 1088*a^
6*c^8*d^4*e^3 + 1088*a^6*c^4*d^8*e^3 - a^6*c^9*d^3*e^3*768i + a^6*c^3*d^9*e
^3*768i - a^6*c^7*d^5*e^3*512i + a^6*c^5*d^7*e^3*512i + 128*a^6*c^10*d^2*e^
3 + 128*a^6*c^2*d^10*e^3 - a^6*c^11*d*e^3*256i + a^6*c*d^11*e^3*256i - 64*a
```

```
^6*d^12*e^3 - 64*a^6*c^12*e^3 - a^2*c*d^7*e*984i - a^2*c^7*d*e*8i + 1020*a^
2*c^2*d^6*e + a^2*c^3*d^5*e*72i + 42*a^2*c^4*d^4*e + a^2*c^5*d^3*e*24i + 28
*a^2*c^6*d^2*e - 273*a^2*d^8*e - a^2*c^8*e - c^2*d^4*22i - 4*c^3*d^3 + 56*c
*d^5 - d^6*34i, e, k)*((a^2*d^6 + a^2*c*d^5*4i - 6*a^2*c^2*d^4 - a^2*c^3*d^
3*4i + a^2*c^4*d^2)*(a^4*c^2*d^8*512i - 128*a^4*c^9*d - 128*a^4*c*d^9 + 512
*a^4*c^3*d^7 + a^4*c^4*d^6*512i + 1280*a^4*c^5*d^5 - a^4*c^6*d^4*512i + 512
*a^4*c^7*d^3 - a^4*c^8*d^2*512i) + tan(e + f*x)*(a^2*d^6 + a^2*c*d^5*4i - 6
*a^2*c^2*d^4 - a^2*c^3*d^3*4i + a^2*c^4*d^2)*(32*a^4*c^10 - 96*a^4*d^10 + a
^4*c*d^9*384i + a^4*c^9*d*128i + 416*a^4*c^2*d^8 + a^4*c^3*d^7*256i + 832*a
^4*c^4*d^6 - a^4*c^5*d^5*512i + 64*a^4*c^6*d^4 - a^4*c^7*d^3*256i - 224*a^4
*c^8*d^2)) + tan(e + f*x)*(a^2*d^6 + a^2*c*d^5*4i - 6*a^2*c^2*d^4 - a^2*c^3
*d^3*4i + a^2*c^4*d^2)*(a^2*d^8*96i + 72*a^2*c*d^7 - 8*a^2*c^7*d + a^2*c^2*
d^6*272i + 264*a^2*c^3*d^5 + a^2*c^4*d^4*128i + 184*a^2*c^5*d^3 - a^2*c^6*d
^2*48i)) + tan(e + f*x)*(a^2*d^6 + a^2*c*d^5*4i - 6*a^2*c^2*d^4 - a^2*c^3*d
^3*4i + a^2*c^4*d^2)*(c*d^5*72i + 81*d^6 + 2*c^2*d^4 + c^3*d^3*8i + c^4*d^2
))*root(1792*a^6*c^6*d^6*e^3 + 1088*a^6*c^8*d^4*e^3 + 1088*a^6*c^4*d^8*e^3
- a^6*c^9*d^3*e^3*768i + a^6*c^3*d^9*e^3*768i - a^6*c^7*d^5*e^3*512i + a^6*
c^5*d^7*e^3*512i + 128*a^6*c^10*d^2*e^3 + 128*a^6*c^2*d^10*e^3 - a^6*c^11*d
*e^3*256i + a^6*c*d^11*e^3*256i - 64*a^6*d^12*e^3 - 64*a^6*c^12*e^3 - a^2*c
*d^7*e*984i - a^2*c^7*d*e*8i + 1020*a^2*c^2*d^6*e + a^2*c^3*d^5*e*72i + 42*
a^2*c^4*d^4*e + a^2*c^5*d^3*e*24i + 28*a^2*c^6*d^2*e - 273*a^2*d^8*e - a^2*
c^8*e - c^2*d^4*22i - 4*c^3*d^3 + 56*c*d^5 - d^6*34i, e, k), k, 1, 3)/f + (
(tan(e + f*x)^2*(c*d*4i + c^2 + 9*d^2))/(4*a^2*(c*d^3*2i + c^3*d*2i + c^4 -
 d^4)) - ((9*c*d^2 + c^2*d*6i + 3*c^3 - d^3*6i)*1i)/(6*a^2*d*(c^2 + d^2)*(c
*d*2i + c^2 - d^2)) + (tan(e + f*x)*(18*c*d^2 + c^2*d*4i + 2*c^3 - d^3*28i)
)/(8*a^2*d*(c*d^3*2i + c^3*d*2i + c^4 - d^4)))/(f*(tan(e + f*x)^2*(c/d - 2i
) - c/d - tan(e + f*x)*((c*2i)/d + 1) + tan(e + f*x)^3))
```

3.1094 $\int \frac{1}{(a+ia\tan(e+fx))^3(c+d\tan(e+fx))^2}\,dx$

Optimal. Leaf size=357

$$\frac{(c^5+5ic^4d-10c^3d^2-10ic^2d^3-35cd^4+25id^5)\,x}{8a^3(c-id)^2(c+id)^5}+\frac{(5c-3id)d^4\log(c\cos(e+fx)+d\sin(e+fx))}{a^3(ic-d)^5(c-id)^2f}+\frac{}{8a^3(c}$$

```
[Out] 1/8*(c^5+5*I*c^4*d-10*c^3*d^2-10*I*c^2*d^3-35*c*d^4+25*I*d^5)*x/a^3/(c-I*d)
^2/(c+I*d)^5+(5*c-3*I*d)*d^4*ln(c*cos(f*x+e)+d*sin(f*x+e))/a^3/(I*c-d)^5/(c
-I*d)^2/f+1/8*d*(c^3+5*I*c^2*d-11*c*d^2+25*I*d^3)/a^3/(c-I*d)/(c+I*d)^4/f/(
c+d*tan(f*x+e))-1/6/(I*c-d)/f/(a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))+1/24*(3
*I*c-11*d)/a/(c+I*d)^2/f/(a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))+1/8*(c^2+5*I
*c*d-12*d^2)/(I*c-d)^3/f/(a^3+I*a^3*tan(f*x+e))/(c+d*tan(f*x+e))
```

Rubi [A]

time = 0.64, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3640, 3677, 3610, 3612, 3611}

$$\frac{c^2+5icd-12d^2}{8f(-d+ic)^3\left(a^3+ia^3\tan(e+fx)\right)(c+d\tan(e+fx))}+\frac{d(c^3+5ic^2d-11cd^2+25id^3)}{8a^3f(c-id)(c+id)^4(c+d\tan(e+fx))}+\frac{x(c^5+5ic^4d-10c^3d^2-10ic^2d^3-35cd^4+25id^5)}{8a^3(c-id)^2(c+id)^5}+\frac{d^4(5c-3id)\log(c\cos(e+fx)+d\sin(e+fx))}{a^3f(-d+ic)^5(c-id)^2}+\frac{-11d+3ic}{24af(c+id)^2(a+ia\tan(e+fx))^2(c+d\tan(e+fx))}-\frac{1}{6f(-d+ic)(a+ia\tan(e+fx))^3(c+d\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^2),x]
```

```
[Out] ((c^5 + (5*I)*c^4*d - 10*c^3*d^2 - (10*I)*c^2*d^3 - 35*c*d^4 + (25*I)*d^5)*
x)/(8*a^3*(c - I*d)^2*(c + I*d)^5) + ((5*c - (3*I)*d)*d^4*Log[c*Cos[e + f*x
] + d*Sin[e + f*x]])/(a^3*(I*c - d)^5*(c - I*d)^2*f) + (d*(c^3 + (5*I)*c^2*
d - 11*c*d^2 + (25*I)*d^3))/(8*a^3*(c - I*d)*(c + I*d)^4*f*(c + d*Tan[e + f
*x])) - 1/(6*(I*c - d)*f*(a + I*a*Tan[e + f*x])^3*(c + d*Tan[e + f*x])) + (
(3*I)*c - 11*d)/(24*a*(c + I*d)^2*f*(a + I*a*Tan[e + f*x])^2*(c + d*Tan[e +
 f*x])) + (c^2 + (5*I)*c*d - 12*d^2)/(8*(I*c - d)^3*f*(a^3 + I*a^3*Tan[e +
f*x])*(c + d*Tan[e + f*x]))
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^3(c+d\tan(e+fx))^2}\,dx = -\frac{1}{6(ic-d)f(a+ia\tan(e+fx))^3(c+d\tan(e+fx))} -$$

$$= -\frac{1}{6(ic-d)f(a+ia\tan(e+fx))^3(c+d\tan(e+fx))} +$$

$$= -\frac{1}{6(ic-d)f(a+ia\tan(e+fx))^3(c+d\tan(e+fx))} +$$

$$= \frac{d(c^3+5ic^2d-11cd^2+25id^3)}{8a^3(c-id)(c+id)^4f(c+d\tan(e+fx))} - \frac{}{6(ic-d)f(}$$

$$= \frac{(c^5+5ic^4d-10c^3d^2-10ic^2d^3-35cd^4+25id^5)\,x}{8a^3(c-id)^2(c+id)^5} + \frac{}{8a}$$

$$= \frac{(c^5+5ic^4d-10c^3d^2-10ic^2d^3-35cd^4+25id^5)\,x}{8a^3(c-id)^2(c+id)^5} - \frac{d^4}{}$$

Mathematica [**A**]
time = 5.22, size = 633, normalized size = 1.77

$$\frac{\sec^3(e+fx)(\cos(fx)+i\sin(fx))^3\left(\frac{6i(c+id)(3c^2+14icd-23d^2)\cos(2fx)(\cos(e)+i\sin(e))}{f}+\frac{3(c+id)^2(3c+7id)\cos(4fx)(i\cos(e)+\sin(e))}{f}+\frac{96(5c-3id)d^4\,\mathrm{ArcTan}\left(\frac{(-3c^2d+d^3)\cos(fx)-c(c^2-3d^2)\sin(fx)}{c(c^2-3d^2)\cos(fx)+d(-3c^2+d^2)\sin(fx)}\right)\left(\cos\left(\frac{3e}{2}\right)+i\sin\left(\frac{3e}{2}\right)\right)^2}{(c-id)^2f}-\frac{48i(5c-3id)d^4\log\left((c\cos(e+fx)+d\sin(e+fx))^2\right)\left(\cos\left(\frac{3e}{2}\right)+i\sin\left(\frac{3e}{2}\right)\right)^2}{(c-id)^2f}+\frac{96(5c-3id)d^4x(\cos(3e)+i\sin(3e))}{(c-id)^2}+\frac{12(c^5+5ic^4d-10c^3d^2-10ic^2d^3-35cd^4+25id^5)x(\cos(3e)+i\sin(3e))}{(c-id)^2}+\frac{2(c+id)^3\cos(6fx)(i\cos(3e)+\sin(3e))}{f}+\frac{6(c+id)(3c^2+14icd-23d^2)(\cos(e)+i\sin(e))\sin(2fx)}{f}+\frac{3(c+id)^2(3c+7id)(\cos(e)-i\sin(e))\sin(4fx)}{f}+\frac{2(c+id)^3(\cos(3e)-i\sin(3e))\sin(6fx)}{f}+\frac{96d^5(-ic+d)(\cos(3e)+i\sin(3e))\sin(fx)}{(c-id)f(c\cos(e)+d\sin(e))(c\cos(e+fx)+d\sin(e+fx))}\right)}{96(c+id)^5(a+ia\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^2),x]
```

```
[Out] (Sec[e + f*x]^3*(Cos[f*x] + I*Sin[f*x])^3*(((6*I)*(c + I*d)*(3*c^2 + (14*I)
*c*d - 23*d^2)*Cos[2*f*x]*(Cos[e] + I*Sin[e]))/f + (3*(c + I*d)^2*(3*c + (7
*I)*d)*Cos[4*f*x]*(I*Cos[e] + Sin[e]))/f + (96*(5*c - (3*I)*d)*d^4*ArcTan[(
(-3*c^2*d + d^3)*Cos[f*x] - c*(c^2 - 3*d^2)*Sin[f*x])/(c*(c^2 - 3*d^2)*Cos[
f*x] + d*(-3*c^2 + d^2)*Sin[f*x])]*(Cos[(3*e)/2] + I*Sin[(3*e)/2])^2)/((c -
 I*d)^2*f) - ((48*I)*(5*c - (3*I)*d)*d^4*Log[(c*Cos[e + f*x] + d*Sin[e + f*
x])^2]*(Cos[(3*e)/2] + I*Sin[(3*e)/2])^2)/((c - I*d)^2*f) + (96*(5*c - (3*I
)*d)*d^4*x*(Cos[3*e] + I*Sin[3*e]))/(c - I*d)^2 + (12*(c^5 + (5*I)*c^4*d -
10*c^3*d^2 - (10*I)*c^2*d^3 - 35*c*d^4 + (25*I)*d^5)*x*(Cos[3*e] + I*Sin[3*
e]))/(c - I*d)^2 + (2*(c + I*d)^3*Cos[6*f*x]*(I*Cos[3*e] + Sin[3*e]))/f + (
6*(c + I*d)*(3*c^2 + (14*I)*c*d - 23*d^2)*(Cos[e] + I*Sin[e])*Sin[2*f*x])/f
 + (3*(c + I*d)^2*(3*c + (7*I)*d)*(Cos[e] - I*Sin[e])*Sin[4*f*x])/f + (2*(c
 + I*d)^3*(Cos[3*e] - I*Sin[3*e])*Sin[6*f*x])/f + (96*d^5*((-I)*c + d)*(Cos
[3*e] + I*Sin[3*e])*Sin[f*x])/((c - I*d)*f*(c*Cos[e] + d*Sin[e])*(c*Cos[e +
 f*x] + d*Sin[e + f*x]))))/(96*(c + I*d)^5*(a + I*a*Tan[e + f*x])^3)
```

Maple [**A**]
time = 1.03, size = 298, normalized size = 0.83 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a^3*(1/16/(c+I*d)^5*(-I*c^3+23*I*c*d^2+7*c^2*d-49*d^3)*ln(tan(f*x+e)-I)
-1/8*(-7*I*c^2*d+17*I*d^3-c^3+23*c*d^2)/(c+I*d)^5/(tan(f*x+e)-I)-1/6*(3*I*c
^2*d-I*d^3+c^3-3*c*d^2)/(c+I*d)^5/(tan(f*x+e)-I)^3-1/8*(I*c^3-11*I*c*d^2-7*
c^2*d+5*d^3)/(c+I*d)^5/(tan(f*x+e)-I)^2+1/16*I/(I*d-c)^2*ln(tan(f*x+e)+I)+I
*d^4*(c^2+d^2)/(I*d-c)^2/(c+I*d)^5/(c+d*tan(f*x+e))-d^4*(5*I*c+3*d)/(I*d-c)
^2/(c+I*d)^5*ln(c+d*tan(f*x+e)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.30, size = 609, normalized size = 1.71

$$-\frac{-2i c^6+4 c^5 d-2i c^4 d^2+8 c^3 d^3+2i c^2 d^4+4 c d^5+2i d^6-12\left(c^6+4i c^5 d-5 c^4 d^2-85 c^2 d^4+124i c d^5+49 d^6\right) f x e^{(8i f x+8i e)}-6\left(3i c^6-8 c^5 d+5i c^4 d^2-40 c^3 d^3+25i c^2 d^4+55i d^6+2\left(c^6+6i c^5 d-15 c^4 d^2-20i c^3 d^3-65 c^2 d^4-26i c d^5-49 d^6\right) f x\right) e^{(6i f x+6i e)}-3\left(9i c^6-32 c^5 d-21i c^4 d^2-64 c^3 d^3-69i c^2 d^4-32 c d^5-39i d^6\right) e^{(4i f x+4i e)}+\left(-11i c^6+30 c^5 d-3i c^4 d^2+60 c^3 d^3+27i c^2 d^4+30 c d^5+19i d^6\right) e^{(2i f x+2i e)}-96\left(\left(-5i c^2 d^4-8 c d^5+3i d^6\right) e^{(8i f x+8i e)}+\left(-5i c^2 d^4+2 c d^5-3i d^6\right) e^{(6i f x+6i e)}\right)\log\left(\frac{(i c+d) e^{(2i f x+2i e)}+i c-d}{i c+d}\right)}{96\left(\left(a^3 c^8+2i a^3 c^7 d+2 a^3 c^6 d^2+6i a^3 c^5 d^3+6i a^3 c^3 d^5-2 a^3 c^2 d^6+2i a^3 c d^7-a^3 d^8\right) f e^{(8i f x+8i e)}+\left(a^3 c^8+4i a^3 c^7 d-4 a^3 c^6 d^2+4i a^3 c^5 d^3-10 a^3 c^4 d^4-4i a^3 c^3 d^5-4 a^3 c^2 d^6-4i a^3 c d^7+a^3 d^8\right) f e^{(6i f x+6i e)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/96*(-2*I*c^6 + 4*c^5*d - 2*I*c^4*d^2 + 8*c^3*d^3 + 2*I*c^2*d^4 + 4*c*d^5
 + 2*I*d^6 - 12*(c^6 + 4*I*c^5*d - 5*c^4*d^2 - 85*c^2*d^4 + 124*I*c*d^5 + 4
9*d^6)*f*x*e^(8*I*f*x + 8*I*e) - 6*(3*I*c^6 - 8*c^5*d + 5*I*c^4*d^2 - 40*c^
3*d^3 + 25*I*c^2*d^4 + 55*I*d^6 + 2*(c^6 + 6*I*c^5*d - 15*c^4*d^2 - 20*I*c^
3*d^3 - 65*c^2*d^4 - 26*I*c*d^5 - 49*d^6)*f*x)*e^(6*I*f*x + 6*I*e) - 3*(9*I
*c^6 - 32*c^5*d - 21*I*c^4*d^2 - 64*c^3*d^3 - 69*I*c^2*d^4 - 32*c*d^5 - 39*
I*d^6)*e^(4*I*f*x + 4*I*e) + (-11*I*c^6 + 30*c^5*d - 3*I*c^4*d^2 + 60*c^3*d
^3 + 27*I*c^2*d^4 + 30*c*d^5 + 19*I*d^6)*e^(2*I*f*x + 2*I*e) - 96*((-5*I*c^
2*d^4 - 8*c*d^5 + 3*I*d^6)*e^(8*I*f*x + 8*I*e) + (-5*I*c^2*d^4 + 2*c*d^5 -
3*I*d^6)*e^(6*I*f*x + 6*I*e))*log(((I*c + d)*e^(2*I*f*x + 2*I*e) + I*c - d)
/(I*c + d)))/((a^3*c^8 + 2*I*a^3*c^7*d + 2*a^3*c^6*d^2 + 6*I*a^3*c^5*d^3 +
6*I*a^3*c^3*d^5 - 2*a^3*c^2*d^6 + 2*I*a^3*c*d^7 - a^3*d^8)*f*e^(8*I*f*x + 8
*I*e) + (a^3*c^8 + 4*I*a^3*c^7*d - 4*a^3*c^6*d^2 + 4*I*a^3*c^5*d^3 - 10*a^3
*c^4*d^4 - 4*I*a^3*c^3*d^5 - 4*a^3*c^2*d^6 - 4*I*a^3*c*d^7 + a^3*d^8)*f*e^(
6*I*f*x + 6*I*e))
```

Sympy [A]
time = 73.49, size = 1792, normalized size = 5.02

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**3/(c+d*tan(f*x+e))**2,x)
```

```
[Out] 2*d**5/(a**3*c**7*f + 3*I*a**3*c**6*d*f - a**3*c**5*d**2*f + 5*I*a**3*c**4*
d**3*f - 5*a**3*c**3*d**4*f + I*a**3*c**2*d**5*f - 3*a**3*c*d**6*f - I*a**3
*d**7*f + (a**3*c**7*f*exp(2*I*e) + I*a**3*c**6*d*f*exp(2*I*e) + 3*a**3*c**
5*d**2*f*exp(2*I*e) + 3*I*a**3*c**4*d**3*f*exp(2*I*e) + 3*a**3*c**3*d**4*f*
exp(2*I*e) + 3*I*a**3*c**2*d**5*f*exp(2*I*e) + a**3*c*d**6*f*exp(2*I*e) + I
*a**3*d**7*f*exp(2*I*e))*exp(2*I*f*x)) + x*(c**3 + 7*I*c**2*d - 23*c*d**2 -
 49*I*d**3)/(8*a**3*c**5 + 40*I*a**3*c**4*d - 80*a**3*c**3*d**2 - 80*I*a**3
*c**2*d**3 + 40*a**3*c*d**4 + 8*I*a**3*d**5) + Piecewise((((512*I*a**6*c**7
*f**2*exp(6*I*e) - 3584*a**6*c**6*d*f**2*exp(6*I*e) - 10752*I*a**6*c**5*d**
2*f**2*exp(6*I*e) + 17920*a**6*c**4*d**3*f**2*exp(6*I*e) + 17920*I*a**6*c**
3*d**4*f**2*exp(6*I*e) - 10752*a**6*c**2*d**5*f**2*exp(6*I*e) - 3584*I*a**6
*c*d**6*f**2*exp(6*I*e) + 512*a**6*d**7*f**2*exp(6*I*e))*exp(-6*I*f*x) + (2
304*I*a**6*c**7*f**2*exp(8*I*e) - 19200*a**6*c**6*d*f**2*exp(8*I*e) - 66816
*I*a**6*c**5*d**2*f**2*exp(8*I*e) + 126720*a**6*c**4*d**3*f**2*exp(8*I*e) +
 142080*I*a**6*c**3*d**4*f**2*exp(8*I*e) - 94464*a**6*c**2*d**5*f**2*exp(8*
I*e) - 34560*I*a**6*c*d**6*f**2*exp(8*I*e) + 5376*a**6*d**7*f**2*exp(8*I*e)
)*exp(-4*I*f*x) + (4608*I*a**6*c**7*f**2*exp(10*I*e) - 44544*a**6*c**6*d*f*
*2*exp(10*I*e) - 188928*I*a**6*c**5*d**2*f**2*exp(10*I*e) + 437760*a**6*c**
4*d**3*f**2*exp(10*I*e) + 591360*I*a**6*c**3*d**4*f**2*exp(10*I*e) - 465408
*a**6*c**2*d**5*f**2*exp(10*I*e) - 198144*I*a**6*c*d**6*f**2*exp(10*I*e) +
35328*a**6*d**7*f**2*exp(10*I*e))*exp(-2*I*f*x))/(24576*a**9*c**9*f**3*exp(
12*I*e) + 221184*I*a**9*c**8*d*f**3*exp(12*I*e) - 884736*a**9*c**7*d**2*f**
3*exp(12*I*e) - 2064384*I*a**9*c**6*d**3*f**3*exp(12*I*e) + 3096576*a**9*c*
*5*d**4*f**3*exp(12*I*e) + 3096576*I*a**9*c**4*d**5*f**3*exp(12*I*e) - 2064
384*a**9*c**3*d**6*f**3*exp(12*I*e) - 884736*I*a**9*c**2*d**7*f**3*exp(12*I
*e) + 221184*a**9*c*d**8*f**3*exp(12*I*e) + 24576*I*a**9*d**9*f**3*exp(12*I
*e)), Ne(24576*a**9*c**9*f**3*exp(12*I*e) + 221184*I*a**9*c**8*d*f**3*exp(1
2*I*e) - 884736*a**9*c**7*d**2*f**3*exp(12*I*e) - 2064384*I*a**9*c**6*d**3*
f**3*exp(12*I*e) + 3096576*a**9*c**5*d**4*f**3*exp(12*I*e) + 3096576*I*a**9
*c**4*d**5*f**3*exp(12*I*e) - 2064384*a**9*c**3*d**6*f**3*exp(12*I*e) - 884
736*I*a**9*c**2*d**7*f**3*exp(12*I*e) + 221184*a**9*c*d**8*f**3*exp(12*I*e)
 + 24576*I*a**9*d**9*f**3*exp(12*I*e), 0)), (x*(-(c**3 + 7*I*c**2*d - 23*c*
d**2 - 49*I*d**3)/(8*a**3*c**5 + 40*I*a**3*c**4*d - 80*a**3*c**3*d**2 - 80*
I*a**3*c**2*d**3 + 40*a**3*c*d**4 + 8*I*a**3*d**5) + (c**3*exp(6*I*e) + 3*c
**3*exp(4*I*e) + 3*c**3*exp(2*I*e) + c**3 + 7*I*c**2*d*exp(6*I*e) + 17*I*c*
*2*d*exp(4*I*e) + 13*I*c**2*d*exp(2*I*e) + 3*I*c**2*d - 23*c*d**2*exp(6*I*e
) - 37*c*d**2*exp(4*I*e) - 17*c*d**2*exp(2*I*e) - 3*c*d**2 - 49*I*d**3*exp(
```

```
6*I*e) - 23*I*d**3*exp(4*I*e) - 7*I*d**3*exp(2*I*e) - I*d**3)/(8*a**3*c**5*
exp(6*I*e) + 40*I*a**3*c**4*d*exp(6*I*e) - 80*a**3*c**3*d**2*exp(6*I*e) - 8
0*I*a**3*c**2*d**3*exp(6*I*e) + 40*a**3*c*d**4*exp(6*I*e) + 8*I*a**3*d**5*e
xp(6*I*e))), True)) - I*d**4*(5*c - 3*I*d)*log((c + I*d)/(c*exp(2*I*e) - I*
d*exp(2*I*e)) + exp(2*I*f*x))/(a**3*f*(c - I*d)**2*(c + I*d)**5)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 648 vs. $2(316) = 632$.
time = 0.83, size = 648, normalized size = 1.82

$$-\frac{2\left(\frac{(5\,cd^5-3i\,d^6)\log(d\tan(fx+e)+c)}{-2i\,a^3c^7d+6\,a^3c^6d^2+2i\,a^3c^5d^3+10\,a^3c^4d^4+10i\,a^3c^3d^5+2\,a^3c^2d^6+6i\,a^3cd^7-2\,a^3d^8}-\frac{(c^3+7i\,c^2d-23\,cd^2-49i\,d^3)\log(i\tan(fx+e)+1)}{32i\,a^3c^5-160\,a^3c^4d-320i\,a^3c^3d^2+320\,a^3c^2d^3+160i\,a^3cd^4-32\,a^3d^5}-\frac{\log(-i\tan(fx+e)+1)}{-32i\,a^3c^2-64\,a^3cd+32i\,a^3d^2}-\frac{5\,cd^5\tan(fx+e)-3i\,d^6\tan(fx+e)+6\,c^2d^4-3i\,cd^5+d^6}{-2\,(i\,a^3c^7-3\,a^3c^6d-i\,a^3c^5d^2-5\,a^3c^4d^3-5i\,a^3c^3d^4-a^3c^2d^5-3i\,a^3cd^6+a^3d^7)(d\tan(fx+e)+c)}+\frac{11\,c^3\tan(fx+e)^3+77i\,c^2d\tan(fx+e)^3-253\,cd^2\tan(fx+e)^3-539i\,d^3\tan(fx+e)^3-45i\,c^3\tan(fx+e)^2+315\,c^2d\tan(fx+e)^2+1035i\,cd^2\tan(fx+e)^2-1821\,d^3\tan(fx+e)^2-69\,c^3\tan(fx+e)-483i\,c^2d\tan(fx+e)+1443\,cd^2\tan(fx+e)+2085i\,d^3\tan(fx+e)+51i\,c^3-293\,c^2d-709i\,cd^2+819\,d^3}{-192\,(-i\,a^3c^5+5\,a^3c^4d+10i\,a^3c^3d^2-10\,a^3c^2d^3-5i\,a^3cd^4+a^3d^5)(\tan(fx+e)-i)^3}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -2*((5*c*d^5 - 3*I*d^6)*log(d*tan(f*x + e) + c)/(-2*I*a^3*c^7*d + 6*a^3*c^6
*d^2 + 2*I*a^3*c^5*d^3 + 10*a^3*c^4*d^4 + 10*I*a^3*c^3*d^5 + 2*a^3*c^2*d^6
+ 6*I*a^3*c*d^7 - 2*a^3*d^8) - (c^3 + 7*I*c^2*d - 23*c*d^2 - 49*I*d^3)*log(
I*tan(f*x + e) + 1)/(32*I*a^3*c^5 - 160*a^3*c^4*d - 320*I*a^3*c^3*d^2 + 320
*a^3*c^2*d^3 + 160*I*a^3*c*d^4 - 32*a^3*d^5) - log(-I*tan(f*x + e) + 1)/(-3
2*I*a^3*c^2 - 64*a^3*c*d + 32*I*a^3*d^2) - (5*c*d^5*tan(f*x + e) - 3*I*d^6*
tan(f*x + e) + 6*c^2*d^4 - 3*I*c*d^5 + d^6)/((-2*I*a^3*c^7 + 6*a^3*c^6*d +
2*I*a^3*c^5*d^2 + 10*a^3*c^4*d^3 + 10*I*a^3*c^3*d^4 + 2*a^3*c^2*d^5 + 6*I*a
^3*c*d^6 - 2*a^3*d^7)*(d*tan(f*x + e) + c)) + (11*c^3*tan(f*x + e)^3 + 77*I
*c^2*d*tan(f*x + e)^3 - 253*c*d^2*tan(f*x + e)^3 - 539*I*d^3*tan(f*x + e)^3
 - 45*I*c^3*tan(f*x + e)^2 + 315*c^2*d*tan(f*x + e)^2 + 1035*I*c*d^2*tan(f*
x + e)^2 - 1821*d^3*tan(f*x + e)^2 - 69*c^3*tan(f*x + e) - 483*I*c^2*d*tan(
f*x + e) + 1443*c*d^2*tan(f*x + e) + 2085*I*d^3*tan(f*x + e) + 51*I*c^3 - 2
93*c^2*d - 709*I*c*d^2 + 819*d^3)/((192*I*a^3*c^5 - 960*a^3*c^4*d - 1920*I*
a^3*c^3*d^2 + 1920*a^3*c^2*d^3 + 960*I*a^3*c*d^4 - 192*a^3*d^5)*(tan(f*x +
e) - I)^3))/f
```

Mupad [B]
time = 11.67, size = 2653, normalized size = 7.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^3*(c + d*tan(e + f*x))^2),x)
```

```
[Out] symsum(log(tan(e + f*x)*(c*d^7*550i + 625*d^8 + 129*c^2*d^6 + c^3*d^5*60i +
 47*c^4*d^4 - c^5*d^3*10i - c^6*d^2)*(a^3*d^8 + a^3*c*d^7*6i - 15*a^3*c^2*d
^6 - a^3*c^3*d^5*20i + 15*a^3*c^4*d^4 + a^3*c^5*d^3*6i - a^3*c^6*d^2) - roo
t(a^9*c^7*d^7*e^3*18432i + 9984*a^9*c^10*d^4*e^3 - 9984*a^9*c^4*d^10*e^3 +
a^9*c^9*d^5*e^3*9728i + a^9*c^5*d^9*e^3*9728i + 6912*a^9*c^8*d^6*e^3 - 6912
*a^9*c^6*d^8*e^3 + 2816*a^9*c^12*d^2*e^3 - 2816*a^9*c^2*d^12*e^3 - a^9*c^11
```

*d^3*e^3*1024i - a^9*c^3*d^11*e^3*1024i - a^9*c^13*d*e^3*1536i - a^9*c*d^13*e^3*1536i + 256*a^9*d^14*e^3 - 256*a^9*c^14*e^3 + a^3*c*d^9*e*7510i - a^3*c^9*d*e*10i - 6525*a^3*c^2*d^8*e - 350*a^3*c^4*d^6*e - a^3*c^3*d^7*e*200i - 130*a^3*c^6*d^4*e + a^3*c^7*d^3*e*120i + a^3*c^5*d^5*e*100i + 45*a^3*c^8*d^2*e + 2353*a^3*d^10*e - a^3*c^10*e + c^2*d^6*94i + 32*c^3*d^5 - c^4*d^4*5i - 176*c*d^7 + d^8*147i, e, k)*((a^3*d^8 + a^3*c*d^7*6i - 15*a^3*c^2*d^6 - a^3*c^3*d^5*20i + 15*a^3*c^4*d^4 + a^3*c^5*d^3*6i - a^3*c^6*d^2)*(8*a^3*c^10 - 200*a^3*d^10 + a^3*c*d^9*704i + a^3*c^9*d*64i + 552*a^3*c^2*d^8 + a^3*c^3*d^7*768i + 1456*a^3*c^4*d^6 - a^3*c^5*d^5*512i + 464*a^3*c^6*d^4 - a^3*c^7*d^3*512i - 232*a^3*c^8*d^2) - root(a^9*c^7*d^7*e^3*18432i + 9984*a^9*c^10*d^4*e^3 - 9984*a^9*c^4*d^10*e^3 + a^9*c^9*d^5*e^3*9728i + a^9*c^5*d^9*e^3*9728i + 6912*a^9*c^8*d^6*e^3 - 6912*a^9*c^6*d^8*e^3 + 2816*a^9*c^12*d^2*e^3 - 2816*a^9*c^2*d^12*e^3 - a^9*c^11*d^3*e^3*1024i - a^9*c^3*d^11*e^3*1024i - a^9*c^13*d*e^3*1536i - a^9*c*d^13*e^3*1536i + 256*a^9*d^14*e^3 - 256*a^9*c^14*e^3 + a^3*c*d^9*e*7510i - a^3*c^9*d*e*10i - 6525*a^3*c^2*d^8*e - 350*a^3*c^4*d^6*e - a^3*c^3*d^7*e*200i - 130*a^3*c^6*d^4*e + a^3*c^7*d^3*e*120i + a^3*c^5*d^5*e*100i + 45*a^3*c^8*d^2*e + 2353*a^3*d^10*e - a^3*c^10*e + c^2*d^6*94i + 32*c^3*d^5 - c^4*d^4*5i - 176*c*d^7 + d^8*147i, e, k)*((a^3*d^8 + a^3*c*d^7*6i - 15*a^3*c^2*d^6 - a^3*c^3*d^5*20i + 15*a^3*c^4*d^4 + a^3*c^5*d^3*6i - a^3*c^6*d^2)*(512*a^6*c*d^11 - 512*a^6*c^11*d - a^6*c^2*d^10*3072i - 6656*a^6*c^3*d^9 + a^6*c^4*d^8*4096i - 7168*a^6*c^5*d^7 + a^6*c^6*d^6*14336i + 7168*a^6*c^7*d^5 + a^6*c^8*d^4*4096i + 6656*a^6*c^9*d^3 - a^6*c^10*d^2*3072i) + tan(e + f*x)*(a^3*d^8 + a^3*c*d^7*6i - 15*a^3*c^2*d^6 - a^3*c^3*d^5*20i + 15*a^3*c^4*d^4 + a^3*c^5*d^3*6i - a^3*c^6*d^2)*(128*a^6*c^12 + 384*a^6*d^12 - a^6*c*d^11*2304i + a^6*c^11*d*768i - 5120*a^6*c^2*d^10 + a^6*c^3*d^9*3840i - 3712*a^6*c^4*d^8 + a^6*c^5*d^7*9728i + 7168*a^6*c^6*d^6 - a^6*c^7*d^5*512i + 3200*a^6*c^8*d^4 - a^6*c^9*d^3*3328i - 2048*a^6*c^10*d^2)) + tan(e + f*x)*(a^3*d^8 + a^3*c*d^7*6i - 15*a^3*c^2*d^6 - a^3*c^3*d^5*20i + 15*a^3*c^4*d^4 + a^3*c^5*d^3*6i - a^3*c^6*d^2)*(a^3*d^10*576i + 1168*a^3*c*d^9 + 16*a^3*c^9*d + a^3*c^2*d^8*704i + 2880*a^3*c^3*d^7 - a^3*c^4*d^6*1216i + 1248*a^3*c^5*d^5 - a^3*c^6*d^4*1216i - 448*a^3*c^7*d^3 + a^3*c^8*d^2*128i)) - (a^3*d^8 + a^3*c*d^7*6i - 15*a^3*c^2*d^6 - a^3*c^3*d^5*20i + 15*a^3*c^4*d^4 + a^3*c^5*d^3*6i - a^3*c^6*d^2)*(639*c*d^7 + c^7*d - d^8*600i - c^2*d^6*230i + 47*c^3*d^5 - c^4*d^4*100i - 47*c^5*d^3 + c^6*d^2*10i))*root(a^9*c^7*d^7*e^3*18432i + 9984*a^9*c^10*d^4*e^3 - 9984*a^9*c^4*d^10*e^3 + a^9*c^9*d^5*e^3*9728i + a^9*c^5*d^9*e^3*9728i + 6912*a^9*c^8*d^6*e^3 - 6912*a^9*c^6*d^8*e^3 + 2816*a^9*c^12*d^2*e^3 - 2816*a^9*c^2*d^12*e^3 - a^9*c^11*d^3*e^3*1024i - a^9*c^3*d^11*e^3*1024i - a^9*c^13*d*e^3*1536i - a^9*c*d^13*e^3*1536i + 256*a^9*d^14*e^3 - 256*a^9*c^14*e^3 + a^3*c*d^9*e*7510i - a^3*c^9*d*e*10i - 6525*a^3*c^2*d^8*e - 350*a^3*c^4*d^6*e - a^3*c^3*d^7*e*200i - 130*a^3*c^6*d^4*e + a^3*c^7*d^3*e*120i + a^3*c^5*d^5*e*100i + 45*a^3*c^8*d^2*e + 2353*a^3*d^10*e - a^3*c^10*e + c^2*d^6*94i + 32*c^3*d^5 - c^4*d^4*5i - 176*c*d^7 + d^8*147i, e, k), k, 1, 3)/f - ((tan(e + f*x)^3*(c^2*d*5i - 11*c*d^2 + c^3 + d^3*25i))/(8*a^3*(3*c*d^4 - c^4*d*3i - c^5 + d^5*1i - c^2*d^3*2i + 2*c^3*d^2)) - (c*d^3*280i + c^3*d*136i + 40*c^4 + 96*d^4 - 104*c^

```
2*d^2)/(96*a^3*d*(3*c*d^4 - c^4*d*3i - c^5 + d^5*1i - c^2*d^3*2i + 2*c^3*d^
2)) + (tan(e + f*x)^2*(c*d^3*76i + c^3*d*4i + 2*c^4 + 126*d^4 + 8*c^2*d^2))
/(16*a^3*d*(3*c*d^4 - c^4*d*3i - c^5 + d^5*1i - c^2*d^3*2i + 2*c^3*d^2)) -
(tan(e + f*x)*(c*d^3*143i + c^3*d*35i + 9*c^4 + 142*d^4 - 29*c^2*d^2)*1i)/(
24*a^3*d*(3*c*d^4 - c^4*d*3i - c^5 + d^5*1i - c^2*d^3*2i + 2*c^3*d^2)))/(f*
(tan(e + f*x)^3*(c/d - 3i) - tan(e + f*x)^2*((c*3i)/d + 3) + (c*1i)/d - tan
(e + f*x)*((3*c)/d - 1i) + tan(e + f*x)^4))
```

3.1095 $\int \frac{(a+ia\tan(e+fx))^3}{(c+d\tan(e+fx))^3}\,dx$

Optimal. Leaf size=134

$$\frac{4a^3x}{(c-id)^3}-\frac{4a^3\log(c\cos(e+fx)+d\sin(e+fx))}{(ic+d)^3f}-\frac{a(a+ia\tan(e+fx))^2}{2(ic+d)f(c+d\tan(e+fx))^2}+\frac{2a^3(c+id)}{(c-id)^2df(c+d\tan(e+fx))}$$

```
[Out] 4*a^3*x/(c-I*d)^3-4*a^3*ln(c*cos(f*x+e)+d*sin(f*x+e))/(I*c+d)^3/f-1/2*a*(a+
I*a*tan(f*x+e))^2/(I*c+d)/f/(c+d*tan(f*x+e))^2+2*a^3*(c+I*d)/(c-I*d)^2/d/f/
(c+d*tan(f*x+e))
```

Rubi [A]

time = 0.18, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3626, 3623, 3612, 3611}

$$\frac{2a^3(c+id)}{df(c-id)^2(c+d\tan(e+fx))}-\frac{4a^3\log(c\cos(e+fx)+d\sin(e+fx))}{f(d+ic)^3}+\frac{4a^3x}{(c-id)^3}-\frac{a(a+ia\tan(e+fx))^2}{2f(d+ic)(c+d\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^3/(c + d*Tan[e + f*x])^3,x]
```

```
[Out] (4*a^3*x)/(c - I*d)^3 - (4*a^3*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((I*c
+ d)^3*f) - (a*(a + I*a*Tan[e + f*x])^2)/(2*(I*c + d)*f*(c + d*Tan[e + f*x]
)^2) + (2*a^3*(c + I*d))/((c - I*d)^2*d*f*(c + d*Tan[e + f*x]))
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
```

```
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3626

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*b*(a + b*Tan[e + f*x])^(m - 1)*((c +
 d*Tan[e + f*x])^(n + 1)/(f*(m - 1)*(a*c - b*d))), x] + Dist[2*(a^2/(a*c -
b*d)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+ia\tan(e+fx))^3}{(c+d\tan(e+fx))^3}\,dx &= -\frac{a(a+ia\tan(e+fx))^2}{2(ic+d)f(c+d\tan(e+fx))^2}+\frac{(2a)\int \frac{(a+ia\tan(e+fx))^2}{(c+d\tan(e+fx))^2}\,dx}{c-id}\\ &= -\frac{a(a+ia\tan(e+fx))^2}{2(ic+d)f(c+d\tan(e+fx))^2}+\frac{2a^3(c+id)}{(c-id)^2df(c+d\tan(e+fx))}+\frac{(2a)\int \frac{2a}{\cdots}}{\cdots}\\ &= \frac{4a^3x}{(c-id)^3}-\frac{a(a+ia\tan(e+fx))^2}{2(ic+d)f(c+d\tan(e+fx))^2}+\frac{2a^3(c+id)}{(c-id)^2df(c+d\tan(e+fx))}\cdots\\ &= \frac{4a^3x}{(c-id)^3}-\frac{4a^3\log(c\cos(e+fx)+d\sin(e+fx))}{(ic+d)^3f}-\frac{a(a+ia\tan(e+fx\cdots}{2(ic+d)f(c+d\tan(e+\cdots}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 595 vs. 2(134) = 268.
time = 2.80, size = 595, normalized size = 4.44

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3/(c + d*Tan[e + f*x])^3,x]
```

```
[Out] (a^3*(2*c^3*f*x*Cos[3*e + 2*f*x] - 6*c*d^2*f*x*Cos[3*e + 2*f*x] - I*c^3*Cos
[3*e + 2*f*x]*Log[(c*Cos[e + f*x] + d*Sin[e + f*x])^2] + (3*I)*c*d^2*Cos[3*
e + 2*f*x]*Log[(c*Cos[e + f*x] + d*Sin[e + f*x])^2] + (c^2 + d^2)*Cos[e + 2
*f*x]*(3*d + 2*c*f*x - I*c*Log[(c*Cos[e + f*x] + d*Sin[e + f*x])^2]) + (c^2
 + d^2)*Cos[e]*((-I)*c - 3*d + 4*c*f*x - (2*I)*c*Log[(c*Cos[e + f*x] + d*Si
n[e + f*x])^2]) + 3*c^3*Sin[e] - I*c^2*d*Sin[e] + 3*c*d^2*Sin[e] - I*d^3*Si
n[e] + 4*c^2*d*f*x*Sin[e] + 4*d^3*f*x*Sin[e] - (2*I)*c^2*d*Log[(c*Cos[e + f
```

```
*x] + d*Sin[e + f*x])^2]*Sin[e] - (2*I)*d^3*Log[(c*Cos[e + f*x] + d*Sin[e +
 f*x])^2]*Sin[e] - 3*c^3*Sin[e + 2*f*x] - 3*c*d^2*Sin[e + 2*f*x] + 2*c^2*d*
f*x*Sin[e + 2*f*x] + 2*d^3*f*x*Sin[e + 2*f*x] - I*c^2*d*Log[(c*Cos[e + f*x]
 + d*Sin[e + f*x])^2]*Sin[e + 2*f*x] - I*d^3*Log[(c*Cos[e + f*x] + d*Sin[e
+ f*x])^2]*Sin[e + 2*f*x] + 6*c^2*d*f*x*Sin[3*e + 2*f*x] - 2*d^3*f*x*Sin[3*
e + 2*f*x] - (3*I)*c^2*d*Log[(c*Cos[e + f*x] + d*Sin[e + f*x])^2]*Sin[3*e +
 2*f*x] + I*d^3*Log[(c*Cos[e + f*x] + d*Sin[e + f*x])^2]*Sin[3*e + 2*f*x]))
/(2*(c - I*d)^3*f*(c*Cos[e] + d*Sin[e])*(c*Cos[e + f*x] + d*Sin[e + f*x])^2
)
```

Maple [A]
time = 0.34, size = 239, normalized size = 1.78

method	result
derivativedivides	$\frac{a^3\left(\frac{\frac{(4ic^3-12icd^2-12c^2d+4d^3)\ln(1+\tan^2(fx+e))}{2}+(12ic^2d-4id^3+4c^3-12cd^2)\arctan(\tan(fx+e))}{(c^2+d^2)^3}-\frac{-ic^4-6ic^2d^2+3id^4+8c\,d\ldots}{(c^2+d^2)^2d^2(c+d\tan(fx+\ldots}\ldots\right.}{f}$
default	$\frac{a^3\left(\frac{\frac{(4ic^3-12icd^2-12c^2d+4d^3)\ln(1+\tan^2(fx+e))}{2}+(12ic^2d-4id^3+4c^3-12cd^2)\arctan(\tan(fx+e))}{(c^2+d^2)^3}-\frac{-ic^4-6ic^2d^2+3id^4+8c\,d\ldots}{(c^2+d^2)^2d^2(c+d\tan(fx+\ldots}\ldots\right.}{f}$
risch	$-\frac{8a^3x}{3ic^2d-id^3-c^3+3cd^2}-\frac{8ia^3x}{ic^3-3icd^2+3c^2d-d^3}-\frac{8ia^3e}{f(ic^3-3icd^2+3c^2d-d^3)}-\frac{2ia^3(4c^2e^{2i(fx+e)}+4d^2e^{2i(fx+e)}+6\ldots}{f(-id+c)^3(-ie^{2i(fx+e)}d+id+e^{2i\ldots}}\ldots$
norman	$\frac{\frac{(ia^3c^2+3ia^3d^2+2a^3cd)\tan(fx+e)}{df(-2icd+c^2-d^2)}+\frac{4a^3c^2x}{(-id+c)^3}+\frac{ia^3c^3+5ia^3cd^2+5a^3c^2d+a^3d^3}{2d^2f(-2icd+c^2-d^2)}+\frac{8cda^3x\tan(fx+e)}{(-id+c)^3}-\frac{4ia^3d^2x(\tan^2(fx+e))}{(ic+d)^3}}{(c+d\tan(fx+e))^2}+\ldots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a^3*(1/(c^2+d^2)^3*(1/2*(4*I*c^3-12*I*c*d^2-12*c^2*d+4*d^3)*ln(1+tan(f*
x+e)^2)+(12*I*c^2*d-4*I*d^3+4*c^3-12*c*d^2)*arctan(tan(f*x+e)))-(-I*c^4-6*I
*c^2*d^2+3*I*d^4+8*c*d^3)/(c^2+d^2)^2/d^2/(c+d*tan(f*x+e))-1/2*(I*c^3-3*I*c
*d^2-3*c^2*d+d^3)/d^2/(c^2+d^2)/(c+d*tan(f*x+e))^2-4*(I*c^3-3*I*c*d^2-3*c^2
*d+d^3)/(c^2+d^2)^3*ln(c+d*tan(f*x+e)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 409 vs. 2(125) = 250.
time = 0.50, size = 409, normalized size = 3.05

$$\frac{\frac{8(a^3c^3+3ia^3c^2d-3a^3cd^2-ia^3d^3)(fx+e)}{c^6+3c^4d^2+3c^2d^4+d^6}-\frac{8(ia^3c^3-3a^3c^2d-3ia^3cd^2+a^3d^3)\log(d\tan(fx+e)+c)}{c^6+3c^4d^2+3c^2d^4+d^6}-\frac{4(-ia^3c^3+3a^3c^2d+3ia^3cd^2-a^3d^3)\log\left(\tan(fx+e)^2+1\right)}{c^6+3c^4d^2+3c^2d^4+d^6}+\frac{ia^3c^5+3a^3c^4d+14ia^3c^3d^2-14a^3c^2d^3-3ia^3cd^4-a^3d^5+2(ia^3c^4d+6ia^3c^2d^3-8a^3cd^4-3ia^3d^5)\tan(fx+e)}{c^6d^2+2c^4d^4+c^2d^6+(c^4d^4+2c^2d^6+d^8)\tan(fx+e)^2+2(c^5d^3+2c^3d^5+cd^7)\tan(fx+e)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(8*(a^3*c^3 + 3*I*a^3*c^2*d - 3*a^3*c*d^2 - I*a^3*d^3)*(f*x + e)/(c^6 +
 3*c^4*d^2 + 3*c^2*d^4 + d^6) - 8*(I*a^3*c^3 - 3*a^3*c^2*d - 3*I*a^3*c*d^2
+ a^3*d^3)*log(d*tan(f*x + e) + c)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) - 4*
(-I*a^3*c^3 + 3*a^3*c^2*d + 3*I*a^3*c*d^2 - a^3*d^3)*log(tan(f*x + e)^2 + 1
)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) + (I*a^3*c^5 + 3*a^3*c^4*d + 14*I*a^3
*c^3*d^2 - 14*a^3*c^2*d^3 - 3*I*a^3*c*d^4 - a^3*d^5 + 2*(I*a^3*c^4*d + 6*I*
a^3*c^2*d^3 - 8*a^3*c*d^4 - 3*I*a^3*d^5)*tan(f*x + e))/(c^6*d^2 + 2*c^4*d^4
 + c^2*d^6 + (c^4*d^4 + 2*c^2*d^6 + d^8)*tan(f*x + e)^2 + 2*(c^5*d^3 + 2*c^
3*d^5 + c*d^7)*tan(f*x + e)))/f
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(125) = 250$.
time = 1.55, size = 311, normalized size = 2.32

$$\frac{2\left(3\,a^3c^2+6i\,a^3cd-3\,a^3d^2+4\,(a^3c^2+a^3d^2)e^{(2i\,fx+2i\,e)}+2\,(a^3c^2+2i\,a^3cd-a^3d^2+(a^3c^2-2i\,a^3cd-a^3d^2)e^{(4i\,fx+4i\,e)}+2\,(a^3c^2+a^3d^2)e^{(2i\,fx+2i\,e)})\log\left(\frac{(i\,c+d)e^{(2i\,fx+2i\,e)}+i\,c-d}{i\,c+d}\right)\right)}{(i\,c^5+5\,c^4d-10i\,c^3d^2-10\,c^2d^3+5i\,cd^4+d^5)fe^{(4i\,fx+4i\,e)}-2\,(-i\,c^5-3\,c^4d+2i\,c^3d^2-2\,c^2d^3+3i\,cd^4+d^5)fe^{(2i\,fx+2i\,e)}+(i\,c^5+c^4d+2i\,c^3d^2+2\,c^2d^3+i\,cd^4+d^5)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 2*(3*a^3*c^2 + 6*I*a^3*c*d - 3*a^3*d^2 + 4*(a^3*c^2 + a^3*d^2)*e^(2*I*f*x +
 2*I*e) + 2*(a^3*c^2 + 2*I*a^3*c*d - a^3*d^2 + (a^3*c^2 - 2*I*a^3*c*d - a^3
*d^2)*e^(4*I*f*x + 4*I*e) + 2*(a^3*c^2 + a^3*d^2)*e^(2*I*f*x + 2*I*e))*log(
((I*c + d)*e^(2*I*f*x + 2*I*e) + I*c - d)/(I*c + d)))/((I*c^5 + 5*c^4*d - 1
0*I*c^3*d^2 - 10*c^2*d^3 + 5*I*c*d^4 + d^5)*f*e^(4*I*f*x + 4*I*e) - 2*(-I*c
^5 - 3*c^4*d + 2*I*c^3*d^2 - 2*c^2*d^3 + 3*I*c*d^4 + d^5)*f*e^(2*I*f*x + 2*
I*e) + (I*c^5 + c^4*d + 2*I*c^3*d^2 + 2*c^2*d^3 + I*c*d^4 + d^5)*f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(110) = 220$.
time = 4.03, size = 377, normalized size = 2.81

$$-\frac{4ia^3\log\left(\frac{c+id}{ce^{2ie}-ide^{2ie}}+e^{2ifx}\right)}{f\left(c-id\right)^3}+\frac{-6ia^3c^2+12a^3cd+6ia^3d^2+\left(-8ia^3c^2e^{2ie}-8ia^3d^2e^{2ie}\right)e^{2ifx}}{c^5f-ic^4df+2c^3d^2f-2ic^2d^3f+cd^4f-id^5f+\left(2c^5fe^{2ie}-6ic^4dfe^{2ie}-4c^3d^2fe^{2ie}-4ic^2d^3fe^{2ie}-6cd^4fe^{2ie}+2id^5fe^{2ie}\right)e^{2ifx}+\left(c^5fe^{4ie}-5ic^4dfe^{4ie}-10c^3d^2fe^{4ie}+10ic^2d^3fe^{4ie}+5cd^4fe^{4ie}-id^5fe^{4ie}\right)e^{4ifx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3/(c+d*tan(f*x+e))**3,x)
```

```
[Out] -4*I*a**3*log((c + I*d)/(c*exp(2*I*e) - I*d*exp(2*I*e)) + exp(2*I*f*x))/(f*
(c - I*d)**3) + (-6*I*a**3*c**2 + 12*a**3*c*d + 6*I*a**3*d**2 + (-8*I*a**3*
c**2*exp(2*I*e) - 8*I*a**3*d**2*exp(2*I*e))*exp(2*I*f*x))/(c**5*f - I*c**4*
d*f + 2*c**3*d**2*f - 2*I*c**2*d**3*f + c*d**4*f - I*d**5*f + (2*c**5*f*exp
(2*I*e) - 6*I*c**4*d*f*exp(2*I*e) - 4*c**3*d**2*f*exp(2*I*e) - 4*I*c**2*d**
3*f*exp(2*I*e) - 6*c*d**4*f*exp(2*I*e) + 2*I*d**5*f*exp(2*I*e))*exp(2*I*f*x
) + (c**5*f*exp(4*I*e) - 5*I*c**4*d*f*exp(4*I*e) - 10*c**3*d**2*f*exp(4*I*e
) + 10*I*c**2*d**3*f*exp(4*I*e) + 5*c*d**4*f*exp(4*I*e) - I*d**5*f*exp(4*I*
e))*exp(4*I*f*x))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 470 vs. 2(125) = 250.
time = 0.80, size = 470, normalized size = 3.51

$$\frac{2\left(\frac{2\,a^3\log\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-2\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-c\right)}{i\,c^3+3\,c^2d-3i\,cd^2-d^3}-\frac{4\,a^3\log\left(-i\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}{i\,c^3+3\,c^2d-3i\,cd^2-d^3}+\frac{3\,a^3c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-3i\,a^3c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-13\,a^3c^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-3i\,a^3c^2d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-a^3cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-7\,a^3c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+6i\,a^3c^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+12\,a^3c^2d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+6i\,a^3cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a^3d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3i\,a^3c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+13\,a^3c^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+3i\,a^3c^2d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+a^3cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+3\,a^3c^4}{\left(-i\,c^5-3\,c^4d+3i\,c^3d^2+c^2d^3\right)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-2\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-c\right)^2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 2*(2*a^3*log(c*tan(1/2*f*x + 1/2*e)^2 - 2*d*tan(1/2*f*x + 1/2*e) - c)/(I*c^
3 + 3*c^2*d - 3*I*c*d^2 - d^3) - 4*a^3*log(-I*tan(1/2*f*x + 1/2*e) + 1)/(I*
c^3 + 3*c^2*d - 3*I*c*d^2 - d^3) + (3*a^3*c^4*tan(1/2*f*x + 1/2*e)^4 - 3*I*
a^3*c^4*tan(1/2*f*x + 1/2*e)^3 - 13*a^3*c^3*d*tan(1/2*f*x + 1/2*e)^3 - 3*I*
a^3*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - a^3*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 7*a
^3*c^4*tan(1/2*f*x + 1/2*e)^2 + 6*I*a^3*c^3*d*tan(1/2*f*x + 1/2*e)^2 + 12*a
^3*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 + 6*I*a^3*c*d^3*tan(1/2*f*x + 1/2*e)^2 +
a^3*d^4*tan(1/2*f*x + 1/2*e)^2 + 3*I*a^3*c^4*tan(1/2*f*x + 1/2*e) + 13*a^3*
c^3*d*tan(1/2*f*x + 1/2*e) + 3*I*a^3*c^2*d^2*tan(1/2*f*x + 1/2*e) + a^3*c*d
^3*tan(1/2*f*x + 1/2*e) + 3*a^3*c^4)/((-I*c^5 - 3*c^4*d + 3*I*c^3*d^2 + c^2
*d^3)*(c*tan(1/2*f*x + 1/2*e)^2 - 2*d*tan(1/2*f*x + 1/2*e) - c)^2))/f
```

Mupad [B]
time = 5.62, size = 314, normalized size = 2.34

$$-\frac{\frac{a^3\,\left(c^3\,1{}\mathrm{i}+5\,c^2\,d+c\,d^2\,5{}\mathrm{i}+d^3\right)}{2\,d^4\,\left(-c^2+c\,d\,2{}\mathrm{i}+d^2\right)}+\frac{a^3\,\tan\left(e+f\,x\right)\,\left(c^2-c\,d\,2{}\mathrm{i}+3\,d^2\right)\,1{}\mathrm{i}}{d^3\,\left(-c^2+c\,d\,2{}\mathrm{i}+d^2\right)}}{f\,\left({\tan\left(e+f\,x\right)}^2+\frac{c^2}{d^2}+\frac{2\,c\,\tan\left(e+f\,x\right)}{d}\right)}+\frac{a^3\,\mathrm{atan}\left(\frac{c^3-c^2\,d\,1{}\mathrm{i}+c\,d^2-d^3\,1{}\mathrm{i}}{{\left(c-d\,1{}\mathrm{i}\right)}^2\,\left(d+c\,1{}\mathrm{i}\right)}-\frac{\tan\left(e+f\,x\right)\,\left(2\,c^8\,d^2+8\,c^6\,d^4+12\,c^4\,d^6+8\,c^2\,d^8+2\,d^{10}\right)\,1{}\mathrm{i}}{{\left(c-d\,1{}\mathrm{i}\right)}^2\,\left(d+c\,1{}\mathrm{i}\right)\,\left(-c^6\,d\,1{}\mathrm{i}+2\,c^5\,d^2-c^4\,d^3\,1{}\mathrm{i}+4\,c^3\,d^4+c^2\,d^5\,1{}\mathrm{i}+2\,c\,d^6+d^7\,1{}\mathrm{i}\right)}\right)\,8{}\mathrm{i}}{f\,{\left(c-d\,1{}\mathrm{i}\right)}^2\,\left(d+c\,1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^3/(c + d*tan(e + f*x))^3,x)
```

```
[Out] (a^3*atan((c*d^2 - c^2*d*1i + c^3 - d^3*1i)/((c - d*1i)^2*(c*1i + d)) - (ta
n(e + f*x)*(2*d^10 + 8*c^2*d^8 + 12*c^4*d^6 + 8*c^6*d^4 + 2*c^8*d^2)*1i)/((
c - d*1i)^2*(c*1i + d)*(2*c*d^6 - c^6*d*1i + d^7*1i + c^2*d^5*1i + 4*c^3*d^
4 - c^4*d^3*1i + 2*c^5*d^2)))*8i)/(f*(c - d*1i)^2*(c*1i + d)) - ((a^3*(c*d^
2*5i + 5*c^2*d + c^3*1i + d^3))/(2*d^4*(c*d*2i - c^2 + d^2)) + (a^3*tan(e +
 f*x)*(c^2 - c*d*2i + 3*d^2)*1i)/(d^3*(c*d*2i - c^2 + d^2)))/(f*(tan(e + f*
x)^2 + c^2/d^2 + (2*c*tan(e + f*x))/d))
```

3.1096 $\int \frac{(a+ia\tan(e+fx))^2}{(c+d\tan(e+fx))^3}\,dx$

Optimal. Leaf size=125

$$\frac{2a^2x}{(c-id)^3}-\frac{2a^2\log(c\cos(e+fx)+d\sin(e+fx))}{(ic+d)^3f}+\frac{a^2(ic-d)}{2d(ic+d)f(c+d\tan(e+fx))^2}+\frac{2ia^2}{(c-id)^2f(c+d\tan(e+$$

```
[Out] 2*a^2*x/(c-I*d)^3-2*a^2*ln(c*cos(f*x+e)+d*sin(f*x+e))/(I*c+d)^3/f+1/2*a^2*(
I*c-d)/d/(I*c+d)/f/(c+d*tan(f*x+e))^2+2*I*a^2/(c-I*d)^2/f/(c+d*tan(f*x+e))
```

Rubi [A]
time = 0.18, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3623, 3610, 3612, 3611}

$$\frac{2ia^2}{f(c-id)^2(c+d\tan(e+fx))}+\frac{a^2(-d+ic)}{2df(d+ic)(c+d\tan(e+fx))^2}-\frac{2a^2\log(c\cos(e+fx)+d\sin(e+fx))}{f(d+ic)^3}+\frac{2a^2x}{(c-id)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2/(c + d*Tan[e + f*x])^3,x]
```

```
[Out] (2*a^2*x)/(c - I*d)^3 - (2*a^2*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((I*c
+ d)^3*f) + (a^2*(I*c - d))/(2*d*(I*c + d)*f*(c + d*Tan[e + f*x])^2) + ((2*
I)*a^2)/((c - I*d)^2*f*(c + d*Tan[e + f*x]))
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
```

```
Q[a*c + b*d, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+ia\tan(e+fx))^2}{(c+d\tan(e+fx))^3}\,dx &= \frac{a^2(ic-d)}{2d(ic+d)f(c+d\tan(e+fx))^2} + \frac{\int \frac{2a^2(c+id)+2a^2(ic-d)\tan(e+fx)}{(c+d\tan(e+fx))^2}\,dx}{c^2+d^2}\\ &= \frac{a^2(ic-d)}{2d(ic+d)f(c+d\tan(e+fx))^2} + \frac{2ia^2}{(c-id)^2f(c+d\tan(e+fx))} + \frac{\int \frac{2a^2(c+i}{}}{}\\ &= \frac{2a^2x}{(c-id)^3} + \frac{a^2(ic-d)}{2d(ic+d)f(c+d\tan(e+fx))^2} + \frac{2ia^2}{(c-id)^2f(c+d\tan(e+fx)}\\ &= \frac{2a^2x}{(c-id)^3} - \frac{2a^2\log(c\cos(e+fx)+d\sin(e+fx))}{(ic+d)^3f} + \frac{a^2(ic-d)}{2d(ic+d)f(c+d\tan(e}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 317 vs. 2(125) = 250.
time = 4.53, size = 317, normalized size = 2.54

$$\frac{a^2(\cos(e+fx)+i\sin(e+fx))^2\left(\frac{\log\left((c\cos(e+fx)+d\sin(e+fx))^2\right)(-i\cos(2e)-\sin(2e))}{f}+4x(\cos(2e)-i\sin(2e))-\frac{2\text{ArcTan}\left(\frac{\left(-3c^2d+d^3\right)\cos(3e+fx)+c\left(c^2-3d^2\right)\sin(3e+fx)}{c\left(c^2-3d^2\right)\cos(3e+fx)-d\left(-3c^2+d^2\right)\sin(3e+fx)}\right)(\cos(2e)-i\sin(2e))}{f}+\frac{(c-id)d(\cos(2e)-i\sin(2e))}{2f(c\cos(e+fx)+d\sin(e+fx))^2}-\frac{(c-id)(c+2id)(\cos(2e)-i\sin(2e))\sin(fx)}{f(c\cos(e)+d\sin(e))(c\cos(e+fx)+d\sin(e+fx))}\right)}{(c-id)^3(\cos(fx)+i\sin(fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2/(c + d*Tan[e + f*x])^3,x]
```

```
[Out] (a^2*(Cos[e + f*x] + I*Sin[e + f*x])^2*((Log[(c*Cos[e + f*x] + d*Sin[e + f*
x])^2]*((-I)*Cos[2*e] - Sin[2*e]))/f + 4*x*(Cos[2*e] - I*Sin[2*e]) - (2*Arc
Tan[((-3*c^2*d + d^3)*Cos[3*e + f*x] + c*(c^2 - 3*d^2)*Sin[3*e + f*x])/(c*(
c^2 - 3*d^2)*Cos[3*e + f*x] - d*(-3*c^2 + d^2)*Sin[3*e + f*x])]*(Cos[2*e] -
 I*Sin[2*e]))/f + ((c - I*d)*d*(Cos[2*e] - I*Sin[2*e]))/(2*f*(c*Cos[e + f*x
] + d*Sin[e + f*x])^2) - ((c - I*d)*(c + (2*I)*d)*(Cos[2*e] - I*Sin[2*e])*S
in[f*x])/(f*(c*Cos[e] + d*Sin[e])*(c*Cos[e + f*x] + d*Sin[e + f*x]))))/((c
- I*d)^3*(Cos[f*x] + I*Sin[f*x])^2)
```

Maple [A]
time = 0.31, size = 216, normalized size = 1.73

method	result
derivativedivides	$\frac{a^{2}\left(\frac{\frac{\left(2ic^{3}-6icd^{2}-6c^{2}d+2d^{3}\right)\ln\left(1+\tan^{2}(fx+e)\right)}{2}+\left(6ic^{2}d-2id^{3}+2c^{3}-6cd^{2}\right)\arctan(\tan(fx+e))}{\left(c^{2}+d^{2}\right)^{3}}-\frac{-2icd-c^{2}+d^{2}}{2\left(c^{2}+d^{2}\right)d(c+d\tan(fx+e))^{2}}+\cdots\right.}{f}$
default	$\frac{a^{2}\left(\frac{\frac{\left(2ic^{3}-6icd^{2}-6c^{2}d+2d^{3}\right)\ln\left(1+\tan^{2}(fx+e)\right)}{2}+\left(6ic^{2}d-2id^{3}+2c^{3}-6cd^{2}\right)\arctan(\tan(fx+e))}{\left(c^{2}+d^{2}\right)^{3}}-\frac{-2icd-c^{2}+d^{2}}{2\left(c^{2}+d^{2}\right)d(c+d\tan(fx+e))^{2}}+\cdots\right.}{f}$
risch	$-\frac{4a^{2}x}{3ic^{2}d-id^{3}-c^{3}+3cd^{2}}-\frac{4ia^{2}x}{ic^{3}-3icd^{2}+3c^{2}d-d^{3}}-\frac{4ia^{2}e}{f(ic^{3}-3icd^{2}+3c^{2}d-d^{3})}-\frac{2i\left(3a^{2}d^{2}e^{2i(fx+e)}+2ia^{2}cd\,e^{2i(fx+e)}-\cdots\right.}{\left(-ie^{2i(fx+e)}d+id+e^{2i\cdots}\right.}$
norman	$\frac{\frac{2ia^{2}cd+a^{2}c^{2}+a^{2}d^{2}}{2f\left(-2icd+c^{2}-d^{2}\right)d}+\frac{2a^{2}c^{2}x}{(-id+c)\left(-2icd+c^{2}-d^{2}\right)}+\frac{id^{2}a^{2}\left(\tan^{2}(fx+e)\right)}{\left(2icd-c^{2}+d^{2}\right)fc}+\frac{4cd\,a^{2}x\tan(fx+e)}{(-id+c)\left(-2icd+c^{2}-d^{2}\right)}-\frac{2ia^{2}d^{2}x\left(\tan^{2}(fx+e)\right)}{(ic+d)\left(2icd-c^{2}+d^{2}\right)}}{(c+d\tan(fx+e))^{2}}+\frac{ia\cdots}{f(-\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a^2*(1/(c^2+d^2)^3*(1/2*(2*I*c^3-6*I*c*d^2-6*c^2*d+2*d^3)*ln(1+tan(f*x+
e)^2)+(6*I*c^2*d-2*I*d^3+2*c^3-6*c*d^2)*arctan(tan(f*x+e)))-1/2*(-2*I*c*d-c
^2+d^2)/(c^2+d^2)/d/(c+d*tan(f*x+e))^2+2*(I*c^2-I*d^2-2*c*d)/(c^2+d^2)^2/(c
+d*tan(f*x+e))-2*(I*c^3-3*I*c*d^2-3*c^2*d+d^3)/(c^2+d^2)^3*ln(c+d*tan(f*x+e
)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(115) = 230.
time = 0.52, size = 386, normalized size = 3.09

$$\frac{\frac{4\left(a^{2}c^{3}+3i\,a^{2}c^{2}d-3\,a^{2}cd^{2}-i\,a^{2}d^{3}\right)(fx+e)}{c^{6}+3\,c^{4}d^{2}+3\,c^{2}d^{4}+d^{6}}-\frac{4\left(i\,a^{2}c^{3}-3\,a^{2}c^{2}d-3i\,a^{2}cd^{2}+a^{2}d^{3}\right)\log(d\tan(fx+e)+c)}{c^{6}+3\,c^{4}d^{2}+3\,c^{2}d^{4}+d^{6}}-\frac{2\left(-i\,a^{2}c^{3}+3\,a^{2}c^{2}d+3i\,a^{2}cd^{2}-a^{2}d^{3}\right)\log\left(\tan(fx+e)^{2}+1\right)}{c^{6}+3\,c^{4}d^{2}+3\,c^{2}d^{4}+d^{6}}+\frac{a^{2}c^{4}+6i\,a^{2}c^{3}d-8\,a^{2}c^{2}d^{2}-2i\,a^{2}cd^{3}-a^{2}d^{4}+4\left(i\,a^{2}c^{2}d^{2}-2\,a^{2}cd^{3}-i\,a^{2}d^{4}\right)\tan(fx+e)}{c^{6}d+2\,c^{4}d^{3}+c^{2}d^{5}+\left(c^{4}d^{3}+2\,c^{2}d^{5}+d^{7}\right)\tan(fx+e)^{2}+2\left(c^{5}d^{2}+2\,c^{3}d^{4}+cd^{6}\right)\tan(fx+e)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(4*(a^2*c^3 + 3*I*a^2*c^2*d - 3*a^2*c*d^2 - I*a^2*d^3)*(f*x + e)/(c^6 +
 3*c^4*d^2 + 3*c^2*d^4 + d^6) - 4*(I*a^2*c^3 - 3*a^2*c^2*d - 3*I*a^2*c*d^2
+ a^2*d^3)*log(d*tan(f*x + e) + c)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) - 2*
(-I*a^2*c^3 + 3*a^2*c^2*d + 3*I*a^2*c*d^2 - a^2*d^3)*log(tan(f*x + e)^2 + 1
)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) + (a^2*c^4 + 6*I*a^2*c^3*d - 8*a^2*c^
2*d^2 - 2*I*a^2*c*d^3 - a^2*d^4 + 4*(I*a^2*c^2*d^2 - 2*a^2*c*d^3 - I*a^2*d^
4)*tan(f*x + e))/(c^6*d + 2*c^4*d^3 + c^2*d^5 + (c^4*d^3 + 2*c^2*d^5 + d^7)
*tan(f*x + e)^2 + 2*(c^5*d^2 + 2*c^3*d^4 + c*d^6)*tan(f*x + e)))/f
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(115) = 230.

time = 0.90, size = 316, normalized size = 2.53

$$\frac{2\left(a^2c^2+3i\,a^2cd-2\,a^2d^2+(a^2c^2+2i\,a^2cd+3\,a^2d^2)e^{(2i\,fx+2i\,e)}+\left(a^2c^2+2i\,a^2cd-a^2d^2+(a^2c^2-2i\,a^2cd-a^2d^2)e^{(4i\,fx+4i\,e)}+2\,(a^2c^2+a^2d^2)e^{(2i\,fx+2i\,e)}\right)\log\left(\frac{(i\,c+d)e^{(2i\,fx+2i\,e)}+i\,c-d}{i\,c+d}\right)\right)}{(i\,c^5+5\,c^4d-10i\,c^3d^2-10\,c^2d^3+5i\,cd^4+d^5)fe^{(4i\,fx+4i\,e)}-2\,(-i\,c^5-3\,c^4d+2i\,c^3d^2-2\,c^2d^3+3i\,cd^4+d^5)fe^{(2i\,fx+2i\,e)}+(i\,c^5+c^4d+2i\,c^3d^2+2\,c^2d^3+i\,cd^4+d^5)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 2*(a^2*c^2 + 3*I*a^2*c*d - 2*a^2*d^2 + (a^2*c^2 + 2*I*a^2*c*d + 3*a^2*d^2)*
e^(2*I*f*x + 2*I*e) + (a^2*c^2 + 2*I*a^2*c*d - a^2*d^2 + (a^2*c^2 - 2*I*a^2
*c*d - a^2*d^2)*e^(4*I*f*x + 4*I*e) + 2*(a^2*c^2 + a^2*d^2)*e^(2*I*f*x + 2*
I*e))*log(((I*c + d)*e^(2*I*f*x + 2*I*e) + I*c - d)/(I*c + d)))/((I*c^5 + 5
*c^4*d - 10*I*c^3*d^2 - 10*c^2*d^3 + 5*I*c*d^4 + d^5)*f*e^(4*I*f*x + 4*I*e)
 - 2*(-I*c^5 - 3*c^4*d + 2*I*c^3*d^2 - 2*c^2*d^3 + 3*I*c*d^4 + d^5)*f*e^(2*
I*f*x + 2*I*e) + (I*c^5 + c^4*d + 2*I*c^3*d^2 + 2*c^2*d^3 + I*c*d^4 + d^5)*
f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(100) = 200$.
time = 4.20, size = 391, normalized size = 3.13

$$-\frac{2ia^2\log\left(\frac{c+id}{ce^{2ie}-ide^{2ie}}+e^{2ifx}\right)}{f\left(c-id\right)^3}+\frac{-2ia^2c^2+6a^2cd+4ia^2d^2+\left(-2ia^2c^2e^{2ie}+4a^2cde^{2ie}-6ia^2d^2e^{2ie}\right)e^{2ifx}}{c^5f-ic^4df+2c^3d^2f-2ic^2d^3f+cd^4f-id^5f+\left(2c^5fe^{2ie}-6ic^4dfe^{2ie}-4c^3d^2fe^{2ie}-4ic^2d^3fe^{2ie}-6cd^4fe^{2ie}+2id^5fe^{2ie}\right)e^{2ifx}+\left(c^5fe^{4ie}-5ic^4dfe^{4ie}-10c^3d^2fe^{4ie}+10ic^2d^3fe^{4ie}+5cd^4fe^{4ie}-id^5fe^{4ie}\right)e^{4ifx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2/(c+d*tan(f*x+e))**3,x)
```

```
[Out] -2*I*a**2*log((c + I*d)/(c*exp(2*I*e) - I*d*exp(2*I*e)) + exp(2*I*f*x))/(f*
(c - I*d)**3) + (-2*I*a**2*c**2 + 6*a**2*c*d + 4*I*a**2*d**2 + (-2*I*a**2*c
**2*exp(2*I*e) + 4*a**2*c*d*exp(2*I*e) - 6*I*a**2*d**2*exp(2*I*e))*exp(2*I*
f*x))/(c**5*f - I*c**4*d*f + 2*c**3*d**2*f - 2*I*c**2*d**3*f + c*d**4*f - I
*d**5*f + (2*c**5*f*exp(2*I*e) - 6*I*c**4*d*f*exp(2*I*e) - 4*c**3*d**2*f*ex
p(2*I*e) - 4*I*c**2*d**3*f*exp(2*I*e) - 6*c*d**4*f*exp(2*I*e) + 2*I*d**5*f*
exp(2*I*e))*exp(2*I*f*x) + (c**5*f*exp(4*I*e) - 5*I*c**4*d*f*exp(4*I*e) - 1
0*c**3*d**2*f*exp(4*I*e) + 10*I*c**2*d**3*f*exp(4*I*e) + 5*c*d**4*f*exp(4*I
*e) - I*d**5*f*exp(4*I*e))*exp(4*I*f*x))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(115) = 230$.
time = 0.74, size = 472, normalized size = 3.78

$$\frac{2\left(\frac{a^2\log\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-2\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-c\right)}{i\,c^3+3\,c^2d-3i\,cd^2-d^3}-\frac{2\,a^2\log\left(-i\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}{i\,c^3+3\,c^2d-3i\,cd^2-d^3}+\frac{3\,a^2c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-2i\,a^2c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-10\,a^2c^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-6i\,a^2c^2d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,a^2cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-6\,a^2c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2i\,a^2c^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+6\,a^2c^2d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+10i\,a^2cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,a^2d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2i\,a^2c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+10\,a^2c^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+6i\,a^2c^2d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\,a^2cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+3\,a^2c^4}{-2\,(i\,c^5+3\,c^4d-3i\,c^3d^2-c^2d^3)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-2\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-c\right)^2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 2*(a^2*log(c*tan(1/2*f*x + 1/2*e)^2 - 2*d*tan(1/2*f*x + 1/2*e) - c)/(I*c^3
+ 3*c^2*d - 3*I*c*d^2 - d^3) - 2*a^2*log(-I*tan(1/2*f*x + 1/2*e) + 1)/(I*c^
```

```
3 + 3*c^2*d - 3*I*c*d^2 - d^3) + (3*a^2*c^4*tan(1/2*f*x + 1/2*e)^4 - 2*I*a^
2*c^4*tan(1/2*f*x + 1/2*e)^3 - 10*a^2*c^3*d*tan(1/2*f*x + 1/2*e)^3 - 6*I*a^
2*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - 2*a^2*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 6*a
^2*c^4*tan(1/2*f*x + 1/2*e)^2 + 2*I*a^2*c^3*d*tan(1/2*f*x + 1/2*e)^2 + 6*a^
2*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 + 10*I*a^2*c*d^3*tan(1/2*f*x + 1/2*e)^2 +
2*a^2*d^4*tan(1/2*f*x + 1/2*e)^2 + 2*I*a^2*c^4*tan(1/2*f*x + 1/2*e) + 10*a^
2*c^3*d*tan(1/2*f*x + 1/2*e) + 6*I*a^2*c^2*d^2*tan(1/2*f*x + 1/2*e) + 2*a^2
*c*d^3*tan(1/2*f*x + 1/2*e) + 3*a^2*c^4)/((-2*I*c^5 - 6*c^4*d + 6*I*c^3*d^2
 + 2*c^2*d^3)*(c*tan(1/2*f*x + 1/2*e)^2 - 2*d*tan(1/2*f*x + 1/2*e) - c)^2))
/f
```

Mupad [B]
time = 5.51, size = 297, normalized size = 2.38

$$-\frac{\frac{a^2\,c^2+a^2\,c\,d\,4{}\mathrm{i}+a^2\,d^2}{2\,d^3\,\left(-c^2+c\,d\,2{}\mathrm{i}+d^2\right)}+\frac{a^2\,\mathrm{tan}\left(e+f\,x\right)\,2{}\mathrm{i}}{d\,\left(-c^2+c\,d\,2{}\mathrm{i}+d^2\right)}}{f\,\left({\mathrm{tan}\left(e+f\,x\right)}^2+\frac{c^2}{d^2}+\frac{2\,c\,\mathrm{tan}\left(e+f\,x\right)}{d}\right)}+\frac{a^2\,\mathrm{atan}\left(\frac{c^3-c^2\,d\,1{}\mathrm{i}+c\,d^2-d^3\,1{}\mathrm{i}}{{\left(c-d\,1{}\mathrm{i}\right)}^2\,\left(d+c\,1{}\mathrm{i}\right)}-\frac{\mathrm{tan}\left(e+f\,x\right)\,\left(2\,c^8\,d^2+8\,c^6\,d^4+12\,c^4\,d^6+8\,c^2\,d^8+2\,d^{10}\right)\,1{}\mathrm{i}}{{\left(c-d\,1{}\mathrm{i}\right)}^2\,\left(d+c\,1{}\mathrm{i}\right)\,\left(-c^6\,d\,1{}\mathrm{i}+2\,c^5\,d^2-c^4\,d^3\,1{}\mathrm{i}+4\,c^3\,d^4+c^2\,d^5\,1{}\mathrm{i}+2\,c\,d^6+d^7\,1{}\mathrm{i}\right)}\right)\,4{}\mathrm{i}}{f\,{\left(c-d\,1{}\mathrm{i}\right)}^2\,\left(d+c\,1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^2/(c + d*tan(e + f*x))^3,x)
```

```
[Out] (a^2*atan((c*d^2 - c^2*d*1i + c^3 - d^3*1i)/((c - d*1i)^2*(c*1i + d)) - (ta
n(e + f*x)*(2*d^10 + 8*c^2*d^8 + 12*c^4*d^6 + 8*c^6*d^4 + 2*c^8*d^2)*1i)/((
c - d*1i)^2*(c*1i + d)*(2*c*d^6 - c^6*d*1i + d^7*1i + c^2*d^5*1i + 4*c^3*d^
4 - c^4*d^3*1i + 2*c^5*d^2)))*4i)/(f*(c - d*1i)^2*(c*1i + d)) - ((a^2*c^2 +
 a^2*d^2 + a^2*c*d*4i)/(2*d^3*(c*d*2i - c^2 + d^2)) + (a^2*tan(e + f*x)*2i)
/(d*(c*d*2i - c^2 + d^2)))/(f*(tan(e + f*x)^2 + c^2/d^2 + (2*c*tan(e + f*x)
)/d))
```

3.1097 $\int \frac{a+ia\tan(e+fx)}{(c+d\tan(e+fx))^3}\,dx$

Optimal. Leaf size=104

$$\frac{ax}{(c-id)^3}-\frac{a\log(c\cos(e+fx)+d\sin(e+fx))}{(ic+d)^3f}-\frac{a}{2(ic+d)f(c+d\tan(e+fx))^2}+\frac{ia}{(c-id)^2f(c+d\tan(e+$$

[Out] a*x/(c-I*d)^3-a*ln(c*cos(f*x+e)+d*sin(f*x+e))/(I*c+d)^3/f-1/2*a/(I*c+d)/f/(c+d*tan(f*x+e))^2+I*a/(c-I*d)^2/f/(c+d*tan(f*x+e))

Rubi [A]
time = 0.15, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3610, 3612, 3611}

$$\frac{ia}{f(c-id)^2(c+d\tan(e+fx))}-\frac{a}{2f(d+ic)(c+d\tan(e+fx))^2}-\frac{a\log(c\cos(e+fx)+d\sin(e+fx))}{f(d+ic)^3}+\frac{ax}{(c-id)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])/(c + d*Tan[e + f*x])^3,x]
```

```
[Out] (a*x)/(c - I*d)^3 - (a*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((I*c + d)^3*f
) - a/(2*(I*c + d)*f*(c + d*Tan[e + f*x])^2) + (I*a)/((c - I*d)^2*f*(c + d*
Tan[e + f*x]))
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
```

```
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{a+ia\tan(e+fx)}{(c+d\tan(e+fx))^3}\,dx = -\frac{a}{2(ic+d)f(c+d\tan(e+fx))^2}+\frac{\int \frac{a(c+id)+a(ic-d)\tan(e+fx)}{(c+d\tan(e+fx))^2}\,dx}{c^2+d^2}$$
$$= -\frac{a}{2(ic+d)f(c+d\tan(e+fx))^2}+\frac{ia}{(c-id)^2f(c+d\tan(e+fx))}+\frac{\int \frac{a(c+id)^2+}{c+}}{(}$$
$$= \frac{ax}{(c-id)^3}-\frac{a}{2(ic+d)f(c+d\tan(e+fx))^2}+\frac{ia}{(c-id)^2f(c+d\tan(e+fx))}-$$
$$= \frac{ax}{(c-id)^3}-\frac{a\log(c\cos(e+fx)+d\sin(e+fx))}{(ic+d)^3f}-\frac{a}{2(ic+d)f(c+d\tan(e+fx}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 315 vs. 2(104) = 208.
time = 3.59, size = 315, normalized size = 3.03

$$\frac{\cos(e+fx)(\cos(fx)-i\sin(fx))\left(2x(\cos(e)-i\sin(e))-\frac{\text{ArcTan}\left(\frac{(-3c^2d+d^3)\cos(2e+fx)+c(c^2-3d^2)\sin(2e+fx)}{c(c^2-3d^2)\cos(2e+fx)-d(-3c^2+d^2)\sin(2e+fx)}\right)(\cos(e)-i\sin(e))}{f}-\frac{i\log((c\cos(e+fx)+d\sin(e+fx))^2)(\cos(e)-i\sin(e))}{2f}+\frac{(c-id)d^2(i\cos(e)+\sin(e))}{2(c+id)f(c\cos(e+fx)+d\sin(e+fx))^2}+\frac{(c-id)d(-2ic+d)(\cos(e)-i\sin(e))\sin(fx)}{(c+id)f(c\cos(e)+d\sin(e))(c\cos(e+fx)+d\sin(e+fx))}\right)(a+ia\tan(e+fx))}{(c-id)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])/(c + d*Tan[e + f*x])^3,x]
```

```
[Out] (Cos[e + f*x]*(Cos[f*x] - I*Sin[f*x])*(2*x*(Cos[e] - I*Sin[e]) - (ArcTan[((
-3*c^2*d + d^3)*Cos[2*e + f*x] + c*(c^2 - 3*d^2)*Sin[2*e + f*x])/(c*(c^2 -
3*d^2)*Cos[2*e + f*x] - d*(-3*c^2 + d^2)*Sin[2*e + f*x])]*(Cos[e] - I*Sin[e
]))/f - ((I/2)*Log[(c*Cos[e + f*x] + d*Sin[e + f*x])^2]*(Cos[e] - I*Sin[e])
)/f + ((c - I*d)*d^2*(I*Cos[e] + Sin[e]))/(2*(c + I*d)*f*(c*Cos[e + f*x] +
d*Sin[e + f*x])^2) + ((c - I*d)*d*((-2*I)*c + d)*(Cos[e] - I*Sin[e])*Sin[f*
x])/((c + I*d)*f*(c*Cos[e] + d*Sin[e])*(c*Cos[e + f*x] + d*Sin[e + f*x])))*
(a + I*a*Tan[e + f*x]))/(c - I*d)^3
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(97) = 194.
time = 0.28, size = 200, normalized size = 1.92

method	result
derivativedivides	$\frac{a\left(\frac{\frac{(ic^3-3icd^2-3c^2d+d^3)\ln(1+\tan^2(fx+e))}{2}+(3ic^2d-id^3+c^3-3cd^2)\arctan(\tan(fx+e))}{(c^2+d^2)^3}+\frac{ic^2-id^2-2cd}{(c^2+d^2)^2(c+d\tan(fx+e))}-\frac{(ic^3-3ic}{}\right.}{f}$

default	$a\left(\frac{\frac{\left(ic^3-3ic\,d^2-3c^2d+d^3\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(3ic^2d-id^3+c^3-3c\,d^2\right)\arctan(\tan(fx+e))}{\left(c^2+d^2\right)^3}+\frac{ic^2-id^2-2cd}{\left(c^2+d^2\right)^2(c+d\tan(fx+e))}-\frac{\left(ic^3-3\right.}{}\right.$ $\overline{\qquad f \qquad}$
risch	$-\frac{2ax}{3ic^2d-id^3-c^3+3c\,d^2}-\frac{2iax}{ic^3-3ic\,d^2+3c^2d-d^3}-\frac{2iae}{f(ic^3-3ic\,d^2+3c^2d-d^3)}+\frac{-4ia\,d^2\mathrm{e}^{2i(fx+e)}+4acd\,\mathrm{e}^{2i(fx+e)}+}{\left(-i\mathrm{e}^{2i(fx+e)}d+id+\mathrm{e}^{2i(fx+e)}c+c\right)^2}$
norman	$\frac{\frac{a\,c^2x}{(-id+c)\left(-2icd+c^2-d^2\right)}+\frac{2iacd+a\,d^2}{2f\left(-2icd+c^2-d^2\right)d}+\frac{id^2a\left(\tan^2(fx+e)\right)}{2\left(2icd-c^2+d^2\right)fc}+\frac{2cdax\tan(fx+e)}{(-id+c)\left(-2icd+c^2-d^2\right)}-\frac{ia\,d^2x\left(\tan^2(fx+e)\right)}{(ic+d)\left(2icd-c^2+d^2\right)}}{(c+d\tan(fx+e))^2}+\frac{i}{2f(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a*(1/(c^2+d^2)^3*(1/2*(I*c^3-3*I*c*d^2-3*c^2*d+d^3)*ln(1+tan(f*x+e)^2)+
(3*I*c^2*d-I*d^3+c^3-3*c*d^2)*arctan(tan(f*x+e)))+(I*c^2-I*d^2-2*c*d)/(c^2+
d^2)^2/(c+d*tan(f*x+e))-(I*c^3-3*I*c*d^2-3*c^2*d+d^3)/(c^2+d^2)^3*ln(c+d*ta
n(f*x+e))+1/2*(I*c-d)/(c^2+d^2)/(c+d*tan(f*x+e))^2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(96) = 192$.
time = 0.54, size = 331, normalized size = 3.18

$$\frac{\frac{2\left(ac^3+3i\,ac^2d-3\,acd^2-i\,ad^3\right)(fx+e)}{c^6+3\,c^4d^2+3\,c^2d^4+d^6}+\frac{2\left(-i\,ac^3+3\,ac^2d+3i\,acd^2-ad^3\right)\log(d\tan(fx+e)+c)}{c^6+3\,c^4d^2+3\,c^2d^4+d^6}+\frac{\left(i\,ac^3-3\,ac^2d-3i\,acd^2+ad^3\right)\log\left(\tan(fx+e)^2+1\right)}{c^6+3\,c^4d^2+3\,c^2d^4+d^6}+\frac{3i\,ac^3-5\,ac^2d-i\,acd^2-ad^3+2\left(i\,ac^2d-2\,acd^2-i\,ad^3\right)\tan(fx+e)}{c^6+2\,c^4d^2+c^2d^4+\left(c^4d^2+2\,c^2d^4+d^6\right)\tan(fx+e)^2+2\left(c^5d+2\,c^3d^3+cd^5\right)\tan(fx+e)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a*c^3 + 3*I*a*c^2*d - 3*a*c*d^2 - I*a*d^3)*(f*x + e)/(c^6 + 3*c^4*d
^2 + 3*c^2*d^4 + d^6) + 2*(-I*a*c^3 + 3*a*c^2*d + 3*I*a*c*d^2 - a*d^3)*log(
d*tan(f*x + e) + c)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) + (I*a*c^3 - 3*a*c^
2*d - 3*I*a*c*d^2 + a*d^3)*log(tan(f*x + e)^2 + 1)/(c^6 + 3*c^4*d^2 + 3*c^2
*d^4 + d^6) + (3*I*a*c^3 - 5*a*c^2*d - I*a*c*d^2 - a*d^3 + 2*(I*a*c^2*d - 2
*a*c*d^2 - I*a*d^3)*tan(f*x + e))/(c^6 + 2*c^4*d^2 + c^2*d^4 + (c^4*d^2 + 2
*c^2*d^4 + d^6)*tan(f*x + e)^2 + 2*(c^5*d + 2*c^3*d^3 + c*d^5)*tan(f*x + e)
))/f
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(96) = 192$.
time = 1.48, size = 278, normalized size = 2.67

$$\frac{4i\,acd-2\,ad^2-4\left(-i\,acd-ad^2\right)e^{(2i\,fx+2i\,e)}+\left(ac^2+2i\,acd-ad^2+\left(ac^2-2i\,acd-ad^2\right)e^{(4i\,fx+4i\,e)}+2\left(ac^2+ad^2\right)e^{(2i\,fx+2i\,e)}\right)\log\left(\frac{(i\,c+d)e^{(2i\,fx+2i\,e)}+i\,c-d}{i\,c+d}\right)}{\left(i\,c^5+5\,c^4d-10i\,c^3d^2-10\,c^2d^3+5i\,cd^4+d^5\right)fe^{(4i\,fx+4i\,e)}-2\left(-i\,c^5-3\,c^4d+2i\,c^3d^2-2\,c^2d^3+3i\,cd^4+d^5\right)fe^{(2i\,fx+2i\,e)}+\left(i\,c^5+c^4d+2i\,c^3d^2+2\,c^2d^3+i\,cd^4+d^5\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] (4*I*a*c*d - 2*a*d^2 - 4*(-I*a*c*d - a*d^2)*e^(2*I*f*x + 2*I*e) + (a*c^2 +
2*I*a*c*d - a*d^2 + (a*c^2 - 2*I*a*c*d - a*d^2)*e^(4*I*f*x + 4*I*e) + 2*(a*
c^2 + a*d^2)*e^(2*I*f*x + 2*I*e))*log(((I*c + d)*e^(2*I*f*x + 2*I*e) + I*c
- d)/(I*c + d)))/((I*c^5 + 5*c^4*d - 10*I*c^3*d^2 - 10*c^2*d^3 + 5*I*c*d^4
+ d^5)*f*e^(4*I*f*x + 4*I*e) - 2*(-I*c^5 - 3*c^4*d + 2*I*c^3*d^2 - 2*c^2*d^
3 + 3*I*c*d^4 + d^5)*f*e^(2*I*f*x + 2*I*e) + (I*c^5 + c^4*d + 2*I*c^3*d^2 +
 2*c^2*d^3 + I*c*d^4 + d^5)*f)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(82) = 164.
time = 4.05, size = 354, normalized size = 3.40

$$-\frac{ia\log\left(\frac{c+id}{ce^{2ie}-ide^{2ie}}+e^{2ifx}\right)}{f(c-id)^3}+\frac{4acd+2iad^2+\left(4acde^{2ie}-4iad^2e^{2ie}\right)e^{2ifx}}{c^5f-ic^4df+2c^3d^2f-2ic^2d^3f+cd^4f-id^5f+\left(2c^5fe^{2ie}-6ic^4dfe^{2ie}-4c^3d^2fe^{2ie}-4ic^2d^3fe^{2ie}-6cd^4fe^{2ie}+2id^5fe^{2ie}\right)e^{2ifx}+\left(c^5fe^{4ie}-5ic^4dfe^{4ie}-10c^3d^2fe^{4ie}+10ic^2d^3fe^{4ie}+5cd^4fe^{4ie}-id^5fe^{4ie}\right)e^{4ifx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))**3,x)
```

```
[Out] -I*a*log((c + I*d)/(c*exp(2*I*e) - I*d*exp(2*I*e)) + exp(2*I*f*x))/(f*(c -
I*d)**3) + (4*a*c*d + 2*I*a*d**2 + (4*a*c*d*exp(2*I*e) - 4*I*a*d**2*exp(2*I
*e))*exp(2*I*f*x))/(c**5*f - I*c**4*d*f + 2*c**3*d**2*f - 2*I*c**2*d**3*f +
 c*d**4*f - I*d**5*f + (2*c**5*f*exp(2*I*e) - 6*I*c**4*d*f*exp(2*I*e) - 4*c
**3*d**2*f*exp(2*I*e) - 4*I*c**2*d**3*f*exp(2*I*e) - 6*c*d**4*f*exp(2*I*e)
+ 2*I*d**5*f*exp(2*I*e))*exp(2*I*f*x) + (c**5*f*exp(4*I*e) - 5*I*c**4*d*f*e
xp(4*I*e) - 10*c**3*d**2*f*exp(4*I*e) + 10*I*c**2*d**3*f*exp(4*I*e) + 5*c*d
**4*f*exp(4*I*e) - I*d**5*f*exp(4*I*e))*exp(4*I*f*x))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(96) = 192.
time = 0.65, size = 364, normalized size = 3.50

$$\frac{2\left(\frac{a\log\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-c\right)}{2ic^3+6c^2d-6icd^2-2d^3}-\frac{a\log\left(-i\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{ic^3+3c^2d-3icd^2-d^3}+\frac{3ac^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-4ac^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-12iac^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-4acd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-6ac^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+16iacd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4ad^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4ac^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+12iac^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+4acd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+3ac^4}{-4\left(ic^5+3c^4d-3ic^3d^2-c^2d^3\right)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-c\right)^2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 2*(a*log(c*tan(1/2*f*x + 1/2*e)^2 - 2*d*tan(1/2*f*x + 1/2*e) - c)/(2*I*c^3
+ 6*c^2*d - 6*I*c*d^2 - 2*d^3) - a*log(-I*tan(1/2*f*x + 1/2*e) + 1)/(I*c^3
+ 3*c^2*d - 3*I*c*d^2 - d^3) + (3*a*c^4*tan(1/2*f*x + 1/2*e)^4 - 4*a*c^3*d*
tan(1/2*f*x + 1/2*e)^3 - 12*I*a*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - 4*a*c*d^3*
tan(1/2*f*x + 1/2*e)^3 - 6*a*c^4*tan(1/2*f*x + 1/2*e)^2 + 16*I*a*c*d^3*tan(
1/2*f*x + 1/2*e)^2 + 4*a*d^4*tan(1/2*f*x + 1/2*e)^2 + 4*a*c^3*d*tan(1/2*f*x
 + 1/2*e) + 12*I*a*c^2*d^2*tan(1/2*f*x + 1/2*e) + 4*a*c*d^3*tan(1/2*f*x + 1
/2*e) + 3*a*c^4)/((-4*I*c^5 - 12*c^4*d + 12*I*c^3*d^2 + 4*c^2*d^3)*(c*tan(1
/2*f*x + 1/2*e)^2 - 2*d*tan(1/2*f*x + 1/2*e) - c)^2))/f
```

Mupad [B]
time = 5.37, size = 281, normalized size = 2.70

$$-\frac{\frac{(3\,a\,c-a\,d\,1\mathrm{i})\,1\mathrm{i}}{2\,d^2\,(-c^2+c\,d\,2\mathrm{i}+d^2)}+\frac{a\tan(e+f\,x)\,1\mathrm{i}}{d\,(-c^2+c\,d\,2\mathrm{i}+d^2)}}{f\,\left(\tan\left(e+f\,x\right)^2+\frac{c^2}{d^2}+\frac{2\,c\tan(e+f\,x)}{d}\right)}+\frac{a\,\mathrm{atan}\left(\frac{c^3-c^2\,d\,1\mathrm{i}+c\,d^2-d^3\,1\mathrm{i}}{(c-d\,1\mathrm{i})^2\,(d+c\,1\mathrm{i})}-\frac{\tan(e+f\,x)\,\left(2\,c^8\,d^2+8\,c^6\,d^4+12\,c^4\,d^6+8\,c^2\,d^8+2\,d^{10}\right)\,1\mathrm{i}}{(c-d\,1\mathrm{i})^2\,(d+c\,1\mathrm{i})\,(-c^6\,d\,1\mathrm{i}+2\,c^5\,d^2-c^4\,d^3\,1\mathrm{i}+4\,c^3\,d^4+c^2\,d^5\,1\mathrm{i}+2\,c\,d^6+d^7\,1\mathrm{i})}\right)\,2\mathrm{i}}{f\,\left(c-d\,1\mathrm{i}\right)^2\,\left(d+c\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)/(c + d*tan(e + f*x))^3,x)
```

```
[Out] (a*atan((c*d^2 - c^2*d*1i + c^3 - d^3*1i)/((c - d*1i)^2*(c*1i + d)) - (tan(
e + f*x)*(2*d^10 + 8*c^2*d^8 + 12*c^4*d^6 + 8*c^6*d^4 + 2*c^8*d^2)*1i)/((c
- d*1i)^2*(c*1i + d)*(2*c*d^6 - c^6*d*1i + d^7*1i + c^2*d^5*1i + 4*c^3*d^4
- c^4*d^3*1i + 2*c^5*d^2)))*2i)/(f*(c - d*1i)^2*(c*1i + d)) - (((3*a*c - a*
d*1i)*1i)/(2*d^2*(c*d*2i - c^2 + d^2)) + (a*tan(e + f*x)*1i)/(d*(c*d*2i - c
^2 + d^2)))/(f*(tan(e + f*x)^2 + c^2/d^2 + (2*c*tan(e + f*x))/d))
```

3.1098 $\int \frac{1}{(a+ia\tan(e+fx))(c+d\tan(e+fx))^3}\,dx$

Optimal. Leaf size=273

$$\frac{(c^4+4ic^3d+6c^2d^2-12icd^3-3d^4)\,x}{2a(c-id)^3(c+id)^4}+\frac{2d^2(3c^2-2icd-d^2)\log(c\cos(e+fx)+d\sin(e+fx))}{a(c+id)^4(ic+d)^3f}+\frac{}{2a(c-id)(}$$

```
[Out] 1/2*(c^4+4*I*c^3*d+6*c^2*d^2-12*I*c*d^3-3*d^4)*x/a/(c-I*d)^3/(c+I*d)^4+2*d^
2*(3*c^2-2*I*c*d-d^2)*ln(c*cos(f*x+e)+d*sin(f*x+e))/a/(c+I*d)^4/(I*c+d)^3/f
+1/2*(c-2*I*d)*d/a/(c-I*d)/(c+I*d)^2/f/(c+d*tan(f*x+e))^2-1/2/(I*c-d)/f/(a+
I*a*tan(f*x+e))/(c+d*tan(f*x+e))^2+1/2*d*(c^2-8*I*c*d-3*d^2)/a/(c-I*d)^2/(c
+I*d)^3/f/(c+d*tan(f*x+e))
```

Rubi [A]

time = 0.33, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3633, 3610, 3612, 3611}

$$\frac{d(c^2-8icd-3d^2)}{2af(c-id)^2(c+id)^3(c+d\tan(e+fx))}+\frac{2d^2(3c^2-2icd-d^2)\log(c\cos(e+fx)+d\sin(e+fx))}{af(c+id)^4(d+ic)^3}+\frac{x(c^4+4ic^3d+6c^2d^2-12icd^3-3d^4)}{2a(c-id)^3(c+id)^4}+\frac{d(c-2id)}{2af(c-id)(c+id)^2(c+d\tan(e+fx))^2}-\frac{1}{2f(-d+ic)(a+ia\tan(e+fx))(c+d\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])*(c + d*Tan[e + f*x])^3),x]
```

```
[Out] ((c^4 + (4*I)*c^3*d + 6*c^2*d^2 - (12*I)*c*d^3 - 3*d^4)*x)/(2*a*(c - I*d)^3
*(c + I*d)^4) + (2*d^2*(3*c^2 - (2*I)*c*d - d^2)*Log[c*Cos[e + f*x] + d*Sin
[e + f*x]])/(a*(c + I*d)^4*(I*c + d)^3*f) + ((c - (2*I)*d)*d)/(2*a*(c - I*d
)*(c + I*d)^2*f*(c + d*Tan[e + f*x])^2) - 1/(2*(I*c - d)*f*(a + I*a*Tan[e +
 f*x])*(c + d*Tan[e + f*x])^2) + (d*(c^2 - (8*I)*c*d - 3*d^2))/(2*a*(c - I*
d)^2*(c + I*d)^3*f*(c + d*Tan[e + f*x]))
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3633

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-a)*((c + d*Tan[e + f*x])^(n + 1)/(2*f*(b*
c - a*d)*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*
Tan[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &
& NeQ[c^2 + d^2, 0] && !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))(c+d\tan(e+fx))^3}\,dx = -\frac{1}{2(ic-d)f(a+ia\tan(e+fx))(c+d\tan(e+fx))^2} + \cdots$$

$$= \frac{(c-2id)d}{2a(c-id)(c+id)^2f(c+d\tan(e+fx))^2} - \frac{\cdots}{2(ic-d)f(a\cdots}$$

$$= \frac{(c-2id)d}{2a(c-id)(c+id)^2f(c+d\tan(e+fx))^2} - \frac{\cdots}{2(ic-d)f(a\cdots}$$

$$= \frac{(c^4+4ic^3d+6c^2d^2-12icd^3-3d^4)\,x}{2a(c-id)^3(c+id)^4} + \frac{(\cdots}{2a(c-id)(c+i\cdots}$$

$$= \frac{(c^4+4ic^3d+6c^2d^2-12icd^3-3d^4)\,x}{2a(c-id)^3(c+id)^4} + \frac{2d^2(3c^2-2icd-\cdots}{\cdots}$$

Mathematica [**A**]
time = 7.46, size = 474, normalized size = 1.74

$$\frac{\sec(e+fx)(\cos(fx)+i\sin(fx))\left(\frac{8d^2(-3c^2+2icd+d^2)\mathrm{ArcTan}\left(\frac{d\cos(fx)+c\sin(fx)}{-c\cos(fx)+d\sin(fx)}\right)\left(\cos\left(\frac{e}{2}\right)+i\sin\left(\frac{e}{2}\right)\right)^2}{(c-id)^3f}+\frac{4d^2(3ic^2+2cd-id^2)\log\left((c\cos(e+fx)+d\sin(e+fx))^2\right)\left(\cos\left(\frac{e}{2}\right)+i\sin\left(\frac{e}{2}\right)\right)^2}{(c-id)^3f}+\frac{8d^2(-3c^2+2icd+d^2)x(\cos(e)+i\sin(e))}{(c-id)^3}+\frac{2(c^4+4ic^3d+6c^2d^2-12icd^3-3d^4)x(\cos(e)+i\sin(e))}{(c-id)^3}+\frac{(c+id)\cos(2fx)(i\cos(e)+\sin(e))}{f}+\frac{(c+id)(\cos(e)-i\sin(e))\sin(2fx)}{f}+\frac{2(c+id)d^4(-i\cos(e)+\sin(e))}{(c-id)^2f(c\cos(e+fx)+d\sin(e+fx))^2}+\frac{4(c+id)d^3(4ic+d)(\cos(e)+i\sin(e))\sin(fx)}{(c-id)^2f(c\cos(e)+d\sin(e))(c\cos(e+fx)+d\sin(e+fx))}\right)}{4(c+id)^4(a+ia\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])*(c + d*Tan[e + f*x])^3),x]
```

```
[Out] (Sec[e + f*x]*(Cos[f*x] + I*Sin[f*x])*((8*d^2*(-3*c^2 + (2*I)*c*d + d^2)*Ar
cTan[(d*Cos[f*x] + c*Sin[f*x])/(-(c*Cos[f*x]) + d*Sin[f*x])]*(Cos[e/2] + I*
Sin[e/2])^2)/((c - I*d)^3*f) + (4*d^2*((3*I)*c^2 + 2*c*d - I*d^2)*Log[(c*Co
```

s[e + f*x] + d*Sin[e + f*x])^2]*(Cos[e/2] + I*Sin[e/2])^2)/((c - I*d)^3*f)
+ (8*d^2*(-3*c^2 + (2*I)*c*d + d^2)*x*(Cos[e] + I*Sin[e]))/(c - I*d)^3 + (2
*(c^4 + (4*I)*c^3*d + 6*c^2*d^2 - (12*I)*c*d^3 - 3*d^4)*x*(Cos[e] + I*Sin[e
]))/(c - I*d)^3 + ((c + I*d)*Cos[2*f*x]*(I*Cos[e] + Sin[e]))/f + ((c + I*d)
*(Cos[e] - I*Sin[e])*Sin[2*f*x])/f + (2*(c + I*d)*d^4*((-I)*Cos[e] + Sin[e]
))/((c - I*d)^2*f*(c*Cos[e + f*x] + d*Sin[e + f*x])^2) + (4*(c + I*d)*d^3*(
(4*I)*c + d)*(Cos[e] + I*Sin[e])*Sin[f*x])/((c - I*d)^2*f*(c*Cos[e] + d*Sin
[e])*(c*Cos[e + f*x] + d*Sin[e + f*x]))))/(4*(c + I*d)^4*(a + I*a*Tan[e + f
*x]))

Maple [A]
time = 0.64, size = 239, normalized size = 0.88 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a*(-2*d^2*(3*I*c^2-I*d^2+2*c*d)/(I*d-c)^3/(c+I*d)^4*ln(c+d*tan(f*x+e))+
1/2*I*d^2*(c^4+2*c^2*d^2+d^4)/(I*d-c)^3/(c+I*d)^4/(c+d*tan(f*x+e))^2+d^2*(3
*I*c^3+3*I*c*d^2+c^2*d+d^3)/(I*d-c)^3/(c+I*d)^4/(c+d*tan(f*x+e))-1/4*I/(I*d
-c)^3*ln(tan(f*x+e)+I)+1/2/(c+I*d)^3/(tan(f*x+e)-I)+1/4/(c+I*d)^4*(-I*c+7*d
)*ln(tan(f*x+e)-I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 719 vs. 2(241) = 482.
time = 1.12, size = 719, normalized size = 2.63

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6 + 2*(-I*c^6 + 2*c^5*d - 25*I*c^4*d^2
 - 60*c^3*d^3 + 65*I*c^2*d^4 + 34*c*d^5 - 7*I*d^6)*f*x*e^(6*I*f*x + 6*I*e)
+ (c^6 - 4*I*c^5*d - 5*c^4*d^2 + 32*I*c^3*d^3 - 5*c^2*d^4 + 36*I*c*d^5 + d^
6 + 4*(-I*c^6 + 4*c^5*d - 19*I*c^4*d^2 - 16*c^3*d^3 - 11*I*c^2*d^4 - 20*c*d
```

```
^5 + 7*I*d^6)*f*x)*e^(4*I*f*x + 4*I*e) + 2*(c^6 - 2*I*c^5*d + c^4*d^2 + 12*
I*c^3*d^3 - 29*c^2*d^4 - 10*I*c*d^5 - 5*d^6 + (-I*c^6 + 6*c^5*d - 9*I*c^4*d
^2 + 12*c^3*d^3 - 15*I*c^2*d^4 + 6*c*d^5 - 7*I*d^6)*f*x)*e^(2*I*f*x + 2*I*e
) + 8*((3*c^4*d^2 - 8*I*c^3*d^3 - 8*c^2*d^4 + 4*I*c*d^5 + d^6)*e^(6*I*f*x +
 6*I*e) + 2*(3*c^4*d^2 - 2*I*c^3*d^3 + 2*c^2*d^4 - 2*I*c*d^5 - d^6)*e^(4*I*
f*x + 4*I*e) + (3*c^4*d^2 + 4*I*c^3*d^3 + d^6)*e^(2*I*f*x + 2*I*e))*log(((I
*c + d)*e^(2*I*f*x + 2*I*e) + I*c - d)/(I*c + d)))/((-I*a*c^9 - a*c^8*d - 4
*I*a*c^7*d^2 - 4*a*c^6*d^3 - 6*I*a*c^5*d^4 - 6*a*c^4*d^5 - 4*I*a*c^3*d^6 -
4*a*c^2*d^7 - I*a*c*d^8 - a*d^9)*f*e^(6*I*f*x + 6*I*e) + 2*(-I*a*c^9 + a*c^
8*d - 4*I*a*c^7*d^2 + 4*a*c^6*d^3 - 6*I*a*c^5*d^4 + 6*a*c^4*d^5 - 4*I*a*c^3
*d^6 + 4*a*c^2*d^7 - I*a*c*d^8 + a*d^9)*f*e^(4*I*f*x + 4*I*e) + (-I*a*c^9 +
 3*a*c^8*d + 8*a*c^6*d^3 + 6*I*a*c^5*d^4 + 6*a*c^4*d^5 + 8*I*a*c^3*d^6 + 3*
I*a*c*d^8 - a*d^9)*f*e^(2*I*f*x + 2*I*e))
```

Sympy [A]
time = 54.15, size = 821, normalized size = 3.01

$$\frac{x(c+7id)}{2ac^4+8iac^3d-12ac^2d^2-8iacd^3+2ad^4}+\frac{-8c^2d^3-6icd^4-2d^5+\left(-8c^2d^3e^{2ie}+8icd^4e^{2ie}\right)e^{2ifx}}{ac^8f+2iac^7df+2ac^6d^2f+6iac^5d^3f+6iac^3d^5f-2ac^2d^6f+2iacd^7f-ad^8f+\left(2ac^8fe^{2ie}+8ac^6d^2fe^{2ie}+12ac^4d^4fe^{2ie}+8ac^2d^6fe^{2ie}+2ad^8fe^{2ie}\right)e^{2ifx}+\left(ac^8fe^{4ie}-2iac^7dfe^{4ie}+2ac^6d^2fe^{4ie}-6iac^5d^3fe^{4ie}-6iac^3d^5fe^{4ie}-2ac^2d^6fe^{4ie}-2iacd^7fe^{4ie}-ad^8fe^{4ie}\right)e^{4ifx}}+\begin{cases}\frac{ie^{-2ifx}}{4ac^3fe^{2ie}+12iac^2dfe^{2ie}-12acd^2fe^{2ie}-4iad^3fe^{2ie}} & \text{for}\ 4ac^3fe^{2ie}+12iac^2dfe^{2ie}-12acd^2fe^{2ie}-4iad^3fe^{2ie}\neq 0\\ x\left(-\frac{c+7id}{2ac^4+8iac^3d-12ac^2d^2-8iacd^3+2ad^4}+\frac{ce^{2ie}+c+7ide^{2ie}+id}{2ac^4e^{2ie}+8iac^3de^{2ie}-12ac^2d^2e^{2ie}-8iacd^3e^{2ie}+2ad^4e^{2ie}}\right) & \text{otherwise}\end{cases}+\frac{2id^2\cdot\left(3c^2-2icd-d^2\right)\log\left(\frac{c+id}{ce^{2ie}-ide^{2ie}}+e^{2ifx}\right)}{af\left(c-id\right)^3\left(c+id\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))**3,x)
```

```
[Out] x*(c + 7*I*d)/(2*a*c**4 + 8*I*a*c**3*d - 12*a*c**2*d**2 - 8*I*a*c*d**3 + 2*
a*d**4) + (-8*c**2*d**3 - 6*I*c*d**4 - 2*d**5 + (-8*c**2*d**3*exp(2*I*e) +
8*I*c*d**4*exp(2*I*e))*exp(2*I*f*x))/(a*c**8*f + 2*I*a*c**7*d*f + 2*a*c**6*
d**2*f + 6*I*a*c**5*d**3*f + 6*I*a*c**3*d**5*f - 2*a*c**2*d**6*f + 2*I*a*c*
d**7*f - a*d**8*f + (2*a*c**8*f*exp(2*I*e) + 8*a*c**6*d**2*f*exp(2*I*e) + 1
2*a*c**4*d**4*f*exp(2*I*e) + 8*a*c**2*d**6*f*exp(2*I*e) + 2*a*d**8*f*exp(2*
I*e))*exp(2*I*f*x) + (a*c**8*f*exp(4*I*e) - 2*I*a*c**7*d*f*exp(4*I*e) + 2*a
*c**6*d**2*f*exp(4*I*e) - 6*I*a*c**5*d**3*f*exp(4*I*e) - 6*I*a*c**3*d**5*f*
exp(4*I*e) - 2*a*c**2*d**6*f*exp(4*I*e) - 2*I*a*c*d**7*f*exp(4*I*e) - a*d**
8*f*exp(4*I*e))*exp(4*I*f*x)) + Piecewise((I*exp(-2*I*f*x)/(4*a*c**3*f*exp(
2*I*e) + 12*I*a*c**2*d*f*exp(2*I*e) - 12*a*c*d**2*f*exp(2*I*e) - 4*I*a*d**3
*f*exp(2*I*e)), Ne(4*a*c**3*f*exp(2*I*e) + 12*I*a*c**2*d*f*exp(2*I*e) - 12*
a*c*d**2*f*exp(2*I*e) - 4*I*a*d**3*f*exp(2*I*e), 0)), (x*(-(c + 7*I*d)/(2*a
*c**4 + 8*I*a*c**3*d - 12*a*c**2*d**2 - 8*I*a*c*d**3 + 2*a*d**4) + (c*exp(2
*I*e) + c + 7*I*d*exp(2*I*e) + I*d)/(2*a*c**4*exp(2*I*e) + 8*I*a*c**3*d*exp
(2*I*e) - 12*a*c**2*d**2*exp(2*I*e) - 8*I*a*c*d**3*exp(2*I*e) + 2*a*d**4*ex
p(2*I*e))), True)) + 2*I*d**2*(3*c**2 - 2*I*c*d - d**2)*log((c + I*d)/(c*ex
p(2*I*e) - I*d*exp(2*I*e)) + exp(2*I*f*x))/(a*f*(c - I*d)**3*(c + I*d)**4)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(241) = 482$.
time = 0.71, size = 496, normalized size = 1.82

$$-\frac{\frac{8\left(3c^2d^3-2icd^4-d^5\right)\log\left(d\tan\left(fx+e\right)+c\right)}{iac^7d-ac^6d^2+3iac^5d^3-3ac^4d^4+3iac^3d^5-3ac^2d^6+iacd^7-ad^8}+\frac{\left(ic-7d\right)\log\left(i\tan\left(fx+e\right)+1\right)}{ac^4+4iac^3d-6ac^2d^2-4iacd^3+ad^4}-\frac{i\log\left(-i\tan\left(fx+e\right)+1\right)}{ac^3-3iac^2d-3acd^2+iad^3}+\frac{-ic\tan\left(fx+e\right)+7d\tan\left(fx+e\right)-3c-9id}{\left(ac^4+4iac^3d-6ac^2d^2-4iacd^3+ad^4\right)\left(\tan\left(fx+e\right)-i\right)}-\frac{8\left(18c^2d^4\tan\left(fx+e\right)^2-12icd^5\tan\left(fx+e\right)^2-6d^6\tan\left(fx+e\right)^2+42c^3d^3\tan\left(fx+e\right)-26ic^2d^4\tan\left(fx+e\right)-6cd^5\tan\left(fx+e\right)-2id^6\tan\left(fx+e\right)+25c^4d^2-14ic^3d^3+2c^2d^4-2icd^5+d^6\right)}{-4\left(-iac^7+ac^6d-3iac^5d^2+3ac^4d^3-3iac^3d^4+3ac^2d^5-iacd^6+ad^7\right)\left(d\tan\left(fx+e\right)+c\right)^2}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^3,x, algorithm="giac")

[Out] -1/4*(8*(3*c^2*d^3 - 2*I*c*d^4 - d^5)*log(d*tan(f*x + e) + c)/(I*a*c^7*d - a*c^6*d^2 + 3*I*a*c^5*d^3 - 3*a*c^4*d^4 + 3*I*a*c^3*d^5 - 3*a*c^2*d^6 + I*a *c*d^7 - a*d^8) + (I*c - 7*d)*log(I*tan(f*x + e) + 1)/(a*c^4 + 4*I*a*c^3*d - 6*a*c^2*d^2 - 4*I*a*c*d^3 + a*d^4) - I*log(-I*tan(f*x + e) + 1)/(a*c^3 - 3*I*a*c^2*d - 3*a*c*d^2 + I*a*d^3) + (-I*c*tan(f*x + e) + 7*d*tan(f*x + e) - 3*c - 9*I*d)/((a*c^4 + 4*I*a*c^3*d - 6*a*c^2*d^2 - 4*I*a*c*d^3 + a*d^4)*(tan(f*x + e) - I)) - 8*(18*c^2*d^4*tan(f*x + e)^2 - 12*I*c*d^5*tan(f*x + e) ^2 - 6*d^6*tan(f*x + e)^2 + 42*c^3*d^3*tan(f*x + e) - 26*I*c^2*d^4*tan(f*x + e) - 6*c*d^5*tan(f*x + e) - 2*I*d^6*tan(f*x + e) + 25*c^4*d^2 - 14*I*c^3* d^3 + 2*c^2*d^4 - 2*I*c*d^5 + d^6)/((4*I*a*c^7 - 4*a*c^6*d + 12*I*a*c^5*d^2 - 12*a*c^4*d^3 + 12*I*a*c^3*d^4 - 12*a*c^2*d^5 + 4*I*a*c*d^6 - 4*a*d^7)*(d *tan(f*x + e) + c)^2))/f

Mupad [B]
time = 10.10, size = 1910, normalized size = 7.00

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a*tan(e + f*x)*1i)*(c + d*tan(e + f*x))^3),x)

[Out] symsum(log((a*d^4 - a*c^2*d^2 + a*c*d^3*2i)*(47*c*d^5 - c^5*d - d^6*12i + c ^2*d^4*56i - 34*c^3*d^3 + c^4*d^2*4i) - root(a^3*c^7*d^7*e^3*640i + a^3*c^9 *d^5*e^3*480i + a^3*c^5*d^9*e^3*480i + a^3*c^11*d^3*e^3*192i + a^3*c^3*d^11 *e^3*192i + 144*a^3*c^10*d^4*e^3 - 144*a^3*c^4*d^10*e^3 + 80*a^3*c^12*d^2*e ^3 + 80*a^3*c^8*d^6*e^3 - 80*a^3*c^6*d^8*e^3 - 80*a^3*c^2*d^12*e^3 + a^3*c^ 13*d*e^3*32i + a^3*c*d^13*e^3*32i - 16*a^3*d^14*e^3 + 16*a^3*c^14*e^3 - a*c ^3*d^5*e*744i - 660*a*c^2*d^6*e + 558*a*c^4*d^4*e + a*c^5*d^3*e*24i - 4*a*c ^6*d^2*e + a*c*d^7*e*264i + a*c^7*d*e*8i + 57*a*d^8*e + a*c^8*e + 38*c^2*d^ 3 - c^3*d^2*6i - c*d^4*26i - 14*d^5, e, k)*((a*d^4 - a*c^2*d^2 + a*c*d^3*2i)*(2*a*c^9 + a*d^9*6i + a*c^2*d^7*28i - 4*a*c^3*d^6 + a*c^4*d^5*48i + a*c^6 *d^3*36i + 4*a*c^7*d^2 - 2*a*c*d^8 + a*c^8*d*10i) + root(a^3*c^7*d^7*e^3*64 0i + a^3*c^9*d^5*e^3*480i + a^3*c^5*d^9*e^3*480i + a^3*c^11*d^3*e^3*192i + a^3*c^3*d^11*e^3*192i + 144*a^3*c^10*d^4*e^3 - 144*a^3*c^4*d^10*e^3 + 80*a^ 3*c^12*d^2*e^3 + 80*a^3*c^8*d^6*e^3 - 80*a^3*c^6*d^8*e^3 - 80*a^3*c^2*d^12* e^3 + a^3*c^13*d*e^3*32i + a^3*c*d^13*e^3*32i - 16*a^3*d^14*e^3 + 16*a^3*c^ 14*e^3 - a*c^3*d^5*e*744i - 660*a*c^2*d^6*e + 558*a*c^4*d^4*e + a*c^5*d^3*e *24i - 4*a*c^6*d^2*e + a*c*d^7*e*264i + a*c^7*d*e*8i + 57*a*d^8*e + a*c^8*e + 38*c^2*d^3 - c^3*d^2*6i - c*d^4*26i - 14*d^5, e, k)*((a*d^4 - a*c^2*d^2 + a*c*d^3*2i)*(32*a^2*c^11*d - 32*a^2*c*d^11 + a^2*c^2*d^10*64i - 96*a^2*c^ 3*d^9 + a^2*c^4*d^8*256i - 64*a^2*c^5*d^7 + a^2*c^6*d^6*384i + 64*a^2*c^7*d ^5 + a^2*c^8*d^4*256i + 96*a^2*c^9*d^3 + a^2*c^10*d^2*64i) - tan(e + f*x)*(

a*d^4 - a*c^2*d^2 + a*c*d^3*2i)*(8*a^2*c^12 + 24*a^2*d^12 - a^2*c*d^11*48i + a^2*c^11*d*16i + 64*a^2*c^2*d^10 - a^2*c^3*d^9*176i + 24*a^2*c^4*d^8 - a^2*c^5*d^7*224i - 64*a^2*c^6*d^6 - a^2*c^7*d^5*96i - 56*a^2*c^8*d^4 + a^2*c^9*d^3*16i)) + tan(e + f*x)*(a*d^4 - a*c^2*d^2 + a*c*d^3*2i)*(24*a*d^9 + 44*a*c^2*d^7 + a*c^3*d^6*68i + 20*a*c^4*d^5 + a*c^5*d^4*100i + 4*a*c^6*d^3 + a*c^7*d^2*44i + a*c*d^8*12i + 4*a*c^8*d)) - tan(e + f*x)*(a*d^4 - a*c^2*d^2 + a*c*d^3*2i)*(c*d^5*48i + 9*d^6 - 70*c^2*d^4 - c^3*d^3*16i + c^4*d^2))*root(a^3*c^7*d^7*e^3*640i + a^3*c^9*d^5*e^3*480i + a^3*c^5*d^9*e^3*480i + a^3*c^11*d^3*e^3*192i + a^3*c^3*d^11*e^3*192i + 144*a^3*c^10*d^4*e^3 - 144*a^3*c^4*d^10*e^3 + 80*a^3*c^12*d^2*e^3 + 80*a^3*c^8*d^6*e^3 - 80*a^3*c^6*d^8*e^3 - 80*a^3*c^2*d^12*e^3 + a^3*c^13*d*e^3*32i + a^3*c*d^13*e^3*32i - 16*a^3*d^14*e^3 + 16*a^3*c^14*e^3 - a*c^3*d^5*e*744i - 660*a*c^2*d^6*e + 558*a*c^4*d^4*e + a*c^5*d^3*e*24i - 4*a*c^6*d^2*e + a*c*d^7*e*264i + a*c^7*d*e*8i + 57*a*d^8*e + a*c^8*e + 38*c^2*d^3 - c^3*d^2*6i - c*d^4*26i - 14*d^5, e, k), k, 1, 3)/f - ((tan(e + f*x)^2*(c*d*8i - c^2 + 3*d^2))/(2*a*(c*d^4 + c^4*d*1i + c^5 + d^5*1i + c^2*d^3*2i + 2*c^3*d^2)) + (tan(e + f*x)*(9*c*d + c^2*2i - d^2*1i))/(2*a*d*(2*c*d^3 + 2*c^3*d - c^4*1i + d^4*1i)) + (c^3*d*6i - c*d^3*6i - 3*c^4 + 3*d^4 + 24*c^2*d^2)/(6*a*d^2*(c^2 + d^2)*(c*d^2 + c^2*d*1i + c^3 + d^3*1i)))/(f*(tan(e + f*x)^2*((2*c)/d - 1i) - tan(e + f*x)*((c*2i)/d - c^2/d^2) + tan(e + f*x)^3 - (c^2*1i)/d^2))

3.1099 $\int \frac{1}{(a+ia\tan(e+fx))^2(c+d\tan(e+fx))^3}\,dx$

Optimal. Leaf size=354

$$\frac{(c^5+5ic^4d-10c^3d^2+30ic^2d^3+45cd^4-15id^5)\,x}{4a^2(c-id)^3(c+id)^5}-\frac{2d^3(5c^2-5icd-2d^2)\log(c\cos(e+fx)+d\sin(e+fx))}{a^2(ic-d)^5(ic+d)^3f}$$

```
[Out] 1/4*(c^5+5*I*c^4*d-10*c^3*d^2+30*I*c^2*d^3+45*c*d^4-15*I*d^5)*x/a^2/(c-I*d)
^3/(c+I*d)^5-2*d^3*(5*c^2-5*I*c*d-2*d^2)*ln(c*cos(f*x+e)+d*sin(f*x+e))/a^2/
(I*c-d)^5/(I*c+d)^3/f+1/4*d*(c^2+5*I*c*d+8*d^2)/a^2/(c-I*d)/(c+I*d)^3/f/(c+
d*tan(f*x+e))^2+1/4*(I*c-5*d)/a^2/(c+I*d)^2/f/(1+I*tan(f*x+e))/(c+d*tan(f*x
+e))^2-1/4/(I*c-d)/f/(a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^2+1/4*(c-3*I*d)*
d*(c^2+8*I*c*d+5*d^2)/a^2/(c-I*d)^2/(c+I*d)^4/f/(c+d*tan(f*x+e))
```

Rubi [A]
time = 0.53, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3640, 3677, 3610, 3612, 3611}

$$\frac{d(c-3id)\,(c^2+8icd+5d^2)}{4a^2f(c-id)^2(c+id)^4(c+d\tan(e+fx))}+\frac{d(c^2+5icd+8d^2)}{4a^2f(c-id)(c+id)^3(c+d\tan(e+fx))^2}-\frac{2d^3(5c^2-5icd-2d^2)\log(c\cos(e+fx)+d\sin(e+fx))}{a^2f(-d+ic)^5(d+ic)^3}+\frac{x(c^5+5ic^4d-10c^3d^2+30ic^2d^3+45cd^4-15id^5)}{4a^2(c-id)^3(c+id)^5}+\frac{-5d+ic}{4a^2f(c+id)^2(1+i\tan(e+fx))(c+d\tan(e+fx))^2}-\frac{1}{4f(-d+ic)(a+ia\tan(e+fx))^2(c+d\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^3),x]
```

```
[Out] ((c^5 + (5*I)*c^4*d - 10*c^3*d^2 + (30*I)*c^2*d^3 + 45*c*d^4 - (15*I)*d^5)*
x)/(4*a^2*(c - I*d)^3*(c + I*d)^5) - (2*d^3*(5*c^2 - (5*I)*c*d - 2*d^2)*Log
[c*Cos[e + f*x] + d*Sin[e + f*x]])/(a^2*(I*c - d)^5*(I*c + d)^3*f) + (d*(c^
2 + (5*I)*c*d + 8*d^2))/(4*a^2*(c - I*d)*(c + I*d)^3*f*(c + d*Tan[e + f*x])
^2) + (I*c - 5*d)/(4*a^2*(c + I*d)^2*f*(1 + I*Tan[e + f*x])*(c + d*Tan[e +
f*x])^2) - 1/(4*(I*c - d)*f*(a + I*a*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^2
) + ((c - (3*I)*d)*d*(c^2 + (8*I)*c*d + 5*d^2))/(4*a^2*(c - I*d)^2*(c + I*d
)^4*f*(c + d*Tan[e + f*x]))
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^2(c+d\tan(e+fx))^3}\,dx = -\frac{1}{4(ic-d)f(a+ia\tan(e+fx))^2(c+d\tan(e+fx))^2} -$$

$$= \frac{ic-5d}{4a^2(c+id)^2f(1+i\tan(e+fx))(c+d\tan(e+fx))^2} - \frac{}{4}$$

$$= \frac{d(c^2+5icd+8d^2)}{4a^2(c+id)^2(c^2+d^2)f(c+d\tan(e+fx))^2} + \frac{}{4a^2(c+id}$$

$$= \frac{d(c^2+5icd+8d^2)}{4a^2(c+id)^2(c^2+d^2)f(c+d\tan(e+fx))^2} + \frac{}{4a^2(c+id}$$

$$= \frac{(c^5+5ic^4d-10c^3d^2+30ic^2d^3+45cd^4-15id^5)\,x}{4a^2(c+id)^2(c^2+d^2)^3} + \frac{}{4a^2(}$$

$$= \frac{(c^5+5ic^4d-10c^3d^2+30ic^2d^3+45cd^4-15id^5)\,x}{4a^2(c+id)^2(c^2+d^2)^3} - \frac{2d^3(}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 4395 vs. $2(354) = 708$.
time = 8.17, size = 4395, normalized size = 12.42

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^3),x]
```

```
[Out] (Sec[e + f*x]^2*(5*c^2*d^3*Cos[e] - (5*I)*c*d^4*Cos[e] - 2*d^5*Cos[e] + (5*
I)*c^2*d^3*Sin[e] + 5*c*d^4*Sin[e] - (2*I)*d^5*Sin[e])*((-2*I)*ArcTan[(-2*c
*d*Cos[f*x] - c^2*Sin[f*x] + d^2*Sin[f*x])/(c^2*Cos[f*x] - d^2*Cos[f*x] - 2
*c*d*Sin[f*x])]*Cos[e] + 2*ArcTan[(-2*c*d*Cos[f*x] - c^2*Sin[f*x] + d^2*Sin
[f*x])/(c^2*Cos[f*x] - d^2*Cos[f*x] - 2*c*d*Sin[f*x])]*Sin[e])*(Cos[f*x] +
I*Sin[f*x])^2)/((c - I*d)^3*(c + I*d)^5*f*(a + I*a*Tan[e + f*x])^2) + (Sec[
e + f*x]^2*(5*c^2*d^3*Cos[e] - (5*I)*c*d^4*Cos[e] - 2*d^5*Cos[e] + (5*I)*c^
2*d^3*Sin[e] + 5*c*d^4*Sin[e] - (2*I)*d^5*Sin[e])*(-(Cos[e]*Log[(c*Cos[e +
f*x] + d*Sin[e + f*x])^2]) - I*Log[(c*Cos[e + f*x] + d*Sin[e + f*x])^2]*Sin
[e])*(Cos[f*x] + I*Sin[f*x])^2)/((c - I*d)^3*(c + I*d)^5*f*(a + I*a*Tan[e +
 f*x])^2) + (x*Sec[e + f*x]^2*(((-10*I)*c^2*d^3*Cos[e])/((c - I*d)^3*(c + I
*d)^4*(c*Cos[e] + d*Sin[e])) - (10*c*d^4*Cos[e])/((c - I*d)^3*(c + I*d)^4*(
c*Cos[e] + d*Sin[e])) + ((4*I)*d^5*Cos[e])/((c - I*d)^3*(c + I*d)^4*(c*Cos[
e] + d*Sin[e])) + (10*c^2*d^3*Sin[e])/((c - I*d)^3*(c + I*d)^4*(c*Cos[e] +
d*Sin[e])) - ((10*I)*c*d^4*Sin[e])/((c - I*d)^3*(c + I*d)^4*(c*Cos[e] + d*S
in[e])) - (4*d^5*Sin[e])/((c - I*d)^3*(c + I*d)^4*(c*Cos[e] + d*Sin[e])) +
```

((2*Cos[2*e] + (2*I)*Sin[2*e])*((5*I)*c^3*d^3 + (3*I)*c*d^5 + 2*d^6 - (5*I)
*c^3*d^3*Cos[2*e] - 10*c^2*d^4*Cos[2*e] + (7*I)*c*d^5*Cos[2*e] + 2*d^6*Cos[
2*e] + 5*c^3*d^3*Sin[2*e] - (10*I)*c^2*d^4*Sin[2*e] - 7*c*d^5*Sin[2*e] + (2
*I)*d^6*Sin[2*e]))/((c - I*d)^3*(c + I*d)^5*(c + I*d + c*Cos[2*e] - I*d*Cos
[2*e] + I*c*Sin[2*e] + d*Sin[2*e])))*(Cos[f*x] + I*Sin[f*x])^2)/(a + I*a*Ta
n[e + f*x])^2 + (Sec[e + f*x]^2*(Cos[f*x] + I*Sin[f*x])^2*(Cos[2*e + 4*f*x]
/64 - (I/64)*Sin[2*e + 4*f*x])*((6*I)*c^8*Cos[e] - 14*c^7*d*Cos[e] + (18*I)
*c^6*d^2*Cos[e] - 42*c^5*d^3*Cos[e] + (18*I)*c^4*d^4*Cos[e] - 42*c^3*d^5*Co
s[e] + (6*I)*c^2*d^6*Cos[e] - 14*c*d^7*Cos[e] + (5*I)*c^8*Cos[e + 2*f*x] -
11*c^7*d*Cos[e + 2*f*x] + (31*I)*c^6*d^2*Cos[e + 2*f*x] - 33*c^5*d^3*Cos[e
+ 2*f*x] + (63*I)*c^4*d^4*Cos[e + 2*f*x] - 33*c^3*d^5*Cos[e + 2*f*x] + (53*
I)*c^2*d^6*Cos[e + 2*f*x] - 11*c*d^7*Cos[e + 2*f*x] + (16*I)*d^8*Cos[e + 2*
f*x] + 2*c^8*f*x*Cos[e + 2*f*x] + (16*I)*c^7*d*f*x*Cos[e + 2*f*x] - 56*c^6*
d^2*f*x*Cos[e + 2*f*x] - (32*I)*c^5*d^3*f*x*Cos[e + 2*f*x] - 20*c^4*d^4*f*x
*Cos[e + 2*f*x] + (80*I)*c^3*d^5*f*x*Cos[e + 2*f*x] - 120*c^2*d^6*f*x*Cos[e
 + 2*f*x] - 30*d^8*f*x*Cos[e + 2*f*x] + (5*I)*c^8*Cos[3*e + 2*f*x] - 3*c^7*
d*Cos[3*e + 2*f*x] + (43*I)*c^6*d^2*Cos[3*e + 2*f*x] + 35*c^5*d^3*Cos[3*e +
 2*f*x] - (25*I)*c^4*d^4*Cos[3*e + 2*f*x] + 127*c^3*d^5*Cos[3*e + 2*f*x] -
(111*I)*c^2*d^6*Cos[3*e + 2*f*x] + 89*c*d^7*Cos[3*e + 2*f*x] - (48*I)*d^8*C
os[3*e + 2*f*x] + 2*c^8*f*x*Cos[3*e + 2*f*x] + (12*I)*c^7*d*f*x*Cos[3*e + 2
*f*x] - 28*c^6*d^2*f*x*Cos[3*e + 2*f*x] + (52*I)*c^5*d^3*f*x*Cos[3*e + 2*f*
x] + (100*I)*c^3*d^5*f*x*Cos[3*e + 2*f*x] + 60*c^2*d^6*f*x*Cos[3*e + 2*f*x]
 + (60*I)*c*d^7*f*x*Cos[3*e + 2*f*x] + 30*d^8*f*x*Cos[3*e + 2*f*x] + (2*I)*
c^8*Cos[3*e + 4*f*x] - 2*c^7*d*Cos[3*e + 4*f*x] + (22*I)*c^6*d^2*Cos[3*e +
4*f*x] + 18*c^5*d^3*Cos[3*e + 4*f*x] + (110*I)*c^4*d^4*Cos[3*e + 4*f*x] + 1
0*c^3*d^5*Cos[3*e + 4*f*x] + (130*I)*c^2*d^6*Cos[3*e + 4*f*x] - 10*c*d^7*Co
s[3*e + 4*f*x] + (40*I)*d^8*Cos[3*e + 4*f*x] + 4*c^8*f*x*Cos[3*e + 4*f*x] +
 (24*I)*c^7*d*f*x*Cos[3*e + 4*f*x] - 56*c^6*d^2*f*x*Cos[3*e + 4*f*x] + (104
*I)*c^5*d^3*f*x*Cos[3*e + 4*f*x] + (200*I)*c^3*d^5*f*x*Cos[3*e + 4*f*x] + 1
20*c^2*d^6*f*x*Cos[3*e + 4*f*x] + (120*I)*c*d^7*f*x*Cos[3*e + 4*f*x] + 60*d
^8*f*x*Cos[3*e + 4*f*x] + (2*I)*c^8*Cos[5*e + 4*f*x] + 2*c^7*d*Cos[5*e + 4*
f*x] + (22*I)*c^6*d^2*Cos[5*e + 4*f*x] + 62*c^5*d^3*Cos[5*e + 4*f*x] - (130
*I)*c^4*d^4*Cos[5*e + 4*f*x] - 74*c^3*d^5*Cos[5*e + 4*f*x] - (126*I)*c^2*d^
6*Cos[5*e + 4*f*x] - 134*c*d^7*Cos[5*e + 4*f*x] + (24*I)*d^8*Cos[5*e + 4*f*
x] + 4*c^8*f*x*Cos[5*e + 4*f*x] + (16*I)*c^7*d*f*x*Cos[5*e + 4*f*x] - 16*c^
6*d^2*f*x*Cos[5*e + 4*f*x] + (176*I)*c^5*d^3*f*x*Cos[5*e + 4*f*x] + 280*c^4
*d^4*f*x*Cos[5*e + 4*f*x] - (80*I)*c^3*d^5*f*x*Cos[5*e + 4*f*x] + 240*c^2*d
^6*f*x*Cos[5*e + 4*f*x] - (240*I)*c*d^7*f*x*Cos[5*e + 4*f*x] - 60*d^8*f*x*C
os[5*e + 4*f*x] + (80*I)*c^4*d^4*Cos[5*e + 6*f*x] + 112*c^3*d^5*Cos[5*e + 6
*f*x] + (48*I)*c^2*d^6*Cos[5*e + 6*f*x] + 112*c*d^7*Cos[5*e + 6*f*x] - (32*
I)*d^8*Cos[5*e + 6*f*x] + 2*c^8*f*x*Cos[5*e + 6*f*x] + (8*I)*c^7*d*f*x*Cos[
5*e + 6*f*x] - 8*c^6*d^2*f*x*Cos[5*e + 6*f*x] + (88*I)*c^5*d^3*f*x*Cos[5*e
+ 6*f*x] + 140*c^4*d^4*f*x*Cos[5*e + 6*f*x] - (40*I)*c^3*d^5*f*x*Cos[5*e +
6*f*x] + 120*c^2*d^6*f*x*Cos[5*e + 6*f*x] - (120*I)*c*d^7*f*x*Cos[5*e + 6*f
*x] - 30*d^8*f*x*Cos[5*e + 6*f*x] + 2*c^8*f*x*Cos[7*e + 6*f*x] + (4*I)*c^7*

d*f*x*Cos[7*e + 6*f*x] + 4*c^6*d^2*f*x*Cos[7*e + 6*f*x] + (92*I)*c^5*d^3*f*
x*Cos[7*e + 6*f*x] + 320*c^4*d^4*f*x*Cos[7*e + 6*f*x] - (500*I)*c^3*d^5*f*x
*Cos[7*e + 6*f*x] - 420*c^2*d^6*f*x*Cos[7*e + 6*f*x] + (180*I)*c*d^7*f*x*Co
s[7*e + 6*f*x] + 30*d^8*f*x*Cos[7*e + 6*f*x] + (6*I)*c^7*d*Sin[e] - 14*c^6*
d^2*Sin[e] + (18*I)*c^5*d^3*Sin[e] - 42*c^4*d^4*Sin[e] + (18*I)*c^3*d^5*Sin
[e] - 42*c^2*d^6*Sin[e] + (6*I)*c*d^7*Sin[e] - ...

Maple [**A**]
time = 1.01, size = 304, normalized size = 0.86 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^3,x,method=_RETURNVERBOSE)

[Out] 1/f/a^2*(1/8*(-I*c^2+31*I*d^2+8*c*d)/(c+I*d)^5*ln(tan(f*x+e)-I)-1/4*(I*c^2-I*d^2-2*c*d)/(c+I*d)^5/(tan(f*x+e)-I)^2-1/4*(-8*I*c*d-c^2+7*d^2)/(c+I*d)^5/(tan(f*x+e)-I)-1/8*I/(I*d-c)^3*ln(tan(f*x+e)+I)+2*d^3*(I*c^2*d+I*d^3-2*c^3-2*c*d^2)/(I*d-c)^3/(c+I*d)^5/(c+d*tan(f*x+e))-1/2*d^3*(c^4+2*c^2*d^2+d^4)/(I*d-c)^3/(c+I*d)^5/(c+d*tan(f*x+e))^2-2*d^3*(5*I*c*d-5*c^2+2*d^2)/(I*d-c)^3/(c+I*d)^5*ln(c+d*tan(f*x+e)))

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^3,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 918 vs. 2(312) = 624.
time = 1.21, size = 918, normalized size = 2.59

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^3,x, algorithm="fricas")

[Out] 1/16*(I*c^7 - c^6*d + 3*I*c^5*d^2 - 3*c^4*d^3 + 3*I*c^3*d^4 - 3*c^2*d^5 + I*c*d^6 - d^7 + 4*(c^7 + 3*I*c^6*d - c^5*d^2 + 85*I*c^4*d^3 + 235*c^3*d^4 - 271*I*c^2*d^5 - 147*c*d^6 + 31*I*d^7)*f*x*e^(8*I*f*x + 8*I*e) - 4*(-I*c^7 - 11*I*c^5*d^2 - 20*c^4*d^3 + 45*I*c^3*d^4 - 8*c^2*d^5 + 55*I*c*d^6 + 12*d^7 - 2*(c^7 + 5*I*c^6*d - 9*c^5*d^2 + 75*I*c^4*d^3 + 75*c^3*d^4 + 39*I*c^2*d^5 + 85*c*d^6 - 31*I*d^7)*f*x)*e^(6*I*f*x + 6*I*e) + (9*I*c^7 - 13*c^6*d + 7

```
1*I*c^5*d^2 + 5*c^4*d^3 - 45*I*c^3*d^4 + 305*c^2*d^5 + 85*I*c*d^6 + 95*d^7
+ 4*(c^7 + 7*I*c^6*d - 21*c^5*d^2 + 45*I*c^4*d^3 - 45*c^3*d^4 + 69*I*c^2*d^
5 - 23*c*d^6 + 31*I*d^7)*f*x)*e^(4*I*f*x + 4*I*e) - 2*(-3*I*c^7 + 7*c^6*d -
 9*I*c^5*d^2 + 21*c^4*d^3 - 9*I*c^3*d^4 + 21*c^2*d^5 - 3*I*c*d^6 + 7*d^7)*e
^(2*I*f*x + 2*I*e) - 32*((5*c^4*d^3 - 15*I*c^3*d^4 - 17*c^2*d^5 + 9*I*c*d^6
 + 2*d^7)*e^(8*I*f*x + 8*I*e) + 2*(5*c^4*d^3 - 5*I*c^3*d^4 + 3*c^2*d^5 - 5*
I*c*d^6 - 2*d^7)*e^(6*I*f*x + 6*I*e) + (5*c^4*d^3 + 5*I*c^3*d^4 + 3*c^2*d^5
 + I*c*d^6 + 2*d^7)*e^(4*I*f*x + 4*I*e))*log(((I*c + d)*e^(2*I*f*x + 2*I*e)
 + I*c - d)/(I*c + d)))/((a^2*c^10 + 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 + 10*a^
2*c^4*d^6 + 5*a^2*c^2*d^8 + a^2*d^10)*f*e^(8*I*f*x + 8*I*e) + 2*(a^2*c^10 +
 2*I*a^2*c^9*d + 3*a^2*c^8*d^2 + 8*I*a^2*c^7*d^3 + 2*a^2*c^6*d^4 + 12*I*a^2
*c^5*d^5 - 2*a^2*c^4*d^6 + 8*I*a^2*c^3*d^7 - 3*a^2*c^2*d^8 + 2*I*a^2*c*d^9
- a^2*d^10)*f*e^(6*I*f*x + 6*I*e) + (a^2*c^10 + 4*I*a^2*c^9*d - 3*a^2*c^8*d
^2 + 8*I*a^2*c^7*d^3 - 14*a^2*c^6*d^4 - 14*a^2*c^4*d^6 - 8*I*a^2*c^3*d^7 -
3*a^2*c^2*d^8 - 4*I*a^2*c*d^9 + a^2*d^10)*f*e^(4*I*f*x + 4*I*e))
```

Sympy [**A**]
time = 102.38, size = 1583, normalized size = 4.47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**2/(c+d*tan(f*x+e))**3,x)
```

```
[Out] x*(c**2 + 8*I*c*d - 31*d**2)/(4*a**2*c**5 + 20*I*a**2*c**4*d - 40*a**2*c**3
*d**2 - 40*I*a**2*c**2*d**3 + 20*a**2*c*d**4 + 4*I*a**2*d**5) + (-10*I*c**2
*d**4 + 6*c*d**5 - 4*I*d**6 + (-10*I*c**2*d**4*exp(2*I*e) - 12*c*d**5*exp(2
*I*e) + 2*I*d**6*exp(2*I*e))*exp(2*I*f*x))/(a**2*c**9*f + 3*I*a**2*c**8*d*f
 + 8*I*a**2*c**6*d**3*f - 6*a**2*c**5*d**4*f + 6*I*a**2*c**4*d**5*f - 8*a**
2*c**3*d**6*f - 3*a**2*c*d**8*f - I*a**2*d**9*f + (2*a**2*c**9*f*exp(2*I*e)
 + 2*I*a**2*c**8*d*f*exp(2*I*e) + 8*a**2*c**7*d**2*f*exp(2*I*e) + 8*I*a**2*
c**6*d**3*f*exp(2*I*e) + 12*a**2*c**5*d**4*f*exp(2*I*e) + 12*I*a**2*c**4*d*
*5*f*exp(2*I*e) + 8*a**2*c**3*d**6*f*exp(2*I*e) + 8*I*a**2*c**2*d**7*f*exp(
2*I*e) + 2*a**2*c*d**8*f*exp(2*I*e) + 2*I*a**2*d**9*f*exp(2*I*e))*exp(2*I*f
*x) + (a**2*c**9*f*exp(4*I*e) - I*a**2*c**8*d*f*exp(4*I*e) + 4*a**2*c**7*d*
*2*f*exp(4*I*e) - 4*I*a**2*c**6*d**3*f*exp(4*I*e) + 6*a**2*c**5*d**4*f*exp(
4*I*e) - 6*I*a**2*c**4*d**5*f*exp(4*I*e) + 4*a**2*c**3*d**6*f*exp(4*I*e) -
4*I*a**2*c**2*d**7*f*exp(4*I*e) + a**2*c*d**8*f*exp(4*I*e) - I*a**2*d**9*f*
exp(4*I*e))*exp(4*I*f*x)) + Piecewise((((4*I*a**2*c**4*f*exp(2*I*e) - 16*a*
*2*c**3*d*f*exp(2*I*e) - 24*I*a**2*c**2*d**2*f*exp(2*I*e) + 16*a**2*c*d**3*
f*exp(2*I*e) + 4*I*a**2*d**4*f*exp(2*I*e))*exp(-4*I*f*x) + (16*I*a**2*c**4*
f*exp(4*I*e) - 112*a**2*c**3*d*f*exp(4*I*e) - 240*I*a**2*c**2*d**2*f*exp(4*
I*e) + 208*a**2*c*d**3*f*exp(4*I*e) + 64*I*a**2*d**4*f*exp(4*I*e))*exp(-2*I
*f*x))/(64*a**4*c**7*f**2*exp(6*I*e) + 448*I*a**4*c**6*d*f**2*exp(6*I*e) -
1344*a**4*c**5*d**2*f**2*exp(6*I*e) - 2240*I*a**4*c**4*d**3*f**2*exp(6*I*e)
```

```
 + 2240*a**4*c**3*d**4*f**2*exp(6*I*e) + 1344*I*a**4*c**2*d**5*f**2*exp(6*I
*e) - 448*a**4*c*d**6*f**2*exp(6*I*e) - 64*I*a**4*d**7*f**2*exp(6*I*e)), Ne
(64*a**4*c**7*f**2*exp(6*I*e) + 448*I*a**4*c**6*d*f**2*exp(6*I*e) - 1344*a*
*4*c**5*d**2*f**2*exp(6*I*e) - 2240*I*a**4*c**4*d**3*f**2*exp(6*I*e) + 2240
*a**4*c**3*d**4*f**2*exp(6*I*e) + 1344*I*a**4*c**2*d**5*f**2*exp(6*I*e) - 4
48*a**4*c*d**6*f**2*exp(6*I*e) - 64*I*a**4*d**7*f**2*exp(6*I*e), 0)), (x*(-
(c**2 + 8*I*c*d - 31*d**2)/(4*a**2*c**5 + 20*I*a**2*c**4*d - 40*a**2*c**3*d
**2 - 40*I*a**2*c**2*d**3 + 20*a**2*c*d**4 + 4*I*a**2*d**5) + (c**2*exp(4*I
*e) + 2*c**2*exp(2*I*e) + c**2 + 8*I*c*d*exp(4*I*e) + 10*I*c*d*exp(2*I*e) +
 2*I*c*d - 31*d**2*exp(4*I*e) - 8*d**2*exp(2*I*e) - d**2)/(4*a**2*c**5*exp(
4*I*e) + 20*I*a**2*c**4*d*exp(4*I*e) - 40*a**2*c**3*d**2*exp(4*I*e) - 40*I*
a**2*c**2*d**3*exp(4*I*e) + 20*a**2*c*d**4*exp(4*I*e) + 4*I*a**2*d**5*exp(4
*I*e))), True)) - 2*d**3*(5*c**2 - 5*I*c*d - 2*d**2)*log((c + I*d)/(c*exp(2
*I*e) - I*d*exp(2*I*e)) + exp(2*I*f*x))/(a**2*f*(c - I*d)**3*(c + I*d)**5)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 806 vs. $2(312) = 624$.
time = 0.83, size = 806, normalized size = 2.28

$$-\frac{\frac{16\,(5\,c^2d^4-5i\,cd^5-2\,d^6)\log(i\,d\tan(fx+e)+i\,c)}{a^2c^8d+2i\,a^2c^7d^2+2\,a^2c^6d^3+6i\,a^2c^5d^4+6i\,a^2c^3d^6-2\,a^2c^2d^7+2i\,a^2cd^8-a^2d^9}+\frac{(i\,c^2-8\,cd-31i\,d^2)\log(\tan(fx+e)-i)}{a^2c^5+5i\,a^2c^4d-10\,a^2c^3d^2-10i\,a^2c^2d^3+5\,a^2cd^4+i\,a^2d^5}+\frac{16\log(-i\tan(fx+e)+1)}{16i\,a^2c^3+48\,a^2c^2d-48i\,a^2cd^2-16\,a^2d^3}-[illegible]}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/8*(16*(5*c^2*d^4 - 5*I*c*d^5 - 2*d^6)*log(I*d*tan(f*x + e) + I*c)/(a^2*c
^8*d + 2*I*a^2*c^7*d^2 + 2*a^2*c^6*d^3 + 6*I*a^2*c^5*d^4 + 6*I*a^2*c^3*d^6
- 2*a^2*c^2*d^7 + 2*I*a^2*c*d^8 - a^2*d^9) + (I*c^2 - 8*c*d - 31*I*d^2)*log
(tan(f*x + e) - I)/(a^2*c^5 + 5*I*a^2*c^4*d - 10*a^2*c^3*d^2 - 10*I*a^2*c^2
*d^3 + 5*a^2*c*d^4 + I*a^2*d^5) + 16*log(-I*tan(f*x + e) + 1)/(16*I*a^2*c^3
 + 48*a^2*c^2*d - 48*I*a^2*c*d^2 - 16*a^2*d^3) - 16*(3*c^4*d^2*tan(f*x + e)
^4 + 12*I*c^3*d^3*tan(f*x + e)^4 - 18*c^2*d^4*tan(f*x + e)^4 - 12*I*c*d^5*t
an(f*x + e)^4 + 3*d^6*tan(f*x + e)^4 + 6*c^5*d*tan(f*x + e)^3 + 10*I*c^4*d^
2*tan(f*x + e)^3 + 20*c^3*d^3*tan(f*x + e)^3 - 260*I*c^2*d^4*tan(f*x + e)^3
 - 370*c*d^5*tan(f*x + e)^3 + 114*I*d^6*tan(f*x + e)^3 + 3*c^6*tan(f*x + e)
^2 - 16*I*c^5*d*tan(f*x + e)^2 + 75*c^4*d^2*tan(f*x + e)^2 - 400*I*c^3*d^3*
tan(f*x + e)^2 - 955*c^2*d^4*tan(f*x + e)^2 + 720*I*c*d^5*tan(f*x + e)^2 +
173*d^6*tan(f*x + e)^2 - 14*I*c^6*tan(f*x + e) + 18*c^5*d*tan(f*x + e) - 16
4*I*c^4*d^2*tan(f*x + e) - 724*c^3*d^3*tan(f*x + e) + 970*I*c^2*d^4*tan(f*x
 + e) + 410*c*d^5*tan(f*x + e) - 32*I*d^6*tan(f*x + e) - 19*c^6 - 28*I*c^5*
d - 126*c^4*d^2 + 332*I*c^3*d^3 + 269*c^2*d^4 - 48*I*c*d^5 + 16*d^6)/((-64*
I*a^2*c^7 + 64*a^2*c^6*d - 192*I*a^2*c^5*d^2 + 192*a^2*c^4*d^3 - 192*I*a^2*
c^3*d^4 + 192*a^2*c^2*d^5 - 64*I*a^2*c*d^6 + 64*a^2*d^7)*(d*tan(f*x + e)^2
+ c*tan(f*x + e) - I*d*tan(f*x + e) - I*c)^2))/f
```

Mupad [B]

time = 12.08, size = 2640, normalized size = 7.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^2*(c + d*tan(e + f*x))^3),x)
```

```
[Out] symsum(log((a^2*d^6 + a^2*c*d^5*4i - 6*a^2*c^2*d^4 - a^2*c^3*d^3*4i + a^2*c
^4*d^2)*(839*c*d^7 + c^7*d - d^8*240i + c^2*d^6*970i - 353*c^3*d^5 + c^4*d^
4*100i - 7*c^5*d^3 + c^6*d^2*10i) - root(5760*a^6*c^8*d^8*e^3 + 4096*a^6*c^
10*d^6*e^3 + 4096*a^6*c^6*d^10*e^3 - a^6*c^11*d^5*e^3*2304i + a^6*c^5*d^11*
e^3*2304i - a^6*c^13*d^3*e^3*1280i - a^6*c^9*d^7*e^3*1280i + a^6*c^7*d^9*e^
3*1280i + a^6*c^3*d^13*e^3*1280i + 1280*a^6*c^12*d^4*e^3 + 1280*a^6*c^4*d^1
2*e^3 - a^6*c^15*d*e^3*256i + a^6*c*d^15*e^3*256i - 64*a^6*d^16*e^3 - 64*a^
6*c^16*e^3 + a^2*c*d^9*e*5190i - a^2*c^9*d*e*10i - a^2*c^3*d^7*e*12600i - 1
1565*a^2*c^2*d^8*e + 6450*a^2*c^4*d^6*e + a^2*c^5*d^5*e*180i + 110*a^2*c^6*
d^4*e + a^2*c^7*d^3*e*40i + 45*a^2*c^8*d^2*e + 993*a^2*d^10*e - a^2*c^10*e
+ 234*c^2*d^5 - c^3*d^4*70i - 10*c^4*d^3 - c*d^6*278i - 124*d^7, e, k)*(roo
t(5760*a^6*c^8*d^8*e^3 + 4096*a^6*c^10*d^6*e^3 + 4096*a^6*c^6*d^10*e^3 - a^
6*c^11*d^5*e^3*2304i + a^6*c^5*d^11*e^3*2304i - a^6*c^13*d^3*e^3*1280i - a^
6*c^9*d^7*e^3*1280i + a^6*c^7*d^9*e^3*1280i + a^6*c^3*d^13*e^3*1280i + 1280
*a^6*c^12*d^4*e^3 + 1280*a^6*c^4*d^12*e^3 - a^6*c^15*d*e^3*256i + a^6*c*d^1
5*e^3*256i - 64*a^6*d^16*e^3 - 64*a^6*c^16*e^3 + a^2*c*d^9*e*5190i - a^2*c^
9*d*e*10i - a^2*c^3*d^7*e*12600i - 11565*a^2*c^2*d^8*e + 6450*a^2*c^4*d^6*e
 + a^2*c^5*d^5*e*180i + 110*a^2*c^6*d^4*e + a^2*c^7*d^3*e*40i + 45*a^2*c^8*
d^2*e + 993*a^2*d^10*e - a^2*c^10*e + 234*c^2*d^5 - c^3*d^4*70i - 10*c^4*d^
3 - c*d^6*278i - 124*d^7, e, k)*((a^2*d^6 + a^2*c*d^5*4i - 6*a^2*c^2*d^4 -
a^2*c^3*d^3*4i + a^2*c^4*d^2)*(a^4*c^2*d^12*512i - 128*a^4*c^13*d - 128*a^4
*c*d^13 + 256*a^4*c^3*d^11 + a^4*c^4*d^10*1536i + 2176*a^4*c^5*d^9 + a^4*c^
6*d^8*1024i + 3584*a^4*c^7*d^7 - a^4*c^8*d^6*1024i + 2176*a^4*c^9*d^5 - a^4
*c^10*d^4*1536i + 256*a^4*c^11*d^3 - a^4*c^12*d^2*512i) + tan(e + f*x)*(a^2
*d^6 + a^2*c*d^5*4i - 6*a^2*c^2*d^4 - a^2*c^3*d^3*4i + a^2*c^4*d^2)*(32*a^4
*c^14 - 96*a^4*d^14 + a^4*c*d^13*384i + a^4*c^13*d*128i + 224*a^4*c^2*d^12
+ a^4*c^3*d^11*1024i + 1568*a^4*c^4*d^10 + a^4*c^5*d^9*384i + 2144*a^4*c^6*
d^8 - a^4*c^7*d^7*1024i + 736*a^4*c^8*d^6 - a^4*c^9*d^5*896i - 352*a^4*c^10
*d^4 - 160*a^4*c^12*d^2)) + (a^2*d^6 + a^2*c*d^5*4i - 6*a^2*c^2*d^4 - a^2*c
^3*d^3*4i + a^2*c^4*d^2)*(a^2*d^11*60i - 4*a^2*c^11 + 68*a^2*c*d^10 - a^2*c
^10*d*28i + a^2*c^2*d^9*244i + 300*a^2*c^3*d^8 + a^2*c^4*d^7*344i + 488*a^2
*c^5*d^6 + a^2*c^6*d^5*168i + 344*a^2*c^7*d^4 - a^2*c^8*d^3*20i + 84*a^2*c^
9*d^2) + tan(e + f*x)*(a^2*d^6 + a^2*c*d^5*4i - 6*a^2*c^2*d^4 - a^2*c^3*d^3
*4i + a^2*c^4*d^2)*(192*a^2*d^11 - a^2*c*d^10*24i - 8*a^2*c^10*d + 664*a^2*
c^2*d^9 + a^2*c^3*d^8*128i + 1088*a^2*c^4*d^7 + a^2*c^5*d^6*272i + 944*a^2*
c^6*d^5 + a^2*c^7*d^4*64i + 320*a^2*c^8*d^3 - a^2*c^9*d^2*56i)) + tan(e + f
*x)*(a^2*d^6 + a^2*c*d^5*4i - 6*a^2*c^2*d^4 - a^2*c^3*d^3*4i + a^2*c^4*d^2)
*(991*c^2*d^6 - 225*d^8 - c*d^7*870i + c^3*d^5*260i + 33*c^4*d^4 + c^5*d^3*
```

10i + c^6*d^2))*root(5760*a^6*c^8*d^8*e^3 + 4096*a^6*c^10*d^6*e^3 + 4096*a^6*c^6*d^10*e^3 - a^6*c^11*d^5*e^3*2304i + a^6*c^5*d^11*e^3*2304i - a^6*c^13*d^3*e^3*1280i - a^6*c^9*d^7*e^3*1280i + a^6*c^7*d^9*e^3*1280i + a^6*c^3*d^13*e^3*1280i + 1280*a^6*c^12*d^4*e^3 + 1280*a^6*c^4*d^12*e^3 - a^6*c^15*d*e^3*256i + a^6*c*d^15*e^3*256i - 64*a^6*d^16*e^3 - 64*a^6*c^16*e^3 + a^2*c*d^9*e*5190i - a^2*c^9*d*e*10i - a^2*c^3*d^7*e*12600i - 11565*a^2*c^2*d^8*e + 6450*a^2*c^4*d^6*e + a^2*c^5*d^5*e*180i + 110*a^2*c^6*d^4*e + a^2*c^7*d^3*e*40i + 45*a^2*c^8*d^2*e + 993*a^2*d^10*e - a^2*c^10*e + 234*c^2*d^5 - c^3*d^4*70i - 10*c^4*d^3 - c*d^6*278i - 124*d^7, e, k), k, 1, 3)/f - ((tan(e + f*x)^3*(29*c*d^2 + c^2*d*5i + c^3 - d^3*15i)*1i)/(4*a^2*(2*c*d^5 + 2*c^5*d - c^6*1i + d^6*1i + c^2*d^4*1i + 4*c^3*d^3 - c^4*d^2*1i)) - ((c^5*8i - 16*c^4*d - c*d^4*32i + 8*d^5 + 104*c^2*d^3 + c^3*d^2*56i)*1i)/(16*a^2*d^2*(2*c*d^5 + 2*c^5*d - c^6*1i + d^6*1i + c^2*d^4*1i + 4*c^3*d^3 - c^4*d^2*1i)) + (tan(e + f*x)^2*(c^3*d*16i - c*d^3*136i + 4*c^4 - 44*d^4 + 96*c^2*d^2)*1i)/(8*a^2*d*(2*c*d^5 + 2*c^5*d - c^6*1i + d^6*1i + c^2*d^4*1i + 4*c^3*d^3 - c^4*d^2*1i)) + (tan(e + f*x)*(c^4*d*3i - 144*c*d^4 + 3*c^5 + d^5*12i - c^2*d^3*213i + 63*c^3*d^2)*1i)/(12*a^2*d^2*(2*c*d^5 + 2*c^5*d - c^6*1i + d^6*1i + c^2*d^4*1i + 4*c^3*d^3 - c^4*d^2*1i)))/(f*(tan(e + f*x)^3*((2*c)/d - 2i) + tan(e + f*x)^2*((c^2 - d^2)/d^2 - (c*4i)/d) - tan(e + f*x)*((2*c)/d + (c^2*2i)/d^2) + tan(e + f*x)^4 - c^2/d^2))

3.1100 $\int \frac{1}{(a+ia\tan(e+fx))^3(c+d\tan(e+fx))^3}\,dx$

Optimal. Leaf size=448

$$\frac{(c^6+6ic^5d-15c^4d^2-20ic^3d^3-105c^2d^4+150icd^5+55d^6)\,x}{8a^3(c-id)^3(c+id)^6}-\frac{d^4(15c^2-18icd-7d^2)\log(c\cos(e+fx)+}{a^3(c+id)^6(ic+d)^3f}$$

```
[Out] 1/8*(c^6+6*I*c^5*d-15*c^4*d^2-20*I*c^3*d^3-105*c^2*d^4+150*I*c*d^5+55*d^6)*
x/a^3/(c-I*d)^3/(c+I*d)^6-d^4*(15*c^2-18*I*c*d-7*d^2)*ln(c*cos(f*x+e)+d*sin
(f*x+e))/a^3/(c+I*d)^6/(I*c+d)^3/f+1/8*d*(c^3+6*I*c^2*d-17*c*d^2+28*I*d^3)/
a^3/(c-I*d)/(c+I*d)^4/f/(c+d*tan(f*x+e))^2-1/6/(I*c-d)/f/(a+I*a*tan(f*x+e))
^3/(c+d*tan(f*x+e))^2+1/24*(3*I*c-13*d)/a/(c+I*d)^2/f/(a+I*a*tan(f*x+e))^2/
(c+d*tan(f*x+e))^2+1/24*(3*c^2+18*I*c*d-55*d^2)/(I*c-d)^3/f/(a^3+I*a^3*tan(
f*x+e))/(c+d*tan(f*x+e))^2+1/8*d*(c^4+6*I*c^3*d-16*c^2*d^2+94*I*c*d^3+55*d^
4)/a^3/(c-I*d)^2/(c+I*d)^5/f/(c+d*tan(f*x+e))
```

Rubi [A]
time = 0.83, antiderivative size = 448, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3640, 3677, 3610, 3612, 3611}

$$\frac{3c^2+18icd-55d^2}{24f(-d+ic)^3(a^3+ia^3\tan(e+fx))(c+d\tan(e+fx))^2}-\frac{d^4(15c^2-18icd-7d^2)\log(c\cos(e+fx)+d\sin(e+fx))}{a^3f(c+id)^6(d+ic)^3}+\frac{d(c^3+6ic^2d-17cd^2+28id^3)}{8a^3f(c-id)(c+id)^4(c+d\tan(e+fx))^2}+\frac{d(c^4+6ic^3d-16c^2d^2+94icd^3+55d^4)}{8a^3f(c-id)^2(c+id)^5(c+d\tan(e+fx))}+\frac{x(c^6+6ic^5d-15c^4d^2-20ic^3d^3-105c^2d^4+150icd^5+55d^6)}{8a^3(c-id)^3(c+id)^6}+\frac{-13d+3ic}{24af(c+id)^2(a+ia\tan(e+fx))^2(c+d\tan(e+fx))^2}-\frac{1}{6f(-d+ic)(a+ia\tan(e+fx))^3(c+d\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^3),x]
```

```
[Out] ((c^6 + (6*I)*c^5*d - 15*c^4*d^2 - (20*I)*c^3*d^3 - 105*c^2*d^4 + (150*I)*c
*d^5 + 55*d^6)*x)/(8*a^3*(c - I*d)^3*(c + I*d)^6) - (d^4*(15*c^2 - (18*I)*c
*d - 7*d^2)*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/(a^3*(c + I*d)^6*(I*c + d
)^3*f) + (d*(c^3 + (6*I)*c^2*d - 17*c*d^2 + (28*I)*d^3))/(8*a^3*(c - I*d)*(
c + I*d)^4*f*(c + d*Tan[e + f*x])^2) - 1/(6*(I*c - d)*f*(a + I*a*Tan[e + f*
x])^3*(c + d*Tan[e + f*x])^2) + ((3*I)*c - 13*d)/(24*a*(c + I*d)^2*f*(a + I
*a*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^2) + (3*c^2 + (18*I)*c*d - 55*d^2)/
(24*(I*c - d)^3*f*(a^3 + I*a^3*Tan[e + f*x])*(c + d*Tan[e + f*x])^2) + (d*(
c^4 + (6*I)*c^3*d - 16*c^2*d^2 + (94*I)*c*d^3 + 55*d^4))/(8*a^3*(c - I*d)^2
*(c + I*d)^5*f*(c + d*Tan[e + f*x]))
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^3(c+d\tan(e+fx))^3}\,dx = -\frac{1}{6(ic-d)f(a+ia\tan(e+fx))^3(c+d\tan(e+fx))^2} -$$

$$= -\frac{1}{6(ic-d)f(a+ia\tan(e+fx))^3(c+d\tan(e+fx))^2} +$$

$$= -\frac{1}{6(ic-d)f(a+ia\tan(e+fx))^3(c+d\tan(e+fx))^2} +$$

$$= \frac{d(c^3+6ic^2d-17cd^2+28id^3)}{8a^3(c-id)(c+id)^4f(c+d\tan(e+fx))^2} - \frac{}{6(ic-d)f(}$$

$$= \frac{d(c^3+6ic^2d-17cd^2+28id^3)}{8a^3(c-id)(c+id)^4f(c+d\tan(e+fx))^2} - \frac{}{6(ic-d)f(}$$

$$= \frac{(c^6+6ic^5d-15c^4d^2-20ic^3d^3-105c^2d^4+150icd^5+5}{8a^3(c-id)^3(c+id)^6}$$

$$= \frac{(c^6+6ic^5d-15c^4d^2-20ic^3d^3-105c^2d^4+150icd^5+5}{8a^3(c-id)^3(c+id)^6}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 5726 vs. $2(448) = 896$.
time = 8.53, size = 5726, normalized size = 12.78

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^3),x]
```

```
[Out] Result too large to show
```

Maple [A]
time = 1.69, size = 374, normalized size = 0.83 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a^3*(-1/16*I/(I*d-c)^3*ln(tan(f*x+e)+I)-1/6*(3*I*c^2*d-I*d^3+c^3-3*c*d^
2)/(c+I*d)^6/(tan(f*x+e)-I)^3+1/16/(c+I*d)^6*(-I*c^3+39*I*c*d^2+9*c^2*d-111
*d^3)*ln(tan(f*x+e)-I)-1/8*(-9*I*c^2*d+31*I*d^3-c^3+39*c*d^2)/(c+I*d)^6/(ta
n(f*x+e)-I)-1/8*(I*c^3-15*I*c*d^2-9*c^2*d+7*d^3)/(c+I*d)^6/(tan(f*x+e)-I)^2
-d^4*(5*I*c^3+5*I*c*d^2+3*c^2*d+3*d^3)/(I*d-c)^3/(c+I*d)^6/(c+d*tan(f*x+e))
-1/2*I*d^4*(c^4+2*c^2*d^2+d^4)/(I*d-c)^3/(c+I*d)^6/(c+d*tan(f*x+e))^2+d^4*(
15*I*c^2-7*I*d^2+18*c*d)/(I*d-c)^3/(c+I*d)^6*ln(c+d*tan(f*x+e)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1151 vs. 2(398) = 796.
time = 1.24, size = 1151, normalized size = 2.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/96*(2*c^8 + 4*I*c^7*d + 4*c^6*d^2 + 12*I*c^5*d^3 + 12*I*c^3*d^5 - 4*c^2*
d^6 + 4*I*c*d^7 - 2*d^8 - 12*(I*c^8 - 4*c^7*d - 4*I*c^6*d^2 - 4*c^5*d^3 - 2
50*I*c^4*d^4 - 764*c^3*d^5 + 924*I*c^2*d^6 + 516*c*d^7 - 111*I*d^8)*f*x*e^(
10*I*f*x + 10*I*e) + 6*(3*c^8 + 6*I*c^7*d + 18*c^6*d^2 + 66*I*c^5*d^3 + 180
*c^4*d^4 - 270*I*c^3*d^5 + 62*c^2*d^6 - 330*I*c*d^7 - 103*d^8 - 4*(I*c^8 -
6*c^7*d - 14*I*c^6*d^2 + 14*c^5*d^3 - 240*I*c^4*d^4 - 274*c^3*d^5 - 114*I*c
^2*d^6 - 294*c*d^7 + 111*I*d^8)*f*x)*e^(8*I*f*x + 8*I*e) + 3*(15*c^8 + 48*I
*c^7*d + 24*c^6*d^2 + 312*I*c^5*d^3 + 150*c^4*d^4 + 96*I*c^3*d^5 + 864*c^2*
d^6 + 216*I*c*d^7 + 339*d^8 - 4*(I*c^8 - 8*c^7*d - 28*I*c^6*d^2 + 56*c^5*d^
3 - 170*I*c^4*d^4 + 136*c^3*d^5 - 252*I*c^2*d^6 + 72*c*d^7 - 111*I*d^8)*f*x
)*e^(6*I*f*x + 6*I*e) + 2*(19*c^8 + 70*I*c^7*d - 34*c^6*d^2 + 210*I*c^5*d^3
 - 216*c^4*d^4 + 210*I*c^3*d^5 - 254*c^2*d^6 + 70*I*c*d^7 - 91*d^8)*e^(4*I*
f*x + 4*I*e) + (13*c^8 + 36*I*c^7*d + 16*c^6*d^2 + 108*I*c^5*d^3 - 30*c^4*d
^4 + 108*I*c^3*d^5 - 56*c^2*d^6 + 36*I*c*d^7 - 23*d^8)*e^(2*I*f*x + 2*I*e)
- 96*((15*c^4*d^4 - 48*I*c^3*d^5 - 58*c^2*d^6 + 32*I*c*d^7 + 7*d^8)*e^(10*I
*f*x + 10*I*e) + 2*(15*c^4*d^4 - 18*I*c^3*d^5 + 8*c^2*d^6 - 18*I*c*d^7 - 7*
d^8)*e^(8*I*f*x + 8*I*e) + (15*c^4*d^4 + 12*I*c^3*d^5 + 14*c^2*d^6 + 4*I*c*
d^7 + 7*d^8)*e^(6*I*f*x + 6*I*e))*log(((I*c + d)*e^(2*I*f*x + 2*I*e) + I*c
- d)/(I*c + d)))/((I*a^3*c^11 - a^3*c^10*d + 5*I*a^3*c^9*d^2 - 5*a^3*c^8*d^
3 + 10*I*a^3*c^7*d^4 - 10*a^3*c^6*d^5 + 10*I*a^3*c^5*d^6 - 10*a^3*c^4*d^7 +
 5*I*a^3*c^3*d^8 - 5*a^3*c^2*d^9 + I*a^3*c*d^10 - a^3*d^11)*f*e^(10*I*f*x +
 10*I*e) + 2*(I*a^3*c^11 - 3*a^3*c^10*d + I*a^3*c^9*d^2 - 11*a^3*c^8*d^3 -
6*I*a^3*c^7*d^4 - 14*a^3*c^6*d^5 - 14*I*a^3*c^5*d^6 - 6*a^3*c^4*d^7 - 11*I*
a^3*c^3*d^8 + a^3*c^2*d^9 - 3*I*a^3*c*d^10 + a^3*d^11)*f*e^(8*I*f*x + 8*I*e
) + (I*a^3*c^11 - 5*a^3*c^10*d - 7*I*a^3*c^9*d^2 - 5*a^3*c^8*d^3 - 22*I*a^3
```

```
*c^7*d^4 + 14*a^3*c^6*d^5 - 14*I*a^3*c^5*d^6 + 22*a^3*c^4*d^7 + 5*I*a^3*c^3
*d^8 + 7*a^3*c^2*d^9 + 5*I*a^3*c*d^10 - a^3*d^11)*f*e^(6*I*f*x + 6*I*e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**3/(c+d*tan(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.03, size = 782, normalized size = 1.75

$$-\frac{\frac{192\,(15\,c^2d^5-18i\,cd^6-7\,d^7)\log(d\tan(fx+e)+c)}{-2i\,a^3c^9d+6\,a^3c^8d^2+16\,a^3c^6d^4+12i\,a^3c^5d^5+12\,a^3c^4d^6+16i\,a^3c^3d^7+6i\,a^3cd^9-2\,a^3d^{10}}-\frac{6\,(-i\,c^3+9\,c^2d+39i\,cd^2-111\,d^3)\log(i\tan(fx+e)+1)}{a^3c^6+6i\,a^3c^5d-15\,a^3c^4d^2-20i\,a^3c^3d^3+15\,a^3c^2d^4+6i\,a^3cd^5-a^3d^6}-\frac{6i\log(-i\tan(fx+e)+1)}{a^3c^3-3i\,a^3c^2d-3\,a^3cd^2+i\,a^3d^3}-\frac{192\left(45\,c^2d^6\tan(fx+e)^2-54i\,cd^7\tan(fx+e)^2-21\,d^8\tan(fx+e)^2+100\,c^3d^5\tan(fx+e)-114i\,c^2d^6\tan(fx+e)-32\,cd^7\tan(fx+e)-6i\,d^8\tan(fx+e)+56\,c^4d^4-60i\,c^3d^5-9\,c^2d^6-6i\,cd^7+d^8\right)}{-4\,(i\,a^3c^9-3\,a^3c^8d-8\,a^3c^6d^3-6i\,a^3c^5d^4-6\,a^3c^4d^5-8i\,a^3c^3d^6-3i\,a^3cd^8+a^3d^9)(d\tan(fx+e)+c)^2}-\frac{11i\,c^3\tan(fx+e)^3-99\,c^2d\tan(fx+e)^3-429i\,cd^2\tan(fx+e)^3+1221\,d^3\tan(fx+e)^3+45\,c^3\tan(fx+e)^2+405i\,c^2d\tan(fx+e)^2-1755\,cd^2\tan(fx+e)^2-4035i\,d^3\tan(fx+e)^2-69i\,c^3\tan(fx+e)+621\,c^2d\tan(fx+e)+2403i\,cd^2\tan(fx+e)-4491\,d^3\tan(fx+e)-51\,c^3-363i\,c^2d+1125\,cd^2+1693i\,d^3}{(a^3c^6+6i\,a^3c^5d-15\,a^3c^4d^2-20i\,a^3c^3d^3+15\,a^3c^2d^4+6i\,a^3cd^5-a^3d^6)(\tan(fx+e)-i)^3}}{96\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/96*(192*(15*c^2*d^5 - 18*I*c*d^6 - 7*d^7)*log(d*tan(f*x + e) + c)/(-2*I*
a^3*c^9*d + 6*a^3*c^8*d^2 + 16*a^3*c^6*d^4 + 12*I*a^3*c^5*d^5 + 12*a^3*c^4*
d^6 + 16*I*a^3*c^3*d^7 + 6*I*a^3*c*d^9 - 2*a^3*d^10) - 6*(-I*c^3 + 9*c^2*d
+ 39*I*c*d^2 - 111*d^3)*log(I*tan(f*x + e) + 1)/(a^3*c^6 + 6*I*a^3*c^5*d -
15*a^3*c^4*d^2 - 20*I*a^3*c^3*d^3 + 15*a^3*c^2*d^4 + 6*I*a^3*c*d^5 - a^3*d^
6) - 6*I*log(-I*tan(f*x + e) + 1)/(a^3*c^3 - 3*I*a^3*c^2*d - 3*a^3*c*d^2 +
I*a^3*d^3) - 192*(45*c^2*d^6*tan(f*x + e)^2 - 54*I*c*d^7*tan(f*x + e)^2 - 2
1*d^8*tan(f*x + e)^2 + 100*c^3*d^5*tan(f*x + e) - 114*I*c^2*d^6*tan(f*x + e
) - 32*c*d^7*tan(f*x + e) - 6*I*d^8*tan(f*x + e) + 56*c^4*d^4 - 60*I*c^3*d^
5 - 9*c^2*d^6 - 6*I*c*d^7 + d^8)/((-4*I*a^3*c^9 + 12*a^3*c^8*d + 32*a^3*c^6
*d^3 + 24*I*a^3*c^5*d^4 + 24*a^3*c^4*d^5 + 32*I*a^3*c^3*d^6 + 12*I*a^3*c*d^
8 - 4*a^3*d^9)*(d*tan(f*x + e) + c)^2) - (11*I*c^3*tan(f*x + e)^3 - 99*c^2*
d*tan(f*x + e)^3 - 429*I*c*d^2*tan(f*x + e)^3 + 1221*d^3*tan(f*x + e)^3 + 4
5*c^3*tan(f*x + e)^2 + 405*I*c^2*d*tan(f*x + e)^2 - 1755*c*d^2*tan(f*x + e)
^2 - 4035*I*d^3*tan(f*x + e)^2 - 69*I*c^3*tan(f*x + e) + 621*c^2*d*tan(f*x
+ e) + 2403*I*c*d^2*tan(f*x + e) - 4491*d^3*tan(f*x + e) - 51*c^3 - 363*I*c
^2*d + 1125*c*d^2 + 1693*I*d^3)/((a^3*c^6 + 6*I*a^3*c^5*d - 15*a^3*c^4*d^2
- 20*I*a^3*c^3*d^3 + 15*a^3*c^2*d^4 + 6*I*a^3*c*d^5 - a^3*d^6)*(tan(f*x + e
) - I)^3))/f
```

Mupad [**B**]
time = 13.52, size = 2500, normalized size = 5.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^3*(c + d*tan(e + f*x))^3),x)
```

```
[Out] symsum(log((a^3*d^8 + a^3*c*d^7*6i - 15*a^3*c^2*d^6 - a^3*c^3*d^5*20i + 15*
a^3*c^4*d^4 + a^3*c^5*d^3*6i - a^3*c^6*d^2)*(10159*c*d^9 - c^9*d - d^10*308
0i + c^2*d^8*10692i - 2652*c^3*d^7 + c^4*d^6*1236i + 186*c^5*d^5 + c^6*d^4*
124i + 68*c^7*d^3 - c^8*d^2*12i) - root(a^9*c^9*d^9*e^3*56320i + a^9*c^11*d
^7*e^3*36864i + a^9*c^7*d^11*e^3*36864i + 29696*a^9*c^12*d^6*e^3 - 29696*a^
9*c^6*d^12*e^3 + 16896*a^9*c^10*d^8*e^3 - 16896*a^9*c^8*d^10*e^3 + 15360*a^
9*c^14*d^4*e^3 - 15360*a^9*c^4*d^14*e^3 + a^9*c^13*d^5*e^3*6144i + a^9*c^5*
d^13*e^3*6144i - a^9*c^15*d^3*e^3*4096i - a^9*c^3*d^15*e^3*4096i + 2304*a^9
*c^16*d^2*e^3 - 2304*a^9*c^2*d^16*e^3 - a^9*c^17*d*e^3*1536i - a^9*c*d^17*e
^3*1536i + 256*a^9*d^18*e^3 - 256*a^9*c^18*e^3 - a^3*c*d^11*e*64884i - a^3*
c^11*d*e*12i + a^3*c^3*d^9*e*137380i + 136578*a^3*c^2*d^10*e - 58575*a^3*c^
4*d^8*e - 1060*a^3*c^6*d^6*e + a^3*c^7*d^5*e*360i - a^3*c^5*d^7*e*360i - 25
5*a^3*c^8*d^4*e + a^3*c^9*d^3*e*220i + 66*a^3*c^10*d^2*e - 12433*a^3*d^12*e
 - a^3*c^12*e - 1026*c^2*d^7 + c^3*d^6*430i + 117*c^4*d^5 - c^5*d^4*15i + c
*d^8*1725i + 777*d^9, e, k)*(root(a^9*c^9*d^9*e^3*56320i + a^9*c^11*d^7*e^3
*36864i + a^9*c^7*d^11*e^3*36864i + 29696*a^9*c^12*d^6*e^3 - 29696*a^9*c^6*
d^12*e^3 + 16896*a^9*c^10*d^8*e^3 - 16896*a^9*c^8*d^10*e^3 + 15360*a^9*c^14
*d^4*e^3 - 15360*a^9*c^4*d^14*e^3 + a^9*c^13*d^5*e^3*6144i + a^9*c^5*d^13*e
^3*6144i - a^9*c^15*d^3*e^3*4096i - a^9*c^3*d^15*e^3*4096i + 2304*a^9*c^16*
d^2*e^3 - 2304*a^9*c^2*d^16*e^3 - a^9*c^17*d*e^3*1536i - a^9*c*d^17*e^3*153
6i + 256*a^9*d^18*e^3 - 256*a^9*c^18*e^3 - a^3*c*d^11*e*64884i - a^3*c^11*d
*e*12i + a^3*c^3*d^9*e*137380i + 136578*a^3*c^2*d^10*e - 58575*a^3*c^4*d^8*
e - 1060*a^3*c^6*d^6*e + a^3*c^7*d^5*e*360i - a^3*c^5*d^7*e*360i - 255*a^3*
c^8*d^4*e + a^3*c^9*d^3*e*220i + 66*a^3*c^10*d^2*e - 12433*a^3*d^12*e - a^3
*c^12*e - 1026*c^2*d^7 + c^3*d^6*430i + 117*c^4*d^5 - c^5*d^4*15i + c*d^8*1
725i + 777*d^9, e, k)*((a^3*d^8 + a^3*c*d^7*6i - 15*a^3*c^2*d^6 - a^3*c^3*d
^5*20i + 15*a^3*c^4*d^4 + a^3*c^5*d^3*6i - a^3*c^6*d^2)*(512*a^6*c^15*d - 5
12*a^6*c*d^15 + a^6*c^2*d^14*3072i + 5632*a^6*c^3*d^13 + a^6*c^4*d^12*2048i
 + 19968*a^6*c^5*d^11 - a^6*c^6*d^10*19456i + 13824*a^6*c^7*d^9 - a^6*c^8*d
^8*36864i - 13824*a^6*c^9*d^7 - a^6*c^10*d^6*19456i - 19968*a^6*c^11*d^5 +
a^6*c^12*d^4*2048i - 5632*a^6*c^13*d^3 + a^6*c^14*d^2*3072i) + tan(e + f*x)
*(a^3*d^8 + a^3*c*d^7*6i - 15*a^3*c^2*d^6 - a^3*c^3*d^5*20i + 15*a^3*c^4*d^
4 + a^3*c^5*d^3*6i - a^3*c^6*d^2)*(a^6*c*d^15*2304i - 384*a^6*d^16 - 128*a^
6*c^16 - a^6*c^15*d*768i + 4352*a^6*c^2*d^14 + a^6*c^3*d^13*768i + 13568*a^
6*c^4*d^12 - a^6*c^5*d^11*15104i + 5376*a^6*c^6*d^10 - a^6*c^7*d^9*22784i -
 13824*a^6*c^8*d^8 - a^6*c^9*d^7*5376i - 11520*a^6*c^10*d^6 + a^6*c^11*d^5*
6400i + 768*a^6*c^12*d^4 + a^6*c^13*d^3*1792i + 1792*a^6*c^14*d^2)) + (a^3*
d^8 + a^3*c*d^7*6i - 15*a^3*c^2*d^6 - a^3*c^3*d^5*20i + 15*a^3*c^4*d^4 + a^
3*c^5*d^3*6i - a^3*c^6*d^2)*(8*a^3*c^13 + a^3*d^13*440i + 1016*a^3*c*d^12 +
 a^3*c^12*d*72i + a^3*c^2*d^11*1056i + 4000*a^3*c^3*d^10 - a^3*c^4*d^9*360i
 + 5592*a^3*c^5*d^8 - a^3*c^6*d^7*2944i + 2944*a^3*c^7*d^6 - a^3*c^8*d^5*27
12i + 40*a^3*c^9*d^4 - a^3*c^10*d^3*672i - 288*a^3*c^11*d^2) + tan(e + f*x)
*(a^3*d^8 + a^3*c*d^7*6i - 15*a^3*c^2*d^6 - a^3*c^3*d^5*20i + 15*a^3*c^4*d^
```

4 + a^3*c^5*d^3*6i - a^3*c^6*d^2)*(1344*a^3*d^13 - a^3*c*d^12*1456i + 16*a^3*c^12*d + 5008*a^3*c^2*d^11 - a^3*c^3*d^10*4912i + 8400*a^3*c^4*d^9 - a^3*c^5*d^8*7904i + 6560*a^3*c^6*d^7 - a^3*c^7*d^6*6752i + 1248*a^3*c^8*d^5 - a^3*c^9*d^4*2160i - 560*a^3*c^10*d^3 + a^3*c^11*d^2*144i)) + tan(e + f*x)*(a^3*d^8 + a^3*c*d^7*6i - 15*a^3*c^2*d^6 - a^3*c^3*d^5*20i + 15*a^3*c^4*d^4 + a^3*c^5*d^3*6i - a^3*c^6*d^2)*(10596*c^2*d^8 - 3025*d^10 - c*d^9*10340i + c^3*d^7*2348i + 762*c^4*d^6 + c^5*d^5*4i + 68*c^6*d^4 - c^7*d^3*12i - c^8*d^2))*root(a^9*c^9*d^9*e^3*56320i + a^9*c^11*d^7*e^3*36864i + a^9*c^7*d^11*e^3*36864i + 29696*a^9*c^12*d^6*e^3 - 29696*a^9*c^6*d^12*e^3 + 16896*a^9*c^10*d^8*e^3 - 16896*a^9*c^8*d^10*e^3 + 15360*a^9*c^14*d^4*e^3 - 15360*a^9*c^4*d^14*e^3 + a^9*c^13*d^5*e^3*6144i + a^9*c^5*d^13*e^3*6144i - a^9*c^15*d^3*e^3*4096i - a^9*c^3*d^15*e^3*4096i + 2304*a^9*c^16*d^2*e^3 - 2304*a^9*c^2*d^16*e^3 - a^9*c^17*d*e^3*1536i - a^9*c*d^17*e^3*1536i + 256*a^9*d^18*e^3 - 256*a^9*c^18*e^3 - a^3*c*d^11*e*64884i - a^3*c^11*d*e*12i + a^3*c^3*d^9*e*137380i + 136578*a^3*c^2*d^10*e - 58575*a^3*c^4*d^8*e - 1060*a^3*c^6*d^6*e + a^3*c^7*d^5*e*360i - a^3*c^5*d^7*e*360i - 255*a^3*c^8*d^4*e + a^3*c^9*d^3*e*220i + 66*a^3*c^10*d^2*e - 12433*a^3*d^12*e - a^3*c^12*e - 1026*c^2*d^7 + c^3*d^6*430i + 117*c^4*d^5 - c^5*d^4*15i + c*d^8*1725i + 777*d^9, e, k), k, 1, 3)/f + ((tan(e + f*x)^2*(c*d^4*872i - 3*c^4*d + c^5*3i + 298*d^5 - 317*c^2*d^3 + c^3*d^2*47i))/(24*a^3*d^2*(4*c*d^5 - 4*c^5*d + c^6*1i + d^6*1i - c^2*d^4*5i - c^4*d^2*5i)) - (tan(e + f*x)^4*(c*d^3*94i + c^3*d*6i + c^4 + 55*d^4 - 16*c^2*d^2))/(8*a^3*(3*c*d^6 - c^6*d*3i - c^7 + d^7*1i - c^2*d^5*1i + 5*c^3*d^4 - c^4*d^3*5i + c^5*d^2)) + (c^5*d*180i - c*d^5*360i + 50*c^6 + 60*d^6 + 1250*c^2*d^4 + c^3*d^3*900i - 80*c^4...

3.1101 $\int (a+ia\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=150

$$-\frac{8ia^3\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{8ia^3\sqrt{c+d\tan(e+fx)}}{f}+\frac{4a^3(ic-6d)(c+d\tan(e+fx)}{15d^2f}$$

```
[Out] -8*I*a^3*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))*(c-I*d)^(1/2)/f+8*I*
a^3*(c+d*tan(f*x+e))^(1/2)/f+4/15*a^3*(I*c-6*d)*(c+d*tan(f*x+e))^(3/2)/d^2/
f-2/5*(a^3+I*a^3*tan(f*x+e))*(c+d*tan(f*x+e))^(3/2)/d/f
```

Rubi [A]
time = 0.29, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3637, 3673, 3609, 3618, 65, 214}

$$\frac{4a^3(-6d+ic)(c+d\tan(e+fx))^{3/2}}{15d^2f}+\frac{8ia^3\sqrt{c+d\tan(e+fx)}}{f}-\frac{2(a^3+ia^3\tan(e+fx))(c+d\tan(e+fx))^{3/2}}{5df}-\frac{8ia^3\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^3*Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((-8*I)*a^3*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/
f + ((8*I)*a^3*Sqrt[c + d*Tan[e + f*x]])/f + (4*a^3*(I*c - 6*d)*(c + d*Tan[
e + f*x])^(3/2))/(15*d^2*f) - (2*(a^3 + I*a^3*Tan[e + f*x])*(c + d*Tan[e +
f*x])^(3/2))/(5*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
```

```
0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

Rubi steps

$$\int (a+ia\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}\,dx = -\frac{2(a^3+ia^3\tan(e+fx))\,(c+d\tan(e+fx))^{3/2}}{5df} + \frac{(2a)\int}{}$$

$$= \frac{4a^3(ic-6d)(c+d\tan(e+fx))^{3/2}}{15d^2f} - \frac{2(a^3+ia^3\tan(e+f}{}$$

$$= \frac{8ia^3\sqrt{c+d\tan(e+fx)}}{f} + \frac{4a^3(ic-6d)(c+d\tan(e+f}{15d^2f}$$

$$= \frac{8ia^3\sqrt{c+d\tan(e+fx)}}{f} + \frac{4a^3(ic-6d)(c+d\tan(e+f}{15d^2f}$$

$$= \frac{8ia^3\sqrt{c+d\tan(e+fx)}}{f} + \frac{4a^3(ic-6d)(c+d\tan(e+f}{15d^2f}$$

$$= -\frac{8ia^3\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f} + \frac{8ia^3\sqrt{}}{}$$

Mathematica [A]
time = 4.55, size = 219, normalized size = 1.46

$$\frac{a^3(\cos(e+fx)+i\sin(e+fx))^3\left(-8i\sqrt{c-id}\,e^{-3ie}\tanh^{-1}\left(\frac{\sqrt{c-\frac{id(-1+e^{2i(e+fx)})}{1+e^{2i(e+fx)}}}}{\sqrt{c-id}}\right)+\frac{\sec^2(e+fx)(i\cos(3e)+\sin(3e))(2c^2+15icd+57d^2+(2c^2+15icd+63d^2)\cos(2(e+fx))-(c-15id)d\sin(2(e+fx)))\sqrt{c+d\tan(e+fx)}}{15d^2}\right)}{f(\cos(fx)+i\sin(fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3*Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] (a^3*(Cos[e + f*x] + I*Sin[e + f*x])^3*(((-8*I)*Sqrt[c - I*d]*ArcTanh[Sqrt[
c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))]/Sqrt[c - I*
d]])/E^((3*I)*e) + (Sec[e + f*x]^2*(I*Cos[3*e] + Sin[3*e])*(2*c^2 + (15*I)*
c*d + 57*d^2 + (2*c^2 + (15*I)*c*d + 63*d^2)*Cos[2*(e + f*x)] - (c - (15*I)
*d)*d*Sin[2*(e + f*x)])*Sqrt[c + d*Tan[e + f*x]])/(15*d^2)))/(f*(Cos[f*x] +
 I*Sin[f*x])^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 739 vs. $2(128) = 256$.
time = 0.38, size = 740, normalized size = 4.93

method	result

derivativedivides	$\frac{2a^3\left(-\frac{i(c+d\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{ic(c+d\tan(fx+e))^{\frac{3}{2}}}{3}+4id^2\sqrt{c+d\tan(fx+e)}-d(c+d\tan(fx+e))^{\frac{3}{2}}-4d^2\left(\frac{\left(-i\sqrt{\right.}}{}\right.\right.}{}$
default	$\frac{2a^3\left(-\frac{i(c+d\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{ic(c+d\tan(fx+e))^{\frac{3}{2}}}{3}+4id^2\sqrt{c+d\tan(fx+e)}-d(c+d\tan(fx+e))^{\frac{3}{2}}-4d^2\left(\frac{\left(-i\sqrt{\right.}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3/d^2*(-1/5*I*(c+d*tan(f*x+e))^(5/2)+1/3*I*c*(c+d*tan(f*x+e))^(3/2)+4
*I*d^2*(c+d*tan(f*x+e))^(1/2)-d*(c+d*tan(f*x+e))^(3/2)-4*d^2*(1/(4*(c^2+d^2
)^(1/2)+4*c)*(1/2*(-I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)-I*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*c-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d)*ln(d*tan(f*x+e)
+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*
(2*I*(c^2+d^2)^(1/2)*c+2*I*c^2+2*I*d^2+1/2*(-I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2
)*(c^2+d^2)^(1/2)-I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c-(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)*d)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arct
an((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1
/2)-2*c)^(1/2)))+1/(4*(c^2+d^2)^(1/2)+4*c)*(1/2*(I*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*(c^2+d^2)^(1/2)+I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c+(2*(c^2+d^2)^(1/2)+
2*c)^(1/2)*d)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)+(c^2+d^2)^(1/2))+2*(2*I*(c^2+d^2)^(1/2)*c+2*I*c^2+2*I*d^2-1/2*(I*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)+I*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*c+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^
2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)
+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^3,x, algorithm="maxima"
)
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^3*sqrt(d*tan(f*x + e) + c), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 532 vs. 2(129) = 258.
time = 1.32, size = 532, normalized size = 3.55

$$\frac{2\left(15\left(d^2fe^{(4ifx+4ie)}+2d^2fe^{(2ifx+2ie)}+d^2f\right)\sqrt{-\frac{a^6c-ia^6d}{f^2}}\log\left(\frac{2\left(a^3c+\left(ife^{(2ifx+2ie)}+if\right)\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}\sqrt{-\frac{a^6c-ia^6d}{f^2}}+\left(a^3c-ia^3d\right)e^{(2ifx+2ie)}\right)e^{(-2ifx-2ie)}}{a^3}\right)-15\left(d^2fe^{(4ifx+4ie)}+2d^2fe^{(2ifx+2ie)}+d^2f\right)\sqrt{-\frac{a^6c-ia^6d}{f^2}}\log\left(\frac{2\left(a^3c+\left(-ife^{(2ifx+2ie)}-if\right)\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}\sqrt{-\frac{a^6c-ia^6d}{f^2}}+\left(a^3c-ia^3d\right)e^{(2ifx+2ie)}\right)e^{(-2ifx-2ie)}}{a^3}\right)-2\left(-ia^3c^2+7a^3cd-24ia^3d^2+\left(-ia^3c^2+8a^3cd-39ia^3d^2\right)e^{(4ifx+4ie)}+\left(-2ia^3c^2+15a^3cd-57ia^3d^2\right)e^{(2ifx+2ie)}\right)\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}\right)}{15\left(d^2fe^{(4ifx+4ie)}+2d^2fe^{(2ifx+2ie)}+d^2f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^3,x, algorithm="fricas"
)
```

```
[Out] 2/15*(15*(d^2*f*e^(4*I*f*x + 4*I*e) + 2*d^2*f*e^(2*I*f*x + 2*I*e) + d^2*f)*
sqrt(-(a^6*c - I*a^6*d)/f^2)*log(2*(a^3*c + (I*f*e^(2*I*f*x + 2*I*e) + I*f)
*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*
sqrt(-(a^6*c - I*a^6*d)/f^2) + (a^3*c - I*a^3*d)*e^(2*I*f*x + 2*I*e))*e^(-2
*I*f*x - 2*I*e)/a^3) - 15*(d^2*f*e^(4*I*f*x + 4*I*e) + 2*d^2*f*e^(2*I*f*x +
 2*I*e) + d^2*f)*sqrt(-(a^6*c - I*a^6*d)/f^2)*log(2*(a^3*c + (-I*f*e^(2*I*f
*x + 2*I*e) - I*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f
*x + 2*I*e) + 1))*sqrt(-(a^6*c - I*a^6*d)/f^2) + (a^3*c - I*a^3*d)*e^(2*I*f
*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/a^3) - 2*(-I*a^3*c^2 + 7*a^3*c*d - 24*I*a
^3*d^2 + (-I*a^3*c^2 + 8*a^3*c*d - 39*I*a^3*d^2)*e^(4*I*f*x + 4*I*e) + (-2*
I*a^3*c^2 + 15*a^3*c*d - 57*I*a^3*d^2)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)
*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))/(d^2*f*e^(4*I*f
*x + 4*I*e) + 2*d^2*f*e^(2*I*f*x + 2*I*e) + d^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia^3\left(\int i\sqrt{c+d\tan(e+fx)}\,dx+\int\left(-3\sqrt{c+d\tan(e+fx)}\tan(e+fx)\right)dx+\int\sqrt{c+d\tan(e+fx)}\tan^3(e+fx)\,dx+\int\left(-3i\sqrt{c+d\tan(e+fx)}\tan^2(e+fx)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] -I*a**3*(Integral(I*sqrt(c + d*tan(e + f*x)), x) + Integral(-3*sqrt(c + d*t
an(e + f*x))*tan(e + f*x), x) + Integral(sqrt(c + d*tan(e + f*x))*tan(e + f
*x)**3, x) + Integral(-3*I*sqrt(c + d*tan(e + f*x))*tan(e + f*x)**2, x))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 277 vs. 2(129) = 258.
time = 0.67, size = 277, normalized size = 1.85

$$-\frac{16\left(-ia^3c-a^3d\right)\arctan\left(\frac{2\left(\sqrt{d\tan(fx+e)+c}\,c-\sqrt{c^2+d^2}\sqrt{d\tan(fx+e)+c}\right)}{c\sqrt{-2c+2\sqrt{c^2+d^2}}-i\sqrt{-2c+2\sqrt{c^2+d^2}}\,d-\sqrt{c^2+d^2}\sqrt{-2c+2\sqrt{c^2+d^2}}}\right)}{\sqrt{-2c+2\sqrt{c^2+d^2}}\,f\left(-\frac{id}{c-\sqrt{c^2+d^2}}+1\right)}-\frac{2\left(3i\left(d\tan(fx+e)+c\right)^{\frac{5}{2}}a^3d^8f^4-5i\left(d\tan(fx+e)+c\right)^{\frac{3}{2}}a^3cd^8f^4+15\left(d\tan(fx+e)+c\right)^{\frac{3}{2}}a^3d^9f^4-60i\sqrt{d\tan(fx+e)+c}\,a^3d^{10}f^4\right)}{15\,d^{10}f^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -16*(-I*a^3*c - a^3*d)*arctan(2*(sqrt(d*tan(f*x + e) + c)*c - sqrt(c^2 + d^
2)*sqrt(d*tan(f*x + e) + c))/(c*sqrt(-2*c + 2*sqrt(c^2 + d^2)) - I*sqrt(-2*
c + 2*sqrt(c^2 + d^2))*d - sqrt(c^2 + d^2)*sqrt(-2*c + 2*sqrt(c^2 + d^2))))
/(sqrt(-2*c + 2*sqrt(c^2 + d^2))*f*(-I*d/(c - sqrt(c^2 + d^2)) + 1)) - 2/15
*(3*I*(d*tan(f*x + e) + c)^(5/2)*a^3*d^8*f^4 - 5*I*(d*tan(f*x + e) + c)^(3/
2)*a^3*c*d^8*f^4 + 15*(d*tan(f*x + e) + c)^(3/2)*a^3*d^9*f^4 - 60*I*sqrt(d*
tan(f*x + e) + c)*a^3*d^10*f^4)/(d^10*f^5)
```

Mupad [B]
time = 8.89, size = 200, normalized size = 1.33

$$-\left(\left(c-d\,1\mathrm{i}\right)\,\left(\frac{a^3\,\left(c-d\,1\mathrm{i}\right)\,2\mathrm{i}}{d^2\,f}-\frac{a^3\,\left(c+d\,1\mathrm{i}\right)\,4\mathrm{i}}{d^2\,f}\right)+\frac{a^3\,{\left(c+d\,1\mathrm{i}\right)}^2\,2\mathrm{i}}{d^2\,f}\right)\,\sqrt{c+d\,\tan\left(e+f\,x\right)}-\left(\frac{a^3\,\left(c-d\,1\mathrm{i}\right)\,2\mathrm{i}}{3\,d^2\,f}-\frac{a^3\,\left(c+d\,1\mathrm{i}\right)\,4\mathrm{i}}{3\,d^2\,f}\right)\,{\left(c+d\,\tan(e+f\,x)\right)}^{3/2}-\frac{a^3\,{\left(c+d\,\tan(e+f\,x)\right)}^{5/2}\,2\mathrm{i}}{5\,d^2\,f}+\frac{\sqrt{16\mathrm{i}}\,a^3\,\mathrm{atan}\left(\frac{\sqrt{16\mathrm{i}}\,\sqrt{c+d\,\tan\left(e+f\,x\right)}\,1\mathrm{i}}{4\,\sqrt{d+c\,1\mathrm{i}}}\right)\,\sqrt{d+c\,1\mathrm{i}}\,2\mathrm{i}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^3*(c + d*tan(e + f*x))^(1/2),x)
```

```
[Out] (16i^(1/2)*a^3*atan((16i^(1/2)*(c + d*tan(e + f*x))^(1/2)*1i)/(4*(c*1i + d)
^(1/2)))*(c*1i + d)^(1/2)*2i)/f - ((a^3*(c - d*1i)*2i)/(3*d^2*f) - (a^3*(c
+ d*1i)*4i)/(3*d^2*f))*(c + d*tan(e + f*x))^(3/2) - (a^3*(c + d*tan(e + f*x
))^(5/2)*2i)/(5*d^2*f) - ((c - d*1i)*((a^3*(c - d*1i)*2i)/(d^2*f) - (a^3*(c
 + d*1i)*4i)/(d^2*f)) + (a^3*(c + d*1i)^2*2i)/(d^2*f))*(c + d*tan(e + f*x))
^(1/2)
```

3.1102 $\int (a+ia\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=100

$$-\frac{4ia^2\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{4ia^2\sqrt{c+d\tan(e+fx)}}{f}-\frac{2a^2(c+d\tan(e+fx))^{3/2}}{3df}$$

```
[Out] -4*I*a^2*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))*(c-I*d)^(1/2)/f+4*I*
a^2*(c+d*tan(f*x+e))^(1/2)/f-2/3*a^2*(c+d*tan(f*x+e))^(3/2)/d/f
```

Rubi [A]
time = 0.17, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3624, 3609, 3618, 65, 214}

$$-\frac{2a^2(c+d\tan(e+fx))^{3/2}}{3df}+\frac{4ia^2\sqrt{c+d\tan(e+fx)}}{f}-\frac{4ia^2\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2*Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((-4*I)*a^2*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/
f + ((4*I)*a^2*Sqrt[c + d*Tan[e + f*x]])/f - (2*a^2*(c + d*Tan[e + f*x])^(3
/2))/(3*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
```

```
0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int (a+ia\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}\,dx = -\frac{2a^2(c+d\tan(e+fx))^{3/2}}{3df}+\int \left(2a^2+2ia^2\tan(e+fx\right.$$

$$= \frac{4ia^2\sqrt{c+d\tan(e+fx)}}{f}-\frac{2a^2(c+d\tan(e+fx))^{3/2}}{3df}+$$

$$= \frac{4ia^2\sqrt{c+d\tan(e+fx)}}{f}-\frac{2a^2(c+d\tan(e+fx))^{3/2}}{3df}+$$

$$= \frac{4ia^2\sqrt{c+d\tan(e+fx)}}{f}-\frac{2a^2(c+d\tan(e+fx))^{3/2}}{3df}-$$

$$= -\frac{4ia^2\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{4ia^2\sqrt{}}{}$$

Mathematica [A]
time = 2.98, size = 155, normalized size = 1.55

$$-\frac{2a^2e^{-2ie}(\cos(2(e+fx))+i\sin(2(e+fx)))\left(6i\sqrt{c-id}\,d\tanh^{-1}\left(\frac{\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}}{\sqrt{c-id}}\right)+\sqrt{c+d\tan(e+fx)}\,(c-6id+d\tan(e+fx))\right)}{3df(\cos(fx)+i\sin(fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2*Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] (-2*a^2*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)])*((6*I)*Sqrt[c - I*d]*d*ArcT
anh[Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))]]/Sq
rt[c - I*d]] + Sqrt[c + d*Tan[e + f*x]]*(c - (6*I)*d + d*Tan[e + f*x])))/(3
*d*E^((2*I)*e)*f*(Cos[f*x] + I*Sin[f*x])^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 703 vs. $2(84) = 168$.

time = 0.25, size = 704, normalized size = 7.04

method	result
derivativedivides	$\frac{2a^2\left(-\frac{(c+d\tan(fx+e))^{\frac{3}{2}}}{3}+2id\sqrt{c+d\tan(fx+e)}-2d\left(\frac{\left(-i\sqrt{2\sqrt{c^2+d^2}+2c}\,\sqrt{c^2+d^2}-i\sqrt{2\sqrt{\cdots}}\right.}{\cdots}\right.\right.}{\cdots}$
default	$\frac{2a^2\left(-\frac{(c+d\tan(fx+e))^{\frac{3}{2}}}{3}+2id\sqrt{c+d\tan(fx+e)}-2d\left(\frac{\left(-i\sqrt{2\sqrt{c^2+d^2}+2c}\,\sqrt{c^2+d^2}-i\sqrt{2\sqrt{\cdots}}\right.}{\cdots}\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^2/d*(-1/3*(c+d*tan(f*x+e))^(3/2)+2*I*d*(c+d*tan(f*x+e))^(1/2)-2*d*(1/
(4*(c^2+d^2)^(1/2)+4*c)*(1/2*(-I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1
/2)-I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d)*ln(d
*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2
)^(1/2))+2*(2*I*(c^2+d^2)^(1/2)*c+2*I*c^2+2*I*d^2+1/2*(-I*(2*(c^2+d^2)^(1/2
)+2*c)^(1/2)*(c^2+d^2)^(1/2)-I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c-(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*d)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)
^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(
c^2+d^2)^(1/2)-2*c)^(1/2)))+1/(4*(c^2+d^2)^(1/2)+4*c)*(1/2*(I*(2*(c^2+d^2)^
```

```
(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)+I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c+(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*d)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(2*I*(c^2+d^2)^(1/2)*c+2*I*c^2+2*I*
d^2-1/2*(I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)+I*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*c+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d)*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2
+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^2,x, algorithm="maxima"
)
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^2*sqrt(d*tan(f*x + e) + c), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 413 vs. 2(83) = 166.
time = 1.04, size = 413, normalized size = 4.13

$$\frac{3\left(dfe^{(2i\,fx+2i\,e)}+df\right)\sqrt{-\frac{a^4c-i\,a^4d}{f^2}}\log\left(\frac{2\left(a^2c+\left(i\,fe^{(2i\,fx+2i\,e)}+i\,f\right)\sqrt{\frac{(c-i\,d)e^{(2i\,fx+2i\,e)}+c+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{-\frac{a^4c-i\,a^4d}{f^2}}+(a^2c-i\,a^2d)e^{(2i\,fx+2i\,e)}\right)e^{(-2i\,fx-2i\,e)}}{a^2}\right)-3\left(dfe^{(2i\,fx+2i\,e)}+df\right)\sqrt{-\frac{a^4c-i\,a^4d}{f^2}}\log\left(\frac{2\left(a^2c+\left(-i\,fe^{(2i\,fx+2i\,e)}-i\,f\right)\sqrt{\frac{(c-i\,d)e^{(2i\,fx+2i\,e)}+c+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{-\frac{a^4c-i\,a^4d}{f^2}}+(a^2c-i\,a^2d)e^{(2i\,fx+2i\,e)}\right)e^{(-2i\,fx-2i\,e)}}{a^2}\right)-2\left(a^2c-5i\,a^2d+(a^2c-7i\,a^2d)e^{(2i\,fx+2i\,e)}\right)\sqrt{\frac{(c-i\,d)e^{(2i\,fx+2i\,e)}+c+i\,d}{e^{(2i\,fx+2i\,e)}+1}}}{3\left(dfe^{(2i\,fx+2i\,e)}+df\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] 1/3*(3*(d*f*e^(2*I*f*x + 2*I*e) + d*f)*sqrt(-(a^4*c - I*a^4*d)/f^2)*log(2*(
a^2*c + (I*f*e^(2*I*f*x + 2*I*e) + I*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e)
 + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(a^4*c - I*a^4*d)/f^2) + (a^2*
c - I*a^2*d)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/a^2) - 3*(d*f*e^(2*I
*f*x + 2*I*e) + d*f)*sqrt(-(a^4*c - I*a^4*d)/f^2)*log(2*(a^2*c + (-I*f*e^(2
*I*f*x + 2*I*e) - I*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2
*I*f*x + 2*I*e) + 1))*sqrt(-(a^4*c - I*a^4*d)/f^2) + (a^2*c - I*a^2*d)*e^(2
*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/a^2) - 2*(a^2*c - 5*I*a^2*d + (a^2*c
- 7*I*a^2*d)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c +
 I*d)/(e^(2*I*f*x + 2*I*e) + 1)))/(d*f*e^(2*I*f*x + 2*I*e) + d*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int\sqrt{c+d\tan(e+fx)}\tan^2(e+fx)\,dx+\int\left(-2i\sqrt{c+d\tan(e+fx)}\tan(e+fx)\right)dx+\int\left(-\sqrt{c+d\tan(e+fx)}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -a**2*(Integral(sqrt(c + d*tan(e + f*x))*tan(e + f*x)**2, x) + Integral(-2*
I*sqrt(c + d*tan(e + f*x))*tan(e + f*x), x) + Integral(-sqrt(c + d*tan(e +
f*x)), x))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(83) = 166.
time = 0.57, size = 227, normalized size = 2.27

$$-\frac{8\left(-i\,a^{2}c-a^{2}d\right)\arctan\left(\frac{2\left(\sqrt{d\tan\left(fx+e\right)+c}\,c-\sqrt{c^{2}+d^{2}}\,\sqrt{d\tan\left(fx+e\right)+c}\right)}{c\sqrt{-2\,c+2\,\sqrt{c^{2}+d^{2}}}-i\,\sqrt{-2\,c+2\,\sqrt{c^{2}+d^{2}}}\,d-\sqrt{c^{2}+d^{2}}\,\sqrt{-2\,c+2\,\sqrt{c^{2}+d^{2}}}}\right)}{\sqrt{-2\,c+2\,\sqrt{c^{2}+d^{2}}}\,f\left(-\frac{i\,d}{c-\sqrt{c^{2}+d^{2}}}+1\right)}-\frac{2\left(\left(d\tan\left(fx+e\right)+c\right)^{\frac{3}{2}}a^{2}d^{2}f^{2}-6i\,\sqrt{d\tan\left(fx+e\right)+c}\,a^{2}d^{3}f^{2}\right)}{3\,d^{3}f^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -8*(-I*a^2*c - a^2*d)*arctan(2*(sqrt(d*tan(f*x + e) + c)*c - sqrt(c^2 + d^2
)*sqrt(d*tan(f*x + e) + c))/(c*sqrt(-2*c + 2*sqrt(c^2 + d^2)) - I*sqrt(-2*c
 + 2*sqrt(c^2 + d^2))*d - sqrt(c^2 + d^2)*sqrt(-2*c + 2*sqrt(c^2 + d^2))))/
(sqrt(-2*c + 2*sqrt(c^2 + d^2))*f*(-I*d/(c - sqrt(c^2 + d^2)) + 1)) - 2/3*(
(d*tan(f*x + e) + c)^(3/2)*a^2*d^2*f^2 - 6*I*sqrt(d*tan(f*x + e) + c)*a^2*d
^3*f^2)/(d^3*f^3)
```

Mupad [B]
time = 7.07, size = 90, normalized size = 0.90

$$\frac{a^{2}\,\sqrt{c+d\tan\left(e+f\,x\right)}\,4\mathrm{i}}{f}-\frac{2\,a^{2}\,{\left(c+d\tan(e+f\,x)\right)}^{3/2}}{3\,d\,f}-\frac{2\,\sqrt{4\mathrm{i}}\,a^{2}\,\mathrm{atanh}\left(\frac{\sqrt{4\mathrm{i}}\,\sqrt{c+d\tan\left(e+f\,x\right)}}{2\,\sqrt{d+c\,1\mathrm{i}}}\right)\,\sqrt{d+c\,1\mathrm{i}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^2*(c + d*tan(e + f*x))^(1/2),x)
```

```
[Out] (a^2*(c + d*tan(e + f*x))^(1/2)*4i)/f - (2*a^2*(c + d*tan(e + f*x))^(3/2))/
(3*d*f) - (2*4i^(1/2)*a^2*atanh((4i^(1/2)*(c + d*tan(e + f*x))^(1/2))/(2*(c
*1i + d)^(1/2)))*(c*1i + d)^(1/2))/f
```

3.1103 $\int (a+ia\tan(e+fx))\sqrt{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=69

$$-\frac{2ia\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{2ia\sqrt{c+d\tan(e+fx)}}{f}$$

```
[Out] -2*I*a*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))*(c-I*d)^(1/2)/f+2*I*a*
(c+d*tan(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.10, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3609, 3618, 65, 214}

$$\frac{2ia\sqrt{c+d\tan(e+fx)}}{f}-\frac{2ia\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])*Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((-2*I)*a*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/f
+ ((2*I)*a*Sqrt[c + d*Tan[e + f*x]])/f
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rubi steps

$$\int (a+ia\tan(e+fx))\sqrt{c+d\tan(e+fx)}\,dx = \frac{2ia\sqrt{c+d\tan(e+fx)}}{f} + \int \frac{a(c-id)+a(ic+d)\tan(e+}{\sqrt{c+d\tan(e+fx)}}$$

$$= \frac{2ia\sqrt{c+d\tan(e+fx)}}{f} + \frac{(ia^2(c-id)^2)\,\mathrm{Subst}\left(\int \frac{}{(a^2(ic+d)}\right.}{}$$

$$= \frac{2ia\sqrt{c+d\tan(e+fx)}}{f} - \frac{(2a^3(c-id)^3)\,\mathrm{Subst}\left(\int \frac{}{-\frac{a^2c(c-i}{}}\right.}{}$$

$$= -\frac{2ia\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f} + \frac{2ia\sqrt{c+}}{}$$

Mathematica [A]
time = 1.22, size = 88, normalized size = 1.28

$$\frac{2ia\left(-\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}}{\sqrt{c-id}}\right)+\sqrt{c+d\tan(e+fx)}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])*Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((2*I)*a*(-(Sqrt[c - I*d]*ArcTanh[Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*x)))
/(1 + E^((2*I)*(e + f*x))))]/Sqrt[c - I*d]]) + Sqrt[c + d*Tan[e + f*x]]))/f
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 684 vs. 2(57) = 114.
time = 0.29, size = 685, normalized size = 9.93

method	result
derivativedivides	$\frac{a\left(2i\sqrt{c+d\tan(fx+e)}+\frac{-\left(-i\sqrt{2\sqrt{c^2+d^2}+2c}\,\sqrt{c^2+d^2}-i\sqrt{2\sqrt{c^2+d^2}+2c}\,c-\sqrt{}\right.}{}\right.}{}$
default	$\frac{a\left(2i\sqrt{c+d\tan(fx+e)}+\frac{-\left(-i\sqrt{2\sqrt{c^2+d^2}+2c}\,\sqrt{c^2+d^2}-i\sqrt{2\sqrt{c^2+d^2}+2c}\,c-\sqrt{}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a*(2*I*(c+d*tan(f*x+e))^(1/2)+2/(4*(c^2+d^2)^(1/2)+4*c)*(-1/2*(-I*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)-I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c-
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d)*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))+2*(2*I*(c^2+d^2)^(1/2)*c+2*I*
c^2+2*I*d^2+1/2*(-I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)-I*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*c-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d)*(2*(c^2+d^2)^(1/2
)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)
^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+2/(4*(c^2+
d^2)^(1/2)+4*c)*(1/2*(-I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)-I*(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*c-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d)*ln(d*tan(f*x
+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))
+2*(-2*I*(c^2+d^2)^(1/2)*c-2*I*c^2-2*I*d^2-1/2*(-I*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*(c^2+d^2)^(1/2)-I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c-(2*(c^2+d^2)^(1/2)+
2*c)^(1/2)*d)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*
arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2
)^(1/2)-2*c)^(1/2))))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4714 vs. $2(55) = 110$.

time = 0.70, size = 4714, normalized size = 68.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] -4*((2*(I*sqrt(2)*a*cos(2*f*x + 2*e) - sqrt(2)*a*sin(2*f*x + 2*e) + I*sqrt(
2)*a)*arctan2(-2*d*cos(2*f*x + 2*e) + 2*c*sin(2*f*x + 2*e) - (4*c^2*cos(2*f
*x + 2*e)^2 + 4*c^2*sin(2*f*x + 2*e)^2 + (c^2 + d^2)*cos(4*f*x + 4*e)^2 + 4
*c^2*cos(2*f*x + 2*e) + (c^2 + d^2)*sin(4*f*x + 4*e)^2 + 4*c*d*sin(2*f*x +
2*e) + c^2 + d^2 + 2*(2*c^2*cos(2*f*x + 2*e) - 2*c*d*sin(2*f*x + 2*e) + c^2
 - d^2)*cos(4*f*x + 4*e) + 4*(c*d*cos(2*f*x + 2*e) + c^2*sin(2*f*x + 2*e) +
 c*d)*sin(4*f*x + 4*e))^(1/4)*(sqrt(2)*sqrt(-c + sqrt(c^2 + d^2))*cos(1/2*a
rctan2(-d*cos(4*f*x + 4*e) + c*sin(4*f*x + 4*e) + 2*c*sin(2*f*x + 2*e) + d,
 c*cos(4*f*x + 4*e) + 2*c*cos(2*f*x + 2*e) + d*sin(4*f*x + 4*e) + c)) - sqr
t(2)*sqrt(c + sqrt(c^2 + d^2))*sin(1/2*arctan2(-d*cos(4*f*x + 4*e) + c*sin(
4*f*x + 4*e) + 2*c*sin(2*f*x + 2*e) + d, c*cos(4*f*x + 4*e) + 2*c*cos(2*f*x
 + 2*e) + d*sin(4*f*x + 4*e) + c))), 2*c*cos(2*f*x + 2*e) + 2*d*sin(2*f*x +
 2*e) + (4*c^2*cos(2*f*x + 2*e)^2 + 4*c^2*sin(2*f*x + 2*e)^2 + (c^2 + d^2)*
cos(4*f*x + 4*e)^2 + 4*c^2*cos(2*f*x + 2*e) + (c^2 + d^2)*sin(4*f*x + 4*e)^
2 + 4*c*d*sin(2*f*x + 2*e) + c^2 + d^2 + 2*(2*c^2*cos(2*f*x + 2*e) - 2*c*d*
sin(2*f*x + 2*e) + c^2 - d^2)*cos(4*f*x + 4*e) + 4*(c*d*cos(2*f*x + 2*e) +
c^2*sin(2*f*x + 2*e) + c*d)*sin(4*f*x + 4*e))^(1/4)*(sqrt(2)*sqrt(c + sqrt(
c^2 + d^2))*cos(1/2*arctan2(-d*cos(4*f*x + 4*e) + c*sin(4*f*x + 4*e) + 2*c*
sin(2*f*x + 2*e) + d, c*cos(4*f*x + 4*e) + 2*c*cos(2*f*x + 2*e) + d*sin(4*f
*x + 4*e) + c)) + sqrt(2)*sqrt(-c + sqrt(c^2 + d^2))*sin(1/2*arctan2(-d*cos
(4*f*x + 4*e) + c*sin(4*f*x + 4*e) + 2*c*sin(2*f*x + 2*e) + d, c*cos(4*f*x
+ 4*e) + 2*c*cos(2*f*x + 2*e) + d*sin(4*f*x + 4*e) + c))) + 2*c) + (sqrt(2)
*a*cos(2*f*x + 2*e) + I*sqrt(2)*a*sin(2*f*x + 2*e) + sqrt(2)*a)*log(8*c^2*c
os(2*f*x + 2*e) + 4*(c^2 + d^2)*cos(2*f*x + 2*e)^2 + 8*c*d*sin(2*f*x + 2*e)
 + 4*(c^2 + d^2)*sin(2*f*x + 2*e)^2 + 4*sqrt(4*c^2*cos(2*f*x + 2*e)^2 + 4*c
^2*sin(2*f*x + 2*e)^2 + (c^2 + d^2)*cos(4*f*x + 4*e)^2 + 4*c^2*cos(2*f*x +
2*e) + (c^2 + d^2)*sin(4*f*x + 4*e)^2 + 4*c*d*sin(2*f*x + 2*e) + c^2 + d^2
+ 2*(2*c^2*cos(2*f*x + 2*e) - 2*c*d*sin(2*f*x + 2*e) + c^2 - d^2)*cos(4*f*x
 + 4*e) + 4*(c*d*cos(2*f*x + 2*e) + c^2*sin(2*f*x + 2*e) + c*d)*sin(4*f*x +
 4*e))*sqrt(c^2 + d^2)*(cos(1/2*arctan2(-d*cos(4*f*x + 4*e) + c*sin(4*f*x +
 4*e) + 2*c*sin(2*f*x + 2*e) + d, c*cos(4*f*x + 4*e) + 2*c*cos(2*f*x + 2*e)
 + d*sin(4*f*x + 4*e) + c))^2 + sin(1/2*arctan2(-d*cos(4*f*x + 4*e) + c*sin
(4*f*x + 4*e) + 2*c*sin(2*f*x + 2*e) + d, c*cos(4*f*x + 4*e) + 2*c*cos(2*f*
x + 2*e) + d*sin(4*f*x + 4*e) + c))^2) + 4*c^2 + 4*(4*c^2*cos(2*f*x + 2*e)^
2 + 4*c^2*sin(2*f*x + 2*e)^2 + (c^2 + d^2)*cos(4*f*x + 4*e)^2 + 4*c^2*cos(2
*f*x + 2*e) + (c^2 + d^2)*sin(4*f*x + 4*e)^2 + 4*c*d*sin(2*f*x + 2*e) + c^2
 + d^2 + 2*(2*c^2*cos(2*f*x + 2*e) - 2*c*d*sin(2*f*x + 2*e) + c^2 - d^2)*co
s(4*f*x + 4*e) + 4*(c*d*cos(2*f*x + 2*e) + c^2*sin(2*f*x + 2*e) + c*d)*sin(
```

```
4*f*x + 4*e))^(1/4)*(((sqrt(2)*c*cos(2*f*x + 2*e) + sqrt(2)*d*sin(2*f*x + 2
*e) + sqrt(2)*c)*cos(1/2*arctan2(-d*cos(4*f*x + 4*e) + c*sin(4*f*x + 4*e) +
 2*c*sin(2*f*x + 2*e) + d, c*cos(4*f*x + 4*e) + 2*c*cos(2*f*x + 2*e) + d*si
n(4*f*x + 4*e) + c)) - (sqrt(2)*d*cos(2*f*x + 2*e) - sqrt(2)*c*sin(2*f*x +
2*e))*sin(1/2*arctan2(-d*cos(4*f*x + 4*e) + c*sin(4*f*x + 4*e) + 2*c*sin(2*
f*x + 2*e) + d, c*cos(4*f*x + 4*e) + 2*c*cos(2*f*x + 2*e) + d*sin(4*f*x + 4
*e) + c)))*sqrt(c + sqrt(c^2 + d^2)) + ((sqrt(2)*d*cos(2*f*x + 2*e) - sqrt(
2)*c*sin(2*f*x + 2*e))*cos(1/2*arctan2(-d*cos(4*f*x + 4*e) + c*sin(4*f*x +
4*e) + 2*c*sin(2*f*x + 2*e) + d, c*cos(4*f*x + 4*e) + 2*c*cos(2*f*x + 2*e)
+ d*sin(4*f*x + 4*e) + c)) + (sqrt(2)*c*cos(2*f*x + 2*e) + sqrt(2)*d*sin(2*
f*x + 2*e) + sqrt(2)*c)*sin(1/2*arctan2(-d*cos(4*f*x + 4*e) + c*sin(4*f*x +
 4*e) + 2*c*sin(2*f*x + 2*e) + d, c*cos(4*f*x + 4*e) + 2*c*cos(2*f*x + 2*e)
 + d*sin(4*f*x + 4*e) + c)))*sqrt(-c + sqrt(c^2 + d^2)))))*sqrt(c + sqrt(c^
2 + d^2)) + (2*(sqrt(2)*a*cos(2*f*x + 2*e) + I*sqrt(2)*a*sin(2*f*x + 2*e) +
 sqrt(2)*a)*arctan2(-2*d*cos(2*f*x + 2*e) + 2*c*sin(2*f*x + 2*e) - (4*c^2*c
os(2*f*x + 2*e)^2 + 4*c^2*sin(2*f*x + 2*e)^2 + (c^2 + d^2)*cos(4*f*x + 4*e)
^2 + 4*c^2*cos(2*f*x + 2*e) + (c^2 + d^2)*sin(4*f*x + 4*e)^2 + 4*c*d*sin(2*
f*x + 2*e) + c^2 + d^2 + 2*(2*c^2*cos(2*f*x + 2*e) - 2*c*d*sin(2*f*x + 2*e)
 + c^2 - d^2)*cos(4*f*x + 4*e) + 4*(c*d*cos(2*f*x + 2*e) + c^2*sin(2*f*x +
2*e) + c*d)*sin(4*f*x + 4*e))^(1/4)*(sqrt(2)*sqrt(-c + sqrt(c^2 + d^2))*cos
(1/2*arctan2(-d*cos(4*f*x + 4*e) + c*sin(4*f*x + 4*e) + 2*c*sin(2*f*x + 2*e
) + d, c*cos(4*f*x + 4*e) + 2*c*cos(2*f*x + 2*e) + d*sin(4*f*x + 4*e) + c))
 - sqrt(2)*sqrt(c + sqrt(c^2 + d^2))*sin(1/2*arctan2(-d*cos(4*f*x + 4*e) +
c*sin(4*f*x + 4*e) + 2*c*sin(2*f*x + 2*e) + d, c*cos(4*f*x + 4*e) + 2*c*cos
(2*f*x + 2*e) + d*sin(4*f*x + 4*e) + c))), 2*c*cos(2*f*x + 2*e) + 2*d*sin(2
*f*x + 2*e) + (4*c^2*cos(2*f*x + 2*e)^2 + 4*c^2*sin(2*f*x + 2*e)^2 + (c^2 +
 d^2)*cos(4*f*x + 4*e)^2 + 4*c^2*cos(2*f*x + 2*e) + (c^2 + d^2)*sin(4*f*x +
 4*e)^2 + 4*c*d*sin(2*f*x + 2*e) + c^2 + d^2 + 2*(2*c^2*cos(2*f*x + 2*e) -
2*c*d*sin(2*f*x + 2*e) + c^2 - d^2)*cos(4*f*x +...
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(55) = 110$.
time = 0.91, size = 318, normalized size = 4.61

$$\frac{f\sqrt{-\frac{a^2c-i\,a^2d}{f^2}}\log\left(\frac{2\left(ac+\left(i\,fe^{(2i\,fx+2i\,e)}+i\,f\right)\sqrt{\frac{(c-i\,d)e^{(2i\,fx+2i\,e)}+c+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{-\frac{a^2c-i\,a^2d}{f^2}}+(ac-i\,ad)e^{(2i\,fx+2i\,e)}\right)e^{(-2i\,fx-2i\,e)}}{a}\right)-f\sqrt{-\frac{a^2c-i\,a^2d}{f^2}}\log\left(\frac{2\left(ac+\left(-i\,fe^{(2i\,fx+2i\,e)}-i\,f\right)\sqrt{\frac{(c-i\,d)e^{(2i\,fx+2i\,e)}+c+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{-\frac{a^2c-i\,a^2d}{f^2}}+(ac-i\,ad)e^{(2i\,fx+2i\,e)}\right)e^{(-2i\,fx-2i\,e)}}{a}\right)+4i\,a\sqrt{\frac{(c-i\,d)e^{(2i\,fx+2i\,e)}+c+i\,d}{e^{(2i\,fx+2i\,e)}+1}}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(f*sqrt(-(a^2*c - I*a^2*d)/f^2)*log(2*(a*c + (I*f*e^(2*I*f*x + 2*I*e) +
 I*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) +
 1))*sqrt(-(a^2*c - I*a^2*d)/f^2) + (a*c - I*a*d)*e^(2*I*f*x + 2*I*e))*e^(-
2*I*f*x - 2*I*e)/a) - f*sqrt(-(a^2*c - I*a^2*d)/f^2)*log(2*(a*c + (-I*f*e^(
2*I*f*x + 2*I*e) - I*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(
```

```
2*I*f*x + 2*I*e) + 1))*sqrt(-(a^2*c - I*a^2*d)/f^2) + (a*c - I*a*d)*e^(2*I*
f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/a) + 4*I*a*sqrt(((c - I*d)*e^(2*I*f*x +
2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-i\sqrt{c+d\tan\left(e+fx\right)}\right)dx+\int\sqrt{c+d\tan\left(e+fx\right)}\tan\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(a+I*a*tan(f*x+e)),x)
```

```
[Out] I*a*(Integral(-I*sqrt(c + d*tan(e + f*x)), x) + Integral(sqrt(c + d*tan(e +
 f*x))*tan(e + f*x), x))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(55) = 110.
time = 0.48, size = 185, normalized size = 2.68

$$\frac{2i\sqrt{d\tan\left(fx+e\right)+c}\,a}{f}+\frac{4\left(iac+ad\right)\arctan\left(\frac{2\left(\sqrt{d\tan\left(fx+e\right)+c}\,c-\sqrt{c^2+d^2}\sqrt{d\tan\left(fx+e\right)+c}\right)}{c\sqrt{-2c+2\sqrt{c^2+d^2}}-i\sqrt{-2c+2\sqrt{c^2+d^2}}\,d-\sqrt{c^2+d^2}\sqrt{-2c+2\sqrt{c^2+d^2}}}\right)}{\sqrt{-2c+2\sqrt{c^2+d^2}}\,f\left(-\frac{id}{c-\sqrt{c^2+d^2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] 2*I*sqrt(d*tan(f*x + e) + c)*a/f + 4*(I*a*c + a*d)*arctan(2*(sqrt(d*tan(f*x
 + e) + c)*c - sqrt(c^2 + d^2)*sqrt(d*tan(f*x + e) + c))/(c*sqrt(-2*c + 2*s
qrt(c^2 + d^2)) - I*sqrt(-2*c + 2*sqrt(c^2 + d^2))*d - sqrt(c^2 + d^2)*sqrt
(-2*c + 2*sqrt(c^2 + d^2))))/(sqrt(-2*c + 2*sqrt(c^2 + d^2))*f*(-I*d/(c - s
qrt(c^2 + d^2)) + 1))
```

Mupad [B]
time = 7.20, size = 854, normalized size = 12.38

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)*(c + d*tan(e + f*x))^(1/2),x)
```

```
[Out] 2*atanh((32*a^2*d^4*((-a^4*d^2*f^4)^(1/2)/(4*f^4) - (a^2*c)/(4*f^2))^(1/2)*
(c + d*tan(e + f*x))^(1/2))/((a*d^4*(-a^4*d^2*f^4)^(1/2)*16i)/f^3 + (a*c^2*
d^2*(-a^4*d^2*f^4)^(1/2)*16i)/f^3) - (32*c*d^2*((-a^4*d^2*f^4)^(1/2)/(4*f^4
) - (a^2*c)/(4*f^2))^(1/2)*(c + d*tan(e + f*x))^(1/2)*(-a^4*d^2*f^4)^(1/2))
/((a*d^4*(-a^4*d^2*f^4)^(1/2)*16i)/f + (a*c^2*d^2*(-a^4*d^2*f^4)^(1/2)*16i)
```

/f))*(((-a^4*d^2*f^4)^(1/2) - a^2*c*f^2)/(4*f^4))^(1/2) - 2*atanh((32*a^2*d^4*(- (-a^4*d^2*f^4)^(1/2)/(4*f^4) - (a^2*c)/(4*f^2))^(1/2)*(c + d*tan(e + f*x))^(1/2))/((a*d^4*(-a^4*d^2*f^4)^(1/2)*16i)/f^3 + (a*c^2*d^2*(-a^4*d^2*f^4)^(1/2)*16i)/f^3) + (32*c*d^2*(- (-a^4*d^2*f^4)^(1/2)/(4*f^4) - (a^2*c)/(4*f^2))^(1/2)*(c + d*tan(e + f*x))^(1/2)*(-a^4*d^2*f^4)^(1/2))/((a*d^4*(-a^4*d^2*f^4)^(1/2)*16i)/f + (a*c^2*d^2*(-a^4*d^2*f^4)^(1/2)*16i)/f))*(-((-a^4*d^2*f^4)^(1/2) + a^2*c*f^2)/(4*f^4))^(1/2) - atanh((f^3*((16*(a^2*d^4 - a^2*c^2*d^2)*(c + d*tan(e + f*x))^(1/2))/f^2 + (16*c*d^2*((-a^4*d^2*f^4)^(1/2) + a^2*c*f^2)*(c + d*tan(e + f*x))^(1/2))/f^4)*(-((-a^4*d^2*f^4)^(1/2) + a^2*c*f^2)/f^4)^(1/2))/(16*(a^3*d^5 + a^3*c^2*d^3)))*(-((-a^4*d^2*f^4)^(1/2) + a^2*c*f^2)/f^4)^(1/2) - atanh((f^3*((16*(a^2*d^4 - a^2*c^2*d^2)*(c + d*tan(e + f*x))^(1/2))/f^2 - (16*c*d^2*((-a^4*d^2*f^4)^(1/2) - a^2*c*f^2)*(c + d*tan(e + f*x))^(1/2))/f^4)*(((-a^4*d^2*f^4)^(1/2) - a^2*c*f^2)/f^4)^(1/2))/(16*(a^3*d^5 + a^3*c^2*d^3)))*(((-a^4*d^2*f^4)^(1/2) - a^2*c*f^2)/f^4)^(1/2) + (a*(c + d*tan(e + f*x))^(1/2)*2i)/f

3.1104 $\int \frac{\sqrt{c+d\tan(e+fx)}}{a+ia\tan(e+fx)}\,dx$

Optimal. Leaf size=140

$$-\frac{i\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{2af}+\frac{ic\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{2a\sqrt{c+id}\,f}+\frac{i\sqrt{c+d\tan(e+fx)}}{2f(a+ia\tan(e+fx))}$$

```
[Out] -1/2*I*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))*(c-I*d)^(1/2)/a/f+1/2*
I*c*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/a/f/(c+I*d)^(1/2)+1/2*I*(
c+d*tan(f*x+e))^(1/2)/f/(a+I*a*tan(f*x+e))
```

Rubi [A]
time = 0.24, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3630, 3620, 3618, 65, 214}

$$\frac{i\sqrt{c+d\tan(e+fx)}}{2f(a+ia\tan(e+fx))}-\frac{i\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{2af}+\frac{ic\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{2af\sqrt{c+id}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Tan[e + f*x]]/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] ((-1/2*I)*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/(a
*f) + ((I/2)*c*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/(a*Sqrt[c +
 I*d]*f) + ((I/2)*Sqrt[c + d*Tan[e + f*x]])/(f*(a + I*a*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
```

```
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3630

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(a*c + b*d))*((c + d*Tan[e + f*x])^n/(2*(
b*c - a*d)*f*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c
+ d*Tan[e + f*x])^(n - 1)*Simp[a*c*d*(n - 1) + b*c^2 + b*d^2*n - d*(b*c - a
*d)*(n - 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[0, n, 1]
```

Rubi steps

$$\int \frac{\sqrt{c+d\tan(e+fx)}}{a+ia\tan(e+fx)}\,dx = \frac{i\sqrt{c+d\tan(e+fx)}}{2f(a+ia\tan(e+fx))} + \frac{\int \frac{\frac{1}{2}a(c+id)(2ic+d)+\frac{1}{2}a(ic-d)d\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx}{2a^2(ic-d)}$$

$$= \frac{i\sqrt{c+d\tan(e+fx)}}{2f(a+ia\tan(e+fx))} + \frac{c\int \frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx}{4a} + \frac{(c-id)\int \frac{1+i\,\text{t}}{\sqrt{c+d\,\text{t}}}}{4a}$$

$$= \frac{i\sqrt{c+d\tan(e+fx)}}{2f(a+ia\tan(e+fx))} - \frac{(ic)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{c+idx}}\,dx, x, -i\tan(e+fx\right.}{4af}$$

$$= \frac{i\sqrt{c+d\tan(e+fx)}}{2f(a+ia\tan(e+fx))} - \frac{c\text{Subst}\left(\int \frac{1}{-1+\frac{ic}{d}-\frac{ix^2}{d}}\,dx, x, \sqrt{c+d\tan(e+fx)}\right)}{2adf}$$

$$= -\frac{i\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{2af} + \frac{ic\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+f}}{\sqrt{c+id}}\right.}{2a\sqrt{c+id}\,f}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 339 vs. 2(140) = 280.

time = 5.23, size = 339, normalized size = 2.42

$$\frac{\sec(e+fx)(\cos(fx)+i\sin(fx))\left(\left(-i\left(\sqrt{c-id}\,\log\left(\frac{2\left(-ide^{2i(e+fx)}+c\left(1+e^{2i(e+fx)}\right)+\sqrt{c-id}\,\left(1+e^{2i(e+fx)}\right)\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)}{\sqrt{c-id}}\right)-\frac{c\log\left(\frac{8ie^{-2ifx}\left(id+c\left(1+e^{2i(e+fx)}\right)+\sqrt{c+id}\,\left(1+e^{2i(e+fx)}\right)\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)}{c\sqrt{c+id}}\right)}{\sqrt{c+id}}\right)\right)(\cos(e)+i\sin(e))+2\cos(e+fx)(i\cos(fx)+\sin(fx))\sqrt{c+d\tan(e+fx)}\right)}{4f(a+ia\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Tan[e + f*x]]/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (Sec[e + f*x]*(Cos[f*x] + I*Sin[f*x])*((-I)*(Sqrt[c - I*d]*Log[(2*((-I)*d*E
^((2*I)*(e + f*x)) + c*(1 + E^((2*I)*(e + f*x))) + Sqrt[c - I*d]*(1 + E^((2
*I)*(e + f*x)))*Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e
+ f*x)))]))/Sqrt[c - I*d]] - (c*Log[((8*I)*(I*d + c*(1 + E^((2*I)*(e + f*x)
)) + Sqrt[c + I*d]*(1 + E^((2*I)*(e + f*x)))*Sqrt[c - (I*d*(-1 + E^((2*I)*(
e + f*x))))/(1 + E^((2*I)*(e + f*x)))]))/(c*Sqrt[c + I*d]*E^((2*I)*f*x))])/
Sqrt[c + I*d])*(Cos[e] + I*Sin[e]) + 2*Cos[e + f*x]*(I*Cos[f*x] + Sin[f*x])
*Sqrt[c + d*Tan[e + f*x]]))/(4*f*(a + I*a*Tan[e + f*x]))
```

Maple [A]

time = 0.80, size = 128, normalized size = 0.91

method	result
derivativedivides	$\frac{2d^2\left(\frac{-\frac{d\sqrt{c+d\tan(fx+e)}}{-d\tan(fx+e)+id}-\frac{ic\arctan\left(\frac{\sqrt{c+d\tan(fx+e)}}{\sqrt{-id-c}}\right)}{\sqrt{-id-c}}}{4d^2}-\frac{i\sqrt{id-c}\,\arctan\left(\frac{\sqrt{c+d\tan}}{\sqrt{id}}\right.}{4d^2}\right.}{fa}$
default	$\frac{2d^2\left(\frac{-\frac{d\sqrt{c+d\tan(fx+e)}}{-d\tan(fx+e)+id}-\frac{ic\arctan\left(\frac{\sqrt{c+d\tan(fx+e)}}{\sqrt{-id-c}}\right)}{\sqrt{-id-c}}}{4d^2}-\frac{i\sqrt{id-c}\,\arctan\left(\frac{\sqrt{c+d\tan}}{\sqrt{id}}\right.}{4d^2}\right.}{fa}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a*d^2*(1/4/d^2*(-d*(c+d*tan(f*x+e))^(1/2)/(-d*tan(f*x+e)+I*d)-I*c/(-I*d
-c)^(1/2)*arctan((c+d*tan(f*x+e))^(1/2)/(-I*d-c)^(1/2)))-1/4*I*(I*d-c)^(1/2
)/d^2*arctan((c+d*tan(f*x+e))^(1/2)/(I*d-c)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 714 vs. $2(108) = 216$.
time = 1.52, size = 714, normalized size = 5.10

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/8*(a*f*sqrt(-(c - I*d)/(a^2*f^2))*e^(2*I*f*x + 2*I*e)*log(-2*((I*a*f*e^(
2*I*f*x + 2*I*e) + I*a*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e
^(2*I*f*x + 2*I*e) + 1))*sqrt(-(c - I*d)/(a^2*f^2)) - (c - I*d)*e^(2*I*f*x
+ 2*I*e) - c)*e^(-2*I*f*x - 2*I*e)) - a*f*sqrt(-(c - I*d)/(a^2*f^2))*e^(2*I
*f*x + 2*I*e)*log(-2*((-I*a*f*e^(2*I*f*x + 2*I*e) - I*a*f)*sqrt(((c - I*d)*
e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(c - I*d)/(
a^2*f^2)) - (c - I*d)*e^(2*I*f*x + 2*I*e) - c)*e^(-2*I*f*x - 2*I*e)) - 2*a*
f*sqrt(-1/4*I*c^2/((I*a^2*c - a^2*d)*f^2))*e^(2*I*f*x + 2*I*e)*log(-1/2*(c^
2*e^(2*I*f*x + 2*I*e) + c^2 + I*c*d - 2*((I*a*c - a*d)*f*e^(2*I*f*x + 2*I*e
) + (I*a*c - a*d)*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I
*f*x + 2*I*e) + 1))*sqrt(-1/4*I*c^2/((I*a^2*c - a^2*d)*f^2)))*e^(-2*I*f*x -
 2*I*e)/((I*a*c - a*d)*f)) + 2*a*f*sqrt(-1/4*I*c^2/((I*a^2*c - a^2*d)*f^2))
*e^(2*I*f*x + 2*I*e)*log(-1/2*(c^2*e^(2*I*f*x + 2*I*e) + c^2 + I*c*d - 2*((
-I*a*c + a*d)*f*e^(2*I*f*x + 2*I*e) + (-I*a*c + a*d)*f)*sqrt(((c - I*d)*e^(
2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-1/4*I*c^2/((I*
a^2*c - a^2*d)*f^2)))*e^(-2*I*f*x - 2*I*e)/((I*a*c - a*d)*f)) - 2*sqrt(((c
- I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*(I*e^(2*I*
f*x + 2*I*e) + I))*e^(-2*I*f*x - 2*I*e)/(a*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{\sqrt{c+d\tan\left(e+fx\right)}}{\tan\left(e+fx\right)-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e)),x)
[Out] -I*Integral(sqrt(c + d*tan(e + f*x))/(tan(e + f*x) - I), x)/a
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(108) = 216$.
time = 0.52, size = 360, normalized size = 2.57

$$\frac{\sqrt{d\tan\left(fx+e\right)+c}\,d}{2\left(d\tan\left(fx+e\right)-i\,d\right)af}+\frac{c\arctan\left(-\frac{2\left(-i\sqrt{d\tan\left(fx+e\right)+c}\,c-i\sqrt{c^2+d^2}\sqrt{d\tan\left(fx+e\right)+c}\right)}{\sqrt{2\,c+2\sqrt{c^2+d^2}}\,c+i\sqrt{2\,c+2\sqrt{c^2+d^2}}\,d+\sqrt{c^2+d^2}\sqrt{2\,c+2\sqrt{c^2+d^2}}}\right)}{a\sqrt{2\,c+2\sqrt{c^2+d^2}}\,f\left(\frac{i\,d}{c+\sqrt{c^2+d^2}}+1\right)}+\frac{\left(i\,c+d\right)\arctan\left(\frac{2\left(\sqrt{d\tan\left(fx+e\right)+c}\,c-\sqrt{c^2+d^2}\sqrt{d\tan\left(fx+e\right)+c}\right)}{c\sqrt{-2\,c+2\sqrt{c^2+d^2}}-i\sqrt{-2\,c+2\sqrt{c^2+d^2}}\,d-\sqrt{c^2+d^2}\sqrt{-2\,c+2\sqrt{c^2+d^2}}}\right)}{a\sqrt{-2\,c+2\sqrt{c^2+d^2}}\,f\left(-\frac{i\,d}{c-\sqrt{c^2+d^2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
[Out] 1/2*sqrt(d*tan(f*x + e) + c)*d/((d*tan(f*x + e) - I*d)*a*f) + c*arctan(-2*(
-I*sqrt(d*tan(f*x + e) + c)*c - I*sqrt(c^2 + d^2)*sqrt(d*tan(f*x + e) + c))
/(sqrt(2*c + 2*sqrt(c^2 + d^2))*c + I*sqrt(2*c + 2*sqrt(c^2 + d^2))*d + sqr
t(c^2 + d^2)*sqrt(2*c + 2*sqrt(c^2 + d^2))))/(a*sqrt(2*c + 2*sqrt(c^2 + d^2
))*f*(I*d/(c + sqrt(c^2 + d^2)) + 1)) + (I*c + d)*arctan(2*(sqrt(d*tan(f*x
+ e) + c)*c - sqrt(c^2 + d^2)*sqrt(d*tan(f*x + e) + c))/(c*sqrt(-2*c + 2*sq
rt(c^2 + d^2)) - I*sqrt(-2*c + 2*sqrt(c^2 + d^2))*d - sqrt(c^2 + d^2)*sqrt(
-2*c + 2*sqrt(c^2 + d^2))))/(a*sqrt(-2*c + 2*sqrt(c^2 + d^2))*f*(-I*d/(c -
sqrt(c^2 + d^2)) + 1))
```

Mupad [B]
time = 7.74, size = 763, normalized size = 5.45

$$-2\,\mathrm{atanh}\left(\frac{4\,a^2\,d^4\,f^2\sqrt{-\frac{c}{16\,a^2\,f^2}+\frac{d\,1\mathrm{i}}{16\,a^2\,f^2}}\,\sqrt{c+d\tan\left(e+f\,x\right)}}{a\,f\,d^5+1\mathrm{i}\,a\,c\,f\,d^4}\right)\sqrt{-\frac{c}{16\,a^2\,f^2}+\frac{d\,1\mathrm{i}}{16\,a^2\,f^2}}-\mathrm{atan}\left(\frac{a^2\,d^4\,f^2\sqrt{c+d\tan\left(e+f\,x\right)}\sqrt{-\frac{c^2\,1\mathrm{i}}{16\left(-a^2\,d\,f^2+a^2\,c\,f^2\,1\mathrm{i}\right)}}\,4\mathrm{i}}{a\,c\,d^4\,f\,1\mathrm{i}-a\,c^2\,d^3\,f+\frac{a^3\,c^2\,d^4\,f^3}{-a^2\,d\,f^2+a^2\,c\,f^2\,1\mathrm{i}}+\frac{a^3\,c^3\,d^3\,f^3\,1\mathrm{i}}{-a^2\,d\,f^2+a^2\,c\,f^2\,1\mathrm{i}}}+\frac{8\,a^4\,c^3\,d^2\,f^4\sqrt{c+d\tan\left(e+f\,x\right)}\sqrt{-\frac{c^2\,1\mathrm{i}}{16\left(-a^2\,d\,f^2+a^2\,c\,f^2\,1\mathrm{i}\right)}}}{a^3\,c\,d^5\,f^3\,1\mathrm{i}+a^3\,c^3\,d^3\,f^3\,1\mathrm{i}+\frac{a^5\,c^2\,d^5\,f^5}{-a^2\,d\,f^2+a^2\,c\,f^2\,1\mathrm{i}}+\frac{a^5\,c^4\,d^3\,f^5}{-a^2\,d\,f^2+a^2\,c\,f^2\,1\mathrm{i}}}-\frac{8\,a^2\,c\,d^3\,f^2\sqrt{c+d\tan\left(e+f\,x\right)}\sqrt{-\frac{c^2\,1\mathrm{i}}{16\left(-a^2\,d\,f^2+a^2\,c\,f^2\,1\mathrm{i}\right)}}}{a\,c\,d^4\,f\,1\mathrm{i}-a\,c^2\,d^3\,f+\frac{a^3\,c^2\,d^4\,f^3}{-a^2\,d\,f^2+a^2\,c\,f^2\,1\mathrm{i}}+\frac{a^3\,c^3\,d^3\,f^3\,1\mathrm{i}}{-a^2\,d\,f^2+a^2\,c\,f^2\,1\mathrm{i}}}-\frac{a^2\,c^2\,d^2\,f^2\sqrt{c+d\tan\left(e+f\,x\right)}\sqrt{-\frac{c^2\,1\mathrm{i}}{16\left(-a^2\,d\,f^2+a^2\,c\,f^2\,1\mathrm{i}\right)}}\,8\mathrm{i}}{a\,c\,d^4\,f\,1\mathrm{i}-a\,c^2\,d^3\,f+\frac{a^3\,c^2\,d^4\,f^3}{-a^2\,d\,f^2+a^2\,c\,f^2\,1\mathrm{i}}+\frac{a^3\,c^3\,d^3\,f^3\,1\mathrm{i}}{-a^2\,d\,f^2+a^2\,c\,f^2\,1\mathrm{i}}}\right)\sqrt{-\frac{c^2\,1\mathrm{i}}{16\left(-a^2\,d\,f^2+a^2\,c\,f^2\,1\mathrm{i}\right)}}\,2\mathrm{i}-\frac{d\sqrt{c+d\tan\left(e+f\,x\right)}}{2\left(-a\,d\,f\tan\left(e+f\,x\right)+a\,d\,f\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(1/2)/(a + a*tan(e + f*x)*1i),x)
[Out] - 2*atanh((4*a^2*d^4*f^2*((d*1i)/(16*a^2*f^2) - c/(16*a^2*f^2))^(1/2)*(c +
d*tan(e + f*x))^(1/2))/(a*d^5*f + a*c*d^4*f*1i))*((d*1i)/(16*a^2*f^2) - c/(
16*a^2*f^2))^(1/2) - atan((a^2*d^4*f^2*(c + d*tan(e + f*x))^(1/2)*(-(c^2*1i
)/(16*(a^2*c*f^2*1i - a^2*d*f^2)))^(1/2)*4i)/(a*c*d^4*f*1i - a*c^2*d^3*f +
(a^3*c^2*d^4*f^3)/(a^2*c*f^2*1i - a^2*d*f^2) + (a^3*c^3*d^3*f^3*1i)/(a^2*c*
f^2*1i - a^2*d*f^2)) + (8*a^4*c^3*d^2*f^4*(c + d*tan(e + f*x))^(1/2)*(-(c^2
*1i)/(16*(a^2*c*f^2*1i - a^2*d*f^2)))^(1/2))/(a^3*c*d^5*f^3*1i + a^3*c^3*d^
3*f^3*1i + (a^5*c^2*d^5*f^5)/(a^2*c*f^2*1i - a^2*d*f^2) + (a^5*c^4*d^3*f^5)
/(a^2*c*f^2*1i - a^2*d*f^2)) - (8*a^2*c*d^3*f^2*(c + d*tan(e + f*x))^(1/2)*
(-(c^2*1i)/(16*(a^2*c*f^2*1i - a^2*d*f^2)))^(1/2))/(a*c*d^4*f*1i - a*c^2*d^
3*f + (a^3*c^2*d^4*f^3)/(a^2*c*f^2*1i - a^2*d*f^2) + (a^3*c^3*d^3*f^3*1i)/(
a^2*c*f^2*1i - a^2*d*f^2)) - (a^2*c^2*d^2*f^2*(c + d*tan(e + f*x))^(1/2)*(-
(c^2*1i)/(16*(a^2*c*f^2*1i - a^2*d*f^2)))^(1/2)*8i)/(a*c*d^4*f*1i - a*c^2*d
^3*f + (a^3*c^2*d^4*f^3)/(a^2*c*f^2*1i - a^2*d*f^2) + (a^3*c^3*d^3*f^3*1i)/
(a^2*c*f^2*1i - a^2*d*f^2)))*(-(c^2*1i)/(16*(a^2*c*f^2*1i - a^2*d*f^2)))^(1
/2)*2i - (d*(c + d*tan(e + f*x))^(1/2))/(2*(a*d*f*1i - a*d*f*tan(e + f*x)))
```

3.1105 $\int \frac{\sqrt{c+d\tan(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=211

$$-\frac{i\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{4a^2f}-\frac{(2cd-i(2c^2+d^2))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{8a^2(c+id)^{3/2}f}+\frac{(2ic-\cdots}{8a^2(c\cdots}$$

```
[Out] -1/8*(2*c*d-I*(2*c^2+d^2))*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/a^
2/(c+I*d)^(3/2)/f-1/4*I*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))*(c-I*
d)^(1/2)/a^2/f+1/8*(2*I*c-d)*(c+d*tan(f*x+e))^(1/2)/a^2/(c+I*d)/f/(1+I*tan(
f*x+e))+1/4*I*(c+d*tan(f*x+e))^(1/2)/f/(a+I*a*tan(f*x+e))^2
```

Rubi [A]
time = 0.41, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3638, 3677, 3620, 3618, 65, 214}

$$-\frac{(2cd-i(2c^2+d^2))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{8a^2f(c+id)^{3/2}}+\frac{(-d+2ic)\sqrt{c+d\tan(e+fx)}}{8a^2f(c+id)(1+i\tan(e+fx))}-\frac{i\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{4a^2f}+\frac{i\sqrt{c+d\tan(e+fx)}}{4f(a+ia\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Tan[e + f*x]]/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] ((-1/4*I)*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/(a
^2*f) - ((2*c*d - I*(2*c^2 + d^2))*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c
+ I*d]])/(8*a^2*(c + I*d)^(3/2)*f) + (((2*I)*c - d)*Sqrt[c + d*Tan[e + f*x]
])/(8*a^2*(c + I*d)*f*(1 + I*Tan[e + f*x])) + ((I/4)*Sqrt[c + d*Tan[e + f*x
]])/(f*(a + I*a*Tan[e + f*x])^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3638

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*Sqrt[(c_.) + (d_.)*tan[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-b)*(a + b*Tan[e + f*x])^m*(Sqrt[c + d
*Tan[e + f*x]]/(2*a*f*m)), x] + Dist[1/(4*a^2*m), Int[(a + b*Tan[e + f*x])^
(m + 1)*(Simp[2*a*c*m + b*d + a*d*(2*m + 1)*Tan[e + f*x], x]/Sqrt[c + d*Tan
[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && IntegersQ[2*m]
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+d\tan(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx = \frac{i\sqrt{c+d\tan(e+fx)}}{4f(a+ia\tan(e+fx))^2} - \frac{\int \frac{-a(4c-id)-3ad\tan(e+fx)}{(a+ia\tan(e+fx))\sqrt{c+d\tan(e+fx)}}\,dx}{8a^2}$$

$$= \frac{(2ic-d)\sqrt{c+d\tan(e+fx)}}{8a^2(c+id)f(1+i\tan(e+fx))} + \frac{i\sqrt{c+d\tan(e+fx)}}{4f(a+ia\tan(e+fx))^2} + \frac{\int \frac{-a^2(2cd-i(4c^2}{\sqrt{c}}}{}$$

$$= \frac{(2ic-d)\sqrt{c+d\tan(e+fx)}}{8a^2(c+id)f(1+i\tan(e+fx))} + \frac{i\sqrt{c+d\tan(e+fx)}}{4f(a+ia\tan(e+fx))^2} + \frac{(c-id)\int \frac{}{\sqrt{c}}}{}$$

$$= \frac{(2ic-d)\sqrt{c+d\tan(e+fx)}}{8a^2(c+id)f(1+i\tan(e+fx))} + \frac{i\sqrt{c+d\tan(e+fx)}}{4f(a+ia\tan(e+fx))^2} + \frac{(ic+d)\text{Subst}}{}$$

$$= \frac{(2ic-d)\sqrt{c+d\tan(e+fx)}}{8a^2(c+id)f(1+i\tan(e+fx))} + \frac{i\sqrt{c+d\tan(e+fx)}}{4f(a+ia\tan(e+fx))^2} - \frac{(c-id)\text{Subst}}{}$$

$$= -\frac{i\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{4a^2f} - \frac{(2cd-i(2c^2+d^2))\tanh^{-1}\Big(}{8a^2(c+id}$$

Mathematica [A]

time = 1.99, size = 281, normalized size = 1.33

$$\frac{\sec^2(e+fx)(\cos(fx)+i\sin(fx))^2\left(-\frac{2\left(-i\sqrt{-c+id}\,(2c^2+2icd+d^2)\text{ArcTan}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{-c-id}}\right)+2i\sqrt{-c-id}\,(c^2+d^2)\text{ArcTan}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{-c+id}}\right)\right)(\cos(2e)+i\sin(2e))}{(-c-id)^{3/2}\sqrt{-c+id}}+\frac{2\cos(e+fx)(i\cos(2fx)+\sin(2fx))((4c+3id)\cos(e+fx)+(2ic-d)\sin(e+fx))\sqrt{c+d\tan(e+fx)}}{c+id}\right)}{16f(a+ia\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Tan[e + f*x]]/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (Sec[e + f*x]^2*(Cos[f*x] + I*Sin[f*x])^2*((-2*((-I)*Sqrt[-c + I*d]*(2*c^2
+ (2*I)*c*d + d^2)*ArcTan[Sqrt[c + d*Tan[e + f*x]]/Sqrt[-c - I*d]] + (2*I)*
Sqrt[-c - I*d]*(c^2 + d^2)*ArcTan[Sqrt[c + d*Tan[e + f*x]]/Sqrt[-c + I*d]])
*(Cos[2*e] + I*Sin[2*e]))/((-c - I*d)^(3/2)*Sqrt[-c + I*d]) + (2*Cos[e + f*
x]*(I*Cos[2*f*x] + Sin[2*f*x])*((4*c + (3*I)*d)*Cos[e + f*x] + ((2*I)*c - d
)*Sin[e + f*x])*Sqrt[c + d*Tan[e + f*x]])/(c + I*d)))/(16*f*(a + I*a*Tan[e
+ f*x])^2)
```

Maple [A]

time = 0.45, size = 269, normalized size = 1.27

method	result

derivativedivides	$\frac{2d^3\left(-\frac{i\sqrt{id-c}\,\arctan\left(\frac{\sqrt{c+d\tan(fx+e)}}{\sqrt{id-c}}\right)}{8d^3}+\frac{\frac{d\left(2ic^2-id^2-3cd\right)(c+d\tan(fx+e))^{\frac{3}{2}}}{2ic^2-2id^2-4cd}-\frac{id\left(7ic^2d-3id^3+2c^3-8c\right.}{2(}}{(-d\tan(fx+e)+id)^2}\right.}{f\,a^2}$
default	$\frac{2d^3\left(-\frac{i\sqrt{id-c}\,\arctan\left(\frac{\sqrt{c+d\tan(fx+e)}}{\sqrt{id-c}}\right)}{8d^3}+\frac{\frac{d\left(2ic^2-id^2-3cd\right)(c+d\tan(fx+e))^{\frac{3}{2}}}{2ic^2-2id^2-4cd}-\frac{id\left(7ic^2d-3id^3+2c^3-8c\right.}{2(}}{(-d\tan(fx+e)+id)^2}\right.}{f\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^2*d^3*(-1/8*I*(I*d-c)^(1/2)/d^3*arctan((c+d*tan(f*x+e))^(1/2)/(I*d-c)
^(1/2))+1/8/d^3*((1/2*d*(2*I*c^2-I*d^2-3*c*d)/(I*c^2-I*d^2-2*c*d)*(c+d*tan(
f*x+e))^(3/2)-1/2*I*d*(7*I*c^2*d-3*I*d^3+2*c^3-8*c*d^2)/(I*c^2-I*d^2-2*c*d)
*(c+d*tan(f*x+e))^(1/2))/(-d*tan(f*x+e)+I*d)^2-1/2*I*(-d^3+2*I*c^3-I*c*d^2-
4*c^2*d)/(I*c^2-I*d^2-2*c*d)/(-I*d-c)^(1/2)*arctan((c+d*tan(f*x+e))^(1/2)/(
-I*d-c)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1115 vs. 2(168) = 336.
time = 1.37, size = 1115, normalized size = 5.28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] 1/32*(2*(-I*a^2*c + a^2*d)*f*sqrt(-(c - I*d)/(a^4*f^2))*e^(4*I*f*x + 4*I*e)
*log(-2*((I*a^2*f*e^(2*I*f*x + 2*I*e) + I*a^2*f)*sqrt(((c - I*d)*e^(2*I*f*x
 + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(c - I*d)/(a^4*f^2))
- (c - I*d)*e^(2*I*f*x + 2*I*e) - c)*e^(-2*I*f*x - 2*I*e)) + 2*(I*a^2*c - a
^2*d)*f*sqrt(-(c - I*d)/(a^4*f^2))*e^(4*I*f*x + 4*I*e)*log(-2*((-I*a^2*f*e^
(2*I*f*x + 2*I*e) - I*a^2*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)
/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(c - I*d)/(a^4*f^2)) - (c - I*d)*e^(2*I*f
*x + 2*I*e) - c)*e^(-2*I*f*x - 2*I*e)) + (-I*a^2*c + a^2*d)*f*sqrt(-(-4*I*c
^4 + 8*c^3*d + 4*c*d^3 - I*d^4)/((-I*a^4*c^3 + 3*a^4*c^2*d + 3*I*a^4*c*d^2
- a^4*d^3)*f^2))*e^(4*I*f*x + 4*I*e)*log(-1/8*(-2*I*c^3 + 4*c^2*d + I*c*d^2
 + d^3 + ((a^2*c^2 + 2*I*a^2*c*d - a^2*d^2)*f*e^(2*I*f*x + 2*I*e) + (a^2*c^
2 + 2*I*a^2*c*d - a^2*d^2)*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d
)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(-4*I*c^4 + 8*c^3*d + 4*c*d^3 - I*d^4)/(
(-I*a^4*c^3 + 3*a^4*c^2*d + 3*I*a^4*c*d^2 - a^4*d^3)*f^2)) + (-2*I*c^3 + 2*
c^2*d - I*c*d^2)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/((a^2*c^2 + 2*I*
a^2*c*d - a^2*d^2)*f)) + (I*a^2*c - a^2*d)*f*sqrt(-(-4*I*c^4 + 8*c^3*d + 4*
c*d^3 - I*d^4)/((-I*a^4*c^3 + 3*a^4*c^2*d + 3*I*a^4*c*d^2 - a^4*d^3)*f^2))*
e^(4*I*f*x + 4*I*e)*log(-1/8*(-2*I*c^3 + 4*c^2*d + I*c*d^2 + d^3 - ((a^2*c^
2 + 2*I*a^2*c*d - a^2*d^2)*f*e^(2*I*f*x + 2*I*e) + (a^2*c^2 + 2*I*a^2*c*d -
 a^2*d^2)*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2
*I*e) + 1))*sqrt(-(-4*I*c^4 + 8*c^3*d + 4*c*d^3 - I*d^4)/((-I*a^4*c^3 + 3*a
^4*c^2*d + 3*I*a^4*c*d^2 - a^4*d^3)*f^2)) + (-2*I*c^3 + 2*c^2*d - I*c*d^2)*
e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/((a^2*c^2 + 2*I*a^2*c*d - a^2*d^2
)*f)) - 2*((3*c + 2*I*d)*e^(4*I*f*x + 4*I*e) + (4*c + 3*I*d)*e^(2*I*f*x + 2
*I*e) + c + I*d)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x
 + 2*I*e) + 1)))*e^(-4*I*f*x - 4*I*e)/((I*a^2*c - a^2*d)*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt{c+d\tan(e+fx)}}{\tan^2(e+fx)-2i\tan(e+fx)-1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -Integral(sqrt(c + d*tan(e + f*x))/(tan(e + f*x)**2 - 2*I*tan(e + f*x) - 1)
, x)/a**2
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 473 vs. 2(168) = 336.
time = 0.57, size = 473, normalized size = 2.24

$$-\frac{(2c^2+2icd+d^2)\arctan\left(\frac{2\left(\sqrt{d\tan(fx+e)+c}\,c-\sqrt{c^2+d^2}\sqrt{d\tan(fx+e)+c}\right)}{c\sqrt{-2c+2\sqrt{c^2+d^2}}+i\sqrt{-2c+2\sqrt{c^2+d^2}}\,d-\sqrt{c^2+d^2}\sqrt{-2c+2\sqrt{c^2+d^2}}}\right)}{-4(ia^2cf-a^2df)\sqrt{-2c+2\sqrt{c^2+d^2}}\left(\frac{id}{c-\sqrt{c^2+d^2}}+1\right)}+\frac{2(d\tan(fx+e)+c)^{\frac{3}{2}}cd-2\sqrt{d\tan(fx+e)+c}\,c^2d+i(d\tan(fx+e)+c)^{\frac{3}{2}}d^2-5i\sqrt{d\tan(fx+e)+c}\,cd^2+3\sqrt{d\tan(fx+e)+c}\,d^3}{8(a^2cf+ia^2df)(d\tan(fx+e)-id)^2}+\frac{(c-id)\arctan\left(-\frac{2\left(i\sqrt{d\tan(fx+e)+c}\,c+i\sqrt{c^2+d^2}\sqrt{d\tan(fx+e)+c}\right)}{\sqrt{2c+2\sqrt{c^2+d^2}}\,c-i\sqrt{2c+2\sqrt{c^2+d^2}}\,d+\sqrt{c^2+d^2}\sqrt{2c+2\sqrt{c^2+d^2}}}\right)}{2a^2\sqrt{2c+2\sqrt{c^2+d^2}}\,f\left(-\frac{id}{c+\sqrt{c^2+d^2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -(2*c^2 + 2*I*c*d + d^2)*arctan(2*(sqrt(d*tan(f*x + e) + c)*c - sqrt(c^2 +
d^2)*sqrt(d*tan(f*x + e) + c))/(c*sqrt(-2*c + 2*sqrt(c^2 + d^2)) + I*sqrt(-
2*c + 2*sqrt(c^2 + d^2))*d - sqrt(c^2 + d^2)*sqrt(-2*c + 2*sqrt(c^2 + d^2))
))/((-4*I*a^2*c*f + 4*a^2*d*f)*sqrt(-2*c + 2*sqrt(c^2 + d^2))*(I*d/(c - sqr
t(c^2 + d^2)) + 1)) + 1/8*(2*(d*tan(f*x + e) + c)^(3/2)*c*d - 2*sqrt(d*tan(
f*x + e) + c)*c^2*d + I*(d*tan(f*x + e) + c)^(3/2)*d^2 - 5*I*sqrt(d*tan(f*x
 + e) + c)*c*d^2 + 3*sqrt(d*tan(f*x + e) + c)*d^3)/((a^2*c*f + I*a^2*d*f)*(
d*tan(f*x + e) - I*d)^2) + 1/2*(c - I*d)*arctan(-2*(I*sqrt(d*tan(f*x + e) +
 c)*c + I*sqrt(c^2 + d^2)*sqrt(d*tan(f*x + e) + c))/(sqrt(2*c + 2*sqrt(c^2
+ d^2))*c - I*sqrt(2*c + 2*sqrt(c^2 + d^2))*d + sqrt(c^2 + d^2)*sqrt(2*c +
2*sqrt(c^2 + d^2))))/(a^2*sqrt(2*c + 2*sqrt(c^2 + d^2))*f*(-I*d/(c + sqrt(c
^2 + d^2)) + 1))
```

Mupad [B]
time = 8.17, size = 2500, normalized size = 11.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(1/2)/(a + a*tan(e + f*x)*1i)^2,x)
```

```
[Out] log(((-(3*d^9 - c*d^8*9i + 12*c^2*d^7 - c^3*d^6*16i + 8*c^4*d^5 - c^5*d^4*8
i - a^4*c^2*f^2*((((3*d^11 + 27*c^2*d^9 + 28*c^4*d^7 + 8*c^6*d^5)*1i)/(a^4*
c^4*f^2 + a^4*d^4*f^2 + 2*a^4*c^2*d^2*f^2) - (17*c^3*d^8 - 3*c*d^10 + 24*c^
5*d^6 + 8*c^7*d^4)/(a^4*c^4*f^2 + a^4*d^4*f^2 + 2*a^4*c^2*d^2*f^2))^2 + 4*(
256*d^6 + 256*c^2*d^4)*((((5*c^3*d^9)/16 - (c*d^11)/8 + (7*c^5*d^7)/16 + (c
^7*d^5)/8)*1i)/(a^8*c^4*f^4 + a^8*d^4*f^4 + 2*a^8*c^2*d^2*f^4) - (d^12/64 -
 (11*c^2*d^10)/32 - (11*c^4*d^8)/64 + (c^6*d^6)/8 + (c^8*d^4)/16)/(a^8*c^4*
f^4 + a^8*d^4*f^4 + 2*a^8*c^2*d^2*f^4)))^(1/2)*1i + a^4*d^2*f^2*((((3*d^11
+ 27*c^2*d^9 + 28*c^4*d^7 + 8*c^6*d^5)*1i)/(a^4*c^4*f^2 + a^4*d^4*f^2 + 2*a
^4*c^2*d^2*f^2) - (17*c^3*d^8 - 3*c*d^10 + 24*c^5*d^6 + 8*c^7*d^4)/(a^4*c^4
*f^2 + a^4*d^4*f^2 + 2*a^4*c^2*d^2*f^2))^2 + 4*(256*d^6 + 256*c^2*d^4)*((((
5*c^3*d^9)/16 - (c*d^11)/8 + (7*c^5*d^7)/16 + (c^7*d^5)/8)*1i)/(a^8*c^4*f^4
 + a^8*d^4*f^4 + 2*a^8*c^2*d^2*f^4) - (d^12/64 - (11*c^2*d^10)/32 - (11*c^4
*d^8)/64 + (c^6*d^6)/8 + (c^8*d^4)/16)/(a^8*c^4*f^4 + a^8*d^4*f^4 + 2*a^8*c
^2*d^2*f^4)))^(1/2)*1i + 2*a^4*c*d*f^2*((((3*d^11 + 27*c^2*d^9 + 28*c^4*d^7
 + 8*c^6*d^5)*1i)/(a^4*c^4*f^2 + a^4*d^4*f^2 + 2*a^4*c^2*d^2*f^2) - (17*c^3
*d^8 - 3*c*d^10 + 24*c^5*d^6 + 8*c^7*d^4)/(a^4*c^4*f^2 + a^4*d^4*f^2 + 2*a^
4*c^2*d^2*f^2))^2 + 4*(256*d^6 + 256*c^2*d^4)*((((5*c^3*d^9)/16 - (c*d^11)/
8 + (7*c^5*d^7)/16 + (c^7*d^5)/8)*1i)/(a^8*c^4*f^4 + a^8*d^4*f^4 + 2*a^8*c^
2*d^2*f^4) - (d^12/64 - (11*c^2*d^10)/32 - (11*c^4*d^8)/64 + (c^6*d^6)/8 +
(c^8*d^4)/16)/(a^8*c^4*f^4 + a^8*d^4*f^4 + 2*a^8*c^2*d^2*f^4)))^(1/2))/(512
```

```
*(d^6 + c^2*d^4)*(a^4*d^2*f^2*1i - a^4*c^2*f^2*1i + 2*a^4*c*d*f^2)))^(1/2)*
(a^6*d^8*f^3*384i - 512*a^6*c*d^7*f^3 + 8*(c + d*tan(e + f*x))^(1/2)*(a^4*c
^2*d^3*f^2*1024i - 512*a^4*c*d^4*f^2 + 512*a^4*c^3*d^2*f^2)*(a^4*d^2*f^2 -
a^4*c^2*f^2 + a^4*c*d*f^2*2i)*(-(3*d^9 - c*d^8*9i + 12*c^2*d^7 - c^3*d^6*16
i + 8*c^4*d^5 - c^5*d^4*8i - a^4*c^2*f^2*((((3*d^11 + 27*c^2*d^9 + 28*c^4*d
^7 + 8*c^6*d^5)*1i)/(a^4*c^4*f^2 + a^4*d^4*f^2 + 2*a^4*c^2*d^2*f^2) - (17*c
^3*d^8 - 3*c*d^10 + 24*c^5*d^6 + 8*c^7*d^4)/(a^4*c^4*f^2 + a^4*d^4*f^2 + 2*
a^4*c^2*d^2*f^2))^2 + 4*(256*d^6 + 256*c^2*d^4)*((((5*c^3*d^9)/16 - (c*d^11
)/8 + (7*c^5*d^7)/16 + (c^7*d^5)/8)*1i)/(a^8*c^4*f^4 + a^8*d^4*f^4 + 2*a^8*
c^2*d^2*f^4) - (d^12/64 - (11*c^2*d^10)/32 - (11*c^4*d^8)/64 + (c^6*d^6)/8
+ (c^8*d^4)/16)/(a^8*c^4*f^4 + a^8*d^4*f^4 + 2*a^8*c^2*d^2*f^4)))^(1/2)*1i
+ a^4*d^2*f^2*((((3*d^11 + 27*c^2*d^9 + 28*c^4*d^7 + 8*c^6*d^5)*1i)/(a^4*c^
4*f^2 + a^4*d^4*f^2 + 2*a^4*c^2*d^2*f^2) - (17*c^3*d^8 - 3*c*d^10 + 24*c^5*
d^6 + 8*c^7*d^4)/(a^4*c^4*f^2 + a^4*d^4*f^2 + 2*a^4*c^2*d^2*f^2))^2 + 4*(25
6*d^6 + 256*c^2*d^4)*((((5*c^3*d^9)/16 - (c*d^11)/8 + (7*c^5*d^7)/16 + (c^7
*d^5)/8)*1i)/(a^8*c^4*f^4 + a^8*d^4*f^4 + 2*a^8*c^2*d^2*f^4) - (d^12/64 - (
11*c^2*d^10)/32 - (11*c^4*d^8)/64 + (c^6*d^6)/8 + (c^8*d^4)/16)/(a^8*c^4*f^
4 + a^8*d^4*f^4 + 2*a^8*c^2*d^2*f^4)))^(1/2)*1i + 2*a^4*c*d*f^2*((((3*d^11
+ 27*c^2*d^9 + 28*c^4*d^7 + 8*c^6*d^5)*1i)/(a^4*c^4*f^2 + a^4*d^4*f^2 + 2*a
^4*c^2*d^2*f^2) - (17*c^3*d^8 - 3*c*d^10 + 24*c^5*d^6 + 8*c^7*d^4)/(a^4*c^4
*f^2 + a^4*d^4*f^2 + 2*a^4*c^2*d^2*f^2))^2 + 4*(256*d^6 + 256*c^2*d^4)*((((
5*c^3*d^9)/16 - (c*d^11)/8 + (7*c^5*d^7)/16 + (c^7*d^5)/8)*1i)/(a^8*c^4*f^4
 + a^8*d^4*f^4 + 2*a^8*c^2*d^2*f^4) - (d^12/64 - (11*c^2*d^10)/32 - (11*c^4
*d^8)/64 + (c^6*d^6)/8 + (c^8*d^4)/16)/(a^8*c^4*f^4 + a^8*d^4*f^4 + 2*a^8*c
^2*d^2*f^4)))^(1/2))/(512*(d^6 + c^2*d^4)*(a^4*d^2*f^2*1i - a^4*c^2*f^2*1i
+ 2*a^4*c*d*f^2)))^(1/2) + a^6*c^2*d^6*f^3*512i - 768*a^6*c^3*d^5*f^3 + a^6
*c^4*d^4*f^3*128i - 256*a^6*c^5*d^3*f^3) + 8*(c + d*tan(e + f*x))^(1/2)*(a^
4*d^2*f^2 - a^4*c^2*f^2 + a^4*c*d*f^2*2i)*(c*d^5*4i + 5*d^6 + 8*c^2*d^4 + c
^3*d^3*8i + 8*c^4*d^2))*(-(3*d^9 - c*d^8*9i + 12*c^2*d^7 - c^3*d^6*16i + 8*
c^4*d^5 - c^5*d^4*8i - a^4*c^2*f^2*((((3*d^11 + 27*c^2*d^9 + 28*c^4*d^7 + 8
*c^6*d^5)*1i)/(a^4*c^4*f^2 + a^4*d^4*f^2 + 2*a^4*c^2*d^2*f^2) - (17*c^3*d^8
 - 3*c*d^10 + 24*c^5*d^6 + 8*c^7*d^4)/(a^4*c^4*f^2 + a^4*d^4*f^2 + 2*a^4*c^
2*d^2*f^2))^2 + 4*(256*d^6 + 256*c^2*d^4)*((((5*c^3*d^9)/16 - (c*d^11)/8 +
(7*c^5*d^7)/16 + (c^7*d^5)/8)*1i)/(a^8*c^4*f^4 + a^8*d^4*f^4 + 2*a^8*c^2*d^
2*f^4) - (d^12/64 - (11*c^2*d^10)/32 - (11*c^4*d^8)/64 + (c^6*d^6)/8 + (c^8
*d^4)/16)/(a^8*c^4*f^4 + a^8*d^4*f^4 + 2*a^8*c^2*d^2*f^4)))^(1/2)*1i + a^4*
d^2*f^2*((((3*d^11 + 27*c^2*d^9 + 28*c^4*d^7 + 8*c^6*d^5)*1i)/(a^4*c^4*f^2
+ a^4*d^4*f^2 + 2*a^4*c^2*d^2*f^2) - (17*c^3*d^8 - 3*c*d^10 + 24*c^5*d^6 +
8*c^7*d^4)/(a^4*c^4*f^2 + a^4*d^4*f^2 + 2*a^4*c^2*d^2*f^2))^2 + 4*(256*d^6
+ 256*c^2*d^4)*((((5*c^3*d^9)/16 - (c*d^11)/8 + (7*c^5*d^7)/16 + (c^7*d^5)/
8)*1i)/(a^8*c^4*f^4 + a^8*d^4*f^4 + 2*a^8*c^2*d^2*f^4) - (d^12/64 - (11*c^2
*d^10)/32 - (11*c^4*d^8)/64 + (c^6*d^6)/8 + (c^8*d^4)/16)/(a^8*c^4*f^4 + a^
8*d^4*f^4 + 2*a^8*c^2*d^2*f^4)))^(1/2)*1i + 2*a^4*c*d*f^2*((((3*d^11 + 27*c
^2*d^9 + 28*c^4*d^7 + 8*c^6*d^5)*1i)/(a^4*c^4*f^2 + a^4*d^4*f^2 + 2*a^4*c^2
*d^2*f^2) - (17*c^3*d^8 - 3*c*d^10 + 24*c^5*d^6 + 8*c^7*d^4)/(a^4*c^4*f^2 +
```

```
a^4*d^4*f^2 + 2*a^4*c^2*d^2*f^2))^2 + 4*(256*d...
```

3.1106 $\int \frac{\sqrt{c+d\tan(e+fx)}}{(a+ia\tan(e+fx))^3}\,dx$

Optimal. Leaf size=280

$$-\frac{i\sqrt{c-id}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{8a^3f}+\frac{(2ic^3-4c^2d-icd^2-2d^3)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{16a^3(c+id)^{5/2}f}+$$

```
[Out] 1/16*(2*I*c^3-4*c^2*d-I*c*d^2-2*d^3)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)
^(1/2))/a^3/(c+I*d)^(5/2)/f-1/8*I*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1
/2))*(c-I*d)^(1/2)/a^3/f+1/6*I*(c+d*tan(f*x+e))^(1/2)/f/(a+I*a*tan(f*x+e))^
3+1/24*(3*I*c-2*d)*(c+d*tan(f*x+e))^(1/2)/a/(c+I*d)/f/(a+I*a*tan(f*x+e))^2+
1/16*c*(2*I*c-3*d)*(c+d*tan(f*x+e))^(1/2)/(c+I*d)^2/f/(a^3+I*a^3*tan(f*x+e)
)
```

Rubi [A]

time = 0.66, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3638, 3677, 3620, 3618, 65, 214}

$$\frac{(2ic^3-4c^2d-icd^2-2d^3)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{16a^3f(c+id)^{5/2}}+\frac{c(-3d+2ic)\sqrt{c+d\tan(e+fx)}}{16f(c+id)^2(a^3+ia^3\tan(e+fx))}-\frac{i\sqrt{c-id}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{8a^3f}+\frac{(-2d+3ic)\sqrt{c+d\tan(e+fx)}}{24af(c+id)(a+ia\tan(e+fx))^2}+\frac{i\sqrt{c+d\tan(e+fx)}}{6f(a+ia\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Tan[e + f*x]]/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] ((-1/8*I)*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/(a
^3*f) + (((2*I)*c^3 - 4*c^2*d - I*c*d^2 - 2*d^3)*ArcTanh[Sqrt[c + d*Tan[e +
 f*x]]/Sqrt[c + I*d]])/(16*a^3*(c + I*d)^(5/2)*f) + ((I/6)*Sqrt[c + d*Tan[e
 + f*x]])/(f*(a + I*a*Tan[e + f*x])^3) + (((3*I)*c - 2*d)*Sqrt[c + d*Tan[e
+ f*x]])/(24*a*(c + I*d)*f*(a + I*a*Tan[e + f*x])^2) + (c*((2*I)*c - 3*d)*S
qrt[c + d*Tan[e + f*x]])/(16*(c + I*d)^2*f*(a^3 + I*a^3*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3638

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*Sqrt[(c_.) + (d_.)*tan[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-b)*(a + b*Tan[e + f*x])^m*(Sqrt[c + d
*Tan[e + f*x]]/(2*a*f*m)), x] + Dist[1/(4*a^2*m), Int[(a + b*Tan[e + f*x])^
(m + 1)*(Simp[2*a*c*m + b*d + a*d*(2*m + 1)*Tan[e + f*x], x]/Sqrt[c + d*Tan
[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && IntegersQ[2*m]
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+d\tan(e+fx)}}{(a+ia\tan(e+fx))^3}\,dx = \frac{i\sqrt{c+d\tan(e+fx)}}{6f(a+ia\tan(e+fx))^3} - \frac{\int \frac{-a(6c-id)-5ad\tan(e+fx)}{(a+ia\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}\,dx}{12a^2}$$

$$= \frac{i\sqrt{c+d\tan(e+fx)}}{6f(a+ia\tan(e+fx))^3} + \frac{(3ic-2d)\sqrt{c+d\tan(e+fx)}}{24a(c+id)f(a+ia\tan(e+fx))^2} + \frac{\int \frac{-3a^2(3cd-i}{(a+ia\tan(}}{}$$

$$= \frac{i\sqrt{c+d\tan(e+fx)}}{6f(a+ia\tan(e+fx))^3} + \frac{(3ic-2d)\sqrt{c+d\tan(e+fx)}}{24a(c+id)f(a+ia\tan(e+fx))^2} + \frac{c(2ic-3c}{16(c+id)^2}$$

$$= \frac{i\sqrt{c+d\tan(e+fx)}}{6f(a+ia\tan(e+fx))^3} + \frac{(3ic-2d)\sqrt{c+d\tan(e+fx)}}{24a(c+id)f(a+ia\tan(e+fx))^2} + \frac{c(2ic-3c}{16(c+id)^2}$$

$$= \frac{i\sqrt{c+d\tan(e+fx)}}{6f(a+ia\tan(e+fx))^3} + \frac{(3ic-2d)\sqrt{c+d\tan(e+fx)}}{24a(c+id)f(a+ia\tan(e+fx))^2} + \frac{c(2ic-3c}{16(c+id)^2}$$

$$= \frac{i\sqrt{c+d\tan(e+fx)}}{6f(a+ia\tan(e+fx))^3} + \frac{(3ic-2d)\sqrt{c+d\tan(e+fx)}}{24a(c+id)f(a+ia\tan(e+fx))^2} + \frac{c(2ic-3c}{16(c+id)^2}$$

$$= -\frac{i\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{8a^3f} - \frac{(4c^2d-i(2c^3-cd^2+2id^3))}{16a^3(c}$$

Mathematica [A]
time = 2.90, size = 329, normalized size = 1.18

$$\frac{\sec^3(e+fx)(\cos(fx)+i\sin(fx))^3\left(\frac{2\left(\sqrt{-c+id}\,(-2ic^3+4c^2d+icd^2+2d^3)\text{ArcTan}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{-c-id}}\right)+2(-c-id)^{5/2}(ic+d)\text{ArcTan}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{-c+id}}\right)\right)(\cos(3e)+i\sin(3e))}{(-c-id)^{5/2}\sqrt{-c+id}} + \frac{2\cos(e+fx)(i\cos(3fx)+\sin(3fx))\left(7c^2+13icd-6d^2+\left(13c^2+22icd-6d^2\right)\cos(2(e+fx))+i\left(9c^2+14icd-2d^2\right)\sin(2(e+fx))\right)\sqrt{c+d\tan(e+fx)}}{3(c+id)^2}\right)}{32f(a+ia\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Tan[e + f*x]]/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] (Sec[e + f*x]^3*(Cos[f*x] + I*Sin[f*x])^3*((2*(Sqrt[-c + I*d]*((-2*I)*c^3 +
 4*c^2*d + I*c*d^2 + 2*d^3)*ArcTan[Sqrt[c + d*Tan[e + f*x]]/Sqrt[-c - I*d]]
 + 2*(-c - I*d)^(5/2)*(I*c + d)*ArcTan[Sqrt[c + d*Tan[e + f*x]]/Sqrt[-c + I
*d]])*(Cos[3*e] + I*Sin[3*e]))/((-c - I*d)^(5/2)*Sqrt[-c + I*d]) + (2*Cos[e
 + f*x]*(I*Cos[3*f*x] + Sin[3*f*x])*(7*c^2 + (13*I)*c*d - 6*d^2 + (13*c^2 +
 (22*I)*c*d - 6*d^2)*Cos[2*(e + f*x)] + I*(9*c^2 + (14*I)*c*d - 2*d^2)*Sin[
2*(e + f*x)])*Sqrt[c + d*Tan[e + f*x]])/(3*(c + I*d)^2)))/(32*f*(a + I*a*Ta
n[e + f*x])^3)
```

Maple [A]
time = 0.44, size = 382, normalized size = 1.36

method	result
derivativedivides	$\frac{2d^4\left(\frac{-\frac{cd\left(5icd+2c^2-3d^2\right)(c+d\tan(fx+e))^{\frac{5}{2}}}{2\left(3ic^2d-id^3+c^3-3c\,d^2\right)}+\frac{2d\left(12ic^3d-8ic\,d^3+3c^4-16c^2d^2+d^4\right)(c+d\tan(fx+e))^{\frac{3}{2}}}{3\left(3ic^2d-id^3+c^3-3c\,d^2\right)}-\frac{d\left(11ic^4d-29ic^2d^3+4id^5+\right.}{}}{(-d\tan(fx+e)+id)^3}\right.}{}$
default	$\frac{2d^4\left(\frac{-\frac{cd\left(5icd+2c^2-3d^2\right)(c+d\tan(fx+e))^{\frac{5}{2}}}{2\left(3ic^2d-id^3+c^3-3c\,d^2\right)}+\frac{2d\left(12ic^3d-8ic\,d^3+3c^4-16c^2d^2+d^4\right)(c+d\tan(fx+e))^{\frac{3}{2}}}{3\left(3ic^2d-id^3+c^3-3c\,d^2\right)}-\frac{d\left(11ic^4d-29ic^2d^3+4id^5+\right.}{}}{(-d\tan(fx+e)+id)^3}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3*d^4*(1/16/d^4*((-1/2*c*d*(5*I*c*d+2*c^2-3*d^2)/(3*I*c^2*d-I*d^3+c^3
-3*c*d^2)*(c+d*tan(f*x+e))^(5/2)+2/3*d*(-16*c^2*d^2+d^4+12*I*c^3*d-8*I*c*d^
3+3*c^4)/(3*I*c^2*d-I*d^3+c^3-3*c*d^2)*(c+d*tan(f*x+e))^(3/2)-1/2*d*(11*I*c
^4*d-29*I*c^2*d^3+4*I*d^5+2*c^5-25*c^3*d^2+17*c*d^4)/(3*I*c^2*d-I*d^3+c^3-3
*c*d^2)*(c+d*tan(f*x+e))^(1/2))/(-d*tan(f*x+e)+I*d)^3-1/2*(-6*c^3*d-c*d^3+2
*I*c^4-5*I*c^2*d^2-2*I*d^4)/(3*I*c^2*d-I*d^3+c^3-3*c*d^2)/(-I*d-c)^(1/2)*ar
ctan((c+d*tan(f*x+e))^(1/2)/(-I*d-c)^(1/2)))-1/16*I*(I*d-c)^(1/2)/d^4*arcta
n((c+d*tan(f*x+e))^(1/2)/(I*d-c)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1478 vs. 2(230) = 460.
time = 1.80, size = 1478, normalized size = 5.28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="fricas"
)
[Out] -1/192*(6*(a^3*c^2 + 2*I*a^3*c*d - a^3*d^2)*f*sqrt(-(c - I*d)/(a^6*f^2))*e^
(6*I*f*x + 6*I*e)*log(-2*((I*a^3*f*e^(2*I*f*x + 2*I*e) + I*a^3*f)*sqrt(((c
- I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(c -
 I*d)/(a^6*f^2)) - (c - I*d)*e^(2*I*f*x + 2*I*e) - c)*e^(-2*I*f*x - 2*I*e))
 - 6*(a^3*c^2 + 2*I*a^3*c*d - a^3*d^2)*f*sqrt(-(c - I*d)/(a^6*f^2))*e^(6*I*
f*x + 6*I*e)*log(-2*((-I*a^3*f*e^(2*I*f*x + 2*I*e) - I*a^3*f)*sqrt(((c - I*
d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(c - I*d
)/(a^6*f^2)) - (c - I*d)*e^(2*I*f*x + 2*I*e) - c)*e^(-2*I*f*x - 2*I*e)) + 3
*(a^3*c^2 + 2*I*a^3*c*d - a^3*d^2)*f*sqrt(-(4*I*c^6 - 16*c^5*d - 20*I*c^4*d
^2 - 15*I*c^2*d^4 + 4*c*d^5 - 4*I*d^6)/((I*a^6*c^5 - 5*a^6*c^4*d - 10*I*a^6
*c^3*d^2 + 10*a^6*c^2*d^3 + 5*I*a^6*c*d^4 - a^6*d^5)*f^2))*e^(6*I*f*x + 6*I
*e)*log(1/16*(2*c^4 + 6*I*c^3*d - 5*c^2*d^2 + I*c*d^3 - 2*d^4 + ((I*a^3*c^3
 - 3*a^3*c^2*d - 3*I*a^3*c*d^2 + a^3*d^3)*f*e^(2*I*f*x + 2*I*e) + (I*a^3*c^
3 - 3*a^3*c^2*d - 3*I*a^3*c*d^2 + a^3*d^3)*f)*sqrt(((c - I*d)*e^(2*I*f*x +
2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(4*I*c^6 - 16*c^5*d - 20
*I*c^4*d^2 - 15*I*c^2*d^4 + 4*c*d^5 - 4*I*d^6)/((I*a^6*c^5 - 5*a^6*c^4*d -
10*I*a^6*c^3*d^2 + 10*a^6*c^2*d^3 + 5*I*a^6*c*d^4 - a^6*d^5)*f^2)) + (2*c^4
 + 4*I*c^3*d - c^2*d^2 + 2*I*c*d^3)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*
e)/((-I*a^3*c^3 + 3*a^3*c^2*d + 3*I*a^3*c*d^2 - a^3*d^3)*f)) - 3*(a^3*c^2 +
 2*I*a^3*c*d - a^3*d^2)*f*sqrt(-(4*I*c^6 - 16*c^5*d - 20*I*c^4*d^2 - 15*I*c
^2*d^4 + 4*c*d^5 - 4*I*d^6)/((I*a^6*c^5 - 5*a^6*c^4*d - 10*I*a^6*c^3*d^2 +
10*a^6*c^2*d^3 + 5*I*a^6*c*d^4 - a^6*d^5)*f^2))*e^(6*I*f*x + 6*I*e)*log(1/1
6*(2*c^4 + 6*I*c^3*d - 5*c^2*d^2 + I*c*d^3 - 2*d^4 + ((-I*a^3*c^3 + 3*a^3*c
^2*d + 3*I*a^3*c*d^2 - a^3*d^3)*f*e^(2*I*f*x + 2*I*e) + (-I*a^3*c^3 + 3*a^3
*c^2*d + 3*I*a^3*c*d^2 - a^3*d^3)*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) +
c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(4*I*c^6 - 16*c^5*d - 20*I*c^4*d^
2 - 15*I*c^2*d^4 + 4*c*d^5 - 4*I*d^6)/((I*a^6*c^5 - 5*a^6*c^4*d - 10*I*a^6*
c^3*d^2 + 10*a^6*c^2*d^3 + 5*I*a^6*c*d^4 - a^6*d^5)*f^2)) + (2*c^4 + 4*I*c^
3*d - c^2*d^2 + 2*I*c*d^3)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/((-I*a
^3*c^3 + 3*a^3*c^2*d + 3*I*a^3*c*d^2 - a^3*d^3)*f)) + 2*(-2*I*c^2 + 4*c*d +
 2*I*d^2 + (-11*I*c^2 + 18*c*d + 4*I*d^2)*e^(6*I*f*x + 6*I*e) + (-18*I*c^2
+ 31*c*d + 10*I*d^2)*e^(4*I*f*x + 4*I*e) + (-9*I*c^2 + 17*c*d + 8*I*d^2)*e^
(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f
*x + 2*I*e) + 1)))*e^(-6*I*f*x - 6*I*e)/((a^3*c^2 + 2*I*a^3*c*d - a^3*d^2)*
f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int\frac{\sqrt{c+d\tan\left(e+fx\right)}}{\tan^{3}\left(e+fx\right)-3i\tan^{2}\left(e+fx\right)-3\tan\left(e+fx\right)+i}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] I*Integral(sqrt(c + d*tan(e + f*x))/(tan(e + f*x)**3 - 3*I*tan(e + f*x)**2
- 3*tan(e + f*x) + I), x)/a**3
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(230) = 460$.
time = 0.69, size = 632, normalized size = 2.26

$$\frac{(2c^3+4ic^2d-cd^2+2id^3)\arctan\left(\frac{2\left(\sqrt{d\tan(fx+e)+c}\,c-\sqrt{c^2+d^2}\sqrt{d\tan(fx+e)+c}\right)}{c\sqrt{-2c+2\sqrt{c^2+d^2}}+i\sqrt{-2c+2\sqrt{c^2+d^2}}\,d-\sqrt{c^2+d^2}\sqrt{-2c+2\sqrt{c^2+d^2}}}\right)}{-8\left(-ia^3c^2f+2a^3cdf+ia^3d^2f\right)\sqrt{-2c+2\sqrt{c^2+d^2}}\left(\frac{id}{c-\sqrt{c^2+d^2}}+1\right)}+\frac{6(d\tan(fx+e)+c)^{\frac{5}{2}}c^2d-12(d\tan(fx+e)+c)^{\frac{3}{2}}c^3d+6\sqrt{d\tan(fx+e)+c}\,c^4d+9i(d\tan(fx+e)+c)^{\frac{5}{2}}cd^2-36i(d\tan(fx+e)+c)^{\frac{3}{2}}c^2d^2+27i\sqrt{d\tan(fx+e)+c}\,c^3d^2+28(d\tan(fx+e)+c)^{\frac{3}{2}}cd^3-48\sqrt{d\tan(fx+e)+c}\,c^2d^3+4i(d\tan(fx+e)+c)^{\frac{3}{2}}d^4-39i\sqrt{d\tan(fx+e)+c}\,cd^4+12\sqrt{d\tan(fx+e)+c}\,d^5}{48\left(a^3c^2f+2ia^3cdf-a^3d^2f\right)(d\tan(fx+e)-id)^3}-\frac{(-ic-d)\arctan\left(\frac{2\left(\sqrt{d\tan(fx+e)+c}\,c-\sqrt{c^2+d^2}\sqrt{d\tan(fx+e)+c}\right)}{c\sqrt{-2c+2\sqrt{c^2+d^2}}-i\sqrt{-2c+2\sqrt{c^2+d^2}}\,d-\sqrt{c^2+d^2}\sqrt{-2c+2\sqrt{c^2+d^2}}}\right)}{4a^3\sqrt{-2c+2\sqrt{c^2+d^2}}\,f\left(-\frac{id}{c-\sqrt{c^2+d^2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] (2*c^3 + 4*I*c^2*d - c*d^2 + 2*I*d^3)*arctan(2*(sqrt(d*tan(f*x + e) + c)*c
- sqrt(c^2 + d^2)*sqrt(d*tan(f*x + e) + c))/(c*sqrt(-2*c + 2*sqrt(c^2 + d^2
)) + I*sqrt(-2*c + 2*sqrt(c^2 + d^2))*d - sqrt(c^2 + d^2)*sqrt(-2*c + 2*sqr
t(c^2 + d^2))))/((8*I*a^3*c^2*f - 16*a^3*c*d*f - 8*I*a^3*d^2*f)*sqrt(-2*c +
 2*sqrt(c^2 + d^2))*(I*d/(c - sqrt(c^2 + d^2)) + 1)) + 1/48*(6*(d*tan(f*x +
 e) + c)^(5/2)*c^2*d - 12*(d*tan(f*x + e) + c)^(3/2)*c^3*d + 6*sqrt(d*tan(f
*x + e) + c)*c^4*d + 9*I*(d*tan(f*x + e) + c)^(5/2)*c*d^2 - 36*I*(d*tan(f*x
 + e) + c)^(3/2)*c^2*d^2 + 27*I*sqrt(d*tan(f*x + e) + c)*c^3*d^2 + 28*(d*ta
n(f*x + e) + c)^(3/2)*c*d^3 - 48*sqrt(d*tan(f*x + e) + c)*c^2*d^3 + 4*I*(d*
tan(f*x + e) + c)^(3/2)*d^4 - 39*I*sqrt(d*tan(f*x + e) + c)*c*d^4 + 12*sqrt
(d*tan(f*x + e) + c)*d^5)/((a^3*c^2*f + 2*I*a^3*c*d*f - a^3*d^2*f)*(d*tan(f
*x + e) - I*d)^3) - 1/4*(-I*c - d)*arctan(2*(sqrt(d*tan(f*x + e) + c)*c - s
qrt(c^2 + d^2)*sqrt(d*tan(f*x + e) + c))/(c*sqrt(-2*c + 2*sqrt(c^2 + d^2))
- I*sqrt(-2*c + 2*sqrt(c^2 + d^2))*d - sqrt(c^2 + d^2)*sqrt(-2*c + 2*sqrt(c
^2 + d^2))))/(a^3*sqrt(-2*c + 2*sqrt(c^2 + d^2))*f*(-I*d/(c - sqrt(c^2 + d^
2)) + 1))
```

Mupad [B]
time = 10.25, size = 2500, normalized size = 8.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(1/2)/(a + a*tan(e + f*x)*1i)^3,x)
```

```
[Out] log(58*a^3*c^2*d^11*f - (((c*d^10*20i + 15*c^2*d^9 + c^3*d^8*35i + 40*c^4*d
^7 + c^5*d^6*8i + 24*c^6*d^5 - c^7*d^4*8i - a^6*c^4*f^2*((((25*c^3*d^12)/4
- 5*c*d^14 + (41*c^5*d^10)/2 + (85*c^7*d^8)/4 + 10*c^9*d^6 + 2*c^11*d^4)/(a
^6*c^8*f^2 + a^6*d^8*f^2 + 4*a^6*c^2*d^6*f^2 + 6*a^6*c^4*d^4*f^2 + 4*a^6*c^
6*d^2*f^2) - (((65*c^2*d^13)/4 + (55*c^4*d^11)/2 + (93*c^6*d^9)/4 + 10*c^8*
```

d^7 + 2*c^10*d^5)*1i)/(a^6*c^8*f^2 + a^6*d^8*f^2 + 4*a^6*c^2*d^6*f^2 + 6*a^6*c^4*d^4*f^2 + 4*a^6*c^6*d^2*f^2))^2 + 4*((((25*c^3*d^13)/512 - (7*c*d^15)/256 + (7*c^5*d^11)/64 + (49*c^7*d^9)/512 + (5*c^9*d^7)/128 + (c^11*d^5)/128)*1i)/(a^12*c^8*f^4 + a^12*d^8*f^4 + 4*a^12*c^2*d^6*f^4 + 6*a^12*c^4*d^4*f^4 + 4*a^12*c^6*d^2*f^4) - (d^16/256 - (69*c^2*d^14)/1024 - (65*c^4*d^12)/1024 - (11*c^6*d^10)/1024 + (25*c^8*d^8)/1024 + (c^10*d^6)/64 + (c^12*d^4)/256)/(a^12*c^8*f^4 + a^12*d^8*f^4 + 4*a^12*c^2*d^6*f^4 + 6*a^12*c^4*d^4*f^4 + 4*a^12*c^6*d^2*f^4))*(256*d^6 + 256*c^2*d^4))^(1/2)*4i - a^6*d^4*f^2*((((25*c^3*d^12)/4 - 5*c*d^14 + (41*c^5*d^10)/2 + (85*c^7*d^8)/4 + 10*c^9*d^6 + 2*c^11*d^4)/(a^6*c^8*f^2 + a^6*d^8*f^2 + 4*a^6*c^2*d^6*f^2 + 6*a^6*c^4*d^4*f^2 + 4*a^6*c^6*d^2*f^2) - (((65*c^2*d^13)/4 + (55*c^4*d^11)/2 + (93*c^6*d^9)/4 + 10*c^8*d^7 + 2*c^10*d^5)*1i)/(a^6*c^8*f^2 + a^6*d^8*f^2 + 4*a^6*c^2*d^6*f^2 + 6*a^6*c^4*d^4*f^2 + 4*a^6*c^6*d^2*f^2))^2 + 4*((((25*c^3*d^13)/512 - (7*c*d^15)/256 + (7*c^5*d^11)/64 + (49*c^7*d^9)/512 + (5*c^9*d^7)/128 + (c^11*d^5)/128)*1i)/(a^12*c^8*f^4 + a^12*d^8*f^4 + 4*a^12*c^2*d^6*f^4 + 6*a^12*c^4*d^4*f^4 + 4*a^12*c^6*d^2*f^4) - (d^16/256 - (69*c^2*d^14)/1024 - (65*c^4*d^12)/1024 - (11*c^6*d^10)/1024 + (25*c^8*d^8)/1024 + (c^10*d^6)/64 + (c^12*d^4)/256)/(a^12*c^8*f^4 + a^12*d^8*f^4 + 4*a^12*c^2*d^6*f^4 + 6*a^12*c^4*d^4*f^4 + 4*a^12*c^6*d^2*f^4))*(256*d^6 + 256*c^2*d^4))^(1/2)*4i + a^6*c^2*d^2*f^2*((((25*c^3*d^12)/4 - 5*c*d^14 + (41*c^5*d^10)/2 + (85*c^7*d^8)/4 + 10*c^9*d^6 + 2*c^11*d^4)/(a^6*c^8*f^2 + a^6*d^8*f^2 + 4*a^6*c^2*d^6*f^2 + 6*a^6*c^4*d^4*f^2 + 4*a^6*c^6*d^2*f^2) - (((65*c^2*d^13)/4 + (55*c^4*d^11)/2 + (93*c^6*d^9)/4 + 10*c^8*d^7 + 2*c^10*d^5)*1i)/(a^6*c^8*f^2 + a^6*d^8*f^2 + 4*a^6*c^2*d^6*f^2 + 6*a^6*c^4*d^4*f^2 + 4*a^6*c^6*d^2*f^2))^2 + 4*((((25*c^3*d^13)/512 - (7*c*d^15)/256 + (7*c^5*d^11)/64 + (49*c^7*d^9)/512 + (5*c^9*d^7)/128 + (c^11*d^5)/128)*1i)/(a^12*c^8*f^4 + a^12*d^8*f^4 + 4*a^12*c^2*d^6*f^4 + 6*a^12*c^4*d^4*f^4 + 4*a^12*c^6*d^2*f^4) - (d^16/256 - (69*c^2*d^14)/1024 - (65*c^4*d^12)/1024 - (11*c^6*d^10)/1024 + (25*c^8*d^8)/1024 + (c^10*d^6)/64 + (c^12*d^4)/256)/(a^12*c^8*f^4 + a^12*d^8*f^4 + 4*a^12*c^2*d^6*f^4 + 6*a^12*c^4*d^4*f^4 + 4*a^12*c^6*d^2*f^4))*(256*d^6 + 256*c^2*d^4))^(1/2)*24i - 16*a^6*c*d^3*f^2*((((25*c^3*d^12)/4 - 5*c*d^14 + (41*c^5*d^10)/2 + (85*c^7*d^8)/4 + 10*c^9*d^6 + 2*c^11*d^4)/(a^6*c^8*f^2 + a^6*d^8*f^2 + 4*a^6*c^2*d^6*f^2 + 6*a^6*c^4*d^4*f^2 + 4*a^6*c^6*d^2*f^2) - (((65*c^2*d^13)/4 + (55*c^4*d^11)/2 + (93*c^6*d^9)/4 + 10*c^8*d^7 + 2*c^10*d^5)*1i)/(a^6*c^8*f^2 + a^6*d^8*f^2 + 4*a^6*c^2*d^6*f^2 + 6*a^6*c^4*d^4*f^2 + 4*a^6*c^6*d^2*f^2))^2 + 4*((((25*c^3*d^13)/512 - (7*c*d^15)/256 + (7*c^5*d^11)/64 + (49*c^7*d^9)/512 + (5*c^9*d^7)/128 + (c^11*d^5)/128)*1i)/(a^12*c^8*f^4 + a^12*d^8*f^4 + 4*a^12*c^2*d^6*f^4 + 6*a^12*c^4*d^4*f^4 + 4*a^12*c^6*d^2*f^4) - (d^16/256 - (69*c^2*d^14)/1024 - (65*c^4*d^12)/1024 - (11*c^6*d^10)/1024 + (25*c^8*d^8)/1024 + (c^10*d^6)/64 + (c^12*d^4)/256)/(a^12*c^8*f^4 + a^12*d^8*f^4 + 4*a^12*c^2*d^6*f^4 + 6*a^12*c^4*d^4*f^4 + 4*a^12*c^6*d^2*f^4))*(256*d^6 + 256*c^2*d^4))^(1/2) + 16*a^6*c^3*d*f^2*((((25*c^3*d^12)/4 - 5*c*d^14 + (41*c^5*d^10)/2 + (85*c^7*d^8)/4 + 10*c^9*d^6 + 2*c^11*d^4)/(a^6*c^8*f^2 + a^6*d^8*f^2 + 4*a^6*c^2*d^6*f^2 + 6*a^6*c^4*d^4*f^2 + 4*a^6*c^6*d^2*f^2) - (((65*c^2*d^13)/4 + (55*c^4*d^11)/2 + (93*c^6*d^9)/4 + 10*c^8*d^7

+ 2*c^10*d^5)*1i)/(a^6*c^8*f^2 + a^6*d^8*f^2 + 4*a^6*c^2*d^6*f^2 + 6*a^6*c^4*d^4*f^2 + 4*a^6*c^6*d^2*f^2))^2 + 4*((((25*c^3*d^13)/512 - (7*c*d^15)/256 + (7*c^5*d^11)/64 + (49*c^7*d^9)/512 + (5*c^9*d^7)/128 + (c^11*d^5)/128)*1i)/(a^12*c^8*f^4 + a^12*d^8*f^4 + 4*a^12*c^2*d^6*f^4 + 6*a^12*c^4*d^4*f^4 + 4*a^12*c^6*d^2*f^4) - (d^16/256 - (69*c^2*d^14)/1024 - (65*c^4*d^12)/1024 - (11*c^6*d^10)/1024 + (25*c^8*d^8)/1024 + (c^10*d^6)/64 + (c^12*d^4)/256)/(a^12*c^8*f^4 + a^12*d^8*f^4 + 4*a^12*c^2*d^6*f^4 + 6*a^12*c^4*d^4*f^4 + 4*a^12*c^6*d^2*f^4))*(256*d^6 + 256*c^2*d^4))^(1/2))/(2048*(d^6 + c^2*d^4)*(a^6*c^4*f^2*1i + a^6*d^4*f^2*1i + 4*a^6*c*d^3*f^2 - 4*a^6*c^3*d*f^2 - a^6*c^2*d^2*f^2*6i)))^(1/2)*(7168*a^9*c*d^11*f^3 - a^9*d^12*f^3*4096i + 32*(c + d*tan(e + f*x))^(1/2)*((c*d^10*20i + 15*c^2*d^9 + c^3*d^8*35i + 40*c^4*d^7 + c^5*d^6*8i + 24*c^6*d^5 - c^7*d^4*8i - a^6*c^4*f^2*((((25*c^3*d^12)/4 - 5*c*d^14 + (41*c^5*d^10)/2 + (85*c^7*d^8)/4 + 10*c^9*d^6 + 2*c^11*d^4)/(a^6*c^8*f^2 + a^6*d^8*f^2 + 4*a^6*c^2*d^6*f^2 + 6*a^6*c^4*d^4*f^2 + 4*a^6*c^6*d^2*f^2) - (((65*c^2*d^13)/4 + (55*c^4*d^11)/2 + (93*c^6*d^9)/4 + 10*c^8*d^7 + 2*c^10*d^5)*1i)/(a^6*c^8*f^2 + a^6*d^8*f^2 + 4*a^6*c^2*d^6*f^2 + 6*a^6*c^4*d^4*f^2 + 4*a^6*c^6*d^2*f^2))^2 + 4*((((25*c...

3.1107 $\int (a+ia\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=181

$$-\frac{8ia^3(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{8a^3(ic+d)\sqrt{c+d\tan(e+fx)}}{f}+\frac{8ia^3(c+d\tan(e+fx}{3f}$$

[Out] -8*I*a^3*(c-I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/f+8*a^3*(I*c+d)*(c+d*tan(f*x+e))^(1/2)/f+8/3*I*a^3*(c+d*tan(f*x+e))^(3/2)/f+4/35*a^3*(I*c-8*d)*(c+d*tan(f*x+e))^(5/2)/d^2/f-2/7*(a^3+I*a^3*tan(f*x+e))*(c+d*tan(f*x+e))^(5/2)/d/f

Rubi [A]
time = 0.36, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3637, 3673, 3609, 3618, 65, 214}

$$\frac{4a^3(-8d+ic)(c+d\tan(e+fx))^{5/2}}{35d^2f}+\frac{8ia^3(c+d\tan(e+fx))^{3/2}}{3f}+\frac{8a^3(d+ic)\sqrt{c+d\tan(e+fx)}}{f}-\frac{2(a^3+ia^3\tan(e+fx))(c+d\tan(e+fx))^{5/2}}{7df}-\frac{8ia^3(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}$$

Antiderivative was successfully verified.

[In] Int[(a + I*a*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^(3/2),x]

[Out] ((-8*I)*a^3*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/f + (8*a^3*(I*c + d)*Sqrt[c + d*Tan[e + f*x]])/f + (((8*I)/3)*a^3*(c + d*Tan[e + f*x])^(3/2))/f + (4*a^3*(I*c - 8*d)*(c + d*Tan[e + f*x])^(5/2))/(35*d^2*f) - (2*(a^3 + I*a^3*Tan[e + f*x])*(c + d*Tan[e + f*x])^(5/2))/(7*d*f)

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
```

```
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (a+ia\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}\,dx = -\frac{2(a^3+ia^3\tan(e+fx))\,(c+d\tan(e+fx))^{5/2}}{7df} + \frac{(2a)}{}$$

$$= \frac{4a^3(ic-8d)(c+d\tan(e+fx))^{5/2}}{35d^2f} - \frac{2(a^3+ia^3\tan(e-}{}$$

$$= \frac{8ia^3(c+d\tan(e+fx))^{3/2}}{3f} + \frac{4a^3(ic-8d)(c+d\tan(e-}{35d^2f}$$

$$= \frac{8a^3(ic+d)\sqrt{c+d\tan(e+fx)}}{f} + \frac{8ia^3(c+d\tan(e+}{3f}$$

$$= \frac{8a^3(ic+d)\sqrt{c+d\tan(e+fx)}}{f} + \frac{8ia^3(c+d\tan(e+}{3f}$$

$$= \frac{8a^3(ic+d)\sqrt{c+d\tan(e+fx)}}{f} + \frac{8ia^3(c+d\tan(e+}{3f}$$

$$= -\frac{8ia^3(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f} + \frac{8a}{}$$

Mathematica [A]
time = 7.61, size = 271, normalized size = 1.50

$$\frac{a^3(\cos(e+fx)+i\sin(e+fx))^3\left(-8i(c-id)^{3/2}e^{-3ie}\tanh^{-1}\left(\frac{\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}}{\sqrt{c-id}}\right)+\frac{\sec^2(e+fx)(i\cos(3e)+\sin(3e))\sqrt{c+d\tan(e+fx)}\,(6c^3+63ic^2d+536cd^2-357id^3+d(-3c^2+126icd+125d^2)\tan(e+fx)+\cos(2(e+fx))(6c^3+63ic^2d+584cd^2-483id^3+d(-3c^2+126icd+155d^2)\tan(e+fx)))}{105d^2}\right)}{f(\cos(fx)+i\sin(fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (a^3*(Cos[e + f*x] + I*Sin[e + f*x])^3*(((-8*I)*(c - I*d)^(3/2)*ArcTanh[Sqr
t[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))]/Sqrt[c -
I*d]])/E^((3*I)*e) + (Sec[e + f*x]^2*(I*Cos[3*e] + Sin[3*e])*Sqrt[c + d*Tan
[e + f*x]]*(6*c^3 + (63*I)*c^2*d + 536*c*d^2 - (357*I)*d^3 + d*(-3*c^2 + (1
26*I)*c*d + 125*d^2)*Tan[e + f*x] + Cos[2*(e + f*x)]*(6*c^3 + (63*I)*c^2*d
+ 584*c*d^2 - (483*I)*d^3 + d*(-3*c^2 + (126*I)*c*d + 155*d^2)*Tan[e + f*x]
)))/(105*d^2)))/(f*(Cos[f*x] + I*Sin[f*x])^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 909 vs. 2(154) = 308.
time = 0.29, size = 910, normalized size = 5.03

method	result
derivativedivides	$\frac{2a^3\left(-\frac{i(c+d\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{ic(c+d\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{4id^2(c+d\tan(fx+e))^{\frac{3}{2}}}{3}-\frac{3d(c+d\tan(fx+e))^{\frac{5}{2}}}{5}+4id^2c\sqrt{c+d\tan(fx+\right.}{}$
default	$\frac{2a^3\left(-\frac{i(c+d\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{ic(c+d\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{4id^2(c+d\tan(fx+e))^{\frac{3}{2}}}{3}-\frac{3d(c+d\tan(fx+e))^{\frac{5}{2}}}{5}+4id^2c\sqrt{c+d\tan(fx+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3*(c+d*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3/d^2*(-1/7*I*(c+d*tan(f*x+e))^(7/2)+1/5*I*c*(c+d*tan(f*x+e))^(5/2)+4
/3*I*d^2*(c+d*tan(f*x+e))^(3/2)-3/5*d*(c+d*tan(f*x+e))^(5/2)+4*I*d^2*c*(c+d
*tan(f*x+e))^(1/2)+4*d^3*(c+d*tan(f*x+e))^(1/2)-4*d^2*(1/2/(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)/(c^2+d^2)^(1/2)*(1/2*(-I*c^2*(c^2+d^2)^(1/2)+I*d^2*(c^2+d^2)^
(1/2)-I*c^3-I*c*d^2-2*c*d*(c^2+d^2)^(1/2)-c^2*d-d^3)*ln(d*tan(f*x+e)+c-(c+d
*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(I*(2*(
c^2+d^2)^(1/2)+2*c)^(1/2)*c^3+I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^2+(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*c^2*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^3+1/2*(-I*c^2*
(c^2+d^2)^(1/2)+I*d^2*(c^2+d^2)^(1/2)-I*c^3-I*c*d^2-2*c*d*(c^2+d^2)^(1/2)-c
^2*d-d^3)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arct
an((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1
/2)-2*c)^(1/2)))+1/2/(2*(c^2+d^2)^(1/2)+2*c)^(1/2)/(c^2+d^2)^(1/2)*(1/2*(I*
c^2*(c^2+d^2)^(1/2)-I*d^2*(c^2+d^2)^(1/2)+I*c^3+I*c*d^2+2*c*d*(c^2+d^2)^(1/
2)+c^2*d+d^3)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)+(c^2+d^2)^(1/2))+2*(I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3+I*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*c*d^2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2*d+(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*d^3-1/2*(I*c^2*(c^2+d^2)^(1/2)-I*d^2*(c^2+d^2)^(1/2)+I*
c^3+I*c*d^2+2*c*d*(c^2+d^2)^(1/2)+c^2*d+d^3)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))
/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2
)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c+d*tan(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^3*(d*tan(f*x + e) + c)^(3/2), x)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 810 vs. 2(155) = 310.
time = 1.86, size = 810, normalized size = 4.48

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c+d*tan(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 2/105*(105*(d^2*f*e^(6*I*f*x + 6*I*e) + 3*d^2*f*e^(4*I*f*x + 4*I*e) + 3*d^2
*f*e^(2*I*f*x + 2*I*e) + d^2*f)*sqrt(-(a^6*c^3 - 3*I*a^6*c^2*d - 3*a^6*c*d^
2 + I*a^6*d^3)/f^2)*log(2*(-I*a^3*c^2 - a^3*c*d + (f*e^(2*I*f*x + 2*I*e) +
f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1)
)*sqrt(-(a^6*c^3 - 3*I*a^6*c^2*d - 3*a^6*c*d^2 + I*a^6*d^3)/f^2) + (-I*a^3*
c^2 - 2*a^3*c*d + I*a^3*d^2)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/(-I*
a^3*c - a^3*d)) - 105*(d^2*f*e^(6*I*f*x + 6*I*e) + 3*d^2*f*e^(4*I*f*x + 4*I
*e) + 3*d^2*f*e^(2*I*f*x + 2*I*e) + d^2*f)*sqrt(-(a^6*c^3 - 3*I*a^6*c^2*d -
 3*a^6*c*d^2 + I*a^6*d^3)/f^2)*log(2*(-I*a^3*c^2 - a^3*c*d - (f*e^(2*I*f*x
+ 2*I*e) + f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x +
2*I*e) + 1))*sqrt(-(a^6*c^3 - 3*I*a^6*c^2*d - 3*a^6*c*d^2 + I*a^6*d^3)/f^2)
 + (-I*a^3*c^2 - 2*a^3*c*d + I*a^3*d^2)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x -
2*I*e)/(-I*a^3*c - a^3*d)) - 2*(-3*I*a^3*c^3 + 30*a^3*c^2*d - 229*I*a^3*c*d
^2 - 164*a^3*d^3 + (-3*I*a^3*c^3 + 33*a^3*c^2*d - 355*I*a^3*c*d^2 - 319*a^3
*d^3)*e^(6*I*f*x + 6*I*e) + (-9*I*a^3*c^3 + 96*a^3*c^2*d - 891*I*a^3*c*d^2
- 646*a^3*d^3)*e^(4*I*f*x + 4*I*e) + (-9*I*a^3*c^3 + 93*a^3*c^2*d - 765*I*a
^3*c*d^2 - 551*a^3*d^3)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2
*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))/(d^2*f*e^(6*I*f*x + 6*I*e) + 3
*d^2*f*e^(4*I*f*x + 4*I*e) + 3*d^2*f*e^(2*I*f*x + 2*I*e) + d^2*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$- i a^{3} \left(\int i c \sqrt{c + d \tan{\left(e + f x \right)}}\, dx + \int \left(- 3 c \sqrt{c + d \tan{\left(e + f x \right)}} \tan{\left(e + f x \right)}\right)\, dx + \int c \sqrt{c + d \tan{\left(e + f x \right)}} \tan^{3}{\left(e + f x \right)}\, dx + \int \left(- 3 d \sqrt{c + d \tan{\left(e + f x \right)}} \tan^{2}{\left(e + f x \right)}\right)\, dx + \int d \sqrt{c + d \tan{\left(e + f x \right)}} \tan^{4}{\left(e + f x \right)}\, dx + \int \left(- 3 i c \sqrt{c + d \tan{\left(e + f x \right)}} \tan^{2}{\left(e + f x \right)}\right)\, dx + \int i d \sqrt{c + d \tan{\left(e + f x \right)}} \tan{\left(e + f x \right)}\, dx + \int \left(- 3 i d \sqrt{c + d \tan{\left(e + f x \right)}} \tan^{3}{\left(e + f x \right)}\right)\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3*(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] -I*a**3*(Integral(I*c*sqrt(c + d*tan(e + f*x)), x) + Integral(-3*c*sqrt(c +
 d*tan(e + f*x))*tan(e + f*x), x) + Integral(c*sqrt(c + d*tan(e + f*x))*tan
(e + f*x)**3, x) + Integral(-3*d*sqrt(c + d*tan(e + f*x))*tan(e + f*x)**2,
x) + Integral(d*sqrt(c + d*tan(e + f*x))*tan(e + f*x)**4, x) + Integral(-3*
I*c*sqrt(c + d*tan(e + f*x))*tan(e + f*x)**2, x) + Integral(I*d*sqrt(c + d*
tan(e + f*x))*tan(e + f*x), x) + Integral(-3*I*d*sqrt(c + d*tan(e + f*x))*t
an(e + f*x)**3, x))
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(155) = 310$.
time = 0.96, size = 337, normalized size = 1.86

$$-\frac{16\left(-i a^3c^2-2a^3cd+i a^3d^2\right)\arctan\left(\frac{2\left(\sqrt{d\tan\left(fx+e\right)+c}\,c-\sqrt{c^2+d^2}\,\sqrt{d\tan\left(fx+e\right)+c}\right)}{c\sqrt{-2c+2\sqrt{c^2+d^2}}-i\sqrt{-2c+2\sqrt{c^2+d^2}}\,d-\sqrt{c^2+d^2}\,\sqrt{-2c+2\sqrt{c^2+d^2}}}\right)}{\sqrt{-2c+2\sqrt{c^2+d^2}}\,f\left(-\frac{id}{c-\sqrt{c^2+d^2}}+1\right)}-\frac{2\left(15i\left(d\tan\left(fx+e\right)+c\right)^{\frac{7}{2}}a^3d^{12}f^6-21i\left(d\tan\left(fx+e\right)+c\right)^{\frac{5}{2}}a^3cd^{12}f^6+63\left(d\tan\left(fx+e\right)+c\right)^{\frac{5}{2}}a^3d^{13}f^6-140i\left(d\tan\left(fx+e\right)+c\right)^{\frac{3}{2}}a^3d^{14}f^6-420i\sqrt{d\tan\left(fx+e\right)+c}\,a^3cd^{14}f^6-420\sqrt{d\tan\left(fx+e\right)+c}\,a^3d^{15}f^6\right)}{105\,d^{14}f^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c+d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] -16*(-I*a^3*c^2 - 2*a^3*c*d + I*a^3*d^2)*arctan(2*(sqrt(d*tan(f*x + e) + c)
*c - sqrt(c^2 + d^2)*sqrt(d*tan(f*x + e) + c))/(c*sqrt(-2*c + 2*sqrt(c^2 +
d^2)) - I*sqrt(-2*c + 2*sqrt(c^2 + d^2))*d - sqrt(c^2 + d^2)*sqrt(-2*c + 2*
sqrt(c^2 + d^2))))/(sqrt(-2*c + 2*sqrt(c^2 + d^2))*f*(-I*d/(c - sqrt(c^2 +
d^2)) + 1)) - 2/105*(15*I*(d*tan(f*x + e) + c)^(7/2)*a^3*d^12*f^6 - 21*I*(d
*tan(f*x + e) + c)^(5/2)*a^3*c*d^12*f^6 + 63*(d*tan(f*x + e) + c)^(5/2)*a^3
*d^13*f^6 - 140*I*(d*tan(f*x + e) + c)^(3/2)*a^3*d^14*f^6 - 420*I*sqrt(d*ta
n(f*x + e) + c)*a^3*c*d^14*f^6 - 420*sqrt(d*tan(f*x + e) + c)*a^3*d^15*f^6)
/(d^14*f^7)
```

Mupad [**B**]
time = 13.96, size = 309, normalized size = 1.71

$$-\left(\frac{(c-d\,1\mathrm{i})\left(\frac{a^3\,(c-d\,1\mathrm{i})\,2\mathrm{i}}{d^2\,f}-\frac{a^3\,(c+d\,1\mathrm{i})\,4\mathrm{i}}{d^2\,f}\right)}{3}+\frac{a^3\,{(c+d\,1\mathrm{i})}^2\,2\mathrm{i}}{3\,d^2\,f}\right)\,{(c+d\tan(e+f\,x))}^{3/2}-\left(\frac{a^3\,(c-d\,1\mathrm{i})\,2\mathrm{i}}{5\,d^2\,f}-\frac{a^3\,(c+d\,1\mathrm{i})\,4\mathrm{i}}{5\,d^2\,f}\right)\,{(c+d\tan(e+f\,x))}^{5/2}-(c-d\,1\mathrm{i})\,\left((c-d\,1\mathrm{i})\,\left(\frac{a^3\,(c-d\,1\mathrm{i})\,2\mathrm{i}}{d^2\,f}-\frac{a^3\,(c+d\,1\mathrm{i})\,4\mathrm{i}}{d^2\,f}\right)+\frac{a^3\,{(c+d\,1\mathrm{i})}^2\,2\mathrm{i}}{d^2\,f}\right)\,\sqrt{c+d\tan\left(e+f\,x\right)}-\frac{a^3\,{(c+d\tan(e+f\,x))}^{7/2}\,2\mathrm{i}}{7\,d^2\,f}+\frac{\sqrt{16\mathrm{i}}\,a^3\,\mathrm{atan}\left(\frac{\sqrt{16\mathrm{i}}\,{(-d-c\,1\mathrm{i})}^{3/2}\,\sqrt{c+d\tan\left(e+f\,x\right)}\,1\mathrm{i}}{4\,(c^2\,1\mathrm{i}+2\,c\,d-d^2\,1\mathrm{i})}\right)\,{(-d-c\,1\mathrm{i})}^{3/2}\,2\mathrm{i}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^3*(c + d*tan(e + f*x))^(3/2),x)
```

```
[Out] (16i^(1/2)*a^3*atan((16i^(1/2)*(- c*1i - d)^(3/2)*(c + d*tan(e + f*x))^(1/2
)*1i)/(4*(2*c*d + c^2*1i - d^2*1i)))*(- c*1i - d)^(3/2)*2i)/f - ((a^3*(c -
d*1i)*2i)/(5*d^2*f) - (a^3*(c + d*1i)*4i)/(5*d^2*f))*(c + d*tan(e + f*x))^(
5/2) - (c - d*1i)*((c - d*1i)*((a^3*(c - d*1i)*2i)/(d^2*f) - (a^3*(c + d*1i
)*4i)/(d^2*f)) + (a^3*(c + d*1i)^2*2i)/(d^2*f))*(c + d*tan(e + f*x))^(1/2)
- (a^3*(c + d*tan(e + f*x))^(7/2)*2i)/(7*d^2*f) - (((c - d*1i)*((a^3*(c - d
*1i)*2i)/(d^2*f) - (a^3*(c + d*1i)*4i)/(d^2*f)))/3 + (a^3*(c + d*1i)^2*2i)/
(3*d^2*f))*(c + d*tan(e + f*x))^(3/2)
```

3.1108 $\int (a+ia\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=131

$$-\frac{4ia^2(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{4a^2(ic+d)\sqrt{c+d\tan(e+fx)}}{f}+\frac{4ia^2(c+d\tan(e+fx}{3f}$$

```
[Out] -4*I*a^2*(c-I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/f+4*a^
2*(I*c+d)*(c+d*tan(f*x+e))^(1/2)/f+4/3*I*a^2*(c+d*tan(f*x+e))^(3/2)/f-2/5*a
^2*(c+d*tan(f*x+e))^(5/2)/d/f
```

Rubi [A]
time = 0.22, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3624, 3609, 3618, 65, 214}

$$-\frac{2a^2(c+d\tan(e+fx))^{5/2}}{5df}+\frac{4ia^2(c+d\tan(e+fx))^{3/2}}{3f}+\frac{4a^2(d+ic)\sqrt{c+d\tan(e+fx)}}{f}-\frac{4ia^2(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-4*I)*a^2*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]]
)/f + (4*a^2*(I*c + d)*Sqrt[c + d*Tan[e + f*x]])/f + (((4*I)/3)*a^2*(c + d*
Tan[e + f*x])^(3/2))/f - (2*a^2*(c + d*Tan[e + f*x])^(5/2))/(5*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
```

```
0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int (a+ia\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}\,dx = -\frac{2a^2(c+d\tan(e+fx))^{5/2}}{5df}+\int \left(2a^2+2ia^2\tan(e+fx\right.$$

$$=\frac{4ia^2(c+d\tan(e+fx))^{3/2}}{3f}-\frac{2a^2(c+d\tan(e+fx))^{5/2}}{5df}$$

$$=\frac{4a^2(ic+d)\sqrt{c+d\tan(e+fx)}}{f}+\frac{4ia^2(c+d\tan(e+fx}{3f}$$

$$=\frac{4a^2(ic+d)\sqrt{c+d\tan(e+fx)}}{f}+\frac{4ia^2(c+d\tan(e+fx}{3f}$$

$$=\frac{4a^2(ic+d)\sqrt{c+d\tan(e+fx)}}{f}+\frac{4ia^2(c+d\tan(e+fx}{3f}$$

$$=-\frac{4ia^2(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{4a^2(}{}$$

Mathematica [**A**]

time = 3.98, size = 221, normalized size = 1.69

$$\frac{a^2(\cos(e+fx)+i\sin(e+fx))^2\left(-4i(c-id)^{3/2}e^{-2ie}\tanh^{-1}\left(\frac{\sqrt{c-\frac{id(-1+e^{2i(e+fx)})}{1+e^{2i(e+fx)}}}}{\sqrt{c-id}}\right)-\frac{\sec^2(e+fx)(\cos(2e)-i\sin(2e))(3c^2-40icd-27d^2+(3c^2-40icd-33d^2)\cos(2(e+fx))+2(3c-5id)d\sin(2(e+fx)))\sqrt{c+d\tan(e+fx)}}{15d}\right)}{f(\cos(fx)+i\sin(fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (a^2*(Cos[e + f*x] + I*Sin[e + f*x])^2*(((-4*I)*(c - I*d)^(3/2)*ArcTanh[Sqr
t[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))]/Sqrt[c -
I*d]])/E^((2*I)*e) - (Sec[e + f*x]^2*(Cos[2*e] - I*Sin[2*e])*(3*c^2 - (40*I
)*c*d - 27*d^2 + (3*c^2 - (40*I)*c*d - 33*d^2)*Cos[2*(e + f*x)] + 2*(3*c -
(5*I)*d)*d*Sin[2*(e + f*x)])*Sqrt[c + d*Tan[e + f*x]])/(15*d)))/(f*(Cos[f*x
] + I*Sin[f*x])^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 871 vs. 2(110) = 220.
time = 0.27, size = 872, normalized size = 6.66

method	result
derivativedivides	$2a^2\left(-\frac{(c+d\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{2id(c+d\tan(fx+e))^{\frac{3}{2}}}{3}+2idc\sqrt{c+d\tan(fx+e)}+2d^2\sqrt{c+d\tan(fx+e)}-\right.$
default	$2a^2\left(-\frac{(c+d\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{2id(c+d\tan(fx+e))^{\frac{3}{2}}}{3}+2idc\sqrt{c+d\tan(fx+e)}+2d^2\sqrt{c+d\tan(fx+e)}-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2*(c+d*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^2/d*(-1/5*(c+d*tan(f*x+e))^(5/2)+2/3*I*d*(c+d*tan(f*x+e))^(3/2)+2*I*d
*c*(c+d*tan(f*x+e))^(1/2)+2*d^2*(c+d*tan(f*x+e))^(1/2)-2*d*(1/2/(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)/(c^2+d^2)^(1/2)*(1/2*(-I*c^2*(c^2+d^2)^(1/2)+I*d^2*(c^2+
```

d^2)^(1/2)-I*c^3-I*c*d^2-2*c*d*(c^2+d^2)^(1/2)-c^2*d-d^3)*ln(d*tan(f*x+e)+c -(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(I *(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3+I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^2+(2 *(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^3+1/2*(-I *c^2*(c^2+d^2)^(1/2)+I*d^2*(c^2+d^2)^(1/2)-I*c^3-I*c*d^2-2*c*d*(c^2+d^2)^(1 /2)-c^2*d-d^3)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2) *arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^ 2)^(1/2)-2*c)^(1/2)))+1/2/(2*(c^2+d^2)^(1/2)+2*c)^(1/2)/(c^2+d^2)^(1/2)*(1/ 2*(I*c^2*(c^2+d^2)^(1/2)-I*d^2*(c^2+d^2)^(1/2)+I*c^3+I*c*d^2+2*c*d*(c^2+d^2)^(1/2)+c^2*d+d^3)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1 /2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3+I*(2 *(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2*d+(2*(c ^2+d^2)^(1/2)+2*c)^(1/2)*d^3-1/2*(I*c^2*(c^2+d^2)^(1/2)-I*d^2*(c^2+d^2)^(1/ 2)+I*c^3+I*c*d^2+2*c*d*(c^2+d^2)^(1/2)+c^2*d+d^3)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^ 2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I*a*tan(f*x+e))^2*(c+d*tan(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate((I*a*tan(f*x + e) + a)^2*(d*tan(f*x + e) + c)^(3/2), x)

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 666 vs. 2(109) = 218.
time = 1.46, size = 666, normalized size = 5.08

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I*a*tan(f*x+e))^2*(c+d*tan(f*x+e))^(3/2),x, algorithm="fricas")

[Out] 1/15*(15*(d*f*e^(4*I*f*x + 4*I*e) + 2*d*f*e^(2*I*f*x + 2*I*e) + d*f)*sqrt(-(a^4*c^3 - 3*I*a^4*c^2*d - 3*a^4*c*d^2 + I*a^4*d^3)/f^2)*log(2*(-I*a^2*c^2 - a^2*c*d + (f*e^(2*I*f*x + 2*I*e) + f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(a^4*c^3 - 3*I*a^4*c^2*d - 3*a ^4*c*d^2 + I*a^4*d^3)/f^2) + (-I*a^2*c^2 - 2*a^2*c*d + I*a^2*d^2)*e^(2*I*f* x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/(-I*a^2*c - a^2*d)) - 15*(d*f*e^(4*I*f*x + 4*I*e) + 2*d*f*e^(2*I*f*x + 2*I*e) + d*f)*sqrt(-(a^4*c^3 - 3*I*a^4*c^2*d -

```
 3*a^4*c*d^2 + I*a^4*d^3)/f^2)*log(2*(-I*a^2*c^2 - a^2*c*d - (f*e^(2*I*f*x
+ 2*I*e) + f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x +
2*I*e) + 1))*sqrt(-(a^4*c^3 - 3*I*a^4*c^2*d - 3*a^4*c*d^2 + I*a^4*d^3)/f^2)
 + (-I*a^2*c^2 - 2*a^2*c*d + I*a^2*d^2)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x -
2*I*e)/(-I*a^2*c - a^2*d)) - 2*(3*a^2*c^2 - 34*I*a^2*c*d - 23*a^2*d^2 + (3*
a^2*c^2 - 46*I*a^2*c*d - 43*a^2*d^2)*e^(4*I*f*x + 4*I*e) + 2*(3*a^2*c^2 - 4
0*I*a^2*c*d - 27*a^2*d^2)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(2*I*f*x +
 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))/(d*f*e^(4*I*f*x + 4*I*e) + 2
*d*f*e^(2*I*f*x + 2*I*e) + d*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int\left(-c\sqrt{c+d\tan(e+fx)}\right)dx+\int c\sqrt{c+d\tan(e+fx)}\tan^2(e+fx)\,dx+\int\left(-d\sqrt{c+d\tan(e+fx)}\tan(e+fx)\right)dx+\int d\sqrt{c+d\tan(e+fx)}\tan^3(e+fx)\,dx+\int\left(-2ic\sqrt{c+d\tan(e+fx)}\tan(e+fx)\right)dx+\int\left(-2id\sqrt{c+d\tan(e+fx)}\tan^2(e+fx)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2*(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] -a**2*(Integral(-c*sqrt(c + d*tan(e + f*x)), x) + Integral(c*sqrt(c + d*tan
(e + f*x))*tan(e + f*x)**2, x) + Integral(-d*sqrt(c + d*tan(e + f*x))*tan(e
 + f*x), x) + Integral(d*sqrt(c + d*tan(e + f*x))*tan(e + f*x)**3, x) + Int
egral(-2*I*c*sqrt(c + d*tan(e + f*x))*tan(e + f*x), x) + Integral(-2*I*d*sq
rt(c + d*tan(e + f*x))*tan(e + f*x)**2, x))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(109) = 218$.
time = 0.81, size = 288, normalized size = 2.20

$$-\frac{8\left(-ia^2c^2-2a^2cd+ia^2d^2\right)\arctan\left(\frac{2\left(\sqrt{d\tan(fx+e)+c}\,c-\sqrt{c^2+d^2}\sqrt{d\tan(fx+e)+c}\right)}{c\sqrt{-2c+2\sqrt{c^2+d^2}}-i\sqrt{-2c+2\sqrt{c^2+d^2}}\,d-\sqrt{c^2+d^2}\sqrt{-2c+2\sqrt{c^2+d^2}}}\right)}{\sqrt{-2c+2\sqrt{c^2+d^2}}\,f\left(-\frac{id}{c-\sqrt{c^2+d^2}}+1\right)}-\frac{2\left(3(d\tan(fx+e)+c)^{\frac{5}{2}}a^2d^4f^4-10i(d\tan(fx+e)+c)^{\frac{3}{2}}a^2d^5f^4-30i\sqrt{d\tan(fx+e)+c}\,a^2cd^5f^4-30\sqrt{d\tan(fx+e)+c}\,a^2d^6f^4\right)}{15d^5f^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c+d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] -8*(-I*a^2*c^2 - 2*a^2*c*d + I*a^2*d^2)*arctan(2*(sqrt(d*tan(f*x + e) + c)*
c - sqrt(c^2 + d^2)*sqrt(d*tan(f*x + e) + c))/(c*sqrt(-2*c + 2*sqrt(c^2 + d
^2)) - I*sqrt(-2*c + 2*sqrt(c^2 + d^2))*d - sqrt(c^2 + d^2)*sqrt(-2*c + 2*s
qrt(c^2 + d^2))))/(sqrt(-2*c + 2*sqrt(c^2 + d^2))*f*(-I*d/(c - sqrt(c^2 + d
^2)) + 1)) - 2/15*(3*(d*tan(f*x + e) + c)^(5/2)*a^2*d^4*f^4 - 10*I*(d*tan(f
*x + e) + c)^(3/2)*a^2*d^5*f^4 - 30*I*sqrt(d*tan(f*x + e) + c)*a^2*c*d^5*f^
4 - 30*sqrt(d*tan(f*x + e) + c)*a^2*d^6*f^4)/(d^5*f^5)
```

Mupad [B]
time = 10.32, size = 196, normalized size = 1.50

$$-\left(\frac{2a^2(c-d\,1\mathrm{i})}{3df}-\frac{2a^2(c+d\,1\mathrm{i})}{3df}\right)(c+d\tan(e+fx))^{3/2}-(c-d\,1\mathrm{i})\left(\frac{2a^2(c-d\,1\mathrm{i})}{df}-\frac{2a^2(c+d\,1\mathrm{i})}{df}\right)\sqrt{c+d\tan(e+fx)}-\frac{2a^2(c+d\tan(e+fx))^{5/2}}{5df}+\frac{\sqrt{4\mathrm{i}}\,a^2\,\mathrm{atan}\left(\frac{\sqrt{4\mathrm{i}}\,(-d-c\,1\mathrm{i})^{3/2}\sqrt{c+d\tan(e+fx)}\,1\mathrm{i}}{2(c^2\,1\mathrm{i}+2cd-d^2\,1\mathrm{i})}\right)(-d-c\,1\mathrm{i})^{3/2}\,2\mathrm{i}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^2*(c + d*tan(e + f*x))^(3/2),x)
```

```
[Out] (4i^(1/2)*a^2*atan((4i^(1/2)*(- c*1i - d)^(3/2)*(c + d*tan(e + f*x))^(1/2)*
1i)/(2*(2*c*d + c^2*1i - d^2*1i)))*(- c*1i - d)^(3/2)*2i)/f - (c - d*1i)*((
2*a^2*(c - d*1i))/(d*f) - (2*a^2*(c + d*1i))/(d*f))*(c + d*tan(e + f*x))^(1
/2) - (2*a^2*(c + d*tan(e + f*x))^(5/2))/(5*d*f) - ((2*a^2*(c - d*1i))/(3*d
*f) - (2*a^2*(c + d*1i))/(3*d*f))*(c + d*tan(e + f*x))^(3/2)
```

3.1109 $\int (a+ia\tan(e+fx))(c+d\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=98

$$-\frac{2ia(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{2a(ic+d)\sqrt{c+d\tan(e+fx)}}{f}+\frac{2ia(c+d\tan(e+fx))^{3/2}}{3f}$$

```
[Out] -2*I*a*(c-I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/f+2*a*(I
*c+d)*(c+d*tan(f*x+e))^(1/2)/f+2/3*I*a*(c+d*tan(f*x+e))^(3/2)/f
```

Rubi [A]
time = 0.15, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3609, 3618, 65, 214}

$$\frac{2ia(c+d\tan(e+fx))^{3/2}}{3f}+\frac{2a(d+ic)\sqrt{c+d\tan(e+fx)}}{f}-\frac{2ia(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])*(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-2*I)*a*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/
f + (2*a*(I*c + d)*Sqrt[c + d*Tan[e + f*x]])/f + (((2*I)/3)*a*(c + d*Tan[e
+ f*x])^(3/2))/f
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
```

```
0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rubi steps

$$\int (a+ia\tan(e+fx))(c+d\tan(e+fx))^{3/2}\,dx = \frac{2ia(c+d\tan(e+fx))^{3/2}}{3f}+\int\sqrt{c+d\tan(e+fx)}\,(a(c-$$

$$=\frac{2a(ic+d)\sqrt{c+d\tan(e+fx)}}{f}+\frac{2ia(c+d\tan(e+fx))^{3}}{3f}$$

$$=\frac{2a(ic+d)\sqrt{c+d\tan(e+fx)}}{f}+\frac{2ia(c+d\tan(e+fx))^{3}}{3f}$$

$$=\frac{2a(ic+d)\sqrt{c+d\tan(e+fx)}}{f}+\frac{2ia(c+d\tan(e+fx))^{3}}{3f}$$

$$=-\frac{2ia(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{2a(ic}{}$$

Mathematica [A]
time = 2.05, size = 111, normalized size = 1.13

$$\frac{2a\left(-3i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}}{\sqrt{c-id}}\right)+(4ic+3d+id\tan(e+fx))\sqrt{c+d\tan(e+fx)}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])*(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*a*((-3*I)*(c - I*d)^(3/2)*ArcTanh[Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*x)
)))/(1 + E^((2*I)*(e + f*x)))]/Sqrt[c - I*d]] + ((4*I)*c + 3*d + I*d*Tan[e
+ f*x])*Sqrt[c + d*Tan[e + f*x]]))/(3*f)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 851 vs. $2(81) = 162$.
time = 0.25, size = 852, normalized size = 8.69

method	result
derivativedivides	$a\left(\frac{2i(c+d\tan(fx+e))^{\frac{3}{2}}}{3}+2ic\sqrt{c+d\tan(fx+e)}+2d\sqrt{c+d\tan(fx+e)}+\frac{\left(-ic^2\sqrt{c^2+d^2}+id^2\sqrt{\ldots}\right.}{\ldots}\right.$
default	$a\left(\frac{2i(c+d\tan(fx+e))^{\frac{3}{2}}}{3}+2ic\sqrt{c+d\tan(fx+e)}+2d\sqrt{c+d\tan(fx+e)}+\frac{\left(-ic^2\sqrt{c^2+d^2}+id^2\sqrt{\ldots}\right.}{\ldots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))*(c+d*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a*(2/3*I*(c+d*tan(f*x+e))^(3/2)+2*I*c*(c+d*tan(f*x+e))^(1/2)+2*d*(c+d*t
an(f*x+e))^(1/2)+1/(2*(c^2+d^2)^(1/2)+2*c)^(1/2)/(c^2+d^2)^(1/2)*(1/2*(-I*c
^2*(c^2+d^2)^(1/2)+I*d^2*(c^2+d^2)^(1/2)-I*c^3-I*c*d^2-2*c*d*(c^2+d^2)^(1/2
)-c^2*d-d^3)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)+(c^2+d^2)^(1/2))+2*(-I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3-I*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*c*d^2-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2*d-(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*d^3-1/2*(-I*c^2*(c^2+d^2)^(1/2)+I*d^2*(c^2+d^2)^(1/2)-I
*c^3-I*c*d^2-2*c*d*(c^2+d^2)^(1/2)-c^2*d-d^3)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^
2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)/(c^2+d^2)^(1/2)*(-1/2*(-I*c^2*(c^2+d^2)^(1/2)+I*d^2*(c^2+d^2)^(1/
2)-I*c^3-I*c*d^2-2*c*d*(c^2+d^2)^(1/2)-c^2*d-d^3)*ln((c+d*tan(f*x+e))^(1/2)
*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))+2*(I*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*c^3+I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^2+(2*(c^2+d^
```

```
2)^(1/2)+2*c)^(1/2)*c^2*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^3+1/2*(-I*c^2*(c^
2+d^2)^(1/2)+I*d^2*(c^2+d^2)^(1/2)-I*c^3-I*c*d^2-2*c*d*(c^2+d^2)^(1/2)-c^2*
d-d^3)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(
((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)
-2*c)^(1/2))))
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c+d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(79) = 158$.
time = 1.12, size = 520, normalized size = 5.31

$$\frac{3\left(fe^{(2ifx+2ie)}+f\right)\sqrt{-\frac{a^2c^3-3i\,a^2c^2d-3\,a^2cd^2+i\,a^2d^3}{f^2}}\log\left(-\frac{2\left(-i\,ac^2-acd+\left(fe^{(2ifx+2ie)}+f\right)\sqrt{\frac{(c-i\,d)e^{(2ifx+2ie)}+c+i\,d}{e^{(2ifx+2ie)}+1}}\sqrt{-\frac{a^2c^3-3i\,a^2c^2d-3\,a^2cd^2+i\,a^2d^3}{f^2}}+\left(-i\,ac^2-2\,acd+i\,ad^2\right)e^{(2ifx+2ie)}\right)e^{(-2ifx-2ie)}}{i\,ac+ad}\right)-3\left(fe^{(2ifx+2ie)}+f\right)\sqrt{-\frac{a^2c^3-3i\,a^2c^2d-3\,a^2cd^2+i\,a^2d^3}{f^2}}\log\left(-\frac{2\left(-i\,ac^2-acd-\left(fe^{(2ifx+2ie)}+f\right)\sqrt{\frac{(c-i\,d)e^{(2ifx+2ie)}+c+i\,d}{e^{(2ifx+2ie)}+1}}\sqrt{-\frac{a^2c^3-3i\,a^2c^2d-3\,a^2cd^2+i\,a^2d^3}{f^2}}+\left(-i\,ac^2-2\,acd+i\,ad^2\right)e^{(2ifx+2ie)}\right)e^{(-2ifx-2ie)}}{i\,ac+ad}\right)-8\left(-2i\,ac-ad+2\left(-i\,ac-ad\right)e^{(2ifx+2ie)}\right)\sqrt{\frac{(c-i\,d)e^{(2ifx+2ie)}+c+i\,d}{e^{(2ifx+2ie)}+1}}}{6\left(fe^{(2ifx+2ie)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c+d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(f*e^(2*I*f*x + 2*I*e) + f)*sqrt(-(a^2*c^3 - 3*I*a^2*c^2*d - 3*a^2*c
*d^2 + I*a^2*d^3)/f^2)*log(-2*(-I*a*c^2 - a*c*d + (f*e^(2*I*f*x + 2*I*e) +
f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1)
)*sqrt(-(a^2*c^3 - 3*I*a^2*c^2*d - 3*a^2*c*d^2 + I*a^2*d^3)/f^2) + (-I*a*c^
2 - 2*a*c*d + I*a*d^2)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/(I*a*c + a
*d)) - 3*(f*e^(2*I*f*x + 2*I*e) + f)*sqrt(-(a^2*c^3 - 3*I*a^2*c^2*d - 3*a^2
*c*d^2 + I*a^2*d^3)/f^2)*log(-2*(-I*a*c^2 - a*c*d - (f*e^(2*I*f*x + 2*I*e)
+ f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) +
1))*sqrt(-(a^2*c^3 - 3*I*a^2*c^2*d - 3*a^2*c*d^2 + I*a^2*d^3)/f^2) + (-I*a*
c^2 - 2*a*c*d + I*a*d^2)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/(I*a*c +
 a*d)) - 8*(-2*I*a*c - a*d + 2*(-I*a*c - a*d)*e^(2*I*f*x + 2*I*e))*sqrt(((c
 - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))/(f*e^(2*
I*f*x + 2*I*e) + f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-ic\sqrt{c+d\tan\left(e+fx\right)}\right)dx+\int c\sqrt{c+d\tan\left(e+fx\right)}\tan\left(e+fx\right)dx+\int d\sqrt{c+d\tan\left(e+fx\right)}\tan^{2}\left(e+fx\right)dx+\int\left(-id\sqrt{c+d\tan\left(e+fx\right)}\tan\left(e+fx\right)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] I*a*(Integral(-I*c*sqrt(c + d*tan(e + f*x)), x) + Integral(c*sqrt(c + d*tan
(e + f*x))*tan(e + f*x), x) + Integral(d*sqrt(c + d*tan(e + f*x))*tan(e + f
*x)**2, x) + Integral(-I*d*sqrt(c + d*tan(e + f*x))*tan(e + f*x), x))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(79) = 158$.
time = 0.56, size = 241, normalized size = 2.46

$$-\frac{4\left(-i\,ac^2-2\,acd+i\,ad^2\right)\arctan\left(\frac{2\left(\sqrt{d\tan\left(fx+e\right)+c}\,c-\sqrt{c^2+d^2}\,\sqrt{d\tan\left(fx+e\right)+c}\right)}{c\sqrt{-2\,c+2\,\sqrt{c^2+d^2}}-i\,\sqrt{-2\,c+2\,\sqrt{c^2+d^2}}\,d-\sqrt{c^2+d^2}\,\sqrt{-2\,c+2\,\sqrt{c^2+d^2}}}\right)}{\sqrt{-2\,c+2\,\sqrt{c^2+d^2}}\,f\left(-\frac{i\,d}{c-\sqrt{c^2+d^2}}+1\right)}-\frac{2\left(-i\left(d\tan\left(fx+e\right)+c\right)^{\frac{3}{2}}af^2-3i\,\sqrt{d\tan\left(fx+e\right)+c}\,acf^2-3\,\sqrt{d\tan\left(fx+e\right)+c}\,adf^2\right)}{3\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c+d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] -4*(-I*a*c^2 - 2*a*c*d + I*a*d^2)*arctan(2*(sqrt(d*tan(f*x + e) + c)*c - sq
rt(c^2 + d^2)*sqrt(d*tan(f*x + e) + c))/(c*sqrt(-2*c + 2*sqrt(c^2 + d^2)) -
 I*sqrt(-2*c + 2*sqrt(c^2 + d^2))*d - sqrt(c^2 + d^2)*sqrt(-2*c + 2*sqrt(c^
2 + d^2))))/(sqrt(-2*c + 2*sqrt(c^2 + d^2))*f*(-I*d/(c - sqrt(c^2 + d^2)) +
 1)) - 2/3*(-I*(d*tan(f*x + e) + c)^(3/2)*a*f^2 - 3*I*sqrt(d*tan(f*x + e) +
 c)*a*c*f^2 - 3*sqrt(d*tan(f*x + e) + c)*a*d*f^2)/f^3
```

Mupad [B]
time = 16.32, size = 2869, normalized size = 29.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)*(c + d*tan(e + f*x))^(3/2),x)
```

```
[Out] log(((((-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) - a^2*c^3*f^2 + 3*a^2*c*d^2*f^2
)/f^4)^(1/2)*((16*c*d^2*(((-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) - a^2*c^3*f^
2 + 3*a^2*c*d^2*f^2)/f^4)^(1/2)*(a*c^2*1i + a*d^2*1i - f*(((-a^4*d^2*f^4*(3
*c^2 - d^2)^2)^(1/2) - a^2*c^3*f^2 + 3*a^2*c*d^2*f^2)/f^4)^(1/2)*(c + d*tan
(e + f*x))^(1/2)))/f - (16*a^2*d^2*(c + d*tan(e + f*x))^(1/2)*(c^4 + d^4 -
6*c^2*d^2))/f^2))/2 - (a^3*d^2*(c^2 - d^2)*(c^2*1i + d^2*1i)^2*8i)/f^3)*((6
*a^4*c^2*d^4*f^4 - a^4*d^6*f^4 - 9*a^4*c^4*d^2*f^4)^(1/2)/(4*f^4) - (a^2*c^
3)/(4*f^2) + (3*a^2*c*d^2)/(4*f^2))^(1/2) - log(((((-a^4*d^2*f^4*(3*c^2 - d
^2)^2)^(1/2) - a^2*c^3*f^2 + 3*a^2*c*d^2*f^2)/f^4)^(1/2)*((16*c*d^2*(((-a^4
*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) - a^2*c^3*f^2 + 3*a^2*c*d^2*f^2)/f^4)^(1/2)
*(a*c^2*1i + a*d^2*1i + f*(((-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) - a^2*c^3*
f^2 + 3*a^2*c*d^2*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2)))/f + (16*a^2*
d^2*(c + d*tan(e + f*x))^(1/2)*(c^4 + d^4 - 6*c^2*d^2))/f^2))/2 - (a^3*d^2*
(c^2 - d^2)*(c^2*1i + d^2*1i)^2*8i)/f^3)*(((6*a^4*c^2*d^4*f^4 - a^4*d^6*f^4
 - 9*a^4*c^4*d^2*f^4)^(1/2) - a^2*c^3*f^2 + 3*a^2*c*d^2*f^2)/(4*f^4))^(1/2)
```

- log(((-((-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + a^2*c^3*f^2 - 3*a^2*c*d^2*f^2)/f^4)^(1/2)*((16*c*d^2*(-((-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + a^2*c^3*f^2 - 3*a^2*c*d^2*f^2)/f^4)^(1/2)*(a*c^2*1i + a*d^2*1i + f*(-((-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + a^2*c^3*f^2 - 3*a^2*c*d^2*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2)))/f + (16*a^2*d^2*(c + d*tan(e + f*x))^(1/2)*(c^4 + d^4 - 6*c^2*d^2))/f^2))/2 - (a^3*d^2*(c^2 - d^2)*(c^2*1i + d^2*1i)^2*8i)/f^3)*(-((6*a^4*c^2*d^4*f^4 - a^4*d^6*f^4 - 9*a^4*c^4*d^2*f^4)^(1/2) + a^2*c^3*f^2 - 3*a^2*c*d^2*f^2)/(4*f^4))^(1/2) + log(((-((-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + a^2*c^3*f^2 - 3*a^2*c*d^2*f^2)/f^4)^(1/2)*((16*c*d^2*(-((-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + a^2*c^3*f^2 - 3*a^2*c*d^2*f^2)/f^4)^(1/2)*(a*c^2*1i + a*d^2*1i - f*(-((-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + a^2*c^3*f^2 - 3*a^2*c*d^2*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2)))/f - (16*a^2*d^2*(c + d*tan(e + f*x))^(1/2)*(c^4 + d^4 - 6*c^2*d^2))/f^2))/2 - (a^3*d^2*(c^2 - d^2)*(c^2*1i + d^2*1i)^2*8i)/f^3)*((3*a^2*c*d^2)/(4*f^2) - (a^2*c^3)/(4*f^2) - (6*a^4*c^2*d^4*f^4 - a^4*d^6*f^4 - 9*a^4*c^4*d^2*f^4)^(1/2)/(4*f^4))^(1/2) - log(((-((-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + a^2*c^3*f^2 - 3*a^2*c*d^2*f^2)/f^4)^(1/2)*((16*d^2*(-((-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + a^2*c^3*f^2 - 3*a^2*c*d^2*f^2)/f^4)^(1/2)*(a*d^3 + a*c^2*d + c*f*(-((-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + a^2*c^3*f^2 - 3*a^2*c*d^2*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2)))/f + (16*a^2*d^2*(c + d*tan(e + f*x))^(1/2)*(c^4 + d^4 - 6*c^2*d^2))/f^2))/2 - (16*a^3*c*d^3*(c^2 + d^2)^2)/f^3)*(-((6*a^4*c^2*d^4*f^4 - a^4*d^6*f^4 - 9*a^4*c^4*d^2*f^4)^(1/2) + a^2*c^3*f^2 - 3*a^2*c*d^2*f^2)/(4*f^4))^(1/2) - log(((((-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) - a^2*c^3*f^2 + 3*a^2*c*d^2*f^2)/f^4)^(1/2)*((16*d^2*(((-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) - a^2*c^3*f^2 + 3*a^2*c*d^2*f^2)/f^4)^(1/2)*(a*d^3 + a*c^2*d + c*f*(((-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) - a^2*c^3*f^2 + 3*a^2*c*d^2*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2)))/f + (16*a^2*d^2*(c + d*tan(e + f*x))^(1/2)*(c^4 + d^4 - 6*c^2*d^2))/f^2))/2 - (16*a^3*c*d^3*(c^2 + d^2)^2)/f^3)*(((6*a^4*c^2*d^4*f^4 - a^4*d^6*f^4 - 9*a^4*c^4*d^2*f^4)^(1/2) - a^2*c^3*f^2 + 3*a^2*c*d^2*f^2)/(4*f^4))^(1/2) + log(((((-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) - a^2*c^3*f^2 + 3*a^2*c*d^2*f^2)/f^4)^(1/2)*((16*d^2*(((-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) - a^2*c^3*f^2 + 3*a^2*c*d^2*f^2)/f^4)^(1/2)*(a*d^3 + a*c^2*d - c*f*(((-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) - a^2*c^3*f^2 + 3*a^2*c*d^2*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2)))/f - (16*a^2*d^2*(c + d*tan(e + f*x))^(1/2)*(c^4 + d^4 - 6*c^2*d^2))/f^2))/2 - (16*a^3*c*d^3*(c^2 + d^2)^2)/f^3)*((6*a^4*c^2*d^4*f^4 - a^4*d^6*f^4 - 9*a^4*c^4*d^2*f^4)^(1/2)/(4*f^4) - (a^2*c^3)/(4*f^2) + (3*a^2*c*d^2)/(4*f^2))^(1/2) + log(((-((-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + a^2*c^3*f^2 - 3*a^2*c*d^2*f^2)/f^4)^(1/2)*((16*d^2*(-((-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + a^2*c^3*f^2 - 3*a^2*c*d^2*f^2)/f^4)^(1/2)*(a*d^3 + a*c^2*d - c*f*(-((-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + a^2*c^3*f^2 - 3*a^2*c*d^2*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2)))/f - (16*a^2*d^2*(c + d*tan(e + f*x))^(1/2)*(c^4 + d^4 - 6*c^2*d^2))/f^2))/2 - (16*a^3*c*d^3*(c^2 + d^2)^2)/f^3)*((3*a^2*c*d^2)/(4*f^2) - (a^2*c^3)/(4*f^2) - (6*a^4*c^2*d^4*f^4 - a^4*d^6*f^4 - 9*a^4*c^4*d^2*f^4)^(1/2)/(4*f^4))^(1/2) + (a*(c + d*tan(e + f*x))^(3/2)*2i)/(3*f) + (a*c*(c + d*

```
tan(e + f*x))^(1/2)*2i)/f + (2*a*d*(c + d*tan(e + f*x))^(1/2))/f
```

3.1110 $\int \frac{(c+d\tan(e+fx))^{3/2}}{a+ia\tan(e+fx)}\,dx$

Optimal. Leaf size=153

$$-\frac{i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{2af}+\frac{\sqrt{c+id}\,(ic+2d)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{2af}+\frac{(ic-d)}{2f(a}$$

```
[Out] -1/2*I*(c-I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/a/f+1/2*
(I*c+2*d)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))*(c+I*d)^(1/2)/a/f+1
/2*(I*c-d)*(c+d*tan(f*x+e))^(1/2)/f/(a+I*a*tan(f*x+e))
```

Rubi [A]
time = 0.23, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3631, 3620, 3618, 65, 214}

$$\frac{(-d+ic)\sqrt{c+d\tan(e+fx)}}{2f(a+ia\tan(e+fx))}-\frac{i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{2af}+\frac{\sqrt{c+id}\,(2d+ic)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^(3/2)/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] ((-1/2*I)*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/
(a*f) + (Sqrt[c + I*d]*(I*c + 2*d)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c
+ I*d]])/(2*a*f) + ((I*c - d)*Sqrt[c + d*Tan[e + f*x]])/(2*f*(a + I*a*Tan[e
 + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
```

```
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3631

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((c + d*Tan[e + f*x])^(n - 1)/(
2*a*f*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a^2), Int[(c + d*Tan[e + f*x])
^(n - 2)*Simp[a*c^2 + a*d^2*(n - 1) - b*c*d*n - d*(a*c*(n - 2) + b*d*n)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+d\tan(e+fx))^{3/2}}{a+ia\tan(e+fx)}\,dx &= \frac{(ic-d)\sqrt{c+d\tan(e+fx)}}{2f(a+ia\tan(e+fx))} + \frac{\int \frac{\frac{1}{2}a(2c^2-3icd+d^2)+\frac{1}{2}a(c-3id)d\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx}{2a^2} \\
&= \frac{(ic-d)\sqrt{c+d\tan(e+fx)}}{2f(a+ia\tan(e+fx))} + \frac{(c-id)^2\int \frac{1+i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx}{4a} + \frac{((c+
\\
&= \frac{(ic-d)\sqrt{c+d\tan(e+fx)}}{2f(a+ia\tan(e+fx))} + \frac{(i(c-id)^2)\operatorname{Subst}\left(\int \frac{1}{(-1+x)\sqrt{c-idx}}\,dx, x
\\
&= \frac{(ic-d)\sqrt{c+d\tan(e+fx)}}{2f(a+ia\tan(e+fx))} - \frac{(c-id)^2\operatorname{Subst}\left(\int \frac{1}{-1-\frac{ic}{d}+\frac{ix^2}{d}}\,dx, x, \sqrt{c+d\,t
\\
&= -\frac{i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{2af} + \frac{\sqrt{c+id}\,(ic+2d)\tanh^{-1}\left(
\end{aligned}
$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 376 vs. 2(153) = 306.

time = 4.52, size = 376, normalized size = 2.46

$$\frac{\sec(e+fx)(\cos(fx)+i\sin(fx))\left(\left(-i(c-id)^{3/2}\log\left(\frac{2\left(-ide^{2i(e+fx)}+c\left(1+e^{2i(e+fx)}\right)+\sqrt{c-id}\left(1+e^{2i(e+fx)}\right)\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)}{\sqrt{c-id}}\right)+\frac{(ic^2+cd+2id^2)\log\left(\frac{8ie^{-2ifx}\left(id+c\left(1+e^{2i(e+fx)}\right)+\sqrt{c+id}\left(1+e^{2i(e+fx)}\right)\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)}{\sqrt{c+id}\,(c^2-icd+2d^2)}\right)}{\sqrt{c+id}}\right)(\cos(e)+i\sin(e))+2(c+id)\cos(e+fx)(i\cos(fx)+\sin(fx))\sqrt{c+d\tan(e+fx)}\right)}{4f(a+ia\tan(e+fx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Tan[e + f*x])^(3/2)/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (Sec[e + f*x]*(Cos[f*x] + I*Sin[f*x])*(((-I)*(c - I*d)^(3/2)*Log[(2*((-I)*d
*E^((2*I)*(e + f*x)) + c*(1 + E^((2*I)*(e + f*x))) + Sqrt[c - I*d]*(1 + E^(
(2*I)*(e + f*x)))*Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(
e + f*x)))]))/Sqrt[c - I*d]] + ((I*c^2 + c*d + (2*I)*d^2)*Log[((8*I)*(I*d +
 c*(1 + E^((2*I)*(e + f*x))) + Sqrt[c + I*d]*(1 + E^((2*I)*(e + f*x)))*Sqrt
[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))]))/(Sqrt[c
+ I*d]*(c^2 - I*c*d + 2*d^2)*E^((2*I)*f*x))])/Sqrt[c + I*d])*(Cos[e] + I*Si
n[e]) + 2*(c + I*d)*Cos[e + f*x]*(I*Cos[f*x] + Sin[f*x])*Sqrt[c + d*Tan[e +
 f*x]]))/(4*f*(a + I*a*Tan[e + f*x]))
```

Maple [A]

time = 0.37, size = 140, normalized size = 0.92

method	result
derivativedivides	$\frac{2d^2\left(\frac{i(id-c)^{\frac{3}{2}}\arctan\left(\frac{\sqrt{c+d\tan(fx+e)}}{\sqrt{id-c}}\right)}{4d^2}+\frac{(id+c)\left(-\frac{d\sqrt{c+d\tan(fx+e)}}{-d\tan(fx+e)+id}-\frac{(ic+2d)\arctan\left(\frac{\sqrt{c+}}{}\right)}{\sqrt{-i}}\right.}{4d^2}\right.}{fa}$
default	$\frac{2d^2\left(\frac{i(id-c)^{\frac{3}{2}}\arctan\left(\frac{\sqrt{c+d\tan(fx+e)}}{\sqrt{id-c}}\right)}{4d^2}+\frac{(id+c)\left(-\frac{d\sqrt{c+d\tan(fx+e)}}{-d\tan(fx+e)+id}-\frac{(ic+2d)\arctan\left(\frac{\sqrt{c+}}{}\right)}{\sqrt{-i}}\right.}{4d^2}\right.}{fa}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

[Out] 2/f/a*d^2*(1/4*I*(I*d-c)^(3/2)/d^2*arctan((c+d*tan(f*x+e))^(1/2)/(I*d-c)^(1/2))+1/4*(c+I*d)/d^2*(-d*(c+d*tan(f*x+e))^(1/2)/(-d*tan(f*x+e)+I*d)-(I*c+2*d)/(-I*d-c)^(1/2)*arctan((c+d*tan(f*x+e))^(1/2)/(-I*d-c)^(1/2))))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 808 vs. 2(119) = 238.
time = 1.08, size = 808, normalized size = 5.28

$$\frac{af\sqrt{-\frac{c^3-3ic^2d-3cd^2+id^3}{a^2f^2}}e^{(2ifx+2ie)}\log\left(-\frac{2\left(-ic^2-cd+\left(afe^{(2ifx+2ie)}+af\right)\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}\sqrt{-\frac{c^3-3ic^2d-3cd^2+id^3}{a^2f^2}}+\left(-ic^2-2cd+id^2\right)e^{(2ifx+2ie)}\right)e^{(-2ifx-2ie)}}{ic+d}\right)-af\sqrt{-\frac{c^3-3ic^2d-3cd^2+id^3}{a^2f^2}}e^{(2ifx+2ie)}\log\left(-\frac{2\left(-ic^2-cd-\left(afe^{(2ifx+2ie)}+af\right)\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}\sqrt{-\frac{c^3-3ic^2d-3cd^2+id^3}{a^2f^2}}+\left(-ic^2-2cd+id^2\right)e^{(2ifx+2ie)}\right)e^{(-2ifx-2ie)}}{ic+d}\right)+af\sqrt{-\frac{c^3-3ic^2d-4id^3}{a^2f^2}}e^{(2ifx+2ie)}\log\left(\frac{\left(ic^2+cd+2id^2+\left(afe^{(2ifx+2ie)}+af\right)\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}\sqrt{-\frac{c^3-3ic^2d-4id^3}{a^2f^2}}+\left(ic^2+2cd\right)e^{(2ifx+2ie)}\right)e^{(-2ifx-2ie)}}{2af}\right)-af\sqrt{-\frac{c^3-3ic^2d-4id^3}{a^2f^2}}e^{(2ifx+2ie)}\log\left(\frac{\left(ic^2+cd+2id^2-\left(afe^{(2ifx+2ie)}+af\right)\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}\sqrt{-\frac{c^3-3ic^2d-4id^3}{a^2f^2}}+\left(ic^2+2cd\right)e^{(2ifx+2ie)}\right)e^{(-2ifx-2ie)}}{2af}\right)+2\left((ic-d)e^{(2ifx+2ie)}+ic-d\right)\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}}{8af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/8*(a*f*sqrt(-(c^3 - 3*I*c^2*d - 3*c*d^2 + I*d^3)/(a^2*f^2))*e^(2*I*f*x +
2*I*e)*log(-2*(-I*c^2 - c*d + (a*f*e^(2*I*f*x + 2*I*e) + a*f)*sqrt(((c - I*
d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(c^3 - 3
*I*c^2*d - 3*c*d^2 + I*d^3)/(a^2*f^2)) + (-I*c^2 - 2*c*d + I*d^2)*e^(2*I*f*
x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/(I*c + d)) - a*f*sqrt(-(c^3 - 3*I*c^2*d -
3*c*d^2 + I*d^3)/(a^2*f^2))*e^(2*I*f*x + 2*I*e)*log(-2*(-I*c^2 - c*d - (a*f
*e^(2*I*f*x + 2*I*e) + a*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/
(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(c^3 - 3*I*c^2*d - 3*c*d^2 + I*d^3)/(a^2*f
^2)) + (-I*c^2 - 2*c*d + I*d^2)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/(
I*c + d)) + a*f*sqrt(-(c^3 - 3*I*c^2*d - 4*I*d^3)/(a^2*f^2))*e^(2*I*f*x + 2
*I*e)*log(1/2*(I*c^2 + c*d + 2*I*d^2 + (a*f*e^(2*I*f*x + 2*I*e) + a*f)*sqrt
(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(
-(c^3 - 3*I*c^2*d - 4*I*d^3)/(a^2*f^2)) + (I*c^2 + 2*c*d)*e^(2*I*f*x + 2*I*
e))*e^(-2*I*f*x - 2*I*e)/(a*f)) - a*f*sqrt(-(c^3 - 3*I*c^2*d - 4*I*d^3)/(a^
2*f^2))*e^(2*I*f*x + 2*I*e)*log(1/2*(I*c^2 + c*d + 2*I*d^2 - (a*f*e^(2*I*f*
x + 2*I*e) + a*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*
x + 2*I*e) + 1))*sqrt(-(c^3 - 3*I*c^2*d - 4*I*d^3)/(a^2*f^2)) + (I*c^2 + 2*
c*d)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/(a*f)) + 2*((I*c - d)*e^(2*I
*f*x + 2*I*e) + I*c - d)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^
(2*I*f*x + 2*I*e) + 1)))*e^(-2*I*f*x - 2*I*e)/(a*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\left(\int\frac{c\sqrt{c+d\tan\left(e+fx\right)}}{\tan\left(e+fx\right)-i}\,dx+\int\frac{d\sqrt{c+d\tan\left(e+fx\right)}\tan\left(e+fx\right)}{\tan\left(e+fx\right)-i}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(3/2)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] -I*(Integral(c*sqrt(c + d*tan(e + f*x))/(tan(e + f*x) - I), x) + Integral(d
*sqrt(c + d*tan(e + f*x))*tan(e + f*x)/(tan(e + f*x) - I), x))/a
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 407 vs. 2(119) = 238.
time = 0.57, size = 407, normalized size = 2.66

$$\frac{\left(-i c^{2}-c d-2 i d^{2}\right)\arctan\left(\frac{2\left(\sqrt{d\tan\left(fx+e\right)+c}\,c-\sqrt{c^{2}+d^{2}}\sqrt{d\tan\left(fx+e\right)+c}\right)}{c\sqrt{-2c+2\sqrt{c^{2}+d^{2}}}+i\sqrt{-2c+2\sqrt{c^{2}+d^{2}}}\,d-\sqrt{c^{2}+d^{2}}\sqrt{-2c+2\sqrt{c^{2}+d^{2}}}}\right)}{a\sqrt{-2c+2\sqrt{c^{2}+d^{2}}}\,f\left(\frac{id}{c-\sqrt{c^{2}+d^{2}}}+1\right)}+\frac{\left(i c^{2}+2cd-i d^{2}\right)\arctan\left(\frac{2\left(\sqrt{d\tan\left(fx+e\right)+c}\,c-\sqrt{c^{2}+d^{2}}\sqrt{d\tan\left(fx+e\right)+c}\right)}{c\sqrt{-2c+2\sqrt{c^{2}+d^{2}}}-i\sqrt{-2c+2\sqrt{c^{2}+d^{2}}}\,d-\sqrt{c^{2}+d^{2}}\sqrt{-2c+2\sqrt{c^{2}+d^{2}}}}\right)}{a\sqrt{-2c+2\sqrt{c^{2}+d^{2}}}\,f\left(-\frac{id}{c-\sqrt{c^{2}+d^{2}}}+1\right)}+\frac{\sqrt{d\tan\left(fx+e\right)+c}\,cd+i\sqrt{d\tan\left(fx+e\right)+c}\,d^{2}}{2\left(d\tan\left(fx+e\right)-id\right)af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] (-I*c^2 - c*d - 2*I*d^2)*arctan(2*(sqrt(d*tan(f*x + e) + c)*c - sqrt(c^2 +
d^2)*sqrt(d*tan(f*x + e) + c))/(c*sqrt(-2*c + 2*sqrt(c^2 + d^2)) + I*sqrt(-
2*c + 2*sqrt(c^2 + d^2))*d - sqrt(c^2 + d^2)*sqrt(-2*c + 2*sqrt(c^2 + d^2))
))/(a*sqrt(-2*c + 2*sqrt(c^2 + d^2))*f*(I*d/(c - sqrt(c^2 + d^2)) + 1)) + (
I*c^2 + 2*c*d - I*d^2)*arctan(2*(sqrt(d*tan(f*x + e) + c)*c - sqrt(c^2 + d^
2)*sqrt(d*tan(f*x + e) + c))/(c*sqrt(-2*c + 2*sqrt(c^2 + d^2)) - I*sqrt(-2*
c + 2*sqrt(c^2 + d^2))*d - sqrt(c^2 + d^2)*sqrt(-2*c + 2*sqrt(c^2 + d^2))))
/(a*sqrt(-2*c + 2*sqrt(c^2 + d^2))*f*(-I*d/(c - sqrt(c^2 + d^2)) + 1)) + 1/
2*(sqrt(d*tan(f*x + e) + c)*c*d + I*sqrt(d*tan(f*x + e) + c)*d^2)/((d*tan(f
*x + e) - I*d)*a*f)
```

Mupad [B]
time = 7.34, size = 847, normalized size = 5.54

$$-2\,\mathrm{atanh}\left(\frac{20\,a^2\,d^6\,f^2\,\sqrt{c+d\tan\left(e+f\,x\right)}\,\sqrt{-\frac{c^3}{16\,a^2\,f^2}+\frac{d^3\,1{}\mathrm{i}}{4\,a^2\,f^2}+\frac{c^2\,d\,3{}\mathrm{i}}{16\,a^2\,f^2}}}{3{}\mathrm{i}\,a\,f\,c^4\,d^4+11\,a\,f\,c^3\,d^5-7{}\mathrm{i}\,a\,f\,c^2\,d^6+11\,a\,f\,c\,d^7-10{}\mathrm{i}\,a\,f\,d^8}+\frac{a^2\,c\,d^5\,f^2\,\sqrt{c+d\tan\left(e+f\,x\right)}\,\sqrt{-\frac{c^3}{16\,a^2\,f^2}+\frac{d^3\,1{}\mathrm{i}}{4\,a^2\,f^2}+\frac{c^2\,d\,3{}\mathrm{i}}{16\,a^2\,f^2}}\,32{}\mathrm{i}}{3{}\mathrm{i}\,a\,f\,c^4\,d^4+11\,a\,f\,c^3\,d^5-7{}\mathrm{i}\,a\,f\,c^2\,d^6+11\,a\,f\,c\,d^7-10{}\mathrm{i}\,a\,f\,d^8}-\frac{12\,a^2\,c^2\,d^4\,f^2\,\sqrt{c+d\tan\left(e+f\,x\right)}\,\sqrt{-\frac{c^3}{16\,a^2\,f^2}+\frac{d^3\,1{}\mathrm{i}}{4\,a^2\,f^2}+\frac{c^2\,d\,3{}\mathrm{i}}{16\,a^2\,f^2}}}{3{}\mathrm{i}\,a\,f\,c^4\,d^4+11\,a\,f\,c^3\,d^5-7{}\mathrm{i}\,a\,f\,c^2\,d^6+11\,a\,f\,c\,d^7-10{}\mathrm{i}\,a\,f\,d^8}\right)\sqrt{\frac{-2\,c^3+c^2\,d\,6{}\mathrm{i}+d^3\,8{}\mathrm{i}}{32\,a^2\,f^2}}-2\,\mathrm{atanh}\left(\frac{20\,a^2\,d^6\,f^2\,\sqrt{c+d\tan\left(e+f\,x\right)}\,\sqrt{\frac{3\,c\,d^2}{16\,a^2\,f^2}-\frac{d^3\,1{}\mathrm{i}}{16\,a^2\,f^2}-\frac{c^3}{16\,a^2\,f^2}+\frac{c^2\,d\,3{}\mathrm{i}}{16\,a^2\,f^2}}}{3{}\mathrm{i}\,a\,f\,c^4\,d^4+8\,a\,f\,c^3\,d^5-2{}\mathrm{i}\,a\,f\,c^2\,d^6+8\,a\,f\,c\,d^7-5{}\mathrm{i}\,a\,f\,d^8}-\frac{a^2\,c\,d^5\,f^2\,\sqrt{c+d\tan\left(e+f\,x\right)}\,\sqrt{\frac{3\,c\,d^2}{16\,a^2\,f^2}-\frac{d^3\,1{}\mathrm{i}}{16\,a^2\,f^2}-\frac{c^3}{16\,a^2\,f^2}+\frac{c^2\,d\,3{}\mathrm{i}}{16\,a^2\,f^2}}\,8{}\mathrm{i}}{3{}\mathrm{i}\,a\,f\,c^4\,d^4+8\,a\,f\,c^3\,d^5-2{}\mathrm{i}\,a\,f\,c^2\,d^6+8\,a\,f\,c\,d^7-5{}\mathrm{i}\,a\,f\,d^8}+\frac{12\,a^2\,c^2\,d^4\,f^2\,\sqrt{c+d\tan\left(e+f\,x\right)}\,\sqrt{\frac{3\,c\,d^2}{16\,a^2\,f^2}-\frac{d^3\,1{}\mathrm{i}}{16\,a^2\,f^2}-\frac{c^3}{16\,a^2\,f^2}+\frac{c^2\,d\,3{}\mathrm{i}}{16\,a^2\,f^2}}}{3{}\mathrm{i}\,a\,f\,c^4\,d^4+8\,a\,f\,c^3\,d^5-2{}\mathrm{i}\,a\,f\,c^2\,d^6+8\,a\,f\,c\,d^7-5{}\mathrm{i}\,a\,f\,d^8}\right)\sqrt{\frac{-2\,c^3+c^2\,d\,6{}\mathrm{i}+6\,c\,d^2-d^3\,2{}\mathrm{i}}{32\,a^2\,f^2}}-\frac{\left(d^2\,1{}\mathrm{i}+c\,d\right)\,\sqrt{c+d\tan\left(e+f\,x\right)}}{2\,a\,f\,\left(-d\tan\left(e+f\,x\right)+d\,1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(3/2)/(a + a*tan(e + f*x)*1i),x)
```

```
[Out] - 2*atanh((20*a^2*d^6*f^2*(c + d*tan(e + f*x))^(1/2)*((d^3*1i)/(4*a^2*f^2)
- c^3/(16*a^2*f^2) + (c^2*d*3i)/(16*a^2*f^2))^(1/2))/(11*a*c*d^7*f - a*d^8*
f*10i - a*c^2*d^6*f*7i + 11*a*c^3*d^5*f + a*c^4*d^4*f*3i) + (a^2*c*d^5*f^2*
(c + d*tan(e + f*x))^(1/2)*((d^3*1i)/(4*a^2*f^2) - c^3/(16*a^2*f^2) + (c^2*
```

```
d*3i)/(16*a^2*f^2))^(1/2)*32i)/(11*a*c*d^7*f - a*d^8*f*10i - a*c^2*d^6*f*7i
 + 11*a*c^3*d^5*f + a*c^4*d^4*f*3i) - (12*a^2*c^2*d^4*f^2*(c + d*tan(e + f*
x))^(1/2)*((d^3*1i)/(4*a^2*f^2) - c^3/(16*a^2*f^2) + (c^2*d*3i)/(16*a^2*f^2
))^(1/2))/(11*a*c*d^7*f - a*d^8*f*10i - a*c^2*d^6*f*7i + 11*a*c^3*d^5*f + a
*c^4*d^4*f*3i))*((c^2*d*6i - 2*c^3 + d^3*8i)/(32*a^2*f^2))^(1/2) - 2*atanh(
(20*a^2*d^6*f^2*(c + d*tan(e + f*x))^(1/2)*((3*c*d^2)/(16*a^2*f^2) - (d^3*1
i)/(16*a^2*f^2) - c^3/(16*a^2*f^2) + (c^2*d*3i)/(16*a^2*f^2))^(1/2))/(8*a*c
*d^7*f - a*d^8*f*5i - a*c^2*d^6*f*2i + 8*a*c^3*d^5*f + a*c^4*d^4*f*3i) - (a
^2*c*d^5*f^2*(c + d*tan(e + f*x))^(1/2)*((3*c*d^2)/(16*a^2*f^2) - (d^3*1i)/
(16*a^2*f^2) - c^3/(16*a^2*f^2) + (c^2*d*3i)/(16*a^2*f^2))^(1/2)*8i)/(8*a*c
*d^7*f - a*d^8*f*5i - a*c^2*d^6*f*2i + 8*a*c^3*d^5*f + a*c^4*d^4*f*3i) + (1
2*a^2*c^2*d^4*f^2*(c + d*tan(e + f*x))^(1/2)*((3*c*d^2)/(16*a^2*f^2) - (d^3
*1i)/(16*a^2*f^2) - c^3/(16*a^2*f^2) + (c^2*d*3i)/(16*a^2*f^2))^(1/2))/(8*a
*c*d^7*f - a*d^8*f*5i - a*c^2*d^6*f*2i + 8*a*c^3*d^5*f + a*c^4*d^4*f*3i))*(
(6*c*d^2 + c^2*d*6i - 2*c^3 - d^3*2i)/(32*a^2*f^2))^(1/2) - ((c*d + d^2*1i)
*(c + d*tan(e + f*x))^(1/2))/(2*a*f*(d*1i - d*tan(e + f*x)))
```

3.1111 $\int \frac{(c+d\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=209

$$-\frac{i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{4a^2f}+\frac{(2cd+i(2c^2+d^2))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{8a^2\sqrt{c+id}\,f}+\frac{(2ic+\ldots}{8a^{\ldots}}$$

[Out] -1/4*I*(c-I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/a^2/f+1/8*(2*c*d+I*(2*c^2+d^2))*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/a^2/f/(c+I*d)^(1/2)+1/8*(2*I*c+3*d)*(c+d*tan(f*x+e))^(1/2)/a^2/f/(1+I*tan(f*x+e))+1/4*(I*c-d)*(c+d*tan(f*x+e))^(1/2)/f/(a+I*a*tan(f*x+e))^2

Rubi [A]

time = 0.44, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3639, 3677, 3620, 3618, 65, 214}

$$\frac{(2cd+i(2c^2+d^2))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{8a^2f\sqrt{c+id}}+\frac{(3d+2ic)\sqrt{c+d\tan(e+fx)}}{8a^2f(1+i\tan(e+fx))}-\frac{i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{4a^2f}+\frac{(-d+ic)\sqrt{c+d\tan(e+fx)}}{4f(a+ia\tan(e+fx))^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d*Tan[e + f*x])^(3/2)/(a + I*a*Tan[e + f*x])^2,x]

[Out] ((-1/4*I)*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/(a^2*f) + ((2*c*d + I*(2*c^2 + d^2))*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/(8*a^2*Sqrt[c + I*d]*f) + (((2*I)*c + 3*d)*Sqrt[c + d*Tan[e + f*x]])/(8*a^2*f*(1 + I*Tan[e + f*x])) + ((I*c - d)*Sqrt[c + d*Tan[e + f*x]])/(4*f*(a + I*a*Tan[e + f*x])^2)

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
```

```
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^2}\,dx = \frac{(ic-d)\sqrt{c+d\tan(e+fx)}}{4f(a+ia\tan(e+fx))^2} - \frac{\int \frac{-\frac{1}{2}a(4c^2-5icd+d^2)-\frac{1}{2}a(3c-5id)d\tan(e+fx)}{(a+ia\tan(e+fx))\sqrt{c+d\tan(e+fx)}}\,dx}{4a^2}$$

$$= \frac{(2ic+3d)\sqrt{c+d\tan(e+fx)}}{8a^2f(1+i\tan(e+fx))} + \frac{(ic-d)\sqrt{c+d\tan(e+fx)}}{4f(a+ia\tan(e+fx))^2} + \frac{\int \frac{1}{2}a^2(4ic^3+}{}$$

$$= \frac{(2ic+3d)\sqrt{c+d\tan(e+fx)}}{8a^2f(1+i\tan(e+fx))} + \frac{(ic-d)\sqrt{c+d\tan(e+fx)}}{4f(a+ia\tan(e+fx))^2} + \frac{(c-id)^2\int}{}$$

$$= \frac{(2ic+3d)\sqrt{c+d\tan(e+fx)}}{8a^2f(1+i\tan(e+fx))} + \frac{(ic-d)\sqrt{c+d\tan(e+fx)}}{4f(a+ia\tan(e+fx))^2} + \frac{(i(c-id)^2}{}$$

$$= \frac{(2ic+3d)\sqrt{c+d\tan(e+fx)}}{8a^2f(1+i\tan(e+fx))} + \frac{(ic-d)\sqrt{c+d\tan(e+fx)}}{4f(a+ia\tan(e+fx))^2} - \frac{(c-id)^2\mathrm{S}}{}$$

$$= -\frac{i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{4a^2f} + \frac{(2ic^2+2cd+id^2)\tanh^{-1}\left(\right.}{8a^2\sqrt{c+}}$$

Mathematica [A]
time = 1.96, size = 272, normalized size = 1.30

$$\frac{\sec^2(e+fx)(\cos(fx)+i\sin(fx))^2\left(\frac{2\left(-i\sqrt{-c+id}\,(2c^2-2icd+d^2)\mathrm{ArcTan}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{-c-id}}\right)+2i\sqrt{-c-id}\,(c-id)^2\mathrm{ArcTan}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{-c+id}}\right)\right)(\cos(2e)+i\sin(2e))}{\sqrt{-c-id}\sqrt{-c+id}}+2\cos(e+fx)(\cos(2fx)-i\sin(2fx))((4ic+d)\cos(e+fx)+(-2c+3id)\sin(e+fx))\sqrt{c+d\tan(e+fx)}\right)}{16f(a+ia\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^(3/2)/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (Sec[e + f*x]^2*(Cos[f*x] + I*Sin[f*x])^2*((2*((-I)*Sqrt[-c + I*d]*(2*c^2 -
 (2*I)*c*d + d^2)*ArcTan[Sqrt[c + d*Tan[e + f*x]]/Sqrt[-c - I*d]] + (2*I)*S
qrt[-c - I*d]*(c - I*d)^2*ArcTan[Sqrt[c + d*Tan[e + f*x]]/Sqrt[-c + I*d]])*
(Cos[2*e] + I*Sin[2*e]))/(Sqrt[-c - I*d]*Sqrt[-c + I*d]) + 2*Cos[e + f*x]*(
Cos[2*f*x] - I*Sin[2*f*x])*(((4*I)*c + d)*Cos[e + f*x] + (-2*c + (3*I)*d)*S
in[e + f*x])*Sqrt[c + d*Tan[e + f*x]]))/(16*f*(a + I*a*Tan[e + f*x])^2)
```

Maple [A]
time = 0.42, size = 284, normalized size = 1.36

method	result

derivativedivides	$\dfrac{2d^3\left(\dfrac{i\left(\dfrac{-\frac{d(2ic^3+4icd^2-c^2d-3d^3)(c+d\tan(fx+e))^{\frac{3}{2}}}{2(2icd+c^2-d^2)}+\frac{d(2ic^4-3ic^2d^2-id^4-5c^3d-cd^3)\sqrt{c+d\tan(fx+e)}}{4icd+2c^2-2d^2}}{(-d\tan(fx+e)+id)^2}-\cdots\right)}{8d^3}\cdots\right)}{f\cdots}$
default	$\dfrac{2d^3\left(\dfrac{i\left(\dfrac{-\frac{d(2ic^3+4icd^2-c^2d-3d^3)(c+d\tan(fx+e))^{\frac{3}{2}}}{2(2icd+c^2-d^2)}+\frac{d(2ic^4-3ic^2d^2-id^4-5c^3d-cd^3)\sqrt{c+d\tan(fx+e)}}{4icd+2c^2-2d^2}}{(-d\tan(fx+e)+id)^2}-\cdots\right)}{8d^3}\cdots\right)}{f\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^2*d^3*(1/8*I/d^3*((-1/2*d*(2*I*c^3+4*I*c*d^2-c^2*d-3*d^3)/(2*I*c*d+c^
2-d^2)*(c+d*tan(f*x+e))^(3/2)+1/2*d*(2*I*c^4-3*I*c^2*d^2-I*d^4-5*c^3*d-c*d^
3)/(2*I*c*d+c^2-d^2)*(c+d*tan(f*x+e))^(1/2))/(-d*tan(f*x+e)+I*d)^2-1/2*(2*I
*c^3*d+4*I*c*d^3+2*c^4+3*c^2*d^2-d^4)/(2*I*c*d+c^2-d^2)/(-I*d-c)^(1/2)*arct
an((c+d*tan(f*x+e))^(1/2)/(-I*d-c)^(1/2)))+1/8*I*(I*d-c)^(3/2)/d^3*arctan((
c+d*tan(f*x+e))^(1/2)/(I*d-c)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1012 vs. $2(168) = 336$.
time = 1.26, size = 1012, normalized size = 4.84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] 1/32*(2*a^2*f*sqrt(-(c^3 - 3*I*c^2*d - 3*c*d^2 + I*d^3)/(a^4*f^2))*e^(4*I*f
*x + 4*I*e)*log(-2*(-I*c^2 - c*d + (a^2*f*e^(2*I*f*x + 2*I*e) + a^2*f)*sqrt
(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(
-(c^3 - 3*I*c^2*d - 3*c*d^2 + I*d^3)/(a^4*f^2)) + (-I*c^2 - 2*c*d + I*d^2)*
e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/(I*c + d)) - 2*a^2*f*sqrt(-(c^3 -
 3*I*c^2*d - 3*c*d^2 + I*d^3)/(a^4*f^2))*e^(4*I*f*x + 4*I*e)*log(-2*(-I*c^2
 - c*d - (a^2*f*e^(2*I*f*x + 2*I*e) + a^2*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2
*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(c^3 - 3*I*c^2*d - 3*c*d^
2 + I*d^3)/(a^4*f^2)) + (-I*c^2 - 2*c*d + I*d^2)*e^(2*I*f*x + 2*I*e))*e^(-2
*I*f*x - 2*I*e)/(I*c + d)) + a^2*f*sqrt(-(4*I*c^4 + 8*c^3*d + 4*c*d^3 + I*d
^4)/((I*a^4*c - a^4*d)*f^2))*e^(4*I*f*x + 4*I*e)*log(-1/8*(2*c^3 + 3*c*d^2
+ I*d^3 - ((I*a^2*c - a^2*d)*f*e^(2*I*f*x + 2*I*e) + (I*a^2*c - a^2*d)*f)*s
qrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sq
rt(-(4*I*c^4 + 8*c^3*d + 4*c*d^3 + I*d^4)/((I*a^4*c - a^4*d)*f^2)) + (2*c^3
 - 2*I*c^2*d + c*d^2)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/((I*a^2*c -
 a^2*d)*f)) - a^2*f*sqrt(-(4*I*c^4 + 8*c^3*d + 4*c*d^3 + I*d^4)/((I*a^4*c -
 a^4*d)*f^2))*e^(4*I*f*x + 4*I*e)*log(-1/8*(2*c^3 + 3*c*d^2 + I*d^3 - ((-I*
a^2*c + a^2*d)*f*e^(2*I*f*x + 2*I*e) + (-I*a^2*c + a^2*d)*f)*sqrt(((c - I*d
)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(4*I*c^4
+ 8*c^3*d + 4*c*d^3 + I*d^4)/((I*a^4*c - a^4*d)*f^2)) + (2*c^3 - 2*I*c^2*d
+ c*d^2)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/((I*a^2*c - a^2*d)*f)) +
 2*((3*I*c + 2*d)*e^(4*I*f*x + 4*I*e) + (4*I*c + d)*e^(2*I*f*x + 2*I*e) + I
*c - d)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e)
 + 1)))*e^(-4*I*f*x - 4*I*e)/(a^2*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{c\sqrt{c+d\tan(e+fx)}}{\tan^2(e+fx)-2i\tan(e+fx)-1}\,dx+\int \frac{d\sqrt{c+d\tan(e+fx)}\tan(e+fx)}{\tan^2(e+fx)-2i\tan(e+fx)-1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(3/2)/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -(Integral(c*sqrt(c + d*tan(e + f*x))/(tan(e + f*x)**2 - 2*I*tan(e + f*x) -
 1), x) + Integral(d*sqrt(c + d*tan(e + f*x))*tan(e + f*x)/(tan(e + f*x)**2
 - 2*I*tan(e + f*x) - 1), x))/a**2
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 462 vs. 2(168) = 336.
time = 0.66, size = 462, normalized size = 2.21

$$-\frac{(c^2-2icd-d^2)\arctan\left(-\frac{2\left(-i\sqrt{d\tan(fx+e)+c}\,c-i\sqrt{c^2+d^2}\sqrt{d\tan(fx+e)+c}\right)}{\sqrt{2c+2\sqrt{c^2+d^2}}\,c-i\sqrt{2c+2\sqrt{c^2+d^2}}\,d+\sqrt{c^2+d^2}\sqrt{2c+2\sqrt{c^2+d^2}}}\right)}{2a^2\sqrt{2c+2\sqrt{c^2+d^2}}\,f\left(-\frac{id}{c+\sqrt{c^2+d^2}}+1\right)}-\frac{(2ic^2+2cd+id^2)\arctan\left(\frac{2\left(\sqrt{d\tan(fx+e)+c}\,c-\sqrt{c^2+d^2}\sqrt{d\tan(fx+e)+c}\right)}{c\sqrt{-2c+2\sqrt{c^2+d^2}}+i\sqrt{-2c+2\sqrt{c^2+d^2}}\,d-\sqrt{c^2+d^2}\sqrt{-2c+2\sqrt{c^2+d^2}}}\right)}{4a^2\sqrt{-2c+2\sqrt{c^2+d^2}}\,f\left(\frac{id}{c-\sqrt{c^2+d^2}}+1\right)}+\frac{2(d\tan(fx+e)+c)^{\frac{3}{2}}cd-2\sqrt{d\tan(fx+e)+c}\,c^2d-3i(d\tan(fx+e)+c)^{\frac{3}{2}}d^2-i\sqrt{d\tan(fx+e)+c}\,cd^2-\sqrt{d\tan(fx+e)+c}\,d^3}{8(d\tan(fx+e)-id)^2a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/2*(c^2 - 2*I*c*d - d^2)*arctan(-2*(-I*sqrt(d*tan(f*x + e) + c)*c - I*sqr
t(c^2 + d^2)*sqrt(d*tan(f*x + e) + c))/(sqrt(2*c + 2*sqrt(c^2 + d^2))*c - I
*sqrt(2*c + 2*sqrt(c^2 + d^2))*d + sqrt(c^2 + d^2)*sqrt(2*c + 2*sqrt(c^2 +
d^2))))/(a^2*sqrt(2*c + 2*sqrt(c^2 + d^2))*f*(-I*d/(c + sqrt(c^2 + d^2)) +
1)) - 1/4*(2*I*c^2 + 2*c*d + I*d^2)*arctan(2*(sqrt(d*tan(f*x + e) + c)*c -
sqrt(c^2 + d^2)*sqrt(d*tan(f*x + e) + c))/(c*sqrt(-2*c + 2*sqrt(c^2 + d^2))
 + I*sqrt(-2*c + 2*sqrt(c^2 + d^2))*d - sqrt(c^2 + d^2)*sqrt(-2*c + 2*sqrt(
c^2 + d^2))))/(a^2*sqrt(-2*c + 2*sqrt(c^2 + d^2))*f*(I*d/(c - sqrt(c^2 + d^
2)) + 1)) + 1/8*(2*(d*tan(f*x + e) + c)^(3/2)*c*d - 2*sqrt(d*tan(f*x + e) +
 c)*c^2*d - 3*I*(d*tan(f*x + e) + c)^(3/2)*d^2 - I*sqrt(d*tan(f*x + e) + c)
*c*d^2 - sqrt(d*tan(f*x + e) + c)*d^3)/((d*tan(f*x + e) - I*d)^2*a^2*f)
```

Mupad [B]
time = 7.76, size = 1580, normalized size = 7.56

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(3/2)/(a + a*tan(e + f*x)*1i)^2,x)
```

```
[Out] - atan((a^4*d^6*f^2*(c + d*tan(e + f*x))^(1/2)*((3*c*d^2)/(64*a^4*f^2) - (d
^3*1i)/(64*a^4*f^2) - c^3/(64*a^4*f^2) + (c^2*d*3i)/(64*a^4*f^2))^(1/2)*80i
)/(a^2*d^8*f*10i - a^2*c^2*d^6*f*26i + 8*a^2*c^3*d^5*f - 28*a^2*c*d^7*f) -
(64*a^4*c*d^5*f^2*(c + d*tan(e + f*x))^(1/2)*((3*c*d^2)/(64*a^4*f^2) - (d^3
*1i)/(64*a^4*f^2) - c^3/(64*a^4*f^2) + (c^2*d*3i)/(64*a^4*f^2))^(1/2))/(a^2
*d^8*f*10i - a^2*c^2*d^6*f*26i + 8*a^2*c^3*d^5*f - 28*a^2*c*d^7*f))*((6*c*d
^2 + c^2*d*6i - 2*c^3 - d^3*2i)/(128*a^4*f^2))^(1/2)*2i - (((c + d*tan(e +
f*x))^(1/2)*(c*d^2*3i + 6*c^2*d + 3*d^3))/(24*a^2*f) + (d*(c*2i + 3*d)*(c +
 d*tan(e + f*x))^(3/2)*1i)/(8*a^2*f))/(c*d*2i - (2*c + d*2i)*(c + d*tan(e +
 f*x)) + (c + d*tan(e + f*x))^2 + c^2 - d^2) - atan((((a^2*f*(128*a^4*d^5*f
^2 + a^4*c*d^4*f^2*384i - 256*a^4*c^2*d^3*f^2) - 4096*a^8*c*d^2*f^4*(c + d*
tan(e + f*x))^(1/2)*(-(4*c*d^3 + 8*c^3*d + c^4*4i + d^4*1i)/(256*a^4*f^2*(c
*1i - d)))^(1/2))*(-(4*c*d^3 + 8*c^3*d + c^4*4i + d^4*1i)/(256*a^4*f^2*(c*1
i - d)))^(1/2) - 8*a^4*f^2*(c + d*tan(e + f*x))^(1/2)*(c*d^5*12i + 5*d^6 -
24*c^2*d^4 - c^3*d^3*24i + 8*c^4*d^2))*(-(4*c*d^3 + 8*c^3*d + c^4*4i + d^4*
1i)/(256*a^4*f^2*(c*1i - d)))^(1/2)*1i - ((a^2*f*(128*a^4*d^5*f^2 + a^4*c*d
^4*f^2*384i - 256*a^4*c^2*d^3*f^2) + 4096*a^8*c*d^2*f^4*(c + d*tan(e + f*x)
)^(1/2)*(-(4*c*d^3 + 8*c^3*d + c^4*4i + d^4*1i)/(256*a^4*f^2*(c*1i - d)))^(
1/2))*(-(4*c*d^3 + 8*c^3*d + c^4*4i + d^4*1i)/(256*a^4*f^2*(c*1i - d)))^(1/
2) + 8*a^4*f^2*(c + d*tan(e + f*x))^(1/2)*(c*d^5*12i + 5*d^6 - 24*c^2*d^4 -
 c^3*d^3*24i + 8*c^4*d^2))*(-(4*c*d^3 + 8*c^3*d + c^4*4i + d^4*1i)/(256*a^4
*f^2*(c*1i - d)))^(1/2)*1i)/(((a^2*f*(128*a^4*d^5*f^2 + a^4*c*d^4*f^2*384i
```

```
- 256*a^4*c^2*d^3*f^2) - 4096*a^8*c*d^2*f^4*(c + d*tan(e + f*x))^(1/2)*(-(4
*c*d^3 + 8*c^3*d + c^4*4i + d^4*1i)/(256*a^4*f^2*(c*1i - d)))^(1/2))*(-(4*c
*d^3 + 8*c^3*d + c^4*4i + d^4*1i)/(256*a^4*f^2*(c*1i - d)))^(1/2) - 8*a^4*f
^2*(c + d*tan(e + f*x))^(1/2)*(c*d^5*12i + 5*d^6 - 24*c^2*d^4 - c^3*d^3*24i
 + 8*c^4*d^2))*(-(4*c*d^3 + 8*c^3*d + c^4*4i + d^4*1i)/(256*a^4*f^2*(c*1i -
 d)))^(1/2) + ((a^2*f*(128*a^4*d^5*f^2 + a^4*c*d^4*f^2*384i - 256*a^4*c^2*d
^3*f^2) + 4096*a^8*c*d^2*f^4*(c + d*tan(e + f*x))^(1/2)*(-(4*c*d^3 + 8*c^3*
d + c^4*4i + d^4*1i)/(256*a^4*f^2*(c*1i - d)))^(1/2))*(-(4*c*d^3 + 8*c^3*d
+ c^4*4i + d^4*1i)/(256*a^4*f^2*(c*1i - d)))^(1/2) + 8*a^4*f^2*(c + d*tan(e
 + f*x))^(1/2)*(c*d^5*12i + 5*d^6 - 24*c^2*d^4 - c^3*d^3*24i + 8*c^4*d^2))*
(-(4*c*d^3 + 8*c^3*d + c^4*4i + d^4*1i)/(256*a^4*f^2*(c*1i - d)))^(1/2) + 2
*a^2*f*(2*c*d^7 - d^8*3i - c^2*d^6*15i + 28*c^3*d^5 + c^4*d^4*18i - 4*c^5*d
^3)))*(-(4*c*d^3 + 8*c^3*d + c^4*4i + d^4*1i)/(256*a^4*f^2*(c*1i - d)))^(1/
2)*2i
```

3.1112 $\int \frac{(c+d\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^3}\,dx$

Optimal. Leaf size=274

$$-\frac{i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{8a^3f}+\frac{ic(2c^2+3d^2)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{16a^3(c+id)^{3/2}f}+\frac{(ic-d)\sqrt{c}}{6f(a+i}$$

```
[Out] -1/8*I*(c-I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/a^3/f+1/
16*I*c*(2*c^2+3*d^2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/a^3/(c+I
*d)^(3/2)/f+1/6*(I*c-d)*(c+d*tan(f*x+e))^(1/2)/f/(a+I*a*tan(f*x+e))^3+1/24*
(3*I*c+4*d)*(c+d*tan(f*x+e))^(1/2)/a/f/(a+I*a*tan(f*x+e))^2-1/16*(2*c^2-I*c
*d+2*d^2)*(c+d*tan(f*x+e))^(1/2)/(I*c-d)/f/(a^3+I*a^3*tan(f*x+e))
```

Rubi [A]
time = 0.68, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3639, 3677, 3620, 3618, 65, 214}

$$-\frac{(2c^2-icd+2d^2)\sqrt{c+d\tan(e+fx)}}{16f(-d+ic)(a^3+ia^3\tan(e+fx))}+\frac{ic(2c^2+3d^2)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{16a^3f(c+id)^{3/2}}-\frac{i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{8a^3f}+\frac{(4d+3ic)\sqrt{c+d\tan(e+fx)}}{24af(a+ia\tan(e+fx))^2}+\frac{(-d+ic)\sqrt{c+d\tan(e+fx)}}{6f(a+ia\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^(3/2)/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] ((-1/8*I)*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/
(a^3*f) + ((I/16)*c*(2*c^2 + 3*d^2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c
 + I*d]])/(a^3*(c + I*d)^(3/2)*f) + ((I*c - d)*Sqrt[c + d*Tan[e + f*x]])/(6
*f*(a + I*a*Tan[e + f*x])^3) + (((3*I)*c + 4*d)*Sqrt[c + d*Tan[e + f*x]])/(
24*a*f*(a + I*a*Tan[e + f*x])^2) - ((2*c^2 - I*c*d + 2*d^2)*Sqrt[c + d*Tan[
e + f*x]])/(16*(I*c - d)*f*(a^3 + I*a^3*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^3}\,dx = \frac{(ic-d)\sqrt{c+d\tan(e+fx)}}{6f(a+ia\tan(e+fx))^3} - \frac{\int \frac{-\frac{1}{2}a(6c^2-7icd+d^2)-\frac{1}{2}a(5c-7id)d\tan(e+fx)}{(a+ia\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}\,dx}{6a^2}$$

$$= \frac{(ic-d)\sqrt{c+d\tan(e+fx)}}{6f(a+ia\tan(e+fx))^3} + \frac{(3ic+4d)\sqrt{c+d\tan(e+fx)}}{24af(a+ia\tan(e+fx))^2} + \frac{\int \frac{\frac{3}{2}a^2c(c+}{(a+i}}{}$$

$$= \frac{(ic-d)\sqrt{c+d\tan(e+fx)}}{6f(a+ia\tan(e+fx))^3} + \frac{(3ic+4d)\sqrt{c+d\tan(e+fx)}}{24af(a+ia\tan(e+fx))^2} - \frac{(2c^2-ic}{16(ic-}$$

$$= \frac{(ic-d)\sqrt{c+d\tan(e+fx)}}{6f(a+ia\tan(e+fx))^3} + \frac{(3ic+4d)\sqrt{c+d\tan(e+fx)}}{24af(a+ia\tan(e+fx))^2} - \frac{(2c^2-ic}{16(ic-}$$

$$= \frac{(ic-d)\sqrt{c+d\tan(e+fx)}}{6f(a+ia\tan(e+fx))^3} + \frac{(3ic+4d)\sqrt{c+d\tan(e+fx)}}{24af(a+ia\tan(e+fx))^2} - \frac{(2c^2-ic}{16(ic-}$$

$$= \frac{(ic-d)\sqrt{c+d\tan(e+fx)}}{6f(a+ia\tan(e+fx))^3} + \frac{(3ic+4d)\sqrt{c+d\tan(e+fx)}}{24af(a+ia\tan(e+fx))^2} - \frac{(2c^2-ic}{16(ic-}$$

$$= -\frac{i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{8a^3f} + \frac{ic(2c^2+3d^2)\tanh^{-1}\left(\frac{\sqrt{c}}{}\right.}{16a^3(c+id}$$

Mathematica [A]

time = 2.85, size = 311, normalized size = 1.14

$$\frac{\sec^3(e+fx)(\cos(fx)+i\sin(fx))^3\left(\frac{2i\left(c\sqrt{-c+id}\,(2c^2+3d^2)\text{ArcTan}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{-c-id}}\right)+2(-c-id)^{3/2}(c-id)^2\text{ArcTan}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{-c+id}}\right)\right)(\cos(3e)+i\sin(3e))}{(-c-id)^{3/2}\sqrt{-c+id}}+\frac{2\cos(e+fx)(i\cos(3fx)+\sin(3fx))(7c(c+id)+(13c^2+4icd+6d^2)\cos(2(e+fx))+(9ic^2+4cd+10id^2)\sin(2(e+fx)))\sqrt{c+d\tan(e+fx)}}{3(c+id)}\right)}{32f(a+ia\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^(3/2)/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] (Sec[e + f*x]^3*(Cos[f*x] + I*Sin[f*x])^3*(((2*I)*(c*Sqrt[-c + I*d]*(2*c^2
+ 3*d^2)*ArcTan[Sqrt[c + d*Tan[e + f*x]]/Sqrt[-c - I*d]] + 2*(-c - I*d)^(3/
2)*(c - I*d)^2*ArcTan[Sqrt[c + d*Tan[e + f*x]]/Sqrt[-c + I*d]])*(Cos[3*e] +
 I*Sin[3*e]))/((-c - I*d)^(3/2)*Sqrt[-c + I*d]) + (2*Cos[e + f*x]*(I*Cos[3*
f*x] + Sin[3*f*x])*(7*c*(c + I*d) + (13*c^2 + (4*I)*c*d + 6*d^2)*Cos[2*(e +
 f*x)] + ((9*I)*c^2 + 4*c*d + (10*I)*d^2)*Sin[2*(e + f*x)])*Sqrt[c + d*Tan[
e + f*x]])/(3*(c + I*d))))/(32*f*(a + I*a*Tan[e + f*x])^3)
```

Maple [A]

time = 0.42, size = 362, normalized size = 1.32

method	result
derivativedivides	$2d^4\left(\frac{-\frac{d\left(3ic^3d+5ic\,d^3+2c^4+2c^2d^2-2d^4\right)(c+d\tan(fx+e))^{\frac{5}{2}}}{2\left(3ic^2d-id^3+c^3-3c\,d^2\right)}+\frac{2\left(3c^2+5d^2\right)d(c+d\tan(fx+e))^{\frac{3}{2}}}{3}-\frac{d\left(9ic^5d-6ic^3d^3-7ic\,d^5+2c^6-14c\right.}{2\left(3i\right.}}{(-d\tan(fx+e)+id)^3}\right.$
default	$2d^4\left(\frac{-\frac{d\left(3ic^3d+5ic\,d^3+2c^4+2c^2d^2-2d^4\right)(c+d\tan(fx+e))^{\frac{5}{2}}}{2\left(3ic^2d-id^3+c^3-3c\,d^2\right)}+\frac{2\left(3c^2+5d^2\right)d(c+d\tan(fx+e))^{\frac{3}{2}}}{3}-\frac{d\left(9ic^5d-6ic^3d^3-7ic\,d^5+2c^6-14c\right.}{2\left(3i\right.}}{(-d\tan(fx+e)+id)^3}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3*d^4*(1/16/d^4*((-1/2*d*(3*I*c^3*d+5*I*c*d^3+2*c^4+2*c^2*d^2-2*d^4)/
(3*I*c^2*d-I*d^3+c^3-3*c*d^2)*(c+d*tan(f*x+e))^(5/2)+2/3*(3*c^2+5*d^2)*d*(c
+d*tan(f*x+e))^(3/2)-1/2*d*(9*I*c^5*d-6*I*c^3*d^3-7*I*c*d^5+2*c^6-14*c^4*d^
2-6*c^2*d^4+2*d^6)/(3*I*c^2*d-I*d^3+c^3-3*c*d^2)*(c+d*tan(f*x+e))^(1/2))/(-
d*tan(f*x+e)+I*d)^3-1/2*c*(2*I*c^4+I*c^2*d^2-3*I*d^4-4*c^3*d-6*c*d^3)/(3*I*
c^2*d-I*d^3+c^3-3*c*d^2)/(-I*d-c)^(1/2)*arctan((c+d*tan(f*x+e))^(1/2)/(-I*d
-c)^(1/2)))+1/16*I*(I*d-c)^(3/2)/d^4*arctan((c+d*tan(f*x+e))^(1/2)/(I*d-c)^
(1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1268 vs. $2(226) = 452$.
time = 1.81, size = 1268, normalized size = 4.63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="fricas"
)
```

```
[Out] 1/192*(6*(I*a^3*c - a^3*d)*f*sqrt(-(c^3 - 3*I*c^2*d - 3*c*d^2 + I*d^3)/(a^6
*f^2))*e^(6*I*f*x + 6*I*e)*log(-2*(-I*c^2 - c*d + (a^3*f*e^(2*I*f*x + 2*I*e
) + a^3*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I
*e) + 1))*sqrt(-(c^3 - 3*I*c^2*d - 3*c*d^2 + I*d^3)/(a^6*f^2)) + (-I*c^2 -
2*c*d + I*d^2)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/(I*c + d)) + 6*(-I
*a^3*c + a^3*d)*f*sqrt(-(c^3 - 3*I*c^2*d - 3*c*d^2 + I*d^3)/(a^6*f^2))*e^(6
*I*f*x + 6*I*e)*log(-2*(-I*c^2 - c*d - (a^3*f*e^(2*I*f*x + 2*I*e) + a^3*f)*
sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*s
qrt(-(c^3 - 3*I*c^2*d - 3*c*d^2 + I*d^3)/(a^6*f^2)) + (-I*c^2 - 2*c*d + I*d
^2)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/(I*c + d)) + 3*(-I*a^3*c + a^
3*d)*f*sqrt(-(-4*I*c^6 - 12*I*c^4*d^2 - 9*I*c^2*d^4)/((-I*a^6*c^3 + 3*a^6*c
^2*d + 3*I*a^6*c*d^2 - a^6*d^3)*f^2))*e^(6*I*f*x + 6*I*e)*log(-1/16*(-2*I*c
^4 + 2*c^3*d - 3*I*c^2*d^2 + 3*c*d^3 + ((a^3*c^2 + 2*I*a^3*c*d - a^3*d^2)*f
*e^(2*I*f*x + 2*I*e) + (a^3*c^2 + 2*I*a^3*c*d - a^3*d^2)*f)*sqrt(((c - I*d)
*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(-4*I*c^6
- 12*I*c^4*d^2 - 9*I*c^2*d^4)/((-I*a^6*c^3 + 3*a^6*c^2*d + 3*I*a^6*c*d^2 -
a^6*d^3)*f^2)) + (-2*I*c^4 - 3*I*c^2*d^2)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x
- 2*I*e)/((a^3*c^2 + 2*I*a^3*c*d - a^3*d^2)*f)) + 3*(I*a^3*c - a^3*d)*f*sqr
t(-(-4*I*c^6 - 12*I*c^4*d^2 - 9*I*c^2*d^4)/((-I*a^6*c^3 + 3*a^6*c^2*d + 3*I
*a^6*c*d^2 - a^6*d^3)*f^2))*e^(6*I*f*x + 6*I*e)*log(-1/16*(-2*I*c^4 + 2*c^3
*d - 3*I*c^2*d^2 + 3*c*d^3 - ((a^3*c^2 + 2*I*a^3*c*d - a^3*d^2)*f*e^(2*I*f*
x + 2*I*e) + (a^3*c^2 + 2*I*a^3*c*d - a^3*d^2)*f)*sqrt(((c - I*d)*e^(2*I*f*
x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(-4*I*c^6 - 12*I*c^4
*d^2 - 9*I*c^2*d^4)/((-I*a^6*c^3 + 3*a^6*c^2*d + 3*I*a^6*c*d^2 - a^6*d^3)*f
^2)) + (-2*I*c^4 - 3*I*c^2*d^2)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/(
(a^3*c^2 + 2*I*a^3*c*d - a^3*d^2)*f)) - 2*(2*c^2 + 4*I*c*d - 2*d^2 + (11*c^
2 + 8*d^2)*e^(6*I*f*x + 6*I*e) + (18*c^2 + 7*I*c*d + 8*d^2)*e^(4*I*f*x + 4*
I*e) + (9*c^2 + 11*I*c*d - 2*d^2)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(2
*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-6*I*f*x - 6*I*e)
/((I*a^3*c - a^3*d)*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\left(\int \frac{c\sqrt{c+d\tan{(e+fx)}}}{\tan^3{(e+fx)}-3i\tan^2{(e+fx)}-3\tan{(e+fx)}+i}\,dx+\int \frac{d\sqrt{c+d\tan{(e+fx)}}\tan{(e+fx)}}{\tan^3{(e+fx)}-3i\tan^2{(e+fx)}-3\tan{(e+fx)}+i}\,dx\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(3/2)/(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] I*(Integral(c*sqrt(c + d*tan(e + f*x))/(tan(e + f*x)**3 - 3*I*tan(e + f*x)*
*2 - 3*tan(e + f*x) + I), x) + Integral(d*sqrt(c + d*tan(e + f*x))*tan(e +
f*x)/(tan(e + f*x)**3 - 3*I*tan(e + f*x)**2 - 3*tan(e + f*x) + I), x))/a**3
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(226) = 452$.
time = 0.75, size = 624, normalized size = 2.28

$$-\frac{\left(2 c^{3}+3 c d^{2}\right) \arctan \left(\frac{2\left(\sqrt{d \tan (f x+e)+c}\, c-\sqrt{c^{2}+d^{2}} \sqrt{d \tan (f x+e)+c}\right)}{c \sqrt{-2 c+2 \sqrt{c^{2}+d^{2}}}+i \sqrt{-2 c+2 \sqrt{c^{2}+d^{2}}}\, d-\sqrt{c^{2}+d^{2}} \sqrt{-2 c+2 \sqrt{c^{2}+d^{2}}}}\right)}{-8\left(i a^{3} c f-a^{3} d f\right) \sqrt{-2 c+2 \sqrt{c^{2}+d^{2}}}\left(\frac{i d}{c-\sqrt{c^{2}+d^{2}}}+1\right)}+\frac{-6 i (d \tan (f x+e)+c)^{\frac{5}{2}} c^{2} d+12 i (d \tan (f x+e)+c)^{\frac{3}{2}} c^{3} d-6 i \sqrt{d \tan (f x+e)+c}\, c^{4} d-3 (d \tan (f x+e)+c)^{\frac{5}{2}} c d^{2}-12 (d \tan (f x+e)+c)^{\frac{3}{2}} c^{2} d^{2}+15 \sqrt{d \tan (f x+e)+c}\, c^{3} d^{2}-6 i (d \tan (f x+e)+c)^{\frac{5}{2}} d^{3}+20 i (d \tan (f x+e)+c)^{\frac{3}{2}} c d^{3}+6 i \sqrt{d \tan (f x+e)+c}\, c^{2} d^{3}-20 (d \tan (f x+e)+c)^{\frac{3}{2}} d^{4}+9 \sqrt{d \tan (f x+e)+c}\, c d^{4}+6 i \sqrt{d \tan (f x+e)+c}\, d^{5}}{-48\left(i a^{3} c f-a^{3} d f\right)(d \tan (f x+e)-i d)^{3}}-\frac{\left(-i c^{2}-2 c d+i d^{2}\right) \arctan \left(\frac{2\left(\sqrt{d \tan (f x+e)+c}\, c-\sqrt{c^{2}+d^{2}} \sqrt{d \tan (f x+e)+c}\right)}{c \sqrt{-2 c+2 \sqrt{c^{2}+d^{2}}}-i \sqrt{-2 c+2 \sqrt{c^{2}+d^{2}}}\, d-\sqrt{c^{2}+d^{2}} \sqrt{-2 c+2 \sqrt{c^{2}+d^{2}}}}\right)}{4 a^{3} \sqrt{-2 c+2 \sqrt{c^{2}+d^{2}}}\, f\left(-\frac{i d}{c-\sqrt{c^{2}+d^{2}}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -(2*c^3 + 3*c*d^2)*arctan(2*(sqrt(d*tan(f*x + e) + c)*c - sqrt(c^2 + d^2)*s
qrt(d*tan(f*x + e) + c))/(c*sqrt(-2*c + 2*sqrt(c^2 + d^2)) + I*sqrt(-2*c +
2*sqrt(c^2 + d^2))*d - sqrt(c^2 + d^2)*sqrt(-2*c + 2*sqrt(c^2 + d^2))))/((-
8*I*a^3*c*f + 8*a^3*d*f)*sqrt(-2*c + 2*sqrt(c^2 + d^2))*(I*d/(c - sqrt(c^2
+ d^2)) + 1)) + (-6*I*(d*tan(f*x + e) + c)^(5/2)*c^2*d + 12*I*(d*tan(f*x +
e) + c)^(3/2)*c^3*d - 6*I*sqrt(d*tan(f*x + e) + c)*c^4*d - 3*(d*tan(f*x + e
) + c)^(5/2)*c*d^2 - 12*(d*tan(f*x + e) + c)^(3/2)*c^2*d^2 + 15*sqrt(d*tan(
f*x + e) + c)*c^3*d^2 - 6*I*(d*tan(f*x + e) + c)^(5/2)*d^3 + 20*I*(d*tan(f*
x + e) + c)^(3/2)*c*d^3 + 6*I*sqrt(d*tan(f*x + e) + c)*c^2*d^3 - 20*(d*tan(
f*x + e) + c)^(3/2)*d^4 + 9*sqrt(d*tan(f*x + e) + c)*c*d^4 + 6*I*sqrt(d*tan
(f*x + e) + c)*d^5)/((-48*I*a^3*c*f + 48*a^3*d*f)*(d*tan(f*x + e) - I*d)^3)
 - 1/4*(-I*c^2 - 2*c*d + I*d^2)*arctan(2*(sqrt(d*tan(f*x + e) + c)*c - sqrt
(c^2 + d^2)*sqrt(d*tan(f*x + e) + c))/(c*sqrt(-2*c + 2*sqrt(c^2 + d^2)) - I
*sqrt(-2*c + 2*sqrt(c^2 + d^2))*d - sqrt(c^2 + d^2)*sqrt(-2*c + 2*sqrt(c^2
+ d^2))))/(a^3*sqrt(-2*c + 2*sqrt(c^2 + d^2))*f*(-I*d/(c - sqrt(c^2 + d^2))
 + 1))
```

Mupad [B]
time = 9.21, size = 2500, normalized size = 9.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(3/2)/(a + a*tan(e + f*x)*1i)^3,x)
```

```
[Out] (((c + d*tan(e + f*x))^(1/2)*(5*c*d^3 + 10*c^3*d + d^4*10i + c^2*d^2*15i))/
(80*a^3*f) - (d*(c + d*tan(e + f*x))^(5/2)*(c*d + c^2*2i + d^2*2i)*1i)/(16*
(a^3*c*f + a^3*d*f*1i)) + (d*(c^2*6i + d^2*10i)*(c + d*tan(e + f*x))^(3/2)*
1i)/(24*a^3*f))/((c + d*tan(e + f*x))*(c*d*6i + 3*c^2 - 3*d^2) + (c + d*tan
(e + f*x))^3 + 3*c*d^2 - c^2*d*3i - (3*c + d*3i)*(c + d*tan(e + f*x))^2 - c
^3 + d^3*1i) - log((((12*c*d^12 - d^13*4i - c^2*d^11*9i + 59*c^3*d^10 + c^4
*d^9*39i + 51*c^5*d^8 + c^6*d^7*64i + c^8*d^5*24i - 8*c^9*d^4 - 4*a^6*c^4*f
^2*((216*c^2*d^24 - 16*d^26 - c*d^25*96i + c^3*d^23*176i + 111*c^4*d^22 + c
```

^5*d^21*330i - 209*c^6*d^20 + c^7*d^19*36i - 111*c^8*d^18 - c^9*d^17*54i + 9*c^10*d^16)/(16*a^12*c^8*f^4 + 16*a^12*d^8*f^4 + 64*a^12*c^2*d^6*f^4 + 96*a^12*c^4*d^4*f^4 + 64*a^12*c^6*d^2*f^4))^(1/2) - 4*a^6*d^4*f^2*((216*c^2*d^24 - 16*d^26 - c*d^25*96i + c^3*d^23*176i + 111*c^4*d^22 + c^5*d^21*330i - 209*c^6*d^20 + c^7*d^19*36i - 111*c^8*d^18 - c^9*d^17*54i + 9*c^10*d^16)/(16*a^12*c^8*f^4 + 16*a^12*d^8*f^4 + 64*a^12*c^2*d^6*f^4 + 96*a^12*c^4*d^4*f^4 + 64*a^12*c^6*d^2*f^4))^(1/2) - 8*a^6*c^2*d^2*f^2*((216*c^2*d^24 - 16*d^26 - c*d^25*96i + c^3*d^23*176i + 111*c^4*d^22 + c^5*d^21*330i - 209*c^6*d^20 + c^7*d^19*36i - 111*c^8*d^18 - c^9*d^17*54i + 9*c^10*d^16)/(16*a^12*c^8*f^4 + 16*a^12*d^8*f^4 + 64*a^12*c^2*d^6*f^4 + 96*a^12*c^4*d^4*f^4 + 64*a^12*c^6*d^2*f^4))^(1/2))/(2048*a^6*d^10*f^2 + 6144*a^6*c^2*d^8*f^2 + 6144*a^6*c^4*d^6*f^2 + 2048*a^6*c^6*d^4*f^2))^(1/2)*(2048*a^9*d^9*f^3 + a^9*c*d^8*f^3*5120i - 32*((12*c*d^12 - d^13*4i - c^2*d^11*9i + 59*c^3*d^10 + c^4*d^9*39i + 51*c^5*d^8 + c^6*d^7*64i + c^8*d^5*24i - 8*c^9*d^4 - 4*a^6*c^4*f^2*((216*c^2*d^24 - 16*d^26 - c*d^25*96i + c^3*d^23*176i + 111*c^4*d^22 + c^5*d^21*330i - 209*c^6*d^20 + c^7*d^19*36i - 111*c^8*d^18 - c^9*d^17*54i + 9*c^10*d^16)/(16*a^12*c^8*f^4 + 16*a^12*d^8*f^4 + 64*a^12*c^2*d^6*f^4 + 96*a^12*c^4*d^4*f^4 + 64*a^12*c^6*d^2*f^4))^(1/2) - 4*a^6*d^4*f^2*((216*c^2*d^24 - 16*d^26 - c*d^25*96i + c^3*d^23*176i + 111*c^4*d^22 + c^5*d^21*330i - 209*c^6*d^20 + c^7*d^19*36i - 111*c^8*d^18 - c^9*d^17*54i + 9*c^10*d^16)/(16*a^12*c^8*f^4 + 16*a^12*d^8*f^4 + 64*a^12*c^2*d^6*f^4 + 96*a^12*c^4*d^4*f^4 + 64*a^12*c^6*d^2*f^4))^(1/2) - 8*a^6*c^2*d^2*f^2*((216*c^2*d^24 - 16*d^26 - c*d^25*96i + c^3*d^23*176i + 111*c^4*d^22 + c^5*d^21*330i - 209*c^6*d^20 + c^7*d^19*36i - 111*c^8*d^18 - c^9*d^17*54i + 9*c^10*d^16)/(16*a^12*c^8*f^4 + 16*a^12*d^8*f^4 + 64*a^12*c^2*d^6*f^4 + 96*a^12*c^4*d^4*f^4 + 64*a^12*c^6*d^2*f^4))^(1/2))/(2048*a^6*d^10*f^2 + 6144*a^6*c^2*d^8*f^2 + 6144*a^6*c^4*d^6*f^2 + 2048*a^6*c^6*d^4*f^2))^(1/2)*(c + d*tan(e + f*x))^(1/2)*(a^6*c^2*d^3*f^2*4096i - 2048*a^6*c*d^4*f^2 + 2048*a^6*c^3*d^2*f^2)*(a^6*d^2*f^2 - a^6*c^2*f^2 + a^6*c*d*f^2*2i) + a^9*c^3*d^6*f^3*8192i - 4096*a^9*c^4*d^5*f^3 + a^9*c^5*d^4*f^3*3072i - 2048*a^9*c^6*d^3*f^3) + 32*(c + d*tan(e + f*x))^(1/2)*(a^6*d^2*f^2 - a^6*c^2*f^2 + a^6*c*d*f^2*2i)*(c*d^7*8i + 4*d^8 - 5*c^2*d^6 + c^3*d^5*16i - 16*c^4*d^4 + c^5*d^3*8i - 8*c^6*d^2))*((12*c*d^12 - d^13*4i - c^2*d^11*9i + 59*c^3*d^10 + c^4*d^9*39i + 51*c^5*d^8 + c^6*d^7*64i + c^8*d^5*24i - 8*c^9*d^4 - 4*a^6*c^4*f^2*((216*c^2*d^24 - 16*d^26 - c*d^25*96i + c^3*d^23*176i + 111*c^4*d^22 + c^5*d^21*330i - 209*c^6*d^20 + c^7*d^19*36i - 111*c^8*d^18 - c^9*d^17*54i + 9*c^10*d^16)/(16*a^12*c^8*f^4 + 16*a^12*d^8*f^4 + 64*a^12*c^2*d^6*f^4 + 96*a^12*c^4*d^4*f^4 + 64*a^12*c^6*d^2*f^4))^(1/2) - 4*a^6*d^4*f^2*((216*c^2*d^24 - 16*d^26 - c*d^25*96i + c^3*d^23*176i + 111*c^4*d^22 + c^5*d^21*330i - 209*c^6*d^20 + c^7*d^19*36i - 111*c^8*d^18 - c^9*d^17*54i + 9*c^10*d^16)/(16*a^12*c^8*f^4 + 16*a^12*d^8*f^4 + 64*a^12*c^2*d^6*f^4 + 96*a^12*c^4*d^4*f^4 + 64*a^12*c^6*d^2*f^4))^(1/2) - 8*a^6*c^2*d^2*f^2*((216*c^2*d^24 - 16*d^26 - c*d^25*96i + c^3*d^23*176i + 111*c^4*d^22 + c^5*d^21*330i - 209*c^6*d^20 + c^7*d^19*36i - 111*c^8*d^18 - c^9*d^17*54i + 9*c^10*d^16)/(16*a^12*c^8*f^4 + 16*a^12*d^8*f^4 + 64*a^12*c^2*d^6*f^4 + 96*a^12*c^4*d^4*f^4 + 64*a^12*c^6*d^2*f^4))^(1/2))/(2048*a^6*d^10*f

```
^2 + 6144*a^6*c^2*d^8*f^2 + 6144*a^6*c^4*d^6*f^2 + 2048*a^6*c^6*d^4*f^2))^(
1/2) - a^3*c^2*d^10*f*42i + 28*a^3*c^3*d^9*f - a^3*c^4*d^8*f*64i + 108*a^3*
c^5*d^7*f + a^3*c^6*d^6*f*30i + 44*a^3*c^7*d^5*f + a^3*c^8*d^4*f*36i - 8*a^
3*c^9*d^3*f - 12*a^3*c*d^11*f)*((12*c*d^12 - d^13*4i - c^2*d^11*9i + 59*c^3
*d^10 + c^4*d^9*39i + 51*c^5*d^8 + c^6*d^7*64i + c^8*d^5*24i - 8*c^9*d^4 -
4*a^6*c^4*f^2*((216*c^2*d^24 - 16*d^26 - c*d^25*96i + c^3*d^23*176i + 111*c
^4*d^22 + c^5*d^21*330i - 209*c^6*d^20 + c^7*d^19*36i - 111*c^8*d^18 - c^9*
d^17*54i + 9*c^10*d^16)/(16*a^12*c^8*f^4 + 16*a^12*d^8*f^4 + 64*a^12*c^2*d^
6*f^4 + 96*a^12*c^4*d^4*f^4 + 64*a^12*c^6*d^2*f^4))^(1/2) - 4*a^6*d^4*f^2*(
(216*c^2*d^24 - 16*d^26 - c*d^25*96i + c^3*d^23*176i + 111*c^4*d^22 + c^5*d
^21*330i - 209*c^6*d^20 + c^7*d^19*36i - 111*c^8*d^18 - c^9*d^17*54i + 9*c^
10*d^16)/(16*a^12*c^8*f^4 + 16*a^12*d^8*f^4 + 64*a^12*c^2*d^6*f^4 + 96*a^12
*c^4*d^4*f^4 + 64*a^12*c^6*d^2*f^4))^(1/2) - 8*a^6*c^2*d^2*f^2*((216*c^2*d^
24 - 16*d^26 - c*d^25*96i + c^3*d^23*176i + 111...
```

3.1113 $\int (a+ia\tan(e+fx))^3(c+d\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=216

$$-\frac{8ia^3(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{8ia^3(c-id)^2\sqrt{c+d\tan(e+fx)}}{f}+\frac{8a^3(ic+d)(c+d\tan(e+fx))^{3/2}}{3f}$$

```
[Out] -8*I*a^3*(c-I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/f+8*I*
a^3*(c-I*d)^2*(c+d*tan(f*x+e))^(1/2)/f+8/3*a^3*(I*c+d)*(c+d*tan(f*x+e))^(3/
2)/f+8/5*I*a^3*(c+d*tan(f*x+e))^(5/2)/f+4/63*a^3*(I*c-10*d)*(c+d*tan(f*x+e)
)^(7/2)/d^2/f-2/9*(a^3+I*a^3*tan(f*x+e))*(c+d*tan(f*x+e))^(7/2)/d/f
```

Rubi [A]

time = 0.43, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3637, 3673, 3609, 3618, 65, 214}

$$\frac{4a^3(-10d+ic)(c+d\tan(e+fx))^{7/2}}{63d^2f}-\frac{2(a^3+ia^3\tan(e+fx))\,(c+d\tan(e+fx))^{7/2}}{9df}+\frac{8ia^3(c+d\tan(e+fx))^{5/2}}{5f}+\frac{8a^3(d+ic)(c+d\tan(e+fx))^{3/2}}{3f}+\frac{8ia^3(c-id)^2\sqrt{c+d\tan(e+fx)}}{f}-\frac{8ia^3(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-8*I)*a^3*(c - I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]]
)/f + ((8*I)*a^3*(c - I*d)^2*Sqrt[c + d*Tan[e + f*x]])/f + (8*a^3*(I*c + d)
*(c + d*Tan[e + f*x])^(3/2))/(3*f) + (((8*I)/5)*a^3*(c + d*Tan[e + f*x])^(5
/2))/f + (4*a^3*(I*c - 10*d)*(c + d*Tan[e + f*x])^(7/2))/(63*d^2*f) - (2*(a
^3 + I*a^3*Tan[e + f*x])*(c + d*Tan[e + f*x])^(7/2))/(9*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
```

```
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (a+ia\tan(e+fx))^3(c+d\tan(e+fx))^{5/2}\,dx = -\frac{2(a^3+ia^3\tan(e+fx))\,(c+d\tan(e+fx))^{7/2}}{9df} + \frac{(2a)}{}$$

$$= \frac{4a^3(ic-10d)(c+d\tan(e+fx))^{7/2}}{63d^2f} - \frac{2(a^3+ia^3\tan(e}{}$$

$$= \frac{8ia^3(c+d\tan(e+fx))^{5/2}}{5f} + \frac{4a^3(ic-10d)(c+d\tan(e}{63d^2f}$$

$$= \frac{8a^3(ic+d)(c+d\tan(e+fx))^{3/2}}{3f} + \frac{8ia^3(c+d\tan(e+}{5f}$$

$$= \frac{8ia^3(c-id)^2\sqrt{c+d\tan(e+fx)}}{f} + \frac{8a^3(ic+d)(c+d}{3f}$$

$$= \frac{8ia^3(c-id)^2\sqrt{c+d\tan(e+fx)}}{f} + \frac{8a^3(ic+d)(c+d}{3f}$$

$$= \frac{8ia^3(c-id)^2\sqrt{c+d\tan(e+fx)}}{f} + \frac{8a^3(ic+d)(c+d}{3f}$$

$$= -\frac{8ia^3(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f} + \frac{8ia}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 528 vs. $2(216) = 432$.
time = 9.92, size = 528, normalized size = 2.44

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-8*I)*(c - I*d)^(5/2)*ArcTanh[Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(
1 + E^((2*I)*(e + f*x)))]/Sqrt[c - I*d]]*Cos[e + f*x]^3*(a + I*a*Tan[e + f*
x])^3)/(E^((3*I)*e)*f*(Cos[f*x] + I*Sin[f*x])^3) + (Cos[e + f*x]^3*(Sec[e]*
Sec[e + f*x]^2*(75*c^2*Cos[e] - (405*I)*c*d*Cos[e] - 322*d^2*Cos[e] + 95*c*
d*Sin[e] - (135*I)*d^2*Sin[e])*(((-2*I)/315)*Cos[3*e] - (2*Sin[3*e])/315) +
 Sec[e]*((10*I)*c^4*Cos[e] - 135*c^3*d*Cos[e] + (2007*I)*c^2*d^2*Cos[e] + 3
345*c*d^3*Cos[e] - (1547*I)*d^4*Cos[e] - (5*I)*c^3*d*Sin[e] - 405*c^2*d^2*S
in[e] + (1019*I)*c*d^3*Sin[e] + 555*d^4*Sin[e])*((2*Cos[3*e])/(315*d^2) - (
```

```
((2*I)/315)*Sin[3*e])/d^2) + Sec[e + f*x]^4*(((-2*I)/9)*d^2*Cos[3*e] - (2*d
^2*Sin[3*e])/9) + Sec[e]*Sec[e + f*x]^3*((2*Cos[3*e])/63 - ((2*I)/63)*Sin[3
*e])*((-19*I)*c*d*Sin[f*x] - 27*d^2*Sin[f*x]) + Sec[e]*Sec[e + f*x]*((2*Cos
[3*e])/(315*d) - (((2*I)/315)*Sin[3*e])/d)*((-5*I)*c^3*Sin[f*x] - 405*c^2*d
*Sin[f*x] + (1019*I)*c*d^2*Sin[f*x] + 555*d^3*Sin[f*x]))*Sqrt[Sec[e + f*x]*
(c*Cos[e + f*x] + d*Sin[e + f*x])]*(a + I*a*Tan[e + f*x])^3)/(f*(Cos[f*x] +
 I*Sin[f*x])^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1042 vs. $2(183) = 366$.
time = 0.32, size = 1043, normalized size = 4.83 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3*(c+d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3/d^2*(-1/9*I*(c+d*tan(f*x+e))^(9/2)+1/7*I*c*(c+d*tan(f*x+e))^(7/2)+4
/5*I*d^2*(c+d*tan(f*x+e))^(5/2)-3/7*d*(c+d*tan(f*x+e))^(7/2)+4/3*I*c*d^2*(c
+d*tan(f*x+e))^(3/2)+4*I*c^2*d^2*(c+d*tan(f*x+e))^(1/2)-4*I*d^4*(c+d*tan(f*
x+e))^(1/2)+4/3*d^3*(c+d*tan(f*x+e))^(3/2)+8*c*d^3*(c+d*tan(f*x+e))^(1/2)-4
*d^2*(1/2/(2*(c^2+d^2)^(1/2)+2*c)^(1/2)/(c^2+d^2)^(1/2)*(1/2*(I*c^3*(c^2+d^
2)^(1/2)-3*I*c*d^2*(c^2+d^2)^(1/2)+I*c^4-I*d^4+3*c^2*d*(c^2+d^2)^(1/2)-d^3*
(c^2+d^2)^(1/2)+2*c^3*d+2*c*d^3)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(I*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*c^4-I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^4+2*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*c^3*d+2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^3-1/2*(I*c^3*(c^2+d^2)^(1/2)-3
*I*c*d^2*(c^2+d^2)^(1/2)+I*c^4-I*d^4+3*c^2*d*(c^2+d^2)^(1/2)-d^3*(c^2+d^2)^
(1/2)+2*c^3*d+2*c*d^3)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*
c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2
*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/2/(2*(c^2+d^2)^(1/2)+2*c)^(1/2)/(c^2+d^2)^(
1/2)*(1/2*(-I*c^3*(c^2+d^2)^(1/2)+3*I*c*d^2*(c^2+d^2)^(1/2)-I*c^4+I*d^4-3*c
^2*d*(c^2+d^2)^(1/2)+d^3*(c^2+d^2)^(1/2)-2*c^3*d-2*c*d^3)*ln(d*tan(f*x+e)+c
-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(I
*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^4-I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^4+2*(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3*d+2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^3+1/2
*(-I*c^3*(c^2+d^2)^(1/2)+3*I*c*d^2*(c^2+d^2)^(1/2)-I*c^4+I*d^4-3*c^2*d*(c^2
+d^2)^(1/2)+d^3*(c^2+d^2)^(1/2)-2*c^3*d-2*c*d^3)*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2
+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c+d*tan(f*x+e))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^3*(d*tan(f*x + e) + c)^(5/2), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1115 vs. $2(183) = 366$.
time = 2.54, size = 1115, normalized size = 5.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c+d*tan(f*x+e))^(5/2),x, algorithm="fricas"
)
```

```
[Out] -2/315*(315*(d^2*f*e^(8*I*f*x + 8*I*e) + 4*d^2*f*e^(6*I*f*x + 6*I*e) + 6*d^
2*f*e^(4*I*f*x + 4*I*e) + 4*d^2*f*e^(2*I*f*x + 2*I*e) + d^2*f)*sqrt(-(a^6*c
^5 - 5*I*a^6*c^4*d - 10*a^6*c^3*d^2 + 10*I*a^6*c^2*d^3 + 5*a^6*c*d^4 - I*a^
6*d^5)/f^2)*log(2*(a^3*c^3 - 2*I*a^3*c^2*d - a^3*c*d^2 - (I*f*e^(2*I*f*x +
2*I*e) + I*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x +
2*I*e) + 1))*sqrt(-(a^6*c^5 - 5*I*a^6*c^4*d - 10*a^6*c^3*d^2 + 10*I*a^6*c^2
*d^3 + 5*a^6*c*d^4 - I*a^6*d^5)/f^2) + (a^3*c^3 - 3*I*a^3*c^2*d - 3*a^3*c*d
^2 + I*a^3*d^3)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/(a^3*c^2 - 2*I*a^
3*c*d - a^3*d^2)) - 315*(d^2*f*e^(8*I*f*x + 8*I*e) + 4*d^2*f*e^(6*I*f*x + 6
*I*e) + 6*d^2*f*e^(4*I*f*x + 4*I*e) + 4*d^2*f*e^(2*I*f*x + 2*I*e) + d^2*f)*
sqrt(-(a^6*c^5 - 5*I*a^6*c^4*d - 10*a^6*c^3*d^2 + 10*I*a^6*c^2*d^3 + 5*a^6*
c*d^4 - I*a^6*d^5)/f^2)*log(2*(a^3*c^3 - 2*I*a^3*c^2*d - a^3*c*d^2 - (-I*f*
e^(2*I*f*x + 2*I*e) - I*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(
e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(a^6*c^5 - 5*I*a^6*c^4*d - 10*a^6*c^3*d^2 +
 10*I*a^6*c^2*d^3 + 5*a^6*c*d^4 - I*a^6*d^5)/f^2) + (a^3*c^3 - 3*I*a^3*c^2*
d - 3*a^3*c*d^2 + I*a^3*d^3)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/(a^3
*c^2 - 2*I*a^3*c*d - a^3*d^2)) + 2*(-5*I*a^3*c^4 + 65*a^3*c^3*d - 801*I*a^3
*c^2*d^2 - 1163*a^3*c*d^3 + 496*I*a^3*d^4 + (-5*I*a^3*c^4 + 70*a^3*c^3*d -
1206*I*a^3*c^2*d^2 - 2182*a^3*c*d^3 + 1051*I*a^3*d^4)*e^(8*I*f*x + 8*I*e) +
 (-20*I*a^3*c^4 + 275*a^3*c^3*d - 4269*I*a^3*c^2*d^2 - 6709*a^3*c*d^3 + 273
5*I*a^3*d^4)*e^(6*I*f*x + 6*I*e) + 3*(-10*I*a^3*c^4 + 135*a^3*c^3*d - 1907*
I*a^3*c^2*d^2 - 2805*a^3*c*d^3 + 1211*I*a^3*d^4)*e^(4*I*f*x + 4*I*e) + (-20
*I*a^3*c^4 + 265*a^3*c^3*d - 3459*I*a^3*c^2*d^2 - 5051*a^3*c*d^3 + 2165*I*a
^3*d^4)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)
/(e^(2*I*f*x + 2*I*e) + 1)))/(d^2*f*e^(8*I*f*x + 8*I*e) + 4*d^2*f*e^(6*I*f*
x + 6*I*e) + 6*d^2*f*e^(4*I*f*x + 4*I*e) + 4*d^2*f*e^(2*I*f*x + 2*I*e) + d^
2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3*(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] -I*a**3*(Integral(I*c**2*sqrt(c + d*tan(e + f*x)), x) + Integral(-3*c**2*sq
rt(c + d*tan(e + f*x))*tan(e + f*x), x) + Integral(c**2*sqrt(c + d*tan(e +
f*x))*tan(e + f*x)**3, x) + Integral(-3*d**2*sqrt(c + d*tan(e + f*x))*tan(e
 + f*x)**3, x) + Integral(d**2*sqrt(c + d*tan(e + f*x))*tan(e + f*x)**5, x)
 + Integral(-3*I*c**2*sqrt(c + d*tan(e + f*x))*tan(e + f*x)**2, x) + Integr
al(I*d**2*sqrt(c + d*tan(e + f*x))*tan(e + f*x)**2, x) + Integral(-3*I*d**2
*sqrt(c + d*tan(e + f*x))*tan(e + f*x)**4, x) + Integral(-6*c*d*sqrt(c + d*
tan(e + f*x))*tan(e + f*x)**2, x) + Integral(2*c*d*sqrt(c + d*tan(e + f*x))
*tan(e + f*x)**4, x) + Integral(2*I*c*d*sqrt(c + d*tan(e + f*x))*tan(e + f*
x), x) + Integral(-6*I*c*d*sqrt(c + d*tan(e + f*x))*tan(e + f*x)**3, x))
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(183) = 366$.
time = 1.35, size = 423, normalized size = 1.96

$$-\frac{16\left(-i a^3c^3 - 3a^3c^2d + 3i a^3cd^2 + a^3d^3\right)\arctan\left(\frac{2\left(\sqrt{d\tan(fx+e)+c}\,c - \sqrt{c^2+d^2}\sqrt{d\tan(fx+e)+c}\right)}{c\sqrt{-2c+2\sqrt{c^2+d^2}} - i\sqrt{-2c+2\sqrt{c^2+d^2}}\,d - \sqrt{c^2+d^2}\sqrt{-2c+2\sqrt{c^2+d^2}}}\right)}{\sqrt{-2c+2\sqrt{c^2+d^2}}\,f\left(-\frac{id}{c-\sqrt{c^2+d^2}}+1\right)} - \frac{2\left(35i\,(d\tan(fx+e)+c)^{\frac{9}{2}}a^3d^{16}f^8 - 45i\,(d\tan(fx+e)+c)^{\frac{7}{2}}a^3cd^{16}f^8 + 135\,(d\tan(fx+e)+c)^{\frac{7}{2}}a^3d^{17}f^8 - 252i\,(d\tan(fx+e)+c)^{\frac{5}{2}}a^3d^{18}f^8 - 420i\,(d\tan(fx+e)+c)^{\frac{3}{2}}a^3cd^{18}f^8 - 1260i\,\sqrt{d\tan(fx+e)+c}\,a^3c^2d^{18}f^8 - 420\,(d\tan(fx+e)+c)^{\frac{3}{2}}a^3d^{19}f^8 - 2520\,\sqrt{d\tan(fx+e)+c}\,a^3cd^{19}f^8 + 1260i\,\sqrt{d\tan(fx+e)+c}\,a^3d^{20}f^8\right)}{315\,d^{18}f^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c+d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] -16*(-I*a^3*c^3 - 3*a^3*c^2*d + 3*I*a^3*c*d^2 + a^3*d^3)*arctan(2*(sqrt(d*t
an(f*x + e) + c)*c - sqrt(c^2 + d^2)*sqrt(d*tan(f*x + e) + c))/(c*sqrt(-2*c
 + 2*sqrt(c^2 + d^2)) - I*sqrt(-2*c + 2*sqrt(c^2 + d^2))*d - sqrt(c^2 + d^2
)*sqrt(-2*c + 2*sqrt(c^2 + d^2))))/(sqrt(-2*c + 2*sqrt(c^2 + d^2))*f*(-I*d/
(c - sqrt(c^2 + d^2)) + 1)) - 2/315*(35*I*(d*tan(f*x + e) + c)^(9/2)*a^3*d^
16*f^8 - 45*I*(d*tan(f*x + e) + c)^(7/2)*a^3*c*d^16*f^8 + 135*(d*tan(f*x +
e) + c)^(7/2)*a^3*d^17*f^8 - 252*I*(d*tan(f*x + e) + c)^(5/2)*a^3*d^18*f^8
- 420*I*(d*tan(f*x + e) + c)^(3/2)*a^3*c*d^18*f^8 - 1260*I*sqrt(d*tan(f*x +
 e) + c)*a^3*c^2*d^18*f^8 - 420*(d*tan(f*x + e) + c)^(3/2)*a^3*d^19*f^8 - 2
520*sqrt(d*tan(f*x + e) + c)*a^3*c*d^19*f^8 + 1260*I*sqrt(d*tan(f*x + e) +
c)*a^3*d^20*f^8)/(d^18*f^9)
```

Mupad [**B**]
time = 27.63, size = 400, normalized size = 1.85

$$-\left(\frac{(c-d\,1\mathrm{i})\left(\frac{a^3(c-d\,1\mathrm{i})\,2\mathrm{i}}{d^2f}-\frac{a^3(c+d\,1\mathrm{i})\,4\mathrm{i}}{d^2f}\right)}{5}+\frac{a^3(c+d\,1\mathrm{i})^2\,2\mathrm{i}}{5\,d^2f}\right)(c+d\tan(e+f\,x))^{5/2}-\left(\frac{a^3(c-d\,1\mathrm{i})\,2\mathrm{i}}{7\,d^2f}-\frac{a^3(c+d\,1\mathrm{i})\,4\mathrm{i}}{7\,d^2f}\right)(c+d\tan(e+f\,x))^{7/2}-(c-d\,1\mathrm{i})^2\left((c-d\,1\mathrm{i})\left(\frac{a^3(c-d\,1\mathrm{i})\,2\mathrm{i}}{d^2f}-\frac{a^3(c+d\,1\mathrm{i})\,4\mathrm{i}}{d^2f}\right)+\frac{a^3(c+d\,1\mathrm{i})^2\,2\mathrm{i}}{d^2f}\right)\sqrt{c+d\tan(e+f\,x)}-\frac{(c-d\,1\mathrm{i})\left((c-d\,1\mathrm{i})\left(\frac{a^3(c-d\,1\mathrm{i})\,2\mathrm{i}}{d^2f}-\frac{a^3(c+d\,1\mathrm{i})\,4\mathrm{i}}{d^2f}\right)+\frac{a^3(c+d\,1\mathrm{i})^2\,2\mathrm{i}}{d^2f}\right)(c+d\tan(e+f\,x))^{3/2}}{3}-\frac{a^3(c+d\tan(e+f\,x))^{9/2}\,2\mathrm{i}}{9\,d^2f}-\frac{\sqrt{16\mathrm{i}}\,a^3\,\mathrm{atan}\left(\frac{\sqrt{16\mathrm{i}}\,(d+c\,1\mathrm{i})^{5/2}\sqrt{c+d\tan(e+f\,x)}\,1\mathrm{i}}{4\left(-c^3\,1\mathrm{i}-3\,c^2\,d+c\,d^2\,3\mathrm{i}+d^3\right)}\right)(d+c\,1\mathrm{i})^{5/2}\,2\mathrm{i}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^3*(c + d*tan(e + f*x))^(5/2),x)
```

```
[Out] - (((c - d*1i)*((a^3*(c - d*1i)*2i)/(d^2*f) - (a^3*(c + d*1i)*4i)/(d^2*f)))
/5 + (a^3*(c + d*1i)^2*2i)/(5*d^2*f))*(c + d*tan(e + f*x))^(5/2) - ((a^3*(c
 - d*1i)*2i)/(7*d^2*f) - (a^3*(c + d*1i)*4i)/(7*d^2*f))*(c + d*tan(e + f*x)
)^(7/2) - (c - d*1i)^2*((c - d*1i)*((a^3*(c - d*1i)*2i)/(d^2*f) - (a^3*(c +
```

```
 d*1i)*4i)/(d^2*f)) + (a^3*(c + d*1i)^2*2i)/(d^2*f))*(c + d*tan(e + f*x))^(
1/2) - ((c - d*1i)*((c - d*1i)*((a^3*(c - d*1i)*2i)/(d^2*f) - (a^3*(c + d*1
i)*4i)/(d^2*f)) + (a^3*(c + d*1i)^2*2i)/(d^2*f))*(c + d*tan(e + f*x))^(3/2)
)/3 - (a^3*(c + d*tan(e + f*x))^(9/2)*2i)/(9*d^2*f) - (16i^(1/2)*a^3*atan((
16i^(1/2)*(c*1i + d)^(5/2)*(c + d*tan(e + f*x))^(1/2)*1i)/(4*(c*d^2*3i - 3*
c^2*d - c^3*1i + d^3)))*(c*1i + d)^(5/2)*2i)/f
```

3.1114 $\int (a+ia\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=166

$$-\frac{4ia^2(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{4ia^2(c-id)^2\sqrt{c+d\tan(e+fx)}}{f}+\frac{4a^2(ic+d)(c+d\tan}{3f}$$

```
[Out] -4*I*a^2*(c-I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/f+4*I*
a^2*(c-I*d)^2*(c+d*tan(f*x+e))^(1/2)/f+4/3*a^2*(I*c+d)*(c+d*tan(f*x+e))^(3/
2)/f+4/5*I*a^2*(c+d*tan(f*x+e))^(5/2)/f-2/7*a^2*(c+d*tan(f*x+e))^(7/2)/d/f
```

Rubi [A]
time = 0.27, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3624, 3609, 3618, 65, 214}

$$-\frac{2a^2(c+d\tan(e+fx))^{7/2}}{7df}+\frac{4ia^2(c+d\tan(e+fx))^{5/2}}{5f}+\frac{4a^2(d+ic)(c+d\tan(e+fx))^{3/2}}{3f}+\frac{4ia^2(c-id)^2\sqrt{c+d\tan(e+fx)}}{f}-\frac{4ia^2(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-4*I)*a^2*(c - I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]]
)/f + ((4*I)*a^2*(c - I*d)^2*Sqrt[c + d*Tan[e + f*x]])/f + (4*a^2*(I*c + d)
*(c + d*Tan[e + f*x])^(3/2))/(3*f) + (((4*I)/5)*a^2*(c + d*Tan[e + f*x])^(5
/2))/f - (2*a^2*(c + d*Tan[e + f*x])^(7/2))/(7*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
```

```
0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int (a+ia\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}\,dx = -\frac{2a^2(c+d\tan(e+fx))^{7/2}}{7df}+\int \left(2a^2+2ia^2\tan(e+f\right.$$

$$= \frac{4ia^2(c+d\tan(e+fx))^{5/2}}{5f}-\frac{2a^2(c+d\tan(e+fx))^{7/2}}{7df}$$

$$= \frac{4a^2(ic+d)(c+d\tan(e+fx))^{3/2}}{3f}+\frac{4ia^2(c+d\tan(e+}{5f}$$

$$= \frac{4ia^2(c-id)^2\sqrt{c+d\tan(e+fx)}}{f}+\frac{4a^2(ic+d)(c+d}{3f}$$

$$= \frac{4ia^2(c-id)^2\sqrt{c+d\tan(e+fx)}}{f}+\frac{4a^2(ic+d)(c+d}{3f}$$

$$= \frac{4ia^2(c-id)^2\sqrt{c+d\tan(e+fx)}}{f}+\frac{4a^2(ic+d)(c+d}{3f}$$

$$= -\frac{4ia^2(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{4ia}{}$$

Mathematica [A]

time = 7.39, size = 271, normalized size = 1.63

$$\frac{a^2(\cos(e+fx)+i\sin(e+fx))^2\left(-4i(c-id)^{5/2}e^{-2ie}\tanh^{-1}\left(\frac{\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}}{\sqrt{c-id}}\right)-\frac{\sec^2(e+fx)(\cos(2e)-i\sin(2e))\sqrt{c+d\tan(e+fx)}\,(15c^3-322ic^2d-445cd^2+168id^3+d(45c^2-154icd-55d^2)\tan(e+fx)+\cos(2(e+fx))(15c^3-322ic^2d-535cd^2+252id^3+d(45c^2-154icd-85d^2)\tan(e+fx)))}{105d}\right)}{f(\cos(fx)+i\sin(fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (a^2*(Cos[e + f*x] + I*Sin[e + f*x])^2*(((-4*I)*(c - I*d)^(5/2)*ArcTanh[Sqr
t[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))]/Sqrt[c -
I*d]])/E^((2*I)*e) - (Sec[e + f*x]^2*(Cos[2*e] - I*Sin[2*e])*Sqrt[c + d*Tan
[e + f*x]]*(15*c^3 - (322*I)*c^2*d - 445*c*d^2 + (168*I)*d^3 + d*(45*c^2 -
(154*I)*c*d - 55*d^2)*Tan[e + f*x] + Cos[2*(e + f*x)]*(15*c^3 - (322*I)*c^2
*d - 535*c*d^2 + (252*I)*d^3 + d*(45*c^2 - (154*I)*c*d - 85*d^2)*Tan[e + f*
x])))/(105*d)))/(f*(Cos[f*x] + I*Sin[f*x])^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1002 vs. 2(139) = 278.
time = 0.28, size = 1003, normalized size = 6.04

method	result
derivativedivides	$2a^2\left(\frac{2id(c+d\tan(fx+e))^{\frac{5}{2}}}{5}-\frac{(c+d\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{2icd(c+d\tan(fx+e))^{\frac{3}{2}}}{3}+2ic^2d\sqrt{c+d\tan(fx+e)}-2id^3\sqrt{c+d}\right.$
default	$2a^2\left(\frac{2id(c+d\tan(fx+e))^{\frac{5}{2}}}{5}-\frac{(c+d\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{2icd(c+d\tan(fx+e))^{\frac{3}{2}}}{3}+2ic^2d\sqrt{c+d\tan(fx+e)}-2id^3\sqrt{c+d}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2*(c+d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^2/d*(2/5*I*d*(c+d*tan(f*x+e))^(5/2)-1/7*(c+d*tan(f*x+e))^(7/2)+2/3*I*
c*d*(c+d*tan(f*x+e))^(3/2)+2*I*c^2*d*(c+d*tan(f*x+e))^(1/2)-2*I*d^3*(c+d*ta
```

```
n(f*x+e))^(1/2)+2/3*d^2*(c+d*tan(f*x+e))^(3/2)+4*c*d^2*(c+d*tan(f*x+e))^(1/
2)-2*d*(1/2/(2*(c^2+d^2)^(1/2)+2*c)^(1/2)/(c^2+d^2)^(1/2)*(1/2*(I*c^3*(c^2+
d^2)^(1/2)-3*I*c*d^2*(c^2+d^2)^(1/2)+I*c^4-I*d^4+3*c^2*d*(c^2+d^2)^(1/2)-d^
3*(c^2+d^2)^(1/2)+2*c^3*d+2*c*d^3)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)
*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(I*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*c^4-I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^4+2*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*c^3*d+2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^3-1/2*(I*c^3*(c^2+d^2)^(1/2)
-3*I*c*d^2*(c^2+d^2)^(1/2)+I*c^4-I*d^4+3*c^2*d*(c^2+d^2)^(1/2)-d^3*(c^2+d^2
)^(1/2)+2*c^3*d+2*c*d^3)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-
2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/
(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/2/(2*(c^2+d^2)^(1/2)+2*c)^(1/2)/(c^2+d^2)
^(1/2)*(1/2*(-I*c^3*(c^2+d^2)^(1/2)+3*I*c*d^2*(c^2+d^2)^(1/2)-I*c^4+I*d^4-3
*c^2*d*(c^2+d^2)^(1/2)+d^3*(c^2+d^2)^(1/2)-2*c^3*d-2*c*d^3)*ln(d*tan(f*x+e)
+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*
(I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^4-I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^4+2*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3*d+2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^3+1
/2*(-I*c^3*(c^2+d^2)^(1/2)+3*I*c*d^2*(c^2+d^2)^(1/2)-I*c^4+I*d^4-3*c^2*d*(c
^2+d^2)^(1/2)+d^3*(c^2+d^2)^(1/2)-2*c^3*d-2*c*d^3)*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c
^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c+d*tan(f*x+e))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^2*(d*tan(f*x + e) + c)^(5/2), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 941 vs. $2(137) = 274$.
time = 1.02, size = 941, normalized size = 5.67

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c+d*tan(f*x+e))^(5/2),x, algorithm="fricas"
)
```

```
[Out] -1/105*(105*(d*f*e^(6*I*f*x + 6*I*e) + 3*d*f*e^(4*I*f*x + 4*I*e) + 3*d*f*e^
(2*I*f*x + 2*I*e) + d*f)*sqrt(-(a^4*c^5 - 5*I*a^4*c^4*d - 10*a^4*c^3*d^2 +
10*I*a^4*c^2*d^3 + 5*a^4*c*d^4 - I*a^4*d^5)/f^2)*log(2*(a^2*c^3 - 2*I*a^2*c
^2*d - a^2*c*d^2 - (I*f*e^(2*I*f*x + 2*I*e) + I*f)*sqrt(((c - I*d)*e^(2*I*f
```

```
*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(a^4*c^5 - 5*I*a^4*
c^4*d - 10*a^4*c^3*d^2 + 10*I*a^4*c^2*d^3 + 5*a^4*c*d^4 - I*a^4*d^5)/f^2) +
 (a^2*c^3 - 3*I*a^2*c^2*d - 3*a^2*c*d^2 + I*a^2*d^3)*e^(2*I*f*x + 2*I*e))*e
^(-2*I*f*x - 2*I*e)/(a^2*c^2 - 2*I*a^2*c*d - a^2*d^2)) - 105*(d*f*e^(6*I*f*
x + 6*I*e) + 3*d*f*e^(4*I*f*x + 4*I*e) + 3*d*f*e^(2*I*f*x + 2*I*e) + d*f)*s
qrt(-(a^4*c^5 - 5*I*a^4*c^4*d - 10*a^4*c^3*d^2 + 10*I*a^4*c^2*d^3 + 5*a^4*c
*d^4 - I*a^4*d^5)/f^2)*log(2*(a^2*c^3 - 2*I*a^2*c^2*d - a^2*c*d^2 - (-I*f*e
^(2*I*f*x + 2*I*e) - I*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e
^(2*I*f*x + 2*I*e) + 1))*sqrt(-(a^4*c^5 - 5*I*a^4*c^4*d - 10*a^4*c^3*d^2 +
10*I*a^4*c^2*d^3 + 5*a^4*c*d^4 - I*a^4*d^5)/f^2) + (a^2*c^3 - 3*I*a^2*c^2*d
 - 3*a^2*c*d^2 + I*a^2*d^3)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/(a^2*
c^2 - 2*I*a^2*c*d - a^2*d^2)) + 2*(15*a^2*c^3 - 277*I*a^2*c^2*d - 381*a^2*c
*d^2 + 167*I*a^2*d^3 + (15*a^2*c^3 - 367*I*a^2*c^2*d - 689*a^2*c*d^2 + 337*
I*a^2*d^3)*e^(6*I*f*x + 6*I*e) + (45*a^2*c^3 - 1011*I*a^2*c^2*d - 1579*a^2*
c*d^2 + 613*I*a^2*d^3)*e^(4*I*f*x + 4*I*e) + (45*a^2*c^3 - 921*I*a^2*c^2*d
- 1271*a^2*c*d^2 + 563*I*a^2*d^3)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(2
*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))/(d*f*e^(6*I*f*x + 6*
I*e) + 3*d*f*e^(4*I*f*x + 4*I*e) + 3*d*f*e^(2*I*f*x + 2*I*e) + d*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^{2}\left(\int \left(-c^{2}\sqrt{c+d\tan \left(e+fx\right)}\right) dx+\int c^{2}\sqrt{c+d\tan \left(e+fx\right)}\tan^{2}\left(e+fx\right) dx+\int \left(-d^{2}\sqrt{c+d\tan \left(e+fx\right)}\tan^{2}\left(e+fx\right)\right) dx+\int d^{2}\sqrt{c+d\tan \left(e+fx\right)}\tan^{4}\left(e+fx\right) dx+\int \left(-2ic^{2}\sqrt{c+d\tan \left(e+fx\right)}\tan \left(e+fx\right)\right) dx+\int \left(-2id^{2}\sqrt{c+d\tan \left(e+fx\right)}\tan^{3}\left(e+fx\right)\right) dx+\int \left(-2cd\sqrt{c+d\tan \left(e+fx\right)}\tan \left(e+fx\right)\right) dx+\int 2cd\sqrt{c+d\tan \left(e+fx\right)}\tan^{3}\left(e+fx\right) dx+\int \left(-4icd\sqrt{c+d\tan \left(e+fx\right)}\tan^{2}\left(e+fx\right)\right) dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2*(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] -a**2*(Integral(-c**2*sqrt(c + d*tan(e + f*x)), x) + Integral(c**2*sqrt(c +
 d*tan(e + f*x))*tan(e + f*x)**2, x) + Integral(-d**2*sqrt(c + d*tan(e + f*
x))*tan(e + f*x)**2, x) + Integral(d**2*sqrt(c + d*tan(e + f*x))*tan(e + f*
x)**4, x) + Integral(-2*I*c**2*sqrt(c + d*tan(e + f*x))*tan(e + f*x), x) +
Integral(-2*I*d**2*sqrt(c + d*tan(e + f*x))*tan(e + f*x)**3, x) + Integral(
-2*c*d*sqrt(c + d*tan(e + f*x))*tan(e + f*x), x) + Integral(2*c*d*sqrt(c +
d*tan(e + f*x))*tan(e + f*x)**3, x) + Integral(-4*I*c*d*sqrt(c + d*tan(e +
f*x))*tan(e + f*x)**2, x))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(137) = 274$.
time = 1.12, size = 374, normalized size = 2.25

$$-\frac{8\left(-i a^{2} c^{3}-3 a^{2} c^{2} d+3 i a^{2} c d^{2}+a^{2} d^{3}\right) \arctan \left(\frac{2\left(\sqrt{d \tan \left(fx+e\right)+c}\, c-\sqrt{c^{2}+d^{2}}\sqrt{d \tan \left(fx+e\right)+c}\right)}{c\sqrt{-2c+2\sqrt{c^{2}+d^{2}}}-i\sqrt{-2c+2\sqrt{c^{2}+d^{2}}}\, d-\sqrt{c^{2}+d^{2}}\sqrt{-2c+2\sqrt{c^{2}+d^{2}}}}\right)}{\sqrt{-2c+2\sqrt{c^{2}+d^{2}}}\, f\left(-\frac{id}{c-\sqrt{c^{2}+d^{2}}}+1\right)}-\frac{2\left(15\left(d\tan \left(fx+e\right)+c\right)^{\frac{7}{2}}a^{2}d^{6}f^{6}-42i\left(d\tan \left(fx+e\right)+c\right)^{\frac{5}{2}}a^{2}d^{7}f^{6}-70i\left(d\tan \left(fx+e\right)+c\right)^{\frac{3}{2}}a^{2}cd^{7}f^{6}-210i\sqrt{d\tan \left(fx+e\right)+c}\, a^{2}c^{2}d^{7}f^{6}-70\left(d\tan \left(fx+e\right)+c\right)^{\frac{3}{2}}a^{2}d^{8}f^{6}-420\sqrt{d\tan \left(fx+e\right)+c}\, a^{2}cd^{8}f^{6}+210i\sqrt{d\tan \left(fx+e\right)+c}\, a^{2}d^{9}f^{6}\right)}{105\, d^{7}f^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c+d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] -8*(-I*a^2*c^3 - 3*a^2*c^2*d + 3*I*a^2*c*d^2 + a^2*d^3)*arctan(2*(sqrt(d*ta
n(f*x + e) + c)*c - sqrt(c^2 + d^2)*sqrt(d*tan(f*x + e) + c))/(c*sqrt(-2*c
+ 2*sqrt(c^2 + d^2)) - I*sqrt(-2*c + 2*sqrt(c^2 + d^2))*d - sqrt(c^2 + d^2)
*sqrt(-2*c + 2*sqrt(c^2 + d^2))))/(sqrt(-2*c + 2*sqrt(c^2 + d^2))*f*(-I*d/(
c - sqrt(c^2 + d^2)) + 1)) - 2/105*(15*(d*tan(f*x + e) + c)^(7/2)*a^2*d^6*f
^6 - 42*I*(d*tan(f*x + e) + c)^(5/2)*a^2*d^7*f^6 - 70*I*(d*tan(f*x + e) + c
)^(3/2)*a^2*c*d^7*f^6 - 210*I*sqrt(d*tan(f*x + e) + c)*a^2*c^2*d^7*f^6 - 70
*(d*tan(f*x + e) + c)^(3/2)*a^2*d^8*f^6 - 420*sqrt(d*tan(f*x + e) + c)*a^2*
c*d^8*f^6 + 210*I*sqrt(d*tan(f*x + e) + c)*a^2*d^9*f^6)/(d^7*f^7)
```

Mupad [B]
time = 19.22, size = 257, normalized size = 1.55

$$-\left(\frac{2\,a^2\,\left(c-d\,1{}\mathrm{i}\right)}{5\,d\,f}-\frac{2\,a^2\,\left(c+d\,1{}\mathrm{i}\right)}{5\,d\,f}\right)\,{\left(c+d\,\tan\left(e+f\,x\right)\right)}^{5/2}-{\left(c-d\,1{}\mathrm{i}\right)}^2\,\left(\frac{2\,a^2\,\left(c-d\,1{}\mathrm{i}\right)}{d\,f}-\frac{2\,a^2\,\left(c+d\,1{}\mathrm{i}\right)}{d\,f}\right)\,\sqrt{c+d\,\tan\left(e+f\,x\right)}-\frac{\left(c-d\,1{}\mathrm{i}\right)\,\left(\frac{2\,a^2\,\left(c-d\,1{}\mathrm{i}\right)}{d\,f}-\frac{2\,a^2\,\left(c+d\,1{}\mathrm{i}\right)}{d\,f}\right)\,{\left(c+d\,\tan\left(e+f\,x\right)\right)}^{3/2}}{3}-\frac{2\,a^2\,{\left(c+d\,\tan\left(e+f\,x\right)\right)}^{7/2}}{7\,d\,f}-\frac{\sqrt{4{}\mathrm{i}}\,a^2\,\mathrm{atan}\left(\frac{\sqrt{4{}\mathrm{i}}\,{\left(d+c\,1{}\mathrm{i}\right)}^{5/2}\,\sqrt{c+d\,\tan\left(e+f\,x\right)}\,1{}\mathrm{i}}{2\,\left(-c^3\,1{}\mathrm{i}-3\,c^2\,d+c\,d^2\,3{}\mathrm{i}+d^3\right)}\right)\,{\left(d+c\,1{}\mathrm{i}\right)}^{5/2}\,2{}\mathrm{i}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^2*(c + d*tan(e + f*x))^(5/2),x)
```

```
[Out] - ((2*a^2*(c - d*1i))/(5*d*f) - (2*a^2*(c + d*1i))/(5*d*f))*(c + d*tan(e +
f*x))^(5/2) - (c - d*1i)^2*((2*a^2*(c - d*1i))/(d*f) - (2*a^2*(c + d*1i))/(
d*f))*(c + d*tan(e + f*x))^(1/2) - ((c - d*1i)*((2*a^2*(c - d*1i))/(d*f) -
(2*a^2*(c + d*1i))/(d*f))*(c + d*tan(e + f*x))^(3/2))/3 - (2*a^2*(c + d*tan
(e + f*x))^(7/2))/(7*d*f) - (4i^(1/2)*a^2*atan((4i^(1/2)*(c*1i + d)^(5/2)*(
c + d*tan(e + f*x))^(1/2)*1i)/(2*(c*d^2*3i - 3*c^2*d - c^3*1i + d^3))*(c*1
i + d)^(5/2)*2i)/f
```

3.1115 $\int (a+ia\tan(e+fx))(c+d\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=131

$$-\frac{2ia(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{2ia(c-id)^2\sqrt{c+d\tan(e+fx)}}{f}+\frac{2a(ic+d)(c+d\tan(e}{3f}$$

[Out] -2*I*a*(c-I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/f+2*I*a*(c-I*d)^2*(c+d*tan(f*x+e))^(1/2)/f+2/3*a*(I*c+d)*(c+d*tan(f*x+e))^(3/2)/f+2/5*I*a*(c+d*tan(f*x+e))^(5/2)/f

Rubi [A]
time = 0.20, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3609, 3618, 65, 214}

$$\frac{2ia(c-id)^2\sqrt{c+d\tan(e+fx)}}{f}+\frac{2ia(c+d\tan(e+fx))^{5/2}}{5f}+\frac{2a(d+ic)(c+d\tan(e+fx))^{3/2}}{3f}-\frac{2ia(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])*(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-2*I)*a*(c - I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/
f + ((2*I)*a*(c - I*d)^2*Sqrt[c + d*Tan[e + f*x]])/f + (2*a*(I*c + d)*(c +
d*Tan[e + f*x])^(3/2))/(3*f) + (((2*I)/5)*a*(c + d*Tan[e + f*x])^(5/2))/f
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
```

```
0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rubi steps

$$\int (a+ia\tan(e+fx))(c+d\tan(e+fx))^{5/2}\,dx = \frac{2ia(c+d\tan(e+fx))^{5/2}}{5f} + \int (c+d\tan(e+fx))^{3/2}(a(\ldots$$

$$= \frac{2a(ic+d)(c+d\tan(e+fx))^{3/2}}{3f} + \frac{2ia(c+d\tan(e+fx\ldots}{5f}$$

$$= \frac{2ia(c-id)^2\sqrt{c+d\tan(e+fx)}}{f} + \frac{2a(ic+d)(c+d\tan\ldots}{3f}$$

$$= \frac{2ia(c-id)^2\sqrt{c+d\tan(e+fx)}}{f} + \frac{2a(ic+d)(c+d\tan\ldots}{3f}$$

$$= \frac{2ia(c-id)^2\sqrt{c+d\tan(e+fx)}}{f} + \frac{2a(ic+d)(c+d\tan\ldots}{3f}$$

$$= -\frac{2ia(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f} + \frac{2ia(\ldots}{}$$

Mathematica [A]
time = 3.29, size = 208, normalized size = 1.59

$$\frac{\cos(e+fx)(\cos(fx)-i\sin(fx))(a+ia\tan(e+fx))\left(-2i(c-id)^{5/2}e^{-ie}\tanh^{-1}\left(\frac{\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}}{\sqrt{c-id}}\right)+\frac{1}{15}\sec^2(e+fx)(i\cos(e)+\sin(e))\left(23c^2-35icd-12d^2+\left(23c^2-35icd-18d^2\right)\cos(2(e+fx))+(11c-5id)d\sin(2(e+fx))\right)\sqrt{c+d\tan(e+fx)}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])*(c + d*Tan[e + f*x])^(5/2),x]
[Out] (Cos[e + f*x]*(Cos[f*x] - I*Sin[f*x])*(a + I*a*Tan[e + f*x])*(((-2*I)*(c -
I*d)^(5/2)*ArcTanh[Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*
```

```
(e + f*x)))]/Sqrt[c - I*d]])/E^(I*e) + (Sec[e + f*x]^2*(I*Cos[e] + Sin[e])*
(23*c^2 - (35*I)*c*d - 12*d^2 + (23*c^2 - (35*I)*c*d - 18*d^2)*Cos[2*(e + f
*x)] + (11*c - (5*I)*d)*d*Sin[2*(e + f*x)])*Sqrt[c + d*Tan[e + f*x]])/15))/
f
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 969 vs. $2(108) = 216$.
time = 0.26, size = 970, normalized size = 7.40

method	result
derivativedivides	$a\left(\frac{2i(c+d\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{2ic(c+d\tan(fx+e))^{\frac{3}{2}}}{3}+2ic^2\sqrt{c+d\tan(fx+e)}-2id^2\sqrt{c+d\tan(fx+e)}+\frac{2d(c\right.$
default	$a\left(\frac{2i(c+d\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{2ic(c+d\tan(fx+e))^{\frac{3}{2}}}{3}+2ic^2\sqrt{c+d\tan(fx+e)}-2id^2\sqrt{c+d\tan(fx+e)}+\frac{2d(c\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))*(c+d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a*(2/5*I*(c+d*tan(f*x+e))^(5/2)+2/3*I*c*(c+d*tan(f*x+e))^(3/2)+2*I*c^2*
(c+d*tan(f*x+e))^(1/2)-2*I*d^2*(c+d*tan(f*x+e))^(1/2)+2/3*d*(c+d*tan(f*x+e)
)^(3/2)+4*c*d*(c+d*tan(f*x+e))^(1/2)+1/(2*(c^2+d^2)^(1/2)+2*c)^(1/2)/(c^2+d
^2)^(1/2)*(-1/2*(-I*c^3*(c^2+d^2)^(1/2)+3*I*c*d^2*(c^2+d^2)^(1/2)-I*c^4+I*d
^4-3*c^2*d*(c^2+d^2)^(1/2)+d^3*(c^2+d^2)^(1/2)-2*c^3*d-2*c*d^3)*ln((c+d*tan
(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2)
)+2*(I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^4-I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^
4+2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3*d+2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d
^3+1/2*(-I*c^3*(c^2+d^2)^(1/2)+3*I*c*d^2*(c^2+d^2)^(1/2)-I*c^4+I*d^4-3*c^2*
d*(c^2+d^2)^(1/2)+d^3*(c^2+d^2)^(1/2)-2*c^3*d-2*c*d^3)*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1
/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)/(c^2+d^2)^(1/2)*(1/2*(-I*c^3*(c^2+d^2)^(1/2)+3*I*c*d^2*(
```

```
c^2+d^2)^(1/2)-I*c^4+I*d^4-3*c^2*d*(c^2+d^2)^(1/2)+d^3*(c^2+d^2)^(1/2)-2*c^
3*d-2*c*d^3)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)+(c^2+d^2)^(1/2))+2*(-I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^4+I*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*d^4-2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3*d-2*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*c*d^3-1/2*(-I*c^3*(c^2+d^2)^(1/2)+3*I*c*d^2*(c^2+d^2)
^(1/2)-I*c^4+I*d^4-3*c^2*d*(c^2+d^2)^(1/2)+d^3*(c^2+d^2)^(1/2)-2*c^3*d-2*c*
d^3)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2
*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2
*c)^(1/2))))
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c+d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 752 vs. 2(105) = 210.
time = 1.10, size = 752, normalized size = 5.74

$$-\frac{15\left(fe^{(4i fx+4i e)}+2 fe^{(2i fx+2i e)}+f\right)\sqrt{-\frac{a^2c^5-5i a^2c^4d-10a^2c^3d^2+10i a^2c^2d^3+5a^2cd^4-i a^2d^5}{f^2}}\log\left(\frac{2\left(ac^3-2i ac^2d-acd^2-\left(i fe^{(2i fx+2i e)}+i f\right)\sqrt{\frac{(c-i d)e^{(2i fx+2i e)}+c+i d}{e^{(2i fx+2i e)}+1}}\sqrt{-\frac{a^2c^5-5i a^2c^4d-10a^2c^3d^2+10i a^2c^2d^3+5a^2cd^4-i a^2d^5}{f^2}}+\left(ac^3-3i ac^2d-3acd^2+i ad^3\right)e^{(2i fx+2i e)}\right)e^{(-2i fx-2i e)}}{ac^2-2i acd-ad^2}\right)-15\left(fe^{(4i fx+4i e)}+2 fe^{(2i fx+2i e)}+f\right)\sqrt{-\frac{a^2c^5-5i a^2c^4d-10a^2c^3d^2+10i a^2c^2d^3+5a^2cd^4-i a^2d^5}{f^2}}\log\left(\frac{2\left(ac^3-2i ac^2d-acd^2-\left(-i fe^{(2i fx+2i e)}-i f\right)\sqrt{\frac{(c-i d)e^{(2i fx+2i e)}+c+i d}{e^{(2i fx+2i e)}+1}}\sqrt{-\frac{a^2c^5-5i a^2c^4d-10a^2c^3d^2+10i a^2c^2d^3+5a^2cd^4-i a^2d^5}{f^2}}+\left(ac^3-3i ac^2d-3acd^2+i ad^3\right)e^{(2i fx+2i e)}\right)e^{(-2i fx-2i e)}}{ac^2-2i acd-ad^2}\right)+4\left(-23i ac^2-24acd+13i ad^2+23\left(-i ac^2-2acd+i ad^2\right)e^{(4i fx+4i e)}+2\left(-23i ac^2-35acd+12i ad^2\right)e^{(2i fx+2i e)}\right)\sqrt{\frac{(c-i d)e^{(2i fx+2i e)}+c+i d}{e^{(2i fx+2i e)}+1}}}{30\left(fe^{(4i fx+4i e)}+2 fe^{(2i fx+2i e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c+d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/30*(15*(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*e) + f)*sqrt(-(a^2*
c^5 - 5*I*a^2*c^4*d - 10*a^2*c^3*d^2 + 10*I*a^2*c^2*d^3 + 5*a^2*c*d^4 - I*a
^2*d^5)/f^2)*log(2*(a*c^3 - 2*I*a*c^2*d - a*c*d^2 - (I*f*e^(2*I*f*x + 2*I*e
) + I*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e
) + 1))*sqrt(-(a^2*c^5 - 5*I*a^2*c^4*d - 10*a^2*c^3*d^2 + 10*I*a^2*c^2*d^3
+ 5*a^2*c*d^4 - I*a^2*d^5)/f^2) + (a*c^3 - 3*I*a*c^2*d - 3*a*c*d^2 + I*a*d^
3)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/(a*c^2 - 2*I*a*c*d - a*d^2)) -
 15*(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*e) + f)*sqrt(-(a^2*c^5 -
5*I*a^2*c^4*d - 10*a^2*c^3*d^2 + 10*I*a^2*c^2*d^3 + 5*a^2*c*d^4 - I*a^2*d^5
)/f^2)*log(2*(a*c^3 - 2*I*a*c^2*d - a*c*d^2 - (-I*f*e^(2*I*f*x + 2*I*e) - I
*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1
))*sqrt(-(a^2*c^5 - 5*I*a^2*c^4*d - 10*a^2*c^3*d^2 + 10*I*a^2*c^2*d^3 + 5*a
^2*c*d^4 - I*a^2*d^5)/f^2) + (a*c^3 - 3*I*a*c^2*d - 3*a*c*d^2 + I*a*d^3)*e^
(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/(a*c^2 - 2*I*a*c*d - a*d^2)) + 4*(-
23*I*a*c^2 - 24*a*c*d + 13*I*a*d^2 + 23*(-I*a*c^2 - 2*a*c*d + I*a*d^2)*e^(4
*I*f*x + 4*I*e) + 2*(-23*I*a*c^2 - 35*a*c*d + 12*I*a*d^2)*e^(2*I*f*x + 2*I*
```

```
e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1
)))/(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-ic^2\sqrt{c+d\tan(e+fx)}\right)dx+\int c^2\sqrt{c+d\tan(e+fx)}\tan(e+fx)\,dx+\int d^2\sqrt{c+d\tan(e+fx)}\tan^3(e+fx)\,dx+\int\left(-id^2\sqrt{c+d\tan(e+fx)}\tan^2(e+fx)\right)dx+\int 2cd\sqrt{c+d\tan(e+fx)}\tan^2(e+fx)\,dx+\int\left(-2icd\sqrt{c+d\tan(e+fx)}\tan(e+fx)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] I*a*(Integral(-I*c**2*sqrt(c + d*tan(e + f*x)), x) + Integral(c**2*sqrt(c +
 d*tan(e + f*x))*tan(e + f*x), x) + Integral(d**2*sqrt(c + d*tan(e + f*x))*
tan(e + f*x)**3, x) + Integral(-I*d**2*sqrt(c + d*tan(e + f*x))*tan(e + f*x
)**2, x) + Integral(2*c*d*sqrt(c + d*tan(e + f*x))*tan(e + f*x)**2, x) + In
tegral(-2*I*c*d*sqrt(c + d*tan(e + f*x))*tan(e + f*x), x))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 315 vs. 2(105) = 210.
time = 0.68, size = 315, normalized size = 2.40

$$\frac{4\left(i\,ac^3+3\,ac^2d-3i\,acd^2-ad^3\right)\arctan\left(\frac{2\left(\sqrt{d\tan(fx+e)+c}\,c-\sqrt{c^2+d^2}\sqrt{d\tan(fx+e)+c}\right)}{c\sqrt{-2\,c+2\,\sqrt{c^2+d^2}}-i\sqrt{-2\,c+2\,\sqrt{c^2+d^2}}\,d-\sqrt{c^2+d^2}\sqrt{-2\,c+2\,\sqrt{c^2+d^2}}}\right)}{\sqrt{-2\,c+2\,\sqrt{c^2+d^2}}\,f\left(-\frac{i\,d}{c-\sqrt{c^2+d^2}}+1\right)}-\frac{2\left(-3i\,(d\tan(fx+e)+c)^{\frac{5}{2}}af^4-5i\,(d\tan(fx+e)+c)^{\frac{3}{2}}acf^4-15i\,\sqrt{d\tan(fx+e)+c}\,ac^2f^4-5\,(d\tan(fx+e)+c)^{\frac{3}{2}}adf^4-30\,\sqrt{d\tan(fx+e)+c}\,acdf^4+15i\,\sqrt{d\tan(fx+e)+c}\,ad^2f^4\right)}{15\,f^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c+d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 4*(I*a*c^3 + 3*a*c^2*d - 3*I*a*c*d^2 - a*d^3)*arctan(2*(sqrt(d*tan(f*x + e)
 + c)*c - sqrt(c^2 + d^2)*sqrt(d*tan(f*x + e) + c))/(c*sqrt(-2*c + 2*sqrt(c
^2 + d^2)) - I*sqrt(-2*c + 2*sqrt(c^2 + d^2))*d - sqrt(c^2 + d^2)*sqrt(-2*c
 + 2*sqrt(c^2 + d^2))))/(sqrt(-2*c + 2*sqrt(c^2 + d^2))*f*(-I*d/(c - sqrt(c
^2 + d^2)) + 1)) - 2/15*(-3*I*(d*tan(f*x + e) + c)^(5/2)*a*f^4 - 5*I*(d*tan
(f*x + e) + c)^(3/2)*a*c*f^4 - 15*I*sqrt(d*tan(f*x + e) + c)*a*c^2*f^4 - 5*
(d*tan(f*x + e) + c)^(3/2)*a*d*f^4 - 30*sqrt(d*tan(f*x + e) + c)*a*c*d*f^4
+ 15*I*sqrt(d*tan(f*x + e) + c)*a*d^2*f^4)/f^5
```

Mupad [B]
time = 28.13, size = 2500, normalized size = 19.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)*(c + d*tan(e + f*x))^(5/2),x)
```

```
[Out] log(((((-a^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) - a^2*c^5*f^2 - 5*
a^2*c*d^4*f^2 + 10*a^2*c^3*d^2*f^2)/f^4)^(1/2)*(((((-a^4*d^2*f^4*(5*c^4 + d
^4 - 10*c^2*d^2)^2)^(1/2) - a^2*c^5*f^2 - 5*a^2*c*d^4*f^2 + 10*a^2*c^3*d^2*
```

```
f^2)/f^4)^(1/2)*(64*a*c^3*d^3 + 64*a*c*d^5 - 32*c*d^2*f*(((-a^4*d^2*f^4*(5*
c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) - a^2*c^5*f^2 - 5*a^2*c*d^4*f^2 + 10*a^2*c
^3*d^2*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2)))/(2*f) - (16*a^2*d^2*(c
+ d*tan(e + f*x))^(1/2)*(c^6 - d^6 + 15*c^2*d^4 - 15*c^4*d^2))/f^2))/2 - (8
*a^3*d^3*(3*c^2 - d^2)*(c^2 + d^2)^3)/f^3)*((20*a^4*c^2*d^8*f^4 - a^4*d^10*
f^4 - 110*a^4*c^4*d^6*f^4 + 100*a^4*c^6*d^4*f^4 - 25*a^4*c^8*d^2*f^4)^(1/2)
/(4*f^4) - (a^2*c^5)/(4*f^2) - (5*a^2*c*d^4)/(4*f^2) + (5*a^2*c^3*d^2)/(2*f
^2))^(1/2) - log(((((-a^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) - a^2
*c^5*f^2 - 5*a^2*c*d^4*f^2 + 10*a^2*c^3*d^2*f^2)/f^4)^(1/2)*(((((-a^4*d^2*f
^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) - a^2*c^5*f^2 - 5*a^2*c*d^4*f^2 + 10
*a^2*c^3*d^2*f^2)/f^4)^(1/2)*(64*a*c^3*d^3 + 64*a*c*d^5 + 32*c*d^2*f*(((-a^
4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) - a^2*c^5*f^2 - 5*a^2*c*d^4*f
^2 + 10*a^2*c^3*d^2*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2)))/(2*f) + (1
6*a^2*d^2*(c + d*tan(e + f*x))^(1/2)*(c^6 - d^6 + 15*c^2*d^4 - 15*c^4*d^2))
/f^2))/2 - (8*a^3*d^3*(3*c^2 - d^2)*(c^2 + d^2)^3)/f^3)*(((20*a^4*c^2*d^8*f
^4 - a^4*d^10*f^4 - 110*a^4*c^4*d^6*f^4 + 100*a^4*c^6*d^4*f^4 - 25*a^4*c^8*
d^2*f^4)^(1/2) - a^2*c^5*f^2 - 5*a^2*c*d^4*f^2 + 10*a^2*c^3*d^2*f^2)/(4*f^4
))^(1/2) - log(((-((-a^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) + a^2*
c^5*f^2 + 5*a^2*c*d^4*f^2 - 10*a^2*c^3*d^2*f^2)/f^4)^(1/2)*(((-((-a^4*d^2*f
^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) + a^2*c^5*f^2 + 5*a^2*c*d^4*f^2 - 10
*a^2*c^3*d^2*f^2)/f^4)^(1/2)*(64*a*c^3*d^3 + 64*a*c*d^5 + 32*c*d^2*f*(-((-a
^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) + a^2*c^5*f^2 + 5*a^2*c*d^4*
f^2 - 10*a^2*c^3*d^2*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2)))/(2*f) + (
16*a^2*d^2*(c + d*tan(e + f*x))^(1/2)*(c^6 - d^6 + 15*c^2*d^4 - 15*c^4*d^2)
)/f^2))/2 - (8*a^3*d^3*(3*c^2 - d^2)*(c^2 + d^2)^3)/f^3)*(-((20*a^4*c^2*d^8
*f^4 - a^4*d^10*f^4 - 110*a^4*c^4*d^6*f^4 + 100*a^4*c^6*d^4*f^4 - 25*a^4*c^
8*d^2*f^4)^(1/2) + a^2*c^5*f^2 + 5*a^2*c*d^4*f^2 - 10*a^2*c^3*d^2*f^2)/(4*f
^4))^(1/2) + log(((-((-a^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) + a^
2*c^5*f^2 + 5*a^2*c*d^4*f^2 - 10*a^2*c^3*d^2*f^2)/f^4)^(1/2)*(((-((-a^4*d^2
*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) + a^2*c^5*f^2 + 5*a^2*c*d^4*f^2 -
10*a^2*c^3*d^2*f^2)/f^4)^(1/2)*(64*a*c^3*d^3 + 64*a*c*d^5 - 32*c*d^2*f*(-((
-a^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) + a^2*c^5*f^2 + 5*a^2*c*d^
4*f^2 - 10*a^2*c^3*d^2*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2)))/(2*f) -
 (16*a^2*d^2*(c + d*tan(e + f*x))^(1/2)*(c^6 - d^6 + 15*c^2*d^4 - 15*c^4*d^
2))/f^2))/2 - (8*a^3*d^3*(3*c^2 - d^2)*(c^2 + d^2)^3)/f^3)*((5*a^2*c^3*d^2)
/(2*f^2) - (a^2*c^5)/(4*f^2) - (5*a^2*c*d^4)/(4*f^2) - (20*a^4*c^2*d^8*f^4
- a^4*d^10*f^4 - 110*a^4*c^4*d^6*f^4 + 100*a^4*c^6*d^4*f^4 - 25*a^4*c^8*d^2
*f^4)^(1/2)/(4*f^4))^(1/2) - log(((((-((-a^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*
d^2)^2)^(1/2) + a^2*c^5*f^2 + 5*a^2*c*d^4*f^2 - 10*a^2*c^3*d^2*f^2)/f^4)^(1
/2)*(a*c^4*d^2*32i - a*d^6*32i + 32*c*d^2*f*(-((-a^4*d^2*f^4*(5*c^4 + d^4 -
 10*c^2*d^2)^2)^(1/2) + a^2*c^5*f^2 + 5*a^2*c*d^4*f^2 - 10*a^2*c^3*d^2*f^2)
/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2)))/(2*f) + (16*a^2*d^2*(c + d*tan(e +
 f*x))^(1/2)*(c^6 - d^6 + 15*c^2*d^4 - 15*c^4*d^2))/f^2)*(-((-a^4*d^2*f^4*(
5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) + a^2*c^5*f^2 + 5*a^2*c*d^4*f^2 - 10*a^2
*c^3*d^2*f^2)/f^4)^(1/2))/2 - (8*a^3*c*d^2*(c^2 - 3*d^2)*(c^2*1i + d^2*1i)^
```

```
3)/f^3)*(-((20*a^4*c^2*d^8*f^4 - a^4*d^10*f^4 - 110*a^4*c^4*d^6*f^4 + 100*a
^4*c^6*d^4*f^4 - 25*a^4*c^8*d^2*f^4)^(1/2) + a^2*c^5*f^2 + 5*a^2*c*d^4*f^2
- 10*a^2*c^3*d^2*f^2)/(4*f^4))^(1/2) - log(((((((-a^4*d^2*f^4*(5*c^4 + d^4
- 10*c^2*d^2)^2)^(1/2) - a^2*c^5*f^2 - 5*a^2*c*d^4*f^2 + 10*a^2*c^3*d^2*f^2
)/f^4)^(1/2)*(a*c^4*d^2*32i - a*d^6*32i + 32*c*d^2*f*(((-a^4*d^2*f^4*(5*c^4
 + d^4 - 10*c^2*d^2)^2)^(1/2) - a^2*c^5*f^2 - 5*a^2*c*d^4*f^2 + 10*a^2*c^3*
d^2*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2)))/(2*f) + (16*a^2*d^2*(c + d
*tan(e + f*x))^(1/2)*(c^6 - d^6 + 15*c^2*d^4 - 15*c^4*d^2))/f^2)*(((-a^4*d^
2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) - a^2*c^5*f^2 - 5*a^2*c*d^4*f^2 +
 10*a^2*c^3*d^2*f^2)/f^4)^(1/2))/2 - (8*a^3*c*d^2*(c^2 - 3*d^2)*(c^2*1i + d
^2*1i)^3)/f^3)*(((20*a^4*c^2*d^8*f^4 - a^4*d^10*f^4 - 110*a^4*c^4*d^6*f^4 +
 100*a^4*c^6*d^4*f^4 - 25*a^4*c^8*d^2*f^4)^(1/2) - a^2*c^5*f^2 - 5*a^2*c*d^
4*f^2 + 10*a^2*c^3*d^2*f^2)/(4*f^4))^(1/2) + log(- ((((((-a^4*d^2*f^4*(5*c^
4 + d^4 - 10*c^2*d^2)^2)^(1/2) - a^2*c^5*f^2 - 5*a^2*c*d^4*f^2 + 10*a^2*c^3
*d^2*f^2)/f^4)^(1/2)*(a*d^6*32i - a*c^4*d^2*32i + 32*c*d^2*f*(((-a^4*d^2*f^
4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) - a^2*c^5*f^2 - 5*a^2*c*d^4*f^2 + 10*
a^2*c^3*d^2*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2)))/(2*f) + (16*a^2*d^
2*(c + d*tan(e + f*x))^(1/2)*(c^6 - d^6 + 15*c^2*d^4 - 15*c^4*d^2))/f^2)*((
(-a^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/...
```

3.1116 $\int \frac{(c+d\tan(e+fx))^{5/2}}{a+ia\tan(e+fx)}\,dx$

Optimal. Leaf size=185

$$-\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{2af}+\frac{(c+id)^{3/2}(ic+4d)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{2af}-\frac{(c+$$

```
[Out] -1/2*I*(c-I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/a/f+1/2*
(c+I*d)^(3/2)*(I*c+4*d)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/a/f-1
/2*(c+5*I*d)*d*(c+d*tan(f*x+e))^(1/2)/a/f+1/2*(I*c-d)*(c+d*tan(f*x+e))^(3/2
)/f/(a+I*a*tan(f*x+e))
```

Rubi [A]

time = 0.29, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3631, 3609, 3620, 3618, 65, 214}

$$\frac{(-d+ic)(c+d\tan(e+fx))^{3/2}}{2f(a+ia\tan(e+fx))}-\frac{d(c+5id)\sqrt{c+d\tan(e+fx)}}{2af}-\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{2af}+\frac{(c+id)^{3/2}(4d+ic)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^(5/2)/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] ((-1/2*I)*(c - I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/
(a*f) + ((c + I*d)^(3/2)*(I*c + 4*d)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[
c + I*d]])/(2*a*f) - ((c + (5*I)*d)*d*Sqrt[c + d*Tan[e + f*x]])/(2*a*f) + (
(I*c - d)*(c + d*Tan[e + f*x])^(3/2))/(2*f*(a + I*a*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
```

```
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3631

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((c + d*Tan[e + f*x])^(n - 1)/(
2*a*f*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a^2), Int[(c + d*Tan[e + f*x])
^(n - 2)*Simp[a*c^2 + a*d^2*(n - 1) - b*c*d*n - d*(a*c*(n - 2) + b*d*n)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{5/2}}{a+ia\tan(e+fx)}\,dx = \frac{(ic-d)(c+d\tan(e+fx))^{3/2}}{2f(a+ia\tan(e+fx))} + \frac{\int \sqrt{c+d\tan(e+fx)}\,\left(\tfrac{1}{2}a(2c+id)(c-3i\right.}{2a^2}$$

$$= -\frac{(c+5id)d\sqrt{c+d\tan(e+fx)}}{2af} + \frac{(ic-d)(c+d\tan(e+fx))^{3/2}}{2f(a+ia\tan(e+fx))} + \frac{\int \tfrac{1}{2}a(2}{}$$

$$= -\frac{(c+5id)d\sqrt{c+d\tan(e+fx)}}{2af} + \frac{(ic-d)(c+d\tan(e+fx))^{3/2}}{2f(a+ia\tan(e+fx))} + \frac{(c-i}{}$$

$$= -\frac{(c+5id)d\sqrt{c+d\tan(e+fx)}}{2af} + \frac{(ic-d)(c+d\tan(e+fx))^{3/2}}{2f(a+ia\tan(e+fx))} - \frac{(ic+}{}$$

$$= -\frac{(c+5id)d\sqrt{c+d\tan(e+fx)}}{2af} + \frac{(ic-d)(c+d\tan(e+fx))^{3/2}}{2f(a+ia\tan(e+fx))} - \frac{(c-i}{}$$

$$= -\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{2af} + \frac{(c+id)^{3/2}(ic+4d)\tanh^{-1}}{2a}$$

Mathematica [A]
time = 2.15, size = 260, normalized size = 1.41

$$\frac{\sec(e+fx)(\cos(fx)+i\sin(fx))\left(\frac{2\left(-i\sqrt{-c+id}\,(c+id)^2(c-4id)\text{ArcTan}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{-c-id}}\right)-\sqrt{-c-id}\,(ic+d)^3\text{ArcTan}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{-c+id}}\right)\right)(\cos(e)+i\sin(e))}{\sqrt{-c-id}\,\sqrt{-c+id}}+2(i\cos(fx)+\sin(fx))\left((c^2+2icd-5d^2)\cos(e+fx)-4id^2\sin(e+fx)\right)\sqrt{c+d\tan(e+fx)}\right)}{4f(a+ia\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^(5/2)/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (Sec[e + f*x]*(Cos[f*x] + I*Sin[f*x])*((2*((-I)*Sqrt[-c + I*d]*(c + I*d)^2*
(c - (4*I)*d)*ArcTan[Sqrt[c + d*Tan[e + f*x]]/Sqrt[-c - I*d]] - Sqrt[-c - I
*d]*(I*c + d)^3*ArcTan[Sqrt[c + d*Tan[e + f*x]]/Sqrt[-c + I*d]])*(Cos[e] +
I*Sin[e]))/(Sqrt[-c - I*d]*Sqrt[-c + I*d]) + 2*(I*Cos[f*x] + Sin[f*x])*((c^
2 + (2*I)*c*d - 5*d^2)*Cos[e + f*x] - (4*I)*d^2*Sin[e + f*x])*Sqrt[c + d*Ta
n[e + f*x]]))/(4*f*(a + I*a*Tan[e + f*x]))
```

Maple [A]
time = 0.32, size = 237, normalized size = 1.28

method	result

derivativedivides	$\frac{2d^2\left(-i\sqrt{c+d\tan(fx+e)}+\frac{(ic^3-3icd^2+3c^2d-d^3)\arctan\left(\frac{\sqrt{c+d\tan(fx+e)}}{\sqrt{id-c}}\right)}{4d^2\sqrt{id-c}}+\frac{-\frac{d(3ic^2d-id^3+c^3-}{(i}}{}\right.}{fa}$
default	$\frac{2d^2\left(-i\sqrt{c+d\tan(fx+e)}+\frac{(ic^3-3icd^2+3c^2d-d^3)\arctan\left(\frac{\sqrt{c+d\tan(fx+e)}}{\sqrt{id-c}}\right)}{4d^2\sqrt{id-c}}+\frac{-\frac{d(3ic^2d-id^3+c^3-}{(i}}{}\right.}{fa}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a*d^2*(-I*(c+d*tan(f*x+e))^(1/2)+1/4*(-3*I*c*d^2-d^3+I*c^3+3*c^2*d)/d^2
/(I*d-c)^(1/2)*arctan((c+d*tan(f*x+e))^(1/2)/(I*d-c)^(1/2))+1/4/d^2*(-d*(3*
I*c^2*d-I*d^3+c^3-3*c*d^2)/(c+I*d)*(c+d*tan(f*x+e))^(1/2)/(-d*tan(f*x+e)+I*
d)-(c^3*d-11*c*d^3+I*c^4+9*I*c^2*d^2-4*I*d^4)/(c+I*d)/(-I*d-c)^(1/2)*arctan
((c+d*tan(f*x+e))^(1/2)/(-I*d-c)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1053 vs. 2(146) = 292.
time = 0.99, size = 1053, normalized size = 5.69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/8*(a*f*sqrt(-(c^5 - 5*I*c^4*d - 10*c^3*d^2 + 10*I*c^2*d^3 + 5*c*d^4 - I*d
^5)/(a^2*f^2))*e^(2*I*f*x + 2*I*e)*log(2*(c^3 - 2*I*c^2*d - c*d^2 + (I*a*f*
e^(2*I*f*x + 2*I*e) + I*a*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)
/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(c^5 - 5*I*c^4*d - 10*c^3*d^2 + 10*I*c^2*
d^3 + 5*c*d^4 - I*d^5)/(a^2*f^2)) + (c^3 - 3*I*c^2*d - 3*c*d^2 + I*d^3)*e^(
2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/(c^2 - 2*I*c*d - d^2)) - a*f*sqrt(-(
c^5 - 5*I*c^4*d - 10*c^3*d^2 + 10*I*c^2*d^3 + 5*c*d^4 - I*d^5)/(a^2*f^2))*e
^(2*I*f*x + 2*I*e)*log(2*(c^3 - 2*I*c^2*d - c*d^2 + (-I*a*f*e^(2*I*f*x + 2*
I*e) - I*a*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x +
2*I*e) + 1))*sqrt(-(c^5 - 5*I*c^4*d - 10*c^3*d^2 + 10*I*c^2*d^3 + 5*c*d^4 -
 I*d^5)/(a^2*f^2)) + (c^3 - 3*I*c^2*d - 3*c*d^2 + I*d^3)*e^(2*I*f*x + 2*I*e
))*e^(-2*I*f*x - 2*I*e)/(c^2 - 2*I*c*d - d^2)) + a*f*sqrt(-(c^5 - 5*I*c^4*d
 + 5*c^3*d^2 - 25*I*c^2*d^3 + 40*c*d^4 + 16*I*d^5)/(a^2*f^2))*e^(2*I*f*x +
2*I*e)*log(1/2*(I*c^3 + 2*c^2*d + 7*I*c*d^2 - 4*d^3 + (a*f*e^(2*I*f*x + 2*I
*e) + a*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I
*e) + 1))*sqrt(-(c^5 - 5*I*c^4*d + 5*c^3*d^2 - 25*I*c^2*d^3 + 40*c*d^4 + 16
*I*d^5)/(a^2*f^2)) + (I*c^3 + 3*c^2*d + 4*I*c*d^2)*e^(2*I*f*x + 2*I*e))*e^(
-2*I*f*x - 2*I*e)/(a*f)) - a*f*sqrt(-(c^5 - 5*I*c^4*d + 5*c^3*d^2 - 25*I*c^
2*d^3 + 40*c*d^4 + 16*I*d^5)/(a^2*f^2))*e^(2*I*f*x + 2*I*e)*log(1/2*(I*c^3
+ 2*c^2*d + 7*I*c*d^2 - 4*d^3 - (a*f*e^(2*I*f*x + 2*I*e) + a*f)*sqrt(((c -
I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(c^5 -
 5*I*c^4*d + 5*c^3*d^2 - 25*I*c^2*d^3 + 40*c*d^4 + 16*I*d^5)/(a^2*f^2)) + (
I*c^3 + 3*c^2*d + 4*I*c*d^2)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/(a*f
)) + 2*(I*c^2 - 2*c*d - I*d^2 + (I*c^2 - 2*c*d - 9*I*d^2)*e^(2*I*f*x + 2*I*
e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1
)))*e^(-2*I*f*x - 2*I*e)/(a*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\left(\int \frac{c^2\sqrt{c+d\tan(e+fx)}}{\tan(e+fx)-i}\,dx+\int \frac{d^2\sqrt{c+d\tan(e+fx)}\tan^2(e+fx)}{\tan(e+fx)-i}\,dx+\int \frac{2cd\sqrt{c+d\tan(e+fx)}\tan(e+fx)}{\tan(e+fx)-i}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(5/2)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] -I*(Integral(c**2*sqrt(c + d*tan(e + f*x))/(tan(e + f*x) - I), x) + Integra
l(d**2*sqrt(c + d*tan(e + f*x))*tan(e + f*x)**2/(tan(e + f*x) - I), x) + In
tegral(2*c*d*sqrt(c + d*tan(e + f*x))*tan(e + f*x)/(tan(e + f*x) - I), x))/
a
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(146) = 292$.
time = 0.62, size = 464, normalized size = 2.51

$$-\frac{2i\sqrt{d\tan(fx+e)+c}\,d^2}{af}+\frac{(c^3-3ic^2d-3cd^2+id^3)\arctan\left(-\frac{2\left(i\sqrt{d\tan(fx+e)+c}\,c+i\sqrt{c^2+d^2}\sqrt{d\tan(fx+e)+c}\right)}{\sqrt{2c+2\sqrt{c^2+d^2}}\,c-i\sqrt{2c+2\sqrt{c^2+d^2}}\,d+\sqrt{c^2+d^2}\sqrt{2c+2\sqrt{c^2+d^2}}}\right)}{a\sqrt{2c+2\sqrt{c^2+d^2}}\,f\left(-\frac{id}{c+\sqrt{c^2+d^2}}+1\right)}+\frac{(-ic^3-2c^2d-7icd^2+4d^3)\arctan\left(\frac{2\left(\sqrt{d\tan(fx+e)+c}\,c-\sqrt{c^2+d^2}\sqrt{d\tan(fx+e)+c}\right)}{c\sqrt{-2c+2\sqrt{c^2+d^2}}+i\sqrt{-2c+2\sqrt{c^2+d^2}}\,d-\sqrt{c^2+d^2}\sqrt{-2c+2\sqrt{c^2+d^2}}}\right)}{a\sqrt{-2c+2\sqrt{c^2+d^2}}\,f\left(\frac{id}{c-\sqrt{c^2+d^2}}+1\right)}+\frac{\sqrt{d\tan(fx+e)+c}\,c^2d+2i\sqrt{d\tan(fx+e)+c}\,cd^2-\sqrt{d\tan(fx+e)+c}\,d^3}{2(d\tan(fx+e)-id)af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] -2*I*sqrt(d*tan(f*x + e) + c)*d^2/(a*f) + (c^3 - 3*I*c^2*d - 3*c*d^2 + I*d^
3)*arctan(-2*(I*sqrt(d*tan(f*x + e) + c)*c + I*sqrt(c^2 + d^2)*sqrt(d*tan(f
*x + e) + c))/(sqrt(2*c + 2*sqrt(c^2 + d^2))*c - I*sqrt(2*c + 2*sqrt(c^2 +
d^2))*d + sqrt(c^2 + d^2)*sqrt(2*c + 2*sqrt(c^2 + d^2))))/(a*sqrt(2*c + 2*s
qrt(c^2 + d^2))*f*(-I*d/(c + sqrt(c^2 + d^2)) + 1)) + (-I*c^3 - 2*c^2*d - 7
*I*c*d^2 + 4*d^3)*arctan(2*(sqrt(d*tan(f*x + e) + c)*c - sqrt(c^2 + d^2)*sq
rt(d*tan(f*x + e) + c))/(c*sqrt(-2*c + 2*sqrt(c^2 + d^2)) + I*sqrt(-2*c + 2
*sqrt(c^2 + d^2))*d - sqrt(c^2 + d^2)*sqrt(-2*c + 2*sqrt(c^2 + d^2))))/(a*s
qrt(-2*c + 2*sqrt(c^2 + d^2))*f*(I*d/(c - sqrt(c^2 + d^2)) + 1)) + 1/2*(sqr
t(d*tan(f*x + e) + c)*c^2*d + 2*I*sqrt(d*tan(f*x + e) + c)*c*d^2 - sqrt(d*t
an(f*x + e) + c)*d^3)/((d*tan(f*x + e) - I*d)*a*f)
```

Mupad [B]
time = 9.36, size = 2500, normalized size = 13.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(5/2)/(a + a*tan(e + f*x)*1i),x)
```

```
[Out] log((a*f*(12*d^11 - c*d^10*25i + 75*c^2*d^9 - c^3*d^8*35i + 113*c^4*d^7 + c
^5*d^6*5i + 49*c^6*d^5 + c^7*d^4*15i - c^8*d^3))/2 - (((a*f*(a^2*d^6*f^2*80
i + 16*a^2*c*d^5*f^2 + a^2*c^2*d^4*f^2*80i + 16*a^2*c^3*d^3*f^2))/2 + 64*a^
4*c*d^2*f^4*(c + d*tan(e + f*x))^(1/2)*(-(720*c*d^10 + d^11*240i + 640*c^3*
d^8 - c^4*d^7*400i - 48*c^5*d^6 - c^6*d^5*160i + 32*c^7*d^4 + a^2*f^2*((((4
00*c^4*d^7 - 240*d^11 + 160*c^6*d^5)*1i)/(a^2*f^2) - (720*c*d^10 + 640*c^3*
d^8 - 48*c^5*d^6 + 32*c^7*d^4)/(a^2*f^2))^2 - 4*(256*d^6 + 256*c^2*d^4)*(((
40*c*d^15 + 150*c^3*d^13 + 200*c^5*d^11 + 100*c^7*d^9 - 10*c^11*d^5)*1i)/(a
^4*f^4) - (35*c^4*d^12 - 31*c^2*d^14 - 16*d^16 + 130*c^6*d^10 + 110*c^8*d^8
 + 29*c^10*d^6 - c^12*d^4)/(a^4*f^4)))^(1/2))/(512*a^2*f^2*(d^6 + c^2*d^4))
)^(1/2))*(-(720*c*d^10 + d^11*240i + 640*c^3*d^8 - c^4*d^7*400i - 48*c^5*d^
6 - c^6*d^5*160i + 32*c^7*d^4 + a^2*f^2*((((400*c^4*d^7 - 240*d^11 + 160*c^
6*d^5)*1i)/(a^2*f^2) - (720*c*d^10 + 640*c^3*d^8 - 48*c^5*d^6 + 32*c^7*d^4)
/(a^2*f^2))^2 - 4*(256*d^6 + 256*c^2*d^4)*(((40*c*d^15 + 150*c^3*d^13 + 200
*c^5*d^11 + 100*c^7*d^9 - 10*c^11*d^5)*1i)/(a^4*f^4) - (35*c^4*d^12 - 31*c^
2*d^14 - 16*d^16 + 130*c^6*d^10 + 110*c^8*d^8 + 29*c^10*d^6 - c^12*d^4)/(a^
4*f^4)))^(1/2))/(512*a^2*f^2*(d^6 + c^2*d^4)))^(1/2) + 2*a^2*f^2*(c + d*tan
(e + f*x))^(1/2)*(c*d^7*50i - 17*d^8 + 80*c^2*d^6 - 5*c^4*d^4 - c^5*d^3*10i
 + 2*c^6*d^2))*(-(720*c*d^10 + d^11*240i + 640*c^3*d^8 - c^4*d^7*400i - 48*
c^5*d^6 - c^6*d^5*160i + 32*c^7*d^4 + a^2*f^2*((((400*c^4*d^7 - 240*d^11 +
160*c^6*d^5)*1i)/(a^2*f^2) - (720*c*d^10 + 640*c^3*d^8 - 48*c^5*d^6 + 32*c^
7*d^4)/(a^2*f^2))^2 - 4*(256*d^6 + 256*c^2*d^4)*(((40*c*d^15 + 150*c^3*d^13
```

+ 200*c^5*d^11 + 100*c^7*d^9 - 10*c^11*d^5)*1i)/(a^4*f^4) - (35*c^4*d^12 - 31*c^2*d^14 - 16*d^16 + 130*c^6*d^10 + 110*c^8*d^8 + 29*c^10*d^6 - c^12*d^4)/(a^4*f^4)))^(1/2))/(512*a^2*f^2*(d^6 + c^2*d^4)))^(1/2))*(-(720*c*d^10 + d^11*240i + 640*c^3*d^8 - c^4*d^7*400i - 48*c^5*d^6 - c^6*d^5*160i + 32*c^7*d^4 + a^2*f^2*((((400*c^4*d^7 - 240*d^11 + 160*c^6*d^5)*1i)/(a^2*f^2) - (720*c*d^10 + 640*c^3*d^8 - 48*c^5*d^6 + 32*c^7*d^4)/(a^2*f^2))^2 - 4*(256*d^6 + 256*c^2*d^4)*(((40*c*d^15 + 150*c^3*d^13 + 200*c^5*d^11 + 100*c^7*d^9 - 10*c^11*d^5)*1i)/(a^4*f^4) - (35*c^4*d^12 - 31*c^2*d^14 - 16*d^16 + 130*c^6*d^10 + 110*c^8*d^8 + 29*c^10*d^6 - c^12*d^4)/(a^4*f^4)))^(1/2))/(512*a^2*f^2*(d^6 + c^2*d^4)))^(1/2) + log((a*f*(12*d^11 - c*d^10*25i + 75*c^2*d^9 - c^3*d^8*35i + 113*c^4*d^7 + c^5*d^6*5i + 49*c^6*d^5 + c^7*d^4*15i - c^8*d^3))/2 - (((a*f*(a^2*d^6*f^2*80i + 16*a^2*c*d^5*f^2 + a^2*c^2*d^4*f^2*80i + 16*a^2*c^3*d^3*f^2))/2 + 64*a^4*c*d^2*f^4*(c + d*tan(e + f*x))^(1/2)*(-(720*c*d^10 + d^11*240i + 640*c^3*d^8 - c^4*d^7*400i - 48*c^5*d^6 - c^6*d^5*160i + 32*c^7*d^4 - a^2*f^2*((((400*c^4*d^7 - 240*d^11 + 160*c^6*d^5)*1i)/(a^2*f^2) - (720*c*d^10 + 640*c^3*d^8 - 48*c^5*d^6 + 32*c^7*d^4)/(a^2*f^2))^2 - 4*(256*d^6 + 256*c^2*d^4)*(((40*c*d^15 + 150*c^3*d^13 + 200*c^5*d^11 + 100*c^7*d^9 - 10*c^11*d^5)*1i)/(a^4*f^4) - (35*c^4*d^12 - 31*c^2*d^14 - 16*d^16 + 130*c^6*d^10 + 110*c^8*d^8 + 29*c^10*d^6 - c^12*d^4)/(a^4*f^4)))^(1/2))/(512*a^2*f^2*(d^6 + c^2*d^4)))^(1/2))*(-(720*c*d^10 + d^11*240i + 640*c^3*d^8 - c^4*d^7*400i - 48*c^5*d^6 - c^6*d^5*160i + 32*c^7*d^4 - a^2*f^2*((((400*c^4*d^7 - 240*d^11 + 160*c^6*d^5)*1i)/(a^2*f^2) - (720*c*d^10 + 640*c^3*d^8 - 48*c^5*d^6 + 32*c^7*d^4)/(a^2*f^2))^2 - 4*(256*d^6 + 256*c^2*d^4)*(((40*c*d^15 + 150*c^3*d^13 + 200*c^5*d^11 + 100*c^7*d^9 - 10*c^11*d^5)*1i)/(a^4*f^4) - (35*c^4*d^12 - 31*c^2*d^14 - 16*d^16 + 130*c^6*d^10 + 110*c^8*d^8 + 29*c^10*d^6 - c^12*d^4)/(a^4*f^4)))^(1/2))/(512*a^2*f^2*(d^6 + c^2*d^4)))^(1/2) + 2*a^2*f^2*(c + d*tan(e + f*x))^(1/2)*(c*d^7*50i - 17*d^8 + 80*c^2*d^6 - 5*c^4*d^4 - c^5*d^3*10i + 2*c^6*d^2))*(-(720*c*d^10 + d^11*240i + 640*c^3*d^8 - c^4*d^7*400i - 48*c^5*d^6 - c^6*d^5*160i + 32*c^7*d^4 - a^2*f^2*((((400*c^4*d^7 - 240*d^11 + 160*c^6*d^5)*1i)/(a^2*f^2) - (720*c*d^10 + 640*c^3*d^8 - 48*c^5*d^6 + 32*c^7*d^4)/(a^2*f^2))^2 - 4*(256*d^6 + 256*c^2*d^4)*(((40*c*d^15 + 150*c^3*d^13 + 200*c^5*d^11 + 100*c^7*d^9 - 10*c^11*d^5)*1i)/(a^4*f^4) - (35*c^4*d^12 - 31*c^2*d^14 - 16*d^16 + 130*c^6*d^10 + 110*c^8*d^8 + 29*c^10*d^6 - c^12*d^4)/(a^4*f^4)))^(1/2))/(512*a^2*f^2*(d^6 + c^2*d^4)))^(1/2))*(-(720*c*d^10 + d^11*240i + 640*c^3*d^8 - c^4*d^7*400i - 48*c^5*d^6 - c^6*d^5*160i + 32*c^7*d^4 - a^2*f^2*((((400*c^4*d^7 - 240*d^11 + 160*c^6*d^5)*1i)/(a^2*f^2) - (720*c*d^10 + 640*c^3*d^8 - 48*c^5*d^6 + 32*c^7*d^4)/(a^2*f^2))^2 - 4*(256*d^6 + 256*c^2*d^4)*(((40*c*d^15 + 150*c^3*d^13 + 200*c^5*d^11 + 100*c^7*d^9 - 10*c^11*d^5)*1i)/(a^4*f^4) - (35*c^4*d^12 - 31*c^2*d^14 - 16*d^16 + 130*c^6*d^10 + 110*c^8*d^8 + 29*c^10*d^6 - c^12*d^4)/(a^4*f^4)))^(1/2))/(512*a^2*f^2*(d^6 + c^2*d^4)))^(1/2) - log((a*f*(12*d^11 - c*d^10*25i + 75*c^2*d^9 - c^3*d^8*35i + 113*c^4*d^7 + c^5*d^6*5i + 49*c^6*d^5 + c^7*d^4*15i - c^8*d^3))/2 - (((a*f*(a^2*d^6*f^2*80i + 16*a^2*c*d^5*f^2 + a^2*c^2*d^4*f^2*80i + 16*a^2*c^3*d^3*f^2))/2 - 64*a^4*c*d^2*f^4*(-(720*c*d^10 + d^11*240i + 640*c^3*d^8 - c^4*d^...

3.1117 $\int \frac{(c+d\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=217

$$-\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{4a^2f}+\frac{\sqrt{c+id}\,(2ic^2+6cd-7id^2)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{8a^2f}$$

```
[Out] -1/4*I*(c-I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/a^2/f+1/
8*(2*I*c^2+6*c*d-7*I*d^2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))*(c+
I*d)^(1/2)/a^2/f+1/8*(c+I*d)*(2*I*c+5*d)*(c+d*tan(f*x+e))^(1/2)/a^2/f/(1+I*
tan(f*x+e))+1/4*(I*c-d)*(c+d*tan(f*x+e))^(3/2)/f/(a+I*a*tan(f*x+e))^2
```

Rubi [A]
time = 0.42, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3639, 3676, 3620, 3618, 65, 214}

$$\frac{\sqrt{c+id}\,(2ic^2+6cd-7id^2)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{8a^2f}+\frac{(c+id)(5d+2ic)\sqrt{c+d\tan(e+fx)}}{8a^2f(1+i\tan(e+fx))}-\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{4a^2f}+\frac{(-d+ic)(c+d\tan(e+fx))^{3/2}}{4f(a+ia\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^(5/2)/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] ((-1/4*I)*(c - I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/
(a^2*f) + (Sqrt[c + I*d]*((2*I)*c^2 + 6*c*d - (7*I)*d^2)*ArcTanh[Sqrt[c + d
*Tan[e + f*x]]/Sqrt[c + I*d]])/(8*a^2*f) + ((c + I*d)*((2*I)*c + 5*d)*Sqrt[
c + d*Tan[e + f*x]])/(8*a^2*f*(1 + I*Tan[e + f*x])) + ((I*c - d)*(c + d*Tan
[e + f*x])^(3/2))/(4*f*(a + I*a*Tan[e + f*x])^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
```

```
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^2}\,dx = \frac{(ic-d)(c+d\tan(e+fx))^{3/2}}{4f(a+ia\tan(e+fx))^2} - \frac{\int \frac{\sqrt{c+d\tan(e+fx)}\,\left(-\frac{1}{2}a(4c^2-7icd+3d^2)-\frac{1}{2}a(\right.}{a+ia\tan(e+fx)}}{4a^2}$$

$$= \frac{(c+id)(2ic+5d)\sqrt{c+d\tan(e+fx)}}{8a^2f(1+i\tan(e+fx))} + \frac{(ic-d)(c+d\tan(e+fx))^{3/2}}{4f(a+ia\tan(e+fx))^2} + \frac{\int}{}$$

$$= \frac{(c+id)(2ic+5d)\sqrt{c+d\tan(e+fx)}}{8a^2f(1+i\tan(e+fx))} + \frac{(ic-d)(c+d\tan(e+fx))^{3/2}}{4f(a+ia\tan(e+fx))^2} + \frac{(c}{}$$

$$= \frac{(c+id)(2ic+5d)\sqrt{c+d\tan(e+fx)}}{8a^2f(1+i\tan(e+fx))} + \frac{(ic-d)(c+d\tan(e+fx))^{3/2}}{4f(a+ia\tan(e+fx))^2} - \frac{(i}{}$$

$$= \frac{(c+id)(2ic+5d)\sqrt{c+d\tan(e+fx)}}{8a^2f(1+i\tan(e+fx))} + \frac{(ic-d)(c+d\tan(e+fx))^{3/2}}{4f(a+ia\tan(e+fx))^2} - \frac{(c}{}$$

$$= -\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{4a^2f} + \frac{\sqrt{c+id}\,(2ic^2+6cd-7id^2)}{}$$

Mathematica [A]
time = 2.12, size = 291, normalized size = 1.34

$$\frac{\sec^2(e+fx)(\cos(fx)+i\sin(fx))^2\left(\frac{2\left(-i\sqrt{-c+id}\,(2c^3-4ic^2d-cd^2-7id^3)\text{ArcTan}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{-c-id}}\right)-2\sqrt{-c-id}\,(ic+d)^3\text{ArcTan}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{-c+id}}\right)\right)(\cos(2e)+i\sin(2e))}{\sqrt{-c-id}\,\sqrt{-c+id}}+2(c+id)\cos(e+fx)(\cos(2fx)-i\sin(2fx))((4ic+5d)\cos(e+fx)+(-2c+7id)\sin(e+fx))\sqrt{c+d\tan(e+fx)}\right)}{16f(a+ia\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^(5/2)/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (Sec[e + f*x]^2*(Cos[f*x] + I*Sin[f*x])^2*((2*((-I)*Sqrt[-c + I*d]*(2*c^3 -
 (4*I)*c^2*d - c*d^2 - (7*I)*d^3)*ArcTan[Sqrt[c + d*Tan[e + f*x]]/Sqrt[-c -
 I*d]] - 2*Sqrt[-c - I*d]*(I*c + d)^3*ArcTan[Sqrt[c + d*Tan[e + f*x]]/Sqrt[
-c + I*d]])*(Cos[2*e] + I*Sin[2*e]))/(Sqrt[-c - I*d]*Sqrt[-c + I*d]) + 2*(c
 + I*d)*Cos[e + f*x]*(Cos[2*f*x] - I*Sin[2*f*x])*(((4*I)*c + 5*d)*Cos[e + f
*x] + (-2*c + (7*I)*d)*Sin[e + f*x])*Sqrt[c + d*Tan[e + f*x]]))/(16*f*(a +
I*a*Tan[e + f*x])^2)
```

Maple [A]
time = 0.36, size = 302, normalized size = 1.39

method	result

derivativedivides	$\dfrac{2d^3\left(-\dfrac{i(id-c)^{\frac{5}{2}}\arctan\left(\frac{\sqrt{c+d\tan(fx+e)}}{\sqrt{id-c}}\right)}{8d^3}+\dfrac{i\left(-\frac{d\left(2ic^4+15ic^2d^2-7id^4+c^3d-19c\,d^3\right)(c+d\tan(fx+e))^{\frac{3}{2}}}{2\left(2icd+c^2-d^2\right)}+\frac{d(2i\cdots}{(-d\tan\cdots}\right.}{\cdots}\right.}{\cdots}$
default	$\dfrac{2d^3\left(-\dfrac{i(id-c)^{\frac{5}{2}}\arctan\left(\frac{\sqrt{c+d\tan(fx+e)}}{\sqrt{id-c}}\right)}{8d^3}+\dfrac{i\left(-\frac{d\left(2ic^4+15ic^2d^2-7id^4+c^3d-19c\,d^3\right)(c+d\tan(fx+e))^{\frac{3}{2}}}{2\left(2icd+c^2-d^2\right)}+\frac{d(2i\cdots}{(-d\tan\cdots}\right.}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^2*d^3*(-1/8*I*(I*d-c)^(5/2)/d^3*arctan((c+d*tan(f*x+e))^(1/2)/(I*d-c)
^(1/2))+1/8*I/d^3*((-1/2*d*(2*I*c^4+15*I*c^2*d^2-7*I*d^4+c^3*d-19*c*d^3)/(2
*I*c*d+c^2-d^2)*(c+d*tan(f*x+e))^(3/2)+1/2*d*(2*I*c^5+8*I*c^3*d^2-18*I*c*d^
4-3*c^4*d-22*c^2*d^3+5*d^5)/(2*I*c*d+c^2-d^2)*(c+d*tan(f*x+e))^(1/2))/(-d*t
an(f*x+e)+I*d)^2+1/2*(-5*c^3*d^2-15*c*d^4+5*I*c^2*d^3-7*I*d^5-2*c^5)/(2*I*c
*d+c^2-d^2)/(-I*d-c)^(1/2)*arctan((c+d*tan(f*x+e))^(1/2)/(-I*d-c)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1112 vs. 2(173) = 346.
time = 0.98, size = 1112, normalized size = 5.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] 1/32*(2*a^2*f*sqrt(-(c^5 - 5*I*c^4*d - 10*c^3*d^2 + 10*I*c^2*d^3 + 5*c*d^4
- I*d^5)/(a^4*f^2))*e^(4*I*f*x + 4*I*e)*log(2*(c^3 - 2*I*c^2*d - c*d^2 + (I
*a^2*f*e^(2*I*f*x + 2*I*e) + I*a^2*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) +
 c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(c^5 - 5*I*c^4*d - 10*c^3*d^2 +
10*I*c^2*d^3 + 5*c*d^4 - I*d^5)/(a^4*f^2)) + (c^3 - 3*I*c^2*d - 3*c*d^2 + I
*d^3)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/(c^2 - 2*I*c*d - d^2)) - 2*
a^2*f*sqrt(-(c^5 - 5*I*c^4*d - 10*c^3*d^2 + 10*I*c^2*d^3 + 5*c*d^4 - I*d^5)
/(a^4*f^2))*e^(4*I*f*x + 4*I*e)*log(2*(c^3 - 2*I*c^2*d - c*d^2 + (-I*a^2*f*
e^(2*I*f*x + 2*I*e) - I*a^2*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*
d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(c^5 - 5*I*c^4*d - 10*c^3*d^2 + 10*I*c^
2*d^3 + 5*c*d^4 - I*d^5)/(a^4*f^2)) + (c^3 - 3*I*c^2*d - 3*c*d^2 + I*d^3)*e
^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/(c^2 - 2*I*c*d - d^2)) + a^2*f*sqr
t(-(4*c^5 - 20*I*c^4*d - 40*c^3*d^2 + 20*I*c^2*d^3 - 35*c*d^4 + 49*I*d^5)/(
a^4*f^2))*e^(4*I*f*x + 4*I*e)*log(1/8*(2*I*c^3 + 4*c^2*d - I*c*d^2 + 7*d^3
+ (a^2*f*e^(2*I*f*x + 2*I*e) + a^2*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) +
 c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(4*c^5 - 20*I*c^4*d - 40*c^3*d^2
 + 20*I*c^2*d^3 - 35*c*d^4 + 49*I*d^5)/(a^4*f^2)) + (2*I*c^3 + 6*c^2*d - 7*
I*c*d^2)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/(a^2*f)) - a^2*f*sqrt(-(
4*c^5 - 20*I*c^4*d - 40*c^3*d^2 + 20*I*c^2*d^3 - 35*c*d^4 + 49*I*d^5)/(a^4*
f^2))*e^(4*I*f*x + 4*I*e)*log(1/8*(2*I*c^3 + 4*c^2*d - I*c*d^2 + 7*d^3 - (a
^2*f*e^(2*I*f*x + 2*I*e) + a^2*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c +
 I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(4*c^5 - 20*I*c^4*d - 40*c^3*d^2 + 2
0*I*c^2*d^3 - 35*c*d^4 + 49*I*d^5)/(a^4*f^2)) + (2*I*c^3 + 6*c^2*d - 7*I*c*
d^2)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/(a^2*f)) + 2*(I*c^2 - 2*c*d
- I*d^2 - 3*(-I*c^2 - c*d - 2*I*d^2)*e^(4*I*f*x + 4*I*e) + (4*I*c^2 + c*d +
 5*I*d^2)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*
d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-4*I*f*x - 4*I*e)/(a^2*f)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{c^2\sqrt{c+d\tan(e+fx)}}{\tan^2(e+fx)-2i\tan(e+fx)-1}\,dx+\int \frac{d^2\sqrt{c+d\tan(e+fx)}\tan^2(e+fx)}{\tan^2(e+fx)-2i\tan(e+fx)-1}\,dx+\int \frac{2cd\sqrt{c+d\tan(e+fx)}\tan(e+fx)}{\tan^2(e+fx)-2i\tan(e+fx)-1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(5/2)/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -(Integral(c**2*sqrt(c + d*tan(e + f*x))/(tan(e + f*x)**2 - 2*I*tan(e + f*x
) - 1), x) + Integral(d**2*sqrt(c + d*tan(e + f*x))*tan(e + f*x)**2/(tan(e
+ f*x)**2 - 2*I*tan(e + f*x) - 1), x) + Integral(2*c*d*sqrt(c + d*tan(e + f
*x))*tan(e + f*x)/(tan(e + f*x)**2 - 2*I*tan(e + f*x) - 1), x))/a**2
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(173) = 346$.
time = 0.78, size = 520, normalized size = 2.40

$$\frac{\left(c^{3}-3 i c^{2} d-3 c d^{2}+i d^{3}\right) \arctan\left(-\frac{2\left(i \sqrt{d \tan\left(f x+e\right)+c}\, c+i \sqrt{c^{2}+d^{2}} \sqrt{d \tan\left(f x+e\right)+c}\right)}{\sqrt{2 c+2 \sqrt{c^{2}+d^{2}}}\, c-i \sqrt{2 c+2 \sqrt{c^{2}+d^{2}}}\, d+\sqrt{c^{2}+d^{2}} \sqrt{2 c+2 \sqrt{c^{2}+d^{2}}}}\right)}{2 a^{2} \sqrt{2 c+2 \sqrt{c^{2}+d^{2}}}\, f\left(-\frac{i d}{c+\sqrt{c^{2}+d^{2}}}+1\right)}-\frac{\left(2 i c^{3}+4 c^{2} d-i c d^{2}+7 d^{3}\right) \arctan\left(\frac{2\left(\sqrt{d \tan\left(f x+e\right)+c}\, c-\sqrt{c^{2}+d^{2}} \sqrt{d \tan\left(f x+e\right)+c}\right)}{c \sqrt{-2 c+2 \sqrt{c^{2}+d^{2}}}+i \sqrt{-2 c+2 \sqrt{c^{2}+d^{2}}}\, d-\sqrt{c^{2}+d^{2}} \sqrt{-2 c+2 \sqrt{c^{2}+d^{2}}}}\right)}{4 a^{2} \sqrt{-2 c+2 \sqrt{c^{2}+d^{2}}}\, f\left(\frac{i d}{c-\sqrt{c^{2}+d^{2}}}+1\right)}+\frac{2\left(d \tan\left(f x+e\right)+c\right)^{\frac{3}{2}} c^{2} d-2 \sqrt{d \tan\left(f x+e\right)+c}\, c^{3} d-5 i\left(d \tan\left(f x+e\right)+c\right)^{\frac{3}{2}} c d^{2}+i \sqrt{d \tan\left(f x+e\right)+c}\, c^{2} d^{2}+7\left(d \tan\left(f x+e\right)+c\right)^{\frac{3}{2}} d^{3}-8 \sqrt{d \tan\left(f x+e\right)+c}\, c d^{3}-5 i \sqrt{d \tan\left(f x+e\right)+c}\, d^{4}}{8\left(d \tan\left(f x+e\right)-i d\right)^{2} a^{2} f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*(c^3 - 3*I*c^2*d - 3*c*d^2 + I*d^3)*arctan(-2*(I*sqrt(d*tan(f*x + e) +
c)*c + I*sqrt(c^2 + d^2)*sqrt(d*tan(f*x + e) + c))/(sqrt(2*c + 2*sqrt(c^2 +
 d^2))*c - I*sqrt(2*c + 2*sqrt(c^2 + d^2))*d + sqrt(c^2 + d^2)*sqrt(2*c + 2
*sqrt(c^2 + d^2))))/(a^2*sqrt(2*c + 2*sqrt(c^2 + d^2))*f*(-I*d/(c + sqrt(c^
2 + d^2)) + 1)) - 1/4*(2*I*c^3 + 4*c^2*d - I*c*d^2 + 7*d^3)*arctan(2*(sqrt(
d*tan(f*x + e) + c)*c - sqrt(c^2 + d^2)*sqrt(d*tan(f*x + e) + c))/(c*sqrt(-
2*c + 2*sqrt(c^2 + d^2)) + I*sqrt(-2*c + 2*sqrt(c^2 + d^2))*d - sqrt(c^2 +
d^2)*sqrt(-2*c + 2*sqrt(c^2 + d^2))))/(a^2*sqrt(-2*c + 2*sqrt(c^2 + d^2))*f
*(I*d/(c - sqrt(c^2 + d^2)) + 1)) + 1/8*(2*(d*tan(f*x + e) + c)^(3/2)*c^2*d
 - 2*sqrt(d*tan(f*x + e) + c)*c^3*d - 5*I*(d*tan(f*x + e) + c)^(3/2)*c*d^2
+ I*sqrt(d*tan(f*x + e) + c)*c^2*d^2 + 7*(d*tan(f*x + e) + c)^(3/2)*d^3 - 8
*sqrt(d*tan(f*x + e) + c)*c*d^3 - 5*I*sqrt(d*tan(f*x + e) + c)*d^4)/((d*tan
(f*x + e) - I*d)^2*a^2*f)
```

Mupad [B]
time = 9.19, size = 2500, normalized size = 11.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(5/2)/(a + a*tan(e + f*x)*1i)^2,x)
```

```
[Out] - atan((((a^2*f*(a^4*d^6*f^2*640i - 256*a^4*c*d^5*f^2 + a^4*c^2*d^4*f^2*640
i - 256*a^4*c^3*d^3*f^2) - 4096*a^8*c*d^2*f^4*(c + d*tan(e + f*x))^(1/2)*(-
(d^11*45i - 15*c*d^10 + c^2*d^9*105i - 95*c^3*d^8 + c^4*d^7*20i - 72*c^5*d^
6 - c^6*d^5*40i + 8*c^7*d^4 + a^4*f^2*(4*(256*d^6 + 256*c^2*d^4)*((((105*c^
3*d^13)/16 - (35*c*d^15)/8 + (295*c^5*d^11)/16 + (5*c^7*d^9)/16 - (105*c^9*
d^7)/16 + (5*c^11*d^5)/8)*1i)/(a^8*f^4) - ((49*d^16)/64 - (149*c^2*d^14)/16
 - (235*c^4*d^12)/32 + (215*c^6*d^10)/16 + (505*c^8*d^8)/64 - (11*c^10*d^6)
/4 + (c^12*d^4)/16)/(a^8*f^4)) + (((45*d^11 + 105*c^2*d^9 + 20*c^4*d^7 - 40
*c^6*d^5)*1i)/(a^4*f^2) - (15*c*d^10 + 95*c^3*d^8 + 72*c^5*d^6 - 8*c^7*d^4)
/(a^4*f^2))^2)^(1/2))/(512*a^4*f^2*(d^6 + c^2*d^4)))^(1/2))*(-(d^11*45i - 1
5*c*d^10 + c^2*d^9*105i - 95*c^3*d^8 + c^4*d^7*20i - 72*c^5*d^6 - c^6*d^5*4
0i + 8*c^7*d^4 + a^4*f^2*(4*(256*d^6 + 256*c^2*d^4)*((((105*c^3*d^13)/16 -
(35*c*d^15)/8 + (295*c^5*d^11)/16 + (5*c^7*d^9)/16 - (105*c^9*d^7)/16 + (5*
c^11*d^5)/8)*1i)/(a^8*f^4) - ((49*d^16)/64 - (149*c^2*d^14)/16 - (235*c^4*d
```

^12)/32 + (215*c^6*d^10)/16 + (505*c^8*d^8)/64 - (11*c^10*d^6)/4 + (c^12*d^4)/16)/(a^8*f^4)) + (((45*d^11 + 105*c^2*d^9 + 20*c^4*d^7 - 40*c^6*d^5)*1i)/(a^4*f^2) - (15*c*d^10 + 95*c^3*d^8 + 72*c^5*d^6 - 8*c^7*d^4)/(a^4*f^2))^2)^(1/2))/(512*a^4*f^2*(d^6 + c^2*d^4)))^(1/2) + 8*a^4*f^2*(c + d*tan(e + f*x))^(1/2)*(c*d^7*10i + 53*d^8 - 5*c^2*d^6 - c^3*d^5*60i + 80*c^4*d^4 + c^5*d^3*40i - 8*c^6*d^2))*(-(d^11*45i - 15*c*d^10 + c^2*d^9*105i - 95*c^3*d^8 + c^4*d^7*20i - 72*c^5*d^6 - c^6*d^5*40i + 8*c^7*d^4 + a^4*f^2*(4*(256*d^6 + 256*c^2*d^4)*((((105*c^3*d^13)/16 - (35*c*d^15)/8 + (295*c^5*d^11)/16 + (5*c^7*d^9)/16 - (105*c^9*d^7)/16 + (5*c^11*d^5)/8)*1i)/(a^8*f^4) - ((49*d^16)/64 - (149*c^2*d^14)/16 - (235*c^4*d^12)/32 + (215*c^6*d^10)/16 + (505*c^8*d^8)/64 - (11*c^10*d^6)/4 + (c^12*d^4)/16)/(a^8*f^4)) + (((45*d^11 + 105*c^2*d^9 + 20*c^4*d^7 - 40*c^6*d^5)*1i)/(a^4*f^2) - (15*c*d^10 + 95*c^3*d^8 + 72*c^5*d^6 - 8*c^7*d^4)/(a^4*f^2))^2)^(1/2))/(512*a^4*f^2*(d^6 + c^2*d^4)))^(1/2)*1i - ((a^2*f*(a^4*d^6*f^2*640i - 256*a^4*c*d^5*f^2 + a^4*c^2*d^4*f^2*640i - 256*a^4*c^3*d^3*f^2) + 4096*a^8*c*d^2*f^4*(c + d*tan(e + f*x))^(1/2)*(-(d^11*45i - 15*c*d^10 + c^2*d^9*105i - 95*c^3*d^8 + c^4*d^7*20i - 72*c^5*d^6 - c^6*d^5*40i + 8*c^7*d^4 + a^4*f^2*(4*(256*d^6 + 256*c^2*d^4)*((((105*c^3*d^13)/16 - (35*c*d^15)/8 + (295*c^5*d^11)/16 + (5*c^7*d^9)/16 - (105*c^9*d^7)/16 + (5*c^11*d^5)/8)*1i)/(a^8*f^4) - ((49*d^16)/64 - (149*c^2*d^14)/16 - (235*c^4*d^12)/32 + (215*c^6*d^10)/16 + (505*c^8*d^8)/64 - (11*c^10*d^6)/4 + (c^12*d^4)/16)/(a^8*f^4)) + (((45*d^11 + 105*c^2*d^9 + 20*c^4*d^7 - 40*c^6*d^5)*1i)/(a^4*f^2) - (15*c*d^10 + 95*c^3*d^8 + 72*c^5*d^6 - 8*c^7*d^4)/(a^4*f^2))^2)^(1/2))/(512*a^4*f^2*(d^6 + c^2*d^4)))^(1/2))*(-(d^11*45i - 15*c*d^10 + c^2*d^9*105i - 95*c^3*d^8 + c^4*d^7*20i - 72*c^5*d^6 - c^6*d^5*40i + 8*c^7*d^4 + a^4*f^2*(4*(256*d^6 + 256*c^2*d^4)*((((105*c^3*d^13)/16 - (35*c*d^15)/8 + (295*c^5*d^11)/16 + (5*c^7*d^9)/16 - (105*c^9*d^7)/16 + (5*c^11*d^5)/8)*1i)/(a^8*f^4) - ((49*d^16)/64 - (149*c^2*d^14)/16 - (235*c^4*d^12)/32 + (215*c^6*d^10)/16 + (505*c^8*d^8)/64 - (11*c^10*d^6)/4 + (c^12*d^4)/16)/(a^8*f^4)) + (((45*d^11 + 105*c^2*d^9 + 20*c^4*d^7 - 40*c^6*d^5)*1i)/(a^4*f^2) - (15*c*d^10 + 95*c^3*d^8 + 72*c^5*d^6 - 8*c^7*d^4)/(a^4*f^2))^2)^(1/2))/(512*a^4*f^2*(d^6 + c^2*d^4)))^(1/2) - 8*a^4*f^2*(c + d*tan(e + f*x))^(1/2)*(c*d^7*10i + 53*d^8 - 5*c^2*d^6 - c^3*d^5*60i + 80*c^4*d^4 + c^5*d^3*40i - 8*c^6*d^2))*(-(d^11*45i - 15*c*d^10 + c^2*d^9*105i - 95*c^3*d^8 + c^4*d^7*20i - 72*c^5*d^6 - c^6*d^5*40i + 8*c^7*d^4 + a^4*f^2*(4*(256*d^6 + 256*c^2*d^4)*((((105*c^3*d^13)/16 - (35*c*d^15)/8 + (295*c^5*d^11)/16 + (5*c^7*d^9)/16 - (105*c^9*d^7)/16 + (5*c^11*d^5)/8)*1i)/(a^8*f^4) - ((49*d^16)/64 - (149*c^2*d^14)/16 - (235*c^4*d^12)/32 + (215*c^6*d^10)/16 + (505*c^8*d^8)/64 - (11*c^10*d^6)/4 + (c^12*d^4)/16)/(a^8*f^4)) + (((45*d^11 + 105*c^2*d^9 + 20*c^4*d^7 - 40*c^6*d^5)*1i)/(a^4*f^2) - (15*c*d^10 + 95*c^3*d^8 + 72*c^5*d^6 - 8*c^7*d^4)/(a^4*f^2))^2)^(1/2))/(512*a^4*f^2*(d^6 + c^2*d^4)))^(1/2)*1i)/(((a^2*f*(a^4*d^6*f^2*640i - 256*a^4*c*d^5*f^2 + a^4*c^2*d^4*f^2*640i - 256*a^4*c^3*d^3*f^2) - 4096*a^8*c*d^2*f^4*(c + d*tan(e + f*x))^(1/2)*(-(d^11*45i - 15*c*d^10 + c^2*d^9*105i - 95*c^3*d^8 + c^4*d^7*20i - 72*c^5*d^6 - c^6*d^5*40i + 8*c^7*d^4 + a^4*f^2*(4*(256*d^6 + 256*c^2*d^4)*((((105*c^3*d^13)/16 - (35*c*d^15)/8 + (295*c^5*d^11)/16 + (5*c^7*d^9)/16 -

```
(105*c^9*d^7)/16 + (5*c^11*d^5)/8)*1i)/(a^8*f^4) - ((49*d^16)/64 - (149*c^
2*d^14)/16 - (235*c^4*d^12)/32 + (215*c^6*d^10)/16 + (505*c^8*d^8)/64 - (11
*c^10*d^6)/4 + (c^12*d^4)/16)/(a^8*f^4)) + (((45*d^11 + 105*c^2*d^9 + 20*c^
4*d^7 - 40*c^6*d^5)*1i)/(a^4*f^2) - (15*c*d^10 + 95*c^3*d^8 + 72*c^5*d^6 -
8*c^7*d^4)/(a^4*f^2))^2)^(1/2))/(512*a^4*f^2*(d^6 + c^2*d^4)))^(1/2))*(-(d^
11*45i - 15*c*d^10 + c^2*d^9*105i - 95*c^3*d^8 + c^4*d^7*20i - 72*c^5*d^6 -
 c^6*d^5*40i + 8*c^7*d^4 + a^4*f^2*(4*(256*d^6 + 256*c^2*d^4)*((((105*c^3*d
^13)/16 - (35*c*d^15)/8 + (295*c^5*d^11)/16 + (...
```

3.1118 $\int \frac{(c+d\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^3}\,dx$

Optimal. Leaf size=285

$$-\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{8a^3f}+\frac{(2ic^3+4c^2d-icd^2+2d^3)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{16a^3\sqrt{c+id}\,f}+$$

```
[Out] -1/8*I*(c-I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/a^3/f+1/
16*(2*I*c^3+4*c^2*d-I*c*d^2+2*d^3)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(
1/2))/a^3/f/(c+I*d)^(1/2)+1/8*(c+I*d)*(I*c+2*d)*(c+d*tan(f*x+e))^(1/2)/a/f/
(a+I*a*tan(f*x+e))^2+1/16*(2*I*c^2+5*c*d-4*I*d^2)*(c+d*tan(f*x+e))^(1/2)/f/
(a^3+I*a^3*tan(f*x+e))+1/6*(I*c-d)*(c+d*tan(f*x+e))^(3/2)/f/(a+I*a*tan(f*x+
e))^3
```

Rubi [A]
time = 0.67, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3639, 3676, 3677, 3620, 3618, 65, 214}

$$\frac{(2ic^2+5cd-4id^2)\sqrt{c+d\tan(e+fx)}}{16f(a^3+ia^3\tan(e+fx))}+\frac{(2ic^3+4c^2d-icd^2+2d^3)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{16a^3f\sqrt{c+id}}-\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{8a^3f}+\frac{(-d+ic)(c+d\tan(e+fx))^{3/2}}{6f(a+ia\tan(e+fx))^3}+\frac{(c+id)(2d+ic)\sqrt{c+d\tan(e+fx)}}{8af(a+ia\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^(5/2)/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] ((-1/8*I)*(c - I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/
(a^3*f) + (((2*I)*c^3 + 4*c^2*d - I*c*d^2 + 2*d^3)*ArcTanh[Sqrt[c + d*Tan[e
 + f*x]]/Sqrt[c + I*d]])/(16*a^3*Sqrt[c + I*d]*f) + ((c + I*d)*(I*c + 2*d)*
Sqrt[c + d*Tan[e + f*x]])/(8*a*f*(a + I*a*Tan[e + f*x])^2) + (((2*I)*c^2 +
5*c*d - (4*I)*d^2)*Sqrt[c + d*Tan[e + f*x]])/(16*f*(a^3 + I*a^3*Tan[e + f*x
])) + ((I*c - d)*(c + d*Tan[e + f*x])^(3/2))/(6*f*(a + I*a*Tan[e + f*x])^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^3}\,dx = \frac{(ic-d)(c+d\tan(e+fx))^{3/2}}{6f(a+ia\tan(e+fx))^3} - \frac{\int \frac{\sqrt{c+d\tan(e+fx)}\,\left(-\frac{3}{2}a\left(2c^2-3icd+d^2\right)-\frac{3}{2}a(c\cdots}{(a+ia\tan(e+fx))^2}}{6a^2}$$

$$= \frac{(c+id)(ic+2d)\sqrt{c+d\tan(e+fx)}}{8af(a+ia\tan(e+fx))^2} + \frac{(ic-d)(c+d\tan(e+fx))^{3/2}}{6f(a+ia\tan(e+fx))^3} + \frac{\int \cdots}{\cdots}$$

$$= \frac{(c+id)(ic+2d)\sqrt{c+d\tan(e+fx)}}{8af(a+ia\tan(e+fx))^2} + \frac{\left(2ic^2+5cd-4id^2\right)\sqrt{c+d\tan(e+\cdots}}{16f\left(a^3+ia^3\tan(e+fx)\right)}$$

$$= \frac{(c+id)(ic+2d)\sqrt{c+d\tan(e+fx)}}{8af(a+ia\tan(e+fx))^2} + \frac{\left(2ic^2+5cd-4id^2\right)\sqrt{c+d\tan(e+\cdots}}{16f\left(a^3+ia^3\tan(e+fx)\right)}$$

$$= \frac{(c+id)(ic+2d)\sqrt{c+d\tan(e+fx)}}{8af(a+ia\tan(e+fx))^2} + \frac{\left(2ic^2+5cd-4id^2\right)\sqrt{c+d\tan(e+\cdots}}{16f\left(a^3+ia^3\tan(e+fx)\right)}$$

$$= \frac{(c+id)(ic+2d)\sqrt{c+d\tan(e+fx)}}{8af(a+ia\tan(e+fx))^2} + \frac{\left(2ic^2+5cd-4id^2\right)\sqrt{c+d\tan(e+\cdots}}{16f\left(a^3+ia^3\tan(e+fx)\right)}$$

$$= -\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{8a^3f} + \frac{\left(2ic^3+4c^2d-icd^2+2d^3\right)\mathrm{t}\cdots}{16a^3\sqrt{\cdots}}$$

Mathematica [A]

time = 2.90, size = 324, normalized size = 1.14

$$\frac{\sec^3(e+fx)(\cos(fx)+i\sin(fx))^3\left(\frac{2\left(-i\sqrt{-c+id}\,\left(2c^3-4ic^2d-cd^2-2id^3\right)\mathrm{ArcTan}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{-c-id}}\right)-2\sqrt{-c-id}\,(ic+d)^3\mathrm{ArcTan}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{-c+id}}\right)\right)(\cos(3e)+i\sin(3e))}{\sqrt{-c-id}\,\sqrt{-c+id}}+\frac{2}{3}\cos(e+fx)(i\cos(3fx)+\sin(3fx))\left(7c^2+icd+6d^2+\left(13c^2-14icd-6d^2\right)\cos(2(e+fx))+\left(9ic^2+22cd-2id^2\right)\sin(2(e+fx))\right)\sqrt{c+d\tan(e+fx)}\right)}{32f(a+ia\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^(5/2)/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] (Sec[e + f*x]^3*(Cos[f*x] + I*Sin[f*x])^3*((2*((-I)*Sqrt[-c + I*d]*(2*c^3 -
 (4*I)*c^2*d - c*d^2 - (2*I)*d^3)*ArcTan[Sqrt[c + d*Tan[e + f*x]]/Sqrt[-c -
 I*d]] - 2*Sqrt[-c - I*d]*(I*c + d)^3*ArcTan[Sqrt[c + d*Tan[e + f*x]]/Sqrt[
-c + I*d]])*(Cos[3*e] + I*Sin[3*e]))/(Sqrt[-c - I*d]*Sqrt[-c + I*d]) + (2*C
os[e + f*x]*(I*Cos[3*f*x] + Sin[3*f*x])*(7*c^2 + I*c*d + 6*d^2 + (13*c^2 -
(14*I)*c*d - 6*d^2)*Cos[2*(e + f*x)] + ((9*I)*c^2 + 22*c*d - (2*I)*d^2)*Sin
[2*(e + f*x)])*Sqrt[c + d*Tan[e + f*x]])/3))/(32*f*(a + I*a*Tan[e + f*x])^3
)
```

Maple [A]

time = 0.42, size = 452, normalized size = 1.59

method	result
derivativedivides	$2d^4\left(-\frac{i(id-c)^{\frac{5}{2}}\arctan\left(\frac{\sqrt{c+d\tan(fx+e)}}{\sqrt{id-c}}\right)}{16d^4}+\frac{-\frac{d\left(ic^4d+ic^2d^3+4id^5+2c^5+5c^3d^2+7c\,d^4\right)(c+d\tan(fx+e))^{\frac{5}{2}}}{2\left(3ic^2d-id^3+c^3-3c\,d^2\right)}+2d(\ldots}{\ldots}\right.$
default	$2d^4\left(-\frac{i(id-c)^{\frac{5}{2}}\arctan\left(\frac{\sqrt{c+d\tan(fx+e)}}{\sqrt{id-c}}\right)}{16d^4}+\frac{-\frac{d\left(ic^4d+ic^2d^3+4id^5+2c^5+5c^3d^2+7c\,d^4\right)(c+d\tan(fx+e))^{\frac{5}{2}}}{2\left(3ic^2d-id^3+c^3-3c\,d^2\right)}+2d(\ldots}{\ldots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3*d^4*(-1/16*I*(I*d-c)^(5/2)/d^4*arctan((c+d*tan(f*x+e))^(1/2)/(I*d-c
)^(1/2))+1/16/d^4*((-1/2*d*(I*c^4*d+I*c^2*d^3+4*I*d^5+2*c^5+5*c^3*d^2+7*c*d
^4)/(3*I*c^2*d-I*d^3+c^3-3*c*d^2)*(c+d*tan(f*x+e))^(5/2)+2/3*d*(5*c^4*d^2-1
5*c^2*d^4-d^6+6*I*c^5*d+20*I*c^3*d^3-2*I*c*d^5+3*c^6)/(3*I*c^2*d-I*d^3+c^3-
3*c*d^2)*(c+d*tan(f*x+e))^(3/2)-1/2*d*c*(-5*c^4*d^2-20*c^2*d^4+3*d^6+7*I*c^
5*d+10*I*c^3*d^3-13*I*c*d^5+2*c^6)/(3*I*c^2*d-I*d^3+c^3-3*c*d^2)*(c+d*tan(f
*x+e))^(1/2))/(-d*tan(f*x+e)+I*d)^3-1/2*(-2*c^5*d-5*c^3*d^3-7*c*d^5+2*I*c^6
+5*I*c^4*d^2+5*I*c^2*d^4-2*I*d^6)/(3*I*c^2*d-I*d^3+c^3-3*c*d^2)/(-I*d-c)^(1
/2)*arctan((c+d*tan(f*x+e))^(1/2)/(-I*d-c)^(1/2))))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1282 vs. $2(233) = 466$.
time = 1.34, size = 1282, normalized size = 4.50

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="fricas"
)
[Out] 1/192*(6*a^3*f*sqrt(-(c^5 - 5*I*c^4*d - 10*c^3*d^2 + 10*I*c^2*d^3 + 5*c*d^4
 - I*d^5)/(a^6*f^2))*e^(6*I*f*x + 6*I*e)*log(2*(c^3 - 2*I*c^2*d - c*d^2 + (
I*a^3*f*e^(2*I*f*x + 2*I*e) + I*a^3*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e)
+ c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(c^5 - 5*I*c^4*d - 10*c^3*d^2 +
 10*I*c^2*d^3 + 5*c*d^4 - I*d^5)/(a^6*f^2)) + (c^3 - 3*I*c^2*d - 3*c*d^2 +
I*d^3)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/(c^2 - 2*I*c*d - d^2)) - 6
*a^3*f*sqrt(-(c^5 - 5*I*c^4*d - 10*c^3*d^2 + 10*I*c^2*d^3 + 5*c*d^4 - I*d^5
)/(a^6*f^2))*e^(6*I*f*x + 6*I*e)*log(2*(c^3 - 2*I*c^2*d - c*d^2 + (-I*a^3*f
*e^(2*I*f*x + 2*I*e) - I*a^3*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I
*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(c^5 - 5*I*c^4*d - 10*c^3*d^2 + 10*I*c
^2*d^3 + 5*c*d^4 - I*d^5)/(a^6*f^2)) + (c^3 - 3*I*c^2*d - 3*c*d^2 + I*d^3)*
e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/(c^2 - 2*I*c*d - d^2)) + 3*a^3*f*
sqrt(-(4*I*c^6 + 16*c^5*d - 20*I*c^4*d^2 - 15*I*c^2*d^4 - 4*c*d^5 - 4*I*d^6
)/((I*a^6*c - a^6*d)*f^2))*e^(6*I*f*x + 6*I*e)*log(-1/16*(2*c^4 - 2*I*c^3*d
 + 3*c^2*d^2 - 3*I*c*d^3 + 2*d^4 - ((I*a^3*c - a^3*d)*f*e^(2*I*f*x + 2*I*e)
 + (I*a^3*c - a^3*d)*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(
2*I*f*x + 2*I*e) + 1))*sqrt(-(4*I*c^6 + 16*c^5*d - 20*I*c^4*d^2 - 15*I*c^2*
d^4 - 4*c*d^5 - 4*I*d^6)/((I*a^6*c - a^6*d)*f^2)) + (2*c^4 - 4*I*c^3*d - c^
2*d^2 - 2*I*c*d^3)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/((I*a^3*c - a^
3*d)*f)) - 3*a^3*f*sqrt(-(4*I*c^6 + 16*c^5*d - 20*I*c^4*d^2 - 15*I*c^2*d^4
- 4*c*d^5 - 4*I*d^6)/((I*a^6*c - a^6*d)*f^2))*e^(6*I*f*x + 6*I*e)*log(-1/16
*(2*c^4 - 2*I*c^3*d + 3*c^2*d^2 - 3*I*c*d^3 + 2*d^4 - ((-I*a^3*c + a^3*d)*f
*e^(2*I*f*x + 2*I*e) + (-I*a^3*c + a^3*d)*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2
*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(4*I*c^6 + 16*c^5*d - 20*
I*c^4*d^2 - 15*I*c^2*d^4 - 4*c*d^5 - 4*I*d^6)/((I*a^6*c - a^6*d)*f^2)) + (2
*c^4 - 4*I*c^3*d - c^2*d^2 - 2*I*c*d^3)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x -
2*I*e)/((I*a^3*c - a^3*d)*f)) + 2*(2*I*c^2 - 4*c*d - 2*I*d^2 + (11*I*c^2 +
18*c*d - 4*I*d^2)*e^(6*I*f*x + 6*I*e) + (18*I*c^2 + 17*c*d + 2*I*d^2)*e^(4*
I*f*x + 4*I*e) + (9*I*c^2 - 5*c*d + 4*I*d^2)*e^(2*I*f*x + 2*I*e))*sqrt(((c
- I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-6*I*f
*x - 6*I*e)/(a^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\left(\int \frac{c^2\sqrt{c+d\tan(e+fx)}}{\tan^3(e+fx)-3i\tan^2(e+fx)-3\tan(e+fx)+i}\,dx+\int \frac{d^2\sqrt{c+d\tan(e+fx)}\tan^2(e+fx)}{\tan^3(e+fx)-3i\tan^2(e+fx)-3\tan(e+fx)+i}\,dx+\int \frac{2cd\sqrt{c+d\tan(e+fx)}\tan(e+fx)}{\tan^3(e+fx)-3i\tan^2(e+fx)-3\tan(e+fx)+i}\,dx\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(5/2)/(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] I*(Integral(c**2*sqrt(c + d*tan(e + f*x))/(tan(e + f*x)**3 - 3*I*tan(e + f*
x)**2 - 3*tan(e + f*x) + I), x) + Integral(d**2*sqrt(c + d*tan(e + f*x))*ta
n(e + f*x)**2/(tan(e + f*x)**3 - 3*I*tan(e + f*x)**2 - 3*tan(e + f*x) + I),
 x) + Integral(2*c*d*sqrt(c + d*tan(e + f*x))*tan(e + f*x)/(tan(e + f*x)**3
 - 3*I*tan(e + f*x)**2 - 3*tan(e + f*x) + I), x))/a**3
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 602 vs. $2(233) = 466$.
time = 0.82, size = 602, normalized size = 2.11

$$-\frac{(2ic^3+4c^2d-icd^2+2d^3)\arctan\left(\frac{2\left(\sqrt{d\tan(fx+e)+c}\,c-\sqrt{c^2+d^2}\sqrt{d\tan(fx+e)+c}\right)}{c\sqrt{-2c+2\sqrt{c^2+d^2}}+i\sqrt{-2c+2\sqrt{c^2+d^2}}\,d-\sqrt{c^2+d^2}\sqrt{-2c+2\sqrt{c^2+d^2}}}\right)}{8a^3\sqrt{-2c+2\sqrt{c^2+d^2}}\,f\left(\frac{id}{c-\sqrt{c^2+d^2}}+1\right)}-\frac{(-ic^3-3c^2d+3icd^2+d^3)\arctan\left(\frac{2\left(\sqrt{d\tan(fx+e)+c}\,c-\sqrt{c^2+d^2}\sqrt{d\tan(fx+e)+c}\right)}{c\sqrt{-2c+2\sqrt{c^2+d^2}}-i\sqrt{-2c+2\sqrt{c^2+d^2}}\,d-\sqrt{c^2+d^2}\sqrt{-2c+2\sqrt{c^2+d^2}}}\right)}{4a^3\sqrt{-2c+2\sqrt{c^2+d^2}}\,f\left(-\frac{id}{c-\sqrt{c^2+d^2}}+1\right)}+\frac{6(d\tan(fx+e)+c)^{\frac{5}{2}}c^2d-12(d\tan(fx+e)+c)^{\frac{3}{2}}c^3d+6\sqrt{d\tan(fx+e)+c}\,c^4d-15i(d\tan(fx+e)+c)^{\frac{5}{2}}cd^2+12i(d\tan(fx+e)+c)^{\frac{3}{2}}c^2d^2+3i\sqrt{d\tan(fx+e)+c}\,c^3d^2-12(d\tan(fx+e)+c)^{\frac{5}{2}}d^3-20(d\tan(fx+e)+c)^{\frac{3}{2}}cd^3+12\sqrt{d\tan(fx+e)+c}\,c^2d^3+4i(d\tan(fx+e)+c)^{\frac{3}{2}}d^4+9i\sqrt{d\tan(fx+e)+c}\,cd^4}{48(d\tan(fx+e)-id)^3a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/8*(2*I*c^3 + 4*c^2*d - I*c*d^2 + 2*d^3)*arctan(2*(sqrt(d*tan(f*x + e) +
c)*c - sqrt(c^2 + d^2)*sqrt(d*tan(f*x + e) + c))/(c*sqrt(-2*c + 2*sqrt(c^2
+ d^2)) + I*sqrt(-2*c + 2*sqrt(c^2 + d^2))*d - sqrt(c^2 + d^2)*sqrt(-2*c +
2*sqrt(c^2 + d^2))))/(a^3*sqrt(-2*c + 2*sqrt(c^2 + d^2))*f*(I*d/(c - sqrt(c
^2 + d^2)) + 1)) - 1/4*(-I*c^3 - 3*c^2*d + 3*I*c*d^2 + d^3)*arctan(2*(sqrt(
d*tan(f*x + e) + c)*c - sqrt(c^2 + d^2)*sqrt(d*tan(f*x + e) + c))/(c*sqrt(-
2*c + 2*sqrt(c^2 + d^2)) - I*sqrt(-2*c + 2*sqrt(c^2 + d^2))*d - sqrt(c^2 +
d^2)*sqrt(-2*c + 2*sqrt(c^2 + d^2))))/(a^3*sqrt(-2*c + 2*sqrt(c^2 + d^2))*f
*(-I*d/(c - sqrt(c^2 + d^2)) + 1)) + 1/48*(6*(d*tan(f*x + e) + c)^(5/2)*c^2
*d - 12*(d*tan(f*x + e) + c)^(3/2)*c^3*d + 6*sqrt(d*tan(f*x + e) + c)*c^4*d
 - 15*I*(d*tan(f*x + e) + c)^(5/2)*c*d^2 + 12*I*(d*tan(f*x + e) + c)^(3/2)*
c^2*d^2 + 3*I*sqrt(d*tan(f*x + e) + c)*c^3*d^2 - 12*(d*tan(f*x + e) + c)^(5
/2)*d^3 - 20*(d*tan(f*x + e) + c)^(3/2)*c*d^3 + 12*sqrt(d*tan(f*x + e) + c)
*c^2*d^3 + 4*I*(d*tan(f*x + e) + c)^(3/2)*d^4 + 9*I*sqrt(d*tan(f*x + e) + c
)*c*d^4)/((d*tan(f*x + e) - I*d)^3*a^3*f)
```

Mupad [B]
time = 9.34, size = 2500, normalized size = 8.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(5/2)/(a + a*tan(e + f*x)*1i)^3,x)
```

```
[Out] - atan((((2*a^3*f*(1536*a^6*c*d^5*f^2 + a^6*c^2*d^4*f^2*2560i - 1024*a^6*c^
3*d^3*f^2) - 65536*a^12*c*d^2*f^4*(c + d*tan(e + f*x))^(1/2)*(-(20*c*d^10 +
 c^2*d^9*55i - 35*c^3*d^8 + c^4*d^7*40i - 72*c^5*d^6 - c^6*d^5*40i + 8*c^7*
```

d^4 - 4*a^6*f^2*(((5*c*d^10 - (35*c^3*d^8)/4 - 18*c^5*d^6 + 2*c^7*d^4)/(a^6*f^2) + (((55*c^2*d^9)/4 + 10*c^4*d^7 - 10*c^6*d^5)*1i)/(a^6*f^2))^2 - 4*(256*d^6 + 256*c^2*d^4)*((((5*c*d^15)/256 + (35*c^3*d^13)/512 - (25*c^5*d^11)/128 - (285*c^7*d^9)/512 + (55*c^9*d^7)/128 - (5*c^11*d^5)/128)*1i)/(a^12*f^4) - ((21*c^2*d^14)/1024 - d^16/256 + (225*c^4*d^12)/1024 + (155*c^6*d^10)/1024 - (665*c^8*d^8)/1024 + (11*c^10*d^6)/64 - (c^12*d^4)/256)/(a^12*f^4)))^(1/2))/(2048*a^6*f^2*(d^6 + c^2*d^4)))^(1/2))*(-(20*c*d^10 + c^2*d^9*55i - 35*c^3*d^8 + c^4*d^7*40i - 72*c^5*d^6 - c^6*d^5*40i + 8*c^7*d^4 - 4*a^6*f^2*(((5*c*d^10 - (35*c^3*d^8)/4 - 18*c^5*d^6 + 2*c^7*d^4)/(a^6*f^2) + (((55*c^2*d^9)/4 + 10*c^4*d^7 - 10*c^6*d^5)*1i)/(a^6*f^2))^2 - 4*(256*d^6 + 256*c^2*d^4)*((((5*c*d^15)/256 + (35*c^3*d^13)/512 - (25*c^5*d^11)/128 - (285*c^7*d^9)/512 + (55*c^9*d^7)/128 - (5*c^11*d^5)/128)*1i)/(a^12*f^4) - ((21*c^2*d^14)/1024 - d^16/256 + (225*c^4*d^12)/1024 + (155*c^6*d^10)/1024 - (665*c^8*d^8)/1024 + (11*c^10*d^6)/64 - (c^12*d^4)/256)/(a^12*f^4)))^(1/2))/(2048*a^6*f^2*(d^6 + c^2*d^4)))^(1/2) + 32*a^6*f^2*(c + d*tan(e + f*x))^(1/2)*(c*d^7*20i + 8*d^8 - 45*c^2*d^6 - c^3*d^5*80i + 80*c^4*d^4 + c^5*d^3*40i - 8*c^6*d^2))*(-(20*c*d^10 + c^2*d^9*55i - 35*c^3*d^8 + c^4*d^7*40i - 72*c^5*d^6 - c^6*d^5*40i + 8*c^7*d^4 - 4*a^6*f^2*(((5*c*d^10 - (35*c^3*d^8)/4 - 18*c^5*d^6 + 2*c^7*d^4)/(a^6*f^2) + (((55*c^2*d^9)/4 + 10*c^4*d^7 - 10*c^6*d^5)*1i)/(a^6*f^2))^2 - 4*(256*d^6 + 256*c^2*d^4)*((((5*c*d^15)/256 + (35*c^3*d^13)/512 - (25*c^5*d^11)/128 - (285*c^7*d^9)/512 + (55*c^9*d^7)/128 - (5*c^11*d^5)/128)*1i)/(a^12*f^4) - ((21*c^2*d^14)/1024 - d^16/256 + (225*c^4*d^12)/1024 + (155*c^6*d^10)/1024 - (665*c^8*d^8)/1024 + (11*c^10*d^6)/64 - (c^12*d^4)/256)/(a^12*f^4)))^(1/2))/(2048*a^6*f^2*(d^6 + c^2*d^4)))^(1/2)*1i - ((2*a^3*f*(1536*a^6*c*d^5*f^2 + a^6*c^2*d^4*f^2*2560i - 1024*a^6*c^3*d^3*f^2) + 65536*a^12*c*d^2*f^4*(c + d*tan(e + f*x))^(1/2)*(-(20*c*d^10 + c^2*d^9*55i - 35*c^3*d^8 + c^4*d^7*40i - 72*c^5*d^6 - c^6*d^5*40i + 8*c^7*d^4 - 4*a^6*f^2*(((5*c*d^10 - (35*c^3*d^8)/4 - 18*c^5*d^6 + 2*c^7*d^4)/(a^6*f^2) + (((55*c^2*d^9)/4 + 10*c^4*d^7 - 10*c^6*d^5)*1i)/(a^6*f^2))^2 - 4*(256*d^6 + 256*c^2*d^4)*((((5*c*d^15)/256 + (35*c^3*d^13)/512 - (25*c^5*d^11)/128 - (285*c^7*d^9)/512 + (55*c^9*d^7)/128 - (5*c^11*d^5)/128)*1i)/(a^12*f^4) - ((21*c^2*d^14)/1024 - d^16/256 + (225*c^4*d^12)/1024 + (155*c^6*d^10)/1024 - (665*c^8*d^8)/1024 + (11*c^10*d^6)/64 - (c^12*d^4)/256)/(a^12*f^4)))^(1/2))/(2048*a^6*f^2*(d^6 + c^2*d^4)))^(1/2))*(-(20*c*d^10 + c^2*d^9*55i - 35*c^3*d^8 + c^4*d^7*40i - 72*c^5*d^6 - c^6*d^5*40i + 8*c^7*d^4 - 4*a^6*f^2*(((5*c*d^10 - (35*c^3*d^8)/4 - 18*c^5*d^6 + 2*c^7*d^4)/(a^6*f^2) + (((55*c^2*d^9)/4 + 10*c^4*d^7 - 10*c^6*d^5)*1i)/(a^6*f^2))^2 - 4*(256*d^6 + 256*c^2*d^4)*((((5*c*d^15)/256 + (35*c^3*d^13)/512 - (25*c^5*d^11)/128 - (285*c^7*d^9)/512 + (55*c^9*d^7)/128 - (5*c^11*d^5)/128)*1i)/(a^12*f^4) - ((21*c^2*d^14)/1024 - d^16/256 + (225*c^4*d^12)/1024 + (155*c^6*d^10)/1024 - (665*c^8*d^8)/1024 + (11*c^10*d^6)/64 - (c^12*d^4)/256)/(a^12*f^4)))^(1/2))/(2048*a^6*f^2*(d^6 + c^2*d^4)))^(1/2) - 32*a^6*f^2*(c + d*tan(e + f*x))^(1/2)*(c*d^7*20i + 8*d^8 - 45*c^2*d^6 - c^3*d^5*80i + 80*c^4*d^4 + c^5*d^3*40i - 8*c^6*d^2))*(-(20*c*d^10 + c^2*d^9*55i - 35*c^3*d^8 + c^4*d^7*40i - 72*c^5*d^6 - c^6*d^5*40i + 8*c^7*d^4 - 4*a^6*f^2*(((5*c*d^10 - (35*c^3*d^8)/4 - 18*c^5*d^

6 + 2*c^7*d^4)/(a^6*f^2) + (((55*c^2*d^9)/4 + 10*c^4*d^7 - 10*c^6*d^5)*1i)/
(a^6*f^2))^2 - 4*(256*d^6 + 256*c^2*d^4)*((((5*c*d^15)/256 + (35*c^3*d^13)/
512 - (25*c^5*d^11)/128 - (285*c^7*d^9)/512 + (55*c^9*d^7)/128 - (5*c^11*d^
5)/128)*1i)/(a^12*f^4) - ((21*c^2*d^14)/1024 - d^16/256 + (225*c^4*d^12)/10
24 + (155*c^6*d^10)/1024 - (665*c^8*d^8)/1024 + (11*c^10*d^6)/64 - (c^12*d^
4)/256)/(a^12*f^4)))^(1/2))/(2048*a^6*f^2*(d^6 + c^2*d^4)))^(1/2)*1i)/(((2*
a^3*f*(1536*a^6*c*d^5*f^2 + a^6*c^2*d^4*f^2*2560i - 1024*a^6*c^3*d^3*f^2) -
 65536*a^12*c*d^2*f^4*(c + d*tan(e + f*x))^(1/2)*(-(20*c*d^10 + c^2*d^9*55i
 - 35*c^3*d^8 + c^4*d^7*40i - 72*c^5*d^6 - c^6*d^5*40i + 8*c^7*d^4 - 4*a^6*
f^2*(((5*c*d^10 - (35*c^3*d^8)/4 - 18*c^5*d^6 + 2*c^7*d^4)/(a^6*f^2) + (((5
5*c^2*d^9)/4 + 10*c^4*d^7 - 10*c^6*d^5)*1i)/(a^6*f^2))^2 - 4*(256*d^6 + 256
*c^2*d^4)*((((5*c*d^15)/256 + (35*c^3*d^13)/512 - (25*c^5*d^11)/128 - (285*
c^7*d^9)/512 + (55*c^9*d^7)/128 - (5*c^11*d^5)/128)*1i)/(a^12*f^4) - ((21*c
^2*d^14)/1024 - d^16/256 + (225*c^4*d^12)/1024 + (155*c^6*d^10)/1024 - (665
*c^8*d^8)/1024 + (11*c^10*d^6)/64 - (c^12*d^4)/256)/(a^12*f^4)))^(1/2))/(20
48*a^6*f^2*(d^6 + c^2*d^4)))^(1/2))*(-(20*c*d^10 + c^2*d^9*55i - 35*c^3*d^8
 + c^4*d^7*40i - 72*c^5*d^6 - c^6*d^5*40i + 8*c^7*d^4 - 4*a^6*f^2*(((5*c*d^
10 - (35*c^3*d^8)/4 - 18*c^5*d^6 + 2*c^7*d^4)/(a^6*f^2) + (((55*c^2*d^9)/4
+ 10*c^4*d^7 - 10*c^6*d^5)*1i)/(a^6*f^2))^2 - 4...

3.1119 $\int \frac{(a+ia\tan(e+fx))^3}{\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=126

$$-\frac{8ia^3\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}\,f}+\frac{4a^3(ic-4d)\sqrt{c+d\tan(e+fx)}}{3d^2f}-\frac{2(a^3+ia^3\tan(e+fx))\sqrt{c+d}}{3df}$$

```
[Out] -8*I*a^3*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/f/(c-I*d)^(1/2)+4/3*
a^3*(I*c-4*d)*(c+d*tan(f*x+e))^(1/2)/d^2/f-2/3*(c+d*tan(f*x+e))^(1/2)*(a^3+
I*a^3*tan(f*x+e))/d/f
```

Rubi [A]
time = 0.20, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3637, 3673, 3618, 65, 214}

$$\frac{4a^3(-4d+ic)\sqrt{c+d\tan(e+fx)}}{3d^2f}-\frac{2(a^3+ia^3\tan(e+fx))\sqrt{c+d\tan(e+fx)}}{3df}-\frac{8ia^3\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f\sqrt{c-id}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^3/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((-8*I)*a^3*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/(Sqrt[c - I*d]
*f) + (4*a^3*(I*c - 4*d)*Sqrt[c + d*Tan[e + f*x]])/(3*d^2*f) - (2*(a^3 + I*
a^3*Tan[e + f*x])*Sqrt[c + d*Tan[e + f*x]])/(3*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
```

```
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^3}{\sqrt{c+d\tan(e+fx)}}\,dx = -\frac{2(a^3+ia^3\tan(e+fx))\sqrt{c+d\tan(e+fx)}}{3df} + \frac{(2a)\int \frac{(a+ia\tan(e+fx))(a(ic+2d}{\sqrt{c+d\tan}}}{3d}$$

$$= \frac{4a^3(ic-4d)\sqrt{c+d\tan(e+fx)}}{3d^2f} - \frac{2(a^3+ia^3\tan(e+fx))\sqrt{c+d\tan(e+}}{3df}$$

$$= \frac{4a^3(ic-4d)\sqrt{c+d\tan(e+fx)}}{3d^2f} - \frac{2(a^3+ia^3\tan(e+fx))\sqrt{c+d\tan(e+}}{3df}$$

$$= \frac{4a^3(ic-4d)\sqrt{c+d\tan(e+fx)}}{3d^2f} - \frac{2(a^3+ia^3\tan(e+fx))\sqrt{c+d\tan(e+}}{3df}$$

$$= -\frac{8ia^3\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}\,f} + \frac{4a^3(ic-4d)\sqrt{c+d\tan(e+fx)}}{3d^2f}$$

Mathematica [A]

time = 3.78, size = 168, normalized size = 1.33

$$\frac{a^3(\cos(e+fx)+i\sin(e+fx))^3\left(-\frac{8ie^{-3ie}\tanh^{-1}\left(\frac{\sqrt{c-\frac{id(-1+e^{2i(e+fx)})}{1+e^{2i(e+fx)}}}}{\sqrt{c-id}}\right)}{\sqrt{c-id}}+\frac{2(i\cos(3e)+\sin(3e))(2c+9id-d\tan(e+fx))\sqrt{c+d\tan(e+fx)}}{3d^2}\right)}{f(\cos(fx)+i\sin(fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] (a^3*(Cos[e + f*x] + I*Sin[e + f*x])^3*(((-8*I)*ArcTanh[Sqrt[c - (I*d*(-1 +
 E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))]/Sqrt[c - I*d]])/(Sqrt[c -
 I*d]*E^((3*I)*e)) + (2*(I*Cos[3*e] + Sin[3*e])*(2*c + (9*I)*d - d*Tan[e +
f*x])*Sqrt[c + d*Tan[e + f*x]])/(3*d^2)))/(f*(Cos[f*x] + I*Sin[f*x])^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 792 vs. 2(107) = 214.
time = 0.31, size = 793, normalized size = 6.29

method	result
derivativedivides	$2a^3\left(-\frac{i(c+d\tan(fx+e))^{\frac{3}{2}}}{3}+ic\sqrt{c+d\tan(fx+e)}-3d\sqrt{c+d\tan(fx+e)}-4d^2\left(\frac{\left(i\sqrt{c^2+d^2}+ic-\right.}{}\right.\right.$
default	$2a^3\left(-\frac{i(c+d\tan(fx+e))^{\frac{3}{2}}}{3}+ic\sqrt{c+d\tan(fx+e)}-3d\sqrt{c+d\tan(fx+e)}-4d^2\left(\frac{\left(i\sqrt{c^2+d^2}+ic-\right.}{}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3/d^2*(-1/3*I*(c+d*tan(f*x+e))^(3/2)+I*c*(c+d*tan(f*x+e))^(1/2)-3*d*(
c+d*tan(f*x+e))^(1/2)-4*d^2*(1/2/(2*(c^2+d^2)^(1/2)+2*c)^(1/2)/(c^2+d^2)^(1
/2)*(1/2*(I*(c^2+d^2)^(1/2)+I*c-d)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)
*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(I*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*c-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d-1/2*(I*(c^2+d^2)^(1/2)+I*c-d)*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*ta
n(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2
)))+1/2/(2*(c^2+d^2)^(1/2)+2*c)^(1/2)/(c^2+d^2)^(1/2)/((c^2+d^2)^(1/2)*c+c^
2+d^2)*(1/2*(-2*I*(c^2+d^2)^(1/2)*c^2-I*d^2*(c^2+d^2)^(1/2)-2*I*c^3-2*I*c*d
^2+c*d*(c^2+d^2)^(1/2)+c^2*d+d^3)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(I*(c^2+d^2)^(1/2)*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*c^2+I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3+I*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*c*d^2-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d
-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^3+1/2*
(-2*I*(c^2+d^2)^(1/2)*c^2-I*d^2*(c^2+d^2)^(1/2)-2*I*c^3-2*I*c*d^2+c*d*(c^2+
d^2)^(1/2)+c^2*d+d^3)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c
)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*
(c^2+d^2)^(1/2)-2*c)^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^3/sqrt(d*tan(f*x + e) + c), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 443 vs. 2(106) = 212.
time = 0.85, size = 443, normalized size = 3.52

$$\frac{3\left(d^2 f e^{(2i fx+2i e)}+d^2 f\right)\sqrt{-\frac{64i a^6}{(ic+d)f^2}}\log\left(\frac{\left(8a^3c+\sqrt{-\frac{64i a^6}{(ic+d)f^2}}\left((ic+d)fe^{(2i fx+2i e)}+(ic+d)f\right)\sqrt{\frac{(c-id)e^{(2i fx+2i e)}+c+id}{e^{(2i fx+2i e)}+1}}+8\left(a^3c-ia^3d\right)e^{(2i fx+2i e)}\right)e^{(-2i fx-2i e)}}{4a^3}\right)-3\left(d^2 f e^{(2i fx+2i e)}+d^2 f\right)\sqrt{-\frac{64i a^6}{(ic+d)f^2}}\log\left(\frac{\left(8a^3c+\sqrt{-\frac{64i a^6}{(ic+d)f^2}}\left((-ic-d)fe^{(2i fx+2i e)}+(-ic-d)f\right)\sqrt{\frac{(c-id)e^{(2i fx+2i e)}+c+id}{e^{(2i fx+2i e)}+1}}+8\left(a^3c-ia^3d\right)e^{(2i fx+2i e)}\right)e^{(-2i fx-2i e)}}{4a^3}\right)-16\left(-ia^3c+4a^3d+\left(-ia^3c+5a^3d\right)e^{(2i fx+2i e)}\right)\sqrt{\frac{(c-id)e^{(2i fx+2i e)}+c+id}{e^{(2i fx+2i e)}+1}}}{12\left(d^2 f e^{(2i fx+2i e)}+d^2 f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/12*(3*(d^2*f*e^(2*I*f*x + 2*I*e) + d^2*f)*sqrt(-64*I*a^6/((I*c + d)*f^2))
*log(1/4*(8*a^3*c + sqrt(-64*I*a^6/((I*c + d)*f^2))*((I*c + d)*f*e^(2*I*f*x
 + 2*I*e) + (I*c + d)*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^
(2*I*f*x + 2*I*e) + 1)) + 8*(a^3*c - I*a^3*d)*e^(2*I*f*x + 2*I*e))*e^(-2*I*
f*x - 2*I*e)/a^3) - 3*(d^2*f*e^(2*I*f*x + 2*I*e) + d^2*f)*sqrt(-64*I*a^6/((
```

```
I*c + d)*f^2))*log(1/4*(8*a^3*c + sqrt(-64*I*a^6/((I*c + d)*f^2))*((-I*c -
d)*f*e^(2*I*f*x + 2*I*e) + (-I*c - d)*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e
) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1)) + 8*(a^3*c - I*a^3*d)*e^(2*I*f*x +
2*I*e))*e^(-2*I*f*x - 2*I*e)/a^3) - 16*(-I*a^3*c + 4*a^3*d + (-I*a^3*c + 5*
a^3*d)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/
(e^(2*I*f*x + 2*I*e) + 1)))/(d^2*f*e^(2*I*f*x + 2*I*e) + d^2*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-ia^3\left(\int \frac{i}{\sqrt{c+d\tan{(e+fx)}}}\,dx+\int\left(-\frac{3\tan{(e+fx)}}{\sqrt{c+d\tan{(e+fx)}}}\right)dx+\int\frac{\tan^3{(e+fx)}}{\sqrt{c+d\tan{(e+fx)}}}\,dx+\int\left(-\frac{3i\tan^2{(e+fx)}}{\sqrt{c+d\tan{(e+fx)}}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3/(c+d*tan(f*x+e))**(1/2),x)
```

```
[Out] -I*a**3*(Integral(I/sqrt(c + d*tan(e + f*x)), x) + Integral(-3*tan(e + f*x)
/sqrt(c + d*tan(e + f*x)), x) + Integral(tan(e + f*x)**3/sqrt(c + d*tan(e +
 f*x)), x) + Integral(-3*I*tan(e + f*x)**2/sqrt(c + d*tan(e + f*x)), x))
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(106) = 212.
time = 0.68, size = 243, normalized size = 1.93

$$\frac{16i\,a^3\arctan\left(\frac{2\left(\sqrt{d\tan{(fx+e)}+c}\,c-\sqrt{c^2+d^2}\,\sqrt{d\tan{(fx+e)}+c}\right)}{c\sqrt{-2c+2\sqrt{c^2+d^2}}-i\sqrt{-2c+2\sqrt{c^2+d^2}}\,d-\sqrt{c^2+d^2}\,\sqrt{-2c+2\sqrt{c^2+d^2}}}\right)}{\sqrt{-2c+2\sqrt{c^2+d^2}}\,f\left(-\frac{id}{c-\sqrt{c^2+d^2}}+1\right)}-\frac{2\left(i\,(d\tan{(fx+e)}+c)^{\frac{3}{2}}a^3d^4f^2-3i\sqrt{d\tan{(fx+e)}+c}\,a^3cd^4f^2+9\sqrt{d\tan{(fx+e)}+c}\,a^3d^5f^2\right)}{3\,d^6f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 16*I*a^3*arctan(2*(sqrt(d*tan(f*x + e) + c)*c - sqrt(c^2 + d^2)*sqrt(d*tan(
f*x + e) + c))/(c*sqrt(-2*c + 2*sqrt(c^2 + d^2)) - I*sqrt(-2*c + 2*sqrt(c^2
 + d^2))*d - sqrt(c^2 + d^2)*sqrt(-2*c + 2*sqrt(c^2 + d^2))))/(sqrt(-2*c +
2*sqrt(c^2 + d^2))*f*(-I*d/(c - sqrt(c^2 + d^2)) + 1)) - 2/3*(I*(d*tan(f*x
+ e) + c)^(3/2)*a^3*d^4*f^2 - 3*I*sqrt(d*tan(f*x + e) + c)*a^3*c*d^4*f^2 +
9*sqrt(d*tan(f*x + e) + c)*a^3*d^5*f^2)/(d^6*f^3)
```

Mupad [**B**]
time = 6.55, size = 119, normalized size = 0.94

$$-\left(\frac{a^3\,\left(c-d\,1\mathrm{i}\right)\,2\mathrm{i}}{d^2\,f}-\frac{a^3\,\left(c+d\,1\mathrm{i}\right)\,4\mathrm{i}}{d^2\,f}\right)\,\sqrt{c+d\,\tan\left(e+f\,x\right)}-\frac{a^3\,{\left(c+d\,\tan\left(e+f\,x\right)\right)}^{3/2}\,2\mathrm{i}}{3\,d^2\,f}+\frac{a^3\,\mathrm{atan}\left(\frac{\sqrt{c+d\,\tan\left(e+f\,x\right)}}{\sqrt{-c+d\,1\mathrm{i}}}\right)\,8\mathrm{i}}{f\,\sqrt{-c+d\,1\mathrm{i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^3/(c + d*tan(e + f*x))^(1/2),x)
```

```
[Out] (a^3*atan((c + d*tan(e + f*x))^(1/2)/(d*1i - c)^(1/2))*8i)/(f*(d*1i - c)^(1
/2)) - (a^3*(c + d*tan(e + f*x))^(3/2)*2i)/(3*d^2*f) - ((a^3*(c - d*1i)*2i)
/(d^2*f) - (a^3*(c + d*1i)*4i)/(d^2*f))*(c + d*tan(e + f*x))^(1/2)
```

3.1120 $\int \frac{(a+ia\tan(e+fx))^2}{\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=74

$$-\frac{4ia^2\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}\,f}-\frac{2a^2\sqrt{c+d\tan(e+fx)}}{df}$$

```
[Out] -4*I*a^2*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/f/(c-I*d)^(1/2)-2*a^
2*(c+d*tan(f*x+e))^(1/2)/d/f
```

Rubi [A]
time = 0.10, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3624, 3618, 65, 214}

$$-\frac{2a^2\sqrt{c+d\tan(e+fx)}}{df}-\frac{4ia^2\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f\sqrt{c-id}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((-4*I)*a^2*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/(Sqrt[c - I*d]
*f) - (2*a^2*Sqrt[c + d*Tan[e + f*x]])/(d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
```

```
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^2}{\sqrt{c+d\tan(e+fx)}}\,dx = -\frac{2a^2\sqrt{c+d\tan(e+fx)}}{df} + \int \frac{2a^2+2ia^2\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx$$

$$= -\frac{2a^2\sqrt{c+d\tan(e+fx)}}{df} + \frac{(4ia^4)\,\mathrm{Subst}\left(\int \frac{1}{(-4a^4+2a^2x)\sqrt{c-\frac{idx}{2a^2}}}\,dx, x, 2ia^2\right.}{f}$$

$$= -\frac{2a^2\sqrt{c+d\tan(e+fx)}}{df} - \frac{(16a^6)\,\mathrm{Subst}\left(\int \frac{1}{-4a^4-\frac{4ia^4c}{d}+\frac{4ia^4x^2}{d}}\,dx, x, \sqrt{c+d\,\mathrm{t}\right.}{df}$$

$$= -\frac{4ia^2\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}\,f} - \frac{2a^2\sqrt{c+d\tan(e+fx)}}{df}$$

Mathematica [A]
time = 2.14, size = 95, normalized size = 1.28

$$\frac{2a^2\left(-\frac{2i\tanh^{-1}\left(\frac{\sqrt{c-\frac{id(-1+e^{2i(e+fx)})}{1+e^{2i(e+fx)}}}}{\sqrt{c-id}}\right)}{\sqrt{c-id}} - \frac{\sqrt{c+d\tan(e+fx)}}{d}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] (2*a^2*(((-2*I)*ArcTanh[Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((
2*I)*(e + f*x)))]/Sqrt[c - I*d]])/Sqrt[c - I*d] - Sqrt[c + d*Tan[e + f*x]]/
d))/f
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 758 vs. $2(63) = 126$.
time = 0.29, size = 759, normalized size = 10.26

method	result
derivativedivides	$2a^2\left(-\sqrt{c+d\tan(fx+e)}-2d\left(\frac{\left(i\sqrt{c^2+d^2}+ic-d\right)\ln\left(d\tan(fx+e)+c+\sqrt{c+d\tan(fx+e)}\,\sqrt{\ }\right.}{2}\right.\right.$
default	$2a^2\left(-\sqrt{c+d\tan(fx+e)}-2d\left(\frac{\left(i\sqrt{c^2+d^2}+ic-d\right)\ln\left(d\tan(fx+e)+c+\sqrt{c+d\tan(fx+e)}\,\sqrt{\ }\right.}{2}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^2/d*(-(c+d*tan(f*x+e))^(1/2)-2*d*(1/2/(2*(c^2+d^2)^(1/2)+2*c)^(1/2)/(
c^2+d^2)^(1/2)*(1/2*(I*(c^2+d^2)^(1/2)+I*c-d)*ln(d*tan(f*x+e)+c+(c+d*tan(f*
x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(I*(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)*c-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d-1/2*(I*(c^2+d^2)^(1/2)
+I*c-d)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan
((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2
)-2*c)^(1/2)))+1/2/(2*(c^2+d^2)^(1/2)+2*c)^(1/2)/(c^2+d^2)^(1/2)/((c^2+d^2)
^(1/2)*c+c^2+d^2)*(1/2*(-2*I*(c^2+d^2)^(1/2)*c^2-I*d^2*(c^2+d^2)^(1/2)-2*I*
c^3-2*I*c*d^2+c*d*(c^2+d^2)^(1/2)+c^2*d+d^3)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x
+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(I*(c^2+d^2)^(1
/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2+I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3+I
```

```
*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^2-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*c*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*d^3+1/2*(-2*I*(c^2+d^2)^(1/2)*c^2-I*d^2*(c^2+d^2)^(1/2)-2*I*c^3-2*I*c*d^
2+c*d*(c^2+d^2)^(1/2)+c^2*d+d^3)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2
)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d^2)^(1/2)+2*c)
^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^2/sqrt(d*tan(f*x + e) + c), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 353 vs. 2(62) = 124.
time = 1.11, size = 353, normalized size = 4.77

$$\frac{d\sqrt{-\frac{16i\,a^4}{(ic+d)f^2}}\,f\log\left(\frac{\left(4a^2c+\left((ic+d)fe^{(2i\,fx+2i\,e)}+(ic+d)f\right)\sqrt{-\frac{16i\,a^4}{(ic+d)f^2}}\sqrt{\frac{(c-id)e^{(2i\,fx+2i\,e)}+c+id}{e^{(2i\,fx+2i\,e)}+1}}+4\left(a^2c-ia^2d\right)e^{(2i\,fx+2i\,e)}\right)e^{(-2i\,fx-2i\,e)}}{2a^2}\right)-d\sqrt{-\frac{16i\,a^4}{(ic+d)f^2}}\,f\log\left(\frac{\left(4a^2c+\left((-ic-d)fe^{(2i\,fx+2i\,e)}+(-ic-d)f\right)\sqrt{-\frac{16i\,a^4}{(ic+d)f^2}}\sqrt{\frac{(c-id)e^{(2i\,fx+2i\,e)}+c+id}{e^{(2i\,fx+2i\,e)}+1}}+4\left(a^2c-ia^2d\right)e^{(2i\,fx+2i\,e)}\right)e^{(-2i\,fx-2i\,e)}}{2a^2}\right)-8a^2\sqrt{\frac{(c-id)e^{(2i\,fx+2i\,e)}+c+id}{e^{(2i\,fx+2i\,e)}+1}}}{4df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/4*(d*sqrt(-16*I*a^4/((I*c + d)*f^2))*f*log(1/2*(4*a^2*c + ((I*c + d)*f*e^
(2*I*f*x + 2*I*e) + (I*c + d)*f)*sqrt(-16*I*a^4/((I*c + d)*f^2))*sqrt(((c -
 I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1)) + 4*(a^2*c
- I*a^2*d)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/a^2) - d*sqrt(-16*I*a^
4/((I*c + d)*f^2))*f*log(1/2*(4*a^2*c + ((-I*c - d)*f*e^(2*I*f*x + 2*I*e) +
 (-I*c - d)*f)*sqrt(-16*I*a^4/((I*c + d)*f^2))*sqrt(((c - I*d)*e^(2*I*f*x +
 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1)) + 4*(a^2*c - I*a^2*d)*e^(2*I*
f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/a^2) - 8*a^2*sqrt(((c - I*d)*e^(2*I*f*x
+ 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))/(d*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int\frac{\tan^2(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx+\int\left(-\frac{2i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\right)dx+\int\left(-\frac{1}{\sqrt{c+d\tan(e+fx)}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2/(c+d*tan(f*x+e))**(1/2),x)
```

```
[Out] -a**2*(Integral(tan(e + f*x)**2/sqrt(c + d*tan(e + f*x)), x) + Integral(-2*
I*tan(e + f*x)/sqrt(c + d*tan(e + f*x)), x) + Integral(-1/sqrt(c + d*tan(e
+ f*x)), x))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(62) = 124$.
time = 0.59, size = 185, normalized size = 2.50

$$-\frac{2\sqrt{d\tan(fx+e)+c}\,a^2}{df}+\frac{8i\,a^2\arctan\left(\frac{2\left(\sqrt{d\tan(fx+e)+c}\,c-\sqrt{c^2+d^2}\sqrt{d\tan(fx+e)+c}\right)}{c\sqrt{-2c+2\sqrt{c^2+d^2}}-i\sqrt{-2c+2\sqrt{c^2+d^2}}\,d-\sqrt{c^2+d^2}\sqrt{-2c+2\sqrt{c^2+d^2}}}\right)}{\sqrt{-2c+2\sqrt{c^2+d^2}}\,f\left(-\frac{id}{c-\sqrt{c^2+d^2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(d*tan(f*x + e) + c)*a^2/(d*f) + 8*I*a^2*arctan(2*(sqrt(d*tan(f*x +
e) + c)*c - sqrt(c^2 + d^2)*sqrt(d*tan(f*x + e) + c))/(c*sqrt(-2*c + 2*sqrt
(c^2 + d^2)) - I*sqrt(-2*c + 2*sqrt(c^2 + d^2))*d - sqrt(c^2 + d^2)*sqrt(-2
*c + 2*sqrt(c^2 + d^2))))/(sqrt(-2*c + 2*sqrt(c^2 + d^2))*f*(-I*d/(c - sqrt
(c^2 + d^2)) + 1))
```

Mupad [B]
time = 5.91, size = 67, normalized size = 0.91

$$-\frac{2\,a^2\,\sqrt{c+d\tan(e+f\,x)}}{d\,f}+\frac{a^2\,\mathrm{atan}\left(\frac{\sqrt{c+d\tan(e+f\,x)}}{\sqrt{-c+d\,1\mathrm{i}}}\right)\,4\mathrm{i}}{f\,\sqrt{-c+d\,1\mathrm{i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^2/(c + d*tan(e + f*x))^(1/2),x)
```

```
[Out] (a^2*atan((c + d*tan(e + f*x))^(1/2)/(d*1i - c)^(1/2))*4i)/(f*(d*1i - c)^(1
/2)) - (2*a^2*(c + d*tan(e + f*x))^(1/2))/(d*f)
```

3.1121 $\int \frac{a+ia\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=46

$$-\frac{2ia\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}\,f}$$

```
[Out] -2*I*a*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/f/(c-I*d)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3618, 65, 214}

$$-\frac{2ia\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f\sqrt{c-id}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((-2*I)*a*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/(Sqrt[c - I*d]*f
)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{a+ia\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx=\frac{(ia^2)\,\text{Subst}\left(\int \frac{1}{(-a^2+ax)\sqrt{c-\frac{idx}{a}}}\,dx,x,ia\tan(e+fx)\right)}{f}$$

$$=-\frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{-a^2-\frac{ia^2c}{d}+\frac{ia^2x^2}{d}}\,dx,x,\sqrt{c+d\tan(e+fx)}\right)}{df}$$

$$=-\frac{2ia\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}\,f}$$

Mathematica [A]
time = 1.10, size = 71, normalized size = 1.54

$$-\frac{2ia\tanh^{-1}\left(\frac{\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}}{\sqrt{c-id}}\right)}{\sqrt{c-id}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((-2*I)*a*ArcTanh[Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(
e + f*x)))]/Sqrt[c - I*d]])/(Sqrt[c - I*d]*f)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 732 vs. 2(37) = 74.
time = 0.36, size = 733, normalized size = 15.93

method	result

derivativedivides	$a\left(\frac{\frac{\left(-i\sqrt{c^2+d^2}-ic+d\right)\ln\left(d\tan(fx+e)+c+\sqrt{c+d\tan(fx+e)}\sqrt{2\sqrt{c^2+d^2}+2c}+\sqrt{c^2+d^2}\right)}{2}}{}\right.$
default	$a\left(\frac{\frac{\left(-i\sqrt{c^2+d^2}-ic+d\right)\ln\left(d\tan(fx+e)+c+\sqrt{c+d\tan(fx+e)}\sqrt{2\sqrt{c^2+d^2}+2c}+\sqrt{c^2+d^2}\right)}{2}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a*(1/(2*(c^2+d^2)^(1/2)+2*c)^(1/2)/(c^2+d^2)^(1/2)*(1/2*(-I*(c^2+d^2)^(
1/2)-I*c+d)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)+(c^2+d^2)^(1/2))+2*(-I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c+(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*d-1/2*(-I*(c^2+d^2)^(1/2)-I*c+d)*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c
^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)/(c^2+d^2)^(1/2)/((c^2+d^2)^(1/2)*c+c^2+d^2)*(1/2*(2*I*(c^2+d
^2)^(1/2)*c^2+I*d^2*(c^2+d^2)^(1/2)+2*I*c^3+2*I*c*d^2-c*d*(c^2+d^2)^(1/2)-c
^2*d-d^3)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)+(c^2+d^2)^(1/2))+2*(-I*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*
c^2-I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3-I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d
^2+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d+(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*c^2*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^3+1/2*(2*I*(c^2+d^2)^(1/2)*c^
2+I*d^2*(c^2+d^2)^(1/2)+2*I*c^3+2*I*c*d^2-c*d*(c^2+d^2)^(1/2)-c^2*d-d^3)*(2
*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*t
an(f*x+e))^(1/2)-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/
2))))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 6772 vs. $2(35) = 70$.
time = 0.70, size = 6772, normalized size = 147.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^(1/2),x, algorithm="maxima")

[Out] -1/4*(sqrt(2*c^2 + 2*d^2)*(2*a*arctan2((d^2*cos(2*f*x + 2*e) - c*d*sin(2*f*x + 2*e) + ((c^4 + 2*c^2*d^2 + d^4)*cos(2*f*x + 2*e)^4 + (c^4 + 2*c^2*d^2 + d^4)*sin(2*f*x + 2*e)^4 + c^4 + 2*c^2*d^2 + d^4 + 4*(c^4 + c^2*d^2)*cos(2*f*x + 2*e)^3 + 4*(c^3*d + c*d^3)*sin(2*f*x + 2*e)^3 + 2*(3*c^4 + 2*c^2*d^2 - d^4)*cos(2*f*x + 2*e)^2 + 2*(c^4 + 2*c^2*d^2 + d^4 + (c^4 + 2*c^2*d^2 + d^4)*cos(2*f*x + 2*e)^2 + 2*(c^4 + c^2*d^2)*cos(2*f*x + 2*e))*sin(2*f*x + 2*e)^2 + 4*(c^4 + c^2*d^2)*cos(2*f*x + 2*e) + 4*(c^3*d + c*d^3 + (c^3*d + c*d^3)*cos(2*f*x + 2*e)^2 + 2*(c^3*d + c*d^3)*cos(2*f*x + 2*e))*sin(2*f*x + 2*e))^(1/4)*d*sin(1/2*arctan2(-2*(c*d*cos(2*f*x + 2*e)^2 - c*d*sin(2*f*x + 2*e)^2 + c*d*cos(2*f*x + 2*e) - (c^2 + (c^2 - d^2)*cos(2*f*x + 2*e))*sin(2*f*x + 2*e))/d^2, (2*c^2*cos(2*f*x + 2*e) + (c^2 - d^2)*cos(2*f*x + 2*e)^2 - (c^2 - d^2)*sin(2*f*x + 2*e)^2 + c^2 + d^2 + 2*(2*c*d*cos(2*f*x + 2*e) + c*d)*sin(2*f*x + 2*e))/d^2)))/d^2, -(c*cos(2*f*x + 2*e) + d*sin(2*f*x + 2*e) - ((c^4 + 2*c^2*d^2 + d^4)*cos(2*f*x + 2*e)^4 + (c^4 + 2*c^2*d^2 + d^4)*sin(2*f*x + 2*e)^4 + c^4 + 2*c^2*d^2 + d^4 + 4*(c^4 + c^2*d^2)*cos(2*f*x + 2*e)^3 + 4*(c^3*d + c*d^3)*sin(2*f*x + 2*e)^3 + 2*(3*c^4 + 2*c^2*d^2 - d^4)*cos(2*f*x + 2*e)^2 + 2*(c^4 + 2*c^2*d^2 + d^4 + (c^4 + 2*c^2*d^2 + d^4)*cos(2*f*x + 2*e)^2 + 2*(c^4 + c^2*d^2)*cos(2*f*x + 2*e))*sin(2*f*x + 2*e)^2 + 4*(c^4 + c^2*d^2)*cos(2*f*x + 2*e) + 4*(c^3*d + c*d^3 + (c^3*d + c*d^3)*cos(2*f*x + 2*e)^2 + 2*(c^3*d + c*d^3)*cos(2*f*x + 2*e))*sin(2*f*x + 2*e))^(1/4)*cos(1/2*arctan2(-2*(c*d*cos(2*f*x + 2*e)^2 - c*d*sin(2*f*x + 2*e)^2 + c*d*cos(2*f*x + 2*e) - (c^2 + (c^2 - d^2)*cos(2*f*x + 2*e))*sin(2*f*x + 2*e))/d^2, (2*c^2*cos(2*f*x + 2*e) + (c^2 - d^2)*cos(2*f*x + 2*e)^2 - (c^2 - d^2)*sin(2*f*x + 2*e)^2 + c^2 + d^2 + 2*(2*c*d*cos(2*f*x + 2*e) + c*d)*sin(2*f*x + 2*e))/d^2)) + c)/d) - I*a*log((2*c^2*cos(2*f*x + 2*e) + (c^2 + d^2)*cos(2*f*x + 2*e)^2 + 2*c*d*sin(2*f*x + 2*e) + (c^2 + d^2)*sin(2*f*x + 2*e)^2 + sqrt((c^4 + 2*c^2*d^2 + d^4)*cos(2*f*x + 2*e)^4 + (c^4 + 2*c^2*d^2 + d^4)*sin(2*f*x + 2*e)^4 + c^4 + 2*c^2*d^2 + d^4 + 4*(c^4 + c^2*d^2)*cos(2*f*x + 2*e)^3 + 4*(c^3*d + c*d^3)*sin(2*f*x + 2*e)^3 + 2*(3*c^4 + 2*c^2*d^2 - d^4)*cos(2*f*x + 2*e)^2 + 2*(c^4 + 2*c^2*d^2 + d^4 + (c^4 + 2*c^2*d^2 + d^4)*cos(2*f*x + 2*e)^2 + 2*(c^4 + c^2*d^2)*cos(2*f*x + 2*e))*sin(2*f*x + 2*e)^2 + 4*(c^4 + c^2*d^2)*cos(2*f*x + 2*e) + 4*(c^3*d + c*d^3 + (c^3*d + c*d^3)*cos(2*f*x + 2*e)^2 + 2*(c^3*d + c*d^3)*cos(2*f*x + 2*e))*sin(2*f*x + 2*e))*cos(1/2*arctan2(-2*(c*d*cos(2*f*x + 2*e)^2 - c*d*sin(2*f*x + 2*e)^2 + c*d*cos(2*f*x + 2*e) - (c^2 + (c^2 - d^2)*cos(2*f*x + 2*e))*sin(2*f*x + 2*e))/d^2, (2*c^2*cos(2*f*x + 2*e) + (c^2 - d^2)*cos(2*f*x + 2*e)^2 - (c^2 - d^2)*sin(2*f*x + 2*e)^2 + c^2 + d^2 + 2*(2*c*d*cos(2*f*x + 2*e) + c*d)*sin(2*f*x + 2*e))/d^2))^2 + sqrt((c^4 + 2*c^2*d^2 + d^4)*cos(2*f*x + 2*e)^4 + (c^4 + 2*c^2*d^2 + d^4)*sin(2*f*x + 2*e)^4 + c^4 + 2*c^2*d^2 + d^4 + 4*(c^4 + c^2*d^2)*cos(2*f*x + 2*e)^3 + 4*(c^3*d + c*d^3)*sin(2*f*x + 2*e)^3 + 2*(3*c^4 + 2*c^2*d^2 - d^4)*cos(2*f*x + 2*e)^2 + 2*(c^4 + 2*c^2*d^2 + d^4 + (c^4 + 2*c^2*

```
d^2 + d^4)*cos(2*f*x + 2*e)^2 + 2*(c^4 + c^2*d^2)*cos(2*f*x + 2*e))*sin(2*f
*x + 2*e)^2 + 4*(c^4 + c^2*d^2)*cos(2*f*x + 2*e) + 4*(c^3*d + c*d^3 + (c^3*
d + c*d^3)*cos(2*f*x + 2*e)^2 + 2*(c^3*d + c*d^3)*cos(2*f*x + 2*e))*sin(2*f
*x + 2*e))*sin(1/2*arctan2(-2*(c*d*cos(2*f*x + 2*e)^2 - c*d*sin(2*f*x + 2*e
)^2 + c*d*cos(2*f*x + 2*e) - (c^2 + (c^2 - d^2)*cos(2*f*x + 2*e))*sin(2*f*x
 + 2*e))/d^2, (2*c^2*cos(2*f*x + 2*e) + (c^2 - d^2)*cos(2*f*x + 2*e)^2 - (c
^2 - d^2)*sin(2*f*x + 2*e)^2 + c^2 + d^2 + 2*(2*c*d*cos(2*f*x + 2*e) + c*d)
*sin(2*f*x + 2*e))/d^2))^2 + c^2 - 2*((c^4 + 2*c^2*d^2 + d^4)*cos(2*f*x + 2
*e)^4 + (c^4 + 2*c^2*d^2 + d^4)*sin(2*f*x + 2*e)^4 + c^4 + 2*c^2*d^2 + d^4
+ 4*(c^4 + c^2*d^2)*cos(2*f*x + 2*e)^3 + 4*(c^3*d + c*d^3)*sin(2*f*x + 2*e)
^3 + 2*(3*c^4 + 2*c^2*d^2 - d^4)*cos(2*f*x + 2*e)^2 + 2*(c^4 + 2*c^2*d^2 +
d^4 + (c^4 + 2*c^2*d^2 + d^4)*cos(2*f*x + 2*e)^2 + 2*(c^4 + c^2*d^2)*cos(2*
f*x + 2*e))*sin(2*f*x + 2*e)^2 + 4*(c^4 + c^2*d^2)*cos(2*f*x + 2*e) + 4*(c^
3*d + c*d^3 + (c^3*d + c*d^3)*cos(2*f*x + 2*e)^2 + 2*(c^3*d + c*d^3)*cos(2*
f*x + 2*e))*sin(2*f*x + 2*e))^(1/4)*(c*d*cos(2*f*x + 2*e) + d^2*sin(2*f*x +
 2*e) + c*d)*cos(1/2*arctan2(-2*(c*d*cos(2*f*x + 2*e)^2 - c*d*sin(2*f*x + 2
*e)^2 + c*d*cos(2*f*x + 2*e) - (c^2 + (c^2 - d^2)*cos(2*f*x + 2*e))*sin(2*f
*x + 2*e))/d^2, (2*c^2*cos(2*f*x + 2*e) + (c^2 - d^2)*cos(2*f*x + 2*e)^2 -
(c^2 - d^2)*sin(2*f*x + 2*e)^2 + c^2 + d^2 + 2*(2*c*d*cos(2*f*x + 2*e) + c*
d)*sin(2*f*x + 2*e))/d^2))/d + 2*((c^4 + 2*c^2*d^2 + d^4)*cos(2*f*x + 2*e)^
4 + (c^4 + 2*c^2*d^2 + d^4)*sin(2*f*x + 2*e)^4 + c^4 + 2*c^2*d^2 + d^4 + 4*
(c^4 + c^2*d^2)*cos(2*f*x + 2*e)^3 + 4*(c^3*d + c*d^3)*sin(2*f*x + 2*e)^3 +
 2*(3*c^4 + 2*c^2*d^2 - d^4)*cos(2*f*x + 2*e)^2 + 2*(c^4 + 2*c^2*d^2 + d^4
+ (c^4 + 2*c^2*d^2 + d^4)*cos(2*f*x + 2*e)^2 + 2*(c^4 + c^2*d^2)*cos(2*f*x
+ 2*e))*sin(2*f*x + 2*e)^2 + 4*(c^4 + c^2*d^2)*cos(2*f*x + 2*e) + 4*(c^3*d
+ c*d^3 + (c^3*d + c*d^3)*cos(2*f*x + 2*e)^2 + ...
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(35) = 70$.
time = 0.81, size = 285, normalized size = 6.20

$$\frac{1}{4}\sqrt{-\frac{4i\,a^2}{(i\,c+d)f^2}}\log\left(\frac{\left(2\,ac+\left((i\,c+d)fe^{(2i\,fx+2i\,e)}+(i\,c+d)f\right)\sqrt{\frac{(c-i\,d)e^{(2i\,fx+2i\,e)}+c+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{-\frac{4i\,a^2}{(i\,c+d)f^2}}+2\,(ac-i\,ad)e^{(2i\,fx+2i\,e)}\right)e^{(-2i\,fx-2i\,e)}}{a}\right)-\frac{1}{4}\sqrt{-\frac{4i\,a^2}{(i\,c+d)f^2}}\log\left(\frac{\left(2\,ac+\left((-i\,c-d)fe^{(2i\,fx+2i\,e)}+(-i\,c-d)f\right)\sqrt{\frac{(c-i\,d)e^{(2i\,fx+2i\,e)}+c+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{-\frac{4i\,a^2}{(i\,c+d)f^2}}+2\,(ac-i\,ad)e^{(2i\,fx+2i\,e)}\right)e^{(-2i\,fx-2i\,e)}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-4*I*a^2/((I*c + d)*f^2))*log((2*a*c + ((I*c + d)*f*e^(2*I*f*x + 2
*I*e) + (I*c + d)*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I
*f*x + 2*I*e) + 1))*sqrt(-4*I*a^2/((I*c + d)*f^2)) + 2*(a*c - I*a*d)*e^(2*I
*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/a) - 1/4*sqrt(-4*I*a^2/((I*c + d)*f^2))
*log((2*a*c + ((-I*c - d)*f*e^(2*I*f*x + 2*I*e) + (-I*c - d)*f)*sqrt(((c -
I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-4*I*a^
2/((I*c + d)*f^2)) + 2*(a*c - I*a*d)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I
*e)/a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-\frac{i}{\sqrt{c+d\tan\left(e+fx\right)}}\right)dx+\int\frac{\tan\left(e+fx\right)}{\sqrt{c+d\tan\left(e+fx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))**(1/2),x)
```

```
[Out] I*a*(Integral(-I/sqrt(c + d*tan(e + f*x)), x) + Integral(tan(e + f*x)/sqrt(
c + d*tan(e + f*x)), x))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(35) = 70.
time = 0.51, size = 156, normalized size = 3.39

$$\frac{4\,a\arctan\left(-\frac{2\left(i\sqrt{d\tan\left(fx+e\right)+c}\,c+i\sqrt{c^2+d^2}\sqrt{d\tan\left(fx+e\right)+c}\right)}{\sqrt{2\,c+2\sqrt{c^2+d^2}}\,c-i\sqrt{2\,c+2\sqrt{c^2+d^2}}\,d+\sqrt{c^2+d^2}\sqrt{2\,c+2\sqrt{c^2+d^2}}}\right)}{\sqrt{2\,c+2\sqrt{c^2+d^2}}\,f\left(-\frac{i\,d}{c+\sqrt{c^2+d^2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 4*a*arctan(-2*(I*sqrt(d*tan(f*x + e) + c)*c + I*sqrt(c^2 + d^2)*sqrt(d*tan(
f*x + e) + c))/(sqrt(2*c + 2*sqrt(c^2 + d^2))*c - I*sqrt(2*c + 2*sqrt(c^2 +
 d^2))*d + sqrt(c^2 + d^2)*sqrt(2*c + 2*sqrt(c^2 + d^2))))/(sqrt(2*c + 2*sq
rt(c^2 + d^2))*f*(-I*d/(c + sqrt(c^2 + d^2)) + 1))
```

Mupad [B]
time = 6.40, size = 2947, normalized size = 64.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)/(c + d*tan(e + f*x))^(1/2),x)
```

```
[Out] 2*atanh((8*c*d^2*(- (-16*a^4*d^2*f^4)^(1/2)/(16*(c^2*f^4 + d^2*f^4)) - (a^2
*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2)*(c + d*tan(e + f*x))^(1/2)*(-16*a^4*
d^2*f^4)^(1/2))/((16*a^3*c*d^5*f^5)/(c^2*f^4 + d^2*f^4) + (4*a*d^5*f^4*(-16
*a^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5) + (16*a^3*c^3*d^3*f^5)/(c^2*f^4 +
d^2*f^4) + (4*a*c^2*d^3*f^4*(-16*a^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5)) -
 (32*a^2*d^2*(- (-16*a^4*d^2*f^4)^(1/2)/(16*(c^2*f^4 + d^2*f^4)) - (a^2*c*f
^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2)*(c + d*tan(e + f*x))^(1/2))/((16*a^3*c*d
```

^3*f^3)/(c^2*f^4 + d^2*f^4) + (4*a*d^3*f^2*(-16*a^4*d^2*f^4)^(1/2))/(c^2*f^
5 + d^2*f^5)) + (32*a^2*c^2*d^2*f^2*(- (-16*a^4*d^2*f^4)^(1/2)/(16*(c^2*f^4
 + d^2*f^4)) - (a^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2)*(c + d*tan(e + f*
x))^(1/2))/((16*a^3*c*d^5*f^5)/(c^2*f^4 + d^2*f^4) + (4*a*d^5*f^4*(-16*a^4*
d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5) + (16*a^3*c^3*d^3*f^5)/(c^2*f^4 + d^2*f
^4) + (4*a*c^2*d^3*f^4*(-16*a^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5)))*(- (-
16*a^4*d^2*f^4)^(1/2)/(16*(c^2*f^4 + d^2*f^4)) - (a^2*c*f^2)/(4*(c^2*f^4 +
d^2*f^4)))^(1/2) - 2*atanh((32*a^2*d^2*((-16*a^4*d^2*f^4)^(1/2)/(16*(c^2*f^
4 + d^2*f^4)) - (a^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2)*(c + d*tan(e + f
*x))^(1/2))/((16*a^3*c*d^3*f^3)/(c^2*f^4 + d^2*f^4) - (4*a*d^3*f^2*(-16*a^4
*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5)) + (8*c*d^2*((-16*a^4*d^2*f^4)^(1/2)/(
16*(c^2*f^4 + d^2*f^4)) - (a^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2)*(c + d
*tan(e + f*x))^(1/2)*(-16*a^4*d^2*f^4)^(1/2))/((16*a^3*c*d^5*f^5)/(c^2*f^4
+ d^2*f^4) - (4*a*d^5*f^4*(-16*a^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5) + (1
6*a^3*c^3*d^3*f^5)/(c^2*f^4 + d^2*f^4) - (4*a*c^2*d^3*f^4*(-16*a^4*d^2*f^4)
^(1/2))/(c^2*f^5 + d^2*f^5)) - (32*a^2*c^2*d^2*f^2*((-16*a^4*d^2*f^4)^(1/2)
/(16*(c^2*f^4 + d^2*f^4)) - (a^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2)*(c +
 d*tan(e + f*x))^(1/2))/((16*a^3*c*d^5*f^5)/(c^2*f^4 + d^2*f^4) - (4*a*d^5*
f^4*(-16*a^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5) + (16*a^3*c^3*d^3*f^5)/(c^
2*f^4 + d^2*f^4) - (4*a*c^2*d^3*f^4*(-16*a^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2
f^5)))((-16*a^4*d^2*f^4)^(1/2)/(16*(c^2*f^4 + d^2*f^4)) - (a^2*c*f^2)/(4*
(c^2*f^4 + d^2*f^4)))^(1/2) - 2*atanh((8*c*d^2*(- (-16*a^4*d^2*f^4)^(1/2)/(
16*(c^2*f^4 + d^2*f^4)) - (a^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2)*(c + d
*tan(e + f*x))^(1/2)*(-16*a^4*d^2*f^4)^(1/2))/((a^3*c^2*d^4*f^5*16i)/(c^2*f
^4 + d^2*f^4) - a^3*c^2*d^2*f*16i - a^3*d^4*f*16i + (a^3*c^4*d^2*f^5*16i)/(
c^2*f^4 + d^2*f^4) + (a*c*d^4*f^4*(-16*a^4*d^2*f^4)^(1/2)*4i)/(c^2*f^5 + d^
2*f^5) + (a*c^3*d^2*f^4*(-16*a^4*d^2*f^4)^(1/2)*4i)/(c^2*f^5 + d^2*f^5)) -
(32*a^2*d^2*(- (-16*a^4*d^2*f^4)^(1/2)/(16*(c^2*f^4 + d^2*f^4)) - (a^2*c*f^
2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2)*(c + d*tan(e + f*x))^(1/2))/((a^3*c^2*d^2
*f^3*16i)/(c^2*f^4 + d^2*f^4) - (a^3*d^2*16i)/f + (a*c*d^2*f^2*(-16*a^4*d^2
*f^4)^(1/2)*4i)/(c^2*f^5 + d^2*f^5)) + (32*a^2*c^2*d^2*f^2*(- (-16*a^4*d^2*
f^4)^(1/2)/(16*(c^2*f^4 + d^2*f^4)) - (a^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^
(1/2)*(c + d*tan(e + f*x))^(1/2))/((a^3*c^2*d^4*f^5*16i)/(c^2*f^4 + d^2*f^4
) - a^3*c^2*d^2*f*16i - a^3*d^4*f*16i + (a^3*c^4*d^2*f^5*16i)/(c^2*f^4 + d^
2*f^4) + (a*c*d^4*f^4*(-16*a^4*d^2*f^4)^(1/2)*4i)/(c^2*f^5 + d^2*f^5) + (a*
c^3*d^2*f^4*(-16*a^4*d^2*f^4)^(1/2)*4i)/(c^2*f^5 + d^2*f^5)))*(- (-16*a^4*d
^2*f^4)^(1/2)/(16*(c^2*f^4 + d^2*f^4)) - (a^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)
))^(1/2) - 2*atanh((32*a^2*d^2*((-16*a^4*d^2*f^4)^(1/2)/(16*(c^2*f^4 + d^2*
f^4)) - (a^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2)*(c + d*tan(e + f*x))^(1/
2))/((a^3*d^2*16i)/f - (a^3*c^2*d^2*f^3*16i)/(c^2*f^4 + d^2*f^4) + (a*c*d^2
f^2(-16*a^4*d^2*f^4)^(1/2)*4i)/(c^2*f^5 + d^2*f^5)) + (8*c*d^2*((-16*a^4*
d^2*f^4)^(1/2)/(16*(c^2*f^4 + d^2*f^4)) - (a^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4
)))^(1/2)*(c + d*tan(e + f*x))^(1/2)*(-16*a^4*d^2*f^4)^(1/2))/(a^3*d^4*f*16
i + a^3*c^2*d^2*f*16i - (a^3*c^2*d^4*f^5*16i)/(c^2*f^4 + d^2*f^4) - (a^3*c^
4*d^2*f^5*16i)/(c^2*f^4 + d^2*f^4) + (a*c*d^4*f^4*(-16*a^4*d^2*f^4)^(1/2)*4

```
i)/(c^2*f^5 + d^2*f^5) + (a*c^3*d^2*f^4*(-16*a^4*d^2*f^4)^(1/2)*4i)/(c^2*f^
5 + d^2*f^5)) - (32*a^2*c^2*d^2*f^2*((-16*a^4*d^2*f^4)^(1/2)/(16*(c^2*f^4 +
 d^2*f^4)) - (a^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2)*(c + d*tan(e + f*x)
)^(1/2))/(a^3*d^4*f*16i + a^3*c^2*d^2*f*16i - (a^3*c^2*d^4*f^5*16i)/(c^2*f^
4 + d^2*f^4) - (a^3*c^4*d^2*f^5*16i)/(c^2*f^4 + d^2*f^4) + (a*c*d^4*f^4*(-1
6*a^4*d^2*f^4)^(1/2)*4i)/(c^2*f^5 + d^2*f^5) + (a*c^3*d^2*f^4*(-16*a^4*d^2*
f^4)^(1/2)*4i)/(c^2*f^5 + d^2*f^5)))*((-16*a^4*d^2*f^4)^(1/2)/(16*(c^2*f^4
+ d^2*f^4)) - (a^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2)
```

3.1122 $\int \frac{1}{(a+ia\tan(e+fx))\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=155

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{2a\sqrt{c-id}\,f}+\frac{(ic-2d)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{2a(c+id)^{3/2}f}-\frac{\sqrt{c+d\tan(e+fx)}}{2(ic-d)f(a+ia\tan(e+fx))}$$

```
[Out] 1/2*(I*c-2*d)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/a/(c+I*d)^(3/2)
/f-1/2*I*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/a/f/(c-I*d)^(1/2)-1/
2*(c+d*tan(f*x+e))^(1/2)/(I*c-d)/f/(a+I*a*tan(f*x+e))
```

Rubi [A]
time = 0.21, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3633, 3620, 3618, 65, 214}

$$-\frac{\sqrt{c+d\tan(e+fx)}}{2f(-d+ic)(a+ia\tan(e+fx))}-\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{2af\sqrt{c-id}}+\frac{(-2d+ic)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{2af(c+id)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])*Sqrt[c + d*Tan[e + f*x]]),x]
```

```
[Out] ((-1/2*I)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/(a*Sqrt[c - I*d]
*f) + ((I*c - 2*d)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/(2*a*(c
 + I*d)^(3/2)*f) - Sqrt[c + d*Tan[e + f*x]]/(2*(I*c - d)*f*(a + I*a*Tan[e +
 f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
```

```
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3633

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-a)*((c + d*Tan[e + f*x])^(n + 1)/(2*f*(b*
c - a*d)*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*
Tan[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &
& NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))\sqrt{c+d\tan(e+fx)}}\,dx = -\frac{\sqrt{c+d\tan(e+fx)}}{2(ic-d)f(a+ia\tan(e+fx))}+\frac{\int \frac{\frac{1}{2}a(2ic-3d)+\frac{1}{2}iad\tan(e}{\sqrt{c+d\tan(e+f}}}{2a^2(ic-d)}$$

$$= -\frac{\sqrt{c+d\tan(e+fx)}}{2(ic-d)f(a+ia\tan(e+fx))}+\frac{\int \frac{1+i\tan(e+fx)}{\sqrt{c+d\tan(e+f}}}{4a}$$

$$= -\frac{\sqrt{c+d\tan(e+fx)}}{2(ic-d)f(a+ia\tan(e+fx))}+\frac{i\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{c}}\right.}{}$$

$$= -\frac{\sqrt{c+d\tan(e+fx)}}{2(ic-d)f(a+ia\tan(e+fx))}-\frac{\text{Subst}\left(\int \frac{1}{-1-\frac{ic}{d}+\frac{ix^2}{d}}\right.}{}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{2a\sqrt{c-id}\,f}+\frac{(ic-2d)\tanh^{-1}\left(\right.}{2a(}$$

Mathematica [A]

time = 1.55, size = 222, normalized size = 1.43

$$\frac{\sec(e+fx)(\cos(fx)+i\sin(fx))\left(-\frac{2\left(\sqrt{-c+id}\,(-ic+2d)\text{ArcTan}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{-c-id}}\right)-i(-c-id)^{3/2}\text{ArcTan}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{-c+id}}\right)\right)(\cos(e)+i\sin(e))}{(-c-id)^{3/2}\sqrt{-c+id}}+\frac{2\cos(e+fx)(i\cos(fx)+\sin(fx))\sqrt{c+d\tan(e+fx)}}{c+id}\right)}{4f(a+ia\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])*Sqrt[c + d*Tan[e + f*x]]),x]
```

```
[Out] (Sec[e + f*x]*(Cos[f*x] + I*Sin[f*x])*((-2*(Sqrt[-c + I*d]*((-I)*c + 2*d)*A
rcTan[Sqrt[c + d*Tan[e + f*x]]/Sqrt[-c - I*d]] - I*(-c - I*d)^(3/2)*ArcTan[
Sqrt[c + d*Tan[e + f*x]]/Sqrt[-c + I*d]])*(Cos[e] + I*Sin[e]))/((-c - I*d)^
(3/2)*Sqrt[-c + I*d]) + (2*Cos[e + f*x]*(I*Cos[f*x] + Sin[f*x])*Sqrt[c + d*
Tan[e + f*x]])/(c + I*d)))/(4*f*(a + I*a*Tan[e + f*x]))
```

Maple [A]

time = 0.40, size = 150, normalized size = 0.97

method	result
derivativedivides	$\frac{2d^2\left(\frac{-\frac{d\sqrt{c+d\tan(fx+e)}}{(id+c)(-d\tan(fx+e)+id)}-\frac{(ic-2d)\arctan\left(\frac{\sqrt{c+d\tan(fx+e)}}{\sqrt{-id-c}}\right)}{(id+c)\sqrt{-id-c}}}{4d^2}+\frac{i\arctan\left(\frac{\sqrt{c+d\tan(fx+}}{\sqrt{id-c}}\right.}{4d^2\sqrt{id-c}}\right.}{fa}$
default	$\frac{2d^2\left(\frac{-\frac{d\sqrt{c+d\tan(fx+e)}}{(id+c)(-d\tan(fx+e)+id)}-\frac{(ic-2d)\arctan\left(\frac{\sqrt{c+d\tan(fx+e)}}{\sqrt{-id-c}}\right)}{(id+c)\sqrt{-id-c}}}{4d^2}+\frac{i\arctan\left(\frac{\sqrt{c+d\tan(fx+}}{\sqrt{id-c}}\right.}{4d^2\sqrt{id-c}}\right.}{fa}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a*d^2*(1/4/d^2*(-1/(c+I*d)*d*(c+d*tan(f*x+e))^(1/2)/(-d*tan(f*x+e)+I*d)
-(I*c-2*d)/(c+I*d)/(-I*d-c)^(1/2)*arctan((c+d*tan(f*x+e))^(1/2)/(-I*d-c)^(1
/2)))+1/4*I/d^2/(I*d-c)^(1/2)*arctan((c+d*tan(f*x+e))^(1/2)/(I*d-c)^(1/2)))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1018 vs. 2(123) = 246.
time = 1.42, size = 1018, normalized size = 6.57

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e)),x, algorithm="fricas"
)
```

```
[Out] -1/8*(2*(I*a*c - a*d)*f*sqrt(1/4*I/((-I*a^2*c - a^2*d)*f^2))*e^(2*I*f*x + 2
*I*e)*log(-2*(2*((I*a*c + a*d)*f*e^(2*I*f*x + 2*I*e) + (I*a*c + a*d)*f)*sqr
t(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt
(1/4*I/((-I*a^2*c - a^2*d)*f^2)) - (c - I*d)*e^(2*I*f*x + 2*I*e) - c)*e^(-2
*I*f*x - 2*I*e)) + 2*(-I*a*c + a*d)*f*sqrt(1/4*I/((-I*a^2*c - a^2*d)*f^2))*
e^(2*I*f*x + 2*I*e)*log(-2*(2*((-I*a*c - a*d)*f*e^(2*I*f*x + 2*I*e) + (-I*a
*c - a*d)*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2
*I*e) + 1))*sqrt(1/4*I/((-I*a^2*c - a^2*d)*f^2)) - (c - I*d)*e^(2*I*f*x + 2
*I*e) - c)*e^(-2*I*f*x - 2*I*e)) + (I*a*c - a*d)*f*sqrt(-(-I*c^2 + 4*c*d +
4*I*d^2)/((-I*a^2*c^3 + 3*a^2*c^2*d + 3*I*a^2*c*d^2 - a^2*d^3)*f^2))*e^(2*I
*f*x + 2*I*e)*log(-1/2*(-I*c^2 + 3*c*d + 2*I*d^2 + ((a*c^2 + 2*I*a*c*d - a*
d^2)*f*e^(2*I*f*x + 2*I*e) + (a*c^2 + 2*I*a*c*d - a*d^2)*f)*sqrt(((c - I*d)
*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(-I*c^2 +
4*c*d + 4*I*d^2)/((-I*a^2*c^3 + 3*a^2*c^2*d + 3*I*a^2*c*d^2 - a^2*d^3)*f^2)
) + (-I*c^2 + 2*c*d)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/((a*c^2 + 2*
I*a*c*d - a*d^2)*f)) + (-I*a*c + a*d)*f*sqrt(-(-I*c^2 + 4*c*d + 4*I*d^2)/((
-I*a^2*c^3 + 3*a^2*c^2*d + 3*I*a^2*c*d^2 - a^2*d^3)*f^2))*e^(2*I*f*x + 2*I*
e)*log(-1/2*(-I*c^2 + 3*c*d + 2*I*d^2 - ((a*c^2 + 2*I*a*c*d - a*d^2)*f*e^(2
*I*f*x + 2*I*e) + (a*c^2 + 2*I*a*c*d - a*d^2)*f)*sqrt(((c - I*d)*e^(2*I*f*x
 + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(-I*c^2 + 4*c*d + 4*I
*d^2)/((-I*a^2*c^3 + 3*a^2*c^2*d + 3*I*a^2*c*d^2 - a^2*d^3)*f^2)) + (-I*c^2
 + 2*c*d)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/((a*c^2 + 2*I*a*c*d - a
*d^2)*f)) + 2*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x +
2*I*e) + 1))*(e^(2*I*f*x + 2*I*e) + 1))*e^(-2*I*f*x - 2*I*e)/((I*a*c - a*d)
*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{1}{\sqrt{c+d\tan\left(e+fx\right)}\tan\left(e+fx\right)-i\sqrt{c+d\tan\left(e+fx\right)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] -I*Integral(1/(sqrt(c + d*tan(e + f*x))*tan(e + f*x) - I*sqrt(c + d*tan(e +
 f*x))), x)/a
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(123) = 246$.
time = 0.57, size = 376, normalized size = 2.43

$$\frac{\sqrt{d\tan\left(fx+e\right)+c}\,d}{2\left(acf+i\,adf\right)\left(d\tan\left(fx+e\right)-i\,d\right)}+\frac{2\left(c+2i\,d\right)\arctan\left(\frac{2\left(\sqrt{d\tan\left(fx+e\right)+c}\,c-\sqrt{c^2+d^2}\,\sqrt{d\tan\left(fx+e\right)+c}\right)}{c\sqrt{-2\,c+2\,\sqrt{c^2+d^2}}+i\sqrt{-2\,c+2\,\sqrt{c^2+d^2}}\,d-\sqrt{c^2+d^2}\,\sqrt{-2\,c+2\,\sqrt{c^2+d^2}}}\right)}{-2\left(-i\,acf+adf\right)\sqrt{-2\,c+2\,\sqrt{c^2+d^2}}\left(\frac{i\,d}{c-\sqrt{c^2+d^2}}+1\right)}+\frac{i\,\arctan\left(\frac{2\left(\sqrt{d\tan\left(fx+e\right)+c}\,c-\sqrt{c^2+d^2}\,\sqrt{d\tan\left(fx+e\right)+c}\right)}{c\sqrt{-2\,c+2\,\sqrt{c^2+d^2}}-i\sqrt{-2\,c+2\,\sqrt{c^2+d^2}}\,d-\sqrt{c^2+d^2}\,\sqrt{-2\,c+2\,\sqrt{c^2+d^2}}}\right)}{a\sqrt{-2\,c+2\,\sqrt{c^2+d^2}}\,f\left(-\frac{i\,d}{c-\sqrt{c^2+d^2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*tan(f*x + e) + c)*d/((a*c*f + I*a*d*f)*(d*tan(f*x + e) - I*d)) +
 2*(c + 2*I*d)*arctan(2*(sqrt(d*tan(f*x + e) + c)*c - sqrt(c^2 + d^2)*sqrt(
d*tan(f*x + e) + c))/(c*sqrt(-2*c + 2*sqrt(c^2 + d^2)) + I*sqrt(-2*c + 2*sq
rt(c^2 + d^2))*d - sqrt(c^2 + d^2)*sqrt(-2*c + 2*sqrt(c^2 + d^2))))/((2*I*a
*c*f - 2*a*d*f)*sqrt(-2*c + 2*sqrt(c^2 + d^2))*(I*d/(c - sqrt(c^2 + d^2)) +
 1)) + I*arctan(2*(sqrt(d*tan(f*x + e) + c)*c - sqrt(c^2 + d^2)*sqrt(d*tan(
f*x + e) + c))/(c*sqrt(-2*c + 2*sqrt(c^2 + d^2)) - I*sqrt(-2*c + 2*sqrt(c^2
 + d^2))*d - sqrt(c^2 + d^2)*sqrt(-2*c + 2*sqrt(c^2 + d^2))))/(a*sqrt(-2*c
+ 2*sqrt(c^2 + d^2))*f*(-I*d/(c - sqrt(c^2 + d^2)) + 1))
```

Mupad [B]
time = 8.31, size = 2500, normalized size = 16.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)*(c + d*tan(e + f*x))^(1/2)),x)
```

```
[Out] log(a*d^6*f*1i - ((-(c*d^6*48i + 48*d^7 + 96*c^2*d^5 - c^3*d^4*32i - a^2*c^
2*f^2*((((48*c^2*d^7 - 48*d^9 + 32*c^4*d^5)*1i)/(a^2*c^4*f^2 + a^2*d^4*f^2
+ 2*a^2*c^2*d^2*f^2) + (144*c*d^8 + 112*c^3*d^6 + 32*c^5*d^4)/(a^2*c^4*f^2
+ a^2*d^4*f^2 + 2*a^2*c^2*d^2*f^2))^2 - 4*((4*d^8 + 3*c^2*d^6 + c^4*d^4)/(a
^4*c^4*f^4 + a^4*d^4*f^4 + 2*a^4*c^2*d^2*f^4) + ((4*c*d^7 + 2*c^3*d^5)*1i)/
(a^4*c^4*f^4 + a^4*d^4*f^4 + 2*a^4*c^2*d^2*f^4))*(256*d^6 + 256*c^2*d^4))^(
1/2)*1i + a^2*d^2*f^2*((((48*c^2*d^7 - 48*d^9 + 32*c^4*d^5)*1i)/(a^2*c^4*f^
2 + a^2*d^4*f^2 + 2*a^2*c^2*d^2*f^2) + (144*c*d^8 + 112*c^3*d^6 + 32*c^5*d^
4)/(a^2*c^4*f^2 + a^2*d^4*f^2 + 2*a^2*c^2*d^2*f^2))^2 - 4*((4*d^8 + 3*c^2*d
^6 + c^4*d^4)/(a^4*c^4*f^4 + a^4*d^4*f^4 + 2*a^4*c^2*d^2*f^4) + ((4*c*d^7 +
 2*c^3*d^5)*1i)/(a^4*c^4*f^4 + a^4*d^4*f^4 + 2*a^4*c^2*d^2*f^4))*(256*d^6 +
```

256*c^2*d^4))^(1/2)*1i + 2*a^2*c*d*f^2*((((48*c^2*d^7 - 48*d^9 + 32*c^4*d^5)*1i)/(a^2*c^4*f^2 + a^2*d^4*f^2 + 2*a^2*c^2*d^2*f^2) + (144*c*d^8 + 112*c^3*d^6 + 32*c^5*d^4)/(a^2*c^4*f^2 + a^2*d^4*f^2 + 2*a^2*c^2*d^2*f^2))^2 - 4*((4*d^8 + 3*c^2*d^6 + c^4*d^4)/(a^4*c^4*f^4 + a^4*d^4*f^4 + 2*a^4*c^2*d^2*f^4) + ((4*c*d^7 + 2*c^3*d^5)*1i)/(a^4*c^4*f^4 + a^4*d^4*f^4 + 2*a^4*c^2*d^2*f^4))*(256*d^6 + 256*c^2*d^4))^(1/2))/(512*(d^6 + c^2*d^4)*(a^2*d^2*f^2*1i - a^2*c^2*f^2*1i + 2*a^2*c*d*f^2)))^(1/2)*(24*a^3*d^7*f^3 + a^3*c*d^6*f^3*16i + 32*a^3*c^2*d^5*f^3 + a^3*c^3*d^4*f^3*16i + 8*a^3*c^4*d^3*f^3 - 2*(c + d*tan(e + f*x))^(1/2)*(a^2*c^2*d^3*f^2*64i - 32*a^2*c*d^4*f^2 + 32*a^2*c^3*d^2*f^2)*(a^2*d^2*f^2 - a^2*c^2*f^2 + a^2*c*d*f^2*2i)*(-(c*d^6*48i + 48*d^7 + 96*c^2*d^5 - c^3*d^4*32i - a^2*c^2*f^2*((((48*c^2*d^7 - 48*d^9 + 32*c^4*d^5)*1i)/(a^2*c^4*f^2 + a^2*d^4*f^2 + 2*a^2*c^2*d^2*f^2) + (144*c*d^8 + 112*c^3*d^6 + 32*c^5*d^4)/(a^2*c^4*f^2 + a^2*d^4*f^2 + 2*a^2*c^2*d^2*f^2))^2 - 4*((4*d^8 + 3*c^2*d^6 + c^4*d^4)/(a^4*c^4*f^4 + a^4*d^4*f^4 + 2*a^4*c^2*d^2*f^4) + ((4*c*d^7 + 2*c^3*d^5)*1i)/(a^4*c^4*f^4 + a^4*d^4*f^4 + 2*a^4*c^2*d^2*f^4))*(256*d^6 + 256*c^2*d^4))^(1/2)*1i + a^2*d^2*f^2*((((48*c^2*d^7 - 48*d^9 + 32*c^4*d^5)*1i)/(a^2*c^4*f^2 + a^2*d^4*f^2 + 2*a^2*c^2*d^2*f^2) + (144*c*d^8 + 112*c^3*d^6 + 32*c^5*d^4)/(a^2*c^4*f^2 + a^2*d^4*f^2 + 2*a^2*c^2*d^2*f^2))^2 - 4*((4*d^8 + 3*c^2*d^6 + c^4*d^4)/(a^4*c^4*f^4 + a^4*d^4*f^4 + 2*a^4*c^2*d^2*f^4) + ((4*c*d^7 + 2*c^3*d^5)*1i)/(a^4*c^4*f^4 + a^4*d^4*f^4 + 2*a^4*c^2*d^2*f^4))*(256*d^6 + 256*c^2*d^4))^(1/2)*1i + 2*a^2*c*d*f^2*((((48*c^2*d^7 - 48*d^9 + 32*c^4*d^5)*1i)/(a^2*c^4*f^2 + a^2*d^4*f^2 + 2*a^2*c^2*d^2*f^2) + (144*c*d^8 + 112*c^3*d^6 + 32*c^5*d^4)/(a^2*c^4*f^2 + a^2*d^4*f^2 + 2*a^2*c^2*d^2*f^2))^2 - 4*((4*d^8 + 3*c^2*d^6 + c^4*d^4)/(a^4*c^4*f^4 + a^4*d^4*f^4 + 2*a^4*c^2*d^2*f^4) + ((4*c*d^7 + 2*c^3*d^5)*1i)/(a^4*c^4*f^4 + a^4*d^4*f^4 + 2*a^4*c^2*d^2*f^4))*(256*d^6 + 256*c^2*d^4))^(1/2))/(512*(d^6 + c^2*d^4)*(a^2*d^2*f^2*1i - a^2*c^2*f^2*1i + 2*a^2*c*d*f^2)))^(1/2)) - 2*(c + d*tan(e + f*x))^(1/2)*(a^2*d^2*f^2 - a^2*c^2*f^2 + a^2*c*d*f^2*2i)*(c*d^3*6i - 5*d^4 + 2*c^2*d^2))*(-(c*d^6*48i + 48*d^7 + 96*c^2*d^5 - c^3*d^4*32i - a^2*c^2*f^2*((((48*c^2*d^7 - 48*d^9 + 32*c^4*d^5)*1i)/(a^2*c^4*f^2 + a^2*d^4*f^2 + 2*a^2*c^2*d^2*f^2) + (144*c*d^8 + 112*c^3*d^6 + 32*c^5*d^4)/(a^2*c^4*f^2 + a^2*d^4*f^2 + 2*a^2*c^2*d^2*f^2))^2 - 4*((4*d^8 + 3*c^2*d^6 + c^4*d^4)/(a^4*c^4*f^4 + a^4*d^4*f^4 + 2*a^4*c^2*d^2*f^4) + ((4*c*d^7 + 2*c^3*d^5)*1i)/(a^4*c^4*f^4 + a^4*d^4*f^4 + 2*a^4*c^2*d^2*f^4))*(256*d^6 + 256*c^2*d^4))^(1/2)*1i + a^2*d^2*f^2*((((48*c^2*d^7 - 48*d^9 + 32*c^4*d^5)*1i)/(a^2*c^4*f^2 + a^2*d^4*f^2 + 2*a^2*c^2*d^2*f^2) + (144*c*d^8 + 112*c^3*d^6 + 32*c^5*d^4)/(a^2*c^4*f^2 + a^2*d^4*f^2 + 2*a^2*c^2*d^2*f^2))^2 - 4*((4*d^8 + 3*c^2*d^6 + c^4*d^4)/(a^4*c^4*f^4 + a^4*d^4*f^4 + 2*a^4*c^2*d^2*f^4) + ((4*c*d^7 + 2*c^3*d^5)*1i)/(a^4*c^4*f^4 + a^4*d^4*f^4 + 2*a^4*c^2*d^2*f^4))*(256*d^6 + 256*c^2*d^4))^(1/2)*1i + 2*a^2*c*d*f^2*((((48*c^2*d^7 - 48*d^9 + 32*c^4*d^5)*1i)/(a^2*c^4*f^2 + a^2*d^4*f^2 + 2*a^2*c^2*d^2*f^2) + (144*c*d^8 + 112*c^3*d^6 + 32*c^5*d^4)/(a^2*c^4*f^2 + a^2*d^4*f^2 + 2*a^2*c^2*d^2*f^2))^2 - 4*((4*d^8 + 3*c^2*d^6 + c^4*d^4)/(a^4*c^4*f^4 + a^4*d^4*f^4 + 2*a^4*c^2*d^2*f^4) + ((4*c*d^7 + 2*c^3*d^5)*1i)/(a^4*c^4*f^4 + a^4*d^4*f^4 + 2*a^4*c^2*d^2*f^4))*(256*d^6 + 256*c^2*d^4))^(1/2))/(512*(d^6 +

c^2*d^4)*(a^2*d^2*f^2*1i - a^2*c^2*f^2*1i + 2*a^2*c*d*f^2)))^(1/2) - (3*a*c *d^5*f)/2 - (a*c^3*d^3*f)/2)*(-(c*d^6*48i + 48*d^7 + 96*c^2*d^5 - c^3*d^4*3 2i - a^2*c^2*f^2*((((48*c^2*d^7 - 48*d^9 + 32*c^4*d^5)*1i)/(a^2*c^4*f^2 + a ^2*d^4*f^2 + 2*a^2*c^2*d^2*f^2) + (144*c*d^8 + 112*c^3*d^6 + 32*c^5*d^4)/(a ^2*c^4*f^2 + a^2*d^4*f^2 + 2*a^2*c^2*d^2*f^2))^2 - 4*((4*d^8 + 3*c^2*d^6 + c^4*d^4)/(a^4*c^4*f^4 + a^4*d^4*f^4 + 2*a^4*c^2*d^2*f^4) + ((4*c*d^7 + 2*c^ 3*d^5)*1i)/(a^4*c^4*f^4 + a^4*d^4*f^4 + 2*a^4*c^2*d^2*f^4))*(256*d^6 + 256* c^2*d^4))^(1/2)*1i + a^2*d^2*f^2*((((48*c^2*d^7 - 48*d^9 + 32*c^4*d^5)*1i)/ (a^2*c^4*f^2 + a^2*d^4*f^2 + 2*a^2*c^2*d^2*f^2) + (144*c*d^8 + 112*c^3*d^6 + 32*c^5*d^4)/(a^2*c^4*f^2 + a^2*d^4*f^2 + 2*a^2*c^2*d^2*f^2))^2 - 4*((4*d^ 8 + 3*c^2*d^6 + c^4*d^4)/(a^4*c^4*f^4 + a^4*d^4...

3.1123 $\int \frac{1}{(a+ia\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=221

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{4a^2\sqrt{c-id}\,f}+\frac{(2ic^2-6cd-7id^2)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{8a^2(c+id)^{5/2}f}+\frac{(2ic-5d)\sqrt{c}}{8a^2(c+id)^2f(1}$$

```
[Out] 1/8*(2*I*c^2-6*c*d-7*I*d^2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/a
^2/(c+I*d)^(5/2)/f-1/4*I*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/a^2/
f/(c-I*d)^(1/2)+1/8*(2*I*c-5*d)*(c+d*tan(f*x+e))^(1/2)/a^2/(c+I*d)^2/f/(1+I
*tan(f*x+e))-1/4*(c+d*tan(f*x+e))^(1/2)/(I*c-d)/f/(a+I*a*tan(f*x+e))^2
```

Rubi [A]
time = 0.42, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3640, 3677, 3620, 3618, 65, 214}

$$\frac{(2ic^2-6cd-7id^2)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{8a^2f(c+id)^{5/2}}+\frac{(-5d+2ic)\sqrt{c+d\tan(e+fx)}}{8a^2f(c+id)^2(1+i\tan(e+fx))}-\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{4a^2f\sqrt{c-id}}-\frac{\sqrt{c+d\tan(e+fx)}}{4f(-d+ic)(a+ia\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^2*Sqrt[c + d*Tan[e + f*x]]),x]
```

```
[Out] ((-1/4*I)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/(a^2*Sqrt[c - I*
d]*f) + (((2*I)*c^2 - 6*c*d - (7*I)*d^2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/S
qrt[c + I*d]])/(8*a^2*(c + I*d)^(5/2)*f) + (((2*I)*c - 5*d)*Sqrt[c + d*Tan[
e + f*x]])/(8*a^2*(c + I*d)^2*f*(1 + I*Tan[e + f*x])) - Sqrt[c + d*Tan[e +
f*x]]/(4*(I*c - d)*f*(a + I*a*Tan[e + f*x])^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}\,dx = -\frac{\sqrt{c+d\tan(e+fx)}}{4(ic-d)f(a+ia\tan(e+fx))^2} - \frac{\int \frac{-\frac{1}{2}a(4ic-7d) \cdots}{(a+ia\tan(e+fx))\sqrt{\cdots}}}{4a^2(\cdots}$$

$$= \frac{(2ic-5d)\sqrt{c+d\tan(e+fx)}}{8a^2(c+id)^2f(1+i\tan(e+fx))} - \frac{\sqrt{c+d\tan(e \cdots}}{4(ic-d)f(a+ia\,\mathrm{ta}\cdots}$$

$$= \frac{(2ic-5d)\sqrt{c+d\tan(e+fx)}}{8a^2(c+id)^2f(1+i\tan(e+fx))} - \frac{\sqrt{c+d\tan(e \cdots}}{4(ic-d)f(a+ia\,\mathrm{ta}\cdots}$$

$$= \frac{(2ic-5d)\sqrt{c+d\tan(e+fx)}}{8a^2(c+id)^2f(1+i\tan(e+fx))} - \frac{\sqrt{c+d\tan(e \cdots}}{4(ic-d)f(a+ia\,\mathrm{ta}\cdots}$$

$$= \frac{(2ic-5d)\sqrt{c+d\tan(e+fx)}}{8a^2(c+id)^2f(1+i\tan(e+fx))} - \frac{\sqrt{c+d\tan(e \cdots}}{4(ic-d)f(a+ia\,\mathrm{ta}\cdots}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{4a^2\sqrt{c-id}\,f} - \frac{(6cd-i(2c^2-7 \cdots}{\cdots}$$

Mathematica [A]

time = 2.24, size = 275, normalized size = 1.24

$$\frac{\sec^2(e+fx)(\cos(fx)+i\sin(fx))^2\left(\frac{2\left(\sqrt{-c+id}\,(-2ic^2+6cd+7id^2)\mathrm{ArcTan}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{-c-id}}\right)+2i(-c-id)^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{-c+id}}\right)\right)(\cos(2e)+i\sin(2e))}{(-c-id)^{5/2}\sqrt{-c+id}}+\frac{2\cos(e+fx)(i\cos(2fx)+\sin(2fx))((4c+7id)\cos(e+fx)+(2ic-5d)\sin(e+fx))\sqrt{c+d\tan(e+fx)}}{(c+id)^2}\right)}{16f(a+ia\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^2*Sqrt[c + d*Tan[e + f*x]]),x]
```

```
[Out] (Sec[e + f*x]^2*(Cos[f*x] + I*Sin[f*x])^2*((2*(Sqrt[-c + I*d]*((-2*I)*c^2 +
 6*c*d + (7*I)*d^2)*ArcTan[Sqrt[c + d*Tan[e + f*x]]/Sqrt[-c - I*d]] + (2*I)
*(-c - I*d)^(5/2)*ArcTan[Sqrt[c + d*Tan[e + f*x]]/Sqrt[-c + I*d]])*(Cos[2*e
] + I*Sin[2*e]))/((-c - I*d)^(5/2)*Sqrt[-c + I*d]) + (2*Cos[e + f*x]*(I*Cos
[2*f*x] + Sin[2*f*x])*((4*c + (7*I)*d)*Cos[e + f*x] + ((2*I)*c - 5*d)*Sin[e
 + f*x])*Sqrt[c + d*Tan[e + f*x]])/(c + I*d)^2))/(16*f*(a + I*a*Tan[e + f*x
])^2)
```

Maple [A]

time = 0.35, size = 232, normalized size = 1.05

method	result

derivativedivides	$\frac{2d^3\left(\frac{i\arctan\left(\frac{\sqrt{c+d\tan(fx+e)}}{\sqrt{id-c}}\right)}{8d^3\sqrt{id-c}}+\frac{\frac{\frac{d(5id+2c)(c+d\tan(fx+e))^{\frac{3}{2}}}{4icd+2c^2-2d^2}-\frac{d(9icd+2c^2-7d^2)\sqrt{c+d\tan(fx+e}}{2(2icd+c^2-d^2)}}{(-d\tan(fx+e)+id)^2}}{8d^3}\right.}{f\,a^2}$
default	$\frac{2d^3\left(\frac{i\arctan\left(\frac{\sqrt{c+d\tan(fx+e)}}{\sqrt{id-c}}\right)}{8d^3\sqrt{id-c}}+\frac{\frac{\frac{d(5id+2c)(c+d\tan(fx+e))^{\frac{3}{2}}}{4icd+2c^2-2d^2}-\frac{d(9icd+2c^2-7d^2)\sqrt{c+d\tan(fx+e}}{2(2icd+c^2-d^2)}}{(-d\tan(fx+e)+id)^2}}{8d^3}\right.}{f\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^2*d^3*(1/8*I/d^3/(I*d-c)^(1/2)*arctan((c+d*tan(f*x+e))^(1/2)/(I*d-c)^
(1/2))+1/8/d^3*((1/2*d*(5*I*d+2*c)/(2*I*c*d+c^2-d^2)*(c+d*tan(f*x+e))^(3/2)
-1/2*d*(9*I*c*d+2*c^2-7*d^2)/(2*I*c*d+c^2-d^2)*(c+d*tan(f*x+e))^(1/2))/(-d*
tan(f*x+e)+I*d)^2-1/2*(2*I*c^2-6*c*d-7*I*d^2)/(2*I*c*d+c^2-d^2)/(-I*d-c)^(1
/2)*arctan((c+d*tan(f*x+e))^(1/2)/(-I*d-c)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1429 vs. 2(179) = 358.
time = 1.57, size = 1429, normalized size = 6.47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="frica
s")
```

```
[Out] -1/32*(8*(a^2*c^2 + 2*I*a^2*c*d - a^2*d^2)*f*sqrt(1/16*I/((-I*a^4*c - a^4*d
)*f^2))*e^(4*I*f*x + 4*I*e)*log(-2*(4*((I*a^2*c + a^2*d)*f*e^(2*I*f*x + 2*I
*e) + (I*a^2*c + a^2*d)*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(
e^(2*I*f*x + 2*I*e) + 1))*sqrt(1/16*I/((-I*a^4*c - a^4*d)*f^2)) - (c - I*d)
*e^(2*I*f*x + 2*I*e) - c)*e^(-2*I*f*x - 2*I*e)) - 8*(a^2*c^2 + 2*I*a^2*c*d
- a^2*d^2)*f*sqrt(1/16*I/((-I*a^4*c - a^4*d)*f^2))*e^(4*I*f*x + 4*I*e)*log(
-2*(4*((-I*a^2*c - a^2*d)*f*e^(2*I*f*x + 2*I*e) + (-I*a^2*c - a^2*d)*f)*sqr
t(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt
(1/16*I/((-I*a^4*c - a^4*d)*f^2)) - (c - I*d)*e^(2*I*f*x + 2*I*e) - c)*e^(-
2*I*f*x - 2*I*e)) + (a^2*c^2 + 2*I*a^2*c*d - a^2*d^2)*f*sqrt(-(4*I*c^4 - 24
*c^3*d - 64*I*c^2*d^2 + 84*c*d^3 + 49*I*d^4)/((I*a^4*c^5 - 5*a^4*c^4*d - 10
*I*a^4*c^3*d^2 + 10*a^4*c^2*d^3 + 5*I*a^4*c*d^4 - a^4*d^5)*f^2))*e^(4*I*f*x
 + 4*I*e)*log(1/8*(2*c^3 + 8*I*c^2*d - 13*c*d^2 - 7*I*d^3 + ((I*a^2*c^3 - 3
*a^2*c^2*d - 3*I*a^2*c*d^2 + a^2*d^3)*f*e^(2*I*f*x + 2*I*e) + (I*a^2*c^3 -
3*a^2*c^2*d - 3*I*a^2*c*d^2 + a^2*d^3)*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*
e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(4*I*c^4 - 24*c^3*d - 64*I*c
^2*d^2 + 84*c*d^3 + 49*I*d^4)/((I*a^4*c^5 - 5*a^4*c^4*d - 10*I*a^4*c^3*d^2
+ 10*a^4*c^2*d^3 + 5*I*a^4*c*d^4 - a^4*d^5)*f^2)) + (2*c^3 + 6*I*c^2*d - 7*
c*d^2)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/((-I*a^2*c^3 + 3*a^2*c^2*d
 + 3*I*a^2*c*d^2 - a^2*d^3)*f)) - (a^2*c^2 + 2*I*a^2*c*d - a^2*d^2)*f*sqrt(
-(4*I*c^4 - 24*c^3*d - 64*I*c^2*d^2 + 84*c*d^3 + 49*I*d^4)/((I*a^4*c^5 - 5*
a^4*c^4*d - 10*I*a^4*c^3*d^2 + 10*a^4*c^2*d^3 + 5*I*a^4*c*d^4 - a^4*d^5)*f^
2))*e^(4*I*f*x + 4*I*e)*log(1/8*(2*c^3 + 8*I*c^2*d - 13*c*d^2 - 7*I*d^3 + (
(-I*a^2*c^3 + 3*a^2*c^2*d + 3*I*a^2*c*d^2 - a^2*d^3)*f*e^(2*I*f*x + 2*I*e)
+ (-I*a^2*c^3 + 3*a^2*c^2*d + 3*I*a^2*c*d^2 - a^2*d^3)*f)*sqrt(((c - I*d)*e
^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(4*I*c^4 - 2
4*c^3*d - 64*I*c^2*d^2 + 84*c*d^3 + 49*I*d^4)/((I*a^4*c^5 - 5*a^4*c^4*d - 1
0*I*a^4*c^3*d^2 + 10*a^4*c^2*d^3 + 5*I*a^4*c*d^4 - a^4*d^5)*f^2)) + (2*c^3
+ 6*I*c^2*d - 7*c*d^2)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/((-I*a^2*c
^3 + 3*a^2*c^2*d + 3*I*a^2*c*d^2 - a^2*d^3)*f)) - 2*(3*(I*c - 2*d)*e^(4*I*f
*x + 4*I*e) - (-4*I*c + 7*d)*e^(2*I*f*x + 2*I*e) + I*c - d)*sqrt(((c - I*d)
*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-4*I*f*x - 4
*I*e)/((a^2*c^2 + 2*I*a^2*c*d - a^2*d^2)*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{\sqrt{c+d\tan{(e+fx)}}\tan^2{(e+fx)}-2i\sqrt{c+d\tan{(e+fx)}}\tan{(e+fx)}-\sqrt{c+d\tan{(e+fx)}}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -Integral(1/(sqrt(c + d*tan(e + f*x))*tan(e + f*x)**2 - 2*I*sqrt(c + d*tan(
e + f*x))*tan(e + f*x) - sqrt(c + d*tan(e + f*x))), x)/a**2
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(179) = 358$.
time = 0.64, size = 494, normalized size = 2.24

$$-\frac{2\,(2\,c^2+6i\,cd-7\,d^2)\arctan\left(\frac{2\left(\sqrt{d\tan(fx+e)+c}\,c-\sqrt{c^2+d^2}\,\sqrt{d\tan(fx+e)+c}\right)}{c\sqrt{-2\,c+2\,\sqrt{c^2+d^2}}+i\sqrt{-2\,c+2\,\sqrt{c^2+d^2}}\,d-\sqrt{c^2+d^2}\,\sqrt{-2\,c+2\,\sqrt{c^2+d^2}}}\right)}{-8\,(i\,a^2c^2f-2\,a^2cdf-i\,a^2d^2f)\sqrt{-2\,c+2\,\sqrt{c^2+d^2}}\left(\frac{i\,d}{c-\sqrt{c^2+d^2}}+1\right)}+\frac{2\,(d\tan(fx+e)+c)^{\frac{3}{2}}cd-2\,\sqrt{d\tan(fx+e)+c}\,c^2d+5i\,(d\tan(fx+e)+c)^{\frac{3}{2}}d^2-9i\,\sqrt{d\tan(fx+e)+c}\,cd^2+7\,\sqrt{d\tan(fx+e)+c}\,d^3}{8\,(a^2c^2f+2i\,a^2cdf-a^2d^2f)(d\tan(fx+e)-i\,d)^2}-\frac{\arctan\left(-\frac{2\left(-i\,\sqrt{d\tan(fx+e)+c}\,c-i\,\sqrt{c^2+d^2}\,\sqrt{d\tan(fx+e)+c}\right)}{\sqrt{2\,c+2\,\sqrt{c^2+d^2}}\,c-i\sqrt{2\,c+2\,\sqrt{c^2+d^2}}\,d+\sqrt{c^2+d^2}\,\sqrt{2\,c+2\,\sqrt{c^2+d^2}}}\right)}{2\,a^2\sqrt{2\,c+2\,\sqrt{c^2+d^2}}\,f\left(-\frac{i\,d}{c+\sqrt{c^2+d^2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac"
)
```

```
[Out] -2*(2*c^2 + 6*I*c*d - 7*d^2)*arctan(2*(sqrt(d*tan(f*x + e) + c)*c - sqrt(c^
2 + d^2)*sqrt(d*tan(f*x + e) + c))/(c*sqrt(-2*c + 2*sqrt(c^2 + d^2)) + I*sq
rt(-2*c + 2*sqrt(c^2 + d^2))*d - sqrt(c^2 + d^2)*sqrt(-2*c + 2*sqrt(c^2 + d
^2))))/((-8*I*a^2*c^2*f + 16*a^2*c*d*f + 8*I*a^2*d^2*f)*sqrt(-2*c + 2*sqrt(
c^2 + d^2))*(I*d/(c - sqrt(c^2 + d^2)) + 1)) + 1/8*(2*(d*tan(f*x + e) + c)^
(3/2)*c*d - 2*sqrt(d*tan(f*x + e) + c)*c^2*d + 5*I*(d*tan(f*x + e) + c)^(3/
2)*d^2 - 9*I*sqrt(d*tan(f*x + e) + c)*c*d^2 + 7*sqrt(d*tan(f*x + e) + c)*d^
3)/((a^2*c^2*f + 2*I*a^2*c*d*f - a^2*d^2*f)*(d*tan(f*x + e) - I*d)^2) - 1/2
*arctan(-2*(-I*sqrt(d*tan(f*x + e) + c)*c - I*sqrt(c^2 + d^2)*sqrt(d*tan(f*
x + e) + c))/(sqrt(2*c + 2*sqrt(c^2 + d^2))*c - I*sqrt(2*c + 2*sqrt(c^2 + d
^2))*d + sqrt(c^2 + d^2)*sqrt(2*c + 2*sqrt(c^2 + d^2))))/(a^2*sqrt(2*c + 2*
sqrt(c^2 + d^2))*f*(-I*d/(c + sqrt(c^2 + d^2)) + 1))
```

Mupad [B]
time = 9.17, size = 2500, normalized size = 11.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^2*(c + d*tan(e + f*x))^(1/2)),x)
```

```
[Out] log(a^2*d^10*f*35i - ((-(45*d^9 - c*d^8*15i + 60*c^2*d^7 - c^3*d^6*80i - 40
*c^4*d^5 + c^5*d^4*8i + a^4*c^4*f^2*(((165*c*d^12 + 70*c^3*d^10 + 73*c^5*d^
8 + 32*c^7*d^6 + 8*c^9*d^4)/(a^4*c^8*f^2 + a^4*d^8*f^2 + 4*a^4*c^2*d^6*f^2
+ 6*a^4*c^4*d^4*f^2 + 4*a^4*c^6*d^2*f^2) + ((150*c^2*d^11 - 45*d^13 + 95*c^
4*d^9 + 52*c^6*d^7 + 8*c^8*d^5)*1i)/(a^4*c^8*f^2 + a^4*d^8*f^2 + 4*a^4*c^2*
d^6*f^2 + 6*a^4*c^4*d^4*f^2 + 4*a^4*c^6*d^2*f^2))^2 - 4*(256*d^6 + 256*c^2*
d^4)*((((7*c*d^11)/4 + (19*c^3*d^9)/16 + (11*c^5*d^7)/16 + (c^7*d^5)/8)*1i)
/(a^8*c^8*f^4 + a^8*d^8*f^4 + 4*a^8*c^2*d^6*f^4 + 6*a^8*c^4*d^4*f^4 + 4*a^8
*c^6*d^2*f^4) + ((49*d^12)/64 - (11*c^2*d^10)/32 + (5*c^4*d^8)/64 + (c^6*d^
6)/8 + (c^8*d^4)/16)/(a^8*c^8*f^4 + a^8*d^8*f^4 + 4*a^8*c^2*d^6*f^4 + 6*a^8
*c^4*d^4*f^4 + 4*a^8*c^6*d^2*f^4)))^(1/2)*1i + a^4*d^4*f^2*(((165*c*d^12 +
```

70*c^3*d^10 + 73*c^5*d^8 + 32*c^7*d^6 + 8*c^9*d^4)/(a^4*c^8*f^2 + a^4*d^8*f
^2 + 4*a^4*c^2*d^6*f^2 + 6*a^4*c^4*d^4*f^2 + 4*a^4*c^6*d^2*f^2) + ((150*c^2
*d^11 - 45*d^13 + 95*c^4*d^9 + 52*c^6*d^7 + 8*c^8*d^5)*1i)/(a^4*c^8*f^2 + a
^4*d^8*f^2 + 4*a^4*c^2*d^6*f^2 + 6*a^4*c^4*d^4*f^2 + 4*a^4*c^6*d^2*f^2))^2
- 4*(256*d^6 + 256*c^2*d^4)*((((7*c*d^11)/4 + (19*c^3*d^9)/16 + (11*c^5*d^7
)/16 + (c^7*d^5)/8)*1i)/(a^8*c^8*f^4 + a^8*d^8*f^4 + 4*a^8*c^2*d^6*f^4 + 6*
a^8*c^4*d^4*f^4 + 4*a^8*c^6*d^2*f^4) + ((49*d^12)/64 - (11*c^2*d^10)/32 + (
5*c^4*d^8)/64 + (c^6*d^6)/8 + (c^8*d^4)/16)/(a^8*c^8*f^4 + a^8*d^8*f^4 + 4*
a^8*c^2*d^6*f^4 + 6*a^8*c^4*d^4*f^4 + 4*a^8*c^6*d^2*f^4)))^(1/2)*1i - a^4*c
^2*d^2*f^2*(((165*c*d^12 + 70*c^3*d^10 + 73*c^5*d^8 + 32*c^7*d^6 + 8*c^9*d^
4)/(a^4*c^8*f^2 + a^4*d^8*f^2 + 4*a^4*c^2*d^6*f^2 + 6*a^4*c^4*d^4*f^2 + 4*a
^4*c^6*d^2*f^2) + ((150*c^2*d^11 - 45*d^13 + 95*c^4*d^9 + 52*c^6*d^7 + 8*c^
8*d^5)*1i)/(a^4*c^8*f^2 + a^4*d^8*f^2 + 4*a^4*c^2*d^6*f^2 + 6*a^4*c^4*d^4*f
^2 + 4*a^4*c^6*d^2*f^2))^2 - 4*(256*d^6 + 256*c^2*d^4)*((((7*c*d^11)/4 + (1
9*c^3*d^9)/16 + (11*c^5*d^7)/16 + (c^7*d^5)/8)*1i)/(a^8*c^8*f^4 + a^8*d^8*f
^4 + 4*a^8*c^2*d^6*f^4 + 6*a^8*c^4*d^4*f^4 + 4*a^8*c^6*d^2*f^4) + ((49*d^12
)/64 - (11*c^2*d^10)/32 + (5*c^4*d^8)/64 + (c^6*d^6)/8 + (c^8*d^4)/16)/(a^8
*c^8*f^4 + a^8*d^8*f^4 + 4*a^8*c^2*d^6*f^4 + 6*a^8*c^4*d^4*f^4 + 4*a^8*c^6*
d^2*f^4)))^(1/2)*6i + 4*a^4*c*d^3*f^2*(((165*c*d^12 + 70*c^3*d^10 + 73*c^5*
d^8 + 32*c^7*d^6 + 8*c^9*d^4)/(a^4*c^8*f^2 + a^4*d^8*f^2 + 4*a^4*c^2*d^6*f^
2 + 6*a^4*c^4*d^4*f^2 + 4*a^4*c^6*d^2*f^2) + ((150*c^2*d^11 - 45*d^13 + 95*
c^4*d^9 + 52*c^6*d^7 + 8*c^8*d^5)*1i)/(a^4*c^8*f^2 + a^4*d^8*f^2 + 4*a^4*c^
2*d^6*f^2 + 6*a^4*c^4*d^4*f^2 + 4*a^4*c^6*d^2*f^2))^2 - 4*(256*d^6 + 256*c^
2*d^4)*((((7*c*d^11)/4 + (19*c^3*d^9)/16 + (11*c^5*d^7)/16 + (c^7*d^5)/8)*1
i)/(a^8*c^8*f^4 + a^8*d^8*f^4 + 4*a^8*c^2*d^6*f^4 + 6*a^8*c^4*d^4*f^4 + 4*a
^8*c^6*d^2*f^4) + ((49*d^12)/64 - (11*c^2*d^10)/32 + (5*c^4*d^8)/64 + (c^6*
d^6)/8 + (c^8*d^4)/16)/(a^8*c^8*f^4 + a^8*d^8*f^4 + 4*a^8*c^2*d^6*f^4 + 6*a
^8*c^4*d^4*f^4 + 4*a^8*c^6*d^2*f^4)))^(1/2) - 4*a^4*c^3*d*f^2*(((165*c*d^12
 + 70*c^3*d^10 + 73*c^5*d^8 + 32*c^7*d^6 + 8*c^9*d^4)/(a^4*c^8*f^2 + a^4*d^
8*f^2 + 4*a^4*c^2*d^6*f^2 + 6*a^4*c^4*d^4*f^2 + 4*a^4*c^6*d^2*f^2) + ((150*
c^2*d^11 - 45*d^13 + 95*c^4*d^9 + 52*c^6*d^7 + 8*c^8*d^5)*1i)/(a^4*c^8*f^2
+ a^4*d^8*f^2 + 4*a^4*c^2*d^6*f^2 + 6*a^4*c^4*d^4*f^2 + 4*a^4*c^6*d^2*f^2))
^2 - 4*(256*d^6 + 256*c^2*d^4)*((((7*c*d^11)/4 + (19*c^3*d^9)/16 + (11*c^5*
d^7)/16 + (c^7*d^5)/8)*1i)/(a^8*c^8*f^4 + a^8*d^8*f^4 + 4*a^8*c^2*d^6*f^4 +
 6*a^8*c^4*d^4*f^4 + 4*a^8*c^6*d^2*f^4) + ((49*d^12)/64 - (11*c^2*d^10)/32
+ (5*c^4*d^8)/64 + (c^6*d^6)/8 + (c^8*d^4)/16)/(a^8*c^8*f^4 + a^8*d^8*f^4 +
 4*a^8*c^2*d^6*f^4 + 6*a^8*c^4*d^4*f^4 + 4*a^8*c^6*d^2*f^4)))^(1/2))/(512*(
d^6 + c^2*d^4)*(a^4*c^4*f^2*1i + a^4*d^4*f^2*1i + 4*a^4*c*d^3*f^2 - 4*a^4*c
^3*d*f^2 - a^4*c^2*d^2*f^2*6i)))^(1/2)*(1152*a^6*d^11*f^3 + a^6*c*d^10*f^3*
1664i + 8*(c + d*tan(e + f*x))^(1/2)*(-(45*d^9 - c*d^8*15i + 60*c^2*d^7 - c
^3*d^6*80i - 40*c^4*d^5 + c^5*d^4*8i + a^4*c^4*f^2*(((165*c*d^12 + 70*c^3*d
^10 + 73*c^5*d^8 + 32*c^7*d^6 + 8*c^9*d^4)/(a^4*c^8*f^2 + a^4*d^8*f^2 + 4*a
^4*c^2*d^6*f^2 + 6*a^4*c^4*d^4*f^2 + 4*a^4*c^6*d^2*f^2) + ((150*c^2*d^11 -
45*d^13 + 95*c^4*d^9 + 52*c^6*d^7 + 8*c^8*d^5)*1i)/(a^4*c^8*f^2 + a^4*d^8*f
^2 + 4*a^4*c^2*d^6*f^2 + 6*a^4*c^4*d^4*f^2 + 4*a^4*c^6*d^2*f^2))^2 - 4*(256

```
*d^6 + 256*c^2*d^4)*((((7*c*d^11)/4 + (19*c^3*d^9)/16 + (11*c^5*d^7)/16 + (
c^7*d^5)/8)*1i)/(a^8*c^8*f^4 + a^8*d^8*f^4 + 4*a^8*c^2*d^6*f^4 + 6*a^8*c^4*
d^4*f^4 + 4*a^8*c^6*d^2*f^4) + ((49*d^12)/64 - (11*c^2*d^10)/32 + (5*c^4*d^
8)/64 + (c^6*d^6)/8 + (c^8*d^4)/16)/(a^8*c^8*f^4 + a^8*d^8*f^4 + 4*a^8*c^2*
d^6*f^4 + 6*a^8*c^4*d^4*f^4 + 4*a^8*c^6*d^2*f^4)))^(1/2)*1i + a^4*d^4*f^2*(
((165*c*d^12 + 70*c^3*d^10 + 73*c^5*d^8 + 32*c^7*d^6 + 8*c^9*d^4)/(a^4*c^8*
f^2 + a^4*d^8*f^2 + 4*a^4*c^2*d^6*f^2 + 6*a^4*c^4*d^4*f^2 + 4*a^4*c^6*d^2*f
^2) + ((150*c^2*d^11 - 45*d^13 + 95*c^4*d^9 + 52*c^6*d^7 + 8*c^8*d^5)*1i)/(
a^4*c^8*f^2 + a^4*d^8*f^2 + 4*a^4*c^2*d^6*f^2 + 6*a^4*c^4*d^4*f^2 + 4*a^4*c
^6*d^2*f^2))^2 - 4*(256*d^6 + 256*c^2*d^4)*((((7*c*d^11)/4 + (19*c^3*d^9)/1
6 + (11*c^5*d^7)/16 + (c^7*d^5)/8)*1i)/(a^8*c^8...
```

3.1124 $\int \frac{1}{(a+ia\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=298

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{8a^3\sqrt{c-id}\,f}+\frac{(2ic^3-8c^2d-13icd^2+12d^3)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{16a^3(c+id)^{7/2}f}-\frac{6(ic}{}$$

```
[Out] 1/16*(2*I*c^3-8*c^2*d-13*I*c*d^2+12*d^3)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+
I*d)^(1/2))/a^3/(c+I*d)^(7/2)/f-1/8*I*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d
)^(1/2))/a^3/f/(c-I*d)^(1/2)-1/6*(c+d*tan(f*x+e))^(1/2)/(I*c-d)/f/(a+I*a*ta
n(f*x+e))^3+1/24*(3*I*c-8*d)*(c+d*tan(f*x+e))^(1/2)/a/(c+I*d)^2/f/(a+I*a*ta
n(f*x+e))^2+1/16*(2*c^2+7*I*c*d-10*d^2)*(c+d*tan(f*x+e))^(1/2)/(I*c-d)^3/f/
(a^3+I*a^3*tan(f*x+e))
```

Rubi [A]
time = 0.66, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3640, 3677, 3620, 3618, 65, 214}

$$\frac{(2c^2+7icd-10d^2)\sqrt{c+d\tan(e+fx)}}{16f(-d+ic)^3(a^3+ia^3\tan(e+fx))}+\frac{(2ic^3-8c^2d-13icd^2+12d^3)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{16a^3f(c+id)^{7/2}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{8a^3f\sqrt{c-id}}+\frac{(-8d+3ic)\sqrt{c+d\tan(e+fx)}}{24af(c+id)^2(a+ia\tan(e+fx))^2}-\frac{\sqrt{c+d\tan(e+fx)}}{6f(-d+ic)(a+ia\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^3*Sqrt[c + d*Tan[e + f*x]]),x]
```

```
[Out] ((-1/8*I)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/(a^3*Sqrt[c - I*
d]*f) + (((2*I)*c^3 - 8*c^2*d - (13*I)*c*d^2 + 12*d^3)*ArcTanh[Sqrt[c + d*T
an[e + f*x]]/Sqrt[c + I*d]])/(16*a^3*(c + I*d)^(7/2)*f) - Sqrt[c + d*Tan[e
+ f*x]]/(6*(I*c - d)*f*(a + I*a*Tan[e + f*x])^3) + (((3*I)*c - 8*d)*Sqrt[c
+ d*Tan[e + f*x]])/(24*a*(c + I*d)^2*f*(a + I*a*Tan[e + f*x])^2) + ((2*c^2
+ (7*I)*c*d - 10*d^2)*Sqrt[c + d*Tan[e + f*x]])/(16*(I*c - d)^3*f*(a^3 + I*
a^3*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}}\,dx = -\frac{\sqrt{c+d\tan(e+fx)}}{6(ic-d)f(a+ia\tan(e+fx))^3} - \frac{\int \frac{-\frac{1}{2}a(6ic-11d)}{(a+ia\tan(e+fx))^2\sqrt{}}}{6a^2}$$

$$= -\frac{\sqrt{c+d\tan(e+fx)}}{6(ic-d)f(a+ia\tan(e+fx))^3} + \frac{(3ic-8d)\sqrt{c+}}{24a(c+id)^2f(a+}$$

$$= -\frac{\sqrt{c+d\tan(e+fx)}}{6(ic-d)f(a+ia\tan(e+fx))^3} + \frac{(3ic-8d)\sqrt{c+}}{24a(c+id)^2f(a+}$$

$$= -\frac{\sqrt{c+d\tan(e+fx)}}{6(ic-d)f(a+ia\tan(e+fx))^3} + \frac{(3ic-8d)\sqrt{c+}}{24a(c+id)^2f(a+}$$

$$= -\frac{\sqrt{c+d\tan(e+fx)}}{6(ic-d)f(a+ia\tan(e+fx))^3} + \frac{(3ic-8d)\sqrt{c+}}{24a(c+id)^2f(a+}$$

$$= -\frac{\sqrt{c+d\tan(e+fx)}}{6(ic-d)f(a+ia\tan(e+fx))^3} + \frac{(3ic-8d)\sqrt{c+}}{24a(c+id)^2f(a+}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{8a^3\sqrt{c-id}\,f} + \frac{(2ic^3-8c^2d-1}{}$$

Mathematica [A]
time = 3.42, size = 324, normalized size = 1.09

$$\frac{\sec^3(e+fx)(\cos(fx)+i\sin(fx))^3\left(-\frac{2\left(\sqrt{-c+id}\,(-2ic^3+8c^2d+13icd^2-12d^3)\text{ArcTan}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{-c-id}}\right)-2i(-c-id)^{7/2}\text{ArcTan}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{-c+id}}\right)\right)(\cos(3e)+i\sin(3e))}{(-c-id)^{7/2}\sqrt{-c+id}}+\frac{2\cos(e+fx)(i\cos(3fx)+\sin(3fx))(7c^2+19icd-12d^2+(13c^2+40icd-42d^2)\cos(2(e+fx))+i(9c^2+32icd-38d^2)\sin(2(e+fx)))\sqrt{c+d\tan(e+fx)}}{3(c+id)^3}\right)}{32f(a+ia\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^3*Sqrt[c + d*Tan[e + f*x]]),x]
```

```
[Out] (Sec[e + f*x]^3*(Cos[f*x] + I*Sin[f*x])^3*((-2*(Sqrt[-c + I*d]*((-2*I)*c^3
+ 8*c^2*d + (13*I)*c*d^2 - 12*d^3)*ArcTan[Sqrt[c + d*Tan[e + f*x]]/Sqrt[-c
- I*d]] - (2*I)*(-c - I*d)^(7/2)*ArcTan[Sqrt[c + d*Tan[e + f*x]]/Sqrt[-c +
I*d]])*(Cos[3*e] + I*Sin[3*e]))/((-c - I*d)^(7/2)*Sqrt[-c + I*d]) + (2*Cos[
e + f*x]*(I*Cos[3*f*x] + Sin[3*f*x])*(7*c^2 + (19*I)*c*d - 12*d^2 + (13*c^2
 + (40*I)*c*d - 42*d^2)*Cos[2*(e + f*x)] + I*(9*c^2 + (32*I)*c*d - 38*d^2)*
Sin[2*(e + f*x)])*Sqrt[c + d*Tan[e + f*x]])/(3*(c + I*d)^3)))/(32*f*(a + I*
a*Tan[e + f*x])^3)
```

Maple [A]
time = 0.36, size = 357, normalized size = 1.20

method	result
derivativedivides	$2d^4\left(\frac{\frac{-\frac{d\left(7icd+2c^2-10d^2\right)(c+d\tan(fx+e))^{\frac{5}{2}}}{2\left(3ic^2d-id^3+c^3-3c\,d^2\right)}+\frac{2d\left(15ic^2d-19id^3+3c^3-31c\,d^2\right)(c+d\tan(fx+e))^{\frac{3}{2}}}{3\left(3ic^2d-id^3+c^3-3c\,d^2\right)}-\frac{d\left(13ic^3d-45ic\,d^3+2c^4-38c^2d^2\right.}{2\left(3ic^2d\right.}}{(-d\tan(fx+e)+id)^3}}{16d^4}\right.$
default	$2d^4\left(\frac{\frac{-\frac{d\left(7icd+2c^2-10d^2\right)(c+d\tan(fx+e))^{\frac{5}{2}}}{2\left(3ic^2d-id^3+c^3-3c\,d^2\right)}+\frac{2d\left(15ic^2d-19id^3+3c^3-31c\,d^2\right)(c+d\tan(fx+e))^{\frac{3}{2}}}{3\left(3ic^2d-id^3+c^3-3c\,d^2\right)}-\frac{d\left(13ic^3d-45ic\,d^3+2c^4-38c^2d^2\right.}{2\left(3ic^2d\right.}}{(-d\tan(fx+e)+id)^3}}{16d^4}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3*d^4*(1/16/d^4*((-1/2*d*(2*c^2+7*I*c*d-10*d^2)/(3*I*c^2*d-I*d^3+c^3-
3*c*d^2)*(c+d*tan(f*x+e))^(5/2)+2/3*d*(-31*c*d^2+15*I*c^2*d-19*I*d^3+3*c^3)
/(3*I*c^2*d-I*d^3+c^3-3*c*d^2)*(c+d*tan(f*x+e))^(3/2)-1/2*d*(13*I*c^3*d-45*
I*c*d^3+2*c^4-38*c^2*d^2+18*d^4)/(3*I*c^2*d-I*d^3+c^3-3*c*d^2)*(c+d*tan(f*x
+e))^(1/2))/(-d*tan(f*x+e)+I*d)^3-1/2*(2*I*c^3-8*c^2*d-13*I*c*d^2+12*d^3)/(
3*I*c^2*d-I*d^3+c^3-3*c*d^2)/(-I*d-c)^(1/2)*arctan((c+d*tan(f*x+e))^(1/2)/(
-I*d-c)^(1/2)))+1/16*I/d^4/(I*d-c)^(1/2)*arctan((c+d*tan(f*x+e))^(1/2)/(I*d
-c)^(1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1777 vs. $2(248) = 496$.
time = 1.42, size = 1777, normalized size = 5.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="frica
s")
```

```
[Out] 1/192*(48*(I*a^3*c^3 - 3*a^3*c^2*d - 3*I*a^3*c*d^2 + a^3*d^3)*f*sqrt(1/64*I
/((-I*a^6*c - a^6*d)*f^2))*e^(6*I*f*x + 6*I*e)*log(-2*(8*((I*a^3*c + a^3*d)
*f*e^(2*I*f*x + 2*I*e) + (I*a^3*c + a^3*d)*f)*sqrt(((c - I*d)*e^(2*I*f*x +
2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(1/64*I/((-I*a^6*c - a^6*d
)*f^2)) - (c - I*d)*e^(2*I*f*x + 2*I*e) - c)*e^(-2*I*f*x - 2*I*e)) + 48*(-I
*a^3*c^3 + 3*a^3*c^2*d + 3*I*a^3*c*d^2 - a^3*d^3)*f*sqrt(1/64*I/((-I*a^6*c
- a^6*d)*f^2))*e^(6*I*f*x + 6*I*e)*log(-2*(8*((-I*a^3*c - a^3*d)*f*e^(2*I*f
*x + 2*I*e) + (-I*a^3*c - a^3*d)*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c
 + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(1/64*I/((-I*a^6*c - a^6*d)*f^2)) -
(c - I*d)*e^(2*I*f*x + 2*I*e) - c)*e^(-2*I*f*x - 2*I*e)) + 3*(-I*a^3*c^3 +
3*a^3*c^2*d + 3*I*a^3*c*d^2 - a^3*d^3)*f*sqrt(-(-4*I*c^6 + 32*c^5*d + 116*I
*c^4*d^2 - 256*c^3*d^3 - 361*I*c^2*d^4 + 312*c*d^5 + 144*I*d^6)/((-I*a^6*c^
7 + 7*a^6*c^6*d + 21*I*a^6*c^5*d^2 - 35*a^6*c^4*d^3 - 35*I*a^6*c^3*d^4 + 21
*a^6*c^2*d^5 + 7*I*a^6*c*d^6 - a^6*d^7)*f^2))*e^(6*I*f*x + 6*I*e)*log(1/16*
(2*I*c^4 - 10*c^3*d - 21*I*c^2*d^2 + 25*c*d^3 + 12*I*d^4 + ((a^3*c^4 + 4*I*
a^3*c^3*d - 6*a^3*c^2*d^2 - 4*I*a^3*c*d^3 + a^3*d^4)*f*e^(2*I*f*x + 2*I*e)
+ (a^3*c^4 + 4*I*a^3*c^3*d - 6*a^3*c^2*d^2 - 4*I*a^3*c*d^3 + a^3*d^4)*f)*sq
rt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqr
t(-(-4*I*c^6 + 32*c^5*d + 116*I*c^4*d^2 - 256*c^3*d^3 - 361*I*c^2*d^4 + 312
*c*d^5 + 144*I*d^6)/((-I*a^6*c^7 + 7*a^6*c^6*d + 21*I*a^6*c^5*d^2 - 35*a^6*
c^4*d^3 - 35*I*a^6*c^3*d^4 + 21*a^6*c^2*d^5 + 7*I*a^6*c*d^6 - a^6*d^7)*f^2)
) + (2*I*c^4 - 8*c^3*d - 13*I*c^2*d^2 + 12*c*d^3)*e^(2*I*f*x + 2*I*e))*e^(-
2*I*f*x - 2*I*e)/((a^3*c^4 + 4*I*a^3*c^3*d - 6*a^3*c^2*d^2 - 4*I*a^3*c*d^3
+ a^3*d^4)*f)) + 3*(I*a^3*c^3 - 3*a^3*c^2*d - 3*I*a^3*c*d^2 + a^3*d^3)*f*sq
rt(-(-4*I*c^6 + 32*c^5*d + 116*I*c^4*d^2 - 256*c^3*d^3 - 361*I*c^2*d^4 + 31
2*c*d^5 + 144*I*d^6)/((-I*a^6*c^7 + 7*a^6*c^6*d + 21*I*a^6*c^5*d^2 - 35*a^6
*c^4*d^3 - 35*I*a^6*c^3*d^4 + 21*a^6*c^2*d^5 + 7*I*a^6*c*d^6 - a^6*d^7)*f^2
))*e^(6*I*f*x + 6*I*e)*log(1/16*(2*I*c^4 - 10*c^3*d - 21*I*c^2*d^2 + 25*c*d
^3 + 12*I*d^4 - ((a^3*c^4 + 4*I*a^3*c^3*d - 6*a^3*c^2*d^2 - 4*I*a^3*c*d^3 +
 a^3*d^4)*f*e^(2*I*f*x + 2*I*e) + (a^3*c^4 + 4*I*a^3*c^3*d - 6*a^3*c^2*d^2
- 4*I*a^3*c*d^3 + a^3*d^4)*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d
)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(-4*I*c^6 + 32*c^5*d + 116*I*c^4*d^2 - 2
56*c^3*d^3 - 361*I*c^2*d^4 + 312*c*d^5 + 144*I*d^6)/((-I*a^6*c^7 + 7*a^6*c^
6*d + 21*I*a^6*c^5*d^2 - 35*a^6*c^4*d^3 - 35*I*a^6*c^3*d^4 + 21*a^6*c^2*d^5
 + 7*I*a^6*c*d^6 - a^6*d^7)*f^2)) + (2*I*c^4 - 8*c^3*d - 13*I*c^2*d^2 + 12*
c*d^3)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/((a^3*c^4 + 4*I*a^3*c^3*d
- 6*a^3*c^2*d^2 - 4*I*a^3*c*d^3 + a^3*d^4)*f)) + 2*(2*c^2 + 4*I*c*d - 2*d^2
 + (11*c^2 + 36*I*c*d - 40*d^2)*e^(6*I*f*x + 6*I*e) + (18*c^2 + 55*I*c*d -
52*d^2)*e^(4*I*f*x + 4*I*e) + (9*c^2 + 23*I*c*d - 14*d^2)*e^(2*I*f*x + 2*I*
e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1
```

```
)))*e^(-6*I*f*x - 6*I*e)/((-I*a^3*c^3 + 3*a^3*c^2*d + 3*I*a^3*c*d^2 - a^3*d
^3)*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int\frac{1}{\sqrt{c+d\tan(e+fx)}\tan^3(e+fx)-3i\sqrt{c+d\tan(e+fx)}\tan^2(e+fx)-3\sqrt{c+d\tan(e+fx)}\tan(e+fx)+i\sqrt{c+d\tan(e+fx)}}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] I*Integral(1/(sqrt(c + d*tan(e + f*x))*tan(e + f*x)**3 - 3*I*sqrt(c + d*tan
(e + f*x))*tan(e + f*x)**2 - 3*sqrt(c + d*tan(e + f*x))*tan(e + f*x) + I*sq
rt(c + d*tan(e + f*x))), x)/a**3
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 669 vs. 2(248) = 496.
time = 0.83, size = 669, normalized size = 2.24

$$-\frac{2\left(2c^3+8ic^2d-13cd^2-12id^3\right)\arctan\left(\frac{2\left(\sqrt{d\tan(fx+e)+c}\,c-\sqrt{c^2+d^2}\sqrt{d\tan(fx+e)+c}\right)}{c\sqrt{-2c+2\sqrt{c^2+d^2}}+i\sqrt{-2c+2\sqrt{c^2+d^2}}\,d-\sqrt{c^2+d^2}\sqrt{-2c+2\sqrt{c^2+d^2}}}\right)}{-16\left(ia^3c^3f-3a^3c^2df-3ia^3cd^2f+a^3d^3f\right)\sqrt{-2c+2\sqrt{c^2+d^2}}\left(\frac{id}{c-\sqrt{c^2+d^2}}+1\right)}+\frac{2\left(-6i(d\tan(fx+e)+c)^{\frac{5}{2}}c^2d+12i(d\tan(fx+e)+c)^{\frac{3}{2}}c^3d-6i\sqrt{d\tan(fx+e)+c}\,c^4d+21(d\tan(fx+e)+c)^{\frac{5}{2}}cd^2-60(d\tan(fx+e)+c)^{\frac{3}{2}}c^2d^2+39\sqrt{d\tan(fx+e)+c}\,c^3d^2+30i(d\tan(fx+e)+c)^{\frac{5}{2}}d^3-124i(d\tan(fx+e)+c)^{\frac{3}{2}}cd^3+114i\sqrt{d\tan(fx+e)+c}\,c^2d^3+76(d\tan(fx+e)+c)^{\frac{3}{2}}d^4-135\sqrt{d\tan(fx+e)+c}\,cd^4-54i\sqrt{d\tan(fx+e)+c}\,d^5\right)}{-96\left(ia^3c^3f-3a^3c^2df-3ia^3cd^2f+a^3d^3f\right)(d\tan(fx+e)-id)^3}+\frac{i\arctan\left(\frac{2\left(\sqrt{d\tan(fx+e)+c}\,c-\sqrt{c^2+d^2}\sqrt{d\tan(fx+e)+c}\right)}{c\sqrt{-2c+2\sqrt{c^2+d^2}}-i\sqrt{-2c+2\sqrt{c^2+d^2}}\,d-\sqrt{c^2+d^2}\sqrt{-2c+2\sqrt{c^2+d^2}}}\right)}{4a^3\sqrt{-2c+2\sqrt{c^2+d^2}}\,f\left(-\frac{id}{c-\sqrt{c^2+d^2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="giac"
)
```

```
[Out] -2*(2*c^3 + 8*I*c^2*d - 13*c*d^2 - 12*I*d^3)*arctan(2*(sqrt(d*tan(f*x + e)
+ c)*c - sqrt(c^2 + d^2)*sqrt(d*tan(f*x + e) + c))/(c*sqrt(-2*c + 2*sqrt(c^
2 + d^2)) + I*sqrt(-2*c + 2*sqrt(c^2 + d^2))*d - sqrt(c^2 + d^2)*sqrt(-2*c
+ 2*sqrt(c^2 + d^2))))/((-16*I*a^3*c^3*f + 48*a^3*c^2*d*f + 48*I*a^3*c*d^2*
f - 16*a^3*d^3*f)*sqrt(-2*c + 2*sqrt(c^2 + d^2))*(I*d/(c - sqrt(c^2 + d^2))
 + 1)) + 2*(-6*I*(d*tan(f*x + e) + c)^(5/2)*c^2*d + 12*I*(d*tan(f*x + e) +
c)^(3/2)*c^3*d - 6*I*sqrt(d*tan(f*x + e) + c)*c^4*d + 21*(d*tan(f*x + e) +
c)^(5/2)*c*d^2 - 60*(d*tan(f*x + e) + c)^(3/2)*c^2*d^2 + 39*sqrt(d*tan(f*x
+ e) + c)*c^3*d^2 + 30*I*(d*tan(f*x + e) + c)^(5/2)*d^3 - 124*I*(d*tan(f*x
+ e) + c)^(3/2)*c*d^3 + 114*I*sqrt(d*tan(f*x + e) + c)*c^2*d^3 + 76*(d*tan(
f*x + e) + c)^(3/2)*d^4 - 135*sqrt(d*tan(f*x + e) + c)*c*d^4 - 54*I*sqrt(d*
tan(f*x + e) + c)*d^5)/((-96*I*a^3*c^3*f + 288*a^3*c^2*d*f + 288*I*a^3*c*d^
2*f - 96*a^3*d^3*f)*(d*tan(f*x + e) - I*d)^3) + 1/4*I*arctan(2*(sqrt(d*tan(
f*x + e) + c)*c - sqrt(c^2 + d^2)*sqrt(d*tan(f*x + e) + c))/(c*sqrt(-2*c +
2*sqrt(c^2 + d^2)) - I*sqrt(-2*c + 2*sqrt(c^2 + d^2))*d - sqrt(c^2 + d^2)*s
qrt(-2*c + 2*sqrt(c^2 + d^2))))/(a^3*sqrt(-2*c + 2*sqrt(c^2 + d^2))*f*(-I*d
/(c - sqrt(c^2 + d^2)) + 1))
```

Mupad [B]
time = 11.14, size = 2500, normalized size = 8.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a*tan(e + f*x)*1i)^3*(c + d*tan(e + f*x))^(1/2)),x)

[Out] log(a^3*d^14*f*240i - ((-(140*d^11 - c*d^10*140i + 35*c^2*d^9 - c^3*d^8*245 i - 280*c^4*d^7 + c^5*d^6*168i + 56*c^6*d^5 - c^7*d^4*8i - a^6*c^6*f^2*(4*(256*d^6 + 256*c^2*d^4)*(((649*c^2*d^14)/1024 - (9*d^16)/64 + (85*c^4*d^12)/ 1024 + (119*c^6*d^10)/1024 + (15*c^8*d^8)/1024 - (c^10*d^6)/64 - (c^12*d^4) /256)/(a^12*c^12*f^4 + a^12*d^12*f^4 + 6*a^12*c^2*d^10*f^4 + 15*a^12*c^4*d^ 8*f^4 + 20*a^12*c^6*d^6*f^4 + 15*a^12*c^8*d^4*f^4 + 6*a^12*c^10*d^2*f^4) - (((69*c*d^15)/128 - (55*c^3*d^13)/512 + (57*c^5*d^11)/256 + (61*c^7*d^9)/51 2 + (5*c^9*d^7)/128 + (c^11*d^5)/128)*1i)/(a^12*c^12*f^4 + a^12*d^12*f^4 + 6*a^12*c^2*d^10*f^4 + 15*a^12*c^4*d^8*f^4 + 20*a^12*c^6*d^6*f^4 + 15*a^12*c ^8*d^4*f^4 + 6*a^12*c^10*d^2*f^4)) + ((((1225*c^2*d^15)/4 - 35*d^17 + (35*c ^4*d^13)/4 + (427*c^6*d^11)/4 + (197*c^8*d^9)/4 + 12*c^10*d^7 + 2*c^12*d^5) *1i)/(a^6*c^12*f^2 + a^6*d^12*f^2 + 6*a^6*c^2*d^10*f^2 + 15*a^6*c^4*d^8*f^2 + 20*a^6*c^6*d^6*f^2 + 15*a^6*c^8*d^4*f^2 + 6*a^6*c^10*d^2*f^2) + (175*c*d ^16 - (735*c^3*d^14)/4 + (203*c^5*d^12)/4 + (83*c^7*d^10)/4 + (85*c^9*d^8)/ 4 + 12*c^11*d^6 + 2*c^13*d^4)/(a^6*c^12*f^2 + a^6*d^12*f^2 + 6*a^6*c^2*d^10 *f^2 + 15*a^6*c^4*d^8*f^2 + 20*a^6*c^6*d^6*f^2 + 15*a^6*c^8*d^4*f^2 + 6*a^6 *c^10*d^2*f^2))^2)^(1/2)*4i + a^6*d^6*f^2*(4*(256*d^6 + 256*c^2*d^4)*(((649 *c^2*d^14)/1024 - (9*d^16)/64 + (85*c^4*d^12)/1024 + (119*c^6*d^10)/1024 + (15*c^8*d^8)/1024 - (c^10*d^6)/64 - (c^12*d^4)/256)/(a^12*c^12*f^4 + a^12*d ^12*f^4 + 6*a^12*c^2*d^10*f^4 + 15*a^12*c^4*d^8*f^4 + 20*a^12*c^6*d^6*f^4 + 15*a^12*c^8*d^4*f^4 + 6*a^12*c^10*d^2*f^4) - (((69*c*d^15)/128 - (55*c^3*d ^13)/512 + (57*c^5*d^11)/256 + (61*c^7*d^9)/512 + (5*c^9*d^7)/128 + (c^11*d ^5)/128)*1i)/(a^12*c^12*f^4 + a^12*d^12*f^4 + 6*a^12*c^2*d^10*f^4 + 15*a^12 *c^4*d^8*f^4 + 20*a^12*c^6*d^6*f^4 + 15*a^12*c^8*d^4*f^4 + 6*a^12*c^10*d^2* f^4)) + ((((1225*c^2*d^15)/4 - 35*d^17 + (35*c^4*d^13)/4 + (427*c^6*d^11)/4 + (197*c^8*d^9)/4 + 12*c^10*d^7 + 2*c^12*d^5)*1i)/(a^6*c^12*f^2 + a^6*d^12 *f^2 + 6*a^6*c^2*d^10*f^2 + 15*a^6*c^4*d^8*f^2 + 20*a^6*c^6*d^6*f^2 + 15*a^ 6*c^8*d^4*f^2 + 6*a^6*c^10*d^2*f^2) + (175*c*d^16 - (735*c^3*d^14)/4 + (203 *c^5*d^12)/4 + (83*c^7*d^10)/4 + (85*c^9*d^8)/4 + 12*c^11*d^6 + 2*c^13*d^4) /(a^6*c^12*f^2 + a^6*d^12*f^2 + 6*a^6*c^2*d^10*f^2 + 15*a^6*c^4*d^8*f^2 + 2 0*a^6*c^6*d^6*f^2 + 15*a^6*c^8*d^4*f^2 + 6*a^6*c^10*d^2*f^2))^2)^(1/2)*4i + 24*a^6*c*d^5*f^2*(4*(256*d^6 + 256*c^2*d^4)*(((649*c^2*d^14)/1024 - (9*d^1 6)/64 + (85*c^4*d^12)/1024 + (119*c^6*d^10)/1024 + (15*c^8*d^8)/1024 - (c^1 0*d^6)/64 - (c^12*d^4)/256)/(a^12*c^12*f^4 + a^12*d^12*f^4 + 6*a^12*c^2*d^1 0*f^4 + 15*a^12*c^4*d^8*f^4 + 20*a^12*c^6*d^6*f^4 + 15*a^12*c^8*d^4*f^4 + 6 *a^12*c^10*d^2*f^4) - (((69*c*d^15)/128 - (55*c^3*d^13)/512 + (57*c^5*d^11) /256 + (61*c^7*d^9)/512 + (5*c^9*d^7)/128 + (c^11*d^5)/128)*1i)/(a^12*c^12* f^4 + a^12*d^12*f^4 + 6*a^12*c^2*d^10*f^4 + 15*a^12*c^4*d^8*f^4 + 20*a^12*c ^6*d^6*f^4 + 15*a^12*c^8*d^4*f^4 + 6*a^12*c^10*d^2*f^4)) + ((((1225*c^2*d^1 5)/4 - 35*d^17 + (35*c^4*d^13)/4 + (427*c^6*d^11)/4 + (197*c^8*d^9)/4 + 12* c^10*d^7 + 2*c^12*d^5)*1i)/(a^6*c^12*f^2 + a^6*d^12*f^2 + 6*a^6*c^2*d^10*f^

2 + 15*a^6*c^4*d^8*f^2 + 20*a^6*c^6*d^6*f^2 + 15*a^6*c^8*d^4*f^2 + 6*a^6*c^10*d^2*f^2) + (175*c*d^16 - (735*c^3*d^14)/4 + (203*c^5*d^12)/4 + (83*c^7*d^10)/4 + (85*c^9*d^8)/4 + 12*c^11*d^6 + 2*c^13*d^4)/(a^6*c^12*f^2 + a^6*d^12*f^2 + 6*a^6*c^2*d^10*f^2 + 15*a^6*c^4*d^8*f^2 + 20*a^6*c^6*d^6*f^2 + 15*a^6*c^8*d^4*f^2 + 6*a^6*c^10*d^2*f^2))^2)^(1/2) + 24*a^6*c^5*d*f^2*(4*(256*d^6 + 256*c^2*d^4)*(((649*c^2*d^14)/1024 - (9*d^16)/64 + (85*c^4*d^12)/1024 + (119*c^6*d^10)/1024 + (15*c^8*d^8)/1024 - (c^10*d^6)/64 - (c^12*d^4)/256)/(a^12*c^12*f^4 + a^12*d^12*f^4 + 6*a^12*c^2*d^10*f^4 + 15*a^12*c^4*d^8*f^4 + 20*a^12*c^6*d^6*f^4 + 15*a^12*c^8*d^4*f^4 + 6*a^12*c^10*d^2*f^4) - (((69*c*d^15)/128 - (55*c^3*d^13)/512 + (57*c^5*d^11)/256 + (61*c^7*d^9)/512 + (5*c^9*d^7)/128 + (c^11*d^5)/128)*1i)/(a^12*c^12*f^4 + a^12*d^12*f^4 + 6*a^12*c^2*d^10*f^4 + 15*a^12*c^4*d^8*f^4 + 20*a^12*c^6*d^6*f^4 + 15*a^12*c^8*d^4*f^4 + 6*a^12*c^10*d^2*f^4)) + ((((1225*c^2*d^15)/4 - 35*d^17 + (35*c^4*d^13)/4 + (427*c^6*d^11)/4 + (197*c^8*d^9)/4 + 12*c^10*d^7 + 2*c^12*d^5)*1i)/(a^6*c^12*f^2 + a^6*d^12*f^2 + 6*a^6*c^2*d^10*f^2 + 15*a^6*c^4*d^8*f^2 + 20*a^6*c^6*d^6*f^2 + 15*a^6*c^8*d^4*f^2 + 6*a^6*c^10*d^2*f^2) + (175*c*d^16 - (735*c^3*d^14)/4 + (203*c^5*d^12)/4 + (83*c^7*d^10)/4 + (85*c^9*d^8)/4 + 12*c^11*d^6 + 2*c^13*d^4)/(a^6*c^12*f^2 + a^6*d^12*f^2 + 6*a^6*c^2*d^10*f^2 + 15*a^6*c^4*d^8*f^2 + 20*a^6*c^6*d^6*f^2 + 15*a^6*c^8*d^4*f^2 + 6*a^6*c^10*d^2*f^2))^2)^(1/2) - a^6*c^2*d^4*f^2*(4*(256*d^6 + 256*c^2*d^4)*(((649*c^2*d^14)/1024 - (9*d^16)/64 + (85*c^4*d^12)/1024 + (119*c^6*d^10)/1024 + (15*c^8*d^8)/1024 - (c^10*d^6)/64 - (c^12*d^4)/256)/(a^12*c^12*f^4 + a^12*d^12*f^4 + 6*a^12*c^2*d^10*f^4 + 15*a^12*c^4*d^8*f^4 + 20*a^12*c^6*d^6*f^4 + 15*a^12*c^8*d^4*f^4 + 6*a^12*c^10*d^2*f^4) - (((69*c*d^15)/128 - (55*c^3*d^13)/512 + (57*c^5*d^11)/256 + (61*c^7*d^9)/512 + (5*c^9*d^7)/128 + (c^11*d^5)/128)*1i)/(a^12*c^12*f^4 + a^12*d^12*f^4 + 6*a^1...

3.1125 $\int \frac{(a+ia\tan(e+fx))^3}{(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=139

$$-\frac{8ia^3\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{3/2}f}+\frac{2(c+id)\left(a^3+ia^3\tan(e+fx)\right)}{(c-id)df\sqrt{c+d\tan(e+fx)}}+\frac{4a^3c\sqrt{c+d\tan(e+fx)}}{d^2(ic+d)f}$$

```
[Out] -8*I*a^3*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(c-I*d)^(3/2)/f+4*a^
3*c*(c+d*tan(f*x+e))^(1/2)/d^2/(I*c+d)/f+2*(c+I*d)*(a^3+I*a^3*tan(f*x+e))/(
c-I*d)/d/f/(c+d*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3634, 3673, 3618, 65, 214}

$$\frac{4a^3c\sqrt{c+d\tan(e+fx)}}{d^2f(d+ic)}+\frac{2(c+id)\left(a^3+ia^3\tan(e+fx)\right)}{df(c-id)\sqrt{c+d\tan(e+fx)}}-\frac{8ia^3\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(c-id)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^3/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-8*I)*a^3*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((c - I*d)^(3/
2)*f) + (2*(c + I*d)*(a^3 + I*a^3*Tan[e + f*x]))/((c - I*d)*d*f*Sqrt[c + d*
Tan[e + f*x]]) + (4*a^3*c*Sqrt[c + d*Tan[e + f*x]])/(d^2*(I*c + d)*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
```

```
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3634

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-a^2)*(b*c - a*d)*(a + b*Tan[e + f*x]
)^(m - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] + Di
st[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[b*(b*c*(m - 2) - a*d*(m - 2*n - 4)) + (a*b*c*(m - 2)
+ b^2*d*(n + 1) - a^2*d*(m + n - 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^3}{(c+d\tan(e+fx))^{3/2}}\,dx = \frac{2(c+id)\left(a^3+ia^3\tan(e+fx)\right)}{(c-id)df\sqrt{c+d\tan(e+fx)}} - \frac{2\int \frac{(a+ia\tan(e+fx))\left(-a^2(c+2id)+ia^2c\tan(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,d}{d(ic+d)}$$

$$= \frac{2(c+id)\left(a^3+ia^3\tan(e+fx)\right)}{(c-id)df\sqrt{c+d\tan(e+fx)}} + \frac{4a^3c\sqrt{c+d\tan(e+fx)}}{d^2(ic+d)f} - \frac{2\int \frac{-2ia^3d+2a}{\sqrt{c+d\,\mathrm{ta}}}}{d(ic}$$

$$= \frac{2(c+id)\left(a^3+ia^3\tan(e+fx)\right)}{(c-id)df\sqrt{c+d\tan(e+fx)}} + \frac{4a^3c\sqrt{c+d\tan(e+fx)}}{d^2(ic+d)f} + \frac{(8a^6d)\,\mathrm{Subst}}{}$$

$$= \frac{2(c+id)\left(a^3+ia^3\tan(e+fx)\right)}{(c-id)df\sqrt{c+d\tan(e+fx)}} + \frac{4a^3c\sqrt{c+d\tan(e+fx)}}{d^2(ic+d)f} + \frac{(32a^9d)\,\mathrm{Subst}}{}$$

$$= -\frac{8ia^3\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{3/2}f} + \frac{2(c+id)\left(a^3+ia^3\tan(e+fx)\right)}{(c-id)df\sqrt{c+d\tan(e+fx)}} +$$

Mathematica [A]
time = 4.37, size = 219, normalized size = 1.58

$$\frac{a^3(\cos(e+fx)+i\sin(e+fx))^3\left(-\frac{8ie^{-3ie}\tanh^{-1}\left(\frac{\sqrt{c-\frac{id(-1+e^{2i(e+fx)})}{1+e^{2i(e+fx)}}}}{\sqrt{c-id}}\right)}{(c-id)^{3/2}}+\frac{2(\cos(3e)-i\sin(3e))((-2ic^2+cd+id^2)\cos(e+fx)+(-ic-d)d\sin(e+fx))\sqrt{c+d\tan(e+fx)}}{(c-id)d^2(c\cos(e+fx)+d\sin(e+fx))}\right)}{f(\cos(fx)+i\sin(fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (a^3*(Cos[e + f*x] + I*Sin[e + f*x])^3*(((-8*I)*ArcTanh[Sqrt[c - (I*d*(-1 +
 E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))]/Sqrt[c - I*d]])/((c - I*d
)^(3/2)*E^((3*I)*e)) + (2*(Cos[3*e] - I*Sin[3*e])*(((-2*I)*c^2 + c*d + I*d^
2)*Cos[e + f*x] + ((-I)*c - d)*d*Sin[e + f*x])*Sqrt[c + d*Tan[e + f*x]])/((
c - I*d)*d^2*(c*Cos[e + f*x] + d*Sin[e + f*x]))))/(f*(Cos[f*x] + I*Sin[f*x]
)^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1055 vs. 2(122) = 244.
time = 0.29, size = 1056, normalized size = 7.60

method	result
derivativedivides	$2a^3\left(-i\sqrt{c+d\tan(fx+e)}-\frac{ic^3-3ic\,d^2-3c^2d+d^3}{(c^2+d^2)\sqrt{c+d\tan(fx+e)}}-\frac{4d^2\left(\frac{\left(i\sqrt{c^2+d^2}\,\sqrt{2\sqrt{c^2+d^2}\right.}{}\right)}{}\right.}{}\right.$

default	$2a^3\left(-i\sqrt{c+d\tan(fx+e)}-\frac{ic^3-3ic\,d^2-3c^2d+d^3}{(c^2+d^2)\sqrt{c+d\tan(fx+e)}}-\frac{4d^2\left(\frac{\left(i\sqrt{c^2+d^2}\,\sqrt{2\sqrt{c^2+d^2}\right.}{}\right.}{}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/f*a^3/d^2*(-I*(c+d*tan(f*x+e))^(1/2)-(I*c^3-3*I*c*d^2-3*c^2*d+d^3)/(c^2+d^2)/(c+d*tan(f*x+e))^(1/2)-4*d^2/(c^2+d^2)*(1/4/((c^2+d^2)^(1/2)+c)/(c^2+d^2)^(1/2)*(1/2*(I*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c+I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2-I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^2-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d-2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(2*I*(c^2+d^2)^(1/2)*c^2-2*I*(c^2+d^2)^(1/2)*d^2+2*I*c^3-2*I*c*d^2-4*c*d*(c^2+d^2)^(1/2)-4*c^2*d-1/2*(I*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c+I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2-I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^2-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d-2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/4/((c^2+d^2)^(1/2)+c)/(c^2+d^2)^(1/2)*(1/2*(-I*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c-I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2+I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^2+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d+2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(2*I*(c^2+d^2)^(1/2)*c^2-2*I*(c^2+d^2)^(1/2)*d^2+2*I*c^3-2*I*c*d^2-4*c*d*(c^2+d^2)^(1/2)-4*c^2*d+1/2*(-I*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c-I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2+I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^2+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d+2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)

```
-2*c)^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^(3/2),x, algorithm="maxima"
)

[Out] integrate((I*a*tan(f*x + e) + a)^3/(d*tan(f*x + e) + c)^(3/2), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 620 vs. 2(119) = 238.
time = 1.24, size = 620, normalized size = 4.46

$$\frac{\sqrt{\frac{64 i a^{6}}{(-i c^{3} - 3 c^{2} d + 3 i c d^{2} + d^{3}) f^{2}}} \left((c^{2} d^{2} - 2 i c d^{3} - d^{4}) f e^{(2 i f x + 2 i e)} + (c^{2} d^{2} + d^{4}) f\right) \log\left(\frac{\left(8 a^{3} c + \sqrt{\frac{64 i a^{6}}{(-i c^{3} - 3 c^{2} d + 3 i c d^{2} + d^{3}) f^{2}}} \left((i c^{2} + 2 c d - i d^{2}) f e^{(2 i f x + 2 i e)} + (i c^{2} + 2 c d - i d^{2}) f\right) \sqrt{\frac{(c - i d) e^{(2 i f x + 2 i e)} + c + i d}{e^{(2 i f x + 2 i e)} + 1}} + 8 (a^{3} c - i a^{3} d) e^{(2 i f x + 2 i e)}\right) e^{(-2 i f x - 2 i e)}}{4 a^{3}}\right) - \sqrt{\frac{64 i a^{6}}{(-i c^{3} - 3 c^{2} d + 3 i c d^{2} + d^{3}) f^{2}}} \left((c^{2} d^{2} - 2 i c d^{3} - d^{4}) f e^{(2 i f x + 2 i e)} + (c^{2} d^{2} + d^{4}) f\right) \log\left(\frac{\left(8 a^{3} c + \sqrt{\frac{64 i a^{6}}{(-i c^{3} - 3 c^{2} d + 3 i c d^{2} + d^{3}) f^{2}}} \left((-i c^{2} - 2 c d + i d^{2}) f e^{(2 i f x + 2 i e)} + (-i c^{2} - 2 c d + i d^{2}) f\right) \sqrt{\frac{(c - i d) e^{(2 i f x + 2 i e)} + c + i d}{e^{(2 i f x + 2 i e)} + 1}} + 8 (a^{3} c - i a^{3} d) e^{(2 i f x + 2 i e)}\right) e^{(-2 i f x - 2 i e)}}{4 a^{3}}\right) + 16 \left(-i a^{3} c^{2} + a^{3} c d + (-i a^{3} c^{2} + i a^{3} d^{2}) e^{(2 i f x + 2 i e)}\right) \sqrt{\frac{(c - i d) e^{(2 i f x + 2 i e)} + c + i d}{e^{(2 i f x + 2 i e)} + 1}}}{4 \left((c^{2} d^{2} - 2 i c d^{3} - d^{4}) f e^{(2 i f x + 2 i e)} + (c^{2} d^{2} + d^{4}) f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^(3/2),x, algorithm="fricas"
)

[Out] 1/4*(sqrt(64*I*a^6/((-I*c^3 - 3*c^2*d + 3*I*c*d^2 + d^3)*f^2))*((c^2*d^2 -
2*I*c*d^3 - d^4)*f*e^(2*I*f*x + 2*I*e) + (c^2*d^2 + d^4)*f)*log(1/4*(8*a^3*
c + sqrt(64*I*a^6/((-I*c^3 - 3*c^2*d + 3*I*c*d^2 + d^3)*f^2))*((I*c^2 + 2*c
*d - I*d^2)*f*e^(2*I*f*x + 2*I*e) + (I*c^2 + 2*c*d - I*d^2)*f)*sqrt(((c - I
*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1)) + 8*(a^3*c -
I*a^3*d)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/a^3) - sqrt(64*I*a^6/((-
I*c^3 - 3*c^2*d + 3*I*c*d^2 + d^3)*f^2))*((c^2*d^2 - 2*I*c*d^3 - d^4)*f*e^(
2*I*f*x + 2*I*e) + (c^2*d^2 + d^4)*f)*log(1/4*(8*a^3*c + sqrt(64*I*a^6/((-I
*c^3 - 3*c^2*d + 3*I*c*d^2 + d^3)*f^2))*((-I*c^2 - 2*c*d + I*d^2)*f*e^(2*I*
f*x + 2*I*e) + (-I*c^2 - 2*c*d + I*d^2)*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I
*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1)) + 8*(a^3*c - I*a^3*d)*e^(2*I*f*x
+ 2*I*e))*e^(-2*I*f*x - 2*I*e)/a^3) + 16*(-I*a^3*c^2 + a^3*c*d + (-I*a^3*c^
2 + I*a^3*d^2)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c
 + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))/((c^2*d^2 - 2*I*c*d^3 - d^4)*f*e^(2*I*f
*x + 2*I*e) + (c^2*d^2 + d^4)*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$- i a^{3} \left(\int \frac{i}{c \sqrt{c + d \tan{\left(e + f x \right)}} + d \sqrt{c + d \tan{\left(e + f x \right)}} \tan{\left(e + f x \right)}}\, dx + \int \left(- \frac{3 \tan{\left(e + f x \right)}}{c \sqrt{c + d \tan{\left(e + f x \right)}} + d \sqrt{c + d \tan{\left(e + f x \right)}} \tan{\left(e + f x \right)}}\right)\, dx + \int \frac{\tan^{3}{\left(e + f x \right)}}{c \sqrt{c + d \tan{\left(e + f x \right)}} + d \sqrt{c + d \tan{\left(e + f x \right)}} \tan{\left(e + f x \right)}}\, dx + \int \left(- \frac{3 i \tan^{2}{\left(e + f x \right)}}{c \sqrt{c + d \tan{\left(e + f x \right)}} + d \sqrt{c + d \tan{\left(e + f x \right)}} \tan{\left(e + f x \right)}}\right)\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3/(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] -I*a**3*(Integral(I/(c*sqrt(c + d*tan(e + f*x)) + d*sqrt(c + d*tan(e + f*x)
)*tan(e + f*x)), x) + Integral(-3*tan(e + f*x)/(c*sqrt(c + d*tan(e + f*x))
+ d*sqrt(c + d*tan(e + f*x))*tan(e + f*x)), x) + Integral(tan(e + f*x)**3/(
c*sqrt(c + d*tan(e + f*x)) + d*sqrt(c + d*tan(e + f*x))*tan(e + f*x)), x) +
 Integral(-3*I*tan(e + f*x)**2/(c*sqrt(c + d*tan(e + f*x)) + d*sqrt(c + d*t
an(e + f*x))*tan(e + f*x)), x))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(119) = 238$.
time = 0.78, size = 247, normalized size = 1.78

$$\frac{16\,a^3\arctan\left(\frac{2\left(\sqrt{d\tan\left(fx+e\right)+c}\,c-\sqrt{c^2+d^2}\,\sqrt{d\tan\left(fx+e\right)+c}\right)}{c\sqrt{-2\,c+2\,\sqrt{c^2+d^2}}-i\sqrt{-2\,c+2\,\sqrt{c^2+d^2}}\,d-\sqrt{c^2+d^2}\,\sqrt{-2\,c+2\,\sqrt{c^2+d^2}}}\right)}{\left(-icf-df\right)\sqrt{-2\,c+2\,\sqrt{c^2+d^2}}\left(-\frac{id}{c-\sqrt{c^2+d^2}}+1\right)}-\frac{2i\,\sqrt{d\tan\left(fx+e\right)+c}\,a^3}{d^2f}+\frac{2\left(-i\,a^3c^2+2\,a^3cd+i\,a^3d^2\right)}{\left(cd^2f-i\,d^3f\right)\sqrt{d\tan\left(fx+e\right)+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] 16*a^3*arctan(2*(sqrt(d*tan(f*x + e) + c)*c - sqrt(c^2 + d^2)*sqrt(d*tan(f*
x + e) + c))/(c*sqrt(-2*c + 2*sqrt(c^2 + d^2)) - I*sqrt(-2*c + 2*sqrt(c^2 +
 d^2))*d - sqrt(c^2 + d^2)*sqrt(-2*c + 2*sqrt(c^2 + d^2))))/((-I*c*f - d*f)
*sqrt(-2*c + 2*sqrt(c^2 + d^2))*(-I*d/(c - sqrt(c^2 + d^2)) + 1)) - 2*I*sqr
t(d*tan(f*x + e) + c)*a^3/(d^2*f) + 2*(-I*a^3*c^2 + 2*a^3*c*d + I*a^3*d^2)/
((c*d^2*f - I*d^3*f)*sqrt(d*tan(f*x + e) + c))
```

Mupad [B]
time = 6.59, size = 182, normalized size = 1.31

$$-\frac{a^3\,\sqrt{c+d\,\tan\left(e+f\,x\right)}\,2{}\mathrm{i}}{d^2\,f}+\frac{a^3\,\mathrm{atan}\left(\frac{\sqrt{c+d\,\tan\left(e+f\,x\right)}\,\left(2\,c^4\,f^2+4\,c^2\,d^2\,f^2+2\,d^4\,f^2\right)}{2\,f\,{\left(-c+d\,1{}\mathrm{i}\right)}^{3/2}\,\left(f\,c^3+1{}\mathrm{i}\,f\,c^2\,d+f\,c\,d^2+1{}\mathrm{i}\,f\,d^3\right)}\right)\,8{}\mathrm{i}}{f\,{\left(-c+d\,1{}\mathrm{i}\right)}^{3/2}}-\frac{\left(a^3\,c^2+a^3\,c\,d\,2{}\mathrm{i}-a^3\,d^2\right)\,2{}\mathrm{i}}{d^2\,f\,\left(c-d\,1{}\mathrm{i}\right)\,\sqrt{c+d\,\tan\left(e+f\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^3/(c + d*tan(e + f*x))^(3/2),x)
```

```
[Out] (a^3*atan(((c + d*tan(e + f*x))^(1/2)*(2*c^4*f^2 + 2*d^4*f^2 + 4*c^2*d^2*f^
2))/(2*f*(d*1i - c)^(3/2)*(c^3*f + d^3*f*1i + c*d^2*f + c^2*d*f*1i)))*8i)/(
f*(d*1i - c)^(3/2)) - (a^3*(c + d*tan(e + f*x))^(1/2)*2i)/(d^2*f) - ((a^3*c
^2 - a^3*d^2 + a^3*c*d*2i)*2i)/(d^2*f*(c - d*1i)*(c + d*tan(e + f*x))^(1/2)
)
```

3.1126 $\int \frac{(a+ia\tan(e+fx))^2}{(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=92

$$-\frac{4ia^2\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{3/2}f}+\frac{2a^2(ic-d)}{d(ic+d)f\sqrt{c+d\tan(e+fx)}}$$

```
[Out] -4*I*a^2*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(c-I*d)^(3/2)/f+2*a^
2*(I*c-d)/d/(I*c+d)/f/(c+d*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3623, 3618, 65, 214}

$$\frac{2a^2(-d+ic)}{df(d+ic)\sqrt{c+d\tan(e+fx)}}-\frac{4ia^2\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(c-id)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-4*I)*a^2*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((c - I*d)^(3/
2)*f) + (2*a^2*(I*c - d))/(d*(I*c + d)*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^2}{(c+d\tan(e+fx))^{3/2}}\,dx = \frac{2a^2(ic-d)}{d(ic+d)f\sqrt{c+d\tan(e+fx)}} + \frac{\int \frac{2a^2(c+id)+2a^2(ic-d)\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx}{c^2+d^2}$$

$$= \frac{2a^2(ic-d)}{d(ic+d)f\sqrt{c+d\tan(e+fx)}} - \frac{(4a^4(c+id))\,\text{Subst}\left(\int \frac{\ldots}{(4a^4(ic-d)^2+2a^2(c+id)x \ldots}\right.}{\ldots}$$

$$= \frac{2a^2(ic-d)}{d(ic+d)f\sqrt{c+d\tan(e+fx)}} - \frac{(16a^6(c+id)^2)\,\text{Subst}\left(\int \frac{\ldots}{4a^4(ic-d)^2-\frac{4a^4c(ic-d)}{d}\ldots}\right.}{(c\ldots}$$

$$= -\frac{4ia^2\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{3/2}f} + \frac{2a^2(ic-d)}{d(ic+d)f\sqrt{c+d\tan(e+fx)}}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 189 vs. 2(92) = 184.
time = 3.52, size = 189, normalized size = 2.05

$$\frac{a^2(\cos(e+fx)+i\sin(e+fx))^2\left(-\frac{4ie^{-2ie}\tanh^{-1}\left(\frac{\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}}{\sqrt{c-id}}\right)}{(c-id)^{3/2}}+\frac{2(c+id)\cos(e+fx)(\cos(2e)-i\sin(2e))\sqrt{c+d\tan(e+fx)}}{(c-id)d(c\cos(e+fx)+d\sin(e+fx))}\right)}{f(\cos(fx)+i\sin(fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (a^2*(Cos[e + f*x] + I*Sin[e + f*x])^2*(((-4*I)*ArcTanh[Sqrt[c - (I*d*(-1 +
 E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))]/Sqrt[c - I*d]])/((c - I*d
)^(3/2)*E^((2*I)*e)) + (2*(c + I*d)*Cos[e + f*x]*(Cos[2*e] - I*Sin[2*e])*Sq
rt[c + d*Tan[e + f*x]])/((c - I*d)*d*(c*Cos[e + f*x] + d*Sin[e + f*x]))))/(
f*(Cos[f*x] + I*Sin[f*x])^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1029 vs. $2(79) = 158$.
time = 0.29, size = 1030, normalized size = 11.20

method	result
derivativedivides	$2a^2\left(-\frac{2d\left(\frac{\left(i\sqrt{c^2+d^2}\,\sqrt{2\sqrt{c^2+d^2}+2c}\,c+i\sqrt{2\sqrt{c^2+d^2}+2c}\,c^2-i\sqrt{2\sqrt{c^2+d^2}+2c}\,\cdots\right.}{\cdots}}{\cdots}\right.$

default	$2a^2\left(-\frac{2d\left(\frac{\left(i\sqrt{c^2+d^2}\,\sqrt{2\sqrt{c^2+d^2}+2c}\,c+i\sqrt{2\sqrt{c^2+d^2}+2c}\,c^2-i\sqrt{2\sqrt{c^2+d^2}+2c}\,d^2\right.}{}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/f*a^2/d*(-2*d/(c^2+d^2)*(1/4/((c^2+d^2)^(1/2)+c)/(c^2+d^2)^(1/2)*(1/2*(I*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c+I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2-I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^2-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d-2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(2*I*(c^2+d^2)^(1/2)*c^2-2*I*(c^2+d^2)^(1/2)*d^2+2*I*c^3-2*I*c*d^2-4*c*d*(c^2+d^2)^(1/2)-4*c^2*d-1/2*(I*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c+I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2-I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^2-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d-2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/4/((c^2+d^2)^(1/2)+c)/(c^2+d^2)^(1/2)*(1/2*(-I*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c-I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2+I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^2+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d+2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(2*I*(c^2+d^2)^(1/2)*c^2-2*I*(c^2+d^2)^(1/2)*d^2+2*I*c^3-2*I*c*d^2-4*c*d*(c^2+d^2)^(1/2)-4*c^2*d+1/2*(-I*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c-I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2+I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^2+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d+2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))))-(-2*I*c*d-c^2+d^2)/(c^2+d^2)/(c+d*tan(f*x+e))^(1/2))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^2/(d*tan(f*x + e) + c)^(3/2), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(76) = 152$.
time = 0.87, size = 600, normalized size = 6.52

$$\frac{\left((c^2d-2icd^2-d^3)fe^{(2ifx+2ie)}+(c^2d+d^3)f\right)\sqrt{\frac{16ia^4}{(-ic^3-3c^2d+3icd^2+d^3)f^2}}\log\left(\frac{\left(4a^2c+\left((ic^2+2cd-id^2)fe^{(2ifx+2ie)}+(ic^2+2cd-id^2)f\right)\sqrt{\frac{16ia^4}{(-ic^3-3c^2d+3icd^2+d^3)f^2}}\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}+4(a^2c-ia^2d)e^{(2ifx+2ie)}\right)e^{(-2ifx-2ie)}}{2a^2}\right)-\left((c^2d-2icd^2-d^3)fe^{(2ifx+2ie)}+(c^2d+d^3)f\right)\sqrt{\frac{16ia^4}{(-ic^3-3c^2d+3icd^2+d^3)f^2}}\log\left(\frac{\left(4a^2c+\left((-ic^2-2cd+id^2)fe^{(2ifx+2ie)}+(-ic^2-2cd+id^2)f\right)\sqrt{\frac{16ia^4}{(-ic^3-3c^2d+3icd^2+d^3)f^2}}\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}+4(a^2c-ia^2d)e^{(2ifx+2ie)}\right)e^{(-2ifx-2ie)}}{2a^2}\right)+8\left(a^2c+ia^2d+(a^2c+ia^2d)e^{(2ifx+2ie)}\right)\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}}{4\left((c^2d-2icd^2-d^3)fe^{(2ifx+2ie)}+(c^2d+d^3)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/4*(((c^2*d - 2*I*c*d^2 - d^3)*f*e^(2*I*f*x + 2*I*e) + (c^2*d + d^3)*f)*sq
rt(16*I*a^4/((-I*c^3 - 3*c^2*d + 3*I*c*d^2 + d^3)*f^2))*log(1/2*(4*a^2*c +
((I*c^2 + 2*c*d - I*d^2)*f*e^(2*I*f*x + 2*I*e) + (I*c^2 + 2*c*d - I*d^2)*f)
*sqrt(16*I*a^4/((-I*c^3 - 3*c^2*d + 3*I*c*d^2 + d^3)*f^2))*sqrt(((c - I*d)*
e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1)) + 4*(a^2*c - I*a^
2*d)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/a^2) - ((c^2*d - 2*I*c*d^2 -
 d^3)*f*e^(2*I*f*x + 2*I*e) + (c^2*d + d^3)*f)*sqrt(16*I*a^4/((-I*c^3 - 3*c
^2*d + 3*I*c*d^2 + d^3)*f^2))*log(1/2*(4*a^2*c + ((-I*c^2 - 2*c*d + I*d^2)*
f*e^(2*I*f*x + 2*I*e) + (-I*c^2 - 2*c*d + I*d^2)*f)*sqrt(16*I*a^4/((-I*c^3
- 3*c^2*d + 3*I*c*d^2 + d^3)*f^2))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c
+ I*d)/(e^(2*I*f*x + 2*I*e) + 1)) + 4*(a^2*c - I*a^2*d)*e^(2*I*f*x + 2*I*e)
)*e^(-2*I*f*x - 2*I*e)/a^2) + 8*(a^2*c + I*a^2*d + (a^2*c + I*a^2*d)*e^(2*I
*f*x + 2*I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x +
 2*I*e) + 1)))/((c^2*d - 2*I*c*d^2 - d^3)*f*e^(2*I*f*x + 2*I*e) + (c^2*d +
d^3)*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int\frac{\tan^2(e+fx)}{c\sqrt{c+d\tan(e+fx)}+d\sqrt{c+d\tan(e+fx)}\tan(e+fx)}\,dx+\int\left(-\frac{2i\tan(e+fx)}{c\sqrt{c+d\tan(e+fx)}+d\sqrt{c+d\tan(e+fx)}\tan(e+fx)}\right)dx+\int\left(-\frac{1}{c\sqrt{c+d\tan(e+fx)}+d\sqrt{c+d\tan(e+fx)}\tan(e+fx)}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2/(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] -a**2*(Integral(tan(e + f*x)**2/(c*sqrt(c + d*tan(e + f*x)) + d*sqrt(c + d*
tan(e + f*x))*tan(e + f*x)), x) + Integral(-2*I*tan(e + f*x)/(c*sqrt(c + d*
tan(e + f*x)) + d*sqrt(c + d*tan(e + f*x))*tan(e + f*x)), x) + Integral(-1/
(c*sqrt(c + d*tan(e + f*x)) + d*sqrt(c + d*tan(e + f*x))*tan(e + f*x)), x))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(76) = 152$.
time = 0.71, size = 209, normalized size = 2.27

$$\frac{8\,a^2\arctan\left(\frac{2\left(\sqrt{d\tan\left(fx+e\right)+c}\,c-\sqrt{c^2+d^2}\,\sqrt{d\tan\left(fx+e\right)+c}\right)}{c\sqrt{-2\,c+2\,\sqrt{c^2+d^2}}-i\,\sqrt{-2\,c+2\,\sqrt{c^2+d^2}}\,d-\sqrt{c^2+d^2}\,\sqrt{-2\,c+2\,\sqrt{c^2+d^2}}}\right)}{\left(-icf-df\right)\sqrt{-2\,c+2\,\sqrt{c^2+d^2}}\left(-\frac{id}{c-\sqrt{c^2+d^2}}+1\right)}+\frac{2\left(a^2c+i\,a^2d\right)}{\left(cdf-i\,d^2f\right)\sqrt{d\tan\left(fx+e\right)+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] 8*a^2*arctan(2*(sqrt(d*tan(f*x + e) + c)*c - sqrt(c^2 + d^2)*sqrt(d*tan(f*x
 + e) + c))/(c*sqrt(-2*c + 2*sqrt(c^2 + d^2)) - I*sqrt(-2*c + 2*sqrt(c^2 +
d^2))*d - sqrt(c^2 + d^2)*sqrt(-2*c + 2*sqrt(c^2 + d^2))))/((-I*c*f - d*f)*
sqrt(-2*c + 2*sqrt(c^2 + d^2))*(-I*d/(c - sqrt(c^2 + d^2)) + 1)) + 2*(a^2*c
 + I*a^2*d)/((c*d*f - I*d^2*f)*sqrt(d*tan(f*x + e) + c))
```

Mupad [B]
time = 6.38, size = 142, normalized size = 1.54

$$\frac{a^2\,\mathrm{atan}\left(\frac{\sqrt{c+d\tan\left(e+f\,x\right)}\,\left(2\,c^4\,f^2+4\,c^2\,d^2\,f^2+2\,d^4\,f^2\right)}{2\,f\,{\left(-c+d\,1\mathrm{i}\right)}^{3/2}\,\left(f\,c^3+1\mathrm{i}\,f\,c^2\,d+f\,c\,d^2+1\mathrm{i}\,f\,d^3\right)}\right)\,4\mathrm{i}}{f\,{\left(-c+d\,1\mathrm{i}\right)}^{3/2}}+\frac{2\,a^2\,\left(c+d\,1\mathrm{i}\right)}{d\,f\,\left(c-d\,1\mathrm{i}\right)\,\sqrt{c+d\tan\left(e+f\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^2/(c + d*tan(e + f*x))^(3/2),x)
```

```
[Out] (a^2*atan(((c + d*tan(e + f*x))^(1/2)*(2*c^4*f^2 + 2*d^4*f^2 + 4*c^2*d^2*f^
2))/(2*f*(d*1i - c)^(3/2)*(c^3*f + d^3*f*1i + c*d^2*f + c^2*d*f*1i)))*4i)/(
f*(d*1i - c)^(3/2)) + (2*a^2*(c + d*1i))/(d*f*(c - d*1i)*(c + d*tan(e + f*x
))^(1/2))
```

3.1127 $\int \frac{a+ia\tan(e+fx)}{(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=76

$$-\frac{2ia\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{3/2}f}-\frac{2a}{(ic+d)f\sqrt{c+d\tan(e+fx)}}$$

```
[Out] -2*I*a*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(c-I*d)^(3/2)/f-2*a/(I
*c+d)/f/(c+d*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3610, 3618, 65, 214}

$$-\frac{2a}{f(d+ic)\sqrt{c+d\tan(e+fx)}}-\frac{2ia\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(c-id)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-2*I)*a*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((c - I*d)^(3/2)
*f) - (2*a)/((I*c + d)*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
```

```
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{a+ia\tan(e+fx)}{(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{2a}{(ic+d)f\sqrt{c+d\tan(e+fx)}} + \frac{\int \frac{a(c+id)+a(ic-d)\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx}{c^2+d^2}$$

$$= -\frac{2a}{(ic+d)f\sqrt{c+d\tan(e+fx)}} - \frac{(a^2(c+id))\,\mathrm{Subst}\left(\int \frac{1}{(a^2(ic-d)^2+a(c+id)x)\sqrt{\cdots}} \cdots\right)}{(ic \cdots}$$

$$= -\frac{2a}{(ic+d)f\sqrt{c+d\tan(e+fx)}} - \frac{(2a^3(c+id)^2)\,\mathrm{Subst}\left(\int \frac{1}{a^2(ic-d)^2-\frac{a^2c(ic-d)(c\cdots}{d}} \cdots\right)}{(c \cdots}$$

$$= -\frac{2ia\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{3/2}f} - \frac{2a}{(ic+d)f\sqrt{c+d\tan(e+fx)}}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 158 vs. 2(76) = 152.
time = 2.62, size = 158, normalized size = 2.08

$$\frac{2iae^{-ie}(\cos(e)+i\sin(e))\left(-\tanh^{-1}\left(\frac{\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}}{\sqrt{c-id}}\right)(c\cos(e+fx)+d\sin(e+fx))+\sqrt{c-id}\,\cos(e+fx)\sqrt{c+d\tan(e+fx)}\right)}{(c-id)^{3/2}f(c\cos(e+fx)+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])/(c + d*Tan[e + f*x])^(3/2),x]
[Out] ((2*I)*a*(Cos[e] + I*Sin[e])*(-(ArcTanh[Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f
*x))))/(1 + E^((2*I)*(e + f*x)))]/Sqrt[c - I*d]]*(c*Cos[e + f*x] + d*Sin[e
```

+ f*x])) + Sqrt[c - I*d]*Cos[e + f*x]*Sqrt[c + d*Tan[e + f*x]]))/((c - I*d)
^(3/2)*E^(I*e)*f*(c*Cos[e + f*x] + d*Sin[e + f*x]))

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1014 vs. $2(64) = 128$.
time = 0.31, size = 1015, normalized size = 13.36

method	result
derivativedivides	$a\left(\frac{\left(i\sqrt{c^2+d^2}\,\sqrt{2\sqrt{c^2+d^2}+2c}\,c+i\sqrt{2\sqrt{c^2+d^2}+2c}\,c^2-i\sqrt{2\sqrt{c^2+d^2}+2c}\,d^2-\sqrt{c\cdots}\right.}{\cdots}\right.$
default	$a\left(\frac{\left(i\sqrt{c^2+d^2}\,\sqrt{2\sqrt{c^2+d^2}+2c}\,c+i\sqrt{2\sqrt{c^2+d^2}+2c}\,c^2-i\sqrt{2\sqrt{c^2+d^2}+2c}\,d^2-\sqrt{c\cdots}\right.}{\cdots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/f*a*(2/(c^2+d^2)*(1/4/((c^2+d^2)^(1/2)+c)/(c^2+d^2)^(1/2)*(1/2*(I*(c^2+d^
2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c+I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^
2-I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^2-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*d-2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d)*ln(d*tan(f*x+e)+c-(c+d*tan(
f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(-2*I*(c^2+d
^2)^(1/2)*c^2+2*I*(c^2+d^2)^(1/2)*d^2-2*I*c^3+2*I*c*d^2+4*c*d*(c^2+d^2)^(1/
2)+4*c^2*d+1/2*(I*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c+I*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*c^2-I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^2-(c^2+d^2)^(1
/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d-2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d)*(2

```
*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*t
an(f*x+e))^(1/2)-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/
2)))+1/4/((c^2+d^2)^(1/2)+c)/(c^2+d^2)^(1/2)*(1/2*(-I*(c^2+d^2)^(1/2)*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*c-I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2+I*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*d^2+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d+2*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(-2*I*(c^2+d^2)^(1/2)*c^2+
2*I*(c^2+d^2)^(1/2)*d^2-2*I*c^3+2*I*c*d^2+4*c*d*(c^2+d^2)^(1/2)+4*c^2*d-1/2
*(-I*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c-I*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)*c^2+I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^2+(c^2+d^2)^(1/2)*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*d+2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d)*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1
/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))))-2*(d-I*
c)/(c^2+d^2)/(c+d*tan(f*x+e))^(1/2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((-(2*c*d^4)/((c^2-d^2)^2>0)', see
`assume
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 550 vs. 2(62) = 124.
time = 1.00, size = 550, normalized size = 7.24

$$\frac{\left(\left(c^{2}-2 i c d-d^{2}\right) f e^{(2 i f x+2 i e)}+\left(c^{2}+d^{2}\right) f\right) \sqrt{\frac{4 i a^{2}}{\left(-i c^{3}-3 c^{2} d+3 i c d^{2}+d^{3}\right) f^{2}}} \log \left(\frac{\left(2 a c+\left(\left(i c^{2}+2 c d-i d^{2}\right) f e^{(2 i f x+2 i e)}+\left(i c^{2}+2 c d-i d^{2}\right) f\right) \sqrt{\frac{(c-i d) e^{(2 i f x+2 i e)}+c+i d}{e^{(2 i f x+2 i e)}+1}} \sqrt{\frac{4 i a^{2}}{\left(-i c^{3}-3 c^{2} d+3 i c d^{2}+d^{3}\right) f^{2}}}+2(a c-i a d) e^{(2 i f x+2 i e)}\right) e^{(-2 i f x-2 i e)}}{a}\right)-\left(\left(c^{2}-2 i c d-d^{2}\right) f e^{(2 i f x+2 i e)}+\left(c^{2}+d^{2}\right) f\right) \sqrt{\frac{4 i a^{2}}{\left(-i c^{3}-3 c^{2} d+3 i c d^{2}+d^{3}\right) f^{2}}} \log \left(\frac{\left(2 a c+\left(\left(-i c^{2}-2 c d+i d^{2}\right) f e^{(2 i f x+2 i e)}+\left(-i c^{2}-2 c d+i d^{2}\right) f\right) \sqrt{\frac{(c-i d) e^{(2 i f x+2 i e)}+c+i d}{e^{(2 i f x+2 i e)}+1}} \sqrt{\frac{4 i a^{2}}{\left(-i c^{3}-3 c^{2} d+3 i c d^{2}+d^{3}\right) f^{2}}}+2(a c-i a d) e^{(2 i f x+2 i e)}\right) e^{(-2 i f x-2 i e)}}{a}\right)+8\left(i a e^{(2 i f x+2 i e)}+i a\right) \sqrt{\frac{(c-i d) e^{(2 i f x+2 i e)}+c+i d}{e^{(2 i f x+2 i e)}+1}}}{4\left(\left(c^{2}-2 i c d-d^{2}\right) f e^{(2 i f x+2 i e)}+\left(c^{2}+d^{2}\right) f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(((c^2 - 2*I*c*d - d^2)*f*e^(2*I*f*x + 2*I*e) + (c^2 + d^2)*f)*sqrt(4*I
*a^2/((-I*c^3 - 3*c^2*d + 3*I*c*d^2 + d^3)*f^2))*log((2*a*c + ((I*c^2 + 2*c
*d - I*d^2)*f*e^(2*I*f*x + 2*I*e) + (I*c^2 + 2*c*d - I*d^2)*f)*sqrt(((c - I
*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(4*I*a^2/
((-I*c^3 - 3*c^2*d + 3*I*c*d^2 + d^3)*f^2)) + 2*(a*c - I*a*d)*e^(2*I*f*x +
2*I*e))*e^(-2*I*f*x - 2*I*e)/a) - ((c^2 - 2*I*c*d - d^2)*f*e^(2*I*f*x + 2*I
*e) + (c^2 + d^2)*f)*sqrt(4*I*a^2/((-I*c^3 - 3*c^2*d + 3*I*c*d^2 + d^3)*f^2
))*log((2*a*c + ((-I*c^2 - 2*c*d + I*d^2)*f*e^(2*I*f*x + 2*I*e) + (-I*c^2 -
```

```
2*c*d + I*d^2)*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f
*x + 2*I*e) + 1))*sqrt(4*I*a^2/((-I*c^3 - 3*c^2*d + 3*I*c*d^2 + d^3)*f^2))
+ 2*(a*c - I*a*d)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/a) + 8*(I*a*e^(
2*I*f*x + 2*I*e) + I*a)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(
2*I*f*x + 2*I*e) + 1)))/((c^2 - 2*I*c*d - d^2)*f*e^(2*I*f*x + 2*I*e) + (c^2
 + d^2)*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-\frac{i}{c\sqrt{c+d\tan(e+fx)}+d\sqrt{c+d\tan(e+fx)}\tan(e+fx)}\right)dx+\int\frac{\tan(e+fx)}{c\sqrt{c+d\tan(e+fx)}+d\sqrt{c+d\tan(e+fx)}\tan(e+fx)}dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] I*a*(Integral(-I/(c*sqrt(c + d*tan(e + f*x)) + d*sqrt(c + d*tan(e + f*x))*t
an(e + f*x)), x) + Integral(tan(e + f*x)/(c*sqrt(c + d*tan(e + f*x)) + d*sq
rt(c + d*tan(e + f*x))*tan(e + f*x)), x))
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(62) = 124.
time = 0.64, size = 192, normalized size = 2.53

$$-2a\left(\frac{1}{(icf+df)\sqrt{d\tan(fx+e)+c}}-\frac{2i\arctan\left(\frac{2\left(\sqrt{d\tan(fx+e)+c}\,c-\sqrt{c^2+d^2}\sqrt{d\tan(fx+e)+c}\right)}{c\sqrt{-2c+2\sqrt{c^2+d^2}}-i\sqrt{-2c+2\sqrt{c^2+d^2}}\,d-\sqrt{c^2+d^2}\sqrt{-2c+2\sqrt{c^2+d^2}}}\right)}{(cf-idf)\sqrt{-2c+2\sqrt{c^2+d^2}}\left(-\frac{id}{c-\sqrt{c^2+d^2}}+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] -2*a*(1/((I*c*f + d*f)*sqrt(d*tan(f*x + e) + c)) - 2*I*arctan(2*(sqrt(d*tan
(f*x + e) + c)*c - sqrt(c^2 + d^2)*sqrt(d*tan(f*x + e) + c))/(c*sqrt(-2*c +
 2*sqrt(c^2 + d^2)) - I*sqrt(-2*c + 2*sqrt(c^2 + d^2))*d - sqrt(c^2 + d^2)*
sqrt(-2*c + 2*sqrt(c^2 + d^2))))/((c*f - I*d*f)*sqrt(-2*c + 2*sqrt(c^2 + d^
2))*(-I*d/(c - sqrt(c^2 + d^2)) + 1)))
```

Mupad [**B**]
time = 14.43, size = 2500, normalized size = 32.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)/(c + d*tan(e + f*x))^(3/2),x)
```

[Out] (log((a^3*c*d^2*8i)/(f^3*(c^2 + d^2)^2) - ((((16*c*d^2*(c + d*tan(e + f*x))
^(1/2)*((4*(-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) - 4*a^2*c^3*f^2 + 12*a^2*c*
d^2*f^2)/(f^4*(c^2 + d^2)^3))^(1/2) - (32*a*d^2*(c^2*1i - d^2*1i))/(f*(c^2
+ d^2)))*((4*(-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) - 4*a^2*c^3*f^2 + 12*a^2*
c*d^2*f^2)/(f^4*(c^2 + d^2)^3))^(1/2))/4 + (16*a^2*d^2*(c^2 - d^2)*(c + d*t
an(e + f*x))^(1/2))/(f^2*(c^2 + d^2)^2))*((4*(-a^4*d^2*f^4*(3*c^2 - d^2)^2)
^(1/2) - 4*a^2*c^3*f^2 + 12*a^2*c*d^2*f^2)/(f^4*(c^2 + d^2)^3))^(1/2))/4)*(
((96*a^4*c^2*d^4*f^4 - 16*a^4*d^6*f^4 - 144*a^4*c^4*d^2*f^4)^(1/2) - 4*a^2*
c^3*f^2 + 12*a^2*c*d^2*f^2)/(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*
f^4))^(1/2))/4 + (log((a^3*c*d^2*8i)/(f^3*(c^2 + d^2)^2) - ((((16*c*d^2*(c
+ d*tan(e + f*x))^(1/2)*(-(4*(-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + 4*a^2*c
^3*f^2 - 12*a^2*c*d^2*f^2)/(f^4*(c^2 + d^2)^3))^(1/2) - (32*a*d^2*(c^2*1i -
d^2*1i))/(f*(c^2 + d^2)))*(-(4*(-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + 4*a^
2*c^3*f^2 - 12*a^2*c*d^2*f^2)/(f^4*(c^2 + d^2)^3))^(1/2))/4 + (16*a^2*d^2*(
c^2 - d^2)*(c + d*tan(e + f*x))^(1/2))/(f^2*(c^2 + d^2)^2))*(-(4*(-a^4*d^2*
f^4*(3*c^2 - d^2)^2)^(1/2) + 4*a^2*c^3*f^2 - 12*a^2*c*d^2*f^2)/(f^4*(c^2 +
d^2)^3))^(1/2))/4)*(-((96*a^4*c^2*d^4*f^4 - 16*a^4*d^6*f^4 - 144*a^4*c^4*d^
2*f^4)^(1/2) + 4*a^2*c^3*f^2 - 12*a^2*c*d^2*f^2)/(c^6*f^4 + d^6*f^4 + 3*c^2
*d^4*f^4 + 3*c^4*d^2*f^4))^(1/2))/4 - log(((((16*c*d^2*(c + d*tan(e + f*x))
^(1/2)*((4*(-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) - 4*a^2*c^3*f^2 + 12*a^2*c*
d^2*f^2)/(f^4*(c^2 + d^2)^3))^(1/2) + (32*a*d^2*(c^2*1i - d^2*1i))/(f*(c^2
+ d^2)))*((4*(-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) - 4*a^2*c^3*f^2 + 12*a^2*
c*d^2*f^2)/(f^4*(c^2 + d^2)^3))^(1/2))/4 + (16*a^2*d^2*(c^2 - d^2)*(c + d*t
an(e + f*x))^(1/2))/(f^2*(c^2 + d^2)^2))*((4*(-a^4*d^2*f^4*(3*c^2 - d^2)^2)
^(1/2) - 4*a^2*c^3*f^2 + 12*a^2*c*d^2*f^2)/(f^4*(c^2 + d^2)^3))^(1/2))/4 +
(a^3*c*d^2*8i)/(f^3*(c^2 + d^2)^2))*(((96*a^4*c^2*d^4*f^4 - 16*a^4*d^6*f^4
- 144*a^4*c^4*d^2*f^4)^(1/2) - 4*a^2*c^3*f^2 + 12*a^2*c*d^2*f^2)/(16*c^6*f^
4 + 16*d^6*f^4 + 48*c^2*d^4*f^4 + 48*c^4*d^2*f^4))^(1/2) - log(((((16*c*d^2
*(c + d*tan(e + f*x))^(1/2)*(-(4*(-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + 4*a
^2*c^3*f^2 - 12*a^2*c*d^2*f^2)/(f^4*(c^2 + d^2)^3))^(1/2) + (32*a*d^2*(c^2*
1i - d^2*1i))/(f*(c^2 + d^2)))*(-(4*(-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) +
4*a^2*c^3*f^2 - 12*a^2*c*d^2*f^2)/(f^4*(c^2 + d^2)^3))^(1/2))/4 + (16*a^2*d
^2*(c^2 - d^2)*(c + d*tan(e + f*x))^(1/2))/(f^2*(c^2 + d^2)^2))*(-(4*(-a^4*
d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + 4*a^2*c^3*f^2 - 12*a^2*c*d^2*f^2)/(f^4*(c^
2 + d^2)^3))^(1/2))/4 + (a^3*c*d^2*8i)/(f^3*(c^2 + d^2)^2))*(-((96*a^4*c^2*
d^4*f^4 - 16*a^4*d^6*f^4 - 144*a^4*c^4*d^2*f^4)^(1/2) + 4*a^2*c^3*f^2 - 12*
a^2*c*d^2*f^2)/(16*c^6*f^4 + 16*d^6*f^4 + 48*c^2*d^4*f^4 + 48*c^4*d^2*f^4))
^(1/2) + (log(((((96*a^4*c^2*d^4*f^4 - 16*a^4*d^6*f^4 - 144*a^4*c^4*d^2*f^4
)^(1/2) - 4*a^2*c^3*f^2 + 12*a^2*c*d^2*f^2)/(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*
f^4 + 3*c^4*d^2*f^4))^(1/2)*((c + d*tan(e + f*x))^(1/2)*(16*a^2*d^10*f^3 +
32*a^2*c^2*d^8*f^3 - 32*a^2*c^6*d^4*f^3 - 16*a^2*c^8*d^2*f^3) - ((((96*a^4*
c^2*d^4*f^4 - 16*a^4*d^6*f^4 - 144*a^4*c^4*d^2*f^4)^(1/2) - 4*a^2*c^3*f^2 +
12*a^2*c*d^2*f^2)/(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4))^(1/
2)*(((((96*a^4*c^2*d^4*f^4 - 16*a^4*d^6*f^4 - 144*a^4*c^4*d^2*f^4)^(1/2) -
4*a^2*c^3*f^2 + 12*a^2*c*d^2*f^2)/(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^

4*d^2*f^4))^(1/2)*(c + d*tan(e + f*x))^(1/2)*(64*c*d^12*f^5 + 320*c^3*d^10*
f^5 + 640*c^5*d^8*f^5 + 640*c^7*d^6*f^5 + 320*c^9*d^4*f^5 + 64*c^11*d^2*f^5
))/4 + 256*a*c^3*d^9*f^4 + 384*a*c^5*d^7*f^4 + 256*a*c^7*d^5*f^4 + 64*a*c^9
*d^3*f^4 + 64*a*c*d^11*f^4))/4))/4 + 8*a^3*d^9*f^2 + 24*a^3*c^2*d^7*f^2 + 2
4*a^3*c^4*d^5*f^2 + 8*a^3*c^6*d^3*f^2)*(((96*a^4*c^2*d^4*f^4 - 16*a^4*d^6*f
^4 - 144*a^4*c^4*d^2*f^4)^(1/2) - 4*a^2*c^3*f^2 + 12*a^2*c*d^2*f^2)/(c^6*f^
4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4))^(1/2))/4 + (log(((-((96*a^4*c
^2*d^4*f^4 - 16*a^4*d^6*f^4 - 144*a^4*c^4*d^2*f^4)^(1/2) + 4*a^2*c^3*f^2 -
12*a^2*c*d^2*f^2)/(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4))^(1/2
)*((c + d*tan(e + f*x))^(1/2)*(16*a^2*d^10*f^3 + 32*a^2*c^2*d^8*f^3 - 32*a^
2*c^6*d^4*f^3 - 16*a^2*c^8*d^2*f^3) - ((-((96*a^4*c^2*d^4*f^4 - 16*a^4*d^6*
f^4 - 144*a^4*c^4*d^2*f^4)^(1/2) + 4*a^2*c^3*f^2 - 12*a^2*c*d^2*f^2)/(c^6*f
^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4))^(1/2)*(((-((96*a^4*c^2*d^4*f
^4 - 16*a^4*d^6*f^4 - 144*a^4*c^4*d^2*f^4)^(1/2) + 4*a^2*c^3*f^2 - 12*a^2*c
*d^2*f^2)/(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4))^(1/2)*(c + d
*tan(e + f*x))^(1/2)*(64*c*d^12*f^5 + 320*c^3*d^10*f^5 + 640*c^5*d^8*f^5 +
640*c^7*d^6*f^5 + 320*c^9*d^4*f^5 + 64*c^11*d^2*f^5))/4 + 256*a*c^3*d^9*f^4
 + 384*a*c^5*d^7*f^4 + 256*a*c^7*d^5*f^4 + 64*a*c^9*d^3*f^4 + 64*a*c*d^11*f
^4))/4))/4 + 8*a^3*d^9*f^2 + 24*a^3*c^2*d^7*f^2 + 24*a^3*c^4*d^5*f^2 + 8*a^
3*c^6*d^3*f^2)*(-((96*a^4*c^2*d^4*f^4 - 16*a^4*d^6*f^4 - 144*a^4*c^4*d^2*f^
4)^(1/2) + 4*a^2*c^3*f^2 - 12*a^2*c*d^2*f^2)/(c^6*f^4 + d^6*f^4 + 3*c^2*d^4
*f^4 + 3*c^4*d^2*f^4))^(1/2))/4 - log(8*a^3*d^9...

3.1128 $\int \frac{1}{(a+ia\tan(e+fx))(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=205

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{2a(c-id)^{3/2}f}+\frac{(ic-4d)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{2a(c+id)^{5/2}f}+\frac{(c-5id)d}{2a(c-id)(c+id)^2f\sqrt{c+}}$$

```
[Out] -1/2*I*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/a/(c-I*d)^(3/2)/f+1/2*
(I*c-4*d)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/a/(c+I*d)^(5/2)/f+1
/2*(c-5*I*d)*d/a/(c-I*d)/(c+I*d)^2/f/(c+d*tan(f*x+e))^(1/2)-1/2/(I*c-d)/f/(
c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))
```

Rubi [A]

time = 0.33, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3633, 3610, 3620, 3618, 65, 214}

$$\frac{d(c-5id)}{2af(c-id)(c+id)^2\sqrt{c+d\tan(e+fx)}}-\frac{1}{2f(-d+ic)(a+ia\tan(e+fx))\sqrt{c+d\tan(e+fx)}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{2af(c-id)^{3/2}}+\frac{(-4d+ic)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{2af(c+id)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])*(c + d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] ((-1/2*I)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/(a*(c - I*d)^(3/
2)*f) + ((I*c - 4*d)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/(2*a*
(c + I*d)^(5/2)*f) + ((c - (5*I)*d)*d)/(2*a*(c - I*d)*(c + I*d)^2*f*Sqrt[c
+ d*Tan[e + f*x]]) - 1/(2*(I*c - d)*f*(a + I*a*Tan[e + f*x])*Sqrt[c + d*Tan
[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
```

```
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3633

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-a)*((c + d*Tan[e + f*x])^(n + 1)/(2*f*(b*
c - a*d)*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*
Tan[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &
& NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{1}{2(ic-d)f(a+ia\tan(e+fx))\sqrt{c+d\tan(e+fx)}} + $$

$$= \frac{(c-5id)d}{2a(c-id)(c+id)^2f\sqrt{c+d\tan(e+fx)}} - \frac{}{2(ic-d)f(a}$$

$$= \frac{(c-5id)d}{2a(c-id)(c+id)^2f\sqrt{c+d\tan(e+fx)}} - \frac{}{2(ic-d)f(a}$$

$$= \frac{(c-5id)d}{2a(c-id)(c+id)^2f\sqrt{c+d\tan(e+fx)}} - \frac{}{2(ic-d)f(a}$$

$$= \frac{(c-5id)d}{2a(c-id)(c+id)^2f\sqrt{c+d\tan(e+fx)}} - \frac{}{2(ic-d)f(a}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{2a(c-id)^{3/2}f} + \frac{(ic-4d)\tanh^{-1}\left(\right.}{2a(c}$$

Mathematica [A]
time = 3.41, size = 297, normalized size = 1.45

$$\frac{\sec(e+fx)(\cos(fx)+i\sin(fx))\left(-\frac{2\left(-i\sqrt{-c+id}\,(c^2+3icd+4d^2)\operatorname{ArcTan}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{-c-id}}\right)+i(-c-id)^{5/2}\operatorname{ArcTan}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{-c+id}}\right)\right)(\cos(e)+i\sin(e))}{(-c-id)^{5/2}(-c+id)^{3/2}}+\frac{2\cos(e+fx)(i\cos(fx)+\sin(fx))((c^2-icd-4d^2)\cos(e+fx)+(c-5id)d\sin(e+fx))\sqrt{c+d\tan(e+fx)}}{(c-id)(c+id)^2(c\cos(e+fx)+d\sin(e+fx))}\right)}{4f(a+ia\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])*(c + d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] (Sec[e + f*x]*(Cos[f*x] + I*Sin[f*x])*((-2*((-I)*Sqrt[-c + I*d]*(c^2 + (3*I
)*c*d + 4*d^2)*ArcTan[Sqrt[c + d*Tan[e + f*x]]/Sqrt[-c - I*d]] + I*(-c - I*
d)^(5/2)*ArcTan[Sqrt[c + d*Tan[e + f*x]]/Sqrt[-c + I*d]])*(Cos[e] + I*Sin[e
]))/((-c - I*d)^(5/2)*(-c + I*d)^(3/2)) + (2*Cos[e + f*x]*(I*Cos[f*x] + Sin
[f*x])*((c^2 - I*c*d - 4*d^2)*Cos[e + f*x] + (c - (5*I)*d)*d*Sin[e + f*x])*
Sqrt[c + d*Tan[e + f*x]])/((c - I*d)*(c + I*d)^2*(c*Cos[e + f*x] + d*Sin[e
+ f*x]))))/(4*f*(a + I*a*Tan[e + f*x]))
```

Maple [A]
time = 0.35, size = 257, normalized size = 1.25

method	result

derivativedivides	$\frac{2d^2\left(-\frac{-\frac{(c^2+d^2)d\sqrt{c+d\tan(fx+e)}}{(id+c)(-d\tan(fx+e)+id)}-\frac{(ic^3+icd^2-4c^2d-4d^3)\arctan\left(\frac{\sqrt{c+d\tan(fx+e)}}{\sqrt{-id-c}}\right)}{(id+c)\sqrt{-id-c}}}{4d^2(id+c)^2(id-c)}+\frac{\cdots}{(ic+\cdots}\right.}{fa}$
default	$\frac{2d^2\left(-\frac{-\frac{(c^2+d^2)d\sqrt{c+d\tan(fx+e)}}{(id+c)(-d\tan(fx+e)+id)}-\frac{(ic^3+icd^2-4c^2d-4d^3)\arctan\left(\frac{\sqrt{c+d\tan(fx+e)}}{\sqrt{-id-c}}\right)}{(id+c)\sqrt{-id-c}}}{4d^2(id+c)^2(id-c)}+\frac{\cdots}{(ic+\cdots}\right.}{fa}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a*d^2*(-1/4/d^2/(c+I*d)^2/(I*d-c)*(-(c^2+d^2)*d/(c+I*d)*(c+d*tan(f*x+e)
)^(1/2)/(-d*tan(f*x+e)+I*d)-(-4*c^2*d-4*d^3+I*c^3+I*c*d^2)/(c+I*d)/(-I*d-c)
^(1/2)*arctan((c+d*tan(f*x+e))^(1/2)/(-I*d-c)^(1/2)))+I/(I*c+d)/(I*c-d)/(c+
I*d)/(c+d*tan(f*x+e))^(1/2)+1/4*(-I*c^2+I*d^2+2*c*d)/(I*d-c)^(3/2)/(c+I*d)^
2/d^2*arctan((c+d*tan(f*x+e))^(1/2)/(I*d-c)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1589 vs. 2(164) = 328.
time = 1.53, size = 1589, normalized size = 7.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] -1/8*(2*((a*c^4 + 2*a*c^2*d^2 + a*d^4)*f*e^(4*I*f*x + 4*I*e) + (a*c^4 + 2*I
*a*c^3*d + 2*I*a*c*d^3 - a*d^4)*f*e^(2*I*f*x + 2*I*e))*sqrt(-1/4*I/((I*a^2*
c^3 + 3*a^2*c^2*d - 3*I*a^2*c*d^2 - a^2*d^3)*f^2))*log(-2*(2*((I*a*c^2 + 2*
a*c*d - I*a*d^2)*f*e^(2*I*f*x + 2*I*e) + (I*a*c^2 + 2*a*c*d - I*a*d^2)*f)*s
qrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sq
rt(-1/4*I/((I*a^2*c^3 + 3*a^2*c^2*d - 3*I*a^2*c*d^2 - a^2*d^3)*f^2)) - (c -
 I*d)*e^(2*I*f*x + 2*I*e) - c)*e^(-2*I*f*x - 2*I*e)) - 2*((a*c^4 + 2*a*c^2*
d^2 + a*d^4)*f*e^(4*I*f*x + 4*I*e) + (a*c^4 + 2*I*a*c^3*d + 2*I*a*c*d^3 - a
*d^4)*f*e^(2*I*f*x + 2*I*e))*sqrt(-1/4*I/((I*a^2*c^3 + 3*a^2*c^2*d - 3*I*a^
2*c*d^2 - a^2*d^3)*f^2))*log(-2*(2*((-I*a*c^2 - 2*a*c*d + I*a*d^2)*f*e^(2*I
*f*x + 2*I*e) + (-I*a*c^2 - 2*a*c*d + I*a*d^2)*f)*sqrt(((c - I*d)*e^(2*I*f*
x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-1/4*I/((I*a^2*c^3 +
3*a^2*c^2*d - 3*I*a^2*c*d^2 - a^2*d^3)*f^2)) - (c - I*d)*e^(2*I*f*x + 2*I*e
) - c)*e^(-2*I*f*x - 2*I*e)) + ((a*c^4 + 2*a*c^2*d^2 + a*d^4)*f*e^(4*I*f*x
+ 4*I*e) + (a*c^4 + 2*I*a*c^3*d + 2*I*a*c*d^3 - a*d^4)*f*e^(2*I*f*x + 2*I*e
))*sqrt(-(I*c^2 - 8*c*d - 16*I*d^2)/((I*a^2*c^5 - 5*a^2*c^4*d - 10*I*a^2*c^
3*d^2 + 10*a^2*c^2*d^3 + 5*I*a^2*c*d^4 - a^2*d^5)*f^2))*log(1/2*(c^2 + 5*I*
c*d - 4*d^2 + ((I*a*c^3 - 3*a*c^2*d - 3*I*a*c*d^2 + a*d^3)*f*e^(2*I*f*x + 2
*I*e) + (I*a*c^3 - 3*a*c^2*d - 3*I*a*c*d^2 + a*d^3)*f)*sqrt(((c - I*d)*e^(2
*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(I*c^2 - 8*c*d
- 16*I*d^2)/((I*a^2*c^5 - 5*a^2*c^4*d - 10*I*a^2*c^3*d^2 + 10*a^2*c^2*d^3 +
 5*I*a^2*c*d^4 - a^2*d^5)*f^2)) + (c^2 + 4*I*c*d)*e^(2*I*f*x + 2*I*e))*e^(-
2*I*f*x - 2*I*e)/((-I*a*c^3 + 3*a*c^2*d + 3*I*a*c*d^2 - a*d^3)*f)) - ((a*c^
4 + 2*a*c^2*d^2 + a*d^4)*f*e^(4*I*f*x + 4*I*e) + (a*c^4 + 2*I*a*c^3*d + 2*I
*a*c*d^3 - a*d^4)*f*e^(2*I*f*x + 2*I*e))*sqrt(-(I*c^2 - 8*c*d - 16*I*d^2)/(
(I*a^2*c^5 - 5*a^2*c^4*d - 10*I*a^2*c^3*d^2 + 10*a^2*c^2*d^3 + 5*I*a^2*c*d^
4 - a^2*d^5)*f^2))*log(1/2*(c^2 + 5*I*c*d - 4*d^2 + ((-I*a*c^3 + 3*a*c^2*d
+ 3*I*a*c*d^2 - a*d^3)*f*e^(2*I*f*x + 2*I*e) + (-I*a*c^3 + 3*a*c^2*d + 3*I*
a*c*d^2 - a*d^3)*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*
f*x + 2*I*e) + 1))*sqrt(-(I*c^2 - 8*c*d - 16*I*d^2)/((I*a^2*c^5 - 5*a^2*c^4
*d - 10*I*a^2*c^3*d^2 + 10*a^2*c^2*d^3 + 5*I*a^2*c*d^4 - a^2*d^5)*f^2)) + (
c^2 + 4*I*c*d)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/((-I*a*c^3 + 3*a*c
^2*d + 3*I*a*c*d^2 - a*d^3)*f)) - 2*(I*c^2 + I*d^2 + (I*c^2 + 2*c*d - 9*I*d
^2)*e^(4*I*f*x + 4*I*e) - 2*(-I*c^2 - c*d + 4*I*d^2)*e^(2*I*f*x + 2*I*e))*s
qrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))/(
(a*c^4 + 2*a*c^2*d^2 + a*d^4)*f*e^(4*I*f*x + 4*I*e) + (a*c^4 + 2*I*a*c^3*d
+ 2*I*a*c*d^3 - a*d^4)*f*e^(2*I*f*x + 2*I*e))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int \frac{1}{c\sqrt{c+d\tan{(e+fx)}}\tan{(e+fx)}-ic\sqrt{c+d\tan{(e+fx)}}+d\sqrt{c+d\tan{(e+fx)}}\tan^2{(e+fx)}-id\sqrt{c+d\tan{(e+fx)}}\tan{(e+fx)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] -I*Integral(1/(c*sqrt(c + d*tan(e + f*x))*tan(e + f*x) - I*c*sqrt(c + d*tan
(e + f*x)) + d*sqrt(c + d*tan(e + f*x))*tan(e + f*x)**2 - I*d*sqrt(c + d*ta
n(e + f*x))*tan(e + f*x)), x)/a
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(164) = 328$.
time = 0.81, size = 476, normalized size = 2.32

$$\frac{(i c-4 d) \arctan\left(-\frac{2\left(\sqrt{d \tan(f x+e)+c}\, c-\sqrt{c^2+d^2} \sqrt{d \tan(f x+e)+c}\right)}{c \sqrt{-2 c+2 \sqrt{c^2+d^2}}+i \sqrt{-2 c+2 \sqrt{c^2+d^2}}\, d-\sqrt{c^2+d^2} \sqrt{-2 c+2 \sqrt{c^2+d^2}}}\right)}{\left(a c^2 f+2 i a c d f-a d^2 f\right) \sqrt{-2 c+2 \sqrt{c^2+d^2}}\left(\frac{i d}{c-\sqrt{c^2+d^2}}+1\right)}-\frac{(-i d \tan(f x+e)-i c) c d-5(d \tan(f x+e)+c) d^2+4 c d^2+4 i d^3}{2\left(a c^3 f+i a c^2 d f+a c d^2 f+i a d^3 f\right)\left(i(d \tan(f x+e)+c)^{\frac{3}{2}}-i \sqrt{d \tan(f x+e)+c}\, c+\sqrt{d \tan(f x+e)+c}\, d\right)}+\frac{i \arctan\left(\frac{2\left(\sqrt{d \tan(f x+e)+c}\, c-\sqrt{c^2+d^2} \sqrt{d \tan(f x+e)+c}\right)}{c \sqrt{-2 c+2 \sqrt{c^2+d^2}}-i \sqrt{-2 c+2 \sqrt{c^2+d^2}}\, d-\sqrt{c^2+d^2} \sqrt{-2 c+2 \sqrt{c^2+d^2}}}\right)}{(a c f-i a d f) \sqrt{-2 c+2 \sqrt{c^2+d^2}}\left(-\frac{i d}{c-\sqrt{c^2+d^2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] (I*c - 4*d)*arctan(-2*(sqrt(d*tan(f*x + e) + c)*c - sqrt(c^2 + d^2)*sqrt(d*
tan(f*x + e) + c))/(c*sqrt(-2*c + 2*sqrt(c^2 + d^2)) + I*sqrt(-2*c + 2*sqrt
(c^2 + d^2))*d - sqrt(c^2 + d^2)*sqrt(-2*c + 2*sqrt(c^2 + d^2))))/((a*c^2*f
 + 2*I*a*c*d*f - a*d^2*f)*sqrt(-2*c + 2*sqrt(c^2 + d^2))*(I*d/(c - sqrt(c^2
 + d^2)) + 1)) - 1/2*((-I*d*tan(f*x + e) - I*c)*c*d - 5*(d*tan(f*x + e) + c
)*d^2 + 4*c*d^2 + 4*I*d^3)/((a*c^3*f + I*a*c^2*d*f + a*c*d^2*f + I*a*d^3*f)
*(I*(d*tan(f*x + e) + c)^(3/2) - I*sqrt(d*tan(f*x + e) + c)*c + sqrt(d*tan(
f*x + e) + c)*d)) + I*arctan(2*(sqrt(d*tan(f*x + e) + c)*c - sqrt(c^2 + d^2
)*sqrt(d*tan(f*x + e) + c))/(c*sqrt(-2*c + 2*sqrt(c^2 + d^2)) - I*sqrt(-2*c
 + 2*sqrt(c^2 + d^2))*d - sqrt(c^2 + d^2)*sqrt(-2*c + 2*sqrt(c^2 + d^2))))/
((a*c*f - I*a*d*f)*sqrt(-2*c + 2*sqrt(c^2 + d^2))*(-I*d/(c - sqrt(c^2 + d^2
)) + 1))
```

Mupad [B]
time = 15.81, size = 2500, normalized size = 12.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)*(c + d*tan(e + f*x))^(3/2)),x)
```

```
[Out] log(10*a*d^7*f - ((-(240*c^2*d^7 - 240*d^9 - c*d^8*720i + c^3*d^6*80i + 160
*c^4*d^5 - c^5*d^4*32i - a^2*c^6*f^2*(((1280*c^3*d^8 - 1200*c*d^10 + 208*c^
5*d^6 + 32*c^7*d^4)/(a^2*c^8*f^2 + a^2*d^8*f^2 + 4*a^2*c^2*d^6*f^2 + 6*a^2*
c^4*d^4*f^2 + 4*a^2*c^6*d^2*f^2) + ((240*d^11 - 1920*c^2*d^9 + 240*c^4*d^7
+ 96*c^6*d^5)*1i)/(a^2*c^8*f^2 + a^2*d^8*f^2 + 4*a^2*c^2*d^6*f^2 + 6*a^2*c^
4*d^4*f^2 + 4*a^2*c^6*d^2*f^2))^2 - 4*(256*d^6 + 256*c^2*d^4)*(((24*c*d^7 +
 6*c^3*d^5)*1i)/(a^4*c^8*f^4 + a^4*d^8*f^4 + 4*a^4*c^2*d^6*f^4 + 6*a^4*c^4*
d^4*f^4 + 4*a^4*c^6*d^2*f^4) + (16*d^8 - c^2*d^6 + c^4*d^4)/(a^4*c^8*f^4 +
a^4*d^8*f^4 + 4*a^4*c^2*d^6*f^4 + 6*a^4*c^4*d^4*f^4 + 4*a^4*c^6*d^2*f^4)))^
(1/2)*1i + a^2*d^6*f^2*(((1280*c^3*d^8 - 1200*c*d^10 + 208*c^5*d^6 + 32*c^7
*d^4)/(a^2*c^8*f^2 + a^2*d^8*f^2 + 4*a^2*c^2*d^6*f^2 + 6*a^2*c^4*d^4*f^2 +
```

4*a^2*c^6*d^2*f^2) + ((240*d^11 - 1920*c^2*d^9 + 240*c^4*d^7 + 96*c^6*d^5)*
1i)/(a^2*c^8*f^2 + a^2*d^8*f^2 + 4*a^2*c^2*d^6*f^2 + 6*a^2*c^4*d^4*f^2 + 4*
a^2*c^6*d^2*f^2))^2 - 4*(256*d^6 + 256*c^2*d^4)*(((24*c*d^7 + 6*c^3*d^5)*1i
)/(a^4*c^8*f^4 + a^4*d^8*f^4 + 4*a^4*c^2*d^6*f^4 + 6*a^4*c^4*d^4*f^4 + 4*a^
4*c^6*d^2*f^4) + (16*d^8 - c^2*d^6 + c^4*d^4)/(a^4*c^8*f^4 + a^4*d^8*f^4 +
4*a^4*c^2*d^6*f^4 + 6*a^4*c^4*d^4*f^4 + 4*a^4*c^6*d^2*f^4)))^(1/2)*1i + 2*a
^2*c*d^5*f^2*(((1280*c^3*d^8 - 1200*c*d^10 + 208*c^5*d^6 + 32*c^7*d^4)/(a^2
*c^8*f^2 + a^2*d^8*f^2 + 4*a^2*c^2*d^6*f^2 + 6*a^2*c^4*d^4*f^2 + 4*a^2*c^6*
d^2*f^2) + ((240*d^11 - 1920*c^2*d^9 + 240*c^4*d^7 + 96*c^6*d^5)*1i)/(a^2*c
^8*f^2 + a^2*d^8*f^2 + 4*a^2*c^2*d^6*f^2 + 6*a^2*c^4*d^4*f^2 + 4*a^2*c^6*d^
2*f^2))^2 - 4*(256*d^6 + 256*c^2*d^4)*(((24*c*d^7 + 6*c^3*d^5)*1i)/(a^4*c^8
*f^4 + a^4*d^8*f^4 + 4*a^4*c^2*d^6*f^4 + 6*a^4*c^4*d^4*f^4 + 4*a^4*c^6*d^2*
f^4) + (16*d^8 - c^2*d^6 + c^4*d^4)/(a^4*c^8*f^4 + a^4*d^8*f^4 + 4*a^4*c^2*
d^6*f^4 + 6*a^4*c^4*d^4*f^4 + 4*a^4*c^6*d^2*f^4)))^(1/2) + 2*a^2*c^5*d*f^2*
(((1280*c^3*d^8 - 1200*c*d^10 + 208*c^5*d^6 + 32*c^7*d^4)/(a^2*c^8*f^2 + a^
2*d^8*f^2 + 4*a^2*c^2*d^6*f^2 + 6*a^2*c^4*d^4*f^2 + 4*a^2*c^6*d^2*f^2) + ((
240*d^11 - 1920*c^2*d^9 + 240*c^4*d^7 + 96*c^6*d^5)*1i)/(a^2*c^8*f^2 + a^2*
d^8*f^2 + 4*a^2*c^2*d^6*f^2 + 6*a^2*c^4*d^4*f^2 + 4*a^2*c^6*d^2*f^2))^2 - 4
*(256*d^6 + 256*c^2*d^4)*(((24*c*d^7 + 6*c^3*d^5)*1i)/(a^4*c^8*f^4 + a^4*d^
8*f^4 + 4*a^4*c^2*d^6*f^4 + 6*a^4*c^4*d^4*f^4 + 4*a^4*c^6*d^2*f^4) + (16*d^
8 - c^2*d^6 + c^4*d^4)/(a^4*c^8*f^4 + a^4*d^8*f^4 + 4*a^4*c^2*d^6*f^4 + 6*a
^4*c^4*d^4*f^4 + 4*a^4*c^6*d^2*f^4)))^(1/2) + a^2*c^2*d^4*f^2*(((1280*c^3*d
^8 - 1200*c*d^10 + 208*c^5*d^6 + 32*c^7*d^4)/(a^2*c^8*f^2 + a^2*d^8*f^2 + 4
*a^2*c^2*d^6*f^2 + 6*a^2*c^4*d^4*f^2 + 4*a^2*c^6*d^2*f^2) + ((240*d^11 - 19
20*c^2*d^9 + 240*c^4*d^7 + 96*c^6*d^5)*1i)/(a^2*c^8*f^2 + a^2*d^8*f^2 + 4*a
^2*c^2*d^6*f^2 + 6*a^2*c^4*d^4*f^2 + 4*a^2*c^6*d^2*f^2))^2 - 4*(256*d^6 + 2
56*c^2*d^4)*(((24*c*d^7 + 6*c^3*d^5)*1i)/(a^4*c^8*f^4 + a^4*d^8*f^4 + 4*a^4
*c^2*d^6*f^4 + 6*a^4*c^4*d^4*f^4 + 4*a^4*c^6*d^2*f^4) + (16*d^8 - c^2*d^6 +
c^4*d^4)/(a^4*c^8*f^4 + a^4*d^8*f^4 + 4*a^4*c^2*d^6*f^4 + 6*a^4*c^4*d^4*f^
4 + 4*a^4*c^6*d^2*f^4)))^(1/2)*1i + 4*a^2*c^3*d^3*f^2*(((1280*c^3*d^8 - 120
0*c*d^10 + 208*c^5*d^6 + 32*c^7*d^4)/(a^2*c^8*f^2 + a^2*d^8*f^2 + 4*a^2*c^2
*d^6*f^2 + 6*a^2*c^4*d^4*f^2 + 4*a^2*c^6*d^2*f^2) + ((240*d^11 - 1920*c^2*d
^9 + 240*c^4*d^7 + 96*c^6*d^5)*1i)/(a^2*c^8*f^2 + a^2*d^8*f^2 + 4*a^2*c^2*d
^6*f^2 + 6*a^2*c^4*d^4*f^2 + 4*a^2*c^6*d^2*f^2))^2 - 4*(256*d^6 + 256*c^2*d
^4)*(((24*c*d^7 + 6*c^3*d^5)*1i)/(a^4*c^8*f^4 + a^4*d^8*f^4 + 4*a^4*c^2*d^6
*f^4 + 6*a^4*c^4*d^4*f^4 + 4*a^4*c^6*d^2*f^4) + (16*d^8 - c^2*d^6 + c^4*d^4
)/(a^4*c^8*f^4 + a^4*d^8*f^4 + 4*a^4*c^2*d^6*f^4 + 6*a^4*c^4*d^4*f^4 + 4*a^
4*c^6*d^2*f^4)))^(1/2) - a^2*c^4*d^2*f^2*(((1280*c^3*d^8 - 1200*c*d^10 + 20
8*c^5*d^6 + 32*c^7*d^4)/(a^2*c^8*f^2 + a^2*d^8*f^2 + 4*a^2*c^2*d^6*f^2 + 6*
a^2*c^4*d^4*f^2 + 4*a^2*c^6*d^2*f^2) + ((240*d^11 - 1920*c^2*d^9 + 240*c^4*
d^7 + 96*c^6*d^5)*1i)/(a^2*c^8*f^2 + a^2*d^8*f^2 + 4*a^2*c^2*d^6*f^2 + 6*a^
2*c^4*d^4*f^2 + 4*a^2*c^6*d^2*f^2))^2 - 4*(256*d^6 + 256*c^2*d^4)*(((24*c*d
^7 + 6*c^3*d^5)*1i)/(a^4*c^8*f^4 + a^4*d^8*f^4 + 4*a^4*c^2*d^6*f^4 + 6*a^4*
c^4*d^4*f^4 + 4*a^4*c^6*d^2*f^4) + (16*d^8 - c^2*d^6 + c^4*d^4)/(a^4*c^8*f^
4 + a^4*d^8*f^4 + 4*a^4*c^2*d^6*f^4 + 6*a^4*c^4*d^4*f^4 + 4*a^4*c^6*d^2*f^4

```
)))^(1/2)*1i)/(512*(d^6 + c^2*d^4)*(a^2*d^6*f^2*1i - a^2*c^6*f^2*1i + 2*a^2
*c*d^5*f^2 + 2*a^2*c^5*d*f^2 + a^2*c^2*d^4*f^2*1i + 4*a^2*c^3*d^3*f^2 - a^2
*c^4*d^2*f^2*1i)))^(1/2)*(104*a^3*c*d^9*f^3 - a^3*d^10*f^3*24i + a^3*c^2*d^
8*f^3*24i + 216*a^3*c^3*d^7*f^3 + a^3*c^4*d^6*f^3*120i + 120*a^3*c^5*d^5*f^
3 + a^3*c^6*d^4*f^3*72i + 8*a^3*c^7*d^3*f^3 - 2*(c + d*tan(e + f*x))^(1/2)*
(a^2*d^2*f^2 - a^2*c^2*f^2 + a^2*c*d*f^2*2i)*(-(240*c^2*d^7 - 240*d^9 - c*d
^8*720i + c^3*d^6*80i + 160*c^4*d^5 - c^5*d^4*32i - a^2*c^6*f^2*(((1280*c^3
*d^8 - 1200*c*d^10 + 208*c^5*d^6 + 32*c^7*d^4)/(a^2*c^8*f^2 + a^2*d^8*f^2 +
 4*a^2*c^2*d^6*f^2 + 6*a^2*c^4*d^4*f^2 + 4*a^2*c^6*d^2*f^2) + ((240*d^11 -
1920*c^2*d^9 + 240*c^4*d^7 + 96*c^6*d^5)*1i)/(a^2*c^8*f^2 + a^2*d^8*f^2 + 4
*a^2*c^2*d^6*f^2 + 6*a^2*c^4*d^4*f^2 + 4*a^2*c^...
```

3.1129 $\int \frac{1}{(a+ia\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=281

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{4a^2(c-id)^{3/2}f}+\frac{(2ic^2-10cd-23id^2)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{8a^2(c+id)^{7/2}f}+\frac{d(2c}{8a^2(c-id)(c-}$$

```
[Out] -1/4*I*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/a^2/(c-I*d)^(3/2)/f+1/
8*(2*I*c^2-10*c*d-23*I*d^2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/a
^2/(c+I*d)^(7/2)/f+1/8*d*(2*c^2+7*I*c*d+25*d^2)/a^2/(c-I*d)/(c+I*d)^3/f/(c+
d*tan(f*x+e))^(1/2)+1/8*(2*I*c-7*d)/a^2/(c+I*d)^2/f/(c+d*tan(f*x+e))^(1/2)/
(1+I*tan(f*x+e))-1/4/(I*c-d)/f/(c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2
```

Rubi [A]
time = 0.54, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3640, 3677, 3610, 3620, 3618, 65, 214}

$$\frac{d(2c^2+7icd+25d^2)}{8a^2f(c-id)(c+id)^3\sqrt{c+d\tan(e+fx)}}+\frac{(2ic^2-10cd-23id^2)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{8a^2f(c+id)^{7/2}}+\frac{-7d+2ic}{8a^2f(c+id)^2(1+i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{4a^2f(c-id)^{3/2}}-\frac{1}{4f(-d+ic)(a+ia\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] ((-1/4*I)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/(a^2*(c - I*d)^(
3/2)*f) + (((2*I)*c^2 - 10*c*d - (23*I)*d^2)*ArcTanh[Sqrt[c + d*Tan[e + f*x
]]/Sqrt[c + I*d]])/(8*a^2*(c + I*d)^(7/2)*f) + (d*(2*c^2 + (7*I)*c*d + 25*d
^2))/(8*a^2*(c - I*d)*(c + I*d)^3*f*Sqrt[c + d*Tan[e + f*x]]) + ((2*I)*c -
7*d)/(8*a^2*(c + I*d)^2*f*(1 + I*Tan[e + f*x])*Sqrt[c + d*Tan[e + f*x]]) -
1/(4*(I*c - d)*f*(a + I*a*Tan[e + f*x])^2*Sqrt[c + d*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{1}{4(ic-d)f(a+ia\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}$$

$$= \frac{2ic-7d}{8a^2(c+id)^2f(1+i\tan(e+fx))\sqrt{c+d\tan(e+fx)}} -$$

$$= \frac{d(2c^2+7icd+25d^2)}{8a^2(c+id)^2\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{}{8a^2(c+}$$

$$= \frac{d(2c^2+7icd+25d^2)}{8a^2(c+id)^2\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{}{8a^2(c+}$$

$$= \frac{d(2c^2+7icd+25d^2)}{8a^2(c+id)^2\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{}{8a^2(c+}$$

$$= \frac{d(2c^2+7icd+25d^2)}{8a^2(c+id)^2\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{}{8a^2(c+}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{4a^2(c-id)^{3/2}f} - \frac{(10cd-i(2c^2-2}{}$$

Mathematica [A]
time = 4.92, size = 388, normalized size = 1.38

$$\frac{\sec^2(e+fx)(\cos(fx)+i\sin(fx))^2\left(\frac{2\left(\sqrt{-c+id}\,(-2ic^3+8c^2d+13icd^2+23d^3)\text{ArcTan}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{-c-id}}\right)-2i(-c-id)^{7/2}\text{ArcTan}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{-c+id}}\right)\right)(\cos(2e)+i\sin(2e))}{(-c-id)^{7/2}(-c+id)^{3/2}}+\frac{(\cos(2fx)-i\sin(2fx))\left((12ic^3-23c^2d+26icd^2+23d^3)\cos(e+fx)+(4ic^3-5c^2d+18icd^2+41d^3)\cos(3(e+fx))-4(2c^3+3ic^2d+16cd^2-43id^3)\cos^2(e+fx)\sin(e+fx)\right)\sqrt{c+d\tan(e+fx)}}{2(c-id)(c+id)^3(c\cos(e+fx)+d\sin(e+fx))}\right)}{16f(a+ia\tan(e+fx))^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] (Sec[e + f*x]^2*(Cos[f*x] + I*Sin[f*x])^2*((2*(Sqrt[-c + I*d]*((-2*I)*c^3 +
 8*c^2*d + (13*I)*c*d^2 + 23*d^3)*ArcTan[Sqrt[c + d*Tan[e + f*x]]/Sqrt[-c -
 I*d]] - (2*I)*(-c - I*d)^(7/2)*ArcTan[Sqrt[c + d*Tan[e + f*x]]/Sqrt[-c + I
*d]])*(Cos[2*e] + I*Sin[2*e]))/((-c - I*d)^(7/2)*(-c + I*d)^(3/2)) + ((Cos[
2*f*x] - I*Sin[2*f*x])*(((12*I)*c^3 - 23*c^2*d + (26*I)*c*d^2 + 23*d^3)*Cos
[e + f*x] + ((4*I)*c^3 - 5*c^2*d + (18*I)*c*d^2 + 41*d^3)*Cos[3*(e + f*x)]
- 4*(2*c^3 + (3*I)*c^2*d + 16*c*d^2 - (43*I)*d^3)*Cos[e + f*x]^2*Sin[e + f*
x])*Sqrt[c + d*Tan[e + f*x]])/(2*(c - I*d)*(c + I*d)^3*(c*Cos[e + f*x] + d*
Sin[e + f*x]))))/(16*f*(a + I*a*Tan[e + f*x])^2)
```

Maple [A]
time = 0.32, size = 399, normalized size = 1.42

method	result
derivativedivides	$\frac{2d^3\left(-\frac{i\left(\frac{-\frac{d\left(2ic^4-7ic^2d^2-9id^4-11c^3d-11c\,d^3\right)(c+d\tan(fx+e))^{\frac{3}{2}}}{2\left(2icd+c^2-d^2\right)}+\frac{d\left(2ic^5-22ic^3d^2-24ic\,d^4-15c^4d-4c^2d^3+11d^5\right)\sqrt{c+d}}{4icd+2c^2-2d^2}}{(-d\tan(fx+e)+id)^2}}{8d^3(id+c)^3}$
default	$\frac{2d^3\left(-\frac{i\left(\frac{-\frac{d\left(2ic^4-7ic^2d^2-9id^4-11c^3d-11c\,d^3\right)(c+d\tan(fx+e))^{\frac{3}{2}}}{2\left(2icd+c^2-d^2\right)}+\frac{d\left(2ic^5-22ic^3d^2-24ic\,d^4-15c^4d-4c^2d^3+11d^5\right)\sqrt{c+d}}{4icd+2c^2-2d^2}}{(-d\tan(fx+e)+id)^2}}{8d^3(id+c)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^2*d^3*(-1/8*I/d^3/(c+I*d)^3/(I*d-c)*((-1/2*d*(2*I*c^4-7*I*c^2*d^2-9*I
*d^4-11*c^3*d-11*c*d^3)/(2*I*c*d+c^2-d^2)*(c+d*tan(f*x+e))^(3/2)+1/2*d*(2*I
*c^5-22*I*c^3*d^2-24*I*c*d^4-15*c^4*d-4*c^2*d^3+11*d^5)/(2*I*c*d+c^2-d^2)*(
c+d*tan(f*x+e))^(1/2))/(-d*tan(f*x+e)+I*d)^2-1/2*(-31*c^3*d^2-33*c*d^4+12*I
*c^4*d-11*I*c^2*d^3-23*I*d^5+2*c^5)/(2*I*c*d+c^2-d^2)/(-I*d-c)^(1/2)*arctan
((c+d*tan(f*x+e))^(1/2)/(-I*d-c)^(1/2)))-1/(I*c-d)/(I*c+d)/(c+I*d)^2/(c+d*t
an(f*x+e))^(1/2)-1/8*I/(I*d-c)^(3/2)/(c+I*d)^3*(3*I*c^2*d-I*d^3+c^3-3*c*d^2
)/d^3*arctan((c+d*tan(f*x+e))^(1/2)/(I*d-c)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2298 vs. $2(230) = 460$.
time = 2.27, size = 2298, normalized size = 8.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/32*(8*((-I*a^2*c^5 + a^2*c^4*d - 2*I*a^2*c^3*d^2 + 2*a^2*c^2*d^3 - I*a^2*
c*d^4 + a^2*d^5)*f*e^(6*I*f*x + 6*I*e) + (-I*a^2*c^5 + 3*a^2*c^4*d + 2*I*a^
2*c^3*d^2 + 2*a^2*c^2*d^3 + 3*I*a^2*c*d^4 - a^2*d^5)*f*e^(4*I*f*x + 4*I*e))
*sqrt(-1/16*I/((I*a^4*c^3 + 3*a^4*c^2*d - 3*I*a^4*c*d^2 - a^4*d^3)*f^2))*lo
g(-2*(4*((I*a^2*c^2 + 2*a^2*c*d - I*a^2*d^2)*f*e^(2*I*f*x + 2*I*e) + (I*a^2
*c^2 + 2*a^2*c*d - I*a^2*d^2)*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c +
I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-1/16*I/((I*a^4*c^3 + 3*a^4*c^2*d - 3*
I*a^4*c*d^2 - a^4*d^3)*f^2)) - (c - I*d)*e^(2*I*f*x + 2*I*e) - c)*e^(-2*I*f
*x - 2*I*e)) + 8*((I*a^2*c^5 - a^2*c^4*d + 2*I*a^2*c^3*d^2 - 2*a^2*c^2*d^3
+ I*a^2*c*d^4 - a^2*d^5)*f*e^(6*I*f*x + 6*I*e) + (I*a^2*c^5 - 3*a^2*c^4*d -
 2*I*a^2*c^3*d^2 - 2*a^2*c^2*d^3 - 3*I*a^2*c*d^4 + a^2*d^5)*f*e^(4*I*f*x +
4*I*e))*sqrt(-1/16*I/((I*a^4*c^3 + 3*a^4*c^2*d - 3*I*a^4*c*d^2 - a^4*d^3)*f
^2))*log(-2*(4*((-I*a^2*c^2 - 2*a^2*c*d + I*a^2*d^2)*f*e^(2*I*f*x + 2*I*e)
+ (-I*a^2*c^2 - 2*a^2*c*d + I*a^2*d^2)*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*
e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-1/16*I/((I*a^4*c^3 + 3*a^4*c
^2*d - 3*I*a^4*c*d^2 - a^4*d^3)*f^2)) - (c - I*d)*e^(2*I*f*x + 2*I*e) - c)*
e^(-2*I*f*x - 2*I*e)) + ((I*a^2*c^5 - a^2*c^4*d + 2*I*a^2*c^3*d^2 - 2*a^2*c
^2*d^3 + I*a^2*c*d^4 - a^2*d^5)*f*e^(6*I*f*x + 6*I*e) + (I*a^2*c^5 - 3*a^2*
c^4*d - 2*I*a^2*c^3*d^2 - 2*a^2*c^2*d^3 - 3*I*a^2*c*d^4 + a^2*d^5)*f*e^(4*I
*f*x + 4*I*e))*sqrt(-(-4*I*c^4 + 40*c^3*d + 192*I*c^2*d^2 - 460*c*d^3 - 529
*I*d^4)/((-I*a^4*c^7 + 7*a^4*c^6*d + 21*I*a^4*c^5*d^2 - 35*a^4*c^4*d^3 - 35
*I*a^4*c^3*d^4 + 21*a^4*c^2*d^5 + 7*I*a^4*c*d^6 - a^4*d^7)*f^2))*log(1/8*(2
*I*c^3 - 12*c^2*d - 33*I*c*d^2 + 23*d^3 + ((a^2*c^4 + 4*I*a^2*c^3*d - 6*a^2
*c^2*d^2 - 4*I*a^2*c*d^3 + a^2*d^4)*f*e^(2*I*f*x + 2*I*e) + (a^2*c^4 + 4*I*
a^2*c^3*d - 6*a^2*c^2*d^2 - 4*I*a^2*c*d^3 + a^2*d^4)*f)*sqrt(((c - I*d)*e^(
2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(-4*I*c^4 + 40
*c^3*d + 192*I*c^2*d^2 - 460*c*d^3 - 529*I*d^4)/((-I*a^4*c^7 + 7*a^4*c^6*d
+ 21*I*a^4*c^5*d^2 - 35*a^4*c^4*d^3 - 35*I*a^4*c^3*d^4 + 21*a^4*c^2*d^5 + 7
*I*a^4*c*d^6 - a^4*d^7)*f^2)) + (2*I*c^3 - 10*c^2*d - 23*I*c*d^2)*e^(2*I*f*
x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/((a^2*c^4 + 4*I*a^2*c^3*d - 6*a^2*c^2*d^2
- 4*I*a^2*c*d^3 + a^2*d^4)*f)) + ((-I*a^2*c^5 + a^2*c^4*d - 2*I*a^2*c^3*d^2
 + 2*a^2*c^2*d^3 - I*a^2*c*d^4 + a^2*d^5)*f*e^(6*I*f*x + 6*I*e) + (-I*a^2*c
```

```
^5 + 3*a^2*c^4*d + 2*I*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + 3*I*a^2*c*d^4 - a^2*d^
5)*f*e^(4*I*f*x + 4*I*e))*sqrt(-(-4*I*c^4 + 40*c^3*d + 192*I*c^2*d^2 - 460*
c*d^3 - 529*I*d^4)/((-I*a^4*c^7 + 7*a^4*c^6*d + 21*I*a^4*c^5*d^2 - 35*a^4*c
^4*d^3 - 35*I*a^4*c^3*d^4 + 21*a^4*c^2*d^5 + 7*I*a^4*c*d^6 - a^4*d^7)*f^2))
*log(1/8*(2*I*c^3 - 12*c^2*d - 33*I*c*d^2 + 23*d^3 - ((a^2*c^4 + 4*I*a^2*c^
3*d - 6*a^2*c^2*d^2 - 4*I*a^2*c*d^3 + a^2*d^4)*f*e^(2*I*f*x + 2*I*e) + (a^2
*c^4 + 4*I*a^2*c^3*d - 6*a^2*c^2*d^2 - 4*I*a^2*c*d^3 + a^2*d^4)*f)*sqrt(((c
 - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(-4
*I*c^4 + 40*c^3*d + 192*I*c^2*d^2 - 460*c*d^3 - 529*I*d^4)/((-I*a^4*c^7 + 7
*a^4*c^6*d + 21*I*a^4*c^5*d^2 - 35*a^4*c^4*d^3 - 35*I*a^4*c^3*d^4 + 21*a^4*
c^2*d^5 + 7*I*a^4*c*d^6 - a^4*d^7)*f^2)) + (2*I*c^3 - 10*c^2*d - 23*I*c*d^2
)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/((a^2*c^4 + 4*I*a^2*c^3*d - 6*a
^2*c^2*d^2 - 4*I*a^2*c*d^3 + a^2*d^4)*f)) - 2*(c^3 + I*c^2*d + c*d^2 + I*d^
3 + (3*c^3 + 4*I*c^2*d + 17*c*d^2 - 42*I*d^3)*e^(6*I*f*x + 6*I*e) + (7*c^3
+ 13*I*c^2*d + 21*c*d^2 - 33*I*d^3)*e^(4*I*f*x + 4*I*e) + 5*(c^3 + 2*I*c^2*
d + c*d^2 + 2*I*d^3)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*
e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1)))/((I*a^2*c^5 - a^2*c^4*d + 2*I*a^2
*c^3*d^2 - 2*a^2*c^2*d^3 + I*a^2*c*d^4 - a^2*d^5)*f*e^(6*I*f*x + 6*I*e) + (
I*a^2*c^5 - 3*a^2*c^4*d - 2*I*a^2*c^3*d^2 - 2*a^2*c^2*d^3 - 3*I*a^2*c*d^4 +
 a^2*d^5)*f*e^(4*I*f*x + 4*I*e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{c\sqrt{c+d\tan(e+fx)}\tan^2(e+fx)-2ic\sqrt{c+d\tan(e+fx)}\tan(e+fx)-c\sqrt{c+d\tan(e+fx)}+d\sqrt{c+d\tan(e+fx)}\tan^3(e+fx)-2id\sqrt{c+d\tan(e+fx)}\tan^2(e+fx)-d\sqrt{c+d\tan(e+fx)}\tan(e+fx)}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**2/(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] -Integral(1/(c*sqrt(c + d*tan(e + f*x))*tan(e + f*x)**2 - 2*I*c*sqrt(c + d*
tan(e + f*x))*tan(e + f*x) - c*sqrt(c + d*tan(e + f*x)) + d*sqrt(c + d*tan(
e + f*x))*tan(e + f*x)**3 - 2*I*d*sqrt(c + d*tan(e + f*x))*tan(e + f*x)**2
- d*sqrt(c + d*tan(e + f*x))*tan(e + f*x)), x)/a**2
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 588 vs. 2(230) = 460.
time = 0.94, size = 588, normalized size = 2.09

$$\frac{2d^3}{(a^2c^4f+2ia^2c^3df+2ia^2cd^3f-a^2d^4f)\sqrt{d\tan(fx+e)+c}}-\frac{(2ic^2-10cd-23id^2)\arctan\left(\frac{2\left(\sqrt{d\tan(fx+e)+c}\,c-\sqrt{c^2+d^2}\sqrt{d\tan(fx+e)+c}\right)}{c\sqrt{-2c+2\sqrt{c^2+d^2}}+i\sqrt{-2c+2\sqrt{c^2+d^2}}\,d-\sqrt{c^2+d^2}\sqrt{-2c+2\sqrt{c^2+d^2}}}\right)}{4(a^2c^3f+3ia^2c^2df-3a^2cd^2f-ia^2d^3f)\sqrt{-2c+2\sqrt{c^2+d^2}}\left(\frac{id}{c-\sqrt{c^2+d^2}}+1\right)}-\frac{2\arctan\left(\frac{2\left(\sqrt{d\tan(fx+e)+c}\,c-\sqrt{c^2+d^2}\sqrt{d\tan(fx+e)+c}\right)}{c\sqrt{-2c+2\sqrt{c^2+d^2}}-i\sqrt{-2c+2\sqrt{c^2+d^2}}\,d-\sqrt{c^2+d^2}\sqrt{-2c+2\sqrt{c^2+d^2}}}\right)}{-4(-ia^2cf-a^2df)\sqrt{-2c+2\sqrt{c^2+d^2}}\left(-\frac{id}{c-\sqrt{c^2+d^2}}+1\right)}+\frac{2(d\tan(fx+e)+c)^{\frac{3}{2}}cd-2\sqrt{d\tan(fx+e)+c}\,c^2d+9i(d\tan(fx+e)+c)^{\frac{3}{2}}d^2-13i\sqrt{d\tan(fx+e)+c}\,cd^2+11\sqrt{d\tan(fx+e)+c}\,d^3}{8(a^2c^3f+3ia^2c^2df-3a^2cd^2f-ia^2d^3f)(d\tan(fx+e)-id)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] 2*d^3/((a^2*c^4*f + 2*I*a^2*c^3*d*f + 2*I*a^2*c*d^3*f - a^2*d^4*f)*sqrt(d*t
an(f*x + e) + c)) - 1/4*(2*I*c^2 - 10*c*d - 23*I*d^2)*arctan(2*(sqrt(d*tan(
```

```
f*x + e) + c)*c - sqrt(c^2 + d^2)*sqrt(d*tan(f*x + e) + c))/(c*sqrt(-2*c +
2*sqrt(c^2 + d^2)) + I*sqrt(-2*c + 2*sqrt(c^2 + d^2))*d - sqrt(c^2 + d^2)*s
qrt(-2*c + 2*sqrt(c^2 + d^2))))/((a^2*c^3*f + 3*I*a^2*c^2*d*f - 3*a^2*c*d^2
*f - I*a^2*d^3*f)*sqrt(-2*c + 2*sqrt(c^2 + d^2))*(I*d/(c - sqrt(c^2 + d^2))
 + 1)) - 2*arctan(2*(sqrt(d*tan(f*x + e) + c)*c - sqrt(c^2 + d^2)*sqrt(d*ta
n(f*x + e) + c))/(c*sqrt(-2*c + 2*sqrt(c^2 + d^2)) - I*sqrt(-2*c + 2*sqrt(c
^2 + d^2))*d - sqrt(c^2 + d^2)*sqrt(-2*c + 2*sqrt(c^2 + d^2))))/((4*I*a^2*c
*f + 4*a^2*d*f)*sqrt(-2*c + 2*sqrt(c^2 + d^2))*(-I*d/(c - sqrt(c^2 + d^2))
+ 1)) + 1/8*(2*(d*tan(f*x + e) + c)^(3/2)*c*d - 2*sqrt(d*tan(f*x + e) + c)*
c^2*d + 9*I*(d*tan(f*x + e) + c)^(3/2)*d^2 - 13*I*sqrt(d*tan(f*x + e) + c)*
c*d^2 + 11*sqrt(d*tan(f*x + e) + c)*d^3)/((a^2*c^3*f + 3*I*a^2*c^2*d*f - 3*
a^2*c*d^2*f - I*a^2*d^3*f)*(d*tan(f*x + e) - I*d)^2)
```

Mupad [B]
time = 15.10, size = 2500, normalized size = 8.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^2*(c + d*tan(e + f*x))^(3/2)),x)
```

```
[Out] log(575*a^2*d^11*f - ((-(c*d^10*1155i + 525*d^11 - 315*c^2*d^9 + c^3*d^8*17
5i - 140*c^4*d^7 + c^5*d^6*168i + 56*c^6*d^5 - c^7*d^4*8i - a^4*c^8*f^2*(((
(525*d^15 - 8085*c^2*d^13 + 6195*c^4*d^11 + 609*c^6*d^9 + 228*c^8*d^7 + 24*
c^10*d^5)*1i)/(a^4*c^12*f^2 + a^4*d^12*f^2 + 6*a^4*c^2*d^10*f^2 + 15*a^4*c^
4*d^8*f^2 + 20*a^4*c^6*d^6*f^2 + 15*a^4*c^8*d^4*f^2 + 6*a^4*c^10*d^2*f^2) +
 (10115*c^3*d^12 - 3255*c*d^14 - 973*c^5*d^10 + 57*c^7*d^8 + 8*c^9*d^6 + 8*
c^11*d^4)/(a^4*c^12*f^2 + a^4*d^12*f^2 + 6*a^4*c^2*d^10*f^2 + 15*a^4*c^4*d^
8*f^2 + 20*a^4*c^6*d^6*f^2 + 15*a^4*c^8*d^4*f^2 + 6*a^4*c^10*d^2*f^2))^2 -
4*((((207*c*d^11)/8 - (21*c^3*d^9)/16 + (21*c^5*d^7)/16 + (3*c^7*d^5)/8)*1i
)/(a^8*c^12*f^4 + a^8*d^12*f^4 + 6*a^8*c^2*d^10*f^4 + 15*a^8*c^4*d^8*f^4 +
20*a^8*c^6*d^6*f^4 + 15*a^8*c^8*d^4*f^4 + 6*a^8*c^10*d^2*f^4) - ((763*c^2*d
^10)/32 - (529*d^12)/64 + (315*c^4*d^8)/64 + (7*c^6*d^6)/8 - (c^8*d^4)/16)/
(a^8*c^12*f^4 + a^8*d^12*f^4 + 6*a^8*c^2*d^10*f^4 + 15*a^8*c^4*d^8*f^4 + 20
*a^8*c^6*d^6*f^4 + 15*a^8*c^8*d^4*f^4 + 6*a^8*c^10*d^2*f^4))*(256*d^6 + 256
*c^2*d^4))^(1/2)*1i - a^4*d^8*f^2*((((525*d^15 - 8085*c^2*d^13 + 6195*c^4*d
^11 + 609*c^6*d^9 + 228*c^8*d^7 + 24*c^10*d^5)*1i)/(a^4*c^12*f^2 + a^4*d^12
*f^2 + 6*a^4*c^2*d^10*f^2 + 15*a^4*c^4*d^8*f^2 + 20*a^4*c^6*d^6*f^2 + 15*a^
4*c^8*d^4*f^2 + 6*a^4*c^10*d^2*f^2) + (10115*c^3*d^12 - 3255*c*d^14 - 973*c
^5*d^10 + 57*c^7*d^8 + 8*c^9*d^6 + 8*c^11*d^4)/(a^4*c^12*f^2 + a^4*d^12*f^2
 + 6*a^4*c^2*d^10*f^2 + 15*a^4*c^4*d^8*f^2 + 20*a^4*c^6*d^6*f^2 + 15*a^4*c^
8*d^4*f^2 + 6*a^4*c^10*d^2*f^2))^2 - 4*((((207*c*d^11)/8 - (21*c^3*d^9)/16
+ (21*c^5*d^7)/16 + (3*c^7*d^5)/8)*1i)/(a^8*c^12*f^4 + a^8*d^12*f^4 + 6*a^8
*c^2*d^10*f^4 + 15*a^8*c^4*d^8*f^4 + 20*a^8*c^6*d^6*f^4 + 15*a^8*c^8*d^4*f^
4 + 6*a^8*c^10*d^2*f^4) - ((763*c^2*d^10)/32 - (529*d^12)/64 + (315*c^4*d^8
```

)/64 + (7*c^6*d^6)/8 - (c^8*d^4)/16)/(a^8*c^12*f^4 + a^8*d^12*f^4 + 6*a^8*c
^2*d^10*f^4 + 15*a^8*c^4*d^8*f^4 + 20*a^8*c^6*d^6*f^4 + 15*a^8*c^8*d^4*f^4
+ 6*a^8*c^10*d^2*f^4))*(256*d^6 + 256*c^2*d^4))^(1/2)*1i + a^4*c^2*d^6*f^2*
((((525*d^15 - 8085*c^2*d^13 + 6195*c^4*d^11 + 609*c^6*d^9 + 228*c^8*d^7 +
24*c^10*d^5)*1i)/(a^4*c^12*f^2 + a^4*d^12*f^2 + 6*a^4*c^2*d^10*f^2 + 15*a^4
*c^4*d^8*f^2 + 20*a^4*c^6*d^6*f^2 + 15*a^4*c^8*d^4*f^2 + 6*a^4*c^10*d^2*f^2
) + (10115*c^3*d^12 - 3255*c*d^14 - 973*c^5*d^10 + 57*c^7*d^8 + 8*c^9*d^6 +
8*c^11*d^4)/(a^4*c^12*f^2 + a^4*d^12*f^2 + 6*a^4*c^2*d^10*f^2 + 15*a^4*c^4
*d^8*f^2 + 20*a^4*c^6*d^6*f^2 + 15*a^4*c^8*d^4*f^2 + 6*a^4*c^10*d^2*f^2))^2
- 4*((((207*c*d^11)/8 - (21*c^3*d^9)/16 + (21*c^5*d^7)/16 + (3*c^7*d^5)/8)
*1i)/(a^8*c^12*f^4 + a^8*d^12*f^4 + 6*a^8*c^2*d^10*f^4 + 15*a^8*c^4*d^8*f^4
+ 20*a^8*c^6*d^6*f^4 + 15*a^8*c^8*d^4*f^4 + 6*a^8*c^10*d^2*f^4) - ((763*c^
2*d^10)/32 - (529*d^12)/64 + (315*c^4*d^8)/64 + (7*c^6*d^6)/8 - (c^8*d^4)/1
6)/(a^8*c^12*f^4 + a^8*d^12*f^4 + 6*a^8*c^2*d^10*f^4 + 15*a^8*c^4*d^8*f^4 +
20*a^8*c^6*d^6*f^4 + 15*a^8*c^8*d^4*f^4 + 6*a^8*c^10*d^2*f^4))*(256*d^6 +
256*c^2*d^4))^(1/2)*4i - 4*a^4*c^3*d^5*f^2*((((525*d^15 - 8085*c^2*d^13 + 6
195*c^4*d^11 + 609*c^6*d^9 + 228*c^8*d^7 + 24*c^10*d^5)*1i)/(a^4*c^12*f^2 +
a^4*d^12*f^2 + 6*a^4*c^2*d^10*f^2 + 15*a^4*c^4*d^8*f^2 + 20*a^4*c^6*d^6*f^
2 + 15*a^4*c^8*d^4*f^2 + 6*a^4*c^10*d^2*f^2) + (10115*c^3*d^12 - 3255*c*d^1
4 - 973*c^5*d^10 + 57*c^7*d^8 + 8*c^9*d^6 + 8*c^11*d^4)/(a^4*c^12*f^2 + a^4
*d^12*f^2 + 6*a^4*c^2*d^10*f^2 + 15*a^4*c^4*d^8*f^2 + 20*a^4*c^6*d^6*f^2 +
15*a^4*c^8*d^4*f^2 + 6*a^4*c^10*d^2*f^2))^2 - 4*((((207*c*d^11)/8 - (21*c^3
*d^9)/16 + (21*c^5*d^7)/16 + (3*c^7*d^5)/8)*1i)/(a^8*c^12*f^4 + a^8*d^12*f^
4 + 6*a^8*c^2*d^10*f^4 + 15*a^8*c^4*d^8*f^4 + 20*a^8*c^6*d^6*f^4 + 15*a^8*c
^8*d^4*f^4 + 6*a^8*c^10*d^2*f^4) - ((763*c^2*d^10)/32 - (529*d^12)/64 + (31
5*c^4*d^8)/64 + (7*c^6*d^6)/8 - (c^8*d^4)/16)/(a^8*c^12*f^4 + a^8*d^12*f^4
+ 6*a^8*c^2*d^10*f^4 + 15*a^8*c^4*d^8*f^4 + 20*a^8*c^6*d^6*f^4 + 15*a^8*c^8
*d^4*f^4 + 6*a^8*c^10*d^2*f^4))*(256*d^6 + 256*c^2*d^4))^(1/2) + a^4*c^4*d^
4*f^2*((((525*d^15 - 8085*c^2*d^13 + 6195*c^4*d^11 + 609*c^6*d^9 + 228*c^8*
d^7 + 24*c^10*d^5)*1i)/(a^4*c^12*f^2 + a^4*d^12*f^2 + 6*a^4*c^2*d^10*f^2 +
15*a^4*c^4*d^8*f^2 + 20*a^4*c^6*d^6*f^2 + 15*a^4*c^8*d^4*f^2 + 6*a^4*c^10*d
^2*f^2) + (10115*c^3*d^12 - 3255*c*d^14 - 973*c^5*d^10 + 57*c^7*d^8 + 8*c^9
*d^6 + 8*c^11*d^4)/(a^4*c^12*f^2 + a^4*d^12*f^2 + 6*a^4*c^2*d^10*f^2 + 15*a
^4*c^4*d^8*f^2 + 20*a^4*c^6*d^6*f^2 + 15*a^4*c^8*d^4*f^2 + 6*a^4*c^10*d^2*f
^2))^2 - 4*((((207*c*d^11)/8 - (21*c^3*d^9)/16 + (21*c^5*d^7)/16 + (3*c^7*d
^5)/8)*1i)/(a^8*c^12*f^4 + a^8*d^12*f^4 + 6*a^8*c^2*d^10*f^4 + 15*a^8*c^4*d
^8*f^4 + 20*a^8*c^6*d^6*f^4 + 15*a^8*c^8*d^4*f^4 + 6*a^8*c^10*d^2*f^4) - ((
763*c^2*d^10)/32 - (529*d^12)/64 + (315*c^4*d^8)/64 + (7*c^6*d^6)/8 - (c^8*
d^4)/16)/(a^8*c^12*f^4 + a^8*d^12*f^4 + 6*a^8*c^2*d^10*f^4 + 15*a^8*c^4*d^8
*f^4 + 20*a^8*c^6*d^6*f^4 + 15*a^8*c^8*d^4*f^4 + 6*a^8*c^10*d^2*f^4))*(256*
d^6 + 256*c^2*d^4))^(1/2)*10i + 4*a^4*c^5*d^3*f^2*((((525*d^15 - 8085*c^2*d
^13 + 6195*c^4*d^11 + 609*c^6*d^9 + 228*c^8*d^7 + 24*c^10*d^5)*1i)/(a^4*c^1
2*f^2 + a^4*d^12*f^2 + 6*a^4*c^2*d^10*f^2 + 15*...

3.1130 $\int \frac{1}{(a+ia\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=368

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{8a^3(c-id)^{3/2}f}+\frac{(2ic^3-12c^2d-33icd^2+58d^3)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{16a^3(c+id)^{9/2}f}+\frac{}{16a^3(}$$

```
[Out] -1/8*I*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/a^3/(c-I*d)^(3/2)/f+1/
16*(2*I*c^3-12*c^2*d-33*I*c*d^2+58*d^3)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I
*d)^(1/2))/a^3/(c+I*d)^(9/2)/f+1/16*d*(2*c^3+9*I*c^2*d-17*c*d^2+60*I*d^3)/a
^3/(c-I*d)/(c+I*d)^4/f/(c+d*tan(f*x+e))^(1/2)-1/6/(I*c-d)/f/(c+d*tan(f*x+e)
)^(1/2)/(a+I*a*tan(f*x+e))^3+1/24*(3*I*c-10*d)/a/(c+I*d)^2/f/(c+d*tan(f*x+e
))^(1/2)/(a+I*a*tan(f*x+e))^2+1/48*(6*c^2+27*I*c*d-56*d^2)/(I*c-d)^3/f/(c+d
*tan(f*x+e))^(1/2)/(a^3+I*a^3*tan(f*x+e))
```

Rubi [A]
time = 0.86, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3640, 3677, 3610, 3620, 3618, 65, 214}

$$\frac{6c^2+27icd-56d^2}{48f(-d+ic)^3\left(a^3+ia^3\tan(e+fx)\right)\sqrt{c+d\tan(e+fx)}}+\frac{d(2c^3+9ic^2d-17cd^2+60id^3)}{16a^3f(c-id)(c+id)^4\sqrt{c+d\tan(e+fx)}}+\frac{(2ic^3-12c^2d-33icd^2+58d^3)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{16a^3f(c+id)^{9/2}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{8a^3f(c-id)^{3/2}}+\frac{-10d+3ic}{24af(c+id)^2(a+ia\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}-\frac{1}{6f(-d+ic)(a+ia\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] ((-1/8*I)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/(a^3*(c - I*d)^(
3/2)*f) + (((2*I)*c^3 - 12*c^2*d - (33*I)*c*d^2 + 58*d^3)*ArcTanh[Sqrt[c +
d*Tan[e + f*x]]/Sqrt[c + I*d]])/(16*a^3*(c + I*d)^(9/2)*f) + (d*(2*c^3 + (9
*I)*c^2*d - 17*c*d^2 + (60*I)*d^3))/(16*a^3*(c - I*d)*(c + I*d)^4*f*Sqrt[c
+ d*Tan[e + f*x]]) - 1/(6*(I*c - d)*f*(a + I*a*Tan[e + f*x])^3*Sqrt[c + d*T
an[e + f*x]]) + ((3*I)*c - 10*d)/(24*a*(c + I*d)^2*f*(a + I*a*Tan[e + f*x])
^2*Sqrt[c + d*Tan[e + f*x]]) + (6*c^2 + (27*I)*c*d - 56*d^2)/(48*(I*c - d)^
3*f*(a^3 + I*a^3*Tan[e + f*x])*Sqrt[c + d*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
```

```
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{1}{6(ic-d)f(a+ia\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{1}{6(ic-d)f(a+ia\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{1}{6(ic-d)f(a+ia\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}}$$

$$= \frac{d(2c^3+9ic^2d-17cd^2+60id^3)}{16a^3(c-id)(c+id)^4f\sqrt{c+d\tan(e+fx)}} - \frac{}{6(ic-d)}$$

$$= \frac{d(2c^3+9ic^2d-17cd^2+60id^3)}{16a^3(c-id)(c+id)^4f\sqrt{c+d\tan(e+fx)}} - \frac{}{6(ic-d)}$$

$$= \frac{d(2c^3+9ic^2d-17cd^2+60id^3)}{16a^3(c-id)(c+id)^4f\sqrt{c+d\tan(e+fx)}} - \frac{}{6(ic-d)}$$

$$= \frac{d(2c^3+9ic^2d-17cd^2+60id^3)}{16a^3(c-id)(c+id)^4f\sqrt{c+d\tan(e+fx)}} - \frac{}{6(ic-d)}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{8a^3(c-id)^{3/2}f} + \frac{(2ic^3-12c^2d-3}{}$$

Mathematica [A]
time = 7.87, size = 468, normalized size = 1.27

$$\frac{\sec^3(e+fx)(\cos(fx)+i\sin(fx))^3\left(-\frac{2\left(\sqrt{-c+id}\,(-2ic^4+10c^3d+21ic^2d^2-25cd^3+58id^4)\,\text{ArcTan}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{-c-id}}\right)+2i(-c-id)^{9/2}\,\text{ArcTan}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{-c+id}}\right)\right)(\cos(3e)+i\sin(3e))}{(-c-id)^{9/2}(-c+id)^{3/2}}+\frac{\cos(e+fx)(i\cos(3fx)+\sin(3fx))\left((27c^4+90ic^3d-71c^2d^2+90icd^3-98d^4)\cos(e+fx)+(13c^4+36ic^3d-3c^2d^2+150icd^3+290d^4)\cos(3(e+fx))+i\left((9c^4+40ic^3d-57c^2d^2+40icd^3-66d^4)\sin(e+fx)+(9c^4+28ic^3d-3c^2d^2+142icd^3+294d^4)\sin(3(e+fx))\right)\right)\sqrt{c+d\tan(e+fx)}}{3(c-id)(c+id)^4(c\cos(e+fx)+d\sin(e+fx))}\right)}{32f(a+ia\tan(e+fx))^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] (Sec[e + f*x]^3*(Cos[f*x] + I*Sin[f*x])^3*((-2*(Sqrt[-c + I*d]*((-2*I)*c^4
+ 10*c^3*d + (21*I)*c^2*d^2 - 25*c*d^3 + (58*I)*d^4)*ArcTan[Sqrt[c + d*Tan[
e + f*x]]/Sqrt[-c - I*d]] + (2*I)*(-c - I*d)^(9/2)*ArcTan[Sqrt[c + d*Tan[e
```

```
+ f*x]]/Sqrt[-c + I*d]])*(Cos[3*e] + I*Sin[3*e]))/((-c - I*d)^(9/2)*(-c + I
*d)^(3/2)) + (Cos[e + f*x]*(I*Cos[3*f*x] + Sin[3*f*x])*((27*c^4 + (90*I)*c^
3*d - 71*c^2*d^2 + (90*I)*c*d^3 - 98*d^4)*Cos[e + f*x] + (13*c^4 + (36*I)*c
^3*d - 3*c^2*d^2 + (150*I)*c*d^3 + 290*d^4)*Cos[3*(e + f*x)] + I*((9*c^4 +
(40*I)*c^3*d - 57*c^2*d^2 + (40*I)*c*d^3 - 66*d^4)*Sin[e + f*x] + (9*c^4 +
(28*I)*c^3*d - 3*c^2*d^2 + (142*I)*c*d^3 + 294*d^4)*Sin[3*(e + f*x)]))*Sqrt
[c + d*Tan[e + f*x]])/(3*(c - I*d)*(c + I*d)^4*(c*Cos[e + f*x] + d*Sin[e +
f*x]))))/(32*f*(a + I*a*Tan[e + f*x])^3)
```

Maple [A]

time = 0.38, size = 596, normalized size = 1.62

method	result
derivativedivides	$2d^4\left(-\frac{i}{(ic+d)(ic-d)(id+c)^3\sqrt{c+d\tan(fx+e)}}+\frac{\left(-ic^4+6ic^2d^2-id^4+4c^3d-4c\,d^3\right)\arctan\left(\frac{\sqrt{c+d\tan(fx}}{\sqrt{id-c}}\right.}{16(id-c)^{\frac{3}{2}}(id+c)^4d^4}\right.$
default	$2d^4\left(-\frac{i}{(ic+d)(ic-d)(id+c)^3\sqrt{c+d\tan(fx+e)}}+\frac{\left(-ic^4+6ic^2d^2-id^4+4c^3d-4c\,d^3\right)\arctan\left(\frac{\sqrt{c+d\tan(fx}}{\sqrt{id-c}}\right.}{16(id-c)^{\frac{3}{2}}(id+c)^4d^4}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3*d^4*(-I/(I*c+d)/(I*c-d)/(c+I*d)^3/(c+d*tan(f*x+e))^(1/2)+1/16*(-I*c
^4+6*I*c^2*d^2-I*d^4+4*c^3*d-4*c*d^3)/(I*d-c)^(3/2)/(c+I*d)^4/d^4*arctan((c
+d*tan(f*x+e))^(1/2)/(I*d-c)^(1/2))-1/16*I/(I*d-c)/(c+I*d)^4/d^4*((1/2*d*(2
*I*c^6-50*I*c^4*d^2-24*I*c^2*d^4+28*I*d^6-15*c^5*d+52*c^3*d^3+67*c*d^5)/(3*
I*c^2*d-I*d^3+c^3-3*c*d^2)*(c+d*tan(f*x+e))^(5/2)-2/3*d*(-27*c^6*d+177*c^4*
d^3+155*c^2*d^5-49*d^7+3*I*c^7-109*I*c^5*d^2+53*I*c^3*d^4+165*I*c*d^6)/(3*I
*c^2*d-I*d^3+c^3-3*c*d^2)*(c+d*tan(f*x+e))^(3/2)+1/2*d*(2*I*c^8-102*I*c^6*d
^2+190*I*c^4*d^4+254*I*c^2*d^6-40*I*d^8-21*c^7*d+225*c^5*d^3+73*c^3*d^5-173
*c*d^7)/(3*I*c^2*d-I*d^3+c^3-3*c*d^2)*(c+d*tan(f*x+e))^(1/2))/(-d*tan(f*x+e
)+I*d)^3-1/2*(-57*c^5*d^2+90*c^3*d^4+149*c*d^6+16*I*c^6*d-120*I*c^4*d^3-78*
```

```
I*c^2*d^5+58*I*d^7+2*c^7)/(3*I*c^2*d-I*d^3+c^3-3*c*d^2)/(-I*d-c)^(1/2)*arct
an((c+d*tan(f*x+e))^(1/2)/(-I*d-c)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2674 vs. 2(307) = 614.
time = 4.60, size = 2674, normalized size = 7.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] -1/192*(48*((a^3*c^6 + 2*I*a^3*c^5*d + a^3*c^4*d^2 + 4*I*a^3*c^3*d^3 - a^3*
c^2*d^4 + 2*I*a^3*c*d^5 - a^3*d^6)*f*e^(8*I*f*x + 8*I*e) + (a^3*c^6 + 4*I*a
^3*c^5*d - 5*a^3*c^4*d^2 - 5*a^3*c^2*d^4 - 4*I*a^3*c*d^5 + a^3*d^6)*f*e^(6*
I*f*x + 6*I*e))*sqrt(-1/64*I/((I*a^6*c^3 + 3*a^6*c^2*d - 3*I*a^6*c*d^2 - a^
6*d^3)*f^2))*log(-2*(8*((I*a^3*c^2 + 2*a^3*c*d - I*a^3*d^2)*f*e^(2*I*f*x +
2*I*e) + (I*a^3*c^2 + 2*a^3*c*d - I*a^3*d^2)*f)*sqrt(((c - I*d)*e^(2*I*f*x
+ 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-1/64*I/((I*a^6*c^3 + 3
*a^6*c^2*d - 3*I*a^6*c*d^2 - a^6*d^3)*f^2)) - (c - I*d)*e^(2*I*f*x + 2*I*e)
 - c)*e^(-2*I*f*x - 2*I*e)) - 48*((a^3*c^6 + 2*I*a^3*c^5*d + a^3*c^4*d^2 +
4*I*a^3*c^3*d^3 - a^3*c^2*d^4 + 2*I*a^3*c*d^5 - a^3*d^6)*f*e^(8*I*f*x + 8*I
*e) + (a^3*c^6 + 4*I*a^3*c^5*d - 5*a^3*c^4*d^2 - 5*a^3*c^2*d^4 - 4*I*a^3*c*
d^5 + a^3*d^6)*f*e^(6*I*f*x + 6*I*e))*sqrt(-1/64*I/((I*a^6*c^3 + 3*a^6*c^2*
d - 3*I*a^6*c*d^2 - a^6*d^3)*f^2))*log(-2*(8*((-I*a^3*c^2 - 2*a^3*c*d + I*a
^3*d^2)*f*e^(2*I*f*x + 2*I*e) + (-I*a^3*c^2 - 2*a^3*c*d + I*a^3*d^2)*f)*sqr
t(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt
(-1/64*I/((I*a^6*c^3 + 3*a^6*c^2*d - 3*I*a^6*c*d^2 - a^6*d^3)*f^2)) - (c -
I*d)*e^(2*I*f*x + 2*I*e) - c)*e^(-2*I*f*x - 2*I*e)) - 3*((a^3*c^6 + 2*I*a^3
*c^5*d + a^3*c^4*d^2 + 4*I*a^3*c^3*d^3 - a^3*c^2*d^4 + 2*I*a^3*c*d^5 - a^3*
d^6)*f*e^(8*I*f*x + 8*I*e) + (a^3*c^6 + 4*I*a^3*c^5*d - 5*a^3*c^4*d^2 - 5*a
^3*c^2*d^4 - 4*I*a^3*c*d^5 + a^3*d^6)*f*e^(6*I*f*x + 6*I*e))*sqrt(-(4*I*c^6
 - 48*c^5*d - 276*I*c^4*d^2 + 1024*c^3*d^3 + 2481*I*c^2*d^4 - 3828*c*d^5 -
```

```
3364*I*d^6)/((I*a^6*c^9 - 9*a^6*c^8*d - 36*I*a^6*c^7*d^2 + 84*a^6*c^6*d^3 +
 126*I*a^6*c^5*d^4 - 126*a^6*c^4*d^5 - 84*I*a^6*c^3*d^6 + 36*a^6*c^2*d^7 +
9*I*a^6*c*d^8 - a^6*d^9)*f^2))*log(-1/16*(2*c^4 + 14*I*c^3*d - 45*c^2*d^2 -
 91*I*c*d^3 + 58*d^4 - ((I*a^3*c^5 - 5*a^3*c^4*d - 10*I*a^3*c^3*d^2 + 10*a^
3*c^2*d^3 + 5*I*a^3*c*d^4 - a^3*d^5)*f*e^(2*I*f*x + 2*I*e) + (I*a^3*c^5 - 5
*a^3*c^4*d - 10*I*a^3*c^3*d^2 + 10*a^3*c^2*d^3 + 5*I*a^3*c*d^4 - a^3*d^5)*f
)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))
*sqrt(-(4*I*c^6 - 48*c^5*d - 276*I*c^4*d^2 + 1024*c^3*d^3 + 2481*I*c^2*d^4
- 3828*c*d^5 - 3364*I*d^6)/((I*a^6*c^9 - 9*a^6*c^8*d - 36*I*a^6*c^7*d^2 + 8
4*a^6*c^6*d^3 + 126*I*a^6*c^5*d^4 - 126*a^6*c^4*d^5 - 84*I*a^6*c^3*d^6 + 36
*a^6*c^2*d^7 + 9*I*a^6*c*d^8 - a^6*d^9)*f^2)) + (2*c^4 + 12*I*c^3*d - 33*c^
2*d^2 - 58*I*c*d^3)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/((I*a^3*c^5 -
 5*a^3*c^4*d - 10*I*a^3*c^3*d^2 + 10*a^3*c^2*d^3 + 5*I*a^3*c*d^4 - a^3*d^5)
*f)) + 3*((a^3*c^6 + 2*I*a^3*c^5*d + a^3*c^4*d^2 + 4*I*a^3*c^3*d^3 - a^3*c^
2*d^4 + 2*I*a^3*c*d^5 - a^3*d^6)*f*e^(8*I*f*x + 8*I*e) + (a^3*c^6 + 4*I*a^3
*c^5*d - 5*a^3*c^4*d^2 - 5*a^3*c^2*d^4 - 4*I*a^3*c*d^5 + a^3*d^6)*f*e^(6*I*
f*x + 6*I*e))*sqrt(-(4*I*c^6 - 48*c^5*d - 276*I*c^4*d^2 + 1024*c^3*d^3 + 24
81*I*c^2*d^4 - 3828*c*d^5 - 3364*I*d^6)/((I*a^6*c^9 - 9*a^6*c^8*d - 36*I*a^
6*c^7*d^2 + 84*a^6*c^6*d^3 + 126*I*a^6*c^5*d^4 - 126*a^6*c^4*d^5 - 84*I*a^6
*c^3*d^6 + 36*a^6*c^2*d^7 + 9*I*a^6*c*d^8 - a^6*d^9)*f^2))*log(-1/16*(2*c^4
 + 14*I*c^3*d - 45*c^2*d^2 - 91*I*c*d^3 + 58*d^4 - ((-I*a^3*c^5 + 5*a^3*c^4
*d + 10*I*a^3*c^3*d^2 - 10*a^3*c^2*d^3 - 5*I*a^3*c*d^4 + a^3*d^5)*f*e^(2*I*
f*x + 2*I*e) + (-I*a^3*c^5 + 5*a^3*c^4*d + 10*I*a^3*c^3*d^2 - 10*a^3*c^2*d^
3 - 5*I*a^3*c*d^4 + a^3*d^5)*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I
*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(4*I*c^6 - 48*c^5*d - 276*I*c^4*d^2 +
1024*c^3*d^3 + 2481*I*c^2*d^4 - 3828*c*d^5 - 3364*I*d^6)/((I*a^6*c^9 - 9*a^
6*c^8*d - 36*I*a^6*c^7*d^2 + 84*a^6*c^6*d^3 + 126*I*a^6*c^5*d^4 - 126*a^6*c
^4*d^5 - 84*I*a^6*c^3*d^6 + 36*a^6*c^2*d^7 + 9*I*a^6*c*d^8 - a^6*d^9)*f^2))
 + (2*c^4 + 12*I*c^3*d - 33*c^2*d^2 - 58*I*c*d^3)*e^(2*I*f*x + 2*I*e))*e^(-
2*I*f*x - 2*I*e)/((I*a^3*c^5 - 5*a^3*c^4*d - 10*I*a^3*c^3*d^2 + 10*a^3*c^2*
d^3 + 5*I*a^3*c*d^4 - a^3*d^5)*f)) + 2*(-2*I*c^4 + 4*c^3*d + 4*c*d^3 + 2*I*
d^4 + (-11*I*c^4 + 32*c^3*d + 3*I*c^2*d^2 + 146*c*d^3 - 292*I*d^4)*e^(8*I*f
*x + 8*I*e) + (-29*I*c^4 + 97*c^3*d + 67*I*c^2*d^2 + 211*c*d^3 - 210*I*d^4)
*e^(6*I*f*x + 6*I*e) + (-27*I*c^4 + 90*c^3*d + 71*I*c^2*d^2 + 90*c*d^3 + 98
*I*d^4)*e^(4*I*f*x + 4*I*e) + (-11*I*c^4 + 29*c^3*d + 7*I*c^2*d^2 + 29*c*d^
3 + 18*I*d^4)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c
+ I*d)/(e^(2*I*f*x + 2*I*e) + 1)))/((a^3*c^6 + 2*I*a^3*c^5*d + a^3*c^4*d^2
+ 4*I*a^3*c^3*d^3 - a^3*c^2*d^4 + 2*I*a^3*c*d^5 - a^3*d^6)*f*e^(8*I*f*x + 8
*I*e) + (a^3*c^6 + 4*I*a^3*c^5*d - 5*a^3*c^4*d^2 - 5*a^3*c^2*d^4 - 4*I*a^3*
c*d^5 + a^3*d^6)*f*e^(6*I*f*x + 6*I*e))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int \frac{1}{c\sqrt{c+d\tan(e+fx)}\tan^3(e+fx)-3ic\sqrt{c+d\tan(e+fx)}\tan^2(e+fx)-3c\sqrt{c+d\tan(e+fx)}\tan(e+fx)+ic\sqrt{c+d\tan(e+fx)}+d\sqrt{c+d\tan(e+fx)}\tan^4(e+fx)-3id\sqrt{c+d\tan(e+fx)}\tan^3(e+fx)-3d\sqrt{c+d\tan(e+fx)}\tan^2(e+fx)+id\sqrt{c+d\tan(e+fx)}\tan(e+fx)}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**3/(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] I*Integral(1/(c*sqrt(c + d*tan(e + f*x))*tan(e + f*x)**3 - 3*I*c*sqrt(c + d
*tan(e + f*x))*tan(e + f*x)**2 - 3*c*sqrt(c + d*tan(e + f*x))*tan(e + f*x)
+ I*c*sqrt(c + d*tan(e + f*x)) + d*sqrt(c + d*tan(e + f*x))*tan(e + f*x)**4
 - 3*I*d*sqrt(c + d*tan(e + f*x))*tan(e + f*x)**3 - 3*d*sqrt(c + d*tan(e +
f*x))*tan(e + f*x)**2 + I*d*sqrt(c + d*tan(e + f*x))*tan(e + f*x)), x)/a**3
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 785 vs. $2(307) = 614$.
time = 1.40, size = 785, normalized size = 2.13

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] 2*d^4/((-I*a^3*c^5*f + 3*a^3*c^4*d*f + 2*I*a^3*c^3*d^2*f + 2*a^3*c^2*d^3*f
+ 3*I*a^3*c*d^4*f - a^3*d^5*f)*sqrt(d*tan(f*x + e) + c)) - 1/8*(2*I*c^3 - 1
2*c^2*d - 33*I*c*d^2 + 58*d^3)*arctan(2*(sqrt(d*tan(f*x + e) + c)*c - sqrt(
c^2 + d^2)*sqrt(d*tan(f*x + e) + c))/(c*sqrt(-2*c + 2*sqrt(c^2 + d^2)) + I*
sqrt(-2*c + 2*sqrt(c^2 + d^2))*d - sqrt(c^2 + d^2)*sqrt(-2*c + 2*sqrt(c^2 +
 d^2))))/((a^3*c^4*f + 4*I*a^3*c^3*d*f - 6*a^3*c^2*d^2*f - 4*I*a^3*c*d^3*f
+ a^3*d^4*f)*sqrt(-2*c + 2*sqrt(c^2 + d^2))*(I*d/(c - sqrt(c^2 + d^2)) + 1)
) + 1/4*I*arctan(2*(sqrt(d*tan(f*x + e) + c)*c - sqrt(c^2 + d^2)*sqrt(d*tan
(f*x + e) + c))/(c*sqrt(-2*c + 2*sqrt(c^2 + d^2)) - I*sqrt(-2*c + 2*sqrt(c^
2 + d^2))*d - sqrt(c^2 + d^2)*sqrt(-2*c + 2*sqrt(c^2 + d^2))))/((a^3*c*f -
I*a^3*d*f)*sqrt(-2*c + 2*sqrt(c^2 + d^2))*(-I*d/(c - sqrt(c^2 + d^2)) + 1))
 + 2*(6*(d*tan(f*x + e) + c)^(5/2)*c^2*d - 12*(d*tan(f*x + e) + c)^(3/2)*c^
3*d + 6*sqrt(d*tan(f*x + e) + c)*c^4*d + 33*I*(d*tan(f*x + e) + c)^(5/2)*c*
d^2 - 84*I*(d*tan(f*x + e) + c)^(3/2)*c^2*d^2 + 51*I*sqrt(d*tan(f*x + e) +
c)*c^3*d^2 - 84*(d*tan(f*x + e) + c)^(5/2)*d^3 + 268*(d*tan(f*x + e) + c)^(
3/2)*c*d^3 - 204*sqrt(d*tan(f*x + e) + c)*c^2*d^3 + 196*I*(d*tan(f*x + e) +
 c)^(3/2)*d^4 - 279*I*sqrt(d*tan(f*x + e) + c)*c*d^4 + 120*sqrt(d*tan(f*x +
 e) + c)*d^5)/((-96*I*a^3*c^4*f + 384*a^3*c^3*d*f + 576*I*a^3*c^2*d^2*f - 3
84*a^3*c*d^3*f - 96*I*a^3*d^4*f)*(-I*d*tan(f*x + e) - d)^3)
```

Mupad [B]
time = 13.88, size = 2500, normalized size = 6.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a*tan(e + f*x)*1i)^3*(c + d*tan(e + f*x))^(3/2)),x)

[Out] log(6960*a^3*d^15*f - ((-(c*d^12*6300i + 3360*d^13 - 945*c^2*d^11 + c^3*d^10*1365i + 315*c^4*d^9 + c^5*d^8*693i + 672*c^6*d^7 - c^7*d^6*288i - 72*c^8*d^5 + c^9*d^4*8i - a^6*c^10*f^2*(4*(((29973*c^2*d^14)/1024 - (841*d^16)/256 - (5247*c^4*d^12)/1024 + (987*c^6*d^10)/1024 + (423*c^8*d^8)/1024 + (3*c^10*d^6)/64 - (c^12*d^4)/256)/(a^12*c^16*f^4 + a^12*d^16*f^4 + 8*a^12*c^2*d^14*f^4 + 28*a^12*c^4*d^12*f^4 + 56*a^12*c^6*d^10*f^4 + 70*a^12*c^8*d^8*f^4 + 56*a^12*c^10*d^6*f^4 + 28*a^12*c^12*d^4*f^4 + 8*a^12*c^14*d^2*f^4) + (((11861*c^3*d^13)/512 - (4089*c*d^15)/256 + (171*c^5*d^11)/128 + (261*c^7*d^9)/512 + (c^9*d^7)/128 - (3*c^11*d^5)/128)*1i)/(a^12*c^16*f^4 + a^12*d^16*f^4 + 8*a^12*c^2*d^14*f^4 + 28*a^12*c^4*d^12*f^4 + 56*a^12*c^6*d^10*f^4 + 70*a^12*c^8*d^8*f^4 + 56*a^12*c^10*d^6*f^4 + 28*a^12*c^12*d^4*f^4 + 8*a^12*c^14*d^2*f^4))*(256*d^6 + 256*c^2*d^4) + (((840*d^19 - (89145*c^2*d^17)/4 + 45675*c^4*d^15 - (18123*c^6*d^13)/2 + 729*c^8*d^11 + (879*c^10*d^9)/4 + 34*c^12*d^7 + 6*c^14*d^5)*1i)/(a^6*c^16*f^2 + a^6*d^16*f^2 + 8*a^6*c^2*d^14*f^2 + 28*a^6*c^4*d^12*f^2 + 56*a^6*c^6*d^10*f^2 + 70*a^6*c^8*d^8*f^2 + 56*a^6*c^10*d^6*f^2 + 28*a^6*c^12*d^4*f^2 + 8*a^6*c^14*d^2*f^2) - (6615*c*d^18 - (166005*c^3*d^16)/4 + 28917*c^5*d^14 - (2223*c^7*d^12)/2 + 344*c^9*d^10 + (339*c^11*d^8)/4 - 6*c^13*d^6 - 2*c^15*d^4)/(a^6*c^16*f^2 + a^6*d^16*f^2 + 8*a^6*c^2*d^14*f^2 + 28*a^6*c^4*d^12*f^2 + 56*a^6*c^6*d^10*f^2 + 70*a^6*c^8*d^8*f^2 + 56*a^6*c^10*d^6*f^2 + 28*a^6*c^12*d^4*f^2 + 8*a^6*c^14*d^2*f^2))^2)^(1/2)*4i + a^6*d^10*f^2*(4*(((29973*c^2*d^14)/1024 - (841*d^16)/256 - (5247*c^4*d^12)/1024 + (987*c^6*d^10)/1024 + (423*c^8*d^8)/1024 + (3*c^10*d^6)/64 - (c^12*d^4)/256)/(a^12*c^16*f^4 + a^12*d^16*f^4 + 8*a^12*c^2*d^14*f^4 + 28*a^12*c^4*d^12*f^4 + 56*a^12*c^6*d^10*f^4 + 70*a^12*c^8*d^8*f^4 + 56*a^12*c^10*d^6*f^4 + 28*a^12*c^12*d^4*f^4 + 8*a^12*c^14*d^2*f^4) + (((11861*c^3*d^13)/512 - (4089*c*d^15)/256 + (171*c^5*d^11)/128 + (261*c^7*d^9)/512 + (c^9*d^7)/128 - (3*c^11*d^5)/128)*1i)/(a^12*c^16*f^4 + a^12*d^16*f^4 + 8*a^12*c^2*d^14*f^4 + 28*a^12*c^4*d^12*f^4 + 56*a^12*c^6*d^10*f^4 + 70*a^12*c^8*d^8*f^4 + 56*a^12*c^10*d^6*f^4 + 28*a^12*c^12*d^4*f^4 + 8*a^12*c^14*d^2*f^4))*(256*d^6 + 256*c^2*d^4) + (((840*d^19 - (89145*c^2*d^17)/4 + 45675*c^4*d^15 - (18123*c^6*d^13)/2 + 729*c^8*d^11 + (879*c^10*d^9)/4 + 34*c^12*d^7 + 6*c^14*d^5)*1i)/(a^6*c^16*f^2 + a^6*d^16*f^2 + 8*a^6*c^2*d^14*f^2 + 28*a^6*c^4*d^12*f^2 + 56*a^6*c^6*d^10*f^2 + 70*a^6*c^8*d^8*f^2 + 56*a^6*c^10*d^6*f^2 + 28*a^6*c^12*d^4*f^2 + 8*a^6*c^14*d^2*f^2) - (6615*c*d^18 - (166005*c^3*d^16)/4 + 28917*c^5*d^14 - (2223*c^7*d^12)/2 + 344*c^9*d^10 + (339*c^11*d^8)/4 - 6*c^13*d^6 - 2*c^15*d^4)/(a^6*c^16*f^2 + a^6*d^16*f^2 + 8*a^6*c^2*d^14*f^2 + 28*a^6*c^4*d^12*f^2 + 56*a^6*c^6*d^10*f^2 + 70*a^6*c^8*d^8*f^2 + 56*a^6*c^10*d^6*f^2 + 28*a^6*c^12*d^4*f^2 + 8*a^6*c^14*d^2*f^2))^2)^(1/2)*4i + 24*a^6*c*d^9*f^2*(4*(((29973*c^2*d^14)/1024 - (841*d^16)/256 - (5247*c^4*d^12)/1024 + (987*c^6*d^10)/1024 + (423*c^8*d^8)/1024 + (3*c^10*d^6)/64 - (c^12*d^4)/256)/(a^12*c^16*f^4 + a^12*d^16*f^4 + 8*a^12*c^2*d^14*f^4 + 28*a^12*c^4*d^12*f^4 + 56*a^12*c^6*d^10*f^4 + 70*a^12*c^8*d^8*f^4 + 56*a^12*c^10*d^6*f^4 + 28*a^12*c^12*d^4*f^4 + 8*a^12*c^14*d^2*f^4) + (((11861*c^3*d^13)/

512 - (4089*c*d^15)/256 + (171*c^5*d^11)/128 + (261*c^7*d^9)/512 + (c^9*d^7
)/128 - (3*c^11*d^5)/128)*1i)/(a^12*c^16*f^4 + a^12*d^16*f^4 + 8*a^12*c^2*d
^14*f^4 + 28*a^12*c^4*d^12*f^4 + 56*a^12*c^6*d^10*f^4 + 70*a^12*c^8*d^8*f^4
 + 56*a^12*c^10*d^6*f^4 + 28*a^12*c^12*d^4*f^4 + 8*a^12*c^14*d^2*f^4))*(256
*d^6 + 256*c^2*d^4) + (((840*d^19 - (89145*c^2*d^17)/4 + 45675*c^4*d^15 - (
18123*c^6*d^13)/2 + 729*c^8*d^11 + (879*c^10*d^9)/4 + 34*c^12*d^7 + 6*c^14*
d^5)*1i)/(a^6*c^16*f^2 + a^6*d^16*f^2 + 8*a^6*c^2*d^14*f^2 + 28*a^6*c^4*d^1
2*f^2 + 56*a^6*c^6*d^10*f^2 + 70*a^6*c^8*d^8*f^2 + 56*a^6*c^10*d^6*f^2 + 28
*a^6*c^12*d^4*f^2 + 8*a^6*c^14*d^2*f^2) - (6615*c*d^18 - (166005*c^3*d^16)/
4 + 28917*c^5*d^14 - (2223*c^7*d^12)/2 + 344*c^9*d^10 + (339*c^11*d^8)/4 -
6*c^13*d^6 - 2*c^15*d^4)/(a^6*c^16*f^2 + a^6*d^16*f^2 + 8*a^6*c^2*d^14*f^2
+ 28*a^6*c^4*d^12*f^2 + 56*a^6*c^6*d^10*f^2 + 70*a^6*c^8*d^8*f^2 + 56*a^6*c
^10*d^6*f^2 + 28*a^6*c^12*d^4*f^2 + 8*a^6*c^14*d^2*f^2))^2)^(1/2) + 24*a^6*
c^9*d*f^2*(4*(((29973*c^2*d^14)/1024 - (841*d^16)/256 - (5247*c^4*d^12)/102
4 + (987*c^6*d^10)/1024 + (423*c^8*d^8)/1024 + (3*c^10*d^6)/64 - (c^12*d^4)
/256)/(a^12*c^16*f^4 + a^12*d^16*f^4 + 8*a^12*c^2*d^14*f^4 + 28*a^12*c^4*d^
12*f^4 + 56*a^12*c^6*d^10*f^4 + 70*a^12*c^8*d^8*f^4 + 56*a^12*c^10*d^6*f^4
+ 28*a^12*c^12*d^4*f^4 + 8*a^12*c^14*d^2*f^4) + (((11861*c^3*d^13)/512 - (4
089*c*d^15)/256 + (171*c^5*d^11)/128 + (261*c^7*d^9)/512 + (c^9*d^7)/128 -
(3*c^11*d^5)/128)*1i)/(a^12*c^16*f^4 + a^12*d^16*f^4 + 8*a^12*c^2*d^14*f^4
+ 28*a^12*c^4*d^12*f^4 + 56*a^12*c^6*d^10*f^4 + 70*a^12*c^8*d^8*f^4 + 56*a^
12*c^10*d^6*f^4 + 28*a^12*c^12*d^4*f^4 + 8*a^12*c^14*d^2*f^4))*(256*d^6 + 2
56*c^2*d^4) + (((840*d^19 - (89145*c^2*d^17)/4 + 45675*c^4*d^15 - (18123*c^
6*d^13)/2 + 729*c^8*d^11 + (879*c^10*d^9)/4 + 3...

3.1131 $\int \frac{(a+ia\tan(e+fx))^3}{(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=158

$$-\frac{8ia^3\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{5/2}f}+\frac{2(c+id)\left(a^3+ia^3\tan(e+fx)\right)}{3(c-id)df(c+d\tan(e+fx))^{3/2}}+\frac{4a^3(ic-d)(c-4id)}{3(c-id)^2d^2f\sqrt{c+d\tan(e+fx)}}$$

```
[Out] -8*I*a^3*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(c-I*d)^(5/2)/f+4/3*
a^3*(I*c-d)*(c-4*I*d)/(c-I*d)^2/d^2/f/(c+d*tan(f*x+e))^(1/2)+2/3*(c+I*d)*(a
^3+I*a^3*tan(f*x+e))/(c-I*d)/d/f/(c+d*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.31, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3634, 3672, 3618, 65, 214}

$$\frac{4a^3(-d+ic)(c-4id)}{3d^2f(c-id)^2\sqrt{c+d\tan(e+fx)}}+\frac{2(c+id)\left(a^3+ia^3\tan(e+fx)\right)}{3df(c-id)(c+d\tan(e+fx))^{3/2}}-\frac{8ia^3\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(c-id)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^3/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-8*I)*a^3*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((c - I*d)^(5/
2)*f) + (2*(c + I*d)*(a^3 + I*a^3*Tan[e + f*x]))/(3*(c - I*d)*d*f*(c + d*Ta
n[e + f*x])^(3/2)) + (4*a^3*(I*c - d)*(c - (4*I)*d))/(3*(c - I*d)^2*d^2*f*S
qrt[c + d*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
```

```
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3634

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-a^2)*(b*c - a*d)*(a + b*Tan[e + f*x]
)^(m - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] + Di
st[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[b*(b*c*(m - 2) - a*d*(m - 2*n - 4)) + (a*b*c*(m - 2)
+ b^2*d*(n + 1) - a^2*d*(m + n - 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3672

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
 b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
 -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^3}{(c+d\tan(e+fx))^{5/2}}\,dx = \frac{2(c+id)\left(a^3+ia^3\tan(e+fx)\right)}{3(c-id)df(c+d\tan(e+fx))^{3/2}} - \frac{2\int \frac{(a+ia\tan(e+fx))\left(-a^2(c+4id)+a^2(ic+2d)\tan(e+\right.}{(c+d\tan(e+fx))^{3/2}}}{3d(ic+d)}$$

$$= \frac{2(c+id)\left(a^3+ia^3\tan(e+fx)\right)}{3(c-id)df(c+d\tan(e+fx))^{3/2}} + \frac{4a^3(ic-d)(c-4id)}{3(c-id)^2d^2f\sqrt{c+d\tan(e+fx)}} - \frac{2\int}{}$$

(24

$$= \frac{2(c+id)\left(a^3+ia^3\tan(e+fx)\right)}{3(c-id)df(c+d\tan(e+fx))^{3/2}} + \frac{4a^3(ic-d)(c-4id)}{3(c-id)^2d^2f\sqrt{c+d\tan(e+fx)}} + -$$

$$= \frac{2(c+id)\left(a^3+ia^3\tan(e+fx)\right)}{3(c-id)df(c+d\tan(e+fx))^{3/2}} + \frac{4a^3(ic-d)(c-4id)}{3(c-id)^2d^2f\sqrt{c+d\tan(e+fx)}} + \frac{(28}{}$$

$$= -\frac{8ia^3\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{5/2}f} + \frac{2(c+id)\left(a^3+ia^3\tan(e+fx)\right)}{3(c-id)df(c+d\tan(e+fx))^{3/2}} +$$

Mathematica [A]
time = 7.37, size = 232, normalized size = 1.47

$$\frac{a^3(\cos(e+fx)+i\sin(e+fx))^3\left(-\frac{8ie^{-3ie}\tanh^{-1}\left(\frac{\sqrt{c-\frac{id(-1+e^{2i(e+fx)})}{1+e^{2i(e+fx)}}}}{\sqrt{c-id}}\right)}{(c-id)^{5/2}}+\frac{2(c+id)\cos(e+fx)(i\cos(3e)+\sin(3e))((2c^2-9icd-d^2)\cos(e+fx)+3(c-3id)d\sin(e+fx))\sqrt{c+d\tan(e+fx)}}{3(c-id)^2d^2(c\cos(e+fx)+d\sin(e+fx))^2}\right)}{f(\cos(fx)+i\sin(fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (a^3*(Cos[e + f*x] + I*Sin[e + f*x])^3*(((-8*I)*ArcTanh[Sqrt[c - (I*d*(-1 +
 E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))]/Sqrt[c - I*d]])/((c - I*d
)^(5/2)*E^((3*I)*e)) + (2*(c + I*d)*Cos[e + f*x]*(I*Cos[3*e] + Sin[3*e])*((
2*c^2 - (9*I)*c*d - d^2)*Cos[e + f*x] + 3*(c - (3*I)*d)*d*Sin[e + f*x])*Sqr
t[c + d*Tan[e + f*x]])/(3*(c - I*d)^2*d^2*(c*Cos[e + f*x] + d*Sin[e + f*x])
^2)))/(f*(Cos[f*x] + I*Sin[f*x])^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 919 vs. 2(135) = 270.
time = 0.30, size = 920, normalized size = 5.82

method	result
derivativedivides	$\frac{2a^3\left(-\frac{4d^2\left(\frac{\left(ic^2\sqrt{c^2+d^2}-id^2\sqrt{c^2+d^2}+ic^3-3icd^2-2cd\sqrt{c^2+d^2}-3c^2d+d^3\right)\ln\left(d\tan(fx+e)+c+\sqrt{c+d}\right.}{2}}{}\right.}{}\right.}{}$

default	$2a^3\left(-\frac{4d^2\left(\frac{\left(ic^2\sqrt{c^2+d^2}-id^2\sqrt{c^2+d^2}+ic^3-3ic\,d^2-2cd\sqrt{c^2+d^2}-3c^2d+d^3\right)\ln\left(d\tan(fx+e)+c+\sqrt{c+d\tan\cdots}\right.}{2}\cdots\right)}{\cdots}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3/d^2*(-4*d^2/(c^2+d^2)^2*(1/2/(2*(c^2+d^2)^(1/2)+2*c)^(1/2)/(c^2+d^2
)^(1/2)*(1/2*(I*c^2*(c^2+d^2)^(1/2)-I*d^2*(c^2+d^2)^(1/2)+I*c^3-3*I*c*d^2-2
*c*d*(c^2+d^2)^(1/2)-3*c^2*d+d^3)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(I*(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)*c^3-3*I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^2-3*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*c^2*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^3-1/2*(I*c^2*(c^2+d^2)^(1/2)-
I*d^2*(c^2+d^2)^(1/2)+I*c^3-3*I*c*d^2-2*c*d*(c^2+d^2)^(1/2)-3*c^2*d+d^3)*(2
*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*t
an(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/
2)))+1/2/(2*(c^2+d^2)^(1/2)+2*c)^(1/2)/(c^2+d^2)^(1/2)*(1/2*(-I*c^2*(c^2+d^
2)^(1/2)+I*d^2*(c^2+d^2)^(1/2)-I*c^3+3*I*c*d^2+2*c*d*(c^2+d^2)^(1/2)+3*c^2*
d-d^3)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)+(c^2+d^2)^(1/2))+2*(I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3-3*I*(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*c*d^2-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2*d+(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*d^3+1/2*(-I*c^2*(c^2+d^2)^(1/2)+I*d^2*(c^2+d^2)^(1/2)-I*c^
3+3*I*c*d^2+2*c*d*(c^2+d^2)^(1/2)+3*c^2*d-d^3)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2
))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d
^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))))-1/(c^2+d^2)^2*(-I*c^
4-6*I*c^2*d^2+3*I*d^4+8*c*d^3)/(c+d*tan(f*x+e))^(1/2)-1/3*(I*c^3-3*I*c*d^2-
3*c^2*d+d^3)/(c^2+d^2)/(c+d*tan(f*x+e))^(3/2))
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^(5/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 983 vs. $2(130) = 260$.
time = 1.50, size = 983, normalized size = 6.22

$$\frac{3\sqrt{-\frac{64i\,a^{6}}{(i\,c^{5}+5\,c^{4}d-10i\,c^{3}d^{2}-10\,c^{2}d^{3}+5i\,cd^{4}+d^{5})f^{2}}}\left((c^{4}d^{2}-4i\,c^{3}d^{3}-6\,c^{2}d^{4}+4i\,cd^{5}+d^{6})fe^{(4i\,fx+4i\,e)}+2\,(c^{4}d^{2}-2i\,c^{3}d^{3}-2i\,cd^{5}-d^{6})fe^{(2i\,fx+2i\,e)}+(c^{4}d^{2}+2\,c^{2}d^{4}+d^{6})f\right)\log\left(\frac{\left(8\,a^{3}c+\sqrt{-\frac{64i\,a^{6}}{(i\,c^{5}+5\,c^{4}d-10i\,c^{3}d^{2}-10\,c^{2}d^{3}+5i\,cd^{4}+d^{5})f^{2}}}\left((i\,c^{3}+3\,c^{2}d-3i\,cd^{2}-d^{3})fe^{(2i\,fx+2i\,e)}+(i\,c^{3}+3\,c^{2}d-3i\,cd^{2}-d^{3})f\right)\sqrt{\frac{(c-i\,d)e^{(2i\,fx+2i\,e)}+c+i\,d}{e^{(2i\,fx+2i\,e)}+1}}+8\,(a^{3}c-i\,a^{3}d)e^{(2i\,fx+2i\,e)}\right)e^{(-2i\,fx-2i\,e)}}{4\,a^{3}}\right)-3\sqrt{-\frac{64i\,a^{6}}{(i\,c^{5}+5\,c^{4}d-10i\,c^{3}d^{2}-10\,c^{2}d^{3}+5i\,cd^{4}+d^{5})f^{2}}}\left((c^{4}d^{2}-4i\,c^{3}d^{3}-6\,c^{2}d^{4}+4i\,cd^{5}+d^{6})fe^{(4i\,fx+4i\,e)}+2\,(c^{4}d^{2}-2i\,c^{3}d^{3}-2i\,cd^{5}-d^{6})fe^{(2i\,fx+2i\,e)}+(c^{4}d^{2}+2\,c^{2}d^{4}+d^{6})f\right)\log\left(\frac{\left(8\,a^{3}c+\sqrt{-\frac{64i\,a^{6}}{(i\,c^{5}+5\,c^{4}d-10i\,c^{3}d^{2}-10\,c^{2}d^{3}+5i\,cd^{4}+d^{5})f^{2}}}\left((-i\,c^{3}-3\,c^{2}d+3i\,cd^{2}+d^{3})fe^{(2i\,fx+2i\,e)}+(-i\,c^{3}-3\,c^{2}d+3i\,cd^{2}+d^{3})f\right)\sqrt{\frac{(c-i\,d)e^{(2i\,fx+2i\,e)}+c+i\,d}{e^{(2i\,fx+2i\,e)}+1}}+8\,(a^{3}c-i\,a^{3}d)e^{(2i\,fx+2i\,e)}\right)e^{(-2i\,fx-2i\,e)}}{4\,a^{3}}\right)-16\left(-i\,a^{3}c^{3}-2\,a^{3}c^{2}d-7i\,a^{3}cd^{2}+4\,a^{3}d^{3}+(-i\,a^{3}c^{3}-5\,a^{3}c^{2}d-i\,a^{3}cd^{2}-5\,a^{3}d^{3})e^{(4i\,fx+4i\,e)}+(-2i\,a^{3}c^{3}-7\,a^{3}c^{2}d-8i\,a^{3}cd^{2}-a^{3}d^{3})e^{(2i\,fx+2i\,e)}\right)\sqrt{\frac{(c-i\,d)e^{(2i\,fx+2i\,e)}+c+i\,d}{e^{(2i\,fx+2i\,e)}+1}}}{12\left((c^{4}d^{2}-4i\,c^{3}d^{3}-6\,c^{2}d^{4}+4i\,cd^{5}+d^{6})fe^{(4i\,fx+4i\,e)}+2\,(c^{4}d^{2}-2i\,c^{3}d^{3}-2i\,cd^{5}-d^{6})fe^{(2i\,fx+2i\,e)}+(c^{4}d^{2}+2\,c^{2}d^{4}+d^{6})f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/12*(3*sqrt(-64*I*a^6/((I*c^5 + 5*c^4*d - 10*I*c^3*d^2 - 10*c^2*d^3 + 5*I*
c*d^4 + d^5)*f^2))*((c^4*d^2 - 4*I*c^3*d^3 - 6*c^2*d^4 + 4*I*c*d^5 + d^6)*f
*e^(4*I*f*x + 4*I*e) + 2*(c^4*d^2 - 2*I*c^3*d^3 - 2*I*c*d^5 - d^6)*f*e^(2*I
*f*x + 2*I*e) + (c^4*d^2 + 2*c^2*d^4 + d^6)*f)*log(1/4*(8*a^3*c + sqrt(-64*
I*a^6/((I*c^5 + 5*c^4*d - 10*I*c^3*d^2 - 10*c^2*d^3 + 5*I*c*d^4 + d^5)*f^2)
)*((I*c^3 + 3*c^2*d - 3*I*c*d^2 - d^3)*f*e^(2*I*f*x + 2*I*e) + (I*c^3 + 3*c
^2*d - 3*I*c*d^2 - d^3)*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(
e^(2*I*f*x + 2*I*e) + 1)) + 8*(a^3*c - I*a^3*d)*e^(2*I*f*x + 2*I*e))*e^(-2*
I*f*x - 2*I*e)/a^3) - 3*sqrt(-64*I*a^6/((I*c^5 + 5*c^4*d - 10*I*c^3*d^2 - 1
0*c^2*d^3 + 5*I*c*d^4 + d^5)*f^2))*((c^4*d^2 - 4*I*c^3*d^3 - 6*c^2*d^4 + 4*
I*c*d^5 + d^6)*f*e^(4*I*f*x + 4*I*e) + 2*(c^4*d^2 - 2*I*c^3*d^3 - 2*I*c*d^5
 - d^6)*f*e^(2*I*f*x + 2*I*e) + (c^4*d^2 + 2*c^2*d^4 + d^6)*f)*log(1/4*(8*a
^3*c + sqrt(-64*I*a^6/((I*c^5 + 5*c^4*d - 10*I*c^3*d^2 - 10*c^2*d^3 + 5*I*c
*d^4 + d^5)*f^2))*((-I*c^3 - 3*c^2*d + 3*I*c*d^2 + d^3)*f*e^(2*I*f*x + 2*I*
e) + (-I*c^3 - 3*c^2*d + 3*I*c*d^2 + d^3)*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2
*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1)) + 8*(a^3*c - I*a^3*d)*e^(2*I*f*
x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/a^3) - 16*(-I*a^3*c^3 - 2*a^3*c^2*d - 7*I*
a^3*c*d^2 + 4*a^3*d^3 + (-I*a^3*c^3 - 5*a^3*c^2*d - I*a^3*c*d^2 - 5*a^3*d^3
)*e^(4*I*f*x + 4*I*e) + (-2*I*a^3*c^3 - 7*a^3*c^2*d - 8*I*a^3*c*d^2 - a^3*d
^3)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^
(2*I*f*x + 2*I*e) + 1)))/((c^4*d^2 - 4*I*c^3*d^3 - 6*c^2*d^4 + 4*I*c*d^5 +
d^6)*f*e^(4*I*f*x + 4*I*e) + 2*(c^4*d^2 - 2*I*c^3*d^3 - 2*I*c*d^5 - d^6)*f*
e^(2*I*f*x + 2*I*e) + (c^4*d^2 + 2*c^2*d^4 + d^6)*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-ia^{3}\left(\int \frac{i}{c^{2}\sqrt{c+d\tan{(e+fx)}}+2cd\sqrt{c+d\tan{(e+fx)}}\tan{(e+fx)}+d^{2}\sqrt{c+d\tan{(e+fx)}}\tan^{2}{(e+fx)}}\,dx+\int\left(-\frac{3\tan{(e+fx)}}{c^{2}\sqrt{c+d\tan{(e+fx)}}+2cd\sqrt{c+d\tan{(e+fx)}}\tan{(e+fx)}+d^{2}\sqrt{c+d\tan{(e+fx)}}\tan^{2}{(e+fx)}}\right)dx+\int \frac{\tan^{3}{(e+fx)}}{c^{2}\sqrt{c+d\tan{(e+fx)}}+2cd\sqrt{c+d\tan{(e+fx)}}\tan{(e+fx)}+d^{2}\sqrt{c+d\tan{(e+fx)}}\tan^{2}{(e+fx)}}\,dx+\int\left(-\frac{3i\tan^{2}{(e+fx)}}{c^{2}\sqrt{c+d\tan{(e+fx)}}+2cd\sqrt{c+d\tan{(e+fx)}}\tan{(e+fx)}+d^{2}\sqrt{c+d\tan{(e+fx)}}\tan^{2}{(e+fx)}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3/(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] -I*a**3*(Integral(I/(c**2*sqrt(c + d*tan(e + f*x)) + 2*c*d*sqrt(c + d*tan(e
 + f*x))*tan(e + f*x) + d**2*sqrt(c + d*tan(e + f*x))*tan(e + f*x)**2), x)
+ Integral(-3*tan(e + f*x)/(c**2*sqrt(c + d*tan(e + f*x)) + 2*c*d*sqrt(c +
d*tan(e + f*x))*tan(e + f*x) + d**2*sqrt(c + d*tan(e + f*x))*tan(e + f*x)**
2), x) + Integral(tan(e + f*x)**3/(c**2*sqrt(c + d*tan(e + f*x)) + 2*c*d*sq
rt(c + d*tan(e + f*x))*tan(e + f*x) + d**2*sqrt(c + d*tan(e + f*x))*tan(e +
 f*x)**2), x) + Integral(-3*I*tan(e + f*x)**2/(c**2*sqrt(c + d*tan(e + f*x)
) + 2*c*d*sqrt(c + d*tan(e + f*x))*tan(e + f*x) + d**2*sqrt(c + d*tan(e + f
*x))*tan(e + f*x)**2), x))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(130) = 260$.
time = 1.02, size = 314, normalized size = 1.99

$$\frac{16\,a^{3}\arctan\left(\frac{2\left(\sqrt{d\tan\left(fx+e\right)+c}\,c-\sqrt{c^{2}+d^{2}}\,\sqrt{d\tan\left(fx+e\right)+c}\right)}{c\sqrt{-2\,c+2\,\sqrt{c^{2}+d^{2}}}-i\,\sqrt{-2\,c+2\,\sqrt{c^{2}+d^{2}}}\,d-\sqrt{c^{2}+d^{2}}\,\sqrt{-2\,c+2\,\sqrt{c^{2}+d^{2}}}}\right)}{\left(-i\,c^{2}f-2\,cdf+i\,d^{2}f\right)\sqrt{-2\,c+2\,\sqrt{c^{2}+d^{2}}}\left(-\frac{i\,d}{c-\sqrt{c^{2}+d^{2}}}+1\right)}-\frac{2\left(3\left(-i\,d\tan\left(fx+e\right)-i\,c\right)a^{3}c^{2}+i\,a^{3}c^{3}-6\left(d\tan\left(fx+e\right)+c\right)a^{3}cd-a^{3}c^{2}d+9\left(-i\,d\tan\left(fx+e\right)-i\,c\right)a^{3}d^{2}+i\,a^{3}cd^{2}-a^{3}d^{3}\right)}{3\left(c^{2}d^{2}f-2i\,cd^{3}f-d^{4}f\right)\left(d\tan\left(fx+e\right)+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 16*a^3*arctan(2*(sqrt(d*tan(f*x + e) + c)*c - sqrt(c^2 + d^2)*sqrt(d*tan(f*
x + e) + c))/(c*sqrt(-2*c + 2*sqrt(c^2 + d^2)) - I*sqrt(-2*c + 2*sqrt(c^2 +
 d^2))*d - sqrt(c^2 + d^2)*sqrt(-2*c + 2*sqrt(c^2 + d^2))))/((-I*c^2*f - 2*
c*d*f + I*d^2*f)*sqrt(-2*c + 2*sqrt(c^2 + d^2))*(-I*d/(c - sqrt(c^2 + d^2))
 + 1)) - 2/3*(3*(-I*d*tan(f*x + e) - I*c)*a^3*c^2 + I*a^3*c^3 - 6*(d*tan(f*
x + e) + c)*a^3*c*d - a^3*c^2*d + 9*(-I*d*tan(f*x + e) - I*c)*a^3*d^2 + I*a
^3*c*d^2 - a^3*d^3)/((c^2*d^2*f - 2*I*c*d^3*f - d^4*f)*(d*tan(f*x + e) + c)
^(3/2))
```

Mupad [B]
time = 9.15, size = 255, normalized size = 1.61

$$-\frac{\frac{\left(a^{3}\,c^{2}+a^{3}\,c\,d\,2{}\mathrm{i}-a^{3}\,d^{2}\right)\,2{}\mathrm{i}}{3\,d^{2}\,f\,\left(c-d\,1{}\mathrm{i}\right)}-\frac{a^{3}\,\left(c+d\,\tan\left(e+f\,x\right)\right)\,\left(c^{2}-c\,d\,2{}\mathrm{i}+3\,d^{2}\right)\,2{}\mathrm{i}}{d^{2}\,f\,{\left(c-d\,1{}\mathrm{i}\right)}^{2}}}{{\left(c+d\,\tan\left(e+f\,x\right)\right)}^{3/2}}+\frac{a^{3}\,\mathrm{atan}\left(\frac{\sqrt{c+d\,\tan\left(e+f\,x\right)}\,\left(2\,c^{8}\,f^{2}+8\,c^{6}\,d^{2}\,f^{2}+12\,c^{4}\,d^{4}\,f^{2}+8\,c^{2}\,d^{6}\,f^{2}+2\,d^{8}\,f^{2}\right)}{2\,f\,{\left(-c+d\,1{}\mathrm{i}\right)}^{5/2}\,\left(f\,c^{6}+2{}\mathrm{i}\,f\,c^{5}\,d+f\,c^{4}\,d^{2}+4{}\mathrm{i}\,f\,c^{3}\,d^{3}-f\,c^{2}\,d^{4}+2{}\mathrm{i}\,f\,c\,d^{5}-f\,d^{6}\right)}\right)\,8{}\mathrm{i}}{f\,{\left(-c+d\,1{}\mathrm{i}\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^3/(c + d*tan(e + f*x))^(5/2),x)
```

```
[Out] (a^3*atan(((c + d*tan(e + f*x))^(1/2)*(2*c^8*f^2 + 2*d^8*f^2 + 8*c^2*d^6*f^
2 + 12*c^4*d^4*f^2 + 8*c^6*d^2*f^2))/(2*f*(d*1i - c)^(5/2)*(c^6*f - d^6*f -
 c^2*d^4*f + c^3*d^3*f*4i + c^4*d^2*f + c*d^5*f*2i + c^5*d*f*2i)))*8i)/(f*(
d*1i - c)^(5/2)) - (((a^3*c^2 - a^3*d^2 + a^3*c*d*2i)*2i)/(3*d^2*f*(c - d*1
i)) - (a^3*(c + d*tan(e + f*x))*(c^2 - c*d*2i + 3*d^2)*2i)/(d^2*f*(c - d*1i
)^2))/(c + d*tan(e + f*x))^(3/2)
```

3.1132 $\int \frac{(a+ia\tan(e+fx))^2}{(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{4ia^2\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{5/2}f}+\frac{2a^2(ic-d)}{3d(ic+d)f(c+d\tan(e+fx))^{3/2}}+\frac{4ia^2}{(c-id)^2f\sqrt{c+d\tan(e+fx)}}$$

```
[Out] -4*I*a^2*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(c-I*d)^(5/2)/f+4*I*
a^2/(c-I*d)^2/f/(c+d*tan(f*x+e))^(1/2)+2/3*a^2*(I*c-d)/d/(I*c+d)/f/(c+d*tan
(f*x+e))^(3/2)
```

Rubi [A]
time = 0.24, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3623, 3610, 3618, 65, 214}

$$\frac{4ia^2}{f(c-id)^2\sqrt{c+d\tan(e+fx)}}+\frac{2a^2(-d+ic)}{3df(d+ic)(c+d\tan(e+fx))^{3/2}}-\frac{4ia^2\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(c-id)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-4*I)*a^2*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((c - I*d)^(5/
2)*f) + (2*a^2*(I*c - d))/(3*d*(I*c + d)*f*(c + d*Tan[e + f*x])^(3/2)) + ((
4*I)*a^2)/((c - I*d)^2*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
```

```
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^2}{(c+d\tan(e+fx))^{5/2}}\,dx = \frac{2a^2(ic-d)}{3d(ic+d)f(c+d\tan(e+fx))^{3/2}} + \frac{\int \frac{2a^2(c+id)+2a^2(ic-d)\tan(e+fx)}{(c+d\tan(e+fx))^{3/2}}\,dx}{c^2+d^2}$$

$$= \frac{2a^2(ic-d)}{3d(ic+d)f(c+d\tan(e+fx))^{3/2}} + \frac{4ia^2}{(c-id)^2f\sqrt{c+d\tan(e+fx)}} + \frac{\int \frac{2a^2(\cdots}{\cdots}}{\cdots}$$

$$(4ia^4(\cdots$$

$$= \frac{2a^2(ic-d)}{3d(ic+d)f(c+d\tan(e+fx))^{3/2}} + \frac{4ia^2}{(c-id)^2f\sqrt{c+d\tan(e+fx)}} + \frac{\cdots}{\cdots}$$

$$= \frac{2a^2(ic-d)}{3d(ic+d)f(c+d\tan(e+fx))^{3/2}} + \frac{4ia^2}{(c-id)^2f\sqrt{c+d\tan(e+fx)}} - \frac{(16a^6\cdots}{\cdots}$$

$$= -\frac{4ia^2\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{5/2}f} + \frac{2a^2(ic-d)}{3d(ic+d)f(c+d\tan(e+fx))^{3/2}} \cdots$$

Mathematica [A]

time = 5.14, size = 218, normalized size = 1.72

$$\frac{a^2(\cos(e+fx)+i\sin(e+fx))^2\left(-\frac{4ie^{-2ie}\tanh^{-1}\left(\frac{\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}}{\sqrt{c-id}}\right)}{(c-id)^{5/2}}+\frac{2\cos(e+fx)(\cos(2e)-i\sin(2e))\left((c^2+6icd+d^2)\cos(e+fx)+6id^2\sin(e+fx)\right)\sqrt{c+d\tan(e+fx)}}{3(c-id)^2d(c\cos(e+fx)+d\sin(e+fx))^2}\right)}{f(\cos(fx)+i\sin(fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (a^2*(Cos[e + f*x] + I*Sin[e + f*x])^2*(((-4*I)*ArcTanh[Sqrt[c - (I*d*(-1 +
 E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))]/Sqrt[c - I*d]])/((c - I*d
)^(5/2)*E^((2*I)*e)) + (2*Cos[e + f*x]*(Cos[2*e] - I*Sin[2*e])*((c^2 + (6*I
)*c*d + d^2)*Cos[e + f*x] + (6*I)*d^2*Sin[e + f*x])*Sqrt[c + d*Tan[e + f*x]
])/(3*(c - I*d)^2*d*(c*Cos[e + f*x] + d*Sin[e + f*x])^2)))/(f*(Cos[f*x] + I
*Sin[f*x])^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 898 vs. 2(108) = 216.
time = 0.29, size = 899, normalized size = 7.08

method	result
derivativedivides	$2a^2\left(-\frac{-2icd-c^2+d^2}{3\left(c^2+d^2\right)(c+d\tan(fx+e))^{\frac{3}{2}}}+\frac{2d\left(ic^2-id^2-2cd\right)}{\left(c^2+d^2\right)^2\sqrt{c+d\tan\left(fx+e\right)}}-\frac{2d\left(\frac{\left(ic^2\sqrt{c^2+d^2}-id^2\sqrt{c^2+d^2}+\right.}{}\right.}{}\right.$

default	$2a^2\left(-\frac{-2icd-c^2+d^2}{3\left(c^2+d^2\right)\left(c+d\tan\left(fx+e\right)\right)^{\frac{3}{2}}}+\frac{2d\left(ic^2-id^2-2cd\right)}{\left(c^2+d^2\right)^2\sqrt{c+d\tan\left(fx+e\right)}}-\frac{2d\left(\frac{\left(ic^2\sqrt{c^2+d^2}-id^2\sqrt{c^2+d^2}+ic\cdots\right.}{\cdots}\right.}{\cdots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^2/d*(-1/3*(-2*I*c*d-c^2+d^2)/(c^2+d^2)/(c+d*tan(f*x+e))^(3/2)+2*d*(I*
c^2-I*d^2-2*c*d)/(c^2+d^2)^2/(c+d*tan(f*x+e))^(1/2)-2*d/(c^2+d^2)^2*(1/2/(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)/(c^2+d^2)^(1/2)*(1/2*(I*c^2*(c^2+d^2)^(1/2)-I*d
^2*(c^2+d^2)^(1/2)+I*c^3-3*I*c*d^2-2*c*d*(c^2+d^2)^(1/2)-3*c^2*d+d^3)*ln(d*
tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)
^(1/2))+2*(I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3-3*I*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*c*d^2-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2*d+(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*d^3-1/2*(I*c^2*(c^2+d^2)^(1/2)-I*d^2*(c^2+d^2)^(1/2)+I*c^3-3*I*c*d^2-2
*c*d*(c^2+d^2)^(1/2)-3*c^2*d+d^3)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^
2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c
)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/2/(2*(c^2+d^2)^(1/2)+2*c)^(1/2)/
(c^2+d^2)^(1/2)*(1/2*(-I*c^2*(c^2+d^2)^(1/2)+I*d^2*(c^2+d^2)^(1/2)-I*c^3+3*
I*c*d^2+2*c*d*(c^2+d^2)^(1/2)+3*c^2*d-d^3)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e
))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(I*(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)*c^3-3*I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^2-3*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*c^2*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^3+1/2*(-I*c^2*(c^2+d
^2)^(1/2)+I*d^2*(c^2+d^2)^(1/2)-I*c^3+3*I*c*d^2+2*c*d*(c^2+d^2)^(1/2)+3*c^2
*d-d^3)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan
((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2
)-2*c)^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^2/(d*tan(f*x + e) + c)^(5/2), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 929 vs. $2(104) = 208$.
time = 1.04, size = 929, normalized size = 7.31

$$\frac{3\left(\left(c^{4}d-4ic^{3}d^{2}-6c^{2}d^{3}+4icd^{4}+d^{5}\right)fe^{(4ifx+4ie)}+2\left(c^{4}d-2ic^{3}d^{2}-2icd^{4}-d^{5}\right)fe^{(2ifx+2ie)}+\left(c^{4}d+2c^{2}d^{3}+d^{5}\right)f\right)\sqrt{-\frac{16ia^{4}}{(ic^{5}+5c^{4}d-10ic^{3}d^{2}-10c^{2}d^{3}+5icd^{4}+d^{5})f^{2}}}\log\left(\frac{\left(4a^{2}c+\left((ic^{3}+3c^{2}d-3icd^{2}-d^{3})fe^{(2ifx+2ie)}+(ic^{3}+3c^{2}d-3icd^{2}-d^{3})f\right)\sqrt{-\frac{16ia^{4}}{(ic^{5}+5c^{4}d-10ic^{3}d^{2}-10c^{2}d^{3}+5icd^{4}+d^{5})f^{2}}}\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}+4(a^{2}c-ia^{2}d)e^{(2ifx+2ie)}\right)e^{(-2ifx-2ie)}}{2a^{2}}\right)-3\left(\left(c^{4}d-4ic^{3}d^{2}-6c^{2}d^{3}+4icd^{4}+d^{5}\right)fe^{(4ifx+4ie)}+2\left(c^{4}d-2ic^{3}d^{2}-2icd^{4}-d^{5}\right)fe^{(2ifx+2ie)}+\left(c^{4}d+2c^{2}d^{3}+d^{5}\right)f\right)\sqrt{-\frac{16ia^{4}}{(ic^{5}+5c^{4}d-10ic^{3}d^{2}-10c^{2}d^{3}+5icd^{4}+d^{5})f^{2}}}\log\left(\frac{\left(4a^{2}c+\left((-ic^{3}-3c^{2}d+3icd^{2}+d^{3})fe^{(2ifx+2ie)}+(-ic^{3}-3c^{2}d+3icd^{2}+d^{3})f\right)\sqrt{-\frac{16ia^{4}}{(ic^{5}+5c^{4}d-10ic^{3}d^{2}-10c^{2}d^{3}+5icd^{4}+d^{5})f^{2}}}\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}+4(a^{2}c-ia^{2}d)e^{(2ifx+2ie)}\right)e^{(-2ifx-2ie)}}{2a^{2}}\right)+8\left(a^{2}c^{2}+6ia^{2}cd-5a^{2}d^{2}+(a^{2}c^{2}+6ia^{2}cd+7a^{2}d^{2})e^{(4ifx+4ie)}+2(a^{2}c^{2}+6ia^{2}cd+a^{2}d^{2})e^{(2ifx+2ie)}\right)\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}}{12\left(\left(c^{4}d-4ic^{3}d^{2}-6c^{2}d^{3}+4icd^{4}+d^{5}\right)fe^{(4ifx+4ie)}+2\left(c^{4}d-2ic^{3}d^{2}-2icd^{4}-d^{5}\right)fe^{(2ifx+2ie)}+\left(c^{4}d+2c^{2}d^{3}+d^{5}\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/12*(3*((c^4*d - 4*I*c^3*d^2 - 6*c^2*d^3 + 4*I*c*d^4 + d^5)*f*e^(4*I*f*x +
 4*I*e) + 2*(c^4*d - 2*I*c^3*d^2 - 2*I*c*d^4 - d^5)*f*e^(2*I*f*x + 2*I*e) +
 (c^4*d + 2*c^2*d^3 + d^5)*f)*sqrt(-16*I*a^4/((I*c^5 + 5*c^4*d - 10*I*c^3*d
^2 - 10*c^2*d^3 + 5*I*c*d^4 + d^5)*f^2))*log(1/2*(4*a^2*c + ((I*c^3 + 3*c^2
*d - 3*I*c*d^2 - d^3)*f*e^(2*I*f*x + 2*I*e) + (I*c^3 + 3*c^2*d - 3*I*c*d^2
- d^3)*f)*sqrt(-16*I*a^4/((I*c^5 + 5*c^4*d - 10*I*c^3*d^2 - 10*c^2*d^3 + 5*
I*c*d^4 + d^5)*f^2))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I
*f*x + 2*I*e) + 1)) + 4*(a^2*c - I*a^2*d)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x
- 2*I*e)/a^2) - 3*((c^4*d - 4*I*c^3*d^2 - 6*c^2*d^3 + 4*I*c*d^4 + d^5)*f*e^
(4*I*f*x + 4*I*e) + 2*(c^4*d - 2*I*c^3*d^2 - 2*I*c*d^4 - d^5)*f*e^(2*I*f*x
+ 2*I*e) + (c^4*d + 2*c^2*d^3 + d^5)*f)*sqrt(-16*I*a^4/((I*c^5 + 5*c^4*d -
10*I*c^3*d^2 - 10*c^2*d^3 + 5*I*c*d^4 + d^5)*f^2))*log(1/2*(4*a^2*c + ((-I*
c^3 - 3*c^2*d + 3*I*c*d^2 + d^3)*f*e^(2*I*f*x + 2*I*e) + (-I*c^3 - 3*c^2*d
+ 3*I*c*d^2 + d^3)*f)*sqrt(-16*I*a^4/((I*c^5 + 5*c^4*d - 10*I*c^3*d^2 - 10*
c^2*d^3 + 5*I*c*d^4 + d^5)*f^2))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c +
I*d)/(e^(2*I*f*x + 2*I*e) + 1)) + 4*(a^2*c - I*a^2*d)*e^(2*I*f*x + 2*I*e))*
e^(-2*I*f*x - 2*I*e)/a^2) + 8*(a^2*c^2 + 6*I*a^2*c*d - 5*a^2*d^2 + (a^2*c^2
 + 6*I*a^2*c*d + 7*a^2*d^2)*e^(4*I*f*x + 4*I*e) + 2*(a^2*c^2 + 6*I*a^2*c*d
+ a^2*d^2)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I
*d)/(e^(2*I*f*x + 2*I*e) + 1)))/((c^4*d - 4*I*c^3*d^2 - 6*c^2*d^3 + 4*I*c*d
^4 + d^5)*f*e^(4*I*f*x + 4*I*e) + 2*(c^4*d - 2*I*c^3*d^2 - 2*I*c*d^4 - d^5)
*f*e^(2*I*f*x + 2*I*e) + (c^4*d + 2*c^2*d^3 + d^5)*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^{2}\left(\int \frac{\tan^{2}(e+fx)}{c^{2}\sqrt{c+d\tan(e+fx)}+2cd\sqrt{c+d\tan(e+fx)}\tan(e+fx)+d^{2}\sqrt{c+d\tan(e+fx)}\tan^{2}(e+fx)}\,dx+\int\left(-\frac{2i\tan(e+fx)}{c^{2}\sqrt{c+d\tan(e+fx)}+2cd\sqrt{c+d\tan(e+fx)}\tan(e+fx)+d^{2}\sqrt{c+d\tan(e+fx)}\tan^{2}(e+fx)}\right)dx+\int\left(-\frac{1}{c^{2}\sqrt{c+d\tan(e+fx)}+2cd\sqrt{c+d\tan(e+fx)}\tan(e+fx)+d^{2}\sqrt{c+d\tan(e+fx)}\tan^{2}(e+fx)}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2/(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] -a**2*(Integral(tan(e + f*x)**2/(c**2*sqrt(c + d*tan(e + f*x)) + 2*c*d*sqrt
(c + d*tan(e + f*x))*tan(e + f*x) + d**2*sqrt(c + d*tan(e + f*x))*tan(e + f
*x)**2), x) + Integral(-2*I*tan(e + f*x)/(c**2*sqrt(c + d*tan(e + f*x)) + 2
*c*d*sqrt(c + d*tan(e + f*x))*tan(e + f*x) + d**2*sqrt(c + d*tan(e + f*x))*
tan(e + f*x)**2), x) + Integral(-1/(c**2*sqrt(c + d*tan(e + f*x)) + 2*c*d*s
qrt(c + d*tan(e + f*x))*tan(e + f*x) + d**2*sqrt(c + d*tan(e + f*x))*tan(e
+ f*x)**2), x))
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(104) = 208$.
time = 0.90, size = 250, normalized size = 1.97

$$\frac{8\,a^2\arctan\left(\frac{2\left(\sqrt{d\tan\left(fx+e\right)+c}\,c-\sqrt{c^2+d^2}\,\sqrt{d\tan\left(fx+e\right)+c}\right)}{c\sqrt{-2\,c+2\,\sqrt{c^2+d^2}}-i\sqrt{-2\,c+2\,\sqrt{c^2+d^2}}\,d-\sqrt{c^2+d^2}\,\sqrt{-2\,c+2\,\sqrt{c^2+d^2}}}\right)}{\left(-i\,c^2f-2\,cdf+i\,d^2f\right)\sqrt{-2\,c+2\,\sqrt{c^2+d^2}}\left(-\frac{i\,d}{c-\sqrt{c^2+d^2}}+1\right)}-\frac{2\left(-i\,a^2c^2+6\left(d\tan\left(fx+e\right)+c\right)a^2d-i\,a^2d^2\right)}{-3\left(-i\,c^2df-2\,cd^2f+i\,d^3f\right)\left(d\tan\left(fx+e\right)+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 8*a^2*arctan(2*(sqrt(d*tan(f*x + e) + c)*c - sqrt(c^2 + d^2)*sqrt(d*tan(f*x
 + e) + c))/(c*sqrt(-2*c + 2*sqrt(c^2 + d^2)) - I*sqrt(-2*c + 2*sqrt(c^2 +
d^2))*d - sqrt(c^2 + d^2)*sqrt(-2*c + 2*sqrt(c^2 + d^2))))/((-I*c^2*f - 2*c
*d*f + I*d^2*f)*sqrt(-2*c + 2*sqrt(c^2 + d^2))*(-I*d/(c - sqrt(c^2 + d^2))
+ 1)) - 2*(-I*a^2*c^2 + 6*(d*tan(f*x + e) + c)*a^2*d - I*a^2*d^2)/((3*I*c^2
*d*f + 6*c*d^2*f - 3*I*d^3*f)*(d*tan(f*x + e) + c)^(3/2))
```

Mupad [**B**]
time = 8.90, size = 221, normalized size = 1.74

$$\frac{\frac{a^2\,\left(c+d\tan\left(e+f\,x\right)\right)\,4{}\mathrm{i}}{f\,{\left(c-d\,1{}\mathrm{i}\right)}^2}+\frac{2\,a^2\,\left(c+d\,1{}\mathrm{i}\right)}{3\,d\,f\,\left(c-d\,1{}\mathrm{i}\right)}}{{\left(c+d\tan\left(e+f\,x\right)\right)}^{3/2}}+\frac{a^2\,\mathrm{atan}\left(\frac{\sqrt{c+d\tan\left(e+f\,x\right)}\,\left(2\,c^8\,f^2+8\,c^6\,d^2\,f^2+12\,c^4\,d^4\,f^2+8\,c^2\,d^6\,f^2+2\,d^8\,f^2\right)}{2\,f\,{\left(-c+d\,1{}\mathrm{i}\right)}^{5/2}\,\left(f\,c^6+2{}\mathrm{i}\,f\,c^5\,d+f\,c^4\,d^2+4{}\mathrm{i}\,f\,c^3\,d^3-f\,c^2\,d^4+2{}\mathrm{i}\,f\,c\,d^5-f\,d^6\right)}\right)\,4{}\mathrm{i}}{f\,{\left(-c+d\,1{}\mathrm{i}\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^2/(c + d*tan(e + f*x))^(5/2),x)
```

```
[Out] ((a^2*(c + d*tan(e + f*x))*4i)/(f*(c - d*1i)^2) + (2*a^2*(c + d*1i))/(3*d*f
*(c - d*1i)))/(c + d*tan(e + f*x))^(3/2) + (a^2*atan(((c + d*tan(e + f*x))^
(1/2)*(2*c^8*f^2 + 2*d^8*f^2 + 8*c^2*d^6*f^2 + 12*c^4*d^4*f^2 + 8*c^6*d^2*f
^2))/(2*f*(d*1i - c)^(5/2)*(c^6*f - d^6*f - c^2*d^4*f + c^3*d^3*f*4i + c^4*
d^2*f + c*d^5*f*2i + c^5*d*f*2i)))*4i)/(f*(d*1i - c)^(5/2))
```

3.1133 $\int \frac{a+ia\tan(e+fx)}{(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=109

$$-\frac{2ia\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{5/2}f}-\frac{2a}{3(ic+d)f(c+d\tan(e+fx))^{3/2}}+\frac{2ia}{(c-id)^2f\sqrt{c+d\tan(e+fx)}}$$

```
[Out] -2*I*a*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(c-I*d)^(5/2)/f+2*I*a/
(c-I*d)^2/f/(c+d*tan(f*x+e))^(1/2)-2/3*a/(I*c+d)/f/(c+d*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.19, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3610, 3618, 65, 214}

$$\frac{2ia}{f(c-id)^2\sqrt{c+d\tan(e+fx)}}-\frac{2a}{3f(d+ic)(c+d\tan(e+fx))^{3/2}}-\frac{2ia\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(c-id)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-2*I)*a*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((c - I*d)^(5/2)
*f) - (2*a)/(3*(I*c + d)*f*(c + d*Tan[e + f*x])^(3/2)) + ((2*I)*a)/((c - I*
d)^2*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
```

```
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{a+ia\tan(e+fx)}{(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{2a}{3(ic+d)f(c+d\tan(e+fx))^{3/2}} + \frac{\int \frac{a(c+id)+a(ic-d)\tan(e+fx)}{(c+d\tan(e+fx))^{3/2}}\,dx}{c^2+d^2}$$

$$= -\frac{2a}{3(ic+d)f(c+d\tan(e+fx))^{3/2}} + \frac{2ia}{(c-id)^2 f\sqrt{c+d\tan(e+fx)}} + \frac{\int \frac{a(c\cdots}{\sqrt{\cdots}}}{\cdots}$$

$$(ia^2(\cdots$$

$$= -\frac{2a}{3(ic+d)f(c+d\tan(e+fx))^{3/2}} + \frac{2ia}{(c-id)^2 f\sqrt{c+d\tan(e+fx)}} + \cdots$$

$$= -\frac{2a}{3(ic+d)f(c+d\tan(e+fx))^{3/2}} + \frac{2ia}{(c-id)^2 f\sqrt{c+d\tan(e+fx)}} - \frac{(2a^3(\cdots}{\cdots}$$

$$= -\frac{2ia\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{5/2}f} - \frac{2a}{3(ic+d)f(c+d\tan(e+fx))^{3/2}} + \cdots$$

Mathematica [A]
time = 4.03, size = 198, normalized size = 1.82

$$\frac{\cos(e+fx)(\cos(fx)-i\sin(fx))(a+ia\tan(e+fx))\left(-\frac{2ie^{-ie}\tanh^{-1}\left(\frac{\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}}{\sqrt{c-id}}\right)}{(c-id)^{5/2}} + \frac{2\cos(e+fx)(\cos(e)-i\sin(e))((4ic+d)\cos(e+fx)+3id\sin(e+fx))\sqrt{c+d\tan(e+fx)}}{3(c-id)^2(c\cos(e+fx)+d\sin(e+fx))^2}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (Cos[e + f*x]*(Cos[f*x] - I*Sin[f*x])*(a + I*a*Tan[e + f*x])*(((-2*I)*ArcTa
nh[Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))]]/Sqr
t[c - I*d]])/((c - I*d)^(5/2)*E^(I*e)) + (2*Cos[e + f*x]*(Cos[e] - I*Sin[e]
)*(((4*I)*c + d)*Cos[e + f*x] + (3*I)*d*Sin[e + f*x])*Sqrt[c + d*Tan[e + f*
x]])/(3*(c - I*d)^2*(c*Cos[e + f*x] + d*Sin[e + f*x])^2)))/f
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 884 vs. $2(91) = 182$.
time = 0.32, size = 885, normalized size = 8.12

method	result
derivativedivides	$a\left(-\frac{2(-ic+d)}{3\left(c^2+d^2\right)(c+d\tan(fx+e))^{\frac{3}{2}}}-\frac{2\left(-ic^2+id^2+2cd\right)}{\left(c^2+d^2\right)^2\sqrt{c+d\tan\left(fx+e\right)}}+\frac{\left(ic^2\sqrt{c^2+d^2}-id^2\sqrt{c^2+d^2}+ic^3-3\right.}{}\right.$
default	$a\left(-\frac{2(-ic+d)}{3\left(c^2+d^2\right)(c+d\tan(fx+e))^{\frac{3}{2}}}-\frac{2\left(-ic^2+id^2+2cd\right)}{\left(c^2+d^2\right)^2\sqrt{c+d\tan\left(fx+e\right)}}+\frac{\left(ic^2\sqrt{c^2+d^2}-id^2\sqrt{c^2+d^2}+ic^3-3\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a*(-2/3*(d-I*c)/(c^2+d^2)/(c+d*tan(f*x+e))^(3/2)-2/(c^2+d^2)^2*(-I*c^2+
I*d^2+2*c*d)/(c+d*tan(f*x+e))^(1/2)+2/(c^2+d^2)^2*(1/2/(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)/(c^2+d^2)^(1/2)*(1/2*(I*c^2*(c^2+d^2)^(1/2)-I*d^2*(c^2+d^2)^(1/2)
+I*c^3-3*I*c*d^2-2*c*d*(c^2+d^2)^(1/2)-3*c^2*d+d^3)*ln(d*tan(f*x+e)+c-(c+d*
tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(-I*(2*(
c^2+d^2)^(1/2)+2*c)^(1/2)*c^3+3*I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^2+3*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^3+1/2*(I*c
```

```
^2*(c^2+d^2)^(1/2)-I*d^2*(c^2+d^2)^(1/2)+I*c^3-3*I*c*d^2-2*c*d*(c^2+d^2)^(1
/2)-3*c^2*d+d^3)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/
2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+
d^2)^(1/2)-2*c)^(1/2)))+1/2/(2*(c^2+d^2)^(1/2)+2*c)^(1/2)/(c^2+d^2)^(1/2)*(
1/2*(-I*c^2*(c^2+d^2)^(1/2)+I*d^2*(c^2+d^2)^(1/2)-I*c^3+3*I*c*d^2+2*c*d*(c^
2+d^2)^(1/2)+3*c^2*d-d^3)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(-I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*
c^3+3*I*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^2+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*c^2*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^3-1/2*(-I*c^2*(c^2+d^2)^(1/2)+I*d^2*
(c^2+d^2)^(1/2)-I*c^3+3*I*c*d^2+2*c*d*(c^2+d^2)^(1/2)+3*c^2*d-d^3)*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x
+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((-(2*c*d^4)/((c^2-d^2)^2>0)', see
`assume
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 843 vs. 2(88) = 176.
time = 1.02, size = 843, normalized size = 7.73

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*((c^4 - 4*I*c^3*d - 6*c^2*d^2 + 4*I*c*d^3 + d^4)*f*e^(4*I*f*x + 4*I
*e) + 2*(c^4 - 2*I*c^3*d - 2*I*c*d^3 - d^4)*f*e^(2*I*f*x + 2*I*e) + (c^4 +
2*c^2*d^2 + d^4)*f)*sqrt(-4*I*a^2/((I*c^5 + 5*c^4*d - 10*I*c^3*d^2 - 10*c^2
*d^3 + 5*I*c*d^4 + d^5)*f^2))*log((2*a*c + ((I*c^3 + 3*c^2*d - 3*I*c*d^2 -
d^3)*f*e^(2*I*f*x + 2*I*e) + (I*c^3 + 3*c^2*d - 3*I*c*d^2 - d^3)*f)*sqrt(((
c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-4*
I*a^2/((I*c^5 + 5*c^4*d - 10*I*c^3*d^2 - 10*c^2*d^3 + 5*I*c*d^4 + d^5)*f^2)
) + 2*(a*c - I*a*d)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/a) - 3*((c^4
- 4*I*c^3*d - 6*c^2*d^2 + 4*I*c*d^3 + d^4)*f*e^(4*I*f*x + 4*I*e) + 2*(c^4 -
 2*I*c^3*d - 2*I*c*d^3 - d^4)*f*e^(2*I*f*x + 2*I*e) + (c^4 + 2*c^2*d^2 + d^
4)*f)*sqrt(-4*I*a^2/((I*c^5 + 5*c^4*d - 10*I*c^3*d^2 - 10*c^2*d^3 + 5*I*c*d
```

```
^4 + d^5)*f^2))*log((2*a*c + ((-I*c^3 - 3*c^2*d + 3*I*c*d^2 + d^3)*f*e^(2*I
*f*x + 2*I*e) + (-I*c^3 - 3*c^2*d + 3*I*c*d^2 + d^3)*f)*sqrt(((c - I*d)*e^(
2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-4*I*a^2/((I*c^
5 + 5*c^4*d - 10*I*c^3*d^2 - 10*c^2*d^3 + 5*I*c*d^4 + d^5)*f^2)) + 2*(a*c -
 I*a*d)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/a) - 16*(-2*I*a*c + a*d +
 2*(-I*a*c - a*d)*e^(4*I*f*x + 4*I*e) + (-4*I*a*c - a*d)*e^(2*I*f*x + 2*I*e
))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1)
))/((c^4 - 4*I*c^3*d - 6*c^2*d^2 + 4*I*c*d^3 + d^4)*f*e^(4*I*f*x + 4*I*e) +
 2*(c^4 - 2*I*c^3*d - 2*I*c*d^3 - d^4)*f*e^(2*I*f*x + 2*I*e) + (c^4 + 2*c^2
*d^2 + d^4)*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-\frac{i}{c^2\sqrt{c+d\tan(e+fx)}+2cd\sqrt{c+d\tan(e+fx)}\tan(e+fx)+d^2\sqrt{c+d\tan(e+fx)}\tan^2(e+fx)}\right)dx+\int\frac{\tan(e+fx)}{c^2\sqrt{c+d\tan(e+fx)}+2cd\sqrt{c+d\tan(e+fx)}\tan(e+fx)+d^2\sqrt{c+d\tan(e+fx)}\tan^2(e+fx)}dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] I*a*(Integral(-I/(c**2*sqrt(c + d*tan(e + f*x)) + 2*c*d*sqrt(c + d*tan(e +
f*x))*tan(e + f*x) + d**2*sqrt(c + d*tan(e + f*x))*tan(e + f*x)**2), x) + I
ntegral(tan(e + f*x)/(c**2*sqrt(c + d*tan(e + f*x)) + 2*c*d*sqrt(c + d*tan(
e + f*x))*tan(e + f*x) + d**2*sqrt(c + d*tan(e + f*x))*tan(e + f*x)**2), x)
)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(88) = 176.
time = 0.81, size = 228, normalized size = 2.09

$$-2a\left(\frac{3d\tan(fx+e)+4c-id}{-3(-ic^2f-2cdf+id^2f)(d\tan(fx+e)+c)^{\frac{3}{2}}}-\frac{2i\arctan\left(\frac{2\left(\sqrt{d\tan(fx+e)+c}\,c-\sqrt{c^2+d^2}\sqrt{d\tan(fx+e)+c}\right)}{c\sqrt{-2c+2\sqrt{c^2+d^2}}-i\sqrt{-2c+2\sqrt{c^2+d^2}}\,d-\sqrt{c^2+d^2}\sqrt{-2c+2\sqrt{c^2+d^2}}}\right)}{(c^2f-2icdf-d^2f)\sqrt{-2c+2\sqrt{c^2+d^2}}\left(-\frac{id}{c-\sqrt{c^2+d^2}}+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] -2*a*((3*d*tan(f*x + e) + 4*c - I*d)/((3*I*c^2*f + 6*c*d*f - 3*I*d^2*f)*(d*
tan(f*x + e) + c)^(3/2)) - 2*I*arctan(2*(sqrt(d*tan(f*x + e) + c)*c - sqrt(
c^2 + d^2)*sqrt(d*tan(f*x + e) + c))/(c*sqrt(-2*c + 2*sqrt(c^2 + d^2)) - I*
sqrt(-2*c + 2*sqrt(c^2 + d^2))*d - sqrt(c^2 + d^2)*sqrt(-2*c + 2*sqrt(c^2 +
 d^2))))/((c^2*f - 2*I*c*d*f - d^2*f)*sqrt(-2*c + 2*sqrt(c^2 + d^2))*(-I*d/
(c - sqrt(c^2 + d^2)) + 1)))
```

Mupad [B]
time = 25.07, size = 2500, normalized size = 22.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*tan(e + f*x)*1i)/(c + d*tan(e + f*x))^(5/2),x)

[Out] ((a*c*2i)/(3*f*(c^2 + d^2)) + (a*(c^2 - d^2)*(c + d*tan(e + f*x))*2i)/(f*(c^2 + d^2)^2))/(c + d*tan(e + f*x))^(3/2) + (log(((((320*a^4*c^2*d^8*f^4 - 16*a^4*d^10*f^4 - 1760*a^4*c^4*d^6*f^4 + 1600*a^4*c^6*d^4*f^4 - 400*a^4*c^8*d^2*f^4)^(1/2) - 4*a^2*c^5*f^2 - 20*a^2*c*d^4*f^2 + 40*a^2*c^3*d^2*f^2)/(c^10*f^4 + d^10*f^4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*c^6*d^4*f^4 + 5*c^8*d^2*f^4))^(1/2)*((c + d*tan(e + f*x))^(1/2)*(320*a^2*c^4*d^14*f^3 - 16*a^2*d^18*f^3 + 1024*a^2*c^6*d^12*f^3 + 1440*a^2*c^8*d^10*f^3 + 1024*a^2*c^10*d^8*f^3 + 320*a^2*c^12*d^6*f^3 - 16*a^2*c^16*d^2*f^3) - ((((320*a^4*c^2*d^8*f^4 - 16*a^4*d^10*f^4 - 1760*a^4*c^4*d^6*f^4 + 1600*a^4*c^6*d^4*f^4 - 400*a^4*c^8*d^2*f^4)^(1/2) - 4*a^2*c^5*f^2 - 20*a^2*c*d^4*f^2 + 40*a^2*c^3*d^2*f^2)/(c^10*f^4 + d^10*f^4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*c^6*d^4*f^4 + 5*c^8*d^2*f^4))^(1/2)*(((((320*a^4*c^2*d^8*f^4 - 16*a^4*d^10*f^4 - 1760*a^4*c^4*d^6*f^4 + 1600*a^4*c^6*d^4*f^4 - 400*a^4*c^8*d^2*f^4)^(1/2) - 4*a^2*c^5*f^2 - 20*a^2*c*d^4*f^2 + 40*a^2*c^3*d^2*f^2)/(c^10*f^4 + d^10*f^4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*c^6*d^4*f^4 + 5*c^8*d^2*f^4))^(1/2)*(c + d*tan(e + f*x))^(1/2)*(64*c*d^22*f^5 + 640*c^3*d^20*f^5 + 2880*c^5*d^18*f^5 + 7680*c^7*d^16*f^5 + 13440*c^9*d^14*f^5 + 16128*c^11*d^12*f^5 + 13440*c^13*d^10*f^5 + 7680*c^15*d^8*f^5 + 2880*c^17*d^6*f^5 + 640*c^19*d^4*f^5 + 64*c^21*d^2*f^5))/4 - 32*a*d^21*f^4 - 160*a*c^2*d^19*f^4 - 128*a*c^4*d^17*f^4 + 896*a*c^6*d^15*f^4 + 3136*a*c^8*d^13*f^4 + 4928*a*c^10*d^11*f^4 + 4480*a*c^12*d^9*f^4 + 2432*a*c^14*d^7*f^4 + 736*a*c^16*d^5*f^4 + 96*a*c^18*d^3*f^4))/4))/4 + 16*a^3*c*d^15*f^2 + 96*a^3*c^3*d^13*f^2 + 240*a^3*c^5*d^11*f^2 + 320*a^3*c^7*d^9*f^2 + 240*a^3*c^9*d^7*f^2 + 96*a^3*c^11*d^5*f^2 + 16*a^3*c^13*d^3*f^2)*(((320*a^4*c^2*d^8*f^4 - 16*a^4*d^10*f^4 - 1760*a^4*c^4*d^6*f^4 + 1600*a^4*c^6*d^4*f^4 - 400*a^4*c^8*d^2*f^4)^(1/2) - 4*a^2*c^5*f^2 - 20*a^2*c*d^4*f^2 + 40*a^2*c^3*d^2*f^2)/(c^10*f^4 + d^10*f^4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*c^6*d^4*f^4 + 5*c^8*d^2*f^4))^(1/2))/4 + (log(((-((320*a^4*c^2*d^8*f^4 - 16*a^4*d^10*f^4 - 1760*a^4*c^4*d^6*f^4 + 1600*a^4*c^6*d^4*f^4 - 400*a^4*c^8*d^2*f^4)^(1/2) + 4*a^2*c^5*f^2 + 20*a^2*c*d^4*f^2 - 40*a^2*c^3*d^2*f^2)/(c^10*f^4 + d^10*f^4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*c^6*d^4*f^4 + 5*c^8*d^2*f^4))^(1/2)*((c + d*tan(e + f*x))^(1/2)*(320*a^2*c^4*d^14*f^3 - 16*a^2*d^18*f^3 + 1024*a^2*c^6*d^12*f^3 + 1440*a^2*c^8*d^10*f^3 + 1024*a^2*c^10*d^8*f^3 + 320*a^2*c^12*d^6*f^3 - 16*a^2*c^16*d^2*f^3) - ((-((320*a^4*c^2*d^8*f^4 - 16*a^4*d^10*f^4 - 1760*a^4*c^4*d^6*f^4 + 1600*a^4*c^6*d^4*f^4 - 400*a^4*c^8*d^2*f^4)^(1/2) + 4*a^2*c^5*f^2 + 20*a^2*c*d^4*f^2 - 40*a^2*c^3*d^2*f^2)/(c^10*f^4 + d^10*f^4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*c^6*d^4*f^4 + 5*c^8*d^2*f^4))^(1/2)*(((-((320*a^4*c^2*d^8*f^4 - 16*a^4*d^10*f^4 - 1760*a^4*c^4*d^6*f^4 + 1600*a^4*c^6*d^4*f^4 - 400*a^4*c^8*d^2*f^4)^(1/2) + 4*a^2*c^5*f^2 + 20*a^2*c*d^4*f^2 - 40*a^2*c^3*d^2*f^2)/(c^10*f^4 + d^10*f^4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*c^6*d^4*f^4 + 5*c^8*d^2*f^4))^(1/2)*(c + d*tan(e + f*x))^(1/2)*(64*c*d^22*f^5 + 640*c^3*d^20*f^5

+ 2880*c^5*d^18*f^5 + 7680*c^7*d^16*f^5 + 13440*c^9*d^14*f^5 + 16128*c^11*d^12*f^5 + 13440*c^13*d^10*f^5 + 7680*c^15*d^8*f^5 + 2880*c^17*d^6*f^5 + 640*c^19*d^4*f^5 + 64*c^21*d^2*f^5))/4 - 32*a*d^21*f^4 - 160*a*c^2*d^19*f^4 - 128*a*c^4*d^17*f^4 + 896*a*c^6*d^15*f^4 + 3136*a*c^8*d^13*f^4 + 4928*a*c^10*d^11*f^4 + 4480*a*c^12*d^9*f^4 + 2432*a*c^14*d^7*f^4 + 736*a*c^16*d^5*f^4 + 96*a*c^18*d^3*f^4))/4))/4 + 16*a^3*c*d^15*f^2 + 96*a^3*c^3*d^13*f^2 + 240*a^3*c^5*d^11*f^2 + 320*a^3*c^7*d^9*f^2 + 240*a^3*c^9*d^7*f^2 + 96*a^3*c^11*d^5*f^2 + 16*a^3*c^13*d^3*f^2)*(-((320*a^4*c^2*d^8*f^4 - 16*a^4*d^10*f^4 - 1760*a^4*c^4*d^6*f^4 + 1600*a^4*c^6*d^4*f^4 - 400*a^4*c^8*d^2*f^4)^(1/2) + 4*a^2*c^5*f^2 + 20*a^2*c*d^4*f^2 - 40*a^2*c^3*d^2*f^2)/(c^10*f^4 + d^10*f^4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*c^6*d^4*f^4 + 5*c^8*d^2*f^4))^(1/2))/4 - log(16*a^3*c*d^15*f^2 - (((320*a^4*c^2*d^8*f^4 - 16*a^4*d^10*f^4 - 1760*a^4*c^4*d^6*f^4 + 1600*a^4*c^6*d^4*f^4 - 400*a^4*c^8*d^2*f^4)^(1/2) - 4*a^2*c^5*f^2 - 20*a^2*c*d^4*f^2 + 40*a^2*c^3*d^2*f^2)/(16*c^10*f^4 + 16*d^10*f^4 + 80*c^2*d^8*f^4 + 160*c^4*d^6*f^4 + 160*c^6*d^4*f^4 + 80*c^8*d^2*f^4))^(1/2)*((c + d*tan(e + f*x))^(1/2)*(320*a^2*c^4*d^14*f^3 - 16*a^2*d^18*f^3 + 1024*a^2*c^6*d^12*f^3 + 1440*a^2*c^8*d^10*f^3 + 1024*a^2*c^10*d^8*f^3 + 320*a^2*c^12*d^6*f^3 - 16*a^2*c^16*d^2*f^3) + (((320*a^4*c^2*d^8*f^4 - 16*a^4*d^10*f^4 - 1760*a^4*c^4*d^6*f^4 + 1600*a^4*c^6*d^4*f^4 - 400*a^4*c^8*d^2*f^4)^(1/2) - 4*a^2*c^5*f^2 - 20*a^2*c*d^4*f^2 + 40*a^2*c^3*d^2*f^2)/(16*c^10*f^4 + 16*d^10*f^4 + 80*c^2*d^8*f^4 + 160*c^4*d^6*f^4 + 160*c^6*d^4*f^4 + 80*c^8*d^2*f^4))^(1/2)*(896*a*c^6*d^15*f^4 - (((320*a^4*c^2*d^8*f^4 - 16*a^4*d^10*f^4 - 1760*a^4*c^4*d^6*f^4 + 1600*a^4*c^6*d^4*f^4 - 400*a^4*c^8*d^2*f^4)^(1/2) - 4*a^2*c^5*f^2 - 20*a^2*c*d^4*f^2 + 40*a^2*c^3*d^2*f^2)/(16*c^10*f^4 + 16*d^10*f^4 + 80*c^2*d^8*f^4 + 160*c^4*d^6*f^4 + 160*c^6*d^4*f^4 + 80*c^8*d^2*f^4))^(1/2)*(c + d*tan(e + f*x))^(1...

3.1134 $\int \frac{1}{(a+ia\tan(e+fx))(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=267

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{2a(c-id)^{5/2}f}+\frac{(ic-6d)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{2a(c+id)^{7/2}f}+\frac{d(3ic+7d)}{6a(ic-d)\left(c^2+d^2\right)f(c+}$$

```
[Out] -1/2*I*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/a/(c-I*d)^(5/2)/f+1/2*
(I*c-6*d)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/a/(c+I*d)^(7/2)/f+1
/2*d*(c^2-14*I*c*d-5*d^2)/a/(c-I*d)^2/(c+I*d)^3/f/(c+d*tan(f*x+e))^(1/2)+1/
6*d*(3*I*c+7*d)/a/(I*c-d)/(c^2+d^2)/f/(c+d*tan(f*x+e))^(3/2)-1/2/(I*c-d)/f/
(a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.45, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3633, 3610, 3620, 3618, 65, 214}

$$\frac{d(7d+3ic)}{6af(-d+ic)\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}+\frac{d\left(c^2-14icd-5d^2\right)}{2af(c-id)^2(c+id)^3\sqrt{c+d\tan(e+fx)}}-\frac{1}{2f(-d+ic)(a+ia\tan(e+fx))(c+d\tan(e+fx))^{3/2}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{2af(c-id)^{5/2}}+\frac{(-6d+ic)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{2af(c+id)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])*(c + d*Tan[e + f*x])^(5/2)),x]
```

```
[Out] ((-1/2*I)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/(a*(c - I*d)^(5/
2)*f) + ((I*c - 6*d)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/(2*a*
(c + I*d)^(7/2)*f) + (d*((3*I)*c + 7*d))/(6*a*(I*c - d)*(c^2 + d^2)*f*(c +
d*Tan[e + f*x])^(3/2)) - 1/(2*(I*c - d)*f*(a + I*a*Tan[e + f*x])*(c + d*Tan
[e + f*x])^(3/2)) + (d*(c^2 - (14*I)*c*d - 5*d^2))/(2*a*(c - I*d)^2*(c + I*
d)^3*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3633

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-a)*((c + d*Tan[e + f*x])^(n + 1)/(2*f*(b*
c - a*d)*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*
Tan[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &
& NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{1}{2(ic-d)f(a+ia\tan(e+fx))(c+d\tan(e+fx))^{3/2}} +$$

$$= \frac{(3c-7id)d}{6a(c-id)(c+id)^2f(c+d\tan(e+fx))^{3/2}} - \frac{}{2(ic-d)f(}$$

$$= \frac{(3c-7id)d}{6a(c-id)(c+id)^2f(c+d\tan(e+fx))^{3/2}} - \frac{}{2(ic-d)f(}$$

$$= \frac{(3c-7id)d}{6a(c-id)(c+id)^2f(c+d\tan(e+fx))^{3/2}} - \frac{}{2(ic-d)f(}$$

$$= \frac{(3c-7id)d}{6a(c-id)(c+id)^2f(c+d\tan(e+fx))^{3/2}} - \frac{}{2(ic-d)f(}$$

$$= \frac{(3c-7id)d}{6a(c-id)(c+id)^2f(c+d\tan(e+fx))^{3/2}} - \frac{}{2(ic-d)f(}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{2a(c-id)^{5/2}f} + \frac{(ic-6d)\tanh^{-1}\left(\right.}{2a(c}$$

Mathematica [A]
time = 6.82, size = 371, normalized size = 1.39

$$\frac{\sec(e+fx)(\cos(fx)+i\sin(fx))\left(-\frac{2\left((-c+id)^{5/2}(-ic+6d)\text{ArcTan}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{-c-id}}\right)-i(-c-id)^{7/2}\text{ArcTan}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{-c+id}}\right)\right)(\cos(e)+i\sin(e))}{(-c-id)^{7/2}(-c+id)^{5/2}}+\frac{(\cos(fx)-i\sin(fx))\left(3(3ic^4+6c^3d-42ic^2d^2+2cd^3-9id^4)\cos(e+fx)+(ic+d)\left((3c^3-3ic^2d-43cd^2+11id^3)\cos(3(e+fx))-8d(-3c^2+23icd+4d^2)\cos^2(e+fx)\sin(e+fx)\right)\right)\sqrt{c+d\tan(e+fx)}}{6(c-id)^2(c+id)^3(c\cos(e+fx)+d\sin(e+fx))^2}\right)}{4f(a+ia\tan(e+fx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])*(c + d*Tan[e + f*x])^(5/2)),x]
```

```
[Out] (Sec[e + f*x]*(Cos[f*x] + I*Sin[f*x])*((-2*((-c + I*d)^(5/2)*((-I)*c + 6*d)
*ArcTan[Sqrt[c + d*Tan[e + f*x]]/Sqrt[-c - I*d]] - I*(-c - I*d)^(7/2)*ArcTa
n[Sqrt[c + d*Tan[e + f*x]]/Sqrt[-c + I*d]])*(Cos[e] + I*Sin[e]))/((-c - I*d
)^(7/2)*(-c + I*d)^(5/2)) + ((Cos[f*x] - I*Sin[f*x])*(3*((3*I)*c^4 + 6*c^3*
d - (42*I)*c^2*d^2 + 2*c*d^3 - (9*I)*d^4)*Cos[e + f*x] + (I*c + d)*((3*c^3
- (3*I)*c^2*d - 43*c*d^2 + (11*I)*d^3)*Cos[3*(e + f*x)] - 8*d*(-3*c^2 + (23
*I)*c*d + 4*d^2)*Cos[e + f*x]^2*Sin[e + f*x]))*Sqrt[c + d*Tan[e + f*x]])/(6
*(c - I*d)^2*(c + I*d)^3*(c*Cos[e + f*x] + d*Sin[e + f*x])^2)))/(4*f*(a + I
*a*Tan[e + f*x]))
```

Maple [A]
time = 0.47, size = 364, normalized size = 1.36

method	result
derivativedivides	$\frac{2d^2\left(\frac{-\frac{(c^4+2c^2d^2+d^4)d\sqrt{c+d\tan(fx+e)}}{(id+c)(-d\tan(fx+e)+id)}-\frac{(ic^5+2ic^3d^2+icd^4-6c^4d-12c^2d^3-6d^5)\arctan\left(\frac{\sqrt{c+d\tan(}}{\sqrt{-id-}}\right.}{(id+c)\sqrt{-id-c}}}{4d^2(id+c)^4(id-c)^2}\right.}{}$
default	$\frac{2d^2\left(\frac{-\frac{(c^4+2c^2d^2+d^4)d\sqrt{c+d\tan(fx+e)}}{(id+c)(-d\tan(fx+e)+id)}-\frac{(ic^5+2ic^3d^2+icd^4-6c^4d-12c^2d^3-6d^5)\arctan\left(\frac{\sqrt{c+d\tan(}}{\sqrt{-id-}}\right.}{(id+c)\sqrt{-id-c}}}{4d^2(id+c)^4(id-c)^2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a*d^2*(1/4/d^2/(c+I*d)^4/(I*d-c)^2*(-(c^4+2*c^2*d^2+d^4)*d/(c+I*d)*(c+d
*tan(f*x+e))^(1/2)/(-d*tan(f*x+e)+I*d)-(-6*c^4*d-12*c^2*d^3-6*d^5+I*c^5+2*I
*c^3*d^2+I*c*d^4)/(c+I*d)/(-I*d-c)^(1/2)*arctan((c+d*tan(f*x+e))^(1/2)/(-I*
d-c)^(1/2)))+1/4/(I*d-c)^(5/2)/(c+I*d)^4*(-6*I*c^2*d^2+I*d^4-4*c^3*d+4*c*d^
3+I*c^4)/d^2*arctan((c+d*tan(f*x+e))^(1/2)/(I*d-c)^(1/2))-1/3/(I*d-c)^2/(c+
I*d)^4*(I*c^3+I*c*d^2-c^2*d-d^3)/(c+d*tan(f*x+e))^(3/2)-1/(I*d-c)^2/(c+I*d)
^4*(3*I*c^2+I*d^2-2*c*d)/(c+d*tan(f*x+e))^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^(5/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2699 vs. 2(219) = 438.
time = 4.03, size = 2699, normalized size = 10.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/24*(6*((I*a*c^7 + a*c^6*d + 3*I*a*c^5*d^2 + 3*a*c^4*d^3 + 3*I*a*c^3*d^4 +
 3*a*c^2*d^5 + I*a*c*d^6 + a*d^7)*f*e^(6*I*f*x + 6*I*e) + 2*(I*a*c^7 - a*c^
6*d + 3*I*a*c^5*d^2 - 3*a*c^4*d^3 + 3*I*a*c^3*d^4 - 3*a*c^2*d^5 + I*a*c*d^6
 - a*d^7)*f*e^(4*I*f*x + 4*I*e) + (I*a*c^7 - 3*a*c^6*d - I*a*c^5*d^2 - 5*a*
c^4*d^3 - 5*I*a*c^3*d^4 - a*c^2*d^5 - 3*I*a*c*d^6 + a*d^7)*f*e^(2*I*f*x + 2
*I*e))*sqrt(1/4*I/((-I*a^2*c^5 - 5*a^2*c^4*d + 10*I*a^2*c^3*d^2 + 10*a^2*c^
2*d^3 - 5*I*a^2*c*d^4 - a^2*d^5)*f^2))*log(-2*(2*((I*a*c^3 + 3*a*c^2*d - 3*
I*a*c*d^2 - a*d^3)*f*e^(2*I*f*x + 2*I*e) + (I*a*c^3 + 3*a*c^2*d - 3*I*a*c*d
^2 - a*d^3)*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x +
 2*I*e) + 1))*sqrt(1/4*I/((-I*a^2*c^5 - 5*a^2*c^4*d + 10*I*a^2*c^3*d^2 + 10
*a^2*c^2*d^3 - 5*I*a^2*c*d^4 - a^2*d^5)*f^2)) - (c - I*d)*e^(2*I*f*x + 2*I*
e) - c)*e^(-2*I*f*x - 2*I*e)) + 6*((-I*a*c^7 - a*c^6*d - 3*I*a*c^5*d^2 - 3*
a*c^4*d^3 - 3*I*a*c^3*d^4 - 3*a*c^2*d^5 - I*a*c*d^6 - a*d^7)*f*e^(6*I*f*x +
 6*I*e) + 2*(-I*a*c^7 + a*c^6*d - 3*I*a*c^5*d^2 + 3*a*c^4*d^3 - 3*I*a*c^3*d
^4 + 3*a*c^2*d^5 - I*a*c*d^6 + a*d^7)*f*e^(4*I*f*x + 4*I*e) + (-I*a*c^7 + 3
*a*c^6*d + I*a*c^5*d^2 + 5*a*c^4*d^3 + 5*I*a*c^3*d^4 + a*c^2*d^5 + 3*I*a*c*
d^6 - a*d^7)*f*e^(2*I*f*x + 2*I*e))*sqrt(1/4*I/((-I*a^2*c^5 - 5*a^2*c^4*d +
 10*I*a^2*c^3*d^2 + 10*a^2*c^2*d^3 - 5*I*a^2*c*d^4 - a^2*d^5)*f^2))*log(-2*
(2*((-I*a*c^3 - 3*a*c^2*d + 3*I*a*c*d^2 + a*d^3)*f*e^(2*I*f*x + 2*I*e) + (-
I*a*c^3 - 3*a*c^2*d + 3*I*a*c*d^2 + a*d^3)*f)*sqrt(((c - I*d)*e^(2*I*f*x +
2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(1/4*I/((-I*a^2*c^5 - 5*a^
2*c^4*d + 10*I*a^2*c^3*d^2 + 10*a^2*c^2*d^3 - 5*I*a^2*c*d^4 - a^2*d^5)*f^2)
) - (c - I*d)*e^(2*I*f*x + 2*I*e) - c)*e^(-2*I*f*x - 2*I*e)) + 3*((-I*a*c^7
 - a*c^6*d - 3*I*a*c^5*d^2 - 3*a*c^4*d^3 - 3*I*a*c^3*d^4 - 3*a*c^2*d^5 - I*
a*c*d^6 - a*d^7)*f*e^(6*I*f*x + 6*I*e) + 2*(-I*a*c^7 + a*c^6*d - 3*I*a*c^5*
d^2 + 3*a*c^4*d^3 - 3*I*a*c^3*d^4 + 3*a*c^2*d^5 - I*a*c*d^6 + a*d^7)*f*e^(4
*I*f*x + 4*I*e) + (-I*a*c^7 + 3*a*c^6*d + I*a*c^5*d^2 + 5*a*c^4*d^3 + 5*I*a
*c^3*d^4 + a*c^2*d^5 + 3*I*a*c*d^6 - a*d^7)*f*e^(2*I*f*x + 2*I*e))*sqrt(-(-
I*c^2 + 12*c*d + 36*I*d^2)/((-I*a^2*c^7 + 7*a^2*c^6*d + 21*I*a^2*c^5*d^2 -
35*a^2*c^4*d^3 - 35*I*a^2*c^3*d^4 + 21*a^2*c^2*d^5 + 7*I*a^2*c*d^6 - a^2*d^
7)*f^2))*log(1/2*(I*c^2 - 7*c*d - 6*I*d^2 + ((a*c^4 + 4*I*a*c^3*d - 6*a*c^2
*d^2 - 4*I*a*c*d^3 + a*d^4)*f*e^(2*I*f*x + 2*I*e) + (a*c^4 + 4*I*a*c^3*d -
6*a*c^2*d^2 - 4*I*a*c*d^3 + a*d^4)*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) +
 c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(-I*c^2 + 12*c*d + 36*I*d^2)/((-
I*a^2*c^7 + 7*a^2*c^6*d + 21*I*a^2*c^5*d^2 - 35*a^2*c^4*d^3 - 35*I*a^2*c^3*
d^4 + 21*a^2*c^2*d^5 + 7*I*a^2*c*d^6 - a^2*d^7)*f^2)) + (I*c^2 - 6*c*d)*e^(
2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/((a*c^4 + 4*I*a*c^3*d - 6*a*c^2*d^2
- 4*I*a*c*d^3 + a*d^4)*f)) + 3*((I*a*c^7 + a*c^6*d + 3*I*a*c^5*d^2 + 3*a*c^
4*d^3 + 3*I*a*c^3*d^4 + 3*a*c^2*d^5 + I*a*c*d^6 + a*d^7)*f*e^(6*I*f*x + 6*I
*e) + 2*(I*a*c^7 - a*c^6*d + 3*I*a*c^5*d^2 - 3*a*c^4*d^3 + 3*I*a*c^3*d^4 -
```

```
3*a*c^2*d^5 + I*a*c*d^6 - a*d^7)*f*e^(4*I*f*x + 4*I*e) + (I*a*c^7 - 3*a*c^6
*d - I*a*c^5*d^2 - 5*a*c^4*d^3 - 5*I*a*c^3*d^4 - a*c^2*d^5 - 3*I*a*c*d^6 +
a*d^7)*f*e^(2*I*f*x + 2*I*e))*sqrt(-(-I*c^2 + 12*c*d + 36*I*d^2)/((-I*a^2*c
^7 + 7*a^2*c^6*d + 21*I*a^2*c^5*d^2 - 35*a^2*c^4*d^3 - 35*I*a^2*c^3*d^4 + 2
1*a^2*c^2*d^5 + 7*I*a^2*c*d^6 - a^2*d^7)*f^2))*log(1/2*(I*c^2 - 7*c*d - 6*I
*d^2 - ((a*c^4 + 4*I*a*c^3*d - 6*a*c^2*d^2 - 4*I*a*c*d^3 + a*d^4)*f*e^(2*I*
f*x + 2*I*e) + (a*c^4 + 4*I*a*c^3*d - 6*a*c^2*d^2 - 4*I*a*c*d^3 + a*d^4)*f)
*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*
sqrt(-(-I*c^2 + 12*c*d + 36*I*d^2)/((-I*a^2*c^7 + 7*a^2*c^6*d + 21*I*a^2*c^
5*d^2 - 35*a^2*c^4*d^3 - 35*I*a^2*c^3*d^4 + 21*a^2*c^2*d^5 + 7*I*a^2*c*d^6
- a^2*d^7)*f^2)) + (I*c^2 - 6*c*d)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e
)/((a*c^4 + 4*I*a*c^3*d - 6*a*c^2*d^2 - 4*I*a*c*d^3 + a*d^4)*f)) + 2*(3*c^4
 + 6*c^2*d^2 + 3*d^4 + (3*c^4 - 12*I*c^3*d - 98*c^2*d^2 + 108*I*c*d^3 + 19*
d^4)*e^(6*I*f*x + 6*I*e) + (9*c^4 - 24*I*c^3*d - 178*c^2*d^2 + 48*I*c*d^3 -
 19*d^4)*e^(4*I*f*x + 4*I*e) + (9*c^4 - 12*I*c^3*d - 74*c^2*d^2 - 60*I*c*d^
3 - 35*d^4)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c +
I*d)/(e^(2*I*f*x + 2*I*e) + 1)))/((-I*a*c^7 - a*c^6*d - 3*I*a*c^5*d^2 - 3*a
*c^4*d^3 - 3*I*a*c^3*d^4 - 3*a*c^2*d^5 - I*a*c*d^6 - a*d^7)*f*e^(6*I*f*x +
6*I*e) + 2*(-I*a*c^7 + a*c^6*d - 3*I*a*c^5*d^2 + 3*a*c^4*d^3 - 3*I*a*c^3*d^
4 + 3*a*c^2*d^5 - I*a*c*d^6 + a*d^7)*f*e^(4*I*f*x + 4*I*e) + (-I*a*c^7 + 3*
a*c^6*d + I*a*c^5*d^2 + 5*a*c^4*d^3 + 5*I*a*c^3*d^4 + a*c^2*d^5 + 3*I*a*c*d
^6 - a*d^7)*f*e^(2*I*f*x + 2*I*e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int \frac{1}{c^{2}\sqrt{c+d\tan(e+fx)}\tan(e+fx)-ic^{2}\sqrt{c+d\tan(e+fx)}+2cd\sqrt{c+d\tan(e+fx)}\tan^{2}(e+fx)-2icd\sqrt{c+d\tan(e+fx)}\tan(e+fx)+d^{2}\sqrt{c+d\tan(e+fx)}\tan^{3}(e+fx)-id^{2}\sqrt{c+d\tan(e+fx)}\tan^{2}(e+fx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] -I*Integral(1/(c**2*sqrt(c + d*tan(e + f*x))*tan(e + f*x) - I*c**2*sqrt(c +
 d*tan(e + f*x)) + 2*c*d*sqrt(c + d*tan(e + f*x))*tan(e + f*x)**2 - 2*I*c*d
*sqrt(c + d*tan(e + f*x))*tan(e + f*x) + d**2*sqrt(c + d*tan(e + f*x))*tan(
e + f*x)**3 - I*d**2*sqrt(c + d*tan(e + f*x))*tan(e + f*x)**2), x)/a
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(219) = 438$.
time = 1.09, size = 547, normalized size = 2.05

$$\frac{2\sqrt{d\tan(fx+e)+c}\,d}{-4\,(i\,ac^{3}f-3\,ac^{2}df-3i\,acd^{2}f+ad^{3}f)(i\,d\tan(fx+e)+d)}+\frac{(-i\,c+6\,d)\arctan\left(\frac{2\left(\sqrt{d\tan(fx+e)+c}\,c-\sqrt{c^{2}+d^{2}}\sqrt{d\tan(fx+e)+c}\right)}{c\sqrt{-2\,c+2\sqrt{c^{2}+d^{2}}}+i\sqrt{-2\,c+2\sqrt{c^{2}+d^{2}}}\,d-\sqrt{c^{2}+d^{2}}\sqrt{-2\,c+2\sqrt{c^{2}+d^{2}}}}\right)}{(ac^{3}f+3i\,ac^{2}df-3\,acd^{2}f-i\,ad^{3}f)\sqrt{-2\,c+2\sqrt{c^{2}+d^{2}}}\left(\frac{i\,d}{c-\sqrt{c^{2}+d^{2}}}+1\right)}+\frac{2\,(9\,(d\tan(fx+e)+c)cd^{2}+c^{2}d^{2}-3\,(i\,d\tan(fx+e)+i\,c)d^{3}+d^{4})}{-3\,(-i\,ac^{5}f+ac^{4}df-2i\,ac^{3}d^{2}f+2\,ac^{2}d^{3}f-i\,acd^{4}f+ad^{5}f)(d\tan(fx+e)+c)^{\frac{3}{2}}}+\frac{i\arctan\left(\frac{2\left(\sqrt{d\tan(fx+e)+c}\,c-\sqrt{c^{2}+d^{2}}\sqrt{d\tan(fx+e)+c}\right)}{c\sqrt{-2\,c+2\sqrt{c^{2}+d^{2}}}-i\sqrt{-2\,c+2\sqrt{c^{2}+d^{2}}}\,d-\sqrt{c^{2}+d^{2}}\sqrt{-2\,c+2\sqrt{c^{2}+d^{2}}}}\right)}{(ac^{2}f-2i\,acdf-ad^{2}f)\sqrt{-2\,c+2\sqrt{c^{2}+d^{2}}}\left(-\frac{i\,d}{c-\sqrt{c^{2}+d^{2}}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))/(c+d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(d*tan(f*x + e) + c)*d/((-4*I*a*c^3*f + 12*a*c^2*d*f + 12*I*a*c*d^2*f
 - 4*a*d^3*f)*(I*d*tan(f*x + e) + d)) + (-I*c + 6*d)*arctan(2*(sqrt(d*tan(f
*x + e) + c)*c - sqrt(c^2 + d^2)*sqrt(d*tan(f*x + e) + c))/(c*sqrt(-2*c + 2
*sqrt(c^2 + d^2)) + I*sqrt(-2*c + 2*sqrt(c^2 + d^2))*d - sqrt(c^2 + d^2)*sq
rt(-2*c + 2*sqrt(c^2 + d^2))))/((a*c^3*f + 3*I*a*c^2*d*f - 3*a*c*d^2*f - I*
a*d^3*f)*sqrt(-2*c + 2*sqrt(c^2 + d^2))*(I*d/(c - sqrt(c^2 + d^2)) + 1)) +
2*(9*(d*tan(f*x + e) + c)*c*d^2 + c^2*d^2 - 3*(I*d*tan(f*x + e) + I*c)*d^3
+ d^4)/((3*I*a*c^5*f - 3*a*c^4*d*f + 6*I*a*c^3*d^2*f - 6*a*c^2*d^3*f + 3*I*
a*c*d^4*f - 3*a*d^5*f)*(d*tan(f*x + e) + c)^(3/2)) + I*arctan(2*(sqrt(d*tan
(f*x + e) + c)*c - sqrt(c^2 + d^2)*sqrt(d*tan(f*x + e) + c))/(c*sqrt(-2*c +
 2*sqrt(c^2 + d^2)) - I*sqrt(-2*c + 2*sqrt(c^2 + d^2))*d - sqrt(c^2 + d^2)*
sqrt(-2*c + 2*sqrt(c^2 + d^2))))/((a*c^2*f - 2*I*a*c*d*f - a*d^2*f)*sqrt(-2
*c + 2*sqrt(c^2 + d^2))*(-I*d/(c - sqrt(c^2 + d^2)) + 1))
```

Mupad [B]
time = 93.43, size = 2500, normalized size = 9.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)*(c + d*tan(e + f*x))^(5/2)),x)
```

```
[Out] log((a*f*(139*c*d^7 - d^8*30i + c^2*d^6*180i - 62*c^3*d^5 + c^4*d^4*10i - c
^5*d^3))/2 - (((a*f*(208*a^2*c^2*d^11*f^2 - a^2*c*d^12*f^2*320i - 112*a^2*d
^13*f^2 - a^2*c^3*d^10*f^2*640i + 1312*a^2*c^4*d^9*f^2 + 1568*a^2*c^6*d^7*f
^2 + a^2*c^7*d^6*f^2*640i + 592*a^2*c^8*d^5*f^2 + a^2*c^9*d^4*f^2*320i + 16
*a^2*c^10*d^3*f^2))/2 - 2*(c + d*tan(e + f*x))^(1/2)*(a^2*d^2*f^2 - a^2*c^2
*f^2 + a^2*c*d*f^2*2i)*((4480*c^2*d^9 - 560*d^11 - c*d^10*2800i + c^3*d^8*4
480i - 560*c^4*d^7 - c^5*d^6*112i - 224*c^6*d^5 + c^7*d^4*32i - a^2*c^10*f^
2*(((3920*c*d^12 - 16240*c^3*d^10 + 5712*c^5*d^8 + 304*c^7*d^6 + 32*c^9*d^4
)/(a^2*c^12*f^2 + a^2*d^12*f^2 + 6*a^2*c^2*d^10*f^2 + 15*a^2*c^4*d^8*f^2 +
20*a^2*c^6*d^6*f^2 + 15*a^2*c^8*d^4*f^2 + 6*a^2*c^10*d^2*f^2) + ((10640*c^2
*d^11 - 560*d^13 - 14000*c^4*d^9 + 560*c^6*d^7 + 160*c^8*d^5)*1i)/(a^2*c^12
*f^2 + a^2*d^12*f^2 + 6*a^2*c^2*d^10*f^2 + 15*a^2*c^4*d^8*f^2 + 20*a^2*c^6*
d^6*f^2 + 15*a^2*c^8*d^4*f^2 + 6*a^2*c^10*d^2*f^2))^2 - 4*(256*d^6 + 256*c^
2*d^4)*(((60*c*d^7 + 10*c^3*d^5)*1i)/(a^4*c^12*f^4 + a^4*d^12*f^4 + 6*a^4*c
^2*d^10*f^4 + 15*a^4*c^4*d^8*f^4 + 20*a^4*c^6*d^6*f^4 + 15*a^4*c^8*d^4*f^4
+ 6*a^4*c^10*d^2*f^4) + (36*d^8 - 13*c^2*d^6 + c^4*d^4)/(a^4*c^12*f^4 + a^4
*d^12*f^4 + 6*a^4*c^2*d^10*f^4 + 15*a^4*c^4*d^8*f^4 + 20*a^4*c^6*d^6*f^4 +
15*a^4*c^8*d^4*f^4 + 6*a^4*c^10*d^2*f^4)))^(1/2)*1i + a^2*d^10*f^2*(((3920*
c*d^12 - 16240*c^3*d^10 + 5712*c^5*d^8 + 304*c^7*d^6 + 32*c^9*d^4)/(a^2*c^1
2*f^2 + a^2*d^12*f^2 + 6*a^2*c^2*d^10*f^2 + 15*a^2*c^4*d^8*f^2 + 20*a^2*c^6
*d^6*f^2 + 15*a^2*c^8*d^4*f^2 + 6*a^2*c^10*d^2*f^2) + ((10640*c^2*d^11 - 56
0*d^13 - 14000*c^4*d^9 + 560*c^6*d^7 + 160*c^8*d^5)*1i)/(a^2*c^12*f^2 + a^2
*d^12*f^2 + 6*a^2*c^2*d^10*f^2 + 15*a^2*c^4*d^8*f^2 + 20*a^2*c^6*d^6*f^2 +
```

15*a^2*c^8*d^4*f^2 + 6*a^2*c^10*d^2*f^2))^2 - 4*(256*d^6 + 256*c^2*d^4)*(((
60*c*d^7 + 10*c^3*d^5)*1i)/(a^4*c^12*f^4 + a^4*d^12*f^4 + 6*a^4*c^2*d^10*f^
4 + 15*a^4*c^4*d^8*f^4 + 20*a^4*c^6*d^6*f^4 + 15*a^4*c^8*d^4*f^4 + 6*a^4*c^
10*d^2*f^4) + (36*d^8 - 13*c^2*d^6 + c^4*d^4)/(a^4*c^12*f^4 + a^4*d^12*f^4
+ 6*a^4*c^2*d^10*f^4 + 15*a^4*c^4*d^8*f^4 + 20*a^4*c^6*d^6*f^4 + 15*a^4*c^8
*d^4*f^4 + 6*a^4*c^10*d^2*f^4)))^(1/2)*1i + 2*a^2*c*d^9*f^2*(((3920*c*d^12
- 16240*c^3*d^10 + 5712*c^5*d^8 + 304*c^7*d^6 + 32*c^9*d^4)/(a^2*c^12*f^2 +
a^2*d^12*f^2 + 6*a^2*c^2*d^10*f^2 + 15*a^2*c^4*d^8*f^2 + 20*a^2*c^6*d^6*f^
2 + 15*a^2*c^8*d^4*f^2 + 6*a^2*c^10*d^2*f^2) + ((10640*c^2*d^11 - 560*d^13
- 14000*c^4*d^9 + 560*c^6*d^7 + 160*c^8*d^5)*1i)/(a^2*c^12*f^2 + a^2*d^12*f
^2 + 6*a^2*c^2*d^10*f^2 + 15*a^2*c^4*d^8*f^2 + 20*a^2*c^6*d^6*f^2 + 15*a^2*
c^8*d^4*f^2 + 6*a^2*c^10*d^2*f^2))^2 - 4*(256*d^6 + 256*c^2*d^4)*(((60*c*d^
7 + 10*c^3*d^5)*1i)/(a^4*c^12*f^4 + a^4*d^12*f^4 + 6*a^4*c^2*d^10*f^4 + 15*
a^4*c^4*d^8*f^4 + 20*a^4*c^6*d^6*f^4 + 15*a^4*c^8*d^4*f^4 + 6*a^4*c^10*d^2*
f^4) + (36*d^8 - 13*c^2*d^6 + c^4*d^4)/(a^4*c^12*f^4 + a^4*d^12*f^4 + 6*a^4
*c^2*d^10*f^4 + 15*a^4*c^4*d^8*f^4 + 20*a^4*c^6*d^6*f^4 + 15*a^4*c^8*d^4*f^
4 + 6*a^4*c^10*d^2*f^4)))^(1/2) + 2*a^2*c^9*d*f^2*(((3920*c*d^12 - 16240*c^
3*d^10 + 5712*c^5*d^8 + 304*c^7*d^6 + 32*c^9*d^4)/(a^2*c^12*f^2 + a^2*d^12*
f^2 + 6*a^2*c^2*d^10*f^2 + 15*a^2*c^4*d^8*f^2 + 20*a^2*c^6*d^6*f^2 + 15*a^2
*c^8*d^4*f^2 + 6*a^2*c^10*d^2*f^2) + ((10640*c^2*d^11 - 560*d^13 - 14000*c^
4*d^9 + 560*c^6*d^7 + 160*c^8*d^5)*1i)/(a^2*c^12*f^2 + a^2*d^12*f^2 + 6*a^2
*c^2*d^10*f^2 + 15*a^2*c^4*d^8*f^2 + 20*a^2*c^6*d^6*f^2 + 15*a^2*c^8*d^4*f^
2 + 6*a^2*c^10*d^2*f^2))^2 - 4*(256*d^6 + 256*c^2*d^4)*(((60*c*d^7 + 10*c^3
*d^5)*1i)/(a^4*c^12*f^4 + a^4*d^12*f^4 + 6*a^4*c^2*d^10*f^4 + 15*a^4*c^4*d^
8*f^4 + 20*a^4*c^6*d^6*f^4 + 15*a^4*c^8*d^4*f^4 + 6*a^4*c^10*d^2*f^4) + (36
*d^8 - 13*c^2*d^6 + c^4*d^4)/(a^4*c^12*f^4 + a^4*d^12*f^4 + 6*a^4*c^2*d^10*
f^4 + 15*a^4*c^4*d^8*f^4 + 20*a^4*c^6*d^6*f^4 + 15*a^4*c^8*d^4*f^4 + 6*a^4*
c^10*d^2*f^4)))^(1/2) + a^2*c^2*d^8*f^2*(((3920*c*d^12 - 16240*c^3*d^10 + 5
712*c^5*d^8 + 304*c^7*d^6 + 32*c^9*d^4)/(a^2*c^12*f^2 + a^2*d^12*f^2 + 6*a^
2*c^2*d^10*f^2 + 15*a^2*c^4*d^8*f^2 + 20*a^2*c^6*d^6*f^2 + 15*a^2*c^8*d^4*f
^2 + 6*a^2*c^10*d^2*f^2) + ((10640*c^2*d^11 - 560*d^13 - 14000*c^4*d^9 + 56
0*c^6*d^7 + 160*c^8*d^5)*1i)/(a^2*c^12*f^2 + a^2*d^12*f^2 + 6*a^2*c^2*d^10*
f^2 + 15*a^2*c^4*d^8*f^2 + 20*a^2*c^6*d^6*f^2 + 15*a^2*c^8*d^4*f^2 + 6*a^2*
c^10*d^2*f^2))^2 - 4*(256*d^6 + 256*c^2*d^4)*(((60*c*d^7 + 10*c^3*d^5)*1i)/
(a^4*c^12*f^4 + a^4*d^12*f^4 + 6*a^4*c^2*d^10*f^4 + 15*a^4*c^4*d^8*f^4 + 20
*a^4*c^6*d^6*f^4 + 15*a^4*c^8*d^4*f^4 + 6*a^4*c^10*d^2*f^4) + (36*d^8 - 13*
c^2*d^6 + c^4*d^4)/(a^4*c^12*f^4 + a^4*d^12*f^4 + 6*a^4*c^2*d^10*f^4 + 15*a
^4*c^4*d^8*f^4 + 20*a^4*c^6*d^6*f^4 + 15*a^4*c^8*d^4*f^4 + 6*a^4*c^10*d^2*f
^4)))^(1/2)*3i + 8*a^2*c^3*d^7*f^2*(((3920*c*d^12 - 16240*c^3*d^10 + 5712*c
^5*d^8 + 304*c^7*d^6 + 32*c^9*d^4)/(a^2*c^12*f^2 + a^2*d^12*f^2 + 6*a^2*c^2
*d^10*f^2 + 15*a^2*c^4*d^8*f^2 + 20*a^2*c^6*d^6*f^2 + 15*a^2*c^8*d^4*f^2 +
6*a^2*c^10*d^2*f^2) + ((10640*c^2*d^11 - 560*d^13 - 14000*c^4*d^9 + 560*c^6
*d^7 + 160*c^8*d^5)*1i)/(a^2*c^12*f^2 + a^2*d^12*f^2 + 6*a^2*c^2*d^10*f^2 +
15*a^2*c^4*d^8*f^2 + 20*a^2*c^6*d^6*f^2 + 15*a...

3.1135 $\int \frac{1}{(a+ia\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=351

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{4a^2(c-id)^{5/2}f}+\frac{(2ic^2-14cd-47id^2)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{8a^2(c+id)^{9/2}f}+\frac{d(6}{24a^2(c-id)(c}$$

```
[Out] -1/4*I*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/a^2/(c-I*d)^(5/2)/f+1/
8*(2*I*c^2-14*c*d-47*I*d^2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/a
^2/(c+I*d)^(9/2)/f+1/8*d*(2*c^3+9*I*c^2*d+88*c*d^2-45*I*d^3)/a^2/(c-I*d)^2/
(c+I*d)^4/f/(c+d*tan(f*x+e))^(1/2)+1/24*d*(6*c^2+27*I*c*d+49*d^2)/a^2/(c-I*
d)/(c+I*d)^3/f/(c+d*tan(f*x+e))^(3/2)+1/8*(2*I*c-9*d)/a^2/(c+I*d)^2/f/(1+I*
tan(f*x+e))/(c+d*tan(f*x+e))^(3/2)-1/4/(I*c-d)/f/(a+I*a*tan(f*x+e))^2/(c+d*
tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.70, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3640, 3677, 3610, 3620, 3618, 65, 214}

$$\frac{d(6c^2+27icd+49d^2)}{24a^2f(c-id)(c+id)^3(c+d\tan(e+fx))^{3/2}}+\frac{(2ic^2-14cd-47id^2)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{8a^2f(c+id)^{9/2}}+\frac{d(2c^3+9ic^2d+88cd^2-45id^3)}{8a^2f(c-id)^2(c+id)^4\sqrt{c+d\tan(e+fx)}}+\frac{-9d+2ic}{8a^2f(c+id)^2(1+i\tan(e+fx))(c+d\tan(e+fx))^{3/2}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{4a^2f(c-id)^{5/2}}-\frac{1}{4f(-d+ic)(a+ia\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(5/2)),x]
```

```
[Out] ((-1/4*I)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/(a^2*(c - I*d)^(
5/2)*f) + (((2*I)*c^2 - 14*c*d - (47*I)*d^2)*ArcTanh[Sqrt[c + d*Tan[e + f*x
]]/Sqrt[c + I*d]])/(8*a^2*(c + I*d)^(9/2)*f) + (d*(6*c^2 + (27*I)*c*d + 49*
d^2))/(24*a^2*(c - I*d)*(c + I*d)^3*f*(c + d*Tan[e + f*x])^(3/2)) + ((2*I)*
c - 9*d)/(8*a^2*(c + I*d)^2*f*(1 + I*Tan[e + f*x])*(c + d*Tan[e + f*x])^(3/
2)) - 1/(4*(I*c - d)*f*(a + I*a*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(3/2))
 + (d*(2*c^3 + (9*I)*c^2*d + 88*c*d^2 - (45*I)*d^3))/(8*a^2*(c - I*d)^2*(c
+ I*d)^4*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
```

```
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{1}{4(ic-d)f(a+ia\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}$$

$$= \frac{2ic-9d}{8a^2(c+id)^2f(1+i\tan(e+fx))(c+d\tan(e+fx))^{3/2}}$$

$$= \frac{d(6c^2+27icd+49d^2)}{24a^2(c+id)^2\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{}{8a^2(c}$$

$$= \frac{d(6c^2+27icd+49d^2)}{24a^2(c+id)^2\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{}{8a^2(c}$$

$$= \frac{d(6c^2+27icd+49d^2)}{24a^2(c+id)^2\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{}{8a^2(c}$$

$$= \frac{d(6c^2+27icd+49d^2)}{24a^2(c+id)^2\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{}{8a^2(c}$$

$$= \frac{d(6c^2+27icd+49d^2)}{24a^2(c+id)^2\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{}{8a^2(c}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{4a^2(c-id)^{5/2}f} - \frac{(14cd-i(2c^2-}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1004 vs. $2(351) = 702$.
time = 9.87, size = 1004, normalized size = 2.86

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(5/2)),x]
```

```
[Out] (Sec[e + f*x]^2*(Cos[f*x] + I*Sin[f*x])^2*Sqrt[Sec[e + f*x]*(c*Cos[e + f*x]
 + d*Sin[e + f*x])]*(((I/16)*(4*c + (15*I)*d)*Cos[2*f*x])/(c + I*d)^4 + (((
```

```
9*I)*c^4*Cos[e] - 24*c^3*d*Cos[e] + (75*I)*c^2*d^2*Cos[e] + 458*c*d^3*Cos[e
] - (192*I)*d^4*Cos[e] + (9*I)*c^3*d*Sin[e] - 24*c^2*d^2*Sin[e] + (75*I)*c*
d^3*Sin[e] + 10*d^4*Sin[e])*(Cos[2*e]/48 + (I/48)*Sin[2*e]))/((c - I*d)^2*(
c + I*d)^4*(c*Cos[e] + d*Sin[e])) + (Cos[4*f*x]*((I/16)*Cos[2*e] + Sin[2*e]
/16))/(c + I*d)^3 + ((4*c + (15*I)*d)*Sin[2*f*x])/(16*(c + I*d)^4) + ((Cos[
2*e]/16 - (I/16)*Sin[2*e])*Sin[4*f*x])/(c + I*d)^3 + ((2*d^5*Cos[2*e])/3 +
((2*I)/3)*d^5*Sin[2*e])/((c - I*d)^2*(c + I*d)^4*(c*Cos[e + f*x] + d*Sin[e
+ f*x])^2) - (4*(((7*I)/2)*c*d^4*Cos[2*e - f*x] + (3*d^5*Cos[2*e - f*x])/2
- ((7*I)/2)*c*d^4*Cos[2*e + f*x] - (3*d^5*Cos[2*e + f*x])/2 - (7*c*d^4*Sin[
2*e - f*x])/2 + ((3*I)/2)*d^5*Sin[2*e - f*x] + (7*c*d^4*Sin[2*e + f*x])/2 -
 ((3*I)/2)*d^5*Sin[2*e + f*x]))/(3*(c - I*d)^2*(c + I*d)^4*(c*Cos[e] + d*Si
n[e])*(c*Cos[e + f*x] + d*Sin[e + f*x]))))/(f*(a + I*a*Tan[e + f*x])^2) + (
Sec[e + f*x]^2*(Cos[2*e] + I*Sin[2*e])*(Cos[f*x] + I*Sin[f*x])^2*(((-I)*(4*
c^4 + (18*I)*c^3*d - 33*c^2*d^2 + (72*I)*c*d^3 + 49*d^4)*(ArcTan[Sqrt[c + d
*Tan[e + f*x]]/Sqrt[-c - I*d]]/Sqrt[-c - I*d] - ArcTan[Sqrt[c + d*Tan[e + f
*x]]/Sqrt[-c + I*d]]/Sqrt[-c + I*d])*Sec[e + f*x]*(c + d*Tan[e + f*x]))/((c
*Cos[e + f*x] + d*Sin[e + f*x])*(1 + Tan[e + f*x]^2)) + (2*(2*c^3*d + (9*I)
*c^2*d^2 + 88*c*d^3 - (45*I)*d^4)*(ArcTan[Sqrt[c + d*Tan[e + f*x]]/Sqrt[-c
- I*d]]/(2*Sqrt[-c - I*d]) + ArcTan[Sqrt[c + d*Tan[e + f*x]]/Sqrt[-c + I*d]
]/(2*Sqrt[-c + I*d]))*Sec[e + f*x]*(c + d*Tan[e + f*x]))/((c*Cos[e + f*x] +
 d*Sin[e + f*x])*(1 + Tan[e + f*x]^2))))/(16*(c - I*d)^2*(c + I*d)^4*f*(a +
 I*a*Tan[e + f*x])^2)
```

Maple [A]

time = 0.48, size = 518, normalized size = 1.48

method	result
derivativedivides	$2d^3\left(i\left(\frac{-\frac{d\left(2ic^6-9ic^4d^2-24ic^2d^4-13id^6-15c^5d-30c^3d^3-15c\,d^5\right)(c+d\tan(fx+e))^{\frac{3}{2}}}{2\left(2icd+c^2-d^2\right)}+\frac{d\left(2ic^7-28ic^5d^2-62ic^3d^4-32ic\,d^6-19c^6\right.}{}}{(-d\tan(fx+e)+id)^2}\right.\right.$
default	$2d^3\left(i\left(\frac{-\frac{d\left(2ic^6-9ic^4d^2-24ic^2d^4-13id^6-15c^5d-30c^3d^3-15c\,d^5\right)(c+d\tan(fx+e))^{\frac{3}{2}}}{2\left(2icd+c^2-d^2\right)}+\frac{d\left(2ic^7-28ic^5d^2-62ic^3d^4-32ic\,d^6-19c^6\right.}{}}{(-d\tan(fx+e)+id)^2}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)

[Out] 2/f/a^2*d^3*(1/8*I/(I*d-c)^2/(c+I*d)^5/d^3*((-1/2*d*(2*I*c^6-9*I*c^4*d^2-24*I*c^2*d^4-13*I*d^6-15*c^5*d-30*c^3*d^3-15*c*d^5)/(2*I*c*d+c^2-d^2)*(c+d*tan(f*x+e))^(3/2)+1/2*d*(2*I*c^7-28*I*c^5*d^2-62*I*c^3*d^4-32*I*c*d^6-19*c^6*d-23*c^4*d^3+11*c^2*d^5+15*d^7)/(2*I*c*d+c^2-d^2)*(c+d*tan(f*x+e))^(1/2))/(-d*tan(f*x+e)+I*d)^2-1/2*(-57*c^5*d^2-120*c^3*d^4-61*c*d^6+16*I*c^6*d-15*I*c^4*d^3-78*I*c^2*d^5-47*I*d^7+2*c^7)/(2*I*c*d+c^2-d^2)/(-I*d-c)^(1/2)*arctan((c+d*tan(f*x+e))^(1/2)/(-I*d-c)^(1/2)))-1/(I*d-c)^2/(c+I*d)^5*(-2*I*c*d-4*c^2-2*d^2)/(c+d*tan(f*x+e))^(1/2)-1/3*(-c^2-d^2)/(c+I*d)^4/(I*d-c)^2/(c+d*tan(f*x+e))^(3/2)+1/8*I/(I*d-c)^(5/2)/(c+I*d)^5*(5*c*d^4-10*I*c^2*d^3+I*d^5-10*c^3*d^2+5*I*c^4*d+c^5)/d^3*arctan((c+d*tan(f*x+e))^(1/2)/(I*d-c)^(1/2)))

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^(5/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3287 vs. $2(289) = 578$.
time = 10.19, size = 3287, normalized size = 9.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^(5/2),x, algorithm="fricas")

[Out] -1/96*(24*((a^2*c^8 + 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 + 4*a^2*c^2*d^6 + a^2*d^8)*f*e^(8*I*f*x + 8*I*e) + 2*(a^2*c^8 + 2*I*a^2*c^7*d + 2*a^2*c^6*d^2 + 6*I*a^2*c^5*d^3 + 6*I*a^2*c^3*d^5 - 2*a^2*c^2*d^6 + 2*I*a^2*c*d^7 - a^2*d^8)*f*e^(6*I*f*x + 6*I*e) + (a^2*c^8 + 4*I*a^2*c^7*d - 4*a^2*c^6*d^2 + 4*I*a^2*c^5*d^3 - 10*a^2*c^4*d^4 - 4*I*a^2*c^3*d^5 - 4*a^2*c^2*d^6 - 4*I*a^2*c*d^7 + a^2*d^8)*f*e^(4*I*f*x + 4*I*e))*sqrt(1/16*I/((-I*a^4*c^5 - 5*a^4*c^4*d +

10*I*a^4*c^3*d^2 + 10*a^4*c^2*d^3 - 5*I*a^4*c*d^4 - a^4*d^5)*f^2))*log(-2*(4*((I*a^2*c^3 + 3*a^2*c^2*d - 3*I*a^2*c*d^2 - a^2*d^3)*f*e^(2*I*f*x + 2*I*e) + (I*a^2*c^3 + 3*a^2*c^2*d - 3*I*a^2*c*d^2 - a^2*d^3)*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(1/16*I/((-I*a^4*c^5 - 5*a^4*c^4*d + 10*I*a^4*c^3*d^2 + 10*a^4*c^2*d^3 - 5*I*a^4*c*d^4 - a^4*d^5)*f^2)) - (c - I*d)*e^(2*I*f*x + 2*I*e) - c)*e^(-2*I*f*x - 2*I*e)) - 24*((a^2*c^8 + 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 + 4*a^2*c^2*d^6 + a^2*d^8)*f*e^(8*I*f*x + 8*I*e) + 2*(a^2*c^8 + 2*I*a^2*c^7*d + 2*a^2*c^6*d^2 + 6*I*a^2*c^5*d^3 + 6*I*a^2*c^3*d^5 - 2*a^2*c^2*d^6 + 2*I*a^2*c*d^7 - a^2*d^8)*f*e^(6*I*f*x + 6*I*e) + (a^2*c^8 + 4*I*a^2*c^7*d - 4*a^2*c^6*d^2 + 4*I*a^2*c^5*d^3 - 10*a^2*c^4*d^4 - 4*I*a^2*c^3*d^5 - 4*a^2*c^2*d^6 - 4*I*a^2*c*d^7 + a^2*d^8)*f*e^(4*I*f*x + 4*I*e))*sqrt(1/16*I/((-I*a^4*c^5 - 5*a^4*c^4*d + 10*I*a^4*c^3*d^2 + 10*a^4*c^2*d^3 - 5*I*a^4*c*d^4 - a^4*d^5)*f^2))*log(-2*(4*((-I*a^2*c^3 - 3*a^2*c^2*d + 3*I*a^2*c*d^2 + a^2*d^3)*f*e^(2*I*f*x + 2*I*e) + (-I*a^2*c^3 - 3*a^2*c^2*d + 3*I*a^2*c*d^2 + a^2*d^3)*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(1/16*I/((-I*a^4*c^5 - 5*a^4*c^4*d + 10*I*a^4*c^3*d^2 + 10*a^4*c^2*d^3 - 5*I*a^4*c*d^4 - a^4*d^5)*f^2)) - (c - I*d)*e^(2*I*f*x + 2*I*e) - c)*e^(-2*I*f*x - 2*I*e)) - 3*((a^2*c^8 + 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 + 4*a^2*c^2*d^6 + a^2*d^8)*f*e^(8*I*f*x + 8*I*e) + 2*(a^2*c^8 + 2*I*a^2*c^7*d + 2*a^2*c^6*d^2 + 6*I*a^2*c^5*d^3 + 6*I*a^2*c^3*d^5 - 2*a^2*c^2*d^6 + 2*I*a^2*c*d^7 - a^2*d^8)*f*e^(6*I*f*x + 6*I*e) + (a^2*c^8 + 4*I*a^2*c^7*d - 4*a^2*c^6*d^2 + 4*I*a^2*c^5*d^3 - 10*a^2*c^4*d^4 - 4*I*a^2*c^3*d^5 - 4*a^2*c^2*d^6 - 4*I*a^2*c*d^7 + a^2*d^8)*f*e^(4*I*f*x + 4*I*e))*sqrt(-(4*I*c^4 - 56*c^3*d - 384*I*c^2*d^2 + 1316*c*d^3 + 2209*I*d^4)/((I*a^4*c^9 - 9*a^4*c^8*d - 36*I*a^4*c^7*d^2 + 84*a^4*c^6*d^3 + 126*I*a^4*c^5*d^4 - 126*a^4*c^4*d^5 - 84*I*a^4*c^3*d^6 + 36*a^4*c^2*d^7 + 9*I*a^4*c*d^8 - a^4*d^9)*f^2))*log(-1/8*(2*c^3 + 16*I*c^2*d - 61*c*d^2 - 47*I*d^3 - ((I*a^2*c^5 - 5*a^2*c^4*d - 10*I*a^2*c^3*d^2 + 10*a^2*c^2*d^3 + 5*I*a^2*c*d^4 - a^2*d^5)*f*e^(2*I*f*x + 2*I*e) + (I*a^2*c^5 - 5*a^2*c^4*d - 10*I*a^2*c^3*d^2 + 10*a^2*c^2*d^3 + 5*I*a^2*c*d^4 - a^2*d^5)*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(4*I*c^4 - 56*c^3*d - 384*I*c^2*d^2 + 1316*c*d^3 + 2209*I*d^4)/((I*a^4*c^9 - 9*a^4*c^8*d - 36*I*a^4*c^7*d^2 + 84*a^4*c^6*d^3 + 126*I*a^4*c^5*d^4 - 126*a^4*c^4*d^5 - 84*I*a^4*c^3*d^6 + 36*a^4*c^2*d^7 + 9*I*a^4*c*d^8 - a^4*d^9)*f^2)) + (2*c^3 + 14*I*c^2*d - 47*c*d^2)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/((I*a^2*c^5 - 5*a^2*c^4*d - 10*I*a^2*c^3*d^2 + 10*a^2*c^2*d^3 + 5*I*a^2*c*d^4 - a^2*d^5)*f)) + 3*((a^2*c^8 + 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 + 4*a^2*c^2*d^6 + a^2*d^8)*f*e^(8*I*f*x + 8*I*e) + 2*(a^2*c^8 + 2*I*a^2*c^7*d + 2*a^2*c^6*d^2 + 6*I*a^2*c^5*d^3 + 6*I*a^2*c^3*d^5 - 2*a^2*c^2*d^6 + 2*I*a^2*c*d^7 - a^2*d^8)*f*e^(6*I*f*x + 6*I*e) + (a^2*c^8 + 4*I*a^2*c^7*d - 4*a^2*c^6*d^2 + 4*I*a^2*c^5*d^3 - 10*a^2*c^4*d^4 - 4*I*a^2*c^3*d^5 - 4*a^2*c^2*d^6 - 4*I*a^2*c*d^7 + a^2*d^8)*f*e^(4*I*f*x + 4*I*e))*sqrt(-(4*I*c^4 - 56*c^3*d - 384*I*c^2*d^2 + 1316*c*d^3 + 2209*I*d^4)/((I*a^4*c^9 - 9*a^4*c^8*d - 36*I*a^4*c^7*d^2 + 84*a^4*c^6*d^3 + 126*I*a^4*c^5*d^4 - 126*a^4*c^4*d^5 - 84*I*a^4*c^3*d^6 + 36*a^4*c^2*d^7 + 9*I*a^4*c*d^8 - a^4*d^9)*f^2))*log(-1/8

```
*(2*c^3 + 16*I*c^2*d - 61*c*d^2 - 47*I*d^3 - ((-I*a^2*c^5 + 5*a^2*c^4*d + 1
0*I*a^2*c^3*d^2 - 10*a^2*c^2*d^3 - 5*I*a^2*c*d^4 + a^2*d^5)*f*e^(2*I*f*x +
2*I*e) + (-I*a^2*c^5 + 5*a^2*c^4*d + 10*I*a^2*c^3*d^2 - 10*a^2*c^2*d^3 - 5*
I*a^2*c*d^4 + a^2*d^5)*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e
^(2*I*f*x + 2*I*e) + 1))*sqrt(-(4*I*c^4 - 56*c^3*d - 384*I*c^2*d^2 + 1316*c
*d^3 + 2209*I*d^4)/((I*a^4*c^9 - 9*a^4*c^8*d - 36*I*a^4*c^7*d^2 + 84*a^4*c^
6*d^3 + 126*I*a^4*c^5*d^4 - 126*a^4*c^4*d^5 - 84*I*a^4*c^3*d^6 + 36*a^4*c^2
*d^7 + 9*I*a^4*c*d^8 - a^4*d^9)*f^2)) + (2*c^3 + 14*I*c^2*d - 47*c*d^2)*e^(
2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/((I*a^2*c^5 - 5*a^2*c^4*d - 10*I*a^2
*c^3*d^2 + 10*a^2*c^2*d^3 + 5*I*a^2*c*d^4 - a^2*d^5)*f)) - 2*(3*I*c^5 - 3*c
^4*d + 6*I*c^3*d^2 - 6*c^2*d^3 + 3*I*c*d^4 - 3*d^5 + (9*I*c^5 - 6*c^4*d + 1
14*I*c^3*d^2 + 632*c^2*d^3 - 735*I*c*d^4 - 202*d^5)*e^(8*I*f*x + 8*I*e) + (
30*I*c^5 - 45*c^4*d + 276*I*c^3*d^2 + 1090*c^2*d^3 - 402*I*c*d^4 + 103*d^5)
*e^(6*I*f*x + 6*I*e) + (36*I*c^5 - 75*c^4*d + 192*I*c^3*d^2 + 386*c^2*d^3 +
 348*I*c*d^4 + 269*d^5)*e^(4*I*f*x + 4*I*e) - 3*(-6*I*c^5 + 13*c^4*d - 12*I
*c^3*d^2 + 26*c^2*d^3 - 6*I*c*d^4 + 13*d^5)*e^(...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**2/(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 701 vs. $2(289) = 578$.
time = 1.33, size = 701, normalized size = 2.00

$$-\frac{(2ic^2-14cd-47id^2)\arctan\left(\frac{2\left(\sqrt{d\tan(fx+e)+c}\,c-\sqrt{c^2+d^2}\sqrt{d\tan(fx+e)+c}\right)}{c\sqrt{-2c+2\sqrt{c^2+d^2}}+i\sqrt{-2c+2\sqrt{c^2+d^2}}\,d-\sqrt{c^2+d^2}\sqrt{-2c+2\sqrt{c^2+d^2}}}\right)}{4\,(a^2c^4f+4ia^2c^3df-6a^2c^2d^2f-4ia^2cd^3f+a^2d^4f)\sqrt{-2c+2\sqrt{c^2+d^2}}\left(\frac{id}{c-\sqrt{c^2+d^2}}+1\right)}+\frac{2\,(12\,(d\tan(fx+e)+c)cd^3+c^2d^3-6\,(id\tan(fx+e)+ic)d^4+d^5)}{3\,(a^2c^6f+2ia^2c^5df+a^2c^4d^2f+4ia^2c^3d^3f-a^2c^2d^4f+2ia^2cd^5f-a^2d^6f)(d\tan(fx+e)+c)^{\frac{3}{2}}}-\frac{2\arctan\left(\frac{2\left(\sqrt{d\tan(fx+e)+c}\,c-\sqrt{c^2+d^2}\sqrt{d\tan(fx+e)+c}\right)}{c\sqrt{-2c+2\sqrt{c^2+d^2}}-i\sqrt{-2c+2\sqrt{c^2+d^2}}\,d-\sqrt{c^2+d^2}\sqrt{-2c+2\sqrt{c^2+d^2}}}\right)}{-4\,(-ia^2c^2f-2a^2cdf+ia^2d^2f)\sqrt{-2c+2\sqrt{c^2+d^2}}\left(-\frac{id}{c-\sqrt{c^2+d^2}}+1\right)}+\frac{2\,(d\tan(fx+e)+c)^{\frac{3}{2}}cd-2\sqrt{d\tan(fx+e)+c}\,c^2d+13i\,(d\tan(fx+e)+c)^{\frac{3}{2}}d^2-17i\sqrt{d\tan(fx+e)+c}\,cd^2+15\sqrt{d\tan(fx+e)+c}\,d^3}{8\,(a^2c^4f+4ia^2c^3df-6a^2c^2d^2f-4ia^2cd^3f+a^2d^4f)(d\tan(fx+e)-id)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] -1/4*(2*I*c^2 - 14*c*d - 47*I*d^2)*arctan(2*(sqrt(d*tan(f*x + e) + c)*c - s
qrt(c^2 + d^2)*sqrt(d*tan(f*x + e) + c))/(c*sqrt(-2*c + 2*sqrt(c^2 + d^2))
+ I*sqrt(-2*c + 2*sqrt(c^2 + d^2))*d - sqrt(c^2 + d^2)*sqrt(-2*c + 2*sqrt(c
^2 + d^2))))/((a^2*c^4*f + 4*I*a^2*c^3*d*f - 6*a^2*c^2*d^2*f - 4*I*a^2*c*d^
3*f + a^2*d^4*f)*sqrt(-2*c + 2*sqrt(c^2 + d^2))*(I*d/(c - sqrt(c^2 + d^2))
+ 1)) + 2/3*(12*(d*tan(f*x + e) + c)*c*d^3 + c^2*d^3 - 6*(I*d*tan(f*x + e)
+ I*c)*d^4 + d^5)/((a^2*c^6*f + 2*I*a^2*c^5*d*f + a^2*c^4*d^2*f + 4*I*a^2*c
^3*d^3*f - a^2*c^2*d^4*f + 2*I*a^2*c*d^5*f - a^2*d^6*f)*(d*tan(f*x + e) + c
)^(3/2)) - 2*arctan(2*(sqrt(d*tan(f*x + e) + c)*c - sqrt(c^2 + d^2)*sqrt(d*
```

```
tan(f*x + e) + c))/(c*sqrt(-2*c + 2*sqrt(c^2 + d^2)) - I*sqrt(-2*c + 2*sqrt
(c^2 + d^2))*d - sqrt(c^2 + d^2)*sqrt(-2*c + 2*sqrt(c^2 + d^2))))/((4*I*a^2
*c^2*f + 8*a^2*c*d*f - 4*I*a^2*d^2*f)*sqrt(-2*c + 2*sqrt(c^2 + d^2))*(-I*d/
(c - sqrt(c^2 + d^2)) + 1)) + 1/8*(2*(d*tan(f*x + e) + c)^(3/2)*c*d - 2*sqr
t(d*tan(f*x + e) + c)*c^2*d + 13*I*(d*tan(f*x + e) + c)^(3/2)*d^2 - 17*I*sq
rt(d*tan(f*x + e) + c)*c*d^2 + 15*sqrt(d*tan(f*x + e) + c)*d^3)/((a^2*c^4*f
 + 4*I*a^2*c^3*d*f - 6*a^2*c^2*d^2*f - 4*I*a^2*c*d^3*f + a^2*d^4*f)*(d*tan(
f*x + e) - I*d)^2)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^2*(c + d*tan(e + f*x))^(5/2)),x)
[Out] \text{Hanged}
```

3.1136 $\int \frac{1}{(a+ia\tan(e+fx))^3(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=446

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{8a^3(c-id)^{5/2}f}+\frac{(2ic^3-16c^2d-61icd^2+152d^3)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{16a^3(c+id)^{11/2}f}+\frac{}{48a}$$

```
[Out] -1/8*I*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/a^3/(c-I*d)^(5/2)/f+1/
16*(2*I*c^3-16*c^2*d-61*I*c*d^2+152*d^3)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+
I*d)^(1/2))/a^3/(c+I*d)^(11/2)/f+1/16*d*(2*c^4+11*I*c^3*d-26*c^2*d^2+253*I*
c*d^3+150*d^4)/a^3/(c-I*d)^2/(c+I*d)^5/f/(c+d*tan(f*x+e))^(1/2)+1/48*d*(6*c
^3+33*I*c^2*d-83*c*d^2+154*I*d^3)/a^3/(c-I*d)/(c+I*d)^4/f/(c+d*tan(f*x+e))^
(3/2)-1/6/(I*c-d)/f/(a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^(3/2)+1/8*(I*c-4*
d)/a/(c+I*d)^2/f/(a+I*a*tan(f*x+e))^2/(c+d*tan(f*x+e))^(3/2)+1/16*(2*c^2+11
*I*c*d-30*d^2)/(I*c-d)^3/f/(a^3+I*a^3*tan(f*x+e))/(c+d*tan(f*x+e))^(3/2)
```

Rubi [A]

time = 1.07, antiderivative size = 446, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3640, 3677, 3610, 3620, 3618, 65, 214}

$$\frac{2c^2+11icd-30d^2}{16f(-d+ic)^3\,(a^3+ia^3\tan(e+fx))\,(c+d\tan(e+fx))^{3/2}}+\frac{d(6c^3+33ic^2d-83cd^2+154id^3)}{48a^3f(c-id)(c+id)^4(c+d\tan(e+fx))^{3/2}}+\frac{(2ic^3-16c^2d-61icd^2+152d^3)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{16a^3f(c+id)^{11/2}}+\frac{d(2c^4+11ic^3d-26c^2d^2+253icd^3+150d^4)}{16a^3f(c-id)^2(c+id)^5\sqrt{c+d\tan(e+fx)}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{8a^3f(c-id)^{5/2}}+\frac{-4d+ic}{8af(c+id)^2(a+ia\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}-\frac{1}{6f(-d+ic)(a+ia\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^(5/2)),x]
```

```
[Out] ((-1/8*I)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/(a^3*(c - I*d)^(
5/2)*f) + (((2*I)*c^3 - 16*c^2*d - (61*I)*c*d^2 + 152*d^3)*ArcTanh[Sqrt[c +
 d*Tan[e + f*x]]/Sqrt[c + I*d]])/(16*a^3*(c + I*d)^(11/2)*f) + (d*(6*c^3 +
(33*I)*c^2*d - 83*c*d^2 + (154*I)*d^3))/(48*a^3*(c - I*d)*(c + I*d)^4*f*(c
+ d*Tan[e + f*x])^(3/2)) - 1/(6*(I*c - d)*f*(a + I*a*Tan[e + f*x])^3*(c + d
*Tan[e + f*x])^(3/2)) + (I*c - 4*d)/(8*a*(c + I*d)^2*f*(a + I*a*Tan[e + f*x
])^2*(c + d*Tan[e + f*x])^(3/2)) + (2*c^2 + (11*I)*c*d - 30*d^2)/(16*(I*c -
 d)^3*f*(a^3 + I*a^3*Tan[e + f*x])*(c + d*Tan[e + f*x])^(3/2)) + (d*(2*c^4
+ (11*I)*c^3*d - 26*c^2*d^2 + (253*I)*c*d^3 + 150*d^4))/(16*a^3*(c - I*d)^2
*(c + I*d)^5*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^3(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{1}{6(ic-d)f(a+ia\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}}$$

$$= -\frac{1}{6(ic-d)f(a+ia\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}}$$

$$= -\frac{1}{6(ic-d)f(a+ia\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}}$$

$$= \frac{d(6c^3+33ic^2d-83cd^2+154id^3)}{48a^3(c-id)(c+id)^4f(c+d\tan(e+fx))^{3/2}} - \frac{}{6(ic-d}$$

$$= \frac{d(6c^3+33ic^2d-83cd^2+154id^3)}{48a^3(c-id)(c+id)^4f(c+d\tan(e+fx))^{3/2}} - \frac{}{6(ic-d}$$

$$= \frac{d(6c^3+33ic^2d-83cd^2+154id^3)}{48a^3(c-id)(c+id)^4f(c+d\tan(e+fx))^{3/2}} - \frac{}{6(ic-d}$$

$$= \frac{d(6c^3+33ic^2d-83cd^2+154id^3)}{48a^3(c-id)(c+id)^4f(c+d\tan(e+fx))^{3/2}} - \frac{}{6(ic-d}$$

$$= \frac{d(6c^3+33ic^2d-83cd^2+154id^3)}{48a^3(c-id)(c+id)^4f(c+d\tan(e+fx))^{3/2}} - \frac{}{6(ic-d}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{8a^3(c-id)^{5/2}f} + \frac{(2ic^3-16c^2d-6}{}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1160 vs. 2(446) = 892.
time = 10.93, size = 1160, normalized size = 2.60

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^(5/2)),x]
```

[Out] (Sec[e + f*x]^3*(Cos[f*x] + I*Sin[f*x])^3*Sqrt[Sec[e + f*x]*(c*Cos[e + f*x] + d*Sin[e + f*x])]*(((18*c^2 + (103*I)*c*d - 208*d^2)*Cos[2*f*x]*((I/96)*Cos[e] - Sin[e]/96))/(c + I*d)^5 + ((9*c + (26*I)*d)*Cos[4*f*x]*((I/96)*Cos[e] + Sin[e]/96))/(c + I*d)^4 + (((11*I)*c^5*Cos[e] - 50*c^4*d*Cos[e] - (51*I)*c^3*d^2*Cos[e] - 296*c^2*d^3*Cos[e] + (1208*I)*c*d^4*Cos[e] + 576*d^5*Cos[e] + (11*I)*c^4*d*Sin[e] - 50*c^3*d^2*Sin[e] - (51*I)*c^2*d^3*Sin[e] - 296*c*d^4*Sin[e] + (120*I)*d^5*Sin[e])*(Cos[3*e]/96 + (I/96)*Sin[3*e]))/((c - I*d)^2*(c + I*d)^5*(c*Cos[e] + d*Sin[e])) + (Cos[6*f*x]*((I/48)*Cos[3*e] + Sin[3*e]/48))/(c + I*d)^3 + ((18*c^2 + (103*I)*c*d - 208*d^2)*(Cos[e]/96 + (I/96)*Sin[e])*Sin[2*f*x])/(c + I*d)^5 + ((9*c + (26*I)*d)*(Cos[e]/96 - (I/96)*Sin[e])*Sin[4*f*x])/(c + I*d)^4 + ((Cos[3*e]/48 - (I/48)*Sin[3*e])*Sin[6*f*x])/(c + I*d)^3 + (((2*I)/3)*d^6*Cos[3*e] - (2*d^6*Sin[3*e])/3)/((c - I*d)^2*(c + I*d)^5*(c*Cos[e + f*x] + d*Sin[e + f*x])^2) + (2*((17*c*d^5*Cos[3*e - f*x])/2 - ((9*I)/2)*d^6*Cos[3*e - f*x] - (17*c*d^5*Cos[3*e + f*x])/2 + ((9*I)/2)*d^6*Cos[3*e + f*x] + ((17*I)/2)*c*d^5*Sin[3*e - f*x] + (9*d^6*Sin[3*e - f*x])/2 - ((17*I)/2)*c*d^5*Sin[3*e + f*x] - (9*d^6*Sin[3*e + f*x])/2))/(3*(c - I*d)^2*(c + I*d)^5*(c*Cos[e] + d*Sin[e])*(c*Cos[e + f*x] + d*Sin[e + f*x]))))/(f*(a + I*a*Tan[e + f*x])^3) + (Sec[e + f*x]^3*(Cos[3*e] + I*Sin[3*e])*(Cos[f*x] + I*Sin[f*x])^3*(((-I)*(4*c^5 + (22*I)*c^4*d - 51*c^3*d^2 - (66*I)*c^2*d^3 - 233*c*d^4 + (154*I)*d^5)*(ArcTan[Sqrt[c + d*Tan[e + f*x]]/Sqrt[-c - I*d]]/Sqrt[-c - I*d] - ArcTan[Sqrt[c + d*Tan[e + f*x]]/Sqrt[-c + I*d]]/Sqrt[-c + I*d])*Sec[e + f*x]*(c + d*Tan[e + f*x]))/((c*Cos[e + f*x] + d*Sin[e + f*x])*(1 + Tan[e + f*x]^2)) + (2*(2*c^4*d + (11*I)*c^3*d^2 - 26*c^2*d^3 + (253*I)*c*d^4 + 150*d^5)*(ArcTan[Sqrt[c + d*Tan[e + f*x]]/Sqrt[-c - I*d]]/(2*Sqrt[-c - I*d]) + ArcTan[Sqrt[c + d*Tan[e + f*x]]/Sqrt[-c + I*d]]/(2*Sqrt[-c + I*d]))*Sec[e + f*x]*(c + d*Tan[e + f*x]))/((c*Cos[e + f*x] + d*Sin[e + f*x])*(1 + Tan[e + f*x]^2))))/(32*(c - I*d)^2*(c + I*d)^5*f*(a + I*a*Tan[e + f*x])^3)

Maple [A]

time = 0.42, size = 743, normalized size = 1.67

method	result
derivativedivides	$\frac{2d^4\left(\frac{\left(ic^6-15ic^4d^2+15ic^2d^4-id^6-6c^5d+20c^3d^3-6c\,d^5\right)\arctan\left(\frac{\sqrt{c+d\tan(fx+e)}}{\sqrt{id-c}}\right)}{16(id-c)^{\frac{5}{2}}(id+c)^6d^4}+\frac{i\left(\frac{d\left(2ic^8-82ic^6d^2-116ic^4 \cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$

default	$2d^4\left(\frac{\left(ic^6-15ic^4d^2+15ic^2d^4-id^6-6c^5d+20c^3d^3-6c\,d^5\right)\arctan\left(\frac{\sqrt{c+d\tan(fx+e)}}{\sqrt{id-c}}\right)}{16(id-c)^{\frac{5}{2}}(id+c)^6d^4}+\frac{i\left(\frac{d\left(2ic^8-82ic^6d^2-116ic^4d\right.}{}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3*d^4*(1/16/(I*d-c)^(5/2)/(c+I*d)^6*(-15*I*c^4*d^2+15*I*c^2*d^4+20*c^
3*d^3-6*c*d^5+I*c^6-I*d^6-6*c^5*d)/d^4*arctan((c+d*tan(f*x+e))^(1/2)/(I*d-c
)^(1/2))+1/16*I/(I*d-c)^2/(c+I*d)^6/d^4*((1/2*d*(2*I*c^8-82*I*c^6*d^2-116*I
*c^4*d^4+22*I*c^2*d^6+54*I*d^8-19*c^7*d+85*c^5*d^3+227*c^3*d^5+123*c*d^7)/(
3*I*c^2*d-I*d^3+c^3-3*c*d^2)*(c+d*tan(f*x+e))^(5/2)-2/3*d*(-33*c^8*d+282*c^
6*d^3+572*c^4*d^5+166*c^2*d^7-91*d^9+3*I*c^9-166*I*c^7*d^2-44*I*c^5*d^4+422
*I*c^3*d^6+297*I*c*d^8)/(3*I*c^2*d-I*d^3+c^3-3*c*d^2)*(c+d*tan(f*x+e))^(3/2
)+1/2*d*(2*I*c^10-146*I*c^8*d^2+192*I*c^6*d^4+760*I*c^4*d^6+350*I*c^2*d^8-7
0*I*d^10-25*c^9*d+340*c^7*d^3+458*c^5*d^5-204*c^3*d^7-297*c*d^9)/(3*I*c^2*d
-I*d^3+c^3-3*c*d^2)*(c+d*tan(f*x+e))^(1/2))/(-d*tan(f*x+e)+I*d)^3-1/2*(-91*
c^7*d^2+177*c^5*d^4+635*c^3*d^6+365*c*d^8+20*I*c^8*d-250*I*c^6*d^3-408*I*c^
4*d^5+14*I*c^2*d^7+152*I*d^9+2*c^9)/(3*I*c^2*d-I*d^3+c^3-3*c*d^2)/(-I*d-c)^
(1/2)*arctan((c+d*tan(f*x+e))^(1/2)/(-I*d-c)^(1/2)))-1/(I*d-c)^2/(c+I*d)^6*
(-5*I*c^2-3*I*d^2+2*c*d)/(c+d*tan(f*x+e))^(1/2)-1/3/(I*d-c)^2/(c+I*d)^6*(-I
*c^3-I*c*d^2+c^2*d+d^3)/(c+d*tan(f*x+e))^(3/2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3845 vs. $2(374) = 748$.
time = 20.58, size = 3845, normalized size = 8.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^(5/2),x, algorithm="frica
s")

[Out] 1/192*(48*((-I*a^3*c^9 + a^3*c^8*d - 4*I*a^3*c^7*d^2 + 4*a^3*c^6*d^3 - 6*I*
a^3*c^5*d^4 + 6*a^3*c^4*d^5 - 4*I*a^3*c^3*d^6 + 4*a^3*c^2*d^7 - I*a^3*c*d^8
 + a^3*d^9)*f*e^(10*I*f*x + 10*I*e) + 2*(-I*a^3*c^9 + 3*a^3*c^8*d + 8*a^3*c
^6*d^3 + 6*I*a^3*c^5*d^4 + 6*a^3*c^4*d^5 + 8*I*a^3*c^3*d^6 + 3*I*a^3*c*d^8
- a^3*d^9)*f*e^(8*I*f*x + 8*I*e) + (-I*a^3*c^9 + 5*a^3*c^8*d + 8*I*a^3*c^7*
d^2 + 14*I*a^3*c^5*d^4 - 14*a^3*c^4*d^5 - 8*a^3*c^2*d^7 - 5*I*a^3*c*d^8 + a
^3*d^9)*f*e^(6*I*f*x + 6*I*e))*sqrt(1/64*I/((-I*a^6*c^5 - 5*a^6*c^4*d + 10*
I*a^6*c^3*d^2 + 10*a^6*c^2*d^3 - 5*I*a^6*c*d^4 - a^6*d^5)*f^2))*log(-2*(8*(
(I*a^3*c^3 + 3*a^3*c^2*d - 3*I*a^3*c*d^2 - a^3*d^3)*f*e^(2*I*f*x + 2*I*e) +
 (I*a^3*c^3 + 3*a^3*c^2*d - 3*I*a^3*c*d^2 - a^3*d^3)*f)*sqrt(((c - I*d)*e^(
2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(1/64*I/((-I*a^6
*c^5 - 5*a^6*c^4*d + 10*I*a^6*c^3*d^2 + 10*a^6*c^2*d^3 - 5*I*a^6*c*d^4 - a^
6*d^5)*f^2)) - (c - I*d)*e^(2*I*f*x + 2*I*e) - c)*e^(-2*I*f*x - 2*I*e)) + 4
8*((I*a^3*c^9 - a^3*c^8*d + 4*I*a^3*c^7*d^2 - 4*a^3*c^6*d^3 + 6*I*a^3*c^5*d
^4 - 6*a^3*c^4*d^5 + 4*I*a^3*c^3*d^6 - 4*a^3*c^2*d^7 + I*a^3*c*d^8 - a^3*d^
9)*f*e^(10*I*f*x + 10*I*e) + 2*(I*a^3*c^9 - 3*a^3*c^8*d - 8*a^3*c^6*d^3 - 6
*I*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 8*I*a^3*c^3*d^6 - 3*I*a^3*c*d^8 + a^3*d^9)
*f*e^(8*I*f*x + 8*I*e) + (I*a^3*c^9 - 5*a^3*c^8*d - 8*I*a^3*c^7*d^2 - 14*I*
a^3*c^5*d^4 + 14*a^3*c^4*d^5 + 8*a^3*c^2*d^7 + 5*I*a^3*c*d^8 - a^3*d^9)*f*e
^(6*I*f*x + 6*I*e))*sqrt(1/64*I/((-I*a^6*c^5 - 5*a^6*c^4*d + 10*I*a^6*c^3*d
^2 + 10*a^6*c^2*d^3 - 5*I*a^6*c*d^4 - a^6*d^5)*f^2))*log(-2*(8*((-I*a^3*c^3
 - 3*a^3*c^2*d + 3*I*a^3*c*d^2 + a^3*d^3)*f*e^(2*I*f*x + 2*I*e) + (-I*a^3*c
^3 - 3*a^3*c^2*d + 3*I*a^3*c*d^2 + a^3*d^3)*f)*sqrt(((c - I*d)*e^(2*I*f*x +
 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(1/64*I/((-I*a^6*c^5 - 5*
a^6*c^4*d + 10*I*a^6*c^3*d^2 + 10*a^6*c^2*d^3 - 5*I*a^6*c*d^4 - a^6*d^5)*f^
2)) - (c - I*d)*e^(2*I*f*x + 2*I*e) - c)*e^(-2*I*f*x - 2*I*e)) + 3*((-I*a^3
*c^9 + a^3*c^8*d - 4*I*a^3*c^7*d^2 + 4*a^3*c^6*d^3 - 6*I*a^3*c^5*d^4 + 6*a^
3*c^4*d^5 - 4*I*a^3*c^3*d^6 + 4*a^3*c^2*d^7 - I*a^3*c*d^8 + a^3*d^9)*f*e^(1
0*I*f*x + 10*I*e) + 2*(-I*a^3*c^9 + 3*a^3*c^8*d + 8*a^3*c^6*d^3 + 6*I*a^3*c
^5*d^4 + 6*a^3*c^4*d^5 + 8*I*a^3*c^3*d^6 + 3*I*a^3*c*d^8 - a^3*d^9)*f*e^(8*
I*f*x + 8*I*e) + (-I*a^3*c^9 + 5*a^3*c^8*d + 8*I*a^3*c^7*d^2 + 14*I*a^3*c^5
*d^4 - 14*a^3*c^4*d^5 - 8*a^3*c^2*d^7 - 5*I*a^3*c*d^8 + a^3*d^9)*f*e^(6*I*f
*x + 6*I*e))*sqrt(-(-4*I*c^6 + 64*c^5*d + 500*I*c^4*d^2 - 2560*c^3*d^3 - 85
85*I*c^2*d^4 + 18544*c*d^5 + 23104*I*d^6)/((-I*a^6*c^11 + 11*a^6*c^10*d + 5
5*I*a^6*c^9*d^2 - 165*a^6*c^8*d^3 - 330*I*a^6*c^7*d^4 + 462*a^6*c^6*d^5 + 4
62*I*a^6*c^5*d^6 - 330*a^6*c^4*d^7 - 165*I*a^6*c^3*d^8 + 55*a^6*c^2*d^9 + 1
1*I*a^6*c*d^10 - a^6*d^11)*f^2))*log(-1/16*(-2*I*c^4 + 18*c^3*d + 77*I*c^2*
d^2 - 213*c*d^3 - 152*I*d^4 + ((a^3*c^6 + 6*I*a^3*c^5*d - 15*a^3*c^4*d^2 -
20*I*a^3*c^3*d^3 + 15*a^3*c^2*d^4 + 6*I*a^3*c*d^5 - a^3*d^6)*f*e^(2*I*f*x +
 2*I*e) + (a^3*c^6 + 6*I*a^3*c^5*d - 15*a^3*c^4*d^2 - 20*I*a^3*c^3*d^3 + 15
```

```
*a^3*c^2*d^4 + 6*I*a^3*c*d^5 - a^3*d^6)*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I
*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(-4*I*c^6 + 64*c^5*d + 500*
I*c^4*d^2 - 2560*c^3*d^3 - 8585*I*c^2*d^4 + 18544*c*d^5 + 23104*I*d^6)/((-I
*a^6*c^11 + 11*a^6*c^10*d + 55*I*a^6*c^9*d^2 - 165*a^6*c^8*d^3 - 330*I*a^6*
c^7*d^4 + 462*a^6*c^6*d^5 + 462*I*a^6*c^5*d^6 - 330*a^6*c^4*d^7 - 165*I*a^6
*c^3*d^8 + 55*a^6*c^2*d^9 + 11*I*a^6*c*d^10 - a^6*d^11)*f^2)) + (-2*I*c^4 +
 16*c^3*d + 61*I*c^2*d^2 - 152*c*d^3)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*
I*e)/((a^3*c^6 + 6*I*a^3*c^5*d - 15*a^3*c^4*d^2 - 20*I*a^3*c^3*d^3 + 15*a^3
*c^2*d^4 + 6*I*a^3*c*d^5 - a^3*d^6)*f)) + 3*((I*a^3*c^9 - a^3*c^8*d + 4*I*a
^3*c^7*d^2 - 4*a^3*c^6*d^3 + 6*I*a^3*c^5*d^4 - 6*a^3*c^4*d^5 + 4*I*a^3*c^3*
d^6 - 4*a^3*c^2*d^7 + I*a^3*c*d^8 - a^3*d^9)*f*e^(10*I*f*x + 10*I*e) + 2*(I
*a^3*c^9 - 3*a^3*c^8*d - 8*a^3*c^6*d^3 - 6*I*a^3*c^5*d^4 - 6*a^3*c^4*d^5 -
8*I*a^3*c^3*d^6 - 3*I*a^3*c*d^8 + a^3*d^9)*f*e^(8*I*f*x + 8*I*e) + (I*a^3*c
^9 - 5*a^3*c^8*d - 8*I*a^3*c^7*d^2 - 14*I*a^3*c^5*d^4 + 14*a^3*c^4*d^5 + 8*
a^3*c^2*d^7 + 5*I*a^3*c*d^8 - a^3*d^9)*f*e^(6*I*f*x + 6*I*e))*sqrt(-(-4*I*c
^6 + 64*c^5*d + 500*I*c^4*d^2 - 2560*c^3*d^3 - 8585*I*c^2*d^4 + 18544*c*d^5
 + 23104*I*d^6)/((-I*a^6*c^11 + 11*a^6*c^10*d + 55*I*a^6*c^9*d^2 - 165*a^6*
c^8*d^3 - 330*I*a^6*c^7*d^4 + 462*a^6*c^6*d^5 + 462*I*a^6*c^5*d^6 - 330*a^6
*c^4*d^7 - 165*I*a^6*c^3*d^8 + 55*a^6*c^2*d^9 + 11*I*a^6*c*d^10 - a^6*d^11)
*f^2))*log(-1/16*(-2*I*c^4 + 18*c^3*d + 77*I*c^2*d^2 - 213*c*d^3 - 152*I*d^
4 - ((a^3*c^6 + 6*I*a^3*c^5*d - 15*a^3*c^4*d^2 - 20*I*a^3*c^3*d^3 + 15*a^3*
c^2*d^4 + 6*I*a^3*c*d^5 - a^3*d^6)*f*e^(2*I*f*x + 2*I*e) + (a^3*c^6 + 6*I*a
^3*c^5*d - 15*a^3*c^4*d^2 - 20*I*a^3*c^3*d^3 + 15*a^3*c^2*d^4 + 6*I*a^3*c*d
^5 - a^3*d^6)*f)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x
 + 2*I*e) + 1))*sqrt(-(-4*I*c^6 + 64*c^5*d + 500*I*c^4*d^2 - 2560*c^3*d^3 -
 8585*I*c^2*d^4 + 18544*c*d^5 + 23104*I*d^6)/((-I*a^6*c^11 + 11*a^6*c^10*d
+ 55*I*a^6*c^9*d^2 - 165*a^6*c^8*d^3 - 330*I*a^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**3/(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1061 vs. $2(374) = 748$.
time = 1.29, size = 1061, normalized size = 2.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^3/(c+d*tan(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] -1/8*(-2*I*c^3 + 16*c^2*d + 61*I*c*d^2 - 152*d^3)*arctan(-2*(sqrt(d*tan(f*x
 + e) + c)*c - sqrt(c^2 + d^2)*sqrt(d*tan(f*x + e) + c))/(c*sqrt(-2*c + 2*s
qrt(c^2 + d^2)) + I*sqrt(-2*c + 2*sqrt(c^2 + d^2))*d - sqrt(c^2 + d^2)*sqrt
(-2*c + 2*sqrt(c^2 + d^2)))))/((a^3*c^5*f + 5*I*a^3*c^4*d*f - 10*a^3*c^3*d^2
*f - 10*I*a^3*c^2*d^3*f + 5*a^3*c*d^4*f + I*a^3*d^5*f)*sqrt(-2*c + 2*sqrt(c
^2 + d^2))*(I*d/(c - sqrt(c^2 + d^2)) + 1)) + 1/4*I*arctan(2*(sqrt(d*tan(f*
x + e) + c)*c - sqrt(c^2 + d^2)*sqrt(d*tan(f*x + e) + c))/(c*sqrt(-2*c + 2*
sqrt(c^2 + d^2)) - I*sqrt(-2*c + 2*sqrt(c^2 + d^2))*d - sqrt(c^2 + d^2)*sqr
t(-2*c + 2*sqrt(c^2 + d^2)))))/((a^3*c^2*f - 2*I*a^3*c*d*f - a^3*d^2*f)*sqrt
(-2*c + 2*sqrt(c^2 + d^2))*(-I*d/(c - sqrt(c^2 + d^2)) + 1)) + 1/48*(6*I*(d
*tan(f*x + e) + c)^4*c^4*d - 12*I*(d*tan(f*x + e) + c)^3*c^5*d + 6*I*(d*tan
(f*x + e) + c)^2*c^6*d - 33*(d*tan(f*x + e) + c)^4*c^3*d^2 + 84*(d*tan(f*x
+ e) + c)^3*c^4*d^2 - 51*(d*tan(f*x + e) + c)^2*c^5*d^2 - 78*I*(d*tan(f*x +
 e) + c)^4*c^2*d^3 + 256*I*(d*tan(f*x + e) + c)^3*c^3*d^3 - 198*I*(d*tan(f*
x + e) + c)^2*c^4*d^3 - 759*(d*tan(f*x + e) + c)^4*c*d^4 + 1856*(d*tan(f*x
+ e) + c)^3*c^2*d^4 - 1446*(d*tan(f*x + e) + c)^2*c^3*d^4 + 384*(d*tan(f*x
+ e) + c)*c^4*d^4 + 32*c^5*d^4 + 450*I*(d*tan(f*x + e) + c)^4*d^5 + 844*I*(
d*tan(f*x + e) + c)^3*c*d^5 - 2334*I*(d*tan(f*x + e) + c)^2*c^2*d^5 - 960*(
-I*d*tan(f*x + e) - I*c)*c^3*d^5 + 96*I*c^4*d^5 + 1196*(d*tan(f*x + e) + c)
^3*d^6 - 243*(d*tan(f*x + e) + c)^2*c*d^6 - 576*(d*tan(f*x + e) + c)*c^2*d^
6 - 64*c^3*d^6 - 978*I*(d*tan(f*x + e) + c)^2*d^7 - 192*(-I*d*tan(f*x + e)
- I*c)*c*d^7 + 64*I*c^2*d^7 - 192*(d*tan(f*x + e) + c)*d^8 - 96*c*d^8 - 32*
I*d^9)/((a^3*c^7*f + 3*I*a^3*c^6*d*f - a^3*c^5*d^2*f + 5*I*a^3*c^4*d^3*f -
5*a^3*c^3*d^4*f + I*a^3*c^2*d^5*f - 3*a^3*c*d^6*f - I*a^3*d^7*f)*(-I*(d*tan
(f*x + e) + c)^(3/2) + I*sqrt(d*tan(f*x + e) + c)*c - sqrt(d*tan(f*x + e) +
 c)*d)^3)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^3*(c + d*tan(e + f*x))^(5/2)),x)
```

```
[Out] \text{Hanged}
```

3.1137 $\int (a+ia\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=263

$$-\frac{\sqrt[4]{-1}\,a^{5/2}(c^2+10icd+23d^2)\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}\right)}{4d^{3/2}f}-\frac{4i\sqrt{2}\,a^{5/2}\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{f}$$

```
[Out] -1/4*(-1)^(1/4)*a^(5/2)*(c^2+10*I*c*d+23*d^2)*arctanh((-1)^(3/4)*d^(1/2)*(a
+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c+d*tan(f*x+e))^(1/2))/d^(3/2)/f-4*I*a^(5/2
)*arctanh(2^(1/2)*a^(1/2)*(c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2)/(a+I*a*tan(f
*x+e))^(1/2))*2^(1/2)*(c-I*d)^(1/2)/f+1/4*a^2*(c+9*I*d)*(a+I*a*tan(f*x+e))^
(1/2)*(c+d*tan(f*x+e))^(1/2)/d/f-1/2*a^2*(a+I*a*tan(f*x+e))^(1/2)*(c+d*tan(
f*x+e))^(3/2)/d/f
```

Rubi [A]
time = 0.68, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {3637, 3678, 3682, 3625, 214, 3680, 65, 223, 212}

$$-\frac{\sqrt[4]{-1}\,a^{5/2}(c^2+10icd+23d^2)\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}\right)}{4d^{3/2}f}-\frac{4i\sqrt{2}\,a^{5/2}\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+fx)}}\right)}{f}-\frac{a^2\sqrt{a+ia\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}}{2df}+\frac{a^2(c+9id)\sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{4df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^(5/2)*Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] -1/4*((-1)^(1/4)*a^(5/2)*(c^2 + (10*I)*c*d + 23*d^2)*ArcTanh[((-1)^(3/4)*Sq
rt[d]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])])/(d^(
3/2)*f) - ((4*I)*Sqrt[2]*a^(5/2)*Sqrt[c - I*d]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqr
t[c + d*Tan[e + f*x]])/(Sqrt[c - I*d]*Sqrt[a + I*a*Tan[e + f*x]])])/f + (a^
2*(c + (9*I)*d)*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(4*d*f
) - (a^2*Sqrt[a + I*a*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))/(2*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3678

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Dist[
1/(a*(m + n)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Simp
[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int (a+ia\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}\,dx = -\frac{a^2\sqrt{a+ia\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}}{2df} + \frac{a\int \cdots}{}$$

$$= \frac{a^2(c+9id)\sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{4df} - \cdots$$

$$= \frac{a^2(c+9id)\sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{4df} - \cdots$$

$$= \frac{a^2(c+9id)\sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{4df} - \cdots$$

$$= -\frac{4i\sqrt{2}\,a^{5/2}\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+\cdots}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+\cdots}}\right.}{f}$$

$$= -\frac{4i\sqrt{2}\,a^{5/2}\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+\cdots}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+\cdots}}\right.}{f}$$

$$= -\frac{\sqrt[4]{-1}\,a^{5/2}(c^2+10icd+23d^2)\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\,\sqrt{a\cdots}}{\sqrt{a}\,\sqrt{c+\cdots}}\right.}{4d^{3/2}f}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 589 vs. $2(263) = 526$.
time = 8.00, size = 589, normalized size = 2.24

$$\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\cos^2(e+fx)(a+ia\tan(e+fx))^{5/2}\left(-\frac{\cos(e+fx)\left((c^2+10icd+23d^2)\left(\log\left(\frac{(2+2i)e^{\frac{ie}{2}}\left(-id+de^{i(e+fx)}+ic\left(i+e^{i(e+fx)}\right)-(1+i)\sqrt{d}\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)}{\sqrt{d}\,(c^2+10icd+23d^2)\left(i+e^{i(e+fx)}\right)}\right)-\log\left(\frac{(2+2i)e^{\frac{ie}{2}}\left(c+id+ice^{i(e+fx)}+de^{i(e+fx)}+(1+i)\sqrt{d}\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)}{\sqrt{d}\,(c^2+10icd+23d^2)\left(-i+e^{i(e+fx)}\right)}\right)\right)+(32+32i)\sqrt{c-id}\,d^{3/2}\log\left(2\left(\sqrt{c-id}\cos(e+fx)+i\sqrt{c-id}\sin(e+fx)+\sqrt{1+\cos(2(e+fx))+i\sin(2(e+fx))}\sqrt{c+d\tan(e+fx)}\right)\right)\right)(\cos(2e)-i\sin(2e))}{d^{3/2}\sqrt{1+\cos(2(e+fx))+i\sin(2(e+fx))}}+\frac{(1+i)(i\cos(2e)+\sin(2e))\sqrt{c+d\tan(e+fx)}(c-9id+2d\tan(e+fx))}{d}\right)}{f(\cos(fx)+i\sin(fx))^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(5/2)*Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((1/8 + I/8)*Cos[e + f*x]^2*(a + I*a*Tan[e + f*x])^(5/2)*(-((Cos[e + f*x]*(
(c^2 + (10*I)*c*d + 23*d^2)*(Log[((2 + 2*I)*E^((I/2)*e)*((-I)*d + d*E^(I*(e
 + f*x)) + I*c*(I + E^(I*(e + f*x))) - (1 + I)*Sqrt[d]*Sqrt[1 + E^((2*I)*(e
 + f*x))]*Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)
))]))/(Sqrt[d]*(c^2 + (10*I)*c*d + 23*d^2)*(I + E^(I*(e + f*x))))] - Log[((
2 + 2*I)*E^((I/2)*e)*(c + I*d + I*c*E^(I*(e + f*x)) + d*E^(I*(e + f*x)) + (
1 + I)*Sqrt[d]*Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^((2*I)*(
e + f*x))))/(1 + E^((2*I)*(e + f*x)))]))/(Sqrt[d]*(c^2 + (10*I)*c*d + 23*d^
2)*(-I + E^(I*(e + f*x))))]) + (32 + 32*I)*Sqrt[c - I*d]*d^(3/2)*Log[2*(Sqr
t[c - I*d]*Cos[e + f*x] + I*Sqrt[c - I*d]*Sin[e + f*x] + Sqrt[1 + Cos[2*(e
+ f*x)] + I*Sin[2*(e + f*x)]]*Sqrt[c + d*Tan[e + f*x]])])*(Cos[2*e] - I*Sin
[2*e]))/(d^(3/2)*Sqrt[1 + Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)]])) + ((1 +
I)*(I*Cos[2*e] + Sin[2*e])*Sqrt[c + d*Tan[e + f*x]]*(c - (9*I)*d + 2*d*Tan[
e + f*x]))/d))/(f*(Cos[f*x] + I*Sin[f*x])^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1078 vs. $2(208) = 416$.
time = 0.72, size = 1079, normalized size = 4.10 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/16/f*(c+d*tan(f*x+e))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)*a^2*(-10*ln(1/2*(2
*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*
d)^(1/2)+a*d)/(I*a*d)^(1/2))*2^(1/2)*(-a*(I*d-c))^(1/2)*a*c*d+I*ln(1/2*(2*I
*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)
^(1/2)+a*d)/(I*a*d)^(1/2))*2^(1/2)*(-a*(I*d-c))^(1/2)*a*c^2+16*I*ln((3*a*c+
I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+
d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*(I*a*d)^(1/2)*a*c*d+
16*I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c
))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*(I*a*
d)^(1/2)*a*d^2-4*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)*
2^(1/2)*(-a*(I*d-c))^(1/2)*d*tan(f*x+e)+32*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c
+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(
1/2))*2^(1/2)*(-a*(I*d-c))^(1/2)*a*d^2+32*I*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*
c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^
(1/2))*2^(1/2)*(-a*(I*d-c))^(1/2)*a*c*d+23*I*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a
*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)
^(1/2))*2^(1/2)*(-a*(I*d-c))^(1/2)*a*d^2-2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x
+e)))^(1/2)*(I*a*d)^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*c+18*I*(a*(c+d*tan(f*x
+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*d-16*
ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1
```

```
/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*(I*a*d)^(1
/2)*a*c*d+16*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-
a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I
))*(I*a*d)^(1/2)*a*d^2)/(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)/d/(I*a*
d)^(1/2)*2^(1/2)/(-a*(I*d-c))^(1/2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

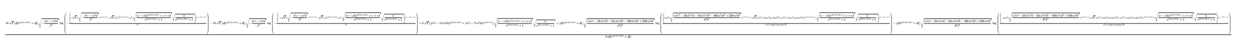
Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(3*d-c>0)', see `assume?` for more d
etails)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1073 vs. 2(207) = 414.
time = 1.00, size = 1073, normalized size = 4.08



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] -1/8*(16*sqrt(2)*(d*f*e^(2*I*f*x + 2*I*e) + d*f)*sqrt(-(a^5*c - I*a^5*d)/f^
2)*log(-(I*sqrt(2)*f*sqrt(-(a^5*c - I*a^5*d)/f^2)*e^(I*f*x + I*e) - sqrt(2)
*(a^2*e^(2*I*f*x + 2*I*e) + a^2)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c +
I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f*
x - I*e)/a^2) - 16*sqrt(2)*(d*f*e^(2*I*f*x + 2*I*e) + d*f)*sqrt(-(a^5*c - I
*a^5*d)/f^2)*log(-(-I*sqrt(2)*f*sqrt(-(a^5*c - I*a^5*d)/f^2)*e^(I*f*x + I*e
) - sqrt(2)*(a^2*e^(2*I*f*x + 2*I*e) + a^2)*sqrt(((c - I*d)*e^(2*I*f*x + 2*
I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)
))*e^(-I*f*x - I*e)/a^2) + 2*sqrt(2)*((a^2*c - 11*I*a^2*d)*e^(3*I*f*x + 3*I
*e) + (a^2*c - 7*I*a^2*d)*e^(I*f*x + I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I
*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))
 + (d*f*e^(2*I*f*x + 2*I*e) + d*f)*sqrt((I*a^5*c^4 - 20*a^5*c^3*d - 54*I*a^
5*c^2*d^2 - 460*a^5*c*d^3 + 529*I*a^5*d^4)/(d^3*f^2))*log((2*I*d^2*f*sqrt((
I*a^5*c^4 - 20*a^5*c^3*d - 54*I*a^5*c^2*d^2 - 460*a^5*c*d^3 + 529*I*a^5*d^4
)/(d^3*f^2))*e^(I*f*x + I*e) + sqrt(2)*(a^2*c^2 + 10*I*a^2*c*d + 23*a^2*d^2
 + (a^2*c^2 + 10*I*a^2*c*d + 23*a^2*d^2)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*
```

```
d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I
*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/(a^2*c^2 + 10*I*a^2*c*d + 23*a^2*d^2)
) - (d*f*e^(2*I*f*x + 2*I*e) + d*f)*sqrt((I*a^5*c^4 - 20*a^5*c^3*d - 54*I*a
^5*c^2*d^2 - 460*a^5*c*d^3 + 529*I*a^5*d^4)/(d^3*f^2))*log((-2*I*d^2*f*sqrt
((I*a^5*c^4 - 20*a^5*c^3*d - 54*I*a^5*c^2*d^2 - 460*a^5*c*d^3 + 529*I*a^5*d
^4)/(d^3*f^2))*e^(I*f*x + I*e) + sqrt(2)*(a^2*c^2 + 10*I*a^2*c*d + 23*a^2*d
^2 + (a^2*c^2 + 10*I*a^2*c*d + 23*a^2*d^2)*e^(2*I*f*x + 2*I*e))*sqrt(((c -
I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2
*I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/(a^2*c^2 + 10*I*a^2*c*d + 23*a^2*d^
2)))/(d*f*e^(2*I*f*x + 2*I*e) + d*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(e+fx)-i))^{\frac{5}{2}}\sqrt{c+d\tan(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(a+I*a*tan(f*x+e))**(5/2),x)
[Out] Integral((I*a*(tan(e + f*x) - I))**(5/2)*sqrt(c + d*tan(e + f*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^(5/2),x, algorithm="gia
c")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 0.65sym2poly/r2sym(c
onst gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+a\tan(e+f\,x)\,1\mathrm{i})^{5/2}\sqrt{c+d\tan(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(5/2)*(c + d*tan(e + f*x))^(1/2),x)
[Out] int((a + a*tan(e + f*x)*1i)^(5/2)*(c + d*tan(e + f*x))^(1/2), x)
```

3.1138 $\int (a+ia\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=250

$$-\frac{\sqrt[4]{-1}\,a^{3/2}(ic+3d)\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{d}\,f}-\frac{2i\sqrt{2}\,a^{3/2}\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c}}{\sqrt{c-id}\,\sqrt{a}}\right.}{f}$$

```
[Out] -2*I*a^(3/2)*arctanh(2^(1/2)*a^(1/2)*(c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2)/(
a+I*a*tan(f*x+e))^(1/2))*2^(1/2)*(c-I*d)^(1/2)/f-(-1)^(1/4)*a^(3/2)*(I*c+3*
d)*arctanh((-1)^(3/4)*d^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c+d*tan(f*x
+e))^(1/2))/f/d^(1/2)+a^2*(c+I*d)*(c+d*tan(f*x+e))^(1/2)/d/f/(a+I*a*tan(f*x
+e))^(1/2)-a^2*(c+d*tan(f*x+e))^(3/2)/d/f/(a+I*a*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.63, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$,
Rules used = {3637, 3676, 3682, 3625, 214, 3680, 65, 223, 212}

$$-\frac{\sqrt[4]{-1}\,a^{3/2}(3d+ic)\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{d}\,f}-\frac{2i\sqrt{2}\,a^{3/2}\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+fx)}}\right)}{f}-\frac{a^2(c+d\tan(e+fx))^{3/2}}{df\sqrt{a+ia\tan(e+fx)}}+\frac{a^2(c+id)\sqrt{c+d\tan(e+fx)}}{df\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^(3/2)*Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] -(((-1)^(1/4)*a^(3/2)*(I*c + 3*d)*ArcTanh[((-1)^(3/4)*Sqrt[d]*Sqrt[a + I*a*
Tan[e + f*x]])/(Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[d]*f)) - ((2*I)*S
qrt[2]*a^(3/2)*Sqrt[c - I*d]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[c + d*Tan[e + f*
x]])/(Sqrt[c - I*d]*Sqrt[a + I*a*Tan[e + f*x]])])/f + (a^2*(c + I*d)*Sqrt[c
 + d*Tan[e + f*x]])/(d*f*Sqrt[a + I*a*Tan[e + f*x]]) - (a^2*(c + d*Tan[e +
f*x])^(3/2))/(d*f*Sqrt[a + I*a*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int (a+ia\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}\,dx = -\frac{a^2(c+d\tan(e+fx))^{3/2}}{df\sqrt{a+ia\tan(e+fx)}} + \frac{a\int\frac{\left(-\frac{1}{2}a(ic-5d)-\frac{1}{2}a(c-3id)\tan(\ldots}{\sqrt{a+ia\ldots}}}{\ldots}$$

$$= \frac{a^2(c+id)\sqrt{c+d\tan(e+fx)}}{df\sqrt{a+ia\tan(e+fx)}} - \frac{a^2(c+d\tan(e+fx))\ldots}{df\sqrt{a+ia\tan(e+f\ldots}}$$

$$= \frac{a^2(c+id)\sqrt{c+d\tan(e+fx)}}{df\sqrt{a+ia\tan(e+fx)}} - \frac{a^2(c+d\tan(e+fx))\ldots}{df\sqrt{a+ia\tan(e+f\ldots}}$$

$$= \frac{a^2(c+id)\sqrt{c+d\tan(e+fx)}}{df\sqrt{a+ia\tan(e+fx)}} - \frac{a^2(c+d\tan(e+fx))\ldots}{df\sqrt{a+ia\tan(e+f\ldots}}$$

$$= -\frac{2i\sqrt{2}\,a^{3/2}\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+\ldots}}{\sqrt{c-id}\sqrt{a+ia\tan(e+\ldots}}\right.}{f}$$

$$= -\frac{2i\sqrt{2}\,a^{3/2}\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+\ldots}}{\sqrt{c-id}\sqrt{a+ia\tan(e+\ldots}}\right.}{f}$$

$$= -\frac{\sqrt[4]{-1}\,a^{3/2}(ic+3d)\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\sqrt{a+ia\tan(e\ldots}}{\sqrt{a}\sqrt{c+d\tan(e+f\ldots}}\right.}{\sqrt{d}\,f}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 559 vs. 2(250) = 500.

time = 6.29, size = 559, normalized size = 2.24

$$\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\cos(e+fx)(\cos(fx)-i\sin(fx))(a+ia\tan(e+fx))^{3/2}\left(\frac{\cos(e+fx)\left((-ic-3d)\log\left(\frac{(2+2i)e^{\frac{ie}{2}}\left(-id+de^{i(e+fx)}+ic\left(i+e^{i(e+fx)}\right)-(1+i)\sqrt{d}\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)}{\sqrt{d}(ic+3d)\left(i+e^{i(e+fx)}\right)}\right)+(ic+3d)\log\left(\frac{(2+2i)e^{\frac{ie}{2}}\left(c+id+ice^{i(e+fx)}+de^{i(e+fx)}+(1+i)\sqrt{d}\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)}{\sqrt{d}(ic+3d)\left(-i+e^{i(e+fx)}\right)}\right)-(4+4i)\sqrt{c-id}\sqrt{d}\log\left(2\left(\sqrt{c-id}\cos(e+fx)+i\sqrt{c-id}\sin(e+fx)+\sqrt{1+\cos(2(e+fx))+i\sin(2(e+fx))}\sqrt{c+d\tan(e+fx)}\right)\right)\right)(\cos(e)-i\sin(e))}{\sqrt{d}\sqrt{1+\cos(2(e+fx))+i\sin(2(e+fx))}}+(1+i)\cos(e)\sqrt{c+d\tan(e+fx)}+(1-i)\sin(e)\sqrt{c+d\tan(e+fx)}\right)}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(3/2)*Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((1/2 + I/2)*Cos[e + f*x]*(Cos[f*x] - I*Sin[f*x])*(a + I*a*Tan[e + f*x])^(3
/2)*((Cos[e + f*x]*(((-I)*c - 3*d)*Log[((2 + 2*I)*E^((I/2)*e)*((-I)*d + d*E
^(I*(e + f*x)) + I*c*(I + E^(I*(e + f*x))) - (1 + I)*Sqrt[d]*Sqrt[1 + E^((2
*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e
+ f*x)))]))/(Sqrt[d]*(I*c + 3*d)*(I + E^(I*(e + f*x))))] + (I*c + 3*d)*Log[
((2 + 2*I)*E^((I/2)*e)*(c + I*d + I*c*E^(I*(e + f*x)) + d*E^(I*(e + f*x)) +
 (1 + I)*Sqrt[d]*Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^((2*I)
*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))]))/(Sqrt[d]*(I*c + 3*d)*(-I + E^(I*
(e + f*x))))] - (4 + 4*I)*Sqrt[c - I*d]*Sqrt[d]*Log[2*(Sqrt[c - I*d]*Cos[e
+ f*x] + I*Sqrt[c - I*d]*Sin[e + f*x] + Sqrt[1 + Cos[2*(e + f*x)] + I*Sin[2
*(e + f*x)]]*Sqrt[c + d*Tan[e + f*x]])])*(Cos[e] - I*Sin[e]))/(Sqrt[d]*Sqrt
[1 + Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)]]) + (1 + I)*Cos[e]*Sqrt[c + d*Ta
n[e + f*x]] + (1 - I)*Sin[e]*Sqrt[c + d*Tan[e + f*x]]))/f
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 865 vs. 2(201) = 402.
time = 0.52, size = 866, normalized size = 3.46 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/4/f*(c+d*tan(f*x+e))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)*a*(4*I*2^(1/2)*(-a*
(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*
tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c+3*I*ln(1/2*(2*I*a*
d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1
/2)+a*d)/(I*a*d)^(1/2))*a*d*2^(1/2)*(-a*(I*d-c))^(1/2)+2*I*(I*a*d)^(1/2)*ln
((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2
)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c+2*I*(I*a
*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(
I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*
a*d+2*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*2^(1/2)*(-a*(I*d-c))^(1
/2)*(I*a*d)^(1/2)-2^(1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a
*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)
^(1/2))*a*c+4*2^(1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2
*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/
2))*a*d-2*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2
```

```
*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(t
an(f*x+e)+I))*a*c+2*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(
1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f
*x+e)+I))*a*d*(I*a*d)^(1/2))/(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)/(I
*a*d)^(1/2)*2^(1/2)/(-a*(I*d-c))^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(3*d-c>0)', see `assume?` for more d
etails)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 773 vs. 2(200) = 400.
time = 0.79, size = 773, normalized size = 3.09

$$\frac{2i\sqrt{2}\,a\sqrt{\frac{(c-i d)e^{(2i f x+2i e)}+c+i d}{e^{(2i f x+2i e)}+1}}\sqrt{\frac{a}{e^{(2i f x+2i e)}+1}}\,e^{(i f x+i e)}-2\sqrt{2}\,f\sqrt{-\frac{a^3c-i a^3d}{f^2}}\log\left(-\frac{\left(i\sqrt{2}\,f\sqrt{-\frac{a^3c-i a^3d}{f^2}}\,e^{(i f x+i e)}-\sqrt{2}\left(a e^{(2i f x+2i e)}+a\right)\sqrt{\frac{(c-i d)e^{(2i f x+2i e)}+c+i d}{e^{(2i f x+2i e)}+1}}\sqrt{\frac{a}{e^{(2i f x+2i e)}+1}}\right)e^{(-i f x-i e)}}{a}\right)+2\sqrt{2}\,f\sqrt{-\frac{a^3c-i a^3d}{f^2}}\log\left(-\frac{\left(-i\sqrt{2}\,f\sqrt{-\frac{a^3c-i a^3d}{f^2}}\,e^{(i f x+i e)}-\sqrt{2}\left(a e^{(2i f x+2i e)}+a\right)\sqrt{\frac{(c-i d)e^{(2i f x+2i e)}+c+i d}{e^{(2i f x+2i e)}+1}}\sqrt{\frac{a}{e^{(2i f x+2i e)}+1}}\right)e^{(-i f x-i e)}}{a}\right)-f\sqrt{\frac{-i a^3c^2-6a^3cd+9i a^3d^2}{df^2}}\log\left(\frac{\left(2i df\sqrt{\frac{-i a^3c^2-6a^3cd+9i a^3d^2}{df^2}}\,e^{(i f x+i e)}+\sqrt{2}\left(i ac+3ad+(i ac+3ad)e^{(2i f x+2i e)}\right)\sqrt{\frac{(c-i d)e^{(2i f x+2i e)}+c+i d}{e^{(2i f x+2i e)}+1}}\sqrt{\frac{a}{e^{(2i f x+2i e)}+1}}\right)e^{(-i f x-i e)}}{i ac+3ad}\right)+f\sqrt{\frac{-i a^3c^2-6a^3cd+9i a^3d^2}{df^2}}\log\left(\frac{\left(-2i df\sqrt{\frac{-i a^3c^2-6a^3cd+9i a^3d^2}{df^2}}\,e^{(i f x+i e)}+\sqrt{2}\left(i ac+3ad+(i ac+3ad)e^{(2i f x+2i e)}\right)\sqrt{\frac{(c-i d)e^{(2i f x+2i e)}+c+i d}{e^{(2i f x+2i e)}+1}}\sqrt{\frac{a}{e^{(2i f x+2i e)}+1}}\right)e^{(-i f x-i e)}}{i ac+3ad}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/2*(2*I*sqrt(2)*a*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f
*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*e^(I*f*x + I*e) - 2*sqr
t(2)*f*sqrt(-(a^3*c - I*a^3*d)/f^2)*log(-(I*sqrt(2)*f*sqrt(-(a^3*c - I*a^3*
d)/f^2)*e^(I*f*x + I*e) - sqrt(2)*(a*e^(2*I*f*x + 2*I*e) + a)*sqrt(((c - I*
d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I
*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/a) + 2*sqrt(2)*f*sqrt(-(a^3*c - I*a^3
*d)/f^2)*log(-(-I*sqrt(2)*f*sqrt(-(a^3*c - I*a^3*d)/f^2)*e^(I*f*x + I*e) -
sqrt(2)*(a*e^(2*I*f*x + 2*I*e) + a)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c
 + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-I
*f*x - I*e)/a) - f*sqrt((-I*a^3*c^2 - 6*a^3*c*d + 9*I*a^3*d^2)/(d*f^2))*log
((2*I*d*f*sqrt((-I*a^3*c^2 - 6*a^3*c*d + 9*I*a^3*d^2)/(d*f^2))*e^(I*f*x + I
*e) + sqrt(2)*(I*a*c + 3*a*d + (I*a*c + 3*a*d)*e^(2*I*f*x + 2*I*e))*sqrt(((
c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(
e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/(I*a*c + 3*a*d)) + f*sqrt((-I*a
^3*c^2 - 6*a^3*c*d + 9*I*a^3*d^2)/(d*f^2))*log((-2*I*d*f*sqrt((-I*a^3*c^2 -
 6*a^3*c*d + 9*I*a^3*d^2)/(d*f^2))*e^(I*f*x + I*e) + sqrt(2)*(I*a*c + 3*a*d
```

```
 + (I*a*c + 3*a*d)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e)
 + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))*e
^(-I*f*x - I*e)/(I*a*c + 3*a*d)))/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(e+fx)-i))^{\frac{3}{2}}\sqrt{c+d\tan(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(a+I*a*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((I*a*(tan(e + f*x) - I))**(3/2)*sqrt(c + d*tan(e + f*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error: Ba
d Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+a\tan(e+f\,x)\,1\mathrm{i})^{3/2}\,\sqrt{c+d\tan(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(3/2)*(c + d*tan(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^(3/2)*(c + d*tan(e + f*x))^(1/2), x)
```

3.1139 $\int \sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=151

$$-\frac{2\sqrt[4]{-1}\,\sqrt{a}\,\sqrt{d}\,\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}\right)}{f}-\frac{i\sqrt{2}\,\sqrt{a}\,\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+fx)}}\right)}{f}$$

```
[Out] -I*arctanh(2^(1/2)*a^(1/2)*(c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2)/(a+I*a*tan(
f*x+e))^(1/2))*2^(1/2)*a^(1/2)*(c-I*d)^(1/2)/f-2*(-1)^(1/4)*arctanh((-1)^(3
/4)*d^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c+d*tan(f*x+e))^(1/2))*a^(1/2
)*d^(1/2)/f
```

Rubi [A]
time = 0.27, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {3644, 3625, 214, 3680, 65, 223, 212}

$$-\frac{2\sqrt[4]{-1}\,\sqrt{a}\,\sqrt{d}\,\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}\right)}{f}-\frac{i\sqrt{2}\,\sqrt{a}\,\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] (-2*(-1)^(1/4)*Sqrt[a]*Sqrt[d]*ArcTanh[((-1)^(3/4)*Sqrt[d]*Sqrt[a + I*a*Tan
[e + f*x]])/(Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])])/f - (I*Sqrt[2]*Sqrt[a]*Sqr
t[c - I*d]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])/(Sqrt[c - I*d
]*Sqrt[a + I*a*Tan[e + f*x]])])/f
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3644

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(a*c - b*d)/a, Int[Sqrt[a + b*Tan[e + f*
x]]/Sqrt[c + d*Tan[e + f*x]], x], x] + Dist[d/a, Int[Sqrt[a + b*Tan[e + f*x
]]*((b + a*Tan[e + f*x])/Sqrt[c + d*Tan[e + f*x]]), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rubi steps

$$\int \sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}\,dx = (c-id)\int \frac{\sqrt{a+ia\tan(e+fx)}}{\sqrt{c+d\tan(e+fx)}}\,dx + \frac{d\int \frac{\sqrt{a+ia\tan(}}{\sqrt{c+}}}{}$$

$$= \frac{(iad)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+iax}\,\sqrt{c+dx}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{i\sqrt{2}\,\sqrt{a}\,\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+f}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+}}\right.}{f}$$

$$= -\frac{i\sqrt{2}\,\sqrt{a}\,\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+f}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+}}\right.}{f}$$

$$= -\frac{2\sqrt[4]{-1}\,\sqrt{a}\,\sqrt{d}\,\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\,\sqrt{a+ia\tan(e+f}}{\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}\right.}{f}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 442 vs. $2(151) = 302$.
time = 3.74, size = 442, normalized size = 2.93

$$-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)e^{-i(e+fx)}\sqrt{1+e^{2i(e+fx)}}\left((1+i)\sqrt{c-id}\log\left(2\left(\sqrt{c-id}\,e^{i(e+fx)}+\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id(-1+e^{2i(e+fx)})}{1+e^{2i(e+fx)}}}\right)\right)+\sqrt{d}\left(\log\left(\frac{(1+i)e^{\frac{ie}{2}}\left(-id+de^{i(e+fx)}+ic(i+e^{i(e+fx)})-(1+i)\sqrt{d}\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id(-1+e^{2i(e+fx)})}{1+e^{2i(e+fx)}}}\right)}{d^{3/2}(i+e^{i(e+fx)})}\right)-\log\left(\frac{(1+i)e^{\frac{ie}{2}}\left(c+id+ice^{i(e+fx)}+de^{i(e+fx)}+(1+i)\sqrt{d}\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id(-1+e^{2i(e+fx)})}{1+e^{2i(e+fx)}}}\right)}{d^{3/2}(-i+e^{i(e+fx)})}\right)\right)\right)\sqrt{a+ia\tan(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((-1/2 - I/2)*Sqrt[1 + E^((2*I)*(e + f*x))]*((1 + I)*Sqrt[c - I*d]*Log[2*(S
qrt[c - I*d]*E^(I*(e + f*x)) + Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*
(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))])] + Sqrt[d]*(Log[((1
 + I)*E^((I/2)*e)*((-I)*d + d*E^(I*(e + f*x)) + I*c*(I + E^(I*(e + f*x))) -
 (1 + I)*Sqrt[d]*Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^((2*I)
*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))]))/(d^(3/2)*(I + E^(I*(e + f*x))))]
 - Log[((1 + I)*E^((I/2)*e)*(c + I*d + I*c*E^(I*(e + f*x)) + d*E^(I*(e + f*
x)) + (1 + I)*Sqrt[d]*Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^(
(2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))]))/(d^(3/2)*(-I + E^(I*(e + f*
x))))]))*Sqrt[a + I*a*Tan[e + f*x]])/(E^(I*(e + f*x))*f)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 865 vs. $2(114) = 228$.
time = 0.52, size = 866, normalized size = 5.74 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/2/f*(a*(1+I*tan(f*x+e)))^(1/2)*(c+d*tan(f*x+e))^(1/2)*a*(-I*ln(1/2*(2*I*
a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^
(1/2)+a*d)/(I*a*d)^(1/2))*2^(1/2)*(-a*(I*d-c))^(1/2)*d^2*tan(f*x+e)+I*ln(1/
2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(
I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*2^(1/2)*(-a*(I*d-c))^(1/2)*c*d-ln(1/2*(2*I
*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)
^(1/2)+a*d)/(I*a*d)^(1/2))*2^(1/2)*(-a*(I*d-c))^(1/2)*c*d*tan(f*x+e)-I*ln((
3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*
(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*(I*a*d)^(1/2)*
c^2-I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-
c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*(I*a
*d)^(1/2)*d^2-ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*t
an(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*2^(1/2)*(-a*(I*d-c))^(1
/2)*d^2+ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*
d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*(I
*a*d)^(1/2)*c^2*tan(f*x+e)+ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e
)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))
/(tan(f*x+e)+I))*(I*a*d)^(1/2)*d^2*tan(f*x+e))*2^(1/2)/(a*(c+d*tan(f*x+e))*
(1+I*tan(f*x+e)))^(1/2)/(I*a*d)^(1/2)/(I*c-d)/(-tan(f*x+e)+I)/(-a*(I*d-c))^
(1/2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 509 vs. 2(113) = 226.
time = 0.86, size = 509, normalized size = 3.37

$$\frac{1}{2}\sqrt{2}\sqrt{-\frac{ac-iad}{f^2}}\log\left(\left(i\sqrt{2}f\sqrt{-\frac{ac-iad}{f^2}}e^{(ifx+ie)}+\sqrt{2}\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}\left(e^{(2ifx+2ie)}+1\right)\right)e^{(-ifx-ie)}\right)-\frac{1}{2}\sqrt{2}\sqrt{-\frac{ac-iad}{f^2}}\log\left(\left(-i\sqrt{2}f\sqrt{-\frac{ac-iad}{f^2}}e^{(ifx+ie)}+\sqrt{2}\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}\left(e^{(2ifx+2ie)}+1\right)\right)e^{(-ifx-ie)}\right)-\frac{1}{2}\sqrt{\frac{4iad}{f^2}}\log\left(\left(\sqrt{2}\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}\left(e^{(2ifx+2ie)}+1\right)+if\sqrt{\frac{4iad}{f^2}}e^{(ifx+ie)}\right)e^{(-ifx-ie)}\right)+\frac{1}{2}\sqrt{\frac{4iad}{f^2}}\log\left(\left(\sqrt{2}\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}\left(e^{(2ifx+2ie)}+1\right)-if\sqrt{\frac{4iad}{f^2}}e^{(ifx+ie)}\right)e^{(-ifx-ie)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/2*sqrt(2)*sqrt(-(a*c - I*a*d)/f^2)*log((I*sqrt(2)*f*sqrt(-(a*c - I*a*d)/f
^2)*e^(I*f*x + I*e) + sqrt(2)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d
)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*(e^(2*I*f*x
+ 2*I*e) + 1))*e^(-I*f*x - I*e)) - 1/2*sqrt(2)*sqrt(-(a*c - I*a*d)/f^2)*log
((-I*sqrt(2)*f*sqrt(-(a*c - I*a*d)/f^2)*e^(I*f*x + I*e) + sqrt(2)*sqrt(((c
- I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^
(2*I*f*x + 2*I*e) + 1))*(e^(2*I*f*x + 2*I*e) + 1))*e^(-I*f*x - I*e)) - 1/2*
sqrt(4*I*a*d/f^2)*log((sqrt(2)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*
d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*(e^(2*I*f*x
 + 2*I*e) + 1) + I*f*sqrt(4*I*a*d/f^2)*e^(I*f*x + I*e))*e^(-I*f*x - I*e)) +
 1/2*sqrt(4*I*a*d/f^2)*log((sqrt(2)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c
 + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*(e^(2*
I*f*x + 2*I*e) + 1) - I*f*sqrt(4*I*a*d/f^2)*e^(I*f*x + I*e))*e^(-I*f*x - I*
e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\left(\tan\left(e+fx\right)-i\right)}\,\sqrt{c+d\tan\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(1/2)*(c+d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(e + f*x) - I))*sqrt(c + d*tan(e + f*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 20.35, size = 2101, normalized size = 13.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*tan(e + f*x)*1i)^(1/2)*(c + d*tan(e + f*x))^(1/2),x)

[Out] (2^(1/2)*a^(1/2)*d^(1/2)*log((2*d^(1/2)*(a + a*tan(e + f*x)*1i)^(1/2) - 2*a^(1/2)*d^(1/2) - 2^(1/2)*a^(1/2)*c^(1/2)*(1 + 1i) + 2^(1/2)*a^(1/2)*(c + d*tan(e + f*x))^(1/2)*(1 + 1i))/((c + d*tan(e + f*x))^(1/2) - c^(1/2)))*(1 + 1i))/f - (a^(1/2)*atan((4*(29*(1i/2)^(1/2)*c^3*(c*1i + d)^(1/2)*(a + a*tan(e + f*x)*1i)^(1/2)*(c + d*tan(e + f*x))^(1/2) - 29*(1i/2)^(1/2)*c^(7/2)*(c*1i + d)^(1/2)*(a + a*tan(e + f*x)*1i)^(1/2) - 6*(1i/2)^(1/2)*a^(1/2)*c^(7/2)*(c*1i + d)^(1/2) + (1i/2)^(1/2)*d^3*(c*1i + d)^(1/2)*(a + a*tan(e + f*x)*1i)^(1/2)*(c + d*tan(e + f*x))^(1/2)*1i + (1i/2)^(1/2)*c^(5/2)*d*(c*1i + d)^(1/2)*(a + a*tan(e + f*x)*1i)^(1/2)*1i - 35*(1i/2)^(1/2)*a^(1/2)*c^(5/2)*(c*1i + d)^(1/2)*(c + d*tan(e + f*x)) + (1i/2)^(1/2)*c^(1/2)*d^3*(c*1i + d)^(1/2)*(a + a*tan(e + f*x)*1i)^(1/2)*5i - 25*(1i/2)^(1/2)*c^(3/2)*d^2*(c*1i + d)^(1/2)*(a + a*tan(e + f*x)*1i)^(1/2) + 41*(1i/2)^(1/2)*a^(1/2)*c^3*(c*1i + d)^(1/2)*(c + d*tan(e + f*x))^(1/2) - (1i/2)^(1/2)*a^(1/2)*d^3*(c*1i + d)^(1/2)*(c + d*tan(e + f*x))^(1/2)*1i + (1i/2)^(1/2)*a^(1/2)*c^(5/2)*d*(c*1i + d)^(1/2)*25i - (1i/2)^(1/2)*a^(1/2)*c^(1/2)*d^3*(c*1i + d)^(1/2)*5i + 28*(1i/2)^(1/2)*a^(1/2)*c^(3/2)*d^2*(c*1i + d)^(1/2) - 35*(1i/2)^(1/2)*a^(1/2)*c^(5/2)*d*tan(e + f*x)*(c*1i + d)^(1/2) + 3*(1i/2)^(1/2)*a^(1/2)*c^(1/2)*d^3*tan(e + f*x)*(c*1i + d)^(1/2) + (1i/2)^(1/2)*a^(1/2)*c^(3/2)*d^2*tan(e + f*x)*(c*1i + d)^(1/2)*26i - 27*(1i/2)^(1/2)*c*d^2*(c*1i + d)^(1/2)*(a + a*tan(e + f*x)*1i)^(1/2)*(c + d*tan(e + f*x))^(1/2) - (1i/2)^(1/2)*c^2*d*(c*1i + d)^(1/2)*(a + a*tan(e + f*x)*1i)^(1/2)*(c + d*tan(e + f*x))^(1/2)*71i + (1i/2)^(1/2)*a^(1/2)*c^(3/2)*d*(c*1i + d)^(1/2)*(c + d*tan(e + f*x))*26i + 21*(1i/2)^(1/2)*a^(1/2)*c*d^2*(c*1i + d)^(1/2)*(c + d*tan(e + f*x))^(1/2) + (1i/2)^(1/2)*a^(1/2)*c^2*d*(c*1i + d)^(1/2)*(c + d*tan(e + f*x))^(1/2)*19i + 3*(1i/2)^(1/2)*a^(1/2)*c^(1/2)*d^2*(c*1i + d)^(1/2)*(c + d*tan(e + f*x))))/(82*c^4*(a + a*tan(e + f*x)*1i)^(1/2) - 2*d^4*(a + a*tan(e + f*x)*1i)^(1/2) + 17*a^(1/2)*c^4 + 2*a^(1/2)*d^4 + 80*c^2*d^2*(a + a*tan(e + f*x)*1i)^(1/2) - 116*a^(1/2)*c^(7/2)*(c + d*tan(e + f*x))^(1/2) + a^(1/2)*c*d^3*45i - a^(1/2)*c^3*d*79i + a^(1/2)*d^4*tan(e + f*x)*1i - 113*a^(1/2)*c^2*d^2 - 82*c^(7/2)*(a + a*tan(e + f*x)*1i)^(1/2)*(c + d*tan(e + f*x))^(1/2) - c*d^3*(a + a*tan(e + f*x)*1i)^(1/2)*44i - c^3*d*(a + a*tan(e + f*x)*1i)^(1/2)*44i + 99*a^(1/2)*c^3*(c + d*tan(e + f*x)) + a^(1/2)*d^3*(c + d*tan(e + f*x))*1i + a^(1/2)*c^(1/2)*d^3*(c + d*tan(e + f*x))^(1/2)*20i - 100*a^(1/2)*c^(3/2)*d^2*(c + d*tan(e + f*x))^(1/2) - a^(1/2)*c^2*d^2*tan(e + f*x)*123i + c^(5/2)*d*(a + a*tan(e + f*x)*1i)^(1/2)*(c + d*tan(e + f*x))^(1/2)*242i - 33*a^(1/2)*c*d^2*(c + d*tan(e + f*x)) - a^(1/2)*c^2*d*(c + d*tan(e + f*x))*123i - c^(1/2)*d^3*(a + a*tan(e + f*x)*1i)^(1/2)*(c + d*tan(e + f*x))^(1/2)*22i + 166*c^(3/2)*d^2*(a + a*tan(e + f*x)*1i)^(1/2)*(c + d*tan(e + f*x))^(1/2) + a^(1/2)*c^(5/2)*d*(c + d*tan(e + f*x))^(1/2)*4i - 33*a^(1/2)*c*d^3*tan(e + f*x) + 99*a^(1/2)*c^3*d*tan(e + f*x)))*(c*1i + d)^(1/2)*(1 - 1i))/f - (2^(1/2)*a^(1/2)*d^(1/2)*log((d^(17/2)*(a + a*tan(e + f*x)*1i)^(1/2) - a^(1/2)*d^(17/2) + c^8*d^(1/2)*(a + a*tan(e + f*x)*1i)^(1/2) + c^7*d^(3/2)*(a + a*tan(e + f*x)*1i)^(1/2)*64i + 324*c^6*d^(5/2)*(a + a*tan(e + f*x)*1i)^(1/

```
2) + c^5*d^(7/2)*(a + a*tan(e + f*x)*1i)^(1/2)*640i + 1670*c^4*d^(9/2)*(a +
 a*tan(e + f*x)*1i)^(1/2) + c^3*d^(11/2)*(a + a*tan(e + f*x)*1i)^(1/2)*1600
i + 324*c^2*d^(13/2)*(a + a*tan(e + f*x)*1i)^(1/2) + (-1)^(1/4)*a^(1/2)*c^(
17/2) - a^(1/2)*c^8*d^(1/2) + a^(1/2)*c^7*d^(3/2)*192i - 324*a^(1/2)*c^6*d^
(5/2) + a^(1/2)*c^5*d^(7/2)*896i - 1670*a^(1/2)*c^4*d^(9/2) - a^(1/2)*c^3*d
^(11/2)*320i - 324*a^(1/2)*c^2*d^(13/2) - a^(1/2)*c^(5/2)*d^(11/2)*(c + d*t
an(e + f*x))^(1/2)*1280i - a^(1/2)*c^(9/2)*d^(7/2)*(c + d*tan(e + f*x))^(1/
2)*1536i - a^(1/2)*c^(13/2)*d^(3/2)*(c + d*tan(e + f*x))^(1/2)*256i + (-1)^
(1/4)*a^(1/2)*c^(1/2)*d^8 - 956*(-1)^(1/4)*a^(1/2)*c^(5/2)*d^6 + 134*(-1)^(
1/4)*a^(1/2)*c^(9/2)*d^4 + 1600*(-1)^(3/4)*a^(1/2)*c^(7/2)*d^5 + 68*(-1)^(1
/4)*a^(1/2)*c^(13/2)*d^2 + 640*(-1)^(3/4)*a^(1/2)*c^(11/2)*d^3 + 1280*(-1)^
(1/4)*c^(5/2)*d^6*(a + a*tan(e + f*x)*1i)^(1/2) + 1536*(-1)^(1/4)*c^(9/2)*d
^4*(a + a*tan(e + f*x)*1i)^(1/2) + 256*(-1)^(1/4)*c^(13/2)*d^2*(a + a*tan(e
 + f*x)*1i)^(1/2) - (-1)^(1/4)*a^(1/2)*c^8*(c + d*tan(e + f*x))^(1/2) - (-1
)^(1/4)*a^(1/2)*d^8*(c + d*tan(e + f*x))^(1/2) + 64*(-1)^(3/4)*a^(1/2)*c^(1
5/2)*d - 64*(-1)^(3/4)*a^(1/2)*c^7*d*(c + d*tan(e + f*x))^(1/2) - 324*(-1)^
(1/4)*a^(1/2)*c^2*d^6*(c + d*tan(e + f*x))^(1/2) - 1670*(-1)^(1/4)*a^(1/2)*
c^4*d^4*(c + d*tan(e + f*x))^(1/2) - 324*(-1)^(1/4)*a^(1/2)*c^6*d^2*(c + d*
tan(e + f*x))^(1/2) - 1600*(-1)^(3/4)*a^(1/2)*c^3*d^5*(c + d*tan(e + f*x))^
(1/2) - 640*(-1)^(3/4)*a^(1/2)*c^5*d^3*(c + d*tan(e + f*x))^(1/2))/((c + d*
tan(e + f*x))^(1/2) - c^(1/2)))*(1 + 1i))/f
```

3.1140 $\int \frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{a+ia\tan(e+fx)}}\,dx$

Optimal. Leaf size=121

$$-\frac{i\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+fx)}}\right)}{\sqrt{2}\,\sqrt{a}\,f}+\frac{i\sqrt{c+d\tan(e+fx)}}{f\sqrt{a+ia\tan(e+fx)}}$$

```
[Out] -1/2*I*arctanh(2^(1/2)*a^(1/2)*(c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2)/(a+I*a*
tan(f*x+e))^(1/2))*(c-I*d)^(1/2)/f*2^(1/2)/a^(1/2)+I*(c+d*tan(f*x+e))^(1/2)
/f/(a+I*a*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3627, 3625, 214}

$$\frac{i\sqrt{c+d\tan(e+fx)}}{f\sqrt{a+ia\tan(e+fx)}}-\frac{i\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+fx)}}\right)}{\sqrt{2}\,\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + I*a*Tan[e + f*x]],x]
```

```
[Out] ((-I)*Sqrt[c - I*d]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])/(Sqr
t[c - I*d]*Sqrt[a + I*a*Tan[e + f*x]])])/(Sqrt[2]*Sqrt[a]*f) + (I*Sqrt[c +
d*Tan[e + f*x]])/(f*Sqrt[a + I*a*Tan[e + f*x]])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3627

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
```

```
 + f*x])^n/(2*b*f*m)), x] - Dist[(a*c - b*d)/(2*b^2), Int[(a + b*Tan[e + f*
x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && EqQ
[m + n, 0] && LeQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{a+ia\tan(e+fx)}}\,dx = \frac{i\sqrt{c+d\tan(e+fx)}}{f\sqrt{a+ia\tan(e+fx)}} + \frac{(c-id)\int \frac{\sqrt{a+ia\tan(e+fx)}}{\sqrt{c+d\tan(e+fx)}}\,dx}{2a}$$

$$= \frac{i\sqrt{c+d\tan(e+fx)}}{f\sqrt{a+ia\tan(e+fx)}} - \frac{(a(ic+d))\text{Subst}\left(\int \frac{1}{ac-iad-2a^2x^2}\,dx, x, \frac{\sqrt{c+d\tan}}{\sqrt{a+ia\tan}}\right.}{f}$$

$$= -\frac{i\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\,f} + \frac{i\sqrt{c+d\tan(e+f}}{f\sqrt{a+ia\tan(e+}}$$

Mathematica [A]
time = 3.44, size = 182, normalized size = 1.50

$$\frac{i\left(-\sqrt{c-id}\,e^{i(e+fx)}\log\left(2\left(\sqrt{c-id}\,e^{i(e+fx)}+\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)\right)+\sqrt{1+e^{2i(e+fx)}}\sqrt{c+d\tan(e+fx)}\right)}{\sqrt{1+e^{2i(e+fx)}}\,f\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + I*a*Tan[e + f*x]],x]
```

```
[Out] (I*(-(Sqrt[c - I*d]*E^(I*(e + f*x))*Log[2*(Sqrt[c - I*d]*E^(I*(e + f*x)) +
Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(1
+ E^((2*I)*(e + f*x)))])]) + Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c + d*Tan[e
 + f*x]]))/(Sqrt[1 + E^((2*I)*(e + f*x))]*f*Sqrt[a + I*a*Tan[e + f*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 876 vs. 2(95) = 190.
time = 4.22, size = 877, normalized size = 7.25 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/4/f*(c+d*tan(f*x+e))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)/a*(-I*2^(1/2)*(-a*
(I*d-c))^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*
```

```
(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)
+I))*d*tan(f*x+e)^2+2*I*2^(1/2)*(-a*(I*d-c))^(1/2)*ln((3*a*c+I*a*tan(f*x+e)
*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*
(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c*tan(f*x+e)-2^(1/2)*(-a*(I*d-c))^
(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-
c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c*ta
n(f*x+e)^2+I*2^(1/2)*(-a*(I*d-c))^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*
a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*
x+e)))^(1/2))/(tan(f*x+e)+I))*d-2*2^(1/2)*(-a*(I*d-c))^(1/2)*ln((3*a*c+I*a*
tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*ta
n(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*d*tan(f*x+e)-4*I*c*(a*(c
+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*tan(f*x+e)+2^(1/2)*(-a*(I*d-c))^(1/2
)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^
(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c-4*I*(a
*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*d+4*(a*(c+d*tan(f*x+e))*(1+I*tan(
f*x+e)))^(1/2)*d*tan(f*x+e)-4*c*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)
)/(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)/(I*c-d)/(-tan(f*x+e)+I)^2
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] Timed out
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(93) = 186$.
time = 0.70, size = 369, normalized size = 3.05

$$\frac{\left(\sqrt{2}\,af\sqrt{-\frac{c-i\,d}{af^2}}\,e^{(i\,fx+i\,e)}\log\left(i\,\sqrt{2}\,af\sqrt{-\frac{c-i\,d}{af^2}}\,e^{(i\,fx+i\,e)}+\sqrt{2}\,\sqrt{\frac{(c-i\,d)e^{(2i\,fx+2i\,e)}+c+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\,\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\,\left(e^{(2i\,fx+2i\,e)}+1\right)\right)-\sqrt{2}\,af\sqrt{-\frac{c-i\,d}{af^2}}\,e^{(i\,fx+i\,e)}\log\left(-i\,\sqrt{2}\,af\sqrt{-\frac{c-i\,d}{af^2}}\,e^{(i\,fx+i\,e)}+\sqrt{2}\,\sqrt{\frac{(c-i\,d)e^{(2i\,fx+2i\,e)}+c+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\,\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\,\left(e^{(2i\,fx+2i\,e)}+1\right)\right)-2\,\sqrt{2}\,\sqrt{\frac{(c-i\,d)e^{(2i\,fx+2i\,e)}+c+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\,\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\,\left(-i\,e^{(2i\,fx+2i\,e)}-i\right)\right)e^{(-i\,fx-i\,e)}}{4\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/4*(sqrt(2)*a*f*sqrt(-(c - I*d)/(a*f^2))*e^(I*f*x + I*e)*log(I*sqrt(2)*a*f
*sqrt(-(c - I*d)/(a*f^2))*e^(I*f*x + I*e) + sqrt(2)*sqrt(((c - I*d)*e^(2*I*
f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I
*e) + 1))*(e^(2*I*f*x + 2*I*e) + 1)) - sqrt(2)*a*f*sqrt(-(c - I*d)/(a*f^2))
*e^(I*f*x + I*e)*log(-I*sqrt(2)*a*f*sqrt(-(c - I*d)/(a*f^2))*e^(I*f*x + I*e
) + sqrt(2)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*
I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*(e^(2*I*f*x + 2*I*e) + 1)) - 2
```

```
*sqrt(2)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e
) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*(-I*e^(2*I*f*x + 2*I*e) - I))*e^(
-I*f*x - I*e)/(a*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + d\tan{(e + fx)}}}{\sqrt{ia(\tan{(e + fx)} - i)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*tan(e + f*x))/sqrt(I*a*(tan(e + f*x) - I)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

Mupad [**B**]
time = 19.83, size = 1724, normalized size = 14.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(1/2)/(a + a*tan(e + f*x)*1i)^(1/2),x)
```

```
[Out] (2*(c + d*1i)*((a + a*tan(e + f*x)*1i)^(1/2) - a^(1/2)))/(d*f*((c + d*tan(e
 + f*x))^(1/2) - c^(1/2))*((a*1i)/d + ((a + a*tan(e + f*x)*1i)^(1/2) - a^(1
/2))^2/((c + d*tan(e + f*x))^(1/2) - c^(1/2))^2 - (a^(1/2)*c^(1/2)*((a + a*
tan(e + f*x)*1i)^(1/2) - a^(1/2))*2i)/(d*((c + d*tan(e + f*x))^(1/2) - c^(1
/2))))) - (2^(1/2)*atan(((2^(1/2)*(d*1i - c)^(1/2)*(4*d^7*(4*a^(3/2)*c^(3/2
)*f - a^(3/2)*c^(1/2)*d*f*4i) + (16*d^7*((a + a*tan(e + f*x)*1i)^(1/2) - a^
(1/2))*(a*d^2*f - a*c^2*f + a*c*d*f*2i))/((c + d*tan(e + f*x))^(1/2) - c^(1
/2)) - (4*d^8*(a^(1/2)*c^(3/2)*f*4i + 4*a^(1/2)*c^(1/2)*d*f)*((a + a*tan(e
+ f*x)*1i)^(1/2) - a^(1/2))^2)/((c + d*tan(e + f*x))^(1/2) - c^(1/2))^2 - (
```

```
2^(1/2)*(d*1i - c)^(1/2)*(4*d^7*(a^2*c*f^2*4i - 4*a^2*d*f^2) - (16*d^7*(a^(
3/2)*c^(3/2)*f^2*2i + 6*a^(3/2)*c^(1/2)*d*f^2)*((a + a*tan(e + f*x)*1i)^(1/
2) - a^(1/2)))/((c + d*tan(e + f*x))^(1/2) - c^(1/2)) + (4*d^8*(20*a*c*f^2
- a*d*f^2*12i)*((a + a*tan(e + f*x)*1i)^(1/2) - a^(1/2))^2)/((c + d*tan(e +
 f*x))^(1/2) - c^(1/2))^2))/(4*a^(1/2)*f))*1i)/(4*a^(1/2)*f) + (2^(1/2)*(d*
1i - c)^(1/2)*(4*d^7*(4*a^(3/2)*c^(3/2)*f - a^(3/2)*c^(1/2)*d*f*4i) + (16*d
^7*((a + a*tan(e + f*x)*1i)^(1/2) - a^(1/2))*(a*d^2*f - a*c^2*f + a*c*d*f*2
i))/((c + d*tan(e + f*x))^(1/2) - c^(1/2)) - (4*d^8*(a^(1/2)*c^(3/2)*f*4i +
 4*a^(1/2)*c^(1/2)*d*f)*((a + a*tan(e + f*x)*1i)^(1/2) - a^(1/2))^2)/((c +
d*tan(e + f*x))^(1/2) - c^(1/2))^2 + (2^(1/2)*(d*1i - c)^(1/2)*(4*d^7*(a^2*
c*f^2*4i - 4*a^2*d*f^2) - (16*d^7*(a^(3/2)*c^(3/2)*f^2*2i + 6*a^(3/2)*c^(1/
2)*d*f^2)*((a + a*tan(e + f*x)*1i)^(1/2) - a^(1/2)))/((c + d*tan(e + f*x))^
(1/2) - c^(1/2)) + (4*d^8*(20*a*c*f^2 - a*d*f^2*12i)*((a + a*tan(e + f*x)*1
i)^(1/2) - a^(1/2))^2)/((c + d*tan(e + f*x))^(1/2) - c^(1/2))^2))/(4*a^(1/2
)*f))*1i)/(4*a^(1/2)*f))/(8*d^7*(a*c^2*1i - a*d^2*1i + 2*a*c*d) + (8*d^8*((
a + a*tan(e + f*x)*1i)^(1/2) - a^(1/2))^2*(c*d*2i - c^2 + d^2))/((c + d*tan
(e + f*x))^(1/2) - c^(1/2))^2 + (2^(1/2)*(d*1i - c)^(1/2)*(4*d^7*(4*a^(3/2)
*c^(3/2)*f - a^(3/2)*c^(1/2)*d*f*4i) + (16*d^7*((a + a*tan(e + f*x)*1i)^(1/
2) - a^(1/2))*(a*d^2*f - a*c^2*f + a*c*d*f*2i))/((c + d*tan(e + f*x))^(1/2)
 - c^(1/2)) - (4*d^8*(a^(1/2)*c^(3/2)*f*4i + 4*a^(1/2)*c^(1/2)*d*f)*((a + a
*tan(e + f*x)*1i)^(1/2) - a^(1/2))^2)/((c + d*tan(e + f*x))^(1/2) - c^(1/2)
)^2 - (2^(1/2)*(d*1i - c)^(1/2)*(4*d^7*(a^2*c*f^2*4i - 4*a^2*d*f^2) - (16*d
^7*(a^(3/2)*c^(3/2)*f^2*2i + 6*a^(3/2)*c^(1/2)*d*f^2)*((a + a*tan(e + f*x)*
1i)^(1/2) - a^(1/2)))/((c + d*tan(e + f*x))^(1/2) - c^(1/2)) + (4*d^8*(20*a
*c*f^2 - a*d*f^2*12i)*((a + a*tan(e + f*x)*1i)^(1/2) - a^(1/2))^2)/((c + d*
tan(e + f*x))^(1/2) - c^(1/2))^2))/(4*a^(1/2)*f)))/(4*a^(1/2)*f) - (2^(1/2)
*(d*1i - c)^(1/2)*(4*d^7*(4*a^(3/2)*c^(3/2)*f - a^(3/2)*c^(1/2)*d*f*4i) + (
16*d^7*((a + a*tan(e + f*x)*1i)^(1/2) - a^(1/2))*(a*d^2*f - a*c^2*f + a*c*d
*f*2i))/((c + d*tan(e + f*x))^(1/2) - c^(1/2)) - (4*d^8*(a^(1/2)*c^(3/2)*f*
4i + 4*a^(1/2)*c^(1/2)*d*f)*((a + a*tan(e + f*x)*1i)^(1/2) - a^(1/2))^2)/((
c + d*tan(e + f*x))^(1/2) - c^(1/2))^2 + (2^(1/2)*(d*1i - c)^(1/2)*(4*d^7*(
a^2*c*f^2*4i - 4*a^2*d*f^2) - (16*d^7*(a^(3/2)*c^(3/2)*f^2*2i + 6*a^(3/2)*c
^(1/2)*d*f^2)*((a + a*tan(e + f*x)*1i)^(1/2) - a^(1/2)))/((c + d*tan(e + f*
x))^(1/2) - c^(1/2)) + (4*d^8*(20*a*c*f^2 - a*d*f^2*12i)*((a + a*tan(e + f*
x)*1i)^(1/2) - a^(1/2))^2)/((c + d*tan(e + f*x))^(1/2) - c^(1/2))^2))/(4*a^
(1/2)*f)))/(4*a^(1/2)*f)))*(d*1i - c)^(1/2)*1i)/(2*a^(1/2)*f)
```

3.1141 $\int \frac{\sqrt{c+d\tan(e+fx)}}{(a+ia\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=177

$$-\frac{i\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f}+\frac{i\sqrt{c+d\tan(e+fx)}}{2af\sqrt{a+ia\tan(e+fx)}}-\frac{(c+d\tan(e+fx)}{3(ic-d)f(a+ia\tan(e}$$

```
[Out] -1/4*I*arctanh(2^(1/2)*a^(1/2)*(c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2)/(a+I*a*
tan(f*x+e))^(1/2))*(c-I*d)^(1/2)/a^(3/2)/f*2^(1/2)+1/2*I*(c+d*tan(f*x+e))^(
1/2)/a/f/(a+I*a*tan(f*x+e))^(1/2)-1/3*(c+d*tan(f*x+e))^(3/2)/(I*c-d)/f/(a+I
*a*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.21, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3628, 3627, 3625, 214}

$$-\frac{i\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f}-\frac{(c+d\tan(e+fx))^{3/2}}{3f(-d+ic)(a+ia\tan(e+fx))^{3/2}}+\frac{i\sqrt{c+d\tan(e+fx)}}{2af\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Tan[e + f*x]]/(a + I*a*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-1/2*I)*Sqrt[c - I*d]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])/
(Sqrt[c - I*d]*Sqrt[a + I*a*Tan[e + f*x]])])/(Sqrt[2]*a^(3/2)*f) + ((I/2)*S
qrt[c + d*Tan[e + f*x]])/(a*f*Sqrt[a + I*a*Tan[e + f*x]]) - (c + d*Tan[e +
f*x])^(3/2)/(3*(I*c - d)*f*(a + I*a*Tan[e + f*x])^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3627

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^n/(2*b*f*m)), x] - Dist[(a*c - b*d)/(2*b^2), Int[(a + b*Tan[e + f*
x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && EqQ
[m + n, 0] && LeQ[m, -2^(-1)]
```

Rule 3628

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a), Int[(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && EqQ[m
 + n + 1, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{c+d\tan(e+fx)}}{(a+ia\tan(e+fx))^{3/2}}\,dx = -\frac{(c+d\tan(e+fx))^{3/2}}{3(ic-d)f(a+ia\tan(e+fx))^{3/2}} + \frac{\int \frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{a+ia\tan(e+fx)}}\,dx}{2a}$$

$$= \frac{i\sqrt{c+d\tan(e+fx)}}{2af\sqrt{a+ia\tan(e+fx)}} - \frac{(c+d\tan(e+fx))^{3/2}}{3(ic-d)f(a+ia\tan(e+fx))^{3/2}} + \frac{(c-id)\ldots}{\ldots}$$

$$= \frac{i\sqrt{c+d\tan(e+fx)}}{2af\sqrt{a+ia\tan(e+fx)}} - \frac{(c+d\tan(e+fx))^{3/2}}{3(ic-d)f(a+ia\tan(e+fx))^{3/2}} - \frac{(ic+d)\ldots}{\ldots}$$

$$= -\frac{i\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f} + \frac{i\sqrt{c+d\tan(e\ldots}}{2af\sqrt{a+ia\tan(\ldots}}$$

Mathematica [A]
time = 4.38, size = 249, normalized size = 1.41

$$\frac{\sec^{\frac{3}{2}}(e+fx)\left(-i\sqrt{2}\sqrt{c-id}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{3/2}\left(1+e^{2i(e+fx)}\right)^{3/2}\log\left(2\left(\sqrt{c-id}\,e^{i(e+fx)}+\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)\right)-\frac{2(-5ic+3d+(3c+id)\tan(e+fx))\sqrt{c+d\tan(e+fx)}}{3(c+id)\sec^{\frac{3}{2}}(e+fx)}\right)}{4f(a+ia\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Tan[e + f*x]]/(a + I*a*Tan[e + f*x])^(3/2),x]
```

```
[Out] (Sec[e + f*x]^(3/2)*((-I)*Sqrt[2]*Sqrt[c - I*d]*(E^(I*(e + f*x))/(1 + E^((2
*I)*(e + f*x))))^(3/2)*(1 + E^((2*I)*(e + f*x)))^(3/2)*Log[2*(Sqrt[c - I*d]
*E^(I*(e + f*x)) + Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^((2*
I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))])] - (2*((-5*I)*c + 3*d + (3*c +
I*d)*Tan[e + f*x])*Sqrt[c + d*Tan[e + f*x]])/(3*(c + I*d)*Sec[e + f*x]^(3/2
))))/(4*f*(a + I*a*Tan[e + f*x])^(3/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1179 vs. $2(139) = 278$.
time = 0.69, size = 1180, normalized size = 6.67

method	result	size
derivativedivides	Expression too large to display	1180
default	Expression too large to display	1180

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/24/f*(c+d*tan(f*x+e))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)/a^2*(9*I*ln((3*a*c
+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c
+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c)
)^(1/2)*c*tan(f*x+e)^2-12*I*c*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*t
an(f*x+e)^2-3*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(
-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+
I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c*tan(f*x+e)^3+4*(a*(c+d*tan(f*x+e))*(1+I*ta
n(f*x+e)))^(1/2)*d*tan(f*x+e)^2+20*I*c*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e))
)^(1/2)-9*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(
I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*
2^(1/2)*(-a*(I*d-c))^(1/2)*d*tan(f*x+e)^2-32*c*(a*(c+d*tan(f*x+e))*(1+I*tan
(f*x+e)))^(1/2)*tan(f*x+e)-3*I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f
*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1
/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*d*tan(f*x+e)^3-16*I*(a*(c+d
*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*d*tan(f*x+e)+9*ln((3*a*c+I*a*tan(f*x+e
)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))
*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c*tan(
f*x+e)+9*I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*
(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))
*2^(1/2)*(-a*(I*d-c))^(1/2)*d*tan(f*x+e)+3*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d
+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan
(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*d-3*I*ln((3*a*c
+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c
+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c)
)^(1/2)*c-12*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*d)/(a*(c+d*tan(f*x
+e))*(1+I*tan(f*x+e)))^(1/2)/(I*c-d)/(-tan(f*x+e)+I)^3
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(137) = 274$.
time = 0.87, size = 437, normalized size = 2.47

$$\frac{\left(3\sqrt{\frac{1}{2}}\left(ia^2c-a^2d\right)f\sqrt{-\frac{c-id}{a^3f^2}}e^{(3ifx+3ie)}\log\left(2i\sqrt{\frac{1}{2}}a^2f\sqrt{-\frac{c-id}{a^3f^2}}e^{(ifx+ie)}+\sqrt{2}\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}\left(e^{(2ifx+2ie)}+1\right)\right)+3\sqrt{\frac{1}{2}}\left(-ia^2c+a^2d\right)f\sqrt{-\frac{c-id}{a^3f^2}}e^{(3ifx+3ie)}\log\left(-2i\sqrt{\frac{1}{2}}a^2f\sqrt{-\frac{c-id}{a^3f^2}}e^{(ifx+ie)}+\sqrt{2}\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}\left(e^{(2ifx+2ie)}+1\right)\right)-\sqrt{2}\left(2(2c+id)e^{(4ifx+4ie)}+(5c+3id)e^{(2ifx+2ie)}+c+id\right)\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}\right)e^{(-3ifx-3ie)}}{12\left(ia^2c-a^2d\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/12*(3*sqrt(1/2)*(I*a^2*c - a^2*d)*f*sqrt(-(c - I*d)/(a^3*f^2))*e^(3*I*f*x
 + 3*I*e)*log(2*I*sqrt(1/2)*a^2*f*sqrt(-(c - I*d)/(a^3*f^2))*e^(I*f*x + I*e
) + sqrt(2)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*
I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*(e^(2*I*f*x + 2*I*e) + 1)) + 3
*sqrt(1/2)*(-I*a^2*c + a^2*d)*f*sqrt(-(c - I*d)/(a^3*f^2))*e^(3*I*f*x + 3*I
*e)*log(-2*I*sqrt(1/2)*a^2*f*sqrt(-(c - I*d)/(a^3*f^2))*e^(I*f*x + I*e) + s
qrt(2)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e)
+ 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*(e^(2*I*f*x + 2*I*e) + 1)) - sqrt(2
)*(2*(2*c + I*d)*e^(4*I*f*x + 4*I*e) + (5*c + 3*I*d)*e^(2*I*f*x + 2*I*e) +
c + I*d)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e
) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-3*I*f*x - 3*I*e)/((I*a^2*c -
 a^2*d)*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + d \tan{\left(e + fx \right)}}}{\left(ia \left(\tan{\left(e + fx \right)} - i\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(c + d*tan(e + f*x))/(I*a*(tan(e + f*x) - I))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c+d\tan\left(e+f\,x\right)}}{{\left(a+a\tan\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(1/2)/(a + a*tan(e + f*x)*1i)^(3/2),x)
```

```
[Out] int((c + d*tan(e + f*x))^(1/2)/(a + a*tan(e + f*x)*1i)^(3/2), x)
```

3.1142 $\int \frac{\sqrt{c+d\tan(e+fx)}}{(a+ia\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=254

$$-\frac{i\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+fx)}}\right)}{4\sqrt{2}\,a^{5/2}f}+\frac{i\sqrt{c+d\tan(e+fx)}}{5f(a+ia\tan(e+fx))^{5/2}}+\frac{(5ic-3d)\sqrt{c+d\,\mathrm{ta}}}{30a(c+id)f(a+ia\,\mathrm{ta}}$$

```
[Out] -1/8*I*arctanh(2^(1/2)*a^(1/2)*(c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2)/(a+I*a*
tan(f*x+e))^(1/2))*(c-I*d)^(1/2)/a^(5/2)/f*2^(1/2)-1/60*(20*c*d-I*(15*c^2+3
*d^2))*(c+d*tan(f*x+e))^(1/2)/a^2/(c+I*d)^2/f/(a+I*a*tan(f*x+e))^(1/2)+1/5*
I*(c+d*tan(f*x+e))^(1/2)/f/(a+I*a*tan(f*x+e))^(5/2)+1/30*(5*I*c-3*d)*(c+d*t
an(f*x+e))^(1/2)/a/(c+I*d)/f/(a+I*a*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.55, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3638, 3677, 12, 3625, 214}

$$-\frac{i\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+fx)}}\right)}{4\sqrt{2}\,a^{5/2}f}-\frac{(20cd-i(15c^2+3d^2))\sqrt{c+d\tan(e+fx)}}{60a^2f(c+id)^2\sqrt{a+ia\tan(e+fx)}}+\frac{(-3d+5ic)\sqrt{c+d\tan(e+fx)}}{30af(c+id)(a+ia\tan(e+fx))^{3/2}}+\frac{i\sqrt{c+d\tan(e+fx)}}{5f(a+ia\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Tan[e + f*x]]/(a + I*a*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-1/4*I)*Sqrt[c - I*d]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])/
(Sqrt[c - I*d]*Sqrt[a + I*a*Tan[e + f*x]])])/(Sqrt[2]*a^(5/2)*f) + ((I/5)*S
qrt[c + d*Tan[e + f*x]])/(f*(a + I*a*Tan[e + f*x])^(5/2)) + (((5*I)*c - 3*d
)*Sqrt[c + d*Tan[e + f*x]])/(30*a*(c + I*d)*f*(a + I*a*Tan[e + f*x])^(3/2))
 - ((20*c*d - I*(15*c^2 + 3*d^2))*Sqrt[c + d*Tan[e + f*x]])/(60*a^2*(c + I*
d)^2*f*Sqrt[a + I*a*Tan[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
```

```
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3638

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*Sqrt[(c_.) + (d_.)*tan[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-b)*(a + b*Tan[e + f*x])^m*(Sqrt[c + d
*Tan[e + f*x]]/(2*a*f*m)), x] + Dist[1/(4*a^2*m), Int[(a + b*Tan[e + f*x])^
(m + 1)*(Simp[2*a*c*m + b*d + a*d*(2*m + 1)*Tan[e + f*x], x]/Sqrt[c + d*Tan
[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && IntegersQ[2*m]
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+d\tan(e+fx)}}{(a+ia\tan(e+fx))^{5/2}}\,dx = \frac{i\sqrt{c+d\tan(e+fx)}}{5f(a+ia\tan(e+fx))^{5/2}} - \frac{\int \frac{-a(5c-id)-4ad\tan(e+fx)}{(a+ia\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}\,dx}{10a^2}$$

$$= \frac{i\sqrt{c+d\tan(e+fx)}}{5f(a+ia\tan(e+fx))^{5/2}} + \frac{(5ic-3d)\sqrt{c+d\tan(e+fx)}}{30a(c+id)f(a+ia\tan(e+fx))^{3/2}} + \frac{\int \frac{-\ldots}{\sqrt{a\ldots}}}{\ldots}$$

$$= \frac{i\sqrt{c+d\tan(e+fx)}}{5f(a+ia\tan(e+fx))^{5/2}} + \frac{(5ic-3d)\sqrt{c+d\tan(e+fx)}}{30a(c+id)f(a+ia\tan(e+fx))^{3/2}} + \frac{(15ic^2\ldots}{60a\ldots}$$

$$= \frac{i\sqrt{c+d\tan(e+fx)}}{5f(a+ia\tan(e+fx))^{5/2}} + \frac{(5ic-3d)\sqrt{c+d\tan(e+fx)}}{30a(c+id)f(a+ia\tan(e+fx))^{3/2}} + \frac{(15ic^2\ldots}{60a\ldots}$$

$$= \frac{i\sqrt{c+d\tan(e+fx)}}{5f(a+ia\tan(e+fx))^{5/2}} + \frac{(5ic-3d)\sqrt{c+d\tan(e+fx)}}{30a(c+id)f(a+ia\tan(e+fx))^{3/2}} + \frac{(15ic^2\ldots}{60a\ldots}$$

$$= -\frac{i\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{4\sqrt{2}\,a^{5/2}f} + \frac{i\sqrt{c+d\tan(e\ldots}}{5f(a+ia\tan(e\ldots}$$

Mathematica [**A**]
time = 5.82, size = 302, normalized size = 1.19

$$\frac{\sec^{\frac{5}{2}}(e+fx)\left(-i\sqrt{2}\sqrt{c-id}\,e^{2i(e+fx)}\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\sqrt{1+e^{2i(e+fx)}}\log\left(2\left(\sqrt{c-id}\,e^{i(e+fx)}+\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)\right)+\frac{2i(11c^2+20icd-9d^2+(26c^2+40icd-6d^2)\cos(2(e+fx))+4ic(5c+7id)\sin(2(e+fx)))\sqrt{c+d\tan(e+fx)}}{15(c+id)^2\sqrt{\sec(e+fx)}}\right)}{8f(a+ia\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Tan[e + f*x]]/(a + I*a*Tan[e + f*x])^(5/2),x]
```

```
[Out] (Sec[e + f*x]^(5/2)*((-I)*Sqrt[2]*Sqrt[c - I*d]*E^((2*I)*(e + f*x))*Sqrt[E^
(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*Sqrt[1 + E^((2*I)*(e + f*x))]*Log[
2*(Sqrt[c - I*d]*E^(I*(e + f*x)) + Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (
I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))])] + (((2*I)/15)*
(11*c^2 + (20*I)*c*d - 9*d^2 + (26*c^2 + (40*I)*c*d - 6*d^2)*Cos[2*(e + f*x
)] + (4*I)*c*(5*c + (7*I)*d)*Sin[2*(e + f*x)])*Sqrt[c + d*Tan[e + f*x]])/((
c + I*d)^2*Sqrt[Sec[e + f*x]])))/(8*f*(a + I*a*Tan[e + f*x])^(5/2))
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2225 vs. 2(206) = 412.
time = 0.75, size = 2226, normalized size = 8.76

method	result	size
derivativedivides	Expression too large to display	2226
default	Expression too large to display	2226

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/240/f*(c+d*tan(f*x+e))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)/a^3*(-15*I*ln((3*
a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a
*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d
-c))^(1/2)*d^2+464*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c*d*tan(f*
x+e)-148*I*c^2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)+60*I*(a*(c+d*tan
(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*d^2-60*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)
))^(1/2)*d^2*tan(f*x+e)+240*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c*d
-12*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*d^2*tan(f*x+e)^3+308*c^2*(a
*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*tan(f*x+e)-60*c^2*(a*(c+d*tan(f*x
+e))*(1+I*tan(f*x+e)))^(1/2)*tan(f*x+e)^3-304*(a*(c+d*tan(f*x+e))*(1+I*tan(
f*x+e)))^(1/2)*c*d*tan(f*x+e)^2+220*I*c^2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+
e)))^(1/2)*tan(f*x+e)^2+12*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*d^
2*tan(f*x+e)^2-90*I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(
1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f
*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c^2*tan(f*x+e)^2+120*I*ln((3*a*c+I*a*t
an(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan
(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2
)*c*d*tan(f*x+e)^3-120*I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+
2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(
tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c*d*tan(f*x+e)-15*I*ln((3*a*c+I*a
*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*t
an(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1
/2)*d^2*tan(f*x+e)^4+15*I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)
+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/
(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c^2*tan(f*x+e)^4-30*ln((3*a*c+I*
a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*
tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(
1/2)*c*d*tan(f*x+e)^4+90*I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e
)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))
/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*d^2*tan(f*x+e)^2+180*ln((3*a*c+
I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+
d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))
^(1/2)*c*d*tan(f*x+e)^2+60*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e
)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))
/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c^2*tan(f*x+e)^3-60*ln((3*a*c+I
```

```
*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d
*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^
(1/2)*d^2*tan(f*x+e)^3-60*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)
+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/
(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c^2*tan(f*x+e)+60*ln((3*a*c+I*a*
tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*ta
n(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/
2)*d^2*tan(f*x+e)-30*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^
(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(
f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c*d-80*I*(a*(c+d*tan(f*x+e))*(1+I*tan
(f*x+e)))^(1/2)*c*d*tan(f*x+e)^3+15*I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*
d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+
e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c^2)/(a*(c+d*tan(f*x
+e))*(1+I*tan(f*x+e)))^(1/2)/(I*c-d)^2/(-tan(f*x+e)+I)^4
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(199) = 398$.
time = 0.86, size = 521, normalized size = 2.05

$$\frac{\left(15\sqrt{\frac{1}{2}}(a^3c^2+2i\,a^3cd-a^3d^2)f\sqrt{-\frac{c-i\,d}{a^5f^2}}e^{(5i\,fx+5i\,e)}\log\left(2i\sqrt{\frac{1}{2}}a^3f\sqrt{-\frac{c-i\,d}{a^5f^2}}e^{(i\,fx+i\,e)}+\sqrt{2}\sqrt{\frac{(c-i\,d)e^{(2i\,fx+2i\,e)}+c+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}(e^{(2i\,fx+2i\,e)}+1)\right)-15\sqrt{\frac{1}{2}}(a^3c^2+2i\,a^3cd-a^3d^2)f\sqrt{-\frac{c-i\,d}{a^5f^2}}e^{(5i\,fx+5i\,e)}\log\left(-2i\sqrt{\frac{1}{2}}a^3f\sqrt{-\frac{c-i\,d}{a^5f^2}}e^{(i\,fx+i\,e)}+\sqrt{2}\sqrt{\frac{(c-i\,d)e^{(2i\,fx+2i\,e)}+c+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}(e^{(2i\,fx+2i\,e)}+1)\right)-\sqrt{2}\left(-3i\,c^2+6\,cd+3i\,d^2+(-23i\,c^2+34\,cd+3i\,d^2)e^{(6i\,fx+6i\,e)}-2\,(17i\,c^2-27\,cd-6i\,d^2)e^{(4i\,fx+4i\,e)}-2\,(7i\,c^2-13\,cd-6i\,d^2)e^{(2i\,fx+2i\,e)}\right)\sqrt{\frac{(c-i\,d)e^{(2i\,fx+2i\,e)}+c+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-5i\,fx-5i\,e)}}{120\,(a^3c^2+2i\,a^3cd-a^3d^2)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] 1/120*(15*sqrt(1/2)*(a^3*c^2 + 2*I*a^3*c*d - a^3*d^2)*f*sqrt(-(c - I*d)/(a^
5*f^2))*e^(5*I*f*x + 5*I*e)*log(2*I*sqrt(1/2)*a^3*f*sqrt(-(c - I*d)/(a^5*f^
2))*e^(I*f*x + I*e) + sqrt(2)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d
)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*(e^(2*I*f*x
+ 2*I*e) + 1)) - 15*sqrt(1/2)*(a^3*c^2 + 2*I*a^3*c*d - a^3*d^2)*f*sqrt(-(c
- I*d)/(a^5*f^2))*e^(5*I*f*x + 5*I*e)*log(-2*I*sqrt(1/2)*a^3*f*sqrt(-(c - I
*d)/(a^5*f^2))*e^(I*f*x + I*e) + sqrt(2)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e
) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*(
e^(2*I*f*x + 2*I*e) + 1)) - sqrt(2)*(-3*I*c^2 + 6*c*d + 3*I*d^2 + (-23*I*c^
```

```
2 + 34*c*d + 3*I*d^2)*e^(6*I*f*x + 6*I*e) - 2*(17*I*c^2 - 27*c*d - 6*I*d^2)
*e^(4*I*f*x + 4*I*e) - 2*(7*I*c^2 - 13*c*d - 6*I*d^2)*e^(2*I*f*x + 2*I*e))*
sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*s
qrt(a/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-5*I*f*x - 5*I*e)/((a^3*c^2 + 2*I*a^3*
c*d - a^3*d^2)*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + d\tan(e + fx)}}{(ia(\tan(e + fx) - i))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(c + d*tan(e + f*x))/(I*a*(tan(e + f*x) - I))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c + d\tan(e + f\,x)}}{(a + a\tan(e + f\,x)\ 1\mathrm{i})^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(1/2)/(a + a*tan(e + f*x)*1i)^(5/2),x)
```

```
[Out] int((c + d*tan(e + f*x))^(1/2)/(a + a*tan(e + f*x)*1i)^(5/2), x)
```

3.1143 $\int (a+ia\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=329

$$-\frac{\sqrt[4]{-1}\,a^{5/2}(c-3id)\left(c^2+18icd+15d^2\right)\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}\right)}{8d^{3/2}f}-\frac{4i\sqrt{2}\,a^{5/2}(c-id)^{3/2}$$

```
[Out] -1/8*(-1)^(1/4)*a^(5/2)*(c-3*I*d)*(c^2+18*I*c*d+15*d^2)*arctanh((-1)^(3/4)*
d^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c+d*tan(f*x+e))^(1/2))/d^(3/2)/f-
4*I*a^(5/2)*(c-I*d)^(3/2)*arctanh(2^(1/2)*a^(1/2)*(c+d*tan(f*x+e))^(1/2)/(c
-I*d)^(1/2)/(a+I*a*tan(f*x+e))^(1/2))*2^(1/2)/f+1/8*a^2*(c^2+14*I*c*d+19*d^
2)*(a+I*a*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/d/f+1/12*a^2*(c+13*I*d)*
(a+I*a*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2)/d/f-1/3*a^2*(a+I*a*tan(f*x+
e))^(1/2)*(c+d*tan(f*x+e))^(5/2)/d/f
```

Rubi [A]
time = 0.89, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {3637, 3678, 3682, 3625, 214, 3680, 65, 223, 212}

$$-\frac{\sqrt[4]{-1}\,a^{5/2}(c-3id)\left(c^2+18icd+15d^2\right)\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}\right)}{8d^{3/2}f}-\frac{4i\sqrt{2}\,a^{5/2}(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+fx)}}\right)}{f}+\frac{a^2\left(c^2+14icd+19d^2\right)\sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{8df}-\frac{a^2\sqrt{a+ia\tan(e+fx)}\,(c+d\tan(e+fx))^{5/2}}{3df}+\frac{a^2(c+13id)\sqrt{a+ia\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}}{12df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^(5/2)*(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] -1/8*((-1)^(1/4)*a^(5/2)*(c - (3*I)*d)*(c^2 + (18*I)*c*d + 15*d^2)*ArcTanh[
((-1)^(3/4)*Sqrt[d]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c + d*Tan[e +
 f*x]])])/(d^(3/2)*f) - ((4*I)*Sqrt[2]*a^(5/2)*(c - I*d)^(3/2)*ArcTanh[(Sqr
t[2]*Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])/(Sqrt[c - I*d]*Sqrt[a + I*a*Tan[e +
f*x]])])/f + (a^2*(c^2 + (14*I)*c*d + 19*d^2)*Sqrt[a + I*a*Tan[e + f*x]]*Sq
rt[c + d*Tan[e + f*x]])/(8*d*f) + (a^2*(c + (13*I)*d)*Sqrt[a + I*a*Tan[e +
f*x]]*(c + d*Tan[e + f*x])^(3/2))/(12*d*f) - (a^2*Sqrt[a + I*a*Tan[e + f*x]
]*(c + d*Tan[e + f*x])^(5/2))/(3*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3678

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Dist[
1/(a*(m + n)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Simp
[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
```

```
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int (a+ia\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}\,dx = -\frac{a^2\sqrt{a+ia\tan(e+fx)}\,(c+d\tan(e+fx))^{5/2}}{3df}+\frac{a}{}$$

$$=\frac{a^2(c+13id)\sqrt{a+ia\tan(e+fx)}\,(c+d\tan(e+fx))}{12df}$$

$$=\frac{a^2(c^2+14icd+19d^2)\sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\,\mathrm{ta}}}{8df}$$

$$=\frac{a^2(c^2+14icd+19d^2)\sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\,\mathrm{ta}}}{8df}$$

$$=\frac{a^2(c^2+14icd+19d^2)\sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\,\mathrm{ta}}}{8df}$$

$$=-\frac{4i\sqrt{2}\,a^{5/2}(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(}}{\sqrt{c-id}\,\sqrt{a+ia\tan}}\right.}{f}$$

$$=-\frac{4i\sqrt{2}\,a^{5/2}(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(}}{\sqrt{c-id}\,\sqrt{a+ia\tan}}\right.}{f}$$

$$=-\frac{\sqrt[4]{-1}\,a^{5/2}(c-3id)\left(c^2+18icd+15d^2\right)\tanh^{-1}\left(\frac{(-1}{}\right.}{8d^{3/2}f}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 686 vs. $2(329) = 658$.
time = 10.30, size = 686, normalized size = 2.09

$$\frac{\left(\frac{1}{16}+\frac{i}{16}\right)\cos^2(e+fx)(a+ia\tan(e+fx))^{5/2}\left(-\frac{\left(i\cos(e+fx)\left(-ic^3+15c^2d-69icd^2-45d^3\right)\left(\log\left(\frac{(2+2i)e^{\frac{ie}{2}}\left(-id+de^{i(e+fx)}+ic\left(i+e^{i(e+fx)}\right)-(1+i)\sqrt{d}\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)}{\sqrt{d}\left(ic^3-15c^2d+69icd^2+45d^3\right)\left(i+e^{i(e+fx)}\right)}\right)-\log\left(\frac{(2+2i)e^{\frac{ie}{2}}\left(c+id+ice^{i(e+fx)}+de^{i(e+fx)}+(1+i)\sqrt{d}\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)}{\sqrt{d}\left(ic^3-15c^2d+69icd^2+45d^3\right)\left(-i+e^{i(e+fx)}\right)}\right)\right)+(64-64i)(c-id)^{3/2}d^{3/2}\log\left(2\left(\sqrt{c-id}\cos(e+fx)+i\sqrt{c-id}\sin(e+fx)+\sqrt{1+\cos(2(e+fx))+i\sin(2(e+fx))}\sqrt{c+d\tan(e+fx)}\right)\right)\right)(\cos(2e)-i\sin(2e))}{d^{3/2}\sqrt{1+\cos(2(e+fx))+i\sin(2(e+fx))}}+\frac{\left(\frac{1}{6}+\frac{i}{6}\right)\sec^2(e+fx)(i\cos(2e)+\sin(2e))\left(3c^2-68icd-49d^2+\left(3c^2-68icd-65d^2\right)\cos(2(e+fx))+2(7c-13id)d\sin(2(e+fx))\right)\sqrt{c+d\tan(e+fx)}}{d}\right)}{f(\cos(fx)+i\sin(fx))^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(5/2)*(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((1/16 + I/16)*Cos[e + f*x]^2*(a + I*a*Tan[e + f*x])^(5/2)*(((-I)*Cos[e + f
*x]*(((-I)*c^3 + 15*c^2*d - (69*I)*c*d^2 - 45*d^3)*(Log[((2 + 2*I)*E^((I/2)
*e)*((-I)*d + d*E^(I*(e + f*x)) + I*c*(I + E^(I*(e + f*x))) - (1 + I)*Sqrt[
d]*Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/
(1 + E^((2*I)*(e + f*x)))]))/(Sqrt[d]*(I*c^3 - 15*c^2*d + (69*I)*c*d^2 + 45
*d^3)*(I + E^(I*(e + f*x))))] - Log[((2 + 2*I)*E^((I/2)*e)*(c + I*d + I*c*E
^(I*(e + f*x)) + d*E^(I*(e + f*x)) + (1 + I)*Sqrt[d]*Sqrt[1 + E^((2*I)*(e +
 f*x))]*Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))
]))/(Sqrt[d]*(I*c^3 - 15*c^2*d + (69*I)*c*d^2 + 45*d^3)*(-I + E^(I*(e + f*x
))))]) + (64 - 64*I)*(c - I*d)^(3/2)*d^(3/2)*Log[2*(Sqrt[c - I*d]*Cos[e + f
*x] + I*Sqrt[c - I*d]*Sin[e + f*x] + Sqrt[1 + Cos[2*(e + f*x)] + I*Sin[2*(e
 + f*x)]]*Sqrt[c + d*Tan[e + f*x]])])*(Cos[2*e] - I*Sin[2*e]))/(d^(3/2)*Sqr
t[1 + Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)]]) + ((1/6 + I/6)*Sec[e + f*x]^2
*(I*Cos[2*e] + Sin[2*e])*(3*c^2 - (68*I)*c*d - 49*d^2 + (3*c^2 - (68*I)*c*d
 - 65*d^2)*Cos[2*(e + f*x)] + 2*(7*c - (13*I)*d)*d*Sin[2*(e + f*x)])*Sqrt[c
 + d*Tan[e + f*x]])/d))/(f*(Cos[f*x] + I*Sin[f*x])^2)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1517 vs. $2(265) = 530$.
time = 0.50, size = 1518, normalized size = 4.61

method	result	size
derivativedivides	Expression too large to display	1518
default	Expression too large to display	1518

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(5/2)*(c+d*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/96/f*(a*(1+I*tan(f*x+e)))^(1/2)*(c+d*tan(f*x+e))^(1/2)*a^2*(-16*2^(1/2)*d
^2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)*(-a*(I*d-c))^(
1/2)*tan(f*x+e)^2-28*2^(1/2)*c*d*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2
)*(I*a*d)^(1/2)*(-a*(I*d-c))^(1/2)*tan(f*x+e)+96*I*(I*a*d)^(1/2)*ln((3*a*c+
I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+
```

```
d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c*d+136*I*2^(1/2)*
(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2
)*c*d+3*I*2^(1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*
(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*
a*c^3+207*I*2^(1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(
a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2)
)*a*c*d^2-96*I*2^(1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+
2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1
/2))*a*d^2-45*2^(1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2
*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/
2))*a*c^2*d+135*2^(1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c
+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(
1/2))*a*d^3+52*I*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*
x+e)))^(1/2)*(I*a*d)^(1/2)*d^2*tan(f*x+e)+96*2^(1/2)*(-a*(I*d-c))^(1/2)*ln(
1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)
*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c*d-96*2^(1/2)*(-a*(I*d-c))^(1/2)*ln(1
/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*
(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*d^2-6*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+
d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)*c^2+114*2^(1/2)*(-a*(I*
d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)*d^2-9
6*I*2^(1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*t
an(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c*d+
96*I*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1
/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*
x+e)+I))*a*d^2-96*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(
f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(
1/2))/(tan(f*x+e)+I))*a*c*d+96*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a
*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*t
an(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*d^2)*2^(1/2)/d/(a*(c+d*tan(f*x+e))*(1+
I*tan(f*x+e)))^(1/2)/(I*a*d)^(1/2)/(-a*(I*d-c))^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)*(c+d*tan(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(3*d-c>0)', see `assume?` for more d
etails)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1512 vs. 2(263) = 526.

time = 1.08, size = 1512, normalized size = 4.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)*(c+d*tan(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/48*(96*sqrt(2)*(d*f*e^(4*I*f*x + 4*I*e) + 2*d*f*e^(2*I*f*x + 2*I*e) + d*f
)*sqrt(-(a^5*c^3 - 3*I*a^5*c^2*d - 3*a^5*c*d^2 + I*a^5*d^3)/f^2)*log((sqrt(
2)*f*sqrt(-(a^5*c^3 - 3*I*a^5*c^2*d - 3*a^5*c*d^2 + I*a^5*d^3)/f^2)*e^(I*f*
x + I*e) + sqrt(2)*(-I*a^2*c - a^2*d + (-I*a^2*c - a^2*d)*e^(2*I*f*x + 2*I*
e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1
))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/(-I*a^2*c - a^2*d))
- 96*sqrt(2)*(d*f*e^(4*I*f*x + 4*I*e) + 2*d*f*e^(2*I*f*x + 2*I*e) + d*f)*sq
rt(-(a^5*c^3 - 3*I*a^5*c^2*d - 3*a^5*c*d^2 + I*a^5*d^3)/f^2)*log(-(sqrt(2)*
f*sqrt(-(a^5*c^3 - 3*I*a^5*c^2*d - 3*a^5*c*d^2 + I*a^5*d^3)/f^2)*e^(I*f*x +
 I*e) - sqrt(2)*(-I*a^2*c - a^2*d + (-I*a^2*c - a^2*d)*e^(2*I*f*x + 2*I*e))
*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*
sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/(-I*a^2*c - a^2*d)) - 2
*sqrt(2)*((3*a^2*c^2 - 82*I*a^2*c*d - 91*a^2*d^2)*e^(5*I*f*x + 5*I*e) + 2*(
3*a^2*c^2 - 68*I*a^2*c*d - 49*a^2*d^2)*e^(3*I*f*x + 3*I*e) + 3*(a^2*c^2 - 1
8*I*a^2*c*d - 13*a^2*d^2)*e^(I*f*x + I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I
*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))
 + 3*(d*f*e^(4*I*f*x + 4*I*e) + 2*d*f*e^(2*I*f*x + 2*I*e) + d*f)*sqrt((I*a^
5*c^6 - 30*a^5*c^5*d - 87*I*a^5*c^4*d^2 - 1980*a^5*c^3*d^3 + 6111*I*a^5*c^2
*d^4 + 6210*a^5*c*d^5 - 2025*I*a^5*d^6)/(d^3*f^2))*log((2*d^2*f*sqrt((I*a^5
*c^6 - 30*a^5*c^5*d - 87*I*a^5*c^4*d^2 - 1980*a^5*c^3*d^3 + 6111*I*a^5*c^2*
d^4 + 6210*a^5*c*d^5 - 2025*I*a^5*d^6)/(d^3*f^2))*e^(I*f*x + I*e) + sqrt(2)
*(I*a^2*c^3 - 15*a^2*c^2*d + 69*I*a^2*c*d^2 + 45*a^2*d^3 + (I*a^2*c^3 - 15*
a^2*c^2*d + 69*I*a^2*c*d^2 + 45*a^2*d^3)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*
d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I
*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/(I*a^2*c^3 - 15*a^2*c^2*d + 69*I*a^2*
c*d^2 + 45*a^2*d^3)) - 3*(d*f*e^(4*I*f*x + 4*I*e) + 2*d*f*e^(2*I*f*x + 2*I*
e) + d*f)*sqrt((I*a^5*c^6 - 30*a^5*c^5*d - 87*I*a^5*c^4*d^2 - 1980*a^5*c^3*
d^3 + 6111*I*a^5*c^2*d^4 + 6210*a^5*c*d^5 - 2025*I*a^5*d^6)/(d^3*f^2))*log(
-(2*d^2*f*sqrt((I*a^5*c^6 - 30*a^5*c^5*d - 87*I*a^5*c^4*d^2 - 1980*a^5*c^3*
d^3 + 6111*I*a^5*c^2*d^4 + 6210*a^5*c*d^5 - 2025*I*a^5*d^6)/(d^3*f^2))*e^(I
*f*x + I*e) - sqrt(2)*(I*a^2*c^3 - 15*a^2*c^2*d + 69*I*a^2*c*d^2 + 45*a^2*d
^3 + (I*a^2*c^3 - 15*a^2*c^2*d + 69*I*a^2*c*d^2 + 45*a^2*d^3)*e^(2*I*f*x +
2*I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e)
 + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/(I*a^2*c^3 - 15*
a^2*c^2*d + 69*I*a^2*c*d^2 + 45*a^2*d^3)))/(d*f*e^(4*I*f*x + 4*I*e) + 2*d*f
*e^(2*I*f*x + 2*I*e) + d*f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(5/2)*(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)*(c+d*tan(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 1.51sym2poly/r2sym(c
onst gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+a\tan(e+f\,x)\ 1\mathrm{i})^{5/2}\,(c+d\tan(e+f\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(5/2)*(c + d*tan(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^(5/2)*(c + d*tan(e + f*x))^(3/2), x)
```

3.1144 $\int (a+ia\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=315

$$-\frac{\sqrt[4]{-1}\,a^{3/2}(3ic^2+18cd-11id^2)\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}\right)}{4\sqrt{d}\,f}-\frac{2i\sqrt{2}\,a^{3/2}(c-id)^{3/2}\tanh^{-1}\left(\right.}{}$$

```
[Out] -2*I*a^(3/2)*(c-I*d)^(3/2)*arctanh(2^(1/2)*a^(1/2)*(c+d*tan(f*x+e))^(1/2)/(
c-I*d)^(1/2)/(a+I*a*tan(f*x+e))^(1/2))*2^(1/2)/f-1/4*(-1)^(1/4)*a^(3/2)*(3*
I*c^2+18*c*d-11*I*d^2)*arctanh((-1)^(3/4)*d^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/
a^(1/2)/(c+d*tan(f*x+e))^(1/2))/f/d^(1/2)+1/4*a*(3*I*c+5*d)*(a+I*a*tan(f*x+
e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/f+1/2*a^2*(c+I*d)*(c+d*tan(f*x+e))^(3/2)/d
/f/(a+I*a*tan(f*x+e))^(1/2)-1/2*a^2*(c+d*tan(f*x+e))^(5/2)/d/f/(a+I*a*tan(f
*x+e))^(1/2)
```

Rubi [A]
time = 0.87, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3637, 3676, 3678, 3682, 3625, 214, 3680, 65, 223, 212}

$$-\frac{\sqrt[4]{-1}\,a^{3/2}(3ic^2+18cd-11id^2)\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}\right)}{4\sqrt{d}\,f}-\frac{2i\sqrt{2}\,a^{3/2}(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+fx)}}\right)}{f}-\frac{a^2(c+d\tan(e+fx))^{5/2}}{2df\sqrt{a+ia\tan(e+fx)}}+\frac{a^2(c+id)(c+d\tan(e+fx))^{3/2}}{2df\sqrt{a+ia\tan(e+fx)}}+\frac{a(5d+3ic)\sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] -1/4*((-1)^(1/4)*a^(3/2)*((3*I)*c^2 + 18*c*d - (11*I)*d^2)*ArcTanh[((-1)^(3
/4)*Sqrt[d]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])]
)/(Sqrt[d]*f) - ((2*I)*Sqrt[2]*a^(3/2)*(c - I*d)^(3/2)*ArcTanh[(Sqrt[2]*Sqr
t[a]*Sqrt[c + d*Tan[e + f*x]])/(Sqrt[c - I*d]*Sqrt[a + I*a*Tan[e + f*x]])])
/f + (a*((3*I)*c + 5*d)*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]]
)/(4*f) + (a^2*(c + I*d)*(c + d*Tan[e + f*x])^(3/2))/(2*d*f*Sqrt[a + I*a*Ta
n[e + f*x]]) - (a^2*(c + d*Tan[e + f*x])^(5/2))/(2*d*f*Sqrt[a + I*a*Tan[e +
 f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rule 3678

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Dist[
```

```
1/(a*(m + n)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Simp
[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int (a+ia\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}\,dx = -\frac{a^2(c+d\tan(e+fx))^{5/2}}{2df\sqrt{a+ia\tan(e+fx)}} + \frac{a\int \frac{(-\frac{1}{2}a(ic-9d)-\frac{1}{2}a(c-7id)}{\sqrt{a+ia}}}{}$$

$$= \frac{a^2(c+id)(c+d\tan(e+fx))^{3/2}}{2df\sqrt{a+ia\tan(e+fx)}} - \frac{a^2(c+d\tan(e+}{2df\sqrt{a+ia\tan(e}}$$

$$= \frac{a(3ic+5d)\sqrt{a+ia\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{4f}$$

$$= \frac{a(3ic+5d)\sqrt{a+ia\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{4f}$$

$$= \frac{a(3ic+5d)\sqrt{a+ia\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{4f}$$

$$= -\frac{2i\sqrt{2}\,a^{3/2}(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(}{\sqrt{c-id}\sqrt{a+ia\tan}}\right.}{f}$$

$$= -\frac{2i\sqrt{2}\,a^{3/2}(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(}{\sqrt{c-id}\sqrt{a+ia\tan}}\right.}{f}$$

$$= -\frac{\sqrt[4]{-1}\,a^{3/2}(3ic^2+18cd-11id^2)\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}}{\sqrt{a}\sqrt{}}\right.}{4\sqrt{d}\,f}$$

Mathematica [A]
time = 7.20, size = 574, normalized size = 1.82

$$\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\cos(e+fx)(\cos(e)-i\sin(e))(\cos(fx)-i\sin(fx))(a+ia\tan(e+fx))^{3/2}\left(-\frac{\cos(e+fx)(3ic^2+18cd-11id^2)\left(\log\left(\frac{(2+2i)e^{\frac{ie}{2}}\left(d+ide^{i(e+fx)}-c\left(i+e^{i(e+fx)}\right)+(1-i)\sqrt{d}\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)}{\sqrt{d}\left(-3c^2+18icd+11d^2\right)\left(i+e^{i(e+fx)}\right)}\right)-\log\left(\frac{(2+2i)e^{\frac{ie}{2}}\left(c+id+ice^{i(e+fx)}+de^{i(e+fx)}+(1+i)\sqrt{d}\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)}{\sqrt{d}\left(3ic^2+18cd-11id^2\right)\left(-i+e^{i(e+fx)}\right)}\right)\right)+(16+16i)(c-id)^{3/2}\sqrt{d}\log\left(2\left(\sqrt{c-id}\cos(e+fx)+i\sqrt{c-id}\sin(e+fx)+\sqrt{1+\cos(2(e+fx))+i\sin(2(e+fx))}\sqrt{c+d\tan(e+fx)}\right)\right)}{\sqrt{d}\sqrt{1+\cos(2(e+fx))+i\sin(2(e+fx))}}+(1+i)\sqrt{c+d\tan(e+fx)}(5c-5id+2d\tan(e+fx))\right)}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((1/8 + I/8)*Cos[e + f*x]*(Cos[e] - I*Sin[e])*(Cos[f*x] - I*Sin[f*x])*(a +
I*a*Tan[e + f*x])^(3/2)*(-((Cos[e + f*x]*(((3*I)*c^2 + 18*c*d - (11*I)*d^2)
```

```
*(Log[((2 + 2*I)*E^((I/2)*e)*(d + I*d*E^(I*(e + f*x)) - c*(I + E^(I*(e + f*
x))) + (1 - I)*Sqrt[d]*Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^
((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))]))/(Sqrt[d]*(-3*c^2 + (18*I)*
c*d + 11*d^2)*(I + E^(I*(e + f*x))))] - Log[((2 + 2*I)*E^((I/2)*e)*(c + I*d
 + I*c*E^(I*(e + f*x)) + d*E^(I*(e + f*x)) + (1 + I)*Sqrt[d]*Sqrt[1 + E^((2
*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e
+ f*x)))]))/(Sqrt[d]*((3*I)*c^2 + 18*c*d - (11*I)*d^2)*(-I + E^(I*(e + f*x)
)))]) + (16 + 16*I)*(c - I*d)^(3/2)*Sqrt[d]*Log[2*(Sqrt[c - I*d]*Cos[e + f*
x] + I*Sqrt[c - I*d]*Sin[e + f*x] + Sqrt[1 + Cos[2*(e + f*x)] + I*Sin[2*(e
+ f*x)]]*Sqrt[c + d*Tan[e + f*x]])]))/(Sqrt[d]*Sqrt[1 + Cos[2*(e + f*x)] +
I*Sin[2*(e + f*x)]])) + (1 + I)*Sqrt[c + d*Tan[e + f*x]]*(5*c - (5*I)*d + 2
*d*Tan[e + f*x])))/f
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1233 vs. $2(251) = 502$.
time = 0.54, size = 1234, normalized size = 3.92

method	result	size
derivativedivides	Expression too large to display	1234
default	Expression too large to display	1234

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/16/f*(a*(1+I*tan(f*x+e)))^(1/2)*(c+d*tan(f*x+e))^(1/2)*a*(4*I*(a*(c+d*tan
(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*(I*a*d)^(1/2)*d
*tan(f*x+e)+18*I*2^(1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*
c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^
(1/2))*a*c*d+10*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*2^(1/2)*(-a*(
I*d-c))^(1/2)*(I*a*d)^(1/2)*c-8*I*2^(1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*
d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1
/2)+a*d)/(I*a*d)^(1/2))*a*c-8*I*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*
tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*d*2
^(1/2)*(-a*(I*d-c))^(1/2)-3*2^(1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(
f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*
d)/(I*a*d)^(1/2))*a*c^2+11*2^(1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f
*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d
)/(I*a*d)^(1/2))*a*d^2+8*I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e
)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))
/(tan(f*x+e)+I))*(I*a*d)^(1/2)*a*c+8*I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a
*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x
+e)))^(1/2))/(tan(f*x+e)+I))*(I*a*d)^(1/2)*a*d+10*(a*(c+d*tan(f*x+e))*(1+I*
tan(f*x+e)))^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*(I*a*d)^(1/2)*d+8*2^(1/2)*(-a
*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I
```

```
*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c-8*2^(1/2)*(-a*(I*
d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan
(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*d-8*(I*a*d)^(1/2)*ln((3
*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(
a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c+8*ln((3*a*c
+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c
+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*d*(I*a*d)^(1/2))/
(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)/(I*a*d)^(1/2)*2^(1/2)/(-a*(I*d-
c))^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

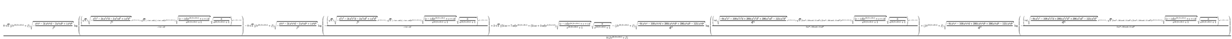
Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(3*d-c>0)', see `assume?` for more d
etails)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1142 vs. 2(249) = 498.
time = 0.91, size = 1142, normalized size = 3.63



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/8*(8*sqrt(2)*(f*e^(2*I*f*x + 2*I*e) + f)*sqrt(-(a^3*c^3 - 3*I*a^3*c^2*d -
 3*a^3*c*d^2 + I*a^3*d^3)/f^2)*log((sqrt(2)*f*sqrt(-(a^3*c^3 - 3*I*a^3*c^2*
d - 3*a^3*c*d^2 + I*a^3*d^3)/f^2)*e^(I*f*x + I*e) + sqrt(2)*(-I*a*c - a*d +
 (-I*a*c - a*d)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) +
c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-
I*f*x - I*e)/(-I*a*c - a*d)) - 8*sqrt(2)*(f*e^(2*I*f*x + 2*I*e) + f)*sqrt(-
(a^3*c^3 - 3*I*a^3*c^2*d - 3*a^3*c*d^2 + I*a^3*d^3)/f^2)*log(-(sqrt(2)*f*sq
rt(-(a^3*c^3 - 3*I*a^3*c^2*d - 3*a^3*c*d^2 + I*a^3*d^3)/f^2)*e^(I*f*x + I*e
) - sqrt(2)*(-I*a*c - a*d + (-I*a*c - a*d)*e^(2*I*f*x + 2*I*e))*sqrt(((c -
I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2
*I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/(-I*a*c - a*d)) + 2*sqrt(2)*((5*I*a
*c + 7*a*d)*e^(3*I*f*x + 3*I*e) + (5*I*a*c + 3*a*d)*e^(I*f*x + I*e))*sqrt((
```

```
(c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/
(e^(2*I*f*x + 2*I*e) + 1)) - (f*e^(2*I*f*x + 2*I*e) + f)*sqrt((-9*I*a^3*c^4
 - 108*a^3*c^3*d + 390*I*a^3*c^2*d^2 + 396*a^3*c*d^3 - 121*I*a^3*d^4)/(d*f^
2))*log((2*d*f*sqrt((-9*I*a^3*c^4 - 108*a^3*c^3*d + 390*I*a^3*c^2*d^2 + 396
*a^3*c*d^3 - 121*I*a^3*d^4)/(d*f^2))*e^(I*f*x + I*e) + sqrt(2)*(3*a*c^2 - 1
8*I*a*c*d - 11*a*d^2 + (3*a*c^2 - 18*I*a*c*d - 11*a*d^2)*e^(2*I*f*x + 2*I*e
))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1)
)*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/(3*a*c^2 - 18*I*a*c*d
 - 11*a*d^2)) + (f*e^(2*I*f*x + 2*I*e) + f)*sqrt((-9*I*a^3*c^4 - 108*a^3*c^
3*d + 390*I*a^3*c^2*d^2 + 396*a^3*c*d^3 - 121*I*a^3*d^4)/(d*f^2))*log(-(2*d
*f*sqrt((-9*I*a^3*c^4 - 108*a^3*c^3*d + 390*I*a^3*c^2*d^2 + 396*a^3*c*d^3 -
 121*I*a^3*d^4)/(d*f^2))*e^(I*f*x + I*e) - sqrt(2)*(3*a*c^2 - 18*I*a*c*d -
11*a*d^2 + (3*a*c^2 - 18*I*a*c*d - 11*a*d^2)*e^(2*I*f*x + 2*I*e))*sqrt(((c
- I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^
(2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/(3*a*c^2 - 18*I*a*c*d - 11*a*d^2)
))/(f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan{(e+fx)}-i))^{\frac{3}{2}}\,(c+d\tan{(e+fx)})^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(3/2)*(c+d*tan(f*x+e))**(3/2),x)
[Out] Integral((I*a*(tan(e + f*x) - I))**(3/2)*(c + d*tan(e + f*x))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(3/2),x, algorithm="gia
c")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 0.61index.cc index_m
 operator + Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+a\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{3/2}\,{\left(c+d\,\mathrm{tan}\left(e+f\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(3/2)*(c + d*tan(e + f*x))^(3/2),x)
[Out] int((a + a*tan(e + f*x)*1i)^(3/2)*(c + d*tan(e + f*x))^(3/2), x)
```

3.1145 $\int \sqrt{a+ia\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=196

$$-\frac{\sqrt[4]{-1}\sqrt{a}\,(3c-id)\sqrt{d}\,\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}\right)}{f}-\frac{i\sqrt{2}\,\sqrt{a}\,(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\cdots}{\sqrt{c-id}\cdots}\right.}{f}$$

```
[Out] -I*(c-I*d)^(3/2)*arctanh(2^(1/2)*a^(1/2)*(c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/
2)/(a+I*a*tan(f*x+e))^(1/2))*2^(1/2)*a^(1/2)/f-(-1)^(1/4)*(3*c-I*d)*arctanh
((-1)^(3/4)*d^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c+d*tan(f*x+e))^(1/2)
)*a^(1/2)*d^(1/2)/f+d*(a+I*a*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.47, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3641, 3682, 3625, 214, 3680, 65, 223, 212}

$$\frac{d\sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{f}-\frac{i\sqrt{2}\,\sqrt{a}\,(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+fx)}}\right)}{f}-\frac{\sqrt[4]{-1}\sqrt{a}\,\sqrt{d}\,(3c-id)\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] -(((-1)^(1/4)*Sqrt[a]*(3*c - I*d)*Sqrt[d]*ArcTanh[((-1)^(3/4)*Sqrt[d]*Sqrt[
a + I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])])/f) - (I*Sqrt[2]
*Sqrt[a]*(c - I*d)^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])
/(Sqrt[c - I*d]*Sqrt[a + I*a*Tan[e + f*x]])])/f + (d*Sqrt[a + I*a*Tan[e + f
*x]]*Sqrt[c + d*Tan[e + f*x]])/f
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3641

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[d*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n - 1)/(f*(m + n - 1))), x] - Dist[1/(a*(m + n - 1)), Int[(a + b*
Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 2)*Simp[d*(b*c*m + a*d*(-1 + n))
- a*c^2*(m + n - 1) + d*(b*d*m - a*c*(m + 2*n - 2))*Tan[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1] && NeQ[m + n - 1, 0] && (IntegerQ[n]
|| IntegersQ[2*m, 2*n])
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \sqrt{a+ia\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}\,dx = \frac{d\sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{f} - \frac{\int \sqrt{a+ \ldots}{\ldots}$$

$$= \frac{d\sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{f} + (c-id)^{\ldots}$$

$$= \frac{d\sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{f} - \frac{(2ia^2(c \ldots}{\ldots}$$

$$= -\frac{i\sqrt{2}\,\sqrt{a}\,(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+\ldots}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e\ldots}}\right.}{f}$$

$$= -\frac{i\sqrt{2}\,\sqrt{a}\,(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+\ldots}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e\ldots}}\right.}{f}$$

$$= -\frac{\sqrt[4]{-1}\,\sqrt{a}\,(3c-id)\sqrt{d}\,\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\,\sqrt{a+ia\tan\ldots}}{\sqrt{a}\,\sqrt{c+d\tan(\ldots}}\right.}{f}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 507 vs. 2(196) = 392.
time = 7.85, size = 507, normalized size = 2.59

$$\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{a+ia\tan(e+fx)}\left(-\frac{\cos(e+fx)\left((3c-id)\sqrt{d}\log\left(\frac{(2+2i)e^{\frac{ie}{2}}\left(d+ide^{i(e+fx)}-c\left(i+e^{i(e+fx)}\right)+(1-i)\sqrt{d}\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)}{d^{3/2}(3ic+d)\left(i+e^{i(e+fx)}\right)}\right)+i\sqrt{d}(3ic+d)\log\left(\frac{(2+2i)e^{\frac{ie}{2}}\left(c+id+ice^{i(e+fx)}+de^{i(e+fx)}+(1+i)\sqrt{d}\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)}{(3c-id)d^{3/2}\left(-i+e^{i(e+fx)}\right)}\right)+(2+2i)(c-id)^{3/2}\log\left(2\left(\sqrt{c-id}\cos(e+fx)+i\sqrt{c-id}\sin(e+fx)+\sqrt{1+\cos(2(e+fx))+i\sin(2(e+fx))}\sqrt{c+d\tan(e+fx)}\right)\right)\right)}{\sqrt{1+\cos(2(e+fx))+i\sin(2(e+fx))}}+(1-i)d\sqrt{c+d\tan(e+fx)}\right)}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + I*a*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((1/2 + I/2)*Sqrt[a + I*a*Tan[e + f*x]]*(-((Cos[e + f*x]*((3*c - I*d)*Sqrt[
d]*Log[((2 + 2*I)*E^((I/2)*e)*(d + I*d*E^(I*(e + f*x)) - c*(I + E^(I*(e + f
*x))) + (1 - I)*Sqrt[d]*Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E
^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))]))/(d^(3/2)*((3*I)*c + d)*(I
 + E^(I*(e + f*x))))] + I*Sqrt[d]*((3*I)*c + d)*Log[((2 + 2*I)*E^((I/2)*e)*
(c + I*d + I*c*E^(I*(e + f*x)) + d*E^(I*(e + f*x)) + (1 + I)*Sqrt[d]*Sqrt[1
 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((
```

```
2*I)*(e + f*x)))]))/((3*c - I*d)*d^(3/2)*(-I + E^(I*(e + f*x))))] + (2 + 2*
I)*(c - I*d)^(3/2)*Log[2*(Sqrt[c - I*d]*Cos[e + f*x] + I*Sqrt[c - I*d]*Sin[
e + f*x] + Sqrt[1 + Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)]]*Sqrt[c + d*Tan[e
 + f*x]])]))/Sqrt[1 + Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)]]) + (1 - I)*d*S
qrt[c + d*Tan[e + f*x]]))/f
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1701 vs. 2(153) = 306.
time = 0.60, size = 1702, normalized size = 8.68

method	result	size
derivativedivides	Expression too large to display	1702
default	Expression too large to display	1702

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/4/f*(a*(1+I*tan(f*x+e)))^(1/2)*(c+d*tan(f*x+e))^(1/2)*(-2*I*(I*a*d)^(1/2
)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^
(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c^3-2*
I*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)
*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e
)+I))*a*c^2*d*tan(f*x+e)-2*I*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d
+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan
(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*d^3*tan(f*x+e)+2*(I*a*d)^(1/2)*ln((3*a*c
+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c
+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c^3*tan(f*x+e)-2*
(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(
-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+
I))*a*c^2*d+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)*2^(
1/2)*(-a*(I*d-c))^(1/2)*c*d+3*I*2^(1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*
tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2
)+a*d)/(I*a*d)^(1/2))*a*c^2*d+I*2^(1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*
tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2
)+a*d)/(I*a*d)^(1/2))*a*d^3+2*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)
*(I*a*d)^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*c*d*tan(f*x+e)-2*I*2^(1/2)*(-a*(I
*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*ta
n(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c*d^2*tan(f*x+e)-2*2^(
1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+
e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c*d^2+2*I*(
a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)*2^(1/2)*(-a*(I*d-c
))^(1/2)*d^2-2*I*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f
*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1
/2))/(tan(f*x+e)+I))*a*c*d^2-2*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a
```

```
*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*t
an(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*d^3-2^(1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*
(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*
a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*d^3*tan(f*x+e)-2*(a*(c+d*tan(f*x+e))*(1+I*
tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*d^2*tan(f*x+e)+
2*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)
*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e
)+I))*a*c*d^2*tan(f*x+e)-3*2^(1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f
*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d
)/(I*a*d)^(1/2))*a*c^2*d*tan(f*x+e))/(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^
(1/2)/(I*a*d)^(1/2)/(I*c-d)/(-tan(f*x+e)+I)*2^(1/2)/(-a*(I*d-c))^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 796 vs. 2(152) = 304.
time = 0.89, size = 796, normalized size = 4.06

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/2*(2*sqrt(2)*d*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x
 + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*e^(I*f*x + I*e) - sqrt(2)
*f*sqrt(-(a*c^3 - 3*I*a*c^2*d - 3*a*c*d^2 + I*a*d^3)/f^2)*log((sqrt(2)*f*sq
rt(-(a*c^3 - 3*I*a*c^2*d - 3*a*c*d^2 + I*a*d^3)/f^2)*e^(I*f*x + I*e) + sqrt
(2)*((I*c + d)*e^(2*I*f*x + 2*I*e) + I*c + d)*sqrt(((c - I*d)*e^(2*I*f*x +
2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) +
1)))*e^(-I*f*x - I*e)/(I*c + d)) + sqrt(2)*f*sqrt(-(a*c^3 - 3*I*a*c^2*d - 3
*a*c*d^2 + I*a*d^3)/f^2)*log(-(sqrt(2)*f*sqrt(-(a*c^3 - 3*I*a*c^2*d - 3*a*c
*d^2 + I*a*d^3)/f^2)*e^(I*f*x + I*e) - sqrt(2)*((I*c + d)*e^(2*I*f*x + 2*I*
e) + I*c + d)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x +
2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/(I*c + d))
 + f*sqrt((9*I*a*c^2*d + 6*a*c*d^2 - I*a*d^3)/f^2)*log((sqrt(2)*((3*I*c + d
```

```
)*e^(2*I*f*x + 2*I*e) + 3*I*c + d)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c
+ I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)) + 2*f*s
qrt((9*I*a*c^2*d + 6*a*c*d^2 - I*a*d^3)/f^2)*e^(I*f*x + I*e))*e^(-I*f*x - I
*e)/(3*I*c + d)) - f*sqrt((9*I*a*c^2*d + 6*a*c*d^2 - I*a*d^3)/f^2)*log((sqr
t(2)*((3*I*c + d)*e^(2*I*f*x + 2*I*e) + 3*I*c + d)*sqrt(((c - I*d)*e^(2*I*f
*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*
e) + 1)) - 2*f*sqrt((9*I*a*c^2*d + 6*a*c*d^2 - I*a*d^3)/f^2)*e^(I*f*x + I*e
))*e^(-I*f*x - I*e)/(3*I*c + d)))/f
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\left(\tan\left(e+fx\right)-i\right)}\,\left(c+d\tan\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(1/2)*(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(e + f*x) - I))*(c + d*tan(e + f*x))**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+a\tan\left(e+f\,x\right)\,1{}\mathrm{i}}\,\left(c+d\tan(e+f\,x)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(1/2)*(c + d*tan(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^(1/2)*(c + d*tan(e + f*x))^(3/2), x)
```

3.1146 $\int \frac{(c+d\tan(e+fx))^{3/2}}{\sqrt{a+ia\tan(e+fx)}}\,dx$

Optimal. Leaf size=195

$$\frac{2(-1)^{3/4}d^{3/2}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a}\,f}-\frac{i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\,f}$$

```
[Out] 2*(-1)^(3/4)*d^(3/2)*arctanh((-1)^(3/4)*d^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^
(1/2)/(c+d*tan(f*x+e))^(1/2))/f/a^(1/2)-1/2*I*(c-I*d)^(3/2)*arctanh(2^(1/2)
*a^(1/2)*(c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2)/(a+I*a*tan(f*x+e))^(1/2))/f*2
^(1/2)/a^(1/2)+(I*c-d)*(c+d*tan(f*x+e))^(1/2)/f/(a+I*a*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3639, 3682, 3625, 214, 3680, 65, 223, 212}

$$\frac{2(-1)^{3/4}d^{3/2}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a}\,f}+\frac{(-d+ic)\sqrt{c+d\tan(e+fx)}}{f\sqrt{a+ia\tan(e+fx)}}-\frac{i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^(3/2)/Sqrt[a + I*a*Tan[e + f*x]],x]
```

```
[Out] (2*(-1)^(3/4)*d^(3/2)*ArcTanh[((-1)^(3/4)*Sqrt[d]*Sqrt[a + I*a*Tan[e + f*x]
])/(Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a]*f) - (I*(c - I*d)^(3/2)*Ar
cTanh[(Sqrt[2]*Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])/(Sqrt[c - I*d]*Sqrt[a + I*
a*Tan[e + f*x]])])/(Sqrt[2]*Sqrt[a]*f) + ((I*c - d)*Sqrt[c + d*Tan[e + f*x]
])/(f*Sqrt[a + I*a*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{3/2}}{\sqrt{a+ia\tan(e+fx)}}\,dx = \frac{(ic-d)\sqrt{c+d\tan(e+fx)}}{f\sqrt{a+ia\tan(e+fx)}} - \frac{\int \frac{\sqrt{a+ia\tan(e+fx)}\left(-\frac{1}{2}a\left(c^2-2icd+d^2\right)+iad^2\,\text{ta}\right.}{\sqrt{c+d\tan(e+fx)}}}{a^2}$$

$$= \frac{(ic-d)\sqrt{c+d\tan(e+fx)}}{f\sqrt{a+ia\tan(e+fx)}} + \frac{(c-id)^2\int \frac{\sqrt{a+ia\tan(e+fx)}}{\sqrt{c+d\tan(e+fx)}}\,dx}{2a} + \frac{d^2\int}{}$$

$$= \frac{(ic-d)\sqrt{c+d\tan(e+fx)}}{f\sqrt{a+ia\tan(e+fx)}} - \frac{\left(ia(c-id)^2\right)\text{Subst}\left(\int \frac{1}{ac-iad-2a^2x^2}\,dx, x, \frac{\sqrt{c}}{\sqrt{a}}\right.}{f}$$

$$= -\frac{i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{\sqrt{2}\sqrt{a}f} + \frac{(ic-d)\sqrt{c+d\,\text{ta}}}{f\sqrt{a+ia\tan(}}$$

$$= -\frac{i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{\sqrt{2}\sqrt{a}f} + \frac{(ic-d)\sqrt{c+d\,\text{ta}}}{f\sqrt{a+ia\tan(}}$$

$$= \frac{2(-1)^{3/4}d^{3/2}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a}f} - \frac{i(c-id)^{3/2}\tanh^{-}}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 518 vs. $2(195) = 390$.
time = 7.88, size = 518, normalized size = 2.66

$$\frac{\sqrt{\sec(e+fx)}\left(\sqrt{2}\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\sqrt{1+e^{2i(e+fx)}}\left(-i(c-id)^{3/2}\log\left(2\left(\sqrt{c-id}\,e^{i(e+fx)}+\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)\right)-(1-i)d^{3/2}\left(\log\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)e^{\frac{ie}{2}}\left(d+ide^{i(e+fx)}-c\left(i+e^{i(e+fx)}\right)+(1-i)\sqrt{d}\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)}{d^{5/2}\left(i+e^{i(e+fx)}\right)}\right)-\log\left(-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)e^{\frac{ie}{2}}\left(c+id+ice^{i(e+fx)}+de^{i(e+fx)}+(1+i)\sqrt{d}\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)}{d^{5/2}\left(-i+e^{i(e+fx)}\right)}\right)\right)\right)+\frac{2i(c+id)\sqrt{c+d\tan(e+fx)}}{\sqrt{\sec(e+fx)}}\right)}{2f\sqrt{a+ia\tan(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Tan[e + f*x])^(3/2)/Sqrt[a + I*a*Tan[e + f*x]],x]
```

```
[Out] (Sqrt[Sec[e + f*x]]*(Sqrt[2]*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))
]*Sqrt[1 + E^((2*I)*(e + f*x))]*((-I)*(c - I*d)^(3/2)*Log[2*(Sqrt[c - I*d]*
E^(I*(e + f*x)) + Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^((2*I
)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))])] - (1 - I)*d^(3/2)*(Log[((1/2 +
I/2)*E^((I/2)*e)*(d + I*d*E^(I*(e + f*x)) - c*(I + E^(I*(e + f*x))) + (1 -
I)*Sqrt[d]*Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^((2*I)*(e +
```

```
f*x))))/(1 + E^((2*I)*(e + f*x)))]))/(d^(5/2)*(I + E^(I*(e + f*x))))] - Log
[((-1/2 + I/2)*E^((I/2)*e)*(c + I*d + I*c*E^(I*(e + f*x)) + d*E^(I*(e + f*x
)) + (1 + I)*Sqrt[d]*Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^((
2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))]))/(d^(5/2)*(-I + E^(I*(e + f*x
))))])) + ((2*I)*(c + I*d)*Sqrt[c + d*Tan[e + f*x]])/Sqrt[Sec[e + f*x]]))/(
2*f*Sqrt[a + I*a*Tan[e + f*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1168 vs. $2(152) = 304$.
time = 0.52, size = 1169, normalized size = 5.99

method	result	size
derivativedivides	Expression too large to display	1169
default	Expression too large to display	1169

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/4/f/a*(I*(I*a*d)^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*ln((3*a*c+I*a*tan(f*x+
e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e)
)*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c*tan(f*x+e)^2-2*I*(I*a*d)^(1/2)
*2^(1/2)*(-a*(I*d-c))^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+
e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)
)/(tan(f*x+e)+I))*d*tan(f*x+e)+(I*a*d)^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*ln(
(3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)
*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*d*tan(f*x+e)^
2-4*c*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)*tan(f*x+e)-
I*(I*a*d)^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d
+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan
(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c+8*I*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(
a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2)
)*a*d^2*tan(f*x+e)+2*(I*a*d)^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*ln((3*a*c+I*a
*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*t
an(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c*tan(f*x+e)-4*ln(1/2*(
2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a
*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*d^2*tan(f*x+e)^2-4*I*(a*(c+d*tan(f*x+e))*(1
+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)*d*tan(f*x+e)-(I*a*d)^(1/2)*2^(1/2)*(-a*
(I*d-c))^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*
(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)
+I))*d+4*I*c*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+4*ln
(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2
)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*d^2-4*(a*(c+d*tan(f*x+e))*(1+I*tan(f*
x+e)))^(1/2)*(I*a*d)^(1/2)*d)*(a*(1+I*tan(f*x+e)))^(1/2)*(c+d*tan(f*x+e))^(
```

```
1/2)/(I*a*d)^(1/2)/(-tan(f*x+e)+I)^2/(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^
(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(1/2),x, algorithm="max
ima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 928 vs. $2(151) = 302$.
time = 0.92, size = 928, normalized size = 4.76

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(1/2),x, algorithm="fri
cas")

[Out] -1/4*(sqrt(2)*a*f*sqrt(-(c^3 - 3*I*c^2*d - 3*c*d^2 + I*d^3)/(a*f^2))*e^(I*f
*x + I*e)*log((sqrt(2)*a*f*sqrt(-(c^3 - 3*I*c^2*d - 3*c*d^2 + I*d^3)/(a*f^2
))*e^(I*f*x + I*e) + sqrt(2)*((I*c + d)*e^(2*I*f*x + 2*I*e) + I*c + d)*sqrt
(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(
a/(e^(2*I*f*x + 2*I*e) + 1)))/(I*c + d)) - sqrt(2)*a*f*sqrt(-(c^3 - 3*I*c^2
*d - 3*c*d^2 + I*d^3)/(a*f^2))*e^(I*f*x + I*e)*log(-(sqrt(2)*a*f*sqrt(-(c^3
 - 3*I*c^2*d - 3*c*d^2 + I*d^3)/(a*f^2))*e^(I*f*x + I*e) - sqrt(2)*((I*c +
d)*e^(2*I*f*x + 2*I*e) + I*c + d)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c +
 I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))/(I*c +
d)) + a*f*sqrt(-4*I*d^3/(a*f^2))*e^(I*f*x + I*e)*log(2*(4*sqrt(2)*(d^3*e^(3
*I*f*x + 3*I*e) + d^3*e^(I*f*x + I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e)
+ c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)) - (
(I*a*c*d + 3*a*d^2)*f*e^(2*I*f*x + 2*I*e) + (I*a*c*d - a*d^2)*f)*sqrt(-4*I*
d^3/(a*f^2)))/(I*c^3 + c^2*d + I*c*d^2 + d^3 + (I*c^3 + c^2*d + I*c*d^2 + d
^3)*e^(2*I*f*x + 2*I*e))) - a*f*sqrt(-4*I*d^3/(a*f^2))*e^(I*f*x + I*e)*log(
2*(4*sqrt(2)*(d^3*e^(3*I*f*x + 3*I*e) + d^3*e^(I*f*x + I*e))*sqrt(((c - I*d
)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*
f*x + 2*I*e) + 1)) - ((-I*a*c*d - 3*a*d^2)*f*e^(2*I*f*x + 2*I*e) + (-I*a*c*
d + a*d^2)*f)*sqrt(-4*I*d^3/(a*f^2)))/(I*c^3 + c^2*d + I*c*d^2 + d^3 + (I*c
^3 + c^2*d + I*c*d^2 + d^3)*e^(2*I*f*x + 2*I*e))) + 2*sqrt(2)*((-I*c + d)*e
^(2*I*f*x + 2*I*e) - I*c + d)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d
```

```
)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x -
 I*e)/(a*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d\tan{(e + fx)})^{\frac{3}{2}}}{\sqrt{ia(\tan{(e + fx)} - i)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(3/2)/(a+I*a*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(3/2)/sqrt(I*a*(tan(e + f*x) - I)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c + d\tan(e + f\,x))^{3/2}}{\sqrt{a + a\tan(e + f\,x)\,1\mathrm{i}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(3/2)/(a + a*tan(e + f*x)*1i)^(1/2),x)
```

```
[Out] int((c + d*tan(e + f*x))^(3/2)/(a + a*tan(e + f*x)*1i)^(1/2), x)
```

3.1147 $\int \frac{(c+d\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=173

$$-\frac{i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f}+\frac{(ic+d)\sqrt{c+d\tan(e+fx)}}{2af\sqrt{a+ia\tan(e+fx)}}+\frac{i(c+d\tan(e+fx))^{3/2}}{3f(a+ia\tan(e+fx))^{3/2}}$$

```
[Out] -1/4*I*(c-I*d)^(3/2)*arctanh(2^(1/2)*a^(1/2)*(c+d*tan(f*x+e))^(1/2)/(c-I*d)
^(1/2)/(a+I*a*tan(f*x+e))^(1/2))/a^(3/2)/f*2^(1/2)+1/2*(I*c+d)*(c+d*tan(f*x
+e))^(1/2)/a/f/(a+I*a*tan(f*x+e))^(1/2)+1/3*I*(c+d*tan(f*x+e))^(3/2)/f/(a+I
*a*tan(f*x+e))^(3/2)
```

Rubi [A]

time = 0.22, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3627, 3625, 214}

$$-\frac{i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f}+\frac{i(c+d\tan(e+fx))^{3/2}}{3f(a+ia\tan(e+fx))^{3/2}}+\frac{(d+ic)\sqrt{c+d\tan(e+fx)}}{2af\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^(3/2)/(a + I*a*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-1/2*I)*(c - I*d)^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[c + d*Tan[e + f*x]]
)/(Sqrt[c - I*d]*Sqrt[a + I*a*Tan[e + f*x]])])/(Sqrt[2]*a^(3/2)*f) + ((I*c
+ d)*Sqrt[c + d*Tan[e + f*x]])/(2*a*f*Sqrt[a + I*a*Tan[e + f*x]]) + ((I/3)*
(c + d*Tan[e + f*x])^(3/2))/(f*(a + I*a*Tan[e + f*x])^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3627

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^n/(2*b*f*m)), x] - Dist[(a*c - b*d)/(2*b^2), Int[(a + b*Tan[e + f*
x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && EqQ
[m + n, 0] && LeQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^{3/2}}\,dx = \frac{i(c+d\tan(e+fx))^{3/2}}{3f(a+ia\tan(e+fx))^{3/2}} + \frac{(c-id)\int \frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{a+ia\tan(e+fx)}}\,dx}{2a}$$

$$= \frac{(ic+d)\sqrt{c+d\tan(e+fx)}}{2af\sqrt{a+ia\tan(e+fx)}} + \frac{i(c+d\tan(e+fx))^{3/2}}{3f(a+ia\tan(e+fx))^{3/2}} + \frac{(c-id)^2\int \ldots}{\ldots}$$

$$= \frac{(ic+d)\sqrt{c+d\tan(e+fx)}}{2af\sqrt{a+ia\tan(e+fx)}} + \frac{i(c+d\tan(e+fx))^{3/2}}{3f(a+ia\tan(e+fx))^{3/2}} - \frac{(i(c-id)^2)\,\mathrm{S}\ldots}{\ldots}$$

$$= -\frac{i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f} + \frac{(ic+d)\sqrt{c+d\ldots}}{2af\sqrt{a+ia\,\mathrm{t}\ldots}}$$

Mathematica [A]
time = 4.48, size = 240, normalized size = 1.39

$$\frac{\sec^{\frac{3}{2}}(e+fx)\left(-i\sqrt{2}\,(c-id)^{3/2}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{3/2}\left(1+e^{2i(e+fx)}\right)^{3/2}\log\left(2\left(\sqrt{c-id}\,e^{i(e+fx)}+\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)\right)-\frac{2(-5ic-3d+(3c-5id)\tan(e+fx))\sqrt{c+d\tan(e+fx)}}{3\sec^{\frac{3}{2}}(e+fx)}\right)}{4f(a+ia\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^(3/2)/(a + I*a*Tan[e + f*x])^(3/2),x]
```

```
[Out] (Sec[e + f*x]^(3/2)*((-I)*Sqrt[2]*(c - I*d)^(3/2)*(E^(I*(e + f*x))/(1 + E^(
(2*I)*(e + f*x))))^(3/2)*(1 + E^((2*I)*(e + f*x)))^(3/2)*Log[2*(Sqrt[c - I*
d]*E^(I*(e + f*x)) + Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^((
2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))])] - (2*((-5*I)*c - 3*d + (3*c
- (5*I)*d)*Tan[e + f*x])*Sqrt[c + d*Tan[e + f*x]])/(3*Sec[e + f*x]^(3/2))))
/(4*f*(a + I*a*Tan[e + f*x])^(3/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1274 vs. 2(135) = 270.
time = 0.70, size = 1275, normalized size = 7.37

method	result	size
derivativedivides	Expression too large to display	1275
default	Expression too large to display	1275

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/24/f*(c+d*tan(f*x+e))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)/a^2*(-3*I*2^(1/2)*
(-a*(I*d-c))^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1
/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*
x+e)+I))*d^2-12*I*c^2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*tan(f*x+e
)^2-3*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-
c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1
/2)*(-a*(I*d-c))^(1/2)*c^2*tan(f*x+e)^3-3*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+
3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(
f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*d^2*tan(f*x+e)^3
-8*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c*d*tan(f*x+e)^2+9*I*2^(1/2)
*(-a*(I*d-c))^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(
1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f
*x+e)+I))*d^2*tan(f*x+e)^2+9*I*2^(1/2)*(-a*(I*d-c))^(1/2)*ln((3*a*c+I*a*tan
(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f
*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c^2*tan(f*x+e)^2+9*ln((3*a*
c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(
c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c
))^(1/2)*c^2*tan(f*x+e)+9*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)
+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/
(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*d^2*tan(f*x+e)-32*c^2*(a*(c+d*ta
n(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*tan(f*x+e)+12*I*(a*(c+d*tan(f*x+e))*(1+I*
tan(f*x+e)))^(1/2)*d^2-3*I*2^(1/2)*(-a*(I*d-c))^(1/2)*ln((3*a*c+I*a*tan(f*x
+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e
))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c^2+20*I*c^2*(a*(c+d*tan(f*x+e)
)*(1+I*tan(f*x+e)))^(1/2)-20*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*
d^2*tan(f*x+e)^2-32*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*d^2*tan(f*x
+e)-8*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c*d)/(a*(c+d*tan(f*x+e))*
(1+I*tan(f*x+e)))^(1/2)/(I*c-d)/(-tan(f*x+e)+I)^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 506 vs. $2(135) = 270$.
time = 0.98, size = 506, normalized size = 2.92

$$-\frac{\left(3\sqrt{\frac{1}{2}}a^2 f\sqrt{-\frac{c^3-3ic^2d-3cd^2+id^3}{a^3f^2}}e^{(3ifx+3ie)}\log\left(\frac{2\sqrt{\frac{1}{2}}a^2f\sqrt{-\frac{c^3-3ic^2d-3cd^2+id^3}{a^3f^2}}e^{(ifx+ie)}+\sqrt{2}\left((ic+d)e^{(2ifx+2ie)}+ic+d\right)\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}}{ic+d}\right)-3\sqrt{\frac{1}{2}}a^2f\sqrt{-\frac{c^3-3ic^2d-3cd^2+id^3}{a^3f^2}}e^{(3ifx+3ie)}\log\left(-\frac{2\sqrt{\frac{1}{2}}a^2f\sqrt{-\frac{c^3-3ic^2d-3cd^2+id^3}{a^3f^2}}e^{(ifx+ie)}-\sqrt{2}\left((ic+d)e^{(2ifx+2ie)}+ic+d\right)\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}}{ic+d}\right)+\sqrt{2}\left(4(-ic-d)e^{(4ifx+4ie)}-(5ic+3d)e^{(2ifx+2ie)}-ic+d\right)\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}\right)e^{(-3ifx-3ie)}}{12a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] -1/12*(3*sqrt(1/2)*a^2*f*sqrt(-(c^3 - 3*I*c^2*d - 3*c*d^2 + I*d^3)/(a^3*f^2
))*e^(3*I*f*x + 3*I*e)*log((2*sqrt(1/2)*a^2*f*sqrt(-(c^3 - 3*I*c^2*d - 3*c*
d^2 + I*d^3)/(a^3*f^2))*e^(I*f*x + I*e) + sqrt(2)*((I*c + d)*e^(2*I*f*x + 2
*I*e) + I*c + d)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x
 + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))/(I*c + d)) - 3*sqrt(1/2)
*a^2*f*sqrt(-(c^3 - 3*I*c^2*d - 3*c*d^2 + I*d^3)/(a^3*f^2))*e^(3*I*f*x + 3*
I*e)*log(-(2*sqrt(1/2)*a^2*f*sqrt(-(c^3 - 3*I*c^2*d - 3*c*d^2 + I*d^3)/(a^3
*f^2))*e^(I*f*x + I*e) - sqrt(2)*((I*c + d)*e^(2*I*f*x + 2*I*e) + I*c + d)*
sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*s
qrt(a/(e^(2*I*f*x + 2*I*e) + 1)))/(I*c + d)) + sqrt(2)*(4*(-I*c - d)*e^(4*I
*f*x + 4*I*e) - (5*I*c + 3*d)*e^(2*I*f*x + 2*I*e) - I*c + d)*sqrt(((c - I*d
)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*
f*x + 2*I*e) + 1)))*e^(-3*I*f*x - 3*I*e)/(a^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\tan(e+fx))^{\frac{3}{2}}}{(ia(\tan(e+fx)-i))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(3/2)/(a+I*a*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(3/2)/(I*a*(tan(e + f*x) - I))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\tan(e+f\,x))^{3/2}}{(a+a\tan\left(e+f\,x\right)\,1\mathrm{i})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(3/2)/(a + a*tan(e + f*x)*1i)^(3/2),x)
```

```
[Out] int((c + d*tan(e + f*x))^(3/2)/(a + a*tan(e + f*x)*1i)^(3/2), x)
```

3.1148 $\int \frac{(c+d\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=225

$$-\frac{i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{4\sqrt{2}\,a^{5/2}f}+\frac{(ic+d)\sqrt{c+d\tan(e+fx)}}{4a^2f\sqrt{a+ia\tan(e+fx)}}+\frac{i(c+d\tan(e+}{6af(a+ia\tan(e-}$$

```
[Out] -1/8*I*(c-I*d)^(3/2)*arctanh(2^(1/2)*a^(1/2)*(c+d*tan(f*x+e))^(1/2)/(c-I*d)
^(1/2)/(a+I*a*tan(f*x+e))^(1/2))/a^(5/2)/f*2^(1/2)+1/4*(I*c+d)*(c+d*tan(f*x
+e))^(1/2)/a^2/f/(a+I*a*tan(f*x+e))^(1/2)+1/6*I*(c+d*tan(f*x+e))^(3/2)/a/f/
(a+I*a*tan(f*x+e))^(3/2)-1/5*(c+d*tan(f*x+e))^(5/2)/(I*c-d)/f/(a+I*a*tan(f*
x+e))^(5/2)
```

Rubi [A]
time = 0.30, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3628, 3627, 3625, 214}

$$-\frac{i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{4\sqrt{2}\,a^{5/2}f}+\frac{(d+ic)\sqrt{c+d\tan(e+fx)}}{4a^2f\sqrt{a+ia\tan(e+fx)}}-\frac{(c+d\tan(e+fx))^{5/2}}{5f(-d+ic)(a+ia\tan(e+fx))^{5/2}}+\frac{i(c+d\tan(e+fx))^{3/2}}{6af(a+ia\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^(3/2)/(a + I*a*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-1/4*I)*(c - I*d)^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[c + d*Tan[e + f*x]]
)/(Sqrt[c - I*d]*Sqrt[a + I*a*Tan[e + f*x]])])/(Sqrt[2]*a^(5/2)*f) + ((I*c
+ d)*Sqrt[c + d*Tan[e + f*x]])/(4*a^2*f*Sqrt[a + I*a*Tan[e + f*x]]) + ((I/6
)*(c + d*Tan[e + f*x])^(3/2))/(a*f*(a + I*a*Tan[e + f*x])^(3/2)) - (c + d*T
an[e + f*x])^(5/2)/(5*(I*c - d)*f*(a + I*a*Tan[e + f*x])^(5/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3627

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^n/(2*b*f*m)), x] - Dist[(a*c - b*d)/(2*b^2), Int[(a + b*Tan[e + f*
x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && EqQ
[m + n, 0] && LeQ[m, -2^(-1)]
```

Rule 3628

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a), Int[(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && EqQ[m
 + n + 1, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx = -\frac{(c+d\tan(e+fx))^{5/2}}{5(ic-d)f(a+ia\tan(e+fx))^{5/2}} + \frac{\int \frac{(c+d\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^{3/2}}\,dx}{2a}$$

$$= \frac{i(c+d\tan(e+fx))^{3/2}}{6af(a+ia\tan(e+fx))^{3/2}} - \frac{(c+d\tan(e+fx))^{5/2}}{5(ic-d)f(a+ia\tan(e+fx))^{5/2}} + \frac{(c-id)\ \int}{}$$

$$= \frac{(ic+d)\sqrt{c+d\tan(e+fx)}}{4a^2f\sqrt{a+ia\tan(e+fx)}} + \frac{i(c+d\tan(e+fx))^{3/2}}{6af(a+ia\tan(e+fx))^{3/2}} - \frac{(c+d\,\mathrm{ta}}{5(ic-d)f(a\,-}$$

$$= \frac{(ic+d)\sqrt{c+d\tan(e+fx)}}{4a^2f\sqrt{a+ia\tan(e+fx)}} + \frac{i(c+d\tan(e+fx))^{3/2}}{6af(a+ia\tan(e+fx))^{3/2}} - \frac{(c+d\,\mathrm{ta}}{5(ic-d)f(a\,-}$$

$$= -\frac{i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{4\sqrt{2}\,a^{5/2}f} + \frac{(ic+d)\sqrt{c+d\,\mathrm{t}}}{4a^2f\sqrt{a+ia\,\mathrm{ta}}}$$

Mathematica [A]
time = 5.97, size = 302, normalized size = 1.34

$$\frac{\sec^{\frac{5}{2}}(e+fx)\left(-i\sqrt{2}\,(c-id)^{3/2}e^{2i(e+fx)}\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\sqrt{1+e^{2i(e+fx)}}\log\left(2\left(\sqrt{c-id}\,e^{i(e+fx)}+\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)\right)+\frac{2i(11c^2+10icd+d^2+2(13c^2+7d^2)\cos(2(e+fx))+4(5ic^2+3cd+5id^2)\sin(2(e+fx)))\sqrt{c+d\tan(e+fx)}}{15(c+id)\sqrt{\sec(e+fx)}}\right)}{8f(a+ia\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^(3/2)/(a + I*a*Tan[e + f*x])^(5/2),x]
```

```
[Out] (Sec[e + f*x]^(5/2)*((-I)*Sqrt[2]*(c - I*d)^(3/2)*E^((2*I)*(e + f*x))*Sqrt[
E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*Sqrt[1 + E^((2*I)*(e + f*x))]*Lo
g[2*(Sqrt[c - I*d]*E^(I*(e + f*x)) + Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c -
 (I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))])] + (((2*I)/15
)*(11*c^2 + (10*I)*c*d + d^2 + 2*(13*c^2 + 7*d^2)*Cos[2*(e + f*x)] + 4*((5*
I)*c^2 + 3*c*d + (5*I)*d^2)*Sin[2*(e + f*x)])*Sqrt[c + d*Tan[e + f*x]])/((c
 + I*d)*Sqrt[Sec[e + f*x]])))/(8*f*(a + I*a*Tan[e + f*x])^(5/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1656 vs. $2(179) = 358$.
time = 0.73, size = 1657, normalized size = 7.36

method	result	size
derivativedivides	Expression too large to display	1657
default	Expression too large to display	1657

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/240/f/a^3*(-15*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/
2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x
+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c^2*tan(f*x+e)^4-15*ln((3*a*c+I*a*tan(f*
x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+
e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*d^2
*tan(f*x+e)^4+90*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2
)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+
e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c^2*tan(f*x+e)^2+90*ln((3*a*c+I*a*tan(f*x
+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e
))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*d^2*
tan(f*x+e)^2-40*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c*d*tan(f*x+e)^
3+56*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c*d*tan(f*x+e)+60*(a*(c+d*
tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*d^2-220*c^2*(a*(c+d*tan(f*x+e))*(1+I*ta
n(f*x+e)))^(1/2)*tan(f*x+e)^2+60*I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*t
an(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e))
)^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c^2*tan(f*x+e)^3+60*I*l
n((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/
2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a
*(I*d-c))^(1/2)*d^2*tan(f*x+e)^3+308*I*c^2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x
+e)))^(1/2)*tan(f*x+e)-60*I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+
e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)
)/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c^2*tan(f*x+e)+136*I*(a*(c+d*t
an(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c*d*tan(f*x+e)^2-15*ln((3*a*c+I*a*tan(f*
```

```
x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+
e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c^2
-15*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c)
)^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2
)*(-a*(I*d-c))^(1/2)*d^2-52*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*d
^2*tan(f*x+e)^3+148*c^2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)+40*I*(a
*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c*d-60*I*ln((3*a*c+I*a*tan(f*x+e)
*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*
(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*d^2*tan
(f*x+e)+220*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*d^2*tan(f*x+e)-60
*I*c^2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*tan(f*x+e)^3-212*(a*(c+d
*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*d^2*tan(f*x+e)^2)*(a*(1+I*tan(f*x+e)))
^(1/2)*(c+d*tan(f*x+e))^(1/2)/(-tan(f*x+e)+I)^4/(I*c-d)/(a*(c+d*tan(f*x+e))
*(1+I*tan(f*x+e)))^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 590 vs. 2(179) = 358.
time = 1.64, size = 590, normalized size = 2.62

$$\frac{\left(15\sqrt{\frac{1}{2}}\left(-i a^3c+a^3d\right)f\sqrt{-\frac{c^3-3ic^2d-3cd^2+id^3}{a^5f^2}}e^{(5ifx+5ie)}\log\left(\frac{2\sqrt{\frac{1}{2}}a^3f\sqrt{-\frac{c^3-3ic^2d-3cd^2+id^3}{a^5f^2}}e^{(ifx+ie)}+\sqrt{2}\left((ic+d)e^{(2ifx+2ie)}+ic+d\right)\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}}{ic+d}\right)+15\sqrt{\frac{1}{2}}\left(i a^3c-a^3d\right)f\sqrt{-\frac{c^3-3ic^2d-3cd^2+id^3}{a^5f^2}}e^{(5ifx+5ie)}\log\left(-\frac{2\sqrt{\frac{1}{2}}a^3f\sqrt{-\frac{c^3-3ic^2d-3cd^2+id^3}{a^5f^2}}e^{(ifx+ie)}-\sqrt{2}\left((ic+d)e^{(2ifx+2ie)}+ic+d\right)\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}}{ic+d}\right)-\sqrt{2}\left(3c^2+6icd-3d^2+\left(23c^2-6icd+17d^2\right)e^{(6ifx+6ie)}+2\left(17c^2+2icd+9d^2\right)e^{(4ifx+4ie)}+2\left(7c^2+8icd-d^2\right)e^{(2ifx+2ie)}\right)\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}\right)e^{(-5ifx-5ie)}}{120\left(i a^3c-a^3d\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] 1/120*(15*sqrt(1/2)*(-I*a^3*c + a^3*d)*f*sqrt(-(c^3 - 3*I*c^2*d - 3*c*d^2 +
 I*d^3)/(a^5*f^2))*e^(5*I*f*x + 5*I*e)*log((2*sqrt(1/2)*a^3*f*sqrt(-(c^3 -
3*I*c^2*d - 3*c*d^2 + I*d^3)/(a^5*f^2))*e^(I*f*x + I*e) + sqrt(2)*((I*c + d
)*e^(2*I*f*x + 2*I*e) + I*c + d)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c +
I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))/(I*c + d
)) + 15*sqrt(1/2)*(I*a^3*c - a^3*d)*f*sqrt(-(c^3 - 3*I*c^2*d - 3*c*d^2 + I*
d^3)/(a^5*f^2))*e^(5*I*f*x + 5*I*e)*log(-(2*sqrt(1/2)*a^3*f*sqrt(-(c^3 - 3*
I*c^2*d - 3*c*d^2 + I*d^3)/(a^5*f^2))*e^(I*f*x + I*e) - sqrt(2)*((I*c + d)*
e^(2*I*f*x + 2*I*e) + I*c + d)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*
```

```
d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))/(I*c + d))
 - sqrt(2)*(3*c^2 + 6*I*c*d - 3*d^2 + (23*c^2 - 6*I*c*d + 17*d^2)*e^(6*I*f*
x + 6*I*e) + 2*(17*c^2 + 2*I*c*d + 9*d^2)*e^(4*I*f*x + 4*I*e) + 2*(7*c^2 +
8*I*c*d - d^2)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c
 + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-5
*I*f*x - 5*I*e)/((I*a^3*c - a^3*d)*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\tan(e+fx))^{\frac{3}{2}}}{(ia(\tan(e+fx)-i))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(3/2)/(a+I*a*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(3/2)/(I*a*(tan(e + f*x) - I))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\tan(e+f\,x))^{3/2}}{(a+a\tan(e+f\,x)\;1\mathrm{i})^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(3/2)/(a + a*tan(e + f*x)*1i)^(5/2),x)
```

```
[Out] int((c + d*tan(e + f*x))^(3/2)/(a + a*tan(e + f*x)*1i)^(5/2), x)
```

3.1149 $\int (a+ia\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=415

$$-\frac{\sqrt[4]{-1}\,a^{5/2}(5c^4+100ic^3d+690c^2d^2-900icd^3-363d^4)\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}\right)}{64d^{3/2}f}-\frac{4i\sqrt{2}}{}$$

```
[Out] -1/64*(-1)^(1/4)*a^(5/2)*(5*c^4+100*I*c^3*d+690*c^2*d^2-900*I*c*d^3-363*d^4
)*arctanh((-1)^(3/4)*d^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c+d*tan(f*x+
e))^(1/2))/d^(3/2)/f-4*I*a^(5/2)*(c-I*d)^(5/2)*arctanh(2^(1/2)*a^(1/2)*(c+d
*tan(f*x+e))^(1/2)/(c-I*d)^(1/2)/(a+I*a*tan(f*x+e))^(1/2))*2^(1/2)/f+1/64*a
^2*(5*c^3+95*I*c^2*d+273*c*d^2-149*I*d^3)*(a+I*a*tan(f*x+e))^(1/2)*(c+d*tan
(f*x+e))^(1/2)/d/f+1/96*a^2*(5*c^2+90*I*c*d+107*d^2)*(a+I*a*tan(f*x+e))^(1/
2)*(c+d*tan(f*x+e))^(3/2)/d/f+1/24*a^2*(c+17*I*d)*(a+I*a*tan(f*x+e))^(1/2)*
(c+d*tan(f*x+e))^(5/2)/d/f-1/4*a^2*(a+I*a*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e)
)^(7/2)/d/f
```

Rubi [A]
time = 1.14, antiderivative size = 415, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {3637, 3678, 3682, 3625, 214, 3680, 65, 223, 212}

$$-\frac{\sqrt[4]{-1}\,a^{5/2}(5c^4+100ic^3d+690c^2d^2-900icd^3-363d^4)\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}\right)}{64d^{3/2}f}-\frac{4i\sqrt{2}\,a^{5/2}(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+fx)}}\right)}{f}+\frac{a^2(5c^2+90icd+107d^2)\sqrt{a+ia\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}}{96df}+\frac{a^2(5c^3+95ic^2d+273cd^2-149id^3)\sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{64df}-\frac{a^2\sqrt{a+ia\tan(e+fx)}\,(c+d\tan(e+fx))^{7/2}}{4df}+\frac{a^2(c+17id)\sqrt{a+ia\tan(e+fx)}\,(c+d\tan(e+fx))^{5/2}}{24df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^(5/2)*(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] -1/64*((-1)^(1/4)*a^(5/2)*(5*c^4 + (100*I)*c^3*d + 690*c^2*d^2 - (900*I)*c*
d^3 - 363*d^4)*ArcTanh[((-1)^(3/4)*Sqrt[d]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqr
t[a]*Sqrt[c + d*Tan[e + f*x]])])/(d^(3/2)*f) - ((4*I)*Sqrt[2]*a^(5/2)*(c -
I*d)^(5/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])/(Sqrt[c - I*d
]*Sqrt[a + I*a*Tan[e + f*x]])])/f + (a^2*(5*c^3 + (95*I)*c^2*d + 273*c*d^2
- (149*I)*d^3)*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(64*d*f
) + (a^2*(5*c^2 + (90*I)*c*d + 107*d^2)*Sqrt[a + I*a*Tan[e + f*x]]*(c + d*T
an[e + f*x])^(3/2))/(96*d*f) + (a^2*(c + (17*I)*d)*Sqrt[a + I*a*Tan[e + f*x
]]*(c + d*Tan[e + f*x])^(5/2))/(24*d*f) - (a^2*Sqrt[a + I*a*Tan[e + f*x]]*(
c + d*Tan[e + f*x])^(7/2))/(4*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3678

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Dist[
1/(a*(m + n)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Simp
[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int (a+ia\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{5/2}\,dx = -\frac{a^2\sqrt{a+ia\tan(e+fx)}\,(c+d\tan(e+fx))^{7/2}}{4df} + \frac{a \ldots}{\ldots}$$

$$= \frac{a^2(c+17id)\sqrt{a+ia\tan(e+fx)}\,(c+d\tan(e+fx))\ldots}{24df}$$

$$= \frac{a^2(5c^2+90icd+107d^2)\sqrt{a+ia\tan(e+fx)}\,(c+d\,\mathrm{t}\ldots}{96df}$$

$$= \frac{a^2(5c^3+95ic^2d+273cd^2-149id^3)\sqrt{a+ia\tan(e+\ldots}}{64df}$$

$$= \frac{a^2(5c^3+95ic^2d+273cd^2-149id^3)\sqrt{a+ia\tan(e+\ldots}}{64df}$$

$$= \frac{a^2(5c^3+95ic^2d+273cd^2-149id^3)\sqrt{a+ia\tan(e+\ldots}}{64df}$$

$$= -\frac{4i\sqrt{2}\,a^{5/2}(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(\ldots}}{\sqrt{c-id}\sqrt{a+ia\tan\ldots}}\right.}{f}$$

$$= -\frac{4i\sqrt{2}\,a^{5/2}(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(\ldots}}{\sqrt{c-id}\sqrt{a+ia\tan\ldots}}\right.}{f}$$

$$= -\frac{\sqrt[4]{-1}\,a^{5/2}(5c^4+100ic^3d+690c^2d^2-900icd^3-36\ldots}{64d^{3/2}\ldots}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 849 vs. 2(415) = 830.
time = 13.20, size = 849, normalized size = 2.05

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(5/2)*(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-1/128 - I/128)*Cos[e + f*x]^3*((5*c^4 + (100*I)*c^3*d + 690*c^2*d^2 - (9
00*I)*c*d^3 - 363*d^4)*(Log[((2 + 2*I)*E^((I/2)*e)*(c + I*d - I*c*E^(I*(e +
```

```
f*x)) - d*E^(I*(e + f*x)) + (1 + I)*Sqrt[d]*Sqrt[1 + E^((2*I)*(e + f*x))]*
Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))]))/(Sqr
t[d]*(-5*c^4 - (100*I)*c^3*d - 690*c^2*d^2 + (900*I)*c*d^3 + 363*d^4)*(I +
E^(I*(e + f*x))))] - Log[((-2 - 2*I)*E^((I/2)*e)*(c + I*d + I*c*E^(I*(e + f
*x)) + d*E^(I*(e + f*x)) + (1 + I)*Sqrt[d]*Sqrt[1 + E^((2*I)*(e + f*x))]*Sq
rt[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))]))/(Sqrt[
d]*(-5*c^4 - (100*I)*c^3*d - 690*c^2*d^2 + (900*I)*c*d^3 + 363*d^4)*(-I + E
^(I*(e + f*x))))]) + (512 + 512*I)*(c - I*d)^(5/2)*d^(3/2)*Log[2*(Sqrt[c -
I*d]*Cos[e + f*x] + I*Sqrt[c - I*d]*Sin[e + f*x] + Sqrt[1 + Cos[2*e + 2*f*x
] + I*Sin[2*e + 2*f*x]]*Sqrt[c + d*Tan[e + f*x]])])*(Cos[2*e] - I*Sin[2*e])
*(a + I*a*Tan[e + f*x])^(5/2))/(d^(3/2)*f*(Cos[f*x] + I*Sin[f*x])^2*Sqrt[1
+ Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)]]) + (Cos[e + f*x]^2*Sqrt[Sec[e + f*
x]*(c*Cos[e + f*x] + d*Sin[e + f*x])]*((-15*c^3 + (719*I)*c^2*d + 1621*c*d^
2 - (845*I)*d^3)*(Cos[2*e]/(192*d) - ((I/192)*Sin[2*e])/d) + ((17*I)*c + 23
*d)*Sec[e + f*x]^2*((I/24)*d*Cos[2*e] + (d*Sin[2*e])/24) + (59*c^2 - (226*I
)*c*d - 131*d^2)*Sec[e + f*x]*((-1/96*I)*Cos[3*e + f*x] - Sin[3*e + f*x]/96
) + Sec[e + f*x]^3*((-1/4*I)*d^2*Cos[3*e + f*x] - (d^2*Sin[3*e + f*x])/4))*
(a + I*a*Tan[e + f*x])^(5/2))/(f*(Cos[f*x] + I*Sin[f*x])^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1850 vs. $2(342) = 684$.
time = 0.53, size = 1851, normalized size = 4.46

method	result	size
derivativedivides	Expression too large to display	1851
default	Expression too large to display	1851

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(5/2)*(c+d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/768/f*(a*(1+I*tan(f*x+e)))^(1/2)*(c+d*tan(f*x+e))^(1/2)*a^2*(-96*2^(1/2)*
d^3*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)*(-a*(I*d-c))^
(1/2)*tan(f*x+e)^3-272*2^(1/2)*c*d^2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^
(1/2)*(I*a*d)^(1/2)*(-a*(I*d-c))^(1/2)*tan(f*x+e)^2-236*2^(1/2)*c^2*d*(a*(c
+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)*(-a*(I*d-c))^(1/2)*tan
(f*x+e)+768*I*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+
e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)
)/(tan(f*x+e)+I))*a*c*d-894*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(
I*a*d)^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*d^3-1089*I*2^(1/2)*(-a*(I*d-c))^(1/
2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))
^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*d^4-768*I*2^(1/2)*(-a*(I*d-c))^(
1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)
))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*d^2+2070*I*2^(1/2)*(-a*(I*d-c)
)^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x
```

```
+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c^2*d^2-300*2^(1/2)*(-a*(I*
d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan
(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c^3*d+2700*2^(1/2)*(-a*
(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*
tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c*d^3+272*I*2^(1/2)*
d^3*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)*(-a*(I*d-c))^
(1/2)*tan(f*x+e)^2+904*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d
)^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*c*d^2*tan(f*x+e)+428*(a*(c+d*tan(f*x+e))
*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*d^3*tan(f
*x+e)+15*I*2^(1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a
*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))
*a*c^4+1202*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)*2^(
1/2)*(-a*(I*d-c))^(1/2)*c^2*d-30*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2
)*(I*a*d)^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*c^3+2066*(a*(c+d*tan(f*x+e))*(1+
I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*c*d^2+768*2^(
1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+
e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c*d-768*2^(
1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+
e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*d^2-768*I*2
^(1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*
x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c*d+768*I
*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*
(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)
+I))*a*d^2-768*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x
+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2
))/(tan(f*x+e)+I))*a*c*d+768*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d
+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan
(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*d^2)*2^(1/2)/d/(a*(c+d*tan(f*x+e))*(1+I*
tan(f*x+e)))^(1/2)/(I*a*d)^(1/2)/(-a*(I*d-c))^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)*(c+d*tan(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(3*d-c>0)', see `assume?` for more d
etails)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1965 vs. 2(339) = 678.

time = 1.37, size = 1965, normalized size = 4.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)*(c+d*tan(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] 1/384*(768*sqrt(2)*(d*f*e^(6*I*f*x + 6*I*e) + 3*d*f*e^(4*I*f*x + 4*I*e) + 3
*d*f*e^(2*I*f*x + 2*I*e) + d*f)*sqrt(-(a^5*c^5 - 5*I*a^5*c^4*d - 10*a^5*c^3
*d^2 + 10*I*a^5*c^2*d^3 + 5*a^5*c*d^4 - I*a^5*d^5)/f^2)*log((I*sqrt(2)*f*sq
rt(-(a^5*c^5 - 5*I*a^5*c^4*d - 10*a^5*c^3*d^2 + 10*I*a^5*c^2*d^3 + 5*a^5*c*
d^4 - I*a^5*d^5)/f^2)*e^(I*f*x + I*e) + sqrt(2)*(a^2*c^2 - 2*I*a^2*c*d - a^
2*d^2 + (a^2*c^2 - 2*I*a^2*c*d - a^2*d^2)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I
*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*
I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/(a^2*c^2 - 2*I*a^2*c*d - a^2*d^2)) -
 768*sqrt(2)*(d*f*e^(6*I*f*x + 6*I*e) + 3*d*f*e^(4*I*f*x + 4*I*e) + 3*d*f*e
^(2*I*f*x + 2*I*e) + d*f)*sqrt(-(a^5*c^5 - 5*I*a^5*c^4*d - 10*a^5*c^3*d^2 +
 10*I*a^5*c^2*d^3 + 5*a^5*c*d^4 - I*a^5*d^5)/f^2)*log((-I*sqrt(2)*f*sqrt(-(
a^5*c^5 - 5*I*a^5*c^4*d - 10*a^5*c^3*d^2 + 10*I*a^5*c^2*d^3 + 5*a^5*c*d^4 -
 I*a^5*d^5)/f^2)*e^(I*f*x + I*e) + sqrt(2)*(a^2*c^2 - 2*I*a^2*c*d - a^2*d^2
 + (a^2*c^2 - 2*I*a^2*c*d - a^2*d^2)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*e
^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x
 + 2*I*e) + 1)))*e^(-I*f*x - I*e)/(a^2*c^2 - 2*I*a^2*c*d - a^2*d^2)) - 2*sq
rt(2)*((15*a^2*c^3 - 719*I*a^2*c^2*d - 1621*a^2*c*d^2 + 845*I*a^2*d^3)*e^(7
*I*f*x + 7*I*e) + (45*a^2*c^3 - 1921*I*a^2*c^2*d - 3415*a^2*c*d^2 + 1275*I*
a^2*d^3)*e^(5*I*f*x + 5*I*e) + (45*a^2*c^3 - 1685*I*a^2*c^2*d - 2511*a^2*c*
d^2 + 1135*I*a^2*d^3)*e^(3*I*f*x + 3*I*e) + 3*(5*a^2*c^3 - 161*I*a^2*c^2*d
- 239*a^2*c*d^2 + 107*I*a^2*d^3)*e^(I*f*x + I*e))*sqrt(((c - I*d)*e^(2*I*f*
x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e
) + 1)) + 3*(d*f*e^(6*I*f*x + 6*I*e) + 3*d*f*e^(4*I*f*x + 4*I*e) + 3*d*f*e^
(2*I*f*x + 2*I*e) + d*f)*sqrt((25*I*a^5*c^8 - 1000*a^5*c^7*d - 3100*I*a^5*c
^6*d^2 - 129000*a^5*c^5*d^3 + 652470*I*a^5*c^4*d^4 + 1314600*a^5*c^3*d^5 -
1310940*I*a^5*c^2*d^6 - 653400*a^5*c*d^7 + 131769*I*a^5*d^8)/(d^3*f^2))*log
(-(2*I*d^2*f*sqrt((25*I*a^5*c^8 - 1000*a^5*c^7*d - 3100*I*a^5*c^6*d^2 - 129
000*a^5*c^5*d^3 + 652470*I*a^5*c^4*d^4 + 1314600*a^5*c^3*d^5 - 1310940*I*a^
5*c^2*d^6 - 653400*a^5*c*d^7 + 131769*I*a^5*d^8)/(d^3*f^2))*e^(I*f*x + I*e)
 - sqrt(2)*(5*a^2*c^4 + 100*I*a^2*c^3*d + 690*a^2*c^2*d^2 - 900*I*a^2*c*d^3
 - 363*a^2*d^4 + (5*a^2*c^4 + 100*I*a^2*c^3*d + 690*a^2*c^2*d^2 - 900*I*a^2
*c*d^3 - 363*a^2*d^4)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I
*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))
)*e^(-I*f*x - I*e)/(5*a^2*c^4 + 100*I*a^2*c^3*d + 690*a^2*c^2*d^2 - 900*I*a
^2*c*d^3 - 363*a^2*d^4)) - 3*(d*f*e^(6*I*f*x + 6*I*e) + 3*d*f*e^(4*I*f*x +
4*I*e) + 3*d*f*e^(2*I*f*x + 2*I*e) + d*f)*sqrt((25*I*a^5*c^8 - 1000*a^5*c^7
*d - 3100*I*a^5*c^6*d^2 - 129000*a^5*c^5*d^3 + 652470*I*a^5*c^4*d^4 + 13146
```

```
00*a^5*c^3*d^5 - 1310940*I*a^5*c^2*d^6 - 653400*a^5*c*d^7 + 131769*I*a^5*d^
8)/(d^3*f^2))*log(-(-2*I*d^2*f*sqrt((25*I*a^5*c^8 - 1000*a^5*c^7*d - 3100*I
*a^5*c^6*d^2 - 129000*a^5*c^5*d^3 + 652470*I*a^5*c^4*d^4 + 1314600*a^5*c^3*
d^5 - 1310940*I*a^5*c^2*d^6 - 653400*a^5*c*d^7 + 131769*I*a^5*d^8)/(d^3*f^2
))*e^(I*f*x + I*e) - sqrt(2)*(5*a^2*c^4 + 100*I*a^2*c^3*d + 690*a^2*c^2*d^2
 - 900*I*a^2*c*d^3 - 363*a^2*d^4 + (5*a^2*c^4 + 100*I*a^2*c^3*d + 690*a^2*c
^2*d^2 - 900*I*a^2*c*d^3 - 363*a^2*d^4)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d
)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*
f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/(5*a^2*c^4 + 100*I*a^2*c^3*d + 690*a^2
*c^2*d^2 - 900*I*a^2*c*d^3 - 363*a^2*d^4)))/(d*f*e^(6*I*f*x + 6*I*e) + 3*d*
f*e^(4*I*f*x + 4*I*e) + 3*d*f*e^(2*I*f*x + 2*I*e) + d*f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(5/2)*(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)*(c+d*tan(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 2.64sym2poly/r2sym(c
onst gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + a\tan(e + f\,x)\;1\mathrm{i})^{5/2}\,(c + d\tan(e + f\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(5/2)*(c + d*tan(e + f*x))^(5/2),x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^(5/2)*(c + d*tan(e + f*x))^(5/2), x)
```

3.1150 $\int (a+ia\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=378

$$-\frac{\sqrt[4]{-1}\,a^{3/2}(5ic^3+45c^2d-55icd^2-23d^3)\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}\right)}{8\sqrt{d}\,f}-\frac{2i\sqrt{2}\,a^{3/2}(c-id)^{5/}}{}$$

```
[Out] -2*I*a^(3/2)*(c-I*d)^(5/2)*arctanh(2^(1/2)*a^(1/2)*(c+d*tan(f*x+e))^(1/2)/(
c-I*d)^(1/2)/(a+I*a*tan(f*x+e))^(1/2))*2^(1/2)/f-1/8*(-1)^(1/4)*a^(3/2)*(5*
I*c^3+45*c^2*d-55*I*c*d^2-23*d^3)*arctanh((-1)^(3/4)*d^(1/2)*(a+I*a*tan(f*x
+e))^(1/2)/a^(1/2)/(c+d*tan(f*x+e))^(1/2))/f/d^(1/2)+1/8*a*(c-3*I*d)*(5*I*c
+3*d)*(a+I*a*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/f+1/12*a*(5*I*c+7*d)*
(a+I*a*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2)/f+1/3*a^2*(c+I*d)*(c+d*tan(
f*x+e))^(5/2)/d/f/(a+I*a*tan(f*x+e))^(1/2)-1/3*a^2*(c+d*tan(f*x+e))^(7/2)/d
/f/(a+I*a*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 1.14, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3637, 3676, 3678, 3682, 3625, 214, 3680, 65, 223, 212}

$$-\frac{\sqrt[4]{-1}\,a^{3/2}(5ic^3+45c^2d-55icd^2-23d^3)\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}\right)}{8\sqrt{d}\,f}-\frac{2i\sqrt{2}\,a^{3/2}(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+fx)}}\right)}{f}-\frac{a^2(c+d\tan(e+fx))^{7/2}}{3df\sqrt{a+ia\tan(e+fx)}}+\frac{a^2(c+id)(c+d\tan(e+fx))^{5/2}}{3df\sqrt{a+ia\tan(e+fx)}}+\frac{a(7d+5ic)\sqrt{a+ia\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}}{12f}+\frac{a(c-3id)(3d+5ic)\sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] -1/8*((-1)^(1/4)*a^(3/2)*((5*I)*c^3 + 45*c^2*d - (55*I)*c*d^2 - 23*d^3)*Arc
Tanh[((-1)^(3/4)*Sqrt[d]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c + d*Ta
n[e + f*x]])])/(Sqrt[d]*f) - ((2*I)*Sqrt[2]*a^(3/2)*(c - I*d)^(5/2)*ArcTanh
[(Sqrt[2]*Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])/(Sqrt[c - I*d]*Sqrt[a + I*a*Tan
[e + f*x]])])/f + (a*(c - (3*I)*d)*((5*I)*c + 3*d)*Sqrt[a + I*a*Tan[e + f*x
]]*Sqrt[c + d*Tan[e + f*x]])/(8*f) + (a*((5*I)*c + 7*d)*Sqrt[a + I*a*Tan[e
+ f*x]]*(c + d*Tan[e + f*x])^(3/2))/(12*f) + (a^2*(c + I*d)*(c + d*Tan[e +
f*x])^(5/2))/(3*d*f*Sqrt[a + I*a*Tan[e + f*x]]) - (a^2*(c + d*Tan[e + f*x])
^(7/2))/(3*d*f*Sqrt[a + I*a*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rule 3678

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Dist[
1/(a*(m + n)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Simp
[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int (a+ia\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}\,dx = -\frac{a^2(c+d\tan(e+fx))^{7/2}}{3df\sqrt{a+ia\tan(e+fx)}} + \frac{a\int \frac{(-\frac{1}{2}a(ic-13d)-\frac{1}{2}a(c-11\cdots}{\sqrt{a+i\cdots}}}{}$$

$$= \frac{a^2(c+id)(c+d\tan(e+fx))^{5/2}}{3df\sqrt{a+ia\tan(e+fx)}} - \frac{a^2(c+d\tan(e+\cdots}{3df\sqrt{a+ia\tan(e\cdots}}$$

$$= \frac{a(5ic+7d)\sqrt{a+ia\tan(e+fx)}\,(c+d\tan(e+fx))^{3\cdots}}{12f}$$

$$= \frac{a(c-3id)(5ic+3d)\sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\tan\cdots}}{8f}$$

$$= \frac{a(c-3id)(5ic+3d)\sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\tan\cdots}}{8f}$$

$$= \frac{a(c-3id)(5ic+3d)\sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\tan\cdots}}{8f}$$

$$= -\frac{2i\sqrt{2}\,a^{3/2}(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(\cdots}}{\sqrt{c-id}\sqrt{a+ia\tan\cdots}}\right.}{f}$$

$$= -\frac{2i\sqrt{2}\,a^{3/2}(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(\cdots}}{\sqrt{c-id}\sqrt{a+ia\tan\cdots}}\right.}{f}$$

$$= -\frac{\sqrt[4]{-1}\,a^{3/2}(5ic^3+45c^2d-55icd^2-23d^3)\tanh^{-1}\left(\cdots\right.}{8\sqrt{d}\,f}$$

Mathematica [A]
time = 8.95, size = 645, normalized size = 1.71

$$\frac{i\sec(e+fx)(\cos(e)-i\sin(e))(\cos(fx)-i\sin(fx))(a+ia\tan(e+fx))^{3/2}\left(-\frac{(3-3i)\cos^3(e+fx)\left((5ic^3+45c^2d-55icd^2-23d^3)\left(\log\left(\frac{(2+2i)e^{\frac{ie}{2}}\left(c+id-ice^{i(e+fx)}-de^{i(e+fx)}+(1+i)\sqrt{d}\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id(-1+e^{2i(e+fx)})}{1+e^{2i(e+fx)}}}\right)}{\sqrt{d}(-5ic^3-45c^2d+55icd^2+23d^3)(i+e^{i(e+fx)})}\right)-\log\left(-\frac{(2+2i)e^{\frac{ie}{2}}\left(c+id+ice^{i(e+fx)}+de^{i(e+fx)}+(1+i)\sqrt{d}\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id(-1+e^{2i(e+fx)})}{1+e^{2i(e+fx)}}}\right)}{\sqrt{d}(-5ic^3-45c^2d+55icd^2+23d^3)(-i+e^{i(e+fx)})}\right)\right)+(32+32i)(c-id)^{5/2}\sqrt{d}\log\left(2\left(\sqrt{c-id}\cos(e+fx)+i\sqrt{c-id}\sin(e+fx)+\sqrt{1+\cos(2(e+fx))+i\sin(2(e+fx))}\sqrt{c+d\tan(e+fx)}\right)\right)\right)}{\sqrt{d}\sqrt{1+\cos(2(e+fx))+i\sin(2(e+fx))}}+(33c^2-68icd-19d^2+(33c^2-68icd-35d^2)\cos(2(e+fx))+2(13c-7id)d\sin(2(e+fx)))\sqrt{c+d\tan(e+fx)}\right)}{48f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((I/48)*Sec[e + f*x]*(Cos[e] - I*Sin[e])*(Cos[f*x] - I*Sin[f*x])*(a + I*a*T
an[e + f*x])^(3/2)*(((-3 + 3*I)*Cos[e + f*x]^3*(((5*I)*c^3 + 45*c^2*d - (55
*I)*c*d^2 - 23*d^3)*(Log[((2 + 2*I)*E^((I/2)*e)*(c + I*d - I*c*E^(I*(e + f*
x)) - d*E^(I*(e + f*x)) + (1 + I)*Sqrt[d]*Sqrt[1 + E^((2*I)*(e + f*x))]*Sqr
t[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))]))/(Sqrt[d
]*((-5*I)*c^3 - 45*c^2*d + (55*I)*c*d^2 + 23*d^3)*(I + E^(I*(e + f*x))))] -
 Log[((-2 - 2*I)*E^((I/2)*e)*(c + I*d + I*c*E^(I*(e + f*x)) + d*E^(I*(e + f
*x)) + (1 + I)*Sqrt[d]*Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^
((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))]))/(Sqrt[d]*((-5*I)*c^3 - 45*
c^2*d + (55*I)*c*d^2 + 23*d^3)*(-I + E^(I*(e + f*x))))]) + (32 + 32*I)*(c -
 I*d)^(5/2)*Sqrt[d]*Log[2*(Sqrt[c - I*d]*Cos[e + f*x] + I*Sqrt[c - I*d]*Sin
[e + f*x] + Sqrt[1 + Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)]]*Sqrt[c + d*Tan[
e + f*x]])]))/(Sqrt[d]*Sqrt[1 + Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)]]) + (
33*c^2 - (68*I)*c*d - 19*d^2 + (33*c^2 - (68*I)*c*d - 35*d^2)*Cos[2*(e + f*
x)] + 2*(13*c - (7*I)*d)*d*Sin[2*(e + f*x)])*Sqrt[c + d*Tan[e + f*x]]))/f
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1502 vs. $2(305) = 610$.
time = 0.53, size = 1503, normalized size = 3.98

method	result	size
derivativedivides	Expression too large to display	1503
default	Expression too large to display	1503

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/96/f*(a*(1+I*tan(f*x+e)))^(1/2)*(c+d*tan(f*x+e))^(1/2)*a*(66*I*(I*a*d)^(1
/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*
c^2-48*I*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*
x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*2^(1/2)*(-a*(I*d-c))^(1/2)*a
*c+135*I*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*
x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*2^(1/2)*(-a*(I*d-c))^(1/2)*a
*c^2*d-69*I*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan
(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*2^(1/2)*(-a*(I*d-c))^(1/2
)*a*d^3-15*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(
f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*2^(1/2)*(-a*(I*d-c))^(1/2)
*a*c^3+165*2^(1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a
*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))
*a*c*d^2+52*I*(I*a*d)^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*
(1+I*tan(f*x+e)))^(1/2)*c*d*tan(f*x+e)-48*I*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*
c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^
(1/2))*2^(1/2)*(-a*(I*d-c))^(1/2)*a*d+28*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e
)))^(1/2)*(I*a*d)^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*d^2*tan(f*x+e)+48*I*ln((
```

```
3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*
(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*(I*a*d)^(1/2)*
a*c-54*I*(I*a*d)^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*
tan(f*x+e)))^(1/2)*d^2+136*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a
*d)^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*c*d+48*2^(1/2)*(-a*(I*d-c))^(1/2)*ln(1
/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*
(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c-48*2^(1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*
(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*
a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*d+48*I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*
a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*
x+e)))^(1/2))/(tan(f*x+e)+I))*(I*a*d)^(1/2)*a*d+16*I*2^(1/2)*d^2*(I*a*d)^(1
/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*tan(f*x+
e)^2-48*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2
^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan
(f*x+e)+I))*a*c+48*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1
/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*
x+e)+I))*a*d*(I*a*d)^(1/2))*2^(1/2)/(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(
1/2)/(I*a*d)^(1/2)/(-a*(I*d-c))^(1/2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(3*d-c>0)', see `assume?` for more d
etails)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1533 vs. $2(302) = 604$.
time = 1.69, size = 1533, normalized size = 4.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] 1/48*(48*sqrt(2)*(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*e) + f)*sqrt
(-(a^3*c^5 - 5*I*a^3*c^4*d - 10*a^3*c^3*d^2 + 10*I*a^3*c^2*d^3 + 5*a^3*c*d^
4 - I*a^3*d^5)/f^2)*log((I*sqrt(2)*f*sqrt(-(a^3*c^5 - 5*I*a^3*c^4*d - 10*a^
```

```
3*c^3*d^2 + 10*I*a^3*c^2*d^3 + 5*a^3*c*d^4 - I*a^3*d^5)/f^2)*e^(I*f*x + I*e
) + sqrt(2)*(a*c^2 - 2*I*a*c*d - a*d^2 + (a*c^2 - 2*I*a*c*d - a*d^2)*e^(2*I
*f*x + 2*I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x +
 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/(a*c^2 -
2*I*a*c*d - a*d^2)) - 48*sqrt(2)*(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x +
2*I*e) + f)*sqrt(-(a^3*c^5 - 5*I*a^3*c^4*d - 10*a^3*c^3*d^2 + 10*I*a^3*c^2*
d^3 + 5*a^3*c*d^4 - I*a^3*d^5)/f^2)*log((-I*sqrt(2)*f*sqrt(-(a^3*c^5 - 5*I*
a^3*c^4*d - 10*a^3*c^3*d^2 + 10*I*a^3*c^2*d^3 + 5*a^3*c*d^4 - I*a^3*d^5)/f^
2)*e^(I*f*x + I*e) + sqrt(2)*(a*c^2 - 2*I*a*c*d - a*d^2 + (a*c^2 - 2*I*a*c*
d - a*d^2)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I
*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x
 - I*e)/(a*c^2 - 2*I*a*c*d - a*d^2)) + 2*sqrt(2)*((33*I*a*c^2 + 94*a*c*d -
49*I*a*d^2)*e^(5*I*f*x + 5*I*e) - 2*(-33*I*a*c^2 - 68*a*c*d + 19*I*a*d^2)*e
^(3*I*f*x + 3*I*e) - 3*(-11*I*a*c^2 - 14*a*c*d + 7*I*a*d^2)*e^(I*f*x + I*e)
)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))
*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)) + 3*(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*
f*x + 2*I*e) + f)*sqrt((-25*I*a^3*c^6 - 450*a^3*c^5*d + 2575*I*a^3*c^4*d^2
+ 5180*a^3*c^3*d^3 - 5095*I*a^3*c^2*d^4 - 2530*a^3*c*d^5 + 529*I*a^3*d^6)/(
d*f^2))*log((2*I*d*f*sqrt((-25*I*a^3*c^6 - 450*a^3*c^5*d + 2575*I*a^3*c^4*d
^2 + 5180*a^3*c^3*d^3 - 5095*I*a^3*c^2*d^4 - 2530*a^3*c*d^5 + 529*I*a^3*d^6
)/(d*f^2))*e^(I*f*x + I*e) + sqrt(2)*(-5*I*a*c^3 - 45*a*c^2*d + 55*I*a*c*d^
2 + 23*a*d^3 + (-5*I*a*c^3 - 45*a*c^2*d + 55*I*a*c*d^2 + 23*a*d^3)*e^(2*I*f
*x + 2*I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2
*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/(-5*I*a*c^3
 - 45*a*c^2*d + 55*I*a*c*d^2 + 23*a*d^3)) - 3*(f*e^(4*I*f*x + 4*I*e) + 2*f*
e^(2*I*f*x + 2*I*e) + f)*sqrt((-25*I*a^3*c^6 - 450*a^3*c^5*d + 2575*I*a^3*c
^4*d^2 + 5180*a^3*c^3*d^3 - 5095*I*a^3*c^2*d^4 - 2530*a^3*c*d^5 + 529*I*a^3
*d^6)/(d*f^2))*log((-2*I*d*f*sqrt((-25*I*a^3*c^6 - 450*a^3*c^5*d + 2575*I*a
^3*c^4*d^2 + 5180*a^3*c^3*d^3 - 5095*I*a^3*c^2*d^4 - 2530*a^3*c*d^5 + 529*I
*a^3*d^6)/(d*f^2))*e^(I*f*x + I*e) + sqrt(2)*(-5*I*a*c^3 - 45*a*c^2*d + 55*
I*a*c*d^2 + 23*a*d^3 + (-5*I*a*c^3 - 45*a*c^2*d + 55*I*a*c*d^2 + 23*a*d^3)*
e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I
*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/(-5
*I*a*c^3 - 45*a*c^2*d + 55*I*a*c*d^2 + 23*a*d^3)))/(f*e^(4*I*f*x + 4*I*e) +
 2*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(3/2)*(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 0.95index.cc index_m
 operator + Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + a\tan(e + f\,x)\ 1\mathrm{i})^{3/2}\,(c + d\tan(e + f\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(3/2)*(c + d*tan(e + f*x))^(5/2),x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^(3/2)*(c + d*tan(e + f*x))^(5/2), x)
```

3.1151 $\int \sqrt{a+ia\tan(e+fx)}\,(c+d\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=257

$$-\frac{\sqrt[4]{-1}\sqrt{a}\sqrt{d}\,(15c^2-10icd-7d^2)\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c+d\tan(e+fx)}}\right)}{4f}-\frac{i\sqrt{2}\sqrt{a}\,(c-id)^{5/2}\tanh^{-1}$$

```
[Out] -I*(c-I*d)^(5/2)*arctanh(2^(1/2)*a^(1/2)*(c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/
2)/(a+I*a*tan(f*x+e))^(1/2))*2^(1/2)*a^(1/2)/f-1/4*(-1)^(1/4)*(15*c^2-10*I*
c*d-7*d^2)*arctanh((-1)^(3/4)*d^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c+d
*tan(f*x+e))^(1/2))*a^(1/2)*d^(1/2)/f+1/4*(7*c-I*d)*d*(a+I*a*tan(f*x+e))^(1
/2)*(c+d*tan(f*x+e))^(1/2)/f+1/2*d*(a+I*a*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e)
)^(3/2)/f
```

Rubi [A]
time = 0.68, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {3641, 3678, 3682, 3625, 214, 3680, 65, 223, 212}

$$-\frac{\sqrt[4]{-1}\sqrt{a}\sqrt{d}\,(15c^2-10icd-7d^2)\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c+d\tan(e+fx)}}\right)}{4f}+\frac{d\sqrt{a+ia\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}}{2f}+\frac{d(7c-id)\sqrt{a+ia\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{4f}-\frac{i\sqrt{2}\sqrt{a}\,(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] -1/4*((-1)^(1/4)*Sqrt[a]*Sqrt[d]*(15*c^2 - (10*I)*c*d - 7*d^2)*ArcTanh[((-1
)^(3/4)*Sqrt[d]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c + d*Tan[e + f*x
]])])/f - (I*Sqrt[2]*Sqrt[a]*(c - I*d)^(5/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[
c + d*Tan[e + f*x]])/(Sqrt[c - I*d]*Sqrt[a + I*a*Tan[e + f*x]])])/f + ((7*c
 - I*d)*d*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(4*f) + (d*S
qrt[a + I*a*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))/(2*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3641

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[d*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n - 1)/(f*(m + n - 1))), x] - Dist[1/(a*(m + n - 1)), Int[(a + b*
Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 2)*Simp[d*(b*c*m + a*d*(-1 + n))
- a*c^2*(m + n - 1) + d*(b*d*m - a*c*(m + 2*n - 2))*Tan[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1] && NeQ[m + n - 1, 0] && (IntegerQ[n]
|| IntegersQ[2*m, 2*n])
```

Rule 3678

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Dist[
1/(a*(m + n)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Simp
[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \sqrt{a+ia\tan(e+fx)}\,(c+d\tan(e+fx))^{5/2}\,dx = \frac{d\sqrt{a+ia\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}}{2f} - \frac{\int \sqrt{a\cdots}}{\cdots}$$

$$= \frac{(7c-id)d\sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{4f} + \cdots$$

$$= \frac{(7c-id)d\sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{4f} + \cdots$$

$$= \frac{(7c-id)d\sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{4f} + \cdots$$

$$= -\frac{i\sqrt{2}\,\sqrt{a}\,(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+\cdots}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e\cdots}}\cdots\right)}{f}\cdots$$

$$= -\frac{i\sqrt{2}\,\sqrt{a}\,(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+\cdots}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e\cdots}}\cdots\right)}{f}\cdots$$

$$= -\frac{\sqrt[4]{-1}\,\sqrt{a}\,\sqrt{d}\,(15c^2-10icd-7d^2)\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\cdots}{\sqrt{a}\,\cdots}\cdots\right)}{4f}\cdots$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 539 vs. 2(257) = 514.
time = 6.30, size = 539, normalized size = 2.10

$$\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\sqrt{a+ia\tan(e+fx)}\left(-\frac{\cos(e+fx)\left(\sqrt{d}\,(15c^2-10icd-7d^2)\left(\log\left(\frac{(2+2i)e^{\frac{ie}{2}}\left(c+id-ice^{i(e+fx)}-de^{i(e+fx)}+(1+i)\sqrt{d}\,\sqrt{1+e^{2i(e+fx)}}\,\sqrt{c-\frac{id(-1+e^{2i(e+fx)})}{1+e^{2i(e+fx)}}}\right)}{d^{3/2}(-15c^2+10icd+7d^2)(i+e^{i(e+fx)})}\right)-\log\left(-\frac{(2+2i)e^{\frac{ie}{2}}\left(c+id+ice^{i(e+fx)}+de^{i(e+fx)}+(1+i)\sqrt{d}\,\sqrt{1+e^{2i(e+fx)}}\,\sqrt{c-\frac{id(-1+e^{2i(e+fx)})}{1+e^{2i(e+fx)}}}\right)}{d^{3/2}(-15c^2+10icd+7d^2)(-i+e^{i(e+fx)})}\right)\right)+(8+8i)(c-id)^{5/2}\log\left(2\left(\sqrt{c-id}\cos(e+fx)+i\sqrt{c-id}\sin(e+fx)+\sqrt{1+\cos(2(e+fx))+i\sin(2(e+fx))}\,\sqrt{c+d\tan(e+fx)}\right)\right)\right)}{\sqrt{1+\cos(2(e+fx))+i\sin(2(e+fx))}}+(1-i)d\sqrt{c+d\tan(e+fx)}\,(9c-id+2d\tan(e+fx))\right)}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + I*a*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((1/8 + I/8)*Sqrt[a + I*a*Tan[e + f*x]]*(-((Cos[e + f*x]*(Sqrt[d]*(15*c^2 -
 (10*I)*c*d - 7*d^2)*(Log[((2 + 2*I)*E^((I/2)*e)*(c + I*d - I*c*E^(I*(e + f
*x)) - d*E^(I*(e + f*x)) + (1 + I)*Sqrt[d]*Sqrt[1 + E^((2*I)*(e + f*x))]*Sq
rt[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))]))/(d^(3/
2)*(-15*c^2 + (10*I)*c*d + 7*d^2)*(I + E^(I*(e + f*x))))] - Log[((-2 - 2*I)
*E^((I/2)*e)*(c + I*d + I*c*E^(I*(e + f*x)) + d*E^(I*(e + f*x)) + (1 + I)*S
qrt[d]*Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*x)
)))/(1 + E^((2*I)*(e + f*x)))]))/(d^(3/2)*(-15*c^2 + (10*I)*c*d + 7*d^2)*(-
I + E^(I*(e + f*x))))]) + (8 + 8*I)*(c - I*d)^(5/2)*Log[2*(Sqrt[c - I*d]*Co
s[e + f*x] + I*Sqrt[c - I*d]*Sin[e + f*x] + Sqrt[1 + Cos[2*(e + f*x)] + I*S
in[2*(e + f*x)]]*Sqrt[c + d*Tan[e + f*x]])]))/Sqrt[1 + Cos[2*(e + f*x)] + I
*Sin[2*(e + f*x)]]) + (1 - I)*d*Sqrt[c + d*Tan[e + f*x]]*(9*c - I*d + 2*d*T
an[e + f*x])))/f
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2129 vs. 2(202) = 404.
time = 0.57, size = 2130, normalized size = 8.29

method	result	size
derivativedivides	Expression too large to display	2130
default	Expression too large to display	2130

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/16/f*(a*(1+I*tan(f*x+e)))^(1/2)*(c+d*tan(f*x+e))^(1/2)*(4*I*(a*(c+d*tan(
f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*c*
d^2*tan(f*x+e)^2+7*2^(1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*
a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d
)^(1/2))*a*d^4+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)*
2^(1/2)*(-a*(I*d-c))^(1/2)*d^3+3*I*2^(1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I*a
*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(
1/2)+a*d)/(I*a*d)^(1/2))*a*c*d^3-5*I*2^(1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I
*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)
^(1/2)+a*d)/(I*a*d)^(1/2))*a*c^2*d^2*tan(f*x+e)+16*I*(a*(c+d*tan(f*x+e))*(1
+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*c*d^2+6*I*(a
*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)*2^(1/2)*(-a*(I*d-c)
)^(1/2)*d^3*tan(f*x+e)-16*I*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+
3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(
f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c^3*d*tan(f*x+e)-15*2^(1/2)*(-a*(I*d-c))^
```

```
(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e
)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c^3*d*tan(f*x+e)-12*(a*(c+d*t
an(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)
*c*d^2*tan(f*x+e)-5*2^(1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I
*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*
d)^(1/2))*a*c^2*d^2+18*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^
(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*c^2*d+8*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*
x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+
e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c^4*tan(f*x+e)-16*(I*a*d)^(1
/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c)
)^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c^3*
d-3*2^(1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*t
an(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c*d^
3*tan(f*x+e)+7*I*2^(1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*
c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^
(1/2))*a*d^4*tan(f*x+e)-16*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3
*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f
*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c*d^3-8*I*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(
f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*
x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c^4-16*I*(I*a*d)^(1/2)*ln(
(3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)
*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c*d^3*tan(f
*x+e)+15*I*2^(1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a
*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))
*a*c^3*d+18*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)*2^(
1/2)*(-a*(I*d-c))^(1/2)*c^2*d*tan(f*x+e)-4*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x
+e)))^(1/2)*(I*a*d)^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*d^3*tan(f*x+e)^2-8*(I*
a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*
(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))
*a*d^4*tan(f*x+e)+8*I*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*
tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)
))^(1/2))/(tan(f*x+e)+I))*a*d^4)/(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2
)/(I*a*d)^(1/2)/(I*c-d)/(-tan(f*x+e)+I)*2^(1/2)/(-a*(I*d-c))^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

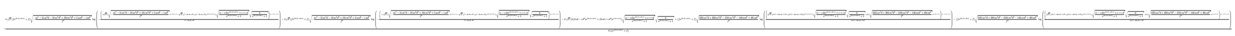
Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1151 vs. 2(201) = 402.
time = 1.31, size = 1151, normalized size = 4.48



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] -1/8*(4*sqrt(2)*(f*e^(2*I*f*x + 2*I*e) + f)*sqrt(-(a*c^5 - 5*I*a*c^4*d - 10
*a*c^3*d^2 + 10*I*a*c^2*d^3 + 5*a*c*d^4 - I*a*d^5)/f^2)*log(-(I*sqrt(2)*f*s
qrt(-(a*c^5 - 5*I*a*c^4*d - 10*a*c^3*d^2 + 10*I*a*c^2*d^3 + 5*a*c*d^4 - I*a
*d^5)/f^2)*e^(I*f*x + I*e) - sqrt(2)*(c^2 - 2*I*c*d - d^2 + (c^2 - 2*I*c*d
- d^2)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/
(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I
*e)/(c^2 - 2*I*c*d - d^2)) - 4*sqrt(2)*(f*e^(2*I*f*x + 2*I*e) + f)*sqrt(-(a
*c^5 - 5*I*a*c^4*d - 10*a*c^3*d^2 + 10*I*a*c^2*d^3 + 5*a*c*d^4 - I*a*d^5)/f
^2)*log(-(-I*sqrt(2)*f*sqrt(-(a*c^5 - 5*I*a*c^4*d - 10*a*c^3*d^2 + 10*I*a*c
^2*d^3 + 5*a*c*d^4 - I*a*d^5)/f^2)*e^(I*f*x + I*e) - sqrt(2)*(c^2 - 2*I*c*d
 - d^2 + (c^2 - 2*I*c*d - d^2)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(2*I*
f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I
*e) + 1)))*e^(-I*f*x - I*e)/(c^2 - 2*I*c*d - d^2)) - 2*sqrt(2)*(3*(3*c*d -
I*d^2)*e^(3*I*f*x + 3*I*e) + (9*c*d + I*d^2)*e^(I*f*x + I*e))*sqrt(((c - I*
d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I
*f*x + 2*I*e) + 1)) + (f*e^(2*I*f*x + 2*I*e) + f)*sqrt((225*I*a*c^4*d + 300
*a*c^3*d^2 - 310*I*a*c^2*d^3 - 140*a*c*d^4 + 49*I*a*d^5)/f^2)*log((sqrt(2)*
(15*c^2 - 10*I*c*d - 7*d^2 + (15*c^2 - 10*I*c*d - 7*d^2)*e^(2*I*f*x + 2*I*e
))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1)
)*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)) + 2*I*f*sqrt((225*I*a*c^4*d + 300*a*c^3
*d^2 - 310*I*a*c^2*d^3 - 140*a*c*d^4 + 49*I*a*d^5)/f^2)*e^(I*f*x + I*e))*e^
(-I*f*x - I*e)/(15*c^2 - 10*I*c*d - 7*d^2)) - (f*e^(2*I*f*x + 2*I*e) + f)*s
qrt((225*I*a*c^4*d + 300*a*c^3*d^2 - 310*I*a*c^2*d^3 - 140*a*c*d^4 + 49*I*a
*d^5)/f^2)*log((sqrt(2)*(15*c^2 - 10*I*c*d - 7*d^2 + (15*c^2 - 10*I*c*d - 7
*d^2)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(
e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)) - 2*I*f*sqrt((2
25*I*a*c^4*d + 300*a*c^3*d^2 - 310*I*a*c^2*d^3 - 140*a*c*d^4 + 49*I*a*d^5)/
f^2)*e^(I*f*x + I*e))*e^(-I*f*x - I*e)/(15*c^2 - 10*I*c*d - 7*d^2)))/(f*e^(
2*I*f*x + 2*I*e) + f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\left(\tan\left(e+fx\right)-i\right)}\left(c+d\tan\left(e+fx\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(1/2)*(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(e + f*x) - I))*(c + d*tan(e + f*x))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+a\tan\left(e+f\,x\right)\,1{}\mathrm{i}}\,\left(c+d\,\tan\left(e+f\,x\right)\right)^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(1/2)*(c + d*tan(e + f*x))^(5/2),x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^(1/2)*(c + d*tan(e + f*x))^(5/2), x)
```

3.1152 $\int \frac{(c+d\tan(e+fx))^{5/2}}{\sqrt{a+ia\tan(e+fx)}}\,dx$

Optimal. Leaf size=250

$$\frac{\sqrt[4]{-1}\,(5ic-d)d^{3/2}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a}\,f}-\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(}{\sqrt{c-id}\,\sqrt{a+ia\tan}}\right.}{\sqrt{2}\,\sqrt{a}\,f}$$

```
[Out] (-1)^(1/4)*(5*I*c-d)*d^(3/2)*arctanh((-1)^(3/4)*d^(1/2)*(a+I*a*tan(f*x+e))^
(1/2)/a^(1/2)/(c+d*tan(f*x+e))^(1/2))/f/a^(1/2)-1/2*I*(c-I*d)^(5/2)*arctanh
(2^(1/2)*a^(1/2)*(c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2)/(a+I*a*tan(f*x+e))^(1
/2))/f*2^(1/2)/a^(1/2)-(c+2*I*d)*d*(a+I*a*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e)
)^(1/2)/a/f+(I*c-d)*(c+d*tan(f*x+e))^(3/2)/f/(a+I*a*tan(f*x+e))^(1/2)
```

Rubi [A]

time = 0.66, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {3639, 3678, 3682, 3625, 214, 3680, 65, 223, 212}

$$\frac{\sqrt[4]{-1}\,d^{3/2}(-d+5ic)\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a}\,f}+\frac{(-d+ic)(c+d\tan(e+fx))^{3/2}}{f\sqrt{a+ia\tan(e+fx)}}-\frac{d(c+2id)\sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{af}-\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+fx)}}\right)}{\sqrt{2}\,\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^(5/2)/Sqrt[a + I*a*Tan[e + f*x]],x]
```

```
[Out] ((-1)^(1/4)*((5*I)*c - d)*d^(3/2)*ArcTanh[((-1)^(3/4)*Sqrt[d]*Sqrt[a + I*a*
Tan[e + f*x]])/(Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a]*f) - (I*(c - I
*d)^(5/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])/(Sqrt[c - I*d]
*Sqrt[a + I*a*Tan[e + f*x]])])/(Sqrt[2]*Sqrt[a]*f) - ((c + (2*I)*d)*d*Sqrt[
a + I*a*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(a*f) + ((I*c - d)*(c + d*T
an[e + f*x])^(3/2))/(f*Sqrt[a + I*a*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3678

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Dist[
1/(a*(m + n)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Simp
[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{5/2}}{\sqrt{a+ia\tan(e+fx)}}\,dx = \frac{(ic-d)(c+d\tan(e+fx))^{3/2}}{f\sqrt{a+ia\tan(e+fx)}} - \frac{\int \sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}$$

$$= -\frac{(c+2id)d\sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{af} + \frac{(ic-d)(c+d\tan}{f\sqrt{a+ia\tan}}$$

$$= -\frac{(c+2id)d\sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{af} + \frac{(ic-d)(c+d\tan}{f\sqrt{a+ia\tan}}$$

$$= -\frac{(c+2id)d\sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{af} + \frac{(ic-d)(c+d\tan}{f\sqrt{a+ia\tan}}$$

$$= -\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+fx)}}\right)}{\sqrt{2}\,\sqrt{a}\,f} - \frac{(c+2id)d\sqrt{a+}}{}$$

$$= -\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+fx)}}\right)}{\sqrt{2}\,\sqrt{a}\,f} - \frac{(c+2id)d\sqrt{a+}}{}$$

$$= \frac{\sqrt[4]{-1}\,(5ic-d)d^{3/2}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a}\,f} - \frac{i(c-id)^{5}}{}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 549 vs. 2(250) = 500.

time = 8.32, size = 549, normalized size = 2.20

$$\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(\cos(fx)+i\sin(fx))\left(-\frac{\left(d^{3/2}(-5ic+d)\left(\log\left(\frac{(1+i)e^{\frac{ie}{2}}\left(-id+de^{i(e+fx)}+ic\left(i+e^{i(e+fx)}\right)-(1+i)\sqrt{d}\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)}{d^{5/2}(-5ic+d)\left(i+e^{i(e+fx)}\right)}\right)-\log\left(\frac{(1+i)e^{\frac{ie}{2}}\left(c+id+ice^{i(e+fx)}+de^{i(e+fx)}+(1+i)\sqrt{d}\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)}{d^{5/2}(-5ic+d)\left(-i+e^{i(e+fx)}\right)}\right)\right)+(1+i)(c-id)^{5/2}\log\left(2\left(\sqrt{c-id}\cos(e+fx)+i\sqrt{c-id}\sin(e+fx)+\sqrt{1+\cos(2(e+fx))+i\sin(2(e+fx))}\sqrt{c+d\tan(e+fx)}\right)\right)\right)(\cos(e)+i\sin(e))}{\sqrt{1+\cos(2(e+fx))+i\sin(2(e+fx))}}+(1+i)(\cos(fx)-i\sin(fx))\sqrt{c+d\tan(e+fx)}\left(c^2+2icd-2d^2-id^2\tan(e+fx)\right)\right)}{f\sqrt{a+ia\tan(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Tan[e + f*x])^(5/2)/Sqrt[a + I*a*Tan[e + f*x]],x]
```

```
[Out] ((1/2 + I/2)*(Cos[f*x] + I*Sin[f*x])*(-(((d^(3/2)*((-5*I)*c + d)*(Log[((1 +
 I)*E^((I/2)*e)*((-I)*d + d*E^(I*(e + f*x)) + I*c*(I + E^(I*(e + f*x))) - (
1 + I)*Sqrt[d]*Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^((2*I)*(
e + f*x))))/(1 + E^((2*I)*(e + f*x)))]))/(d^(5/2)*((-5*I)*c + d)*(I + E^(I*
(e + f*x))))] - Log[((1 + I)*E^((I/2)*e)*(c + I*d + I*c*E^(I*(e + f*x)) + d
*E^(I*(e + f*x)) + (1 + I)*Sqrt[d]*Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (
I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))]))/(d^(5/2)*((-5*
I)*c + d)*(-I + E^(I*(e + f*x))))]) + (1 + I)*(c - I*d)^(5/2)*Log[2*(Sqrt[c
 - I*d]*Cos[e + f*x] + I*Sqrt[c - I*d]*Sin[e + f*x] + Sqrt[1 + Cos[2*(e + f
*x)] + I*Sin[2*(e + f*x)]]*Sqrt[c + d*Tan[e + f*x]])])*(Cos[e] + I*Sin[e]))
/Sqrt[1 + Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)]]) + (1 + I)*(Cos[f*x] - I*S
in[f*x])*Sqrt[c + d*Tan[e + f*x]]*(c^2 + (2*I)*c*d - 2*d^2 - I*d^2*Tan[e +
f*x])))/(f*Sqrt[a + I*a*Tan[e + f*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2646 vs. 2(200) = 400.
time = 0.56, size = 2647, normalized size = 10.59

method	result	size
derivativedivides	Expression too large to display	2647
default	Expression too large to display	2647

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/4/f*(c+d*tan(f*x+e))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)*(I*2^(1/2)*(-a*(I*
d-c))^(1/2)*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)
+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/
(tan(f*x+e)+I))*c^2*d*tan(f*x+e)^2+2*I*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(
a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2)
)*a*d^4*tan(f*x+e)^2-4*c^3*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a
*d)^(1/2)+8*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)*d^3
+16*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)*c*d^2-12*(a*(
c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)*d^3*tan(f*x+e)+4*ln(1
/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*
```

(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*d^4*tan(f*x+e)-12*ln(1/2*(2*I*a*d*tan(f
*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d
)/(I*a*d)^(1/2))*a*c*d^3-2*I*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan
(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*d^4+2^
(1/2)*(-a*(I*d-c))^(1/2)*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a
*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x
+e)))^(1/2))/(tan(f*x+e)+I))*c^3+2*I*2^(1/2)*(-a*(I*d-c))^(1/2)*(I*a*d)^(1/
2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))
^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c^3*tan
(f*x+e)-I*2^(1/2)*(-a*(I*d-c))^(1/2)*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)
*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*
(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c^2*d+I*2^(1/2)*(-a*(I*d-c))^(1/2)
*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*
(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)
+I))*d^3*tan(f*x+e)^2-2^(1/2)*(-a*(I*d-c))^(1/2)*(I*a*d)^(1/2)*ln((3*a*c+I*
a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*
tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c*d^2*tan(f*x+e)^2+2*2
^(1/2)*(-a*(I*d-c))^(1/2)*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*
a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*
x+e)))^(1/2))/(tan(f*x+e)+I))*c^2*d*tan(f*x+e)-I*2^(1/2)*(-a*(I*d-c))^(1/2)
*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*
(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)
+I))*d^3-24*I*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*t
an(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c*d^3*tan(f*x+e)+20*I
(a(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)*c*d^2*tan(f*x+e)
+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*
d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*ta
n(f*x+e)))^(1/2))/(tan(f*x+e)+I))*d^3*tan(f*x+e)+2^(1/2)*(-a*(I*d-c))^(1/2)
*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*
(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)
+I))*c*d^2-2^(1/2)*(-a*(I*d-c))^(1/2)*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e
)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))
*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c^3*tan(f*x+e)^2-10*I*ln(1/2*(2*I
*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)
^(1/2)+a*d)/(I*a*d)^(1/2))*a*c^2*d^2*tan(f*x+e)^2-4*I*(a*(c+d*tan(f*x+e))*(
1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)*d^3*tan(f*x+e)^2+10*I*ln(1/2*(2*I*a*d*
tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2
)+a*d)/(I*a*d)^(1/2))*a*c^2*d^2-4*I*c^3*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)
))^(1/2)*(I*a*d)^(1/2)*tan(f*x+e)-12*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e))
)^(1/2)*(I*a*d)^(1/2)*c^2*d+12*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*t
an(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c*d^
3*tan(f*x+e)^2-20*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1
+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c^2*d^2*tan(f*x+e
)-4*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)*c*d^2*tan(f*x
+e)^2+12*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)*c^2*d*ta

```
n(f*x+e)+2*I*2^(1/2)*(-a*(I*d-c))^(1/2)*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x
+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e
))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c*d^2*tan(f*x+e))/a/(a*(c+d*tan
(f*x+e))*(1+I*tan(f*x+e)))^(1/2)/(I*c-d)/(-tan(f*x+e)+I)^2/(I*a*d)^(1/2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

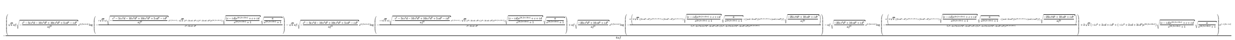
Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1196 vs. 2(198) = 396.
time = 1.83, size = 1196, normalized size = 4.78



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] -1/4*(sqrt(2)*a*f*sqrt(-(c^5 - 5*I*c^4*d - 10*c^3*d^2 + 10*I*c^2*d^3 + 5*c*
d^4 - I*d^5)/(a*f^2))*e^(I*f*x + I*e)*log(-(I*sqrt(2)*a*f*sqrt(-(c^5 - 5*I*
c^4*d - 10*c^3*d^2 + 10*I*c^2*d^3 + 5*c*d^4 - I*d^5)/(a*f^2))*e^(I*f*x + I*
e) - sqrt(2)*(c^2 - 2*I*c*d - d^2 + (c^2 - 2*I*c*d - d^2)*e^(2*I*f*x + 2*I*
e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1
))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))/(c^2 - 2*I*c*d - d^2)) - sqrt(2)*a*f*
sqrt(-(c^5 - 5*I*c^4*d - 10*c^3*d^2 + 10*I*c^2*d^3 + 5*c*d^4 - I*d^5)/(a*f^
2))*e^(I*f*x + I*e)*log(-(-I*sqrt(2)*a*f*sqrt(-(c^5 - 5*I*c^4*d - 10*c^3*d^
2 + 10*I*c^2*d^3 + 5*c*d^4 - I*d^5)/(a*f^2))*e^(I*f*x + I*e) - sqrt(2)*(c^2
 - 2*I*c*d - d^2 + (c^2 - 2*I*c*d - d^2)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*
d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I
*f*x + 2*I*e) + 1)))/(c^2 - 2*I*c*d - d^2)) + a*f*sqrt((-25*I*c^2*d^3 + 10*
c*d^4 + I*d^5)/(a*f^2))*e^(I*f*x + I*e)*log(-4*(2*sqrt(2)*((5*I*c*d^3 - d^4
)*e^(3*I*f*x + 3*I*e) + (5*I*c*d^3 - d^4)*e^(I*f*x + I*e))*sqrt(((c - I*d)*
e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*
x + 2*I*e) + 1)) + ((a*c*d - 3*I*a*d^2)*f*e^(2*I*f*x + 2*I*e) + (a*c*d + I*
a*d^2)*f)*sqrt((-25*I*c^2*d^3 + 10*c*d^4 + I*d^5)/(a*f^2)))/(5*c^4 - 4*I*c^
3*d + 6*c^2*d^2 - 4*I*c*d^3 + d^4 + (5*c^4 - 4*I*c^3*d + 6*c^2*d^2 - 4*I*c*
d^3 + d^4)*e^(2*I*f*x + 2*I*e))) - a*f*sqrt((-25*I*c^2*d^3 + 10*c*d^4 + I*d
```

```
^5)/(a*f^2))*e^(I*f*x + I*e)*log(-4*(2*sqrt(2)*((5*I*c*d^3 - d^4)*e^(3*I*f*
x + 3*I*e) + (5*I*c*d^3 - d^4)*e^(I*f*x + I*e))*sqrt(((c - I*d)*e^(2*I*f*x
+ 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e)
+ 1)) - ((a*c*d - 3*I*a*d^2)*f*e^(2*I*f*x + 2*I*e) + (a*c*d + I*a*d^2)*f)*s
qrt((-25*I*c^2*d^3 + 10*c*d^4 + I*d^5)/(a*f^2)))/(5*c^4 - 4*I*c^3*d + 6*c^2
*d^2 - 4*I*c*d^3 + d^4 + (5*c^4 - 4*I*c^3*d + 6*c^2*d^2 - 4*I*c*d^3 + d^4)*
e^(2*I*f*x + 2*I*e))) + 2*sqrt(2)*(-I*c^2 + 2*c*d + I*d^2 + (-I*c^2 + 2*c*d
 + 3*I*d^2)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c +
I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f*
x - I*e)/(a*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\tan{(e+fx)})^{\frac{5}{2}}}{\sqrt{ia\,(\tan{(e+fx)}-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(5/2)/(a+I*a*tan(f*x+e))**(1/2),x)
[Out] Integral((c + d*tan(e + f*x))**(5/2)/sqrt(I*a*(tan(e + f*x) - I)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(1/2),x, algorithm="gia
c")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\tan(e+f\,x))^{5/2}}{\sqrt{a+a\tan\left(e+f\,x\right)\,1{}\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(5/2)/(a + a*tan(e + f*x)*1i)^(1/2),x)
[Out] int((c + d*tan(e + f*x))^(5/2)/(a + a*tan(e + f*x)*1i)^(1/2), x)
```

3.1153 $\int \frac{(c+d\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=257

$$\frac{2\sqrt[4]{-1}\,d^{5/2}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}\right)}{a^{3/2}f}-\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f}$$

```
[Out] 2*(-1)^(1/4)*d^(5/2)*arctanh((-1)^(3/4)*d^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^
(1/2)/(c+d*tan(f*x+e))^(1/2))/a^(3/2)/f-1/4*I*(c-I*d)^(5/2)*arctanh(2^(1/2)
*a^(1/2)*(c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2)/(a+I*a*tan(f*x+e))^(1/2))/a^(
3/2)/f*2^(1/2)+1/2*(c+I*d)*(I*c+3*d)*(c+d*tan(f*x+e))^(1/2)/a/f/(a+I*a*tan(
f*x+e))^(1/2)+1/3*(I*c-d)*(c+d*tan(f*x+e))^(3/2)/f/(a+I*a*tan(f*x+e))^(3/2)
```

Rubi [A]

time = 0.65, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {3639, 3676, 3682, 3625, 214, 3680, 65, 223, 212}

$$\frac{2\sqrt[4]{-1}\,d^{5/2}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}\right)}{a^{3/2}f}-\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f}+\frac{(-d+ic)(c+d\tan(e+fx))^{3/2}}{3f(a+ia\tan(e+fx))^{3/2}}+\frac{(c+id)(3d+ic)\sqrt{c+d\tan(e+fx)}}{2af\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^(5/2)/(a + I*a*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*(-1)^(1/4)*d^(5/2)*ArcTanh[((-1)^(3/4)*Sqrt[d]*Sqrt[a + I*a*Tan[e + f*x]
])/(Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])])/(a^(3/2)*f) - ((I/2)*(c - I*d)^(5/2
)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])/(Sqrt[c - I*d]*Sqrt[a
+ I*a*Tan[e + f*x]])])/(Sqrt[2]*a^(3/2)*f) + ((c + I*d)*(I*c + 3*d)*Sqrt[c
+ d*Tan[e + f*x]])/(2*a*f*Sqrt[a + I*a*Tan[e + f*x]]) + ((I*c - d)*(c + d*T
an[e + f*x])^(3/2))/(3*f*(a + I*a*Tan[e + f*x])^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3639

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*(a + b*Tan[e + f*x])^m*
((c + d*Tan[e + f*x])^(n - 1)/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b
*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[c*(a*c*m + b*d*(n
- 1)) - d*(b*c*m + a*d*(n - 1)) - d*(b*d*(m - n + 1) - a*c*(m + n - 1))*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, 0] && GtQ[n, 1] && (Int
egerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^{3/2}}\,dx = \frac{(ic-d)(c+d\tan(e+fx))^{3/2}}{3f(a+ia\tan(e+fx))^{3/2}} - \frac{\int \frac{\sqrt{c+d\tan(e+fx)}\,\left(-\frac{3}{2}a(c^2-2icd+d^2)+3iad^2\right.}{\sqrt{a+ia\tan(e+fx)}}}{3a^2}$$

$$= \frac{(c+id)(ic+3d)\sqrt{c+d\tan(e+fx)}}{2af\sqrt{a+ia\tan(e+fx)}} + \frac{(ic-d)(c+d\tan(e+fx))^{3/2}}{3f(a+ia\tan(e+fx))^{3/2}} + \frac{\int}{}$$

$$= \frac{(c+id)(ic+3d)\sqrt{c+d\tan(e+fx)}}{2af\sqrt{a+ia\tan(e+fx)}} + \frac{(ic-d)(c+d\tan(e+fx))^{3/2}}{3f(a+ia\tan(e+fx))^{3/2}} + \frac{(c}{}$$

$$= \frac{(c+id)(ic+3d)\sqrt{c+d\tan(e+fx)}}{2af\sqrt{a+ia\tan(e+fx)}} + \frac{(ic-d)(c+d\tan(e+fx))^{3/2}}{3f(a+ia\tan(e+fx))^{3/2}} - \frac{(i}{}$$

$$= -\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f} + \frac{(c+id)(ic+3d)}{2af\sqrt{a+i}}$$

$$= -\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f} + \frac{(c+id)(ic+3d)}{2af\sqrt{a+i}}$$

$$= \frac{2\sqrt[4]{-1}\,d^{5/2}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c+d\tan(e+fx)}}\right)}{a^{3/2}f} - \frac{i(c-id)^{5/2}\tanh^{-}}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 560 vs. 2(257) = 514.

time = 8.69, size = 560, normalized size = 2.18

$$\frac{\sec(e+fx)(\cos(fx)+i\sin(fx))^2\left(\frac{\left((2+2i)d^{5/2}\log\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)e^{\frac{ie}{2}}\left(c+id-ice^{i(e+fx)}-de^{i(e+fx)}+(1+i)\sqrt{d}\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)}{d^{7/2}\left(i+e^{i(e+fx)}\right)}\right)-(2+2i)d^{5/2}\log\left(-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)e^{\frac{ie}{2}}\left(c+id+ice^{i(e+fx)}+de^{i(e+fx)}+(1+i)\sqrt{d}\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)}{d^{7/2}\left(-i+e^{i(e+fx)}\right)}\right)-i(c-id)^{5/2}\log\left(2\left(\sqrt{c-id}\cos(e+fx)+i\sqrt{c-id}\sin(e+fx)+\sqrt{1+\cos(2(e+fx))+i\sin(2(e+fx))}\sqrt{c+d\tan(e+fx)}\right)\right)\right)(\cos(2e)+i\sin(2e))}{\sqrt{1+\cos(2(e+fx))+i\sin(2(e+fx))}}+\frac{1}{3}(c+id)(i\cos(2fx)+\sin(2fx))((5c-9id)\cos(e+fx)+(3ic+11d)\sin(e+fx))\sqrt{c+d\tan(e+fx)}\right)}{2f(a+ia\tan(e+fx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Tan[e + f*x])^(5/2)/(a + I*a*Tan[e + f*x])^(3/2),x]
```

```
[Out] (Sec[e + f*x]*(Cos[f*x] + I*Sin[f*x])^2*((((2 + 2*I)*d^(5/2)*Log[((1/4 + I/
4)*E^((I/2)*e)*(c + I*d - I*c*E^(I*(e + f*x)) - d*E^(I*(e + f*x)) + (1 + I)
*Sqrt[d]*Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*
x))))/(1 + E^((2*I)*(e + f*x)))]))/(d^(7/2)*(I + E^(I*(e + f*x))))] - (2 +
2*I)*d^(5/2)*Log[((-1/4 - I/4)*E^((I/2)*e)*(c + I*d + I*c*E^(I*(e + f*x)) +
 d*E^(I*(e + f*x)) + (1 + I)*Sqrt[d]*Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c -
 (I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))]))/(d^(7/2)*(-I
 + E^(I*(e + f*x))))] - I*(c - I*d)^(5/2)*Log[2*(Sqrt[c - I*d]*Cos[e + f*x]
 + I*Sqrt[c - I*d]*Sin[e + f*x] + Sqrt[1 + Cos[2*(e + f*x)] + I*Sin[2*(e +
f*x)]]*Sqrt[c + d*Tan[e + f*x]])])*(Cos[2*e] + I*Sin[2*e]))/Sqrt[1 + Cos[2*
(e + f*x)] + I*Sin[2*(e + f*x)]] + ((c + I*d)*(I*Cos[2*f*x] + Sin[2*f*x])*(
(5*c - (9*I)*d)*Cos[e + f*x] + ((3*I)*c + 11*d)*Sin[e + f*x])*Sqrt[c + d*Ta
n[e + f*x]])/3))/(2*f*(a + I*a*Tan[e + f*x])^(3/2))
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3160 vs. 2(201) = 402.
time = 0.51, size = 3161, normalized size = 12.30

method	result	size
derivativedivides	Expression too large to display	3161
default	Expression too large to display	3161

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/24*I/f*(c+d*tan(f*x+e))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)/a^2*(-9*I*2^(1/2
)*(-a*(I*d-c))^(1/2)*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*t
an(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e))
)^(1/2))/(tan(f*x+e)+I))*c*d^2*tan(f*x+e)+3*I*2^(1/2)*(-a*(I*d-c))^(1/2)*(I
*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a
*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I)
)*c*d^2*tan(f*x+e)^3-24*I*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*
x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*d^4+36*I*
(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)*d^3+20*c^3*(a*(c+
d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)-24*ln(1/2*(2*I*a*d*tan(
```

f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*
d)/(I*a*d)^(1/2))*a*d^4*tan(f*x+e)^3-12*c^3*(a*(c+d*tan(f*x+e))*(1+I*tan(f*
x+e)))^(1/2)*(I*a*d)^(1/2)*tan(f*x+e)^2+52*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x
+e)))^(1/2)*(I*a*d)^(1/2)*c*d^2-80*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1
/2)*(I*a*d)^(1/2)*d^3*tan(f*x+e)+72*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(
c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a
*d^4*tan(f*x+e)-24*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(
1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c*d^3-3*2^(1/2)*
(-a*(I*d-c))^(1/2)*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan
(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^
(1/2))/(tan(f*x+e)+I))*c^3+72*I*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*
tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*d^4
*tan(f*x+e)^2-44*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2
)*d^3*tan(f*x+e)^2+32*I*c^3*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*
a*d)^(1/2)*tan(f*x+e)+4*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*
d)^(1/2)*c^2*d+9*2^(1/2)*(-a*(I*d-c))^(1/2)*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan
(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f
x+e))(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c*d^2*tan(f*x+e)^2-9*2^(1/2
)*(-a*(I*d-c))^(1/2)*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*t
an(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e))
)^(1/2))/(tan(f*x+e)+I))*c^2*d*tan(f*x+e)-9*I*2^(1/2)*(-a*(I*d-c))^(1/2)*(I
*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a
*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I)
)*c^3*tan(f*x+e)+3*I*2^(1/2)*(-a*(I*d-c))^(1/2)*(I*a*d)^(1/2)*ln((3*a*c+I*a
*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*t
an(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c^2*d+3*2^(1/2)*(-a*(I*
d-c))^(1/2)*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)
+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/
(tan(f*x+e)+I))*c^2*d*tan(f*x+e)^3+3*I*2^(1/2)*(-a*(I*d-c))^(1/2)*(I*a*d)^(
1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c
))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c^3*t
an(f*x+e)^3-9*I*2^(1/2)*(-a*(I*d-c))^(1/2)*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(
f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*
x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*d^3*tan(f*x+e)^2+24*I*ln(1/2
*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I
*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c*d^3*tan(f*x+e)^3+3*I*2^(1/2)*(-a*(I*d-c
))^(1/2)*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*
2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(ta
n(f*x+e)+I))*d^3-72*I*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e)
)*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c*d^3*tan(f*x
+e)+20*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)*c^2*d*ta
n(f*x+e)^2+128*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)*
c*d^2*tan(f*x+e)-9*2^(1/2)*(-a*(I*d-c))^(1/2)*(I*a*d)^(1/2)*ln((3*a*c+I*a*t
an(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan
(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*d^3*tan(f*x+e)-3*2^(1/2)*

```
(-a*(I*d-c))^(1/2)*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan
(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^
(1/2))/(tan(f*x+e)+I))*c*d^2+9*2^(1/2)*(-a*(I*d-c))^(1/2)*(I*a*d)^(1/2)*ln(
(3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)
*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c^3*tan(f*x+e
)^2+3*2^(1/2)*(-a*(I*d-c))^(1/2)*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I
*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I
*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*d^3*tan(f*x+e)^3-9*I*2^(1/2)*(-a*(I*d-
c))^(1/2)*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2
*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(t
an(f*x+e)+I))*c^2*d*tan(f*x+e)^2+72*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(
c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)...
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

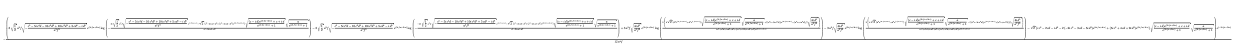
Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1105 vs. $2(197) = 394$.
time = 1.22, size = 1105, normalized size = 4.30



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] -1/12*(3*sqrt(1/2)*a^2*f*sqrt(-(c^5 - 5*I*c^4*d - 10*c^3*d^2 + 10*I*c^2*d^3
 + 5*c*d^4 - I*d^5)/(a^3*f^2))*e^(3*I*f*x + 3*I*e)*log(-(2*I*sqrt(1/2)*a^2*
f*sqrt(-(c^5 - 5*I*c^4*d - 10*c^3*d^2 + 10*I*c^2*d^3 + 5*c*d^4 - I*d^5)/(a^
3*f^2))*e^(I*f*x + I*e) - sqrt(2)*(c^2 - 2*I*c*d - d^2 + (c^2 - 2*I*c*d - d
^2)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^
(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))/(c^2 - 2*I*c*d -
 d^2)) - 3*sqrt(1/2)*a^2*f*sqrt(-(c^5 - 5*I*c^4*d - 10*c^3*d^2 + 10*I*c^2*d
^3 + 5*c*d^4 - I*d^5)/(a^3*f^2))*e^(3*I*f*x + 3*I*e)*log(-(-2*I*sqrt(1/2)*a
^2*f*sqrt(-(c^5 - 5*I*c^4*d - 10*c^3*d^2 + 10*I*c^2*d^3 + 5*c*d^4 - I*d^5)/
(a^3*f^2))*e^(I*f*x + I*e) - sqrt(2)*(c^2 - 2*I*c*d - d^2 + (c^2 - 2*I*c*d
- d^2)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/
```

```
(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))/(c^2 - 2*I*c*
d - d^2)) + 3*a^2*f*sqrt(4*I*d^5/(a^3*f^2))*e^(3*I*f*x + 3*I*e)*log(2*(4*sq
rt(2)*(d^3*e^(3*I*f*x + 3*I*e) + d^3*e^(I*f*x + I*e))*sqrt(((c - I*d)*e^(2*
I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2
*I*e) + 1)) + ((a^2*c - 3*I*a^2*d)*f*e^(2*I*f*x + 2*I*e) + (a^2*c + I*a^2*d
)*f)*sqrt(4*I*d^5/(a^3*f^2)))/(I*c^3 + c^2*d + I*c*d^2 + d^3 + (I*c^3 + c^2
*d + I*c*d^2 + d^3)*e^(2*I*f*x + 2*I*e))) - 3*a^2*f*sqrt(4*I*d^5/(a^3*f^2))
*e^(3*I*f*x + 3*I*e)*log(2*(4*sqrt(2)*(d^3*e^(3*I*f*x + 3*I*e) + d^3*e^(I*f
*x + I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I
*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)) - ((a^2*c - 3*I*a^2*d)*f*e^(2*I
*f*x + 2*I*e) + (a^2*c + I*a^2*d)*f)*sqrt(4*I*d^5/(a^3*f^2)))/(I*c^3 + c^2*
d + I*c*d^2 + d^3 + (I*c^3 + c^2*d + I*c*d^2 + d^3)*e^(2*I*f*x + 2*I*e))) -
 sqrt(2)*(I*c^2 - 2*c*d - I*d^2 - 2*(-2*I*c^2 - 3*c*d - 5*I*d^2)*e^(4*I*f*x
 + 4*I*e) + (5*I*c^2 + 4*c*d + 9*I*d^2)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d
)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*
f*x + 2*I*e) + 1)))*e^(-3*I*f*x - 3*I*e)/(a^2*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\tan(e+fx))^{\frac{5}{2}}}{(ia(\tan(e+fx)-i))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(5/2)/(a+I*a*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(5/2)/(I*a*(tan(e + f*x) - I))**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\tan(e+f\,x))^{5/2}}{(a+a\tan(e+f\,x)\;1\mathrm{i})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(5/2)/(a + a*tan(e + f*x)*1i)^(3/2),x)
[Out] int((c + d*tan(e + f*x))^(5/2)/(a + a*tan(e + f*x)*1i)^(3/2), x)
```

3.1154 $\int \frac{(c+d\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=225

$$-\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{4\sqrt{2}\,a^{5/2}f}+\frac{i(c-id)^2\sqrt{c+d\tan(e+fx)}}{4a^2f\sqrt{a+ia\tan(e+fx)}}+\frac{(ic+d)(c+d\tan}{6af(a+ia\tan(e}$$

```
[Out] -1/8*I*(c-I*d)^(5/2)*arctanh(2^(1/2)*a^(1/2)*(c+d*tan(f*x+e))^(1/2)/(c-I*d)
^(1/2)/(a+I*a*tan(f*x+e))^(1/2))/a^(5/2)/f*2^(1/2)+1/4*I*(c-I*d)^2*(c+d*tan
(f*x+e))^(1/2)/a^2/f/(a+I*a*tan(f*x+e))^(1/2)+1/6*(I*c+d)*(c+d*tan(f*x+e))^
(3/2)/a/f/(a+I*a*tan(f*x+e))^(3/2)+1/5*I*(c+d*tan(f*x+e))^(5/2)/f/(a+I*a*ta
n(f*x+e))^(5/2)
```

Rubi [A]
time = 0.31, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3627, 3625, 214}

$$-\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{4\sqrt{2}\,a^{5/2}f}+\frac{i(c-id)^2\sqrt{c+d\tan(e+fx)}}{4a^2f\sqrt{a+ia\tan(e+fx)}}+\frac{i(c+d\tan(e+fx))^{5/2}}{5f(a+ia\tan(e+fx))^{5/2}}+\frac{(d+ic)(c+d\tan(e+fx))^{3/2}}{6af(a+ia\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^(5/2)/(a + I*a*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-1/4*I)*(c - I*d)^(5/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[c + d*Tan[e + f*x]]
)/(Sqrt[c - I*d]*Sqrt[a + I*a*Tan[e + f*x]])])/(Sqrt[2]*a^(5/2)*f) + ((I/4)
*(c - I*d)^2*Sqrt[c + d*Tan[e + f*x]])/(a^2*f*Sqrt[a + I*a*Tan[e + f*x]]) +
 ((I*c + d)*(c + d*Tan[e + f*x])^(3/2))/(6*a*f*(a + I*a*Tan[e + f*x])^(3/2)
) + ((I/5)*(c + d*Tan[e + f*x])^(5/2))/(f*(a + I*a*Tan[e + f*x])^(5/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3627

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^n/(2*b*f*m)), x] - Dist[(a*c - b*d)/(2*b^2), Int[(a + b*Tan[e + f*
x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && EqQ
[m + n, 0] && LeQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx = \frac{i(c+d\tan(e+fx))^{5/2}}{5f(a+ia\tan(e+fx))^{5/2}} + \frac{(c-id)\int \frac{(c+d\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^{3/2}}\,dx}{2a}$$

$$= \frac{(ic+d)(c+d\tan(e+fx))^{3/2}}{6af(a+ia\tan(e+fx))^{3/2}} + \frac{i(c+d\tan(e+fx))^{5/2}}{5f(a+ia\tan(e+fx))^{5/2}} + \frac{(c-id)^2\int -}{}$$

$$= \frac{i(c-id)^2\sqrt{c+d\tan(e+fx)}}{4a^2f\sqrt{a+ia\tan(e+fx)}} + \frac{(ic+d)(c+d\tan(e+fx))^{3/2}}{6af(a+ia\tan(e+fx))^{3/2}} + \frac{i(c+}{5f(a+}$$

$$= \frac{i(c-id)^2\sqrt{c+d\tan(e+fx)}}{4a^2f\sqrt{a+ia\tan(e+fx)}} + \frac{(ic+d)(c+d\tan(e+fx))^{3/2}}{6af(a+ia\tan(e+fx))^{3/2}} + \frac{i(c+}{5f(a+}$$

$$= -\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{4\sqrt{2}a^{5/2}f} + \frac{i(c-id)^2\sqrt{c+}}{4a^2f\sqrt{a+ia}}$$

Mathematica [A]
time = 6.70, size = 292, normalized size = 1.30

$$\frac{\sec^{\frac{5}{2}}(e+fx)\left(-i\sqrt{2}(c-id)^{5/2}e^{2i(e+fx)}\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\sqrt{1+e^{2i(e+fx)}}\log\left(2\left(\sqrt{c-id}\,e^{i(e+fx)}+\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)\right)+\frac{2(11i(c^2+d^2)+(26ic^2+40cd-26id^2)\cos(2(e+fx))+(-20c^2+52icd+20d^2)\sin(2(e+fx)))\sqrt{c+d\tan(e+fx)}}{15\sqrt{\sec(e+fx)}}\right)}{8f(a+ia\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^(5/2)/(a + I*a*Tan[e + f*x])^(5/2),x]
```

```
[Out] (Sec[e + f*x]^(5/2)*((-I)*Sqrt[2]*(c - I*d)^(5/2)*E^((2*I)*(e + f*x))*Sqrt[
E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*Sqrt[1 + E^((2*I)*(e + f*x))]*Lo
g[2*(Sqrt[c - I*d]*E^(I*(e + f*x)) + Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c -
 (I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))])] + (2*((11*I)
*(c^2 + d^2) + ((26*I)*c^2 + 40*c*d - (26*I)*d^2)*Cos[2*(e + f*x)] + (-20*c
^2 + (52*I)*c*d + 20*d^2)*Sin[2*(e + f*x)])*Sqrt[c + d*Tan[e + f*x]])/(15*S
qrt[Sec[e + f*x]])))/(8*f*(a + I*a*Tan[e + f*x])^(5/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2558 vs. 2(178) = 356.
time = 0.53, size = 2559, normalized size = 11.37

method	result	size
derivativedivides	Expression too large to display	2559
default	Expression too large to display	2559

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/240/f*(c+d*tan(f*x+e))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)/a^3*(40*(a*(c+d*t
an(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c*d^3+308*c^4*(a*(c+d*tan(f*x+e))*(1+I*t
an(f*x+e)))^(1/2)*tan(f*x+e)-60*c^4*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(
1/2)*tan(f*x+e)^3-148*I*c^4*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)-60*
I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*d^4-148*(a*(c+d*tan(f*x+e))*(
1+I*tan(f*x+e)))^(1/2)*d^4*tan(f*x+e)^3+220*(a*(c+d*tan(f*x+e))*(1+I*tan(f*
x+e)))^(1/2)*d^4*tan(f*x+e)+136*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)
*c^3*d+136*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c*d^3*tan(f*x+e)+4
0*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c*d^3*tan(f*x+e)^2+624*(a*(c+
d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c^2*d^2*tan(f*x+e)+220*I*c^4*(a*(c+d*
tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*tan(f*x+e)^2+308*I*(a*(c+d*tan(f*x+e))*
(1+I*tan(f*x+e)))^(1/2)*d^4*tan(f*x+e)^2-112*I*(a*(c+d*tan(f*x+e))*(1+I*tan
(f*x+e)))^(1/2)*c^2*d^2-112*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c^2
*d^2*tan(f*x+e)^3+136*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c^3*d*tan
(f*x+e)^2+30*I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*
(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)
+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c^2*d^2*tan(f*x+e)^4-180*I*ln((3*a*c+I*a*ta
n(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(
f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)
*c^2*d^2*tan(f*x+e)^2-90*I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e
)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))
/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*d^4*tan(f*x+e)^2+30*I*ln((3*a*c
+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c
+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c)
)^(1/2)*c^2*d^2+120*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(
1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f
*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c^2*d^2*tan(f*x+e)^3-120*ln((3*a*c+I*a
*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*t
an(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1
/2)*c^2*d^2*tan(f*x+e)+15*I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+
e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)
)/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c^4*tan(f*x+e)^4+15*I*ln((3*a*
```

```
c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(
c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c
))^(1/2)*d^4*tan(f*x+e)^4+60*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x
+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2
))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c^4*tan(f*x+e)^3+60*ln((3*a*c
+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c
+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c)
)^(1/2)*d^4*tan(f*x+e)^3-60*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+
e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)
)/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c^4*tan(f*x+e)-60*ln((3*a*c+I*
a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*
tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(
1/2)*d^4*tan(f*x+e)+40*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c^3*d*
tan(f*x+e)^3+136*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c*d^3*tan(f*
x+e)^3+15*I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a
*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I)
)*2^(1/2)*(-a*(I*d-c))^(1/2)*c^4+15*I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*
d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+
e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*d^4+624*I*(a*(c+d*ta
n(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c^2*d^2*tan(f*x+e)^2+40*I*(a*(c+d*tan(f*x
+e))*(1+I*tan(f*x+e)))^(1/2)*c^3*d*tan(f*x+e)-90*I*ln((3*a*c+I*a*tan(f*x+e)
*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*
(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c^4*tan
(f*x+e)^2)/(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)/(I*c-d)^2/(-tan(f*x+
e)+I)^4
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

$$\text{Exception raised: RuntimeError}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 674 vs. 2(177) = 354.
time = 1.43, size = 674, normalized size = 3.00

$$-\frac{\left(15\sqrt{\frac{1}{2}}a^3f\sqrt{-\frac{c^5-5ic^4d-10c^3d^2+10ic^2d^3+5cd^4-id^5}{a^5f^2}}e^{(5ifx+5ie)}\log\left(-\frac{2i\sqrt{\frac{1}{2}}a^3f\sqrt{-\frac{c^5-5ic^4d-10c^3d^2+10ic^2d^3+5cd^4-id^5}{a^5f^2}}e^{(ifx+ie)}-\sqrt{2}\left(c^2-2icd-d^2+(c^2-2icd-d^2)e^{(2ifx+2ie)}\right)\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}}{c^2-2icd-d^2}\right)-15\sqrt{\frac{1}{2}}a^3f\sqrt{-\frac{c^5-5ic^4d-10c^3d^2+10ic^2d^3+5cd^4-id^5}{a^5f^2}}e^{(5ifx+5ie)}\log\left(-\frac{-2i\sqrt{\frac{1}{2}}a^3f\sqrt{-\frac{c^5-5ic^4d-10c^3d^2+10ic^2d^3+5cd^4-id^5}{a^5f^2}}e^{(ifx+ie)}-\sqrt{2}\left(c^2-2icd-d^2+(c^2-2icd-d^2)e^{(2ifx+2ie)}\right)\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}}{c^2-2icd-d^2}\right)-\sqrt{2}\left(3ic^2-6cd-3id^2-23(-ic^2-2cd+id^2)e^{(6ifx+6ie)}-2(-17ic^2-23cd+6id^2)e^{(4ifx+4ie)}-2(-7ic^2+3cd-4id^2)e^{(2ifx+2ie)}\right)\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}\right)e^{(-5ifx-5ie)}}{120a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] -1/120*(15*sqrt(1/2)*a^3*f*sqrt(-(c^5 - 5*I*c^4*d - 10*c^3*d^2 + 10*I*c^2*d
^3 + 5*c*d^4 - I*d^5)/(a^5*f^2))*e^(5*I*f*x + 5*I*e)*log(-(2*I*sqrt(1/2)*a^
3*f*sqrt(-(c^5 - 5*I*c^4*d - 10*c^3*d^2 + 10*I*c^2*d^3 + 5*c*d^4 - I*d^5)/(
a^5*f^2))*e^(I*f*x + I*e) - sqrt(2)*(c^2 - 2*I*c*d - d^2 + (c^2 - 2*I*c*d -
 d^2)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(
e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))/(c^2 - 2*I*c*d
 - d^2)) - 15*sqrt(1/2)*a^3*f*sqrt(-(c^5 - 5*I*c^4*d - 10*c^3*d^2 + 10*I*c^
2*d^3 + 5*c*d^4 - I*d^5)/(a^5*f^2))*e^(5*I*f*x + 5*I*e)*log(-(-2*I*sqrt(1/2
)*a^3*f*sqrt(-(c^5 - 5*I*c^4*d - 10*c^3*d^2 + 10*I*c^2*d^3 + 5*c*d^4 - I*d^
5)/(a^5*f^2))*e^(I*f*x + I*e) - sqrt(2)*(c^2 - 2*I*c*d - d^2 + (c^2 - 2*I*c
*d - d^2)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*
d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))/(c^2 - 2*I
*c*d - d^2)) - sqrt(2)*(3*I*c^2 - 6*c*d - 3*I*d^2 - 23*(-I*c^2 - 2*c*d + I*
d^2)*e^(6*I*f*x + 6*I*e) - 2*(-17*I*c^2 - 23*c*d + 6*I*d^2)*e^(4*I*f*x + 4*
I*e) - 2*(-7*I*c^2 + 3*c*d - 4*I*d^2)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*
e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*
x + 2*I*e) + 1)))*e^(-5*I*f*x - 5*I*e)/(a^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d\tan{(e + fx)})^{\frac{5}{2}}}{(ia(\tan{(e + fx)} - i))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(5/2)/(a+I*a*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(5/2)/(I*a*(tan(e + f*x) - I))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\tan(e+f\,x))^{5/2}}{(a+a\tan\left(e+f\,x\right)\,1\mathrm{i})^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(5/2)/(a + a*tan(e + f*x)*1i)^(5/2),x)
[Out] int((c + d*tan(e + f*x))^(5/2)/(a + a*tan(e + f*x)*1i)^(5/2), x)
```

3.1155 $\int \frac{(a+ia\tan(e+fx))^{5/2}}{\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=200

$$-\frac{\sqrt[4]{-1}\,a^{5/2}(c+5id)\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}\right)}{d^{3/2}f}-\frac{4i\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e}}{\sqrt{c-id}\,\sqrt{a+ia\tan(}}\right.}{\sqrt{c-id}\,f}$$

```
[Out] -(-1)^(1/4)*a^(5/2)*(c+5*I*d)*arctanh((-1)^(3/4)*d^(1/2)*(a+I*a*tan(f*x+e))
^(1/2)/a^(1/2)/(c+d*tan(f*x+e))^(1/2))/d^(3/2)/f-4*I*a^(5/2)*arctanh(2^(1/2
)*a^(1/2)*(c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2)/(a+I*a*tan(f*x+e))^(1/2))*2^
(1/2)/f/(c-I*d)^(1/2)-a^2*(a+I*a*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/d
/f
```

Rubi [A]
time = 0.44, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3637, 3682, 3625, 214, 3680, 65, 223, 212}

$$-\frac{\sqrt[4]{-1}\,a^{5/2}(c+5id)\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}\right)}{d^{3/2}f}-\frac{4i\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+fx)}}\right)}{f\sqrt{c-id}}-\frac{a^2\sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^(5/2)/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] -(((-1)^(1/4)*a^(5/2)*(c + (5*I)*d)*ArcTanh[((-1)^(3/4)*Sqrt[d]*Sqrt[a + I*
a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])])/(d^(3/2)*f)) - ((4*I)
*Sqrt[2]*a^(5/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])/(Sqrt[c
 - I*d]*Sqrt[a + I*a*Tan[e + f*x]])])/(Sqrt[c - I*d]*f) - (a^2*Sqrt[a + I*a
*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^{5/2}}{\sqrt{c+d\tan(e+fx)}}\,dx = -\frac{a^2\sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{df} + \frac{a\int \frac{\sqrt{a+ia\tan(e+fx)}}{\sqrt{c+d}}}{}$$

$$= -\frac{a^2\sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{df} + \left(4a^2\right)\int \frac{\sqrt{a+ia\tan(e+}}{\sqrt{c+d\tan(e+}}$$

$$= -\frac{a^2\sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{df} - \frac{\left(8ia^4\right)\text{Subst}\left(\int \frac{1}{ac-iad-2a^2}\right.}{}$$

$$= -\frac{4i\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+fx)}}\right)}{\sqrt{c-id}\,f} - \frac{a^2\sqrt{a+ia\tan(e-}}{}$$

$$= -\frac{4i\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+fx)}}\right)}{\sqrt{c-id}\,f} - \frac{a^2\sqrt{a+ia\tan(e-}}{}$$

$$= -\frac{\sqrt[4]{-1}\,a^{5/2}(c+5id)\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}\right)}{d^{3/2}f} - \frac{4i\sqrt{2}\,a^{5}}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 602 vs. 2(200) = 400.
time = 7.59, size = 602, normalized size = 3.01

$$\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\cos^2(e+fx)(a+ia\tan(e+fx))^{5/2}\left(\frac{\cos(e+fx)\left(\sqrt{c-id}\,(c+5id)\log\left(\frac{(2+2i)e^{\frac{ie}{2}}\left(-id+de^{i(e+fx)}+ic\left(i+e^{i(e+fx)}\right)-(1+i)\sqrt{d}\,\sqrt{1+e^{2i(e+fx)}}\,\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)}{\sqrt{d}\,(-ic+5d)\left(i+e^{i(e+fx)}\right)}\right)-\sqrt{c-id}\,(c+5id)\log\left(\frac{(2+2i)e^{\frac{ie}{2}}\left(c+id+ice^{i(e+fx)}+de^{i(e+fx)}+(1+i)\sqrt{d}\,\sqrt{1+e^{2i(e+fx)}}\,\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)}{\sqrt{d}\,(-ic+5d)\left(-i+e^{i(e+fx)}\right)}\right)+(8+8i)d^{3/2}\log\left(2\left(\sqrt{c-id}\,\cos(e+fx)+i\sqrt{c-id}\,\sin(e+fx)+\sqrt{1+\cos(2(e+fx))+i\sin(2(e+fx))}\,\sqrt{c+d\tan(e+fx)}\right)\right)\right)(-\cos(2e)+i\sin(2e))}{\sqrt{c-id}\,\sqrt{1+\cos(2(e+fx))+i\sin(2(e+fx))}}-(1-i)\sqrt{d}\cos(2e)\sqrt{c+d\tan(e+fx)}+(1+i)\sqrt{d}\sin(2e)\sqrt{c+d\tan(e+fx)}\right)}{d^{3/2}f(\cos(fx)+i\sin(fx))^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(5/2)/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((1/2 + I/2)*Cos[e + f*x]^2*(a + I*a*Tan[e + f*x])^(5/2)*((Cos[e + f*x]*(Sq
rt[c - I*d]*(c + (5*I)*d)*Log[((2 + 2*I)*E^((I/2)*e)*((-I)*d + d*E^(I*(e +
f*x)) + I*c*(I + E^(I*(e + f*x))) - (1 + I)*Sqrt[d]*Sqrt[1 + E^((2*I)*(e +
f*x))]*Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))]
))/(Sqrt[d]*((-I)*c + 5*d)*(I + E^(I*(e + f*x))))] - Sqrt[c - I*d]*(c + (5*
```

```
I)*d)*Log[((2 + 2*I)*E^((I/2)*e)*(c + I*d + I*c*E^(I*(e + f*x)) + d*E^(I*(e
 + f*x)) + (1 + I)*Sqrt[d]*Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1
+ E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))]))/(Sqrt[d]*((-I)*c + 5*d
)*(-I + E^(I*(e + f*x))))] + (8 + 8*I)*d^(3/2)*Log[2*(Sqrt[c - I*d]*Cos[e +
 f*x] + I*Sqrt[c - I*d]*Sin[e + f*x] + Sqrt[1 + Cos[2*(e + f*x)] + I*Sin[2*
(e + f*x)]]*Sqrt[c + d*Tan[e + f*x]])]*(-Cos[2*e] + I*Sin[2*e]))/(Sqrt[c -
 I*d]*Sqrt[1 + Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)]]) - (1 - I)*Sqrt[d]*Co
s[2*e]*Sqrt[c + d*Tan[e + f*x]] + (1 + I)*Sqrt[d]*Sin[2*e]*Sqrt[c + d*Tan[e
 + f*x]]))/(d^(3/2)*f*(Cos[f*x] + I*Sin[f*x])^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1294 vs. $2(157) = 314$.
time = 0.66, size = 1295, normalized size = 6.48

method	result	size
derivativedivides	Expression too large to display	1295
default	Expression too large to display	1295

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/4/f*(a*(1+I*tan(f*x+e)))^(1/2)*(c+d*tan(f*x+e))^(1/2)*a^2*(-4*I*d^2*ln(1/
2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(
I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*2^(1/2)*(-a*(I*d-c))^(1/2)+4*I*ln((3*a*c
+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c
+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*(I*a*d)^(1/2)*a*c*d
+I*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))
^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*2^(1/2)*(-a*(I*d-c))^(1/2)*a*c*d^2
+I*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))
^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*2^(1/2)*(-a*(I*d-c))^(1/2)*a*c^3+4
*I*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))
^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*2^(1/2)*(-a*(I*d-c))^(1/2)*a*c*d+4
*I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))
^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*(I*a*d)
^(1/2)*a*d^2+3*2^(1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+
2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1
/2))*a*c^2*d+3*2^(1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+
2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1
/2))*a*d^3-2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)
))^(1/2)*(I*a*d)^(1/2)*d^2+4*2^(1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan
(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a
*d)/(I*a*d)^(1/2))*a*c*d+4*2^(1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f
*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d
)/(I*a*d)^(1/2))*a*d^2-4*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a
```

```
*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x
+e)))^(1/2))/(tan(f*x+e)+I))*a*c*d+4*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)
*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*
(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*d^2-2*2^(1/2)*(-a*(I*d-c))^(1/2)
*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)*c^2)/(a*(c+d*tan
(f*x+e))*(1+I*tan(f*x+e)))^(1/2)/(c^2+d^2)/d/(I*a*d)^(1/2)*2^(1/2)/(-a*(I*d
-c))^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((d^2-2*c*d-c^2)>0)', see `assume?`
for mor
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 809 vs. 2(156) = 312.
time = 1.55, size = 809, normalized size = 4.04

$$-\frac{2\sqrt{2}a^2\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}e^{(ifx+ie)}+df\sqrt{\frac{ia^5c^2-10a^5cd-25ia^5d^2}{d^3f^2}}\log\left(\frac{\left(2d^2f\sqrt{\frac{ia^5c^2-10a^5cd-25ia^5d^2}{d^3f^2}}e^{(ifx+ie)}+\sqrt{2}(-ia^2c+5a^2d+(-ia^2c+5a^2d)e^{(2ifx+2ie)})\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}\right)e^{(-ifx-ie)}}{-ia^2c+5a^2d}\right)-df\sqrt{\frac{ia^5c^2-10a^5cd-25ia^5d^2}{d^3f^2}}\log\left(-\frac{\left(2d^2f\sqrt{\frac{ia^5c^2-10a^5cd-25ia^5d^2}{d^3f^2}}e^{(ifx+ie)}-\sqrt{2}(-ia^2c+5a^2d+(-ia^2c+5a^2d)e^{(2ifx+2ie)})\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}\right)e^{(-ifx-ie)}}{-ia^2c+5a^2d}\right)-\sqrt{-\frac{32ia^5}{(ic+d)f^2}}df\log\left(\frac{\left(\sqrt{-\frac{32ia^5}{(ic+d)f^2}}(ic+d)fe^{(ifx+ie)}+4\sqrt{2}(a^2e^{(2ifx+2ie)}+a^2)\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}\right)e^{(-ifx-ie)}}{4a^2}\right)+\sqrt{-\frac{32ia^5}{(ic+d)f^2}}df\log\left(\frac{\left(\sqrt{-\frac{32ia^5}{(ic+d)f^2}}(-ic-d)fe^{(ifx+ie)}+4\sqrt{2}(a^2e^{(2ifx+2ie)}+a^2)\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}\right)e^{(-ifx-ie)}}{4a^2}\right)}{2df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] -1/2*(2*sqrt(2)*a^2*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*
f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*e^(I*f*x + I*e) + d*f*
sqrt((I*a^5*c^2 - 10*a^5*c*d - 25*I*a^5*d^2)/(d^3*f^2))*log((2*d^2*f*sqrt((
I*a^5*c^2 - 10*a^5*c*d - 25*I*a^5*d^2)/(d^3*f^2))*e^(I*f*x + I*e) + sqrt(2)
*(-I*a^2*c + 5*a^2*d + (-I*a^2*c + 5*a^2*d)*e^(2*I*f*x + 2*I*e))*sqrt(((c -
 I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(
2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/(-I*a^2*c + 5*a^2*d)) - d*f*sqrt((
I*a^5*c^2 - 10*a^5*c*d - 25*I*a^5*d^2)/(d^3*f^2))*log(-(2*d^2*f*sqrt((I*a^5
*c^2 - 10*a^5*c*d - 25*I*a^5*d^2)/(d^3*f^2))*e^(I*f*x + I*e) - sqrt(2)*(-I*
a^2*c + 5*a^2*d + (-I*a^2*c + 5*a^2*d)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)
*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f
*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/(-I*a^2*c + 5*a^2*d)) - sqrt(-32*I*a^5/
((I*c + d)*f^2))*d*f*log(1/4*(sqrt(-32*I*a^5/((I*c + d)*f^2))*(I*c + d)*f*e
^(I*f*x + I*e) + 4*sqrt(2)*(a^2*e^(2*I*f*x + 2*I*e) + a^2)*sqrt(((c - I*d)*
```

```
e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*
x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/a^2) + sqrt(-32*I*a^5/((I*c + d)*f^2))*d
*f*log(1/4*(sqrt(-32*I*a^5/((I*c + d)*f^2))*(-I*c - d)*f*e^(I*f*x + I*e) +
4*sqrt(2)*(a^2*e^(2*I*f*x + 2*I*e) + a^2)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*
e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))
*e^(-I*f*x - I*e)/a^2))/(d*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan{(e+fx)}-i))^{\frac{5}{2}}}{\sqrt{c+d\tan{(e+fx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(5/2)/(c+d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral((I*a*(tan(e + f*x) - I))**(5/2)/sqrt(c + d*tan(e + f*x)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 0.47sym2poly/r2sym(c
onst gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+a\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{5/2}}{\sqrt{c+d\,\mathrm{tan}\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(5/2)/(c + d*tan(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^(5/2)/(c + d*tan(e + f*x))^(1/2), x)
```

3.1156 $\int \frac{(a+ia\tan(e+fx))^{3/2}}{\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=151

$$-\frac{2(-1)^{3/4}a^{3/2}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{d}\,f}-\frac{2i\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{\sqrt{c-id}\,f}$$

```
[Out] -2*I*a^(3/2)*arctanh(2^(1/2)*a^(1/2)*(c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2)/(
a+I*a*tan(f*x+e))^(1/2))*2^(1/2)/f/(c-I*d)^(1/2)-2*(-1)^(3/4)*a^(3/2)*arcta
nh((-1)^(3/4)*d^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c+d*tan(f*x+e))^(1/
2))/f/d^(1/2)
```

Rubi [A]

time = 0.28, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {3636, 3625, 214, 3680, 65, 223, 212}

$$-\frac{2(-1)^{3/4}a^{3/2}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{d}\,f}-\frac{2i\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{f\sqrt{c-id}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^(3/2)/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] (-2*(-1)^(3/4)*a^(3/2)*ArcTanh[((-1)^(3/4)*Sqrt[d]*Sqrt[a + I*a*Tan[e + f*x
]])/(Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[d]*f) - ((2*I)*Sqrt[2]*a^(3/
2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])/(Sqrt[c - I*d]*Sqrt[a
 + I*a*Tan[e + f*x]])])/(Sqrt[c - I*d]*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3636

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(c_.) + (d_.)*tan[(e_
.) + (f_.)*(x_)]], x_Symbol] :> Dist[2*a, Int[Sqrt[a + b*Tan[e + f*x]]/Sqrt
[c + d*Tan[e + f*x]], x], x] + Dist[b/a, Int[(b + a*Tan[e + f*x])*(Sqrt[a +
 b*Tan[e + f*x]]/Sqrt[c + d*Tan[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^{3/2}}{\sqrt{c+d\tan(e+fx)}}\,dx = i\int \frac{\sqrt{a+ia\tan(e+fx)}\,(ia+a\tan(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx + (2a)\int \frac{\sqrt{a+ia\tan(e-}}{\sqrt{c+d\tan(e+}}$$

$$= -\frac{a^2\text{Subst}\left(\int \frac{1}{\sqrt{a+iax}\,\sqrt{c+dx}}\,dx, x, \tan(e+fx)\right)}{f} - \frac{(4ia^3)\,\text{Subst}\left(\int \frac{}{ac-}\right.}{}$$

$$= -\frac{2i\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{\sqrt{c-id}\,f} + \frac{(2ia)\text{Subst}\left(\int \frac{}{\sqrt{c}}\right.}{}$$

$$= -\frac{2i\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{\sqrt{c-id}\,f} + \frac{(2ia)\text{Subst}\left(\int \frac{1}{1+\frac{i}{}}\right.}{}$$

$$= -\frac{2(-1)^{3/4}a^{3/2}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{d}\,f} - \frac{2i\sqrt{2}\,a^{3/2}\tanh}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 505 vs. $2(151) = 302$.

time = 5.73, size = 505, normalized size = 3.34

$$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\cos(e+fx)\left(\sqrt{c-id}\log\left(\frac{(2-2i)e^{\frac{3ie}{2}}\left(-id+de^{i(e+fx)}+ic\left(i+e^{i(e+fx)}\right)-(1+i)\sqrt{d}\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)}{\sqrt{d}\left(i+e^{i(e+fx)}\right)}\right)-\sqrt{c-id}\log\left(\frac{(2+2i)e^{\frac{3ie}{2}}\left(d-ide^{i(e+fx)}+c\left(-i+e^{i(e+fx)}\right)+(1-i)\sqrt{d}\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)}{\sqrt{d}\left(-i+e^{i(e+fx)}\right)}\right)+(2-2i)\sqrt{d}\log\left(2\left(\sqrt{c-id}\cos(e+fx)+i\sqrt{c-id}\sin(e+fx)+\sqrt{1+\cos(2(e+fx))+i\sin(2(e+fx))}\sqrt{c+d\tan(e+fx)}\right)\right)\right)(\cos(fx)-i\sin(fx))\sqrt{1+\cos(2(e+fx))+i\sin(2(e+fx))}(\cos(2e+fx)-i\sin(2e+fx))(a+ia\tan(e+fx))^{3/2}}{\sqrt{c-id}\sqrt{d}\,f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(3/2)/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((1/2 - I/2)*Cos[e + f*x]*(Sqrt[c - I*d]*Log[((2 - 2*I)*E^(((3*I)/2)*e)*((-
I)*d + d*E^(I*(e + f*x)) + I*c*(I + E^(I*(e + f*x))) - (1 + I)*Sqrt[d]*Sqrt
[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E^
((2*I)*(e + f*x)))]))/(Sqrt[d]*(I + E^(I*(e + f*x))))] - Sqrt[c - I*d]*Log[
((2 + 2*I)*E^(((3*I)/2)*e)*(d - I*d*E^(I*(e + f*x)) + c*(-I + E^(I*(e + f*x
))) + (1 - I)*Sqrt[d]*Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^(
(2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))]))/(Sqrt[d]*(-I + E^(I*(e + f*
x))))] + (2 - 2*I)*Sqrt[d]*Log[2*(Sqrt[c - I*d]*Cos[e + f*x] + I*Sqrt[c - I
*d]*Sin[e + f*x] + Sqrt[1 + Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)]]*Sqrt[c +
 d*Tan[e + f*x]])])*(Cos[f*x] - I*Sin[f*x])*Sqrt[1 + Cos[2*(e + f*x)] + I*S
in[2*(e + f*x)]]*(Cos[2*e + f*x] - I*Sin[2*e + f*x])*(a + I*a*Tan[e + f*x])
^(3/2))/(Sqrt[c - I*d]*Sqrt[d]*f)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 982 vs. $2(114) = 228$.
time = 0.65, size = 983, normalized size = 6.51 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/2/f*(a*(1+I*tan(f*x+e)))^(1/2)*(c+d*tan(f*x+e))^(1/2)*a^2*(I*(I*a*d)^(1/2
)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^
(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c+I*(I*a
*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(
I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*
d+I*2^(1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*t
an(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*c-I*2^
(1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x
+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*d+2^(1/2)*(-
a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+
I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*c^2+2^(1/2)*(-a*(I*d
-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(
f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*d^2-(I*a*d)^(1/2)*ln((3*a*
c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(
c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c+(I*a*d)^(1/2)*ln
((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2
)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*d+2^(1/2)*(-
a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+
I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*c+2^(1/2)*(-a*(I*d-c
))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*
x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*d)/(a*(c+d*tan(f*x+e))*(1+I*
tan(f*x+e)))^(1/2)/(c^2+d^2)/(I*a*d)^(1/2)*2^(1/2)/(-a*(I*d-c))^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((d^2-2*c*d-c^2)>0)', see `assume?`
for mor
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 557 vs. $2(113) = 226$.

time = 1.04, size = 557, normalized size = 3.69

$$\frac{1}{2}\sqrt{-\frac{8i\,a^3}{(i\,c+d)f^2}}\log\left(\frac{\left((i\,c+d)f\sqrt{-\frac{8i\,a^3}{(i\,c+d)f^2}}e^{(i\,fx+i\,e)}+2\sqrt{2}\left(ae^{(2i\,fx+2i\,e)}+a\right)\sqrt{\frac{(c-i\,d)e^{(2i\,fx+2i\,e)}+c+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-i\,fx-i\,e)}}{2\,a}\right)-\frac{1}{2}\sqrt{-\frac{8i\,a^3}{(i\,c+d)f^2}}\log\left(\frac{\left((-i\,c-d)f\sqrt{-\frac{8i\,a^3}{(i\,c+d)f^2}}e^{(i\,fx+i\,e)}+2\sqrt{2}\left(ae^{(2i\,fx+2i\,e)}+a\right)\sqrt{\frac{(c-i\,d)e^{(2i\,fx+2i\,e)}+c+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-i\,fx-i\,e)}}{2\,a}\right)-\frac{1}{2}\sqrt{-\frac{4i\,a^3}{df^2}}\log\left(\frac{\left(df\sqrt{-\frac{4i\,a^3}{df^2}}e^{(i\,fx+i\,e)}+\sqrt{2}\left(ae^{(2i\,fx+2i\,e)}+a\right)\sqrt{\frac{(c-i\,d)e^{(2i\,fx+2i\,e)}+c+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-i\,fx-i\,e)}}{a}\right)+\frac{1}{2}\sqrt{-\frac{4i\,a^3}{df^2}}\log\left(-\frac{\left(df\sqrt{-\frac{4i\,a^3}{df^2}}e^{(i\,fx+i\,e)}-\sqrt{2}\left(ae^{(2i\,fx+2i\,e)}+a\right)\sqrt{\frac{(c-i\,d)e^{(2i\,fx+2i\,e)}+c+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-i\,fx-i\,e)}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/2*sqrt(-8*I*a^3/((I*c + d)*f^2))*log(1/2*((I*c + d)*f*sqrt(-8*I*a^3/((I*c
 + d)*f^2))*e^(I*f*x + I*e) + 2*sqrt(2)*(a*e^(2*I*f*x + 2*I*e) + a)*sqrt(((
c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(
e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/a) - 1/2*sqrt(-8*I*a^3/((I*c +
d)*f^2))*log(1/2*((-I*c - d)*f*sqrt(-8*I*a^3/((I*c + d)*f^2))*e^(I*f*x + I*
e) + 2*sqrt(2)*(a*e^(2*I*f*x + 2*I*e) + a)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I
*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))
)*e^(-I*f*x - I*e)/a) - 1/2*sqrt(-4*I*a^3/(d*f^2))*log((d*f*sqrt(-4*I*a^3/(
d*f^2))*e^(I*f*x + I*e) + sqrt(2)*(a*e^(2*I*f*x + 2*I*e) + a)*sqrt(((c - I*
d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I
*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/a) + 1/2*sqrt(-4*I*a^3/(d*f^2))*log(-
(d*f*sqrt(-4*I*a^3/(d*f^2))*e^(I*f*x + I*e) - sqrt(2)*(a*e^(2*I*f*x + 2*I*e
) + a)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e)
+ 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(e+fx)-i))^{\frac{3}{2}}}{\sqrt{c+d\tan(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(3/2)/(c+d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral((I*a*(tan(e + f*x) - I))**(3/2)/sqrt(c + d*tan(e + f*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error: Ba
d Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\tan(e+f\,x)\;1\mathrm{i})^{3/2}}{\sqrt{c+d\tan\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(3/2)/(c + d*tan(e + f*x))^(1/2),x)
[Out] int((a + a*tan(e + f*x)*1i)^(3/2)/(c + d*tan(e + f*x))^(1/2), x)
```

3.1157 $\int \frac{\sqrt{a+ia\tan(e+fx)}}{\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=82

$$-\frac{i\sqrt{2}\,\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+fx)}}\right)}{\sqrt{c-id}\,f}$$

```
[Out] -I*arctanh(2^(1/2)*a^(1/2)*(c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2)/(a+I*a*tan(
f*x+e))^(1/2))*2^(1/2)*a^(1/2)/f/(c-I*d)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {3625, 214}

$$-\frac{i\sqrt{2}\,\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+fx)}}\right)}{f\sqrt{c-id}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[e + f*x]]/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((-I)*Sqrt[2]*Sqrt[a]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])/(S
qrt[c - I*d]*Sqrt[a + I*a*Tan[e + f*x]])])/(Sqrt[c - I*d]*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+ia\tan(e+fx)}}{\sqrt{c+d\tan(e+fx)}}\,dx = -\frac{(2ia^2)\,\text{Subst}\left(\int \frac{1}{ac-iad-2a^2x^2}\,dx, x, \frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{a+ia\tan(e+fx)}}\right)}{f}$$
$$= -\frac{i\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{\sqrt{c-id}\,f}$$

Mathematica [A]
time = 2.92, size = 147, normalized size = 1.79

$$-\frac{ie^{-i(e+fx)}\sqrt{1+e^{2i(e+fx)}}\log\left(2\left(\sqrt{c-id}\,e^{i(e+fx)}+\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)\right)\sqrt{a+ia\tan(e+fx)}}{\sqrt{c-id}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Tan[e + f*x]]/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((-I)*Sqrt[1 + E^((2*I)*(e + f*x))]*Log[2*(Sqrt[c - I*d]*E^(I*(e + f*x)) +
Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(1
+ E^((2*I)*(e + f*x)))])]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[c - I*d]*E^(I*(
e + f*x))*f)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(62) = 124.
time = 0.85, size = 212, normalized size = 2.59

method	result
derivativedivides	$\frac{(-id\tan(fx+e)+ic-c\tan(fx+e)-d)a\ln\left(\frac{3ac+ia\tan(fx+e)c-iad+3ad\tan(fx+e)+2\sqrt{2}\sqrt{-a(id-c)}\sqrt{a(c-}}{\tan(fx+e)+i}\right.}{2f(-\tan(fx+e)+i)(ic-d)\sqrt{-a(id-c)}\sqrt{a}}$
default	$\frac{(-id\tan(fx+e)+ic-c\tan(fx+e)-d)a\ln\left(\frac{3ac+ia\tan(fx+e)c-iad+3ad\tan(fx+e)+2\sqrt{2}\sqrt{-a(id-c)}\sqrt{a(c-}}{\tan(fx+e)+i}\right.}{2f(-\tan(fx+e)+i)(ic-d)\sqrt{-a(id-c)}\sqrt{a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/2/f*(-I*d*tan(f*x+e)+I*c-c*tan(f*x+e)-d)*a*ln((3*a*c+I*a*tan(f*x+e)*c-I*a
*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*t
```

an(f*x+e)))^(1/2))/(tan(f*x+e)+I))*(c+d*tan(f*x+e))^(1/2)*(a*(1+I*tan(f*x+e
)))^(1/2)/(-tan(f*x+e)+I)/(I*c-d)*2^(1/2)/(-a*(I*d-c))^(1/2)/(a*(c+d*tan(f*
x+e))*(1+I*tan(f*x+e)))^(1/2)

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3834 vs. 2(60) = 120.
time = 0.71, size = 3834, normalized size = 46.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] -1/4*(sqrt(2*c^2 + 2*d^2)*(2*sqrt(2)*arctan2((sqrt(c^2 + d^2)*d*cos(f*x + e
) - sqrt(c^2 + d^2)*c*sin(f*x + e) + (c^2 + d^2)*(((c^2 + d^2)*cos(f*x + e)
^4 + (c^2 + d^2)*sin(f*x + e)^4 + 8*c*d*cos(f*x + e)*sin(f*x + e) + 2*(c^2
- d^2)*cos(f*x + e)^2 + 2*((c^2 + d^2)*cos(f*x + e)^2 - c^2 + d^2)*sin(f*x
+ e)^2 + c^2 + d^2)/(c^2 + d^2))^(1/4)*sin(1/2*arctan2(-2*(c*d*cos(f*x + e)
^2 - c*d*sin(f*x + e)^2 - (c^2 - d^2)*cos(f*x + e)*sin(f*x + e))/(c^2 + d^2
), (4*c*d*cos(f*x + e)*sin(f*x + e) + (c^2 - d^2)*cos(f*x + e)^2 - (c^2 - d
^2)*sin(f*x + e)^2 + c^2 + d^2)/(c^2 + d^2))))/(c^2 + d^2), -(sqrt(c^2 + d^
2)*c*cos(f*x + e) + sqrt(c^2 + d^2)*d*sin(f*x + e) - (c^2 + d^2)*(((c^2 + d
^2)*cos(f*x + e)^4 + (c^2 + d^2)*sin(f*x + e)^4 + 8*c*d*cos(f*x + e)*sin(f*
x + e) + 2*(c^2 - d^2)*cos(f*x + e)^2 + 2*((c^2 + d^2)*cos(f*x + e)^2 - c^2
 + d^2)*sin(f*x + e)^2 + c^2 + d^2)/(c^2 + d^2))^(1/4)*cos(1/2*arctan2(-2*(
c*d*cos(f*x + e)^2 - c*d*sin(f*x + e)^2 - (c^2 - d^2)*cos(f*x + e)*sin(f*x
+ e))/(c^2 + d^2), (4*c*d*cos(f*x + e)*sin(f*x + e) + (c^2 - d^2)*cos(f*x +
 e)^2 - (c^2 - d^2)*sin(f*x + e)^2 + c^2 + d^2)/(c^2 + d^2))))/(c^2 + d^2))
 - I*sqrt(2)*log(((c^2 + d^2)*sqrt(((c^2 + d^2)*cos(f*x + e)^4 + (c^2 + d^2
)*sin(f*x + e)^4 + 8*c*d*cos(f*x + e)*sin(f*x + e) + 2*(c^2 - d^2)*cos(f*x
+ e)^2 + 2*((c^2 + d^2)*cos(f*x + e)^2 - c^2 + d^2)*sin(f*x + e)^2 + c^2 +
d^2)/(c^2 + d^2))*cos(1/2*arctan2(-2*(c*d*cos(f*x + e)^2 - c*d*sin(f*x + e)
^2 - (c^2 - d^2)*cos(f*x + e)*sin(f*x + e))/(c^2 + d^2), (4*c*d*cos(f*x + e
)*sin(f*x + e) + (c^2 - d^2)*cos(f*x + e)^2 - (c^2 - d^2)*sin(f*x + e)^2 +
c^2 + d^2)/(c^2 + d^2)))^2 + (c^2 + d^2)*sqrt(((c^2 + d^2)*cos(f*x + e)^4 +
 (c^2 + d^2)*sin(f*x + e)^4 + 8*c*d*cos(f*x + e)*sin(f*x + e) + 2*(c^2 - d^
2)*cos(f*x + e)^2 + 2*((c^2 + d^2)*cos(f*x + e)^2 - c^2 + d^2)*sin(f*x + e)
^2 + c^2 + d^2)/(c^2 + d^2))*sin(1/2*arctan2(-2*(c*d*cos(f*x + e)^2 - c*d*s
in(f*x + e)^2 - (c^2 - d^2)*cos(f*x + e)*sin(f*x + e))/(c^2 + d^2), (4*c*d*
cos(f*x + e)*sin(f*x + e) + (c^2 - d^2)*cos(f*x + e)^2 - (c^2 - d^2)*sin(f*
x + e)^2 + c^2 + d^2)/(c^2 + d^2)))^2 + (c^2 + d^2)*cos(f*x + e)^2 + (c^2 +
 d^2)*sin(f*x + e)^2 - 2*(sqrt(c^2 + d^2)*c*cos(f*x + e) + sqrt(c^2 + d^2)*
d*sin(f*x + e))*(((c^2 + d^2)*cos(f*x + e)^4 + (c^2 + d^2)*sin(f*x + e)^4 +
 8*c*d*cos(f*x + e)*sin(f*x + e) + 2*(c^2 - d^2)*cos(f*x + e)^2 + 2*((c^2 +
```

```
d^2)*cos(f*x + e)^2 - c^2 + d^2)*sin(f*x + e)^2 + c^2 + d^2)/(c^2 + d^2))^
(1/4)*cos(1/2*arctan2(-2*(c*d*cos(f*x + e)^2 - c*d*sin(f*x + e)^2 - (c^2 -
d^2)*cos(f*x + e)*sin(f*x + e))/(c^2 + d^2), (4*c*d*cos(f*x + e)*sin(f*x +
e) + (c^2 - d^2)*cos(f*x + e)^2 - (c^2 - d^2)*sin(f*x + e)^2 + c^2 + d^2)/(
c^2 + d^2))) + 2*(sqrt(c^2 + d^2)*d*cos(f*x + e) - sqrt(c^2 + d^2)*c*sin(f*
x + e))*(((c^2 + d^2)*cos(f*x + e)^4 + (c^2 + d^2)*sin(f*x + e)^4 + 8*c*d*c
os(f*x + e)*sin(f*x + e) + 2*(c^2 - d^2)*cos(f*x + e)^2 + 2*((c^2 + d^2)*co
s(f*x + e)^2 - c^2 + d^2)*sin(f*x + e)^2 + c^2 + d^2)/(c^2 + d^2))^(1/4)*si
n(1/2*arctan2(-2*(c*d*cos(f*x + e)^2 - c*d*sin(f*x + e)^2 - (c^2 - d^2)*cos
(f*x + e)*sin(f*x + e))/(c^2 + d^2), (4*c*d*cos(f*x + e)*sin(f*x + e) + (c^
2 - d^2)*cos(f*x + e)^2 - (c^2 - d^2)*sin(f*x + e)^2 + c^2 + d^2)/(c^2 + d^
2))))/(c^2 + d^2)))*sqrt(a)*sqrt(c + sqrt(c^2 + d^2)) - sqrt(2*c^2 + 2*d^2)
*(-2*I*sqrt(2)*arctan2((sqrt(c^2 + d^2)*d*cos(f*x + e) - sqrt(c^2 + d^2)*c*
sin(f*x + e) + (c^2 + d^2)*(((c^2 + d^2)*cos(f*x + e)^4 + (c^2 + d^2)*sin(f
*x + e)^4 + 8*c*d*cos(f*x + e)*sin(f*x + e) + 2*(c^2 - d^2)*cos(f*x + e)^2
+ 2*((c^2 + d^2)*cos(f*x + e)^2 - c^2 + d^2)*sin(f*x + e)^2 + c^2 + d^2)/(c
^2 + d^2))^(1/4)*sin(1/2*arctan2(-2*(c*d*cos(f*x + e)^2 - c*d*sin(f*x + e)^
2 - (c^2 - d^2)*cos(f*x + e)*sin(f*x + e))/(c^2 + d^2), (4*c*d*cos(f*x + e)
*sin(f*x + e) + (c^2 - d^2)*cos(f*x + e)^2 - (c^2 - d^2)*sin(f*x + e)^2 + c
^2 + d^2)/(c^2 + d^2))))/(c^2 + d^2), -(sqrt(c^2 + d^2)*c*cos(f*x + e) + sq
rt(c^2 + d^2)*d*sin(f*x + e) - (c^2 + d^2)*(((c^2 + d^2)*cos(f*x + e)^4 + (
c^2 + d^2)*sin(f*x + e)^4 + 8*c*d*cos(f*x + e)*sin(f*x + e) + 2*(c^2 - d^2)
*cos(f*x + e)^2 + 2*((c^2 + d^2)*cos(f*x + e)^2 - c^2 + d^2)*sin(f*x + e)^2
 + c^2 + d^2)/(c^2 + d^2))^(1/4)*cos(1/2*arctan2(-2*(c*d*cos(f*x + e)^2 - c
*d*sin(f*x + e)^2 - (c^2 - d^2)*cos(f*x + e)*sin(f*x + e))/(c^2 + d^2), (4*
c*d*cos(f*x + e)*sin(f*x + e) + (c^2 - d^2)*cos(f*x + e)^2 - (c^2 - d^2)*si
n(f*x + e)^2 + c^2 + d^2)/(c^2 + d^2))))/(c^2 + d^2)) - sqrt(2)*log(((c^2 +
 d^2)*sqrt(((c^2 + d^2)*cos(f*x + e)^4 + (c^2 + d^2)*sin(f*x + e)^4 + 8*c*d
*cos(f*x + e)*sin(f*x + e) + 2*(c^2 - d^2)*cos(f*x + e)^2 + 2*((c^2 + d^2)*
cos(f*x + e)^2 - c^2 + d^2)*sin(f*x + e)^2 + c^2 + d^2)/(c^2 + d^2))*cos(1/
2*arctan2(-2*(c*d*cos(f*x + e)^2 - c*d*sin(f*x + e)^2 - (c^2 - d^2)*cos(f*x
 + e)*sin(f*x + e))/(c^2 + d^2), (4*c*d*cos(f*x + e)*sin(f*x + e) + (c^2 -
d^2)*cos(f*x + e)^2 - (c^2 - d^2)*sin(f*x + e)^2 + c^2 + d^2)/(c^2 + d^2)))
^2 + (c^2 + d^2)*sqrt(((c^2 + d^2)*cos(f*x + e)^4 + (c^2 + d^2)*sin(f*x + e
)^4 + 8*c*d*cos(f*x + e)*sin(f*x + e) + 2*(c^2 - d^2)*cos(f*x + e)^2 + 2*((
c^2 + d^2)*cos(f*x + e)^2 - c^2 + d^2)*sin(f*x ...
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(60) = 120$.
time = 1.23, size = 271, normalized size = 3.30

$$\frac{1}{2} \sqrt{-\frac{2 i a}{(i c+d) f^{2}}} \log \left(\left((i c+d) f \sqrt{-\frac{2 i a}{(i c+d) f^{2}}} e^{(i f x+i e)}+\sqrt{2} \sqrt{\frac{(c-i d) e^{(2 i f x+2 i e)}+c+i d}{e^{(2 i f x+2 i e)}+1}} \sqrt{\frac{a}{e^{(2 i f x+2 i e)}+1}}\left(e^{(2 i f x+2 i e)}+1\right)\right) e^{(-i f x-i e)}\right)-\frac{1}{2} \sqrt{-\frac{2 i a}{(i c+d) f^{2}}} \log \left(\left((-i c-d) f \sqrt{-\frac{2 i a}{(i c+d) f^{2}}} e^{(i f x+i e)}+\sqrt{2} \sqrt{\frac{(c-i d) e^{(2 i f x+2 i e)}+c+i d}{e^{(2 i f x+2 i e)}+1}} \sqrt{\frac{a}{e^{(2 i f x+2 i e)}+1}}\left(e^{(2 i f x+2 i e)}+1\right)\right) e^{(-i f x-i e)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(1/2),x, algorithm="fri
cas")
```

[Out] 1/2*sqrt(-2*I*a/((I*c + d)*f^2))*log(((I*c + d)*f*sqrt(-2*I*a/((I*c + d)*f^
2))*e^(I*f*x + I*e) + sqrt(2)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d
)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*(e^(2*I*f*x
+ 2*I*e) + 1))*e^(-I*f*x - I*e)) - 1/2*sqrt(-2*I*a/((I*c + d)*f^2))*log(((-
I*c - d)*f*sqrt(-2*I*a/((I*c + d)*f^2))*e^(I*f*x + I*e) + sqrt(2)*sqrt(((c
- I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^
(2*I*f*x + 2*I*e) + 1))*(e^(2*I*f*x + 2*I*e) + 1))*e^(-I*f*x - I*e))

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ia\left(\tan\left(e+fx\right)-i\right)}}{\sqrt{c+d\tan\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I*a*tan(f*x+e))**(1/2)/(c+d*tan(f*x+e))**(1/2),x)

[Out] Integral(sqrt(I*a*(tan(e + f*x) - I))/sqrt(c + d*tan(e + f*x)), x)

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I*a*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(1/2),x, algorithm="gia
c")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done

Mupad [B]
time = 10.05, size = 473, normalized size = 5.77

$$\frac{\sqrt{2}\,\sqrt{a}\,\mathrm{atan}\left(\frac{2\,\sqrt{-c+d\,1{}\mathrm{i}}\,\left(\sqrt{2}\,c^{3/2}\,\sqrt{a+a\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}}\,1{}\mathrm{i}-\sqrt{2}\,\sqrt{c}\,d\,\sqrt{a+a\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}}-\sqrt{2}\,\sqrt{a}\,c\,\sqrt{c+d\,\tan\left(e+f\,x\right)}\,1{}\mathrm{i}+\sqrt{2}\,\sqrt{a}\,\sqrt{c}\,\left(c+d\,\tan\left(e+f\,x\right)\right)\,1{}\mathrm{i}+\sqrt{2}\,\sqrt{a}\,d\,\sqrt{c+d\,\tan\left(e+f\,x\right)}+\sqrt{2}\,\sqrt{a}\,\sqrt{c}\,d-\sqrt{2}\,c\,\sqrt{a+a\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}}\,\sqrt{c+d\,\tan\left(e+f\,x\right)}\,1{}\mathrm{i}-\sqrt{2}\,d\,\sqrt{a+a\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}}\,\sqrt{c+d\,\tan\left(e+f\,x\right)}+\sqrt{2}\,\sqrt{a}\,\sqrt{c}\,d\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}\right)}{\sqrt{a}\,d^2\,\tan\left(e+f\,x\right)+\sqrt{a}\,d\,\left(c+d\,\tan\left(e+f\,x\right)\right)+\sqrt{a}\,c\,d-6\,\sqrt{c}\,d\,\sqrt{a+a\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}}\,\sqrt{c+d\,\tan\left(e+f\,x\right)}+4\,\sqrt{a}\,\sqrt{c}\,d\,\sqrt{c+d\,\tan\left(e+f\,x\right)}+c^2\,\sqrt{a+a\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}}\,2{}\mathrm{i}+d^2\,\sqrt{a+a\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}}\,2{}\mathrm{i}+\sqrt{a}\,c^2\,1{}\mathrm{i}-\sqrt{a}\,d^2\,2{}\mathrm{i}-\sqrt{a}\,c^{3/2}\,\sqrt{c+d\,\tan\left(e+f\,x\right)}\,4{}\mathrm{i}+\sqrt{a}\,c\,\left(c+d\,\tan\left(e+f\,x\right)\right)\,3{}\mathrm{i}-c^{3/2}\,\sqrt{a+a\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}}\,\sqrt{c+d\,\tan\left(e+f\,x\right)}\,2{}\mathrm{i}+\sqrt{a}\,c\,d\,\tan\left(e+f\,x\right)\,3{}\mathrm{i}}\right)\,1{}\mathrm{i}}{f\,\sqrt{-c+d\,1{}\mathrm{i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*tan(e + f*x)*1i)^(1/2)/(c + d*tan(e + f*x))^(1/2),x)

[Out] (2^(1/2)*a^(1/2)*atan((2*(d*1i - c)^(1/2)*(2^(1/2)*c^(3/2)*(a + a*tan(e + f
*x)*1i)^(1/2)*1i - 2^(1/2)*c^(1/2)*d*(a + a*tan(e + f*x)*1i)^(1/2) - 2^(1/2
)*a^(1/2)*c*(c + d*tan(e + f*x))^(1/2)*1i + 2^(1/2)*a^(1/2)*c^(1/2)*(c + d*
tan(e + f*x))*1i + 2^(1/2)*a^(1/2)*d*(c + d*tan(e + f*x))^(1/2) + 2^(1/2)*a
^(1/2)*c^(1/2)*d - 2^(1/2)*c*(a + a*tan(e + f*x)*1i)^(1/2)*(c + d*tan(e + f

```
*x))^(1/2)*1i - 2^(1/2)*d*(a + a*tan(e + f*x)*1i)^(1/2)*(c + d*tan(e + f*x)
)^(1/2) + 2^(1/2)*a^(1/2)*c^(1/2)*d*tan(e + f*x)*1i))/(c^2*(a + a*tan(e + f
*x)*1i)^(1/2)*2i + d^2*(a + a*tan(e + f*x)*1i)^(1/2)*2i + a^(1/2)*c^2*1i -
a^(1/2)*d^2*2i - a^(1/2)*c^(3/2)*(c + d*tan(e + f*x))^(1/2)*4i + a^(1/2)*d^
2*tan(e + f*x) + a^(1/2)*c*(c + d*tan(e + f*x))*3i + a^(1/2)*d*(c + d*tan(e
 + f*x)) - c^(3/2)*(a + a*tan(e + f*x)*1i)^(1/2)*(c + d*tan(e + f*x))^(1/2)
*2i + a^(1/2)*c*d - 6*c^(1/2)*d*(a + a*tan(e + f*x)*1i)^(1/2)*(c + d*tan(e
+ f*x))^(1/2) + a^(1/2)*c*d*tan(e + f*x)*3i + 4*a^(1/2)*c^(1/2)*d*(c + d*ta
n(e + f*x))^(1/2)))*1i)/(f*(d*1i - c)^(1/2))
```

3.1158 $\int \frac{1}{\sqrt{a+ia\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=174

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{c-id}\,f}-\frac{\sqrt{c+d\tan(e+fx)}}{(ic+d)f\sqrt{a+ia\tan(e+fx)}}+\frac{2d\sqrt{c+d\tan(e+fx)}}{(c^2+d^2)\,f\sqrt{a+ia\tan(e+fx)}}$$

```
[Out] -1/2*I*arctanh(2^(1/2)*a^(1/2)*(c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2)/(a+I*a*
tan(f*x+e))^(1/2))/f*2^(1/2)/a^(1/2)/(c-I*d)^(1/2)-(c+d*tan(f*x+e))^(1/2)/(
I*c+d)/f/(a+I*a*tan(f*x+e))^(1/2)+2*d*(c+d*tan(f*x+e))^(1/2)/(c^2+d^2)/f/(a
+I*a*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3629, 3627, 3625, 214}

$$\frac{2d\sqrt{c+d\tan(e+fx)}}{f\left(c^2+d^2\right)\sqrt{a+ia\tan(e+fx)}}-\frac{\sqrt{c+d\tan(e+fx)}}{f(d+ic)\sqrt{a+ia\tan(e+fx)}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\,f\sqrt{c-id}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]]),x]
```

```
[Out] ((-I)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])/(Sqrt[c - I*d]*Sqr
t[a + I*a*Tan[e + f*x]])])/(Sqrt[2]*Sqrt[a]*Sqrt[c - I*d]*f) - Sqrt[c + d*T
an[e + f*x]]/((I*c + d)*f*Sqrt[a + I*a*Tan[e + f*x]]) + (2*d*Sqrt[c + d*Tan
[e + f*x]])/((c^2 + d^2)*f*Sqrt[a + I*a*Tan[e + f*x]])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3627

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^n/(2*b*f*m)), x] - Dist[(a*c - b*d)/(2*b^2), Int[(a + b*Tan[e + f*
x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && EqQ
[m + n, 0] && LeQ[m, -2^(-1)]
```

Rule 3629

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-d)*(a + b*Tan[e + f*x])^m*((c + d*Ta
n[e + f*x])^(n + 1)/(f*m*(c^2 + d^2))), x] + Dist[a/(a*c - b*d), Int[(a + b
*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+ia\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}\,dx = \frac{2d\sqrt{c+d\tan(e+fx)}}{(c^2+d^2)\,f\sqrt{a+ia\tan(e+fx)}} + \frac{\int \frac{\sqrt{c+d\tan(e+}}{\sqrt{a+ia\tan(e+}}}{c-id}$$

$$= -\frac{\sqrt{c+d\tan(e+fx)}}{(ic+d)f\sqrt{a+ia\tan(e+fx)}} + \frac{2d\sqrt{c+d\tan}}{(c^2+d^2)\,f\sqrt{a+i}}$$

$$= -\frac{\sqrt{c+d\tan(e+fx)}}{(ic+d)f\sqrt{a+ia\tan(e+fx)}} + \frac{2d\sqrt{c+d\tan}}{(c^2+d^2)\,f\sqrt{a+i}}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{c-id}\,f} - \frac{\sqrt{}}{(ic+}$$

Mathematica [A]
time = 3.42, size = 195, normalized size = 1.12

$$\frac{\frac{2(-ic+d)e^{i(e+fx)}\log\left(2\left(\sqrt{c-id}\,e^{i(e+fx)}+\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)\right)}{\sqrt{1+e^{2i(e+fx)}}}+2i\sqrt{c-id}\,\sqrt{c+d\tan(e+fx)}}{2\sqrt{c-id}\,(c+id)f\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]]),x]
```

```
[Out] ((2*((-I)*c + d)*E^(I*(e + f*x))*Log[2*(Sqrt[c - I*d]*E^(I*(e + f*x)) + Sqr
t[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E
^((2*I)*(e + f*x)))])])/Sqrt[1 + E^((2*I)*(e + f*x))] + (2*I)*Sqrt[c - I*d]
*Sqrt[c + d*Tan[e + f*x]])/(2*Sqrt[c - I*d]*(c + I*d)*f*Sqrt[a + I*a*Tan[e
+ f*x]])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1737 vs. $2(143) = 286$.
time = 0.91, size = 1738, normalized size = 9.99

method	result	size
derivativedivides	Expression too large to display	1738
default	Expression too large to display	1738

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/4/f*(a*(1+I*tan(f*x+e)))^(1/2)*(c+d*tan(f*x+e))^(1/2)/a*(-I*ln((3*a*c+I*a
*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*t
an(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1
/2)*d^3-4*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*d^3+2*I*ln((3*a*c+I
*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d
*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^
(1/2)*c^3*tan(f*x+e)-4*I*c*d^2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*
tan(f*x+e)-ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*
(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))
*2^(1/2)*(-a*(I*d-c))^(1/2)*c^3*tan(f*x+e)^2+3*ln((3*a*c+I*a*tan(f*x+e)*c-I
*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I
*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c*d^2*tan(f
*x+e)^2-3*I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a
*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I)
)*2^(1/2)*(-a*(I*d-c))^(1/2)*c^2*d*tan(f*x+e)^2+3*I*ln((3*a*c+I*a*tan(f*x+e
)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))
*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c^2*d-
6*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^
(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*
(-a*(I*d-c))^(1/2)*c^2*d*tan(f*x+e)+2*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*
d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+
e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*d^3*tan(f*x+e)-4*I*(
a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c^2*d-6*I*ln((3*a*c+I*a*tan(f*x+
e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e)
)*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c*d^2
```

```
*tan(f*x+e)+ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a
*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I)
)*2^(1/2)*(-a*(I*d-c))^(1/2)*c^3-3*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*t
an(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e))
)^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c*d^2-4*I*c^3*(a*(c+d*t
an(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*tan(f*x+e)+I*ln((3*a*c+I*a*tan(f*x+e)*c-
I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+
I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*d^3*tan(f*
x+e)^2+4*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c^2*d*tan(f*x+e)+4*(a*
(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*d^3*tan(f*x+e)-4*c^3*(a*(c+d*tan(f
*x+e))*(1+I*tan(f*x+e)))^(1/2)-4*c*d^2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e))
)^(1/2))/(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)/(I*d-c)/(c+I*d)^2/(I*c
-d)/(-tan(f*x+e)+I)^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(142) = 284$.
time = 1.48, size = 398, normalized size = 2.29

$$-\frac{\left((-i\,ac+ad)f\sqrt{-\frac{2i}{(i\,ac+ad)f^2}}\,e^{(i\,fx+i\,e)}\log\left((i\,ac+ad)f\sqrt{-\frac{2i}{(i\,ac+ad)f^2}}\,e^{(i\,fx+i\,e)}+\sqrt{2}\sqrt{\frac{(c-i\,d)e^{(2i\,fx+2i\,e)}+c+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\left(e^{(2i\,fx+2i\,e)}+1\right)\right)+(i\,ac-ad)f\sqrt{-\frac{2i}{(i\,ac+ad)f^2}}\,e^{(i\,fx+i\,e)}\log\left((-i\,ac-ad)f\sqrt{-\frac{2i}{(i\,ac+ad)f^2}}\,e^{(i\,fx+i\,e)}+\sqrt{2}\sqrt{\frac{(c-i\,d)e^{(2i\,fx+2i\,e)}+c+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\left(e^{(2i\,fx+2i\,e)}+1\right)\right)+2\sqrt{2}\sqrt{\frac{(c-i\,d)e^{(2i\,fx+2i\,e)}+c+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\left(e^{(2i\,fx+2i\,e)}+1\right)\right)e^{(-i\,fx-i\,e)}}{4\,(i\,ac-ad)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(1/2),x, algorithm="f
ricas")
```

```
[Out] -1/4*((-I*a*c + a*d)*f*sqrt(-2*I/((I*a*c + a*d)*f^2))*e^(I*f*x + I*e)*log((
I*a*c + a*d)*f*sqrt(-2*I/((I*a*c + a*d)*f^2))*e^(I*f*x + I*e) + sqrt(2)*sqr
t(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt
(a/(e^(2*I*f*x + 2*I*e) + 1))*(e^(2*I*f*x + 2*I*e) + 1)) + (I*a*c - a*d)*f*
sqrt(-2*I/((I*a*c + a*d)*f^2))*e^(I*f*x + I*e)*log((-I*a*c - a*d)*f*sqrt(-2
*I/((I*a*c + a*d)*f^2))*e^(I*f*x + I*e) + sqrt(2)*sqrt(((c - I*d)*e^(2*I*f*
x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e
) + 1))*(e^(2*I*f*x + 2*I*e) + 1)) + 2*sqrt(2)*sqrt(((c - I*d)*e^(2*I*f*x +
 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) +
 1))*(e^(2*I*f*x + 2*I*e) + 1))*e^(-I*f*x - I*e)/((I*a*c - a*d)*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ia\left(\tan\left(e+fx\right)-i\right)}\,\sqrt{c+d\tan\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**(1/2)/(c+d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(I*a*(tan(e + f*x) - I))*sqrt(c + d*tan(e + f*x))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

Mupad [B]
time = 19.62, size = 1508, normalized size = 8.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^(1/2)*(c + d*tan(e + f*x))^(1/2)),x)
```

```
[Out] (2*((a + a*tan(e + f*x)*1i)^(1/2) - a^(1/2)))/(d*f*((c + d*tan(e + f*x))^(1
/2) - c^(1/2))*((a*1i)/d + ((a + a*tan(e + f*x)*1i)^(1/2) - a^(1/2))^2/((c
+ d*tan(e + f*x))^(1/2) - c^(1/2))^2 - (a^(1/2)*c^(1/2)*((a + a*tan(e + f*x
)*1i)^(1/2) - a^(1/2))*2i)/(d*((c + d*tan(e + f*x))^(1/2) - c^(1/2))))) - (
2^(1/2)*atan(((2^(1/2)*((4*d^7*f*(a*c - a*d*1i)*((a + a*tan(e + f*x)*1i)^(1
/2) - a^(1/2)))/((c + d*tan(e + f*x))^(1/2) - c^(1/2)) - 4*a^(3/2)*c^(1/2)*
d^7*f + (2^(1/2)*(d^7*(a^2*c*f^2*1i - a^2*d*f^2) + (d^8*(5*a*c*f^2 - a*d*f^
2*3i)*((a + a*tan(e + f*x)*1i)^(1/2) - a^(1/2))^2)/((c + d*tan(e + f*x))^(1
/2) - c^(1/2))^2 - (d^7*f*(a^(3/2)*c^(3/2)*f*2i + 6*a^(3/2)*c^(1/2)*d*f)*((
a + a*tan(e + f*x)*1i)^(1/2) - a^(1/2)))/((c + d*tan(e + f*x))^(1/2) - c^(1
/2))))/(a^(1/2)*f*(d*1i - c)^(1/2)) + (a^(1/2)*c^(1/2)*d^8*f*((a + a*tan(e
+ f*x)*1i)^(1/2) - a^(1/2))^2*4i)/((c + d*tan(e + f*x))^(1/2) - c^(1/2))^2)
```

*1i)/(a^(1/2)*f*(d*1i - c)^(1/2)) - (2^(1/2)*(4*a^(3/2)*c^(1/2)*d^7*f - (4*d^7*f*(a*c - a*d*1i)*((a + a*tan(e + f*x)*1i)^(1/2) - a^(1/2)))/((c + d*tan(e + f*x))^(1/2) - c^(1/2)) + (2^(1/2)*(d^7*(a^2*c*f^2*1i - a^2*d*f^2) + (d^8*(5*a*c*f^2 - a*d*f^2*3i)*((a + a*tan(e + f*x)*1i)^(1/2) - a^(1/2))^2)/((c + d*tan(e + f*x))^(1/2) - c^(1/2))^2 - (d^7*f*(a^(3/2)*c^(3/2)*f*2i + 6*a^(3/2)*c^(1/2)*d*f)*((a + a*tan(e + f*x)*1i)^(1/2) - a^(1/2)))/((c + d*tan(e + f*x))^(1/2) - c^(1/2))))/(a^(1/2)*f*(d*1i - c)^(1/2)) - (a^(1/2)*c^(1/2)*d^8*f*((a + a*tan(e + f*x)*1i)^(1/2) - a^(1/2))^2*4i)/((c + d*tan(e + f*x))^(1/2) - c^(1/2))^2)*1i)/(a^(1/2)*f*(d*1i - c)^(1/2)))/((8*d^8*((a + a*tan(e + f*x)*1i)^(1/2) - a^(1/2))^2)/((c + d*tan(e + f*x))^(1/2) - c^(1/2))^2 - a*d^7*8i + (2^(1/2)*((4*d^7*f*(a*c - a*d*1i)*((a + a*tan(e + f*x)*1i)^(1/2) - a^(1/2)))/((c + d*tan(e + f*x))^(1/2) - c^(1/2)) - 4*a^(3/2)*c^(1/2)*d^7*f + (2^(1/2)*(d^7*(a^2*c*f^2*1i - a^2*d*f^2) + (d^8*(5*a*c*f^2 - a*d*f^2*3i)*((a + a*tan(e + f*x)*1i)^(1/2) - a^(1/2))^2)/((c + d*tan(e + f*x))^(1/2) - c^(1/2))^2 - (d^7*f*(a^(3/2)*c^(3/2)*f*2i + 6*a^(3/2)*c^(1/2)*d*f)*((a + a*tan(e + f*x)*1i)^(1/2) - a^(1/2)))/((c + d*tan(e + f*x))^(1/2) - c^(1/2))))/(a^(1/2)*f*(d*1i - c)^(1/2)) + (a^(1/2)*c^(1/2)*d^8*f*((a + a*tan(e + f*x)*1i)^(1/2) - a^(1/2))^2*4i)/((c + d*tan(e + f*x))^(1/2) - c^(1/2))^2))/(a^(1/2)*f*(d*1i - c)^(1/2)) + (2^(1/2)*(4*a^(3/2)*c^(1/2)*d^7*f - (4*d^7*f*(a*c - a*d*1i)*((a + a*tan(e + f*x)*1i)^(1/2) - a^(1/2)))/((c + d*tan(e + f*x))^(1/2) - c^(1/2)) + (2^(1/2)*(d^7*(a^2*c*f^2*1i - a^2*d*f^2) + (d^8*(5*a*c*f^2 - a*d*f^2*3i)*((a + a*tan(e + f*x)*1i)^(1/2) - a^(1/2))^2)/((c + d*tan(e + f*x))^(1/2) - c^(1/2))^2 - (d^7*f*(a^(3/2)*c^(3/2)*f*2i + 6*a^(3/2)*c^(1/2)*d*f)*((a + a*tan(e + f*x)*1i)^(1/2) - a^(1/2)))/((c + d*tan(e + f*x))^(1/2) - c^(1/2))))/(a^(1/2)*f*(d*1i - c)^(1/2)) - (a^(1/2)*c^(1/2)*d^8*f*((a + a*tan(e + f*x)*1i)^(1/2) - a^(1/2))^2*4i)/((c + d*tan(e + f*x))^(1/2) - c^(1/2))^2))/(a^(1/2)*f*(d*1i - c)^(1/2))))*1i)/(2*a^(1/2)*f*(d*1i - c)^(1/2))

3.1159 $\int \frac{1}{(a+ia\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=193

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}\sqrt{c-id}\,f}-\frac{\sqrt{c+d\tan(e+fx)}}{3(ic-d)f(a+ia\tan(e+fx))^{3/2}}+\frac{(3ic-7d)\sqrt{c+d\tan(e+fx)}}{6a(c+id)^2f\sqrt{a+ia\tan(e+fx)}}$$

```
[Out] -1/4*I*arctanh(2^(1/2)*a^(1/2)*(c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2)/(a+I*a*
tan(f*x+e))^(1/2))/a^(3/2)/f*2^(1/2)/(c-I*d)^(1/2)+1/6*(3*I*c-7*d)*(c+d*tan
(f*x+e))^(1/2)/a/(c+I*d)^2/f/(a+I*a*tan(f*x+e))^(1/2)-1/3*(c+d*tan(f*x+e))^
(1/2)/(I*c-d)/f/(a+I*a*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.31, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3640, 3677, 12, 3625, 214}

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f\sqrt{c-id}}+\frac{(-7d+3ic)\sqrt{c+d\tan(e+fx)}}{6af(c+id)^2\sqrt{a+ia\tan(e+fx)}}-\frac{\sqrt{c+d\tan(e+fx)}}{3f(-d+ic)(a+ia\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^(3/2)*Sqrt[c + d*Tan[e + f*x]]),x]
```

```
[Out] ((-1/2*I)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])/(Sqrt[c - I*d]
*Sqrt[a + I*a*Tan[e + f*x]])])/(Sqrt[2]*a^(3/2)*Sqrt[c - I*d]*f) - Sqrt[c +
 d*Tan[e + f*x]]/(3*(I*c - d)*f*(a + I*a*Tan[e + f*x])^(3/2)) + (((3*I)*c -
 7*d)*Sqrt[c + d*Tan[e + f*x]])/(6*a*(c + I*d)^2*f*Sqrt[a + I*a*Tan[e + f*x
]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
```

```
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}\,dx = -\frac{\sqrt{c+d\tan(e+fx)}}{3(ic-d)f(a+ia\tan(e+fx))^{3/2}} - \frac{\int \frac{-\frac{1}{2}}{\sqrt{a+ia\tan}}}{}$$

$$= -\frac{\sqrt{c+d\tan(e+fx)}}{3(ic-d)f(a+ia\tan(e+fx))^{3/2}} + \frac{(3ic-7d)\sqrt{c}}{6a(c+id)^2f\sqrt{}}$$

$$= -\frac{\sqrt{c+d\tan(e+fx)}}{3(ic-d)f(a+ia\tan(e+fx))^{3/2}} + \frac{(3ic-7d)\sqrt{c}}{6a(c+id)^2f\sqrt{}}$$

$$= -\frac{\sqrt{c+d\tan(e+fx)}}{3(ic-d)f(a+ia\tan(e+fx))^{3/2}} + \frac{(3ic-7d)\sqrt{c}}{6a(c+id)^2f\sqrt{}}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{2\sqrt{2}a^{3/2}\sqrt{c-id}f} - \frac{}{3(ic}$$

Mathematica [A]
time = 4.01, size = 249, normalized size = 1.29

$$\frac{\sec^{\frac{3}{2}}(e+fx)\left(-\frac{i\sqrt{2}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{3/2}\left(1+e^{2i(e+fx)}\right)^{3/2}\log\left(2\left(\sqrt{c-id}\,e^{i(e+fx)}+\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)\right)}{\sqrt{c-id}}-\frac{2(-5ic+9d+(3c+7id)\tan(e+fx))\sqrt{c+d\tan(e+fx)}}{3(c+id)^2\sec^{\frac{3}{2}}(e+fx)}\right)}{4f(a+ia\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^(3/2)*Sqrt[c + d*Tan[e + f*x]]),x]
```

```
[Out] (Sec[e + f*x]^(3/2)*(((-I)*Sqrt[2]*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)
)))^(3/2)*(1 + E^((2*I)*(e + f*x)))^(3/2)*Log[2*(Sqrt[c - I*d]*E^(I*(e + f*
x)) + Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*x))
))/(1 + E^((2*I)*(e + f*x)))])])/Sqrt[c - I*d] - (2*((-5*I)*c + 9*d + (3*c
+ (7*I)*d)*Tan[e + f*x])*Sqrt[c + d*Tan[e + f*x]])/(3*(c + I*d)^2*Sec[e + f
*x]^(3/2))))/(4*f*(a + I*a*Tan[e + f*x])^(3/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2945 vs. $2(154) = 308$.
time = 0.82, size = 2946, normalized size = 15.26

method	result	size
derivativedivides	Expression too large to display	2946
default	Expression too large to display	2946

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(3/2),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/24/f*(c+d*tan(f*x+e))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)/a^2*(9*2^(1/2)*(-
a*(I*d-c))^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2
)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+
e)+I))*d^4*tan(f*x+e)+12*2^(1/2)*(-a*(I*d-c))^(1/2)*ln((3*a*c+I*a*tan(f*x+e
)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))
*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c^3*d-12*2^(1/2)*(-a*(I*d-c))^(1/
2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))
^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c*d^3-3
*I*2^(1/2)*(-a*(I*d-c))^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*
x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/
2))/(tan(f*x+e)+I))*c^4-3*I*2^(1/2)*(-a*(I*d-c))^(1/2)*ln((3*a*c+I*a*tan(f*
x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+
e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*d^4+16*I*(a*(c+d*tan(f*x+e))*(
1+I*tan(f*x+e)))^(1/2)*c^2*d^2*tan(f*x+e)^2-96*I*(a*(c+d*tan(f*x+e))*(1+I*t
an(f*x+e)))^(1/2)*c^3*d*tan(f*x+e)-96*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)
```

))^(1/2)*c*d^3*tan(f*x+e)-3*2^(1/2)*(-a*(I*d-c))^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c^4*tan(f*x+e)^3-3*2^(1/2)*(-a*(I*d-c))^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*d^4*tan(f*x+e)^3-12*I*c^4*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*tan(f*x+e)^2+28*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*d^4*tan(f*x+e)^2-16*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c^2*d^2+40*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c^3*d*tan(f*x+e)^2+40*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c*d^3*tan(f*x+e)^2+32*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c^2*d^2*tan(f*x+e)+64*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*d^4*tan(f*x+e)-56*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c^3*d-56*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c*d^3+20*I*c^4*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)-36*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*d^4-32*c^4*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*tan(f*x+e)-12*I*2^(1/2)*(-a*(I*d-c))^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c^3*d*tan(f*x+e)^3+12*I*2^(1/2)*(-a*(I*d-c))^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c*d^3*tan(f*x+e)^3-54*I*2^(1/2)*(-a*(I*d-c))^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c^2*d^2*tan(f*x+e)^2+36*I*2^(1/2)*(-a*(I*d-c))^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c^3*d*tan(f*x+e)-36*I*2^(1/2)*(-a*(I*d-c))^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c*d^3*tan(f*x+e)+9*I*2^(1/2)*(-a*(I*d-c))^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c^4*tan(f*x+e)^2+18*2^(1/2)*(-a*(I*d-c))^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c^2*d^2*tan(f*x+e)^3-36*2^(1/2)*(-a*(I*d-c))^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c^3*d*tan(f*x+e)^2+36*2^(1/2)*(-a*(I*d-c))^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c*d^3*tan(f*x+e)^2-54*2^(1/2)*(-a*(I*d-c))^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c^2*d^2*tan(f*x+e)+9*I*2^(1/2)*(-a*(I*d-c))^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*d^4*tan(f*x+e)^2+18*I*2^(1/2)*(-a*(I*d-c))^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)

```
+I))*c^2*d^2+9*2^(1/2)*(-a*(I*d-c))^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+
3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(
f*x+e)))^(1/2))/(tan(f*x+e)+I))*c^4*tan(f*x+e))/(a*(c+d*tan(f*x+e))*(1+I*ta
n(f*x+e)))^(1/2)/(c+I*d)^3/(I*c-d)/(-tan(f*x+e)+I)^3/(I*d-c)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(3/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 506 vs. 2(151) = 302.
time = 1.35, size = 506, normalized size = 2.62

$$-\frac{3\,(a^2c^2+2i\,a^2cd-a^2d^2)f\sqrt{\frac{i}{2\,(-i\,a^3c-a^3d)f^2}}\,e^{(3i\,fx+3i\,e)}\log\left(-2\,(i\,a^2c+a^2d)f\sqrt{\frac{i}{2\,(-i\,a^3c-a^3d)f^2}}\,e^{(i\,fx+i\,e)}+\sqrt{2}\sqrt{\frac{(c-i\,d)e^{(2i\,fx+2i\,e)}+c+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\,(e^{(2i\,fx+2i\,e)}+1)\right)-3\,(a^2c^2+2i\,a^2cd-a^2d^2)f\sqrt{\frac{i}{2\,(-i\,a^3c-a^3d)f^2}}\,e^{(3i\,fx+3i\,e)}\log\left(-2\,(-i\,a^2c-a^2d)f\sqrt{\frac{i}{2\,(-i\,a^3c-a^3d)f^2}}\,e^{(i\,fx+i\,e)}+\sqrt{2}\sqrt{\frac{(c-i\,d)e^{(2i\,fx+2i\,e)}+c+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\,(e^{(2i\,fx+2i\,e)}+1)\right)-\sqrt{2}\,(4\,(i\,c-2\,d)e^{(4i\,fx+4i\,e)}-(-5i\,c+9\,d)e^{(2i\,fx+2i\,e)}+i\,c-d)\sqrt{\frac{(c-i\,d)e^{(2i\,fx+2i\,e)}+c+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}}{12\,(a^2c^2+2i\,a^2cd-a^2d^2)f}e^{(-3i\,fx-3i\,e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(3/2),x, algorithm="f
ricas")
```

```
[Out] -1/12*(3*(a^2*c^2 + 2*I*a^2*c*d - a^2*d^2)*f*sqrt(1/2*I/((-I*a^3*c - a^3*d)
*f^2))*e^(3*I*f*x + 3*I*e)*log(-2*(I*a^2*c + a^2*d)*f*sqrt(1/2*I/((-I*a^3*c
 - a^3*d)*f^2))*e^(I*f*x + I*e) + sqrt(2)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*
e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*
(e^(2*I*f*x + 2*I*e) + 1)) - 3*(a^2*c^2 + 2*I*a^2*c*d - a^2*d^2)*f*sqrt(1/2
*I/((-I*a^3*c - a^3*d)*f^2))*e^(3*I*f*x + 3*I*e)*log(-2*(-I*a^2*c - a^2*d)*
f*sqrt(1/2*I/((-I*a^3*c - a^3*d)*f^2))*e^(I*f*x + I*e) + sqrt(2)*sqrt(((c -
 I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(
2*I*f*x + 2*I*e) + 1))*(e^(2*I*f*x + 2*I*e) + 1)) - sqrt(2)*(4*(I*c - 2*d)*
e^(4*I*f*x + 4*I*e) - (-5*I*c + 9*d)*e^(2*I*f*x + 2*I*e) + I*c - d)*sqrt(((
c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(
e^(2*I*f*x + 2*I*e) + 1)))*e^(-3*I*f*x - 3*I*e)/((a^2*c^2 + 2*I*a^2*c*d - a
^2*d^2)*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(\tan\left(e+fx\right)-i\right)\right)^{\frac{3}{2}}\sqrt{c+d\tan\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/((I*a*(tan(e + f*x) - I))**(3/2)*sqrt(c + d*tan(e + f*x))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(3/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+a\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^{3/2}\,\sqrt{c+d\tan\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^(3/2)*(c + d*tan(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + a*tan(e + f*x)*1i)^(3/2)*(c + d*tan(e + f*x))^(1/2)), x)
```

3.1160 $\int \frac{1}{(a+ia\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=262

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{4\sqrt{2}\,a^{5/2}\sqrt{c-id}\,f}-\frac{\sqrt{c+d\tan(e+fx)}}{5(ic-d)f(a+ia\tan(e+fx))^{5/2}}+\frac{(5ic-13d)\sqrt{c+d\tan}}{30a(c+id)^2f(a+ia\tan(}$$

```
[Out] -1/8*I*arctanh(2^(1/2)*a^(1/2)*(c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2)/(a+I*a*
tan(f*x+e))^(1/2))/a^(5/2)/f*2^(1/2)/(c-I*d)^(1/2)+1/60*(15*c^2+50*I*c*d-67
*d^2)*(c+d*tan(f*x+e))^(1/2)/a^2/(I*c-d)^3/f/(a+I*a*tan(f*x+e))^(1/2)-1/5*(
c+d*tan(f*x+e))^(1/2)/(I*c-d)/f/(a+I*a*tan(f*x+e))^(5/2)+1/30*(5*I*c-13*d)*
(c+d*tan(f*x+e))^(1/2)/a/(c+I*d)^2/f/(a+I*a*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.55, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3640, 3677, 12, 3625, 214}

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{4\sqrt{2}\,a^{5/2}f\sqrt{c-id}}+\frac{(15c^2+50icd-67d^2)\sqrt{c+d\tan(e+fx)}}{60a^2f(-d+ic)^3\sqrt{a+ia\tan(e+fx)}}+\frac{(-13d+5ic)\sqrt{c+d\tan(e+fx)}}{30af(c+id)^2(a+ia\tan(e+fx))^{3/2}}-\frac{\sqrt{c+d\tan(e+fx)}}{5f(-d+ic)(a+ia\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^(5/2)*Sqrt[c + d*Tan[e + f*x]]),x]
```

```
[Out] ((-1/4*I)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])/(Sqrt[c - I*d]
*Sqrt[a + I*a*Tan[e + f*x]])])/(Sqrt[2]*a^(5/2)*Sqrt[c - I*d]*f) - Sqrt[c +
 d*Tan[e + f*x]]/(5*(I*c - d)*f*(a + I*a*Tan[e + f*x])^(5/2)) + (((5*I)*c -
 13*d)*Sqrt[c + d*Tan[e + f*x]])/(30*a*(c + I*d)^2*f*(a + I*a*Tan[e + f*x])
^(3/2)) + ((15*c^2 + (50*I)*c*d - 67*d^2)*Sqrt[c + d*Tan[e + f*x]])/(60*a^2
*(I*c - d)^3*f*Sqrt[a + I*a*Tan[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
```

```
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}\,dx = -\frac{\sqrt{c+d\tan(e+fx)}}{5(ic-d)f(a+ia\tan(e+fx))^{5/2}} - \frac{\int \frac{-\frac{1}{2}a(5ic-}{(a+ia\tan(e+fx))^{3}}}{5}$$

$$= -\frac{\sqrt{c+d\tan(e+fx)}}{5(ic-d)f(a+ia\tan(e+fx))^{5/2}} + \frac{(5ic-13d)\sqrt{}}{30a(c+id)^2f(a}$$

$$= -\frac{\sqrt{c+d\tan(e+fx)}}{5(ic-d)f(a+ia\tan(e+fx))^{5/2}} + \frac{(5ic-13d)\sqrt{}}{30a(c+id)^2f(a}$$

$$= -\frac{\sqrt{c+d\tan(e+fx)}}{5(ic-d)f(a+ia\tan(e+fx))^{5/2}} + \frac{(5ic-13d)\sqrt{}}{30a(c+id)^2f(a}$$

$$= -\frac{\sqrt{c+d\tan(e+fx)}}{5(ic-d)f(a+ia\tan(e+fx))^{5/2}} + \frac{(5ic-13d)\sqrt{}}{30a(c+id)^2f(a}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{4\sqrt{2}a^{5/2}\sqrt{c-id}\,f} - \frac{}{5(ic-}$$

Mathematica [A]
time = 5.60, size = 309, normalized size = 1.18

$$\frac{\sec^{\frac{5}{2}}(e+fx)\left(-\frac{i\sqrt{2}e^{2i(e+fx)}\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\sqrt{1+e^{2i(e+fx)}}\log\left(2\left(\sqrt{c-id}\,e^{i(e+fx)}+\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id(-1+e^{2i(e+fx)})}{1+e^{2i(e+fx)}}}\right)\right)}{\sqrt{c-id}}+\frac{2i(11c^2+30icd-19d^2+(26c^2+80icd-86d^2)\cos(2(e+fx))+4i(5c^2+17icd-20d^2)\sin(2(e+fx)))\sqrt{c+d\tan(e+fx)}}{15(c+id)^3\sqrt{\sec(e+fx)}}\right)}{8f(a+ia\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^(5/2)*Sqrt[c + d*Tan[e + f*x]]),x]
```

```
[Out] (Sec[e + f*x]^(5/2)*(((-I)*Sqrt[2]*E^((2*I)*(e + f*x))*Sqrt[E^(I*(e + f*x))
/(1 + E^((2*I)*(e + f*x)))]*Sqrt[1 + E^((2*I)*(e + f*x))]*Log[2*(Sqrt[c - I
*d]*E^(I*(e + f*x)) + Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^(
(2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))])])/Sqrt[c - I*d] + (((2*I)/15
)*(11*c^2 + (30*I)*c*d - 19*d^2 + (26*c^2 + (80*I)*c*d - 86*d^2)*Cos[2*(e +
 f*x)] + (4*I)*(5*c^2 + (17*I)*c*d - 20*d^2)*Sin[2*(e + f*x)])*Sqrt[c + d*T
an[e + f*x]])/((c + I*d)^3*Sqrt[Sec[e + f*x]])))/(8*f*(a + I*a*Tan[e + f*x]
)^(5/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5217 vs. 2(214) = 428.
time = 0.82, size = 5218, normalized size = 19.92

method	result	size
derivativedivides	Expression too large to display	5218
default	Expression too large to display	5218

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(5/2),x,method=_RETURNVERBO
SE)
```

```
[Out] result too large to display
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(5/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 590 vs. 2(210) = 420.
time = 1.06, size = 590, normalized size = 2.25

$$\frac{\left(30\,(i\,a^3c^3-3\,a^3c^2d-3i\,a^3cd^2+a^3d^3)f\sqrt{\frac{i}{8\,(-i\,a^5c-a^5d)f^2}}\,e^{(5i\,fx+5i\,e)}\log\left(-4\,(i\,a^3c+a^3d)f\sqrt{\frac{i}{8\,(-i\,a^5c-a^5d)f^2}}\,e^{(i\,fx+i\,e)}+\sqrt{2}\sqrt{\frac{(c-i\,d)e^{(2i\,fx+2i\,e)}+c+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\,(e^{(2i\,fx+2i\,e)}+1)\right)+30\,(-i\,a^3c^3+3\,a^3c^2d+3i\,a^3cd^2-a^3d^3)f\sqrt{\frac{i}{8\,(-i\,a^5c-a^5d)f^2}}\,e^{(5i\,fx+5i\,e)}\log\left(-4\,(-i\,a^3c-a^3d)f\sqrt{\frac{i}{8\,(-i\,a^5c-a^5d)f^2}}\,e^{(i\,fx+i\,e)}+\sqrt{2}\sqrt{\frac{(c-i\,d)e^{(2i\,fx+2i\,e)}+c+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\,(e^{(2i\,fx+2i\,e)}+1)\right)+\sqrt{2}\,(3\,c^2+6i\,cd-3\,d^2+(23\,c^2+74i\,cd-83\,d^2)e^{(6i\,fx+6i\,e)}+2\,(17\,c^2+52i\,cd-51\,d^2)e^{(4i\,fx+4i\,e)}+2\,(7\,c^2+18i\,cd-11\,d^2)e^{(2i\,fx+2i\,e)})\sqrt{\frac{(c-i\,d)e^{(2i\,fx+2i\,e)}+c+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-5i\,fx-5i\,e)}}{120\,(-i\,a^3c^3+3\,a^3c^2d+3i\,a^3cd^2-a^3d^3)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(5/2),x, algorithm="f
ricas")
```

```
[Out] 1/120*(30*(I*a^3*c^3 - 3*a^3*c^2*d - 3*I*a^3*c*d^2 + a^3*d^3)*f*sqrt(1/8*I/
((-I*a^5*c - a^5*d)*f^2))*e^(5*I*f*x + 5*I*e)*log(-4*(I*a^3*c + a^3*d)*f*sq
rt(1/8*I/((-I*a^5*c - a^5*d)*f^2))*e^(I*f*x + I*e) + sqrt(2)*sqrt(((c - I*d
)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*
f*x + 2*I*e) + 1))*(e^(2*I*f*x + 2*I*e) + 1)) + 30*(-I*a^3*c^3 + 3*a^3*c^2*
d + 3*I*a^3*c*d^2 - a^3*d^3)*f*sqrt(1/8*I/((-I*a^5*c - a^5*d)*f^2))*e^(5*I*
f*x + 5*I*e)*log(-4*(-I*a^3*c - a^3*d)*f*sqrt(1/8*I/((-I*a^5*c - a^5*d)*f^2
))*e^(I*f*x + I*e) + sqrt(2)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)
/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*(e^(2*I*f*x +
 2*I*e) + 1)) + sqrt(2)*(3*c^2 + 6*I*c*d - 3*d^2 + (23*c^2 + 74*I*c*d - 83*
d^2)*e^(6*I*f*x + 6*I*e) + 2*(17*c^2 + 52*I*c*d - 51*d^2)*e^(4*I*f*x + 4*I*
e) + 2*(7*c^2 + 18*I*c*d - 11*d^2)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(
```

```
2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x +
 2*I*e) + 1)))*e^(-5*I*f*x - 5*I*e)/((-I*a^3*c^3 + 3*a^3*c^2*d + 3*I*a^3*c*
d^2 - a^3*d^3)*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia(\tan(e+fx)-i))^{\frac{5}{2}}\sqrt{c+d\tan(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral(1/((I*a*(tan(e + f*x) - I))**(5/2)*sqrt(c + d*tan(e + f*x))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(5/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+a\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{5/2}\,\sqrt{c+d\,\tan\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^(5/2)*(c + d*tan(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + a*tan(e + f*x)*1i)^(5/2)*(c + d*tan(e + f*x))^(1/2)), x)
```

3.1161 $\int \frac{(a+ia\tan(e+fx))^{5/2}}{(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=209

$$\frac{2\sqrt[4]{-1}\,a^{5/2}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}\right)}{d^{3/2}f}-\frac{4i\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+fx)}}\right)}{(c-id)^{3/2}f}$$

```
[Out] 2*(-1)^(1/4)*a^(5/2)*arctanh((-1)^(3/4)*d^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^
(1/2)/(c+d*tan(f*x+e))^(1/2))/d^(3/2)/f-4*I*a^(5/2)*arctanh(2^(1/2)*a^(1/2)
*(c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2)/(a+I*a*tan(f*x+e))^(1/2))*2^(1/2)/(c-
I*d)^(3/2)/f+2*a^2*(c+I*d)*(a+I*a*tan(f*x+e))^(1/2)/(c-I*d)/d/f/(c+d*tan(f*
x+e))^(1/2)
```

Rubi [A]

time = 0.49, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3634, 3682, 3625, 214, 3680, 65, 223, 212}

$$\frac{2\sqrt[4]{-1}\,a^{5/2}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\,\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}\right)}{d^{3/2}f}-\frac{4i\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+fx)}}\right)}{f(c-id)^{3/2}}+\frac{2a^2(c+id)\sqrt{a+ia\tan(e+fx)}}{df(c-id)\sqrt{c+d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^(5/2)/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*(-1)^(1/4)*a^(5/2)*ArcTanh[((-1)^(3/4)*Sqrt[d]*Sqrt[a + I*a*Tan[e + f*x]
])/(Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])])/(d^(3/2)*f) - ((4*I)*Sqrt[2]*a^(5/2
)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])/(Sqrt[c - I*d]*Sqrt[a
+ I*a*Tan[e + f*x]])])/((c - I*d)^(3/2)*f) + (2*a^2*(c + I*d)*Sqrt[a + I*a*
Tan[e + f*x]])/((c - I*d)*d*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3634

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-a^2)*(b*c - a*d)*(a + b*Tan[e + f*x]
)^(m - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] + Di
st[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[b*(b*c*(m - 2) - a*d*(m - 2*n - 4)) + (a*b*c*(m - 2)
+ b^2*d*(n + 1) - a^2*d*(m + n - 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^{5/2}}{(c+d\tan(e+fx))^{3/2}}\,dx = \frac{2a^2(c+id)\sqrt{a+ia\tan(e+fx)}}{(c-id)df\sqrt{c+d\tan(e+fx)}} - \frac{2\int \frac{\sqrt{a+ia\tan(e+fx)}\,(-\frac{1}{2}a^2(c+3id)+}{\sqrt{c+d\tan(e+fx}}}{d(ic+d)}$$

$$= \frac{2a^2(c+id)\sqrt{a+ia\tan(e+fx)}}{(c-id)df\sqrt{c+d\tan(e+fx)}} + \frac{(4a^2)\int \frac{\sqrt{a+ia\tan(e+fx)}}{\sqrt{c+d\tan(e+fx)}}\,dx}{c-id} - (i$$

$$= \frac{2a^2(c+id)\sqrt{a+ia\tan(e+fx)}}{(c-id)df\sqrt{c+d\tan(e+fx)}} - \frac{(ia^3)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+iax}\sqrt{c+dx}}\,dx\right.}{df}$$

$$= -\frac{4i\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{(c-id)^{3/2}f} + \frac{2a^2(c+id)\sqrt{a+}}{(c-id)df\sqrt{c+}}$$

$$= -\frac{4i\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{(c-id)^{3/2}f} + \frac{2a^2(c+id)\sqrt{a+}}{(c-id)df\sqrt{c+}}$$

$$= \frac{2\sqrt[4]{-1}\,a^{5/2}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{d}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c+d\tan(e+fx)}}\right)}{d^{3/2}f} - \frac{4i\sqrt{2}\,a^{5/2}\tanh^{-}}{}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 718 vs. 2(209) = 418.
time = 11.64, size = 718, normalized size = 3.44

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(5/2)/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (Cos[e + f*x]^2*Sqrt[Sec[e + f*x]*(c*Cos[e + f*x] + d*Sin[e + f*x])]*(((c +
 I*d)*Cos[e]*((2*Cos[2*e])/d - ((2*I)*Sin[2*e])/d))/((c - I*d)*(c*Cos[e] +
d*Sin[e])) + ((-2*Cos[2*e] + (2*I)*Sin[2*e])*(c*Sin[f*x] + I*d*Sin[f*x]))/(
(c - I*d)*(c*Cos[e] + d*Sin[e])*(c*Cos[e + f*x] + d*Sin[e + f*x])))*(a + I*
a*Tan[e + f*x])^(5/2))/(f*(Cos[f*x] + I*Sin[f*x])^2) + ((1 + I)*Cos[e + f*x
]^3*((c - I*d)^(3/2)*Log[((2 - 2*I)*E^((I/2)*e)*((-I)*d + d*E^(I*(e + f*x))
```

```
 + I*c*(I + E^(I*(e + f*x))) - (1 + I)*Sqrt[d]*Sqrt[1 + E^((2*I)*(e + f*x))
]*Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))]))/(S
qrt[d]*(I*c + d)*(I + E^(I*(e + f*x))))] - (c - I*d)^(3/2)*Log[((2 + 2*I)*E
^((I/2)*e)*(c + I*d + I*c*E^(I*(e + f*x)) + d*E^(I*(e + f*x)) + (1 + I)*Sqr
t[d]*Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*x)))
)/(1 + E^((2*I)*(e + f*x)))]))/(Sqrt[d]*(I*c + d)*(1 + I*E^(I*(e + f*x))))]
 - (4 + 4*I)*d^(3/2)*Log[2*(Sqrt[c - I*d]*Cos[e + f*x] + I*Sqrt[c - I*d]*Si
n[e + f*x] + Sqrt[1 + Cos[2*e + 2*f*x] + I*Sin[2*e + 2*f*x]]*Sqrt[c + d*Tan
[e + f*x]])])*(Cos[2*e] - I*Sin[2*e])*(a + I*a*Tan[e + f*x])^(5/2))/((c - I
*d)^(3/2)*d^(3/2)*f*(Cos[f*x] + I*Sin[f*x])^2*Sqrt[1 + Cos[2*e + 2*f*x] + I
*Sin[2*e + 2*f*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1814 vs. $2(165) = 330$.
time = 0.59, size = 1815, normalized size = 8.68

method	result	size
derivativedivides	Expression too large to display	1815
default	Expression too large to display	1815

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/2/f*2^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)*(-2*I*ln((3*a*c+I*a*tan(f*x+e)*c-I
*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I
*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c*d*(I*a*d)^(1/2)+2*I*2^(1/2)*c^3*(-
a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)-
2*I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c)
)^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*d^2*
(I*a*d)^(1/2)*tan(f*x+e)-3*I*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*
(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*
a*c*d^3*(-a*(I*d-c))^(1/2)+2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c
+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*
c^3*d*(-a*(I*d-c))^(1/2)*tan(f*x+e)-7*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*
a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d
)^(1/2))*a*c*d^3*(-a*(I*d-c))^(1/2)*tan(f*x+e)-6*I*(I*a*d)^(1/2)*(a*(c+d*ta
n(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*c*d^2+2^(1/2)*
ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1
/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c^4*(-a*(I*d-c))^(1/2)-7*2^(1/2)*ln
(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2
)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c^2*d^2*(-a*(I*d-c))^(1/2)+5*I*2^(1/2
)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^
(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c^2*d^2*(-a*(I*d-c))^(1/2)*tan(f*
x+e)+5*I*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+
```

```
I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c^3*d*(-a*(I*d-c))
^(1/2)-3*I*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(
1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*d^4*(-a*(I*d-c))
^(1/2)*tan(f*x+e)-2*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(
f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*d^2*(-a
*(I*d-c))^(1/2)*tan(f*x+e)-6*(I*a*d)^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x
+e)))^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*c^2*d+2*(I*a*d)^(1/2)*(a*(c+d*tan(f*
x+e))*(1+I*tan(f*x+e)))^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*d^3+2*I*2^(1/2)*ln
(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2
)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c*d*(-a*(I*d-c))^(1/2)-2*2^(1/2)*(-a*
(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*
tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c*d+2*I*2^(1/2)*ln(1
/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*
(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*d^2*(-a*(I*d-c))^(1/2)*tan(f*x+e)-2*ln(
(3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)
*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*d^2*(I*a*d)
^(1/2)*tan(f*x+e)-2*(I*a*d)^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*ta
n(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))
^(1/2))/(tan(f*x+e)+I))*a*c*d)*a^2/(c+d*tan(f*x+e))^(1/2)/d/(a*(c+d*tan(f*x
+e))*(1+I*tan(f*x+e)))^(1/2)/(c^2+d^2)/(I*c-d)/(I*a*d)^(1/2)/(-a*(I*d-c))^(
1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((d^2-2*c*d-c^2)>0)', see `assume?`
for mor
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 974 vs. 2(163) = 326.
time = 1.73, size = 974, normalized size = 4.66

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/2*(4*sqrt(2)*((a^2*c + I*a^2*d)*e^(3*I*f*x + 3*I*e) + (a^2*c + I*a^2*d)*e
^(I*f*x + I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x
+ 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)) + ((c^2*d - 2*I*c*d^2 - d^
3)*f*e^(2*I*f*x + 2*I*e) + (c^2*d + d^3)*f)*sqrt(4*I*a^5/(d^3*f^2))*log((I*
d^2*f*sqrt(4*I*a^5/(d^3*f^2))*e^(I*f*x + I*e) + sqrt(2)*(a^2*e^(2*I*f*x + 2
*I*e) + a^2)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2
*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/a^2) - ((c^
2*d - 2*I*c*d^2 - d^3)*f*e^(2*I*f*x + 2*I*e) + (c^2*d + d^3)*f)*sqrt(4*I*a^
5/(d^3*f^2))*log((-I*d^2*f*sqrt(4*I*a^5/(d^3*f^2))*e^(I*f*x + I*e) + sqrt(2
)*(a^2*e^(2*I*f*x + 2*I*e) + a^2)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c +
 I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f
*x - I*e)/a^2) + ((c^2*d - 2*I*c*d^2 - d^3)*f*e^(2*I*f*x + 2*I*e) + (c^2*d
+ d^3)*f)*sqrt(32*I*a^5/((-I*c^3 - 3*c^2*d + 3*I*c*d^2 + d^3)*f^2))*log(1/4
*((I*c^2 + 2*c*d - I*d^2)*sqrt(32*I*a^5/((-I*c^3 - 3*c^2*d + 3*I*c*d^2 + d^
3)*f^2))*f*e^(I*f*x + I*e) + 4*sqrt(2)*(a^2*e^(2*I*f*x + 2*I*e) + a^2)*sqrt
(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(
a/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/a^2) - ((c^2*d - 2*I*c*d^2 -
 d^3)*f*e^(2*I*f*x + 2*I*e) + (c^2*d + d^3)*f)*sqrt(32*I*a^5/((-I*c^3 - 3*c
^2*d + 3*I*c*d^2 + d^3)*f^2))*log(1/4*((-I*c^2 - 2*c*d + I*d^2)*sqrt(32*I*a
^5/((-I*c^3 - 3*c^2*d + 3*I*c*d^2 + d^3)*f^2))*f*e^(I*f*x + I*e) + 4*sqrt(2
)*(a^2*e^(2*I*f*x + 2*I*e) + a^2)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c +
 I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f
*x - I*e)/a^2))/((c^2*d - 2*I*c*d^2 - d^3)*f*e^(2*I*f*x + 2*I*e) + (c^2*d +
 d^3)*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan{(e+fx)}-i))^{\frac{5}{2}}}{(c+d\tan{(e+fx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(5/2)/(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((I*a*(tan(e + f*x) - I))**(5/2)/(c + d*tan(e + f*x))**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 0.83index.cc index_m
 operator + Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + a\tan(e + f\,x)\ 1\mathrm{i})^{5/2}}{(c + d\tan{(e + f\,x)})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(5/2)/(c + d*tan(e + f*x))^(3/2),x)
[Out] int((a + a*tan(e + f*x)*1i)^(5/2)/(c + d*tan(e + f*x))^(3/2), x)
```

3.1162 $\int \frac{(a+ia\tan(e+fx))^{3/2}}{(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=129

$$-\frac{2i\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+fx)}}\right)}{(c-id)^{3/2}f}-\frac{2a\sqrt{a+ia\tan(e+fx)}}{(ic+d)f\sqrt{c+d\tan(e+fx)}}$$

```
[Out] -2*I*a^(3/2)*arctanh(2^(1/2)*a^(1/2)*(c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2)/(
a+I*a*tan(f*x+e))^(1/2))*2^(1/2)/(c-I*d)^(3/2)/f-2*a*(a+I*a*tan(f*x+e))^(1/
2)/(I*c+d)/f/(c+d*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3626, 3625, 214}

$$-\frac{2i\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+fx)}}\right)}{f(c-id)^{3/2}}-\frac{2a\sqrt{a+ia\tan(e+fx)}}{f(d+ic)\sqrt{c+d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^(3/2)/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-2*I)*Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])/
(Sqrt[c - I*d]*Sqrt[a + I*a*Tan[e + f*x]])])/((c - I*d)^(3/2)*f) - (2*a*Sqr
t[a + I*a*Tan[e + f*x]])/((I*c + d)*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3626

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*b*(a + b*Tan[e + f*x])^(m - 1)*((c +
```

```
 d*Tan[e + f*x])^(n + 1)/(f*(m - 1)*(a*c - b*d))), x] + Dist[2*(a^2/(a*c -
b*d)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^{3/2}}{(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{2a\sqrt{a+ia\tan(e+fx)}}{(ic+d)f\sqrt{c+d\tan(e+fx)}} + \frac{(2a)\int \frac{\sqrt{a+ia\tan(e+fx)}}{\sqrt{c+d\tan(e+fx)}}\,dx}{c-id}$$

$$= -\frac{2a\sqrt{a+ia\tan(e+fx)}}{(ic+d)f\sqrt{c+d\tan(e+fx)}} + \frac{(4a^3)\,\mathrm{Subst}\left(\int \frac{1}{ac-iad-2a^2x^2}\,dx, x, \frac{\sqrt{c+\cdots}}{\sqrt{a+\cdots}}\right.}{(ic+d)f}$$

$$= -\frac{2i\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{(c-id)^{3/2}f} - \frac{2a\sqrt{a+ia\tan\cdots}}{(ic+d)f\sqrt{c+d\cdots}}$$

Mathematica [A]
time = 5.18, size = 209, normalized size = 1.62

$$\frac{2iae^{-i(e+fx)}\sqrt{a+ia\tan(e+fx)}\left(\sqrt{c-id}\,e^{i(e+fx)}-\sqrt{1+e^{2i(e+fx)}}\log\left(2e^{-ie}\left(\sqrt{c-id}\,e^{i(e+fx)}+\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)\right)\sqrt{c+d\tan(e+fx)}\right)}{(c-id)^{3/2}f\sqrt{c+d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(3/2)/(c + d*Tan[e + f*x])^(3/2),x]
[Out] ((2*I)*a*Sqrt[a + I*a*Tan[e + f*x]]*(Sqrt[c - I*d]*E^(I*(e + f*x)) - Sqrt[1
 + E^((2*I)*(e + f*x))]*Log[(2*(Sqrt[c - I*d]*E^(I*(e + f*x)) + Sqrt[1 + E^
((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*
(e + f*x)))]))/E^(I*e)]*Sqrt[c + d*Tan[e + f*x]]))/((c - I*d)^(3/2)*E^(I*(e
 + f*x))*f*Sqrt[c + d*Tan[e + f*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2563 vs. 2(103) = 206.
time = 0.59, size = 2564, normalized size = 19.88

method	result	size
derivativedivides	Expression too large to display	2564
default	Expression too large to display	2564

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/2/f*2^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)*(I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*
d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*ta
n(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c^2*(I*a*d)^(1/2)+2^(1/2)*ln(1/2*(2*I*a
*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(
1/2)+a*d)/(I*a*d)^(1/2))*a*c^2*(-a*(I*d-c))^(1/2)-ln((3*a*c+I*a*tan(f*x+e)*
c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(
1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c^2*(I*a*d)^(1/2)+2*2^(1/2)*ln(1/
2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(
I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c^2*d^2*(-a*(I*d-c))^(1/2)+2*I*2^(1/2)*c
^3*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(
1/2)+2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*ta
n(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*d^2*(-a*(I*d-c))^(1/2)
*tan(f*x+e)-2*(I*a*d)^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*2^(
1/2)*(-a*(I*d-c))^(1/2)*c^2*d+2^(1/2)*(-a*(I*d-c))^(1/2)*ln(1/2*(2*I*a*d*ta
n(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+
a*d)/(I*a*d)^(1/2))*a*c*d+2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+
d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c
*d*(-a*(I*d-c))^(1/2)*tan(f*x+e)+I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*t
an(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e))
)^(1/2))/(tan(f*x+e)+I))*a*c*d*(I*a*d)^(1/2)*tan(f*x+e)+2*I*2^(1/2)*ln(1/2*
(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*
a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*d^4*(-a*(I*d-c))^(1/2)*tan(f*x+e)+2*I*2^(1
/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e))
)^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c^3*d*(-a*(I*d-c))^(1/2)+2*I*2^
(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e
)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c*d^3*(-a*(I*d-c))^(1/2)-I*2^
(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e
)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*d^2*(-a*(I*d-c))^(1/2)*tan(f*
x+e)+2*I*2^(1/2)*c*d^2*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+
e)))^(1/2)*(I*a*d)^(1/2)-I*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c
+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*
c*d*(-a*(I*d-c))^(1/2)+I*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d
*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c^
2*(-a*(I*d-c))^(1/2)+I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*
2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(ta
n(f*x+e)+I))*a*d^2*(I*a*d)^(1/2)*tan(f*x+e)+I*ln((3*a*c+I*a*tan(f*x+e)*c-I*
a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*
tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c*d*(I*a*d)^(1/2)-ln((3*a*c+I*a*tan(f
*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x
```

```
+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c*d*(I*a*d)^(1/2)*tan(f*x+e
)+2*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan
(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c^3*d*(-a*(I*d-c))^(1/2
)*tan(f*x+e)+2*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e
))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c*d^3*(-a*(I
*d-c))^(1/2)*tan(f*x+e)+I*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+
d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c
*d*(-a*(I*d-c))^(1/2)*tan(f*x+e)+2*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c
+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(
1/2))*a*c^4*(-a*(I*d-c))^(1/2)-2*(I*a*d)^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan
(f*x+e)))^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*d^3+ln((3*a*c+I*a*tan(f*x+e)*c-I
*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I
*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*d^2*(I*a*d)^(1/2)*tan(f*x+e)+(I*a*d)
^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d
-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c
*d+2*I*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*
tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c^2*d^2*(-a*(I*d-c))
^(1/2)*tan(f*x+e))*a/(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)/(c^2+d^2)^
2/(I*a*d)^(1/2)/(-a*(I*d-c))^(1/2)/(c+d*tan(f*x+e))^(1/2)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] Timed out
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 571 vs. 2(101) = 202.
time = 1.32, size = 571, normalized size = 4.43

$$\frac{4\sqrt{2}\left(i\,ae^{(3i\,fx+3i\,e)}+i\,ae^{(i\,fx+i\,e)}\right)\sqrt{\frac{(c-i\,d)e^{(2i\,fx+2i\,e)}+c+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}+\left((c^2-2i\,cd-d^2)fe^{(2i\,fx+2i\,e)}+(c^2+d^2)f\right)\sqrt{\frac{8i\,a^3}{(-i\,c^3-3\,c^2d+3i\,cd^2+d^3)f^2}}\log\left(\frac{\left((i\,c^2+2\,cd-i\,d^2)f\sqrt{\frac{8i\,a^3}{(-i\,c^3-3\,c^2d+3i\,cd^2+d^3)f^2}}e^{(i\,fx+i\,e)}+2\sqrt{2}\left(ae^{(2i\,fx+2i\,e)}+a\right)\sqrt{\frac{(c-i\,d)e^{(2i\,fx+2i\,e)}+c+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-i\,fx-i\,e)}}{2a}\right)-\left((c^2-2i\,cd-d^2)fe^{(2i\,fx+2i\,e)}+(c^2+d^2)f\right)\sqrt{\frac{8i\,a^3}{(-i\,c^3-3\,c^2d+3i\,cd^2+d^3)f^2}}\log\left(\frac{\left((-i\,c^2-2\,cd+i\,d^2)f\sqrt{\frac{8i\,a^3}{(-i\,c^3-3\,c^2d+3i\,cd^2+d^3)f^2}}e^{(i\,fx+i\,e)}+2\sqrt{2}\left(ae^{(2i\,fx+2i\,e)}+a\right)\sqrt{\frac{(c-i\,d)e^{(2i\,fx+2i\,e)}+c+i\,d}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\right)e^{(-i\,fx-i\,e)}}{2a}\right)}{2\left((c^2-2i\,cd-d^2)fe^{(2i\,fx+2i\,e)}+(c^2+d^2)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/2*(4*sqrt(2)*(I*a*e^(3*I*f*x + 3*I*e) + I*a*e^(I*f*x + I*e))*sqrt(((c - I
*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*
I*f*x + 2*I*e) + 1)) + ((c^2 - 2*I*c*d - d^2)*f*e^(2*I*f*x + 2*I*e) + (c^2
+ d^2)*f)*sqrt(8*I*a^3/((-I*c^3 - 3*c^2*d + 3*I*c*d^2 + d^3)*f^2))*log(1/2*
```

```
((I*c^2 + 2*c*d - I*d^2)*f*sqrt(8*I*a^3/((-I*c^3 - 3*c^2*d + 3*I*c*d^2 + d^
3)*f^2))*e^(I*f*x + I*e) + 2*sqrt(2)*(a*e^(2*I*f*x + 2*I*e) + a)*sqrt(((c -
 I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(
2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/a) - ((c^2 - 2*I*c*d - d^2)*f*e^(2
*I*f*x + 2*I*e) + (c^2 + d^2)*f)*sqrt(8*I*a^3/((-I*c^3 - 3*c^2*d + 3*I*c*d^
2 + d^3)*f^2))*log(1/2*((-I*c^2 - 2*c*d + I*d^2)*f*sqrt(8*I*a^3/((-I*c^3 -
3*c^2*d + 3*I*c*d^2 + d^3)*f^2))*e^(I*f*x + I*e) + 2*sqrt(2)*(a*e^(2*I*f*x
+ 2*I*e) + a)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x +
2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/a))/((c^2
- 2*I*c*d - d^2)*f*e^(2*I*f*x + 2*I*e) + (c^2 + d^2)*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(e+fx)-i))^{\frac{3}{2}}}{(c+d\tan(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(3/2)/(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((I*a*(tan(e + f*x) - I))**(3/2)/(c + d*tan(e + f*x))**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad Ar
gument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Argum
ent Ty
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\tan(e+f\,x)\,1\mathrm{i})^{3/2}}{(c+d\tan(e+f\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(3/2)/(c + d*tan(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^(3/2)/(c + d*tan(e + f*x))^(3/2), x)
```

3.1163 $\int \frac{\sqrt{a+ia\tan(e+fx)}}{(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=129

$$-\frac{i\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{(c-id)^{3/2}f}-\frac{2d\sqrt{a+ia\tan(e+fx)}}{(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}}$$

```
[Out] -I*arctanh(2^(1/2)*a^(1/2)*(c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2)/(a+I*a*tan(
f*x+e))^(1/2))*2^(1/2)*a^(1/2)/(c-I*d)^(3/2)/f-2*d*(a+I*a*tan(f*x+e))^(1/2)
/(c^2+d^2)/f/(c+d*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3629, 3625, 214}

$$-\frac{2d\sqrt{a+ia\tan(e+fx)}}{f\,(c^2+d^2)\sqrt{c+d\tan(e+fx)}}-\frac{i\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{f(c-id)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[e + f*x]]/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-I)*Sqrt[2]*Sqrt[a]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])/(S
qrt[c - I*d]*Sqrt[a + I*a*Tan[e + f*x]])])/((c - I*d)^(3/2)*f) - (2*d*Sqrt[
a + I*a*Tan[e + f*x]])/((c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3629

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-d)*(a + b*Tan[e + f*x])^m*((c + d*Ta
```

```
n[e + f*x])^(n + 1)/(f*m*(c^2 + d^2))), x] + Dist[a/(a*c - b*d), Int[(a + b
*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+ia\tan(e+fx)}}{(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{2d\sqrt{a+ia\tan(e+fx)}}{(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} + \frac{\int \frac{\sqrt{a+ia\tan(e+fx)}}{\sqrt{c+d\tan(e+fx)}}\,dx}{c-id}$$

$$= -\frac{2d\sqrt{a+ia\tan(e+fx)}}{(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{ac-iad-2a^2x^2}\,dx, x, \frac{\sqrt{c+\cdots}}{\sqrt{a+\cdots}}\right.}{(ic+d)f}$$

$$= -\frac{i\sqrt{2}\,\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+fx)}}\right)}{(c-id)^{3/2}f} - \frac{2d\sqrt{a+ia\tan(\cdots}}{(c^2+d^2)\,f\sqrt{c+d\tan\cdots}}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 337 vs. $2(129) = 258$.
time = 4.64, size = 337, normalized size = 2.61

$$\frac{\sqrt{2}\,\sqrt{e^{ifx}}\,\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\,\sqrt{1+e^{2i(e+fx)}}\left(-\frac{2d\sqrt{1+e^{2i(e+fx)}}\,\sqrt{c-\frac{id(-1+e^{2i(e+fx)})}{1+e^{2i(e+fx)}}}}{(c-id)(c+id)\left(-id(-1+e^{2i(e+fx)})+c(1+e^{2i(e+fx)})\right)} - \frac{ie^{-i(e+fx)}\log\left(2\left(\sqrt{c-id}\,e^{i(e+fx)}+\sqrt{1+e^{2i(e+fx)}}\,\sqrt{c-\frac{id(-1+e^{2i(e+fx)})}{1+e^{2i(e+fx)}}}\right)\right)}{(c-id)^{3/2}}\right)\sqrt{a+ia\tan(e+fx)}}{f\sqrt{\sec(e+fx)}\,\sqrt{\cos(fx)+i\sin(fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Tan[e + f*x]]/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (Sqrt[2]*Sqrt[E^(I*f*x)]*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*Sq
rt[1 + E^((2*I)*(e + f*x))]*((-2*d*Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (
I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))])/((c - I*d)*(c +
 I*d)*((-I)*d*(-1 + E^((2*I)*(e + f*x))) + c*(1 + E^((2*I)*(e + f*x))))) -
(I*Log[2*(Sqrt[c - I*d]*E^(I*(e + f*x)) + Sqrt[1 + E^((2*I)*(e + f*x))]*Sqr
t[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))])])/((c -
I*d)^(3/2)*E^(I*(e + f*x))))*Sqrt[a + I*a*Tan[e + f*x]])/(f*Sqrt[Sec[e + f*
x]]*Sqrt[Cos[f*x] + I*Sin[f*x]])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1290 vs. $2(104) = 208$.
time = 0.67, size = 1291, normalized size = 10.01

method	result	size
derivativedivides	Expression too large to display	1291
default	Expression too large to display	1291

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/2/f*2^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)*(-2*2^(1/2)*d^2*(-a*(I*d-c))^(1/2)
*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*tan(f*x+e)-2*I*ln((3*a*c+I*a*t
an(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan
(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c*d^2*tan(f*x+e)^2+2*I*
2^(1/2)*c*d*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*
tan(f*x+e)-I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-
a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I
))*a*c^2*d*tan(f*x+e)-I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2
*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(t
an(f*x+e)+I))*a*d^3*tan(f*x+e)-ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f
*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1
/2))/(tan(f*x+e)+I))*a*c^2*d*tan(f*x+e)^2+ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+
3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(
f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*d^3*tan(f*x+e)^2+2*I*2^(1/2)*(-a*(I*d-c))
^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*d^2+I*ln((3*a*c+I*a*tan(
f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*
x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c^3-I*ln((3*a*c+I*a*tan(f*
x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+
e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c*d^2-ln((3*a*c+I*a*tan(f*x+
e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e)
)*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c^3*tan(f*x+e)-ln((3*a*c+I*a*t
an(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan
(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c*d^2*tan(f*x+e)+2*2^(1
/2)*c*d*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)-2*ln
((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2
)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c^2*d)/(c+
d*tan(f*x+e))^(1/2)/(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)/(c^2+d^2)/(
-a*(I*d-c))^(1/2)/(I*c-d)/(-tan(f*x+e)+I)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 8970 vs. $2(103) = 206$.
time = 1.05, size = 8970, normalized size = 69.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I*a*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="max
ima")

[Out] -1/4*(8*((I*sqrt(2)*c^2*d + I*sqrt(2)*d^3)*cos(f*x + e) - (sqrt(2)*c^2*d +
sqrt(2)*d^3)*sin(f*x + e))*sqrt(a) + ((c^2 + d^2)*cos(2*f*x + 2*e)^2 + 4*c*
d*sin(2*f*x + 2*e) + (c^2 + d^2)*sin(2*f*x + 2*e)^2 + c^2 + d^2 + 2*(c^2 -
d^2)*cos(2*f*x + 2*e))^(1/4)*((sqrt(c^2 + d^2)*(2*(-I*sqrt(2)*cos(1/2*arcta
n2(-d*cos(2*f*x + 2*e) + c*sin(2*f*x + 2*e) + d, c*cos(2*f*x + 2*e) + d*sin
(2*f*x + 2*e) + c)) + sqrt(2)*sin(1/2*arctan2(-d*cos(2*f*x + 2*e) + c*sin(2
*f*x + 2*e) + d, c*cos(2*f*x + 2*e) + d*sin(2*f*x + 2*e) + c)))*arctan2((sq
rt(c^2 + d^2)*d*cos(f*x + e) - sqrt(c^2 + d^2)*c*sin(f*x + e) + (c^2 + d^2)
*(((c^2 + d^2)*cos(f*x + e)^4 + (c^2 + d^2)*sin(f*x + e)^4 + 8*c*d*cos(f*x
+ e)*sin(f*x + e) + 2*(c^2 - d^2)*cos(f*x + e)^2 + 2*((c^2 + d^2)*cos(f*x +
e)^2 - c^2 + d^2)*sin(f*x + e)^2 + c^2 + d^2)/(c^2 + d^2))^(1/4)*sin(1/2*a
rctan2(-2*(c*d*cos(f*x + e)^2 - c*d*sin(f*x + e)^2 - (c^2 - d^2)*cos(f*x +
e)*sin(f*x + e))/(c^2 + d^2), (4*c*d*cos(f*x + e)*sin(f*x + e) + (c^2 - d^2
)*cos(f*x + e)^2 - (c^2 - d^2)*sin(f*x + e)^2 + c^2 + d^2)/(c^2 + d^2))))/(
c^2 + d^2), -(sqrt(c^2 + d^2)*c*cos(f*x + e) + sqrt(c^2 + d^2)*d*sin(f*x +
e) - (c^2 + d^2)*(((c^2 + d^2)*cos(f*x + e)^4 + (c^2 + d^2)*sin(f*x + e)^4
+ 8*c*d*cos(f*x + e)*sin(f*x + e) + 2*(c^2 - d^2)*cos(f*x + e)^2 + 2*((c^2
+ d^2)*cos(f*x + e)^2 - c^2 + d^2)*sin(f*x + e)^2 + c^2 + d^2)/(c^2 + d^2))
^(1/4)*cos(1/2*arctan2(-2*(c*d*cos(f*x + e)^2 - c*d*sin(f*x + e)^2 - (c^2 -
d^2)*cos(f*x + e)*sin(f*x + e))/(c^2 + d^2), (4*c*d*cos(f*x + e)*sin(f*x +
e) + (c^2 - d^2)*cos(f*x + e)^2 - (c^2 - d^2)*sin(f*x + e)^2 + c^2 + d^2)/
(c^2 + d^2))))/(c^2 + d^2)) - (sqrt(2)*cos(1/2*arctan2(-d*cos(2*f*x + 2*e)
+ c*sin(2*f*x + 2*e) + d, c*cos(2*f*x + 2*e) + d*sin(2*f*x + 2*e) + c)) + I
*sqrt(2)*sin(1/2*arctan2(-d*cos(2*f*x + 2*e) + c*sin(2*f*x + 2*e) + d, c*co
s(2*f*x + 2*e) + d*sin(2*f*x + 2*e) + c)))*log(((c^2 + d^2)*sqrt(((c^2 + d^
2)*cos(f*x + e)^4 + (c^2 + d^2)*sin(f*x + e)^4 + 8*c*d*cos(f*x + e)*sin(f*x
+ e) + 2*(c^2 - d^2)*cos(f*x + e)^2 + 2*((c^2 + d^2)*cos(f*x + e)^2 - c^2
+ d^2)*sin(f*x + e)^2 + c^2 + d^2)/(c^2 + d^2))*cos(1/2*arctan2(-2*(c*d*cos
(f*x + e)^2 - c*d*sin(f*x + e)^2 - (c^2 - d^2)*cos(f*x + e)*sin(f*x + e))/(
c^2 + d^2), (4*c*d*cos(f*x + e)*sin(f*x + e) + (c^2 - d^2)*cos(f*x + e)^2 -
(c^2 - d^2)*sin(f*x + e)^2 + c^2 + d^2)/(c^2 + d^2)))^2 + (c^2 + d^2)*sqrt
(((c^2 + d^2)*cos(f*x + e)^4 + (c^2 + d^2)*sin(f*x + e)^4 + 8*c*d*cos(f*x +
e)*sin(f*x + e) + 2*(c^2 - d^2)*cos(f*x + e)^2 + 2*((c^2 + d^2)*cos(f*x +
e)^2 - c^2 + d^2)*sin(f*x + e)^2 + c^2 + d^2)/(c^2 + d^2))*sin(1/2*arctan2(
-2*(c*d*cos(f*x + e)^2 - c*d*sin(f*x + e)^2 - (c^2 - d^2)*cos(f*x + e)*sin(
f*x + e))/(c^2 + d^2), (4*c*d*cos(f*x + e)*sin(f*x + e) + (c^2 - d^2)*cos(f
*x + e)^2 - (c^2 - d^2)*sin(f*x + e)^2 + c^2 + d^2)/(c^2 + d^2)))^2 + (c^2
+ d^2)*cos(f*x + e)^2 + (c^2 + d^2)*sin(f*x + e)^2 - 2*(sqrt(c^2 + d^2)*c*c
os(f*x + e) + sqrt(c^2 + d^2)*d*sin(f*x + e))*(((c^2 + d^2)*cos(f*x + e)^4
+ (c^2 + d^2)*sin(f*x + e)^4 + 8*c*d*cos(f*x + e)*sin(f*x + e) + 2*(c^2 - d
^2)*cos(f*x + e)^2 + 2*((c^2 + d^2)*cos(f*x + e)^2 - c^2 + d^2)*sin(f*x + e
)^2 + c^2 + d^2)/(c^2 + d^2))^(1/4)*cos(1/2*arctan2(-2*(c*d*cos(f*x + e)^2

```
- c*d*sin(f*x + e)^2 - (c^2 - d^2)*cos(f*x + e)*sin(f*x + e))/(c^2 + d^2),
(4*c*d*cos(f*x + e)*sin(f*x + e) + (c^2 - d^2)*cos(f*x + e)^2 - (c^2 - d^2)
*sin(f*x + e)^2 + c^2 + d^2)/(c^2 + d^2))) + 2*(sqrt(c^2 + d^2)*d*cos(f*x +
 e) - sqrt(c^2 + d^2)*c*sin(f*x + e))*(((c^2 + d^2)*cos(f*x + e)^4 + (c^2 +
 d^2)*sin(f*x + e)^4 + 8*c*d*cos(f*x + e)*sin(f*x + e) + 2*(c^2 - d^2)*cos(
f*x + e)^2 + 2*((c^2 + d^2)*cos(f*x + e)^2 - c^2 + d^2)*sin(f*x + e)^2 + c^
2 + d^2)/(c^2 + d^2))^(1/4)*sin(1/2*arctan2(-2*(c*d*cos(f*x + e)^2 - c*d*si
n(f*x + e)^2 - (c^2 - d^2)*cos(f*x + e)*sin(f*x + e))/(c^2 + d^2), (4*c*d*c
os(f*x + e)*sin(f*x + e) + (c^2 - d^2)*cos(f*x + e)^2 - (c^2 - d^2)*sin(f*x
 + e)^2 + c^2 + d^2)/(c^2 + d^2))))/(c^2 + d^2)))*sqrt(a) + 2*(2*(I*sqrt(2)
*c*cos(1/2*arctan2(-d*cos(2*f*x + 2*e) + c*sin(2*f*x + 2*e) + d, c*cos(2*f*
x + 2*e) + d*sin(2*f*x + 2*e) + c)) - sqrt(2)*c*sin(1/2*arctan2(-d*cos(2*f*
x + 2*e) + c*sin(2*f*x + 2*e) + d, c*cos(2*f*x + 2*e) + d*sin(2*f*x + 2*e)
+ c)))*arctan2((sqrt(c^2 + d^2)*d*cos(f*x + e) - sqrt(c^2 + d^2)*c*sin(f*x
+ e) + (c^2 + d^2)*(((c^2 + d^2)*cos(f*x + e)^4 + (c^2 + d^2)*sin(f*x + e)^
4 + 8*c*d*cos(f*x + e)*sin(f*x + e) + 2*(c^2 - d^2)*cos(f*x + e)^2 + 2*((c^
2 + d^2)*cos(f*x + e)^2 - c^2 + d^2)*sin(f*x + e)^2 + c^2 + d^2)/(c^2 + d^2
))^(1/4)*sin(1/2*arctan2(-2*(c*d*cos(f*x + e)^2 - c*d*sin(f*x + e)^2 - (c^2
 - d^2)*cos(f*x + e)*sin(f*x + e))/(c^2 + d^2), (4*c*d*cos(f*x + e)*sin(f*x
 + e) + (c^2 - d^2)*cos(f*x + e)^2 - (c^2 - d^2)*sin(f*x + e)^2 + c^2 + d^2
)/(c^2 + d^2))))/(c^2 + d^2), -(sqrt(c^2 + d^2)*c*cos(f*x + e) + sqrt(c^2 +
 d^2)*d*sin(f*x + e) - (c^2 + d^2)*(((c^2 + d^2)*cos(f*x + e)^4 + (c^2 + d^
2)*sin(f*x + e)^4 + 8*c*d*cos(f*x + e)*sin(f*x + e) + 2*(c^2 - d^2)*cos(f*x
 + e)^2 + 2*((c^2 + d^2)*cos(f*x + e)^2 - c^2 + d^2)*sin(f*x + e)^2 + c^2 +
 d^2)/(c^2 + d^2))^(1/4)*cos(1/2*arctan2(-2*(c*...
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(103) = 206$.
time = 0.82, size = 619, normalized size = 4.80

$$\frac{4\sqrt{2}\left(ide^{(3ifx+3ie)}+ide^{(ifx+ie)}\right)\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}-\left((ic^3+c^2d+icd^2+d^3)fe^{(2ifx+2ie)}+(ic^3-c^2d+icd^2-d^3)f\right)\sqrt{\frac{2ia}{(-ic^3-3c^2d+3icd^2+d^3)f^2}}\log\left(\left((ic^2+2cd-id^2)f\sqrt{\frac{2ia}{(-ic^3-3c^2d+3icd^2+d^3)f^2}}e^{(ifx+ie)}+\sqrt{2}\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}\left(e^{(2ifx+2ie)}+1\right)\right)e^{(-ifx-ie)}\right)-\left((-ic^3-c^2d-icd^2-d^3)fe^{(2ifx+2ie)}+(-ic^3+c^2d-icd^2+d^3)f\right)\sqrt{\frac{2ia}{(-ic^3-3c^2d+3icd^2+d^3)f^2}}\log\left(\left((-ic^2-2cd+id^2)f\sqrt{\frac{2ia}{(-ic^3-3c^2d+3icd^2+d^3)f^2}}e^{(ifx+ie)}+\sqrt{2}\sqrt{\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}\left(e^{(2ifx+2ie)}+1\right)\right)e^{(-ifx-ie)}\right)}{2\left((-ic^3-c^2d-icd^2-d^3)fe^{(2ifx+2ie)}+(-ic^3+c^2d-icd^2+d^3)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/2*(4*sqrt(2)*(I*d*e^(3*I*f*x + 3*I*e) + I*d*e^(I*f*x + I*e))*sqrt(((c - I
*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*
I*f*x + 2*I*e) + 1)) - ((I*c^3 + c^2*d + I*c*d^2 + d^3)*f*e^(2*I*f*x + 2*I*
e) + (I*c^3 - c^2*d + I*c*d^2 - d^3)*f)*sqrt(2*I*a/((-I*c^3 - 3*c^2*d + 3*I
*c*d^2 + d^3)*f^2))*log(((I*c^2 + 2*c*d - I*d^2)*f*sqrt(2*I*a/((-I*c^3 - 3*
c^2*d + 3*I*c*d^2 + d^3)*f^2))*e^(I*f*x + I*e) + sqrt(2)*sqrt(((c - I*d)*e^
(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x
+ 2*I*e) + 1))*(e^(2*I*f*x + 2*I*e) + 1))*e^(-I*f*x - I*e)) - ((-I*c^3 - c^
2*d - I*c*d^2 - d^3)*f*e^(2*I*f*x + 2*I*e) + (-I*c^3 + c^2*d - I*c*d^2 + d^
```

```
3)*f)*sqrt(2*I*a/((-I*c^3 - 3*c^2*d + 3*I*c*d^2 + d^3)*f^2))*log(((-I*c^2 -
 2*c*d + I*d^2)*f*sqrt(2*I*a/((-I*c^3 - 3*c^2*d + 3*I*c*d^2 + d^3)*f^2))*e^
(I*f*x + I*e) + sqrt(2)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(
2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*(e^(2*I*f*x + 2*I*
e) + 1))*e^(-I*f*x - I*e)))/((-I*c^3 - c^2*d - I*c*d^2 - d^3)*f*e^(2*I*f*x
+ 2*I*e) + (-I*c^3 + c^2*d - I*c*d^2 + d^3)*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ia\left(\tan\left(e+fx\right)-i\right)}}{\left(c+d\tan\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(1/2)/(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(e + f*x) - I))/(c + d*tan(e + f*x))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\tan\left(e+f\,x\right)\,1{}\mathrm{i}}}{{\left(c+d\tan\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(1/2)/(c + d*tan(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^(1/2)/(c + d*tan(e + f*x))^(3/2), x)
```

3.1164 $\int \frac{1}{\sqrt{a+ia\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=194

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\,(c-id)^{3/2}f}-\frac{1}{(ic-d)f\sqrt{a+ia\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}+\frac{(\cdots}{a(c-\cdots}$$

```
[Out] -1/2*I*arctanh(2^(1/2)*a^(1/2)*(c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2)/(a+I*a*
tan(f*x+e))^(1/2))/(c-I*d)^(3/2)/f*2^(1/2)/a^(1/2)-1/(I*c-d)/f/(a+I*a*tan(f
*x+e))^(1/2)/(c+d*tan(f*x+e))^(1/2)+(c-3*I*d)*d*(a+I*a*tan(f*x+e))^(1/2)/a/
(c-I*d)/(c+I*d)^2/f/(c+d*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3640, 3679, 12, 3625, 214}

$$\frac{d(c-3id)\sqrt{a+ia\tan(e+fx)}}{af(c-id)(c+id)^2\sqrt{c+d\tan(e+fx)}}-\frac{1}{f(-d+ic)\sqrt{a+ia\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\,f(c-id)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + I*a*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] ((-I)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])/(Sqrt[c - I*d]*Sqr
t[a + I*a*Tan[e + f*x]])])/(Sqrt[2]*Sqrt[a]*(c - I*d)^(3/2)*f) - 1/((I*c -
d)*f*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]]) + ((c - (3*I)*d)*
d*Sqrt[a + I*a*Tan[e + f*x]])/(a*(c - I*d)*(c + I*d)^2*f*Sqrt[c + d*Tan[e +
 f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
```

```
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+ia\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{1}{(ic-d)f\sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{1}{(ic-d)f\sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{1}{(ic-d)f\sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{1}{(ic-d)f\sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+fx)}}\right)}{\sqrt{2}\,\sqrt{a}\,(c-id)^{3/2}f} - \frac{}{(ic-d}$$

Mathematica [**A**]
time = 4.64, size = 267, normalized size = 1.38

$$\frac{\sqrt{\sec(e+fx)}\left(-\frac{i\sqrt{2}\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\sqrt{1+e^{2i(e+fx)}}\log\left(2\left(\sqrt{c-id}\,e^{i(e+fx)}+\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)\right)}{(c-id)^{3/2}}+\frac{2ic^2+2cd-4id^2+2d(ic+3d)\tan(e+fx)}{(c-id)(c+id)^2\sqrt{\sec(e+fx)}\sqrt{c+d\tan(e+fx)}}\right)}{2f\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + I*a*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] (Sqrt[Sec[e + f*x]]*(((-I)*Sqrt[2]*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e +
f*x)))]*Sqrt[1 + E^((2*I)*(e + f*x))]*Log[2*(Sqrt[c - I*d]*E^(I*(e + f*x))
+ Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(
1 + E^((2*I)*(e + f*x)))])])/(c - I*d)^(3/2) + ((2*I)*c^2 + 2*c*d - (4*I)*d
^2 + 2*d*(I*c + 3*d)*Tan[e + f*x])/((c - I*d)*(c + I*d)^2*Sqrt[Sec[e + f*x]
]*Sqrt[c + d*Tan[e + f*x]])))/(2*f*Sqrt[a + I*a*Tan[e + f*x]])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3195 vs. 2(160) = 320.
time = 0.72, size = 3196, normalized size = 16.47

method	result	size
derivativedivides	Expression too large to display	3196
default	Expression too large to display	3196

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(3/2),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/4/f*(-6*2^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1
/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*
x+e)+I))*c^3*d^2*(-a*(I*d-c))^(1/2)+2^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*
d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*ta
n(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c*d^4*(-a*(I*d-c))^(1/2)-4*I*c^4*d*(a*(c+
d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*tan(f*x+e)^2-16*I*(a*(c+d*tan(f*x+e))
*(1+I*tan(f*x+e)))^(1/2)*c^2*d^3*tan(f*x+e)^2+8*I*(a*(c+d*tan(f*x+e))*(1+I*
tan(f*x+e)))^(1/2)*c^3*d^2*tan(f*x+e)+12*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x
+e)))^(1/2)*c*d^4*tan(f*x+e)+8*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2
)*d^5-20*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*d^5*tan(f*x+e)+4*c*d^4
*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)-12*I*(a*(c+d*tan(f*x+e))*(1+I*
tan(f*x+e)))^(1/2)*d^5*tan(f*x+e)^2-4*I*c^5*(a*(c+d*tan(f*x+e))*(1+I*tan(f*
x+e)))^(1/2)*tan(f*x+e)+8*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c^2
*d^3-8*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c^3*d^2*tan(f*x+e)^2-8*(
a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c*d^4*tan(f*x+e)^2-4*(a*(c+d*tan
```

(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c^4*d*tan(f*x+e)-24*(a*(c+d*tan(f*x+e))*(1
+I*tan(f*x+e)))^(1/2)*c^2*d^3*tan(f*x+e)+2^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c
-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1
+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c^5*(-a*(I*d-c))^(1/2)-2^(1/2)*ln((3
*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(
a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c^5*(-a*(I*d-c)
)^(1/2)*tan(f*x+e)^2-2^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x
+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2
))/(tan(f*x+e)+I))*d^5*(-a*(I*d-c))^(1/2)*tan(f*x+e)^3+2^(1/2)*ln((3*a*c+I*
a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*
tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*d^5*(-a*(I*d-c))^(1/2)
*tan(f*x+e)-4*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c^5+4*I*2^(1/2)*l
n((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/
2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c^4*d*(-a*(
I*d-c))^(1/2)-4*I*2^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)
+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/
(tan(f*x+e)+I))*c^2*d^3*(-a*(I*d-c))^(1/2)+6*2^(1/2)*ln((3*a*c+I*a*tan(f*x+
e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e)
)*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c^2*d^3*(-a*(I*d-c))^(1/2)*tan(f
*x+e)^3-2*2^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/
2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x
+e)+I))*c^3*d^2*(-a*(I*d-c))^(1/2)*tan(f*x+e)^2+7*2^(1/2)*ln((3*a*c+I*a*tan
(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f
x+e))(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c*d^4*(-a*(I*d-c))^(1/2)*ta
n(f*x+e)^2-7*2^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^
(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(
f*x+e)+I))*c^4*d*(-a*(I*d-c))^(1/2)*tan(f*x+e)+2*2^(1/2)*ln((3*a*c+I*a*tan(
f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*
x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c^2*d^3*(-a*(I*d-c))^(1/2)*t
an(f*x+e)+2*I*2^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2
^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan
(f*x+e)+I))*d^5*(-a*(I*d-c))^(1/2)*tan(f*x+e)^2+2*I*2^(1/2)*ln((3*a*c+I*a*t
an(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan
(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c^5*(-a*(I*d-c))^(1/2)*ta
n(f*x+e)-4*I*2^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^
(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(
f*x+e)+I))*c^3*d^2*(-a*(I*d-c))^(1/2)*tan(f*x+e)^3+4*I*2^(1/2)*ln((3*a*c+I*
a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*
tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c*d^4*(-a*(I*d-c))^(1/
2)*tan(f*x+e)^3-2*I*2^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+
e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)
)/(tan(f*x+e)+I))*c^4*d*(-a*(I*d-c))^(1/2)*tan(f*x+e)^2-8*I*2^(1/2)*ln((3*a
*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*
(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c^2*d^3*(-a*(I*d-
c))^(1/2)*tan(f*x+e)^2-8*I*2^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*t

```
an(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e))
)^(1/2))/(tan(f*x+e)+I))*c^3*d^2*(-a*(I*d-c))^(1/2)*tan(f*x+e)-2*I*2^(1/2)*
ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1
/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c*d^4*(-a*
(I*d-c))^(1/2)*tan(f*x+e)-2^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*ta
n(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))
^(1/2))/(tan(f*x+e)+I))*c^4*d*(-a*(I*d-c))^(1/2...
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 694 vs. 2(156) = 312.
time = 1.38, size = 694, normalized size = 3.58

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="f
ricas")
```

```
[Out] -1/4*(2*sqrt(2)*(-I*c^2 - I*d^2 + (-I*c^2 - 2*c*d + 5*I*d^2)*e^(4*I*f*x + 4
*I*e) + 2*(-I*c^2 - c*d + 2*I*d^2)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(
2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x +
 2*I*e) + 1)) - ((a*c^4 + 2*a*c^2*d^2 + a*d^4)*f*e^(3*I*f*x + 3*I*e) + (a*c
^4 + 2*I*a*c^3*d + 2*I*a*c*d^3 - a*d^4)*f*e^(I*f*x + I*e))*sqrt(2*I/((-I*a*
c^3 - 3*a*c^2*d + 3*I*a*c*d^2 + a*d^3)*f^2))*log((I*a*c^2 + 2*a*c*d - I*a*d
^2)*f*sqrt(2*I/((-I*a*c^3 - 3*a*c^2*d + 3*I*a*c*d^2 + a*d^3)*f^2))*e^(I*f*x
 + I*e) + sqrt(2)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*
x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*(e^(2*I*f*x + 2*I*e) + 1
)) + ((a*c^4 + 2*a*c^2*d^2 + a*d^4)*f*e^(3*I*f*x + 3*I*e) + (a*c^4 + 2*I*a*
c^3*d + 2*I*a*c*d^3 - a*d^4)*f*e^(I*f*x + I*e))*sqrt(2*I/((-I*a*c^3 - 3*a*c
^2*d + 3*I*a*c*d^2 + a*d^3)*f^2))*log((-I*a*c^2 - 2*a*c*d + I*a*d^2)*f*sqrt
(2*I/((-I*a*c^3 - 3*a*c^2*d + 3*I*a*c*d^2 + a*d^3)*f^2))*e^(I*f*x + I*e) +
sqrt(2)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e)
 + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*(e^(2*I*f*x + 2*I*e) + 1)))/((a*c^
```

```
4 + 2*a*c^2*d^2 + a*d^4)*f*e^(3*I*f*x + 3*I*e) + (a*c^4 + 2*I*a*c^3*d + 2*I
*a*c*d^3 - a*d^4)*f*e^(I*f*x + I*e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ia\left(\tan\left(e+fx\right)-i\right)}\,\left(c+d\tan\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**(1/2)/(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/(sqrt(I*a*(tan(e + f*x) - I))*(c + d*tan(e + f*x))**(3/2)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad Ar
gument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Argum
ent Ty
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+a\tan\left(e+f\,x\right)\,1{}\mathrm{i}}\,{\left(c+d\tan\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^(1/2)*(c + d*tan(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + a*tan(e + f*x)*1i)^(1/2)*(c + d*tan(e + f*x))^(3/2)), x)
```

3.1165 $\int \frac{1}{(a+ia\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=269

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}(c-id)^{3/2}f}-\frac{1}{3(ic-d)f(a+ia\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}+\frac{}{6a}$$

```
[Out] -1/4*I*arctanh(2^(1/2)*a^(1/2)*(c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2)/(a+I*a*
tan(f*x+e))^(1/2))/a^(3/2)/(c-I*d)^(3/2)/f*2^(1/2)+1/6*(3*I*c-11*d)/a/(c+I*
d)^2/f/(a+I*a*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(1/2)+1/6*(3*c-5*I*d)*(c+5
*I*d)*d*(a+I*a*tan(f*x+e))^(1/2)/a^2/(c-I*d)/(c+I*d)^3/f/(c+d*tan(f*x+e))^(
1/2)-1/3/(I*c-d)/f/(c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.58, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3640, 3677, 3679, 12, 3625, 214}

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\,\sqrt{a+ia\tan(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f(c-id)^{3/2}}+\frac{d(3c-5id)(c+5id)\sqrt{a+ia\tan(e+fx)}}{6a^2f(c-id)(c+id)^3\sqrt{c+d\tan(e+fx)}}+\frac{-11d+3ic}{6af(c+id)^2\sqrt{a+ia\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}-\frac{1}{3f(-d+ic)(a+ia\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] ((-1/2*I)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])/(Sqrt[c - I*d]
*Sqrt[a + I*a*Tan[e + f*x]])])/(Sqrt[2]*a^(3/2)*(c - I*d)^(3/2)*f) - 1/(3*(
I*c - d)*f*(a + I*a*Tan[e + f*x])^(3/2)*Sqrt[c + d*Tan[e + f*x]]) + ((3*I)*
c - 11*d)/(6*a*(c + I*d)^2*f*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c + d*Tan[e +
f*x]]) + ((3*c - (5*I)*d)*(c + (5*I)*d)*d*Sqrt[a + I*a*Tan[e + f*x]])/(6*a^
2*(c - I*d)*(c + I*d)^3*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
```

```
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{1}{3(ic-d)f(a+ia\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+}$$

$$= -\frac{1}{3(ic-d)f(a+ia\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+}$$

$$= -\frac{1}{3(ic-d)f(a+ia\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+}$$

$$= -\frac{1}{3(ic-d)f(a+ia\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+}$$

$$= -\frac{1}{3(ic-d)f(a+ia\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{2\sqrt{2}a^{3/2}(c-id)^{3/2}f} - \frac{}{3(ic}$$

Mathematica [A]
time = 7.30, size = 333, normalized size = 1.24

$$\frac{\sec^{\frac{3}{2}}(e+fx)\left(-\frac{i\sqrt{2}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{3/2}(1+e^{2i(e+fx)})^{3/2}\log\left(2\left(\sqrt{c-id}\,e^{i(e+fx)}+\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id(-1+e^{2i(e+fx)})}{1+e^{2i(e+fx)}}}\right)\right)}{(c-id)^{3/2}}+\frac{\sqrt{\sec(e+fx)}\,(5ic^3-13c^2d+5icd^2-13d^3+(5ic^3-7c^2d+25icd^2+37d^3)\cos(2(e+fx))-(3c^3+5ic^2d+23cd^2-39id^3)\sin(2(e+fx)))}{3(c-id)(c+id)^3\sqrt{c+d\tan(e+fx)}}\right)}{4f(a+ia\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] (Sec[e + f*x]^(3/2)*(((-I)*Sqrt[2]*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)
)))^(3/2)*(1 + E^((2*I)*(e + f*x)))^(3/2)*Log[2*(Sqrt[c - I*d]*E^(I*(e + f*
x)) + Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*x))
))/(1 + E^((2*I)*(e + f*x)))])])/(c - I*d)^(3/2) + (Sqrt[Sec[e + f*x]]*((5*
I)*c^3 - 13*c^2*d + (5*I)*c*d^2 - 13*d^3 + ((5*I)*c^3 - 7*c^2*d + (25*I)*c*
d^2 + 37*d^3)*Cos[2*(e + f*x)] - (3*c^3 + (5*I)*c^2*d + 23*c*d^2 - (39*I)*d
^3)*Sin[2*(e + f*x)]))/(3*(c - I*d)*(c + I*d)^3*Sqrt[c + d*Tan[e + f*x]])))
/(4*f*(a + I*a*Tan[e + f*x])^(3/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4834 vs. 2(219) = 438.
time = 0.66, size = 4835, normalized size = 17.97

method	result	size
derivativedivides	Expression too large to display	4835
default	Expression too large to display	4835

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+I*a*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/24/f*(-136*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c^4*d^2*tan(f*x+e)^2-380*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c^2*d^4*tan(f*x+e)^2-92*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c^5*d*tan(f*x+e)-40*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c^3*d^3*tan(f*x+e)+52*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c*d^5*tan(f*x+e)-12*I*c^5*d*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*tan(f*x+e)^3-72*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c^3*d^3*tan(f*x+e)^3-60*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c*d^5*tan(f*x+e)^3-3*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c^6*tan(f*x+e)^3-9*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*d^6*tan(f*x+e)^3+9*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c^6*tan(f*x+e)+3*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*d^6*tan(f*x+e)+15*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c^5*d-30*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c^3*d^3+3*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c*d^5+52*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c^4*d^2*tan(f*x+e)^3+152*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c^2*d^4*tan(f*x+e)^3-296*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c^2*d^4*tan(f*x+e)-124*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c*d^5*tan(f*x+e)^2+20*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c^5*d*tan(f*x+e)^2-104*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c^3*d^3*tan(f*x+e)^2-124*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c^4*d^2*tan(f*x+e)-3*I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*c^6*(-a*(I*d-c))^(1/2)-15*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*

```
(-a*(I*d-c))^(1/2)*c^4*d^2*tan(f*x+e)^3+75*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d
+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan
(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c^2*d^4*tan(f*x
+e)^3-36*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I
*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2
^(1/2)*(-a*(I*d-c))^(1/2)*c^5*d*tan(f*x+e)^2+36*ln((3*a*c+I*a*tan(f*x+e)*c-
I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+
I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c*d^5*tan(
f*x+e)^2-75*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a
*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I)
)*2^(1/2)*(-a*(I*d-c))^(1/2)*c^4*d^2*tan(f*x+e)+15*ln((3*a*c+I*a*tan(f*x+e)
*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*
(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c^2*d^4
*tan(f*x+e)-3*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(
-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+
I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c^5*d*tan(f*x+e)^4+30*ln((3*a*c+I*a*tan(f*x+
e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e)
)*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c^3*d
^3*tan(f*x+e)^4-15*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1
/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*
x+e)+I))*2^(1/2)*(-a*(I*d-c))^(1/2)*c*d^5*tan(f*x+e)^4-3*I*ln((3*a*c+I*a*ta
n(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(
f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*d^6*(-a*(I*d-c))^(
1/2)*tan(f*x+e)^4+9*I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2
^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan
(f*x+e)+I))*2^(1/2)*c^6*(-a*(I*d-c))^(1/2)*tan(f*x+e)^2+9*I*ln((3*a*c+I*a*t
an(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan
(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*2^(1/2)*d^6*(-a*(I*d-c))^
(1/2)*tan(f*x+e)^2+30*I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2
*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(t
an(f*x+e)+I))*2^(1/2)*c^4*d^2*(-a*(I*d-c))^(1/2)-15*I*ln((3*a*c+I*a*tan(f*x
+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d...
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1004 vs. 2(213) = 426.

time = 1.44, size = 1004, normalized size = 3.73

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="f
ricas")
```

```
[Out] -1/12*(sqrt(2)*(c^3 + I*c^2*d + c*d^2 + I*d^3 + 2*(2*c^3 + 3*I*c^2*d + 12*c
*d^2 - 19*I*d^3)*e^(6*I*f*x + 6*I*e) + (9*c^3 + 19*I*c^2*d + 29*c*d^2 - 25*
I*d^3)*e^(4*I*f*x + 4*I*e) + 2*(3*c^3 + 7*I*c^2*d + 3*c*d^2 + 7*I*d^3)*e^(2
*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x
 + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)) - 3*((-I*a^2*c^5 + a^2*c^
4*d - 2*I*a^2*c^3*d^2 + 2*a^2*c^2*d^3 - I*a^2*c*d^4 + a^2*d^5)*f*e^(5*I*f*x
 + 5*I*e) + (-I*a^2*c^5 + 3*a^2*c^4*d + 2*I*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + 3
*I*a^2*c*d^4 - a^2*d^5)*f*e^(3*I*f*x + 3*I*e))*sqrt(-1/2*I/((I*a^3*c^3 + 3*
a^3*c^2*d - 3*I*a^3*c*d^2 - a^3*d^3)*f^2))*log(-2*(I*a^2*c^2 + 2*a^2*c*d -
I*a^2*d^2)*f*sqrt(-1/2*I/((I*a^3*c^3 + 3*a^3*c^2*d - 3*I*a^3*c*d^2 - a^3*d^
3)*f^2))*e^(I*f*x + I*e) + sqrt(2)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c
+ I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*(e^(2*I
*f*x + 2*I*e) + 1)) - 3*((I*a^2*c^5 - a^2*c^4*d + 2*I*a^2*c^3*d^2 - 2*a^2*c
^2*d^3 + I*a^2*c*d^4 - a^2*d^5)*f*e^(5*I*f*x + 5*I*e) + (I*a^2*c^5 - 3*a^2*
c^4*d - 2*I*a^2*c^3*d^2 - 2*a^2*c^2*d^3 - 3*I*a^2*c*d^4 + a^2*d^5)*f*e^(3*I
*f*x + 3*I*e))*sqrt(-1/2*I/((I*a^3*c^3 + 3*a^3*c^2*d - 3*I*a^3*c*d^2 - a^3*
d^3)*f^2))*log(-2*(-I*a^2*c^2 - 2*a^2*c*d + I*a^2*d^2)*f*sqrt(-1/2*I/((I*a^
3*c^3 + 3*a^3*c^2*d - 3*I*a^3*c*d^2 - a^3*d^3)*f^2))*e^(I*f*x + I*e) + sqrt
(2)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1
))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*(e^(2*I*f*x + 2*I*e) + 1)))/((I*a^2*c^
5 - a^2*c^4*d + 2*I*a^2*c^3*d^2 - 2*a^2*c^2*d^3 + I*a^2*c*d^4 - a^2*d^5)*f*
e^(5*I*f*x + 5*I*e) + (I*a^2*c^5 - 3*a^2*c^4*d - 2*I*a^2*c^3*d^2 - 2*a^2*c^
2*d^3 - 3*I*a^2*c*d^4 + a^2*d^5)*f*e^(3*I*f*x + 3*I*e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(\tan\left(e+fx\right)-i\right)\right)^{\frac{3}{2}}\left(c+d\tan\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**(3/2)/(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/((I*a*(tan(e + f*x) - I))**(3/2)*(c + d*tan(e + f*x))**(3/2)), x
)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad Ar
gument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Argum
ent Ty
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a + a\tan(e + f\,x)\ 1\mathrm{i})^{3/2}\,(c + d\tan(e + f\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^(3/2)*(c + d*tan(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + a*tan(e + f*x)*1i)^(3/2)*(c + d*tan(e + f*x))^(3/2)), x)
```

3.1166 $\int \frac{1}{(a+ia\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=349

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{4\sqrt{2}a^{5/2}(c-id)^{3/2}f}-\frac{1}{5(ic-d)f(a+ia\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}+\frac{}{30a(}$$

[Out] -1/8*I*arctanh(2^(1/2)*a^(1/2)*(c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2)/(a+I*a*tan(f*x+e))^(1/2))/a^(5/2)/(c-I*d)^(3/2)/f*2^(1/2)+1/60*(15*c^2+70*I*c*d-151*d^2)/a^2/(I*c-d)^3/f/(a+I*a*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(1/2)+1/60*d*(15*c^3+65*I*c^2*d-117*c*d^2+317*I*d^3)*(a+I*a*tan(f*x+e))^(1/2)/a^3/(c-I*d)/(c+I*d)^4/f/(c+d*tan(f*x+e))^(1/2)-1/5/(I*c-d)/f/(c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(5/2)+1/30*(5*I*c-17*d)/a/(c+I*d)^2/f/(c+d*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(3/2)

Rubi [A]
time = 0.87, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3640, 3677, 3679, 12, 3625, 214}

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{4\sqrt{2}a^{5/2}f(c-id)^{3/2}}+\frac{d\left(15c^3+65ic^2d-117cd^2+317id^3\right)\sqrt{a+ia\tan(e+fx)}}{60a^3f(c-id)(c+id)^4\sqrt{c+d\tan(e+fx)}}+\frac{15c^2+70icd-151d^2}{60a^2f(-d+ic)^3\sqrt{a+ia\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}+\frac{-17d+5ic}{30af(c+id)^2(a+ia\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}-\frac{1}{5f(-d+ic)(a+ia\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + I*a*Tan[e + f*x])^(5/2)*(c + d*Tan[e + f*x])^(3/2)),x]

[Out] ((-1/4*I)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])/(Sqrt[c - I*d]*Sqrt[a + I*a*Tan[e + f*x]])])/(Sqrt[2]*a^(5/2)*(c - I*d)^(3/2)*f) - 1/(5*(I*c - d)*f*(a + I*a*Tan[e + f*x])^(5/2)*Sqrt[c + d*Tan[e + f*x]]) + ((5*I)*c - 17*d)/(30*a*(c + I*d)^2*f*(a + I*a*Tan[e + f*x])^(3/2)*Sqrt[c + d*Tan[e + f*x]]) + (15*c^2 + (70*I)*c*d - 151*d^2)/(60*a^2*(I*c - d)^3*f*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]]) + (d*(15*c^3 + (65*I)*c^2*d - 117*c*d^2 + (317*I)*d^3)*Sqrt[a + I*a*Tan[e + f*x]])/(60*a^3*(c - I*d)*(c + I*d)^4*f*Sqrt[c + d*Tan[e + f*x]])

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{1}{5(ic-d)f(a+ia\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx}}$$

$$= -\frac{1}{5(ic-d)f(a+ia\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx}}$$

$$= -\frac{1}{5(ic-d)f(a+ia\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx}}$$

$$= -\frac{1}{5(ic-d)f(a+ia\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx}}$$

$$= -\frac{1}{5(ic-d)f(a+ia\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx}}$$

$$= -\frac{1}{5(ic-d)f(a+ia\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx}}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{4\sqrt{2}a^{5/2}(c-id)^{3/2}f} - \frac{}{5(ic-}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 788 vs. 2(349) = 698.
time = 9.35, size = 788, normalized size = 2.26

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^(5/2)*(c + d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] ((-1/4*I)*E^((3*I)*e)*Sqrt[E^(I*f*x)]*Log[2*(Sqrt[c - I*d]*E^(I*(e + f*x))
+ Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(
1 + E^((2*I)*(e + f*x)))])]*Sec[e + f*x]^(5/2)*(Cos[f*x] + I*Sin[f*x])^(5/2
))/(Sqrt[2]*(c - I*d)^(3/2)*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]
*Sqrt[1 + E^((2*I)*(e + f*x))]*f*(a + I*a*Tan[e + f*x])^(5/2)) + (Sec[e + f
*x]^3*(Cos[f*x] + I*Sin[f*x])^3*Sqrt[Sec[e + f*x]*(c*Cos[e + f*x] + d*Sin[e
 + f*x])]*(((17*c^2 + (77*I)*c*d - 126*d^2)*Cos[2*f*x]*((I/60)*Cos[e] - Sin
[e]/60))/(c + I*d)^4 + ((7*c + (16*I)*d)*Cos[4*f*x]*((I/60)*Cos[e] + Sin[e]
```

```
/60))/(c + I*d)^3 + ((23*c^4*Cos[e] + (91*I)*c^3*d*Cos[e] - 109*c^2*d^2*Cos
[e] + (223*I)*c*d^3*Cos[e] + 240*d^4*Cos[e] + 23*c^3*d*Sin[e] + (91*I)*c^2*
d^2*Sin[e] - 109*c*d^3*Sin[e] + (223*I)*d^4*Sin[e])*(Cos[3*e]/120 + (I/120)
*Sin[3*e]))/((c - I*d)*(c + I*d)^4*((-I)*c*Cos[e] - I*d*Sin[e])) + (Cos[6*f
*x]*((I/40)*Cos[3*e] + Sin[3*e]/40))/(c + I*d)^2 + ((17*c^2 + (77*I)*c*d -
126*d^2)*(Cos[e]/60 + (I/60)*Sin[e])*Sin[2*f*x])/(c + I*d)^4 + ((7*c + (16*
I)*d)*(Cos[e]/60 - (I/60)*Sin[e])*Sin[4*f*x])/(c + I*d)^3 + ((Cos[3*e]/40 -
 (I/40)*Sin[3*e])*Sin[6*f*x])/(c + I*d)^2 + (2*((d^5*Cos[3*e - f*x])/2 - (d
^5*Cos[3*e + f*x])/2 + (I/2)*d^5*Sin[3*e - f*x] - (I/2)*d^5*Sin[3*e + f*x])
)/((c - I*d)*(c + I*d)^4*(c*Cos[e] + d*Sin[e])*(c*Cos[e + f*x] + d*Sin[e +
f*x]))))/(f*(a + I*a*Tan[e + f*x])^(5/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 7869 vs. $2(290) = 580$.
time = 0.64, size = 7870, normalized size = 22.55

method	result	size
derivativedivides	Expression too large to display	7870
default	Expression too large to display	7870

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(3/2),x,method=_RETURNVERBO
SE)
```

```
[Out] result too large to display
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1097 vs. $2(283) = 566$.
time = 2.22, size = 1097, normalized size = 3.14

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="f
ricas")
```

```
[Out] -1/120*(sqrt(2)*(-3*I*c^4 + 6*c^3*d + 6*c*d^3 + 3*I*d^4 + (-23*I*c^4 + 68*c
^3*d + 18*I*c^2*d^2 + 332*c*d^3 - 463*I*d^4)*e^(8*I*f*x + 8*I*e) + (-57*I*c
^4 + 200*c^3*d + 178*I*c^2*d^2 + 464*c*d^3 - 269*I*d^4)*e^(6*I*f*x + 6*I*e)
 - 4*(12*I*c^4 - 43*c^3*d - 43*I*c^2*d^2 - 43*c*d^3 - 55*I*d^4)*e^(4*I*f*x
+ 4*I*e) + (-17*I*c^4 + 46*c^3*d + 12*I*c^2*d^2 + 46*c*d^3 + 29*I*d^4)*e^(2
*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x
 + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)) + 30*((a^3*c^6 + 2*I*a^3*
c^5*d + a^3*c^4*d^2 + 4*I*a^3*c^3*d^3 - a^3*c^2*d^4 + 2*I*a^3*c*d^5 - a^3*d
^6)*f*e^(7*I*f*x + 7*I*e) + (a^3*c^6 + 4*I*a^3*c^5*d - 5*a^3*c^4*d^2 - 5*a^
3*c^2*d^4 - 4*I*a^3*c*d^5 + a^3*d^6)*f*e^(5*I*f*x + 5*I*e))*sqrt(-1/8*I/((I
*a^5*c^3 + 3*a^5*c^2*d - 3*I*a^5*c*d^2 - a^5*d^3)*f^2))*log(-4*(I*a^3*c^2 +
 2*a^3*c*d - I*a^3*d^2)*f*sqrt(-1/8*I/((I*a^5*c^3 + 3*a^5*c^2*d - 3*I*a^5*c
*d^2 - a^5*d^3)*f^2))*e^(I*f*x + I*e) + sqrt(2)*sqrt(((c - I*d)*e^(2*I*f*x
+ 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e)
+ 1))*(e^(2*I*f*x + 2*I*e) + 1)) - 30*((a^3*c^6 + 2*I*a^3*c^5*d + a^3*c^4*d
^2 + 4*I*a^3*c^3*d^3 - a^3*c^2*d^4 + 2*I*a^3*c*d^5 - a^3*d^6)*f*e^(7*I*f*x
+ 7*I*e) + (a^3*c^6 + 4*I*a^3*c^5*d - 5*a^3*c^4*d^2 - 5*a^3*c^2*d^4 - 4*I*a
^3*c*d^5 + a^3*d^6)*f*e^(5*I*f*x + 5*I*e))*sqrt(-1/8*I/((I*a^5*c^3 + 3*a^5*
c^2*d - 3*I*a^5*c*d^2 - a^5*d^3)*f^2))*log(-4*(-I*a^3*c^2 - 2*a^3*c*d + I*a
^3*d^2)*f*sqrt(-1/8*I/((I*a^5*c^3 + 3*a^5*c^2*d - 3*I*a^5*c*d^2 - a^5*d^3)*
f^2))*e^(I*f*x + I*e) + sqrt(2)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I
*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*(e^(2*I*f*
x + 2*I*e) + 1)))/((a^3*c^6 + 2*I*a^3*c^5*d + a^3*c^4*d^2 + 4*I*a^3*c^3*d^3
 - a^3*c^2*d^4 + 2*I*a^3*c*d^5 - a^3*d^6)*f*e^(7*I*f*x + 7*I*e) + (a^3*c^6
+ 4*I*a^3*c^5*d - 5*a^3*c^4*d^2 - 5*a^3*c^2*d^4 - 4*I*a^3*c*d^5 + a^3*d^6)*
f*e^(5*I*f*x + 5*I*e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia(\tan(e+fx)-i))^{\frac{5}{2}}(c+d\tan(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**(5/2)/(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/((I*a*(tan(e + f*x) - I))**(5/2)*(c + d*tan(e + f*x))**(3/2)), x
)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad Ar
gument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Argum
ent Ty
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a + a\tan(e + f\,x)\ 1\mathrm{i})^{5/2}\,(c + d\tan(e + f\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^(5/2)*(c + d*tan(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + a*tan(e + f*x)*1i)^(5/2)*(c + d*tan(e + f*x))^(3/2)), x)
```

3.1167 $\int \frac{(a+ia\tan(e+fx))^{5/2}}{(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=181

$$-\frac{4i\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{(c-id)^{5/2}f}-\frac{2a(a+ia\tan(e+fx))^{3/2}}{3(ic+d)f(c+d\tan(e+fx))^{3/2}}+\frac{4ia^2\sqrt{a+ia\,\mathrm{ta}}}{(c-id)^2f\sqrt{c+}}$$

```
[Out] -4*I*a^(5/2)*arctanh(2^(1/2)*a^(1/2)*(c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2)/(
a+I*a*tan(f*x+e))^(1/2))*2^(1/2)/(c-I*d)^(5/2)/f+4*I*a^2*(a+I*a*tan(f*x+e))
^(1/2)/(c-I*d)^2/f/(c+d*tan(f*x+e))^(1/2)-2/3*a*(a+I*a*tan(f*x+e))^(3/2)/(I
*c+d)/f/(c+d*tan(f*x+e))^(3/2)
```

Rubi [A]

time = 0.26, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3626, 3625, 214}

$$-\frac{4i\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{f(c-id)^{5/2}}+\frac{4ia^2\sqrt{a+ia\tan(e+fx)}}{f(c-id)^2\sqrt{c+d\tan(e+fx)}}-\frac{2a(a+ia\tan(e+fx))^{3/2}}{3f(d+ic)(c+d\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^(5/2)/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-4*I)*Sqrt[2]*a^(5/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])/
(Sqrt[c - I*d]*Sqrt[a + I*a*Tan[e + f*x]])])/((c - I*d)^(5/2)*f) - (2*a*(a
+ I*a*Tan[e + f*x])^(3/2))/(3*(I*c + d)*f*(c + d*Tan[e + f*x])^(3/2)) + ((4
*I)*a^2*Sqrt[a + I*a*Tan[e + f*x]])/((c - I*d)^2*f*Sqrt[c + d*Tan[e + f*x]]
)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3626

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*b*(a + b*Tan[e + f*x])^(m - 1)*((c +
 d*Tan[e + f*x])^(n + 1)/(f*(m - 1)*(a*c - b*d))), x] + Dist[2*(a^2/(a*c -
b*d)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^{5/2}}{(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{2a(a+ia\tan(e+fx))^{3/2}}{3(ic+d)f(c+d\tan(e+fx))^{3/2}} + \frac{(2a)\int \frac{(a+ia\tan(e+fx))^{3/2}}{(c+d\tan(e+fx))^{3/2}}\,dx}{c-id}$$

$$= -\frac{2a(a+ia\tan(e+fx))^{3/2}}{3(ic+d)f(c+d\tan(e+fx))^{3/2}} + \frac{4ia^2\sqrt{a+ia\tan(e+fx)}}{(c-id)^2f\sqrt{c+d\tan(e+fx)}} + \frac{(4\ldots}{\ldots}$$

$$= -\frac{2a(a+ia\tan(e+fx))^{3/2}}{3(ic+d)f(c+d\tan(e+fx))^{3/2}} + \frac{4ia^2\sqrt{a+ia\tan(e+fx)}}{(c-id)^2f\sqrt{c+d\tan(e+fx)}} - \frac{(8\ldots}{\ldots}$$

$$= -\frac{4i\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{(c-id)^{5/2}f} - \frac{2a(a+ia\tan\ldots}{3(ic+d)f(c+d\ldots}$$

Mathematica [A]
time = 7.05, size = 283, normalized size = 1.56

$$\frac{(a+ia\tan(e+fx))^{5/2}\left(-\frac{4i\sqrt{2}\,e^{-3i(e+fx)}\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\sqrt{1+e^{2i(e+fx)}}\log\left(2e^{-ie}\left(\sqrt{c-id}\,e^{i(e+fx)}+\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)\right)}{(c-id)^{5/2}} - \frac{2\sqrt{\sec(e+fx)}\,(\cos(2(e+fx))-i\sin(2(e+fx)))(-7ic-d+(c-7id)\tan(e+fx))}{3(c-id)^2(c+d\tan(e+fx))^{3/2}}\right)}{f\sec^{\frac{5}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(5/2)/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((a + I*a*Tan[e + f*x])^(5/2)*(((-4*I)*Sqrt[2]*Sqrt[E^(I*(e + f*x))/(1 + E^
((2*I)*(e + f*x)))]*Sqrt[1 + E^((2*I)*(e + f*x))]*Log[(2*(Sqrt[c - I*d]*E^(
I*(e + f*x)) + Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^((2*I)*(
e + f*x))))/(1 + E^((2*I)*(e + f*x)))]))/E^(I*e)])/((c - I*d)^(5/2)*E^((3*I
)*(e + f*x))) - (2*Sqrt[Sec[e + f*x]]*(Cos[2*(e + f*x)] - I*Sin[2*(e + f*x)
])*((-7*I)*c - d + (c - (7*I)*d)*Tan[e + f*x]))/(3*(c - I*d)^2*(c + d*Tan[e
 + f*x])^(3/2))))/(f*Sec[e + f*x]^(5/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2967 vs. $2(146) = 292$.
time = 0.62, size = 2968, normalized size = 16.40

method	result	size
derivativedivides	Expression too large to display	2968
default	Expression too large to display	2968

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/3/f*2^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)*(6*I*2^(1/2)*ln(1/2*(2*I*a*d*tan(f
*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d
)/(I*a*d)^(1/2))*a*c*d*(-a*(I*d-c))^(1/2)*tan(f*x+e)-7*(I*a*d)^(1/2)*(a*(c+
d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*c^4-3*I*ln
((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2
)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c^2*(I*a*d
)^(1/2)+6*I*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*
(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c^3*d^2*(-a*(I*
d-c))^(1/2)*tan(f*x+e)^2-18*I*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a
*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))
*a*c*d^4*(-a*(I*d-c))^(1/2)*tan(f*x+e)^2+12*I*2^(1/2)*ln(1/2*(2*I*a*d*tan(f
*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d
)/(I*a*d)^(1/2))*a*c^4*d*(-a*(I*d-c))^(1/2)*tan(f*x+e)-36*I*2^(1/2)*ln(1/2*
(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*
a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c^2*d^3*(-a*(I*d-c))^(1/2)*tan(f*x+e)-18*I
*(I*a*d)^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*2^(1/2)*(-a*(I*d
-c))^(1/2)*c^2*d^2*tan(f*x+e)-3*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*
(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2
))*a*c^2*(-a*(I*d-c))^(1/2)+(I*a*d)^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+
e)))^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*d^4-3*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*
d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*ta
n(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*d^2*(I*a*d)^(1/2)*tan(f*x+e)^2-3*ln((3*
a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a
*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c^2*(I*a*d)^(1
/2)-6*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*t
an(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c*d*(-a*(I*d-c))^(1/2
)*tan(f*x+e)-6*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*
(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)
+I))*a*c*d*(I*a*d)^(1/2)*tan(f*x+e)-3*I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*
a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*
x+e)))^(1/2))/(tan(f*x+e)+I))*a*d^2*(I*a*d)^(1/2)*tan(f*x+e)^2+3*I*2^(1/2)*
ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1
```

```
/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c^2*(-a*(I*d-c))^(1/2)+6*2^(1/2)*ln
(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2
)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*d^5*(-a*(I*d-c))^(1/2)*tan(f*x+e)^2-1
8*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f
*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c^4*d*(-a*(I*d-c))^(1/2)+
6*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f
*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c^2*d^3*(-a*(I*d-c))^(1/2
)-3*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan
(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*d^2*(-a*(I*d-c))^(1/2)*
tan(f*x+e)^2+18*(I*a*d)^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*2
^(1/2)*(-a*(I*d-c))^(1/2)*c^2*d^2-18*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a
*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)
^(1/2))*a*c^2*d^3*(-a*(I*d-c))^(1/2)*tan(f*x+e)^2-4*2^(1/2)*c^3*d*(I*a*d)^(
1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(-a*(I*d-c))^(1/2)*tan(f*x
+e)+20*2^(1/2)*c*d^3*(I*a*d)^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1
/2)*(-a*(I*d-c))^(1/2)*tan(f*x+e)-36*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a
*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)
^(1/2))*a*c^3*d^2*(-a*(I*d-c))^(1/2)*tan(f*x+e)+12*2^(1/2)*ln(1/2*(2*I*a*d*
tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2
)+a*d)/(I*a*d)^(1/2))*a*c*d^4*(-a*(I*d-c))^(1/2)*tan(f*x+e)-18*I*2^(1/2)*ln
(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2
)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c^3*d^2*(-a*(I*d-c))^(1/2)-I*2^(1/2)*
c^4*(I*a*d)^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(-a*(I*d-c))^
(1/2)*tan(f*x+e)+7*I*(I*a*d)^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1
/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*d^4*tan(f*x+e)+3*I*2^(1/2)*ln(1/2*(2*I*a*d*t
an(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)
+a*d)/(I*a*d)^(1/2))*a*d^2*(-a*(I*d-c))^(1/2)*tan(f*x+e)^2-20*I*(I*a*d)^(1/
2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*c
^3*d+4*I*(I*a*d)^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*2^(1/2)*
(-a*(I*d-c))^(1/2)*c*d^3-6*I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x
+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2
))/(tan(f*x+e)+I))*a*c*d*(I*a*d)^(1/2)*tan(f*x+e)+6*I*2^(1/2)*ln(1/2*(2*I*a
*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(
1/2)+a*d)/(I*a*d)^(1/2))*a*c^5*(-a*(I*d-c))^(1/2))*a^2/(c+d*tan(f*x+e))^(3/
2)/(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)/...
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] Timed out
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 873 vs. $2(143) = 286$.
time = 1.22, size = 873, normalized size = 4.82

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] -1/6*(8*sqrt(2)*(4*(-I*a^2*c - a^2*d)*e^(5*I*f*x + 5*I*e) + (-7*I*a^2*c - a
^2*d)*e^(3*I*f*x + 3*I*e) + 3*(-I*a^2*c + a^2*d)*e^(I*f*x + I*e))*sqrt(((c
- I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^
(2*I*f*x + 2*I*e) + 1)) - 3*((c^4 - 4*I*c^3*d - 6*c^2*d^2 + 4*I*c*d^3 + d^4
)*f*e^(4*I*f*x + 4*I*e) + 2*(c^4 - 2*I*c^3*d - 2*I*c*d^3 - d^4)*f*e^(2*I*f*
x + 2*I*e) + (c^4 + 2*c^2*d^2 + d^4)*f)*sqrt(-32*I*a^5/((I*c^5 + 5*c^4*d -
10*I*c^3*d^2 - 10*c^2*d^3 + 5*I*c*d^4 + d^5)*f^2))*log(1/4*((I*c^3 + 3*c^2*
d - 3*I*c*d^2 - d^3)*sqrt(-32*I*a^5/((I*c^5 + 5*c^4*d - 10*I*c^3*d^2 - 10*c
^2*d^3 + 5*I*c*d^4 + d^5)*f^2))*f*e^(I*f*x + I*e) + 4*sqrt(2)*(a^2*e^(2*I*f
*x + 2*I*e) + a^2)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f
*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/a^2)
+ 3*((c^4 - 4*I*c^3*d - 6*c^2*d^2 + 4*I*c*d^3 + d^4)*f*e^(4*I*f*x + 4*I*e)
+ 2*(c^4 - 2*I*c^3*d - 2*I*c*d^3 - d^4)*f*e^(2*I*f*x + 2*I*e) + (c^4 + 2*c^
2*d^2 + d^4)*f)*sqrt(-32*I*a^5/((I*c^5 + 5*c^4*d - 10*I*c^3*d^2 - 10*c^2*d^
3 + 5*I*c*d^4 + d^5)*f^2))*log(1/4*((-I*c^3 - 3*c^2*d + 3*I*c*d^2 + d^3)*sq
rt(-32*I*a^5/((I*c^5 + 5*c^4*d - 10*I*c^3*d^2 - 10*c^2*d^3 + 5*I*c*d^4 + d^
5)*f^2))*f*e^(I*f*x + I*e) + 4*sqrt(2)*(a^2*e^(2*I*f*x + 2*I*e) + a^2)*sqrt
(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(
a/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/a^2))/((c^4 - 4*I*c^3*d - 6*
c^2*d^2 + 4*I*c*d^3 + d^4)*f*e^(4*I*f*x + 4*I*e) + 2*(c^4 - 2*I*c^3*d - 2*I
*c*d^3 - d^4)*f*e^(2*I*f*x + 2*I*e) + (c^4 + 2*c^2*d^2 + d^4)*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan{(e+fx)}-i))^{\frac{5}{2}}}{(c+d\tan{(e+fx)})^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(5/2)/(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral((I*a*(tan(e + f*x) - I))**(5/2)/(c + d*tan(e + f*x))**(5/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\tan(e + f\,x)\ 1\mathrm{i})^{5/2}}{(c + d\tan(e + f\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(5/2)/(c + d*tan(e + f*x))^(5/2),x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^(5/2)/(c + d*tan(e + f*x))^(5/2), x)
```

3.1168 $\int \frac{(a+ia\tan(e+fx))^{3/2}}{(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=179

$$-\frac{2i\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{(c-id)^{5/2}f}-\frac{2d(a+ia\tan(e+fx))^{3/2}}{3\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}}+\frac{2ia\sqrt{a+ia\tan(e+fx)}}{(c-id)^2f\sqrt{c+d\tan(e+fx)}}$$

```
[Out] -2*I*a^(3/2)*arctanh(2^(1/2)*a^(1/2)*(c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2)/(
a+I*a*tan(f*x+e))^(1/2))*2^(1/2)/(c-I*d)^(5/2)/f+2*I*a*(a+I*a*tan(f*x+e))^(
1/2)/(c-I*d)^2/f/(c+d*tan(f*x+e))^(1/2)-2/3*d*(a+I*a*tan(f*x+e))^(3/2)/(c^2
+d^2)/f/(c+d*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.23, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3629, 3626, 3625, 214}

$$-\frac{2i\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{f(c-id)^{5/2}}-\frac{2d(a+ia\tan(e+fx))^{3/2}}{3f\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}+\frac{2ia\sqrt{a+ia\tan(e+fx)}}{f(c-id)^2\sqrt{c+d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^(3/2)/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-2*I)*Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])/
(Sqrt[c - I*d]*Sqrt[a + I*a*Tan[e + f*x]])])/((c - I*d)^(5/2)*f) - (2*d*(a
+ I*a*Tan[e + f*x])^(3/2))/(3*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2)) + (
(2*I)*a*Sqrt[a + I*a*Tan[e + f*x]])/((c - I*d)^2*f*Sqrt[c + d*Tan[e + f*x]]
)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3626

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*b*(a + b*Tan[e + f*x])^(m - 1)*((c +
 d*Tan[e + f*x])^(n + 1)/(f*(m - 1)*(a*c - b*d))), x] + Dist[2*(a^2/(a*c -
b*d)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2]
```

Rule 3629

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-d)*(a + b*Tan[e + f*x])^m*((c + d*Ta
n[e + f*x])^(n + 1)/(f*m*(c^2 + d^2))), x] + Dist[a/(a*c - b*d), Int[(a + b
*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^{3/2}}{(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{2d(a+ia\tan(e+fx))^{3/2}}{3(c^2+d^2)f(c+d\tan(e+fx))^{3/2}} + \frac{\int \frac{(a+ia\tan(e+fx))^{3/2}}{(c+d\tan(e+fx))^{3/2}}\,dx}{c-id}$$

$$= -\frac{2d(a+ia\tan(e+fx))^{3/2}}{3(c^2+d^2)f(c+d\tan(e+fx))^{3/2}} + \frac{2ia\sqrt{a+ia\tan(e+fx)}}{(c-id)^2f\sqrt{c+d\tan(e+fx)}} + \cdots$$

$$= -\frac{2d(a+ia\tan(e+fx))^{3/2}}{3(c^2+d^2)f(c+d\tan(e+fx))^{3/2}} + \frac{2ia\sqrt{a+ia\tan(e+fx)}}{(c-id)^2f\sqrt{c+d\tan(e+fx)}} - \cdots$$

$$= -\frac{2i\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{(c-id)^{5/2}f} - \frac{2d(a+ia\tan\cdots}{3(c^2+d^2)f(c+\cdots}$$

Mathematica [A]
time = 6.41, size = 278, normalized size = 1.55

$$\frac{(a+ia\tan(e+fx))^{3/2}\left(-\frac{2i\sqrt{2}\log\left(2e^{-ie}\left(\sqrt{c-id}\,e^{i(e+fx)}+\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)\right)}{(c-id)^{5/2}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{3/2}\left(1+e^{2i(e+fx)}\right)^{3/2}}+\frac{2(i+\tan(e+fx))(3c^2+4icd+d^2+2(c+2id)d\tan(e+fx))}{3(c-id)^2(c+id)\sqrt{\sec(e+fx)}(c+d\tan(e+fx))^{3/2}}\right)}{f\sec^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(3/2)/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((a + I*a*Tan[e + f*x])^(3/2)*(((-2*I)*Sqrt[2]*Log[(2*(Sqrt[c - I*d]*E^(I*(
e + f*x)) + Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^((2*I)*(e +
 f*x))))/(1 + E^((2*I)*(e + f*x)))]))/E^(I*e)])/((c - I*d)^(5/2)*(E^(I*(e +
 f*x))/(1 + E^((2*I)*(e + f*x))))^(3/2)*(1 + E^((2*I)*(e + f*x)))^(3/2)) +
(2*(I + Tan[e + f*x])*(3*c^2 + (4*I)*c*d + d^2 + 2*(c + (2*I)*d)*d*Tan[e +
f*x]))/(3*(c - I*d)^2*(c + I*d)*Sqrt[Sec[e + f*x]]*(c + d*Tan[e + f*x])^(3/
2))))/(f*Sec[e + f*x]^(3/2))
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2906 vs. $2(145) = 290$.
time = 0.61, size = 2907, normalized size = 16.24

method	result	size
derivativedivides	Expression too large to display	2907
default	Expression too large to display	2907

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/6/f*2^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)*(6*I*2^(1/2)*ln(1/2*(2*I*a*d*tan(f
*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d
)/(I*a*d)^(1/2))*a*c*d*(-a*(I*d-c))^(1/2)*tan(f*x+e)-6*(I*a*d)^(1/2)*(a*(c+
d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*c^4-3*I*ln
((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2
)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c^2*(I*a*d
)^(1/2)-16*I*(I*a*d)^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*2^(1
/2)*(-a*(I*d-c))^(1/2)*c^2*d^2*tan(f*x+e)+6*I*2^(1/2)*ln(1/2*(2*I*a*d*tan(f
*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d
)/(I*a*d)^(1/2))*a*c^3*d^2*(-a*(I*d-c))^(1/2)*tan(f*x+e)^2-18*I*2^(1/2)*ln(
1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)
*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c*d^4*(-a*(I*d-c))^(1/2)*tan(f*x+e)^2+
12*I*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*ta
n(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c^4*d*(-a*(I*d-c))^(1/
2)*tan(f*x+e)-36*I*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f
*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c^2*d^3*
(-a*(I*d-c))^(1/2)*tan(f*x+e)-3*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*
(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2
))*a*c^2*(-a*(I*d-c))^(1/2)+2*(I*a*d)^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*
x+e)))^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*d^4-3*ln((3*a*c+I*a*tan(f*x+e)*c-I*
a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*
tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*d^2*(I*a*d)^(1/2)*tan(f*x+e)^2-3*ln((
3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*
```

(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c^2*(I*a*d)^(1/2)-6*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c*d*(-a*(I*d-c))^(1/2)*tan(f*x+e)+8*I*(I*a*d)^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*d^4*tan(f*x+e)-6*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c*d*(I*a*d)^(1/2)*tan(f*x+e)-3*I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*d^2*(I*a*d)^(1/2)*tan(f*x+e)^2+3*I*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c^2*(-a*(I*d-c))^(1/2)+6*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*d^5*(-a*(I*d-c))^(1/2)*tan(f*x+e)^2-18*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c^4*d*(-a*(I*d-c))^(1/2)+6*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c^2*d^3*(-a*(I*d-c))^(1/2)-3*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*d^2*(-a*(I*d-c))^(1/2)*tan(f*x+e)^2+20*(I*a*d)^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*c^2*d^2-18*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c^2*d^3*(-a*(I*d-c))^(1/2)*tan(f*x+e)^2-4*2^(1/2)*c^3*d*(I*a*d)^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(-a*(I*d-c))^(1/2)*tan(f*x+e)+20*2^(1/2)*c*d^3*(I*a*d)^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(-a*(I*d-c))^(1/2)*tan(f*x+e)-36*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c^3*d^2*(-a*(I*d-c))^(1/2)*tan(f*x+e)+12*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c*d^4*(-a*(I*d-c))^(1/2)*tan(f*x+e)-18*I*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c^3*d^2*(-a*(I*d-c))^(1/2)+3*I*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*d^2*(-a*(I*d-c))^(1/2)*tan(f*x+e)^2-20*I*(I*a*d)^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*c^3*d+4*I*(I*a*d)^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*c*d^3-6*I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c*d*(I*a*d)^(1/2)*tan(f*x+e)+6*I*2^(1/2)*ln(1/2*(2*I*a*d*tan(f*x+e)+I*a*c+2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*(I*a*d)^(1/2)+a*d)/(I*a*d)^(1/2))*a*c^5*(-a*(I*d-c))^(1/2))*a/(c+d*tan(f*x+e))^(3/2)/(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)/(c^2+d^2)^2/(I*c-d)/(I*a*d)^(1/2)/(-a*(I*d-c))^(1/2)

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((-(2*c*d^4)/((c^2-d^2)^2>0)', see
`assume
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1030 vs. $2(143) = 286$.
time = 1.57, size = 1030, normalized size = 5.75

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] -1/6*(4*sqrt(2)*((3*a*c^2 + 2*I*a*c*d + 5*a*d^2)*e^(5*I*f*x + 5*I*e) + 2*(3
*a*c^2 + 4*I*a*c*d + a*d^2)*e^(3*I*f*x + 3*I*e) + 3*(a*c^2 + 2*I*a*c*d - a*
d^2)*e^(I*f*x + I*e))*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*
I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)) - 3*((I*c^5 + 3*c^4*
d - 2*I*c^3*d^2 + 2*c^2*d^3 - 3*I*c*d^4 - d^5)*f*e^(4*I*f*x + 4*I*e) + 2*(I
*c^5 + c^4*d + 2*I*c^3*d^2 + 2*c^2*d^3 + I*c*d^4 + d^5)*f*e^(2*I*f*x + 2*I*
e) + (I*c^5 - c^4*d + 2*I*c^3*d^2 - 2*c^2*d^3 + I*c*d^4 - d^5)*f)*sqrt(-8*I
*a^3/((I*c^5 + 5*c^4*d - 10*I*c^3*d^2 - 10*c^2*d^3 + 5*I*c*d^4 + d^5)*f^2))
*log(1/2*((I*c^3 + 3*c^2*d - 3*I*c*d^2 - d^3)*f*sqrt(-8*I*a^3/((I*c^5 + 5*c
^4*d - 10*I*c^3*d^2 - 10*c^2*d^3 + 5*I*c*d^4 + d^5)*f^2))*e^(I*f*x + I*e) +
 2*sqrt(2)*(a*e^(2*I*f*x + 2*I*e) + a)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e)
+ c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))*e^
(-I*f*x - I*e)/a) - 3*((-I*c^5 - 3*c^4*d + 2*I*c^3*d^2 - 2*c^2*d^3 + 3*I*c*
d^4 + d^5)*f*e^(4*I*f*x + 4*I*e) + 2*(-I*c^5 - c^4*d - 2*I*c^3*d^2 - 2*c^2*
d^3 - I*c*d^4 - d^5)*f*e^(2*I*f*x + 2*I*e) + (-I*c^5 + c^4*d - 2*I*c^3*d^2
+ 2*c^2*d^3 - I*c*d^4 + d^5)*f)*sqrt(-8*I*a^3/((I*c^5 + 5*c^4*d - 10*I*c^3*
d^2 - 10*c^2*d^3 + 5*I*c*d^4 + d^5)*f^2))*log(1/2*((-I*c^3 - 3*c^2*d + 3*I*
c*d^2 + d^3)*f*sqrt(-8*I*a^3/((I*c^5 + 5*c^4*d - 10*I*c^3*d^2 - 10*c^2*d^3
+ 5*I*c*d^4 + d^5)*f^2))*e^(I*f*x + I*e) + 2*sqrt(2)*(a*e^(2*I*f*x + 2*I*e)
 + a)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) +
 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/a))/((I*c^5 + 3*c^
4*d - 2*I*c^3*d^2 + 2*c^2*d^3 - 3*I*c*d^4 - d^5)*f*e^(4*I*f*x + 4*I*e) + 2*
```

```
(I*c^5 + c^4*d + 2*I*c^3*d^2 + 2*c^2*d^3 + I*c*d^4 + d^5)*f*e^(2*I*f*x + 2*
I*e) + (I*c^5 - c^4*d + 2*I*c^3*d^2 - 2*c^2*d^3 + I*c*d^4 - d^5)*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(e+fx)-i))^{\frac{3}{2}}}{(c+d\tan(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(3/2)/(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral((I*a*(tan(e + f*x) - I))**(3/2)/(c + d*tan(e + f*x))**(5/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\tan(e+f\,x)\ 1\mathrm{i})^{3/2}}{(c+d\tan(e+f\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(3/2)/(c + d*tan(e + f*x))^(5/2),x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^(3/2)/(c + d*tan(e + f*x))^(5/2), x)
```

3.1169 $\int \frac{\sqrt{a+ia\tan(e+fx)}}{(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=188

$$-\frac{i\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{(c-id)^{5/2}f}-\frac{2d\sqrt{a+ia\tan(e+fx)}}{3\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}}-\frac{2(5c+id)d\sqrt{a}}{3\left(c^2+d^2\right)^2f\sqrt{}}$$

```
[Out] -I*arctanh(2^(1/2)*a^(1/2)*(c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2)/(a+I*a*tan(
f*x+e))^(1/2))*2^(1/2)*a^(1/2)/(c-I*d)^(5/2)/f-2/3*(5*c+I*d)*d*(a+I*a*tan(f
*x+e))^(1/2)/(c^2+d^2)^2/f/(c+d*tan(f*x+e))^(1/2)-2/3*d*(a+I*a*tan(f*x+e))^
(1/2)/(c^2+d^2)/f/(c+d*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.31, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3642, 3679, 12, 3625, 214}

$$-\frac{2d(5c+id)\sqrt{a+ia\tan(e+fx)}}{3f\left(c^2+d^2\right)^2\sqrt{c+d\tan(e+fx)}}-\frac{2d\sqrt{a+ia\tan(e+fx)}}{3f\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}-\frac{i\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{f(c-id)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[e + f*x]]/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-I)*Sqrt[2]*Sqrt[a]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])/(S
qrt[c - I*d]*Sqrt[a + I*a*Tan[e + f*x]])])/((c - I*d)^(5/2)*f) - (2*d*Sqrt[
a + I*a*Tan[e + f*x]])/(3*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2)) - (2*(5
*c + I*d)*d*Sqrt[a + I*a*Tan[e + f*x]])/(3*(c^2 + d^2)^2*f*Sqrt[c + d*Tan[e
 + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
```

```
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3642

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[d*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(a*(c^2 + d^2)*(n +
1)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[b*d*m - a
*c*(n + 1) + a*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
 e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0
] && LtQ[n, -1] && (IntegerQ[n] || IntegersQ[2*m, 2*n])
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+ia\tan(e+fx)}}{(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{2d\sqrt{a+ia\tan(e+fx)}}{3\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{2\int \frac{\sqrt{a+ia\tan(e+fx)}\left(\frac{1}{2}a(3c+id)-ad\right.}{(c+d\tan(e+fx))^{3/2}}}{3a\left(c^2+d^2\right)}$$

$$= -\frac{2d\sqrt{a+ia\tan(e+fx)}}{3\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - \frac{2(5c+id)d\sqrt{a+ia\tan(e+fx)}}{3\left(c^2+d^2\right)^2f\sqrt{c+d\tan(e+fx)}} +$$

$$= -\frac{2d\sqrt{a+ia\tan(e+fx)}}{3\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - \frac{2(5c+id)d\sqrt{a+ia\tan(e+fx)}}{3\left(c^2+d^2\right)^2f\sqrt{c+d\tan(e+fx)}} +$$

$$= -\frac{2d\sqrt{a+ia\tan(e+fx)}}{3\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - \frac{2(5c+id)d\sqrt{a+ia\tan(e+fx)}}{3\left(c^2+d^2\right)^2f\sqrt{c+d\tan(e+fx)}} -$$

$$= -\frac{i\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{(c-id)^{5/2}f} - \frac{2d\sqrt{a+ia\tan}}{3\left(c^2+d^2\right)f(c+d\,\mathrm{ta}}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 394 vs. 2(188) = 376.
time = 4.97, size = 394, normalized size = 2.10

$$\frac{\sqrt{2}\sqrt{e^{ifx}}\left(-\frac{4de^{i(e+fx)}\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\left(d^2e^{2i(e+fx)}+3c^2\left(1+e^{2i(e+fx)}\right)-icd\left(-3+2e^{2i(e+fx)}\right)\right)}{3(c-id)^2(c+id)^2\left(-id\left(-1+e^{2i(e+fx)}\right)+c\left(1+e^{2i(e+fx)}\right)\right)^2}-\frac{i\log\left(2\left(\sqrt{c-id}\,e^{i(e+fx)}+\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)\right)}{(c-id)^{5/2}}\right)\sqrt{a+ia\tan(e+fx)}}{\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\sqrt{1+e^{2i(e+fx)}}\,f\sqrt{\sec(e+fx)}\sqrt{\cos(fx)+i\sin(fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Tan[e + f*x]]/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[2]*Sqrt[E^(I*f*x)]*((-4*d*E^(I*(e + f*x))*Sqrt[1 + E^((2*I)*(e + f*x)
)]*Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))]*(d^
2*E^((2*I)*(e + f*x)) + 3*c^2*(1 + E^((2*I)*(e + f*x))) - I*c*d*(-3 + 2*E^(
(2*I)*(e + f*x)))))/(3*(c - I*d)^2*(c + I*d)^2*((-I)*d*(-1 + E^((2*I)*(e +
f*x))) + c*(1 + E^((2*I)*(e + f*x))))^2) - (I*Log[2*(Sqrt[c - I*d]*E^(I*(e
+ f*x)) + Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f
*x))))/(1 + E^((2*I)*(e + f*x)))])])/(c - I*d)^(5/2))*Sqrt[a + I*a*Tan[e +
f*x]])/(Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*Sqrt[1 + E^((2*I)*(
e + f*x))]*f*Sqrt[Sec[e + f*x]]*Sqrt[Cos[f*x] + I*Sin[f*x]])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2447 vs. 2(153) = 306.
time = 0.73, size = 2448, normalized size = 13.02

method	result	size
derivativedivides	Expression too large to display	2448
default	Expression too large to display	2448

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/6/f*2^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)*(-9*ln((3*a*c+I*a*tan(f*x+e)*c-I*a
*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*t
an(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c^3*d^2*tan(f*x+e)+9*ln((3*a*c+I*a*tan
(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f
*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c*d^4*tan(f*x+e)^3-6*ln((
3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*
(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c^4*d*tan(f*
x+e)^2+9*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I
*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a
*c^2*d^3*tan(f*x+e)^2+3*I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)
+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/
(tan(f*x+e)+I))*a*d^5*tan(f*x+e)^3-9*I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a
*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x
+e)))^(1/2))/(tan(f*x+e)+I))*a*c^3*d^2+2*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x
+e)))^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*d^4+6*ln((3*a*c+I*a*tan(f*x+e)*c-I*a
*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*t
an(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c*d^4*tan(f*x+e)+12*2^(1/2)*c^3*d*(-a*
(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)-4*2^(1/2)*d^4*(-
a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*tan(f*x+e)-3*l
n((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/
2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c^3*d^2*t
an(f*x+e)^3+3*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(
-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+
I))*a*d^5*tan(f*x+e)^2-3*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+
2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(
tan(f*x+e)+I))*a*c^5*tan(f*x+e)-9*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*ta
n(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))
^(1/2))/(tan(f*x+e)+I))*a*c^4*d+3*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*ta
n(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))
^(1/2))/(tan(f*x+e)+I))*a*c^2*d^3+3*I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*
d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+
e)))^(1/2))/(tan(f*x+e)+I))*a*c^5+12*I*2^(1/2)*c^3*d*(-a*(I*d-c))^(1/2)*(a*
(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*tan(f*x+e)+12*I*(a*(c+d*tan(f*x+e)
)*(1+I*tan(f*x+e)))^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*c*d^3*tan(f*x+e)+10*I*
2^(1/2)*c^2*d^2*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1
```

```
/2)*tan(f*x+e)^2-2*I*2^(1/2)*d^4*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+
I*tan(f*x+e)))^(1/2)*tan(f*x+e)^2-3*I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*
d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+
e)))^(1/2))/(tan(f*x+e)+I))*a*c^4*d*tan(f*x+e)-15*I*ln((3*a*c+I*a*tan(f*x+e
)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))
*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c^2*d^3*tan(f*x+e)+14*I*(a*(c+d
*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*2^(1/2)*(-a*(I*d-c))^(1/2)*c^2*d^2-12*
2^(1/2)*c*d^3*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2
)*tan(f*x+e)^2-9*I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1
/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*
x+e)+I))*a*c^2*d^3*tan(f*x+e)^3-15*I*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d
*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e
)))^(1/2))/(tan(f*x+e)+I))*a*c^3*d^2*tan(f*x+e)^2-3*I*ln((3*a*c+I*a*tan(f*x
+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e
))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*a*c*d^4*tan(f*x+e)^2-4*2^(1/2)*
c^2*d^2*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*tan(
f*x+e))/(c+d*tan(f*x+e))^(3/2)/(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)/
(c^2+d^2)^2/(I*c-d)/(-tan(f*x+e)+I)/(-a*(I*d-c))^(1/2)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] Timed out
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 992 vs. $2(152) = 304$.
time = 1.05, size = 992, normalized size = 5.28

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] -1/6*(8*sqrt(2)*((3*c^2*d - 2*I*c*d^2 + d^3)*e^(5*I*f*x + 5*I*e) + (6*c^2*d
 + I*c*d^2 + d^3)*e^(3*I*f*x + 3*I*e) + 3*(c^2*d + I*c*d^2)*e^(I*f*x + I*e)
)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))
*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)) - 3*((c^6 - 2*I*c^5*d + c^4*d^2 - 4*I*c^
3*d^3 - c^2*d^4 - 2*I*c*d^5 - d^6)*f*e^(4*I*f*x + 4*I*e) + 2*(c^6 + 3*c^4*d
^2 + 3*c^2*d^4 + d^6)*f*e^(2*I*f*x + 2*I*e) + (c^6 + 2*I*c^5*d + c^4*d^2 +
```

```
4*I*c^3*d^3 - c^2*d^4 + 2*I*c*d^5 - d^6)*f)*sqrt(-2*I*a/((I*c^5 + 5*c^4*d -
 10*I*c^3*d^2 - 10*c^2*d^3 + 5*I*c*d^4 + d^5)*f^2))*log(((I*c^3 + 3*c^2*d -
 3*I*c*d^2 - d^3)*f*sqrt(-2*I*a/((I*c^5 + 5*c^4*d - 10*I*c^3*d^2 - 10*c^2*d
^3 + 5*I*c*d^4 + d^5)*f^2))*e^(I*f*x + I*e) + sqrt(2)*sqrt(((c - I*d)*e^(2*
I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2
*I*e) + 1))*(e^(2*I*f*x + 2*I*e) + 1))*e^(-I*f*x - I*e)) + 3*((c^6 - 2*I*c^
5*d + c^4*d^2 - 4*I*c^3*d^3 - c^2*d^4 - 2*I*c*d^5 - d^6)*f*e^(4*I*f*x + 4*I
*e) + 2*(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6)*f*e^(2*I*f*x + 2*I*e) + (c^6 +
2*I*c^5*d + c^4*d^2 + 4*I*c^3*d^3 - c^2*d^4 + 2*I*c*d^5 - d^6)*f)*sqrt(-2*I
*a/((I*c^5 + 5*c^4*d - 10*I*c^3*d^2 - 10*c^2*d^3 + 5*I*c*d^4 + d^5)*f^2))*l
og(((-I*c^3 - 3*c^2*d + 3*I*c*d^2 + d^3)*f*sqrt(-2*I*a/((I*c^5 + 5*c^4*d -
10*I*c^3*d^2 - 10*c^2*d^3 + 5*I*c*d^4 + d^5)*f^2))*e^(I*f*x + I*e) + sqrt(2
)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))
*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*(e^(2*I*f*x + 2*I*e) + 1))*e^(-I*f*x - I
*e)))/((c^6 - 2*I*c^5*d + c^4*d^2 - 4*I*c^3*d^3 - c^2*d^4 - 2*I*c*d^5 - d^6
)*f*e^(4*I*f*x + 4*I*e) + 2*(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6)*f*e^(2*I*f*
x + 2*I*e) + (c^6 + 2*I*c^5*d + c^4*d^2 + 4*I*c^3*d^3 - c^2*d^4 + 2*I*c*d^5
 - d^6)*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ia\left(\tan\left(e+fx\right)-i\right)}}{\left(c+d\tan\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(1/2)/(c+d*tan(f*x+e))**(5/2),x)
[Out] Integral(sqrt(I*a*(tan(e + f*x) - I))/(c + d*tan(e + f*x))**(5/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="gia
c")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}}}{{\left(c+d\,\tan\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^(1/2)/(c + d*tan(e + f*x))^(5/2),x)
[Out] int((a + a*tan(e + f*x)*1i)^(1/2)/(c + d*tan(e + f*x))^(5/2), x)
```

3.1170 $\int \frac{1}{\sqrt{a+ia\tan(e+fx)}\,(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=277

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\,(c-id)^{5/2}f}-\frac{1}{(ic-d)f\sqrt{a+ia\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}}+\frac{}{3a(i}$$

```
[Out] -1/2*I*arctanh(2^(1/2)*a^(1/2)*(c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2)/(a+I*a*
tan(f*x+e))^(1/2))/(c-I*d)^(5/2)/f*2^(1/2)/a^(1/2)+1/3*(3*c-I*d)*(c-7*I*d)*
d*(a+I*a*tan(f*x+e))^(1/2)/a/(c-I*d)^2/(c+I*d)^3/f/(c+d*tan(f*x+e))^(1/2)-1
/(I*c-d)/f/(a+I*a*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(3/2)+1/3*d*(3*I*c+5*d
)*(a+I*a*tan(f*x+e))^(1/2)/a/(I*c-d)/(c^2+d^2)/f/(c+d*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.59, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3640, 3679, 12, 3625, 214}

$$\frac{d(5d+3ic)\sqrt{a+ia\tan(e+fx)}}{3af(-d+ic)\,(c^2+d^2)\,(c+d\tan(e+fx))^{3/2}}+\frac{d(3c-id)(c-7id)\sqrt{a+ia\tan(e+fx)}}{3af(c-id)^2(c+id)^3\sqrt{c+d\tan(e+fx)}}-\frac{1}{f(-d+ic)\sqrt{a+ia\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\,f(c-id)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + I*a*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(5/2)),x]
```

```
[Out] ((-I)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])/(Sqrt[c - I*d]*Sqr
t[a + I*a*Tan[e + f*x]])])/(Sqrt[2]*Sqrt[a]*(c - I*d)^(5/2)*f) - 1/((I*c -
d)*f*Sqrt[a + I*a*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2)) + (d*((3*I)*c +
 5*d)*Sqrt[a + I*a*Tan[e + f*x]])/(3*a*(I*c - d)*(c^2 + d^2)*f*(c + d*Tan[e
 + f*x])^(3/2)) + ((3*c - I*d)*(c - (7*I)*d)*d*Sqrt[a + I*a*Tan[e + f*x]])/
(3*a*(c - I*d)^2*(c + I*d)^3*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
```

```
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+ia\tan(e+fx)}\,(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{1}{(ic-d)f\sqrt{a+ia\tan(e+fx)}\,(c+d\tan(e+fx))^{3/}}$$

$$= -\frac{1}{(ic-d)f\sqrt{a+ia\tan(e+fx)}\,(c+d\tan(e+fx))^{3/}}$$

$$= -\frac{1}{(ic-d)f\sqrt{a+ia\tan(e+fx)}\,(c+d\tan(e+fx))^{3/}}$$

$$= -\frac{1}{(ic-d)f\sqrt{a+ia\tan(e+fx)}\,(c+d\tan(e+fx))^{3/}}$$

$$= -\frac{1}{(ic-d)f\sqrt{a+ia\tan(e+fx)}\,(c+d\tan(e+fx))^{3/}}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\,(c-id)^{5/2}f} - \frac{}{(ic-}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 687 vs. 2(277) = 554.
time = 8.97, size = 687, normalized size = 2.48

$$-\frac{ie^{ie}\sqrt{e^{ifx}}\log\left(2\left(\sqrt{c-id}\,e^{i(e+fx)}+\sqrt{1+e^{2i(e+fx)}}\sqrt{c-\frac{id\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}}\right)\right)\sqrt{\sec(e+fx)}\sqrt{\cos(fx)+i\sin(fx)}}{\sqrt{2}\,(c-id)^{5/2}\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\sqrt{1+e^{2i(e+fx)}}\,f\sqrt{a+ia\tan(e+fx)}}+\frac{\sec(e+fx)(\cos(fx)+i\sin(fx))\sqrt{\sec(e+fx)(c\cos(e+fx)+d\sin(e+fx))}\left(\frac{\cos(2fx)\left(\frac{1}{2}i\cos(e)+\frac{\sin(e)}{2}\right)}{(c+id)^3}+\frac{\left(\frac{\cos(e)}{6}+\frac{1}{6}i\sin(e)\right)(3ic^3\cos(e)+6c^2d\cos(e)-39icd^2\cos(e)-8d^3\cos(e)+3ic^2d\sin(e)+6cd^2\sin(e)+id^3\sin(e))}{(c-id)^2(c+id)^3(c\cos(e)+d\sin(e))}+\frac{\left(\frac{\cos(e)}{2}-\frac{1}{2}i\sin(e)\right)\sin(2fx)}{(c+id)^3}+\frac{-\frac{2}{3}id^4\cos(e)+\frac{2}{3}d^4\sin(e)}{(c-id)^2(c+id)^3(c\cos(e+fx)+d\sin(e+fx))^2}+\frac{4\left(-\frac{5}{2}cd^3\cos(e-fx)+\frac{1}{2}id^4\cos(e-fx)+\frac{5}{2}cd^3\cos(e+fx)-\frac{1}{2}id^4\cos(e+fx)-\frac{5}{2}icd^3\sin(e-fx)-\frac{1}{2}d^4\sin(e-fx)+\frac{5}{2}icd^3\sin(e+fx)+\frac{1}{2}d^4\sin(e+fx)\right)}{3(c-id)^2(c+id)^3(c\cos(e)+d\sin(e))(c\cos(e+fx)+d\sin(e+fx))}\right)}{f\sqrt{a+ia\tan(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + I*a*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(5/2)),x]
```

```
[Out] ((-I)*E^(I*e)*Sqrt[E^(I*f*x)]*Log[2*(Sqrt[c - I*d]*E^(I*(e + f*x)) + Sqrt[1
 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((
2*I)*(e + f*x)))])]*Sqrt[Sec[e + f*x]]*Sqrt[Cos[f*x] + I*Sin[f*x]])/(Sqrt[2
]*(c - I*d)^(5/2)*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*Sqrt[1 +
E^((2*I)*(e + f*x))]*f*Sqrt[a + I*a*Tan[e + f*x]]) + (Sec[e + f*x]*(Cos[f*x
] + I*Sin[f*x])*Sqrt[Sec[e + f*x]*(c*Cos[e + f*x] + d*Sin[e + f*x])]*((Cos[
2*f*x]*((I/2)*Cos[e] + Sin[e]/2))/(c + I*d)^3 + ((Cos[e]/6 + (I/6)*Sin[e])*
((3*I)*c^3*Cos[e] + 6*c^2*d*Cos[e] - (39*I)*c*d^2*Cos[e] - 8*d^3*Cos[e] + (
3*I)*c^2*d*Sin[e] + 6*c*d^2*Sin[e] + I*d^3*Sin[e]))/((c - I*d)^2*(c + I*d)^
3*(c*Cos[e] + d*Sin[e])) + ((Cos[e]/2 - (I/2)*Sin[e])*Sin[2*f*x])/(c + I*d)
^3 + (((-2*I)/3)*d^4*Cos[e] + (2*d^4*Sin[e])/3)/((c - I*d)^2*(c + I*d)^3*(c
```

```
*Cos[e + f*x] + d*Sin[e + f*x])^2) + (4*((-5*c*d^3*Cos[e - f*x])/2 + (I/2)*
d^4*Cos[e - f*x] + (5*c*d^3*Cos[e + f*x])/2 - (I/2)*d^4*Cos[e + f*x] - ((5*
I)/2)*c*d^3*Sin[e - f*x] - (d^4*Sin[e - f*x])/2 + ((5*I)/2)*c*d^3*Sin[e + f
*x] + (d^4*Sin[e + f*x])/2))/(3*(c - I*d)^2*(c + I*d)^3*(c*Cos[e] + d*Sin[e
])*(c*Cos[e + f*x] + d*Sin[e + f*x]))))/(f*Sqrt[a + I*a*Tan[e + f*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4888 vs. $2(231) = 462$.
time = 0.71, size = 4889, normalized size = 17.65

method	result	size
derivativedivides	Expression too large to display	4889
default	Expression too large to display	4889

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(5/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/12/f*(-28*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*d^7*tan(f*x+e)^3+84
*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c^3*d^4+24*(a*(c+d*tan(f*x+e))
*(1+I*tan(f*x+e)))^(1/2)*c*d^6+48*c^5*d^2*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+
e)))^(1/2)+8*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*d^7-30*2^(1/2)*l
n((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/
2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c^5*d^2*(-a
*(I*d-c))^(1/2)+15*2^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e
)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))
/(tan(f*x+e)+I))*c^3*d^4*(-a*(I*d-c))^(1/2)-12*I*c^5*d^2*(a*(c+d*tan(f*x+e)
)*(1+I*tan(f*x+e)))^(1/2)*tan(f*x+e)^3+80*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*
x+e)))^(1/2)*c^4*d^3+76*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c^2*d
^5-108*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c^5*d^2*tan(f*x+e)^2-264
*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c^3*d^4*tan(f*x+e)^2-156*(a*(c
+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c*d^6*tan(f*x+e)^2-36*(a*(c+d*tan(f*
x+e))*(1+I*tan(f*x+e)))^(1/2)*c^6*d*tan(f*x+e)-120*(a*(c+d*tan(f*x+e))*(1+I
*tan(f*x+e)))^(1/2)*c^4*d^3*tan(f*x+e)-84*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+
e)))^(1/2)*c^2*d^5*tan(f*x+e)-76*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2
)*c^4*d^3*tan(f*x+e)^3-104*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c^2*
d^5*tan(f*x+e)^3+3*2^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e
)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))
/(tan(f*x+e)+I))*c^7*(-a*(I*d-c))^(1/2)+36*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f
*x+e)))^(1/2)*d^7*tan(f*x+e)^2-12*I*c^7*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)
))^(1/2)*tan(f*x+e)+12*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c^6*d-
72*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c^3*d^4*tan(f*x+e)^3-60*I*
(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c*d^6*tan(f*x+e)^3-24*I*c^6*d*(
a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*tan(f*x+e)^2+36*I*(a*(c+d*tan(f*
x+e))*(1+I*tan(f*x+e)))^(1/2)*c^4*d^3*tan(f*x+e)^2+96*I*(a*(c+d*tan(f*x+e))
```

```
*(1+I*tan(f*x+e)))^(1/2)*c^2*d^5*tan(f*x+e)^2+144*I*(a*(c+d*tan(f*x+e))*(1+
I*tan(f*x+e)))^(1/2)*c^5*d^2*tan(f*x+e)+276*I*(a*(c+d*tan(f*x+e))*(1+I*tan(
f*x+e)))^(1/2)*c^3*d^4*tan(f*x+e)+120*I*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)
))^(1/2)*c*d^6*tan(f*x+e)-6*2^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*
tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)
))^(1/2))/(tan(f*x+e)+I))*d^7*(-a*(I*d-c))^(1/2)*tan(f*x+e)^3-3*2^(1/2)*ln(
(3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)
*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c^7*(-a*(I*d-
c))^(1/2)*tan(f*x+e)^2-12*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2)*c^7-1
5*I*2^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a
*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I)
)*c^4*d^3*(-a*(I*d-c))^(1/2)*tan(f*x+e)^4+30*I*2^(1/2)*ln((3*a*c+I*a*tan(f*
x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+
e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c^2*d^5*(-a*(I*d-c))^(1/2)*tan
(f*x+e)^4+3*I*2^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2
^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan
(f*x+e)+I))*c^2*d^5*(-a*(I*d-c))^(1/2)+30*2^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*
c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(
1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c^3*d^4*(-a*(I*d-c))^(1/2)*tan(f*x+
e)^4-15*2^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)
*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e
)+I))*c*d^6*(-a*(I*d-c))^(1/2)*tan(f*x+e)^4-6*2^(1/2)*ln((3*a*c+I*a*tan(f*x
+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e
))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c^6*d*(-a*(I*d-c))^(1/2)*tan(f*
x+e)^3+30*2^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/
2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x
+e)+I))*c^4*d^3*(-a*(I*d-c))^(1/2)*tan(f*x+e)^3+30*2^(1/2)*ln((3*a*c+I*a*ta
n(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(
f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c^2*d^5*(-a*(I*d-c))^(1/2)
*tan(f*x+e)^3-27*2^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+
2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(
tan(f*x+e)+I))*c^5*d^2*(-a*(I*d-c))^(1/2)*tan(f*x+e)^2+75*2^(1/2)*ln((3*a*c
+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c
+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c^3*d^4*(-a*(I*d-c)
)^(1/2)*tan(f*x+e)^2+3*2^(1/2)*ln((3*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f
*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1
/2))/(tan(f*x+e)+I))*c*d^6*(-a*(I*d-c))^(1/2)*tan(f*x+e)^2-24*2^(1/2)*ln((3
*a*c+I*a*tan(f*x+e)*c-I*a*d+3*a*d*tan(f*x+e)+2*2^(1/2)*(-a*(I*d-c))^(1/2)*(
a*(c+d*tan(f*x+e))*(1+I*tan(f*x+e)))^(1/2))/(tan(f*x+e)+I))*c^6*d*(-a*(I*d-
c))^(1/2)*tan(f*x+e)+24*2^(1/2)*ln((3*a*c+I*a*t...
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1325 vs. $2(225) = 450$.
time = 1.39, size = 1325, normalized size = 4.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="f
ricas")
```

```
[Out] 1/12*(2*sqrt(2)*(3*c^4 + 6*c^2*d^2 + 3*d^4 + (3*c^4 - 12*I*c^3*d - 54*c^2*d
^2 + 52*I*c*d^3 + 7*d^4)*e^(6*I*f*x + 6*I*e) + (9*c^4 - 24*I*c^3*d - 90*c^2
*d^2 + 16*I*c*d^3 - 11*d^4)*e^(4*I*f*x + 4*I*e) + 3*(3*c^4 - 4*I*c^3*d - 10
*c^2*d^2 - 12*I*c*d^3 - 5*d^4)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(2*I*
f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I
*e) + 1)) + 3*((-I*a*c^7 - a*c^6*d - 3*I*a*c^5*d^2 - 3*a*c^4*d^3 - 3*I*a*c^
3*d^4 - 3*a*c^2*d^5 - I*a*c*d^6 - a*d^7)*f*e^(5*I*f*x + 5*I*e) + 2*(-I*a*c^
7 + a*c^6*d - 3*I*a*c^5*d^2 + 3*a*c^4*d^3 - 3*I*a*c^3*d^4 + 3*a*c^2*d^5 - I
*a*c*d^6 + a*d^7)*f*e^(3*I*f*x + 3*I*e) + (-I*a*c^7 + 3*a*c^6*d + I*a*c^5*d
^2 + 5*a*c^4*d^3 + 5*I*a*c^3*d^4 + a*c^2*d^5 + 3*I*a*c*d^6 - a*d^7)*f*e^(I*
f*x + I*e))*sqrt(-2*I/((I*a*c^5 + 5*a*c^4*d - 10*I*a*c^3*d^2 - 10*a*c^2*d^3
 + 5*I*a*c*d^4 + a*d^5)*f^2))*log((I*a*c^3 + 3*a*c^2*d - 3*I*a*c*d^2 - a*d^
3)*f*sqrt(-2*I/((I*a*c^5 + 5*a*c^4*d - 10*I*a*c^3*d^2 - 10*a*c^2*d^3 + 5*I*
a*c*d^4 + a*d^5)*f^2))*e^(I*f*x + I*e) + sqrt(2)*sqrt(((c - I*d)*e^(2*I*f*x
 + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e)
 + 1))*(e^(2*I*f*x + 2*I*e) + 1)) + 3*((I*a*c^7 + a*c^6*d + 3*I*a*c^5*d^2 +
 3*a*c^4*d^3 + 3*I*a*c^3*d^4 + 3*a*c^2*d^5 + I*a*c*d^6 + a*d^7)*f*e^(5*I*f*
x + 5*I*e) + 2*(I*a*c^7 - a*c^6*d + 3*I*a*c^5*d^2 - 3*a*c^4*d^3 + 3*I*a*c^3
*d^4 - 3*a*c^2*d^5 + I*a*c*d^6 - a*d^7)*f*e^(3*I*f*x + 3*I*e) + (I*a*c^7 -
3*a*c^6*d - I*a*c^5*d^2 - 5*a*c^4*d^3 - 5*I*a*c^3*d^4 - a*c^2*d^5 - 3*I*a*c
*d^6 + a*d^7)*f*e^(I*f*x + I*e))*sqrt(-2*I/((I*a*c^5 + 5*a*c^4*d - 10*I*a*c
^3*d^2 - 10*a*c^2*d^3 + 5*I*a*c*d^4 + a*d^5)*f^2))*log((-I*a*c^3 - 3*a*c^2*
d + 3*I*a*c*d^2 + a*d^3)*f*sqrt(-2*I/((I*a*c^5 + 5*a*c^4*d - 10*I*a*c^3*d^2
 - 10*a*c^2*d^3 + 5*I*a*c*d^4 + a*d^5)*f^2))*e^(I*f*x + I*e) + sqrt(2)*sqrt
(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(
a/(e^(2*I*f*x + 2*I*e) + 1))*(e^(2*I*f*x + 2*I*e) + 1)))/((-I*a*c^7 - a*c^6
*d - 3*I*a*c^5*d^2 - 3*a*c^4*d^3 - 3*I*a*c^3*d^4 - 3*a*c^2*d^5 - I*a*c*d^6
- a*d^7)*f*e^(5*I*f*x + 5*I*e) + 2*(-I*a*c^7 + a*c^6*d - 3*I*a*c^5*d^2 + 3*
```

```
a*c^4*d^3 - 3*I*a*c^3*d^4 + 3*a*c^2*d^5 - I*a*c*d^6 + a*d^7)*f*e^(3*I*f*x +
 3*I*e) + (-I*a*c^7 + 3*a*c^6*d + I*a*c^5*d^2 + 5*a*c^4*d^3 + 5*I*a*c^3*d^4
 + a*c^2*d^5 + 3*I*a*c*d^6 - a*d^7)*f*e^(I*f*x + I*e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ia\left(\tan\left(e+fx\right)-i\right)}\,\left(c+d\tan\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**(1/2)/(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral(1/(sqrt(I*a*(tan(e + f*x) - I))*(c + d*tan(e + f*x))**(5/2)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="g
iac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a+a\tan\left(e+f\,x\right)\,1{}\mathrm{i}}\,{\left(c+d\tan\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^(1/2)*(c + d*tan(e + f*x))^(5/2)),x)
```

```
[Out] int(1/((a + a*tan(e + f*x)*1i)^(1/2)*(c + d*tan(e + f*x))^(5/2)), x)
```

3.1171 $\int \frac{1}{(a+ia\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=354

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}(c-id)^{5/2}f}-\frac{1}{3(ic-d)f(a+ia\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}+\frac{}{2a(}$$

```
[Out] -1/4*I*arctanh(2^(1/2)*a^(1/2)*(c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2)/(a+I*a*
tan(f*x+e))^(1/2))/a^(3/2)/(c-I*d)^(5/2)/f*2^(1/2)+1/6*(c-3*I*d)*d*(3*c^2+2
2*I*c*d+13*d^2)*(a+I*a*tan(f*x+e))^(1/2)/a^2/(c-I*d)^2/(c+I*d)^4/f/(c+d*tan
(f*x+e))^(1/2)+1/2*(I*c-5*d)/a/(c+I*d)^2/f/(a+I*a*tan(f*x+e))^(1/2)/(c+d*ta
n(f*x+e))^(3/2)+1/6*d*(3*c^2+14*I*c*d+21*d^2)*(a+I*a*tan(f*x+e))^(1/2)/a^2/
(c-I*d)/(c+I*d)^3/f/(c+d*tan(f*x+e))^(3/2)-1/3/(I*c-d)/f/(a+I*a*tan(f*x+e))
^(3/2)/(c+d*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.85, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3640, 3677, 3679, 12, 3625, 214}

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f(c-id)^{5/2}}+\frac{d(c-3id)\left(3c^2+22icd+13d^2\right)\sqrt{a+ia\tan(e+fx)}}{6a^2f(c-id)^2(c+id)^4\sqrt{c+d\tan(e+fx)}}+\frac{d\left(3c^2+14icd+21d^2\right)\sqrt{a+ia\tan(e+fx)}}{6a^2f(c-id)(c+id)^3(c+d\tan(e+fx))^{3/2}}+\frac{-5d+ic}{2af(c+id)^2\sqrt{a+ia\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}}-\frac{1}{3f(-d+ic)(a+ia\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(5/2)),x]
```

```
[Out] ((-1/2*I)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])/(Sqrt[c - I*d]
*Sqrt[a + I*a*Tan[e + f*x]])])/(Sqrt[2]*a^(3/2)*(c - I*d)^(5/2)*f) - 1/(3*(
I*c - d)*f*(a + I*a*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(3/2)) + (I*c
- 5*d)/(2*a*(c + I*d)^2*f*Sqrt[a + I*a*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(
3/2)) + (d*(3*c^2 + (14*I)*c*d + 21*d^2)*Sqrt[a + I*a*Tan[e + f*x]])/(6*a^2
*(c - I*d)*(c + I*d)^3*f*(c + d*Tan[e + f*x])^(3/2)) + ((c - (3*I)*d)*d*(3*
c^2 + (22*I)*c*d + 13*d^2)*Sqrt[a + I*a*Tan[e + f*x]])/(6*a^2*(c - I*d)^2*(
c + I*d)^4*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{1}{3(ic-d)f(a+ia\tan(e+fx))^{3/2}(c+d\tan(e+fx)}$$

$$= -\frac{1}{3(ic-d)f(a+ia\tan(e+fx))^{3/2}(c+d\tan(e+fx)}$$

$$= -\frac{1}{3(ic-d)f(a+ia\tan(e+fx))^{3/2}(c+d\tan(e+fx)}$$

$$= -\frac{1}{3(ic-d)f(a+ia\tan(e+fx))^{3/2}(c+d\tan(e+fx)}$$

$$= -\frac{1}{3(ic-d)f(a+ia\tan(e+fx))^{3/2}(c+d\tan(e+fx)}$$

$$= -\frac{1}{3(ic-d)f(a+ia\tan(e+fx))^{3/2}(c+d\tan(e+fx)}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}(c-id)^{5/2}f} - \frac{}{3(ic-}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 803 vs. $2(354) = 708$.
time = 9.37, size = 803, normalized size = 2.27

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(5/2)),x]
```

```
[Out] ((-1/2*I)*E^((2*I)*e)*Sqrt[E^(I*f*x)]*Log[2*(Sqrt[c - I*d]*E^(I*(e + f*x))
+ Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(
1 + E^((2*I)*(e + f*x)))])]*Sec[e + f*x]^(3/2)*(Cos[f*x] + I*Sin[f*x])^(3/2
))/(Sqrt[2]*(c - I*d)^(5/2)*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]
*Sqrt[1 + E^((2*I)*(e + f*x))]*f*(a + I*a*Tan[e + f*x])^(3/2)) + (Sec[e + f
*x]^2*(Cos[f*x] + I*Sin[f*x])^2*Sqrt[Sec[e + f*x]*(c*Cos[e + f*x] + d*Sin[e
 + f*x])]*(((I/12)*(5*c + (21*I)*d)*Cos[2*f*x])/(c + I*d)^4 + ((I*c^4*Cos[e
```

```
] - 3*c^3*d*Cos[e] + (9*I)*c^2*d^2*Cos[e] + 29*c*d^3*Cos[e] - (10*I)*d^4*Co
s[e] + I*c^3*d*Sin[e] - 3*c^2*d^2*Sin[e] + (9*I)*c*d^3*Sin[e] + 3*d^4*Sin[e
])*(Cos[2*e]/3 + (I/3)*Sin[2*e]))/((c - I*d)^2*(c + I*d)^4*(c*Cos[e] + d*Si
n[e])) + (Cos[4*f*x]*((I/12)*Cos[2*e] + Sin[2*e]/12))/(c + I*d)^3 + ((5*c +
 (21*I)*d)*Sin[2*f*x])/(12*(c + I*d)^4) + ((Cos[2*e]/12 - (I/12)*Sin[2*e])*
Sin[4*f*x])/(c + I*d)^3 + ((2*d^5*Cos[2*e])/3 + ((2*I)/3)*d^5*Sin[2*e])/((c
 - I*d)^2*(c + I*d)^4*(c*Cos[e + f*x] + d*Sin[e + f*x])^2) - (2*(((13*I)/2)
*c*d^4*Cos[2*e - f*x] + (5*d^5*Cos[2*e - f*x])/2 - ((13*I)/2)*c*d^4*Cos[2*e
 + f*x] - (5*d^5*Cos[2*e + f*x])/2 - (13*c*d^4*Sin[2*e - f*x])/2 + ((5*I)/2
)*d^5*Sin[2*e - f*x] + (13*c*d^4*Sin[2*e + f*x])/2 - ((5*I)/2)*d^5*Sin[2*e
+ f*x]))/(3*(c - I*d)^2*(c + I*d)^4*(c*Cos[e] + d*Sin[e])*(c*Cos[e + f*x] +
 d*Sin[e + f*x]))))/(f*(a + I*a*Tan[e + f*x])^(3/2))
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 7060 vs. 2(294) = 588.
time = 0.65, size = 7061, normalized size = 19.95

method	result	size
derivativedivides	Expression too large to display	7061
default	Expression too large to display	7061

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(5/2),x,method=_RETURNVERBO
SE)
```

```
[Out] result too large to display
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1572 vs. 2(286) = 572.
time = 1.27, size = 1572, normalized size = 4.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="f
ricas")
```

```
[Out] 1/12*(sqrt(2)*(I*c^5 - c^4*d + 2*I*c^3*d^2 - 2*c^2*d^3 + I*c*d^4 - d^5 - 4*
(-I*c^5 + c^4*d - 14*I*c^3*d^2 - 50*c^2*d^3 + 51*I*c*d^4 + 13*d^5)*e^(8*I*f
*x + 8*I*e) + (13*I*c^5 - 25*c^4*d + 134*I*c^3*d^2 + 314*c^2*d^3 - 87*I*c*d
^4 + 35*d^5)*e^(6*I*f*x + 6*I*e) - 3*(-5*I*c^5 + 13*c^4*d - 30*I*c^3*d^2 -
26*c^2*d^3 - 41*I*c*d^4 - 23*d^5)*e^(4*I*f*x + 4*I*e) + (7*I*c^5 - 19*c^4*d
 + 14*I*c^3*d^2 - 38*c^2*d^3 + 7*I*c*d^4 - 19*d^5)*e^(2*I*f*x + 2*I*e))*sqr
t(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt
(a/(e^(2*I*f*x + 2*I*e) + 1)) - 3*((a^2*c^8 + 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4
 + 4*a^2*c^2*d^6 + a^2*d^8)*f*e^(7*I*f*x + 7*I*e) + 2*(a^2*c^8 + 2*I*a^2*c^
7*d + 2*a^2*c^6*d^2 + 6*I*a^2*c^5*d^3 + 6*I*a^2*c^3*d^5 - 2*a^2*c^2*d^6 + 2
*I*a^2*c*d^7 - a^2*d^8)*f*e^(5*I*f*x + 5*I*e) + (a^2*c^8 + 4*I*a^2*c^7*d -
4*a^2*c^6*d^2 + 4*I*a^2*c^5*d^3 - 10*a^2*c^4*d^4 - 4*I*a^2*c^3*d^5 - 4*a^2*
c^2*d^6 - 4*I*a^2*c*d^7 + a^2*d^8)*f*e^(3*I*f*x + 3*I*e))*sqrt(1/2*I/((-I*a
^3*c^5 - 5*a^3*c^4*d + 10*I*a^3*c^3*d^2 + 10*a^3*c^2*d^3 - 5*I*a^3*c*d^4 -
a^3*d^5)*f^2))*log(-2*(I*a^2*c^3 + 3*a^2*c^2*d - 3*I*a^2*c*d^2 - a^2*d^3)*f
*sqrt(1/2*I/((-I*a^3*c^5 - 5*a^3*c^4*d + 10*I*a^3*c^3*d^2 + 10*a^3*c^2*d^3
- 5*I*a^3*c*d^4 - a^3*d^5)*f^2))*e^(I*f*x + I*e) + sqrt(2)*sqrt(((c - I*d)*
e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*
x + 2*I*e) + 1))*(e^(2*I*f*x + 2*I*e) + 1)) + 3*((a^2*c^8 + 4*a^2*c^6*d^2 +
 6*a^2*c^4*d^4 + 4*a^2*c^2*d^6 + a^2*d^8)*f*e^(7*I*f*x + 7*I*e) + 2*(a^2*c^
8 + 2*I*a^2*c^7*d + 2*a^2*c^6*d^2 + 6*I*a^2*c^5*d^3 + 6*I*a^2*c^3*d^5 - 2*a
^2*c^2*d^6 + 2*I*a^2*c*d^7 - a^2*d^8)*f*e^(5*I*f*x + 5*I*e) + (a^2*c^8 + 4*
I*a^2*c^7*d - 4*a^2*c^6*d^2 + 4*I*a^2*c^5*d^3 - 10*a^2*c^4*d^4 - 4*I*a^2*c^
3*d^5 - 4*a^2*c^2*d^6 - 4*I*a^2*c*d^7 + a^2*d^8)*f*e^(3*I*f*x + 3*I*e))*sqr
t(1/2*I/((-I*a^3*c^5 - 5*a^3*c^4*d + 10*I*a^3*c^3*d^2 + 10*a^3*c^2*d^3 - 5*
I*a^3*c*d^4 - a^3*d^5)*f^2))*log(-2*(-I*a^2*c^3 - 3*a^2*c^2*d + 3*I*a^2*c*d
^2 + a^2*d^3)*f*sqrt(1/2*I/((-I*a^3*c^5 - 5*a^3*c^4*d + 10*I*a^3*c^3*d^2 +
10*a^3*c^2*d^3 - 5*I*a^3*c*d^4 - a^3*d^5)*f^2))*e^(I*f*x + I*e) + sqrt(2)*s
qrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sq
rt(a/(e^(2*I*f*x + 2*I*e) + 1))*(e^(2*I*f*x + 2*I*e) + 1)))/((a^2*c^8 + 4*a
^2*c^6*d^2 + 6*a^2*c^4*d^4 + 4*a^2*c^2*d^6 + a^2*d^8)*f*e^(7*I*f*x + 7*I*e)
 + 2*(a^2*c^8 + 2*I*a^2*c^7*d + 2*a^2*c^6*d^2 + 6*I*a^2*c^5*d^3 + 6*I*a^2*c
^3*d^5 - 2*a^2*c^2*d^6 + 2*I*a^2*c*d^7 - a^2*d^8)*f*e^(5*I*f*x + 5*I*e) + (
a^2*c^8 + 4*I*a^2*c^7*d - 4*a^2*c^6*d^2 + 4*I*a^2*c^5*d^3 - 10*a^2*c^4*d^4
- 4*I*a^2*c^3*d^5 - 4*a^2*c^2*d^6 - 4*I*a^2*c*d^7 + a^2*d^8)*f*e^(3*I*f*x +
 3*I*e))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia(\tan(e+fx)-i))^{\frac{3}{2}}(c+d\tan(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**(3/2)/(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral(1/((I*a*(tan(e + f*x) - I))**(3/2)*(c + d*tan(e + f*x))**(5/2)), x
)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="g
iac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+a\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{3/2}\,{\left(c+d\,\mathrm{tan}\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^(3/2)*(c + d*tan(e + f*x))^(5/2)),x)
```

```
[Out] int(1/((a + a*tan(e + f*x)*1i)^(3/2)*(c + d*tan(e + f*x))^(5/2)), x)
```

3.1172 $\int \frac{1}{(a+ia\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=444

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{4\sqrt{2}a^{5/2}(c-id)^{5/2}f}-\frac{1}{5(ic-d)f(a+ia\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}+\frac{}{30a}$$

```
[Out] -1/8*I*arctanh(2^(1/2)*a^(1/2)*(c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2)/(a+I*a*
tan(f*x+e))^(1/2))/a^(5/2)/(c-I*d)^(5/2)/f*2^(1/2)+1/60*d*(15*c^4+80*I*c^3*
d-182*c^2*d^2+1224*I*c*d^3+707*d^4)*(a+I*a*tan(f*x+e))^(1/2)/a^3/(c-I*d)^2/
(c+I*d)^5/f/(c+d*tan(f*x+e))^(1/2)+1/20*(5*c^2+30*I*c*d-89*d^2)/a^2/(I*c-d)
^3/f/(a+I*a*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(3/2)+1/60*d*(15*c^3+85*I*c^
2*d-221*c*d^2+361*I*d^3)*(a+I*a*tan(f*x+e))^(1/2)/a^3/(c-I*d)/(c+I*d)^4/f/(
c+d*tan(f*x+e))^(3/2)-1/5/(I*c-d)/f/(a+I*a*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e
))^(3/2)+1/30*(5*I*c-21*d)/a/(c+I*d)^2/f/(a+I*a*tan(f*x+e))^(3/2)/(c+d*tan(
f*x+e))^(3/2)
```

Rubi [A]
time = 1.19, antiderivative size = 444, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$,
Rules used = {3640, 3677, 3679, 12, 3625, 214}

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{4\sqrt{2}a^{5/2}f(c-id)^{5/2}}+\frac{d\left(15c^3+85ic^2d-221cd^2+361id^3\right)\sqrt{a+ia\tan(e+fx)}}{60a^3f(c-id)(c+id)^4(c+d\tan(e+fx))^{3/2}}+\frac{d\left(15c^4+80ic^3d-182c^2d^2+1224icd^3+707d^4\right)\sqrt{a+ia\tan(e+fx)}}{60a^3f(c-id)^2(c+id)^5\sqrt{c+d\tan(e+fx)}}+\frac{5c^2+30icd-89d^2}{20a^2f(-d+ic)^3\sqrt{a+ia\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}+\frac{-21d+5ic}{30af(c+id)^2(a+ia\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}-\frac{1}{5f(-d+ic)(a+ia\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + I*a*Tan[e + f*x])^(5/2)*(c + d*Tan[e + f*x])^(5/2)),x]
```

```
[Out] ((-1/4*I)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[c + d*Tan[e + f*x]])/(Sqrt[c - I*d]
*Sqrt[a + I*a*Tan[e + f*x]])]])/(Sqrt[2]*a^(5/2)*(c - I*d)^(5/2)*f) - 1/(5*(
I*c - d)*f*(a + I*a*Tan[e + f*x])^(5/2)*(c + d*Tan[e + f*x])^(3/2)) + ((5*I
)*c - 21*d)/(30*a*(c + I*d)^2*f*(a + I*a*Tan[e + f*x])^(3/2)*(c + d*Tan[e +
 f*x])^(3/2)) + (5*c^2 + (30*I)*c*d - 89*d^2)/(20*a^2*(I*c - d)^3*f*Sqrt[a
+ I*a*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2)) + (d*(15*c^3 + (85*I)*c^2*d
 - 221*c*d^2 + (361*I)*d^3)*Sqrt[a + I*a*Tan[e + f*x]])/(60*a^3*(c - I*d)*(
c + I*d)^4*f*(c + d*Tan[e + f*x])^(3/2)) + (d*(15*c^4 + (80*I)*c^3*d - 182*
c^2*d^2 + (1224*I)*c*d^3 + 707*d^4)*Sqrt[a + I*a*Tan[e + f*x]])/(60*a^3*(c
- I*d)^2*(c + I*d)^5*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{1}{5(ic-d)f(a+ia\tan(e+fx))^{5/2}(c+d\tan(e+fx)}$$

$$= -\frac{1}{5(ic-d)f(a+ia\tan(e+fx))^{5/2}(c+d\tan(e+fx)}$$

$$= -\frac{1}{5(ic-d)f(a+ia\tan(e+fx))^{5/2}(c+d\tan(e+fx)}$$

$$= -\frac{1}{5(ic-d)f(a+ia\tan(e+fx))^{5/2}(c+d\tan(e+fx)}$$

$$= -\frac{1}{5(ic-d)f(a+ia\tan(e+fx))^{5/2}(c+d\tan(e+fx)}$$

$$= -\frac{1}{5(ic-d)f(a+ia\tan(e+fx))^{5/2}(c+d\tan(e+fx)}$$

$$= -\frac{1}{5(ic-d)f(a+ia\tan(e+fx))^{5/2}(c+d\tan(e+fx)}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}\sqrt{a+ia\tan(e+fx)}}\right)}{4\sqrt{2}a^{5/2}(c-id)^{5/2}f} - \frac{}{5(ic-}$$

<u>Mathematica</u> [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 928 vs. $2(444) = 888$.
time = 10.28, size = 928, normalized size = 2.09

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + I*a*Tan[e + f*x])^(5/2)*(c + d*Tan[e + f*x])^(5/2)),x]
```

```
[Out] ((-1/4*I)*E^((3*I)*e)*Sqrt[E^(I*f*x)]*Log[2*(Sqrt[c - I*d]*E^(I*(e + f*x))
+ Sqrt[1 + E^((2*I)*(e + f*x))]*Sqrt[c - (I*d*(-1 + E^((2*I)*(e + f*x))))/(
1 + E^((2*I)*(e + f*x)))])]*Sec[e + f*x]^(5/2)*(Cos[f*x] + I*Sin[f*x])^(5/2
))/(Sqrt[2]*(c - I*d)^(5/2)*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]
*Sqrt[1 + E^((2*I)*(e + f*x))]*f*(a + I*a*Tan[e + f*x])^(5/2)) + (Sec[e + f
```

```
*x]^3*(Cos[f*x] + I*Sin[f*x])^3*Sqrt[Sec[e + f*x]*(c*Cos[e + f*x] + d*Sin[e
 + f*x])]*(((17*c^2 + (102*I)*c*d - 231*d^2)*Cos[2*f*x]*((I/60)*Cos[e] - Si
n[e]/60))/(c + I*d)^5 + ((c + (3*I)*d)*Cos[4*f*x]*(((7*I)/60)*Cos[e] + (7*S
in[e])/60))/(c + I*d)^4 + (((23*I)*c^5*Cos[e] - 108*c^4*d*Cos[e] - (138*I)*
c^3*d^2*Cos[e] - 692*c^2*d^3*Cos[e] + (1623*I)*c*d^4*Cos[e] + 640*d^5*Cos[e
] + (23*I)*c^4*d*Sin[e] - 108*c^3*d^2*Sin[e] - (138*I)*c^2*d^3*Sin[e] - 692
*c*d^4*Sin[e] + (343*I)*d^5*Sin[e])*(Cos[3*e]/120 + (I/120)*Sin[3*e]))/((c
- I*d)^2*(c + I*d)^5*(c*Cos[e] + d*Sin[e])) + (Cos[6*f*x]*((I/40)*Cos[3*e]
+ Sin[3*e]/40))/(c + I*d)^3 + ((17*c^2 + (102*I)*c*d - 231*d^2)*(Cos[e]/60
+ (I/60)*Sin[e])*Sin[2*f*x])/(c + I*d)^5 + ((c + (3*I)*d)*((7*Cos[e])/60 -
((7*I)/60)*Sin[e])*Sin[4*f*x])/(c + I*d)^4 + ((Cos[3*e]/40 - (I/40)*Sin[3*e
])*Sin[6*f*x])/(c + I*d)^3 + (((2*I)/3)*d^6*Cos[3*e] - (2*d^6*Sin[3*e])/3)/
((c - I*d)^2*(c + I*d)^5*(c*Cos[e + f*x] + d*Sin[e + f*x])^2) + (16*(c*d^5*
Cos[3*e - f*x] - (I/2)*d^6*Cos[3*e - f*x] - c*d^5*Cos[3*e + f*x] + (I/2)*d^
6*Cos[3*e + f*x] + I*c*d^5*Sin[3*e - f*x] + (d^6*Sin[3*e - f*x])/2 - I*c*d^
5*Sin[3*e + f*x] - (d^6*Sin[3*e + f*x])/2))/(3*(c - I*d)^2*(c + I*d)^5*(c*C
os[e] + d*Sin[e])*(c*Cos[e + f*x] + d*Sin[e + f*x]))))/(f*(a + I*a*Tan[e +
f*x])^(5/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 10144 vs. $2(374) = 748$.
time = 0.73, size = 10145, normalized size = 22.85

method	result	size
derivativedivides	Expression too large to display	10145
default	Expression too large to display	10145

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(5/2),x,method=_RETURNVERBO
SE)
```

```
[Out] result too large to display
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1801 vs. 2(364) = 728.
time = 1.40, size = 1801, normalized size = 4.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="f
ricas")
```

```
[Out] -1/120*(sqrt(2)*(3*c^6 + 6*I*c^5*d + 3*c^4*d^2 + 12*I*c^3*d^3 - 3*c^2*d^4 +
 6*I*c*d^5 - 3*d^6 + (23*c^6 + 62*I*c^5*d + 55*c^4*d^2 + 860*I*c^3*d^3 + 31
45*c^2*d^4 - 3298*I*c*d^5 - 983*d^6)*e^(10*I*f*x + 10*I*e) + 4*(20*c^6 + 71
*I*c^5*d - 20*c^4*d^2 + 590*I*c^3*d^3 + 1240*c^2*d^4 - 385*I*c*d^5 + 136*d^
6)*e^(8*I*f*x + 8*I*e) + 3*(35*c^6 + 142*I*c^5*d - 129*c^4*d^2 + 636*I*c^3*
d^3 + 389*c^2*d^4 + 654*I*c*d^5 + 393*d^6)*e^(6*I*f*x + 6*I*e) + (65*c^6 +
254*I*c^5*d - 251*c^4*d^2 + 508*I*c^3*d^3 - 697*c^2*d^4 + 254*I*c*d^5 - 381
*d^6)*e^(4*I*f*x + 4*I*e) + 4*(5*c^6 + 14*I*c^5*d + c^4*d^2 + 28*I*c^3*d^3
- 13*c^2*d^4 + 14*I*c*d^5 - 9*d^6)*e^(2*I*f*x + 2*I*e))*sqrt(((c - I*d)*e^(
2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x +
 2*I*e) + 1)) - 30*((-I*a^3*c^9 + a^3*c^8*d - 4*I*a^3*c^7*d^2 + 4*a^3*c^6*d
^3 - 6*I*a^3*c^5*d^4 + 6*a^3*c^4*d^5 - 4*I*a^3*c^3*d^6 + 4*a^3*c^2*d^7 - I*
a^3*c*d^8 + a^3*d^9)*f*e^(9*I*f*x + 9*I*e) + 2*(-I*a^3*c^9 + 3*a^3*c^8*d +
8*a^3*c^6*d^3 + 6*I*a^3*c^5*d^4 + 6*a^3*c^4*d^5 + 8*I*a^3*c^3*d^6 + 3*I*a^3
*c*d^8 - a^3*d^9)*f*e^(7*I*f*x + 7*I*e) + (-I*a^3*c^9 + 5*a^3*c^8*d + 8*I*a
^3*c^7*d^2 + 14*I*a^3*c^5*d^4 - 14*a^3*c^4*d^5 - 8*a^3*c^2*d^7 - 5*I*a^3*c*
d^8 + a^3*d^9)*f*e^(5*I*f*x + 5*I*e))*sqrt(1/8*I/((-I*a^5*c^5 - 5*a^5*c^4*d
 + 10*I*a^5*c^3*d^2 + 10*a^5*c^2*d^3 - 5*I*a^5*c*d^4 - a^5*d^5)*f^2))*log(-
4*(I*a^3*c^3 + 3*a^3*c^2*d - 3*I*a^3*c*d^2 - a^3*d^3)*f*sqrt(1/8*I/((-I*a^5
*c^5 - 5*a^5*c^4*d + 10*I*a^5*c^3*d^2 + 10*a^5*c^2*d^3 - 5*I*a^5*c*d^4 - a^
5*d^5)*f^2))*e^(I*f*x + I*e) + sqrt(2)*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e)
+ c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*(e^
(2*I*f*x + 2*I*e) + 1)) - 30*((I*a^3*c^9 - a^3*c^8*d + 4*I*a^3*c^7*d^2 - 4*
a^3*c^6*d^3 + 6*I*a^3*c^5*d^4 - 6*a^3*c^4*d^5 + 4*I*a^3*c^3*d^6 - 4*a^3*c^2
*d^7 + I*a^3*c*d^8 - a^3*d^9)*f*e^(9*I*f*x + 9*I*e) + 2*(I*a^3*c^9 - 3*a^3*
c^8*d - 8*a^3*c^6*d^3 - 6*I*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 8*I*a^3*c^3*d^6 -
 3*I*a^3*c*d^8 + a^3*d^9)*f*e^(7*I*f*x + 7*I*e) + (I*a^3*c^9 - 5*a^3*c^8*d
- 8*I*a^3*c^7*d^2 - 14*I*a^3*c^5*d^4 + 14*a^3*c^4*d^5 + 8*a^3*c^2*d^7 + 5*I
*a^3*c*d^8 - a^3*d^9)*f*e^(5*I*f*x + 5*I*e))*sqrt(1/8*I/((-I*a^5*c^5 - 5*a^
5*c^4*d + 10*I*a^5*c^3*d^2 + 10*a^5*c^2*d^3 - 5*I*a^5*c*d^4 - a^5*d^5)*f^2)
)*log(-4*(-I*a^3*c^3 - 3*a^3*c^2*d + 3*I*a^3*c*d^2 + a^3*d^3)*f*sqrt(1/8*I/
((-I*a^5*c^5 - 5*a^5*c^4*d + 10*I*a^5*c^3*d^2 + 10*a^5*c^2*d^3 - 5*I*a^5*c*
d^4 - a^5*d^5)*f^2))*e^(I*f*x + I*e) + sqrt(2)*sqrt(((c - I*d)*e^(2*I*f*x +
 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(a/(e^(2*I*f*x + 2*I*e) +
 1))*(e^(2*I*f*x + 2*I*e) + 1)))/((I*a^3*c^9 - a^3*c^8*d + 4*I*a^3*c^7*d^2
```

```
- 4*a^3*c^6*d^3 + 6*I*a^3*c^5*d^4 - 6*a^3*c^4*d^5 + 4*I*a^3*c^3*d^6 - 4*a^3
*c^2*d^7 + I*a^3*c*d^8 - a^3*d^9)*f*e^(9*I*f*x + 9*I*e) + 2*(I*a^3*c^9 - 3*
a^3*c^8*d - 8*a^3*c^6*d^3 - 6*I*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 8*I*a^3*c^3*d
^6 - 3*I*a^3*c*d^8 + a^3*d^9)*f*e^(7*I*f*x + 7*I*e) + (I*a^3*c^9 - 5*a^3*c^
8*d - 8*I*a^3*c^7*d^2 - 14*I*a^3*c^5*d^4 + 14*a^3*c^4*d^5 + 8*a^3*c^2*d^7 +
 5*I*a^3*c*d^8 - a^3*d^9)*f*e^(5*I*f*x + 5*I*e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(\tan\left(e+fx\right)-i\right)\right)^{\frac{5}{2}}\left(c+d\tan\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))**(5/2)/(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral(1/((I*a*(tan(e + f*x) - I))**(5/2)*(c + d*tan(e + f*x))**(5/2)), x
)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="g
iac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+a\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{5/2}\,{\left(c+d\,\mathrm{tan}\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^(5/2)*(c + d*tan(e + f*x))^(5/2)),x)
```

```
[Out] int(1/((a + a*tan(e + f*x)*1i)^(5/2)*(c + d*tan(e + f*x))^(5/2)), x)
```

3.1173 $\int (a+ia\tan(e+fx))^m(c+d\tan(e+fx))^n\,dx$

Optimal. Leaf size=114

$$-\frac{iF_1\left(m;-n,1;1+m;-\frac{d(1+i\tan(e+fx))}{ic-d},\frac{1}{2}(1+i\tan(e+fx))\right)(a+ia\tan(e+fx))^m(c+d\tan(e+fx))^n\left(\frac{c+}{}\right.}{2fm}$$

```
[Out] -1/2*I*AppellF1(m,-n,1,1+m,-d*(1+I*tan(f*x+e))/(I*c-d),1/2+1/2*I*tan(f*x+e)
)*(a+I*a*tan(f*x+e))^m*(c+d*tan(f*x+e))^n/f/m/(((c+d*tan(f*x+e))/(c+I*d))^n
)
```

Rubi [A]
time = 0.13, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3645, 142, 141}

$$-\frac{i(a+ia\tan(e+fx))^m(c+d\tan(e+fx))^n\left(\frac{c+d\tan(e+fx)}{c+id}\right)^{-n}F_1\left(m;-n,1;m+1;-\frac{d(i\tan(e+fx)+1)}{ic-d},\frac{1}{2}(i\tan(e+fx)+1)\right)}{2fm}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n,x]
```

```
[Out] ((-1/2*I)*AppellF1[m, -n, 1, 1 + m, -((d*(1 + I*Tan[e + f*x]))/(I*c - d)),
(1 + I*Tan[e + f*x])/2]*(a + I*a*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n)/(f
*m*((c + d*Tan[e + f*x])/(c + I*d))^n)
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\int (a+ia\tan(e+fx))^m(c+d\tan(e+fx))^n\,dx = \frac{(ia^2)\,\text{Subst}\left(\int \frac{(a+x)^{-1+m}\left(c-\frac{idx}{a}\right)^n}{-a^2+ax}\,dx, x, ia\tan(e+fx)\right)}{f}$$

$$= \frac{\left(ia^2(c+d\tan(e+fx))^n\left(\frac{c+d\tan(e+fx)}{c+id}\right)^{-n}\right)\text{Subst}\left(\int \frac{(a\cdots}{\cdots}\right.}{f}$$

$$= -\frac{iF_1\left(m;-n,1;1+m;-\frac{d(1+i\tan(e+fx))}{ic-d},\frac{1}{2}(1+i\tan(e+\cdots\right.}{\cdots}$$

Mathematica [F]
time = 7.82, size = 0, normalized size = 0.00

$$\int (a+ia\tan(e+fx))^m(c+d\tan(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n,x]
[Out] Integrate[(a + I*a*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x]
```

Maple [F]
time = 0.58, size = 0, normalized size = 0.00

$$\int \left(a+ia\tan\left(fx+e\right)\right)^m\left(c+d\tan\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^m*(c+d*tan(f*x+e))^n,x)
[Out] int((a+I*a*tan(f*x+e))^m*(c+d*tan(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c+d*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^m*(d*tan(f*x + e) + c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c+d*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((2*a*e^(2*I*f*x + 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1))^m*(((c - I*d)*
e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan (e+fx)-i))^m \left(c+d\tan (e+fx)\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**m*(c+d*tan(f*x+e))**n,x)
```

```
[Out] Integral((I*a*(tan(e + f*x) - I))**m*(c + d*tan(e + f*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c+d*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^m*(d*tan(f*x + e) + c)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+a\tan(e+f\,x)\,1\mathrm{i}\right)^m\,\left(c+d\tan(e+f\,x)\right)^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^m*(c + d*tan(e + f*x))^n,x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^m*(c + d*tan(e + f*x))^n, x)
```

3.1174 $\int (a+ia\tan(e+fx))^3(c+d\tan(e+fx))^n\,dx$

Optimal. Leaf size=157

$$\frac{a^3(ic-d(5+2n))(c+d\tan(e+fx))^{1+n}}{d^2f(1+n)(2+n)}+\frac{4a^3\,{}_2F_1\left(1,1+n;2+n;\frac{c+d\tan(e+fx)}{c-id}\right)(c+d\tan(e+fx))^{1+n}}{(ic+d)f(1+n)}-\frac{(\ldots}{}$$

```
[Out] a^3*(I*c-d*(5+2*n))*(c+d*tan(f*x+e))^(1+n)/d^2/f/(1+n)/(2+n)+4*a^3*hypergeo
m([1, 1+n],[2+n],(c+d*tan(f*x+e))/(c-I*d))*(c+d*tan(f*x+e))^(1+n)/(I*c+d)/f
/(1+n)-(a^3+I*a^3*tan(f*x+e))*(c+d*tan(f*x+e))^(1+n)/d/f/(2+n)
```

Rubi [A]
time = 0.24, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3637, 3673, 3618, 70}

$$\frac{a^3(-d(2n+5)+ic)(c+d\tan(e+fx))^{n+1}}{d^2f(n+1)(n+2)}+\frac{4a^3(c+d\tan(e+fx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{c+d\tan(e+fx)}{c-id}\right)}{f(n+1)(d+ic)}-\frac{(a^3+ia^3\tan(e+fx))(c+d\tan(e+fx))^{n+1}}{df(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^n,x]
```

```
[Out] (a^3*(I*c - d*(5 + 2*n))*(c + d*Tan[e + f*x])^(1 + n))/(d^2*f*(1 + n)*(2 +
n)) + (4*a^3*Hypergeometric2F1[1, 1 + n, 2 + n, (c + d*Tan[e + f*x])/(c - I
*d)]*(c + d*Tan[e + f*x])^(1 + n))/((I*c + d)*f*(1 + n)) - ((a^3 + I*a^3*Ta
n[e + f*x])*(c + d*Tan[e + f*x])^(1 + n))/(d*f*(2 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c +
 d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[a/(d*(m + n - 1)), I
```

```
nt[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[b*c*(m - 2) + a
*d*(m + 2*n) + (a*c*(m - 2) + b*d*(3*m + 2*n - 4))*Tan[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 1] && NeQ[m + n - 1, 0]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (a+ia\tan(e+fx))^3(c+d\tan(e+fx))^n\,dx = -\frac{(a^3+ia^3\tan(e+fx))\,(c+d\tan(e+fx))^{1+n}}{df(2+n)} + \frac{a\int(a+$$

$$= \frac{a^3(ic-d(5+2n))(c+d\tan(e+fx))^{1+n}}{d^2f(1+n)(2+n)} - \frac{(a^3+ia^3\tan(}{}$$

$$= \frac{a^3(ic-d(5+2n))(c+d\tan(e+fx))^{1+n}}{d^2f(1+n)(2+n)} - \frac{(a^3+ia^3\tan(}{}$$

$$= \frac{a^3(ic-d(5+2n))(c+d\tan(e+fx))^{1+n}}{d^2f(1+n)(2+n)} + \frac{4a^3\,{}_2F_1\big(1,1+}{}$$

Mathematica [F]
time = 16.17, size = 0, normalized size = 0.00

$$\int (a+ia\tan(e+fx))^3(c+d\tan(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^n,x]
[Out] Integrate[(a + I*a*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^n, x]
```

Maple [F]
time = 2.93, size = 0, normalized size = 0.00

$$\int (a+ia\tan(fx+e))^3\,(c+d\tan(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3*(c+d*tan(f*x+e))^n,x)
```

```
[Out] int((a+I*a*tan(f*x+e))^3*(c+d*tan(f*x+e))^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c+d*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^3*(d*tan(f*x + e) + c)^n, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c+d*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral(8*a^3*(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I
*e) + 1))^n*e^(6*I*f*x + 6*I*e)/(e^(6*I*f*x + 6*I*e) + 3*e^(4*I*f*x + 4*I*e
) + 3*e^(2*I*f*x + 2*I*e) + 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3*(c+d*tan(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(c+d*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^3*(d*tan(f*x + e) + c)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a\tan(e + f\,x)\ 1\mathrm{i})^3\,(c + d\tan(e + f\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^3*(c + d*tan(e + f*x))^n,x)
[Out] int((a + a*tan(e + f*x)*1i)^3*(c + d*tan(e + f*x))^n, x)
```

3.1175 $\int (a+ia\tan(e+fx))^2(c+d\tan(e+fx))^n\,dx$

Optimal. Leaf size=95

$$-\frac{a^2(c+d\tan(e+fx))^{1+n}}{df(1+n)}+\frac{2a^2\,{}_2F_1\left(1,1+n;2+n;\frac{c+d\tan(e+fx)}{c-id}\right)(c+d\tan(e+fx))^{1+n}}{(ic+d)f(1+n)}$$

```
[Out] -a^2*(c+d*tan(f*x+e))^(1+n)/d/f/(1+n)+2*a^2*hypergeom([1, 1+n],[2+n],(c+d*t
an(f*x+e))/(c-I*d))*(c+d*tan(f*x+e))^(1+n)/(I*c+d)/f/(1+n)
```

Rubi [A]

time = 0.09, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3624, 3618, 70}

$$-\frac{a^2(c+d\tan(e+fx))^{n+1}}{df(n+1)}+\frac{2a^2(c+d\tan(e+fx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{c+d\tan(e+fx)}{c-id}\right)}{f(n+1)(d+ic)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^n,x]
```

```
[Out] -((a^2*(c + d*Tan[e + f*x])^(1 + n))/(d*f*(1 + n))) + (2*a^2*Hypergeometric
2F1[1, 1 + n, 2 + n, (c + d*Tan[e + f*x])/(c - I*d)]*(c + d*Tan[e + f*x])^(
1 + n))/((I*c + d)*f*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
```

```
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int (a+ia\tan(e+fx))^2(c+d\tan(e+fx))^n\,dx = -\frac{a^2(c+d\tan(e+fx))^{1+n}}{df(1+n)}+\int (2a^2+2ia^2\tan(e+fx))$$
$$= -\frac{a^2(c+d\tan(e+fx))^{1+n}}{df(1+n)}+\frac{(4ia^4)\,\text{Subst}\left(\int \frac{\left(c-\frac{idx}{2a^2}\right)^n}{-4a^4+2a^2x}\,dx\right.}{f}$$
$$= -\frac{a^2(c+d\tan(e+fx))^{1+n}}{df(1+n)}+\frac{2a^2\,{}_2F_1\left(1,1+n;2+n;\frac{c+d\,t}{}\right.}{(ic+}$$

Mathematica [F]
time = 4.02, size = 0, normalized size = 0.00

$$\int (a+ia\tan(e+fx))^2(c+d\tan(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^n,x]
[Out] Integrate[(a + I*a*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^n, x]
```

Maple [F]
time = 0.56, size = 0, normalized size = 0.00

$$\int (a+ia\tan(fx+e))^2\,(c+d\tan(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2*(c+d*tan(f*x+e))^n,x)
[Out] int((a+I*a*tan(f*x+e))^2*(c+d*tan(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c+d*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^2*(d*tan(f*x + e) + c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c+d*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral(4*a^2*(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I
*e) + 1))^n*e^(4*I*f*x + 4*I*e)/(e^(4*I*f*x + 4*I*e) + 2*e^(2*I*f*x + 2*I*e
) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^{2}\left(\int (c+d\tan{(e+fx)})^{n}\tan^{2}{(e+fx)}\,dx+\int \left(-2i(c+d\tan{(e+fx)})^{n}\tan{(e+fx)}\right)dx+\int \left(-(c+d\tan{(e+fx)})^{n}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2*(c+d*tan(f*x+e))**n,x)
```

```
[Out] -a**2*(Integral((c + d*tan(e + f*x))**n*tan(e + f*x)**2, x) + Integral(-2*I
*(c + d*tan(e + f*x))**n*tan(e + f*x), x) + Integral(-(c + d*tan(e + f*x))*
*n, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(c+d*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^2*(d*tan(f*x + e) + c)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\tan(e+f\,x)\,1\mathrm{i})^2\,(c+d\tan(e+f\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^2*(c + d*tan(e + f*x))^n,x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^2*(c + d*tan(e + f*x))^n, x)
```

3.1176 $\int (a+ia\tan(e+fx))(c+d\tan(e+fx))^n\,dx$

Optimal. Leaf size=61

$$\frac{a\,{}_2F_1\left(1,1+n;2+n;\frac{c+d\tan(e+fx)}{c-id}\right)(c+d\tan(e+fx))^{1+n}}{(ic+d)f(1+n)}$$

```
[Out] a*hypergeom([1, 1+n],[2+n],(c+d*tan(f*x+e))/(c-I*d))*(c+d*tan(f*x+e))^(1+n)
/(I*c+d)/f/(1+n)
```

Rubi [A]

time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3618, 70}

$$\frac{a(c+d\tan(e+fx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{c+d\tan(e+fx)}{c-id}\right)}{f(n+1)(d+ic)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])*(c + d*Tan[e + f*x])^n,x]
```

```
[Out] (a*Hypergeometric2F1[1, 1 + n, 2 + n, (c + d*Tan[e + f*x])/(c - I*d)]*(c +
d*Tan[e + f*x])^(1 + n))/((I*c + d)*f*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+ia\tan(e+fx))(c+d\tan(e+fx))^n\,dx &= \frac{(ia^2)\,\text{Subst}\left(\int\frac{\left(c-\frac{idx}{a}\right)^n}{-a^2+ax}\,dx,x,ia\tan(e+fx)\right)}{f}\\ &= \frac{a\,{}_2F_1\left(1,1+n;2+n;\frac{c+d\tan(e+fx)}{c-id}\right)(c+d\tan(e+fx))^{1+n}}{(ic+d)f(1+n)}\end{aligned}$$

Mathematica [F]
time = 1.75, size = 0, normalized size = 0.00

$$\int (a+ia\tan(e+fx))(c+d\tan(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + I*a*Tan[e + f*x])*(c + d*Tan[e + f*x])^n,x]
[Out] Integrate[(a + I*a*Tan[e + f*x])*(c + d*Tan[e + f*x])^n, x]
```

Maple [F]
time = 0.51, size = 0, normalized size = 0.00

$$\int (a+ia\tan(fx+e))(c+d\tan(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))*(c+d*tan(f*x+e))^n,x)
[Out] int((a+I*a*tan(f*x+e))*(c+d*tan(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c+d*tan(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((I*a*tan(f*x + e) + a)*(d*tan(f*x + e) + c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c+d*tan(f*x+e))^n,x, algorithm="fricas")
[Out] integral(2*a*(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e
) + 1))^n*e^(2*I*f*x + 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-i(c+d\tan(e+fx))^n\right)dx+\int(c+d\tan(e+fx))^n\tan(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c+d*tan(f*x+e))**n,x)
```

```
[Out] I*a*(Integral(-I*(c + d*tan(e + f*x))**n, x) + Integral((c + d*tan(e + f*x)
)**n*tan(e + f*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(c+d*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)*(d*tan(f*x + e) + c)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (a + a \tan(e + f\,x)\ 1\mathrm{i})\ (c + d\tan(e + f\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)*(c + d*tan(e + f*x))^n,x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)*(c + d*tan(e + f*x))^n, x)
```

3.1177 $\int \frac{(c+d\tan(e+fx))^n}{a+ia\tan(e+fx)}\,dx$

Optimal. Leaf size=193

$$\frac{{}_2F_1\left(1,1+n;2+n;\frac{c+d\tan(e+fx)}{c-id}\right)(c+d\tan(e+fx))^{1+n}}{4a(ic+d)f(1+n)}+\frac{(ic-d+2dn)\,{}_2F_1\left(1,1+n;2+n;\frac{c+d\tan(e+fx)}{c+id}\right)}{4a(c+id)^2f(1+n)}$$

```
[Out] 1/4*hypergeom([1, 1+n],[2+n],(c+d*tan(f*x+e))/(c-I*d))*(c+d*tan(f*x+e))^(1+
n)/a/(I*c+d)/f/(1+n)+1/4*(I*c-d+2*d*n)*hypergeom([1, 1+n],[2+n],(c+d*tan(f*
x+e))/(c+I*d))*(c+d*tan(f*x+e))^(1+n)/a/(c+I*d)^2/f/(1+n)-1/2*(c+d*tan(f*x+
e))^(1+n)/(I*c-d)/f/(a+I*a*tan(f*x+e))
```

Rubi [A]
time = 0.20, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3633, 3620, 3618, 70}

$$\frac{(c+d\tan(e+fx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{c+d\tan(e+fx)}{c-id}\right)}{4af(n+1)(d+ic)}+\frac{(ic+2dn-d)(c+d\tan(e+fx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{c+d\tan(e+fx)}{c+id}\right)}{4af(n+1)(c+id)^2}-\frac{(c+d\tan(e+fx))^{n+1}}{2f(-d+ic)(a+ia\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^n/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (Hypergeometric2F1[1, 1 + n, 2 + n, (c + d*Tan[e + f*x])/(c - I*d)]*(c + d*
Tan[e + f*x])^(1 + n))/(4*a*(I*c + d)*f*(1 + n)) + ((I*c - d + 2*d*n)*Hyper
geometric2F1[1, 1 + n, 2 + n, (c + d*Tan[e + f*x])/(c + I*d)]*(c + d*Tan[e
+ f*x])^(1 + n))/(4*a*(c + I*d)^2*f*(1 + n)) - (c + d*Tan[e + f*x])^(1 + n)
/(2*(I*c - d)*f*(a + I*a*Tan[e + f*x]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
```

```
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3633

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-a)*((c + d*Tan[e + f*x])^(n + 1)/(2*f*(b*
c - a*d)*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*
Tan[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &
& NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^n}{a+ia\tan(e+fx)}\,dx = -\frac{(c+d\tan(e+fx))^{1+n}}{2(ic-d)f(a+ia\tan(e+fx))} + \frac{\int (c+d\tan(e+fx))^n(a(ic-d(1-n))-}{2a^2(ic-d)}$$

$$= -\frac{(c+d\tan(e+fx))^{1+n}}{2(ic-d)f(a+ia\tan(e+fx))} + \frac{\int (1+i\tan(e+fx))(c+d\tan(e+fx))^n\,d}{4a}$$

$$= -\frac{(c+d\tan(e+fx))^{1+n}}{2(ic-d)f(a+ia\tan(e+fx))} + \frac{i\text{Subst}\left(\int \frac{(c-idx)^n}{-1+x}\,dx, x, i\tan(e+fx)\right)}{4af} - $$

$$= \frac{{}_2F_1\left(1,1+n;2+n;\frac{c+d\tan(e+fx)}{c-id}\right)(c+d\tan(e+fx))^{1+n}}{4a(ic+d)f(1+n)} + \frac{(ic-d(1-2n))\,{}_2F}{}$$

Mathematica [F]
time = 26.78, size = 0, normalized size = 0.00

$$\int \frac{(c+d\tan(e+fx))^n}{a+ia\tan(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*Tan[e + f*x])^n/(a + I*a*Tan[e + f*x]),x]
[Out] Integrate[(c + d*Tan[e + f*x])^n/(a + I*a*Tan[e + f*x]), x]
```

Maple [F]
time = 3.27, size = 0, normalized size = 0.00

$$\int \frac{(c+d\tan(fx+e))^n}{a+ia\tan(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^n/(a+I*a*tan(f*x+e)),x)
```

```
[Out] int((c+d*tan(f*x+e))^n/(a+I*a*tan(f*x+e)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^n/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^n/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/2*(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e
) + 1))^n*(e^(2*I*f*x + 2*I*e) + 1)*e^(-2*I*f*x - 2*I*e)/a, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{(c+d\tan{(e+fx)})^n}{\tan{(e+fx)}-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**n/(a+I*a*tan(f*x+e)),x)
```

```
[Out] -I*Integral((c + d*tan(e + f*x))**n/(tan(e + f*x) - I), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^n/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*tan(f*x + e) + c)^n/(I*a*tan(f*x + e) + a), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\tan(e+f\,x))^n}{a+a\tan\left(e+f\,x\right)\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^n/(a + a*tan(e + f*x)*1i),x)
[Out] int((c + d*tan(e + f*x))^n/(a + a*tan(e + f*x)*1i), x)
```

3.1178 $\int \frac{(c+d\tan(e+fx))^n}{(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=273

$$\frac{{}_2F_1\left(1,1+n;2+n;\frac{c+d\tan(e+fx)}{c-id}\right)(c+d\tan(e+fx))^{1+n}}{8a^2(ic+d)f(1+n)}+\frac{(c^2+2icd(1-n)-d^2(1-4n+2n^2))\ {}_2F_1\left(1,1\right.}{8a^2(ic-d)}$$

```
[Out] 1/8*hypergeom([1, 1+n],[2+n],(c+d*tan(f*x+e))/(c-I*d))*(c+d*tan(f*x+e))^(1+
n)/a^2/(I*c+d)/f/(1+n)+1/8*(c^2+2*I*c*d*(1-n)-d^2*(2*n^2-4*n+1))*hypergeom(
[1, 1+n],[2+n],(c+d*tan(f*x+e))/(c+I*d))*(c+d*tan(f*x+e))^(1+n)/a^2/(I*c-d)
^3/f/(1+n)+1/4*(I*c-d*(2-n))*(c+d*tan(f*x+e))^(1+n)/a^2/(c+I*d)^2/f/(1+I*ta
n(f*x+e))-1/4*(c+d*tan(f*x+e))^(1+n)/(I*c-d)/f/(a+I*a*tan(f*x+e))^2
```

Rubi [A]

time = 0.38, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3640, 3677, 3620, 3618, 70}

$$\frac{(c^2+2icd(1-n)-d^2(2n^2-4n+1))\,(c+d\tan(e+fx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{c+d\tan(e+fx)}{c+id}\right)}{8a^2f(n+1)(-d+ic)^3}+\frac{(c+d\tan(e+fx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{c+d\tan(e+fx)}{c-id}\right)}{8a^2f(n+1)(d+ic)}+\frac{(-d(2-n)+ic)(c+d\tan(e+fx))^{n+1}}{4a^2f(c+id)^2(1+i\tan(e+fx))}-\frac{(c+d\tan(e+fx))^{n+1}}{4f(-d+ic)(a+ia\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^n/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (Hypergeometric2F1[1, 1 + n, 2 + n, (c + d*Tan[e + f*x])/(c - I*d)]*(c + d*
Tan[e + f*x])^(1 + n))/(8*a^2*(I*c + d)*f*(1 + n)) + ((c^2 + (2*I)*c*d*(1 -
 n) - d^2*(1 - 4*n + 2*n^2))*Hypergeometric2F1[1, 1 + n, 2 + n, (c + d*Tan[
e + f*x])/(c + I*d)]*(c + d*Tan[e + f*x])^(1 + n))/(8*a^2*(I*c - d)^3*f*(1
+ n)) + ((I*c - d*(2 - n))*(c + d*Tan[e + f*x])^(1 + n))/(4*a^2*(c + I*d)^2
*f*(1 + I*Tan[e + f*x])) - (c + d*Tan[e + f*x])^(1 + n)/(4*(I*c - d)*f*(a +
 I*a*Tan[e + f*x])^2)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+d\tan(e+fx))^n}{(a+ia\tan(e+fx))^2}\,dx &= -\frac{(c+d\tan(e+fx))^{1+n}}{4(ic-d)f(a+ia\tan(e+fx))^2} - \frac{\int \frac{(c+d\tan(e+fx))^n(-a(2ic-d(3-n))-iad(1-n)\tan(\ldots}{a+ia\tan(e+fx)}}{4a^2(ic-d)} \ldots \\
&= \frac{(ic-d(2-n))(c+d\tan(e+fx))^{1+n}}{4a^2(c+id)^2f(1+i\tan(e+fx))} - \frac{(c+d\tan(e+fx))^{1+n}}{4(ic-d)f(a+ia\tan(e+fx))^2} - \ldots \\
&= \frac{(ic-d(2-n))(c+d\tan(e+fx))^{1+n}}{4a^2(c+id)^2f(1+i\tan(e+fx))} - \frac{(c+d\tan(e+fx))^{1+n}}{4(ic-d)f(a+ia\tan(e+fx))^2} + \ldots \\
&= \frac{(ic-d(2-n))(c+d\tan(e+fx))^{1+n}}{4a^2(c+id)^2f(1+i\tan(e+fx))} - \frac{(c+d\tan(e+fx))^{1+n}}{4(ic-d)f(a+ia\tan(e+fx))^2} + \ldots \\
&= \frac{{}_2F_1\left(1,1+n;2+n;\frac{c+d\tan(e+fx)}{c-id}\right)(c+d\tan(e+fx))^{1+n}}{8a^2(ic+d)f(1+n)} + \frac{(c^2+2icd(1-n)\ldots}{\ldots}
\end{aligned}
$$

Mathematica [F]
time = 8.13, size = 0, normalized size = 0.00

$$\int \frac{(c+d\tan(e+fx))^n}{(a+ia\tan(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*Tan[e + f*x])^n/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] Integrate[(c + d*Tan[e + f*x])^n/(a + I*a*Tan[e + f*x])^2, x]
```

Maple [F]
time = 3.48, size = 0, normalized size = 0.00

$$\int \frac{(c+d\tan(fx+e))^n}{(a+ia\tan(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^2,x)
```

```
[Out] int((c+d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/4*(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e
) + 1))^n*(e^(4*I*f*x + 4*I*e) + 2*e^(2*I*f*x + 2*I*e) + 1)*e^(-4*I*f*x - 4
*I*e)/a^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{(c+d\tan(e+fx))^n}{\tan^2(e+fx)-2i\tan(e+fx)-1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**n/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -Integral((c + d*tan(e + f*x))**n/(tan(e + f*x)**2 - 2*I*tan(e + f*x) - 1),
 x)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*tan(f*x + e) + c)^n/(I*a*tan(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\tan(e+f\,x))^n}{(a+a\tan(e+f\,x)\ 1\mathrm{i})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^n/(a + a*tan(e + f*x)*1i)^2,x)
```

```
[Out] int((c + d*tan(e + f*x))^n/(a + a*tan(e + f*x)*1i)^2, x)
```

3.1179 $\int \frac{(c+d\tan(e+fx))^n}{(a+ia\tan(e+fx))^3}\,dx$

Optimal. Leaf size=381

$$\frac{{}_2F_1\left(1,1+n;2+n;\frac{c+d\tan(e+fx)}{c-id}\right)(c+d\tan(e+fx))^{1+n}}{16a^3(ic+d)f(1+n)}+\frac{(3ic^3-c^2d(9-6n)-3icd^2(3-6n+2n^2)+d^3(\cdots}{\cdots}$$

```
[Out] 1/16*hypergeom([1, 1+n],[2+n],(c+d*tan(f*x+e))/(c-I*d))*(c+d*tan(f*x+e))^(1
+n)/a^3/(I*c+d)/f/(1+n)+1/48*(3*I*c^3-c^2*d*(9-6*n)-3*I*c*d^2*(2*n^2-6*n+3)
+d^3*(-4*n^3+18*n^2-20*n+3))*hypergeom([1, 1+n],[2+n],(c+d*tan(f*x+e))/(c+I
*d))*(c+d*tan(f*x+e))^(1+n)/a^3/(c+I*d)^4/f/(1+n)-1/6*(c+d*tan(f*x+e))^(1+n
)/(I*c-d)/f/(a+I*a*tan(f*x+e))^3+1/24*(3*I*c-d*(7-2*n))*(c+d*tan(f*x+e))^(1
+n)/a/(c+I*d)^2/f/(a+I*a*tan(f*x+e))^2+1/24*(3*I*c^2-3*c*d*(3-n)-I*d^2*(2*n
^2-9*n+10))*(c+d*tan(f*x+e))^(1+n)/(c+I*d)^3/f/(a^3+I*a^3*tan(f*x+e))
```

Rubi [A]
time = 0.75, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3640, 3677, 3620, 3618, 70}

$$\frac{(3ic^2-3cd(3-n)-id^2(2n^2-9n+10))(c+d\tan(e+fx))^{n+1}}{24f(c+id)^3(a^3+ia^3\tan(e+fx))}+\frac{(3ic^3-c^2d(9-6n)-3icd^2(2n^2-6n+3)+d^3(-4n^3+18n^2-20n+3))(c+d\tan(e+fx))^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{c+d\tan(e+fx)}{c+id}\right)}{48a^3f(n+1)(c+id)^4}+\frac{(c+d\tan(e+fx))^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{c+d\tan(e+fx)}{c-id}\right)}{16a^3f(n+1)(d+ic)}+\frac{(-d(7-2n)+3ic)(c+d\tan(e+fx))^{n+1}}{24af(c+id)^2(a+ia\tan(e+fx))^2}-\frac{(c+d\tan(e+fx))^{n+1}}{6f(-d+ic)(a+ia\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^n/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] (Hypergeometric2F1[1, 1 + n, 2 + n, (c + d*Tan[e + f*x])/(c - I*d)]*(c + d*
Tan[e + f*x])^(1 + n))/(16*a^3*(I*c + d)*f*(1 + n)) + (((3*I)*c^3 - c^2*d*(
9 - 6*n) - (3*I)*c*d^2*(3 - 6*n + 2*n^2) + d^3*(3 - 20*n + 18*n^2 - 4*n^3))
*Hypergeometric2F1[1, 1 + n, 2 + n, (c + d*Tan[e + f*x])/(c + I*d)]*(c + d*
Tan[e + f*x])^(1 + n))/(48*a^3*(c + I*d)^4*f*(1 + n)) - (c + d*Tan[e + f*x]
)^(1 + n)/(6*(I*c - d)*f*(a + I*a*Tan[e + f*x])^3) + (((3*I)*c - d*(7 - 2*n
))*(c + d*Tan[e + f*x])^(1 + n))/(24*a*(c + I*d)^2*f*(a + I*a*Tan[e + f*x])
^2) + (((3*I)*c^2 - 3*c*d*(3 - n) - I*d^2*(10 - 9*n + 2*n^2))*(c + d*Tan[e
+ f*x])^(1 + n))/(24*(c + I*d)^3*f*(a^3 + I*a^3*Tan[e + f*x]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
```

```
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3640

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int
[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*c*m - a*d*(2*m
+ n + 1) + b*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
&& LtQ[m, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
 b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^n}{(a+ia\tan(e+fx))^3}\,dx = -\frac{(c+d\tan(e+fx))^{1+n}}{6(ic-d)f(a+ia\tan(e+fx))^3} - \frac{\int \frac{(c+d\tan(e+fx))^n(-a(3ic-d(5-n))-iad(2-n)\tan}{(a+ia\tan(e+fx))^2}}{6a^2(ic-d)}$$

$$= -\frac{(c+d\tan(e+fx))^{1+n}}{6(ic-d)f(a+ia\tan(e+fx))^3} + \frac{(3ic-d(7-2n))(c+d\tan(e+fx))^{1+}}{24a(c+id)^2f(a+ia\tan(e+fx))^2}$$

$$= -\frac{(c+d\tan(e+fx))^{1+n}}{6(ic-d)f(a+ia\tan(e+fx))^3} + \frac{(3ic-d(7-2n))(c+d\tan(e+fx))^{1+}}{24a(c+id)^2f(a+ia\tan(e+fx))^2}$$

$$= -\frac{(c+d\tan(e+fx))^{1+n}}{6(ic-d)f(a+ia\tan(e+fx))^3} + \frac{(3ic-d(7-2n))(c+d\tan(e+fx))^{1+}}{24a(c+id)^2f(a+ia\tan(e+fx))^2}$$

$$= -\frac{(c+d\tan(e+fx))^{1+n}}{6(ic-d)f(a+ia\tan(e+fx))^3} + \frac{(3ic-d(7-2n))(c+d\tan(e+fx))^{1+}}{24a(c+id)^2f(a+ia\tan(e+fx))^2}$$

$$= \frac{{}_2F_1\left(1,1+n;2+n;\frac{c+d\tan(e+fx)}{c-id}\right)(c+d\tan(e+fx))^{1+n}}{16a^3(ic+d)f(1+n)} + \frac{(3ic^3-c^2d(9-}{}$$

Mathematica [F]
time = 27.94, size = 0, normalized size = 0.00

$$\int \frac{(c+d\tan(e+fx))^n}{(a+ia\tan(e+fx))^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*Tan[e + f*x])^n/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] Integrate[(c + d*Tan[e + f*x])^n/(a + I*a*Tan[e + f*x])^3, x]
```

Maple [F]
time = 3.37, size = 0, normalized size = 0.00

$$\int \frac{(c+d\tan(fx+e))^n}{(a+ia\tan(fx+e))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^3,x)
```

```
[Out] int((c+d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^3,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(1/8*(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e
) + 1))^n*(e^(6*I*f*x + 6*I*e) + 3*e^(4*I*f*x + 4*I*e) + 3*e^(2*I*f*x + 2*I
*e) + 1)*e^(-6*I*f*x - 6*I*e)/a^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i \int \frac{(c+d\tan(e+fx))^n}{\tan^3(e+fx)-3i\tan^2(e+fx)-3\tan(e+fx)+i}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**n/(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] I*Integral((c + d*tan(e + f*x))**n/(tan(e + f*x)**3 - 3*I*tan(e + f*x)**2 -
 3*tan(e + f*x) + I), x)/a**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*tan(f*x + e) + c)^n/(I*a*tan(f*x + e) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\tan(e+f\,x))^n}{(a+a\tan(e+f\,x)\,1\mathrm{i})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^n/(a + a*tan(e + f*x)*1i)^3,x)
```

```
[Out] int((c + d*tan(e + f*x))^n/(a + a*tan(e + f*x)*1i)^3, x)
```

3.1180 $\int (a+ia\tan(e+fx))^m(c+d\tan(e+fx))^3\,dx$

Optimal. Leaf size=192

$$-\frac{2d(d^2+icdm-c^2(3+m))\,(a+ia\tan(e+fx))^m}{fm(2+m)}+\frac{(ic+d)^3\,{}_2F_1\left(1,m;1+m;\frac{1}{2}(1+i\tan(e+fx))\right)(a+$$

```
[Out] -2*d*(d^2+I*c*d*m-c^2*(3+m))*(a+I*a*tan(f*x+e))^m/f/m/(2+m)+1/2*(I*c+d)^3*h
ypergeom([1, m],[1+m],1/2+1/2*I*tan(f*x+e))*(a+I*a*tan(f*x+e))^m/f/m-d^2*(d
*m+I*c*(4+m))*(a+I*a*tan(f*x+e))^(1+m)/a/f/(1+m)/(2+m)+d*(a+I*a*tan(f*x+e))
^m*(c+d*tan(f*x+e))^2/f/(2+m)
```

Rubi [A]
time = 0.30, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3641, 3673, 3608, 3562, 70}

$$-\frac{2d(-(c^2(m+3))+icdm+d^2)\,(a+ia\tan(e+fx))^m}{fm(m+2)}-\frac{d^2(dm+ic(m+4))(a+ia\tan(e+fx))^{m+1}}{af(m+1)(m+2)}+\frac{(d+ic)^3(a+ia\tan(e+fx))^m\,{}_2F_1\left(1,m;m+1;\frac{1}{2}(i\tan(e+fx)+1)\right)}{2fm}+\frac{d(a+ia\tan(e+fx))^m(c+d\tan(e+fx))^2}{f(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^3,x]
```

```
[Out] (-2*d*(d^2 + I*c*d*m - c^2*(3 + m))*(a + I*a*Tan[e + f*x])^m)/(f*m*(2 + m))
 + ((I*c + d)^3*Hypergeometric2F1[1, m, 1 + m, (1 + I*Tan[e + f*x])/2]*(a +
 I*a*Tan[e + f*x])^m)/(2*f*m) - (d^2*(d*m + I*c*(4 + m))*(a + I*a*Tan[e + f
*x])^(1 + m))/(a*f*(1 + m)*(2 + m)) + (d*(a + I*a*Tan[e + f*x])^m*(c + d*Ta
n[e + f*x])^2)/(f*(2 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rule 3641

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[d*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n - 1)/(f*(m + n - 1))), x] - Dist[1/(a*(m + n - 1)), Int[(a + b*
Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 2)*Simp[d*(b*c*m + a*d*(-1 + n))
- a*c^2*(m + n - 1) + d*(b*d*m - a*c*(m + 2*n - 2))*Tan[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1] && NeQ[m + n - 1, 0] && (IntegerQ[n]
|| IntegersQ[2*m, 2*n])
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (a+ia\tan(e+fx))^m(c+d\tan(e+fx))^3\,dx = \frac{d(a+ia\tan(e+fx))^m(c+d\tan(e+fx))^2}{f(2+m)} - \frac{\int (a+ia\,\mathrm{ta}\cdots}{\cdots}$$

$$= -\frac{d^2(dm+ic(4+m))(a+ia\tan(e+fx))^{1+m}}{af(1+m)(2+m)} + \frac{d(a+ia\cdots}{\cdots}$$

$$= -\frac{2d\left(d^2+icdm-c^2(3+m)\right)(a+ia\tan(e+fx))^m}{fm(2+m)} - \frac{d^2\cdots}{\cdots}$$

$$= -\frac{2d\left(d^2+icdm-c^2(3+m)\right)(a+ia\tan(e+fx))^m}{fm(2+m)} - \frac{d^2\cdots}{\cdots}$$

$$= -\frac{2d\left(d^2+icdm-c^2(3+m)\right)(a+ia\tan(e+fx))^m}{fm(2+m)} + \frac{(ic\cdots}{\cdots}$$

Mathematica [F]
time = 50.22, size = 0, normalized size = 0.00

$$\int (a+ia\tan(e+fx))^m(c+d\tan(e+fx))^3\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^3,x]
[Out] Integrate[(a + I*a*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^3, x]
```

Maple [F]
time = 3.31, size = 0, normalized size = 0.00

$$\int (a+ia\tan(fx+e))^m (c+d\tan(fx+e))^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^m*(c+d*tan(f*x+e))^3,x)
[Out] int((a+I*a*tan(f*x+e))^m*(c+d*tan(f*x+e))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c+d*tan(f*x+e))^3,x, algorithm="maxima")
[Out] integrate((d*tan(f*x + e) + c)^3*(I*a*tan(f*x + e) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c+d*tan(f*x+e))^3,x, algorithm="fricas")
[Out] integral((c^3 + 3*I*c^2*d - 3*c*d^2 - I*d^3 + (c^3 - 3*I*c^2*d - 3*c*d^2 +
I*d^3)*e^(6*I*f*x + 6*I*e) + 3*(c^3 - I*c^2*d + c*d^2 - I*d^3)*e^(4*I*f*x +
 4*I*e) + 3*(c^3 + I*c^2*d + c*d^2 + I*d^3)*e^(2*I*f*x + 2*I*e))*(2*a*e^(2*
I*f*x + 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1))^m/(e^(6*I*f*x + 6*I*e) + 3*e^(4*I
*f*x + 4*I*e) + 3*e^(2*I*f*x + 2*I*e) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(e+fx)-i))^m (c+d\tan(e+fx))^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**m*(c+d*tan(f*x+e))**3,x)
```

[Out] Integral((I*a*(tan(e + f*x) - I))**m*(c + d*tan(e + f*x))**3, x)

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I*a*tan(f*x+e))^m*(c+d*tan(f*x+e))^3,x, algorithm="giac")

[Out] integrate((d*tan(f*x + e) + c)^3*(I*a*tan(f*x + e) + a)^m, x)

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a\tan(e + f\,x)\ 1\mathrm{i})^m\,(c + d\tan(e + f\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*tan(e + f*x)*1i)^m*(c + d*tan(e + f*x))^3,x)

[Out] int((a + a*tan(e + f*x)*1i)^m*(c + d*tan(e + f*x))^3, x)

3.1181 $\int (a+ia\tan(e+fx))^m(c+d\tan(e+fx))^2\,dx$

Optimal. Leaf size=119

$$\frac{2cd(a+ia\tan(e+fx))^m}{fm}-\frac{i(c-id)^2\,{}_2F_1\left(1,m;1+m;\frac{1}{2}(1+i\tan(e+fx))\right)(a+ia\tan(e+fx))^m}{2fm}-\frac{id^2(a\cdots}{\cdots}$$

```
[Out] 2*c*d*(a+I*a*tan(f*x+e))^m/f/m-1/2*I*(c-I*d)^2*hypergeom([1, m],[1+m],1/2+1
/2*I*tan(f*x+e))*(a+I*a*tan(f*x+e))^m/f/m-I*d^2*(a+I*a*tan(f*x+e))^(1+m)/a/
f/(1+m)
```

Rubi [A]
time = 0.12, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3624, 3608, 3562, 70}

$$-\frac{i(c-id)^2(a+ia\tan(e+fx))^m\,{}_2F_1\left(1,m;m+1;\frac{1}{2}(i\tan(e+fx)+1)\right)}{2fm}+\frac{2cd(a+ia\tan(e+fx))^m}{fm}-\frac{id^2(a+ia\tan(e+fx))^{m+1}}{af(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^2,x]
```

```
[Out] (2*c*d*(a + I*a*Tan[e + f*x])^m)/(f*m) - ((I/2)*(c - I*d)^2*Hypergeometric2
F1[1, m, 1 + m, (1 + I*Tan[e + f*x])/2]*(a + I*a*Tan[e + f*x])^m)/(f*m) - (
I*d^2*(a + I*a*Tan[e + f*x])^(1 + m))/(a*f*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (a+ia\tan(e+fx))^m(c+d\tan(e+fx))^2\,dx &= -\frac{id^2(a+ia\tan(e+fx))^{1+m}}{af(1+m)}+\int (a+ia\tan(e+fx))^m \\ &= \frac{2cd(a+ia\tan(e+fx))^m}{fm}-\frac{id^2(a+ia\tan(e+fx))^{1+m}}{af(1+m)} \\ &= \frac{2cd(a+ia\tan(e+fx))^m}{fm}-\frac{id^2(a+ia\tan(e+fx))^{1+m}}{af(1+m)} \\ &= \frac{2cd(a+ia\tan(e+fx))^m}{fm}-\frac{i(c-id)^2\,{}_2F_1\big(1,m;1+m;\tfrac{1}{2}\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 422 vs. 2(119) = 238.
time = 18.18, size = 422, normalized size = 3.55

$$\frac{2^{-1+m}e^{-2ifmx}\left(e^{ifx}\right)^m\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^m\left(-\frac{i(c+id)^2e^{2ifmx}\left(1+e^{2i(e+fx)}\right)^m{}_2F_1\left(m,2+m;1+m;-e^{2i(e+fx)}\right)}{m}-\frac{ie^{2ie}\left(2c^2e^{2if(1+m)x}(2+m)+2d^2e^{2if(1+m)x}(2+m)+c^2e^{2i(e+f(2+m)x)}\left(1+e^{2i(e+fx)}\right)^{1+m}(1+m)\,{}_2F_1\left(2+m,2+m;3+m;-e^{2i(e+fx)}\right)-2icde^{2i(e+f(2+m)x)}\left(1+e^{2i(e+fx)}\right)^{1+m}(1+m)\,{}_2F_1\left(2+m,2+m;3+m;-e^{2i(e+fx)}\right)-d^2e^{2i(e+f(2+m)x)}\left(1+e^{2i(e+fx)}\right)^{1+m}(1+m)\,{}_2F_1\left(2+m,2+m;3+m;-e^{2i(e+fx)}\right)\right)}{\left(1+e^{2i(e+fx)}\right)(1+m)(2+m)}\right)\sec^{-m}(e+fx)(\cos(fx)+i\sin(fx))^{-m}(a+ia\tan(e+fx))^m}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^2,x]
```

```
[Out] (2^(-1 + m)*(E^(I*f*x))^m*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^m*(((
-I)*(c + I*d)^2*E^((2*I)*f*m*x)*(1 + E^((2*I)*(e + f*x)))^m*Hypergeometric2
F1[m, 2 + m, 1 + m, -E^((2*I)*(e + f*x))])/m - (I*E^((2*I)*e)*(2*c^2*E^((2*
I)*f*(1 + m)*x)*(2 + m) + 2*d^2*E^((2*I)*f*(1 + m)*x)*(2 + m) + c^2*E^((2*I
)*(e + f*(2 + m)*x))*(1 + E^((2*I)*(e + f*x)))^(1 + m)*(1 + m)*Hypergeometr
ic2F1[2 + m, 2 + m, 3 + m, -E^((2*I)*(e + f*x))] - (2*I)*c*d*E^((2*I)*(e +
f*(2 + m)*x))*(1 + E^((2*I)*(e + f*x)))^(1 + m)*(1 + m)*Hypergeometric2F1[2
 + m, 2 + m, 3 + m, -E^((2*I)*(e + f*x))] - d^2*E^((2*I)*(e + f*(2 + m)*x))
*(1 + E^((2*I)*(e + f*x)))^(1 + m)*(1 + m)*Hypergeometric2F1[2 + m, 2 + m,
3 + m, -E^((2*I)*(e + f*x))]))/((1 + E^((2*I)*(e + f*x)))*(1 + m)*(2 + m)))
*(a + I*a*Tan[e + f*x])^m)/(E^((2*I)*f*m*x)*f*Sec[e + f*x]^m*(Cos[f*x] + I*
Sin[f*x])^m)
```

Maple [**F**]
time = 3.01, size = 0, normalized size = 0.00

$$\int \left(a+ia\tan\left(fx+e\right)\right)^{m}\left(c+d\tan\left(fx+e\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^m*(c+d*tan(f*x+e))^2,x)
```

```
[Out] int((a+I*a*tan(f*x+e))^m*(c+d*tan(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c+d*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*tan(f*x + e) + c)^2*(I*a*tan(f*x + e) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c+d*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((c^2 + 2*I*c*d - d^2 + (c^2 - 2*I*c*d - d^2)*e^(4*I*f*x + 4*I*e) +
 2*(c^2 + d^2)*e^(2*I*f*x + 2*I*e))*(2*a*e^(2*I*f*x + 2*I*e)/(e^(2*I*f*x +
2*I*e) + 1))^m/(e^(4*I*f*x + 4*I*e) + 2*e^(2*I*f*x + 2*I*e) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan (e + fx) - i))^m (c + d\tan (e + fx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**m*(c+d*tan(f*x+e))**2,x)
```

```
[Out] Integral((I*a*(tan(e + f*x) - I))**m*(c + d*tan(e + f*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c+d*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*tan(f*x + e) + c)^2*(I*a*tan(f*x + e) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a\tan(e + f\,x)\ 1\mathrm{i})^m\,(c + d\tan(e + f\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^m*(c + d*tan(e + f*x))^2,x)
[Out] int((a + a*tan(e + f*x)*1i)^m*(c + d*tan(e + f*x))^2, x)
```

3.1182 $\int (a+ia\tan(e+fx))^m(c+d\tan(e+fx))\,dx$

Optimal. Leaf size=78

$$\frac{d(a+ia\tan(e+fx))^m}{fm}-\frac{(ic+d)\,{}_2F_1\left(1,m;1+m;\frac{1}{2}(1+i\tan(e+fx))\right)(a+ia\tan(e+fx))^m}{2fm}$$

```
[Out] d*(a+I*a*tan(f*x+e))^m/f/m-1/2*(I*c+d)*hypergeom([1, m],[1+m],1/2+1/2*I*tan
(f*x+e))*(a+I*a*tan(f*x+e))^m/f/m
```

Rubi [A]
time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3608, 3562, 70}

$$\frac{d(a+ia\tan(e+fx))^m}{fm}-\frac{(d+ic)(a+ia\tan(e+fx))^m\,{}_2F_1\left(1,m;m+1;\frac{1}{2}(i\tan(e+fx)+1)\right)}{2fm}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^m*(c + d*Tan[e + f*x]),x]
```

```
[Out] (d*(a + I*a*Tan[e + f*x])^m)/(f*m) - ((I*c + d)*Hypergeometric2F1[1, m, 1 +
 m, (1 + I*Tan[e + f*x])/2]*(a + I*a*Tan[e + f*x])^m)/(2*f*m)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rubi steps

$$\int (a+ia\tan(e+fx))^m(c+d\tan(e+fx))\,dx = \frac{d(a+ia\tan(e+fx))^m}{fm} - (-c+id)\int (a+ia\tan(e+fx)$$
$$= \frac{d(a+ia\tan(e+fx))^m}{fm} - \frac{(a(ic+d))\text{Subst}\left(\int \frac{(a+x)^{-1+m}}{a-x}\,dx\right.}{f}$$
$$= \frac{d(a+ia\tan(e+fx))^m}{fm} - \frac{(ic+d)\,{}_2F_1\left(1,m;1+m;\frac{1}{2}(1+\right.}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 171 vs. 2(78) = 156.
time = 6.29, size = 171, normalized size = 2.19

$$\frac{2^{-1+m}\left(e^{ifx}\right)^m\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^m\left((-ic+d)(1+m)-i(c-id)e^{2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^m m\,{}_2F_1\left(1+m,1+m;2+m;-e^{2i(e+fx)}\right)\right)\sec^{-m}(e+fx)(\cos(fx)+i\sin(fx))^{-m}(a+ia\tan(e+fx))^m}{fm(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^m*(c + d*Tan[e + f*x]),x]
```

```
[Out] (2^(-1 + m)*(E^(I*f*x))^m*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^m*(((
-I)*c + d)*(1 + m) - I*(c - I*d)*E^((2*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x
)))^m*m*Hypergeometric2F1[1 + m, 1 + m, 2 + m, -E^((2*I)*(e + f*x))])*(a +
I*a*Tan[e + f*x])^m)/(f*m*(1 + m)*Sec[e + f*x]^m*(Cos[f*x] + I*Sin[f*x])^m)
```

Maple [F]
time = 0.64, size = 0, normalized size = 0.00

$$\int (a+ia\tan(fx+e))^m\,(c+d\tan(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^m*(c+d*tan(f*x+e)),x)
```

```
[Out] int((a+I*a*tan(f*x+e))^m*(c+d*tan(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c+d*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(f*x + e) + c)*(I*a*tan(f*x + e) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c+d*tan(f*x+e)),x, algorithm="fricas")
[Out] integral(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)*(2*a*e^(2*I*f*x + 2*I*e)
/(e^(2*I*f*x + 2*I*e) + 1))^m/(e^(2*I*f*x + 2*I*e) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(e+fx)-i))^m (c+d\tan(e+fx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**m*(c+d*tan(f*x+e)),x)
[Out] Integral((I*a*(tan(e + f*x) - I))**m*(c + d*tan(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c+d*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((d*tan(f*x + e) + c)*(I*a*tan(f*x + e) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\tan(e+f\,x)\,1\mathrm{i})^m\,(c+d\tan(e+f\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^m*(c + d*tan(e + f*x)),x)
[Out] int((a + a*tan(e + f*x)*1i)^m*(c + d*tan(e + f*x)), x)
```

3.1183 $\int \frac{(a+ia\tan(e+fx))^m}{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=122

$$\frac{{}_2F_1\left(1,m;1+m;\frac{1}{2}(1+i\tan(e+fx))\right)(a+ia\tan(e+fx))^m}{2(ic+d)fm}-\frac{d\,{}_2F_1\left(1,m;1+m;-\frac{d(1+i\tan(e+fx))}{ic-d}\right)(a+ia\text{t}}{(c^2+d^2)\,fm}$$

```
[Out] 1/2*hypergeom([1, m],[1+m],1/2+1/2*I*tan(f*x+e))*(a+I*a*tan(f*x+e))^m/(I*c+
d)/f/m-d*hypergeom([1, m],[1+m],-d*(1+I*tan(f*x+e))/(I*c-d))*(a+I*a*tan(f*x
+e))^m/(c^2+d^2)/f/m
```

Rubi [A]
time = 0.18, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3643, 3562, 70, 3680}

$$\frac{(a+ia\tan(e+fx))^m\,{}_2F_1\left(1,m;m+1;\frac{1}{2}(i\tan(e+fx)+1)\right)}{2fm(d+ic)}-\frac{d(a+ia\tan(e+fx))^m\,{}_2F_1\left(1,m;m+1;-\frac{d(i\tan(e+fx)+1)}{ic-d}\right)}{fm\left(c^2+d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^m/(c + d*Tan[e + f*x]),x]
```

```
[Out] (Hypergeometric2F1[1, m, 1 + m, (1 + I*Tan[e + f*x])/2]*(a + I*a*Tan[e + f*
x])^m)/(2*(I*c + d)*f*m) - (d*Hypergeometric2F1[1, m, 1 + m, -((d*(1 + I*Ta
n[e + f*x]))/(I*c - d))]*(a + I*a*Tan[e + f*x])^m)/((c^2 + d^2)*f*m)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3643

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)/((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[a/(a*c - b*d), Int[(a + b*Tan[e + f*x])^m,
x], x] - Dist[d/(a*c - b*d), Int[(a + b*Tan[e + f*x])^m*((b + a*Tan[e + f*x
])/(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
```

```
b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^m}{c+d\tan(e+fx)}\,dx = \frac{\int (a+ia\tan(e+fx))^m\,dx}{c-id} - \frac{d\int \frac{(a+ia\tan(e+fx))^m(ia+a\tan(e+fx))}{c+d\tan(e+fx)}\,dx}{a(c-id)}$$
$$= \frac{a\text{Subst}\left(\int \frac{(a+x)^{-1+m}}{a-x}\,dx, x, ia\tan(e+fx)\right)}{(ic+d)f} + \frac{(ad)\text{Subst}\left(\int \frac{(a+iax)^{-1+m}}{c+dx}\,dx, x\right.}{(ic+d)f}$$
$$= \frac{{}_2F_1\left(1,m;1+m;\frac{1}{2}(1+i\tan(e+fx))\right)(a+ia\tan(e+fx))^m}{2(ic+d)fm} - \frac{d\,{}_2F_1\left(1,m\right.}{}$$

Mathematica [F]
time = 14.66, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(e+fx))^m}{c+d\tan(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^m/(c + d*Tan[e + f*x]),x]
[Out] Integrate[(a + I*a*Tan[e + f*x])^m/(c + d*Tan[e + f*x]), x]
```

Maple [F]
time = 1.23, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(fx+e))^m}{c+d\tan(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^m/(c+d*tan(f*x+e)),x)
[Out] int((a+I*a*tan(f*x+e))^m/(c+d*tan(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c+d*tan(f*x+e)),x, algorithm="maxima")
[Out] integrate((I*a*tan(f*x + e) + a)^m/(d*tan(f*x + e) + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c+d*tan(f*x+e)),x, algorithm="fricas")
[Out] integral((2*a*e^(2*I*f*x + 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1))^m*(I*e^(2*I*f*
x + 2*I*e) + I)/((I*c + d)*e^(2*I*f*x + 2*I*e) + I*c - d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan{(e+fx)}-i))^m}{c+d\tan{(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**m/(c+d*tan(f*x+e)),x)
[Out] Integral((I*a*(tan(e + f*x) - I))**m/(c + d*tan(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c+d*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((I*a*tan(f*x + e) + a)^m/(d*tan(f*x + e) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\tan(e+f\,x)\;1\mathrm{i})^m}{c+d\tan{(e+f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^m/(c + d*tan(e + f*x)),x)
[Out] int((a + a*tan(e + f*x)*1i)^m/(c + d*tan(e + f*x)), x)
```

3.1184 $\int \frac{(a+ia\tan(e+fx))^m}{(c+d\tan(e+fx))^2} \, dx$

Optimal. Leaf size=180

$$-\frac{i\,{}_2F_1\left(1,m;1+m;\frac{1}{2}(1+i\tan(e+fx))\right)(a+ia\tan(e+fx))^m}{2(c-id)^2fm}-\frac{d(c(2-m)+idm)\,{}_2F_1\left(1,m;1+m;-\frac{d(}{}\right.}{(c^2+d^2)^2}$$

```
[Out] -1/2*I*hypergeom([1, m],[1+m],1/2+1/2*I*tan(f*x+e))*(a+I*a*tan(f*x+e))^m/(c
-I*d)^2/f/m-d*(c*(2-m)+I*d*m)*hypergeom([1, m],[1+m],-d*(1+I*tan(f*x+e))/(I
*c-d))*(a+I*a*tan(f*x+e))^m/(c^2+d^2)^2/f/m-d*(a+I*a*tan(f*x+e))^m/(c^2+d^2
)/f/(c+d*tan(f*x+e))
```

Rubi [A]

time = 0.34, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3642, 3681, 3562, 70, 3680}

$$-\frac{d(c(2-m)+idm)(a+ia\tan(e+fx))^m\,{}_2F_1\left(1,m;m+1;-\frac{d(i\tan(e+fx)+1)}{ic-d}\right)}{fm\left(c^2+d^2\right)^2}-\frac{d(a+ia\tan(e+fx))^m}{f\left(c^2+d^2\right)(c+d\tan(e+fx))}-\frac{i(a+ia\tan(e+fx))^m\,{}_2F_1\left(1,m;m+1;\frac{1}{2}(i\tan(e+fx)+1)\right)}{2fm(c-id)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^m/(c + d*Tan[e + f*x])^2,x]
```

```
[Out] ((-1/2*I)*Hypergeometric2F1[1, m, 1 + m, (1 + I*Tan[e + f*x])/2]*(a + I*a*T
an[e + f*x])^m)/((c - I*d)^2*f*m) - (d*(c*(2 - m) + I*d*m)*Hypergeometric2F
1[1, m, 1 + m, -((d*(1 + I*Tan[e + f*x]))/(I*c - d))]*(a + I*a*Tan[e + f*x]
)^m)/((c^2 + d^2)^2*f*m) - (d*(a + I*a*Tan[e + f*x])^m)/((c^2 + d^2)*f*(c +
 d*Tan[e + f*x]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3642

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[d*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
```

```
 + f*x])^(n + 1)/(f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(a*(c^2 + d^2)*(n +
1)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[b*d*m - a
*c*(n + 1) + a*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
 e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0
] && LtQ[n, -1] && (IntegerQ[n] || IntegersQ[2*m, 2*n])
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3681

```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(
A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^m}{(c+d\tan(e+fx))^2}\,dx = -\frac{d(a+ia\tan(e+fx))^m}{(c^2+d^2)\,f(c+d\tan(e+fx))} + \frac{\int \frac{(a+ia\tan(e+fx))^m(a(c+idm)-ad(1-m)\tan(e+fx))}{c+d\tan(e+fx)}}{a\left(c^2+d^2\right)}$$

$$= -\frac{d(a+ia\tan(e+fx))^m}{(c^2+d^2)\,f(c+d\tan(e+fx))} + \frac{\int (a+ia\tan(e+fx))^m\,dx}{(c-id)^2} - \frac{(d(ic(2-m}{}$$

$$= -\frac{d(a+ia\tan(e+fx))^m}{(c^2+d^2)\,f(c+d\tan(e+fx))} - \frac{(ia)\text{Subst}\left(\int \frac{(a+x)^{-1+m}}{a-x}\,dx, x, ia\tan(e+f}{(c-id)^2 f}$$

$$= -\frac{i\,{}_2F_1\left(1,m;1+m;\frac{1}{2}(1+i\tan(e+fx))\right)(a+ia\tan(e+fx))^m}{2(c-id)^2 fm} - \frac{d(c(2-m}{}$$

Mathematica [F]
time = 28.49, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(e+fx))^m}{(c+d\tan(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^m/(c + d*Tan[e + f*x])^2,x]
```

```
[Out] Integrate[(a + I*a*Tan[e + f*x])^m/(c + d*Tan[e + f*x])^2, x]
```

Maple [F]
time = 3.53, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(fx+e))^{m}}{(c+d\tan(fx+e))^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^m/(c+d*tan(f*x+e))^2,x)
```

```
[Out] int((a+I*a*tan(f*x+e))^m/(c+d*tan(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c+d*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^m/(d*tan(f*x + e) + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c+d*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((2*a*e^(2*I*f*x + 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1))^m*(e^(4*I*f*x
+ 4*I*e) + 2*e^(2*I*f*x + 2*I*e) + 1)/(c^2 + 2*I*c*d - d^2 + (c^2 - 2*I*c*d
 - d^2)*e^(4*I*f*x + 4*I*e) + 2*(c^2 + d^2)*e^(2*I*f*x + 2*I*e)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(e+fx)-i))^{m}}{(c+d\tan(e+fx))^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**m/(c+d*tan(f*x+e))**2,x)
```

```
[Out] Integral((I*a*(tan(e + f*x) - I))**m/(c + d*tan(e + f*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c+d*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^m/(d*tan(f*x + e) + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\tan(e+f\,x)\ 1\mathrm{i})^m}{(c+d\tan\left(e+f\,x\right))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^m/(c + d*tan(e + f*x))^2,x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^m/(c + d*tan(e + f*x))^2, x)
```

3.1185 $\int \frac{(a+ia\tan(e+fx))^m}{(c+d\tan(e+fx))^3}\,dx$

Optimal. Leaf size=264

$$-\frac{{}_2F_1\left(1,m;1+m;\frac{1}{2}(1+i\tan(e+fx))\right)(a+ia\tan(e+fx))^m}{2(ic+d)^3fm}-\frac{d(2icd(3-m)m+c^2(6-5m+m^2)-d^2}{}$$

```
[Out] -1/2*hypergeom([1, m],[1+m],1/2+1/2*I*tan(f*x+e))*(a+I*a*tan(f*x+e))^m/(I*c
+d)^3/f/m-1/2*d*(2*I*c*d*(3-m)*m+c^2*(m^2-5*m+6)-d^2*(m^2-m+2))*hypergeom([
1, m],[1+m],-d*(1+I*tan(f*x+e))/(I*c-d))*(a+I*a*tan(f*x+e))^m/(c^2+d^2)^3/f
/m-1/2*d*(a+I*a*tan(f*x+e))^m/(c^2+d^2)/f/(c+d*tan(f*x+e))^2-1/2*d*(c*(4-m)
+I*d*m)*(a+I*a*tan(f*x+e))^m/(c^2+d^2)^2/f/(c+d*tan(f*x+e))
```

Rubi [A]
time = 0.63, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3642, 3679, 3681, 3562, 70, 3680}

$$-\frac{d\left(c^2(m^2-5m+6)+2icd(3-m)m-d^2(m^2-m+2)\right)(a+ia\tan(e+fx))^m\,{}_2F_1\left(1,m;m+1;-\frac{d(i\tan(e+fx)+1)}{ic-d}\right)}{2fm\left(c^2+d^2\right)^3}-\frac{d(c(4-m)+idm)(a+ia\tan(e+fx))^m}{2f\left(c^2+d^2\right)^2(c+d\tan(e+fx))}-\frac{d(a+ia\tan(e+fx))^m}{2f\left(c^2+d^2\right)(c+d\tan(e+fx))^2}-\frac{(a+ia\tan(e+fx))^m\,{}_2F_1\left(1,m;m+1;\frac{1}{2}(i\tan(e+fx)+1)\right)}{2fm(d+ic)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^m/(c + d*Tan[e + f*x])^3,x]
```

```
[Out] -1/2*(Hypergeometric2F1[1, m, 1 + m, (1 + I*Tan[e + f*x])/2]*(a + I*a*Tan[e
 + f*x])^m)/((I*c + d)^3*f*m) - (d*((2*I)*c*d*(3 - m)*m + c^2*(6 - 5*m + m^
2) - d^2*(2 - m + m^2))*Hypergeometric2F1[1, m, 1 + m, -((d*(1 + I*Tan[e +
f*x]))/(I*c - d))]*(a + I*a*Tan[e + f*x])^m)/(2*(c^2 + d^2)^3*f*m) - (d*(a
+ I*a*Tan[e + f*x])^m)/(2*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^2) - (d*(c*(4
- m) + I*d*m)*(a + I*a*Tan[e + f*x])^m)/(2*(c^2 + d^2)^2*f*(c + d*Tan[e + f
*x]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3642

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[d*(a + b*Tan[e + f*x])^m*((c + d*Tan[e
 + f*x])^(n + 1)/(f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(a*(c^2 + d^2)*(n +
1)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[b*d*m - a
*c*(n + 1) + a*d*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
 e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0
] && LtQ[n, -1] && (IntegerQ[n] || IntegersQ[2*m, 2*n])
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
 + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3681

```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(
A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^m}{(c+d\tan(e+fx))^3}\,dx = -\frac{d(a+ia\tan(e+fx))^m}{2(c^2+d^2)f(c+d\tan(e+fx))^2} + \frac{\int \frac{(a+ia\tan(e+fx))^m(a(2c+idm)-ad(2-m)\tan(e}{(c+d\tan(e+fx))^2}}{2a(c^2+d^2)}$$

$$= -\frac{d(a+ia\tan(e+fx))^m}{2(c^2+d^2)f(c+d\tan(e+fx))^2} - \frac{d(c(4-m)+idm)(a+ia\tan(e+fx}{2(c^2+d^2)^2f(c+d\tan(e+fx))}$$

$$= -\frac{d(a+ia\tan(e+fx))^m}{2(c^2+d^2)f(c+d\tan(e+fx))^2} - \frac{d(c(4-m)+idm)(a+ia\tan(e+fx}{2(c^2+d^2)^2f(c+d\tan(e+fx))}$$

$$= -\frac{d(a+ia\tan(e+fx))^m}{2(c^2+d^2)f(c+d\tan(e+fx))^2} - \frac{d(c(4-m)+idm)(a+ia\tan(e+fx}{2(c^2+d^2)^2f(c+d\tan(e+fx))}$$

$$= -\frac{{}_2F_1\left(1,m;1+m;\frac{1}{2}(1+i\tan(e+fx))\right)(a+ia\tan(e+fx))^m}{2(ic+d)^3fm} - \frac{d(2icd(3}{}$$

Mathematica [F]
time = 47.35, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(e+fx))^m}{(c+d\tan(e+fx))^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^m/(c + d*Tan[e + f*x])^3,x]
```

```
[Out] Integrate[(a + I*a*Tan[e + f*x])^m/(c + d*Tan[e + f*x])^3, x]
```

Maple [F]
time = 3.99, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(fx+e))^m}{(c+d\tan(fx+e))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^m/(c+d*tan(f*x+e))^3,x)
```

```
[Out] int((a+I*a*tan(f*x+e))^m/(c+d*tan(f*x+e))^3,x)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c+d*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c+d*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral((2*a*e^(2*I*f*x + 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1))^m*(-I*e^(6*I*f
*x + 6*I*e) - 3*I*e^(4*I*f*x + 4*I*e) - 3*I*e^(2*I*f*x + 2*I*e) - I)/(-I*c^
3 + 3*c^2*d + 3*I*c*d^2 - d^3 + (-I*c^3 - 3*c^2*d + 3*I*c*d^2 + d^3)*e^(6*I
*f*x + 6*I*e) - 3*(I*c^3 + c^2*d + I*c*d^2 + d^3)*e^(4*I*f*x + 4*I*e) - 3*(
I*c^3 - c^2*d + I*c*d^2 - d^3)*e^(2*I*f*x + 2*I*e)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan{(e+fx)}-i))^m}{(c+d\tan{(e+fx)})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**m/(c+d*tan(f*x+e))**3,x)
```

```
[Out] Integral((I*a*(tan(e + f*x) - I))**m/(c + d*tan(e + f*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c+d*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^m/(d*tan(f*x + e) + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+a\tan(e+f\,x)\;1\mathrm{i})^m}{(c+d\tan{(e+f\,x)})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^m/(c + d*tan(e + f*x))^3,x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^m/(c + d*tan(e + f*x))^3, x)
```

3.1186 $\int (a+ia\tan(e+fx))^m(c+d\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=123

$$-\frac{(ic-d)F_1\left(m;-\frac{3}{2},1;1+m;-\frac{d(1+i\tan(e+fx))}{ic-d},\frac{1}{2}(1+i\tan(e+fx))\right)(a+ia\tan(e+fx))^m\sqrt{c+d\tan(e+fx)}}{2fm\sqrt{\frac{c+d\tan(e+fx)}{c+id}}}$$

```
[Out] -1/2*(I*c-d)*AppellF1(m,-3/2,1,1+m,-d*(1+I*tan(f*x+e))/(I*c-d),1/2+1/2*I*ta
n(f*x+e))*(c+d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^m/f/m/((c+d*tan(f*x+e))
/(c+I*d))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3645, 142, 141}

$$-\frac{(-d+ic)(a+ia\tan(e+fx))^m\sqrt{c+d\tan(e+fx)}\,F_1\left(m;-\frac{3}{2},1;m+1;-\frac{d(i\tan(e+fx)+1)}{ic-d},\frac{1}{2}(i\tan(e+fx)+1)\right)}{2fm\sqrt{\frac{c+d\tan(e+fx)}{c+id}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] -1/2*((I*c - d)*AppellF1[m, -3/2, 1, 1 + m, -((d*(1 + I*Tan[e + f*x]))/(I*c
 - d)), (1 + I*Tan[e + f*x])/2]*(a + I*a*Tan[e + f*x])^m*Sqrt[c + d*Tan[e +
 f*x]])/(f*m*Sqrt[(c + d*Tan[e + f*x])/(c + I*d)])
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\int (a+ia\tan(e+fx))^m(c+d\tan(e+fx))^{3/2}\,dx = \frac{(ia^2)\,\text{Subst}\left(\int \frac{(a+x)^{-1+m}\left(c-\frac{idx}{a}\right)^{3/2}}{-a^2+ax}\,dx, x, ia\tan(e+fx)\right)}{f}$$

$$= \frac{\left(ia^2(c+id)\sqrt{c+d\tan(e+fx)}\right)\,\text{Subst}\left(\int \frac{(a+x)^{-1+m}\left(\frac{c}{\ldots}\right.}{-a\ldots}\right.}{f\sqrt{\frac{c+d\tan(e+fx)}{c+id}}}$$

$$= -\frac{(ic-d)F_1\left(m;-\frac{3}{2},1;1+m;-\frac{d(1+i\tan(e+fx))}{ic-d},\frac{1}{2}(1+i\,\text{ta}\ldots\right.}{2fm\sqrt{\frac{c+\ldots}{\ldots}}}$$

Mathematica [F]
time = 27.04, size = 0, normalized size = 0.00

$$\int (a+ia\tan(e+fx))^m(c+d\tan(e+fx))^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(3/2),x]
[Out] Integrate[(a + I*a*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(3/2), x]
```

Maple [F]
time = 3.00, size = 0, normalized size = 0.00

$$\int \left(a+ia\tan\left(fx+e\right)\right)^m\left(c+d\tan\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^m*(c+d*tan(f*x+e))^(3/2),x)
```

```
[Out] int((a+I*a*tan(f*x+e))^m*(c+d*tan(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c+d*tan(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((d*tan(f*x + e) + c)^(3/2)*(I*a*tan(f*x + e) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c+d*tan(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d)*(2*a*e^(2*I*f*x + 2*I*e)
/(e^(2*I*f*x + 2*I*e) + 1))^m*sqrt(((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d
)/(e^(2*I*f*x + 2*I*e) + 1))/(e^(2*I*f*x + 2*I*e) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(e+fx)-i))^m (c+d\tan(e+fx))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**m*(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((I*a*(tan(e + f*x) - I))**m*(c + d*tan(e + f*x))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c+d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
```

```
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\tan(e+f\,x)\ 1\mathrm{i})^m\,(c+d\tan(e+f\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^m*(c + d*tan(e + f*x))^(3/2),x)
[Out] int((a + a*tan(e + f*x)*1i)^m*(c + d*tan(e + f*x))^(3/2), x)
```

3.1187 $\int (a+ia\tan(e+fx))^m\sqrt{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=116

$$-\frac{iF_1\left(m;-\frac{1}{2},1;1+m;-\frac{d(1+i\tan(e+fx))}{ic-d},\frac{1}{2}(1+i\tan(e+fx))\right)(a+ia\tan(e+fx))^m\sqrt{c+d\tan(e+fx)}}{2fm\sqrt{\frac{c+d\tan(e+fx)}{c+id}}}$$

[Out] -1/2*I*AppellF1(m,-1/2,1,1+m,-d*(1+I*tan(f*x+e))/(I*c-d),1/2+1/2*I*tan(f*x+e))*(c+d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^m/f/m/((c+d*tan(f*x+e))/(c+I*d))^(1/2)

Rubi [A]
time = 0.10, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3645, 142, 141}

$$-\frac{i(a+ia\tan(e+fx))^m\sqrt{c+d\tan(e+fx)}\,F_1\left(m;-\frac{1}{2},1;m+1;-\frac{d(i\tan(e+fx)+1)}{ic-d},\frac{1}{2}(i\tan(e+fx)+1)\right)}{2fm\sqrt{\frac{c+d\tan(e+fx)}{c+id}}}$$

Antiderivative was successfully verified.

[In] Int[(a + I*a*Tan[e + f*x])^m*Sqrt[c + d*Tan[e + f*x]],x]

[Out] ((-1/2*I)*AppellF1[m, -1/2, 1, 1 + m, -((d*(1 + I*Tan[e + f*x]))/(I*c - d)), (1 + I*Tan[e + f*x])/2]*(a + I*a*Tan[e + f*x])^m*Sqrt[c + d*Tan[e + f*x]])/(f*m*Sqrt[(c + d*Tan[e + f*x])/(c + I*d)])

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\int (a+ia\tan(e+fx))^m\sqrt{c+d\tan(e+fx)}\,dx=\frac{(ia^2)\,\mathrm{Subst}\left(\int\frac{(a+x)^{-1+m}\sqrt{c-\frac{idx}{a}}}{-a^2+ax}\,dx,x,ia\tan(e+fx)\right.}{f}$$

$$=\frac{\left(ia^2\sqrt{c+d\tan(e+fx)}\right)\mathrm{Subst}\left(\int\frac{(a+x)^{-1+m}\sqrt{\frac{c}{c+id}-}}{-a^2+ax}\right.}{f\sqrt{\frac{c+d\tan(e+fx)}{c+id}}}$$

$$=-\frac{iF_1\left(m;-\frac{1}{2},1;1+m;-\frac{d(1+i\tan(e+fx))}{ic-d},\frac{1}{2}(1+i\tan(e+f\right.}{2fm\sqrt{\frac{c+d\tan}{c+}}}$$

Mathematica [F]
time = 2.54, size = 0, normalized size = 0.00

$$\int (a+ia\tan(e+fx))^m\sqrt{c+d\tan(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^m*Sqrt[c + d*Tan[e + f*x]],x]
[Out] Integrate[(a + I*a*Tan[e + f*x])^m*Sqrt[c + d*Tan[e + f*x]], x]
```

Maple [F]
time = 2.68, size = 0, normalized size = 0.00

$$\int (a+ia\tan(fx+e))^m\sqrt{c+d\tan(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^m*(c+d*tan(f*x+e))^(1/2),x)
```

```
[Out] int((a+I*a*tan(f*x+e))^m*(c+d*tan(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c+d*tan(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(d*tan(f*x + e) + c)*(I*a*tan(f*x + e) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c+d*tan(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((2*a*e^(2*I*f*x + 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1))^m*sqrt(((c - I
*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(e+fx)-i))^m \sqrt{c+d\tan(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**m*(c+d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral((I*a*(tan(e + f*x) - I))**m*sqrt(c + d*tan(e + f*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(c+d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a\tan(e + f\,x)\ 1\mathrm{i})^m\,\sqrt{c + d\tan\left(e + f\,x\right)}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^m*(c + d*tan(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^m*(c + d*tan(e + f*x))^(1/2), x)
```

3.1188 $\int \frac{(a+ia\tan(e+fx))^m}{\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=116

$$-\frac{iF_1\left(m;\frac{1}{2},1;1+m;-\frac{d(1+i\tan(e+fx))}{ic-d},\frac{1}{2}(1+i\tan(e+fx))\right)(a+ia\tan(e+fx))^m\sqrt{\frac{c+d\tan(e+fx)}{c+id}}}{2fm\sqrt{c+d\tan(e+fx)}}$$

```
[Out] -1/2*I*AppellF1(m,1/2,1,1+m,-d*(1+I*tan(f*x+e))/(I*c-d),1/2+1/2*I*tan(f*x+e
))*((c+d*tan(f*x+e))/(c+I*d))^(1/2)*(a+I*a*tan(f*x+e))^m/f/m/(c+d*tan(f*x+e
))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3645, 142, 141}

$$-\frac{i(a+ia\tan(e+fx))^m\sqrt{\frac{c+d\tan(e+fx)}{c+id}}\,F_1\left(m;\frac{1}{2},1;m+1;-\frac{d(i\tan(e+fx)+1)}{ic-d},\frac{1}{2}(i\tan(e+fx)+1)\right)}{2fm\sqrt{c+d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^m/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((-1/2*I)*AppellF1[m, 1/2, 1, 1 + m, -((d*(1 + I*Tan[e + f*x]))/(I*c - d)),
 (1 + I*Tan[e + f*x])/2]*(a + I*a*Tan[e + f*x])^m*Sqrt[(c + d*Tan[e + f*x])
/(c + I*d)])/(f*m*Sqrt[c + d*Tan[e + f*x]])
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^m}{\sqrt{c+d\tan(e+fx)}}\,dx = \frac{(ia^2)\,\mathrm{Subst}\left(\int \frac{(a+x)^{-1+m}}{(-a^2+ax)\sqrt{c-\frac{idx}{a}}}\,dx, x, ia\tan(e+fx)\right)}{f}$$

$$= \frac{\left(ia^2\sqrt{\frac{c+d\tan(e+fx)}{c+id}}\right)\mathrm{Subst}\left(\int \frac{(a+x)^{-1+m}}{(-a^2+ax)\sqrt{\frac{c}{c+id}-\frac{idx}{a(c+id)}}}\,dx, x, ia$$

$$= -\frac{iF_1\left(m;\frac{1}{2},1;1+m;-\frac{d(1+i\tan(e+fx))}{ic-d},\frac{1}{2}(1+i\tan(e+fx))\right)(a+ia\tan(e+f}{2fm\sqrt{c+d\tan(e+fx)}}$$

Mathematica [F]
time = 20.45, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(e+fx))^m}{\sqrt{c+d\tan(e+fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^m/Sqrt[c + d*Tan[e + f*x]],x]
[Out] Integrate[(a + I*a*Tan[e + f*x])^m/Sqrt[c + d*Tan[e + f*x]], x]
```

Maple [F]
time = 2.78, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(fx+e))^m}{\sqrt{c+d\tan(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^m/(c+d*tan(f*x+e))^(1/2),x)
```

```
[Out] int((a+I*a*tan(f*x+e))^m/(c+d*tan(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c+d*tan(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^m/sqrt(d*tan(f*x + e) + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c+d*tan(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((2*a*e^(2*I*f*x + 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1))^m*sqrt(((c - I
*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*(I*e^(2*I*f*x
 + 2*I*e) + I)/((I*c + d)*e^(2*I*f*x + 2*I*e) + I*c - d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan{(e + fx)} - i))^m}{\sqrt{c + d\tan{(e + fx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**m/(c+d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral((I*a*(tan(e + f*x) - I))**m/sqrt(c + d*tan(e + f*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c+d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\tan(e + f\,x)\ 1\mathrm{i})^m}{\sqrt{c + d\tan\left(e + f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^m/(c + d*tan(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^m/(c + d*tan(e + f*x))^(1/2), x)
```

3.1189 $\int \frac{(a+ia\tan(e+fx))^m}{(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=125

$$\frac{F_1\left(m;\frac{3}{2},1;1+m;-\frac{d(1+i\tan(e+fx))}{ic-d},\frac{1}{2}(1+i\tan(e+fx))\right)(a+ia\tan(e+fx))^m\sqrt{\frac{c+d\tan(e+fx)}{c+id}}}{2(ic-d)fm\sqrt{c+d\tan(e+fx)}}$$

```
[Out] 1/2*AppellF1(m,3/2,1,1+m,-d*(1+I*tan(f*x+e))/(I*c-d),1/2+1/2*I*tan(f*x+e))*
((c+d*tan(f*x+e))/(c+I*d))^(1/2)*(a+I*a*tan(f*x+e))^m/(I*c-d)/f/m/(c+d*tan(
f*x+e))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3645, 142, 141}

$$\frac{(a+ia\tan(e+fx))^m\sqrt{\frac{c+d\tan(e+fx)}{c+id}}\,F_1\left(m;\frac{3}{2},1;m+1;-\frac{d(i\tan(e+fx)+1)}{ic-d},\frac{1}{2}(i\tan(e+fx)+1)\right)}{2fm(-d+ic)\sqrt{c+d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^m/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (AppellF1[m, 3/2, 1, 1 + m, -((d*(1 + I*Tan[e + f*x]))/(I*c - d)), (1 + I*T
an[e + f*x])/2]*(a + I*a*Tan[e + f*x])^m*Sqrt[(c + d*Tan[e + f*x])/(c + I*d
)])/(2*(I*c - d)*f*m*Sqrt[c + d*Tan[e + f*x]])
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^m}{(c+d\tan(e+fx))^{3/2}}\,dx = \frac{(ia^2)\,\text{Subst}\left(\int \frac{(a+x)^{-1+m}}{(-a^2+ax)\left(c-\frac{idx}{a}\right)^{3/2}}\,dx, x, ia\tan(e+fx)\right)}{f}$$

$$= \frac{\left(ia^2\sqrt{\frac{c+d\tan(e+fx)}{c+id}}\right)\text{Subst}\left(\int \frac{(a+x)^{-1+m}}{(-a^2+ax)\left(\frac{c}{c+id}-\frac{idx}{a(c+id)}\right)^{3/2}}\,dx, x, ia\tan(e+\right.}{(c+id)f\sqrt{c+d\tan(e+fx)}}$$

$$= \frac{F_1\left(m;\frac{3}{2},1;1+m;-\frac{d(1+i\tan(e+fx))}{ic-d},\frac{1}{2}(1+i\tan(e+fx))\right)(a+ia\tan(e+fx)}{2(ic-d)fm\sqrt{c+d\tan(e+fx)}}$$

Mathematica [**F**]
time = 11.14, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(e+fx))^m}{(c+d\tan(e+fx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^m/(c + d*Tan[e + f*x])^(3/2),x]
[Out] Integrate[(a + I*a*Tan[e + f*x])^m/(c + d*Tan[e + f*x])^(3/2), x]
```

Maple [**F**]
time = 2.84, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(fx+e))^m}{(c+d\tan(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^m/(c+d*tan(f*x+e))^(3/2),x)
[Out] int((a+I*a*tan(f*x+e))^m/(c+d*tan(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c+d*tan(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^m/(d*tan(f*x + e) + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c+d*tan(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral((2*a*e^(2*I*f*x + 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1))^m*sqrt(((c - I
*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*(e^(4*I*f*x +
 4*I*e) + 2*e^(2*I*f*x + 2*I*e) + 1)/(c^2 + 2*I*c*d - d^2 + (c^2 - 2*I*c*d
- d^2)*e^(4*I*f*x + 4*I*e) + 2*(c^2 + d^2)*e^(2*I*f*x + 2*I*e)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(e+fx)-i))^m}{(c+d\tan(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**m/(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((I*a*(tan(e + f*x) - I))**m/(c + d*tan(e + f*x))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c+d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
```

```
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\tan(e + f\,x)\ 1\mathrm{i})^m}{(c + d\tan(e + f\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^m/(c + d*tan(e + f*x))^(3/2),x)
[Out] int((a + a*tan(e + f*x)*1i)^m/(c + d*tan(e + f*x))^(3/2), x)
```

3.1190 $\int \frac{(a+ia\tan(e+fx))^m}{(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=125

$$-\frac{iF_1\left(m;\frac{5}{2},1;1+m;-\frac{d(1+i\tan(e+fx))}{ic-d},\frac{1}{2}(1+i\tan(e+fx))\right)(a+ia\tan(e+fx))^m\sqrt{\frac{c+d\tan(e+fx)}{c+id}}}{2(c+id)^2fm\sqrt{c+d\tan(e+fx)}}$$

```
[Out] -1/2*I*AppellF1(m,5/2,1,1+m,-d*(1+I*tan(f*x+e))/(I*c-d),1/2+1/2*I*tan(f*x+e
))*((c+d*tan(f*x+e))/(c+I*d))^(1/2)*(a+I*a*tan(f*x+e))^m/(c+I*d)^2/f/m/(c+d
*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3645, 142, 141}

$$-\frac{i(a+ia\tan(e+fx))^m\sqrt{\frac{c+d\tan(e+fx)}{c+id}}\,F_1\left(m;\frac{5}{2},1;m+1;-\frac{d(i\tan(e+fx)+1)}{ic-d},\frac{1}{2}(i\tan(e+fx)+1)\right)}{2fm(c+id)^2\sqrt{c+d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^m/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-1/2*I)*AppellF1[m, 5/2, 1, 1 + m, -((d*(1 + I*Tan[e + f*x]))/(I*c - d)),
 (1 + I*Tan[e + f*x])/2]*(a + I*a*Tan[e + f*x])^m*Sqrt[(c + d*Tan[e + f*x])
/(c + I*d)])/((c + I*d)^2*f*m*Sqrt[c + d*Tan[e + f*x]])
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^m}{(c+d\tan(e+fx))^{5/2}}\,dx = \frac{(ia^2)\,\mathrm{Subst}\left(\int \frac{(a+x)^{-1+m}}{(-a^2+ax)\left(c-\frac{idx}{a}\right)^{5/2}}\,dx, x, ia\tan(e+fx)\right)}{f}$$

$$= \frac{\left(ia^2\sqrt{\frac{c+d\tan(e+fx)}{c+id}}\right)\mathrm{Subst}\left(\int \frac{(a+x)^{-1+m}}{(-a^2+ax)\left(\frac{c}{c+id}-\frac{idx}{a(c+id)}\right)^{5/2}}\,dx, x, ia\tan(e+\right.}{(c+id)^2 f\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{iF_1\left(m;\frac{5}{2},1;1+m;-\frac{d(1+i\tan(e+fx))}{ic-d},\frac{1}{2}(1+i\tan(e+fx))\right)(a+ia\tan(e+f}{2(c+id)^2 fm\sqrt{c+d\tan(e+fx)}}$$

Mathematica [F]
time = 17.14, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(e+fx))^m}{(c+d\tan(e+fx))^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^m/(c + d*Tan[e + f*x])^(5/2),x]
[Out] Integrate[(a + I*a*Tan[e + f*x])^m/(c + d*Tan[e + f*x])^(5/2), x]
```

Maple [F]
time = 2.71, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(fx+e))^m}{(c+d\tan(fx+e))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^m/(c+d*tan(f*x+e))^(5/2),x)
[Out] int((a+I*a*tan(f*x+e))^m/(c+d*tan(f*x+e))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c+d*tan(f*x+e))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^m/(d*tan(f*x + e) + c)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c+d*tan(f*x+e))^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral((2*a*e^(2*I*f*x + 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1))^m*sqrt(((c - I
*d)*e^(2*I*f*x + 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))*(-I*e^(6*I*f*
x + 6*I*e) - 3*I*e^(4*I*f*x + 4*I*e) - 3*I*e^(2*I*f*x + 2*I*e) - I)/(-I*c^3
 + 3*c^2*d + 3*I*c*d^2 - d^3 + (-I*c^3 - 3*c^2*d + 3*I*c*d^2 + d^3)*e^(6*I*
f*x + 6*I*e) - 3*(I*c^3 + c^2*d + I*c*d^2 + d^3)*e^(4*I*f*x + 4*I*e) - 3*(I
*c^3 - c^2*d + I*c*d^2 - d^3)*e^(2*I*f*x + 2*I*e)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ia\left(\tan\left(e+fx\right)-i\right)\right)^{m}}{\left(c+d\tan\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**m/(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral((I*a*(tan(e + f*x) - I))**m/(c + d*tan(e + f*x))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m/(c+d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\tan(e+f\,x)\ 1\mathrm{i})^m}{(c+d\tan{(e+f\,x)})^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^m/(c + d*tan(e + f*x))^(5/2),x)
[Out] int((a + a*tan(e + f*x)*1i)^m/(c + d*tan(e + f*x))^(5/2), x)
```

3.1191 $\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))\,dx$

Optimal. Leaf size=140

$$\left(a^3c-3ab^2c-3a^2bd+b^3d\right)x-\frac{\left(3a^2bc-b^3c+a^3d-3ab^2d\right)\log(\cos(e+fx))}{f}+\frac{b\left(2abc+a^2d-b^2d\right)\tan(e+}{f}$$

```
[Out] (a^3*c-3*a^2*b*d-3*a*b^2*c+b^3*d)*x-(a^3*d+3*a^2*b*c-3*a*b^2*d-b^3*c)*ln(co
s(f*x+e))/f+b*(a^2*d+2*a*b*c-b^2*d)*tan(f*x+e)/f+1/2*(a*d+b*c)*(a+b*tan(f*x
+e))^2/f+1/3*d*(a+b*tan(f*x+e))^3/f
```

Rubi [A]

time = 0.11, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3609, 3606, 3556}

$$\frac{b\left(a^2d+2abc-b^2d\right)\tan(e+fx)}{f}-\frac{\left(a^3d+3a^2bc-3ab^2d-b^3c\right)\log(\cos(e+fx))}{f}+x\left(a^3c-3a^2bd-3ab^2c+b^3d\right)+\frac{(ad+bc)(a+b\tan(e+fx))^2}{2f}+\frac{d(a+b\tan(e+fx))^3}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x]),x]
```

```
[Out] (a^3*c - 3*a*b^2*c - 3*a^2*b*d + b^3*d)*x - ((3*a^2*b*c - b^3*c + a^3*d - 3
*a*b^2*d)*Log[Cos[e + f*x]])/f + (b*(2*a*b*c + a^2*d - b^2*d)*Tan[e + f*x])
/f + ((b*c + a*d)*(a + b*Tan[e + f*x])^2)/(2*f) + (d*(a + b*Tan[e + f*x])^3
)/(3*f)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))\,dx = \frac{d(a+b\tan(e+fx))^3}{3f} + \int (a+b\tan(e+fx))^2(ac-bd+(b$$
$$= \frac{(bc+ad)(a+b\tan(e+fx))^2}{2f} + \frac{d(a+b\tan(e+fx))^3}{3f} + \int$$
$$= \left(a^3c-3ab^2c-3a^2bd+b^3d\right)x + \frac{b(2abc+a^2d-b^2d)\tan(e+}{f}$$
$$= \left(a^3c-3ab^2c-3a^2bd+b^3d\right)x - \frac{(3a^2bc-b^3c+a^3d-3ab^2d}{f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.11, size = 130, normalized size = 0.93

$$\frac{3(a+ib)^3(-ic+d)\log(i-\tan(e+fx))+3(a-ib)^3(ic+d)\log(i+\tan(e+fx))+6b(3abc+3a^2d-b^2d)\tan(e+fx)+3b^2(bc+3ad)\tan^2(e+fx)+2b^3d\tan^3(e+fx)}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x]),x]
```

```
[Out] (3*(a + I*b)^3*((-I)*c + d)*Log[I - Tan[e + f*x]] + 3*(a - I*b)^3*(I*c + d)
*Log[I + Tan[e + f*x]] + 6*b*(3*a*b*c + 3*a^2*d - b^2*d)*Tan[e + f*x] + 3*b
^2*(b*c + 3*a*d)*Tan[e + f*x]^2 + 2*b^3*d*Tan[e + f*x]^3)/(6*f)
```

Maple [A]
time = 0.09, size = 159, normalized size = 1.14

method	result
norman	$\left(a^3c-3a^2bd-3a\,b^2c+b^3d\right)x+\frac{b\left(3a^2d+3abc-b^2d\right)\tan(fx+e)}{f}+\frac{b^2(3ad+bc)\left(\tan^2(fx+e)\right)}{2f}+\frac{b^3d\left(\tan^3(\right.}{3f}$
derivativedivides	$\frac{\frac{b^3d\left(\tan^3(fx+e)\right)}{3}+\frac{3a\,b^2d\left(\tan^2(fx+e)\right)}{2}+\frac{b^3c\left(\tan^2(fx+e)\right)}{2}+3a^2bd\tan(fx+e)+3a\,b^2c\tan(fx+e)-b^3d\tan(fx+e)+\frac{\left(a^3d+3a\right.}{}}{f}$
default	$\frac{\frac{b^3d\left(\tan^3(fx+e)\right)}{3}+\frac{3a\,b^2d\left(\tan^2(fx+e)\right)}{2}+\frac{b^3c\left(\tan^2(fx+e)\right)}{2}+3a^2bd\tan(fx+e)+3a\,b^2c\tan(fx+e)-b^3d\tan(fx+e)+\frac{\left(a^3d+3a\right.}{}}{f}$
risch	$a^3cx-3a^2bdx-3a\,b^2cx+b^3dx+3ia^2bcx+\frac{2ia^3de}{f}-3ia\,b^2dx-\frac{2ib^3ce}{f}-ib^3cx+ia^3dx-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/3*b^3*d*tan(f*x+e)^3+3/2*a*b^2*d*tan(f*x+e)^2+1/2*b^3*c*tan(f*x+e)^2
+3*a^2*b*d*tan(f*x+e)+3*a*b^2*c*tan(f*x+e)-b^3*d*tan(f*x+e)+1/2*(a^3*d+3*a^
```

```
2*b*c-3*a*b^2*d-b^3*c)*ln(1+tan(f*x+e)^2)+(a^3*c-3*a^2*b*d-3*a*b^2*c+b^3*d)
*arctan(tan(f*x+e)))
```

Maxima [A]
time = 0.54, size = 153, normalized size = 1.09

$$\frac{2 b^3 d \tan (fx+e)^3 + 3 (b^3 c + 3 ab^2 d) \tan (fx+e)^2 + 6 ((a^3 - 3 ab^2)c - (3 a^2 b - b^3)d)(fx+e) + 3 ((3 a^2 b - b^3)c + (a^3 - 3 ab^2)d) \log (\tan (fx+e)^2 + 1) + 6 (3 ab^2 c + (3 a^2 b - b^3)d) \tan (fx+e)}{6 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^3*d*tan(f*x + e)^3 + 3*(b^3*c + 3*a*b^2*d)*tan(f*x + e)^2 + 6*((a^
3 - 3*a*b^2)*c - (3*a^2*b - b^3)*d)*(f*x + e) + 3*((3*a^2*b - b^3)*c + (a^3
 - 3*a*b^2)*d)*log(tan(f*x + e)^2 + 1) + 6*(3*a*b^2*c + (3*a^2*b - b^3)*d)*
tan(f*x + e))/f
```

Fricas [A]
time = 1.13, size = 151, normalized size = 1.08

$$\frac{2 b^3 d \tan (fx+e)^3 + 6 ((a^3 - 3 ab^2)c - (3 a^2 b - b^3)d) fx + 3 (b^3 c + 3 ab^2 d) \tan (fx+e)^2 - 3 ((3 a^2 b - b^3)c + (a^3 - 3 ab^2)d) \log \left(\frac{1}{\tan(fx+e)^2+1}\right) + 6 (3 ab^2 c + (3 a^2 b - b^3)d) \tan (fx+e)}{6 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*d*tan(f*x + e)^3 + 6*((a^3 - 3*a*b^2)*c - (3*a^2*b - b^3)*d)*f*x
 + 3*(b^3*c + 3*a*b^2*d)*tan(f*x + e)^2 - 3*((3*a^2*b - b^3)*c + (a^3 - 3*a
*b^2)*d)*log(1/(tan(f*x + e)^2 + 1)) + 6*(3*a*b^2*c + (3*a^2*b - b^3)*d)*ta
n(f*x + e))/f
```

Sympy [A]
time = 0.15, size = 240, normalized size = 1.71

$$\begin{cases} a^3 cx + \frac{a^3 d \log (\tan^2 (e+fx)+1)}{2f} + \frac{3a^2 bc \log (\tan^2 (e+fx)+1)}{2f} - 3a^2 bdx + \frac{3a^2 bd \tan (e+fx)}{f} - 3ab^2 cx + \frac{3ab^2 c \tan (e+fx)}{f} - \frac{3ab^2 d \log (\tan^2 (e+fx)+1)}{2f} + \frac{3ab^2 d \tan^2 (e+fx)}{2f} - \frac{b^3 c \log (\tan^2 (e+fx)+1)}{2f} + \frac{b^3 c \tan^2 (e+fx)}{2f} + b^3 dx + \frac{b^3 d \tan^3 (e+fx)}{3f} - \frac{b^3 d \tan (e+fx)}{f} & \text{for } f \neq 0 \\ x(a + b \tan (e))^3 (c + d \tan (e)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3*(c+d*tan(f*x+e)),x)
```

```
[Out] Piecewise((a**3*c*x + a**3*d*log(tan(e + f*x)**2 + 1)/(2*f) + 3*a**2*b*c*lo
g(tan(e + f*x)**2 + 1)/(2*f) - 3*a**2*b*d*x + 3*a**2*b*d*tan(e + f*x)/f - 3
*a*b**2*c*x + 3*a*b**2*c*tan(e + f*x)/f - 3*a*b**2*d*log(tan(e + f*x)**2 +
1)/(2*f) + 3*a*b**2*d*tan(e + f*x)**2/(2*f) - b**3*c*log(tan(e + f*x)**2 +
1)/(2*f) + b**3*c*tan(e + f*x)**2/(2*f) + b**3*d*x + b**3*d*tan(e + f*x)**3
/(3*f) - b**3*d*tan(e + f*x)/f, Ne(f, 0)), (x*(a + b*tan(e))**3*(c + d*tan(
e)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2046 vs. $2(140) = 280$.

time = 1.35, size = 2046, normalized size = 14.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/6*(6*a^3*c*f*x*tan(f*x)^3*tan(e)^3 - 18*a*b^2*c*f*x*tan(f*x)^3*tan(e)^3 -
 18*a^2*b*d*f*x*tan(f*x)^3*tan(e)^3 + 6*b^3*d*f*x*tan(f*x)^3*tan(e)^3 - 9*a
^2*b*c*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)
^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^
3 + 3*b^3*c*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*t
an(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*ta
n(e)^3 - 3*a^3*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x
)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)
^3*tan(e)^3 + 9*a*b^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) +
tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*t
an(f*x)^3*tan(e)^3 - 18*a^3*c*f*x*tan(f*x)^2*tan(e)^2 + 54*a*b^2*c*f*x*tan(
f*x)^2*tan(e)^2 + 54*a^2*b*d*f*x*tan(f*x)^2*tan(e)^2 - 18*b^3*d*f*x*tan(f*x
)^2*tan(e)^2 + 3*b^3*c*tan(f*x)^3*tan(e)^3 + 9*a*b^2*d*tan(f*x)^3*tan(e)^3
+ 27*a^2*b*c*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*
tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*t
an(e)^2 - 9*b^3*c*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*
x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x
)^2*tan(e)^2 + 9*a^3*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + t
an(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*ta
n(f*x)^2*tan(e)^2 - 27*a*b^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*ta
n(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2
+ 1))*tan(f*x)^2*tan(e)^2 - 18*a*b^2*c*tan(f*x)^3*tan(e)^2 - 18*a^2*b*d*tan
(f*x)^3*tan(e)^2 + 6*b^3*d*tan(f*x)^3*tan(e)^2 - 18*a*b^2*c*tan(f*x)^2*tan(
e)^3 - 18*a^2*b*d*tan(f*x)^2*tan(e)^3 + 6*b^3*d*tan(f*x)^2*tan(e)^3 + 18*a^
3*c*f*x*tan(f*x)*tan(e) - 54*a*b^2*c*f*x*tan(f*x)*tan(e) - 54*a^2*b*d*f*x*t
an(f*x)*tan(e) + 18*b^3*d*f*x*tan(f*x)*tan(e) + 3*b^3*c*tan(f*x)^3*tan(e) +
 9*a*b^2*d*tan(f*x)^3*tan(e) - 3*b^3*c*tan(f*x)^2*tan(e)^2 - 9*a*b^2*d*tan(
f*x)^2*tan(e)^2 + 3*b^3*c*tan(f*x)*tan(e)^3 + 9*a*b^2*d*tan(f*x)*tan(e)^3 -
 2*b^3*d*tan(f*x)^3 - 27*a^2*b*c*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*
tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^
2 + 1))*tan(f*x)*tan(e) + 9*b^3*c*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3
*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)
^2 + 1))*tan(f*x)*tan(e) - 9*a^3*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^
3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e
)^2 + 1))*tan(f*x)*tan(e) + 27*a*b^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f
*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(t
```

```
an(e)^2 + 1))*tan(f*x)*tan(e) + 36*a*b^2*c*tan(f*x)^2*tan(e) + 36*a^2*b*d*t
an(f*x)^2*tan(e) - 18*b^3*d*tan(f*x)^2*tan(e) + 36*a*b^2*c*tan(f*x)*tan(e)^
2 + 36*a^2*b*d*tan(f*x)*tan(e)^2 - 18*b^3*d*tan(f*x)*tan(e)^2 - 2*b^3*d*tan
(e)^3 - 6*a^3*c*f*x + 18*a*b^2*c*f*x + 18*a^2*b*d*f*x - 6*b^3*d*f*x - 3*b^3
*c*tan(f*x)^2 - 9*a*b^2*d*tan(f*x)^2 + 3*b^3*c*tan(f*x)*tan(e) + 9*a*b^2*d*
tan(f*x)*tan(e) - 3*b^3*c*tan(e)^2 - 9*a*b^2*d*tan(e)^2 + 9*a^2*b*c*log(4*(
tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^
2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1)) - 3*b^3*c*log(4*(tan(f*x)^4*tan(
e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*
tan(e) + 1)/(tan(e)^2 + 1)) + 3*a^3*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*
x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(ta
n(e)^2 + 1)) - 9*a*b^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) +
 tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))
- 18*a*b^2*c*tan(f*x) - 18*a^2*b*d*tan(f*x) + 6*b^3*d*tan(f*x) - 18*a*b^2*c
*tan(e) - 18*a^2*b*d*tan(e) + 6*b^3*d*tan(e) - 3*b^3*c - 9*a*b^2*d)/(f*tan(
f*x)^3*tan(e)^3 - 3*f*tan(f*x)^2*tan(e)^2 + 3*f*tan(f*x)*tan(e) - f)
```

Mupad [B]
time = 5.27, size = 141, normalized size = 1.01

$$x\,\left(c\,a^3-3\,d\,a^2\,b-3\,c\,a\,b^2+d\,b^3\right)-\frac{\mathrm{tan}\left(e+f\,x\right)\,\left(b^3\,d-3\,a\,b\,\left(a\,d+b\,c\right)\right)}{f}+\frac{{\mathrm{tan}\left(e+f\,x\right)}^2\,\left(\frac{c\,b^3}{2}+\frac{3\,a\,d\,b^2}{2}\right)}{f}+\frac{\ln\left({\mathrm{tan}\left(e+f\,x\right)}^2+1\right)\,\left(\frac{d\,a^3}{2}+\frac{3\,c\,a^2\,b}{2}-\frac{3\,d\,a\,b^2}{2}-\frac{c\,b^3}{2}\right)}{f}+\frac{b^3\,d\,{\mathrm{tan}\left(e+f\,x\right)}^3}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^3*(c + d*tan(e + f*x)),x)
```

```
[Out] x*(a^3*c + b^3*d - 3*a*b^2*c - 3*a^2*b*d) - (tan(e + f*x)*(b^3*d - 3*a*b*(a
*d + b*c)))/f + (tan(e + f*x)^2*((b^3*c)/2 + (3*a*b^2*d)/2))/f + (log(tan(e
 + f*x)^2 + 1)*((a^3*d)/2 - (b^3*c)/2 + (3*a^2*b*c)/2 - (3*a*b^2*d)/2))/f +
 (b^3*d*tan(e + f*x)^3)/(3*f)
```

3.1192 $\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))\,dx$

Optimal. Leaf size=87

$$\left(a^2c-b^2c-2abd\right)x-\frac{(2abc+a^2d-b^2d)\log(\cos(e+fx))}{f}+\frac{b(bc+ad)\tan(e+fx)}{f}+\frac{d(a+b\tan(e+fx))^2}{2f}$$

[Out] (a^2*c-2*a*b*d-b^2*c)*x-(a^2*d+2*a*b*c-b^2*d)*ln(cos(f*x+e))/f+b*(a*d+b*c)*tan(f*x+e)/f+1/2*d*(a+b*tan(f*x+e))^2/f

Rubi [A]
time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3609, 3606, 3556}

$$-\frac{(a^2d+2abc-b^2d)\log(\cos(e+fx))}{f}+x\left(a^2c-2abd-b^2c\right)+\frac{b(ad+bc)\tan(e+fx)}{f}+\frac{d(a+b\tan(e+fx))^2}{2f}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x]),x]

[Out] (a^2*c - b^2*c - 2*a*b*d)*x - ((2*a*b*c + a^2*d - b^2*d)*Log[Cos[e + f*x]])/f + (b*(b*c + a*d)*Tan[e + f*x])/f + (d*(a + b*Tan[e + f*x])^2)/(2*f)

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))\,dx = \frac{d(a+b\tan(e+fx))^2}{2f} + \int (a+b\tan(e+fx))(ac-bd+($$
$$= \left(a^2c-b^2c-2abd\right)x + \frac{b(bc+ad)\tan(e+fx)}{f} + \frac{d(a+b\,\mathrm{ta}}{}$$
$$= \left(a^2c-b^2c-2abd\right)x - \frac{(2abc+a^2d-b^2d)\log(\cos(e+fx))}{f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.46, size = 96, normalized size = 1.10

$$\frac{(a+ib)^2(-ic+d)\log(i-\tan(e+fx))+(a-ib)^2(ic+d)\log(i+\tan(e+fx))+2b(bc+2ad)\tan(e+fx)+b^2d\tan^2(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x]),x]
```

```
[Out] ((a + I*b)^2*((-I)*c + d)*Log[I - Tan[e + f*x]] + (a - I*b)^2*(I*c + d)*Log
[I + Tan[e + f*x]] + 2*b*(b*c + 2*a*d)*Tan[e + f*x] + b^2*d*Tan[e + f*x]^2)
/(2*f)
```

Maple [A]
time = 0.08, size = 97, normalized size = 1.11

method	result
norman	$\left(a^2c-2abd-b^2c\right)x+\frac{b(2ad+bc)\tan(fx+e)}{f}+\frac{b^2d\left(\tan^2(fx+e)\right)}{2f}+\frac{\left(a^2d+2abc-b^2d\right)\ln\left(1+\tan^2(fx+e)\right)}{2f}$
derivativedivides	$\frac{\frac{b^2d\left(\tan^2(fx+e)\right)}{2}+2abd\tan(fx+e)+b^2c\tan(fx+e)+\frac{\left(a^2d+2abc-b^2d\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(a^2c-2abd-b^2c\right)\arctan(\tan(f}{f}$
default	$\frac{\frac{b^2d\left(\tan^2(fx+e)\right)}{2}+2abd\tan(fx+e)+b^2c\tan(fx+e)+\frac{\left(a^2d+2abc-b^2d\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(a^2c-2abd-b^2c\right)\arctan(\tan(f}{f}$
risch	$\frac{2ib\left(2ad\,e^{2i(fx+e)}+bc\,e^{2i(fx+e)}-ibd\,e^{2i(fx+e)}+2ad+bc\right)}{f\left(e^{2i(fx+e)}+1\right)^2}-\frac{2ib^2de}{f}+ia^2dx+a^2cx-2abdx-b^2cx+\frac{4i}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2*b^2*d*tan(f*x+e)^2+2*a*b*d*tan(f*x+e)+b^2*c*tan(f*x+e)+1/2*(a^2*d+
2*a*b*c-b^2*d)*ln(1+tan(f*x+e)^2)+(a^2*c-2*a*b*d-b^2*c)*arctan(tan(f*x+e)))
```

Maxima [A]
time = 0.66, size = 96, normalized size = 1.10

$$\frac{b^2d\tan\left(fx+e\right)^2-2\left(2\,abd-(a^2-b^2)c\right)(fx+e)+\left(2\,abc+(a^2-b^2)d\right)\log\left(\tan\left(fx+e\right)^2+1\right)+2\left(b^2c+2\,abd\right)\tan\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e)),x, algorithm="maxima")

[Out] 1/2*(b^2*d*tan(f*x + e)^2 - 2*(2*a*b*d - (a^2 - b^2)*c)*(f*x + e) + (2*a*b*c + (a^2 - b^2)*d)*log(tan(f*x + e)^2 + 1) + 2*(b^2*c + 2*a*b*d)*tan(f*x + e))/f

Fricas [A]
time = 1.17, size = 95, normalized size = 1.09

$$\frac{b^2d\tan(fx+e)^2 - 2\left(2\,abd - (a^2-b^2)c\right)fx - \left(2\,abc + (a^2-b^2)d\right)\log\left(\frac{1}{\tan(fx+e)^2+1}\right) + 2\left(b^2c + 2\,abd\right)\tan(fx+e)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e)),x, algorithm="fricas")

[Out] 1/2*(b^2*d*tan(f*x + e)^2 - 2*(2*a*b*d - (a^2 - b^2)*c)*f*x - (2*a*b*c + (a^2 - b^2)*d)*log(1/(tan(f*x + e)^2 + 1)) + 2*(b^2*c + 2*a*b*d)*tan(f*x + e))/f

Sympy [A]
time = 0.11, size = 143, normalized size = 1.64

$$\begin{cases} a^2cx + \frac{a^2d\log(\tan^2(e+fx)+1)}{2f} + \frac{abc\log(\tan^2(e+fx)+1)}{f} - 2abdx + \frac{2abd\tan(e+fx)}{f} - b^2cx + \frac{b^2c\tan(e+fx)}{f} - \frac{b^2d\log(\tan^2(e+fx)+1)}{2f} + \frac{b^2d\tan^2(e+fx)}{2f} & \text{for } f \neq 0 \\ x(a+b\tan(e))^2(c+d\tan(e)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(f*x+e))**2*(c+d*tan(f*x+e)),x)

[Out] Piecewise((a**2*c*x + a**2*d*log(tan(e + f*x)**2 + 1)/(2*f) + a*b*c*log(tan(e + f*x)**2 + 1)/f - 2*a*b*d*x + 2*a*b*d*tan(e + f*x)/f - b**2*c*x + b**2*c*tan(e + f*x)/f - b**2*d*log(tan(e + f*x)**2 + 1)/(2*f) + b**2*d*tan(e + f*x)**2/(2*f), Ne(f, 0)), (x*(a + b*tan(e))**2*(c + d*tan(e)), True))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 968 vs. 2(88) = 176.
time = 0.75, size = 968, normalized size = 11.13

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e)),x, algorithm="giac")

[Out] 1/2*(2*a^2*c*f*x*tan(f*x)^2*tan(e)^2 - 2*b^2*c*f*x*tan(f*x)^2*tan(e)^2 - 4*a*b*d*f*x*tan(f*x)^2*tan(e)^2 - 2*a*b*c*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(

```
tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 - a^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*t
an(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1
)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 + b^2*d*log(4*(tan(f*x)^4*tan(e)^2 -
2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e)
+ 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 - 4*a^2*c*f*x*tan(f*x)*tan(e) + 4*
b^2*c*f*x*tan(f*x)*tan(e) + 8*a*b*d*f*x*tan(f*x)*tan(e) + b^2*d*tan(f*x)^2*
tan(e)^2 + 4*a*b*c*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f
*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*
x)*tan(e) + 2*a^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(
f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f
*x)*tan(e) - 2*b^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan
(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(
f*x)*tan(e) - 2*b^2*c*tan(f*x)^2*tan(e) - 4*a*b*d*tan(f*x)^2*tan(e) - 2*b^2
*c*tan(f*x)*tan(e)^2 - 4*a*b*d*tan(f*x)*tan(e)^2 + 2*a^2*c*f*x - 2*b^2*c*f*
x - 4*a*b*d*f*x + b^2*d*tan(f*x)^2 + b^2*d*tan(e)^2 - 2*a*b*c*log(4*(tan(f*
x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*
tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1)) - a^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2
*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) +
 1)/(tan(e)^2 + 1)) + b^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e
) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1
)) + 2*b^2*c*tan(f*x) + 4*a*b*d*tan(f*x) + 2*b^2*c*tan(e) + 4*a*b*d*tan(e)
+ b^2*d)/(f*tan(f*x)^2*tan(e)^2 - 2*f*tan(f*x)*tan(e) + f)
```

Mupad [B]
time = 5.22, size = 91, normalized size = 1.05

$$\frac{\tan(e+f\,x)\,\left(c\,b^2+2\,a\,d\,b\right)}{f}-x\,\left(-c\,a^2+2\,d\,a\,b+c\,b^2\right)+\frac{\ln\left(\tan(e+f\,x)^2+1\right)\,\left(\frac{d\,a^2}{2}+c\,a\,b-\frac{d\,b^2}{2}\right)}{f}+\frac{b^2\,d\,\tan(e+f\,x)^2}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^2*(c + d*tan(e + f*x)),x)
```

```
[Out] (tan(e + f*x)*(b^2*c + 2*a*b*d))/f - x*(b^2*c - a^2*c + 2*a*b*d) + (log(tan
(e + f*x)^2 + 1)*((a^2*d)/2 - (b^2*d)/2 + a*b*c))/f + (b^2*d*tan(e + f*x)^2
)/(2*f)
```

3.1193 $\int (a+b\tan(e+fx))(c+d\tan(e+fx))\,dx$

Optimal. Leaf size=42

$$(ac-bd)x-\frac{(bc+ad)\log(\cos(e+fx))}{f}+\frac{bd\tan(e+fx)}{f}$$

```
[Out] (a*c-b*d)*x-(a*d+b*c)*ln(cos(f*x+e))/f+b*d*tan(f*x+e)/f
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {3606, 3556}

$$-\frac{(ad+bc)\log(\cos(e+fx))}{f}+x(ac-bd)+\frac{bd\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x]),x]
[Out] (a*c - b*d)*x - ((b*c + a*d)*Log[Cos[e + f*x]])/f + (b*d*Tan[e + f*x])/f
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rubi steps

$$\begin{aligned}\int (a+b\tan(e+fx))(c+d\tan(e+fx))\,dx &= (ac-bd)x+\frac{bd\tan(e+fx)}{f}+(bc+ad)\int \tan(e+fx)\,dx\\ &= (ac-bd)x-\frac{(bc+ad)\log(\cos(e+fx))}{f}+\frac{bd\tan(e+fx)}{f}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 59, normalized size = 1.40

$$acx-\frac{bd\text{ArcTan}(\tan(e+fx))}{f}-\frac{bc\log(\cos(e+fx))}{f}-\frac{ad\log(\cos(e+fx))}{f}+\frac{bd\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x]),x]
```

```
[Out] a*c*x - (b*d*ArcTan[Tan[e + f*x]])/f - (b*c*Log[Cos[e + f*x]])/f - (a*d*Log
[Cos[e + f*x]])/f + (b*d*Tan[e + f*x])/f
```

Maple [A]
time = 0.06, size = 51, normalized size = 1.21

method	result
norman	$(ac-bd)\,x+\frac{bd\tan(fx+e)}{f}+\frac{(ad+bc)\ln(1+\tan^2(fx+e))}{2f}$
derivativedivides	$\frac{\tan(fx+e)bd+\frac{(ad+bc)\ln\left(1+\tan^2(fx+e)\right)}{2}+(ac-bd)\arctan(\tan(fx+e))}{f}$
default	$\frac{\tan(fx+e)bd+\frac{(ad+bc)\ln\left(1+\tan^2(fx+e)\right)}{2}+(ac-bd)\arctan(\tan(fx+e))}{f}$
risch	$iadx+ibcx+acx-bdx+\frac{2iade}{f}+\frac{2ibce}{f}+\frac{2ibd}{f\left(e^{2i(fx+e)}+1\right)}-\frac{\ln\left(e^{2i(fx+e)}+1\right)ad}{f}-\frac{\ln\left(e^{2i(fx+e)}+1\right)}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))*(c+d*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(tan(f*x+e)*b*d+1/2*(a*d+b*c)*ln(1+tan(f*x+e)^2)+(a*c-b*d)*arctan(tan(f
*x+e)))
```

Maxima [A]
time = 0.58, size = 53, normalized size = 1.26

$$\frac{2\,bd\tan\left(fx+e\right)+2\,(ac-bd)(fx+e)+(bc+ad)\log\left(\tan\left(fx+e\right)^2+1\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*d*tan(f*x + e) + 2*(a*c - b*d)*(f*x + e) + (b*c + a*d)*log(tan(f*x
 + e)^2 + 1))/f
```

Fricas [A]
time = 0.95, size = 52, normalized size = 1.24

$$\frac{2\,(ac-bd)fx+2\,bd\tan\left(fx+e\right)-(bc+ad)\log\left(\frac{1}{\tan(fx+e)^2+1}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a*c - b*d)*f*x + 2*b*d*tan(f*x + e) - (b*c + a*d)*log(1/(tan(f*x +
e)^2 + 1)))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(36) = 72$.
time = 0.10, size = 73, normalized size = 1.74

$$\begin{cases} acx + \frac{ad\log\left(\tan^2\left(e+fx\right)+1\right)}{2f} + \frac{bc\log\left(\tan^2\left(e+fx\right)+1\right)}{2f} - bdx + \frac{bd\tan\left(e+fx\right)}{f} & \text{for } f \neq 0 \\ x(a + b\tan\left(e\right))\left(c + d\tan\left(e\right)\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e)),x)
```

```
[Out] Piecewise((a*c*x + a*d*log(tan(e + f*x)**2 + 1)/(2*f) + b*c*log(tan(e + f*x
)**2 + 1)/(2*f) - b*d*x + b*d*tan(e + f*x)/f, Ne(f, 0)), (x*(a + b*tan(e))*
(c + d*tan(e)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(44) = 88$.
time = 0.48, size = 355, normalized size = 8.45

$$\frac{2\,acfx\tan\left(fx\right)\tan\left(e\right) - 2\,bdfx\tan\left(fx\right)\tan\left(e\right) - bc\log\left(\frac{4\left(\tan(fx)^4\tan(e)^2 - 2\tan(fx)^3\tan(e) + \tan(fx)^2\tan(e)^2 + \tan(fx)^2 - 2\tan(fx)\tan(e) + 1\right)}{\tan(e)^2 + 1}\right)\tan\left(fx\right)\tan\left(e\right) - ad\log\left(\frac{4\left(\tan(fx)^4\tan(e)^2 - 2\tan(fx)^3\tan(e) + \tan(fx)^2\tan(e)^2 + \tan(fx)^2 - 2\tan(fx)\tan(e) + 1\right)}{\tan(e)^2 + 1}\right)\tan\left(fx\right)\tan\left(e\right) - 2\,acfx + 2\,bdfx + bc\log\left(\frac{4\left(\tan(fx)^4\tan(e)^2 - 2\tan(fx)^3\tan(e) + \tan(fx)^2\tan(e)^2 + \tan(fx)^2 - 2\tan(fx)\tan(e) + 1\right)}{\tan(e)^2 + 1}\right) + ad\log\left(\frac{4\left(\tan(fx)^4\tan(e)^2 - 2\tan(fx)^3\tan(e) + \tan(fx)^2\tan(e)^2 + \tan(fx)^2 - 2\tan(fx)\tan(e) + 1\right)}{\tan(e)^2 + 1}\right) - 2\,bd\tan\left(fx\right) - 2\,bd\tan\left(e\right)}{2\left(f\tan\left(fx\right)\tan\left(e\right) - f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*(2*a*c*f*x*tan(f*x)*tan(e) - 2*b*d*f*x*tan(f*x)*tan(e) - b*c*log(4*(tan
(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 -
 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)*tan(e) - a*d*log(4*(tan(f*
x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*
tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)*tan(e) - 2*a*c*f*x + 2*b*d*f*
x + b*c*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e
)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1)) + a*d*log(4*(tan(
f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 -
2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1)) - 2*b*d*tan(f*x) - 2*b*d*tan(e))/(f*
tan(f*x)*tan(e) - f)
```

Mupad [B]
time = 5.17, size = 55, normalized size = 1.31

$$\frac{b\,d\tan(e + f\,x) + \frac{a\,d\,\ln\left(\tan(e+f\,x)^2+1\right)}{2} + \frac{b\,c\,\ln\left(\tan(e+f\,x)^2+1\right)}{2} + a\,c\,f\,x - b\,d\,f\,x}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))*(c + d*tan(e + f*x)),x)
```

```
[Out] (b*d*tan(e + f*x) + (a*d*log(tan(e + f*x)^2 + 1))/2 + (b*c*log(tan(e + f*x)
^2 + 1))/2 + a*c*f*x - b*d*f*x)/f
```

3.1194 $\int \frac{c+d\tan(e+fx)}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=58

$$\frac{(ac+bd)x}{a^2+b^2}+\frac{(bc-ad)\log(a\cos(e+fx)+b\sin(e+fx))}{(a^2+b^2)\,f}$$

```
[Out] (a*c+b*d)*x/(a^2+b^2)+(-a*d+b*c)*ln(a*cos(f*x+e)+b*sin(f*x+e))/(a^2+b^2)/f
```

Rubi [A]
time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3612, 3611}

$$\frac{(bc-ad)\log(a\cos(e+fx)+b\sin(e+fx))}{f\,(a^2+b^2)}+\frac{x(ac+bd)}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])/(a + b*Tan[e + f*x]),x]
```

```
[Out] ((a*c + b*d)*x)/(a^2 + b^2) + ((b*c - a*d)*Log[a*Cos[e + f*x] + b*Sin[e + f
*x]])/((a^2 + b^2)*f)
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{c+d\tan(e+fx)}{a+b\tan(e+fx)}\,dx &= \frac{(ac+bd)x}{a^2+b^2}+\frac{(bc-ad)\int \frac{b-a\tan(e+fx)}{a+b\tan(e+fx)}\,dx}{a^2+b^2}\\ &= \frac{(ac+bd)x}{a^2+b^2}+\frac{(bc-ad)\log(a\cos(e+fx)+b\sin(e+fx))}{(a^2+b^2)\,f}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 66, normalized size = 1.14

$$\frac{2(ac+bd)\text{ArcTan}(\tan(e+fx))-(bc-ad)\left(\log\left(\sec^2(e+fx)\right)-2\log(a+b\tan(e+fx))\right)}{2\left(a^2+b^2\right)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])/(a + b*Tan[e + f*x]),x]
```

```
[Out] (2*(a*c + b*d)*ArcTan[Tan[e + f*x]] - (b*c - a*d)*(Log[Sec[e + f*x]^2] - 2*
Log[a + b*Tan[e + f*x]]))/(2*(a^2 + b^2)*f)
```

Maple [A]
time = 0.17, size = 83, normalized size = 1.43

method	result
derivativedivides	$\frac{-\frac{(ad-bc)\ln(a+b\tan(fx+e))}{a^2+b^2}+\frac{\frac{(ad-bc)\ln\left(1+\tan^2(fx+e)\right)}{2}+(ac+bd)\arctan(\tan(fx+e))}{a^2+b^2}}{f}$
default	$\frac{-\frac{(ad-bc)\ln(a+b\tan(fx+e))}{a^2+b^2}+\frac{\frac{(ad-bc)\ln\left(1+\tan^2(fx+e)\right)}{2}+(ac+bd)\arctan(\tan(fx+e))}{a^2+b^2}}{f}$
norman	$\frac{(ac+bd)x}{a^2+b^2}+\frac{(ad-bc)\ln\left(1+\tan^2(fx+e)\right)}{2f(a^2+b^2)}-\frac{(ad-bc)\ln(a+b\tan(fx+e))}{f(a^2+b^2)}$
risch	$\frac{ixd}{ib-a}-\frac{xc}{ib-a}+\frac{2iadx}{a^2+b^2}-\frac{2ibcx}{a^2+b^2}+\frac{2iade}{f(a^2+b^2)}-\frac{2ibce}{f(a^2+b^2)}-\frac{\ln\left(e^{2i(fx+e)}-\frac{ib+a}{ib-a}\right)ad}{f(a^2+b^2)}+\frac{\ln\left(e^{2i(fx+e)}-\frac{ib+a}{ib-a}\right)bc}{f(a^2+b^2)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-(a*d-b*c)/(a^2+b^2)*ln(a+b*tan(f*x+e))+1/(a^2+b^2)*(1/2*(a*d-b*c)*ln(
1+tan(f*x+e)^2)+(a*c+b*d)*arctan(tan(f*x+e))))
```

Maxima [A]
time = 0.53, size = 92, normalized size = 1.59

$$\frac{\frac{2\,(ac+bd)(fx+e)}{a^2+b^2}+\frac{2\,(bc-ad)\log(b\tan(fx+e)+a)}{a^2+b^2}-\frac{(bc-ad)\log\left(\tan(fx+e)^2+1\right)}{a^2+b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a*c + b*d)*(f*x + e)/(a^2 + b^2) + 2*(b*c - a*d)*log(b*tan(f*x + e)
 + a)/(a^2 + b^2) - (b*c - a*d)*log(tan(f*x + e)^2 + 1)/(a^2 + b^2))/f
```

Fricas [A]
time = 1.12, size = 78, normalized size = 1.34

$$\frac{2\,(ac+bd)fx+(bc-ad)\log\left(\frac{b^2\tan(fx+e)^2+2\,ab\tan(fx+e)+a^2}{\tan(fx+e)^2+1}\right)}{2\,(a^2+b^2)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a*c + b*d)*f*x + (b*c - a*d)*log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*
x + e) + a^2)/(tan(f*x + e)^2 + 1)))/((a^2 + b^2)*f)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.47, size = 524, normalized size = 9.03

$$\begin{cases} \frac{\tilde{\infty}x(c+d\tan(e))}{\tan(e)} & \text{for } a=0\wedge b=0\wedge f=0\\ \frac{cx+\frac{d\log\left(\tan^2(e+fx)+1\right)}{2f}}{a} & \text{for } b=0\\ \frac{icfx\tan(e+fx)}{2bf\tan(e+fx)-2ibf}+\frac{cfx}{2bf\tan(e+fx)-2ibf}+\frac{ic}{2bf\tan(e+fx)-2ibf}+\frac{dfx\tan(e+fx)}{2bf\tan(e+fx)-2ibf}-\frac{idfx}{2bf\tan(e+fx)-2ibf}-\frac{d}{2bf\tan(e+fx)-2ibf} & \text{for } a=-ib\\ -\frac{icfx\tan(e+fx)}{2bf\tan(e+fx)+2ibf}+\frac{cfx}{2bf\tan(e+fx)+2ibf}-\frac{ic}{2bf\tan(e+fx)+2ibf}+\frac{dfx\tan(e+fx)}{2bf\tan(e+fx)+2ibf}+\frac{idfx}{2bf\tan(e+fx)+2ibf}-\frac{d}{2bf\tan(e+fx)+2ibf} & \text{for } a=ib\\ \frac{x(c+d\tan(e))}{a+b\tan(e)} & \text{for } f=0\\ \frac{2acfx}{2a^2f+2b^2f}-\frac{2ad\log\left(\frac{a}{b}+\tan(e+fx)\right)}{2a^2f+2b^2f}+\frac{ad\log\left(\tan^2(e+fx)+1\right)}{2a^2f+2b^2f}+\frac{2bc\log\left(\frac{a}{b}+\tan(e+fx)\right)}{2a^2f+2b^2f}-\frac{bc\log\left(\tan^2(e+fx)+1\right)}{2a^2f+2b^2f}+\frac{2bdfx}{2a^2f+2b^2f} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))/(a+b*tan(f*x+e)),x)
```

```
[Out] Piecewise((zoo*x*(c + d*tan(e))/tan(e), Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((
c*x + d*log(tan(e + f*x)**2 + 1)/(2*f))/a, Eq(b, 0)), (I*c*f*x*tan(e + f*x)
/(2*b*f*tan(e + f*x) - 2*I*b*f) + c*f*x/(2*b*f*tan(e + f*x) - 2*I*b*f) + I*
c/(2*b*f*tan(e + f*x) - 2*I*b*f) + d*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) -
 2*I*b*f) - I*d*f*x/(2*b*f*tan(e + f*x) - 2*I*b*f) - d/(2*b*f*tan(e + f*x)
- 2*I*b*f), Eq(a, -I*b)), (-I*c*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) + 2*I*
b*f) + c*f*x/(2*b*f*tan(e + f*x) + 2*I*b*f) - I*c/(2*b*f*tan(e + f*x) + 2*I
*b*f) + d*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) + 2*I*b*f) + I*d*f*x/(2*b*f*
tan(e + f*x) + 2*I*b*f) - d/(2*b*f*tan(e + f*x) + 2*I*b*f), Eq(a, I*b)), (x
*(c + d*tan(e))/(a + b*tan(e)), Eq(f, 0)), (2*a*c*f*x/(2*a**2*f + 2*b**2*f)
 - 2*a*d*log(a/b + tan(e + f*x))/(2*a**2*f + 2*b**2*f) + a*d*log(tan(e + f*
x)**2 + 1)/(2*a**2*f + 2*b**2*f) + 2*b*c*log(a/b + tan(e + f*x))/(2*a**2*f
+ 2*b**2*f) - b*c*log(tan(e + f*x)**2 + 1)/(2*a**2*f + 2*b**2*f) + 2*b*d*f*
x/(2*a**2*f + 2*b**2*f), True))
```

Giac [A]
time = 0.48, size = 98, normalized size = 1.69

$$\frac{\frac{2\,(ac+bd)(fx+e)}{a^2+b^2}-\frac{(bc-ad)\log\left(\tan(fx+e)^2+1\right)}{a^2+b^2}+\frac{2\,(b^2c-abd)\log(|b\tan(fx+e)+a|)}{a^2b+b^3}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))/(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*(2*(a*c + b*d)*(f*x + e)/(a^2 + b^2) - (b*c - a*d)*log(tan(f*x + e)^2 +
 1)/(a^2 + b^2) + 2*(b^2*c - a*b*d)*log(abs(b*tan(f*x + e) + a))/(a^2*b + b
^3))/f
```

Mupad [B]
time = 5.68, size = 94, normalized size = 1.62

$$-\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(-d+c\,1\mathrm{i}\right)}{2\,f\,\left(a+b\,1\mathrm{i}\right)}-\frac{\ln\left(a+b\tan(e+f\,x)\right)\,\left(a\,d-b\,c\right)}{f\,\left(a^2+b^2\right)}-\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(c-d\,1\mathrm{i}\right)}{2\,f\,\left(b+a\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))/(a + b*tan(e + f*x)),x)
```

```
[Out] - (log(tan(e + f*x) - 1i)*(c*1i - d))/(2*f*(a + b*1i)) - (log(a + b*tan(e +
 f*x))*(a*d - b*c))/(f*(a^2 + b^2)) - (log(tan(e + f*x) + 1i)*(c - d*1i))/(
2*f*(a*1i + b))
```

3.1195 $\int \frac{c+d\tan(e+fx)}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=111

$$\frac{(a^2c-b^2c+2abd)\,x}{(a^2+b^2)^2}+\frac{(2abc-a^2d+b^2d)\log(a\cos(e+fx)+b\sin(e+fx))}{(a^2+b^2)^2\,f}-\frac{bc-ad}{(a^2+b^2)\,f(a+b\tan(e+fx)}$$

[Out] (a^2*c+2*a*b*d-b^2*c)*x/(a^2+b^2)^2+(-a^2*d+2*a*b*c+b^2*d)*ln(a*cos(f*x+e)+b*sin(f*x+e))/(a^2+b^2)^2/f+(a*d-b*c)/(a^2+b^2)/f/(a+b*tan(f*x+e))

Rubi [A]
time = 0.10, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3610, 3612, 3611}

$$-\frac{bc-ad}{f\,(a^2+b^2)\,(a+b\tan(e+fx))}+\frac{(a^2(-d)+2abc+b^2d)\log(a\cos(e+fx)+b\sin(e+fx))}{f\,(a^2+b^2)^2}+\frac{x(a^2c+2abd-b^2c)}{(a^2+b^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d*Tan[e + f*x])/(a + b*Tan[e + f*x])^2,x]

[Out] ((a^2*c - b^2*c + 2*a*b*d)*x)/(a^2 + b^2)^2 + ((2*a*b*c - a^2*d + b^2*d)*Log[a*Cos[e + f*x] + b*Sin[e + f*x]])/((a^2 + b^2)^2*f) - (b*c - a*d)/((a^2 + b^2)*f*(a + b*Tan[e + f*x]))

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
```

```
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{c+d\tan(e+fx)}{(a+b\tan(e+fx))^2}\,dx = -\frac{bc-ad}{(a^2+b^2)\,f(a+b\tan(e+fx))} + \frac{\int \frac{ac+bd-(bc-ad)\tan(e+fx)}{a+b\tan(e+fx)}\,dx}{a^2+b^2}$$

$$= \frac{(a^2c-b^2c+2abd)\,x}{(a^2+b^2)^2} - \frac{bc-ad}{(a^2+b^2)\,f(a+b\tan(e+fx))} + \frac{(2abc-a^2d+b^2d)\int}{(a^2+b^2)}$$

$$= \frac{(a^2c-b^2c+2abd)\,x}{(a^2+b^2)^2} + \frac{(2abc-a^2d+b^2d)\log(a\cos(e+fx)+b\sin(e+fx))}{(a^2+b^2)^2\,f} -$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.29, size = 190, normalized size = 1.71

$$\frac{\frac{d((-ia-b)\log(i-\tan(e+fx))+i(a+ib)\log(i+\tan(e+fx))+2b\log(a+b\tan(e+fx)))}{a^2+b^2}-(bc-ad)\left(\frac{i\log(i-\tan(e+fx))}{(a+ib)^2}-\frac{i\log(i+\tan(e+fx))}{(a-ib)^2}+\frac{2b\left(-2a\log(a+b\tan(e+fx))+\frac{a^2+b^2}{a+b\tan(e+fx)}\right)}{(a^2+b^2)^2}\right)}{2bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] ((d*(((-I)*a - b)*Log[I - Tan[e + f*x]] + I*(a + I*b)*Log[I + Tan[e + f*x]]
 + 2*b*Log[a + b*Tan[e + f*x]]))/(a^2 + b^2) - (b*c - a*d)*((I*Log[I - Tan[
e + f*x]])/(a + I*b)^2 - (I*Log[I + Tan[e + f*x]])/(a - I*b)^2 + (2*b*(-2*a
*Log[a + b*Tan[e + f*x]] + (a^2 + b^2)/(a + b*Tan[e + f*x])))/(a^2 + b^2)^2
))/(2*b*f)
```

Maple [A]
time = 0.18, size = 141, normalized size = 1.27

method	result
derivativedivides	$\frac{\frac{ad-bc}{\left(a^2+b^2\right)(a+b\tan(fx+e))}-\frac{\left(a^2d-2abc-b^2d\right)\ln(a+b\tan(fx+e))}{\left(a^2+b^2\right)^2}+\frac{\frac{\left(a^2d-2abc-b^2d\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(a^2c+2abd-b^2c\right)\arctan(\mathrm{ta}}{\left(a^2+b^2\right)^2}}{f}$
default	$\frac{\frac{ad-bc}{\left(a^2+b^2\right)(a+b\tan(fx+e))}-\frac{\left(a^2d-2abc-b^2d\right)\ln(a+b\tan(fx+e))}{\left(a^2+b^2\right)^2}+\frac{\frac{\left(a^2d-2abc-b^2d\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(a^2c+2abd-b^2c\right)\arctan(\mathrm{ta}}{\left(a^2+b^2\right)^2}}{f}$
norman	$\frac{\frac{a\left(a^2c+2abd-b^2c\right)x}{a^4+2a^2b^2+b^4}+\frac{b\left(a^2c+2abd-b^2c\right)x\tan(fx+e)}{a^4+2a^2b^2+b^4}+\frac{(-ad+bc)b\tan(fx+e)}{af\left(a^2+b^2\right)}}{a+b\tan(fx+e)}+\frac{(a^2d-2abc-b^2d)\ln(1+\tan^2(fx+e))}{2f(a^4+2a^2b^2+b^4)}-\frac{(a^2}{}$

risch	$\frac{ixd}{2iab-a^2+b^2} - \frac{xc}{2iab-a^2+b^2} + \frac{2ia^2dx}{a^4+2a^2b^2+b^4} - \frac{4iabcx}{a^4+2a^2b^2+b^4} - \frac{2ib^2dx}{a^4+2a^2b^2+b^4} + \frac{2ia^2de}{f(a^4+2a^2b^2+b^4)} - \frac{4}{f(a^4+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*((a*d-b*c)/(a^2+b^2)/(a+b*tan(f*x+e))-(a^2*d-2*a*b*c-b^2*d)/(a^2+b^2)^2
*ln(a+b*tan(f*x+e))+1/(a^2+b^2)^2*(1/2*(a^2*d-2*a*b*c-b^2*d)*ln(1+tan(f*x+e
)^2)+(a^2*c+2*a*b*d-b^2*c)*arctan(tan(f*x+e))))
```

Maxima [A]
time = 0.53, size = 184, normalized size = 1.66

$$\frac{\frac{2\,(2\,abd+(a^2-b^2)c)(fx+e)}{a^4+2\,a^2b^2+b^4} + \frac{2\,(2\,abc-(a^2-b^2)d)\log(b\tan(fx+e)+a)}{a^4+2\,a^2b^2+b^4} - \frac{(2\,abc-(a^2-b^2)d)\log\left(\tan(fx+e)^2+1\right)}{a^4+2\,a^2b^2+b^4} - \frac{2\,(bc-ad)}{a^3+ab^2+(a^2b+b^3)\tan(fx+e)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(2*a*b*d + (a^2 - b^2)*c)*(f*x + e)/(a^4 + 2*a^2*b^2 + b^4) + 2*(2*a
*b*c - (a^2 - b^2)*d)*log(b*tan(f*x + e) + a)/(a^4 + 2*a^2*b^2 + b^4) - (2*
a*b*c - (a^2 - b^2)*d)*log(tan(f*x + e)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*
(b*c - a*d)/(a^3 + a*b^2 + (a^2*b + b^3)*tan(f*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(114) = 228$.
time = 1.45, size = 231, normalized size = 2.08

$$-\frac{2\,b^3c - 2\,ab^2d - 2\,(2\,a^2bd + (a^3 - ab^2)c)fx - (2\,a^2bc - (a^3 - ab^2)d + (2\,ab^2c - (a^2b - b^3)d)\tan(fx+e))\log\left(\frac{b^2\tan(fx+e)^2 + 2\,ab\tan(fx+e) + a^2}{\tan(fx+e)^2+1}\right) - 2\,(ab^2c - a^2bd + (2\,ab^2d + (a^2b - b^3)c)fx)\tan(fx+e)}{2\,((a^4b + 2\,a^2b^3 + b^5)f\tan(fx+e) + (a^5 + 2\,a^3b^2 + ab^4)f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*c - 2*a*b^2*d - 2*(2*a^2*b*d + (a^3 - a*b^2)*c)*f*x - (2*a^2*b*
c - (a^3 - a*b^2)*d + (2*a*b^2*c - (a^2*b - b^3)*d)*tan(f*x + e))*log((b^2*
tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(tan(f*x + e)^2 + 1)) - 2*(a*b^2
*c - a^2*b*d + (2*a*b^2*d + (a^2*b - b^3)*c)*f*x)*tan(f*x + e))/((a^4*b + 2
*a^2*b^3 + b^5)*f*tan(f*x + e) + (a^5 + 2*a^3*b^2 + a*b^4)*f)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.87, size = 2878, normalized size = 25.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))/(a+b*tan(f*x+e))**2,x)

[Out] Piecewise((zoo*x*(c + d*tan(e))/tan(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)),
 ((c*x + d*log(tan(e + f*x)**2 + 1)/(2*f))/a**2, Eq(b, 0)), (-c*f*x*tan(e +
 f*x)**2/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) +
2*I*c*f*x*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x)
- 4*b**2*f) + c*f*x/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4
*b**2*f) - c*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*
x) - 4*b**2*f) + 2*I*c/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x)
- 4*b**2*f) + I*d*f*x*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*
f*tan(e + f*x) - 4*b**2*f) + 2*d*f*x*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2
 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - I*d*f*x/(4*b**2*f*tan(e + f*x)**2
- 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + I*d*tan(e + f*x)/(4*b**2*f*tan(e +
f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f), Eq(a, -I*b)), (-c*f*x*tan(e
+ f*x)**2/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) -
 2*I*c*f*x*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x)
 - 4*b**2*f) + c*f*x/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x) -
4*b**2*f) - c*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e + f
*x) - 4*b**2*f) - 2*I*c/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x)
 - 4*b**2*f) - I*d*f*x*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2
*f*tan(e + f*x) - 4*b**2*f) + 2*d*f*x*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**
2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + I*d*f*x/(4*b**2*f*tan(e + f*x)**2
 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - I*d*tan(e + f*x)/(4*b**2*f*tan(e +
 f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f), Eq(a, I*b)), (x*(c + d*tan(
e))/(a + b*tan(e))**2, Eq(f, 0)), (2*a**3*c*f*x/(2*a**5*f + 2*a**4*b*f*tan(
e + f*x) + 4*a**3*b**2*f + 4*a**2*b**3*f*tan(e + f*x) + 2*a*b**4*f + 2*b**5
*f*tan(e + f*x)) - 2*a**3*d*log(a/b + tan(e + f*x))/(2*a**5*f + 2*a**4*b*f*
tan(e + f*x) + 4*a**3*b**2*f + 4*a**2*b**3*f*tan(e + f*x) + 2*a*b**4*f + 2*
b**5*f*tan(e + f*x)) + a**3*d*log(tan(e + f*x)**2 + 1)/(2*a**5*f + 2*a**4*b
*f*tan(e + f*x) + 4*a**3*b**2*f + 4*a**2*b**3*f*tan(e + f*x) + 2*a*b**4*f +
 2*b**5*f*tan(e + f*x)) + 2*a**3*d/(2*a**5*f + 2*a**4*b*f*tan(e + f*x) + 4*
a**3*b**2*f + 4*a**2*b**3*f*tan(e + f*x) + 2*a*b**4*f + 2*b**5*f*tan(e + f*
x)) + 2*a**2*b*c*f*x*tan(e + f*x)/(2*a**5*f + 2*a**4*b*f*tan(e + f*x) + 4*a
**3*b**2*f + 4*a**2*b**3*f*tan(e + f*x) + 2*a*b**4*f + 2*b**5*f*tan(e + f*x
)) + 4*a**2*b*c*log(a/b + tan(e + f*x))/(2*a**5*f + 2*a**4*b*f*tan(e + f*x)
 + 4*a**3*b**2*f + 4*a**2*b**3*f*tan(e + f*x) + 2*a*b**4*f + 2*b**5*f*tan(e
 + f*x)) - 2*a**2*b*c*log(tan(e + f*x)**2 + 1)/(2*a**5*f + 2*a**4*b*f*tan(e
 + f*x) + 4*a**3*b**2*f + 4*a**2*b**3*f*tan(e + f*x) + 2*a*b**4*f + 2*b**5*
f*tan(e + f*x)) - 2*a**2*b*c/(2*a**5*f + 2*a**4*b*f*tan(e + f*x) + 4*a**3*b
**2*f + 4*a**2*b**3*f*tan(e + f*x) + 2*a*b**4*f + 2*b**5*f*tan(e + f*x)) +
4*a**2*b*d*f*x/(2*a**5*f + 2*a**4*b*f*tan(e + f*x) + 4*a**3*b**2*f + 4*a**2
*b**3*f*tan(e + f*x) + 2*a*b**4*f + 2*b**5*f*tan(e + f*x)) - 2*a**2*b*d*log
(a/b + tan(e + f*x))*tan(e + f*x)/(2*a**5*f + 2*a**4*b*f*tan(e + f*x) + 4*a
**3*b**2*f + 4*a**2*b**3*f*tan(e + f*x) + 2*a*b**4*f + 2*b**5*f*tan(e + f*x
)) + a**2*b*d*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*a**5*f + 2*a**4*b*f*
```

```
tan(e + f*x) + 4*a**3*b**2*f + 4*a**2*b**3*f*tan(e + f*x) + 2*a*b**4*f + 2*
b**5*f*tan(e + f*x)) - 2*a*b**2*c*f*x/(2*a**5*f + 2*a**4*b*f*tan(e + f*x) +
 4*a**3*b**2*f + 4*a**2*b**3*f*tan(e + f*x) + 2*a*b**4*f + 2*b**5*f*tan(e +
 f*x)) + 4*a*b**2*c*log(a/b + tan(e + f*x))*tan(e + f*x)/(2*a**5*f + 2*a**4
*b*f*tan(e + f*x) + 4*a**3*b**2*f + 4*a**2*b**3*f*tan(e + f*x) + 2*a*b**4*f
 + 2*b**5*f*tan(e + f*x)) - 2*a*b**2*c*log(tan(e + f*x)**2 + 1)*tan(e + f*x
)/(2*a**5*f + 2*a**4*b*f*tan(e + f*x) + 4*a**3*b**2*f + 4*a**2*b**3*f*tan(e
 + f*x) + 2*a*b**4*f + 2*b**5*f*tan(e + f*x)) + 4*a*b**2*d*f*x*tan(e + f*x)
/(2*a**5*f + 2*a**4*b*f*tan(e + f*x) + 4*a**3*b**2*f + 4*a**2*b**3*f*tan(e
+ f*x) + 2*a*b**4*f + 2*b**5*f*tan(e + f*x)) + 2*a*b**2*d*log(a/b + tan(e +
 f*x))/(2*a**5*f + 2*a**4*b*f*tan(e + f*x) + 4*a**3*b**2*f + 4*a**2*b**3*f*
tan(e + f*x) + 2*a*b**4*f + 2*b**5*f*tan(e + f*x)) - a*b**2*d*log(tan(e + f
*x)**2 + 1)/(2*a**5*f + 2*a**4*b*f*tan(e + f*x) + 4*a**3*b**2*f + 4*a**2*b*
*3*f*tan(e + f*x) + 2*a*b**4*f + 2*b**5*f*tan(e + f*x)) + 2*a*b**2*d/(2*a**
5*f + 2*a**4*b*f*tan(e + f*x) + 4*a**3*b**2*f + 4*a**2*b**3*f*tan(e + f*x)
+ 2*a*b**4*f + 2*b**5*f*tan(e + f*x)) - 2*b**3*c*f*x*tan(e + f*x)/(2*a**5*f
 + 2*a**4*b*f*tan(e + f*x) + 4*a**3*b**2*f + 4*a**2*b**3*f*tan(e + f*x) + 2
*a*b**4*f + 2*b**5*f*tan(e + f*x)) - 2*b**3*c/(2*a**5*f + 2*a**4*b*f*tan(e
+ f*x) + 4*a**3*b**2*f + 4*a**2*b**3*f*tan(e + f*x) + 2*a*b**4*f + 2*b**5*f
*tan(e + f*x)) + 2*b**3*d*log(a/b + tan(e + f*x))*tan(e + f*x)/(2*a**5*f +
2*a**4*b*f*tan(e + f*x) + 4*a**3*b**2*f + 4*a**2*b**3*f*tan(e + f*x) + 2*a*
b**4*f + 2*b**5*f*tan(e + f*x)) - b**3*d*log(tan(e + f*x)**2 + 1)*tan(e + f
*x)/(2*a**5*f + 2*a**4*b*f*tan(e + f*x) + 4*a**3*b**2*f + 4*a**2*b**3*f*tan
(e + f*x) + 2*a*b**4*f + 2*b**5*f*tan(e + f*x))...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(114) = 228$.

time = 0.54, size = 241, normalized size = 2.17

$$\frac{\frac{2\,(a^2c-b^2c+2\,abd)(fx+e)}{a^4+2\,a^2b^2+b^4}-\frac{(2\,abc-a^2d+b^2d)\log\left(\tan(fx+e)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\,(2\,ab^2c-a^2bd+b^3d)\log(|b\tan(fx+e)+a|)}{a^4b+2\,a^2b^3+b^5}-\frac{2\,(2\,ab^2c\tan(fx+e)-a^2bd\tan(fx+e)+b^3d\tan(fx+e)+3\,a^2bc+b^3c-2\,a^3d)}{(a^4+2\,a^2b^2+b^4)(b\tan(fx+e)+a)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(a^2*c - b^2*c + 2*a*b*d)*(f*x + e)/(a^4 + 2*a^2*b^2 + b^4) - (2*a*b
*c - a^2*d + b^2*d)*log(tan(f*x + e)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 2*(2*
a*b^2*c - a^2*b*d + b^3*d)*log(abs(b*tan(f*x + e) + a))/(a^4*b + 2*a^2*b^3
+ b^5) - 2*(2*a*b^2*c*tan(f*x + e) - a^2*b*d*tan(f*x + e) + b^3*d*tan(f*x +
 e) + 3*a^2*b*c + b^3*c - 2*a^3*d)/((a^4 + 2*a^2*b^2 + b^4)*(b*tan(f*x + e)
 + a)))/f
```

Mupad [B]

time = 5.48, size = 152, normalized size = 1.37

$$\frac{a\,d-b\,c}{f\,\left(a^2+b^2\right)\,\left(a+b\,\tan\left(e+f\,x\right)\right)}-\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(c+d\,1\mathrm{i}\right)}{2\,f\,\left(-a^2\,1\mathrm{i}+2\,a\,b+b^2\,1\mathrm{i}\right)}-\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(d+c\,1\mathrm{i}\right)}{2\,f\,\left(-a^2+a\,b\,2\mathrm{i}+b^2\right)}+\frac{\ln\left(a+b\,\tan(e+f\,x)\right)\,\left(-d\,a^2+2\,c\,a\,b+d\,b^2\right)}{f\,{\left(a^2+b^2\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))/(a + b*tan(e + f*x))^2,x)
```

```
[Out] (a*d - b*c)/(f*(a^2 + b^2)*(a + b*tan(e + f*x))) - (log(tan(e + f*x) - 1i)*
(c + d*1i))/(2*f*(2*a*b - a^2*1i + b^2*1i)) - (log(tan(e + f*x) + 1i)*(c*1i
 + d))/(2*f*(a*b*2i - a^2 + b^2)) + (log(a + b*tan(e + f*x))*(b^2*d - a^2*d
 + 2*a*b*c))/(f*(a^2 + b^2)^2)
```

3.1196 $\int \frac{c+d\tan(e+fx)}{(a+b\tan(e+fx))^3}\,dx$

Optimal. Leaf size=175

$$\frac{(a^3c-3ab^2c+3a^2bd-b^3d)\,x}{(a^2+b^2)^3}+\frac{(3a^2bc-b^3c-a^3d+3ab^2d)\log(a\cos(e+fx)+b\sin(e+fx))}{(a^2+b^2)^3\,f}-\frac{}{2\,(a^2+b^2)}$$

```
[Out] (a^3*c+3*a^2*b*d-3*a*b^2*c-b^3*d)*x/(a^2+b^2)^3+(-a^3*d+3*a^2*b*c+3*a*b^2*d
-b^3*c)*ln(a*cos(f*x+e)+b*sin(f*x+e))/(a^2+b^2)^3/f+1/2*(a*d-b*c)/(a^2+b^2)
/f/(a+b*tan(f*x+e))^2+(a^2*d-2*a*b*c-b^2*d)/(a^2+b^2)^2/f/(a+b*tan(f*x+e))
```

Rubi [A]

time = 0.18, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3610, 3612, 3611}

$$-\frac{bc-ad}{2f\,(a^2+b^2)\,(a+b\tan(e+fx))^2}-\frac{a^2(-d)+2abc+b^2d}{f\,(a^2+b^2)^2\,(a+b\tan(e+fx))}+\frac{(a^3(-d)+3a^2bc+3ab^2d-b^3c)\log(a\cos(e+fx)+b\sin(e+fx))}{f\,(a^2+b^2)^3}+\frac{x(a^3c+3a^2bd-3ab^2c-b^3d)}{(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] ((a^3*c - 3*a*b^2*c + 3*a^2*b*d - b^3*d)*x)/(a^2 + b^2)^3 + ((3*a^2*b*c - b
^3*c - a^3*d + 3*a*b^2*d)*Log[a*Cos[e + f*x] + b*Sin[e + f*x]])/((a^2 + b^2
)^3*f) - (b*c - a*d)/(2*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^2) - (2*a*b*c -
a^2*d + b^2*d)/((a^2 + b^2)^2*f*(a + b*Tan[e + f*x]))
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
```

```
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+d\tan(e+fx)}{(a+b\tan(e+fx))^3}\,dx &= -\frac{bc-ad}{2\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}+\frac{\int \frac{ac+bd-(bc-ad)\tan(e+fx)}{(a+b\tan(e+fx))^2}\,dx}{a^2+b^2} \\
&= -\frac{bc-ad}{2\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}-\frac{2abc-a^2d+b^2d}{\left(a^2+b^2\right)^2f(a+b\tan(e+fx))}+\frac{\int \frac{a^2c-b}{}}{} \\
&= \frac{\left(a^3c-3ab^2c+3a^2bd-b^3d\right)x}{\left(a^2+b^2\right)^3}-\frac{bc-ad}{2\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}-\frac{2a}{\left(a^2+b^2\right)^2} \\
&= \frac{\left(a^3c-3ab^2c+3a^2bd-b^3d\right)x}{\left(a^2+b^2\right)^3}+\frac{\left(3a^2bc-b^3c-a^3d+3ab^2d\right)\log(a\cos(e+fx)}{\left(a^2+b^2\right)^3f}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.58, size = 243, normalized size = 1.39

$$-\frac{d\left(\frac{i\log(i-\tan(e+fx))}{(a+ib)^2}-\frac{i\log(i+\tan(e+fx))}{(a-ib)^2}+\frac{2b\left(-2a\log(a+b\tan(e+fx))+\frac{a^2+b^2}{a+b\tan(e+fx)}\right)}{(a^2+b^2)^2}\right)+(bc-ad)\left(\frac{i\log(i-\tan(e+fx))}{(a+ib)^3}-\frac{\log(i+\tan(e+fx))}{(ia+b)^3}+\frac{b\left((-6a^2+2b^2)\log(a+b\tan(e+fx))+\frac{(a^2+b^2)(5a^2+b^2+4ab\tan(e+fx))}{(a+b\tan(e+fx))^2}\right)}{(a^2+b^2)^3}\right)}{2bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] -1/2*(d*((I*Log[I - Tan[e + f*x]])/(a + I*b)^2 - (I*Log[I + Tan[e + f*x]])/
(a - I*b)^2 + (2*b*(-2*a*Log[a + b*Tan[e + f*x]] + (a^2 + b^2)/(a + b*Tan[e
 + f*x])))/(a^2 + b^2)^2) + (b*c - a*d)*((I*Log[I - Tan[e + f*x]])/(a + I*b
)^3 - Log[I + Tan[e + f*x]]/(I*a + b)^3 + (b*((-6*a^2 + 2*b^2)*Log[a + b*Ta
n[e + f*x]] + ((a^2 + b^2)*(5*a^2 + b^2 + 4*a*b*Tan[e + f*x]))/(a + b*Tan[e
 + f*x])^2))/(a^2 + b^2)^3))/(b*f)
```

Maple [A]
time = 0.27, size = 206, normalized size = 1.18

method	result
derivativedivides	$\frac{-\frac{\left(a^3d-3a^2bc-3a\,b^2d+b^3c\right)\ln(a+b\tan(fx+e))}{\left(a^2+b^2\right)^3}+\frac{ad-bc}{2\left(a^2+b^2\right)(a+b\tan(fx+e))^2}+\frac{a^2d-2abc-b^2d}{\left(a^2+b^2\right)^2(a+b\tan(fx+e))}+\frac{\left(a^3d-3a^2bc-3a\,b^2d+\right.}{}}{f}$

default	$\frac{-\frac{\left(a^{3}d-3a^{2}bc-3a\,b^{2}d+b^{3}c\right)\ln(a+b\tan(fx+e))}{\left(a^{2}+b^{2}\right)^{3}}+\frac{ad-bc}{2\left(a^{2}+b^{2}\right)(a+b\tan(fx+e))^{2}}+\frac{a^{2}d-2abc-b^{2}d}{\left(a^{2}+b^{2}\right)^{2}(a+b\tan(fx+e))}+\frac{\left(a^{3}d-3a^{2}bc-3a\,b^{2}d\right.}{}}{f}$
norman	$\frac{\frac{\left(a^{3}c+3a^{2}bd-3a\,b^{2}c-b^{3}d\right)a^{2}x}{\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\left(a^{2}+b^{2}\right)}+\frac{b^{2}\left(a^{3}c+3a^{2}bd-3a\,b^{2}c-b^{3}d\right)x\left(\tan^{2}(fx+e)\right)}{\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\left(a^{2}+b^{2}\right)}+\frac{2a^{3}bd-3a^{2}b^{2}c-b^{4}c}{2bf\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}+\frac{b\left(-a^{2}bd+2a\,b^{2}c+b^{3}d\right)\left(\tan\right.}{2fa\left(a^{4}+2a^{2}b^{2}+b^{4}\right.}}{(a+b\tan(fx+e))^{2}}$
risch	$\frac{ixd}{3ia^{2}b-ib^{3}-a^{3}+3a\,b^{2}}-\frac{xc}{3ia^{2}b-ib^{3}-a^{3}+3a\,b^{2}}+\frac{2ia^{3}dx}{a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}}-\frac{6ia^{2}bcx}{a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}}-\frac{6ia\,b^{2}dx}{a^{6}+3a^{4}b^{2}+3a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))/(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-(a^3*d-3*a^2*b*c-3*a*b^2*d+b^3*c)/(a^2+b^2)^3*ln(a+b*tan(f*x+e))+1/2*
(a*d-b*c)/(a^2+b^2)/(a+b*tan(f*x+e))^2+(a^2*d-2*a*b*c-b^2*d)/(a^2+b^2)^2/(a
+b*tan(f*x+e))+1/(a^2+b^2)^3*(1/2*(a^3*d-3*a^2*b*c-3*a*b^2*d+b^3*c)*ln(1+ta
n(f*x+e)^2)+(a^3*c+3*a^2*b*d-3*a*b^2*c-b^3*d)*arctan(tan(f*x+e))))
```

Maxima [A]
time = 0.61, size = 339, normalized size = 1.94

$$\frac{\frac{2\left(\left(a^{3}-3\,ab^{2}\right)c+\left(3\,a^{2}b-b^{3}\right)d\right)(fx+e)}{a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}}+\frac{2\left(\left(3\,a^{2}b-b^{3}\right)c-\left(a^{3}-3\,ab^{2}\right)d\right)\log\left(b\tan\left(fx+e\right)+a\right)}{a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}}-\frac{\left(\left(3\,a^{2}b-b^{3}\right)c-\left(a^{3}-3\,ab^{2}\right)d\right)\log\left(\tan\left(fx+e\right)^{2}+1\right)}{a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}}-\frac{\left(5\,a^{2}b+b^{3}\right)c-\left(3\,a^{3}-ab^{2}\right)d+2\left(2\,ab^{2}c-\left(a^{2}b-b^{3}\right)d\right)\tan\left(fx+e\right)}{a^{6}+2\,a^{4}b^{2}+a^{2}b^{4}+\left(a^{4}b^{2}+2\,a^{2}b^{4}+b^{6}\right)\tan\left(fx+e\right)^{2}+2\left(a^{5}b+2\,a^{3}b^{3}+ab^{5}\right)\tan\left(fx+e\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*((a^3 - 3*a*b^2)*c + (3*a^2*b - b^3)*d)*(f*x + e)/(a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6) + 2*((3*a^2*b - b^3)*c - (a^3 - 3*a*b^2)*d)*log(b*tan(f*x
 + e) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - ((3*a^2*b - b^3)*c - (a^3
- 3*a*b^2)*d)*log(tan(f*x + e)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) -
 ((5*a^2*b + b^3)*c - (3*a^3 - a*b^2)*d + 2*(2*a*b^2*c - (a^2*b - b^3)*d)*t
an(f*x + e))/(a^6 + 2*a^4*b^2 + a^2*b^4 + (a^4*b^2 + 2*a^2*b^4 + b^6)*tan(f
*x + e)^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*tan(f*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 510 vs. 2(177) = 354.
time = 1.49, size = 510, normalized size = 2.91

$$\frac{2\left(\left(a^{5}-3\,a^{3}b^{2}\right)c+\left(3\,a^{4}b-a^{2}b^{3}\right)d\right)fx+\left(2\left(\left(a^{3}b^{2}-3\,ab^{4}\right)c+\left(3\,a^{2}b^{3}-b^{5}\right)d\right)fx+\left(5\,a^{2}b^{3}-b^{5}\right)c-3\left(a^{3}b^{2}-ab^{4}\right)d\right)\tan\left(fx+e\right)^{2}-\left(7\,a^{2}b^{3}+b^{5}\right)c+\left(5\,a^{3}b^{2}-ab^{4}\right)d+\left(\left(\left(3\,a^{2}b^{3}-b^{5}\right)c-\left(a^{3}b^{2}-3\,ab^{4}\right)d\right)\tan\left(fx+e\right)^{2}+\left(3\,a^{4}b-a^{2}b^{3}\right)c-\left(a^{5}-3\,a^{3}b^{2}\right)d+2\left(\left(3\,a^{3}b^{2}-ab^{4}\right)c-\left(a^{4}b-3\,a^{2}b^{3}\right)d\right)\tan\left(fx+e\right)\right)\log\left(\frac{b^{2}\tan\left(fx+e\right)^{2}+2\,ab\tan\left(fx+e\right)+a^{2}}{\tan\left(fx+e\right)^{2}+1}\right)+2\left(2\left(\left(a^{4}b-3\,a^{2}b^{3}\right)c+\left(3\,a^{3}b^{2}-ab^{4}\right)d\right)fx+3\left(a^{3}b^{2}-ab^{4}\right)c-\left(2\,a^{4}b-3\,a^{2}b^{3}+b^{5}\right)d\right)\tan\left(fx+e\right)}{2\left(\left(a^{6}b^{2}+3\,a^{4}b^{4}+3\,a^{2}b^{6}+b^{8}\right)f\tan\left(fx+e\right)^{2}+2\left(a^{7}b+3\,a^{5}b^{3}+3\,a^{3}b^{5}+ab^{7}\right)f\tan\left(fx+e\right)+\left(a^{8}+3\,a^{6}b^{2}+3\,a^{4}b^{4}+a^{2}b^{6}\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*((a^5 - 3*a^3*b^2)*c + (3*a^4*b - a^2*b^3)*d)*f*x + (2*((a^3*b^2 - 3
*a*b^4)*c + (3*a^2*b^3 - b^5)*d)*f*x + (5*a^2*b^3 - b^5)*c - 3*(a^3*b^2 - a
*b^4)*d)*tan(f*x + e)^2 - (7*a^2*b^3 + b^5)*c + (5*a^3*b^2 - a*b^4)*d + (((
```

```
3*a^2*b^3 - b^5)*c - (a^3*b^2 - 3*a*b^4)*d)*tan(f*x + e)^2 + (3*a^4*b - a^2
*b^3)*c - (a^5 - 3*a^3*b^2)*d + 2*((3*a^3*b^2 - a*b^4)*c - (a^4*b - 3*a^2*b
^3)*d)*tan(f*x + e))*log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(t
an(f*x + e)^2 + 1)) + 2*(2*((a^4*b - 3*a^2*b^3)*c + (3*a^3*b^2 - a*b^4)*d)*
f*x + 3*(a^3*b^2 - a*b^4)*c - (2*a^4*b - 3*a^2*b^3 + b^5)*d)*tan(f*x + e))/
((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*f*tan(f*x + e)^2 + 2*(a^7*b + 3*a^
5*b^3 + 3*a^3*b^5 + a*b^7)*f*tan(f*x + e) + (a^8 + 3*a^6*b^2 + 3*a^4*b^4 +
a^2*b^6)*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))/(a+b*tan(f*x+e))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 426 vs. 2(177) = 354.
time = 0.65, size = 426, normalized size = 2.43

$$\frac{\frac{2\left(a^{3}c-3\,ab^{2}c+3\,a^{2}bd-b^{3}d\right)(fx+e)}{a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}}-\frac{\left(3\,a^{2}bc-b^{3}c-a^{3}d+3\,ab^{2}d\right)\log\left(\tan(fx+e)^{2}+1\right)}{a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}}+\frac{2\left(3\,a^{2}b^{2}c-b^{4}c-a^{3}bd+3\,ab^{3}d\right)\log(|b\tan(fx+e)+a|)}{a^{6}b+3\,a^{4}b^{3}+3\,a^{2}b^{5}+b^{7}}-\frac{9\,a^{2}b^{3}c\tan(fx+e)^{2}-3\,b^{5}c\tan(fx+e)^{2}-3\,a^{3}b^{2}d\tan(fx+e)^{2}+9\,ab^{4}d\tan(fx+e)^{2}+22\,a^{3}b^{2}c\tan(fx+e)-2\,ab^{4}c\tan(fx+e)-8\,a^{4}bd\tan(fx+e)+18\,a^{2}b^{3}d\tan(fx+e)+2\,b^{5}d\tan(fx+e)+14\,a^{4}bc+3\,a^{2}b^{3}c+b^{5}c-6\,a^{5}d+7\,a^{3}b^{2}d+ab^{4}d}{\left(a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}\right)(b\tan(fx+e)+a)^{2}}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))/(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(a^3*c - 3*a*b^2*c + 3*a^2*b*d - b^3*d)*(f*x + e)/(a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6) - (3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*log(tan(f*x + e
)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(3*a^2*b^2*c - b^4*c - a^3
*b*d + 3*a*b^3*d)*log(abs(b*tan(f*x + e) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b
^5 + b^7) - (9*a^2*b^3*c*tan(f*x + e)^2 - 3*b^5*c*tan(f*x + e)^2 - 3*a^3*b^
2*d*tan(f*x + e)^2 + 9*a*b^4*d*tan(f*x + e)^2 + 22*a^3*b^2*c*tan(f*x + e) -
 2*a*b^4*c*tan(f*x + e) - 8*a^4*b*d*tan(f*x + e) + 18*a^2*b^3*d*tan(f*x + e
) + 2*b^5*d*tan(f*x + e) + 14*a^4*b*c + 3*a^2*b^3*c + b^5*c - 6*a^5*d + 7*a
^3*b^2*d + a*b^4*d)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(b*tan(f*x + e) +
a)^2))/f
```

Mupad [B]
time = 5.65, size = 279, normalized size = 1.59

$$-\frac{\ln\left(a+b\tan\left(e+f\,x\right)\right)\,\left(\frac{a\,d-3\,b\,c}{{\left(a^2+b^2\right)}^2}-\frac{4\,b^2\,\left(a\,d-b\,c\right)}{{\left(a^2+b^2\right)}^3}\right)}{f}-\frac{\frac{-3\,d\,a^3+5\,c\,a^2\,b+d\,a\,b^2+c\,b^3}{2\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{\mathrm{tan}\left(e+f\,x\right)\,\left(-d\,a^2\,b+2\,c\,a\,b^2+d\,b^3\right)}{a^4+2\,a^2\,b^2+b^4}}{f\,\left(a^2+2\,a\,b\,\mathrm{tan}\left(e+f\,x\right)+b^2\,{\mathrm{tan}\left(e+f\,x\right)}^2\right)}+\frac{\ln\left(\mathrm{tan}\left(e+f\,x\right)-\mathrm{i}\right)\,\left(-d+c\,1{}\mathrm{i}\right)}{2\,f\,\left(-a^3-a^2\,b\,3{}\mathrm{i}+3\,a\,b^2+b^3\,1{}\mathrm{i}\right)}+\frac{\ln\left(\mathrm{tan}\left(e+f\,x\right)+1{}\mathrm{i}\right)\,\left(c-d\,1{}\mathrm{i}\right)}{2\,f\,\left(-a^3\,1{}\mathrm{i}-3\,a^2\,b+a\,b^2\,3{}\mathrm{i}+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))/(a + b*tan(e + f*x))^3,x)
```

```
[Out] (log(tan(e + f*x) - 1i)*(c*1i - d))/(2*f*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i
)) - ((b^3*c - 3*a^3*d + 5*a^2*b*c + a*b^2*d)/(2*(a^4 + b^4 + 2*a^2*b^2)) +
 (tan(e + f*x)*(b^3*d + 2*a*b^2*c - a^2*b*d))/(a^4 + b^4 + 2*a^2*b^2))/(f*(
a^2 + b^2*tan(e + f*x)^2 + 2*a*b*tan(e + f*x))) - (log(a + b*tan(e + f*x))*
((a*d - 3*b*c)/(a^2 + b^2)^2 - (4*b^2*(a*d - b*c))/(a^2 + b^2)^3))/f + (log
(tan(e + f*x) + 1i)*(c - d*1i))/(2*f*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3))
```

3.1197 $\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^2\,dx$

Optimal. Leaf size=215

$$-\left(\left(6a^2bcd-2b^3cd-a^3\left(c^2-d^2\right)+3ab^2\left(c^2-d^2\right)\right)x\right)-\frac{\left(2a^3cd-6ab^2cd+3a^2b(c^2-d^2)-b^3(c^2-d^2)\right)\log(\mathrm{c}$$

```
[Out] -(6*a^2*b*c*d-2*b^3*c*d-a^3*(c^2-d^2)+3*a*b^2*(c^2-d^2))*x-(2*a^3*c*d-6*a*b
^2*c*d+3*a^2*b*(c^2-d^2)-b^3*(c^2-d^2))*ln(cos(f*x+e))/f+2*b*(a*d+b*c)*(a*c
-b*d)*tan(f*x+e)/f+1/2*(2*a*c*d+b*(c^2-d^2))*(a+b*tan(f*x+e))^2/f+2/3*c*d*(
a+b*tan(f*x+e))^3/f+1/4*d^2*(a+b*tan(f*x+e))^4/b/f
```

Rubi [A]
time = 0.18, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3624, 3609, 3606, 3556}

$$-\frac{\left(2a^3cd+3a^2b(c^2-d^2)-6ab^2cd-b^3(c^2-d^2)\right)\log(\cos(e+fx))}{f}-x\left(-\left(a^3\left(c^2-d^2\right)\right)+6a^2bcd+3ab^2\left(c^2-d^2\right)-2b^3cd\right)+\frac{\left(2acd+b(c^2-d^2)\right)(a+b\tan(e+fx))^2}{2f}+\frac{2cd(a+b\tan(e+fx))^3}{3f}+\frac{2b(ad+bc)(ac-bd)\tan(e+fx)}{f}+\frac{d^2(a+b\tan(e+fx))^4}{4bf}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^2,x]
```

```
[Out] -((6*a^2*b*c*d - 2*b^3*c*d - a^3*(c^2 - d^2) + 3*a*b^2*(c^2 - d^2))*x) - ((
2*a^3*c*d - 6*a*b^2*c*d + 3*a^2*b*(c^2 - d^2) - b^3*(c^2 - d^2))*Log[Cos[e
+ f*x]])/f + (2*b*(b*c + a*d)*(a*c - b*d)*Tan[e + f*x])/f + ((2*a*c*d + b*(
c^2 - d^2))*(a + b*Tan[e + f*x])^2)/(2*f) + (2*c*d*(a + b*Tan[e + f*x])^3)/
(3*f) + (d^2*(a + b*Tan[e + f*x])^4)/(4*b*f)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
```

```
0] && GtQ[m, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^2\,dx &= \frac{d^2(a+b\tan(e+fx))^4}{4bf}+\int (a+b\tan(e+fx))^3\left(c^2-d^2\right.\\ &= \frac{2cd(a+b\tan(e+fx))^3}{3f}+\frac{d^2(a+b\tan(e+fx))^4}{4bf}+\int (a\\ &= \frac{(2acd+b(c^2-d^2))\,(a+b\tan(e+fx))^2}{2f}+\frac{2cd(a+b\tan(}{3f}\\ &= -\left(6a^2bcd-2b^3cd-a^3\left(c^2-d^2\right)+3ab^2\left(c^2-d^2\right)\right)x+\frac{2b(}{}\\ &= -\left(6a^2bcd-2b^3cd-a^3\left(c^2-d^2\right)+3ab^2\left(c^2-d^2\right)\right)x-\frac{(2a}{}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.55, size = 221, normalized size = 1.03

$$\frac{3d^2(a+b\tan(e+fx))^4-6(2acd+b(-c^2+d^2))\left((ia-b)^3\log(i-\tan(e+fx))-(ia+b)^3\log(i+\tan(e+fx))+6ab^2\tan(e+fx)+b^3\tan^2(e+fx)\right)-4cd\left(3i(a+ib)^4\log(i-\tan(e+fx))-3i(a-ib)^4\log(i+\tan(e+fx))+6b^2(-6a^2+b^2)\tan(e+fx)-12ab^3\tan^2(e+fx)-2b^4\tan^3(e+fx)\right)}{12bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^2,x]
```

```
[Out] (3*d^2*(a + b*Tan[e + f*x])^4 - 6*(2*a*c*d + b*(-c^2 + d^2))*((I*a - b)^3*L
og[I - Tan[e + f*x]] - (I*a + b)^3*Log[I + Tan[e + f*x]] + 6*a*b^2*Tan[e +
f*x] + b^3*Tan[e + f*x]^2) - 4*c*d*((3*I)*(a + I*b)^4*Log[I - Tan[e + f*x]]
 - (3*I)*(a - I*b)^4*Log[I + Tan[e + f*x]] + 6*b^2*(-6*a^2 + b^2)*Tan[e + f
*x] - 12*a*b^3*Tan[e + f*x]^2 - 2*b^4*Tan[e + f*x]^3))/(12*b*f)
```

Maple [A]
time = 0.11, size = 307, normalized size = 1.43 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/4*b^3*d^2*tan(f*x+e)^4+a*b^2*d^2*tan(f*x+e)^3+2/3*b^3*c*d*tan(f*x+e)
^3+3/2*a^2*b*d^2*tan(f*x+e)^2+3*a*b^2*c*d*tan(f*x+e)^2+1/2*b^3*c^2*tan(f*x+
e)^2-1/2*b^3*d^2*tan(f*x+e)^2+a^3*d^2*tan(f*x+e)+6*a^2*b*c*d*tan(f*x+e)+3*a
*b^2*c^2*tan(f*x+e)-3*a*b^2*d^2*tan(f*x+e)-2*b^3*c*d*tan(f*x+e)+1/2*(2*a^3*
c*d+3*a^2*b*c^2-3*a^2*b*d^2-6*a*b^2*c*d-b^3*c^2+b^3*d^2)*ln(1+tan(f*x+e)^2)
+(a^3*c^2-a^3*d^2-6*a^2*b*c*d-3*a*b^2*c^2+3*a*b^2*d^2+2*b^3*c*d)*arctan(tan
(f*x+e)))
```

Maxima [A]
time = 0.63, size = 259, normalized size = 1.20

$$\frac{3\,b^3d^2\tan\left(fx+e\right)^4+4\left(2\,b^3cd+3\,ab^2d^2\right)\tan\left(fx+e\right)^3+6\left(b^3c^2+6\,ab^2cd+\left(3\,a^2b-b^3\right)d^2\right)\tan\left(fx+e\right)^2+12\left(\left(a^3-3\,ab^2\right)c^2-2\left(3\,a^2b-b^3\right)cd-\left(a^3-3\,ab^2\right)d^2\right)\left(fx+e\right)+6\left(\left(3\,a^2b-b^3\right)c^2+2\left(a^3-3\,ab^2\right)cd-\left(3\,a^2b-b^3\right)d^2\right)\log\left(\tan\left(fx+e\right)^2+1\right)+12\left(3\,ab^2c^2+2\left(3\,a^2b-b^3\right)cd+\left(a^3-3\,ab^2\right)d^2\right)\tan\left(fx+e\right)}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/12*(3*b^3*d^2*tan(f*x + e)^4 + 4*(2*b^3*c*d + 3*a*b^2*d^2)*tan(f*x + e)^3
 + 6*(b^3*c^2 + 6*a*b^2*c*d + (3*a^2*b - b^3)*d^2)*tan(f*x + e)^2 + 12*((a^
3 - 3*a*b^2)*c^2 - 2*(3*a^2*b - b^3)*c*d - (a^3 - 3*a*b^2)*d^2)*(f*x + e) +
 6*((3*a^2*b - b^3)*c^2 + 2*(a^3 - 3*a*b^2)*c*d - (3*a^2*b - b^3)*d^2)*log(
tan(f*x + e)^2 + 1) + 12*(3*a*b^2*c^2 + 2*(3*a^2*b - b^3)*c*d + (a^3 - 3*a*
b^2)*d^2)*tan(f*x + e))/f
```

Fricas [A]
time = 0.97, size = 257, normalized size = 1.20

$$\frac{3\,b^3d^2\tan\left(fx+e\right)^4+4\left(2\,b^3cd+3\,ab^2d^2\right)\tan\left(fx+e\right)^3+12\left(\left(a^3-3\,ab^2\right)c^2-2\left(3\,a^2b-b^3\right)cd-\left(a^3-3\,ab^2\right)d^2\right)fx+6\left(b^3c^2+6\,ab^2cd+\left(3\,a^2b-b^3\right)d^2\right)\tan\left(fx+e\right)^2-6\left(\left(3\,a^2b-b^3\right)c^2+2\left(a^3-3\,ab^2\right)cd-\left(3\,a^2b-b^3\right)d^2\right)\log\left(\frac{1}{\tan\left(fx+e\right)^2+1}\right)+12\left(3\,ab^2c^2+2\left(3\,a^2b-b^3\right)cd+\left(a^3-3\,ab^2\right)d^2\right)\tan\left(fx+e\right)}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^3*d^2*tan(f*x + e)^4 + 4*(2*b^3*c*d + 3*a*b^2*d^2)*tan(f*x + e)^3
 + 12*((a^3 - 3*a*b^2)*c^2 - 2*(3*a^2*b - b^3)*c*d - (a^3 - 3*a*b^2)*d^2)*f
*x + 6*(b^3*c^2 + 6*a*b^2*c*d + (3*a^2*b - b^3)*d^2)*tan(f*x + e)^2 - 6*((3
*a^2*b - b^3)*c^2 + 2*(a^3 - 3*a*b^2)*c*d - (3*a^2*b - b^3)*d^2)*log(1/(tan
(f*x + e)^2 + 1)) + 12*(3*a*b^2*c^2 + 2*(3*a^2*b - b^3)*c*d + (a^3 - 3*a*b^
2)*d^2)*tan(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(190) = 380$.
time = 0.20, size = 445, normalized size = 2.07

$$\begin{cases} a^3c^2x+\frac{a^3cd\log\left(\tan^2\left(e+fx\right)+1\right)}{f}-a^3d^2x+\frac{a^3d^2\tan\left(e+fx\right)}{f}+\frac{3a^2bc^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}-6a^2bcdx+\frac{6a^2bcd\tan\left(e+fx\right)}{f}-\frac{3a^2bd^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{3a^2bd^2\tan^2\left(e+fx\right)}{2f}-3ab^2c^2x+\frac{3ab^2c^2\tan\left(e+fx\right)}{f}-\frac{3ab^2cd\log\left(\tan^2\left(e+fx\right)+1\right)}{f}+\frac{3ab^2cd\tan^2\left(e+fx\right)}{f}+3ab^2d^2x+\frac{ab^2d^2\tan^3\left(e+fx\right)}{f}-\frac{3ab^2d^2\tan\left(e+fx\right)}{f}-\frac{b^3c^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{b^3c^2\tan^2\left(e+fx\right)}{2f}+2b^3cdx+\frac{2b^3cd\tan^3\left(e+fx\right)}{3f}-\frac{2b^3cd\tan\left(e+fx\right)}{f}+\frac{b^3d^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{b^3d^2\tan^4\left(e+fx\right)}{4f}-\frac{b^3d^2\tan^2\left(e+fx\right)}{2f} & \text{for}\: f \neq 0 \\ x\left(a+b\tan\left(e\right)\right)^3\left(c+d\tan\left(e\right)\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3*(c+d*tan(f*x+e))**2,x)
```

```
[Out] Piecewise((a**3*c**2*x + a**3*c*d*log(tan(e + f*x)**2 + 1)/f - a**3*d**2*x
+ a**3*d**2*tan(e + f*x)/f + 3*a**2*b*c**2*log(tan(e + f*x)**2 + 1)/(2*f) -
 6*a**2*b*c*d*x + 6*a**2*b*c*d*tan(e + f*x)/f - 3*a**2*b*d**2*log(tan(e + f
*x)**2 + 1)/(2*f) + 3*a**2*b*d**2*tan(e + f*x)**2/(2*f) - 3*a*b**2*c**2*x +
 3*a*b**2*c**2*tan(e + f*x)/f - 3*a*b**2*c*d*log(tan(e + f*x)**2 + 1)/f + 3
*a*b**2*c*d*tan(e + f*x)**2/f + 3*a*b**2*d**2*x + a*b**2*d**2*tan(e + f*x)*
*3/f - 3*a*b**2*d**2*tan(e + f*x)/f - b**3*c**2*log(tan(e + f*x)**2 + 1)/(2
*f) + b**3*c**2*tan(e + f*x)**2/(2*f) + 2*b**3*c*d*x + 2*b**3*c*d*tan(e + f
*x)**3/(3*f) - 2*b**3*c*d*tan(e + f*x)/f + b**3*d**2*log(tan(e + f*x)**2 +
1)/(2*f) + b**3*d**2*tan(e + f*x)**4/(4*f) - b**3*d**2*tan(e + f*x)**2/(2*f
), Ne(f, 0)), (x*(a + b*tan(e))**3*(c + d*tan(e))**2, True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4557 vs. $2(214) = 428$.
time = 2.68, size = 4557, normalized size = 21.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/12*(12*a^3*c^2*f*x*tan(f*x)^4*tan(e)^4 - 36*a*b^2*c^2*f*x*tan(f*x)^4*tan(
e)^4 - 72*a^2*b*c*d*f*x*tan(f*x)^4*tan(e)^4 + 24*b^3*c*d*f*x*tan(f*x)^4*tan
(e)^4 - 12*a^3*d^2*f*x*tan(f*x)^4*tan(e)^4 + 36*a*b^2*d^2*f*x*tan(f*x)^4*ta
n(e)^4 - 18*a^2*b*c^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + ta
n(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan
(f*x)^4*tan(e)^4 + 6*b^3*c^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(
e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 +
1))*tan(f*x)^4*tan(e)^4 - 12*a^3*c*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x
)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan
(e)^2 + 1))*tan(f*x)^4*tan(e)^4 + 36*a*b^2*c*d*log(4*(tan(f*x)^4*tan(e)^2 -
 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e)
 + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 + 18*a^2*b*d^2*log(4*(tan(f*x)^4*
tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f
*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 - 6*b^3*d^2*log(4*(tan(
f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 -
2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 - 48*a^3*c^2*f*x
*tan(f*x)^3*tan(e)^3 + 144*a*b^2*c^2*f*x*tan(f*x)^3*tan(e)^3 + 288*a^2*b*c*
d*f*x*tan(f*x)^3*tan(e)^3 - 96*b^3*c*d*f*x*tan(f*x)^3*tan(e)^3 + 48*a^3*d^2
*f*x*tan(f*x)^3*tan(e)^3 - 144*a*b^2*d^2*f*x*tan(f*x)^3*tan(e)^3 + 6*b^3*c^
2*tan(f*x)^4*tan(e)^4 + 36*a*b^2*c*d*tan(f*x)^4*tan(e)^4 + 18*a^2*b*d^2*tan
(f*x)^4*tan(e)^4 - 9*b^3*d^2*tan(f*x)^4*tan(e)^4 + 72*a^2*b*c^2*log(4*(tan(
f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 -
2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 - 24*b^3*c^2*log
(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f
```

```
*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 + 48*a^3
*c*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2
 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3
- 144*a*b^2*c*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)
^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^
3*tan(e)^3 - 72*a^2*b*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e)
+ tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))
*tan(f*x)^3*tan(e)^3 + 24*b^3*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3
*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)
^2 + 1))*tan(f*x)^3*tan(e)^3 - 36*a*b^2*c^2*tan(f*x)^4*tan(e)^3 - 72*a^2*b*
c*d*tan(f*x)^4*tan(e)^3 + 24*b^3*c*d*tan(f*x)^4*tan(e)^3 - 12*a^3*d^2*tan(f
*x)^4*tan(e)^3 + 36*a*b^2*d^2*tan(f*x)^4*tan(e)^3 - 36*a*b^2*c^2*tan(f*x)^3
*tan(e)^4 - 72*a^2*b*c*d*tan(f*x)^3*tan(e)^4 + 24*b^3*c*d*tan(f*x)^3*tan(e)
^4 - 12*a^3*d^2*tan(f*x)^3*tan(e)^4 + 36*a*b^2*d^2*tan(f*x)^3*tan(e)^4 + 72
*a^3*c^2*f*x*tan(f*x)^2*tan(e)^2 - 216*a*b^2*c^2*f*x*tan(f*x)^2*tan(e)^2 -
432*a^2*b*c*d*f*x*tan(f*x)^2*tan(e)^2 + 144*b^3*c*d*f*x*tan(f*x)^2*tan(e)^2
 - 72*a^3*d^2*f*x*tan(f*x)^2*tan(e)^2 + 216*a*b^2*d^2*f*x*tan(f*x)^2*tan(e)
^2 + 6*b^3*c^2*tan(f*x)^4*tan(e)^2 + 36*a*b^2*c*d*tan(f*x)^4*tan(e)^2 + 18*
a^2*b*d^2*tan(f*x)^4*tan(e)^2 - 6*b^3*d^2*tan(f*x)^4*tan(e)^2 - 12*b^3*c^2*
tan(f*x)^3*tan(e)^3 - 72*a*b^2*c*d*tan(f*x)^3*tan(e)^3 - 36*a^2*b*d^2*tan(f
*x)^3*tan(e)^3 + 24*b^3*d^2*tan(f*x)^3*tan(e)^3 + 6*b^3*c^2*tan(f*x)^2*tan(
e)^4 + 36*a*b^2*c*d*tan(f*x)^2*tan(e)^4 + 18*a^2*b*d^2*tan(f*x)^2*tan(e)^4
- 6*b^3*d^2*tan(f*x)^2*tan(e)^4 - 8*b^3*c*d*tan(f*x)^4*tan(e) - 12*a*b^2*d^
2*tan(f*x)^4*tan(e) - 108*a^2*b*c^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)
^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(
e)^2 + 1))*tan(f*x)^2*tan(e)^2 + 36*b^3*c^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*
tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) +
1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 - 72*a^3*c*d*log(4*(tan(f*x)^4*tan(e
)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*t
an(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 + 216*a*b^2*c*d*log(4*(tan(f
*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2
*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 + 108*a^2*b*d^2*l
og(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan
(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 - 36*b
^3*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)
^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^
2 + 108*a*b^2*c^2*tan(f*x)^3*tan(e)^2 + 216*a^2*b*c*d*tan(f*x)^3*tan(e)^2 -
 96*b^3*c*d*tan(f*x)^3*tan(e)^2 + 36*a^3*d^2*tan(f*x)^3*tan(e)^2 - 144*a*b^
2*d^2*tan(f*x)^3*tan(e)^2 + 108*a*b^2*c^2*tan(f*x)^2*tan(e)^3 + 216*a^2*b*c
*d*tan(f*x)^2*tan(e)^3 - 96*b^3*c*d*tan(f*x)^2*tan(e)^3 + 36*a^3*d^2*tan(f*
x)^2*tan(e)^3 - 144*a*b^2*d^2*tan(f*x)^2*tan(e)^3 - 8*b^3*c*d*tan(f*x)*tan(
e)^4 - 12*a*b^2*d^2*tan(f*x)*tan(e)^4 + 3*b^3*d^2*tan(f*x)^4 - 48*a^3*c^2*f
*x*tan(f*x)*tan(e) + 144*a*b^2*c^2*f*x*tan(f*x)...
```

Mupad [B]

time = 5.35, size = 259, normalized size = 1.20

$$x\left(a^3c^2-a^3d^2-6a^2bcd-3ab^2c^2+3ab^2d^2+2b^3cd\right)+\frac{\tan(e+fx)\left(a^3d^2-b^2d(3ad+2bc)+3ab^2c^2+6a^2bcd\right)}{f}-\frac{\ln\left(\tan(e+fx)^2+1\right)\left(-a^3cd-\frac{3a^2bc^2}{2}+\frac{3a^2bd^2}{2}+3ab^2cd+\frac{b^3c^2}{2}-\frac{b^3d^2}{2}\right)}{f}+\frac{\tan(e+fx)^2\left(\frac{3a^2bd^2}{2}+3ab^2cd+\frac{b^3c^2}{2}-\frac{b^3d^2}{2}\right)}{f}+\frac{b^3d^2\tan(e+fx)^4}{4f}+\frac{b^2d\tan(e+fx)^3(3ad+2bc)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^3*(c + d*tan(e + f*x))^2,x)
```

```
[Out] x*(a^3*c^2 - a^3*d^2 - 3*a*b^2*c^2 + 3*a*b^2*d^2 + 2*b^3*c*d - 6*a^2*b*c*d)
 + (tan(e + f*x)*(a^3*d^2 - b^2*d*(3*a*d + 2*b*c) + 3*a*b^2*c^2 + 6*a^2*b*c
*d))/f - (log(tan(e + f*x)^2 + 1)*((b^3*c^2)/2 - (b^3*d^2)/2 - (3*a^2*b*c^2
)/2 + (3*a^2*b*d^2)/2 - a^3*c*d + 3*a*b^2*c*d))/f + (tan(e + f*x)^2*((b^3*c
^2)/2 - (b^3*d^2)/2 + (3*a^2*b*d^2)/2 + 3*a*b^2*c*d))/f + (b^3*d^2*tan(e +
f*x)^4)/(4*f) + (b^2*d*tan(e + f*x)^3*(3*a*d + 2*b*c))/(3*f)
```

3.1198 $\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^2\,dx$

Optimal. Leaf size=131

$$(ac-bc-ad-bd)(ac+bc+ad-bd)x-\frac{2(bc+ad)(ac-bd)\log(\cos(e+fx))}{f}+\frac{b(2acd+b(c^2-d^2))\tan(e+fx)}{f}$$

```
[Out] (a*c-a*d-b*c-b*d)*(a*c+a*d+b*c-b*d)*x-2*(a*d+b*c)*(a*c-b*d)*ln(cos(f*x+e))/
f+b*(2*a*c*d+b*(c^2-d^2))*tan(f*x+e)/f+c*d*(a+b*tan(f*x+e))^2/f+1/3*d^2*(a+
b*tan(f*x+e))^3/b/f
```

Rubi [A]
time = 0.13, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3624, 3609, 3606, 3556}

$$\frac{b(2acd+b(c^2-d^2))\tan(e+fx)}{f}+\frac{cd(a+b\tan(e+fx))^2}{f}-\frac{2(ad+bc)(ac-bd)\log(\cos(e+fx))}{f}+x(ac-ad-bc-bd)(ac+ad+bc-bd)+\frac{d^2(a+b\tan(e+fx))^3}{3bf}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^2,x]
```

```
[Out] (a*c - b*c - a*d - b*d)*(a*c + b*c + a*d - b*d)*x - (2*(b*c + a*d)*(a*c - b
*d)*Log[Cos[e + f*x]])/f + (b*(2*a*c*d + b*(c^2 - d^2))*Tan[e + f*x])/f + (
c*d*(a + b*Tan[e + f*x])^2)/f + (d^2*(a + b*Tan[e + f*x])^3)/(3*b*f)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^2\,dx = \frac{d^2(a+b\tan(e+fx))^3}{3bf} + \int (a+b\tan(e+fx))^2\left(c^2-d^2\right.$$

$$= \frac{cd(a+b\tan(e+fx))^2}{f} + \frac{d^2(a+b\tan(e+fx))^3}{3bf} + \int (a+$$

$$= (ac-bc-ad-bd)(ac+bc+ad-bd)x + \frac{b(2acd+b(c^2}{}$$

$$= (ac-bc-ad-bd)(ac+bc+ad-bd)x - \frac{2(bc+ad)(ac}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.13, size = 185, normalized size = 1.41

$$\frac{2d^2(a+b\tan(e+fx))^3+3(2acd+b(-c^2+d^2))\left(i((a+ib)^2\log(i-\tan(e+fx))-(a-ib)^2\log(i+\tan(e+fx)))-2b^2\tan(e+fx)\right)+6cd((ia-b)^3\log(i-\tan(e+fx))-(ia+b)^3\log(i+\tan(e+fx))+6ab^2\tan(e+fx)+b^3\tan^2(e+fx))}{6bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^2,x]
```

```
[Out] (2*d^2*(a + b*Tan[e + f*x])^3 + 3*(2*a*c*d + b*(-c^2 + d^2))*(I*((a + I*b)^
2*Log[I - Tan[e + f*x]] - (a - I*b)^2*Log[I + Tan[e + f*x]]) - 2*b^2*Tan[e
+ f*x]) + 6*c*d*((I*a - b)^3*Log[I - Tan[e + f*x]] - (I*a + b)^3*Log[I + Ta
n[e + f*x]] + 6*a*b^2*Tan[e + f*x] + b^3*Tan[e + f*x]^2))/(6*b*f)
```

Maple [A]
time = 0.10, size = 189, normalized size = 1.44

method	result
norman	$\left(a^2c^2-a^2d^2-4abcd-b^2c^2+b^2d^2\right)x+\frac{\left(a^2d^2+4abcd+b^2c^2-b^2d^2\right)\tan(fx+e)}{f}+\frac{bd(ad+bc)(\tan^2(fx}{f}$
derivativedivides	$\frac{\frac{b^2d^2\left(\tan^3(fx+e)\right)}{3}+ab\,d^2\left(\tan^2(fx+e)\right)+b^2cd\left(\tan^2(fx+e)\right)+a^2d^2\tan(fx+e)+4abcd\tan(fx+e)+b^2c^2\tan(fx+e)-b^2d^2}{f}$
default	$\frac{\frac{b^2d^2\left(\tan^3(fx+e)\right)}{3}+ab\,d^2\left(\tan^2(fx+e)\right)+b^2cd\left(\tan^2(fx+e)\right)+a^2d^2\tan(fx+e)+4abcd\tan(fx+e)+b^2c^2\tan(fx+e)-b^2d^2}{f}$

risch	$-\frac{4ib^2cde}{f}+\frac{4iab\,c^2e}{f}-2ib^2cdx-\frac{4iab\,d^2e}{f}+a^2c^2x-a^2d^2x-4abcdx-b^2c^2x+b^2d^2x-2iab$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/3*b^2*d^2*tan(f*x+e)^3+a*b*d^2*tan(f*x+e)^2+b^2*c*d*tan(f*x+e)^2+a^2
*d^2*tan(f*x+e)+4*a*b*c*d*tan(f*x+e)+b^2*c^2*tan(f*x+e)-b^2*d^2*tan(f*x+e)+
1/2*(2*a^2*c*d+2*a*b*c^2-2*a*b*d^2-2*b^2*c*d)*ln(1+tan(f*x+e)^2)+(a^2*c^2-a
^2*d^2-4*a*b*c*d-b^2*c^2+b^2*d^2)*arctan(tan(f*x+e)))
```

Maxima [A]
time = 0.56, size = 164, normalized size = 1.25

$$\frac{b^2d^2\tan(fx+e)^3+3\,(b^2cd+abd^2)\tan(fx+e)^2-3\,(4\,abcd-(a^2-b^2)c^2+(a^2-b^2)d^2)(fx+e)+3\,(abc^2-abd^2+(a^2-b^2)cd)\log\left(\tan(fx+e)^2+1\right)+3\,(b^2c^2+4\,abcd+(a^2-b^2)d^2)\tan(fx+e)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*d^2*tan(f*x + e)^3 + 3*(b^2*c*d + a*b*d^2)*tan(f*x + e)^2 - 3*(4*a
*b*c*d - (a^2 - b^2)*c^2 + (a^2 - b^2)*d^2)*(f*x + e) + 3*(a*b*c^2 - a*b*d^
2 + (a^2 - b^2)*c*d)*log(tan(f*x + e)^2 + 1) + 3*(b^2*c^2 + 4*a*b*c*d + (a^
2 - b^2)*d^2)*tan(f*x + e))/f
```

Fricas [A]
time = 0.63, size = 162, normalized size = 1.24

$$\frac{b^2d^2\tan(fx+e)^3-3\,(4\,abcd-(a^2-b^2)c^2+(a^2-b^2)d^2)fx+3\,(b^2cd+abd^2)\tan(fx+e)^2-3\,(abc^2-abd^2+(a^2-b^2)cd)\log\left(\frac{1}{\tan(fx+e)^2+1}\right)+3\,(b^2c^2+4\,abcd+(a^2-b^2)d^2)\tan(fx+e)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*d^2*tan(f*x + e)^3 - 3*(4*a*b*c*d - (a^2 - b^2)*c^2 + (a^2 - b^2)*
d^2)*f*x + 3*(b^2*c*d + a*b*d^2)*tan(f*x + e)^2 - 3*(a*b*c^2 - a*b*d^2 + (a
^2 - b^2)*c*d)*log(1/(tan(f*x + e)^2 + 1)) + 3*(b^2*c^2 + 4*a*b*c*d + (a^2
- b^2)*d^2)*tan(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(117) = 234$.
time = 0.14, size = 258, normalized size = 1.97

$$\begin{cases} a^2c^2x+\frac{a^2cd\log(\tan^2(e+fx)+1)}{f}-a^2d^2x+\frac{a^2d^2\tan(e+fx)}{f}+\frac{abc^2\log(\tan^2(e+fx)+1)}{f}-4abcdx+\frac{4abcd\tan(e+fx)}{f}-\frac{abd^2\log(\tan^2(e+fx)+1)}{f}+\frac{abd^2\tan^2(e+fx)}{f}-b^2c^2x+\frac{b^2c^2\tan(e+fx)}{f}-\frac{b^2cd\log(\tan^2(e+fx)+1)}{f}+\frac{b^2cd\tan^2(e+fx)}{f}+b^2d^2x+\frac{b^2d^2\tan^3(e+fx)}{3f}-\frac{b^2d^2\tan(e+fx)}{f} & \text{for}\: f\neq 0 \\ x(a+b\tan(e))^2\,(c+d\tan(e))^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2*(c+d*tan(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c**2*x + a**2*c*d*log(tan(e + f*x)**2 + 1)/f - a**2*d**2*x
+ a**2*d**2*tan(e + f*x)/f + a*b*c**2*log(tan(e + f*x)**2 + 1)/f - 4*a*b*c*
d*x + 4*a*b*c*d*tan(e + f*x)/f - a*b*d**2*log(tan(e + f*x)**2 + 1)/f + a*b*
d**2*tan(e + f*x)**2/f - b**2*c**2*x + b**2*c**2*tan(e + f*x)/f - b**2*c*d*
log(tan(e + f*x)**2 + 1)/f + b**2*c*d*tan(e + f*x)**2/f + b**2*d**2*x + b**
2*d**2*tan(e + f*x)**3/(3*f) - b**2*d**2*tan(e + f*x)/f, Ne(f, 0)), (x*(a +
 b*tan(e))**2*(c + d*tan(e))**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2258 vs. $2(133) = 266$.
time = 1.28, size = 2258, normalized size = 17.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*c^2*f*x*tan(f*x)^3*tan(e)^3 - 3*b^2*c^2*f*x*tan(f*x)^3*tan(e)^3
- 12*a*b*c*d*f*x*tan(f*x)^3*tan(e)^3 - 3*a^2*d^2*f*x*tan(f*x)^3*tan(e)^3 +
3*b^2*d^2*f*x*tan(f*x)^3*tan(e)^3 - 3*a*b*c^2*log(4*(tan(f*x)^4*tan(e)^2 -
2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e)
+ 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 - 3*a^2*c*d*log(4*(tan(f*x)^4*tan(
e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*
tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 + 3*b^2*c*d*log(4*(tan(f*x)
^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*ta
n(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 + 3*a*b*d^2*log(4*(t
an(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2
 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 - 9*a^2*c^2*f
*x*tan(f*x)^2*tan(e)^2 + 9*b^2*c^2*f*x*tan(f*x)^2*tan(e)^2 + 36*a*b*c*d*f*x
*tan(f*x)^2*tan(e)^2 + 9*a^2*d^2*f*x*tan(f*x)^2*tan(e)^2 - 9*b^2*d^2*f*x*ta
n(f*x)^2*tan(e)^2 + 3*b^2*c*d*tan(f*x)^3*tan(e)^3 + 3*a*b*d^2*tan(f*x)^3*ta
n(e)^3 + 9*a*b*c^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f
*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*
x)^2*tan(e)^2 + 9*a^2*c*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e)
+ tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))
*tan(f*x)^2*tan(e)^2 - 9*b^2*c*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*
tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^
2 + 1))*tan(f*x)^2*tan(e)^2 - 9*a*b*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(
f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(
tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 - 3*b^2*c^2*tan(f*x)^3*tan(e)^2 - 12*a*b
*c*d*tan(f*x)^3*tan(e)^2 - 3*a^2*d^2*tan(f*x)^3*tan(e)^2 + 3*b^2*d^2*tan(f*
x)^3*tan(e)^2 - 3*b^2*c^2*tan(f*x)^2*tan(e)^3 - 12*a*b*c*d*tan(f*x)^2*tan(e
)^3 - 3*a^2*d^2*tan(f*x)^2*tan(e)^3 + 3*b^2*d^2*tan(f*x)^2*tan(e)^3 + 9*a^2
*c^2*f*x*tan(f*x)*tan(e) - 9*b^2*c^2*f*x*tan(f*x)*tan(e) - 36*a*b*c*d*f*x*t
an(f*x)*tan(e) - 9*a^2*d^2*f*x*tan(f*x)*tan(e) + 9*b^2*d^2*f*x*tan(f*x)*tan
```

```
(e) + 3*b^2*c*d*tan(f*x)^3*tan(e) + 3*a*b*d^2*tan(f*x)^3*tan(e) - 3*b^2*c*d
*tan(f*x)^2*tan(e)^2 - 3*a*b*d^2*tan(f*x)^2*tan(e)^2 + 3*b^2*c*d*tan(f*x)*t
an(e)^3 + 3*a*b*d^2*tan(f*x)*tan(e)^3 - b^2*d^2*tan(f*x)^3 - 9*a*b*c^2*log(
4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*
x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)*tan(e) - 9*a^2*c*d*l
og(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan
(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)*tan(e) + 9*b^2*c*
d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 +
tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)*tan(e) + 9*a*b
*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2
 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)*tan(e) + 6*
b^2*c^2*tan(f*x)^2*tan(e) + 24*a*b*c*d*tan(f*x)^2*tan(e) + 6*a^2*d^2*tan(f*
x)^2*tan(e) - 9*b^2*d^2*tan(f*x)^2*tan(e) + 6*b^2*c^2*tan(f*x)*tan(e)^2 + 2
4*a*b*c*d*tan(f*x)*tan(e)^2 + 6*a^2*d^2*tan(f*x)*tan(e)^2 - 9*b^2*d^2*tan(f
*x)*tan(e)^2 - b^2*d^2*tan(e)^3 - 3*a^2*c^2*f*x + 3*b^2*c^2*f*x + 12*a*b*c*
d*f*x + 3*a^2*d^2*f*x - 3*b^2*d^2*f*x - 3*b^2*c*d*tan(f*x)^2 - 3*a*b*d^2*ta
n(f*x)^2 + 3*b^2*c*d*tan(f*x)*tan(e) + 3*a*b*d^2*tan(f*x)*tan(e) - 3*b^2*c*
d*tan(e)^2 - 3*a*b*d^2*tan(e)^2 + 3*a*b*c^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*
tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) +
1)/(tan(e)^2 + 1)) + 3*a^2*c*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*ta
n(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2
+ 1)) - 3*b^2*c*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*
x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1)) - 3*a*b
*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2
 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1)) - 3*b^2*c^2*tan(f*x)
 - 12*a*b*c*d*tan(f*x) - 3*a^2*d^2*tan(f*x) + 3*b^2*d^2*tan(f*x) - 3*b^2*c^
2*tan(e) - 12*a*b*c*d*tan(e) - 3*a^2*d^2*tan(e) + 3*b^2*d^2*tan(e) - 3*b^2*
c*d - 3*a*b*d^2)/(f*tan(f*x)^3*tan(e)^3 - 3*f*tan(f*x)^2*tan(e)^2 + 3*f*tan
(f*x)*tan(e) - f)
```

Mupad [B]
time = 5.28, size = 230, normalized size = 1.76

$$\frac{\tan(e+fx)\,(a^2d^2+4abcd+b^2c^2-b^2d^2)}{f}+\frac{\ln\left(\tan(e+fx)^2+1\right)\,(a^2cd+abc^2-abd^2-b^2cd)}{f}-\frac{\mathrm{atan}\left(\frac{\tan(e+fx)\,(ac+ad+bc-bd)\,(ad-ac+bc+bd)}{-a^2c^2+a^2d^2+4abcd+b^2c^2-b^2d^2}\right)\,(ac+ad+bc-bd)\,(ad-ac+bc+bd)}{f}+\frac{b^2d^2\tan(e+fx)^3}{3f}+\frac{bd\tan(e+fx)^2\,(ad+bc)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^2*(c + d*tan(e + f*x))^2,x)
```

```
[Out] (tan(e + f*x)*(a^2*d^2 + b^2*c^2 - b^2*d^2 + 4*a*b*c*d))/f + (log(tan(e + f
*x)^2 + 1)*(a*b*c^2 - a*b*d^2 + a^2*c*d - b^2*c*d))/f - (atan((tan(e + f*x)
*(a*c + a*d + b*c - b*d)*(a*d - a*c + b*c + b*d))/(a^2*d^2 - a^2*c^2 + b^2*
c^2 - b^2*d^2 + 4*a*b*c*d))*(a*c + a*d + b*c - b*d)*(a*d - a*c + b*c + b*d)
)/f + (b^2*d^2*tan(e + f*x)^3)/(3*f) + (b*d*tan(e + f*x)^2*(a*d + b*c))/f
```

3.1199 $\int (a+b\tan(e+fx))(c+d\tan(e+fx))^2\,dx$

Optimal. Leaf size=89

$$-\left(\left(2bcd-a\left(c^2-d^2\right)\right)x\right)-\frac{\left(2acd+b\left(c^2-d^2\right)\right)\log(\cos(e+fx))}{f}+\frac{d(bc+ad)\tan(e+fx)}{f}+\frac{b(c+d\tan(e+$$

[Out] -(2*b*c*d-a*(c^2-d^2))*x-(2*a*c*d+b*(c^2-d^2))*ln(cos(f*x+e))/f+d*(a*d+b*c)*tan(f*x+e)/f+1/2*b*(c+d*tan(f*x+e))^2/f

Rubi [A]
time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3609, 3606, 3556}

$$-\frac{\left(2acd+b\left(c^2-d^2\right)\right)\log(\cos(e+fx))}{f}-x\left(2bcd-a\left(c^2-d^2\right)\right)+\frac{d(ad+bc)\tan(e+fx)}{f}+\frac{b(c+d\tan(e+fx))^2}{2f}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^2,x]

[Out] -((2*b*c*d - a*(c^2 - d^2))*x) - ((2*a*c*d + b*(c^2 - d^2))*Log[Cos[e + f*x]])/f + (d*(b*c + a*d)*Tan[e + f*x])/f + (b*(c + d*Tan[e + f*x])^2)/(2*f)

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^2\,dx = \frac{b(c+d\tan(e+fx))^2}{2f} + \int (c+d\tan(e+fx))(ac-bd+(bc$$
$$= -\left(2bcd-a\left(c^2-d^2\right)\right)x + \frac{d(bc+ad)\tan(e+fx)}{f} + \frac{b(c+d$$
$$= -\left(2bcd-a\left(c^2-d^2\right)\right)x - \frac{(2acd+b(c^2-d^2))\log(\cos(e+f$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.46, size = 96, normalized size = 1.08

$$\frac{(-ia+b)(c+id)^2\log(i-\tan(e+fx))+(ia+b)(c-id)^2\log(i+\tan(e+fx))+2d(2bc+ad)\tan(e+fx)+bd^2\tan^2(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^2,x]
```

```
[Out] (((-I)*a + b)*(c + I*d)^2*Log[I - Tan[e + f*x]] + (I*a + b)*(c - I*d)^2*Log
[I + Tan[e + f*x]] + 2*d*(2*b*c + a*d)*Tan[e + f*x] + b*d^2*Tan[e + f*x]^2)
/(2*f)
```

Maple [A]
time = 0.08, size = 97, normalized size = 1.09

method	result
norman	$\left(a\,c^2-a\,d^2-2bcd\right)x+\frac{d(ad+2bc)\tan(fx+e)}{f}+\frac{b\,d^2\left(\tan^2(fx+e)\right)}{2f}+\frac{\left(2acd+b\,c^2-b\,d^2\right)\ln\left(1+\tan^2(fx+e)\right)}{2f}$
derivativedivides	$\frac{\frac{b\,d^2\left(\tan^2(fx+e)\right)}{2}+a\,d^2\tan(fx+e)+2bcd\tan(fx+e)+\frac{\left(2acd+b\,c^2-b\,d^2\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(a\,c^2-a\,d^2-2bcd\right)\arctan(\tan(f}{f}$
default	$\frac{\frac{b\,d^2\left(\tan^2(fx+e)\right)}{2}+a\,d^2\tan(fx+e)+2bcd\tan(fx+e)+\frac{\left(2acd+b\,c^2-b\,d^2\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(a\,c^2-a\,d^2-2bcd\right)\arctan(\tan(f}{f}$
risch	$\frac{4iacde}{f}-\frac{2ib\,d^2e}{f}-ib\,d^2x+a\,c^2x-a\,d^2x-2bcdx+2iacdx+\frac{2ib\,c^2e}{f}+ib\,c^2x+\frac{2id\left(ad\,e^{2i(fx+e}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2*b*d^2*tan(f*x+e)^2+a*d^2*tan(f*x+e)+2*b*c*d*tan(f*x+e)+1/2*(2*a*c*
d+b*c^2-b*d^2)*ln(1+tan(f*x+e)^2)+(a*c^2-a*d^2-2*b*c*d)*arctan(tan(f*x+e)))
```

Maxima [A]
time = 0.53, size = 95, normalized size = 1.07

$$\frac{bd^2\tan(fx+e)^2+2\left(ac^2-2\,bcd-ad^2\right)(fx+e)+\left(bc^2+2\,acd-bd^2\right)\log\left(\tan(fx+e)^2+1\right)+2\left(2\,bcd+ad^2\right)\tan(fx+e)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*d^2*tan(f*x + e)^2 + 2*(a*c^2 - 2*b*c*d - a*d^2)*(f*x + e) + (b*c^2
+ 2*a*c*d - b*d^2)*log(tan(f*x + e)^2 + 1) + 2*(2*b*c*d + a*d^2)*tan(f*x +
e))/f
```

Fricas [A]
time = 0.76, size = 94, normalized size = 1.06

$$\frac{bd^2\tan(fx+e)^2 + 2(ac^2 - 2bcd - ad^2)fx - (bc^2 + 2acd - bd^2)\log\left(\frac{1}{\tan(fx+e)^2+1}\right) + 2(2bcd + ad^2)\tan(fx+e)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*d^2*tan(f*x + e)^2 + 2*(a*c^2 - 2*b*c*d - a*d^2)*f*x - (b*c^2 + 2*a*
c*d - b*d^2)*log(1/(tan(f*x + e)^2 + 1)) + 2*(2*b*c*d + a*d^2)*tan(f*x + e)
)/f
```

Sympy [A]
time = 0.12, size = 143, normalized size = 1.61

$$\begin{cases} ac^2x + \frac{acd\log(\tan^2(e+fx)+1)}{f} - ad^2x + \frac{ad^2\tan(e+fx)}{f} + \frac{bc^2\log(\tan^2(e+fx)+1)}{2f} - 2bcdx + \frac{2bcd\tan(e+fx)}{f} - \frac{bd^2\log(\tan^2(e+fx)+1)}{2f} + \frac{bd^2\tan^2(e+fx)}{2f} & \text{for } f \neq 0 \\ x(a+b\tan(e))(c+d\tan(e))^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))**2,x)
```

```
[Out] Piecewise((a*c**2*x + a*c*d*log(tan(e + f*x)**2 + 1)/f - a*d**2*x + a*d**2*
tan(e + f*x)/f + b*c**2*log(tan(e + f*x)**2 + 1)/(2*f) - 2*b*c*d*x + 2*b*c*
d*tan(e + f*x)/f - b*d**2*log(tan(e + f*x)**2 + 1)/(2*f) + b*d**2*tan(e + f
*x)**2/(2*f), Ne(f, 0)), (x*(a + b*tan(e))*(c + d*tan(e))**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 968 vs. 2(90) = 180.
time = 0.75, size = 968, normalized size = 10.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*a*c^2*f*x*tan(f*x)^2*tan(e)^2 - 4*b*c*d*f*x*tan(f*x)^2*tan(e)^2 - 2*
a*d^2*f*x*tan(f*x)^2*tan(e)^2 - b*c^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*
x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(ta
```

```
n(e)^2 + 1))*tan(f*x)^2*tan(e)^2 - 2*a*c*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*t
an(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1
)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 + b*d^2*log(4*(tan(f*x)^4*tan(e)^2 -
2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e)
+ 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 - 4*a*c^2*f*x*tan(f*x)*tan(e) + 8*
b*c*d*f*x*tan(f*x)*tan(e) + 4*a*d^2*f*x*tan(f*x)*tan(e) + b*d^2*tan(f*x)^2*
tan(e)^2 + 2*b*c^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f
*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*
x)*tan(e) + 4*a*c*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(
f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f
*x)*tan(e) - 2*b*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan
(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(
f*x)*tan(e) - 4*b*c*d*tan(f*x)^2*tan(e) - 2*a*d^2*tan(f*x)^2*tan(e) - 4*b*c
*d*tan(f*x)*tan(e)^2 - 2*a*d^2*tan(f*x)*tan(e)^2 + 2*a*c^2*f*x - 4*b*c*d*f*
x - 2*a*d^2*f*x + b*d^2*tan(f*x)^2 + b*d^2*tan(e)^2 - b*c^2*log(4*(tan(f*x)
^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*ta
n(f*x)*tan(e) + 1)/(tan(e)^2 + 1)) - 2*a*c*d*log(4*(tan(f*x)^4*tan(e)^2 - 2
*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) +
 1)/(tan(e)^2 + 1)) + b*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e
) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1
)) + 4*b*c*d*tan(f*x) + 2*a*d^2*tan(f*x) + 4*b*c*d*tan(e) + 2*a*d^2*tan(e)
+ b*d^2)/(f*tan(f*x)^2*tan(e)^2 - 2*f*tan(f*x)*tan(e) + f)
```

Mupad [B]
time = 5.18, size = 91, normalized size = 1.02

$$\frac{\tan(e+f\,x)\,\left(a\,d^2+2\,b\,c\,d\right)}{f}-x\,\left(-a\,c^2+2\,b\,c\,d+a\,d^2\right)+\frac{\ln\left(\tan(e+f\,x)^2+1\right)\,\left(\frac{b\,c^2}{2}+a\,c\,d-\frac{b\,d^2}{2}\right)}{f}+\frac{b\,d^2\,\tan(e+f\,x)^2}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))*(c + d*tan(e + f*x))^2,x)
[Out] (tan(e + f*x)*(a*d^2 + 2*b*c*d))/f - x*(a*d^2 - a*c^2 + 2*b*c*d) + (log(tan
(e + f*x)^2 + 1)*((b*c^2)/2 - (b*d^2)/2 + a*c*d))/f + (b*d^2*tan(e + f*x)^2
)/(2*f)
```

3.1200 $\int \frac{(c+d\tan(e+fx))^2}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=103

$$\frac{a(bc-ad)^2x}{b^2(a^2+b^2)}+\frac{d(2bc-ad)x}{b^2}-\frac{d^2\log(\cos(e+fx))}{bf}+\frac{(bc-ad)^2\log(a\cos(e+fx)+b\sin(e+fx))}{b(a^2+b^2)f}$$

```
[Out] a*(-a*d+b*c)^2*x/b^2/(a^2+b^2)+d*(-a*d+2*b*c)*x/b^2-d^2*ln(cos(f*x+e))/b/f+
(-a*d+b*c)^2*ln(a*cos(f*x+e)+b*sin(f*x+e))/b/(a^2+b^2)/f
```

Rubi [A]
time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3622, 3556, 3565, 3611}

$$\frac{(bc-ad)^2\log(a\cos(e+fx)+b\sin(e+fx))}{bf(a^2+b^2)}+\frac{ax(bc-ad)^2}{b^2(a^2+b^2)}+\frac{dx(2bc-ad)}{b^2}-\frac{d^2\log(\cos(e+fx))}{bf}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^2/(a + b*Tan[e + f*x]),x]
```

```
[Out] (a*(b*c - a*d)^2*x)/(b^2*(a^2 + b^2)) + (d*(2*b*c - a*d)*x)/b^2 - (d^2*Log[
Cos[e + f*x]])/(b*f) + ((b*c - a*d)^2*Log[a*Cos[e + f*x] + b*Sin[e + f*x]])
/(b*(a^2 + b^2)*f)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3565

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3622

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^2/((a_.) + (b_.)*tan[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[d*(2*b*c - a*d)*(x/b^2), x] + (Dist[d^2/b, In
```

```
t[Tan[e + f*x], x], x] + Dist[(b*c - a*d)^2/b^2, Int[1/(a + b*Tan[e + f*x])
, x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 +
 b^2, 0]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^2}{a+b\tan(e+fx)}\,dx = \frac{d(2bc-ad)x}{b^2} + \frac{d^2\int\tan(e+fx)\,dx}{b} + \frac{(bc-ad)^2\int\frac{1}{a+b\tan(e+fx)}\,dx}{b^2}$$

$$= \frac{a(bc-ad)^2x}{b^2(a^2+b^2)} + \frac{d(2bc-ad)x}{b^2} - \frac{d^2\log(\cos(e+fx))}{bf} + \frac{(bc-ad)^2\int\frac{b-a\tan(e+fx)}{a+b\tan(e+fx)}}{b(a^2+b^2)}$$

$$= \frac{a(bc-ad)^2x}{b^2(a^2+b^2)} + \frac{d(2bc-ad)x}{b^2} - \frac{d^2\log(\cos(e+fx))}{bf} + \frac{(bc-ad)^2\log(a\cos(e+}{b(a^2+}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.17, size = 108, normalized size = 1.05

$$\frac{\frac{(c+id)^2\log(i-\tan(e+fx))}{ia-b} - \frac{(c-id)^2\log(i+\tan(e+fx))}{ia+b} + \frac{2(bc-ad)^2\log(a+b\tan(e+fx))}{b(a^2+b^2)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^2/(a + b*Tan[e + f*x]),x]
[Out] (((c + I*d)^2*Log[I - Tan[e + f*x]])/(I*a - b) - ((c - I*d)^2*Log[I + Tan[e
 + f*x]])/(I*a + b) + (2*(b*c - a*d)^2*Log[a + b*Tan[e + f*x]])/(b*(a^2 + b
^2)))/(2*f)
```

Maple [A]
time = 0.20, size = 117, normalized size = 1.14

method	result
derivativedivides	$\frac{\frac{(a^2d^2-2abcd+b^2c^2)\ln(a+b\tan(fx+e))}{(a^2+b^2)b} + \frac{\frac{(2acd-bc^2+bd^2)\ln(1+\tan^2(fx+e))}{2} + (ac^2-ad^2+2bcd)\arctan(\tan(fx+e))}{a^2+b^2}}{f}$
default	$\frac{\frac{(a^2d^2-2abcd+b^2c^2)\ln(a+b\tan(fx+e))}{(a^2+b^2)b} + \frac{\frac{(2acd-bc^2+bd^2)\ln(1+\tan^2(fx+e))}{2} + (ac^2-ad^2+2bcd)\arctan(\tan(fx+e))}{a^2+b^2}}{f}$
norman	$\frac{(ac^2-ad^2+2bcd)x}{a^2+b^2} + \frac{(a^2d^2-2abcd+b^2c^2)\ln(a+b\tan(fx+e))}{(a^2+b^2)bf} + \frac{(2acd-bc^2+bd^2)\ln(1+\tan^2(fx+e))}{2f(a^2+b^2)}$
risch	$\frac{2ixcd}{ib-a} - \frac{xc^2}{ib-a} + \frac{xd^2}{ib-a} - \frac{2ia^2d^2x}{(a^2+b^2)b} - \frac{2ia^2d^2e}{(a^2+b^2)bf} + \frac{4iacdx}{a^2+b^2} + \frac{4iacde}{(a^2+b^2)f} - \frac{2ibc^2x}{a^2+b^2} - \frac{2ibc^2e}{(a^2+b^2)f} + \frac{2id^2x}{b} +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^2/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*((a^2*d^2-2*a*b*c*d+b^2*c^2)/(a^2+b^2)/b*ln(a+b*tan(f*x+e))+1/(a^2+b^2)
*(1/2*(2*a*c*d-b*c^2+b*d^2)*ln(1+tan(f*x+e)^2)+(a*c^2-a*d^2+2*b*c*d)*arctan
(tan(f*x+e))))
```

Maxima [A]
time = 0.56, size = 126, normalized size = 1.22

$$\frac{\frac{2\,(ac^2+2\,bcd-ad^2)(fx+e)}{a^2+b^2}+\frac{2\,(b^2c^2-2\,abcd+a^2d^2)\log(b\tan(fx+e)+a)}{a^2b+b^3}-\frac{(bc^2-2\,acd-bd^2)\log\left(\tan(fx+e)^2+1\right)}{a^2+b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a*c^2 + 2*b*c*d - a*d^2)*(f*x + e)/(a^2 + b^2) + 2*(b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*log(b*tan(f*x + e) + a)/(a^2*b + b^3) - (b*c^2 - 2*a*c*d -
b*d^2)*log(tan(f*x + e)^2 + 1)/(a^2 + b^2))/f
```

Fricas [A]
time = 0.70, size = 133, normalized size = 1.29

$$-\frac{(a^2+b^2)d^2\log\left(\frac{1}{\tan(fx+e)^2+1}\right)-2\,(abc^2+2\,b^2cd-abd^2)fx-(b^2c^2-2\,abcd+a^2d^2)\log\left(\frac{b^2\tan(fx+e)^2+2\,ab\tan(fx+e)+a^2}{\tan(fx+e)^2+1}\right)}{2\,(a^2b+b^3)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/2*((a^2 + b^2)*d^2*log(1/(tan(f*x + e)^2 + 1)) - 2*(a*b*c^2 + 2*b^2*c*d
- a*b*d^2)*f*x - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log((b^2*tan(f*x + e)^2 +
2*a*b*tan(f*x + e) + a^2)/(tan(f*x + e)^2 + 1)))/((a^2*b + b^3)*f)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.64, size = 1025, normalized size = 9.95

$$\begin{cases} \frac{\tilde{\infty}x(c+d\tan(e))^2}{\tan(e)} & \text{for}\ a=0\wedge b=0\wedge f=0 \\ \frac{c^2x+\frac{cd\log\left(\tan^2(e+fx)+1\right)}{f}-d^2x+\frac{d^2\tan(e+fx)}{f}}{a} & \text{for}\ b=0 \\ \frac{ic^2fx\tan(e+fx)}{2bf\tan(e+fx)-2ibf}+\frac{c^2fx}{2bf\tan(e+fx)-2ibf}+\frac{ic^2}{2bf\tan(e+fx)-2ibf}+\frac{2cdfx\tan(e+fx)}{2bf\tan(e+fx)-2ibf}-\frac{2icdfx}{2bf\tan(e+fx)-2ibf}-\frac{2cd}{2bf\tan(e+fx)-2ibf}+\frac{id^2fx\tan(e+fx)}{2bf\tan(e+fx)-2ibf}+\frac{d^2fx}{2bf\tan(e+fx)-2ibf}+\frac{d^2\log\left(\tan^2(e+fx)+1\right)\tan(e+fx)}{2bf\tan(e+fx)-2ibf}-\frac{id^2\log\left(\tan^2(e+fx)+1\right)}{2bf\tan(e+fx)-2ibf}-\frac{id^2}{2bf\tan(e+fx)-2ibf} & \text{for}\ a=-ib \\ -\frac{ic^2fx\tan(e+fx)}{2bf\tan(e+fx)+2ibf}+\frac{c^2fx}{2bf\tan(e+fx)+2ibf}-\frac{ic^2}{2bf\tan(e+fx)+2ibf}+\frac{2cdfx\tan(e+fx)}{2bf\tan(e+fx)+2ibf}+\frac{2icdfx}{2bf\tan(e+fx)+2ibf}-\frac{2cd}{2bf\tan(e+fx)+2ibf}-\frac{id^2fx\tan(e+fx)}{2bf\tan(e+fx)+2ibf}+\frac{d^2fx}{2bf\tan(e+fx)+2ibf}+\frac{d^2\log\left(\tan^2(e+fx)+1\right)\tan(e+fx)}{2bf\tan(e+fx)+2ibf}+\frac{id^2\log\left(\tan^2(e+fx)+1\right)}{2bf\tan(e+fx)+2ibf}+\frac{id^2}{2bf\tan(e+fx)+2ibf} & \text{for}\ a=ib \\ \frac{x(c+d\tan(e))^2}{a+b\tan(e)} & \text{for}\ f=0 \\ \frac{2a^2d^2\log\left(\frac{a}{b}+\tan(e+fx)\right)}{2a^2bf+2b^3f}+\frac{2abc^2fx}{2a^2bf+2b^3f}-\frac{4abcd\log\left(\frac{a}{b}+\tan(e+fx)\right)}{2a^2bf+2b^3f}+\frac{2abcd\log\left(\tan^2(e+fx)+1\right)}{2a^2bf+2b^3f}-\frac{2abd^2fx}{2a^2bf+2b^3f}+\frac{2b^2c^2\log\left(\frac{a}{b}+\tan(e+fx)\right)}{2a^2bf+2b^3f}-\frac{b^2c^2\log\left(\tan^2(e+fx)+1\right)}{2a^2bf+2b^3f}+\frac{4b^2cdfx}{2a^2bf+2b^3f}+\frac{b^2d^2\log\left(\tan^2(e+fx)+1\right)}{2a^2bf+2b^3f} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**2/(a+b*tan(f*x+e)),x)
```

```
[Out] Piecewise((zoo*x*(c + d*tan(e))**2/tan(e), Eq(a, 0) & Eq(b, 0) & Eq(f, 0)),
 ((c**2*x + c*d*log(tan(e + f*x)**2 + 1)/f - d**2*x + d**2*tan(e + f*x)/f)/
a, Eq(b, 0)), (I*c**2*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) - 2*I*b*f) + c**
2*f*x/(2*b*f*tan(e + f*x) - 2*I*b*f) + I*c**2/(2*b*f*tan(e + f*x) - 2*I*b*f
) + 2*c*d*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) - 2*I*b*f) - 2*I*c*d*f*x/(2*
b*f*tan(e + f*x) - 2*I*b*f) - 2*c*d/(2*b*f*tan(e + f*x) - 2*I*b*f) + I*d**2
*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) - 2*I*b*f) + d**2*f*x/(2*b*f*tan(e +
f*x) - 2*I*b*f) + d**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*b*f*tan(e +
 f*x) - 2*I*b*f) - I*d**2*log(tan(e + f*x)**2 + 1)/(2*b*f*tan(e + f*x) - 2*
I*b*f) - I*d**2/(2*b*f*tan(e + f*x) - 2*I*b*f), Eq(a, -I*b)), (-I*c**2*f*x*
tan(e + f*x)/(2*b*f*tan(e + f*x) + 2*I*b*f) + c**2*f*x/(2*b*f*tan(e + f*x)
+ 2*I*b*f) - I*c**2/(2*b*f*tan(e + f*x) + 2*I*b*f) + 2*c*d*f*x*tan(e + f*x)
/(2*b*f*tan(e + f*x) + 2*I*b*f) + 2*I*c*d*f*x/(2*b*f*tan(e + f*x) + 2*I*b*f
) - 2*c*d/(2*b*f*tan(e + f*x) + 2*I*b*f) - I*d**2*f*x*tan(e + f*x)/(2*b*f*t
an(e + f*x) + 2*I*b*f) + d**2*f*x/(2*b*f*tan(e + f*x) + 2*I*b*f) + d**2*log
(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*b*f*tan(e + f*x) + 2*I*b*f) + I*d**2*
log(tan(e + f*x)**2 + 1)/(2*b*f*tan(e + f*x) + 2*I*b*f) + I*d**2/(2*b*f*tan
(e + f*x) + 2*I*b*f), Eq(a, I*b)), (x*(c + d*tan(e))**2/(a + b*tan(e)), Eq(
f, 0)), (2*a**2*d**2*log(a/b + tan(e + f*x))/(2*a**2*b*f + 2*b**3*f) + 2*a*
b*c**2*f*x/(2*a**2*b*f + 2*b**3*f) - 4*a*b*c*d*log(a/b + tan(e + f*x))/(2*a
**2*b*f + 2*b**3*f) + 2*a*b*c*d*log(tan(e + f*x)**2 + 1)/(2*a**2*b*f + 2*b*
*3*f) - 2*a*b*d**2*f*x/(2*a**2*b*f + 2*b**3*f) + 2*b**2*c**2*log(a/b + tan(
e + f*x))/(2*a**2*b*f + 2*b**3*f) - b**2*c**2*log(tan(e + f*x)**2 + 1)/(2*a
**2*b*f + 2*b**3*f) + 4*b**2*c*d*f*x/(2*a**2*b*f + 2*b**3*f) + b**2*d**2*lo
g(tan(e + f*x)**2 + 1)/(2*a**2*b*f + 2*b**3*f), True))
```

Giac [A]
time = 0.57, size = 127, normalized size = 1.23

$$\frac{\frac{2\,(ac^2+2\,bcd-ad^2)(fx+e)}{a^2+b^2}-\frac{(bc^2-2\,acd-bd^2)\log\left(\tan(fx+e)^2+1\right)}{a^2+b^2}+\frac{2\,(b^2c^2-2\,abcd+a^2d^2)\log(|b\tan(fx+e)+a|)}{a^2b+b^3}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2/(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*(2*(a*c^2 + 2*b*c*d - a*d^2)*(f*x + e)/(a^2 + b^2) - (b*c^2 - 2*a*c*d -
 b*d^2)*log(tan(f*x + e)^2 + 1)/(a^2 + b^2) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*
d^2)*log(abs(b*tan(f*x + e) + a))/(a^2*b + b^3))/f
```

Mupad [B]
time = 5.73, size = 115, normalized size = 1.12

$$\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(-c^2\,1\mathrm{i}+2\,c\,d+d^2\,1\mathrm{i}\right)}{2\,f\,\left(a+b\,1\mathrm{i}\right)}+\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(-c^2+c\,d\,2\mathrm{i}+d^2\right)}{2\,f\,\left(b+a\,1\mathrm{i}\right)}+\frac{\ln\left(a+b\tan(e+f\,x)\right)\,{\left(a\,d-b\,c\right)}^2}{b\,f\,\left(a^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^2/(a + b*tan(e + f*x)),x)
```

```
[Out] (log(tan(e + f*x) - 1i)*(2*c*d - c^2*1i + d^2*1i))/(2*f*(a + b*1i)) + (log(
tan(e + f*x) + 1i)*(c*d*2i - c^2 + d^2))/(2*f*(a*1i + b)) + (log(a + b*tan(
e + f*x))*(a*d - b*c)^2)/(b*f*(a^2 + b^2))
```

3.1201 $\int \frac{(c+d\tan(e+fx))^2}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=126

$$-\frac{(b(c-d)-a(c+d))(a(c-d)+b(c+d))x}{(a^2+b^2)^2}+\frac{2(bc-ad)(ac+bd)\log(a\cos(e+fx)+b\sin(e+fx))}{(a^2+b^2)^2 f}-\frac{}{b\,(a^2}$$

```
[Out] -(b*(c-d)-a*(c+d))*(a*(c-d)+b*(c+d))*x/(a^2+b^2)^2+2*(-a*d+b*c)*(a*c+b*d)*l
n(a*cos(f*x+e)+b*sin(f*x+e))/(a^2+b^2)^2/f-(-a*d+b*c)^2/b/(a^2+b^2)/f/(a+b*
tan(f*x+e))
```

Rubi [A]
time = 0.16, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3623, 3612, 3611}

$$-\frac{(bc-ad)^2}{bf\,(a^2+b^2)\,(a+b\tan(e+fx))}+\frac{2(ac+bd)(bc-ad)\log(a\cos(e+fx)+b\sin(e+fx))}{f\,(a^2+b^2)^2}-\frac{x(b(c-d)-a(c+d))(a(c-d)+b(c+d))}{(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^2/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] -(((b*(c - d) - a*(c + d))*(a*(c - d) + b*(c + d))*x)/(a^2 + b^2)^2) + (2*(
b*c - a*d)*(a*c + b*d)*Log[a*Cos[e + f*x] + b*Sin[e + f*x]])/((a^2 + b^2)^2
*f) - (b*c - a*d)^2/(b*(a^2 + b^2)*f*(a + b*Tan[e + f*x]))
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
```

```
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^2}{(a+b\tan(e+fx))^2}\,dx = -\frac{(bc-ad)^2}{b\,(a^2+b^2)\,f(a+b\tan(e+fx))} + \frac{\int \frac{2bcd+a(c^2-d^2)+(2acd-b(c^2-d^2))\tan(e+fx)}{a+b\tan(e+fx)}\,dx}{a^2+b^2}$$

$$= -\frac{(b(c-d)-a(c+d))(a(c-d)+b(c+d))x}{(a^2+b^2)^2} - \frac{(bc-ad)^2}{b\,(a^2+b^2)\,f(a+b\tan(e+f$$

$$= -\frac{(b(c-d)-a(c+d))(a(c-d)+b(c+d))x}{(a^2+b^2)^2} + \frac{2(bc-ad)(ac+bd)\log(a\cos}{(a^2+b^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.15, size = 321, normalized size = 2.55

$$\frac{(a\cos(e+fx)+b\sin(e+fx))\left(\frac{(a^2+b^2)(bc-ad)^2\sin(e+fx)}{a}+(b(-c+d)+a(c+d))(a(c-d)+b(c+d))(e+fx)(a\cos(e+fx)+b\sin(e+fx))-2i(a^2cd-b^2cd+ab(-c^2+d^2))(e+fx)(a\cos(e+fx)+b\sin(e+fx))+2i(a^2cd-b^2cd+ab(-c^2+d^2))\operatorname{ArcTan}(\tan(e+fx))(a\cos(e+fx)+b\sin(e+fx))-(a^2cd-b^2cd+ab(-c^2+d^2))\log\left((a\cos(e+fx)+b\sin(e+fx))^2\right)(a\cos(e+fx)+b\sin(e+fx))\right)(c+d\tan(e+fx))^2}{(a^2+b^2)^2 f(c\cos(e+fx)+d\sin(e+fx))^2(a+b\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^2/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] ((a*Cos[e + f*x] + b*Sin[e + f*x])*(((a^2 + b^2)*(b*c - a*d)^2*Sin[e + f*x]
)/a + (b*(-c + d) + a*(c + d))*(a*(c - d) + b*(c + d))*(e + f*x)*(a*Cos[e +
 f*x] + b*Sin[e + f*x]) - (2*I)*(a^2*c*d - b^2*c*d + a*b*(-c^2 + d^2))*(e +
 f*x)*(a*Cos[e + f*x] + b*Sin[e + f*x]) + (2*I)*(a^2*c*d - b^2*c*d + a*b*(-
c^2 + d^2))*ArcTan[Tan[e + f*x]]*(a*Cos[e + f*x] + b*Sin[e + f*x]) - (a^2*c
*d - b^2*c*d + a*b*(-c^2 + d^2))*Log[(a*Cos[e + f*x] + b*Sin[e + f*x])^2]*(
a*Cos[e + f*x] + b*Sin[e + f*x]))*(c + d*Tan[e + f*x])^2)/((a^2 + b^2)^2*f*
(c*Cos[e + f*x] + d*Sin[e + f*x])^2*(a + b*Tan[e + f*x])^2)
```

Maple [A]
time = 0.22, size = 200, normalized size = 1.59

method	result
derivativedivides	$\frac{-\frac{a^2d^2-2abcd+b^2c^2}{(a^2+b^2)b(a+b\tan(fx+e))}-\frac{2(a^2cd-ab\,c^2+ab\,d^2-b^2cd)\ln(a+b\tan(fx+e))}{(a^2+b^2)^2}+\frac{\frac{(2a^2cd-2ab\,c^2+2ab\,d^2-2b^2cd)\ln(1+\tan^2(fx+e)}{2}}{(a}}{f}$
default	$\frac{-\frac{a^2d^2-2abcd+b^2c^2}{(a^2+b^2)b(a+b\tan(fx+e))}-\frac{2(a^2cd-ab\,c^2+ab\,d^2-b^2cd)\ln(a+b\tan(fx+e))}{(a^2+b^2)^2}+\frac{\frac{(2a^2cd-2ab\,c^2+2ab\,d^2-2b^2cd)\ln(1+\tan^2(fx+e)}{2}}{(a}}{f}$

norman	$\frac{\frac{a\left(a^2c^2-a^2d^2+4abcd-b^2c^2+b^2d^2\right)x}{a^4+2a^2b^2+b^4}+\frac{b\left(a^2c^2-a^2d^2+4abcd-b^2c^2+b^2d^2\right)x\tan(fx+e)}{a^4+2a^2b^2+b^4}+\frac{\left(a^2d^2-2abcd+b^2c^2\right)\tan(fx+e)}{af\left(a^2+b^2\right)}}{a+b\tan(fx+e)}+\frac{(a^2cd}{}$
risch	$\frac{4ia^2cde}{f(a^4+2a^2b^2+b^4)}-\frac{x\,c^2}{2iab-a^2+b^2}+\frac{x\,d^2}{2iab-a^2+b^2}+\frac{2ixcd}{2iab-a^2+b^2}+\frac{2ia^2d^2}{(ib+a)f(-ib+a)^2(-ie^{2i(fx+e)}b+e^{2i(fx+e)}a+ib}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^2/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a^2+b^2)/b/(a+b*tan(f*x+e))-2*(a^2*c*d-a
*b*c^2+a*b*d^2-b^2*c*d)/(a^2+b^2)^2*ln(a+b*tan(f*x+e))+1/(a^2+b^2)^2*(1/2*(
2*a^2*c*d-2*a*b*c^2+2*a*b*d^2-2*b^2*c*d)*ln(1+tan(f*x+e)^2)+(a^2*c^2-a^2*d^
2+4*a*b*c*d-b^2*c^2+b^2*d^2)*arctan(tan(f*x+e))))
```

Maxima [A]
time = 0.57, size = 234, normalized size = 1.86

$$\frac{\frac{(4\,abcd+(a^2-b^2)c^2-(a^2-b^2)d^2)(fx+e)}{a^4+2\,a^2b^2+b^4}+\frac{2\,(abc^2-abd^2-(a^2-b^2)cd)\log(b\tan(fx+e)+a)}{a^4+2\,a^2b^2+b^4}-\frac{(abc^2-abd^2-(a^2-b^2)cd)\log\left(\tan(fx+e)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{b^2c^2-2\,abcd+a^2d^2}{a^3b+ab^3+(a^2b^2+b^4)\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] ((4*a*b*c*d + (a^2 - b^2)*c^2 - (a^2 - b^2)*d^2)*(f*x + e)/(a^4 + 2*a^2*b^2
 + b^4) + 2*(a*b*c^2 - a*b*d^2 - (a^2 - b^2)*c*d)*log(b*tan(f*x + e) + a)/(
a^4 + 2*a^2*b^2 + b^4) - (a*b*c^2 - a*b*d^2 - (a^2 - b^2)*c*d)*log(tan(f*x
+ e)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(a^3*
b + a*b^3 + (a^2*b^2 + b^4)*tan(f*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(128) = 256.
time = 0.93, size = 310, normalized size = 2.46

$$-\frac{b^3c^2-2\,ab^2cd+a^2bd^2-(4\,a^2bcd+(a^3-ab^2)c^2-(a^3-ab^2)d^2)fx-(a^2bc^2-a^2bd^2-(a^3-ab^2)cd+(ab^2c^2-ab^2d^2-(a^2b-b^3)cd)\tan(fx+e))\log\left(\frac{b^2\tan(fx+e)^2+2\,ab\tan(fx+e)+a^2}{\tan(fx+e)^2+1}\right)-(ab^2c^2-2\,a^2bcd+a^3d^2+(4\,ab^2cd+(a^2b-b^3)c^2-(a^2b-b^3)d^2)fx)\tan(fx+e)}{(a^4b+2\,a^2b^3+b^5)f\tan(fx+e)+(a^5+2\,a^3b^2+ab^4)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2 - (4*a^2*b*c*d + (a^3 - a*b^2)*c^2 - (a
^3 - a*b^2)*d^2)*f*x - (a^2*b*c^2 - a^2*b*d^2 - (a^3 - a*b^2)*c*d + (a*b^2*
c^2 - a*b^2*d^2 - (a^2*b - b^3)*c*d)*tan(f*x + e))*log((b^2*tan(f*x + e)^2
+ 2*a*b*tan(f*x + e) + a^2)/(tan(f*x + e)^2 + 1)) - (a*b^2*c^2 - 2*a^2*b*c*
d + a^3*d^2 + (4*a*b^2*c*d + (a^2*b - b^3)*c^2 - (a^2*b - b^3)*d^2)*f*x)*ta
n(f*x + e))/((a^4*b + 2*a^2*b^3 + b^5)*f*tan(f*x + e) + (a^5 + 2*a^3*b^2 +
a*b^4)*f)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.96, size = 4258, normalized size = 33.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**2/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Piecewise((zoo*x*(c + d*tan(e))**2/tan(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0
)), ((c**2*x + c*d*log(tan(e + f*x)**2 + 1)/f - d**2*x + d**2*tan(e + f*x)/
f)/a**2, Eq(b, 0)), (-c**2*f*x*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 -
8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 2*I*c**2*f*x*tan(e + f*x)/(4*b**2*f*t
an(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + c**2*f*x/(4*b**2*f*t
an(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - c**2*tan(e + f*x)/(4
*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 2*I*c**2/(4
*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 2*I*c*d*f*x
*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b*
*2*f) + 4*c*d*f*x*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e
 + f*x) - 4*b**2*f) - 2*I*c*d*f*x/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*ta
n(e + f*x) - 4*b**2*f) + 2*I*c*d*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8
*I*b**2*f*tan(e + f*x) - 4*b**2*f) + d**2*f*x*tan(e + f*x)**2/(4*b**2*f*tan
(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 2*I*d**2*f*x*tan(e + f
*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - d**2*
f*x/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 3*d**
2*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2
*f) + 2*I*d**2/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2
*f), Eq(a, -I*b)), (-c**2*f*x*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 + 8
*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 2*I*c**2*f*x*tan(e + f*x)/(4*b**2*f*ta
n(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + c**2*f*x/(4*b**2*f*ta
n(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - c**2*tan(e + f*x)/(4*
b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 2*I*c**2/(4*
b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 2*I*c*d*f*x*
tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**
2*f) + 4*c*d*f*x*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e
+ f*x) - 4*b**2*f) + 2*I*c*d*f*x/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan
(e + f*x) - 4*b**2*f) - 2*I*c*d*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 + 8*
I*b**2*f*tan(e + f*x) - 4*b**2*f) + d**2*f*x*tan(e + f*x)**2/(4*b**2*f*tan(
e + f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 2*I*d**2*f*x*tan(e + f*
x)/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - d**2*f
*x/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 3*d**2
*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*
f) - 2*I*d**2/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*
f), Eq(a, I*b)), (x*(c + d*tan(e))**2/(a + b*tan(e))**2, Eq(f, 0)), (-a**4*
d**2/(a**5*b*f + a**4*b**2*f*tan(e + f*x) + 2*a**3*b**3*f + 2*a**2*b**4*f*t
an(e + f*x) + a*b**5*f + b**6*f*tan(e + f*x)) + a**3*b*c**2*f*x/(a**5*b*f +
```

```
 a**4*b**2*f*tan(e + f*x) + 2*a**3*b**3*f + 2*a**2*b**4*f*tan(e + f*x) + a*
b**5*f + b**6*f*tan(e + f*x)) - 2*a**3*b*c*d*log(a/b + tan(e + f*x))/(a**5*
b*f + a**4*b**2*f*tan(e + f*x) + 2*a**3*b**3*f + 2*a**2*b**4*f*tan(e + f*x)
 + a*b**5*f + b**6*f*tan(e + f*x)) + a**3*b*c*d*log(tan(e + f*x)**2 + 1)/(a
**5*b*f + a**4*b**2*f*tan(e + f*x) + 2*a**3*b**3*f + 2*a**2*b**4*f*tan(e +
f*x) + a*b**5*f + b**6*f*tan(e + f*x)) + 2*a**3*b*c*d/(a**5*b*f + a**4*b**2
*f*tan(e + f*x) + 2*a**3*b**3*f + 2*a**2*b**4*f*tan(e + f*x) + a*b**5*f + b
**6*f*tan(e + f*x)) - a**3*b*d**2*f*x/(a**5*b*f + a**4*b**2*f*tan(e + f*x)
+ 2*a**3*b**3*f + 2*a**2*b**4*f*tan(e + f*x) + a*b**5*f + b**6*f*tan(e + f*
x)) + a**2*b**2*c**2*f*x*tan(e + f*x)/(a**5*b*f + a**4*b**2*f*tan(e + f*x)
+ 2*a**3*b**3*f + 2*a**2*b**4*f*tan(e + f*x) + a*b**5*f + b**6*f*tan(e + f*
x)) + 2*a**2*b**2*c**2*log(a/b + tan(e + f*x))/(a**5*b*f + a**4*b**2*f*tan(
e + f*x) + 2*a**3*b**3*f + 2*a**2*b**4*f*tan(e + f*x) + a*b**5*f + b**6*f*t
an(e + f*x)) - a**2*b**2*c**2*log(tan(e + f*x)**2 + 1)/(a**5*b*f + a**4*b**
2*f*tan(e + f*x) + 2*a**3*b**3*f + 2*a**2*b**4*f*tan(e + f*x) + a*b**5*f +
b**6*f*tan(e + f*x)) - a**2*b**2*c**2/(a**5*b*f + a**4*b**2*f*tan(e + f*x)
+ 2*a**3*b**3*f + 2*a**2*b**4*f*tan(e + f*x) + a*b**5*f + b**6*f*tan(e + f*
x)) + 4*a**2*b**2*c*d*f*x/(a**5*b*f + a**4*b**2*f*tan(e + f*x) + 2*a**3*b**
3*f + 2*a**2*b**4*f*tan(e + f*x) + a*b**5*f + b**6*f*tan(e + f*x)) - 2*a**2
*b**2*c*d*log(a/b + tan(e + f*x))*tan(e + f*x)/(a**5*b*f + a**4*b**2*f*tan(
e + f*x) + 2*a**3*b**3*f + 2*a**2*b**4*f*tan(e + f*x) + a*b**5*f + b**6*f*t
an(e + f*x)) + a**2*b**2*c*d*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(a**5*b*
f + a**4*b**2*f*tan(e + f*x) + 2*a**3*b**3*f + 2*a**2*b**4*f*tan(e + f*x) +
 a*b**5*f + b**6*f*tan(e + f*x)) - a**2*b**2*d**2*f*x*tan(e + f*x)/(a**5*b*
f + a**4*b**2*f*tan(e + f*x) + 2*a**3*b**3*f + 2*a**2*b**4*f*tan(e + f*x) +
 a*b**5*f + b**6*f*tan(e + f*x)) - 2*a**2*b**2*d**2*log(a/b + tan(e + f*x))
/(a**5*b*f + a**4*b**2*f*tan(e + f*x) + 2*a**3*b**3*f + 2*a**2*b**4*f*tan(e
 + f*x) + a*b**5*f + b**6*f*tan(e + f*x)) + a**2*b**2*d**2*log(tan(e + f*x)
**2 + 1)/(a**5*b*f + a**4*b**2*f*tan(e + f*x) + 2*a**3*b**3*f + 2*a**2*b**4
*f*tan(e + f*x) + a*b**5*f + b**6*f*tan(e + f*x...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 332 vs. 2(128) = 256.
time = 0.63, size = 332, normalized size = 2.63

$$\frac{\frac{(a^2c^2-b^2c^2+4\,abcd-a^2d^2+b^2d^2)(fx+e)}{a^4+2\,a^2b^2+b^4}-\frac{(abc^2-a^2cd+b^2cd-abd^2)\log\left(\tan(fx+e)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\,(ab^2c^2-a^2bcd+b^3cd-ab^2d^2)\log(|b\tan(fx+e)+a|)}{a^4b+2\,a^2b^3+b^5}-\frac{2\,ab^3c^2\tan(fx+e)-2\,a^2b^2cd\tan(fx+e)+2\,b^4cd\tan(fx+e)-2\,ab^3d^2\tan(fx+e)+3\,a^2b^2c^2+b^4c^2-4\,a^3bcd+a^4d^2-a^2b^2d^2}{(a^4b+2\,a^2b^3+b^5)(b\tan(fx+e)+a)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] ((a^2*c^2 - b^2*c^2 + 4*a*b*c*d - a^2*d^2 + b^2*d^2)*(f*x + e)/(a^4 + 2*a^2
*b^2 + b^4) - (a*b*c^2 - a^2*c*d + b^2*c*d - a*b*d^2)*log(tan(f*x + e)^2 +
1)/(a^4 + 2*a^2*b^2 + b^4) + 2*(a*b^2*c^2 - a^2*b*c*d + b^3*c*d - a*b^2*d^2
)*log(abs(b*tan(f*x + e) + a))/(a^4*b + 2*a^2*b^3 + b^5) - (2*a*b^3*c^2*tan
(f*x + e) - 2*a^2*b^2*c*d*tan(f*x + e) + 2*b^4*c*d*tan(f*x + e) - 2*a*b^3*d
```

```
^2*tan(f*x + e) + 3*a^2*b^2*c^2 + b^4*c^2 - 4*a^3*b*c*d + a^4*d^2 - a^2*b^2
*d^2)/((a^4*b + 2*a^2*b^3 + b^5)*(b*tan(f*x + e) + a)))/f
```

Mupad [B]
time = 7.12, size = 208, normalized size = 1.65

$$\frac{\ln\left(a+b\tan(e+f\,x)\right)\,\left(-2\,c\,d\,a^2+\left(2\,c^2-2\,d^2\right)\,a\,b+2\,c\,d\,b^2\right)}{f\,\left(a^4+2\,a^2\,b^2+b^4\right)}-\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(c^2+c\,d\,2\mathrm{i}-d^2\right)}{2\,f\,\left(-a^2\,1\mathrm{i}+2\,a\,b+b^2\,1\mathrm{i}\right)}-\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(c^2\,1\mathrm{i}+2\,c\,d-d^2\,1\mathrm{i}\right)}{2\,f\,\left(-a^2+a\,b\,2\mathrm{i}+b^2\right)}-\frac{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}{b\,f\,\left(a^2+b^2\right)\,\left(a+b\tan\left(e+f\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^2/(a + b*tan(e + f*x))^2,x)
```

```
[Out] (log(a + b*tan(e + f*x))*(a*b*(2*c^2 - 2*d^2) - 2*a^2*c*d + 2*b^2*c*d))/(f*
(a^4 + b^4 + 2*a^2*b^2)) - (log(tan(e + f*x) - 1i)*(c*d*2i + c^2 - d^2))/(2
*f*(2*a*b - a^2*1i + b^2*1i)) - (log(tan(e + f*x) + 1i)*(2*c*d + c^2*1i - d
^2*1i))/(2*f*(a*b*2i - a^2 + b^2)) - (a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(b*f*(
a^2 + b^2)*(a + b*tan(e + f*x)))
```

3.1202 $\int \frac{(c+d\tan(e+fx))^2}{(a+b\tan(e+fx))^3}\,dx$

Optimal. Leaf size=214

$$\frac{(6a^2bcd-2b^3cd+a^3(c^2-d^2)-3ab^2(c^2-d^2))\,x}{(a^2+b^2)^3}-\frac{(2a^3cd-6ab^2cd-3a^2b(c^2-d^2)+b^3(c^2-d^2))\log(a\cos($$

```
[Out] (6*a^2*b*c*d-2*b^3*c*d+a^3*(c^2-d^2)-3*a*b^2*(c^2-d^2))*x/(a^2+b^2)^3-(2*a^
3*c*d-6*a*b^2*c*d-3*a^2*b*(c^2-d^2)+b^3*(c^2-d^2))*ln(a*cos(f*x+e)+b*sin(f*
x+e))/(a^2+b^2)^3/f-1/2*(-a*d+b*c)^2/b/(a^2+b^2)/f/(a+b*tan(f*x+e))^2-2*(-a
*d+b*c)*(a*c+b*d)/(a^2+b^2)^2/f/(a+b*tan(f*x+e))
```

Rubi [A]
time = 0.25, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3623, 3610, 3612, 3611}

$$-\frac{(bc-ad)^2}{2bf\,(a^2+b^2)\,(a+b\tan(e+fx))^2}-\frac{2(ac+bd)(bc-ad)}{f\,(a^2+b^2)^2\,(a+b\tan(e+fx))}-\frac{(2a^3cd-3a^2b(c^2-d^2)-6ab^2cd+b^3(c^2-d^2))\log(a\cos(e+fx)+b\sin(e+fx))}{f\,(a^2+b^2)^3}+\frac{x(a^3(c^2-d^2)+6a^2bcd-3ab^2(c^2-d^2)-2b^3cd)}{(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^2/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] ((6*a^2*b*c*d - 2*b^3*c*d + a^3*(c^2 - d^2) - 3*a*b^2*(c^2 - d^2))*x)/(a^2
+ b^2)^3 - ((2*a^3*c*d - 6*a*b^2*c*d - 3*a^2*b*(c^2 - d^2) + b^3*(c^2 - d^2
))*Log[a*Cos[e + f*x] + b*Sin[e + f*x]])/((a^2 + b^2)^3*f) - (b*c - a*d)^2/
(2*b*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^2) - (2*(b*c - a*d)*(a*c + b*d))/((
a^2 + b^2)^2*f*(a + b*Tan[e + f*x]))
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
```

```
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^2}{(a+b\tan(e+fx))^3}\,dx = -\frac{(bc-ad)^2}{2b\,(a^2+b^2)\,f(a+b\tan(e+fx))^2} + \frac{\int \frac{2bcd+a(c^2-d^2)+(2acd-b(c^2-d^2))\tan(e+fx)}{(a+b\tan(e+fx))^2}\,d}{a^2+b^2}$$

$$= -\frac{(bc-ad)^2}{2b\,(a^2+b^2)\,f(a+b\tan(e+fx))^2} - \frac{2(bc-ad)(ac+bd)}{(a^2+b^2)^2\,f(a+b\tan(e+fx))} + \frac{\int (a}{}$$

$$= \frac{(6a^2bcd-2b^3cd+a^3(c^2-d^2)-3ab^2(c^2-d^2))\,x}{(a^2+b^2)^3} - \frac{(bc-ad)^2}{2b\,(a^2+b^2)\,f(a+b\tan(e}$$

$$= \frac{(6a^2bcd-2b^3cd+a^3(c^2-d^2)-3ab^2(c^2-d^2))\,x}{(a^2+b^2)^3} - (2a^3cd-6ab^2cd-3a^2b(c$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.74, size = 291, normalized size = 1.36

$$\frac{\frac{bd(c+d\tan(e+fx))^2}{a+b\tan(e+fx)} - \frac{b^2(c+d\tan(e+fx))^3}{(a+b\tan(e+fx))^2} + (bc-ad)\left(\frac{(ia+b)^3(c+id)^2\log(i-\tan(e+fx))}{(a^2+b^2)^2} + \frac{i(a+ib)(c-id)^2\log(i+\tan(e+fx))}{(a-ib)^2} - \frac{2\left(2a^3cd-6ab^2cd+b^3(c^2-d^2)+3a^2b(-c^2+d^2)\right)\log(a+b\tan(e+fx))}{(a^2+b^2)^2} - \frac{2(bc-ad)(2abc-a^2d+b^2d)}{b(a^2+b^2)(a+b\tan(e+fx))}\right)}{2\,(a^2+b^2)\,(bc-ad)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^2/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] ((b*d*(c + d*Tan[e + f*x])^2)/(a + b*Tan[e + f*x]) - (b^2*(c + d*Tan[e + f*
x])^3)/(a + b*Tan[e + f*x])^2 + (b*c - a*d)*(((I*a + b)^3*(c + I*d)^2*Log[I
 - Tan[e + f*x]])/(a^2 + b^2)^2 + (I*(a + I*b)*(c - I*d)^2*Log[I + Tan[e +
f*x]])/(a - I*b)^2 - (2*(2*a^3*c*d - 6*a*b^2*c*d + b^3*(c^2 - d^2) + 3*a^2*
b*(-c^2 + d^2))*Log[a + b*Tan[e + f*x]])/(a^2 + b^2)^2 - (2*(b*c - a*d)*(2*
a*b*c - a^2*d + b^2*d))/(b*(a^2 + b^2)*(a + b*Tan[e + f*x]))))/(2*(a^2 + b^
2)*(b*c - a*d)*f)
```

Maple [A]
time = 0.26, size = 304, normalized size = 1.42 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^2/(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(a^2+b^2)^3*(1/2*(2*a^3*c*d-3*a^2*b*c^2+3*a^2*b*d^2-6*a*b^2*c*d+b^3*
c^2-b^3*d^2)*ln(1+tan(f*x+e)^2)+(a^3*c^2-a^3*d^2+6*a^2*b*c*d-3*a*b^2*c^2+3*
a*b^2*d^2-2*b^3*c*d)*arctan(tan(f*x+e)))-1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a
^2+b^2)/b/(a+b*tan(f*x+e))^2+2*(a^2*c*d-a*b*c^2+a*b*d^2-b^2*c*d)/(a^2+b^2)^
2/(a+b*tan(f*x+e))-(2*a^3*c*d-3*a^2*b*c^2+3*a^2*b*d^2-6*a*b^2*c*d+b^3*c^2-b
^3*d^2)/(a^2+b^2)^3*ln(a+b*tan(f*x+e)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 437 vs. 2(216) = 432.
time = 0.53, size = 437, normalized size = 2.04

$$\frac{\frac{2\left(\left(a^{3}-3ab^{2}\right)c^{2}+2\left(3a^{2}b-b^{3}\right)cd-\left(a^{3}-3ab^{2}\right)d^{2}\right)(fx+e)}{a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}}+\frac{2\left(\left(3a^{2}b-b^{3}\right)c^{2}-2\left(a^{3}-3ab^{2}\right)cd-\left(3a^{2}b-b^{3}\right)d^{2}\right)\log\left(b\tan\left(fx+e\right)+a\right)}{a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}}-\frac{\left(\left(3a^{2}b-b^{3}\right)c^{2}-2\left(a^{3}-3ab^{2}\right)cd-\left(3a^{2}b-b^{3}\right)d^{2}\right)\log\left(\tan\left(fx+e\right)^{2}+1\right)}{a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}}-\frac{\left(5a^{2}b^{2}+b^{4}\right)c^{2}-2\left(3a^{3}b-ab^{3}\right)cd+\left(a^{4}-3a^{2}b^{2}\right)d^{2}+4\left(ab^{3}c^{2}-ab^{3}d^{2}-\left(a^{2}b^{2}-b^{4}\right)cd\right)\tan\left(fx+e\right)}{a^{6}b+2a^{4}b^{3}+a^{2}b^{5}+\left(a^{4}b^{3}+2a^{2}b^{5}+b^{7}\right)\tan\left(fx+e\right)^{2}+2\left(a^{5}b^{2}+2a^{3}b^{4}+ab^{6}\right)\tan\left(fx+e\right)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*((a^3 - 3*a*b^2)*c^2 + 2*(3*a^2*b - b^3)*c*d - (a^3 - 3*a*b^2)*d^2)*
(f*x + e)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*((3*a^2*b - b^3)*c^2 - 2*
(a^3 - 3*a*b^2)*c*d - (3*a^2*b - b^3)*d^2)*log(b*tan(f*x + e) + a)/(a^6 + 3
*a^4*b^2 + 3*a^2*b^4 + b^6) - ((3*a^2*b - b^3)*c^2 - 2*(a^3 - 3*a*b^2)*c*d
- (3*a^2*b - b^3)*d^2)*log(tan(f*x + e)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6) - ((5*a^2*b^2 + b^4)*c^2 - 2*(3*a^3*b - a*b^3)*c*d + (a^4 - 3*a^2*b
^2)*d^2 + 4*(a*b^3*c^2 - a*b^3*d^2 - (a^2*b^2 - b^4)*c*d)*tan(f*x + e))/(a^
6*b + 2*a^4*b^3 + a^2*b^5 + (a^4*b^3 + 2*a^2*b^5 + b^7)*tan(f*x + e)^2 + 2*
(a^5*b^2 + 2*a^3*b^4 + a*b^6)*tan(f*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 715 vs. 2(216) = 432.
time = 1.03, size = 715, normalized size = 3.34

$$-\frac{\left(7a^{2}b^{3}+b^{5}\right)c^{2}-2\left(5a^{3}b^{2}-ab^{4}\right)cd+3\left(a^{4}b-a^{2}b^{3}\right)d^{2}-2\left(\left(a^{5}-3a^{3}b^{2}\right)c^{2}+2\left(3a^{4}b-a^{2}b^{3}\right)cd-\left(a^{5}-3a^{3}b^{2}\right)d^{2}\right)fx-\left(\left(5a^{2}b^{3}-b^{5}\right)c^{2}-6\left(a^{3}b^{2}-ab^{4}\right)cd+\left(a^{4}b-5a^{2}b^{3}\right)d^{2}+2\left(\left(a^{3}b^{2}-3ab^{4}\right)c^{2}+2\left(3a^{2}b^{3}-b^{5}\right)cd-\left(a^{3}b^{2}-3ab^{4}\right)d^{2}\right)fx\right)\tan\left(fx+e\right)^{2}-\left(\left(3a^{4}b-a^{2}b^{3}\right)c^{2}-2\left(a^{5}-3a^{3}b^{2}\right)cd-\left(3a^{4}b-a^{2}b^{3}\right)d^{2}+\left(\left(3a^{2}b^{3}-b^{5}\right)c^{2}-2\left(a^{3}b^{2}-3ab^{4}\right)cd-\left(3a^{2}b^{3}-b^{5}\right)d^{2}\right)\tan\left(fx+e\right)^{2}+2\left(\left(3a^{3}b^{2}-ab^{4}\right)c^{2}-2\left(a^{4}b-3a^{2}b^{3}\right)cd-\left(3a^{3}b^{2}-ab^{4}\right)d^{2}\right)\tan\left(fx+e\right)\right)\log\left(\frac{b^{2}\tan\left(fx+e\right)^{2}+2ab\tan\left(fx+e\right)+a^{2}}{\tan\left(fx+e\right)^{2}+1}\right)-2\left(3\left(a^{3}b^{2}-ab^{4}\right)c^{2}-2\left(2a^{4}b-3a^{2}b^{3}+b^{5}\right)cd+\left(a^{5}-3a^{3}b^{2}+2ab^{4}\right)d^{2}+2\left(\left(a^{4}b-3a^{2}b^{3}\right)c^{2}+2\left(3a^{3}b^{2}-ab^{4}\right)cd-\left(a^{4}b-3a^{2}b^{3}\right)d^{2}\right)fx\right)\tan\left(fx+e\right)}{2\left(\left(a^{6}b^{2}+3a^{4}b^{4}+3a^{2}b^{6}+b^{8}\right)f\tan\left(fx+e\right)^{2}+2\left(a^{7}b+3a^{5}b^{3}+3a^{3}b^{5}+ab^{7}\right)f\tan\left(fx+e\right)+\left(a^{8}+3a^{6}b^{2}+3a^{4}b^{4}+a^{2}b^{6}\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/2*((7*a^2*b^3 + b^5)*c^2 - 2*(5*a^3*b^2 - a*b^4)*c*d + 3*(a^4*b - a^2*b^
3)*d^2 - 2*((a^5 - 3*a^3*b^2)*c^2 + 2*(3*a^4*b - a^2*b^3)*c*d - (a^5 - 3*a^
3*b^2)*d^2)*f*x - ((5*a^2*b^3 - b^5)*c^2 - 6*(a^3*b^2 - a*b^4)*c*d + (a^4*b
 - 5*a^2*b^3)*d^2 + 2*((a^3*b^2 - 3*a*b^4)*c^2 + 2*(3*a^2*b^3 - b^5)*c*d -
(a^3*b^2 - 3*a*b^4)*d^2)*f*x)*tan(f*x + e)^2 - ((3*a^4*b - a^2*b^3)*c^2 - 2
```

```
*(a^5 - 3*a^3*b^2)*c*d - (3*a^4*b - a^2*b^3)*d^2 + ((3*a^2*b^3 - b^5)*c^2 -
 2*(a^3*b^2 - 3*a*b^4)*c*d - (3*a^2*b^3 - b^5)*d^2)*tan(f*x + e)^2 + 2*((3*
a^3*b^2 - a*b^4)*c^2 - 2*(a^4*b - 3*a^2*b^3)*c*d - (3*a^3*b^2 - a*b^4)*d^2)
*tan(f*x + e))*log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(tan(f*x
 + e)^2 + 1)) - 2*(3*(a^3*b^2 - a*b^4)*c^2 - 2*(2*a^4*b - 3*a^2*b^3 + b^5)*
c*d + (a^5 - 3*a^3*b^2 + 2*a*b^4)*d^2 + 2*((a^4*b - 3*a^2*b^3)*c^2 + 2*(3*a
^3*b^2 - a*b^4)*c*d - (a^4*b - 3*a^2*b^3)*d^2)*f*x)*tan(f*x + e))/((a^6*b^2
 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*f*tan(f*x + e)^2 + 2*(a^7*b + 3*a^5*b^3 + 3
*a^3*b^5 + a*b^7)*f*tan(f*x + e) + (a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*
f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**2/(a+b*tan(f*x+e))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 614 vs. 2(216) = 432.
time = 0.73, size = 614, normalized size = 2.87

$$\frac{\frac{2\,(a^3c^2-3\,ab^2c^2+6\,a^2bcd-2\,b^3cd-a^3d^2+3\,ab^2d^2)(fx+e)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{(3\,a^2bc^2-b^3c^2-2\,a^3cd+6\,ab^2cd-3\,a^2bd^2+b^3d^2)\log\left(\tan(fx+e)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\,(3\,a^2b^2c^2-b^4c^2-2\,a^3bcd+6\,ab^3cd-3\,a^2b^2d^2+b^4d^2)\log(|b\tan(fx+e)+a|)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}-\frac{9\,a^2b^4c^2\tan(fx+e)^2-3\,b^6c^2\tan(fx+e)^2-6\,a^3b^3cd\tan(fx+e)^2+18\,ab^5cd\tan(fx+e)^2-9\,a^2b^4d^2\tan(fx+e)^2+3\,b^6d^2\tan(fx+e)^2+22\,a^3b^3c^2\tan(fx+e)-2\,ab^5c^2\tan(fx+e)-16\,a^4b^2cd\tan(fx+e)+36\,a^2b^4cd\tan(fx+e)+4\,b^6cd\tan(fx+e)-22\,a^3b^3d^2\tan(fx+e)+2\,ab^5d^2\tan(fx+e)+14\,a^4b^2c^2+3\,a^2b^4c^2+b^6c^2-12\,a^5bcd+14\,a^3b^3cd+2\,ab^5cd+a^6d^2-11\,a^4b^2d^2}{(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)(b\tan(fx+e)+a)^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2/(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(a^3*c^2 - 3*a*b^2*c^2 + 6*a^2*b*c*d - 2*b^3*c*d - a^3*d^2 + 3*a*b^2
*d^2)*(f*x + e)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (3*a^2*b*c^2 - b^3*c^
2 - 2*a^3*c*d + 6*a*b^2*c*d - 3*a^2*b*d^2 + b^3*d^2)*log(tan(f*x + e)^2 + 1
)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(3*a^2*b^2*c^2 - b^4*c^2 - 2*a^3*
b*c*d + 6*a*b^3*c*d - 3*a^2*b^2*d^2 + b^4*d^2)*log(abs(b*tan(f*x + e) + a))
/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) - (9*a^2*b^4*c^2*tan(f*x + e)^2 - 3*
b^6*c^2*tan(f*x + e)^2 - 6*a^3*b^3*c*d*tan(f*x + e)^2 + 18*a*b^5*c*d*tan(f*
x + e)^2 - 9*a^2*b^4*d^2*tan(f*x + e)^2 + 3*b^6*d^2*tan(f*x + e)^2 + 22*a^3
*b^3*c^2*tan(f*x + e) - 2*a*b^5*c^2*tan(f*x + e) - 16*a^4*b^2*c*d*tan(f*x +
 e) + 36*a^2*b^4*c*d*tan(f*x + e) + 4*b^6*c*d*tan(f*x + e) - 22*a^3*b^3*d^2
*tan(f*x + e) + 2*a*b^5*d^2*tan(f*x + e) + 14*a^4*b^2*c^2 + 3*a^2*b^4*c^2 +
 b^6*c^2 - 12*a^5*b*c*d + 14*a^3*b^3*c*d + 2*a*b^5*c*d + a^6*d^2 - 11*a^4*b
^2*d^2)/((a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*(b*tan(f*x + e) + a)^2))/f
```

Mupad [B]

time = 7.30, size = 367, normalized size = 1.71

$$-\frac{\frac{2\tan(e+f\,x)\,\left(-a^2\,b\,c\,d+a\,b^2\,c^2-a\,b^2\,d^2+b^3\,c\,d\right)}{a^4+2\,a^2\,b^2+b^4}+\frac{a^4\,d^2-6\,a^3\,b\,c\,d+5\,a^2\,b^2\,c^2-3\,a^2\,b^2\,d^2+2\,a\,b^3\,c\,d+b^4\,c^2}{2\,b\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{f\,\left(a^2+2\,a\,b\tan\left(e+f\,x\right)+b^2\tan\left(e+f\,x\right)^2\right)}-\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(-c^2\,1\mathrm{i}+2\,c\,d+d^2\,1\mathrm{i}\right)}{2\,f\,\left(-a^3-a^2\,b\,3\mathrm{i}+3\,a\,b^2+b^3\,1\mathrm{i}\right)}-\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(-c^2+c\,d\,2\mathrm{i}+d^2\right)}{2\,f\,\left(-a^3\,1\mathrm{i}-3\,a^2\,b+a\,b^2\,3\mathrm{i}+b^3\right)}-\frac{\ln\left(a+b\tan(e+f\,x)\right)\,\left(2\,c\,d\,a^3+\left(3\,d^2-3\,c^2\right)\,a^2\,b-6\,c\,d\,a\,b^2+\left(c^2-d^2\right)\,b^3\right)}{f\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^2/(a + b*tan(e + f*x))^3,x)
```

```
[Out] - ((2*tan(e + f*x)*(a*b^2*c^2 - a*b^2*d^2 + b^3*c*d - a^2*b*c*d))/(a^4 + b^
4 + 2*a^2*b^2) + (a^4*d^2 + b^4*c^2 + 5*a^2*b^2*c^2 - 3*a^2*b^2*d^2 + 2*a*b
^3*c*d - 6*a^3*b*c*d)/(2*b*(a^4 + b^4 + 2*a^2*b^2)))/(f*(a^2 + b^2*tan(e +
f*x)^2 + 2*a*b*tan(e + f*x))) - (log(tan(e + f*x) - 1i)*(2*c*d - c^2*1i + d
^2*1i))/(2*f*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)) - (log(tan(e + f*x) + 1i)
*(c*d*2i - c^2 + d^2))/(2*f*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3)) - (log(a +
 b*tan(e + f*x))*(b^3*(c^2 - d^2) - a^2*b*(3*c^2 - 3*d^2) + 2*a^3*c*d - 6*a
*b^2*c*d))/(f*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))
```

3.1203 $\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^3\,dx$

Optimal. Leaf size=302

$$-\left((ac-bd)\left(8abcd-a^2\left(c^2-3d^2\right)+b^2\left(3c^2-d^2\right)\right)x\right)+\frac{(bc+ad)\left(8abcd+b^2(c^2-3d^2)-a^2(3c^2-d^2)\right)\log}{f}$$

```
[Out] -(a*c-b*d)*(8*a*b*c*d-a^2*(c^2-3*d^2)+b^2*(3*c^2-d^2))*x+(a*d+b*c)*(8*a*b*c
*d+b^2*(c^2-3*d^2)-a^2*(3*c^2-d^2))*ln(cos(f*x+e))/f+d*(2*a^3*c*d-6*a*b^2*c
*d+3*a^2*b*(c^2-d^2)-b^3*(c^2-d^2))*tan(f*x+e)/f+1/2*(a^3*d+3*a^2*b*c-3*a*b
^2*d-b^3*c)*(c+d*tan(f*x+e))^2/f+1/3*b*(3*a^2-b^2)*(c+d*tan(f*x+e))^3/f-1/2
0*b^2*(-11*a*d+b*c)*(c+d*tan(f*x+e))^4/d^2/f+1/5*b^2*(a+b*tan(f*x+e))*(c+d*
tan(f*x+e))^4/d/f
```

Rubi [A]
time = 0.36, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3647, 3711, 3609, 3606, 3556}

$$\frac{(ad+bc)\left(-\left(a^2(3c^2-d^2)\right)+8abcd+b^2(c^2-3d^2)\right)\log(\cos(e+fx))}{f}-x(ac-bd)\left(-\left(a^2\left(c^2-3d^2\right)\right)+8abcd+b^2\left(3c^2-d^2\right)\right)+\frac{b(3a^2-b^2)\,(c+d\tan(e+fx))^3}{3f}+\frac{d(2a^3cd+3a^2b(c^2-d^2)-6ab^2cd-b^3(c^2-d^2))\tan(e+fx)}{f}+\frac{(a^3d+3a^2bc-3ab^2d-b^3c)\,(c+d\tan(e+fx))^2}{2f}-\frac{b^2(bc-11ad)(c+d\tan(e+fx))^4}{20d^2f}+\frac{b^2(a+b\tan(e+fx))(c+d\tan(e+fx))^4}{5df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^3,x]
```

```
[Out] -((a*c - b*d)*(8*a*b*c*d - a^2*(c^2 - 3*d^2) + b^2*(3*c^2 - d^2))*x) + ((b*
c + a*d)*(8*a*b*c*d + b^2*(c^2 - 3*d^2) - a^2*(3*c^2 - d^2))*Log[Cos[e + f*
x]])/f + (d*(2*a^3*c*d - 6*a*b^2*c*d + 3*a^2*b*(c^2 - d^2) - b^3*(c^2 - d^2
))*Tan[e + f*x])/f + ((3*a^2*b*c - b^3*c + a^3*d - 3*a*b^2*d)*(c + d*Tan[e
+ f*x])^2)/(2*f) + (b*(3*a^2 - b^2)*(c + d*Tan[e + f*x])^3)/(3*f) - (b^2*(b
*c - 11*a*d)*(c + d*Tan[e + f*x])^4)/(20*d^2*f) + (b^2*(a + b*Tan[e + f*x])
*(c + d*Tan[e + f*x])^4)/(5*d*f)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^3\,dx=\frac{b^2(a+b\tan(e+fx))(c+d\tan(e+fx))^4}{5df}+\frac{\int (c+d\tan(e\ldots}{\ldots}$$

$$=-\frac{b^2(bc-11ad)(c+d\tan(e+fx))^4}{20d^2f}+\frac{b^2(a+b\tan(e+fx)\ldots}{5\ldots}$$

$$=\frac{b(3a^2-b^2)\,(c+d\tan(e+fx))^3}{3f}-\frac{b^2(bc-11ad)(c+d\tan(\ldots}{20d^2f}$$

$$=\frac{(3a^2bc-b^3c+a^3d-3ab^2d)\,(c+d\tan(e+fx))^2}{2f}+\frac{b(3a^2-\ldots}{\ldots}$$

$$=-(ac-bd)\left(8abcd-a^2\left(c^2-3d^2\right)+b^2\left(3c^2-d^2\right)\right)x+\frac{d(2a\ldots}{\ldots}$$

$$=-(ac-bd)\left(8abcd-a^2\left(c^2-3d^2\right)+b^2\left(3c^2-d^2\right)\right)x-\frac{(bc-\ldots}{\ldots}$$

Mathematica [C] Result contains complex when optimal does not.

time = 6.43, size = 299, normalized size = 0.99

$$\frac{b^2(a+b\tan(e+fx))(c+d\tan(e+fx))^4}{5df}+\frac{-\frac{b^2(bc-11ad)(c+d\tan(e+fx))^4}{4df}-\frac{5(3(3a^2bc-b^3c-a^3d+3ab^2d)((ic-d)^3\log(i-\tan(e+fx))-(ic+d)^3\log(i+\tan(e+fx))+6cd^2\tan(e+fx)+d^3\tan^2(e+fx))+b(3a^2-b^2)(3i(c+id)^4\log(i-\tan(e+fx))-3i(c-id)^4\log(i+\tan(e+fx))-6d^2(6c^2-d^2)\tan(e+fx)-12cd^3\tan^2(e+fx)-2d^4\tan^3(e+fx)))}{6f}}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^3,x]
```

```
[Out] (b^2*(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^4)/(5*d*f) + (-1/4*(b^2*(b*c
 - 11*a*d)*(c + d*Tan[e + f*x])^4)/(d*f) - (5*(3*(3*a^2*b*c - b^3*c - a^3*d
 + 3*a*b^2*d)*((I*c - d)^3*Log[I - Tan[e + f*x]] - (I*c + d)^3*Log[I + Tan[
e + f*x]] + 6*c*d^2*Tan[e + f*x] + d^3*Tan[e + f*x]^2) + b*(3*a^2 - b^2)*((
3*I)*(c + I*d)^4*Log[I - Tan[e + f*x]] - (3*I)*(c - I*d)^4*Log[I + Tan[e +
f*x]] - 6*d^2*(6*c^2 - d^2)*Tan[e + f*x] - 12*c*d^3*Tan[e + f*x]^2 - 2*d^4*
Tan[e + f*x]^3)))/(6*f))/(5*d)
```

Maple [A]
time = 0.14, size = 495, normalized size = 1.64

method	result
norman	$\left(a^3c^3-3a^3c\,d^2-9a^2b\,c^2d+3a^2b\,d^3-3a\,b^2c^3+9a\,b^2c\,d^2+3b^3c^2d-b^3d^3\right)x+\frac{(3a^3c\,d^2+\cdots}{}$
derivativedivides	$\frac{b^3d^3\left(\tan^5(fx+e)\right)}{5}+\frac{3a\,b^2d^3\left(\tan^4(fx+e)\right)}{4}+\frac{3b^3c\,d^2\left(\tan^4(fx+e)\right)}{4}+a^2b\,d^3\left(\tan^3(fx+e)\right)+3a\,b^2c\,d^2\left(\tan^3(fx+e)\right)+b^3c^2d(\text{ta}\cdots$
default	$\frac{b^3d^3\left(\tan^5(fx+e)\right)}{5}+\frac{3a\,b^2d^3\left(\tan^4(fx+e)\right)}{4}+\frac{3b^3c\,d^2\left(\tan^4(fx+e)\right)}{4}+a^2b\,d^3\left(\tan^3(fx+e)\right)+3a\,b^2c\,d^2\left(\tan^3(fx+e)\right)+b^3c^2d(\text{ta}\cdots$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/5*b^3*d^3*tan(f*x+e)^5+3/4*a*b^2*d^3*tan(f*x+e)^4+3/4*b^3*c*d^2*tan(
f*x+e)^4+a^2*b*d^3*tan(f*x+e)^3+3*a*b^2*c*d^2*tan(f*x+e)^3+b^3*c^2*d*tan(f*
x+e)^3-1/3*b^3*d^3*tan(f*x+e)^3+1/2*a^3*d^3*tan(f*x+e)^2+9/2*a^2*b*c*d^2*ta
n(f*x+e)^2+9/2*a*b^2*c^2*d*tan(f*x+e)^2-3/2*a*b^2*d^3*tan(f*x+e)^2+1/2*b^3*
c^3*tan(f*x+e)^2-3/2*b^3*c*d^2*tan(f*x+e)^2+3*a^3*c*d^2*tan(f*x+e)+9*a^2*b*
c^2*d*tan(f*x+e)-3*a^2*b*d^3*tan(f*x+e)+3*a*b^2*c^3*tan(f*x+e)-9*a*b^2*c*d^
2*tan(f*x+e)-3*b^3*c^2*d*tan(f*x+e)+b^3*d^3*tan(f*x+e)+1/2*(3*a^3*c^2*d-a^3
*d^3+3*a^2*b*c^3-9*a^2*b*c*d^2-9*a*b^2*c^2*d+3*a*b^2*d^3-b^3*c^3+3*b^3*c*d^
2)*ln(1+tan(f*x+e)^2)+(a^3*c^3-3*a^3*c*d^2-9*a^2*b*c^2*d+3*a^2*b*d^3-3*a*b^
2*c^3+9*a*b^2*c*d^2+3*b^3*c^2*d-b^3*d^3)*arctan(tan(f*x+e)))
```

Maxima [A]
time = 0.59, size = 382, normalized size = 1.26

$$\frac{12\,b^3d^3\tan(fx+e)^5+45\,(b^3cd^2+ab^2d^3)\tan(fx+e)^4+20\,(3\,b^3c^2d+9\,ab^2cd^2+(3\,a^2b-b^3)d^3)\tan(fx+e)^3+30\,(b^3c^3+9\,ab^2c^2d+3\,(3\,a^2b-b^3)cd^2+(a^3-3\,ab^2)d^3)\tan(fx+e)^2+60\,((a^3-3\,ab^2)c^3-3\,(3\,a^2b-b^3)c^2d-3\,(a^3-3\,ab^2)cd^2+(3\,a^2b-b^3)d^3)(fx+e)+30\,((3\,a^2b-b^3)c^3+3\,(a^3-3\,ab^2)c^2d-3\,(3\,a^2b-b^3)cd^2-(a^3-3\,ab^2)d^3)\log(\tan(fx+e)^2+1)+60\,(3\,ab^2c^3+3\,(3\,a^2b-b^3)c^2d+3\,(a^3-3\,ab^2)cd^2-(3\,a^2b-b^3)d^3)\tan(fx+e)}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(12*b^3*d^3*tan(f*x + e)^5 + 45*(b^3*c*d^2 + a*b^2*d^3)*tan(f*x + e)^4
 + 20*(3*b^3*c^2*d + 9*a*b^2*c*d^2 + (3*a^2*b - b^3)*d^3)*tan(f*x + e)^3 +
30*(b^3*c^3 + 9*a*b^2*c^2*d + 3*(3*a^2*b - b^3)*c*d^2 + (a^3 - 3*a*b^2)*d^3
)*tan(f*x + e)^2 + 60*((a^3 - 3*a*b^2)*c^3 - 3*(3*a^2*b - b^3)*c^2*d - 3*(a
^3 - 3*a*b^2)*c*d^2 + (3*a^2*b - b^3)*d^3)*(f*x + e) + 30*((3*a^2*b - b^3)*
c^3 + 3*(a^3 - 3*a*b^2)*c^2*d - 3*(3*a^2*b - b^3)*c*d^2 - (a^3 - 3*a*b^2)*d
^3)*log(tan(f*x + e)^2 + 1) + 60*(3*a*b^2*c^3 + 3*(3*a^2*b - b^3)*c^2*d + 3
*(a^3 - 3*a*b^2)*c*d^2 - (3*a^2*b - b^3)*d^3)*tan(f*x + e))/f
```

Fricas [A]
time = 1.32, size = 380, normalized size = 1.26

$$\frac{12 b^3 d^3 \tan(fx+e)^5 + 45 (b^3 c d^2 + a b^2 d^3) \tan(fx+e)^4 + 20 (3 b^3 c^2 d + 9 a b^2 c d^2 + (3a^2 b - b^3) d^3) \tan(fx+e)^3 + 60 ((a^3 - 3ab^2) c^3 - 3 (3a^2 b - b^3) c^2 d - 3 (a^3 - 3ab^2) c d^2 + (3a^2 b - b^3) d^3) fx + 30 (b^3 c^3 + 9ab^2 c^2 d + 3 (3a^2 b - b^3) c d^2 + (a^3 - 3ab^2) d^3) \tan(fx+e)^2 - 30 ((3a^2 b - b^3) c^3 + 3 (a^3 - 3ab^2) c^2 d - 3 (3a^2 b - b^3) c d^2 - (a^3 - 3ab^2) d^3) \log\left(\frac{1}{\tan(fx+e)^2 + 1}\right) + 60 (3ab^2 c^3 + 3 (3a^2 b - b^3) c^2 d + 3 (a^3 - 3ab^2) c d^2 - (3a^2 b - b^3) d^3) \tan(fx+e)}{60 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(12*b^3*d^3*tan(f*x + e)^5 + 45*(b^3*c*d^2 + a*b^2*d^3)*tan(f*x + e)^4
 + 20*(3*b^3*c^2*d + 9*a*b^2*c*d^2 + (3*a^2*b - b^3)*d^3)*tan(f*x + e)^3 +
60*((a^3 - 3*a*b^2)*c^3 - 3*(3*a^2*b - b^3)*c^2*d - 3*(a^3 - 3*a*b^2)*c*d^2
 + (3*a^2*b - b^3)*d^3)*f*x + 30*(b^3*c^3 + 9*a*b^2*c^2*d + 3*(3*a^2*b - b^
3)*c*d^2 + (a^3 - 3*a*b^2)*d^3)*tan(f*x + e)^2 - 30*((3*a^2*b - b^3)*c^3 +
3*(a^3 - 3*a*b^2)*c^2*d - 3*(3*a^2*b - b^3)*c*d^2 - (a^3 - 3*a*b^2)*d^3)*lo
g(1/(tan(f*x + e)^2 + 1)) + 60*(3*a*b^2*c^3 + 3*(3*a^2*b - b^3)*c^2*d + 3*(
a^3 - 3*a*b^2)*c*d^2 - (3*a^2*b - b^3)*d^3)*tan(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 711 vs. $2(275) = 550$.
time = 0.27, size = 711, normalized size = 2.35

$$\begin{cases} a^3c^3x + \frac{3a^3c^2d\log(\tan^2(e+fx)+1)}{2f} - 3a^3cd^2x + \frac{3a^3cd^2\tan(e+fx)}{f} - \frac{a^3d^3\log(\tan^2(e+fx)+1)}{2f} + \frac{a^3d^3\tan^2(e+fx)}{2f} + \frac{3a^2bc^3\log(\tan^2(e+fx)+1)}{2f} - 9a^2bc^2dx + \frac{9a^2bc^2d\tan(e+fx)}{f} - \frac{9a^2bcd^2\log(\tan^2(e+fx)+1)}{2f} + \frac{9a^2bcd^2\tan^2(e+fx)}{2f} + 3a^2bd^3x + \frac{a^2bd^3\tan^3(e+fx)}{f} - \frac{3a^2bd^3\tan(e+fx)}{f} - 3ab^2c^3x + \frac{3ab^2c^3\tan(e+fx)}{f} - \frac{9ab^2c^2d\log(\tan^2(e+fx)+1)}{2f} + \frac{9ab^2c^2d\tan^2(e+fx)}{2f} + 9ab^2cd^2x + \frac{3ab^2cd^2\tan^3(e+fx)}{f} - \frac{9ab^2cd^2\tan(e+fx)}{f} + \frac{3ab^2d^3\log(\tan^2(e+fx)+1)}{2f} + \frac{3ab^2d^3\tan^4(e+fx)}{4f} - \frac{3ab^2d^3\tan^2(e+fx)}{2f} - \frac{b^3c^3\log(\tan^2(e+fx)+1)}{2f} + \frac{b^3c^3\tan^2(e+fx)}{2f} + 3b^3c^2dx + \frac{b^3c^2d\tan^3(e+fx)}{f} - \frac{3b^3c^2d\tan(e+fx)}{f} + \frac{3b^3cd^2\log(\tan^2(e+fx)+1)}{2f} + \frac{3b^3cd^2\tan^4(e+fx)}{4f} - \frac{3b^3cd^2\tan^2(e+fx)}{2f} - b^3d^3x + \frac{b^3d^3\tan^5(e+fx)}{5f} - \frac{b^3d^3\tan^3(e+fx)}{3f} + \frac{b^3d^3\tan(e+fx)}{f} & \text{for}\: f \neq 0 \\ x(a+b\tan(e))^3(c+d\tan(e))^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3*(c+d*tan(f*x+e))**3,x)
```

```
[Out] Piecewise((a**3*c**3*x + 3*a**3*c**2*d*log(tan(e + f*x)**2 + 1)/(2*f) - 3*a
**3*c*d**2*x + 3*a**3*c*d**2*tan(e + f*x)/f - a**3*d**3*log(tan(e + f*x)**2
 + 1)/(2*f) + a**3*d**3*tan(e + f*x)**2/(2*f) + 3*a**2*b*c**3*log(tan(e + f
*x)**2 + 1)/(2*f) - 9*a**2*b*c**2*d*x + 9*a**2*b*c**2*d*tan(e + f*x)/f - 9*
a**2*b*c*d**2*log(tan(e + f*x)**2 + 1)/(2*f) + 9*a**2*b*c*d**2*tan(e + f*x)
**2/(2*f) + 3*a**2*b*d**3*x + a**2*b*d**3*tan(e + f*x)**3/f - 3*a**2*b*d**3
*tan(e + f*x)/f - 3*a*b**2*c**3*x + 3*a*b**2*c**3*tan(e + f*x)/f - 9*a*b**2
```

```
*c**2*d*log(tan(e + f*x)**2 + 1)/(2*f) + 9*a*b**2*c**2*d*tan(e + f*x)**2/(2
*f) + 9*a*b**2*c*d**2*x + 3*a*b**2*c*d**2*tan(e + f*x)**3/f - 9*a*b**2*c*d*
*2*tan(e + f*x)/f + 3*a*b**2*d**3*log(tan(e + f*x)**2 + 1)/(2*f) + 3*a*b**2
*d**3*tan(e + f*x)**4/(4*f) - 3*a*b**2*d**3*tan(e + f*x)**2/(2*f) - b**3*c*
*3*log(tan(e + f*x)**2 + 1)/(2*f) + b**3*c**3*tan(e + f*x)**2/(2*f) + 3*b**
3*c**2*d*x + b**3*c**2*d*tan(e + f*x)**3/f - 3*b**3*c**2*d*tan(e + f*x)/f +
 3*b**3*c*d**2*log(tan(e + f*x)**2 + 1)/(2*f) + 3*b**3*c*d**2*tan(e + f*x)*
*4/(4*f) - 3*b**3*c*d**2*tan(e + f*x)**2/(2*f) - b**3*d**3*x + b**3*d**3*ta
n(e + f*x)**5/(5*f) - b**3*d**3*tan(e + f*x)**3/(3*f) + b**3*d**3*tan(e + f
*x)/f, Ne(f, 0)), (x*(a + b*tan(e))**3*(c + d*tan(e))**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 8276 vs. $2(301) = 602$.
time = 5.78, size = 8276, normalized size = 27.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/60*(60*a^3*c^3*f*x*tan(f*x)^5*tan(e)^5 - 180*a*b^2*c^3*f*x*tan(f*x)^5*tan
(e)^5 - 540*a^2*b*c^2*d*f*x*tan(f*x)^5*tan(e)^5 + 180*b^3*c^2*d*f*x*tan(f*x
)^5*tan(e)^5 - 180*a^3*c*d^2*f*x*tan(f*x)^5*tan(e)^5 + 540*a*b^2*c*d^2*f*x*
tan(f*x)^5*tan(e)^5 + 180*a^2*b*d^3*f*x*tan(f*x)^5*tan(e)^5 - 60*b^3*d^3*f*
x*tan(f*x)^5*tan(e)^5 - 90*a^2*b*c^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x
)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan
(e)^2 + 1))*tan(f*x)^5*tan(e)^5 + 30*b^3*c^3*log(4*(tan(f*x)^4*tan(e)^2 - 2
*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) +
 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 - 90*a^3*c^2*d*log(4*(tan(f*x)^4*ta
n(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x
)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 + 270*a*b^2*c^2*d*log(4*(
tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^
2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 + 270*a^2*b*
c*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^
2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5
 - 90*b^3*c*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)
^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^
5*tan(e)^5 + 30*a^3*d^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) +
tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*t
an(f*x)^5*tan(e)^5 - 90*a*b^2*d^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3
*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)
^2 + 1))*tan(f*x)^5*tan(e)^5 - 300*a^3*c^3*f*x*tan(f*x)^4*tan(e)^4 + 900*a*
b^2*c^3*f*x*tan(f*x)^4*tan(e)^4 + 2700*a^2*b*c^2*d*f*x*tan(f*x)^4*tan(e)^4
- 900*b^3*c^2*d*f*x*tan(f*x)^4*tan(e)^4 + 900*a^3*c*d^2*f*x*tan(f*x)^4*tan(
e)^4 - 2700*a*b^2*c*d^2*f*x*tan(f*x)^4*tan(e)^4 - 900*a^2*b*d^3*f*x*tan(f*x
```

```
)^4*tan(e)^4 + 300*b^3*d^3*f*x*tan(f*x)^4*tan(e)^4 + 30*b^3*c^3*tan(f*x)^5*
tan(e)^5 + 270*a*b^2*c^2*d*tan(f*x)^5*tan(e)^5 + 270*a^2*b*c*d^2*tan(f*x)^5
*tan(e)^5 - 135*b^3*c*d^2*tan(f*x)^5*tan(e)^5 + 30*a^3*d^3*tan(f*x)^5*tan(e
)^5 - 135*a*b^2*d^3*tan(f*x)^5*tan(e)^5 + 450*a^2*b*c^3*log(4*(tan(f*x)^4*t
an(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*
x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 - 150*b^3*c^3*log(4*(tan
(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 -
 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 + 450*a^3*c^2*d
*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + t
an(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 - 13
50*a*b^2*c^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^
2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4
*tan(e)^4 - 1350*a^2*b*c*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(
e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 +
1))*tan(f*x)^4*tan(e)^4 + 450*b^3*c*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(
f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(
tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 - 150*a^3*d^3*log(4*(tan(f*x)^4*tan(e)^2
 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(
e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 + 450*a*b^2*d^3*log(4*(tan(f*x)
^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*ta
n(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 - 180*a*b^2*c^3*tan(
f*x)^5*tan(e)^4 - 540*a^2*b*c^2*d*tan(f*x)^5*tan(e)^4 + 180*b^3*c^2*d*tan(f
*x)^5*tan(e)^4 - 180*a^3*c*d^2*tan(f*x)^5*tan(e)^4 + 540*a*b^2*c*d^2*tan(f*
x)^5*tan(e)^4 + 180*a^2*b*d^3*tan(f*x)^5*tan(e)^4 - 60*b^3*d^3*tan(f*x)^5*t
an(e)^4 - 180*a*b^2*c^3*tan(f*x)^4*tan(e)^5 - 540*a^2*b*c^2*d*tan(f*x)^4*ta
n(e)^5 + 180*b^3*c^2*d*tan(f*x)^4*tan(e)^5 - 180*a^3*c*d^2*tan(f*x)^4*tan(e
)^5 + 540*a*b^2*c*d^2*tan(f*x)^4*tan(e)^5 + 180*a^2*b*d^3*tan(f*x)^4*tan(e)
^5 - 60*b^3*d^3*tan(f*x)^4*tan(e)^5 + 600*a^3*c^3*f*x*tan(f*x)^3*tan(e)^3 -
 1800*a*b^2*c^3*f*x*tan(f*x)^3*tan(e)^3 - 5400*a^2*b*c^2*d*f*x*tan(f*x)^3*t
an(e)^3 + 1800*b^3*c^2*d*f*x*tan(f*x)^3*tan(e)^3 - 1800*a^3*c*d^2*f*x*tan(f
*x)^3*tan(e)^3 + 5400*a*b^2*c*d^2*f*x*tan(f*x)^3*tan(e)^3 + 1800*a^2*b*d^3*
f*x*tan(f*x)^3*tan(e)^3 - 600*b^3*d^3*f*x*tan(f*x)^3*tan(e)^3 + 30*b^3*c^3*
tan(f*x)^5*tan(e)^3 + 270*a*b^2*c^2*d*tan(f*x)^5*tan(e)^3 + 270*a^2*b*c*d^2
*tan(f*x)^5*tan(e)^3 - 90*b^3*c*d^2*tan(f*x)^5*tan(e)^3 + 30*a^3*d^3*tan(f*
x)^5*tan(e)^3 - 90*a*b^2*d^3*tan(f*x)^5*tan(e)^3 - 90*b^3*c^3*tan(f*x)^4*ta
n(e)^4 - 810*a*b^2*c^2*d*tan(f*x)^4*tan(e)^4 - 810*a^2*b*c*d^2*tan(f*x)^4*t
an(e)^4 + 495*b^3*c*d^2*tan(f*x)^4*tan(e)^4 - 90*a^3*d^3*tan(f*x)^4*tan(e)^
4 + 495*a*b^2*d^3*tan(f*x)^4*tan(e)^4 + 30*b^3*c^3*tan(f*x)^3*tan(e)^5 + 27
0*a*b^2*c^2*d*tan(f*x)^3*tan(e)^5 + 270*a^2*b*c*d^2*tan(f*x)^3*tan(e)^5 - 9
0*b^3*c*d^2*tan(f*x)^3*tan(e)^5 + 30*a^3*d^3*tan(f*x)^3*tan(e)^5 - 90*a*b^2
*d^3*tan(f*x)^3*tan(e)^5 - 60*b^3*c^2*d*tan(f*x)^5*tan(e)^2 - 180*a*b^2*c*d
^2*tan(f*x)^5*tan(e)^2 - 60*a^2*b*d^3*tan(f*x)^...
```

Mupad [B]

time = 5.38, size = 494, normalized size = 1.64

$$\frac{\tan(e+fx)\left(b^3d^3+3ac\left(a^2d^2+3abcd+b^2c^2\right)-3bd\left(a^2d^2+3abcd+b^2c^2\right)\right)}{f}-\frac{\ln\left(\tan(e+fx)^2+1\right)\left(-\frac{3a^3c^2d}{2}+\frac{a^3d^3}{2}-\frac{3a^2bc^3}{2}+\frac{9a^2bcd^2}{2}+\frac{9ab^2c^2d}{2}-\frac{3ab^2d^3}{2}+\frac{b^3c^3}{2}-\frac{3b^3cd^2}{2}\right)}{f}-\frac{\tan(e+fx)^3\left(\frac{b^3d^3}{3}-bd\left(a^2d^2+3abcd+b^2c^2\right)\right)}{f}+\frac{\tan(e+fx)^2\left(\frac{a^3d^3}{2}+\frac{b^3c^3}{2}-\frac{3b^2d^2(ad+bc)}{2}+\frac{9ab^2c^2d}{2}+\frac{9a^2bcd^2}{2}\right)}{f}+\frac{\operatorname{atan}\left(\frac{\tan(e+fx)(ac-bd)\left(-a^2c^2+3a^2d^2+8abcd+3b^2c^2-b^2d^2\right)}{a^3c^3-3a^3cd^2-9a^2bc^2d+3a^2bd^3-3ab^2c^3+9ab^2cd^2+3b^3c^2d-b^3d^3}\right)(ac-bd)\left(-a^2c^2+3a^2d^2+8abcd+3b^2c^2-b^2d^2\right)}{f}+\frac{b^3d^3\tan(e+fx)^5}{5f}+\frac{3b^2d^2\tan(e+fx)^4(ad+bc)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^3*(c + d*tan(e + f*x))^3,x)
```

```
[Out] (tan(e + f*x)*(b^3*d^3 + 3*a*c*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d) - 3*b*d*(a^2
*d^2 + b^2*c^2 + 3*a*b*c*d)))/f - (log(tan(e + f*x)^2 + 1)*((a^3*d^3)/2 + (
b^3*c^3)/2 - (3*a^2*b*c^3)/2 - (3*a*b^2*d^3)/2 - (3*a^3*c^2*d)/2 - (3*b^3*c
*d^2)/2 + (9*a*b^2*c^2*d)/2 + (9*a^2*b*c*d^2)/2))/f - (tan(e + f*x)^3*((b^3
*d^3)/3 - b*d*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d)))/f + (tan(e + f*x)^2*((a^3*d
^3)/2 + (b^3*c^3)/2 - (3*b^2*d^2*(a*d + b*c))/2 + (9*a*b^2*c^2*d)/2 + (9*a^
2*b*c*d^2)/2))/f + (atan((tan(e + f*x)*(a*c - b*d)*(3*a^2*d^2 - a^2*c^2 + 3
*b^2*c^2 - b^2*d^2 + 8*a*b*c*d))/(a^3*c^3 - b^3*d^3 - 3*a*b^2*c^3 + 3*a^2*b
*d^3 - 3*a^3*c*d^2 + 3*b^3*c^2*d + 9*a*b^2*c*d^2 - 9*a^2*b*c^2*d))*(a*c - b
*d)*(3*a^2*d^2 - a^2*c^2 + 3*b^2*c^2 - b^2*d^2 + 8*a*b*c*d))/f + (b^3*d^3*t
an(e + f*x)^5)/(5*f) + (3*b^2*d^2*tan(e + f*x)^4*(a*d + b*c))/(4*f)
```

3.1204 $\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^3\,dx$

Optimal. Leaf size=219

$$-\left(\left(b^2c\left(c^2-3d^2\right)+2abd\left(3c^2-d^2\right)-a^2\left(c^3-3cd^2\right)\right)x\right)-\frac{\left(2abc(c^2-3d^2)-b^2d(3c^2-d^2)+a^2(3c^2d-d^3)\right)\text{lo}}{f}$$

```
[Out] -(b^2*c*(c^2-3*d^2)+2*a*b*d*(3*c^2-d^2)-a^2*(c^3-3*c*d^2))*x-(2*a*b*c*(c^2-
3*d^2)-b^2*d*(3*c^2-d^2)+a^2*(3*c^2*d-d^3))*ln(cos(f*x+e))/f+2*d*(a*d+b*c)*
(a*c-b*d)*tan(f*x+e)/f+1/2*(a^2*d+2*a*b*c-b^2*d)*(c+d*tan(f*x+e))^2/f+2/3*a
*b*(c+d*tan(f*x+e))^3/f+1/4*b^2*(c+d*tan(f*x+e))^4/d/f
```

Rubi [A]
time = 0.18, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3624, 3609, 3606, 3556}

$$-\frac{\left(a^2(3c^2d-d^3)+2abc(c^2-3d^2)-b^2d(3c^2-d^2)\right)\log(\cos(e+fx))}{f}-x\left(-\left(a^2\left(c^3-3cd^2\right)\right)+2abd\left(3c^2-d^2\right)+b^2c\left(c^2-3d^2\right)\right)+\frac{\left(a^2d+2abc-b^2d\right)(c+d\tan(e+fx))^2}{2f}+\frac{2ab(c+d\tan(e+fx))^3}{3f}+\frac{2d(ad+bc)(ac-bd)\tan(e+fx)}{f}+\frac{b^2(c+d\tan(e+fx))^4}{4df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^3,x]
```

```
[Out] -((b^2*c*(c^2 - 3*d^2) + 2*a*b*d*(3*c^2 - d^2) - a^2*(c^3 - 3*c*d^2))*x) -
((2*a*b*c*(c^2 - 3*d^2) - b^2*d*(3*c^2 - d^2) + a^2*(3*c^2*d - d^3))*Log[Co
s[e + f*x]])/f + (2*d*(b*c + a*d)*(a*c - b*d)*Tan[e + f*x])/f + ((2*a*b*c +
 a^2*d - b^2*d)*(c + d*Tan[e + f*x])^2)/(2*f) + (2*a*b*(c + d*Tan[e + f*x])
^3)/(3*f) + (b^2*(c + d*Tan[e + f*x])^4)/(4*d*f)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
```

```
 0] && GtQ[m, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^3\,dx &= \frac{b^2(c+d\tan(e+fx))^4}{4df} + \int \left(a^2-b^2+2ab\tan(e+fx)\right) (\\
&= \frac{2ab(c+d\tan(e+fx))^3}{3f} + \frac{b^2(c+d\tan(e+fx))^4}{4df} + \int (c \\
&= \frac{\left(2abc+a^2d-b^2d\right)(c+d\tan(e+fx))^2}{2f} + \frac{2ab(c+d\tan(e}{3f} \\
&= -\left(b^2c\left(c^2-3d^2\right)+2abd\left(3c^2-d^2\right)-a^2\left(c^3-3cd^2\right)\right)x + 2 \\
&= -\left(b^2c\left(c^2-3d^2\right)+2abd\left(3c^2-d^2\right)-a^2\left(c^3-3cd^2\right)\right)x - (
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.30, size = 221, normalized size = 1.01

$$\frac{3b^2(c+d\tan(e+fx))^4-6\left(2abc-a^2d+b^2d\right)\left((ic-d)^3\log(i-\tan(e+fx))-(ic+d)^3\log(i+\tan(e+fx))+6cd^2\tan(e+fx)+d^3\tan^2(e+fx)\right)-4ab\left(3i(c+id)^4\log(i-\tan(e+fx))-3i(c-id)^4\log(i+\tan(e+fx))+6d^2\left(-6c^2+d^2\right)\tan(e+fx)-12cd^3\tan^2(e+fx)-2d^4\tan^3(e+fx)\right)}{12df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^3,x]
```

```
[Out] (3*b^2*(c + d*Tan[e + f*x])^4 - 6*(2*a*b*c - a^2*d + b^2*d)*((I*c - d)^3*Lo
g[I - Tan[e + f*x]] - (I*c + d)^3*Log[I + Tan[e + f*x]] + 6*c*d^2*Tan[e + f
*x] + d^3*Tan[e + f*x]^2) - 4*a*b*((3*I)*(c + I*d)^4*Log[I - Tan[e + f*x]]
- (3*I)*(c - I*d)^4*Log[I + Tan[e + f*x]] + 6*d^2*(-6*c^2 + d^2)*Tan[e + f*
x] - 12*c*d^3*Tan[e + f*x]^2 - 2*d^4*Tan[e + f*x]^3))/(12*d*f)
```

Maple [A]
time = 0.13, size = 307, normalized size = 1.40 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/4*b^2*d^3*tan(f*x+e)^4+2/3*a*b*d^3*tan(f*x+e)^3+b^2*c*d^2*tan(f*x+e)
^3+1/2*a^2*d^3*tan(f*x+e)^2+3*a*b*c*d^2*tan(f*x+e)^2+3/2*b^2*c^2*d*tan(f*x+
e)^2-1/2*b^2*d^3*tan(f*x+e)^2+3*a^2*c*d^2*tan(f*x+e)+6*a*b*c^2*d*tan(f*x+e)
-2*a*b*d^3*tan(f*x+e)+b^2*c^3*tan(f*x+e)-3*b^2*c*d^2*tan(f*x+e)+1/2*(3*a^2*
c^2*d-a^2*d^3+2*a*b*c^3-6*a*b*c*d^2-3*b^2*c^2*d+b^2*d^3)*ln(1+tan(f*x+e)^2)
+(a^2*c^3-3*a^2*c*d^2-6*a*b*c^2*d+2*a*b*d^3-b^2*c^3+3*b^2*c*d^2)*arctan(tan
(f*x+e)))
```

Maxima [A]
time = 0.54, size = 252, normalized size = 1.15

$$\frac{3\,b^2d^3\tan\left(fx+e\right)^4+4\,(3\,b^2cd^2+2\,abd^3)\tan\left(fx+e\right)^3+6\,(3\,b^2c^2d+6\,abcd^2+(a^2-b^2)d^3)\tan\left(fx+e\right)^2-12\,(6\,abc^2d-2\,abd^3-(a^2-b^2)c^3+3\,(a^2-b^2)cd^2)(fx+e)+6\,(2\,abc^3-6\,abcd^2+3\,(a^2-b^2)c^2d-(a^2-b^2)d^3)\log\left(\tan\left(fx+e\right)^2+1\right)+12\,(b^2c^3+6\,abc^2d-2\,abd^3+3\,(a^2-b^2)cd^2)\tan\left(fx+e\right)}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/12*(3*b^2*d^3*tan(f*x + e)^4 + 4*(3*b^2*c*d^2 + 2*a*b*d^3)*tan(f*x + e)^3
 + 6*(3*b^2*c^2*d + 6*a*b*c*d^2 + (a^2 - b^2)*d^3)*tan(f*x + e)^2 - 12*(6*a
*b*c^2*d - 2*a*b*d^3 - (a^2 - b^2)*c^3 + 3*(a^2 - b^2)*c*d^2)*(f*x + e) + 6
*(2*a*b*c^3 - 6*a*b*c*d^2 + 3*(a^2 - b^2)*c^2*d - (a^2 - b^2)*d^3)*log(tan(
f*x + e)^2 + 1) + 12*(b^2*c^3 + 6*a*b*c^2*d - 2*a*b*d^3 + 3*(a^2 - b^2)*c*d
^2)*tan(f*x + e))/f
```

Fricas [A]
time = 0.95, size = 250, normalized size = 1.14

$$\frac{3\,b^2d^3\tan\left(fx+e\right)^4+4\,(3\,b^2cd^2+2\,abd^3)\tan\left(fx+e\right)^3-12\,(6\,abc^2d-2\,abd^3-(a^2-b^2)c^3+3\,(a^2-b^2)cd^2)fx+6\,(3\,b^2c^2d+6\,abcd^2+(a^2-b^2)d^3)\tan\left(fx+e\right)^2-6\,(2\,abc^3-6\,abcd^2+3\,(a^2-b^2)c^2d-(a^2-b^2)d^3)\log\left(\frac{1}{\tan\left(fx+e\right)^2+1}\right)+12\,(b^2c^3+6\,abc^2d-2\,abd^3+3\,(a^2-b^2)cd^2)\tan\left(fx+e\right)}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*d^3*tan(f*x + e)^4 + 4*(3*b^2*c*d^2 + 2*a*b*d^3)*tan(f*x + e)^3
 - 12*(6*a*b*c^2*d - 2*a*b*d^3 - (a^2 - b^2)*c^3 + 3*(a^2 - b^2)*c*d^2)*f*x
 + 6*(3*b^2*c^2*d + 6*a*b*c*d^2 + (a^2 - b^2)*d^3)*tan(f*x + e)^2 - 6*(2*a*
b*c^3 - 6*a*b*c*d^2 + 3*(a^2 - b^2)*c^2*d - (a^2 - b^2)*d^3)*log(1/(tan(f*x
 + e)^2 + 1)) + 12*(b^2*c^3 + 6*a*b*c^2*d - 2*a*b*d^3 + 3*(a^2 - b^2)*c*d^2
)*tan(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(194) = 388$.
time = 0.18, size = 445, normalized size = 2.03

$$\begin{cases} a^2c^3x + \frac{3a^2c^2d\log\left(\tan^2\left(e+fx\right)+1\right)}{2f} - 3a^2cd^2x + \frac{3a^2cd^2\tan\left(e+fx\right)}{f} - \frac{a^2d^3\log\left(\tan^2\left(e+fx\right)+1\right)}{2f} + \frac{a^2d^3\tan^2\left(e+fx\right)}{2f} + \frac{abc^3\log\left(\tan^2\left(e+fx\right)+1\right)}{f} - 6abc^2dx + \frac{6abc^2d\tan\left(e+fx\right)}{f} - \frac{3abcd^2\log\left(\tan^2\left(e+fx\right)+1\right)}{f} + \frac{3abcd^2\tan^2\left(e+fx\right)}{f} + 2abd^3x + \frac{2abd^3\tan^3\left(e+fx\right)}{3f} - \frac{2abd^3\tan\left(e+fx\right)}{f} - b^2c^3x + \frac{b^2c^3\tan\left(e+fx\right)}{f} - \frac{3b^2c^2d\log\left(\tan^2\left(e+fx\right)+1\right)}{2f} + \frac{3b^2c^2d\tan^2\left(e+fx\right)}{2f} + 3b^2cd^2x + \frac{b^2cd^2\tan^3\left(e+fx\right)}{f} - \frac{3b^2cd^2\tan\left(e+fx\right)}{f} + \frac{b^2d^3\log\left(\tan^2\left(e+fx\right)+1\right)}{2f} + \frac{b^2d^3\tan^4\left(e+fx\right)}{4f} - \frac{b^2d^3\tan^2\left(e+fx\right)}{2f} & \text{for}\: f \neq 0 \\ x\left(a+b\tan\left(e\right)\right)^2\left(c+d\tan\left(e\right)\right)^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2*(c+d*tan(f*x+e))**3,x)
```

```
[Out] Piecewise((a**2*c**3*x + 3*a**2*c**2*d*log(tan(e + f*x)**2 + 1)/(2*f) - 3*a
**2*c*d**2*x + 3*a**2*c*d**2*tan(e + f*x)/f - a**2*d**3*log(tan(e + f*x)**2
 + 1)/(2*f) + a**2*d**3*tan(e + f*x)**2/(2*f) + a*b*c**3*log(tan(e + f*x)**
2 + 1)/f - 6*a*b*c**2*d*x + 6*a*b*c**2*d*tan(e + f*x)/f - 3*a*b*c*d**2*log(
tan(e + f*x)**2 + 1)/f + 3*a*b*c*d**2*tan(e + f*x)**2/f + 2*a*b*d**3*x + 2*
a*b*d**3*tan(e + f*x)**3/(3*f) - 2*a*b*d**3*tan(e + f*x)/f - b**2*c**3*x +
b**2*c**3*tan(e + f*x)/f - 3*b**2*c**2*d*log(tan(e + f*x)**2 + 1)/(2*f) + 3
*b**2*c**2*d*tan(e + f*x)**2/(2*f) + 3*b**2*c*d**2*x + b**2*c*d**2*tan(e +
f*x)**3/f - 3*b**2*c*d**2*tan(e + f*x)/f + b**2*d**3*log(tan(e + f*x)**2 +
1)/(2*f) + b**2*d**3*tan(e + f*x)**4/(4*f) - b**2*d**3*tan(e + f*x)**2/(2*f
), Ne(f, 0)), (x*(a + b*tan(e))**2*(c + d*tan(e))**3, True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4557 vs. $2(218) = 436$.
time = 2.67, size = 4557, normalized size = 20.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/12*(12*a^2*c^3*f*x*tan(f*x)^4*tan(e)^4 - 12*b^2*c^3*f*x*tan(f*x)^4*tan(e)
^4 - 72*a*b*c^2*d*f*x*tan(f*x)^4*tan(e)^4 - 36*a^2*c*d^2*f*x*tan(f*x)^4*tan
(e)^4 + 36*b^2*c*d^2*f*x*tan(f*x)^4*tan(e)^4 + 24*a*b*d^3*f*x*tan(f*x)^4*ta
n(e)^4 - 12*a*b*c^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(
f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f
*x)^4*tan(e)^4 - 18*a^2*c^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan
(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 +
 1))*tan(f*x)^4*tan(e)^4 + 18*b^2*c^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(
f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(
tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 + 36*a*b*c*d^2*log(4*(tan(f*x)^4*tan(e)^
2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan
(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 + 6*a^2*d^3*log(4*(tan(f*x)^4*
tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f
*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 - 6*b^2*d^3*log(4*(tan(
f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 -
2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 - 48*a^2*c^3*f*x
*tan(f*x)^3*tan(e)^3 + 48*b^2*c^3*f*x*tan(f*x)^3*tan(e)^3 + 288*a*b*c^2*d*f
*x*tan(f*x)^3*tan(e)^3 + 144*a^2*c*d^2*f*x*tan(f*x)^3*tan(e)^3 - 144*b^2*c*
d^2*f*x*tan(f*x)^3*tan(e)^3 - 96*a*b*d^3*f*x*tan(f*x)^3*tan(e)^3 + 18*b^2*c
^2*d*tan(f*x)^4*tan(e)^4 + 36*a*b*c*d^2*tan(f*x)^4*tan(e)^4 + 6*a^2*d^3*tan
(f*x)^4*tan(e)^4 - 9*b^2*d^3*tan(f*x)^4*tan(e)^4 + 48*a*b*c^3*log(4*(tan(f*
x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*
tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 + 72*a^2*c^2*d*log
(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f
```

```
*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 - 72*b^2
*c^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)
^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^
3 - 144*a*b*c*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*
x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x
)^3*tan(e)^3 - 24*a^2*d^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e)
+ tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))
*tan(f*x)^3*tan(e)^3 + 24*b^2*d^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3
*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)
^2 + 1))*tan(f*x)^3*tan(e)^3 - 12*b^2*c^3*tan(f*x)^4*tan(e)^3 - 72*a*b*c^2*
d*tan(f*x)^4*tan(e)^3 - 36*a^2*c*d^2*tan(f*x)^4*tan(e)^3 + 36*b^2*c*d^2*tan
(f*x)^4*tan(e)^3 + 24*a*b*d^3*tan(f*x)^4*tan(e)^3 - 12*b^2*c^3*tan(f*x)^3*t
an(e)^4 - 72*a*b*c^2*d*tan(f*x)^3*tan(e)^4 - 36*a^2*c*d^2*tan(f*x)^3*tan(e)
^4 + 36*b^2*c*d^2*tan(f*x)^3*tan(e)^4 + 24*a*b*d^3*tan(f*x)^3*tan(e)^4 + 72
*a^2*c^3*f*x*tan(f*x)^2*tan(e)^2 - 72*b^2*c^3*f*x*tan(f*x)^2*tan(e)^2 - 432
*a*b*c^2*d*f*x*tan(f*x)^2*tan(e)^2 - 216*a^2*c*d^2*f*x*tan(f*x)^2*tan(e)^2
+ 216*b^2*c*d^2*f*x*tan(f*x)^2*tan(e)^2 + 144*a*b*d^3*f*x*tan(f*x)^2*tan(e)
^2 + 18*b^2*c^2*d*tan(f*x)^4*tan(e)^2 + 36*a*b*c*d^2*tan(f*x)^4*tan(e)^2 +
6*a^2*d^3*tan(f*x)^4*tan(e)^2 - 6*b^2*d^3*tan(f*x)^4*tan(e)^2 - 36*b^2*c^2*
d*tan(f*x)^3*tan(e)^3 - 72*a*b*c*d^2*tan(f*x)^3*tan(e)^3 - 12*a^2*d^3*tan(f
*x)^3*tan(e)^3 + 24*b^2*d^3*tan(f*x)^3*tan(e)^3 + 18*b^2*c^2*d*tan(f*x)^2*t
an(e)^4 + 36*a*b*c*d^2*tan(f*x)^2*tan(e)^4 + 6*a^2*d^3*tan(f*x)^2*tan(e)^4
- 6*b^2*d^3*tan(f*x)^2*tan(e)^4 - 12*b^2*c*d^2*tan(f*x)^4*tan(e) - 8*a*b*d^
3*tan(f*x)^4*tan(e) - 72*a*b*c^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*
tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^
2 + 1))*tan(f*x)^2*tan(e)^2 - 108*a^2*c^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*
tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) +
1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 + 108*b^2*c^2*d*log(4*(tan(f*x)^4*ta
n(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x
)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 + 216*a*b*c*d^2*log(4*(ta
n(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2
- 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 + 36*a^2*d^3*l
og(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan
(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 - 36*b
^2*d^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)
^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^
2 + 36*b^2*c^3*tan(f*x)^3*tan(e)^2 + 216*a*b*c^2*d*tan(f*x)^3*tan(e)^2 + 10
8*a^2*c*d^2*tan(f*x)^3*tan(e)^2 - 144*b^2*c*d^2*tan(f*x)^3*tan(e)^2 - 96*a*
b*d^3*tan(f*x)^3*tan(e)^2 + 36*b^2*c^3*tan(f*x)^2*tan(e)^3 + 216*a*b*c^2*d*
tan(f*x)^2*tan(e)^3 + 108*a^2*c*d^2*tan(f*x)^2*tan(e)^3 - 144*b^2*c*d^2*tan
(f*x)^2*tan(e)^3 - 96*a*b*d^3*tan(f*x)^2*tan(e)^3 - 12*b^2*c*d^2*tan(f*x)*t
an(e)^4 - 8*a*b*d^3*tan(f*x)*tan(e)^4 + 3*b^2*d^3*tan(f*x)^4 - 48*a^2*c^3*f
*x*tan(f*x)*tan(e) + 48*b^2*c^3*f*x*tan(f*x)*ta...
```

Mupad [B]

time = 5.27, size = 259, normalized size = 1.18

$$x\left(a^2c^3-3a^2cd^2-6abc^2d+2abd^3-b^2c^3+3b^2cd^2\right)+\frac{\tan(e+fx)\left(b^2c^3-bd^2(2ad+3bc)+3a^2cd^2+6abc^2d\right)}{f}-\frac{\ln\left(\tan(e+fx)^2+1\right)\left(-\frac{3a^2c^2d}{2}+\frac{a^2d^3}{2}-abc^3+3abcd^2+\frac{3b^2c^2d}{2}-\frac{b^2d^3}{2}\right)}{f}+\frac{\tan(e+fx)^2\left(\frac{a^2d^3}{2}+3abcd^2+\frac{3b^2c^2d}{2}-\frac{b^2d^3}{2}\right)}{f}+\frac{b^2d^3\tan(e+fx)^4}{4f}+\frac{bd^2\tan(e+fx)^3(2ad+3bc)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^2*(c + d*tan(e + f*x))^3,x)
```

```
[Out] x*(a^2*c^3 - b^2*c^3 - 3*a^2*c*d^2 + 3*b^2*c*d^2 + 2*a*b*d^3 - 6*a*b*c^2*d)
 + (tan(e + f*x)*(b^2*c^3 - b*d^2*(2*a*d + 3*b*c) + 3*a^2*c*d^2 + 6*a*b*c^2
*d))/f - (log(tan(e + f*x)^2 + 1)*((a^2*d^3)/2 - (b^2*d^3)/2 - (3*a^2*c^2*d
)/2 + (3*b^2*c^2*d)/2 - a*b*c^3 + 3*a*b*c*d^2))/f + (tan(e + f*x)^2*((a^2*d
^3)/2 - (b^2*d^3)/2 + (3*b^2*c^2*d)/2 + 3*a*b*c*d^2))/f + (b^2*d^3*tan(e +
f*x)^4)/(4*f) + (b*d^2*tan(e + f*x)^3*(2*a*d + 3*b*c))/(3*f)
```

3.1205 $\int (a+b\tan(e+fx))(c+d\tan(e+fx))^3\,dx$

Optimal. Leaf size=144

$$-\left(\left(bd\left(3c^2-d^2\right)-a\left(c^3-3cd^2\right)\right)x\right)-\frac{\left(bc^3+3ac^2d-3bcd^2-ad^3\right)\log(\cos(e+fx))}{f}+\frac{d(2acd+b(c^2-d^2))\,\text{ta}}{f}$$

```
[Out] -(b*d*(3*c^2-d^2)-a*(c^3-3*c*d^2))*x-(3*a*c^2*d-a*d^3+b*c^3-3*b*c*d^2)*ln(c
os(f*x+e))/f+d*(2*a*c*d+b*(c^2-d^2))*tan(f*x+e)/f+1/2*(a*d+b*c)*(c+d*tan(f*
x+e))^2/f+1/3*b*(c+d*tan(f*x+e))^3/f
```

Rubi [A]
time = 0.12, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3609, 3606, 3556}

$$\frac{d\left(2acd+b\left(c^2-d^2\right)\right)\tan(e+fx)}{f}-x\left(bd\left(3c^2-d^2\right)-a\left(c^3-3cd^2\right)\right)-\frac{\left(3ac^2d-ad^3+bc^3-3bcd^2\right)\log(\cos(e+fx))}{f}+\frac{(ad+bc)(c+d\tan(e+fx))^2}{2f}+\frac{b(c+d\tan(e+fx))^3}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^3,x]
```

```
[Out] -((b*d*(3*c^2 - d^2) - a*(c^3 - 3*c*d^2))*x) - ((b*c^3 + 3*a*c^2*d - 3*b*c*
d^2 - a*d^3)*Log[Cos[e + f*x]])/f + (d*(2*a*c*d + b*(c^2 - d^2))*Tan[e + f*
x])/f + ((b*c + a*d)*(c + d*Tan[e + f*x])^2)/(2*f) + (b*(c + d*Tan[e + f*x]
)^3)/(3*f)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^3\,dx = \frac{b(c+d\tan(e+fx))^3}{3f} + \int (c+d\tan(e+fx))^2(ac-bd+$$

$$= \frac{(bc+ad)(c+d\tan(e+fx))^2}{2f} + \frac{b(c+d\tan(e+fx))^3}{3f} + \int$$

$$= -\left(bd\left(3c^2-d^2\right)-a\left(c^3-3cd^2\right)\right)x + \frac{d(2acd+b(c^2-d^2))\,\mathrm{t}}{f}$$

$$= -\left(bd\left(3c^2-d^2\right)-a\left(c^3-3cd^2\right)\right)x - \frac{(bc^3+3ac^2d-3bcd^2}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.03, size = 130, normalized size = 0.90

$$\frac{3(-ia+b)(c+id)^3\log(i-\tan(e+fx))+3(ia+b)(c-id)^3\log(i+\tan(e+fx))+6d(3bc^2+3acd-bd^2)\tan(e+fx)+3d^2(3bc+ad)\tan^2(e+fx)+2bd^3\tan^3(e+fx)}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^3,x]
```

```
[Out] (3*((-I)*a + b)*(c + I*d)^3*Log[I - Tan[e + f*x]] + 3*(I*a + b)*(c - I*d)^3
*Log[I + Tan[e + f*x]] + 6*d*(3*b*c^2 + 3*a*c*d - b*d^2)*Tan[e + f*x] + 3*d
^2*(3*b*c + a*d)*Tan[e + f*x]^2 + 2*b*d^3*Tan[e + f*x]^3)/(6*f)
```

Maple [A]
time = 0.09, size = 159, normalized size = 1.10

method	result
norman	$\left(a\,c^3-3ac\,d^2-3b\,c^2d+b\,d^3\right)x+\frac{d\left(3acd+3b\,c^2-b\,d^2\right)\tan(fx+e)}{f}+\frac{b\,d^3\left(\tan^3(fx+e)\right)}{3f}+\frac{d^2(ad+3bc)(}{2}$
derivativedivides	$\frac{\frac{b\,d^3\left(\tan^3(fx+e)\right)}{3}+\frac{a\,d^3\left(\tan^2(fx+e)\right)}{2}+\frac{3bc\,d^2\left(\tan^2(fx+e)\right)}{2}+3ac\,d^2\tan(fx+e)+3b\,c^2d\tan(fx+e)-b\,d^3\tan(fx+e)+\frac{(3a\,c}{}}{f}$
default	$\frac{\frac{b\,d^3\left(\tan^3(fx+e)\right)}{3}+\frac{a\,d^3\left(\tan^2(fx+e)\right)}{2}+\frac{3bc\,d^2\left(\tan^2(fx+e)\right)}{2}+3ac\,d^2\tan(fx+e)+3b\,c^2d\tan(fx+e)-b\,d^3\tan(fx+e)+\frac{(3a\,c}{}}{f}$
risch	$a\,c^3x-3ac\,d^2x-3b\,c^2dx+b\,d^3x+\frac{6ia\,c^2de}{f}-3ibc\,d^2x-\frac{6ibc\,d^2e}{f}+ib\,c^3x-ia\,d^3x+\frac{2id(9}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/3*b*d^3*tan(f*x+e)^3+1/2*a*d^3*tan(f*x+e)^2+3/2*b*c*d^2*tan(f*x+e)^2
+3*a*c*d^2*tan(f*x+e)+3*b*c^2*d*tan(f*x+e)-b*d^3*tan(f*x+e)+1/2*(3*a*c^2*d-
```

```
a*d^3+b*c^3-3*b*c*d^2)*ln(1+tan(f*x+e)^2)+(a*c^3-3*a*c*d^2-3*b*c^2*d+b*d^3)
*arctan(tan(f*x+e)))
```

Maxima [A]
time = 0.71, size = 148, normalized size = 1.03

$$\frac{2\,bd^3\tan\left(fx+e\right)^3+3\,\left(3\,bcd^2+ad^3\right)\tan\left(fx+e\right)^2+6\,\left(ac^3-3\,bc^2d-3\,acd^2+bd^3\right)\left(fx+e\right)+3\,\left(bc^3+3\,ac^2d-3\,bcd^2-ad^3\right)\log\left(\tan\left(fx+e\right)^2+1\right)+6\,\left(3\,bc^2d+3\,acd^2-bd^3\right)\tan\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b*d^3*tan(f*x + e)^3 + 3*(3*b*c*d^2 + a*d^3)*tan(f*x + e)^2 + 6*(a*c
^3 - 3*b*c^2*d - 3*a*c*d^2 + b*d^3)*(f*x + e) + 3*(b*c^3 + 3*a*c^2*d - 3*b*
c*d^2 - a*d^3)*log(tan(f*x + e)^2 + 1) + 6*(3*b*c^2*d + 3*a*c*d^2 - b*d^3)*
tan(f*x + e))/f
```

Fricas [A]
time = 1.19, size = 146, normalized size = 1.01

$$\frac{2\,bd^3\tan\left(fx+e\right)^3+6\,\left(ac^3-3\,bc^2d-3\,acd^2+bd^3\right)fx+3\,\left(3\,bcd^2+ad^3\right)\tan\left(fx+e\right)^2-3\,\left(bc^3+3\,ac^2d-3\,bcd^2-ad^3\right)\log\left(\frac{1}{\tan\left(fx+e\right)^2+1}\right)+6\,\left(3\,bc^2d+3\,acd^2-bd^3\right)\tan\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*d^3*tan(f*x + e)^3 + 6*(a*c^3 - 3*b*c^2*d - 3*a*c*d^2 + b*d^3)*f*x
 + 3*(3*b*c*d^2 + a*d^3)*tan(f*x + e)^2 - 3*(b*c^3 + 3*a*c^2*d - 3*b*c*d^2
- a*d^3)*log(1/(tan(f*x + e)^2 + 1)) + 6*(3*b*c^2*d + 3*a*c*d^2 - b*d^3)*ta
n(f*x + e))/f
```

Sympy [A]
time = 0.15, size = 240, normalized size = 1.67

$$\begin{cases} ac^3x+\frac{3ac^2d\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}-3acd^2x+\frac{3acd^2\tan\left(e+fx\right)}{f}-\frac{ad^3\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{ad^3\tan^2\left(e+fx\right)}{2f}+\frac{bc^3\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}-3bc^2dx+\frac{3bc^2d\tan\left(e+fx\right)}{f}-\frac{3bcd^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{3bcd^2\tan^2\left(e+fx\right)}{2f}+bd^3x+\frac{bd^3\tan^3\left(e+fx\right)}{3f}-\frac{bd^3\tan\left(e+fx\right)}{f} & \text{for}\ f\neq 0 \\ x(a+b\tan\left(e\right))\left(c+d\tan\left(e\right)\right)^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))**3,x)
```

```
[Out] Piecewise((a*c**3*x + 3*a*c**2*d*log(tan(e + f*x)**2 + 1)/(2*f) - 3*a*c*d**
2*x + 3*a*c*d**2*tan(e + f*x)/f - a*d**3*log(tan(e + f*x)**2 + 1)/(2*f) + a
*d**3*tan(e + f*x)**2/(2*f) + b*c**3*log(tan(e + f*x)**2 + 1)/(2*f) - 3*b*c
**2*d*x + 3*b*c**2*d*tan(e + f*x)/f - 3*b*c*d**2*log(tan(e + f*x)**2 + 1)/(
2*f) + 3*b*c*d**2*tan(e + f*x)**2/(2*f) + b*d**3*x + b*d**3*tan(e + f*x)**3
/(3*f) - b*d**3*tan(e + f*x)/f, Ne(f, 0)), (x*(a + b*tan(e))*(c + d*tan(e))
**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2046 vs. $2(144) = 288$.

time = 1.34, size = 2046, normalized size = 14.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/6*(6*a*c^3*f*x*tan(f*x)^3*tan(e)^3 - 18*b*c^2*d*f*x*tan(f*x)^3*tan(e)^3 -
 18*a*c*d^2*f*x*tan(f*x)^3*tan(e)^3 + 6*b*d^3*f*x*tan(f*x)^3*tan(e)^3 - 3*b
*c^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2
 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3
- 9*a*c^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*t
an(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*ta
n(e)^3 + 9*b*c*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f
*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*
x)^3*tan(e)^3 + 3*a*d^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) +
tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*t
an(f*x)^3*tan(e)^3 - 18*a*c^3*f*x*tan(f*x)^2*tan(e)^2 + 54*b*c^2*d*f*x*tan(
f*x)^2*tan(e)^2 + 54*a*c*d^2*f*x*tan(f*x)^2*tan(e)^2 - 18*b*d^3*f*x*tan(f*x
)^2*tan(e)^2 + 9*b*c*d^2*tan(f*x)^3*tan(e)^3 + 3*a*d^3*tan(f*x)^3*tan(e)^3
+ 9*b*c^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan
(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(
e)^2 + 27*a*c^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*
x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x
)^2*tan(e)^2 - 27*b*c*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e)
+ tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))
*tan(f*x)^2*tan(e)^2 - 9*a*d^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*ta
n(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2
+ 1))*tan(f*x)^2*tan(e)^2 - 18*b*c^2*d*tan(f*x)^3*tan(e)^2 - 18*a*c*d^2*tan
(f*x)^3*tan(e)^2 + 6*b*d^3*tan(f*x)^3*tan(e)^2 - 18*b*c^2*d*tan(f*x)^2*tan(
e)^3 - 18*a*c*d^2*tan(f*x)^2*tan(e)^3 + 6*b*d^3*tan(f*x)^2*tan(e)^3 + 18*a*
c^3*f*x*tan(f*x)*tan(e) - 54*b*c^2*d*f*x*tan(f*x)*tan(e) - 54*a*c*d^2*f*x*t
an(f*x)*tan(e) + 18*b*d^3*f*x*tan(f*x)*tan(e) + 9*b*c*d^2*tan(f*x)^3*tan(e)
 + 3*a*d^3*tan(f*x)^3*tan(e) - 9*b*c*d^2*tan(f*x)^2*tan(e)^2 - 3*a*d^3*tan(
f*x)^2*tan(e)^2 + 9*b*c*d^2*tan(f*x)*tan(e)^3 + 3*a*d^3*tan(f*x)*tan(e)^3 -
 2*b*d^3*tan(f*x)^3 - 9*b*c^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan
(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 +
 1))*tan(f*x)*tan(e) - 27*a*c^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3
*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)
^2 + 1))*tan(f*x)*tan(e) + 27*b*c*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*
x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(ta
n(e)^2 + 1))*tan(f*x)*tan(e) + 9*a*d^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f
*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(t
```

```
an(e)^2 + 1))*tan(f*x)*tan(e) + 36*b*c^2*d*tan(f*x)^2*tan(e) + 36*a*c*d^2*t
an(f*x)^2*tan(e) - 18*b*d^3*tan(f*x)^2*tan(e) + 36*b*c^2*d*tan(f*x)*tan(e)^
2 + 36*a*c*d^2*tan(f*x)*tan(e)^2 - 18*b*d^3*tan(f*x)*tan(e)^2 - 2*b*d^3*tan
(e)^3 - 6*a*c^3*f*x + 18*b*c^2*d*f*x + 18*a*c*d^2*f*x - 6*b*d^3*f*x - 9*b*c
*d^2*tan(f*x)^2 - 3*a*d^3*tan(f*x)^2 + 9*b*c*d^2*tan(f*x)*tan(e) + 3*a*d^3*
tan(f*x)*tan(e) - 9*b*c*d^2*tan(e)^2 - 3*a*d^3*tan(e)^2 + 3*b*c^3*log(4*(ta
n(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2
- 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1)) + 9*a*c^2*d*log(4*(tan(f*x)^4*tan(
e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*
tan(e) + 1)/(tan(e)^2 + 1)) - 9*b*c*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(
f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(
tan(e)^2 + 1)) - 3*a*d^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) +
 tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))
- 18*b*c^2*d*tan(f*x) - 18*a*c*d^2*tan(f*x) + 6*b*d^3*tan(f*x) - 18*b*c^2*d
*tan(e) - 18*a*c*d^2*tan(e) + 6*b*d^3*tan(e) - 9*b*c*d^2 - 3*a*d^3)/(f*tan(
f*x)^3*tan(e)^3 - 3*f*tan(f*x)^2*tan(e)^2 + 3*f*tan(f*x)*tan(e) - f)
```

Mupad [B]
time = 5.23, size = 142, normalized size = 0.99

$$x\,\left(a\,c^3-3\,b\,c^2\,d-3\,a\,c\,d^2+b\,d^3\right)-\frac{\mathrm{tan}\left(e+f\,x\right)\,\left(b\,d^3-3\,c\,d\,\left(a\,d+b\,c\right)\right)}{f}+\frac{{\mathrm{tan}\left(e+f\,x\right)}^2\,\left(\frac{a\,d^3}{2}+\frac{3\,b\,c\,d^2}{2}\right)}{f}-\frac{\ln\left({\mathrm{tan}\left(e+f\,x\right)}^2+1\right)\,\left(-\frac{b\,c^3}{2}-\frac{3\,a\,c^2\,d}{2}+\frac{3\,b\,c\,d^2}{2}+\frac{a\,d^3}{2}\right)}{f}+\frac{b\,d^3\,{\mathrm{tan}\left(e+f\,x\right)}^3}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))*(c + d*tan(e + f*x))^3,x)
```

```
[Out] x*(a*c^3 + b*d^3 - 3*a*c*d^2 - 3*b*c^2*d) - (tan(e + f*x)*(b*d^3 - 3*c*d*(a
*d + b*c)))/f + (tan(e + f*x)^2*((a*d^3)/2 + (3*b*c*d^2)/2))/f - (log(tan(e
 + f*x)^2 + 1)*((a*d^3)/2 - (b*c^3)/2 - (3*a*c^2*d)/2 + (3*b*c*d^2)/2))/f +
 (b*d^3*tan(e + f*x)^3)/(3*f)
```

3.1206 $\int \frac{(c+d\tan(e+fx))^3}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=140

$$\frac{(ac^3+3bc^2d-3acd^2-bd^3)\,x}{a^2+b^2}+\frac{(bc^3-3ac^2d-3bcd^2+ad^3)\log(\cos(e+fx))}{(a^2+b^2)\,f}+\frac{(bc-ad)^3\log(a+b\tan(e+$$

```
[Out] (a*c^3-3*a*c*d^2+3*b*c^2*d-b*d^3)*x/(a^2+b^2)+(-3*a*c^2*d+a*d^3+b*c^3-3*b*c
*d^2)*ln(cos(f*x+e))/(a^2+b^2)/f+(-a*d+b*c)^3*ln(a+b*tan(f*x+e))/b^2/(a^2+b
^2)/f+d^2*(c+d*tan(f*x+e))/b/f
```

Rubi [A]
time = 0.19, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3647, 3707, 3698, 31, 3556}

$$\frac{(-3ac^2d+ad^3+bc^3-3bcd^2)\log(\cos(e+fx))}{f\,(a^2+b^2)}+\frac{x(ac^3-3acd^2+3bc^2d-bd^3)}{a^2+b^2}+\frac{(bc-ad)^3\log(a+b\tan(e+fx))}{b^2f\,(a^2+b^2)}+\frac{d^2(c+d\tan(e+fx))}{bf}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^3/(a + b*Tan[e + f*x]),x]
```

```
[Out] ((a*c^3 + 3*b*c^2*d - 3*a*c*d^2 - b*d^3)*x)/(a^2 + b^2) + ((b*c^3 - 3*a*c^2
*d - 3*b*c*d^2 + a*d^3)*Log[Cos[e + f*x]])/((a^2 + b^2)*f) + ((b*c - a*d)^3
*Log[a + b*Tan[e + f*x]])/(b^2*(a^2 + b^2)*f) + (d^2*(c + d*Tan[e + f*x]))/
(b*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
```

```
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+d\tan(e+fx))^3}{a+b\tan(e+fx)}\,dx &= \frac{d^2(c+d\tan(e+fx))}{bf}+\frac{\int \frac{bc^3-ad^3+bd(3c^2-d^2)\tan(e+fx)+d^2(3bc-ad)\tan^2(e+fx)}{a+b\tan(e+fx)}\,dx}{b}\\
&= \frac{(ac^3+3bc^2d-3acd^2-bd^3)\,x}{a^2+b^2}+\frac{d^2(c+d\tan(e+fx))}{bf}+\frac{(bc-ad)^3\int \frac{1+\tan^2(e+}{a+b\tan(e+}}{b\,(a^2+b^2)}\\
&= \frac{(ac^3+3bc^2d-3acd^2-bd^3)\,x}{a^2+b^2}+\frac{(bc^3-3ac^2d-3bcd^2+ad^3)\log(\cos(e+fx))}{(a^2+b^2)\,f}\\
&= \frac{(ac^3+3bc^2d-3acd^2-bd^3)\,x}{a^2+b^2}+\frac{(bc^3-3ac^2d-3bcd^2+ad^3)\log(\cos(e+fx))}{(a^2+b^2)\,f}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.73, size = 126, normalized size = 0.90

$$\frac{\frac{(c+id)^3\log(i-\tan(e+fx))}{ia-b}-\frac{(c-id)^3\log(i+\tan(e+fx))}{ia+b}+\frac{2(bc-ad)^3\log(a+b\tan(e+fx))}{b^2(a^2+b^2)}+\frac{2d^2(c+d\tan(e+fx))}{b}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^3/(a + b*Tan[e + f*x]),x]
```

```
[Out] (((c + I*d)^3*Log[I - Tan[e + f*x]])/(I*a - b) - ((c - I*d)^3*Log[I + Tan[e
 + f*x]])/(I*a + b) + (2*(b*c - a*d)^3*Log[a + b*Tan[e + f*x]])/(b^2*(a^2 +
 b^2)) + (2*d^2*(c + d*Tan[e + f*x]))/b)/(2*f)
```

Maple [A]

time = 0.20, size = 164, normalized size = 1.17

method	result
derivativedivides	$\frac{\frac{d^3\tan(fx+e)}{b}+\frac{\frac{\left(3a\,c^2d-a\,d^3-b\,c^3+3bc\,d^2\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(a\,c^3-3ac\,d^2+3b\,c^2d-b\,d^3\right)\arctan(\tan(fx+e))}{a^2+b^2}+\frac{\left(-a^3d^3+3a^2bc\right.}{}}{f}$
default	$\frac{\frac{d^3\tan(fx+e)}{b}+\frac{\frac{\left(3a\,c^2d-a\,d^3-b\,c^3+3bc\,d^2\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(a\,c^3-3ac\,d^2+3b\,c^2d-b\,d^3\right)\arctan(\tan(fx+e))}{a^2+b^2}+\frac{\left(-a^3d^3+3a^2bc\right.}{}}{f}$
norman	$\frac{(a\,c^3-3ac\,d^2+3b\,c^2d-b\,d^3)x}{a^2+b^2}+\frac{d^3\tan(fx+e)}{bf}+\frac{(3a\,c^2d-a\,d^3-b\,c^3+3bc\,d^2)\ln(1+\tan^2(fx+e))}{2f(a^2+b^2)}-\frac{(a^3d^3-3a^2bc\,d^2}{}$
risch	$-\frac{6ia^2c\,d^2e}{(a^2+b^2)bf}-\frac{ix\,d^3}{ib-a}-\frac{x\,c^3}{ib-a}+\frac{3xc\,d^2}{ib-a}+\frac{2id^3}{fb(e^{2i(fx+e)}+1)}+\frac{6ia\,c^2dx}{a^2+b^2}+\frac{3ix\,c^2d}{ib-a}+\frac{2ia^3d^3e}{(a^2+b^2)b^2f}-\frac{2ib\,c^3e}{(a^2+b^2)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^3/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(d^3/b*tan(f*x+e)+1/(a^2+b^2)*(1/2*(3*a*c^2*d-a*d^3-b*c^3+3*b*c*d^2)*ln
(1+tan(f*x+e)^2)+(a*c^3-3*a*c*d^2+3*b*c^2*d-b*d^3)*arctan(tan(f*x+e)))+(-a^
3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/(a^2+b^2)/b^2*ln(a+b*tan(f*x+e))
)
```

Maxima [A]

time = 0.62, size = 175, normalized size = 1.25

$$\frac{\frac{2\,d^3\tan(fx+e)}{b}+\frac{2\,(ac^3+3\,bc^2d-3\,acd^2-bd^3)(fx+e)}{a^2+b^2}+\frac{2\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log(b\tan(fx+e)+a)}{a^2b^2+b^4}-\frac{(bc^3-3\,ac^2d-3\,bcd^2+ad^3)\log\left(\tan(fx+e)^2+1\right)}{a^2+b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*d^3*tan(f*x + e)/b + 2*(a*c^3 + 3*b*c^2*d - 3*a*c*d^2 - b*d^3)*(f*x
 + e)/(a^2 + b^2) + 2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*lo
g(b*tan(f*x + e) + a)/(a^2*b^2 + b^4) - (b*c^3 - 3*a*c^2*d - 3*b*c*d^2 + a*
d^3)*log(tan(f*x + e)^2 + 1)/(a^2 + b^2))/f
```

Fricas [A]

time = 1.38, size = 206, normalized size = 1.47

$$\frac{2\,(a^2b+b^3)d^3\tan\left(fx+e\right)+2\,(ab^2c^3+3\,b^3c^2d-3\,ab^2cd^2-b^3d^3)fx+(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log\left(\frac{b^2\tan(fx+e)^2+2\,ab\tan(fx+e)+a^2}{\tan(fx+e)^2+1}\right)-(3\,(a^2b+b^3)cd^2-(a^3+ab^2)d^3)\log\left(\frac{1}{\tan(fx+e)^2+1}\right)}{2\,(a^2b^2+b^4)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a^2*b + b^3)*d^3*tan(f*x + e) + 2*(a*b^2*c^3 + 3*b^3*c^2*d - 3*a*b^
2*c*d^2 - b^3*d^3)*f*x + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3
)*log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(tan(f*x + e)^2 + 1))
 - (3*(a^2*b + b^3)*c*d^2 - (a^3 + a*b^2)*d^3)*log(1/(tan(f*x + e)^2 + 1)))
/((a^2*b^2 + b^4)*f)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.85, size = 1712, normalized size = 12.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**3/(a+b*tan(f*x+e)),x)
```

```
[Out] Piecewise((zoo*x*(c + d*tan(e))**3/tan(e), Eq(a, 0) & Eq(b, 0) & Eq(f, 0)),
 ((c**3*x + 3*c**2*d*log(tan(e + f*x)**2 + 1)/(2*f) - 3*c*d**2*x + 3*c*d**2
*tan(e + f*x)/f - d**3*log(tan(e + f*x)**2 + 1)/(2*f) + d**3*tan(e + f*x)**
2/(2*f))/a, Eq(b, 0)), (I*c**3*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) - 2*I*b
*f) + c**3*f*x/(2*b*f*tan(e + f*x) - 2*I*b*f) + I*c**3/(2*b*f*tan(e + f*x)
- 2*I*b*f) + 3*c**2*d*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) - 2*I*b*f) - 3*I
*c**2*d*f*x/(2*b*f*tan(e + f*x) - 2*I*b*f) - 3*c**2*d/(2*b*f*tan(e + f*x) -
 2*I*b*f) + 3*I*c*d**2*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) - 2*I*b*f) + 3*
c*d**2*f*x/(2*b*f*tan(e + f*x) - 2*I*b*f) + 3*c*d**2*log(tan(e + f*x)**2 +
1)*tan(e + f*x)/(2*b*f*tan(e + f*x) - 2*I*b*f) - 3*I*c*d**2*log(tan(e + f*x
)**2 + 1)/(2*b*f*tan(e + f*x) - 2*I*b*f) - 3*I*c*d**2/(2*b*f*tan(e + f*x) -
 2*I*b*f) - 3*d**3*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) - 2*I*b*f) + 3*I*d*
*3*f*x/(2*b*f*tan(e + f*x) - 2*I*b*f) + I*d**3*log(tan(e + f*x)**2 + 1)*tan
(e + f*x)/(2*b*f*tan(e + f*x) - 2*I*b*f) + d**3*log(tan(e + f*x)**2 + 1)/(2
*b*f*tan(e + f*x) - 2*I*b*f) + 2*d**3*tan(e + f*x)**2/(2*b*f*tan(e + f*x) -
 2*I*b*f) + 3*d**3/(2*b*f*tan(e + f*x) - 2*I*b*f), Eq(a, -I*b)), (-I*c**3*f
*x*tan(e + f*x)/(2*b*f*tan(e + f*x) + 2*I*b*f) + c**3*f*x/(2*b*f*tan(e + f*
x) + 2*I*b*f) - I*c**3/(2*b*f*tan(e + f*x) + 2*I*b*f) + 3*c**2*d*f*x*tan(e
+ f*x)/(2*b*f*tan(e + f*x) + 2*I*b*f) + 3*I*c**2*d*f*x/(2*b*f*tan(e + f*x)
+ 2*I*b*f) - 3*c**2*d/(2*b*f*tan(e + f*x) + 2*I*b*f) - 3*I*c*d**2*f*x*tan(e
 + f*x)/(2*b*f*tan(e + f*x) + 2*I*b*f) + 3*c*d**2*f*x/(2*b*f*tan(e + f*x) +
 2*I*b*f) + 3*c*d**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*b*f*tan(e + f
*x) + 2*I*b*f) + 3*I*c*d**2*log(tan(e + f*x)**2 + 1)/(2*b*f*tan(e + f*x) +
2*I*b*f) + 3*I*c*d**2/(2*b*f*tan(e + f*x) + 2*I*b*f) - 3*d**3*f*x*tan(e + f
*x)/(2*b*f*tan(e + f*x) + 2*I*b*f) - 3*I*d**3*f*x/(2*b*f*tan(e + f*x) + 2*I
*b*f) - I*d**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*b*f*tan(e + f*x) +
2*I*b*f) + d**3*log(tan(e + f*x)**2 + 1)/(2*b*f*tan(e + f*x) + 2*I*b*f) + 2
*d**3*tan(e + f*x)**2/(2*b*f*tan(e + f*x) + 2*I*b*f) + 3*d**3/(2*b*f*tan(e
+ f*x) + 2*I*b*f), Eq(a, I*b)), (x*(c + d*tan(e))**3/(a + b*tan(e)), Eq(f,
0)), (-2*a**3*d**3*log(a/b + tan(e + f*x))/(2*a**2*b**2*f + 2*b**4*f) + 6*a
**2*b*c*d**2*log(a/b + tan(e + f*x))/(2*a**2*b**2*f + 2*b**4*f) + 2*a**2*b*
```

```
d**3*tan(e + f*x)/(2*a**2*b**2*f + 2*b**4*f) + 2*a*b**2*c**3*f*x/(2*a**2*b*
*2*f + 2*b**4*f) - 6*a*b**2*c**2*d*log(a/b + tan(e + f*x))/(2*a**2*b**2*f +
 2*b**4*f) + 3*a*b**2*c**2*d*log(tan(e + f*x)**2 + 1)/(2*a**2*b**2*f + 2*b*
*4*f) - 6*a*b**2*c*d**2*f*x/(2*a**2*b**2*f + 2*b**4*f) - a*b**2*d**3*log(ta
n(e + f*x)**2 + 1)/(2*a**2*b**2*f + 2*b**4*f) + 2*b**3*c**3*log(a/b + tan(e
 + f*x))/(2*a**2*b**2*f + 2*b**4*f) - b**3*c**3*log(tan(e + f*x)**2 + 1)/(2
*a**2*b**2*f + 2*b**4*f) + 6*b**3*c**2*d*f*x/(2*a**2*b**2*f + 2*b**4*f) + 3
*b**3*c*d**2*log(tan(e + f*x)**2 + 1)/(2*a**2*b**2*f + 2*b**4*f) - 2*b**3*d
**3*f*x/(2*a**2*b**2*f + 2*b**4*f) + 2*b**3*d**3*tan(e + f*x)/(2*a**2*b**2*
f + 2*b**4*f), True))
```

Giac [A]
time = 0.72, size = 176, normalized size = 1.26

$$\frac{\frac{2\,d^3\tan(fx+e)}{b}+\frac{2\,(ac^3+3\,bc^2d-3\,acd^2-bd^3)(fx+e)}{a^2+b^2}-\frac{(bc^3-3\,ac^2d-3\,bcd^2+ad^3)\log\left(\tan(fx+e)^2+1\right)}{a^2+b^2}+\frac{2\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log(|b\tan(fx+e)+a|)}{a^2b^2+b^4}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3/(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*(2*d^3*tan(f*x + e)/b + 2*(a*c^3 + 3*b*c^2*d - 3*a*c*d^2 - b*d^3)*(f*x
+ e)/(a^2 + b^2) - (b*c^3 - 3*a*c^2*d - 3*b*c*d^2 + a*d^3)*log(tan(f*x + e)
^2 + 1)/(a^2 + b^2) + 2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
*log(abs(b*tan(f*x + e) + a))/(a^2*b^2 + b^4))/f
```

Mupad [B]
time = 5.59, size = 178, normalized size = 1.27

$$\frac{d^3\tan(e+f\,x)}{b\,f}-\frac{\ln\left(a+b\tan(e+f\,x)\right)\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{f\,\left(a^2\,b^2+b^4\right)}+\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(-c^3\,1\mathrm{i}+3\,c^2\,d+c\,d^2\,3\mathrm{i}-d^3\right)}{2\,f\,\left(a+b\,1\mathrm{i}\right)}+\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(-c^3+c^2\,d\,3\mathrm{i}+3\,c\,d^2-d^3\,1\mathrm{i}\right)}{2\,f\,\left(b+a\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^3/(a + b*tan(e + f*x)),x)
```

```
[Out] (d^3*tan(e + f*x))/(b*f) - (log(a + b*tan(e + f*x))*(a^3*d^3 - b^3*c^3 + 3*
a*b^2*c^2*d - 3*a^2*b*c*d^2))/(f*(b^4 + a^2*b^2)) + (log(tan(e + f*x) - 1i)
*(c*d^2*3i + 3*c^2*d - c^3*1i - d^3))/(2*f*(a + b*1i)) + (log(tan(e + f*x)
+ 1i)*(3*c*d^2 + c^2*d*3i - c^3 - d^3*1i))/(2*f*(a*1i + b))
```

3.1207 $\int \frac{(c+d\tan(e+fx))^3}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=230

$$-\frac{(b^2c(c^2-3d^2)-2abd(3c^2-d^2)-a^2(c^3-3cd^2))\,x}{(a^2+b^2)^2}+\frac{(2abc(c^2-3d^2)+b^2d(3c^2-d^2)-a^2(3c^2d-d^3))\log(c$$

```
[Out] -(b^2*c*(c^2-3*d^2)-2*a*b*d*(3*c^2-d^2)-a^2*(c^3-3*c*d^2))*x/(a^2+b^2)^2+(2
*a*b*c*(c^2-3*d^2)+b^2*d*(3*c^2-d^2)-a^2*(3*c^2*d-d^3))*ln(cos(f*x+e))/(a^2
+b^2)^2/f+(-a*d+b*c)^2*(a^2*d+2*a*b*c+3*b^2*d)*ln(a+b*tan(f*x+e))/b^2/(a^2+
b^2)^2/f-(-a*d+b*c)^2*(c+d*tan(f*x+e))/b/(a^2+b^2)/f/(a+b*tan(f*x+e))
```

Rubi [A]
time = 0.24, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3646, 3707, 3698, 31, 3556}

$$\frac{(-(a^2(3c^2d-d^3))+2abc(c^2-3d^2)+b^2d(3c^2-d^2))\log(\cos(e+fx))}{f(a^2+b^2)^2}-\frac{x(-(a^2(c^3-3cd^2))-2abd(3c^2-d^2)+b^2c(c^2-3d^2))}{(a^2+b^2)^2}-\frac{(bc-ad)^2(c+d\tan(e+fx))}{bf(a^2+b^2)(a+b\tan(e+fx))}+\frac{(a^2d+2abc+3b^2d)(bc-ad)^2\log(a+b\tan(e+fx))}{b^2f(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^3/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] -(((b^2*c*(c^2 - 3*d^2) - 2*a*b*d*(3*c^2 - d^2) - a^2*(c^3 - 3*c*d^2))*x)/(
a^2 + b^2)^2) + ((2*a*b*c*(c^2 - 3*d^2) + b^2*d*(3*c^2 - d^2) - a^2*(3*c^2*
d - d^3))*Log[Cos[e + f*x]])/((a^2 + b^2)^2*f) + ((b*c - a*d)^2*(2*a*b*c +
a^2*d + 3*b^2*d)*Log[a + b*Tan[e + f*x]])/(b^2*(a^2 + b^2)^2*f) - ((b*c - a
*d)^2*(c + d*Tan[e + f*x]))/(b*(a^2 + b^2)*f*(a + b*Tan[e + f*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
```

```
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^3}{(a+b\tan(e+fx))^2}\,dx = -\frac{(bc-ad)^2(c+d\tan(e+fx))}{b\left(a^2+b^2\right)f(a+b\tan(e+fx))} + \frac{\int \frac{3b^2c^2d+a^2d^3+abc(c^2-3d^2)+b(ad(3c^2-d^2)-b(c^3-3\cdots}{a+b\tan(e+f\cdots}}{b\left(a^2+b\cdots\right.}$$

$$= -\frac{(b^2c(c^2-3d^2)-2abd(3c^2-d^2)-a^2(c^3-3cd^2))\,x}{\left(a^2+b^2\right)^2} - \frac{(bc-ad)^2(c+d\tan\cdots}{b\left(a^2+b^2\right)f(a+b\tan\cdots}$$

$$= -\frac{(b^2c(c^2-3d^2)-2abd(3c^2-d^2)-a^2(c^3-3cd^2))\,x}{\left(a^2+b^2\right)^2} + \frac{(2abc(c^2-3d^2)+b^2d\cdots}{\cdots}$$

$$= -\frac{(b^2c(c^2-3d^2)-2abd(3c^2-d^2)-a^2(c^3-3cd^2))\,x}{\left(a^2+b^2\right)^2} + \frac{(2abc(c^2-3d^2)+b^2d\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.62, size = 535, normalized size = 2.33

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^3/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (Cos[e + f*x]*(a*Cos[e + f*x] + b*Sin[e + f*x])*(a^2*Cos[e + f*x]*(2*(a + I
*b)^2*(I*a^2*d^3 + 2*a*b*d^3 + b^2*c*(c^2 - (3*I)*c*d - 3*d^2))*(e + f*x) -
 2*(a^2 + b^2)^2*d^3*Log[Cos[e + f*x]] + (b*c - a*d)^2*(2*a*b*c + a^2*d + 3
*b^2*d)*Log[(a*Cos[e + f*x] + b*Sin[e + f*x])^2]) + b*(2*(a + I*b)*((-I)*b^
4*c^3 + I*a^4*d^3*(I + e + f*x) + a^3*b*d^2*(3*c + d*(I + e + f*x)) + a*b^3
*c*(c^2*(1 + I*e + I*f*x) - (3*I)*d^2*(e + f*x) + 3*c*d*(I + e + f*x)) + a^
2*b^2*(c^3*(e + f*x) + (2*I)*d^3*(e + f*x) - (3*I)*c^2*d*(-I + e + f*x) - 3
*c*d^2*(I + e + f*x))) - 2*a*(a^2 + b^2)^2*d^3*Log[Cos[e + f*x]] + a*(b*c -
 a*d)^2*(2*a*b*c + a^2*d + 3*b^2*d)*Log[(a*Cos[e + f*x] + b*Sin[e + f*x])^2
])*Sin[e + f*x] - (2*I)*a*(b*c - a*d)^2*(2*a*b*c + a^2*d + 3*b^2*d)*ArcTan[
Tan[e + f*x]]*(a*Cos[e + f*x] + b*Sin[e + f*x]))*(c + d*Tan[e + f*x])^3)/(2
*a*(a - I*b)^2*(a + I*b)^2*b^2*f*(c*Cos[e + f*x] + d*Sin[e + f*x])^3*(a + b
*Tan[e + f*x])^2)
```

Maple [A]
time = 0.30, size = 281, normalized size = 1.22 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^3/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(a^2+b^2)^2*(1/2*(3*a^2*c^2*d-a^2*d^3-2*a*b*c^3+6*a*b*c*d^2-3*b^2*c^
2*d+b^2*d^3)*ln(1+tan(f*x+e)^2)+(a^2*c^3-3*a^2*c*d^2+6*a*b*c^2*d-2*a*b*d^3-
b^2*c^3+3*b^2*c*d^2)*arctan(tan(f*x+e)))-(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^
2*d+b^3*c^3)/(a^2+b^2)/b^2/(a+b*tan(f*x+e))+(a^4*d^3-3*a^2*b^2*c^2*d+3*a^2*
b^2*d^3+2*a*b^3*c^3-6*a*b^3*c*d^2+3*b^4*c^2*d)/(a^2+b^2)^2/b^2*ln(a+b*tan(f
*x+e)))
```

Maxima [A]
time = 0.60, size = 312, normalized size = 1.36

$$\frac{\frac{2\,(6\,abc^2d-2\,abd^3+(a^2-b^2)c^3-3\,(a^2-b^2)cd^2)(fx+e)}{a^4+2\,a^2b^2+b^4}+\frac{2\,(2\,ab^3c^3-6\,ab^3cd^2-3\,(a^2b^2-b^4)c^2d+(a^4+3\,a^2b^2)d^3)\log(b\tan(fx+e)+a)}{a^4b^2+2\,a^2b^4+b^6}-\frac{(2\,abc^3-6\,abcd^2-3\,(a^2-b^2)c^2d+(a^2-b^2)d^3)\log\left(\tan(fx+e)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)}{a^3b^2+ab^4+(a^2b^3+b^5)\tan(fx+e)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(6*a*b*c^2*d - 2*a*b*d^3 + (a^2 - b^2)*c^3 - 3*(a^2 - b^2)*c*d^2)*(f
*x + e)/(a^4 + 2*a^2*b^2 + b^4) + 2*(2*a*b^3*c^3 - 6*a*b^3*c*d^2 - 3*(a^2*b
^2 - b^4)*c^2*d + (a^4 + 3*a^2*b^2)*d^3)*log(b*tan(f*x + e) + a)/(a^4*b^2 +
 2*a^2*b^4 + b^6) - (2*a*b*c^3 - 6*a*b*c*d^2 - 3*(a^2 - b^2)*c^2*d + (a^2 -
 b^2)*d^3)*log(tan(f*x + e)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*(b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(a^3*b^2 + a*b^4 + (a^2*b^3 + b^5)*
tan(f*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 509 vs. 2(234) = 468.
time = 1.41, size = 509, normalized size = 2.21

$$-\frac{2\,b^5c^3-6\,ab^4c^2d+6\,a^2b^3cd^2-2\,a^3b^2d^3-2\,(6\,a^2b^3c^2d-2\,a^2b^3d^3+(a^3b^2-ab^4)c^3-3\,(a^3b^2-ab^4)cd^2)fx-(2\,a^2b^3c^3-6\,a^2b^3cd^2-3\,(a^3b^2-ab^4)c^2d+(a^5+3\,a^3b^2)d^3+(2\,ab^4c^3-6\,ab^4cd^2-3\,(a^2b^3-b^5)c^2d+(a^4b+3\,a^2b^3)d^3)\tan(fx+e))\log\left(\frac{b^2\tan(fx+e)^2+2\,ab\tan(fx+e)+a^2}{\tan(fx+e)^2+1}\right)+((a^4b+2\,a^2b^3+b^5)d^3\tan(fx+e)+(a^5+2\,a^3b^2+ab^4)d^3)\log\left(\frac{1}{\tan(fx+e)^2+1}\right)-2\,(ab^4c^3-3\,a^2b^3c^2d+3\,a^3b^2cd^2-a^4bd^3+(6\,ab^4c^2d-2\,ab^4d^3+(a^2b^3-b^5)c^3-3\,(a^2b^3-b^5)cd^2)fx)\tan(fx+e)}{2\,((a^4b^3+2\,a^2b^5+b^7)f\tan(fx+e)+(a^5b^2+2\,a^3b^4+ab^6)f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^5*c^3 - 6*a*b^4*c^2*d + 6*a^2*b^3*c*d^2 - 2*a^3*b^2*d^3 - 2*(6*a^
2*b^3*c^2*d - 2*a^2*b^3*d^3 + (a^3*b^2 - a*b^4)*c^3 - 3*(a^3*b^2 - a*b^4)*c
*d^2)*f*x - (2*a^2*b^3*c^3 - 6*a^2*b^3*c*d^2 - 3*(a^3*b^2 - a*b^4)*c^2*d +
(a^5 + 3*a^3*b^2)*d^3 + (2*a*b^4*c^3 - 6*a*b^4*c*d^2 - 3*(a^2*b^3 - b^5)*c^
2*d + (a^4*b + 3*a^2*b^3)*d^3)*tan(f*x + e))*log((b^2*tan(f*x + e)^2 + 2*a*
b*tan(f*x + e) + a^2)/(tan(f*x + e)^2 + 1)) + ((a^4*b + 2*a^2*b^3 + b^5)*d^
3*tan(f*x + e) + (a^5 + 2*a^3*b^2 + a*b^4)*d^3)*log(1/(tan(f*x + e)^2 + 1))
 - 2*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - a^4*b*d^3 + (6*a*b^4*
c^2*d - 2*a*b^4*d^3 + (a^2*b^3 - b^5)*c^3 - 3*(a^2*b^3 - b^5)*c*d^2)*f*x)*t
an(f*x + e))/((a^4*b^3 + 2*a^2*b^5 + b^7)*f*tan(f*x + e) + (a^5*b^2 + 2*a^3
*b^4 + a*b^6)*f)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.38, size = 6730, normalized size = 29.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**3/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Piecewise((zoo*x*(c + d*tan(e))**3/tan(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0
)), ((c**3*x + 3*c**2*d*log(tan(e + f*x)**2 + 1)/(2*f) - 3*c*d**2*x + 3*c*d
**2*tan(e + f*x)/f - d**3*log(tan(e + f*x)**2 + 1)/(2*f) + d**3*tan(e + f*x
)**2/(2*f))/a**2, Eq(b, 0)), (-c**3*f*x*tan(e + f*x)**2/(4*b**2*f*tan(e + f
*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 2*I*c**3*f*x*tan(e + f*x)/(4
*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + c**3*f*x/(4
*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - c**3*tan(e
+ f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 2*
I*c**3/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 3*
I*c**2*d*f*x*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e +
 f*x) - 4*b**2*f) + 6*c**2*d*f*x*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8
*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 3*I*c**2*d*f*x/(4*b**2*f*tan(e + f*x)*
*2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 3*I*c**2*d*tan(e + f*x)/(4*b**2*
f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 3*c*d**2*f*x*tan(
e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f)
 - 6*I*c*d**2*f*x*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e
 + f*x) - 4*b**2*f) - 3*c*d**2*f*x/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*t
an(e + f*x) - 4*b**2*f) - 9*c*d**2*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 -
 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 6*I*c*d**2/(4*b**2*f*tan(e + f*x)**2
 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 3*I*d**3*f*x*tan(e + f*x)**2/(4*b*
*2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 6*d**3*f*x*tan
(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) -
```

```
3*I*d**3*f*x/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*
f) + 2*d**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)
**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 4*I*d**3*log(tan(e + f*x)**2 +
1)*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**
2*f) - 2*d**3*log(tan(e + f*x)**2 + 1)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2
*f*tan(e + f*x) - 4*b**2*f) - 5*I*d**3*tan(e + f*x)/(4*b**2*f*tan(e + f*x)*
*2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 4*d**3/(4*b**2*f*tan(e + f*x)**2
 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f), Eq(a, -I*b)), (-c**3*f*x*tan(e + f*
x)**2/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 2*I
*c**3*f*x*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x)
- 4*b**2*f) + c**3*f*x/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x)
- 4*b**2*f) - c**3*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(
e + f*x) - 4*b**2*f) - 2*I*c**3/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(
e + f*x) - 4*b**2*f) - 3*I*c**2*d*f*x*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x
)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 6*c**2*d*f*x*tan(e + f*x)/(4*b
**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 3*I*c**2*d*f*
x/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 3*I*c**
2*d*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b*
*2*f) + 3*c*d**2*f*x*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f
*tan(e + f*x) - 4*b**2*f) + 6*I*c*d**2*f*x*tan(e + f*x)/(4*b**2*f*tan(e + f
*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 3*c*d**2*f*x/(4*b**2*f*tan(e
 + f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 9*c*d**2*tan(e + f*x)/(4
*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 6*I*c*d**2/
(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 3*I*d**3*
f*x*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4
*b**2*f) + 6*d**3*f*x*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*t
an(e + f*x) - 4*b**2*f) + 3*I*d**3*f*x/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2
*f*tan(e + f*x) - 4*b**2*f) + 2*d**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)*
*2/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 4*I*d*
*3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 + 8*I*b*
*2*f*tan(e + f*x) - 4*b**2*f) - 2*d**3*log(tan(e + f*x)**2 + 1)/(4*b**2*f*t
an(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 5*I*d**3*tan(e + f*x
)/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 4*d**3/
(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f), Eq(a, I*b)
), (x*(c + d*tan(e))**3/(a + b*tan(e))**2, Eq(f, 0)), (2*a**5*d**3*log(a/b
+ tan(e + f*x))/(2*a**5*b**2*f + 2*a**4*b**3*f*tan(e + f*x) + 4*a**3*b**4*f
 + 4*a**2*b**5*f*tan(e + f*x) + 2*a*b**6*f + 2*b**7*f*tan(e + f*x)) + 2*a**
5*d**3/(2*a**5*b**2*f + 2*a**4*b**3*f*tan(e + f*x) + 4*a**3*b**4*f + 4*a**2
*b**5*f*tan(e + f*x) + 2*a*b**6*f + 2*b**7*f*tan(e + f*x)) - 6*a**4*b*c*d**
2/(2*a**5*b**2*f + 2*a**4*b**3*f*tan(e + f*x) + 4*a**3*b**4*f + 4*a**2*b**5
*f*tan(e + f*x) + 2*a*b**6*f + 2*b**7*f*tan(e + f*x)) + 2*a**4*b*d**3*log(a
/b + tan(e + f*x))*tan(e + f*x)/(2*a**5*b**2*f + 2*a**4*b**3*f*tan(e + f*x)
 + 4*a**3*b**4*f + 4*a**2*b**5*f*tan(e + f*x) + 2*a*b**6*f + 2*b**7*f*tan(e
 + f*x)) + 2*a**3*b**2*c**3*f*x/(2*a**5*b**2*f + 2*a**4*b**3*f*tan(e + f*x)
 + 4*a**3*b**4*f + 4*a**2*b**5*f*tan(e + f*x) +...
```

Giac [A]
time = 0.79, size = 448, normalized size = 1.95

$$\frac{\frac{2\,(a^2c^3-b^2c^3+6\,abc^2d-3\,a^2cd^2+3\,b^2cd^2-2\,abd^3)(fx+e)}{a^4+2\,a^2b^2+b^4}-\frac{(2\,abc^3-3\,a^2c^2d+3\,b^2c^2d-6\,abcd^2+a^2d^3-b^2d^3)\log\left(\tan(fx+e)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\,(2\,ab^3c^3-3\,a^2b^2c^2d+3\,b^4c^2d-6\,ab^3cd^2+a^4d^3+3\,a^2b^2d^3)\log(|b\tan(fx+e)+a|)}{a^4b^2+2\,a^2b^4+b^6}-\frac{2\,(2\,ab^3c^3\tan(fx+e)-3\,a^2b^2c^2d\tan(fx+e)+3\,b^4c^2d\tan(fx+e)-6\,ab^3cd^2\tan(fx+e)+a^4d^3\tan(fx+e)+3\,a^2b^2d^3\tan(fx+e)+3\,a^2b^2c^3+b^4c^3-6\,a^3bc^2d+3\,a^4cd^2-3\,a^2b^2cd^2+2\,a^3bd^3)}{(a^4b+2\,a^2b^3+b^5)(b\tan(fx+e)+a)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(a^2*c^3 - b^2*c^3 + 6*a*b*c^2*d - 3*a^2*c*d^2 + 3*b^2*c*d^2 - 2*a*b
*d^3)*(f*x + e)/(a^4 + 2*a^2*b^2 + b^4) - (2*a*b*c^3 - 3*a^2*c^2*d + 3*b^2*
c^2*d - 6*a*b*c*d^2 + a^2*d^3 - b^2*d^3)*log(tan(f*x + e)^2 + 1)/(a^4 + 2*a
^2*b^2 + b^4) + 2*(2*a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*b^4*c^2*d - 6*a*b^3*c*
d^2 + a^4*d^3 + 3*a^2*b^2*d^3)*log(abs(b*tan(f*x + e) + a))/(a^4*b^2 + 2*a^
2*b^4 + b^6) - 2*(2*a*b^3*c^3*tan(f*x + e) - 3*a^2*b^2*c^2*d*tan(f*x + e) +
 3*b^4*c^2*d*tan(f*x + e) - 6*a*b^3*c*d^2*tan(f*x + e) + a^4*d^3*tan(f*x +
e) + 3*a^2*b^2*d^3*tan(f*x + e) + 3*a^2*b^2*c^3 + b^4*c^3 - 6*a^3*b*c^2*d +
 3*a^4*c*d^2 - 3*a^2*b^2*c*d^2 + 2*a^3*b*d^3)/((a^4*b + 2*a^2*b^3 + b^5)*(b
*tan(f*x + e) + a)))/f
```

Mupad [B]
time = 7.82, size = 271, normalized size = 1.18

$$\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(-c^3-c^2\,d\,3\mathrm{i}+3\,c\,d^2+d^3\,1\mathrm{i}\right)}{2\,f\,\left(-a^2\,1\mathrm{i}+2\,a\,b+b^2\,1\mathrm{i}\right)}+\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(-c^3\,1\mathrm{i}-3\,c^2\,d+c\,d^2\,3\mathrm{i}+d^3\right)}{2\,f\,\left(-a^2+a\,b\,2\mathrm{i}+b^2\right)}+\frac{\ln\left(a+b\tan(e+f\,x)\right)\,\left(b^2\,\left(3\,a^2\,d^3-3\,a^2\,c^2\,d\right)+a^4\,d^3+b^3\,\left(2\,a\,c^3-6\,a\,c\,d^2\right)+3\,b^4\,c^2\,d\right)}{f\,\left(a^4\,b^2+2\,a^2\,b^4+b^6\right)}+\frac{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}{b^2\,f\,\left(a^2+b^2\right)\,\left(a+b\tan\left(e+f\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^3/(a + b*tan(e + f*x))^2,x)
```

```
[Out] (log(tan(e + f*x) - 1i)*(3*c*d^2 - c^2*d*3i - c^3 + d^3*1i))/(2*f*(2*a*b -
a^2*1i + b^2*1i)) + (log(tan(e + f*x) + 1i)*(c*d^2*3i - 3*c^2*d - c^3*1i +
d^3))/(2*f*(a*b*2i - a^2 + b^2)) + (log(a + b*tan(e + f*x))*(b^2*(3*a^2*d^3
 - 3*a^2*c^2*d) + a^4*d^3 + b^3*(2*a*c^3 - 6*a*c*d^2) + 3*b^4*c^2*d))/(f*(b
^6 + 2*a^2*b^4 + a^4*b^2)) + (a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c
*d^2)/(b^2*f*(a^2 + b^2)*(a + b*tan(e + f*x)))
```

3.1208 $\int \frac{(c+d\tan(e+fx))^3}{(a+b\tan(e+fx))^3}\,dx$

Optimal. Leaf size=239

$$\frac{(ac+bd)\left(8abcd+a^2(c^2-3d^2)-b^2(3c^2-d^2)\right)x}{\left(a^2+b^2\right)^3}+\frac{(bc-ad)\left(8abcd-b^2(c^2-3d^2)+a^2(3c^2-d^2)\right)\log(a\cos($$

```
[Out] (a*c+b*d)*(8*a*b*c*d+a^2*(c^2-3*d^2)-b^2*(3*c^2-d^2))*x/(a^2+b^2)^3+(-a*d+b
*c)*(8*a*b*c*d-b^2*(c^2-3*d^2)+a^2*(3*c^2-d^2))*ln(a*cos(f*x+e)+b*sin(f*x+e
))/(a^2+b^2)^3/f-1/2*(-a*d+b*c)^2*(a^2*d+4*a*b*c+5*b^2*d)/b^2/(a^2+b^2)^2/f
/(a+b*tan(f*x+e))-1/2*(-a*d+b*c)^2*(c+d*tan(f*x+e))/b/(a^2+b^2)/f/(a+b*tan(
f*x+e))^2
```

Rubi [A]
time = 0.36, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3646, 3709, 3612, 3611}

$$\frac{\left(a^2(3c^2-d^2)+8abcd-b^2(c^2-3d^2)\right)(bc-ad)\log(a\cos(e+fx)+b\sin(e+fx))}{f\left(a^2+b^2\right)^3}+\frac{x(ac+bd)\left(a^2(c^2-3d^2)+8abcd-b^2(3c^2-d^2)\right)}{\left(a^2+b^2\right)^3}-\frac{(bc-ad)^2(c+d\tan(e+fx))}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}-\frac{\left(a^2d+4abc+5b^2d\right)(bc-ad)^2}{2b^2f\left(a^2+b^2\right)^2(a+b\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^3/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] ((a*c + b*d)*(8*a*b*c*d + a^2*(c^2 - 3*d^2) - b^2*(3*c^2 - d^2))*x)/(a^2 +
b^2)^3 + ((b*c - a*d)*(8*a*b*c*d - b^2*(c^2 - 3*d^2) + a^2*(3*c^2 - d^2))*L
og[a*Cos[e + f*x] + b*Sin[e + f*x]])/((a^2 + b^2)^3*f) - ((b*c - a*d)^2*(4*
a*b*c + a^2*d + 5*b^2*d))/(2*b^2*(a^2 + b^2)^2*f*(a + b*Tan[e + f*x])) - ((
b*c - a*d)^2*(c + d*Tan[e + f*x]))/(2*b*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^
2)
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^3}{(a+b\tan(e+fx))^3}\,dx = -\frac{(bc-ad)^2(c+d\tan(e+fx))}{2b\,(a^2+b^2)\,f(a+b\tan(e+fx))^2} + \frac{\int \frac{d(2bc-ad)^2+bc^2(2ac+bd)+2b\left(ad(3c^2-d^2)-b(c^3-\cdots\right.}{(a+\cdots}}{2\cdots}$$

$$= -\frac{(bc-ad)^2\left(4abc+a^2d+5b^2d\right)}{2b^2\left(a^2+b^2\right)^2 f(a+b\tan(e+fx))} - \frac{(bc-ad)^2(c+d\tan(e+fx))}{2b\,(a^2+b^2)\,f(a+b\tan(e+fx))^2} + \cdots$$

$$= \frac{(ac+bd)\left(8abcd+a^2(c^2-3d^2)-b^2(3c^2-d^2)\right)x}{\left(a^2+b^2\right)^3} - \frac{(bc-ad)^2\left(4abc+a^2d\cdots\right.}{2b^2\left(a^2+b^2\right)^2 f(a+b\tan\cdots}$$

$$= \frac{(ac+bd)\left(8abcd+a^2(c^2-3d^2)-b^2(3c^2-d^2)\right)x}{\left(a^2+b^2\right)^3} + \frac{(bc-ad)\left(3a^2c^2-b^2c^2+\cdots\right.}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.77, size = 327, normalized size = 1.37

$$\frac{-\frac{d^2(bc+ad)}{(a+b\tan(e+fx))^2} - \frac{2bd^2(c+d\tan(e+fx))}{(a+b\tan(e+fx))^2} + 2bd(3c^2-d^2)\left(-\frac{i\log(i-\tan(e+fx))}{2(a+ib)^2} + \frac{i\log(i+\tan(e+fx))}{2(a-ib)^2} + \frac{b\left(2a\log(a+b\tan(e+fx)) - \frac{a^2+b^2}{a+b\tan(e+fx)}\right)}{(a^2+b^2)^2}\right) + b(ad(-3c^2+d^2)+b(c^3-3cd^2))\left(\frac{\log(i-\tan(e+fx))}{(-ia+b)^3} + \frac{\log(i+\tan(e+fx))}{(ia+b)^3} + \frac{b\left((6a^2-2b^2)\log(a+b\tan(e+fx)) - \frac{(a^2+b^2)(5a^2+b^2+4ab\tan(e+fx))}{(a+b\tan(e+fx))^2}\right)}{(a^2+b^2)^3}\right)}{2b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^3/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (-((d^2*(b*c + a*d))/(a + b*Tan[e + f*x])^2) - (2*b*d^2*(c + d*Tan[e + f*x]
))/(a + b*Tan[e + f*x])^2 + 2*b*d*(3*c^2 - d^2)*(((-1/2*I)*Log[I - Tan[e +
f*x]])/(a + I*b)^2 + ((I/2)*Log[I + Tan[e + f*x]])/(a - I*b)^2 + (b*(2*a*Lo
g[a + b*Tan[e + f*x]] - (a^2 + b^2)/(a + b*Tan[e + f*x])))/(a^2 + b^2)^2) +
 b*(a*d*(-3*c^2 + d^2) + b*(c^3 - 3*c*d^2))*(Log[I - Tan[e + f*x]]/((-I)*a
+ b)^3 + Log[I + Tan[e + f*x]]/(I*a + b)^3 + (b*((6*a^2 - 2*b^2)*Log[a + b*
Tan[e + f*x]] - ((a^2 + b^2)*(5*a^2 + b^2 + 4*a*b*Tan[e + f*x]))/(a + b*Tan
[e + f*x])^2))/(a^2 + b^2)^3))/(2*b^2*f)
```

Maple [A]
time = 0.32, size = 422, normalized size = 1.77

method	result
derivativedivides	$\frac{\frac{\left(3a^3c^2d-a^3d^3-3a^2b\,c^3+9a^2bc\,d^2-9a\,b^2c^2d+3a\,b^2d^3+b^3c^3-3b^3c\,d^2\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(a^3c^3-3a^3c\,d^2+9a^2b\,c^2d-3a^2b\,d^3-3a\,b^2\right.}{\left(a^2+b^2\right)^3}$
default	$\frac{\frac{\left(3a^3c^2d-a^3d^3-3a^2b\,c^3+9a^2bc\,d^2-9a\,b^2c^2d+3a\,b^2d^3+b^3c^3-3b^3c\,d^2\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(a^3c^3-3a^3c\,d^2+9a^2b\,c^2d-3a^2b\,d^3-3a\,b^2\right.}{\left(a^2+b^2\right)^3}$
norman	$\frac{\frac{\left(-a^4d^3+3a^2b^2c^2d-3a^2b^2d^3-2a\,b^3c^3+6a\,b^3c\,d^2-3b^4c^2d\right)\tan(fx+e)}{fb\left(a^4+2a^2b^2+b^4\right)}+\frac{\left(a^3c^3-3a^3c\,d^2+9a^2b\,c^2d-3a^2b\,d^3-3a\,b^2c^3+9a\,b^2c\,d^2-3b^3c\right.}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^3/(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(a^2+b^2)^3*(1/2*(3*a^3*c^2*d-a^3*d^3-3*a^2*b*c^3+9*a^2*b*c*d^2-9*a*
b^2*c^2*d+3*a*b^2*d^3+b^3*c^3-3*b^3*c*d^2)*ln(1+tan(f*x+e)^2)+(a^3*c^3-3*a^
3*c*d^2+9*a^2*b*c^2*d-3*a^2*b*d^3-3*a*b^2*c^3+9*a*b^2*c*d^2-3*b^3*c^2*d+b^3
*d^3)*arctan(tan(f*x+e)))-1/2*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3
)/(a^2+b^2)/b^2/(a+b*tan(f*x+e))^2-(a^4*d^3-3*a^2*b^2*c^2*d+3*a^2*b^2*d^3+2
*a*b^3*c^3-6*a*b^3*c*d^2+3*b^4*c^2*d)/(a^2+b^2)^2/b^2/(a+b*tan(f*x+e))-(3*a
^3*c^2*d-a^3*d^3-3*a^2*b*c^3+9*a^2*b*c*d^2-9*a*b^2*c^2*d+3*a*b^2*d^3+b^3*c^
3-3*b^3*c*d^2)/(a^2+b^2)^3*ln(a+b*tan(f*x+e)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 539 vs. 2(240) = 480.
time = 0.66, size = 539, normalized size = 2.26

$$\frac{\frac{2\left((a^3-3ab^2)c^3+3(3a^2b-b^3)c^2d-3(a^3-3ab^2)cd^2-(3a^2b-b^3)d^3\right)(fx+e)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{2\left((3a^2b-b^3)c^3-3(a^3-3ab^2)c^2d-3(3a^2b-b^3)cd^2+(a^3-3ab^2)d^3\right)\log(b\tan(fx+e)+a)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{\left((3a^2b-b^3)c^3-3(a^3-3ab^2)c^2d-3(3a^2b-b^3)cd^2+(a^3-3ab^2)d^3\right)\log\left(\tan(fx+e)^2+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{(5a^2b^3+b^5)c^3-3(3a^3b^2-ab^4)c^2d+3(a^4b-3a^2b^3)cd^2+(a^5+5a^3b^2)d^3+2\left(2ab^4c^3-6ab^4cd^2-3(a^2b^3-b^5)c^2d+(a^4b+3a^2b^3)d^3\right)\tan(fx+e)}{a^6b^2+2a^4b^4+a^2b^6+(a^4b^4+2a^2b^6+b^8)\tan(fx+e)^2+2(a^5b^3+2a^3b^5+ab^7)\tan(fx+e)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*((a^3 - 3*a*b^2)*c^3 + 3*(3*a^2*b - b^3)*c^2*d - 3*(a^3 - 3*a*b^2)*c
*d^2 - (3*a^2*b - b^3)*d^3)*(f*x + e)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) +
 2*((3*a^2*b - b^3)*c^3 - 3*(a^3 - 3*a*b^2)*c^2*d - 3*(3*a^2*b - b^3)*c*d^2
 + (a^3 - 3*a*b^2)*d^3)*log(b*tan(f*x + e) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^
4 + b^6) - ((3*a^2*b - b^3)*c^3 - 3*(a^3 - 3*a*b^2)*c^2*d - 3*(3*a^2*b - b^
3)*c*d^2 + (a^3 - 3*a*b^2)*d^3)*log(tan(f*x + e)^2 + 1)/(a^6 + 3*a^4*b^2 +
3*a^2*b^4 + b^6) - ((5*a^2*b^3 + b^5)*c^3 - 3*(3*a^3*b^2 - a*b^4)*c^2*d + 3
*(a^4*b - 3*a^2*b^3)*c*d^2 + (a^5 + 5*a^3*b^2)*d^3 + 2*(2*a*b^4*c^3 - 6*a*b
^4*c*d^2 - 3*(a^2*b^3 - b^5)*c^2*d + (a^4*b + 3*a^2*b^3)*d^3)*tan(f*x + e))
/(a^6*b^2 + 2*a^4*b^4 + a^2*b^6 + (a^4*b^4 + 2*a^2*b^6 + b^8)*tan(f*x + e)^
2 + 2*(a^5*b^3 + 2*a^3*b^5 + a*b^7)*tan(f*x + e)))/f
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 907 vs. $2(240) = 480$.
time = 1.43, size = 907, normalized size = 3.79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/2*((7*a^2*b^3 + b^5)*c^3 - 3*(5*a^3*b^2 - a*b^4)*c^2*d + 9*(a^4*b - a^2*
b^3)*c*d^2 - (a^5 - 5*a^3*b^2)*d^3 - 2*((a^5 - 3*a^3*b^2)*c^3 + 3*(3*a^4*b
- a^2*b^3)*c^2*d - 3*(a^5 - 3*a^3*b^2)*c*d^2 - (3*a^4*b - a^2*b^3)*d^3)*f*x
 - ((5*a^2*b^3 - b^5)*c^3 - 9*(a^3*b^2 - a*b^4)*c^2*d + 3*(a^4*b - 5*a^2*b^
3)*c*d^2 + (a^5 + 7*a^3*b^2)*d^3 + 2*((a^3*b^2 - 3*a*b^4)*c^3 + 3*(3*a^2*b^
3 - b^5)*c^2*d - 3*(a^3*b^2 - 3*a*b^4)*c*d^2 - (3*a^2*b^3 - b^5)*d^3)*f*x)*
tan(f*x + e)^2 - ((3*a^4*b - a^2*b^3)*c^3 - 3*(a^5 - 3*a^3*b^2)*c^2*d - 3*(
3*a^4*b - a^2*b^3)*c*d^2 + (a^5 - 3*a^3*b^2)*d^3 + ((3*a^2*b^3 - b^5)*c^3 -
 3*(a^3*b^2 - 3*a*b^4)*c^2*d - 3*(3*a^2*b^3 - b^5)*c*d^2 + (a^3*b^2 - 3*a*b
^4)*d^3)*tan(f*x + e)^2 + 2*((3*a^3*b^2 - a*b^4)*c^3 - 3*(a^4*b - 3*a^2*b^3
)*c^2*d - 3*(3*a^3*b^2 - a*b^4)*c*d^2 + (a^4*b - 3*a^2*b^3)*d^3)*tan(f*x +
e))*log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(tan(f*x + e)^2 + 1
)) - 2*(3*(a^3*b^2 - a*b^4)*c^3 - 3*(2*a^4*b - 3*a^2*b^3 + b^5)*c^2*d + 3*(
a^5 - 3*a^3*b^2 + 2*a*b^4)*c*d^2 + 3*(a^4*b - a^2*b^3)*d^3 + 2*((a^4*b - 3*
a^2*b^3)*c^3 + 3*(3*a^3*b^2 - a*b^4)*c^2*d - 3*(a^4*b - 3*a^2*b^3)*c*d^2 -
(3*a^3*b^2 - a*b^4)*d^3)*f*x)*tan(f*x + e))/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b
^6 + b^8)*f*tan(f*x + e)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*f*ta
n(f*x + e) + (a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**3/(a+b*tan(f*x+e))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 830 vs. $2(240) = 480$.
time = 0.95, size = 830, normalized size = 3.47

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3/(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(a^3*c^3 - 3*a*b^2*c^3 + 9*a^2*b*c^2*d - 3*b^3*c^2*d - 3*a^3*c*d^2 +
 9*a*b^2*c*d^2 - 3*a^2*b*d^3 + b^3*d^3)*(f*x + e)/(a^6 + 3*a^4*b^2 + 3*a^2*
b^4 + b^6) - (3*a^2*b*c^3 - b^3*c^3 - 3*a^3*c^2*d + 9*a*b^2*c^2*d - 9*a^2*b
*c*d^2 + 3*b^3*c*d^2 + a^3*d^3 - 3*a*b^2*d^3)*log(tan(f*x + e)^2 + 1)/(a^6
+ 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(3*a^2*b^2*c^3 - b^4*c^3 - 3*a^3*b*c^2*d
 + 9*a*b^3*c^2*d - 9*a^2*b^2*c*d^2 + 3*b^4*c*d^2 + a^3*b*d^3 - 3*a*b^3*d^3)
*log(abs(b*tan(f*x + e) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) - (9*a^
2*b^5*c^3*tan(f*x + e)^2 - 3*b^7*c^3*tan(f*x + e)^2 - 9*a^3*b^4*c^2*d*tan(f
*x + e)^2 + 27*a*b^6*c^2*d*tan(f*x + e)^2 - 27*a^2*b^5*c*d^2*tan(f*x + e)^2
 + 9*b^7*c*d^2*tan(f*x + e)^2 + 3*a^3*b^4*d^3*tan(f*x + e)^2 - 9*a*b^6*d^3*
tan(f*x + e)^2 + 22*a^3*b^4*c^3*tan(f*x + e) - 2*a*b^6*c^3*tan(f*x + e) - 2
4*a^4*b^3*c^2*d*tan(f*x + e) + 54*a^2*b^5*c^2*d*tan(f*x + e) + 6*b^7*c^2*d*
tan(f*x + e) - 66*a^3*b^4*c*d^2*tan(f*x + e) + 6*a*b^6*c*d^2*tan(f*x + e) +
 2*a^6*b*d^3*tan(f*x + e) + 14*a^4*b^3*d^3*tan(f*x + e) - 12*a^2*b^5*d^3*ta
n(f*x + e) + 14*a^4*b^3*c^3 + 3*a^2*b^5*c^3 + b^7*c^3 - 18*a^5*b^2*c^2*d +
21*a^3*b^4*c^2*d + 3*a*b^6*c^2*d + 3*a^6*b*c*d^2 - 33*a^4*b^3*c*d^2 + a^7*d
^3 + 9*a^5*b^2*d^3 - 4*a^3*b^4*d^3)/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8
)*(b*tan(f*x + e) + a)^2))/f
```

Mupad [B]
time = 8.00, size = 466, normalized size = 1.95

$$-\frac{\frac{a^5\,d^3+3\,a^4\,b\,c\,d^2-9\,a^3\,b^2\,c^2\,d+5\,a^3\,b^2\,d^3+5\,a^2\,b^3\,c^3-9\,a^2\,b^3\,c\,d^2+3\,a\,b^4\,c^2\,d+b^5\,c^3}{2\,b^2\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{\mathrm{tan}\left(e+f\,x\right)\,\left(a^4\,d^3-3\,a^2\,b^2\,c^2\,d+3\,a^2\,b^2\,d^3+2\,a\,b^3\,c^3-6\,a\,b^3\,c\,d^2+3\,b^4\,c^2\,d\right)}{b\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{f\,\left(a^2+2\,a\,b\,\mathrm{tan}\left(e+f\,x\right)+b^2\,{\mathrm{tan}\left(e+f\,x\right)}^2\right)}-\frac{\ln\left(\mathrm{tan}(e+f\,x)-\mathrm{i}\right)\,\left(-c^3\,1{}\mathrm{i}+3\,c^2\,d+c\,d^2\,3{}\mathrm{i}-d^3\right)}{2\,f\,\left(-a^3-a^2\,b\,3{}\mathrm{i}+3\,a\,b^2+b^3\,1{}\mathrm{i}\right)}-\frac{\ln\left(a+b\,\mathrm{tan}(e+f\,x)\right)\,\left(\left(3\,c^2\,d-d^3\right)\,a^3+\left(9\,c\,d^2-3\,c^3\right)\,a^2\,b+\left(3\,d^3-9\,c^2\,d\right)\,a\,b^2+\left(c^3-3\,c\,d^2\right)\,b^3\right)}{f\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}-\frac{\ln\left(\mathrm{tan}(e+f\,x)+1{}\mathrm{i}\right)\,\left(-c^3+c^2\,d\,3{}\mathrm{i}+3\,c\,d^2-d^3\,1{}\mathrm{i}\right)}{2\,f\,\left(-a^3\,1{}\mathrm{i}-3\,a^2\,b+a\,b^2\,3{}\mathrm{i}+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^3/(a + b*tan(e + f*x))^3,x)
```

```
[Out] - ((a^5*d^3 + b^5*c^3 + 5*a^2*b^3*c^3 + 5*a^3*b^2*d^3 - 9*a^2*b^3*c*d^2 - 9
*a^3*b^2*c^2*d + 3*a*b^4*c^2*d + 3*a^4*b*c*d^2)/(2*b^2*(a^4 + b^4 + 2*a^2*b
^2)) + (tan(e + f*x)*(a^4*d^3 + 2*a*b^3*c^3 + 3*b^4*c^2*d + 3*a^2*b^2*d^3 -
 3*a^2*b^2*c^2*d - 6*a*b^3*c*d^2))/(b*(a^4 + b^4 + 2*a^2*b^2)))/(f*(a^2 + b
^2*tan(e + f*x)^2 + 2*a*b*tan(e + f*x))) - (log(tan(e + f*x) - 1i)*(c*d^2*3
```

```
i + 3*c^2*d - c^3*1i - d^3))/(2*f*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)) - (l
og(a + b*tan(e + f*x))*(a^3*(3*c^2*d - d^3) - b^3*(3*c*d^2 - c^3) + a^2*b*(
9*c*d^2 - 3*c^3) - a*b^2*(9*c^2*d - 3*d^3)))/(f*(a^6 + b^6 + 3*a^2*b^4 + 3*
a^4*b^2)) - (log(tan(e + f*x) + 1i)*(3*c*d^2 + c^2*d*3i - c^3 - d^3*1i))/(2
*f*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3))
```

3.1209 $\int \frac{(a+b\tan(e+fx))^4}{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=190

$$\frac{(a^4c-6a^2b^2c+b^4c+4a^3bd-4ab^3d)\,x}{c^2+d^2}-\frac{(4a^3bc-4ab^3c-a^4d+6a^2b^2d-b^4d)\log(\cos(e+fx))}{(c^2+d^2)\,f}+\frac{(bc-ad)^4}{}$$

```
[Out] (a^4*c+4*a^3*b*d-6*a^2*b^2*c-4*a*b^3*d+b^4*c)*x/(c^2+d^2)-(-a^4*d+4*a^3*b*c
+6*a^2*b^2*d-4*a*b^3*c-b^4*d)*ln(cos(f*x+e))/(c^2+d^2)/f+(-a*d+b*c)^4*ln(c+
d*tan(f*x+e))/d^3/(c^2+d^2)/f-b^3*(-3*a*d+b*c)*tan(f*x+e)/d^2/f+1/2*b^2*(a+
b*tan(f*x+e))^2/d/f
```

Rubi [A]
time = 0.33, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3647, 3718, 3707, 3698, 31, 3556}

$$-\frac{(a^4(-d)+4a^3bc+6a^2b^2d-4ab^3c-b^4d)\log(\cos(e+fx))}{f\,(c^2+d^2)}+\frac{x(a^4c+4a^3bd-6a^2b^2c-4ab^3d+b^4c)}{c^2+d^2}-\frac{b^3(bc-3ad)\tan(e+fx)}{d^2f}+\frac{b^2(a+b\tan(e+fx))^2}{2df}+\frac{(bc-ad)^4\log(c+d\tan(e+fx))}{d^3f\,(c^2+d^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^4/(c + d*Tan[e + f*x]),x]
```

```
[Out] ((a^4*c - 6*a^2*b^2*c + b^4*c + 4*a^3*b*d - 4*a*b^3*d)*x)/(c^2 + d^2) - ((4
*a^3*b*c - 4*a*b^3*c - a^4*d + 6*a^2*b^2*d - b^4*d)*Log[Cos[e + f*x]])/((c^
2 + d^2)*f) + ((b*c - a*d)^4*Log[c + d*Tan[e + f*x]])/(d^3*(c^2 + d^2)*f) -
 (b^3*(b*c - 3*a*d)*Tan[e + f*x])/(d^2*f) + (b^2*(a + b*Tan[e + f*x])^2)/(2
*d*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
```

```
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^4}{c+d\tan(e+fx)}\,dx = \frac{b^2(a+b\tan(e+fx))^2}{2df} + \frac{\int \frac{(a+b\tan(e+fx))\left(-2(b^3c-a^3d)+2b(3a^2-b^2)d\tan(e+fx)-2b^2(bc-3a\right.}{c+d\tan(e+fx)}}{2d}$$

$$= -\frac{b^3(bc-3ad)\tan(e+fx)}{d^2f} + \frac{b^2(a+b\tan(e+fx))^2}{2df} - \frac{\int \frac{-2(b^4c^2-4ab^3cd+a^4d^2)-8a}{}}{}$$

$$= \frac{(a^4c-6a^2b^2c+b^4c+4a^3bd-4ab^3d)\,x}{c^2+d^2} - \frac{b^3(bc-3ad)\tan(e+fx)}{d^2f} + \frac{b^2(a+b}{}$$

$$= \frac{(a^4c-6a^2b^2c+b^4c+4a^3bd-4ab^3d)\,x}{c^2+d^2} - \frac{(4a^3bc-4ab^3c-a^4d+6a^2b^2d-b^4d}{(c^2+d^2)\,f}$$

$$= \frac{(a^4c-6a^2b^2c+b^4c+4a^3bd-4ab^3d)\,x}{c^2+d^2} - \frac{(4a^3bc-4ab^3c-a^4d+6a^2b^2d-b^4d}{(c^2+d^2)\,f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.30, size = 160, normalized size = 0.84

$$\frac{\frac{\frac{(a+ib)^4d^2\log(i-\tan(e+fx))}{ic-d}-\frac{(a-ib)^4d^2\log(i+\tan(e+fx))}{ic+d}+\frac{2(bc-ad)^4\log(c+d\tan(e+fx))}{d(c^2+d^2)}}{d}-\frac{2b^3(bc-3ad)\tan(e+fx)}{d}+b^2(a+b\tan(e+fx))^2}{2df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^4/(c + d*Tan[e + f*x]),x]
```

```
[Out] ((((a + I*b)^4*d^2*Log[I - Tan[e + f*x]])/(I*c - d) - ((a - I*b)^4*d^2*Log[
I + Tan[e + f*x]])/(I*c + d) + (2*(b*c - a*d)^4*Log[c + d*Tan[e + f*x]])/(d
*(c^2 + d^2)))/d - (2*b^3*(b*c - 3*a*d)*Tan[e + f*x])/d + b^2*(a + b*Tan[e
+ f*x])^2)/(2*d*f)
```

Maple [A]
time = 0.23, size = 221, normalized size = 1.16

method	result
derivativedivides	$\frac{\frac{b^3\left(\frac{bd\left(\tan^2(fx+e)\right)}{2}+4ad\tan(fx+e)-bc\tan(fx+e)\right)}{d^2}+\frac{\frac{\left(-a^4d+4a^3bc+6a^2b^2d-4a\,b^3c-b^4d\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(a^4c+4a^3bd-6a^2\right.}{c^2+d^2}}{f}$
default	$\frac{\frac{b^3\left(\frac{bd\left(\tan^2(fx+e)\right)}{2}+4ad\tan(fx+e)-bc\tan(fx+e)\right)}{d^2}+\frac{\frac{\left(-a^4d+4a^3bc+6a^2b^2d-4a\,b^3c-b^4d\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(a^4c+4a^3bd-6a^2\right.}{c^2+d^2}}{f}$
norman	$\frac{(a^4c+4a^3bd-6a^2b^2c-4b^3da+b^4c)x}{c^2+d^2}+\frac{b^3(4ad-bc)\tan(fx+e)}{d^2f}+\frac{b^4\left(\tan^2(fx+e)\right)}{2df}+\frac{(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a}{(c^2+}$

risch	$-\frac{12ia^2b^2c^2e}{(c^2+d^2)df}+\frac{8ia\,b^3c^3e}{(c^2+d^2)d^2f}-\frac{4ix\,b^3a}{id-c}+\frac{12ib^2a^2x}{d}+\frac{2ib^4c^2x}{d^3}-\frac{2ib^4e}{df}-\frac{2id\,a^4x}{c^2+d^2}+\frac{d\ln\left(\mathrm{e}^{2i(fx+e)}-\frac{id+c}{id-c}\right)a^4}{(c^2+d^2)f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^4/(c+d*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^3/d^2*(1/2*b*d*tan(f*x+e)^2+4*a*d*tan(f*x+e)-b*c*tan(f*x+e))+1/(c^2+
d^2)*(1/2*(-a^4*d+4*a^3*b*c+6*a^2*b^2*d-4*a*b^3*c-b^4*d)*ln(1+tan(f*x+e)^2)
+(a^4*c+4*a^3*b*d-6*a^2*b^2*c-4*a*b^3*d+b^4*c)*arctan(tan(f*x+e)))+1/d^3*(a
^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(c^2+d^2)*ln(
c+d*tan(f*x+e)))
```

Maxima [A]
time = 0.64, size = 229, normalized size = 1.21

$$\frac{\frac{2\left(\left(a^4-6a^2b^2+b^4\right)c+4\left(a^3b-ab^3\right)d\right)(fx+e)}{c^2+d^2}+\frac{2\left(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4\right)\log(d\tan(fx+e)+c)}{c^2d^3+d^5}+\frac{\left(4\left(a^3b-ab^3\right)c-\left(a^4-6a^2b^2+b^4\right)d\right)\log\left(\tan(fx+e)^2+1\right)}{c^2+d^2}+\frac{b^4d\tan(fx+e)^2-2\left(b^4c-4ab^3d\right)\tan(fx+e)}{d^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^4/(c+d*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*((a^4 - 6*a^2*b^2 + b^4)*c + 4*(a^3*b - a*b^3)*d)*(f*x + e)/(c^2 + d
^2) + 2*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*
d^4)*log(d*tan(f*x + e) + c)/(c^2*d^3 + d^5) + (4*(a^3*b - a*b^3)*c - (a^4
- 6*a^2*b^2 + b^4)*d)*log(tan(f*x + e)^2 + 1)/(c^2 + d^2) + (b^4*d*tan(f*x
+ e)^2 - 2*(b^4*c - 4*a*b^3*d)*tan(f*x + e))/d^2)/f
```

Fricas [A]
time = 1.61, size = 305, normalized size = 1.61

$$\frac{2\left(\left(a^4-6a^2b^2+b^4\right)cd^3+4\left(a^3b-ab^3\right)d^4\right)fx+\left(b^4c^2d^2+b^4d^4\right)\tan\left(fx+e\right)^2+\left(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4\right)\log\left(\frac{d^2\tan(fx+e)^2+2cd\tan(fx+e)+c^2}{\tan(fx+e)^2+1}\right)-\left(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4ab^3cd^3+\left(6a^2b^2-b^4\right)d^4\right)\log\left(\frac{1}{\tan(fx+e)^2+1}\right)-2\left(b^4c^3d-4ab^3c^2d^2+b^4cd^3-4ab^3d^4\right)\tan\left(fx+e\right)}{2\left(c^2d^3+d^5\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^4/(c+d*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*((a^4 - 6*a^2*b^2 + b^4)*c*d^3 + 4*(a^3*b - a*b^3)*d^4)*f*x + (b^4*c
^2*d^2 + b^4*d^4)*tan(f*x + e)^2 + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2
*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e
) + c^2)/(tan(f*x + e)^2 + 1)) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d
^2 - 4*a*b^3*c*d^3 + (6*a^2*b^2 - b^4)*d^4)*log(1/(tan(f*x + e)^2 + 1)) - 2
*(b^4*c^3*d - 4*a*b^3*c^2*d^2 + b^4*c*d^3 - 4*a*b^3*d^4)*tan(f*x + e))/((c^
2*d^3 + d^5)*f)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.37, size = 2516, normalized size = 13.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**4/(c+d*tan(f*x+e)),x)
```

```
[Out] Piecewise((zoo*x*(a + b*tan(e))**4/tan(e), Eq(c, 0) & Eq(d, 0) & Eq(f, 0)),
 (I*a**4*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) + a**4*f*x/(2*d*f*
tan(e + f*x) - 2*I*d*f) + I*a**4/(2*d*f*tan(e + f*x) - 2*I*d*f) + 4*a**3*b*
f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) - 4*I*a**3*b*f*x/(2*d*f*tan
(e + f*x) - 2*I*d*f) - 4*a**3*b/(2*d*f*tan(e + f*x) - 2*I*d*f) + 6*I*a**2*b
**2*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) + 6*a**2*b**2*f*x/(2*d*
f*tan(e + f*x) - 2*I*d*f) + 6*a**2*b**2*log(tan(e + f*x)**2 + 1)*tan(e + f*
x)/(2*d*f*tan(e + f*x) - 2*I*d*f) - 6*I*a**2*b**2*log(tan(e + f*x)**2 + 1)/
(2*d*f*tan(e + f*x) - 2*I*d*f) - 6*I*a**2*b**2/(2*d*f*tan(e + f*x) - 2*I*d*
f) - 12*a*b**3*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) + 12*I*a*b**
3*f*x/(2*d*f*tan(e + f*x) - 2*I*d*f) + 4*I*a*b**3*log(tan(e + f*x)**2 + 1)*
tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) + 4*a*b**3*log(tan(e + f*x)**2
+ 1)/(2*d*f*tan(e + f*x) - 2*I*d*f) + 8*a*b**3*tan(e + f*x)**2/(2*d*f*tan(e
 + f*x) - 2*I*d*f) + 12*a*b**3/(2*d*f*tan(e + f*x) - 2*I*d*f) - 3*I*b**4*f*
x*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) - 3*b**4*f*x/(2*d*f*tan(e + f
*x) - 2*I*d*f) - 2*b**4*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(e
+ f*x) - 2*I*d*f) + 2*I*b**4*log(tan(e + f*x)**2 + 1)/(2*d*f*tan(e + f*x) -
 2*I*d*f) + b**4*tan(e + f*x)**3/(2*d*f*tan(e + f*x) - 2*I*d*f) + I*b**4*ta
n(e + f*x)**2/(2*d*f*tan(e + f*x) - 2*I*d*f) + 3*I*b**4/(2*d*f*tan(e + f*x)
 - 2*I*d*f), Eq(c, -I*d)), (-I*a**4*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) +
2*I*d*f) + a**4*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) - I*a**4/(2*d*f*tan(e +
f*x) + 2*I*d*f) + 4*a**3*b*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f)
+ 4*I*a**3*b*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) - 4*a**3*b/(2*d*f*tan(e + f
*x) + 2*I*d*f) - 6*I*a**2*b**2*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d
*f) + 6*a**2*b**2*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) + 6*a**2*b**2*log(tan(
e + f*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) + 6*I*a**2*b**
2*log(tan(e + f*x)**2 + 1)/(2*d*f*tan(e + f*x) + 2*I*d*f) + 6*I*a**2*b**2/(
2*d*f*tan(e + f*x) + 2*I*d*f) - 12*a*b**3*f*x*tan(e + f*x)/(2*d*f*tan(e + f
*x) + 2*I*d*f) - 12*I*a*b**3*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) - 4*I*a*b**
3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) + 4*
a*b**3*log(tan(e + f*x)**2 + 1)/(2*d*f*tan(e + f*x) + 2*I*d*f) + 8*a*b**3*t
an(e + f*x)**2/(2*d*f*tan(e + f*x) + 2*I*d*f) + 12*a*b**3/(2*d*f*tan(e + f*
x) + 2*I*d*f) + 3*I*b**4*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) -
3*b**4*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) - 2*b**4*log(tan(e + f*x)**2 + 1)
*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) - 2*I*b**4*log(tan(e + f*x)**2
 + 1)/(2*d*f*tan(e + f*x) + 2*I*d*f) + b**4*tan(e + f*x)**3/(2*d*f*tan(e +
f*x) + 2*I*d*f) - I*b**4*tan(e + f*x)**2/(2*d*f*tan(e + f*x) + 2*I*d*f) - 3
*I*b**4/(2*d*f*tan(e + f*x) + 2*I*d*f), Eq(c, I*d)), ((a**4*x + 2*a**3*b*lo
g(tan(e + f*x)**2 + 1)/f - 6*a**2*b**2*x + 6*a**2*b**2*tan(e + f*x)/f - 2*a
*b**3*log(tan(e + f*x)**2 + 1)/f + 2*a*b**3*tan(e + f*x)**2/f + b**4*x + b*
*4*tan(e + f*x)**3/(3*f) - b**4*tan(e + f*x)/f)/c, Eq(d, 0)), (x*(a + b*tan
```

```
(e))**4/(c + d*tan(e)), Eq(f, 0)), (2*a**4*c*d**3*f*x/(2*c**2*d**3*f + 2*d*
*5*f) + 2*a**4*d**4*log(c/d + tan(e + f*x))/(2*c**2*d**3*f + 2*d**5*f) - a*
*4*d**4*log(tan(e + f*x)**2 + 1)/(2*c**2*d**3*f + 2*d**5*f) - 8*a**3*b*c*d*
*3*log(c/d + tan(e + f*x))/(2*c**2*d**3*f + 2*d**5*f) + 4*a**3*b*c*d**3*log
(tan(e + f*x)**2 + 1)/(2*c**2*d**3*f + 2*d**5*f) + 8*a**3*b*d**4*f*x/(2*c**
2*d**3*f + 2*d**5*f) + 12*a**2*b**2*c**2*d**2*log(c/d + tan(e + f*x))/(2*c*
*2*d**3*f + 2*d**5*f) - 12*a**2*b**2*c*d**3*f*x/(2*c**2*d**3*f + 2*d**5*f)
+ 6*a**2*b**2*d**4*log(tan(e + f*x)**2 + 1)/(2*c**2*d**3*f + 2*d**5*f) - 8*
a*b**3*c**3*d*log(c/d + tan(e + f*x))/(2*c**2*d**3*f + 2*d**5*f) + 8*a*b**3
*c**2*d**2*tan(e + f*x)/(2*c**2*d**3*f + 2*d**5*f) - 4*a*b**3*c*d**3*log(ta
n(e + f*x)**2 + 1)/(2*c**2*d**3*f + 2*d**5*f) - 8*a*b**3*d**4*f*x/(2*c**2*d
**3*f + 2*d**5*f) + 8*a*b**3*d**4*tan(e + f*x)/(2*c**2*d**3*f + 2*d**5*f) +
 2*b**4*c**4*log(c/d + tan(e + f*x))/(2*c**2*d**3*f + 2*d**5*f) - 2*b**4*c*
*3*d*tan(e + f*x)/(2*c**2*d**3*f + 2*d**5*f) + b**4*c**2*d**2*tan(e + f*x)*
*2/(2*c**2*d**3*f + 2*d**5*f) + 2*b**4*c*d**3*f*x/(2*c**2*d**3*f + 2*d**5*f
) - 2*b**4*c*d**3*tan(e + f*x)/(2*c**2*d**3*f + 2*d**5*f) - b**4*d**4*log(t
an(e + f*x)**2 + 1)/(2*c**2*d**3*f + 2*d**5*f) + b**4*d**4*tan(e + f*x)**2/
(2*c**2*d**3*f + 2*d**5*f), True))
```

Giac [**A**]
time = 0.99, size = 238, normalized size = 1.25

$$\frac{\frac{2\,(a^4c-6\,a^2b^2c+b^4c+4\,a^3bd-4\,ab^3d)(fx+e)}{c^2+d^2}+\frac{(4\,a^3bc-4\,ab^3c-a^4d+6\,a^2b^2d-b^4d)\log\left(\tan(fx+e)^2+1\right)}{c^2+d^2}+\frac{2\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\log(|d\tan(fx+e)+c|)}{c^2d^3+d^5}+\frac{b^4d\tan(fx+e)^2-2\,b^4c\tan(fx+e)+8\,ab^3d\tan(fx+e)}{d^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^4/(c+d*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*(2*(a^4*c - 6*a^2*b^2*c + b^4*c + 4*a^3*b*d - 4*a*b^3*d)*(f*x + e)/(c^2
 + d^2) + (4*a^3*b*c - 4*a*b^3*c - a^4*d + 6*a^2*b^2*d - b^4*d)*log(tan(f*x
 + e)^2 + 1)/(c^2 + d^2) + 2*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
 4*a^3*b*c*d^3 + a^4*d^4)*log(abs(d*tan(f*x + e) + c))/(c^2*d^3 + d^5) + (b
^4*d*tan(f*x + e)^2 - 2*b^4*c*tan(f*x + e) + 8*a*b^3*d*tan(f*x + e))/d^2)/f
```

Mupad [**B**]
time = 5.78, size = 235, normalized size = 1.24

$$\frac{\tan(e+f\,x)\,\left(\frac{4\,a\,b^3}{d}-\frac{b^4\,c}{d^2}\right)}{f}-\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(a^4-a^3\,b\,4\mathrm{i}-6\,a^2\,b^2+a\,b^3\,4\mathrm{i}+b^4\right)}{2\,f\,\left(d+c\,1\mathrm{i}\right)}+\frac{b^4\tan(e+f\,x)^2}{2\,d\,f}-\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(a^4\,1\mathrm{i}-4\,a^3\,b-a^2\,b^2\,6\mathrm{i}+4\,a\,b^3+b^4\,1\mathrm{i}\right)}{2\,f\,\left(c+d\,1\mathrm{i}\right)}+\frac{\ln\left(c+d\tan(e+f\,x)\right)\,\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{f\,\left(c^2\,d^3+d^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^4/(c + d*tan(e + f*x)),x)
```

```
[Out] (tan(e + f*x)*((4*a*b^3)/d - (b^4*c)/d^2))/f - (log(tan(e + f*x) + 1i)*(a*b
^3*4i - a^3*b*4i + a^4 + b^4 - 6*a^2*b^2))/(2*f*(c*1i + d)) + (b^4*tan(e +
f*x)^2)/(2*d*f) - (log(tan(e + f*x) - 1i)*(4*a*b^3 - 4*a^3*b + a^4*1i + b^4
*1i - a^2*b^2*6i))/(2*f*(c + d*1i)) + (log(c + d*tan(e + f*x))*(a^4*d^4 + b
^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/(f*(d^5 + c^2*
d^3))
```

3.1210 $\int \frac{(a+b\tan(e+fx))^3}{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=144

$$\frac{(a^3c-3ab^2c+3a^2bd-b^3d)\,x}{c^2+d^2}-\frac{(3a^2bc-b^3c-a^3d+3ab^2d)\log(\cos(e+fx))}{(c^2+d^2)\,f}-\frac{(bc-ad)^3\log(c+d\tan(e+$$

```
[Out] (a^3*c+3*a^2*b*d-3*a*b^2*c-b^3*d)*x/(c^2+d^2)-(-a^3*d+3*a^2*b*c+3*a*b^2*d-b
^3*c)*ln(cos(f*x+e))/(c^2+d^2)/f-(-a*d+b*c)^3*ln(c+d*tan(f*x+e))/d^2/(c^2+d
^2)/f+b^2*(a+b*tan(f*x+e))/d/f
```

Rubi [A]
time = 0.18, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3647, 3707, 3698, 31, 3556}

$$-\frac{(a^3(-d)+3a^2bc+3ab^2d-b^3c)\log(\cos(e+fx))}{f\,(c^2+d^2)}+\frac{x(a^3c+3a^2bd-3ab^2c-b^3d)}{c^2+d^2}+\frac{b^2(a+b\tan(e+fx))}{df}-\frac{(bc-ad)^3\log(c+d\tan(e+fx))}{d^2f\,(c^2+d^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^3/(c + d*Tan[e + f*x]),x]
```

```
[Out] ((a^3*c - 3*a*b^2*c + 3*a^2*b*d - b^3*d)*x)/(c^2 + d^2) - ((3*a^2*b*c - b^3
*c - a^3*d + 3*a*b^2*d)*Log[Cos[e + f*x]])/((c^2 + d^2)*f) - ((b*c - a*d)^3
*Log[c + d*Tan[e + f*x]])/(d^2*(c^2 + d^2)*f) + (b^2*(a + b*Tan[e + f*x]))/
(d*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
```

```
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\tan(e+fx))^3}{c+d\tan(e+fx)}\,dx &= \frac{b^2(a+b\tan(e+fx))}{df}+\frac{\int \frac{-b^3c+a^3d+b(3a^2-b^2)d\tan(e+fx)-b^2(bc-3ad)\tan^2(e+fx)}{c+d\tan(e+fx)}\,dx}{d}\\
&= \frac{(a^3c-3ab^2c+3a^2bd-b^3d)\,x}{c^2+d^2}+\frac{b^2(a+b\tan(e+fx))}{df}-\frac{(bc-ad)^3\int \frac{1+\tan^2(\cdots}{c+d\tan(\cdots}}{d\,(c^2+d^2)}\\
&= \frac{(a^3c-3ab^2c+3a^2bd-b^3d)\,x}{c^2+d^2}-\frac{(3a^2bc-b^3c-a^3d+3ab^2d)\log(\cos(e+fx)}{(c^2+d^2)\,f}\\
&= \frac{(a^3c-3ab^2c+3a^2bd-b^3d)\,x}{c^2+d^2}-\frac{(3a^2bc-b^3c-a^3d+3ab^2d)\log(\cos(e+fx)}{(c^2+d^2)\,f}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.77, size = 126, normalized size = 0.88

$$\frac{\frac{(a+ib)^3\log(i-\tan(e+fx))}{ic-d}-\frac{(ia+b)^3\log(i+\tan(e+fx))}{c-id}+\frac{2(-bc+ad)^3\log(c+d\tan(e+fx))}{d^2(c^2+d^2)}+\frac{2b^2(a+b\tan(e+fx))}{d}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^3/(c + d*Tan[e + f*x]),x]
```

```
[Out] (((a + I*b)^3*Log[I - Tan[e + f*x]])/(I*c - d) - ((I*a + b)^3*Log[I + Tan[e
 + f*x]])/(c - I*d) + (2*(-(b*c) + a*d)^3*Log[c + d*Tan[e + f*x]])/(d^2*(c^
2 + d^2)) + (2*b^2*(a + b*Tan[e + f*x]))/d)/(2*f)
```

Maple [A]

time = 0.23, size = 164, normalized size = 1.14

method	result
derivativedivides	$\frac{\frac{b^3\tan(fx+e)}{d}+\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\ln(c+d\tan(fx+e))}{d^2\left(c^2+d^2\right)}+\frac{\frac{\left(-a^3d+3a^2bc+3a\,b^2d-b^3c\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(a^3c+3a^2b\right.}{c^2+d^2}}{f}$
default	$\frac{\frac{b^3\tan(fx+e)}{d}+\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\ln(c+d\tan(fx+e))}{d^2\left(c^2+d^2\right)}+\frac{\frac{\left(-a^3d+3a^2bc+3a\,b^2d-b^3c\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(a^3c+3a^2b\right.}{c^2+d^2}}{f}$
norman	$\frac{(a^3c+3a^2bd-3a\,b^2c-b^3d)x}{c^2+d^2}+\frac{b^3\tan(fx+e)}{df}+\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\ln(c+d\tan(fx+e))}{(c^2+d^2)d^2f}-\frac{(a^3d-3a^2bc-}{}$
risch	$\frac{6ia^2bce}{(c^2+d^2)f}+\frac{3ix\,a^2b}{id-c}-\frac{a^3x}{id-c}+\frac{3xa\,b^2}{id-c}-\frac{2ib^3cx}{d^2}+\frac{6ib^2ax}{d}-\frac{ix\,b^3}{id-c}-\frac{2ib^3ce}{d^2f}-\frac{2id\,a^3e}{(c^2+d^2)f}+\frac{6ib^2ae}{df}+\frac{2ib^3}{(c^2+d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3/(c+d*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^3/d*tan(f*x+e)+1/d^2*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(
c^2+d^2)*ln(c+d*tan(f*x+e))+1/(c^2+d^2)*(1/2*(-a^3*d+3*a^2*b*c+3*a*b^2*d-b^
3*c)*ln(1+tan(f*x+e)^2)+(a^3*c+3*a^2*b*d-3*a*b^2*c-b^3*d)*arctan(tan(f*x+e)
)))
```

Maxima [A]

time = 0.57, size = 178, normalized size = 1.24

$$\frac{\frac{2\,b^3\tan(fx+e)}{d}+\frac{2\,((a^3-3\,ab^2)c+(3\,a^2b-b^3)d)(fx+e)}{c^2+d^2}-\frac{2\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log(d\tan(fx+e)+c)}{c^2d^2+d^4}+\frac{((3\,a^2b-b^3)c-(a^3-3\,ab^2)d)\log\left(\tan(fx+e)^2+1\right)}{c^2+d^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(c+d*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^3*tan(f*x + e)/d + 2*((a^3 - 3*a*b^2)*c + (3*a^2*b - b^3)*d)*(f*x
+ e)/(c^2 + d^2) - 2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*lo
g(d*tan(f*x + e) + c)/(c^2*d^2 + d^4) + ((3*a^2*b - b^3)*c - (a^3 - 3*a*b^2
)*d)*log(tan(f*x + e)^2 + 1)/(c^2 + d^2))/f
```

Fricas [A]

time = 1.41, size = 211, normalized size = 1.47

$$\frac{2\,((a^3-3\,ab^2)cd^2+(3\,a^2b-b^3)d^3)fx-(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log\left(\frac{d^2\tan(fx+e)^2+2\,cd\tan(fx+e)+c^2}{\tan(fx+e)^2+1}\right)+(b^3c^3-3\,ab^2c^2d+b^3cd^2-3\,ab^2d^3)\log\left(\frac{1}{\tan(fx+e)^2+1}\right)+2\,(b^3c^2d+b^3d^3)\tan(fx+e)}{2\,(c^2d^2+d^4)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(c+d*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*((a^3 - 3*a*b^2)*c*d^2 + (3*a^2*b - b^3)*d^3)*f*x - (b^3*c^3 - 3*a*b
^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x
 + e) + c^2)/(tan(f*x + e)^2 + 1)) + (b^3*c^3 - 3*a*b^2*c^2*d + b^3*c*d^2 -
 3*a*b^2*d^3)*log(1/(tan(f*x + e)^2 + 1)) + 2*(b^3*c^2*d + b^3*d^3)*tan(f*x
 + e))/((c^2*d^2 + d^4)*f)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.87, size = 1712, normalized size = 11.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3/(c+d*tan(f*x+e)),x)
```

```
[Out] Piecewise((zoo*x*(a + b*tan(e))**3/tan(e), Eq(c, 0) & Eq(d, 0) & Eq(f, 0)),
 (I*a**3*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) + a**3*f*x/(2*d*f*
tan(e + f*x) - 2*I*d*f) + I*a**3/(2*d*f*tan(e + f*x) - 2*I*d*f) + 3*a**2*b*
f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) - 3*I*a**2*b*f*x/(2*d*f*tan
(e + f*x) - 2*I*d*f) - 3*a**2*b/(2*d*f*tan(e + f*x) - 2*I*d*f) + 3*I*a*b**2
*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) + 3*a*b**2*f*x/(2*d*f*tan(
e + f*x) - 2*I*d*f) + 3*a*b**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*d*f
*tan(e + f*x) - 2*I*d*f) - 3*I*a*b**2*log(tan(e + f*x)**2 + 1)/(2*d*f*tan(e
 + f*x) - 2*I*d*f) - 3*I*a*b**2/(2*d*f*tan(e + f*x) - 2*I*d*f) - 3*b**3*f*x
*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) + 3*I*b**3*f*x/(2*d*f*tan(e +
f*x) - 2*I*d*f) + I*b**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(e
 + f*x) - 2*I*d*f) + b**3*log(tan(e + f*x)**2 + 1)/(2*d*f*tan(e + f*x) - 2*
I*d*f) + 2*b**3*tan(e + f*x)**2/(2*d*f*tan(e + f*x) - 2*I*d*f) + 3*b**3/(2*
d*f*tan(e + f*x) - 2*I*d*f), Eq(c, -I*d)), (-I*a**3*f*x*tan(e + f*x)/(2*d*f
*tan(e + f*x) + 2*I*d*f) + a**3*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) - I*a**3
/(2*d*f*tan(e + f*x) + 2*I*d*f) + 3*a**2*b*f*x*tan(e + f*x)/(2*d*f*tan(e +
f*x) + 2*I*d*f) + 3*I*a**2*b*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) - 3*a**2*b/
(2*d*f*tan(e + f*x) + 2*I*d*f) - 3*I*a*b**2*f*x*tan(e + f*x)/(2*d*f*tan(e +
 f*x) + 2*I*d*f) + 3*a*b**2*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) + 3*a*b**2*l
og(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) + 3*I*a
*b**2*log(tan(e + f*x)**2 + 1)/(2*d*f*tan(e + f*x) + 2*I*d*f) + 3*I*a*b**2/
(2*d*f*tan(e + f*x) + 2*I*d*f) - 3*b**3*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x
) + 2*I*d*f) - 3*I*b**3*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) - I*b**3*log(tan
(e + f*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) + b**3*log(ta
n(e + f*x)**2 + 1)/(2*d*f*tan(e + f*x) + 2*I*d*f) + 2*b**3*tan(e + f*x)**2/
(2*d*f*tan(e + f*x) + 2*I*d*f) + 3*b**3/(2*d*f*tan(e + f*x) + 2*I*d*f), Eq(
c, I*d)), ((a**3*x + 3*a**2*b*log(tan(e + f*x)**2 + 1)/(2*f) - 3*a*b**2*x +
 3*a*b**2*tan(e + f*x)/f - b**3*log(tan(e + f*x)**2 + 1)/(2*f) + b**3*tan(e
 + f*x)**2/(2*f))/c, Eq(d, 0)), (x*(a + b*tan(e))**3/(c + d*tan(e)), Eq(f,
0)), (2*a**3*c*d**2*f*x/(2*c**2*d**2*f + 2*d**4*f) + 2*a**3*d**3*log(c/d +
tan(e + f*x))/(2*c**2*d**2*f + 2*d**4*f) - a**3*d**3*log(tan(e + f*x)**2 +
```

```
1)/(2*c**2*d**2*f + 2*d**4*f) - 6*a**2*b*c*d**2*log(c/d + tan(e + f*x))/(2*
c**2*d**2*f + 2*d**4*f) + 3*a**2*b*c*d**2*log(tan(e + f*x)**2 + 1)/(2*c**2*
d**2*f + 2*d**4*f) + 6*a**2*b*d**3*f*x/(2*c**2*d**2*f + 2*d**4*f) + 6*a*b**
2*c**2*d*log(c/d + tan(e + f*x))/(2*c**2*d**2*f + 2*d**4*f) - 6*a*b**2*c*d*
*2*f*x/(2*c**2*d**2*f + 2*d**4*f) + 3*a*b**2*d**3*log(tan(e + f*x)**2 + 1)/
(2*c**2*d**2*f + 2*d**4*f) - 2*b**3*c**3*log(c/d + tan(e + f*x))/(2*c**2*d*
*2*f + 2*d**4*f) + 2*b**3*c**2*d*tan(e + f*x)/(2*c**2*d**2*f + 2*d**4*f) -
b**3*c*d**2*log(tan(e + f*x)**2 + 1)/(2*c**2*d**2*f + 2*d**4*f) - 2*b**3*d*
*3*f*x/(2*c**2*d**2*f + 2*d**4*f) + 2*b**3*d**3*tan(e + f*x)/(2*c**2*d**2*f
 + 2*d**4*f), True))
```

Giac [A]
time = 0.74, size = 177, normalized size = 1.23

$$\frac{\frac{2\,b^3\tan(fx+e)}{d}+\frac{2\,(a^3c-3\,ab^2c+3\,a^2bd-b^3d)(fx+e)}{c^2+d^2}+\frac{(3\,a^2bc-b^3c-a^3d+3\,ab^2d)\log\left(\tan(fx+e)^2+1\right)}{c^2+d^2}-\frac{2\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log(|d\tan(fx+e)+c|)}{c^2d^2+d^4}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(c+d*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^3*tan(f*x + e)/d + 2*(a^3*c - 3*a*b^2*c + 3*a^2*b*d - b^3*d)*(f*x
+ e)/(c^2 + d^2) + (3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*log(tan(f*x + e)
^2 + 1)/(c^2 + d^2) - 2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
*log(abs(d*tan(f*x + e) + c))/(c^2*d^2 + d^4))/f
```

Mupad [B]
time = 5.49, size = 177, normalized size = 1.23

$$\frac{b^3\tan(e+f\,x)}{d\,f}+\frac{\ln\left(c+d\tan(e+f\,x)\right)\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{f\,\left(c^2\,d^2+d^4\right)}+\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(-a^3\,1\mathrm{i}+3\,a^2\,b+a\,b^2\,3\mathrm{i}-b^3\right)}{2\,f\,\left(c+d\,1\mathrm{i}\right)}+\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(-a^3+a^2\,b\,3\mathrm{i}+3\,a\,b^2-b^3\,1\mathrm{i}\right)}{2\,f\,\left(d+c\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^3/(c + d*tan(e + f*x)),x)
```

```
[Out] (b^3*tan(e + f*x))/(d*f) + (log(c + d*tan(e + f*x))*(a^3*d^3 - b^3*c^3 + 3*
a*b^2*c^2*d - 3*a^2*b*c*d^2))/(f*(d^4 + c^2*d^2)) + (log(tan(e + f*x) - 1i)
*(a*b^2*3i + 3*a^2*b - a^3*1i - b^3))/(2*f*(c + d*1i)) + (log(tan(e + f*x)
+ 1i)*(3*a*b^2 + a^2*b*3i - a^3 - b^3*1i))/(2*f*(c*1i + d))
```

3.1211 $\int \frac{(a+b\tan(e+fx))^2}{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=103

$$-\frac{b(bc-2ad)x}{d^2}+\frac{c(bc-ad)^2x}{d^2\left(c^2+d^2\right)}-\frac{b^2\log(\cos(e+fx))}{df}+\frac{(bc-ad)^2\log(c\cos(e+fx)+d\sin(e+fx))}{d\left(c^2+d^2\right)f}$$

```
[Out] -b*(-2*a*d+b*c)*x/d^2+c*(-a*d+b*c)^2*x/d^2/(c^2+d^2)-b^2*ln(cos(f*x+e))/d/f
+(-a*d+b*c)^2*ln(c*cos(f*x+e)+d*sin(f*x+e))/d/(c^2+d^2)/f
```

Rubi [A]
time = 0.09, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3622, 3556, 3565, 3611}

$$\frac{(bc-ad)^2\log(c\cos(e+fx)+d\sin(e+fx))}{df\left(c^2+d^2\right)}+\frac{cx(bc-ad)^2}{d^2\left(c^2+d^2\right)}-\frac{bx(bc-2ad)}{d^2}-\frac{b^2\log(\cos(e+fx))}{df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2/(c + d*Tan[e + f*x]),x]
```

```
[Out] -((b*(b*c - 2*a*d)*x)/d^2) + (c*(b*c - a*d)^2*x)/(d^2*(c^2 + d^2)) - (b^2*L
og[Cos[e + f*x]])/(d*f) + ((b*c - a*d)^2*Log[c*Cos[e + f*x] + d*Sin[e + f*x
]])/(d*(c^2 + d^2)*f)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3565

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3622

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^2/((a_.) + (b_.)*tan[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[d*(2*b*c - a*d)*(x/b^2), x] + (Dist[d^2/b, In
```

```
t[Tan[e + f*x], x], x] + Dist[(b*c - a*d)^2/b^2, Int[1/(a + b*Tan[e + f*x])
, x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 +
 b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^2}{c+d\tan(e+fx)}\,dx = -\frac{b(bc-2ad)x}{d^2}+\frac{b^2\int\tan(e+fx)\,dx}{d}+\frac{(bc-ad)^2\int\frac{1}{c+d\tan(e+fx)}\,dx}{d^2}$$
$$= -\frac{b(bc-2ad)x}{d^2}+\frac{c(bc-ad)^2x}{d^2(c^2+d^2)}-\frac{b^2\log(\cos(e+fx))}{df}+\frac{(bc-ad)^2\int\frac{d-c\tan(e+f}{c+d\tan(e+f}}{d(c^2+d^2)}$$
$$= -\frac{b(bc-2ad)x}{d^2}+\frac{c(bc-ad)^2x}{d^2(c^2+d^2)}-\frac{b^2\log(\cos(e+fx))}{df}+\frac{(bc-ad)^2\log(c\cos(e-}{d(c^2+}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.16, size = 108, normalized size = 1.05

$$\frac{\frac{(a+ib)^2\log(i-\tan(e+fx))}{ic-d}-\frac{(a-ib)^2\log(i+\tan(e+fx))}{ic+d}+\frac{2(bc-ad)^2\log(c+d\tan(e+fx))}{d(c^2+d^2)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2/(c + d*Tan[e + f*x]),x]
```

```
[Out] (((a + I*b)^2*Log[I - Tan[e + f*x]])/(I*c - d) - ((a - I*b)^2*Log[I + Tan[e
 + f*x]])/(I*c + d) + (2*(b*c - a*d)^2*Log[c + d*Tan[e + f*x]])/(d*(c^2 + d
^2)))/(2*f)
```

Maple [A]
time = 0.22, size = 117, normalized size = 1.14

method	result
derivativedivides	$\frac{\frac{\left(a^2d^2-2abcd+b^2c^2\right)\ln(c+d\tan(fx+e))}{\left(c^2+d^2\right)d}+\frac{\frac{\left(-a^2d+2abc+b^2d\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(a^2c+2abd-b^2c\right)\arctan(\tan(fx+e))}{c^2+d^2}}{f}$
default	$\frac{\frac{\left(a^2d^2-2abcd+b^2c^2\right)\ln(c+d\tan(fx+e))}{\left(c^2+d^2\right)d}+\frac{\frac{\left(-a^2d+2abc+b^2d\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(a^2c+2abd-b^2c\right)\arctan(\tan(fx+e))}{c^2+d^2}}{f}$
norman	$\frac{(a^2c+2abd-b^2c)x}{c^2+d^2}+\frac{(a^2d^2-2abcd+b^2c^2)\ln(c+d\tan(fx+e))}{(c^2+d^2)df}-\frac{(a^2d-2abc-b^2d)\ln(1+\tan^2(fx+e))}{2f(c^2+d^2)}$
risch	$\frac{2ixab}{id-c}-\frac{a^2x}{id-c}+\frac{x\,b^2}{id-c}-\frac{2id\,a^2x}{c^2+d^2}-\frac{2id\,a^2e}{(c^2+d^2)f}+\frac{4iabcx}{c^2+d^2}+\frac{4iabce}{(c^2+d^2)f}-\frac{2ib^2c^2x}{(c^2+d^2)d}-\frac{2ib^2c^2e}{(c^2+d^2)df}+\frac{2ib^2x}{d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2/(c+d*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*((a^2*d^2-2*a*b*c*d+b^2*c^2)/(c^2+d^2)/d*ln(c+d*tan(f*x+e))+1/(c^2+d^2)
*(1/2*(-a^2*d+2*a*b*c+b^2*d)*ln(1+tan(f*x+e)^2)+(a^2*c+2*a*b*d-b^2*c)*arcta
n(tan(f*x+e))))
```

Maxima [A]
time = 0.53, size = 126, normalized size = 1.22

$$\frac{\frac{2\left(2\,abd+\left(a^2-b^2\right)c\right)\left(fx+e\right)}{c^2+d^2}+\frac{2\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(d\tan\left(fx+e\right)+c\right)}{c^2d+d^3}+\frac{\left(2\,abc-\left(a^2-b^2\right)d\right)\log\left(\tan\left(fx+e\right)^2+1\right)}{c^2+d^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(c+d*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(2*a*b*d + (a^2 - b^2)*c)*(f*x + e)/(c^2 + d^2) + 2*(b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*log(d*tan(f*x + e) + c)/(c^2*d + d^3) + (2*a*b*c - (a^2 - b
^2)*d)*log(tan(f*x + e)^2 + 1)/(c^2 + d^2))/f
```

Fricas [A]
time = 0.81, size = 137, normalized size = 1.33

$$\frac{2\left(2\,abd^2+\left(a^2-b^2\right)cd\right)fx+\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(\frac{d^2\tan\left(fx+e\right)^2+2\,cd\tan\left(fx+e\right)+c^2}{\tan\left(fx+e\right)^2+1}\right)-\left(b^2c^2+b^2d^2\right)\log\left(\frac{1}{\tan\left(fx+e\right)^2+1}\right)}{2\left(c^2d+d^3\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(c+d*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(2*a*b*d^2 + (a^2 - b^2)*c*d)*f*x + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*
log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) -
 (b^2*c^2 + b^2*d^2)*log(1/(tan(f*x + e)^2 + 1)))/((c^2*d + d^3)*f)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.63, size = 1025, normalized size = 9.95

$$\begin{cases} \frac{\tilde{\infty}x\left(a+b\tan\left(e\right)\right)^2}{\tan\left(e\right)} & \text{for}\: c=0\wedge d=0\wedge f=0 \\ \frac{ia^2fx\tan\left(e+fx\right)}{2df\tan\left(e+fx\right)-2idf}+\frac{a^2fx}{2df\tan\left(e+fx\right)-2idf}+\frac{ia^2}{2df\tan\left(e+fx\right)-2idf}+\frac{2abfx\tan\left(e+fx\right)}{2df\tan\left(e+fx\right)-2idf}-\frac{2iabfx}{2df\tan\left(e+fx\right)-2idf}-\frac{2ab}{2df\tan\left(e+fx\right)-2idf}+\frac{ib^2fx\tan\left(e+fx\right)}{2df\tan\left(e+fx\right)-2idf}+\frac{b^2fx}{2df\tan\left(e+fx\right)-2idf}+\frac{b^2\log\left(\tan^2\left(e+fx\right)+1\right)\tan\left(e+fx\right)}{2df\tan\left(e+fx\right)-2idf}-\frac{ib^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2df\tan\left(e+fx\right)-2idf}-\frac{ib^2}{2df\tan\left(e+fx\right)-2idf} & \text{for}\: c=-id \\ -\frac{ia^2fx\tan\left(e+fx\right)}{2df\tan\left(e+fx\right)+2idf}+\frac{a^2fx}{2df\tan\left(e+fx\right)+2idf}-\frac{ia^2}{2df\tan\left(e+fx\right)+2idf}+\frac{2abfx\tan\left(e+fx\right)}{2df\tan\left(e+fx\right)+2idf}+\frac{2iabfx}{2df\tan\left(e+fx\right)+2idf}-\frac{2ab}{2df\tan\left(e+fx\right)+2idf}-\frac{ib^2fx\tan\left(e+fx\right)}{2df\tan\left(e+fx\right)+2idf}+\frac{b^2fx}{2df\tan\left(e+fx\right)+2idf}+\frac{b^2\log\left(\tan^2\left(e+fx\right)+1\right)\tan\left(e+fx\right)}{2df\tan\left(e+fx\right)+2idf}+\frac{ib^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2df\tan\left(e+fx\right)+2idf}+\frac{ib^2}{2df\tan\left(e+fx\right)+2idf} & \text{for}\: c=id \\ \frac{a^2x+\frac{ab\log\left(\tan^2\left(e+fx\right)+1\right)}{f}-b^2x+\frac{b^2\tan\left(e+fx\right)}{f}}{c} & \text{for}\: d=0 \\ \frac{x\left(a+b\tan\left(e\right)\right)^2}{c+d\tan\left(e\right)} & \text{for}\: f=0 \\ \frac{2a^2cdfx}{2c^2df+2d^3f}+\frac{2a^2d^2\log\left(\frac{c}{d}+\tan\left(e+fx\right)\right)}{2c^2df+2d^3f}-\frac{a^2d^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2c^2df+2d^3f}-\frac{4abcd\log\left(\frac{c}{d}+\tan\left(e+fx\right)\right)}{2c^2df+2d^3f}+\frac{2abcd\log\left(\tan^2\left(e+fx\right)+1\right)}{2c^2df+2d^3f}+\frac{4abd^2fx}{2c^2df+2d^3f}+\frac{2b^2c^2\log\left(\frac{c}{d}+\tan\left(e+fx\right)\right)}{2c^2df+2d^3f}-\frac{2b^2cdfx}{2c^2df+2d^3f}+\frac{b^2d^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2c^2df+2d^3f} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2/(c+d*tan(f*x+e)),x)
```

```
[Out] Piecewise((zoo*x*(a + b*tan(e))**2/tan(e), Eq(c, 0) & Eq(d, 0) & Eq(f, 0)),
 (I*a**2*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) + a**2*f*x/(2*d*f*
tan(e + f*x) - 2*I*d*f) + I*a**2/(2*d*f*tan(e + f*x) - 2*I*d*f) + 2*a*b*f*x
*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) - 2*I*a*b*f*x/(2*d*f*tan(e + f
*x) - 2*I*d*f) - 2*a*b/(2*d*f*tan(e + f*x) - 2*I*d*f) + I*b**2*f*x*tan(e +
f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) + b**2*f*x/(2*d*f*tan(e + f*x) - 2*I*d*
f) + b**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d
*f) - I*b**2*log(tan(e + f*x)**2 + 1)/(2*d*f*tan(e + f*x) - 2*I*d*f) - I*b*
*2/(2*d*f*tan(e + f*x) - 2*I*d*f), Eq(c, -I*d)), (-I*a**2*f*x*tan(e + f*x)/
(2*d*f*tan(e + f*x) + 2*I*d*f) + a**2*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) -
I*a**2/(2*d*f*tan(e + f*x) + 2*I*d*f) + 2*a*b*f*x*tan(e + f*x)/(2*d*f*tan(e
 + f*x) + 2*I*d*f) + 2*I*a*b*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) - 2*a*b/(2*
d*f*tan(e + f*x) + 2*I*d*f) - I*b**2*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) +
 2*I*d*f) + b**2*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) + b**2*log(tan(e + f*x)
**2 + 1)*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) + I*b**2*log(tan(e + f
*x)**2 + 1)/(2*d*f*tan(e + f*x) + 2*I*d*f) + I*b**2/(2*d*f*tan(e + f*x) + 2
*I*d*f), Eq(c, I*d)), ((a**2*x + a*b*log(tan(e + f*x)**2 + 1)/f - b**2*x +
b**2*tan(e + f*x)/f)/c, Eq(d, 0)), (x*(a + b*tan(e))**2/(c + d*tan(e)), Eq(
f, 0)), (2*a**2*c*d*f*x/(2*c**2*d*f + 2*d**3*f) + 2*a**2*d**2*log(c/d + tan
(e + f*x))/(2*c**2*d*f + 2*d**3*f) - a**2*d**2*log(tan(e + f*x)**2 + 1)/(2*
c**2*d*f + 2*d**3*f) - 4*a*b*c*d*log(c/d + tan(e + f*x))/(2*c**2*d*f + 2*d*
*3*f) + 2*a*b*c*d*log(tan(e + f*x)**2 + 1)/(2*c**2*d*f + 2*d**3*f) + 4*a*b*
d**2*f*x/(2*c**2*d*f + 2*d**3*f) + 2*b**2*c**2*log(c/d + tan(e + f*x))/(2*c
**2*d*f + 2*d**3*f) - 2*b**2*c*d*f*x/(2*c**2*d*f + 2*d**3*f) + b**2*d**2*lo
g(tan(e + f*x)**2 + 1)/(2*c**2*d*f + 2*d**3*f), True))
```

Giac [A]
time = 0.57, size = 126, normalized size = 1.22

$$\frac{\frac{2\,(a^2c-b^2c+2\,abd)(fx+e)}{c^2+d^2}+\frac{(2\,abc-a^2d+b^2d)\log\left(\tan(fx+e)^2+1\right)}{c^2+d^2}+\frac{2\,(b^2c^2-2\,abcd+a^2d^2)\log(|d\tan(fx+e)+c|)}{c^2d+d^3}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(c+d*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*(2*(a^2*c - b^2*c + 2*a*b*d)*(f*x + e)/(c^2 + d^2) + (2*a*b*c - a^2*d +
 b^2*d)*log(tan(f*x + e)^2 + 1)/(c^2 + d^2) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*
d^2)*log(abs(d*tan(f*x + e) + c))/(c^2*d + d^3))/f
```

Mupad [B]
time = 5.60, size = 115, normalized size = 1.12

$$\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(-a^2\,1\mathrm{i}+2\,a\,b+b^2\,1\mathrm{i}\right)}{2\,f\,\left(c+d\,1\mathrm{i}\right)}+\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(-a^2+a\,b\,2\mathrm{i}+b^2\right)}{2\,f\,\left(d+c\,1\mathrm{i}\right)}+\frac{\ln\left(c+d\tan(e+f\,x)\right)\,\left(a\,d-b\,c\right)^2}{d\,f\,\left(c^2+d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^2/(c + d*tan(e + f*x)),x)
```

```
[Out] (log(tan(e + f*x) - 1i)*(2*a*b - a^2*1i + b^2*1i))/(2*f*(c + d*1i)) + (log(
tan(e + f*x) + 1i)*(a*b*2i - a^2 + b^2))/(2*f*(c*1i + d)) + (log(c + d*tan(
e + f*x))*(a*d - b*c)^2)/(d*f*(c^2 + d^2))
```

3.1212 $\int \frac{a+b\tan(e+fx)}{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=59

$$\frac{(ac+bd)x}{c^2+d^2}-\frac{(bc-ad)\log(c\cos(e+fx)+d\sin(e+fx))}{(c^2+d^2)\,f}$$

```
[Out] (a*c+b*d)*x/(c^2+d^2)-(-a*d+b*c)*ln(c*cos(f*x+e)+d*sin(f*x+e))/(c^2+d^2)/f
```

Rubi [A]
time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3612, 3611}

$$\frac{x(ac+bd)}{c^2+d^2}-\frac{(bc-ad)\log(c\cos(e+fx)+d\sin(e+fx))}{f\,(c^2+d^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])/(c + d*Tan[e + f*x]),x]
```

```
[Out] ((a*c + b*d)*x)/(c^2 + d^2) - ((b*c - a*d)*Log[c*Cos[e + f*x] + d*Sin[e + f
*x]])/((c^2 + d^2)*f)
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan(e+fx)}{c+d\tan(e+fx)}\,dx &= \frac{(ac+bd)x}{c^2+d^2}-\frac{(bc-ad)\int \frac{d-c\tan(e+fx)}{c+d\tan(e+fx)}\,dx}{c^2+d^2}\\ &= \frac{(ac+bd)x}{c^2+d^2}-\frac{(bc-ad)\log(c\cos(e+fx)+d\sin(e+fx))}{(c^2+d^2)\,f}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 65, normalized size = 1.10

$$\frac{2(ac+bd)\text{ArcTan}(\tan(e+fx))+(bc-ad)\left(\log\left(\sec^2(e+fx)\right)-2\log(c+d\tan(e+fx))\right)}{2\left(c^2+d^2\right)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])/(c + d*Tan[e + f*x]),x]
```

```
[Out] (2*(a*c + b*d)*ArcTan[Tan[e + f*x]] + (b*c - a*d)*(Log[Sec[e + f*x]^2] - 2*
Log[c + d*Tan[e + f*x]]))/(2*(c^2 + d^2)*f)
```

Maple [A]
time = 0.18, size = 82, normalized size = 1.39

method	result
derivativedivides	$\frac{\frac{\frac{(-ad+bc)\ln\left(1+\tan^2(fx+e)\right)}{2}+(ac+bd)\arctan(\tan(fx+e))}{c^2+d^2}+\frac{(ad-bc)\ln(c+d\tan(fx+e))}{c^2+d^2}}{f}$
default	$\frac{\frac{\frac{(-ad+bc)\ln\left(1+\tan^2(fx+e)\right)}{2}+(ac+bd)\arctan(\tan(fx+e))}{c^2+d^2}+\frac{(ad-bc)\ln(c+d\tan(fx+e))}{c^2+d^2}}{f}$
norman	$\frac{(ac+bd)x}{c^2+d^2}+\frac{(ad-bc)\ln(c+d\tan(fx+e))}{f(c^2+d^2)}-\frac{(ad-bc)\ln\left(1+\tan^2(fx+e)\right)}{2f(c^2+d^2)}$
risch	$\frac{ixb}{id-c}-\frac{ax}{id-c}-\frac{2iadx}{c^2+d^2}+\frac{2ibcx}{c^2+d^2}-\frac{2iade}{f(c^2+d^2)}+\frac{2ibce}{f(c^2+d^2)}+\frac{\ln\left(e^{2i(fx+e)}-\frac{id+c}{id-c}\right)ad}{f(c^2+d^2)}-\frac{\ln\left(e^{2i(fx+e)}-\frac{id+c}{id-c}\right)}{f(c^2+d^2)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))/(c+d*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(c^2+d^2)*(1/2*(-a*d+b*c)*ln(1+tan(f*x+e)^2)+(a*c+b*d)*arctan(tan(f*
x+e)))+(a*d-b*c)/(c^2+d^2)*ln(c+d*tan(f*x+e)))
```

Maxima [A]
time = 0.56, size = 91, normalized size = 1.54

$$\frac{\frac{2\,(ac+bd)(fx+e)}{c^2+d^2}-\frac{2\,(bc-ad)\log(d\tan(fx+e)+c)}{c^2+d^2}+\frac{(bc-ad)\log\left(\tan(fx+e)^2+1\right)}{c^2+d^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(c+d*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a*c + b*d)*(f*x + e)/(c^2 + d^2) - 2*(b*c - a*d)*log(d*tan(f*x + e)
 + c)/(c^2 + d^2) + (b*c - a*d)*log(tan(f*x + e)^2 + 1)/(c^2 + d^2))/f
```

Fricas [A]
time = 1.24, size = 79, normalized size = 1.34

$$\frac{2\,(ac+bd)fx-(bc-ad)\log\left(\frac{d^2\tan(fx+e)^2+2\,cd\tan(fx+e)+c^2}{\tan(fx+e)^2+1}\right)}{2\,(c^2+d^2)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(c+d*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a*c + b*d)*f*x - (b*c - a*d)*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*
x + e) + c^2)/(tan(f*x + e)^2 + 1)))/((c^2 + d^2)*f)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.47, size = 524, normalized size = 8.88

$$\begin{cases} \frac{\tilde{\infty}x(a+b\tan(e))}{\tan(e)} & \text{for}\ c=0\wedge d=0\wedge f=0 \\ \frac{iafx\tan(e+fx)}{2df\tan(e+fx)-2idf}+\frac{afx}{2df\tan(e+fx)-2idf}+\frac{ia}{2df\tan(e+fx)-2idf}+\frac{bfx\tan(e+fx)}{2df\tan(e+fx)-2idf}-\frac{ibfx}{2df\tan(e+fx)-2idf}-\frac{b}{2df\tan(e+fx)-2idf} & \text{for}\ c=-id \\ -\frac{iafx\tan(e+fx)}{2df\tan(e+fx)+2idf}+\frac{afx}{2df\tan(e+fx)+2idf}-\frac{ia}{2df\tan(e+fx)+2idf}+\frac{bfx\tan(e+fx)}{2df\tan(e+fx)+2idf}+\frac{ibfx}{2df\tan(e+fx)+2idf}-\frac{b}{2df\tan(e+fx)+2idf} & \text{for}\ c=id \\ \frac{x(a+b\tan(e))}{c+d\tan(e)} & \text{for}\ f=0 \\ \frac{ax+\frac{b\log\left(\tan^2(e+fx)+1\right)}{2f}}{c} & \text{for}\ d=0 \\ \frac{2acfx}{2c^2f+2d^2f}+\frac{2ad\log\left(\frac{c}{d}+\tan(e+fx)\right)}{2c^2f+2d^2f}-\frac{ad\log\left(\tan^2(e+fx)+1\right)}{2c^2f+2d^2f}-\frac{2bc\log\left(\frac{c}{d}+\tan(e+fx)\right)}{2c^2f+2d^2f}+\frac{bc\log\left(\tan^2(e+fx)+1\right)}{2c^2f+2d^2f}+\frac{2bdfx}{2c^2f+2d^2f} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(c+d*tan(f*x+e)),x)
```

```
[Out] Piecewise((zoo*x*(a + b*tan(e))/tan(e), Eq(c, 0) & Eq(d, 0) & Eq(f, 0)), (I
*a*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) + a*f*x/(2*d*f*tan(e + f
*x) - 2*I*d*f) + I*a/(2*d*f*tan(e + f*x) - 2*I*d*f) + b*f*x*tan(e + f*x)/(2
*d*f*tan(e + f*x) - 2*I*d*f) - I*b*f*x/(2*d*f*tan(e + f*x) - 2*I*d*f) - b/(
2*d*f*tan(e + f*x) - 2*I*d*f), Eq(c, -I*d)), (-I*a*f*x*tan(e + f*x)/(2*d*f*
tan(e + f*x) + 2*I*d*f) + a*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) - I*a/(2*d*f
*tan(e + f*x) + 2*I*d*f) + b*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f
) + I*b*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) - b/(2*d*f*tan(e + f*x) + 2*I*d*
f), Eq(c, I*d)), (x*(a + b*tan(e))/(c + d*tan(e)), Eq(f, 0)), ((a*x + b*log
(tan(e + f*x)**2 + 1)/(2*f))/c, Eq(d, 0)), (2*a*c*f*x/(2*c**2*f + 2*d**2*f)
 + 2*a*d*log(c/d + tan(e + f*x))/(2*c**2*f + 2*d**2*f) - a*d*log(tan(e + f*
x)**2 + 1)/(2*c**2*f + 2*d**2*f) - 2*b*c*log(c/d + tan(e + f*x))/(2*c**2*f
+ 2*d**2*f) + b*c*log(tan(e + f*x)**2 + 1)/(2*c**2*f + 2*d**2*f) + 2*b*d*f*
x/(2*c**2*f + 2*d**2*f), True))
```

Giac [A]
time = 0.47, size = 97, normalized size = 1.64

$$\frac{\frac{2\,(ac+bd)(fx+e)}{c^2+d^2}+\frac{(bc-ad)\log\left(\tan(fx+e)^2+1\right)}{c^2+d^2}-\frac{2\,(bcd-ad^2)\log(|d\tan(fx+e)+c|)}{c^2d+d^3}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(c+d*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*(2*(a*c + b*d)*(f*x + e)/(c^2 + d^2) + (b*c - a*d)*log(tan(f*x + e)^2 +
 1)/(c^2 + d^2) - 2*(b*c*d - a*d^2)*log(abs(d*tan(f*x + e) + c))/(c^2*d + d
^3))/f
```

Mupad [B]
time = 5.49, size = 93, normalized size = 1.58

$$\frac{\ln\left(c+d\tan(e+f\,x)\right)\,\left(a\,d-b\,c\right)}{f\,\left(c^2+d^2\right)}-\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(-b+a\,1\mathrm{i}\right)}{2\,f\,\left(c+d\,1\mathrm{i}\right)}-\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(a-b\,1\mathrm{i}\right)}{2\,f\,\left(d+c\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))/(c + d*tan(e + f*x)),x)
```

```
[Out] (log(c + d*tan(e + f*x))*(a*d - b*c))/(f*(c^2 + d^2)) - (log(tan(e + f*x) -
 1i)*(a*1i - b))/(2*f*(c + d*1i)) - (log(tan(e + f*x) + 1i)*(a - b*1i))/(2*
f*(c*1i + d))
```

3.1213 $\int \frac{1}{(a+b\tan(e+fx))(c+d\tan(e+fx))}\,dx$

Optimal. Leaf size=118

$$\frac{(ac-bd)x}{(a^2+b^2)(c^2+d^2)}+\frac{b^2\log(a\cos(e+fx)+b\sin(e+fx))}{(a^2+b^2)(bc-ad)f}-\frac{d^2\log(c\cos(e+fx)+d\sin(e+fx))}{(bc-ad)(c^2+d^2)f}$$

```
[Out] (a*c-b*d)*x/(a^2+b^2)/(c^2+d^2)+b^2*ln(a*cos(f*x+e)+b*sin(f*x+e))/(a^2+b^2)
/(-a*d+b*c)/f-d^2*ln(c*cos(f*x+e)+d*sin(f*x+e))/(-a*d+b*c)/(c^2+d^2)/f
```

Rubi [A]
time = 0.11, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = $\{3652, 3611\}$

$$\frac{x(ac-bd)}{(a^2+b^2)(c^2+d^2)}+\frac{b^2\log(a\cos(e+fx)+b\sin(e+fx))}{f(a^2+b^2)(bc-ad)}-\frac{d^2\log(c\cos(e+fx)+d\sin(e+fx))}{f(c^2+d^2)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])),x]
```

```
[Out] ((a*c - b*d)*x)/((a^2 + b^2)*(c^2 + d^2)) + (b^2*Log[a*Cos[e + f*x] + b*Sin
[e + f*x]])/((a^2 + b^2)*(b*c - a*d)*f) - (d^2*Log[c*Cos[e + f*x] + d*Sin[e
 + f*x]])/((b*c - a*d)*(c^2 + d^2)*f)
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3652

```
Int[1/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f
_.)*(x_)])), x_Symbol] :> Simp[(a*c - b*d)*(x/((a^2 + b^2)*(c^2 + d^2))), x
] + (Dist[b^2/((b*c - a*d)*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Ta
n[e + f*x]), x], x] - Dist[d^2/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e
+ f*x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\tan(e+fx))(c+d\tan(e+fx))}\,dx = \frac{(ac-bd)x}{(a^2+b^2)(c^2+d^2)} + \frac{b^2\int \frac{b-a\tan(e+fx)}{a+b\tan(e+fx)}\,dx}{(a^2+b^2)(bc-ad)} - \frac{d^2\int \frac{d-c\tan(e+fx)}{c+d\tan(e+fx)}}{(bc-ad)(c^2+}$$
$$= \frac{(ac-bd)x}{(a^2+b^2)(c^2+d^2)} + \frac{b^2\log(a\cos(e+fx)+b\sin(e+fx))}{(a^2+b^2)(bc-ad)f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.37, size = 143, normalized size = 1.21

$$\frac{\frac{\log(i-\tan(e+fx))}{(a+ib)(ic-d)} - \frac{\log(i+\tan(e+fx))}{(ia+b)(c-id)} + \frac{2b^2\log(a+b\tan(e+fx))}{(a^2+b^2)(bc-ad)} + \frac{2d^2\log(c+d\tan(e+fx))}{(-bc+ad)(c^2+d^2)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])),x]
```

```
[Out] (Log[I - Tan[e + f*x]]/((a + I*b)*(I*c - d)) - Log[I + Tan[e + f*x]]/((I*a
+ b)*(c - I*d)) + (2*b^2*Log[a + b*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*d))
 + (2*d^2*Log[c + d*Tan[e + f*x]])/((-(b*c) + a*d)*(c^2 + d^2)))/(2*f)
```

Maple [A]
time = 0.26, size = 133, normalized size = 1.13

method	result
derivativedivides	$\frac{\frac{\frac{(-ad-bc)\ln\left(1+\tan^2(fx+e)\right)}{2}+(ac-bd)\arctan(\tan(fx+e))}{\left(a^2+b^2\right)\left(c^2+d^2\right)}+\frac{d^2\ln(c+d\tan(fx+e))}{(ad-bc)\left(c^2+d^2\right)}-\frac{b^2\ln(a+b\tan(fx+e))}{\left(a^2+b^2\right)(ad-bc)}}{f}$
default	$\frac{\frac{\frac{(-ad-bc)\ln\left(1+\tan^2(fx+e)\right)}{2}+(ac-bd)\arctan(\tan(fx+e))}{\left(a^2+b^2\right)\left(c^2+d^2\right)}+\frac{d^2\ln(c+d\tan(fx+e))}{(ad-bc)\left(c^2+d^2\right)}-\frac{b^2\ln(a+b\tan(fx+e))}{\left(a^2+b^2\right)(ad-bc)}}{f}$
norman	$\frac{(ac-bd)x}{(a^2+b^2)(c^2+d^2)}+\frac{d^2\ln(c+d\tan(fx+e))}{f(a c^2d+a d^3-b c^3-bc d^2)}-\frac{b^2\ln(a+b\tan(fx+e))}{(ad-bc)f(a^2+b^2)}-\frac{(ad+bc)\ln(1+\tan^2(fx+e))}{2(a^2+b^2)(c^2+d^2)f}$
risch	$-\frac{x}{iad+ibc-ac+bd}+\frac{2ib^2x}{a^3d-a^2bc+a b^2d-b^3c}+\frac{2ib^2e}{f(a^3d-a^2bc+a b^2d-b^3c)}-\frac{2id^2x}{a c^2d+a d^3-b c^3-bc d^2}-\frac{2}{f(a c^2d+a d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e))/(c+d*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(a^2+b^2)/(c^2+d^2)*(1/2*(-a*d-b*c)*ln(1+tan(f*x+e)^2)+(a*c-b*d)*arc
tan(tan(f*x+e)))+d^2/(a*d-b*c)/(c^2+d^2)*ln(c+d*tan(f*x+e))-b^2/(a^2+b^2)/(
a*d-b*c)*ln(a+b*tan(f*x+e)))
```

Maxima [A]
time = 0.55, size = 180, normalized size = 1.53

$$\frac{\frac{2\,b^2\log(b\tan(fx+e)+a)}{(a^2b+b^3)c-(a^3+ab^2)d}-\frac{2\,d^2\log(d\tan(fx+e)+c)}{bc^3-ac^2d+bcd^2-ad^3}+\frac{2\,(ac-bd)(fx+e)}{(a^2+b^2)c^2+(a^2+b^2)d^2}-\frac{(bc+ad)\log\left(\tan(fx+e)^2+1\right)}{(a^2+b^2)c^2+(a^2+b^2)d^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))/(c+d*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^2*log(b*tan(f*x + e) + a)/((a^2*b + b^3)*c - (a^3 + a*b^2)*d) - 2*
d^2*log(d*tan(f*x + e) + c)/(b*c^3 - a*c^2*d + b*c*d^2 - a*d^3) + 2*(a*c -
b*d)*(f*x + e)/((a^2 + b^2)*c^2 + (a^2 + b^2)*d^2) - (b*c + a*d)*log(tan(f*
x + e)^2 + 1)/((a^2 + b^2)*c^2 + (a^2 + b^2)*d^2))/f
```

Fricas [A]
time = 1.06, size = 207, normalized size = 1.75

$$-\frac{(a^2+b^2)d^2\log\left(\frac{d^2\tan(fx+e)^2+2\,cd\tan(fx+e)+c^2}{\tan(fx+e)^2+1}\right)-2\,(abc^2+abd^2-(a^2+b^2)cd)fx-(b^2c^2+b^2d^2)\log\left(\frac{b^2\tan(fx+e)^2+2\,ab\tan(fx+e)+a^2}{\tan(fx+e)^2+1}\right)}{2\,((a^2b+b^3)c^3-(a^3+ab^2)c^2d+(a^2b+b^3)cd^2-(a^3+ab^2)d^3)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))/(c+d*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/2*((a^2 + b^2)*d^2*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2)/(
tan(f*x + e)^2 + 1)) - 2*(a*b*c^2 + a*b*d^2 - (a^2 + b^2)*c*d)*f*x - (b^2*c
^2 + b^2*d^2)*log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(tan(f*x
+ e)^2 + 1)))/(((a^2*b + b^3)*c^3 - (a^3 + a*b^2)*c^2*d + (a^2*b + b^3)*c*d
^2 - (a^3 + a*b^2)*d^3)*f)
```

Sympy [C] Result contains complex when optimal does not.
time = 11.75, size = 8053, normalized size = 68.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))/(c+d*tan(f*x+e)),x)
```

```
[Out] Piecewise(((2*c*f*x/(2*c**2*f + 2*d**2*f) + 2*d*log(c/d + tan(e + f*x))/(2*
c**2*f + 2*d**2*f) - d*log(tan(e + f*x)**2 + 1)/(2*c**2*f + 2*d**2*f))/a, E
q(b, 0)), ((2*a*f*x/(2*a**2*f + 2*b**2*f) + 2*b*log(a/b + tan(e + f*x))/(2*
a**2*f + 2*b**2*f) - b*log(tan(e + f*x)**2 + 1)/(2*a**2*f + 2*b**2*f))/c, E
q(d, 0)), (I*c**2*f*x*tan(e + f*x)/(2*b*c**3*f*tan(e + f*x) - 2*I*b*c**3*f
+ 2*I*b*c**2*d*f*tan(e + f*x) + 2*b*c**2*d*f + 2*b*c*d**2*f*tan(e + f*x) -
2*I*b*c*d**2*f + 2*I*b*d**3*f*tan(e + f*x) + 2*b*d**3*f) + c**2*f*x/(2*b*c*
*3*f*tan(e + f*x) - 2*I*b*c**3*f + 2*I*b*c**2*d*f*tan(e + f*x) + 2*b*c**2*d
```

```
*f + 2*b*c*d**2*f*tan(e + f*x) - 2*I*b*c*d**2*f + 2*I*b*d**3*f*tan(e + f*x)
 + 2*b*d**3*f) + I*c**2/(2*b*c**3*f*tan(e + f*x) - 2*I*b*c**3*f + 2*I*b*c**
2*d*f*tan(e + f*x) + 2*b*c**2*d*f + 2*b*c*d**2*f*tan(e + f*x) - 2*I*b*c*d**
2*f + 2*I*b*d**3*f*tan(e + f*x) + 2*b*d**3*f) - 2*c*d*f*x*tan(e + f*x)/(2*b
*c**3*f*tan(e + f*x) - 2*I*b*c**3*f + 2*I*b*c**2*d*f*tan(e + f*x) + 2*b*c**
2*d*f + 2*b*c*d**2*f*tan(e + f*x) - 2*I*b*c*d**2*f + 2*I*b*d**3*f*tan(e + f
*x) + 2*b*d**3*f) + 2*I*c*d*f*x/(2*b*c**3*f*tan(e + f*x) - 2*I*b*c**3*f + 2
*I*b*c**2*d*f*tan(e + f*x) + 2*b*c**2*d*f + 2*b*c*d**2*f*tan(e + f*x) - 2*I
*b*c*d**2*f + 2*I*b*d**3*f*tan(e + f*x) + 2*b*d**3*f) + I*d**2*f*x*tan(e +
f*x)/(2*b*c**3*f*tan(e + f*x) - 2*I*b*c**3*f + 2*I*b*c**2*d*f*tan(e + f*x)
+ 2*b*c**2*d*f + 2*b*c*d**2*f*tan(e + f*x) - 2*I*b*c*d**2*f + 2*I*b*d**3*f*
tan(e + f*x) + 2*b*d**3*f) + d**2*f*x/(2*b*c**3*f*tan(e + f*x) - 2*I*b*c**3
*f + 2*I*b*c**2*d*f*tan(e + f*x) + 2*b*c**2*d*f + 2*b*c*d**2*f*tan(e + f*x)
 - 2*I*b*c*d**2*f + 2*I*b*d**3*f*tan(e + f*x) + 2*b*d**3*f) - 2*d**2*log(c/
d + tan(e + f*x))*tan(e + f*x)/(2*b*c**3*f*tan(e + f*x) - 2*I*b*c**3*f + 2*
I*b*c**2*d*f*tan(e + f*x) + 2*b*c**2*d*f + 2*b*c*d**2*f*tan(e + f*x) - 2*I*
b*c*d**2*f + 2*I*b*d**3*f*tan(e + f*x) + 2*b*d**3*f) + 2*I*d**2*log(c/d + t
an(e + f*x))/(2*b*c**3*f*tan(e + f*x) - 2*I*b*c**3*f + 2*I*b*c**2*d*f*tan(e
 + f*x) + 2*b*c**2*d*f + 2*b*c*d**2*f*tan(e + f*x) - 2*I*b*c*d**2*f + 2*I*b
*d**3*f*tan(e + f*x) + 2*b*d**3*f) + d**2*log(tan(e + f*x)**2 + 1)*tan(e +
f*x)/(2*b*c**3*f*tan(e + f*x) - 2*I*b*c**3*f + 2*I*b*c**2*d*f*tan(e + f*x)
+ 2*b*c**2*d*f + 2*b*c*d**2*f*tan(e + f*x) - 2*I*b*c*d**2*f + 2*I*b*d**3*f*
tan(e + f*x) + 2*b*d**3*f) - I*d**2*log(tan(e + f*x)**2 + 1)/(2*b*c**3*f*ta
n(e + f*x) - 2*I*b*c**3*f + 2*I*b*c**2*d*f*tan(e + f*x) + 2*b*c**2*d*f + 2*
b*c*d**2*f*tan(e + f*x) - 2*I*b*c*d**2*f + 2*I*b*d**3*f*tan(e + f*x) + 2*b*
d**3*f) + I*d**2/(2*b*c**3*f*tan(e + f*x) - 2*I*b*c**3*f + 2*I*b*c**2*d*f*t
an(e + f*x) + 2*b*c**2*d*f + 2*b*c*d**2*f*tan(e + f*x) - 2*I*b*c*d**2*f + 2
*I*b*d**3*f*tan(e + f*x) + 2*b*d**3*f), Eq(a, -I*b)), (-I*c**2*f*x*tan(e +
f*x)/(2*b*c**3*f*tan(e + f*x) + 2*I*b*c**3*f - 2*I*b*c**2*d*f*tan(e + f*x)
+ 2*b*c**2*d*f + 2*b*c*d**2*f*tan(e + f*x) + 2*I*b*c*d**2*f - 2*I*b*d**3*f*
tan(e + f*x) + 2*b*d**3*f) + c**2*f*x/(2*b*c**3*f*tan(e + f*x) + 2*I*b*c**3
*f - 2*I*b*c**2*d*f*tan(e + f*x) + 2*b*c**2*d*f + 2*b*c*d**2*f*tan(e + f*x)
 + 2*I*b*c*d**2*f - 2*I*b*d**3*f*tan(e + f*x) + 2*b*d**3*f) - I*c**2/(2*b*c
**3*f*tan(e + f*x) + 2*I*b*c**3*f - 2*I*b*c**2*d*f*tan(e + f*x) + 2*b*c**2*
d*f + 2*b*c*d**2*f*tan(e + f*x) + 2*I*b*c*d**2*f - 2*I*b*d**3*f*tan(e + f*x
) + 2*b*d**3*f) - 2*c*d*f*x*tan(e + f*x)/(2*b*c**3*f*tan(e + f*x) + 2*I*b*c
**3*f - 2*I*b*c**2*d*f*tan(e + f*x) + 2*b*c**2*d*f + 2*b*c*d**2*f*tan(e + f
*x) + 2*I*b*c*d**2*f - 2*I*b*d**3*f*tan(e + f*x) + 2*b*d**3*f) - 2*I*c*d*f*
x/(2*b*c**3*f*tan(e + f*x) + 2*I*b*c**3*f - 2*I*b*c**2*d*f*tan(e + f*x) + 2
*b*c**2*d*f + 2*b*c*d**2*f*tan(e + f*x) + 2*I*b*c*d**2*f - 2*I*b*d**3*f*tan
(e + f*x) + 2*b*d**3*f) - I*d**2*f*x*tan(e + f*x)/(2*b*c**3*f*tan(e + f*x)
+ 2*I*b*c**3*f - 2*I*b*c**2*d*f*tan(e + f*x) + 2*b*c**2*d*f + 2*b*c*d**2*f*
tan(e + f*x) + 2*I*b*c*d**2*f - 2*I*b*d**3*f*tan(e + f*x) + 2*b*d**3*f) + d
**2*f*x/(2*b*c**3*f*tan(e + f*x) + 2*I*b*c**3*f - 2*I*b*c**2*d*f*tan(e + f*
x) + 2*b*c**2*d*f + 2*b*c*d**2*f*tan(e + f*x) + 2*I*b*c*d**2*f - 2*I*b*d**3
```

```
*f*tan(e + f*x) + 2*b*d**3*f) - 2*d**2*log(c/d + tan(e + f*x))*tan(e + f*x)
/(2*b*c**3*f*tan(e + f*x) + 2*I*b*c**3*f - 2*I*b*c**2*d*f*tan(e + f*x) + 2*
b*c**2*d*f + 2*b*c*d**2*f*tan(e + f*x) + 2*I*b*c*d**2*f - 2*I*b*d**3*f*tan(
e + f*x) + 2*b*d**3*f) - 2*I*d**2*log(c/d + tan(e + f*x))/(2*b*c**3*f*tan(e
 + f*x) + 2*I*b*c**3*f - 2*I*b*c**2*d*f*tan(e + f*x) + 2*b*c**2*d*f + 2*b*c
*d**2*f*tan(e + f*x) + 2*I*b*c*d**2*f - 2*I*b*d**3*f*tan(e + f*x) + 2*b*d**
3*f) + d**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*b*c**3*f*tan(e + f*x)
+ 2*I*b*c**3*f - 2*I*b*c**2*d*f*tan(e + f*x) + 2*b*c**2*d*f + 2*b*c*d**2*f*
tan(e + f*x) + 2*I*b*c*d**2*f - 2*I*b*d**3*f*tan(e + f*x) + 2*b*d**3*f) + I
*d**2*log(tan(e + f*x)**2 + 1)/(2*b*c**3*f*tan(e + f*x) + 2*I*b*c**3*f - 2*
I*b*c**2*d*f*tan(e + f*x) + 2*b*c**2*d*f + 2*b*c*d**2*f*tan(e + f*x) + 2*I*
b*c*d**2*f - 2*I*b*d**3*f*tan(e + f*x) + 2*b*d**3*f) - I*d**2/(2*b*c**3*f*t
an(e + f*x) + 2*I*b*c**3*f - 2*I*b*c**2*d*f*tan(e + f*x) + 2*b*c**2*d*f + 2
*b*c*d**2*f*tan(e + f*x) + 2*I*b*c*d**2*f - 2*I...
```

Giac [A]
time = 0.51, size = 201, normalized size = 1.70

$$\frac{\frac{2\,b^3\log(|b\tan(fx+e)+a|)}{a^2b^2c+b^4c-a^3bd-ab^3d}-\frac{2\,d^3\log(|d\tan(fx+e)+c|)}{bc^3d-ac^2d^2+bcd^3-ad^4}+\frac{2\,(ac-bd)(fx+e)}{a^2c^2+b^2c^2+a^2d^2+b^2d^2}-\frac{(bc+ad)\log\left(\tan(fx+e)^2+1\right)}{a^2c^2+b^2c^2+a^2d^2+b^2d^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))/(c+d*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^3*log(abs(b*tan(f*x + e) + a))/(a^2*b^2*c + b^4*c - a^3*b*d - a*b^
3*d) - 2*d^3*log(abs(d*tan(f*x + e) + c))/(b*c^3*d - a*c^2*d^2 + b*c*d^3 -
a*d^4) + 2*(a*c - b*d)*(f*x + e)/(a^2*c^2 + b^2*c^2 + a^2*d^2 + b^2*d^2) -
(b*c + a*d)*log(tan(f*x + e)^2 + 1)/(a^2*c^2 + b^2*c^2 + a^2*d^2 + b^2*d^2)
)/f
```

Mupad [B]
time = 5.75, size = 173, normalized size = 1.47

$$\frac{d^2\,\ln\left(c+d\tan(e+f\,x)\right)}{f\,\left(a\,d-b\,c\right)\,\left(c^2+d^2\right)}-\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)}{2\,f\,\left(a\,c\,1\mathrm{i}+a\,d+b\,c-b\,d\,1\mathrm{i}\right)}-\frac{\ln\left(a+b\tan(e+f\,x)\right)\,\left(\frac{d^2}{\left(a\,d-b\,c\right)\,\left(c^2+d^2\right)}-\frac{a\,d+b\,c}{\left(a^2+b^2\right)\,\left(c^2+d^2\right)}\right)}{f}-\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)}{2\,f\,\left(a\,d-a\,c\,1\mathrm{i}+b\,c+b\,d\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*tan(e + f*x))*(c + d*tan(e + f*x))),x)
```

```
[Out] (d^2*log(c + d*tan(e + f*x)))/(f*(a*d - b*c)*(c^2 + d^2)) - log(tan(e + f*x
) + 1i)/(2*f*(a*c*1i + a*d + b*c - b*d*1i)) - (log(a + b*tan(e + f*x))*(d^2
/((a*d - b*c)*(c^2 + d^2)) - (a*d + b*c)/((a^2 + b^2)*(c^2 + d^2))))/f - lo
g(tan(e + f*x) - 1i)/(2*f*(a*d - a*c*1i + b*c + b*d*1i))
```

3.1214 $\int \frac{1}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))}\,dx$

Optimal. Leaf size=183

$$\frac{(a^2c-b^2c-2abd)\,x}{(a^2+b^2)^2\,(c^2+d^2)}+\frac{b^2(2abc-3a^2d-b^2d)\log(a\cos(e+fx)+b\sin(e+fx))}{(a^2+b^2)^2\,(bc-ad)^2f}+\frac{d^3\log(c\cos(e+fx)+d\sin}{(bc-ad)^2\,(c^2+d^2}$$

```
[Out] (a^2*c-2*a*b*d-b^2*c)*x/(a^2+b^2)^2/(c^2+d^2)+b^2*(-3*a^2*d+2*a*b*c-b^2*d)*
ln(a*cos(f*x+e)+b*sin(f*x+e))/(a^2+b^2)^2/(-a*d+b*c)^2/f+d^3*ln(c*cos(f*x+e
)+d*sin(f*x+e))/(-a*d+b*c)^2/(c^2+d^2)/f-b^2/(a^2+b^2)/(-a*d+b*c)/f/(a+b*ta
n(f*x+e))
```

Rubi [A]

time = 0.32, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3650, 3732, 3611}

$$\frac{x(a^2c-2abd-b^2c)}{(a^2+b^2)^2\,(c^2+d^2)}-\frac{b^2}{f\,(a^2+b^2)\,(bc-ad)(a+b\tan(e+fx))}+\frac{b^2(-3a^2d+2abc-b^2d)\log(a\cos(e+fx)+b\sin(e+fx))}{f\,(a^2+b^2)^2\,(bc-ad)^2}+\frac{d^3\log(c\cos(e+fx)+d\sin(e+fx))}{f\,(c^2+d^2)\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])),x]
```

```
[Out] ((a^2*c - b^2*c - 2*a*b*d)*x)/((a^2 + b^2)^2*(c^2 + d^2)) + (b^2*(2*a*b*c -
 3*a^2*d - b^2*d)*Log[a*Cos[e + f*x] + b*Sin[e + f*x]])/((a^2 + b^2)^2*(b*c
 - a*d)^2*f) + (d^3*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((b*c - a*d)^2*(c
^2 + d^2)*f) - b^2/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x]))
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))}\,dx = -\frac{b^2}{(a^2+b^2)\,(bc-ad)f(a+b\tan(e+fx))} - \frac{\int \frac{-abc+a^2d+b^2d+}{(a+b\,\mathrm{ta}}}{(}$$

$$= \frac{(a^2c-b^2c-2abd)\,x}{(a^2+b^2)^2\,(c^2+d^2)} - \frac{b^2}{(a^2+b^2)\,(bc-ad)f(a+b\tan(e+f}$$

$$= \frac{(a^2c-b^2c-2abd)\,x}{(a^2+b^2)^2\,(c^2+d^2)} + \frac{b^2(2abc-3a^2d-b^2d)\log(a\cos(e+}{(a^2+b^2)^2\,(bc-ad}$$

Mathematica [A]
time = 3.35, size = 302, normalized size = 1.65

$$-\frac{\frac{\left(2abc+a^2d-b^2d+\frac{\sqrt{-b^2}\,(a^2c-b^2c-2abd)}{b}\right)\log\left(\sqrt{-b^2}-b\tan(e+fx)\right)}{(a^2+b^2)^2(c^2+d^2)}+\frac{2b^2(-2abc+3a^2d+b^2d)\log(a+b\tan(e+fx))}{(a^2+b^2)^2(bc-ad)^2}+\frac{\left(2abc+a^2d-b^2d+\frac{\sqrt{-b^2}\,(-a^2c+b^2c+2abd)}{b}\right)\log\left(\sqrt{-b^2}+b\tan(e+fx)\right)}{(a^2+b^2)^2(c^2+d^2)}-\frac{2d^3\log(c+d\tan(e+fx))}{(bc-ad)^2(c^2+d^2)}+\frac{2b^2}{(a^2+b^2)(bc-ad)(a+b\tan(e+fx))}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])),x]
```

```
[Out] -1/2*(((2*a*b*c + a^2*d - b^2*d + (Sqrt[-b^2]*(a^2*c - b^2*c - 2*a*b*d))/b)
*Log[Sqrt[-b^2] - b*Tan[e + f*x]])/((a^2 + b^2)^2*(c^2 + d^2)) + (2*b^2*(-2
*a*b*c + 3*a^2*d + b^2*d)*Log[a + b*Tan[e + f*x]])/((a^2 + b^2)^2*(b*c - a*
d)^2) + ((2*a*b*c + a^2*d - b^2*d + (Sqrt[-b^2]*(-(a^2*c) + b^2*c + 2*a*b*d
))/b)*Log[Sqrt[-b^2] + b*Tan[e + f*x]])/((a^2 + b^2)^2*(c^2 + d^2)) - (2*d^
3*Log[c + d*Tan[e + f*x]])/((b*c - a*d)^2*(c^2 + d^2)) + (2*b^2)/((a^2 + b^
2)*(b*c - a*d)*(a + b*Tan[e + f*x])))/f
```

Maple [A]
time = 0.46, size = 202, normalized size = 1.10

method	result

derivativedivides	$\frac{\frac{b^2}{(ad-bc)\left(a^2+b^2\right)(a+b\tan(fx+e))}-\frac{b^2\left(3a^2d-2abc+b^2d\right)\ln(a+b\tan(fx+e))}{(ad-bc)^2\left(a^2+b^2\right)^2}+\frac{\frac{\left(-a^2d-2abc+b^2d\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(a^2c-2abd\right.}{\left(a^2+b^2\right)^2\left(c^2+d^2\right)}}{f}$
default	$\frac{\frac{b^2}{(ad-bc)\left(a^2+b^2\right)(a+b\tan(fx+e))}-\frac{b^2\left(3a^2d-2abc+b^2d\right)\ln(a+b\tan(fx+e))}{(ad-bc)^2\left(a^2+b^2\right)^2}+\frac{\frac{\left(-a^2d-2abc+b^2d\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(a^2c-2abd\right.}{\left(a^2+b^2\right)^2\left(c^2+d^2\right)}}{f}$
norman	$\frac{\frac{a\left(a^2c-2abd-b^2c\right)x}{\left(a^4+2a^2b^2+b^4\right)\left(c^2+d^2\right)}+\frac{b^2}{(ad-bc)f\left(a^2+b^2\right)}+\frac{b\left(a^2c-2abd-b^2c\right)x\tan(fx+e)}{\left(a^4+2a^2b^2+b^4\right)\left(c^2+d^2\right)}}{a+b\tan(fx+e)}+\frac{d^3\ln(c+d\tan(fx+e))}{f(a^2c^2d^2+a^2d^4-2abc^3d-2abcd^3+b^2c^4+}$
risch	$-\frac{x}{ia^2d+2iabc-ib^2d-a^2c+2abd+b^2c}-\frac{2id^3x}{a^2c^2d^2+a^2d^4-2abc^3d-2abcd^3+b^2c^4+b^2c^2d^2}-\frac{2id^3}{f(a^2c^2d^2+a^2d^4-2abc^3d-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^2/(a*d-b*c)/(a^2+b^2)/(a+b*tan(f*x+e))-b^2*(3*a^2*d-2*a*b*c+b^2*d)/(
a*d-b*c)^2/(a^2+b^2)^2*ln(a+b*tan(f*x+e))+1/(a^2+b^2)^2/(c^2+d^2)*(1/2*(-a^
2*d-2*a*b*c+b^2*d)*ln(1+tan(f*x+e)^2)+(a^2*c-2*a*b*d-b^2*c)*arctan(tan(f*x+
e)))+d^3/(a*d-b*c)^2/(c^2+d^2)*ln(c+d*tan(f*x+e)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 389 vs. 2(188) = 376.
time = 0.58, size = 389, normalized size = 2.13

$$\frac{\frac{2d^3\log(d\tan(fx+e)+c)}{b^2c^4-2abc^3d-2abcd^3+a^2d^4+(a^2+b^2)c^2d^2}-\frac{2(2abd-(a^2-b^2)c)(fx+e)}{(a^4+2a^2b^2+b^4)c^2+(a^4+2a^2b^2+b^4)d^2}-\frac{2b^2}{(a^3b+ab^3)c-(a^4+a^2b^2)d+((a^2b^2+b^4)c-(a^3b+ab^3)d)\tan(fx+e)}+\frac{2(2ab^3c-(3a^2b^2+b^4)d)\log(b\tan(fx+e)+a)}{(a^4b^2+2a^2b^4+b^6)c^2-2(a^5b+2a^3b^3+ab^5)cd+(a^6+2a^4b^2+a^2b^4)d^2}-\frac{(2abc+(a^2-b^2)d)\log\left(\tan(fx+e)^2+1\right)}{(a^4+2a^2b^2+b^4)c^2+(a^4+2a^2b^2+b^4)d^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*d^3*log(d*tan(f*x + e) + c)/(b^2*c^4 - 2*a*b*c^3*d - 2*a*b*c*d^3 + a
^2*d^4 + (a^2 + b^2)*c^2*d^2) - 2*(2*a*b*d - (a^2 - b^2)*c)*(f*x + e)/((a^4
 + 2*a^2*b^2 + b^4)*c^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2) - 2*b^2/((a^3*b + a*
b^3)*c - (a^4 + a^2*b^2)*d + ((a^2*b^2 + b^4)*c - (a^3*b + a*b^3)*d)*tan(f*
x + e)) + 2*(2*a*b^3*c - (3*a^2*b^2 + b^4)*d)*log(b*tan(f*x + e) + a)/((a^4
*b^2 + 2*a^2*b^4 + b^6)*c^2 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*c*d + (a^6 + 2*
a^4*b^2 + a^2*b^4)*d^2) - (2*a*b*c + (a^2 - b^2)*d)*log(tan(f*x + e)^2 + 1)
/((a^4 + 2*a^2*b^2 + b^4)*c^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 766 vs. 2(188) = 376.
time = 1.51, size = 766, normalized size = 4.19

$$-\frac{2b^5c^3-2ab^4c^2d+2b^5cd^2-2ab^4d^3+2(2a^4bc^2d+2a^4bd^3-(a^3b^2-ab^4)c^3-(a^5+3a^3b^2)cd^2)fx-(2a^2b^3c^3+2a^2b^3cd^2-(3a^3b^2+ab^4)c^2d-(3a^3b^2+ab^4)d^3+(2ab^4c^3+2ab^4cd^2-(3a^2b^3+b^5)c^2d-(3a^2b^3+b^5)d^3)\tan(fx+e))\log\left(\frac{b^2\tan(fx+e)^2+2ab\tan(fx+e)+a^2}{\tan(fx+e)^2+1}\right)-((a^4b+2a^2b^3+b^5)d^3\tan(fx+e)+(a^5+2a^3b^2+ab^4)d^3)\log\left(\frac{d^2\tan(fx+e)^2+2cd\tan(fx+e)+c^2}{\tan(fx+e)^2+1}\right)-2(ab^4c^3-a^2b^3c^2d+ab^4cd^2-a^2b^3d^3-(2a^3b^2c^2d+2a^3b^2d^3-(a^2b^3-b^5)c^3-(a^4b+3a^2b^3)cd^2)fx)\tan(fx+e)}{2(((a^4b^3+2a^2b^5+b^7)c^4-2(a^5b^2+2a^3b^4+ab^6)c^3d+(a^6b+3a^4b^3+3a^2b^5+b^7)c^2d^2-2(a^5b^2+2a^3b^4+ab^6)cd^3+(a^6b+2a^4b^3+a^2b^5)d^4)f\tan(fx+e)+((a^5b^2+2a^3b^4+ab^6)c^4-2(a^6b+2a^4b^3+a^2b^5)c^3d+(a^7+3a^5b^2+3a^3b^4+ab^6)c^2d^2-2(a^6b+2a^4b^3+a^2b^5)cd^3+(a^7+2a^5b^2+a^3b^4)d^4)f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e)),x, algorithm="fricas")

[Out] -1/2*(2*b^5*c^3 - 2*a*b^4*c^2*d + 2*b^5*c*d^2 - 2*a*b^4*d^3 + 2*(2*a^4*b*c^2*d + 2*a^4*b*d^3 - (a^3*b^2 - a*b^4)*c^3 - (a^5 + 3*a^3*b^2)*c*d^2)*f*x - (2*a^2*b^3*c^3 + 2*a^2*b^3*c*d^2 - (3*a^3*b^2 + a*b^4)*c^2*d - (3*a^3*b^2 + a*b^4)*d^3 + (2*a*b^4*c^3 + 2*a*b^4*c*d^2 - (3*a^2*b^3 + b^5)*c^2*d - (3*a^2*b^3 + b^5)*d^3)*tan(f*x + e))*log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(tan(f*x + e)^2 + 1)) - ((a^4*b + 2*a^2*b^3 + b^5)*d^3*tan(f*x + e) + (a^5 + 2*a^3*b^2 + a*b^4)*d^3)*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) - 2*(a*b^4*c^3 - a^2*b^3*c^2*d + a*b^4*c*d^2 - a^2*b^3*d^3 - (2*a^3*b^2*c^2*d + 2*a^3*b^2*d^3 - (a^2*b^3 - b^5)*c^3 - (a^4*b + 3*a^2*b^3)*c*d^2)*f*x)*tan(f*x + e))/(((a^4*b^3 + 2*a^2*b^5 + b^7)*c^4 - 2*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*c^3*d + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*c^2*d^2 - 2*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*c*d^3 + (a^6*b + 2*a^4*b^3 + a^2*b^5)*d^4)*f*tan(f*x + e) + ((a^5*b^2 + 2*a^3*b^4 + a*b^6)*c^4 - 2*(a^6*b + 2*a^4*b^3 + a^2*b^5)*c^3*d + (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*c^2*d^2 - 2*(a^6*b + 2*a^4*b^3 + a^2*b^5)*c*d^3 + (a^7 + 2*a^5*b^2 + a^3*b^4)*d^4)*f)

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tan(f*x+e))**2/(c+d*tan(f*x+e)),x)

[Out] Exception raised: NotImplementedError >> no valid subset found

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 542 vs. 2(188) = 376.
time = 0.57, size = 542, normalized size = 2.96

$$\frac{\frac{2d^4\log(|d\tan(fx+e)+c|)}{b^2c^4d-2abc^3d^2+a^2c^2d^3+b^2c^2d^3-2abcd^4+a^2d^5}+\frac{2(a^2c-b^2c-2abd)(fx+e)}{a^4c^2+2a^2b^2c^2+b^4c^2+a^4d^2+2a^2b^2d^2+b^4d^2}-\frac{(2abc+a^2d-b^2d)\log\left(\tan(fx+e)^2+1\right)}{a^4c^2+2a^2b^2c^2+b^4c^2+a^4d^2+2a^2b^2d^2+b^4d^2}+\frac{2(2ab^4c-3a^2b^3d-b^5d)\log(|b\tan(fx+e)+a|)}{a^4b^3c^2+2a^2b^5c^2+b^7c^2-2a^5b^2cd-4a^3b^4cd-2ab^6cd+a^6bd^2+2a^4b^3d^2+a^2b^5d^2}-\frac{2(2ab^4c\tan(fx+e)-3a^2b^3d\tan(fx+e)-b^5d\tan(fx+e)+3a^2b^3c+b^5c-4a^3b^2d-2ab^4d)}{(a^4b^2c^2+2a^2b^4c^2+b^6c^2-2a^5bcd-4a^3b^3cd-2ab^5cd+a^6d^2+2a^4b^2d^2+a^2b^4d^2)(b\tan(fx+e)+a)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e)),x, algorithm="giac")

[Out] 1/2*(2*d^4*log(abs(d*tan(f*x + e) + c))/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3 + b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5) + 2*(a^2*c - b^2*c - 2*a*b*d)*(f*x + e)/(a^4*c^2 + 2*a^2*b^2*c^2 + b^4*c^2 + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2) - (2*a*b*c + a^2*d - b^2*d)*log(tan(f*x + e)^2 + 1)/(a^4*c^2 + 2*a^2*b^2*c^2 + b^4*c^2 + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2) + 2*(2*a*b^4*c - 3*a^2*b^3*d - b^5*d)*log(abs(b*tan(f*x + e) + a))/(a^4*b^3*c^2 + 2*a^2*b^5*c^2 + b^7*c^2 - 2*a^5*b^2*c*d - 4*a^3*b^4*c*d - 2*a*b^6*c*d + a^6*b*d^2 + 2*a^4*b^3*d^2 + a^2*b^5*d^2) - 2*(2*a*b^4*c*tan(f*x + e) - 3*a^2*b^3*d*tan(f*x

```
 + e) - b^5*d*tan(f*x + e) + 3*a^2*b^3*c + b^5*c - 4*a^3*b^2*d - 2*a*b^4*d)
/((a^4*b^2*c^2 + 2*a^2*b^4*c^2 + b^6*c^2 - 2*a^5*b*c*d - 4*a^3*b^3*c*d - 2*
a*b^5*c*d + a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(b*tan(f*x + e) + a)))/f
```

Mupad [B]
time = 6.97, size = 309, normalized size = 1.69

$$\frac{b^2}{f\,\left(a\,d-b\,c\right)\,\left(a^2+b^2\right)\,\left(a+b\,\tan\left(e+f\,x\right)\right)}-\frac{\ln\left(a+b\,\tan(e+f\,x)\right)\,\left(d\,\left(3\,a^2\,b^2+b^4\right)-2\,a\,b^3\,c\right)}{f\,\left(a^6\,d^2-2\,a^5\,b\,c\,d+a^4\,b^2\,c^2+2\,a^4\,b^2\,d^2-4\,a^3\,b^3\,c\,d+2\,a^2\,b^4\,c^2+a^2\,b^4\,d^2-2\,a\,b^5\,c\,d+b^6\,c^2\right)}+\frac{d^3\,\ln\left(c+d\tan(e+f\,x)\right)}{f\,\left(a\,d-b\,c\right)^2\,\left(c^2+d^2\right)}-\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,1\mathrm{i}}{2\,f\,\left(a^2\,c-b^2\,c-2\,a\,b\,d+a^2\,d\,1\mathrm{i}-b^2\,d\,1\mathrm{i}+a\,b\,c\,2\mathrm{i}\right)}-\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{2\,f\,\left(b^2\,c-a^2\,c+2\,a\,b\,d+a^2\,d\,1\mathrm{i}-b^2\,d\,1\mathrm{i}+a\,b\,c\,2\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*tan(e + f*x))^2*(c + d*tan(e + f*x))),x)
```

```
[Out] b^2/(f*(a*d - b*c)*(a^2 + b^2)*(a + b*tan(e + f*x))) - (log(tan(e + f*x) +
1i)*1i)/(2*f*(a^2*d*1i - a^2*c + b^2*c - b^2*d*1i + a*b*c*2i + 2*a*b*d)) -
(log(a + b*tan(e + f*x))*(d*(b^4 + 3*a^2*b^2) - 2*a*b^3*c))/(f*(a^6*d^2 + b
^6*c^2 + 2*a^2*b^4*c^2 + a^4*b^2*c^2 + a^2*b^4*d^2 + 2*a^4*b^2*d^2 - 2*a*b^
5*c*d - 2*a^5*b*c*d - 4*a^3*b^3*c*d)) - (log(tan(e + f*x) - 1i)*1i)/(2*f*(a
^2*c + a^2*d*1i - b^2*c - b^2*d*1i + a*b*c*2i - 2*a*b*d)) + (d^3*log(c + d*
tan(e + f*x)))/(f*(a*d - b*c)^2*(c^2 + d^2))
```

3.1215 $\int \frac{1}{(a+b\tan(e+fx))^3(c+d\tan(e+fx))}\,dx$

Optimal. Leaf size=279

$$\frac{(a^3c-3ab^2c-3a^2bd+b^3d)\,x}{(a^2+b^2)^3\,(c^2+d^2)}-\frac{b^2(8a^3bcd-6a^4d^2+b^4(c^2-d^2)-3a^2b^2(c^2+d^2))\log(a\cos(e+fx)+b\sin(e$$

```
[Out] (a^3*c-3*a^2*b*d-3*a*b^2*c+b^3*d)*x/(a^2+b^2)^3/(c^2+d^2)-b^2*(8*a^3*b*c*d-
6*a^4*d^2+b^4*(c^2-d^2)-3*a^2*b^2*(c^2+d^2))*ln(a*cos(f*x+e)+b*sin(f*x+e))/
(a^2+b^2)^3/(-a*d+b*c)^3/f-d^4*ln(c*cos(f*x+e)+d*sin(f*x+e))/(-a*d+b*c)^3/(
c^2+d^2)/f-1/2*b^2/(a^2+b^2)/(-a*d+b*c)/f/(a+b*tan(f*x+e))^2-b^2*(-3*a^2*d+
2*a*b*c-b^2*d)/(a^2+b^2)^2/(-a*d+b*c)^2/f/(a+b*tan(f*x+e))
```

Rubi [A]
time = 0.63, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3650, 3730, 3732, 3611}

$$-\frac{b^2(-3a^2d+2abc-b^2d)}{f\,(a^2+b^2)^2\,(bc-ad)^2(a+b\tan(e+fx))}-\frac{b^2}{2f\,(a^2+b^2)\,(bc-ad)(a+b\tan(e+fx))^2}+\frac{x(a^3c-3a^2bd-3ab^2c+b^3d)}{(a^2+b^2)^3\,(c^2+d^2)}-\frac{b^2(-6a^4d^2+8a^3bcd-3a^2b^2(c^2+d^2)+b^4(c^2-d^2))\log(a\cos(e+fx)+b\sin(e+fx))}{f\,(a^2+b^2)^3\,(bc-ad)^3}-\frac{d^4\log(c\cos(e+fx)+d\sin(e+fx))}{f\,(c^2+d^2)\,(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])),x]
```

```
[Out] ((a^3*c - 3*a*b^2*c - 3*a^2*b*d + b^3*d)*x)/((a^2 + b^2)^3*(c^2 + d^2)) - (
b^2*(8*a^3*b*c*d - 6*a^4*d^2 + b^4*(c^2 - d^2) - 3*a^2*b^2*(c^2 + d^2))*Log
[a*Cos[e + f*x] + b*Sin[e + f*x]])/((a^2 + b^2)^3*(b*c - a*d)^3*f) - (d^4*L
og[c*Cos[e + f*x] + d*Sin[e + f*x]])/((b*c - a*d)^3*(c^2 + d^2)*f) - b^2/(2
*(a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])^2) - (b^2*(2*a*b*c - 3*a^2*
d - b^2*d))/((a^2 + b^2)^2*(b*c - a*d)^2*f*(a + b*Tan[e + f*x]))
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
 + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
```

```
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+b\tan(e+fx))^3(c+d\tan(e+fx))}\,dx &= -\frac{b^2}{2\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^2} - \frac{\int \frac{-2(abc-a^2\cdots}{\cdots}}{\cdots} \\
&= -\frac{b^2}{2\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^2} - \frac{b\cdots}{\left(a^2+b^2\right)^2(\cdots} \\
&= \frac{\left(a^3c-3ab^2c-3a^2bd+b^3d\right)x}{\left(a^2+b^2\right)^3\left(c^2+d^2\right)} - \frac{b^2}{2\left(a^2+b^2\right)(bc-ad)f(a\cdots} \\
&= \frac{\left(a^3c-3ab^2c-3a^2bd+b^3d\right)x}{\left(a^2+b^2\right)^3\left(c^2+d^2\right)} - \frac{b^2(8a^3bcd-6a^4d^2+b^4(c\cdots}{\cdots}
\end{aligned}
$$

Mathematica [**A**]

time = 6.90, size = 529, normalized size = 1.90

$$-\frac{b^2}{2\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^2}-\frac{-\frac{-\frac{b(bc-ad)^2\left(3a^2bc-b^3c+a^3d-3ab^2d+\frac{\sqrt{-b^2}\left(a^3c-3ab^2c-3a^2bd+b^3d\right)}{b}\right)\log\left(\sqrt{-b^2}-b\tan(e+fx)\right)}{\left(a^2+b^2\right)\left(c^2+d^2\right)}-\frac{2b^3\left(8a^3bcd-6a^4d^2+b^4\left(c^2-d^2\right)-3a^2b^2\left(c^2+d^2\right)\right)\log(a+b\tan(e+fx))}{\left(a^2+b^2\right)(bc-ad)}-\frac{b(bc-ad)^2\left(3a^2bc-b^3c+a^3d-3ab^2d+\frac{b\left(a^3c-3ab^2c-3a^2bd+b^3d\right)}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}+b\tan(e+fx)\right)}{\left(a^2+b^2\right)\left(c^2+d^2\right)}-\frac{2b\left(a^2+b^2\right)^2d^4\log(c+d\tan(e+fx))}{(bc-ad)\left(c^2+d^2\right)}}{b(a^2+b^2)(bc-ad)f}-\frac{-2b^2\left(abc-a^2d-b^2d\right)-a\left(-2ab^2d+2b^2(bc-ad)\right)}{(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))}}{2\left(a^2+b^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])),x]
```

```
[Out] -1/2*b^2/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])^2) - (-((-((b*(b*c
 - a*d)^2*(3*a^2*b*c - b^3*c + a^3*d - 3*a*b^2*d + (Sqrt[-b^2]*(a^3*c - 3*a
*b^2*c - 3*a^2*b*d + b^3*d))/b)*Log[Sqrt[-b^2] - b*Tan[e + f*x]])/((a^2 + b
^2)*(c^2 + d^2))) - (2*b^3*(8*a^3*b*c*d - 6*a^4*d^2 + b^4*(c^2 - d^2) - 3*a
^2*b^2*(c^2 + d^2))*Log[a + b*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*d)) - (b
*(b*c - a*d)^2*(3*a^2*b*c - b^3*c + a^3*d - 3*a*b^2*d + (b*(a^3*c - 3*a*b^2
*c - 3*a^2*b*d + b^3*d))/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[e + f*x]])/((a^
2 + b^2)*(c^2 + d^2)) - (2*b*(a^2 + b^2)^2*d^4*Log[c + d*Tan[e + f*x]])/((b
*c - a*d)*(c^2 + d^2)))/(b*(a^2 + b^2)*(b*c - a*d)*f)) - (-2*b^2*(a*b*c - a
^2*d - b^2*d) - a*(-2*a*b^2*d + 2*b^2*(b*c - a*d)))/((a^2 + b^2)*(b*c - a*d
)*f*(a + b*Tan[e + f*x])))/(2*(a^2 + b^2)*(b*c - a*d))
```

Maple [A]

time = 0.81, size = 309, normalized size = 1.11

method	result
derivativedivides	$\frac{\frac{\left(-a^3d-3a^2bc+3a\,b^2d+b^3c\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(a^3c-3a^2bd-3a\,b^2c+b^3d\right)\arctan(\tan(fx+e))}{\left(a^2+b^2\right)^3\left(c^2+d^2\right)}+\frac{b^2}{2(ad-bc)\left(a^2+b^2\right)(a+b\tan(fx+e))}$
default	$\frac{\frac{\left(-a^3d-3a^2bc+3a\,b^2d+b^3c\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(a^3c-3a^2bd-3a\,b^2c+b^3d\right)\arctan(\tan(fx+e))}{\left(a^2+b^2\right)^3\left(c^2+d^2\right)}+\frac{b^2}{2(ad-bc)\left(a^2+b^2\right)(a+b\tan(fx+e))}$
norman	$\frac{\frac{a^2\left(a^3c-3a^2bd-3a\,b^2c+b^3d\right)x}{\left(c^2+d^2\right)\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}+\frac{b^2\left(a^3c-3a^2bd-3a\,b^2c+b^3d\right)x\left(\tan^2(fx+e)\right)}{\left(c^2+d^2\right)\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}+\frac{\left(3a^2b^4d-2a\,b^5c+b^6d\right)\tan(fx+e)}{fb\left(a^2d^2-2abcd+b^2c^2\right)\left(a^4+2a^2b^2+b^4\right)}+2f\,b^2(}{(a+b\tan(fx+e))^2}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e))^3/(c+d*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(a^2+b^2)^3/(c^2+d^2)*(1/2*(-a^3*d-3*a^2*b*c+3*a*b^2*d+b^3*c)*ln(1+t
an(f*x+e)^2)+(a^3*c-3*a^2*b*d-3*a*b^2*c+b^3*d)*arctan(tan(f*x+e)))+1/2*b^2/
(a*d-b*c)/(a^2+b^2)/(a+b*tan(f*x+e))^2+b^2*(3*a^2*d-2*a*b*c+b^2*d)/(a*d-b*c
)^2/(a^2+b^2)^2/(a+b*tan(f*x+e))-b^2*(6*a^4*d^2-8*a^3*b*c*d+3*a^2*b^2*c^2+3
*a^2*b^2*d^2-b^4*c^2+b^4*d^2)/(a*d-b*c)^3/(a^2+b^2)^3*ln(a+b*tan(f*x+e))+d^
4/(c^2+d^2)/(a*d-b*c)^3*ln(c+d*tan(f*x+e)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(283) = 566$.

time = 0.65, size = 808, normalized size = 2.90

$$-\frac{\frac{2\,d^4\log\left(d\tan\left(fx+e\right)+c\right)}{b^3c^5-3\,ab^2c^4d+3\,a^2bcd^4-a^3d^5+\left(3\,a^2b+b^3\right)c^3d^2-\left(a^3+3\,ab^2\right)c^2d^3}-\frac{2\,\left(\left(a^3-3\,ab^2\right)c-\left(3\,a^2b-b^3\right)d\right)\left(fx+e\right)}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)c^2+\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)d^2}+\frac{2\,\left(8\,a^3b^3cd-\left(3\,a^2b^4-b^6\right)c^2-\left(6\,a^4b^2+3\,a^2b^4+b^6\right)d^2\right)\log\left(b\tan\left(fx+e\right)+a\right)}{\left(a^6b^3+3\,a^4b^5+3\,a^2b^7+b^9\right)c^3-3\,\left(a^7b^2+3\,a^5b^4+3\,a^3b^6+ab^8\right)c^2d+3\,\left(a^8b+3\,a^6b^3+3\,a^4b^5+a^2b^7\right)cd^2-\left(a^9+3\,a^7b^2+3\,a^5b^4+a^3b^6\right)d^3}+\frac{\left(\left(3\,a^2b-b^3\right)c+\left(a^3-3\,ab^2\right)d\right)\log\left(\tan\left(fx+e\right)^2+1\right)}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)c^2+\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)d^2}+\frac{\left(5\,a^2b^3+b^5\right)c-\left(7\,a^3b^2+3\,ab^4\right)d+2\,\left(2\,ab^4c-\left(3\,a^2b^3+b^5\right)d\right)\tan\left(fx+e\right)}{\left(a^6b^2+2\,a^4b^4+a^2b^6\right)c^2-2\,\left(a^7b+2\,a^5b^3+a^3b^5\right)cd+\left(a^8+2\,a^6b^2+a^4b^4\right)d^2+\left(\left(a^4b^4+2\,a^2b^6+b^8\right)c^2-2\,\left(a^5b^3+2\,a^3b^5+ab^7\right)cd+\left(a^6b^2+2\,a^4b^4+a^2b^6\right)d^2\right)\tan\left(fx+e\right)^2+2\,\left(\left(a^5b^3+2\,a^3b^5+ab^7\right)c^2-2\,\left(a^6b^2+2\,a^4b^4+a^2b^6\right)cd+\left(a^7b+2\,a^5b^3+a^3b^5\right)d^2\right)\tan\left(fx+e\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^3/(c+d*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*d^4*log(d*tan(f*x + e) + c)/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c*d^
4 - a^3*d^5 + (3*a^2*b + b^3)*c^3*d^2 - (a^3 + 3*a*b^2)*c^2*d^3) - 2*((a^3
- 3*a*b^2)*c - (3*a^2*b - b^3)*d)*(f*x + e)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
 b^6)*c^2 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2) + 2*(8*a^3*b^3*c*d - (
3*a^2*b^4 - b^6)*c^2 - (6*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2)*log(b*tan(f*x + e
) + a)/((a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9)*c^3 - 3*(a^7*b^2 + 3*a^5*b^
4 + 3*a^3*b^6 + a*b^8)*c^2*d + 3*(a^8*b + 3*a^6*b^3 + 3*a^4*b^5 + a^2*b^7)*
c*d^2 - (a^9 + 3*a^7*b^2 + 3*a^5*b^4 + a^3*b^6)*d^3) + ((3*a^2*b - b^3)*c +
 (a^3 - 3*a*b^2)*d)*log(tan(f*x + e)^2 + 1)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
 b^6)*c^2 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2) + ((5*a^2*b^3 + b^5)*c
 - (7*a^3*b^2 + 3*a*b^4)*d + 2*(2*a*b^4*c - (3*a^2*b^3 + b^5)*d)*tan(f*x +
e))/((a^6*b^2 + 2*a^4*b^4 + a^2*b^6)*c^2 - 2*(a^7*b + 2*a^5*b^3 + a^3*b^5)*
c*d + (a^8 + 2*a^6*b^2 + a^4*b^4)*d^2 + ((a^4*b^4 + 2*a^2*b^6 + b^8)*c^2 -
2*(a^5*b^3 + 2*a^3*b^5 + a*b^7)*c*d + (a^6*b^2 + 2*a^4*b^4 + a^2*b^6)*d^2)*
tan(f*x + e)^2 + 2*((a^5*b^3 + 2*a^3*b^5 + a*b^7)*c^2 - 2*(a^6*b^2 + 2*a^4*
b^4 + a^2*b^6)*c*d + (a^7*b + 2*a^5*b^3 + a^3*b^5)*d^2)*tan(f*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1941 vs. $2(283) = 566$.

time = 2.76, size = 1941, normalized size = 6.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^3/(c+d*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/2*((7*a^2*b^6 + b^8)*c^4 - 4*(4*a^3*b^5 + a*b^7)*c^3*d + (9*a^4*b^4 + 10
*a^2*b^6 + b^8)*c^2*d^2 - 4*(4*a^3*b^5 + a*b^7)*c*d^3 + 3*(3*a^4*b^4 + a^2*
b^6)*d^4 - 2*((a^5*b^3 - 3*a^3*b^5)*c^4 - (3*a^6*b^2 - 6*a^4*b^4 - a^2*b^6)
*c^3*d + 3*(a^7*b - a^3*b^5)*c^2*d^2 - (a^8 + 6*a^6*b^2 - 3*a^4*b^4)*c*d^3
+ (3*a^7*b - a^5*b^3)*d^4)*f*x + (12*a^3*b^5*c^3*d + 12*a^3*b^5*c*d^3 - (5*
a^2*b^6 - b^8)*c^4 - (7*a^4*b^4 + 6*a^2*b^6 - b^8)*c^2*d^2 - (7*a^4*b^4 + a
^2*b^6)*d^4 - 2*((a^3*b^5 - 3*a*b^7)*c^4 - (3*a^4*b^4 - 6*a^2*b^6 - b^8)*c^
3*d + 3*(a^5*b^3 - a*b^7)*c^2*d^2 - (a^6*b^2 + 6*a^4*b^4 - 3*a^2*b^6)*c*d^3
 + (3*a^5*b^3 - a^3*b^5)*d^4)*f*x)*tan(f*x + e)^2 + (8*a^5*b^3*c^3*d + 8*a^
5*b^3*c*d^3 - (3*a^4*b^4 - a^2*b^6)*c^4 - 6*(a^6*b^2 + a^4*b^4)*c^2*d^2 - (
```

```
6*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*d^4 + (8*a^3*b^5*c^3*d + 8*a^3*b^5*c*d^3 -
 (3*a^2*b^6 - b^8)*c^4 - 6*(a^4*b^4 + a^2*b^6)*c^2*d^2 - (6*a^4*b^4 + 3*a^2
*b^6 + b^8)*d^4)*tan(f*x + e)^2 + 2*(8*a^4*b^4*c^3*d + 8*a^4*b^4*c*d^3 - (3
*a^3*b^5 - a*b^7)*c^4 - 6*(a^5*b^3 + a^3*b^5)*c^2*d^2 - (6*a^5*b^3 + 3*a^3*
b^5 + a*b^7)*d^4)*tan(f*x + e))*log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e
) + a^2)/(tan(f*x + e)^2 + 1)) + ((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d
^4*tan(f*x + e)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d^4*tan(f*x +
 e) + (a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*d^4)*log((d^2*tan(f*x + e)^2
+ 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) - 2*(3*(a^3*b^5 - a*b^7)*
c^4 - (7*a^4*b^4 - 6*a^2*b^6 - b^8)*c^3*d + 4*(a^5*b^3 - a*b^7)*c^2*d^2 - (
7*a^4*b^4 - 6*a^2*b^6 - b^8)*c*d^3 + (4*a^5*b^3 - 3*a^3*b^5 - a*b^7)*d^4 +
2*((a^4*b^4 - 3*a^2*b^6)*c^4 - (3*a^5*b^3 - 6*a^3*b^5 - a*b^7)*c^3*d + 3*(a
^6*b^2 - a^2*b^6)*c^2*d^2 - (a^7*b + 6*a^5*b^3 - 3*a^3*b^5)*c*d^3 + (3*a^6*
b^2 - a^4*b^4)*d^4)*f*x)*tan(f*x + e))/(((a^6*b^5 + 3*a^4*b^7 + 3*a^2*b^9 +
 b^11)*c^5 - 3*(a^7*b^4 + 3*a^5*b^6 + 3*a^3*b^8 + a*b^10)*c^4*d + (3*a^8*b^
3 + 10*a^6*b^5 + 12*a^4*b^7 + 6*a^2*b^9 + b^11)*c^3*d^2 - (a^9*b^2 + 6*a^7*
b^4 + 12*a^5*b^6 + 10*a^3*b^8 + 3*a*b^10)*c^2*d^3 + 3*(a^8*b^3 + 3*a^6*b^5
+ 3*a^4*b^7 + a^2*b^9)*c*d^4 - (a^9*b^2 + 3*a^7*b^4 + 3*a^5*b^6 + a^3*b^8)*
d^5)*f*tan(f*x + e)^2 + 2*((a^7*b^4 + 3*a^5*b^6 + 3*a^3*b^8 + a*b^10)*c^5 -
 3*(a^8*b^3 + 3*a^6*b^5 + 3*a^4*b^7 + a^2*b^9)*c^4*d + (3*a^9*b^2 + 10*a^7*
b^4 + 12*a^5*b^6 + 6*a^3*b^8 + a*b^10)*c^3*d^2 - (a^10*b + 6*a^8*b^3 + 12*a
^6*b^5 + 10*a^4*b^7 + 3*a^2*b^9)*c^2*d^3 + 3*(a^9*b^2 + 3*a^7*b^4 + 3*a^5*b
^6 + a^3*b^8)*c*d^4 - (a^10*b + 3*a^8*b^3 + 3*a^6*b^5 + a^4*b^7)*d^5)*f*tan
(f*x + e) + ((a^8*b^3 + 3*a^6*b^5 + 3*a^4*b^7 + a^2*b^9)*c^5 - 3*(a^9*b^2 +
 3*a^7*b^4 + 3*a^5*b^6 + a^3*b^8)*c^4*d + (3*a^10*b + 10*a^8*b^3 + 12*a^6*b
^5 + 6*a^4*b^7 + a^2*b^9)*c^3*d^2 - (a^11 + 6*a^9*b^2 + 12*a^7*b^4 + 10*a^5
*b^6 + 3*a^3*b^8)*c^2*d^3 + 3*(a^10*b + 3*a^8*b^3 + 3*a^6*b^5 + a^4*b^7)*c*
d^4 - (a^11 + 3*a^9*b^2 + 3*a^7*b^4 + a^5*b^6)*d^5)*f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))**3/(c+d*tan(f*x+e)),x)
```

```
[Out] Exception raised: NotImplementedError >> no valid subset found
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1111 vs. 2(283) = 566.
time = 0.70, size = 1111, normalized size = 3.98

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tan(f*x+e))^3/(c+d*tan(f*x+e)),x, algorithm="giac")

[Out] -1/2*(2*d^5*log(abs(d*tan(f*x + e) + c))/(b^3*c^5*d - 3*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 + b^3*c^3*d^3 - a^3*c^2*d^4 - 3*a*b^2*c^2*d^4 + 3*a^2*b*c*d^5 - a^3*d^6) - 2*(a^3*c - 3*a*b^2*c - 3*a^2*b*d + b^3*d)*(f*x + e)/(a^6*c^2 + 3*a^4*b^2*c^2 + 3*a^2*b^4*c^2 + b^6*c^2 + a^6*d^2 + 3*a^4*b^2*d^2 + 3*a^2*b^4*d^2 + b^6*d^2) + (3*a^2*b*c - b^3*c + a^3*d - 3*a*b^2*d)*log(tan(f*x + e)^2 + 1)/(a^6*c^2 + 3*a^4*b^2*c^2 + 3*a^2*b^4*c^2 + b^6*c^2 + a^6*d^2 + 3*a^4*b^2*d^2 + 3*a^2*b^4*d^2 + b^6*d^2) - 2*(3*a^2*b^5*c^2 - b^7*c^2 - 8*a^3*b^4*c*d + 6*a^4*b^3*d^2 + 3*a^2*b^5*d^2 + b^7*d^2)*log(abs(b*tan(f*x + e) + a))/(a^6*b^4*c^3 + 3*a^4*b^6*c^3 + 3*a^2*b^8*c^3 + b^10*c^3 - 3*a^7*b^3*c^2*d - 9*a^5*b^5*c^2*d - 9*a^3*b^7*c^2*d - 3*a*b^9*c^2*d + 3*a^8*b^2*c*d^2 + 9*a^6*b^4*c*d^2 + 9*a^4*b^6*c*d^2 + 3*a^2*b^8*c*d^2 - a^9*b*d^3 - 3*a^7*b^3*d^3 - 3*a^5*b^5*d^3 - a^3*b^7*d^3) + (9*a^2*b^6*c^2*tan(f*x + e)^2 - 3*b^8*c^2*tan(f*x + e)^2 - 24*a^3*b^5*c*d*tan(f*x + e)^2 + 18*a^4*b^4*d^2*tan(f*x + e)^2 + 9*a^2*b^6*d^2*tan(f*x + e)^2 + 3*b^8*d^2*tan(f*x + e)^2 + 22*a^3*b^5*c^2*tan(f*x + e) - 2*a*b^7*c^2*tan(f*x + e) - 58*a^4*b^4*c*d*tan(f*x + e) - 12*a^2*b^6*c*d*tan(f*x + e) - 2*b^8*c*d*tan(f*x + e) + 42*a^5*b^3*d^2*tan(f*x + e) + 26*a^3*b^5*d^2*tan(f*x + e) + 8*a*b^7*d^2*tan(f*x + e) + 14*a^4*b^4*c^2 + 3*a^2*b^6*c^2 + b^8*c^2 - 36*a^5*b^3*c*d - 16*a^3*b^5*c*d - 4*a*b^7*c*d + 25*a^6*b^2*d^2 + 19*a^4*b^4*d^2 + 6*a^2*b^6*d^2)/((a^6*b^3*c^3 + 3*a^4*b^5*c^3 + 3*a^2*b^7*c^3 + b^9*c^3 - 3*a^7*b^2*c^2*d - 9*a^5*b^4*c^2*d - 9*a^3*b^6*c^2*d - 3*a*b^8*c^2*d + 3*a^8*b*c*d^2 + 9*a^6*b^3*c*d^2 + 9*a^4*b^5*c*d^2 + 3*a^2*b^7*c*d^2 - a^9*d^3 - 3*a^7*b^2*d^3 - 3*a^5*b^4*d^3 - a^3*b^6*d^3)*(b*tan(f*x + e) + a)^2))/f

Mupad [B]
time = 12.02, size = 609, normalized size = 2.18

$$\frac{d^4\ln\left(c+d\tan(e+f\,x)\right)}{f\,{\left(a\,d-b\,c\right)}^3\,\left(c^2+d^2\right)}+\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)}{2\,f\,\left(a^3\,c\,1\mathrm{i}-a^3\,d+b^3\,c+b^3\,d\,1\mathrm{i}-a\,b^2\,c\,3\mathrm{i}-3\,a^2\,b\,c+3\,a\,b^2\,d-a^2\,b\,d\,3\mathrm{i}\right)}-\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)}{2\,f\,\left(a^3\,c\,1\mathrm{i}+a^3\,d-b^3\,c+b^3\,d\,1\mathrm{i}-a\,b^2\,c\,3\mathrm{i}+3\,a^2\,b\,c-3\,a\,b^2\,d-a^2\,b\,d\,3\mathrm{i}\right)}-\frac{\ln\left(a+b\tan(e+f\,x)\right)\,\left(b^4\,\left(3\,a^2\,c^2+3\,a^2\,d^2\right)-b^6\,\left(c^2-d^2\right)+6\,a^4\,b^2\,d^2-8\,a^3\,b^3\,c\,d\right)}{f\,\left(a^9\,d^3-3\,a^8\,b\,c\,d^2+3\,a^7\,b^2\,c^2\,d+3\,a^7\,b^2\,d^3-a^6\,b^3\,c^3-9\,a^6\,b^3\,c\,d^2+9\,a^5\,b^4\,c^2\,d+3\,a^5\,b^4\,d^3-3\,a^4\,b^5\,c^3-9\,a^4\,b^5\,c\,d^2+9\,a^3\,b^6\,c^2\,d+a^3\,b^6\,d^3-3\,a^2\,b^7\,c^3-3\,a^2\,b^7\,c\,d^2+3\,a\,b^8\,c^2\,d-b^9\,c^3\right)}-\frac{\frac{-7\,d\,a^3\,b^2+5\,c\,a^2\,b^3-3\,d\,a\,b^4+c\,b^5}{2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}-\frac{\tan(e+f\,x)\,\left(3\,d\,a^2\,b^3-2\,c\,a\,b^4+d\,b^5\right)}{\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{f\,\left(a^2+2\,a\,b\tan\left(e+f\,x\right)+b^2\tan\left(e+f\,x\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b*tan(e + f*x))^3*(c + d*tan(e + f*x))),x)

[Out] log(tan(e + f*x) - 1i)/(2*f*(a^3*c*1i - a^3*d + b^3*c + b^3*d*1i - a*b^2*c*3i - 3*a^2*b*c + 3*a*b^2*d - a^2*b*d*3i)) - ((b^5*c + 5*a^2*b^3*c - 7*a^3*b^2*d - 3*a*b^4*d)/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^4 + b^4 + 2*a^2*b^2)) - (tan(e + f*x)*(b^5*d + 3*a^2*b^3*d - 2*a*b^4*c))/((a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^4 + b^4 + 2*a^2*b^2)))/(f*(a^2 + b^2*tan(e + f*x)^2 + 2*a*b*tan(e + f*x))) - log(tan(e + f*x) + 1i)/(2*f*(a^3*c*1i + a^3*d - b^3*c + b^3*d*1i - a*b^2*c*3i + 3*a^2*b*c - 3*a*b^2*d - a^2*b*d*3i)) - (log(a + b*tan(e + f*x))*(b^4*(3*a^2*c^2 + 3*a^2*d^2) - b^6*(c^2 - d^2) + 6*a^4*b^2*d^2 - 8*a^3*b^3*c*d))/(f*(a^9*d^3 - b^9*c^3 - 3*a^2*b^7*c^3 - 3*a^4*b^5*c^3 - a^6*b^3*c^3 + a^3*b^6*d^3 + 3*a^5*b^4*d^3 + 3*a^7*b^2*d^3 - 3*a^2*b^7*c*d^2 + 9*a^3*b^6*c^2*d - 9*a^4*b^5*c*d^2 + 9*a^5*b^4*c^2*d - 9*a^6*b^3*c*d^2 + 3*a^7*b^2*c^2*d + 3*a*b^8*c^2*d - 3*a^8*b*c*d^2)) + (d^4*log(c + d*tan(e + f*x)))/(f*(a*d - b*c)^3*(c^2 + d^2))

3.1216 $\int \frac{(a+b\tan(e+fx))^4}{(c+d\tan(e+fx))^2}\,dx$

Optimal. Leaf size=285

$$\frac{(8a^3bcd-8ab^3cd+a^4(c^2-d^2)-6a^2b^2(c^2-d^2)+b^4(c^2-d^2))\,x}{(c^2+d^2)^2}-\frac{2(a^2c-b^2c+2abd)\,(2abc-a^2d+b^2d)\log}{(c^2+d^2)^2\,f}$$

```
[Out] (8*a^3*b*c*d-8*a*b^3*c*d+a^4*(c^2-d^2)-6*a^2*b^2*(c^2-d^2)+b^4*(c^2-d^2))*x
/(c^2+d^2)^2-2*(a^2*c+2*a*b*d-b^2*c)*(-a^2*d+2*a*b*c+b^2*d)*ln(cos(f*x+e))/
(c^2+d^2)^2/f-2*(-a*d+b*c)^3*(a*c*d+b*(c^2+2*d^2))*ln(c+d*tan(f*x+e))/d^3/(
c^2+d^2)^2/f-b^2*(a*d*(-a*d+2*b*c)-b^2*(2*c^2+d^2))*tan(f*x+e)/d^2/(c^2+d^2
)/f-(-a*d+b*c)^2*(a+b*tan(f*x+e))^2/d/(c^2+d^2)/f/(c+d*tan(f*x+e))
```

Rubi [A]

time = 0.54, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3646, 3718, 3707, 3698, 31, 3556}

$$-\frac{2(a^2c+2abd-b^2c)\,(a^2(-d)+2abc+b^2d)\log(\cos(e+fx))}{f\,(c^2+d^2)^2}+\frac{x(a^4(c^2-d^2)+8a^3bcd-6a^2b^2(c^2-d^2)-8ab^3cd+b^4(c^2-d^2))}{(c^2+d^2)^2}-\frac{b^2(ad(2bc-ad)-b^2(2c^2+d^2))\tan(e+fx)}{d^2f\,(c^2+d^2)}-\frac{(bc-ad)^2(a+b\tan(e+fx))^2}{df\,(c^2+d^2)\,(c+d\tan(e+fx))}-\frac{2(acd+b(c^2+2d^2))\,(bc-ad)^3\log(c+d\tan(e+fx))}{d^3f\,(c^2+d^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^4/(c + d*Tan[e + f*x])^2,x]
```

```
[Out] ((8*a^3*b*c*d - 8*a*b^3*c*d + a^4*(c^2 - d^2) - 6*a^2*b^2*(c^2 - d^2) + b^4
*(c^2 - d^2))*x)/(c^2 + d^2)^2 - (2*(a^2*c - b^2*c + 2*a*b*d)*(2*a*b*c - a^
2*d + b^2*d)*Log[Cos[e + f*x]])/((c^2 + d^2)^2*f) - (2*(b*c - a*d)^3*(a*c*d
 + b*(c^2 + 2*d^2))*Log[c + d*Tan[e + f*x]])/(d^3*(c^2 + d^2)^2*f) - (b^2*(
a*d*(2*b*c - a*d) - b^2*(2*c^2 + d^2))*Tan[e + f*x])/(d^2*(c^2 + d^2)*f) -
((b*c - a*d)^2*(a + b*Tan[e + f*x])^2)/(d*(c^2 + d^2)*f*(c + d*Tan[e + f*x]
))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
```

```
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^4}{(c+d\tan(e+fx))^2}\,dx = -\frac{(bc-ad)^2(a+b\tan(e+fx))^2}{d\,(c^2+d^2)\,f(c+d\tan(e+fx))} + \frac{\int (a+b\tan(e+fx))(2b^3c^2+a^3cd-5ab^2cd+4a^2bd^2+d(3a^2\cdots}{\cdots}$$

$$= -\frac{b^2(ad(2bc-ad)-b^2(2c^2+d^2))\tan(e+fx)}{d^2\,(c^2+d^2)\,f} - \frac{(bc-ad)^2(a+b\tan(e+fx)\cdots}{d\,(c^2+d^2)\,f(c+d\tan(e+f\cdots}$$

$$= \frac{(8a^3bcd-8ab^3cd+a^4(c^2-d^2)-6a^2b^2(c^2-d^2)+b^4(c^2-d^2))\,x}{(c^2+d^2)^2} - \frac{b^2(ad(2bc\cdots}{\cdots}$$

$$= \frac{(8a^3bcd-8ab^3cd+a^4(c^2-d^2)-6a^2b^2(c^2-d^2)+b^4(c^2-d^2))\,x}{(c^2+d^2)^2} - \frac{2(a^2c-b^2\cdots}{\cdots}$$

$$= \frac{(8a^3bcd-8ab^3cd+a^4(c^2-d^2)-6a^2b^2(c^2-d^2)+b^4(c^2-d^2))\,x}{(c^2+d^2)^2} - \frac{2(a^2c-b^2\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.10, size = 1789, normalized size = 6.28

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Tan[e + f*x])^4/(c + d*Tan[e + f*x])^2,x]
```

```
[Out] (2*((-I)*b^4*c^10*d^2 + (2*I)*a*b^3*c^9*d^3 - b^4*c^9*d^3 + 2*a*b^3*c^8*d^4
 - (3*I)*b^4*c^8*d^4 - (2*I)*a^3*b*c^7*d^5 + (8*I)*a*b^3*c^7*d^5 - 3*b^4*c^
7*d^5 + I*a^4*c^6*d^6 - 2*a^3*b*c^6*d^6 - (6*I)*a^2*b^2*c^6*d^6 + 8*a*b^3*c
^6*d^6 - (2*I)*b^4*c^6*d^6 + a^4*c^5*d^7 - 6*a^2*b^2*c^5*d^7 + (6*I)*a*b^3*
c^5*d^7 - 2*b^4*c^5*d^7 + I*a^4*c^4*d^8 - (6*I)*a^2*b^2*c^4*d^8 + 6*a*b^3*c
^4*d^8 + a^4*c^3*d^9 + (2*I)*a^3*b*c^3*d^9 - 6*a^2*b^2*c^3*d^9 + 2*a^3*b*c^
2*d^10)*(e + f*x)*Cos[e + f*x]^2*(c*Cos[e + f*x] + d*Sin[e + f*x])^2*(a + b
*Tan[e + f*x])^4)/(c^2*(c - I*d)^4*(c + I*d)^3*d^5*f*(a*Cos[e + f*x] + b*Si
n[e + f*x])^4*(c + d*Tan[e + f*x])^2) - ((2*I)*(-(b^4*c^5) + 2*a*b^3*c^4*d
 - 2*b^4*c^3*d^2 - 2*a^3*b*c^2*d^3 + 6*a*b^3*c^2*d^3 + a^4*c*d^4 - 6*a^2*b^2
*c*d^4 + 2*a^3*b*d^5)*ArcTan[Tan[e + f*x]]*Cos[e + f*x]^2*(c*Cos[e + f*x] +
 d*Sin[e + f*x])^2*(a + b*Tan[e + f*x])^4)/(d^3*(c^2 + d^2)^2*f*(a*Cos[e +
f*x] + b*Sin[e + f*x])^4*(c + d*Tan[e + f*x])^2) - (2*(-(b^4*c) + 2*a*b^3*d
)*Cos[e + f*x]^2*Log[Cos[e + f*x]]*(c*Cos[e + f*x] + d*Sin[e + f*x])^2*(a +
 b*Tan[e + f*x])^4)/(d^3*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^4*(c + d*Tan[e
 + f*x])^2) + ((-(b^4*c^5) + 2*a*b^3*c^4*d - 2*b^4*c^3*d^2 - 2*a^3*b*c^2*d^
3 + 6*a*b^3*c^2*d^3 + a^4*c*d^4 - 6*a^2*b^2*c*d^4 + 2*a^3*b*d^5)*Cos[e + f*
x]^2*Log[(c*Cos[e + f*x] + d*Sin[e + f*x])^2]*(c*Cos[e + f*x] + d*Sin[e + f
*x])^2*(a + b*Tan[e + f*x])^4)/(d^3*(c^2 + d^2)^2*f*(a*Cos[e + f*x] + b*Sin
[e + f*x])^4*(c + d*Tan[e + f*x])^2) + (Cos[e + f*x]*(c*Cos[e + f*x] + d*Si
```

```
n[e + f*x])*(b^4*c^5*d + 2*b^4*c^3*d^3 + b^4*c*d^5 + a^4*c^4*d^2*(e + f*x)
- 6*a^2*b^2*c^4*d^2*(e + f*x) + b^4*c^4*d^2*(e + f*x) + 8*a^3*b*c^3*d^3*(e
+ f*x) - 8*a*b^3*c^3*d^3*(e + f*x) - a^4*c^2*d^4*(e + f*x) + 6*a^2*b^2*c^2*
d^4*(e + f*x) - b^4*c^2*d^4*(e + f*x) - b^4*c^5*d*Cos[2*(e + f*x)] - 2*b^4*
c^3*d^3*Cos[2*(e + f*x)] - b^4*c*d^5*Cos[2*(e + f*x)] + a^4*c^4*d^2*(e + f*
x)*Cos[2*(e + f*x)] - 6*a^2*b^2*c^4*d^2*(e + f*x)*Cos[2*(e + f*x)] + b^4*c^
4*d^2*(e + f*x)*Cos[2*(e + f*x)] + 8*a^3*b*c^3*d^3*(e + f*x)*Cos[2*(e + f*x
)] - 8*a*b^3*c^3*d^3*(e + f*x)*Cos[2*(e + f*x)] - a^4*c^2*d^4*(e + f*x)*Cos
[2*(e + f*x)] + 6*a^2*b^2*c^2*d^4*(e + f*x)*Cos[2*(e + f*x)] - b^4*c^2*d^4*
(e + f*x)*Cos[2*(e + f*x)] + 2*b^4*c^6*Sin[2*(e + f*x)] - 4*a*b^3*c^5*d*Sin
[2*(e + f*x)] + 6*a^2*b^2*c^4*d^2*Sin[2*(e + f*x)] + 3*b^4*c^4*d^2*Sin[2*(e
 + f*x)] - 4*a^3*b*c^3*d^3*Sin[2*(e + f*x)] - 4*a*b^3*c^3*d^3*Sin[2*(e + f*
x)] + a^4*c^2*d^4*Sin[2*(e + f*x)] + 6*a^2*b^2*c^2*d^4*Sin[2*(e + f*x)] + b
^4*c^2*d^4*Sin[2*(e + f*x)] - 4*a^3*b*c*d^5*Sin[2*(e + f*x)] + a^4*d^6*Sin[
2*(e + f*x)] + a^4*c^3*d^3*(e + f*x)*Sin[2*(e + f*x)] - 6*a^2*b^2*c^3*d^3*(
e + f*x)*Sin[2*(e + f*x)] + b^4*c^3*d^3*(e + f*x)*Sin[2*(e + f*x)] + 8*a^3*
b*c^2*d^4*(e + f*x)*Sin[2*(e + f*x)] - 8*a*b^3*c^2*d^4*(e + f*x)*Sin[2*(e +
 f*x)] - a^4*c*d^5*(e + f*x)*Sin[2*(e + f*x)] + 6*a^2*b^2*c*d^5*(e + f*x)*S
in[2*(e + f*x)] - b^4*c*d^5*(e + f*x)*Sin[2*(e + f*x)])*(a + b*Tan[e + f*x]
)^4)/(2*c*(c - I*d)^2*(c + I*d)^2*d^2*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^4
*(c + d*Tan[e + f*x])^2)
```

Maple [A]
time = 0.36, size = 364, normalized size = 1.28

method	result
derivativedivides	$\frac{\frac{b^4\tan(fx+e)}{d^2}-\frac{a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4}{d^3\left(c^2+d^2\right)\left(c+d\tan(fx+e)\right)}+\frac{\left(2a^4c\,d^4-4a^3b\,c^2d^3+4a^3b\,d^5-12a^2b^2c\,d^4+4a\,b^3c^4d+12a\,b^3c^2d^3-2\right.}{d^3\left(c^2+d^2\right)^2}}{}$
default	$\frac{\frac{b^4\tan(fx+e)}{d^2}-\frac{a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4}{d^3\left(c^2+d^2\right)\left(c+d\tan(fx+e)\right)}+\frac{\left(2a^4c\,d^4-4a^3b\,c^2d^3+4a^3b\,d^5-12a^2b^2c\,d^4+4a\,b^3c^4d+12a\,b^3c^2d^3-2\right.}{d^3\left(c^2+d^2\right)^2}}{}$
norman	$\frac{\frac{b^4\left(\tan^2(fx+e)\right)}{df}+\frac{c\left(a^4c^2-a^4d^2+8a^3bcd-6a^2b^2c^2+6a^2b^2d^2-8a\,b^3cd+b^4c^2-b^4d^2\right)x}{c^4+2c^2d^2+d^4}+\frac{d\left(a^4c^2-a^4d^2+8a^3bcd-6a^2b^2c^2+6a^2b^2d^2\right.}{c^4+2c^2d^2}}{c+d\tan(fx+e)}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^4/(c+d*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^4/d^2*tan(f*x+e)-1/d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*
b^3*c^3*d+b^4*c^4)/(c^2+d^2)/(c+d*tan(f*x+e))+(2*a^4*c*d^4-4*a^3*b*c^2*d^3+
4*a^3*b*d^5-12*a^2*b^2*c*d^4+4*a*b^3*c^4*d+12*a*b^3*c^2*d^3-2*b^4*c^5-4*b^4
*c^3*d^2)/d^3/(c^2+d^2)^2*ln(c+d*tan(f*x+e))+1/(c^2+d^2)^2*(1/2*(-2*a^4*c*d
+4*a^3*b*c^2-4*a^3*b*d^2+12*a^2*b^2*c*d-4*a*b^3*c^2+4*a*b^3*d^2-2*b^4*c*d)*
```

```
ln(1+tan(f*x+e)^2)+(a^4*c^2-a^4*d^2+8*a^3*b*c*d-6*a^2*b^2*c^2+6*a^2*b^2*d^2
-8*a*b^3*c*d+b^4*c^2-b^4*d^2)*arctan(tan(f*x+e))))
```

Maxima [A]
time = 0.56, size = 379, normalized size = 1.33

$$\frac{\frac{b^4\tan(fx+e)}{d^2}+\frac{\left((a^4-6a^2b^2+b^4)c^2+8\,(a^3b-ab^3)cd-(a^4-6a^2b^2+b^4)d^2\right)(fx+e)}{c^4+2c^2d^2+d^4}-\frac{2\,\left(b^4c^5-2\,ab^3c^4d+2\,b^4c^3d^2-2\,a^3bd^5+2\,(a^3b-3\,ab^3)c^2d^3-(a^4-6\,a^2b^2)cd^4\right)\log(d\tan(fx+e)+c)}{c^4d^3+2\,c^2d^5+d^7}+\frac{\left(2\,(a^3b-ab^3)c^2-(a^4-6\,a^2b^2+b^4)cd-2\,(a^3b-ab^3)d^2\right)\log\left(\tan(fx+e)^2+1\right)}{c^4+2\,c^2d^2+d^4}-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{c^3d^3+cd^5+(c^2d^4+d^6)\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^4/(c+d*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] (b^4*tan(f*x + e)/d^2 + ((a^4 - 6*a^2*b^2 + b^4)*c^2 + 8*(a^3*b - a*b^3)*c*
d - (a^4 - 6*a^2*b^2 + b^4)*d^2)*(f*x + e)/(c^4 + 2*c^2*d^2 + d^4) - 2*(b^4
*c^5 - 2*a*b^3*c^4*d + 2*b^4*c^3*d^2 - 2*a^3*b*d^5 + 2*(a^3*b - 3*a*b^3)*c^
2*d^3 - (a^4 - 6*a^2*b^2)*c*d^4)*log(d*tan(f*x + e) + c)/(c^4*d^3 + 2*c^2*d
^5 + d^7) + (2*(a^3*b - a*b^3)*c^2 - (a^4 - 6*a^2*b^2 + b^4)*c*d - 2*(a^3*b
 - a*b^3)*d^2)*log(tan(f*x + e)^2 + 1)/(c^4 + 2*c^2*d^2 + d^4) - (b^4*c^4 -
 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^3 + c*
d^5 + (c^2*d^4 + d^6)*tan(f*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 713 vs. 2(289) = 578.
time = 1.63, size = 713, normalized size = 2.50

$$-\frac{b^4c^4d^2-4\,ab^3c^3d^3+6\,a^2b^2c^2d^4-4\,a^3bcd^5+a^4d^6-\left((a^4-6\,a^2b^2+b^4)c^3d^3+8\,(a^3b-ab^3)c^2d^4-(a^4-6\,a^2b^2+b^4)cd^5\right)fx-(b^4c^4d^2+2\,b^4c^2d^4+b^4d^6)\tan(fx+e)^2+\left(b^4c^6-2\,ab^3c^5d+2\,b^4c^4d^2-2\,a^3bcd^5+2\,(a^3b-3\,ab^3)c^3d^3-(a^4-6\,a^2b^2)c^2d^4+(b^4c^5d-2\,ab^3c^4d^2+2\,b^4c^3d^3-2\,a^3bd^6+2\,(a^3b-3\,ab^3)c^2d^4-(a^4-6\,a^2b^2)cd^5)\tan(fx+e)\right)\log\left(\frac{d^2\tan(fx+e)^2+2\,cd\tan(fx+e)+c^2}{\tan(fx+e)^2+1}\right)-\left(b^4c^6-2\,ab^3c^5d+2\,b^4c^4d^2-4\,ab^3c^3d^3+b^4c^2d^4-2\,ab^3cd^5+(b^4c^5d-2\,ab^3c^4d^2+2\,b^4c^3d^3-4\,ab^3c^2d^4+b^4cd^5-2\,ab^3d^6)\tan(fx+e)\right)\log\left(\frac{1}{\tan(fx+e)^2+1}\right)-\left(2\,b^4c^5d-4\,ab^3c^4d^2-4\,a^3bc^2d^4+2\,(3\,a^2b^2+b^4)c^3d^3+(a^4+b^4)cd^5+\left((a^4-6\,a^2b^2+b^4)c^2d^4+8\,(a^3b-ab^3)cd^5-(a^4-6\,a^2b^2+b^4)d^6\right)fx\right)\tan(fx+e)}{(c^4d^4+2\,c^2d^6+d^8)f\tan(fx+e)+(c^5d^3+2\,c^3d^5+cd^7)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^4/(c+d*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -(b^4*c^4*d^2 - 4*a*b^3*c^3*d^3 + 6*a^2*b^2*c^2*d^4 - 4*a^3*b*c*d^5 + a^4*d
^6 - ((a^4 - 6*a^2*b^2 + b^4)*c^3*d^3 + 8*(a^3*b - a*b^3)*c^2*d^4 - (a^4 -
6*a^2*b^2 + b^4)*c*d^5)*f*x - (b^4*c^4*d^2 + 2*b^4*c^2*d^4 + b^4*d^6)*tan(f
*x + e)^2 + (b^4*c^6 - 2*a*b^3*c^5*d + 2*b^4*c^4*d^2 - 2*a^3*b*c*d^5 + 2*(a
^3*b - 3*a*b^3)*c^3*d^3 - (a^4 - 6*a^2*b^2)*c^2*d^4 + (b^4*c^5*d - 2*a*b^3*
c^4*d^2 + 2*b^4*c^3*d^3 - 2*a^3*b*d^6 + 2*(a^3*b - 3*a*b^3)*c^2*d^4 - (a^4
- 6*a^2*b^2)*c*d^5)*tan(f*x + e))*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x +
 e) + c^2)/(tan(f*x + e)^2 + 1)) - (b^4*c^6 - 2*a*b^3*c^5*d + 2*b^4*c^4*d^2
 - 4*a*b^3*c^3*d^3 + b^4*c^2*d^4 - 2*a*b^3*c*d^5 + (b^4*c^5*d - 2*a*b^3*c^4
*d^2 + 2*b^4*c^3*d^3 - 4*a*b^3*c^2*d^4 + b^4*c*d^5 - 2*a*b^3*d^6)*tan(f*x +
 e))*log(1/(tan(f*x + e)^2 + 1)) - (2*b^4*c^5*d - 4*a*b^3*c^4*d^2 - 4*a^3*b
*c^2*d^4 + 2*(3*a^2*b^2 + b^4)*c^3*d^3 + (a^4 + b^4)*c*d^5 + ((a^4 - 6*a^2*
b^2 + b^4)*c^2*d^4 + 8*(a^3*b - a*b^3)*c*d^5 - (a^4 - 6*a^2*b^2 + b^4)*d^6)
*f*x)*tan(f*x + e))/((c^4*d^4 + 2*c^2*d^6 + d^8)*f*tan(f*x + e) + (c^5*d^3
+ 2*c^3*d^5 + c*d^7)*f)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.08, size = 8928, normalized size = 31.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**4/(c+d*tan(f*x+e))**2,x)
```

```
[Out] Piecewise((zoo*x*(a + b*tan(e))**4/tan(e)**2, Eq(c, 0) & Eq(d, 0) & Eq(f, 0
)), ((a**4*x + 2*a**3*b*log(tan(e + f*x)**2 + 1)/f - 6*a**2*b**2*x + 6*a**2
*b**2*tan(e + f*x)/f - 2*a*b**3*log(tan(e + f*x)**2 + 1)/f + 2*a*b**3*tan(e
 + f*x)**2/f + b**4*x + b**4*tan(e + f*x)**3/(3*f) - b**4*tan(e + f*x)/f)/c
**2, Eq(d, 0)), (-a**4*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*
d**2*f*tan(e + f*x) - 4*d**2*f) + 2*I*a**4*f*x*tan(e + f*x)/(4*d**2*f*tan(e
 + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + a**4*f*x/(4*d**2*f*tan(e
 + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - a**4*tan(e + f*x)/(4*d**
2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*I*a**4/(4*d**
2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 4*I*a**3*b*f*x*
tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**
2*f) + 8*a**3*b*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan
(e + f*x) - 4*d**2*f) - 4*I*a**3*b*f*x/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2
*f*tan(e + f*x) - 4*d**2*f) + 4*I*a**3*b*tan(e + f*x)/(4*d**2*f*tan(e + f*x
)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 6*a**2*b**2*f*x*tan(e + f*x)**
2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 12*I*a*
*2*b**2*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x
) - 4*d**2*f) - 6*a**2*b**2*f*x/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(
e + f*x) - 4*d**2*f) - 18*a**2*b**2*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2
 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 12*I*a**2*b**2/(4*d**2*f*tan(e + f*
x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 12*I*a*b**3*f*x*tan(e + f*x)*
*2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 24*a*b
**3*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) -
4*d**2*f) - 12*I*a*b**3*f*x/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e +
f*x) - 4*d**2*f) + 8*a*b**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(4*d**
2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 16*I*a*b**3*log
(tan(e + f*x)**2 + 1)*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*t
an(e + f*x) - 4*d**2*f) - 8*a*b**3*log(tan(e + f*x)**2 + 1)/(4*d**2*f*tan(e
 + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 20*I*a*b**3*tan(e + f*x)
/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 16*a*b**
3/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 9*b**4*
f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4
*d**2*f) + 18*I*b**4*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*
f*tan(e + f*x) - 4*d**2*f) + 9*b**4*f*x/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**
2*f*tan(e + f*x) - 4*d**2*f) + 4*I*b**4*log(tan(e + f*x)**2 + 1)*tan(e + f*
x)**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 8*b
**4*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d
```

```
**2*f*tan(e + f*x) - 4*d**2*f) - 4*I*b**4*log(tan(e + f*x)**2 + 1)/(4*d**2*
f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 4*b**4*tan(e + f*
x)**3/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 19*
b**4*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d
**2*f) - 14*I*b**4/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*
d**2*f), Eq(c, -I*d)), (-a**4*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2
 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 2*I*a**4*f*x*tan(e + f*x)/(4*d**2*
f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + a**4*f*x/(4*d**2*
f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - a**4*tan(e + f*x)
/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 2*I*a**4
/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 4*I*a**3
*b*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x)
- 4*d**2*f) + 8*a**3*b*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**
2*f*tan(e + f*x) - 4*d**2*f) + 4*I*a**3*b*f*x/(4*d**2*f*tan(e + f*x)**2 + 8
*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 4*I*a**3*b*tan(e + f*x)/(4*d**2*f*tan(
e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 6*a**2*b**2*f*x*tan(e +
 f*x)**2/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) +
12*I*a**2*b**2*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(
e + f*x) - 4*d**2*f) - 6*a**2*b**2*f*x/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2
*f*tan(e + f*x) - 4*d**2*f) - 18*a**2*b**2*tan(e + f*x)/(4*d**2*f*tan(e + f
*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 12*I*a**2*b**2/(4*d**2*f*tan
(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 12*I*a*b**3*f*x*tan(e
+ f*x)**2/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) +
 24*a*b**3*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e +
f*x) - 4*d**2*f) + 12*I*a*b**3*f*x/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*t
an(e + f*x) - 4*d**2*f) + 8*a*b**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2
/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 16*I*a*b
**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 + 8*I*d
**2*f*tan(e + f*x) - 4*d**2*f) - 8*a*b**3*log(tan(e + f*x)**2 + 1)/(4*d**2*
f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 589 vs. $2(289) = 578$.
time = 1.06, size = 589, normalized size = 2.07

$$\frac{\frac{b^4\tan(fx+e)}{d^2} + \frac{(a^4c^2-6a^2b^2c^2+b^4c^2+8a^3bcd-8ab^3cd-a^4d^2+6a^2b^2d^2-b^4d^2)(fx+e)}{c^4+2c^2d^2+d^4} + \frac{(2a^3bc^2-2ab^3c^2-a^4cd+6a^2b^2cd-b^4cd-2a^3bd^2+2ab^3d^2)\log\left(\tan(fx+e)^2+1\right)}{c^4+2c^2d^2+d^4} - \frac{2(b^4c^5-2ab^3c^4d+2b^4c^3d^2+2a^3bc^2d^3-6ab^3c^2d^3-a^4cd^4+6a^2b^2cd^4-2a^3bd^5)\log(|d\tan(fx+e)+c|)}{c^4d^3+2c^2d^5+d^7} + \frac{2b^4c^5d\tan(fx+e)-4ab^3c^4d^2\tan(fx+e)+4b^4c^3d^3\tan(fx+e)+4a^3bc^2d^4\tan(fx+e)-12ab^3c^2d^4\tan(fx+e)-2a^4cd^5\tan(fx+e)+12a^2b^2cd^5\tan(fx+e)-4a^3bd^6\tan(fx+e)+b^4c^6-6a^2b^2c^4d^2+3b^4c^4d^2+8a^3bc^3d^3-8ab^3c^3d^3-3a^4c^2d^4+6a^2b^2c^2d^4-a^4d^6}{(c^4d^3+2c^2d^5+d^7)(d\tan(fx+e)+c)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^4/(c+d*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] (b^4*tan(f*x + e)/d^2 + (a^4*c^2 - 6*a^2*b^2*c^2 + b^4*c^2 + 8*a^3*b*c*d -
8*a*b^3*c*d - a^4*d^2 + 6*a^2*b^2*d^2 - b^4*d^2)*(f*x + e)/(c^4 + 2*c^2*d^2
 + d^4) + (2*a^3*b*c^2 - 2*a*b^3*c^2 - a^4*c*d + 6*a^2*b^2*c*d - b^4*c*d -
2*a^3*b*d^2 + 2*a*b^3*d^2)*log(tan(f*x + e)^2 + 1)/(c^4 + 2*c^2*d^2 + d^4)
- 2*(b^4*c^5 - 2*a*b^3*c^4*d + 2*b^4*c^3*d^2 + 2*a^3*b*c^2*d^3 - 6*a*b^3*c^
```

```
2*d^3 - a^4*c*d^4 + 6*a^2*b^2*c*d^4 - 2*a^3*b*d^5)*log(abs(d*tan(f*x + e) +
 c))/(c^4*d^3 + 2*c^2*d^5 + d^7) + (2*b^4*c^5*d*tan(f*x + e) - 4*a*b^3*c^4*
d^2*tan(f*x + e) + 4*b^4*c^3*d^3*tan(f*x + e) + 4*a^3*b*c^2*d^4*tan(f*x + e
) - 12*a*b^3*c^2*d^4*tan(f*x + e) - 2*a^4*c*d^5*tan(f*x + e) + 12*a^2*b^2*c
*d^5*tan(f*x + e) - 4*a^3*b*d^6*tan(f*x + e) + b^4*c^6 - 6*a^2*b^2*c^4*d^2
+ 3*b^4*c^4*d^2 + 8*a^3*b*c^3*d^3 - 8*a*b^3*c^3*d^3 - 3*a^4*c^2*d^4 + 6*a^2
*b^2*c^2*d^4 - a^4*d^6)/((c^4*d^3 + 2*c^2*d^5 + d^7)*(d*tan(f*x + e) + c)))
/f
```

Mupad [B]
time = 9.38, size = 347, normalized size = 1.22

$$\frac{\ln\left(c+d\tan(e+f\,x)\right)\,\left(d^3\,\left(12\,a\,b^3\,c^2-4\,a^3\,b\,c^2\right)+d^4\,\left(2\,a^4\,c-12\,a^2\,b^2\,c\right)-2\,b^4\,c^5+4\,a^3\,b\,d^5-4\,b^4\,c^3\,d^2+4\,a\,b^3\,c^4\,d\right)}{f\,\left(c^4\,d^3+2\,c^2\,d^5+d^7\right)}+\frac{b^4\tan(e+f\,x)}{d^2\,f}-\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(a^4+a^3\,b\,4\mathrm{i}-6\,a^2\,b^2-a\,b^3\,4\mathrm{i}+b^4\right)}{2\,f\,\left(-c^2\,1\mathrm{i}+2\,c\,d+d^2\,1\mathrm{i}\right)}-\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(a^4\,1\mathrm{i}+4\,a^3\,b-a^2\,b^2\,6\mathrm{i}-4\,a\,b^3+b^4\,1\mathrm{i}\right)}{2\,f\,\left(-c^2+c\,d\,2\mathrm{i}+d^2\right)}-\frac{a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4}{d\,f\,\left(\tan\left(e+f\,x\right)\,d^3+c\,d^2\right)\,\left(c^2+d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^4/(c + d*tan(e + f*x))^2,x)
```

```
[Out] (log(c + d*tan(e + f*x))*(d^3*(12*a*b^3*c^2 - 4*a^3*b*c^2) + d^4*(2*a^4*c -
 12*a^2*b^2*c) - 2*b^4*c^5 + 4*a^3*b*d^5 - 4*b^4*c^3*d^2 + 4*a*b^3*c^4*d))/
(f*(d^7 + 2*c^2*d^5 + c^4*d^3)) + (b^4*tan(e + f*x))/(d^2*f) - (log(tan(e +
 f*x) - 1i)*(a^3*b*4i - a*b^3*4i + a^4 + b^4 - 6*a^2*b^2))/(2*f*(2*c*d - c^
2*1i + d^2*1i)) - (log(tan(e + f*x) + 1i)*(4*a^3*b - 4*a*b^3 + a^4*1i + b^4
*1i - a^2*b^2*6i))/(2*f*(c*d*2i - c^2 + d^2)) - (a^4*d^4 + b^4*c^4 + 6*a^2*
b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)/(d*f*(c*d^2 + d^3*tan(e + f*x)
)*(c^2 + d^2))
```

3.1217 $\int \frac{(a+b\tan(e+fx))^3}{(c+d\tan(e+fx))^2}\,dx$

Optimal. Leaf size=223

$$\frac{\left(6a^2bcd-2b^3cd+a^3(c^2-d^2)-3ab^2(c^2-d^2)\right)x}{\left(c^2+d^2\right)^2}+\frac{\left(2a^3cd-6ab^2cd-3a^2b(c^2-d^2)+b^3(c^2-d^2)\right)\log(\cos(e+$$

```
[Out] (6*a^2*b*c*d-2*b^3*c*d+a^3*(c^2-d^2)-3*a*b^2*(c^2-d^2))*x/(c^2+d^2)^2+(2*a^
3*c*d-6*a*b^2*c*d-3*a^2*b*(c^2-d^2)+b^3*(c^2-d^2))*ln(cos(f*x+e))/(c^2+d^2)
^2/f+(-a*d+b*c)^2*(2*a*c*d+b*(c^2+3*d^2))*ln(c+d*tan(f*x+e))/d^2/(c^2+d^2)^
2/f-(-a*d+b*c)^2*(a+b*tan(f*x+e))/d/(c^2+d^2)/f/(c+d*tan(f*x+e))
```

Rubi [A]
time = 0.25, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3646, 3707, 3698, 31, 3556}

$$\frac{\left(2a^3cd-3a^2b(c^2-d^2)-6ab^2cd+b^3(c^2-d^2)\right)\log(\cos(e+fx))}{f\left(c^2+d^2\right)^2}+\frac{x\left(a^3(c^2-d^2)+6a^2bcd-3ab^2(c^2-d^2)-2b^3cd\right)}{\left(c^2+d^2\right)^2}-\frac{(bc-ad)^2(a+b\tan(e+fx))}{df\left(c^2+d^2\right)(c+d\tan(e+fx))}+\frac{\left(2acd+b(c^2+3d^2)\right)(bc-ad)^2\log(c+d\tan(e+fx))}{d^2f\left(c^2+d^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^3/(c + d*Tan[e + f*x])^2,x]
```

```
[Out] ((6*a^2*b*c*d - 2*b^3*c*d + a^3*(c^2 - d^2) - 3*a*b^2*(c^2 - d^2))*x)/(c^2
+ d^2)^2 + ((2*a^3*c*d - 6*a*b^2*c*d - 3*a^2*b*(c^2 - d^2) + b^3*(c^2 - d^2
))*Log[Cos[e + f*x]])/((c^2 + d^2)^2*f) + ((b*c - a*d)^2*(2*a*c*d + b*(c^2
+ 3*d^2))*Log[c + d*Tan[e + f*x]])/(d^2*(c^2 + d^2)^2*f) - ((b*c - a*d)^2*(
a + b*Tan[e + f*x]))/(d*(c^2 + d^2)*f*(c + d*Tan[e + f*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
```

```
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+b\tan(e+fx))^3}{(c+d\tan(e+fx))^2}\,dx &= -\frac{(bc-ad)^2(a+b\tan(e+fx))}{d\,(c^2+d^2)\,f(c+d\tan(e+fx))} + \frac{\int \frac{b^3c^2+a^3cd-3ab^2cd+3a^2bd^2+d(3a^2bc-b^3c-a^3d+3a\cdots}{c+d\tan(e+f\cdots}}{d\,(c^2+d^2\cdots} \\
&= \frac{(6a^2bcd-2b^3cd+a^3(c^2-d^2)-3ab^2(c^2-d^2))\,x}{(c^2+d^2)^2} - \frac{(bc-ad)^2(a+b\tan(e+\cdots}{d\,(c^2+d^2)\,f(c+d\tan(e\cdots} \\
&= \frac{(6a^2bcd-2b^3cd+a^3(c^2-d^2)-3ab^2(c^2-d^2))\,x}{(c^2+d^2)^2} + \frac{(2a^3cd-6ab^2cd-3a^2b(c\cdots}{} \\
&= \frac{(6a^2bcd-2b^3cd+a^3(c^2-d^2)-3ab^2(c^2-d^2))\,x}{(c^2+d^2)^2} + \frac{(2a^3cd-6ab^2cd-3a^2b(c\cdots}{}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.79, size = 538, normalized size = 2.41

$$\frac{\cos(e+fx)(c\cos(e+fx)+d\sin(e+fx))\left(c^2\cos(e+fx)\left(2(c+id)^2\left(a^3d^2-3ia^2bd^2-3ab^2d^2+b^3c(ic+2d)\right)(e+fx)-2b^3(c^2+d^2)^2\log(\cos(e+fx))+(bc-ad)^2\left(2acd+b(c^2+3d^2)\right)\log\left((c\cos(e+fx)+d\sin(e+fx))^2\right)\right)+d\left(2(c+id)\left(a^3d^2(-id^2+cd(1+ie+ifx)+c^2(e+fx))+3a^2bcd^2(-ic(-i+e+fx)+d(i+e+fx))+3ab^2cd(c^2-id^2(e+fx)-cd(i+e+fx))+b^3c^2(2id^2(e+fx)+ic^2(i+e+fx)+cd(i+e+fx))\right)-2b^3c(c^2+d^2)^2\log(\cos(e+fx))+c(bc-ad)^2\left(2acd+b(c^2+3d^2)\right)\log\left((c\cos(e+fx)+d\sin(e+fx))^2\right)\right)\sin(e+fx)-2ic(bc-ad)^2\left(2acd+b(c^2+3d^2)\right)\mathrm{ArcTan}(\tan(e+fx))(c\cos(e+fx)+d\sin(e+fx))\right)(a+b\tan(e+fx))^3}{2c(c-id)^2(c+id)^2d^2f(a\cos(e+fx)+b\sin(e+fx))^3(c+d\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^3/(c + d*Tan[e + f*x])^2,x]
```

```
[Out] (Cos[e + f*x]*(c*Cos[e + f*x] + d*Sin[e + f*x])*(c^2*Cos[e + f*x]*(2*(c + I
*d)^2*(a^3*d^2 - (3*I)*a^2*b*d^2 - 3*a*b^2*d^2 + b^3*c*(I*c + 2*d))*(e + f*
x) - 2*b^3*(c^2 + d^2)^2*Log[Cos[e + f*x]] + (b*c - a*d)^2*(2*a*c*d + b*(c^
2 + 3*d^2))*Log[(c*Cos[e + f*x] + d*Sin[e + f*x])^2]) + d*(2*(c + I*d)*(a^3
*d^2*((-I)*d^2 + c*d*(1 + I*e + I*f*x) + c^2*(e + f*x)) + 3*a^2*b*c*d^2*((-
I)*c*(-I + e + f*x) + d*(I + e + f*x)) + 3*a*b^2*c*d*(c^2 - I*d^2*(e + f*x)
 - c*d*(I + e + f*x)) + b^3*c^2*((2*I)*d^2*(e + f*x) + I*c^2*(I + e + f*x)
+ c*d*(I + e + f*x))) - 2*b^3*c*(c^2 + d^2)^2*Log[Cos[e + f*x]] + c*(b*c -
a*d)^2*(2*a*c*d + b*(c^2 + 3*d^2))*Log[(c*Cos[e + f*x] + d*Sin[e + f*x])^2]
)*Sin[e + f*x] - (2*I)*c*(b*c - a*d)^2*(2*a*c*d + b*(c^2 + 3*d^2))*ArcTan[T
an[e + f*x]]*(c*Cos[e + f*x] + d*Sin[e + f*x]))*(a + b*Tan[e + f*x])^3)/(2*
c*(c - I*d)^2*(c + I*d)^2*d^2*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^3*(c + d*
Tan[e + f*x])^2)
```

Maple [A]
time = 0.28, size = 281, normalized size = 1.26 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3/(c+d*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/d^2/(c^2+d^2)/(c+d*tan(
f*x+e))+(2*a^3*c*d^3-3*a^2*b*c^2*d^2+3*a^2*b*d^4-6*a*b^2*c*d^3+b^3*c^4+3*b^
3*c^2*d^2)/(c^2+d^2)^2/d^2*ln(c+d*tan(f*x+e))+1/(c^2+d^2)^2*(1/2*(-2*a^3*c*
d+3*a^2*b*c^2-3*a^2*b*d^2+6*a*b^2*c*d-b^3*c^2+b^3*d^2)*ln(1+tan(f*x+e)^2)+(
a^3*c^2-a^3*d^2+6*a^2*b*c*d-3*a*b^2*c^2+3*a*b^2*d^2-2*b^3*c*d)*arctan(tan(f
*x+e))))
```

Maxima [A]
time = 0.55, size = 314, normalized size = 1.41

$$\frac{\frac{2\left(\left(a^3-3ab^2\right)c^2+2\left(3a^2b-b^3\right)cd-\left(a^3-3ab^2\right)d^2\right)(fx+e)}{c^4+2c^2d^2+d^4}+\frac{2\left(b^3c^4+3a^2bd^4-3\left(a^2b-b^3\right)c^2d^2+2\left(a^3-3ab^2\right)cd^3\right)\log(d\tan(fx+e)+c)}{c^4d^2+2c^2d^4+d^6}+\frac{\left(\left(3a^2b-b^3\right)c^2-2\left(a^3-3ab^2\right)cd-\left(3a^2b-b^3\right)d^2\right)\log\left(\tan(fx+e)^2+1\right)}{c^4+2c^2d^2+d^4}+\frac{2\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)}{c^3d^2+cd^4+\left(c^2d^3+d^5\right)\tan(fx+e)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(c+d*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*((a^3 - 3*a*b^2)*c^2 + 2*(3*a^2*b - b^3)*c*d - (a^3 - 3*a*b^2)*d^2)*
(f*x + e)/(c^4 + 2*c^2*d^2 + d^4) + 2*(b^3*c^4 + 3*a^2*b*d^4 - 3*(a^2*b - b
^3)*c^2*d^2 + 2*(a^3 - 3*a*b^2)*c*d^3)*log(d*tan(f*x + e) + c)/(c^4*d^2 + 2
*c^2*d^4 + d^6) + ((3*a^2*b - b^3)*c^2 - 2*(a^3 - 3*a*b^2)*c*d - (3*a^2*b -
 b^3)*d^2)*log(tan(f*x + e)^2 + 1)/(c^4 + 2*c^2*d^2 + d^4) + 2*(b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^2 + c*d^4 + (c^2*d^3 + d^5)*
tan(f*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(227) = 454$.
time = 1.66, size = 500, normalized size = 2.24

$$\frac{2b^3c^3d^2-6ab^2c^2d^3+6a^2bcd^4-2a^3d^5+2\left(\left(a^3-3ab^2\right)c^3d^2+2\left(3a^2b-b^3\right)c^2d^3-\left(a^3-3ab^2\right)cd^4\right)fx+\left(b^3c^5+3a^2bcd^4-3\left(a^2b-b^3\right)c^3d^2+2\left(a^3-3ab^2\right)c^2d^3+\left(b^3c^4d+3a^2bd^5-3\left(a^2b-b^3\right)c^2d^3+2\left(a^3-3ab^2\right)cd^4\right)\tan(fx+e)\right)\log\left(\frac{d^2\tan(fx+e)^2+2cd\tan(fx+e)+c^2}{\tan(fx+e)^2+1}\right)-\left(b^3c^5+2b^3c^3d^2+b^3cd^4+\left(b^3c^4d+2b^3c^2d^3+b^3d^5\right)\tan(fx+e)\right)\log\left(\frac{1}{\tan(fx+e)^2+1}\right)-2\left(b^3c^4d-3ab^2c^3d^2+3a^2bc^2d^3-a^3cd^4-\left(\left(a^3-3ab^2\right)c^2d^3+2\left(3a^2b-b^3\right)cd^4-\left(a^3-3ab^2\right)d^5\right)fx\right)\tan(fx+e)}{2\left(\left(c^4d^3+2c^2d^5+d^7\right)f\tan(fx+e)+\left(c^5d^2+2c^3d^4+cd^6\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(c+d*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*c^3*d^2 - 6*a*b^2*c^2*d^3 + 6*a^2*b*c*d^4 - 2*a^3*d^5 + 2*((a^3
- 3*a*b^2)*c^3*d^2 + 2*(3*a^2*b - b^3)*c^2*d^3 - (a^3 - 3*a*b^2)*c*d^4)*f*x
 + (b^3*c^5 + 3*a^2*b*c*d^4 - 3*(a^2*b - b^3)*c^3*d^2 + 2*(a^3 - 3*a*b^2)*c
^2*d^3 + (b^3*c^4*d + 3*a^2*b*d^5 - 3*(a^2*b - b^3)*c^2*d^3 + 2*(a^3 - 3*a*
b^2)*c*d^4)*tan(f*x + e))*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^
2)/(tan(f*x + e)^2 + 1)) - (b^3*c^5 + 2*b^3*c^3*d^2 + b^3*c*d^4 + (b^3*c^4*
d + 2*b^3*c^2*d^3 + b^3*d^5)*tan(f*x + e))*log(1/(tan(f*x + e)^2 + 1)) - 2*
(b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^3 - a^3*c*d^4 - ((a^3 - 3*a*b^
2)*c^2*d^3 + 2*(3*a^2*b - b^3)*c*d^4 - (a^3 - 3*a*b^2)*d^5)*f*x)*tan(f*x +
e))/((c^4*d^3 + 2*c^2*d^5 + d^7)*f*tan(f*x + e) + (c^5*d^2 + 2*c^3*d^4 + c*
d^6)*f)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.50, size = 6730, normalized size = 30.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3/(c+d*tan(f*x+e))**2,x)
```

```
[Out] Piecewise((zoo*x*(a + b*tan(e))**3/tan(e)**2, Eq(c, 0) & Eq(d, 0) & Eq(f, 0
)), (-a**3*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e
 + f*x) - 4*d**2*f) + 2*I*a**3*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 -
 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + a**3*f*x/(4*d**2*f*tan(e + f*x)**2 -
 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - a**3*tan(e + f*x)/(4*d**2*f*tan(e +
f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*I*a**3/(4*d**2*f*tan(e +
f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 3*I*a**2*b*f*x*tan(e + f*x)
**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 6*a**
2*b*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) -
4*d**2*f) - 3*I*a**2*b*f*x/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f
*x) - 4*d**2*f) + 3*I*a**2*b*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d
**2*f*tan(e + f*x) - 4*d**2*f) + 3*a*b**2*f*x*tan(e + f*x)**2/(4*d**2*f*tan
(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 6*I*a*b**2*f*x*tan(e +
 f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 3*a
*b**2*f*x/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) -
 9*a*b**2*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x)
- 4*d**2*f) + 6*I*a*b**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x
) - 4*d**2*f) + 3*I*b**3*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 - 8*
I*d**2*f*tan(e + f*x) - 4*d**2*f) + 6*b**3*f*x*tan(e + f*x)/(4*d**2*f*tan(e
 + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 3*I*b**3*f*x/(4*d**2*f*t
an(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*b**3*log(tan(e + f
*x)**2 + 1)*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e +
```

f*x) - 4*d**2*f) - 4*I*b**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(4*d**2*f *tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 2*b**3*log(tan(e + f*x)**2 + 1)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2* f) - 5*I*b**3*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f *x) - 4*d**2*f) - 4*b**3/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f), Eq(c, -I*d)), (-a**3*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f *x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 2*I*a**3*f*x*tan(e + f*x)/(4 *d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + a**3*f*x/(4 *d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - a**3*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 2* I*a**3/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 3* I*a**2*b*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 6*a**2*b*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 + 8 *I*d**2*f*tan(e + f*x) - 4*d**2*f) + 3*I*a**2*b*f*x/(4*d**2*f*tan(e + f*x)* *2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 3*I*a**2*b*tan(e + f*x)/(4*d**2* f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 3*a*b**2*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 6*I*a*b**2*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 3*a*b**2*f*x/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*t an(e + f*x) - 4*d**2*f) - 9*a*b**2*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 6*I*a*b**2/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 3*I*b**3*f*x*tan(e + f*x)**2/(4*d* *2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 6*b**3*f*x*tan (e + f*x)/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 3*I*b**3*f*x/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2* f) + 2*b**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x) **2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 4*I*b**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d** 2*f) - 2*b**3*log(tan(e + f*x)**2 + 1)/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2 *f*tan(e + f*x) - 4*d**2*f) + 5*I*b**3*tan(e + f*x)/(4*d**2*f*tan(e + f*x)* *2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 4*b**3/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f), Eq(c, I*d)), ((a**3*x + 3*a**2*b*lo g(tan(e + f*x)**2 + 1)/(2*f) - 3*a*b**2*x + 3*a*b**2*tan(e + f*x)/f - b**3* log(tan(e + f*x)**2 + 1)/(2*f) + b**3*tan(e + f*x)**2/(2*f))/c**2, Eq(d, 0)), (x*(a + b*tan(e))**3/(c + d*tan(e))**2, Eq(f, 0)), (2*a**3*c**3*d**2*f*x /(2*c**5*d**2*f + 2*c**4*d**3*f*tan(e + f*x) + 4*c**3*d**4*f + 4*c**2*d**5* f*tan(e + f*x) + 2*c*d**6*f + 2*d**7*f*tan(e + f*x)) + 2*a**3*c**2*d**3*f*x *tan(e + f*x)/(2*c**5*d**2*f + 2*c**4*d**3*f*tan(e + f*x) + 4*c**3*d**4*f + 4*c**2*d**5*f*tan(e + f*x) + 2*c*d**6*f + 2*d**7*f*tan(e + f*x)) + 4*a**3* c**2*d**3*log(c/d + tan(e + f*x))/(2*c**5*d**2*f + 2*c**4*d**3*f*tan(e + f* x) + 4*c**3*d**4*f + 4*c**2*d**5*f*tan(e + f*x) + 2*c*d**6*f + 2*d**7*f*tan (e + f*x)) - 2*a**3*c**2*d**3*log(tan(e + f*x)**2 + 1)/(2*c**5*d**2*f + 2*c **4*d**3*f*tan(e + f*x) + 4*c**3*d**4*f + 4*c**2*d**5*f*tan(e + f*x) + 2*c* d**6*f + 2*d**7*f*tan(e + f*x)) - 2*a**3*c**2*d**3/(2*c**5*d**2*f + 2*c**4* d**3*f*tan(e + f*x) + 4*c**3*d**4*f + 4*c**2*d*...

Giac [A]
time = 0.83, size = 447, normalized size = 2.00

$$\frac{\frac{2\,(a^3c^2-3\,ab^2c^2+6\,a^2bcd-2\,b^3cd-a^3d^2+3\,ab^2d^2)(fx+e)}{c^4+2\,c^2d^2+d^4}+\frac{(3\,a^2bc^2-b^3c^2-2\,a^3cd+6\,ab^2cd-3\,a^2bd^2+b^3d^2)\log\left(\tan(fx+e)^2+1\right)}{c^4+2\,c^2d^2+d^4}+\frac{2\,(b^3c^4-3\,a^2bc^2d^2+3\,b^3c^2d^2+2\,a^3cd^3-6\,ab^2cd^3+3\,a^2bd^4)\log(|d\tan(fx+e)+c|)}{c^4d^2+2\,c^2d^4+d^6}-\frac{2\,(b^3c^4\tan(fx+e)-3\,a^2bc^2d^2\tan(fx+e)+3\,b^3c^2d^2\tan(fx+e)+2\,a^3cd^3\tan(fx+e)-6\,ab^2cd^3\tan(fx+e)+3\,a^2bd^4\tan(fx+e)+3\,ab^2c^4-6\,a^2bc^3d+2\,b^3c^3d+3\,a^3c^2d^2-3\,ab^2c^2d^2+a^3d^4)}{(c^4d+2\,c^2d^3+d^5)(d\tan(fx+e)+c)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(c+d*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(a^3*c^2 - 3*a*b^2*c^2 + 6*a^2*b*c*d - 2*b^3*c*d - a^3*d^2 + 3*a*b^2
*d^2)*(f*x + e)/(c^4 + 2*c^2*d^2 + d^4) + (3*a^2*b*c^2 - b^3*c^2 - 2*a^3*c*
d + 6*a*b^2*c*d - 3*a^2*b*d^2 + b^3*d^2)*log(tan(f*x + e)^2 + 1)/(c^4 + 2*c
^2*d^2 + d^4) + 2*(b^3*c^4 - 3*a^2*b*c^2*d^2 + 3*b^3*c^2*d^2 + 2*a^3*c*d^3
- 6*a*b^2*c*d^3 + 3*a^2*b*d^4)*log(abs(d*tan(f*x + e) + c))/(c^4*d^2 + 2*c^
2*d^4 + d^6) - 2*(b^3*c^4*tan(f*x + e) - 3*a^2*b*c^2*d^2*tan(f*x + e) + 3*b
^3*c^2*d^2*tan(f*x + e) + 2*a^3*c*d^3*tan(f*x + e) - 6*a*b^2*c*d^3*tan(f*x
+ e) + 3*a^2*b*d^4*tan(f*x + e) + 3*a*b^2*c^4 - 6*a^2*b*c^3*d + 2*b^3*c^3*d
 + 3*a^3*c^2*d^2 - 3*a*b^2*c^2*d^2 + a^3*d^4)/((c^4*d + 2*c^2*d^3 + d^5)*(d
*tan(f*x + e) + c)))/f
```

Mupad [B]
time = 7.75, size = 272, normalized size = 1.22

$$\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(-a^3-a^2\,b\,3\mathrm{i}+3\,a\,b^2+b^3\,1\mathrm{i}\right)}{2\,f\,\left(-c^2\,1\mathrm{i}+2\,c\,d+d^2\,1\mathrm{i}\right)}+\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(-a^3\,1\mathrm{i}-3\,a^2\,b+a\,b^2\,3\mathrm{i}+b^3\right)}{2\,f\,\left(-c^2+c\,d\,2\mathrm{i}+d^2\right)}+\frac{\ln\left(c+d\tan(e+f\,x)\right)\,\left(d^2\,\left(3\,b^3\,c^2-3\,a^2\,b\,c^2\right)+b^3\,c^4+d^3\,\left(2\,a^3\,c-6\,a\,b^2\,c\right)+3\,a^2\,b\,d^4\right)}{f\,\left(c^4\,d^2+2\,c^2\,d^4+d^6\right)}-\frac{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}{d^2\,f\,\left(c^2+d^2\right)\,\left(c+d\tan\left(e+f\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^3/(c + d*tan(e + f*x))^2,x)
```

```
[Out] (log(tan(e + f*x) - 1i)*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i))/(2*f*(2*c*d -
c^2*1i + d^2*1i)) + (log(tan(e + f*x) + 1i)*(a*b^2*3i - 3*a^2*b - a^3*1i +
b^3))/(2*f*(c*d*2i - c^2 + d^2)) + (log(c + d*tan(e + f*x))*(d^2*(3*b^3*c^2
 - 3*a^2*b*c^2) + b^3*c^4 + d^3*(2*a^3*c - 6*a*b^2*c) + 3*a^2*b*d^4))/(f*(d
^6 + 2*c^2*d^4 + c^4*d^2)) - (a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c
*d^2)/(d^2*f*(c^2 + d^2)*(c + d*tan(e + f*x)))
```

3.1218 $\int \frac{(a+b\tan(e+fx))^2}{(c+d\tan(e+fx))^2}\,dx$

Optimal. Leaf size=126

$$-\frac{(b(c-d)-a(c+d))(a(c-d)+b(c+d))x}{(c^2+d^2)^2}-\frac{2(bc-ad)(ac+bd)\log(c\cos(e+fx)+d\sin(e+fx))}{(c^2+d^2)^2 f}-\frac{}{d\,(c^2}$$

```
[Out] -(b*(c-d)-a*(c+d))*(a*(c-d)+b*(c+d))*x/(c^2+d^2)^2-2*(-a*d+b*c)*(a*c+b*d)*l
n(c*cos(f*x+e)+d*sin(f*x+e))/(c^2+d^2)^2/f-(-a*d+b*c)^2/d/(c^2+d^2)/f/(c+d*
tan(f*x+e))
```

Rubi [A]
time = 0.16, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3623, 3612, 3611}

$$-\frac{(bc-ad)^2}{df\,(c^2+d^2)\,(c+d\tan(e+fx))}-\frac{2(ac+bd)(bc-ad)\log(c\cos(e+fx)+d\sin(e+fx))}{f\,(c^2+d^2)^2}-\frac{x(b(c-d)-a(c+d))(a(c-d)+b(c+d))}{(c^2+d^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2/(c + d*Tan[e + f*x])^2,x]
```

```
[Out] -(((b*(c - d) - a*(c + d))*(a*(c - d) + b*(c + d))*x)/(c^2 + d^2)^2) - (2*(
b*c - a*d)*(a*c + b*d)*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((c^2 + d^2)^2
*f) - (b*c - a*d)^2/(d*(c^2 + d^2)*f*(c + d*Tan[e + f*x]))
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
```

```
f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^2}{(c+d\tan(e+fx))^2}\,dx = -\frac{(bc-ad)^2}{d\,(c^2+d^2)\,f(c+d\tan(e+fx))} + \frac{\int \frac{a^2c-b^2c+2abd+(2abc-a^2d+b^2d)\tan(e+fx)}{c+d\tan(e+fx)}\,dx}{c^2+d^2}$$

$$= -\frac{(b(c-d)-a(c+d))(a(c-d)+b(c+d))x}{(c^2+d^2)^2} - \frac{(bc-ad)^2}{d\,(c^2+d^2)\,f(c+d\tan(e+f$$

$$= -\frac{(b(c-d)-a(c+d))(a(c-d)+b(c+d))x}{(c^2+d^2)^2} - \frac{2(bc-ad)(ac+bd)\log(c\cos}{(c^2+d^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.17, size = 320, normalized size = 2.54

$$\frac{(c\cos(e+fx)+d\sin(e+fx))\left(\frac{(bc-ad)^2(c^2+d^2)\sin(e+fx)}{c}+(b(-c+d)+a(c+d))(a(c-d)+b(c+d))(e+fx)(c\cos(e+fx)+d\sin(e+fx))+2i(a^2cd-b^2cd+ab(-c^2+d^2))(e+fx)(c\cos(e+fx)+d\sin(e+fx))+2i(-a^2cd+b^2cd+ab(c^2-d^2))\,\mathrm{ArcTan}(\tan(e+fx))(c\cos(e+fx)+d\sin(e+fx))+(a^2cd-b^2cd+ab(-c^2+d^2))\log\left((c\cos(e+fx)+d\sin(e+fx))^2\right)(c\cos(e+fx)+d\sin(e+fx))\right)(a+b\tan(e+fx))^2}{(c^2+d^2)^2 f(a\cos(e+fx)+b\sin(e+fx))^2(c+d\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2/(c + d*Tan[e + f*x])^2,x]
```

```
[Out] ((c*Cos[e + f*x] + d*Sin[e + f*x])*(((b*c - a*d)^2*(c^2 + d^2)*Sin[e + f*x]
)/c + (b*(-c + d) + a*(c + d))*(a*(c - d) + b*(c + d))*(e + f*x)*(c*Cos[e +
 f*x] + d*Sin[e + f*x]) + (2*I)*(a^2*c*d - b^2*c*d + a*b*(-c^2 + d^2))*(e +
 f*x)*(c*Cos[e + f*x] + d*Sin[e + f*x]) + (2*I)*(-(a^2*c*d) + b^2*c*d + a*b
*(c^2 - d^2))*ArcTan[Tan[e + f*x]]*(c*Cos[e + f*x] + d*Sin[e + f*x]) + (a^2
*c*d - b^2*c*d + a*b*(-c^2 + d^2))*Log[(c*Cos[e + f*x] + d*Sin[e + f*x])^2]
*(c*Cos[e + f*x] + d*Sin[e + f*x]))*(a + b*Tan[e + f*x])^2)/((c^2 + d^2)^2*
f*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(c + d*Tan[e + f*x])^2)
```

Maple [A]
time = 0.21, size = 200, normalized size = 1.59

method	result
derivativedivides	$\frac{-\frac{a^2d^2-2abcd+b^2c^2}{(c^2+d^2)d(c+d\tan(fx+e))}+\frac{2(a^2cd-ab\,c^2+ab\,d^2-b^2cd)\ln(c+d\tan(fx+e))}{(c^2+d^2)^2}+\frac{\frac{(-2a^2cd+2ab\,c^2-2ab\,d^2+2b^2cd)\ln(1+\tan^2(fx+}{2}}{(}}{f}$
default	$\frac{-\frac{a^2d^2-2abcd+b^2c^2}{(c^2+d^2)d(c+d\tan(fx+e))}+\frac{2(a^2cd-ab\,c^2+ab\,d^2-b^2cd)\ln(c+d\tan(fx+e))}{(c^2+d^2)^2}+\frac{\frac{(-2a^2cd+2ab\,c^2-2ab\,d^2+2b^2cd)\ln(1+\tan^2(fx+}{2}}{(}}{f}$

norman	$\frac{\frac{c\left(a^2c^2-a^2d^2+4abcd-b^2c^2+b^2d^2\right)x}{c^4+2c^2d^2+d^4}+\frac{d\left(a^2c^2-a^2d^2+4abcd-b^2c^2+b^2d^2\right)x\tan(fx+e)}{c^4+2c^2d^2+d^4}+\frac{\left(a^2d^2-2abcd+b^2c^2\right)\tan(fx+e)}{cf\left(c^2+d^2\right)}}{c+d\tan(fx+e)}-\frac{(a^2cd}{}$
risch	$\frac{2ib^2c^2}{(id+c)f(-id+c)^2\left(-ie^{2i(fx+e)}d+id+e^{2i(fx+e)}c+c\right)}-\frac{a^2x}{2icd-c^2+d^2}+\frac{x\,b^2}{2icd-c^2+d^2}+\frac{4ib^2cde}{f(c^4+2c^2d^2+d^4)}+\frac{}{(id+c)f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-(a^2*d^2-2*a*b*c*d+b^2*c^2)/(c^2+d^2)/d/(c+d*tan(f*x+e))+2*(a^2*c*d-a
*b*c^2+a*b*d^2-b^2*c*d)/(c^2+d^2)^2*ln(c+d*tan(f*x+e))+1/(c^2+d^2)^2*(1/2*(
-2*a^2*c*d+2*a*b*c^2-2*a*b*d^2+2*b^2*c*d)*ln(1+tan(f*x+e)^2)+(a^2*c^2-a^2*d
^2+4*a*b*c*d-b^2*c^2+b^2*d^2)*arctan(tan(f*x+e))))
```

Maxima [A]
time = 0.54, size = 233, normalized size = 1.85

$$\frac{\frac{(4\,abcd+(a^2-b^2)c^2-(a^2-b^2)d^2)(fx+e)}{c^4+2\,c^2d^2+d^4}-\frac{2\,(abc^2-abd^2-(a^2-b^2)cd)\log(d\tan(fx+e)+c)}{c^4+2\,c^2d^2+d^4}+\frac{(abc^2-abd^2-(a^2-b^2)cd)\log\left(\tan(fx+e)^2+1\right)}{c^4+2\,c^2d^2+d^4}-\frac{b^2c^2-2\,abcd+a^2d^2}{c^3d+cd^3+(c^2d^2+d^4)\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] ((4*a*b*c*d + (a^2 - b^2)*c^2 - (a^2 - b^2)*d^2)*(f*x + e)/(c^4 + 2*c^2*d^2
 + d^4) - 2*(a*b*c^2 - a*b*d^2 - (a^2 - b^2)*c*d)*log(d*tan(f*x + e) + c)/(
c^4 + 2*c^2*d^2 + d^4) + (a*b*c^2 - a*b*d^2 - (a^2 - b^2)*c*d)*log(tan(f*x
+ e)^2 + 1)/(c^4 + 2*c^2*d^2 + d^4) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(c^3*
d + c*d^3 + (c^2*d^2 + d^4)*tan(f*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(128) = 256$.
time = 1.18, size = 300, normalized size = 2.38

$$-\frac{b^2c^2d-2\,abcd^2+a^2d^3-(4\,abc^2d+(a^2-b^2)c^3-(a^2-b^2)cd^2)fx+(abc^3-abcd^2-(a^2-b^2)c^2d+(abc^2d-abd^3-(a^2-b^2)cd^2)\tan(fx+e))\log\left(\frac{d^2\tan(fx+e)^2+2\,cd\tan(fx+e)+c^2}{\tan(fx+e)^2+1}\right)-(b^2c^3-2\,abc^2d+a^2cd^2+(4\,abcd^2+(a^2-b^2)c^2d-(a^2-b^2)d^3)fx)\tan(fx+e)}{(c^4d+2\,c^2d^3+d^5)f\tan(fx+e)+(c^5+2\,c^3d^2+cd^4)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3 - (4*a*b*c^2*d + (a^2 - b^2)*c^3 - (a^2
 - b^2)*c*d^2)*f*x + (a*b*c^3 - a*b*c*d^2 - (a^2 - b^2)*c^2*d + (a*b*c^2*d
- a*b*d^3 - (a^2 - b^2)*c*d^2)*tan(f*x + e))*log((d^2*tan(f*x + e)^2 + 2*c*
d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) - (b^2*c^3 - 2*a*b*c^2*d + a^2*
c*d^2 + (4*a*b*c*d^2 + (a^2 - b^2)*c^2*d - (a^2 - b^2)*d^3)*f*x)*tan(f*x +
e))/((c^4*d + 2*c^2*d^3 + d^5)*f*tan(f*x + e) + (c^5 + 2*c^3*d^2 + c*d^4)*f
)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.01, size = 4258, normalized size = 33.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2/(c+d*tan(f*x+e))**2,x)
```

```
[Out] Piecewise((zoo*x*(a + b*tan(e))**2/tan(e)**2, Eq(c, 0) & Eq(d, 0) & Eq(f, 0
)), ((a**2*x + a*b*log(tan(e + f*x)**2 + 1)/f - b**2*x + b**2*tan(e + f*x)/
f)/c**2, Eq(d, 0)), (-a**2*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 -
8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*I*a**2*f*x*tan(e + f*x)/(4*d**2*f*t
an(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + a**2*f*x/(4*d**2*f*t
an(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - a**2*tan(e + f*x)/(4
*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*I*a**2/(4
*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*I*a*b*f*x
*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d*
*2*f) + 4*a*b*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e
 + f*x) - 4*d**2*f) - 2*I*a*b*f*x/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*ta
n(e + f*x) - 4*d**2*f) + 2*I*a*b*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8
*I*d**2*f*tan(e + f*x) - 4*d**2*f) + b**2*f*x*tan(e + f*x)**2/(4*d**2*f*tan
(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 2*I*b**2*f*x*tan(e + f
*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - b**2*
f*x/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 3*b**
2*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2
*f) + 2*I*b**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2
*f), Eq(c, -I*d)), (-a**2*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 + 8
*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 2*I*a**2*f*x*tan(e + f*x)/(4*d**2*f*ta
n(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + a**2*f*x/(4*d**2*f*ta
n(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - a**2*tan(e + f*x)/(4*
d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 2*I*a**2/(4*
d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 2*I*a*b*f*x*
tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**
2*f) + 4*a*b*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e
+ f*x) - 4*d**2*f) + 2*I*a*b*f*x/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan
(e + f*x) - 4*d**2*f) - 2*I*a*b*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 + 8*
I*d**2*f*tan(e + f*x) - 4*d**2*f) + b**2*f*x*tan(e + f*x)**2/(4*d**2*f*tan(
e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*I*b**2*f*x*tan(e + f*
x)/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - b**2*f
*x/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 3*b**2
*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*
f) - 2*I*b**2/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*
f), Eq(c, I*d)), (x*(a + b*tan(e))**2/(c + d*tan(e))**2, Eq(f, 0)), (a**2*c
**3*d*f*x/(c**5*d*f + c**4*d**2*f*tan(e + f*x) + 2*c**3*d**3*f + 2*c**2*d**
4*f*tan(e + f*x) + c*d**5*f + d**6*f*tan(e + f*x)) + a**2*c**2*d**2*f*x*tan
```

```
(e + f*x)/(c**5*d*f + c**4*d**2*f*tan(e + f*x) + 2*c**3*d**3*f + 2*c**2*d**
4*f*tan(e + f*x) + c*d**5*f + d**6*f*tan(e + f*x)) + 2*a**2*c**2*d**2*log(c
/d + tan(e + f*x))/(c**5*d*f + c**4*d**2*f*tan(e + f*x) + 2*c**3*d**3*f + 2
*c**2*d**4*f*tan(e + f*x) + c*d**5*f + d**6*f*tan(e + f*x)) - a**2*c**2*d**
2*log(tan(e + f*x)**2 + 1)/(c**5*d*f + c**4*d**2*f*tan(e + f*x) + 2*c**3*d*
*3*f + 2*c**2*d**4*f*tan(e + f*x) + c*d**5*f + d**6*f*tan(e + f*x)) - a**2*
c**2*d**2/(c**5*d*f + c**4*d**2*f*tan(e + f*x) + 2*c**3*d**3*f + 2*c**2*d**
4*f*tan(e + f*x) + c*d**5*f + d**6*f*tan(e + f*x)) - a**2*c*d**3*f*x/(c**5*
d*f + c**4*d**2*f*tan(e + f*x) + 2*c**3*d**3*f + 2*c**2*d**4*f*tan(e + f*x)
 + c*d**5*f + d**6*f*tan(e + f*x)) + 2*a**2*c*d**3*log(c/d + tan(e + f*x))*
tan(e + f*x)/(c**5*d*f + c**4*d**2*f*tan(e + f*x) + 2*c**3*d**3*f + 2*c**2*
d**4*f*tan(e + f*x) + c*d**5*f + d**6*f*tan(e + f*x)) - a**2*c*d**3*log(tan
(e + f*x)**2 + 1)*tan(e + f*x)/(c**5*d*f + c**4*d**2*f*tan(e + f*x) + 2*c**
3*d**3*f + 2*c**2*d**4*f*tan(e + f*x) + c*d**5*f + d**6*f*tan(e + f*x)) - a
**2*d**4*f*x*tan(e + f*x)/(c**5*d*f + c**4*d**2*f*tan(e + f*x) + 2*c**3*d**
3*f + 2*c**2*d**4*f*tan(e + f*x) + c*d**5*f + d**6*f*tan(e + f*x)) - a**2*d
**4/(c**5*d*f + c**4*d**2*f*tan(e + f*x) + 2*c**3*d**3*f + 2*c**2*d**4*f*ta
n(e + f*x) + c*d**5*f + d**6*f*tan(e + f*x)) - 2*a*b*c**3*d*log(c/d + tan(e
 + f*x))/(c**5*d*f + c**4*d**2*f*tan(e + f*x) + 2*c**3*d**3*f + 2*c**2*d**4
*f*tan(e + f*x) + c*d**5*f + d**6*f*tan(e + f*x)) + a*b*c**3*d*log(tan(e +
f*x)**2 + 1)/(c**5*d*f + c**4*d**2*f*tan(e + f*x) + 2*c**3*d**3*f + 2*c**2*
d**4*f*tan(e + f*x) + c*d**5*f + d**6*f*tan(e + f*x)) + 2*a*b*c**3*d/(c**5*
d*f + c**4*d**2*f*tan(e + f*x) + 2*c**3*d**3*f + 2*c**2*d**4*f*tan(e + f*x)
 + c*d**5*f + d**6*f*tan(e + f*x)) + 4*a*b*c**2*d**2*f*x/(c**5*d*f + c**4*d
**2*f*tan(e + f*x) + 2*c**3*d**3*f + 2*c**2*d**4*f*tan(e + f*x) + c*d**5*f
+ d**6*f*tan(e + f*x)) - 2*a*b*c**2*d**2*log(c/d + tan(e + f*x))*tan(e + f*
x)/(c**5*d*f + c**4*d**2*f*tan(e + f*x) + 2*c**3*d**3*f + 2*c**2*d**4*f*tan
(e + f*x) + c*d**5*f + d**6*f*tan(e + f*x)) + a*b*c**2*d**2*log(tan(e + f*x
)**2 + 1)*tan(e + f*x)/(c**5*d*f + c**4*d**2*f*tan(e + f*x) + 2*c**3*d**3*f
 + 2*c**2*d**4*f*tan(e + f*x) + c*d**5*f + d**6...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(128) = 256.
time = 0.65, size = 331, normalized size = 2.63

$$\frac{\frac{(a^2c^2-b^2c^2+4\,abcd-a^2d^2+b^2d^2)(fx+e)}{c^4+2\,c^2d^2+d^4}+\frac{(abc^2-a^2cd+b^2cd-abd^2)\log\left(\tan(fx+e)^2+1\right)}{c^4+2\,c^2d^2+d^4}-\frac{2\,(abc^2d-a^2cd^2+b^2cd^2-abd^3)\log(|d\tan(fx+e)+c|)}{c^4d+2\,c^2d^3+d^5}+\frac{2\,abc^2d^2\tan(fx+e)-2\,a^2cd^3\tan(fx+e)+2\,b^2cd^3\tan(fx+e)-2\,abd^4\tan(fx+e)-b^2c^4+4\,abc^3d-3\,a^2c^2d^2+b^2c^2d^2-a^2d^4}{(c^4d+2\,c^2d^3+d^5)(d\tan(fx+e)+c)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] ((a^2*c^2 - b^2*c^2 + 4*a*b*c*d - a^2*d^2 + b^2*d^2)*(f*x + e)/(c^4 + 2*c^2
*d^2 + d^4) + (a*b*c^2 - a^2*c*d + b^2*c*d - a*b*d^2)*log(tan(f*x + e)^2 +
1)/(c^4 + 2*c^2*d^2 + d^4) - 2*(a*b*c^2*d - a^2*c*d^2 + b^2*c*d^2 - a*b*d^3
)*log(abs(d*tan(f*x + e) + c))/(c^4*d + 2*c^2*d^3 + d^5) + (2*a*b*c^2*d^2*t
an(f*x + e) - 2*a^2*c*d^3*tan(f*x + e) + 2*b^2*c*d^3*tan(f*x + e) - 2*a*b*d
```

```
^4*tan(f*x + e) - b^2*c^4 + 4*a*b*c^3*d - 3*a^2*c^2*d^2 + b^2*c^2*d^2 - a^2
*d^4)/((c^4*d + 2*c^2*d^3 + d^5)*(d*tan(f*x + e) + c)))/f
```

Mupad [B]
time = 6.95, size = 208, normalized size = 1.65

$$\frac{\ln(c+d\tan(e+fx))\ (-2abc^2+(2a^2-2b^2)\ cd+2abd^2)}{f\ (c^4+2c^2d^2+d^4)}-\frac{\ln(\tan(e+fx)-\mathrm{i})\ (a^2+ab\,2\mathrm{i}-b^2)}{2f\ (-c^2\,1\mathrm{i}+2cd+d^2\,1\mathrm{i})}-\frac{\ln(\tan(e+fx)+1\mathrm{i})\ (a^2\,1\mathrm{i}+2ab-b^2\,1\mathrm{i})}{2f\ (-c^2+cd\,2\mathrm{i}+d^2)}-\frac{a^2d^2-2abcd+b^2c^2}{df\ (c^2+d^2)\ (c+d\tan(e+fx))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^2/(c + d*tan(e + f*x))^2,x)
```

```
[Out] (log(c + d*tan(e + f*x))*(c*d*(2*a^2 - 2*b^2) - 2*a*b*c^2 + 2*a*b*d^2))/(f*
(c^4 + d^4 + 2*c^2*d^2)) - (log(tan(e + f*x) - 1i)*(a*b*2i + a^2 - b^2))/(2
*f*(2*c*d - c^2*1i + d^2*1i)) - (log(tan(e + f*x) + 1i)*(2*a*b + a^2*1i - b
^2*1i))/(2*f*(c*d*2i - c^2 + d^2)) - (a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(d*f*(
c^2 + d^2)*(c + d*tan(e + f*x)))
```

3.1219 $\int \frac{a+b\tan(e+fx)}{(c+d\tan(e+fx))^2}\,dx$

Optimal. Leaf size=111

$$\frac{(2bcd+a(c^2-d^2))\,x}{(c^2+d^2)^2}+\frac{(2acd-b(c^2-d^2))\log(c\cos(e+fx)+d\sin(e+fx))}{(c^2+d^2)^2\,f}+\frac{bc-ad}{(c^2+d^2)\,f(c+d\tan(e+fx)}$$

```
[Out] (2*b*c*d+a*(c^2-d^2))*x/(c^2+d^2)^2+(2*a*c*d-b*(c^2-d^2))*ln(c*cos(f*x+e)+d
*sin(f*x+e))/(c^2+d^2)^2/f+(-a*d+b*c)/(c^2+d^2)/f/(c+d*tan(f*x+e))
```

Rubi [A]
time = 0.11, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3610, 3612, 3611}

$$\frac{bc-ad}{f\,(c^2+d^2)\,(c+d\tan(e+fx))}+\frac{(2acd-b(c^2-d^2))\log(c\cos(e+fx)+d\sin(e+fx))}{f\,(c^2+d^2)^2}+\frac{x(a(c^2-d^2)+2bcd)}{(c^2+d^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])/(c + d*Tan[e + f*x])^2,x]
```

```
[Out] ((2*b*c*d + a*(c^2 - d^2))*x)/(c^2 + d^2)^2 + ((2*a*c*d - b*(c^2 - d^2))*Lo
g[c*Cos[e + f*x] + d*Sin[e + f*x]])/((c^2 + d^2)^2*f) + (b*c - a*d)/((c^2 +
 d^2)*f*(c + d*Tan[e + f*x]))
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
```

```
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{a+b\tan(e+fx)}{(c+d\tan(e+fx))^2}\,dx = \frac{bc-ad}{(c^2+d^2)\,f(c+d\tan(e+fx))} + \frac{\int \frac{ac+bd+(bc-ad)\tan(e+fx)}{c+d\tan(e+fx)}\,dx}{c^2+d^2}$$
$$= \frac{(2bcd+a(c^2-d^2))\,x}{(c^2+d^2)^2} + \frac{bc-ad}{(c^2+d^2)\,f(c+d\tan(e+fx))} + \frac{(2acd-b(c^2-d^2))}{(c^2+}$$
$$= \frac{(2bcd+a(c^2-d^2))\,x}{(c^2+d^2)^2} + \frac{(2acd-b(c^2-d^2))\log(c\cos(e+fx)+d\sin(e+fx)}{(c^2+d^2)^2\,f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.25, size = 189, normalized size = 1.70

$$\frac{\frac{b((-ic-d)\log(i-\tan(e+fx))+i(c+id)\log(i+\tan(e+fx))+2d\log(c+d\tan(e+fx)))}{c^2+d^2}+(bc-ad)\left(\frac{i\log(i-\tan(e+fx))}{(c+id)^2}-\frac{i\log(i+\tan(e+fx))}{(c-id)^2}+\frac{2d\left(-2c\log(c+d\tan(e+fx))+\frac{c^2+d^2}{c+d\tan(e+fx)}\right)}{(c^2+d^2)^2}\right)}{2df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])/(c + d*Tan[e + f*x])^2,x]
```

```
[Out] ((b*(((-I)*c - d)*Log[I - Tan[e + f*x]] + I*(c + I*d)*Log[I + Tan[e + f*x]]
 + 2*d*Log[c + d*Tan[e + f*x]]))/(c^2 + d^2) + (b*c - a*d)*((I*Log[I - Tan[
e + f*x]])/(c + I*d)^2 - (I*Log[I + Tan[e + f*x]])/(c - I*d)^2 + (2*d*(-2*c
*Log[c + d*Tan[e + f*x]] + (c^2 + d^2)/(c + d*Tan[e + f*x])))/(c^2 + d^2)^2
))/(2*d*f)
```

Maple [A]
time = 0.21, size = 141, normalized size = 1.27

method	result
derivativedivides	$\frac{\frac{\frac{\left(-2acd+b\,c^2-b\,d^2\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(a\,c^2-a\,d^2+2bcd\right)\arctan(\tan(fx+e))}{\left(c^2+d^2\right)^2}-\frac{ad-bc}{\left(c^2+d^2\right)(c+d\tan(fx+e))}+\frac{\left(2acd-b\,c^2+b\,d^2\right)\ln(}{\left(c^2+d^2\right.}}{f}$
default	$\frac{\frac{\frac{\left(-2acd+b\,c^2-b\,d^2\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(a\,c^2-a\,d^2+2bcd\right)\arctan(\tan(fx+e))}{\left(c^2+d^2\right)^2}-\frac{ad-bc}{\left(c^2+d^2\right)(c+d\tan(fx+e))}+\frac{\left(2acd-b\,c^2+b\,d^2\right)\ln(}{\left(c^2+d^2\right.}}{f}$
norman	$\frac{\frac{c\left(a\,c^2-a\,d^2+2bcd\right)x}{c^4+2c^2d^2+d^4}+\frac{d\left(a\,c^2-a\,d^2+2bcd\right)x\tan(fx+e)}{c^4+2c^2d^2+d^4}+\frac{(ad-bc)d\tan(fx+e)}{cf\left(c^2+d^2\right)}}{c+d\tan(fx+e)}+\frac{\left(2acd-b\,c^2+b\,d^2\right)\ln(c+d\tan(fx+e))}{f(c^4+2c^2d^2+d^4)}-\ ($

risch	$\frac{ixb}{2icd-c^2+d^2} - \frac{ax}{2icd-c^2+d^2} - \frac{4iacdx}{c^4+2c^2d^2+d^4} + \frac{2ib\,c^2x}{c^4+2c^2d^2+d^4} - \frac{2ib\,d^2x}{c^4+2c^2d^2+d^4} - \frac{4iacde}{f(c^4+2c^2d^2+d^4)} +$ 2ib … / f(c^4+2c^…

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))/(c+d*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(c^2+d^2)^2*(1/2*(-2*a*c*d+b*c^2-b*d^2)*ln(1+tan(f*x+e)^2)+(a*c^2-a*
d^2+2*b*c*d)*arctan(tan(f*x+e)))-(a*d-b*c)/(c^2+d^2)/(c+d*tan(f*x+e))+(2*a*
c*d-b*c^2+b*d^2)/(c^2+d^2)^2*ln(c+d*tan(f*x+e)))
```

Maxima [A]
time = 0.53, size = 181, normalized size = 1.63

$$\frac{\frac{2\,(ac^2+2\,bcd-ad^2)(fx+e)}{c^4+2\,c^2d^2+d^4} - \frac{2\,(bc^2-2\,acd-bd^2)\log(d\tan(fx+e)+c)}{c^4+2\,c^2d^2+d^4} + \frac{(bc^2-2\,acd-bd^2)\log\left(\tan(fx+e)^2+1\right)}{c^4+2\,c^2d^2+d^4} + \frac{2\,(bc-ad)}{c^3+cd^2+(c^2d+d^3)\tan(fx+e)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(c+d*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a*c^2 + 2*b*c*d - a*d^2)*(f*x + e)/(c^4 + 2*c^2*d^2 + d^4) - 2*(b*c
^2 - 2*a*c*d - b*d^2)*log(d*tan(f*x + e) + c)/(c^4 + 2*c^2*d^2 + d^4) + (b*
c^2 - 2*a*c*d - b*d^2)*log(tan(f*x + e)^2 + 1)/(c^4 + 2*c^2*d^2 + d^4) + 2*
(b*c - a*d)/(c^3 + c*d^2 + (c^2*d + d^3)*tan(f*x + e)))/f
```

Fricas [A]
time = 1.33, size = 228, normalized size = 2.05

$$\frac{2\,bcd^2 - 2\,ad^3 + 2\,(ac^3 + 2\,bc^2d - acd^2)fx - (bc^3 - 2\,ac^2d - bcd^2 + (bc^2d - 2\,acd^2 - bd^3)\tan(fx+e))\log\left(\frac{d^2\tan(fx+e)^2 + 2\,cd\tan(fx+e) + c^2}{\tan(fx+e)^2+1}\right) - 2\,(bc^2d - acd^2 - (ac^2d + 2\,bcd^2 - ad^3)fx)\tan(fx+e)}{2\,((c^4d + 2\,c^2d^3 + d^5)f\tan(fx+e) + (c^5 + 2\,c^3d^2 + cd^4)f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(c+d*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*c*d^2 - 2*a*d^3 + 2*(a*c^3 + 2*b*c^2*d - a*c*d^2)*f*x - (b*c^3 - 2
*a*c^2*d - b*c*d^2 + (b*c^2*d - 2*a*c*d^2 - b*d^3)*tan(f*x + e))*log((d^2*t
an(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) - 2*(b*c^2*
d - a*c*d^2 - (a*c^2*d + 2*b*c*d^2 - a*d^3)*f*x)*tan(f*x + e))/((c^4*d + 2*
c^2*d^3 + d^5)*f*tan(f*x + e) + (c^5 + 2*c^3*d^2 + c*d^4)*f)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.89, size = 2878, normalized size = 25.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(c+d*tan(f*x+e))**2,x)
```

```
[Out] Piecewise((zoo*x*(a + b*tan(e))/tan(e)**2, Eq(c, 0) & Eq(d, 0) & Eq(f, 0)),
 (-a*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x
) - 4*d**2*f) + 2*I*a*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2
*f*tan(e + f*x) - 4*d**2*f) + a*f*x/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*
tan(e + f*x) - 4*d**2*f) - a*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d
**2*f*tan(e + f*x) - 4*d**2*f) + 2*I*a/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2
*f*tan(e + f*x) - 4*d**2*f) + I*b*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x
)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*b*f*x*tan(e + f*x)/(4*d**2*f
*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - I*b*f*x/(4*d**2*f*
tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + I*b*tan(e + f*x)/(4
*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f), Eq(c, -I*d))
, (-a*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*
x) - 4*d**2*f) - 2*I*a*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**
2*f*tan(e + f*x) - 4*d**2*f) + a*f*x/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f
*tan(e + f*x) - 4*d**2*f) - a*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 + 8*I*
d**2*f*tan(e + f*x) - 4*d**2*f) - 2*I*a/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**
2*f*tan(e + f*x) - 4*d**2*f) - I*b*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*
x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*b*f*x*tan(e + f*x)/(4*d**2*
f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + I*b*f*x/(4*d**2*f
*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - I*b*tan(e + f*x)/(
4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f), Eq(c, I*d))
, (x*(a + b*tan(e))/(c + d*tan(e))**2, Eq(f, 0)), ((a*x + b*log(tan(e + f*x
)**2 + 1)/(2*f))/c**2, Eq(d, 0)), (2*a*c**3*f*x/(2*c**5*f + 2*c**4*d*f*tan(
e + f*x) + 4*c**3*d**2*f + 4*c**2*d**3*f*tan(e + f*x) + 2*c*d**4*f + 2*d**5
*f*tan(e + f*x)) + 2*a*c**2*d*f*x*tan(e + f*x)/(2*c**5*f + 2*c**4*d*f*tan(e
 + f*x) + 4*c**3*d**2*f + 4*c**2*d**3*f*tan(e + f*x) + 2*c*d**4*f + 2*d**5*
f*tan(e + f*x)) + 4*a*c**2*d*log(c/d + tan(e + f*x))/(2*c**5*f + 2*c**4*d*f
*tan(e + f*x) + 4*c**3*d**2*f + 4*c**2*d**3*f*tan(e + f*x) + 2*c*d**4*f + 2
*d**5*f*tan(e + f*x)) - 2*a*c**2*d*log(tan(e + f*x)**2 + 1)/(2*c**5*f + 2*c
**4*d*f*tan(e + f*x) + 4*c**3*d**2*f + 4*c**2*d**3*f*tan(e + f*x) + 2*c*d**
4*f + 2*d**5*f*tan(e + f*x)) - 2*a*c**2*d/(2*c**5*f + 2*c**4*d*f*tan(e + f*
x) + 4*c**3*d**2*f + 4*c**2*d**3*f*tan(e + f*x) + 2*c*d**4*f + 2*d**5*f*tan
(e + f*x)) - 2*a*c*d**2*f*x/(2*c**5*f + 2*c**4*d*f*tan(e + f*x) + 4*c**3*d*
*2*f + 4*c**2*d**3*f*tan(e + f*x) + 2*c*d**4*f + 2*d**5*f*tan(e + f*x)) + 4
*a*c*d**2*log(c/d + tan(e + f*x))*tan(e + f*x)/(2*c**5*f + 2*c**4*d*f*tan(e
 + f*x) + 4*c**3*d**2*f + 4*c**2*d**3*f*tan(e + f*x) + 2*c*d**4*f + 2*d**5*
f*tan(e + f*x)) - 2*a*c*d**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*c**5*
f + 2*c**4*d*f*tan(e + f*x) + 4*c**3*d**2*f + 4*c**2*d**3*f*tan(e + f*x) +
2*c*d**4*f + 2*d**5*f*tan(e + f*x)) - 2*a*d**3*f*x*tan(e + f*x)/(2*c**5*f +
 2*c**4*d*f*tan(e + f*x) + 4*c**3*d**2*f + 4*c**2*d**3*f*tan(e + f*x) + 2*c
*d**4*f + 2*d**5*f*tan(e + f*x)) - 2*a*d**3/(2*c**5*f + 2*c**4*d*f*tan(e +
f*x) + 4*c**3*d**2*f + 4*c**2*d**3*f*tan(e + f*x) + 2*c*d**4*f + 2*d**5*f*t
an(e + f*x)) - 2*b*c**3*log(c/d + tan(e + f*x))/(2*c**5*f + 2*c**4*d*f*tan(
e + f*x) + 4*c**3*d**2*f + 4*c**2*d**3*f*tan(e + f*x) + 2*c*d**4*f + 2*d**5
*f*tan(e + f*x)) + b*c**3*log(tan(e + f*x)**2 + 1)/(2*c**5*f + 2*c**4*d*f*t
```

```
an(e + f*x) + 4*c**3*d**2*f + 4*c**2*d**3*f*tan(e + f*x) + 2*c*d**4*f + 2*d
**5*f*tan(e + f*x)) + 2*b*c**3/(2*c**5*f + 2*c**4*d*f*tan(e + f*x) + 4*c**3
*d**2*f + 4*c**2*d**3*f*tan(e + f*x) + 2*c*d**4*f + 2*d**5*f*tan(e + f*x))
+ 4*b*c**2*d*f*x/(2*c**5*f + 2*c**4*d*f*tan(e + f*x) + 4*c**3*d**2*f + 4*c*
*2*d**3*f*tan(e + f*x) + 2*c*d**4*f + 2*d**5*f*tan(e + f*x)) - 2*b*c**2*d*l
og(c/d + tan(e + f*x))*tan(e + f*x)/(2*c**5*f + 2*c**4*d*f*tan(e + f*x) + 4
*c**3*d**2*f + 4*c**2*d**3*f*tan(e + f*x) + 2*c*d**4*f + 2*d**5*f*tan(e + f
*x)) + b*c**2*d*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*c**5*f + 2*c**4*d*
f*tan(e + f*x) + 4*c**3*d**2*f + 4*c**2*d**3*f*tan(e + f*x) + 2*c*d**4*f +
2*d**5*f*tan(e + f*x)) + 4*b*c*d**2*f*x*tan(e + f*x)/(2*c**5*f + 2*c**4*d*f
*tan(e + f*x) + 4*c**3*d**2*f + 4*c**2*d**3*f*tan(e + f*x) + 2*c*d**4*f + 2
*d**5*f*tan(e + f*x)) + 2*b*c*d**2*log(c/d + tan(e + f*x))/(2*c**5*f + 2*c*
*4*d*f*tan(e + f*x) + 4*c**3*d**2*f + 4*c**2*d**3*f*tan(e + f*x) + 2*c*d**4
*f + 2*d**5*f*tan(e + f*x)) - b*c*d**2*log(tan(e + f*x)**2 + 1)/(2*c**5*f +
 2*c**4*d*f*tan(e + f*x) + 4*c**3*d**2*f + 4*c**2*d**3*f*tan(e + f*x) + 2*c
*d**4*f + 2*d**5*f*tan(e + f*x)) + 2*b*c*d**2/(2*c**5*f + 2*c**4*d*f*tan(e
+ f*x) + 4*c**3*d**2*f + 4*c**2*d**3*f*tan(e + f*x) + 2*c*d**4*f + 2*d**5*f
*tan(e + f*x)) + 2*b*d**3*log(c/d + tan(e + f*x))*tan(e + f*x)/(2*c**5*f +
2*c**4*d*f*tan(e + f*x) + 4*c**3*d**2*f + 4*c**2*d**3*f*tan(e + f*x) + 2*c*
d**4*f + 2*d**5*f*tan(e + f*x)) - b*d**3*log(tan(e + f*x)**2 + 1)*tan(e + f
*x)/(2*c**5*f + 2*c**4*d*f*tan(e + f*x) + 4*c**3*d**2*f + 4*c**2*d**3*f*tan
(e + f*x) + 2*c*d**4*f + 2*d**5*f*tan(e + f*x))...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(114) = 228$.

time = 0.54, size = 241, normalized size = 2.17

$$\frac{\frac{2\,(ac^2+2\,bcd-ad^2)(fx+e)}{c^4+2\,c^2d^2+d^4}+\frac{(bc^2-2\,acd-bd^2)\log\left(\tan(fx+e)^2+1\right)}{c^4+2\,c^2d^2+d^4}-\frac{2\,(bc^2d-2\,acd^2-bd^3)\log(|d\tan(fx+e)+c|)}{c^4d+2\,c^2d^3+d^5}+\frac{2\,(bc^2d\tan(fx+e)-2\,acd^2\tan(fx+e)-bd^3\tan(fx+e)+2\,bc^3-3\,ac^2d-ad^3)}{(c^4+2\,c^2d^2+d^4)(d\tan(fx+e)+c)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(c+d*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(a*c^2 + 2*b*c*d - a*d^2)*(f*x + e)/(c^4 + 2*c^2*d^2 + d^4) + (b*c^2
 - 2*a*c*d - b*d^2)*log(tan(f*x + e)^2 + 1)/(c^4 + 2*c^2*d^2 + d^4) - 2*(b*
c^2*d - 2*a*c*d^2 - b*d^3)*log(abs(d*tan(f*x + e) + c))/(c^4*d + 2*c^2*d^3
+ d^5) + 2*(b*c^2*d*tan(f*x + e) - 2*a*c*d^2*tan(f*x + e) - b*d^3*tan(f*x +
 e) + 2*b*c^3 - 3*a*c^2*d - a*d^3)/((c^4 + 2*c^2*d^2 + d^4)*(d*tan(f*x + e)
 + c)))/f
```

Mupad [B]

time = 5.61, size = 153, normalized size = 1.38

$$\frac{\ln\left(c+d\tan(e+f\,x)\right)\,\left(-b\,c^2+2\,a\,c\,d+b\,d^2\right)}{f\,{\left(c^2+d^2\right)}^2}-\frac{a\,d-b\,c}{f\,\left(c^2+d^2\right)\,\left(c+d\tan\left(e+f\,x\right)\right)}-\frac{\ln\left(\tan(e+f\,x)+1{}\mathrm{i}\right)\,\left(b+a\,1{}\mathrm{i}\right)}{2\,f\,\left(-c^2+c\,d\,2{}\mathrm{i}+d^2\right)}-\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(a+b\,1{}\mathrm{i}\right)}{2\,f\,\left(-c^2\,1{}\mathrm{i}+2\,c\,d+d^2\,1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))/(c + d*tan(e + f*x))^2,x)
```

```
[Out] (log(c + d*tan(e + f*x))*(b*d^2 - b*c^2 + 2*a*c*d))/(f*(c^2 + d^2)^2) - (a*
d - b*c)/(f*(c^2 + d^2)*(c + d*tan(e + f*x))) - (log(tan(e + f*x) + 1i)*(a*
1i + b))/(2*f*(c*d*2i - c^2 + d^2)) - (log(tan(e + f*x) - 1i)*(a + b*1i))/(
2*f*(2*c*d - c^2*1i + d^2*1i))
```

3.1220 $\int \frac{1}{(a+b\tan(e+fx))(c+d\tan(e+fx))^2}\,dx$

Optimal. Leaf size=184

$$-\frac{(2bcd-a(c^2-d^2))\,x}{(a^2+b^2)\,(c^2+d^2)^2}+\frac{b^3\log(a\cos(e+fx)+b\sin(e+fx))}{(a^2+b^2)\,(bc-ad)^2f}+\frac{d^2(2acd-b(3c^2+d^2))\log(c\cos(e+fx)+d}{(bc-ad)^2\,(c^2+d^2)^2\,f}$$

```
[Out] -(2*b*c*d-a*(c^2-d^2))*x/(a^2+b^2)/(c^2+d^2)^2+b^3*ln(a*cos(f*x+e)+b*sin(f*
x+e))/(a^2+b^2)/(-a*d+b*c)^2/f+d^2*(2*a*c*d-b*(3*c^2+d^2))*ln(c*cos(f*x+e)+
d*sin(f*x+e))/(-a*d+b*c)^2/(c^2+d^2)^2/f+d^2/(-a*d+b*c)/(c^2+d^2)/f/(c+d*ta
n(f*x+e))
```

Rubi [A]
time = 0.36, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3650, 3732, 3611}

$$-\frac{x(2bcd-a(c^2-d^2))}{(a^2+b^2)\,(c^2+d^2)^2}+\frac{b^3\log(a\cos(e+fx)+b\sin(e+fx))}{f\,(a^2+b^2)\,(bc-ad)^2}+\frac{d^2}{f\,(c^2+d^2)\,(bc-ad)(c+d\tan(e+fx))}+\frac{d^2(2acd-b(3c^2+d^2))\log(c\cos(e+fx)+d\sin(e+fx))}{f\,(c^2+d^2)^2\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^2),x]
```

```
[Out] -(((2*b*c*d - a*(c^2 - d^2))*x)/((a^2 + b^2)*(c^2 + d^2)^2)) + (b^3*Log[a*C
os[e + f*x] + b*Sin[e + f*x]])/((a^2 + b^2)*(b*c - a*d)^2*f) + (d^2*(2*a*c*
d - b*(3*c^2 + d^2))*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((b*c - a*d)^2*(
c^2 + d^2)^2*f) + d^2/((b*c - a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x]))
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\tan(e+fx))(c+d\tan(e+fx))^2}\,dx = \frac{d^2}{(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))} + \frac{\int \frac{-acd+b\left(c^2+d^2\right)-}{(a+b\,\mathrm{t}}}{(}$$

$$= -\frac{\left(2bcd-a\left(c^2-d^2\right)\right)x}{\left(a^2+b^2\right)\left(c^2+d^2\right)^2} + \frac{d^2}{(bc-ad)\left(c^2+d^2\right)f(c+d\tan(}$$

$$= -\frac{\left(2bcd-a\left(c^2-d^2\right)\right)x}{\left(a^2+b^2\right)\left(c^2+d^2\right)^2} + \frac{b^3\log(a\cos(e+fx)+b\sin(e+}{\left(a^2+b^2\right)(bc-ad)^2f}$$

Mathematica [A]
time = 2.24, size = 306, normalized size = 1.66

$$\frac{\frac{(bc-ad)^2\left(2b\left(a-\sqrt{-b^2}\right)cd+b^2\left(c^2-d^2\right)+a\sqrt{-b^2}\left(c^2-d^2\right)\right)\log\left(\sqrt{-b^2}-b\tan(e+fx)\right)-2b^4\left(c^2+d^2\right)^2\log(a+b\tan(e+fx))+(bc-ad)^2\left(2b\left(a+\sqrt{-b^2}\right)cd+b^2\left(c^2-d^2\right)+a\sqrt{-b^2}\left(-c^2+d^2\right)\right)\log\left(\sqrt{-b^2}+b\tan(e+fx)\right)+2b\left(a^2+b^2\right)d^2\left(-2acd+b\left(3c^2+d^2\right)\right)\log(c+d\tan(e+fx))}{2b(a^2+b^2)(bc-ad)(c^2+d^2)}-\frac{d^2}{c+d\tan(e+fx)}}{(-bc+ad)\left(c^2+d^2\right)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^2),x]
```

```
[Out] (((b*c - a*d)^2*(2*b*(a - Sqrt[-b^2])*c*d + b^2*(c^2 - d^2) + a*Sqrt[-b^2]*
(c^2 - d^2))*Log[Sqrt[-b^2] - b*Tan[e + f*x]] - 2*b^4*(c^2 + d^2)^2*Log[a +
 b*Tan[e + f*x]] + (b*c - a*d)^2*(2*b*(a + Sqrt[-b^2])*c*d + b^2*(c^2 - d^2
) + a*Sqrt[-b^2]*(-c^2 + d^2))*Log[Sqrt[-b^2] + b*Tan[e + f*x]] + 2*b*(a^2
+ b^2)*d^2*(-2*a*c*d + b*(3*c^2 + d^2))*Log[c + d*Tan[e + f*x]])/(2*b*(a^2
+ b^2)*(b*c - a*d)*(c^2 + d^2)) - d^2/(c + d*Tan[e + f*x]))/((-(b*c) + a*d)
*(c^2 + d^2)*f)
```

Maple [A]
time = 0.39, size = 203, normalized size = 1.10 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(a^2+b^2)/(c^2+d^2)^2*(1/2*(-2*a*c*d-b*c^2+b*d^2)*ln(1+tan(f*x+e)^2)
+(a*c^2-a*d^2-2*b*c*d)*arctan(tan(f*x+e)))-d^2/(a*d-b*c)/(c^2+d^2)/(c+d*tan
(f*x+e))+d^2*(2*a*c*d-3*b*c^2-b*d^2)/(a*d-b*c)^2/(c^2+d^2)^2*ln(c+d*tan(f*x
+e))+b^3/(a^2+b^2)/(a*d-b*c)^2*ln(a+b*tan(f*x+e)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(189) = 378$.
time = 0.54, size = 389, normalized size = 2.11

$$\frac{\frac{2\,b^3\log\left(b\tan\left(fx+e\right)+a\right)}{(a^2b^2+b^4)c^2-2\,(a^3b+ab^3)cd+(a^4+a^2b^2)d^2}+\frac{2\,(ac^2-2\,bcd-ad^2)(fx+e)}{(a^2+b^2)c^4+2\,(a^2+b^2)c^2d^2+(a^2+b^2)d^4}+\frac{2\,d^2}{bc^4-ac^3d+bc^2d^2-acd^3+(bc^3d-ac^2d^2+bcd^3-ad^4)\tan\left(fx+e\right)}-\frac{2\,(3\,bc^2d^2-2\,acd^3+bd^4)\log\left(d\tan\left(fx+e\right)+c\right)}{b^2c^6-2\,abc^5d-4\,abc^3d^3-2\,abcd^5+a^2d^6+(a^2+2\,b^2)c^4d^2+(2\,a^2+b^2)c^2d^4}-\frac{(bc^2+2\,acd-bd^2)\log\left(\tan\left(fx+e\right)^2+1\right)}{(a^2+b^2)c^4+2\,(a^2+b^2)c^2d^2+(a^2+b^2)d^4}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^3*log(b*tan(f*x + e) + a)/((a^2*b^2 + b^4)*c^2 - 2*(a^3*b + a*b^3)
*c*d + (a^4 + a^2*b^2)*d^2) + 2*(a*c^2 - 2*b*c*d - a*d^2)*(f*x + e)/((a^2 +
 b^2)*c^4 + 2*(a^2 + b^2)*c^2*d^2 + (a^2 + b^2)*d^4) + 2*d^2/(b*c^4 - a*c^3
*d + b*c^2*d^2 - a*c*d^3 + (b*c^3*d - a*c^2*d^2 + b*c*d^3 - a*d^4)*tan(f*x
+ e)) - 2*(3*b*c^2*d^2 - 2*a*c*d^3 + b*d^4)*log(d*tan(f*x + e) + c)/(b^2*c^
6 - 2*a*b*c^5*d - 4*a*b*c^3*d^3 - 2*a*b*c*d^5 + a^2*d^6 + (a^2 + 2*b^2)*c^4
*d^2 + (2*a^2 + b^2)*c^2*d^4) - (b*c^2 + 2*a*c*d - b*d^2)*log(tan(f*x + e)^
2 + 1)/((a^2 + b^2)*c^4 + 2*(a^2 + b^2)*c^2*d^2 + (a^2 + b^2)*d^4))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 721 vs. $2(189) = 378$.
time = 1.63, size = 721, normalized size = 3.92

$$\frac{2\,(a^2b+b^3)cd^4-2\,(a^3+ab^2)d^5+2\,(ab^2c^5-a^3cd^4-2\,(a^2b+b^3)c^4d+(a^3+3\,ab^2)c^3d^2)fx+(b^3c^5+2\,b^3c^3d^2+b^3cd^4+(b^3c^4d+2\,b^3c^2d^3+b^3d^5)\tan\left(fx+e\right))\log\left(\frac{b^2\tan\left(fx+e\right)^2+2\,ab\tan\left(fx+e\right)+a^2}{\tan\left(fx+e\right)^2+1}\right)-(3\,(a^2b+b^3)c^3d^2-2\,(a^3+ab^2)c^2d^3+(a^2b+b^3)cd^4+(3\,(a^2b+b^3)c^2d^3-2\,(a^3+ab^2)cd^4+(a^2b+b^3)d^5)\tan\left(fx+e\right))\log\left(\frac{d^2\tan\left(fx+e\right)^2+2\,cd\tan\left(fx+e\right)+c^2}{\tan\left(fx+e\right)^2+1}\right)-2\,((a^2b+b^3)c^2d^3-(a^3+ab^2)cd^4-(ab^2c^4d-a^3d^5-2\,(a^2b+b^3)c^3d^2+(a^3+3\,ab^2)c^2d^3)fx)\tan\left(fx+e\right)}{2\,(((a^2b^2+b^4)c^6d-2\,(a^3b+ab^3)c^5d^2+(a^4+3\,a^2b^2+2\,b^4)c^4d^3-4\,(a^3b+ab^3)c^3d^4+(2\,a^4+3\,a^2b^2+b^4)c^2d^5-2\,(a^3b+ab^3)cd^6+(a^4+a^2b^2)d^7)f\tan\left(fx+e\right)+((a^2b^2+b^4)c^7-2\,(a^3b+ab^3)c^6d+(a^4+3\,a^2b^2+2\,b^4)c^5d^2-4\,(a^3b+ab^3)c^4d^3+(2\,a^4+3\,a^2b^2+b^4)c^3d^4-2\,(a^3b+ab^3)c^2d^5+(a^4+a^2b^2)cd^6)f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a^2*b + b^3)*c*d^4 - 2*(a^3 + a*b^2)*d^5 + 2*(a*b^2*c^5 - a^3*c*d^4
 - 2*(a^2*b + b^3)*c^4*d + (a^3 + 3*a*b^2)*c^3*d^2)*f*x + (b^3*c^5 + 2*b^3*
c^3*d^2 + b^3*c*d^4 + (b^3*c^4*d + 2*b^3*c^2*d^3 + b^3*d^5)*tan(f*x + e))*l
og((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(tan(f*x + e)^2 + 1)) -
(3*(a^2*b + b^3)*c^3*d^2 - 2*(a^3 + a*b^2)*c^2*d^3 + (a^2*b + b^3)*c*d^4 +
(3*(a^2*b + b^3)*c^2*d^3 - 2*(a^3 + a*b^2)*c*d^4 + (a^2*b + b^3)*d^5)*tan(f
*x + e))*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^
2 + 1)) - 2*((a^2*b + b^3)*c^2*d^3 - (a^3 + a*b^2)*c*d^4 - (a*b^2*c^4*d - a
^3*d^5 - 2*(a^2*b + b^3)*c^3*d^2 + (a^3 + 3*a*b^2)*c^2*d^3)*f*x)*tan(f*x +
e))/(((a^2*b^2 + b^4)*c^6*d - 2*(a^3*b + a*b^3)*c^5*d^2 + (a^4 + 3*a^2*b^2
+ 2*b^4)*c^4*d^3 - 4*(a^3*b + a*b^3)*c^3*d^4 + (2*a^4 + 3*a^2*b^2 + b^4)*c^
2*d^5 - 2*(a^3*b + a*b^3)*c*d^6 + (a^4 + a^2*b^2)*d^7)*f*tan(f*x + e) + ((a
^2*b^2 + b^4)*c^7 - 2*(a^3*b + a*b^3)*c^6*d + (a^4 + 3*a^2*b^2 + 2*b^4)*c^5
```

```
*d^2 - 4*(a^3*b + a*b^3)*c^4*d^3 + (2*a^4 + 3*a^2*b^2 + b^4)*c^3*d^4 - 2*(a
^3*b + a*b^3)*c^2*d^5 + (a^4 + a^2*b^2)*c*d^6)*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))**2,x)
```

```
[Out] Exception raised: NotImplementedError >> no valid subset found
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 541 vs. 2(189) = 378.
time = 0.58, size = 541, normalized size = 2.94

$$\frac{\frac{2\,b^4\log(|b\tan(fx+e)+a|)}{a^2b^3c^2+b^5c^2-2\,a^3b^2cd-2\,ab^4cd+a^4bd^2+a^2b^3d^2}+\frac{2\,(ac^2-2\,bcd-ad^2)(fx+e)}{a^2c^4+b^2c^4+2\,a^2c^2d^2+2\,b^2c^2d^2+a^2d^4+b^2d^4}-\frac{(bc^2+2\,acd-bd^2)\log\left(\tan(fx+e)^2+1\right)}{a^2c^4+b^2c^4+2\,a^2c^2d^2+2\,b^2c^2d^2+a^2d^4+b^2d^4}-\frac{2\,(3\,bc^2d^3-2\,acd^4+bd^5)\log(|d\tan(fx+e)+c|)}{b^2c^6d-2\,abc^5d^2+a^2c^4d^3+2\,b^2c^4d^3-4\,abc^3d^4+2\,a^2c^2d^5+b^2c^2d^5-2\,abcd^6+a^2d^7}+\frac{2\,(3\,bc^2d^3\tan(fx+e)-2\,acd^4\tan(fx+e)+bd^5\tan(fx+e)+4\,bc^3d^2-3\,ac^2d^3+2\,bcd^4-ad^5)}{(b^2c^6-2\,abc^5d+a^2c^4d^2+2\,b^2c^4d^2-4\,abc^3d^3+2\,a^2c^2d^4+b^2c^2d^4-2\,abcd^5+a^2d^6)(d\tan(fx+e)+c)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^4*log(abs(b*tan(f*x + e) + a))/(a^2*b^3*c^2 + b^5*c^2 - 2*a^3*b^2*
c*d - 2*a*b^4*c*d + a^4*b*d^2 + a^2*b^3*d^2) + 2*(a*c^2 - 2*b*c*d - a*d^2)*
(f*x + e)/(a^2*c^4 + b^2*c^4 + 2*a^2*c^2*d^2 + 2*b^2*c^2*d^2 + a^2*d^4 + b^
2*d^4) - (b*c^2 + 2*a*c*d - b*d^2)*log(tan(f*x + e)^2 + 1)/(a^2*c^4 + b^2*c
^4 + 2*a^2*c^2*d^2 + 2*b^2*c^2*d^2 + a^2*d^4 + b^2*d^4) - 2*(3*b*c^2*d^3 -
2*a*c*d^4 + b*d^5)*log(abs(d*tan(f*x + e) + c))/(b^2*c^6*d - 2*a*b*c^5*d^2
+ a^2*c^4*d^3 + 2*b^2*c^4*d^3 - 4*a*b*c^3*d^4 + 2*a^2*c^2*d^5 + b^2*c^2*d^5
 - 2*a*b*c*d^6 + a^2*d^7) + 2*(3*b*c^2*d^3*tan(f*x + e) - 2*a*c*d^4*tan(f*x
 + e) + b*d^5*tan(f*x + e) + 4*b*c^3*d^2 - 3*a*c^2*d^3 + 2*b*c*d^4 - a*d^5)
/((b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2 + 2*b^2*c^4*d^2 - 4*a*b*c^3*d^3 + 2*
a^2*c^2*d^4 + b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6)*(d*tan(f*x + e) + c)))/f
```

Mupad [B]
time = 7.36, size = 374, normalized size = 2.03

$$\frac{\ln\left(a+b\tan(e+f\,x)\right)\,\left(\frac{b\,c^2+2\,a\,c\,d-b\,d^2}{\left(a^2+b^2\right)\,{\left(c^2+d^2\right)}^2}+\frac{b\,d^2}{{\left(a\,d-b\,c\right)}^2\,\left(c^2+d^2\right)}-\frac{2\,c\,d^2}{\left(a\,d-b\,c\right)\,{\left(c^2+d^2\right)}^2}\right)}{f}-\frac{\ln\left(c+d\tan(e+f\,x)\right)\,\left(b\,\left(3\,c^2\,d^2+d^4\right)-2\,a\,c\,d^3\right)}{f\,\left(a^2\,c^4\,d^2+2\,a^2\,c^2\,d^4+a^2\,d^6-2\,a\,b\,c^5\,d-4\,a\,b\,c^3\,d^3-2\,a\,b\,c\,d^5+b^2\,c^6+2\,b^2\,c^4\,d^2+b^2\,c^2\,d^4\right)}-\frac{d^2}{f\,\left(a\,d-b\,c\right)\,\left(c^2+d^2\right)\,\left(c+d\tan\left(e+f\,x\right)\right)}-\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,1{}\mathrm{i}}{2\,f\,\left(a\,c^2-a\,d^2-2\,b\,c\,d+b\,c^2\,1{}\mathrm{i}-b\,d^2\,1{}\mathrm{i}+a\,c\,d\,2{}\mathrm{i}\right)}-\frac{\ln\left(\tan(e+f\,x)+1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2\,f\,\left(a\,d^2-a\,c^2+2\,b\,c\,d+b\,c^2\,1{}\mathrm{i}-b\,d^2\,1{}\mathrm{i}+a\,c\,d\,2{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*tan(e + f*x))*(c + d*tan(e + f*x))^2),x)
```

```
[Out] (log(a + b*tan(e + f*x))*((b*c^2 - b*d^2 + 2*a*c*d)/((a^2 + b^2)*(c^2 + d^2
)^2) + (b*d^2)/((a*d - b*c)^2*(c^2 + d^2)) - (2*c*d^2)/((a*d - b*c)*(c^2 +
d^2)^2)))/f - (log(tan(e + f*x) + 1i)*1i)/(2*f*(a*d^2 - a*c^2 + b*c^2*1i -
b*d^2*1i + a*c*d*2i + 2*b*c*d)) - (log(tan(e + f*x) - 1i)*1i)/(2*f*(a*c^2 -
 a*d^2 + b*c^2*1i - b*d^2*1i + a*c*d*2i - 2*b*c*d)) - (log(c + d*tan(e + f*
x))*(b*(d^4 + 3*c^2*d^2) - 2*a*c*d^3))/(f*(a^2*d^6 + b^2*c^6 + 2*a^2*c^2*d^
4 + a^2*c^4*d^2 + b^2*c^2*d^4 + 2*b^2*c^4*d^2 - 2*a*b*c*d^5 - 2*a*b*c^5*d -
 4*a*b*c^3*d^3)) - d^2/(f*(a*d - b*c)*(c^2 + d^2)*(c + d*tan(e + f*x)))
```

3.1221 $\int \frac{1}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^2}\,dx$

Optimal. Leaf size=290

$$\frac{(b(c-d)+a(c+d))(a(c-d)-b(c+d))x}{(a^2+b^2)^2(c^2+d^2)^2}+\frac{2b^3(abc-2a^2d-b^2d)\log(a\cos(e+fx)+b\sin(e+fx))}{(a^2+b^2)^2(bc-ad)^3f}-\frac{2d^3(}{}$$

```
[Out] (b*(c-d)+a*(c+d))*(a*(c-d)-b*(c+d))*x/(a^2+b^2)^2/(c^2+d^2)^2+2*b^3*(-2*a^2
*d+a*b*c-b^2*d)*ln(a*cos(f*x+e)+b*sin(f*x+e))/(a^2+b^2)^2/(-a*d+b*c)^3/f-2*
d^3*(a*c*d-b*(2*c^2+d^2))*ln(c*cos(f*x+e)+d*sin(f*x+e))/(-a*d+b*c)^3/(c^2+d
^2)^2/f-d*(a^2*d^2+b^2*(c^2+2*d^2))/(a^2+b^2)/(-a*d+b*c)^2/(c^2+d^2)/f/(c+d
*tan(f*x+e))-b^2/(a^2+b^2)/(-a*d+b*c)/f/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))
```

Rubi [A]
time = 0.71, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3650, 3730, 3732, 3611}

$$-\frac{d(a^2d^2+b^2(c^2+2d^2))}{f(a^2+b^2)(c^2+d^2)(bc-ad)^2(c+d\tan(e+fx))}+\frac{x(a(c+d)+b(c-d))(a(c-d)-b(c+d))}{(a^2+b^2)^2(c^2+d^2)^2}-\frac{b^2}{f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))(c+d\tan(e+fx))}+\frac{2b^3(-2a^2d+abc-b^2d)\log(a\cos(e+fx)+b\sin(e+fx))}{f(a^2+b^2)^2(bc-ad)^3}-\frac{2d^3(acd-b(2c^2+d^2))\log(c\cos(e+fx)+d\sin(e+fx))}{f(c^2+d^2)^2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^2),x]
```

```
[Out] ((b*(c - d) + a*(c + d))*(a*(c - d) - b*(c + d))*x)/((a^2 + b^2)^2*(c^2 + d
^2)^2) + (2*b^3*(a*b*c - 2*a^2*d - b^2*d)*Log[a*Cos[e + f*x] + b*Sin[e + f*
x]])/((a^2 + b^2)^2*(b*c - a*d)^3*f) - (2*d^3*(a*c*d - b*(2*c^2 + d^2))*Log
[c*Cos[e + f*x] + d*Sin[e + f*x]])/((b*c - a*d)^3*(c^2 + d^2)^2*f) - (d*(a^
2*d^2 + b^2*(c^2 + 2*d^2)))/((a^2 + b^2)*(b*c - a*d)^2*(c^2 + d^2)*f*(c + d
*Tan[e + f*x])) - b^2/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])*(c +
d*Tan[e + f*x]))
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
```

```
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^2}\,dx = -\frac{b^2}{(a^2+b^2)\,(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+f}$$

$$= -\frac{d(a^2d^2+b^2(c^2+2d^2))}{(a^2+b^2)\,(bc-ad)^2\,(c^2+d^2)\,f(c+d\tan(e+fx))} - \frac{}{(a}$$

$$= \frac{(b(c-d)+a(c+d))(a(c-d)-b(c+d))x}{(a^2+b^2)^2\,(c^2+d^2)^2} - \frac{}{(a^2+b^2)\,(}$$

$$= \frac{(b(c-d)+a(c+d))(a(c-d)-b(c+d))x}{(a^2+b^2)^2\,(c^2+d^2)^2} + \frac{2b^3(abc-}{}$$

Mathematica [A]

time = 6.95, size = 556, normalized size = 1.92

$$-\frac{b^2}{(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+fx))}-\frac{-\frac{\frac{b(bc-ad)^2\left(2abc^2+2a^2cd-2b^2cd-2abd^2+\frac{b\left(4abcd-a^2(c^2-d^2)+b^2(c^2-d^2)\right)}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\tan(e+fx)\right)}{2(a^2+b^2)(c^2+d^2)}-\frac{2b^4(abc-2a^2d-b^2d)(c^2+d^2)\log(a+b\tan(e+fx))}{(a^2+b^2)(bc-ad)}+\frac{b(bc-ad)^2\left(2abc^2+2a^2cd-2b^2cd-2abd^2+\frac{\sqrt{-b^2}\left(4abcd-a^2(c^2-d^2)+b^2(c^2-d^2)\right)}{b}\right)\log\left(\sqrt{-b^2}+b\tan(e+fx)\right)}{2(a^2+b^2)(c^2+d^2)}+\frac{2b(a^2+b^2)d^3\left(acd-b(2c^2+d^2)\right)\log(c+d\tan(e+fx))}{(bc-ad)(c^2+d^2)}}{b(-bc+ad)(c^2+d^2)f}-\frac{d^2(-abc+a^2d+2b^2d)-c(-2b^2cd+bd(bc-ad))}{(-bc+ad)(c^2+d^2)f(c+d\tan(e+fx))}}{(a^2+b^2)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^2),x]
```

```
[Out] -(b^2/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x]))
) - (-(((b*(b*c - a*d)^2*(2*a*b*c^2 + 2*a^2*c*d - 2*b^2*c*d - 2*a*b*d^2 + (
b*(4*a*b*c*d - a^2*(c^2 - d^2) + b^2*(c^2 - d^2))))/Sqrt[-b^2])*Log[Sqrt[-b^
2] - b*Tan[e + f*x]])/(2*(a^2 + b^2)*(c^2 + d^2)) - (2*b^4*(a*b*c - 2*a^2*d
 - b^2*d)*(c^2 + d^2)*Log[a + b*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*d)) +
(b*(b*c - a*d)^2*(2*a*b*c^2 + 2*a^2*c*d - 2*b^2*c*d - 2*a*b*d^2 + (Sqrt[-b^
2]*(4*a*b*c*d - a^2*(c^2 - d^2) + b^2*(c^2 - d^2)))/b)*Log[Sqrt[-b^2] + b*T
an[e + f*x]])/(2*(a^2 + b^2)*(c^2 + d^2)) + (2*b*(a^2 + b^2)*d^3*(a*c*d - b
*(2*c^2 + d^2))*Log[c + d*Tan[e + f*x]])/((b*c - a*d)*(c^2 + d^2)))/(b*(-(b
*c) + a*d)*(c^2 + d^2)*f)) - (d^2*(-(a*b*c) + a^2*d + 2*b^2*d) - c*(-2*b^2*
c*d + b*d*(b*c - a*d)))/((-(b*c) + a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x]))
)/((a^2 + b^2)*(b*c - a*d))
```

Maple [A]

time = 0.68, size = 289, normalized size = 1.00

method	result
derivativedivides	$\frac{\frac{\left(-2a^2cd-2abc^2+2abd^2+2b^2cd\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(a^2c^2-a^2d^2-4abcd-b^2c^2+b^2d^2\right)\arctan(\tan(fx+e))}{\left(a^2+b^2\right)^2\left(c^2+d^2\right)^2}-\frac{d^3}{(ad-bc)^2\left(c^2+d^2\right)(c+\cdots}$
default	$\frac{\frac{\left(-2a^2cd-2abc^2+2abd^2+2b^2cd\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(a^2c^2-a^2d^2-4abcd-b^2c^2+b^2d^2\right)\arctan(\tan(fx+e))}{\left(a^2+b^2\right)^2\left(c^2+d^2\right)^2}-\frac{d^3}{(ad-bc)^2\left(c^2+d^2\right)(c+\cdots}$
norman	$\frac{\frac{(ad+bc)\left(a^2c^2-a^2d^2-4abcd-b^2c^2+b^2d^2\right)x\tan(fx+e)}{\left(c^4+2c^2d^2+d^4\right)\left(a^4+2a^2b^2+b^4\right)}+\frac{\left(a^2c^2-a^2d^2-4abcd-b^2c^2+b^2d^2\right)acx}{\left(c^4+2c^2d^2+d^4\right)\left(a^4+2a^2b^2+b^4\right)}+\frac{bd\left(a^2c^2-a^2d^2-4abcd-b^2c^2+b^2d^2\right)\cdots}{\left(c^4+2c^2d^2+d^4\right)\left(a^4+2a^2b\cdots\right.}}{(a+b\tan(fx+e))(c+d\tan(\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(a^2+b^2)^2/(c^2+d^2)^2*(1/2*(-2*a^2*c*d-2*a*b*c^2+2*a*b*d^2+2*b^2*c
*d)*ln(1+tan(f*x+e)^2)+(a^2*c^2-a^2*d^2-4*a*b*c*d-b^2*c^2+b^2*d^2)*arctan(t
an(f*x+e)))-d^3/(a*d-b*c)^2/(c^2+d^2)/(c+d*tan(f*x+e))+2*d^3*(a*c*d-2*b*c^2
-b*d^2)/(a*d-b*c)^3/(c^2+d^2)^2*ln(c+d*tan(f*x+e))-b^3/(a^2+b^2)/(a*d-b*c)^
2/(a+b*tan(f*x+e))+2*b^3*(2*a^2*d-a*b*c+b^2*d)/(a^2+b^2)^2/(a*d-b*c)^3*ln(a
+b*tan(f*x+e)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 885 vs. $2(298) = 596$.
time = 0.63, size = 885, normalized size = 3.05

$$-\frac{\frac{(4abcd-(a^2-b^2)c^2+(a^2-b^2)d^2)(fx+e)}{(a^4+2a^2b^2+b^4)c^4+2(a^4+2a^2b^2+b^4)c^2d^2+(a^4+2a^2b^2+b^4)d^4} - \frac{2(ab^4c-(2a^2b^3+b^5)d)\log(b\tan(fx+e)+a)}{(a^4b^3+2a^2b^5+b^7)c^3-3(a^5b^2+2a^3b^4+ab^6)c^2d+3(a^6b+2a^4b^3+a^2b^5)cd^2-(a^7+2a^5b^2+a^3b^4)d^3} - \frac{2(2bc^2d^3-acd^4+bd^5)\log(d\tan(fx+e)+c)}{b^3c^7-3ab^2c^6d+3a^2bcd^6-a^3d^7+(3a^2b+2b^3)c^5d^2-(a^3+6ab^2)c^4d^3+(6a^2b+b^3)c^3d^4-(2a^3+3ab^2)c^2d^5} + \frac{(abc^2-abd^2+(a^2-b^2)cd)\log\left(\tan(fx+e)^2+1\right)}{(a^4+2a^2b^2+b^4)c^4+2(a^4+2a^2b^2+b^4)c^2d^2+(a^4+2a^2b^2+b^4)d^4} + \frac{b^3c^3+b^3cd^2+(a^3+ab^2)d^3+(b^3c^2d+(a^2b+2b^3)d^3)\tan(fx+e)}{(a^3b^2+ab^4)c^5-2(a^4b+a^2b^3)c^4d+(a^5+2a^3b^2+ab^4)c^3d^2-2(a^4b+a^2b^3)c^2d^3+(a^5+a^3b^2)cd^4+((a^2b^3+b^5)c^4d-2(a^3b^2+ab^4)c^3d^2+(a^4b+2a^2b^3+b^5)c^2d^3-2(a^3b^2+ab^4)cd^4+(a^4b+a^2b^3)d^5)\tan(fx+e)^2+((a^2b^3+b^5)c^5-(a^3b^2+ab^4)c^4d-(a^4b-b^5)c^3d^2+(a^5-ab^4)c^2d^3-(a^4b+a^2b^3)cd^4+(a^5+a^3b^2)d^5)\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -((4*a*b*c*d - (a^2 - b^2)*c^2 + (a^2 - b^2)*d^2)*(f*x + e)/((a^4 + 2*a^2*b
^2 + b^4)*c^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*c^2*d^2 + (a^4 + 2*a^2*b^2 + b^4)
*d^4) - 2*(a*b^4*c - (2*a^2*b^3 + b^5)*d)*log(b*tan(f*x + e) + a)/((a^4*b^3
 + 2*a^2*b^5 + b^7)*c^3 - 3*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*c^2*d + 3*(a^6*b
+ 2*a^4*b^3 + a^2*b^5)*c*d^2 - (a^7 + 2*a^5*b^2 + a^3*b^4)*d^3) - 2*(2*b*c^
2*d^3 - a*c*d^4 + b*d^5)*log(d*tan(f*x + e) + c)/(b^3*c^7 - 3*a*b^2*c^6*d +
 3*a^2*b*c*d^6 - a^3*d^7 + (3*a^2*b + 2*b^3)*c^5*d^2 - (a^3 + 6*a*b^2)*c^4*
d^3 + (6*a^2*b + b^3)*c^3*d^4 - (2*a^3 + 3*a*b^2)*c^2*d^5) + (a*b*c^2 - a*b
*d^2 + (a^2 - b^2)*c*d)*log(tan(f*x + e)^2 + 1)/((a^4 + 2*a^2*b^2 + b^4)*c^
4 + 2*(a^4 + 2*a^2*b^2 + b^4)*c^2*d^2 + (a^4 + 2*a^2*b^2 + b^4)*d^4) + (b^3
*c^3 + b^3*c*d^2 + (a^3 + a*b^2)*d^3 + (b^3*c^2*d + (a^2*b + 2*b^3)*d^3)*ta
n(f*x + e))/((a^3*b^2 + a*b^4)*c^5 - 2*(a^4*b + a^2*b^3)*c^4*d + (a^5 + 2*a
^3*b^2 + a*b^4)*c^3*d^2 - 2*(a^4*b + a^2*b^3)*c^2*d^3 + (a^5 + a^3*b^2)*c*d
^4 + ((a^2*b^3 + b^5)*c^4*d - 2*(a^3*b^2 + a*b^4)*c^3*d^2 + (a^4*b + 2*a^2*
b^3 + b^5)*c^2*d^3 - 2*(a^3*b^2 + a*b^4)*c*d^4 + (a^4*b + a^2*b^3)*d^5)*tan
(f*x + e)^2 + ((a^2*b^3 + b^5)*c^5 - (a^3*b^2 + a*b^4)*c^4*d - (a^4*b - b^5
)*c^3*d^2 + (a^5 - a*b^4)*c^2*d^3 - (a^4*b + a^2*b^3)*c*d^4 + (a^5 + a^3*b^
2)*d^5)*tan(f*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2231 vs. $2(298) = 596$.
time = 2.91, size = 2231, normalized size = 7.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -(b^6*c^6 - a*b^5*c^5*d + 2*b^6*c^4*d^2 - 2*a*b^5*c^3*d^3 + b^6*c^2*d^4 + (
a^5*b + 2*a^3*b^3)*c*d^5 - (a^6 + 2*a^4*b^2 + a^2*b^4)*d^6 - ((a^3*b^3 - a*
b^5)*c^6 - (3*a^4*b^2 + a^2*b^4)*c^5*d + (3*a^5*b + 8*a^3*b^3 + a*b^5)*c^4*
d^2 - (a^6 + 8*a^4*b^2 + 3*a^2*b^4)*c^3*d^3 + (a^5*b + 3*a^3*b^3)*c^2*d^4 +
 (a^6 - a^4*b^2)*c*d^5)*f*x - (a*b^5*c^5*d - a^2*b^4*c^4*d^2 + 2*a*b^5*c^3*
d^3 - a^2*b^4*d^6 + (a^4*b^2 + b^6)*c^2*d^4 - (a^5*b + 2*a^3*b^3)*c*d^5 + (
(a^2*b^4 - b^6)*c^5*d - (3*a^3*b^3 + a*b^5)*c^4*d^2 + (3*a^4*b^2 + 8*a^2*b^
4 + b^6)*c^3*d^3 - (a^5*b + 8*a^3*b^3 + 3*a*b^5)*c^2*d^4 + (a^4*b^2 + 3*a^2
```

```
*b^4)*c*d^5 + (a^5*b - a^3*b^3)*d^6)*f*x)*tan(f*x + e)^2 - (a^2*b^4*c^6 + 2
*a^2*b^4*c^4*d^2 + a^2*b^4*c^2*d^4 - (2*a^3*b^3 + a*b^5)*c^5*d - 2*(2*a^3*b
^3 + a*b^5)*c^3*d^3 - (2*a^3*b^3 + a*b^5)*c*d^5 + (a*b^5*c^5*d + 2*a*b^5*c^
3*d^3 + a*b^5*c*d^5 - (2*a^2*b^4 + b^6)*c^4*d^2 - 2*(2*a^2*b^4 + b^6)*c^2*d
^4 - (2*a^2*b^4 + b^6)*d^6)*tan(f*x + e)^2 + (a*b^5*c^6 - (a^2*b^4 + b^6)*c
^5*d - (2*a^3*b^3 - a*b^5)*c^4*d^2 - 2*(a^2*b^4 + b^6)*c^3*d^3 - (4*a^3*b^3
 + a*b^5)*c^2*d^4 - (a^2*b^4 + b^6)*c*d^5 - (2*a^3*b^3 + a*b^5)*d^6)*tan(f*
x + e))*log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(tan(f*x + e)^2
 + 1)) - (2*(a^5*b + 2*a^3*b^3 + a*b^5)*c^3*d^3 - (a^6 + 2*a^4*b^2 + a^2*b^
4)*c^2*d^4 + (a^5*b + 2*a^3*b^3 + a*b^5)*c*d^5 + (2*(a^4*b^2 + 2*a^2*b^4 +
b^6)*c^2*d^4 - (a^5*b + 2*a^3*b^3 + a*b^5)*c*d^5 + (a^4*b^2 + 2*a^2*b^4 + b
^6)*d^6)*tan(f*x + e)^2 + (2*(a^4*b^2 + 2*a^2*b^4 + b^6)*c^3*d^3 + (a^5*b +
 2*a^3*b^3 + a*b^5)*c^2*d^4 - (a^6 + a^4*b^2 - a^2*b^4 - b^6)*c*d^5 + (a^5*
b + 2*a^3*b^3 + a*b^5)*d^6)*tan(f*x + e))*log((d^2*tan(f*x + e)^2 + 2*c*d*t
an(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) - (a*b^5*c^6 + 3*a*b^5*c^4*d^2 - (
a^2*b^4 + b^6)*c^5*d - 2*(a^2*b^4 + b^6)*c^3*d^3 + (a^5*b + 2*a^3*b^3 + 4*a
*b^5)*c^2*d^4 - (a^6 + 3*a^4*b^2 + 4*a^2*b^4 + 2*b^6)*c*d^5 + (a^5*b + 2*a^
3*b^3 + 2*a*b^5)*d^6 + ((a^2*b^4 - b^6)*c^6 - 2*(a^3*b^3 + a*b^5)*c^5*d + (
7*a^2*b^4 + b^6)*c^4*d^2 + 2*(a^5*b - a*b^5)*c^3*d^3 - (a^6 + 7*a^4*b^2)*c^
2*d^4 + 2*(a^5*b + a^3*b^3)*c*d^5 + (a^6 - a^4*b^2)*d^6)*f*x)*tan(f*x + e))
/(((a^4*b^4 + 2*a^2*b^6 + b^8)*c^7*d - 3*(a^5*b^3 + 2*a^3*b^5 + a*b^7)*c^6*
d^2 + (3*a^6*b^2 + 8*a^4*b^4 + 7*a^2*b^6 + 2*b^8)*c^5*d^3 - (a^7*b + 8*a^5*
b^3 + 13*a^3*b^5 + 6*a*b^7)*c^4*d^4 + (6*a^6*b^2 + 13*a^4*b^4 + 8*a^2*b^6 +
 b^8)*c^3*d^5 - (2*a^7*b + 7*a^5*b^3 + 8*a^3*b^5 + 3*a*b^7)*c^2*d^6 + 3*(a^
6*b^2 + 2*a^4*b^4 + a^2*b^6)*c*d^7 - (a^7*b + 2*a^5*b^3 + a^3*b^5)*d^8)*f*t
an(f*x + e)^2 + ((a^4*b^4 + 2*a^2*b^6 + b^8)*c^8 - 2*(a^5*b^3 + 2*a^3*b^5 +
 a*b^7)*c^7*d + 2*(a^4*b^4 + 2*a^2*b^6 + b^8)*c^6*d^2 + 2*(a^7*b - 3*a^3*b^
5 - 2*a*b^7)*c^5*d^3 - (a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*c^4*d^4 + 2*(2*a
^7*b + 3*a^5*b^3 - a*b^7)*c^3*d^5 - 2*(a^8 + 2*a^6*b^2 + a^4*b^4)*c^2*d^6 +
 2*(a^7*b + 2*a^5*b^3 + a^3*b^5)*c*d^7 - (a^8 + 2*a^6*b^2 + a^4*b^4)*d^8)*f
*tan(f*x + e) + ((a^5*b^3 + 2*a^3*b^5 + a*b^7)*c^8 - 3*(a^6*b^2 + 2*a^4*b^4
 + a^2*b^6)*c^7*d + (3*a^7*b + 8*a^5*b^3 + 7*a^3*b^5 + 2*a*b^7)*c^6*d^2 - (
a^8 + 8*a^6*b^2 + 13*a^4*b^4 + 6*a^2*b^6)*c^5*d^3 + (6*a^7*b + 13*a^5*b^3 +
 8*a^3*b^5 + a*b^7)*c^4*d^4 - (2*a^8 + 7*a^6*b^2 + 8*a^4*b^4 + 3*a^2*b^6)*c
^3*d^5 + 3*(a^7*b + 2*a^5*b^3 + a^3*b^5)*c^2*d^6 - (a^8 + 2*a^6*b^2 + a^4*b
^4)*c*d^7)*f)
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))**2/(c+d*tan(f*x+e))**2,x)
[Out] Exception raised: NotImplementedError >> no valid subset found
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1395 vs. 2(298) = 596.

time = 0.68, size = 1395, normalized size = 4.81

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] ((a^2*c^2 - b^2*c^2 - 4*a*b*c*d - a^2*d^2 + b^2*d^2)*(f*x + e)/(a^4*c^4 + 2
*a^2*b^2*c^4 + b^4*c^4 + 2*a^4*c^2*d^2 + 4*a^2*b^2*c^2*d^2 + 2*b^4*c^2*d^2
+ a^4*d^4 + 2*a^2*b^2*d^4 + b^4*d^4) - (a*b*c^2 + a^2*c*d - b^2*c*d - a*b*d
^2)*log(tan(f*x + e)^2 + 1)/(a^4*c^4 + 2*a^2*b^2*c^4 + b^4*c^4 + 2*a^4*c^2*
d^2 + 4*a^2*b^2*c^2*d^2 + 2*b^4*c^2*d^2 + a^4*d^4 + 2*a^2*b^2*d^4 + b^4*d^4
) + 2*(a*b^5*c - 2*a^2*b^4*d - b^6*d)*log(abs(b*tan(f*x + e) + a))/(a^4*b^4
*c^3 + 2*a^2*b^6*c^3 + b^8*c^3 - 3*a^5*b^3*c^2*d - 6*a^3*b^5*c^2*d - 3*a*b^
7*c^2*d + 3*a^6*b^2*c*d^2 + 6*a^4*b^4*c*d^2 + 3*a^2*b^6*c*d^2 - a^7*b*d^3 -
 2*a^5*b^3*d^3 - a^3*b^5*d^3) + 2*(2*b*c^2*d^4 - a*c*d^5 + b*d^6)*log(abs(d
*tan(f*x + e) + c))/(b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 + 2*b^3*
c^5*d^3 - a^3*c^4*d^4 - 6*a*b^2*c^4*d^4 + 6*a^2*b*c^3*d^5 + b^3*c^3*d^5 - 2
*a^3*c^2*d^6 - 3*a*b^2*c^2*d^6 + 3*a^2*b*c*d^7 - a^3*d^8) - (a*b^4*c^4*d*ta
n(f*x + e)^2 - a^2*b^3*c^3*d^2*tan(f*x + e)^2 - b^5*c^3*d^2*tan(f*x + e)^2
- a^3*b^2*c^2*d^3*tan(f*x + e)^2 + a*b^4*c^2*d^3*tan(f*x + e)^2 + a^4*b*c*d
^4*tan(f*x + e)^2 + a^2*b^3*c*d^4*tan(f*x + e)^2 - a^3*b^2*d^5*tan(f*x + e)
^2 + a*b^4*c^5*tan(f*x + e) + a^2*b^3*c^4*d*tan(f*x + e) - 2*a^3*b^2*c^3*d^
2*tan(f*x + e) + a^4*b*c^2*d^3*tan(f*x + e) + 6*a^2*b^3*c^2*d^3*tan(f*x + e
) + 3*b^5*c^2*d^3*tan(f*x + e) + a^5*c*d^4*tan(f*x + e) + 3*a^2*b^3*d^5*tan
(f*x + e) + 2*b^5*d^5*tan(f*x + e) + 2*a^2*b^3*c^5 + b^5*c^5 - a^3*b^2*c^4*
d - a*b^4*c^4*d - a^4*b*c^3*d^2 + 3*a^2*b^3*c^3*d^2 + 2*b^5*c^3*d^2 + 2*a^5
*c^2*d^3 + 3*a^3*b^2*c^2*d^3 + a*b^4*c^2*d^3 - a^4*b*c*d^4 + a^2*b^3*c*d^4
+ b^5*c*d^4 + a^5*d^5 + 2*a^3*b^2*d^5 + a*b^4*d^5)/((a^4*b^2*c^6 + 2*a^2*b^
4*c^6 + b^6*c^6 - 2*a^5*b*c^5*d - 4*a^3*b^3*c^5*d - 2*a*b^5*c^5*d + a^6*c^4
*d^2 + 4*a^4*b^2*c^4*d^2 + 5*a^2*b^4*c^4*d^2 + 2*b^6*c^4*d^2 - 4*a^5*b*c^3*
d^3 - 8*a^3*b^3*c^3*d^3 - 4*a*b^5*c^3*d^3 + 2*a^6*c^2*d^4 + 5*a^4*b^2*c^2*d
^4 + 4*a^2*b^4*c^2*d^4 + b^6*c^2*d^4 - 2*a^5*b*c*d^5 - 4*a^3*b^3*c*d^5 - 2*
a*b^5*c*d^5 + a^6*d^6 + 2*a^4*b^2*d^6 + a^2*b^4*d^6)*(b*d*tan(f*x + e)^2 +
b*c*tan(f*x + e) + a*d*tan(f*x + e) + a*c)))/f
```

Mupad [B]

time = 10.38, size = 725, normalized size = 2.50

$$\frac{\ln\left(a+b\tan\left(e+f\,x\right)\right)\,\left(d\,\left(4\,a^2\,b^3+2\,b^5\right)-2\,a\,b^4\,c\right)}{f\,\left(a^7\,d^3-3\,a^6\,b\,c\,d^2+3\,a^5\,b^2\,c^2\,d+2\,a^5\,b^2\,d^3-a^4\,b^3\,c^3-6\,a^4\,b^3\,c\,d^2+6\,a^3\,b^4\,c^2\,d+a^3\,b^4\,d^3-2\,a^2\,b^5\,c^3-3\,a^2\,b^5\,c\,d^2+3\,a\,b^6\,c^2\,d-b^7\,c^3\right)}-\frac{\ln\left(\tan\left(e+f\,x\right)-\mathrm{i}\right)}{2\,f\,\left(-a^2\,c^2\,1{}\mathrm{i}+2\,a^2\,c\,d+a^2\,d^2\,1{}\mathrm{i}+2\,a\,b\,c^2+a\,b\,c\,d\,4{}\mathrm{i}-2\,a\,b\,d^2+b^2\,c^2\,1{}\mathrm{i}-2\,b^2\,c\,d-b^2\,d^2\,1{}\mathrm{i}\right)}+\frac{\ln\left(\tan\left(e+f\,x\right)+1{}\mathrm{i}\right)}{2\,f\,\left(-a^2\,c^2\,1{}\mathrm{i}-2\,a^2\,c\,d+a^2\,d^2\,1{}\mathrm{i}-2\,a\,b\,c^2+a\,b\,c\,d\,4{}\mathrm{i}+2\,a\,b\,d^2+b^2\,c^2\,1{}\mathrm{i}+2\,b^2\,c\,d-b^2\,d^2\,1{}\mathrm{i}\right)}-\frac{\frac{a^3\,d^3+a\,b^2\,d^3+b^3\,c^3+b^3\,c\,d^2}{\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)\,\left(a^2\,c^2+a^2\,d^2+b^2\,c^2+b^2\,d^2\right)}+\frac{\tan\left(e+f\,x\right)\,\left(a^2\,b\,d^3+b^3\,c^2\,d+2\,b^3\,d^3\right)}{\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)\,\left(a^2\,c^2+a^2\,d^2+b^2\,c^2+b^2\,d^2\right)}}{f\,\left(b\,d\,{\tan\left(e+f\,x\right)}^2+\left(a\,d+b\,c\right)\,\tan\left(e+f\,x\right)+a\,c\right)}-\frac{\ln\left(c+d\tan\left(e+f\,x\right)\right)\,\left(b\,\left(4\,c^2\,d^3+2\,d^5\right)-2\,a\,c\,d^4\right)}{f\,\left(a^3\,c^4\,d^3+2\,a^3\,c^2\,d^5+a^3\,d^7-3\,a^2\,b\,c^5\,d^2-6\,a^2\,b\,c^3\,d^4-3\,a^2\,b\,c\,d^6+3\,a\,b^2\,c^6\,d+6\,a\,b^2\,c^4\,d^3+3\,a\,b^2\,c^2\,d^5-b^3\,c^7-2\,b^3\,c^5\,d^2-b^3\,c^3\,d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*tan(e + f*x))^2*(c + d*tan(e + f*x))^2),x)
```

```
[Out] log(tan(e + f*x) + 1i)/(2*f*(a^2*d^2*1i - a^2*c^2*1i + b^2*c^2*1i - b^2*d^2
*1i - 2*a*b*c^2 + 2*a*b*d^2 - 2*a^2*c*d + 2*b^2*c*d + a*b*c*d*4i)) - log(ta
n(e + f*x) - 1i)/(2*f*(a^2*d^2*1i - a^2*c^2*1i + b^2*c^2*1i - b^2*d^2*1i +
2*a*b*c^2 - 2*a*b*d^2 + 2*a^2*c*d - 2*b^2*c*d + a*b*c*d*4i)) - ((a^3*d^3 +
b^3*c^3 + a*b^2*d^3 + b^3*c*d^2)/((a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^2*c^2
+ a^2*d^2 + b^2*c^2 + b^2*d^2)) + (tan(e + f*x)*(2*b^3*d^3 + a^2*b*d^3 + b^
3*c^2*d))/((a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^2*c^2 + a^2*d^2 + b^2*c^2 + b
^2*d^2)))/(f*(a*c + tan(e + f*x)*(a*d + b*c) + b*d*tan(e + f*x)^2)) + (log(
a + b*tan(e + f*x))*(d*(2*b^5 + 4*a^2*b^3) - 2*a*b^4*c))/(f*(a^7*d^3 - b^7*
c^3 - 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 + 2*a^5*b^2*d^3 - 3*a^2*b^5
*c*d^2 + 6*a^3*b^4*c^2*d - 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*
d - 3*a^6*b*c*d^2)) - (log(c + d*tan(e + f*x))*(b*(2*d^5 + 4*c^2*d^3) - 2*a
*c*d^4))/(f*(a^3*d^7 - b^3*c^7 + 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4
- 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 + 6*a*b^2*c^4*d^3 - 6*a^2*b*c^3*d^4 - 3*a
^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6))
```

3.1222 $\int \frac{1}{(a+b\tan(e+fx))^3(c+d\tan(e+fx))^2}\,dx$

Optimal. Leaf size=457

$$-\frac{(6a^2bcd-2b^3cd-a^3(c^2-d^2)+3ab^2(c^2-d^2))\,x}{(a^2+b^2)^3\,(c^2+d^2)^2}-\frac{b^3(10a^3bcd+2ab^3cd-10a^4d^2+b^4(c^2-3d^2)-3a^2b^2(}{(a^2+b^2)^3\,(bc-}$$

```
[Out] -(6*a^2*b*c*d-2*b^3*c*d-a^3*(c^2-d^2)+3*a*b^2*(c^2-d^2))*x/(a^2+b^2)^3/(c^2
+d^2)^2-b^3*(10*a^3*b*c*d+2*a*b^3*c*d-10*a^4*d^2+b^4*(c^2-3*d^2)-3*a^2*b^2*
(c^2+3*d^2))*ln(a*cos(f*x+e)+b*sin(f*x+e))/(a^2+b^2)^3/(-a*d+b*c)^4/f-d^4*(
-2*a*c*d+5*b*c^2+3*b*d^2)*ln(c*cos(f*x+e)+d*sin(f*x+e))/(-a*d+b*c)^4/(c^2+d
^2)^2/f+d*(a^4*d^3-2*a*b^3*c*(c^2+d^2)+2*a^2*b^2*d*(2*c^2+3*d^2)+b^4*d*(2*c
^2+3*d^2))/(a^2+b^2)^2/(-a*d+b*c)^3/(c^2+d^2)/f/(c+d*tan(f*x+e))-1/2*b^2/(a
^2+b^2)/(-a*d+b*c)/f/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))-1/2*b^2*(-7*a^2*d+
4*a*b*c-3*b^2*d)/(a^2+b^2)^2/(-a*d+b*c)^2/f/(a+b*tan(f*x+e))/(c+d*tan(f*x+e
))
```

Rubi [A]

time = 1.24, antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3650, 3730, 3732, 3611}

$$-\frac{b^2(-7a^2d+4abc-3b^2d)}{2f\,(a^2+b^2)^2\,(bc-ad)^2(a+b\tan(e+fx))(c+d\tan(e+fx))}-\frac{b^2}{2f\,(a^2+b^2)\,(bc-ad)(a+b\tan(e+fx))^2(c+d\tan(e+fx))}+\frac{d(a^4d^3+2a^2b^2d(2c^2+3d^2)-2ab^3c(c^2+d^2)+b^4d(2c^2+3d^2))}{f\,(a^2+b^2)^2\,(c^2+d^2)\,(bc-ad)^3(c+d\tan(e+fx))}-\frac{x(-(a^3(c^2-d^2))+6a^2bcd+3ab^2(c^2-d^2)-2b^3cd)}{(a^2+b^2)^3\,(c^2+d^2)^2}-\frac{b^3(-10a^4d^2+10a^3bcd-3a^2b^2(c^2+3d^2)+2ab^3cd+b^4(c^2-3d^2))\log(a\cos(e+fx)+b\sin(e+fx))}{f\,(a^2+b^2)^3\,(bc-ad)^4}-\frac{d^4(-2acd+5bc^2+3bd^2)\log(c\cos(e+fx)+d\sin(e+fx))}{f\,(c^2+d^2)^2\,(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^2),x]
```

```
[Out] -(((6*a^2*b*c*d - 2*b^3*c*d - a^3*(c^2 - d^2) + 3*a*b^2*(c^2 - d^2))*x)/((a
^2 + b^2)^3*(c^2 + d^2)^2)) - (b^3*(10*a^3*b*c*d + 2*a*b^3*c*d - 10*a^4*d^2
 + b^4*(c^2 - 3*d^2) - 3*a^2*b^2*(c^2 + 3*d^2))*Log[a*Cos[e + f*x] + b*Sin[
e + f*x]])/((a^2 + b^2)^3*(b*c - a*d)^4*f) - (d^4*(5*b*c^2 - 2*a*c*d + 3*b*
d^2)*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((b*c - a*d)^4*(c^2 + d^2)^2*f)
+ (d*(a^4*d^3 - 2*a*b^3*c*(c^2 + d^2) + 2*a^2*b^2*d*(2*c^2 + 3*d^2) + b^4*d
*(2*c^2 + 3*d^2)))/((a^2 + b^2)^2*(b*c - a*d)^3*(c^2 + d^2)*f*(c + d*Tan[e
+ f*x])) - b^2/(2*(a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])^2*(c + d*T
an[e + f*x])) - (b^2*(4*a*b*c - 7*a^2*d - 3*b^2*d))/(2*(a^2 + b^2)^2*(b*c -
 a*d)^2*f*(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x]))
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\tan(e+fx))^3(c+d\tan(e+fx))^2}\,dx = -\frac{b^2}{2\,(a^2+b^2)\,(bc-ad)f(a+b\tan(e+fx))^2(c+d\tan(e-$$

$$= -\frac{b^2}{2\,(a^2+b^2)\,(bc-ad)f(a+b\tan(e+fx))^2(c+d\tan(e-$$

$$= \frac{d(a^4d^3-2ab^3c(c^2+d^2)+2a^2b^2d(2c^2+3d^2)+b^4d(2c^2+}{(a^2+b^2)^2\,(bc-ad)^3\,(c^2+d^2)\,f(c+d\tan(e+fx))}$$

$$= -\frac{(6a^2bcd-2b^3cd-a^3(c^2-d^2)+3ab^2(c^2-d^2))\,x}{(a^2+b^2)^3\,(c^2+d^2)^2} + \frac{d(a}{}$$

$$= -\frac{(6a^2bcd-2b^3cd-a^3(c^2-d^2)+3ab^2(c^2-d^2))\,x}{(a^2+b^2)^3\,(c^2+d^2)^2} - \frac{b^3(}{}$$

Mathematica [A]
time = 7.30, size = 840, normalized size = 1.84

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^2),x]
```

```
[Out] -1/2*b^2/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f
*x])) - (-((b^2*(-2*a*b*c + 2*a^2*d + 3*b^2*d) - a*(-3*a*b^2*d + 2*b^2*(b*c
 - a*d)))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])*(c + d*Tan[e + f*
x]))) - (-((-((b*(b*c - a*d)^3*(3*a^2*b*c^2 - b^3*c^2 + 2*a^3*c*d - 6*a*b^2
*c*d - 3*a^2*b*d^2 + b^3*d^2 - (Sqrt[-b^2]*(6*a^2*b*c*d - 2*b^3*c*d - a^3*(
c^2 - d^2) + 3*a*b^2*(c^2 - d^2)))/b)*Log[Sqrt[-b^2] - b*Tan[e + f*x]])/((a
^2 + b^2)*(c^2 + d^2))) - (2*b^4*(c^2 + d^2)*(10*a^3*b*c*d + 2*a*b^3*c*d -
10*a^4*d^2 + b^4*(c^2 - 3*d^2) - 3*a^2*b^2*(c^2 + 3*d^2))*Log[a + b*Tan[e +
 f*x]])/((a^2 + b^2)*(b*c - a*d)) - (b*(b*c - a*d)^3*(3*a^2*b*c^2 - b^3*c^2
 + 2*a^3*c*d - 6*a*b^2*c*d - 3*a^2*b*d^2 + b^3*d^2 + (Sqrt[-b^2]*(6*a^2*b*c
*d - 2*b^3*c*d - a^3*(c^2 - d^2) + 3*a*b^2*(c^2 - d^2)))/b)*Log[Sqrt[-b^2]
+ b*Tan[e + f*x]])/((a^2 + b^2)*(c^2 + d^2)) - (2*b*(a^2 + b^2)^2*d^4*(5*b*
c^2 - 2*a*c*d + 3*b*d^2)*Log[c + d*Tan[e + f*x]])/((b*c - a*d)*(c^2 + d^2))
)/(b*(-(b*c) + a*d)*(c^2 + d^2)*f)) - (-(c*(-4*a*b*d*(b*c - a*d)^2 + 2*b^2*
c*d*(4*a*b*c - 7*a^2*d - 3*b^2*d))) - 2*d^2*(2*a^3*b*c*d + 2*a*b^3*c*d - a^
4*d^2 + b^4*(c^2 - 3*d^2) - a^2*b^2*(c^2 + 6*d^2)))/((-(b*c) + a*d)*(c^2 +
d^2)*f*(c + d*Tan[e + f*x])))/((a^2 + b^2)*(b*c - a*d)))/(2*(a^2 + b^2)*(b*
c - a*d))
```

Maple [A]
time = 1.39, size = 419, normalized size = 0.92

method	result
derivativedivides	$\frac{\frac{\left(-2a^3cd-3a^2b\,c^2+3a^2b\,d^2+6a\,b^2cd+b^3c^2-b^3d^2\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(a^3c^2-a^3d^2-6a^2bcd-3a\,b^2c^2+3a\,b^2d^2+2b^3cd\right)\arctan(\tan(f}{\left(a^2+b^2\right)^3\left(c^2+d^2\right)^2}$
default	$\frac{\frac{\left(-2a^3cd-3a^2b\,c^2+3a^2b\,d^2+6a\,b^2cd+b^3c^2-b^3d^2\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(a^3c^2-a^3d^2-6a^2bcd-3a\,b^2c^2+3a\,b^2d^2+2b^3cd\right)\arctan(\tan(f}{\left(a^2+b^2\right)^3\left(c^2+d^2\right)^2}$
norman	Expression too large to display
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e))^3/(c+d*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(a^2+b^2)^3/(c^2+d^2)^2*(1/2*(-2*a^3*c*d-3*a^2*b*c^2+3*a^2*b*d^2+6*a
*b^2*c*d+b^3*c^2-b^3*d^2)*ln(1+tan(f*x+e)^2)+(a^3*c^2-a^3*d^2-6*a^2*b*c*d-3
*a*b^2*c^2+3*a*b^2*d^2+2*b^3*c*d)*arctan(tan(f*x+e)))-1/2*b^3/(a*d-b*c)^2/(
a^2+b^2)/(a+b*tan(f*x+e))^2+b^3*(10*a^4*d^2-10*a^3*b*c*d+3*a^2*b^2*c^2+9*a^
2*b^2*d^2-2*a*b^3*c*d-b^4*c^2+3*b^4*d^2)/(a*d-b*c)^4/(a^2+b^2)^3*ln(a+b*tan
(f*x+e))-2*b^3*(2*a^2*d-a*b*c+b^2*d)/(a*d-b*c)^3/(a^2+b^2)^2/(a+b*tan(f*x+e
))-d^4/(c^2+d^2)/(a*d-b*c)^3/(c+d*tan(f*x+e))+d^4*(2*a*c*d-5*b*c^2-3*b*d^2)
/(c^2+d^2)^2/(a*d-b*c)^4*ln(c+d*tan(f*x+e)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1842 vs. $2(462) = 924$.
time = 0.69, size = 1842, normalized size = 4.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^3/(c+d*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*((a^3 - 3*a*b^2)*c^2 - 2*(3*a^2*b - b^3)*c*d - (a^3 - 3*a*b^2)*d^2)*
(f*x + e)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c^4 + 2*(a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*c^2*d^2 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4) + 2*((3*
a^2*b^5 - b^7)*c^2 - 2*(5*a^3*b^4 + a*b^6)*c*d + (10*a^4*b^3 + 9*a^2*b^5 +
3*b^7)*d^2)*log(b*tan(f*x + e) + a)/((a^6*b^4 + 3*a^4*b^6 + 3*a^2*b^8 + b^1
0)*c^4 - 4*(a^7*b^3 + 3*a^5*b^5 + 3*a^3*b^7 + a*b^9)*c^3*d + 6*(a^8*b^2 + 3
*a^6*b^4 + 3*a^4*b^6 + a^2*b^8)*c^2*d^2 - 4*(a^9*b + 3*a^7*b^3 + 3*a^5*b^5
+ a^3*b^7)*c*d^3 + (a^10 + 3*a^8*b^2 + 3*a^6*b^4 + a^4*b^6)*d^4) - 2*(5*b*c
^2*d^4 - 2*a*c*d^5 + 3*b*d^6)*log(d*tan(f*x + e) + c)/(b^4*c^8 - 4*a*b^3*c^
7*d - 4*a^3*b*c*d^7 + a^4*d^8 + 2*(3*a^2*b^2 + b^4)*c^6*d^2 - 4*(a^3*b + 2*
a*b^3)*c^5*d^3 + (a^4 + 12*a^2*b^2 + b^4)*c^4*d^4 - 4*(2*a^3*b + a*b^3)*c^3
*d^5 + 2*(a^4 + 3*a^2*b^2)*c^2*d^6) - ((3*a^2*b - b^3)*c^2 + 2*(a^3 - 3*a*b
```

```
^2)*c*d - (3*a^2*b - b^3)*d^2)*log(tan(f*x + e)^2 + 1)/((a^6 + 3*a^4*b^2 +
3*a^2*b^4 + b^6)*c^4 + 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c^2*d^2 + (a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4) - ((5*a^2*b^4 + b^6)*c^4 - (9*a^3*b^3
+ 5*a*b^5)*c^3*d + (5*a^2*b^4 + b^6)*c^2*d^2 - (9*a^3*b^3 + 5*a*b^5)*c*d^3
- 2*(a^6 + 2*a^4*b^2 + a^2*b^4)*d^4 + 2*(2*a*b^5*c^3*d + 2*a*b^5*c*d^3 - 2*
(2*a^2*b^4 + b^6)*c^2*d^2 - (a^4*b^2 + 6*a^2*b^4 + 3*b^6)*d^4)*tan(f*x + e)
^2 + (4*a*b^5*c^4 - 3*(a^2*b^4 + b^6)*c^3*d - (9*a^3*b^3 + a*b^5)*c^2*d^2 -
 3*(a^2*b^4 + b^6)*c*d^3 - (4*a^5*b + 17*a^3*b^3 + 9*a*b^5)*d^4)*tan(f*x +
e))/((a^6*b^3 + 2*a^4*b^5 + a^2*b^7)*c^6 - 3*(a^7*b^2 + 2*a^5*b^4 + a^3*b^6
)*c^5*d + (3*a^8*b + 7*a^6*b^3 + 5*a^4*b^5 + a^2*b^7)*c^4*d^2 - (a^9 + 5*a^
7*b^2 + 7*a^5*b^4 + 3*a^3*b^6)*c^3*d^3 + 3*(a^8*b + 2*a^6*b^3 + a^4*b^5)*c^
2*d^4 - (a^9 + 2*a^7*b^2 + a^5*b^4)*c*d^5 + ((a^4*b^5 + 2*a^2*b^7 + b^9)*c^
5*d - 3*(a^5*b^4 + 2*a^3*b^6 + a*b^8)*c^4*d^2 + (3*a^6*b^3 + 7*a^4*b^5 + 5*
a^2*b^7 + b^9)*c^3*d^3 - (a^7*b^2 + 5*a^5*b^4 + 7*a^3*b^6 + 3*a*b^8)*c^2*d^
4 + 3*(a^6*b^3 + 2*a^4*b^5 + a^2*b^7)*c*d^5 - (a^7*b^2 + 2*a^5*b^4 + a^3*b^
6)*d^6)*tan(f*x + e)^3 + ((a^4*b^5 + 2*a^2*b^7 + b^9)*c^6 - (a^5*b^4 + 2*a^
3*b^6 + a*b^8)*c^5*d - (3*a^6*b^3 + 5*a^4*b^5 + a^2*b^7 - b^9)*c^4*d^2 + (5
*a^7*b^2 + 9*a^5*b^4 + 3*a^3*b^6 - a*b^8)*c^3*d^3 - (2*a^8*b + 7*a^6*b^3 +
8*a^4*b^5 + 3*a^2*b^7)*c^2*d^4 + 5*(a^7*b^2 + 2*a^5*b^4 + a^3*b^6)*c*d^5 -
2*(a^8*b + 2*a^6*b^3 + a^4*b^5)*d^6)*tan(f*x + e)^2 + (2*(a^5*b^4 + 2*a^3*b
^6 + a*b^8)*c^6 - 5*(a^6*b^3 + 2*a^4*b^5 + a^2*b^7)*c^5*d + (3*a^7*b^2 + 8*
a^5*b^4 + 7*a^3*b^6 + 2*a*b^8)*c^4*d^2 + (a^8*b - 3*a^6*b^3 - 9*a^4*b^5 - 5
*a^2*b^7)*c^3*d^3 - (a^9 - a^7*b^2 - 5*a^5*b^4 - 3*a^3*b^6)*c^2*d^4 + (a^8*
b + 2*a^6*b^3 + a^4*b^5)*c*d^5 - (a^9 + 2*a^7*b^2 + a^5*b^4)*d^6)*tan(f*x +
 e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4767 vs. $2(462) = 924$.
time = 4.39, size = 4767, normalized size = 10.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^3/(c+d*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/2*((7*a^2*b^7 + b^9)*c^7 - 6*(3*a^3*b^6 + a*b^8)*c^6*d + (11*a^4*b^5 + 1
9*a^2*b^7 + 2*b^9)*c^5*d^2 - 12*(3*a^3*b^6 + a*b^8)*c^4*d^3 + (22*a^4*b^5 +
 17*a^2*b^7 + b^9)*c^3*d^4 - 6*(3*a^3*b^6 + a*b^8)*c^2*d^5 - (2*a^8*b + 6*a
^6*b^3 - 5*a^4*b^5 - 3*a^2*b^7)*c*d^6 + 2*(a^9 + 3*a^7*b^2 + 3*a^5*b^4 + a^
3*b^6)*d^7 - ((5*a^2*b^7 - b^9)*c^6*d - 2*(7*a^3*b^6 + a*b^8)*c^5*d^2 + (9*
a^4*b^5 + 13*a^2*b^7 - 2*b^9)*c^4*d^3 - 4*(7*a^3*b^6 + a*b^8)*c^3*d^4 - (2*
a^6*b^3 - 12*a^4*b^5 - 5*a^2*b^7 + 3*b^9)*c^2*d^5 + 2*(a^7*b^2 + 3*a^5*b^4
- 4*a^3*b^6)*c*d^6 + 3*(3*a^4*b^5 + a^2*b^7)*d^7 + 2*((a^3*b^6 - 3*a*b^8)*c
^6*d - 2*(2*a^4*b^5 - 3*a^2*b^7 - b^9)*c^5*d^2 + (6*a^5*b^4 + 5*a^3*b^6 - 5
*a*b^8)*c^4*d^3 - 4*(a^6*b^3 + 5*a^4*b^5)*c^3*d^4 + (a^7*b^2 + 15*a^5*b^4 +
```

10*a^3*b^6)*c^2*d^5 - 2*(a^6*b^3 + 5*a^4*b^5)*c*d^6 - (a^7*b^2 - 3*a^5*b^4)*d^7)*f*x)*tan(f*x + e)^3 - 2*((a^5*b^4 - 3*a^3*b^6)*c^7 - 2*(2*a^6*b^3 - 3*a^4*b^5 - a^2*b^7)*c^6*d + (6*a^7*b^2 + 5*a^5*b^4 - 5*a^3*b^6)*c^5*d^2 - 4*(a^8*b + 5*a^6*b^3)*c^4*d^3 + (a^9 + 15*a^7*b^2 + 10*a^5*b^4)*c^3*d^4 - 2 *(a^8*b + 5*a^6*b^3)*c^2*d^5 - (a^9 - 3*a^7*b^2)*c*d^6)*f*x - ((5*a^2*b^7 - b^9)*c^7 - 8*(a^3*b^6 + a*b^8)*c^6*d - (7*a^4*b^5 - 25*a^2*b^7 - 2*b^9)*c^ 5*d^2 + 2*(5*a^5*b^4 - 11*a^3*b^6 - 10*a*b^8)*c^4*d^3 - 7*(2*a^4*b^5 - 5*a^ 2*b^7 - b^9)*c^3*d^4 - 4*(a^7*b^2 - 2*a^5*b^4 + 8*a^3*b^6 + 5*a*b^8)*c^2*d^ 5 + (4*a^8*b + 14*a^6*b^3 + 11*a^4*b^5 + 25*a^2*b^7 + 6*b^9)*c*d^6 - 2*(a^7 *b^2 - 2*a^5*b^4 + 6*a^3*b^6 + 3*a*b^8)*d^7 + 2*((a^3*b^6 - 3*a*b^8)*c^7 - 2*(a^4*b^5 - b^9)*c^6*d - (2*a^5*b^4 - 17*a^3*b^6 + a*b^8)*c^5*d^2 + 2*(4*a ^6*b^3 - 5*a^4*b^5 - 5*a^2*b^7)*c^4*d^3 - (7*a^7*b^2 + 25*a^5*b^4 - 10*a^3* b^6)*c^3*d^4 + 2*(a^8*b + 14*a^6*b^3 + 5*a^4*b^5)*c^2*d^5 - (5*a^7*b^2 + 17 *a^5*b^4)*c*d^6 - 2*(a^8*b - 3*a^6*b^3)*d^7)*f*x)*tan(f*x + e)^2 - ((3*a^4* b^5 - a^2*b^7)*c^7 - 2*(5*a^5*b^4 + a^3*b^6)*c^6*d + (10*a^6*b^3 + 15*a^4*b ^5 + a^2*b^7)*c^5*d^2 - 4*(5*a^5*b^4 + a^3*b^6)*c^4*d^3 + (20*a^6*b^3 + 21* a^4*b^5 + 5*a^2*b^7)*c^3*d^4 - 2*(5*a^5*b^4 + a^3*b^6)*c^2*d^5 + (10*a^6*b^ 3 + 9*a^4*b^5 + 3*a^2*b^7)*c*d^6 + ((3*a^2*b^7 - b^9)*c^6*d - 2*(5*a^3*b^6 + a*b^8)*c^5*d^2 + (10*a^4*b^5 + 15*a^2*b^7 + b^9)*c^4*d^3 - 4*(5*a^3*b^6 + a*b^8)*c^3*d^4 + (20*a^4*b^5 + 21*a^2*b^7 + 5*b^9)*c^2*d^5 - 2*(5*a^3*b^6 + a*b^8)*c*d^6 + (10*a^4*b^5 + 9*a^2*b^7 + 3*b^9)*d^7)*tan(f*x + e)^3 + ((3 *a^2*b^7 - b^9)*c^7 - 4*(a^3*b^6 + a*b^8)*c^6*d - (10*a^4*b^5 - 11*a^2*b^7 - b^9)*c^5*d^2 + 2*(10*a^5*b^4 + 5*a^3*b^6 - a*b^8)*c^4*d^3 - (20*a^4*b^5 - 13*a^2*b^7 - 5*b^9)*c^3*d^4 + 8*(5*a^5*b^4 + 4*a^3*b^6 + a*b^8)*c^2*d^5 - (10*a^4*b^5 - 5*a^2*b^7 - 3*b^9)*c*d^6 + 2*(10*a^5*b^4 + 9*a^3*b^6 + 3*a*b^ 8)*d^7)*tan(f*x + e)^2 + (2*(3*a^3*b^6 - a*b^8)*c^7 - (17*a^4*b^5 + 5*a^2*b ^7)*c^6*d + 2*(5*a^5*b^4 + 14*a^3*b^6 + a*b^8)*c^5*d^2 + (10*a^6*b^3 - 25*a ^4*b^5 - 7*a^2*b^7)*c^4*d^3 + 2*(10*a^5*b^4 + 19*a^3*b^6 + 5*a*b^8)*c^3*d^4 + (20*a^6*b^3 + a^4*b^5 + a^2*b^7)*c^2*d^5 + 2*(5*a^5*b^4 + 8*a^3*b^6 + 3* a*b^8)*c*d^6 + (10*a^6*b^3 + 9*a^4*b^5 + 3*a^2*b^7)*d^7)*tan(f*x + e))*log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(tan(f*x + e)^2 + 1)) + (5* (a^8*b + 3*a^6*b^3 + 3*a^4*b^5 + a^2*b^7)*c^3*d^4 - 2*(a^9 + 3*a^7*b^2 + 3* a^5*b^4 + a^3*b^6)*c^2*d^5 + 3*(a^8*b + 3*a^6*b^3 + 3*a^4*b^5 + a^2*b^7)*c* d^6 + (5*(a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9)*c^2*d^5 - 2*(a^7*b^2 + 3*a ^5*b^4 + 3*a^3*b^6 + a*b^8)*c*d^6 + 3*(a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^ 9)*d^7)*tan(f*x + e)^3 + (5*(a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9)*c^3*d^4 + 8*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*c^2*d^5 - (4*a^8*b + 9*a^6*b ^3 + 3*a^4*b^5 - 5*a^2*b^7 - 3*b^9)*c*d^6 + 6*(a^7*b^2 + 3*a^5*b^4 + 3*a^3* b^6 + a*b^8)*d^7)*tan(f*x + e)^2 + (10*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a *b^8)*c^3*d^4 + (a^8*b + 3*a^6*b^3 + 3*a^4*b^5 + a^2*b^7)*c^2*d^5 - 2*(a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*c*d^6 + 3*(a^8*b + 3*a^6*b^3 + 3*a^4*b^5 + a^2*b^7)*d^7)*tan(f*x + e))*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) - (6*(a^3*b^6 - a*b^8)*c^7 - (16*a^4*b^5 - 5* a^2*b^7 - 3*b^9)*c^6*d + 2*(5*a^5*b^4 + 12*a^3*b^6 - 5*a*b^8)*c^5*d^2 - (43 *a^4*b^5 - 5*a^2*b^7 - 6*b^9)*c^4*d^3 + 2*(10*a^5*b^4 + 15*a^3*b^6 - a*b^8)

```
*c^3*d^4 - (2*a^8*b + 6*a^6*b^3 + 44*a^4*b^5 + 7*a^2*b^7 - 3*b^9)*c^2*d^5 +
 2*(a^9 + 5*a^7*b^2 + 14*a^5*b^4 + 13*a^3*b^6 + 3*a*b^8)*c*d^6 - (4*a^8*b +
 12*a^6*b^3 + 23*a^4*b^5 + 9*a^2*b^7)*d^7 + 2*(2*(a^4*b^5 - 3*a^2*b^7)*c^7
- (7*a^5*b^4 - 9*a^3*b^6 - 4*a*b^8)*c^6*d + 8*(a^6*b^3 + 2*a^4*b^5 - a^2*b^
7)*c^5*d^2 - (2*a^7*b^2 + 35*a^5*b^4 + 5*a^3*b^6)*c^4*d^3 - 2*(a^8*b - 5*a^
6*b^3 - 10*a^4*b^5)*c^3*d^4 + (a^9 + 11*a^7*b^2 - 10*a^5*b^4)*c^2*d^5 - 4*(
a^8*b + a^6*b^3)*c*d^6 - (a^9 - 3*a^7*b^2)*d^7)*f*x)*tan(f*x + e))/(((a^6*b
^6 + 3*a^4*b^8 + 3*a^2*b^10 + b^12)*c^8*d - 4*(a^7*b^5 + 3*a^5*b^7 + 3*a^3*
b^9 + a*b^11)*c^7*d^2 + 2*(3*a^8*b^4 + 10*a^6*b^6 + 12*a^4*b^8 + 6*a^2*b^10
 + b^12)*c^6*d^3 - 4*(a^9*b^3 + 5*a^7*b^5 + 9*a^5*b^7 + 7*a^3*b^9 + 2*a*b^1
1)*c^5*d^4 + (a^10*b^2 + 15*a^8*b^4 + 40*a^6*b^6 + 40*a^4*b^8 + 15*a^2*b^10
 + b^12)*c^4*d^5 - 4*(2*a^9*b^3 + 7*a^7*b^5 + 9...
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))**3/(c+d*tan(f*x+e))**2,x)
```

```
[Out] Exception raised: NotImplementedError >> no valid subset found
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1759 vs. $2(462) = 924$.
time = 0.87, size = 1759, normalized size = 3.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^3/(c+d*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(a^3*c^2 - 3*a*b^2*c^2 - 6*a^2*b*c*d + 2*b^3*c*d - a^3*d^2 + 3*a*b^2
*d^2)*(f*x + e)/(a^6*c^4 + 3*a^4*b^2*c^4 + 3*a^2*b^4*c^4 + b^6*c^4 + 2*a^6*
c^2*d^2 + 6*a^4*b^2*c^2*d^2 + 6*a^2*b^4*c^2*d^2 + 2*b^6*c^2*d^2 + a^6*d^4 +
 3*a^4*b^2*d^4 + 3*a^2*b^4*d^4 + b^6*d^4) - (3*a^2*b*c^2 - b^3*c^2 + 2*a^3*
c*d - 6*a*b^2*c*d - 3*a^2*b*d^2 + b^3*d^2)*log(tan(f*x + e)^2 + 1)/(a^6*c^4
 + 3*a^4*b^2*c^4 + 3*a^2*b^4*c^4 + b^6*c^4 + 2*a^6*c^2*d^2 + 6*a^4*b^2*c^2*
d^2 + 6*a^2*b^4*c^2*d^2 + 2*b^6*c^2*d^2 + a^6*d^4 + 3*a^4*b^2*d^4 + 3*a^2*b
^4*d^4 + b^6*d^4) + 2*(3*a^2*b^6*c^2 - b^8*c^2 - 10*a^3*b^5*c*d - 2*a*b^7*c
*d + 10*a^4*b^4*d^2 + 9*a^2*b^6*d^2 + 3*b^8*d^2)*log(abs(b*tan(f*x + e) + a
))/(a^6*b^5*c^4 + 3*a^4*b^7*c^4 + 3*a^2*b^9*c^4 + b^11*c^4 - 4*a^7*b^4*c^3*
d - 12*a^5*b^6*c^3*d - 12*a^3*b^8*c^3*d - 4*a*b^10*c^3*d + 6*a^8*b^3*c^2*d^
2 + 18*a^6*b^5*c^2*d^2 + 18*a^4*b^7*c^2*d^2 + 6*a^2*b^9*c^2*d^2 - 4*a^9*b^2
*c*d^3 - 12*a^7*b^4*c*d^3 - 12*a^5*b^6*c*d^3 - 4*a^3*b^8*c*d^3 + a^10*b*d^4
 + 3*a^8*b^3*d^4 + 3*a^6*b^5*d^4 + a^4*b^7*d^4) - 2*(5*b*c^2*d^5 - 2*a*c*d^
```

```
6 + 3*b*d^7)*log(abs(d*tan(f*x + e) + c))/(b^4*c^8*d - 4*a*b^3*c^7*d^2 + 6*
a^2*b^2*c^6*d^3 + 2*b^4*c^6*d^3 - 4*a^3*b*c^5*d^4 - 8*a*b^3*c^5*d^4 + a^4*c
^4*d^5 + 12*a^2*b^2*c^4*d^5 + b^4*c^4*d^5 - 8*a^3*b*c^3*d^6 - 4*a*b^3*c^3*d
^6 + 2*a^4*c^2*d^7 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9) + 2*(5*b*
c^2*d^5*tan(f*x + e) - 2*a*c*d^6*tan(f*x + e) + 3*b*d^7*tan(f*x + e) + 6*b*
c^3*d^4 - 3*a*c^2*d^5 + 4*b*c*d^6 - a*d^7)/((b^4*c^8 - 4*a*b^3*c^7*d + 6*a^
2*b^2*c^6*d^2 + 2*b^4*c^6*d^2 - 4*a^3*b*c^5*d^3 - 8*a*b^3*c^5*d^3 + a^4*c^4
*d^4 + 12*a^2*b^2*c^4*d^4 + b^4*c^4*d^4 - 8*a^3*b*c^3*d^5 - 4*a*b^3*c^3*d^5
 + 2*a^4*c^2*d^6 + 6*a^2*b^2*c^2*d^6 - 4*a^3*b*c*d^7 + a^4*d^8)*(d*tan(f*x
+ e) + c)) - (9*a^2*b^7*c^2*tan(f*x + e)^2 - 3*b^9*c^2*tan(f*x + e)^2 - 30*
a^3*b^6*c*d*tan(f*x + e)^2 - 6*a*b^8*c*d*tan(f*x + e)^2 + 30*a^4*b^5*d^2*ta
n(f*x + e)^2 + 27*a^2*b^7*d^2*tan(f*x + e)^2 + 9*b^9*d^2*tan(f*x + e)^2 + 2
2*a^3*b^6*c^2*tan(f*x + e) - 2*a*b^8*c^2*tan(f*x + e) - 72*a^4*b^5*c*d*tan(
f*x + e) - 28*a^2*b^7*c*d*tan(f*x + e) - 4*b^9*c*d*tan(f*x + e) + 68*a^5*b^
4*d^2*tan(f*x + e) + 66*a^3*b^6*d^2*tan(f*x + e) + 22*a*b^8*d^2*tan(f*x + e
) + 14*a^4*b^5*c^2 + 3*a^2*b^7*c^2 + b^9*c^2 - 44*a^5*b^4*c*d - 26*a^3*b^6*
c*d - 6*a*b^8*c*d + 39*a^6*b^3*d^2 + 41*a^4*b^5*d^2 + 14*a^2*b^7*d^2)/((a^6
*b^4*c^4 + 3*a^4*b^6*c^4 + 3*a^2*b^8*c^4 + b^10*c^4 - 4*a^7*b^3*c^3*d - 12*
a^5*b^5*c^3*d - 12*a^3*b^7*c^3*d - 4*a*b^9*c^3*d + 6*a^8*b^2*c^2*d^2 + 18*a
^6*b^4*c^2*d^2 + 18*a^4*b^6*c^2*d^2 + 6*a^2*b^8*c^2*d^2 - 4*a^9*b*c*d^3 - 1
2*a^7*b^3*c*d^3 - 12*a^5*b^5*c*d^3 - 4*a^3*b^7*c*d^3 + a^10*d^4 + 3*a^8*b^2
*d^4 + 3*a^6*b^4*d^4 + a^4*b^6*d^4)*(b*tan(f*x + e) + a)^2))/f
```

Mupad [B]
time = 34.61, size = 1421, normalized size = 3.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*tan(e + f*x))^3*(c + d*tan(e + f*x))^2),x)
```

```
[Out] (log(tan(e + f*x) + 1i)*1i)/(2*f*(a^3*c^2 - a^3*d^2 + b^3*c^2*1i - b^3*d^2*
1i - 3*a*b^2*c^2 - a^2*b*c^2*3i + 3*a*b^2*d^2 + a^2*b*d^2*3i - a^3*c*d*2i +
 2*b^3*c*d + a*b^2*c*d*6i - 6*a^2*b*c*d)) - (log(tan(e + f*x) - 1i)*1i)/(2*
f*(a^3*c^2 - a^3*d^2 - b^3*c^2*1i + b^3*d^2*1i - 3*a*b^2*c^2 + a^2*b*c^2*3i
 + 3*a*b^2*d^2 - a^2*b*d^2*3i + a^3*c*d*2i + 2*b^3*c*d - a*b^2*c*d*6i - 6*a
^2*b*c*d)) - ((2*a^6*d^4 - b^6*c^4 - 5*a^2*b^4*c^4 + 2*a^2*b^4*d^4 + 4*a^4*
b^2*d^4 - b^6*c^2*d^2 + 9*a^3*b^3*c*d^3 + 9*a^3*b^3*c^3*d - 5*a^2*b^4*c^2*d
^2 + 5*a*b^5*c*d^3 + 5*a*b^5*c^3*d)/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d -
 3*a^2*b*c*d^2)*(a^4*c^2 + a^4*d^2 + b^4*c^2 + b^4*d^2 + 2*a^2*b^2*c^2 + 2*
a^2*b^2*d^2)) + (tan(e + f*x)*(9*a*b^5*d^4 - 4*a*b^5*c^4 + 4*a^5*b*d^4 + 3*
b^6*c*d^3 + 3*b^6*c^3*d + 17*a^3*b^3*d^4 + a*b^5*c^2*d^2 + 3*a^2*b^4*c*d^3
+ 3*a^2*b^4*c^3*d + 9*a^3*b^3*c^2*d^2))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2)*(a^4*c^2 + a^4*d^2 + b^4*c^2 + b^4*d^2 + 2*a^2*b^2*c^2
+ 2*a^2*b^2*d^2)) + (tan(e + f*x)^2*(3*b^6*d^4 + 6*a^2*b^4*d^4 + a^4*b^2*d^
```

4 + 2*b^6*c^2*d^2 + 4*a^2*b^4*c^2*d^2 - 2*a*b^5*c*d^3 - 2*a*b^5*c^3*d))/((a
^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(a^4*c^2 + a^4*d^2 + b^4*
c^2 + b^4*d^2 + 2*a^2*b^2*c^2 + 2*a^2*b^2*d^2)))/(f*(tan(e + f*x)*(a^2*d +
2*a*b*c) + a^2*c + tan(e + f*x)^2*(b^2*c + 2*a*b*d) + b^2*d*tan(e + f*x)^3)
) - (log(a + b*tan(e + f*x))*(d*(10*a^3*b^4*c + 2*a*b^6*c) - d^2*(3*b^7 + 9
*a^2*b^5 + 10*a^4*b^3) + b^7*c^2 - 3*a^2*b^5*c^2))/(f*(a^10*d^4 + b^10*c^4
+ 3*a^2*b^8*c^4 + 3*a^4*b^6*c^4 + a^6*b^4*c^4 + a^4*b^6*d^4 + 3*a^6*b^4*d^4
 + 3*a^8*b^2*d^4 - 4*a^3*b^7*c*d^3 - 12*a^3*b^7*c^3*d - 12*a^5*b^5*c*d^3 -
12*a^5*b^5*c^3*d - 12*a^7*b^3*c*d^3 - 4*a^7*b^3*c^3*d + 6*a^2*b^8*c^2*d^2 +
 18*a^4*b^6*c^2*d^2 + 18*a^6*b^4*c^2*d^2 + 6*a^8*b^2*c^2*d^2 - 4*a*b^9*c^3*
d - 4*a^9*b*c*d^3)) - (log(c + d*tan(e + f*x))*(b*(3*d^6 + 5*c^2*d^4) - 2*a
*c*d^5))/(f*(a^4*d^8 + b^4*c^8 + 2*a^4*c^2*d^6 + a^4*c^4*d^4 + b^4*c^4*d^4
+ 2*b^4*c^6*d^2 - 4*a*b^3*c^3*d^5 - 8*a*b^3*c^5*d^3 - 8*a^3*b*c^3*d^5 - 4*a
^3*b*c^5*d^3 + 6*a^2*b^2*c^2*d^6 + 12*a^2*b^2*c^4*d^4 + 6*a^2*b^2*c^6*d^2 -
 4*a*b^3*c^7*d - 4*a^3*b*c*d^7))

3.1223 $\int \frac{(a+b\tan(e+fx))^4}{(c+d\tan(e+fx))^3}\,dx$

Optimal. Leaf size=406

$$-\frac{\left(6a^2b^2c(c^2-3d^2)-b^4c(c^2-3d^2)-4a^3bd(3c^2-d^2)+4ab^3d(3c^2-d^2)-a^4(c^3-3cd^2)\right)x}{(c^2+d^2)^3}-\frac{(4a^3bc(c^2-3d^2)$$

```
[Out] -(6*a^2*b^2*c*(c^2-3*d^2)-b^4*c*(c^2-3*d^2)-4*a^3*b*d*(3*c^2-d^2)+4*a*b^3*d
*(3*c^2-d^2)-a^4*(c^3-3*c*d^2))*x/(c^2+d^2)^3-(4*a^3*b*c*(c^2-3*d^2)-4*a*b^
3*c*(c^2-3*d^2)+6*a^2*b^2*d*(3*c^2-d^2)-b^4*d*(3*c^2-d^2)-a^4*(3*c^2*d-d^3)
)*ln(cos(f*x+e))/(c^2+d^2)^3/f+(-a*d+b*c)^2*(a^2*d^2*(3*c^2-d^2)+2*a*b*c*d*
(c^2+5*d^2)+b^2*(c^4+3*c^2*d^2+6*d^4))*ln(c+d*tan(f*x+e))/d^3/(c^2+d^2)^3/f
-1/2*(-a*d+b*c)^2*(a+b*tan(f*x+e))^2/d/(c^2+d^2)/f/(c+d*tan(f*x+e))^2+(-a*d
+b*c)^3*(2*a*c*d+b*(c^2+3*d^2))/d^3/(c^2+d^2)^2/f/(c+d*tan(f*x+e))
```

Rubi [A]
time = 0.53, antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3646, 3716, 3707, 3698, 31, 3556}

$$\frac{\left(a^2d^2(3c^2-d^2)+2abcd(c^2+5d^2)+b^2(c^4+3c^2d^2+6d^4)\right)(bc-ad)^2\log(c+d\tan(e+fx))}{d^3f\left(c^2+d^2\right)^3}-\frac{\left(-\left(a^4(3c^2d-d^3)\right)+4a^3bc(c^2-3d^2)+6a^2b^2d(3c^2-d^2)-4ab^3c(c^2-3d^2)-b^4d(3c^2-d^2)\right)\log(\cos(e+fx))}{f\left(c^2+d^2\right)^3}-\frac{x\left(-\left(a^4(c^3-3cd^2)\right)-4a^3bd(3c^2-d^2)+6a^2b^2c(c^2-3d^2)+4ab^3d(3c^2-d^2)-b^4c(c^2-3d^2)\right)}{\left(c^2+d^2\right)^3}-\frac{(bc-ad)^2(a+b\tan(e+fx))^2}{2df\left(c^2+d^2\right)(c+d\tan(e+fx))^2}+\frac{\left(2acd+b(c^2+3d^2)\right)(bc-ad)^3}{d^3f\left(c^2+d^2\right)^2(c+d\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^4/(c + d*Tan[e + f*x])^3,x]
```

```
[Out] -(((6*a^2*b^2*c*(c^2 - 3*d^2) - b^4*c*(c^2 - 3*d^2) - 4*a^3*b*d*(3*c^2 - d^
2) + 4*a*b^3*d*(3*c^2 - d^2) - a^4*(c^3 - 3*c*d^2))*x)/(c^2 + d^2)^3) - ((4
*a^3*b*c*(c^2 - 3*d^2) - 4*a*b^3*c*(c^2 - 3*d^2) + 6*a^2*b^2*d*(3*c^2 - d^2
) - b^4*d*(3*c^2 - d^2) - a^4*(3*c^2*d - d^3))*Log[Cos[e + f*x]])/((c^2 + d
^2)^3*f) + ((b*c - a*d)^2*(a^2*d^2*(3*c^2 - d^2) + 2*a*b*c*d*(c^2 + 5*d^2)
+ b^2*(c^4 + 3*c^2*d^2 + 6*d^4))*Log[c + d*Tan[e + f*x]])/(d^3*(c^2 + d^2)^
3*f) - ((b*c - a*d)^2*(a + b*Tan[e + f*x])^2)/(2*d*(c^2 + d^2)*f*(c + d*Tan
[e + f*x])^2) + ((b*c - a*d)^3*(2*a*c*d + b*(c^2 + 3*d^2)))/(d^3*(c^2 + d^2
)^2*f*(c + d*Tan[e + f*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3716

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
 -1]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^4}{(c+d\tan(e+fx))^3}\,dx = -\frac{(bc-ad)^2(a+b\tan(e+fx))^2}{2d\,(c^2+d^2)\,f(c+d\tan(e+fx))^2} + \frac{\int \frac{(a+b\tan(e+fx))(2(b^3c^2+a^3cd-3ab^2cd+3a^2bd^2)+}{(c}}{}$$

$$= -\frac{(bc-ad)^2(a+b\tan(e+fx))^2}{2d\,(c^2+d^2)\,f(c+d\tan(e+fx))^2} + \frac{(bc-ad)^3\,(2acd+b(c^2+3d^2))}{d^3\,(c^2+d^2)^2\,f(c+d\tan(e+fx))} + \frac{\int 2}{}$$

$$= -\frac{(6a^2b^2c(c^2-3d^2)-b^4c(c^2-3d^2)-4a^3bd(3c^2-d^2)+4ab^3d(3c^2-d^2)-a^4(}{(c^2+d^2)^3}$$

$$= -\frac{(6a^2b^2c(c^2-3d^2)-b^4c(c^2-3d^2)-4a^3bd(3c^2-d^2)+4ab^3d(3c^2-d^2)-a^4(}{(c^2+d^2)^3}$$

$$= -\frac{(6a^2b^2c(c^2-3d^2)-b^4c(c^2-3d^2)-4a^3bd(3c^2-d^2)+4ab^3d(3c^2-d^2)-a^4(}{(c^2+d^2)^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.05, size = 2775, normalized size = 6.83

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^4/(c + d*Tan[e + f*x])^3,x]
```

```
[Out] ((I*b^4*c^13*d^2 + b^4*c^12*d^3 + (5*I)*b^4*c^11*d^4 - (4*I)*a^3*b*c^10*d^5
 + (4*I)*a*b^3*c^10*d^5 + 5*b^4*c^10*d^5 + (3*I)*a^4*c^9*d^6 - 4*a^3*b*c^9*
d^6 - (18*I)*a^2*b^2*c^9*d^6 + 4*a*b^3*c^9*d^6 + (13*I)*b^4*c^9*d^6 + 3*a^4
*c^8*d^7 + (4*I)*a^3*b*c^8*d^7 - 18*a^2*b^2*c^8*d^7 - (4*I)*a*b^3*c^8*d^7 +
 13*b^4*c^8*d^7 + (5*I)*a^4*c^7*d^8 + 4*a^3*b*c^7*d^8 - (30*I)*a^2*b^2*c^7*
d^8 - 4*a*b^3*c^7*d^8 + (15*I)*b^4*c^7*d^8 + 5*a^4*c^6*d^9 + (20*I)*a^3*b*c
^6*d^9 - 30*a^2*b^2*c^6*d^9 - (20*I)*a*b^3*c^6*d^9 + 15*b^4*c^6*d^9 + I*a^4
*c^5*d^10 + 20*a^3*b*c^5*d^10 - (6*I)*a^2*b^2*c^5*d^10 - 20*a*b^3*c^5*d^10
+ (6*I)*b^4*c^5*d^10 + a^4*c^4*d^11 + (12*I)*a^3*b*c^4*d^11 - 6*a^2*b^2*c^4
*d^11 - (12*I)*a*b^3*c^4*d^11 + 6*b^4*c^4*d^11 - I*a^4*c^3*d^12 + 12*a^3*b*
c^3*d^12 + (6*I)*a^2*b^2*c^3*d^12 - 12*a*b^3*c^3*d^12 - a^4*c^2*d^13 + 6*a^
2*b^2*c^2*d^13)*(e + f*x)*Cos[e + f*x]*(c*Cos[e + f*x] + d*Sin[e + f*x])^3*
(a + b*Tan[e + f*x])^4)/(c^2*(c - I*d)^6*(c + I*d)^5*d^5*f*(a*Cos[e + f*x]
+ b*Sin[e + f*x])^4*(c + d*Tan[e + f*x])^3) - (I*(b^4*c^6 + 3*b^4*c^4*d^2 -
 4*a^3*b*c^3*d^3 + 4*a*b^3*c^3*d^3 + 3*a^4*c^2*d^4 - 18*a^2*b^2*c^2*d^4 + 6
*b^4*c^2*d^4 + 12*a^3*b*c*d^5 - 12*a*b^3*c*d^5 - a^4*d^6 + 6*a^2*b^2*d^6)*A
rcTan[Tan[e + f*x]]*Cos[e + f*x]*(c*Cos[e + f*x] + d*Sin[e + f*x])^3*(a + b
*Tan[e + f*x])^4)/(d^3*(c^2 + d^2)^3*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^4*
(c + d*Tan[e + f*x])^3) - (b^4*Cos[e + f*x]*Log[Cos[e + f*x]]*(c*Cos[e + f*
x] + d*Sin[e + f*x])^3*(a + b*Tan[e + f*x])^4)/(d^3*f*(a*Cos[e + f*x] + b*S
```

```
in[e + f*x])^4*(c + d*Tan[e + f*x])^3) + ((b^4*c^6 + 3*b^4*c^4*d^2 - 4*a^3*
b*c^3*d^3 + 4*a*b^3*c^3*d^3 + 3*a^4*c^2*d^4 - 18*a^2*b^2*c^2*d^4 + 6*b^4*c^
2*d^4 + 12*a^3*b*c*d^5 - 12*a*b^3*c*d^5 - a^4*d^6 + 6*a^2*b^2*d^6)*Cos[e +
f*x]*Log[(c*Cos[e + f*x] + d*Sin[e + f*x])^2]*(c*Cos[e + f*x] + d*Sin[e + f
*x])^3*(a + b*Tan[e + f*x])^4)/(2*d^3*(c^2 + d^2)^3*f*(a*Cos[e + f*x] + b*S
in[e + f*x])^4*(c + d*Tan[e + f*x])^3) + (Cos[e + f*x]*(c*Cos[e + f*x] + d*
Sin[e + f*x])*(-2*b^4*c^7*d + 4*a*b^3*c^6*d^2 - 6*b^4*c^5*d^3 - 4*a^3*b*c^4
*d^4 + 16*a*b^3*c^4*d^4 + 2*a^4*c^3*d^5 - 12*a^2*b^2*c^3*d^5 - 4*b^4*c^3*d^
5 + 12*a*b^3*c^2*d^6 + 2*a^4*c*d^7 - 12*a^2*b^2*c*d^7 + 4*a^3*b*d^8 + a^4*c
^6*d^2*(e + f*x) - 6*a^2*b^2*c^6*d^2*(e + f*x) + b^4*c^6*d^2*(e + f*x) + 12
*a^3*b*c^5*d^3*(e + f*x) - 12*a*b^3*c^5*d^3*(e + f*x) - 2*a^4*c^4*d^4*(e +
f*x) + 12*a^2*b^2*c^4*d^4*(e + f*x) - 2*b^4*c^4*d^4*(e + f*x) + 8*a^3*b*c^3
*d^5*(e + f*x) - 8*a*b^3*c^3*d^5*(e + f*x) - 3*a^4*c^2*d^6*(e + f*x) + 18*a
^2*b^2*c^2*d^6*(e + f*x) - 3*b^4*c^2*d^6*(e + f*x) - 4*a^3*b*c*d^7*(e + f*x
) + 4*a*b^3*c*d^7*(e + f*x) + b^4*c^7*d*Cos[2*(e + f*x)] - 6*a^2*b^2*c^5*d^
3*Cos[2*(e + f*x)] + 5*b^4*c^5*d^3*Cos[2*(e + f*x)] + 8*a^3*b*c^4*d^4*Cos[2
*(e + f*x)] - 12*a*b^3*c^4*d^4*Cos[2*(e + f*x)] - 3*a^4*c^3*d^5*Cos[2*(e +
f*x)] + 6*a^2*b^2*c^3*d^5*Cos[2*(e + f*x)] + 4*b^4*c^3*d^5*Cos[2*(e + f*x)]
 + 4*a^3*b*c^2*d^6*Cos[2*(e + f*x)] - 12*a*b^3*c^2*d^6*Cos[2*(e + f*x)] - 3
*a^4*c*d^7*Cos[2*(e + f*x)] + 12*a^2*b^2*c*d^7*Cos[2*(e + f*x)] - 4*a^3*b*d
^8*Cos[2*(e + f*x)] + a^4*c^6*d^2*(e + f*x)*Cos[2*(e + f*x)] - 6*a^2*b^2*c^
6*d^2*(e + f*x)*Cos[2*(e + f*x)] + b^4*c^6*d^2*(e + f*x)*Cos[2*(e + f*x)] +
 12*a^3*b*c^5*d^3*(e + f*x)*Cos[2*(e + f*x)] - 12*a*b^3*c^5*d^3*(e + f*x)*C
os[2*(e + f*x)] - 4*a^4*c^4*d^4*(e + f*x)*Cos[2*(e + f*x)] + 24*a^2*b^2*c^4
*d^4*(e + f*x)*Cos[2*(e + f*x)] - 4*b^4*c^4*d^4*(e + f*x)*Cos[2*(e + f*x)]
- 16*a^3*b*c^3*d^5*(e + f*x)*Cos[2*(e + f*x)] + 16*a*b^3*c^3*d^5*(e + f*x)*
Cos[2*(e + f*x)] + 3*a^4*c^2*d^6*(e + f*x)*Cos[2*(e + f*x)] - 18*a^2*b^2*c^
2*d^6*(e + f*x)*Cos[2*(e + f*x)] + 3*b^4*c^2*d^6*(e + f*x)*Cos[2*(e + f*x)]
 + 4*a^3*b*c*d^7*(e + f*x)*Cos[2*(e + f*x)] - 4*a*b^3*c*d^7*(e + f*x)*Cos[2
*(e + f*x)] - b^4*c^8*Sin[2*(e + f*x)] + 6*a^2*b^2*c^6*d^2*Sin[2*(e + f*x)]
 - 5*b^4*c^6*d^2*Sin[2*(e + f*x)] - 8*a^3*b*c^5*d^3*Sin[2*(e + f*x)] + 12*a
*b^3*c^5*d^3*Sin[2*(e + f*x)] + 3*a^4*c^4*d^4*Sin[2*(e + f*x)] - 6*a^2*b^2*
c^4*d^4*Sin[2*(e + f*x)] - 4*b^4*c^4*d^4*Sin[2*(e + f*x)] - 4*a^3*b*c^3*d^5
*Sin[2*(e + f*x)] + 12*a*b^3*c^3*d^5*Sin[2*(e + f*x)] + 3*a^4*c^2*d^6*Sin[2
*(e + f*x)] - 12*a^2*b^2*c^2*d^6*Sin[2*(e + f*x)] + 4*a^3*b*c*d^7*Sin[2*(e
+ f*x)] + 2*a^4*c^5*d^3*(e + f*x)*Sin[2*(e + f*x)] - 12*a^2*b^2*c^5*d^3*(e
+ f*x)*Sin[2*(e + f*x)] + 2*b^4*c^5*d^3*(e + f*x)*Sin[2*(e + f*x)] + 24*a^3
*b*c^4*d^4*(e + f*x)*Sin[2*(e + f*x)] - 24*a*b^3*c^4*d^4*(e + f*x)*Sin[2*(e
 + f*x)] - 6*a^4*c^3*d^5*(e + f*x)*Sin[2*(e + f*x)] + 36*a^2*b^2*c^3*d^5*(e
 + f*x)*Sin[2*(e + f*x)] - 6*b^4*c^3*d^5*(e + f*x)*Sin[2*(e + f*x)] - 8*a^3
*b*c^2*d^6*(e + f*x)*Sin[2*(e + f*x)] + 8*a*b^3*c^2*d^6*(e + f*x)*Sin[2*(e
+ f*x)])*(a + b*Tan[e + f*x])^4)/(2*c*(c - I*d)^3*(c + I*d)^3*d^2*f*(a*Cos[
e + f*x] + b*Sin[e + f*x])^4*(c + d*Tan[e + f*x])^3)
```

Maple [A]

time = 0.50, size = 553, normalized size = 1.36

method	result
derivativedivides	$-\frac{a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4}{2d^3(c^2+d^2)(c+d\tan(fx+e))^2}-\frac{2a^4c\,d^4-4a^3b\,c^2d^3+4a^3b\,d^5-12a^2b^2c\,d^4+4a\,b^3c^4d+12a\,b^3c^2d^3-2b^4c^5-4b^4c^3d^2}{d^3(c^2+d^2)^2(c+d\tan(fx+e))}+$
default	$-\frac{a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4}{2d^3(c^2+d^2)(c+d\tan(fx+e))^2}-\frac{2a^4c\,d^4-4a^3b\,c^2d^3+4a^3b\,d^5-12a^2b^2c\,d^4+4a\,b^3c^4d+12a\,b^3c^2d^3-2b^4c^5-4b^4c^3d^2}{d^3(c^2+d^2)^2(c+d\tan(fx+e))}+$
norman	$\frac{(a^4c^3-3a^4c\,d^2+12a^3b\,c^2d-4a^3b\,d^3-6a^2b^2c^3+18a^2b^2c\,d^2-12a\,b^3c^2d+4a\,b^3d^3+b^4c^3-3b^4c\,d^2)c^2x}{(c^4+2c^2d^2+d^4)(c^2+d^2)}+\frac{d^2(a^4c^3-3a^4c\,d^2+12a^3b\,c^2d-}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^4/(c+d*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/2*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/d
^3/(c^2+d^2)/(c+d*tan(f*x+e))^2-(2*a^4*c*d^4-4*a^3*b*c^2*d^3+4*a^3*b*d^5-12
*a^2*b^2*c*d^4+4*a*b^3*c^4*d+12*a*b^3*c^2*d^3-2*b^4*c^5-4*b^4*c^3*d^2)/d^3/
(c^2+d^2)^2/(c+d*tan(f*x+e))+(3*a^4*c^2*d^4-a^4*d^6-4*a^3*b*c^3*d^3+12*a^3*
b*c*d^5-18*a^2*b^2*c^2*d^4+6*a^2*b^2*d^6+4*a*b^3*c^3*d^3-12*a*b^3*c*d^5+b^4
*c^6+3*b^4*c^4*d^2+6*b^4*c^2*d^4)/(c^2+d^2)^3/d^3*ln(c+d*tan(f*x+e))+1/(c^2
+d^2)^3*(1/2*(-3*a^4*c^2*d+a^4*d^3+4*a^3*b*c^3-12*a^3*b*c*d^2+18*a^2*b^2*c^
2*d-6*a^2*b^2*d^3-4*a*b^3*c^3+12*a*b^3*c*d^2-3*b^4*c^2*d+b^4*d^3)*ln(1+tan(
f*x+e)^2)+(a^4*c^3-3*a^4*c*d^2+12*a^3*b*c^2*d-4*a^3*b*d^3-6*a^2*b^2*c^3+18*
a^2*b^2*c*d^2-12*a*b^3*c^2*d+4*a*b^3*d^3+b^4*c^3-3*b^4*c*d^2)*arctan(tan(f*
x+e))))
```

Maxima [A]

time = 0.62, size = 646, normalized size = 1.59

$$\frac{\frac{2((a^4-6a^2b^2+b^4)c^3+12(a^3b-ab^3)c^2d-3(a^4-6a^2b^2+b^4)cd^2-4(a^3b-ab^3)d^3)(fx+e)}{c^6+3c^4d^2+3c^2d^4+d^6}+\frac{2(b^4c^6+3b^4c^4d^2-4(a^3b-ab^3)c^3d^3+3(a^4-6a^2b^2+2b^4)c^2d^4+12(a^3b-ab^3)cd^5-(a^4-6a^2b^2)d^6)\log(d\tan(fx+e)+c)}{c^6d^3+3c^4d^5+3c^2d^7+d^9}+\frac{(4(a^3b-ab^3)c^3-3(a^4-6a^2b^2+b^4)c^2d-12(a^3b-ab^3)cd^2+(a^4-6a^2b^2+b^4)d^3)\log(\tan(fx+e)^2+1)}{c^6+3c^4d^2+3c^2d^4+d^6}+\frac{3b^4c^6-4ab^3c^5d-4a^3bcd^5-a^4d^6-(6a^2b^2-7b^4)c^4d^2+4(3a^3b-5ab^3)c^3d^3-(5a^4-18a^2b^2)c^2d^4+4(b^4c^5d-2ab^3c^4d^2+2b^4c^3d^3-2a^3bd^6+2(a^3b-3ab^3)c^2d^4-(a^4-6a^2b^2)cd^5)\tan(fx+e)}{c^6d^3+2c^4d^5+c^2d^7+(c^4d^5+2c^2d^7+d^9)\tan(fx+e)^2+2(c^5d^4+2c^3d^6+cd^8)\tan(fx+e)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^4/(c+d*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*((a^4 - 6*a^2*b^2 + b^4)*c^3 + 12*(a^3*b - a*b^3)*c^2*d - 3*(a^4 - 6
*a^2*b^2 + b^4)*c*d^2 - 4*(a^3*b - a*b^3)*d^3)*(f*x + e)/(c^6 + 3*c^4*d^2 +
 3*c^2*d^4 + d^6) + 2*(b^4*c^6 + 3*b^4*c^4*d^2 - 4*(a^3*b - a*b^3)*c^3*d^3
+ 3*(a^4 - 6*a^2*b^2 + 2*b^4)*c^2*d^4 + 12*(a^3*b - a*b^3)*c*d^5 - (a^4 - 6
*a^2*b^2)*d^6)*log(d*tan(f*x + e) + c)/(c^6*d^3 + 3*c^4*d^5 + 3*c^2*d^7 + d
^9) + (4*(a^3*b - a*b^3)*c^3 - 3*(a^4 - 6*a^2*b^2 + b^4)*c^2*d - 12*(a^3*b
- a*b^3)*c*d^2 + (a^4 - 6*a^2*b^2 + b^4)*d^3)*log(tan(f*x + e)^2 + 1)/(c^6
+ 3*c^4*d^2 + 3*c^2*d^4 + d^6) + (3*b^4*c^6 - 4*a*b^3*c^5*d - 4*a^3*b*c*d^5
```

```
- a^4*d^6 - (6*a^2*b^2 - 7*b^4)*c^4*d^2 + 4*(3*a^3*b - 5*a*b^3)*c^3*d^3 -
(5*a^4 - 18*a^2*b^2)*c^2*d^4 + 4*(b^4*c^5*d - 2*a*b^3*c^4*d^2 + 2*b^4*c^3*d
^3 - 2*a^3*b*d^6 + 2*(a^3*b - 3*a*b^3)*c^2*d^4 - (a^4 - 6*a^2*b^2)*c*d^5)*t
an(f*x + e))/(c^6*d^3 + 2*c^4*d^5 + c^2*d^7 + (c^4*d^5 + 2*c^2*d^7 + d^9)*t
an(f*x + e)^2 + 2*(c^5*d^4 + 2*c^3*d^6 + c*d^8)*tan(f*x + e)))/f
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1259 vs. $2(409) = 818$.
time = 1.24, size = 1259, normalized size = 3.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^4/(c+d*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*c^6*d^2 + 4*a*b^3*c^5*d^3 - 4*a^3*b*c*d^7 - a^4*d^8 - (18*a^2*b^2
- 7*b^4)*c^4*d^4 + 20*(a^3*b - a*b^3)*c^3*d^5 - (7*a^4 - 18*a^2*b^2)*c^2*d^
6 + 2*((a^4 - 6*a^2*b^2 + b^4)*c^5*d^3 + 12*(a^3*b - a*b^3)*c^4*d^4 - 3*(a^
4 - 6*a^2*b^2 + b^4)*c^3*d^5 - 4*(a^3*b - a*b^3)*c^2*d^6)*f*x - (3*b^4*c^6*
d^2 - 4*a*b^3*c^5*d^3 - 12*a^3*b*c*d^7 + a^4*d^8 - 3*(2*a^2*b^2 - 3*b^4)*c^
4*d^4 + 4*(3*a^3*b - 7*a*b^3)*c^3*d^5 - 5*(a^4 - 6*a^2*b^2)*c^2*d^6 - 2*((a
^4 - 6*a^2*b^2 + b^4)*c^3*d^5 + 12*(a^3*b - a*b^3)*c^2*d^6 - 3*(a^4 - 6*a^2
*b^2 + b^4)*c*d^7 - 4*(a^3*b - a*b^3)*d^8)*f*x)*tan(f*x + e)^2 + (b^4*c^8 +
 3*b^4*c^6*d^2 - 4*(a^3*b - a*b^3)*c^5*d^3 + 3*(a^4 - 6*a^2*b^2 + 2*b^4)*c^
4*d^4 + 12*(a^3*b - a*b^3)*c^3*d^5 - (a^4 - 6*a^2*b^2)*c^2*d^6 + (b^4*c^6*d
^2 + 3*b^4*c^4*d^4 - 4*(a^3*b - a*b^3)*c^3*d^5 + 3*(a^4 - 6*a^2*b^2 + 2*b^4
)*c^2*d^6 + 12*(a^3*b - a*b^3)*c*d^7 - (a^4 - 6*a^2*b^2)*d^8)*tan(f*x + e)^
2 + 2*(b^4*c^7*d + 3*b^4*c^5*d^3 - 4*(a^3*b - a*b^3)*c^4*d^4 + 3*(a^4 - 6*a
^2*b^2 + 2*b^4)*c^3*d^5 + 12*(a^3*b - a*b^3)*c^2*d^6 - (a^4 - 6*a^2*b^2)*c*
d^7)*tan(f*x + e))*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2)/(tan
(f*x + e)^2 + 1)) - (b^4*c^8 + 3*b^4*c^6*d^2 + 3*b^4*c^4*d^4 + b^4*c^2*d^6
+ (b^4*c^6*d^2 + 3*b^4*c^4*d^4 + 3*b^4*c^2*d^6 + b^4*d^8)*tan(f*x + e)^2 +
2*(b^4*c^7*d + 3*b^4*c^5*d^3 + 3*b^4*c^3*d^5 + b^4*c*d^7)*tan(f*x + e))*log
(1/(tan(f*x + e)^2 + 1)) - 2*(b^4*c^7*d + 4*a^3*b*d^8 - 3*(2*a^2*b^2 - b^4)
*c^5*d^3 + 4*(2*a^3*b - 3*a*b^3)*c^4*d^4 - (3*a^4 - 18*a^2*b^2 + 4*b^4)*c^3
*d^5 - 12*(a^3*b - a*b^3)*c^2*d^6 + 3*(a^4 - 4*a^2*b^2)*c*d^7 - 2*((a^4 - 6
*a^2*b^2 + b^4)*c^4*d^4 + 12*(a^3*b - a*b^3)*c^3*d^5 - 3*(a^4 - 6*a^2*b^2 +
 b^4)*c^2*d^6 - 4*(a^3*b - a*b^3)*c*d^7)*f*x)*tan(f*x + e))/((c^6*d^5 + 3*c
^4*d^7 + 3*c^2*d^9 + d^11)*f*tan(f*x + e)^2 + 2*(c^7*d^4 + 3*c^5*d^6 + 3*c^
3*d^8 + c*d^10)*f*tan(f*x + e) + (c^8*d^3 + 3*c^6*d^5 + 3*c^4*d^7 + c^2*d^9
)*f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**4/(c+d*tan(f*x+e))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1066 vs. $2(409) = 818$.
time = 1.21, size = 1066, normalized size = 2.63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^4/(c+d*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(a^4*c^3 - 6*a^2*b^2*c^3 + b^4*c^3 + 12*a^3*b*c^2*d - 12*a*b^3*c^2*d
 - 3*a^4*c*d^2 + 18*a^2*b^2*c*d^2 - 3*b^4*c*d^2 - 4*a^3*b*d^3 + 4*a*b^3*d^3
)*(f*x + e)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) + (4*a^3*b*c^3 - 4*a*b^3*c^
3 - 3*a^4*c^2*d + 18*a^2*b^2*c^2*d - 3*b^4*c^2*d - 12*a^3*b*c*d^2 + 12*a*b^
3*c*d^2 + a^4*d^3 - 6*a^2*b^2*d^3 + b^4*d^3)*log(tan(f*x + e)^2 + 1)/(c^6 +
 3*c^4*d^2 + 3*c^2*d^4 + d^6) + 2*(b^4*c^6 + 3*b^4*c^4*d^2 - 4*a^3*b*c^3*d^
3 + 4*a*b^3*c^3*d^3 + 3*a^4*c^2*d^4 - 18*a^2*b^2*c^2*d^4 + 6*b^4*c^2*d^4 +
12*a^3*b*c*d^5 - 12*a*b^3*c*d^5 - a^4*d^6 + 6*a^2*b^2*d^6)*log(abs(d*tan(f*
x + e) + c))/(c^6*d^3 + 3*c^4*d^5 + 3*c^2*d^7 + d^9) - (3*b^4*c^6*d*tan(f*x
 + e)^2 + 9*b^4*c^4*d^3*tan(f*x + e)^2 - 12*a^3*b*c^3*d^4*tan(f*x + e)^2 +
12*a*b^3*c^3*d^4*tan(f*x + e)^2 + 9*a^4*c^2*d^5*tan(f*x + e)^2 - 54*a^2*b^2
*c^2*d^5*tan(f*x + e)^2 + 18*b^4*c^2*d^5*tan(f*x + e)^2 + 36*a^3*b*c*d^6*ta
n(f*x + e)^2 - 36*a*b^3*c*d^6*tan(f*x + e)^2 - 3*a^4*d^7*tan(f*x + e)^2 + 1
8*a^2*b^2*d^7*tan(f*x + e)^2 + 2*b^4*c^7*tan(f*x + e) + 8*a*b^3*c^6*d*tan(f
*x + e) + 6*b^4*c^5*d^2*tan(f*x + e) - 32*a^3*b*c^4*d^3*tan(f*x + e) + 56*a
*b^3*c^4*d^3*tan(f*x + e) + 22*a^4*c^3*d^4*tan(f*x + e) - 132*a^2*b^2*c^3*d
^4*tan(f*x + e) + 28*b^4*c^3*d^4*tan(f*x + e) + 72*a^3*b*c^2*d^5*tan(f*x +
e) - 48*a*b^3*c^2*d^5*tan(f*x + e) - 2*a^4*c*d^6*tan(f*x + e) + 12*a^2*b^2*
c*d^6*tan(f*x + e) + 8*a^3*b*d^7*tan(f*x + e) + 4*a*b^3*c^7 + 6*a^2*b^2*c^6
*d - b^4*c^6*d - 24*a^3*b*c^5*d^2 + 36*a*b^3*c^5*d^2 + 14*a^4*c^4*d^3 - 66*
a^2*b^2*c^4*d^3 + 11*b^4*c^4*d^3 + 28*a^3*b*c^3*d^4 - 16*a*b^3*c^3*d^4 + 3*
a^4*c^2*d^5 + 4*a^3*b*c*d^6 + a^4*d^7)/((c^6*d^2 + 3*c^4*d^4 + 3*c^2*d^6 +
d^8)*(d*tan(f*x + e) + c)^2))/f
```

Mupad [B]
time = 10.25, size = 578, normalized size = 1.42

$$-\frac{\ln\left(c+d\tan(e+f\,x)\right)\,\left(\frac{d^6\,\left(a^4-6\,a^2\,b^2\right)-c^2\,\left(d^4\,\left(3\,a^4-18\,a^2\,b^2+6\,b^4\right)-3\,b^4\,d^4\right)+b^4\,d^6-c^3\,d^3\,\left(4\,a\,b^3-4\,a^3\,b\right)+c\,d^5\,\left(12\,a\,b^3-12\,a^3\,b\right)}{c^6\,d^3+3\,c^4\,d^5+3\,c^2\,d^7+d^9}-\frac{b^4}{d^3}\right)}{f}-\frac{\frac{5\,a^4\,c^2\,d^4+a^4\,d^6-12\,a^3\,b\,c^3\,d^3+4\,a^3\,b\,c\,d^5+6\,a^2\,b^2\,c^4\,d^2-18\,a^2\,b^2\,c^2\,d^4+4\,a\,b^3\,c^5\,d+20\,a\,b^3\,c^3\,d^3-3\,b^4\,c^6-7\,b^4\,c^4\,d^2}{2\,d^3\,\left(c^4+2\,c^2\,d^2+d^4\right)}-\frac{2\tan(e+f\,x)\,\left(-a^4\,c\,d^4+2\,a^3\,b\,c^2\,d^3-2\,a^3\,b\,d^5+6\,a^2\,b^2\,c\,d^4-2\,a\,b^3\,c^4\,d-6\,a\,b^3\,c^2\,d^3+b^4\,c^5+2\,b^4\,c^3\,d^2\right)}{d^2\,\left(c^4+2\,c^2\,d^2+d^4\right)}}{f\,\left(c^2+2\,c\,d\tan\left(e+f\,x\right)+d^2\tan\left(e+f\,x\right)^2\right)}+\frac{\ln\left(\tan(e+f\,x)+1{}\mathrm{i}\right)\,\left(a^4-a^3\,b\,4{}\mathrm{i}-6\,a^2\,b^2+a\,b^3\,4{}\mathrm{i}+b^4\right)}{2\,f\,\left(-c^3\,1{}\mathrm{i}-3\,c^2\,d+c\,d^2\,3{}\mathrm{i}+d^3\right)}+\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(a^4\,1{}\mathrm{i}-4\,a^3\,b-a^2\,b^2\,6{}\mathrm{i}+4\,a\,b^3+b^4\,1{}\mathrm{i}\right)}{2\,f\,\left(-c^3-c^2\,d\,3{}\mathrm{i}+3\,c\,d^2+d^3\,1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^4/(c + d*tan(e + f*x))^3,x)

[Out] (log(tan(e + f*x) + 1i)*(a*b^3*4i - a^3*b*4i + a^4 + b^4 - 6*a^2*b^2))/(2*f
*(c*d^2*3i - 3*c^2*d - c^3*1i + d^3)) - ((a^4*d^6 - 3*b^4*c^6 + 5*a^4*c^2*d
^4 - 7*b^4*c^4*d^2 + 20*a*b^3*c^3*d^3 - 12*a^3*b*c^3*d^3 - 18*a^2*b^2*c^2*d
^4 + 6*a^2*b^2*c^4*d^2 + 4*a*b^3*c^5*d + 4*a^3*b*c*d^5)/(2*d^3*(c^4 + d^4 +
 2*c^2*d^2)) - (2*tan(e + f*x)*(b^4*c^5 - 2*a^3*b*d^5 - a^4*c*d^4 + 2*b^4*c
^3*d^2 - 6*a*b^3*c^2*d^3 + 6*a^2*b^2*c*d^4 + 2*a^3*b*c^2*d^3 - 2*a*b^3*c^4*
d))/(d^2*(c^4 + d^4 + 2*c^2*d^2)))/(f*(c^2 + d^2*tan(e + f*x)^2 + 2*c*d*tan
(e + f*x))) - (log(c + d*tan(e + f*x))*((d^6*(a^4 - 6*a^2*b^2) - c^2*(d^4*(
3*a^4 + 6*b^4 - 18*a^2*b^2) - 3*b^4*d^4) + b^4*d^6 - c^3*d^3*(4*a*b^3 - 4*a
^3*b) + c*d^5*(12*a*b^3 - 12*a^3*b))/(d^9 + 3*c^2*d^7 + 3*c^4*d^5 + c^6*d^3
) - b^4/d^3))/f + (log(tan(e + f*x) - 1i)*(4*a*b^3 - 4*a^3*b + a^4*1i + b^4
*1i - a^2*b^2*6i))/(2*f*(3*c*d^2 - c^2*d*3i - c^3 + d^3*1i))
```

3.1224 $\int \frac{(a+b\tan(e+fx))^3}{(c+d\tan(e+fx))^3}\,dx$

Optimal. Leaf size=240

$$\frac{(ac+bd)\left(8abcd+a^2(c^2-3d^2)-b^2(3c^2-d^2)\right)x}{\left(c^2+d^2\right)^3}-\frac{(bc-ad)\left(8abcd-b^2(c^2-3d^2)+a^2(3c^2-d^2)\right)\log(c\cos(}{\left(c^2+d^2\right)^3 f}$$

```
[Out] (a*c+b*d)*(8*a*b*c*d+a^2*(c^2-3*d^2)-b^2*(3*c^2-d^2))*x/(c^2+d^2)^3-(-a*d+b
*c)*(8*a*b*c*d-b^2*(c^2-3*d^2)+a^2*(3*c^2-d^2))*ln(c*cos(f*x+e)+d*sin(f*x+e
))/(c^2+d^2)^3/f-1/2*(-a*d+b*c)^2*(a+b*tan(f*x+e))/d/(c^2+d^2)/f/(c+d*tan(f
*x+e))^2-1/2*(-a*d+b*c)^2*(4*a*c*d+b*(c^2+5*d^2))/d^2/(c^2+d^2)^2/f/(c+d*ta
n(f*x+e))
```

Rubi [A]
time = 0.36, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3646, 3709, 3612, 3611}

$$-\frac{\left(a^2(3c^2-d^2)+8abcd-b^2(c^2-3d^2)\right)(bc-ad)\log(c\cos(e+fx)+d\sin(e+fx))}{f\left(c^2+d^2\right)^3}+\frac{x(ac+bd)\left(a^2(c^2-3d^2)+8abcd-b^2(3c^2-d^2)\right)}{\left(c^2+d^2\right)^3}-\frac{\left(4acd+b(c^2+5d^2)\right)(bc-ad)^2}{2d^2f\left(c^2+d^2\right)^2(c+d\tan(e+fx))}-\frac{(bc-ad)^2(a+b\tan(e+fx))}{2df\left(c^2+d^2\right)(c+d\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^3/(c + d*Tan[e + f*x])^3,x]
```

```
[Out] ((a*c + b*d)*(8*a*b*c*d + a^2*(c^2 - 3*d^2) - b^2*(3*c^2 - d^2))*x)/(c^2 +
d^2)^3 - ((b*c - a*d)*(8*a*b*c*d - b^2*(c^2 - 3*d^2) + a^2*(3*c^2 - d^2))*L
og[c*Cos[e + f*x] + d*Sin[e + f*x]])/((c^2 + d^2)^3*f) - ((b*c - a*d)^2*(a
+ b*Tan[e + f*x]))/(2*d*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^2) - ((b*c - a*d
)^2*(4*a*c*d + b*(c^2 + 5*d^2)))/(2*d^2*(c^2 + d^2)^2*f*(c + d*Tan[e + f*x]
))
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^3}{(c+d\tan(e+fx))^3}\,dx = -\frac{(bc-ad)^2(a+b\tan(e+fx))}{2d\,(c^2+d^2)\,f(c+d\tan(e+fx))^2} + \frac{\int \frac{b(bc-2ad)^2+a^2d(2ac+bd)+2d(3a^2bc-b^3c-a^3d+\cdots}{(c+\cdots}}{2\cdots}$$

$$= -\frac{(bc-ad)^2(a+b\tan(e+fx))}{2d\,(c^2+d^2)\,f(c+d\tan(e+fx))^2} - \frac{(bc-ad)^2\,(4acd+b(c^2+5d^2))}{2d^2\,(c^2+d^2)^2\,f(c+d\tan(e+fx))} + \cdots$$

$$= \frac{(ac+bd)\,(8abcd+a^2(c^2-3d^2)-b^2(3c^2-d^2))\,x}{(c^2+d^2)^3} - \frac{(bc-ad)^2(a+b\tan(e\cdots}{2d\,(c^2+d^2)\,f(c+d\tan(e\cdots}$$

$$= \frac{(ac+bd)\,(8abcd+a^2(c^2-3d^2)-b^2(3c^2-d^2))\,x}{(c^2+d^2)^3} - \frac{(bc-ad)\,(3a^2c^2-b^2c^2+\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.74, size = 327, normalized size = 1.36

$$\frac{-\frac{b^2(bc+ad)}{(c+d\tan(e+fx))^2}-\frac{2b^2d(a+b\tan(e+fx))}{(c+d\tan(e+fx))^2}+2b(3a^2-b^2)\,d\left(-\frac{i\log(i-\tan(e+fx))}{2(c+id)^2}+\frac{i\log(i+\tan(e+fx))}{2(c-id)^2}+\frac{d\left(2c\log(c+d\tan(e+fx))-\frac{c^2+d^2}{c+d\tan(e+fx)}\right)}{(c^2+d^2)^2}\right)+d(-3a^2bc+b^3c+a^3d-3ab^2d)\left(\frac{\log(i-\tan(e+fx))}{(-ic+d)^3}+\frac{\log(i+\tan(e+fx))}{(ic+d)^3}+\frac{d\left((6c^2-2d^2)\log(c+d\tan(e+fx))-\frac{(c^2+d^2)(5c^2+d^2+4cd\tan(e+fx))}{(c+d\tan(e+fx))^2}\right)}{(c^2+d^2)^3}\right)}{2d^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^3/(c + d*Tan[e + f*x])^3,x]
```

```
[Out] (-((b^2*(b*c + a*d))/(c + d*Tan[e + f*x])^2) - (2*b^2*d*(a + b*Tan[e + f*x]
))/(c + d*Tan[e + f*x])^2 + 2*b*(3*a^2 - b^2)*d*(((-1/2*I)*Log[I - Tan[e +
f*x]])/(c + I*d)^2 + ((I/2)*Log[I + Tan[e + f*x]])/(c - I*d)^2 + (d*(2*c*Lo
g[c + d*Tan[e + f*x]] - (c^2 + d^2)/(c + d*Tan[e + f*x])))/(c^2 + d^2)^2) +
 d*(-3*a^2*b*c + b^3*c + a^3*d - 3*a*b^2*d)*(Log[I - Tan[e + f*x]]/((-I)*c
+ d)^3 + Log[I + Tan[e + f*x]]/(I*c + d)^3 + (d*((6*c^2 - 2*d^2)*Log[c + d*
Tan[e + f*x]] - ((c^2 + d^2)*(5*c^2 + d^2 + 4*c*d*Tan[e + f*x]))/(c + d*Tan
[e + f*x])^2))/(c^2 + d^2)^3))/(2*d^2*f)
```

Maple [A]
time = 0.30, size = 421, normalized size = 1.75

method	result
derivativedivides	$\frac{\frac{\left(3a^3c^2d-a^3d^3-3a^2bc^3+9a^2bcd^2-9ab^2c^2d+3ab^2d^3+b^3c^3-3b^3cd^2\right)\ln(c+d\tan(fx+e))}{\left(c^2+d^2\right)^3}-\frac{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}{2d^2\left(c^2+d^2\right)(c+d\tan(fx+e))^2}-\frac{2a^3cd}{}}{}$
default	$\frac{\frac{\left(3a^3c^2d-a^3d^3-3a^2bc^3+9a^2bcd^2-9ab^2c^2d+3ab^2d^3+b^3c^3-3b^3cd^2\right)\ln(c+d\tan(fx+e))}{\left(c^2+d^2\right)^3}-\frac{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}{2d^2\left(c^2+d^2\right)(c+d\tan(fx+e))^2}-\frac{2a^3cd}{}}{}$
norman	$\frac{\frac{\left(a^3c^3-3a^3cd^2+9a^2bc^2d-3a^2bd^3-3ab^2c^3+9ab^2cd^2-3b^3c^2d+b^3d^3\right)c^2x}{\left(c^4+2c^2d^2+d^4\right)\left(c^2+d^2\right)}+\frac{d^2\left(a^3c^3-3a^3cd^2+9a^2bc^2d-3a^2bd^3-3ab^2c^3+9ab^2cd\right.}{\left(c^4+2c^2d^2+d^4\right)\left(c^2+d^2\right.}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3/(c+d*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*((3*a^3*c^2*d-a^3*d^3-3*a^2*b*c^3+9*a^2*b*c*d^2-9*a*b^2*c^2*d+3*a*b^2*d
^3+b^3*c^3-3*b^3*c*d^2)/(c^2+d^2)^3*ln(c+d*tan(f*x+e))-1/2*(a^3*d^3-3*a^2*b
*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/d^2/(c^2+d^2)/(c+d*tan(f*x+e))^2-(2*a^3*c*d^3
-3*a^2*b*c^2*d^2+3*a^2*b*d^4-6*a*b^2*c*d^3+b^3*c^4+3*b^3*c^2*d^2)/(c^2+d^2)
^2/d^2/(c+d*tan(f*x+e))+1/(c^2+d^2)^3*(1/2*(-3*a^3*c^2*d+a^3*d^3+3*a^2*b*c^
3-9*a^2*b*c*d^2+9*a*b^2*c^2*d-3*a*b^2*d^3-b^3*c^3+3*b^3*c*d^2)*ln(1+tan(f*x
+e)^2)+(a^3*c^3-3*a^3*c*d^2+9*a^2*b*c^2*d-3*a^2*b*d^3-3*a*b^2*c^3+9*a*b^2*c
*d^2-3*b^3*c^2*d+b^3*d^3)*arctan(tan(f*x+e))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 534 vs. 2(241) = 482.
time = 0.56, size = 534, normalized size = 2.22

$$\frac{\frac{2\left(\left(a^3-3ab^2\right)c^3+3\left(3a^2b-b^3\right)c^2d-3\left(a^3-3ab^2\right)cd^2-\left(3a^2b-b^3\right)d^3\right)(fx+e)}{c^6+3c^4d^2+3c^2d^4+d^6}-\frac{2\left(\left(3a^2b-b^3\right)c^3-3\left(a^3-3ab^2\right)c^2d-3\left(3a^2b-b^3\right)cd^2+\left(a^3-3ab^2\right)d^3\right)\log(d\tan(fx+e)+c)}{c^6+3c^4d^2+3c^2d^4+d^6}+\frac{\left(\left(3a^2b-b^3\right)c^3-3\left(a^3-3ab^2\right)c^2d-3\left(3a^2b-b^3\right)cd^2+\left(a^3-3ab^2\right)d^3\right)\log\left(\tan(fx+e)^2+1\right)}{c^6+3c^4d^2+3c^2d^4+d^6}-\frac{b^3c^5+3ab^2c^4d+3a^2bcd^4+a^3d^5-\left(9a^2b-5b^3\right)c^3d^2+\left(5a^3-9ab^2\right)c^2d^3+2\left(b^3c^4d+3a^2bd^5-3\left(a^2b-b^3\right)c^2d^3+2\left(a^3-3ab^2\right)cd^4\right)\tan(fx+e)}{c^6d^2+2c^4d^4+c^2d^6+\left(c^4d^4+2c^2d^6+d^8\right)\tan(fx+e)^2+2\left(c^5d^3+2c^3d^5+cd^7\right)\tan(fx+e)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(c+d*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*((a^3 - 3*a*b^2)*c^3 + 3*(3*a^2*b - b^3)*c^2*d - 3*(a^3 - 3*a*b^2)*c
*d^2 - (3*a^2*b - b^3)*d^3)*(f*x + e)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) -
 2*((3*a^2*b - b^3)*c^3 - 3*(a^3 - 3*a*b^2)*c^2*d - 3*(3*a^2*b - b^3)*c*d^2
 + (a^3 - 3*a*b^2)*d^3)*log(d*tan(f*x + e) + c)/(c^6 + 3*c^4*d^2 + 3*c^2*d^
4 + d^6) + ((3*a^2*b - b^3)*c^3 - 3*(a^3 - 3*a*b^2)*c^2*d - 3*(3*a^2*b - b^
3)*c*d^2 + (a^3 - 3*a*b^2)*d^3)*log(tan(f*x + e)^2 + 1)/(c^6 + 3*c^4*d^2 +
3*c^2*d^4 + d^6) - (b^3*c^5 + 3*a*b^2*c^4*d + 3*a^2*b*c*d^4 + a^3*d^5 - (9*
a^2*b - 5*b^3)*c^3*d^2 + (5*a^3 - 9*a*b^2)*c^2*d^3 + 2*(b^3*c^4*d + 3*a^2*b
*d^5 - 3*(a^2*b - b^3)*c^2*d^3 + 2*(a^3 - 3*a*b^2)*c*d^4)*tan(f*x + e))/(c^
6*d^2 + 2*c^4*d^4 + c^2*d^6 + (c^4*d^4 + 2*c^2*d^6 + d^8)*tan(f*x + e)^2 +
2*(c^5*d^3 + 2*c^3*d^5 + c*d^7)*tan(f*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 870 vs. $2(241) = 482$.
time = 1.27, size = 870, normalized size = 3.62

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(c+d*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*c^5 - 9*a*b^2*c^4*d - 3*a^2*b*c*d^4 - a^3*d^5 + 5*(3*a^2*b - b^3)*
c^3*d^2 - (7*a^3 - 9*a*b^2)*c^2*d^3 + 2*((a^3 - 3*a*b^2)*c^5 + 3*(3*a^2*b -
 b^3)*c^4*d - 3*(a^3 - 3*a*b^2)*c^3*d^2 - (3*a^2*b - b^3)*c^2*d^3)*f*x + (b
^3*c^5 + 3*a*b^2*c^4*d + 9*a^2*b*c*d^4 - a^3*d^5 - (9*a^2*b - 7*b^3)*c^3*d^
2 + 5*(a^3 - 3*a*b^2)*c^2*d^3 + 2*((a^3 - 3*a*b^2)*c^3*d^2 + 3*(3*a^2*b - b
^3)*c^2*d^3 - 3*(a^3 - 3*a*b^2)*c*d^4 - (3*a^2*b - b^3)*d^5)*f*x)*tan(f*x +
 e)^2 - ((3*a^2*b - b^3)*c^5 - 3*(a^3 - 3*a*b^2)*c^4*d - 3*(3*a^2*b - b^3)*
c^3*d^2 + (a^3 - 3*a*b^2)*c^2*d^3 + ((3*a^2*b - b^3)*c^3*d^2 - 3*(a^3 - 3*a
*b^2)*c^2*d^3 - 3*(3*a^2*b - b^3)*c*d^4 + (a^3 - 3*a*b^2)*d^5)*tan(f*x + e)
^2 + 2*((3*a^2*b - b^3)*c^4*d - 3*(a^3 - 3*a*b^2)*c^3*d^2 - 3*(3*a^2*b - b^
3)*c^2*d^3 + (a^3 - 3*a*b^2)*c*d^4)*tan(f*x + e))*log((d^2*tan(f*x + e)^2 +
 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) + 2*(3*a*b^2*c^5 - 3*a^2*b
*d^5 - 3*(2*a^2*b - b^3)*c^4*d + 3*(a^3 - 3*a*b^2)*c^3*d^2 + 3*(3*a^2*b - b
^3)*c^2*d^3 - 3*(a^3 - 2*a*b^2)*c*d^4 + 2*((a^3 - 3*a*b^2)*c^4*d + 3*(3*a^2
*b - b^3)*c^3*d^2 - 3*(a^3 - 3*a*b^2)*c^2*d^3 - (3*a^2*b - b^3)*c*d^4)*f*x)
*tan(f*x + e))/((c^6*d^2 + 3*c^4*d^4 + 3*c^2*d^6 + d^8)*f*tan(f*x + e)^2 +
2*(c^7*d + 3*c^5*d^3 + 3*c^3*d^5 + c*d^7)*f*tan(f*x + e) + (c^8 + 3*c^6*d^2
 + 3*c^4*d^4 + c^2*d^6)*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3/(c+d*tan(f*x+e))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 830 vs. $2(241) = 482$.
time = 0.95, size = 830, normalized size = 3.46

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(c+d*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(a^3*c^3 - 3*a*b^2*c^3 + 9*a^2*b*c^2*d - 3*b^3*c^2*d - 3*a^3*c*d^2 +
 9*a*b^2*c*d^2 - 3*a^2*b*d^3 + b^3*d^3)*(f*x + e)/(c^6 + 3*c^4*d^2 + 3*c^2*
d^4 + d^6) + (3*a^2*b*c^3 - b^3*c^3 - 3*a^3*c^2*d + 9*a*b^2*c^2*d - 9*a^2*b
*c*d^2 + 3*b^3*c*d^2 + a^3*d^3 - 3*a*b^2*d^3)*log(tan(f*x + e)^2 + 1)/(c^6
+ 3*c^4*d^2 + 3*c^2*d^4 + d^6) - 2*(3*a^2*b*c^3*d - b^3*c^3*d - 3*a^3*c^2*d
^2 + 9*a*b^2*c^2*d^2 - 9*a^2*b*c*d^3 + 3*b^3*c*d^3 + a^3*d^4 - 3*a*b^2*d^4)
*log(abs(d*tan(f*x + e) + c))/(c^6*d + 3*c^4*d^3 + 3*c^2*d^5 + d^7) + (9*a^
2*b*c^3*d^4*tan(f*x + e)^2 - 3*b^3*c^3*d^4*tan(f*x + e)^2 - 9*a^3*c^2*d^5*t
an(f*x + e)^2 + 27*a*b^2*c^2*d^5*tan(f*x + e)^2 - 27*a^2*b*c*d^6*tan(f*x +
e)^2 + 9*b^3*c*d^6*tan(f*x + e)^2 + 3*a^3*d^7*tan(f*x + e)^2 - 9*a*b^2*d^7*
tan(f*x + e)^2 - 2*b^3*c^6*d*tan(f*x + e) + 24*a^2*b*c^4*d^3*tan(f*x + e) -
 14*b^3*c^4*d^3*tan(f*x + e) - 22*a^3*c^3*d^4*tan(f*x + e) + 66*a*b^2*c^3*d
^4*tan(f*x + e) - 54*a^2*b*c^2*d^5*tan(f*x + e) + 12*b^3*c^2*d^5*tan(f*x +
e) + 2*a^3*c*d^6*tan(f*x + e) - 6*a*b^2*c*d^6*tan(f*x + e) - 6*a^2*b*d^7*ta
n(f*x + e) - b^3*c^7 - 3*a*b^2*c^6*d + 18*a^2*b*c^5*d^2 - 9*b^3*c^5*d^2 - 1
4*a^3*c^4*d^3 + 33*a*b^2*c^4*d^3 - 21*a^2*b*c^3*d^4 + 4*b^3*c^3*d^4 - 3*a^3
*c^2*d^5 - 3*a^2*b*c*d^6 - a^3*d^7)/((c^6*d^2 + 3*c^4*d^4 + 3*c^2*d^6 + d^8
)*(d*tan(f*x + e) + c)^2))/f
```

Mupad [B]
time = 8.29, size = 466, normalized size = 1.94

$$-\frac{\frac{5\,a^3\,c^2\,d^3+a^3\,d^5-9\,a^2\,b\,c^3\,d^2+3\,a^2\,b\,c\,d^4+3\,a\,b^2\,c^4\,d-9\,a\,b^2\,c^2\,d^3+b^3\,c^5+5\,b^3\,c^3\,d^2}{2\,d^2\,\left(c^4+2\,c^2\,d^2+d^4\right)}+\frac{\tan(e+f\,x)\,\left(2\,a^3\,c\,d^3-3\,a^2\,b\,c^2\,d^2+3\,a^2\,b\,d^4-6\,a\,b^2\,c\,d^3+b^3\,c^4+3\,b^3\,c^2\,d^2\right)}{d\,\left(c^4+2\,c^2\,d^2+d^4\right)}}{f\,\left(c^2+2\,c\,d\,\tan\left(e+f\,x\right)+d^2\,\tan\left(e+f\,x\right)^2\right)}-\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(-a^3\,1\mathrm{i}+3\,a^2\,b+a\,b^2\,3\mathrm{i}-b^3\right)}{2\,f\,\left(-c^3-c^2\,d\,3\mathrm{i}+3\,c\,d^2+d^3\,1\mathrm{i}\right)}-\frac{\ln\left(c+d\tan(e+f\,x)\right)\,\left(\left(3\,a^2\,b-b^3\right)\,c^3+\left(9\,a\,b^2-3\,a^3\right)\,c^2\,d+\left(3\,b^3-9\,a^2\,b\right)\,c\,d^2+\left(a^3-3\,a\,b^2\right)\,d^3\right)}{f\,\left(c^6+3\,c^4\,d^2+3\,c^2\,d^4+d^6\right)}-\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(-a^3+a^2\,b\,3\mathrm{i}+3\,a\,b^2-b^3\,1\mathrm{i}\right)}{2\,f\,\left(-c^3\,1\mathrm{i}-3\,c^2\,d+c\,d^2\,3\mathrm{i}+d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^3/(c + d*tan(e + f*x))^3,x)
```

```
[Out] - ((a^3*d^5 + b^3*c^5 + 5*a^3*c^2*d^3 + 5*b^3*c^3*d^2 - 9*a*b^2*c^2*d^3 - 9
*a^2*b*c^3*d^2 + 3*a*b^2*c^4*d + 3*a^2*b*c*d^4)/(2*d^2*(c^4 + d^4 + 2*c^2*d
^2)) + (tan(e + f*x)*(b^3*c^4 + 3*a^2*b*d^4 + 2*a^3*c*d^3 + 3*b^3*c^2*d^2 -
 3*a^2*b*c^2*d^2 - 6*a*b^2*c*d^3))/(d*(c^4 + d^4 + 2*c^2*d^2)))/(f*(c^2 + d
^2*tan(e + f*x)^2 + 2*c*d*tan(e + f*x))) - (log(tan(e + f*x) - 1i)*(a*b^2*3
```

```
i + 3*a^2*b - a^3*1i - b^3))/(2*f*(3*c*d^2 - c^2*d*3i - c^3 + d^3*1i)) - (l
og(c + d*tan(e + f*x))*(c^3*(3*a^2*b - b^3) - d^3*(3*a*b^2 - a^3) + c^2*d*(
9*a*b^2 - 3*a^3) - c*d^2*(9*a^2*b - 3*b^3)))/(f*(c^6 + d^6 + 3*c^2*d^4 + 3*
c^4*d^2)) - (log(tan(e + f*x) + 1i)*(3*a*b^2 + a^2*b*3i - a^3 - b^3*1i))/(2
*f*(c*d^2*3i - 3*c^2*d - c^3*1i + d^3))
```

3.1225 $\int \frac{(a+b\tan(e+fx))^2}{(c+d\tan(e+fx))^3}\,dx$

Optimal. Leaf size=221

$$-\frac{(b^2c(c^2-3d^2)-2abd(3c^2-d^2)-a^2(c^3-3cd^2))\,x}{(c^2+d^2)^3}-\frac{(2abc(c^2-3d^2)+b^2d(3c^2-d^2)-a^2(3c^2d-d^3))\log(}{(c^2+d^2)^3\,f}$$

```
[Out] -(b^2*c*(c^2-3*d^2)-2*a*b*d*(3*c^2-d^2)-a^2*(c^3-3*c*d^2))*x/(c^2+d^2)^3-(2
*a*b*c*(c^2-3*d^2)+b^2*d*(3*c^2-d^2)-a^2*(3*c^2*d-d^3))*ln(c*cos(f*x+e)+d*s
in(f*x+e))/(c^2+d^2)^3/f-1/2*(-a*d+b*c)^2/d/(c^2+d^2)/f/(c+d*tan(f*x+e))^2+
2*(-a*d+b*c)*(a*c+b*d)/(c^2+d^2)^2/f/(c+d*tan(f*x+e))
```

Rubi [A]
time = 0.28, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3623, 3610, 3612, 3611}

$$-\frac{(-(a^2(3c^2d-d^3))+2abc(c^2-3d^2)+b^2d(3c^2-d^2))\log(c\cos(e+fx)+d\sin(e+fx))}{f(c^2+d^2)^3}-\frac{x(-(a^2(c^3-3cd^2))-2abd(3c^2-d^2)+b^2c(c^2-3d^2))}{(c^2+d^2)^3}-\frac{(bc-ad)^2}{2df(c^2+d^2)(c+d\tan(e+fx))^2}+\frac{2(ac+bd)(bc-ad)}{f(c^2+d^2)^2(c+d\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2/(c + d*Tan[e + f*x])^3,x]
```

```
[Out] -(((b^2*c*(c^2 - 3*d^2) - 2*a*b*d*(3*c^2 - d^2) - a^2*(c^3 - 3*c*d^2))*x)/(
c^2 + d^2)^3) - ((2*a*b*c*(c^2 - 3*d^2) + b^2*d*(3*c^2 - d^2) - a^2*(3*c^2*
d - d^3))*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((c^2 + d^2)^3*f) - (b*c -
a*d)^2/(2*d*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^2) + (2*(b*c - a*d)*(a*c + b
*d))/((c^2 + d^2)^2*f*(c + d*Tan[e + f*x]))
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
```

```
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^2}{(c+d\tan(e+fx))^3}\,dx = -\frac{(bc-ad)^2}{2d\left(c^2+d^2\right)f(c+d\tan(e+fx))^2} + \frac{\int \frac{a^2c-b^2c+2abd+\left(2abc-a^2d+b^2d\right)\tan(e+fx)}{(c+d\tan(e+fx))^2}\,dx}{c^2+d^2}$$

$$= -\frac{(bc-ad)^2}{2d\left(c^2+d^2\right)f(c+d\tan(e+fx))^2} + \frac{2(bc-ad)(ac+bd)}{\left(c^2+d^2\right)^2f(c+d\tan(e+fx))} + \frac{\int (a$$

$$= -\frac{\left(b^2c(c^2-3d^2)-2abd(3c^2-d^2)-a^2(c^3-3cd^2)\right)x}{\left(c^2+d^2\right)^3} - \frac{(bc-ad)^2}{2d\left(c^2+d^2\right)f(c+d\,\mathrm{ta}}$$

$$= -\frac{\left(b^2c(c^2-3d^2)-2abd(3c^2-d^2)-a^2(c^3-3cd^2)\right)x}{\left(c^2+d^2\right)^3} - \frac{(2abc(c^2-3d^2)+b^2d}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.31, size = 292, normalized size = 1.32

$$-\frac{\frac{d^2(a+b\tan(e+fx))^3}{(c+d\tan(e+fx))^2}-\frac{bd(a+b\tan(e+fx))^2}{c+d\tan(e+fx)}+(bc-ad)\left(\frac{(a+ib)^2(ic+d)^3\log(i-\tan(e+fx))}{(c^2+d^2)^2}+\frac{i(a-ib)^2(c+id)\log(i+\tan(e+fx))}{(c-id)^2}+\frac{2\left(-2abc(c^2-3d^2)+b^2d(-3c^2+d^2)+a^2(3c^2d-d^3)\right)\log(c+d\tan(e+fx))}{(c^2+d^2)^2}-\frac{2(-bc+ad)\left(2acd+b(-c^2+d^2)\right)}{d(c^2+d^2)(c+d\tan(e+fx))}\right)}{2(-bc+ad)\left(c^2+d^2\right)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2/(c + d*Tan[e + f*x])^3,x]
```

```
[Out] -1/2*((d^2*(a + b*Tan[e + f*x])^3)/(c + d*Tan[e + f*x])^2 - (b*d*(a + b*Tan
[e + f*x])^2)/(c + d*Tan[e + f*x]) + (b*c - a*d)*(((a + I*b)^2*(I*c + d)^3*
Log[I - Tan[e + f*x]])/(c^2 + d^2)^2 + (I*(a - I*b)^2*(c + I*d)*Log[I + Tan
[e + f*x]])/(c - I*d)^2 + (2*(-2*a*b*c*(c^2 - 3*d^2) + b^2*d*(-3*c^2 + d^2)
 + a^2*(3*c^2*d - d^3))*Log[c + d*Tan[e + f*x]])/(c^2 + d^2)^2 - (2*(-(b*c)
 + a*d)*(2*a*c*d + b*(-c^2 + d^2)))/(d*(c^2 + d^2)*(c + d*Tan[e + f*x]))))/
((-(b*c) + a*d)*(c^2 + d^2)*f)
```

Maple [A]
time = 0.27, size = 303, normalized size = 1.37 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(c^2+d^2)/d/(c+d*tan(f*x+e))^2+(3*a^2
*c^2*d-a^2*d^3-2*a*b*c^3+6*a*b*c*d^2-3*b^2*c^2*d+b^2*d^3)/(c^2+d^2)^3*ln(c+
d*tan(f*x+e))-2*(a^2*c*d-a*b*c^2+a*b*d^2-b^2*c*d)/(c^2+d^2)^2/(c+d*tan(f*x+
e))+1/(c^2+d^2)^3*(1/2*(-3*a^2*c^2*d+a^2*d^3+2*a*b*c^3-6*a*b*c*d^2+3*b^2*c^
2*d-b^2*d^3)*ln(1+tan(f*x+e)^2)+(a^2*c^3-3*a^2*c*d^2+6*a*b*c^2*d-2*a*b*d^3-
b^2*c^3+3*b^2*c*d^2)*arctan(tan(f*x+e))))
```

Maxima [A]
time = 0.57, size = 422, normalized size = 1.91

$$\frac{\frac{2\,(6\,abc^2d-2\,abd^3+(a^2-b^2)c^3-3\,(a^2-b^2)cd^2)(fx+e)}{c^6+3\,c^4d^2+3\,c^2d^4+d^6}-\frac{2\,(2\,abc^3-6\,abcd^2-3\,(a^2-b^2)c^2d+(a^2-b^2)d^3)\log(d\tan(fx+e)+c)}{c^6+3\,c^4d^2+3\,c^2d^4+d^6}+\frac{(2\,abc^3-6\,abcd^2-3\,(a^2-b^2)c^2d+(a^2-b^2)d^3)\log\left(\tan(fx+e)^2+1\right)}{c^6+3\,c^4d^2+3\,c^2d^4+d^6}-\frac{b^2c^4-6\,abc^3d+2\,abcd^3+a^2d^4+(5\,a^2-3\,b^2)c^2d^2-4\,(abc^2d^2-abd^4-(a^2-b^2)cd^3)\tan(fx+e)}{c^6d+2\,c^4d^3+c^2d^5+(c^4d^3+2\,c^2d^5+d^7)\tan(fx+e)^2+2\,(c^5d^2+2\,c^3d^4+cd^6)\tan(fx+e)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(6*a*b*c^2*d - 2*a*b*d^3 + (a^2 - b^2)*c^3 - 3*(a^2 - b^2)*c*d^2)*(f
*x + e)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) - 2*(2*a*b*c^3 - 6*a*b*c*d^2 -
3*(a^2 - b^2)*c^2*d + (a^2 - b^2)*d^3)*log(d*tan(f*x + e) + c)/(c^6 + 3*c^4
*d^2 + 3*c^2*d^4 + d^6) + (2*a*b*c^3 - 6*a*b*c*d^2 - 3*(a^2 - b^2)*c^2*d +
(a^2 - b^2)*d^3)*log(tan(f*x + e)^2 + 1)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6
) - (b^2*c^4 - 6*a*b*c^3*d + 2*a*b*c*d^3 + a^2*d^4 + (5*a^2 - 3*b^2)*c^2*d^
2 - 4*(a*b*c^2*d^2 - a*b*d^4 - (a^2 - b^2)*c*d^3)*tan(f*x + e))/(c^6*d + 2*
c^4*d^3 + c^2*d^5 + (c^4*d^3 + 2*c^2*d^5 + d^7)*tan(f*x + e)^2 + 2*(c^5*d^2
 + 2*c^3*d^4 + c*d^6)*tan(f*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 672 vs. 2(223) = 446.
time = 1.14, size = 672, normalized size = 3.04

$$-\frac{3\,b^2c^4d-10\,abc^3d^2+2\,abcd^4+a^2d^5+(7\,a^2-3\,b^2)c^2d^3-2\,(6\,abc^4d-2\,abc^2d^3+(a^2-b^2)c^5-3\,(a^2-b^2)c^3d^2)fx-(b^2c^4d-6\,abc^3d^2+6\,abcd^4-a^2d^5+5\,(a^2-b^2)c^2d^3+2\,(6\,abc^2d^3-2\,abd^5+(a^2-b^2)c^3d^2-3\,(a^2-b^2)cd^4)fx)\tan(fx+e)^2+(2\,abc^5-6\,abc^3d^2-3\,(a^2-b^2)c^4d+(a^2-b^2)c^2d^3+(2\,abc^3d^2-6\,abcd^4-3\,(a^2-b^2)c^2d^3+(a^2-b^2)d^5)\tan(fx+e)^2+2\,(2\,abc^4d-6\,abc^2d^3-3\,(a^2-b^2)c^3d^2+(a^2-b^2)cd^4)\tan(fx+e))\log\left(\frac{d^2\tan(fx+e)^2+2\,cd\tan(fx+e)+c^2}{\tan(fx+e)^2+1}\right)-2\,(b^2c^5-4\,abc^4d+6\,abc^2d^3-2\,abd^5+3\,(a^2-b^2)c^3d^2-(3\,a^2-2\,b^2)cd^4+2\,(6\,abc^3d^2-2\,abcd^4+(a^2-b^2)c^4d-3\,(a^2-b^2)c^2d^3)fx)\tan(fx+e)}{2\,((c^6d^2+3\,c^4d^4+3\,c^2d^6+d^8)f\tan(fx+e)^2+2\,(c^7d+3\,c^5d^3+3\,c^3d^5+cd^7)f\tan(fx+e)+(c^8+3\,c^6d^2+3\,c^4d^4+c^2d^6)f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(3*b^2*c^4*d - 10*a*b*c^3*d^2 + 2*a*b*c*d^4 + a^2*d^5 + (7*a^2 - 3*b^2
)*c^2*d^3 - 2*(6*a*b*c^4*d - 2*a*b*c^2*d^3 + (a^2 - b^2)*c^5 - 3*(a^2 - b^2
)*c^3*d^2)*f*x - (b^2*c^4*d - 6*a*b*c^3*d^2 + 6*a*b*c*d^4 - a^2*d^5 + 5*(a^
2 - b^2)*c^2*d^3 + 2*(6*a*b*c^2*d^3 - 2*a*b*d^5 + (a^2 - b^2)*c^3*d^2 - 3*(
a^2 - b^2)*c*d^4)*f*x)*tan(f*x + e)^2 + (2*a*b*c^5 - 6*a*b*c^3*d^2 - 3*(a^2
 - b^2)*c^4*d + (a^2 - b^2)*c^2*d^3 + (2*a*b*c^3*d^2 - 6*a*b*c*d^4 - 3*(a^2
```

```
 - b^2)*c^2*d^3 + (a^2 - b^2)*d^5)*tan(f*x + e)^2 + 2*(2*a*b*c^4*d - 6*a*b*
c^2*d^3 - 3*(a^2 - b^2)*c^3*d^2 + (a^2 - b^2)*c*d^4)*tan(f*x + e))*log((d^2
*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) - 2*(b^2*
c^5 - 4*a*b*c^4*d + 6*a*b*c^2*d^3 - 2*a*b*d^5 + 3*(a^2 - b^2)*c^3*d^2 - (3*
a^2 - 2*b^2)*c*d^4 + 2*(6*a*b*c^3*d^2 - 2*a*b*c*d^4 + (a^2 - b^2)*c^4*d - 3
*(a^2 - b^2)*c^2*d^3)*f*x)*tan(f*x + e))/((c^6*d^2 + 3*c^4*d^4 + 3*c^2*d^6
+ d^8)*f*tan(f*x + e)^2 + 2*(c^7*d + 3*c^5*d^3 + 3*c^3*d^5 + c*d^7)*f*tan(f
*x + e) + (c^8 + 3*c^6*d^2 + 3*c^4*d^4 + c^2*d^6)*f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2/(c+d*tan(f*x+e))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 614 vs. 2(223) = 446.
time = 0.74, size = 614, normalized size = 2.78

$$\frac{\frac{2\,(a^2c^3-b^2c^3+6\,abc^2d-3\,a^2cd^2+3\,b^2cd^2-2\,abd^3)(fx+e)}{c^6+3\,c^4d^2+3\,c^2d^4+d^6}+\frac{(2\,abc^3-3\,a^2c^2d+3\,b^2c^2d-6\,abcd^2+a^2d^3-b^2d^3)\log\left(\tan(fx+e)^2+1\right)}{c^6+3\,c^4d^2+3\,c^2d^4+d^6}-\frac{2\,(2\,abc^3d-3\,a^2c^2d^2+3\,b^2c^2d^2-6\,abcd^3+a^2d^4-b^2d^4)\log(|d\tan(fx+e)+c|)}{c^6d+3\,c^4d^3+3\,c^2d^5+d^7}+\frac{6\,abc^3d^3\tan(fx+e)^2-9\,a^2c^2d^4\tan(fx+e)^2+9\,b^2c^2d^4\tan(fx+e)^2-18\,abcd^5\tan(fx+e)^2+3\,a^2d^6\tan(fx+e)^2-3\,b^2d^6\tan(fx+e)^2+16\,abc^4d^2\tan(fx+e)-22\,a^2c^3d^3\tan(fx+e)+22\,b^2c^3d^3\tan(fx+e)-36\,abc^2d^4\tan(fx+e)+2\,a^2cd^5\tan(fx+e)-2\,b^2cd^5\tan(fx+e)-4\,abd^6\tan(fx+e)-b^2c^6+12\,abc^5d-14\,a^2c^4d^2+11\,b^2c^4d^2-14\,abc^3d^3-3\,a^2c^2d^4-2\,abcd^5-a^2d^6}{(c^6d+3\,c^4d^3+3\,c^2d^5+d^7)(d\tan(fx+e)+c)^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(a^2*c^3 - b^2*c^3 + 6*a*b*c^2*d - 3*a^2*c*d^2 + 3*b^2*c*d^2 - 2*a*b
*d^3)*(f*x + e)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) + (2*a*b*c^3 - 3*a^2*c^
2*d + 3*b^2*c^2*d - 6*a*b*c*d^2 + a^2*d^3 - b^2*d^3)*log(tan(f*x + e)^2 + 1
)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) - 2*(2*a*b*c^3*d - 3*a^2*c^2*d^2 + 3*
b^2*c^2*d^2 - 6*a*b*c*d^3 + a^2*d^4 - b^2*d^4)*log(abs(d*tan(f*x + e) + c))
/(c^6*d + 3*c^4*d^3 + 3*c^2*d^5 + d^7) + (6*a*b*c^3*d^3*tan(f*x + e)^2 - 9*
a^2*c^2*d^4*tan(f*x + e)^2 + 9*b^2*c^2*d^4*tan(f*x + e)^2 - 18*a*b*c*d^5*ta
n(f*x + e)^2 + 3*a^2*d^6*tan(f*x + e)^2 - 3*b^2*d^6*tan(f*x + e)^2 + 16*a*b
*c^4*d^2*tan(f*x + e) - 22*a^2*c^3*d^3*tan(f*x + e) + 22*b^2*c^3*d^3*tan(f*
x + e) - 36*a*b*c^2*d^4*tan(f*x + e) + 2*a^2*c*d^5*tan(f*x + e) - 2*b^2*c*d
^5*tan(f*x + e) - 4*a*b*d^6*tan(f*x + e) - b^2*c^6 + 12*a*b*c^5*d - 14*a^2*
c^4*d^2 + 11*b^2*c^4*d^2 - 14*a*b*c^3*d^3 - 3*a^2*c^2*d^4 - 2*a*b*c*d^5 - a
^2*d^6)/((c^6*d + 3*c^4*d^3 + 3*c^2*d^5 + d^7)*(d*tan(f*x + e) + c)^2))/f
```

Mupad [**B**]
time = 7.42, size = 367, normalized size = 1.66

$$-\frac{\frac{2\tan(e+f\,x)\,(a^2\,c\,d^2-a\,b\,c^2\,d+a\,b\,d^3-b^2\,c\,d^2)}{c^4+2\,c^2\,d^2+d^4}+\frac{5\,a^2\,c^2\,d^2+a^2\,d^4-6\,a\,b\,c^3\,d+2\,a\,b\,c\,d^3+b^2\,c^4-3\,b^2\,c^2\,d^2}{2\,d\,(c^4+2\,c^2\,d^2+d^4)}}{f\,\left(c^2+2\,c\,d\tan\left(e+f\,x\right)+d^2\tan\left(e+f\,x\right)^2\right)}-\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(-a^2\,1\mathrm{i}+2\,a\,b+b^2\,1\mathrm{i}\right)}{2\,f\,\left(-c^3-c^2\,d\,3\mathrm{i}+3\,c\,d^2+d^3\,1\mathrm{i}\right)}-\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(-a^2+a\,b\,2\mathrm{i}+b^2\right)}{2\,f\,\left(-c^3\,1\mathrm{i}-3\,c^2\,d+c\,d^2\,3\mathrm{i}+d^3\right)}-\frac{\ln\left(c+d\tan(e+f\,x)\right)\,\left(2\,a\,b\,c^3+\left(3\,b^2-3\,a^2\right)\,c^2\,d-6\,a\,b\,c\,d^2+\left(a^2-b^2\right)\,d^3\right)}{f\,\left(c^6+3\,c^4\,d^2+3\,c^2\,d^4+d^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^2/(c + d*tan(e + f*x))^3,x)
```

```
[Out] - ((2*tan(e + f*x)*(a^2*c*d^2 - b^2*c*d^2 + a*b*d^3 - a*b*c^2*d))/(c^4 + d^
4 + 2*c^2*d^2) + (a^2*d^4 + b^2*c^4 + 5*a^2*c^2*d^2 - 3*b^2*c^2*d^2 + 2*a*b
*c*d^3 - 6*a*b*c^3*d)/(2*d*(c^4 + d^4 + 2*c^2*d^2)))/(f*(c^2 + d^2*tan(e +
f*x)^2 + 2*c*d*tan(e + f*x))) - (log(tan(e + f*x) - 1i)*(2*a*b - a^2*1i + b
^2*1i))/(2*f*(3*c*d^2 - c^2*d*3i - c^3 + d^3*1i)) - (log(tan(e + f*x) + 1i)
*(a*b*2i - a^2 + b^2))/(2*f*(c*d^2*3i - 3*c^2*d - c^3*1i + d^3)) - (log(c +
 d*tan(e + f*x))*(d^3*(a^2 - b^2) - c^2*d*(3*a^2 - 3*b^2) + 2*a*b*c^3 - 6*a
*b*c*d^2))/(f*(c^6 + d^6 + 3*c^2*d^4 + 3*c^4*d^2))
```

3.1226 $\int \frac{a+b\tan(e+fx)}{(c+d\tan(e+fx))^3}\,dx$

Optimal. Leaf size=177

$$\frac{(ac^3+3bc^2d-3acd^2-bd^3)\,x}{(c^2+d^2)^3}+\frac{(ad(3c^2-d^2)-b(c^3-3cd^2))\log(c\cos(e+fx)+d\sin(e+fx))}{(c^2+d^2)^3\,f}+\frac{}{2\,(c^2+d}$$

```
[Out] (a*c^3-3*a*c*d^2+3*b*c^2*d-b*d^3)*x/(c^2+d^2)^3+(a*d*(3*c^2-d^2)-b*(c^3-3*c
*d^2))*ln(c*cos(f*x+e)+d*sin(f*x+e))/(c^2+d^2)^3/f+1/2*(-a*d+b*c)/(c^2+d^2)
/f/(c+d*tan(f*x+e))^2+(-2*a*c*d+b*(c^2-d^2))/(c^2+d^2)^2/f/(c+d*tan(f*x+e))
```

Rubi [A]
time = 0.20, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3610, 3612, 3611}

$$\frac{bc-ad}{2f\,(c^2+d^2)\,(c+d\tan(e+fx))^2}-\frac{2acd-b(c^2-d^2)}{f\,(c^2+d^2)^2\,(c+d\tan(e+fx))}+\frac{(ad(3c^2-d^2)-b(c^3-3cd^2))\log(c\cos(e+fx)+d\sin(e+fx))}{f\,(c^2+d^2)^3}+\frac{x(ac^3-3acd^2+3bc^2d-bd^3)}{(c^2+d^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])/(c + d*Tan[e + f*x])^3,x]
```

```
[Out] ((a*c^3 + 3*b*c^2*d - 3*a*c*d^2 - b*d^3)*x)/(c^2 + d^2)^3 + ((a*d*(3*c^2 -
d^2) - b*(c^3 - 3*c*d^2))*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((c^2 + d^2
)^3*f) + (b*c - a*d)/(2*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^2) - (2*a*c*d -
b*(c^2 - d^2))/((c^2 + d^2)^2*f*(c + d*Tan[e + f*x]))
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
```

```
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{a+b\tan(e+fx)}{(c+d\tan(e+fx))^3}\,dx = \frac{bc-ad}{2\left(c^2+d^2\right)f(c+d\tan(e+fx))^2} + \frac{\int \frac{ac+bd+(bc-ad)\tan(e+fx)}{(c+d\tan(e+fx))^2}\,dx}{c^2+d^2}$$

$$= \frac{bc-ad}{2\left(c^2+d^2\right)f(c+d\tan(e+fx))^2} - \frac{2acd-b(c^2-d^2)}{\left(c^2+d^2\right)^2 f(c+d\tan(e+fx))} + \frac{\int \frac{2bcd+a(}{}}{}$$

$$= \frac{\left(ac^3+3bc^2d-3acd^2-bd^3\right)x}{\left(c^2+d^2\right)^3} + \frac{bc-ad}{2\left(c^2+d^2\right)f(c+d\tan(e+fx))^2} - \frac{2ac}{\left(c^2+d^2\right)^2}$$

$$= \frac{\left(ac^3+3bc^2d-3acd^2-bd^3\right)x}{\left(c^2+d^2\right)^3} + \frac{\left(ad(3c^2-d^2)-b(c^3-3cd^2)\right)\log(c\cos(e+fx}{\left(c^2+d^2\right)^3 f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.52, size = 244, normalized size = 1.38

$$\frac{-b\left(\frac{i\log(i-\tan(e+fx))}{(c+id)^2}-\frac{i\log(i+\tan(e+fx))}{(c-id)^2}+\frac{2d\left(-2c\log(c+d\tan(e+fx))+\frac{c^2+d^2}{c+d\tan(e+fx)}\right)}{\left(c^2+d^2\right)^2}\right)+(bc-ad)\left(\frac{i\log(i-\tan(e+fx))}{(c+id)^3}-\frac{\log(i+\tan(e+fx))}{(ic+d)^3}+\frac{d\left(\left(-6c^2+2d^2\right)\log(c+d\tan(e+fx))+\frac{\left(c^2+d^2\right)\left(5c^2+d^2+4cd\tan(e+fx)\right)}{(c+d\tan(e+fx))^2}\right)}{\left(c^2+d^2\right)^3}\right)}{2df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])/(c + d*Tan[e + f*x])^3,x]
```

```
[Out] (-(b*((I*Log[I - Tan[e + f*x]])/(c + I*d)^2 - (I*Log[I + Tan[e + f*x]])/(c
- I*d)^2 + (2*d*(-2*c*Log[c + d*Tan[e + f*x]] + (c^2 + d^2)/(c + d*Tan[e +
f*x])))/(c^2 + d^2)^2)) + (b*c - a*d)*((I*Log[I - Tan[e + f*x]])/(c + I*d)^
3 - Log[I + Tan[e + f*x]]/(I*c + d)^3 + (d*((-6*c^2 + 2*d^2)*Log[c + d*Tan[
e + f*x]] + ((c^2 + d^2)*(5*c^2 + d^2 + 4*c*d*Tan[e + f*x]))/(c + d*Tan[e +
 f*x])^2))/(c^2 + d^2)^3))/(2*d*f)
```

Maple [A]
time = 0.28, size = 208, normalized size = 1.18

method	result
derivativedivides	$\frac{\frac{\left(3a\,c^2d-a\,d^3-b\,c^3+3bc\,d^2\right)\ln(c+d\tan(fx+e))}{\left(c^2+d^2\right)^3}-\frac{ad-bc}{2\left(c^2+d^2\right)(c+d\tan(fx+e))^2}-\frac{2acd-b\,c^2+b\,d^2}{\left(c^2+d^2\right)^2(c+d\tan(fx+e))}+\frac{\left(-3a\,c^2d+a\,d^3+b\,c^3-3\right.}{}}{f}$

default	$\frac{\frac{\left(3a\,c^2d-a\,d^3-b\,c^3+3bc\,d^2\right)\ln(c+d\tan(fx+e))}{\left(c^2+d^2\right)^3}-\frac{ad-bc}{2\left(c^2+d^2\right)(c+d\tan(fx+e))^2}-\frac{2acd-b\,c^2+b\,d^2}{\left(c^2+d^2\right)^2(c+d\tan(fx+e))}+\frac{\left(-3a\,c^2d+a\,d^3+b\,c^3\cdots\right.}{}}{f}$
norman	$\frac{\frac{\left(a\,c^3-3ac\,d^2+3b\,c^2d-b\,d^3\right)c^2x}{\left(c^4+2c^2d^2+d^4\right)\left(c^2+d^2\right)}+\frac{d^2\left(a\,c^3-3ac\,d^2+3b\,c^2d-b\,d^3\right)x\left(\tan^2(fx+e)\right)}{\left(c^4+2c^2d^2+d^4\right)\left(c^2+d^2\right)}-\frac{3a\,c^2d^2+a\,d^4-2b\,c^3d}{2df\left(c^4+2c^2d^2+d^4\right)}+\frac{d\left(2ac\,d^2-b\,c^2d+b\,d^3\right)\left(\mathrm{t}\cdots\right.}{2fc\left(c^4+2c^2d^2+\cdots\right.}}{(c+d\tan(fx+e))^2}$
risch	$\frac{ixb}{3ic^2d-id^3-c^3+3c\,d^2}-\frac{ax}{3ic^2d-id^3-c^3+3c\,d^2}-\frac{6ia\,c^2dx}{c^6+3c^4d^2+3c^2d^4+d^6}+\frac{2ia\,d^3x}{c^6+3c^4d^2+3c^2d^4+d^6}+\frac{2ib\,c^3x}{c^6+3c^4d^2+3c^2\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))/(c+d*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*((3*a*c^2*d-a*d^3-b*c^3+3*b*c*d^2)/(c^2+d^2)^3*ln(c+d*tan(f*x+e))-1/2*(
a*d-b*c)/(c^2+d^2)/(c+d*tan(f*x+e))^2-(2*a*c*d-b*c^2+b*d^2)/(c^2+d^2)^2/(c+
d*tan(f*x+e))+1/(c^2+d^2)^3*(1/2*(-3*a*c^2*d+a*d^3+b*c^3-3*b*c*d^2)*ln(1+ta
n(f*x+e)^2)+(a*c^3-3*a*c*d^2+3*b*c^2*d-b*d^3)*arctan(tan(f*x+e))))
```

Maxima [A]
time = 0.67, size = 327, normalized size = 1.85

$$\frac{\frac{2\,(ac^3+3\,bc^2d-3\,acd^2-bd^3)(fx+e)}{c^6+3\,c^4d^2+3\,c^2d^4+d^6}-\frac{2\,(bc^3-3\,ac^2d-3\,bcd^2+ad^3)\log\left(d\tan\left(fx+e\right)+c\right)}{c^6+3\,c^4d^2+3\,c^2d^4+d^6}+\frac{(bc^3-3\,ac^2d-3\,bcd^2+ad^3)\log\left(\tan\left(fx+e\right)^2+1\right)}{c^6+3\,c^4d^2+3\,c^2d^4+d^6}+\frac{3\,bc^3-5\,ac^2d-bcd^2-ad^3+2\,(bc^2d-2\,acd^2-bd^3)\tan\left(fx+e\right)}{c^6+2\,c^4d^2+c^2d^4+(c^4d^2+2\,c^2d^4+d^6)\tan\left(fx+e\right)^2+2\,(c^5d+2\,c^3d^3+cd^5)\tan\left(fx+e\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(c+d*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a*c^3 + 3*b*c^2*d - 3*a*c*d^2 - b*d^3)*(f*x + e)/(c^6 + 3*c^4*d^2 +
 3*c^2*d^4 + d^6) - 2*(b*c^3 - 3*a*c^2*d - 3*b*c*d^2 + a*d^3)*log(d*tan(f*x
 + e) + c)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) + (b*c^3 - 3*a*c^2*d - 3*b*c
*d^2 + a*d^3)*log(tan(f*x + e)^2 + 1)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) +
 (3*b*c^3 - 5*a*c^2*d - b*c*d^2 - a*d^3 + 2*(b*c^2*d - 2*a*c*d^2 - b*d^3)*t
an(f*x + e))/(c^6 + 2*c^4*d^2 + c^2*d^4 + (c^4*d^2 + 2*c^2*d^4 + d^6)*tan(f
*x + e)^2 + 2*(c^5*d + 2*c^3*d^3 + c*d^5)*tan(f*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(179) = 358$.
time = 1.06, size = 491, normalized size = 2.77

$$\frac{5\,bc^3d^2-7\,ac^2d^3-bcd^4-ad^5+2\,(ac^5+3\,bc^4d-3\,ac^3d^2-bc^2d^3)fx-(3\,bc^3d^2-5\,ac^2d^3-3\,bcd^4+ad^5-2\,(ac^3d^2+3\,bc^2d^3-3\,acd^4-bd^5)fx)\tan\left(fx+e\right)^2-(bc^5-3\,ac^4d-3\,bc^3d^2+ac^2d^3+(bc^3d^2-3\,ac^2d^3-3\,bcd^4+ad^5)\tan\left(fx+e\right)^2+2\,(bc^4d-3\,ac^3d^2-3\,bc^2d^3+acd^4)\tan\left(fx+e\right))\log\left(\frac{d^2\tan(fx+e)^2+2\,cd\tan(fx+e)+c^2}{\tan(fx+e)^2+1}\right)-2\,(2\,bc^4d-3\,ac^3d^2-3\,bc^2d^3+3\,acd^4+bd^5-2\,(ac^4d+3\,bc^3d^2-3\,ac^2d^3-bcd^4)fx)\tan\left(fx+e\right)}{2\,((c^6d^2+3\,c^4d^4+3\,c^2d^6+d^8)f\tan\left(fx+e\right)^2+2\,(c^7d+3\,c^5d^3+3\,c^3d^5+cd^7)f\tan\left(fx+e\right)+(c^8+3\,c^6d^2+3\,c^4d^4+c^2d^6)f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(c+d*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(5*b*c^3*d^2 - 7*a*c^2*d^3 - b*c*d^4 - a*d^5 + 2*(a*c^5 + 3*b*c^4*d - 3
*a*c^3*d^2 - b*c^2*d^3)*f*x - (3*b*c^3*d^2 - 5*a*c^2*d^3 - 3*b*c*d^4 + a*d^
```

```
5 - 2*(a*c^3*d^2 + 3*b*c^2*d^3 - 3*a*c*d^4 - b*d^5)*f*x)*tan(f*x + e)^2 - (
b*c^5 - 3*a*c^4*d - 3*b*c^3*d^2 + a*c^2*d^3 + (b*c^3*d^2 - 3*a*c^2*d^3 - 3*
b*c*d^4 + a*d^5)*tan(f*x + e)^2 + 2*(b*c^4*d - 3*a*c^3*d^2 - 3*b*c^2*d^3 +
a*c*d^4)*tan(f*x + e))*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2)/
(tan(f*x + e)^2 + 1)) - 2*(2*b*c^4*d - 3*a*c^3*d^2 - 3*b*c^2*d^3 + 3*a*c*d^
4 + b*d^5 - 2*(a*c^4*d + 3*b*c^3*d^2 - 3*a*c^2*d^3 - b*c*d^4)*f*x)*tan(f*x
+ e))/((c^6*d^2 + 3*c^4*d^4 + 3*c^2*d^6 + d^8)*f*tan(f*x + e)^2 + 2*(c^7*d
+ 3*c^5*d^3 + 3*c^3*d^5 + c*d^7)*f*tan(f*x + e) + (c^8 + 3*c^6*d^2 + 3*c^4*
d^4 + c^2*d^6)*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(c+d*tan(f*x+e))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(179) = 358.
time = 0.65, size = 422, normalized size = 2.38

$$\frac{\frac{2\,(ac^3+3\,bc^2d-3\,acd^2-bd^3)(fx+e)}{c^6+3\,c^4d^2+3\,c^2d^4+d^6}+\frac{(bc^3-3\,ac^2d-3\,bcd^2+ad^3)\log\left(\tan(fx+e)^2+1\right)}{c^6+3\,c^4d^2+3\,c^2d^4+d^6}-\frac{2\,(bc^3d-3\,ac^2d^2-3\,bcd^3+ad^4)\log(|d\tan(fx+e)+c|)}{c^6d+3\,c^4d^3+3\,c^2d^5+d^7}+\frac{3\,bc^3d^2\tan(fx+e)^2-9\,ac^2d^3\tan(fx+e)^2-9\,bcd^4\tan(fx+e)^2+3\,ad^5\tan(fx+e)^2+8\,bc^4d\tan(fx+e)-22\,ac^3d^2\tan(fx+e)-18\,bc^2d^3\tan(fx+e)+2\,acd^4\tan(fx+e)-2\,bd^5\tan(fx+e)+6\,bc^5-14\,ac^4d-7\,bc^3d^2-3\,ac^2d^3-bcd^4-ad^5}{(c^6+3\,c^4d^2+3\,c^2d^4+d^6)(d\tan(fx+e)+c)^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(c+d*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(a*c^3 + 3*b*c^2*d - 3*a*c*d^2 - b*d^3)*(f*x + e)/(c^6 + 3*c^4*d^2 +
 3*c^2*d^4 + d^6) + (b*c^3 - 3*a*c^2*d - 3*b*c*d^2 + a*d^3)*log(tan(f*x + e
)^2 + 1)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) - 2*(b*c^3*d - 3*a*c^2*d^2 - 3
*b*c*d^3 + a*d^4)*log(abs(d*tan(f*x + e) + c))/(c^6*d + 3*c^4*d^3 + 3*c^2*d
^5 + d^7) + (3*b*c^3*d^2*tan(f*x + e)^2 - 9*a*c^2*d^3*tan(f*x + e)^2 - 9*b*
c*d^4*tan(f*x + e)^2 + 3*a*d^5*tan(f*x + e)^2 + 8*b*c^4*d*tan(f*x + e) - 22
*a*c^3*d^2*tan(f*x + e) - 18*b*c^2*d^3*tan(f*x + e) + 2*a*c*d^4*tan(f*x + e
) - 2*b*d^5*tan(f*x + e) + 6*b*c^5 - 14*a*c^4*d - 7*b*c^3*d^2 - 3*a*c^2*d^3
 - b*c*d^4 - a*d^5)/((c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6)*(d*tan(f*x + e) +
c)^2))/f
```

Mupad [B]
time = 5.66, size = 279, normalized size = 1.58

$$\frac{\ln\left(c+d\tan(e+f\,x)\right)\,\left(\frac{3\,a\,d-b\,c}{{\left(c^2+d^2\right)}^2}-\frac{4\,d^2\,\left(a\,d-b\,c\right)}{{\left(c^2+d^2\right)}^3}\right)}{f}-\frac{\frac{-3\,b\,c^3+5\,a\,c^2\,d+b\,c\,d^2+a\,d^3}{2\,\left(c^4+2\,c^2\,d^2+d^4\right)}+\frac{\tan(e+f\,x)\,\left(-b\,c^2\,d+2\,a\,c\,d^2+b\,d^3\right)}{c^4+2\,c^2\,d^2+d^4}}{f\,\left(c^2+2\,c\,d\,\tan\left(e+f\,x\right)+d^2\,{\tan\left(e+f\,x\right)}^2\right)}+\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(-b+a\,1{}\mathrm{i}\right)}{2\,f\,\left(-c^3-c^2\,d\,3{}\mathrm{i}+3\,c\,d^2+d^3\,1{}\mathrm{i}\right)}+\frac{\ln\left(\tan(e+f\,x)+1{}\mathrm{i}\right)\,\left(a-b\,1{}\mathrm{i}\right)}{2\,f\,\left(-c^3\,1{}\mathrm{i}-3\,c^2\,d+c\,d^2\,3{}\mathrm{i}+d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))/(c + d*tan(e + f*x))^3,x)
```

```
[Out] (log(c + d*tan(e + f*x))*((3*a*d - b*c)/(c^2 + d^2)^2 - (4*d^2*(a*d - b*c))
/(c^2 + d^2)^3))/f - ((a*d^3 - 3*b*c^3 + 5*a*c^2*d + b*c*d^2)/(2*(c^4 + d^4
 + 2*c^2*d^2)) + (tan(e + f*x)*(b*d^3 + 2*a*c*d^2 - b*c^2*d))/(c^4 + d^4 +
2*c^2*d^2))/(f*(c^2 + d^2*tan(e + f*x)^2 + 2*c*d*tan(e + f*x))) + (log(tan(
e + f*x) - 1i)*(a*1i - b))/(2*f*(3*c*d^2 - c^2*d*3i - c^3 + d^3*1i)) + (log
(tan(e + f*x) + 1i)*(a - b*1i))/(2*f*(c*d^2*3i - 3*c^2*d - c^3*1i + d^3))
```

3.1227 $\int \frac{1}{(a+b\tan(e+fx))(c+d\tan(e+fx))^3}\,dx$

Optimal. Leaf size=286

$$-\frac{(bd(3c^2-d^2)-a(c^3-3cd^2))\,x}{(a^2+b^2)\,(c^2+d^2)^3}+\frac{b^4\log(a\cos(e+fx)+b\sin(e+fx))}{(a^2+b^2)\,(bc-ad)^3f}+\frac{d^2(8abc^3d-a^2d^2(3c^2-d^2)-b^2(6c$$

```
[Out] -(b*d*(3*c^2-d^2)-a*(c^3-3*c*d^2))*x/(a^2+b^2)/(c^2+d^2)^3+b^4*ln(a*cos(f*x
+e)+b*sin(f*x+e))/(a^2+b^2)/(-a*d+b*c)^3/f+d^2*(8*a*b*c^3*d-a^2*d^2*(3*c^2-
d^2)-b^2*(6*c^4+3*c^2*d^2+d^4))*ln(c*cos(f*x+e)+d*sin(f*x+e))/(-a*d+b*c)^3/
(c^2+d^2)^3/f+1/2*d^2/(-a*d+b*c)/(c^2+d^2)/f/(c+d*tan(f*x+e))^2-d^2*(2*a*c*
d-b*(3*c^2+d^2))/(-a*d+b*c)^2/(c^2+d^2)^2/f/(c+d*tan(f*x+e))
```

Rubi [A]
time = 0.67, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$,
Rules used = {3650, 3730, 3732, 3611}

$$-\frac{x(bd(3c^2-d^2)-a(c^3-3cd^2))}{(a^2+b^2)\,(c^2+d^2)^3}+\frac{d^2(-a^2d^2(3c^2-d^2)+8abc^3d-b^2(6c^4+3c^2d^2+d^4))\log(c\cos(e+fx)+d\sin(e+fx))}{f\,(c^2+d^2)^3\,(bc-ad)^3}+\frac{b^4\log(a\cos(e+fx)+b\sin(e+fx))}{f\,(a^2+b^2)\,(bc-ad)^3}-\frac{d^2(2acd-b(3c^2+d^2))}{f\,(c^2+d^2)^2\,(bc-ad)^2(c+d\tan(e+fx))}+\frac{d^2}{2f\,(c^2+d^2)\,(bc-ad)(c+d\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^3),x]
```

```
[Out] -(((b*d*(3*c^2 - d^2) - a*(c^3 - 3*c*d^2))*x)/((a^2 + b^2)*(c^2 + d^2)^3))
+ (b^4*Log[a*Cos[e + f*x] + b*Sin[e + f*x]])/((a^2 + b^2)*(b*c - a*d)^3*f)
+ (d^2*(8*a*b*c^3*d - a^2*d^2*(3*c^2 - d^2) - b^2*(6*c^4 + 3*c^2*d^2 + d^4)
)*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((b*c - a*d)^3*(c^2 + d^2)^3*f) + d
^2/(2*(b*c - a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^2) - (d^2*(2*a*c*d - b
*(3*c^2 + d^2)))/((b*c - a*d)^2*(c^2 + d^2)^2*f*(c + d*Tan[e + f*x]))
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
```

```
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\tan(e+fx))(c+d\tan(e+fx))^3}\,dx = \frac{d^2}{2(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^2} + \frac{\int \frac{-2(acd-b(c^2-}{(}}{}$$

$$= \frac{d^2}{2(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^2} - \frac{d^2(}{(bc-ad)^2(c}$$

$$= -\frac{\left(bd(3c^2-d^2)-a(c^3-3cd^2)\right)x}{\left(a^2+b^2\right)\left(c^2+d^2\right)^3} + \frac{d^2}{2(bc-ad)\left(c^2+d^2\right)f}$$

$$= -\frac{\left(bd(3c^2-d^2)-a(c^3-3cd^2)\right)x}{\left(a^2+b^2\right)\left(c^2+d^2\right)^3} + \frac{b^4\log(a\cos(e+fx)+}{\left(a^2+b^2\right)(bc-}$$

Mathematica [A]

time = 5.60, size = 409, normalized size = 1.43

$$\frac{\frac{(bc-ad)^3\left(a\sqrt{-b^2}\,c(c^2-3d^2)+b\left(-a+\sqrt{-b^2}\right)d(-3c^2+d^2)+b^2(c^3-3cd^2)\right)\log\left(\sqrt{-b^2}-b\tan(e+fx)\right)-2b^5(c^2+d^2)^3\log(a+b\tan(e+fx))-(bc-ad)^3\left(a\sqrt{-b^2}\,c(c^2-3d^2)+b\left(a+\sqrt{-b^2}\right)d(-3c^2+d^2)-b^2(c^3-3cd^2)\right)\log\left(\sqrt{-b^2}+b\tan(e+fx)\right)-2b(a^2+b^2)d^2(8abc^3d+a^2d^2(-3c^2+d^2)-b^2(6c^4+3c^2d^2+d^4))\log(c+d\tan(e+fx))}{b(a^2+b^2)(bc-ad)^2(c^2+d^2)^2}-\frac{d^2}{(c+d\tan(e+fx))^2}+\frac{2d^2(-2acd+b(3c^2+d^2))}{(-bc+ad)(c^2+d^2)(c+d\tan(e+fx))}}{2(-bc+ad)\,(c^2+d^2)\,f}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^3),x]

[Out] (((b*c - a*d)^3*(a*Sqrt[-b^2]*c*(c^2 - 3*d^2) + b*(-a + Sqrt[-b^2])*d*(-3*c
^2 + d^2) + b^2*(c^3 - 3*c*d^2))*Log[Sqrt[-b^2] - b*Tan[e + f*x]] - 2*b^5*(
c^2 + d^2)^3*Log[a + b*Tan[e + f*x]] - (b*c - a*d)^3*(a*Sqrt[-b^2]*c*(c^2 -
 3*d^2) + b*(a + Sqrt[-b^2])*d*(-3*c^2 + d^2) - b^2*(c^3 - 3*c*d^2))*Log[Sq
rt[-b^2] + b*Tan[e + f*x]] - 2*b*(a^2 + b^2)*d^2*(8*a*b*c^3*d + a^2*d^2*(-3
c^2 + d^2) - b^2(6*c^4 + 3*c^2*d^2 + d^4))*Log[c + d*Tan[e + f*x]])/(b*(a
^2 + b^2)*(b*c - a*d)^2*(c^2 + d^2)^2) - d^2/(c + d*Tan[e + f*x])^2 + (2*d^
2*(-2*a*c*d + b*(3*c^2 + d^2)))/((-(b*c) + a*d)*(c^2 + d^2)*(c + d*Tan[e +
f*x])))/(2*(-(b*c) + a*d)*(c^2 + d^2)*f)

Maple [A]
time = 0.72, size = 311, normalized size = 1.09

method	result
derivativedivides	$\frac{\frac{\left(-3a\,c^2 d+a\,d^3-b\,c^3+3bc\,d^2\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(a\,c^3-3ac\,d^2-3b\,c^2 d+b\,d^3\right)\arctan(\tan(fx+e))}{\left(a^2+b^2\right)\left(c^2+d^2\right)^3}-\frac{d^2}{2(ad-bc)\left(c^2+d^2\right)(c+d\tan(fx+e}$
default	$\frac{\frac{\left(-3a\,c^2 d+a\,d^3-b\,c^3+3bc\,d^2\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(a\,c^3-3ac\,d^2-3b\,c^2 d+b\,d^3\right)\arctan(\tan(fx+e))}{\left(a^2+b^2\right)\left(c^2+d^2\right)^3}-\frac{d^2}{2(ad-bc)\left(c^2+d^2\right)(c+d\tan(fx+e}$
norman	$\frac{\frac{\left(a\,c^3-3ac\,d^2-3b\,c^2 d+b\,d^3\right)c^2 x}{\left(c^6+3c^4 d^2+3c^2 d^4+d^6\right)\left(a^2+b^2\right)}+\frac{\left(-2a\,d^5 c+3b\,c^2 d^4+b\,d^6\right)\tan(fx+e)}{fd\left(c^4+2c^2 d^2+d^4\right)\left(a^2 d^2-2abcd+b^2 c^2\right)}+\frac{\left(a\,c^3-3ac\,d^2-3b\,c^2 d+b\,d^3\right)d^2 x\left(\tan^2(fx+e)\right)}{\left(c^6+3c^4 d^2+3c^2 d^4+d^6\right)\left(a^2+b^2\right)}+\frac{}{2f\,d^2}}{(c+d\tan(fx+e))^2}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))^3,x,method=_RETURNVERBOSE)

[Out] 1/f*(1/(a^2+b^2)/(c^2+d^2)^3*(1/2*(-3*a*c^2*d+a*d^3-b*c^3+3*b*c*d^2)*ln(1+t
an(f*x+e)^2)+(a*c^3-3*a*c*d^2-3*b*c^2*d+b*d^3)*arctan(tan(f*x+e)))-1/2*d^2/
(a*d-b*c)/(c^2+d^2)/(c+d*tan(f*x+e))^2-d^2*(2*a*c*d-3*b*c^2-b*d^2)/(a*d-b*c
)^2/(c^2+d^2)^2/(c+d*tan(f*x+e))+d^2*(3*a^2*c^2*d^2-a^2*d^4-8*a*b*c^3*d+6*b
^2*c^4+3*b^2*c^2*d^2+b^2*d^4)/(a*d-b*c)^3/(c^2+d^2)^3*ln(c+d*tan(f*x+e))-b^
4/(a^2+b^2)/(a*d-b*c)^3*ln(a+b*tan(f*x+e)))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 806 vs. 2(290) = 580.
time = 0.60, size = 806, normalized size = 2.82

$$\frac{\frac{2b^4\log(b\tan(fx+e)+a)}{(a^2b^3+b^5)c^3-3(a^3b^2+ab^4)c^2d+3(a^4b+a^2b^3)cd^2-(a^5+a^3b^2)d^3}+\frac{2(ac^3-3bc^2d-3acd^2+bd^3)(fx+e)}{(a^2+b^2)c^6+3(a^2+b^2)c^4d^2+3(a^2+b^2)c^2d^4+(a^2+b^2)d^6}-\frac{2(6b^2c^4d^2-8abc^3d^3+3(a^2+b^2)c^2d^4-(a^2-b^2)d^6)\log(d\tan(fx+e)+c)}{b^3c^9-3ab^2c^8d+3a^2bcd^8-a^3d^9+3(a^2b+b^3)c^7d^2-(a^3+9ab^2)c^6d^3+3(3a^2b+b^3)c^5d^4-3(a^3+3ab^2)c^4d^5+(9a^2b+b^3)c^3d^6-3(a^3+ab^2)c^2d^7}-\frac{(bc^3+3ac^2d-3bcd^2-ad^3)\log\left(\tan(fx+e)^2+1\right)}{(a^2+b^2)c^6+3(a^2+b^2)c^4d^2+3(a^2+b^2)c^2d^4+(a^2+b^2)d^6}+\frac{7bc^3d^2-5ac^2d^3+3bcd^4-ad^5+2(3bc^2d^3-2acd^4+bd^5)\tan(fx+e)}{b^2c^8-2abc^7d-4abc^5d^3-2abc^3d^5+a^2c^2d^6+(a^2+2b^2)c^6d^2+(2a^2+b^2)c^4d^4+(b^2c^6d^2-2abc^5d^3-4abc^3d^5-2abcd^7+a^2d^8+(a^2+2b^2)c^4d^4+(2a^2+b^2)c^2d^6)\tan(fx+e)^2+2(b^2c^7d-2abc^6d^2-4abc^4d^4-2abc^2d^6+a^2cd^7+(a^2+2b^2)c^5d^3+(2a^2+b^2)c^3d^5)\tan(fx+e)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^4*log(b*tan(f*x + e) + a)/((a^2*b^3 + b^5)*c^3 - 3*(a^3*b^2 + a*b^
4)*c^2*d + 3*(a^4*b + a^2*b^3)*c*d^2 - (a^5 + a^3*b^2)*d^3) + 2*(a*c^3 - 3*
b*c^2*d - 3*a*c*d^2 + b*d^3)*(f*x + e)/((a^2 + b^2)*c^6 + 3*(a^2 + b^2)*c^4
*d^2 + 3*(a^2 + b^2)*c^2*d^4 + (a^2 + b^2)*d^6) - 2*(6*b^2*c^4*d^2 - 8*a*b*
c^3*d^3 + 3*(a^2 + b^2)*c^2*d^4 - (a^2 - b^2)*d^6)*log(d*tan(f*x + e) + c)/
(b^3*c^9 - 3*a*b^2*c^8*d + 3*a^2*b*c*d^8 - a^3*d^9 + 3*(a^2*b + b^3)*c^7*d^
2 - (a^3 + 9*a*b^2)*c^6*d^3 + 3*(3*a^2*b + b^3)*c^5*d^4 - 3*(a^3 + 3*a*b^2)
*c^4*d^5 + (9*a^2*b + b^3)*c^3*d^6 - 3*(a^3 + a*b^2)*c^2*d^7) - (b*c^3 + 3*
a*c^2*d - 3*b*c*d^2 - a*d^3)*log(tan(f*x + e)^2 + 1)/((a^2 + b^2)*c^6 + 3*(
a^2 + b^2)*c^4*d^2 + 3*(a^2 + b^2)*c^2*d^4 + (a^2 + b^2)*d^6) + (7*b*c^3*d^
2 - 5*a*c^2*d^3 + 3*b*c*d^4 - a*d^5 + 2*(3*b*c^2*d^3 - 2*a*c*d^4 + b*d^5)*t
an(f*x + e))/(b^2*c^8 - 2*a*b*c^7*d - 4*a*b*c^5*d^3 - 2*a*b*c^3*d^5 + a^2*c
^2*d^6 + (a^2 + 2*b^2)*c^6*d^2 + (2*a^2 + b^2)*c^4*d^4 + (b^2*c^6*d^2 - 2*a
*b*c^5*d^3 - 4*a*b*c^3*d^5 - 2*a*b*c*d^7 + a^2*d^8 + (a^2 + 2*b^2)*c^4*d^4
+ (2*a^2 + b^2)*c^2*d^6)*tan(f*x + e)^2 + 2*(b^2*c^7*d - 2*a*b*c^6*d^2 - 4*
a*b*c^4*d^4 - 2*a*b*c^2*d^6 + a^2*c*d^7 + (a^2 + 2*b^2)*c^5*d^3 + (2*a^2 +
b^2)*c^3*d^5)*tan(f*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1855 vs. $2(290) = 580$.
time = 2.90, size = 1855, normalized size = 6.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(9*(a^2*b^2 + b^4)*c^4*d^4 - 16*(a^3*b + a*b^3)*c^3*d^5 + (7*a^4 + 10*a
^2*b^2 + 3*b^4)*c^2*d^6 - 4*(a^3*b + a*b^3)*c*d^7 + (a^4 + a^2*b^2)*d^8 + 2
*(a*b^3*c^8 - a^3*b*c^2*d^6 - 3*(a^2*b^2 + b^4)*c^7*d + 3*(a^3*b + 2*a*b^3)
*c^6*d^2 - (a^4 - b^4)*c^5*d^3 - 3*(2*a^3*b + a*b^3)*c^4*d^4 + 3*(a^4 + a^2
*b^2)*c^3*d^5)*f*x - (7*(a^2*b^2 + b^4)*c^4*d^4 - 12*(a^3*b + a*b^3)*c^3*d^
5 + (5*a^4 + 6*a^2*b^2 + b^4)*c^2*d^6 - (a^4 + a^2*b^2)*d^8 - 2*(a*b^3*c^6*
d^2 - a^3*b*d^8 - 3*(a^2*b^2 + b^4)*c^5*d^3 + 3*(a^3*b + 2*a*b^3)*c^4*d^4 -
 (a^4 - b^4)*c^3*d^5 - 3*(2*a^3*b + a*b^3)*c^2*d^6 + 3*(a^4 + a^2*b^2)*c*d^
7)*f*x)*tan(f*x + e)^2 + (b^4*c^8 + 3*b^4*c^6*d^2 + 3*b^4*c^4*d^4 + b^4*c^2
*d^6 + (b^4*c^6*d^2 + 3*b^4*c^4*d^4 + 3*b^4*c^2*d^6 + b^4*d^8)*tan(f*x + e)
^2 + 2*(b^4*c^7*d + 3*b^4*c^5*d^3 + 3*b^4*c^3*d^5 + b^4*c*d^7)*tan(f*x + e)
)*log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(tan(f*x + e)^2 + 1))
 - (6*(a^2*b^2 + b^4)*c^6*d^2 - 8*(a^3*b + a*b^3)*c^5*d^3 + 3*(a^4 + 2*a^2*
b^2 + b^4)*c^4*d^4 - (a^4 - b^4)*c^2*d^6 + (6*(a^2*b^2 + b^4)*c^4*d^4 - 8*(
a^3*b + a*b^3)*c^3*d^5 + 3*(a^4 + 2*a^2*b^2 + b^4)*c^2*d^6 - (a^4 - b^4)*d^
```

```
8)*tan(f*x + e)^2 + 2*(6*(a^2*b^2 + b^4)*c^5*d^3 - 8*(a^3*b + a*b^3)*c^4*d^
4 + 3*(a^4 + 2*a^2*b^2 + b^4)*c^3*d^5 - (a^4 - b^4)*c*d^7)*tan(f*x + e))*lo
g((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) - 2
*(4*(a^2*b^2 + b^4)*c^5*d^3 - 7*(a^3*b + a*b^3)*c^4*d^4 + 3*(a^4 - b^4)*c^3
*d^5 + 6*(a^3*b + a*b^3)*c^2*d^6 - (3*a^4 + 4*a^2*b^2 + b^4)*c*d^7 + (a^3*b
 + a*b^3)*d^8 - 2*(a*b^3*c^7*d - a^3*b*c*d^7 - 3*(a^2*b^2 + b^4)*c^6*d^2 +
3*(a^3*b + 2*a*b^3)*c^5*d^3 - (a^4 - b^4)*c^4*d^4 - 3*(2*a^3*b + a*b^3)*c^3
*d^5 + 3*(a^4 + a^2*b^2)*c^2*d^6)*f*x)*tan(f*x + e))/(((a^2*b^3 + b^5)*c^9*
d^2 - 3*(a^3*b^2 + a*b^4)*c^8*d^3 + 3*(a^4*b + 2*a^2*b^3 + b^5)*c^7*d^4 - (
a^5 + 10*a^3*b^2 + 9*a*b^4)*c^6*d^5 + 3*(3*a^4*b + 4*a^2*b^3 + b^5)*c^5*d^6
 - 3*(a^5 + 4*a^3*b^2 + 3*a*b^4)*c^4*d^7 + (9*a^4*b + 10*a^2*b^3 + b^5)*c^3
*d^8 - 3*(a^5 + 2*a^3*b^2 + a*b^4)*c^2*d^9 + 3*(a^4*b + a^2*b^3)*c*d^10 - (
a^5 + a^3*b^2)*d^11)*f*tan(f*x + e)^2 + 2*((a^2*b^3 + b^5)*c^10*d - 3*(a^3*
b^2 + a*b^4)*c^9*d^2 + 3*(a^4*b + 2*a^2*b^3 + b^5)*c^8*d^3 - (a^5 + 10*a^3*
b^2 + 9*a*b^4)*c^7*d^4 + 3*(3*a^4*b + 4*a^2*b^3 + b^5)*c^6*d^5 - 3*(a^5 + 4
*a^3*b^2 + 3*a*b^4)*c^5*d^6 + (9*a^4*b + 10*a^2*b^3 + b^5)*c^4*d^7 - 3*(a^5
 + 2*a^3*b^2 + a*b^4)*c^3*d^8 + 3*(a^4*b + a^2*b^3)*c^2*d^9 - (a^5 + a^3*b^
2)*c*d^10)*f*tan(f*x + e) + ((a^2*b^3 + b^5)*c^11 - 3*(a^3*b^2 + a*b^4)*c^1
0*d + 3*(a^4*b + 2*a^2*b^3 + b^5)*c^9*d^2 - (a^5 + 10*a^3*b^2 + 9*a*b^4)*c^
8*d^3 + 3*(3*a^4*b + 4*a^2*b^3 + b^5)*c^7*d^4 - 3*(a^5 + 4*a^3*b^2 + 3*a*b^
4)*c^6*d^5 + (9*a^4*b + 10*a^2*b^3 + b^5)*c^5*d^6 - 3*(a^5 + 2*a^3*b^2 + a*
b^4)*c^4*d^7 + 3*(a^4*b + a^2*b^3)*c^3*d^8 - (a^5 + a^3*b^2)*c^2*d^9)*f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))**3,x)
```

```
[Out] Exception raised: NotImplementedError >> no valid subset found
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1112 vs. 2(290) = 580.
time = 0.68, size = 1112, normalized size = 3.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^5*log(abs(b*tan(f*x + e) + a))/(a^2*b^4*c^3 + b^6*c^3 - 3*a^3*b^3*
c^2*d - 3*a*b^5*c^2*d + 3*a^4*b^2*c*d^2 + 3*a^2*b^4*c*d^2 - a^5*b*d^3 - a^3
*b^3*d^3) + 2*(a*c^3 - 3*b*c^2*d - 3*a*c*d^2 + b*d^3)*(f*x + e)/(a^2*c^6 +
b^2*c^6 + 3*a^2*c^4*d^2 + 3*b^2*c^4*d^2 + 3*a^2*c^2*d^4 + 3*b^2*c^2*d^4 + a
```

```
^2*d^6 + b^2*d^6) - (b*c^3 + 3*a*c^2*d - 3*b*c*d^2 - a*d^3)*log(tan(f*x + e
)^2 + 1)/(a^2*c^6 + b^2*c^6 + 3*a^2*c^4*d^2 + 3*b^2*c^4*d^2 + 3*a^2*c^2*d^4
 + 3*b^2*c^2*d^4 + a^2*d^6 + b^2*d^6) - 2*(6*b^2*c^4*d^3 - 8*a*b*c^3*d^4 +
3*a^2*c^2*d^5 + 3*b^2*c^2*d^5 - a^2*d^7 + b^2*d^7)*log(abs(d*tan(f*x + e) +
 c))/(b^3*c^9*d - 3*a*b^2*c^8*d^2 + 3*a^2*b*c^7*d^3 + 3*b^3*c^7*d^3 - a^3*c
^6*d^4 - 9*a*b^2*c^6*d^4 + 9*a^2*b*c^5*d^5 + 3*b^3*c^5*d^5 - 3*a^3*c^4*d^6
- 9*a*b^2*c^4*d^6 + 9*a^2*b*c^3*d^7 + b^3*c^3*d^7 - 3*a^3*c^2*d^8 - 3*a*b^2
*c^2*d^8 + 3*a^2*b*c*d^9 - a^3*d^10) + (18*b^2*c^4*d^4*tan(f*x + e)^2 - 24*
a*b*c^3*d^5*tan(f*x + e)^2 + 9*a^2*c^2*d^6*tan(f*x + e)^2 + 9*b^2*c^2*d^6*t
an(f*x + e)^2 - 3*a^2*d^8*tan(f*x + e)^2 + 3*b^2*d^8*tan(f*x + e)^2 + 42*b^
2*c^5*d^3*tan(f*x + e) - 58*a*b*c^4*d^4*tan(f*x + e) + 22*a^2*c^3*d^5*tan(f
*x + e) + 26*b^2*c^3*d^5*tan(f*x + e) - 12*a*b*c^2*d^6*tan(f*x + e) - 2*a^2
*c*d^7*tan(f*x + e) + 8*b^2*c*d^7*tan(f*x + e) - 2*a*b*d^8*tan(f*x + e) + 2
5*b^2*c^6*d^2 - 36*a*b*c^5*d^3 + 14*a^2*c^4*d^4 + 19*b^2*c^4*d^4 - 16*a*b*c
^3*d^5 + 3*a^2*c^2*d^6 + 6*b^2*c^2*d^6 - 4*a*b*c*d^7 + a^2*d^8)/((b^3*c^9 -
 3*a*b^2*c^8*d + 3*a^2*b*c^7*d^2 + 3*b^3*c^7*d^2 - a^3*c^6*d^3 - 9*a*b^2*c^
6*d^3 + 9*a^2*b*c^5*d^4 + 3*b^3*c^5*d^4 - 3*a^3*c^4*d^5 - 9*a*b^2*c^4*d^5 +
 9*a^2*b*c^3*d^6 + b^3*c^3*d^6 - 3*a^3*c^2*d^7 - 3*a*b^2*c^2*d^7 + 3*a^2*b*
c*d^8 - a^3*d^9)*(d*tan(f*x + e) + c)^2))/f
```

Mupad [B]

time = 12.64, size = 719, normalized size = 2.51

$$\frac{\ln\left(c+d\tan(e+f\,x)\right)\,\left(d^4\,\left(3\,a^2\,c^2+3\,b^2\,c^2\right)-d^6\,\left(a^2-b^2\right)+6\,b^2\,c^4\,d^2-8\,a\,b\,c^3\,d^3\right)}{f\,\left(a^3\,c^6\,d^3+3\,a^3\,c^4\,d^5+3\,a^3\,c^2\,d^7+a^3\,d^9-3\,a^2\,b\,c^7\,d^2-9\,a^2\,b\,c^5\,d^4-9\,a^2\,b\,c^3\,d^6-3\,a^2\,b\,c\,d^8+3\,a\,b^2\,c^8\,d+9\,a\,b^2\,c^6\,d^3+9\,a\,b^2\,c^4\,d^5+3\,a\,b^2\,c^2\,d^7-b^3\,c^9-3\,b^3\,c^7\,d^2-3\,b^3\,c^5\,d^4-b^3\,c^3\,d^6\right)}-\frac{\ln\left(a+b\tan(e+f\,x)\right)\,\left(\frac{-b\,c^3-3\,a\,c^2\,d+3\,b\,c\,d^2+a\,d^3}{\left(a^2+b^2\right)\,{\left(c^2+d^2\right)}^3}+\frac{d^2\,\left(3\,c^2-d^2\right)}{\left(a\,d-b\,c\right)\,{\left(c^2+d^2\right)}^3}+\frac{b^2\,d^2}{{\left(a\,d-b\,c\right)}^3\,\left(c^2+d^2\right)}-\frac{2\,b\,c\,d^2}{{\left(a\,d-b\,c\right)}^2\,{\left(c^2+d^2\right)}^2}\right)}{f}+\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)}{2\,f\,\left(a\,c^3\,1\mathrm{i}+a\,d^3-b\,c^3+b\,d^3\,1\mathrm{i}-a\,c\,d^2\,3\mathrm{i}-3\,a\,c^2\,d+3\,b\,c\,d^2-b\,c^2\,d\,3\mathrm{i}\right)}-\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)}{2\,f\,\left(a\,c^3\,1\mathrm{i}-a\,d^3+b\,c^3+b\,d^3\,1\mathrm{i}-a\,c\,d^2\,3\mathrm{i}+3\,a\,c^2\,d-3\,b\,c\,d^2-b\,c^2\,d\,3\mathrm{i}\right)}-\frac{\frac{-7\,b\,c^3\,d^2+5\,a\,c^2\,d^3-3\,b\,c\,d^4+a\,d^5}{2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)\,\left(c^4+2\,c^2\,d^2+d^4\right)}-\frac{\tan(e+f\,x)\,\left(3\,b\,c^2\,d^3-2\,a\,c\,d^4+b\,d^5\right)}{\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)\,\left(c^4+2\,c^2\,d^2+d^4\right)}}{f\,\left(c^2+2\,c\,d\tan\left(e+f\,x\right)+d^2\tan\left(e+f\,x\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*tan(e + f*x))*(c + d*tan(e + f*x))^3),x)
```

```
[Out] log(tan(e + f*x) - 1i)/(2*f*(a*c^3*1i + a*d^3 - b*c^3 + b*d^3*1i - a*c*d^2*
3i - 3*a*c^2*d + 3*b*c*d^2 - b*c^2*d*3i)) - (log(a + b*tan(e + f*x))*((a*d^
3 - b*c^3 - 3*a*c^2*d + 3*b*c*d^2)/((a^2 + b^2)*(c^2 + d^2)^3) + (d^2*(3*c^
2 - d^2))/((a*d - b*c)*(c^2 + d^2)^3) + (b^2*d^2)/((a*d - b*c)^3*(c^2 + d^2
)) - (2*b*c*d^2)/((a*d - b*c)^2*(c^2 + d^2)^2)))/f - ((a*d^5 + 5*a*c^2*d^3
- 7*b*c^3*d^2 - 3*b*c*d^4)/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(c^4 + d^4 +
2*c^2*d^2)) - (tan(e + f*x)*(b*d^5 + 3*b*c^2*d^3 - 2*a*c*d^4))/((a^2*d^2 +
b^2*c^2 - 2*a*b*c*d)*(c^4 + d^4 + 2*c^2*d^2)))/(f*(c^2 + d^2*tan(e + f*x)^2
 + 2*c*d*tan(e + f*x))) - log(tan(e + f*x) + 1i)/(2*f*(a*c^3*1i - a*d^3 + b
*c^3 + b*d^3*1i - a*c*d^2*3i + 3*a*c^2*d - 3*b*c*d^2 - b*c^2*d*3i)) + (log(
c + d*tan(e + f*x))*(d^4*(3*a^2*c^2 + 3*b^2*c^2) - d^6*(a^2 - b^2) + 6*b^2*
c^4*d^2 - 8*a*b*c^3*d^3))/(f*(a^3*d^9 - b^3*c^9 + 3*a^3*c^2*d^7 + 3*a^3*c^4
*d^5 + a^3*c^6*d^3 - b^3*c^3*d^6 - 3*b^3*c^5*d^4 - 3*b^3*c^7*d^2 + 3*a*b^2*
c^2*d^7 + 9*a*b^2*c^4*d^5 + 9*a*b^2*c^6*d^3 - 9*a^2*b*c^3*d^6 - 9*a^2*b*c^5
*d^4 - 3*a^2*b*c^7*d^2 + 3*a*b^2*c^8*d - 3*a^2*b*c*d^8))
```

3.1228 $\int \frac{1}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}\,dx$

Optimal. Leaf size=457

$$-\frac{(b^2c(c^2-3d^2)-a^2(c^3-3cd^2)+ab(6c^2d-2d^3))\,x}{(a^2+b^2)^2\,(c^2+d^2)^3}+\frac{b^4(2abc-5a^2d-3b^2d)\log(a\cos(e+fx)+b\sin(e+f$$

```
[Out] -(b^2*c*(c^2-3*d^2)-a^2*(c^3-3*c*d^2)+a*b*(6*c^2*d-2*d^3))*x/(a^2+b^2)^2/(c
^2+d^2)^3+b^4*(-5*a^2*d+2*a*b*c-3*b^2*d)*ln(a*cos(f*x+e)+b*sin(f*x+e))/(a^2
+b^2)^2/(-a*d+b*c)^4/f+d^3*(a^2*d^2*(3*c^2-d^2)-2*a*b*c*d*(5*c^2+d^2)+b^2*(
10*c^4+9*c^2*d^2+3*d^4))*ln(c*cos(f*x+e)+d*sin(f*x+e))/(-a*d+b*c)^4/(c^2+d^
2)^3/f-1/2*d*(a^2*d^2+b^2*(2*c^2+3*d^2))/(a^2+b^2)/(-a*d+b*c)^2/(c^2+d^2)/f
/(c+d*tan(f*x+e))^2-b^2/(a^2+b^2)/(-a*d+b*c)/f/(a+b*tan(f*x+e))/(c+d*tan(f*
x+e))^2+d*(2*a^3*c*d^3+2*a*b^2*c*d^3-2*a^2*b*d^2*(2*c^2+d^2)-b^3*(c^4+6*c^2
*d^2+3*d^4))/(a^2+b^2)/(-a*d+b*c)^3/(c^2+d^2)^2/f/(c+d*tan(f*x+e))
```

Rubi [A]
time = 1.28, antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3650, 3730, 3732, 3611}

$$-\frac{d(a^2d^2+b^2(2c^2+3d^2))}{2f\,(a^2+b^2)\,(c^2+d^2)\,(bc-ad)^2(c+d\tan(e+fx))^2}+\frac{d^3(a^2d^2(3c^2-d^2)-2abcd(5c^2+d^2)+b^2(10c^4+9c^2d^2+3d^4))\log(c\cos(e+fx)+d\sin(e+fx))}{f\,(c^2+d^2)^3\,(bc-ad)^4}-\frac{x(-(a^2(c^3-3cd^2))+ab(6c^2d-2d^3)+b^2c(c^2-3d^2))}{(a^2+b^2)^2\,(c^2+d^2)^3}-\frac{b^2}{f\,(a^2+b^2)\,(bc-ad)(a+b\tan(e+fx))(c+d\tan(e+fx))^2}+\frac{b^4(-5a^2d+2abc-3b^2d)\log(a\cos(e+fx)+b\sin(e+fx))}{f\,(a^2+b^2)^2\,(bc-ad)^4}+\frac{d(2a^3cd^3-2a^2bd^2(2c^2+d^2)+2ab^2cd^3-(b^3(c^4+6c^2d^2+3d^4)))}{f\,(a^2+b^2)\,(c^2+d^2)^2\,(bc-ad)^3(c+d\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^3),x]
```

```
[Out] -(((b^2*c*(c^2 - 3*d^2) - a^2*(c^3 - 3*c*d^2) + a*b*(6*c^2*d - 2*d^3))*x)/(
(a^2 + b^2)^2*(c^2 + d^2)^3)) + (b^4*(2*a*b*c - 5*a^2*d - 3*b^2*d)*Log[a*Co
s[e + f*x] + b*Sin[e + f*x]])/((a^2 + b^2)^2*(b*c - a*d)^4*f) + (d^3*(a^2*d
^2*(3*c^2 - d^2) - 2*a*b*c*d*(5*c^2 + d^2) + b^2*(10*c^4 + 9*c^2*d^2 + 3*d^
4))*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((b*c - a*d)^4*(c^2 + d^2)^3*f) -
 (d*(a^2*d^2 + b^2*(2*c^2 + 3*d^2)))/(2*(a^2 + b^2)*(b*c - a*d)^2*(c^2 + d^
2)*f*(c + d*Tan[e + f*x])^2) - b^2/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e
+ f*x])*(c + d*Tan[e + f*x])^2) + (d*(2*a^3*c*d^3 + 2*a*b^2*c*d^3 - 2*a^2*b
*d^2*(2*c^2 + d^2) - b^3*(c^4 + 6*c^2*d^2 + 3*d^4)))/((a^2 + b^2)*(b*c - a*
d)^3*(c^2 + d^2)^2*f*(c + d*Tan[e + f*x]))
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}\,dx = -\frac{b^2}{(a^2+b^2)\,(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+fx}$$

$$= -\frac{d(a^2d^2+b^2(2c^2+3d^2))}{2\,(a^2+b^2)\,(bc-ad)^2\,(c^2+d^2)\,f(c+d\tan(e+fx))^2} - $$

$$= -\frac{d(a^2d^2+b^2(2c^2+3d^2))}{2\,(a^2+b^2)\,(bc-ad)^2\,(c^2+d^2)\,f(c+d\tan(e+fx))^2} - $$

$$= -\frac{(b^2c(c^2-3d^2)-a^2(c^3-3cd^2)+ab(6c^2d-2d^3))\,x}{(a^2+b^2)^2\,(c^2+d^2)^3} - \frac{}{2\,(}$$

$$= -\frac{(b^2c(c^2-3d^2)-a^2(c^3-3cd^2)+ab(6c^2d-2d^3))\,x}{(a^2+b^2)^2\,(c^2+d^2)^3} + \frac{b^4(}{}$$

Mathematica [A]
time = 7.30, size = 833, normalized size = 1.82

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^3),x]
```

```
[Out] -(b^2/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^
2)) - (-1/2*(d^2*(-(a*b*c) + a^2*d + 3*b^2*d) - c*(-3*b^2*c*d + b*d*(b*c -
a*d)))/((-(b*c) + a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^2) - (-(((b*(b*c
- a*d)^3*(2*a*b*c^3 + 3*a^2*c^2*d - 3*b^2*c^2*d - 6*a*b*c*d^2 - a^2*d^3 + b
^2*d^3 + (b*(b^2*c*(c^2 - 3*d^2) - a^2*(c^3 - 3*c*d^2) + a*b*(6*c^2*d - 2*d
^3)))/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[e + f*x]])/((a^2 + b^2)*(c^2 + d^2
)) - (2*b^5*(2*a*b*c - 5*a^2*d - 3*b^2*d)*(c^2 + d^2)^2*Log[a + b*Tan[e + f
*x]])/((a^2 + b^2)*(b*c - a*d)) + (b*(b*c - a*d)^3*(2*a*b*c^3 + 3*a^2*c^2*d
 - 3*b^2*c^2*d - 6*a*b*c*d^2 - a^2*d^3 + b^2*d^3 + (Sqrt[-b^2]*(b^2*c*(c^2
- 3*d^2) - a^2*(c^3 - 3*c*d^2) + a*b*(6*c^2*d - 2*d^3)))/b)*Log[Sqrt[-b^2]
+ b*Tan[e + f*x]])/((a^2 + b^2)*(c^2 + d^2)) - (2*b*(a^2 + b^2)*d^3*(10*b^2
*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + 9*b^2*c^2*d^2 - 2*a*b*c*d^3 - a^2*d^4
 + 3*b^2*d^4)*Log[c + d*Tan[e + f*x]])/((b*c - a*d)*(c^2 + d^2)))/(b*(-(b*c
) + a*d)*(c^2 + d^2)*f)) - (2*d^2*(a*b*d^2*(2*b*c + a*d) + ((a*b*c - a^2*d
- 3*b^2*d)*(2*a*c*d - 2*b*(c^2 + d^2)))/2) - c*(2*d*(b*c - a*d)^2*(b*c + a*
d) - 2*b*c*d*(a^2*d^2 + b^2*(2*c^2 + 3*d^2))))/((-(b*c) + a*d)*(c^2 + d^2)*
f*(c + d*Tan[e + f*x])))/(2*(-(b*c) + a*d)*(c^2 + d^2)))/((a^2 + b^2)*(b*c
- a*d))
```

Maple [A]
time = 1.32, size = 419, normalized size = 0.92

method	result
derivativedivides	$\frac{\frac{b^4}{(ad-bc)^3(a^2+b^2)(a+b\tan(fx+e))}-\frac{b^4(5a^2d-2abc+3b^2d)\ln(a+b\tan(fx+e))}{(ad-bc)^4(a^2+b^2)^2}+\frac{\frac{(-3a^2c^2d+a^2d^3-2abc^3+6abcd^2+3b^2c^2d-b^2d^3)}{2}}{}}{}$
default	$\frac{\frac{b^4}{(ad-bc)^3(a^2+b^2)(a+b\tan(fx+e))}-\frac{b^4(5a^2d-2abc+3b^2d)\ln(a+b\tan(fx+e))}{(ad-bc)^4(a^2+b^2)^2}+\frac{\frac{(-3a^2c^2d+a^2d^3-2abc^3+6abcd^2+3b^2c^2d-b^2d^3)}{2}}{}}{}$
norman	Expression too large to display
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^4/(a*d-b*c)^3/(a^2+b^2)/(a+b*tan(f*x+e))-b^4*(5*a^2*d-2*a*b*c+3*b^2*
d)/(a*d-b*c)^4/(a^2+b^2)^2*ln(a+b*tan(f*x+e))+1/(a^2+b^2)^2/(c^2+d^2)^3*(1/
2*(-3*a^2*c^2*d+a^2*d^3-2*a*b*c^3+6*a*b*c*d^2+3*b^2*c^2*d-b^2*d^3)*ln(1+tan
(f*x+e)^2)+(a^2*c^3-3*a^2*c*d^2-6*a*b*c^2*d+2*a*b*d^3-b^2*c^3+3*b^2*c*d^2)*
arctan(tan(f*x+e)))-1/2*d^3/(a*d-b*c)^2/(c^2+d^2)/(c+d*tan(f*x+e))^2+d^3*(3
*a^2*c^2*d^2-a^2*d^4-10*a*b*c^3*d-2*a*b*c*d^3+10*b^2*c^4+9*b^2*c^2*d^2+3*b^
2*d^4)/(a*d-b*c)^4/(c^2+d^2)^3*ln(c+d*tan(f*x+e))-2*d^3*(a*c*d-2*b*c^2-b*d^
2)/(a*d-b*c)^3/(c^2+d^2)^2/(c+d*tan(f*x+e)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1858 vs. $2(467) = 934$.
time = 0.70, size = 1858, normalized size = 4.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(6*a*b*c^2*d - 2*a*b*d^3 - (a^2 - b^2)*c^3 + 3*(a^2 - b^2)*c*d^2)*(
f*x + e)/((a^4 + 2*a^2*b^2 + b^4)*c^6 + 3*(a^4 + 2*a^2*b^2 + b^4)*c^4*d^2 +
 3*(a^4 + 2*a^2*b^2 + b^4)*c^2*d^4 + (a^4 + 2*a^2*b^2 + b^4)*d^6) - 2*(2*a*
b^5*c - (5*a^2*b^4 + 3*b^6)*d)*log(b*tan(f*x + e) + a)/((a^4*b^4 + 2*a^2*b^
6 + b^8)*c^4 - 4*(a^5*b^3 + 2*a^3*b^5 + a*b^7)*c^3*d + 6*(a^6*b^2 + 2*a^4*b
^4 + a^2*b^6)*c^2*d^2 - 4*(a^7*b + 2*a^5*b^3 + a^3*b^5)*c*d^3 + (a^8 + 2*a^
6*b^2 + a^4*b^4)*d^4) - 2*(10*b^2*c^4*d^3 - 10*a*b*c^3*d^4 - 2*a*b*c*d^6 +
3*(a^2 + 3*b^2)*c^2*d^5 - (a^2 - 3*b^2)*d^7)*log(d*tan(f*x + e) + c)/(b^4*c
^10 - 4*a*b^3*c^9*d - 4*a^3*b*c*d^9 + a^4*d^10 + 3*(2*a^2*b^2 + b^4)*c^8*d^
2 - 4*(a^3*b + 3*a*b^3)*c^7*d^3 + (a^4 + 18*a^2*b^2 + 3*b^4)*c^6*d^4 - 12*(
a^3*b + a*b^3)*c^5*d^5 + (3*a^4 + 18*a^2*b^2 + b^4)*c^4*d^6 - 4*(3*a^3*b +
a*b^3)*c^3*d^7 + 3*(a^4 + 2*a^2*b^2)*c^2*d^8) + (2*a*b*c^3 - 6*a*b*c*d^2 +
```

```
3*(a^2 - b^2)*c^2*d - (a^2 - b^2)*d^3)*log(tan(f*x + e)^2 + 1)/((a^4 + 2*a^
2*b^2 + b^4)*c^6 + 3*(a^4 + 2*a^2*b^2 + b^4)*c^4*d^2 + 3*(a^4 + 2*a^2*b^2 +
 b^4)*c^2*d^4 + (a^4 + 2*a^2*b^2 + b^4)*d^6) + (2*b^4*c^6 + 4*b^4*c^4*d^2 +
 9*(a^3*b + a*b^3)*c^3*d^3 - (5*a^4 + 5*a^2*b^2 - 2*b^4)*c^2*d^4 + 5*(a^3*b
 + a*b^3)*c*d^5 - (a^4 + a^2*b^2)*d^6 + 2*(b^4*c^4*d^2 + 2*(2*a^2*b^2 + 3*b
^4)*c^2*d^4 - 2*(a^3*b + a*b^3)*c*d^5 + (2*a^2*b^2 + 3*b^4)*d^6)*tan(f*x +
e)^2 + (4*b^4*c^5*d + (9*a^2*b^2 + 17*b^4)*c^3*d^3 + 3*(a^3*b + a*b^3)*c^2*
d^4 - (4*a^4 - a^2*b^2 - 9*b^4)*c*d^5 + 3*(a^3*b + a*b^3)*d^6)*tan(f*x + e)
)/((a^3*b^3 + a*b^5)*c^9 - 3*(a^4*b^2 + a^2*b^4)*c^8*d + (3*a^5*b + 5*a^3*b
^3 + 2*a*b^5)*c^7*d^2 - (a^6 + 7*a^4*b^2 + 6*a^2*b^4)*c^6*d^3 + (6*a^5*b +
7*a^3*b^3 + a*b^5)*c^5*d^4 - (2*a^6 + 5*a^4*b^2 + 3*a^2*b^4)*c^4*d^5 + 3*(a
^5*b + a^3*b^3)*c^3*d^6 - (a^6 + a^4*b^2)*c^2*d^7 + ((a^2*b^4 + b^6)*c^7*d^
2 - 3*(a^3*b^3 + a*b^5)*c^6*d^3 + (3*a^4*b^2 + 5*a^2*b^4 + 2*b^6)*c^5*d^4 -
 (a^5*b + 7*a^3*b^3 + 6*a*b^5)*c^4*d^5 + (6*a^4*b^2 + 7*a^2*b^4 + b^6)*c^3*
d^6 - (2*a^5*b + 5*a^3*b^3 + 3*a*b^5)*c^2*d^7 + 3*(a^4*b^2 + a^2*b^4)*c*d^8
 - (a^5*b + a^3*b^3)*d^9)*tan(f*x + e)^3 + (2*(a^2*b^4 + b^6)*c^8*d - 5*(a^
3*b^3 + a*b^5)*c^7*d^2 + (3*a^4*b^2 + 7*a^2*b^4 + 4*b^6)*c^6*d^3 + (a^5*b -
 9*a^3*b^3 - 10*a*b^5)*c^5*d^4 - (a^6 - 5*a^4*b^2 - 8*a^2*b^4 - 2*b^6)*c^4*
d^5 + (2*a^5*b - 3*a^3*b^3 - 5*a*b^5)*c^3*d^6 - (2*a^6 - a^4*b^2 - 3*a^2*b^
4)*c^2*d^7 + (a^5*b + a^3*b^3)*c*d^8 - (a^6 + a^4*b^2)*d^9)*tan(f*x + e)^2
+ ((a^2*b^4 + b^6)*c^9 - (a^3*b^3 + a*b^5)*c^8*d - (3*a^4*b^2 + a^2*b^4 - 2
*b^6)*c^7*d^2 + (5*a^5*b + 3*a^3*b^3 - 2*a*b^5)*c^6*d^3 - (2*a^6 + 8*a^4*b^
2 + 5*a^2*b^4 - b^6)*c^5*d^4 + (10*a^5*b + 9*a^3*b^3 - a*b^5)*c^4*d^5 - (4*
a^6 + 7*a^4*b^2 + 3*a^2*b^4)*c^3*d^6 + 5*(a^5*b + a^3*b^3)*c^2*d^7 - 2*(a^6
 + a^4*b^2)*c*d^8)*tan(f*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4817 vs. $2(467) = 934$.
time = 6.13, size = 4817, normalized size = 10.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^7*c^9 - 2*a*b^6*c^8*d + 6*b^7*c^7*d^2 - 6*a*b^6*c^6*d^3 + 6*b^7*c
^5*d^4 + (11*a^5*b^2 + 22*a^3*b^4 + 5*a*b^6)*c^4*d^5 - 2*(9*a^6*b + 18*a^4*
b^3 + 9*a^2*b^5 - b^7)*c^3*d^6 + (7*a^7 + 19*a^5*b^2 + 17*a^3*b^4 + 3*a*b^6
)*c^2*d^7 - 6*(a^6*b + 2*a^4*b^3 + a^2*b^5)*c*d^8 + (a^7 + 2*a^5*b^2 + a^3*
b^4)*d^9 - (2*a*b^6*c^7*d^2 - 2*a^2*b^5*c^6*d^3 + 6*a*b^6*c^5*d^4 + 3*(3*a^
4*b^3 + 4*a^2*b^5 + 3*b^7)*c^4*d^5 - 2*(7*a^5*b^2 + 14*a^3*b^4 + 4*a*b^6)*c
^3*d^6 + (5*a^6*b + 13*a^4*b^3 + 5*a^2*b^5 + 3*b^7)*c^2*d^7 - 2*(a^5*b^2 +
2*a^3*b^4)*c*d^8 - (a^6*b + 2*a^4*b^3 + 3*a^2*b^5)*d^9 + 2*(6*a^5*b^2*c^2*d
^7 + 2*a^5*b^2*d^9 + (a^2*b^5 - b^7)*c^7*d^2 - 2*(2*a^3*b^4 + a*b^6)*c^6*d^
3 + 3*(2*a^4*b^3 + 5*a^2*b^5 + b^7)*c^5*d^4 - 2*(2*a^5*b^2 + 10*a^3*b^4 + 5
```

*a*b^6)*c^4*d^5 + (a^6*b + 5*a^4*b^3 + 10*a^2*b^5)*c^3*d^6 - (3*a^6*b + 5*a^4*b^3)*c*d^8)*f*x)*tan(f*x + e)^3 - 2*(6*a^6*b*c^4*d^5 + 2*a^6*b*c^2*d^7 + (a^3*b^4 - a*b^6)*c^9 - 2*(2*a^4*b^3 + a^2*b^5)*c^8*d + 3*(2*a^5*b^2 + 5*a^3*b^4 + a*b^6)*c^7*d^2 - 2*(2*a^6*b + 10*a^4*b^3 + 5*a^2*b^5)*c^6*d^3 + (a^7 + 5*a^5*b^2 + 10*a^3*b^4)*c^5*d^4 - (3*a^7 + 5*a^5*b^2)*c^3*d^6)*f*x - (4*a*b^6*c^8*d + 14*a*b^6*c^6*d^3 - 2*(2*a^2*b^5 + b^7)*c^7*d^2 + 2*(5*a^4*b^3 + 4*a^2*b^5 + 2*b^7)*c^5*d^4 - (7*a^5*b^2 + 14*a^3*b^4 - 11*a*b^6)*c^4*d^5 - 2*(4*a^6*b + 11*a^4*b^3 + 16*a^2*b^5 + 6*b^7)*c^3*d^6 + 5*(a^7 + 5*a^5*b^2 + 7*a^3*b^4 + 5*a*b^6)*c^2*d^7 - 2*(4*a^6*b + 10*a^4*b^3 + 10*a^2*b^5 + 3*b^7)*c*d^8 - (a^7 - 2*a^5*b^2 - 7*a^3*b^4 - 6*a*b^6)*d^9 - 2*(10*a^4*b^3*c^2*d^7 - 2*a^6*b*d^9 - 2*(a^2*b^5 - b^7)*c^8*d + (7*a^3*b^4 + 5*a*b^6)*c^7*d^2 - 2*(4*a^4*b^3 + 14*a^2*b^5 + 3*b^7)*c^6*d^3 + (2*a^5*b^2 + 25*a^3*b^4 + 17*a*b^6)*c^5*d^4 + 2*(a^6*b + 5*a^4*b^3 - 5*a^2*b^5)*c^4*d^5 - (a^7 + 17*a^5*b^2 + 10*a^3*b^4)*c^3*d^6 + (3*a^7 + a^5*b^2)*c*d^8)*f*x)*tan(f*x + e)^2 - (2*a^2*b^5*c^9 + 6*a^2*b^5*c^7*d^2 + 6*a^2*b^5*c^5*d^4 + 2*a^2*b^5*c^3*d^6 - (5*a^3*b^4 + 3*a*b^6)*c^8*d - 3*(5*a^3*b^4 + 3*a*b^6)*c^6*d^3 - 3*(5*a^3*b^4 + 3*a*b^6)*c^4*d^5 - (5*a^3*b^4 + 3*a*b^6)*c^2*d^7 + (2*a*b^6*c^7*d^2 + 6*a*b^6*c^5*d^4 + 6*a*b^6*c^3*d^6 + 2*a*b^6*c*d^8 - (5*a^2*b^5 + 3*b^7)*c^6*d^3 - 3*(5*a^2*b^5 + 3*b^7)*c^4*d^5 - 3*(5*a^2*b^5 + 3*b^7)*c^2*d^7 - (5*a^2*b^5 + 3*b^7)*d^9)*tan(f*x + e)^3 + (4*a*b^6*c^8*d - 2*(4*a^2*b^5 + 3*b^7)*c^7*d^2 - (5*a^3*b^4 - 9*a*b^6)*c^6*d^3 - 6*(4*a^2*b^5 + 3*b^7)*c^5*d^4 - 3*(5*a^3*b^4 - a*b^6)*c^4*d^5 - 6*(4*a^2*b^5 + 3*b^7)*c^3*d^6 - 5*(3*a^3*b^4 + a*b^6)*c^2*d^7 - 2*(4*a^2*b^5 + 3*b^7)*c*d^8 - (5*a^3*b^4 + 3*a*b^6)*d^9)*tan(f*x + e)^2 + (2*a*b^6*c^9 - 10*a^3*b^4*c^7*d^2 - (a^2*b^5 + 3*b^7)*c^8*d - 3*(a^2*b^5 + 3*b^7)*c^6*d^3 - 6*(5*a^3*b^4 + 2*a*b^6)*c^5*d^4 - 3*(a^2*b^5 + 3*b^7)*c^4*d^5 - 2*(15*a^3*b^4 + 8*a*b^6)*c^3*d^6 - (a^2*b^5 + 3*b^7)*c^2*d^7 - 2*(5*a^3*b^4 + 3*a*b^6)*c*d^8)*tan(f*x + e))*log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(tan(f*x + e)^2 + 1)) - (10*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*c^6*d^3 - 10*(a^6*b + 2*a^4*b^3 + a^2*b^5)*c^5*d^4 + 3*(a^7 + 5*a^5*b^2 + 7*a^3*b^4 + 3*a*b^6)*c^4*d^5 - 2*(a^6*b + 2*a^4*b^3 + a^2*b^5)*c^3*d^6 - (a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^6)*c^2*d^7 + (10*(a^4*b^3 + 2*a^2*b^5 + b^7)*c^4*d^5 - 10*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*c^3*d^6 + 3*(a^6*b + 5*a^4*b^3 + 7*a^2*b^5 + 3*b^7)*c^2*d^7 - 2*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*c*d^8 - (a^6*b - a^4*b^3 - 5*a^2*b^5 - 3*b^7)*d^9)*tan(f*x + e)^3 + (20*(a^4*b^3 + 2*a^2*b^5 + b^7)*c^5*d^4 - 10*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*c^4*d^5 - 2*(2*a^6*b - 5*a^4*b^3 - 16*a^2*b^5 - 9*b^7)*c^3*d^6 + (3*a^7 + 11*a^5*b^2 + 13*a^3*b^4 + 5*a*b^6)*c^2*d^7 - 2*(2*a^6*b + a^4*b^3 - 4*a^2*b^5 - 3*b^7)*c*d^8 - (a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^6)*d^9)*tan(f*x + e)^2 + (10*(a^4*b^3 + 2*a^2*b^5 + b^7)*c^6*d^3 + 10*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*c^5*d^4 - (17*a^6*b + 25*a^4*b^3 - a^2*b^5 - 9*b^7)*c^4*d^5 + 2*(3*a^7 + 14*a^5*b^2 + 19*a^3*b^4 + 8*a*b^6)*c^3*d^6 - (5*a^6*b + 7*a^4*b^3 - a^2*b^5 - 3*b^7)*c^2*d^7 - 2*(a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^6)*c*d^8)*tan(f*x + e))*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) - (2*a*b^6*c^9 + 10*a*b^6*c^7*d^2 - 2*(a^2*b^5 + 2*b^7)*c^8*d - 6*(a^2*b^5 + 2*b^7)*c^6*d^3 + 2*(5*a^5*b^2 + 10*a^3*b^4 + 14*a*b^6)*

```
c^5*d^4 - (16*a^6*b + 43*a^4*b^3 + 44*a^2*b^5 + 23*b^7)*c^4*d^5 + 2*(3*a^7
+ 12*a^5*b^2 + 15*a^3*b^4 + 13*a*b^6)*c^3*d^6 + (5*a^6*b + 5*a^4*b^3 - 7*a^
2*b^5 - 9*b^7)*c^2*d^7 - 2*(3*a^7 + 5*a^5*b^2 + a^3*b^4 - 3*a*b^6)*c*d^8 +
3*(a^6*b + 2*a^4*b^3 + a^2*b^5)*d^9 + 2*(4*a^6*b*c*d^8 + (a^2*b^5 - b^7)*c^
9 - 2*(a^3*b^4 + 2*a*b^6)*c^8*d - (2*a^4*b^3 - 11*a^2*b^5 - 3*b^7)*c^7*d^2
+ 2*(4*a^5*b^2 + 5*a^3*b^4 - 2*a*b^6)*c^6*d^3 - (7*a^6*b + 35*a^4*b^3 + 10*
a^2*b^5)*c^5*d^4 + 2*(a^7 + 8*a^5*b^2 + 10*a^3*b^4)*c^4*d^5 + (9*a^6*b - 5*
a^4*b^3)*c^3*d^6 - 2*(3*a^7 + 4*a^5*b^2)*c^2*d^7)*f*x)*tan(f*x + e))/(((a^4
*b^5 + 2*a^2*b^7 + b^9)*c^10*d^2 - 4*(a^5*b^4 + 2*a^3*b^6 + a*b^8)*c^9*d^3
+ 3*(2*a^6*b^3 + 5*a^4*b^5 + 4*a^2*b^7 + b^9)*c^8*d^4 - 4*(a^7*b^2 + 5*a^5*
b^4 + 7*a^3*b^6 + 3*a*b^8)*c^7*d^5 + (a^8*b + 20*a^6*b^3 + 40*a^4*b^5 + 24*
a^2*b^7 + 3*b^9)*c^6*d^6 - 12*(a^7*b^2 + 3*a^5*...
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))**2/(c+d*tan(f*x+e))**3,x)
```

```
[Out] Exception raised: NotImplementedError >> no valid subset found
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1758 vs. 2(467) = 934.
time = 0.90, size = 1758, normalized size = 3.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(a^2*c^3 - b^2*c^3 - 6*a*b*c^2*d - 3*a^2*c*d^2 + 3*b^2*c*d^2 + 2*a*b
*d^3)*(f*x + e)/(a^4*c^6 + 2*a^2*b^2*c^6 + b^4*c^6 + 3*a^4*c^4*d^2 + 6*a^2*
b^2*c^4*d^2 + 3*b^4*c^4*d^2 + 3*a^4*c^2*d^4 + 6*a^2*b^2*c^2*d^4 + 3*b^4*c^2
*d^4 + a^4*d^6 + 2*a^2*b^2*d^6 + b^4*d^6) - (2*a*b*c^3 + 3*a^2*c^2*d - 3*b^
2*c^2*d - 6*a*b*c*d^2 - a^2*d^3 + b^2*d^3)*log(tan(f*x + e)^2 + 1)/(a^4*c^6
 + 2*a^2*b^2*c^6 + b^4*c^6 + 3*a^4*c^4*d^2 + 6*a^2*b^2*c^4*d^2 + 3*b^4*c^4*
d^2 + 3*a^4*c^2*d^4 + 6*a^2*b^2*c^2*d^4 + 3*b^4*c^2*d^4 + a^4*d^6 + 2*a^2*b
^2*d^6 + b^4*d^6) + 2*(2*a*b^6*c - 5*a^2*b^5*d - 3*b^7*d)*log(abs(b*tan(f*x
 + e) + a))/(a^4*b^5*c^4 + 2*a^2*b^7*c^4 + b^9*c^4 - 4*a^5*b^4*c^3*d - 8*a^
3*b^6*c^3*d - 4*a*b^8*c^3*d + 6*a^6*b^3*c^2*d^2 + 12*a^4*b^5*c^2*d^2 + 6*a^
2*b^7*c^2*d^2 - 4*a^7*b^2*c*d^3 - 8*a^5*b^4*c*d^3 - 4*a^3*b^6*c*d^3 + a^8*b
*d^4 + 2*a^6*b^3*d^4 + a^4*b^5*d^4) + 2*(10*b^2*c^4*d^4 - 10*a*b*c^3*d^5 +
3*a^2*c^2*d^6 + 9*b^2*c^2*d^6 - 2*a*b*c*d^7 - a^2*d^8 + 3*b^2*d^8)*log(abs(
d*tan(f*x + e) + c))/(b^4*c^10*d - 4*a*b^3*c^9*d^2 + 6*a^2*b^2*c^8*d^3 + 3*
```

```
b^4*c^8*d^3 - 4*a^3*b*c^7*d^4 - 12*a*b^3*c^7*d^4 + a^4*c^6*d^5 + 18*a^2*b^2
*c^6*d^5 + 3*b^4*c^6*d^5 - 12*a^3*b*c^5*d^6 - 12*a*b^3*c^5*d^6 + 3*a^4*c^4*
d^7 + 18*a^2*b^2*c^4*d^7 + b^4*c^4*d^7 - 12*a^3*b*c^3*d^8 - 4*a*b^3*c^3*d^8
 + 3*a^4*c^2*d^9 + 6*a^2*b^2*c^2*d^9 - 4*a^3*b*c*d^10 + a^4*d^11) - 2*(2*a*
b^6*c*tan(f*x + e) - 5*a^2*b^5*d*tan(f*x + e) - 3*b^7*d*tan(f*x + e) + 3*a^
2*b^5*c + b^7*c - 6*a^3*b^4*d - 4*a*b^6*d)/((a^4*b^4*c^4 + 2*a^2*b^6*c^4 +
b^8*c^4 - 4*a^5*b^3*c^3*d - 8*a^3*b^5*c^3*d - 4*a*b^7*c^3*d + 6*a^6*b^2*c^2
*d^2 + 12*a^4*b^4*c^2*d^2 + 6*a^2*b^6*c^2*d^2 - 4*a^7*b*c*d^3 - 8*a^5*b^3*c
*d^3 - 4*a^3*b^5*c*d^3 + a^8*d^4 + 2*a^6*b^2*d^4 + a^4*b^4*d^4)*(b*tan(f*x
+ e) + a)) - (30*b^2*c^4*d^5*tan(f*x + e)^2 - 30*a*b*c^3*d^6*tan(f*x + e)^2
 + 9*a^2*c^2*d^7*tan(f*x + e)^2 + 27*b^2*c^2*d^7*tan(f*x + e)^2 - 6*a*b*c*d
^8*tan(f*x + e)^2 - 3*a^2*d^9*tan(f*x + e)^2 + 9*b^2*d^9*tan(f*x + e)^2 + 6
8*b^2*c^5*d^4*tan(f*x + e) - 72*a*b*c^4*d^5*tan(f*x + e) + 22*a^2*c^3*d^6*t
an(f*x + e) + 66*b^2*c^3*d^6*tan(f*x + e) - 28*a*b*c^2*d^7*tan(f*x + e) - 2
*a^2*c*d^8*tan(f*x + e) + 22*b^2*c*d^8*tan(f*x + e) - 4*a*b*d^9*tan(f*x + e
) + 39*b^2*c^6*d^3 - 44*a*b*c^5*d^4 + 14*a^2*c^4*d^5 + 41*b^2*c^4*d^5 - 26*
a*b*c^3*d^6 + 3*a^2*c^2*d^7 + 14*b^2*c^2*d^7 - 6*a*b*c*d^8 + a^2*d^9)/((b^4
*c^10 - 4*a*b^3*c^9*d + 6*a^2*b^2*c^8*d^2 + 3*b^4*c^8*d^2 - 4*a^3*b*c^7*d^3
 - 12*a*b^3*c^7*d^3 + a^4*c^6*d^4 + 18*a^2*b^2*c^6*d^4 + 3*b^4*c^6*d^4 - 12
*a^3*b*c^5*d^5 - 12*a*b^3*c^5*d^5 + 3*a^4*c^4*d^6 + 18*a^2*b^2*c^4*d^6 + b^
4*c^4*d^6 - 12*a^3*b*c^3*d^7 - 4*a*b^3*c^3*d^7 + 3*a^4*c^2*d^8 + 6*a^2*b^2*
c^2*d^8 - 4*a^3*b*c*d^9 + a^4*d^10)*(d*tan(f*x + e) + c)^2))/f
```

Mupad [B]
time = 18.89, size = 1417, normalized size = 3.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*tan(e + f*x))^2*(c + d*tan(e + f*x))^3),x)
```

```
[Out] ((2*b^4*c^6 - a^4*d^6 - a^2*b^2*d^6 - 5*a^4*c^2*d^4 + 2*b^4*c^2*d^4 + 4*b^4
*c^4*d^2 + 9*a*b^3*c^3*d^3 + 9*a^3*b*c^3*d^3 - 5*a^2*b^2*c^2*d^4 + 5*a*b^3*
c*d^5 + 5*a^3*b*c*d^5)/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^
2)*(a^2*c^4 + a^2*d^4 + b^2*c^4 + b^2*d^4 + 2*a^2*c^2*d^2 + 2*b^2*c^2*d^2))
 + (tan(e + f*x)*(3*a*b^3*d^6 + 3*a^3*b*d^6 - 4*a^4*c*d^5 + 9*b^4*c*d^5 + 4
*b^4*c^5*d + 17*b^4*c^3*d^3 + 3*a*b^3*c^2*d^4 + a^2*b^2*c*d^5 + 3*a^3*b*c^2
*d^4 + 9*a^2*b^2*c^3*d^3))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*
c*d^2)*(a^2*c^4 + a^2*d^4 + b^2*c^4 + b^2*d^4 + 2*a^2*c^2*d^2 + 2*b^2*c^2*d
^2)) + (tan(e + f*x)^2*(3*b^4*d^6 + 2*a^2*b^2*d^6 + 6*b^4*c^2*d^4 + b^4*c^4
*d^2 + 4*a^2*b^2*c^2*d^4 - 2*a*b^3*c*d^5 - 2*a^3*b*c*d^5))/((a^3*d^3 - b^3*
c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(a^2*c^4 + a^2*d^4 + b^2*c^4 + b^2*d^4
 + 2*a^2*c^2*d^2 + 2*b^2*c^2*d^2)))/(f*(tan(e + f*x)*(b*c^2 + 2*a*c*d) + a*
c^2 + tan(e + f*x)^2*(a*d^2 + 2*b*c*d) + b*d^2*tan(e + f*x)^3)) - (log(tan(
e + f*x) - 1i)*1i)/(2*f*(a^2*c^3 - a^2*d^3*1i - b^2*c^3 + b^2*d^3*1i - 3*a^
```

```
2*c*d^2 + a^2*c^2*d*3i + 3*b^2*c*d^2 - b^2*c^2*d*3i + a*b*c^3*2i + 2*a*b*d^
3 - a*b*c*d^2*6i - 6*a*b*c^2*d)) + (log(tan(e + f*x) + 1i)*1i)/(2*f*(a^2*c^
3 + a^2*d^3*1i - b^2*c^3 - b^2*d^3*1i - 3*a^2*c*d^2 - a^2*c^2*d*3i + 3*b^2*
c*d^2 + b^2*c^2*d*3i - a*b*c^3*2i + 2*a*b*d^3 + a*b*c*d^2*6i - 6*a*b*c^2*d)
) - (log(a + b*tan(e + f*x))*(d*(3*b^6 + 5*a^2*b^4) - 2*a*b^5*c))/(f*(a^8*d
^4 + b^8*c^4 + 2*a^2*b^6*c^4 + a^4*b^4*c^4 + a^4*b^4*d^4 + 2*a^6*b^2*d^4 -
4*a^3*b^5*c*d^3 - 8*a^3*b^5*c^3*d - 8*a^5*b^3*c*d^3 - 4*a^5*b^3*c^3*d + 6*a
^2*b^6*c^2*d^2 + 12*a^4*b^4*c^2*d^2 + 6*a^6*b^2*c^2*d^2 - 4*a*b^7*c^3*d - 4
*a^7*b*c*d^3)) - (log(c + d*tan(e + f*x))*(a^2*(d^7 - 3*c^2*d^5) - b^2*(3*d
^7 + 9*c^2*d^5 + 10*c^4*d^3) + a*b*(2*c*d^6 + 10*c^3*d^4)))/(f*(a^4*d^10 +
b^4*c^10 + 3*a^4*c^2*d^8 + 3*a^4*c^4*d^6 + a^4*c^6*d^4 + b^4*c^4*d^6 + 3*b^
4*c^6*d^4 + 3*b^4*c^8*d^2 - 4*a*b^3*c^3*d^7 - 12*a*b^3*c^5*d^5 - 12*a*b^3*c
^7*d^3 - 12*a^3*b*c^3*d^7 - 12*a^3*b*c^5*d^5 - 4*a^3*b*c^7*d^3 + 6*a^2*b^2*
c^2*d^8 + 18*a^2*b^2*c^4*d^6 + 18*a^2*b^2*c^6*d^4 + 6*a^2*b^2*c^8*d^2 - 4*a
*b^3*c^9*d - 4*a^3*b*c*d^9))
```

3.1229 $\int (a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=209

$$\frac{(ia+b)^3\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}-\frac{(ia-b)^3\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}+\frac{2b}{}$$

```
[Out] (I*a+b)^3*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))*(c-I*d)^(1/2)/f-(I*
a-b)^3*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))*(c+I*d)^(1/2)/f+2*b*(3
*a^2-b^2)*(c+d*tan(f*x+e))^(1/2)/f-4/15*b^2*(-6*a*d+b*c)*(c+d*tan(f*x+e))^(
3/2)/d^2/f+2/5*b^2*(a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(3/2)/d/f
```

Rubi [A]
time = 0.39, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3647, 3711, 3609, 3620, 3618, 65, 214}

$$\frac{2b(3a^2-b^2)\sqrt{c+d\tan(e+fx)}}{f}-\frac{4b^2(bc-6ad)(c+d\tan(e+fx))^{3/2}}{15d^2f}+\frac{2b^2(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}{5df}-\frac{(-b+ia)^3\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}+\frac{(b+ia)^3\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^3*Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((I*a + b)^3*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])
/f - ((I*a - b)^3*Sqrt[c + I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I
*d]])/f + (2*b*(3*a^2 - b^2)*Sqrt[c + d*Tan[e + f*x]])/f - (4*b^2*(b*c - 6*
a*d)*(c + d*Tan[e + f*x])^(3/2))/(15*d^2*f) + (2*b^2*(a + b*Tan[e + f*x])*(
c + d*Tan[e + f*x])^(3/2))/(5*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
```

```
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int (a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}\,dx &= \frac{2b^2(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}{5df} + \frac{2\int \sqrt{c+ \cdots}}{\cdots} \\
&= -\frac{4b^2(bc-6ad)(c+d\tan(e+fx))^{3/2}}{15d^2 f} + \frac{2b^2(a+b\tan(e \cdots}{\cdots} \\
&= \frac{2b(3a^2-b^2)\sqrt{c+d\tan(e+fx)}}{f} - \frac{4b^2(bc-6ad)(c+d \cdots}{15d^2 f \cdots} \\
&= \frac{2b(3a^2-b^2)\sqrt{c+d\tan(e+fx)}}{f} - \frac{4b^2(bc-6ad)(c+d \cdots}{15d^2 f \cdots} \\
&= \frac{2b(3a^2-b^2)\sqrt{c+d\tan(e+fx)}}{f} - \frac{4b^2(bc-6ad)(c+d \cdots}{15d^2 f \cdots} \\
&= \frac{2b(3a^2-b^2)\sqrt{c+d\tan(e+fx)}}{f} - \frac{4b^2(bc-6ad)(c+d \cdots}{15d^2 f \cdots} \\
&= \frac{(ia+b)^3\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f} - \frac{(ia \cdots}{\cdots}
\end{aligned}
$$

Mathematica [A]
time = 2.32, size = 194, normalized size = 0.93

$$
\frac{-15i(a-ib)^3\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)+15i(a+ib)^3\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)+\frac{2b\sqrt{c+d\tan(e+fx)}\,(15abcd+45a^2d^2-b^2(2c^2+15d^2)+bd(bc+15ad)\tan(e+fx)+3b^2d^2\tan^2(e+fx))}{d^2}}{15f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^3*Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((-15*I)*(a - I*b)^3*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c
- I*d]] + (15*I)*(a + I*b)^3*Sqrt[c + I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]
/Sqrt[c + I*d]] + (2*b*Sqrt[c + d*Tan[e + f*x]]*(15*a*b*c*d + 45*a^2*d^2 -
b^2*(2*c^2 + 15*d^2) + b*d*(b*c + 15*a*d)*Tan[e + f*x] + 3*b^2*d^2*Tan[e +
f*x]^2))/d^2)/(15*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1095 vs. $2(181) = 362$.
time = 0.52, size = 1096, normalized size = 5.24 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/d^2*(1/5*b^3*(c+d*tan(f*x+e))^(5/2)+a*b^2*d*(c+d*tan(f*x+e))^(3/2)-1/3*
b^3*c*(c+d*tan(f*x+e))^(3/2)+3*a^2*b*d^2*(c+d*tan(f*x+e))^(1/2)-b^3*d^2*(c+
d*tan(f*x+e))^(1/2)+d^2*(1/4/d*(1/2*((c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*a^3-3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2-(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*a^3*c+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d+3*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*d)*ln(d*t
an(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^
(1/2))+2*(-6*(c^2+d^2)^(1/2)*a^2*b*d+2*(c^2+d^2)^(1/2)*b^3*d+1/2*((c^2+d^2)
^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3-3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*a*b^2-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c+3*(2*(c^2+d^2)^(1/2
)+2*c)^(1/2)*a^2*b*d+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c-(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)*b^3*d)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*
c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2
*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/4/d*(1/2*(-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*a^3+3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2+(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d-3*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*d)*
ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2
+d^2)^(1/2))+2*(-6*(c^2+d^2)^(1/2)*a^2*b*d+2*(c^2+d^2)^(1/2)*b^3*d-1/2*(-(c
^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3+3*(c^2+d^2)^(1/2)*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*a*b^2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c-3*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*a^2*b*d-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c+(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*b^3*d)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(
1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1
/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^3*sqrt(d*tan(f*x + e) + c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(e+fx))^3 \sqrt{c+d\tan(e+fx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(a+b*tan(f*x+e))**3,x)
```

```
[Out] Integral((a + b*tan(e + f*x))**3*sqrt(c + d*tan(e + f*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 21.26, size = 2500, normalized size = 11.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^3*(c + d*tan(e + f*x))^(1/2),x)
```

```
[Out] atan(((((8*(4*b^3*d^4*f^2 - 12*a^2*b*d^4*f^2 + 4*b^3*c^2*d^2*f^2 - 12*a^2*b
*c^2*d^2*f^2))/f^3 - 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*(-(((8*b^6*c*f^2 -
 8*a^6*c*f^2 - 120*a^2*b^4*c*f^2 + 120*a^4*b^2*c*f^2 - 160*a^3*b^3*d*f^2 +
48*a*b^5*d*f^2 + 48*a^5*b*d*f^2)^2/64 - f^4*(a^12*c^2 + a^12*d^2 + b^12*c^2
 + b^12*d^2 + 6*a^2*b^10*c^2 + 15*a^4*b^8*c^2 + 20*a^6*b^6*c^2 + 15*a^8*b^4
*c^2 + 6*a^10*b^2*c^2 + 6*a^2*b^10*d^2 + 15*a^4*b^8*d^2 + 20*a^6*b^6*d^2 +
15*a^8*b^4*d^2 + 6*a^10*b^2*d^2))^(1/2) + a^6*c*f^2 - b^6*c*f^2 + 15*a^2*b^
4*c*f^2 - 15*a^4*b^2*c*f^2 + 20*a^3*b^3*d*f^2 - 6*a*b^5*d*f^2 - 6*a^5*b*d*f
^2)/(4*f^4))^(1/2))*(-(((8*b^6*c*f^2 - 8*a^6*c*f^2 - 120*a^2*b^4*c*f^2 + 12
0*a^4*b^2*c*f^2 - 160*a^3*b^3*d*f^2 + 48*a*b^5*d*f^2 + 48*a^5*b*d*f^2)^2/64
 - f^4*(a^12*c^2 + a^12*d^2 + b^12*c^2 + b^12*d^2 + 6*a^2*b^10*c^2 + 15*a^4
*b^8*c^2 + 20*a^6*b^6*c^2 + 15*a^8*b^4*c^2 + 6*a^10*b^2*c^2 + 6*a^2*b^10*d^
2 + 15*a^4*b^8*d^2 + 20*a^6*b^6*d^2 + 15*a^8*b^4*d^2 + 6*a^10*b^2*d^2))^(1/
2) + a^6*c*f^2 - b^6*c*f^2 + 15*a^2*b^4*c*f^2 - 15*a^4*b^2*c*f^2 + 20*a^3*b
```

^3*d*f^2 - 6*a*b^5*d*f^2 - 6*a^5*b*d*f^2)/(4*f^4))^(1/2) + (16*(c + d*tan(e + f*x))^(1/2)*(a^6*d^4 - b^6*d^4 + 15*a^2*b^4*d^4 - 15*a^4*b^2*d^4 - a^6*c^2*d^2 + b^6*c^2*d^2 - 40*a^3*b^3*c*d^3 - 15*a^2*b^4*c^2*d^2 + 15*a^4*b^2*c^2*d^2 + 12*a*b^5*c*d^3 + 12*a^5*b*c*d^3))/f^2)*(-(((8*b^6*c*f^2 - 8*a^6*c*f^2 - 120*a^2*b^4*c*f^2 + 120*a^4*b^2*c*f^2 - 160*a^3*b^3*d*f^2 + 48*a*b^5*d*f^2 + 48*a^5*b*d*f^2)^2/64 - f^4*(a^12*c^2 + a^12*d^2 + b^12*c^2 + b^12*d^2 + 6*a^2*b^10*c^2 + 15*a^4*b^8*c^2 + 20*a^6*b^6*c^2 + 15*a^8*b^4*c^2 + 6*a^10*b^2*c^2 + 6*a^2*b^10*d^2 + 15*a^4*b^8*d^2 + 20*a^6*b^6*d^2 + 15*a^8*b^4*d^2 + 6*a^10*b^2*d^2))^(1/2) + a^6*c*f^2 - b^6*c*f^2 + 15*a^2*b^4*c*f^2 - 15*a^4*b^2*c*f^2 + 20*a^3*b^3*d*f^2 - 6*a*b^5*d*f^2 - 6*a^5*b*d*f^2)/(4*f^4))^(1/2)*1i - (((8*(4*b^3*d^4*f^2 - 12*a^2*b*d^4*f^2 + 4*b^3*c^2*d^2*f^2 - 12*a^2*b*c^2*d^2*f^2))/f^3 + 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*(-(((8*b^6*c*f^2 - 8*a^6*c*f^2 - 120*a^2*b^4*c*f^2 + 120*a^4*b^2*c*f^2 - 160*a^3*b^3*d*f^2 + 48*a*b^5*d*f^2 + 48*a^5*b*d*f^2)^2/64 - f^4*(a^12*c^2 + a^12*d^2 + b^12*c^2 + b^12*d^2 + 6*a^2*b^10*c^2 + 15*a^4*b^8*c^2 + 20*a^6*b^6*c^2 + 15*a^8*b^4*c^2 + 6*a^10*b^2*c^2 + 6*a^2*b^10*d^2 + 15*a^4*b^8*d^2 + 20*a^6*b^6*d^2 + 15*a^8*b^4*d^2 + 6*a^10*b^2*d^2))^(1/2) + a^6*c*f^2 - b^6*c*f^2 + 15*a^2*b^4*c*f^2 - 15*a^4*b^2*c*f^2 + 20*a^3*b^3*d*f^2 - 6*a*b^5*d*f^2 - 6*a^5*b*d*f^2)/(4*f^4))^(1/2))*(-(((8*b^6*c*f^2 - 8*a^6*c*f^2 - 120*a^2*b^4*c*f^2 + 120*a^4*b^2*c*f^2 - 160*a^3*b^3*d*f^2 + 48*a*b^5*d*f^2 + 48*a^5*b*d*f^2)^2/64 - f^4*(a^12*c^2 + a^12*d^2 + b^12*c^2 + b^12*d^2 + 6*a^2*b^10*c^2 + 15*a^4*b^8*c^2 + 20*a^6*b^6*c^2 + 15*a^8*b^4*c^2 + 6*a^10*b^2*c^2 + 6*a^2*b^10*d^2 + 15*a^4*b^8*d^2 + 20*a^6*b^6*d^2 + 15*a^8*b^4*d^2 + 6*a^10*b^2*d^2))^(1/2) + a^6*c*f^2 - b^6*c*f^2 + 15*a^2*b^4*c*f^2 - 15*a^4*b^2*c*f^2 + 20*a^3*b^3*d*f^2 - 6*a*b^5*d*f^2 - 6*a^5*b*d*f^2)/(4*f^4))^(1/2) - (16*(c + d*tan(e + f*x))^(1/2)*(a^6*d^4 - b^6*d^4 + 15*a^2*b^4*d^4 - 15*a^4*b^2*d^4 - a^6*c^2*d^2 + b^6*c^2*d^2 - 40*a^3*b^3*c*d^3 - 15*a^2*b^4*c^2*d^2 + 15*a^4*b^2*c^2*d^2 + 12*a*b^5*c*d^3 + 12*a^5*b*c*d^3))/f^2)*(-(((8*b^6*c*f^2 - 8*a^6*c*f^2 - 120*a^2*b^4*c*f^2 + 120*a^4*b^2*c*f^2 - 160*a^3*b^3*d*f^2 + 48*a*b^5*d*f^2 + 48*a^5*b*d*f^2)^2/64 - f^4*(a^12*c^2 + a^12*d^2 + b^12*c^2 + b^12*d^2 + 6*a^2*b^10*c^2 + 15*a^4*b^8*c^2 + 20*a^6*b^6*c^2 + 15*a^8*b^4*c^2 + 6*a^10*b^2*c^2 + 6*a^2*b^10*d^2 + 15*a^4*b^8*d^2 + 20*a^6*b^6*d^2 + 15*a^8*b^4*d^2 + 6*a^10*b^2*d^2))^(1/2) + a^6*c*f^2 - b^6*c*f^2 + 15*a^2*b^4*c*f^2 - 15*a^4*b^2*c*f^2 + 20*a^3*b^3*d*f^2 - 6*a*b^5*d*f^2 - 6*a^5*b*d*f^2)/(4*f^4))^(1/2)*1i)/((((8*(4*b^3*d^4*f^2 - 12*a^2*b*d^4*f^2 + 4*b^3*c^2*d^2*f^2 - 12*a^2*b*c^2*d^2*f^2))/f^3 - 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*(-(((8*b^6*c*f^2 - 8*a^6*c*f^2 - 120*a^2*b^4*c*f^2 + 120*a^4*b^2*c*f^2 - 160*a^3*b^3*d*f^2 + 48*a*b^5*d*f^2 + 48*a^5*b*d*f^2)^2/64 - f^4*(a^12*c^2 + a^12*d^2 + b^12*c^2 + b^12*d^2 + 6*a^2*b^10*c^2 + 15*a^4*b^8*c^2 + 20*a^6*b^6*c^2 + 15*a^8*b^4*c^2 + 6*a^10*b^2*c^2 + 6*a^2*b^10*d^2 + 15*a^4*b^8*d^2 + 20*a^6*b^6*d^2 + 15*a^8*b^4*d^2 + 6*a^10*b^2*d^2))^(1/2) + a^6*c*f^2 - b^6*c*f^2 + 15*a^2*b^4*c*f^2 - 15*a^4*b^2*c*f^2 + 20*a^3*b^3*d*f^2 - 6*a*b^5*d*f^2 - 6*a^5*b*d*f^2)/(4*f^4))^(1/2))*(-(((8*b^6*c*f^2 - 8*a^6*c*f^2 - 120*a^2*b^4*c*f^2 + 120*a^4*b^2*c*f^2 - 160*a^3*b^3*d*f^2 + 48*a*b^5*d*f^2 + 48*a^5*b*d*f^2)^2/64 - f^4*(a^12*c^2 + a^12*d^2 + b^12*c^2 + b^12*d^2 + 6*a^2

```
*b^10*c^2 + 15*a^4*b^8*c^2 + 20*a^6*b^6*c^2 + 15*a^8*b^4*c^2 + 6*a^10*b^2*c
^2 + 6*a^2*b^10*d^2 + 15*a^4*b^8*d^2 + 20*a^6*b^6*d^2 + 15*a^8*b^4*d^2 + 6*
a^10*b^2*d^2))^(1/2) + a^6*c*f^2 - b^6*c*f^2 + 15*a^2*b^4*c*f^2 - 15*a^4*b^
2*c*f^2 + 20*a^3*b^3*d*f^2 - 6*a*b^5*d*f^2 - 6*a^5*b*d*f^2)/(4*f^4))^(1/2)
+ (16*(c + d*tan(e + f*x))^(1/2)*(a^6*d^4 - b^6*d^4 + 15*a^2*b^4*d^4 - 15*a
^4*b^2*d^4 - a^6*c^2*d^2 + b^6*c^2*d^2 - 40*a^3*b^3*c*d^3 - 15*a^2*b^4*c^2*
d^2 + 15*a^4*b^2*c^2*d^2 + 12*a*b^5*c*d^3 + 12*a^5*b*c*d^3))/f^2)*(-(((8*b^
6*c*f^2 - 8*a^6*c*f^2 - 120*a^2*b^4*c*f^2 + 120...
```

3.1230 $\int (a+b\tan(e+fx))^2 \sqrt{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=157

$$-\frac{i(a-ib)^2\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{i(a+ib)^2\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}+$$

```
[Out] -I*(a-I*b)^2*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))*(c-I*d)^(1/2)/f+
I*(a+I*b)^2*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))*(c+I*d)^(1/2)/f+4
*a*b*(c+d*tan(f*x+e))^(1/2)/f+2/3*b^2*(c+d*tan(f*x+e))^(3/2)/d/f
```

Rubi [A]
time = 0.24, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3624, 3609, 3620, 3618, 65, 214}

$$\frac{4ab\sqrt{c+d\tan(e+fx)}}{f}-\frac{i(a-ib)^2\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{i(a+ib)^2\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}+\frac{2b^2(c+d\tan(e+fx))^{3/2}}{3df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2*Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((-I)*(a - I*b)^2*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I
*d]])/f + (I*(a + I*b)^2*Sqrt[c + I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqr
t[c + I*d]])/f + (4*a*b*Sqrt[c + d*Tan[e + f*x]])/f + (2*b^2*(c + d*Tan[e +
 f*x])^(3/2))/(3*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
```

```
0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int (a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}\,dx = \frac{2b^2(c+d\tan(e+fx))^{3/2}}{3df}+\int \left(a^2-b^2+2ab\tan(e+fx\right.$$

$$= \frac{4ab\sqrt{c+d\tan(e+fx)}}{f}+\frac{2b^2(c+d\tan(e+fx))^{3/2}}{3df}+\int$$

$$= \frac{4ab\sqrt{c+d\tan(e+fx)}}{f}+\frac{2b^2(c+d\tan(e+fx))^{3/2}}{3df}+\frac{1}{2}$$

$$= \frac{4ab\sqrt{c+d\tan(e+fx)}}{f}+\frac{2b^2(c+d\tan(e+fx))^{3/2}}{3df}-\frac{(}{}$$

$$= \frac{4ab\sqrt{c+d\tan(e+fx)}}{f}+\frac{2b^2(c+d\tan(e+fx))^{3/2}}{3df}-\frac{(}{}$$

$$= -\frac{i(a-ib)^2\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{i}{}$$

Mathematica [A]
time = 0.59, size = 149, normalized size = 0.95

$$\frac{-3i(a-ib)^2\sqrt{c-id}\,d\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)+3i(a+ib)^2\sqrt{c+id}\,d\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)+2b\sqrt{c+d\tan(e+fx)}\,(bc+6ad+bd\tan(e+fx))}{3df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2*Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((-3*I)*(a - I*b)^2*Sqrt[c - I*d]*d*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c
 - I*d]] + (3*I)*(a + I*b)^2*Sqrt[c + I*d]*d*ArcTanh[Sqrt[c + d*Tan[e + f*x
]]/Sqrt[c + I*d]] + 2*b*Sqrt[c + d*Tan[e + f*x]]*(b*c + 6*a*d + b*d*Tan[e +
 f*x]))/(3*d*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 889 vs. 2(131) = 262.
time = 0.43, size = 890, normalized size = 5.67

method	result
derivativedivides	$\frac{\frac{2b^2(c+d\tan(fx+e))^{\frac{3}{2}}}{3}+4abd\sqrt{c+d\tan(fx+e)}+2d\left(\frac{\left(-\sqrt{c^2+d^2}\sqrt{2\sqrt{c^2+d^2}+2c}\,a^2+\sqrt{c^2+\right.}{}\right.}{}$
default	$\frac{\frac{2b^2(c+d\tan(fx+e))^{\frac{3}{2}}}{3}+4abd\sqrt{c+d\tan(fx+e)}+2d\left(\frac{\left(-\sqrt{c^2+d^2}\sqrt{2\sqrt{c^2+d^2}+2c}\,a^2+\sqrt{c^2+\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/d*(1/3*b^2*(c+d*tan(f*x+e))^(3/2)+2*a*b*d*(c+d*tan(f*x+e))^(1/2)+d*(1/4
/d*(1/2*(-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2+(c^2+d^2)^(1/2)
*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c-2*(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c)*ln(d
```

```
*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2
)^(1/2))+2*(-4*(c^2+d^2)^(1/2)*a*b*d-1/2*(-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*a^2+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2+(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*a^2*c-2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d-(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*b^2*c)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1
/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/
2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/4/d*(1/2*((c^2+d^2)^(1/2)*(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)*a^2-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2-(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c+2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d+(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(-4*(c^2+d^2)^(1/2)*a*b*d+
1/2*((c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2-(c^2+d^2)^(1/2)*(2*(
c^2+d^2)^(1/2)+2*c)^(1/2)*b^2-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c+2*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*a*b*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c)*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+
e))^(1/2)-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)^2*sqrt(d*tan(f*x + e) + c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 16199 vs. $2(127) = 254$.
time = 99.74, size = 16199, normalized size = 103.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
[Out] -1/12*(12*sqrt(2)*d*f^5*sqrt((((a^4 - 6*a^2*b^2 + b^4)*c - 4*(a^3*b - a*b^3
)*d)*f^2*sqrt(((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*c^2 + (a^8 +
 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^2)/f^4) + (a^8 + 4*a^6*b^2 + 6*
a^4*b^4 + 4*a^2*b^6 + b^8)*c^2 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 +
 b^8)*d^2)/(16*(a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*c^2 + 8*(a^7*b - 7*a^5*b^3 +
 7*a^3*b^5 - a*b^7)*c*d + (a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8
)*d^2))*(((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*c^2 + (a^8 + 4*a^
6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^2)/f^4)^(3/4)*sqrt((16*(a^6*b^2 - 2*
a^4*b^4 + a^2*b^6)*c^2 + 8*(a^7*b - 7*a^5*b^3 + 7*a^3*b^5 - a*b^7)*c*d + (a
^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*d^2)/f^4)*arctan(-((4*(a^1
```

5*b + 5*a^13*b^3 + 9*a^11*b^5 + 5*a^9*b^7 - 5*a^7*b^9 - 9*a^5*b^11 - 5*a^3*b^13 - a*b^15)*c^3 + (a^16 - 20*a^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 - 64*a^6*b^10 - 20*a^4*b^12 + b^16)*c^2*d + 4*(a^15*b + 5*a^13*b^3 + 9*a^11*b^5 + 5*a^9*b^7 - 5*a^7*b^9 - 9*a^5*b^11 - 5*a^3*b^13 - a*b^15)*c*d^2 + (a^16 - 20*a^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 - 64*a^6*b^10 - 20*a^4*b^12 + b^16)*d^3)*f^4*sqrt(((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*c^2 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^2)/f^4)*sqrt((16*(a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*c^2 + 8*(a^7*b - 7*a^5*b^3 + 7*a^3*b^5 - a*b^7)*c*d + (a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*d^2)/f^4) + (4*(a^19*b + 7*a^17*b^3 + 20*a^15*b^5 + 28*a^13*b^7 + 14*a^11*b^9 - 14*a^9*b^11 - 28*a^7*b^13 - 20*a^5*b^15 - 7*a^3*b^17 - a*b^19)*c^4 + (a^20 + 2*a^18*b^2 - 19*a^16*b^4 - 104*a^14*b^6 - 238*a^12*b^8 - 308*a^10*b^10 - 238*a^8*b^12 - 104*a^6*b^14 - 19*a^4*b^16 + 2*a^2*b^18 + b^20)*c^3*d + 4*(a^19*b + 7*a^17*b^3 + 20*a^15*b^5 + 28*a^13*b^7 + 14*a^11*b^9 - 14*a^9*b^11 - 28*a^7*b^13 - 20*a^5*b^15 - 7*a^3*b^17 - a*b^19)*c^2*d^2 + (a^20 + 2*a^18*b^2 - 19*a^16*b^4 - 104*a^14*b^6 - 238*a^12*b^8 - 308*a^10*b^10 - 238*a^8*b^12 - 104*a^6*b^14 - 19*a^4*b^16 + 2*a^2*b^18 + b^20)*c*d^3)*f^2*sqrt((16*(a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*c^2 + 8*(a^7*b - 7*a^5*b^3 + 7*a^3*b^5 - a*b^7)*c*d + (a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*d^2)/f^4) + sqrt(2)*(2*(4*(a^8*b^2 + a^6*b^4 - a^4*b^6 - a^2*b^8)*c + (a^9*b - 4*a^7*b^3 - 10*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d)*f^7*sqrt(((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*c^2 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^2)/f^4)*sqrt((16*(a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*c^2 + 8*(a^7*b - 7*a^5*b^3 + 7*a^3*b^5 - a*b^7)*c*d + (a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*d^2)/f^4) + (8*(a^12*b^2 + 3*a^10*b^4 + 2*a^8*b^6 - 2*a^6*b^8 - 3*a^4*b^10 - a^2*b^12)*c^2 + 2*(3*a^13*b + 2*a^11*b^3 - 19*a^9*b^5 - 36*a^7*b^7 - 19*a^5*b^9 + 2*a^3*b^11 + 3*a*b^13)*c*d + (a^14 - 3*a^12*b^2 - 15*a^10*b^4 - 11*a^8*b^6 + 11*a^6*b^8 + 15*a^4*b^10 + 3*a^2*b^12 - b^14)*d^2)*f^5*sqrt((16*(a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*c^2 + 8*(a^7*b - 7*a^5*b^3 + 7*a^3*b^5 - a*b^7)*c*d + (a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*d^2)/f^4))*sqrt((((a^4 - 6*a^2*b^2 + b^4)*c - 4*(a^3*b - a*b^3)*d)*f^2*sqrt(((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*c^2 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^2)/f^4) + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*c^2 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^2)/(16*(a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*c^2 + 8*(a^7*b - 7*a^5*b^3 + 7*a^3*b^5 - a*b^7)*c*d + (a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*d^2))*sqrt((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))*(((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*c^2 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^2)/f^4)^(3/4) - sqrt(2)*(2*a*b*f^7*sqrt(((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*c^2 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^2)/f^4)*sqrt((16*(a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*c^2 + 8*(a^7*b - 7*a^5*b^3 + 7*a^3*b^5 - a*b^7)*c*d + (a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*d^2)/f^4) + (2*(a^5*b + 2*a^3*b^3 + a*b^5)*c + (a^6 + a^4*b^2 - a^2*b^4 - b^6)*d)*f^5*sqrt((16*(a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*c^2 + 8*(a^7*b - 7*a^5*b^3 + 7*a^3*b^5 - a*b^7)*c*d + (a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*d^2)/f^4))*sqrt((((a

```
^4 - 6*a^2*b^2 + b^4)*c - 4*(a^3*b - a*b^3)*d)*f^2*sqrt(((a^8 + 4*a^6*b^2 +
 6*a^4*b^4 + 4*a^2*b^6 + b^8)*c^2 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^
6 + b^8)*d^2)/f^4) + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*c^2 +
(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^2)/(16*(a^6*b^2 - 2*a^4*b
^4 + a^2*b^6)*c^2 + 8*(a^7*b - 7*a^5*b^3 + 7*a^3*b^5 - a*b^7)*c*d + (a^8 -
12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*d^2))*sqrt(((16*(a^10*b^2 - 2*a
^6*b^6 + a^2*b^10)*c^4 + 8*(a^11*b - 5*a^9*b^3 - 6*a^7*b^5 + 6*a^5*b^7 + 5*
a^3*b^9 - a*b^11)*c^3*d + (a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15
*a^4*b^8 + 6*a^2*b^10 + b^12)*c^2*d^2 + 8*(a^11*b - 5*a^9*b^3 - 6*a^7*b^5 +
 6*a^5*b^7 + 5*a^3*b^9 - a*b^11)*c*d^3 + (a^12 - 10*a^10*b^2 + 15*a^8*b^4 +
 52*a^6*b^6 + 15*a^4*b^8 - 10*a^2*b^10 + b^12)*d^4)*f^2*sqrt(((a^8 + 4*a^6*
b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*c^2 + (a^8 +...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral((a + b*tan(e + f*x))**2*sqrt(c + d*tan(e + f*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 9.66, size = 2500, normalized size = 15.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^2*(c + d*tan(e + f*x))^(1/2),x)
```

```
[Out] (2*b^2*(c + d*tan(e + f*x))^(3/2))/(3*d*f) - atan(((((8*(8*a*b*d^4*f^2 + 8*
a*b*c^2*d^2*f^2))/f^3 - 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*(-(a^4*c - a^4*
```

d*1i + b^4*c - b^4*d*1i - 6*a^2*b^2*c + a^2*b^2*d*6i + a*b^3*c*4i - a^3*b*c
*4i + 4*a*b^3*d - 4*a^3*b*d)/(4*f^2))^(1/2))*(-(a^4*c - a^4*d*1i + b^4*c -
b^4*d*1i - 6*a^2*b^2*c + a^2*b^2*d*6i + a*b^3*c*4i - a^3*b*c*4i + 4*a*b^3*d
 - 4*a^3*b*d)/(4*f^2))^(1/2) + (16*(c + d*tan(e + f*x))^(1/2)*(a^4*d^4 + b^
4*d^4 - 6*a^2*b^2*d^4 - a^4*c^2*d^2 - b^4*c^2*d^2 + 6*a^2*b^2*c^2*d^2 - 8*a
*b^3*c*d^3 + 8*a^3*b*c*d^3))/f^2)*(-(a^4*c - a^4*d*1i + b^4*c - b^4*d*1i -
6*a^2*b^2*c + a^2*b^2*d*6i + a*b^3*c*4i - a^3*b*c*4i + 4*a*b^3*d - 4*a^3*b*
d)/(4*f^2))^(1/2)*1i - (((8*(8*a*b*d^4*f^2 + 8*a*b*c^2*d^2*f^2))/f^3 + 64*c
d^2(c + d*tan(e + f*x))^(1/2)*(-(a^4*c - a^4*d*1i + b^4*c - b^4*d*1i - 6*
a^2*b^2*c + a^2*b^2*d*6i + a*b^3*c*4i - a^3*b*c*4i + 4*a*b^3*d - 4*a^3*b*d)
/(4*f^2))^(1/2))*(-(a^4*c - a^4*d*1i + b^4*c - b^4*d*1i - 6*a^2*b^2*c + a^2
*b^2*d*6i + a*b^3*c*4i - a^3*b*c*4i + 4*a*b^3*d - 4*a^3*b*d)/(4*f^2))^(1/2)
 - (16*(c + d*tan(e + f*x))^(1/2)*(a^4*d^4 + b^4*d^4 - 6*a^2*b^2*d^4 - a^4*
c^2*d^2 - b^4*c^2*d^2 + 6*a^2*b^2*c^2*d^2 - 8*a*b^3*c*d^3 + 8*a^3*b*c*d^3))
/f^2)*(-(a^4*c - a^4*d*1i + b^4*c - b^4*d*1i - 6*a^2*b^2*c + a^2*b^2*d*6i +
 a*b^3*c*4i - a^3*b*c*4i + 4*a*b^3*d - 4*a^3*b*d)/(4*f^2))^(1/2)*1i)/((((8*
(8*a*b*d^4*f^2 + 8*a*b*c^2*d^2*f^2))/f^3 - 64*c*d^2*(c + d*tan(e + f*x))^(1
/2)*(-(a^4*c - a^4*d*1i + b^4*c - b^4*d*1i - 6*a^2*b^2*c + a^2*b^2*d*6i + a
*b^3*c*4i - a^3*b*c*4i + 4*a*b^3*d - 4*a^3*b*d)/(4*f^2))^(1/2))*(-(a^4*c -
a^4*d*1i + b^4*c - b^4*d*1i - 6*a^2*b^2*c + a^2*b^2*d*6i + a*b^3*c*4i - a^3
*b*c*4i + 4*a*b^3*d - 4*a^3*b*d)/(4*f^2))^(1/2) + (16*(c + d*tan(e + f*x))^
(1/2)*(a^4*d^4 + b^4*d^4 - 6*a^2*b^2*d^4 - a^4*c^2*d^2 - b^4*c^2*d^2 + 6*a^
2*b^2*c^2*d^2 - 8*a*b^3*c*d^3 + 8*a^3*b*c*d^3))/f^2)*(-(a^4*c - a^4*d*1i +
b^4*c - b^4*d*1i - 6*a^2*b^2*c + a^2*b^2*d*6i + a*b^3*c*4i - a^3*b*c*4i + 4
*a*b^3*d - 4*a^3*b*d)/(4*f^2))^(1/2) - (16*(a^6*d^5 - b^6*d^5 - a^2*b^4*d^5
 + a^4*b^2*d^5 + a^6*c^2*d^3 - b^6*c^2*d^3 + 2*a*b^5*c^3*d^2 + 4*a^3*b^3*c*
d^4 + 2*a^5*b*c^3*d^2 - a^2*b^4*c^2*d^3 + 4*a^3*b^3*c^3*d^2 + a^4*b^2*c^2*d
^3 + 2*a*b^5*c*d^4 + 2*a^5*b*c*d^4))/f^3 + (((8*(8*a*b*d^4*f^2 + 8*a*b*c^2*
d^2*f^2))/f^3 + 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*(-(a^4*c - a^4*d*1i + b
^4*c - b^4*d*1i - 6*a^2*b^2*c + a^2*b^2*d*6i + a*b^3*c*4i - a^3*b*c*4i + 4*
a*b^3*d - 4*a^3*b*d)/(4*f^2))^(1/2))*(-(a^4*c - a^4*d*1i + b^4*c - b^4*d*1i
 - 6*a^2*b^2*c + a^2*b^2*d*6i + a*b^3*c*4i - a^3*b*c*4i + 4*a*b^3*d - 4*a^3
*b*d)/(4*f^2))^(1/2) - (16*(c + d*tan(e + f*x))^(1/2)*(a^4*d^4 + b^4*d^4 -
6*a^2*b^2*d^4 - a^4*c^2*d^2 - b^4*c^2*d^2 + 6*a^2*b^2*c^2*d^2 - 8*a*b^3*c*d
^3 + 8*a^3*b*c*d^3))/f^2)*(-(a^4*c - a^4*d*1i + b^4*c - b^4*d*1i - 6*a^2*b^
2*c + a^2*b^2*d*6i + a*b^3*c*4i - a^3*b*c*4i + 4*a*b^3*d - 4*a^3*b*d)/(4*f^
2))^(1/2)))*(-(a^4*c - a^4*d*1i + b^4*c - b^4*d*1i - 6*a^2*b^2*c + a^2*b^2*
d*6i + a*b^3*c*4i - a^3*b*c*4i + 4*a*b^3*d - 4*a^3*b*d)/(4*f^2))^(1/2)*2i -
 atan(((((8*(8*a*b*d^4*f^2 + 8*a*b*c^2*d^2*f^2))/f^3 - 64*c*d^2*(c + d*tan(
e + f*x))^(1/2)*(-(a^4*c + a^4*d*1i + b^4*c + b^4*d*1i - 6*a^2*b^2*c - a^2*
b^2*d*6i - a*b^3*c*4i + a^3*b*c*4i + 4*a*b^3*d - 4*a^3*b*d)/(4*f^2))^(1/2))
*(-(a^4*c + a^4*d*1i + b^4*c + b^4*d*1i - 6*a^2*b^2*c - a^2*b^2*d*6i - a*b^
3*c*4i + a^3*b*c*4i + 4*a*b^3*d - 4*a^3*b*d)/(4*f^2))^(1/2) + (16*(c + d*ta
n(e + f*x))^(1/2)*(a^4*d^4 + b^4*d^4 - 6*a^2*b^2*d^4 - a^4*c^2*d^2 - b^4*c^
2*d^2 + 6*a^2*b^2*c^2*d^2 - 8*a*b^3*c*d^3 + 8*a^3*b*c*d^3))/f^2)*(-(a^4*c +

a^4*d*1i + b^4*c + b^4*d*1i - 6*a^2*b^2*c - a^2*b^2*d*6i - a*b^3*c*4i + a^3*b*c*4i + 4*a*b^3*d - 4*a^3*b*d)/(4*f^2))^(1/2)*1i - (((8*(8*a*b*d^4*f^2 + 8*a*b*c^2*d^2*f^2))/f^3 + 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*(-(a^4*c + a^4*d*1i + b^4*c + b^4*d*1i - 6*a^2*b^2*c - a^2*b^2*d*6i - a*b^3*c*4i + a^3*b*c*4i + 4*a*b^3*d - 4*a^3*b*d)/(4*f^2))^(1/2))*(-(a^4*c + a^4*d*1i + b^4*c + b^4*d*1i - 6*a^2*b^2*c - a^2*b^2*d*6i - a*b^3*c*4i + a^3*b*c*4i + 4*a*b^3*d - 4*a^3*b*d)/(4*f^2))^(1/2) - (16*(c + d*tan(e + f*x))^(1/2)*(a^4*d^4 + b^4*d^4 - 6*a^2*b^2*d^4 - a^4*c^2*d^2 - b^4*c^2*d^2 + 6*a^2*b^2*c^2*d^2 - 8*a*b^3*c*d^3 + 8*a^3*b*c*d^3))/f^2)*(-(a^4*c + a^4*d*1i + b^4*c + b^4*d*1i - 6*a^2*b^2*c - a^2*b^2*d*6i - a*b^3*c*4i + a^3*b*c*4i + 4*a*b^3*d - 4*a^3*b*d)/(4*f^2))^(1/2)*1i)/((((8*(8*a*b*d^4*f^2 + 8*a*b*c^2*d^2*f^2))/f^3 - 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*(-(a^4*c + a^4*d*1i + b^4*c + b^4*d*1i - 6*a^2*b^2*c - a^2*b^2*d*6i - a*b^3*c*4i + a^3*b*c*4i + 4*a*b^3*d - 4*a^3*b*d)/(4*f^2))^(1/2))*(-(a^4*c + a^4*d*1i + b^4*c + b^4*d*1i - 6*a^2*b^2*c - a^2*b^2*d*6i - a*b^3*c*4i + a^3*b*c*4i + 4*a*b^3*d - 4*a^3*b*d)/(4*f^2))^(1/2) + (16*(c + d*tan(e + f*x))^(1/2)*(a^4*d^4 + b^4*d^4 - 6*a^2*b^2*d^4 - a^4*c^2*d^2 - b^4*c^2*d^2 + 6*a^2*b^2*c^2*d^2 - 8*a*b^3*c*d^3 + 8*a^3*b*c*d^3))/f^2)*(-(a^4*c + a^4*d*1i + b^4*c + b^4*d*1i - 6*a^2*b^2*c - a^2*b^2*d*6i - a*b^3*c*4i + a^3*b*c*4i + 4*a*b^3*d - 4*a^3*b*d)/(4*f^2))^(1/2) - (16*(a^6*d^5 - b^6*d^5 - a^2*b^4*d^5 + a^4*b^2*d^5 +...

3.1231 $\int (a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=122

$$-\frac{(ia+b)\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{(ia-b)\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}+\frac{2b\sqrt{}}{}$$

```
[Out] -(I*a+b)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))*(c-I*d)^(1/2)/f+(I*a
-b)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))*(c+I*d)^(1/2)/f+2*b*(c+d*
tan(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.15, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3609, 3620, 3618, 65, 214}

$$-\frac{(b+ia)\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{(-b+ia)\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}+\frac{2b\sqrt{c+d\tan(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])*Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] -(((I*a + b)*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])
/f) + ((I*a - b)*Sqrt[c + I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*
d]])/f + (2*b*Sqrt[c + d*Tan[e + f*x]])/f
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
```

```
0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}\,dx = \frac{2b\sqrt{c+d\tan(e+fx)}}{f} + \int \frac{ac-bd+(bc+ad)\tan(e+ \cdots}{\sqrt{c+d\tan(e+fx)}}$$

$$= \frac{2b\sqrt{c+d\tan(e+fx)}}{f} + \frac{1}{2}((a-ib)(c-id)) \int \frac{1+i\tan \cdots}{\sqrt{c+d\tan \cdots}}$$

$$= \frac{2b\sqrt{c+d\tan(e+fx)}}{f} + \frac{(i(a-ib)(c-id))\text{Subst}\left(\int \frac{\cdots}{(-1+\cdots}\right.}{\cdots}$$

$$= \frac{2b\sqrt{c+d\tan(e+fx)}}{f} - \frac{((a-ib)(c-id))\text{Subst}\left(\int \frac{\cdots}{-1-\frac{i}{c}\cdots}\right.}{\cdots}$$

$$= -\frac{(ia+b)\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f} + \frac{(ia \cdots}{\cdots}$$

Mathematica [A]
time = 0.15, size = 120, normalized size = 0.98

$$\frac{-i(a-ib)\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)+i(a+ib)\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)+2b\sqrt{c+d\tan(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])*Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((-I)*(a - I*b)*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d
]] + I*(a + I*b)*Sqrt[c + I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*
d]] + 2*b*Sqrt[c + d*Tan[e + f*x]])/f
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(102) = 204$.
time = 0.45, size = 632, normalized size = 5.18

method	result
derivativedivides	$\frac{2b\sqrt{c+d\tan(fx+e)}+\frac{\left(-\sqrt{2\sqrt{c^2+d^2}+2c}\,\sqrt{c^2+d^2}\,a+\sqrt{2\sqrt{c^2+d^2}+2c}\,ac-\sqrt{2\sqrt{c}}\right.}{}}{}$
default	$\frac{2b\sqrt{c+d\tan(fx+e)}+\frac{\left(-\sqrt{2\sqrt{c^2+d^2}+2c}\,\sqrt{c^2+d^2}\,a+\sqrt{2\sqrt{c^2+d^2}+2c}\,ac-\sqrt{2\sqrt{c}}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*b*(c+d*tan(f*x+e))^(1/2)+1/2/d*(1/2*(-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*
(c^2+d^2)^(1/2)*a+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c-(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)*b*d)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)+(c^2+d^2)^(1/2))+2*(-2*(c^2+d^2)^(1/2)*b*d-1/2*(-(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*(c^2+d^2)^(1/2)*a+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c-(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)*b*d)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2
*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(
2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/2/d*(-1/2*(-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*
(c^2+d^2)^(1/2)*a+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c-(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)*b*d)*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f
*x+e)-c-(c^2+d^2)^(1/2))+2*(2*(c^2+d^2)^(1/2)*b*d+1/2*(-(2*(c^2+d^2)^(1/2)+
2*c)^(1/2)*(c^2+d^2)^(1/2)*a+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c-(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*b*d)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*
c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2
*(c^2+d^2)^(1/2)-2*c)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-c>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 8653 vs. $2(100) = 200$.
time = 11.89, size = 8653, normalized size = 70.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*f^5*sqrt(-((2*a*b*d - (a^2 - b^2)*c)*f^2*sqrt(((a^4 + 2*a^2*
b^2 + b^4)*c^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2)/f^4) - (a^4 + 2*a^2*b^2 + b^4
)*c^2 - (a^4 + 2*a^2*b^2 + b^4)*d^2)/(4*a^2*b^2*c^2 + 4*(a^3*b - a*b^3)*c*d
 + (a^4 - 2*a^2*b^2 + b^4)*d^2))*sqrt((4*a^2*b^2*c^2 + 4*(a^3*b - a*b^3)*c*
d + (a^4 - 2*a^2*b^2 + b^4)*d^2)/f^4)*(((a^4 + 2*a^2*b^2 + b^4)*c^2 + (a^4
+ 2*a^2*b^2 + b^4)*d^2)/f^4)^(3/4)*arctan(((2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^
5 + a*b^7)*c^3 + (a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*c^2*d + 2*(a^7*b + 3*a
^5*b^3 + 3*a^3*b^5 + a*b^7)*c*d^2 + (a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^3
)*f^4*sqrt((4*a^2*b^2*c^2 + 4*(a^3*b - a*b^3)*c*d + (a^4 - 2*a^2*b^2 + b^4)
*d^2)/f^4)*sqrt(((a^4 + 2*a^2*b^2 + b^4)*c^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2)
/f^4) + (2*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*c^4 + (a^10
+ 3*a^8*b^2 + 2*a^6*b^4 - 2*a^4*b^6 - 3*a^2*b^8 - b^10)*c^3*d + 2*(a^9*b +
4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*c^2*d^2 + (a^10 + 3*a^8*b^2 + 2*
a^6*b^4 - 2*a^4*b^6 - 3*a^2*b^8 - b^10)*c*d^3)*f^2*sqrt((4*a^2*b^2*c^2 + 4*
(a^3*b - a*b^3)*c*d + (a^4 - 2*a^2*b^2 + b^4)*d^2)/f^4) + sqrt(2)*((2*(a^3*
b^2 + a*b^4)*c + (a^4*b - b^5)*d)*f^7*sqrt((4*a^2*b^2*c^2 + 4*(a^3*b - a*b^
3)*c*d + (a^4 - 2*a^2*b^2 + b^4)*d^2)/f^4)*sqrt(((a^4 + 2*a^2*b^2 + b^4)*c^
2 + (a^4 + 2*a^2*b^2 + b^4)*d^2)/f^4) + (2*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*c^
2 + (3*a^6*b + 5*a^4*b^3 + a^2*b^5 - b^7)*c*d + (a^7 + a^5*b^2 - a^3*b^4 -
a*b^6)*d^2)*f^5*sqrt((4*a^2*b^2*c^2 + 4*(a^3*b - a*b^3)*c*d + (a^4 - 2*a^2*
b^2 + b^4)*d^2)/f^4))*sqrt(-((2*a*b*d - (a^2 - b^2)*c)*f^2*sqrt(((a^4 + 2*a
^2*b^2 + b^4)*c^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2)/f^4) - (a^4 + 2*a^2*b^2 +
b^4)*c^2 - (a^4 + 2*a^2*b^2 + b^4)*d^2)/(4*a^2*b^2*c^2 + 4*(a^3*b - a*b^3)*
c*d + (a^4 - 2*a^2*b^2 + b^4)*d^2))*sqrt((c*cos(f*x + e) + d*sin(f*x + e))/
cos(f*x + e))*(((a^4 + 2*a^2*b^2 + b^4)*c^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2)/
f^4)^(3/4) + sqrt(2)*(b*f^7*sqrt((4*a^2*b^2*c^2 + 4*(a^3*b - a*b^3)*c*d + (
a^4 - 2*a^2*b^2 + b^4)*d^2)/f^4)*sqrt(((a^4 + 2*a^2*b^2 + b^4)*c^2 + (a^4 +
 2*a^2*b^2 + b^4)*d^2)/f^4) + ((a^2*b + b^3)*c + (a^3 + a*b^2)*d)*f^5*sqrt(
```

```
(4*a^2*b^2*c^2 + 4*(a^3*b - a*b^3)*c*d + (a^4 - 2*a^2*b^2 + b^4)*d^2)/f^4))
*sqrt(-((2*a*b*d - (a^2 - b^2)*c)*f^2*sqrt(((a^4 + 2*a^2*b^2 + b^4)*c^2 + (
a^4 + 2*a^2*b^2 + b^4)*d^2)/f^4) - (a^4 + 2*a^2*b^2 + b^4)*c^2 - (a^4 + 2*a
^2*b^2 + b^4)*d^2)/(4*a^2*b^2*c^2 + 4*(a^3*b - a*b^3)*c*d + (a^4 - 2*a^2*b^
2 + b^4)*d^2))*sqrt(((4*(a^4*b^2 + a^2*b^4)*c^4 + 4*(a^5*b - a*b^5)*c^3*d +
 (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c^2*d^2 + 4*(a^5*b - a*b^5)*c*d^3 + (a
^6 - a^4*b^2 - a^2*b^4 + b^6)*d^4)*f^2*sqrt(((a^4 + 2*a^2*b^2 + b^4)*c^2 +
(a^4 + 2*a^2*b^2 + b^4)*d^2)/f^4)*cos(f*x + e) + sqrt(2)*((4*a^2*b^3*c^3 +
4*(2*a^3*b^2 - a*b^4)*c^2*d + (5*a^4*b - 6*a^2*b^3 + b^5)*c*d^2 + (a^5 - 2*
a^3*b^2 + a*b^4)*d^3)*f^3*sqrt(((a^4 + 2*a^2*b^2 + b^4)*c^2 + (a^4 + 2*a^2*
b^2 + b^4)*d^2)/f^4)*cos(f*x + e) + (4*(a^4*b^3 + a^2*b^5)*c^4 + 4*(a^5*b^2
 - a*b^6)*c^3*d + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*c^2*d^2 + 4*(a^5*b^
2 - a*b^6)*c*d^3 + (a^6*b - a^4*b^3 - a^2*b^5 + b^7)*d^4)*f*cos(f*x + e))*s
qrt(-((2*a*b*d - (a^2 - b^2)*c)*f^2*sqrt(((a^4 + 2*a^2*b^2 + b^4)*c^2 + (a^
4 + 2*a^2*b^2 + b^4)*d^2)/f^4) - (a^4 + 2*a^2*b^2 + b^4)*c^2 - (a^4 + 2*a^2
*b^2 + b^4)*d^2)/(4*a^2*b^2*c^2 + 4*(a^3*b - a*b^3)*c*d + (a^4 - 2*a^2*b^2
+ b^4)*d^2))*sqrt((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))*(((a^4 +
2*a^2*b^2 + b^4)*c^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2)/f^4)^(1/4) + (4*(a^6*b^
2 + 2*a^4*b^4 + a^2*b^6)*c^5 + 4*(a^7*b + a^5*b^3 - a^3*b^5 - a*b^7)*c^4*d
+ (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*c^3*d^2 + 4*(a^7*b + a^5*
b^3 - a^3*b^5 - a*b^7)*c^2*d^3 + (a^8 - 2*a^4*b^4 + b^8)*c*d^4)*cos(f*x + e
) + (4*(a^6*b^2 + 2*a^4*b^4 + a^2*b^6)*c^4*d + 4*(a^7*b + a^5*b^3 - a^3*b^5
 - a*b^7)*c^3*d^2 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*c^2*d^3
 + 4*(a^7*b + a^5*b^3 - a^3*b^5 - a*b^7)*c*d^4 + (a^8 - 2*a^4*b^4 + b^8)*d^
5)*sin(f*x + e))/((c^2 + d^2)*cos(f*x + e)))*(((a^4 + 2*a^2*b^2 + b^4)*c^2
+ (a^4 + 2*a^2*b^2 + b^4)*d^2)/f^4)^(3/4))/(4*(a^10*b^2 + 4*a^8*b^4 + 6*a^6
*b^6 + 4*a^4*b^8 + a^2*b^10)*c^4*d + 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 - 2*
a^5*b^7 - 3*a^3*b^9 - a*b^11)*c^3*d^2 + (a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 2
0*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*c^2*d^3 + 4*(a^11*b + 3*a^9*b^3
 + 2*a^7*b^5 - 2*a^5*b^7 - 3*a^3*b^9 - a*b^11)*c*d^4 + (a^12 + 2*a^10*b^2 -
 a^8*b^4 - 4*a^6*b^6 - a^4*b^8 + 2*a^2*b^10 + b^12)*d^5)) + 4*sqrt(2)*f^5*s
qrt(-((2*a*b*d - (a^2 - b^2)*c)*f^2*sqrt(((a^4 + 2*a^2*b^2 + b^4)*c^2 + (a^
4 + 2*a^2*b^2 + b^4)*d^2)/f^4) - (a^4 + 2*a^2*b^2 + b^4)*c^2 - (a^4 + 2*a^2
*b^2 + b^4)*d^2)/(4*a^2*b^2*c^2 + 4*(a^3*b - a*b^3)*c*d + (a^4 - 2*a^2*b^2
+ b^4)*d^2))*sqrt((4*a^2*b^2*c^2 + 4*(a^3*b - a*b^3)*c*d + (a^4 - 2*a^2*b^2
 + b^4)*d^2)/f^4)*(((a^4 + 2*a^2*b^2 + b^4)*c^2 + (a^4 + 2*a^2*b^2 + b^4)*d
^2)/f^4)^(3/4)*arctan(-((2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*c^3 + (a
^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*c^2*d + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5
+ a*b^7)*c*d^2 + (a^8 + 2*a^6*b^2 - 2*a^2*b^6 -...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(a+b*tan(f*x+e)),x)
```

```
[Out] Integral((a + b*tan(e + f*x))*sqrt(c + d*tan(e + f*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 7.65, size = 845, normalized size = 6.93

$$2\,\mathrm{atanh}\left(\frac{32\,b^2\,d^4\,\sqrt{\frac{b^2\,c}{4\,f^2}-\frac{\sqrt{-b^4\,d^2\,f^4}}{4\,f^4}}\,\sqrt{c+d\,\tan\left(e+f\,x\right)}}{\frac{16\,b\,d^4\,\sqrt{-b^4\,d^2\,f^4}}{f^3}+\frac{16\,b\,c^2\,d^2\,\sqrt{-b^4\,d^2\,f^4}}{f^3}}-\frac{32\,c\,d^2\,\sqrt{\frac{b^2\,c}{4\,f^2}-\frac{\sqrt{-b^4\,d^2\,f^4}}{4\,f^4}}\,\sqrt{c+d\,\tan\left(e+f\,x\right)}\,\sqrt{-b^4\,d^2\,f^4}}{\frac{16\,b\,d^4\,\sqrt{-b^4\,d^2\,f^4}}{f}+\frac{16\,b\,c^2\,d^2\,\sqrt{-b^4\,d^2\,f^4}}{f}}\right)\,\sqrt{-\frac{\sqrt{-b^4\,d^2\,f^4}-b^2\,c\,f^2}{4\,f^4}}-2\,\mathrm{atanh}\left(\frac{32\,b^2\,d^4\,\sqrt{\frac{\sqrt{-b^4\,d^2\,f^4}}{4\,f^4}+\frac{b^2\,c}{4\,f^2}}\,\sqrt{c+d\,\tan\left(e+f\,x\right)}}{\frac{16\,b\,d^4\,\sqrt{-b^4\,d^2\,f^4}}{f^3}+\frac{16\,b\,c^2\,d^2\,\sqrt{-b^4\,d^2\,f^4}}{f^3}}+\frac{32\,c\,d^2\,\sqrt{\frac{\sqrt{-b^4\,d^2\,f^4}}{4\,f^4}+\frac{b^2\,c}{4\,f^2}}\,\sqrt{c+d\,\tan\left(e+f\,x\right)}\,\sqrt{-b^4\,d^2\,f^4}}{\frac{16\,b\,d^4\,\sqrt{-b^4\,d^2\,f^4}}{f}+\frac{16\,b\,c^2\,d^2\,\sqrt{-b^4\,d^2\,f^4}}{f}}\right)\,\sqrt{\frac{\sqrt{-b^4\,d^2\,f^4}+b^2\,c\,f^2}{4\,f^4}}-\mathrm{atanh}\left(\frac{f^3\,\left(\frac{16\,\left(a^2\,d^4-a^2\,c^2\,d^2\right)\,\sqrt{c+d\,\tan\left(e+f\,x\right)}}{f^2}+\frac{16\,c\,d^2\,\left(\sqrt{-a^4\,d^2\,f^4}+a^2\,c\,f^2\right)\,\sqrt{c+d\,\tan\left(e+f\,x\right)}}{f^4}\right)\,\sqrt{-\frac{\sqrt{-a^4\,d^2\,f^4}+a^2\,c\,f^2}{f^4}}}{16\,\left(a^3\,c^2\,d^3+a^3\,d^5\right)}\right)\,\sqrt{-\frac{\sqrt{-a^4\,d^2\,f^4}+a^2\,c\,f^2}{f^4}}-\mathrm{atanh}\left(\frac{f^3\,\left(\frac{16\,\left(a^2\,d^4-a^2\,c^2\,d^2\right)\,\sqrt{c+d\,\tan\left(e+f\,x\right)}}{f^2}-\frac{16\,c\,d^2\,\left(\sqrt{-a^4\,d^2\,f^4}-a^2\,c\,f^2\right)\,\sqrt{c+d\,\tan\left(e+f\,x\right)}}{f^4}\right)\,\sqrt{\frac{\sqrt{-a^4\,d^2\,f^4}-a^2\,c\,f^2}{f^4}}}{16\,\left(a^3\,c^2\,d^3+a^3\,d^5\right)}\right)\,\sqrt{\frac{\sqrt{-a^4\,d^2\,f^4}-a^2\,c\,f^2}{f^4}}+\frac{2\,b\,\sqrt{c+d\,\tan\left(e+f\,x\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))*(c + d*tan(e + f*x))^(1/2),x)
```

```
[Out] 2*atanh((32*b^2*d^4*((b^2*c)/(4*f^2) - (-b^4*d^2*f^4)^(1/2)/(4*f^4))^(1/2)*
(c + d*tan(e + f*x))^(1/2))/((16*b*d^4*(-b^4*d^2*f^4)^(1/2))/f^3 + (16*b*c^
2*d^2*(-b^4*d^2*f^4)^(1/2))/f^3) - (32*c*d^2*((b^2*c)/(4*f^2) - (-b^4*d^2*f
^4)^(1/2)/(4*f^4))^(1/2)*(c + d*tan(e + f*x))^(1/2)*(-b^4*d^2*f^4)^(1/2))/(
(16*b*d^4*(-b^4*d^2*f^4)^(1/2))/f + (16*b*c^2*d^2*(-b^4*d^2*f^4)^(1/2))/f))
*(-((-b^4*d^2*f^4)^(1/2) - b^2*c*f^2)/(4*f^4))^(1/2) - 2*atanh((32*b^2*d^4*
((-b^4*d^2*f^4)^(1/2)/(4*f^4) + (b^2*c)/(4*f^2))^(1/2)*(c + d*tan(e + f*x))
^(1/2))/((16*b*d^4*(-b^4*d^2*f^4)^(1/2))/f^3 + (16*b*c^2*d^2*(-b^4*d^2*f^4)
^(1/2))/f^3) + (32*c*d^2*((-b^4*d^2*f^4)^(1/2)/(4*f^4) + (b^2*c)/(4*f^2))^(
1/2)*(c + d*tan(e + f*x))^(1/2)*(-b^4*d^2*f^4)^(1/2))/((16*b*d^4*(-b^4*d^2*
f^4)^(1/2))/f + (16*b*c^2*d^2*(-b^4*d^2*f^4)^(1/2))/f))*(((-b^4*d^2*f^4)^(1
/2) + b^2*c*f^2)/(4*f^4))^(1/2) - atanh((f^3*((16*(a^2*d^4 - a^2*c^2*d^2)*(
c + d*tan(e + f*x))^(1/2))/f^2 + (16*c*d^2*((-a^4*d^2*f^4)^(1/2) + a^2*c*f^
2)*(c + d*tan(e + f*x))^(1/2))/f^4)*(-((-a^4*d^2*f^4)^(1/2) + a^2*c*f^2)/f^
4)^(1/2))/(16*(a^3*d^5 + a^3*c^2*d^3)))*(-((-a^4*d^2*f^4)^(1/2) + a^2*c*f^2
)/f^4)^(1/2) - atanh((f^3*((16*(a^2*d^4 - a^2*c^2*d^2)*(c + d*tan(e + f*x))
^(1/2))/f^2 - (16*c*d^2*((-a^4*d^2*f^4)^(1/2) - a^2*c*f^2)*(c + d*tan(e + f
*x))^(1/2))/f^4)*(((-a^4*d^2*f^4)^(1/2) - a^2*c*f^2)/f^4)^(1/2))/(16*(a^3*d
^5 + a^3*c^2*d^3)))*(((-a^4*d^2*f^4)^(1/2) - a^2*c*f^2)/f^4)^(1/2) + (2*b*(
c + d*tan(e + f*x))^(1/2))/f
```

3.1232 $\int \frac{\sqrt{c+d\tan(e+fx)}}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=170

$$\frac{\sqrt{c-id}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(ia+b)f}-\frac{\sqrt{c+id}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(ia-b)f}-\frac{2\sqrt{b}\sqrt{bc-ad}\tanh$$

[Out] arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))*(c-I*d)^(1/2)/(I*a+b)/f-arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))*(c+I*d)^(1/2)/(I*a-b)/f-2*arctanh(b^(1/2)*(c+d*tan(f*x+e))^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)*(-a*d+b*c)^(1/2)/(a^2+b^2)/f

Rubi [A]
time = 0.33, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3653, 3620, 3618, 65, 214, 3715}

$$-\frac{2\sqrt{b}\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{f\left(a^2+b^2\right)}+\frac{\sqrt{c-id}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(b+ia)}-\frac{\sqrt{c+id}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(-b+ia)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Tan[e + f*x]]/(a + b*Tan[e + f*x]),x]
```

```
[Out] (Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((I*a + b)*
f) - (Sqrt[c + I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/((I*a
- b)*f) - (2*Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*
x]])/Sqrt[b*c - a*d]])/((a^2 + b^2)*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
```

```
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3653

```
Int[Sqrt[(a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[Simp[a*c + b*d + (b*c -
a*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x]], x], x] - Dist[d*((b*c - a*d
)/(c^2 + d^2)), Int[(1 + Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]]*(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{\sqrt{c+d\tan(e+fx)}}{a+b\tan(e+fx)}\,dx = \frac{\int \frac{ac+bd-(bc-ad)\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx}{a^2+b^2} + \frac{(b(bc-ad))\int \frac{1+\tan^2(e+fx)}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}}{a^2+b^2}$$

$$= \frac{(c-id)\int \frac{1+i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx}{2(a-ib)} + \frac{(c+id)\int \frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx}{2(a+ib)} + \frac{(b(}{}$$

$$= -\frac{(i(c+id))\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{c+idx}}\,dx, x, -i\tan(e+fx)\right)}{2(a+ib)f} + \frac{(ic+d)\text{Subst}}{}$$

$$= -\frac{2\sqrt{b}\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{(a^2+b^2)f} - \frac{(c+id)\text{Subst}\left(\int \frac{}{-1+}\right.}{}$$

$$= \frac{\sqrt{c-id}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(ia+b)f} - \frac{\sqrt{c+id}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e}}{\sqrt{c+id}}\right.}{(ia-b)f}$$

Mathematica [A]
time = 0.34, size = 158, normalized size = 0.93

$$\frac{(-ia+b)\sqrt{c-id}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)+(ia+b)\sqrt{c+id}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)-2\sqrt{b}\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{(a^2+b^2)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Tan[e + f*x]]/(a + b*Tan[e + f*x]),x]
```

```
[Out] (((-I)*a + b)*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]]
 + (I*a + b)*Sqrt[c + I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]]
- 2*Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt
[b*c - a*d]])/((a^2 + b^2)*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 692 vs. $2(142) = 284$.
time = 0.55, size = 693, normalized size = 4.08

method	result

derivativedivides	$\frac{2d^2\left(-\frac{(ad-bc)b\arctan\left(\frac{b\sqrt{c+d\tan(fx+e)}}{\sqrt{(ad-bc)b}}\right)}{d^2(a^2+b^2)\sqrt{(ad-bc)b}}+\frac{-\frac{\left(-\sqrt{2\sqrt{c^2+d^2}+2c}\,\sqrt{c^2+d^2}\,a+\sqrt{2\sqrt{c^2}}\right.}{}}{}\right.}{}$
default	$\frac{2d^2\left(-\frac{(ad-bc)b\arctan\left(\frac{b\sqrt{c+d\tan(fx+e)}}{\sqrt{(ad-bc)b}}\right)}{d^2(a^2+b^2)\sqrt{(ad-bc)b}}+\frac{-\frac{\left(-\sqrt{2\sqrt{c^2+d^2}+2c}\,\sqrt{c^2+d^2}\,a+\sqrt{2\sqrt{c^2}}\right.}{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*d^2*(-(a*d-b*c)*b/d^2/(a^2+b^2)/((a*d-b*c)*b)^(1/2)*arctan(b*(c+d*tan(f
*x+e))^(1/2)/((a*d-b*c)*b)^(1/2))+1/(a^2+b^2)/d^2*(1/4/d*(-1/2*(-(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c+(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d)*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2
)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))+2*(-2*(c^2+d^2)^(1/2)*b*d+1/2*
(-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a+(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*a*c+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/
(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*
tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/4/d*(1/2*(-(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c+(2*(
c^2+d^2)^(1/2)+2*c)^(1/2)*b*d)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(2*(c^2+d^2)^(1/2)*b*d-1/2*(-
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a+(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*a*c+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2
*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(
1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 8361 vs. $2(139) = 278$.
time = 102.60, size = 16728, normalized size = 98.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(2)*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*f^5*sqrt((((a^6 + a^4*
b^2 - a^2*b^4 - b^6)*c + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d)*f^2*sqrt((c^2 + d
^2)/((a^4 + 2*a^2*b^2 + b^4)*f^4)) + (a^4 + 2*a^2*b^2 + b^4)*c^2 + (a^4 + 2
*a^2*b^2 + b^4)*d^2)/(4*a^2*b^2*c^2 - 4*(a^3*b - a*b^3)*c*d + (a^4 - 2*a^2*
b^2 + b^4)*d^2))*sqrt((4*a^2*b^2*c^2 - 4*(a^3*b - a*b^3)*c*d + (a^4 - 2*a^2
*b^2 + b^4)*d^2)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*f^4))*((c
^2 + d^2)/((a^4 + 2*a^2*b^2 + b^4)*f^4))^(3/4)*arctan(-((2*(a^7*b + 3*a^5*b
^3 + 3*a^3*b^5 + a*b^7)*c^3 - (a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*c^2*d + 2
*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*c*d^2 - (a^8 + 2*a^6*b^2 - 2*a^2*b
^6 - b^8)*d^3)*f^4*sqrt((4*a^2*b^2*c^2 - 4*(a^3*b - a*b^3)*c*d + (a^4 - 2*a
^2*b^2 + b^4)*d^2)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*f^4))*s
qrt((c^2 + d^2)/((a^4 + 2*a^2*b^2 + b^4)*f^4)) + (2*(a^5*b + 2*a^3*b^3 + a*
b^5)*c^4 - (a^6 + a^4*b^2 - a^2*b^4 - b^6)*c^3*d + 2*(a^5*b + 2*a^3*b^3 + a
*b^5)*c^2*d^2 - (a^6 + a^4*b^2 - a^2*b^4 - b^6)*c*d^3)*f^2*sqrt((4*a^2*b^2*
c^2 - 4*(a^3*b - a*b^3)*c*d + (a^4 - 2*a^2*b^2 + b^4)*d^2)/((a^8 + 4*a^6*b^
2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*f^4)) + sqrt(2)*((2*(a^9*b^2 + 4*a^7*b^4 +
 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*c - (a^10*b + 3*a^8*b^3 + 2*a^6*b^5 - 2*a^
4*b^7 - 3*a^2*b^9 - b^11)*d)*f^7*sqrt((4*a^2*b^2*c^2 - 4*(a^3*b - a*b^3)*c*
d + (a^4 - 2*a^2*b^2 + b^4)*d^2)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6
+ b^8)*f^4))*sqrt((c^2 + d^2)/((a^4 + 2*a^2*b^2 + b^4)*f^4)) + (2*(a^7*b^2
+ 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*c^2 - (3*a^8*b + 8*a^6*b^3 + 6*a^4*b^5 - b
^9)*c*d + (a^9 + 2*a^7*b^2 - 2*a^3*b^6 - a*b^8)*d^2)*f^5*sqrt((4*a^2*b^2*c^
2 - 4*(a^3*b - a*b^3)*c*d + (a^4 - 2*a^2*b^2 + b^4)*d^2)/((a^8 + 4*a^6*b^2
+ 6*a^4*b^4 + 4*a^2*b^6 + b^8)*f^4)))*sqrt((((a^6 + a^4*b^2 - a^2*b^4 - b^6
)*c + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d)*f^2*sqrt((c^2 + d^2)/((a^4 + 2*a^2*b
^2 + b^4)*f^4)) + (a^4 + 2*a^2*b^2 + b^4)*c^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2
)/(4*a^2*b^2*c^2 - 4*(a^3*b - a*b^3)*c*d + (a^4 - 2*a^2*b^2 + b^4)*d^2))*sq
rt((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))*((c^2 + d^2)/((a^4 + 2*a
```

```
^2*b^2 + b^4)*f^4))^(3/4) - sqrt(2)*((a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2
*b^7 + b^9)*f^7*sqrt((4*a^2*b^2*c^2 - 4*(a^3*b - a*b^3)*c*d + (a^4 - 2*a^2*
b^2 + b^4)*d^2)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*f^4))*sqrt
((c^2 + d^2)/((a^4 + 2*a^2*b^2 + b^4)*f^4)) + ((a^6*b + 3*a^4*b^3 + 3*a^2*b
^5 + b^7)*c - (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*d)*f^5*sqrt((4*a^2*b^2*
c^2 - 4*(a^3*b - a*b^3)*c*d + (a^4 - 2*a^2*b^2 + b^4)*d^2)/((a^8 + 4*a^6*b^
2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*f^4)))*sqrt((((a^6 + a^4*b^2 - a^2*b^4 - b
^6)*c + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d)*f^2*sqrt((c^2 + d^2)/((a^4 + 2*a^2
*b^2 + b^4)*f^4)) + (a^4 + 2*a^2*b^2 + b^4)*c^2 + (a^4 + 2*a^2*b^2 + b^4)*d
^2)/(4*a^2*b^2*c^2 - 4*(a^3*b - a*b^3)*c*d + (a^4 - 2*a^2*b^2 + b^4)*d^2))*
sqrt(((4*(a^4*b^2 + a^2*b^4)*c^4 - 4*(a^5*b - a*b^5)*c^3*d + (a^6 + 3*a^4*b
^2 + 3*a^2*b^4 + b^6)*c^2*d^2 - 4*(a^5*b - a*b^5)*c*d^3 + (a^6 - a^4*b^2 -
a^2*b^4 + b^6)*d^4)*f^2*sqrt((c^2 + d^2)/((a^4 + 2*a^2*b^2 + b^4)*f^4))*cos
(f*x + e) + sqrt(2)*((4*(a^4*b^3 + a^2*b^5)*c^3 - 4*(2*a^5*b^2 + a^3*b^4 -
a*b^6)*c^2*d + (5*a^6*b - a^4*b^3 - 5*a^2*b^5 + b^7)*c*d^2 - (a^7 - a^5*b^2
 - a^3*b^4 + a*b^6)*d^3)*f^3*sqrt((c^2 + d^2)/((a^4 + 2*a^2*b^2 + b^4)*f^4)
)*cos(f*x + e) + (4*a^2*b^3*c^4 - 4*(a^3*b^2 - a*b^4)*c^3*d + (a^4*b + 2*a^
2*b^3 + b^5)*c^2*d^2 - 4*(a^3*b^2 - a*b^4)*c*d^3 + (a^4*b - 2*a^2*b^3 + b^5
)*d^4)*f*cos(f*x + e))*sqrt((((a^6 + a^4*b^2 - a^2*b^4 - b^6)*c + 2*(a^5*b
+ 2*a^3*b^3 + a*b^5)*d)*f^2*sqrt((c^2 + d^2)/((a^4 + 2*a^2*b^2 + b^4)*f^4))
 + (a^4 + 2*a^2*b^2 + b^4)*c^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2)/(4*a^2*b^2*c^
2 - 4*(a^3*b - a*b^3)*c*d + (a^4 - 2*a^2*b^2 + b^4)*d^2))*sqrt((c*cos(f*x +
 e) + d*sin(f*x + e))/cos(f*x + e))*((c^2 + d^2)/((a^4 + 2*a^2*b^2 + b^4)*f
^4))^(1/4) + (4*a^2*b^2*c^5 - 4*(a^3*b - a*b^3)*c^4*d + (a^4 + 2*a^2*b^2 +
b^4)*c^3*d^2 - 4*(a^3*b - a*b^3)*c^2*d^3 + (a^4 - 2*a^2*b^2 + b^4)*c*d^4)*c
os(f*x + e) + (4*a^2*b^2*c^4*d - 4*(a^3*b - a*b^3)*c^3*d^2 + (a^4 + 2*a^2*b
^2 + b^4)*c^2*d^3 - 4*(a^3*b - a*b^3)*c*d^4 + (a^4 - 2*a^2*b^2 + b^4)*d^5)*
sin(f*x + e))/((c^2 + d^2)*cos(f*x + e)))*((c^2 + d^2)/((a^4 + 2*a^2*b^2 +
b^4)*f^4))^(3/4))/(4*a^2*b^2*c^4*d - 4*(a^3*b - a*b^3)*c^3*d^2 + (a^4 + 2*a
^2*b^2 + b^4)*c^2*d^3 - 4*(a^3*b - a*b^3)*c*d^4 + (a^4 - 2*a^2*b^2 + b^4)*d
^5)) + 4*sqrt(2)*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*f^5*sqrt((((a^6 + a^4*
b^2 - a^2*b^4 - b^6)*c + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d)*f^2*sqrt((c^2 + d
^2)/((a^4 + 2*a^2*b^2 + b^4)*f^4)) + (a^4 + 2*a^2*b^2 + b^4)*c^2 + (a^4 + 2
*a^2*b^2 + b^4)*d^2)/(4*a^2*b^2*c^2 - 4*(a^3*b - a*b^3)*c*d + (a^4 - 2*a^2*
b^2 + b^4)*d^2))*sqrt((4*a^2*b^2*c^2 - 4*(a^3*b - a*b^3)*c*d + (a^4 - 2*a^2
*b^2 + b^4)*d^2)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*f^4))*((c
^2 + d^2)/((a^4 + 2*a^2*b^2 + b^4)*f^4))^(3/4)*arctan(((2*(a^7*b + 3*a^5*b^
3 + 3*a^3*b^5 + a*b^7)*c^3 - (a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*c^2*d + 2*
(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*c*d^2 -...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+d\tan\left(e+fx\right)}}{a+b\tan\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)/(a+b*tan(f*x+e)),x)
```

```
[Out] Integral(sqrt(c + d*tan(e + f*x))/(a + b*tan(e + f*x)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**B**]
time = 9.76, size = 2500, normalized size = 14.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(1/2)/(a + b*tan(e + f*x)),x)
```

```
[Out] atan((((((32*(12*a*b^7*d^11*f^4 - 12*b^8*c*d^10*f^4 + 24*a^3*b^5*d^11*f^4
+ 12*a^5*b^3*d^11*f^4 - 12*b^8*c^3*d^8*f^4 - 24*a^2*b^6*c^3*d^8*f^4 + 24*a^
3*b^5*c^2*d^9*f^4 - 12*a^4*b^4*c^3*d^8*f^4 + 12*a^5*b^3*c^2*d^9*f^4 + 12*a*
b^7*c^2*d^9*f^4 - 24*a^2*b^6*c*d^10*f^4 - 12*a^4*b^4*c*d^10*f^4))/f^5 - (32
*(c + d*tan(e + f*x))^(1/2)*(-(c + d*1i)/(4*(a^2*f^2 - b^2*f^2 + a*b*f^2*2i
)))^(1/2)*(16*b^9*d^10*f^4 + 16*a^2*b^7*d^10*f^4 - 16*a^4*b^5*d^10*f^4 - 16
*a^6*b^3*d^10*f^4 + 24*b^9*c^2*d^8*f^4 + 40*a^2*b^7*c^2*d^8*f^4 + 8*a^4*b^5
*c^2*d^8*f^4 - 8*a^6*b^3*c^2*d^8*f^4 + 8*a*b^8*c*d^9*f^4 + 24*a^3*b^6*c*d^9
*f^4 + 24*a^5*b^4*c*d^9*f^4 + 8*a^7*b^2*c*d^9*f^4))/f^4)*(-(c + d*1i)/(4*(a
^2*f^2 - b^2*f^2 + a*b*f^2*2i)))^(1/2) - (32*(c + d*tan(e + f*x))^(1/2)*(14
*a*b^6*d^11*f^2 - 6*b^7*c*d^10*f^2 - 20*a^3*b^4*d^11*f^2 - 2*a^5*b^2*d^11*f
^2 - 18*b^7*c^3*d^8*f^2 + 12*a^2*b^5*c^3*d^8*f^2 - 12*a^3*b^4*c^2*d^9*f^2 -
 2*a^4*b^3*c^3*d^8*f^2 + 2*a^5*b^2*c^2*d^9*f^2 + 18*a*b^6*c^2*d^9*f^2 + 36*
a^2*b^5*c*d^10*f^2 + 10*a^4*b^3*c*d^10*f^2))/f^4)*(-(c + d*1i)/(4*(a^2*f^2
- b^2*f^2 + a*b*f^2*2i)))^(1/2) + (32*(13*a^2*b^4*d^12*f^2 + a^4*b^2*d^12*f
^2 + 3*b^6*c^2*d^10*f^2 + 3*b^6*c^4*d^8*f^2 + 12*a^2*b^4*c^2*d^10*f^2 - a^2
*b^4*c^4*d^8*f^2 + a^4*b^2*c^2*d^10*f^2 - 16*a*b^5*c*d^11*f^2 - 16*a*b^5*c^
3*d^9*f^2))/f^5)*(-(c + d*1i)/(4*(a^2*f^2 - b^2*f^2 + a*b*f^2*2i)))^(1/2) -
 (32*(c + d*tan(e + f*x))^(1/2)*(b^5*d^12 - 2*a^2*b^3*d^12 + 3*b^5*c^4*d^8
```

- 4*a*b^4*c^3*d^9 + 2*a^2*b^3*c^2*d^10 + 4*a*b^4*c*d^11))/f^4)*(-(c + d*1i)
/(4*(a^2*f^2 - b^2*f^2 + a*b*f^2*2i)))^(1/2)*1i - (((((32*(12*a*b^7*d^11*f^
4 - 12*b^8*c*d^10*f^4 + 24*a^3*b^5*d^11*f^4 + 12*a^5*b^3*d^11*f^4 - 12*b^8*
c^3*d^8*f^4 - 24*a^2*b^6*c^3*d^8*f^4 + 24*a^3*b^5*c^2*d^9*f^4 - 12*a^4*b^4*
c^3*d^8*f^4 + 12*a^5*b^3*c^2*d^9*f^4 + 12*a*b^7*c^2*d^9*f^4 - 24*a^2*b^6*c*
d^10*f^4 - 12*a^4*b^4*c*d^10*f^4))/f^5 + (32*(c + d*tan(e + f*x))^(1/2)*(-(
c + d*1i)/(4*(a^2*f^2 - b^2*f^2 + a*b*f^2*2i)))^(1/2)*(16*b^9*d^10*f^4 + 16
*a^2*b^7*d^10*f^4 - 16*a^4*b^5*d^10*f^4 - 16*a^6*b^3*d^10*f^4 + 24*b^9*c^2*
d^8*f^4 + 40*a^2*b^7*c^2*d^8*f^4 + 8*a^4*b^5*c^2*d^8*f^4 - 8*a^6*b^3*c^2*d^
8*f^4 + 8*a*b^8*c*d^9*f^4 + 24*a^3*b^6*c*d^9*f^4 + 24*a^5*b^4*c*d^9*f^4 + 8
*a^7*b^2*c*d^9*f^4))/f^4)*(-(c + d*1i)/(4*(a^2*f^2 - b^2*f^2 + a*b*f^2*2i))
)^(1/2) + (32*(c + d*tan(e + f*x))^(1/2)*(14*a*b^6*d^11*f^2 - 6*b^7*c*d^10*
f^2 - 20*a^3*b^4*d^11*f^2 - 2*a^5*b^2*d^11*f^2 - 18*b^7*c^3*d^8*f^2 + 12*a^
2*b^5*c^3*d^8*f^2 - 12*a^3*b^4*c^2*d^9*f^2 - 2*a^4*b^3*c^3*d^8*f^2 + 2*a^5*
b^2*c^2*d^9*f^2 + 18*a*b^6*c^2*d^9*f^2 + 36*a^2*b^5*c*d^10*f^2 + 10*a^4*b^3
*c*d^10*f^2))/f^4)*(-(c + d*1i)/(4*(a^2*f^2 - b^2*f^2 + a*b*f^2*2i)))^(1/2)
 + (32*(13*a^2*b^4*d^12*f^2 + a^4*b^2*d^12*f^2 + 3*b^6*c^2*d^10*f^2 + 3*b^6
*c^4*d^8*f^2 + 12*a^2*b^4*c^2*d^10*f^2 - a^2*b^4*c^4*d^8*f^2 + a^4*b^2*c^2*
d^10*f^2 - 16*a*b^5*c*d^11*f^2 - 16*a*b^5*c^3*d^9*f^2))/f^5)*(-(c + d*1i)/(
4*(a^2*f^2 - b^2*f^2 + a*b*f^2*2i)))^(1/2) + (32*(c + d*tan(e + f*x))^(1/2)
*(b^5*d^12 - 2*a^2*b^3*d^12 + 3*b^5*c^4*d^8 - 4*a*b^4*c^3*d^9 + 2*a^2*b^3*c
^2*d^10 + 4*a*b^4*c*d^11))/f^4)*(-(c + d*1i)/(4*(a^2*f^2 - b^2*f^2 + a*b*f^
2*2i)))^(1/2)*1i)/((((((32*(12*a*b^7*d^11*f^4 - 12*b^8*c*d^10*f^4 + 24*a^3*
b^5*d^11*f^4 + 12*a^5*b^3*d^11*f^4 - 12*b^8*c^3*d^8*f^4 - 24*a^2*b^6*c^3*d^
8*f^4 + 24*a^3*b^5*c^2*d^9*f^4 - 12*a^4*b^4*c^3*d^8*f^4 + 12*a^5*b^3*c^2*d^
9*f^4 + 12*a*b^7*c^2*d^9*f^4 - 24*a^2*b^6*c*d^10*f^4 - 12*a^4*b^4*c*d^10*f^
4))/f^5 - (32*(c + d*tan(e + f*x))^(1/2)*(-(c + d*1i)/(4*(a^2*f^2 - b^2*f^2
 + a*b*f^2*2i)))^(1/2)*(16*b^9*d^10*f^4 + 16*a^2*b^7*d^10*f^4 - 16*a^4*b^5*
d^10*f^4 - 16*a^6*b^3*d^10*f^4 + 24*b^9*c^2*d^8*f^4 + 40*a^2*b^7*c^2*d^8*f^
4 + 8*a^4*b^5*c^2*d^8*f^4 - 8*a^6*b^3*c^2*d^8*f^4 + 8*a*b^8*c*d^9*f^4 + 24*
a^3*b^6*c*d^9*f^4 + 24*a^5*b^4*c*d^9*f^4 + 8*a^7*b^2*c*d^9*f^4))/f^4)*(-(c
+ d*1i)/(4*(a^2*f^2 - b^2*f^2 + a*b*f^2*2i)))^(1/2) - (32*(c + d*tan(e + f*
x))^(1/2)*(14*a*b^6*d^11*f^2 - 6*b^7*c*d^10*f^2 - 20*a^3*b^4*d^11*f^2 - 2*a
^5*b^2*d^11*f^2 - 18*b^7*c^3*d^8*f^2 + 12*a^2*b^5*c^3*d^8*f^2 - 12*a^3*b^4*
c^2*d^9*f^2 - 2*a^4*b^3*c^3*d^8*f^2 + 2*a^5*b^2*c^2*d^9*f^2 + 18*a*b^6*c^2*
d^9*f^2 + 36*a^2*b^5*c*d^10*f^2 + 10*a^4*b^3*c*d^10*f^2))/f^4)*(-(c + d*1i)
/(4*(a^2*f^2 - b^2*f^2 + a*b*f^2*2i)))^(1/2) + (32*(13*a^2*b^4*d^12*f^2 + a
^4*b^2*d^12*f^2 + 3*b^6*c^2*d^10*f^2 + 3*b^6*c^4*d^8*f^2 + 12*a^2*b^4*c^2*d
^10*f^2 - a^2*b^4*c^4*d^8*f^2 + a^4*b^2*c^2*d^10*f^2 - 16*a*b^5*c*d^11*f^2
- 16*a*b^5*c^3*d^9*f^2))/f^5)*(-(c + d*1i)/(4*(a^2*f^2 - b^2*f^2 + a*b*f^2*
2i)))^(1/2) - (32*(c + d*tan(e + f*x))^(1/2)*(b^5*d^12 - 2*a^2*b^3*d^12 + 3
*b^5*c^4*d^8 - 4*a*b^4*c^3*d^9 + 2*a^2*b^3*c^2*d^10 + 4*a*b^4*c*d^11))/f^4)
*(-(c + d*1i)/(4*(a^2*f^2 - b^2*f^2 + a*b*f^2*2i)))^(1/2) + (((((32*(12*a*b
^7*d^11*f^4 - 12*b^8*c*d^10*f^4 + 24*a^3*b^5*d^11*f^4 + 12*a^5*b^3*d^11*f^4
 - 12*b^8*c^3*d^8*f^4 - 24*a^2*b^6*c^3*d^8*f^4 + 24*a^3*b^5*c^2*d^9*f^4 - 1

```
2*a^4*b^4*c^3*d^8*f^4 + 12*a^5*b^3*c^2*d^9*f^4 + 12*a*b^7*c^2*d^9*f^4 - 24*
a^2*b^6*c*d^10*f^4 - 12*a^4*b^4*c*d^10*f^4))/f^5 + (32*(c + d*tan(e + f*x))
^(1/2)*(-(c + d*1i)/(4*(a^2*f^2 - b^2*f^2 + a*b...
```

3.1233 $\int \frac{\sqrt{c+d\tan(e+fx)}}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=231

$$-\frac{i\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)^2f}+\frac{i\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(a+ib)^2f}-\frac{\sqrt{b}\,(4abc-3a^2}{}$$

```
[Out] -I*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))*(c-I*d)^(1/2)/(a-I*b)^2/f+
I*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))*(c+I*d)^(1/2)/(a+I*b)^2/f-(
-3*a^2*d+4*a*b*c+b^2*d)*arctanh(b^(1/2)*(c+d*tan(f*x+e))^(1/2)/(-a*d+b*c)^(
1/2))*b^(1/2)/(a^2+b^2)^2/f/(-a*d+b*c)^(1/2)-b*(c+d*tan(f*x+e))^(1/2)/(a^2+
b^2)/f/(a+b*tan(f*x+e))
```

Rubi [A]
time = 0.54, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3649, 3734, 3620, 3618, 65, 214, 3715}

$$-\frac{b\sqrt{c+d\tan(e+fx)}}{f(a^2+b^2)(a+b\tan(e+fx))}-\frac{\sqrt{b}\,(-3a^2d+4abc+b^2d)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{f(a^2+b^2)^2\sqrt{bc-ad}}-\frac{i\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(a-ib)^2}+\frac{i\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(a+ib)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Tan[e + f*x]]/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] ((-I)*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((a -
I*b)^2*f) + (I*Sqrt[c + I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]
])/((a + I*b)^2*f) - (Sqrt[b]*(4*a*b*c - 3*a^2*d + b^2*d)*ArcTanh[(Sqrt[b]*
Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/((a^2 + b^2)^2*Sqrt[b*c - a*d]*
f) - (b*Sqrt[c + d*Tan[e + f*x]])/((a^2 + b^2)*f*(a + b*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{c+d\tan(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = -\frac{b\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)\,f(a+b\tan(e+fx))} - \frac{\int \frac{\frac{1}{2}(-2ac-bd)+(bc-ad)\tan(e+fx)+\frac{1}{2}bd\tan^2(e+fx)}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}\,d}{a^2+b^2}$$

$$= -\frac{b\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)\,f(a+b\tan(e+fx))} - \frac{\int \frac{-a^2c+b^2c-2abd+(2abc-a^2d+b^2d)\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx}{(a^2+b^2)^2} +$$

$$= -\frac{b\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)\,f(a+b\tan(e+fx))} + \frac{(c-id)\int \frac{1+i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx}{2(a-ib)^2} + \frac{(c+}{}$$

$$= -\frac{b\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)\,f(a+b\tan(e+fx))} - \frac{(ic-d)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{c+idx}}\,dx, x, -\right.}{2(a+ib)^2f}$$

$$= -\frac{\sqrt{b}\,(4abc-3a^2d+b^2d)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{(a^2+b^2)^2\sqrt{bc-ad}\,f} - \frac{b\sqrt{c+}}{(a^2+b^2)\,f}$$

$$= -\frac{i\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)^2f} + \frac{i\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\,\mathrm{ta}}}{\sqrt{c}}\right.}{(a+ib)^2f}$$

Mathematica [A]
time = 2.08, size = 276, normalized size = 1.19

$$-\frac{\frac{i\left((a+ib)^2\sqrt{c-id}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)+(a-ib)^2\sqrt{c+id}\,(-bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)\right)}{a^2+b^2}+\frac{\sqrt{b}\,\sqrt{bc-ad}\,(4abc-3a^2d+b^2d)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{a^2+b^2}-bd\sqrt{c+d\tan(e+fx)}+\frac{b^2(c+d\tan(e+fx))^{3/2}}{a+b\tan(e+fx)}}{(a^2+b^2)\,(bc-ad)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Tan[e + f*x]]/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] -(((I*((a + I*b)^2*Sqrt[c - I*d]*(b*c - a*d)*ArcTanh[Sqrt[c + d*Tan[e + f*x
]]/Sqrt[c - I*d]] + (a - I*b)^2*Sqrt[c + I*d]*(-(b*c) + a*d)*ArcTanh[Sqrt[c
 + d*Tan[e + f*x]]/Sqrt[c + I*d]]))/(a^2 + b^2) + (Sqrt[b]*Sqrt[b*c - a*d]*
(4*a*b*c - 3*a^2*d + b^2*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt
[b*c - a*d]])/(a^2 + b^2) - b*d*Sqrt[c + d*Tan[e + f*x]] + (b^2*(c + d*Tan[
e + f*x])^(3/2))/(a + b*Tan[e + f*x]))/((a^2 + b^2)*(b*c - a*d)*f))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 991 vs. $2(199) = 398$.
time = 0.52, size = 992, normalized size = 4.29

method	result

derivativedivides	$2d^3 \left(-\frac{\left(-\sqrt{c^2+d^2}\,\sqrt{2\sqrt{c^2+d^2}+2c}\,a^2+\sqrt{c^2+d^2}\,\sqrt{2\sqrt{c^2+d^2}+2c}\,b^2+\sqrt{2\sqrt{c^2+d^2}\right.}{}\right.$
default	$2d^3 \left(-\frac{\left(-\sqrt{c^2+d^2}\,\sqrt{2\sqrt{c^2+d^2}+2c}\,a^2+\sqrt{c^2+d^2}\,\sqrt{2\sqrt{c^2+d^2}+2c}\,b^2+\sqrt{2\sqrt{c^2+d^2}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*d^3*(1/d^3/(a^2+b^2)^2*(1/4/d*(-1/2*(-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2
)+2*c)^(1/2)*a^2+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2+(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*a^2*c+2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d-(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*b^2*c)*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))+2*(-4*(c^2+d^2)^(1/2)*a*b*d+1/2*(-
(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2+(c^2+d^2)^(1/2)*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*b^2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c+2*(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*a*b*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c)*(2*(c^2+d^2)^(
1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2
*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/4/d*(
1/2*(-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2+(c^2+d^2)^(1/2)*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c+2*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)*a*b*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c)*ln(d*tan
(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1
/2))+2*(4*(c^2+d^2)^(1/2)*a*b*d-1/2*(-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*a^2+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2+(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*a^2*c+2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d-(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*b^2*c)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2
*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(
2*(c^2+d^2)^(1/2)-2*c)^(1/2))))-b/d^3/(a^2+b^2)^2*((1/2*a^2*d+1/2*b^2*d)*(c
```

```
+d*tan(f*x+e))^(1/2)/((c+d*tan(f*x+e))*b+a*d-b*c)+1/2*(3*a^2*d-4*a*b*c-b^2*
d)/((a*d-b*c)*b)^(1/2)*arctan(b*(c+d*tan(f*x+e))^(1/2)/((a*d-b*c)*b)^(1/2))
))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 16347 vs. 2(196) = 392.
time = 240.14, size = 32681, normalized size = 141.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(2)*((a^14 + 5*a^12*b^2 + 9*a^10*b^4 + 5*a^8*b^6 - 5*a^6*b^8 -
9*a^4*b^10 - 5*a^2*b^12 - b^14)*f^5*cos(f*x + e)^2 + 2*(a^13*b + 6*a^11*b^3
 + 15*a^9*b^5 + 20*a^7*b^7 + 15*a^5*b^9 + 6*a^3*b^11 + a*b^13)*f^5*cos(f*x
+ e)*sin(f*x + e) + (a^12*b^2 + 6*a^10*b^4 + 15*a^8*b^6 + 20*a^6*b^8 + 15*a
^4*b^10 + 6*a^2*b^12 + b^14)*f^5)*sqrt((((a^12 - 2*a^10*b^2 - 17*a^8*b^4 -
28*a^6*b^6 - 17*a^4*b^8 - 2*a^2*b^10 + b^12)*c + 4*(a^11*b + 3*a^9*b^3 + 2*
a^7*b^5 - 2*a^5*b^7 - 3*a^3*b^9 - a*b^11)*d)*f^2*sqrt((c^2 + d^2)/((a^8 + 4
*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*f^4)) + (a^8 + 4*a^6*b^2 + 6*a^4*b^
4 + 4*a^2*b^6 + b^8)*c^2 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*
d^2)/(16*(a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*c^2 - 8*(a^7*b - 7*a^5*b^3 + 7*a^3
*b^5 - a*b^7)*c*d + (a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*d^2)
)*sqrt((16*(a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*c^2 - 8*(a^7*b - 7*a^5*b^3 + 7*a
^3*b^5 - a*b^7)*c*d + (a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*d^
2)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^
10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*f^4))*((c^2 + d^2)/((a^8 + 4*a^6*b^2
+ 6*a^4*b^4 + 4*a^2*b^6 + b^8)*f^4))^(3/4)*arctan(((4*(a^15*b + 5*a^13*b^3
+ 9*a^11*b^5 + 5*a^9*b^7 - 5*a^7*b^9 - 9*a^5*b^11 - 5*a^3*b^13 - a*b^15)*c^
3 - (a^16 - 20*a^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 - 64*a^6*b^10 - 20*a^4*b
^12 + b^16)*c^2*d + 4*(a^15*b + 5*a^13*b^3 + 9*a^11*b^5 + 5*a^9*b^7 - 5*a^7
*b^9 - 9*a^5*b^11 - 5*a^3*b^13 - a*b^15)*c*d^2 - (a^16 - 20*a^12*b^4 - 64*a
```

^10*b^6 - 90*a^8*b^8 - 64*a^6*b^10 - 20*a^4*b^12 + b^16)*d^3)*f^4*sqrt((16*
(a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*c^2 - 8*(a^7*b - 7*a^5*b^3 + 7*a^3*b^5 - a*
b^7)*c*d + (a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*d^2)/((a^16 +
 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4
*b^12 + 8*a^2*b^14 + b^16)*f^4))*sqrt((c^2 + d^2)/((a^8 + 4*a^6*b^2 + 6*a^4
*b^4 + 4*a^2*b^6 + b^8)*f^4)) + (4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 - 2*a^5*
b^7 - 3*a^3*b^9 - a*b^11)*c^4 - (a^12 - 2*a^10*b^2 - 17*a^8*b^4 - 28*a^6*b^
6 - 17*a^4*b^8 - 2*a^2*b^10 + b^12)*c^3*d + 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b
^5 - 2*a^5*b^7 - 3*a^3*b^9 - a*b^11)*c^2*d^2 - (a^12 - 2*a^10*b^2 - 17*a^8*
b^4 - 28*a^6*b^6 - 17*a^4*b^8 - 2*a^2*b^10 + b^12)*c*d^3)*f^2*sqrt((16*(a^6
*b^2 - 2*a^4*b^4 + a^2*b^6)*c^2 - 8*(a^7*b - 7*a^5*b^3 + 7*a^3*b^5 - a*b^7)
*c*d + (a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*d^2)/((a^16 + 8*a
^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^1
2 + 8*a^2*b^14 + b^16)*f^4)) + sqrt(2)*(2*(4*(a^20*b^2 + 7*a^18*b^4 + 20*a^
16*b^6 + 28*a^14*b^8 + 14*a^12*b^10 - 14*a^10*b^12 - 28*a^8*b^14 - 20*a^6*b
^16 - 7*a^4*b^18 - a^2*b^20)*c - (a^21*b + 2*a^19*b^3 - 19*a^17*b^5 - 104*a
^15*b^7 - 238*a^13*b^9 - 308*a^11*b^11 - 238*a^9*b^13 - 104*a^7*b^15 - 19*a
^5*b^17 + 2*a^3*b^19 + a*b^21)*d)*f^7*sqrt((16*(a^6*b^2 - 2*a^4*b^4 + a^2*b
^6)*c^2 - 8*(a^7*b - 7*a^5*b^3 + 7*a^3*b^5 - a*b^7)*c*d + (a^8 - 12*a^6*b^2
 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*d^2)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 +
56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*f
^4))*sqrt((c^2 + d^2)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*f^4)
) + (8*(a^16*b^2 + 5*a^14*b^4 + 9*a^12*b^6 + 5*a^10*b^8 - 5*a^8*b^10 - 9*a^
6*b^12 - 5*a^4*b^14 - a^2*b^16)*c^2 - 2*(3*a^17*b + 8*a^15*b^3 - 12*a^13*b^
5 - 72*a^11*b^7 - 110*a^9*b^9 - 72*a^7*b^11 - 12*a^5*b^13 + 8*a^3*b^15 + 3*
a*b^17)*c*d + (a^18 - a^16*b^2 - 20*a^14*b^4 - 44*a^12*b^6 - 26*a^10*b^8 +
26*a^8*b^10 + 44*a^6*b^12 + 20*a^4*b^14 + a^2*b^16 - b^18)*d^2)*f^5*sqrt((1
6*(a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*c^2 - 8*(a^7*b - 7*a^5*b^3 + 7*a^3*b^5 -
a*b^7)*c*d + (a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*d^2)/((a^16
 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a
^4*b^12 + 8*a^2*b^14 + b^16)*f^4)))*sqrt((((a^12 - 2*a^10*b^2 - 17*a^8*b^4
 - 28*a^6*b^6 - 17*a^4*b^8 - 2*a^2*b^10 + b^12)*c + 4*(a^11*b + 3*a^9*b^3 +
2*a^7*b^5 - 2*a^5*b^7 - 3*a^3*b^9 - a*b^11)*d)*f^2*sqrt((c^2 + d^2)/((a^8 +
 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*f^4)) + (a^8 + 4*a^6*b^2 + 6*a^4*
b^4 + 4*a^2*b^6 + b^8)*c^2 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8
)*d^2)/(16*(a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*c^2 - 8*(a^7*b - 7*a^5*b^3 + 7*a
^3*b^5 - a*b^7)*c*d + (a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*d^
2))*sqrt((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))*((c^2 + d^2)/((a^8
 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*f^4))^(3/4) + sqrt(2)*(2*(a^17*
b + 8*a^15*b^3 + 28*a^13*b^5 + 56*a^11*b^7 + 70*a^9*b^9 + 56*a^7*b^11 + 28*
a^5*b^13 + 8*a^3*b^15 + a*b^17)*f^7*sqrt((16*(a^6*b^2 - 2*a^4*b^4 + a^2*b^6
)*c^2 - 8*(a^7*b - 7*a^5*b^3 + 7*a^3*b^5 - a*b^7)*c*d + (a^8 - 12*a^6*b^2 +
 38*a^4*b^4 - 12*a^2*b^6 + b^8)*d^2)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56
*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*f^4
))*sqrt((c^2 + d^2)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*f^4))

```
+ (2*(a^13*b + 6*a^11*b^3 + 15*a^9*b^5 + 20*a^7*b^7 + 15*a^5*b^9 + 6*a^3*b^
11 + a*b^13)*c - (a^14 + 5*a^12*b^2 + 9*a^10*b^...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + d\tan{(e + fx)}}}{(a + b\tan{(e + fx)})^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral(sqrt(c + d*tan(e + f*x))/(a + b*tan(e + f*x))**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 11.43, size = 2500, normalized size = 10.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(1/2)/(a + b*tan(e + f*x))^2,x)
```

```
[Out] (atan((((-b*(a*d - b*c))^(1/2)*((16*(c + d*tan(e + f*x))^(1/2)*(3*b^9*d^12
- 3*a^2*b^7*d^12 + 17*a^4*b^5*d^12 - 9*a^6*b^3*d^12 + 3*b^9*c^2*d^10 + 2*b^
9*c^4*d^8 - 8*a*b^8*c^3*d^9 - 56*a^3*b^6*c*d^11 + 60*a^5*b^4*c*d^11 + 63*a^
2*b^7*c^2*d^10 - 12*a^2*b^7*c^4*d^8 + 96*a^3*b^6*c^3*d^9 - 123*a^4*b^5*c^2*
d^10 + 18*a^4*b^5*c^4*d^8 - 24*a^5*b^4*c^3*d^9 + 9*a^6*b^3*c^2*d^10 + 12*a*
b^8*c*d^11))/(a^8*f^4 + b^8*f^4 + 4*a^2*b^6*f^4 + 6*a^4*b^4*f^4 + 4*a^6*b^2
*f^4) - (((8*(20*b^11*c*d^11*f^2 - 52*a*b^10*d^12*f^2 + 128*a^3*b^8*d^12*f^
2 + 24*a^5*b^6*d^12*f^2 - 160*a^7*b^4*d^12*f^2 - 4*a^9*b^2*d^12*f^2 + 20*b^
11*c^3*d^9*f^2 - 256*a^2*b^9*c^3*d^9*f^2 - 128*a^3*b^8*c^4*d^8*f^2 + 72*a^4
*b^7*c^3*d^9*f^2 - 168*a^5*b^6*c^2*d^10*f^2 - 192*a^5*b^6*c^4*d^8*f^2 + 352
*a^6*b^5*c^3*d^9*f^2 - 160*a^7*b^4*c^2*d^10*f^2 + 4*a^8*b^3*c^3*d^9*f^2 - 4
```

*a^9*b^2*c^2*d^10*f^2 + 12*a*b^10*c^2*d^10*f^2 + 64*a*b^10*c^4*d^8*f^2 - 256*a^2*b^9*c*d^11*f^2 + 72*a^4*b^7*c*d^11*f^2 + 352*a^6*b^5*c*d^11*f^2 + 4*a^8*b^3*c*d^11*f^2))/(a^8*f^5 + b^8*f^5 + 4*a^2*b^6*f^5 + 6*a^4*b^4*f^5 + 4*a^6*b^2*f^5) + ((-b*(a*d - b*c))^(1/2)*((16*(c + d*tan(e + f*x))^(1/2)*(68*a*b^12*d^11*f^2 - 8*b^13*c*d^10*f^2 + 20*a^3*b^10*d^11*f^2 - 88*a^5*b^8*d^11*f^2 + 40*a^7*b^6*d^11*f^2 + 84*a^9*b^4*d^11*f^2 + 4*a^11*b^2*d^11*f^2 - 20*b^13*c^3*d^8*f^2 + 116*a^2*b^11*c^3*d^8*f^2 + 204*a^3*b^10*c^2*d^9*f^2 + 216*a^4*b^9*c^3*d^8*f^2 + 168*a^5*b^8*c^2*d^9*f^2 + 8*a^6*b^7*c^3*d^8*f^2 + 184*a^7*b^6*c^2*d^9*f^2 - 68*a^8*b^5*c^3*d^8*f^2 + 100*a^9*b^4*c^2*d^9*f^2 + 4*a^10*b^3*c^3*d^8*f^2 - 4*a^11*b^2*c^2*d^9*f^2 + 116*a*b^12*c^2*d^9*f^2 + 104*a^2*b^11*c*d^10*f^2 + 48*a^4*b^9*c*d^10*f^2 - 304*a^6*b^7*c*d^10*f^2 - 296*a^8*b^5*c*d^10*f^2 - 56*a^10*b^3*c*d^10*f^2))/(a^8*f^4 + b^8*f^4 + 4*a^2*b^6*f^4 + 6*a^4*b^4*f^4 + 4*a^6*b^2*f^4) + ((-b*(a*d - b*c))^(1/2)*((8*(32*b^15*d^11*f^4 + 96*a^2*b^13*d^11*f^4 - 320*a^6*b^9*d^11*f^4 - 480*a^8*b^7*d^11*f^4 - 288*a^10*b^5*d^11*f^4 - 64*a^12*b^3*d^11*f^4 + 32*b^15*c^2*d^9*f^4 + 96*a^2*b^13*c^2*d^9*f^4 + 320*a^3*b^12*c^3*d^8*f^4 + 640*a^5*b^10*c^3*d^8*f^4 - 320*a^6*b^9*c^2*d^9*f^4 + 640*a^7*b^8*c^3*d^8*f^4 - 480*a^8*b^7*c^2*d^9*f^4 + 320*a^9*b^6*c^3*d^8*f^4 - 288*a^10*b^5*c^2*d^9*f^4 + 64*a^11*b^4*c^3*d^8*f^4 - 64*a^12*b^3*c^2*d^9*f^4 + 64*a*b^14*c*d^10*f^4 + 64*a*b^14*c^3*d^8*f^4 + 320*a^3*b^12*c*d^10*f^4 + 640*a^5*b^10*c*d^10*f^4 + 640*a^7*b^8*c*d^10*f^4 + 320*a^9*b^6*c*d^10*f^4 + 64*a^11*b^4*c*d^10*f^4))/(a^8*f^5 + b^8*f^5 + 4*a^2*b^6*f^5 + 6*a^4*b^4*f^5 + 4*a^6*b^2*f^5) - (8*(-b*(a*d - b*c))^(1/2)*(c + d*tan(e + f*x))^(1/2)*(b^2*d - 3*a^2*d + 4*a*b*c)*(32*b^17*d^10*f^4 + 160*a^2*b^15*d^10*f^4 + 288*a^4*b^13*d^10*f^4 + 160*a^6*b^11*d^10*f^4 - 160*a^8*b^9*d^10*f^4 - 288*a^10*b^7*d^10*f^4 - 160*a^12*b^5*d^10*f^4 - 32*a^14*b^3*d^10*f^4 + 48*b^17*c^2*d^8*f^4 + 272*a^2*b^15*c^2*d^8*f^4 + 624*a^4*b^13*c^2*d^8*f^4 + 720*a^6*b^11*c^2*d^8*f^4 + 400*a^8*b^9*c^2*d^8*f^4 + 48*a^10*b^7*c^2*d^8*f^4 - 48*a^12*b^5*c^2*d^8*f^4 - 16*a^14*b^3*c^2*d^8*f^4 + 16*a*b^16*c*d^9*f^4 + 112*a^3*b^14*c*d^9*f^4 + 336*a^5*b^12*c*d^9*f^4 + 560*a^7*b^10*c*d^9*f^4 + 560*a^9*b^8*c*d^9*f^4 + 336*a^11*b^6*c*d^9*f^4 + 112*a^13*b^4*c*d^9*f^4 + 16*a^15*b^2*c*d^9*f^4))/((a^8*f^4 + b^8*f^4 + 4*a^2*b^6*f^4 + 6*a^4*b^4*f^4 + 4*a^6*b^2*f^4)*(a^5*d*f - b^5*c*f - a^4*b*c*f + a*b^4*d*f - 2*a^2*b^3*c*f + 2*a^3*b^2*d*f)))*(b^2*d - 3*a^2*d + 4*a*b*c))/(2*(a^5*d*f - b^5*c*f - a^4*b*c*f + a*b^4*d*f - 2*a^2*b^3*c*f + 2*a^3*b^2*d*f)))*(b^2*d - 3*a^2*d + 4*a*b*c))/(2*(a^5*d*f - b^5*c*f - a^4*b*c*f + a*b^4*d*f - 2*a^2*b^3*c*f + 2*a^3*b^2*d*f)))*(-b*(a*d - b*c))^(1/2)*(b^2*d - 3*a^2*d + 4*a*b*c))/(2*(a^5*d*f - b^5*c*f - a^4*b*c*f + a*b^4*d*f - 2*a^2*b^3*c*f + 2*a^3*b^2*d*f)))*(b^2*d - 3*a^2*d + 4*a*b*c)*1i)/(2*(a^5*d*f - b^5*c*f - a^4*b*c*f + a*b^4*d*f - 2*a^2*b^3*c*f + 2*a^3*b^2*d*f)) + ((-b*(a*d - b*c))^(1/2)*((16*(c + d*tan(e + f*x))^(1/2)*(3*b^9*d^12 - 3*a^2*b^7*d^12 + 17*a^4*b^5*d^12 - 9*a^6*b^3*d^12 + 3*b^9*c^2*d^10 + 2*b^9*c^4*d^8 - 8*a*b^8*c^3*d^9 - 56*a^3*b^6*c*d^11 + 60*a^5*b^4*c*d^11 + 63*a^2*b^7*c^2*d^10 - 12*a^2*b^7*c^4*d^8 + 96*a^3*b^6*c^3*d^9 - 123*a^4*b^5*c^2*d^10 + 18*a^4*b^5*c^4*d^8 - 24*a^5*b^4*c^3*d^9 + 9*a^6*b^3*c^2*d^10 + 12*a*b^8*c*d^11))/(a^8*f^4 + b^8*f^4 + 4*a^2*b^6*f^4 + 6*a^4*b^4*f^4 + 4*a^6*b^2*f^4) +

```
(((8*(20*b^11*c*d^11*f^2 - 52*a*b^10*d^12*f^2 + 128*a^3*b^8*d^12*f^2 + 24*a
^5*b^6*d^12*f^2 - 160*a^7*b^4*d^12*f^2 - 4*a^9*b^2*d^12*f^2 + 20*b^11*c^3*d
^9*f^2 - 256*a^2*b^9*c^3*d^9*f^2 - 128*a^3*b^8*c^4*d^8*f^2 + 72*a^4*b^7*c^3
*d^9*f^2 - 168*a^5*b^6*c^2*d^10*f^2 - 192*a^5*b^6*c^4*d^8*f^2 + 352*a^6*b^5
*c^3*d^9*f^2 - 160*a^7*b^4*c^2*d^10*f^2 + 4*a^8*b^3*c^3*d^9*f^2 - 4*a^9*b^2
*c^2*d^10*f^2 + 12*a*b^10*c^2*d^10*f^2 + 64*a*b^10*c^4*d^8*f^2 - 256*a^2*b^
9*c*d^11*f^2 + 72*a^4*b^7*c*d^11*f^2 + 352*a^6*b^5*c*d^11*f^2 + 4*a^8*b^3*c
*d^11*f^2))/(a^8*f^5 + b^8*f^5 + 4*a^2*b^6*f^5 + 6*a^4*b^4*f^5 + 4*a^6*b^2*
f^5) - ((-b*(a*d - b*c))^(1/2)*((16*(c + d*tan(e + f*x))^(1/2)*(68*a*b^12*d
^11*f^2 - 8*b^13*c*d^10*f^2 + 20*a^3*b^10*d^11*f^2 - 88*a^5*b^8*d^11*f^2 +
40*a^7*b^6*d^11*f^2 + 84*a^9*b^4*d^11*f^2 + 4*a...
```

3.1234 $\int \frac{\sqrt{c+d\tan(e+fx)}}{(a+b\tan(e+fx))^3}\,dx$

Optimal. Leaf size=342

$$-\frac{\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(ia+b)^3f}+\frac{\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(ia-b)^3f}+\frac{\sqrt{b}\,(40a^3bcd-24a$$

```
[Out] 1/4*(40*a^3*b*c*d-24*a*b^3*c*d-15*a^4*d^2-6*a^2*b^2*(4*c^2-3*d^2)+b^4*(8*c^
2+d^2))*arctanh(b^(1/2)*(c+d*tan(f*x+e))^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/(a
^2+b^2)^3/(-a*d+b*c)^(3/2)/f-arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))*
(c-I*d)^(1/2)/(I*a+b)^3/f+arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))*(c+
I*d)^(1/2)/(I*a-b)^3/f-1/2*b*(c+d*tan(f*x+e))^(1/2)/(a^2+b^2)/f/(a+b*tan(f*
x+e))^2-1/4*b*(-7*a^2*d+8*a*b*c+b^2*d)*(c+d*tan(f*x+e))^(1/2)/(a^2+b^2)^2/(
-a*d+b*c)/f/(a+b*tan(f*x+e))
```

Rubi [A]

time = 1.07, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {3649, 3730, 3734, 3620, 3618, 65, 214, 3715}

$$-\frac{b\left(-7a^2d+8abc+b^2d\right)\sqrt{c+d\tan(e+fx)}}{4f\left(a^2+b^2\right)^2(bc-ad)(a+b\tan(e+fx))}-\frac{b\sqrt{c+d\tan(e+fx)}}{2f\left(a^2+b^2\right)(a+b\tan(e+fx))^2}+\frac{\sqrt{b}\left(-15a^4d^2+40a^3bcd-6a^2b^2\left(4c^2-3d^2\right)-24ab^3cd+b^4\left(8c^2+d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{4f\left(a^2+b^2\right)^3(bc-ad)^{3/2}}-\frac{\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(b+ia)^3}+\frac{\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(-b+ia)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Tan[e + f*x]]/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] -((Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((I*a + b
)^3*f)) + (Sqrt[c + I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/(
(I*a - b)^3*f) + (Sqrt[b]*(40*a^3*b*c*d - 24*a*b^3*c*d - 15*a^4*d^2 - 6*a^2
*b^2*(4*c^2 - 3*d^2) + b^4*(8*c^2 + d^2))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e
 + f*x]])/Sqrt[b*c - a*d]])/(4*(a^2 + b^2)^3*(b*c - a*d)^(3/2)*f) - (b*Sqrt
[c + d*Tan[e + f*x]])/(2*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^2) - (b*(8*a*b*
c - 7*a^2*d + b^2*d)*Sqrt[c + d*Tan[e + f*x]])/(4*(a^2 + b^2)^2*(b*c - a*d)
*f*(a + b*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
```

```
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{c+d\tan(e+fx)}}{(a+b\tan(e+fx))^3}\,dx = -\frac{b\sqrt{c+d\tan(e+fx)}}{2\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} - \frac{\int \frac{\frac{1}{2}(-4ac-bd)+2(bc-ad)\tan(e+fx)+\frac{3}{2}bd\tan^2(e+fx}{(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}}{2\left(a^2+b^2\right)}$$

$$= -\frac{b\sqrt{c+d\tan(e+fx)}}{2\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} - \frac{b(8abc-7a^2d+b^2d)\sqrt{c+d\tan(e+fx}}{4\left(a^2+b^2\right)^2(bc-ad)f(a+b\tan(e+fx}$$

$$= -\frac{b\sqrt{c+d\tan(e+fx)}}{2\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} - \frac{b(8abc-7a^2d+b^2d)\sqrt{c+d\tan(e+fx}}{4\left(a^2+b^2\right)^2(bc-ad)f(a+b\tan(e+fx}$$

$$= -\frac{b\sqrt{c+d\tan(e+fx)}}{2\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} - \frac{b(8abc-7a^2d+b^2d)\sqrt{c+d\tan(e+fx}}{4\left(a^2+b^2\right)^2(bc-ad)f(a+b\tan(e+fx}$$

$$= -\frac{b\sqrt{c+d\tan(e+fx)}}{2\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} - \frac{b(8abc-7a^2d+b^2d)\sqrt{c+d\tan(e+fx}}{4\left(a^2+b^2\right)^2(bc-ad)f(a+b\tan(e+fx}$$

$$= \frac{\sqrt{b}\left(40a^3bcd-24ab^3cd-15a^4d^2-6a^2b^2(4c^2-3d^2)+b^4(8c^2+d^2)\right)\tanh^{-1}\Big(}{4\left(a^2+b^2\right)^3(bc-ad)^{3/2}f}$$

$$= -\frac{\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(ia+b)^3f} + \frac{\sqrt{c+id}\,\tanh^{-1}\Big(\frac{\sqrt{c+d\tan(}}{\sqrt{c+}}}{(ia-b)^3f}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 747 vs. 2(342) = 684.

time = 6.35, size = 747, normalized size = 2.18

$$-\frac{b^2(c+d\tan(e+fx))^{3/2}}{2(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))^2}-\frac{-\frac{bd\sqrt{c+d\tan(e+fx)}}{f(a+b\tan(e+fx))}-\frac{2\left(-\frac{-\frac{\frac{i\sqrt{c-id}\left(-ib(bc-ad)^2(3a^2bc-b^3c-a^3d+3ab^2d)-b(bc-ad)^2(a^3c-3ab^2c+3a^2bd-b^3d)\right)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(-c+id)f}-\frac{i\sqrt{c+id}\left(ib(bc-ad)^2(3a^2bc-b^3c-a^3d+3ab^2d)-b(bc-ad)^2(a^3c-3ab^2c+3a^2bd-b^3d)\right)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(-c-id)f}}{a^2+b^2}+\frac{2\sqrt{bc-ad}\left(\frac{1}{8}a^2b^2d(bc-ad)(8abc-7a^2d+b^2d)-ab^2(bc-ad)^2(2abc-a^2d+b^2d)-\frac{1}{8}b^3(bc-ad)(8a^2bc^2-8b^3c^2-8a^3cd+16ab^2cd-9a^2bd^2-b^3d^2)\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{\sqrt{b}(a^2+b^2)(-bc+ad)f}}{(a^2+b^2)(bc-ad)}-\frac{\left(\frac{1}{4}b^3(bc-ad)(4ac+bd)-a\left(\frac{3}{4}ab^2d(bc-ad)-b^2(bc-ad)^2\right)\right)\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))}\right)}{b}}{2(a^2+b^2)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Tan[e + f*x]]/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] -1/2*(b^2*(c + d*Tan[e + f*x])^(3/2))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan
[e + f*x])^2) - (-((b*d*Sqrt[c + d*Tan[e + f*x]])/(f*(a + b*Tan[e + f*x])))
 - (2*(-((((I*Sqrt[c - I*d]*((-I)*b*(b*c - a*d)^2*(3*a^2*b*c - b^3*c - a^3*
d + 3*a*b^2*d) - b*(b*c - a*d)^2*(a^3*c - 3*a*b^2*c + 3*a^2*b*d - b^3*d))*A
rcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((-c + I*d)*f) - (I*Sqrt[c
+ I*d]*(I*b*(b*c - a*d)^2*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d) - b*(b*c
- a*d)^2*(a^3*c - 3*a*b^2*c + 3*a^2*b*d - b^3*d))*ArcTanh[Sqrt[c + d*Tan[e
+ f*x]]/Sqrt[c + I*d]])/((-c - I*d)*f))/(a^2 + b^2) + (2*Sqrt[b*c - a*d]*((
a^2*b^2*d*(b*c - a*d)*(8*a*b*c - 7*a^2*d + b^2*d))/8 - a*b^2*(b*c - a*d)^2*
(2*a*b*c - a^2*d + b^2*d) - (b^3*(b*c - a*d)*(8*a^2*b*c^2 - 8*b^3*c^2 - 8*a
^3*c*d + 16*a*b^2*c*d - 9*a^2*b*d^2 - b^3*d^2))/8)*ArcTanh[(Sqrt[b]*Sqrt[c
+ d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(Sqrt[b]*(a^2 + b^2)*(-(b*c) + a*d)*f)
)/((a^2 + b^2)*(b*c - a*d))) - (((b^3*(b*c - a*d)*(4*a*c + b*d))/4 - a*((3*
a*b^2*d*(b*c - a*d))/4 - b^2*(b*c - a*d)^2))*Sqrt[c + d*Tan[e + f*x]])/((a^
2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x]))))/b)/(2*(a^2 + b^2)*(b*c - a*d
))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1276 vs. $2(304) = 608$.

time = 0.54, size = 1277, normalized size = 3.73 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*d^4*(-b/d^4/(a^2+b^2)^3*((1/8*b*d*(7*a^4*d-8*a^3*b*c+6*a^2*b^2*d-8*a*b^
3*c-b^4*d)/(a*d-b*c)*(c+d*tan(f*x+e))^(3/2)+1/8*(9*a^4*d-8*a^3*b*c+10*a^2*b
^2*d-8*a*b^3*c+b^4*d)*d*(c+d*tan(f*x+e))^(1/2))/((c+d*tan(f*x+e))*b+a*d-b*c
)^2+1/8*(15*a^4*d^2-40*a^3*b*c*d+24*a^2*b^2*c^2-18*a^2*b^2*d^2+24*a*b^3*c*d
-8*b^4*c^2-b^4*d^2)/(a*d-b*c)/((a*d-b*c)*b)^(1/2)*arctan(b*(c+d*tan(f*x+e))
^(1/2)/((a*d-b*c)*b)^(1/2)))+1/d^4/(a^2+b^2)^3*(1/4/d*(1/2*(-(c^2+d^2)^(1/2
)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3+3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*a*b^2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c+3*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*a^2*b*d-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c-(2*(c^2+d^2)^(1/2)+
2*c)^(1/2)*b^3*d)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(6*(c^2+d^2)^(1/2)*a^2*b*d-2*(c^2+d^2)^(1/
2)*b^3*d-1/2*(-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3+3*(c^2+d^2
)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a
^3*c+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2
```

```
)*a*b^2*c-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*d)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2
))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d
^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/4/d*(1/2*((c^2+d^2)
^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3-3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*a*b^2-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c-3*(2*(c^2+d^2)^(1/2
)+2*c)^(1/2)*a^2*b*d+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c+(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)*b^3*d)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(6*(c^2+d^2)^(1/2)*a^2*b*d-2*(c^2+d^2
)^(1/2)*b^3*d+1/2*((c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3-3*(c^2
+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2-(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*a^3*c-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d+3*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*a*b^2*c+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*d)*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c
^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + d \tan{\left(e + f x \right)}}}{\left(a + b \tan{\left(e + f x \right)}\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)/(a+b*tan(f*x+e))**3,x)
```

```
[Out] Integral(sqrt(c + d*tan(e + f*x))/(a + b*tan(e + f*x))**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 35.52, size = 2500, normalized size = 7.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(1/2)/(a + b*tan(e + f*x))^3,x)
```

```
[Out] (atan((((((c + d*tan(e + f*x))^(1/2)*(2*a^2*b^13*d^14 - b^15*d^14 + 49*a^4*
b^11*d^14 + 2460*a^6*b^9*d^14 - 3631*a^8*b^7*d^14 + 1922*a^10*b^5*d^14 - 22
5*a^12*b^3*d^14 + 17*b^15*c^2*d^12 + 16*b^15*c^4*d^10 + 96*b^15*c^6*d^8 + 8
0*a*b^14*c^3*d^11 - 960*a*b^14*c^5*d^9 - 40*a^3*b^12*c*d^13 - 9264*a^5*b^10
*c*d^13 + 21360*a^7*b^8*c*d^13 - 15544*a^9*b^6*c*d^13 + 3000*a^11*b^4*c*d^1
3 - 114*a^2*b^13*c^2*d^12 + 4848*a^2*b^13*c^4*d^10 - 640*a^2*b^13*c^6*d^8 -
 11504*a^3*b^12*c^3*d^11 + 7424*a^3*b^12*c^5*d^9 + 14319*a^4*b^11*c^2*d^12
- 27744*a^4*b^11*c^4*d^10 + 3136*a^4*b^11*c^6*d^8 + 49824*a^5*b^10*c^3*d^11
 - 21120*a^5*b^10*c^5*d^9 - 46588*a^6*b^9*c^2*d^12 + 55264*a^6*b^9*c^4*d^10
 - 3712*a^6*b^9*c^6*d^8 - 71520*a^7*b^8*c^3*d^11 + 17664*a^7*b^8*c^5*d^9 +
47871*a^8*b^7*c^2*d^12 - 32688*a^8*b^7*c^4*d^10 + 608*a^8*b^7*c^6*d^8 + 297
12*a^9*b^6*c^3*d^11 - 1984*a^9*b^6*c^5*d^9 - 13746*a^10*b^5*c^2*d^12 + 2352
*a^10*b^5*c^4*d^10 - 1200*a^11*b^4*c^3*d^11 + 225*a^12*b^3*c^2*d^12 - 24*a*
b^14*c*d^13))/(a^18*d^2*f^4 + b^18*c^2*f^4 + 8*a^2*b^16*c^2*f^4 + 28*a^4*b^
14*c^2*f^4 + 56*a^6*b^12*c^2*f^4 + 70*a^8*b^10*c^2*f^4 + 56*a^10*b^8*c^2*f^
4 + 28*a^12*b^6*c^2*f^4 + 8*a^14*b^4*c^2*f^4 + a^16*b^2*c^2*f^4 + a^2*b^16*
d^2*f^4 + 8*a^4*b^14*d^2*f^4 + 28*a^6*b^12*d^2*f^4 + 56*a^8*b^10*d^2*f^4 +
70*a^10*b^8*d^2*f^4 + 56*a^12*b^6*d^2*f^4 + 28*a^14*b^4*d^2*f^4 + 8*a^16*b^
2*d^2*f^4 - 2*a*b^17*c*d*f^4 - 2*a^17*b*c*d*f^4 - 16*a^3*b^15*c*d*f^4 - 56*
a^5*b^13*c*d*f^4 - 112*a^7*b^11*c*d*f^4 - 140*a^9*b^9*c*d*f^4 - 112*a^11*b^
7*c*d*f^4 - 56*a^13*b^5*c*d*f^4 - 16*a^15*b^3*c*d*f^4) + (((4*b^18*d^14*f^2
 - 276*a^2*b^16*d^14*f^2 - 6092*a^4*b^14*d^14*f^2 + 9724*a^6*b^12*d^14*f^2
+ 18444*a^8*b^10*d^14*f^2 - 10492*a^10*b^8*d^14*f^2 - 8580*a^12*b^6*d^14*f^
```

2 + 4884*a^14*b^4*d^14*f^2 + 64*a^16*b^2*d^14*f^2 + 4*b^18*c^2*d^12*f^2 - 1
92*b^18*c^4*d^10*f^2 - 192*b^18*c^6*d^8*f^2 - 11284*a^2*b^16*c^2*d^12*f^2 -
 5760*a^2*b^16*c^4*d^10*f^2 + 5248*a^2*b^16*c^6*d^8*f^2 - 15872*a^3*b^15*c^
3*d^11*f^2 - 29696*a^3*b^15*c^5*d^9*f^2 + 48820*a^4*b^14*c^2*d^12*f^2 + 492
16*a^4*b^14*c^4*d^10*f^2 - 5696*a^4*b^14*c^6*d^8*f^2 - 37120*a^5*b^13*c^3*d
^11*f^2 + 3328*a^5*b^13*c^5*d^9*f^2 + 38780*a^6*b^12*c^2*d^12*f^2 + 11392*a
^6*b^12*c^4*d^10*f^2 - 17664*a^6*b^12*c^6*d^8*f^2 + 28416*a^7*b^11*c^3*d^11
*f^2 + 73728*a^7*b^11*c^5*d^9*f^2 - 87796*a^8*b^10*c^2*d^12*f^2 - 102464*a^
8*b^10*c^4*d^10*f^2 + 3776*a^8*b^10*c^6*d^8*f^2 + 62464*a^9*b^9*c^3*d^11*f^
2 + 1792*a^9*b^9*c^5*d^9*f^2 - 35068*a^10*b^8*c^2*d^12*f^2 - 14208*a^10*b^8
*c^4*d^10*f^2 + 10368*a^10*b^8*c^6*d^8*f^2 - 8192*a^11*b^7*c^3*d^11*f^2 - 3
5840*a^11*b^7*c^5*d^9*f^2 + 36604*a^12*b^6*c^2*d^12*f^2 + 45248*a^12*b^6*c^
4*d^10*f^2 + 64*a^12*b^6*c^6*d^8*f^2 - 24832*a^13*b^5*c^3*d^11*f^2 - 256*a^
13*b^5*c^5*d^9*f^2 + 5268*a^14*b^4*c^2*d^12*f^2 + 384*a^14*b^4*c^4*d^10*f^2
 - 256*a^15*b^3*c^3*d^11*f^2 + 64*a^16*b^2*c^2*d^12*f^2 + 256*a*b^17*c*d^13
*f^2 + 3584*a*b^17*c^3*d^11*f^2 + 3328*a*b^17*c^5*d^9*f^2 + 13824*a^3*b^15*
c*d^13*f^2 - 40448*a^5*b^13*c*d^13*f^2 - 45312*a^7*b^11*c*d^13*f^2 + 60672*
a^9*b^9*c*d^13*f^2 + 27648*a^11*b^7*c*d^13*f^2 - 24576*a^13*b^5*c*d^13*f^2
- 256*a^15*b^3*c*d^13*f^2)/(2*(a^18*d^2*f^5 + b^18*c^2*f^5 + 8*a^2*b^16*c^2
*f^5 + 28*a^4*b^14*c^2*f^5 + 56*a^6*b^12*c^2*f^5 + 70*a^8*b^10*c^2*f^5 + 56
*a^10*b^8*c^2*f^5 + 28*a^12*b^6*c^2*f^5 + 8*a^14*b^4*c^2*f^5 + a^16*b^2*c^2
*f^5 + a^2*b^16*d^2*f^5 + 8*a^4*b^14*d^2*f^5 + 28*a^6*b^12*d^2*f^5 + 56*a^8
*b^10*d^2*f^5 + 70*a^10*b^8*d^2*f^5 + 56*a^12*b^6*d^2*f^5 + 28*a^14*b^4*d^2
*f^5 + 8*a^16*b^2*d^2*f^5 - 2*a*b^17*c*d*f^5 - 2*a^17*b*c*d*f^5 - 16*a^3*b^
15*c*d*f^5 - 56*a^5*b^13*c*d*f^5 - 112*a^7*b^11*c*d*f^5 - 140*a^9*b^9*c*d*f
^5 - 112*a^11*b^7*c*d*f^5 - 56*a^13*b^5*c*d*f^5 - 16*a^15*b^3*c*d*f^5)) - (
(((c + d*tan(e + f*x))^(1/2)*(8*a*b^20*d^13*f^2 + 4*b^21*c*d^12*f^2 - 1152*
a^3*b^18*d^13*f^2 + 2528*a^5*b^16*d^13*f^2 + 15296*a^7*b^14*d^13*f^2 + 1412
8*a^9*b^12*d^13*f^2 - 5056*a^11*b^10*d^13*f^2 - 9248*a^13*b^8*d^13*f^2 + 64
*a^15*b^6*d^13*f^2 + 1800*a^17*b^4*d^13*f^2 + 64*a^19*b^2*d^13*f^2 + 256*b^
21*c^3*d^10*f^2 + 576*b^21*c^5*d^8*f^2 + 2624*a^2*b^19*c^3*d^10*f^2 - 3584*
a^2*b^19*c^5*d^8*f^2 + 4800*a^3*b^18*c^2*d^11*f^2 + 3584*a^3*b^18*c^4*d^9*f
^2 - 1920*a^4*b^17*c^3*d^10*f^2 - 13056*a^4*b^17*c^5*d^8*f^2 + 18688*a^5*b^
16*c^2*d^11*f^2 + 35072*a^5*b^16*c^4*d^9*f^2 - 19328*a^6*b^15*c^3*d^10*f^2
- 7680*a^6*b^15*c^5*d^8*f^2 - 6144*a^7*b^14*c^2*d^11*f^2 + 38400*a^7*b^14*c
^4*d^9*f^2 - 26496*a^8*b^13*c^3*d^10*f^2 + 8064*a^8*b^13*c^5*d^8*f^2 - 4147
2*a^9*b^12*c^2*d^11*f^2 + 6528*a^9*b^12*c^4*d^9*f^2 - 28416*a^10*b^11*c^3*d
^10*f^2 + 5632*a^10*b^11*c^5*d^8*f^2 - 6272*a^11*b^10*c^2*d^11*f^2 + 6656*a
^11*b^10*c^4*d^9*f^2 - 42624*a^12*b^9*c^3*d^10*f^2 - 3840*a^12*b^9*c^5*d^8*
f^2 + 42752*a^13*b^8*c^2*d^11*f^2 + 19712*a^13*b^8*c^4*d^9*f^2 - 36992*a^14
*b^7*c^3*d^10*f^2 - 2560*a^14*b^7*c^5*d^8*f^2 + 32256*a^15*b^6*c^2*d^11*f^2
 + 8704*a^15*b^6*c^4*d^9*f^2 - 11136*a^16*b^5*c^3*d^10*f^2 + 64*a^16*b^5*c^
5*d^8*f^2 + 6528*a^17*b^4*c^2*d^11*f^2 - 192*a^17*b^4*c^4*d^9*f^2 + 192*a^1
8*b^3*c^3*d^10*f^2 - 64*a^19*b^2*c^2*d^11*f^2 -...

3.1235 $\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=256

$$\frac{(ia+b)^3(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}-\frac{(ia-b)^3(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}-$$

```
[Out] (I*a+b)^3*(c-I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/f-(I*
a-b)^3*(c+I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/f+2*(a^3
*d+3*a^2*b*c-3*a*b^2*d-b^3*c)*(c+d*tan(f*x+e))^(1/2)/f+2/3*b*(3*a^2-b^2)*(c
+d*tan(f*x+e))^(3/2)/f-4/35*b^2*(-8*a*d+b*c)*(c+d*tan(f*x+e))^(5/2)/d^2/f+2
/7*b^2*(a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(5/2)/d/f
```

Rubi [A]
time = 0.51, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3647, 3711, 3609, 3620, 3618, 65, 214}

$$\frac{2b(3a^2-b^2)(c+d\tan(e+fx))^{3/2}}{3f}+\frac{2(a^3d+3a^2bc-3ab^2d-b^3c)\sqrt{c+d\tan(e+fx)}}{f}-\frac{4b^2(bc-8ad)(c+d\tan(e+fx))^{5/2}}{35d^2f}+\frac{2b^2(a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}}{7df}-\frac{(-b+ia)^3(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}+\frac{(b+ia)^3(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((I*a + b)^3*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]
])/f - ((I*a - b)^3*(c + I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c
 + I*d]])/f + (2*(3*a^2*b*c - b^3*c + a^3*d - 3*a*b^2*d)*Sqrt[c + d*Tan[e +
 f*x]])/f + (2*b*(3*a^2 - b^2)*(c + d*Tan[e + f*x])^(3/2))/(3*f) - (4*b^2*(
b*c - 8*a*d)*(c + d*Tan[e + f*x])^(5/2))/(35*d^2*f) + (2*b^2*(a + b*Tan[e +
 f*x])*(c + d*Tan[e + f*x])^(5/2))/(7*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}\,dx = \frac{2b^2(a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}}{7df} + \frac{2\int(c+\cdots}{\cdots}$$

$$= -\frac{4b^2(bc-8ad)(c+d\tan(e+fx))^{5/2}}{35d^2f} + \frac{2b^2(a+b\tan(e\cdots}{\cdots}$$

$$= \frac{2b(3a^2-b^2)(c+d\tan(e+fx))^{3/2}}{3f} - \frac{4b^2(bc-8ad)(c+\cdots}{35d\cdots}$$

$$= \frac{2(3a^2bc-b^3c+a^3d-3ab^2d)\sqrt{c+d\tan(e+fx)}}{f} + \frac{2b\cdots}{\cdots}$$

$$= \frac{2(3a^2bc-b^3c+a^3d-3ab^2d)\sqrt{c+d\tan(e+fx)}}{f} + \frac{2b\cdots}{\cdots}$$

$$= \frac{2(3a^2bc-b^3c+a^3d-3ab^2d)\sqrt{c+d\tan(e+fx)}}{f} + \frac{2b\cdots}{\cdots}$$

$$= \frac{2(3a^2bc-b^3c+a^3d-3ab^2d)\sqrt{c+d\tan(e+fx)}}{f} + \frac{2b\cdots}{\cdots}$$

$$= \frac{(ia+b)^3(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f} - \frac{(\cdots}{\cdots}$$

Mathematica [A]
time = 3.46, size = 247, normalized size = 0.96

$$\frac{\frac{4b^2(-bc+8ad)(c+d\tan(e+fx))^{5/2}}{d}+10b^2(a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}-\frac{35}{3}(ia+b)^3d\left(-3(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)+\sqrt{c+d\tan(e+fx)}\,(4c-3id+d\tan(e+fx))\right)+\frac{35}{3}(ia-b)^3d\left(-3(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)+\sqrt{c+d\tan(e+fx)}\,(4c+3id+d\tan(e+fx))\right)}{35df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((4*b^2*(-(b*c) + 8*a*d)*(c + d*Tan[e + f*x])^(5/2))/d + 10*b^2*(a + b*Tan[
e + f*x])*(c + d*Tan[e + f*x])^(5/2) - (35*(I*a + b)^3*d*(-3*(c - I*d)^(3/2
)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]] + Sqrt[c + d*Tan[e + f*x]
]*(4*c - (3*I)*d + d*Tan[e + f*x])))/3 + (35*(I*a - b)^3*d*(-3*(c + I*d)^(3
/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]] + Sqrt[c + d*Tan[e + f*
x]]*(4*c + (3*I)*d + d*Tan[e + f*x])))/3)/(35*d*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1747 vs. $2(224) = 448$.
time = 0.53, size = 1748, normalized size = 6.83

method	result	size
derivativedivides	Expression too large to display	1748
default	Expression too large to display	1748

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/d^2*(1/7*b^3*(c+d*tan(f*x+e))^(7/2)+3/5*a*b^2*d*(c+d*tan(f*x+e))^(5/2)-
1/5*b^3*c*(c+d*tan(f*x+e))^(5/2)+a^2*b*d^2*(c+d*tan(f*x+e))^(3/2)-1/3*b^3*d
^2*(c+d*tan(f*x+e))^(3/2)+a^3*d^3*(c+d*tan(f*x+e))^(1/2)+3*a^2*b*c*d^2*(c+d
*tan(f*x+e))^(1/2)-3*a*b^2*d^3*(c+d*tan(f*x+e))^(1/2)-b^3*c*d^2*(c+d*tan(f*
x+e))^(1/2)-d^2*(1/4/d*(1/2*((c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*
a^3*c-3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d-3*(c^2+d^2)^(
1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*b^3*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c^2+(2*(c^2+d^2)^(1/2
)+2*c)^(1/2)*a^3*d^2+6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c*d+3*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*a*b^2*c^2-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*d^2-2*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c*d)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1
/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(2*(c^2+d^2)^(1/2)*a^3
*d^2+6*(c^2+d^2)^(1/2)*a^2*b*c*d-6*(c^2+d^2)^(1/2)*a*b^2*d^2-2*(c^2+d^2)^(1
/2)*b^3*c*d-1/2*((c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c-3*(c^2
+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d-3*(c^2+d^2)^(1/2)*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2
)*b^3*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c^2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*a^3*d^2+6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c*d+3*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*a*b^2*c^2-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*d^2-2*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*b^3*c*d)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)
-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))
/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/4/d*(1/2*(-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*a^3*c+3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*
b*d+3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c-(c^2+d^2)^(1/2)
*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c^2-
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*d^2-6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b
*c*d-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c^2+3*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)*a*b^2*d^2+2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c*d)*ln(d*tan(f*x+e)+c-(c
+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(2*(c
^2+d^2)^(1/2)*a^3*d^2+6*(c^2+d^2)^(1/2)*a^2*b*c*d-6*(c^2+d^2)^(1/2)*a*b^2*d
^2-2*(c^2+d^2)^(1/2)*b^3*c*d+1/2*(-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*a^3*c+3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d+3*(c^2+
d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c-(c^2+d^2)^(1/2)*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*b^3*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c^2-(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)*a^3*d^2-6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c*d-3*(2*(
c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c^2+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*d
```

```
^2+2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c*d)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/
(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d^2)
^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))))
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^(3/2),x, algorithm="maxima")
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(e+fx))^3 (c+d\tan(e+fx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3*(c+d*tan(f*x+e))**(3/2),x)
[Out] Integral((a + b*tan(e + f*x))**3*(c + d*tan(e + f*x))**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 79.38, size = 2500, normalized size = 9.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^3*(c + d*tan(e + f*x))^(3/2),x)
```

```
[Out] (c + d*tan(e + f*x))^(1/2)*(((6*b^3*c - 6*a*b^2*d)/(d^2*f) - (4*b^3*c)/(d^2
*f))*(c^2 + d^2) - 2*c*(2*c*((6*b^3*c - 6*a*b^2*d)/(d^2*f) - (4*b^3*c)/(d^2
*f)) - (6*b*(a*d - b*c)^2)/(d^2*f) + (2*b^3*(c^2 + d^2))/(d^2*f)) + (2*(a*d
 - b*c)^3)/(d^2*f)) - atan(((((8*(4*a^3*d^5*f^2 - 12*a*b^2*d^5*f^2 - 4*b^3*
c*d^4*f^2 + 4*a^3*c^2*d^3*f^2 - 4*b^3*c^3*d^2*f^2 + 12*a^2*b*c*d^4*f^2 - 12
*a*b^2*c^2*d^3*f^2 + 12*a^2*b*c^3*d^2*f^2))/f^3 - 64*c*d^2*(c + d*tan(e + f
*x))^(1/2)*(-(((8*a^6*c^3*f^2 - 8*b^6*c^3*f^2 + 48*a*b^5*d^3*f^2 + 48*a^5*b
*d^3*f^2 - 24*a^6*c*d^2*f^2 + 24*b^6*c*d^2*f^2 + 120*a^2*b^4*c^3*f^2 - 120*
a^4*b^2*c^3*f^2 - 160*a^3*b^3*d^3*f^2 - 144*a*b^5*c^2*d*f^2 - 144*a^5*b*c^2
*d*f^2 - 360*a^2*b^4*c*d^2*f^2 + 480*a^3*b^3*c^2*d*f^2 + 360*a^4*b^2*c*d^2*
f^2)^2/64 - f^4*(a^12*c^6 + a^12*d^6 + b^12*c^6 + b^12*d^6 + 6*a^2*b^10*c^6
 + 15*a^4*b^8*c^6 + 20*a^6*b^6*c^6 + 15*a^8*b^4*c^6 + 6*a^10*b^2*c^6 + 6*a^
2*b^10*d^6 + 15*a^4*b^8*d^6 + 20*a^6*b^6*d^6 + 15*a^8*b^4*d^6 + 6*a^10*b^2*
d^6 + 3*a^12*c^2*d^4 + 3*a^12*c^4*d^2 + 3*b^12*c^2*d^4 + 3*b^12*c^4*d^2 + 1
8*a^2*b^10*c^2*d^4 + 18*a^2*b^10*c^4*d^2 + 45*a^4*b^8*c^2*d^4 + 45*a^4*b^8*
c^4*d^2 + 60*a^6*b^6*c^2*d^4 + 60*a^6*b^6*c^4*d^2 + 45*a^8*b^4*c^2*d^4 + 45
*a^8*b^4*c^4*d^2 + 18*a^10*b^2*c^2*d^4 + 18*a^10*b^2*c^4*d^2))^(1/2) + a^6*
c^3*f^2 - b^6*c^3*f^2 + 6*a*b^5*d^3*f^2 + 6*a^5*b*d^3*f^2 - 3*a^6*c*d^2*f^2
 + 3*b^6*c*d^2*f^2 + 15*a^2*b^4*c^3*f^2 - 15*a^4*b^2*c^3*f^2 - 20*a^3*b^3*d
^3*f^2 - 18*a*b^5*c^2*d*f^2 - 18*a^5*b*c^2*d*f^2 - 45*a^2*b^4*c*d^2*f^2 + 6
0*a^3*b^3*c^2*d*f^2 + 45*a^4*b^2*c*d^2*f^2)/(4*f^4))^(1/2))*(-(((8*a^6*c^3*
f^2 - 8*b^6*c^3*f^2 + 48*a*b^5*d^3*f^2 + 48*a^5*b*d^3*f^2 - 24*a^6*c*d^2*f^
2 + 24*b^6*c*d^2*f^2 + 120*a^2*b^4*c^3*f^2 - 120*a^4*b^2*c^3*f^2 - 160*a^3*
b^3*d^3*f^2 - 144*a*b^5*c^2*d*f^2 - 144*a^5*b*c^2*d*f^2 - 360*a^2*b^4*c*d^2
*f^2 + 480*a^3*b^3*c^2*d*f^2 + 360*a^4*b^2*c*d^2*f^2)^2/64 - f^4*(a^12*c^6
+ a^12*d^6 + b^12*c^6 + b^12*d^6 + 6*a^2*b^10*c^6 + 15*a^4*b^8*c^6 + 20*a^6
*b^6*c^6 + 15*a^8*b^4*c^6 + 6*a^10*b^2*c^6 + 6*a^2*b^10*d^6 + 15*a^4*b^8*d^
6 + 20*a^6*b^6*d^6 + 15*a^8*b^4*d^6 + 6*a^10*b^2*d^6 + 3*a^12*c^2*d^4 + 3*a
^12*c^4*d^2 + 3*b^12*c^2*d^4 + 3*b^12*c^4*d^2 + 18*a^2*b^10*c^2*d^4 + 18*a^
2*b^10*c^4*d^2 + 45*a^4*b^8*c^2*d^4 + 45*a^4*b^8*c^4*d^2 + 60*a^6*b^6*c^2*d
^4 + 60*a^6*b^6*c^4*d^2 + 45*a^8*b^4*c^2*d^4 + 45*a^8*b^4*c^4*d^2 + 18*a^10
*b^2*c^2*d^4 + 18*a^10*b^2*c^4*d^2))^(1/2) + a^6*c^3*f^2 - b^6*c^3*f^2 + 6*
a*b^5*d^3*f^2 + 6*a^5*b*d^3*f^2 - 3*a^6*c*d^2*f^2 + 3*b^6*c*d^2*f^2 + 15*a^
2*b^4*c^3*f^2 - 15*a^4*b^2*c^3*f^2 - 20*a^3*b^3*d^3*f^2 - 18*a*b^5*c^2*d*f^
2 - 18*a^5*b*c^2*d*f^2 - 45*a^2*b^4*c*d^2*f^2 + 60*a^3*b^3*c^2*d*f^2 + 45*a
^4*b^2*c*d^2*f^2)/(4*f^4))^(1/2) - (16*(c + d*tan(e + f*x))^(1/2)*(a^6*d^6
- b^6*d^6 + 15*a^2*b^4*d^6 - 15*a^4*b^2*d^6 - 6*a^6*c^2*d^4 + a^6*c^4*d^2 +
```

6*b^6*c^2*d^4 - b^6*c^4*d^2 - 24*a*b^5*c^3*d^3 - 80*a^3*b^3*c*d^5 - 24*a^5*b*c^3*d^3 - 90*a^2*b^4*c^2*d^4 + 15*a^2*b^4*c^4*d^2 + 80*a^3*b^3*c^3*d^3 + 90*a^4*b^2*c^2*d^4 - 15*a^4*b^2*c^4*d^2 + 24*a*b^5*c*d^5 + 24*a^5*b*c*d^5))/f^2)*(-(((8*a^6*c^3*f^2 - 8*b^6*c^3*f^2 + 48*a*b^5*d^3*f^2 + 48*a^5*b*d^3*f^2 - 24*a^6*c*d^2*f^2 + 24*b^6*c*d^2*f^2 + 120*a^2*b^4*c^3*f^2 - 120*a^4*b^2*c^3*f^2 - 160*a^3*b^3*d^3*f^2 - 144*a*b^5*c^2*d*f^2 - 144*a^5*b*c^2*d*f^2 - 360*a^2*b^4*c*d^2*f^2 + 480*a^3*b^3*c^2*d*f^2 + 360*a^4*b^2*c*d^2*f^2)^2/64 - f^4*(a^12*c^6 + a^12*d^6 + b^12*c^6 + b^12*d^6 + 6*a^2*b^10*c^6 + 15*a^4*b^8*c^6 + 20*a^6*b^6*c^6 + 15*a^8*b^4*c^6 + 6*a^10*b^2*c^6 + 6*a^2*b^10*d^6 + 15*a^4*b^8*d^6 + 20*a^6*b^6*d^6 + 15*a^8*b^4*d^6 + 6*a^10*b^2*d^6 + 3*a^12*c^2*d^4 + 3*a^12*c^4*d^2 + 3*b^12*c^2*d^4 + 3*b^12*c^4*d^2 + 18*a^2*b^10*c^2*d^4 + 18*a^2*b^10*c^4*d^2 + 45*a^4*b^8*c^2*d^4 + 45*a^4*b^8*c^4*d^2 + 60*a^6*b^6*c^2*d^4 + 60*a^6*b^6*c^4*d^2 + 45*a^8*b^4*c^2*d^4 + 45*a^8*b^4*c^4*d^2 + 18*a^10*b^2*c^2*d^4 + 18*a^10*b^2*c^4*d^2))^(1/2) + a^6*c^3*f^2 - b^6*c^3*f^2 + 6*a*b^5*d^3*f^2 + 6*a^5*b*d^3*f^2 - 3*a^6*c*d^2*f^2 + 3*b^6*c*d^2*f^2 + 15*a^2*b^4*c^3*f^2 - 15*a^4*b^2*c^3*f^2 - 20*a^3*b^3*d^3*f^2 - 18*a*b^5*c^2*d*f^2 - 18*a^5*b*c^2*d*f^2 - 45*a^2*b^4*c*d^2*f^2 + 60*a^3*b^3*c^2*d*f^2 + 45*a^4*b^2*c*d^2*f^2)/(4*f^4))^(1/2)*1i - (((8*(4*a^3*d^5*f^2 - 12*a*b^2*d^5*f^2 - 4*b^3*c*d^4*f^2 + 4*a^3*c^2*d^3*f^2 - 4*b^3*c^3*d^2*f^2 + 12*a^2*b*c*d^4*f^2 - 12*a*b^2*c^2*d^3*f^2 + 12*a^2*b*c^3*d^2*f^2))/f^3 + 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*(-(((8*a^6*c^3*f^2 - 8*b^6*c^3*f^2 + 48*a*b^5*d^3*f^2 + 48*a^5*b*d^3*f^2 - 24*a^6*c*d^2*f^2 + 24*b^6*c*d^2*f^2 + 120*a^2*b^4*c^3*f^2 - 120*a^4*b^2*c^3*f^2 - 160*a^3*b^3*d^3*f^2 - 144*a*b^5*c^2*d*f^2 - 144*a^5*b*c^2*d*f^2 - 360*a^2*b^4*c*d^2*f^2 + 480*a^3*b^3*c^2*d*f^2 + 360*a^4*b^2*c*d^2*f^2)^2/64 - f^4*(a^12*c^6 + a^12*d^6 + b^12*c^6 + b^12*d^6 + 6*a^2*b^10*c^6 + 15*a^4*b^8*c^6 + 20*a^6*b^6*c^6 + 15*a^8*b^4*c^6 + 6*a^10*b^2*c^6 + 6*a^2*b^10*d^6 + 15*a^4*b^8*d^6 + 20*a^6*b^6*d^6 + 15*a^8*b^4*d^6 + 6*a^10*b^2*d^6 + 3*a^12*c^2*d^4 + 3*a^12*c^4*d^2 + 3*b^12*c^2*d^4 + 3*b^12*c^4*d^2 + 18*a^2*b^10*c^2*d^4 + 18*a^2*b^10*c^4*d^2 + 45*a^4*b^8*c^2*d^4 + 45*a^4*b^8*c^4*d^2 + 60*a^...

3.1236 $\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=195

$$-\frac{i(a-ib)^2(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{i(a+ib)^2(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}$$

```
[Out] -I*(a-I*b)^2*(c-I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/f+
I*(a+I*b)^2*(c+I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/f+2
*(a^2*d+2*a*b*c-b^2*d)*(c+d*tan(f*x+e))^(1/2)/f+4/3*a*b*(c+d*tan(f*x+e))^(3
/2)/f+2/5*b^2*(c+d*tan(f*x+e))^(5/2)/d/f
```

Rubi [A]

time = 0.31, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3624, 3609, 3620, 3618, 65, 214}

$$\frac{2(a^2d+2abc-b^2d)\sqrt{c+d\tan(e+fx)}}{f}+\frac{4ab(c+d\tan(e+fx))^{3/2}}{3f}-\frac{i(a-ib)^2(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{i(a+ib)^2(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}+\frac{2b^2(c+d\tan(e+fx))^{5/2}}{5df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-I)*(a - I*b)^2*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c -
 I*d]])/f + (I*(a + I*b)^2*(c + I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]
/Sqrt[c + I*d]])/f + (2*(2*a*b*c + a^2*d - b^2*d)*Sqrt[c + d*Tan[e + f*x]])
/f + (4*a*b*(c + d*Tan[e + f*x])^(3/2))/(3*f) + (2*b^2*(c + d*Tan[e + f*x])
^(5/2))/(5*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
```

```
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}\,dx &= \frac{2b^2(c+d\tan(e+fx))^{5/2}}{5df} + \int (a^2-b^2+2ab\tan(e+fx) \\ &= \frac{4ab(c+d\tan(e+fx))^{3/2}}{3f} + \frac{2b^2(c+d\tan(e+fx))^{5/2}}{5df} + \\ &= \frac{2(2abc+a^2d-b^2d)\sqrt{c+d\tan(e+fx)}}{f} + \frac{4ab(c+d\tan}{3} \\ &= \frac{2(2abc+a^2d-b^2d)\sqrt{c+d\tan(e+fx)}}{f} + \frac{4ab(c+d\tan}{3} \\ &= \frac{2(2abc+a^2d-b^2d)\sqrt{c+d\tan(e+fx)}}{f} + \frac{4ab(c+d\tan}{3} \\ &= \frac{2(2abc+a^2d-b^2d)\sqrt{c+d\tan(e+fx)}}{f} + \frac{4ab(c+d\tan}{3} \\ &= -\frac{i(a-ib)^2(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f} + \end{aligned}$$

Mathematica [A]
time = 1.71, size = 202, normalized size = 1.04

$$\frac{\frac{6b^2(c+d\tan(e+fx))^{5/2}}{d}+5i(a-ib)^2\left(-3(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)+\sqrt{c+d\tan(e+fx)}\,(4c-3id+d\tan(e+fx))\right)-5i(a+ib)^2\left(-3(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)+\sqrt{c+d\tan(e+fx)}\,(4c+3id+d\tan(e+fx))\right)}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((6*b^2*(c + d*Tan[e + f*x])^(5/2))/d + (5*I)*(a - I*b)^2*(-3*(c - I*d)^(3/
2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]] + Sqrt[c + d*Tan[e + f*x
]]*(4*c - (3*I)*d + d*Tan[e + f*x])) - (5*I)*(a + I*b)^2*(-3*(c + I*d)^(3/2
)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]] + Sqrt[c + d*Tan[e + f*x]
]*(4*c + (3*I)*d + d*Tan[e + f*x])))/(15*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1365 vs. 2(165) = 330.
time = 0.45, size = 1366, normalized size = 7.01 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/d*(1/5*b^2*(c+d*tan(f*x+e))^(5/2)+2/3*a*b*d*(c+d*tan(f*x+e))^(3/2)+a^2*
d^2*(c+d*tan(f*x+e))^(1/2)+2*a*b*c*d*(c+d*tan(f*x+e))^(1/2)-b^2*d^2*(c+d*ta
```

```
n(f*x+e))^(1/2)-d*(1/4/d*(1/2*((c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2
)*a^2*c-2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d-(c^2+d^2)^(1/
2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^
2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d^2+4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b
*c*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^2-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^
2*d^2)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)+(c^2+d^2)^(1/2))+2*(2*(c^2+d^2)^(1/2)*a^2*d^2+4*(c^2+d^2)^(1/2)*a*b*c*d-
2*(c^2+d^2)^(1/2)*b^2*d^2-1/2*((c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2
)*a^2*c-2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d-(c^2+d^2)^(1/
2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^
2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d^2+4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b
*c*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^2-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^
2*d^2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(
(2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)
-2*c)^(1/2)))+1/4/d*(1/2*(-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^
2*c+2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d+(c^2+d^2)^(1/2)*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^2-(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d^2-4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c*d
-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d^
2)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(
c^2+d^2)^(1/2))+2*(2*(c^2+d^2)^(1/2)*a^2*d^2+4*(c^2+d^2)^(1/2)*a*b*c*d-2*(c
^2+d^2)^(1/2)*b^2*d^2+1/2*(-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a
^2*c+2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d+(c^2+d^2)^(1/2)*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^2-(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d^2-4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c*
d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d
^2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*
(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*
c)^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)^2*(d*tan(f*x + e) + c)^(3/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(3/2),x, algorithm="fricas")

[Out] Timed out

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(e+fx))^2 (c+d\tan(e+fx))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(f*x+e))**2*(c+d*tan(f*x+e))**(3/2),x)

[Out] Integral((a + b*tan(e + f*x))**2*(c + d*tan(e + f*x))**(3/2), x)

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done

Mupad [**B**]
time = 21.86, size = 2500, normalized size = 12.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tan(e + f*x))^2*(c + d*tan(e + f*x))^(3/2),x)

[Out] (2*b^2*(c + d*tan(e + f*x))^(5/2))/(5*d*f) - atan(((((16*(2*a^2*d^5*f^2 - 2*b^2*d^5*f^2 + 2*a^2*c^2*d^3*f^2 - 2*b^2*c^2*d^3*f^2 + 4*a*b*c*d^4*f^2 + 4*a*b*c^3*d^2*f^2))/f^3 - 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*(-(((8*a^4*c^3*f^2 + 8*b^4*c^3*f^2 - 32*a*b^3*d^3*f^2 + 32*a^3*b*d^3*f^2 - 24*a^4*c*d^2*f^2 - 24*b^4*c*d^2*f^2 - 48*a^2*b^2*c^3*f^2 + 96*a*b^3*c^2*d*f^2 - 96*a^3*b*c^2*d*f^2 + 144*a^2*b^2*c*d^2*f^2)^2/64 - f^4*(a^8*c^6 + a^8*d^6 + b^8*c^6 + b^8*d^6 + 4*a^2*b^6*c^6 + 6*a^4*b^4*c^6 + 4*a^6*b^2*c^6 + 4*a^2*b^6*d^6 + 6*a^4*b^4*d^6 + 4*a^6*b^2*d^6 + 3*a^8*c^2*d^4 + 3*a^8*c^4*d^2 + 3*b^8*c^2*d^4 + 3*b^8*c^4*d^2 + 12*a^2*b^6*c^2*d^4 + 12*a^2*b^6*c^4*d^2 + 18*a^4*b^4*c^2*d^4 + 18*a^4*b^4*c^4*d^2 + 12*a^6*b^2*c^2*d^4 + 12*a^6*b^2*c^4*d^2))^(1/2) + a^4*c^3*f^2 + b^4*c^3*f^2 - 4*a*b^3*d^3*f^2 + 4*a^3*b*d^3*f^2 - 3*a^4* c*d^2*f^2 - 3*b^4*c*d^2*f^2 - 6*a^2*b^2*c^3*f^2 + 12*a*b^3*c^2*d*f^2 - 12*a^3*b*c^2*d*f^2 + 18*a^2*b^2*c*d^2*f^2)/(4*f^4))^(1/2))*(-(((8*a^4*c^3*f^2 + 8*b^4*c^3*f^2 - 32*a*b^3*d^3*f^2 + 32*a^3*b*d^3*f^2 - 24*a^4*c*d^2*f^2 - 24*b^4*c*d^2*f^2 - 48*a^2*b^2*c^3*f^2 + 96*a*b^3*c^2*d*f^2 - 96*a^3*b*c^2*d*f^2 + 144*a^2*b^2*c*d^2*f^2)^2/64 - f^4*(a^8*c^6 + a^8*d^6 + b^8*c^6 + b^8*d^6 + 4*a^2*b^6*c^6 + 6*a^4*b^4*c^6 + 4*a^6*b^2*c^6 + 4*a^2*b^6*d^6 + 6*a^4*b^4*d^6 + 4*a^6*b^2*d^6 + 3*a^8*c^2*d^4 + 3*a^8*c^4*d^2 + 3*b^8*c^2*d^4 + 3*b^8*c^4*d^2 + 12*a^2*b^6*c^2*d^4 + 12*a^2*b^6*c^4*d^2 + 18*a^4*b^4*c^2*d^4 + 18*a^4*b^4*c^4*d^2 + 12*a^6*b^2*c^2*d^4 + 12*a^6*b^2*c^4*d^2))^(1/2) + a^4*c^3*f^2 + b^4*c^3*f^2 - 4*a*b^3*d^3*f^2 + 4*a^3*b*d^3*f^2 - 3*a^4*c*d^2*f^2 - 3*b^4*c*d^2*f^2 - 6*a^2*b^2*c^3*f^2 + 12*a*b^3*c^2*d*f^2 - 12*a^3*b*c^2*d*f^2 + 18*a^2*b^2*c*d^2*f^2)/(4*f^4))^(1/2) - (16*(c + d*tan(e + f*x))^(1/2)*(a^4*d^6 + b^4*d^6 - 6*a^2*b^2*d^6 - 6*a^4*c^2*d^4 + a^4*c^4*d^2 - 6*b^4*c^2*d^4 + b^4*c^4*d^2 + 16*a*b^3*c^3*d^3 - 16*a^3*b*c^3*d^3 + 36*a^2*b^2*c^2*d^4 - 6*a^2*b^2*c^4*d^2 - 16*a*b^3*c*d^5 + 16*a^3*b*c*d^5))/f^2)*(-(((8*a^4*c^3*f^2 + 8*b^4*c^3*f^2 - 32*a*b^3*d^3*f^2 + 32*a^3*b*d^3*f^2 - 24*a^4*c*d^2*f^2 - 24*b^4*c*d^2*f^2 - 48*a^2*b^2*c^3*f^2 + 96*a*b^3*c^2*d*f^2 - 96*a^3*b*c^2*d*f^2 + 144*a^2*b^2*c*d^2*f^2)^2/64 - f^4*(a^8*c^6 + a^8*d^6 + b^8*c^6 + b^8*d^6 + 4*a^2*b^6*c^6 + 6*a^4*b^4*c^6 + 4*a^6*b^2*c^6 + 4*a^2*b^6*d^6 + 6*a^4*b^4*d^6 + 4*a^6*b^2*d^6 + 3*a^8*c^2*d^4 + 3*a^8*c^4*d^2 + 3*b^8*c^2*d^4 + 3*b^8*c^4*d^2 + 12*a^2*b^6*c^2*d^4 + 12*a^2*b^6*c^4*d^2 + 18*a^4*b^4*c^2*d^4 + 18*a^4*b^4*c^4*d^2 + 12*a^6*b^2*c^2*d^4 + 12*a^6*b^2*c^4*d^2))^(1/2) + a^4*c^3*f^2 + b^4*c^3*f^2 - 4*a*b^3*d^3*f^2 + 4*a^3*b*d^3*f^2 - 3*a^4*c*d^2*f^2 - 3*b^4*c*d^2*f^2 - 6*a^2*b^2*c^3*f^2 + 12*a*b^3*c^2*d*f^2 - 12*a^3*b*c^2*d*f^2 + 18*a^2*b^2*c*d^2*f^2)/(4*f^4))^(1/2)*1i - (((16*(2*a^2*d^5*f^2 - 2*b^2*d^5*f^2 + 2*a^2*c^2*d^3*f^2 - 2*b^2*c^2*d^3*f^2 + 4*a*b*c*d^4*f^2 + 4*a*b*c^3*d^2*f^2))/f^3 + 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*(-(((8*a^4*c^3*f^2 + 8*b^4*c^3*f^2 - 32*a*b^3*d^3*f^2 + 32*a^3*b*d^3*f^2 - 24*a^4*c*d^2*f^2 - 24*b^4*c*d^2*f^2 - 48*a^2*b^2*c^3*f^2 + 96*a*b^3*c^2*d*f^2 - 96*a^3*b*c^2*d*f^2 + 144*a^2*b^2*c*d^2*f^2)^2/64 - f^4*(a^8*c^6 + a^8*d^6 + b^8*c^6 + b^8*d^6 + 4*a^2*b^6*c^6 + 6*a^4*b^4*c^6 + 4*a^6*b^2*c^6 + 4*a^2*b^6*d^6 + 6*a^4*b^4*d^6 + 4*a^6*b^2*d^6 + 3*a^8*c^2*d^4 + 3*a^8*c^4*d^2 + 3*b^8*c^2*d^4 + 3*b^8*c^4*d^2 + 12*a^2*b^6*c^2*d^4 + 12*a^2*b^6*c^4*d^2 + 18*a^4*b^4*c^2*d^4 + 18*a^4*b^4*c^4*d^2 + 12*a^6*b^2*c^2*d^4 + 12*a^6*b^2*c^4*d^2))^(1/2) + a^4*c^3*f^2 + b^4*c^3*f^2 - 4*a*b^3*d^3*f^2 + 4*a^3*b*d^3*f^2 - 3*a^4*c*d^2*f^2 - 3*b^4*c*d^2*f^2 - 6*a^2*b^2*c^3*f^2 + 12*a*b^3*c^2*d*f^2 - 12*a^3*b*c^2*d*f^2 + 18*a^2*b^2*c*d^2*f^2)/(4*f^4))^(1/2))*(-(((8*a^4*c^3*f^2 + 8*b^4*c^3*f^2 - 32*a*b^3*d^3*f^2 + 32*a^3*b*d^3*f^2 - 24*a^4*c*d^2*f^2 - 24*b^4*c*d^2*f^2 - 48*a^2*b^2*c^3*f^2 + 96*a*b^3*c^2*d*f^2 - 96*a^3*b*c^2*d*f^2 + 144*a^2*b^2*c*d^2*f^2)^2/64 - f^4*(a^8*c^6 + a^8*d^6 + b^8*c^6 + b^8*d^6 + 4*a^2*b^6*c^6 + 6*a^4*b^4*c^6 + 4*a^6*b^2*c^6 + 4*a^2*b^6*d^6 + 6*a^4*b^4*d^6 + 4*a^6*b^2*d^6 + 3*a^8*c^2*d^4 + 3*a^8*c^4*d^2 + 3*b^8*c^2*d^4 + 3*b^8*c^4*d^2 + 12*a^2*b^6*c^2*d^4 + 12*a^2*b^6*c^4*d^2 + 18*a^4*b^4*c^2*d^4 + 18*a^4*b^4*c^4*d^2 + 12*a^6*b^2*c^2*d^4 + 12*a^6*b^2*c^4*d^2))^(1/2) + a^4*c^3*f^2 + b^4*c^3*f^2 - 4*a*b^3*d^3*f^2 + 4*a^3*b*d

^3*f^2 - 3*a^4*c*d^2*f^2 - 3*b^4*c*d^2*f^2 - 6*a^2*b^2*c^3*f^2 + 12*a*b^3*c ^2*d*f^2 - 12*a^3*b*c^2*d*f^2 + 18*a^2*b^2*c*d^2*f^2)/(4*f^4))^(1/2) + (16* (c + d*tan(e + f*x))^(1/2)*(a^4*d^6 + b^4*d^6 - 6*a^2*b^2*d^6 - 6*a^4*c^2*d ^4 + a^4*c^4*d^2 - 6*b^4*c^2*d^4 + b^4*c^4*d^2 + 16*a*b^3*c^3*d^3 - 16*a^3* b*c^3*d^3 + 36*a^2*b^2*c^2*d^4 - 6*a^2*b^2*c^4*d^2 - 16*a*b^3*c*d^5 + 16*a^ 3*b*c*d^5))/f^2)*(-(((8*a^4*c^3*f^2 + 8*b^4*c^3*f^2 - 32*a*b^3*d^3*f^2 + 32 *a^3*b*d^3*f^2 - 24*a^4*c*d^2*f^2 - 24*b^4*c*d^2*f^2 - 48*a^2*b^2*c^3*f^2 + 96*a*b^3*c^2*d*f^2 - 96*a^3*b*c^2*d*f^2 + 144*a^2*b^2*c*d^2*f^2)^2/64 - f^ 4*(a^8*c^6 + a^8*d^6 + b^8*c^6 + b^8*d^6 + 4*a^2*b^6*c^6 + 6*a^4*b^4*c^6 + 4*a^6*b^2*c^6 + 4*a^2*b^6*d^6 + 6*a^4*b^4*d^6 + 4*a^6*b^2*d^6 + 3*a^8*c^2*d ^4 + 3*a^8*c^4*d^2 + 3*b^8*c^2*d^4 + 3*b^8*c^4*...

3.1237 $\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=150

$$-\frac{(ia+b)(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{(ia-b)(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}-$$

```
[Out] -(I*a+b)*(c-I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/f+(I*a
-b)*(c+I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/f+2*(a*d+b*
c)*(c+d*tan(f*x+e))^(1/2)/f+2/3*b*(c+d*tan(f*x+e))^(3/2)/f
```

Rubi [A]
time = 0.22, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3609, 3620, 3618, 65, 214}

$$\frac{2(ad+bc)\sqrt{c+d\tan(e+fx)}}{f}-\frac{(b+ia)(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{(-b+ia)(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}+\frac{2b(c+d\tan(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] -(((I*a + b)*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]
])/f) + ((I*a - b)*(c + I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c
+ I*d]])/f + (2*(b*c + a*d)*Sqrt[c + d*Tan[e + f*x]])/f + (2*b*(c + d*Tan[e
 + f*x])^(3/2))/(3*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
```

```
0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}\,dx = \frac{2b(c+d\tan(e+fx))^{3/2}}{3f}+\int\sqrt{c+d\tan(e+fx)}\,(ac-b$$

$$= \frac{2(bc+ad)\sqrt{c+d\tan(e+fx)}}{f}+\frac{2b(c+d\tan(e+fx))^{3/2}}{3f}$$

$$= \frac{2(bc+ad)\sqrt{c+d\tan(e+fx)}}{f}+\frac{2b(c+d\tan(e+fx))^{3/2}}{3f}$$

$$= \frac{2(bc+ad)\sqrt{c+d\tan(e+fx)}}{f}+\frac{2b(c+d\tan(e+fx))^{3/2}}{3f}$$

$$= \frac{2(bc+ad)\sqrt{c+d\tan(e+fx)}}{f}+\frac{2b(c+d\tan(e+fx))^{3/2}}{3f}$$

$$= -\frac{(ia+b)(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{(ia$$

Mathematica [A]
time = 0.56, size = 140, normalized size = 0.93

$$\frac{-3i(a-ib)(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)+3i(a+ib)(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)+2\sqrt{c+d\tan(e+fx)}\,(4bc+3ad+bd\tan(e+fx))}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-3*I)*(a - I*b)*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c -
 I*d]] + (3*I)*(a + I*b)*(c + I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/S
qrt[c + I*d]] + 2*Sqrt[c + d*Tan[e + f*x]]*(4*b*c + 3*a*d + b*d*Tan[e + f*x
]))/(3*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 917 vs. $2(126) = 252$.
time = 0.42, size = 918, normalized size = 6.12

method	result
derivativedivides	$\frac{\frac{2b(c+d\tan(fx+e))^{\frac{3}{2}}}{3}+2ad\sqrt{c+d\tan(fx+e)}+2bc\sqrt{c+d\tan(fx+e)}+\frac{\left(-\sqrt{c^2+d^2}\,\sqrt{2\sqrt{c^2}}\right.}{}}{}$
default	$\frac{\frac{2b(c+d\tan(fx+e))^{\frac{3}{2}}}{3}+2ad\sqrt{c+d\tan(fx+e)}+2bc\sqrt{c+d\tan(fx+e)}+\frac{\left(-\sqrt{c^2+d^2}\,\sqrt{2\sqrt{c^2}}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2/3*b*(c+d*tan(f*x+e))^(3/2)+2*a*d*(c+d*tan(f*x+e))^(1/2)+2*b*c*(c+d*t
an(f*x+e))^(1/2)+1/2/d*(1/2*(-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*a*c+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d+(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)*a*c^2-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^2-2*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*b*c*d)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(-2*(c^2+d^2)^(1/2)*a*d^2-2*(c^2+d^2)^(1/2)*
b*c*d-1/2*(-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c+(c^2+d^2)^(1/
2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2-(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^2-2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d)*(2
*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*t
an(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/
2)))+1/2/d*(1/2*((c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c-(c^2+d^2
)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c
^2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^2+2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*
d)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(
c^2+d^2)^(1/2))+2*(-2*(c^2+d^2)^(1/2)*a*d^2-2*(c^2+d^2)^(1/2)*b*c*d+1/2*((c
^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c-(c^2+d^2)^(1/2)*(2*(c^2+d^2
```

```
)^(1/2)+2*c)^(1/2)*b*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2+(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*a*d^2+2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d)*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/
2)-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-c>0)', see `assume?` for more det
ails)Is
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(e + fx))(c + d\tan(e + fx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))*(c + d*tan(e + f*x))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 17.41, size = 2823, normalized size = 18.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))*(c + d*tan(e + f*x))^(3/2),x)
```

```
[Out] log(((((-b^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + b^2*c^3*f^2 - 3*b^2*c*d^2*f^2
)/f^4)^(1/2)*((16*c*d^2*(((-b^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + b^2*c^3*f^
2 - 3*b^2*c*d^2*f^2)/f^4)^(1/2)*(b*c^2 + b*d^2 - f*(((-b^4*d^2*f^4*(3*c^2 -
 d^2)^2)^(1/2) + b^2*c^3*f^2 - 3*b^2*c*d^2*f^2)/f^4)^(1/2)*(c + d*tan(e + f
*x))^(1/2)))/f + (16*b^2*d^2*(c + d*tan(e + f*x))^(1/2)*(c^4 + d^4 - 6*c^2*
d^2))/f^2))/2 - (8*b^3*d^2*(c^2 - d^2)*(c^2 + d^2)^2)/f^3)*((6*b^4*c^2*d^4*
f^4 - b^4*d^6*f^4 - 9*b^4*c^4*d^2*f^4)^(1/2)/(4*f^4) + (b^2*c^3)/(4*f^2) -
(3*b^2*c*d^2)/(4*f^2))^(1/2) - log(((-((-b^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2)
 - b^2*c^3*f^2 + 3*b^2*c*d^2*f^2)/f^4)^(1/2)*((16*c*d^2*(-((-b^4*d^2*f^4*(3
*c^2 - d^2)^2)^(1/2) - b^2*c^3*f^2 + 3*b^2*c*d^2*f^2)/f^4)^(1/2)*(b*c^2 + b
*d^2 + f*(-((-b^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) - b^2*c^3*f^2 + 3*b^2*c*d^
2*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2)))/f - (16*b^2*d^2*(c + d*tan(e
 + f*x))^(1/2)*(c^4 + d^4 - 6*c^2*d^2))/f^2))/2 - (8*b^3*d^2*(c^2 - d^2)*(c
^2 + d^2)^2)/f^3)*(-((6*b^4*c^2*d^4*f^4 - b^4*d^6*f^4 - 9*b^4*c^4*d^2*f^4)^
(1/2) - b^2*c^3*f^2 + 3*b^2*c*d^2*f^2)/(4*f^4))^(1/2) - log(((((-b^4*d^2*f^
4*(3*c^2 - d^2)^2)^(1/2) + b^2*c^3*f^2 - 3*b^2*c*d^2*f^2)/f^4)^(1/2)*((16*c
*d^2*(((-b^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + b^2*c^3*f^2 - 3*b^2*c*d^2*f^2
)/f^4)^(1/2)*(b*c^2 + b*d^2 + f*(((-b^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + b^
2*c^3*f^2 - 3*b^2*c*d^2*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2)))/f - (1
6*b^2*d^2*(c + d*tan(e + f*x))^(1/2)*(c^4 + d^4 - 6*c^2*d^2))/f^2))/2 - (8*
b^3*d^2*(c^2 - d^2)*(c^2 + d^2)^2)/f^3)*(((6*b^4*c^2*d^4*f^4 - b^4*d^6*f^4
- 9*b^4*c^4*d^2*f^4)^(1/2) + b^2*c^3*f^2 - 3*b^2*c*d^2*f^2)/(4*f^4))^(1/2)
+ log(((-((-b^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) - b^2*c^3*f^2 + 3*b^2*c*d^2*
f^2)/f^4)^(1/2)*((16*c*d^2*(-((-b^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) - b^2*c^
3*f^2 + 3*b^2*c*d^2*f^2)/f^4)^(1/2)*(b*c^2 + b*d^2 - f*(-((-b^4*d^2*f^4*(3*
c^2 - d^2)^2)^(1/2) - b^2*c^3*f^2 + 3*b^2*c*d^2*f^2)/f^4)^(1/2)*(c + d*tan(
e + f*x))^(1/2)))/f + (16*b^2*d^2*(c + d*tan(e + f*x))^(1/2)*(c^4 + d^4 - 6
*c^2*d^2))/f^2))/2 - (8*b^3*d^2*(c^2 - d^2)*(c^2 + d^2)^2)/f^3)*((b^2*c^3)/
(4*f^2) - (6*b^4*c^2*d^4*f^4 - b^4*d^6*f^4 - 9*b^4*c^4*d^2*f^4)^(1/2)/(4*f^
4) - (3*b^2*c*d^2)/(4*f^2))^(1/2) - log(((-((-a^4*d^2*f^4*(3*c^2 - d^2)^2)^
(1/2) + a^2*c^3*f^2 - 3*a^2*c*d^2*f^2)/f^4)^(1/2)*((16*d^2*(-((-a^4*d^2*f^4
*(3*c^2 - d^2)^2)^(1/2) + a^2*c^3*f^2 - 3*a^2*c*d^2*f^2)/f^4)^(1/2)*(a*d^3
```

```
+ a*c^2*d + c*f*(-((-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + a^2*c^3*f^2 - 3*a
^2*c*d^2*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2)))/f + (16*a^2*d^2*(c +
d*tan(e + f*x))^(1/2)*(c^4 + d^4 - 6*c^2*d^2))/f^2))/2 - (16*a^3*c*d^3*(c^2
 + d^2)^2)/f^3)*(-((6*a^4*c^2*d^4*f^4 - a^4*d^6*f^4 - 9*a^4*c^4*d^2*f^4)^(1
/2) + a^2*c^3*f^2 - 3*a^2*c*d^2*f^2)/(4*f^4))^(1/2) - log(((((-a^4*d^2*f^4*
(3*c^2 - d^2)^2)^(1/2) - a^2*c^3*f^2 + 3*a^2*c*d^2*f^2)/f^4)^(1/2)*((16*d^2
*(((-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) - a^2*c^3*f^2 + 3*a^2*c*d^2*f^2)/f^
4)^(1/2)*(a*d^3 + a*c^2*d + c*f*(((-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) - a^
2*c^3*f^2 + 3*a^2*c*d^2*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2)))/f + (1
6*a^2*d^2*(c + d*tan(e + f*x))^(1/2)*(c^4 + d^4 - 6*c^2*d^2))/f^2))/2 - (16
*a^3*c*d^3*(c^2 + d^2)^2)/f^3)*(((6*a^4*c^2*d^4*f^4 - a^4*d^6*f^4 - 9*a^4*c
^4*d^2*f^4)^(1/2) - a^2*c^3*f^2 + 3*a^2*c*d^2*f^2)/(4*f^4))^(1/2) + log((((
(-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) - a^2*c^3*f^2 + 3*a^2*c*d^2*f^2)/f^4)^
(1/2)*((16*d^2*(((-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) - a^2*c^3*f^2 + 3*a^2
*c*d^2*f^2)/f^4)^(1/2)*(a*d^3 + a*c^2*d - c*f*(((-a^4*d^2*f^4*(3*c^2 - d^2)
^2)^(1/2) - a^2*c^3*f^2 + 3*a^2*c*d^2*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^
(1/2)))/f - (16*a^2*d^2*(c + d*tan(e + f*x))^(1/2)*(c^4 + d^4 - 6*c^2*d^2))
/f^2))/2 - (16*a^3*c*d^3*(c^2 + d^2)^2)/f^3)*((6*a^4*c^2*d^4*f^4 - a^4*d^6*
f^4 - 9*a^4*c^4*d^2*f^4)^(1/2)/(4*f^4) - (a^2*c^3)/(4*f^2) + (3*a^2*c*d^2)/
(4*f^2))^(1/2) + log(((-((-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + a^2*c^3*f^2
 - 3*a^2*c*d^2*f^2)/f^4)^(1/2)*((16*d^2*(-((-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(
1/2) + a^2*c^3*f^2 - 3*a^2*c*d^2*f^2)/f^4)^(1/2)*(a*d^3 + a*c^2*d - c*f*(-(
(-a^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + a^2*c^3*f^2 - 3*a^2*c*d^2*f^2)/f^4)^
(1/2)*(c + d*tan(e + f*x))^(1/2)))/f - (16*a^2*d^2*(c + d*tan(e + f*x))^(1/
2)*(c^4 + d^4 - 6*c^2*d^2))/f^2))/2 - (16*a^3*c*d^3*(c^2 + d^2)^2)/f^3)*((3
*a^2*c*d^2)/(4*f^2) - (a^2*c^3)/(4*f^2) - (6*a^4*c^2*d^4*f^4 - a^4*d^6*f^4
 - 9*a^4*c^4*d^2*f^4)^(1/2)/(4*f^4))^(1/2) + (2*b*(c + d*tan(e + f*x))^(3/2)
)/(3*f) + (2*a*d*(c + d*tan(e + f*x))^(1/2))/f + (2*b*c*(c + d*tan(e + f*x)
)^(1/2))/f
```

3.1238 $\int \frac{(c+d\tan(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=170

$$\frac{(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(ia+b)f}-\frac{(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(ia-b)f}-\frac{2(bc-ad)^{3/2}\tan$$

```
[Out] (c-I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(I*a+b)/f-(c+I*
d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(I*a-b)/f-2*(-a*d+b*
c)^(3/2)*arctanh(b^(1/2)*(c+d*tan(f*x+e))^(1/2)/(-a*d+b*c)^(1/2))/(a^2+b^2)
/f/b^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3654, 3620, 3618, 65, 214, 3715}

$$-\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\,f\,(a^2+b^2)}+\frac{(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(b+ia)}-\frac{(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(-b+ia)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^(3/2)/(a + b*Tan[e + f*x]),x]
```

```
[Out] ((c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((I*a + b
)*f) - ((c + I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/((
I*a - b)*f) - (2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]
])/Sqrt[b*c - a*d]])/(Sqrt[b]*(a^2 + b^2)*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
```

```
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3654

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[Simp[a^2*c - b^2*c + 2
*a*b*d + (2*a*b*c - a^2*d + b^2*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x]
], x], x] + Dist[(b*c - a*d)^2/(c^2 + d^2), Int[(1 + Tan[e + f*x]^2)/(Sqrt[
a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx = \frac{\int \frac{2bcd+a(c^2-d^2)+(2acd-b(c^2-d^2))\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx}{a^2+b^2} + \frac{(bc-ad)^2\int \frac{1+\tan^2(e+}{(a+b\tan(e+fx))\sqrt{c+d}}}{a^2+b^2}$$

$$= \frac{(c-id)^2\int \frac{1+i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx}{2(a-ib)} + \frac{(c+id)^2\int \frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx}{2(a+ib)}$$

$$= -\frac{(c-id)^2\mathrm{Subst}\left(\int \frac{1}{(-1+x)\sqrt{c-idx}}\,dx,x,i\tan(e+fx)\right)}{2(ia+b)f} + \frac{(c+id)^2\mathrm{Subst}}{}$$

$$= -\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\,(a^2+b^2)\,f} - \frac{(c-id)^2\mathrm{Subst}\left(\int \frac{}{-1}\right.}{}$$

$$= \frac{(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(ia+b)f} - \frac{(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\,\mathrm{t}}}{\sqrt{c}}\right.}{(ia-b)f}$$

Mathematica [A]
time = 0.41, size = 168, normalized size = 0.99

$$\frac{\sqrt{b}\,(-ia+b)(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)+\sqrt{b}\,(ia+b)(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)-2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\,(a^2+b^2)\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^(3/2)/(a + b*Tan[e + f*x]),x]
```

```
[Out] (Sqrt[b]*((-I)*a + b)*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt
[c - I*d]] + Sqrt[b]*(I*a + b)*(c + I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f
*x]]/Sqrt[c + I*d]] - 2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e
 + f*x]])/Sqrt[b*c - a*d]])/(Sqrt[b]*(a^2 + b^2)*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 956 vs. 2(142) = 284.
time = 0.56, size = 957, normalized size = 5.63

method	result

derivativedivides	$2d^2\left(\frac{\left(-\sqrt{c^2+d^2}\,\sqrt{2\sqrt{c^2+d^2}+2c}\,ac-\sqrt{c^2+d^2}\,\sqrt{2\sqrt{c^2+d^2}+2c}\,bd+\sqrt{2\sqrt{c^2+d^2}+\cdots\right.}{\cdots}\right.$
default	$2d^2\left(\frac{\left(-\sqrt{c^2+d^2}\,\sqrt{2\sqrt{c^2+d^2}+2c}\,ac-\sqrt{c^2+d^2}\,\sqrt{2\sqrt{c^2+d^2}+2c}\,bd+\sqrt{2\sqrt{c^2+d^2}+\cdots\right.}{\cdots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*d^2*(1/d^2/(a^2+b^2)*(1/4/d*(1/2*(-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)*a*c-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d+(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)*a*c^2-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^2+2*(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*b*c*d)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(-2*(c^2+d^2)^(1/2)*a*d^2+2*(c^2+d^
2)^(1/2)*b*c*d-1/2*(-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c-(c^2
+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*a*c^2-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^2+2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*
b*c*d)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(
(2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)
-2*c)^(1/2)))+1/4/d*(-1/2*(-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a
*c-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d+(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*a*c^2-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^2+2*(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)*b*c*d)*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan
(f*x+e)-c-(c^2+d^2)^(1/2))+2*(2*(c^2+d^2)^(1/2)*a*d^2-2*(c^2+d^2)^(1/2)*b*c
*d+1/2*(-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c-(c^2+d^2)^(1/2)*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2-(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*a*d^2+2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d)*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2
)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))
```

```
))+(a*d-b*c)^2/d^2/(a^2+b^2)/((a*d-b*c)*b)^(1/2)*arctan(b*(c+d*tan(f*x+e))^
(1/2)/((a*d-b*c)*b)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d\tan{(e + fx)})^{\frac{3}{2}}}{a + b\tan{(e + fx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(3/2)/(a+b*tan(f*x+e)),x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(3/2)/(a + b*tan(e + f*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
```

```
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 13.01, size = 2500, normalized size = 14.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(3/2)/(a + b*tan(e + f*x)),x)
[Out] (atan(-((((((32*(a*b^5*d^15*f^2 + 4*a^5*b*d^15*f^2 - b^6*c*d^14*f^2 - 15*a^
3*b^3*d^15*f^2 + 23*b^6*c^3*d^12*f^2 + 21*b^6*c^5*d^10*f^2 - 3*b^6*c^7*d^8*
f^2 - 29*a^2*b^4*c^3*d^12*f^2 - 81*a^2*b^4*c^5*d^10*f^2 + a^2*b^4*c^7*d^8*f
^2 + 59*a^3*b^3*c^2*d^13*f^2 + 75*a^3*b^3*c^4*d^11*f^2 + a^3*b^3*c^6*d^9*f^
2 - 32*a^4*b^2*c^3*d^12*f^2 - 2*a^4*b^2*c^5*d^10*f^2 - 61*a*b^5*c^2*d^13*f^
2 - 25*a*b^5*c^4*d^11*f^2 + 37*a*b^5*c^6*d^9*f^2 + 53*a^2*b^4*c*d^14*f^2 -
30*a^4*b^2*c*d^14*f^2 + 4*a^5*b*c^2*d^13*f^2))/f^5 + ((((-b*(a*d - b*c)^3)^
(1/2)*((32*(4*a^2*b^6*d^12*f^4 + 8*a^4*b^4*d^12*f^4 + 4*a^6*b^2*d^12*f^4 +
12*b^8*c^2*d^10*f^4 + 12*b^8*c^4*d^8*f^4 + 28*a^2*b^6*c^2*d^10*f^4 + 24*a^2
*b^6*c^4*d^8*f^4 - 32*a^3*b^5*c^3*d^9*f^4 + 20*a^4*b^4*c^2*d^10*f^4 + 12*a^
4*b^4*c^4*d^8*f^4 - 16*a^5*b^3*c^3*d^9*f^4 + 4*a^6*b^2*c^2*d^10*f^4 - 16*a*
b^7*c*d^11*f^4 - 16*a*b^7*c^3*d^9*f^4 - 32*a^3*b^5*c*d^11*f^4 - 16*a^5*b^3*
c*d^11*f^4))/f^5 - (32*(-b*(a*d - b*c)^3)^(1/2)*(c + d*tan(e + f*x))^(1/2)*
(16*b^9*d^10*f^4 + 16*a^2*b^7*d^10*f^4 - 16*a^4*b^5*d^10*f^4 - 16*a^6*b^3*d
^10*f^4 + 24*b^9*c^2*d^8*f^4 + 40*a^2*b^7*c^2*d^8*f^4 + 8*a^4*b^5*c^2*d^8*f
^4 - 8*a^6*b^3*c^2*d^8*f^4 + 8*a*b^8*c*d^9*f^4 + 24*a^3*b^6*c*d^9*f^4 + 24*
a^5*b^4*c*d^9*f^4 + 8*a^7*b^2*c*d^9*f^4))/(f^4*(b^3*f + a^2*b*f))))/(b^3*f
+ a^2*b*f) - (32*(c + d*tan(e + f*x))^(1/2)*(22*b^7*c*d^12*f^2 - 14*a*b^6*d
^13*f^2 + 4*a^3*b^4*d^13*f^2 - 14*a^5*b^2*d^13*f^2 + 28*b^7*c^3*d^10*f^2 -
18*b^7*c^5*d^8*f^2 + 24*a^2*b^5*c^3*d^10*f^2 + 12*a^2*b^5*c^5*d^8*f^2 - 88*
a^3*b^4*c^2*d^11*f^2 - 28*a^3*b^4*c^4*d^9*f^2 + 60*a^4*b^3*c^3*d^10*f^2 - 2
*a^4*b^3*c^5*d^8*f^2 - 44*a^5*b^2*c^2*d^11*f^2 + 2*a^5*b^2*c^4*d^9*f^2 + 8*
a^6*b*c*d^12*f^2 + 20*a*b^6*c^2*d^11*f^2 + 66*a*b^6*c^4*d^9*f^2 - 28*a^2*b^
5*c*d^12*f^2 + 54*a^4*b^3*c*d^12*f^2))/f^4)*(-b*(a*d - b*c)^3)^(1/2))/(b^3*
f + a^2*b*f))*(-b*(a*d - b*c)^3)^(1/2))/(b^3*f + a^2*b*f) - (32*(c + d*tan(
e + f*x))^(1/2)*(b^5*d^16 + 2*a^4*b*d^16 + 4*b^5*c^2*d^14 + 8*b^5*c^4*d^12
- 8*b^5*c^6*d^10 + 3*b^5*c^8*d^8 - 8*a*b^4*c^3*d^13 + 48*a*b^4*c^5*d^11 - 8
*a*b^4*c^7*d^9 - 8*a^3*b^2*c*d^15 - 12*a^4*b*c^2*d^14 + 2*a^4*b*c^4*d^12 +
12*a^2*b^3*c^2*d^14 - 72*a^2*b^3*c^4*d^12 + 12*a^2*b^3*c^6*d^10 + 48*a^3*b^
2*c^3*d^13 - 8*a^3*b^2*c^5*d^11))/f^4)*(-b*(a*d - b*c)^3)^(1/2)*1i)/(b^3*f
+ a^2*b*f) - (((((32*(a*b^5*d^15*f^2 + 4*a^5*b*d^15*f^2 - b^6*c*d^14*f^2 -
15*a^3*b^3*d^15*f^2 + 23*b^6*c^3*d^12*f^2 + 21*b^6*c^5*d^10*f^2 - 3*b^6*c^7
*d^8*f^2 - 29*a^2*b^4*c^3*d^12*f^2 - 81*a^2*b^4*c^5*d^10*f^2 + a^2*b^4*c^7*
d^8*f^2 + 59*a^3*b^3*c^2*d^13*f^2 + 75*a^3*b^3*c^4*d^11*f^2 + a^3*b^3*c^6*d
```

^9*f^2 - 32*a^4*b^2*c^3*d^12*f^2 - 2*a^4*b^2*c^5*d^10*f^2 - 61*a*b^5*c^2*d^13*f^2 - 25*a*b^5*c^4*d^11*f^2 + 37*a*b^5*c^6*d^9*f^2 + 53*a^2*b^4*c*d^14*f^2 - 30*a^4*b^2*c*d^14*f^2 + 4*a^5*b*c^2*d^13*f^2))/f^5 + ((((-b*(a*d - b*c)^3)^(1/2)*((32*(4*a^2*b^6*d^12*f^4 + 8*a^4*b^4*d^12*f^4 + 4*a^6*b^2*d^12*f^4 + 12*b^8*c^2*d^10*f^4 + 12*b^8*c^4*d^8*f^4 + 28*a^2*b^6*c^2*d^10*f^4 + 24*a^2*b^6*c^4*d^8*f^4 - 32*a^3*b^5*c^3*d^9*f^4 + 20*a^4*b^4*c^2*d^10*f^4 + 12*a^4*b^4*c^4*d^8*f^4 - 16*a^5*b^3*c^3*d^9*f^4 + 4*a^6*b^2*c^2*d^10*f^4 - 16*a*b^7*c*d^11*f^4 - 16*a*b^7*c^3*d^9*f^4 - 32*a^3*b^5*c*d^11*f^4 - 16*a^5*b^3*c*d^11*f^4))/f^5 + (32*(-b*(a*d - b*c)^3)^(1/2)*(c + d*tan(e + f*x))^(1/2)*(16*b^9*d^10*f^4 + 16*a^2*b^7*d^10*f^4 - 16*a^4*b^5*d^10*f^4 - 16*a^6*b^3*d^10*f^4 + 24*b^9*c^2*d^8*f^4 + 40*a^2*b^7*c^2*d^8*f^4 + 8*a^4*b^5*c^2*d^8*f^4 - 8*a^6*b^3*c^2*d^8*f^4 + 8*a*b^8*c*d^9*f^4 + 24*a^3*b^6*c*d^9*f^4 + 24*a^5*b^4*c*d^9*f^4 + 8*a^7*b^2*c*d^9*f^4))/(f^4*(b^3*f + a^2*b*f))))/(b^3*f + a^2*b*f) + (32*(c + d*tan(e + f*x))^(1/2)*(22*b^7*c*d^12*f^2 - 14*a*b^6*d^13*f^2 + 4*a^3*b^4*d^13*f^2 - 14*a^5*b^2*d^13*f^2 + 28*b^7*c^3*d^10*f^2 - 18*b^7*c^5*d^8*f^2 + 24*a^2*b^5*c^3*d^10*f^2 + 12*a^2*b^5*c^5*d^8*f^2 - 88*a^3*b^4*c^2*d^11*f^2 - 28*a^3*b^4*c^4*d^9*f^2 + 60*a^4*b^3*c^3*d^10*f^2 - 2*a^4*b^3*c^5*d^8*f^2 - 44*a^5*b^2*c^2*d^11*f^2 + 2*a^5*b^2*c^4*d^9*f^2 + 8*a^6*b*c*d^12*f^2 + 20*a*b^6*c^2*d^11*f^2 + 66*a*b^6*c^4*d^9*f^2 - 28*a^2*b^5*c*d^12*f^2 + 54*a^4*b^3*c*d^12*f^2))/f^4)*(-b*(a*d - b*c)^3)^(1/2))/(b^3*f + a^2*b*f))*(-b*(a*d - b*c)^3)^(1/2))/(b^3*f + a^2*b*f) + (32*(c + d*tan(e + f*x))^(1/2)*(b^5*d^16 + 2*a^4*b*d^16 + 4*b^5*c^2*d^14 + 8*b^5*c^4*d^12 - 8*b^5*c^6*d^10 + 3*b^5*c^8*d^8 - 8*a*b^4*c^3*d^13 + 48*a*b^4*c^5*d^11 - 8*a*b^4*c^7*d^9 - 8*a^3*b^2*c*d^15 - 12*a^4*b*c^2*d^14 + 2*a^4*b*c^4*d^12 + 12*a^2*b^3*c^2*d^14 - 72*a^2*b^3*c^4*d^12 + 12*a^2*b^3*c^6*d^10 + 48*a^3*b^2*c^3*d^13 - 8*a^3*b^2*c^5*d^11))/f^4)*(-b*(a*d - b*c)^3)^(1/2)*1i)/(b^3*f + a^2*b*f))/((64*(a^2*b^2*d^18 + b^4*c^2*d^16 + 5*b^4*c^4*d^14 + 7*b^4*c^6*d^12 + 3*b^4*c^8*d^10 - 12*a*b^3*c^3*d^15 - 18*a*b^3*c^5*d^13 - 8*a*b^3*c^7*d^11 - 4*a^3*b*c^3*d^15 - 2*a^3*b*c^5*d^13 + 9*a^2*b^2*c^2*d^16 + 15*a^2*b^2*c^4*d^14 + 7*a^2*b^2*c^6*d^12 - 2*a*b^3*c*d^17 - 2*a^3*b*c*d^17))/f^5 + (((((32*(a*b^5*d^15*f^2 + 4*a^5*b*d^15*f^2 - b^6*c*d^14*f^2 - 15*a^3*b^3*d^15*f^2 + 23*b^6*c^3*d^12*f^2 + 21*b^6*c^5*d^10*f^2 - 3*b^6*c^7*d^8*f^2 - 29*a^2*b^4*c^3*d^12*f^2 - 81*a^2*b^4*c^5*d^1...

3.1239 $\int \frac{(c+d\tan(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=239

$$-\frac{i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)^2 f}+\frac{i(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(a+ib)^2 f}-\frac{\sqrt{bc-ad}\,(4a\cdots}{}$$

```
[Out] -I*(c-I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(a-I*b)^2/f+
I*(c+I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(a+I*b)^2/f-(
-a^2*d+4*a*b*c+3*b^2*d)*arctanh(b^(1/2)*(c+d*tan(f*x+e))^(1/2)/(-a*d+b*c)^(
1/2))*(-a*d+b*c)^(1/2)/(a^2+b^2)^2/f/b^(1/2)-(-a*d+b*c)*(c+d*tan(f*x+e))^(1
/2)/(a^2+b^2)/f/(a+b*tan(f*x+e))
```

Rubi [A]
time = 0.62, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3648, 3734, 3620, 3618, 65, 214, 3715}

$$-\frac{(bc-ad)\sqrt{c+d\tan(e+fx)}}{f\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{\sqrt{bc-ad}\left(a^2(-d)+4abc+3b^2d\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\,f\left(a^2+b^2\right)^2}-\frac{i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(a-ib)^2}+\frac{i(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(a+ib)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^(3/2)/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] ((-I)*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((a
 - I*b)^2*f) + (I*(c + I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c +
I*d]])/((a + I*b)^2*f) - (Sqrt[b*c - a*d]*(4*a*b*c - a^2*d + 3*b^2*d)*ArcTa
nh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(Sqrt[b]*(a^2 + b^2
)^2*f) - ((b*c - a*d)*Sqrt[c + d*Tan[e + f*x]])/((a^2 + b^2)*f*(a + b*Tan[e
 + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx = -\frac{(bc-ad)\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)\,f(a+b\tan(e+fx))} - \frac{\int \frac{\frac{1}{2}\left(-3bcd-a(2c^2-d^2)\right)-\left(2acd-b(c^2-d^2)\right)\tan(e+fx)}{(a+b\tan(e+fx))\sqrt{c+d\tan(e-}}}{a^2+b^2}$$

$$= -\frac{(bc-ad)\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)\,f(a+b\tan(e+fx))} - \frac{\int \frac{-(ac+bc-ad+bd)(ac-bc+ad+bd)+2(bc-ad)(ac+bd)}{\sqrt{c+d\tan(e+fx)}}}{(a^2+b^2)^2}$$

$$= -\frac{(bc-ad)\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)\,f(a+b\tan(e+fx))} + \frac{(c-id)^2\int \frac{1+i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx}{2(a-ib)^2} + \frac{(c}{}$$

$$= -\frac{(bc-ad)\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)\,f(a+b\tan(e+fx))} + \frac{(i(c-id)^2)\,\mathrm{Subst}\left(\int \frac{1}{(-1+x)\sqrt{c-idx}}\,dx,\right.}{2(a-ib)^2f}$$

$$= -\frac{\sqrt{bc-ad}\,(4abc-a^2d+3b^2d)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\,(a^2+b^2)^2\,f} - \frac{(bc-}{(a^2+}$$

$$= -\frac{i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)^2f} + \frac{i(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+}}{}\right.}{(a+ib)^2f}$$

Mathematica [A]
time = 3.29, size = 316, normalized size = 1.32

$$-\frac{\frac{4\left(\frac{3}{4}i(a+ib)^2b^2(c-id)^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)-\frac{3}{4}i(a-ib)^2b^2(c+id)^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)+\frac{3}{4}b^{3/2}(bc-ad)^{3/2}(4abc-a^2d+3b^2d)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)\right)}{b^2(a^2+b^2)}+3d(bc-ad)\sqrt{c+d\tan(e+fx)}+3bd(c+d\tan(e+fx))^{3/2}-\frac{3b^2(c+d\tan(e+fx))^{5/2}}{a+b\tan(e+fx)}}{3\,(a^2+b^2)\,(bc-ad)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^(3/2)/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] ((-4*(((3*I)/4)*(a + I*b)^2*b^2*(c - I*d)^(3/2)*(b*c - a*d)*ArcTanh[Sqrt[c
+ d*Tan[e + f*x]]/Sqrt[c - I*d]] - ((3*I)/4)*(a - I*b)^2*b^2*(c + I*d)^(3/2
)*(b*c - a*d)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]] + (3*b^(3/2)*
(b*c - a*d)^(3/2)*(4*a*b*c - a^2*d + 3*b^2*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*T
an[e + f*x]])/Sqrt[b*c - a*d]])/4))/(b^2*(a^2 + b^2)) + 3*d*(b*c - a*d)*Sqr
t[c + d*Tan[e + f*x]] + 3*b*d*(c + d*Tan[e + f*x])^(3/2) - (3*b^2*(c + d*Ta
n[e + f*x])^(5/2))/(a + b*Tan[e + f*x]))/(3*(a^2 + b^2)*(b*c - a*d)*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1412 vs. $2(207) = 414$.
time = 0.54, size = 1413, normalized size = 5.91 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*d^3*((a*d-b*c)/d^3/(a^2+b^2)^2*((1/2*a^2*d+1/2*b^2*d)*(c+d*tan(f*x+e))^
(1/2)/((c+d*tan(f*x+e))*b+a*d-b*c)+1/2*(a^2*d-4*a*b*c-3*b^2*d)/((a*d-b*c)*b
)^(1/2)*arctan(b*(c+d*tan(f*x+e))^(1/2)/((a*d-b*c)*b)^(1/2)))+1/d^3/(a^2+b^
2)^2*(1/4/d*(1/2*((c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c+2*(c^
2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d-(c^2+d^2)^(1/2)*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*b^2*c-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^2+(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)*a^2*d^2-4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c*d+(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*b^2*c^2-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d^2)*ln(d*t
an(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^
(1/2))+2*(-2*(c^2+d^2)^(1/2)*a^2*d^2+4*(c^2+d^2)^(1/2)*a*b*c*d+2*(c^2+d^2)^
(1/2)*b^2*d^2+1/2*((c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c+2*(c
^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d-(c^2+d^2)^(1/2)*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*b^2*c-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^2+(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*a^2*d^2-4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c*d+(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*b^2*c^2-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d^2)*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(
f*x+e))^(1/2)-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))
)+1/4/d*(1/2*(-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c-2*(c^2+d
^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*b^2*c+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^2-(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)*a^2*d^2+4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c*d-(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)*b^2*c^2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d^2)*ln(d*tan(
f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/
2))+2*(-2*(c^2+d^2)^(1/2)*a^2*d^2+4*(c^2+d^2)^(1/2)*a*b*c*d+2*(c^2+d^2)^(1/
2)*b^2*d^2-1/2*(-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c-2*(c^2
+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d+(c^2+d^2)^(1/2)*(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)*b^2*c+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^2-(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*a^2*d^2+4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c*d-(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*b^2*c^2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d^2)*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*
x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))))
)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d\tan{(e + fx)})^{\frac{3}{2}}}{(a + b\tan{(e + fx)})^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(3/2)/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(3/2)/(a + b*tan(e + f*x))**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**B**]
time = 12.95, size = 2500, normalized size = 10.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(3/2)/(a + b*tan(e + f*x))^2,x)
```

```
[Out] (atan(((((16*(c + d*tan(e + f*x))^(1/2)*(2*b^9*d^16 + a^8*b*d^16 - 5*a^2*b^
7*d^16 + 17*a^4*b^5*d^16 - 7*a^6*b^3*d^16 - b^9*c^2*d^14 + 66*b^9*c^4*d^12
- b^9*c^6*d^10 + 2*b^9*c^8*d^8 - 204*a*b^8*c^3*d^13 + 234*a*b^8*c^5*d^11 -
24*a*b^8*c^7*d^9 - 126*a^3*b^6*c*d^15 + 94*a^5*b^4*c*d^15 - 18*a^7*b^2*c*d^
```

15 - 6*a^8*b*c^2*d^14 + a^8*b*c^4*d^12 + 277*a^2*b^7*c^2*d^14 - 715*a^2*b^7*c^4*d^12 + 367*a^2*b^7*c^6*d^10 - 12*a^2*b^7*c^8*d^8 + 892*a^3*b^6*c^3*d^13 - 998*a^3*b^6*c^5*d^11 + 192*a^3*b^6*c^7*d^9 - 457*a^4*b^5*c^2*d^14 + 1173*a^4*b^5*c^4*d^12 - 495*a^4*b^5*c^6*d^10 + 18*a^4*b^5*c^8*d^8 - 628*a^5*b^4*c^3*d^13 + 550*a^5*b^4*c^5*d^11 - 40*a^5*b^4*c^7*d^9 + 155*a^6*b^3*c^2*d^14 - 285*a^6*b^3*c^4*d^12 + 33*a^6*b^3*c^6*d^10 + 68*a^7*b^2*c^3*d^13 - 10*a^7*b^2*c^5*d^11 + 18*a*b^8*c*d^15))/(a^8*f^4 + b^8*f^4 + 4*a^2*b^6*f^4 + 6*a^4*b^4*f^4 + 4*a^6*b^2*f^4) + (((16*(78*a^2*b^9*d^15*f^2 - 2*a^10*b*d^15*f^2 - 8*a^4*b^7*d^15*f^2 - 60*a^6*b^5*d^15*f^2 + 24*a^8*b^3*d^15*f^2 + 50*b^11*c^2*d^13*f^2 + 22*b^11*c^4*d^11*f^2 - 28*b^11*c^6*d^9*f^2 - 546*a^2*b^9*c^2*d^13*f^2 - 296*a^2*b^9*c^4*d^11*f^2 + 328*a^2*b^9*c^6*d^9*f^2 - 240*a^3*b^8*c^3*d^12*f^2 - 544*a^3*b^8*c^5*d^10*f^2 + 64*a^3*b^8*c^7*d^8*f^2 + 108*a^4*b^7*c^2*d^13*f^2 + 148*a^4*b^7*c^4*d^11*f^2 + 32*a^4*b^7*c^6*d^9*f^2 - 296*a^5*b^6*c^3*d^12*f^2 - 456*a^5*b^6*c^5*d^10*f^2 + 96*a^5*b^6*c^7*d^8*f^2 + 580*a^6*b^5*c^2*d^13*f^2 + 312*a^6*b^5*c^4*d^11*f^2 - 328*a^6*b^5*c^6*d^9*f^2 + 144*a^7*b^4*c^3*d^12*f^2 + 352*a^7*b^4*c^5*d^10*f^2 - 126*a^8*b^3*c^2*d^13*f^2 - 154*a^8*b^3*c^4*d^11*f^2 - 4*a^8*b^3*c^6*d^9*f^2 + 36*a^9*b^2*c^3*d^12*f^2 + 4*a^9*b^2*c^5*d^10*f^2 - 128*a*b^10*c*d^14*f^2 + 164*a*b^10*c^3*d^12*f^2 + 260*a*b^10*c^5*d^10*f^2 - 32*a*b^10*c^7*d^8*f^2 + 368*a^3*b^8*c*d^14*f^2 + 256*a^5*b^6*c*d^14*f^2 - 208*a^7*b^4*c*d^14*f^2 + 32*a^9*b^2*c*d^14*f^2 - 2*a^10*b*c^2*d^13*f^2))/(a^8*f^5 + b^8*f^5 + 4*a^2*b^6*f^5 + 6*a^4*b^4*f^5 + 4*a^6*b^2*f^5) - ((-b*(a*d - b*c))^(1/2)*((16*(c + d*tan(e + f*x))^(1/2)*(44*b^13*c*d^12*f^2 - 60*a*b^12*d^13*f^2 + 20*a^3*b^10*d^13*f^2 + 168*a^5*b^8*d^13*f^2 + 40*a^7*b^6*d^13*f^2 - 44*a^9*b^4*d^13*f^2 + 4*a^11*b^2*d^13*f^2 + 92*b^13*c^3*d^10*f^2 - 20*b^13*c^5*d^8*f^2 - 76*a^2*b^11*c^3*d^10*f^2 + 116*a^2*b^11*c^5*d^8*f^2 - 288*a^3*b^10*c^2*d^11*f^2 + 396*a^3*b^10*c^4*d^9*f^2 - 456*a^4*b^9*c^3*d^10*f^2 + 216*a^4*b^9*c^5*d^8*f^2 - 720*a^5*b^8*c^2*d^11*f^2 + 8*a^5*b^8*c^4*d^9*f^2 - 504*a^6*b^7*c^3*d^10*f^2 + 8*a^6*b^7*c^5*d^8*f^2 - 224*a^7*b^6*c^2*d^11*f^2 + 120*a^7*b^6*c^4*d^9*f^2 - 404*a^8*b^5*c^3*d^10*f^2 - 68*a^8*b^5*c^5*d^8*f^2 + 200*a^9*b^4*c^2*d^11*f^2 + 180*a^9*b^4*c^4*d^9*f^2 - 188*a^10*b^3*c^3*d^10*f^2 + 4*a^10*b^3*c^5*d^8*f^2 + 64*a^11*b^2*c^2*d^11*f^2 - 4*a^11*b^2*c^4*d^9*f^2 - 4*a^12*b*c*d^12*f^2 + 72*a*b^12*c^2*d^11*f^2 + 324*a*b^12*c^4*d^9*f^2 - 280*a^2*b^11*c*d^12*f^2 - 348*a^4*b^9*c*d^12*f^2 + 304*a^6*b^7*c*d^12*f^2 + 308*a^8*b^5*c*d^12*f^2 - 24*a^10*b^3*c*d^12*f^2))/(a^8*f^4 + b^8*f^4 + 4*a^2*b^6*f^4 + 6*a^4*b^4*f^4 + 4*a^6*b^2*f^4) + ((-b*(a*d - b*c))^(1/2)*((16*(40*b^15*c*d^11*f^4 - 40*a*b^14*d^12*f^4 - 192*a^3*b^12*d^12*f^4 - 360*a^5*b^10*d^12*f^4 - 320*a^7*b^8*d^12*f^4 - 120*a^9*b^6*d^12*f^4 + 8*a^13*b^2*d^12*f^4 + 40*b^15*c^3*d^9*f^4 + 160*a^2*b^13*c^3*d^9*f^4 - 32*a^3*b^12*c^2*d^10*f^4 + 160*a^3*b^12*c^4*d^8*f^4 + 200*a^4*b^11*c^3*d^9*f^4 - 40*a^5*b^10*c^2*d^10*f^4 + 320*a^5*b^10*c^4*d^8*f^4 + 320*a^7*b^8*c^4*d^8*f^4 - 200*a^8*b^7*c^3*d^9*f^4 + 40*a^9*b^6*c^2*d^10*f^4 + 160*a^9*b^6*c^4*d^8*f^4 - 160*a^10*b^5*c^3*d^9*f^4 + 32*a^11*b^4*c^2*d^10*f^4 + 32*a^11*b^4*c^4*d^8*f^4 - 40*a^12*b^3*c^3*d^9*f^4 + 8*a^13*b^2*c^2*d^10*f^4 - 8*a*b^14*c^2*d^10*f^4 + 32*a*b^14*c^4*d^8*f^4 + 160*a^2*b^13*c*d^11*f^4 + 200*a^4*b^11*c*d^11*f^4 - 200*

a^8*b^7*c*d^11*f^4 - 160*a^10*b^5*c*d^11*f^4 - 40*a^12*b^3*c*d^11*f^4))/(a^
8*f^5 + b^8*f^5 + 4*a^2*b^6*f^5 + 6*a^4*b^4*f^5 + 4*a^6*b^2*f^5) - (8*(-b*(
a*d - b*c))^(1/2)*(c + d*tan(e + f*x))^(1/2)*(3*b^2*d - a^2*d + 4*a*b*c)*(3
2*b^17*d^10*f^4 + 160*a^2*b^15*d^10*f^4 + 288*a^4*b^13*d^10*f^4 + 160*a^6*b
^11*d^10*f^4 - 160*a^8*b^9*d^10*f^4 - 288*a^10*b^7*d^10*f^4 - 160*a^12*b^5*
d^10*f^4 - 32*a^14*b^3*d^10*f^4 + 48*b^17*c^2*d^8*f^4 + 272*a^2*b^15*c^2*d^
8*f^4 + 624*a^4*b^13*c^2*d^8*f^4 + 720*a^6*b^11*c^2*d^8*f^4 + 400*a^8*b^9*c
^2*d^8*f^4 + 48*a^10*b^7*c^2*d^8*f^4 - 48*a^12*b^5*c^2*d^8*f^4 - 16*a^14*b^
3*c^2*d^8*f^4 + 16*a*b^16*c*d^9*f^4 + 112*a^3*b^14*c*d^9*f^4 + 336*a^5*b^12
*c*d^9*f^4 + 560*a^7*b^10*c*d^9*f^4 + 560*a^9*b^8*c*d^9*f^4 + 336*a^11*b^6*
c*d^9*f^4 + 112*a^13*b^4*c*d^9*f^4 + 16*a^15*b^2*c*d^9*f^4))/((b^5*f + 2*a^
2*b^3*f + a^4*b*f)*(a^8*f^4 + b^8*f^4 + 4*a^2*b^6*f^4 + 6*a^4*b^4*f^4 + 4*a
^6*b^2*f^4)))*(3*b^2*d - a^2*d + 4*a*b*c))/(2*(b^5*f + 2*a^2*b^3*f + a^4*b*
f)))*(3*b^2*d - a^2*d + 4*a*b*c))/(2*(b^5*f + 2*a^2*b^3*f + a^4*b*f)))*(-b*
(a*d - b*c))^(1/2)*(3*b^2*d - a^2*d + 4*a*b*c))/(2*(b^5*f + 2*a^2*b^3*f + a
^4*b*f)))*(-b*(a*d - b*c))^(1/2)*(3*b^2*d - a^2*d + 4*a*b*c)*1i)/(2*(b^5*f
+ 2*a^2*b^3*f + a^4*b*f)) + (((16*(c + d*tan(e + f*x))^(1/2)*(2*b^9*d^16 +
a^8*b*d^16 - 5*a^2*b^7*d^16 + 17*a^4*b^5*d^16 -...

3.1240 $\int \frac{(c+d\tan(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx$

Optimal. Leaf size=341

$$-\frac{(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(ia+b)^3f}+\frac{(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(ia-b)^3f}+\frac{(24a^3bcd-40a}{}$$

```
[Out] -(c-I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(I*a+b)^3/f+(c
+I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(I*a-b)^3/f+1/4*(
24*a^3*b*c*d-40*a*b^3*c*d-3*a^4*d^2-2*a^2*b^2*(12*c^2-13*d^2)+b^4*(8*c^2-3*
d^2))*arctanh(b^(1/2)*(c+d*tan(f*x+e))^(1/2)/(-a*d+b*c)^(1/2))/(a^2+b^2)^3/
f/b^(1/2)/(-a*d+b*c)^(1/2)-1/2*(-a*d+b*c)*(c+d*tan(f*x+e))^(1/2)/(a^2+b^2)/
f/(a+b*tan(f*x+e))^2-1/4*(-3*a^2*d+8*a*b*c+5*b^2*d)*(c+d*tan(f*x+e))^(1/2)/
(a^2+b^2)^2/f/(a+b*tan(f*x+e))
```

Rubi [A]
time = 1.30, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {3648, 3730, 3734, 3620, 3618, 65, 214, 3715}

$$-\frac{\left(-3a^2d+8abc+5b^2d\right)\sqrt{c+d\tan(e+fx)}}{4f\left(a^2+b^2\right)^2(a+b\tan(e+fx))}-\frac{(bc-ad)\sqrt{c+d\tan(e+fx)}}{2f\left(a^2+b^2\right)(a+b\tan(e+fx))^2}+\frac{\left(-3a^4d^2+24a^3bcd-2a^2b^2(12c^2-13d^2)-40ab^3cd+b^4(8c^2-3d^2)\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}f\left(a^2+b^2\right)^3\sqrt{bc-ad}}-\frac{(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(b+ia)^3}+\frac{(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(-b+ia)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^(3/2)/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] -(((c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((I*a +
 b)^3*f)) + ((c + I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]
])/((I*a - b)^3*f) + ((24*a^3*b*c*d - 40*a*b^3*c*d - 3*a^4*d^2 - 2*a^2*b^2*
(12*c^2 - 13*d^2) + b^4*(8*c^2 - 3*d^2))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e
+ f*x]])/Sqrt[b*c - a*d]])/(4*Sqrt[b]*(a^2 + b^2)^3*Sqrt[b*c - a*d]*f) - ((
b*c - a*d)*Sqrt[c + d*Tan[e + f*x]])/(2*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^
2) - ((8*a*b*c - 3*a^2*d + 5*b^2*d)*Sqrt[c + d*Tan[e + f*x]])/(4*(a^2 + b^2
)^2*f*(a + b*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
```

```
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx = -\frac{(bc-ad)\sqrt{c+d\tan(e+fx)}}{2(a^2+b^2)f(a+b\tan(e+fx))^2} - \frac{\int \frac{\frac{1}{2}\left(-5bcd-a\left(4c^2-d^2\right)\right)-2\left(2acd-b\left(c^2-d^2\right)\right)\tan(}{(a+b\tan(e+fx))^2\sqrt{c+d\tan}}}{2(a^2+b^2)}$$

$$= -\frac{(bc-ad)\sqrt{c+d\tan(e+fx)}}{2(a^2+b^2)f(a+b\tan(e+fx))^2} - \frac{(8abc-3a^2d+5b^2d)\sqrt{c+d\tan(e+}}{4(a^2+b^2)^2f(a+b\tan(e+fx))}$$

$$= -\frac{(bc-ad)\sqrt{c+d\tan(e+fx)}}{2(a^2+b^2)f(a+b\tan(e+fx))^2} - \frac{(8abc-3a^2d+5b^2d)\sqrt{c+d\tan(e+}}{4(a^2+b^2)^2f(a+b\tan(e+fx))}$$

$$= -\frac{(bc-ad)\sqrt{c+d\tan(e+fx)}}{2(a^2+b^2)f(a+b\tan(e+fx))^2} - \frac{(8abc-3a^2d+5b^2d)\sqrt{c+d\tan(e+}}{4(a^2+b^2)^2f(a+b\tan(e+fx))}$$

$$= -\frac{(bc-ad)\sqrt{c+d\tan(e+fx)}}{2(a^2+b^2)f(a+b\tan(e+fx))^2} - \frac{(8abc-3a^2d+5b^2d)\sqrt{c+d\tan(e+}}{4(a^2+b^2)^2f(a+b\tan(e+fx))}$$

$$= \frac{(24a^3bcd-40ab^3cd-3a^4d^2-2a^2b^2(12c^2-13d^2)+b^4(8c^2-3d^2))\tanh^{-1}\Big(}{4\sqrt{b}(a^2+b^2)^3\sqrt{bc-ad}\,f}$$

$$= -\frac{(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(ia+b)^3f} + \frac{(c+id)^{3/2}\tanh^{-1}\Big(\frac{\sqrt{c+}}{\sqrt{}}}{(ia-b)^3f}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2093 vs. 2(341) = 682.
time = 6.34, size = 2093, normalized size = 6.14

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^(3/2)/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] -1/2*(b^2*(c + d*Tan[e + f*x])^(5/2))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan
[e + f*x])^2) - (-((b*d*(c + d*Tan[e + f*x])^(3/2))/(f*(a + b*Tan[e + f*x])
)) + (2*(-1/2*(b*d*(b*c - a*d)*Sqrt[c + d*Tan[e + f*x]])/(f*(a + b*Tan[e +
f*x])) - (2*(-((((I*Sqrt[c - I*d]*(b*(b*c - a*d)*((3*b^3*d*(b*c - a*d)^2)/8
 + (b^3*(b*c - a*d)*(4*a*c^2 + 5*b*c*d - a*d^2))/8 + (a*b^2*(b*c - a*d)*(b*
c^2 - 2*a*c*d - b*d^2))/2) + a*((b^2*(b*c - a*d)*(4*a*c^2 + 5*b*c*d - a*d^2
)*((b^2*d)/2 - a*(b*c - a*d)))/8 + (-(b*c) + (a*d)/2)*((3*a*b^2*d*(b*c - a*
d)^2)/8 - (b^3*(b*c - a*d)*(b*c^2 - 2*a*c*d - b*d^2))/2) - (d*((b^4*(b*c -
a*d)*(4*a*c^2 + 5*b*c*d - a*d^2))/8 - a*((3*a*b^2*d*(b*c - a*d)^2)/8 - (b^3
*(b*c - a*d)*(b*c^2 - 2*a*c*d - b*d^2))/2)))/2) - I*(a*(b*c - a*d)*((3*b^3*
d*(b*c - a*d)^2)/8 + (b^3*(b*c - a*d)*(4*a*c^2 + 5*b*c*d - a*d^2))/8 + (a*b
^2*(b*c - a*d)*(b*c^2 - 2*a*c*d - b*d^2))/2) - b*((b^2*(b*c - a*d)*(4*a*c^2
 + 5*b*c*d - a*d^2)*((b^2*d)/2 - a*(b*c - a*d)))/8 + (-(b*c) + (a*d)/2)*((3
*a*b^2*d*(b*c - a*d)^2)/8 - (b^3*(b*c - a*d)*(b*c^2 - 2*a*c*d - b*d^2))/2)
- (d*((b^4*(b*c - a*d)*(4*a*c^2 + 5*b*c*d - a*d^2))/8 - a*((3*a*b^2*d*(b*c
- a*d)^2)/8 - (b^3*(b*c - a*d)*(b*c^2 - 2*a*c*d - b*d^2))/2)))/2)))*ArcTanh
[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((-c + I*d)*f) - (I*Sqrt[c + I*d]
*(b*(b*c - a*d)*((3*b^3*d*(b*c - a*d)^2)/8 + (b^3*(b*c - a*d)*(4*a*c^2 + 5*
b*c*d - a*d^2))/8 + (a*b^2*(b*c - a*d)*(b*c^2 - 2*a*c*d - b*d^2))/2) + a*((
b^2*(b*c - a*d)*(4*a*c^2 + 5*b*c*d - a*d^2)*((b^2*d)/2 - a*(b*c - a*d)))/8
+ (-(b*c) + (a*d)/2)*((3*a*b^2*d*(b*c - a*d)^2)/8 - (b^3*(b*c - a*d)*(b*c^2
 - 2*a*c*d - b*d^2))/2) - (d*((b^4*(b*c - a*d)*(4*a*c^2 + 5*b*c*d - a*d^2))
/8 - a*((3*a*b^2*d*(b*c - a*d)^2)/8 - (b^3*(b*c - a*d)*(b*c^2 - 2*a*c*d - b
*d^2))/2)))/2) + I*(a*(b*c - a*d)*((3*b^3*d*(b*c - a*d)^2)/8 + (b^3*(b*c -
a*d)*(4*a*c^2 + 5*b*c*d - a*d^2))/8 + (a*b^2*(b*c - a*d)*(b*c^2 - 2*a*c*d -
 b*d^2))/2) - b*((b^2*(b*c - a*d)*(4*a*c^2 + 5*b*c*d - a*d^2)*((b^2*d)/2 -
a*(b*c - a*d)))/8 + (-(b*c) + (a*d)/2)*((3*a*b^2*d*(b*c - a*d)^2)/8 - (b^3*
(b*c - a*d)*(b*c^2 - 2*a*c*d - b*d^2))/2) - (d*((b^4*(b*c - a*d)*(4*a*c^2 +
 5*b*c*d - a*d^2))/8 - a*((3*a*b^2*d*(b*c - a*d)^2)/8 - (b^3*(b*c - a*d)*(b
*c^2 - 2*a*c*d - b*d^2))/2)))/2)))*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c
+ I*d]])/((-c - I*d)*f))/(a^2 + b^2) + (2*Sqrt[b*c - a*d]*(-(a*b*(b*c - a*d
)*((3*b^3*d*(b*c - a*d)^2)/8 + (b^3*(b*c - a*d)*(4*a*c^2 + 5*b*c*d - a*d^2)
)/8 + (a*b^2*(b*c - a*d)*(b*c^2 - 2*a*c*d - b*d^2))/2)) + (a^2*d*((b^4*(b*c
 - a*d)*(4*a*c^2 + 5*b*c*d - a*d^2))/8 - a*((3*a*b^2*d*(b*c - a*d)^2)/8 - (
b^3*(b*c - a*d)*(b*c^2 - 2*a*c*d - b*d^2))/2)))/2 + b^2*((b^2*(b*c - a*d)*(
4*a*c^2 + 5*b*c*d - a*d^2)*((b^2*d)/2 - a*(b*c - a*d)))/8 + (-(b*c) + (a*d)
```

```
/2)*((3*a*b^2*d*(b*c - a*d)^2)/8 - (b^3*(b*c - a*d)*(b*c^2 - 2*a*c*d - b*d^
2))/2)))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(Sqrt
[b]*(a^2 + b^2)*(-(b*c) + a*d)*f))/((a^2 + b^2)*(b*c - a*d))) - (((b^4*(b*c
 - a*d)*(4*a*c^2 + 5*b*c*d - a*d^2))/8 - a*((3*a*b^2*d*(b*c - a*d)^2)/8 - (
b^3*(b*c - a*d)*(b*c^2 - 2*a*c*d - b*d^2))/2))*Sqrt[c + d*Tan[e + f*x]])/((
a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x]))))/b))/b)/(2*(a^2 + b^2)*(b*c
 - a*d))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1871 vs. $2(303) = 606$.

time = 0.61, size = 1872, normalized size = 5.49

method	result	size
derivativedivides	Expression too large to display	1872
default	Expression too large to display	1872

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*d^4*(1/d^4/(a^2+b^2)^3*(((3/8*a^4*b*d^2-a^3*c*d*b^2-1/4*a^2*b^3*d^2-c*d
*a*b^4-5/8*b^5*d^2)*(c+d*tan(f*x+e))^(3/2)+1/8*d*(5*a^5*d^2-13*a^4*b*c*d+8*
a^3*b^2*c^2+2*a^3*b^2*d^2-10*a^2*b^3*c*d+8*a*b^4*c^2-3*a*b^4*d^2+3*b^5*c*d)
*(c+d*tan(f*x+e))^(1/2))/((c+d*tan(f*x+e))*b+a*d-b*c)^2+1/8*(3*a^4*d^2-24*a
^3*b*c*d+24*a^2*b^2*c^2-26*a^2*b^2*d^2+40*a*b^3*c*d-8*b^4*c^2+3*b^4*d^2)/((
a*d-b*c)*b)^(1/2)*arctan(b*(c+d*tan(f*x+e))^(1/2)/((a*d-b*c)*b)^(1/2)))+1/d
^4/(a^2+b^2)^3*(1/4/d*(1/2*(-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*
a^3*c-3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d+3*(c^2+d^2)^(
1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*b^3*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c^2-(2*(c^2+d^2)^(1/2
)+2*c)^(1/2)*a^3*d^2+6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c*d-3*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*a*b^2*c^2+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*d^2-2*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c*d)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1
/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(-2*(c^2+d^2)^(1/2)*a^
3*d^2+6*(c^2+d^2)^(1/2)*a^2*b*c*d+6*(c^2+d^2)^(1/2)*a*b^2*d^2-2*(c^2+d^2)^(
1/2)*b^3*c*d-1/2*(-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c-3*(c
^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d+3*(c^2+d^2)^(1/2)*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)*b^3*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c^2-(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*a^3*d^2+6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c*d-3*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)*a*b^2*c^2+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*d^2-2*(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)*b^3*c*d)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/
2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2
))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/4/d*(-1/2*(-(c^2+d^2)^(1/2)*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*a^3*c-3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a
^2*b*d+3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c+(c^2+d^2)^(1
```

```
/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c
^2-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*d^2+6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^
2*b*c*d-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c^2+3*(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)*a*b^2*d^2-2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c*d)*ln((c+d*tan(f*x+e
))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))+2*(2
*(c^2+d^2)^(1/2)*a^3*d^2-6*(c^2+d^2)^(1/2)*a^2*b*c*d-6*(c^2+d^2)^(1/2)*a*b^
2*d^2+2*(c^2+d^2)^(1/2)*b^3*c*d+1/2*(-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*a^3*c-3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d+3*(c
^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c+(c^2+d^2)^(1/2)*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*b^3*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c^2-(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*a^3*d^2+6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c*d-3*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c^2+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^
2*d^2-2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c*d)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2
))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c
+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + d\tan\left(e + fx\right)\right)^{\frac{3}{2}}}{\left(a + b\tan\left(e + fx\right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(3/2)/(a+b*tan(f*x+e))**3,x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(3/2)/(a + b*tan(e + f*x))**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 36.70, size = 2500, normalized size = 7.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(3/2)/(a + b*tan(e + f*x))^3,x)
```

```
[Out] (((c + d*tan(e + f*x))^(1/2)*(5*a^3*d^3 - 3*a*b^2*d^3 + 3*b^3*c*d^2 + 8*a*b
^2*c^2*d - 13*a^2*b*c*d^2))/(4*(a^4 + b^4 + 2*a^2*b^2)) - (b*(c + d*tan(e +
 f*x))^(3/2)*(5*b^2*d^2 - 3*a^2*d^2 + 8*a*b*c*d))/(4*(a^4 + b^4 + 2*a^2*b^2
)))/(a^2*d^2*f - (2*b^2*c*f - 2*a*b*d*f)*(c + d*tan(e + f*x)) + b^2*c^2*f +
 b^2*f*(c + d*tan(e + f*x))^2 - 2*a*b*c*d*f) - atan(((((1036*a*b^15*d^15*f^
2 - 36*a^15*b*d^15*f^2 - 604*b^16*c*d^14*f^2 - 8988*a^3*b^13*d^15*f^2 + 604
4*a^5*b^11*d^15*f^2 + 34388*a^7*b^9*d^15*f^2 + 10596*a^9*b^7*d^15*f^2 - 667
6*a^11*b^5*d^15*f^2 + 1012*a^13*b^3*d^15*f^2 + 932*b^16*c^3*d^12*f^2 + 1344
*b^16*c^5*d^10*f^2 - 192*b^16*c^7*d^8*f^2 - 30836*a^2*b^14*c^3*d^12*f^2 - 4
8000*a^2*b^14*c^5*d^10*f^2 + 5248*a^2*b^14*c^7*d^8*f^2 + 95076*a^3*b^13*c^2
*d^13*f^2 + 57600*a^3*b^13*c^4*d^11*f^2 - 46464*a^3*b^13*c^6*d^9*f^2 + 1717
2*a^4*b^12*c^3*d^12*f^2 + 69696*a^4*b^12*c^5*d^10*f^2 - 5696*a^4*b^12*c^7*d
^8*f^2 + 47004*a^5*b^11*c^2*d^13*f^2 + 10944*a^5*b^11*c^4*d^11*f^2 - 30016*
a^5*b^11*c^6*d^9*f^2 + 85404*a^6*b^10*c^3*d^12*f^2 + 169344*a^6*b^10*c^5*d^
10*f^2 - 17664*a^6*b^10*c^7*d^8*f^2 - 171180*a^7*b^9*c^2*d^13*f^2 - 119808*
a^7*b^9*c^4*d^11*f^2 + 85760*a^7*b^9*c^6*d^9*f^2 - 4308*a^8*b^8*c^3*d^12*f^
2 - 49728*a^8*b^8*c^5*d^10*f^2 + 3776*a^8*b^8*c^7*d^8*f^2 - 50972*a^9*b^7*c
^2*d^13*f^2 - 24768*a^9*b^7*c^4*d^11*f^2 + 36800*a^9*b^7*c^6*d^9*f^2 - 3535
6*a^10*b^6*c^3*d^12*f^2 - 80512*a^10*b^6*c^5*d^10*f^2 + 10368*a^10*b^6*c^7*
d^8*f^2 + 55916*a^11*b^5*c^2*d^13*f^2 + 37632*a^11*b^5*c^4*d^11*f^2 - 24960
*a^11*b^5*c^6*d^9*f^2 + 6428*a^12*b^4*c^3*d^12*f^2 + 19648*a^12*b^4*c^5*d^1
```

0*f^2 + 64*a^12*b^4*c^7*d^8*f^2 - 4876*a^13*b^3*c^2*d^13*f^2 - 6080*a^13*b^3*c^4*d^11*f^2 - 192*a^13*b^3*c^6*d^9*f^2 + 1012*a^14*b^2*c^3*d^12*f^2 + 128*a^14*b^2*c^5*d^10*f^2 - 11380*a*b^15*c^2*d^13*f^2 - 4672*a*b^15*c^4*d^11*f^2 + 7744*a*b^15*c^6*d^9*f^2 + 22412*a^2*b^14*c*d^14*f^2 - 58220*a^4*b^12*c*d^14*f^2 - 101604*a^6*b^10*c*d^14*f^2 + 49196*a^8*b^8*c*d^14*f^2 + 55524*a^10*b^6*c*d^14*f^2 - 13156*a^12*b^4*c*d^14*f^2 + 884*a^14*b^2*c*d^14*f^2 - 36*a^15*b*c^2*d^13*f^2)/(2*(a^16*f^5 + b^16*f^5 + 8*a^2*b^14*f^5 + 28*a^4*b^12*f^5 + 56*a^6*b^10*f^5 + 70*a^8*b^8*f^5 + 56*a^10*b^6*f^5 + 28*a^12*b^4*f^5 + 8*a^14*b^2*f^5)) + (((8448*a^4*b^18*d^12*f^4 - 640*a^2*b^20*d^12*f^4 - 384*b^22*d^12*f^4 + 44544*a^6*b^16*d^12*f^4 + 102144*a^8*b^14*d^12*f^4 + 134400*a^10*b^12*d^12*f^4 + 107520*a^12*b^10*d^12*f^4 + 50688*a^14*b^8*d^12*f^4 + 11904*a^16*b^6*d^12*f^4 + 384*a^18*b^4*d^12*f^4 - 256*a^20*b^2*d^12*f^4 + 384*b^22*c^2*d^10*f^4 + 768*b^22*c^4*d^8*f^4 + 4224*a^2*b^20*c^2*d^10*f^4 + 4864*a^2*b^20*c^4*d^8*f^4 - 27136*a^3*b^19*c^3*d^9*f^4 + 19712*a^4*b^18*c^2*d^10*f^4 + 11264*a^4*b^18*c^4*d^8*f^4 - 88064*a^5*b^17*c^3*d^9*f^4 + 51712*a^6*b^16*c^2*d^10*f^4 + 7168*a^6*b^16*c^4*d^8*f^4 - 157696*a^7*b^15*c^3*d^9*f^4 + 84224*a^8*b^14*c^2*d^10*f^4 - 17920*a^8*b^14*c^4*d^8*f^4 - 164864*a^9*b^13*c^3*d^9*f^4 + 87808*a^10*b^12*c^2*d^10*f^4 - 46592*a^10*b^12*c^4*d^8*f^4 - 93184*a^11*b^11*c^3*d^9*f^4 + 57344*a^12*b^10*c^2*d^10*f^4 - 50176*a^12*b^10*c^4*d^8*f^4 - 14336*a^13*b^9*c^3*d^9*f^4 + 20992*a^14*b^8*c^2*d^10*f^4 - 29696*a^14*b^8*c^4*d^8*f^4 + 14336*a^15*b^7*c^3*d^9*f^4 + 2432*a^16*b^6*c^2*d^10*f^4 - 9472*a^16*b^6*c^4*d^8*f^4 + 8704*a^17*b^5*c^3*d^9*f^4 - 896*a^18*b^4*c^2*d^10*f^4 - 1280*a^18*b^4*c^4*d^8*f^4 + 1536*a^19*b^3*c^3*d^9*f^4 - 256*a^20*b^2*c^2*d^10*f^4 - 3584*a*b^21*c*d^11*f^4 - 3584*a*b^21*c^3*d^9*f^4 - 27136*a^3*b^19*c*d^11*f^4 - 88064*a^5*b^17*c*d^11*f^4 - 157696*a^7*b^15*c*d^11*f^4 - 164864*a^9*b^13*c*d^11*f^4 - 93184*a^11*b^11*c*d^11*f^4 - 14336*a^13*b^9*c*d^11*f^4 + 14336*a^15*b^7*c*d^11*f^4 + 8704*a^17*b^5*c*d^11*f^4 + 1536*a^19*b^3*c*d^11*f^4)/(2*(a^16*f^5 + b^16*f^5 + 8*a^2*b^14*f^5 + 28*a^4*b^12*f^5 + 56*a^6*b^10*f^5 + 70*a^8*b^8*f^5 + 56*a^10*b^6*f^5 + 28*a^12*b^4*f^5 + 8*a^14*b^2*f^5)) + ((c + d*tan(e + f*x))^(1/2)*(-(((8*a^6*c^3*f^2 - 8*b^6*c^3*f^2 - 48*a*b^5*d^3*f^2 - 48*a^5*b*d^3*f^2 - 24*a^6*c*d^2*f^2 + 24*b^6*c*d^2*f^2 + 120*a^2*b^4*c^3*f^2 - 120*a^4*b^2*c^3*f^2 + 160*a^3*b^3*d^3*f^2 + 144*a*b^5*c^2*d*f^2 + 144*a^5*b*c^2*d*f^2 - 360*a^2*b^4*c*d^2*f^2 - 480*a^3*b^3*c^2*d*f^2 + 360*a^4*b^2*c*d^2*f^2)^2/4 - (c^6 + d^6 + 3*c^2*d^4 + 3*c^4*d^2)*(16*a^12*f^4 + 16*b^12*f^4 + 96*a^2*b^10*f^4 + 240*a^4*b^8*f^4 + 320*a^6*b^6*f^4 + 240*a^8*b^4*f^4 + 96*a^10*b^2*f^4))^(1/2) + 4*a^6*c^3*f^2 - 4*b^6*c^3*f^2 - 24*a*b^5*d^3*f^2 - 24*a^5*b*d^3*f^2 - 12*a^6*c*d^2*f^2 + 12*b^6*c*d^2*f^2 + 60*a^2*b^4*c^3*f^2 - 60*a^4*b^2*c^3*f^2 + 80*a^3*b^3*d^3*f^2 + 72*a*b^5*c^2*d*f^2 + 72*a^5*b*c^2*d*f^2 - 180*a^2*b^4*c*d^2*f^2 - 240*a^3*b^3*c^2*d*f^2 + 180*a^4*b^2*c*d^2*f^2)/(16*(a^12*f^4 + b^12*f^4 + 6*a^2*b^10*f^4 + 15*a^4*b^8*f^4 + 20*a^6*b^6*f^4 + 15*a^8*b^4*f^4 + 6*a^10*b^2*f^4)))^(1/2)*(512*b^25*d^10*f^4 + 4608*a^2*b^23*d^10*f^4 + 17920*a^4*b^21*d^10*f^4 + 38400*a^6*b^19*d^10*f^4 + 46080*a^8*b^17*d^10*f^4 + 21504*a^10*b^15*d^10*f^4 - 21504*a^12*b^13*d^10*f^4 - 46080*a^14*b^11*d^10*f^4 - 38400*a^16*b^9*d^10*f^4 - 17920*a^18*b^7*d^10*f^4 -

```
4608*a^20*b^5*d^10*f^4 - 512*a^22*b^3*d^10*f^4...
```

3.1241 $\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=322

$$\frac{(ia+b)^3(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}-\frac{(ia-b)^3(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}+$$

```
[Out] (I*a+b)^3*(c-I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/f-(I*
a-b)^3*(c+I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/f+2*(2*a
^3*c*d-6*a*b^2*c*d+3*a^2*b*(c^2-d^2)-b^3*(c^2-d^2))*(c+d*tan(f*x+e))^(1/2)/
f+2/3*(a^3*d+3*a^2*b*c-3*a*b^2*d-b^3*c)*(c+d*tan(f*x+e))^(3/2)/f+2/5*b*(3*a
^2-b^2)*(c+d*tan(f*x+e))^(5/2)/f-4/63*b^2*(-10*a*d+b*c)*(c+d*tan(f*x+e))^(7
/2)/d^2/f+2/9*b^2*(a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(7/2)/d/f
```

Rubi [A]
time = 0.63, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3647, 3711, 3609, 3620, 3618, 65, 214}

$$\frac{2b(3a^2-b^2)(c+d\tan(e+fx))^{5/2}}{5f}+\frac{2(2a^3cd+3a^2b(c^2-d^2)-6ab^2cd-b^3(c^2-d^2))\sqrt{c+d\tan(e+fx)}}{f}+\frac{2(a^3d+3a^2bc-3ab^2d-b^3c)(c+d\tan(e+fx))^{3/2}}{3f}-\frac{4b^2(bc-10ad)(c+d\tan(e+fx))^{7/2}}{63d^2f}+\frac{2b^2(a+b\tan(e+fx))(c+d\tan(e+fx))^{7/2}}{9df}+\frac{(b+ia)^3(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}-\frac{(-b+ia)^3(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((I*a + b)^3*(c - I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]
])/f - ((I*a - b)^3*(c + I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c
 + I*d]])/f + (2*(2*a^3*c*d - 6*a*b^2*c*d + 3*a^2*b*(c^2 - d^2) - b^3*(c^2
- d^2))*Sqrt[c + d*Tan[e + f*x]])/f + (2*(3*a^2*b*c - b^3*c + a^3*d - 3*a*b
^2*d)*(c + d*Tan[e + f*x])^(3/2))/(3*f) + (2*b*(3*a^2 - b^2)*(c + d*Tan[e +
 f*x])^(5/2))/(5*f) - (4*b^2*(b*c - 10*a*d)*(c + d*Tan[e + f*x])^(7/2))/(63
*d^2*f) + (2*b^2*(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^(7/2))/(9*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^{5/2}\,dx = \frac{2b^2(a+b\tan(e+fx))(c+d\tan(e+fx))^{7/2}}{9df} + \frac{2\int(c+d \cdots}{\cdots}$$

$$= -\frac{4b^2(bc-10ad)(c+d\tan(e+fx))^{7/2}}{63d^2f} + \frac{2b^2(a+b\tan(e \cdots}{\cdots}$$

$$= \frac{2b(3a^2-b^2)\,(c+d\tan(e+fx))^{5/2}}{5f} - \frac{4b^2(bc-10ad)(c+ \cdots}{63d^2 \cdots}$$

$$= \frac{2(3a^2bc-b^3c+a^3d-3ab^2d)\,(c+d\tan(e+fx))^{3/2}}{3f} + \frac{2b(\cdots}{\cdots}$$

$$= \frac{2(2a^3cd-6ab^2cd+3a^2b(c^2-d^2)-b^3(c^2-d^2))\,\sqrt{c+d\,\mathrm{t}\cdots}}{f}$$

$$= \frac{2(2a^3cd-6ab^2cd+3a^2b(c^2-d^2)-b^3(c^2-d^2))\,\sqrt{c+d\,\mathrm{t}\cdots}}{f}$$

$$= \frac{2(2a^3cd-6ab^2cd+3a^2b(c^2-d^2)-b^3(c^2-d^2))\,\sqrt{c+d\,\mathrm{t}\cdots}}{f}$$

$$= \frac{2(2a^3cd-6ab^2cd+3a^2b(c^2-d^2)-b^3(c^2-d^2))\,\sqrt{c+d\,\mathrm{t}\cdots}}{f}$$

$$= \frac{(ia+b)^3(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f} - \frac{(ia \cdots}{\cdots}$$

Mathematica [A]

time = 6.21, size = 413, normalized size = 1.28

$$\frac{2b^2(a+b\tan(e+fx))(c+d\tan(e+fx))^{7/2}}{9df}+\frac{2\left(-\frac{2b^2(bc-10ad)(c+d\tan(e+fx))^{7/2}}{7df}+\frac{i\left(\frac{9}{2}a(a^2-3b^2)d-\frac{9}{2}ib(3a^2-b^2)d\right)\left(\frac{2}{5}(c+d\tan(e+fx))^{5/2}+(c-id)\left(\frac{2}{3}(c+d\tan(e+fx))^{3/2}+(c-id)\left(\frac{2(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{-c+id}+2\sqrt{c+d\tan(e+fx)}\right)\right)\right)}{2f}-\frac{i\left(\frac{9}{2}a(a^2-3b^2)d+\frac{9}{2}ib(3a^2-b^2)d\right)\left(\frac{2}{5}(c+d\tan(e+fx))^{5/2}+(c+id)\left(\frac{2}{3}(c+d\tan(e+fx))^{3/2}+(c+id)\left(\frac{2(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{-c-id}+2\sqrt{c+d\tan(e+fx)}\right)\right)\right)}{2f}\right)}{9d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (2*b^2*(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^(7/2))/(9*d*f) + (2*((-2*b
^2*(b*c - 10*a*d)*(c + d*Tan[e + f*x])^(7/2))/(7*d*f) + ((I/2)*((9*a*(a^2 -
 3*b^2)*d)/2 - ((9*I)/2)*b*(3*a^2 - b^2)*d)*((2*(c + d*Tan[e + f*x])^(5/2))
/5 + (c - I*d)*((2*(c + d*Tan[e + f*x])^(3/2))/3 + (c - I*d)*((2*(c - I*d)^
(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/(-c + I*d) + 2*Sqrt[
c + d*Tan[e + f*x]]))))/f - ((I/2)*((9*a*(a^2 - 3*b^2)*d)/2 + ((9*I)/2)*b*(
```

```
3*a^2 - b^2)*d)*((2*(c + d*Tan[e + f*x])^(5/2))/5 + (c + I*d)*((2*(c + d*Ta
n[e + f*x])^(3/2))/3 + (c + I*d)*((2*(c + I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan
[e + f*x]]/Sqrt[c + I*d]])/(-c - I*d) + 2*Sqrt[c + d*Tan[e + f*x]]))))/f))/
(9*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2490 vs. $2(286) = 572$.

time = 0.50, size = 2491, normalized size = 7.74

method	result	size
derivativedivides	Expression too large to display	2491
default	Expression too large to display	2491

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/d^2*(1/9*b^3*(c+d*tan(f*x+e))^(9/2)+3/7*a*b^2*d*(c+d*tan(f*x+e))^(7/2)-
1/7*b^3*c*(c+d*tan(f*x+e))^(7/2)+3/5*a^2*b*d^2*(c+d*tan(f*x+e))^(5/2)-1/5*b
^3*d^2*(c+d*tan(f*x+e))^(5/2)+1/3*a^3*d^3*(c+d*tan(f*x+e))^(3/2)+a^2*b*c*d^
2*(c+d*tan(f*x+e))^(3/2)-a*b^2*d^3*(c+d*tan(f*x+e))^(3/2)-1/3*b^3*c*d^2*(c+
d*tan(f*x+e))^(3/2)+2*a^3*c*d^3*(c+d*tan(f*x+e))^(1/2)+3*a^2*b*c^2*d^2*(c+d
*tan(f*x+e))^(1/2)-3*a^2*b*d^4*(c+d*tan(f*x+e))^(1/2)-6*a*b^2*c*d^3*(c+d*ta
n(f*x+e))^(1/2)-b^3*c^2*d^2*(c+d*tan(f*x+e))^(1/2)+b^3*d^4*(c+d*tan(f*x+e))
^(1/2)-d^2*(1/4/d*(1/2*((2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^3*c
^2-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^3*d^2-6*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^2*b*c*d-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^
2+d^2)^(1/2)*a*b^2*c^2+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*b^
2*d^2+2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b^3*c*d-(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*a^3*c^3+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c*d^2+9*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c^2*d-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d
^3+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c^3-9*(2*(c^2+d^2)^(1/2)+2*c)^(1/2
)*a*b^2*c*d^2-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c^2*d+(2*(c^2+d^2)^(1/2)+
2*c)^(1/2)*b^3*d^3)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(4*(c^2+d^2)^(1/2)*a^3*c*d^2+6*(c^2+d^2)
^(1/2)*a^2*b*c^2*d-6*(c^2+d^2)^(1/2)*a^2*b*d^3-12*(c^2+d^2)^(1/2)*a*b^2*c*d
^2-2*(c^2+d^2)^(1/2)*b^3*c^2*d+2*(c^2+d^2)^(1/2)*b^3*d^3-1/2*((2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^3*c^2-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2
+d^2)^(1/2)*a^3*d^2-6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^2*b*c
*d-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*b^2*c^2+3*(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*b^2*d^2+2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*(c^2+d^2)^(1/2)*b^3*c*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c^3+3*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*a^3*c*d^2+9*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c^2*d-3
*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d^3+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*
b^2*c^3-9*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c*d^2-3*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*b^3*c^2*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*d^3)*(2*(c^2+d^2)^(1/2
```

```
)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2
)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/4/d*(-1/
2*((2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^3*c^2-(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^3*d^2-6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^
2)^(1/2)*a^2*b*c*d-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*b^2*c^
2+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*b^2*d^2+2*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b^3*c*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*
c^3+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c*d^2+9*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*a^2*b*c^2*d-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d^3+3*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*a*b^2*c^3-9*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c*d^2-3*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c^2*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*d^3)*l
n((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+
d^2)^(1/2))+2*(-4*(c^2+d^2)^(1/2)*a^3*c*d^2-6*(c^2+d^2)^(1/2)*a^2*b*c^2*d+6
*(c^2+d^2)^(1/2)*a^2*b*d^3+12*(c^2+d^2)^(1/2)*a*b^2*c*d^2+2*(c^2+d^2)^(1/2)
*b^3*c^2*d-2*(c^2+d^2)^(1/2)*b^3*d^3+1/2*((2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^
2+d^2)^(1/2)*a^3*c^2-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^3*d^2-
6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^2*b*c*d-3*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*b^2*c^2+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c
^2+d^2)^(1/2)*a*b^2*d^2+2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b^3
*c*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c^3+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*
a^3*c*d^2+9*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c^2*d-3*(2*(c^2+d^2)^(1/2)+
2*c)^(1/2)*a^2*b*d^3+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c^3-9*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*a*b^2*c*d^2-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c^2*d+(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*d^3)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^
2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*
x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))))
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3*(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**3*(c + d*tan(e + f*x))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^3*(c + d*tan(e + f*x))^(5/2),x)
```

```
[Out] \text{Hanged}
```

3.1242 $\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=231

$$-\frac{i(a-ib)^2(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{i(a+ib)^2(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}$$

```
[Out] -I*(a-I*b)^2*(c-I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/f+
I*(a+I*b)^2*(c+I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/f+4
*(a*d+b*c)*(a*c-b*d)*(c+d*tan(f*x+e))^(1/2)/f+2/3*(a^2*d+2*a*b*c-b^2*d)*(c+
d*tan(f*x+e))^(3/2)/f+4/5*a*b*(c+d*tan(f*x+e))^(5/2)/f+2/7*b^2*(c+d*tan(f*x
+e))^(7/2)/d/f
```

Rubi [A]
time = 0.41, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3624, 3609, 3620, 3618, 65, 214}

$$\frac{2(a^2d+2abc-b^2d)(c+d\tan(e+fx))^{3/2}}{3f}+\frac{4ab(c+d\tan(e+fx))^{5/2}}{5f}+\frac{4(ad+bc)(ac-bd)\sqrt{c+d\tan(e+fx)}}{f}-\frac{i(a-ib)^2(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{i(a+ib)^2(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}+\frac{2b^2(c+d\tan(e+fx))^{7/2}}{7df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-I)*(a - I*b)^2*(c - I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c -
 I*d]])/f + (I*(a + I*b)^2*(c + I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]
/Sqrt[c + I*d]])/f + (4*(b*c + a*d)*(a*c - b*d)*Sqrt[c + d*Tan[e + f*x]])/f
 + (2*(2*a*b*c + a^2*d - b^2*d)*(c + d*Tan[e + f*x])^(3/2))/(3*f) + (4*a*b*
(c + d*Tan[e + f*x])^(5/2))/(5*f) + (2*b^2*(c + d*Tan[e + f*x])^(7/2))/(7*d
*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}\,dx = \frac{2b^2(c+d\tan(e+fx))^{7/2}}{7df} + \int (a^2-b^2+2ab\tan(e+fx)$$

$$= \frac{4ab(c+d\tan(e+fx))^{5/2}}{5f} + \frac{2b^2(c+d\tan(e+fx))^{7/2}}{7df} +$$

$$= \frac{2(2abc+a^2d-b^2d)\,(c+d\tan(e+fx))^{3/2}}{3f} + \frac{4ab(c+d\,\mathrm{ta}}{5}$$

$$= \frac{4(bc+ad)(ac-bd)\sqrt{c+d\tan(e+fx)}}{f} + \frac{2(2abc+a^2d}{}$$

$$= \frac{4(bc+ad)(ac-bd)\sqrt{c+d\tan(e+fx)}}{f} + \frac{2(2abc+a^2d}{}$$

$$= \frac{4(bc+ad)(ac-bd)\sqrt{c+d\tan(e+fx)}}{f} + \frac{2(2abc+a^2d}{}$$

$$= \frac{4(bc+ad)(ac-bd)\sqrt{c+d\tan(e+fx)}}{f} + \frac{2(2abc+a^2d}{}$$

$$= -\frac{i(a-ib)^2(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f} +$$

Mathematica [A]
time = 2.14, size = 262, normalized size = 1.13

$$\frac{\frac{4b^2(c+d\tan(e+fx))^{7/2}}{d}+7i(a-ib)^2\left(\frac{2}{5}(c+d\tan(e+fx))^{5/2}+\frac{2}{3}(c-id)\left(-3(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)+\sqrt{c+d\tan(e+fx)}\,(4c-3id+d\tan(e+fx))\right)\right)-7i(a+ib)^2\left(\frac{2}{5}(c+d\tan(e+fx))^{5/2}+\frac{2}{3}(c+id)\left(-3(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)+\sqrt{c+d\tan(e+fx)}\,(4c+3id+d\tan(e+fx))\right)\right)}{14f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((4*b^2*(c + d*Tan[e + f*x])^(7/2))/d + (7*I)*(a - I*b)^2*((2*(c + d*Tan[e
+ f*x])^(5/2))/5 + (2*(c - I*d)*(-3*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[
e + f*x]]/Sqrt[c - I*d]] + Sqrt[c + d*Tan[e + f*x]]*(4*c - (3*I)*d + d*Tan[
e + f*x])))/3) - (7*I)*(a + I*b)^2*((2*(c + d*Tan[e + f*x])^(5/2))/5 + (2*(
c + I*d)*(-3*(c + I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]
] + Sqrt[c + d*Tan[e + f*x]]*(4*c + (3*I)*d + d*Tan[e + f*x])))/3))/(14*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1891 vs. 2(197) = 394.
time = 0.48, size = 1892, normalized size = 8.19

method	result	size
derivativedivides	Expression too large to display	1892
default	Expression too large to display	1892

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/d*(1/7*b^2*(c+d*tan(f*x+e))^(7/2)+2/5*a*b*d*(c+d*tan(f*x+e))^(5/2)+1/3*
a^2*d^2*(c+d*tan(f*x+e))^(3/2)+2/3*a*b*c*d*(c+d*tan(f*x+e))^(3/2)-1/3*b^2*d
^2*(c+d*tan(f*x+e))^(3/2)+2*a^2*c*d^2*(c+d*tan(f*x+e))^(1/2)+2*a*b*c^2*d*(c
+d*tan(f*x+e))^(1/2)-2*a*b*d^3*(c+d*tan(f*x+e))^(1/2)-2*b^2*c*d^2*(c+d*tan(
f*x+e))^(1/2)-d*(1/4/d*(1/2*(-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*a^2*c^2+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d^2+4*(c^2+d^2)^
(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c*d+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*b^2*c^2-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d^
2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^3-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2
*c*d^2-6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^2*d+2*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*a*b*d^3-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^3+3*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*b^2*c*d^2)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(4*(c^2+d^2)^(1/2)*a^2*c*d^2+4*(c^2+d^2)
^(1/2)*a*b*c^2*d-4*(c^2+d^2)^(1/2)*a*b*d^3-4*(c^2+d^2)^(1/2)*b^2*c*d^2+1/2*
(-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^2+(c^2+d^2)^(1/2)*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d^2+4*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*a*b*c*d+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^2-(c^2+
d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d^2+(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)*a^2*c^3-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c*d^2-6*(2*(c^2+d^2)^(1/2)+
2*c)^(1/2)*a*b*c^2*d+2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d^3-(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)*b^2*c^3+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c*d^2)*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x
+e))^(1/2)-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1
/4/d*(1/2*((c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^2-(c^2+d^2)^
(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d^2-4*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*a*b*c*d-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*
c^2+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d^2-(2*(c^2+d^2)^(1/2
)+2*c)^(1/2)*a^2*c^3+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c*d^2+6*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*a*b*c^2*d-2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d^3+(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^3-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c*d^2)
*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^
2+d^2)^(1/2))+2*(4*(c^2+d^2)^(1/2)*a^2*c*d^2+4*(c^2+d^2)^(1/2)*a*b*c^2*d-4*
(c^2+d^2)^(1/2)*a*b*d^3-4*(c^2+d^2)^(1/2)*b^2*c*d^2-1/2*((c^2+d^2)^(1/2)*(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^2-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)*a^2*d^2-4*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c*d-(c^2
+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^2+(c^2+d^2)^(1/2)*(2*(c^2+d
```

```
^2)^(1/2)+2*c)^(1/2)*b^2*d^2-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^3+3*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)*a^2*c*d^2+6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^2*d
-2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d^3+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*
c^3-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c*d^2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2
))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d
^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2*(d*tan(f*x + e) + c)^(5/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tan(e + fx))^2 (c + d \tan(e + fx))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2*(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**2*(c + d*tan(e + f*x))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 68.22, size = 2500, normalized size = 10.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^2*(c + d*tan(e + f*x))^(5/2),x)
```

```
[Out] (((4*b^2*c - 4*a*b*d)/(d*f) - (4*b^2*c)/(d*f))*(c^2 + d^2) - 2*c*(2*c*((4*b
^2*c - 4*a*b*d)/(d*f) - (4*b^2*c)/(d*f)) - (2*(a*d - b*c)^2)/(d*f) + (2*b^2
*(c^2 + d^2))/(d*f)))*(c + d*tan(e + f*x))^(1/2) - (c + d*tan(e + f*x))^(3/
2)*((2*c*((4*b^2*c - 4*a*b*d)/(d*f) - (4*b^2*c)/(d*f)))/3 - (2*(a*d - b*c)^
2)/(3*d*f) + (2*b^2*(c^2 + d^2))/(3*d*f)) - ((4*b^2*c - 4*a*b*d)/(5*d*f) -
(4*b^2*c)/(5*d*f))*(c + d*tan(e + f*x))^(5/2) - atan(((((8*(8*a^2*c*d^5*f^2
 - 8*b^2*c*d^5*f^2 + 8*a^2*c^3*d^3*f^2 - 8*b^2*c^3*d^3*f^2 - 8*a*b*d^6*f^2
+ 8*a*b*c^4*d^2*f^2))/f^3 - 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*(-(((8*a^4*
c^5*f^2 + 8*b^4*c^5*f^2 + 32*a*b^3*d^5*f^2 - 32*a^3*b*d^5*f^2 + 40*a^4*c*d^
4*f^2 + 40*b^4*c*d^4*f^2 - 48*a^2*b^2*c^5*f^2 - 80*a^4*c^3*d^2*f^2 - 80*b^4
*c^3*d^2*f^2 + 480*a^2*b^2*c^3*d^2*f^2 + 160*a*b^3*c^4*d*f^2 - 160*a^3*b*c^
4*d*f^2 - 320*a*b^3*c^2*d^3*f^2 - 240*a^2*b^2*c*d^4*f^2 + 320*a^3*b*c^2*d^3
*f^2)^2/64 - f^4*(a^8*c^10 + a^8*d^10 + b^8*c^10 + b^8*d^10 + 4*a^2*b^6*c^1
0 + 6*a^4*b^4*c^10 + 4*a^6*b^2*c^10 + 4*a^2*b^6*d^10 + 6*a^4*b^4*d^10 + 4*a
^6*b^2*d^10 + 5*a^8*c^2*d^8 + 10*a^8*c^4*d^6 + 10*a^8*c^6*d^4 + 5*a^8*c^8*d
^2 + 5*b^8*c^2*d^8 + 10*b^8*c^4*d^6 + 10*b^8*c^6*d^4 + 5*b^8*c^8*d^2 + 20*a
^2*b^6*c^2*d^8 + 40*a^2*b^6*c^4*d^6 + 40*a^2*b^6*c^6*d^4 + 20*a^2*b^6*c^8*d
^2 + 30*a^4*b^4*c^2*d^8 + 60*a^4*b^4*c^4*d^6 + 60*a^4*b^4*c^6*d^4 + 30*a^4*
b^4*c^8*d^2 + 20*a^6*b^2*c^2*d^8 + 40*a^6*b^2*c^4*d^6 + 40*a^6*b^2*c^6*d^4
+ 20*a^6*b^2*c^8*d^2))^(1/2) + a^4*c^5*f^2 + b^4*c^5*f^2 + 4*a*b^3*d^5*f^2
- 4*a^3*b*d^5*f^2 + 5*a^4*c*d^4*f^2 + 5*b^4*c*d^4*f^2 - 6*a^2*b^2*c^5*f^2 -
 10*a^4*c^3*d^2*f^2 - 10*b^4*c^3*d^2*f^2 + 60*a^2*b^2*c^3*d^2*f^2 + 20*a*b^
3*c^4*d*f^2 - 20*a^3*b*c^4*d*f^2 - 40*a*b^3*c^2*d^3*f^2 - 30*a^2*b^2*c*d^4*
f^2 + 40*a^3*b*c^2*d^3*f^2)/(4*f^4))^(1/2))*(-(((8*a^4*c^5*f^2 + 8*b^4*c^5*
f^2 + 32*a*b^3*d^5*f^2 - 32*a^3*b*d^5*f^2 + 40*a^4*c*d^4*f^2 + 40*b^4*c*d^4
*f^2 - 48*a^2*b^2*c^5*f^2 - 80*a^4*c^3*d^2*f^2 - 80*b^4*c^3*d^2*f^2 + 480*a
^2*b^2*c^3*d^2*f^2 + 160*a*b^3*c^4*d*f^2 - 160*a^3*b*c^4*d*f^2 - 320*a*b^3*
c^2*d^3*f^2 - 240*a^2*b^2*c*d^4*f^2 + 320*a^3*b*c^2*d^3*f^2)^2/64 - f^4*(a^
8*c^10 + a^8*d^10 + b^8*c^10 + b^8*d^10 + 4*a^2*b^6*c^10 + 6*a^4*b^4*c^10 +
 4*a^6*b^2*c^10 + 4*a^2*b^6*d^10 + 6*a^4*b^4*d^10 + 4*a^6*b^2*d^10 + 5*a^8*
c^2*d^8 + 10*a^8*c^4*d^6 + 10*a^8*c^6*d^4 + 5*a^8*c^8*d^2 + 5*b^8*c^2*d^8 +
 10*b^8*c^4*d^6 + 10*b^8*c^6*d^4 + 5*b^8*c^8*d^2 + 20*a^2*b^6*c^2*d^8 + 40*
```

a^2*b^6*c^4*d^6 + 40*a^2*b^6*c^6*d^4 + 20*a^2*b^6*c^8*d^2 + 30*a^4*b^4*c^2*d^8 + 60*a^4*b^4*c^4*d^6 + 60*a^4*b^4*c^6*d^4 + 30*a^4*b^4*c^8*d^2 + 20*a^6*b^2*c^2*d^8 + 40*a^6*b^2*c^4*d^6 + 40*a^6*b^2*c^6*d^4 + 20*a^6*b^2*c^8*d^2))^(1/2) + a^4*c^5*f^2 + b^4*c^5*f^2 + 4*a*b^3*d^5*f^2 - 4*a^3*b*d^5*f^2 + 5*a^4*c*d^4*f^2 + 5*b^4*c*d^4*f^2 - 6*a^2*b^2*c^5*f^2 - 10*a^4*c^3*d^2*f^2 - 10*b^4*c^3*d^2*f^2 + 60*a^2*b^2*c^3*d^2*f^2 + 20*a*b^3*c^4*d*f^2 - 20*a^3*b*c^4*d*f^2 - 40*a*b^3*c^2*d^3*f^2 - 30*a^2*b^2*c*d^4*f^2 + 40*a^3*b*c^2*d^3*f^2)/(4*f^4))^(1/2) + (16*(c + d*tan(e + f*x))^(1/2)*(a^4*d^8 + b^4*d^8 - 6*a^2*b^2*d^8 - 15*a^4*c^2*d^6 + 15*a^4*c^4*d^4 - a^4*c^6*d^2 - 15*b^4*c^2*d^6 + 15*b^4*c^4*d^4 - b^4*c^6*d^2 + 80*a*b^3*c^3*d^5 - 24*a*b^3*c^5*d^3 - 80*a^3*b*c^3*d^5 + 24*a^3*b*c^5*d^3 + 90*a^2*b^2*c^2*d^6 - 90*a^2*b^2*c^4*d^4 + 6*a^2*b^2*c^6*d^2 - 24*a*b^3*c*d^7 + 24*a^3*b*c*d^7))/f^2)*(-(((8*a^4*c^5*f^2 + 8*b^4*c^5*f^2 + 32*a*b^3*d^5*f^2 - 32*a^3*b*d^5*f^2 + 40*a^4*c*d^4*f^2 + 40*b^4*c*d^4*f^2 - 48*a^2*b^2*c^5*f^2 - 80*a^4*c^3*d^2*f^2 - 80*b^4*c^3*d^2*f^2 + 480*a^2*b^2*c^3*d^2*f^2 + 160*a*b^3*c^4*d*f^2 - 160*a^3*b*c^4*d*f^2 - 320*a*b^3*c^2*d^3*f^2 - 240*a^2*b^2*c*d^4*f^2 + 320*a^3*b*c^2*d^3*f^2)^2/64 - f^4*(a^8*c^10 + a^8*d^10 + b^8*c^10 + b^8*d^10 + 4*a^2*b^6*c^10 + 6*a^4*b^4*c^10 + 4*a^6*b^2*c^10 + 4*a^2*b^6*d^10 + 6*a^4*b^4*d^10 + 4*a^6*b^2*d^10 + 5*a^8*c^2*d^8 + 10*a^8*c^4*d^6 + 10*a^8*c^6*d^4 + 5*a^8*c^8*d^2 + 5*b^8*c^2*d^8 + 10*b^8*c^4*d^6 + 10*b^8*c^6*d^4 + 5*b^8*c^8*d^2 + 20*a^2*b^6*c^2*d^8 + 40*a^2*b^6*c^4*d^6 + 40*a^2*b^6*c^6*d^4 + 20*a^2*b^6*c^8*d^2 + 30*a^4*b^4*c^2*d^8 + 60*a^4*b^4*c^4*d^6 + 60*a^4*b^4*c^6*d^4 + 30*a^4*b^4*c^8*d^2 + 20*a^6*b^2*c^2*d^8 + 40*a^6*b^2*c^4*d^6 + 40*a^6*b^2*c^6*d^4 + 20*a^6*b^2*c^8*d^2))^(1/2) + a^4*c^5*f^2 + b^4*c^5*f^2 + 4*a*b^3*d^5*f^2 - 4*a^3*b*d^5*f^2 + 5*a^4*c*d^4*f^2 + 5*b^4*c*d^4*f^2 - 6*a^2*b^2*c^5*f^2 - 10*a^4*c^3*d^2*f^2 - 10*b^4*c^3*d^2*f^2 + 60*a^2*b^2*c^3*d^2*f^2 + 20*a*b^3*c^4*d*f^2 - 20*a^3*b*c^4*d*f^2 - 40*a*b^3*c^2*d^3*f^2 - 30*a^2*b^2*c*d^4*f^2 + 40*a^3*b*c^2*d^3*f^2)/(4*f^4))^(1/2)*1i - (((8*(8*a^2*c*d^5*f^2 - 8*b^2*c*d^5*f^2 + 8*a^2*c^3*d^3*f^2 - 8*b^2*c^3*d^3*f^2 - 8*a*b*d^6*f^2 + 8*a*b*c^4*d^2*f^2))/f^3 + 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*(-(((8*a^4*c^5*f^2 + 8*b^4*c^5*f^2 + 32*a*b^3*d^5*f^2 - 32*a^3*b*d^5*f^2 + 40*a^4*c*d^4*f^2 + 40*b^4*c*d^4*f^2 - 48*a^2*b^2*c^5*f^2 - 80*a^4*c^3*d^2*f^2 - 80*b^4*c^3*d^2*f^2 + 480*a^2*b^2*c^3*d^2*f^2 + 160*a*b^3*c^4*d*f^2 - 160*a^3*b*c^4*d*f^2 - 320*a*b^3*c^2*d^3*f^2 - 240*a^2*b^2*c*d^4*f^2 + 320*a^3*b*c^2*d^3*f^2)^2/64 - f^4*(a^8*c^10 + a^8*d^10 + b^8*c^10 + ...

3.1243 $\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=188

$$-\frac{(ia+b)(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{(ia-b)(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}-$$

```
[Out] -(I*a+b)*(c-I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/f+(I*a
-b)*(c+I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/f+2*(2*a*c*
d+b*(c^2-d^2))*(c+d*tan(f*x+e))^(1/2)/f+2/3*(a*d+b*c)*(c+d*tan(f*x+e))^(3/2
)/f+2/5*b*(c+d*tan(f*x+e))^(5/2)/f
```

Rubi [A]
time = 0.30, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3609, 3620, 3618, 65, 214}

$$\frac{2(2acd+b(c^2-d^2))\sqrt{c+d\tan(e+fx)}}{f}+\frac{2(ad+bc)(c+d\tan(e+fx))^{3/2}}{3f}-\frac{(b+ia)(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{(-b+ia)(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}+\frac{2b(c+d\tan(e+fx))^{5/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] -(((I*a + b)*(c - I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]
])/f) + ((I*a - b)*(c + I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c
+ I*d]])/f + (2*(2*a*c*d + b*(c^2 - d^2))*Sqrt[c + d*Tan[e + f*x]])/f + (2*
(b*c + a*d)*(c + d*Tan[e + f*x])^(3/2))/(3*f) + (2*b*(c + d*Tan[e + f*x])^(
5/2))/(5*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
```

```
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}\,dx &= \frac{2b(c+d\tan(e+fx))^{5/2}}{5f}+\int (c+d\tan(e+fx))^{3/2}(ac-b\\
&= \frac{2(bc+ad)(c+d\tan(e+fx))^{3/2}}{3f}+\frac{2b(c+d\tan(e+fx))^{5/}}{5f}\\
&= \frac{2(2acd+b(c^2-d^2))\sqrt{c+d\tan(e+fx)}}{f}+\frac{2(bc+ad)(c}{}\\
&= \frac{2(2acd+b(c^2-d^2))\sqrt{c+d\tan(e+fx)}}{f}+\frac{2(bc+ad)(c}{}\\
&= \frac{2(2acd+b(c^2-d^2))\sqrt{c+d\tan(e+fx)}}{f}+\frac{2(bc+ad)(c}{}\\
&= \frac{2(2acd+b(c^2-d^2))\sqrt{c+d\tan(e+fx)}}{f}+\frac{2(bc+ad)(c}{}\\
&= -\frac{(ia+b)(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{(ia}{}
\end{aligned}$$

Mathematica [A]

time = 1.13, size = 233, normalized size = 1.24

$$\frac{i\left((a-ib)\left(\frac{2}{5}(c+d\tan(e+fx))^{5/2}+\frac{2}{3}(c-id)\left(-3(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)+\sqrt{c+d\tan(e+fx)}\,(4c-3id+d\tan(e+fx))\right)\right)-(a+ib)\left(\frac{2}{5}(c+d\tan(e+fx))^{5/2}+\frac{2}{3}(c+id)\left(-3(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)+\sqrt{c+d\tan(e+fx)}\,(4c+3id+d\tan(e+fx))\right)\right)\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((I/2)*((a - I*b)*((2*(c + d*Tan[e + f*x])^(5/2))/5 + (2*(c - I*d)*(-3*(c -
 I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]] + Sqrt[c + d*Ta
n[e + f*x]]*(4*c - (3*I)*d + d*Tan[e + f*x])))/3) - (a + I*b)*((2*(c + d*Ta
n[e + f*x])^(5/2))/5 + (2*(c + I*d)*(-3*(c + I*d)^(3/2)*ArcTanh[Sqrt[c + d*
Tan[e + f*x]]/Sqrt[c + I*d]] + Sqrt[c + d*Tan[e + f*x]]*(4*c + (3*I)*d + d*
Tan[e + f*x])))/3)))/f
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1250 vs. $2(160) = 320$.

time = 0.45, size = 1251, normalized size = 6.65 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2/5*b*(c+d*tan(f*x+e))^(5/2)+2/3*a*d*(c+d*tan(f*x+e))^(3/2)+2/3*b*c*(c
+d*tan(f*x+e))^(3/2)+4*a*c*d*(c+d*tan(f*x+e))^(1/2)+2*b*c^2*(c+d*tan(f*x+e)
)^(1/2)-2*b*d^2*(c+d*tan(f*x+e))^(1/2)+1/2/d*(1/2*((c^2+d^2)^(1/2)*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*a*c^2-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a
*d^2-2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d-(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*a*c^3+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d^2+3*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*b*c^2*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^3)*ln(d*tan(f*x+
e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+
2*(-4*(c^2+d^2)^(1/2)*a*c*d^2-2*(c^2+d^2)^(1/2)*b*c^2*d+2*(c^2+d^2)^(1/2)*b
*d^3+1/2*((c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2-(c^2+d^2)^(1/
2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^2-2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*b*c*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3+3*(2*(c^2+d^2)^(1/2)+
2*c)^(1/2)*a*c*d^2+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d-(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*b*d^3)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)
^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(
c^2+d^2)^(1/2)-2*c)^(1/2)))+1/2/d*(1/2*(-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*a*c^2+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^2+2*(c^
2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d+(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)*a*c^3-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d^2-3*(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)*b*c^2*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^3)*ln(d*tan(f*x+e)+c+(c+d*
tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(-4*(c^2
+d^2)^(1/2)*a*c*d^2-2*(c^2+d^2)^(1/2)*b*c^2*d+2*(c^2+d^2)^(1/2)*b*d^3-1/2*(
-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2+(c^2+d^2)^(1/2)*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)*a*d^2+2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*b*c*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2
```

```
)*a*c*d^2-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d+(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*b*d^3)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*ar
ctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^
(1/2)-2*c)^(1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-c>0)', see `assume?` for more det
ails)Is
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tan (e + fx)) (c + d \tan (e + fx))^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))*(c + d*tan(e + f*x))**(5/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 34.41, size = 2500, normalized size = 13.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))*(c + d*tan(e + f*x))^(5/2),x)
```

```
[Out] log(- ((((-b^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) + b^2*c^5*f^2 +
5*b^2*c*d^4*f^2 - 10*b^2*c^3*d^2*f^2)/f^4)^(1/2)*(((((-b^4*d^2*f^4*(5*c^4 +
 d^4 - 10*c^2*d^2)^2)^(1/2) + b^2*c^5*f^2 + 5*b^2*c*d^4*f^2 - 10*b^2*c^3*d^
2*f^2)/f^4)^(1/2)*(32*b*d^6 - 32*b*c^4*d^2 + 32*c*d^2*f*(((-b^4*d^2*f^4*(5*
c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) + b^2*c^5*f^2 + 5*b^2*c*d^4*f^2 - 10*b^2*c
^3*d^2*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2)))/(2*f) - (16*b^2*d^2*(c
+ d*tan(e + f*x))^(1/2)*(c^6 - d^6 + 15*c^2*d^4 - 15*c^4*d^2))/f^2))/2 - (8
*b^3*c*d^2*(c^2 - 3*d^2)*(c^2 + d^2)^3)/f^3)*((20*b^4*c^2*d^8*f^4 - b^4*d^1
0*f^4 - 110*b^4*c^4*d^6*f^4 + 100*b^4*c^6*d^4*f^4 - 25*b^4*c^8*d^2*f^4)^(1/
2)/(4*f^4) + (b^2*c^5)/(4*f^2) + (5*b^2*c*d^4)/(4*f^2) - (5*b^2*c^3*d^2)/(2
*f^2))^(1/2) - log(((((-b^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) + b
^2*c^5*f^2 + 5*b^2*c*d^4*f^2 - 10*b^2*c^3*d^2*f^2)/f^4)^(1/2)*(((((-b^4*d^2
*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) + b^2*c^5*f^2 + 5*b^2*c*d^4*f^2 -
10*b^2*c^3*d^2*f^2)/f^4)^(1/2)*(32*b*c^4*d^2 - 32*b*d^6 + 32*c*d^2*f*(((-b^
4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) + b^2*c^5*f^2 + 5*b^2*c*d^4*f
^2 - 10*b^2*c^3*d^2*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2)))/(2*f) - (1
6*b^2*d^2*(c + d*tan(e + f*x))^(1/2)*(c^6 - d^6 + 15*c^2*d^4 - 15*c^4*d^2))
/f^2))/2 - (8*b^3*c*d^2*(c^2 - 3*d^2)*(c^2 + d^2)^3)/f^3)*(((20*b^4*c^2*d^8
*f^4 - b^4*d^10*f^4 - 110*b^4*c^4*d^6*f^4 + 100*b^4*c^6*d^4*f^4 - 25*b^4*c^
8*d^2*f^4)^(1/2) + b^2*c^5*f^2 + 5*b^2*c*d^4*f^2 - 10*b^2*c^3*d^2*f^2)/(4*f
^4))^(1/2) - log(((-((-b^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) - b^
2*c^5*f^2 - 5*b^2*c*d^4*f^2 + 10*b^2*c^3*d^2*f^2)/f^4)^(1/2)*(((-((-b^4*d^2
*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) - b^2*c^5*f^2 - 5*b^2*c*d^4*f^2 +
10*b^2*c^3*d^2*f^2)/f^4)^(1/2)*(32*b*c^4*d^2 - 32*b*d^6 + 32*c*d^2*f*(-((-b
^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) - b^2*c^5*f^2 - 5*b^2*c*d^4*
f^2 + 10*b^2*c^3*d^2*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2)))/(2*f) - (
16*b^2*d^2*(c + d*tan(e + f*x))^(1/2)*(c^6 - d^6 + 15*c^2*d^4 - 15*c^4*d^2)
)/f^2))/2 - (8*b^3*c*d^2*(c^2 - 3*d^2)*(c^2 + d^2)^3)/f^3)*(-((20*b^4*c^2*d
^8*f^4 - b^4*d^10*f^4 - 110*b^4*c^4*d^6*f^4 + 100*b^4*c^6*d^4*f^4 - 25*b^4*
c^8*d^2*f^4)^(1/2) - b^2*c^5*f^2 - 5*b^2*c*d^4*f^2 + 10*b^2*c^3*d^2*f^2)/(4
*f^4))^(1/2) + log(- ((-((-b^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2)
- b^2*c^5*f^2 - 5*b^2*c*d^4*f^2 + 10*b^2*c^3*d^2*f^2)/f^4)^(1/2)*(((-((-b^4
```

```
*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) - b^2*c^5*f^2 - 5*b^2*c*d^4*f^
2 + 10*b^2*c^3*d^2*f^2)/f^4)^(1/2)*(32*b*d^6 - 32*b*c^4*d^2 + 32*c*d^2*f*(-
((-b^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) - b^2*c^5*f^2 - 5*b^2*c*
d^4*f^2 + 10*b^2*c^3*d^2*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2)))/(2*f)
 - (16*b^2*d^2*(c + d*tan(e + f*x))^(1/2)*(c^6 - d^6 + 15*c^2*d^4 - 15*c^4*
d^2))/f^2))/2 - (8*b^3*c*d^2*(c^2 - 3*d^2)*(c^2 + d^2)^3)/f^3)*((b^2*c^5)/(
4*f^2) - (20*b^4*c^2*d^8*f^4 - b^4*d^10*f^4 - 110*b^4*c^4*d^6*f^4 + 100*b^4
*c^6*d^4*f^4 - 25*b^4*c^8*d^2*f^4)^(1/2)/(4*f^4) + (5*b^2*c*d^4)/(4*f^2) -
(5*b^2*c^3*d^2)/(2*f^2))^(1/2) - log(((-((-a^4*d^2*f^4*(5*c^4 + d^4 - 10*c^
2*d^2)^2)^(1/2) + a^2*c^5*f^2 + 5*a^2*c*d^4*f^2 - 10*a^2*c^3*d^2*f^2)/f^4)^
(1/2)*(((-((-a^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) + a^2*c^5*f^2
+ 5*a^2*c*d^4*f^2 - 10*a^2*c^3*d^2*f^2)/f^4)^(1/2)*(64*a*c^3*d^3 + 64*a*c*d
^5 + 32*c*d^2*f*(-((-a^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) + a^2*
c^5*f^2 + 5*a^2*c*d^4*f^2 - 10*a^2*c^3*d^2*f^2)/f^4)^(1/2)*(c + d*tan(e + f
*x))^(1/2)))/(2*f) + (16*a^2*d^2*(c + d*tan(e + f*x))^(1/2)*(c^6 - d^6 + 15
*c^2*d^4 - 15*c^4*d^2))/f^2))/2 - (8*a^3*d^3*(3*c^2 - d^2)*(c^2 + d^2)^3)/f
^3)*(-((20*a^4*c^2*d^8*f^4 - a^4*d^10*f^4 - 110*a^4*c^4*d^6*f^4 + 100*a^4*c
^6*d^4*f^4 - 25*a^4*c^8*d^2*f^4)^(1/2) + a^2*c^5*f^2 + 5*a^2*c*d^4*f^2 - 10
*a^2*c^3*d^2*f^2)/(4*f^4))^(1/2) - log(((((-a^4*d^2*f^4*(5*c^4 + d^4 - 10*c
^2*d^2)^2)^(1/2) - a^2*c^5*f^2 - 5*a^2*c*d^4*f^2 + 10*a^2*c^3*d^2*f^2)/f^4)
^(1/2)*(((((-a^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) - a^2*c^5*f^2
- 5*a^2*c*d^4*f^2 + 10*a^2*c^3*d^2*f^2)/f^4)^(1/2)*(64*a*c^3*d^3 + 64*a*c*d
^5 + 32*c*d^2*f*(((-a^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) - a^2*c
^5*f^2 - 5*a^2*c*d^4*f^2 + 10*a^2*c^3*d^2*f^2)/f^4)^(1/2)*(c + d*tan(e + f*
x))^(1/2)))/(2*f) + (16*a^2*d^2*(c + d*tan(e + f*x))^(1/2)*(c^6 - d^6 + 15*
c^2*d^4 - 15*c^4*d^2))/f^2))/2 - (8*a^3*d^3*(3*c^2 - d^2)*(c^2 + d^2)^3)/f^
3)*(((20*a^4*c^2*d^8*f^4 - a^4*d^10*f^4 - 110*a^4*c^4*d^6*f^4 + 100*a^4*c^6
*d^4*f^4 - 25*a^4*c^8*d^2*f^4)^(1/2) - a^2*c^5*f^2 - 5*a^2*c*d^4*f^2 + 10*a
^2*c^3*d^2*f^2)/(4*f^4))^(1/2) + log(((((-a^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2
*d^2)^2)^(1/2) - a^2*c^5*f^2 - 5*a^2*c*d^4*f^2 + 10*a^2*c^3*d^2*f^2)/f^4)^(
1/2)*(((((-a^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) - a^2*c^5*f^2 -
5*a^2*c*d^4*f^2 + 10*a^2*c^3*d^2*f^2)/f^4)^(1/2)*(64*a*c^3*d^3 + 64*a*c*d^5
 - 32*c*d^2*f*(((-a^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) - a^2*c^5
*f^2 - 5*a^2*c*d^4*f^2 + 10*a^2*c^3*d^2*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x)
)^(1/2)))/(2*f) - (16*a^2*d^2*(c + d*tan(e + f*...
```

3.1244 $\int \frac{(c+d\tan(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=195

$$\frac{(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(ia+b)f}-\frac{(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(ia-b)f}-\frac{2(bc-ad)^{5/2}\tan\cdots}{}$$

```
[Out] (c-I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(I*a+b)/f-(c+I*
d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(I*a-b)/f-2*(-a*d+b*
c)^(5/2)*arctanh(b^(1/2)*(c+d*tan(f*x+e))^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(
a^2+b^2)/f+2*d^2*(c+d*tan(f*x+e))^(1/2)/b/f
```

Rubi [A]
time = 0.61, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3647, 3734, 3620, 3618, 65, 214, 3715}

$$-\frac{2(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{b^{3/2}f\left(a^2+b^2\right)}+\frac{(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(b+ia)}-\frac{(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(-b+ia)}+\frac{2d^2\sqrt{c+d\tan(e+fx)}}{bf}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^(5/2)/(a + b*Tan[e + f*x]),x]
```

```
[Out] ((c - I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((I*a + b
)*f) - ((c + I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/((
I*a - b)*f) - (2*(b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]
])/Sqrt[b*c - a*d]])/(b^(3/2)*(a^2 + b^2)*f) + (2*d^2*Sqrt[c + d*Tan[e + f*
x]])/(b*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
```

```
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx = \frac{2d^2\sqrt{c+d\tan(e+fx)}}{bf} + \frac{2\int \frac{\frac{1}{2}(bc^3-ad^3)+\frac{1}{2}bd(3c^2-d^2)\tan(e+fx)+\frac{1}{2}d^2(3bc-ad)\tan^2(e}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}}{b}$$

$$= \frac{2d^2\sqrt{c+d\tan(e+fx)}}{bf} + \frac{2\int \frac{\frac{1}{2}b(ac^3+3bc^2d-3acd^2-bd^3)+\frac{1}{2}b(ad(3c^2-d^2)-b(c^3-3cd^2))}{\sqrt{c+d\tan(e+fx)}}}{b(a^2+b^2)}$$

$$= \frac{2d^2\sqrt{c+d\tan(e+fx)}}{bf} + \frac{(c-id)^3\int \frac{1+i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx}{2(a-ib)} + \frac{(c+id)^3}{}$$

$$= \frac{2d^2\sqrt{c+d\tan(e+fx)}}{bf} - \frac{(i(c+id)^3)\,\mathrm{Subst}\left(\int \frac{1}{(-1+x)\sqrt{c+idx}}\,dx, x, -i\,t\right.}{2(a+ib)f}$$

$$= -\frac{2(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{b^{3/2}(a^2+b^2)f} + \frac{2d^2\sqrt{c+d\tan(e+f}}{bf}$$

$$= \frac{(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(ia+b)f} - \frac{(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\,t}}{\sqrt{c}}\right.}{(ia-b)f}$$

Mathematica [**A**]
time = 0.59, size = 199, normalized size = 1.02

$$\frac{b^{3/2}(-ia+b)(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)+b^{3/2}(ia+b)(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)-2(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)+2\sqrt{b}(a^2+b^2)d^2\sqrt{c+d\tan(e+fx)}}{b^{3/2}(a^2+b^2)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^(5/2)/(a + b*Tan[e + f*x]),x]
```

```
[Out] (b^(3/2)*((-I)*a + b)*(c - I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt
[c - I*d]] + b^(3/2)*(I*a + b)*(c + I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f
*x]]/Sqrt[c + I*d]] - 2*(b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e
 + f*x]])/Sqrt[b*c - a*d]] + 2*Sqrt[b]*(a^2 + b^2)*d^2*Sqrt[c + d*Tan[e + f
*x]])/(b^(3/2)*(a^2 + b^2)*f)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1277 vs. $2(165) = 330$.
time = 0.56, size = 1278, normalized size = 6.55 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*d^2*(1/b*(c+d*tan(f*x+e))^(1/2)+1/(a^2+b^2)/d^2*(1/4/d*(-1/2*(-(c^2+d^2
)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*a*d^2-2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d+
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d^2
+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^
3)*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(
c^2+d^2)^(1/2))+2*(4*(c^2+d^2)^(1/2)*a*c*d^2-2*(c^2+d^2)^(1/2)*b*c^2*d+2*(c
^2+d^2)^(1/2)*b*d^3+1/2*(-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c
^2+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^2-2*(c^2+d^2)^(1/2)*(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3-3*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d^2+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d-
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^3)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2
+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x
+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/4/d*(1/2*(-(c^2+d^2)^(1/2)*(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*a*d^2-2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d+(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*a*c^3-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d^2+3*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^3)*ln(d*ta
n(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(
1/2))+2*(-4*(c^2+d^2)^(1/2)*a*c*d^2+2*(c^2+d^2)^(1/2)*b*c^2*d-2*(c^2+d^2)^(
1/2)*b*d^3-1/2*(-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2+(c^2+d
^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^2-2*(c^2+d^2)^(1/2)*(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)*b*c*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3-3*(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*a*c*d^2+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d-(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*b*d^3)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/
2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2
))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))))+(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b
^3*c^3)/b/d^2/(a^2+b^2)/((a*d-b*c)*b)^(1/2)*arctan(b*(c+d*tan(f*x+e))^(1/2)
/((a*d-b*c)*b)^(1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")

[Out] Timed out

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\tan(e+fx))^{\frac{5}{2}}}{a+b\tan(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*tan(f*x+e))**(5/2)/(a+b*tan(f*x+e)),x)

[Out] Integral((c + d*tan(e + f*x))**(5/2)/(a + b*tan(e + f*x)), x)

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done

Mupad [B]
time = 13.25, size = 2500, normalized size = 12.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*tan(e + f*x))^(5/2)/(a + b*tan(e + f*x)),x)

[Out] (2*d^2*(c + d*tan(e + f*x))^(1/2))/(b*f) - atan((((((32*(4*b^9*c*d^12*f^4 - 4*a*b^8*d^13*f^4 - 8*a^3*b^6*d^13*f^4 - 4*a^5*b^4*d^13*f^4 + 16*b^9*c^3*d^10*f^4 + 12*b^9*c^5*d^8*f^4 + 40*a^2*b^7*c^3*d^10*f^4 + 24*a^2*b^7*c^5*d^8*f^4 - 48*a^3*b^6*c^2*d^11*f^4 - 40*a^3*b^6*c^4*d^9*f^4 + 32*a^4*b^5*c^3*d^10*f^4 + 12*a^4*b^5*c^5*d^8*f^4 - 24*a^5*b^4*c^2*d^11*f^4 - 20*a^5*b^4*c^4*d^9*f^4 + 8*a^6*b^3*c^3*d^10*f^4 - 24*a*b^8*c^2*d^11*f^4 - 20*a*b^8*c^4*d^9*f^4 + 16*a^2*b^7*c*d^12*f^4 + 20*a^4*b^5*c*d^12*f^4 + 8*a^6*b^3*c*d^12*f^4))/(b*f^5) - (32*(c + d*tan(e + f*x))^(1/2)*(-(c*d^4*5i + 5*c^4*d + c^5*1i + d^5 - 10*c^2*d^3 - c^3*d^2*10i)/(4*(a^2*f^2*1i - b^2*f^2*1i + 2*a*b*f^2))

)^(1/2)*(16*b^10*d^10*f^4 + 16*a^2*b^8*d^10*f^4 - 16*a^4*b^6*d^10*f^4 - 16*a^6*b^4*d^10*f^4 + 24*b^10*c^2*d^8*f^4 + 40*a^2*b^8*c^2*d^8*f^4 + 8*a^4*b^6*c^2*d^8*f^4 - 8*a^6*b^4*c^2*d^8*f^4 + 8*a*b^9*c*d^9*f^4 + 24*a^3*b^7*c*d^9*f^4 + 24*a^5*b^5*c*d^9*f^4 + 8*a^7*b^3*c*d^9*f^4))/(b*f^4))*(-(c*d^4*5i + 5*c^4*d + c^5*1i + d^5 - 10*c^2*d^3 - c^3*d^2*10i)/(4*(a^2*f^2*1i - b^2*f^2*1i + 2*a*b*f^2)))^(1/2) + (32*(c + d*tan(e + f*x))^(1/2)*(16*a^7*b*d^15*f^2 - 14*a*b^7*d^15*f^2 - 8*a^8*c*d^14*f^2 + 38*b^8*c*d^14*f^2 + 4*a^3*b^5*d^15*f^2 + 2*a^5*b^3*d^15*f^2 - 10*b^8*c^3*d^12*f^2 - 102*b^8*c^5*d^10*f^2 + 18*b^8*c^7*d^8*f^2 + 100*a^2*b^6*c^3*d^12*f^2 + 36*a^2*b^6*c^5*d^10*f^2 - 12*a^2*b^6*c^7*d^8*f^2 - 60*a^3*b^5*c^2*d^13*f^2 + 140*a^3*b^5*c^4*d^11*f^2 + 44*a^3*b^5*c^6*d^9*f^2 - 170*a^4*b^4*c^3*d^12*f^2 - 150*a^4*b^4*c^5*d^10*f^2 + 2*a^4*b^4*c^7*d^8*f^2 + 162*a^5*b^3*c^2*d^13*f^2 + 190*a^5*b^3*c^4*d^11*f^2 - 2*a^5*b^3*c^6*d^9*f^2 - 120*a^6*b^2*c^3*d^12*f^2 + 114*a*b^7*c^2*d^13*f^2 + 110*a*b^7*c^4*d^11*f^2 - 114*a*b^7*c^6*d^9*f^2 - 44*a^2*b^6*c*d^14*f^2 - 2*a^4*b^4*c*d^14*f^2 - 88*a^6*b^2*c*d^14*f^2 + 48*a^7*b*c^2*d^13*f^2))/(b*f^4))*(-(c*d^4*5i + 5*c^4*d + c^5*1i + d^5 - 10*c^2*d^3 - c^3*d^2*10i)/(4*(a^2*f^2*1i - b^2*f^2*1i + 2*a*b*f^2)))^(1/2) + (32*(12*a^6*b*d^18*f^2 + 8*a^7*c*d^17*f^2 + a^2*b^5*d^18*f^2 - 15*a^4*b^3*d^18*f^2 + 8*a^7*c^3*d^15*f^2 - 3*b^7*c^2*d^16*f^2 - 48*b^7*c^4*d^14*f^2 + 30*b^7*c^6*d^12*f^2 + 72*b^7*c^8*d^10*f^2 - 3*b^7*c^10*d^8*f^2 - 171*a^2*b^5*c^2*d^16*f^2 + 558*a^2*b^5*c^4*d^14*f^2 + 522*a^2*b^5*c^6*d^12*f^2 - 207*a^2*b^5*c^8*d^10*f^2 + a^2*b^5*c^10*d^8*f^2 - 640*a^3*b^4*c^3*d^15*f^2 - 372*a^3*b^4*c^5*d^13*f^2 + 360*a^3*b^4*c^7*d^11*f^2 + 2*a^3*b^4*c^9*d^9*f^2 + 372*a^4*b^3*c^2*d^16*f^2 + 42*a^4*b^3*c^4*d^14*f^2 - 348*a^4*b^3*c^6*d^12*f^2 - 3*a^4*b^3*c^8*d^10*f^2 + 88*a^5*b^2*c^3*d^15*f^2 + 192*a^5*b^2*c^5*d^13*f^2 + 2*a*b^6*c*d^17*f^2 + 144*a*b^6*c^3*d^15*f^2 - 228*a*b^6*c^5*d^13*f^2 - 312*a*b^6*c^7*d^11*f^2 + 58*a*b^6*c^9*d^9*f^2 + 90*a^3*b^4*c*d^17*f^2 - 104*a^5*b^2*c*d^17*f^2 - 48*a^6*b*c^2*d^16*f^2 - 60*a^6*b*c^4*d^14*f^2))/(b*f^5))*(-(c*d^4*5i + 5*c^4*d + c^5*1i + d^5 - 10*c^2*d^3 - c^3*d^2*10i)/(4*(a^2*f^2*1i - b^2*f^2*1i + 2*a*b*f^2)))^(1/2) - (32*(c + d*tan(e + f*x))^(1/2)*(b^6*d^20 - 2*a^6*d^20 + 30*a^6*c^2*d^18 - 30*a^6*c^4*d^16 + 2*a^6*c^6*d^14 + 6*b^6*c^2*d^18 + 15*b^6*c^4*d^16 + 18*b^6*c^6*d^14 + 45*b^6*c^8*d^12 - 24*b^6*c^10*d^10 + 3*b^6*c^12*d^8 + 12*a*b^5*c^5*d^15 - 180*a*b^5*c^7*d^13 + 180*a*b^5*c^9*d^11 - 12*a*b^5*c^11*d^9 - 180*a^5*b*c^3*d^17 + 180*a^5*b*c^5*d^15 - 12*a^5*b*c^7*d^13 - 30*a^2*b^4*c^4*d^16 + 450*a^2*b^4*c^6*d^14 - 450*a^2*b^4*c^8*d^12 + 30*a^2*b^4*c^10*d^10 + 40*a^3*b^3*c^3*d^17 - 600*a^3*b^3*c^5*d^15 + 600*a^3*b^3*c^7*d^13 - 40*a^3*b^3*c^9*d^11 - 30*a^4*b^2*c^2*d^18 + 450*a^4*b^2*c^4*d^16 - 450*a^4*b^2*c^6*d^14 + 30*a^4*b^2*c^8*d^12 + 12*a^5*b*c*d^19))/(b*f^4))*(-(c*d^4*5i + 5*c^4*d + c^5*1i + d^5 - 10*c^2*d^3 - c^3*d^2*10i)/(4*(a^2*f^2*1i - b^2*f^2*1i + 2*a*b*f^2)))^(1/2)*1i - (((((32*(4*b^9*c*d^12*f^4 - 4*a*b^8*d^13*f^4 - 8*a^3*b^6*d^13*f^4 - 4*a^5*b^4*d^13*f^4 + 16*b^9*c^3*d^10*f^4 + 12*b^9*c^5*d^8*f^4 + 40*a^2*b^7*c^3*d^10*f^4 + 24*a^2*b^7*c^5*d^8*f^4 - 48*a^3*b^6*c^2*d^11*f^4 - 40*a^3*b^6*c^4*d^9*f^4 + 32*a^4*b^5*c^3*d^10*f^4 + 12*a^4*b^5*c^5*d^8*f^4 - 24*a^5*b^4*c^2*d^11*f^4 - 20*a^5*b^4*c^4*d^9*f^4 + 8*a^6*b^3*c^3*d^10*f^4 - 24*a*b^8*c^2*d^11*f^4 - 20*a*b^8*c

^4*d^9*f^4 + 16*a^2*b^7*c*d^12*f^4 + 20*a^4*b^5*c*d^12*f^4 + 8*a^6*b^3*c*d^12*f^4))/(b*f^5) + (32*(c + d*tan(e + f*x))^(1/2)*(-(c*d^4*5i + 5*c^4*d + c^5*1i + d^5 - 10*c^2*d^3 - c^3*d^2*10i)/(4*(a^2*f^2*1i - b^2*f^2*1i + 2*a*b*f^2)))^(1/2)*(16*b^10*d^10*f^4 + 16*a^2*b^8*d^10*f^4 - 16*a^4*b^6*d^10*f^4 - 16*a^6*b^4*d^10*f^4 + 24*b^10*c^2*d^8*f^4 + 40*a^2*b^8*c^2*d^8*f^4 + 8*a^4*b^6*c^2*d^8*f^4 - 8*a^6*b^4*c^2*d^8*f^4 + 8*a*b^9*c*d^9*f^4 + 24*a^3*b^7*c*d^9*f^4 + 24*a^5*b^5*c*d^9*f^4 + 8*a^7*b^3*c*d^9*f^4))/(b*f^4))*(-(c*d^4*5i + 5*c^4*d + c^5*1i + d^5 - 10*c^2*d^3 - c^3*d^2*10i)/(4*(a^2*f^2*1i - b^2*f^2*1i + 2*a*b*f^2)))^(1/2) - (32*(c + d*tan(e + f*x))^(1/2)*(16*a^7*b*d^15*f^2 - 14*a*b^7*d^15*f^2 - 8*a^8*c*d^14*f^2 + 38*b^8*c*d^14*f^2 + 4*a^3*b^5*d^15*f^2 + 2*a^5*b^3*d^15*f^2 - 10*b^8*c^3*d^12*f^2 - 102*b^8*c^5*d^10*f^2 + 18*b^8*c^7*d^8*f^2 + 100*a^2*b^6*c^3*d^12*f^2 + 36*a^2*b^6*c^5*d^10*f^2 - 12*a^2*b^6*c^7*d^8*f^2 - 60*a^3*b^5*c^2*d^...

3.1245 $\int \frac{(c+d\tan(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=243

$$-\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)^2f}+\frac{i(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(a+ib)^2f}-\frac{(bc-ad)^{3/2}\,(4\cdots}{\cdots}$$

```
[Out] -I*(c-I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(a-I*b)^2/f+
I*(c+I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(a+I*b)^2/f-(
-a*d+b*c)^(3/2)*(a^2*d+4*a*b*c+5*b^2*d)*arctanh(b^(1/2)*(c+d*tan(f*x+e))^(1
/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(a^2+b^2)^2/f-(-a*d+b*c)^2*(c+d*tan(f*x+e))^(
1/2)/b/(a^2+b^2)/f/(a+b*tan(f*x+e))
```

Rubi [A]
time = 0.77, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3646, 3734, 3620, 3618, 65, 214, 3715}

$$-\frac{(bc-ad)^2\sqrt{c+d\tan(e+fx)}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{(bc-ad)^{3/2}\left(a^2d+4abc+5b^2d\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{b^{3/2}f\left(a^2+b^2\right)^2}-\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(a-ib)^2}+\frac{i(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(a+ib)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^(5/2)/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] ((-I)*(c - I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((a
- I*b)^2*f) + (I*(c + I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c +
I*d]])/((a + I*b)^2*f) - ((b*c - a*d)^(3/2)*(4*a*b*c + a^2*d + 5*b^2*d)*Arc
Tanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(b^(3/2)*(a^2 + b
^2)^2*f) - ((b*c - a*d)^2*Sqrt[c + d*Tan[e + f*x]])/(b*(a^2 + b^2)*f*(a + b
*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx = -\frac{(bc-ad)^2\sqrt{c+d\tan(e+fx)}}{b(a^2+b^2)f(a+b\tan(e+fx))} + \frac{\int \frac{\frac{1}{2}(5b^2c^2d+a^2d^3+2abc(c^2-2d^2))+b(ad(3c^2-d^2)-b\cdots}{(a+b\tan(e+fx)\cdots}}{\cdots}$$

$$= -\frac{(bc-ad)^2\sqrt{c+d\tan(e+fx)}}{b(a^2+b^2)f(a+b\tan(e+fx))} + \frac{\int \frac{-b(b^2c(c^2-3d^2)-a^2(c^3-3cd^2)-ab(6c^2d-2d^3))-b\cdots}{\sqrt{c+d\,\mathrm{t}\cdots}}}{b(a\cdots}$$

$$= -\frac{(bc-ad)^2\sqrt{c+d\tan(e+fx)}}{b(a^2+b^2)f(a+b\tan(e+fx))} + \frac{(c-id)^3\int\frac{1+i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx}{2(a-ib)^2} + \cdots$$

$$= -\frac{(bc-ad)^2\sqrt{c+d\tan(e+fx)}}{b(a^2+b^2)f(a+b\tan(e+fx))} + \frac{(ic-d)^3\mathrm{Subst}\left(\int\frac{1}{(-1+x)\sqrt{c+idx}}\,dx, x\cdots\right.}{2(a+ib)^2f}$$

$$= -\frac{(bc-ad)^{3/2}(4abc+a^2d+5b^2d)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{b^{3/2}(a^2+b^2)^2f} - \frac{(bc\cdots}{b(a\cdots}$$

$$= -\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)^2f} + \frac{i(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+\cdots}}{\cdots}\right.}{(a+ib)^2f}$$

Mathematica [A]
time = 5.12, size = 333, normalized size = 1.37

$$-\frac{\frac{i(a+ib)^2b^{3/2}(c-id)^{5/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)+ib^{3/2}(ia+b)^2(c+id)^{5/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)+(bc-ad)^{5/2}(4abc+a^2d+5b^2d)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{b^{3/2}(a^2+b^2)}+\frac{d(bc-ad)^2\sqrt{c+d\tan(e+fx)}}{b}+d(bc-ad)(c+d\tan(e+fx))^{3/2}+bd(c+d\tan(e+fx))^{5/2}-\frac{b^2(c+d\tan(e+fx))^{7/2}}{a+b\tan(e+fx)}}{(a^2+b^2)(bc-ad)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^(5/2)/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (-((I*(a + I*b)^2*b^(3/2)*(c - I*d)^(5/2)*(b*c - a*d)*ArcTanh[Sqrt[c + d*Ta
n[e + f*x]]/Sqrt[c - I*d]] + I*b^(3/2)*(I*a + b)^2*(c + I*d)^(5/2)*(b*c - a
*d)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]] + (b*c - a*d)^(5/2)*(4*
a*b*c + a^2*d + 5*b^2*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*
c - a*d]])/(b^(3/2)*(a^2 + b^2))) + (d*(b*c - a*d)^2*Sqrt[c + d*Tan[e + f*x
]])/b + d*(b*c - a*d)*(c + d*Tan[e + f*x])^(3/2) + b*d*(c + d*Tan[e + f*x])
^(5/2) - (b^2*(c + d*Tan[e + f*x])^(7/2))/(a + b*Tan[e + f*x]))/((a^2 + b^2
)*(b*c - a*d)*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1862 vs. $2(211) = 422$.
time = 0.54, size = 1863, normalized size = 7.67

method	result	size
derivativedivides	Expression too large to display	1863
default	Expression too large to display	1863

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*d^3*(1/d^3/(a^2+b^2)^2*(1/4/d*(-1/2*(-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2
)+2*c)^(1/2)*a^2*c^2+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d^2-
4*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c*d+(c^2+d^2)^(1/2)*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^2-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*b^2*d^2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^3-3*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*a^2*c*d^2+6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^2*d-2*(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*a*b*d^3-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^3+3*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*b^2*c*d^2)*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2
)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))+2*(4*(c^2+d^2)^(1/2)*a^2*c*d^2
-4*(c^2+d^2)^(1/2)*a*b*c^2*d+4*(c^2+d^2)^(1/2)*a*b*d^3-4*(c^2+d^2)^(1/2)*b^
2*c*d^2+1/2*(-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^2+(c^2+d^
2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d^2-4*(c^2+d^2)^(1/2)*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*a*b*c*d+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b
^2*c^2-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d^2+(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)*a^2*c^3-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c*d^2+6*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*a*b*c^2*d-2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d^3-(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^3+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c*d
^2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2
*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*
c)^(1/2)))+1/4/d*(1/2*(-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c
^2+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d^2-4*(c^2+d^2)^(1/2)*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c*d+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*b^2*c^2-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d^2+(2*(
c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^3-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c*d^2
+6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^2*d-2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*
a*b*d^3-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^3+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*b^2*c*d^2)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)+(c^2+d^2)^(1/2))+2*(-4*(c^2+d^2)^(1/2)*a^2*c*d^2+4*(c^2+d^2)^(1/2
)*a*b*c^2*d-4*(c^2+d^2)^(1/2)*a*b*d^3+4*(c^2+d^2)^(1/2)*b^2*c*d^2-1/2*(-(c^
2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^2+(c^2+d^2)^(1/2)*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*a^2*d^2-4*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*a*b*c*d+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^2-(c^2+d^2)^
(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d^2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a
^2*c^3-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c*d^2+6*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*a*b*c^2*d-2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d^3-(2*(c^2+d^2)^(1/2)+
2*c)^(1/2)*b^2*c^3+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c*d^2)*(2*(c^2+d^2)^
```

```
(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^
(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))))+(a*d-
b*c)^2/d^3/(a^2+b^2)^2*(-1/2*d*(a^2+b^2)/b*(c+d*tan(f*x+e))^(1/2)/((c+d*tan
(f*x+e))*b+a*d-b*c)+1/2*(a^2*d+4*a*b*c+5*b^2*d)/b/((a*d-b*c)*b)^(1/2)*arcta
n(b*(c+d*tan(f*x+e))^(1/2)/((a*d-b*c)*b)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(5/2)/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 16.40, size = 2500, normalized size = 10.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(5/2)/(a + b*tan(e + f*x))^2,x)
```

```
[Out] - atan(((((8*(4*b^12*c*d^17*f^2 - 16*a^11*b*d^18*f^2 - 8*a^12*c*d^17*f^2 -
4*a*b^11*d^18*f^2 + 304*a^3*b^9*d^18*f^2 + 120*a^5*b^7*d^18*f^2 - 320*a^7*b
^5*d^18*f^2 - 148*a^9*b^3*d^18*f^2 - 8*a^12*c^3*d^15*f^2 - 400*b^12*c^3*d^1
5*f^2 + 176*b^12*c^5*d^13*f^2 + 488*b^12*c^7*d^11*f^2 - 92*b^12*c^9*d^9*f^2
 + 6576*a^2*b^10*c^3*d^15*f^2 + 400*a^2*b^10*c^5*d^13*f^2 - 6144*a^2*b^10*c
^7*d^11*f^2 + 1056*a^2*b^10*c^9*d^9*f^2 - 5408*a^3*b^9*c^2*d^16*f^2 + 3888*
a^3*b^9*c^4*d^14*f^2 + 6400*a^3*b^9*c^6*d^12*f^2 - 3072*a^3*b^9*c^8*d^10*f^
2 + 128*a^3*b^9*c^10*d^8*f^2 - 648*a^4*b^8*c^3*d^15*f^2 - 1952*a^4*b^8*c^5*
d^13*f^2 + 208*a^4*b^8*c^7*d^11*f^2 + 200*a^4*b^8*c^9*d^9*f^2 - 4288*a^5*b^
7*c^2*d^16*f^2 + 4112*a^5*b^7*c^4*d^14*f^2 + 5120*a^5*b^7*c^6*d^12*f^2 - 32
08*a^5*b^7*c^8*d^10*f^2 + 192*a^5*b^7*c^10*d^8*f^2 - 6688*a^6*b^6*c^3*d^15*
f^2 - 2464*a^6*b^6*c^5*d^13*f^2 + 5952*a^6*b^6*c^7*d^11*f^2 - 960*a^6*b^6*c
^9*d^9*f^2 + 2624*a^7*b^5*c^2*d^16*f^2 - 2016*a^7*b^5*c^4*d^14*f^2 - 3456*a
^7*b^5*c^6*d^12*f^2 + 1504*a^7*b^5*c^8*d^10*f^2 + 992*a^8*b^4*c^3*d^15*f^2
- 144*a^8*b^4*c^5*d^13*f^2 - 888*a^8*b^4*c^7*d^11*f^2 - 12*a^8*b^4*c^9*d^9*
f^2 + 352*a^9*b^3*c^2*d^16*f^2 + 520*a^9*b^3*c^4*d^14*f^2 + 32*a^9*b^3*c^6*
d^12*f^2 + 12*a^9*b^3*c^8*d^10*f^2 + 48*a^10*b^2*c^3*d^15*f^2 + 144*a^10*b^
2*c^5*d^13*f^2 + 1120*a*b^11*c^2*d^16*f^2 - 2776*a*b^11*c^4*d^14*f^2 - 2208
*a*b^11*c^6*d^12*f^2 + 1628*a*b^11*c^8*d^10*f^2 - 64*a*b^11*c^10*d^8*f^2 -
1024*a^2*b^10*c*d^17*f^2 + 1312*a^4*b^8*c*d^17*f^2 + 2688*a^6*b^6*c*d^17*f^
2 + 260*a^8*b^4*c*d^17*f^2 - 96*a^10*b^2*c*d^17*f^2 - 32*a^11*b*c^2*d^16*f^
2 - 16*a^11*b*c^4*d^14*f^2))/(b^9*f^5 + a^8*b*f^5 + 4*a^2*b^7*f^5 + 6*a^4*b
^5*f^5 + 4*a^6*b^3*f^5) - (((8*(96*a^2*b^14*d^13*f^4 + 480*a^4*b^12*d^13*f^
4 + 960*a^6*b^10*d^13*f^4 + 960*a^8*b^8*d^13*f^4 + 480*a^10*b^6*d^13*f^4 +
96*a^12*b^4*d^13*f^4 + 128*b^16*c^2*d^11*f^4 + 128*b^16*c^4*d^9*f^4 + 640*a
^2*b^14*c^2*d^11*f^4 + 544*a^2*b^14*c^4*d^9*f^4 - 768*a^3*b^13*c^3*d^10*f^4
 + 320*a^3*b^13*c^5*d^8*f^4 + 1280*a^4*b^12*c^2*d^11*f^4 + 800*a^4*b^12*c^4
*d^9*f^4 - 1440*a^5*b^11*c^3*d^10*f^4 + 640*a^5*b^11*c^5*d^8*f^4 + 1280*a^6
*b^10*c^2*d^11*f^4 + 320*a^6*b^10*c^4*d^9*f^4 - 1280*a^7*b^9*c^3*d^10*f^4 +
 640*a^7*b^9*c^5*d^8*f^4 + 640*a^8*b^8*c^2*d^11*f^4 - 320*a^8*b^8*c^4*d^9*f
^4 - 480*a^9*b^7*c^3*d^10*f^4 + 320*a^9*b^7*c^5*d^8*f^4 + 128*a^10*b^6*c^2*
d^11*f^4 - 352*a^10*b^6*c^4*d^9*f^4 + 64*a^11*b^5*c^5*d^8*f^4 - 96*a^12*b^4
```

*c^4*d^9*f^4 + 32*a^13*b^3*c^3*d^10*f^4 - 224*a*b^15*c*d^12*f^4 - 160*a*b^15*c^3*d^10*f^4 + 64*a*b^15*c^5*d^8*f^4 - 1088*a^3*b^13*c*d^12*f^4 - 2080*a^5*b^11*c*d^12*f^4 - 1920*a^7*b^9*c*d^12*f^4 - 800*a^9*b^7*c*d^12*f^4 - 64*a^11*b^5*c*d^12*f^4 + 32*a^13*b^3*c*d^12*f^4))/(b^9*f^5 + a^8*b*f^5 + 4*a^2*b^7*f^5 + 6*a^4*b^5*f^5 + 4*a^6*b^3*f^5) - (16*(c + d*tan(e + f*x))^(1/2)*((((8*a^4*c^5*f^2 + 8*b^4*c^5*f^2 - 32*a*b^3*d^5*f^2 + 32*a^3*b*d^5*f^2 + 40*a^4*c*d^4*f^2 + 40*b^4*c*d^4*f^2 - 48*a^2*b^2*c^5*f^2 - 80*a^4*c^3*d^2*f^2 - 80*b^4*c^3*d^2*f^2 + 480*a^2*b^2*c^3*d^2*f^2 - 160*a*b^3*c^4*d*f^2 + 160*a^3*b*c^4*d*f^2 + 320*a*b^3*c^2*d^3*f^2 - 240*a^2*b^2*c*d^4*f^2 - 320*a^3*b*c^2*d^3*f^2)^2/4 - (16*a^8*f^4 + 16*b^8*f^4 + 64*a^2*b^6*f^4 + 96*a^4*b^4*f^4 + 64*a^6*b^2*f^4)*(c^10 + d^10 + 5*c^2*d^8 + 10*c^4*d^6 + 10*c^6*d^4 + 5*c^8*d^2))^(1/2) - 4*a^4*c^5*f^2 - 4*b^4*c^5*f^2 + 16*a*b^3*d^5*f^2 - 16*a^3*b*d^5*f^2 - 20*a^4*c*d^4*f^2 - 20*b^4*c*d^4*f^2 + 24*a^2*b^2*c^5*f^2 + 40*a^4*c^3*d^2*f^2 + 40*b^4*c^3*d^2*f^2 - 240*a^2*b^2*c^3*d^2*f^2 + 80*a*b^3*c^4*d*f^2 - 80*a^3*b*c^4*d*f^2 - 160*a*b^3*c^2*d^3*f^2 + 120*a^2*b^2*c*d^4*f^2 + 160*a^3*b*c^2*d^3*f^2)/(16*(a^8*f^4 + b^8*f^4 + 4*a^2*b^6*f^4 + 6*a^4*b^4*f^4 + 4*a^6*b^2*f^4)))^(1/2)*(32*b^18*d^10*f^4 + 160*a^2*b^16*d^10*f^4 + 288*a^4*b^14*d^10*f^4 + 160*a^6*b^12*d^10*f^4 - 160*a^8*b^10*d^10*f^4 - 288*a^10*b^8*d^10*f^4 - 160*a^12*b^6*d^10*f^4 - 32*a^14*b^4*d^10*f^4 + 48*b^18*c^2*d^8*f^4 + 272*a^2*b^16*c^2*d^8*f^4 + 624*a^4*b^14*c^2*d^8*f^4 + 720*a^6*b^12*c^2*d^8*f^4 + 400*a^8*b^10*c^2*d^8*f^4 + 48*a^10*b^8*c^2*d^8*f^4 - 48*a^12*b^6*c^2*d^8*f^4 - 16*a^14*b^4*c^2*d^8*f^4 + 16*a*b^17*c*d^9*f^4 + 112*a^3*b^15*c*d^9*f^4 + 336*a^5*b^13*c*d^9*f^4 + 560*a^7*b^11*c*d^9*f^4 + 560*a^9*b^9*c*d^9*f^4 + 336*a^11*b^7*c*d^9*f^4 + 112*a^13*b^5*c*d^9*f^4 + 16*a^15*b^3*c*d^9*f^4))/(b^9*f^4 + a^8*b*f^4 + 4*a^2*b^7*f^4 + 6*a^4*b^5*f^4 + 4*a^6*b^3*f^4))*((((8*a^4*c^5*f^2 + 8*b^4*c^5*f^2 - 32*a*b^3*d^5*f^2 + 32*a^3*b*d^5*f^2 + 40*a^4*c*d^4*f^2 + 40*b^4*c*d^4*f^2 - 48*a^2*b^2*c^5*f^2 - 80*a^4*c^3*d^2*f^2 - 80*b^4*c^3*d^2*f^2 + 480*a^2*b^2*c^3*d^2*f^2 - 160*a*b^3*c^4*d*f^2 + 160*a^3*b*c^4*d*f^2 + 320*a*b^3*c^2*d^3*f^2 - 240*a^2*b^2*c*d^4*f^2 - 320*a^3*b*c^2*d^3*f^2)^2/4 - (16*a^8*f^4 + 16*b^8*f^4 + 64*a^2*b^6*f^4 + 96*a^4*b^4*f^4 + 64*a^6*b^2*f^4)*(c^10 + d^10 + 5*c^2*d^8 + 10*c^4*d^6 + 10*c^6*d^4 + 5*c^8*d^2))^(1/2) - 4*a^4*c^5*f^2 - 4*b^4*c^5*f^2 + 16*a*b^3*d^5*f^2 - 16*a^3*b*d^5*f^2 - 20*a^4*c*d^4*f^2 - 20*b^4*c*d^4*f^2 + 24*a^2*b^2*c^5*f^2 + 40*a^4*c^3*d^2*f^2 + 40*b^4*c^3*d^2*f^2 - 240*a^2*b^2*c^3*d^2*f^2 + 80*a*b^3*c^4*d*f^2 - 80*a^3*b*c...

3.1246 $\int \frac{(c+d\tan(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx$

Optimal. Leaf size=355

$$-\frac{(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(ia+b)^3f}+\frac{(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(ia-b)^3f}+\frac{\sqrt{bc-ad}\,(8a^3\cdots}{\cdots}$$

```
[Out] -(c-I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(I*a+b)^3/f+(c
+I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(I*a-b)^3/f+1/4*(
8*a^3*b*c*d-56*a*b^3*c*d+a^4*d^2+b^4*(8*c^2-15*d^2)-6*a^2*b^2*(4*c^2-3*d^2)
)*arctanh(b^(1/2)*(c+d*tan(f*x+e))^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)
/b^(3/2)/(a^2+b^2)^3/f-1/2*(-a*d+b*c)^2*(c+d*tan(f*x+e))^(1/2)/b/(a^2+b^2)/
f/(a+b*tan(f*x+e))^2-1/4*(-a*d+b*c)*(a^2*d+8*a*b*c+9*b^2*d)*(c+d*tan(f*x+e)
)^(1/2)/b/(a^2+b^2)^2/f/(a+b*tan(f*x+e))
```

Rubi [A]
time = 1.29, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {3646, 3730, 3734, 3620, 3618, 65, 214, 3715}

$$-\frac{(bc-ad)\left(a^2d+8abc+9b^2d\right)\sqrt{c+d\tan(e+fx)}}{4bf\left(a^2+b^2\right)^2(a+b\tan(e+fx))}-\frac{(bc-ad)^2\sqrt{c+d\tan(e+fx)}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}+\frac{\sqrt{bc-ad}\left(a^4d^2+8a^3bcd-6a^2b^2(4c^2-3d^2)-56ab^3cd+b^4(8c^2-15d^2)\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{4b^{3/2}f\left(a^2+b^2\right)^3}-\frac{(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(b+ia)^3}+\frac{(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(-b+ia)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^(5/2)/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] -(((c - I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((I*a +
 b)^3*f)) + ((c + I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]
])/((I*a - b)^3*f) + (Sqrt[b*c - a*d]*(8*a^3*b*c*d - 56*a*b^3*c*d + a^4*d^2
 + b^4*(8*c^2 - 15*d^2) - 6*a^2*b^2*(4*c^2 - 3*d^2))*ArcTanh[(Sqrt[b]*Sqrt[
c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(4*b^(3/2)*(a^2 + b^2)^3*f) - ((b*c
- a*d)^2*Sqrt[c + d*Tan[e + f*x]])/(2*b*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^
2) - ((b*c - a*d)*(8*a*b*c + a^2*d + 9*b^2*d)*Sqrt[c + d*Tan[e + f*x]])/(4*
b*(a^2 + b^2)^2*f*(a + b*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
```

```
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx = -\frac{(bc-ad)^2\sqrt{c+d\tan(e+fx)}}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}+\frac{\int \frac{\frac{1}{2}\left(9b^2c^2d+a^2d^3+ab\left(4c^3-6cd^2\right)\right)+2b\left(ad\left(3c^2-\right.\right.}{(a+b\tan(}}{}$$

$$= -\frac{(bc-ad)^2\sqrt{c+d\tan(e+fx)}}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}-\frac{(bc-ad)\left(8abc+a^2d+9b^2d\right)\sqrt{c+}}{4b\left(a^2+b^2\right)^2f(a+b\tan(e+}$$

$$= -\frac{(bc-ad)^2\sqrt{c+d\tan(e+fx)}}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}-\frac{(bc-ad)\left(8abc+a^2d+9b^2d\right)\sqrt{c+}}{4b\left(a^2+b^2\right)^2f(a+b\tan(e+}$$

$$= -\frac{(bc-ad)^2\sqrt{c+d\tan(e+fx)}}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}-\frac{(bc-ad)\left(8abc+a^2d+9b^2d\right)\sqrt{c+}}{4b\left(a^2+b^2\right)^2f(a+b\tan(e+}$$

$$= -\frac{(bc-ad)^2\sqrt{c+d\tan(e+fx)}}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}-\frac{(bc-ad)\left(8abc+a^2d+9b^2d\right)\sqrt{c+}}{4b\left(a^2+b^2\right)^2f(a+b\tan(e+}$$

$$= \frac{\sqrt{bc-ad}\left(8a^3bcd-56ab^3cd+a^4d^2+b^4\left(8c^2-15d^2\right)-6a^2b^2\left(4c^2-3d^2\right)\right)\tan}{4b^{3/2}\left(a^2+b^2\right)^3f}$$

$$= -\frac{(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(ia+b)^3f}+\frac{(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+}}{\sqrt{}}\right.}{(ia-b)^3f}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2946 vs. $2(355) = 710$.
time = 6.47, size = 2946, normalized size = 8.30

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^(5/2)/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] -1/2*(b^2*(c + d*Tan[e + f*x])^(7/2))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan
[e + f*x])^2) - (-((b*d*(c + d*Tan[e + f*x])^(5/2))/(f*(a + b*Tan[e + f*x])
)) + (2*((-3*b*d*(b*c - a*d)*(c + d*Tan[e + f*x])^(3/2))/(2*f*(a + b*Tan[e
+ f*x])) + (2*((-3*b*d*(b*c - a*d)^2*Sqrt[c + d*Tan[e + f*x]])/(4*f*(a + b*
Tan[e + f*x])) - (2*(-((((I*Sqrt[c - I*d]*(b*(b*c - a*d)*((3*a*b^3*(b*c - a
*d)*(b*c^3 - 3*a*c^2*d - 3*b*c*d^2 + a*d^3))/4 - (3*b^3*d*(b*c - a*d)*(6*a*
b*c*d + a^2*d^2 - b^2*(3*c^2 - 4*d^2)))/16 + (3*b^3*(b*c - a*d)*(9*b^2*c^2*
d + a^2*d^3 + a*b*(4*c^3 - 6*c*d^2)))/16) + a*((3*b^2*(b*c - a*d)*((b^2*d)/
2 - a*(b*c - a*d))*(9*b^2*c^2*d + a^2*d^3 + a*b*(4*c^3 - 6*c*d^2)))/16 + (-
(b*c) + (a*d)/2)*((-3*b^4*(b*c - a*d)*(b*c^3 - 3*a*c^2*d - 3*b*c*d^2 + a*d^
3))/4 - (3*a*b^2*d*(b*c - a*d)*(6*a*b*c*d + a^2*d^2 - b^2*(3*c^2 - 4*d^2)))
/16) - (d*((3*b^4*(b*c - a*d)*(9*b^2*c^2*d + a^2*d^3 + a*b*(4*c^3 - 6*c*d^2
)))/16 - a*((-3*b^4*(b*c - a*d)*(b*c^3 - 3*a*c^2*d - 3*b*c*d^2 + a*d^3))/4
- (3*a*b^2*d*(b*c - a*d)*(6*a*b*c*d + a^2*d^2 - b^2*(3*c^2 - 4*d^2)))/16)))
/2) - I*(a*(b*c - a*d)*((3*a*b^3*(b*c - a*d)*(b*c^3 - 3*a*c^2*d - 3*b*c*d^2
 + a*d^3))/4 - (3*b^3*d*(b*c - a*d)*(6*a*b*c*d + a^2*d^2 - b^2*(3*c^2 - 4*d
^2)))/16 + (3*b^3*(b*c - a*d)*(9*b^2*c^2*d + a^2*d^3 + a*b*(4*c^3 - 6*c*d^2
)))/16) - b*((3*b^2*(b*c - a*d)*((b^2*d)/2 - a*(b*c - a*d))*(9*b^2*c^2*d +
a^2*d^3 + a*b*(4*c^3 - 6*c*d^2)))/16 + (-(b*c) + (a*d)/2)*((-3*b^4*(b*c - a
*d)*(b*c^3 - 3*a*c^2*d - 3*b*c*d^2 + a*d^3))/4 - (3*a*b^2*d*(b*c - a*d)*(6*
a*b*c*d + a^2*d^2 - b^2*(3*c^2 - 4*d^2)))/16) - (d*((3*b^4*(b*c - a*d)*(9*b
^2*c^2*d + a^2*d^3 + a*b*(4*c^3 - 6*c*d^2)))/16 - a*((-3*b^4*(b*c - a*d)*(b
*c^3 - 3*a*c^2*d - 3*b*c*d^2 + a*d^3))/4 - (3*a*b^2*d*(b*c - a*d)*(6*a*b*c*
d + a^2*d^2 - b^2*(3*c^2 - 4*d^2)))/16)))/2)))*ArcTanh[Sqrt[c + d*Tan[e + f
*x]]/Sqrt[c - I*d]])/((-c + I*d)*f) - (I*Sqrt[c + I*d]*(b*(b*c - a*d)*((3*a
*b^3*(b*c - a*d)*(b*c^3 - 3*a*c^2*d - 3*b*c*d^2 + a*d^3))/4 - (3*b^3*d*(b*c
 - a*d)*(6*a*b*c*d + a^2*d^2 - b^2*(3*c^2 - 4*d^2)))/16 + (3*b^3*(b*c - a*d
)*(9*b^2*c^2*d + a^2*d^3 + a*b*(4*c^3 - 6*c*d^2)))/16) + a*((3*b^2*(b*c - a
*d)*((b^2*d)/2 - a*(b*c - a*d))*(9*b^2*c^2*d + a^2*d^3 + a*b*(4*c^3 - 6*c*d
^2)))/16 + (-(b*c) + (a*d)/2)*((-3*b^4*(b*c - a*d)*(b*c^3 - 3*a*c^2*d - 3*b
*c*d^2 + a*d^3))/4 - (3*a*b^2*d*(b*c - a*d)*(6*a*b*c*d + a^2*d^2 - b^2*(3*c
^2 - 4*d^2)))/16) - (d*((3*b^4*(b*c - a*d)*(9*b^2*c^2*d + a^2*d^3 + a*b*(4*
c^3 - 6*c*d^2)))/16 - a*((-3*b^4*(b*c - a*d)*(b*c^3 - 3*a*c^2*d - 3*b*c*d^2
 + a*d^3))/4 - (3*a*b^2*d*(b*c - a*d)*(6*a*b*c*d + a^2*d^2 - b^2*(3*c^2 - 4
*d^2)))/16)))/2) + I*(a*(b*c - a*d)*((3*a*b^3*(b*c - a*d)*(b*c^3 - 3*a*c^2*
d - 3*b*c*d^2 + a*d^3))/4 - (3*b^3*d*(b*c - a*d)*(6*a*b*c*d + a^2*d^2 - b^2
```

```
*(3*c^2 - 4*d^2)))/16 + (3*b^3*(b*c - a*d)*(9*b^2*c^2*d + a^2*d^3 + a*b*(4*
c^3 - 6*c*d^2)))/16) - b*((3*b^2*(b*c - a*d)*((b^2*d)/2 - a*(b*c - a*d))*(9
*b^2*c^2*d + a^2*d^3 + a*b*(4*c^3 - 6*c*d^2)))/16 + (-(b*c) + (a*d)/2)*((-3
*b^4*(b*c - a*d)*(b*c^3 - 3*a*c^2*d - 3*b*c*d^2 + a*d^3))/4 - (3*a*b^2*d*(b
*c - a*d)*(6*a*b*c*d + a^2*d^2 - b^2*(3*c^2 - 4*d^2)))/16) - (d*((3*b^4*(b*
c - a*d)*(9*b^2*c^2*d + a^2*d^3 + a*b*(4*c^3 - 6*c*d^2)))/16 - a*((-3*b^4*(
b*c - a*d)*(b*c^3 - 3*a*c^2*d - 3*b*c*d^2 + a*d^3))/4 - (3*a*b^2*d*(b*c - a
*d)*(6*a*b*c*d + a^2*d^2 - b^2*(3*c^2 - 4*d^2)))/16)))/2)))*ArcTanh[Sqrt[c
+ d*Tan[e + f*x]]/Sqrt[c + I*d]])/((-c - I*d)*f))/(a^2 + b^2) + (2*Sqrt[b*c
 - a*d]*(-(a*b*(b*c - a*d)*((3*a*b^3*(b*c - a*d)*(b*c^3 - 3*a*c^2*d - 3*b*c
*d^2 + a*d^3))/4 - (3*b^3*d*(b*c - a*d)*(6*a*b*c*d + a^2*d^2 - b^2*(3*c^2 -
 4*d^2)))/16 + (3*b^3*(b*c - a*d)*(9*b^2*c^2*d + a^2*d^3 + a*b*(4*c^3 - 6*c
*d^2)))/16)) + (a^2*d*((3*b^4*(b*c - a*d)*(9*b^2*c^2*d + a^2*d^3 + a*b*(4*c
^3 - 6*c*d^2)))/16 - a*((-3*b^4*(b*c - a*d)*(b*c^3 - 3*a*c^2*d - 3*b*c*d^2
+ a*d^3))/4 - (3*a*b^2*d*(b*c - a*d)*(6*a*b*c*d + a^2*d^2 - b^2*(3*c^2 - 4*
d^2)))/16)))/2 + b^2*((3*b^2*(b*c - a*d)*((b^2*d)/2 - a*(b*c - a*d))*(9*b^2
*c^2*d + a^2*d^3 + a*b*(4*c^3 - 6*c*d^2)))/16 + (-(b*c) + (a*d)/2)*((-3*b^4
*(b*c - a*d)*(b*c^3 - 3*a*c^2*d - 3*b*c*d^2 + a*d^3))/4 - (3*a*b^2*d*(b*c -
 a*d)*(6*a*b*c*d + a^2*d^2 - b^2*(3*c^2 - 4*d^2)))/16)))*ArcTanh[(Sqrt[b]*S
qrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(Sqrt[b]*(a^2 + b^2)*(-(b*c) + a
*d)*f))/((a^2 + b^2)*(b*c - a*d))) - (((3*b^4*(b*c - a*d)*(9*b^2*c^2*d + a^
2*d^3 + a*b*(4*c^3 - 6*c*d^2)))/16 - a*((-3*b^4*(b*c - a*d)*(b*c^3 - 3*a*c^
2*d - 3*b*c*d^2 + a*d^3))/4 - (3*a*b^2*d*(b*c - a*d)*(6*a*b*c*d + a^2*d^2 -
 b^2*(3*c^2 - 4*d^2)))/16))*Sqrt[c + d*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a
*d)*f*(a + b*Tan[e + f*x])))))/b))/b))/(3*b))/(2*(a^2 + b^2)*(b*c - a*d))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2484 vs. $2(317) = 634$.
time = 0.55, size = 2485, normalized size = 7.00

method	result	size
derivativedivides	Expression too large to display	2485
default	Expression too large to display	2485

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*d^4*(1/d^4/(a^2+b^2)^3*(1/4/d*(1/2*(-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2
+d^2)^(1/2)*a^3*c^2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^3*d^2-6
*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^2*b*c*d+3*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*b^2*c^2-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^
2+d^2)^(1/2)*a*b^2*d^2+2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b^3*
c*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c^3-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a
^3*c*d^2+9*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c^2*d-3*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)*a^2*b*d^3-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c^3+9*(2*(c^2+d^2
```

)^(1/2)+2*c)^(1/2)*a*b^2*c*d^2-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c^2*d+(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*d^3)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/
2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(-4*(c^2+d^2)^(1/2)*a^3
*c*d^2+6*(c^2+d^2)^(1/2)*a^2*b*c^2*d-6*(c^2+d^2)^(1/2)*a^2*b*d^3+12*(c^2+d^
2)^(1/2)*a*b^2*c*d^2-2*(c^2+d^2)^(1/2)*b^3*c^2*d+2*(c^2+d^2)^(1/2)*b^3*d^3-
1/2*(-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^3*c^2+(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^3*d^2-6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2
+d^2)^(1/2)*a^2*b*c*d+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*b^2
c^2-3(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*b^2*d^2+2*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b^3*c*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a
^3*c^3-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c*d^2+9*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*a^2*b*c^2*d-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d^3-3*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*a*b^2*c^3+9*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c*d^2-3*(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c^2*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*d^3
)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c
+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)
^(1/2)))+1/4/d*(-1/2*(-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^3*c^
2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^3*d^2-6*(2*(c^2+d^2)^(1/2
)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^2*b*c*d+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2
+d^2)^(1/2)*a*b^2*c^2-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*b^2
d^2+2(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b^3*c*d+(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)*a^3*c^3-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c*d^2+9*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c^2*d-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d^
3-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c^3+9*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*a*b^2*c*d^2-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c^2*d+(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)*b^3*d^3)*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-
d*tan(f*x+e)-c-(c^2+d^2)^(1/2))+2*(4*(c^2+d^2)^(1/2)*a^3*c*d^2-6*(c^2+d^2)^
(1/2)*a^2*b*c^2*d+6*(c^2+d^2)^(1/2)*a^2*b*d^3-12*(c^2+d^2)^(1/2)*a*b^2*c*d^
2+2*(c^2+d^2)^(1/2)*b^3*c^2*d-2*(c^2+d^2)^(1/2)*b^3*d^3+1/2*(-(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^3*c^2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2
+d^2)^(1/2)*a^3*d^2-6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^2*b*c
d+3(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*b^2*c^2-3*(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*b^2*d^2+2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*(c^2+d^2)^(1/2)*b^3*c*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c^3-3*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*a^3*c*d^2+9*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c^2*d-3
(2(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d^3-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*
b^2*c^3+9*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c*d^2-3*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*b^3*c^2*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*d^3)*(2*(c^2+d^2)^(1/2
)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)
^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))))+(a*d-b*c)
/d^4/(a^2+b^2)^3*(((1/8*a^4*d^2+a^3*b*c*d+5/4*a^2*b^2*d^2+a*b^3*c*d+9/8*b^4
d^2)(c+d*tan(f*x+e))^(3/2)-1/8*d*(a^5*d^2-9*a^4*b*c*d+8*a^3*b^2*c^2-6*a^3
*b^2*d^2-2*a^2*b^3*c*d+8*a*b^4*c^2-7*a*b^4*d^2+7*b^5*c*d)/b*(c+d*tan(f*x+e)
)^(1/2))/((c+d*tan(f*x+e))*b+a*d-b*c)^2+1/8*(a^4*d^2+8*a^3*b*c*d-24*a^2*b^2
*c^2+18*a^2*b^2*d^2-56*a*b^3*c*d+8*b^4*c^2-15*b^4*d^2)/b/((a*d-b*c)*b)^(1/2

```
)*arctan(b*(c+d*tan(f*x+e))^(1/2)/((a*d-b*c)*b)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(5/2)/(a+b*tan(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 44.60, size = 2500, normalized size = 7.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(5/2)/(a + b*tan(e + f*x))^3,x)
[Out] atan(((((20*a^16*b*d^18*f^2 + 8*a^17*c*d^17*f^2 - 4796*a^2*b^15*d^18*f^2 +
10476*a^4*b^13*d^18*f^2 + 14772*a^6*b^11*d^18*f^2 - 16644*a^8*b^9*d^18*f^2
- 10996*a^10*b^7*d^18*f^2 + 5892*a^12*b^5*d^18*f^2 + 764*a^14*b^3*d^18*f^2
+ 8*a^17*c^3*d^15*f^2 - 3708*b^17*c^2*d^16*f^2 + 6912*b^17*c^4*d^14*f^2 + 5
820*b^17*c^6*d^12*f^2 - 4608*b^17*c^8*d^10*f^2 + 192*b^17*c^10*d^8*f^2 + 12
5788*a^2*b^15*c^2*d^16*f^2 - 223956*a^2*b^15*c^4*d^14*f^2 - 203692*a^2*b^15
*c^6*d^12*f^2 + 145600*a^2*b^15*c^8*d^10*f^2 - 5248*a^2*b^15*c^10*d^8*f^2 +
 470816*a^3*b^14*c^3*d^15*f^2 - 4848*a^3*b^14*c^5*d^13*f^2 - 482048*a^3*b^1
4*c^7*d^11*f^2 + 73728*a^3*b^14*c^9*d^9*f^2 - 274940*a^4*b^13*c^2*d^16*f^2
+ 324004*a^4*b^13*c^4*d^14*f^2 + 420684*a^4*b^13*c^6*d^12*f^2 - 183040*a^4*
b^13*c^8*d^10*f^2 + 5696*a^4*b^13*c^10*d^8*f^2 + 125696*a^5*b^12*c^3*d^15*f
^2 - 188624*a^5*b^12*c^5*d^13*f^2 - 262656*a^5*b^12*c^7*d^11*f^2 + 51968*a^
5*b^12*c^9*d^9*f^2 - 474836*a^6*b^11*c^2*d^16*f^2 + 859132*a^6*b^11*c^4*d^1
4*f^2 + 822084*a^6*b^11*c^6*d^12*f^2 - 508992*a^6*b^11*c^8*d^10*f^2 + 17664
*a^6*b^11*c^10*d^8*f^2 - 1071584*a^7*b^10*c^3*d^15*f^2 - 235184*a^7*b^10*c^
5*d^13*f^2 + 891392*a^7*b^10*c^7*d^11*f^2 - 133120*a^7*b^10*c^9*d^9*f^2 + 3
25404*a^8*b^9*c^2*d^16*f^2 - 108972*a^8*b^9*c^4*d^14*f^2 - 350476*a^8*b^9*c
^6*d^12*f^2 + 96768*a^8*b^9*c^8*d^10*f^2 - 3776*a^8*b^9*c^10*d^8*f^2 - 3463
52*a^9*b^8*c^3*d^15*f^2 + 31056*a^9*b^8*c^5*d^13*f^2 + 327936*a^9*b^8*c^7*d
^11*f^2 - 64256*a^9*b^8*c^9*d^9*f^2 + 323668*a^10*b^7*c^2*d^16*f^2 - 342716
*a^10*b^7*c^4*d^14*f^2 - 434500*a^10*b^7*c^6*d^12*f^2 + 232512*a^10*b^7*c^8
*d^10*f^2 - 10368*a^10*b^7*c^10*d^8*f^2 + 292448*a^11*b^6*c^3*d^15*f^2 + 83
760*a^11*b^6*c^5*d^13*f^2 - 257792*a^11*b^6*c^7*d^11*f^2 + 34816*a^11*b^6*c
^9*d^9*f^2 - 72724*a^12*b^5*c^2*d^16*f^2 + 65964*a^12*b^5*c^4*d^14*f^2 + 10
4452*a^12*b^5*c^6*d^12*f^2 - 40192*a^12*b^5*c^8*d^10*f^2 - 64*a^12*b^5*c^10
*d^8*f^2 - 24512*a^13*b^4*c^3*d^15*f^2 - 6896*a^13*b^4*c^5*d^13*f^2 + 19456
*a^13*b^4*c^7*d^11*f^2 + 256*a^13*b^4*c^9*d^9*f^2 - 476*a^14*b^3*c^2*d^16*f
^2 - 4460*a^14*b^3*c^4*d^14*f^2 - 3412*a^14*b^3*c^6*d^12*f^2 - 192*a^14*b^3
*c^8*d^10*f^2 + 96*a^15*b^2*c^3*d^15*f^2 - 400*a^15*b^2*c^5*d^13*f^2 + 8504
*a*b^16*c*d^17*f^2 - 63064*a*b^16*c^3*d^15*f^2 + 7792*a*b^16*c^5*d^13*f^2 +
 66816*a*b^16*c^7*d^11*f^2 - 12544*a*b^16*c^9*d^9*f^2 - 80112*a^3*b^14*c*d^
17*f^2 - 304*a^5*b^12*c*d^17*f^2 + 188112*a^7*b^10*c*d^17*f^2 + 14784*a^9*b
^8*c*d^17*f^2 - 83920*a^11*b^6*c*d^17*f^2 + 1584*a^13*b^4*c*d^17*f^2 + 496*
a^15*b^2*c*d^17*f^2 + 112*a^16*b*c^2*d^16*f^2 + 92*a^16*b*c^4*d^14*f^2)/(2*
(b^17*f^5 + a^16*b*f^5 + 8*a^2*b^15*f^5 + 28*a^4*b^13*f^5 + 56*a^6*b^11*f^5
 + 70*a^8*b^9*f^5 + 56*a^10*b^7*f^5 + 28*a^12*b^5*f^5 + 8*a^14*b^3*f^5)) +
(((1664*b^23*c*d^12*f^4 - 1664*a*b^22*d^13*f^4 - 11904*a^3*b^20*d^13*f^4 -
```

35328*a^5*b^18*d^13*f^4 - 53760*a^7*b^16*d^13*f^4 - 37632*a^9*b^14*d^13*f^4 + 5376*a^11*b^12*d^13*f^4 + 32256*a^13*b^10*d^13*f^4 + 26112*a^15*b^8*d^13*f^4 + 9600*a^17*b^6*d^13*f^4 + 1408*a^19*b^4*d^13*f^4 + 896*b^23*c^3*d^10*f^4 - 768*b^23*c^5*d^8*f^4 + 2432*a^2*b^21*c^3*d^10*f^4 - 4864*a^2*b^21*c^5*d^8*f^4 + 29312*a^3*b^20*c^2*d^11*f^4 + 41216*a^3*b^20*c^4*d^9*f^4 - 12288*a^4*b^19*c^3*d^10*f^4 - 11264*a^4*b^19*c^5*d^8*f^4 + 100864*a^5*b^18*c^2*d^11*f^4 + 136192*a^5*b^18*c^4*d^9*f^4 - 78336*a^6*b^17*c^3*d^10*f^4 - 7168*a^6*b^17*c^5*d^8*f^4 + 197120*a^7*b^16*c^2*d^11*f^4 + 250880*a^7*b^16*c^4*d^9*f^4 - 188160*a^8*b^15*c^3*d^10*f^4 + 17920*a^8*b^15*c^5*d^8*f^4 + 238336*a^9*b^14*c^2*d^11*f^4 + 275968*a^9*b^14*c^4*d^9*f^4 - 252672*a^10*b^13*c^3*d^10*f^4 + 46592*a^10*b^13*c^5*d^8*f^4 + 180992*a^11*b^12*c^2*d^11*f^4 + 175616*a^11*b^12*c^4*d^9*f^4 - 204288*a^12*b^11*c^3*d^10*f^4 + 50176*a^12*b^11*c^5*d^8*f^4 + 82432*a^13*b^10*c^2*d^11*f^4 + 50176*a^13*b^10*c^4*d^9*f^4 - 96768*a^14*b^9*c^3*d^10*f^4 + 29696*a^14*b^9*c^5*d^8*f^4 + 18944*a^15*b^8*c^2*d^11*f^4 - 7168*a^15*b^8*c^4*d^9*f^4 - 22656*a^16*b^7*c^3*d^10*f^4 + 9472*a^16*b^7*c^5*d^8*f^4 + 640*a^17*b^6*c^2*d^11*f^4 - 8960*a^17*b^6*c^4*d^9*f^4 - 640*a^18*b^5*c^3*d^10*f^4 + 1280*a^18*b^5*c^5*d^8*f^4 - 384*a^19*b^4*c^2*d^11*f^4 - 1792*a^19*b^4*c^4*d^9*f^4 + 512*a^20*b^3*c^3*d^10*f^4 + 3712*a*b^22*c^2*d^11*f^4 + 5376*a*b^22*c^4*d^9*f^4 + 7296*a^2*b^21*c*d^12*f^4 - 1024*a^4*b^19*c*d^12*f^4 - 71168*a^6*b^17*c*d^12*f^4 - 206080*a^8*b^15*c*d^12*f^4 - 299264*a^10*b^13*c*d^12*f^4 - 254464*a^12*b^11*c*d^12*f^4 - 126464*a^14*b^9*c*d^12*f^4 - 32128*a^16*b^7*c*d^12*f^4 - 1920*a^18*b^5*c*d^12*f^4 + 512*a^20*b^3*c*d^12*f^4)/(2*(b^17*f^5 + a^16*b*f^5 + 8*a^2*b^15*f^5 + 28*a^4*b^13*f^5 + 56*a^6*b^11*f^5 + 70*a^8*b^9*f^5 + 56*a^10*b^7*f^5 + 28*a^12*b^5*f^5 + 8*a^14*b^3*f^5)) + ((c + d*tan(e + f*x))^(1/2)*((((8*a^6*c^5*f^2 - 8*b^6*c^5*f^2 + 48*a*b^5*d^5*f^2 + 48*a^5*b*d^5*f^2 + 40*a^6*c*d^4*f^2 - 40*b^6*c*d^4*f^2 + 120*a^2*b^4*c^5*f^2 - 120*a^4*b^2*c^5*f^2 - 160*a^3*b^3*d^5*f^2 - 80*a^6*c^3*d^2*f^2 + 80*b^6*c^3*d^2*f^2 - 1200*a^2*b^4*c^3*d^2*f^2 + 1600*a^3*b^3*c^2*d^3*f^2 + 1200*a^4*b^2*c^3*d^2*f^2 + 240*a*b^5*c^4*d*f^2 + 240*a^5*b*c^4*d*f^2 - 480*a*b^5*c^2*...

3.1247 $\int \frac{(a+b\tan(e+fx))^4}{\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=248

$$-\frac{i(a-ib)^4\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}\,f}+\frac{i(a+ib)^4\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{\sqrt{c+id}\,f}-\frac{2b^2(40abcd-87a$$

```
[Out] -I*(a-I*b)^4*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/f/(c-I*d)^(1/2)+
I*(a+I*b)^4*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/f/(c+I*d)^(1/2)-2
/15*b^2*(40*a*b*c*d-87*a^2*d^2-b^2*(8*c^2-15*d^2))*(c+d*tan(f*x+e))^(1/2)/d
^3/f-4/15*b^3*(-7*a*d+2*b*c)*(c+d*tan(f*x+e))^(1/2)*tan(f*x+e)/d^2/f+2/5*b^
2*(c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2/d/f
```

Rubi [A]

time = 0.49, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3647, 3718, 3711, 3620, 3618, 65, 214}

$$-\frac{2b^2(-87a^2d^2+40abcd-(b^2(8c^2-15d^2)))\sqrt{c+d\tan(e+fx)}}{15d^3f}-\frac{4b^3(2bc-7ad)\tan(e+fx)\sqrt{c+d\tan(e+fx)}}{15d^2f}+\frac{2b^2(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}-\frac{i(a-ib)^4\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f\sqrt{c-id}}+\frac{i(a+ib)^4\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f\sqrt{c+id}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^4/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((-I)*(a - I*b)^4*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/(Sqrt[c
- I*d]*f) + (I*(a + I*b)^4*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])
/(Sqrt[c + I*d]*f) - (2*b^2*(40*a*b*c*d - 87*a^2*d^2 - b^2*(8*c^2 - 15*d^2)
)*Sqrt[c + d*Tan[e + f*x]])/(15*d^3*f) - (4*b^3*(2*b*c - 7*a*d)*Tan[e + f*x
]*Sqrt[c + d*Tan[e + f*x]])/(15*d^2*f) + (2*b^2*(a + b*Tan[e + f*x])^2*Sqrt
[c + d*Tan[e + f*x]])/(5*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^4}{\sqrt{c+d\tan(e+fx)}}\,dx = \frac{2b^2(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df} + \frac{2\int \frac{(a+b\tan(e+fx))\left(\frac{1}{2}(-4b^3c+5a^3d-a}{\sqrt{}}}{}$$

$$= -\frac{4b^3(2bc-7ad)\tan(e+fx)\sqrt{c+d\tan(e+fx)}}{15d^2f} + \frac{2b^2(a+b\tan(e+fx))^2\sqrt{}}{5df}$$

$$= -\frac{2b^2(40abcd-87a^2d^2-b^2(8c^2-15d^2))\sqrt{c+d\tan(e+fx)}}{15d^3f} - \frac{4b^3(2bc-7a}{}$$

$$= -\frac{2b^2(40abcd-87a^2d^2-b^2(8c^2-15d^2))\sqrt{c+d\tan(e+fx)}}{15d^3f} - \frac{4b^3(2bc-7a}{}$$

$$= -\frac{2b^2(40abcd-87a^2d^2-b^2(8c^2-15d^2))\sqrt{c+d\tan(e+fx)}}{15d^3f} - \frac{4b^3(2bc-7a}{}$$

$$= -\frac{2b^2(40abcd-87a^2d^2-b^2(8c^2-15d^2))\sqrt{c+d\tan(e+fx)}}{15d^3f} - \frac{4b^3(2bc-7a}{}$$

$$= -\frac{i(a-ib)^4\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}\,f} + \frac{i(a+ib)^4\tanh^{-1}\left(\frac{\sqrt{c+d\tan}}{\sqrt{c+}}\right.}{\sqrt{c+id}\,f}$$

Mathematica [A]
time = 3.45, size = 235, normalized size = 0.95

$$\frac{-\frac{15i(a-ib)^4d\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}} + \frac{15i(a+ib)^4d\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{\sqrt{c+id}} + \frac{2b^2(-40abcd+87a^2d^2+b^2(8c^2-15d^2))\sqrt{c+d\tan(e+fx)}}{d^2} + \frac{4b^3(-2bc+7ad)\tan(e+fx)\sqrt{c+d\tan(e+fx)}}{d} + 6b^2(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{15df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^4/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] (((-15*I)*(a - I*b)^4*d*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/Sq
rt[c - I*d] + ((15*I)*(a + I*b)^4*d*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c
 + I*d]])/Sqrt[c + I*d] + (2*b^2*(-40*a*b*c*d + 87*a^2*d^2 + b^2*(8*c^2 - 1
5*d^2))*Sqrt[c + d*Tan[e + f*x]])/d^2 + (4*b^3*(-2*b*c + 7*a*d)*Tan[e + f*x
]*Sqrt[c + d*Tan[e + f*x]])/d + 6*b^2*(a + b*Tan[e + f*x])^2*Sqrt[c + d*Tan
[e + f*x]])/(15*d*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3443 vs. 2(216) = 432.
time = 0.43, size = 3444, normalized size = 13.89

method	result	size
derivativedivides	Expression too large to display	3444
default	Expression too large to display	3444

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^4/(c+d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/d^3*(1/5*b^4*(c+d*tan(f*x+e))^(5/2)+4/3*a*b^3*d*(c+d*tan(f*x+e))^(3/2)-
2/3*b^4*c*(c+d*tan(f*x+e))^(3/2)+6*a^2*b^2*d^2*(c+d*tan(f*x+e))^(1/2)-4*a*b
^3*c*d*(c+d*tan(f*x+e))^(1/2)+b^4*c^2*(c+d*tan(f*x+e))^(1/2)-b^4*d^2*(c+d*t
an(f*x+e))^(1/2)+d^3*(1/4/d^2/(c^2+d^2)^(3/2)*(-1/2*(4*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)*(c^2+d^2)^(3/2)*a^3*b*c-4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)
^(3/2)*a*b^3*c+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^4*c^2*d+(2*(
c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^4*d^3-4*(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)*(c^2+d^2)^(1/2)*a^3*b*c^3-4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^
(1/2)*a^3*b*c*d^2-6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^2*b^2*c
^2*d-6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^2*b^2*d^3+4*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*b^3*c^3+4*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*(c^2+d^2)^(1/2)*a*b^3*c*d^2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1
/2)*b^4*c^2*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b^4*d^3-(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*a^4*c^3*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^4*c*d^3-4*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*b*c^2*d^2-4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*a^3*b*d^4+6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b^2*c^3*d+6*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*a^2*b^2*c*d^3+4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^3*c^2*d^2+4
*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^3*d^4-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^4*
c^3*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^4*c*d^3)*ln((c+d*tan(f*x+e))^(1/2)*(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))+2*(-2*a^4*c^2*d
^3-2*a^4*d^5+8*a^3*b*c^3*d^2+8*a^3*b*c*d^4+12*a^2*b^2*c^2*d^3+12*a^2*b^2*d^
5-8*a*b^3*c^3*d^2-8*a*b^3*c*d^4-2*b^4*c^2*d^3-2*b^4*d^5+1/2*(4*(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*(c^2+d^2)^(3/2)*a^3*b*c-4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(
c^2+d^2)^(3/2)*a*b^3*c+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^4*c^
2*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^4*d^3-4*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^3*b*c^3-4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c
^2+d^2)^(1/2)*a^3*b*c*d^2-6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a
^2*b^2*c^2*d-6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^2*b^2*d^3+4*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*b^3*c^3+4*(2*(c^2+d^2)^(1/2
)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*b^3*c*d^2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2
+d^2)^(1/2)*b^4*c^2*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b^4*d^3
-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^4*c^3*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^4*
c*d^3-4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*b*c^2*d^2-4*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*a^3*b*d^4+6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b^2*c^3*d+6*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*a^2*b^2*c*d^3+4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^3*c
^2*d^2+4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^3*d^4-(2*(c^2+d^2)^(1/2)+2*c)^(1
```

```
/2)*b^4*c^3*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^4*c*d^3)*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1
/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/4/d^2/(c^2+
d^2)^(3/2)*(1/2*(4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(3/2)*a^3*b*c-4*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(3/2)*a*b^3*c+(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*(c^2+d^2)^(1/2)*a^4*c^2*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^
(1/2)*a^4*d^3-4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^3*b*c^3-4*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^3*b*c*d^2-6*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^2*b^2*c^2*d-6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*(c^2+d^2)^(1/2)*a^2*b^2*d^3+4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2
)*a*b^3*c^3+4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*b^3*c*d^2+(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b^4*c^2*d+(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*(c^2+d^2)^(1/2)*b^4*d^3-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^4*c^3*d-(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*a^4*c*d^3-4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*b*
c^2*d^2-4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*b*d^4+6*(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)*a^2*b^2*c^3*d+6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b^2*c*d^3+4*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)*a*b^3*c^2*d^2+4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^3
*d^4-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^4*c^3*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*
b^4*c*d^3)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)+(c^2+d^2)^(1/2))+2*(2*a^4*c^2*d^3+2*a^4*d^5-8*a^3*b*c^3*d^2-8*a^3*b*
c*d^4-12*a^2*b^2*c^2*d^3-12*a^2*b^2*d^5+8*a*b^3*c^3*d^2+8*a*b^3*c*d^4+2*b^4
*c^2*d^3+2*b^4*d^5-1/2*(4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(3/2)*a^3
*b*c-4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(3/2)*a*b^3*c+(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^4*c^2*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^
2+d^2)^(1/2)*a^4*d^3-4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^3*b*
c^3-4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^3*b*c*d^2-6*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^2*b^2*c^2*d-6*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*(c^2+d^2)^(1/2)*a^2*b^2*d^3+4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^
2)^(1/2)*a*b^3*c^3+4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*b^3*c*
d^2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b^4*c^2*d+(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b^4*d^3-(2*(c^2+...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^4/(c+d*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)^4/sqrt(d*tan(f*x + e) + c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^4/(c+d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan{(e+fx)})^4}{\sqrt{c+d\tan{(e+fx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**4/(c+d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**4/sqrt(c + d*tan(e + f*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^4/(c+d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 16.98, size = 2500, normalized size = 10.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^4/(c + d*tan(e + f*x))^(1/2),x)
```

```
[Out] atan(((((32*(a^4*d^3*f^2 + b^4*d^3*f^2 - 6*a^2*b^2*d^3*f^2 + 4*a*b^3*c*d^2*
f^2 - 4*a^3*b*c*d^2*f^2))/f^3 - 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*(-(a*b^
7*8i - a^7*b*8i + a^8 + b^8 - 28*a^2*b^6 - a^3*b^5*56i + 70*a^4*b^4 + a^5*b
^3*56i - 28*a^6*b^2)/(4*(c*f^2 - d*f^2*1i)))^(1/2))*(-(a*b^7*8i - a^7*b*8i
+ a^8 + b^8 - 28*a^2*b^6 - a^3*b^5*56i + 70*a^4*b^4 + a^5*b^3*56i - 28*a^6*
b^2)/(4*(c*f^2 - d*f^2*1i)))^(1/2) - (16*(c + d*tan(e + f*x))^(1/2)*(a^8*d^
2 + b^8*d^2 - 28*a^2*b^6*d^2 + 70*a^4*b^4*d^2 - 28*a^6*b^2*d^2))/f^2)*(-(a*
b^7*8i - a^7*b*8i + a^8 + b^8 - 28*a^2*b^6 - a^3*b^5*56i + 70*a^4*b^4 + a^5
*b^3*56i - 28*a^6*b^2)/(4*(c*f^2 - d*f^2*1i)))^(1/2)*1i - (((32*(a^4*d^3*f^
```

2 + b^4*d^3*f^2 - 6*a^2*b^2*d^3*f^2 + 4*a*b^3*c*d^2*f^2 - 4*a^3*b*c*d^2*f^2
))/f^3 + 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*(-(a*b^7*8i - a^7*b*8i + a^8 +
 b^8 - 28*a^2*b^6 - a^3*b^5*56i + 70*a^4*b^4 + a^5*b^3*56i - 28*a^6*b^2)/(4
*(c*f^2 - d*f^2*1i)))^(1/2))*(-(a*b^7*8i - a^7*b*8i + a^8 + b^8 - 28*a^2*b^
6 - a^3*b^5*56i + 70*a^4*b^4 + a^5*b^3*56i - 28*a^6*b^2)/(4*(c*f^2 - d*f^2*
1i)))^(1/2) + (16*(c + d*tan(e + f*x))^(1/2)*(a^8*d^2 + b^8*d^2 - 28*a^2*b^
6*d^2 + 70*a^4*b^4*d^2 - 28*a^6*b^2*d^2))/f^2)*(-(a*b^7*8i - a^7*b*8i + a^8
 + b^8 - 28*a^2*b^6 - a^3*b^5*56i + 70*a^4*b^4 + a^5*b^3*56i - 28*a^6*b^2)/
(4*(c*f^2 - d*f^2*1i)))^(1/2)*1i)/((((32*(a^4*d^3*f^2 + b^4*d^3*f^2 - 6*a^2
*b^2*d^3*f^2 + 4*a*b^3*c*d^2*f^2 - 4*a^3*b*c*d^2*f^2))/f^3 - 64*c*d^2*(c +
d*tan(e + f*x))^(1/2)*(-(a*b^7*8i - a^7*b*8i + a^8 + b^8 - 28*a^2*b^6 - a^3
*b^5*56i + 70*a^4*b^4 + a^5*b^3*56i - 28*a^6*b^2)/(4*(c*f^2 - d*f^2*1i)))^(
1/2))*(-(a*b^7*8i - a^7*b*8i + a^8 + b^8 - 28*a^2*b^6 - a^3*b^5*56i + 70*a^
4*b^4 + a^5*b^3*56i - 28*a^6*b^2)/(4*(c*f^2 - d*f^2*1i)))^(1/2) - (16*(c +
d*tan(e + f*x))^(1/2)*(a^8*d^2 + b^8*d^2 - 28*a^2*b^6*d^2 + 70*a^4*b^4*d^2
- 28*a^6*b^2*d^2))/f^2)*(-(a*b^7*8i - a^7*b*8i + a^8 + b^8 - 28*a^2*b^6 - a
^3*b^5*56i + 70*a^4*b^4 + a^5*b^3*56i - 28*a^6*b^2)/(4*(c*f^2 - d*f^2*1i)))
^(1/2) + (((32*(a^4*d^3*f^2 + b^4*d^3*f^2 - 6*a^2*b^2*d^3*f^2 + 4*a*b^3*c*d
^2*f^2 - 4*a^3*b*c*d^2*f^2))/f^3 + 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*(-(a
*b^7*8i - a^7*b*8i + a^8 + b^8 - 28*a^2*b^6 - a^3*b^5*56i + 70*a^4*b^4 + a^
5*b^3*56i - 28*a^6*b^2)/(4*(c*f^2 - d*f^2*1i)))^(1/2))*(-(a*b^7*8i - a^7*b*
8i + a^8 + b^8 - 28*a^2*b^6 - a^3*b^5*56i + 70*a^4*b^4 + a^5*b^3*56i - 28*a
^6*b^2)/(4*(c*f^2 - d*f^2*1i)))^(1/2) + (16*(c + d*tan(e + f*x))^(1/2)*(a^8
*d^2 + b^8*d^2 - 28*a^2*b^6*d^2 + 70*a^4*b^4*d^2 - 28*a^6*b^2*d^2))/f^2)*(-
(a*b^7*8i - a^7*b*8i + a^8 + b^8 - 28*a^2*b^6 - a^3*b^5*56i + 70*a^4*b^4 +
a^5*b^3*56i - 28*a^6*b^2)/(4*(c*f^2 - d*f^2*1i)))^(1/2) - (64*(a*b^11*d^2 -
 a^11*b*d^2 + 3*a^3*b^9*d^2 + 2*a^5*b^7*d^2 - 2*a^7*b^5*d^2 - 3*a^9*b^3*d^2
))/f^3))*(-(a*b^7*8i - a^7*b*8i + a^8 + b^8 - 28*a^2*b^6 - a^3*b^5*56i + 70
*a^4*b^4 + a^5*b^3*56i - 28*a^6*b^2)/(4*(c*f^2 - d*f^2*1i)))^(1/2)*2i - (c
+ d*tan(e + f*x))^(1/2)*(2*c*((8*b^4*c - 8*a*b^3*d)/(d^3*f) - (4*b^4*c)/(d^
3*f)) + (2*b^4*(c^2 + d^2))/(d^3*f) - (12*b^2*(a*d - b*c)^2)/(d^3*f)) - ((8
*b^4*c - 8*a*b^3*d)/(3*d^3*f) - (4*b^4*c)/(3*d^3*f))*(c + d*tan(e + f*x))^(
3/2) + atan(((((32*(a^4*d^3*f^2 + b^4*d^3*f^2 - 6*a^2*b^2*d^3*f^2 + 4*a*b^3
*c*d^2*f^2 - 4*a^3*b*c*d^2*f^2))/f^3 - 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*
(-(8*a*b^7 - 8*a^7*b + a^8*1i + b^8*1i - a^2*b^6*28i - 56*a^3*b^5 + a^4*b^4
*70i + 56*a^5*b^3 - a^6*b^2*28i)/(4*(c*f^2*1i - d*f^2)))^(1/2))*(-(8*a*b^7
- 8*a^7*b + a^8*1i + b^8*1i - a^2*b^6*28i - 56*a^3*b^5 + a^4*b^4*70i + 56*a
^5*b^3 - a^6*b^2*28i)/(4*(c*f^2*1i - d*f^2)))^(1/2) - (16*(c + d*tan(e + f*
x))^(1/2)*(a^8*d^2 + b^8*d^2 - 28*a^2*b^6*d^2 + 70*a^4*b^4*d^2 - 28*a^6*b^2
d^2))/f^2)(-(8*a*b^7 - 8*a^7*b + a^8*1i + b^8*1i - a^2*b^6*28i - 56*a^3*b
^5 + a^4*b^4*70i + 56*a^5*b^3 - a^6*b^2*28i)/(4*(c*f^2*1i - d*f^2)))^(1/2)*
1i - (((32*(a^4*d^3*f^2 + b^4*d^3*f^2 - 6*a^2*b^2*d^3*f^2 + 4*a*b^3*c*d^2*f
^2 - 4*a^3*b*c*d^2*f^2))/f^3 + 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*(-(8*a*b
^7 - 8*a^7*b + a^8*1i + b^8*1i - a^2*b^6*28i - 56*a^3*b^5 + a^4*b^4*70i + 5
6*a^5*b^3 - a^6*b^2*28i)/(4*(c*f^2*1i - d*f^2)))^(1/2))*(-(8*a*b^7 - 8*a^7*

```
b + a^8*1i + b^8*1i - a^2*b^6*28i - 56*a^3*b^5 + a^4*b^4*70i + 56*a^5*b^3 -
 a^6*b^2*28i)/(4*(c*f^2*1i - d*f^2)))^(1/2) + (16*(c + d*tan(e + f*x))^(1/2
)*(a^8*d^2 + b^8*d^2 - 28*a^2*b^6*d^2 + 70*a^4*b^4*d^2 - 28*a^6*b^2*d^2))/f
^2)*(-(8*a*b^7 - 8*a^7*b + a^8*1i + b^8*1i - a^2*b^6*28i - 56*a^3*b^5 + a^4
*b^4*70i + 56*a^5*b^3 - a^6*b^2*28i)/(4*(c*f^2*1i - d*f^2)))^(1/2)*1i)/((((
32*(a^4*d^3*f^2 + b^4*d^3*f^2 - 6*a^2*b^2*d^3*f^2 + 4*a*b^3*c*d^2*f^2 - 4*a
^3*b*c*d^2*f^2))/f^3 - 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*(-(8*a*b^7 - 8*a
^7*b + a^8*1i + b^8*1i - a^2*b^6*28i - 56*a^3*b^5 + a^4*b^4*70i + 56*a^5*b^
3 - a^6*b^2*28i)/(4*(c*f^2*1i - d*f^2)))^(1/2))*(-(8*a*b^7 - 8*a^7*b + a^8*
1i + b^8*1i - a^2*b^6*28i - 56*a^3*b^5 + a^4*b^4*70i + 56*a^5*b^3 - a^6*b^2
*28i)/(4*(c*f^2*1i - d*f^2)))^(1/2) - (16*(c + d*tan(e + f*x))^(1/2)*(a^8*d
^2 + b^8*d^2 - 28*a^2*b^6*d^2 + 70*a^4*b^4*d^2 - 28*a^6*b^2*d^2))/f^2)*(-(8
*a*b^7 - 8*a^7*b + a^8*1i + b^8*1i - a^2*b^6*28...
```

3.1248 $\int \frac{(a+b\tan(e+fx))^3}{\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=178

$$\frac{(ia+b)^3\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}\,f}-\frac{(ia-b)^3\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{\sqrt{c+id}\,f}-\frac{4b^2(bc-4ad)\sqrt{c+d}}{3d^2f}$$

```
[Out] (I*a+b)^3*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/f/(c-I*d)^(1/2)-(I*
a-b)^3*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/f/(c+I*d)^(1/2)-4/3*b^
2*(-4*a*d+b*c)*(c+d*tan(f*x+e))^(1/2)/d^2/f+2/3*b^2*(c+d*tan(f*x+e))^(1/2)*
(a+b*tan(f*x+e))/d/f
```

Rubi [A]

time = 0.29, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3647, 3711, 3620, 3618, 65, 214}

$$-\frac{4b^2(bc-4ad)\sqrt{c+d\tan(e+fx)}}{3d^2f}+\frac{2b^2(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}{3df}-\frac{(-b+ia)^3\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f\sqrt{c+id}}+\frac{(b+ia)^3\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f\sqrt{c-id}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^3/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((I*a + b)^3*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/(Sqrt[c - I*d
]*f) - ((I*a - b)^3*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/(Sqrt[
c + I*d]*f) - (4*b^2*(b*c - 4*a*d)*Sqrt[c + d*Tan[e + f*x]])/(3*d^2*f) + (2
*b^2*(a + b*Tan[e + f*x])*Sqrt[c + d*Tan[e + f*x]])/(3*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
```

```
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^3}{\sqrt{c+d\tan(e+fx)}}\,dx = \frac{2b^2(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}{3df} + \frac{2\int \frac{\frac{1}{2}(3a^3d-b^2(2bc+ad))+\frac{3}{2}b(3a^2-b^2)d}{\sqrt{c+d\tan}}}{3a}$$

$$= -\frac{4b^2(bc-4ad)\sqrt{c+d\tan(e+fx)}}{3d^2f} + \frac{2b^2(a+b\tan(e+fx))\sqrt{c+d\tan(e+}}{3df}$$

$$= -\frac{4b^2(bc-4ad)\sqrt{c+d\tan(e+fx)}}{3d^2f} + \frac{2b^2(a+b\tan(e+fx))\sqrt{c+d\tan(e+}}{3df}$$

$$= -\frac{4b^2(bc-4ad)\sqrt{c+d\tan(e+fx)}}{3d^2f} + \frac{2b^2(a+b\tan(e+fx))\sqrt{c+d\tan(e+}}{3df}$$

$$= -\frac{4b^2(bc-4ad)\sqrt{c+d\tan(e+fx)}}{3d^2f} + \frac{2b^2(a+b\tan(e+fx))\sqrt{c+d\tan(e+}}{3df}$$

$$= \frac{(ia+b)^3\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}\,f} - \frac{(ia-b)^3\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e-}}{\sqrt{c+id}}\right.}{\sqrt{c+id}\,f}$$

Mathematica [A]
time = 1.11, size = 178, normalized size = 1.00

$$\frac{2\left(-\frac{3i(a-ib)^3d\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{2\sqrt{c-id}}+\frac{3i(a+ib)^3d\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{2\sqrt{c+id}}+\frac{2b^2(-bc+4ad)\sqrt{c+d\tan(e+fx)}}{d}+b^2(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}\right)}{3df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^3/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] (2*((((-3*I)/2)*(a - I*b)^3*d*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d
]])/Sqrt[c - I*d] + (((3*I)/2)*(a + I*b)^3*d*ArcTanh[Sqrt[c + d*Tan[e + f*x
]]/Sqrt[c + I*d]])/Sqrt[c + I*d] + (2*b^2*(-(b*c) + 4*a*d)*Sqrt[c + d*Tan[e
 + f*x]])/d + b^2*(a + b*Tan[e + f*x])*Sqrt[c + d*Tan[e + f*x]]))/(3*d*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2771 vs. $2(152) = 304$.
time = 0.47, size = 2772, normalized size = 15.57

method	result	size
derivativedivides	Expression too large to display	2772
default	Expression too large to display	2772

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*tan(f*x+e))^3/(c+d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/f/d^2*(1/3*b^3*(c+d*tan(f*x+e))^(3/2)+3*a*b^2*d*(c+d*tan(f*x+e))^(1/2)-b^3*c*(c+d*tan(f*x+e))^(1/2)+d^2*(1/4/(c^2+d^2)^(3/2)/d^2*(1/2*(3*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c-(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c^2*d+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*d^3-3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c^3-3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c*d^2-3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c^2*d-3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*d^3+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c^3+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c*d^2-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c^3*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c*d^3-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c^2*d^2-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d^4+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c^3*d+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c*d^3+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c^2*d^2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*d^4)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(2*a^3*c^2*d^3+2*a^3*d^5-6*a^2*b*c^3*d^2-6*a^2*b*c*d^4-6*a*b^2*c^2*d^3-6*a*b^2*d^5+2*b^3*c^3*d^2+2*b^3*c*d^4-1/2*(3*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c-(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c^2*d+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*d^3-3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c^3-3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c*d^2-3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c^2*d-3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*d^3+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c^3+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c*d^2-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c^3*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c*d^3-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c^2*d^2-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d^4+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c^3*d+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c*d^3+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c^2*d^2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*d^4)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/4/(c^2+d^2)^(3/2)/d^2*(1/2*(-3*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c+(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c^2*d-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*d^3+3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c^3+3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c*d^2+3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c^2*d+3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*d^3-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c^3-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c*d^2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c^3*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c*d^3+3*(2*(c^2+d^2)^(1/2)+2*c)

```
^(1/2)*a^2*b*c^2*d^2+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d^4-3*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*a*b^2*c^3*d-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c*d^3
-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c^2*d^2-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^
3*d^4)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)+(c^2+d^2)^(1/2))+2*(2*a^3*c^2*d^3+2*a^3*d^5-6*a^2*b*c^3*d^2-6*a^2*b*c*d^
4-6*a*b^2*c^2*d^3-6*a*b^2*d^5+2*b^3*c^3*d^2+2*b^3*c*d^4+1/2*(-3*(c^2+d^2)^(
3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c+(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*b^3*c-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c^2*d
-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*d^3+3*(c^2+d^2)^(1/2)*(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c^3+3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+
2*c)^(1/2)*a^2*b*c*d^2+3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^
2*c^2*d+3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*d^3-(c^2+d^2)
^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c^3-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)*b^3*c*d^2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c^3*d+(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*a^3*c*d^3+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c^2*d^
2+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d^4-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*a*b^2*c^3*d-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c*d^3-(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*b^3*c^2*d^2-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*d^4)*(2*(c^2+d^2)
^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))
^(1/2)-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(c+d*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)^3/sqrt(d*tan(f*x + e) + c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(c+d*tan(f*x+e))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan{(e + fx)})^3}{\sqrt{c + d\tan{(e + fx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3/(c+d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**3/sqrt(c + d*tan(e + f*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(c+d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 9.57, size = 3017, normalized size = 16.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^3/(c + d*tan(e + f*x))^(1/2),x)
```

```
[Out] atan(((((8*(4*a^3*d^3*f^2 - 12*a*b^2*d^3*f^2 + 4*b^3*c*d^2*f^2 - 12*a^2*b*c
*d^2*f^2))/f^3 - 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*((6*a*b^5 + 6*a^5*b -
a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*(c*f^2*1i - d*
f^2)))^(1/2))*((6*a*b^5 + 6*a^5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^3*
b^3 + a^4*b^2*15i)/(4*(c*f^2*1i - d*f^2)))^(1/2) - (16*(c + d*tan(e + f*x))
^(1/2)*(a^6*d^2 - b^6*d^2 + 15*a^2*b^4*d^2 - 15*a^4*b^2*d^2))/f^2)*((6*a*b^
5 + 6*a^5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*
(c*f^2*1i - d*f^2)))^(1/2)*1i - (((8*(4*a^3*d^3*f^2 - 12*a*b^2*d^3*f^2 + 4*
b^3*c*d^2*f^2 - 12*a^2*b*c*d^2*f^2))/f^3 + 64*c*d^2*(c + d*tan(e + f*x))^(1
/2)*((6*a*b^5 + 6*a^5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*
b^2*15i)/(4*(c*f^2*1i - d*f^2)))^(1/2))*((6*a*b^5 + 6*a^5*b - a^6*1i + b^6*
1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*(c*f^2*1i - d*f^2)))^(1/2)
+ (16*(c + d*tan(e + f*x))^(1/2)*(a^6*d^2 - b^6*d^2 + 15*a^2*b^4*d^2 - 15*a
^4*b^2*d^2))/f^2)*((6*a*b^5 + 6*a^5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*
a^3*b^3 + a^4*b^2*15i)/(4*(c*f^2*1i - d*f^2)))^(1/2)*1i)/((((8*(4*a^3*d^3*f
^2 - 12*a*b^2*d^3*f^2 + 4*b^3*c*d^2*f^2 - 12*a^2*b*c*d^2*f^2))/f^3 - 64*c*d
^2*(c + d*tan(e + f*x))^(1/2)*((6*a*b^5 + 6*a^5*b - a^6*1i + b^6*1i - a^2*b
^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*(c*f^2*1i - d*f^2)))^(1/2))*((6*a*b^5
 + 6*a^5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*(
```

c*f^2*1i - d*f^2)))^(1/2) - (16*(c + d*tan(e + f*x))^(1/2)*(a^6*d^2 - b^6*d
^2 + 15*a^2*b^4*d^2 - 15*a^4*b^2*d^2))/f^2)*((6*a*b^5 + 6*a^5*b - a^6*1i +
b^6*1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*(c*f^2*1i - d*f^2)))^(1
/2) + (((8*(4*a^3*d^3*f^2 - 12*a*b^2*d^3*f^2 + 4*b^3*c*d^2*f^2 - 12*a^2*b*c
*d^2*f^2))/f^3 + 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*((6*a*b^5 + 6*a^5*b -
a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*(c*f^2*1i - d*
f^2)))^(1/2))*((6*a*b^5 + 6*a^5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^3*
b^3 + a^4*b^2*15i)/(4*(c*f^2*1i - d*f^2)))^(1/2) + (16*(c + d*tan(e + f*x))
^(1/2)*(a^6*d^2 - b^6*d^2 + 15*a^2*b^4*d^2 - 15*a^4*b^2*d^2))/f^2)*((6*a*b^
5 + 6*a^5*b - a^6*1i + b^6*1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)/(4*
(c*f^2*1i - d*f^2)))^(1/2) + (16*(3*a^8*b*d^2 - b^9*d^2 + 6*a^4*b^5*d^2 + 8
*a^6*b^3*d^2))/f^3))*((6*a*b^5 + 6*a^5*b - a^6*1i + b^6*1i - a^2*b^4*15i -
20*a^3*b^3 + a^4*b^2*15i)/(4*(c*f^2*1i - d*f^2)))^(1/2)*2i + atan(((((8*(4*
a^3*d^3*f^2 - 12*a*b^2*d^3*f^2 + 4*b^3*c*d^2*f^2 - 12*a^2*b*c*d^2*f^2))/f^3
 - 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*((a*b^5*6i + a^5*b*6i - a^6 + b^6 -
15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)/(4*(c*f^2 - d*f^2*1i)))^(1/2))*((a*b
^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)/(4*(c
*f^2 - d*f^2*1i)))^(1/2) - (16*(c + d*tan(e + f*x))^(1/2)*(a^6*d^2 - b^6*d^
2 + 15*a^2*b^4*d^2 - 15*a^4*b^2*d^2))/f^2)*((a*b^5*6i + a^5*b*6i - a^6 + b^
6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)/(4*(c*f^2 - d*f^2*1i)))^(1/2)*1i
 - (((8*(4*a^3*d^3*f^2 - 12*a*b^2*d^3*f^2 + 4*b^3*c*d^2*f^2 - 12*a^2*b*c*d^
2*f^2))/f^3 + 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*((a*b^5*6i + a^5*b*6i - a
^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)/(4*(c*f^2 - d*f^2*1i)))^(
1/2))*((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4
b^2)/(4(c*f^2 - d*f^2*1i)))^(1/2) + (16*(c + d*tan(e + f*x))^(1/2)*(a^6*d
^2 - b^6*d^2 + 15*a^2*b^4*d^2 - 15*a^4*b^2*d^2))/f^2)*((a*b^5*6i + a^5*b*6i
 - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)/(4*(c*f^2 - d*f^2*1i)
))^(1/2)*1i)/((((8*(4*a^3*d^3*f^2 - 12*a*b^2*d^3*f^2 + 4*b^3*c*d^2*f^2 - 12
*a^2*b*c*d^2*f^2))/f^3 - 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*((a*b^5*6i + a
^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)/(4*(c*f^2 - d*
f^2*1i)))^(1/2))*((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*2
0i + 15*a^4*b^2)/(4*(c*f^2 - d*f^2*1i)))^(1/2) - (16*(c + d*tan(e + f*x))^(
1/2)*(a^6*d^2 - b^6*d^2 + 15*a^2*b^4*d^2 - 15*a^4*b^2*d^2))/f^2)*((a*b^5*6i
 + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)/(4*(c*f^2
- d*f^2*1i)))^(1/2) + (((8*(4*a^3*d^3*f^2 - 12*a*b^2*d^3*f^2 + 4*b^3*c*d^2*
f^2 - 12*a^2*b*c*d^2*f^2))/f^3 + 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*((a*b^
5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)/(4*(c*
f^2 - d*f^2*1i)))^(1/2))*((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a
^3*b^3*20i + 15*a^4*b^2)/(4*(c*f^2 - d*f^2*1i)))^(1/2) + (16*(c + d*tan(e +
 f*x))^(1/2)*(a^6*d^2 - b^6*d^2 + 15*a^2*b^4*d^2 - 15*a^4*b^2*d^2))/f^2)*((
a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)/(4
*(c*f^2 - d*f^2*1i)))^(1/2) + (16*(3*a^8*b*d^2 - b^9*d^2 + 6*a^4*b^5*d^2 +
8*a^6*b^3*d^2))/f^3))*((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*
b^3*20i + 15*a^4*b^2)/(4*(c*f^2 - d*f^2*1i)))^(1/2)*2i - ((6*b^3*c - 6*a*b^
2*d)/(d^2*f) - (4*b^3*c)/(d^2*f))*(c + d*tan(e + f*x))^(1/2) + (2*b^3*(c +

```
d*tan(e + f*x))^(3/2))/(3*d^2*f)
```

3.1249 $\int \frac{(a+b\tan(e+fx))^2}{\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=134

$$-\frac{i(a-ib)^2\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}\,f}+\frac{i(a+ib)^2\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{\sqrt{c+id}\,f}+\frac{2b^2\sqrt{c+d\tan(e+fx)}}{df}$$

```
[Out] -I*(a-I*b)^2*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/f/(c-I*d)^(1/2)+
I*(a+I*b)^2*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/f/(c+I*d)^(1/2)+2
*b^2*(c+d*tan(f*x+e))^(1/2)/d/f
```

Rubi [A]
time = 0.17, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3624, 3620, 3618, 65, 214}

$$-\frac{i(a-ib)^2\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f\sqrt{c-id}}+\frac{i(a+ib)^2\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f\sqrt{c+id}}+\frac{2b^2\sqrt{c+d\tan(e+fx)}}{df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((-I)*(a - I*b)^2*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/(Sqrt[c
- I*d]*f) + (I*(a + I*b)^2*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])
/(Sqrt[c + I*d]*f) + (2*b^2*Sqrt[c + d*Tan[e + f*x]])/(d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
```

```
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt{c+d\tan(e+fx)}}\,dx = \frac{2b^2\sqrt{c+d\tan(e+fx)}}{df}+\int \frac{a^2-b^2+2ab\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx$$

$$= \frac{2b^2\sqrt{c+d\tan(e+fx)}}{df}+\frac{1}{2}(a-ib)^2\int \frac{1+i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx+\frac{1}{2}(a+ib$$

$$= \frac{2b^2\sqrt{c+d\tan(e+fx)}}{df}+\frac{(i(a-ib)^2)\,\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{c-idx}}\,dx,x,i\tan(\right.}{2f}$$

$$= \frac{2b^2\sqrt{c+d\tan(e+fx)}}{df}-\frac{(a-ib)^2\text{Subst}\left(\int \frac{1}{-1-\frac{ic}{d}+\frac{ix^2}{d}}\,dx,x,\sqrt{c+d\tan(e\right.}{df}$$

$$= -\frac{i(a-ib)^2\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}\,f}+\frac{i(a+ib)^2\tanh^{-1}\left(\frac{\sqrt{c+d\,\text{ta}}}{\sqrt{c}}\right.}{\sqrt{c+id}\,f}$$

Mathematica [A]
time = 0.24, size = 129, normalized size = 0.96

$$\frac{-\frac{i(a-ib)^2\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}}+\frac{i(a+ib)^2\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{\sqrt{c+id}}+\frac{2b^2\sqrt{c+d\tan(e+fx)}}{d}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] (((-I)*(a - I*b)^2*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/Sqrt[c
- I*d] + (I*(a + I*b)^2*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/Sq
rt[c + I*d] + (2*b^2*Sqrt[c + d*Tan[e + f*x]])/d)/f
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2009 vs. $2(112) = 224$.
time = 0.43, size = 2010, normalized size = 15.00

method	result	size
derivativedivides	Expression too large to display	2010
default	Expression too large to display	2010

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/d*((c+d*tan(f*x+e))^(1/2)*b^2+d*(1/4/(c^2+d^2)^(3/2)/d^2*(1/2*(-2*(c^2+
d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c-(c^2+d^2)^(1/2)*(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*a^2*c^2*d-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a
^2*d^3+2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^3+2*(c^2+d^2)^
(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c*d^2+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*b^2*c^2*d+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^
2*d^3+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^3*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*a^2*c*d^3+2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^2*d^2+2*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*a*b*d^4-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^3*d-(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*b^2*c*d^3)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(2*a^2*c^2*d^3+2*a^2*d^5-4*a*b*c^3
*d^2-4*a*b*c*d^4-2*b^2*c^2*d^3-2*b^2*d^5+1/2*(-2*(c^2+d^2)^(3/2)*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*a*b*c-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2
*c^2*d-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d^3+2*(c^2+d^2)^(1
/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^3+2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*a*b*c*d^2+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*
c^2*d+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d^3+(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*a^2*c^3*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c*d^3+2*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*a*b*c^2*d^2+2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d^4-(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^3*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c*
d^3)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2
*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2
*c)^(1/2)))+1/4/(c^2+d^2)^(3/2)/d^2*(1/2*(2*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*a*b*c+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^2*
d+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d^3-2*(c^2+d^2)^(1/2)*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^3-2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2
```

```
*c)^(1/2)*a*b*c*d^2-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^2*d
-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d^3-(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)*a^2*c^3*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c*d^3-2*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*a*b*c^2*d^2-2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d^4+(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^3*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c*d^3)*
ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2
+d^2)^(1/2))+2*(2*a^2*c^2*d^3+2*a^2*d^5-4*a*b*c^3*d^2-4*a*b*c*d^4-2*b^2*c^2
*d^3-2*b^2*d^5-1/2*(2*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c+(
c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^2*d+(c^2+d^2)^(1/2)*(2*(
c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d^3-2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)*a*b*c^3-2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c*d^2-(c
^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^2*d-(c^2+d^2)^(1/2)*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d^3-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^3*d-(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c*d^3-2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c
^2*d^2-2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d^4+(2*(c^2+d^2)^(1/2)+2*c)^(1/2
)*b^2*c^3*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c*d^3)*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*
(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2/sqrt(d*tan(f*x + e) + c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 14968 vs. $2(107) = 214$.
time = 55.51, size = 14968, normalized size = 111.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*(c^2*d + d^3)*f^5*sqrt((((a^4 - 6*a^2*b^2 + b^4)*c^3 + 4*(a^
3*b - a*b^3)*c^2*d + (a^4 - 6*a^2*b^2 + b^4)*c*d^2 + 4*(a^3*b - a*b^3)*d^3)
*f^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/((c^2 + d^2)*f^4)
) + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*c^2 + (a^8 + 4*a^6*b^2
+ 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^2)/(16*(a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*c^2
 - 8*(a^7*b - 7*a^5*b^3 + 7*a^3*b^5 - a*b^7)*c*d + (a^8 - 12*a^6*b^2 + 38*a
^4*b^4 - 12*a^2*b^6 + b^8)*d^2))*sqrt((16*(a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*c
^2 - 8*(a^7*b - 7*a^5*b^3 + 7*a^3*b^5 - a*b^7)*c*d + (a^8 - 12*a^6*b^2 + 38
```

*a^4*b^4 - 12*a^2*b^6 + b^8)*d^2)/((c^4 + 2*c^2*d^2 + d^4)*f^4))*((a^8 + 4*
a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/((c^2 + d^2)*f^4))^(3/4)*arctan(((4*
(a^15*b + 5*a^13*b^3 + 9*a^11*b^5 + 5*a^9*b^7 - 5*a^7*b^9 - 9*a^5*b^11 - 5*
a^3*b^13 - a*b^15)*c^5 - (a^16 - 20*a^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 - 6
4*a^6*b^10 - 20*a^4*b^12 + b^16)*c^4*d + 8*(a^15*b + 5*a^13*b^3 + 9*a^11*b^
5 + 5*a^9*b^7 - 5*a^7*b^9 - 9*a^5*b^11 - 5*a^3*b^13 - a*b^15)*c^3*d^2 - 2*(
a^16 - 20*a^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 - 64*a^6*b^10 - 20*a^4*b^12 +
b^16)*c^2*d^3 + 4*(a^15*b + 5*a^13*b^3 + 9*a^11*b^5 + 5*a^9*b^7 - 5*a^7*b^
9 - 9*a^5*b^11 - 5*a^3*b^13 - a*b^15)*c*d^4 - (a^16 - 20*a^12*b^4 - 64*a^10
*b^6 - 90*a^8*b^8 - 64*a^6*b^10 - 20*a^4*b^12 + b^16)*d^5)*f^4*sqrt((16*(a^
6*b^2 - 2*a^4*b^4 + a^2*b^6)*c^2 - 8*(a^7*b - 7*a^5*b^3 + 7*a^3*b^5 - a*b^7
)*c*d + (a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*d^2)/((c^4 + 2*c
^2*d^2 + d^4)*f^4))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/((
c^2 + d^2)*f^4)) + (4*(a^19*b + 7*a^17*b^3 + 20*a^15*b^5 + 28*a^13*b^7 + 14
*a^11*b^9 - 14*a^9*b^11 - 28*a^7*b^13 - 20*a^5*b^15 - 7*a^3*b^17 - a*b^19)*
c^4 - (a^20 + 2*a^18*b^2 - 19*a^16*b^4 - 104*a^14*b^6 - 238*a^12*b^8 - 308*
a^10*b^10 - 238*a^8*b^12 - 104*a^6*b^14 - 19*a^4*b^16 + 2*a^2*b^18 + b^20)*
c^3*d + 4*(a^19*b + 7*a^17*b^3 + 20*a^15*b^5 + 28*a^13*b^7 + 14*a^11*b^9 -
14*a^9*b^11 - 28*a^7*b^13 - 20*a^5*b^15 - 7*a^3*b^17 - a*b^19)*c^2*d^2 - (a
^20 + 2*a^18*b^2 - 19*a^16*b^4 - 104*a^14*b^6 - 238*a^12*b^8 - 308*a^10*b^1
0 - 238*a^8*b^12 - 104*a^6*b^14 - 19*a^4*b^16 + 2*a^2*b^18 + b^20)*c*d^3)*f
^2*sqrt((16*(a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*c^2 - 8*(a^7*b - 7*a^5*b^3 + 7*
a^3*b^5 - a*b^7)*c*d + (a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*d
^2)/((c^4 + 2*c^2*d^2 + d^4)*f^4)) + sqrt(2)*((2*a*b*c^5 + 4*a*b*c^3*d^2 +
2*a*b*c*d^4 - (a^2 - b^2)*c^4*d - 2*(a^2 - b^2)*c^2*d^3 - (a^2 - b^2)*d^5)*
f^7*sqrt((16*(a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*c^2 - 8*(a^7*b - 7*a^5*b^3 + 7
*a^3*b^5 - a*b^7)*c*d + (a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*
d^2)/((c^4 + 2*c^2*d^2 + d^4)*f^4))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a
^2*b^6 + b^8)/((c^2 + d^2)*f^4)) + 2*((a^5*b + 2*a^3*b^3 + a*b^5)*c^4 + 2*(
a^5*b + 2*a^3*b^3 + a*b^5)*c^2*d^2 + (a^5*b + 2*a^3*b^3 + a*b^5)*d^4)*f^5*s
qrt((16*(a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*c^2 - 8*(a^7*b - 7*a^5*b^3 + 7*a^3*
b^5 - a*b^7)*c*d + (a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*d^2)/
((c^4 + 2*c^2*d^2 + d^4)*f^4)))*sqrt((((a^4 - 6*a^2*b^2 + b^4)*c^3 + 4*(a^3
*b - a*b^3)*c^2*d + (a^4 - 6*a^2*b^2 + b^4)*c*d^2 + 4*(a^3*b - a*b^3)*d^3)*
f^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/((c^2 + d^2)*f^4))
+ (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*c^2 + (a^8 + 4*a^6*b^2 +
6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^2)/(16*(a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*c^2
- 8*(a^7*b - 7*a^5*b^3 + 7*a^3*b^5 - a*b^7)*c*d + (a^8 - 12*a^6*b^2 + 38*a^
4*b^4 - 12*a^2*b^6 + b^8)*d^2))*sqrt(((16*(a^10*b^2 - 2*a^6*b^6 + a^2*b^10)
c^4 - 8(a^11*b - 5*a^9*b^3 - 6*a^7*b^5 + 6*a^5*b^7 + 5*a^3*b^9 - a*b^11)*
c^3*d + (a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b
^10 + b^12)*c^2*d^2 - 8*(a^11*b - 5*a^9*b^3 - 6*a^7*b^5 + 6*a^5*b^7 + 5*a^3
*b^9 - a*b^11)*c*d^3 + (a^12 - 10*a^10*b^2 + 15*a^8*b^4 + 52*a^6*b^6 + 15*a
^4*b^8 - 10*a^2*b^10 + b^12)*d^4)*f^2*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4
*a^2*b^6 + b^8)/((c^2 + d^2)*f^4))*cos(f*x + e) + sqrt(2)*(2*(16*(a^7*b^3 -

```
 2*a^5*b^5 + a^3*b^7)*c^4 - 8*(a^8*b^2 - 7*a^6*b^4 + 7*a^4*b^6 - a^2*b^8)*c
^3*d + (a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*c^2*d^2 - 8*(a^8
*b^2 - 7*a^6*b^4 + 7*a^4*b^6 - a^2*b^8)*c*d^3 + (a^9*b - 12*a^7*b^3 + 38*a^
5*b^5 - 12*a^3*b^7 + a*b^9)*d^4)*f^3*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*
a^2*b^6 + b^8)/((c^2 + d^2)*f^4))*cos(f*x + e) + (32*(a^11*b^3 - 2*a^7*b^7
+ a^3*b^11)*c^3 - 32*(a^12*b^2 - 3*a^10*b^4 - 4*a^8*b^6 + 4*a^6*b^8 + 3*a^4
*b^10 - a^2*b^12)*c^2*d + 2*(5*a^13*b - 34*a^11*b^3 + 11*a^9*b^5 + 100*a^7*
b^7 + 11*a^5*b^9 - 34*a^3*b^11 + 5*a*b^13)*c*d^2 - (a^14 - 11*a^12*b^2 + 25
*a^10*b^4 + 37*a^8*b^6 - 37*a^6*b^8 - 25*a^4*b^10 + 11*a^2*b^12 - b^14)*d^3
)*f*cos(f*x + e))*sqrt((((a^4 - 6*a^2*b^2 + b^4)*c^3 + 4*(a^3*b - a*b^3)*c^
2*d + (a^4 - 6*a^2*b^2 + b^4)*c*d^2 + 4*(a^3*b - a*b^3)*d^3)*f^2*sqrt((a^8
+ 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/((c^2 + d^2)*f^4)) + (a^8 + 4*a^
6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*c^2 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4
*a^2*b^6 + b^8)*d^2)/(16*(a^6*b^2 - 2*a^4*b^4 +...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan(e + fx))^2}{\sqrt{c + d\tan(e + fx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2/(c+d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**2/sqrt(c + d*tan(e + f*x)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**B**]
time = 7.23, size = 2287, normalized size = 17.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^2/(c + d*tan(e + f*x))^(1/2),x)
```

[Out] (2*b^2*(c + d*tan(e + f*x))^(1/2))/(d*f) - atan((((((16*(2*b^2*d^3*f^2 - 2*a
^2*d^3*f^2 + 4*a*b*c*d^2*f^2))/f^3 - 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*(-
(a*b^3*4i - a^3*b*4i + a^4 + b^4 - 6*a^2*b^2)/(4*(c*f^2 - d*f^2*1i)))^(1/2)
)*(-(a*b^3*4i - a^3*b*4i + a^4 + b^4 - 6*a^2*b^2)/(4*(c*f^2 - d*f^2*1i)))^(
1/2) - (16*(c + d*tan(e + f*x))^(1/2)*(a^4*d^2 + b^4*d^2 - 6*a^2*b^2*d^2))/
f^2)*(-(a*b^3*4i - a^3*b*4i + a^4 + b^4 - 6*a^2*b^2)/(4*(c*f^2 - d*f^2*1i))
)^(1/2)*1i - (((16*(2*b^2*d^3*f^2 - 2*a^2*d^3*f^2 + 4*a*b*c*d^2*f^2))/f^3 +
64*c*d^2*(c + d*tan(e + f*x))^(1/2)*(-(a*b^3*4i - a^3*b*4i + a^4 + b^4 - 6
*a^2*b^2)/(4*(c*f^2 - d*f^2*1i)))^(1/2))*(-(a*b^3*4i - a^3*b*4i + a^4 + b^4
- 6*a^2*b^2)/(4*(c*f^2 - d*f^2*1i)))^(1/2) + (16*(c + d*tan(e + f*x))^(1/2
)*(a^4*d^2 + b^4*d^2 - 6*a^2*b^2*d^2))/f^2)*(-(a*b^3*4i - a^3*b*4i + a^4 +
b^4 - 6*a^2*b^2)/(4*(c*f^2 - d*f^2*1i)))^(1/2)*1i)/((((16*(2*b^2*d^3*f^2 -
2*a^2*d^3*f^2 + 4*a*b*c*d^2*f^2))/f^3 - 64*c*d^2*(c + d*tan(e + f*x))^(1/2)
*(-(a*b^3*4i - a^3*b*4i + a^4 + b^4 - 6*a^2*b^2)/(4*(c*f^2 - d*f^2*1i)))^(1
/2))*(-(a*b^3*4i - a^3*b*4i + a^4 + b^4 - 6*a^2*b^2)/(4*(c*f^2 - d*f^2*1i))
)^(1/2) - (16*(c + d*tan(e + f*x))^(1/2)*(a^4*d^2 + b^4*d^2 - 6*a^2*b^2*d^2
))/f^2)*(-(a*b^3*4i - a^3*b*4i + a^4 + b^4 - 6*a^2*b^2)/(4*(c*f^2 - d*f^2*1
i)))^(1/2) + (((16*(2*b^2*d^3*f^2 - 2*a^2*d^3*f^2 + 4*a*b*c*d^2*f^2))/f^3 +
64*c*d^2*(c + d*tan(e + f*x))^(1/2)*(-(a*b^3*4i - a^3*b*4i + a^4 + b^4 - 6
*a^2*b^2)/(4*(c*f^2 - d*f^2*1i)))^(1/2))*(-(a*b^3*4i - a^3*b*4i + a^4 + b^4
- 6*a^2*b^2)/(4*(c*f^2 - d*f^2*1i)))^(1/2) + (16*(c + d*tan(e + f*x))^(1/2
)*(a^4*d^2 + b^4*d^2 - 6*a^2*b^2*d^2))/f^2)*(-(a*b^3*4i - a^3*b*4i + a^4 +
b^4 - 6*a^2*b^2)/(4*(c*f^2 - d*f^2*1i)))^(1/2) - (32*(a*b^5*d^2 + a^5*b*d^2
+ 2*a^3*b^3*d^2))/f^3))*(-(a*b^3*4i - a^3*b*4i + a^4 + b^4 - 6*a^2*b^2)/(4
*(c*f^2 - d*f^2*1i)))^(1/2)*2i - atan(((((16*(2*b^2*d^3*f^2 - 2*a^2*d^3*f^2
+ 4*a*b*c*d^2*f^2))/f^3 - 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*(-(4*a*b^3 -
4*a^3*b + a^4*1i + b^4*1i - a^2*b^2*6i)/(4*(c*f^2*1i - d*f^2)))^(1/2))*(-(
4*a*b^3 - 4*a^3*b + a^4*1i + b^4*1i - a^2*b^2*6i)/(4*(c*f^2*1i - d*f^2)))^(
1/2) - (16*(c + d*tan(e + f*x))^(1/2)*(a^4*d^2 + b^4*d^2 - 6*a^2*b^2*d^2))/
f^2)*(-(4*a*b^3 - 4*a^3*b + a^4*1i + b^4*1i - a^2*b^2*6i)/(4*(c*f^2*1i - d*
f^2)))^(1/2)*1i - (((16*(2*b^2*d^3*f^2 - 2*a^2*d^3*f^2 + 4*a*b*c*d^2*f^2))/
f^3 + 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*(-(4*a*b^3 - 4*a^3*b + a^4*1i + b
^4*1i - a^2*b^2*6i)/(4*(c*f^2*1i - d*f^2)))^(1/2))*(-(4*a*b^3 - 4*a^3*b + a
^4*1i + b^4*1i - a^2*b^2*6i)/(4*(c*f^2*1i - d*f^2)))^(1/2) + (16*(c + d*tan
(e + f*x))^(1/2)*(a^4*d^2 + b^4*d^2 - 6*a^2*b^2*d^2))/f^2)*(-(4*a*b^3 - 4*a
^3*b + a^4*1i + b^4*1i - a^2*b^2*6i)/(4*(c*f^2*1i - d*f^2)))^(1/2)*1i)/((((
16*(2*b^2*d^3*f^2 - 2*a^2*d^3*f^2 + 4*a*b*c*d^2*f^2))/f^3 - 64*c*d^2*(c + d
*tan(e + f*x))^(1/2)*(-(4*a*b^3 - 4*a^3*b + a^4*1i + b^4*1i - a^2*b^2*6i)/(
4*(c*f^2*1i - d*f^2)))^(1/2))*(-(4*a*b^3 - 4*a^3*b + a^4*1i + b^4*1i - a^2*
b^2*6i)/(4*(c*f^2*1i - d*f^2)))^(1/2) - (16*(c + d*tan(e + f*x))^(1/2)*(a^4
*d^2 + b^4*d^2 - 6*a^2*b^2*d^2))/f^2)*(-(4*a*b^3 - 4*a^3*b + a^4*1i + b^4*1
i - a^2*b^2*6i)/(4*(c*f^2*1i - d*f^2)))^(1/2) - (32*(a*b^5*d^2 + a^5*b*d^2
+ 2*a^3*b^3*d^2))/f^3 + (((16*(2*b^2*d^3*f^2 - 2*a^2*d^3*f^2 + 4*a*b*c*d^2*
f^2))/f^3 + 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*(-(4*a*b^3 - 4*a^3*b + a^4*
1i + b^4*1i - a^2*b^2*6i)/(4*(c*f^2*1i - d*f^2)))^(1/2))*(-(4*a*b^3 - 4*a^3

```
*b + a^4*1i + b^4*1i - a^2*b^2*6i)/(4*(c*f^2*1i - d*f^2)))^(1/2) + (16*(c +
 d*tan(e + f*x))^(1/2)*(a^4*d^2 + b^4*d^2 - 6*a^2*b^2*d^2))/f^2)*(-(4*a*b^3
 - 4*a^3*b + a^4*1i + b^4*1i - a^2*b^2*6i)/(4*(c*f^2*1i - d*f^2)))^(1/2)))*
(-(4*a*b^3 - 4*a^3*b + a^4*1i + b^4*1i - a^2*b^2*6i)/(4*(c*f^2*1i - d*f^2))
)^(1/2)*2i
```

3.1250 $\int \frac{a+b\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=102

$$-\frac{(ia+b)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}\,f}+\frac{(ia-b)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{\sqrt{c+id}\,f}$$

```
[Out] -(I*a+b)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/f/(c-I*d)^(1/2)+(I*a
-b)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/f/(c+I*d)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3620, 3618, 65, 214}

$$\frac{(-b+ia)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f\sqrt{c+id}}-\frac{(b+ia)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f\sqrt{c-id}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] -(((I*a + b)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/(Sqrt[c - I*d
]*f)) + ((I*a - b)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/(Sqrt[c
 + I*d]*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
```

```
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{a+b\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx = \frac{1}{2}(a-ib)\int \frac{1+i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx + \frac{1}{2}(a+ib)\int \frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx$$
$$= -\frac{(ia-b)\mathrm{Subst}\left(\int \frac{1}{(-1+x)\sqrt{c+idx}}\,dx, x, -i\tan(e+fx)\right)}{2f} + \frac{(ia+b)\mathrm{Subst}\left(\right.}{}$$
$$= -\frac{(a-ib)\mathrm{Subst}\left(\int \frac{1}{-1-\frac{ic}{d}+\frac{ix^2}{d}}\,dx, x, \sqrt{c+d\tan(e+fx)}\right)}{df} - \frac{(a+ib)\mathrm{Subst}\left(\right.}{}$$
$$= -\frac{(ia+b)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}\,f} + \frac{(ia-b)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e}}{\sqrt{c+ic}}\right.}{\sqrt{c+id}\,f}$$

Mathematica [A]
time = 0.15, size = 101, normalized size = 0.99

$$\frac{i\left(-\frac{(a-ib)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}}+\frac{(a+ib)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{\sqrt{c+id}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] (I*(-(((a - I*b)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/Sqrt[c -
I*d]) + ((a + I*b)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/Sqrt[c
+ I*d]))/f
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1398 vs. 2(84) = 168.
time = 0.46, size = 1399, normalized size = 13.72 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*tan(f*x+e))/(c+d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/f*(1/2/(c^2+d^2)^(3/2)/d^2*(1/2*(-(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2*d-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^3+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^3+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d^2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d^3+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d^2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^4)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(2*a*c^2*d^3+2*a*d^5-2*b*c^3*d^2-2*b*c*d^4+1/2*(-(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2*d-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^3+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^3+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d^2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d^3+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d^2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^4)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/2/(c^2+d^2)^(3/2)/d^2*(1/2*((c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2*d+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^3-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^3-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d^2-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d^3-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d^2-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^4)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(2*a*c^2*d^3+2*a*d^5-2*b*c^3*d^2-2*b*c*d^4-1/2*((c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2*d+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^3-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^3-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d^2-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d^3-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d^2-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^4)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(f*x+e))/(c+d*tan(f*x+e))^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* h

```
elp (example of legal syntax is 'assume(d-c>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 8324 vs. $2(81) = 162$.
time = 4.68, size = 8324, normalized size = 81.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(c+d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*(c^2 + d^2)*f^4*sqrt(((2*a*b*c^2*d + 2*a*b*d^3 + (a^2 - b^2)
*c^3 + (a^2 - b^2)*c*d^2)*f^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/((c^2 + d^2)*f^4
)) + (a^4 + 2*a^2*b^2 + b^4)*c^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2)/(4*a^2*b^2*
c^2 - 4*(a^3*b - a*b^3)*c*d + (a^4 - 2*a^2*b^2 + b^4)*d^2))*sqrt((4*a^2*b^2
*c^2 - 4*(a^3*b - a*b^3)*c*d + (a^4 - 2*a^2*b^2 + b^4)*d^2)/((c^4 + 2*c^2*d
^2 + d^4)*f^4))*((a^4 + 2*a^2*b^2 + b^4)/((c^2 + d^2)*f^4))^(3/4)*arctan(((
2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*c^5 - (a^8 + 2*a^6*b^2 - 2*a^2*b^
6 - b^8)*c^4*d + 4*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*c^3*d^2 - 2*(a^8
 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*c^2*d^3 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5
+ a*b^7)*c*d^4 - (a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^5)*f^4*sqrt((4*a^2*b
^2*c^2 - 4*(a^3*b - a*b^3)*c*d + (a^4 - 2*a^2*b^2 + b^4)*d^2)/((c^4 + 2*c^2
*d^2 + d^4)*f^4))*sqrt((a^4 + 2*a^2*b^2 + b^4)/((c^2 + d^2)*f^4)) + (2*(a^9
*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*c^4 - (a^10 + 3*a^8*b^2 + 2
*a^6*b^4 - 2*a^4*b^6 - 3*a^2*b^8 - b^10)*c^3*d + 2*(a^9*b + 4*a^7*b^3 + 6*a
^5*b^5 + 4*a^3*b^7 + a*b^9)*c^2*d^2 - (a^10 + 3*a^8*b^2 + 2*a^6*b^4 - 2*a^4
*b^6 - 3*a^2*b^8 - b^10)*c*d^3)*f^2*sqrt((4*a^2*b^2*c^2 - 4*(a^3*b - a*b^3)
*c*d + (a^4 - 2*a^2*b^2 + b^4)*d^2)/((c^4 + 2*c^2*d^2 + d^4)*f^4)) - sqrt(2
)*((b*c^5 - a*c^4*d + 2*b*c^3*d^2 - 2*a*c^2*d^3 + b*c*d^4 - a*d^5)*f^7*sqrt
((4*a^2*b^2*c^2 - 4*(a^3*b - a*b^3)*c*d + (a^4 - 2*a^2*b^2 + b^4)*d^2)/((c^
4 + 2*c^2*d^2 + d^4)*f^4))*sqrt((a^4 + 2*a^2*b^2 + b^4)/((c^2 + d^2)*f^4))
+ ((a^2*b + b^3)*c^4 + 2*(a^2*b + b^3)*c^2*d^2 + (a^2*b + b^3)*d^4)*f^5*sqr
t((4*a^2*b^2*c^2 - 4*(a^3*b - a*b^3)*c*d + (a^4 - 2*a^2*b^2 + b^4)*d^2)/((c
^4 + 2*c^2*d^2 + d^4)*f^4)))*sqrt(((2*a*b*c^2*d + 2*a*b*d^3 + (a^2 - b^2)*c
^3 + (a^2 - b^2)*c*d^2)*f^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/((c^2 + d^2)*f^4))
 + (a^4 + 2*a^2*b^2 + b^4)*c^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2)/(4*a^2*b^2*c^
2 - 4*(a^3*b - a*b^3)*c*d + (a^4 - 2*a^2*b^2 + b^4)*d^2))*sqrt(((4*(a^4*b^2
 + a^2*b^4)*c^4 - 4*(a^5*b - a*b^5)*c^3*d + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
b^6)*c^2*d^2 - 4*(a^5*b - a*b^5)*c*d^3 + (a^6 - a^4*b^2 - a^2*b^4 + b^6)*d^
4)*f^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/((c^2 + d^2)*f^4))*cos(f*x + e) + sqrt(
2)*((4*a^2*b^3*c^4 - 4*(a^3*b^2 - a*b^4)*c^3*d + (a^4*b + 2*a^2*b^3 + b^5)*
c^2*d^2 - 4*(a^3*b^2 - a*b^4)*c*d^3 + (a^4*b - 2*a^2*b^3 + b^5)*d^4)*f^3*sq
rt((a^4 + 2*a^2*b^2 + b^4)/((c^2 + d^2)*f^4))*cos(f*x + e) + (4*(a^4*b^3 +
a^2*b^5)*c^3 - 4*(2*a^5*b^2 + a^3*b^4 - a*b^6)*c^2*d + (5*a^6*b - a^4*b^3 -
```

```
 5*a^2*b^5 + b^7)*c*d^2 - (a^7 - a^5*b^2 - a^3*b^4 + a*b^6)*d^3)*f*cos(f*x
+ e))*sqrt(((2*a*b*c^2*d + 2*a*b*d^3 + (a^2 - b^2)*c^3 + (a^2 - b^2)*c*d^2)
*f^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/((c^2 + d^2)*f^4)) + (a^4 + 2*a^2*b^2 + b
^4)*c^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2)/(4*a^2*b^2*c^2 - 4*(a^3*b - a*b^3)*c
*d + (a^4 - 2*a^2*b^2 + b^4)*d^2))*sqrt((c*cos(f*x + e) + d*sin(f*x + e))/c
os(f*x + e))*((a^4 + 2*a^2*b^2 + b^4)/((c^2 + d^2)*f^4))^(1/4) + (4*(a^6*b^
2 + 2*a^4*b^4 + a^2*b^6)*c^3 - 4*(a^7*b + a^5*b^3 - a^3*b^5 - a*b^7)*c^2*d
+ (a^8 - 2*a^4*b^4 + b^8)*c*d^2)*cos(f*x + e) + (4*(a^6*b^2 + 2*a^4*b^4 + a
^2*b^6)*c^2*d - 4*(a^7*b + a^5*b^3 - a^3*b^5 - a*b^7)*c*d^2 + (a^8 - 2*a^4*
b^4 + b^8)*d^3)*sin(f*x + e))/cos(f*x + e))*((a^4 + 2*a^2*b^2 + b^4)/((c^2
+ d^2)*f^4))^(3/4) + sqrt(2)*((2*(a^3*b^2 + a*b^4)*c^6 - (3*a^4*b + 2*a^2*b
^3 - b^5)*c^5*d + (a^5 + 4*a^3*b^2 + 3*a*b^4)*c^4*d^2 - 2*(3*a^4*b + 2*a^2*
b^3 - b^5)*c^3*d^3 + 2*(a^5 + a^3*b^2)*c^2*d^4 - (3*a^4*b + 2*a^2*b^3 - b^5
)*c*d^5 + (a^5 - a*b^4)*d^6)*f^7*sqrt((4*a^2*b^2*c^2 - 4*(a^3*b - a*b^3)*c*
d + (a^4 - 2*a^2*b^2 + b^4)*d^2)/((c^4 + 2*c^2*d^2 + d^4)*f^4))*sqrt((a^4 +
 2*a^2*b^2 + b^4)/((c^2 + d^2)*f^4)) + (2*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*c^5
 - (a^6*b + a^4*b^3 - a^2*b^5 - b^7)*c^4*d + 4*(a^5*b^2 + 2*a^3*b^4 + a*b^6
)*c^3*d^2 - 2*(a^6*b + a^4*b^3 - a^2*b^5 - b^7)*c^2*d^3 + 2*(a^5*b^2 + 2*a^
3*b^4 + a*b^6)*c*d^4 - (a^6*b + a^4*b^3 - a^2*b^5 - b^7)*d^5)*f^5*sqrt((4*a
^2*b^2*c^2 - 4*(a^3*b - a*b^3)*c*d + (a^4 - 2*a^2*b^2 + b^4)*d^2)/((c^4 + 2
*c^2*d^2 + d^4)*f^4)))*sqrt(((2*a*b*c^2*d + 2*a*b*d^3 + (a^2 - b^2)*c^3 + (
a^2 - b^2)*c*d^2)*f^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/((c^2 + d^2)*f^4)) + (a^
4 + 2*a^2*b^2 + b^4)*c^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2)/(4*a^2*b^2*c^2 - 4*
(a^3*b - a*b^3)*c*d + (a^4 - 2*a^2*b^2 + b^4)*d^2))*sqrt((c*cos(f*x + e) +
d*sin(f*x + e))/cos(f*x + e))*((a^4 + 2*a^2*b^2 + b^4)/((c^2 + d^2)*f^4))^(
3/4))/(4*(a^10*b^2 + 4*a^8*b^4 + 6*a^6*b^6 + 4*a^4*b^8 + a^2*b^10)*c^2*d -
4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 - 2*a^5*b^7 - 3*a^3*b^9 - a*b^11)*c*d^2 +
 (a^12 + 2*a^10*b^2 - a^8*b^4 - 4*a^6*b^6 - a^4*b^8 + 2*a^2*b^10 + b^12)*d^
3)) + 4*sqrt(2)*(c^2 + d^2)*f^4*sqrt(((2*a*b*c^2*d + 2*a*b*d^3 + (a^2 - b^2
)*c^3 + (a^2 - b^2)*c*d^2)*f^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/((c^2 + d^2)*f^
4)) + (a^4 + 2*a^2*b^2 + b^4)*c^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2)/(4*a^2*b^2
*c^2 - 4*(a^3*b - a*b^3)*c*d + (a^4 - 2*a^2*b^2 + b^4)*d^2))*sqrt((4*a^2*b^
2*c^2 - 4*(a^3*b - a*b^3)*c*d + (a^4 - 2*a^2*b^2 + b^4)*d^2)/((c^4 + 2*c^2*
d^2 + d^4)*f^4))*((a^4 + 2*a^2*b^2 + b^4)/((c^2...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\tan(e + fx)}{\sqrt{c + d\tan(e + fx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(c+d*tan(f*x+e))**(1/2),x)
[Out] Integral((a + b*tan(e + f*x))/sqrt(c + d*tan(e + f*x)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(c+d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**B**]
time = 7.35, size = 2909, normalized size = 28.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))/(c + d*tan(e + f*x))^(1/2),x)
```

```
[Out] 2*atanh((8*c*d^2*(- (-16*a^4*d^2*f^4)^(1/2)/(16*(c^2*f^4 + d^2*f^4)) - (a^2
*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2)*(c + d*tan(e + f*x))^(1/2)*(-16*a^4*
d^2*f^4)^(1/2))/((16*a^3*c*d^5*f^5)/(c^2*f^4 + d^2*f^4) + (4*a*d^5*f^4*(-16
*a^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5) + (16*a^3*c^3*d^3*f^5)/(c^2*f^4 +
d^2*f^4) + (4*a*c^2*d^3*f^4*(-16*a^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5)) -
 (32*a^2*d^2*(- (-16*a^4*d^2*f^4)^(1/2)/(16*(c^2*f^4 + d^2*f^4)) - (a^2*c*f
^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2)*(c + d*tan(e + f*x))^(1/2))/((16*a^3*c*d
^3*f^3)/(c^2*f^4 + d^2*f^4) + (4*a*d^3*f^2*(-16*a^4*d^2*f^4)^(1/2))/(c^2*f^
5 + d^2*f^5)) + (32*a^2*c^2*d^2*f^2*(- (-16*a^4*d^2*f^4)^(1/2)/(16*(c^2*f^4
 + d^2*f^4)) - (a^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2)*(c + d*tan(e + f*
x))^(1/2))/((16*a^3*c*d^5*f^5)/(c^2*f^4 + d^2*f^4) + (4*a*d^5*f^4*(-16*a^4*
d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5) + (16*a^3*c^3*d^3*f^5)/(c^2*f^4 + d^2*f
^4) + (4*a*c^2*d^3*f^4*(-16*a^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5)))*(- (-
16*a^4*d^2*f^4)^(1/2)/(16*(c^2*f^4 + d^2*f^4)) - (a^2*c*f^2)/(4*(c^2*f^4 +
d^2*f^4)))^(1/2) - 2*atanh((32*a^2*d^2*((-16*a^4*d^2*f^4)^(1/2)/(16*(c^2*f^
4 + d^2*f^4)) - (a^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2)*(c + d*tan(e + f
*x))^(1/2))/((16*a^3*c*d^3*f^3)/(c^2*f^4 + d^2*f^4) - (4*a*d^3*f^2*(-16*a^4
*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5)) + (8*c*d^2*((-16*a^4*d^2*f^4)^(1/2)/(
16*(c^2*f^4 + d^2*f^4)) - (a^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2)*(c + d
*tan(e + f*x))^(1/2)*(-16*a^4*d^2*f^4)^(1/2))/((16*a^3*c*d^5*f^5)/(c^2*f^4
+ d^2*f^4) - (4*a*d^5*f^4*(-16*a^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5) + (1
6*a^3*c^3*d^3*f^5)/(c^2*f^4 + d^2*f^4) - (4*a*c^2*d^3*f^4*(-16*a^4*d^2*f^4)
^(1/2))/(c^2*f^5 + d^2*f^5)) - (32*a^2*c^2*d^2*f^2*((-16*a^4*d^2*f^4)^(1/2)
/(16*(c^2*f^4 + d^2*f^4)) - (a^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2)*(c +
```

d*tan(e + f*x))^(1/2))/((16*a^3*c*d^5*f^5)/(c^2*f^4 + d^2*f^4) - (4*a*d^5*f^4*(-16*a^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5) + (16*a^3*c^3*d^3*f^5)/(c^2*f^4 + d^2*f^4) - (4*a*c^2*d^3*f^4*(-16*a^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5)))*((-16*a^4*d^2*f^4)^(1/2)/(16*(c^2*f^4 + d^2*f^4)) - (a^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2) - 2*atanh((32*b^2*d^2*((b^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)) - (-16*b^4*d^2*f^4)^(1/2)/(16*(c^2*f^4 + d^2*f^4)))^(1/2)*(c + d*tan(e + f*x))^(1/2))/((16*b^3*d^2)/f - (16*b^3*c^2*d^2*f^3)/(c^2*f^4 + d^2*f^4) + (4*b*c*d^2*f^2*(-16*b^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5)) + (8*c*d^2*((b^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)) - (-16*b^4*d^2*f^4)^(1/2)/(16*(c^2*f^4 + d^2*f^4)))^(1/2)*(c + d*tan(e + f*x))^(1/2)*(-16*b^4*d^2*f^4)^(1/2))/(16*b^3*d^4*f + 16*b^3*c^2*d^2*f - (16*b^3*c^2*d^4*f^5)/(c^2*f^4 + d^2*f^4) - (16*b^3*c^4*d^2*f^5)/(c^2*f^4 + d^2*f^4) + (4*b*c*d^4*f^4*(-16*b^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5) + (4*b*c^3*d^2*f^4*(-16*b^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5)) - (32*b^2*c^2*d^2*f^2*((b^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)) - (-16*b^4*d^2*f^4)^(1/2)/(16*(c^2*f^4 + d^2*f^4)))^(1/2)*(c + d*tan(e + f*x))^(1/2))/(16*b^3*d^4*f + 16*b^3*c^2*d^2*f - (16*b^3*c^2*d^4*f^5)/(c^2*f^4 + d^2*f^4) - (16*b^3*c^4*d^2*f^5)/(c^2*f^4 + d^2*f^4) + (4*b*c*d^4*f^4*(-16*b^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5) + (4*b*c^3*d^2*f^4*(-16*b^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5)))*((b^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)) - (-16*b^4*d^2*f^4)^(1/2)/(16*(c^2*f^4 + d^2*f^4)))^(1/2) - 2*atanh((8*c*d^2*((-16*b^4*d^2*f^4)^(1/2)/(16*(c^2*f^4 + d^2*f^4)) + (b^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2)*(c + d*tan(e + f*x))^(1/2)*(-16*b^4*d^2*f^4)^(1/2))/((16*b^3*c^2*d^4*f^5)/(c^2*f^4 + d^2*f^4) - 16*b^3*c^2*d^2*f - 16*b^3*d^4*f + (16*b^3*c^4*d^2*f^5)/(c^2*f^4 + d^2*f^4) + (4*b*c*d^4*f^4*(-16*b^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5) + (4*b*c^3*d^2*f^4*(-16*b^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5)) - (32*b^2*d^2*((-16*b^4*d^2*f^4)^(1/2)/(16*(c^2*f^4 + d^2*f^4)) + (b^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2)*(c + d*tan(e + f*x))^(1/2))/((16*b^3*c^2*d^2*f^3)/(c^2*f^4 + d^2*f^4) - (16*b^3*d^2)/f + (4*b*c*d^2*f^2*(-16*b^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5)) + (32*b^2*c^2*d^2*f^2*((-16*b^4*d^2*f^4)^(1/2)/(16*(c^2*f^4 + d^2*f^4)) + (b^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2)*(c + d*tan(e + f*x))^(1/2))/((16*b^3*c^2*d^4*f^5)/(c^2*f^4 + d^2*f^4) - 16*b^3*c^2*d^2*f - 16*b^3*d^4*f + (16*b^3*c^4*d^2*f^5)/(c^2*f^4 + d^2*f^4) + (4*b*c*d^4*f^4*(-16*b^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5) + (4*b*c^3*d^2*f^4*(-16*b^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5)))*((-16*b^4*d^2*f^4)^(1/2)/(16*(c^2*f^4 + d^2*f^4)) + (b^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2)

3.1251 $\int \frac{1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=170

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(ia+b)\sqrt{c-id}\,f}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(ia-b)\sqrt{c+id}\,f}-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{(a^2+b^2)\sqrt{bc-ad}\,f}$$

```
[Out] arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(I*a+b)/f/(c-I*d)^(1/2)-arcta
nh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(I*a-b)/f/(c+I*d)^(1/2)-2*b^(3/2)*
arctanh(b^(1/2)*(c+d*tan(f*x+e))^(1/2)/(-a*d+b*c)^(1/2))/(a^2+b^2)/f/(-a*d+
b*c)^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3655, 3620, 3618, 65, 214, 3715}

$$-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{f(a^2+b^2)\sqrt{bc-ad}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(b+ia)\sqrt{c-id}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(-b+ia)\sqrt{c+id}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Tan[e + f*x])*Sqrt[c + d*Tan[e + f*x]]),x]
```

```
[Out] ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]]/((I*a + b)*Sqrt[c - I*d]*f)
 - ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]]/((I*a - b)*Sqrt[c + I*d]
*f) - (2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]
])/((a^2 + b^2)*Sqrt[b*c - a*d]*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)/((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[(a + b*Tan[e + f*x])^m*
(c - d*Tan[e + f*x]), x], x] + Dist[d^2/(c^2 + d^2), Int[(a + b*Tan[e + f*x
])^m*((1 + Tan[e + f*x]^2)/(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}\,dx = \frac{\int \frac{a-b\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx}{a^2+b^2} + \frac{b^2\int \frac{1+\tan^2(e+fx)}{(a+b\tan(e+fx))\sqrt{c+d\tan}}}{a^2+b^2}$$

$$= \frac{\int \frac{1+i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx}{2(a-ib)} + \frac{\int \frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx}{2(a+ib)}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{c+idx}}\,dx, x, -i\tan(e+fx)\right)}{2(ia-b)f} - \text{Subs}$$

$$= -\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{(a^2+b^2)\sqrt{bc-ad}\,f} - \text{Subst}\left(\int\right.$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(ia+b)\sqrt{c-id}\,f} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+d\tan(}}{\sqrt{c+}}\right.}{(ia-b)\sqrt{c+i}}$$

Mathematica [**A**]
time = 0.44, size = 158, normalized size = 0.93

$$\frac{\frac{(-ia+b)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}} + \frac{(ia+b)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{\sqrt{c+id}} - \frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}}{(a^2+b^2)\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Tan[e + f*x])*Sqrt[c + d*Tan[e + f*x]]),x]
```

```
[Out] ((((-I)*a + b)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/Sqrt[c - I*
d] + ((I*a + b)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/Sqrt[c + I
*d] - (2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]
])/Sqrt[b*c - a*d])/((a^2 + b^2)*f)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1467 vs. $2(142) = 284$.
time = 0.52, size = 1468, normalized size = 8.64 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*d^2*(b^2/d^2/(a^2+b^2)/((a*d-b*c)*b)^(1/2)*arctan(b*(c+d*tan(f*x+e))^(1
/2)/((a*d-b*c)*b)^(1/2))+1/d^2/(a^2+b^2)*(1/4/(c^2+d^2)^(3/2)/d^2*(-1/2*(-(
c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c+(c^2+d^2)^(1/2)*(2*(c^2+d^
```

```
2)^(1/2)+2*c)^(1/2)*a*c^2*d+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a
*d^3+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^3+(c^2+d^2)^(1/2)*(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d^2-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3*d-(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d^3+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d^
2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^4)*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))+2*(-2*a*c^2*d^3-2*a*d^5
-2*b*c^3*d^2-2*b*c*d^4+1/2*(-(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*
b*c+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2*d+(c^2+d^2)^(1/2)*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^3+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*b*c^3+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d^2-(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*a*c^3*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d^3+(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*b*c^2*d^2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^4)*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)
^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))
+1/4/(c^2+d^2)^(3/2)/d^2*(1/2*(-(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*b*c+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2*d+(c^2+d^2)^(1/2
)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^3+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*b*c^3+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d^2-(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)*a*c^3*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d^3+(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d^2+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^4)*ln(
d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^
2)^(1/2))+2*(2*a*c^2*d^3+2*a*d^5+2*b*c^3*d^2+2*b*c*d^4-1/2*(-(c^2+d^2)^(3/2
)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)*a*c^2*d+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^3+(c^2+d^2
)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^3+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*b*c*d^2-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3*d-(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)*a*c*d^3+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d^2+(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)*b*d^4)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)
-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))
/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11714 vs. $2(139) = 278$.

time = 95.66, size = 23416, normalized size = 137.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(2)*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c^2 + (a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6)*d^2)*f^5*sqrt((((a^6 + a^4*b^2 - a^2*b^4 - b^6)*c^3 - 2
*(a^5*b + 2*a^3*b^3 + a*b^5)*c^2*d + (a^6 + a^4*b^2 - a^2*b^4 - b^6)*c*d^2
- 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^3)*f^2*sqrt(1/(((a^4 + 2*a^2*b^2 + b^4)*c
^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2)*f^4)) + (a^4 + 2*a^2*b^2 + b^4)*c^2 + (a^
4 + 2*a^2*b^2 + b^4)*d^2)/(4*a^2*b^2*c^2 + 4*(a^3*b - a*b^3)*c*d + (a^4 - 2
*a^2*b^2 + b^4)*d^2))*sqrt((4*a^2*b^2*c^2 + 4*(a^3*b - a*b^3)*c*d + (a^4 -
2*a^2*b^2 + b^4)*d^2)/(((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*c^4
 + 2*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*c^2*d^2 + (a^8 + 4*a^6
*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)*f^4))*(1/(((a^4 + 2*a^2*b^2 + b^4)
*c^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2)*f^4))^(3/4)*arctan(((2*(a^7*b + 3*a^5*b
^3 + 3*a^3*b^5 + a*b^7)*c^5 + (a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*c^4*d + 4
*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*c^3*d^2 + 2*(a^8 + 2*a^6*b^2 - 2*a
^2*b^6 - b^8)*c^2*d^3 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*c*d^4 + (
a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^5)*f^4*sqrt((4*a^2*b^2*c^2 + 4*(a^3*b
- a*b^3)*c*d + (a^4 - 2*a^2*b^2 + b^4)*d^2)/(((a^8 + 4*a^6*b^2 + 6*a^4*b^4
+ 4*a^2*b^6 + b^8)*c^4 + 2*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*
c^2*d^2 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)*f^4))*sqrt(1
/(((a^4 + 2*a^2*b^2 + b^4)*c^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2)*f^4)) + (2*(a
^5*b + 2*a^3*b^3 + a*b^5)*c^4 + (a^6 + a^4*b^2 - a^2*b^4 - b^6)*c^3*d + 2*(
a^5*b + 2*a^3*b^3 + a*b^5)*c^2*d^2 + (a^6 + a^4*b^2 - a^2*b^4 - b^6)*c*d^3)
*f^2*sqrt((4*a^2*b^2*c^2 + 4*(a^3*b - a*b^3)*c*d + (a^4 - 2*a^2*b^2 + b^4)*
d^2)/(((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*c^4 + 2*(a^8 + 4*a^6
*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*c^2*d^2 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4
+ 4*a^2*b^6 + b^8)*d^4)*f^4)) + sqrt(2)*(((a^8*b + 4*a^6*b^3 + 6*a^4*b^5 +
4*a^2*b^7 + b^9)*c^5 + (a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*c^
4*d + 2*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*c^3*d^2 + 2*(a^9
+ 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*c^2*d^3 + (a^8*b + 4*a^6*b^3 +
 6*a^4*b^5 + 4*a^2*b^7 + b^9)*c*d^4 + (a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*
b^6 + a*b^8)*d^5)*f^7*sqrt((4*a^2*b^2*c^2 + 4*(a^3*b - a*b^3)*c*d + (a^4 -
2*a^2*b^2 + b^4)*d^2)/(((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*c^4
 + 2*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*c^2*d^2 + (a^8 + 4*a^6
*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)*f^4))*sqrt(1/(((a^4 + 2*a^2*b^2 +
b^4)*c^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2)*f^4)) + ((a^6*b + 3*a^4*b^3 + 3*a^2
*b^5 + b^7)*c^4 + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*c^2*d^2 + (a^6*b
+ 3*a^4*b^3 + 3*a^2*b^5 + b^7)*d^4)*f^5*sqrt((4*a^2*b^2*c^2 + 4*(a^3*b - a*
b^3)*c*d + (a^4 - 2*a^2*b^2 + b^4)*d^2)/(((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*
a^2*b^6 + b^8)*c^4 + 2*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*c^2*
```

```
d^2 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)*f^4)))*sqrt((((a
^6 + a^4*b^2 - a^2*b^4 - b^6)*c^3 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*c^2*d + (
a^6 + a^4*b^2 - a^2*b^4 - b^6)*c*d^2 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^3)*f
^2*sqrt(1/(((a^4 + 2*a^2*b^2 + b^4)*c^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2)*f^4)
) + (a^4 + 2*a^2*b^2 + b^4)*c^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2)/(4*a^2*b^2*c
^2 + 4*(a^3*b - a*b^3)*c*d + (a^4 - 2*a^2*b^2 + b^4)*d^2))*sqrt(((4*(a^4*b^
2 + a^2*b^4)*c^4 + 4*(a^5*b - a*b^5)*c^3*d + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
 b^6)*c^2*d^2 + 4*(a^5*b - a*b^5)*c*d^3 + (a^6 - a^4*b^2 - a^2*b^4 + b^6)*d
^4)*f^2*sqrt(1/(((a^4 + 2*a^2*b^2 + b^4)*c^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2)
*f^4))*cos(f*x + e) + sqrt(2)*((4*(a^4*b^3 + a^2*b^5)*c^4 + 4*(a^5*b^2 - a*
b^6)*c^3*d + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*c^2*d^2 + 4*(a^5*b^2 - a
*b^6)*c*d^3 + (a^6*b - a^4*b^3 - a^2*b^5 + b^7)*d^4)*f^3*sqrt(1/(((a^4 + 2*
a^2*b^2 + b^4)*c^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2)*f^4))*cos(f*x + e) + (4*a
^2*b^3*c^3 + 4*(2*a^3*b^2 - a*b^4)*c^2*d + (5*a^4*b - 6*a^2*b^3 + b^5)*c*d^
2 + (a^5 - 2*a^3*b^2 + a*b^4)*d^3)*f*cos(f*x + e))*sqrt((((a^6 + a^4*b^2 -
a^2*b^4 - b^6)*c^3 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*c^2*d + (a^6 + a^4*b^2 -
 a^2*b^4 - b^6)*c*d^2 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^3)*f^2*sqrt(1/(((a^
4 + 2*a^2*b^2 + b^4)*c^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2)*f^4)) + (a^4 + 2*a^
2*b^2 + b^4)*c^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2)/(4*a^2*b^2*c^2 + 4*(a^3*b -
 a*b^3)*c*d + (a^4 - 2*a^2*b^2 + b^4)*d^2))*sqrt((c*cos(f*x + e) + d*sin(f*
x + e))/cos(f*x + e))*(1/(((a^4 + 2*a^2*b^2 + b^4)*c^2 + (a^4 + 2*a^2*b^2 +
 b^4)*d^2)*f^4))^(1/4) + (4*a^2*b^2*c^3 + 4*(a^3*b - a*b^3)*c^2*d + (a^4 -
2*a^2*b^2 + b^4)*c*d^2)*cos(f*x + e) + (4*a^2*b^2*c^2*d + 4*(a^3*b - a*b^3)
*c*d^2 + (a^4 - 2*a^2*b^2 + b^4)*d^3)*sin(f*x + e))/cos(f*x + e))*(1/(((a^4
 + 2*a^2*b^2 + b^4)*c^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2)*f^4))^(3/4) + sqrt(2
)*((2*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*c^6 + (3*a^10*
b + 11*a^8*b^3 + 14*a^6*b^5 + 6*a^4*b^7 - a^2*b^9 - b^11)*c^5*d + (a^11 + 7
*a^9*b^2 + 18*a^7*b^4 + 22*a^5*b^6 + 13*a^3*b^8 + 3*a*b^10)*c^4*d^2 + 2*(3*
a^10*b + 11*a^8*b^3 + 14*a^6*b^5 + 6*a^4*b^7 - ...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))**(1/2)/(a+b*tan(f*x+e)),x)
```

```
[Out] Integral(1/((a + b*tan(e + f*x))*sqrt(c + d*tan(e + f*x))), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 10.27, size = 2500, normalized size = 14.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*tan(e + f*x))*(c + d*tan(e + f*x))^(1/2)),x)
```

```
[Out] - atan((((((((32*(16*b^8*d^10*f^2 + 28*a^2*b^6*d^10*f^2 + 8*a^4*b^4*d^10*f^2
 - 4*a^6*b^2*d^10*f^2 + 12*b^8*c^2*d^8*f^2 + 24*a^2*b^6*c^2*d^8*f^2 + 12*a^
4*b^4*c^2*d^8*f^2 - 8*a*b^7*c*d^9*f^2 - 16*a^3*b^5*c*d^9*f^2 - 8*a^5*b^3*c*
d^9*f^2))/f^3 - (32*(1i/(4*(a^2*d*f^2 - a^2*c*f^2*1i + b^2*c*f^2*1i - b^2*d
*f^2 + 2*a*b*c*f^2 + a*b*d*f^2*2i)))^(1/2)*(c + d*tan(e + f*x))^(1/2)*(16*b
^9*d^10*f^4 + 16*a^2*b^7*d^10*f^4 - 16*a^4*b^5*d^10*f^4 - 16*a^6*b^3*d^10*f
^4 + 24*b^9*c^2*d^8*f^4 + 40*a^2*b^7*c^2*d^8*f^4 + 8*a^4*b^5*c^2*d^8*f^4 -
8*a^6*b^3*c^2*d^8*f^4 + 8*a*b^8*c*d^9*f^4 + 24*a^3*b^6*c*d^9*f^4 + 24*a^5*b
^4*c*d^9*f^4 + 8*a^7*b^2*c*d^9*f^4))/f^4)*(1i/(4*(a^2*d*f^2 - a^2*c*f^2*1i
+ b^2*c*f^2*1i - b^2*d*f^2 + 2*a*b*c*f^2 + a*b*d*f^2*2i)))^(1/2) + (32*(c +
 d*tan(e + f*x))^(1/2)*(30*a*b^6*d^9*f^2 + 18*b^7*c*d^8*f^2 - 4*a^3*b^4*d^9
*f^2 - 2*a^5*b^2*d^9*f^2 - 12*a^2*b^5*c*d^8*f^2 + 2*a^4*b^3*c*d^8*f^2))/f^4
)*(1i/(4*(a^2*d*f^2 - a^2*c*f^2*1i + b^2*c*f^2*1i - b^2*d*f^2 + 2*a*b*c*f^2
 + a*b*d*f^2*2i)))^(1/2) - (32*(5*a*b^5*d^9 + 3*b^6*c*d^8 + a^3*b^3*d^9 - a
^2*b^4*c*d^8))/f^3)*(1i/(4*(a^2*d*f^2 - a^2*c*f^2*1i + b^2*c*f^2*1i - b^2*d
*f^2 + 2*a*b*c*f^2 + a*b*d*f^2*2i)))^(1/2) - (96*b^5*d^8*(c + d*tan(e + f*x
))^(1/2))/f^4)*(1i/(4*(a^2*d*f^2 - a^2*c*f^2*1i + b^2*c*f^2*1i - b^2*d*f^2
+ 2*a*b*c*f^2 + a*b*d*f^2*2i)))^(1/2)*1i - (((((32*(16*b^8*d^10*f^2 + 28*a^
2*b^6*d^10*f^2 + 8*a^4*b^4*d^10*f^2 - 4*a^6*b^2*d^10*f^2 + 12*b^8*c^2*d^8*f
^2 + 24*a^2*b^6*c^2*d^8*f^2 + 12*a^4*b^4*c^2*d^8*f^2 - 8*a*b^7*c*d^9*f^2 -
16*a^3*b^5*c*d^9*f^2 - 8*a^5*b^3*c*d^9*f^2))/f^3 + (32*(1i/(4*(a^2*d*f^2 -
a^2*c*f^2*1i + b^2*c*f^2*1i - b^2*d*f^2 + 2*a*b*c*f^2 + a*b*d*f^2*2i)))^(1/
2)*(c + d*tan(e + f*x))^(1/2)*(16*b^9*d^10*f^4 + 16*a^2*b^7*d^10*f^4 - 16*a
^4*b^5*d^10*f^4 - 16*a^6*b^3*d^10*f^4 + 24*b^9*c^2*d^8*f^4 + 40*a^2*b^7*c^2
*d^8*f^4 + 8*a^4*b^5*c^2*d^8*f^4 - 8*a^6*b^3*c^2*d^8*f^4 + 8*a*b^8*c*d^9*f^
4 + 24*a^3*b^6*c*d^9*f^4 + 24*a^5*b^4*c*d^9*f^4 + 8*a^7*b^2*c*d^9*f^4))/f^4
)*(1i/(4*(a^2*d*f^2 - a^2*c*f^2*1i + b^2*c*f^2*1i - b^2*d*f^2 + 2*a*b*c*f^2
 + a*b*d*f^2*2i)))^(1/2) - (32*(c + d*tan(e + f*x))^(1/2)*(30*a*b^6*d^9*f^2
 + 18*b^7*c*d^8*f^2 - 4*a^3*b^4*d^9*f^2 - 2*a^5*b^2*d^9*f^2 - 12*a^2*b^5*c*
d^8*f^2 + 2*a^4*b^3*c*d^8*f^2))/f^4)*(1i/(4*(a^2*d*f^2 - a^2*c*f^2*1i + b^2
*c*f^2*1i - b^2*d*f^2 + 2*a*b*c*f^2 + a*b*d*f^2*2i)))^(1/2) - (32*(5*a*b^5*
```

```
d^9 + 3*b^6*c*d^8 + a^3*b^3*d^9 - a^2*b^4*c*d^8))/f^3)*(1i/(4*(a^2*d*f^2 -
a^2*c*f^2*1i + b^2*c*f^2*1i - b^2*d*f^2 + 2*a*b*c*f^2 + a*b*d*f^2*2i)))^(1/
2) + (96*b^5*d^8*(c + d*tan(e + f*x))^(1/2))/f^4)*(1i/(4*(a^2*d*f^2 - a^2*c
*f^2*1i + b^2*c*f^2*1i - b^2*d*f^2 + 2*a*b*c*f^2 + a*b*d*f^2*2i)))^(1/2)*1i
)/((((((32*(16*b^8*d^10*f^2 + 28*a^2*b^6*d^10*f^2 + 8*a^4*b^4*d^10*f^2 - 4*
a^6*b^2*d^10*f^2 + 12*b^8*c^2*d^8*f^2 + 24*a^2*b^6*c^2*d^8*f^2 + 12*a^4*b^4
*c^2*d^8*f^2 - 8*a*b^7*c*d^9*f^2 - 16*a^3*b^5*c*d^9*f^2 - 8*a^5*b^3*c*d^9*f
^2))/f^3 - (32*(1i/(4*(a^2*d*f^2 - a^2*c*f^2*1i + b^2*c*f^2*1i - b^2*d*f^2
+ 2*a*b*c*f^2 + a*b*d*f^2*2i)))^(1/2)*(c + d*tan(e + f*x))^(1/2)*(16*b^9*d^
10*f^4 + 16*a^2*b^7*d^10*f^4 - 16*a^4*b^5*d^10*f^4 - 16*a^6*b^3*d^10*f^4 +
24*b^9*c^2*d^8*f^4 + 40*a^2*b^7*c^2*d^8*f^4 + 8*a^4*b^5*c^2*d^8*f^4 - 8*a^6
*b^3*c^2*d^8*f^4 + 8*a*b^8*c*d^9*f^4 + 24*a^3*b^6*c*d^9*f^4 + 24*a^5*b^4*c*
d^9*f^4 + 8*a^7*b^2*c*d^9*f^4))/f^4)*(1i/(4*(a^2*d*f^2 - a^2*c*f^2*1i + b^2
*c*f^2*1i - b^2*d*f^2 + 2*a*b*c*f^2 + a*b*d*f^2*2i)))^(1/2) + (32*(c + d*ta
n(e + f*x))^(1/2)*(30*a*b^6*d^9*f^2 + 18*b^7*c*d^8*f^2 - 4*a^3*b^4*d^9*f^2
- 2*a^5*b^2*d^9*f^2 - 12*a^2*b^5*c*d^8*f^2 + 2*a^4*b^3*c*d^8*f^2))/f^4)*(1i
/(4*(a^2*d*f^2 - a^2*c*f^2*1i + b^2*c*f^2*1i - b^2*d*f^2 + 2*a*b*c*f^2 + a*
b*d*f^2*2i)))^(1/2) - (32*(5*a*b^5*d^9 + 3*b^6*c*d^8 + a^3*b^3*d^9 - a^2*b^
4*c*d^8))/f^3)*(1i/(4*(a^2*d*f^2 - a^2*c*f^2*1i + b^2*c*f^2*1i - b^2*d*f^2
+ 2*a*b*c*f^2 + a*b*d*f^2*2i)))^(1/2) - (96*b^5*d^8*(c + d*tan(e + f*x))^(1
/2))/f^4)*(1i/(4*(a^2*d*f^2 - a^2*c*f^2*1i + b^2*c*f^2*1i - b^2*d*f^2 + 2*a
*b*c*f^2 + a*b*d*f^2*2i)))^(1/2) + (((((32*(16*b^8*d^10*f^2 + 28*a^2*b^6*d^
10*f^2 + 8*a^4*b^4*d^10*f^2 - 4*a^6*b^2*d^10*f^2 + 12*b^8*c^2*d^8*f^2 + 24*
a^2*b^6*c^2*d^8*f^2 + 12*a^4*b^4*c^2*d^8*f^2 - 8*a*b^7*c*d^9*f^2 - 16*a^3*b
^5*c*d^9*f^2 - 8*a^5*b^3*c*d^9*f^2))/f^3 + (32*(1i/(4*(a^2*d*f^2 - a^2*c*f^
2*1i + b^2*c*f^2*1i - b^2*d*f^2 + 2*a*b*c*f^2 + a*b*d*f^2*2i)))^(1/2)*(c +
d*tan(e + f*x))^(1/2)*(16*b^9*d^10*f^4 + 16*a^2*b^7*d^10*f^4 - 16*a^4*b^5*d
^10*f^4 - 16*a^6*b^3*d^10*f^4 + 24*b^9*c^2*d^8*f^4 + 40*a^2*b^7*c^2*d^8*f^4
 + 8*a^4*b^5*c^2*d^8*f^4 - 8*a^6*b^3*c^2*d^8*f^4 + 8*a*b^8*c*d^9*f^4 + 24*a
^3*b^6*c*d^9*f^4 + 24*a^5*b^4*c*d^9*f^4 + 8*a^7*b^2*c*d^9*f^4))/f^4)*(1i/(4
*(a^2*d*f^2 - a^2*c*f^2*1i + b^2*c*f^2*1i - b^2*d*f^2 + 2*a*b*c*f^2 + a*b*d
*f^2*2i)))^(1/2) - (32*(c + d*tan(e + f*x))^(1/2)*(30*a*b^6*d^9*f^2 + 18*b^
7*c*d^8*f^2 - 4*a^3*b^4*d^9*f^2 - 2*a^5*b^2*d^9*f^2 - 12*a^2*b^5*c*d^8*f^2
+ 2*a^4*b^3*c*d^8*f^2))/f^4)*(1i/(4*(a^2*d*f^2 - a^2*c*f^2*1i + b^2*c*f^2*1
i - b^2*d*f^2 + 2*a*b*c*f^2 + a*b*d*f^2*2i)))^(1/2) - (32*(5*a*b^5*d^9 + 3*
b^6*c*d^8 + a^3*b^3*d^9 - a^2*b^4*c*d^8))/f^3)*...
```

3.1252 $\int \frac{1}{(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=244

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)^2\sqrt{c-id}\,f}+\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(a+ib)^2\sqrt{c+id}\,f}-\frac{b^{3/2}(4abc-5a^2d-b^2d)\tanh^{-1}\left(\right.}{(a^2+b^2)^2\,(bc-}$$

```
[Out] -b^(3/2)*(-5*a^2*d+4*a*b*c-b^2*d)*arctanh(b^(1/2)*(c+d*tan(f*x+e))^(1/2)/(-
a*d+b*c)^(1/2))/(a^2+b^2)^2/(-a*d+b*c)^(3/2)/f-I*arctanh((c+d*tan(f*x+e))^(
1/2)/(c-I*d)^(1/2))/(a-I*b)^2/f/(c-I*d)^(1/2)+I*arctanh((c+d*tan(f*x+e))^(1
/2)/(c+I*d)^(1/2))/(a+I*b)^2/f/(c+I*d)^(1/2)-b^2*(c+d*tan(f*x+e))^(1/2)/(a^
2+b^2)/(-a*d+b*c)/f/(a+b*tan(f*x+e))
```

Rubi [A]
time = 0.61, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3650, 3734, 3620, 3618, 65, 214, 3715}

$$-\frac{b^2\sqrt{c+d\tan(e+fx)}}{f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))}-\frac{b^{3/2}(-5a^2d+4abc-b^2d)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{f(a^2+b^2)^2(bc-ad)^{3/2}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(a-ib)^2\sqrt{c-id}}+\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(a+ib)^2\sqrt{c+id}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Tan[e + f*x])^2*Sqrt[c + d*Tan[e + f*x]]),x]
```

```
[Out] ((-I)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((a - I*b)^2*Sqrt[c
- I*d]*f) + (I*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/((a + I*b)^
2*Sqrt[c + I*d]*f) - (b^(3/2)*(4*a*b*c - 5*a^2*d - b^2*d)*ArcTanh[(Sqrt[b]*
Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/((a^2 + b^2)^2*(b*c - a*d)^(3/2
)*f) - (b^2*Sqrt[c + d*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan
[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}\,dx = -\frac{b^2\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)\,(bc-ad)f(a+b\tan(e+fx))} - \frac{\int \frac{\frac{1}{2}(-2abc+2a\cdots}{(a+\cdots}}{\cdots}$$

$$= -\frac{b^2\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)\,(bc-ad)f(a+b\tan(e+fx))} - \frac{\int \frac{-(a^2-b^2)(bc\cdots}{\sqrt{c\cdots}}}{(a^2\cdots}$$

$$= -\frac{b^2\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)\,(bc-ad)f(a+b\tan(e+fx))} + \frac{\int \frac{1+i\tan\cdots}{\sqrt{c+d\tan\cdots}}}{2(a\cdots}$$

$$= -\frac{b^2\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)\,(bc-ad)f(a+b\tan(e+fx))} + \frac{i\text{Subst}\left(\int \frac{\cdots}{(\cdots}\right.}{\cdots}$$

$$= -\frac{b^{3/2}(4abc-5a^2d-b^2d)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+\cdots}}{\sqrt{bc-ad}}\right.}{(a^2+b^2)^2\,(bc-ad)^{3/2}f}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)^2\sqrt{c-id}\,f} + \frac{i\tanh^{-1}\left(\frac{\sqrt{c+\cdots}}{\cdots}\right.}{(a+ib)^2\sqrt{\cdots}}$$

Mathematica [A]
time = 2.43, size = 258, normalized size = 1.06

$$\frac{-\frac{i\left(\frac{(a+ib)^2(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}}+\frac{(a-ib)^2(-bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{\sqrt{c+id}}\right)}{a^2+b^2}+\frac{b^{3/2}(-4abc+5a^2d+b^2d)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{(a^2+b^2)\sqrt{bc-ad}}-\frac{b^2\sqrt{c+d\tan(e+fx)}}{a+b\tan(e+fx)}}{(a^2+b^2)\,(bc-ad)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Tan[e + f*x])^2*Sqrt[c + d*Tan[e + f*x]]),x]
```

```
[Out] (((-I)*(((a + I*b)^2*(b*c - a*d)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c -
I*d]])/Sqrt[c - I*d] + ((a - I*b)^2*(-(b*c) + a*d)*ArcTanh[Sqrt[c + d*Tan[e
 + f*x]]/Sqrt[c + I*d]])/Sqrt[c + I*d]))/(a^2 + b^2) + (b^(3/2)*(-4*a*b*c +
 5*a^2*d + b^2*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d
]])/((a^2 + b^2)*Sqrt[b*c - a*d]) - (b^2*Sqrt[c + d*Tan[e + f*x]])/(a + b*T
an[e + f*x]))/((a^2 + b^2)*(b*c - a*d)*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2143 vs. $2(212) = 424$.
time = 0.56, size = 2144, normalized size = 8.79

method	result	size
derivativedivides	Expression too large to display	2144
default	Expression too large to display	2144

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*d^3*(1/(a^2+b^2)^2/d^3*(1/4/(c^2+d^2)^(3/2)/d^2*(1/2*(-2*(c^2+d^2)^(3/2
)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*a^2*c^2*d+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d^3+2*
(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^3+2*(c^2+d^2)^(1/2)*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c*d^2-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*b^2*c^2*d-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d^3-(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^3*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c*d^
3+2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^2*d^2+2*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*a*b*d^4+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^3*d+(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*b^2*c*d^3)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2
)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(2*a^2*c^2*d^3+2*a^2*d^5+4*a*b*c^3*d^2+4*a*
b*c*d^4-2*b^2*c^2*d^3-2*b^2*d^5-1/2*(-2*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+
2*c)^(1/2)*a*b*c+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^2*d+(c
^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d^3+2*(c^2+d^2)^(1/2)*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^3+2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*a*b*c*d^2-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^2*d-(c^
2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d^3-(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*a^2*c^3*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c*d^3+2*(2*(c^2+d^2)^(1/2
)+2*c)^(1/2)*a*b*c^2*d^2+2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d^4+(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*b^2*c^3*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c*d^3)*(2*(
c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan
(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)
))+1/4/(c^2+d^2)^(3/2)/d^2*(1/2*(2*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*a*b*c-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^2*d-(c^2+d^
2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d^3-2*(c^2+d^2)^(1/2)*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*a*b*c^3-2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*a*b*c*d^2+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^2*d+(c^2+d^2
)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d^3+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*a^2*c^3*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c*d^3-2*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*a*b*c^2*d^2-2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d^4-(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*b^2*c^3*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c*d^3)*ln(d*tan(
f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/
2))+2*(2*a^2*c^2*d^3+2*a^2*d^5+4*a*b*c^3*d^2+4*a*b*c*d^4-2*b^2*c^2*d^3-2*b^
2*d^5+1/2*(2*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c-(c^2+d^2)^
(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^2*d-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*a^2*d^3-2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*
```

```
b*c^3-2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c*d^2+(c^2+d^2)^(
1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^2*d+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)*b^2*d^3+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^3*d+(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)*a^2*c*d^3-2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^2*d^2-2*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d^4-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^3
*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c*d^3)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/
(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d^2)
^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))))+b^2/d^3/(a^2+b^2)^2*(1/
2*d*(a^2+b^2)/(a*d-b*c)*(c+d*tan(f*x+e))^(1/2)/((c+d*tan(f*x+e))*b+a*d-b*c)
+1/2*(5*a^2*d-4*a*b*c+b^2*d)/(a*d-b*c)/((a*d-b*c)*b)^(1/2)*arctan(b*(c+d*ta
n(f*x+e))^(1/2)/((a*d-b*c)*b)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 23867 vs. $2(209) = 418$.
time = 174.50, size = 47721, normalized size = 195.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] [1/4*(4*sqrt(2)*(((a^14*b + 5*a^12*b^3 + 9*a^10*b^5 + 5*a^8*b^7 - 5*a^6*b^9
 - 9*a^4*b^11 - 5*a^2*b^13 - b^15)*c^3 - (a^15 + 5*a^13*b^2 + 9*a^11*b^4 +
5*a^9*b^6 - 5*a^7*b^8 - 9*a^5*b^10 - 5*a^3*b^12 - a*b^14)*c^2*d + (a^14*b +
 5*a^12*b^3 + 9*a^10*b^5 + 5*a^8*b^7 - 5*a^6*b^9 - 9*a^4*b^11 - 5*a^2*b^13
- b^15)*c*d^2 - (a^15 + 5*a^13*b^2 + 9*a^11*b^4 + 5*a^9*b^6 - 5*a^7*b^8 - 9
*a^5*b^10 - 5*a^3*b^12 - a*b^14)*d^3)*f^5*cos(f*x + e)^2 + 2*((a^13*b^2 + 6
*a^11*b^4 + 15*a^9*b^6 + 20*a^7*b^8 + 15*a^5*b^10 + 6*a^3*b^12 + a*b^14)*c^
3 - (a^14*b + 6*a^12*b^3 + 15*a^10*b^5 + 20*a^8*b^7 + 15*a^6*b^9 + 6*a^4*b^
11 + a^2*b^13)*c^2*d + (a^13*b^2 + 6*a^11*b^4 + 15*a^9*b^6 + 20*a^7*b^8 + 1
5*a^5*b^10 + 6*a^3*b^12 + a*b^14)*c*d^2 - (a^14*b + 6*a^12*b^3 + 15*a^10*b^
```

5 + 20*a^8*b^7 + 15*a^6*b^9 + 6*a^4*b^11 + a^2*b^13)*d^3)*f^5*cos(f*x + e)*
sin(f*x + e) + ((a^12*b^3 + 6*a^10*b^5 + 15*a^8*b^7 + 20*a^6*b^9 + 15*a^4*b
^11 + 6*a^2*b^13 + b^15)*c^3 - (a^13*b^2 + 6*a^11*b^4 + 15*a^9*b^6 + 20*a^7
*b^8 + 15*a^5*b^10 + 6*a^3*b^12 + a*b^14)*c^2*d + (a^12*b^3 + 6*a^10*b^5 +
15*a^8*b^7 + 20*a^6*b^9 + 15*a^4*b^11 + 6*a^2*b^13 + b^15)*c*d^2 - (a^13*b^
2 + 6*a^11*b^4 + 15*a^9*b^6 + 20*a^7*b^8 + 15*a^5*b^10 + 6*a^3*b^12 + a*b^1
4)*d^3)*f^5)*sqrt((((a^12 - 2*a^10*b^2 - 17*a^8*b^4 - 28*a^6*b^6 - 17*a^4*b
^8 - 2*a^2*b^10 + b^12)*c^3 - 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 - 2*a^5*b^7
 - 3*a^3*b^9 - a*b^11)*c^2*d + (a^12 - 2*a^10*b^2 - 17*a^8*b^4 - 28*a^6*b^6
 - 17*a^4*b^8 - 2*a^2*b^10 + b^12)*c*d^2 - 4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^
5 - 2*a^5*b^7 - 3*a^3*b^9 - a*b^11)*d^3)*f^2*sqrt(1/(((a^8 + 4*a^6*b^2 + 6*
a^4*b^4 + 4*a^2*b^6 + b^8)*c^2 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 +
 b^8)*d^2)*f^4)) + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*c^2 + (a
^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^2)/(16*(a^6*b^2 - 2*a^4*b^4
 + a^2*b^6)*c^2 + 8*(a^7*b - 7*a^5*b^3 + 7*a^3*b^5 - a*b^7)*c*d + (a^8 - 12
*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*d^2))*sqrt((16*(a^6*b^2 - 2*a^4*b
^4 + a^2*b^6)*c^2 + 8*(a^7*b - 7*a^5*b^3 + 7*a^3*b^5 - a*b^7)*c*d + (a^8 -
12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*d^2)/(((a^16 + 8*a^14*b^2 + 28*
a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^1
4 + b^16)*c^4 + 2*(a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b
^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*c^2*d^2 + (a^16 + 8*a^1
4*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12
+ 8*a^2*b^14 + b^16)*d^4)*f^4))*(1/(((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b
^6 + b^8)*c^2 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^2)*f^4))^
(3/4)*arctan(((4*(a^15*b + 5*a^13*b^3 + 9*a^11*b^5 + 5*a^9*b^7 - 5*a^7*b^9
- 9*a^5*b^11 - 5*a^3*b^13 - a*b^15)*c^5 + (a^16 - 20*a^12*b^4 - 64*a^10*b^6
 - 90*a^8*b^8 - 64*a^6*b^10 - 20*a^4*b^12 + b^16)*c^4*d + 8*(a^15*b + 5*a^1
3*b^3 + 9*a^11*b^5 + 5*a^9*b^7 - 5*a^7*b^9 - 9*a^5*b^11 - 5*a^3*b^13 - a*b^
15)*c^3*d^2 + 2*(a^16 - 20*a^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 - 64*a^6*b^1
0 - 20*a^4*b^12 + b^16)*c^2*d^3 + 4*(a^15*b + 5*a^13*b^3 + 9*a^11*b^5 + 5*a
^9*b^7 - 5*a^7*b^9 - 9*a^5*b^11 - 5*a^3*b^13 - a*b^15)*c*d^4 + (a^16 - 20*a
^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 - 64*a^6*b^10 - 20*a^4*b^12 + b^16)*d^5)
*f^4*sqrt((16*(a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*c^2 + 8*(a^7*b - 7*a^5*b^3 +
7*a^3*b^5 - a*b^7)*c*d + (a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)
*d^2)/(((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^
6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*c^4 + 2*(a^16 + 8*a^14*b^2 + 28*a
^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14
 + b^16)*c^2*d^2 + (a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*
b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4)*f^4))*sqrt(1/(((a
^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*c^2 + (a^8 + 4*a^6*b^2 + 6*a^
4*b^4 + 4*a^2*b^6 + b^8)*d^2)*f^4)) + (4*(a^11*b + 3*a^9*b^3 + 2*a^7*b^5 -
2*a^5*b^7 - 3*a^3*b^9 - a*b^11)*c^4 + (a^12 - 2*a^10*b^2 - 17*a^8*b^4 - 28*
a^6*b^6 - 17*a^4*b^8 - 2*a^2*b^10 + b^12)*c^3*d + 4*(a^11*b + 3*a^9*b^3 + 2
*a^7*b^5 - 2*a^5*b^7 - 3*a^3*b^9 - a*b^11)*c^2*d^2 + (a^12 - 2*a^10*b^2 - 1
7*a^8*b^4 - 28*a^6*b^6 - 17*a^4*b^8 - 2*a^2*b^10 + b^12)*c*d^3)*f^2*sqrt((1

```
6*(a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*c^2 + 8*(a^7*b - 7*a^5*b^3 + 7*a^3*b^5 -
a*b^7)*c*d + (a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*d^2)/(((a^1
6 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*
a^4*b^12 + 8*a^2*b^14 + b^16)*c^4 + 2*(a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56
*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*c^2
*d^2 + (a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6
*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4)*f^4)) - sqrt(2)*((2*(a^17*b +
 8*a^15*b^3 + 28*a^13*b^5 + 56*a^11*b^7 + 70*a^9*b^9 + 56*a^7*b^11 + 28*a^5
*b^13 + 8*a^3*b^15 + a*b^17)*c^5 + (a^18 + 7*a^16*b^2 + 20*a^14*b^4 + 28*a^
12*b^6 + 14*a^10*b^8 - 14*a^8*b^10 - 28*a^6*b^12 - 20*a^4*b^14 - 7*a^2*b^16
 - b^18)*c^4*d + 4*(a^17*b + 8*a^15*b^3 + 28*a^13*b^5 + 56*a^11*b^7 + 70*a^
9*b^9 + 56*a^7*b^11 + 28*a^5*b^13 + 8*a^3*b^15 ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\tan\left(e + fx\right)\right)^{2}\sqrt{c + d\tan\left(e + fx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))**(1/2)/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral(1/((a + b*tan(e + f*x))**2*sqrt(c + d*tan(e + f*x))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 15.26, size = 2500, normalized size = 10.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*tan(e + f*x))^2*(c + d*tan(e + f*x))^(1/2)),x)
```

```
[Out] (atan((((((16*(2*b^13*d^11*f^2 - 24*a^2*b^11*d^11*f^2 - 196*a^4*b^9*d^11*f
^2 - 120*a^6*b^7*d^11*f^2 + 50*a^8*b^5*d^11*f^2 + 8*b^13*c^2*d^9*f^2 - 8*a^
```

2*b^11*c^2*d^9*f^2 + 64*a^3*b^10*c^3*d^8*f^2 - 232*a^4*b^9*c^2*d^9*f^2 + 96
*a^5*b^8*c^3*d^8*f^2 - 216*a^6*b^7*c^2*d^9*f^2 - 32*a*b^12*c^3*d^8*f^2 + 20
8*a^3*b^10*c*d^10*f^2 + 288*a^5*b^8*c*d^10*f^2 + 80*a^7*b^6*c*d^10*f^2))/(a
^10*d^2*f^5 + b^10*c^2*f^5 + 4*a^2*b^8*c^2*f^5 + 6*a^4*b^6*c^2*f^5 + 4*a^6*
b^4*c^2*f^5 + a^8*b^2*c^2*f^5 + a^2*b^8*d^2*f^5 + 4*a^4*b^6*d^2*f^5 + 6*a^6
*b^4*d^2*f^5 + 4*a^8*b^2*d^2*f^5 - 2*a*b^9*c*d*f^5 - 2*a^9*b*c*d*f^5 - 8*a^
3*b^7*c*d*f^5 - 12*a^5*b^5*c*d*f^5 - 8*a^7*b^3*c*d*f^5) + (((16*(c + d*tan(
e + f*x))^(1/2)*(8*a*b^14*d^11*f^2 + 4*b^15*c*d^10*f^2 + 36*a^3*b^12*d^11*f
^2 + 316*a^5*b^10*d^11*f^2 + 552*a^7*b^8*d^11*f^2 + 256*a^9*b^6*d^11*f^2 -
12*a^11*b^4*d^11*f^2 - 4*a^13*b^2*d^11*f^2 - 20*b^15*c^3*d^8*f^2 + 116*a^2*
b^13*c^3*d^8*f^2 - 220*a^3*b^12*c^2*d^9*f^2 + 216*a^4*b^11*c^3*d^8*f^2 - 10
4*a^5*b^10*c^2*d^9*f^2 + 8*a^6*b^9*c^3*d^8*f^2 + 232*a^7*b^8*c^2*d^9*f^2 -
68*a^8*b^7*c^3*d^8*f^2 + 156*a^9*b^6*c^2*d^9*f^2 + 4*a^10*b^5*c^3*d^8*f^2 -
12*a^11*b^4*c^2*d^9*f^2 - 52*a*b^14*c^2*d^9*f^2 + 80*a^2*b^13*c*d^10*f^2 -
156*a^4*b^11*c*d^10*f^2 - 640*a^6*b^9*c*d^10*f^2 - 500*a^8*b^7*c*d^10*f^2
- 80*a^10*b^5*c*d^10*f^2 + 12*a^12*b^3*c*d^10*f^2))/(a^10*d^2*f^4 + b^10*c^
2*f^4 + 4*a^2*b^8*c^2*f^4 + 6*a^4*b^6*c^2*f^4 + 4*a^6*b^4*c^2*f^4 + a^8*b^2
*c^2*f^4 + a^2*b^8*d^2*f^4 + 4*a^4*b^6*d^2*f^4 + 6*a^6*b^4*d^2*f^4 + 4*a^8*
b^2*d^2*f^4 - 2*a*b^9*c*d*f^4 - 2*a^9*b*c*d*f^4 - 8*a^3*b^7*c*d*f^4 - 12*a^
5*b^5*c*d*f^4 - 8*a^7*b^3*c*d*f^4) - (((16*(16*a*b^16*d^12*f^4 - 16*b^17*c*
d^11*f^4 + 136*a^3*b^14*d^12*f^4 + 432*a^5*b^12*d^12*f^4 + 680*a^7*b^10*d^1
2*f^4 + 560*a^9*b^8*d^12*f^4 + 216*a^11*b^6*d^12*f^4 + 16*a^13*b^4*d^12*f^4
- 8*a^15*b^2*d^12*f^4 - 8*b^17*c^3*d^9*f^4 - 128*a^2*b^15*c^3*d^9*f^4 + 35
2*a^3*b^14*c^2*d^10*f^4 + 160*a^3*b^14*c^4*d^8*f^4 - 520*a^4*b^13*c^3*d^9*f
^4 + 920*a^5*b^12*c^2*d^10*f^4 + 320*a^5*b^12*c^4*d^8*f^4 - 960*a^6*b^11*c^
3*d^9*f^4 + 1280*a^7*b^10*c^2*d^10*f^4 + 320*a^7*b^10*c^4*d^8*f^4 - 920*a^8
*b^9*c^3*d^9*f^4 + 1000*a^9*b^8*c^2*d^10*f^4 + 160*a^9*b^8*c^4*d^8*f^4 - 44
8*a^10*b^7*c^3*d^9*f^4 + 416*a^11*b^6*c^2*d^10*f^4 + 32*a^11*b^6*c^4*d^8*f^
4 - 88*a^12*b^5*c^3*d^9*f^4 + 72*a^13*b^4*c^2*d^10*f^4 + 56*a*b^16*c^2*d^10
*f^4 + 32*a*b^16*c^4*d^8*f^4 - 184*a^2*b^15*c*d^11*f^4 - 688*a^4*b^13*c*d^1
1*f^4 - 1240*a^6*b^11*c*d^11*f^4 - 1200*a^8*b^9*c*d^11*f^4 - 616*a^10*b^7*c
*d^11*f^4 - 144*a^12*b^5*c*d^11*f^4 - 8*a^14*b^3*c*d^11*f^4))/(a^10*d^2*f^5
+ b^10*c^2*f^5 + 4*a^2*b^8*c^2*f^5 + 6*a^4*b^6*c^2*f^5 + 4*a^6*b^4*c^2*f^5
+ a^8*b^2*c^2*f^5 + a^2*b^8*d^2*f^5 + 4*a^4*b^6*d^2*f^5 + 6*a^6*b^4*d^2*f^
5 + 4*a^8*b^2*d^2*f^5 - 2*a*b^9*c*d*f^5 - 2*a^9*b*c*d*f^5 - 8*a^3*b^7*c*d*f
^5 - 12*a^5*b^5*c*d*f^5 - 8*a^7*b^3*c*d*f^5) + (16*(-(b^7*d^2 + 16*a^2*b^5*
c^2 + 10*a^2*b^5*d^2 + 25*a^4*b^3*d^2 - 8*a*b^6*c*d - 40*a^3*b^4*c*d)*(a^11
*d^3*f^2 - b^11*c^3*f^2 - 4*a^2*b^9*c^3*f^2 - 6*a^4*b^7*c^3*f^2 - 4*a^6*b^5
*c^3*f^2 - a^8*b^3*c^3*f^2 + a^3*b^8*d^3*f^2 + 4*a^5*b^6*d^3*f^2 + 6*a^7*b^
4*d^3*f^2 + 4*a^9*b^2*d^3*f^2 + 3*a*b^10*c^2*d*f^2 - 3*a^10*b*c*d^2*f^2 - 3
*a^2*b^9*c*d^2*f^2 + 12*a^3*b^8*c^2*d*f^2 - 12*a^4*b^7*c*d^2*f^2 + 18*a^5*b
^6*c^2*d*f^2 - 18*a^6*b^5*c*d^2*f^2 + 12*a^7*b^4*c^2*d*f^2 - 12*a^8*b^3*c*d
^2*f^2 + 3*a^9*b^2*c^2*d*f^2))^(1/2)*(c + d*tan(e + f*x))^(1/2)*(32*a^2*b^1
7*d^12*f^4 + 160*a^4*b^15*d^12*f^4 + 288*a^6*b^13*d^12*f^4 + 160*a^8*b^11*d
^12*f^4 - 160*a^10*b^9*d^12*f^4 - 288*a^12*b^7*d^12*f^4 - 160*a^14*b^5*d^12

*f^4 - 32*a^16*b^3*d^12*f^4 + 32*b^19*c^2*d^10*f^4 + 48*b^19*c^4*d^8*f^4 + 176*a^2*b^17*c^2*d^10*f^4 + 272*a^2*b^17*c^4*d^8*f^4 - 432*a^3*b^16*c^3*d^9*f^4 + 336*a^4*b^15*c^2*d^10*f^4 + 624*a^4*b^15*c^4*d^8*f^4 - 912*a^5*b^14*c^3*d^9*f^4 + 112*a^6*b^13*c^2*d^10*f^4 + 720*a^6*b^13*c^4*d^8*f^4 - 880*a^7*b^12*c^3*d^9*f^4 - 560*a^8*b^11*c^2*d^10*f^4 + 400*a^8*b^11*c^4*d^8*f^4 - 240*a^9*b^10*c^3*d^9*f^4 - 1008*a^10*b^9*c^2*d^10*f^4 + 48*a^10*b^9*c^4*d^8*f^4 + 240*a^11*b^8*c^3*d^9*f^4 - 784*a^12*b^7*c^2*d^10*f^4 - 48*a^12*b^7*c^4*d^8*f^4 + 208*a^13*b^6*c^3*d^9*f^4 - 304*a^14*b^5*c^2*d^10*f^4 - 16*a^14*b^5*c^4*d^8*f^4 + 48*a^15*b^4*c^3*d^9*f^4 - 48*a^16*b^3*c^2*d^10*f^4 - 64*a*b^18*c*d^11*f^4 - 80*a*b^18*c^3*d^9*f^4 - 304*a^3*b^16*c*d^11*f^4 - 464*a^5*b^14*c*d^11*f^4 + 16*a^7*b^12*c*d^11*f^4 + 880*a^9*b^10*c*d^11*f^4 + 1136*a^11*b^8*c*d^11*f^4 + 656*a^13*b^6*c*d^11*f^4 + 176*a^15*b^4*c*d^11*f^4 + 16*a^17*b^2*c*d^11*f^4))/((b^9*(8*a^2*c^3*f^2 + 6*a^2*c*d^2*f^2) + b^3*(2*a^8*c^3*f^2 + 24*a^8*c*d^2*f^2) + b^7*(12*a^4*c^3*f^2 + 24*a^4*c*d^2*f^2) + b^5*(8*a^6*c^3*f^2 + 36*a^6*c*d^2*f^2) - b^2*(8*a^9*d^3*f^2 + 6*a^9*c^2*d*f^2) - b^8*(2*a^3*d^3*f^2 + 24*a^3*c^2*d*f^2) - b^4*(12*a^7*d^3*f^2 + 24*a^7*c^2*d*f^2) - b^6*(8*a^5*d^3*f^2 + 36*a^5*c^2*d*f^2) - 2*a^11*d^3*f^2 + 2*b^11*c^3*f^2 - 6*a*b^10*c^2*d*f^2 + 6*a^10*b*c*d^2*f^2)*(a^10*d^2*f^4 + b^10*c^2*f^4 + 4*a^2*b^8*c^2*f^4 + 6*a^4*b^6*c^2*f^4 + 4*a^6*b^4*c^2*f^4 + a^8*b^2*c^2*f^4 + a^2*b^8*d^2*f^4 + 4*a^4*b^6*d^2...

3.1253 $\int \frac{(a+b\tan(e+fx))^4}{(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=317

$$-\frac{i(a-ib)^4\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{3/2}f}+\frac{i(a+ib)^4\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(c+id)^{3/2}f}-\frac{2(bc-ad)^2(a+\ldots}{d\,(c^2+d^2)\,f\sqrt{c+\ldots}}$$

```
[Out] -I*(a-I*b)^4*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(c-I*d)^(3/2)/f+
I*(a+I*b)^4*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(c+I*d)^(3/2)/f-2
/3*b*(15*a^2*b*c*d^2-6*a^3*d^3-12*a*b^2*d*(2*c^2+d^2)+b^3*(8*c^3+5*c*d^2))*
(c+d*tan(f*x+e))^(1/2)/d^3/(c^2+d^2)/f-2/3*b^2*(3*a*d*(-a*d+2*b*c)-b^2*(4*c
^2+d^2))*(c+d*tan(f*x+e))^(1/2)*tan(f*x+e)/d^2/(c^2+d^2)/f-2*(-a*d+b*c)^2*(
a+b*tan(f*x+e))^2/d/(c^2+d^2)/f/(c+d*tan(f*x+e))^(1/2)
```

Rubi [A]

time = 0.63, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {3646, 3718, 3711, 3620, 3618, 65, 214}

$$-\frac{2b\left(-6a^3d^3+15a^2bcd^2-12ab^2d(2c^2+d^2)+b^3(8c^3+5cd^2)\right)\sqrt{c+d\tan(e+fx)}}{3d^3f\,(c^2+d^2)}-\frac{2b^2\left(3ad(2bc-ad)-b^2(4c^2+d^2)\right)\tan(e+fx)\sqrt{c+d\tan(e+fx)}}{3d^2f\,(c^2+d^2)}-\frac{2(bc-ad)^2(a+b\tan(e+fx))^2}{df\,(c^2+d^2)\sqrt{c+d\tan(e+fx)}}-\frac{i(a-ib)^4\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(c-id)^{3/2}}+\frac{i(a+ib)^4\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(c+id)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^4/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-I)*(a - I*b)^4*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((c - I*
d)^(3/2)*f) + (I*(a + I*b)^4*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]
])/((c + I*d)^(3/2)*f) - (2*(b*c - a*d)^2*(a + b*Tan[e + f*x])^2)/(d*(c^2 +
 d^2)*f*Sqrt[c + d*Tan[e + f*x]]) - (2*b*(15*a^2*b*c*d^2 - 6*a^3*d^3 - 12*a
*b^2*d*(2*c^2 + d^2) + b^3*(8*c^3 + 5*c*d^2))*Sqrt[c + d*Tan[e + f*x]])/(3*
d^3*(c^2 + d^2)*f) - (2*b^2*(3*a*d*(2*b*c - a*d) - b^2*(4*c^2 + d^2))*Tan[e
 + f*x]*Sqrt[c + d*Tan[e + f*x]])/(3*d^2*(c^2 + d^2)*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^4}{(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{2(bc-ad)^2(a+b\tan(e+fx))^2}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{2\int \frac{(a+b\tan(e+fx))\left(\frac{1}{2}\left(4b^3c^2+a^3cd-9ab^2cd+6a^2b\right.\right.}{}}{}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))^2}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} - \frac{2b^2\left(3ad(2bc-ad)-b^2\left(4c^2+d^2\right)\right)\tan(}{3d^2\left(c^2+d^2\right)}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))^2}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} - \frac{2b\left(15a^2bcd^2-6a^3d^3-12ab^2d\left(2c^2+d\right.\right.}{3d^3\left(c\right.}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))^2}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} - \frac{2b\left(15a^2bcd^2-6a^3d^3-12ab^2d\left(2c^2+d\right.\right.}{3d^3\left(c\right.}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))^2}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} - \frac{2b\left(15a^2bcd^2-6a^3d^3-12ab^2d\left(2c^2+d\right.\right.}{3d^3\left(c\right.}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))^2}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} - \frac{2b\left(15a^2bcd^2-6a^3d^3-12ab^2d\left(2c^2+d\right.\right.}{3d^3\left(c\right.}$$

$$= -\frac{i(a-ib)^4\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{3/2}f} + \frac{i(a+ib)^4\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-}}\right.}{(c+id)^{3/2}f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.36, size = 415, normalized size = 1.31

$$\frac{2b^2(a+b\tan(e+fx))^2}{3df\sqrt{c+d\tan(e+fx)}}+\frac{2\left(-\frac{2b^2(2bc-5ad)(a+b\tan(e+fx))}{df\sqrt{c+d\tan(e+fx)}}+\frac{-\frac{2\left(8b^4c^2-28ab^3cd+29a^2b^2d^2-3b^4d^2\right)}{d\sqrt{c+d\tan(e+fx)}}+\frac{2\left(6a(a-b)b(a+b)d^2\left(-\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}}+\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{\sqrt{c+id}}\right)+\frac{\left(-6a(a-b)b(a+b)cd^3+\frac{3}{2}\left(a^4-6a^2b^2+b^4\right)d^4\right)\left(-\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{c+d\tan(e+fx)}{c-id}\right)}{(ic+d)\sqrt{c+d\tan(e+fx)}}+\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{c+d\tan(e+fx)}{c+id}\right)}{(ic-d)\sqrt{c+d\tan(e+fx)}}\right)}{d}\right)}{d}}{2df}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^4/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*b^2*(a + b*Tan[e + f*x])^2)/(3*d*f*Sqrt[c + d*Tan[e + f*x]]) + (2*((-2*b
^2*(2*b*c - 5*a*d)*(a + b*Tan[e + f*x]))/(d*f*Sqrt[c + d*Tan[e + f*x]]) + (
(-2*(8*b^4*c^2 - 28*a*b^3*c*d + 29*a^2*b^2*d^2 - 3*b^4*d^2))/(d*Sqrt[c + d*
```

```
Tan[e + f*x]]) + (2*(6*a*(a - b)*b*(a + b)*d^2*(((-I)*ArcTanh[Sqrt[c + d*Ta
n[e + f*x]]/Sqrt[c - I*d]])/Sqrt[c - I*d] + (I*ArcTanh[Sqrt[c + d*Tan[e + f
*x]]/Sqrt[c + I*d]])/Sqrt[c + I*d]) + ((-6*a*(a - b)*b*(a + b)*c*d^3 + (3*(
a^4 - 6*a^2*b^2 + b^4)*d^4)/2)*(-(Hypergeometric2F1[-1/2, 1, 1/2, (c + d*Ta
n[e + f*x])/(c - I*d)]/((I*c + d)*Sqrt[c + d*Tan[e + f*x]])) + Hypergeometr
ic2F1[-1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c + I*d)]/((I*c - d)*Sqrt[c + d*T
an[e + f*x]]))))/d))/d)/(2*d*f)))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5654 vs. $2(287) = 574$.
time = 0.51, size = 5655, normalized size = 17.84

method	result	size
derivativedivides	Expression too large to display	5655
default	Expression too large to display	5655

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^4/(c+d*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^4/(c+d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^4/(c+d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(e+fx))^4}{(c+d\tan(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**4/(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**4/(c + d*tan(e + f*x))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^4/(c+d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 23.38, size = 2500, normalized size = 7.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^4/(c + d*tan(e + f*x))^(3/2),x)
```

```
[Out] (2*b^4*(c + d*tan(e + f*x))^(3/2))/(3*d^3*f) - atan(-(((c + d*tan(e + f*x))
^(1/2)*(16*a^8*d^10*f^3 + 16*b^8*d^10*f^3 - 448*a^2*b^6*d^10*f^3 + 1120*a^4
*b^4*d^10*f^3 - 448*a^6*b^2*d^10*f^3 + 32*a^8*c^2*d^8*f^3 - 32*a^8*c^6*d^4*
f^3 - 16*a^8*c^8*d^2*f^3 + 32*b^8*c^2*d^8*f^3 - 32*b^8*c^6*d^4*f^3 - 16*b^8
*c^8*d^2*f^3 - 896*a^2*b^6*c^2*d^8*f^3 + 896*a^2*b^6*c^6*d^4*f^3 + 448*a^2*
b^6*c^8*d^2*f^3 - 5376*a^3*b^5*c^3*d^7*f^3 - 5376*a^3*b^5*c^5*d^5*f^3 - 179
2*a^3*b^5*c^7*d^3*f^3 + 2240*a^4*b^4*c^2*d^8*f^3 - 2240*a^4*b^4*c^6*d^4*f^3
 - 1120*a^4*b^4*c^8*d^2*f^3 + 5376*a^5*b^3*c^3*d^7*f^3 + 5376*a^5*b^3*c^5*d
^5*f^3 + 1792*a^5*b^3*c^7*d^3*f^3 - 896*a^6*b^2*c^2*d^8*f^3 + 896*a^6*b^2*c
^6*d^4*f^3 + 448*a^6*b^2*c^8*d^2*f^3 + 256*a*b^7*c*d^9*f^3 - 256*a^7*b*c*d^
9*f^3 + 768*a*b^7*c^3*d^7*f^3 + 768*a*b^7*c^5*d^5*f^3 + 256*a*b^7*c^7*d^3*f
^3 - 1792*a^3*b^5*c*d^9*f^3 + 1792*a^5*b^3*c*d^9*f^3 - 768*a^7*b*c^3*d^7*f^
3 - 768*a^7*b*c^5*d^5*f^3 - 256*a^7*b*c^7*d^3*f^3) + (-(((8*a^8*c^3*f^2 + 8
*b^8*c^3*f^2 + 64*a*b^7*d^3*f^2 - 64*a^7*b*d^3*f^2 - 24*a^8*c*d^2*f^2 - 24*
b^8*c*d^2*f^2 - 224*a^2*b^6*c^3*f^2 + 560*a^4*b^4*c^3*f^2 - 224*a^6*b^2*c^3
*f^2 - 448*a^3*b^5*d^3*f^2 + 448*a^5*b^3*d^3*f^2 - 192*a*b^7*c^2*d*f^2 + 19
2*a^7*b*c^2*d*f^2 + 672*a^2*b^6*c*d^2*f^2 + 1344*a^3*b^5*c^2*d*f^2 - 1680*a
^4*b^4*c*d^2*f^2 - 1344*a^5*b^3*c^2*d*f^2 + 672*a^6*b^2*c*d^2*f^2)^2/4 - (1
6*c^6*f^4 + 16*d^6*f^4 + 48*c^2*d^4*f^4 + 48*c^4*d^2*f^4)*(a^16 + b^16 + 8*
```

a^2*b^14 + 28*a^4*b^12 + 56*a^6*b^10 + 70*a^8*b^8 + 56*a^10*b^6 + 28*a^12*b^4 + 8*a^14*b^2))^(1/2) + 4*a^8*c^3*f^2 + 4*b^8*c^3*f^2 + 32*a*b^7*d^3*f^2 - 32*a^7*b*d^3*f^2 - 12*a^8*c*d^2*f^2 - 12*b^8*c*d^2*f^2 - 112*a^2*b^6*c^3*f^2 + 280*a^4*b^4*c^3*f^2 - 112*a^6*b^2*c^3*f^2 - 224*a^3*b^5*d^3*f^2 + 224*a^5*b^3*d^3*f^2 - 96*a*b^7*c^2*d*f^2 + 96*a^7*b*c^2*d*f^2 + 336*a^2*b^6*c*d^2*f^2 + 672*a^3*b^5*c^2*d*f^2 - 840*a^4*b^4*c*d^2*f^2 - 672*a^5*b^3*c^2*d*f^2 + 336*a^6*b^2*c*d^2*f^2)/(16*(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4)))^(1/2)*(128*a^3*b*d^12*f^4 - 128*a*b^3*d^12*f^4 - (c + d*tan(e + f*x))^(1/2)*(-(((8*a^8*c^3*f^2 + 8*b^8*c^3*f^2 + 64*a*b^7*d^3*f^2 - 64*a^7*b*d^3*f^2 - 24*a^8*c*d^2*f^2 - 24*b^8*c*d^2*f^2 - 224*a^2*b^6*c^3*f^2 + 560*a^4*b^4*c^3*f^2 - 224*a^6*b^2*c^3*f^2 - 448*a^3*b^5*d^3*f^2 + 448*a^5*b^3*d^3*f^2 - 192*a*b^7*c^2*d*f^2 + 192*a^7*b*c^2*d*f^2 + 672*a^2*b^6*c*d^2*f^2 + 1344*a^3*b^5*c^2*d*f^2 - 1680*a^4*b^4*c*d^2*f^2 - 1344*a^5*b^3*c^2*d*f^2 + 672*a^6*b^2*c*d^2*f^2)^2/4 - (16*c^6*f^4 + 16*d^6*f^4 + 48*c^2*d^4*f^4 + 48*c^4*d^2*f^4)*(a^16 + b^16 + 8*a^2*b^14 + 28*a^4*b^12 + 56*a^6*b^10 + 70*a^8*b^8 + 56*a^10*b^6 + 28*a^12*b^4 + 8*a^14*b^2))^(1/2) + 4*a^8*c^3*f^2 + 4*b^8*c^3*f^2 + 32*a*b^7*d^3*f^2 - 32*a^7*b*d^3*f^2 - 12*a^8*c*d^2*f^2 - 12*b^8*c*d^2*f^2 - 112*a^2*b^6*c^3*f^2 + 280*a^4*b^4*c^3*f^2 - 112*a^6*b^2*c^3*f^2 - 224*a^3*b^5*d^3*f^2 + 224*a^5*b^3*d^3*f^2 - 96*a*b^7*c^2*d*f^2 + 96*a^7*b*c^2*d*f^2 + 336*a^2*b^6*c*d^2*f^2 + 672*a^3*b^5*c^2*d*f^2 - 840*a^4*b^4*c*d^2*f^2 - 672*a^5*b^3*c^2*d*f^2 + 336*a^6*b^2*c*d^2*f^2)/(16*(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4)))^(1/2)*(64*c*d^12*f^5 + 320*c^3*d^10*f^5 + 640*c^5*d^8*f^5 + 640*c^7*d^6*f^5 + 320*c^9*d^4*f^5 + 64*c^11*d^2*f^5) + 64*a^4*c*d^11*f^4 + 64*b^4*c*d^11*f^4 + 256*a^4*c^3*d^9*f^4 + 384*a^4*c^5*d^7*f^4 + 256*a^4*c^7*d^5*f^4 + 64*a^4*c^9*d^3*f^4 + 256*b^4*c^3*d^9*f^4 + 384*b^4*c^5*d^7*f^4 + 256*b^4*c^7*d^5*f^4 + 64*b^4*c^9*d^3*f^4 - 1536*a^2*b^2*c^3*d^9*f^4 - 2304*a^2*b^2*c^5*d^7*f^4 - 1536*a^2*b^2*c^7*d^5*f^4 - 384*a^2*b^2*c^9*d^3*f^4 - 384*a*b^3*c^2*d^10*f^4 - 256*a*b^3*c^4*d^8*f^4 + 256*a*b^3*c^6*d^6*f^4 + 384*a*b^3*c^8*d^4*f^4 + 128*a*b^3*c^10*d^2*f^4 - 384*a^2*b^2*c*d^11*f^4 + 384*a^3*b*c^2*d^10*f^4 + 256*a^3*b*c^4*d^8*f^4 - 256*a^3*b*c^6*d^6*f^4 - 384*a^3*b*c^8*d^4*f^4 - 128*a^3*b*c^10*d^2*f^4))*(-(((8*a^8*c^3*f^2 + 8*b^8*c^3*f^2 + 64*a*b^7*d^3*f^2 - 64*a^7*b*d^3*f^2 - 24*a^8*c*d^2*f^2 - 24*b^8*c*d^2*f^2 - 224*a^2*b^6*c^3*f^2 + 560*a^4*b^4*c^3*f^2 - 224*a^6*b^2*c^3*f^2 - 448*a^3*b^5*d^3*f^2 + 448*a^5*b^3*d^3*f^2 - 192*a*b^7*c^2*d*f^2 + 192*a^7*b*c^2*d*f^2 + 672*a^2*b^6*c*d^2*f^2 + 1344*a^3*b^5*c^2*d*f^2 - 1680*a^4*b^4*c*d^2*f^2 - 1344*a^5*b^3*c^2*d*f^2 + 672*a^6*b^2*c*d^2*f^2)^2/4 - (16*c^6*f^4 + 16*d^6*f^4 + 48*c^2*d^4*f^4 + 48*c^4*d^2*f^4)*(a^16 + b^16 + 8*a^2*b^14 + 28*a^4*b^12 + 56*a^6*b^10 + 70*a^8*b^8 + 56*a^10*b^6 + 28*a^12*b^4 + 8*a^14*b^2))^(1/2) + 4*a^8*c^3*f^2 + 4*b^8*c^3*f^2 + 32*a*b^7*d^3*f^2 - 32*a^7*b*d^3*f^2 - 12*a^8*c*d^2*f^2 - 12*b^8*c*d^2*f^2 - 112*a^2*b^6*c^3*f^2 + 280*a^4*b^4*c^3*f^2 - 112*a^6*b^2*c^3*f^2 - 224*a^3*b^5*d^3*f^2 + 224*a^5*b^3*d^3*f^2 - 96*a*b^7*c^2*d*f^2 + 96*a^7*b*c^2*d*f^2 + 336*a^2*b^6*c*d^2*f^2 + 672*a^3*b^5*c^2*d*f^2 - 840*a^4*b^4*c*d^2*f^2 - 672*a^5*b^3*c^2*d*f^2 + 336*a^6*b^2*c*d^2*f^2)/(16*(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4)))^(1/2)*1i + ((c + d*tan(e + f*x))^(1/

```
2)*(16*a^8*d^10*f^3 + 16*b^8*d^10*f^3 - 448*a^2*b^6*d^10*f^3 + 1120*a^4*b^4
*d^10*f^3 - 448*a^6*b^2*d^10*f^3 + 32*a^8*c^2*d^8*f^3 - 32*a^8*c^6*d^4*f^3
- 16*a^8*c^8*d^2*f^3 + 32*b^8*c^2*d^8*f^3 - 32*...
```

3.1254 $\int \frac{(a+b\tan(e+fx))^3}{(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=216

$$\frac{(ia+b)^3\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{3/2}f}-\frac{(ia-b)^3\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(c+id)^{3/2}f}-\frac{2(bc-ad)^2(a+b\tan}{d\,(c^2+d^2)\,f\sqrt{c+d}}$$

[Out] (I*a+b)^3*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(c-I*d)^(3/2)/f-(I*a-b)^3*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(c+I*d)^(3/2)/f-2*b*(a*d*(-a*d+2*b*c)-b^2*(2*c^2+d^2))*(c+d*tan(f*x+e))^(1/2)/d^2/(c^2+d^2)/f-2*(-a*d+b*c)^2*(a+b*tan(f*x+e))/d/(c^2+d^2)/f/(c+d*tan(f*x+e))^(1/2)

Rubi [A]
time = 0.35, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3646, 3711, 3620, 3618, 65, 214}

$$-\frac{2b\left(ad(2bc-ad)-b^2\left(2c^2+d^2\right)\right)\sqrt{c+d\tan(e+fx)}}{d^2f\left(c^2+d^2\right)}-\frac{2(bc-ad)^2(a+b\tan(e+fx))}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}-\frac{(-b+ia)^3\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(c+id)^{3/2}}+\frac{(b+ia)^3\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(c-id)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Tan[e + f*x])^3/(c + d*Tan[e + f*x])^(3/2),x]

[Out] ((I*a + b)^3*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((c - I*d)^(3/2)*f) - ((I*a - b)^3*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/((c + I*d)^(3/2)*f) - (2*(b*c - a*d)^2*(a + b*Tan[e + f*x]))/(d*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]]) - (2*b*(a*d*(2*b*c - a*d) - b^2*(2*c^2 + d^2))*Sqrt[c + d*Tan[e + f*x]])/(d^2*(c^2 + d^2)*f)

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
```

```
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^3}{(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{2(bc-ad)^2(a+b\tan(e+fx))}{d\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} + \frac{2\int \frac{\frac{1}{2}\left(2b^3c^2+a^3cd-5ab^2cd+4a^2bd^2\right)+\frac{1}{2}d\left(3a^2bc-\cdots\right.}{\cdots}}{\cdots}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))}{d\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} - \frac{2b(ad(2bc-ad)-b^2(2c^2+d^2))\sqrt{c\cdots}}{d^2\,(c^2+d^2)\,f}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))}{d\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} - \frac{2b(ad(2bc-ad)-b^2(2c^2+d^2))\sqrt{c\cdots}}{d^2\,(c^2+d^2)\,f}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))}{d\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} - \frac{2b(ad(2bc-ad)-b^2(2c^2+d^2))\sqrt{c\cdots}}{d^2\,(c^2+d^2)\,f}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))}{d\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} - \frac{2b(ad(2bc-ad)-b^2(2c^2+d^2))\sqrt{c\cdots}}{d^2\,(c^2+d^2)\,f}$$

$$= \frac{(ia+b)^3\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{3/2}f} - \frac{(ia-b)^3\tanh^{-1}\left(\frac{\sqrt{c+d\tan(\cdots}}{\sqrt{c+\cdots}}\right.}{(c+id)^{3/2}f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.84, size = 287, normalized size = 1.33

$$\frac{-ib(3a^2-b^2)\left(\frac{\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{\sqrt{c+id}}\right)+\frac{4b^2(bc-2ad)}{d\sqrt{c+d\tan(e+fx)}}+\frac{(3a^2bc-b^3c-a^3d+3ab^2d)\left((-ic+d)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{c+d\tan(e+fx)}{c-id}\right)+(ic+d)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{c+d\tan(e+fx)}{c+id}\right)\right)}{(c^2+d^2)\sqrt{c+d\tan(e+fx)}}+\frac{2b^2(a+b\tan(e+fx))}{\sqrt{c+d\tan(e+fx)}}}{df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^3/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-I)*b*(3*a^2 - b^2)*(ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]]/Sqrt
[c - I*d] - ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]]/Sqrt[c + I*d])
+ (4*b^2*(b*c - 2*a*d))/(d*Sqrt[c + d*Tan[e + f*x]]) + ((3*a^2*b*c - b^3*c
- a^3*d + 3*a*b^2*d)*(((-I)*c + d)*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*T
an[e + f*x])/(c - I*d)] + (I*c + d)*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*
Tan[e + f*x])/(c + I*d)]))/((c^2 + d^2)*Sqrt[c + d*Tan[e + f*x]]) + (2*b^2*
(a + b*Tan[e + f*x]))/Sqrt[c + d*Tan[e + f*x]])/(d*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4571 vs. $2(194) = 388$.
time = 0.53, size = 4572, normalized size = 21.17

method	result	size
derivativedivides	Expression too large to display	4572
default	Expression too large to display	4572

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*tan(f*x+e))^3/(c+d*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/f/d^2*(b^3*(c+d*tan(f*x+e))^(1/2)+d^2/(c^2+d^2)*(1/4/d^2/(3*c^2-d^2)/(c^2
+d^2)^(3/2)*(1/2*(-9*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c^2*d^5+3*(c^2+d^2
)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c^4-2*(c^2+d^2)^(1/2)*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*b^3*c^4*d^2-3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*b^3*c^2*d^4-18*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c^5*d^2-12*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c^3*d^4+6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*
c*d^6+9*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c^6*d-3*(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)*a*b^2*c^4*d^3+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c^6-
3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c^6*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3
*c^4*d^3+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c^2*d^5+3*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)*a*b^2*d^7+6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c^5*d^2+4*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*b^3*c^3*d^4-2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c*d^6-
3*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d^4+3*(c^2+d^2)^(1/2)
(2(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c^5*d+2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*a^3*c^3*d^3-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3
*c*d^5-3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c^6-(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*a^3*d^7+9*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*a^2*b*c^2*d^4-9*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c^5*d-
6*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c^3*d^3-(c^2+d^2)^(3/
2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c^4+(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*b^3*d^4+6*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c
^4*d^2+3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c*d^5)*ln(d*ta
n(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(
1/2))+2*(12*a^3*c^5*d^3+8*a^3*c^3*d^5-4*a^3*c*d^7-18*a^2*b*c^6*d^2+6*a^2*b*
c^4*d^4+18*a^2*b*c^2*d^6-6*a^2*b*d^8-36*a*b^2*c^5*d^3-24*a*b^2*c^3*d^5+12*a
*b^2*c*d^7+6*b^3*c^6*d^2-2*b^3*c^4*d^4-6*b^3*c^2*d^6+2*b^3*d^8-1/2*(-9*(2*(
c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c^2*d^5+3*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2
)+2*c)^(1/2)*a^2*b*c^4-2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*
c^4*d^2-3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c^2*d^4-18*(2*(
c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c^5*d^2-12*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^
2*b*c^3*d^4+6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c*d^6+9*(2*(c^2+d^2)^(1/2
)+2*c)^(1/2)*a*b^2*c^6*d-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c^4*d^3+(c^2
+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c^6-3*(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)*a^3*c^6*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c^4*d^3+3*(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)*a^3*c^2*d^5+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*d^7+6*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c^5*d^2+4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*

```
c^3*d^4-2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c*d^6-3*(c^2+d^2)^(3/2)*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d^4+3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*a^3*c^5*d+2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c^3*d^3
-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c*d^5-3*(c^2+d^2)^(1/2)*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c^6-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*d
^7+9*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c^2*d^4-9*(c^2+d^2
)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c^5*d-6*(c^2+d^2)^(1/2)*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c^3*d^3-(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*b^3*c^4+(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*d^4+6*(c^
2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c^4*d^2+3*(c^2+d^2)^(1/2)*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c*d^5)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(
2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^
(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/4/d^2/(3*c^2-d^2)/(c^2+
d^2)^(3/2)*(1/2*(9*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c^2*d^5-3*(c^2+d^2)^
(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c^4+2*(c^2+d^2)^(1/2)*(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)*b^3*c^4*d^2+3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)*b^3*c^2*d^4+18*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c^5*d^2+12*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*a^2*b*c^3*d^4-6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c*
d^6-9*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c^6*d+3*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*a*b^2*c^4*d^3-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c^6+3*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c^6*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c
^4*d^3-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c^2*d^5-3*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*a*b^2*d^7-6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c^5*d^2-4*(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)*b^3*c^3*d^4+2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c*d^6+3*
(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d^4-3*(c^2+d^2)^(1/2)*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c^5*d-2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*a^3*c^3*d^3+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c
*d^5+3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c^6+(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*a^3*d^7-9*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a
^2*b*c^2*d^4+9*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c^5*d+6*
(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a...
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(c+d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(c+d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(e+fx))^3}{(c+d\tan(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3/(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**3/(c + d*tan(e + f*x))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(c+d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 13.18, size = 2500, normalized size = 11.57

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^3/(c + d*tan(e + f*x))^(3/2),x)
```

```
[Out] (2*b^3*(c + d*tan(e + f*x))^(1/2))/(d^2*f) - atan((((-(((8*a^6*c^3*f^2 - 8*
b^6*c^3*f^2 - 48*a*b^5*d^3*f^2 - 48*a^5*b*d^3*f^2 - 24*a^6*c*d^2*f^2 + 24*b
^6*c*d^2*f^2 + 120*a^2*b^4*c^3*f^2 - 120*a^4*b^2*c^3*f^2 + 160*a^3*b^3*d^3*
f^2 + 144*a*b^5*c^2*d*f^2 + 144*a^5*b*c^2*d*f^2 - 360*a^2*b^4*c*d^2*f^2 - 4
80*a^3*b^3*c^2*d*f^2 + 360*a^4*b^2*c*d^2*f^2)^2/4 - (16*c^6*f^4 + 16*d^6*f^
4 + 48*c^2*d^4*f^4 + 48*c^4*d^2*f^4)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8
 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))^(1/2) + 4*a^6*c^3*f^2 - 4*b^6*c^3
*f^2 - 24*a*b^5*d^3*f^2 - 24*a^5*b*d^3*f^2 - 12*a^6*c*d^2*f^2 + 12*b^6*c*d^
```

2*f^2 + 60*a^2*b^4*c^3*f^2 - 60*a^4*b^2*c^3*f^2 + 80*a^3*b^3*d^3*f^2 + 72*a*b^5*c^2*d*f^2 + 72*a^5*b*c^2*d*f^2 - 180*a^2*b^4*c*d^2*f^2 - 240*a^3*b^3*c^2*d*f^2 + 180*a^4*b^2*c*d^2*f^2)/(16*(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4)))^(1/2)*((c + d*tan(e + f*x))^(1/2)*(-(((8*a^6*c^3*f^2 - 8*b^6*c^3*f^2 - 48*a*b^5*d^3*f^2 - 48*a^5*b*d^3*f^2 - 24*a^6*c*d^2*f^2 + 24*b^6*c*d^2*f^2 + 120*a^2*b^4*c^3*f^2 - 120*a^4*b^2*c^3*f^2 + 160*a^3*b^3*d^3*f^2 + 144*a*b^5*c^2*d*f^2 + 144*a^5*b*c^2*d*f^2 - 360*a^2*b^4*c*d^2*f^2 - 480*a^3*b^3*c^2*d*f^2 + 360*a^4*b^2*c*d^2*f^2)^2/4 - (16*c^6*f^4 + 16*d^6*f^4 + 48*c^2*d^4*f^4 + 48*c^4*d^2*f^4)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))^(1/2) + 4*a^6*c^3*f^2 - 4*b^6*c^3*f^2 - 24*a*b^5*d^3*f^2 - 24*a^5*b*d^3*f^2 - 12*a^6*c*d^2*f^2 + 12*b^6*c*d^2*f^2 + 60*a^2*b^4*c^3*f^2 - 60*a^4*b^2*c^3*f^2 + 80*a^3*b^3*d^3*f^2 + 72*a*b^5*c^2*d*f^2 + 72*a^5*b*c^2*d*f^2 - 180*a^2*b^4*c*d^2*f^2 - 240*a^3*b^3*c^2*d*f^2 + 180*a^4*b^2*c*d^2*f^2)/(16*(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4)))^(1/2)*(64*c*d^12*f^5 + 320*c^3*d^10*f^5 + 640*c^5*d^8*f^5 + 640*c^7*d^6*f^5 + 320*c^9*d^4*f^5 + 64*c^11*d^2*f^5) - 32*b^3*d^12*f^4 + 96*a^2*b*d^12*f^4 + 64*a^3*c*d^11*f^4 + 256*a^3*c^3*d^9*f^4 + 384*a^3*c^5*d^7*f^4 + 256*a^3*c^7*d^5*f^4 + 64*a^3*c^9*d^3*f^4 - 96*b^3*c^2*d^10*f^4 - 64*b^3*c^4*d^8*f^4 + 64*b^3*c^6*d^6*f^4 + 96*b^3*c^8*d^4*f^4 + 32*b^3*c^10*d^2*f^4 - 192*a*b^2*c*d^11*f^4 - 768*a*b^2*c^3*d^9*f^4 - 1152*a*b^2*c^5*d^7*f^4 - 768*a*b^2*c^7*d^5*f^4 - 192*a*b^2*c^9*d^3*f^4 + 288*a^2*b*c^2*d^10*f^4 + 192*a^2*b*c^4*d^8*f^4 - 192*a^2*b*c^6*d^6*f^4 - 288*a^2*b*c^8*d^4*f^4 - 96*a^2*b*c^10*d^2*f^4) + (c + d*tan(e + f*x))^(1/2)*(16*b^6*d^10*f^3 - 16*a^6*d^10*f^3 - 240*a^2*b^4*d^10*f^3 + 240*a^4*b^2*d^10*f^3 - 32*a^6*c^2*d^8*f^3 + 32*a^6*c^6*d^4*f^3 + 16*a^6*c^8*d^2*f^3 + 32*b^6*c^2*d^8*f^3 - 32*b^6*c^6*d^4*f^3 - 16*b^6*c^8*d^2*f^3 - 480*a^2*b^4*c^2*d^8*f^3 + 480*a^2*b^4*c^6*d^4*f^3 + 240*a^2*b^4*c^8*d^2*f^3 - 1920*a^3*b^3*c^3*d^7*f^3 - 1920*a^3*b^3*c^5*d^5*f^3 - 640*a^3*b^3*c^7*d^3*f^3 + 480*a^4*b^2*c^2*d^8*f^3 - 480*a^4*b^2*c^6*d^4*f^3 - 240*a^4*b^2*c^8*d^2*f^3 + 192*a*b^5*c*d^9*f^3 + 192*a^5*b*c*d^9*f^3 + 576*a*b^5*c^3*d^7*f^3 + 576*a*b^5*c^5*d^5*f^3 + 192*a*b^5*c^7*d^3*f^3 - 640*a^3*b^3*c*d^9*f^3 + 576*a^5*b*c^3*d^7*f^3 + 576*a^5*b*c^5*d^5*f^3 + 192*a^5*b*c^7*d^3*f^3))*(-(((8*a^6*c^3*f^2 - 8*b^6*c^3*f^2 - 48*a*b^5*d^3*f^2 - 48*a^5*b*d^3*f^2 - 24*a^6*c*d^2*f^2 + 24*b^6*c*d^2*f^2 + 120*a^2*b^4*c^3*f^2 - 120*a^4*b^2*c^3*f^2 + 160*a^3*b^3*d^3*f^2 + 144*a*b^5*c^2*d*f^2 + 144*a^5*b*c^2*d*f^2 - 360*a^2*b^4*c*d^2*f^2 - 480*a^3*b^3*c^2*d*f^2 + 360*a^4*b^2*c*d^2*f^2)^2/4 - (16*c^6*f^4 + 16*d^6*f^4 + 48*c^2*d^4*f^4 + 48*c^4*d^2*f^4)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))^(1/2) + 4*a^6*c^3*f^2 - 4*b^6*c^3*f^2 - 24*a*b^5*d^3*f^2 - 24*a^5*b*d^3*f^2 - 12*a^6*c*d^2*f^2 + 12*b^6*c*d^2*f^2 + 60*a^2*b^4*c^3*f^2 - 60*a^4*b^2*c^3*f^2 + 80*a^3*b^3*d^3*f^2 + 72*a*b^5*c^2*d*f^2 + 72*a^5*b*c^2*d*f^2 - 180*a^2*b^4*c*d^2*f^2 - 240*a^3*b^3*c^2*d*f^2 + 180*a^4*b^2*c*d^2*f^2)/(16*(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4)))^(1/2)*1i + ((-(((8*a^6*c^3*f^2 - 8*b^6*c^3*f^2 - 48*a*b^5*d^3*f^2 - 48*a^5*b*d^3*f^2 - 24*a^6*c*d^2*f^2 + 24*b^6*c*d^2*f^2 + 120*a^2*b^4*c^3*f^2 - 120*a^4*b^2*c^3*f^2 + 160*a^3*b^3*d^3*f^2 + 144*a*b^5*c^2*d*f^2 + 144*a^5*b*c^2

```
*d*f^2 - 360*a^2*b^4*c*d^2*f^2 - 480*a^3*b^3*c^2*d*f^2 + 360*a^4*b^2*c*d^2*
f^2)^2/4 - (16*c^6*f^4 + 16*d^6*f^4 + 48*c^2*d^4*f^4 + 48*c^4*d^2*f^4)*(a^1
2 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))
^(1/2) + 4*a^6*c^3*f^2 - 4*b^6*c^3*f^2 - 24*a*b^5*d^3*f^2 - 24*a^5*b*d^3*f^
2 - 12*a^6*c*d^2*f^2 + 12*b^6*c*d^2*f^2 + 60*a^2*b^4*c^3*f^2 - 60*a^4*b^2*c
^3*f^2 + 80*a^3*b^3*d^3*f^2 + 72*a*b^5*c^2*d*f^2 + 72*a^5*b*c^2*d*f^2 - 180
*a^2*b^4*c*d^2*f^2 - 240*a^3*b^3*c^2*d*f^2 + 180*a^4*b^2*c*d^2*f^2)/(16*(c^
6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4)))^(1/2)*(32*b^3*d^12*f^4 +
 (c + d*tan(e + f*x))^(1/2)*(-(((8*a^6*c^3*f^2 - 8*b^6*c^3*f^2 - 48*a*b^5*d
^3*f^2 - 48*a^5*b*d^3*f^2 - 24*a^6*c*d^2*f^2 + 24*b^6*c*d^2*f^2 + 120*a^2*b
^4*c^3*f^2 - 120*a^4*b^2*c^3*f^2 + 160*a^3*b^3*d^3*f^2 + 144*a*b^5*c^2*d*f^
2 + 144*a^5*b*c^2*d*f^2 - 360*a^2*b^4*c*d^2*f^2 - 480*a^3*b^3*c^2*d*f^2 + 3
60*a^4*b^2*c*d^2*f^2)^2/4 - (16*c^6*f^4 + 16*d^6*f^4 + 48*c^2*d^4*f^4 + 48*
c^4*d^2*f^4)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4...
```

3.1255 $\int \frac{(a+b\tan(e+fx))^2}{(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=150

$$-\frac{i(a-ib)^2\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{3/2}f}+\frac{i(a+ib)^2\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(c+id)^{3/2}f}-\frac{2(bc-}{d\left(c^2+d^2\right)f\sqrt{c}}$$

```
[Out] -I*(a-I*b)^2*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(c-I*d)^(3/2)/f+
I*(a+I*b)^2*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(c+I*d)^(3/2)/f-2
*(-a*d+b*c)^2/d/(c^2+d^2)/f/(c+d*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3623, 3620, 3618, 65, 214}

$$-\frac{2(bc-ad)^2}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}-\frac{i(a-ib)^2\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(c-id)^{3/2}}+\frac{i(a+ib)^2\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(c+id)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-I)*(a - I*b)^2*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((c - I*
d)^(3/2)*f) + (I*(a + I*b)^2*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]
])/((c + I*d)^(3/2)*f) - (2*(b*c - a*d)^2)/(d*(c^2 + d^2)*f*Sqrt[c + d*Tan[
e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
```

```
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^2}{(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{2(bc-ad)^2}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{\int \frac{a^2c-b^2c+2abd+\left(2abc-a^2d+b^2d\right)\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx}{c^2+d^2}$$

$$= -\frac{2(bc-ad)^2}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{(a-ib)^2\int \frac{1+i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx}{2(c-id)} +$$

$$= -\frac{2(bc-ad)^2}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{(a+ib)^2\mathrm{Subst}\left(\int \frac{1}{(-1+x)\sqrt{c+idx}}\,dx,\right.}{2(ic-d)f}$$

$$= -\frac{2(bc-ad)^2}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} - \frac{(a-ib)^2\mathrm{Subst}\left(\int \frac{1}{-1-\frac{ic}{d}+\frac{ix^2}{d}}\,dx,x,\sqrt{c\right.}{(c-id)df}$$

$$= -\frac{i(a-ib)^2\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{3/2}f} + \frac{i(a+ib)^2\tanh^{-1}\left(\frac{\sqrt{c+d\,\mathrm{ta}}}{\sqrt{c-}}\right.}{(c+id)^{3/2}f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.19, size = 124, normalized size = 0.83

$$\frac{-\frac{2b^2}{d}-\frac{(a-ib)^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{c+d\tan(e+fx)}{c-id}\right)}{ic+d}+\frac{(a+ib)^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{c+d\tan(e+fx)}{c+id}\right)}{ic-d}}{f\sqrt{c+d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-2*b^2)/d - ((a - I*b)^2*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*Tan[e + f
*x])/(c - I*d)])/(I*c + d) + ((a + I*b)^2*Hypergeometric2F1[-1/2, 1, 1/2, (
c + d*Tan[e + f*x])/(c + I*d)])/(I*c - d))/(f*Sqrt[c + d*Tan[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3310 vs. 2(128) = 256.
time = 0.46, size = 3311, normalized size = 22.07

method	result	size
derivativedivides	Expression too large to display	3311
default	Expression too large to display	3311

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/d*(d/(c^2+d^2)*(1/4/d^2/(3*c^2-d^2)/(c^2+d^2)^(3/2)*(1/2*(-2*(c^2+d^2)^
(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^4+2*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*a*b*d^4-3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^
2*c^5*d-2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^3*d^3+(c^2+d^
2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c*d^5+2*(c^2+d^2)^(1/2)*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*a*b*c^6-4*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*a*b*c^4*d^2-6*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^2*d^4+
3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^5*d+2*(c^2+d^2)^(1/2)
*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^3*d^3-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*b^2*c*d^5+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^6*d-(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*a^2*c^4*d^3-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^2*d^
5+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d^7+12*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*
b*c^5*d^2+8*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^3*d^4-4*(2*(c^2+d^2)^(1/2)+
2*c)^(1/2)*a*b*c*d^6-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^6*d+(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*b^2*c^4*d^3+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^2*d^5-(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d^7)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1
/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(12*a^2*c^5*d^3+8*a^2*
c^3*d^5-4*a^2*c*d^7-12*a*b*c^6*d^2+4*a*b*c^4*d^4+12*a*b*c^2*d^6-4*a*b*d^8-1
2*b^2*c^5*d^3-8*b^2*c^3*d^5+4*b^2*c*d^7+1/2*(-2*(c^2+d^2)^(3/2)*(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)*a*b*c^4+2*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*
a*b*d^4-3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^5*d-2*(c^2+d^
```

2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^3*d^3+(c^2+d^2)^(1/2)*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*a^2*c*d^5+2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*a*b*c^6-4*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^4*d^2-6*
(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^2*d^4+3*(c^2+d^2)^(1/2)
(2(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^5*d+2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*b^2*c^3*d^3-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2
*c*d^5+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^6*d-(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)*a^2*c^4*d^3-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^2*d^5+(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*a^2*d^7+12*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^5*d^2+8*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^3*d^4-4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c*
d^6-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^6*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*b^2*c^4*d^3+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^2*d^5-(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*b^2*d^7)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)
^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(
c^2+d^2)^(1/2)-2*c)^(1/2)))+1/4/d^2/(3*c^2-d^2)/(c^2+d^2)^(3/2)*(1/2*(2*(c^
2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^4-2*(c^2+d^2)^(3/2)*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)*a*b*d^4+3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*a^2*c^5*d+2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^3*d^3-
(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c*d^5-2*(c^2+d^2)^(1/2)*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^6+4*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)*a*b*c^4*d^2+6*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c
^2*d^4-3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^5*d-2*(c^2+d^2
)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^3*d^3+(c^2+d^2)^(1/2)*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*b^2*c*d^5-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^6*d+(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^4*d^3+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2
*c^2*d^5-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d^7-12*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*a*b*c^5*d^2-8*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^3*d^4+4*(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*a*b*c*d^6+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^6*d-(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^4*d^3-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^
2*d^5+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d^7)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x
+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(12*a^2*c^5*d^3
+8*a^2*c^3*d^5-4*a^2*c*d^7-12*a*b*c^6*d^2+4*a*b*c^4*d^4+12*a*b*c^2*d^6-4*a*
b*d^8-12*b^2*c^5*d^3-8*b^2*c^3*d^5+4*b^2*c*d^7-1/2*(2*(c^2+d^2)^(3/2)*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^4-2*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*a*b*d^4+3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^5*d+2*(
c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^3*d^3-(c^2+d^2)^(1/2)*(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c*d^5-2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+
2*c)^(1/2)*a*b*c^6+4*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^4*
d^2+6*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^2*d^4-3*(c^2+d^2)
^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^5*d-2*(c^2+d^2)^(1/2)*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*b^2*c^3*d^3+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*b^2*c*d^5-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^6*d+(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)*a^2*c^4*d^3+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^2*d^5-(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*a^2*d^7-12*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^5*d^2-8
(2(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^3*d^4+4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*

```
a*b*c*d^6+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c...
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^(3/2),x, algorithm="maxima")
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 35751 vs. 2(123) = 246.
time = 240.23, size = 35751, normalized size = 238.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^(3/2),x, algorithm="fricas")
[Out] 1/4*(4*sqrt(2)*((c^10*d + 3*c^8*d^3 + 2*c^6*d^5 - 2*c^4*d^7 - 3*c^2*d^9 - d
^11)*f^5*cos(f*x + e)^2 + 2*(c^9*d^2 + 4*c^7*d^4 + 6*c^5*d^6 + 4*c^3*d^8 +
c*d^10)*f^5*cos(f*x + e)*sin(f*x + e) + (c^8*d^3 + 4*c^6*d^5 + 6*c^4*d^7 +
4*c^2*d^9 + d^11)*f^5)*sqrt(((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8
)*c^6 + 3*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*c^4*d^2 + 3*(a^8
+ 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*c^2*d^4 + (a^8 + 4*a^6*b^2 + 6*a
^4*b^4 + 4*a^2*b^6 + b^8)*d^6 + ((a^4 - 6*a^2*b^2 + b^4)*c^9 + 12*(a^3*b -
a*b^3)*c^8*d + 32*(a^3*b - a*b^3)*c^6*d^3 - 6*(a^4 - 6*a^2*b^2 + b^4)*c^5*d
^4 + 24*(a^3*b - a*b^3)*c^4*d^5 - 8*(a^4 - 6*a^2*b^2 + b^4)*c^3*d^6 - 3*(a^
4 - 6*a^2*b^2 + b^4)*c*d^8 - 4*(a^3*b - a*b^3)*d^9)*f^2*sqrt((a^8 + 4*a^6*b
^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/((c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6)*f^4)
))/(16*(a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*c^6 - 24*(a^7*b - 7*a^5*b^3 + 7*a^3*
b^5 - a*b^7)*c^5*d + 3*(3*a^8 - 68*a^6*b^2 + 178*a^4*b^4 - 68*a^2*b^6 + 3*b
^8)*c^4*d^2 + 80*(a^7*b - 7*a^5*b^3 + 7*a^3*b^5 - a*b^7)*c^3*d^3 - 6*(a^8 -
 36*a^6*b^2 + 86*a^4*b^4 - 36*a^2*b^6 + b^8)*c^2*d^4 - 24*(a^7*b - 7*a^5*b^
3 + 7*a^3*b^5 - a*b^7)*c*d^5 + (a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6
+ b^8)*d^6))*sqrt((16*(a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*c^6 - 24*(a^7*b - 7*a
^5*b^3 + 7*a^3*b^5 - a*b^7)*c^5*d + 3*(3*a^8 - 68*a^6*b^2 + 178*a^4*b^4 - 6
8*a^2*b^6 + 3*b^8)*c^4*d^2 + 80*(a^7*b - 7*a^5*b^3 + 7*a^3*b^5 - a*b^7)*c^3
*d^3 - 6*(a^8 - 36*a^6*b^2 + 86*a^4*b^4 - 36*a^2*b^6 + b^8)*c^2*d^4 - 24*(a
^7*b - 7*a^5*b^3 + 7*a^3*b^5 - a*b^7)*c*d^5 + (a^8 - 12*a^6*b^2 + 38*a^4*b^
4 - 12*a^2*b^6 + b^8)*d^6)/((c^12 + 6*c^10*d^2 + 15*c^8*d^4 + 20*c^6*d^6 +
15*c^4*d^8 + 6*c^2*d^10 + d^12)*f^4))*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2
*b^6 + b^8)/((c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6)*f^4))^(3/4)*arctan(((4*(a^
15*b + 5*a^13*b^3 + 9*a^11*b^5 + 5*a^9*b^7 - 5*a^7*b^9 - 9*a^5*b^11 - 5*a^3
```

```
*b^13 - a*b^15)*c^13 - 3*(a^16 - 20*a^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 - 6
4*a^6*b^10 - 20*a^4*b^12 + b^16)*c^12*d + 8*(a^15*b + 5*a^13*b^3 + 9*a^11*b
^5 + 5*a^9*b^7 - 5*a^7*b^9 - 9*a^5*b^11 - 5*a^3*b^13 - a*b^15)*c^11*d^2 - 1
4*(a^16 - 20*a^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 - 64*a^6*b^10 - 20*a^4*b^1
2 + b^16)*c^10*d^3 - 20*(a^15*b + 5*a^13*b^3 + 9*a^11*b^5 + 5*a^9*b^7 - 5*a
^7*b^9 - 9*a^5*b^11 - 5*a^3*b^13 - a*b^15)*c^9*d^4 - 25*(a^16 - 20*a^12*b^4
 - 64*a^10*b^6 - 90*a^8*b^8 - 64*a^6*b^10 - 20*a^4*b^12 + b^16)*c^8*d^5 - 8
0*(a^15*b + 5*a^13*b^3 + 9*a^11*b^5 + 5*a^9*b^7 - 5*a^7*b^9 - 9*a^5*b^11 -
5*a^3*b^13 - a*b^15)*c^7*d^6 - 20*(a^16 - 20*a^12*b^4 - 64*a^10*b^6 - 90*a^
8*b^8 - 64*a^6*b^10 - 20*a^4*b^12 + b^16)*c^6*d^7 - 100*(a^15*b + 5*a^13*b^
3 + 9*a^11*b^5 + 5*a^9*b^7 - 5*a^7*b^9 - 9*a^5*b^11 - 5*a^3*b^13 - a*b^15)*
c^5*d^8 - 5*(a^16 - 20*a^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 - 64*a^6*b^10 -
20*a^4*b^12 + b^16)*c^4*d^9 - 56*(a^15*b + 5*a^13*b^3 + 9*a^11*b^5 + 5*a^9*
b^7 - 5*a^7*b^9 - 9*a^5*b^11 - 5*a^3*b^13 - a*b^15)*c^3*d^10 + 2*(a^16 - 20
*a^12*b^4 - 64*a^10*b^6 - 90*a^8*b^8 - 64*a^6*b^10 - 20*a^4*b^12 + b^16)*c^
2*d^11 - 12*(a^15*b + 5*a^13*b^3 + 9*a^11*b^5 + 5*a^9*b^7 - 5*a^7*b^9 - 9*a
^5*b^11 - 5*a^3*b^13 - a*b^15)*c*d^12 + (a^16 - 20*a^12*b^4 - 64*a^10*b^6 -
 90*a^8*b^8 - 64*a^6*b^10 - 20*a^4*b^12 + b^16)*d^13)*f^4*sqrt((16*(a^6*b^2
 - 2*a^4*b^4 + a^2*b^6)*c^6 - 24*(a^7*b - 7*a^5*b^3 + 7*a^3*b^5 - a*b^7)*c^
5*d + 3*(3*a^8 - 68*a^6*b^2 + 178*a^4*b^4 - 68*a^2*b^6 + 3*b^8)*c^4*d^2 + 8
0*(a^7*b - 7*a^5*b^3 + 7*a^3*b^5 - a*b^7)*c^3*d^3 - 6*(a^8 - 36*a^6*b^2 + 8
6*a^4*b^4 - 36*a^2*b^6 + b^8)*c^2*d^4 - 24*(a^7*b - 7*a^5*b^3 + 7*a^3*b^5 -
 a*b^7)*c*d^5 + (a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*d^6)/((c
^12 + 6*c^10*d^2 + 15*c^8*d^4 + 20*c^6*d^6 + 15*c^4*d^8 + 6*c^2*d^10 + d^12
)*f^4))*sqrt((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)/((c^6 + 3*c^4*
d^2 + 3*c^2*d^4 + d^6)*f^4)) + (4*(a^19*b + 7*a^17*b^3 + 20*a^15*b^5 + 28*a
^13*b^7 + 14*a^11*b^9 - 14*a^9*b^11 - 28*a^7*b^13 - 20*a^5*b^15 - 7*a^3*b^1
7 - a*b^19)*c^10 - 3*(a^20 + 2*a^18*b^2 - 19*a^16*b^4 - 104*a^14*b^6 - 238*
a^12*b^8 - 308*a^10*b^10 - 238*a^8*b^12 - 104*a^6*b^14 - 19*a^4*b^16 + 2*a^
2*b^18 + b^20)*c^9*d - 8*(a^20 + 2*a^18*b^2 - 19*a^16*b^4 - 104*a^14*b^6 -
238*a^12*b^8 - 308*a^10*b^10 - 238*a^8*b^12 - 104*a^6*b^14 - 19*a^4*b^16 +
2*a^2*b^18 + b^20)*c^7*d^3 - 24*(a^19*b + 7*a^17*b^3 + 20*a^15*b^5 + 28*a^1
3*b^7 + 14*a^11*b^9 - 14*a^9*b^11 - 28*a^7*b^13 - 20*a^5*b^15 - 7*a^3*b^17
- a*b^19)*c^6*d^4 - 6*(a^20 + 2*a^18*b^2 - 19*a^16*b^4 - 104*a^14*b^6 - 238
*a^12*b^8 - 308*a^10*b^10 - 238*a^8*b^12 - 104*a^6*b^14 - 19*a^4*b^16 + 2*a
^2*b^18 + b^20)*c^5*d^5 - 32*(a^19*b + 7*a^17*b^3 + 20*a^15*b^5 + 28*a^13*b
^7 + 14*a^11*b^9 - 14*a^9*b^11 - 28*a^7*b^13 - 20*a^5*b^15 - 7*a^3*b^17 - a
*b^19)*c^4*d^6 - 12*(a^19*b + 7*a^17*b^3 + 20*a^15*b^5 + 28*a^13*b^7 + 14*a
^11*b^9 - 14*a^9*b^11 - 28*a^7*b^13 - 20*a^5*b^15 - 7*a^3*b^17 - a*b^19)*c^
2*d^8 + (a^20 + 2*a^18*b^2 - 19*a^16*b^4 - 104*a^14*b^6 - 238*a^12*b^8 - 30
8*a^10*b^10 - 238*a^8*b^12 - 104*a^6*b^14 - 19*a^4*b^16 + 2*a^2*b^18 + b^20
)*c*d^9)*f^2*sqrt((16*(a^6*b^2 - 2*a^4*b^4 + a^...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan(e + fx))^2}{(c + d\tan(e + fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2/(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**2/(c + d*tan(e + f*x))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 12.22, size = 2500, normalized size = 16.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^2/(c + d*tan(e + f*x))^(3/2),x)
```

```
[Out] atan((((c + d*tan(e + f*x))^(1/2)*(16*a^4*d^10*f^3 + 16*b^4*d^10*f^3 - 96*a
^2*b^2*d^10*f^3 + 32*a^4*c^2*d^8*f^3 - 32*a^4*c^6*d^4*f^3 - 16*a^4*c^8*d^2*
f^3 + 32*b^4*c^2*d^8*f^3 - 32*b^4*c^6*d^4*f^3 - 16*b^4*c^8*d^2*f^3 - 192*a^
2*b^2*c^2*d^8*f^3 + 192*a^2*b^2*c^6*d^4*f^3 + 96*a^2*b^2*c^8*d^2*f^3 + 128*
a*b^3*c*d^9*f^3 - 128*a^3*b*c*d^9*f^3 + 384*a*b^3*c^3*d^7*f^3 + 384*a*b^3*c
^5*d^5*f^3 + 128*a*b^3*c^7*d^3*f^3 - 384*a^3*b*c^3*d^7*f^3 - 384*a^3*b*c^5*
d^5*f^3 - 128*a^3*b*c^7*d^3*f^3) - (-(4*a*b^3 - 4*a^3*b + a^4*1i + b^4*1i -
 a^2*b^2*6i)/(4*(c^3*f^2*1i + d^3*f^2 - c*d^2*f^2*3i - 3*c^2*d*f^2)))^(1/2)
*((c + d*tan(e + f*x))^(1/2)*(-(4*a*b^3 - 4*a^3*b + a^4*1i + b^4*1i - a^2*b
^2*6i)/(4*(c^3*f^2*1i + d^3*f^2 - c*d^2*f^2*3i - 3*c^2*d*f^2)))^(1/2)*(64*c
*d^12*f^5 + 320*c^3*d^10*f^5 + 640*c^5*d^8*f^5 + 640*c^7*d^6*f^5 + 320*c^9*
d^4*f^5 + 64*c^11*d^2*f^5) - 64*a^2*c*d^11*f^4 + 64*b^2*c*d^11*f^4 - 256*a^
2*c^3*d^9*f^4 - 384*a^2*c^5*d^7*f^4 - 256*a^2*c^7*d^5*f^4 - 64*a^2*c^9*d^3*
f^4 + 256*b^2*c^3*d^9*f^4 + 384*b^2*c^5*d^7*f^4 + 256*b^2*c^7*d^5*f^4 + 64*
```

b^2*c^9*d^3*f^4 - 64*a*b*d^12*f^4 - 192*a*b*c^2*d^10*f^4 - 128*a*b*c^4*d^8*f^4 + 128*a*b*c^6*d^6*f^4 + 192*a*b*c^8*d^4*f^4 + 64*a*b*c^10*d^2*f^4))*(-(4*a*b^3 - 4*a^3*b + a^4*1i + b^4*1i - a^2*b^2*6i)/(4*(c^3*f^2*1i + d^3*f^2 - c*d^2*f^2*3i - 3*c^2*d*f^2)))^(1/2)*1i + ((c + d*tan(e + f*x))^(1/2)*(16*a^4*d^10*f^3 + 16*b^4*d^10*f^3 - 96*a^2*b^2*d^10*f^3 + 32*a^4*c^2*d^8*f^3 - 32*a^4*c^6*d^4*f^3 - 16*a^4*c^8*d^2*f^3 + 32*b^4*c^2*d^8*f^3 - 32*b^4*c^6*d^4*f^3 - 16*b^4*c^8*d^2*f^3 - 192*a^2*b^2*c^2*d^8*f^3 + 192*a^2*b^2*c^6*d^4*f^3 + 96*a^2*b^2*c^8*d^2*f^3 + 128*a*b^3*c*d^9*f^3 - 128*a^3*b*c*d^9*f^3 + 384*a*b^3*c^3*d^7*f^3 + 384*a*b^3*c^5*d^5*f^3 + 128*a*b^3*c^7*d^3*f^3 - 384*a^3*b*c^3*d^7*f^3 - 384*a^3*b*c^5*d^5*f^3 - 128*a^3*b*c^7*d^3*f^3) - (-(4*a*b^3 - 4*a^3*b + a^4*1i + b^4*1i - a^2*b^2*6i)/(4*(c^3*f^2*1i + d^3*f^2 - c*d^2*f^2*3i - 3*c^2*d*f^2)))^(1/2)*((c + d*tan(e + f*x))^(1/2)*(-(4*a*b^3 - 4*a^3*b + a^4*1i + b^4*1i - a^2*b^2*6i)/(4*(c^3*f^2*1i + d^3*f^2 - c*d^2*f^2*3i - 3*c^2*d*f^2)))^(1/2)*(64*c*d^12*f^5 + 320*c^3*d^10*f^5 + 640*c^5*d^8*f^5 + 640*c^7*d^6*f^5 + 320*c^9*d^4*f^5 + 64*c^11*d^2*f^5) + 64*a^2*c*d^11*f^4 - 64*b^2*c*d^11*f^4 + 256*a^2*c^3*d^9*f^4 + 384*a^2*c^5*d^7*f^4 + 256*a^2*c^7*d^5*f^4 + 64*a^2*c^9*d^3*f^4 - 256*b^2*c^3*d^9*f^4 - 384*b^2*c^5*d^7*f^4 - 256*b^2*c^7*d^5*f^4 - 64*b^2*c^9*d^3*f^4 + 64*a*b*d^12*f^4 + 192*a*b*c^2*d^10*f^4 + 128*a*b*c^4*d^8*f^4 - 128*a*b*c^6*d^6*f^4 - 192*a*b*c^8*d^4*f^4 - 64*a*b*c^10*d^2*f^4))*(-(4*a*b^3 - 4*a^3*b + a^4*1i + b^4*1i - a^2*b^2*6i)/(4*(c^3*f^2*1i + d^3*f^2 - c*d^2*f^2*3i - 3*c^2*d*f^2)))^(1/2)*1i)/(((c + d*tan(e + f*x))^(1/2)*(16*a^4*d^10*f^3 + 16*b^4*d^10*f^3 - 96*a^2*b^2*d^10*f^3 + 32*a^4*c^2*d^8*f^3 - 32*a^4*c^6*d^4*f^3 - 16*a^4*c^8*d^2*f^3 + 32*b^4*c^2*d^8*f^3 - 32*b^4*c^6*d^4*f^3 - 16*b^4*c^8*d^2*f^3 - 192*a^2*b^2*c^2*d^8*f^3 + 192*a^2*b^2*c^6*d^4*f^3 + 96*a^2*b^2*c^8*d^2*f^3 + 128*a*b^3*c*d^9*f^3 - 128*a^3*b*c*d^9*f^3 + 384*a*b^3*c^3*d^7*f^3 + 384*a*b^3*c^5*d^5*f^3 + 128*a*b^3*c^7*d^3*f^3 - 384*a^3*b*c^3*d^7*f^3 - 384*a^3*b*c^5*d^5*f^3 - 128*a^3*b*c^7*d^3*f^3) - (-(4*a*b^3 - 4*a^3*b + a^4*1i + b^4*1i - a^2*b^2*6i)/(4*(c^3*f^2*1i + d^3*f^2 - c*d^2*f^2*3i - 3*c^2*d*f^2)))^(1/2)*((c + d*tan(e + f*x))^(1/2)*(-(4*a*b^3 - 4*a^3*b + a^4*1i + b^4*1i - a^2*b^2*6i)/(4*(c^3*f^2*1i + d^3*f^2 - c*d^2*f^2*3i - 3*c^2*d*f^2)))^(1/2)*(64*c*d^12*f^5 + 320*c^3*d^10*f^5 + 640*c^5*d^8*f^5 + 640*c^7*d^6*f^5 + 320*c^9*d^4*f^5 + 64*c^11*d^2*f^5) - 64*a^2*c*d^11*f^4 + 64*b^2*c*d^11*f^4 - 256*a^2*c^3*d^9*f^4 - 384*a^2*c^5*d^7*f^4 - 256*a^2*c^7*d^5*f^4 - 64*a^2*c^9*d^3*f^4 + 256*b^2*c^3*d^9*f^4 + 384*b^2*c^5*d^7*f^4 + 256*b^2*c^7*d^5*f^4 + 64*b^2*c^9*d^3*f^4 - 64*a*b*d^12*f^4 - 192*a*b*c^2*d^10*f^4 - 128*a*b*c^4*d^8*f^4 + 128*a*b*c^6*d^6*f^4 + 192*a*b*c^8*d^4*f^4 + 64*a*b*c^10*d^2*f^4))*(-(4*a*b^3 - 4*a^3*b + a^4*1i + b^4*1i - a^2*b^2*6i)/(4*(c^3*f^2*1i + d^3*f^2 - c*d^2*f^2*3i - 3*c^2*d*f^2)))^(1/2) - ((c + d*tan(e + f*x))^(1/2)*(16*a^4*d^10*f^3 + 16*b^4*d^10*f^3 - 96*a^2*b^2*d^10*f^3 + 32*a^4*c^2*d^8*f^3 - 32*a^4*c^6*d^4*f^3 - 16*a^4*c^8*d^2*f^3 + 32*b^4*c^2*d^8*f^3 - 32*b^4*c^6*d^4*f^3 - 16*b^4*c^8*d^2*f^3 - 192*a^2*b^2*c^2*d^8*f^3 + 192*a^2*b^2*c^6*d^4*f^3 + 96*a^2*b^2*c^8*d^2*f^3 + 128*a*b^3*c*d^9*f^3 - 128*a^3*b*c*d^9*f^3 + 384*a*b^3*c^3*d^7*f^3 + 384*a*b^3*c^5*d^5*f^3 + 128*a*b^3*c^7*d^3*f^3 - 384*a^3*b*c^3*d^7*f^3 - 384*a^3*b*c^5*d^5*f^3 - 128*a^3*b*c^7*d^3*f^3) - (-(4*a*

```
b^3 - 4*a^3*b + a^4*1i + b^4*1i - a^2*b^2*6i)/(4*(c^3*f^2*1i + d^3*f^2 - c*
d^2*f^2*3i - 3*c^2*d*f^2)))^(1/2)*((c + d*tan(e + f*x))^(1/2)*(-(4*a*b^3 -
4*a^3*b + a^4*1i + b^4*1i - a^2*b^2*6i)/(4*(c^3*f^2*1i + d^3*f^2 - c*d^2*f^
2*3i - 3*c^2*d*f^2)))^(1/2)*(64*c*d^12*f^5 + 320*c^3*d^10*f^5 + 640*c^5*d^8
*f^5 + 640*c^7*d^6*f^5 + 320*c^9*d^4*f^5 + 64*c^11*d^2*f^5) + 64*a^2*c*d^11
*f^4 - 64*b^2*c*d^11*f^4 + 256*a^2*c^3*d^9*f^4 + 384*a^2*c^5*d^7*f^4 + 256*
a^2*c^7*d^5*f^4 + 64*a^2*c^9*d^3*f^4 - 256*b^2*c^3*d^9*f^4 - 384*b^2*c^5*d^
7*f^4 - 256*b^2*c^7*d^5*f^4 - 64*b^2*c^9*d^3*f^...
```

3.1256 $\int \frac{a+b\tan(e+fx)}{(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=138

$$-\frac{(ia+b)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{3/2}f}+\frac{(ia-b)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(c+id)^{3/2}f}+\frac{2(bc-ad)}{(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}}$$

```
[Out] -(I*a+b)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(c-I*d)^(3/2)/f+(I*a
-b)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(c+I*d)^(3/2)/f+2*(-a*d+b
*c)/(c^2+d^2)/f/(c+d*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3610, 3620, 3618, 65, 214}

$$\frac{2(bc-ad)}{f\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}-\frac{(b+ia)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(c-id)^{3/2}}+\frac{(-b+ia)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(c+id)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] -(((I*a + b)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((c - I*d)^(3
/2)*f)) + ((I*a - b)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/((c +
 I*d)^(3/2)*f) + (2*(b*c - a*d))/((c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
```

```
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{a+b\tan(e+fx)}{(c+d\tan(e+fx))^{3/2}}\,dx = \frac{2(bc-ad)}{(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} + \frac{\int \frac{ac+bd+(bc-ad)\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx}{c^2+d^2}$$

$$= \frac{2(bc-ad)}{(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} + \frac{(a-ib)\int \frac{1+i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx}{2(c-id)} + \frac{(a}{}$$

$$= \frac{2(bc-ad)}{(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} + \frac{(ia+b)\mathrm{Subst}\left(\int \frac{1}{(-1+x)\sqrt{c-idx}}\,dx, x,\right.}{2(c-id)f}$$

$$= \frac{2(bc-ad)}{(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} - \frac{(a-ib)\mathrm{Subst}\left(\int \frac{1}{-1-\frac{ic}{d}+\frac{ix^2}{d}}\,dx, x, \sqrt{c+}\right.}{(c-id)df}$$

$$= -\frac{(ia+b)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{3/2}f} + \frac{(ia-b)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(}{\sqrt{c+}}\right.}{(c+id)^{3/2}f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.20, size = 113, normalized size = 0.82

$$\frac{i\left(\frac{(a-ib)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{c+d\tan(e+fx)}{c-id}\right)}{c-id}-\frac{(a+ib)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{c+d\tan(e+fx)}{c+id}\right)}{c+id}\right)}{f\sqrt{c+d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (I*(((a - I*b)*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c - I*
d)])/(c - I*d) - ((a + I*b)*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*Tan[e +
f*x])/(c + I*d)])/(c + I*d)))/(f*Sqrt[c + d*Tan[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2267 vs. $2(118) = 236$.
time = 0.44, size = 2268, normalized size = 16.43

method	result	size
derivativedivides	Expression too large to display	2268
default	Expression too large to display	2268

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))/(c+d*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2/(c^2+d^2)*(1/4/d^2/(3*c^2-d^2)/(c^2+d^2)^(3/2)*(1/2*((c^2+d^2)^(3/2)
*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^4-(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*b*d^4+3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^5*d+2*(c^
2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3*d^3-(c^2+d^2)^(1/2)*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)*a*c*d^5-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*b*c^6+2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^4*d^2+3*(c^2+d
^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d^4-3*(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)*a*c^6*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^4*d^3+3*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*a*c^2*d^5-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^7-6*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*b*c^5*d^2-4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^3*d^4+2*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)*b*c*d^6)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(12*a*c^5*d^3+8*a*c^3*d^5-4
*a*c*d^7-6*b*c^6*d^2+2*b*c^4*d^4+6*b*c^2*d^6-2*b*d^8-1/2*((c^2+d^2)^(3/2)*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^4-(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*b*d^4+3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^5*d+2*(c^2+
d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3*d^3-(c^2+d^2)^(1/2)*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*a*c*d^5-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*b*c^6+2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^4*d^2+3*(c^2+d^2
)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d^4-3*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*a*c^6*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^4*d^3+3*(2*(c^2+d^2)^(1/2)+2
```

```
*c)^(1/2)*a*c^2*d^5-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^7-6*(2*(c^2+d^2)^(1/2
)+2*c)^(1/2)*b*c^5*d^2-4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^3*d^4+2*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*b*c*d^6)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^
(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(
1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/4/d^2/(3*c^2-d^2)/(c^2+d^2)^(3/2)*(
1/2*(-(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^4+(c^2+d^2)^(3/2)*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^4-3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*a*c^5*d-2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3*d^3+(
c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d^5+(c^2+d^2)^(1/2)*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)*b*c^6-2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*b*c^4*d^2-3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d^4+3*(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^6*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^4*d^3
-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2*d^5+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*
d^7+6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^5*d^2+4*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*b*c^3*d^4-2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d^6)*ln(d*tan(f*x+e)+c-(c+
d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(12*a*
c^5*d^3+8*a*c^3*d^5-4*a*c*d^7-6*b*c^6*d^2+2*b*c^4*d^4+6*b*c^2*d^6-2*b*d^8+1
/2*(-(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^4+(c^2+d^2)^(3/2)*(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^4-3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)*a*c^5*d-2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3*d^3+(c
^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d^5+(c^2+d^2)^(1/2)*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*b*c^6-2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2
)*b*c^4*d^2-3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d^4+3*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^6*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^4*d^3-
3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2*d^5+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d
^7+6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^5*d^2+4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2
)*b*c^3*d^4-2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d^6)*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*
(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))))-2*(a*d-b*c)/(c
^2+d^2)/(c+d*tan(f*x+e))^(1/2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(c+d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-c>0)', see `assume?` for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 20798 vs. $2(116) = 232$.

time = 36.06, size = 20798, normalized size = 150.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(f*x+e))/(c+d*tan(f*x+e))^(3/2),x, algorithm="fricas")

[Out] -1/4*(4*sqrt(2)*((c^10 + 3*c^8*d^2 + 2*c^6*d^4 - 2*c^4*d^6 - 3*c^2*d^8 - d^10)*f^5*cos(f*x + e)^2 + 2*(c^9*d + 4*c^7*d^3 + 6*c^5*d^5 + 4*c^3*d^7 + c*d^9)*f^5*cos(f*x + e)*sin(f*x + e) + (c^8*d^2 + 4*c^6*d^4 + 6*c^4*d^6 + 4*c^2*d^8 + d^10)*f^5)*sqrt(((a^4 + 2*a^2*b^2 + b^4)*c^6 + 3*(a^4 + 2*a^2*b^2 + b^4)*c^4*d^2 + 3*(a^4 + 2*a^2*b^2 + b^4)*c^2*d^4 + (a^4 + 2*a^2*b^2 + b^4)*d^6 + (6*a*b*c^8*d + 16*a*b*c^6*d^3 + 12*a*b*c^4*d^5 - 2*a*b*d^9 + (a^2 - b^2)*c^9 - 6*(a^2 - b^2)*c^5*d^4 - 8*(a^2 - b^2)*c^3*d^6 - 3*(a^2 - b^2)*c*d^8)*f^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/((c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6)*f^4)))/(4*a^2*b^2*c^6 - 12*(a^3*b - a*b^3)*c^5*d + 3*(3*a^4 - 14*a^2*b^2 + 3*b^4)*c^4*d^2 + 40*(a^3*b - a*b^3)*c^3*d^3 - 6*(a^4 - 8*a^2*b^2 + b^4)*c^2*d^4 - 12*(a^3*b - a*b^3)*c*d^5 + (a^4 - 2*a^2*b^2 + b^4)*d^6))*sqrt((4*a^2*b^2*c^6 - 12*(a^3*b - a*b^3)*c^5*d + 3*(3*a^4 - 14*a^2*b^2 + 3*b^4)*c^4*d^2 + 40*(a^3*b - a*b^3)*c^3*d^3 - 6*(a^4 - 8*a^2*b^2 + b^4)*c^2*d^4 - 12*(a^3*b - a*b^3)*c*d^5 + (a^4 - 2*a^2*b^2 + b^4)*d^6)/((c^12 + 6*c^10*d^2 + 15*c^8*d^4 + 20*c^6*d^6 + 15*c^4*d^8 + 6*c^2*d^10 + d^12)*f^4))*((a^4 + 2*a^2*b^2 + b^4)/((c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6)*f^4))^(3/4)*arctan(-((2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*c^13 - 3*(a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*c^12*d + 4*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*c^11*d^2 - 14*(a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*c^10*d^3 - 10*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*c^9*d^4 - 25*(a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*c^8*d^5 - 40*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*c^7*d^6 - 20*(a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*c^6*d^7 - 50*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*c^5*d^8 - 5*(a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*c^4*d^9 - 28*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*c^3*d^10 + 2*(a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*c^2*d^11 - 6*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*c*d^12 + (a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^13)*f^4*sqrt((4*a^2*b^2*c^6 - 12*(a^3*b - a*b^3)*c^5*d + 3*(3*a^4 - 14*a^2*b^2 + 3*b^4)*c^4*d^2 + 40*(a^3*b - a*b^3)*c^3*d^3 - 6*(a^4 - 8*a^2*b^2 + b^4)*c^2*d^4 - 12*(a^3*b - a*b^3)*c*d^5 + (a^4 - 2*a^2*b^2 + b^4)*d^6)/((c^12 + 6*c^10*d^2 + 15*c^8*d^4 + 20*c^6*d^6 + 15*c^4*d^8 + 6*c^2*d^10 + d^12)*f^4))*sqrt((a^4 + 2*a^2*b^2 + b^4)/((c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6)*f^4)) + (2*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*c^10 - 3*(a^10 + 3*a^8*b^2 + 2*a^6*b^4 - 2*a^4*b^6 - 3*a^2*b^8 - b^10)*c^9*d - 8*(a^10 + 3*a^8*b^2 + 2*a^6*b^4 - 2*a^4*b^6 - 3*a^2*b^8 - b^10)*c^7*d^3 - 12*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*c^6*d^4 - 6*(a^10 + 3*a^8*b^2 + 2*a^6*b^4 - 2*a^4*b^6 - 3*a^2*b^8 - b^10)*c^5*d^5 - 16*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*c^4*d^6 - 6*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*c^2*d^8 + (a^10 + 3*a^8*b^2 + 2*a^6*b^4 - 2*a^4*b^6 - 3*a^2*b^8 - b^10)*c*d^9)*f^2*sqrt((4*a^2*b^2*c^6 - 12*(a^3*b

```
 - a*b^3)*c^5*d + 3*(3*a^4 - 14*a^2*b^2 + 3*b^4)*c^4*d^2 + 40*(a^3*b - a*b^
3)*c^3*d^3 - 6*(a^4 - 8*a^2*b^2 + b^4)*c^2*d^4 - 12*(a^3*b - a*b^3)*c*d^5 +
 (a^4 - 2*a^2*b^2 + b^4)*d^6)/((c^12 + 6*c^10*d^2 + 15*c^8*d^4 + 20*c^6*d^6
 + 15*c^4*d^8 + 6*c^2*d^10 + d^12)*f^4)) + sqrt(2)*((b*c^14 - 2*a*c^13*d +
5*b*c^12*d^2 - 12*a*c^11*d^3 + 9*b*c^10*d^4 - 30*a*c^9*d^5 + 5*b*c^8*d^6 -
40*a*c^7*d^7 - 5*b*c^6*d^8 - 30*a*c^5*d^9 - 9*b*c^4*d^10 - 12*a*c^3*d^11 -
5*b*c^2*d^12 - 2*a*c*d^13 - b*d^14)*f^7*sqrt((4*a^2*b^2*c^6 - 12*(a^3*b - a
*b^3)*c^5*d + 3*(3*a^4 - 14*a^2*b^2 + 3*b^4)*c^4*d^2 + 40*(a^3*b - a*b^3)*c
^3*d^3 - 6*(a^4 - 8*a^2*b^2 + b^4)*c^2*d^4 - 12*(a^3*b - a*b^3)*c*d^5 + (a^
4 - 2*a^2*b^2 + b^4)*d^6)/((c^12 + 6*c^10*d^2 + 15*c^8*d^4 + 20*c^6*d^6 + 1
5*c^4*d^8 + 6*c^2*d^10 + d^12)*f^4))*sqrt((a^4 + 2*a^2*b^2 + b^4)/((c^6 + 3
*c^4*d^2 + 3*c^2*d^4 + d^6)*f^4)) + ((a^2*b + b^3)*c^11 - (a^3 + a*b^2)*c^1
0*d + 5*(a^2*b + b^3)*c^9*d^2 - 5*(a^3 + a*b^2)*c^8*d^3 + 10*(a^2*b + b^3)*
c^7*d^4 - 10*(a^3 + a*b^2)*c^6*d^5 + 10*(a^2*b + b^3)*c^5*d^6 - 10*(a^3 + a
*b^2)*c^4*d^7 + 5*(a^2*b + b^3)*c^3*d^8 - 5*(a^3 + a*b^2)*c^2*d^9 + (a^2*b
+ b^3)*c*d^10 - (a^3 + a*b^2)*d^11)*f^5*sqrt((4*a^2*b^2*c^6 - 12*(a^3*b - a
*b^3)*c^5*d + 3*(3*a^4 - 14*a^2*b^2 + 3*b^4)*c^4*d^2 + 40*(a^3*b - a*b^3)*c
^3*d^3 - 6*(a^4 - 8*a^2*b^2 + b^4)*c^2*d^4 - 12*(a^3*b - a*b^3)*c*d^5 + (a^
4 - 2*a^2*b^2 + b^4)*d^6)/((c^12 + 6*c^10*d^2 + 15*c^8*d^4 + 20*c^6*d^6 + 1
5*c^4*d^8 + 6*c^2*d^10 + d^12)*f^4)))*sqrt(((a^4 + 2*a^2*b^2 + b^4)*c^6 + 3
*(a^4 + 2*a^2*b^2 + b^4)*c^4*d^2 + 3*(a^4 + 2*a^2*b^2 + b^4)*c^2*d^4 + (a^4
 + 2*a^2*b^2 + b^4)*d^6 + (6*a*b*c^8*d + 16*a*b*c^6*d^3 + 12*a*b*c^4*d^5 -
2*a*b*d^9 + (a^2 - b^2)*c^9 - 6*(a^2 - b^2)*c^5*d^4 - 8*(a^2 - b^2)*c^3*d^6
 - 3*(a^2 - b^2)*c*d^8)*f^2*sqrt((a^4 + 2*a^2*b^2 + b^4)/((c^6 + 3*c^4*d^2
+ 3*c^2*d^4 + d^6)*f^4)))/(4*a^2*b^2*c^6 - 12*(a^3*b - a*b^3)*c^5*d + 3*(3*
a^4 - 14*a^2*b^2 + 3*b^4)*c^4*d^2 + 40*(a^3*b - a*b^3)*c^3*d^3 - 6*(a^4 - 8
*a^2*b^2 + b^4)*c^2*d^4 - 12*(a^3*b - a*b^3)*c*d^5 + (a^4 - 2*a^2*b^2 + b^4
)*d^6))*sqrt(((4*(a^4*b^2 + a^2*b^4)*c^10 - 12*(a^5*b - a*b^5)*c^9*d + (9*a
^6 - 25*a^4*b^2 - 25*a^2*b^4 + 9*b^6)*c^8*d^2 +...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\tan(e + fx)}{(c + d\tan(e + fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))/(c + d*tan(e + f*x))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(f*x+e))/(c+d*tan(f*x+e))^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done

Mupad [B]
time = 11.43, size = 2500, normalized size = 18.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tan(e + f*x))/(c + d*tan(e + f*x))^(3/2),x)

[Out] (log(- (((c + d*tan(e + f*x))^(1/2)*(16*b^2*d^10*f^3 + 32*b^2*c^2*d^8*f^3 - 32*b^2*c^6*d^4*f^3 - 16*b^2*c^8*d^2*f^3) + ((((96*b^4*c^2*d^4*f^4 - 16*b^4*d^6*f^4 - 144*b^4*c^4*d^2*f^4)^(1/2) + 4*b^2*c^3*f^2 - 12*b^2*c*d^2*f^2)/(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4))^(1/2)*(32*b*d^12*f^4 + ((((96*b^4*c^2*d^4*f^4 - 16*b^4*d^6*f^4 - 144*b^4*c^4*d^2*f^4)^(1/2) + 4*b^2*c^3*f^2 - 12*b^2*c*d^2*f^2)/(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4))^(1/2)*(c + d*tan(e + f*x))^(1/2)*(64*c*d^12*f^5 + 320*c^3*d^10*f^5 + 640*c^5*d^8*f^5 + 640*c^7*d^6*f^5 + 320*c^9*d^4*f^5 + 64*c^11*d^2*f^5))/4 + 96*b*c^2*d^10*f^4 + 64*b*c^4*d^8*f^4 - 64*b*c^6*d^6*f^4 - 96*b*c^8*d^4*f^4 - 32*b*c^10*d^2*f^4))/4)*(((96*b^4*c^2*d^4*f^4 - 16*b^4*d^6*f^4 - 144*b^4*c^4*d^2*f^4)^(1/2) + 4*b^2*c^3*f^2 - 12*b^2*c*d^2*f^2)/(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4))^(1/2))/4 - 8*b^3*c*d^8*f^2 - 24*b^3*c^3*d^6*f^2 - 24*b^3*c^5*d^4*f^2 - 8*b^3*c^7*d^2*f^2)*(((96*b^4*c^2*d^4*f^4 - 16*b^4*d^6*f^4 - 144*b^4*c^4*d^2*f^4)^(1/2) + 4*b^2*c^3*f^2 - 12*b^2*c*d^2*f^2)/(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4))^(1/2))/4 + (log(- (((c + d*tan(e + f*x))^(1/2)*(16*b^2*d^10*f^3 + 32*b^2*c^2*d^8*f^3 - 32*b^2*c^6*d^4*f^3 - 16*b^2*c^8*d^2*f^3) + ((-((96*b^4*c^2*d^4*f^4 - 16*b^4*d^6*f^4 - 144*b^4*c^4*d^2*f^4)^(1/2) - 4*b^2*c^3*f^2 + 12*b^2*c*d^2*f^2)/(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4))^(1/2)*(32*b*d^12*f^4 + ((-((96*b^4*c^2*d^4*f^4 - 16*b^4*d^6*f^4 - 144*b^4*c^4*d^2*f^4)^(1/2) - 4*b^2*c^3*f^2 + 12*b^2*c*d^2*f^2)/(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4))^(1/2)*(c + d*tan(e + f*x))^(1/2)*(64*c*d^12*f^5 + 320*c^3*d^10*f^5 + 640*c^5*d^8*f^5 + 640*c^7*d^6*f^5 + 320*c^9*d^4*f^5 + 64*c^11*d^2*f^5))/4 + 96*b*c^2*d^10*f^4 + 64*b*c^4*d^8*f^4 - 64*b*c^6*d^6*f^4 - 96*b*c^8*d^4*f^4 - 32*b*c^10*d^2*f^4))/4)*(-((96*b^4*c^2*d^4*f^4 - 16*b^4*d^6*f^4 - 144*b^4*c^4*d^2*f^4)^(1/2) - 4*b^2*c^3*f^2 + 12*b^2*c*d^2*f^2)/(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4))^(1/2))/4 - 8*b^3*c*d^8*f^2 - 24*b^3*c^3*d^6*f^2 - 24*b^3*c^5*d^4*f^2 - 8*b^3*c^7*d^2*f^2)*(-((96*b^4*c^2*d^4*f^4 - 16*b^4*d^6*f^4 - 144*b^4*c^4*d^2*f^4)^(1/2) - 4*b^2*c^3*f^2 + 12*b^2*c*d^2*f^2)/(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4))^(1/2))/4 - log(((c + d*tan(e + f*x))^(1/2)*(16*b^2*d^10*f^3 + 32*b^2*c^2*d^8*f^3 - 32*b^2*c^6*d^

```
4*f^3 - 16*b^2*c^8*d^2*f^3) + (((96*b^4*c^2*d^4*f^4 - 16*b^4*d^6*f^4 - 144*
b^4*c^4*d^2*f^4)^(1/2) + 4*b^2*c^3*f^2 - 12*b^2*c*d^2*f^2)/(16*c^6*f^4 + 16
*d^6*f^4 + 48*c^2*d^4*f^4 + 48*c^4*d^2*f^4))^(1/2)*((((96*b^4*c^2*d^4*f^4 -
 16*b^4*d^6*f^4 - 144*b^4*c^4*d^2*f^4)^(1/2) + 4*b^2*c^3*f^2 - 12*b^2*c*d^2
*f^2)/(16*c^6*f^4 + 16*d^6*f^4 + 48*c^2*d^4*f^4 + 48*c^4*d^2*f^4))^(1/2)*(c
 + d*tan(e + f*x))^(1/2)*(64*c*d^12*f^5 + 320*c^3*d^10*f^5 + 640*c^5*d^8*f^
5 + 640*c^7*d^6*f^5 + 320*c^9*d^4*f^5 + 64*c^11*d^2*f^5) - 32*b*d^12*f^4 -
96*b*c^2*d^10*f^4 - 64*b*c^4*d^8*f^4 + 64*b*c^6*d^6*f^4 + 96*b*c^8*d^4*f^4
+ 32*b*c^10*d^2*f^4))*(((96*b^4*c^2*d^4*f^4 - 16*b^4*d^6*f^4 - 144*b^4*c^4*
d^2*f^4)^(1/2) + 4*b^2*c^3*f^2 - 12*b^2*c*d^2*f^2)/(16*c^6*f^4 + 16*d^6*f^4
 + 48*c^2*d^4*f^4 + 48*c^4*d^2*f^4))^(1/2) - 8*b^3*c*d^8*f^2 - 24*b^3*c^3*d
^6*f^2 - 24*b^3*c^5*d^4*f^2 - 8*b^3*c^7*d^2*f^2)*(((96*b^4*c^2*d^4*f^4 - 16
*b^4*d^6*f^4 - 144*b^4*c^4*d^2*f^4)^(1/2) + 4*b^2*c^3*f^2 - 12*b^2*c*d^2*f^
2)/(16*c^6*f^4 + 16*d^6*f^4 + 48*c^2*d^4*f^4 + 48*c^4*d^2*f^4))^(1/2) - log
(((c + d*tan(e + f*x))^(1/2)*(16*b^2*d^10*f^3 + 32*b^2*c^2*d^8*f^3 - 32*b^2
*c^6*d^4*f^3 - 16*b^2*c^8*d^2*f^3) + (-((96*b^4*c^2*d^4*f^4 - 16*b^4*d^6*f^
4 - 144*b^4*c^4*d^2*f^4)^(1/2) - 4*b^2*c^3*f^2 + 12*b^2*c*d^2*f^2)/(16*c^6*
f^4 + 16*d^6*f^4 + 48*c^2*d^4*f^4 + 48*c^4*d^2*f^4))^(1/2)*((-((96*b^4*c^2*
d^4*f^4 - 16*b^4*d^6*f^4 - 144*b^4*c^4*d^2*f^4)^(1/2) - 4*b^2*c^3*f^2 + 12*
b^2*c*d^2*f^2)/(16*c^6*f^4 + 16*d^6*f^4 + 48*c^2*d^4*f^4 + 48*c^4*d^2*f^4))
^(1/2)*(c + d*tan(e + f*x))^(1/2)*(64*c*d^12*f^5 + 320*c^3*d^10*f^5 + 640*c
^5*d^8*f^5 + 640*c^7*d^6*f^5 + 320*c^9*d^4*f^5 + 64*c^11*d^2*f^5) - 32*b*d^
12*f^4 - 96*b*c^2*d^10*f^4 - 64*b*c^4*d^8*f^4 + 64*b*c^6*d^6*f^4 + 96*b*c^8
*d^4*f^4 + 32*b*c^10*d^2*f^4))*(-((96*b^4*c^2*d^4*f^4 - 16*b^4*d^6*f^4 - 14
4*b^4*c^4*d^2*f^4)^(1/2) - 4*b^2*c^3*f^2 + 12*b^2*c*d^2*f^2)/(16*c^6*f^4 +
16*d^6*f^4 + 48*c^2*d^4*f^4 + 48*c^4*d^2*f^4))^(1/2) - 8*b^3*c*d^8*f^2 - 24
*b^3*c^3*d^6*f^2 - 24*b^3*c^5*d^4*f^2 - 8*b^3*c^7*d^2*f^2)*(-((96*b^4*c^2*d
^4*f^4 - 16*b^4*d^6*f^4 - 144*b^4*c^4*d^2*f^4)^(1/2) - 4*b^2*c^3*f^2 + 12*b
^2*c*d^2*f^2)/(16*c^6*f^4 + 16*d^6*f^4 + 48*c^2*d^4*f^4 + 48*c^4*d^2*f^4))^
(1/2) + (log(((((96*a^4*c^2*d^4*f^4 - 16*a^4*d^6*f^4 - 144*a^4*c^4*d^2*f^4)
^(1/2) - 4*a^2*c^3*f^2 + 12*a^2*c*d^2*f^2)/(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f
^4 + 3*c^4*d^2*f^4))^(1/2)*((c + d*tan(e + f*x))^(1/2)*(16*a^2*d^10*f^3 + 3
2*a^2*c^2*d^8*f^3 - 32*a^2*c^6*d^4*f^3 - 16*a^2*c^8*d^2*f^3) - ((((96*a^4*c
^2*d^4*f^4 - 16*a^4*d^6*f^4 - 144*a^4*c^4*d^2*f^4)^(1/2) - 4*a^2*c^3*f^2 +
12*a^2*c*d^2*f^2)/(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4))^(1/2
)*(((((96*a^4*c^2*d^4*f^4 - 16*a^4*d^6*f^4 - 14...
```

3.1257 $\int \frac{1}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=211

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(ia+b)(c-id)^{3/2}f}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(ia-b)(c+id)^{3/2}f}-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{(a^2+b^2)(bc-ad)^{3/2}f}$$

```
[Out] arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(I*a+b)/(c-I*d)^(3/2)/f-arcta
nh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(I*a-b)/(c+I*d)^(3/2)/f-2*b^(5/2)*
arctanh(b^(1/2)*(c+d*tan(f*x+e))^(1/2)/(-a*d+b*c)^(1/2))/(a^2+b^2)/(-a*d+b*
c)^(3/2)/f+2*d^2/(-a*d+b*c)/(c^2+d^2)/f/(c+d*tan(f*x+e))^(1/2)
```

Rubi [A]

time = 0.65, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3650, 3734, 3620, 3618, 65, 214, 3715}

$$-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{f(a^2+b^2)(bc-ad)^{3/2}}+\frac{2d^2}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(b+ia)(c-id)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(-b+ia)(c+id)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]]/((I*a + b)*(c - I*d)^(3/2)*
f) - ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]]/((I*a - b)*(c + I*d)^(
3/2)*f) - (2*b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c -
a*d]])/((a^2 + b^2)*(b*c - a*d)^(3/2)*f) + (2*d^2)/((b*c - a*d)*(c^2 + d^2)
*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
```

```
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}\,dx = \frac{2d^2}{(bc-ad)\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{2\int \frac{\frac{1}{2}(-acd+b(c \cdots}{(a+ \cdots}}{\cdots}$$

$$= \frac{2d^2}{(bc-ad)\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{b^3\int \frac{\cdots}{(a+b\tan(e- \cdots}}{(a \cdots}$$

$$= \frac{2d^2}{(bc-ad)\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{\int \frac{1+i\tan(\cdots}{\sqrt{c+d\tan \cdots}}}{2(a-ib \cdots}$$

$$= \frac{2d^2}{(bc-ad)\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{i\text{Subst}\left(\int \frac{\cdots}{(-1 \cdots}\right.}{\cdots}$$

$$= -\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{\left(a^2+b^2\right)(bc-ad)^{3/2}f} + \frac{\cdots}{(bc-ad)\,(c \cdots}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(ia+b)(c-id)^{3/2}f} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+d\tan(\cdots}}{\sqrt{c+i \cdots}}\right)}{(ia-b)(c+id) \cdots}$$

Mathematica [A]
time = 2.13, size = 247, normalized size = 1.17

$$\frac{-\frac{i\left(\frac{(a+ib)(c+id)(-bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}}+\frac{(a-ib)(c-id)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{\sqrt{c+id}}\right)}{a^2+b^2}+\frac{2b^{5/2}\left(c^2+d^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{\left(a^2+b^2\right)\sqrt{bc-ad}}-\frac{2d^2}{\sqrt{c+d\tan(e+fx)}}}{(-bc+ad)\left(c^2+d^2\right)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] (((-I)*(((a + I*b)*(c + I*d)*(-(b*c) + a*d)*ArcTanh[Sqrt[c + d*Tan[e + f*x]
]/Sqrt[c - I*d]])/Sqrt[c - I*d] + ((a - I*b)*(c - I*d)*(b*c - a*d)*ArcTanh[
Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/Sqrt[c + I*d]))/(a^2 + b^2) + (2*b
^(5/2)*(c^2 + d^2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*
d]])/((a^2 + b^2)*Sqrt[b*c - a*d]) - (2*d^2)/Sqrt[c + d*Tan[e + f*x]])/((-(
b*c) + a*d)*(c^2 + d^2)*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2352 vs. $2(181) = 362$.
time = 0.57, size = 2353, normalized size = 11.15

method	result	size
derivativedivides	Expression too large to display	2353
default	Expression too large to display	2353

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/f*d^2*(-1/(a*d-b*c)/(c^2+d^2)/(c+d*tan(f*x+e))^(1/2)-1/(a*d-b*c)*b^3/d^2/(a^2+b^2)/((a*d-b*c)*b)^(1/2)*arctan(b*(c+d*tan(f*x+e))^(1/2)/((a*d-b*c)*b)^(1/2))+1/(c^2+d^2)/d^2/(a^2+b^2)*(1/4/d^2/(3*c^2-d^2)/(c^2+d^2)^(3/2)*(-1/2*(-(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^4+(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^4+3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^5*d+2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3*d^3-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d^5+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^6-2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^4*d^2-3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d^4-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^6*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^4*d^3+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2*d^5-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^7+6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^5*d^2+4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^3*d^4-2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d^6)*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))+2*(-12*a*c^5*d^3-8*a*c^3*d^5+4*a*c*d^7-6*b*c^6*d^2+2*b*c^4*d^4+6*b*c^2*d^6-2*b*d^8+1/2*(-(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^4+(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^4+3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^5*d+2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3*d^3-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d^5+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^6-2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^4*d^2-3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d^4-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^6*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^4*d^3+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2*d^5-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^7+6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^5*d^2+4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^3*d^4-2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d^6)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/4/d^2/(3*c^2-d^2)/(c^2+d^2)^(3/2)*(1/2*(-(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^4+(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^4+3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^5*d+2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3*d^3-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d^5+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^6-2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^4*d^2-3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d^4-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^6*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^4*d^3+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2*d^5-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^7+6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^5*d^2+4*(2*(c^

```
2+d^2)^(1/2)+2*c)^(1/2)*b*c^3*d^4-2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d^6)*
ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2
+d^2)^(1/2))+2*(12*a*c^5*d^3+8*a*c^3*d^5-4*a*c*d^7+6*b*c^6*d^2-2*b*c^4*d^4-
6*b*c^2*d^6+2*b*d^8-1/2*(-(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c
^4+(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^4+3*(c^2+d^2)^(1/2)*(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^5*d+2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*a*c^3*d^3-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d^5+(c
^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^6-2*(c^2+d^2)^(1/2)*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*b*c^4*d^2-3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*b*c^2*d^4-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^6*d+(2*(c^2+d^2)^(1/2)+
2*c)^(1/2)*a*c^4*d^3+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2*d^5-(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*a*d^7+6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^5*d^2+4*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*b*c^3*d^4-2*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d^6)*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*
tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1
/2)))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 31213 vs. $2(179) = 358$.
time = 208.29, size = 62414, normalized size = 295.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(2)*(((a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*c^11 - (a^7 + 3*a^5
*b^2 + 3*a^3*b^4 + a*b^6)*c^10*d + 3*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*
c^9*d^2 - 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*c^8*d^3 + 2*(a^6*b + 3*a^
4*b^3 + 3*a^2*b^5 + b^7)*c^7*d^4 - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*
c^6*d^5 - 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*c^5*d^6 + 2*(a^7 + 3*a^5*
b^2 + 3*a^3*b^4 + a*b^6)*c^4*d^7 - 3*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*
c^3*d^8 + 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*c^2*d^9 - (a^6*b + 3*a^4*
```

b^3 + 3*a^2*b^5 + b^7)*c*d^10 + (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*d^11)
*f^5*cos(f*x + e)^2 + 2*((a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*c^10*d - (a^
7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*c^9*d^2 + 4*(a^6*b + 3*a^4*b^3 + 3*a^2*b
^5 + b^7)*c^8*d^3 - 4*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*c^7*d^4 + 6*(a^
6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*c^6*d^5 - 6*(a^7 + 3*a^5*b^2 + 3*a^3*b^4
 + a*b^6)*c^5*d^6 + 4*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*c^4*d^7 - 4*(a^
7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*c^3*d^8 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5
 + b^7)*c^2*d^9 - (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*c*d^10)*f^5*cos(f*x
 + e)*sin(f*x + e) + ((a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*c^9*d^2 - (a^7
+ 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*c^8*d^3 + 4*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5
 + b^7)*c^7*d^4 - 4*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*c^6*d^5 + 6*(a^6*
b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*c^5*d^6 - 6*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 +
 a*b^6)*c^4*d^7 + 4*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*c^3*d^8 - 4*(a^7
+ 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*c^2*d^9 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 +
 b^7)*c*d^10 - (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*d^11)*f^5)*sqrt(((a^4
+ 2*a^2*b^2 + b^4)*c^6 + 3*(a^4 + 2*a^2*b^2 + b^4)*c^4*d^2 + 3*(a^4 + 2*a^2
*b^2 + b^4)*c^2*d^4 + (a^4 + 2*a^2*b^2 + b^4)*d^6 + ((a^6 + a^4*b^2 - a^2*b
^4 - b^6)*c^9 - 6*(a^5*b + 2*a^3*b^3 + a*b^5)*c^8*d - 16*(a^5*b + 2*a^3*b^3
 + a*b^5)*c^6*d^3 - 6*(a^6 + a^4*b^2 - a^2*b^4 - b^6)*c^5*d^4 - 12*(a^5*b +
 2*a^3*b^3 + a*b^5)*c^4*d^5 - 8*(a^6 + a^4*b^2 - a^2*b^4 - b^6)*c^3*d^6 - 3
*(a^6 + a^4*b^2 - a^2*b^4 - b^6)*c*d^8 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^9)
*f^2*sqrt(1/(((a^4 + 2*a^2*b^2 + b^4)*c^6 + 3*(a^4 + 2*a^2*b^2 + b^4)*c^4*d
^2 + 3*(a^4 + 2*a^2*b^2 + b^4)*c^2*d^4 + (a^4 + 2*a^2*b^2 + b^4)*d^6)*f^4))
)/(4*a^2*b^2*c^6 + 12*(a^3*b - a*b^3)*c^5*d + 3*(3*a^4 - 14*a^2*b^2 + 3*b^4
)*c^4*d^2 - 40*(a^3*b - a*b^3)*c^3*d^3 - 6*(a^4 - 8*a^2*b^2 + b^4)*c^2*d^4
+ 12*(a^3*b - a*b^3)*c*d^5 + (a^4 - 2*a^2*b^2 + b^4)*d^6))*sqrt((4*a^2*b^2*
c^6 + 12*(a^3*b - a*b^3)*c^5*d + 3*(3*a^4 - 14*a^2*b^2 + 3*b^4)*c^4*d^2 - 4
0*(a^3*b - a*b^3)*c^3*d^3 - 6*(a^4 - 8*a^2*b^2 + b^4)*c^2*d^4 + 12*(a^3*b -
 a*b^3)*c*d^5 + (a^4 - 2*a^2*b^2 + b^4)*d^6)/(((a^8 + 4*a^6*b^2 + 6*a^4*b^4
 + 4*a^2*b^6 + b^8)*c^12 + 6*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8
)*c^10*d^2 + 15*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*c^8*d^4 + 2
0*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*c^6*d^6 + 15*(a^8 + 4*a^6
*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*c^4*d^8 + 6*(a^8 + 4*a^6*b^2 + 6*a^4*b^
4 + 4*a^2*b^6 + b^8)*c^2*d^10 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 +
b^8)*d^12)*f^4))*(1/(((a^4 + 2*a^2*b^2 + b^4)*c^6 + 3*(a^4 + 2*a^2*b^2 + b^
4)*c^4*d^2 + 3*(a^4 + 2*a^2*b^2 + b^4)*c^2*d^4 + (a^4 + 2*a^2*b^2 + b^4)*d^
6)*f^4))^(3/4)*arctan(((2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*c^13 + 3*
(a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*c^12*d + 4*(a^7*b + 3*a^5*b^3 + 3*a^3*b
^5 + a*b^7)*c^11*d^2 + 14*(a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*c^10*d^3 - 10
*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*c^9*d^4 + 25*(a^8 + 2*a^6*b^2 - 2*
a^2*b^6 - b^8)*c^8*d^5 - 40*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*c^7*d^6
 + 20*(a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*c^6*d^7 - 50*(a^7*b + 3*a^5*b^3 +
 3*a^3*b^5 + a*b^7)*c^5*d^8 + 5*(a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*c^4*d^9
 - 28*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*c^3*d^10 - 2*(a^8 + 2*a^6*b^2
 - 2*a^2*b^6 - b^8)*c^2*d^11 - 6*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*c*

```
d^12 - (a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^13)*f^4*sqrt((4*a^2*b^2*c^6 +
12*(a^3*b - a*b^3)*c^5*d + 3*(3*a^4 - 14*a^2*b^2 + 3*b^4)*c^4*d^2 - 40*(a^3
*b - a*b^3)*c^3*d^3 - 6*(a^4 - 8*a^2*b^2 + b^4)*c^2*d^4 + 12*(a^3*b - a*b^3
)*c*d^5 + (a^4 - 2*a^2*b^2 + b^4)*d^6)/(((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a
^2*b^6 + b^8)*c^12 + 6*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*c^10
*d^2 + 15*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*c^8*d^4 + 20*(a^8
 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*c^6*d^6 + 15*(a^8 + 4*a^6*b^2 +
 6*a^4*b^4 + 4*a^2*b^6 + b^8)*c^4*d^8 + 6*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*
a^2*b^6 + b^8)*c^2*d^10 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d
^12)*f^4))*sqrt(1/(((a^4 + 2*a^2*b^2 + b^4)*c^6 + 3*(a^4 + 2*a^2*b^2 + b^4)
*c^4*d^2 + 3*(a^4 + 2*a^2*b^2 + b^4)*c^2*d^4 + (a^4 + 2*a^2*b^2 + b^4)*d^6)
*f^4)) + (2*(a^5*b + 2*a^3*b^3 + a*b^5)*c^10 + 3*(a^6 + a^4*b^2 - a^2*b^4 -
 b^6)*c^9*d + 8*(a^6 + a^4*b^2 - a^2*b^4 - b^6)*c^7*d^3 - 12*(a^5*b + 2*a^3
*b^3 + a*b^5)*c^6*d^4 + 6*(a^6 + a^4*b^2 - a^2*b^4 - b^6)*c^5*d^5 - 16*(a^5
*b + 2*a^3*b^3 + a*b^5)*c^4*d^6 - 6*(a^5*b + 2*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\tan{(e+fx)})\,(c+d\tan{(e+fx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/((a + b*tan(e + f*x))*(c + d*tan(e + f*x))**(3/2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 20.91, size = 2500, normalized size = 11.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*tan(e + f*x))*(c + d*tan(e + f*x))^(3/2)),x)
```

```
[Out] (log(((((2*a^2*b^2*d^6*f^4 - b^4*d^6*f^4 - 4*a^2*b^2*c^6*f^4 - a^4*d^6*f^4
+ 6*a^4*c^2*d^4*f^4 - 9*a^4*c^4*d^2*f^4 + 6*b^4*c^2*d^4*f^4 - 9*b^4*c^4*d^2
*f^4 - 48*a^2*b^2*c^2*d^4*f^4 + 42*a^2*b^2*c^4*d^2*f^4 + 12*a*b^3*c*d^5*f^4
 + 12*a*b^3*c^5*d*f^4 - 12*a^3*b*c*d^5*f^4 - 12*a^3*b*c^5*d*f^4 - 40*a*b^3*
c^3*d^3*f^4 + 40*a^3*b*c^3*d^3*f^4)^(1/2) - a^2*c^3*f^2 + b^2*c^3*f^2 + 3*a
^2*c*d^2*f^2 - 3*b^2*c*d^2*f^2 - 2*a*b*d^3*f^2 + 6*a*b*c^2*d*f^2)/(a^4*c^6*
f^4 + a^4*d^6*f^4 + b^4*c^6*f^4 + b^4*d^6*f^4 + 2*a^2*b^2*c^6*f^4 + 2*a^2*b
^2*d^6*f^4 + 3*a^4*c^2*d^4*f^4 + 3*a^4*c^4*d^2*f^4 + 3*b^4*c^2*d^4*f^4 + 3*
b^4*c^4*d^2*f^4 + 6*a^2*b^2*c^2*d^4*f^4 + 6*a^2*b^2*c^4*d^2*f^4))^(1/2)*(((
((2*a^2*b^2*d^6*f^4 - b^4*d^6*f^4 - 4*a^2*b^2*c^6*f^4 - a^4*d^6*f^4 + 6*a^4
*c^2*d^4*f^4 - 9*a^4*c^4*d^2*f^4 + 6*b^4*c^2*d^4*f^4 - 9*b^4*c^4*d^2*f^4 -
48*a^2*b^2*c^2*d^4*f^4 + 42*a^2*b^2*c^4*d^2*f^4 + 12*a*b^3*c*d^5*f^4 + 12*a
*b^3*c^5*d*f^4 - 12*a^3*b*c*d^5*f^4 - 12*a^3*b*c^5*d*f^4 - 40*a*b^3*c^3*d^3
*f^4 + 40*a^3*b*c^3*d^3*f^4)^(1/2) - a^2*c^3*f^2 + b^2*c^3*f^2 + 3*a^2*c*d^
2*f^2 - 3*b^2*c*d^2*f^2 - 2*a*b*d^3*f^2 + 6*a*b*c^2*d*f^2)/(a^4*c^6*f^4 + a
^4*d^6*f^4 + b^4*c^6*f^4 + b^4*d^6*f^4 + 2*a^2*b^2*c^6*f^4 + 2*a^2*b^2*d^6*
f^4 + 3*a^4*c^2*d^4*f^4 + 3*a^4*c^4*d^2*f^4 + 3*b^4*c^2*d^4*f^4 + 3*b^4*c^4
*d^2*f^4 + 6*a^2*b^2*c^2*d^4*f^4 + 6*a^2*b^2*c^4*d^2*f^4))^(1/2)*(((((2*a^2
*b^2*d^6*f^4 - b^4*d^6*f^4 - 4*a^2*b^2*c^6*f^4 - a^4*d^6*f^4 + 6*a^4*c^2*d^
4*f^4 - 9*a^4*c^4*d^2*f^4 + 6*b^4*c^2*d^4*f^4 - 9*b^4*c^4*d^2*f^4 - 48*a^2*
b^2*c^2*d^4*f^4 + 42*a^2*b^2*c^4*d^2*f^4 + 12*a*b^3*c*d^5*f^4 + 12*a*b^3*c^
5*d*f^4 - 12*a^3*b*c*d^5*f^4 - 12*a^3*b*c^5*d*f^4 - 40*a*b^3*c^3*d^3*f^4 +
40*a^3*b*c^3*d^3*f^4)^(1/2) - a^2*c^3*f^2 + b^2*c^3*f^2 + 3*a^2*c*d^2*f^2 -
 3*b^2*c*d^2*f^2 - 2*a*b*d^3*f^2 + 6*a*b*c^2*d*f^2)/(a^4*c^6*f^4 + a^4*d^6*
f^4 + b^4*c^6*f^4 + b^4*d^6*f^4 + 2*a^2*b^2*c^6*f^4 + 2*a^2*b^2*d^6*f^4 + 3
*a^4*c^2*d^4*f^4 + 3*a^4*c^4*d^2*f^4 + 3*b^4*c^2*d^4*f^4 + 3*b^4*c^4*d^2*f^
4 + 6*a^2*b^2*c^2*d^4*f^4 + 6*a^2*b^2*c^4*d^2*f^4))^(1/2)*(((((2*a^2*b^2*d^
6*f^4 - b^4*d^6*f^4 - 4*a^2*b^2*c^6*f^4 - a^4*d^6*f^4 + 6*a^4*c^2*d^4*f^4 -
 9*a^4*c^4*d^2*f^4 + 6*b^4*c^2*d^4*f^4 - 9*b^4*c^4*d^2*f^4 - 48*a^2*b^2*c^2
*d^4*f^4 + 42*a^2*b^2*c^4*d^2*f^4 + 12*a*b^3*c*d^5*f^4 + 12*a*b^3*c^5*d*f^4
 - 12*a^3*b*c*d^5*f^4 - 12*a^3*b*c^5*d*f^4 - 40*a*b^3*c^3*d^3*f^4 + 40*a^3*
b*c^3*d^3*f^4)^(1/2) - a^2*c^3*f^2 + b^2*c^3*f^2 + 3*a^2*c*d^2*f^2 - 3*b^2*
c*d^2*f^2 - 2*a*b*d^3*f^2 + 6*a*b*c^2*d*f^2)/(a^4*c^6*f^4 + a^4*d^6*f^4 + b
^4*c^6*f^4 + b^4*d^6*f^4 + 2*a^2*b^2*c^6*f^4 + 2*a^2*b^2*d^6*f^4 + 3*a^4*c^
2*d^4*f^4 + 3*a^4*c^4*d^2*f^4 + 3*b^4*c^2*d^4*f^4 + 3*b^4*c^4*d^2*f^4 + 6*a
^2*b^2*c^2*d^4*f^4 + 6*a^2*b^2*c^4*d^2*f^4))^(1/2)*(c + d*tan(e + f*x))^(1/
2)*(512*a^9*b^9*d^37*f^9 + 512*a^11*b^7*d^37*f^9 - 512*a^13*b^5*d^37*f^9 -
512*a^15*b^3*d^37*f^9 - 512*b^18*c^9*d^28*f^9 - 5376*b^18*c^11*d^26*f^9 - 2
5344*b^18*c^13*d^24*f^9 - 70656*b^18*c^15*d^22*f^9 - 129024*b^18*c^17*d^20*
f^9 - 161280*b^18*c^19*d^18*f^9 - 139776*b^18*c^21*d^16*f^9 - 82944*b^18*c^
23*d^14*f^9 - 32256*b^18*c^25*d^12*f^9 - 7424*b^18*c^27*d^10*f^9 - 768*b^18
*c^29*d^8*f^9 - 18432*a^2*b^16*c^7*d^30*f^9 - 191744*a^2*b^16*c^9*d^28*f^9
- 897536*a^2*b^16*c^11*d^26*f^9 - 2490624*a^2*b^16*c^13*d^24*f^9 - 4540416*
```

a^2*b^16*c^15*d^22*f^9 - 5687808*a^2*b^16*c^17*d^20*f^9 - 4967424*a^2*b^16*c^19*d^18*f^9 - 2996736*a^2*b^16*c^21*d^16*f^9 - 1204224*a^2*b^16*c^23*d^14*f^9 - 297216*a^2*b^16*c^25*d^12*f^9 - 37376*a^2*b^16*c^27*d^10*f^9 - 1280*a^2*b^16*c^29*d^8*f^9 + 43008*a^3*b^15*c^6*d^31*f^9 + 446976*a^3*b^15*c^8*d^29*f^9 + 2098176*a^3*b^15*c^10*d^27*f^9 + 5865984*a^3*b^15*c^12*d^25*f^9 + 10838016*a^3*b^15*c^14*d^23*f^9 + 13870080*a^3*b^15*c^16*d^21*f^9 + 12515328*a^3*b^15*c^18*d^19*f^9 + 7934976*a^3*b^15*c^20*d^17*f^9 + 3446784*a^3*b^15*c^22*d^15*f^9 + 969216*a^3*b^15*c^24*d^13*f^9 + 156672*a^3*b^15*c^26*d^11*f^9 + 10752*a^3*b^15*c^28*d^9*f^9 - 64512*a^4*b^14*c^5*d^32*f^9 - 674304*a^4*b^14*c^7*d^30*f^9 - 3204352*a^4*b^14*c^9*d^28*f^9 - 9140224*a^4*b^14*c^11*d^26*f^9 - 17392896*a^4*b^14*c^13*d^24*f^9 - 23190528*a^4*b^14*c^15*d^22*f^9 - 22116864*a^4*b^14*c^17*d^20*f^9 - 15095808*a^4*b^14*c^19*d^18*f^9 - 7233024*a^4*b^14*c^21*d^16*f^9 - 2320896*a^4*b^14*c^23*d^14*f^9 - 450816*a^4*b^14*c^25*d^12*f^9 - 40960*a^4*b^14*c^27*d^10*f^9 - 256*a^4*b^14*c^29*d^8*f^9 + 64512*a^5*b^13*c^4*d^33*f^9 + 688128*a^5*b^13*c^6*d^31*f^9 + 3365376*a^5*b^13*c^8*d^29*f^9 + 9968640*a^5*b^13*c^10*d^27*f^9 + 19883520*a^5*b^13*c^12*d^25*f^9 + 28053504*a^5*b^13*c^14*d^23*f^9 + 28578816*a^5*b^13*c^16*d^21*f^9 + 21030912*a^5*b^13*c^18*d^19*f^9 + 10967040*a^5*b^13*c^20*d^17*f^9 + 3870720*a^5*b^13*c^22*d^15*f^9 + 840192*a^5*b^13*c^24*d^13*f^9 + 89088*a^5*b^13*c^26*d^11*f^9 + 1536*a^5*b^13*c^28*d^9*f^9 - 43008*a^6*b^12*c^3*d^34*f^9 - 483840*a^6*b^12*c^5*d^32*f^9 - 2497536*a^6*b^12*c^7*d^30*f^9 - 7803136*a^6*b^12*c^9*d^28*f^9 - 16383488*a^6*b^12*c^11*d^26*f^9 - 24254208*a^6*b^12*c^13*d^24*f^9 - 25817088*a^6*b^12*c^15*d^22*f^9 - 19751424*a^6*b^12*c^17*d^20*f^9 - 10644480*a^6*b^12*c^19*d^18*f^9 -...

3.1258 $\int \frac{1}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=314

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)^2(c-id)^{3/2}f}+\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(a+ib)^2(c+id)^{3/2}f}-\frac{b^{5/2}(4abc-7a^2d-3b^2d)\tanh^{-1}\left(\right.}{(a^2+b^2)^2(bc-}$$

```
[Out] -I*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(a-I*b)^2/(c-I*d)^(3/2)/f+
I*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(a+I*b)^2/(c+I*d)^(3/2)/f-b
^(5/2)*(-7*a^2*d+4*a*b*c-3*b^2*d)*arctanh(b^(1/2)*(c+d*tan(f*x+e))^(1/2)/(-
a*d+b*c)^(1/2))/(a^2+b^2)^2/(-a*d+b*c)^(5/2)/f-d*(2*a^2*d^2+b^2*(c^2+3*d^2)
)/(a^2+b^2)/(-a*d+b*c)^2/(c^2+d^2)/f/(c+d*tan(f*x+e))^(1/2)-b^2/(a^2+b^2)/(
-a*d+b*c)/f/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))
```

Rubi [A]
time = 1.02, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {3650, 3730, 3734, 3620, 3618, 65, 214, 3715}

$$-\frac{d\left(2a^2d^2+b^2\left(c^2+3d^2\right)\right)}{f\left(a^2+b^2\right)\left(c^2+d^2\right)(bc-ad)^2\sqrt{c+d\tan(e+fx)}}-\frac{b^2}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}-\frac{b^{5/2}\left(-7a^2d+4abc-3b^2d\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{f\left(a^2+b^2\right)^2(bc-ad)^{5/2}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(a-ib)^2(c-id)^{3/2}}+\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(a+ib)^2(c+id)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] ((-I)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((a - I*b)^2*(c - I*
d)^(3/2)*f) + (I*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/((a + I*b
)^2*(c + I*d)^(3/2)*f) - (b^(5/2)*(4*a*b*c - 7*a^2*d - 3*b^2*d)*ArcTanh[(Sq
rt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/((a^2 + b^2)^2*(b*c - a*d
)^(5/2)*f) - (d*(2*a^2*d^2 + b^2*(c^2 + 3*d^2)))/((a^2 + b^2)*(b*c - a*d)^2
*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]]) - b^2/((a^2 + b^2)*(b*c - a*d)*f*(
a + b*Tan[e + f*x])*Sqrt[c + d*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
```

```
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{b^2}{(a^2+b^2)\,(bc-ad)f(a+b\tan(e+fx))\sqrt{c+d\tan(e}$$

$$= -\frac{d(2a^2d^2+b^2(c^2+3d^2))}{(a^2+b^2)\,(bc-ad)^2\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{d(2a^2d^2+b^2(c^2+3d^2))}{(a^2+b^2)\,(bc-ad)^2\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{d(2a^2d^2+b^2(c^2+3d^2))}{(a^2+b^2)\,(bc-ad)^2\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{d(2a^2d^2+b^2(c^2+3d^2))}{(a^2+b^2)\,(bc-ad)^2\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{b^{5/2}(4abc-7a^2d-3b^2d)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+d\tan(e}}{\sqrt{bc-ad}}\right.}{\left(a^2+b^2\right)^2(bc-ad)^{5/2}f}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)^2(c-id)^{3/2}f} + \frac{i\tanh^{-1}\left(\frac{\sqrt{c+}}{}\right.}{(a+ib)^2(}$$

Mathematica [A]

time = 6.27, size = 628, normalized size = 2.00

$$-\frac{b^2}{(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}-\frac{\frac{2\left(\frac{\frac{i\sqrt{c-id}\left(-\frac{1}{2}(bc-ad)^2(a^2c-b^2c-2abd)-\frac{1}{2}i(bc-ad)^2(2abc+a^2d-b^2d)\right)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(-c+id)f}-\frac{i\sqrt{c+id}\left(-\frac{1}{2}(bc-ad)^2(a^2c-b^2c-2abd)+\frac{1}{2}i(bc-ad)^2(2abc+a^2d-b^2d)\right)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(-c-id)f}}{a^2+b^2}+\frac{2\sqrt{bc-ad}\left(-\frac{1}{2}ab(bc-ad)^2(bc+ad)+\frac{1}{4}b^2\left(-2a^3cd^2+4a^2bd(c^2+d^2)+3b^3d(c^2+d^2)-2ab^2c(c^2+2d^2)\right)+\frac{1}{4}a^2bd\left(2a^2d^2+b^2(c^2+3d^2)\right)\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{\sqrt{b}(a^2+b^2)(-bc+ad)f}\right)}{(-bc+ad)(c^2+d^2)}-\frac{2\left(\frac{1}{2}d^2(-2abc+2a^2d+3b^2d)-c\left(-\frac{3}{2}b^2cd+bd(bc-ad)\right)\right)}{(-bc+ad)(c^2+d^2)f\sqrt{c+d\tan(e+fx)}}}{(a^2+b^2)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] -(b^2/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])*Sqrt[c + d*Tan[e + f*
x]])) - ((-2*(((I*Sqrt[c - I*d]*(-1/2*((b*c - a*d)^2*(a^2*c - b^2*c - 2*a*b
*d)) - (I/2)*(b*c - a*d)^2*(2*a*b*c + a^2*d - b^2*d))*ArcTanh[Sqrt[c + d*Ta
n[e + f*x]]/Sqrt[c - I*d]])/((-c + I*d)*f) - (I*Sqrt[c + I*d]*(-1/2*((b*c -
 a*d)^2*(a^2*c - b^2*c - 2*a*b*d)) + (I/2)*(b*c - a*d)^2*(2*a*b*c + a^2*d -
 b^2*d))*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/((-c - I*d)*f))/(
a^2 + b^2) + (2*Sqrt[b*c - a*d]*(-1/2*(a*b*(b*c - a*d)^2*(b*c + a*d)) + (b^
2*(-2*a^3*c*d^2 + 4*a^2*b*d*(c^2 + d^2) + 3*b^3*d*(c^2 + d^2) - 2*a*b^2*c*(
c^2 + 2*d^2)))/4 + (a^2*b*d*(2*a^2*d^2 + b^2*(c^2 + 3*d^2)))/4)*ArcTanh[(Sq
rt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(Sqrt[b]*(a^2 + b^2)*(-(b
*c) + a*d)*f)))/((-(b*c) + a*d)*(c^2 + d^2)) - (2*((d^2*(-2*a*b*c + 2*a^2*d
 + 3*b^2*d))/2 - c*((-3*b^2*c*d)/2 + b*d*(b*c - a*d))))/((-(b*c) + a*d)*(c^
2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]]))/((a^2 + b^2)*(b*c - a*d))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3447 vs. $2(280) = 560$.
time = 0.60, size = 3448, normalized size = 10.98

method	result	size
derivativedivides	Expression too large to display	3448
default	Expression too large to display	3448

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*d^3*(1/(c^2+d^2)/d^3/(a^2+b^2)^2*(1/4/d^2/(3*c^2-d^2)/(c^2+d^2)^(3/2)*(
1/2*(-2*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^4+2*(c^2+d^2)^(
3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d^4+3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)*a^2*c^5*d+2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a
^2*c^3*d^3-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c*d^5+2*(c^2+d
^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^6-4*(c^2+d^2)^(1/2)*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*a*b*c^4*d^2-6*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*a*b*c^2*d^4-3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^5*d
-2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^3*d^3+(c^2+d^2)^(1/2
)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c*d^5-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a
^2*c^6*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^4*d^3+3*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*a^2*c^2*d^5-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d^7+12*(2*(c^2+d^2)^(
```

1/2)+2*c)^(1/2)*a*b*c^5*d^2+8*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^3*d^4-4*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c*d^6+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*
c^6*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^4*d^3-3*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*b^2*c^2*d^5+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d^7)*ln(d*tan(f*x+e)+c+(
c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(12*
a^2*c^5*d^3+8*a^2*c^3*d^5-4*a^2*c*d^7+12*a*b*c^6*d^2-4*a*b*c^4*d^4-12*a*b*c
^2*d^6+4*a*b*d^8-12*b^2*c^5*d^3-8*b^2*c^3*d^5+4*b^2*c*d^7-1/2*(-2*(c^2+d^2)
^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^4+2*(c^2+d^2)^(3/2)*(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*a*b*d^4+3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a
^2*c^5*d+2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^3*d^3-(c^2+d
^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c*d^5+2*(c^2+d^2)^(1/2)*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*a*b*c^6-4*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)*a*b*c^4*d^2-6*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^2*d^4
-3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^5*d-2*(c^2+d^2)^(1/2
)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^3*d^3+(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*b^2*c*d^5-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^6*d+(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*a^2*c^4*d^3+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^2*d
^5-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d^7+12*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a
*b*c^5*d^2+8*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^3*d^4-4*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*a*b*c*d^6+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^6*d-(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)*b^2*c^4*d^3-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^2*d^5+
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d^7)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c
^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2
)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/4/d^2/(3*c^2-d^2)/(c^2+d^2)
^(3/2)*(-1/2*(-2*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^4+2*(c
^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d^4+3*(c^2+d^2)^(1/2)*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^5*d+2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*a^2*c^3*d^3-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c*d^5
+2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^6-4*(c^2+d^2)^(1/2)*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^4*d^2-6*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*a*b*c^2*d^4-3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*
b^2*c^5*d-2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^3*d^3+(c^2+
d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c*d^5-3*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*a^2*c^6*d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^4*d^3+3*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*a^2*c^2*d^5-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d^7+12*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^5*d^2+8*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^
3*d^4-4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c*d^6+3*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*b^2*c^6*d-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^4*d^3-3*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*b^2*c^2*d^5+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d^7)*ln((c+d*ta
n(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2
))+2*(-12*a^2*c^5*d^3-8*a^2*c^3*d^5+4*a^2*c*d^7-12*a*b*c^6*d^2+4*a*b*c^4*d^
4+12*a*b*c^2*d^6-4*a*b*d^8+12*b^2*c^5*d^3+8*b^2*c^3*d^5-4*b^2*c*d^7+1/2*(-2
(c^2+d^2)^(3/2)(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^4+2*(c^2+d^2)^(3/2)*(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d^4+3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*a^2*c^5*d+2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^3*

```
d^3-(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c*d^5+2*(c^2+d^2)^(1/
2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^6-4*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*a*b*c^4*d^2-6*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a
*b*c^2*d^4-3*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^5*d-2*(c^2
+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^3*d^3+(c^2+d^2)^(1/2)*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c*d^5-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^6*
d+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^4*d^3+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*a^2*c^2*d^5-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d^7+12*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*a*b*c^5*d^2+8*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^3*d^4-4*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*a*b*c*d^6+3*(2*(c^2+d^2)^...
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\tan(e+fx))^2 (c+d\tan(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))**2/(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/((a + b*tan(e + f*x))**2*(c + d*tan(e + f*x))**(3/2)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*tan(e + f*x))^2*(c + d*tan(e + f*x))^(3/2)),x)
```

```
[Out] \text{Hanged}
```

3.1259 $\int \frac{(a+b\tan(e+fx))^4}{(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=290

$$-\frac{i(a-ib)^4\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{5/2}f}+\frac{i(a+ib)^4\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(c+id)^{5/2}f}-\frac{2(bc-ad)^2(a+}{3d\,(c^2+d^2)\,f(c+}$$

```
[Out] -I*(a-I*b)^4*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(c-I*d)^(5/2)/f+
I*(a+I*b)^4*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(c+I*d)^(5/2)/f+4
/3*(-a*d+b*c)^3*(3*a*c*d+2*b*c^2+5*b*d^2)/d^3/(c^2+d^2)^2/f/(c+d*tan(f*x+e)
)^(1/2)-2/3*b^2*(a*d*(-a*d+2*b*c)-b^2*(4*c^2+3*d^2))*(c+d*tan(f*x+e))^(1/2)
/d^3/(c^2+d^2)/f-2/3*(-a*d+b*c)^2*(a+b*tan(f*x+e))^2/d/(c^2+d^2)/f/(c+d*tan
(f*x+e))^(3/2)
```

Rubi [A]

time = 0.72, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {3646, 3716, 3711, 3620, 3618, 65, 214}

$$-\frac{2b^2\left(ad(2bc-ad)-b^2\left(4c^2+3d^2\right)\right)\sqrt{c+d\tan(e+fx)}}{3d^3f\left(c^2+d^2\right)}-\frac{2(bc-ad)^2(a+b\tan(e+fx))^2}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}+\frac{4(bc-ad)^3\left(3acd+2bc^2+5bd^2\right)}{3d^3f\left(c^2+d^2\right)^2\sqrt{c+d\tan(e+fx)}}-\frac{i(a-ib)^4\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(c-id)^{5/2}}+\frac{i(a+ib)^4\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(c+id)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^4/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-I)*(a - I*b)^4*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((c - I*
d)^(5/2)*f) + (I*(a + I*b)^4*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]
])/((c + I*d)^(5/2)*f) - (2*(b*c - a*d)^2*(a + b*Tan[e + f*x])^2)/(3*d*(c^2
 + d^2)*f*(c + d*Tan[e + f*x])^(3/2)) + (4*(b*c - a*d)^3*(2*b*c^2 + 3*a*c*d
 + 5*b*d^2))/(3*d^3*(c^2 + d^2)^2*f*Sqrt[c + d*Tan[e + f*x]]) - (2*b^2*(a*d
*(2*b*c - a*d) - b^2*(4*c^2 + 3*d^2))*Sqrt[c + d*Tan[e + f*x]])/(3*d^3*(c^2
 + d^2)*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3716

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
```

```
-1]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^4}{(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{2(bc-ad)^2(a+b\tan(e+fx))^2}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{2\int \frac{(a+b\tan(e+fx))\left(\frac{1}{2}\left(4b^3c^2+3a^3cd-11ab^2cd-\right.\right.}{}}{}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))^2}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{4(bc-ad)^3\left(2bc^2+3acd+5bd^2\right)}{3d^3\left(c^2+d^2\right)^2f\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))^2}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{4(bc-ad)^3\left(2bc^2+3acd+5bd^2\right)}{3d^3\left(c^2+d^2\right)^2f\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))^2}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{4(bc-ad)^3\left(2bc^2+3acd+5bd^2\right)}{3d^3\left(c^2+d^2\right)^2f\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))^2}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{4(bc-ad)^3\left(2bc^2+3acd+5bd^2\right)}{3d^3\left(c^2+d^2\right)^2f\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))^2}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{4(bc-ad)^3\left(2bc^2+3acd+5bd^2\right)}{3d^3\left(c^2+d^2\right)^2f\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{i(a-ib)^4\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{5/2}f} + \frac{i(a+ib)^4\tanh^{-1}\left(\frac{\sqrt{c+d\tan}}{\sqrt{c-}}\right.}{(c+id)^{5/2}f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 3.66, size = 368, normalized size = 1.27

$$-\frac{-2b^2(c-id)(c+id)\left(-20abcd+9a^2d^2+b^2(8c^2+d^2)\right)+d^2(-4a^3bc+4ab^3c+a^4d-6a^2b^2d+b^4d)\left((-ic+d)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{c+d\tan(e+fx)}{c-id}\right)+(ic+d)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{c+d\tan(e+fx)}{c+id}\right)\right)-12b^2(c-id)(c+id)d(2bc-3ad)(a+b\tan(e+fx))-6b^2(c-id)(c+id)d^2(a+b\tan(e+fx))^2-12ab(a^2-b^2)\,d^2\left(i(c+id)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{c+d\tan(e+fx)}{c-id}\right)-(ic+d)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{c+d\tan(e+fx)}{c+id}\right)\right)(c+d\tan(e+fx))}{3d^3\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^4/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] -1/3*(-2*b^2*(c - I*d)*(c + I*d)*(-20*a*b*c*d + 9*a^2*d^2 + b^2*(8*c^2 + d^
2)) + d^2*(-4*a^3*b*c + 4*a*b^3*c + a^4*d - 6*a^2*b^2*d + b^4*d)*(((-I)*c +
 d)*Hypergeometric2F1[-3/2, 1, -1/2, (c + d*Tan[e + f*x])/(c - I*d)] + (I*c
 + d)*Hypergeometric2F1[-3/2, 1, -1/2, (c + d*Tan[e + f*x])/(c + I*d)]) - 1
2*b^2*(c - I*d)*(c + I*d)*d*(2*b*c - 3*a*d)*(a + b*Tan[e + f*x]) - 6*b^2*(c
```

```
 - I*d)*(c + I*d)*d^2*(a + b*Tan[e + f*x])^2 - 12*a*b*(a^2 - b^2)*d^2*(I*(c
 + I*d)*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c - I*d)] - (
I*c + d)*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c + I*d)])*(
c + d*Tan[e + f*x]))/(d^3*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 8857 vs. $2(258) = 516$.
time = 0.55, size = 8858, normalized size = 30.54

method	result	size
derivativedivides	Expression too large to display	8858
default	Expression too large to display	8858

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^4/(c+d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^4/(c+d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^4/(c+d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan{(e + fx)})^4}{(c + d\tan{(e + fx)})^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**4/(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**4/(c + d*tan(e + f*x))**(5/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^4/(c+d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**B**]
time = 35.67, size = 2500, normalized size = 8.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^4/(c + d*tan(e + f*x))^(5/2),x)
```

```
[Out] (2*b^4*(c + d*tan(e + f*x))^(1/2))/(d^3*f) - atan((((-(((8*a^8*c^5*f^2 + 8*
b^8*c^5*f^2 - 64*a*b^7*d^5*f^2 + 64*a^7*b*d^5*f^2 + 40*a^8*c*d^4*f^2 + 40*b
^8*c*d^4*f^2 - 224*a^2*b^6*c^5*f^2 + 560*a^4*b^4*c^5*f^2 - 224*a^6*b^2*c^5*
f^2 + 448*a^3*b^5*d^5*f^2 - 448*a^5*b^3*d^5*f^2 - 80*a^8*c^3*d^2*f^2 - 80*b
^8*c^3*d^2*f^2 + 2240*a^2*b^6*c^3*d^2*f^2 - 4480*a^3*b^5*c^2*d^3*f^2 - 5600
*a^4*b^4*c^3*d^2*f^2 + 4480*a^5*b^3*c^2*d^3*f^2 + 2240*a^6*b^2*c^3*d^2*f^2
- 320*a*b^7*c^4*d*f^2 + 320*a^7*b*c^4*d*f^2 + 640*a*b^7*c^2*d^3*f^2 - 1120*
a^2*b^6*c*d^4*f^2 + 2240*a^3*b^5*c^4*d*f^2 + 2800*a^4*b^4*c*d^4*f^2 - 2240*
a^5*b^3*c^4*d*f^2 - 1120*a^6*b^2*c*d^4*f^2 - 640*a^7*b*c^2*d^3*f^2)^2/4 - (
16*c^10*f^4 + 16*d^10*f^4 + 80*c^2*d^8*f^4 + 160*c^4*d^6*f^4 + 160*c^6*d^4*
f^4 + 80*c^8*d^2*f^4)*(a^16 + b^16 + 8*a^2*b^14 + 28*a^4*b^12 + 56*a^6*b^10
 + 70*a^8*b^8 + 56*a^10*b^6 + 28*a^12*b^4 + 8*a^14*b^2))^(1/2) + 4*a^8*c^5*
f^2 + 4*b^8*c^5*f^2 - 32*a*b^7*d^5*f^2 + 32*a^7*b*d^5*f^2 + 20*a^8*c*d^4*f^
2 + 20*b^8*c*d^4*f^2 - 112*a^2*b^6*c^5*f^2 + 280*a^4*b^4*c^5*f^2 - 112*a^6*
b^2*c^5*f^2 + 224*a^3*b^5*d^5*f^2 - 224*a^5*b^3*d^5*f^2 - 40*a^8*c^3*d^2*f^
2 - 40*b^8*c^3*d^2*f^2 + 1120*a^2*b^6*c^3*d^2*f^2 - 2240*a^3*b^5*c^2*d^3*f^
2 - 2800*a^4*b^4*c^3*d^2*f^2 + 2240*a^5*b^3*c^2*d^3*f^2 + 1120*a^6*b^2*c^3*
d^2*f^2 - 160*a*b^7*c^4*d*f^2 + 160*a^7*b*c^4*d*f^2 + 320*a*b^7*c^2*d^3*f^2
 - 560*a^2*b^6*c*d^4*f^2 + 1120*a^3*b^5*c^4*d*f^2 + 1400*a^4*b^4*c*d^4*f^2
- 1120*a^5*b^3*c^4*d*f^2 - 560*a^6*b^2*c*d^4*f^2 - 320*a^7*b*c^2*d^3*f^2)/(
16*(c^10*f^4 + d^10*f^4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*c^6*d^4*f^4 +
```

5*c^8*d^2*f^4)))^(1/2)*((c + d*tan(e + f*x))^(1/2)*(-(((8*a^8*c^5*f^2 + 8*b^8*c^5*f^2 - 64*a*b^7*d^5*f^2 + 64*a^7*b*d^5*f^2 + 40*a^8*c*d^4*f^2 + 40*b^8*c*d^4*f^2 - 224*a^2*b^6*c^5*f^2 + 560*a^4*b^4*c^5*f^2 - 224*a^6*b^2*c^5*f^2 + 448*a^3*b^5*d^5*f^2 - 448*a^5*b^3*d^5*f^2 - 80*a^8*c^3*d^2*f^2 - 80*b^8*c^3*d^2*f^2 + 2240*a^2*b^6*c^3*d^2*f^2 - 4480*a^3*b^5*c^2*d^3*f^2 - 5600*a^4*b^4*c^3*d^2*f^2 + 4480*a^5*b^3*c^2*d^3*f^2 + 2240*a^6*b^2*c^3*d^2*f^2 - 320*a*b^7*c^4*d*f^2 + 320*a^7*b*c^4*d*f^2 + 640*a*b^7*c^2*d^3*f^2 - 1120*a^2*b^6*c*d^4*f^2 + 2240*a^3*b^5*c^4*d*f^2 + 2800*a^4*b^4*c*d^4*f^2 - 2240*a^5*b^3*c^4*d*f^2 - 1120*a^6*b^2*c*d^4*f^2 - 640*a^7*b*c^2*d^3*f^2)^2/4 - (16*c^10*f^4 + 16*d^10*f^4 + 80*c^2*d^8*f^4 + 160*c^4*d^6*f^4 + 160*c^6*d^4*f^4 + 80*c^8*d^2*f^4)*(a^16 + b^16 + 8*a^2*b^14 + 28*a^4*b^12 + 56*a^6*b^10 + 70*a^8*b^8 + 56*a^10*b^6 + 28*a^12*b^4 + 8*a^14*b^2))^(1/2) + 4*a^8*c^5*f^2 + 4*b^8*c^5*f^2 - 32*a*b^7*d^5*f^2 + 32*a^7*b*d^5*f^2 + 20*a^8*c*d^4*f^2 + 20*b^8*c*d^4*f^2 - 112*a^2*b^6*c^5*f^2 + 280*a^4*b^4*c^5*f^2 - 112*a^6*b^2*c^5*f^2 + 224*a^3*b^5*d^5*f^2 - 224*a^5*b^3*d^5*f^2 - 40*a^8*c^3*d^2*f^2 - 40*b^8*c^3*d^2*f^2 + 1120*a^2*b^6*c^3*d^2*f^2 - 2240*a^3*b^5*c^2*d^3*f^2 - 2800*a^4*b^4*c^3*d^2*f^2 + 2240*a^5*b^3*c^2*d^3*f^2 + 1120*a^6*b^2*c^3*d^2*f^2 - 160*a*b^7*c^4*d*f^2 + 160*a^7*b*c^4*d*f^2 + 320*a*b^7*c^2*d^3*f^2 - 560*a^2*b^6*c*d^4*f^2 + 1120*a^3*b^5*c^4*d*f^2 + 1400*a^4*b^4*c*d^4*f^2 - 1120*a^5*b^3*c^4*d*f^2 - 560*a^6*b^2*c*d^4*f^2 - 320*a^7*b*c^2*d^3*f^2)/(16*(c^10*f^4 + d^10*f^4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*c^6*d^4*f^4 + 5*c^8*d^2*f^4)))^(1/2)*(64*c*d^22*f^5 + 640*c^3*d^20*f^5 + 2880*c^5*d^18*f^5 + 7680*c^7*d^16*f^5 + 13440*c^9*d^14*f^5 + 16128*c^11*d^12*f^5 + 13440*c^13*d^10*f^5 + 7680*c^15*d^8*f^5 + 2880*c^17*d^6*f^5 + 640*c^19*d^4*f^5 + 64*c^21*d^2*f^5) - 32*a^4*d^21*f^4 - 32*b^4*d^21*f^4 + 192*a^2*b^2*d^21*f^4 - 160*a^4*c^2*d^19*f^4 - 128*a^4*c^4*d^17*f^4 + 896*a^4*c^6*d^15*f^4 + 3136*a^4*c^8*d^13*f^4 + 4928*a^4*c^10*d^11*f^4 + 4480*a^4*c^12*d^9*f^4 + 2432*a^4*c^14*d^7*f^4 + 736*a^4*c^16*d^5*f^4 + 96*a^4*c^18*d^3*f^4 - 160*b^4*c^2*d^19*f^4 - 128*b^4*c^4*d^17*f^4 + 896*b^4*c^6*d^15*f^4 + 3136*b^4*c^8*d^13*f^4 + 4928*b^4*c^10*d^11*f^4 + 4480*b^4*c^12*d^9*f^4 + 2432*b^4*c^14*d^7*f^4 + 736*b^4*c^16*d^5*f^4 + 96*b^4*c^18*d^3*f^4 + 960*a^2*b^2*c^2*d^19*f^4 + 768*a^2*b^2*c^4*d^17*f^4 - 5376*a^2*b^2*c^6*d^15*f^4 - 18816*a^2*b^2*c^8*d^13*f^4 - 29568*a^2*b^2*c^10*d^11*f^4 - 26880*a^2*b^2*c^12*d^9*f^4 - 14592*a^2*b^2*c^14*d^7*f^4 - 4416*a^2*b^2*c^16*d^5*f^4 - 576*a^2*b^2*c^18*d^3*f^4 - 384*a*b^3*c*d^20*f^4 + 384*a^3*b*c*d^20*f^4 - 2944*a*b^3*c^3*d^18*f^4 - 9728*a*b^3*c^5*d^16*f^4 - 17920*a*b^3*c^7*d^14*f^4 - 19712*a*b^3*c^9*d^12*f^4 - 12544*a*b^3*c^11*d^10*f^4 - 3584*a*b^3*c^13*d^8*f^4 + 512*a*b^3*c^15*d^6*f^4 + 640*a*b^3*c^17*d^4*f^4 + 128*a*b^3*c^19*d^2*f^4 + 2944*a^3*b*c^3*d^18*f^4 + 9728*a^3*b*c^5*d^16*f^4 + 17920*a^3*b*c^7*d^14*f^4 + 19712*a^3*b*c^9*d^12*f^4 + 12544*a^3*b*c^11*d^10*f^4 + 3584*a^3*b*c^13*d^8*f^4 - 512*a^3*b*c^15*d^6*f^4 - 640*a^3*b*c^17*d^4*f^4 - 128*a^3*b*c^19*d^2*f^4) - (c + d*tan(e + f*x))^(1/2)*(448*a^2*b^6*d^18*f^3 - 16*b^8*d^18*f^3 - 16*a^8*d^18*f^3 - 1120*a^4*b^4*d^18*f^3 + 448*a^6*b^2*d^18*f^3 + 320*a^8*c^4*d^14*f^3 + 1024*a^8*c^6*d^12*f^3 + 1440*a^8*c^8*d^10*f^3 + 1024*a^8*c^10*d^8*f^3 + 320*a^8*c^12*d^6*f^3 - 16*a^8*c^16*d^2*f^3 + 320*b^8*c^4*d^14*f^3 + 1024*b^8

```
*c^6*d^12*f^3 + 1440*b^8*c^8*d^10*f^3 + 1024*b^...
```

3.1260 $\int \frac{(a+b\tan(e+fx))^3}{(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=219

$$\frac{(ia+b)^3\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{5/2}f}-\frac{(ia-b)^3\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(c+id)^{5/2}f}-\frac{2(bc-ad)^2(a+b}{3d\left(c^2+d^2\right)f(c+d}$$

```
[Out] (I*a+b)^3*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(c-I*d)^(5/2)/f-(I*
a-b)^3*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(c+I*d)^(5/2)/f-4/3*(-
a*d+b*c)^2*(3*a*c*d+b*(c^2+4*d^2))/d^2/(c^2+d^2)^2/f/(c+d*tan(f*x+e))^(1/2)
-2/3*(-a*d+b*c)^2*(a+b*tan(f*x+e))/d/(c^2+d^2)/f/(c+d*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.46, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3646, 3709, 3620, 3618, 65, 214}

$$-\frac{4(bc-ad)^2\left(3acd+b(c^2+4d^2)\right)}{3d^2f\left(c^2+d^2\right)^2\sqrt{c+d\tan(e+fx)}}-\frac{2(bc-ad)^2(a+b\tan(e+fx))}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}-\frac{(-b+ia)^3\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(c+id)^{5/2}}+\frac{(b+ia)^3\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(c-id)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^3/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((I*a + b)^3*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((c - I*d)^(5
/2)*f) - ((I*a - b)^3*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/((c
+ I*d)^(5/2)*f) - (2*(b*c - a*d)^2*(a + b*Tan[e + f*x]))/(3*d*(c^2 + d^2)*f
*(c + d*Tan[e + f*x])^(3/2)) - (4*(b*c - a*d)^2*(3*a*c*d + b*(c^2 + 4*d^2))
)/(3*d^2*(c^2 + d^2)^2*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
```

```
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^3}{(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{2(bc-ad)^2(a+b\tan(e+fx))}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{2\int \frac{\frac{1}{2}\left(2b^3c^2+3a^3cd-7ab^2cd+8a^2bd^2\right)+\frac{3}{2}d(3a}{}}{}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - \frac{4(bc-ad)^2\left(3acd+b\left(c^2+4d^2\right)\right)}{3d^2\left(c^2+d^2\right)^2 f\sqrt{c+d\tan(e+fx}}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - \frac{4(bc-ad)^2\left(3acd+b\left(c^2+4d^2\right)\right)}{3d^2\left(c^2+d^2\right)^2 f\sqrt{c+d\tan(e+fx}}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - \frac{4(bc-ad)^2\left(3acd+b\left(c^2+4d^2\right)\right)}{3d^2\left(c^2+d^2\right)^2 f\sqrt{c+d\tan(e+fx}}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - \frac{4(bc-ad)^2\left(3acd+b\left(c^2+4d^2\right)\right)}{3d^2\left(c^2+d^2\right)^2 f\sqrt{c+d\tan(e+fx}}$$

$$= \frac{(ia+b)^3\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{5/2}f} - \frac{(ia-b)^3\tanh^{-1}\left(\frac{\sqrt{c+d\tan(}}{\sqrt{c+}}\right.}{(c+id)^{5/2}f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.59, size = 284, normalized size = 1.30

$$-\frac{4b^3c(c^2+d^2)-d(-3a^2bc+b^3c+a^3d-3ab^2d)\left(i(c+id)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{c+d\tan(e+fx)}{c-id}\right)-(ic+d)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{c+d\tan(e+fx)}{c+id}\right)\right)+6b^2(c-id)(c+id)d(a+b\tan(e+fx))-3b(3a^2-b^2)\,d\left(i(c+id)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{c+d\tan(e+fx)}{c-id}\right)-(ic+d)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{c+d\tan(e+fx)}{c+id}\right)\right)(c+d\tan(e+fx))}{3d^2\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^3/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] -1/3*(4*b^3*c*(c^2 + d^2) - d*(-3*a^2*b*c + b^3*c + a^3*d - 3*a*b^2*d)*(I*(
c + I*d)*Hypergeometric2F1[-3/2, 1, -1/2, (c + d*Tan[e + f*x])/(c - I*d)] -
 (I*c + d)*Hypergeometric2F1[-3/2, 1, -1/2, (c + d*Tan[e + f*x])/(c + I*d)]
) + 6*b^2*(c - I*d)*(c + I*d)*d*(a + b*Tan[e + f*x]) - 3*b*(3*a^2 - b^2)*d*
(I*(c + I*d)*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c - I*d)
] - (I*c + d)*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c + I*d
)])*(c + d*Tan[e + f*x]))/(d^2*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 6645 vs. 2(193) = 386.
time = 0.49, size = 6646, normalized size = 30.35

method	result	size
derivativedivides	Expression too large to display	6646
default	Expression too large to display	6646

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3/(c+d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(c+d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(c+d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan{(e + fx)})^3}{(c + d\tan{(e + fx)})^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3/(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**3/(c + d*tan(e + f*x))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(c+d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 24.50, size = 2500, normalized size = 11.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^3/(c + d*tan(e + f*x))^(5/2),x)
```

```
[Out] - atan((((((8*a^6*c^5*f^2 - 8*b^6*c^5*f^2 + 48*a*b^5*d^5*f^2 + 48*a^5*b*d^
5*f^2 + 40*a^6*c*d^4*f^2 - 40*b^6*c*d^4*f^2 + 120*a^2*b^4*c^5*f^2 - 120*a^4
*b^2*c^5*f^2 - 160*a^3*b^3*d^5*f^2 - 80*a^6*c^3*d^2*f^2 + 80*b^6*c^3*d^2*f^
2 - 1200*a^2*b^4*c^3*d^2*f^2 + 1600*a^3*b^3*c^2*d^3*f^2 + 1200*a^4*b^2*c^3*
d^2*f^2 + 240*a*b^5*c^4*d*f^2 + 240*a^5*b*c^4*d*f^2 - 480*a*b^5*c^2*d^3*f^2
 + 600*a^2*b^4*c*d^4*f^2 - 800*a^3*b^3*c^4*d*f^2 - 600*a^4*b^2*c*d^4*f^2 -
480*a^5*b*c^2*d^3*f^2)^2/4 - (16*c^10*f^4 + 16*d^10*f^4 + 80*c^2*d^8*f^4 +
160*c^4*d^6*f^4 + 160*c^6*d^4*f^4 + 80*c^8*d^2*f^4)*(a^12 + b^12 + 6*a^2*b^
10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))^(1/2) - 4*a^6*c^5*
f^2 + 4*b^6*c^5*f^2 - 24*a*b^5*d^5*f^2 - 24*a^5*b*d^5*f^2 - 20*a^6*c*d^4*f^
2 + 20*b^6*c*d^4*f^2 - 60*a^2*b^4*c^5*f^2 + 60*a^4*b^2*c^5*f^2 + 80*a^3*b^3
*d^5*f^2 + 40*a^6*c^3*d^2*f^2 - 40*b^6*c^3*d^2*f^2 + 600*a^2*b^4*c^3*d^2*f^
2 - 800*a^3*b^3*c^2*d^3*f^2 - 600*a^4*b^2*c^3*d^2*f^2 - 120*a*b^5*c^4*d*f^2
 - 120*a^5*b*c^4*d*f^2 + 240*a*b^5*c^2*d^3*f^2 - 300*a^2*b^4*c*d^4*f^2 + 40
0*a^3*b^3*c^4*d*f^2 + 300*a^4*b^2*c*d^4*f^2 + 240*a^5*b*c^2*d^3*f^2)/(16*(c
^10*f^4 + d^10*f^4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*c^6*d^4*f^4 + 5*c^
8*d^2*f^4)))^(1/2)*((c + d*tan(e + f*x))^(1/2)*((((8*a^6*c^5*f^2 - 8*b^6*c^
5*f^2 + 48*a*b^5*d^5*f^2 + 48*a^5*b*d^5*f^2 + 40*a^6*c*d^4*f^2 - 40*b^6*c*d
^4*f^2 + 120*a^2*b^4*c^5*f^2 - 120*a^4*b^2*c^5*f^2 - 160*a^3*b^3*d^5*f^2 -
80*a^6*c^3*d^2*f^2 + 80*b^6*c^3*d^2*f^2 - 1200*a^2*b^4*c^3*d^2*f^2 + 1600*a
^3*b^3*c^2*d^3*f^2 + 1200*a^4*b^2*c^3*d^2*f^2 + 240*a*b^5*c^4*d*f^2 + 240*a
^5*b*c^4*d*f^2 - 480*a*b^5*c^2*d^3*f^2 + 600*a^2*b^4*c*d^4*f^2 - 800*a^3*b^
3*c^4*d*f^2 - 600*a^4*b^2*c*d^4*f^2 - 480*a^5*b*c^2*d^3*f^2)^2/4 - (16*c^10
*f^4 + 16*d^10*f^4 + 80*c^2*d^8*f^4 + 160*c^4*d^6*f^4 + 160*c^6*d^4*f^4 + 8
0*c^8*d^2*f^4)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8
*b^4 + 6*a^10*b^2))^(1/2) - 4*a^6*c^5*f^2 + 4*b^6*c^5*f^2 - 24*a*b^5*d^5*f^
2 - 24*a^5*b*d^5*f^2 - 20*a^6*c*d^4*f^2 + 20*b^6*c*d^4*f^2 - 60*a^2*b^4*c^5
*f^2 + 60*a^4*b^2*c^5*f^2 + 80*a^3*b^3*d^5*f^2 + 40*a^6*c^3*d^2*f^2 - 40*b^
6*c^3*d^2*f^2 + 600*a^2*b^4*c^3*d^2*f^2 - 800*a^3*b^3*c^2*d^3*f^2 - 600*a^4
```

*b^2*c^3*d^2*f^2 - 120*a*b^5*c^4*d*f^2 - 120*a^5*b*c^4*d*f^2 + 240*a*b^5*c^2*d^3*f^2 - 300*a^2*b^4*c*d^4*f^2 + 400*a^3*b^3*c^4*d*f^2 + 300*a^4*b^2*c*d^4*f^2 + 240*a^5*b*c^2*d^3*f^2)/(16*(c^10*f^4 + d^10*f^4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*c^6*d^4*f^4 + 5*c^8*d^2*f^4)))^(1/2)*(64*c*d^22*f^5 + 640*c^3*d^20*f^5 + 2880*c^5*d^18*f^5 + 7680*c^7*d^16*f^5 + 13440*c^9*d^14*f^5 + 16128*c^11*d^12*f^5 + 13440*c^13*d^10*f^5 + 7680*c^15*d^8*f^5 + 2880*c^17*d^6*f^5 + 640*c^19*d^4*f^5 + 64*c^21*d^2*f^5) - 32*a^3*d^21*f^4 + 96*a*b^2*d^21*f^4 - 96*b^3*c*d^20*f^4 - 160*a^3*c^2*d^19*f^4 - 128*a^3*c^4*d^17*f^4 + 896*a^3*c^6*d^15*f^4 + 3136*a^3*c^8*d^13*f^4 + 4928*a^3*c^10*d^11*f^4 + 4480*a^3*c^12*d^9*f^4 + 2432*a^3*c^14*d^7*f^4 + 736*a^3*c^16*d^5*f^4 + 96*a^3*c^18*d^3*f^4 - 736*b^3*c^3*d^18*f^4 - 2432*b^3*c^5*d^16*f^4 - 4480*b^3*c^7*d^14*f^4 - 4928*b^3*c^9*d^12*f^4 - 3136*b^3*c^11*d^10*f^4 - 896*b^3*c^13*d^8*f^4 + 128*b^3*c^15*d^6*f^4 + 160*b^3*c^17*d^4*f^4 + 32*b^3*c^19*d^2*f^4 + 288*a^2*b*c*d^20*f^4 + 480*a*b^2*c^2*d^19*f^4 + 384*a*b^2*c^4*d^17*f^4 - 2688*a*b^2*c^6*d^15*f^4 - 9408*a*b^2*c^8*d^13*f^4 - 14784*a*b^2*c^10*d^11*f^4 - 13440*a*b^2*c^12*d^9*f^4 - 7296*a*b^2*c^14*d^7*f^4 - 2208*a*b^2*c^16*d^5*f^4 - 288*a*b^2*c^18*d^3*f^4 + 2208*a^2*b*c^3*d^18*f^4 + 7296*a^2*b*c^5*d^16*f^4 + 13440*a^2*b*c^7*d^14*f^4 + 14784*a^2*b*c^9*d^12*f^4 + 9408*a^2*b*c^11*d^10*f^4 + 2688*a^2*b*c^13*d^8*f^4 - 384*a^2*b*c^15*d^6*f^4 - 480*a^2*b*c^17*d^4*f^4 - 96*a^2*b*c^19*d^2*f^4) + (c + d*tan(e + f*x))^(1/2)*(16*a^6*d^18*f^3 - 16*b^6*d^18*f^3 + 240*a^2*b^4*d^18*f^3 - 240*a^4*b^2*d^18*f^3 - 320*a^6*c^4*d^14*f^3 - 1024*a^6*c^6*d^12*f^3 - 1440*a^6*c^8*d^10*f^3 - 1024*a^6*c^10*d^8*f^3 - 320*a^6*c^12*d^6*f^3 + 16*a^6*c^16*d^2*f^3 + 320*b^6*c^4*d^14*f^3 + 1024*b^6*c^6*d^12*f^3 + 1440*b^6*c^8*d^10*f^3 + 1024*b^6*c^10*d^8*f^3 + 320*b^6*c^12*d^6*f^3 - 16*b^6*c^16*d^2*f^3 - 4800*a^2*b^4*c^4*d^14*f^3 - 15360*a^2*b^4*c^6*d^12*f^3 - 21600*a^2*b^4*c^8*d^10*f^3 - 15360*a^2*b^4*c^10*d^8*f^3 - 4800*a^2*b^4*c^12*d^6*f^3 + 240*a^2*b^4*c^16*d^2*f^3 + 6400*a^3*b^3*c^3*d^15*f^3 + 11520*a^3*b^3*c^5*d^13*f^3 + 6400*a^3*b^3*c^7*d^11*f^3 - 6400*a^3*b^3*c^9*d^9*f^3 - 11520*a^3*b^3*c^11*d^7*f^3 - 6400*a^3*b^3*c^13*d^5*f^3 - 1280*a^3*b^3*c^15*d^3*f^3 + 4800*a^4*b^2*c^4*d^14*f^3 + 15360*a^4*b^2*c^6*d^12*f^3 + 21600*a^4*b^2*c^8*d^10*f^3 + 15360*a^4*b^2*c^10*d^8*f^3 + 4800*a^4*b^2*c^12*d^6*f^3 - 240*a^4*b^2*c^16*d^2*f^3 - 384*a*b^5*c*d^17*f^3 - 384*a^5*b*c*d^17*f^3 - 1920*a*b^5*c^3*d^15*f^3 - 3456*a*b^5*c^5*d^13*f^3 - 1920*a*b^5*c^7*d^11*f^3 + 1920*a*b^5*c^9*d^9*f^3 + 3456*a*b^5*c^11*d^7*f^3 + 1920*a*b^5*c^13*d^5*f^3 + 384*a*b^5*c^15*d^3*f^3 + 1280*a^3*b^3*c*d^17*f^3 - 1920*a^5*b*c^3*d^15*f^3 - 3456*a^5*b*c^5*d^13*f^3 - 1920*a^5*b*c^7*d^11*f^3 + 1920*a^5*b*c^9*d^9*f^3 + 3456*a^5*b*c^11*d^7*f^3 + 1920*a^5*b*c^13*d^5*f^3 + 384*a^5*b*c^15*d...

3.1261 $\int \frac{(a+b\tan(e+fx))^2}{(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=195

$$-\frac{i(a-ib)^2\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{5/2}f}+\frac{i(a+ib)^2\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(c+id)^{5/2}f}-\frac{2(bc}{3d\left(c^2+d^2\right)f(c}$$

[Out] -I*(a-I*b)^2*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(c-I*d)^(5/2)/f+I*(a+I*b)^2*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(c+I*d)^(5/2)/f+4*(-a*d+b*c)*(a*c+b*d)/(c^2+d^2)^2/f/(c+d*tan(f*x+e))^(1/2)-2/3*(-a*d+b*c)^2/d/(c^2+d^2)/f/(c+d*tan(f*x+e))^(3/2)

Rubi [A]
time = 0.34, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3623, 3610, 3620, 3618, 65, 214}

$$\frac{4(bc-ad)(ac+bd)}{f\left(c^2+d^2\right)^2\sqrt{c+d\tan(e+fx)}}-\frac{2(bc-ad)^2}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}-\frac{i(a-ib)^2\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(c-id)^{5/2}}+\frac{i(a+ib)^2\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(c+id)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Tan[e + f*x])^2/(c + d*Tan[e + f*x])^(5/2),x]

[Out] ((-I)*(a - I*b)^2*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((c - I*d)^(5/2)*f) + (I*(a + I*b)^2*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/((c + I*d)^(5/2)*f) - (2*(b*c - a*d)^2)/(3*d*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2)) + (4*(b*c - a*d)*(a*c + b*d))/((c^2 + d^2)^2*f*Sqrt[c + d*Tan[e + f*x]])

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^2}{(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{2(bc-ad)^2}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} + \frac{\int \frac{a^2c-b^2c+2abd+(2abc-a^2d+b^2d)\tan(e+fx)}{(c+d\tan(e+fx))^{3/2}}}{c^2+d^2}$$

$$= -\frac{2(bc-ad)^2}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} + \frac{4(bc-ad)(ac+bd)}{(c^2+d^2)^2\,f\sqrt{c+d\tan(e+fx)}} +$$

$$= -\frac{2(bc-ad)^2}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} + \frac{4(bc-ad)(ac+bd)}{(c^2+d^2)^2\,f\sqrt{c+d\tan(e+fx)}} +$$

$$= -\frac{2(bc-ad)^2}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} + \frac{4(bc-ad)(ac+bd)}{(c^2+d^2)^2\,f\sqrt{c+d\tan(e+fx)}} +$$

$$= -\frac{2(bc-ad)^2}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} + \frac{4(bc-ad)(ac+bd)}{(c^2+d^2)^2\,f\sqrt{c+d\tan(e+fx)}} -$$

$$= -\frac{i(a-ib)^2\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{5/2}f} + \frac{i(a+ib)^2\tanh^{-1}\left(\frac{\sqrt{c+d\,\mathrm{t}}}{\sqrt{c}}\right.}{(c+id)^{5/2}f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.22, size = 127, normalized size = 0.65

$$\frac{-\frac{2b^2}{d} - \frac{(a-ib)^2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{c+d\tan(e+fx)}{c-id}\right)}{ic+d} + \frac{(a+ib)^2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{c+d\tan(e+fx)}{c+id}\right)}{ic-d}}{3f(c+d\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-2*b^2)/d - ((a - I*b)^2*Hypergeometric2F1[-3/2, 1, -1/2, (c + d*Tan[e +
f*x])/(c - I*d)])/(I*c + d) + ((a + I*b)^2*Hypergeometric2F1[-3/2, 1, -1/2,
 (c + d*Tan[e + f*x])/(c + I*d)])/(I*c - d))/(3*f*(c + d*Tan[e + f*x])^(3/2
))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5087 vs. 2(169) = 338.
time = 0.51, size = 5088, normalized size = 26.09

method	result	size

derivativedivides	Expression too large to display	5088
default	Expression too large to display	5088

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan(e + fx))^2}{(c + d\tan(e + fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2/(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**2/(c + d*tan(e + f*x))**(5/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 22.06, size = 2500, normalized size = 12.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^2/(c + d*tan(e + f*x))^(5/2),x)
```

```
[Out] - atan(-(((c + d*tan(e + f*x))^(1/2)*(96*a^2*b^2*d^18*f^3 - 16*b^4*d^18*f^3
 - 16*a^4*d^18*f^3 + 320*a^4*c^4*d^14*f^3 + 1024*a^4*c^6*d^12*f^3 + 1440*a^
4*c^8*d^10*f^3 + 1024*a^4*c^10*d^8*f^3 + 320*a^4*c^12*d^6*f^3 - 16*a^4*c^16
*d^2*f^3 + 320*b^4*c^4*d^14*f^3 + 1024*b^4*c^6*d^12*f^3 + 1440*b^4*c^8*d^10
*f^3 + 1024*b^4*c^10*d^8*f^3 + 320*b^4*c^12*d^6*f^3 - 16*b^4*c^16*d^2*f^3 -
 1920*a^2*b^2*c^4*d^14*f^3 - 6144*a^2*b^2*c^6*d^12*f^3 - 8640*a^2*b^2*c^8*d
^10*f^3 - 6144*a^2*b^2*c^10*d^8*f^3 - 1920*a^2*b^2*c^12*d^6*f^3 + 96*a^2*b^
2*c^16*d^2*f^3 - 256*a*b^3*c*d^17*f^3 + 256*a^3*b*c*d^17*f^3 - 1280*a*b^3*c
^3*d^15*f^3 - 2304*a*b^3*c^5*d^13*f^3 - 1280*a*b^3*c^7*d^11*f^3 + 1280*a*b^
3*c^9*d^9*f^3 + 2304*a*b^3*c^11*d^7*f^3 + 1280*a*b^3*c^13*d^5*f^3 + 256*a*b
^3*c^15*d^3*f^3 + 1280*a^3*b*c^3*d^15*f^3 + 2304*a^3*b*c^5*d^13*f^3 + 1280*
a^3*b*c^7*d^11*f^3 - 1280*a^3*b*c^9*d^9*f^3 - 2304*a^3*b*c^11*d^7*f^3 - 128
0*a^3*b*c^13*d^5*f^3 - 256*a^3*b*c^15*d^3*f^3) + ((((8*a^4*c^5*f^2 + 8*b^4*
c^5*f^2 - 32*a*b^3*d^5*f^2 + 32*a^3*b*d^5*f^2 + 40*a^4*c*d^4*f^2 + 40*b^4*c
*d^4*f^2 - 48*a^2*b^2*c^5*f^2 - 80*a^4*c^3*d^2*f^2 - 80*b^4*c^3*d^2*f^2 + 4
80*a^2*b^2*c^3*d^2*f^2 - 160*a*b^3*c^4*d*f^2 + 160*a^3*b*c^4*d*f^2 + 320*a*
b^3*c^2*d^3*f^2 - 240*a^2*b^2*c*d^4*f^2 - 320*a^3*b*c^2*d^3*f^2)^2/4 - (a^8
 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)*(16*c^10*f^4 + 16*d^10*f^4 + 80
*c^2*d^8*f^4 + 160*c^4*d^6*f^4 + 160*c^6*d^4*f^4 + 80*c^8*d^2*f^4))^(1/2) -
 4*a^4*c^5*f^2 - 4*b^4*c^5*f^2 + 16*a*b^3*d^5*f^2 - 16*a^3*b*d^5*f^2 - 20*a
^4*c*d^4*f^2 - 20*b^4*c*d^4*f^2 + 24*a^2*b^2*c^5*f^2 + 40*a^4*c^3*d^2*f^2 +
 40*b^4*c^3*d^2*f^2 - 240*a^2*b^2*c^3*d^2*f^2 + 80*a*b^3*c^4*d*f^2 - 80*a^3
*b*c^4*d*f^2 - 160*a*b^3*c^2*d^3*f^2 + 120*a^2*b^2*c*d^4*f^2 + 160*a^3*b*c^
2*d^3*f^2)/(16*(c^10*f^4 + d^10*f^4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*c
^6*d^4*f^4 + 5*c^8*d^2*f^4)))^(1/2)*(32*b^2*d^21*f^4 - 32*a^2*d^21*f^4 - (c
 + d*tan(e + f*x))^(1/2)*((((8*a^4*c^5*f^2 + 8*b^4*c^5*f^2 - 32*a*b^3*d^5*f
^2 + 32*a^3*b*d^5*f^2 + 40*a^4*c*d^4*f^2 + 40*b^4*c*d^4*f^2 - 48*a^2*b^2*c^
5*f^2 - 80*a^4*c^3*d^2*f^2 - 80*b^4*c^3*d^2*f^2 + 480*a^2*b^2*c^3*d^2*f^2 -
 160*a*b^3*c^4*d*f^2 + 160*a^3*b*c^4*d*f^2 + 320*a*b^3*c^2*d^3*f^2 - 240*a^
2*b^2*c*d^4*f^2 - 320*a^3*b*c^2*d^3*f^2)^2/4 - (a^8 + b^8 + 4*a^2*b^6 + 6*a
^4*b^4 + 4*a^6*b^2)*(16*c^10*f^4 + 16*d^10*f^4 + 80*c^2*d^8*f^4 + 160*c^4*d
```

^6*f^4 + 160*c^6*d^4*f^4 + 80*c^8*d^2*f^4))^(1/2) - 4*a^4*c^5*f^2 - 4*b^4*c
^5*f^2 + 16*a*b^3*d^5*f^2 - 16*a^3*b*d^5*f^2 - 20*a^4*c*d^4*f^2 - 20*b^4*c*
d^4*f^2 + 24*a^2*b^2*c^5*f^2 + 40*a^4*c^3*d^2*f^2 + 40*b^4*c^3*d^2*f^2 - 24
0*a^2*b^2*c^3*d^2*f^2 + 80*a*b^3*c^4*d*f^2 - 80*a^3*b*c^4*d*f^2 - 160*a*b^3
*c^2*d^3*f^2 + 120*a^2*b^2*c*d^4*f^2 + 160*a^3*b*c^2*d^3*f^2)/(16*(c^10*f^4
 + d^10*f^4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*c^6*d^4*f^4 + 5*c^8*d^2*f
^4)))^(1/2)*(64*c*d^22*f^5 + 640*c^3*d^20*f^5 + 2880*c^5*d^18*f^5 + 7680*c^
7*d^16*f^5 + 13440*c^9*d^14*f^5 + 16128*c^11*d^12*f^5 + 13440*c^13*d^10*f^5
 + 7680*c^15*d^8*f^5 + 2880*c^17*d^6*f^5 + 640*c^19*d^4*f^5 + 64*c^21*d^2*f
^5) - 160*a^2*c^2*d^19*f^4 - 128*a^2*c^4*d^17*f^4 + 896*a^2*c^6*d^15*f^4 +
3136*a^2*c^8*d^13*f^4 + 4928*a^2*c^10*d^11*f^4 + 4480*a^2*c^12*d^9*f^4 + 24
32*a^2*c^14*d^7*f^4 + 736*a^2*c^16*d^5*f^4 + 96*a^2*c^18*d^3*f^4 + 160*b^2*
c^2*d^19*f^4 + 128*b^2*c^4*d^17*f^4 - 896*b^2*c^6*d^15*f^4 - 3136*b^2*c^8*d
^13*f^4 - 4928*b^2*c^10*d^11*f^4 - 4480*b^2*c^12*d^9*f^4 - 2432*b^2*c^14*d^
7*f^4 - 736*b^2*c^16*d^5*f^4 - 96*b^2*c^18*d^3*f^4 + 192*a*b*c*d^20*f^4 + 1
472*a*b*c^3*d^18*f^4 + 4864*a*b*c^5*d^16*f^4 + 8960*a*b*c^7*d^14*f^4 + 9856
*a*b*c^9*d^12*f^4 + 6272*a*b*c^11*d^10*f^4 + 1792*a*b*c^13*d^8*f^4 - 256*a*
b*c^15*d^6*f^4 - 320*a*b*c^17*d^4*f^4 - 64*a*b*c^19*d^2*f^4))*((((8*a^4*c^5
*f^2 + 8*b^4*c^5*f^2 - 32*a*b^3*d^5*f^2 + 32*a^3*b*d^5*f^2 + 40*a^4*c*d^4*f
^2 + 40*b^4*c*d^4*f^2 - 48*a^2*b^2*c^5*f^2 - 80*a^4*c^3*d^2*f^2 - 80*b^4*c^
3*d^2*f^2 + 480*a^2*b^2*c^3*d^2*f^2 - 160*a*b^3*c^4*d*f^2 + 160*a^3*b*c^4*d
*f^2 + 320*a*b^3*c^2*d^3*f^2 - 240*a^2*b^2*c*d^4*f^2 - 320*a^3*b*c^2*d^3*f^
2)^2/4 - (a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)*(16*c^10*f^4 + 16*
d^10*f^4 + 80*c^2*d^8*f^4 + 160*c^4*d^6*f^4 + 160*c^6*d^4*f^4 + 80*c^8*d^2*
f^4))^(1/2) - 4*a^4*c^5*f^2 - 4*b^4*c^5*f^2 + 16*a*b^3*d^5*f^2 - 16*a^3*b*d
^5*f^2 - 20*a^4*c*d^4*f^2 - 20*b^4*c*d^4*f^2 + 24*a^2*b^2*c^5*f^2 + 40*a^4*
c^3*d^2*f^2 + 40*b^4*c^3*d^2*f^2 - 240*a^2*b^2*c^3*d^2*f^2 + 80*a*b^3*c^4*d
*f^2 - 80*a^3*b*c^4*d*f^2 - 160*a*b^3*c^2*d^3*f^2 + 120*a^2*b^2*c*d^4*f^2 +
 160*a^3*b*c^2*d^3*f^2)/(16*(c^10*f^4 + d^10*f^4 + 5*c^2*d^8*f^4 + 10*c^4*d
^6*f^4 + 10*c^6*d^4*f^4 + 5*c^8*d^2*f^4)))^(1/2)*1i + ((c + d*tan(e + f*x))
^(1/2)*(96*a^2*b^2*d^18*f^3 - 16*b^4*d^18*f^3 - 16*a^4*d^18*f^3 + 320*a^4*c
^4*d^14*f^3 + 1024*a^4*c^6*d^12*f^3 + 1440*a^4*c^8*d^10*f^3 + 1024*a^4*c^10
*d^8*f^3 + 320*a^4*c^12*d^6*f^3 - 16*a^4*c^16*d^2*f^3 + 320*b^4*c^4*d^14*f^
3 + 1024*b^4*c^6*d^12*f^3 + 1440*b^4*c^8*d^10*f^3 + 1024*b^4*c^10*d^8*f^3 +
 320*b^4*c^12*d^6*f^3 - 16*b^4*c^16*d^2*f^3 - 1920*a^2*b^2*c^4*d^14*f^3 - 6
144*a^2*b^2*c^6*d^12*f^3 - 8640*a^2*b^2*c^8*d^1...

3.1262 $\int \frac{a+b\tan(e+fx)}{(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=186

$$-\frac{(ia+b)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{5/2}f}+\frac{(ia-b)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(c+id)^{5/2}f}+\frac{2(bc-c}{3(c^2+d^2)f(c+d\,t}$$

```
[Out] -(I*a+b)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(c-I*d)^(5/2)/f+(I*a
-b)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(c+I*d)^(5/2)/f-2*(2*a*c*
d-b*(c^2-d^2))/(c^2+d^2)^2/f/(c+d*tan(f*x+e))^(1/2)+2/3*(-a*d+b*c)/(c^2+d^2
)/f/(c+d*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.28, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3610, 3620, 3618, 65, 214}

$$\frac{2(bc-ad)}{3f(c^2+d^2)(c+d\tan(e+fx))^{3/2}}-\frac{2(2acd-b(c^2-d^2))}{f(c^2+d^2)^2\sqrt{c+d\tan(e+fx)}}-\frac{(b+ia)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(c-id)^{5/2}}+\frac{(-b+ia)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(c+id)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] -(((I*a + b)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((c - I*d)^(5
/2)*f)) + ((I*a - b)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/((c +
 I*d)^(5/2)*f) + (2*(b*c - a*d))/(3*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2
)) - (2*(2*a*c*d - b*(c^2 - d^2)))/((c^2 + d^2)^2*f*Sqrt[c + d*Tan[e + f*x]
])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
```

```
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{a+b\tan(e+fx)}{(c+d\tan(e+fx))^{5/2}}\,dx = \frac{2(bc-ad)}{3\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{\int \frac{ac+bd+(bc-ad)\tan(e+fx)}{(c+d\tan(e+fx))^{3/2}}\,dx}{c^2+d^2}$$

$$= \frac{2(bc-ad)}{3\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - \frac{2(2acd-b(c^2-d^2))}{\left(c^2+d^2\right)^2 f\sqrt{c+d\tan(e+fx)}} + \ldots$$

$$= \frac{2(bc-ad)}{3\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - \frac{2(2acd-b(c^2-d^2))}{\left(c^2+d^2\right)^2 f\sqrt{c+d\tan(e+fx)}} + \frac{(a \ldots}{\ldots}$$

$$= \frac{2(bc-ad)}{3\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - \frac{2(2acd-b(c^2-d^2))}{\left(c^2+d^2\right)^2 f\sqrt{c+d\tan(e+fx)}} + \frac{(ia \ldots}{\ldots}$$

$$= \frac{2(bc-ad)}{3\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - \frac{2(2acd-b(c^2-d^2))}{\left(c^2+d^2\right)^2 f\sqrt{c+d\tan(e+fx)}} + \frac{(a \ldots}{\ldots}$$

$$= -\frac{(ia+b)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{5/2}f} + \frac{(ia-b)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e \ldots}}{\sqrt{c+i \ldots}}\right)}{(c+id)^{5/2}f} \ldots$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.18, size = 115, normalized size = 0.62

$$-\frac{i\left(-\frac{(a-ib)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{c+d\tan(e+fx)}{c-id}\right)}{c-id}+\frac{(a+ib)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{c+d\tan(e+fx)}{c+id}\right)}{c+id}\right)}{3f(c+d\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-1/3*I)*(-(((a - I*b)*Hypergeometric2F1[-3/2, 1, -1/2, (c + d*Tan[e + f*x
])/(c - I*d)])/(c - I*d)) + ((a + I*b)*Hypergeometric2F1[-3/2, 1, -1/2, (c
+ d*Tan[e + f*x])/(c + I*d)])/(c + I*d)))/(f*(c + d*Tan[e + f*x])^(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3235 vs. $2(162) = 324$.
time = 0.51, size = 3236, normalized size = 17.40

method	result	size
derivativedivides	Expression too large to display	3236
default	Expression too large to display	3236

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))/(c+d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-2/3*(a*d-b*c)/(c^2+d^2)/(c+d*tan(f*x+e))^(3/2)-2*(2*a*c*d-b*c^2+b*d^2
)/(c^2+d^2)^2/(c+d*tan(f*x+e))^(1/2)+2/(c^2+d^2)^2*(1/4/d/(5*c^4-10*c^2*d^2
+d^4)/(c^2+d^2)^(3/2)*(1/2*(-3*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2
)*a*c^6-5*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^4*d^2-(c^2+d^2)
^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2*d^4+(c^2+d^2)^(3/2)*(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*a*d^6-2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c
^8+18*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^6*d^2+10*(c^2+d^2)^
(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^4*d^4-10*(c^2+d^2)^(1/2)*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*a*c^2*d^6-10*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)*b*c^7*d+10*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^5*d^3+18*(
c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^3*d^5-2*(c^2+d^2)^(1/2)*(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d^7+5*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^9-2
0*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^7*d^2+6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a
*c^5*d^4+28*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3*d^6-3*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*a*c*d^8+15*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^8*d-20*(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)*b*c^6*d^3-22*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^4*d^5+12*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d^7-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^9)*l
n(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+
d^2)^(1/2))+2*(30*a*c^8*d^2-40*a*c^6*d^4-44*a*c^4*d^6+24*a*c^2*d^8-2*a*d^10
-10*b*c^9*d+40*b*c^7*d^3-12*b*c^5*d^5-56*b*c^3*d^7+6*b*c*d^9+1/2*(-3*(c^2+d
^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^6-5*(c^2+d^2)^(3/2)*(2*(c^2+d^2
```

)^(1/2)+2*c)^(1/2)*a*c^4*d^2-(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*
a*c^2*d^4+(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^6-2*(c^2+d^2)^(
1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^8+18*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*a*c^6*d^2+10*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a
*c^4*d^4-10*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2*d^6-10*(c^2
+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^7*d+10*(c^2+d^2)^(1/2)*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)*b*c^5*d^3+18*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*b*c^3*d^5-2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d^7+5
(2(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^9-20*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^7
d^2+6(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^5*d^4+28*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*a*c^3*d^6-3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d^8+15*(2*(c^2+d^2)^(1/2
)+2*c)^(1/2)*b*c^8*d-20*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^6*d^3-22*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*b*c^4*d^5+12*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d^7-
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^9)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2
+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d^2)^(1/2)+
2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/4/d/(5*c^4-10*c^2*d^2+d^4)/(c
^2+d^2)^(3/2)*(1/2*(3*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^6+5
(c^2+d^2)^(3/2)(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^4*d^2+(c^2+d^2)^(3/2)*(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2*d^4-(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*a*d^6+2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^8-18*(c^
2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^6*d^2-10*(c^2+d^2)^(1/2)*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^4*d^4+10*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+
2*c)^(1/2)*a*c^2*d^6+10*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^7
d-10(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^5*d^3-18*(c^2+d^2)^
(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^3*d^5+2*(c^2+d^2)^(1/2)*(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)*b*c*d^7-5*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^9+20*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*a*c^7*d^2-6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^5*d^4-
28*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3*d^6+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*
a*c*d^8-15*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^8*d+20*(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)*b*c^6*d^3+22*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^4*d^5-12*(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*b*c^2*d^7+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^9)*ln(d*tan(f
*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2
))+2*(30*a*c^8*d^2-40*a*c^6*d^4-44*a*c^4*d^6+24*a*c^2*d^8-2*a*d^10-10*b*c^9
*d+40*b*c^7*d^3-12*b*c^5*d^5-56*b*c^3*d^7+6*b*c*d^9-1/2*(3*(c^2+d^2)^(3/2)*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^6+5*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*a*c^4*d^2+(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2*d^4-
(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^6+2*(c^2+d^2)^(1/2)*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*a*c^8-18*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*a*c^6*d^2-10*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^4*d^4+1
0*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2*d^6+10*(c^2+d^2)^(1/2
)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^7*d-10*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*b*c^5*d^3-18*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*
c^3*d^5+2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d^7-5*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*a*c^9+20*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^7*d^2-6*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^5*d^4-28*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3

```
*d^6+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d^8-15...
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(c+d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-c>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 35738 vs. $2(161) = 322$.
time = 148.69, size = 35738, normalized size = 192.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(c+d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(12*sqrt(2)*((c^18 + c^16*d^2 - 20*c^14*d^4 - 84*c^12*d^6 - 154*c^10*
d^8 - 154*c^8*d^10 - 84*c^6*d^12 - 20*c^4*d^14 + c^2*d^16 + d^18)*f^5*cos(f
*x + e)^4 + 2*(3*c^16*d^2 + 20*c^14*d^4 + 56*c^12*d^6 + 84*c^10*d^8 + 70*c^
8*d^10 + 28*c^6*d^12 - 4*c^2*d^16 - d^18)*f^5*cos(f*x + e)^2 + (c^14*d^4 +
7*c^12*d^6 + 21*c^10*d^8 + 35*c^8*d^10 + 35*c^6*d^12 + 21*c^4*d^14 + 7*c^2*
d^16 + d^18)*f^5 + 4*((c^17*d + 6*c^15*d^3 + 14*c^13*d^5 + 14*c^11*d^7 - 14
*c^7*d^11 - 14*c^5*d^13 - 6*c^3*d^15 - c*d^17)*f^5*cos(f*x + e)^3 + (c^15*d
^3 + 7*c^13*d^5 + 21*c^11*d^7 + 35*c^9*d^9 + 35*c^7*d^11 + 21*c^5*d^13 + 7*
c^3*d^15 + c*d^17)*f^5*cos(f*x + e))*sin(f*x + e))*sqrt(((a^4 + 2*a^2*b^2 +
 b^4)*c^10 + 5*(a^4 + 2*a^2*b^2 + b^4)*c^8*d^2 + 10*(a^4 + 2*a^2*b^2 + b^4)
*c^6*d^4 + 10*(a^4 + 2*a^2*b^2 + b^4)*c^4*d^6 + 5*(a^4 + 2*a^2*b^2 + b^4)*c
^2*d^8 + (a^4 + 2*a^2*b^2 + b^4)*d^10 + (10*a*b*c^14*d + 30*a*b*c^12*d^3 +
2*a*b*c^10*d^5 - 90*a*b*c^8*d^7 - 130*a*b*c^6*d^9 - 70*a*b*c^4*d^11 - 10*a*
b*c^2*d^13 + 2*a*b*d^15 + (a^2 - b^2)*c^15 - 5*(a^2 - b^2)*c^13*d^2 - 35*(a
^2 - b^2)*c^11*d^4 - 65*(a^2 - b^2)*c^9*d^6 - 45*(a^2 - b^2)*c^7*d^8 + (a^2
 - b^2)*c^5*d^10 + 15*(a^2 - b^2)*c^3*d^12 + 5*(a^2 - b^2)*c*d^14)*f^2*sqrt
((a^4 + 2*a^2*b^2 + b^4)/((c^10 + 5*c^8*d^2 + 10*c^6*d^4 + 10*c^4*d^6 + 5*c
^2*d^8 + d^10)*f^4)))/(4*a^2*b^2*c^10 - 20*(a^3*b - a*b^3)*c^9*d + 5*(5*a^4
 - 26*a^2*b^2 + 5*b^4)*c^8*d^2 + 240*(a^3*b - a*b^3)*c^7*d^3 - 20*(5*a^4 -
32*a^2*b^2 + 5*b^4)*c^6*d^4 - 504*(a^3*b - a*b^3)*c^5*d^5 + 10*(11*a^4 - 62
*a^2*b^2 + 11*b^4)*c^4*d^6 + 240*(a^3*b - a*b^3)*c^3*d^7 - 20*(a^4 - 7*a^2*
b^2 + b^4)*c^2*d^8 - 20*(a^3*b - a*b^3)*c*d^9 + (a^4 - 2*a^2*b^2 + b^4)*d^1
```

0))*sqrt((4*a^2*b^2*c^10 - 20*(a^3*b - a*b^3)*c^9*d + 5*(5*a^4 - 26*a^2*b^2 + 5*b^4)*c^8*d^2 + 240*(a^3*b - a*b^3)*c^7*d^3 - 20*(5*a^4 - 32*a^2*b^2 + 5*b^4)*c^6*d^4 - 504*(a^3*b - a*b^3)*c^5*d^5 + 10*(11*a^4 - 62*a^2*b^2 + 11 *b^4)*c^4*d^6 + 240*(a^3*b - a*b^3)*c^3*d^7 - 20*(a^4 - 7*a^2*b^2 + b^4)*c^ 2*d^8 - 20*(a^3*b - a*b^3)*c*d^9 + (a^4 - 2*a^2*b^2 + b^4)*d^10)/((c^20 + 1 0*c^18*d^2 + 45*c^16*d^4 + 120*c^14*d^6 + 210*c^12*d^8 + 252*c^10*d^10 + 21 0*c^8*d^12 + 120*c^6*d^14 + 45*c^4*d^16 + 10*c^2*d^18 + d^20)*f^4))*((a^4 + 2*a^2*b^2 + b^4)/((c^10 + 5*c^8*d^2 + 10*c^6*d^4 + 10*c^4*d^6 + 5*c^2*d^8 + d^10)*f^4))^(3/4)*arctan(-((2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*c^2 1 - 5*(a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*c^20*d - 4*(a^7*b + 3*a^5*b^3 + 3 *a^3*b^5 + a*b^7)*c^19*d^2 - 30*(a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*c^18*d^ 3 - 94*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*c^17*d^4 - 61*(a^8 + 2*a^6*b ^2 - 2*a^2*b^6 - b^8)*c^16*d^5 - 368*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*c^15*d^6 - 8*(a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*c^14*d^7 - 700*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*c^13*d^8 + 182*(a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*c^12*d^9 - 728*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*c^11*d^10 + 3 64*(a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*c^10*d^11 - 364*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*c^9*d^12 + 350*(a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*c^8* d^13 + 16*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*c^7*d^14 + 184*(a^8 + 2*a ^6*b^2 - 2*a^2*b^6 - b^8)*c^6*d^15 + 122*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a *b^7)*c^5*d^16 + 47*(a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*c^4*d^17 + 60*(a^7* b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*c^3*d^18 + 2*(a^8 + 2*a^6*b^2 - 2*a^2*b^ 6 - b^8)*c^2*d^19 + 10*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*c*d^20 - (a^ 8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^21)*f^4*sqrt((4*a^2*b^2*c^10 - 20*(a^3*b - a*b^3)*c^9*d + 5*(5*a^4 - 26*a^2*b^2 + 5*b^4)*c^8*d^2 + 240*(a^3*b - a*b ^3)*c^7*d^3 - 20*(5*a^4 - 32*a^2*b^2 + 5*b^4)*c^6*d^4 - 504*(a^3*b - a*b^3) *c^5*d^5 + 10*(11*a^4 - 62*a^2*b^2 + 11*b^4)*c^4*d^6 + 240*(a^3*b - a*b^3)* c^3*d^7 - 20*(a^4 - 7*a^2*b^2 + b^4)*c^2*d^8 - 20*(a^3*b - a*b^3)*c*d^9 + (a^4 - 2*a^2*b^2 + b^4)*d^10)/((c^20 + 10*c^18*d^2 + 45*c^16*d^4 + 120*c^14* d^6 + 210*c^12*d^8 + 252*c^10*d^10 + 210*c^8*d^12 + 120*c^6*d^14 + 45*c^4*d ^16 + 10*c^2*d^18 + d^20)*f^4))*sqrt((a^4 + 2*a^2*b^2 + b^4)/((c^10 + 5*c^8 *d^2 + 10*c^6*d^4 + 10*c^4*d^6 + 5*c^2*d^8 + d^10)*f^4)) + (2*(a^9*b + 4*a^ 7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*c^16 - 5*(a^10 + 3*a^8*b^2 + 2*a^6*b ^4 - 2*a^4*b^6 - 3*a^2*b^8 - b^10)*c^15*d - 10*(a^9*b + 4*a^7*b^3 + 6*a^5*b ^5 + 4*a^3*b^7 + a*b^9)*c^14*d^2 - 15*(a^10 + 3*a^8*b^2 + 2*a^6*b^4 - 2*a^4 *b^6 - 3*a^2*b^8 - b^10)*c^13*d^3 - 70*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a ^3*b^7 + a*b^9)*c^12*d^4 - (a^10 + 3*a^8*b^2 + 2*a^6*b^4 - 2*a^4*b^6 - 3*a^ 2*b^8 - b^10)*c^11*d^5 - 130*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a *b^9)*c^10*d^6 + 45*(a^10 + 3*a^8*b^2 + 2*a^6*b^4 - 2*a^4*b^6 - 3*a^2*b^8 - b^10)*c^9*d^7 - 90*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*c^8 *d^8 + 65*(a^10 + 3*a^8*b^2 + 2*a^6*b^4 - 2*a^4*b^6 - 3*a^2*b^8 - b^10)*c^7 *d^9 + 2*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*c^6*d^10 + 35* (a^10 + 3*a^8*b^2 + 2*a^6*b^4 - 2*a^4*b^6 - 3*a^2*b^8 - b^10)*c^5*d^11 + 30 *(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*c^4*d^12 + 5*(a^10 + 3 *a^8*b^2 + 2*a^6*b^4 - 2*a^4*b^6 - 3*a^2*b^8 - ...

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\tan(e + fx)}{(c + d\tan(e + fx))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))/(c + d*tan(e + f*x))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(c+d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 22.93, size = 2500, normalized size = 13.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))/(c + d*tan(e + f*x))^(5/2),x)
```

```
[Out] (log(8*b^3*d^16*f^2 - ((((320*b^4*c^2*d^8*f^4 - 16*b^4*d^10*f^4 - 1760*b^4*
c^4*d^6*f^4 + 1600*b^4*c^6*d^4*f^4 - 400*b^4*c^8*d^2*f^4)^(1/2) + 4*b^2*c^5
*f^2 + 20*b^2*c*d^4*f^2 - 40*b^2*c^3*d^2*f^2)/(c^10*f^4 + d^10*f^4 + 5*c^2*
d^8*f^4 + 10*c^4*d^6*f^4 + 10*c^6*d^4*f^4 + 5*c^8*d^2*f^4))^(1/2)*(((((320*
b^4*c^2*d^8*f^4 - 16*b^4*d^10*f^4 - 1760*b^4*c^4*d^6*f^4 + 1600*b^4*c^6*d^4
*f^4 - 400*b^4*c^8*d^2*f^4)^(1/2) + 4*b^2*c^5*f^2 + 20*b^2*c*d^4*f^2 - 40*b
^2*c^3*d^2*f^2)/(c^10*f^4 + d^10*f^4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*
c^6*d^4*f^4 + 5*c^8*d^2*f^4))^(1/2)*(((((320*b^4*c^2*d^8*f^4 - 16*b^4*d^10*
f^4 - 1760*b^4*c^4*d^6*f^4 + 1600*b^4*c^6*d^4*f^4 - 400*b^4*c^8*d^2*f^4)^(1
/2) + 4*b^2*c^5*f^2 + 20*b^2*c*d^4*f^2 - 40*b^2*c^3*d^2*f^2)/(c^10*f^4 + d^
10*f^4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*c^6*d^4*f^4 + 5*c^8*d^2*f^4))^
(1/2)*(c + d*tan(e + f*x))^(1/2)*(64*c*d^22*f^5 + 640*c^3*d^20*f^5 + 2880*c
^5*d^18*f^5 + 7680*c^7*d^16*f^5 + 13440*c^9*d^14*f^5 + 16128*c^11*d^12*f^5
```

+ 13440*c^13*d^10*f^5 + 7680*c^15*d^8*f^5 + 2880*c^17*d^6*f^5 + 640*c^19*d^4*f^5 + 64*c^21*d^2*f^5))/4 + 736*b*c^3*d^18*f^4 + 2432*b*c^5*d^16*f^4 + 4480*b*c^7*d^14*f^4 + 4928*b*c^9*d^12*f^4 + 3136*b*c^11*d^10*f^4 + 896*b*c^13*d^8*f^4 - 128*b*c^15*d^6*f^4 - 160*b*c^17*d^4*f^4 - 32*b*c^19*d^2*f^4 + 96*b*c*d^20*f^4))/4 + (c + d*tan(e + f*x))^(1/2)*(320*b^2*c^4*d^14*f^3 - 16*b^2*d^18*f^3 + 1024*b^2*c^6*d^12*f^3 + 1440*b^2*c^8*d^10*f^3 + 1024*b^2*c^10*d^8*f^3 + 320*b^2*c^12*d^6*f^3 - 16*b^2*c^16*d^2*f^3)))/4 + 40*b^3*c^2*d^14*f^2 + 72*b^3*c^4*d^12*f^2 + 40*b^3*c^6*d^10*f^2 - 40*b^3*c^8*d^8*f^2 - 72*b^3*c^10*d^6*f^2 - 40*b^3*c^12*d^4*f^2 - 8*b^3*c^14*d^2*f^2)*(((320*b^4*c^2*d^8*f^4 - 16*b^4*d^10*f^4 - 1760*b^4*c^4*d^6*f^4 + 1600*b^4*c^6*d^4*f^4 - 400*b^4*c^8*d^2*f^4)^(1/2) + 4*b^2*c^5*f^2 + 20*b^2*c*d^4*f^2 - 40*b^2*c^3*d^2*f^2)/(c^10*f^4 + d^10*f^4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*c^6*d^4*f^4 + 5*c^8*d^2*f^4))^(1/2))/4 + (log(8*b^3*d^16*f^2 - ((-((320*b^4*c^2*d^8*f^4 - 16*b^4*d^10*f^4 - 1760*b^4*c^4*d^6*f^4 + 1600*b^4*c^6*d^4*f^4 - 400*b^4*c^8*d^2*f^4)^(1/2) - 4*b^2*c^5*f^2 - 20*b^2*c*d^4*f^2 + 40*b^2*c^3*d^2*f^2)/(c^10*f^4 + d^10*f^4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*c^6*d^4*f^4 + 5*c^8*d^2*f^4))^(1/2)*(((-((320*b^4*c^2*d^8*f^4 - 16*b^4*d^10*f^4 - 1760*b^4*c^4*d^6*f^4 + 1600*b^4*c^6*d^4*f^4 - 400*b^4*c^8*d^2*f^4)^(1/2) - 4*b^2*c^5*f^2 - 20*b^2*c*d^4*f^2 + 40*b^2*c^3*d^2*f^2)/(c^10*f^4 + d^10*f^4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*c^6*d^4*f^4 + 5*c^8*d^2*f^4))^(1/2)*(((-((320*b^4*c^2*d^8*f^4 - 16*b^4*d^10*f^4 - 1760*b^4*c^4*d^6*f^4 + 1600*b^4*c^6*d^4*f^4 - 400*b^4*c^8*d^2*f^4)^(1/2) - 4*b^2*c^5*f^2 - 20*b^2*c*d^4*f^2 + 40*b^2*c^3*d^2*f^2)/(c^10*f^4 + d^10*f^4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*c^6*d^4*f^4 + 5*c^8*d^2*f^4))^(1/2)*(c + d*tan(e + f*x))^(1/2)*(64*c*d^22*f^5 + 640*c^3*d^20*f^5 + 2880*c^5*d^18*f^5 + 7680*c^7*d^16*f^5 + 13440*c^9*d^14*f^5 + 16128*c^11*d^12*f^5 + 13440*c^13*d^10*f^5 + 7680*c^15*d^8*f^5 + 2880*c^17*d^6*f^5 + 640*c^19*d^4*f^5 + 64*c^21*d^2*f^5))/4 + 736*b*c^3*d^18*f^4 + 2432*b*c^5*d^16*f^4 + 4480*b*c^7*d^14*f^4 + 4928*b*c^9*d^12*f^4 + 3136*b*c^11*d^10*f^4 + 896*b*c^13*d^8*f^4 - 128*b*c^15*d^6*f^4 - 160*b*c^17*d^4*f^4 - 32*b*c^19*d^2*f^4 + 96*b*c*d^20*f^4))/4 + (c + d*tan(e + f*x))^(1/2)*(320*b^2*c^4*d^14*f^3 - 16*b^2*d^18*f^3 + 1024*b^2*c^6*d^12*f^3 + 1440*b^2*c^8*d^10*f^3 + 1024*b^2*c^10*d^8*f^3 + 320*b^2*c^12*d^6*f^3 - 16*b^2*c^16*d^2*f^3)))/4 + 40*b^3*c^2*d^14*f^2 + 72*b^3*c^4*d^12*f^2 + 40*b^3*c^6*d^10*f^2 - 40*b^3*c^8*d^8*f^2 - 72*b^3*c^10*d^6*f^2 - 40*b^3*c^12*d^4*f^2 - 8*b^3*c^14*d^2*f^2)*(-((320*b^4*c^2*d^8*f^4 - 16*b^4*d^10*f^4 - 1760*b^4*c^4*d^6*f^4 + 1600*b^4*c^6*d^4*f^4 - 400*b^4*c^8*d^2*f^4)^(1/2) - 4*b^2*c^5*f^2 - 20*b^2*c*d^4*f^2 + 40*b^2*c^3*d^2*f^2)/(c^10*f^4 + d^10*f^4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*c^6*d^4*f^4 + 5*c^8*d^2*f^4))^(1/2))/4 - log(8*b^3*d^16*f^2 - (((320*b^4*c^2*d^8*f^4 - 16*b^4*d^10*f^4 - 1760*b^4*c^4*d^6*f^4 + 1600*b^4*c^6*d^4*f^4 - 400*b^4*c^8*d^2*f^4)^(1/2) + 4*b^2*c^5*f^2 + 20*b^2*c*d^4*f^2 - 40*b^2*c^3*d^2*f^2)/(16*c^10*f^4 + 16*d^10*f^4 + 80*c^2*d^8*f^4 + 160*c^4*d^6*f^4 + 160*c^6*d^4*f^4 + 80*c^8*d^2*f^4))^(1/2)*((((320*b^4*c^2*d^8*f^4 - 16*b^4*d^10*f^4 - 1760*b^4*c^4*d^6*f^4 + 1600*b^4*c^6*d^4*f^4 - 400*b^4*c^8*d^2*f^4)^(1/2) + 4*b^2*c^5*f^2 + 20*b^2*c*d^4*f^2 - 40*b^2*c^3*d^2*f^2)/(16*c^10*f^4 + 16*d^10*f^4 + 80*c^2*d^8*f^4 + 160*c^4

*d^6*f^4 + 160*c^6*d^4*f^4 + 80*c^8*d^2*f^4))^(1/2)*(736*b*c^3*d^18*f^4 - (
((320*b^4*c^2*d^8*f^4 - 16*b^4*d^10*f^4 - 1760*b^4*c^4*d^6*f^4 + 1600*b^4*c
^6*d^4*f^4 - 400*b^4*c^8*d^2*f^4)^(1/2) + 4*b^2*c^5*f^2 + 20*b^2*c*d^4*f^2
- 40*b^2*c^3*d^2*f^2)/(16*c^10*f^4 + 16*d^10*f^4 + 80*c^2*d^8*f^4 + 160*c^4
*d^6*f^4 + 160*c^6*d^4*f^4 + 80*c^8*d^2*f^4))^(1/2)*(c + d*tan(e + f*x))^(1
/2)*(64*c*d^22*f^5 + 640*c^3*d^20*f^5 + 2880*c^5*d^18*f^5 + 7680*c^7*d^16*f
^5 + 13440*c^9*d^14*f^5 + 16128*c^11*d^12*f^5 + 13440*c^13*d^10*f^5 + 7680*
c^15*d^8*f^5 + 2880*c^17*d^6*f^5 + 640*c^19*d^4*f^5 + 64*c^21*d^2*f^5) + 24
32*b*c^5*d^16*f^4 + 4480*b*c^7*d^14*f^4 + 4928*...

3.1263 $\int \frac{1}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=272

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(ia+b)(c-id)^{5/2}f}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(ia-b)(c+id)^{5/2}f}-\frac{2b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right.}{(a^2+b^2)\,(bc-ad)^{5/2}f}$$

```
[Out] arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(I*a+b)/(c-I*d)^(5/2)/f-arcta
nh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(I*a-b)/(c+I*d)^(5/2)/f-2*b^(7/2)*
arctanh(b^(1/2)*(c+d*tan(f*x+e))^(1/2)/(-a*d+b*c)^(1/2))/(a^2+b^2)/(-a*d+b*
c)^(5/2)/f-2*d^2*(2*a*c*d-b*(3*c^2+d^2))/(-a*d+b*c)^2/(c^2+d^2)^2/f/(c+d*ta
n(f*x+e))^(1/2)+2/3*d^2/(-a*d+b*c)/(c^2+d^2)/f/(c+d*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 1.03, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {3650, 3730, 3734, 3620, 3618, 65, 214, 3715}

$$-\frac{2b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{f\,(a^2+b^2)\,(bc-ad)^{5/2}}-\frac{2d^2(2acd-b(3c^2+d^2))}{f\,(c^2+d^2)^2\,(bc-ad)^2\sqrt{c+d\tan(e+fx)}}+\frac{2d^2}{3f\,(c^2+d^2)\,(bc-ad)(c+d\tan(e+fx))^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(b+ia)(c-id)^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(-b+ia)(c+id)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^(5/2)),x]
```

```
[Out] ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]]/((I*a + b)*(c - I*d)^(5/2)*
f) - ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]]/((I*a - b)*(c + I*d)^(
5/2)*f) - (2*b^(7/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c -
a*d]])/((a^2 + b^2)*(b*c - a*d)^(5/2)*f) + (2*d^2)/(3*(b*c - a*d)*(c^2 + d^
2)*f*(c + d*Tan[e + f*x])^(3/2)) - (2*d^2*(2*a*c*d - b*(3*c^2 + d^2)))/((b*
c - a*d)^2*(c^2 + d^2)^2*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}}\,dx = \frac{2d^2}{3(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{2\int \frac{-\frac{3}{2}(acd-}{}}{}$$

$$= \frac{2d^2}{3(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - \frac{2}{(bc-ad)^2}$$

$$= \frac{2d^2}{3(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - \frac{2}{(bc-ad)^2}$$

$$= \frac{2d^2}{3(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - \frac{2}{(bc-ad)^2}$$

$$= \frac{2d^2}{3(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - \frac{2}{(bc-ad)^2}$$

$$= -\frac{2b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{\left(a^2+b^2\right)(bc-ad)^{5/2}f} + \frac{}{3(bc-ad)}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(ia+b)(c-id)^{5/2}f} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+d\tan(}{\sqrt{c+i}}\right)}{(ia-b)(c+id)}$$

Mathematica [A]

time = 5.21, size = 323, normalized size = 1.19

$$\frac{\frac{3\left(\frac{(-ia+b)(c+id)^2(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}}+\frac{(ia+b)(c-id)^2(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{\sqrt{c+id}}-\frac{2b^{7/2}\left(c^2+d^2\right)^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}\right)}{(a^2+b^2)(-bc+ad)(c^2+d^2)}-\frac{2d^2}{(c+d\tan(e+fx))^{3/2}}-\frac{6d^2\left(-2acd+b\left(3c^2+d^2\right)\right)}{(bc-ad)(c^2+d^2)\sqrt{c+d\tan(e+fx)}}}{3(-bc+ad)\left(c^2+d^2\right)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^(5/2)),x]
```

```
[Out] ((3*((((-I)*a + b)*(c + I*d)^2*(b*c - a*d)^2*ArcTanh[Sqrt[c + d*Tan[e + f*x
]]/Sqrt[c - I*d]])/Sqrt[c - I*d] + ((I*a + b)*(c - I*d)^2*(b*c - a*d)^2*Arc
Tanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/Sqrt[c + I*d] - (2*b^(7/2)*(c
^2 + d^2)^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/Sq
rt[b*c - a*d]))/((a^2 + b^2)*(-(b*c) + a*d)*(c^2 + d^2)) - (2*d^2)/(c + d*T
an[e + f*x])^(3/2) - (6*d^2*(-2*a*c*d + b*(3*c^2 + d^2)))/((b*c - a*d)*(c^2
 + d^2)*Sqrt[c + d*Tan[e + f*x]]))/(3*(-(b*c) + a*d)*(c^2 + d^2)*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3328 vs. $2(238) = 476$.
time = 0.58, size = 3329, normalized size = 12.24

method	result	size
derivativedivides	Expression too large to display	3329
default	Expression too large to display	3329

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*d^2*(b^4/(a*d-b*c)^2/d^2/(a^2+b^2)/((a*d-b*c)*b)^(1/2)*arctan(b*(c+d*ta
n(f*x+e))^(1/2)/((a*d-b*c)*b)^(1/2))+1/(c^2+d^2)^2/d^2/(a^2+b^2)*(1/4/d/(5*
c^4-10*c^2*d^2+d^4)/(c^2+d^2)^(3/2)*(1/2*(3*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*a*c^6+5*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^4*
d^2+(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2*d^4-(c^2+d^2)^(3/2)
*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^6+2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)*a*c^8-18*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^6*d^2-
10*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^4*d^4+10*(c^2+d^2)^(1/
2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2*d^6-10*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*b*c^7*d+10*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b
*c^5*d^3+18*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^3*d^5-2*(c^2+
d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d^7-5*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*a*c^9+20*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^7*d^2-6*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*a*c^5*d^4-28*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3*d^6+3*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*a*c*d^8+15*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^8*d-20*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^6*d^3-22*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c
^4*d^5+12*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d^7-(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*b*d^9)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)+(c^2+d^2)^(1/2))+2*(30*a*c^8*d^2-40*a*c^6*d^4-44*a*c^4*d^6+24*a*c^2
*d^8-2*a*d^10+10*b*c^9*d-40*b*c^7*d^3+12*b*c^5*d^5+56*b*c^3*d^7-6*b*c*d^9-1
/2*(3*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^6+5*(c^2+d^2)^(3/2)
*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^4*d^2+(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)
```

+2*c)^(1/2)*a*c^2*d^4-(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^6+2
(c^2+d^2)^(1/2)(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^8-18*(c^2+d^2)^(1/2)*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^6*d^2-10*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+
2*c)^(1/2)*a*c^4*d^4+10*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2
d^6-10(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^7*d+10*(c^2+d^2)^
(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^5*d^3+18*(c^2+d^2)^(1/2)*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*b*c^3*d^5-2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*b*c*d^7-5*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^9+20*(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)*a*c^7*d^2-6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^5*d^4-28*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*a*c^3*d^6+3*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d^8+15*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*b*c^8*d-20*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^6*d^3
-22*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^4*d^5+12*(2*(c^2+d^2)^(1/2)+2*c)^(1/2
)*b*c^2*d^7-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^9)*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2
+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/4/d/(5*c^4-10*c^2
d^2+d^4)/(c^2+d^2)^(3/2)(1/2*(-3*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*a*c^6-5*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^4*d^2-(c^2+
d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2*d^4+(c^2+d^2)^(3/2)*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*a*d^6-2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*a*c^8+18*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^6*d^2+10*(c^2+d
^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^4*d^4-10*(c^2+d^2)^(1/2)*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)*a*c^2*d^6+10*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*b*c^7*d-10*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^5*d^3-
18*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^3*d^5+2*(c^2+d^2)^(1/2
)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d^7+5*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c
^9-20*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^7*d^2+6*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*a*c^5*d^4+28*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3*d^6-3*(2*(c^2+d^2)^(1/2
)+2*c)^(1/2)*a*c*d^8-15*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^8*d+20*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*b*c^6*d^3+22*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^4*d^5-12
(2(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d^7+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^
9)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(
c^2+d^2)^(1/2))+2*(30*a*c^8*d^2-40*a*c^6*d^4-44*a*c^4*d^6+24*a*c^2*d^8-2*a*
d^10+10*b*c^9*d-40*b*c^7*d^3+12*b*c^5*d^5+56*b*c^3*d^7-6*b*c*d^9+1/2*(-3*(c
^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^6-5*(c^2+d^2)^(3/2)*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*a*c^4*d^2-(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)*a*c^2*d^4+(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^6-2*(c^2+d^
2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^8+18*(c^2+d^2)^(1/2)*(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)*a*c^6*d^2+10*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*a*c^4*d^4-10*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2*d^6+10*
(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^7*d-10*(c^2+d^2)^(1/2)*(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^5*d^3-18*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*b*c^3*d^5+2*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d
^7+5*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^9-20*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a
*c^7*d^2+6*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^5*d^4+28*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*a*c^3*d^6-3*(2*(c^2+d^2)^(1/2)+2*c)^(1...

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\tan{(e + fx)})\,(c + d\tan{(e + fx)})^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral(1/((a + b*tan(e + f*x))*(c + d*tan(e + f*x))**(5/2)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*tan(e + f*x))*(c + d*tan(e + f*x))^(5/2)),x)
[Out] \text{Hanged}
```

3.1264 $\int \frac{1}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=425

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)^2(c-id)^{5/2}f}+\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(a+ib)^2(c+id)^{5/2}f}-\frac{b^{7/2}(4abc-9a^2d-5b^2d)\tanh^{-1}\left(\right.}{(a^2+b^2)^2\,(bc-}$$

```
[Out] -I*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(a-I*b)^2/(c-I*d)^(5/2)/f+
I*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(a+I*b)^2/(c+I*d)^(5/2)/f-b
^(7/2)*(-9*a^2*d+4*a*b*c-5*b^2*d)*arctanh(b^(1/2)*(c+d*tan(f*x+e))^(1/2)/(-
a*d+b*c)^(1/2))/(a^2+b^2)^2/(-a*d+b*c)^(7/2)/f+d*(4*a^3*c*d^3+4*a*b^2*c*d^3
-4*a^2*b*d^2*(2*c^2+d^2)-b^3*(c^4+10*c^2*d^2+5*d^4))/(a^2+b^2)/(-a*d+b*c)^3
/(c^2+d^2)^2/f/(c+d*tan(f*x+e))^(1/2)-1/3*d*(2*a^2*d^2+b^2*(3*c^2+5*d^2))/(
a^2+b^2)/(-a*d+b*c)^2/(c^2+d^2)/f/(c+d*tan(f*x+e))^(3/2)-b^2/(a^2+b^2)/(-a*
d+b*c)/f/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 1.79, antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {3650, 3730, 3734, 3620, 3618, 65, 214, 3715}

$$-\frac{d(2a^2d^2+b^2(3c^2+5d^2))}{3f(a^2+b^2)(c^2+d^2)(bc-ad)^2(c+d\tan(e+fx))^{3/2}}-\frac{b^2}{f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}-\frac{b^{7/2}(-9a^2d+4abc-5b^2d)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{f(a^2+b^2)^2(bc-ad)^{7/2}}+\frac{d(4a^3cd^3-4a^2bd^2(2c^2+d^2)+4ab^2cd^3-(b^3(c^4+10c^2d^2+5d^4)))}{f(a^2+b^2)(c^2+d^2)^2(bc-ad)^3\sqrt{c+d\tan(e+fx)}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(a-ib)^2(c-id)^{5/2}}+\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(a+ib)^2(c+id)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(5/2)),x]
```

```
[Out] ((-I)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((a - I*b)^2*(c - I*
d)^(5/2)*f) + (I*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/((a + I*b
)^2*(c + I*d)^(5/2)*f) - (b^(7/2)*(4*a*b*c - 9*a^2*d - 5*b^2*d)*ArcTanh[(Sq
rt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/((a^2 + b^2)^2*(b*c - a*d
)^(7/2)*f) - (d*(2*a^2*d^2 + b^2*(3*c^2 + 5*d^2)))/(3*(a^2 + b^2)*(b*c - a*
d)^2*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2)) - b^2/((a^2 + b^2)*(b*c - a*
d)*f*(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^(3/2)) + (d*(4*a^3*c*d^3 + 4
*a*b^2*c*d^3 - 4*a^2*b*d^2*(2*c^2 + d^2) - b^3*(c^4 + 10*c^2*d^2 + 5*d^4)))
/((a^2 + b^2)*(b*c - a*d)^3*(c^2 + d^2)^2*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
```

```
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{b^2}{(a^2+b^2)\,(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+}$$

$$= -\frac{d(2a^2d^2+b^2(3c^2+5d^2))}{3\,(a^2+b^2)\,(bc-ad)^2\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}}$$

$$= -\frac{d(2a^2d^2+b^2(3c^2+5d^2))}{3\,(a^2+b^2)\,(bc-ad)^2\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}}$$

$$= -\frac{d(2a^2d^2+b^2(3c^2+5d^2))}{3\,(a^2+b^2)\,(bc-ad)^2\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}}$$

$$= -\frac{d(2a^2d^2+b^2(3c^2+5d^2))}{3\,(a^2+b^2)\,(bc-ad)^2\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}}$$

$$= -\frac{d(2a^2d^2+b^2(3c^2+5d^2))}{3\,(a^2+b^2)\,(bc-ad)^2\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}}$$

$$= -\frac{b^{7/2}(4abc-9a^2d-5b^2d)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+d\tan(e+}}{\sqrt{bc-ad}}\right.}{\left(a^2+b^2\right)^2(bc-ad)^{7/2}f}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)^2(c-id)^{5/2}f} + \frac{i\tanh^{-1}\left(\frac{\sqrt{c+d}}{\sqrt{}}\right.}{(a+ib)^2(c}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2536 vs. 2(425) = 850.
time = 6.27, size = 2536, normalized size = 5.97

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(5/2)),x]
```

```
[Out] -(b^2/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^
(3/2))) - ((-2*((d^2*(-2*a*b*c + 2*a^2*d + 5*b^2*d))/2 - c*((-5*b^2*c*d)/2
+ b*d*(b*c - a*d))))/(3*(-(b*c) + a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(
3/2)) - (2*((-2*(((I*Sqrt[c - I*d]*((b*(-(b*c) + a*d)*((-3*c*(b*c - a*d)^2*
(b*c + a*d))/2 - (3*d*(2*a^3*c*d^2 - 4*a^2*b*d*(c^2 + d^2) - 5*b^3*d*(c^2 +
```

d^2) + 2*a*b^2*c*(c^2 + 2*d^2)))/4 - (3*b*d*(2*a^2*d^3 + b^2*(3*c^2*d + 5*d^3)))/4))/2 + a*((-3*((b*d^2)/2 - (c*(-(b*c) + a*d))/2)*(2*a^3*c*d^2 - 4*a^2*b*d*(c^2 + d^2) - 5*b^3*d*(c^2 + d^2) + 2*a*b^2*c*(c^2 + 2*d^2)))/4 - (a*d*((3*d*(b*c - a*d)^2*(b*c + a*d))/2 - (3*b*c*(2*a^2*d^3 + b^2*(3*c^2*d + 5*d^3)))/4))/2 - (b*((-3*d^2*(2*a^3*c*d^2 - 4*a^2*b*d*(c^2 + d^2) - 5*b^3*d*(c^2 + d^2) + 2*a*b^2*c*(c^2 + 2*d^2)))/4 - c*((3*d*(b*c - a*d)^2*(b*c + a*d))/2 - (3*b*c*(2*a^2*d^3 + b^2*(3*c^2*d + 5*d^3)))/4)))/2) - I*((a*(-(b*c) + a*d)*((-3*c*(b*c - a*d)^2*(b*c + a*d))/2 - (3*d*(2*a^3*c*d^2 - 4*a^2*b*d*(c^2 + d^2) - 5*b^3*d*(c^2 + d^2) + 2*a*b^2*c*(c^2 + 2*d^2)))/4 - (3*b*d*(2*a^2*d^3 + b^2*(3*c^2*d + 5*d^3)))/4))/2 - b*((-3*((b*d^2)/2 - (c*(-(b*c) + a*d))/2)*(2*a^3*c*d^2 - 4*a^2*b*d*(c^2 + d^2) - 5*b^3*d*(c^2 + d^2) + 2*a*b^2*c*(c^2 + 2*d^2)))/4 - (a*d*((3*d*(b*c - a*d)^2*(b*c + a*d))/2 - (3*b*c*(2*a^2*d^3 + b^2*(3*c^2*d + 5*d^3)))/4))/2 - (b*((-3*d^2*(2*a^3*c*d^2 - 4*a^2*b*d*(c^2 + d^2) - 5*b^3*d*(c^2 + d^2) + 2*a*b^2*c*(c^2 + 2*d^2)))/4 - c*((3*d*(b*c - a*d)^2*(b*c + a*d))/2 - (3*b*c*(2*a^2*d^3 + b^2*(3*c^2*d + 5*d^3)))/4)))/2)))*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((-c + I*d)*f) - (I*Sqrt[c + I*d]*((b*(-(b*c) + a*d)*((-3*c*(b*c - a*d)^2*(b*c + a*d))/2 - (3*d*(2*a^3*c*d^2 - 4*a^2*b*d*(c^2 + d^2) - 5*b^3*d*(c^2 + d^2) + 2*a*b^2*c*(c^2 + 2*d^2)))/4 - (3*b*d*(2*a^2*d^3 + b^2*(3*c^2*d + 5*d^3)))/4))/2 + a*((-3*((b*d^2)/2 - (c*(-(b*c) + a*d))/2)*(2*a^3*c*d^2 - 4*a^2*b*d*(c^2 + d^2) - 5*b^3*d*(c^2 + d^2) + 2*a*b^2*c*(c^2 + 2*d^2)))/4 - (a*d*((3*d*(b*c - a*d)^2*(b*c + a*d))/2 - (3*b*c*(2*a^2*d^3 + b^2*(3*c^2*d + 5*d^3)))/4))/2 - (b*((-3*d^2*(2*a^3*c*d^2 - 4*a^2*b*d*(c^2 + d^2) - 5*b^3*d*(c^2 + d^2) + 2*a*b^2*c*(c^2 + 2*d^2)))/4 - c*((3*d*(b*c - a*d)^2*(b*c + a*d))/2 - (3*b*c*(2*a^2*d^3 + b^2*(3*c^2*d + 5*d^3)))/4)))/2) + I*((a*(-(b*c) + a*d)*((-3*c*(b*c - a*d)^2*(b*c + a*d))/2 - (3*d*(2*a^3*c*d^2 - 4*a^2*b*d*(c^2 + d^2) - 5*b^3*d*(c^2 + d^2) + 2*a*b^2*c*(c^2 + 2*d^2)))/4 - (3*b*d*(2*a^2*d^3 + b^2*(3*c^2*d + 5*d^3)))/4))/2 - b*((-3*((b*d^2)/2 - (c*(-(b*c) + a*d))/2)*(2*a^3*c*d^2 - 4*a^2*b*d*(c^2 + d^2) - 5*b^3*d*(c^2 + d^2) + 2*a*b^2*c*(c^2 + 2*d^2)))/4 - (a*d*((3*d*(b*c - a*d)^2*(b*c + a*d))/2 - (3*b*c*(2*a^2*d^3 + b^2*(3*c^2*d + 5*d^3)))/4))/2 - (b*((-3*d^2*(2*a^3*c*d^2 - 4*a^2*b*d*(c^2 + d^2) - 5*b^3*d*(c^2 + d^2) + 2*a*b^2*c*(c^2 + 2*d^2)))/4 - c*((3*d*(b*c - a*d)^2*(b*c + a*d))/2 - (3*b*c*(2*a^2*d^3 + b^2*(3*c^2*d + 5*d^3)))/4)))/2)))*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/((-c - I*d)*f))/(a^2 + b^2) + (2*Sqrt[b*c - a*d]*(-1/2*(a*b*(-(b*c) + a*d)*((-3*c*(b*c - a*d)^2*(b*c + a*d))/2 - (3*d*(2*a^3*c*d^2 - 4*a^2*b*d*(c^2 + d^2) - 5*b^3*d*(c^2 + d^2) + 2*a*b^2*c*(c^2 + 2*d^2)))/4 - (3*b*d*(2*a^2*d^3 + b^2*(3*c^2*d + 5*d^3)))/4)) + (a^2*b*((-3*d^2*(2*a^3*c*d^2 - 4*a^2*b*d*(c^2 + d^2) - 5*b^3*d*(c^2 + d^2) + 2*a*b^2*c*(c^2 + 2*d^2)))/4 - c*((3*d*(b*c - a*d)^2*(b*c + a*d))/2 - (3*b*c*(2*a^2*d^3 + b^2*(3*c^2*d + 5*d^3)))/4)))/2 + b^2*((-3*((b*d^2)/2 - (c*(-(b*c) + a*d))/2)*(2*a^3*c*d^2 - 4*a^2*b*d*(c^2 + d^2) - 5*b^3*d*(c^2 + d^2) + 2*a*b^2*c*(c^2 + 2*d^2)))/4 - (a*d*((3*d*(b*c - a*d)^2*(b*c + a*d))/2 - (3*b*c*(2*a^2*d^3 + b^2*(3*c^2*d + 5*d^3)))/4))/2))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(Sqrt[b]*(a^2 + b^2)*(-(b*c) + a*d)*f)))/((-(b*c) + a*d)*(c^2 + d^2)) - (2*((-3*d^2*(2*a^3*c*d

```
^2 - 4*a^2*b*d*(c^2 + d^2) - 5*b^3*d*(c^2 + d^2) + 2*a*b^2*c*(c^2 + 2*d^2))
)/4 - c*((3*d*(b*c - a*d)^2*(b*c + a*d))/2 - (3*b*c*(2*a^2*d^3 + b^2*(3*c^2
*d + 5*d^3)))/4)))/((-(b*c) + a*d)*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]]))
)/(3*(-(b*c) + a*d)*(c^2 + d^2)))/((a^2 + b^2)*(b*c - a*d))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5221 vs. $2(387) = 774$.
time = 0.66, size = 5222, normalized size = 12.29

method	result	size
derivativedivides	Expression too large to display	5222
default	Expression too large to display	5222

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^(5/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^(5/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\tan\left(e + fx\right)\right)^{2}\left(c + d\tan\left(e + fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))**2/(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral(1/((a + b*tan(e + f*x))**2*(c + d*tan(e + f*x))**(5/2)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*tan(e + f*x))^2*(c + d*tan(e + f*x))^(5/2)),x)
```

```
[Out] \text{Hanged}
```

3.1265 $\int (a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=337

$$-\frac{i(a-ib)^{5/2}\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f}+\frac{i(a+ib)^{5/2}\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a}}{\sqrt{a+ib}\,\sqrt{c}}\right.}{f}$$

```
[Out] 1/4*(10*a*b*c*d+15*a^2*d^2-b^2*(c^2+8*d^2))*arctanh(d^(1/2)*(a+b*tan(f*x+e)
)^(1/2)/b^(1/2)/(c+d*tan(f*x+e))^(1/2))*b^(1/2)/d^(3/2)/f-I*(a-I*b)^(5/2)*a
rctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c+d*tan(f*x+e))^
(1/2))*(c-I*d)^(1/2)/f+I*(a+I*b)^(5/2)*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e
))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))*(c+I*d)^(1/2)/f-1/4*b*(-9*a*
d+b*c)*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/d/f+1/2*b^2*(a+b*tan(f
*x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2)/d/f
```

Rubi [A]
time = 2.84, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {3647, 3728, 3736, 6857, 65, 223, 212, 95, 214}

$$\frac{\sqrt{b}\,(15a^2d^2+10abcd-(b^2(c^2+8d^2)))\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{4d^{3/2}f}+\frac{b^2\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}}{2df}-\frac{b(bc-9ad)\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{4df}-\frac{i(a-ib)^{5/2}\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f}+\frac{i(a+ib)^{5/2}\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^(5/2)*Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((-I)*(a - I*b)^(5/2)*Sqrt[c - I*d]*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
 + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/f + (I*(a + I*b)^(5/2)
*Sqrt[c + I*d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I
*b]*Sqrt[c + d*Tan[e + f*x]])])/f + (Sqrt[b]*(10*a*b*c*d + 15*a^2*d^2 - b^2
*(c^2 + 8*d^2))*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c
+ d*Tan[e + f*x]])])/(4*d^(3/2)*f) - (b*(b*c - 9*a*d)*Sqrt[a + b*Tan[e + f*
x]]*Sqrt[c + d*Tan[e + f*x]])/(4*d*f) + (b^2*Sqrt[a + b*Tan[e + f*x]]*(c +
d*Tan[e + f*x])^(3/2))/(2*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int (a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}\,dx = \frac{b^2\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}}{2df} + \frac{\int \sqrt{c-\cdots}}{\cdots}$$

$$= -\frac{b(bc-9ad)\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{4df}\cdots$$

$$= -\frac{b(bc-9ad)\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{4df}\cdots$$

$$= -\frac{b(bc-9ad)\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{4df}\cdots$$

$$= -\frac{b(bc-9ad)\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{4df}\cdots$$

$$= -\frac{b(bc-9ad)\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{4df}\cdots$$

$$= -\frac{b(bc-9ad)\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{4df}\cdots$$

$$= \frac{\sqrt{b}\,(10abcd+15a^2d^2-b^2(c^2+8d^2))\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a\cdots}}{\sqrt{b}\,\sqrt{c\cdots}}\right.}{4d^{3/2}f}\cdots$$

$$= -\frac{i(a-ib)^{5/2}\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e\cdots}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e\cdots}}\right.}{f}\cdots$$

<u>Mathematica</u> [A]

time = 6.03, size = 565, normalized size = 1.68

$$\frac{\frac{4bd\left(b(3a^2bc-b^3c+a^3d-3ab^2d)+\sqrt{-b^2}\,(a^3c-3ab^2c-3a^2bd+b^3d)\right)\tanh^{-1}\left(\frac{\sqrt{-c+\frac{\sqrt{-b^2}\,d}{b}}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+\sqrt{-b^2}}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+\sqrt{-b^2}}\,\sqrt{-c+\frac{\sqrt{-b^2}\,d}{b}}} - \frac{4bd\left(b(3a^2bc-b^3c+a^3d-3ab^2d)-\sqrt{-b^2}\,(a^3c-3ab^2c-3a^2bd+b^3d)\right)\tanh^{-1}\left(\frac{\sqrt{c+\frac{\sqrt{-b^2}\,d}{b}}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+\sqrt{-b^2}}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+\sqrt{-b^2}}\,\sqrt{c+\frac{\sqrt{-b^2}\,d}{b}}} + b^3(-bc+9ad)\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)} + 2b^4\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2} - \frac{b^{5/2}\sqrt{c-\frac{ad}{b}}\,(-10abcd-15a^2d^2+b^2(c^2+8d^2))\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c-\frac{ad}{b}}}\right)\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}}{\sqrt{d}\,\sqrt{c+d\tan(e+fx)}}}{4b^2df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^(5/2)*Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((4*b*d*(b*(3*a^2*b*c - b^3*c + a^3*d - 3*a*b^2*d) + Sqrt[-b^2]*(a^3*c - 3*
a*b^2*c - 3*a^2*b*d + b^3*d))*ArcTanh[(Sqrt[-c + (Sqrt[-b^2]*d)/b]*Sqrt[a +
 b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[
-a + Sqrt[-b^2]]*Sqrt[-c + (Sqrt[-b^2]*d)/b]) - (4*b*d*(b*(3*a^2*b*c - b^3*
c + a^3*d - 3*a*b^2*d) - Sqrt[-b^2]*(a^3*c - 3*a*b^2*c - 3*a^2*b*d + b^3*d)
)*ArcTanh[(Sqrt[c + (Sqrt[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + S
qrt[-b^2]]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + (Sqrt
[-b^2]*d)/b]) + b^3*(-(b*c) + 9*a*d)*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Ta
n[e + f*x]] + 2*b^4*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2) - (
b^(5/2)*Sqrt[c - (a*d)/b]*(-10*a*b*c*d - 15*a^2*d^2 + b^2*(c^2 + 8*d^2))*Ar
cSinh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c - (a*d)/b])]*Sqrt[
(b*(c + d*Tan[e + f*x]))/(b*c - a*d)])/(Sqrt[d]*Sqrt[c + d*Tan[e + f*x]]))/
(4*b^2*d*f)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+d\tan(fx+e)}\,(a+b\tan(fx+e))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(5/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*tan(f*x + e) + a)^(5/2)*sqrt(d*tan(f*x + e) + c), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(e + fx))^{\frac{5}{2}} \sqrt{c + d\tan(e + fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(a+b*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**(5/2)*sqrt(c + d*tan(e + f*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\tan(e + f\,x))^{5/2}\,\sqrt{c + d\tan(e + f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^(5/2)*(c + d*tan(e + f*x))^(1/2),x)
```

```
[Out] int((a + b*tan(e + f*x))^(5/2)*(c + d*tan(e + f*x))^(1/2), x)
```

3.1266 $\int (a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=258

$$-\frac{i(a-ib)^{3/2}\sqrt{c-id}\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{f}+\frac{i(a+ib)^{3/2}\sqrt{c+id}\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a}}{\sqrt{a+ib}\sqrt{c}}\right.}{f}$$

```
[Out] -I*(a-I*b)^(3/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)
/(c+d*tan(f*x+e))^(1/2))*(c-I*d)^(1/2)/f+I*(a+I*b)^(3/2)*arctanh((c+I*d)^(1
/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))*(c+I*d)^(1
/2)/f+(3*a*d+b*c)*arctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f
*x+e))^(1/2))*b^(1/2)/f/d^(1/2)+b*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(
1/2)/f
```

Rubi [A]
time = 1.64, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {3651, 3736, 6857, 65, 223, 212, 95, 214}

$$\frac{b\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{f}-\frac{i(a-ib)^{3/2}\sqrt{c-id}\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{f}+\frac{i(a+ib)^{3/2}\sqrt{c+id}\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{f}+\frac{\sqrt{b}(3ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{d}f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^(3/2)*Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((-I)*(a - I*b)^(3/2)*Sqrt[c - I*d]*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
 + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/f + (I*(a + I*b)^(3/2)
*Sqrt[c + I*d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I
*b]*Sqrt[c + d*Tan[e + f*x]])])/f + (Sqrt[b]*(b*c + 3*a*d)*ArcTanh[(Sqrt[d]
*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[d]*f)
 + (b*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/f
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3651

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m - 1)*((c +
d*Tan[e + f*x])^n/(f*(m + n - 1))), x] + Dist[1/(m + n - 1), Int[(a + b*Tan
[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a^2*c*(m + n - 1) - b*
(b*c*(m - 1) + a*d*n) + (2*a*b*c + a^2*d - b^2*d)*(m + n - 1)*Tan[e + f*x]
+ b*(b*c*n + a*d*(2*m + n - 2))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
 0] && GtQ[m, 1] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int (a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}\,dx = \frac{b\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{f} + \int \frac{1}{2}(2a^2c$$

$$= \frac{b\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{f} + \text{Subst}\Big(\int \frac{1}{2}$$

$$= \frac{b\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{f} + \text{Subst}\Big(\int \Big($$

$$= \frac{b\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{f} + \text{Subst}\Big(\int \frac{a}{}$$

$$= \frac{b\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{f} + \text{Subst}\Big(\int \Big($$

$$= \frac{b\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{f} + ((a+ib)^2($$

$$= \frac{\sqrt{b}\,(bc+3ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{d}\,f} + b\sqrt{}$$

$$= -\frac{i(a-ib)^{3/2}\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+}}{\sqrt{a-ib}\sqrt{c+d\tan(e+}}\right.}{f}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1518 vs. $2(258) = 516$.
time = 6.12, size = 1518, normalized size = 5.88

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^(3/2)*Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((-1/2*I)*(-a - I*b)*(-((-a - I*b)*((-2*(-c - I*d)*ArcTanh[(Sqrt[c + I*d]*S
qrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a
```

```
 + I*b]*Sqrt[c + I*d]) - (2*Sqrt[d]*Sqrt[b*c - a*d]*Sqrt[b/((b^2*c)/(b*c -
a*d) - (a*b*d)/(b*c - a*d))]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)
]*ArcSinh[(Sqrt[b]*Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b*c - a*d]*Sqrt[
(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)])]*Sqrt[(b*(c + d*Tan[e + f*x]))/
(b*c - a*d)])/(b^(3/2)*Sqrt[c + d*Tan[e + f*x]]))) - (2*Sqrt[a + b*Tan[e +
f*x]]*Sqrt[c + d*Tan[e + f*x]]*(1 + (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)
*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))^(3/2)*((Sqrt[b*c - a*d]*Sqrt
[(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)]*ArcSinh[(Sqrt[b]*Sqrt[d]*Sqrt[a
 + b*Tan[e + f*x]])/(Sqrt[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*d)/(b*
c - a*d)])])/(2*Sqrt[b]*Sqrt[d]*Sqrt[a + b*Tan[e + f*x]]*(1 + (b*d*(a + b*T
an[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))^(3
/2)) + 1/(2*(1 + (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*
d) - (a*b*d)/(b*c - a*d)))))))/(Sqrt[b/((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c
- a*d))]*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c - a*d)])))/f - ((I/2)*(-a + I*b
)*(-((-a + I*b)*((-2*Sqrt[-c + I*d]*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[
e + f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[-a + I*b] + (2*
Sqrt[d]*Sqrt[b*c - a*d]*Sqrt[b/((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))]
*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)]*ArcSinh[(Sqrt[b]*Sqrt[d]*S
qrt[a + b*Tan[e + f*x]])/(Sqrt[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*d
)/(b*c - a*d)])]*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c - a*d)])/(b^(3/2)*Sqrt[
c + d*Tan[e + f*x]]))) + (2*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x
]]*(1 + (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b
*d)/(b*c - a*d))))^(3/2)*((Sqrt[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*
d)/(b*c - a*d)]*ArcSinh[(Sqrt[b]*Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b*
c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)])])/(2*Sqrt[b]*Sqrt
[d]*Sqrt[a + b*Tan[e + f*x]]*(1 + (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*(
(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))^(3/2)) + 1/(2*(1 + (b*d*(a + b
*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)))))
))/(Sqrt[b/((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))]*Sqrt[(b*(c + d*Tan[
e + f*x]))/(b*c - a*d)])))/f
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+d\tan(fx+e)}\,(a+b\tan(fx+e))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(3/2),x)
[Out] int((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*tan(f*x + e) + a)^(3/2)*sqrt(d*tan(f*x + e) + c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan{(e+fx)})^{\frac{3}{2}}\sqrt{c+d\tan{(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(a+b*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**(3/2)*sqrt(c + d*tan(e + f*x)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\mathrm{tan}\left(e+f\,x\right)\right)}^{3/2}\,\sqrt{c+d\,\mathrm{tan}\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^(3/2)*(c + d*tan(e + f*x))^(1/2),x)
```

```
[Out] int((a + b*tan(e + f*x))^(3/2)*(c + d*tan(e + f*x))^(1/2), x)
```

3.1267 $\int \sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=218

$$-\frac{i\sqrt{a-ib}\,\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f}+\frac{i\sqrt{a+ib}\,\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a-}}{\sqrt{a+ib}\,\sqrt{c+}}\right.}{f}$$

```
[Out] -I*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c+d*tan(f*x+
e))^(1/2))*(a-I*b)^(1/2)*(c-I*d)^(1/2)/f+I*arctanh((c+I*d)^(1/2)*(a+b*tan(f
*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))*(a+I*b)^(1/2)*(c+I*d)^(1
/2)/f+2*arctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x+e))^(1/
2))*b^(1/2)*d^(1/2)/f
```

Rubi [A]
time = 0.69, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {3656, 920, 65, 223, 212, 6857, 95, 214}

$$-\frac{i\sqrt{a-ib}\,\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f}+\frac{i\sqrt{a+ib}\,\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f}+\frac{2\sqrt{b}\,\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((-I)*Sqrt[a - I*b]*Sqrt[c - I*d]*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e +
 f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/f + (I*Sqrt[a + I*b]*Sqr
t[c + I*d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*
Sqrt[c + d*Tan[e + f*x]])])/f + (2*Sqrt[b]*Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[a
+ b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/f
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 920

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Dist[e*(g/c), Int[(d + e*x)^(m - 1)*(f + g*x)^(n - 1), x
], x] + Dist[1/c, Int[Simp[c*d*f - a*e*g + (c*e*f + c*d*g)*x, x]*(d + e*x)^
(m - 1)*((f + g*x)^(n - 1)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[m, 0
] && GtQ[n, 0]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{ac-bd+(bc+ad)x}{\sqrt{a+bx}\,\sqrt{c+dx}\,(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f} +$$

$$= \frac{\text{Subst}\left(\int \left(\frac{-bc-ad+i(ac-bd)}{2(i-x)\sqrt{a+bx}\,\sqrt{c+dx}} + \frac{bc+ad+i(ac-bd}{2(i+x)\sqrt{a+bx}\,\sqrt{c}}\right.\right.}{f}$$

$$= \frac{((ia-b)(c+id))\text{Subst}\left(\int \frac{1}{(i-x)\sqrt{a+bx}\,\sqrt{c+dx}}\,dx, x,\right.}{2f}$$

$$= \frac{2\sqrt{b}\,\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{f} + \frac{((ia-}{}$$

$$= -\frac{i\sqrt{a-ib}\,\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+}}\right.}{f}$$

Mathematica [A]
time = 2.15, size = 261, normalized size = 1.20

$$\frac{i\sqrt{-a+ib}\,\sqrt{-c+id}\,\tanh^{-1}\left(\frac{\sqrt{-c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\,\sqrt{c+d\tan(e+fx)}}\right) + i\sqrt{a+ib}\,\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right) + \frac{2\sqrt{d}\,\sqrt{bc-ad}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{bc-ad}}\right)\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}}{\sqrt{c+d\tan(e+fx)}}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] (I*Sqrt[-a + I*b]*Sqrt[-c + I*d]*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e +
 f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])] + I*Sqrt[a + I*b]*Sqrt[c
 + I*d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqr
t[c + d*Tan[e + f*x]])] + (2*Sqrt[d]*Sqrt[b*c - a*d]*ArcSinh[(Sqrt[d]*Sqrt[
a + b*Tan[e + f*x]])/Sqrt[b*c - a*d]]*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c -
a*d)])/Sqrt[c + d*Tan[e + f*x]])/f
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan(fx+e)}\,\sqrt{c+d\tan(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(b*tan(f*x + e) + a)*sqrt(d*tan(f*x + e) + c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan{(e+fx)}}\,\sqrt{c+d\tan{(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(1/2)*(c+d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x))*sqrt(c + d*tan(e + f*x)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^(1/2)*(c + d*tan(e + f*x))^(1/2),x)
```

```
[Out] \text{Hanged}
```

3.1268 $\int \frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{a+b\tan(e+fx)}}\,dx$

Optimal. Leaf size=163

$$-\frac{i\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a-ib}\,f}+\frac{i\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}\,f}$$

```
[Out] -I*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c+d*tan(f*x+
e))^(1/2))*(c-I*d)^(1/2)/f/(a-I*b)^(1/2)+I*arctanh((c+I*d)^(1/2)*(a+b*tan(f
*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))*(c+I*d)^(1/2)/f/(a+I*b)^
(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {3656, 924, 95, 214}

$$\frac{i\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a+ib}}-\frac{i\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a-ib}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]],x]
```

```
[Out] ((-I)*Sqrt[c - I*d]*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[
a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a - I*b]*f) + (I*Sqrt[c + I*d]*A
rcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*T
an[e + f*x]])])/(Sqrt[a + I*b]*f)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{a+b\tan(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}\,(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{ic-d}{2(i-x)\sqrt{a+bx}\,\sqrt{c+dx}} + \frac{ic+d}{2(i+x)\sqrt{a+bx}\,\sqrt{c+dx}}\right)dx, x, \tan(e\right.}{f}$$

$$= \frac{(ic-d)\text{Subst}\left(\int \frac{1}{(i-x)\sqrt{a+bx}\,\sqrt{c+dx}}\,dx, x, \tan(e+fx)\right)}{2f} + \frac{(ic+d)\text{Subs}}{}$$

$$= \frac{(ic-d)\text{Subst}\left(\int \frac{1}{a+ib-(c+id)x^2}\,dx, x, \frac{\sqrt{a+b\tan(e+fx)}}{\sqrt{c+d\tan(e+fx)}}\right)}{f} + \frac{(ic+d)\text{Subst}\left(\right.}{}$$

$$= -\frac{i\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a-ib}\,f} + \frac{i\sqrt{c+id}\,\tanh^{-1}\left(\right.}{}$$

Mathematica [A]
time = 0.19, size = 167, normalized size = 1.02

$$-\frac{i\left(\frac{\sqrt{-c+id}\,\tanh^{-1}\left(\frac{\sqrt{-c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+ib}} - \frac{\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]],x]
```

```
[Out] ((-I)*((Sqrt[-c + I*d]*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(S
qrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[-a + I*b] - (Sqrt[c + I*d]*A
rcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*T
an[e + f*x]])])/Sqrt[a + I*b]))/f
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 291.36, size = 1278217828, normalized size = 7841827.17

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((2*b*d+2*a*c)^2>0)', see `assume?`
 for mo
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + d \tan{\left(e + fx \right)}}}{\sqrt{a + b \tan{\left(e + fx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)/(a+b*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*tan(e + f*x))/sqrt(a + b*tan(e + f*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(1/2)/(a + b*tan(e + f*x))^(1/2),x)
```

```
[Out] \text{Hanged}
```

3.1269 $\int \frac{\sqrt{c+d\tan(e+fx)}}{(a+b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=206

$$-\frac{i\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{3/2}f}+\frac{i\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{3/2}f}-$$

```
[Out] -I*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c+d*tan(f*x+
e))^(1/2))*(c-I*d)^(1/2)/(a-I*b)^(3/2)/f+I*arctanh((c+I*d)^(1/2)*(a+b*tan(f
*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))*(c+I*d)^(1/2)/(a+I*b)^(3
/2)/f-2*b*(c+d*tan(f*x+e))^(1/2)/(a^2+b^2)/f/(a+b*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.51, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3649, 3697, 3696, 95, 214}

$$-\frac{2b\sqrt{c+d\tan(e+fx)}}{f\left(a^2+b^2\right)\sqrt{a+b\tan(e+fx)}}-\frac{i\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a-ib)^{3/2}}+\frac{i\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a+ib)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Tan[e + f*x]]/(a + b*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-I)*Sqrt[c - I*d]*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[
a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(3/2)*f) + (I*Sqrt[c + I*d]
*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d
*Tan[e + f*x]])])/((a + I*b)^(3/2)*f) - (2*b*Sqrt[c + d*Tan[e + f*x]])/((a^
2 + b^2)*f*Sqrt[a + b*Tan[e + f*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+d\tan(e+fx)}}{(a+b\tan(e+fx))^{3/2}}\,dx = -\frac{2b\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)\,f\sqrt{a+b\tan(e+fx)}} - \frac{2\int \frac{\frac{1}{2}(-ac-bd)+\frac{1}{2}(bc-ad)\tan(e+fx)}{\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+}}}{a^2+b^2}$$

$$= -\frac{2b\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)\,f\sqrt{a+b\tan(e+fx)}} + \frac{(c-id)\int \frac{1+i\tan(e+fx)}{\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\,\mathrm{t}}}}{2(a-ib)}$$

$$= -\frac{2b\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)\,f\sqrt{a+b\tan(e+fx)}} + \frac{(c-id)\mathrm{Subst}\left(\int \frac{1}{(1-ix)\sqrt{a+bx}\,\sqrt{c+dx}}\right.}{2(a-ib)f}$$

$$= -\frac{2b\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)\,f\sqrt{a+b\tan(e+fx)}} + \frac{(c-id)\mathrm{Subst}\left(\int \frac{1}{ia+b-(ic+d)x^2}\,dx, x, \frac{\sqrt{a}}{\sqrt{c}}\right.}{(a-ib)f}$$

$$= -\frac{i\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{3/2}f} + \frac{i\sqrt{c+id}\,\tanh^{-1}\left(\cdots\right.}{(}$$

Mathematica [A]
time = 1.74, size = 224, normalized size = 1.09

$$\frac{\frac{i\sqrt{-c+id}\,\tanh^{-1}\left(\frac{\sqrt{-c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(-a+ib)^{3/2}} + \frac{\frac{(ia+b)\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{3/2}} - \frac{2b\sqrt{c+d\tan(e+fx)}}{(a+ib)\sqrt{a+b\tan(e+fx)}}}{a-ib}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Tan[e + f*x]]/(a + b*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((I*Sqrt[-c + I*d]*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[
-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(-a + I*b)^(3/2) + (((I*a + b)*Sqrt[c
 + I*d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqr
t[c + d*Tan[e + f*x]])])/(a + I*b)^(3/2) - (2*b*Sqrt[c + d*Tan[e + f*x]])/(
(a + I*b)*Sqrt[a + b*Tan[e + f*x]]))/(a - I*b))/f
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+d\tan\left(fx+e\right)}}{\left(a+b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^(3/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((2*b*d+2*a*c)^2>0)', see `assume?`
 for mo
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + d\tan{(e + fx)}}}{(a + b\tan{(e + fx)})^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)/(a+b*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(c + d*tan(e + f*x))/(a + b*tan(e + f*x))**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c+d\tan\left(e+f\,x\right)}}{{\left(a+b\tan\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(1/2)/(a + b*tan(e + f*x))^(3/2),x)
```

```
[Out] int((c + d*tan(e + f*x))^(1/2)/(a + b*tan(e + f*x))^(3/2), x)
```

3.1270 $\int \frac{\sqrt{c+d\tan(e+fx)}}{(a+b\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=280

$$-\frac{i\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{5/2}f}+\frac{i\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{5/2}f}$$

```
[Out] -I*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c+d*tan(f*x+
e))^(1/2))*(c-I*d)^(1/2)/(a-I*b)^(5/2)/f+I*arctanh((c+I*d)^(1/2)*(a+b*tan(f
*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))*(c+I*d)^(1/2)/(a+I*b)^(5
/2)/f-2/3*b*(-5*a^2*d+6*a*b*c+b^2*d)*(c+d*tan(f*x+e))^(1/2)/(a^2+b^2)^2/(-a
*d+b*c)/f/(a+b*tan(f*x+e))^(1/2)-2/3*b*(c+d*tan(f*x+e))^(1/2)/(a^2+b^2)/f/(
a+b*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.81, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3649, 3730, 3697, 3696, 95, 214}

$$-\frac{2b\left(-5a^2d+6abc+b^2d\right)\sqrt{c+d\tan(e+fx)}}{3f\left(a^2+b^2\right)^2(bc-ad)\sqrt{a+b\tan(e+fx)}}-\frac{2b\sqrt{c+d\tan(e+fx)}}{3f\left(a^2+b^2\right)(a+b\tan(e+fx))^{3/2}}-\frac{i\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a-ib)^{5/2}}+\frac{i\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a+ib)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Tan[e + f*x]]/(a + b*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-I)*Sqrt[c - I*d]*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[
a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(5/2)*f) + (I*Sqrt[c + I*d]
*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d
*Tan[e + f*x]])])/((a + I*b)^(5/2)*f) - (2*b*Sqrt[c + d*Tan[e + f*x]])/(3*(
a^2 + b^2)*f*(a + b*Tan[e + f*x])^(3/2)) - (2*b*(6*a*b*c - 5*a^2*d + b^2*d)
*Sqrt[c + d*Tan[e + f*x]])/(3*(a^2 + b^2)^2*(b*c - a*d)*f*Sqrt[a + b*Tan[e
+ f*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{\sqrt{c+d\tan(e+fx)}}{(a+b\tan(e+fx))^{5/2}}\,dx = -\frac{2b\sqrt{c+d\tan(e+fx)}}{3(a^2+b^2)f(a+b\tan(e+fx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}(-3ac-bd)+\frac{3}{2}(bc-ad)\tan(e+fx)+bd\tan^2}{(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+}}}{3(a^2+b^2)}$$

$$= -\frac{2b\sqrt{c+d\tan(e+fx)}}{3(a^2+b^2)f(a+b\tan(e+fx))^{3/2}} - \frac{2b(6abc-5a^2d+b^2d)\sqrt{c+d\tan(e}}{3(a^2+b^2)^2(bc-ad)f\sqrt{a+b\tan(e}}$$

$$= -\frac{2b\sqrt{c+d\tan(e+fx)}}{3(a^2+b^2)f(a+b\tan(e+fx))^{3/2}} - \frac{2b(6abc-5a^2d+b^2d)\sqrt{c+d\tan(e}}{3(a^2+b^2)^2(bc-ad)f\sqrt{a+b\tan(e}}$$

$$= -\frac{2b\sqrt{c+d\tan(e+fx)}}{3(a^2+b^2)f(a+b\tan(e+fx))^{3/2}} - \frac{2b(6abc-5a^2d+b^2d)\sqrt{c+d\tan(e}}{3(a^2+b^2)^2(bc-ad)f\sqrt{a+b\tan(e}}$$

$$= -\frac{2b\sqrt{c+d\tan(e+fx)}}{3(a^2+b^2)f(a+b\tan(e+fx))^{3/2}} - \frac{2b(6abc-5a^2d+b^2d)\sqrt{c+d\tan(e}}{3(a^2+b^2)^2(bc-ad)f\sqrt{a+b\tan(e}}$$

$$= -\frac{i\sqrt{c-id}\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{5/2}f} + \frac{i\sqrt{c+id}\tanh^{-1}\Big(}{}$$

Mathematica [A]
time = 4.86, size = 266, normalized size = 0.95

$$\frac{-\frac{3i\sqrt{-c+id}\tanh^{-1}\left(\frac{\sqrt{-c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(-a+ib)^{5/2}}+\frac{3i\sqrt{c+id}\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{5/2}}+\frac{2b\sqrt{c+d\tan(e+fx)}\,(7a^2bc+b^3c-6a^3d+b(6abc-5a^2d+b^2d)\tan(e+fx))}{(a^2+b^2)^2(-bc+ad)(a+b\tan(e+fx))^{3/2}}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Tan[e + f*x]]/(a + b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (((-3*I)*Sqrt[-c + I*d]*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(
Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(-a + I*b)^(5/2) + ((3*I)*Sqrt[c
 + I*d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqr
t[c + d*Tan[e + f*x]])])/(a + I*b)^(5/2) + (2*b*Sqrt[c + d*Tan[e + f*x]]*(7
*a^2*b*c + b^3*c - 6*a^3*d + b*(6*a*b*c - 5*a^2*d + b^2*d)*Tan[e + f*x]))/(
(a^2 + b^2)^2*(-(b*c) + a*d)*(a + b*Tan[e + f*x])^(3/2)))/(3*f)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+d\tan(fx+e)}}{(a+b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^(5/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((2*b*d+2*a*c)^2>0)', see `assume?`
 for mo
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + d\tan{(e + fx)}}}{(a + b\tan{(e + fx)})^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)/(a+b*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(c + d*tan(e + f*x))/(a + b*tan(e + f*x))**(5/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(1/2)/(a + b*tan(e + f*x))^(5/2),x)
```

```
[Out] \text{Hanged}
```

3.1271 $\int (a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=330

$$-\frac{i(a-ib)^{3/2}(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f}+\frac{i(a+ib)^{3/2}(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+id}}{\sqrt{a+ib}}\right.}{f}$$

```
[Out] -I*(a-I*b)^(3/2)*(c-I*d)^(3/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)
/(a-I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/f+I*(a+I*b)^(3/2)*(c+I*d)^(3/2)*arct
anh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/
2))/f+1/4*(18*a*b*c*d+3*a^2*d^2+b^2*(3*c^2-8*d^2))*arctanh(d^(1/2)*(a+b*tan
(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x+e))^(1/2))/f/b^(1/2)/d^(1/2)+1/4*(5*a*d
+3*b*c)*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/f+1/2*b*(a+b*tan(f*x+
e))^(1/2)*(c+d*tan(f*x+e))^(3/2)/f
```

Rubi [A]
time = 3.19, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {3651, 3728, 3736, 6857, 65, 223, 212, 95, 214}

$$\frac{(3a^2d^2+18abcd+b^2(3c^2-8d^2))\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{4\sqrt{b}\,\sqrt{d}\,f}+\frac{b\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}}{2f}+\frac{(5ad+3bc)\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{4f}-\frac{i(a-ib)^{3/2}(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f}+\frac{i(a+ib)^{3/2}(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-I)*(a - I*b)^(3/2)*(c - I*d)^(3/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan
[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/f + (I*(a + I*b)^(3/
2)*(c + I*d)^(3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a
 + I*b]*Sqrt[c + d*Tan[e + f*x]])])/f + ((18*a*b*c*d + 3*a^2*d^2 + b^2*(3*c
^2 - 8*d^2))*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d
*Tan[e + f*x]])])/(4*Sqrt[b]*Sqrt[d]*f) + ((3*b*c + 5*a*d)*Sqrt[a + b*Tan[e
 + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(4*f) + (b*Sqrt[a + b*Tan[e + f*x]]*(c +
 d*Tan[e + f*x])^(3/2))/(2*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3651

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m - 1)*((c +
d*Tan[e + f*x])^n/(f*(m + n - 1))), x] + Dist[1/(m + n - 1), Int[(a + b*Tan
[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a^2*c*(m + n - 1) - b*
(b*c*(m - 1) + a*d*n) + (2*a*b*c + a^2*d - b^2*d)*(m + n - 1)*Tan[e + f*x]
+ b*(b*c*n + a*d*(2*m + n - 2))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
 0] && GtQ[m, 1] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int (a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}\,dx = \frac{b\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}}{2f} + \frac{1}{2}\int \sqrt{}$$

$$= \frac{(3bc+5ad)\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{4f}$$

$$= \frac{(3bc+5ad)\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{4f}$$

$$= \frac{(3bc+5ad)\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{4f}$$

$$= \frac{(3bc+5ad)\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{4f}$$

$$= \frac{(3bc+5ad)\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{4f}$$

$$= \frac{(3bc+5ad)\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{4f}$$

$$= \frac{(18abcd+3a^2d^2+b^2(3c^2-8d^2))\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b}}{\sqrt{b}\,\sqrt{c+d}}\right.}{4\sqrt{b}\,\sqrt{d}\,f}$$

$$= -\frac{i(a-ib)^{3/2}(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(}}{\sqrt{a-ib}\,\sqrt{c+d\tan(}}\right.}{f}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2566 vs. $2(330) = 660$.
time = 6.14, size = 2566, normalized size = 7.78

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-1/2*I)*(-a - I*b)*(-((-a - I*b)*(-((-c - I*d)*((-2*(-c - I*d)*ArcTanh[(S
qrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*
x]])])/(Sqrt[a + I*b]*Sqrt[c + I*d]) - (2*Sqrt[d]*Sqrt[b*c - a*d]*Sqrt[b/((
b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*
d)/(b*c - a*d)]*ArcSinh[(Sqrt[b]*Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b*
c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)])]*Sqrt[(b*(c + d*T
an[e + f*x]))/(b*c - a*d)])/(b^(3/2)*Sqrt[c + d*Tan[e + f*x]]))) - (2*d*Sqr
t[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]]*(1 + (b*d*(a + b*Tan[e + f*x
]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))^(3/2)*((Sqrt
[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)]*ArcSinh[(Sqrt[b
]*Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*
d) - (a*b*d)/(b*c - a*d)])])/(2*Sqrt[b]*Sqrt[d]*Sqrt[a + b*Tan[e + f*x]]*(1
 + (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(
b*c - a*d))))^(3/2)) + 1/(2*(1 + (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((
b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))))/(b*Sqrt[b/((b^2*c)/(b*c - a*
d) - (a*b*d)/(b*c - a*d))]*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c - a*d)]))) -
(2*(b*c - a*d)*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]]*(1 + (b*d*
(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*
d))))^(5/2)*((3*Sqrt[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a
*d)]*ArcSinh[(Sqrt[b]*Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b*c - a*d]*Sq
rt[(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)])])/(8*Sqrt[b]*Sqrt[d]*Sqrt[a
+ b*Tan[e + f*x]]*(1 + (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*
c - a*d) - (a*b*d)/(b*c - a*d))))^(5/2)) + (3/(2*(1 + (b*d*(a + b*Tan[e + f
*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))^2) + (1 +
(b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c
 - a*d))))^(-1))/4))/(b*(b/((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)))^(3/
2)*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c - a*d)])))/f - ((I/2)*(-a + I*b)*(-((
-a + I*b)*(-((-c + I*d)*((-2*Sqrt[-c + I*d]*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a
+ b*Tan[e + f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[-a + I*
b] + (2*Sqrt[d]*Sqrt[b*c - a*d]*Sqrt[b/((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c
- a*d))]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)]*ArcSinh[(Sqrt[b]*S
qrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d)
- (a*b*d)/(b*c - a*d)])]*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c - a*d)])/(b^(3/
2)*Sqrt[c + d*Tan[e + f*x]]))) + (2*d*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*T
an[e + f*x]]*(1 + (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a
*d) - (a*b*d)/(b*c - a*d))))^(3/2)*((Sqrt[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*
d) - (a*b*d)/(b*c - a*d)]*ArcSinh[(Sqrt[b]*Sqrt[d]*Sqrt[a + b*Tan[e + f*x]]
)/(Sqrt[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)])])/(2*Sq
rt[b]*Sqrt[d]*Sqrt[a + b*Tan[e + f*x]]*(1 + (b*d*(a + b*Tan[e + f*x]))/((b*
c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))^(3/2)) + 1/(2*(1 + (
b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c
- a*d))))))/(b*Sqrt[b/((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))]*Sqrt[(b
*(c + d*Tan[e + f*x]))/(b*c - a*d)]))) + (2*(b*c - a*d)*Sqrt[a + b*Tan[e +
f*x]]*Sqrt[c + d*Tan[e + f*x]]*(1 + (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)
*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))^(5/2)*((3*Sqrt[b*c - a*d]*Sq
```

```
rt[(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)]*ArcSinh[(Sqrt[b]*Sqrt[d]*Sqrt
[a + b*Tan[e + f*x]])/(Sqrt[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*d)/(
b*c - a*d)])])/(8*Sqrt[b]*Sqrt[d]*Sqrt[a + b*Tan[e + f*x]]*(1 + (b*d*(a + b
*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))^
(5/2)) + (3/(2*(1 + (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c -
 a*d) - (a*b*d)/(b*c - a*d))))^2) + (1 + (b*d*(a + b*Tan[e + f*x]))/((b*c -
 a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))^(-1))/4))/(b*(b/((b^2*c
)/(b*c - a*d) - (a*b*d)/(b*c - a*d)))^(3/2)*Sqrt[(b*(c + d*Tan[e + f*x]))/(
b*c - a*d)])))/f
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan\left(fx+e\right)\right)^{\frac{3}{2}}\left(c+d\tan\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(3/2),x)
[Out] int((a+b*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(3/2),x, algorithm="maxim
a")
[Out] integrate((b*tan(f*x + e) + a)^(3/2)*(d*tan(f*x + e) + c)^(3/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(3/2),x, algorithm="frica
s")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan\left(e+fx\right)\right)^{\frac{3}{2}}\left(c+d\tan\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(3/2)*(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**(3/2)*(c + d*tan(e + f*x))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\tan(e + f\,x))^{3/2}\,(c + d\tan(e + f\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^(3/2)*(c + d*tan(e + f*x))^(3/2),x)
```

```
[Out] int((a + b*tan(e + f*x))^(3/2)*(c + d*tan(e + f*x))^(3/2), x)
```

3.1272 $\int \sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=258

$$-\frac{i\sqrt{a-ib}\,(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f}+\frac{i\sqrt{a+ib}\,(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{}}{\sqrt{a+ib}\,\sqrt{}}\right.}{f}$$

```
[Out] -I*(c-I*d)^(3/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)
/(c+d*tan(f*x+e))^(1/2))*(a-I*b)^(1/2)/f+I*(c+I*d)^(3/2)*arctanh((c+I*d)^(1
/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))*(a+I*b)^(1
/2)/f+(a*d+3*b*c)*arctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f
*x+e))^(1/2))*d^(1/2)/f/b^(1/2)+d*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(
1/2)/f
```

Rubi [A]
time = 1.57, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {3651, 3736, 6857, 65, 223, 212, 95, 214}

$$\frac{d\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{f}-\frac{i\sqrt{a-ib}\,(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f}+\frac{i\sqrt{a+ib}\,(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f}+\frac{\sqrt{d}\,(ad+3bc)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{b}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-I)*Sqrt[a - I*b]*(c - I*d)^(3/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
 + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/f + (I*Sqrt[a + I*b]*(
c + I*d)^(3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I
*b]*Sqrt[c + d*Tan[e + f*x]])])/f + (Sqrt[d]*(3*b*c + a*d)*ArcTanh[(Sqrt[d]
*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[b]*f)
 + (d*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/f
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3651

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m - 1)*((c +
d*Tan[e + f*x])^n/(f*(m + n - 1))), x] + Dist[1/(m + n - 1), Int[(a + b*Tan
[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a^2*c*(m + n - 1) - b*
(b*c*(m - 1) + a*d*n) + (2*a*b*c + a^2*d - b^2*d)*(m + n - 1)*Tan[e + f*x]
+ b*(b*c*n + a*d*(2*m + n - 2))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
 0] && GtQ[m, 1] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}\,dx = \frac{d\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{f} + \int \frac{\frac{1}{2}(2ac}{}$$

$$= \frac{d\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{f} + \frac{\text{Subst}\Big(\int}{}$$

$$= \frac{d\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{f} + \frac{\text{Subst}\Big(\int}{}$$

$$= \frac{d\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{f} + \frac{\text{Subst}\Big(\int}{}$$

$$= \frac{d\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{f} + \frac{\text{Subst}\Big(\int}{}$$

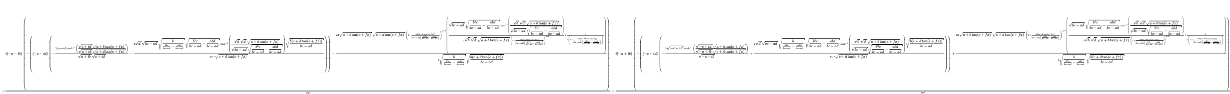
$$= \frac{d\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{f} + \frac{((ia+b)}{}$$

$$= \frac{\sqrt{d}\,(3bc+ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{b}\,f} + $$

$$= -\frac{i\sqrt{a-ib}\,(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e}{\sqrt{a-ib}\,\sqrt{c+d\tan(e}\right.}{f}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1526 vs. $2(258) = 516$.

time = 6.09, size = 1526, normalized size = 5.91



Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-1/2*I)*(-a - I*b)*(-((-c - I*d)*((-2*(-c - I*d)*ArcTanh[(Sqrt[c + I*d]*S
qrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a
```

```
 + I*b]*Sqrt[c + I*d]) - (2*Sqrt[d]*Sqrt[b*c - a*d]*Sqrt[b/((b^2*c)/(b*c -
a*d) - (a*b*d)/(b*c - a*d))]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)
]*ArcSinh[(Sqrt[b]*Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b*c - a*d]*Sqrt[
(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)])]*Sqrt[(b*(c + d*Tan[e + f*x]))/
(b*c - a*d)])/(b^(3/2)*Sqrt[c + d*Tan[e + f*x]]))) - (2*d*Sqrt[a + b*Tan[e
+ f*x]]*Sqrt[c + d*Tan[e + f*x]]*(1 + (b*d*(a + b*Tan[e + f*x]))/((b*c - a*
d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))^(3/2)*((Sqrt[b*c - a*d]*Sq
rt[(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)]*ArcSinh[(Sqrt[b]*Sqrt[d]*Sqrt
[a + b*Tan[e + f*x]])/(Sqrt[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*d)/(
b*c - a*d)])])/(2*Sqrt[b]*Sqrt[d]*Sqrt[a + b*Tan[e + f*x]]*(1 + (b*d*(a + b
*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))^
(3/2)) + 1/(2*(1 + (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c -
a*d) - (a*b*d)/(b*c - a*d)))))))/(b*Sqrt[b/((b^2*c)/(b*c - a*d) - (a*b*d)/(
b*c - a*d))]*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c - a*d)])))/f - ((I/2)*(-a +
 I*b)*(-((-c + I*d)*((-2*Sqrt[-c + I*d]*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*
Tan[e + f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[-a + I*b] +
 (2*Sqrt[d]*Sqrt[b*c - a*d]*Sqrt[b/((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*
d))]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)]*ArcSinh[(Sqrt[b]*Sqrt[
d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) - (a
*b*d)/(b*c - a*d)])]*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c - a*d)])/(b^(3/2)*S
qrt[c + d*Tan[e + f*x]]))) + (2*d*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e
 + f*x]]*(1 + (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d)
- (a*b*d)/(b*c - a*d))))^(3/2)*((Sqrt[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) -
 (a*b*d)/(b*c - a*d)]*ArcSinh[(Sqrt[b]*Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(S
qrt[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)])])/(2*Sqrt[b
]*Sqrt[d]*Sqrt[a + b*Tan[e + f*x]]*(1 + (b*d*(a + b*Tan[e + f*x]))/((b*c -
a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))^(3/2)) + 1/(2*(1 + (b*d*
(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*
d)))))))/(b*Sqrt[b/((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))]*Sqrt[(b*(c
+ d*Tan[e + f*x]))/(b*c - a*d)])))/f
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 305.83, size = 1613618305, normalized size = 6254334.52

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(b*tan(f*x + e) + a)*(d*tan(f*x + e) + c)^(3/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan{(e+fx)}}\,(c+d\tan{(e+fx)})^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(1/2)*(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x))*(c + d*tan(e + f*x))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+b\,\tan\left(e+f\,x\right)}\,{\left(c+d\,\mathrm{tan}\left(e+f\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^(1/2)*(c + d*tan(e + f*x))^(3/2),x)
```

```
[Out] int((a + b*tan(e + f*x))^(1/2)*(c + d*tan(e + f*x))^(3/2), x)
```

3.1273 $\int \frac{(c+d\tan(e+fx))^{3/2}}{\sqrt{a+b\tan(e+fx)}}\,dx$

Optimal. Leaf size=218

$$-\frac{i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a-ib}\,f}+\frac{i(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}\,f}$$

```
[Out] -I*(c-I*d)^(3/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)
/(c+d*tan(f*x+e))^(1/2))/f/(a-I*b)^(1/2)+I*(c+I*d)^(3/2)*arctanh((c+I*d)^(1
/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/f/(a+I*b)^
(1/2)+2*d^(3/2)*arctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x
+e))^(1/2))/f/b^(1/2)
```

Rubi [A]
time = 0.82, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {3656, 924, 65, 223, 212, 6857, 95, 214}

$$\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{b}\,f}-\frac{i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a-ib}}+\frac{i(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a+ib}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^(3/2)/Sqrt[a + b*Tan[e + f*x]],x]
```

```
[Out] ((-I)*(c - I*d)^(3/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqr
t[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a - I*b]*f) + (I*(c + I*d)^(3/
2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c +
 d*Tan[e + f*x]])])/(Sqrt[a + I*b]*f) + (2*d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a
+ b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[b]*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{3/2}}{\sqrt{a+b\tan(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{\sqrt{a+bx}\,(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{d^2}{\sqrt{a+bx}\,\sqrt{c+dx}} + \frac{c^2-d^2+2cdx}{\sqrt{a+bx}\,\sqrt{c+dx}\,(1+x^2)}\right) dx, x, \tan(e+fx\right.}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{c^2-d^2+2cdx}{\sqrt{a+bx}\,\sqrt{c+dx}\,(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f} + \frac{d^2\text{Subst}\left(\int \frac{}{\sqrt{a+b}}\right.}{}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{-2cd+i(c^2-d^2)}{2(i-x)\sqrt{a+bx}\,\sqrt{c+dx}} + \frac{2cd+i(c^2-d^2)}{2(i+x)\sqrt{a+bx}\,\sqrt{c+dx}}\right) dx, x, \tan(e\right.}{f}$$

$$= \frac{(i(c-id)^2)\,\text{Subst}\left(\int \frac{1}{(i+x)\sqrt{a+bx}\,\sqrt{c+dx}}\,dx, x, \tan(e+fx)\right)}{2f} + \frac{(i(c+id}{}$$

$$= \frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{b}\,f} + \frac{(i(c-id)^2)\,\text{Subst}\left(\int \frac{1}{-a+ib-(-c+}\right.}{}$$

$$= -\frac{i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a-ib}\,f} + \frac{i(c+id)^{3/2}\tanh^{-1}}{}$$

Mathematica [A]

time = 1.67, size = 292, normalized size = 1.34

$$\frac{\frac{i(-c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{-c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+ib}} + \frac{ib(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)\sqrt{c+d\tan(e+fx)} + 2\sqrt{a+ib}\,d^{3/2}\sqrt{bc-ad}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{bc-ad}}\right)\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}}{\sqrt{a+ib}\,b\sqrt{c+d\tan(e+fx)}}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^(3/2)/Sqrt[a + b*Tan[e + f*x]],x]
```

```
[Out] ((I*(-c + I*d)^(3/2)*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqr
t[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[-a + I*b] + (I*b*(c + I*d)^(3/
2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c +
 d*Tan[e + f*x]])]*Sqrt[c + d*Tan[e + f*x]] + 2*Sqrt[a + I*b]*d^(3/2)*Sqrt[
b*c - a*d]*ArcSinh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/Sqrt[b*c - a*d]]*Sqrt
```

```
[(b*(c + d*Tan[e + f*x]))/(b*c - a*d)])/(Sqrt[a + I*b]*b*Sqrt[c + d*Tan[e +
 f*x]]))/f
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+d\tan\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{a+b\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(3/2)/(a+b*tan(f*x+e))^(1/2),x)
[Out] int((c+d*tan(f*x+e))^(3/2)/(a+b*tan(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+b*tan(f*x+e))^(1/2),x, algorithm="maxim
a")
[Out] integrate((d*tan(f*x + e) + c)^(3/2)/sqrt(b*tan(f*x + e) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+b*tan(f*x+e))^(1/2),x, algorithm="frica
s")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+d\tan\left(e+fx\right)\right)^{\frac{3}{2}}}{\sqrt{a+b\tan\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(3/2)/(a+b*tan(f*x+e))**(1/2),x)
[Out] Integral((c + d*tan(e + f*x))**(3/2)/sqrt(a + b*tan(e + f*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+b*tan(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\tan(e+fx))^{3/2}}{\sqrt{a+b\tan(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(3/2)/(a + b*tan(e + f*x))^(1/2),x)
```

```
[Out] int((c + d*tan(e + f*x))^(3/2)/(a + b*tan(e + f*x))^(1/2), x)
```

3.1274 $\int \frac{(c+d\tan(e+fx))^{3/2}}{(a+b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=213

$$-\frac{i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{3/2}f}+\frac{i(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{3/2}f}$$

```
[Out] -I*(c-I*d)^(3/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)
/(c+d*tan(f*x+e))^(1/2))/(a-I*b)^(3/2)/f+I*(c+I*d)^(3/2)*arctanh((c+I*d)^(1
/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(a+I*b)^(3
/2)/f-2*(-a*d+b*c)*(c+d*tan(f*x+e))^(1/2)/(a^2+b^2)/f/(a+b*tan(f*x+e))^(1/2
)
```

Rubi [A]
time = 0.56, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3648, 3697, 3696, 95, 214}

$$-\frac{2(bc-ad)\sqrt{c+d\tan(e+fx)}}{f\left(a^2+b^2\right)\sqrt{a+b\tan(e+fx)}}-\frac{i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a-ib)^{3/2}}+\frac{i(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a+ib)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^(3/2)/(a + b*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-I)*(c - I*d)^(3/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqr
t[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(3/2)*f) + (I*(c + I*d)^(
3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c
 + d*Tan[e + f*x]])])/((a + I*b)^(3/2)*f) - (2*(b*c - a*d)*Sqrt[c + d*Tan[e
 + f*x]])/((a^2 + b^2)*f*Sqrt[a + b*Tan[e + f*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{3/2}}{(a+b\tan(e+fx))^{3/2}}\,dx = -\frac{2(bc-ad)\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)\,f\sqrt{a+b\tan(e+fx)}} - \frac{2\int \frac{\frac{1}{2}(-2bcd-a(c^2-d^2))-\frac{1}{2}(2acd-b(c^2-d^2))\tan}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e}}}{a^2+b^2}$$

$$= -\frac{2(bc-ad)\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)\,f\sqrt{a+b\tan(e+fx)}} + \frac{(c-id)^2\int \frac{1+i\tan(e+fx)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+}}}{2(a-ib)}$$

$$= -\frac{2(bc-ad)\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)\,f\sqrt{a+b\tan(e+fx)}} + \frac{(c-id)^2\text{Subst}\left(\int \frac{1}{(1-ix)\sqrt{a+bx}\sqrt{c+}}\right.}{2(a-ib)f}$$

$$= -\frac{2(bc-ad)\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)\,f\sqrt{a+b\tan(e+fx)}} + \frac{(c-id)^2\text{Subst}\left(\int \frac{1}{ia+b-(ic+d)x^2}\,dx, x, \frac{\sqrt{}}{\sqrt{}}\right.}{(a-ib)f}$$

$$= -\frac{i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{3/2}f} + \frac{i(c+id)^{3/2}\tanh^{-}}{}$$

Mathematica [A]
time = 1.93, size = 231, normalized size = 1.08

$$\frac{-\frac{i(-c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{-c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(-a+ib)^{3/2}} + \frac{\frac{(ia+b)(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{3/2}} + \frac{2(-bc+ad)\sqrt{c+d\tan(e+fx)}}{(a+ib)\sqrt{a+b\tan(e+fx)}}}{a-ib}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^(3/2)/(a + b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (((-I)*(-c + I*d)^(3/2)*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(
Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(-a + I*b)^(3/2) + (((I*a + b)*(
c + I*d)^(3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I
*b]*Sqrt[c + d*Tan[e + f*x]])])/(a + I*b)^(3/2) + (2*(-(b*c) + a*d)*Sqrt[c
+ d*Tan[e + f*x]])/((a + I*b)*Sqrt[a + b*Tan[e + f*x]]))/(a - I*b))/f
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\tan(fx+e))^{\frac{3}{2}}}{(a+b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(3/2)/(a+b*tan(f*x+e))^(3/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(3/2)/(a+b*tan(f*x+e))^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+b*tan(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((2*b*d+2*a*c)^2>0)', see `assume?`
 for mo
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+b*tan(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d\tan{(e + fx)})^{\frac{3}{2}}}{(a + b\tan{(e + fx)})^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(3/2)/(a+b*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(3/2)/(a + b*tan(e + f*x))**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+b*tan(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\tan(e+f\,x))^{3/2}}{(a+b\tan\left(e+f\,x\right))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(3/2)/(a + b*tan(e + f*x))^(3/2),x)
```

```
[Out] int((c + d*tan(e + f*x))^(3/2)/(a + b*tan(e + f*x))^(3/2), x)
```

3.1275 $\int \frac{(c+d\tan(e+fx))^{3/2}}{(a+b\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=277

$$-\frac{i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{5/2}f}+\frac{i(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{5/2}f}$$

```
[Out] -I*(c-I*d)^(3/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)
/(c+d*tan(f*x+e))^(1/2))/(a-I*b)^(5/2)/f+I*(c+I*d)^(3/2)*arctanh((c+I*d)^(1
/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(a+I*b)^(5
/2)/f-4/3*(-a^2*d+3*a*b*c+2*b^2*d)*(c+d*tan(f*x+e))^(1/2)/(a^2+b^2)^2/f/(a+
b*tan(f*x+e))^(1/2)-2/3*(-a*d+b*c)*(c+d*tan(f*x+e))^(1/2)/(a^2+b^2)/f/(a+b*
tan(f*x+e))^(3/2)
```

Rubi [A]

time = 0.89, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3648, 3730, 3697, 3696, 95, 214}

$$-\frac{4\left(a^2(-d)+3abc+2b^2d\right)\sqrt{c+d\tan(e+fx)}}{3f\left(a^2+b^2\right)^2\sqrt{a+b\tan(e+fx)}}-\frac{2(bc-ad)\sqrt{c+d\tan(e+fx)}}{3f\left(a^2+b^2\right)(a+b\tan(e+fx))^{3/2}}-\frac{i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a-ib)^{5/2}}+\frac{i(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a+ib)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^(3/2)/(a + b*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-I)*(c - I*d)^(3/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqr
t[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(5/2)*f) + (I*(c + I*d)^(
3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c
 + d*Tan[e + f*x]])])/((a + I*b)^(5/2)*f) - (2*(b*c - a*d)*Sqrt[c + d*Tan[e
 + f*x]])/(3*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^(3/2)) - (4*(3*a*b*c - a^2*
d + 2*b^2*d)*Sqrt[c + d*Tan[e + f*x]])/(3*(a^2 + b^2)^2*f*Sqrt[a + b*Tan[e
+ f*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{3/2}}{(a+b\tan(e+fx))^{5/2}}\,dx = -\frac{2(bc-ad)\sqrt{c+d\tan(e+fx)}}{3\left(a^2+b^2\right)f(a+b\tan(e+fx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(-4bcd-a\left(3c^2-d^2\right)\right)-\frac{3}{2}\left(2acd-b\left(c^2-d^2\right)\right)\,\text{ta}}{(a+b\tan(e+fx))^{3/2}\sqrt{c+d}}}{3\left(a^2+b^2\right)}$$

$$= -\frac{2(bc-ad)\sqrt{c+d\tan(e+fx)}}{3\left(a^2+b^2\right)f(a+b\tan(e+fx))^{3/2}} - \frac{4(3abc-a^2d+2b^2d)\sqrt{c+d\tan(e+}}{3\left(a^2+b^2\right)^2 f\sqrt{a+b\tan(e+fx)}}$$

$$= -\frac{2(bc-ad)\sqrt{c+d\tan(e+fx)}}{3\left(a^2+b^2\right)f(a+b\tan(e+fx))^{3/2}} - \frac{4(3abc-a^2d+2b^2d)\sqrt{c+d\tan(e+}}{3\left(a^2+b^2\right)^2 f\sqrt{a+b\tan(e+fx)}}$$

$$= -\frac{2(bc-ad)\sqrt{c+d\tan(e+fx)}}{3\left(a^2+b^2\right)f(a+b\tan(e+fx))^{3/2}} - \frac{4(3abc-a^2d+2b^2d)\sqrt{c+d\tan(e+}}{3\left(a^2+b^2\right)^2 f\sqrt{a+b\tan(e+fx)}}$$

$$= -\frac{2(bc-ad)\sqrt{c+d\tan(e+fx)}}{3\left(a^2+b^2\right)f(a+b\tan(e+fx))^{3/2}} - \frac{4(3abc-a^2d+2b^2d)\sqrt{c+d\tan(e+}}{3\left(a^2+b^2\right)^2 f\sqrt{a+b\tan(e+fx)}}$$

$$= -\frac{i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{5/2}f} + \frac{i(c+id)^{3/2}\tanh^{-1}}{}$$

Mathematica [A]
time = 3.35, size = 264, normalized size = 0.95

$$\frac{\frac{3i(-c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{-c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(-a+ib)^{5/2}} + \frac{3i(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{5/2}} - \frac{2\sqrt{c+d\tan(e+fx)}\left(7a^2bc+b^3c-3a^3d+3ab^2d+2b(3abc-a^2d+2b^2d)\tan(e+fx)\right)}{(a^2+b^2)^2(a+b\tan(e+fx))^{3/2}}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^(3/2)/(a + b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (((3*I)*(-c + I*d)^(3/2)*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x]])/
(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(-a + I*b)^(5/2) + ((3*I)*(c +
I*d)^(3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*
Sqrt[c + d*Tan[e + f*x]])])/(a + I*b)^(5/2) - (2*Sqrt[c + d*Tan[e + f*x]]*(
7*a^2*b*c + b^3*c - 3*a^3*d + 3*a*b^2*d + 2*b*(3*a*b*c - a^2*d + 2*b^2*d)*T
an[e + f*x]))/((a^2 + b^2)^2*(a + b*Tan[e + f*x])^(3/2)))/(3*f)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+d\tan\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a+b\tan\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(3/2)/(a+b*tan(f*x+e))^(5/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(3/2)/(a+b*tan(f*x+e))^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+b*tan(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((2*b*d+2*a*c)^2>0)', see `assume?`
 for mo
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+b*tan(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d \tan(e + fx))^{\frac{3}{2}}}{(a + b \tan(e + fx))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(3/2)/(a+b*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(3/2)/(a + b*tan(e + f*x))**(5/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+b*tan(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\tan(e+f\,x))^{3/2}}{(a+b\tan(e+f\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(3/2)/(a + b*tan(e + f*x))^(5/2),x)
```

```
[Out] int((c + d*tan(e + f*x))^(3/2)/(a + b*tan(e + f*x))^(5/2), x)
```

3.1276 $\int \frac{(c+d\tan(e+fx))^{3/2}}{(a+b\tan(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=391

$$-\frac{i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{7/2}f}+\frac{i(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{7/2}f}$$

```
[Out] -I*(c-I*d)^(3/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)
/(c+d*tan(f*x+e))^(1/2))/(a-I*b)^(7/2)/f+I*(c+I*d)^(3/2)*arctanh((c+I*d)^(1
/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(a+I*b)^(7
/2)/f+2/15*(50*a^3*b*c*d-70*a*b^3*c*d-8*a^4*d^2-a^2*b^2*(45*c^2-49*d^2)+3*b
^4*(5*c^2-d^2))*(c+d*tan(f*x+e))^(1/2)/(a^2+b^2)^3/(-a*d+b*c)/f/(a+b*tan(f*
x+e))^(1/2)-2/5*(-a*d+b*c)*(c+d*tan(f*x+e))^(1/2)/(a^2+b^2)/f/(a+b*tan(f*x+
e))^(5/2)-4/15*(-2*a^2*d+5*a*b*c+3*b^2*d)*(c+d*tan(f*x+e))^(1/2)/(a^2+b^2)^
2/f/(a+b*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 1.42, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3648, 3730, 3697, 3696, 95, 214}

$$-\frac{4(-2a^2d+5abc+3b^2d)\sqrt{c+d\tan(e+fx)}}{15f(a^2+b^2)^2(a+b\tan(e+fx))^{3/2}}-\frac{2(bc-ad)\sqrt{c+d\tan(e+fx)}}{5f(a^2+b^2)(a+b\tan(e+fx))^{5/2}}+\frac{2(-8a^4d^2+50a^3bcd-a^2b^2(45c^2-49d^2)-70ab^3cd+3b^4(5c^2-d^2))\sqrt{c+d\tan(e+fx)}}{15f(a^2+b^2)^3(bc-ad)\sqrt{a+b\tan(e+fx)}}-\frac{i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a-ib)^{7/2}}+\frac{i(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a+ib)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^(3/2)/(a + b*Tan[e + f*x])^(7/2),x]
```

```
[Out] ((-I)*(c - I*d)^(3/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqr
t[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(7/2)*f) + (I*(c + I*d)^(
3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c
 + d*Tan[e + f*x]])])/((a + I*b)^(7/2)*f) - (2*(b*c - a*d)*Sqrt[c + d*Tan[e
 + f*x]])/(5*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^(5/2)) - (4*(5*a*b*c - 2*a^
2*d + 3*b^2*d)*Sqrt[c + d*Tan[e + f*x]])/(15*(a^2 + b^2)^2*f*(a + b*Tan[e +
 f*x])^(3/2)) + (2*(50*a^3*b*c*d - 70*a*b^3*c*d - 8*a^4*d^2 - a^2*b^2*(45*c
^2 - 49*d^2) + 3*b^4*(5*c^2 - d^2))*Sqrt[c + d*Tan[e + f*x]])/(15*(a^2 + b^
2)^3*(b*c - a*d)*f*Sqrt[a + b*Tan[e + f*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
```

```
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{3/2}}{(a+b\tan(e+fx))^{7/2}}\,dx = -\frac{2(bc-ad)\sqrt{c+d\tan(e+fx)}}{5\left(a^2+b^2\right)f(a+b\tan(e+fx))^{5/2}} - \frac{2\int \frac{\frac{1}{2}\left(-6bcd-a\left(5c^2-d^2\right)\right)-\frac{5}{2}\left(2acd-b\left(c^2-d^2\right)\right)}{(a+b\tan(e+fx))^{5/2}\sqrt{c+}}}{5\left(a^2+b\right.}$$

$$= -\frac{2(bc-ad)\sqrt{c+d\tan(e+fx)}}{5\left(a^2+b^2\right)f(a+b\tan(e+fx))^{5/2}} - \frac{4\left(5abc-2a^2d+3b^2d\right)\sqrt{c+d\tan(e}}{15\left(a^2+b^2\right)^2 f(a+b\tan(e+fx)}$$

$$= -\frac{2(bc-ad)\sqrt{c+d\tan(e+fx)}}{5\left(a^2+b^2\right)f(a+b\tan(e+fx))^{5/2}} - \frac{4\left(5abc-2a^2d+3b^2d\right)\sqrt{c+d\tan(e}}{15\left(a^2+b^2\right)^2 f(a+b\tan(e+fx)}$$

$$= -\frac{2(bc-ad)\sqrt{c+d\tan(e+fx)}}{5\left(a^2+b^2\right)f(a+b\tan(e+fx))^{5/2}} - \frac{4\left(5abc-2a^2d+3b^2d\right)\sqrt{c+d\tan(e}}{15\left(a^2+b^2\right)^2 f(a+b\tan(e+fx)}$$

$$= -\frac{2(bc-ad)\sqrt{c+d\tan(e+fx)}}{5\left(a^2+b^2\right)f(a+b\tan(e+fx))^{5/2}} - \frac{4\left(5abc-2a^2d+3b^2d\right)\sqrt{c+d\tan(e}}{15\left(a^2+b^2\right)^2 f(a+b\tan(e+fx)}$$

$$= -\frac{2(bc-ad)\sqrt{c+d\tan(e+fx)}}{5\left(a^2+b^2\right)f(a+b\tan(e+fx))^{5/2}} - \frac{4\left(5abc-2a^2d+3b^2d\right)\sqrt{c+d\tan(e}}{15\left(a^2+b^2\right)^2 f(a+b\tan(e+fx)}$$

$$= -\frac{i(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{7/2}f} + \frac{i(c+id)^{3/2}\tanh^{-}}{}$$

Mathematica [A]
time = 6.27, size = 498, normalized size = 1.27

$$\frac{b(c+d\tan(e+fx))^{5/2}}{5(ia-b)(bc-ad)f(a+b\tan(e+fx))^{5/2}} - \frac{b(c+d\tan(e+fx))^{5/2}}{5(ia+b)(bc-ad)f(a+b\tan(e+fx))^{5/2}} - \frac{\frac{(c+d\tan(e+fx))^{3/2}}{(a-ib)(a+b\tan(e+fx))^{3/2}} + \frac{3(c-id)\left(\frac{\sqrt{-c+id}\tanh^{-1}\left(\frac{\sqrt{-c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(-a+ib)^{3/2}} + \frac{\sqrt{c+d\tan(e+fx)}}{(a-ib)\sqrt{a+b\tan(e+fx)}}\right)}{a-ib}}{3(ia+b)f} + \frac{\frac{(c+d\tan(e+fx))^{3/2}}{(a+ib)(a+b\tan(e+fx))^{3/2}} - \frac{3(c+id)\left(\frac{\sqrt{c+id}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{3/2}} - \frac{\sqrt{c+d\tan(e+fx)}}{(a+ib)\sqrt{a+b\tan(e+fx)}}\right)}{a+ib}}{3(ia-b)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^(3/2)/(a + b*Tan[e + f*x])^(7/2),x]
```

```
[Out] (b*(c + d*Tan[e + f*x])^(5/2))/(5*(I*a - b)*(b*c - a*d)*f*(a + b*Tan[e + f*
x])^(5/2)) - (b*(c + d*Tan[e + f*x])^(5/2))/(5*(I*a + b)*(b*c - a*d)*f*(a +
 b*Tan[e + f*x])^(5/2)) - ((c + d*Tan[e + f*x])^(3/2)/((a - I*b)*(a + b*Tan
[e + f*x])^(3/2)) + (3*(c - I*d)*((Sqrt[-c + I*d]*ArcTanh[(Sqrt[-c + I*d]*S
```

```
qrt[a + b*Tan[e + f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(-a +
I*b)^(3/2) + Sqrt[c + d*Tan[e + f*x]]/((a - I*b)*Sqrt[a + b*Tan[e + f*x]]))
)/(a - I*b))/(3*(I*a + b)*f) + ((c + d*Tan[e + f*x])^(3/2)/((a + I*b)*(a +
b*Tan[e + f*x])^(3/2)) - (3*(c + I*d)*((Sqrt[c + I*d]*ArcTanh[(Sqrt[c + I*d
]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(a +
 I*b)^(3/2) - Sqrt[c + d*Tan[e + f*x]]/((a + I*b)*Sqrt[a + b*Tan[e + f*x]])
))/(a + I*b))/(3*(I*a - b)*f)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+d\tan\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a+b\tan\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(3/2)/(a+b*tan(f*x+e))^(7/2),x)
[Out] int((c+d*tan(f*x+e))^(3/2)/(a+b*tan(f*x+e))^(7/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+b*tan(f*x+e))^(7/2),x, algorithm="maxim
a")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((2*b*d+2*a*c)^2>0)', see `assume?`
 for mo
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+b*tan(f*x+e))^(7/2),x, algorithm="frica
s")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+d\tan\left(e+fx\right)\right)^{\frac{3}{2}}}{\left(a+b\tan\left(e+fx\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(3/2)/(a+b*tan(f*x+e))**(7/2),x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(3/2)/(a + b*tan(e + f*x))**(7/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)/(a+b*tan(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(3/2)/(a + b*tan(e + f*x))^(7/2),x)
```

```
[Out] \text{Hanged}
```

3.1277 $\int (a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=429

$$-\frac{i(a-ib)^{3/2}(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f}+\frac{i(a+ib)^{3/2}(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\cdots}{\sqrt{a+ib}\,\cdots}\right)}{f}$$

```
[Out] -I*(a-I*b)^(3/2)*(c-I*d)^(5/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)
/(a-I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/f+I*(a+I*b)^(3/2)*(c+I*d)^(5/2)*arct
anh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/
2))/f+1/8*(15*a^2*b*c*d^2-a^3*d^3+3*a*b^2*d*(15*c^2-8*d^2)+5*b^3*(c^3-8*c*d
^2))*arctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x+e))^(1/2))
/b^(3/2)/f/d^(1/2)+1/8*(14*a*b*c*d-a^2*d^2+b^2*(11*c^2-8*d^2))*(a+b*tan(f*x
+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/b/f+1/12*d*(-a*d+13*b*c)*(c+d*tan(f*x+e))
^(1/2)*(a+b*tan(f*x+e))^(3/2)/b/f+1/3*d^2*(c+d*tan(f*x+e))^(1/2)*(a+b*tan(f
*x+e))^(5/2)/b/f
```

Rubi [A]

time = 4.09, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {3647, 3728, 3736, 6857, 65, 223, 212, 95, 214}

$$\frac{(-a^2d^2+14abcd+b^2(11c^2-8d^2))\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{8bf}+\frac{(-a^3d^3+15a^2bcd^2+3ab^2d(15c^2-8d^2)+5b^3(c^3-8cd^2))\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{8b^{3/2}\sqrt{d}\,f}+\frac{d^2(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}{3bf}+\frac{d(13bc-ad)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{12bf}-\frac{i(a-ib)^{3/2}(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{f}+\frac{i(a+ib)^{3/2}(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-I)*(a - I*b)^(3/2)*(c - I*d)^(5/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan
[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/f + (I*(a + I*b)^(3/
2)*(c + I*d)^(5/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a
 + I*b]*Sqrt[c + d*Tan[e + f*x]])])/f + ((15*a^2*b*c*d^2 - a^3*d^3 + 3*a*b^
2*d*(15*c^2 - 8*d^2) + 5*b^3*(c^3 - 8*c*d^2))*ArcTanh[(Sqrt[d]*Sqrt[a + b*T
an[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(8*b^(3/2)*Sqrt[d]*f) +
((14*a*b*c*d - a^2*d^2 + b^2*(11*c^2 - 8*d^2))*Sqrt[a + b*Tan[e + f*x]]*Sqr
t[c + d*Tan[e + f*x]])/(8*b*f) + (d*(13*b*c - a*d)*(a + b*Tan[e + f*x])^(3/
2)*Sqrt[c + d*Tan[e + f*x]])/(12*b*f) + (d^2*(a + b*Tan[e + f*x])^(5/2)*Sqr
t[c + d*Tan[e + f*x]])/(3*b*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
```

```
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int (a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}\,dx = \frac{d^2(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}{3bf} + \frac{\int \frac{(a+b \cdots}{\cdots}}{\cdots}$$

$$= \frac{d(13bc-ad)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{12bf} \cdots$$

$$= \frac{(14abcd-a^2d^2+b^2(11c^2-8d^2))\sqrt{a+b\tan(e+fx)} \cdots}{8bf} \cdots$$

$$= \frac{(14abcd-a^2d^2+b^2(11c^2-8d^2))\sqrt{a+b\tan(e+fx)} \cdots}{8bf} \cdots$$

$$= \frac{(14abcd-a^2d^2+b^2(11c^2-8d^2))\sqrt{a+b\tan(e+fx)} \cdots}{8bf} \cdots$$

$$= \frac{(14abcd-a^2d^2+b^2(11c^2-8d^2))\sqrt{a+b\tan(e+fx)} \cdots}{8bf} \cdots$$

$$= \frac{(14abcd-a^2d^2+b^2(11c^2-8d^2))\sqrt{a+b\tan(e+fx)} \cdots}{8bf} \cdots$$

$$= \frac{(14abcd-a^2d^2+b^2(11c^2-8d^2))\sqrt{a+b\tan(e+fx)} \cdots}{8bf} \cdots$$

$$= \frac{(15a^2bcd^2-a^3d^3+3ab^2d(15c^2-8d^2)+5b^3(c^3-8cd^2 \cdots}{8b^{3/2}\sqrt{d}\,f} \cdots$$

$$= -\frac{i(a-ib)^{3/2}(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(\cdots}{\sqrt{a-ib}\,\sqrt{c+d\tan(\cdots}\right.}{f} \cdots$$

Mathematica [A]
time = 8.21, size = 773, normalized size = 1.80

$$\frac{d^2(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}{3bf} + \frac{\frac{d(13bc-ad)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{4f} + \frac{\frac{3d(14abcd-a^2d^2+b^2(11c^2-8d^2))\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{4f} + \cdots}{\cdots}}{3b}$$

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (d^2*(a + b*Tan[e + f*x])^(5/2)*Sqrt[c + d*Tan[e + f*x]])/(3*b*f) + ((d*(13
*b*c - a*d)*(a + b*Tan[e + f*x])^(3/2)*Sqrt[c + d*Tan[e + f*x]])/(4*f) + ((
3*d*(14*a*b*c*d - a^2*d^2 + b^2*(11*c^2 - 8*d^2))*Sqrt[a + b*Tan[e + f*x]]*
Sqrt[c + d*Tan[e + f*x]])/(4*f) + ((-6*b*d^2*(Sqrt[-b^2]*(b^2*c*(c^2 - 3*d^
2) - a^2*(c^3 - 3*c*d^2) + a*b*(6*c^2*d - 2*d^3)) - b*(2*a*b*c*(c^2 - 3*d^2
) - b^2*d*(3*c^2 - d^2) + a^2*(3*c^2*d - d^3)))*ArcTanh[(Sqrt[-c + (Sqrt[-b
^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e
 + f*x]])])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[-c + (Sqrt[-b^2]*d)/b]) - (6*b*d^2*
(Sqrt[-b^2]*(b^2*c*(c^2 - 3*d^2) - a^2*(c^3 - 3*c*d^2) + a*b*(6*c^2*d - 2*d
^3)) + b*(2*a*b*c*(c^2 - 3*d^2) - b^2*d*(3*c^2 - d^2) + a^2*(3*c^2*d - d^3)
))*ArcTanh[(Sqrt[c + (Sqrt[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a +
Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + (Sqr
t[-b^2]*d)/b]) + (3*Sqrt[b]*d^(3/2)*Sqrt[c - (a*d)/b]*(15*a^2*b*c*d^2 - a^3
*d^3 + 3*a*b^2*d*(15*c^2 - 8*d^2) + 5*b^3*(c^3 - 8*c*d^2))*ArcSinh[(Sqrt[d]
*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c - (a*d)/b])]*Sqrt[(b*c + b*d*Tan
[e + f*x])/(b*c - a*d)])/(4*Sqrt[c + d*Tan[e + f*x]]))/(b*d*f))/(2*d))/(3*b
)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(fx+e))^{\frac{3}{2}} (c+d\tan(fx+e))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(5/2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*tan(f*x + e) + a)^(3/2)*(d*tan(f*x + e) + c)^(5/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan (e+fx))^{\frac{3}{2}} (c+d\tan (e+fx))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(3/2)*(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**(3/2)*(c + d*tan(e + f*x))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\mathrm{tan}\left(e+f\,x\right)\right)}^{3/2}\,{\left(c+d\,\mathrm{tan}\left(e+f\,x\right)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^(3/2)*(c + d*tan(e + f*x))^(5/2),x)
```

```
[Out] int((a + b*tan(e + f*x))^(3/2)*(c + d*tan(e + f*x))^(5/2), x)
```

3.1278 $\int \sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=339

$$-\frac{i\sqrt{a-ib}\,(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f}+\frac{i\sqrt{a+ib}\,(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a}}{\sqrt{a+ib}\,\sqrt{c}}\right.}{f}$$

```
[Out] -I*(c-I*d)^(5/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)
/(c+d*tan(f*x+e))^(1/2))*(a-I*b)^(1/2)/f+I*(c+I*d)^(5/2)*arctanh((c+I*d)^(1
/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))*(a+I*b)^(1
/2)/f+1/4*(10*a*b*c*d-a^2*d^2+b^2*(15*c^2-8*d^2))*arctanh(d^(1/2)*(a+b*tan(
f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x+e))^(1/2))*d^(1/2)/b^(3/2)/f+1/4*d*(-a*d
+9*b*c)*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/b/f+1/2*d^2*(c+d*tan(
f*x+e))^(1/2)*(a+b*tan(f*x+e))^(3/2)/b/f
```

Rubi [A]
time = 2.84, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {3647, 3728, 3736, 6857, 65, 223, 212, 95, 214}

$$\frac{\sqrt{d}\,(-a^2d^2+10abcd+b^2(15c^2-8d^2))\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{4b^{3/2}f}+\frac{d^2(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{2bf}+\frac{d(9bc-ad)\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{4bf}-\frac{i\sqrt{a-ib}\,(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f}+\frac{i\sqrt{a+ib}\,(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-I)*Sqrt[a - I*b]*(c - I*d)^(5/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
 + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/f + (I*Sqrt[a + I*b]*(
c + I*d)^(5/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I
*b]*Sqrt[c + d*Tan[e + f*x]])])/f + (Sqrt[d]*(10*a*b*c*d - a^2*d^2 + b^2*(1
5*c^2 - 8*d^2))*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c
+ d*Tan[e + f*x]])])/(4*b^(3/2)*f) + (d*(9*b*c - a*d)*Sqrt[a + b*Tan[e + f*
x]]*Sqrt[c + d*Tan[e + f*x]])/(4*b*f) + (d^2*(a + b*Tan[e + f*x])^(3/2)*Sqr
t[c + d*Tan[e + f*x]])/(2*b*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{5/2}\,dx = \frac{d^2(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{2bf} + \frac{\int \sqrt{a}}{}$$

$$= \frac{d(9bc-ad)\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{4bf}$$

$$= \frac{d(9bc-ad)\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{4bf}$$

$$= \frac{d(9bc-ad)\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{4bf}$$

$$= \frac{d(9bc-ad)\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{4bf}$$

$$= \frac{d(9bc-ad)\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{4bf}$$

$$= \frac{d(9bc-ad)\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{4bf}$$

$$= \frac{\sqrt{d}\,(10abcd-a^2d^2+b^2(15c^2-8d^2))\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a}}{\sqrt{b}\,\sqrt{c}}\right.}{4b^{3/2}f}$$

$$= -\frac{i\sqrt{a-ib}\,(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e}}\right.}{f}$$

Mathematica [A]
time = 6.70, size = 550, normalized size = 1.62

$$\frac{\frac{4\left(a\sqrt{-b^2}\,c(c^2-3d^2)+b\left(-a+\sqrt{-b^2}\right)d(-3c^2+d^2)+b^2(c^3-3cd^2)\right)\tanh^{-1}\left(\frac{\sqrt{-c+\frac{\sqrt{-b^2}\,d}{b}}\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+\sqrt{-b^2}}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+\sqrt{-b^2}}\sqrt{-c+\frac{\sqrt{-b^2}\,d}{b}}}-\frac{4\left(-a\sqrt{-b^2}\,c(c^2-3d^2)-b\left(a+\sqrt{-b^2}\right)d(-3c^2+d^2)+b^2(c^3-3cd^2)\right)\tanh^{-1}\left(\frac{\sqrt{c+\frac{\sqrt{-b^2}\,d}{b}}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+\sqrt{-b^2}}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+\sqrt{-b^2}}\sqrt{c+\frac{\sqrt{-b^2}\,d}{b}}}+d(9bc-ad)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}+2d^2(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}+\frac{\sqrt{d}\sqrt{c-\frac{ad}{b}}\,(10abcd-a^2d^2+b^2(15c^2-8d^2))\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c-\frac{ad}{b}}}\right)\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}}{4bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((4*(a*Sqrt[-b^2]*c*(c^2 - 3*d^2) + b*(-a + Sqrt[-b^2])*d*(-3*c^2 + d^2) +
b^2*(c^3 - 3*c*d^2))*ArcTanh[(Sqrt[-c + (Sqrt[-b^2]*d)/b]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f*x]])]])/(Sqrt[-a + Sqrt
[-b^2]]*Sqrt[-c + (Sqrt[-b^2]*d)/b]) - (4*(-(a*Sqrt[-b^2]*c*(c^2 - 3*d^2))
- b*(a + Sqrt[-b^2])*d*(-3*c^2 + d^2) + b^2*(c^3 - 3*c*d^2))*ArcTanh[(Sqrt[
c + (Sqrt[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[
c + d*Tan[e + f*x]])]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + (Sqrt[-b^2]*d)/b]) +
d*(9*b*c - a*d)*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]] + 2*d^2*(
a + b*Tan[e + f*x])^(3/2)*Sqrt[c + d*Tan[e + f*x]] + (Sqrt[d]*Sqrt[c - (a*d
)/b]*(10*a*b*c*d - a^2*d^2 + b^2*(15*c^2 - 8*d^2))*ArcSinh[(Sqrt[d]*Sqrt[a
+ b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c - (a*d)/b])]*Sqrt[(b*(c + d*Tan[e + f*x]
))/(b*c - a*d)])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]]))/(4*b*f)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan\left(fx+e\right)}\,\left(c+d\tan\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(5/2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(b*tan(f*x + e) + a)*(d*tan(f*x + e) + c)^(5/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan{(e+fx)}}\,(c+d\tan{(e+fx)})^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(1/2)*(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x))*(c + d*tan(e + f*x))**(5/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+b\tan\left(e+f\,x\right)}\,{\left(c+d\,\mathrm{tan}\left(e+f\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^(1/2)*(c + d*tan(e + f*x))^(5/2),x)
```

```
[Out] int((a + b*tan(e + f*x))^(1/2)*(c + d*tan(e + f*x))^(5/2), x)
```

3.1279 $\int \frac{(c+d\tan(e+fx))^{5/2}}{\sqrt{a+b\tan(e+fx)}}\,dx$

Optimal. Leaf size=264

$$-\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a-ib}\,f}+\frac{i(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}\,f}$$

```
[Out] d^(3/2)*(-a*d+5*b*c)*arctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^(1/2)/(c+d*ta
n(f*x+e))^(1/2))/b^(3/2)/f-I*(c-I*d)^(5/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f
*x+e))^(1/2)/(a-I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/f/(a-I*b)^(1/2)+I*(c+I*d
)^(5/2)*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan
(f*x+e))^(1/2))/f/(a+I*b)^(1/2)+d^2*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))
^(1/2)/b/f
```

Rubi [A]
time = 1.89, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {3647, 3736, 6857, 65, 223, 212, 95, 214}

$$\frac{d^{3/2}(5bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{b^{3/2}f}+\frac{d^2\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{bf}-\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a-ib}}+\frac{i(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a+ib}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^(5/2)/Sqrt[a + b*Tan[e + f*x]],x]
```

```
[Out] ((-I)*(c - I*d)^(5/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqr
t[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a - I*b]*f) + (I*(c + I*d)^(5/
2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c +
 d*Tan[e + f*x]])])/(Sqrt[a + I*b]*f) + (d^(3/2)*(5*b*c - a*d)*ArcTanh[(Sqr
t[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(b^(3/2
)*f) + (d^2*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(b*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

```
[n, 0]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{5/2}}{\sqrt{a+b\tan(e+fx)}}\,dx = \frac{d^2\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{bf} + \frac{\int \frac{\frac{1}{2}\left(2bc^3-d^2(bc+ad)\right)+bd(3c^2-d^2)\tan}{\sqrt{a+b\tan(e+fx)}\,\sqrt{}}}{b}$$

$$= \frac{d^2\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{bf} + \frac{\mathrm{Subst}\Big(\int \frac{\frac{1}{2}\left(2bc^3-d^2(bc+ad)\right)+bd(3c}{\sqrt{a+bx}\,\sqrt{c}}}{}$$

$$= \frac{d^2\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{bf} + \frac{\mathrm{Subst}\Big(\int \Big(\frac{d^2(5bc-ad)}{2\sqrt{a+bx}\,\sqrt{c+dx}}}{}$$

$$= \frac{d^2\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{bf} + \frac{\mathrm{Subst}\Big(\int \frac{bc(c^2-3d^2)+bd(3c^2-d^2}{\sqrt{a+bx}\,\sqrt{c+dx}\,(}}{bf}$$

$$= \frac{d^2\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{bf} + \frac{\mathrm{Subst}\Big(\int \Big(\frac{ibc(c^2-3d^2)-bd(3c^2}{2(i-x)\sqrt{a+bx}\,\sqrt{c}}}{}$$

$$= \frac{d^2\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{bf} - \frac{(ic-d)^3\mathrm{Subst}\Big(\int \frac{}{(i-x)\sqrt{a+b}}}{}$$

$$= \frac{d^{3/2}(5bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{b^{3/2}f} + \frac{d^2\sqrt{a+b\tan(e+fx)}}{bf}$$

$$= -\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a-ib}\,f} + \frac{i(c+id)^{5/2}\tanh^{-1}}{}$$

Mathematica [A]
time = 2.99, size = 432, normalized size = 1.64

$$\frac{\frac{b\left(\sqrt{-b^2}\,c(c^2-3d^2)-bd(-3c^2+d^2)\right)\tanh^{-1}\left(\frac{\sqrt{-c+\frac{\sqrt{-b^2}\,d}{b}}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+\sqrt{-b^2}}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+\sqrt{-b^2}}\,\sqrt{-c+\frac{\sqrt{-b^2}\,d}{b}}} + \frac{b\left(\sqrt{-b^2}\,c(c^2-3d^2)+bd(-3c^2+d^2)\right)\tanh^{-1}\left(\frac{\sqrt{c+\frac{\sqrt{-b^2}\,d}{b}}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+\sqrt{-b^2}}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+\sqrt{-b^2}}\,\sqrt{c+\frac{\sqrt{-b^2}\,d}{b}}} + bd^2\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)} + \frac{\sqrt{b}\,d^{3/2}(5bc-ad)\sqrt{c-\frac{ad}{b}}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c-\frac{ad}{b}}}\right)\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}}{\sqrt{c+d\tan(e+fx)}}}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^(5/2)/Sqrt[a + b*Tan[e + f*x]],x]
```

```
[Out] ((b*(Sqrt[-b^2]*c*(c^2 - 3*d^2) - b*d*(-3*c^2 + d^2))*ArcTanh[(Sqrt[-c + (S
qrt[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + d
*Tan[e + f*x]])]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[-c + (Sqrt[-b^2]*d)/b]) + (b*
(Sqrt[-b^2]*c*(c^2 - 3*d^2) + b*d*(-3*c^2 + d^2))*ArcTanh[(Sqrt[c + (Sqrt[-
b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e
 + f*x]])]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + (Sqrt[-b^2]*d)/b]) + b*d^2*Sqrt[
a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]] + (Sqrt[b]*d^(3/2)*(5*b*c - a*
d)*Sqrt[c - (a*d)/b]*ArcSinh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sq
rt[c - (a*d)/b])]*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c - a*d)])/Sqrt[c + d*Ta
n[e + f*x]])/(b^2*f)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\tan(fx+e))^{\frac{5}{2}}}{\sqrt{a+b\tan(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(5/2)/(a+b*tan(f*x+e))^(1/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(5/2)/(a+b*tan(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+b*tan(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*tan(f*x + e) + c)^(5/2)/sqrt(b*tan(f*x + e) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+b*tan(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d\tan{(e + fx)})^{\frac{5}{2}}}{\sqrt{a + b\tan{(e + fx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(5/2)/(a+b*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(5/2)/sqrt(a + b*tan(e + f*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+b*tan(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c+d\,\mathrm{tan}\left(e+f\,x\right)\right)}^{5/2}}{\sqrt{a+b\,\mathrm{tan}\left(e+f\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(5/2)/(a + b*tan(e + f*x))^(1/2),x)
```

```
[Out] int((c + d*tan(e + f*x))^(5/2)/(a + b*tan(e + f*x))^(1/2), x)
```

3.1280 $\int \frac{(c+d\tan(e+fx))^{5/2}}{(a+b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=273

$$-\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{3/2}f}+\frac{i(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{3/2}f}$$

```
[Out] -I*(c-I*d)^(5/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)
/(c+d*tan(f*x+e))^(1/2))/(a-I*b)^(3/2)/f+I*(c+I*d)^(5/2)*arctanh((c+I*d)^(1
/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(a+I*b)^(3
/2)/f+2*d^(5/2)*arctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x
+e))^(1/2))/b^(3/2)/f-2*(-a*d+b*c)^2*(c+d*tan(f*x+e))^(1/2)/b/(a^2+b^2)/f/(
a+b*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 2.38, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {3646, 3736, 6857, 65, 223, 212, 95, 214}

$$-\frac{2(bc-ad)^2\sqrt{c+d\tan(e+fx)}}{bf\left(a^2+b^2\right)\sqrt{a+b\tan(e+fx)}}+\frac{2d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{b^{3/2}f}-\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a-ib)^{3/2}}+\frac{i(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a+ib)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^(5/2)/(a + b*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-I)*(c - I*d)^(5/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqr
t[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(3/2)*f) + (I*(c + I*d)^(
5/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c
 + d*Tan[e + f*x]])])/((a + I*b)^(3/2)*f) + (2*d^(5/2)*ArcTanh[(Sqrt[d]*Sqr
t[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(b^(3/2)*f) - (
2*(b*c - a*d)^2*Sqrt[c + d*Tan[e + f*x]])/(b*(a^2 + b^2)*f*Sqrt[a + b*Tan[e
 + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

`[n, 0]`

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{5/2}}{(a+b\tan(e+fx))^{3/2}}\,dx = -\frac{2(bc-ad)^2\sqrt{c+d\tan(e+fx)}}{b\left(a^2+b^2\right)f\sqrt{a+b\tan(e+fx)}} + \frac{2\int \frac{\frac{1}{2}\left(3b^2c^2d+a^2d^3+abc\left(c^2-3d^2\right)\right)-\frac{1}{2}b\left(bc^3-3ac \cdots\right.}{\sqrt{a+b\tan(e+f \cdots}}}{b\left(\cdots\right.}$$

$$= -\frac{2(bc-ad)^2\sqrt{c+d\tan(e+fx)}}{b\left(a^2+b^2\right)f\sqrt{a+b\tan(e+fx)}} + \frac{2\mathrm{Subst}\left(\int \frac{\frac{1}{2}\left(3b^2c^2d+a^2d^3+abc\left(c^2-3d^2\right)\right)-\frac{1}{2}b \cdots}{\sqrt{a+bx} \cdots}\right.}{\cdots}$$

$$= -\frac{2(bc-ad)^2\sqrt{c+d\tan(e+fx)}}{b\left(a^2+b^2\right)f\sqrt{a+b\tan(e+fx)}} + \frac{2\mathrm{Subst}\left(\int \left(\frac{\left(a^2+b^2\right)d^3}{2\sqrt{a+bx}\sqrt{c+dx}} + \frac{b(\cdots}{\cdots}\right.\right.}{\cdots}$$

$$= -\frac{2(bc-ad)^2\sqrt{c+d\tan(e+fx)}}{b\left(a^2+b^2\right)f\sqrt{a+b\tan(e+fx)}} + \frac{\mathrm{Subst}\left(\int \frac{b\left(ac^3+3bc^2d-3acd^2-bd^3\right)-b\left(bc^3-3a \cdots\right.}{\sqrt{a+bx}\sqrt{c+dx} \cdots}\right.}{b\left(a^2+ \cdots\right.}$$

$$= -\frac{2(bc-ad)^2\sqrt{c+d\tan(e+fx)}}{b\left(a^2+b^2\right)f\sqrt{a+b\tan(e+fx)}} + \frac{\mathrm{Subst}\left(\int \left(\frac{b\left(bc^3-3ac^2d-3bcd^2+ad^3\right)+ib\left(ac^3 \cdots\right.}{2(i-x)\sqrt{a+bx}\sqrt{c \cdots}}\right.\right.}{\cdots}$$

$$= -\frac{2(bc-ad)^2\sqrt{c+d\tan(e+fx)}}{b\left(a^2+b^2\right)f\sqrt{a+b\tan(e+fx)}} - \frac{(c-id)^3\mathrm{Subst}\left(\int \frac{1}{(i+x)\sqrt{a+bx}\sqrt{c+ \cdots}}\right.}{2(ia+b)f}$$

$$= \frac{2d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{b^{3/2}f} - \frac{2(bc-ad)^2\sqrt{c+d\tan(e+f \cdots}}{b\left(a^2+b^2\right)f\sqrt{a+b\tan(e+f \cdots}}$$

$$= -\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{3/2}f} + \frac{i(c+id)^{5/2}\tanh^{-1} \cdots}{\cdots}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1187 vs. 2(273) = 546.
time = 6.21, size = 1187, normalized size = 4.35

$$-\frac{i(-c-id)\left(-\left((-c-id)\left(-\frac{2\sqrt{c+id}\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(-a-ib)\sqrt{a+ib}}+\frac{2\sqrt{c+d\tan(e+fx)}}{(-a-ib)\sqrt{a+b\tan(e+fx)}}\right)\right)-\frac{2d\sqrt{c+d\tan(e+fx)}\left(1+\frac{bd(a+b\tan(e+fx))}{(bc-ad)\left(\frac{b^2c}{bc-ad}-\frac{abd}{bc-ad}\right)}\right)^{3/2}\left(1-\frac{\sqrt{b}\sqrt{d}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{bc-ad}\sqrt{\frac{b^2c}{bc-ad}-\frac{abd}{bc-ad}}}\right)\sqrt{a+b\tan(e+fx)}}{\sqrt{bc-ad}\sqrt{\frac{b^2c}{bc-ad}-\frac{abd}{bc-ad}}\sqrt{1+\frac{bd(a+b\tan(e+fx))}{(bc-ad)\left(\frac{b^2c}{bc-ad}-\frac{abd}{bc-ad}\right)}}}\right)}{b\sqrt{\frac{b}{\frac{b^2c}{bc-ad}-\frac{abd}{bc-ad}}}\sqrt{a+b\tan(e+fx)}\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}\left(-1-\frac{bd(a+b\tan(e+fx))}{(bc-ad)\left(\frac{b^2c}{bc-ad}-\frac{abd}{bc-ad}\right)}\right)}\right)}{2f}-\frac{i(-c+id)\left(-\left((-c+id)\left(-\frac{2\sqrt{-c+id}\tanh^{-1}\left(\frac{\sqrt{-c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)\sqrt{-a+ib}}+\frac{2\sqrt{c+d\tan(e+fx)}}{(a-ib)\sqrt{a+b\tan(e+fx)}}\right)\right)+\frac{2d\sqrt{c+d\tan(e+fx)}\left(1+\frac{bd(a+b\tan(e+fx))}{(bc-ad)\left(\frac{b^2c}{bc-ad}-\frac{abd}{bc-ad}\right)}\right)^{3/2}\left(1-\frac{\sqrt{b}\sqrt{d}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{bc-ad}\sqrt{\frac{b^2c}{bc-ad}-\frac{abd}{bc-ad}}}\right)\sqrt{a+b\tan(e+fx)}}{\sqrt{bc-ad}\sqrt{\frac{b^2c}{bc-ad}-\frac{abd}{bc-ad}}\sqrt{1+\frac{bd(a+b\tan(e+fx))}{(bc-ad)\left(\frac{b^2c}{bc-ad}-\frac{abd}{bc-ad}\right)}}}\right)}{b\sqrt{\frac{b}{\frac{b^2c}{bc-ad}-\frac{abd}{bc-ad}}}\sqrt{a+b\tan(e+fx)}\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}\left(-1-\frac{bd(a+b\tan(e+fx))}{(bc-ad)\left(\frac{b^2c}{bc-ad}-\frac{abd}{bc-ad}\right)}\right)}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^(5/2)/(a + b*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-1/2*I)*(-c - I*d)*(-((-c - I*d)*((-2*Sqrt[c + I*d]*ArcTanh[(Sqrt[c + I*d
]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((-a
 - I*b)*Sqrt[a + I*b]) + (2*Sqrt[c + d*Tan[e + f*x]])/((-a - I*b)*Sqrt[a +
b*Tan[e + f*x]]))) - (2*d*Sqrt[c + d*Tan[e + f*x]]*(1 + (b*d*(a + b*Tan[e +
 f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))^(3/2)*(1
 - (Sqrt[b]*Sqrt[d]*ArcSinh[(Sqrt[b]*Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqr
t[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)])]*Sqrt[a + b*T
an[e + f*x]])/(Sqrt[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*
d)]*Sqrt[1 + (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) -
 (a*b*d)/(b*c - a*d)))])))/(b*Sqrt[b/((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c -
a*d))]*Sqrt[a + b*Tan[e + f*x]]*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c - a*d)]*
(-1 - (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d
)/(b*c - a*d)))))))/f - ((I/2)*(-c + I*d)*(-((-c + I*d)*((-2*Sqrt[-c + I*d]
*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + I*b]*Sqrt[c +
 d*Tan[e + f*x]])])/((a - I*b)*Sqrt[-a + I*b]) + (2*Sqrt[c + d*Tan[e + f*x]
])/((a - I*b)*Sqrt[a + b*Tan[e + f*x]]))) + (2*d*Sqrt[c + d*Tan[e + f*x]]*(
1 + (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/
(b*c - a*d))))^(3/2)*(1 - (Sqrt[b]*Sqrt[d]*ArcSinh[(Sqrt[b]*Sqrt[d]*Sqrt[a
+ b*Tan[e + f*x]])/(Sqrt[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c
 - a*d)])]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a
*d) - (a*b*d)/(b*c - a*d)]*Sqrt[1 + (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)
*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)))])))/(b*Sqrt[b/((b^2*c)/(b*c -
 a*d) - (a*b*d)/(b*c - a*d))]*Sqrt[a + b*Tan[e + f*x]]*Sqrt[(b*(c + d*Tan[e
 + f*x]))/(b*c - a*d)]*(-1 - (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*
c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)))))))/f
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+d\tan \left(fx+e\right)\right)^{\frac{5}{2}}}{\left(a+b\tan \left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(5/2)/(a+b*tan(f*x+e))^(3/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(5/2)/(a+b*tan(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+b*tan(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*tan(f*x + e) + c)^(5/2)/(b*tan(f*x + e) + a)^(3/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+b*tan(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d \tan{(e + fx)})^{\frac{5}{2}}}{(a + b \tan{(e + fx)})^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(5/2)/(a+b*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(5/2)/(a + b*tan(e + f*x))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+b*tan(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c+d\,\mathrm{tan}\left(e+f\,x\right)\right)}^{5/2}}{{\left(a+b\,\mathrm{tan}\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(5/2)/(a + b*tan(e + f*x))^(3/2),x)
```

```
[Out] int((c + d*tan(e + f*x))^(5/2)/(a + b*tan(e + f*x))^(3/2), x)
```

3.1281 $\int \frac{(c+d\tan(e+fx))^{5/2}}{(a+b\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=292

$$-\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{5/2}f}+\frac{i(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{5/2}f}$$

```
[Out] -I*(c-I*d)^(5/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)
/(c+d*tan(f*x+e))^(1/2))/(a-I*b)^(5/2)/f+I*(c+I*d)^(5/2)*arctanh((c+I*d)^(1
/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(a+I*b)^(5
/2)/f-2/3*(-a*d+b*c)*(a^2*d+6*a*b*c+7*b^2*d)*(c+d*tan(f*x+e))^(1/2)/b/(a^2+
b^2)^2/f/(a+b*tan(f*x+e))^(1/2)-2/3*(-a*d+b*c)^2*(c+d*tan(f*x+e))^(1/2)/b/(
a^2+b^2)/f/(a+b*tan(f*x+e))^(3/2)
```

Rubi [A]

time = 1.08, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3646, 3730, 3697, 3696, 95, 214}

$$-\frac{2(bc-ad)\left(a^2d+6abc+7b^2d\right)\sqrt{c+d\tan(e+fx)}}{3bf\left(a^2+b^2\right)^2\sqrt{a+b\tan(e+fx)}}-\frac{2(bc-ad)^2\sqrt{c+d\tan(e+fx)}}{3bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{3/2}}-\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a-ib)^{5/2}}+\frac{i(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a+ib)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^(5/2)/(a + b*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-I)*(c - I*d)^(5/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqr
t[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(5/2)*f) + (I*(c + I*d)^(
5/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c
 + d*Tan[e + f*x]])])/((a + I*b)^(5/2)*f) - (2*(b*c - a*d)^2*Sqrt[c + d*Tan
[e + f*x]])/(3*b*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^(3/2)) - (2*(b*c - a*d)
*(6*a*b*c + a^2*d + 7*b^2*d)*Sqrt[c + d*Tan[e + f*x]])/(3*b*(a^2 + b^2)^2*f
*Sqrt[a + b*Tan[e + f*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{5/2}}{(a+b\tan(e+fx))^{5/2}}\,dx = -\frac{2(bc-ad)^2\sqrt{c+d\tan(e+fx)}}{3b\,(a^2+b^2)\,f(a+b\tan(e+fx))^{3/2}} + \frac{2\int \frac{\frac{1}{2}\left(7b^2c^2d+a^2d^3+abc\left(3c^2-5d^2\right)\right)-\frac{3}{2}b\left(bc^3-3\cdots\right.}{(a+b\tan(\cdots}}{\cdots}$$

$$= -\frac{2(bc-ad)^2\sqrt{c+d\tan(e+fx)}}{3b\,(a^2+b^2)\,f(a+b\tan(e+fx))^{3/2}} - \frac{2(bc-ad)\left(6abc+a^2d+7b^2d\right)\sqrt{c+\cdots}}{3b\,(a^2+b^2)^2\,f\sqrt{a+b\tan(e\cdots}}$$

$$= -\frac{2(bc-ad)^2\sqrt{c+d\tan(e+fx)}}{3b\,(a^2+b^2)\,f(a+b\tan(e+fx))^{3/2}} - \frac{2(bc-ad)\left(6abc+a^2d+7b^2d\right)\sqrt{c+\cdots}}{3b\,(a^2+b^2)^2\,f\sqrt{a+b\tan(e\cdots}}$$

$$= -\frac{2(bc-ad)^2\sqrt{c+d\tan(e+fx)}}{3b\,(a^2+b^2)\,f(a+b\tan(e+fx))^{3/2}} - \frac{2(bc-ad)\left(6abc+a^2d+7b^2d\right)\sqrt{c+\cdots}}{3b\,(a^2+b^2)^2\,f\sqrt{a+b\tan(e\cdots}}$$

$$= -\frac{2(bc-ad)^2\sqrt{c+d\tan(e+fx)}}{3b\,(a^2+b^2)\,f(a+b\tan(e+fx))^{3/2}} - \frac{2(bc-ad)\left(6abc+a^2d+7b^2d\right)\sqrt{c+\cdots}}{3b\,(a^2+b^2)^2\,f\sqrt{a+b\tan(e\cdots}}$$

$$= -\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{5/2}f} + \frac{i(c+id)^{5/2}\tanh^{-1}\cdots}{\cdots}$$

Mathematica [A]
time = 5.58, size = 350, normalized size = 1.20

$$\frac{-\left((ic-d)\left(-\frac{3(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{5/2}}+\frac{\sqrt{c+d\tan(e+fx)}\,(4ac+ibc+3iad+(3bc+ad+4ibd)\tan(e+fx))}{(a+ib)^2(a+b\tan(e+fx))^{3/2}}\right)\right)+\frac{(ic+d)\left(\frac{(c+d\tan(e+fx))^{3/2}}{(a+b\tan(e+fx))^{3/2}}+3(c-id)\left(\frac{\sqrt{-c+id}\,\tanh^{-1}\left(\frac{\sqrt{-c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(-a+ib)^{3/2}}+\frac{\sqrt{c+d\tan(e+fx)}}{(a-ib)\sqrt{a+b\tan(e+fx)}}\right)\right)}{a-ib}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^(5/2)/(a + b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (-((I*c - d)*((-3*(c + I*d)^(3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
 f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(a + I*b)^(5/2) + (Sqrt[
c + d*Tan[e + f*x]]*(4*a*c + I*b*c + (3*I)*a*d + (3*b*c + a*d + (4*I)*b*d)*
Tan[e + f*x]))/((a + I*b)^2*(a + b*Tan[e + f*x])^(3/2)))) + ((I*c + d)*((c
+ d*Tan[e + f*x])^(3/2)/(a + b*Tan[e + f*x])^(3/2) + 3*(c - I*d)*((Sqrt[-c
+ I*d]*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + I*b]*Sq
rt[c + d*Tan[e + f*x]])])/(-a + I*b)^(3/2) + Sqrt[c + d*Tan[e + f*x]]/((a -
 I*b)*Sqrt[a + b*Tan[e + f*x]]))))/(a - I*b))/(3*f)
```

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d\tan(fx + e))^{\frac{5}{2}}}{(a + b\tan(fx + e))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(5/2)/(a+b*tan(f*x+e))^(5/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(5/2)/(a+b*tan(f*x+e))^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+b*tan(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((2*b*d+2*a*c)^2>0)', see `assume?`
 for mo
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+b*tan(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d\tan(e + fx))^{\frac{5}{2}}}{(a + b\tan(e + fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(5/2)/(a+b*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(5/2)/(a + b*tan(e + f*x))**(5/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+b*tan(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c + d\tan(e + f\,x))^{5/2}}{(a + b\tan\left(e + f\,x\right))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(5/2)/(a + b*tan(e + f*x))^(5/2),x)
```

```
[Out] int((c + d*tan(e + f*x))^(5/2)/(a + b*tan(e + f*x))^(5/2), x)
```

3.1282 $\int \frac{(c+d\tan(e+fx))^{5/2}}{(a+b\tan(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=398

$$-\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{7/2}f}+\frac{i(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{7/2}f}$$

```
[Out] -I*(c-I*d)^(5/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)
/(c+d*tan(f*x+e))^(1/2))/(a-I*b)^(7/2)/f+I*(c+I*d)^(5/2)*arctanh((c+I*d)^(1
/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(a+I*b)^(7
/2)/f+2/15*(20*a^3*b*c*d-100*a*b^3*c*d+2*a^4*d^2+b^4*(15*c^2-23*d^2)-3*a^2*
b^2*(15*c^2-13*d^2))*(c+d*tan(f*x+e))^(1/2)/b/(a^2+b^2)^3/f/(a+b*tan(f*x+e)
)^(1/2)-2/5*(-a*d+b*c)^2*(c+d*tan(f*x+e))^(1/2)/b/(a^2+b^2)/f/(a+b*tan(f*x+
e))^(5/2)-2/15*(-a*d+b*c)*(a^2*d+10*a*b*c+11*b^2*d)*(c+d*tan(f*x+e))^(1/2)/
b/(a^2+b^2)^2/f/(a+b*tan(f*x+e))^(3/2)
```

Rubi [A]

time = 1.59, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3646, 3730, 3697, 3696, 95, 214}

$$-\frac{2(bc-ad)\left(a^2d+10abc+11b^2d\right)\sqrt{c+d\tan(e+fx)}}{15bf\left(a^2+b^2\right)^2(a+b\tan(e+fx))^{3/2}}-\frac{2(bc-ad)^2\sqrt{c+d\tan(e+fx)}}{5bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{5/2}}+\frac{2\left(2a^4d^2+20a^3bcd-3a^2b^2\left(15c^2-13d^2\right)-100ab^3cd+b^4\left(15c^2-23d^2\right)\right)\sqrt{c+d\tan(e+fx)}}{15bf\left(a^2+b^2\right)^3\sqrt{a+b\tan(e+fx)}}-\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a-ib)^{7/2}}+\frac{i(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a+ib)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^(5/2)/(a + b*Tan[e + f*x])^(7/2),x]
```

```
[Out] ((-I)*(c - I*d)^(5/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqr
t[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(7/2)*f) + (I*(c + I*d)^(
5/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c
 + d*Tan[e + f*x]])])/((a + I*b)^(7/2)*f) - (2*(b*c - a*d)^2*Sqrt[c + d*Tan
[e + f*x]])/(5*b*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^(5/2)) - (2*(b*c - a*d)
*(10*a*b*c + a^2*d + 11*b^2*d)*Sqrt[c + d*Tan[e + f*x]])/(15*b*(a^2 + b^2)^
2*f*(a + b*Tan[e + f*x])^(3/2)) + (2*(20*a^3*b*c*d - 100*a*b^3*c*d + 2*a^4*
d^2 + b^4*(15*c^2 - 23*d^2) - 3*a^2*b^2*(15*c^2 - 13*d^2))*Sqrt[c + d*Tan[e
 + f*x]])/(15*b*(a^2 + b^2)^3*f*Sqrt[a + b*Tan[e + f*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
```

```
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{5/2}}{(a+b\tan(e+fx))^{7/2}}\,dx = -\frac{2(bc-ad)^2\sqrt{c+d\tan(e+fx)}}{5b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{5/2}} + \frac{2\int \frac{\frac{1}{2}\left(11b^2c^2d+a^2d^3+abc\left(5c^2-7d^2\right)\right)-\frac{5}{2}b\left(bc^3\right.}{(a+b\,\mathrm{ta}}}{}$$

$$= -\frac{2(bc-ad)^2\sqrt{c+d\tan(e+fx)}}{5b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{5/2}} - \frac{2(bc-ad)\left(10abc+a^2d+11b^2d\right)\sqrt{}}{15b\left(a^2+b^2\right)^2 f(a+b\tan(}$$

$$= -\frac{2(bc-ad)^2\sqrt{c+d\tan(e+fx)}}{5b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{5/2}} - \frac{2(bc-ad)\left(10abc+a^2d+11b^2d\right)\sqrt{}}{15b\left(a^2+b^2\right)^2 f(a+b\tan(}$$

$$= -\frac{2(bc-ad)^2\sqrt{c+d\tan(e+fx)}}{5b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{5/2}} - \frac{2(bc-ad)\left(10abc+a^2d+11b^2d\right)\sqrt{}}{15b\left(a^2+b^2\right)^2 f(a+b\tan(}$$

$$= -\frac{2(bc-ad)^2\sqrt{c+d\tan(e+fx)}}{5b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{5/2}} - \frac{2(bc-ad)\left(10abc+a^2d+11b^2d\right)\sqrt{}}{15b\left(a^2+b^2\right)^2 f(a+b\tan(}$$

$$= -\frac{2(bc-ad)^2\sqrt{c+d\tan(e+fx)}}{5b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{5/2}} - \frac{2(bc-ad)\left(10abc+a^2d+11b^2d\right)\sqrt{}}{15b\left(a^2+b^2\right)^2 f(a+b\tan(}$$

$$= -\frac{i(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{7/2}f} + \frac{i(c+id)^{5/2}\tanh^{-}}{}$$

Mathematica [A]
time = 6.04, size = 440, normalized size = 1.11

$$\frac{\frac{3(c+d\tan(e+fx))^{5/2}}{(ia-b)(a+b\tan(e+fx))^{5/2}}-\frac{3(c+d\tan(e+fx))^{5/2}}{(ia+b)(a+b\tan(e+fx))^{5/2}}-\frac{5(ic-d)\left(-\frac{3(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{5/2}}+\frac{\sqrt{c+d\tan(e+fx)}\,(4ac+ibc+3iad+(3bc+ad+4ibd)\tan(e+fx))}{(a+ib)^2(a+b\tan(e+fx))^{3/2}}\right)}{a+ib}+\frac{5(ic+d)\left(\frac{(c+d\tan(e+fx))^{3/2}}{(a+b\tan(e+fx))^{3/2}}+3(c-id)\left(\frac{\sqrt{-c+id}\tanh^{-1}\left(\frac{\sqrt{-c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(-a+ib)^{3/2}}+\frac{\sqrt{c+d\tan(e+fx)}}{(a-ib)\sqrt{a+b\tan(e+fx)}}\right)\right)}{(a-ib)^2}}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^(5/2)/(a + b*Tan[e + f*x])^(7/2),x]
```

```
[Out] ((3*(c + d*Tan[e + f*x])^(5/2))/((I*a - b)*(a + b*Tan[e + f*x])^(5/2)) - (3
*(c + d*Tan[e + f*x])^(5/2))/((I*a + b)*(a + b*Tan[e + f*x])^(5/2)) - (5*(I
*c - d)*((-3*(c + I*d)^(3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]
```

```
])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(a + I*b)^(5/2) + (Sqrt[c + d
*Tan[e + f*x]]*(4*a*c + I*b*c + (3*I)*a*d + (3*b*c + a*d + (4*I)*b*d)*Tan[e
 + f*x]))/((a + I*b)^2*(a + b*Tan[e + f*x])^(3/2))))/(a + I*b) + (5*(I*c +
d)*((c + d*Tan[e + f*x])^(3/2)/(a + b*Tan[e + f*x])^(3/2) + 3*(c - I*d)*((S
qrt[-c + I*d]*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a +
I*b]*Sqrt[c + d*Tan[e + f*x]])])/(-a + I*b)^(3/2) + Sqrt[c + d*Tan[e + f*x]
]/((a - I*b)*Sqrt[a + b*Tan[e + f*x]]))))/(a - I*b)^2)/(15*f)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\tan(fx+e))^{\frac{5}{2}}}{(a+b\tan(fx+e))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(5/2)/(a+b*tan(f*x+e))^(7/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(5/2)/(a+b*tan(f*x+e))^(7/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+b*tan(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((2*b*d+2*a*c)^2>0)', see `assume?`
 for mo
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+b*tan(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\tan(e+fx))^{\frac{5}{2}}}{(a+b\tan(e+fx))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(5/2)/(a+b*tan(f*x+e))**(7/2),x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(5/2)/(a + b*tan(e + f*x))**(7/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)/(a+b*tan(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\tan(e+f\,x))^{5/2}}{(a+b\tan(e+f\,x))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(5/2)/(a + b*tan(e + f*x))^(7/2),x)
```

```
[Out] int((c + d*tan(e + f*x))^(5/2)/(a + b*tan(e + f*x))^(7/2), x)
```

3.1283 $\int \frac{(a+b\tan(e+fx))^{5/2}}{\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=264

$$-\frac{i(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{c-id}\,f}+\frac{i(a+ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{c+id}\,f}$$

```
[Out] -b^(3/2)*(-5*a*d+b*c)*arctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^(1/2)/(c+d*t
an(f*x+e))^(1/2))/d^(3/2)/f-I*(a-I*b)^(5/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(
f*x+e))^(1/2)/(a-I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/f/(c-I*d)^(1/2)+I*(a+I*
b)^(5/2)*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*ta
n(f*x+e))^(1/2))/f/(c+I*d)^(1/2)+b^2*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e)
)^(1/2)/d/f
```

Rubi [A]
time = 1.85, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {3647, 3736, 6857, 65, 223, 212, 95, 214}

$$-\frac{b^{3/2}(bc-5ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{d^{3/2}f}+\frac{b^2\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{df}-\frac{i(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{c-id}}+\frac{i(a+ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{c+id}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^(5/2)/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((-I)*(a - I*b)^(5/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqr
t[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[c - I*d]*f) + (I*(a + I*b)^(5/
2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c +
 d*Tan[e + f*x]])])/(Sqrt[c + I*d]*f) - (b^(3/2)*(b*c - 5*a*d)*ArcTanh[(Sqr
t[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(d^(3/2
)*f) + (b^2*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

`[n, 0]`

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^{5/2}}{\sqrt{c+d\tan(e+fx)}}\,dx = \frac{b^2\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{df} + \frac{\int \frac{\frac{1}{2}\left(2a^3d-b^2(bc+ad)\right)+b\left(3a^2-b^2\right)d\tan \cdots}{\sqrt{a+b\tan(e+fx)}\,\cdots}}{d \cdots}$$

$$= \frac{b^2\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{df} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}\left(2a^3d-b^2(bc+ad)\right)+b\left(3a^2 \cdots\right.}{\sqrt{a+bx}\,\sqrt{c \cdots}}\right.}{\cdots}$$

$$= \frac{b^2\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{df} + \frac{\text{Subst}\left(\int\left(-\frac{b^2(bc-5ad)}{2\sqrt{a+bx}\,\sqrt{c+ \cdots}}\right.\right.}{\cdots}$$

$$= \frac{b^2\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{df} + \frac{\text{Subst}\left(\int \frac{a\left(a^2-3b^2\right)d+b\left(3a^2-b^2\right) \cdots}{\sqrt{a+bx}\,\sqrt{c+dx}\,(1 \cdots}\right.}{df}$$

$$= \frac{b^2\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{df} + \frac{\text{Subst}\left(\int\left(\frac{ia\left(a^2-3b^2\right)d-b\left(3a^2- \cdots\right.}{2(i-x)\sqrt{a+bx}\,\sqrt{c \cdots}}\right.\right.}{\cdots}$$

$$= \frac{b^2\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{df} - \frac{(ia-b)^3\text{Subst}\left(\int \frac{\cdots}{(i-x)\sqrt{a+b \cdots}}\right.}{\cdots}$$

$$= -\frac{b^{3/2}(bc-5ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{d^{3/2}f} + \frac{b^2\sqrt{a+b\tan(e+fx \cdots}}{\cdots}$$

$$= -\frac{i(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{c-id}\,f} + \frac{i(a+ib)^{5/2}\tanh^{-1} \cdots}{\cdots}$$

Mathematica [A]
time = 3.43, size = 438, normalized size = 1.66

$$\frac{\frac{\left(3a^2b^2-b^4+a\sqrt{-b^2}\left(a^2-3b^2\right)\right)d\tanh^{-1}\left(\frac{\sqrt{-c+\frac{\sqrt{-b^2}\,d}{b}}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+\sqrt{-b^2}}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+\sqrt{-b^2}}\,\sqrt{-c+\frac{\sqrt{-b^2}\,d}{b}}}+\frac{\left(-3a^2b^2+b^4+a^3\sqrt{-b^2}+3a\left(-b^2\right)^{3/2}\right)d\tanh^{-1}\left(\frac{\sqrt{c+\frac{\sqrt{-b^2}\,d}{b}}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+\sqrt{-b^2}}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+\sqrt{-b^2}}\,\sqrt{c+\frac{\sqrt{-b^2}\,d}{b}}}+b^3\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}-\frac{b^{5/2}(bc-5ad)\sqrt{c-\frac{ad}{b}}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c-\frac{ad}{b}}}\right)\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}}{\sqrt{d}\,\sqrt{c+d\tan(e+fx)}}}{bdf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^(5/2)/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] (((3*a^2*b^2 - b^4 + a*Sqrt[-b^2]*(a^2 - 3*b^2))*d*ArcTanh[(Sqrt[-c + (Sqrt
[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + d*Ta
n[e + f*x]])])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[-c + (Sqrt[-b^2]*d)/b]) + ((-3*a
^2*b^2 + b^4 + a^3*Sqrt[-b^2] + 3*a*(-b^2)^(3/2))*d*ArcTanh[(Sqrt[c + (Sqrt
[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + d*Tan
[e + f*x]])])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + (Sqrt[-b^2]*d)/b]) + b^3*Sqrt[
a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]] - (b^(5/2)*(b*c - 5*a*d)*Sqrt[
c - (a*d)/b]*ArcSinh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c - (
a*d)/b])]*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c - a*d)])/(Sqrt[d]*Sqrt[c + d*T
an[e + f*x]]))/(b*d*f)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(fx+e))^{\frac{5}{2}}}{\sqrt{c+d\tan(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(1/2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*tan(f*x + e) + a)^(5/2)/sqrt(d*tan(f*x + e) + c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan{(e + fx)})^{\frac{5}{2}}}{\sqrt{c + d\tan{(e + fx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(5/2)/(c+d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**(5/2)/sqrt(c + d*tan(e + f*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\tan(e + f\,x))^{5/2}}{\sqrt{c + d\tan{(e + f\,x)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^(5/2)/(c + d*tan(e + f*x))^(1/2),x)
```

```
[Out] int((a + b*tan(e + f*x))^(5/2)/(c + d*tan(e + f*x))^(1/2), x)
```

3.1284 $\int \frac{(a+b\tan(e+fx))^{3/2}}{\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=218

$$-\frac{i(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{c-id}\,f}+\frac{i(a+ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{c+id}\,f}$$

```
[Out] -I*(a-I*b)^(3/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)
/(c+d*tan(f*x+e))^(1/2))/f/(c-I*d)^(1/2)+I*(a+I*b)^(3/2)*arctanh((c+I*d)^(1
/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/f/(c+I*d)^
(1/2)+2*b^(3/2)*arctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x
+e))^(1/2))/f/d^(1/2)
```

Rubi [A]

time = 0.85, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {3656, 924, 65, 223, 212, 6857, 95, 214}

$$\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{d}\,f}-\frac{i(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{c-id}}+\frac{i(a+ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{c+id}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^(3/2)/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((-I)*(a - I*b)^(3/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqr
t[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[c - I*d]*f) + (I*(a + I*b)^(3/
2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c +
 d*Tan[e + f*x]])])/(Sqrt[c + I*d]*f) + (2*b^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a
+ b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[d]*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+b\tan(e+fx))^{3/2}}{\sqrt{c+d\tan(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}\,(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \left(\frac{b^2}{\sqrt{a+bx}\,\sqrt{c+dx}} + \frac{a^2-b^2+2abx}{\sqrt{a+bx}\,\sqrt{c+dx}\,(1+x^2)}\right) dx, x, \tan(e+ \right.}{f} \\
&= \frac{\text{Subst}\left(\int \frac{a^2-b^2+2abx}{\sqrt{a+bx}\,\sqrt{c+dx}\,(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f} + \frac{b^2\text{Subst}\left(\int \frac{}{\sqrt{a+}}\right.}{} \\
&= \frac{\text{Subst}\left(\int \left(\frac{-2ab+i(a^2-b^2)}{2(i-x)\sqrt{a+bx}\,\sqrt{c+dx}} + \frac{2ab+i(a^2-b^2)}{2(i+x)\sqrt{a+bx}\,\sqrt{c+dx}}\right) dx, x, \tan(\right.}{f} \\
&= \frac{(i(a-ib)^2)\,\text{Subst}\left(\int \frac{1}{(i+x)\sqrt{a+bx}\,\sqrt{c+dx}}\,dx, x, \tan(e+fx)\right)}{2f} + \frac{(i(a+}{} \\
&= \frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{d}\,f} + \frac{(i(a-ib)^2)\,\text{Subst}\left(\int \frac{1}{-a+ib-(-c}\right.}{} \\
&= -\frac{i(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{c-id}\,f} + \frac{i(a+ib)^{3/2}\tanh^{-}}{}
\end{aligned}
$$

Mathematica [A]
time = 1.99, size = 294, normalized size = 1.35

$$
\frac{\frac{i(-a+ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{-c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-c+id}} + \frac{i(a+ib)^{3/2}\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)\sqrt{c+d\tan(e+fx)} + 2b\sqrt{c+id}\,\sqrt{bc-ad}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{bc-ad}}\right)\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}}{\sqrt{c+id}\,\sqrt{d}\,\sqrt{c+d\tan(e+fx)}}}{f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^(3/2)/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((I*(-a + I*b)^(3/2)*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqr
t[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[-c + I*d] + (I*(a + I*b)^(3/2)
*Sqrt[d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sq
rt[c + d*Tan[e + f*x]])]*Sqrt[c + d*Tan[e + f*x]] + 2*b*Sqrt[c + I*d]*Sqrt[
b*c - a*d]*ArcSinh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/Sqrt[b*c - a*d]]*Sqrt
```

```
[(b*(c + d*Tan[e + f*x]))/(b*c - a*d)])/(Sqrt[c + I*d]*Sqrt[d]*Sqrt[c + d*T
an[e + f*x]]))/f
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(fx+e))^{\frac{3}{2}}}{\sqrt{c+d\tan(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(1/2),x)
[Out] int((a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(1/2),x, algorithm="maxim
a")
[Out] integrate((b*tan(f*x + e) + a)^(3/2)/sqrt(d*tan(f*x + e) + c), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(1/2),x, algorithm="frica
s")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(e+fx))^{\frac{3}{2}}}{\sqrt{c+d\tan(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(3/2)/(c+d*tan(f*x+e))**(1/2),x)
[Out] Integral((a + b*tan(e + f*x))**(3/2)/sqrt(c + d*tan(e + f*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\tan(e+f\,x))^{3/2}}{\sqrt{c+d\tan\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^(3/2)/(c + d*tan(e + f*x))^(1/2),x)
```

```
[Out] int((a + b*tan(e + f*x))^(3/2)/(c + d*tan(e + f*x))^(1/2), x)
```

3.1285 $\int \frac{\sqrt{a+b\tan(e+fx)}}{\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=163

$$-\frac{i\sqrt{a-ib}\,\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{c-id}\,f}+\frac{i\sqrt{a+ib}\,\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{c+id}\,f}$$

```
[Out] -I*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c+d*tan(f*x+
e))^(1/2))*(a-I*b)^(1/2)/f/(c-I*d)^(1/2)+I*arctanh((c+I*d)^(1/2)*(a+b*tan(f
*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))*(a+I*b)^(1/2)/f/(c+I*d)^
(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {3656, 924, 95, 214}

$$\frac{i\sqrt{a+ib}\,\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{c+id}}-\frac{i\sqrt{a-ib}\,\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{c-id}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Tan[e + f*x]]/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((-I)*Sqrt[a - I*b]*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[
a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[c - I*d]*f) + (I*Sqrt[a + I*b]*A
rcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*T
an[e + f*x]])])/(Sqrt[c + I*d]*f)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\tan(e+fx)}}{\sqrt{c+d\tan(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\,(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{ia-b}{2(i-x)\sqrt{a+bx}\,\sqrt{c+dx}} + \frac{ia+b}{2(i+x)\sqrt{a+bx}\,\sqrt{c+dx}}\right) dx, x, \tan(e$$

$$= \frac{(ia-b)\text{Subst}\left(\int \frac{1}{(i-x)\sqrt{a+bx}\,\sqrt{c+dx}}\,dx, x, \tan(e+fx)\right)}{2f} + \frac{(ia+b)\text{Subs}}{}$$

$$= \frac{(ia-b)\text{Subst}\left(\int \frac{1}{a+ib-(c+id)x^2}\,dx, x, \frac{\sqrt{a+b\tan(e+fx)}}{\sqrt{c+d\tan(e+fx)}}\right)}{f} + \frac{(ia+b)\text{Subst}\Big(}{}$$

$$= -\frac{i\sqrt{a-ib}\,\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{c-id}\,f} + \frac{i\sqrt{a+ib}\,\tanh^{-1}\Big(}{}$$

Mathematica [A]
time = 0.17, size = 167, normalized size = 1.02

$$-\frac{i\left(\frac{\sqrt{-a+ib}\,\tanh^{-1}\left(\frac{\sqrt{-c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-c+id}} - \frac{\sqrt{a+ib}\,\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{c+id}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Tan[e + f*x]]/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((-I)*((Sqrt[-a + I*b]*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(S
qrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[-c + I*d] - (Sqrt[a + I*b]*A
rcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*T
an[e + f*x]])])/Sqrt[c + I*d]))/f
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\tan\left(fx+e\right)}}{\sqrt{c+d\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(1/2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((2*b*d+2*a*c)^2>0)', see `assume?`
 for mo
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\tan\left(e+fx\right)}}{\sqrt{c+d\tan\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(1/2)/(c+d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x))/sqrt(c + d*tan(e + f*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^(1/2)/(c + d*tan(e + f*x))^(1/2),x)
```

```
[Out] \text{Hanged}
```

3.1286 $\int \frac{1}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=163

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a-ib}\sqrt{c-id}\,f}+\frac{i\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}\sqrt{c+id}\,f}$$

```
[Out] -I*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c+d*tan(f*x+
e))^(1/2))/f/(a-I*b)^(1/2)/(c-I*d)^(1/2)+I*arctanh((c+I*d)^(1/2)*(a+b*tan(f
*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/f/(a+I*b)^(1/2)/(c+I*d)^
(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {3656, 926, 95, 214}

$$\frac{i\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a+ib}\sqrt{c+id}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a-ib}\sqrt{c-id}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]]),x]
```

```
[Out] ((-I)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[
c + d*Tan[e + f*x]])])/(Sqrt[a - I*b]*Sqrt[c - I*d]*f) + (I*ArcTanh[(Sqrt[c
 + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])]
)/(Sqrt[a + I*b]*Sqrt[c + I*d]*f)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}\,(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{i}{2(i-x)\sqrt{a+bx}\,\sqrt{c+dx}} + \frac{i}{2(i+x)\sqrt{a+bx}\,\sqrt{c}$$

$$= \frac{i\text{Subst}\left(\int \frac{1}{(i-x)\sqrt{a+bx}\,\sqrt{c+dx}}\,dx, x, \tan(e+fx)\right)}{2f} +$$

$$= \frac{i\text{Subst}\left(\int \frac{1}{-a+ib-(-c+id)x^2}\,dx, x, \frac{\sqrt{a+b\tan(e+fx)}}{\sqrt{c+d\tan(e+fx)}}\right)}{f}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a-ib}\,\sqrt{c-id}\,f} + \frac{i\tanh^{-1}}{}$$

Mathematica [A]
time = 0.17, size = 166, normalized size = 1.02

$$\frac{i\left(\frac{\tanh^{-1}\left(\frac{\sqrt{-c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+ib}\,\sqrt{-c+id}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}\,\sqrt{c+id}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]]),x]
```

```
[Out] (I*(ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + I*b]*Sqrt[
c + d*Tan[e + f*x]])]/(Sqrt[-a + I*b]*Sqrt[-c + I*d]) + ArcTanh[(Sqrt[c + I
*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])]/(Sq
rt[a + I*b]*Sqrt[c + I*d])))/f
```

Maple [**F(-1)**] grade_annotation
time = 180.00, size = 0, normalized size = 0.00 hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(1/2),x)
```

```
[Out] int(1/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((2*b*d+2*a*c)^2>0)', see `assume?`
 for mo
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\tan{\left(e + fx \right)}}\sqrt{c + d\tan{\left(e + fx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))**(1/2)/(c+d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*tan(e + f*x))*sqrt(c + d*tan(e + f*x))), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*tan(e + f*x))^(1/2)*(c + d*tan(e + f*x))^(1/2)),x)
```

```
[Out] \text{Hanged}
```

3.1287 $\int \frac{1}{(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=218

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{3/2}\sqrt{c-id}\,f}+\frac{i\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{3/2}\sqrt{c+id}\,f}-\frac{2b^2\sqrt{c+}}{(a^2+b^2)(bc-ad}$$

```
[Out] -I*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c+d*tan(f*x+
e))^(1/2))/(a-I*b)^(3/2)/f/(c-I*d)^(1/2)+I*arctanh((c+I*d)^(1/2)*(a+b*tan(f
*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(a+I*b)^(3/2)/f/(c+I*d)^
(1/2)-2*b^2*(c+d*tan(f*x+e))^(1/2)/(a^2+b^2)/(-a*d+b*c)/f/(a+b*tan(f*x+e))^
(1/2)
```

Rubi [A]
time = 0.51, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3650, 3697, 3696, 95, 214}

$$-\frac{2b^2\sqrt{c+d\tan(e+fx)}}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{f(a-ib)^{3/2}\sqrt{c-id}}+\frac{i\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{f(a+ib)^{3/2}\sqrt{c+id}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Tan[e + f*x])^(3/2)*Sqrt[c + d*Tan[e + f*x]]),x]
```

```
[Out] ((-I)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[
c + d*Tan[e + f*x]])])/((a - I*b)^(3/2)*Sqrt[c - I*d]*f) + (I*ArcTanh[(Sqrt
[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]]
)])/((a + I*b)^(3/2)*Sqrt[c + I*d]*f) - (2*b^2*Sqrt[c + d*Tan[e + f*x]])/((
a^2 + b^2)*(b*c - a*d)*f*Sqrt[a + b*Tan[e + f*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}\,dx = -\frac{2b^2\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)(bc-ad)f\sqrt{a+b\tan(e+fx)}} - \frac{2\int \frac{}{\sqrt{a+}}}{}$$

$$= -\frac{2b^2\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)(bc-ad)f\sqrt{a+b\tan(e+fx)}} + \frac{\int \frac{}{\sqrt{a+b}}}{}$$

$$= -\frac{2b^2\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)(bc-ad)f\sqrt{a+b\tan(e+fx)}} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2b^2\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)(bc-ad)f\sqrt{a+b\tan(e+fx)}} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{3/2}\sqrt{c-id}\,f} + \frac{i\tanh^{-1}}{}$$

Mathematica [A]
time = 1.51, size = 232, normalized size = 1.06

$$\frac{\frac{i(a+ib)\tanh^{-1}\left(\frac{\sqrt{-c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+ib}\sqrt{-c+id}} + \frac{(ia+b)\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}\sqrt{c+id}} + \frac{2b^2\sqrt{c+d\tan(e+fx)}}{(-bc+ad)\sqrt{a+b\tan(e+fx)}}}{(a^2+b^2)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Tan[e + f*x])^(3/2)*Sqrt[c + d*Tan[e + f*x]]),x]
```

```
[Out] ((I*(a + I*b)*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a +
I*b]*Sqrt[c + d*Tan[e + f*x]])]])/(Sqrt[-a + I*b]*Sqrt[-c + I*d]) + ((I*a +
b)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c +
 d*Tan[e + f*x]])]])/(Sqrt[a + I*b]*Sqrt[c + I*d]) + (2*b^2*Sqrt[c + d*Tan[e
 + f*x]])/((-(b*c) + a*d)*Sqrt[a + b*Tan[e + f*x]]))/((a^2 + b^2)*f)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c+d\tan(fx+e)}\,(a+b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^(3/2),x)
```

```
[Out] int(1/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((b*tan(f*x + e) + a)^(3/2)*sqrt(d*tan(f*x + e) + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\tan\left(e+fx\right)\right)^{\frac{3}{2}}\sqrt{c+d\tan\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))**(1/2)/(a+b*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/((a + b*tan(e + f*x))**(3/2)*sqrt(c + d*tan(e + f*x))), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*tan(e + f*x))^(3/2)*(c + d*tan(e + f*x))^(1/2)),x)
[Out] \text{Hanged}
```

3.1288 $\int \frac{1}{(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=295

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{5/2}\sqrt{c-id}\,f}+\frac{i\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{5/2}\sqrt{c+id}\,f}-\frac{2b^2\sqrt{c}}{3\,(a^2+b^2)\,(bc-}$$

```
[Out] -I*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c+d*tan(f*x+
e))^(1/2))/(a-I*b)^(5/2)/f/(c-I*d)^(1/2)+I*arctanh((c+I*d)^(1/2)*(a+b*tan(f
*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(a+I*b)^(5/2)/f/(c+I*d)^
(1/2)-4/3*b^2*(-4*a^2*d+3*a*b*c-b^2*d)*(c+d*tan(f*x+e))^(1/2)/(a^2+b^2)^2/(
-a*d+b*c)^2/f/(a+b*tan(f*x+e))^(1/2)-2/3*b^2*(c+d*tan(f*x+e))^(1/2)/(a^2+b^
2)/(-a*d+b*c)/f/(a+b*tan(f*x+e))^(3/2)
```

Rubi [A]

time = 0.85, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3650, 3730, 3697, 3696, 95, 214}

$$-\frac{4b^2(-4a^2d+3abc-b^2d)\sqrt{c+d\tan(e+fx)}}{3f(a^2+b^2)^2(bc-ad)^2\sqrt{a+b\tan(e+fx)}}-\frac{2b^2\sqrt{c+d\tan(e+fx)}}{3f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))^{3/2}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a-ib)^{5/2}\sqrt{c-id}}+\frac{i\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a+ib)^{5/2}\sqrt{c+id}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Tan[e + f*x])^(5/2)*Sqrt[c + d*Tan[e + f*x]]),x]
```

```
[Out] ((-I)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[
c + d*Tan[e + f*x]])])/((a - I*b)^(5/2)*Sqrt[c - I*d]*f) + (I*ArcTanh[(Sqrt
[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]]
)])/((a + I*b)^(5/2)*Sqrt[c + I*d]*f) - (2*b^2*Sqrt[c + d*Tan[e + f*x]])/(3
*(a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])^(3/2)) - (4*b^2*(3*a*b*c -
4*a^2*d - b^2*d)*Sqrt[c + d*Tan[e + f*x]])/(3*(a^2 + b^2)^2*(b*c - a*d)^2*f
*Sqrt[a + b*Tan[e + f*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}\,dx = -\frac{2b^2\sqrt{c+d\tan(e+fx)}}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}} - \frac{2\int \frac{1}{2}(\cdots}{\cdots}$$

$$= -\frac{2b^2\sqrt{c+d\tan(e+fx)}}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}} - \frac{4b^2(3a\cdots}{3\left(a^2\cdots\right.}$$

$$= -\frac{2b^2\sqrt{c+d\tan(e+fx)}}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}} - \frac{4b^2(3a\cdots}{3\left(a^2\cdots\right.}$$

$$= -\frac{2b^2\sqrt{c+d\tan(e+fx)}}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}} - \frac{4b^2(3a\cdots}{3\left(a^2\cdots\right.}$$

$$= -\frac{2b^2\sqrt{c+d\tan(e+fx)}}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}} - \frac{4b^2(3a\cdots}{3\left(a^2\cdots\right.}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{5/2}\sqrt{c-id}\,f} + \frac{i\tanh^{-1}\cdots}{\cdots}$$

Mathematica [A]
time = 2.22, size = 308, normalized size = 1.04

$$\frac{3i\left(\frac{(a+ib)^2\tanh^{-1}\left(\frac{\sqrt{-c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+ib}\sqrt{-c+id}}+\frac{(a-ib)^2\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}\sqrt{c+id}}\right)+\frac{2b^2(a^2+b^2)\sqrt{c+d\tan(e+fx)}}{(-bc+ad)(a+b\tan(e+fx))^{3/2}}+\frac{4b^2(-3abc+4a^2d+b^2d)\sqrt{c+d\tan(e+fx)}}{(bc-ad)^2\sqrt{a+b\tan(e+fx)}}}{3\left(a^2+b^2\right)^2 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Tan[e + f*x])^(5/2)*Sqrt[c + d*Tan[e + f*x]]),x]
```

```
[Out] ((3*I)*(((a + I*b)^2*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqr
t[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[-a + I*b]*Sqrt[-c + I*d]) + (
(a - I*b)^2*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]
*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*Sqrt[c + I*d])) + (2*b^2*(a^2 +
 b^2)*Sqrt[c + d*Tan[e + f*x]])/((-(b*c) + a*d)*(a + b*Tan[e + f*x])^(3/2))
 + (4*b^2*(-3*a*b*c + 4*a^2*d + b^2*d)*Sqrt[c + d*Tan[e + f*x]])/((b*c - a*
d)^2*Sqrt[a + b*Tan[e + f*x]]))/(3*(a^2 + b^2)^2*f)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c+d\tan\left(fx+e\right)}\,\left(a+b\tan\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^(5/2),x)
```

```
[Out] int(1/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((b*tan(f*x + e) + a)^(5/2)*sqrt(d*tan(f*x + e) + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\tan(e+fx))^{\frac{5}{2}}\sqrt{c+d\tan(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))**(1/2)/(a+b*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral(1/((a + b*tan(e + f*x))**(5/2)*sqrt(c + d*tan(e + f*x))), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*tan(e + f*x))^(5/2)*(c + d*tan(e + f*x))^(1/2)),x)
[Out] int(1/((a + b*tan(e + f*x))^(5/2)*(c + d*tan(e + f*x))^(1/2)), x)
```

3.1289 $\int \frac{(a+b\tan(e+fx))^{7/2}}{(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=356

$$-\frac{i(a-ib)^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(c-id)^{3/2}f}+\frac{i(a+ib)^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(c+id)^{3/2}f}$$

```
[Out] -I*(a-I*b)^(7/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)
/(c+d*tan(f*x+e))^(1/2))/(c-I*d)^(3/2)/f+I*(a+I*b)^(7/2)*arctanh((c+I*d)^(1
/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(c+I*d)^(3
/2)/f-b^(5/2)*(-7*a*d+3*b*c)*arctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^(1/2)
/(c+d*tan(f*x+e))^(1/2))/d^(5/2)/f-b*(2*a*d*(-a*d+2*b*c)-b^2*(3*c^2+d^2))*(
a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/d^2/(c^2+d^2)/f-2*(-a*d+b*c)^2
*(a+b*tan(f*x+e))^(3/2)/d/(c^2+d^2)/f/(c+d*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 2.91, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {3646, 3728, 3736, 6857, 65, 223, 212, 95, 214}

$$-\frac{b^{5/2}(3bc-7ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{d^{5/2}f}-\frac{b(2ad(2bc-ad)-b^2(3c^2+d^2))\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{d^2f(c^2+d^2)}-\frac{2(bc-ad)^2(a+b\tan(e+fx))^{3/2}}{df(c^2+d^2)\sqrt{c+d\tan(e+fx)}}-\frac{i(a-ib)^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(c-id)^{3/2}}+\frac{i(a+ib)^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(c+id)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^(7/2)/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-I)*(a - I*b)^(7/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqr
t[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c - I*d)^(3/2)*f) + (I*(a + I*b)^(
7/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c
 + d*Tan[e + f*x]])])/((c + I*d)^(3/2)*f) - (b^(5/2)*(3*b*c - 7*a*d)*ArcTan
h[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(
d^(5/2)*f) - (2*(b*c - a*d)^2*(a + b*Tan[e + f*x])^(3/2))/(d*(c^2 + d^2)*f*
Sqrt[c + d*Tan[e + f*x]]) - (b*(2*a*d*(2*b*c - a*d) - b^2*(3*c^2 + d^2))*Sq
rt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(d^2*(c^2 + d^2)*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
```

```
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^{7/2}}{(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{2(bc-ad)^2(a+b\tan(e+fx))^{3/2}}{d\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} + \frac{2\int \sqrt{a+b\tan(e+fx)}\,\left(\frac{1}{2}\left(3b^3c^2+a^3cd-\right.\right.}{}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))^{3/2}}{d\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} - \frac{b(2ad(2bc-ad)-b^2(3c^2+d^2))\,\sqrt{a}}{d^2\,(c^2}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))^{3/2}}{d\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} - \frac{b(2ad(2bc-ad)-b^2(3c^2+d^2))\,\sqrt{a}}{d^2\,(c^2}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))^{3/2}}{d\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} - \frac{b(2ad(2bc-ad)-b^2(3c^2+d^2))\,\sqrt{a}}{d^2\,(c^2}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))^{3/2}}{d\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} - \frac{b(2ad(2bc-ad)-b^2(3c^2+d^2))\,\sqrt{a}}{d^2\,(c^2}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))^{3/2}}{d\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} - \frac{b(2ad(2bc-ad)-b^2(3c^2+d^2))\,\sqrt{a}}{d^2\,(c^2}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))^{3/2}}{d\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} - \frac{b(2ad(2bc-ad)-b^2(3c^2+d^2))\,\sqrt{a}}{d^2\,(c^2}$$

$$= -\frac{b^{5/2}(3bc-7ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{d^{5/2}f} - \frac{2(bc-ad)^2(a+b}{d\,(c^2+d^2)\,f\sqrt{c+}}$$

$$= -\frac{i(a-ib)^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(c-id)^{3/2}f} + \frac{i(a+ib)^{7/2}\tanh^{-}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.36, size = 1877, normalized size = 5.27

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^(7/2)/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-1/2*I)*(-a - I*b)*((-2*b*(b/((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)))
^(3/2)*Hypergeometric2F1[3/2, 5/2, 7/2, -((b*d*(a + b*Tan[e + f*x]))/((b*c
- a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))]*(a + b*Tan[e + f*x])^
(5/2)*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c - a*d)])/(5*(b*c - a*d)*Sqrt[c + d
*Tan[e + f*x]]) - (-a - I*b)*(-((-a - I*b)*((-2*Sqrt[a + I*b]*ArcTanh[(Sqrt
[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]]
)])/((-c - I*d)*Sqrt[c + I*d]) + (2*Sqrt[a + b*Tan[e + f*x]])/((-c - I*d)*S
qrt[c + d*Tan[e + f*x]]))) - (2*(b*c - a*d)*(b/((b^2*c)/(b*c - a*d) - (a*b*
d)/(b*c - a*d)))^(3/2)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))^2*Sqrt[(
b*(c + d*Tan[e + f*x]))/(b*c - a*d)]*(-1 - (b*d*(a + b*Tan[e + f*x]))/((b*c
 - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))*(-((b*d*(a + b*Tan[e
+ f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))*(-1 - (b*
d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c -
a*d)))))) - (Sqrt[b]*Sqrt[d]*ArcSinh[(Sqrt[b]*Sqrt[d]*Sqrt[a + b*Tan[e + f*
x]])/(Sqrt[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)])]*Sqr
t[a + b*Tan[e + f*x]])/(Sqrt[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*d)/
(b*c - a*d)]*Sqrt[1 + (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c
 - a*d) - (a*b*d)/(b*c - a*d)))])))/(b*d^2*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c
+ d*Tan[e + f*x]]*Sqrt[1 + (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)
/(b*c - a*d) - (a*b*d)/(b*c - a*d)))]))))/f - ((I/2)*(-a + I*b)*((2*b*(b/((
b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)))^(3/2)*Hypergeometric2F1[3/2, 5/2
, 7/2, -((b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*
b*d)/(b*c - a*d))))]*(a + b*Tan[e + f*x])^(5/2)*Sqrt[(b*(c + d*Tan[e + f*x]
))/(b*c - a*d)])/(5*(b*c - a*d)*Sqrt[c + d*Tan[e + f*x]]) - (-a + I*b)*(-((
-a + I*b)*((-2*Sqrt[-a + I*b]*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*
x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c - I*d)*Sqrt[-c + I*d])
 + (2*Sqrt[a + b*Tan[e + f*x]])/((c - I*d)*Sqrt[c + d*Tan[e + f*x]]))) + (2
*(b*c - a*d)*(b/((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)))^(3/2)*((b^2*c)
/(b*c - a*d) - (a*b*d)/(b*c - a*d))^2*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c -
a*d)]*(-1 - (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) -
(a*b*d)/(b*c - a*d))))*(-((b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/
(b*c - a*d) - (a*b*d)/(b*c - a*d))*(-1 - (b*d*(a + b*Tan[e + f*x]))/((b*c -
 a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)))))) - (Sqrt[b]*Sqrt[d]*Ar
cSinh[(Sqrt[b]*Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b*c - a*d]*Sqrt[(b^2
*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)])]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b*
c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)]*Sqrt[1 + (b*d*(a +
 b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)))
])))/(b*d^2*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]]*Sqrt[1 + (b*d
*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a
*d)))]))))/f
```

Maple [**F**]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(fx+e))^{\frac{7}{2}}}{(c+d\tan(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(7/2)/(c+d*tan(f*x+e))^(3/2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(7/2)/(c+d*tan(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(7/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*tan(f*x + e) + a)^(7/2)/(d*tan(f*x + e) + c)^(3/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(7/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(7/2)/(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(7/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\tan(e+f\,x))^{7/2}}{(c+d\tan(e+f\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^(7/2)/(c + d*tan(e + f*x))^(3/2),x)
```

```
[Out] int((a + b*tan(e + f*x))^(7/2)/(c + d*tan(e + f*x))^(3/2), x)
```

3.1290 $\int \frac{(a+b\tan(e+fx))^{5/2}}{(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=273

$$-\frac{i(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(c-id)^{3/2}f}+\frac{i(a+ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(c+id)^{3/2}f}$$

```
[Out] -I*(a-I*b)^(5/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)
/(c+d*tan(f*x+e))^(1/2))/(c-I*d)^(3/2)/f+I*(a+I*b)^(5/2)*arctanh((c+I*d)^(1
/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(c+I*d)^(3
/2)/f+2*b^(5/2)*arctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x
+e))^(1/2))/d^(3/2)/f-2*(-a*d+b*c)^2*(a+b*tan(f*x+e))^(1/2)/d/(c^2+d^2)/f/(
c+d*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 2.19, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {3646, 3736, 6857, 65, 223, 212, 95, 214}

$$\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{d^{3/2}f}-\frac{2(bc-ad)^2\sqrt{a+b\tan(e+fx)}}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}-\frac{i(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(c-id)^{3/2}}+\frac{i(a+ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(c+id)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^(5/2)/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-I)*(a - I*b)^(5/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqr
t[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c - I*d)^(3/2)*f) + (I*(a + I*b)^(
5/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c
 + d*Tan[e + f*x]])])/((c + I*d)^(3/2)*f) + (2*b^(5/2)*ArcTanh[(Sqrt[d]*Sqr
t[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(d^(3/2)*f) - (
2*(b*c - a*d)^2*Sqrt[a + b*Tan[e + f*x]])/(d*(c^2 + d^2)*f*Sqrt[c + d*Tan[e
 + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

```
[n, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^{5/2}}{(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{2(bc-ad)^2\sqrt{a+b\tan(e+fx)}}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{2\int \frac{\frac{1}{2}\left(b^3c^2+a^3cd-3ab^2cd+3a^2bd^2\right)+\frac{1}{2}d\left(3a^2bc-b\right.}{\sqrt{a+b\tan(e+f}}}{d\left(\right.}$$

$$= -\frac{2(bc-ad)^2\sqrt{a+b\tan(e+fx)}}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{2\text{Subst}\left(\int \frac{\frac{1}{2}\left(b^3c^2+a^3cd-3ab^2cd+3a^2bd^2\right)+\frac{1}{2}c}{\sqrt{a+bx}}\right.}{}$$

$$= -\frac{2(bc-ad)^2\sqrt{a+b\tan(e+fx)}}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{2\text{Subst}\left(\int \left(\frac{b^3\left(c^2+d^2\right)}{2\sqrt{a+bx}\sqrt{c+dx}} + \frac{d(}{}\right.\right.}{}$$

$$= -\frac{2(bc-ad)^2\sqrt{a+b\tan(e+fx)}}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{b^3\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x,\right.}{df}$$

$$= -\frac{2(bc-ad)^2\sqrt{a+b\tan(e+fx)}}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{\left(2b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,\right.}{df}$$

$$= -\frac{2(bc-ad)^2\sqrt{a+b\tan(e+fx)}}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} - \frac{(ia+b)^3\text{Subst}\left(\int \frac{1}{(i+x)\sqrt{a+bx}\sqrt{c+}}\right.}{2(c-id)f}$$

$$= \frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{d^{3/2}f} - \frac{2(bc-ad)^2\sqrt{a+b\tan(e+fx}}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+f}}$$

$$= -\frac{i(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(c-id)^{3/2}f} + \frac{i(a+ib)^{5/2}\tanh^{-}}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1503 vs. 2(273) = 546.
time = 6.19, size = 1503, normalized size = 5.51

$$-\frac{i(-a-ib)\left(-\left((-a-ib)\left(-\frac{2\sqrt{a+ib}\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(-c-id)\sqrt{c+id}}+\frac{2\sqrt{a+b\tan(e+fx)}}{(-c-id)\sqrt{c+d\tan(e+fx)}}\right)\right)-\frac{2(bc-ad)\left(\frac{b}{\frac{b^2c}{bc-ad}-\frac{abd}{bc-ad}}\right)^{3/2}\left(\frac{b^2c}{bc-ad}-\frac{abd}{bc-ad}\right)^2\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}\left(-1-\frac{bd(a+b\tan(e+fx))}{(bc-ad)\left(\frac{b^2c}{bc-ad}-\frac{abd}{bc-ad}\right)}\right)\left(-\frac{bd(a+b\tan(e+fx))}{(bc-ad)\left(\frac{b^2c}{bc-ad}-\frac{abd}{bc-ad}\right)\left(-1-\frac{bd(a+b\tan(e+fx))}{(bc-ad)\left(\frac{b^2c}{bc-ad}-\frac{abd}{bc-ad}\right)}\right)}-\frac{\sqrt{b}\sqrt{d}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{bc-ad}\sqrt{\frac{b^2c}{bc-ad}-\frac{abd}{bc-ad}}}\right)\sqrt{a+b\tan(e+fx)}}{\sqrt{bc-ad}\sqrt{\frac{b^2c}{bc-ad}-\frac{abd}{bc-ad}}\sqrt{1+\frac{bd(a+b\tan(e+fx))}{(bc-ad)\left(\frac{b^2c}{bc-ad}-\frac{abd}{bc-ad}\right)}}}\right)}{bd^2\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\sqrt{1+\frac{bd(a+b\tan(e+fx))}{(bc-ad)\left(\frac{b^2c}{bc-ad}-\frac{abd}{bc-ad}\right)}}}\right)}{2f}-\frac{i(-a+ib)\left(-\left((-a+ib)\left(-\frac{2\sqrt{-a+ib}\tanh^{-1}\left(\frac{\sqrt{-c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(c-id)\sqrt{-c+id}}+\frac{2\sqrt{a+b\tan(e+fx)}}{(c-id)\sqrt{c+d\tan(e+fx)}}\right)\right)+\frac{2(bc-ad)\left(\frac{b}{\frac{b^2c}{bc-ad}-\frac{abd}{bc-ad}}\right)^{3/2}\left(\frac{b^2c}{bc-ad}-\frac{abd}{bc-ad}\right)^2\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}\left(-1-\frac{bd(a+b\tan(e+fx))}{(bc-ad)\left(\frac{b^2c}{bc-ad}-\frac{abd}{bc-ad}\right)}\right)\left(-\frac{bd(a+b\tan(e+fx))}{(bc-ad)\left(\frac{b^2c}{bc-ad}-\frac{abd}{bc-ad}\right)\left(-1-\frac{bd(a+b\tan(e+fx))}{(bc-ad)\left(\frac{b^2c}{bc-ad}-\frac{abd}{bc-ad}\right)}\right)}-\frac{\sqrt{b}\sqrt{d}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{bc-ad}\sqrt{\frac{b^2c}{bc-ad}-\frac{abd}{bc-ad}}}\right)\sqrt{a+b\tan(e+fx)}}{\sqrt{bc-ad}\sqrt{\frac{b^2c}{bc-ad}-\frac{abd}{bc-ad}}\sqrt{1+\frac{bd(a+b\tan(e+fx))}{(bc-ad)\left(\frac{b^2c}{bc-ad}-\frac{abd}{bc-ad}\right)}}}\right)}{bd^2\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\sqrt{1+\frac{bd(a+b\tan(e+fx))}{(bc-ad)\left(\frac{b^2c}{bc-ad}-\frac{abd}{bc-ad}\right)}}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^(5/2)/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-1/2*I)*(-a - I*b)*(-((-a - I*b)*((-2*Sqrt[a + I*b]*ArcTanh[(Sqrt[c + I*d
]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((-c
 - I*d)*Sqrt[c + I*d]) + (2*Sqrt[a + b*Tan[e + f*x]])/((-c - I*d)*Sqrt[c +
d*Tan[e + f*x]]))) - (2*(b*c - a*d)*(b/((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c
- a*d)))^(3/2)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))^2*Sqrt[(b*(c + d
*Tan[e + f*x]))/(b*c - a*d)]*(-1 - (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*
((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))*(-((b*d*(a + b*Tan[e + f*x]))
/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))*(-1 - (b*d*(a + b
*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)))))
) - (Sqrt[b]*Sqrt[d]*ArcSinh[(Sqrt[b]*Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sq
rt[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)])]*Sqrt[a + b*
Tan[e + f*x]])/(Sqrt[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a
*d)]*Sqrt[1 + (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d)
- (a*b*d)/(b*c - a*d)))])))/(b*d^2*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[
e + f*x]]*Sqrt[1 + (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c -
a*d) - (a*b*d)/(b*c - a*d)))])))/f - ((I/2)*(-a + I*b)*(-((-a + I*b)*((-2*S
qrt[-a + I*b]*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a +
I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c - I*d)*Sqrt[-c + I*d]) + (2*Sqrt[a + b
*Tan[e + f*x]])/((c - I*d)*Sqrt[c + d*Tan[e + f*x]]))) + (2*(b*c - a*d)*(b/
((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)))^(3/2)*((b^2*c)/(b*c - a*d) - (
a*b*d)/(b*c - a*d))^2*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c - a*d)]*(-1 - (b*d
*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a
*d))))*(-((b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a
*b*d)/(b*c - a*d))*(-1 - (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(
b*c - a*d) - (a*b*d)/(b*c - a*d)))))) - (Sqrt[b]*Sqrt[d]*ArcSinh[(Sqrt[b]*S
qrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d)
- (a*b*d)/(b*c - a*d)])]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b*c - a*d]*Sqrt[(b
^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)]*Sqrt[1 + (b*d*(a + b*Tan[e + f*x])
)/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)))])))/(b*d^2*Sqrt
[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]]*Sqrt[1 + (b*d*(a + b*Tan[e +
f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)))])))/f
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan(fx + e))^{\frac{5}{2}}}{(c + d\tan(fx + e))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(3/2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="maxim
a")
[Out] integrate((b*tan(f*x + e) + a)^(5/2)/(d*tan(f*x + e) + c)^(3/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="frica
s")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(e+fx))^{\frac{5}{2}}}{(c+d\tan(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(5/2)/(c+d*tan(f*x+e))**(3/2),x)
[Out] Integral((a + b*tan(e + f*x))**(5/2)/(c + d*tan(e + f*x))**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="giac"
)
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\tan(e+f\,x))^{5/2}}{(c+d\tan\left(e+f\,x\right))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^(5/2)/(c + d*tan(e + f*x))^(3/2),x)
[Out] int((a + b*tan(e + f*x))^(5/2)/(c + d*tan(e + f*x))^(3/2), x)
```

3.1291 $\int \frac{(a+b\tan(e+fx))^{3/2}}{(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=213

$$-\frac{i(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(c-id)^{3/2}f}+\frac{i(a+ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(c+id)^{3/2}f}$$

```
[Out] -I*(a-I*b)^(3/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)
/(c+d*tan(f*x+e))^(1/2))/(c-I*d)^(3/2)/f+I*(a+I*b)^(3/2)*arctanh((c+I*d)^(1
/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(c+I*d)^(3
/2)/f+2*(-a*d+b*c)*(a+b*tan(f*x+e))^(1/2)/(c^2+d^2)/f/(c+d*tan(f*x+e))^(1/2
)
```

Rubi [A]
time = 0.54, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3648, 3697, 3696, 95, 214}

$$\frac{2(bc-ad)\sqrt{a+b\tan(e+fx)}}{f\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}-\frac{i(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(c-id)^{3/2}}+\frac{i(a+ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(c+id)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^(3/2)/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-I)*(a - I*b)^(3/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqr
t[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c - I*d)^(3/2)*f) + (I*(a + I*b)^(
3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c
 + d*Tan[e + f*x]])])/((c + I*d)^(3/2)*f) + (2*(b*c - a*d)*Sqrt[a + b*Tan[e
 + f*x]])/((c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^{3/2}}{(c+d\tan(e+fx))^{3/2}}\,dx = \frac{2(bc-ad)\sqrt{a+b\tan(e+fx)}}{(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} - \frac{2\int \frac{\frac{1}{2}(-a^2c+b^2c-2abd)-\frac{1}{2}(2abc-a^2d+b^2d)\tan(e+}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+}}}{c^2+d^2}$$

$$= \frac{2(bc-ad)\sqrt{a+b\tan(e+fx)}}{(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} + \frac{(a-ib)^2\int \frac{1+i\tan(e+fx)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d}}}{2(c-id)}$$

$$= \frac{2(bc-ad)\sqrt{a+b\tan(e+fx)}}{(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} + \frac{(a-ib)^2\text{Subst}\left(\int \frac{1}{(1-ix)\sqrt{a+bx}\sqrt{c+dx}}\right.}{2(c-id)f}$$

$$= \frac{2(bc-ad)\sqrt{a+b\tan(e+fx)}}{(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} + \frac{(a-ib)^2\text{Subst}\left(\int \frac{1}{ia+b-(ic+d)x^2}\,dx, x, \frac{\sqrt{a}}{\sqrt{c}}\right.}{(c-id)f}$$

$$= -\frac{i(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(c-id)^{3/2}f} + \frac{i(a+ib)^{3/2}\tanh^{-}}{}$$

Mathematica [A]
time = 2.06, size = 231, normalized size = 1.08

$$-\frac{\frac{i(-a+ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{-c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(-c+id)^{3/2}} + \frac{\frac{(a+ib)^{3/2}(ic+d)\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(c+id)^{3/2}} + \frac{2(bc-ad)\sqrt{a+b\tan(e+fx)}}{(c+id)\sqrt{c+d\tan(e+fx)}}}{c-id}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^(3/2)/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (((-I)*(-a + I*b)^(3/2)*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(
Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(-c + I*d)^(3/2) + (((a + I*b)^(
3/2)*(I*c + d)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I
*b]*Sqrt[c + d*Tan[e + f*x]])])/(c + I*d)^(3/2) + (2*(b*c - a*d)*Sqrt[a + b
*Tan[e + f*x]])/((c + I*d)*Sqrt[c + d*Tan[e + f*x]]))/(c - I*d))/f
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(fx+e))^{\frac{3}{2}}}{(c+d\tan(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(3/2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((2*b*d+2*a*c)^2>0)', see `assume?`
 for mo
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan{(e+fx)})^{\frac{3}{2}}}{(c+d\tan{(e+fx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(3/2)/(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**(3/2)/(c + d*tan(e + f*x))**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\tan(e+f\,x))^{3/2}}{(c+d\tan\left(e+f\,x\right))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^(3/2)/(c + d*tan(e + f*x))^(3/2),x)
```

```
[Out] int((a + b*tan(e + f*x))^(3/2)/(c + d*tan(e + f*x))^(3/2), x)
```

3.1292 $\int \frac{\sqrt{a+b\tan(e+fx)}}{(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=206

$$-\frac{i\sqrt{a-ib}\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(c-id)^{3/2}f}+\frac{i\sqrt{a+ib}\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(c+id)^{3/2}f}-$$

```
[Out] -I*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c+d*tan(f*x+
e))^(1/2))*(a-I*b)^(1/2)/(c-I*d)^(3/2)/f+I*arctanh((c+I*d)^(1/2)*(a+b*tan(f
*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))*(a+I*b)^(1/2)/(c+I*d)^(3
/2)/f-2*d*(a+b*tan(f*x+e))^(1/2)/(c^2+d^2)/f/(c+d*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3649, 3697, 3696, 95, 214}

$$-\frac{2d\sqrt{a+b\tan(e+fx)}}{f(c^2+d^2)\sqrt{c+d\tan(e+fx)}}-\frac{i\sqrt{a-ib}\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{f(c-id)^{3/2}}+\frac{i\sqrt{a+ib}\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{f(c+id)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Tan[e + f*x]]/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-I)*Sqrt[a - I*b]*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[
a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c - I*d)^(3/2)*f) + (I*Sqrt[a + I*b]
*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d
*Tan[e + f*x]])])/((c + I*d)^(3/2)*f) - (2*d*Sqrt[a + b*Tan[e + f*x]])/((c^
2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\tan(e+fx)}}{(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{2d\sqrt{a+b\tan(e+fx)}}{(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} - \frac{2\int \frac{\frac{1}{2}(-ac-bd)-\frac{1}{2}(bc-ad)\tan(e+fx)}{\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+}}}{c^2+d^2}$$

$$= -\frac{2d\sqrt{a+b\tan(e+fx)}}{(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} + \frac{(a-ib)\int \frac{1+i\tan(e+fx)}{\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\,t}}}{2(c-id)}$$

$$= -\frac{2d\sqrt{a+b\tan(e+fx)}}{(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} + \frac{(a-ib)\text{Subst}\Big(\int \frac{1}{(1-ix)\sqrt{a+bx}\,\sqrt{c+dx}}}{2(c-id)f}$$

$$= -\frac{2d\sqrt{a+b\tan(e+fx)}}{(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} + \frac{(a-ib)\text{Subst}\Big(\int \frac{1}{ia+b-(ic+d)x^2}\,dx, x, \frac{\sqrt{a}}{\sqrt{c}}}{(c-id)f}$$

$$= -\frac{i\sqrt{a-ib}\,\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(c-id)^{3/2}f} + \frac{i\sqrt{a+ib}\,\tanh^{-1}\Big(}{(}$$

Mathematica [A]
time = 1.69, size = 224, normalized size = 1.09

$$\frac{\frac{i\sqrt{-a+ib}\,\tanh^{-1}\left(\frac{\sqrt{-c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(-c+id)^{3/2}} + \frac{\frac{\sqrt{a+ib}\,(ic+d)\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(c+id)^{3/2}} - \frac{2d\sqrt{a+b\tan(e+fx)}}{(c+id)\sqrt{c+d\tan(e+fx)}}}{c-id}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Tan[e + f*x]]/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((I*Sqrt[-a + I*b]*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[
-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(-c + I*d)^(3/2) + ((Sqrt[a + I*b]*(I
*c + d)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqr
t[c + d*Tan[e + f*x]])])/(c + I*d)^(3/2) - (2*d*Sqrt[a + b*Tan[e + f*x]])/(
(c + I*d)*Sqrt[c + d*Tan[e + f*x]]))/(c - I*d))/f
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\tan(fx+e)}}{(c+d\tan(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(3/2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((2*b*d+2*a*c)^2>0)', see `assume?`
 for mo
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\tan{(e + fx)}}}{(c + d\tan{(e + fx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(1/2)/(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x))/(c + d*tan(e + f*x))**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\tan\left(e+f\,x\right)}}{{\left(c+d\tan\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^(1/2)/(c + d*tan(e + f*x))^(3/2),x)
```

```
[Out] int((a + b*tan(e + f*x))^(1/2)/(c + d*tan(e + f*x))^(3/2), x)
```

3.1293 $\int \frac{1}{\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=218

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a-ib}\,(c-id)^{3/2}f}+\frac{i\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}\,(c+id)^{3/2}f}+\frac{2d^2\sqrt{a}}{(bc-ad)\,(c^2+}$$

```
[Out] -I*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c+d*tan(f*x+
e))^(1/2))/(c-I*d)^(3/2)/f/(a-I*b)^(1/2)+I*arctanh((c+I*d)^(1/2)*(a+b*tan(f
*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(c+I*d)^(3/2)/f/(a+I*b)^
(1/2)+2*d^2*(a+b*tan(f*x+e))^(1/2)/(-a*d+b*c)/(c^2+d^2)/f/(c+d*tan(f*x+e))^
(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3650, 3697, 3696, 95, 214}

$$\frac{2d^2\sqrt{a+b\tan(e+fx)}}{f\,(c^2+d^2)\,(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a-ib}\,(c-id)^{3/2}}+\frac{i\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a+ib}\,(c+id)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] ((-I)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[
c + d*Tan[e + f*x]])])/(Sqrt[a - I*b]*(c - I*d)^(3/2)*f) + (I*ArcTanh[(Sqrt
[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]]
)])/(Sqrt[a + I*b]*(c + I*d)^(3/2)*f) + (2*d^2*Sqrt[a + b*Tan[e + f*x]])/((
b*c - a*d)*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}}\,dx = \frac{2d^2\sqrt{a+b\tan(e+fx)}}{(bc-ad)(c^2+d^2)f\sqrt{c+d\tan(e+fx)}} + \frac{2\int \frac{}{\sqrt{a+}}}{}$$

$$= \frac{2d^2\sqrt{a+b\tan(e+fx)}}{(bc-ad)(c^2+d^2)f\sqrt{c+d\tan(e+fx)}} + \frac{\int \frac{}{\sqrt{a+b}}}{}$$

$$= \frac{2d^2\sqrt{a+b\tan(e+fx)}}{(bc-ad)(c^2+d^2)f\sqrt{c+d\tan(e+fx)}} + \frac{\text{Subst}\Big(\int}{}$$

$$= \frac{2d^2\sqrt{a+b\tan(e+fx)}}{(bc-ad)(c^2+d^2)f\sqrt{c+d\tan(e+fx)}} + \frac{\text{Subst}\Big(\int}{}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a-ib}\,(c-id)^{3/2}f} + \frac{i\tanh^{-}}{}$$

Mathematica [A]
time = 1.11, size = 243, normalized size = 1.11

$$-\frac{(bc-ad)\left(\frac{i(c+id)\tanh^{-1}\left(\frac{\sqrt{-c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+ib}\sqrt{-c+id}} + \frac{(ic+d)\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}\sqrt{c+id}}\right) + \frac{2d^2\sqrt{a+b\tan(e+fx)}}{\sqrt{c+d\tan(e+fx)}}}{(-bc+ad)(c^2+d^2)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] -(((b*c - a*d)*((I*(c + I*d)*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x
]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[-a + I*b]*Sqrt[-c + I
*d]) + ((I*c + d)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a
+ I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*Sqrt[c + I*d])) + (2*d^2*
Sqrt[a + b*Tan[e + f*x]])/Sqrt[c + d*Tan[e + f*x]])/((-(b*c) + a*d)*(c^2 +
d^2)*f))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\tan(fx+e)}\,(c+d\tan(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(3/2),x)
```

```
[Out] int(1/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(b*tan(f*x + e) + a)*(d*tan(f*x + e) + c)^(3/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\tan{\left(e + fx \right)}}\, \left(c + d\tan{\left(e + fx \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))**(1/2)/(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*tan(e + f*x))*(c + d*tan(e + f*x))**(3/2)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*tan(e + f*x))^(1/2)*(c + d*tan(e + f*x))^(3/2)),x)
[Out] \text{Hanged}
```

3.1294 $\int \frac{1}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=301

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{3/2}(c-id)^{3/2}f}+\frac{i\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{3/2}(c+id)^{3/2}f}-\frac{}{(a^2+b^2)\,(bc-ad}$$

```
[Out] -I*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c+d*tan(f*x+
e))^(1/2))/(a-I*b)^(3/2)/(c-I*d)^(3/2)/f+I*arctanh((c+I*d)^(1/2)*(a+b*tan(f
*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(a+I*b)^(3/2)/(c+I*d)^(3
/2)/f-2*b^2/(a^2+b^2)/(-a*d+b*c)/f/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^
(1/2)-2*d*(a^2*d^2+b^2*(c^2+2*d^2))*(a+b*tan(f*x+e))^(1/2)/(a^2+b^2)/(-a*d+
b*c)^2/(c^2+d^2)/f/(c+d*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.85, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3650, 3730, 3697, 3696, 95, 214}

$$-\frac{2d(a^2d^2+b^2(c^2+2d^2))\sqrt{a+b\tan(e+fx)}}{f(a^2+b^2)(c^2+d^2)(bc-ad)^2\sqrt{c+d\tan(e+fx)}}-\frac{2b^2}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a-ib)^{3/2}(c-id)^{3/2}}+\frac{i\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a+ib)^{3/2}(c+id)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] ((-I)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[
c + d*Tan[e + f*x]])])/((a - I*b)^(3/2)*(c - I*d)^(3/2)*f) + (I*ArcTanh[(Sq
rt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x
]])])/((a + I*b)^(3/2)*(c + I*d)^(3/2)*f) - (2*b^2)/((a^2 + b^2)*(b*c - a*d
)*f*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]]) - (2*d*(a^2*d^2 + b^
2*(c^2 + 2*d^2))*Sqrt[a + b*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*d)^2*(c^2
+ d^2)*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{2b^2}{(a^2+b^2)\,(bc-ad)f\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(}}$$

$$= -\frac{2b^2}{(a^2+b^2)\,(bc-ad)f\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(}}$$

$$= -\frac{2b^2}{(a^2+b^2)\,(bc-ad)f\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(}}$$

$$= -\frac{2b^2}{(a^2+b^2)\,(bc-ad)f\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(}}$$

$$= -\frac{2b^2}{(a^2+b^2)\,(bc-ad)f\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(}}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{3/2}(c-id)^{3/2}f} + \frac{i\tanh^{-}}{}$$

Mathematica [A]
time = 4.95, size = 349, normalized size = 1.16

$$-\frac{\frac{(bc-ad)^2\left(\frac{i(a+ib)(c+id)\tanh^{-1}\left(\frac{\sqrt{-c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+ib}\,\sqrt{-c+id}}+\frac{(ia+b)(c-id)\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}\,\sqrt{c+id}}\right)}{(-bc+ad)(c^2+d^2)}+\frac{2b^2}{\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}+\frac{2d(a^2d^2+b^2(c^2+2d^2))\sqrt{a+b\tan(e+fx)}}{(bc-ad)(c^2+d^2)\sqrt{c+d\tan(e+fx)}}}{(a^2+b^2)\,(bc-ad)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] -((((b*c - a*d)^2*((I*(a + I*b)*(c + I*d)*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a +
b*Tan[e + f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[-a + I*b
]*Sqrt[-c + I*d]) + ((I*a + b)*(c - I*d)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*
Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*Sq
rt[c + I*d])))/((-(b*c) + a*d)*(c^2 + d^2)) + (2*b^2)/(Sqrt[a + b*Tan[e + f
*x]]*Sqrt[c + d*Tan[e + f*x]]) + (2*d*(a^2*d^2 + b^2*(c^2 + 2*d^2))*Sqrt[a
+ b*Tan[e + f*x]])/((b*c - a*d)*(c^2 + d^2)*Sqrt[c + d*Tan[e + f*x]]))/((a^
2 + b^2)*(b*c - a*d)*f))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\tan(fx+e))^{\frac{3}{2}}\,(c+d\tan(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(3/2),x)
```

```
[Out] int(1/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((2*b*d+2*a*c)^2>0)', see `assume?`
 for mo
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\tan\left(e + fx\right)\right)^{\frac{3}{2}} \left(c + d\tan\left(e + fx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))**(3/2)/(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/((a + b*tan(e + f*x))**(3/2)*(c + d*tan(e + f*x))**(3/2)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\tan(e+f\,x))^{3/2}\,(c+d\tan(e+f\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*tan(e + f*x))^(3/2)*(c + d*tan(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + b*tan(e + f*x))^(3/2)*(c + d*tan(e + f*x))^(3/2)), x)
```

3.1295 $\int \frac{1}{(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=417

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{5/2}(c-id)^{3/2}f}+\frac{i\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{5/2}(c+id)^{3/2}f}-\frac{}{3\left(a^2+b^2\right)(bc-}$$

```
[Out] -I*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c+d*tan(f*x+
e))^(1/2))/(a-I*b)^(5/2)/(c-I*d)^(3/2)/f+I*arctanh((c+I*d)^(1/2)*(a+b*tan(f
*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(a+I*b)^(5/2)/(c+I*d)^(3
/2)/f-4/3*b^2*(-5*a^2*d+3*a*b*c-2*b^2*d)/(a^2+b^2)^2/(-a*d+b*c)^2/f/(a+b*ta
n(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(1/2)+2/3*d*(3*a^4*d^3-6*a*b^3*c*(c^2+d^2)
+b^4*d*(5*c^2+8*d^2)+a^2*b^2*d*(11*c^2+17*d^2))*(a+b*tan(f*x+e))^(1/2)/(a^2
+b^2)^2/(-a*d+b*c)^3/(c^2+d^2)/f/(c+d*tan(f*x+e))^(1/2)-2/3*b^2/(a^2+b^2)/(
-a*d+b*c)/f/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^(3/2)
```

Rubi [A]

time = 1.32, antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3650, 3730, 3697, 3696, 95, 214}

$$-\frac{4b^2(-5a^2d+3abc-2b^2d)}{3f\left(a^2+b^2\right)^2(bc-ad)^2\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}-\frac{2b^2}{3f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}+\frac{2d(3a^4d^3+a^2b^2d(11c^2+17d^2)-6ab^3c(c^2+d^2)+b^4d(5c^2+8d^2))\sqrt{a+b\tan(e+fx)}}{3f\left(a^2+b^2\right)^2(c^2+d^2)(bc-ad)^3\sqrt{c+d\tan(e+fx)}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a-ib)^{5/2}(c-id)^{3/2}}+\frac{i\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a+ib)^{5/2}(c+id)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Tan[e + f*x])^(5/2)*(c + d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] ((-I)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[
c + d*Tan[e + f*x]])])/((a - I*b)^(5/2)*(c - I*d)^(3/2)*f) + (I*ArcTanh[(Sq
rt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x
]])])/((a + I*b)^(5/2)*(c + I*d)^(3/2)*f) - (2*b^2)/(3*(a^2 + b^2)*(b*c - a
*d)*f*(a + b*Tan[e + f*x])^(3/2)*Sqrt[c + d*Tan[e + f*x]]) - (4*b^2*(3*a*b*
c - 5*a^2*d - 2*b^2*d))/(3*(a^2 + b^2)^2*(b*c - a*d)^2*f*Sqrt[a + b*Tan[e +
 f*x]]*Sqrt[c + d*Tan[e + f*x]]) + (2*d*(3*a^4*d^3 - 6*a*b^3*c*(c^2 + d^2)
+ b^4*d*(5*c^2 + 8*d^2) + a^2*b^2*d*(11*c^2 + 17*d^2))*Sqrt[a + b*Tan[e + f
*x]])/(3*(a^2 + b^2)^2*(b*c - a*d)^3*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]]
)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
```

```
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{2b^2}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}\sqrt{c+d}}$$

$$= -\frac{2b^2}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}\sqrt{c+d}}$$

$$= -\frac{2b^2}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}\sqrt{c+d}}$$

$$= -\frac{2b^2}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}\sqrt{c+d}}$$

$$= -\frac{2b^2}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}\sqrt{c+d}}$$

$$= -\frac{2b^2}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}\sqrt{c+d}}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{5/2}(c-id)^{3/2}f} + \frac{i\tanh}{}$$

Mathematica [A]
time = 6.48, size = 601, normalized size = 1.44

$$-\frac{2b^2}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}-\frac{2\left(-\frac{2\left(\frac{1}{2}b^2\left(4b^2d-3a(bc-ad)\right)-a\left(-2ab^2d+\frac{3}{2}b^2(bc-ad)\right)\right)}{\left(a^2+b^2\right)(bc-ad)f\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}-\frac{2\left(-\frac{3(bc-ad)^3\left(\frac{(a+ib)^2(ic-d)\tanh^{-1}\left(\frac{\sqrt{-c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+ib}\,\sqrt{-c+id}}+\frac{(a-ib)^2(ic+d)\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}\,\sqrt{c+id}}\right)}{4(-bc+ad)\left(c^2+d^2\right)f}-\frac{2\left(-c\left(-\frac{3}{2}abd(bc-ad)^2+b^2cd\left(3abc-5a^2d-2b^2d\right)\right)+\frac{1}{4}d^2\left(-6a^3bcd-6ab^3cd+3a^4d^2-b^4\left(3c^2-8d^2\right)+a^2b^2\left(3c^2+17d^2\right)\right)\right)\sqrt{a+b\tan(e+fx)}}{(-bc+ad)\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}}\right)}{\left(a^2+b^2\right)(bc-ad)}\right)}{3\left(a^2+b^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Tan[e + f*x])^(5/2)*(c + d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] (-2*b^2)/(3*(a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])^(3/2)*Sqrt[c + d
*Tan[e + f*x]]) - (2*((-2*((b^2*(4*b^2*d - 3*a*(b*c - a*d)))/2 - a*(-2*a*b^
```

```
2*d + (3*b^2*(b*c - a*d))/2)))/((a^2 + b^2)*(b*c - a*d)*f*Sqrt[a + b*Tan[e
+ f*x]]*Sqrt[c + d*Tan[e + f*x]]) - (2*((-3*(b*c - a*d)^3*(((a + I*b)^2*(I*
c - d)*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + I*b]*Sq
rt[c + d*Tan[e + f*x]])]])/(Sqrt[-a + I*b]*Sqrt[-c + I*d]) + ((a - I*b)^2*(I
*c + d)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqr
t[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*Sqrt[c + I*d])))/(4*(-(b*c) + a*d)*
(c^2 + d^2)*f) - (2*(-(c*((-3*a*b*d*(b*c - a*d)^2)/2 + b^2*c*d*(3*a*b*c - 5
*a^2*d - 2*b^2*d))) + (d^2*(-6*a^3*b*c*d - 6*a*b^3*c*d + 3*a^4*d^2 - b^4*(3
*c^2 - 8*d^2) + a^2*b^2*(3*c^2 + 17*d^2)))/4)*Sqrt[a + b*Tan[e + f*x]])/((-
(b*c) + a*d)*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]])))/((a^2 + b^2)*(b*c -
a*d))))/(3*(a^2 + b^2)*(b*c - a*d))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\tan(fx+e))^{\frac{5}{2}} (c+d\tan(fx+e))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(3/2),x)
```

```
[Out] int(1/(a+b*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((b*tan(f*x + e) + a)^(5/2)*(d*tan(f*x + e) + c)^(3/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\tan(e+fx))^{\frac{5}{2}}(c+d\tan(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))**(5/2)/(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/((a + b*tan(e + f*x))**(5/2)*(c + d*tan(e + f*x))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\tan(e+f\,x))^{5/2}\,(c+d\tan(e+f\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*tan(e + f*x))^(5/2)*(c + d*tan(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + b*tan(e + f*x))^(5/2)*(c + d*tan(e + f*x))^(3/2)), x)
```

3.1296 $\int \frac{(a+b\tan(e+fx))^{9/2}}{(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=470

$$-\frac{i(a-ib)^{9/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(c-id)^{5/2}f}+\frac{i(a+ib)^{9/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(c+id)^{5/2}f}$$

```
[Out] -I*(a-I*b)^(9/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)
/(c+d*tan(f*x+e))^(1/2))/(c-I*d)^(5/2)/f+I*(a+I*b)^(9/2)*arctanh((c+I*d)^(1
/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(c+I*d)^(5
/2)/f-b^(7/2)*(-9*a*d+5*b*c)*arctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^(1/2)
/(c+d*tan(f*x+e))^(1/2))/d^(7/2)/f+b*(4*a^3*c*d^3-4*a^2*b*d^2*(c^2-2*d^2)-4
*a*b^2*c*d*(c^2+4*d^2)+b^3*(5*c^4+10*c^2*d^2+d^4))*(a+b*tan(f*x+e))^(1/2)*(
c+d*tan(f*x+e))^(1/2)/d^3/(c^2+d^2)^2/f-2/3*(-a*d+b*c)^2*(6*a*c*d+5*b*c^2+1
1*b*d^2)*(a+b*tan(f*x+e))^(3/2)/d^2/(c^2+d^2)^2/f/(c+d*tan(f*x+e))^(1/2)-2/
3*(-a*d+b*c)^2*(a+b*tan(f*x+e))^(5/2)/d/(c^2+d^2)/f/(c+d*tan(f*x+e))^(3/2)
```

Rubi [A]

time = 4.62, antiderivative size = 470, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {3646, 3726, 3728, 3736, 6857, 65, 223, 212, 95, 214}

$$\frac{b(4a^3cd^3-4a^2bd^2(c^2-2d^2)-4ab^2cd(c^2+4d^2)+b^3(5c^4+10c^2d^2+d^4))\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{d^3f(c^2+d^2)^2}-\frac{b^{7/2}(5bc-9ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{d^{7/2}f}-\frac{2(bc-ad)^2(6acd+5bc^2+11bd^2)(a+b\tan(e+fx))^{3/2}}{3d^2f(c^2+d^2)^2\sqrt{c+d\tan(e+fx)}}-\frac{2(bc-ad)^2(a+b\tan(e+fx))^{5/2}}{3df(c^2+d^2)(c+d\tan(e+fx))^{3/2}}-\frac{i(a-ib)^{9/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(c-id)^{5/2}}+\frac{i(a+ib)^{9/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(c+id)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^(9/2)/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-I)*(a - I*b)^(9/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqr
t[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c - I*d)^(5/2)*f) + (I*(a + I*b)^(
9/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c
 + d*Tan[e + f*x]])])/((c + I*d)^(5/2)*f) - (b^(7/2)*(5*b*c - 9*a*d)*ArcTan
h[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(
d^(7/2)*f) - (2*(b*c - a*d)^2*(a + b*Tan[e + f*x])^(5/2))/(3*d*(c^2 + d^2)*
f*(c + d*Tan[e + f*x])^(3/2)) - (2*(b*c - a*d)^2*(5*b*c^2 + 6*a*c*d + 11*b*
d^2)*(a + b*Tan[e + f*x])^(3/2))/(3*d^2*(c^2 + d^2)^2*f*Sqrt[c + d*Tan[e +
f*x]]) + (b*(4*a^3*c*d^3 - 4*a^2*b*d^2*(c^2 - 2*d^2) - 4*a*b^2*c*d*(c^2 + 4
*d^2) + b^3*(5*c^4 + 10*c^2*d^2 + d^4))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d
*Tan[e + f*x]])/(d^3*(c^2 + d^2)^2*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
```

```
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^{9/2}}{(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{2(bc-ad)^2(a+b\tan(e+fx))^{5/2}}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} + \frac{2\int \frac{(a+b\tan(e+fx))^{3/2}\left(\frac{1}{2}\left(5b^3c^2+3a^3cd-13a\right.\right.}{}}{}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))^{5/2}}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} - \frac{2(bc-ad)^2\,(5bc^2+6acd+11bd^2)\,(}{3d^2\,(c^2+d^2)^2\,f\sqrt{c+d\,\mathrm{ta}}}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))^{5/2}}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} - \frac{2(bc-ad)^2\,(5bc^2+6acd+11bd^2)\,(}{3d^2\,(c^2+d^2)^2\,f\sqrt{c+d\,\mathrm{ta}}}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))^{5/2}}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} - \frac{2(bc-ad)^2\,(5bc^2+6acd+11bd^2)\,(}{3d^2\,(c^2+d^2)^2\,f\sqrt{c+d\,\mathrm{ta}}}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))^{5/2}}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} - \frac{2(bc-ad)^2\,(5bc^2+6acd+11bd^2)\,(}{3d^2\,(c^2+d^2)^2\,f\sqrt{c+d\,\mathrm{ta}}}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))^{5/2}}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} - \frac{2(bc-ad)^2\,(5bc^2+6acd+11bd^2)\,(}{3d^2\,(c^2+d^2)^2\,f\sqrt{c+d\,\mathrm{ta}}}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))^{5/2}}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} - \frac{2(bc-ad)^2\,(5bc^2+6acd+11bd^2)\,(}{3d^2\,(c^2+d^2)^2\,f\sqrt{c+d\,\mathrm{ta}}}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))^{5/2}}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} - \frac{2(bc-ad)^2\,(5bc^2+6acd+11bd^2)\,(}{3d^2\,(c^2+d^2)^2\,f\sqrt{c+d\,\mathrm{ta}}}$$

$$= -\frac{b^{7/2}(5bc-9ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{d^{7/2}f} - \frac{2(bc-ad)^2(a+b}{3d\,(c^2+d^2)\,f(c+}$$

$$= -\frac{i(a-ib)^{9/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(c-id)^{5/2}f} + \frac{i(a+ib)^{9/2}\tanh^{-}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 6.75, size = 2261, normalized size = 4.81

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[(a + b*Tan[e + f*x])^(9/2)/(c + d*Tan[e + f*x])^(5/2),x]

[Out] ((-1/2*I)*(-a - I*b)*((-2*b^2*(b/((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)
))^(5/2)*Hypergeometric2F1[5/2, 7/2, 9/2, -((b*d*(a + b*Tan[e + f*x]))/((b*
c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))]*(a + b*Tan[e + f*x]
)^(7/2)*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c - a*d)])/(7*(b*c - a*d)^2*Sqrt[c
 + d*Tan[e + f*x]]) - (-a - I*b)*((2*(b*c - a*d)*(b/((b^2*c)/(b*c - a*d) -
(a*b*d)/(b*c - a*d)))^(5/2)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))^3*S
qrt[(b*(c + d*Tan[e + f*x]))/(b*c - a*d)]*(-1 - (b*d*(a + b*Tan[e + f*x]))/
((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))^2*((b^2*d^2*(a +
 b*Tan[e + f*x])^2)/(3*(b*c - a*d)^2*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c -
a*d))^2*(-1 - (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d)
- (a*b*d)/(b*c - a*d))))^2) - (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2
*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))*(-1 - (b*d*(a + b*Tan[e + f*x]))/((b
*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))) - (Sqrt[b]*Sqrt[d]
*ArcSinh[(Sqrt[b]*Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b*c - a*d]*Sqrt[(
b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)])]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt
[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)]*Sqrt[1 + (b*d*(
a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d
)))])))/(b*d^3*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]]*(1 + (b*d*
(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*
d))))^(3/2)) - (-a - I*b)*((-2*(a + b*Tan[e + f*x])^(3/2))/(3*(c + I*d)*(c
+ d*Tan[e + f*x])^(3/2)) + ((a + I*b)*((-2*Sqrt[a + I*b]*ArcTanh[(Sqrt[c +
I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(
(-c - I*d)*Sqrt[c + I*d]) + (2*Sqrt[a + b*Tan[e + f*x]])/((-c - I*d)*Sqrt[c
 + d*Tan[e + f*x]])))/(c + I*d))))/f - ((I/2)*(-a + I*b)*((2*b^2*(b/((b^2*
c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)))^(5/2)*Hypergeometric2F1[5/2, 7/2, 9/
2, -((b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)
/(b*c - a*d))))]*(a + b*Tan[e + f*x])^(7/2)*Sqrt[(b*(c + d*Tan[e + f*x]))/(
b*c - a*d)])/(7*(b*c - a*d)^2*Sqrt[c + d*Tan[e + f*x]]) - (-a + I*b)*((-2*(
b*c - a*d)*(b/((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)))^(5/2)*((b^2*c)/(
b*c - a*d) - (a*b*d)/(b*c - a*d))^3*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c - a*
d)]*(-1 - (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a
*b*d)/(b*c - a*d))))^2*((b^2*d^2*(a + b*Tan[e + f*x])^2)/(3*(b*c - a*d)^2*(
(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))^2*(-1 - (b*d*(a + b*Tan[e + f*x]
))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))^2) - (b*d*(a
+ b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))
*(-1 - (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*
d)/(b*c - a*d))))) - (Sqrt[b]*Sqrt[d]*ArcSinh[(Sqrt[b]*Sqrt[d]*Sqrt[a + b*T
an[e + f*x]])/(Sqrt[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*
d)])]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) -
 (a*b*d)/(b*c - a*d)]*Sqrt[1 + (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^
2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))])))/(b*d^3*Sqrt[a + b*Tan[e + f*x]

```
]*Sqrt[c + d*Tan[e + f*x]]*(1 + (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b
^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))^(3/2)) - (-a + I*b)*((-2*(a + b*
Tan[e + f*x])^(3/2))/(3*(-c + I*d)*(c + d*Tan[e + f*x])^(3/2)) + ((-a + I*b
)*((-2*Sqrt[-a + I*b]*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sq
rt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c - I*d)*Sqrt[-c + I*d]) + (2*Sq
rt[a + b*Tan[e + f*x]])/((c - I*d)*Sqrt[c + d*Tan[e + f*x]])))/(-c + I*d)))
))/f
```

Maple [**F(-1)**] grade_annotation
time = 180.00, size = 0, normalized size = 0.00 hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(9/2)/(c+d*tan(f*x+e))^(5/2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(9/2)/(c+d*tan(f*x+e))^(5/2),x)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(9/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(9/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(9/2)/(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7317 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(9/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\tan(e+f\,x))^{9/2}}{(c+d\tan\left(e+f\,x\right))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^(9/2)/(c + d*tan(e + f*x))^(5/2),x)
```

```
[Out] int((a + b*tan(e + f*x))^(9/2)/(c + d*tan(e + f*x))^(5/2), x)
```

3.1297 $\int \frac{(a+b\tan(e+fx))^{7/2}}{(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=347

$$-\frac{i(a-ib)^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(c-id)^{5/2}f}+\frac{i(a+ib)^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(c+id)^{5/2}f}$$

```
[Out] -I*(a-I*b)^(7/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)
/(c+d*tan(f*x+e))^(1/2))/(c-I*d)^(5/2)/f+I*(a+I*b)^(7/2)*arctanh((c+I*d)^(1
/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(c+I*d)^(5
/2)/f+2*b^(7/2)*arctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x
+e))^(1/2))/d^(5/2)/f-2*(-a*d+b*c)^2*(2*a*c*d+b*(c^2+3*d^2))*(a+b*tan(f*x+e
))^(1/2)/d^2/(c^2+d^2)^2/f/(c+d*tan(f*x+e))^(1/2)-2/3*(-a*d+b*c)^2*(a+b*tan
(f*x+e))^(3/2)/d/(c^2+d^2)/f/(c+d*tan(f*x+e))^(3/2)
```

Rubi [A]

time = 4.21, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {3646, 3726, 3736, 6857, 65, 223, 212, 95, 214}

$$\frac{2b^{7/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{d^{5/2}f}-\frac{2(bc-ad)^2\left(2acd+b(c^2+3d^2)\right)\sqrt{a+b\tan(e+fx)}}{d^2f\left(c^2+d^2\right)^2\sqrt{c+d\tan(e+fx)}}-\frac{2(bc-ad)^2(a+b\tan(e+fx))^{3/2}}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}-\frac{i(a-ib)^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(c-id)^{5/2}}+\frac{i(a+ib)^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(c+id)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^(7/2)/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-I)*(a - I*b)^(7/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqr
t[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c - I*d)^(5/2)*f) + (I*(a + I*b)^(
7/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c
 + d*Tan[e + f*x]])])/((c + I*d)^(5/2)*f) + (2*b^(7/2)*ArcTanh[(Sqrt[d]*Sqr
t[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(d^(5/2)*f) - (
2*(b*c - a*d)^2*(a + b*Tan[e + f*x])^(3/2))/(3*d*(c^2 + d^2)*f*(c + d*Tan[e
 + f*x])^(3/2)) - (2*(b*c - a*d)^2*(2*a*c*d + b*(c^2 + 3*d^2))*Sqrt[a + b*T
an[e + f*x]])/(d^2*(c^2 + d^2)^2*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
```

```
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^{7/2}}{(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{2(bc-ad)^2(a+b\tan(e+fx))^{3/2}}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} + \frac{2\int \sqrt{a+b\tan(e+fx)}\,\left(\frac{3}{2}(b^3c^2+a^3cd-\right.}{}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))^{3/2}}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} - \frac{2(bc-ad)^2\,(2acd+b(c^2+3d^2))\,\sqrt{a}}{d^2\,(c^2+d^2)^2\,f\sqrt{c+d\tan(}}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))^{3/2}}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} - \frac{2(bc-ad)^2\,(2acd+b(c^2+3d^2))\,\sqrt{a}}{d^2\,(c^2+d^2)^2\,f\sqrt{c+d\tan(}}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))^{3/2}}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} - \frac{2(bc-ad)^2\,(2acd+b(c^2+3d^2))\,\sqrt{a}}{d^2\,(c^2+d^2)^2\,f\sqrt{c+d\tan(}}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))^{3/2}}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} - \frac{2(bc-ad)^2\,(2acd+b(c^2+3d^2))\,\sqrt{a}}{d^2\,(c^2+d^2)^2\,f\sqrt{c+d\tan(}}$$

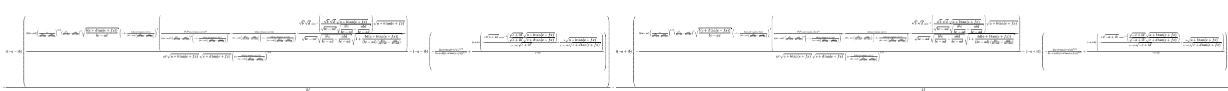
$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))^{3/2}}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} - \frac{2(bc-ad)^2\,(2acd+b(c^2+3d^2))\,\sqrt{a}}{d^2\,(c^2+d^2)^2\,f\sqrt{c+d\tan(}}$$

$$= -\frac{2(bc-ad)^2(a+b\tan(e+fx))^{3/2}}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} - \frac{2(bc-ad)^2\,(2acd+b(c^2+3d^2))\,\sqrt{a}}{d^2\,(c^2+d^2)^2\,f\sqrt{c+d\tan(}}$$

$$= \frac{2b^{7/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{d^{5/2}f} - \frac{2(bc-ad)^2(a+b\tan(e+fx))}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx)}$$

$$= -\frac{i(a-ib)^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(c-id)^{5/2}f} + \frac{i(a+ib)^{7/2}\tanh^{-1}}{}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1883 vs. $2(347) = 694$.
time = 6.43, size = 1883, normalized size = 5.43



Antiderivative was successfully verified.

[In] Integrate[(a + b*Tan[e + f*x])^(7/2)/(c + d*Tan[e + f*x])^(5/2),x]

[Out] ((-1/2*I)*(-a - I*b)*((2*(b*c - a*d)*(b/((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)))^(5/2)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))^3*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c - a*d)]*(-1 - (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))^2*((b^2*d^2*(a + b*Tan[e + f*x])^2)/(3*(b*c - a*d)^2*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))^2*(-1 - (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))^2) - (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))*(-1 - (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))) - (Sqrt[b]*Sqrt[d]*ArcSinh[(Sqrt[b]*Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)])]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)]*Sqrt[1 + (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)))])))/(b*d^3*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]]*(1 + (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))^(3/2)) - (-a - I*b)*((-2*(a + b*Tan[e + f*x])^(3/2))/(3*(c + I*d)*(c + d*Tan[e + f*x])^(3/2)) + ((a + I*b)*((-2*Sqrt[a + I*b]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((-c - I*d)*Sqrt[c + I*d]) + (2*Sqrt[a + b*Tan[e + f*x]])/((-c - I*d)*Sqrt[c + d*Tan[e + f*x]])))/(c + I*d))))/f - ((I/2)*(-a + I*b)*((-2*(b*c - a*d)*(b/((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)))^(5/2)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))^3*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c - a*d)]*(-1 - (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))^2*((b^2*d^2*(a + b*Tan[e + f*x])^2)/(3*(b*c - a*d)^2*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))^2*(-1 - (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))^2) - (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))*(-1 - (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))) - (Sqrt[b]*Sqrt[d]*ArcSinh[(Sqrt[b]*Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)])]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)]*Sqrt[1 + (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)))])))/(b*d^3*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]]*(1 + (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))^(3/2)) - (-a + I*b)*((-2*(a + b*Tan[e + f*x])^(3/2))/(3*(-c + I*d)*(c + d*Tan[e + f*x])^(3/2)) + ((-a + I*b)*((-2*Sqrt[-a + I*b]*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c - I*d)*Sqrt[-c + I*d]) + (2*Sqrt[a + b*Tan[e + f*x]])/((c - I*d)*Sqrt[c + d*Tan[e + f*x]])))/(-c + I*d))))/f

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(fx+e))^{\frac{7}{2}}}{(c+d\tan(fx+e))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(7/2)/(c+d*tan(f*x+e))^(5/2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(7/2)/(c+d*tan(f*x+e))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(7/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*tan(f*x + e) + a)^(7/2)/(d*tan(f*x + e) + c)^(5/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(7/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(7/2)/(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(7/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\tan(e+f\,x))^{7/2}}{(c+d\tan\left(e+f\,x\right))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^(7/2)/(c + d*tan(e + f*x))^(5/2),x)
```

```
[Out] int((a + b*tan(e + f*x))^(7/2)/(c + d*tan(e + f*x))^(5/2), x)
```

3.1298 $\int \frac{(a+b\tan(e+fx))^{5/2}}{(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=292

$$-\frac{i(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(c-id)^{5/2}f}+\frac{i(a+ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(c+id)^{5/2}f}$$

```
[Out] -I*(a-I*b)^(5/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)
/(c+d*tan(f*x+e))^(1/2))/(c-I*d)^(5/2)/f+I*(a+I*b)^(5/2)*arctanh((c+I*d)^(1
/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(c+I*d)^(5
/2)/f+2/3*(-a*d+b*c)*(6*a*c*d+b*(c^2+7*d^2))*(a+b*tan(f*x+e))^(1/2)/d/(c^2+
d^2)^2/f/(c+d*tan(f*x+e))^(1/2)-2/3*(-a*d+b*c)^2*(a+b*tan(f*x+e))^(1/2)/d/(
c^2+d^2)/f/(c+d*tan(f*x+e))^(3/2)
```

Rubi [A]

time = 1.05, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3646, 3730, 3697, 3696, 95, 214}

$$\frac{2(bc-ad)\left(6acd+b(c^2+7d^2)\right)\sqrt{a+b\tan(e+fx)}}{3df\left(c^2+d^2\right)^2\sqrt{c+d\tan(e+fx)}}-\frac{2(bc-ad)^2\sqrt{a+b\tan(e+fx)}}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}-\frac{i(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(c-id)^{5/2}}+\frac{i(a+ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(c+id)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^(5/2)/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-I)*(a - I*b)^(5/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqr
t[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c - I*d)^(5/2)*f) + (I*(a + I*b)^(
5/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c
 + d*Tan[e + f*x]])])/((c + I*d)^(5/2)*f) - (2*(b*c - a*d)^2*Sqrt[a + b*Tan
[e + f*x]])/(3*d*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2)) + (2*(b*c - a*d)
*(6*a*c*d + b*(c^2 + 7*d^2))*Sqrt[a + b*Tan[e + f*x]])/(3*d*(c^2 + d^2)^2*f
*Sqrt[c + d*Tan[e + f*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^{5/2}}{(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{2(bc-ad)^2\sqrt{a+b\tan(e+fx)}}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} + \frac{2\int \frac{\frac{1}{2}\left(b^3c^2+3a^3cd-5ab^2cd+7a^2bd^2\right)+\frac{3}{2}d(3a^2bc}{\sqrt{a+}}}{}$$

$$= -\frac{2(bc-ad)^2\sqrt{a+b\tan(e+fx)}}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} + \frac{2(bc-ad)\left(6acd+b(c^2+7d^2)\right)\sqrt{a}}{3d\,(c^2+d^2)^2 f\sqrt{c+d\tan(e}}$$

$$= -\frac{2(bc-ad)^2\sqrt{a+b\tan(e+fx)}}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} + \frac{2(bc-ad)\left(6acd+b(c^2+7d^2)\right)\sqrt{a}}{3d\,(c^2+d^2)^2 f\sqrt{c+d\tan(e}}$$

$$= -\frac{2(bc-ad)^2\sqrt{a+b\tan(e+fx)}}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} + \frac{2(bc-ad)\left(6acd+b(c^2+7d^2)\right)\sqrt{a}}{3d\,(c^2+d^2)^2 f\sqrt{c+d\tan(e}}$$

$$= -\frac{2(bc-ad)^2\sqrt{a+b\tan(e+fx)}}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} + \frac{2(bc-ad)\left(6acd+b(c^2+7d^2)\right)\sqrt{a}}{3d\,(c^2+d^2)^2 f\sqrt{c+d\tan(e}}$$

$$= -\frac{i(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(c-id)^{5/2}f} + \frac{i(a+ib)^{5/2}\tanh^{-1}}{}$$

Mathematica [A]
time = 5.84, size = 350, normalized size = 1.20

$$\frac{-\left((ia-b)\left(-\frac{3(a+ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(c+id)^{5/2}}+\frac{\sqrt{a+b\tan(e+fx)}\,(4ac+3ibc+iad+(b(c+4id)+3ad)\tan(e+fx))}{(c+id)^2(c+d\tan(e+fx))^{3/2}}\right)\right)+\frac{(ia+b)\left(\frac{(a+b\tan(e+fx))^{3/2}}{(c+d\tan(e+fx))^{3/2}}+3(a-ib)\left(\frac{\sqrt{-a+ib}\tanh^{-1}\left(\frac{\sqrt{-c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(-c+id)^{3/2}}+\frac{\sqrt{a+b\tan(e+fx)}}{(c-id)\sqrt{c+d\tan(e+fx)}}\right)\right)}{c-id}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^(5/2)/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (-((I*a - b)*((-3*(a + I*b)^(3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
 f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(c + I*d)^(5/2) + (Sqrt[
a + b*Tan[e + f*x]]*(4*a*c + (3*I)*b*c + I*a*d + (b*(c + (4*I)*d) + 3*a*d)*
Tan[e + f*x]))/((c + I*d)^2*(c + d*Tan[e + f*x])^(3/2)))) + ((I*a + b)*((a
+ b*Tan[e + f*x])^(3/2)/(c + d*Tan[e + f*x])^(3/2) + 3*(a - I*b)*((Sqrt[-a
+ I*b]*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + I*b]*Sq
rt[c + d*Tan[e + f*x]])])/(-c + I*d)^(3/2) + Sqrt[a + b*Tan[e + f*x]]/((c -
 I*d)*Sqrt[c + d*Tan[e + f*x]]))))/(c - I*d))/(3*f)
```

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(fx+e))^{\frac{5}{2}}}{(c+d\tan(fx+e))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(5/2),x)
[Out] int((a+b*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="maxim
a")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((2*b*d+2*a*c)^2>0)', see `assume?`
 for mo
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="frica
s")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(e+fx))^{\frac{5}{2}}}{(c+d\tan(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(5/2)/(c+d*tan(f*x+e))**(5/2),x)
[Out] Integral((a + b*tan(e + f*x))**(5/2)/(c + d*tan(e + f*x))**(5/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\tan(e+f\,x))^{5/2}}{(c+d\tan\left(e+f\,x\right))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^(5/2)/(c + d*tan(e + f*x))^(5/2),x)
```

```
[Out] int((a + b*tan(e + f*x))^(5/2)/(c + d*tan(e + f*x))^(5/2), x)
```

3.1299 $\int \frac{(a+b\tan(e+fx))^{3/2}}{(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=276

$$-\frac{i(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(c-id)^{5/2}f}+\frac{i(a+ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(c+id)^{5/2}f}$$

```
[Out] -I*(a-I*b)^(3/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)
/(c+d*tan(f*x+e))^(1/2))/(c-I*d)^(5/2)/f+I*(a+I*b)^(3/2)*arctanh((c+I*d)^(1
/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(c+I*d)^(5
/2)/f+4/3*(-3*a*c*d+b*c^2-2*b*d^2)*(a+b*tan(f*x+e))^(1/2)/(c^2+d^2)^2/f/(c+
d*tan(f*x+e))^(1/2)+2/3*(-a*d+b*c)*(a+b*tan(f*x+e))^(1/2)/(c^2+d^2)/f/(c+d*
tan(f*x+e))^(3/2)
```

Rubi [A]

time = 0.88, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3648, 3730, 3697, 3696, 95, 214}

$$\frac{4(-3acd+bc^2-2bd^2)\sqrt{a+b\tan(e+fx)}}{3f(c^2+d^2)^2\sqrt{c+d\tan(e+fx)}}+\frac{2(bc-ad)\sqrt{a+b\tan(e+fx)}}{3f(c^2+d^2)(c+d\tan(e+fx))^{3/2}}-\frac{i(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(c-id)^{5/2}}+\frac{i(a+ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(c+id)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^(3/2)/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-I)*(a - I*b)^(3/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqr
t[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c - I*d)^(5/2)*f) + (I*(a + I*b)^(
3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c
 + d*Tan[e + f*x]])])/((c + I*d)^(5/2)*f) + (2*(b*c - a*d)*Sqrt[a + b*Tan[e
 + f*x]])/(3*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2)) + (4*(b*c^2 - 3*a*c*
d - 2*b*d^2)*Sqrt[a + b*Tan[e + f*x]])/(3*(c^2 + d^2)^2*f*Sqrt[c + d*Tan[e
+ f*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^{3/2}}{(c+d\tan(e+fx))^{5/2}}\,dx = \frac{2(bc-ad)\sqrt{a+b\tan(e+fx)}}{3\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(-3a^2c+b^2c-4abd\right)-\frac{3}{2}\left(2abc-a^2d+b^2d\right)\tan(}{\sqrt{a+b\tan(e+fx)}\,(c+}}{3\left(c^2+d^2\right)}$$

$$= \frac{2(bc-ad)\sqrt{a+b\tan(e+fx)}}{3\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{4\left(bc^2-3acd-2bd^2\right)\sqrt{a+b\tan(e+}}{3\left(c^2+d^2\right)^2 f\sqrt{c+d\tan(e+fx)}}$$

$$= \frac{2(bc-ad)\sqrt{a+b\tan(e+fx)}}{3\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{4\left(bc^2-3acd-2bd^2\right)\sqrt{a+b\tan(e+}}{3\left(c^2+d^2\right)^2 f\sqrt{c+d\tan(e+fx)}}$$

$$= \frac{2(bc-ad)\sqrt{a+b\tan(e+fx)}}{3\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{4\left(bc^2-3acd-2bd^2\right)\sqrt{a+b\tan(e+}}{3\left(c^2+d^2\right)^2 f\sqrt{c+d\tan(e+fx)}}$$

$$= \frac{2(bc-ad)\sqrt{a+b\tan(e+fx)}}{3\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{4\left(bc^2-3acd-2bd^2\right)\sqrt{a+b\tan(e+}}{3\left(c^2+d^2\right)^2 f\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{i(a-ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(c-id)^{5/2}f} + \frac{i(a+ib)^{3/2}\tanh^{-}}{}$$

Mathematica [A]
time = 3.15, size = 264, normalized size = 0.96

$$\frac{\frac{3i(-a+ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{-c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(-c+id)^{5/2}} + \frac{3i(a+ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(c+id)^{5/2}} - \frac{2\sqrt{a+b\tan(e+fx)}\left(-3bc^3+7ac^2d+3bcd^2+ad^3+2d\left(-bc^2+3acd+2bd^2\right)\tan(e+fx)\right)}{\left(c^2+d^2\right)^2(c+d\tan(e+fx))^{3/2}}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^(3/2)/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (((3*I)*(-a + I*b)^(3/2)*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x]])/
(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(-c + I*d)^(5/2) + ((3*I)*(a +
I*b)^(3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*
Sqrt[c + d*Tan[e + f*x]])])/(c + I*d)^(5/2) - (2*Sqrt[a + b*Tan[e + f*x]]*(
-3*b*c^3 + 7*a*c^2*d + 3*b*c*d^2 + a*d^3 + 2*d*(-(b*c^2) + 3*a*c*d + 2*b*d^
2)*Tan[e + f*x]))/((c^2 + d^2)^2*(c + d*Tan[e + f*x])^(3/2)))/(3*f)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(fx+e))^{\frac{3}{2}}}{(c+d\tan(fx+e))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(5/2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((2*b*d+2*a*c)^2>0)', see `assume?`
 for mo
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan{(e + fx)})^{\frac{3}{2}}}{(c + d\tan{(e + fx)})^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(3/2)/(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**(3/2)/(c + d*tan(e + f*x))**(5/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\tan(e+fx))^{3/2}}{(c+d\tan(e+fx))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^(3/2)/(c + d*tan(e + f*x))^(5/2),x)
```

```
[Out] int((a + b*tan(e + f*x))^(3/2)/(c + d*tan(e + f*x))^(5/2), x)
```

3.1300 $\int \frac{\sqrt{a+b\tan(e+fx)}}{(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=283

$$-\frac{i\sqrt{a-ib}\,\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(c-id)^{5/2}f}+\frac{i\sqrt{a+ib}\,\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(c+id)^{5/2}f}-$$

```
[Out] -I*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c+d*tan(f*x+
e))^(1/2))*(a-I*b)^(1/2)/(c-I*d)^(5/2)/f+I*arctanh((c+I*d)^(1/2)*(a+b*tan(f
*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))*(a+I*b)^(1/2)/(c+I*d)^(5
/2)/f+2/3*d*(6*a*c*d-b*(5*c^2-d^2))*(a+b*tan(f*x+e))^(1/2)/(-a*d+b*c)/(c^2+
d^2)^2/f/(c+d*tan(f*x+e))^(1/2)-2/3*d*(a+b*tan(f*x+e))^(1/2)/(c^2+d^2)/f/(c
+d*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.80, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3649, 3730, 3697, 3696, 95, 214}

$$\frac{2d(6acd-b(5c^2-d^2))\sqrt{a+b\tan(e+fx)}}{3f(c^2+d^2)^2(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{2d\sqrt{a+b\tan(e+fx)}}{3f(c^2+d^2)(c+d\tan(e+fx))^{3/2}}-\frac{i\sqrt{a-ib}\,\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(c-id)^{5/2}}+\frac{i\sqrt{a+ib}\,\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(c+id)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Tan[e + f*x]]/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-I)*Sqrt[a - I*b]*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[
a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c - I*d)^(5/2)*f) + (I*Sqrt[a + I*b]
*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d
*Tan[e + f*x]])])/((c + I*d)^(5/2)*f) - (2*d*Sqrt[a + b*Tan[e + f*x]])/(3*(
c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2)) + (2*d*(6*a*c*d - b*(5*c^2 - d^2))
*Sqrt[a + b*Tan[e + f*x]])/(3*(b*c - a*d)*(c^2 + d^2)^2*f*Sqrt[c + d*Tan[e
+ f*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{\sqrt{a+b\tan(e+fx)}}{(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{2d\sqrt{a+b\tan(e+fx)}}{3\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}(-3ac-bd)-\frac{3}{2}(bc-ad)\tan(e+fx)+bd\tan^2(e}{\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx)}}{3\left(c^2+d^2\right)}$$

$$= -\frac{2d\sqrt{a+b\tan(e+fx)}}{3\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{2d(6acd-b(5c^2-d^2))\sqrt{a+b\tan(e}}{3(bc-ad)\left(c^2+d^2\right)^2 f\sqrt{c+d\tan(e}}$$

$$= -\frac{2d\sqrt{a+b\tan(e+fx)}}{3\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{2d(6acd-b(5c^2-d^2))\sqrt{a+b\tan(e}}{3(bc-ad)\left(c^2+d^2\right)^2 f\sqrt{c+d\tan(e}}$$

$$= -\frac{2d\sqrt{a+b\tan(e+fx)}}{3\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{2d(6acd-b(5c^2-d^2))\sqrt{a+b\tan(e}}{3(bc-ad)\left(c^2+d^2\right)^2 f\sqrt{c+d\tan(e}}$$

$$= -\frac{2d\sqrt{a+b\tan(e+fx)}}{3\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{2d(6acd-b(5c^2-d^2))\sqrt{a+b\tan(e}}{3(bc-ad)\left(c^2+d^2\right)^2 f\sqrt{c+d\tan(e}}$$

$$= -\frac{i\sqrt{a-ib}\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(c-id)^{5/2}f} + \frac{i\sqrt{a+ib}\tanh^{-1}\left(\right.}{(}$$

Mathematica [A]

time = 4.93, size = 266, normalized size = 0.94

$$\frac{-\frac{3i\sqrt{-a+ib}\tanh^{-1}\left(\frac{\sqrt{-c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(-c+id)^{5/2}} + \frac{3i\sqrt{a+ib}\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(c+id)^{5/2}} + \frac{2d\sqrt{a+b\tan(e+fx)}\,(-6bc^3+ad(7c^2+d^2)+d(6acd+b(-5c^2+d^2))\tan(e+fx))}{(bc-ad)(c^2+d^2)^2(c+d\tan(e+fx))^{3/2}}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Tan[e + f*x]]/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (((-3*I)*Sqrt[-a + I*b]*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(
Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(-c + I*d)^(5/2) + ((3*I)*Sqrt[a
 + I*b]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqr
t[c + d*Tan[e + f*x]])])/(c + I*d)^(5/2) + (2*d*Sqrt[a + b*Tan[e + f*x]]*(-
6*b*c^3 + a*d*(7*c^2 + d^2) + d*(6*a*c*d + b*(-5*c^2 + d^2))*Tan[e + f*x]))
/((b*c - a*d)*(c^2 + d^2)^2*(c + d*Tan[e + f*x])^(3/2)))/(3*f)
```

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\tan(fx+e)}}{(c+d\tan(fx+e))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(5/2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((2*b*d+2*a*c)^2>0)', see `assume?`
 for mo
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\tan{(e + fx)}}}{(c + d\tan{(e + fx)})^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(1/2)/(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x))/(c + d*tan(e + f*x))**(5/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\tan\left(e+f\,x\right)}}{{\left(c+d\tan\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^(1/2)/(c + d*tan(e + f*x))^(5/2),x)
```

```
[Out] int((a + b*tan(e + f*x))^(1/2)/(c + d*tan(e + f*x))^(5/2), x)
```

3.1301 $\int \frac{1}{\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=295

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a-ib}\,(c-id)^{5/2}f}+\frac{i\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}\,(c+id)^{5/2}f}+\frac{2d^2\sqrt{c}}{3(bc-ad)\,(c^2+}$$

```
[Out] -I*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c+d*tan(f*x+
e))^(1/2))/(c-I*d)^(5/2)/f/(a-I*b)^(1/2)+I*arctanh((c+I*d)^(1/2)*(a+b*tan(f
*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(c+I*d)^(5/2)/f/(a+I*b)^
(1/2)-4/3*d^2*(3*a*c*d-b*(4*c^2+d^2))*(a+b*tan(f*x+e))^(1/2)/(-a*d+b*c)^2/(
c^2+d^2)^2/f/(c+d*tan(f*x+e))^(1/2)+2/3*d^2*(a+b*tan(f*x+e))^(1/2)/(-a*d+b*
c)/(c^2+d^2)/f/(c+d*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.85, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3650, 3730, 3697, 3696, 95, 214}

$$-\frac{4d^2(3acd-b(4c^2+d^2))\sqrt{a+b\tan(e+fx)}}{3f(c^2+d^2)^2(bc-ad)^2\sqrt{c+d\tan(e+fx)}}+\frac{2d^2\sqrt{a+b\tan(e+fx)}}{3f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))^{3/2}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a-ib}\,(c-id)^{5/2}}+\frac{i\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a+ib}\,(c+id)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(5/2)),x]
```

```
[Out] ((-I)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[
c + d*Tan[e + f*x]])])/(Sqrt[a - I*b]*(c - I*d)^(5/2)*f) + (I*ArcTanh[(Sqrt
[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]]
)])/(Sqrt[a + I*b]*(c + I*d)^(5/2)*f) + (2*d^2*Sqrt[a + b*Tan[e + f*x]])/(3
*(b*c - a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2)) - (4*d^2*(3*a*c*d -
b*(4*c^2 + d^2))*Sqrt[a + b*Tan[e + f*x]])/(3*(b*c - a*d)^2*(c^2 + d^2)^2*f
*Sqrt[c + d*Tan[e + f*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{5/2}}\,dx = \frac{2d^2\sqrt{a+b\tan(e+fx)}}{3(bc-ad)\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} + \frac{2\int \frac{\frac{1}{2}(2bc}{}}{}$$

$$= \frac{2d^2\sqrt{a+b\tan(e+fx)}}{3(bc-ad)\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} - \frac{4d^2(3ac}{3(bc-}$$

$$= \frac{2d^2\sqrt{a+b\tan(e+fx)}}{3(bc-ad)\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} - \frac{4d^2(3ac}{3(bc-}$$

$$= \frac{2d^2\sqrt{a+b\tan(e+fx)}}{3(bc-ad)\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} - \frac{4d^2(3ac}{3(bc-}$$

$$= \frac{2d^2\sqrt{a+b\tan(e+fx)}}{3(bc-ad)\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} - \frac{4d^2(3ac}{3(bc-}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a-ib}\,(c-id)^{5/2}f} + \frac{i\tanh^{-}}{}$$

Mathematica [A]
time = 2.10, size = 316, normalized size = 1.07

$$\frac{3i(bc-ad)^2\left(\frac{(c+id)^2\tanh^{-1}\left(\frac{\sqrt{-c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+ib}\,\sqrt{-c+id}} + \frac{(c-id)^2\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}\,\sqrt{c+id}}\right) + \frac{2d^2(bc-ad)(c^2+d^2)\sqrt{a+b\tan(e+fx)}}{(c+d\tan(e+fx))^{3/2}} + \frac{4d^2(-3acd+b(4c^2+d^2))\sqrt{a+b\tan(e+fx)}}{\sqrt{c+d\tan(e+fx)}}}{3(bc-ad)^2\,(c^2+d^2)^2\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(5/2)),x]
```

```
[Out] ((3*I)*(b*c - a*d)^2*(((c + I*d)^2*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e
 + f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[-a + I*b]*Sqrt[
-c + I*d]) + ((c - I*d)^2*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/
(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*Sqrt[c + I*d])) +
 (2*d^2*(b*c - a*d)*(c^2 + d^2)*Sqrt[a + b*Tan[e + f*x]])/(c + d*Tan[e + f*
x])^(3/2) + (4*d^2*(-3*a*c*d + b*(4*c^2 + d^2))*Sqrt[a + b*Tan[e + f*x]])/S
qrt[c + d*Tan[e + f*x]])/(3*(b*c - a*d)^2*(c^2 + d^2)^2*f)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\tan\left(fx+e\right)}\,\left(c+d\tan\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(5/2),x)
```

```
[Out] int(1/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(5/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(b*tan(f*x + e) + a)*(d*tan(f*x + e) + c)^(5/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\tan\left(e+fx\right)}\,\left(c+d\tan\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))**(1/2)/(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*tan(e + f*x))*(c + d*tan(e + f*x))**(5/2)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*tan(e + f*x))^(1/2)*(c + d*tan(e + f*x))^(5/2)),x)
[Out] \text{Hanged}
```

3.1302 $\int \frac{1}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=433

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{3/2}(c-id)^{5/2}f}+\frac{i\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{3/2}(c+id)^{5/2}f}-\frac{\quad}{(a^2+b^2)\,(bc-ad}$$

```
[Out] -I*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c+d*tan(f*x+
e))^(1/2))/(a-I*b)^(3/2)/(c-I*d)^(5/2)/f+I*arctanh((c+I*d)^(1/2)*(a+b*tan(f
*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(a+I*b)^(3/2)/(c+I*d)^(5
/2)/f+2/3*(6*a^3*c*d^4+6*a*b^2*c*d^4-a^2*b*d^3*(11*c^2+5*d^2)-b^3*(3*c^4*d+
17*c^2*d^3+8*d^5))*(a+b*tan(f*x+e))^(1/2)/(a^2+b^2)/(-a*d+b*c)^3/(c^2+d^2)^
2/f/(c+d*tan(f*x+e))^(1/2)-2*b^2/(a^2+b^2)/(-a*d+b*c)/f/(a+b*tan(f*x+e))^(1
/2)/(c+d*tan(f*x+e))^(3/2)-2/3*d*(a^2*d^2+b^2*(3*c^2+4*d^2))*(a+b*tan(f*x+e
))^(1/2)/(a^2+b^2)/(-a*d+b*c)^2/(c^2+d^2)/f/(c+d*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 1.33, antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3650, 3730, 3697, 3696, 95, 214}

$$-\frac{2d(a^2d^2+b^2(3c^2+4d^2))\,\sqrt{a+b\tan(e+fx)}}{3f\,(a^2+b^2)\,(c^2+d^2)\,(bc-ad)^2(c+d\tan(e+fx))^{3/2}}-\frac{2b^2}{f\,(a^2+b^2)\,(bc-ad)\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}}+\frac{2(6a^3cd^4-a^2bd^3(11c^2+5d^2)+6ab^2cd^4-b^3(3c^4d+17c^2d^3+8d^5))\,\sqrt{a+b\tan(e+fx)}}{3f\,(a^2+b^2)\,(c^2+d^2)^2\,(bc-ad)^3\sqrt{c+d\tan(e+fx)}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a-ib)^{3/2}(c-id)^{5/2}}+\frac{i\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a+ib)^{3/2}(c+id)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(5/2)),x]
```

```
[Out] ((-I)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[
c + d*Tan[e + f*x]])])/((a - I*b)^(3/2)*(c - I*d)^(5/2)*f) + (I*ArcTanh[(Sq
rt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x
]])])/((a + I*b)^(3/2)*(c + I*d)^(5/2)*f) - (2*b^2)/((a^2 + b^2)*(b*c - a*d
)*f*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2)) - (2*d*(a^2*d^2 +
b^2*(3*c^2 + 4*d^2))*Sqrt[a + b*Tan[e + f*x]])/(3*(a^2 + b^2)*(b*c - a*d)^2
*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2)) + (2*(6*a^3*c*d^4 + 6*a*b^2*c*d^
4 - a^2*b*d^3*(11*c^2 + 5*d^2) - b^3*(3*c^4*d + 17*c^2*d^3 + 8*d^5))*Sqrt[a
 + b*Tan[e + f*x]])/(3*(a^2 + b^2)*(b*c - a*d)^3*(c^2 + d^2)^2*f*Sqrt[c + d
*Tan[e + f*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
```

```
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{2b^2}{(a^2+b^2)(bc-ad)f\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e \cdots}$$

$$= -\frac{2b^2}{(a^2+b^2)(bc-ad)f\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e \cdots}$$

$$= -\frac{2b^2}{(a^2+b^2)(bc-ad)f\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e \cdots}$$

$$= -\frac{2b^2}{(a^2+b^2)(bc-ad)f\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e \cdots}$$

$$= -\frac{2b^2}{(a^2+b^2)(bc-ad)f\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e \cdots}$$

$$= -\frac{2b^2}{(a^2+b^2)(bc-ad)f\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e \cdots}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{3/2}(c-id)^{5/2}f} + \frac{i\tanh^{-} \cdots}{\cdots}$$

Mathematica [A]
time = 6.48, size = 610, normalized size = 1.41

$$-\frac{2b^2}{(a^2+b^2)(bc-ad)f\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}}-\frac{2\left(-\frac{2\left(\frac{1}{2}d^2(-abc+a^2d+4b^2d)-c\left(-2b^2cd+\frac{1}{2}bd(bc-ad)\right)\right)\sqrt{a+b\tan(e+fx)}}{3(-bc+ad)(c^2+d^2)f(c+d\tan(e+fx))^{3/2}}-\frac{2\left(\frac{3(bc-ad)^3\left(\frac{(ia-b)(c+id)^2\tanh^{-1}\left(\frac{\sqrt{-c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+ib}\,\sqrt{-c+id}}+\frac{(ia+b)(c-id)^2\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}\,\sqrt{c+id}}\right)}{4(-bc+ad)(c^2+d^2)f}-\frac{2\left(\frac{1}{4}d^2\left(-3a^3cd^2-3ab^2c(c^2+2d^2)+a^2bd(6c^2+5d^2)+b^3d(9c^2+8d^2)\right)-c\left(\frac{3}{4}d(bc-ad)^2(bc+ad)-\frac{1}{2}bc\left(a^2d^3+b^2(3c^2d+4d^3)\right)\right)\right)\sqrt{a+b\tan(e+fx)}}{(-bc+ad)(c^2+d^2)f\sqrt{c+d\tan(e+fx)}}\right)}{3(-bc+ad)(c^2+d^2)}\right)}{(a^2+b^2)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(5/2)),x]
```

```
[Out] (-2*b^2)/((a^2 + b^2)*(b*c - a*d)*f*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e +
 f*x])^(3/2)) - (2*((-2*((d^2*(-(a*b*c) + a^2*d + 4*b^2*d))/2 - c*(-2*b^2*c
```

```
*d + (b*d*(b*c - a*d))/2))*Sqrt[a + b*Tan[e + f*x]])/(3*(-(b*c) + a*d)*(c^2
 + d^2)*f*(c + d*Tan[e + f*x])^(3/2)) - (2*((3*(b*c - a*d)^3*(((I*a - b)*(c
 + I*d)^2*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + I*b]
*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[-a + I*b]*Sqrt[-c + I*d]) + ((I*a + b)*(
c - I*d)^2*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*
Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*Sqrt[c + I*d])))/(4*(-(b*c) + a*
d)*(c^2 + d^2)*f) - (2*((d^2*(-3*a^3*c*d^2 - 3*a*b^2*c*(c^2 + 2*d^2) + a^2*
b*d*(6*c^2 + 5*d^2) + b^3*d*(9*c^2 + 8*d^2)))/4 - c*((3*d*(b*c - a*d)^2*(b*
c + a*d))/4 - (b*c*(a^2*d^3 + b^2*(3*c^2*d + 4*d^3)))/2))*Sqrt[a + b*Tan[e
+ f*x]])/((-(b*c) + a*d)*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]])))/(3*(-(b*
c) + a*d)*(c^2 + d^2))))/((a^2 + b^2)*(b*c - a*d))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\tan\left(fx+e\right)\right)^{\frac{3}{2}}\left(c+d\tan\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(5/2),x)
```

```
[Out] int(1/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((b*tan(f*x + e) + a)^(3/2)*(d*tan(f*x + e) + c)^(5/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\tan(e+fx))^{\frac{3}{2}}(c+d\tan(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))**(3/2)/(c+d*tan(f*x+e))**(5/2),x)
[Out] Integral(1/((a + b*tan(e + f*x))**(3/2)*(c + d*tan(e + f*x))**(5/2)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="gia
c")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,\tan\left(e+f\,x\right)\right)}^{3/2}\,{\left(c+d\,\tan\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*tan(e + f*x))^(3/2)*(c + d*tan(e + f*x))^(5/2)),x)
[Out] int(1/((a + b*tan(e + f*x))^(3/2)*(c + d*tan(e + f*x))^(5/2)), x)
```

3.1303 $\int \frac{1}{(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=596

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{5/2}(c-id)^{5/2}f}+\frac{i\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{5/2}(c+id)^{5/2}f}-\frac{}{3\left(a^2+b^2\right)(bc-}$$

```
[Out] -I*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c+d*tan(f*x+
e))^(1/2))/(a-I*b)^(5/2)/(c-I*d)^(5/2)/f+I*arctanh((c+I*d)^(1/2)*(a+b*tan(f
*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(a+I*b)^(5/2)/(c+I*d)^(5
/2)/f-4/3*d*(3*a^5*c*d^4+6*a^3*b^2*c*d^4-a^4*b*d^3*(7*c^2+4*d^2)+3*a*b^4*c*
(c^4+2*c^2*d^2+2*d^4)-b^5*d*(4*c^4+15*c^2*d^2+8*d^4)-a^2*b^3*d*(7*c^4+28*c^
2*d^2+15*d^4))*(a+b*tan(f*x+e))^(1/2)/(a^2+b^2)^2/(-a*d+b*c)^4/(c^2+d^2)^2/
f/(c+d*tan(f*x+e))^(1/2)-4*b^2*(-2*a^2*d+a*b*c-b^2*d)/(a^2+b^2)^2/(-a*d+b*c
)^2/f/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(3/2)+2/3*d*(a^4*d^3-6*a*b^3*
c*(c^2+d^2)+b^4*d*(7*c^2+8*d^2)+a^2*b^2*d*(13*c^2+15*d^2))*(a+b*tan(f*x+e))
^(1/2)/(a^2+b^2)^2/(-a*d+b*c)^3/(c^2+d^2)/f/(c+d*tan(f*x+e))^(3/2)-2/3*b^2/
(a^2+b^2)/(-a*d+b*c)/f/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(3/2)
```

Rubi [A]

time = 2.08, antiderivative size = 596, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3650, 3730, 3697, 3696, 95, 214}

$$-\frac{4b^2(-2a^2d+abc-b^2d)}{f\left(a^2+b^2\right)^2(bc-ad)^2\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}}-\frac{2b^2}{3f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}+\frac{2d\left(a^4d^3+a^2b^2d(13c^2+15d^2)-6ab^3c(c^2+d^2)+b^4d(7c^2+8d^2)\right)\sqrt{a+b\tan(e+fx)}}{3f\left(a^2+b^2\right)^2\left(c^2+d^2\right)(bc-ad)^3(c+d\tan(e+fx))^{3/2}}-\frac{4d\left(3a^5cd^4-a^4bd^3(7c^2+4d^2)+6a^3b^2cd^4-a^2b^3d(7c^4+28c^2d^2+15d^4)+3ab^4c(c^4+2c^2d^2+2d^4)-b^5d(4c^4+15c^2d^2+8d^4)\right)\sqrt{a+b\tan(e+fx)}}{3f\left(a^2+b^2\right)^2\left(c^2+d^2\right)^2(bc-ad)^4\sqrt{c+d\tan(e+fx)}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a-ib)^{5/2}(c-id)^{5/2}}+\frac{i\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a+ib)^{5/2}(c+id)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Tan[e + f*x])^(5/2)*(c + d*Tan[e + f*x])^(5/2)),x]
```

```
[Out] ((-I)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[
c + d*Tan[e + f*x]])])/((a - I*b)^(5/2)*(c - I*d)^(5/2)*f) + (I*ArcTanh[(Sq
rt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x
]])])/((a + I*b)^(5/2)*(c + I*d)^(5/2)*f) - (2*b^2)/(3*(a^2 + b^2)*(b*c - a
*d)*f*(a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(3/2)) - (4*b^2*(a*b*
c - 2*a^2*d - b^2*d))/((a^2 + b^2)^2*(b*c - a*d)^2*f*Sqrt[a + b*Tan[e + f*x
]]*(c + d*Tan[e + f*x])^(3/2)) + (2*d*(a^4*d^3 - 6*a*b^3*c*(c^2 + d^2) + b^
4*d*(7*c^2 + 8*d^2) + a^2*b^2*d*(13*c^2 + 15*d^2))*Sqrt[a + b*Tan[e + f*x]]
)/(3*(a^2 + b^2)^2*(b*c - a*d)^3*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2))
- (4*d*(3*a^5*c*d^4 + 6*a^3*b^2*c*d^4 - a^4*b*d^3*(7*c^2 + 4*d^2) + 3*a*b^4
*c*(c^4 + 2*c^2*d^2 + 2*d^4) - b^5*d*(4*c^4 + 15*c^2*d^2 + 8*d^4) - a^2*b^3
*d*(7*c^4 + 28*c^2*d^2 + 15*d^4))*Sqrt[a + b*Tan[e + f*x]])/(3*(a^2 + b^2)^
2*(b*c - a*d)^4*(c^2 + d^2)^2*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
```

```
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{2b^2}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}(c+d\,\mathrm{ta}}$$

$$= -\frac{2b^2}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}(c+d\,\mathrm{ta}}$$

$$= -\frac{2b^2}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}(c+d\,\mathrm{ta}}$$

$$= -\frac{2b^2}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}(c+d\,\mathrm{ta}}$$

$$= -\frac{2b^2}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}(c+d\,\mathrm{ta}}$$

$$= -\frac{2b^2}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}(c+d\,\mathrm{ta}}$$

$$= -\frac{2b^2}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}(c+d\,\mathrm{ta}}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{5/2}(c-id)^{5/2}f} + \frac{i\,\mathrm{tanh}}{}$$

Mathematica [A]

time = 6.64, size = 1050, normalized size = 1.76

$$-\frac{2b^2}{3(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}-\frac{2\left(-\frac{2\left(-\frac{3}{2}b^2(abc-a^2d-2b^2d)-a\left(-3ab^2d+\frac{3}{2}b^2(bc-ad)\right)\right)}{(a^2+b^2)(bc-ad)f\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}}-\frac{2\left(-\frac{2\left(-c\left(-\frac{3}{2}abd(bc-ad)^2+6b^2cd(abc-2a^2d-b^2d)\right)-\frac{3}{4}d^2\left(2a^3bcd+6ab^3cd-a^4d^2+b^4(c^2-8d^2)-a^2b^2(c^2+15d^2)\right)\right)\sqrt{a+b\tan(e+fx)}}{3(-bc+ad)(c^2+d^2)f(c+d\tan(e+fx))^{3/2}}-\frac{2\left(-\frac{9i(bc-ad)^4\left(\frac{(a+ib)^2(c+id)^2\tanh^{-1}\left(\frac{\sqrt{-c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+ib}\sqrt{-c+id}}+\frac{(a-ib)^2(c-id)^2\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}\sqrt{c+id}}\right)}{8(-bc+ad)(c^2+d^2)f}-\frac{2\left(d^2\left(\left(\frac{bc}{2}-\frac{3ad}{2}\right)\left(-\frac{3}{2}abd(bc-ad)^2+6b^2cd(abc-2a^2d-b^2d)\right)-\frac{3}{4}\left(bd^2-\frac{3}{2}c(-bc+ad)\right)\left(2a^3bcd+6ab^3cd-a^4d^2+b^4(c^2-8d^2)-a^2b^2(c^2+15d^2)\right)\right)-c\left(\frac{3}{2}d(-bc+ad)\left(\frac{3}{2}abc(bc-ad)^2+6b^2d^2(abc-2a^2d-b^2d)-\frac{3}{4}d\left(2a^3bcd+6ab^3cd-a^4d^2+b^4(c^2-8d^2)-a^2b^2(c^2+15d^2)\right)\right)-bc\left(-c\left(-\frac{3}{2}abd(bc-ad)^2+6b^2cd(abc-2a^2d-b^2d)\right)-\frac{3}{4}d^2\left(2a^3bcd+6ab^3cd-a^4d^2+b^4(c^2-8d^2)-a^2b^2(c^2+15d^2)\right)\right)\right)\right)\sqrt{a+b\tan(e+fx)}}{(-bc+ad)(c^2+d^2)f\sqrt{c+d\tan(e+fx)}}\right)}{3(-bc+ad)(c^2+d^2)}\right)}{(a^2+b^2)(bc-ad)}\right)}{3(a^2+b^2)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Tan[e + f*x])^(5/2)*(c + d*Tan[e + f*x])^(5/2)),x]
```

```
[Out] (-2*b^2)/(3*(a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])^(3/2)*(c + d*Tan
[e + f*x])^(3/2)) - (2*((-2*((-3*b^2*(a*b*c - a^2*d - 2*b^2*d))/2 - a*(-3*a
*b^2*d + (3*b^2*(b*c - a*d))/2)))/((a^2 + b^2)*(b*c - a*d)*f*Sqrt[a + b*Tan
[e + f*x]]*(c + d*Tan[e + f*x])^(3/2)) - (2*((-2*(-(c*((-3*a*b*d*(b*c - a*d
)^2)/2 + 6*b^2*c*d*(a*b*c - 2*a^2*d - b^2*d))) - (3*d^2*(2*a^3*b*c*d + 6*a*
b^3*c*d - a^4*d^2 + b^4*(c^2 - 8*d^2) - a^2*b^2*(c^2 + 15*d^2)))/4)*Sqrt[a
+ b*Tan[e + f*x]])/(3*(-(b*c) + a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/
2)) - (2*((((-9*I)/8)*(b*c - a*d)^4*(((a + I*b)^2*(c + I*d)^2*ArcTanh[(Sqrt
[-c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x
]])])/(Sqrt[-a + I*b]*Sqrt[-c + I*d]) + ((a - I*b)^2*(c - I*d)^2*ArcTanh[(S
qrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*
x]])])/(Sqrt[a + I*b]*Sqrt[c + I*d])))/((-(b*c) + a*d)*(c^2 + d^2)*f) - (2*
(d^2*(((b*c)/2 - (3*a*d)/2)*((-3*a*b*d*(b*c - a*d)^2)/2 + 6*b^2*c*d*(a*b*c
- 2*a^2*d - b^2*d)) - (3*(b*d^2 - (3*c*(-(b*c) + a*d))/2)*(2*a^3*b*c*d + 6*
a*b^3*c*d - a^4*d^2 + b^4*(c^2 - 8*d^2) - a^2*b^2*(c^2 + 15*d^2)))/4) - c*(
(3*d*(-(b*c) + a*d)*((3*a*b*c*(b*c - a*d)^2)/2 + 6*b^2*d^2*(a*b*c - 2*a^2*d
 - b^2*d) - (3*d*(2*a^3*b*c*d + 6*a*b^3*c*d - a^4*d^2 + b^4*(c^2 - 8*d^2) -
 a^2*b^2*(c^2 + 15*d^2)))/4))/2 - b*c*(-(c*((-3*a*b*d*(b*c - a*d)^2)/2 + 6*
b^2*c*d*(a*b*c - 2*a^2*d - b^2*d))) - (3*d^2*(2*a^3*b*c*d + 6*a*b^3*c*d - a
^4*d^2 + b^4*(c^2 - 8*d^2) - a^2*b^2*(c^2 + 15*d^2)))/4)))*Sqrt[a + b*Tan[e
 + f*x]])/((-(b*c) + a*d)*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]])))/(3*(-(b
*c) + a*d)*(c^2 + d^2))))/((a^2 + b^2)*(b*c - a*d))))/(3*(a^2 + b^2)*(b*c -
 a*d))
```

Maple [**F(-1)**] grade_annotation
time = 180.00, size = 0, normalized size = 0.00 hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(5/2),x)
```

```
[Out] int(1/(a+b*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((2*b*d+2*a*c)^2>0)', see `assume?`
 for mo
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\tan\left(e+fx\right)\right)^{\frac{5}{2}}\left(c+d\tan\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))**(5/2)/(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral(1/((a + b*tan(e + f*x))**(5/2)*(c + d*tan(e + f*x))**(5/2)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*tan(e + f*x))^(5/2)*(c + d*tan(e + f*x))^(5/2)),x)
```

```
[Out] \text{Hanged}
```

3.1304 $\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^n\,dx$

Optimal. Leaf size=257

$$\frac{F_1\left(1+m;-n,1;2+m;-\frac{d(a+b\tan(e+fx))}{bc-ad},\frac{a+b\tan(e+fx)}{a-ib}\right)(a+b\tan(e+fx))^{1+m}(c+d\tan(e+fx))^n\left(\frac{b(c+d\tan(}{bc-a}\right.}{2(ia+b)f(1+m)}$$

```
[Out] 1/2*AppellF1(1+m,1,-n,2+m,(a+b*tan(f*x+e))/(a-I*b),-d*(a+b*tan(f*x+e))/(-a*
d+b*c))*(a+b*tan(f*x+e))^(1+m)*(c+d*tan(f*x+e))^n/(I*a+b)/f/(1+m)/((b*(c+d*
tan(f*x+e))/(-a*d+b*c))^n)-1/2*AppellF1(1+m,1,-n,2+m,(a+b*tan(f*x+e))/(a+I*
b),-d*(a+b*tan(f*x+e))/(-a*d+b*c))*(a+b*tan(f*x+e))^(1+m)*(c+d*tan(f*x+e))^
n/(I*a-b)/f/(1+m)/((b*(c+d*tan(f*x+e))/(-a*d+b*c))^n)
```

Rubi [A]

time = 0.23, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3656, 926, 142, 141}

$$\frac{(a+b\tan(e+fx))^{m+1}(c+d\tan(e+fx))^n\left(\frac{b(c+d\tan(e+fx))}{bc-ad}\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{d(a+b\tan(e+fx))}{bc-ad},\frac{a+b\tan(e+fx)}{a-ib}\right)}{2f(m+1)(b+ia)}-\frac{(a+b\tan(e+fx))^{m+1}(c+d\tan(e+fx))^n\left(\frac{b(c+d\tan(e+fx))}{bc-ad}\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{d(a+b\tan(e+fx))}{bc-ad},\frac{a+b\tan(e+fx)}{a+ib}\right)}{2f(m+1)(-b+ia)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n,x]
```

```
[Out] (AppellF1[1 + m, -n, 1, 2 + m, -((d*(a + b*Tan[e + f*x]))/(b*c - a*d)), (a
+ b*Tan[e + f*x])/(a - I*b)]*(a + b*Tan[e + f*x])^(1 + m)*(c + d*Tan[e + f*
x])^n)/(2*(I*a + b)*f*(1 + m)*((b*(c + d*Tan[e + f*x]))/(b*c - a*d))^n) - (
AppellF1[1 + m, -n, 1, 2 + m, -((d*(a + b*Tan[e + f*x]))/(b*c - a*d)), (a +
 b*Tan[e + f*x])/(a + I*b)]*(a + b*Tan[e + f*x])^(1 + m)*(c + d*Tan[e + f*x
])^n)/(2*(I*a - b)*f*(1 + m)*((b*(c + d*Tan[e + f*x]))/(b*c - a*d))^n)
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
```

```
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^n\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx)^m(c+dx)^n}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{i(a+bx)^m(c+dx)^n}{2(i-x)} + \frac{i(a+bx)^m(c+dx)^n}{2(i+x)}\right)\,dx, x, \tan(e+\ldots}{f}$$

$$= \frac{i\text{Subst}\left(\int \frac{(a+bx)^m(c+dx)^n}{i-x}\,dx, x, \tan(e+fx)\right)}{2f} + \frac{i\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= \frac{\left(i(c+d\tan(e+fx))^n\left(\frac{b(c+d\tan(e+fx))}{bc-ad}\right)^{-n}\right)\text{Subst}\left(\int \frac{(a+\ldots}{\ldots}\right.}{2f}$$

$$= \frac{F_1\left(1+m; -n, 1; 2+m; -\frac{d(a+b\tan(e+fx))}{bc-ad}, \frac{a+b\tan(e+fx)}{a-ib}\right)(a\ldots}{2(ia+\ldots}$$

Mathematica [F]
time = 2.13, size = 0, normalized size = 0.00

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n,x]
```

```
[Out] Integrate[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x]
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \left(a+b\tan\left(fx+e\right)\right)^{m}\left(c+d\tan\left(fx+e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^n,x)
```

```
[Out] int((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^m*(d*tan(f*x + e) + c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e) + a)^m*(d*tan(f*x + e) + c)^n, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**m*(c+d*tan(f*x+e))**n,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)^m*(d*tan(f*x + e) + c)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\tan(e+f\,x))^m\,(c+d\tan(e+f\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^m*(c + d*tan(e + f*x))^n,x)
```

```
[Out] int((a + b*tan(e + f*x))^m*(c + d*tan(e + f*x))^n, x)
```

3.1305 $\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^3\,dx$

Optimal. Leaf size=214

$$\frac{d^2(3bc-ad)(a+b\tan(e+fx))^{1+m}}{b^2f(1+m)}+\frac{(ic+d)^3\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a-ib}\right)(a+b\tan(e+fx))^{1+m}}{2(a-ib)f(1+m)}-\cdots$$

```
[Out] d^2*(-a*d+3*b*c)*(a+b*tan(f*x+e))^(1+m)/b^2/f/(1+m)+1/2*(I*c+d)^3*hypergeom
([1, 1+m],[2+m],(a+b*tan(f*x+e))/(a-I*b))*(a+b*tan(f*x+e))^(1+m)/(a-I*b)/f/
(1+m)-1/2*(I*c-d)^3*hypergeom([1, 1+m],[2+m],(a+b*tan(f*x+e))/(a+I*b))*(a+b
*tan(f*x+e))^(1+m)/(a+I*b)/f/(1+m)+d^3*(a+b*tan(f*x+e))^(2+m)/b^2/f/(2+m)
```

Rubi [A]
time = 0.33, antiderivative size = 234, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3647, 3711, 3620, 3618, 70}

$$-\frac{d^2(ad-bc(2m+5))(a+b\tan(e+fx))^{m+1}}{b^2f(m+1)(m+2)}+\frac{d^2(c+d\tan(e+fx))(a+b\tan(e+fx))^{m+1}}{bf(m+2)}+\frac{(c-id)^3(a+b\tan(e+fx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\tan(e+fx)}{a-ib}\right)}{2f(m+1)(b+ia)}-\frac{(-d+ic)^3(a+b\tan(e+fx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\tan(e+fx)}{a+ib}\right)}{2f(m+1)(a+ib)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^3,x]
```

```
[Out] -((d^2*(a*d - b*c*(5 + 2*m))*(a + b*Tan[e + f*x])^(1 + m))/(b^2*f*(1 + m)*(
2 + m))) + ((c - I*d)^3*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f
*x])/(a - I*b)]*(a + b*Tan[e + f*x])^(1 + m))/(2*(I*a + b)*f*(1 + m)) - ((I
*c - d)^3*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a + I*b)
]*(a + b*Tan[e + f*x])^(1 + m))/(2*(a + I*b)*f*(1 + m)) + (d^2*(a + b*Tan[e
 + f*x])^(1 + m)*(c + d*Tan[e + f*x]))/(b*f*(2 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
```

```
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^3\,dx=\frac{d^2(a+b\tan(e+fx))^{1+m}(c+d\tan(e+fx))}{bf(2+m)}+\frac{\int (a+b\,\mathrm{t}}{}$$

$$=-\frac{d^2(ad-bc(5+2m))(a+b\tan(e+fx))^{1+m}}{b^2f(1+m)(2+m)}+\frac{d^2(a+b}{}$$

$$=-\frac{d^2(ad-bc(5+2m))(a+b\tan(e+fx))^{1+m}}{b^2f(1+m)(2+m)}+\frac{d^2(a+b}{}$$

$$=-\frac{d^2(ad-bc(5+2m))(a+b\tan(e+fx))^{1+m}}{b^2f(1+m)(2+m)}+\frac{d^2(a+b}{}$$

$$=-\frac{d^2(ad-bc(5+2m))(a+b\tan(e+fx))^{1+m}}{b^2f(1+m)(2+m)}+\frac{(ic+d)^3}{}$$

Mathematica [A]
time = 2.07, size = 189, normalized size = 0.88

$$\frac{(a+b\tan(e+fx))^{1+m}\left(\frac{2d^2(-ad+bc(5+2m))}{b(1+m)}-\frac{ib(c-id)^3(2+m)\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a-ib}\right)}{(a-ib)(1+m)}+\frac{ib(c+id)^3(2+m)\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a+ib}\right)}{(a+ib)(1+m)}+2d^2(c+d\tan(e+fx))\right)}{2bf(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^3,x]
```

```
[Out] ((a + b*Tan[e + f*x])^(1 + m)*((2*d^2*(-(a*d) + b*c*(5 + 2*m)))/(b*(1 + m))
 - (I*b*(c - I*d)^3*(2 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e
 + f*x])/(a - I*b)])/((a - I*b)*(1 + m)) + (I*b*(c + I*d)^3*(2 + m)*Hyperge
ometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a + I*b)])/((a + I*b)*(1
+ m)) + 2*d^2*(c + d*Tan[e + f*x])))/(2*b*f*(2 + m))
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int (a+b\tan (fx+e))^m (c+d\tan (fx+e))^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^3,x)
```

```
[Out] int((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((d*tan(f*x + e) + c)^3*(b*tan(f*x + e) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral((d^3*tan(f*x + e)^3 + 3*c*d^2*tan(f*x + e)^2 + 3*c^2*d*tan(f*x + e
) + c^3)*(b*tan(f*x + e) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan (e+fx))^m (c+d\tan (e+fx))^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**m*(c+d*tan(f*x+e))**3,x)
[Out] Integral((a + b*tan(e + f*x))**m*(c + d*tan(e + f*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^3,x, algorithm="giac")
[Out] integrate((d*tan(f*x + e) + c)^3*(b*tan(f*x + e) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\tan(e+f\,x))^m\,(c+d\tan(e+f\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^m*(c + d*tan(e + f*x))^3,x)
[Out] int((a + b*tan(e + f*x))^m*(c + d*tan(e + f*x))^3, x)
```

3.1306 $\int (a+b\tan(e+fx))^m (c+d\tan(e+fx))^2\,dx$

Optimal. Leaf size=176

$$\frac{d^2(a+b\tan(e+fx))^{1+m}}{bf(1+m)}+\frac{(c-id)^2\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a-ib}\right)(a+b\tan(e+fx))^{1+m}}{2(ia+b)f(1+m)}-\frac{(c+id)^2\,{}_2F$$

```
[Out] d^2*(a+b*tan(f*x+e))^(1+m)/b/f/(1+m)+1/2*(c-I*d)^2*hypergeom([1, 1+m],[2+m]
,(a+b*tan(f*x+e))/(a-I*b))*(a+b*tan(f*x+e))^(1+m)/(I*a+b)/f/(1+m)-1/2*(c+I*
d)^2*hypergeom([1, 1+m],[2+m],(a+b*tan(f*x+e))/(a+I*b))*(a+b*tan(f*x+e))^(1
+m)/(I*a-b)/f/(1+m)
```

Rubi [A]
time = 0.15, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3624, 3620, 3618, 70}

$$\frac{(c-id)^2(a+b\tan(e+fx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\tan(e+fx)}{a-ib}\right)}{2f(m+1)(b+ia)}-\frac{(c+id)^2(a+b\tan(e+fx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\tan(e+fx)}{a+ib}\right)}{2f(m+1)(-b+ia)}+\frac{d^2(a+b\tan(e+fx))^{m+1}}{bf(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^2,x]
```

```
[Out] (d^2*(a + b*Tan[e + f*x])^(1 + m))/(b*f*(1 + m)) + ((c - I*d)^2*Hypergeomet
ric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a - I*b)]*(a + b*Tan[e + f*x]
)^(1 + m))/(2*(I*a + b)*f*(1 + m)) - ((c + I*d)^2*Hypergeometric2F1[1, 1 +
m, 2 + m, (a + b*Tan[e + f*x])/(a + I*b)]*(a + b*Tan[e + f*x])^(1 + m))/(2*
(I*a - b)*f*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
```

```
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^2\,dx = \frac{d^2(a+b\tan(e+fx))^{1+m}}{bf(1+m)} + \int (a+b\tan(e+fx))^m\left(c^2 -\right.$$
$$= \frac{d^2(a+b\tan(e+fx))^{1+m}}{bf(1+m)} + \frac{1}{2}(c-id)^2\int (1+i\tan(e+$$
$$= \frac{d^2(a+b\tan(e+fx))^{1+m}}{bf(1+m)} + \frac{(i(c-id)^2)\operatorname{Subst}\left(\int \frac{(a-ibx)^m}{-1+x}\right.}{2f}$$
$$= \frac{d^2(a+b\tan(e+fx))^{1+m}}{bf(1+m)} + \frac{(c-id)^2\,{}_2F_1\left(1,1+m;2+\right.}{2(ia}$$

Mathematica [A]
time = 0.27, size = 135, normalized size = 0.77

$$\frac{\left(\frac{2d^2}{b} - \frac{i(c-id)^2\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a-ib}\right)}{a-ib} + \frac{i(c+id)^2\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a+ib}\right)}{a+ib}\right)(a+b\tan(e+fx))^{1+m}}{2f(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^2,x]
```

```
[Out] (((2*d^2)/b - (I*(c - I*d)^2*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[
e + f*x])/(a - I*b)])/(a - I*b) + (I*(c + I*d)^2*Hypergeometric2F1[1, 1 + m
, 2 + m, (a + b*Tan[e + f*x])/(a + I*b)])/(a + I*b))*(a + b*Tan[e + f*x])^(
1 + m))/(2*f*(1 + m))
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int (a+b\tan(fx+e))^m (c+d\tan(fx+e))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^2,x)
```

```
[Out] int((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*tan(f*x + e) + c)^2*(b*tan(f*x + e) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2)*(b*tan(f*x + e) +
a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(e + fx))^m (c + d\tan(e + fx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**m*(c+d*tan(f*x+e))**2,x)
```

```
[Out] Integral((a + b*tan(e + f*x))**m*(c + d*tan(e + f*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*tan(f*x + e) + c)^2*(b*tan(f*x + e) + a)^m, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\tan(e+f\,x))^m\,(c+d\tan(e+f\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^m*(c + d*tan(e + f*x))^2,x)
[Out] int((a + b*tan(e + f*x))^m*(c + d*tan(e + f*x))^2, x)
```

3.1307 $\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))\,dx$

Optimal. Leaf size=143

$$\frac{(c-id)\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a-ib}\right)(a+b\tan(e+fx))^{1+m}}{2(ia+b)f(1+m)}+\frac{(ic-d)\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a+ib}\right)(a+b\tan(e+fx))^{1+m}}{2(a+ib)f(1+m)}$$

```
[Out] 1/2*(c-I*d)*hypergeom([1, 1+m],[2+m],(a+b*tan(f*x+e))/(a-I*b))*(a+b*tan(f*x
+e))^(1+m)/(I*a+b)/f/(1+m)+1/2*(I*c-d)*hypergeom([1, 1+m],[2+m],(a+b*tan(f*
x+e))/(a+I*b))*(a+b*tan(f*x+e))^(1+m)/(a+I*b)/f/(1+m)
```

Rubi [A]
time = 0.10, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3620, 3618, 70}

$$\frac{(c-id)(a+b\tan(e+fx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\tan(e+fx)}{a-ib}\right)}{2f(m+1)(b+ia)}+\frac{(-d+ic)(a+b\tan(e+fx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\tan(e+fx)}{a+ib}\right)}{2f(m+1)(a+ib)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x]),x]
```

```
[Out] ((c - I*d)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a - I*b
)]*(a + b*Tan[e + f*x])^(1 + m))/(2*(I*a + b)*f*(1 + m)) + ((I*c - d)*Hyper
geometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a + I*b)]*(a + b*Tan[e
+ f*x])^(1 + m))/(2*(a + I*b)*f*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
```

```
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))\,dx = \frac{1}{2}(c-id)\int (1+i\tan(e+fx))(a+b\tan(e+fx))^m\,dx +$$
$$= -\frac{(ic-d)\text{Subst}\left(\int \frac{(a+ibx)^m}{-1+x}\,dx, x, -i\tan(e+fx)\right)}{2f} + \frac{(ic+}{}$$
$$= -\frac{(ic+d)\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a-ib}\right)(a+b\tan(e}{2(a-ib)f(1+m)}$$

Mathematica [A]
time = 0.23, size = 120, normalized size = 0.84

$$\frac{i\left(-\frac{(c-id)\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a-ib}\right)}{a-ib}+\frac{(c+id)\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a+ib}\right)}{a+ib}\right)(a+b\tan(e+fx))^{1+m}}{2f(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x]),x]
[Out] ((I/2)*(-(((c - I*d)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x]
)/(a - I*b)])/(a - I*b)) + ((c + I*d)*Hypergeometric2F1[1, 1 + m, 2 + m, (a
 + b*Tan[e + f*x])/(a + I*b)])/(a + I*b))*(a + b*Tan[e + f*x])^(1 + m))/(f*
(1 + m))
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(a+b\tan\left(fx+e\right)\right)^{m}\left(c+d\tan\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e)),x)
[Out] int((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*tan(f*x + e) + c)*(b*tan(f*x + e) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e)),x, algorithm="fricas")
[Out] integral((d*tan(f*x + e) + c)*(b*tan(f*x + e) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan{(e + fx)})^m (c + d\tan{(e + fx)})\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**m*(c+d*tan(f*x+e)),x)
[Out] Integral((a + b*tan(e + f*x))**m*(c + d*tan(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((d*tan(f*x + e) + c)*(b*tan(f*x + e) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\mathrm{tan}\left(e+f\,x\right)\right)}^m\,\left(c+d\,\mathrm{tan}\left(e+f\,x\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^m*(c + d*tan(e + f*x)),x)
[Out] int((a + b*tan(e + f*x))^m*(c + d*tan(e + f*x)), x)
```

3.1308 $\int (a+b\tan(e+fx))^m\,dx$

Optimal. Leaf size=167

$$\frac{b\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}}\right)(a+b\tan(e+fx))^{1+m}}{2\sqrt{-b^2}\left(a-\sqrt{-b^2}\right)f(1+m)}-\frac{b\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a+\sqrt{-b^2}}\right)(a+b\tan(e+fx))^{1+m}}{2\sqrt{-b^2}\left(a+\sqrt{-b^2}\right)f(1+m)}$$

```
[Out] 1/2*b*hypergeom([1, 1+m],[2+m],(a+b*tan(f*x+e))/(a-(-b^2)^(1/2)))*(a+b*tan(
f*x+e))^(1+m)/f/(1+m)/(a-(-b^2)^(1/2))/(-b^2)^(1/2)-1/2*b*hypergeom([1, 1+m
],[2+m],(a+b*tan(f*x+e))/(a+(-b^2)^(1/2)))*(a+b*tan(f*x+e))^(1+m)/f/(1+m)/(
-b^2)^(1/2)/(a+(-b^2)^(1/2))
```

Rubi [A]
time = 0.20, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3566, 726, 70}

$$\frac{b(a+b\tan(e+fx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}}\right)}{2\sqrt{-b^2}\,f(m+1)\left(a-\sqrt{-b^2}\right)}-\frac{b(a+b\tan(e+fx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\tan(e+fx)}{a+\sqrt{-b^2}}\right)}{2\sqrt{-b^2}\,f(m+1)\left(a+\sqrt{-b^2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^m,x]
```

```
[Out] (b*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a - Sqrt[-b^2])
]*(a + b*Tan[e + f*x])^(1 + m))/(2*Sqrt[-b^2]*(a - Sqrt[-b^2])*f*(1 + m)) -
 (b*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a + Sqrt[-b^2]
)]*(a + b*Tan[e + f*x])^(1 + m))/(2*Sqrt[-b^2]*(a + Sqrt[-b^2])*f*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 726

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
```

```
d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a+b\tan(e+fx))^m\,dx = \frac{b\text{Subst}\left(\int \frac{(a+x)^m}{b^2+x^2}\,dx, x, b\tan(e+fx)\right)}{f}$$
$$= \frac{b\text{Subst}\left(\int \left(\frac{\sqrt{-b^2}\,(a+x)^m}{2b^2\left(\sqrt{-b^2}-x\right)} + \frac{\sqrt{-b^2}\,(a+x)^m}{2b^2\left(\sqrt{-b^2}+x\right)}\right)dx, x, b\tan(e+fx)\right)}{f}$$
$$= -\frac{b\text{Subst}\left(\int \frac{(a+x)^m}{\sqrt{-b^2}-x}\,dx, x, b\tan(e+fx)\right)}{2\sqrt{-b^2}\,f} - \frac{b\text{Subst}\left(\int \frac{(a+x)^m}{\sqrt{-b^2}+x}\,dx, x, b\tan(\right.}{2\sqrt{-b^2}\,f}$$
$$= \frac{b\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}}\right)(a+b\tan(e+fx))^{1+m}}{2\sqrt{-b^2}\left(a-\sqrt{-b^2}\right)f(1+m)} - \frac{b\,{}_2F_1\left(1,1+m;\right.}{2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.17, size = 118, normalized size = 0.71

$$\frac{\left((a+ib)\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a-ib}\right)-(a-ib)\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a+ib}\right)\right)(a+b\tan(e+fx))^{1+m}}{2(a+ib)(ia+b)f(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^m,x]
[Out] (((a + I*b)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a - I*
b)] - (a - I*b)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a
+ I*b)])*(a + b*Tan[e + f*x])^(1 + m))/(2*(a + I*b)*(I*a + b)*f*(1 + m))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (a+b\tan(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^m,x)
[Out] int((a+b*tan(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(e + fx))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**m,x)
[Out] Integral((a + b*tan(e + f*x))**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\tan(e + f\,x))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^m,x)
[Out] int((a + b*tan(e + f*x))^m, x)
```

3.1309 $\int \frac{(a+b\tan(e+fx))^m}{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=223

$$\frac{{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a-ib}\right)(a+b\tan(e+fx))^{1+m}}{2(ia+b)(c-id)f(1+m)}-\frac{{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a+ib}\right)(a+b\tan(e}{2(a+ib)(ic-d)f(1+m)}$$

```
[Out] 1/2*hypergeom([1, 1+m],[2+m],(a+b*tan(f*x+e))/(a-I*b))*(a+b*tan(f*x+e))^(1+
m)/(I*a+b)/(c-I*d)/f/(1+m)-1/2*hypergeom([1, 1+m],[2+m],(a+b*tan(f*x+e))/(a
+I*b))*(a+b*tan(f*x+e))^(1+m)/(I*a-b)/(c+I*d)/f/(1+m)+d^2*hypergeom([1, 1+m
],[2+m],-d*(a+b*tan(f*x+e))/(-a*d+b*c))*(a+b*tan(f*x+e))^(1+m)/(-a*d+b*c)/(
c^2+d^2)/f/(1+m)
```

Rubi [A]
time = 0.27, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3655, 3620, 3618, 70, 3715}

$$\frac{d^2(a+b\tan(e+fx))^{m+1}{}_2F_1\left(1,m+1;m+2;-\frac{d(a+b\tan(e+fx))}{bc-ad}\right)}{f(m+1)\left(c^2+d^2\right)(bc-ad)}+\frac{(a+b\tan(e+fx))^{m+1}{}_2F_1\left(1,m+1;m+2;\frac{a+b\tan(e+fx)}{a-ib}\right)}{2f(m+1)(b+ia)(c-id)}-\frac{(a+b\tan(e+fx))^{m+1}{}_2F_1\left(1,m+1;m+2;\frac{a+b\tan(e+fx)}{a+ib}\right)}{2f(m+1)(a+ib)(-d+ic)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^m/(c + d*Tan[e + f*x]),x]
```

```
[Out] (Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a - I*b)]*(a + b*
Tan[e + f*x])^(1 + m))/(2*(I*a + b)*(c - I*d)*f*(1 + m)) - (Hypergeometric2
F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a + I*b)]*(a + b*Tan[e + f*x])^(1
 + m))/(2*(a + I*b)*(I*c - d)*f*(1 + m)) + (d^2*Hypergeometric2F1[1, 1 + m,
 2 + m, -((d*(a + b*Tan[e + f*x]))/(b*c - a*d))]*(a + b*Tan[e + f*x])^(1 +
m))/((b*c - a*d)*(c^2 + d^2)*f*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)/((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[(a + b*Tan[e + f*x])^m*
(c - d*Tan[e + f*x]), x], x] + Dist[d^2/(c^2 + d^2), Int[(a + b*Tan[e + f*x
])^m*((1 + Tan[e + f*x]^2)/(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^m}{c+d\tan(e+fx)}\,dx = \frac{\int (a+b\tan(e+fx))^m(c-d\tan(e+fx))\,dx}{c^2+d^2} + \frac{d^2\int \frac{(a+b\tan(e+fx))^m(1+\tan^2(e+f}{c+d\tan(e+fx)}}{c^2+d^2}$$

$$= \frac{\int (1+i\tan(e+fx))(a+b\tan(e+fx))^m\,dx}{2(c-id)} + \frac{\int (1-i\tan(e+fx))(a+b\,\mathrm{ta}}{2(c+id)}$$

$$= \frac{d^2\,{}_2F_1\left(1,1+m;2+m;-\frac{d(a+b\tan(e+fx))}{bc-ad}\right)(a+b\tan(e+fx))^{1+m}}{(bc-ad)\,(c^2+d^2)\,f(1+m)} + \frac{\mathrm{Subst}\Big(\int}{}$$

$$= \frac{{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a-ib}\right)(a+b\tan(e+fx))^{1+m}}{2(ia+b)(c-id)f(1+m)} - \frac{{}_2F_1\Big(1,1+m;}{}$$

Mathematica [A]
time = 0.60, size = 178, normalized size = 0.80

$$\frac{\left(\frac{{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a-ib}\right)}{(ia+b)(c-id)}+\frac{i\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a+ib}\right)}{(a+ib)(c+id)}-\frac{2d^2\,{}_2F_1\left(1,1+m;2+m;\frac{d(a+b\tan(e+fx))}{-bc+ad}\right)}{(-bc+ad)(c^2+d^2)}\right)(a+b\tan(e+fx))^{1+m}}{2f(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^m/(c + d*Tan[e + f*x]),x]
```

```
[Out] ((Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a - I*b)]/((I*a
+ b)*(c - I*d)) + (I*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x]
)/(a + I*b)])/((a + I*b)*(c + I*d)) - (2*d^2*Hypergeometric2F1[1, 1 + m, 2
+ m, (d*(a + b*Tan[e + f*x]))/(-(b*c) + a*d)])/((-(b*c) + a*d)*(c^2 + d^2))
)*(a + b*Tan[e + f*x])^(1 + m))/(2*f*(1 + m))
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan(fx + e))^m}{c + d\tan(fx + e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^m/(c+d*tan(f*x+e)),x)
```

```
[Out] int((a+b*tan(f*x+e))^m/(c+d*tan(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m/(c+d*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^m/(d*tan(f*x + e) + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m/(c+d*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e) + a)^m/(d*tan(f*x + e) + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan(e + fx))^m}{c + d\tan(e + fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**m/(c+d*tan(f*x+e)),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**m/(c + d*tan(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m/(c+d*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^m/(d*tan(f*x + e) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\tan(e+f\,x))^m}{c+d\tan\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^m/(c + d*tan(e + f*x)),x)
[Out] int((a + b*tan(e + f*x))^m/(c + d*tan(e + f*x)), x)
```

3.1310 $\int \frac{(a+b\tan(e+fx))^m}{(c+d\tan(e+fx))^2}\,dx$

Optimal. Leaf size=301

$$\frac{{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a-ib}\right)(a+b\tan(e+fx))^{1+m}}{2(ia+b)(c-id)^2f(1+m)}-\frac{{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a+ib}\right)(a+b\tan(e$$

```
[Out] 1/2*hypergeom([1, 1+m],[2+m],(a+b*tan(f*x+e))/(a-I*b))*(a+b*tan(f*x+e))^(1+
m)/(I*a+b)/(c-I*d)^2/f/(1+m)-1/2*hypergeom([1, 1+m],[2+m],(a+b*tan(f*x+e))/
(a+I*b))*(a+b*tan(f*x+e))^(1+m)/(I*a-b)/(c+I*d)^2/f/(1+m)-d^2*(2*a*c*d-b*(c
^2*(2-m)-d^2*m))*hypergeom([1, 1+m],[2+m],-d*(a+b*tan(f*x+e))/(-a*d+b*c))*(
a+b*tan(f*x+e))^(1+m)/(-a*d+b*c)^2/(c^2+d^2)^2/f/(1+m)+d^2*(a+b*tan(f*x+e))
^(1+m)/(-a*d+b*c)/(c^2+d^2)/f/(c+d*tan(f*x+e))
```

Rubi [A]
time = 0.56, antiderivative size = 299, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3650, 3734, 3620, 3618, 70, 3715}

$$-\frac{d^2(2acd-bc^2(2-m)+bd^2m)(a+b\tan(e+fx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{d(a+b\tan(e+fx))}{bc-ad}\right)}{f(m+1)(c^2+d^2)^2(bc-ad)^2}+\frac{d^2(a+b\tan(e+fx))^{m+1}}{f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))}+\frac{(a+b\tan(e+fx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\tan(e+fx)}{a-ib}\right)}{2f(m+1)(b+ia)(c-id)^2}-\frac{(a+b\tan(e+fx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\tan(e+fx)}{a+ib}\right)}{2f(m+1)(-b+ia)(c+id)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^m/(c + d*Tan[e + f*x])^2,x]
```

```
[Out] (Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a - I*b)]*(a + b*
Tan[e + f*x])^(1 + m))/(2*(I*a + b)*(c - I*d)^2*f*(1 + m)) - (Hypergeometri
c2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a + I*b)]*(a + b*Tan[e + f*x])^
(1 + m))/(2*(I*a - b)*(c + I*d)^2*f*(1 + m)) - (d^2*(2*a*c*d - b*c^2*(2 - m
) + b*d^2*m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((d*(a + b*Tan[e + f*x]))/
(b*c - a*d))]*(a + b*Tan[e + f*x])^(1 + m))/((b*c - a*d)^2*(c^2 + d^2)^2*f*
(1 + m)) + (d^2*(a + b*Tan[e + f*x])^(1 + m))/((b*c - a*d)*(c^2 + d^2)*f*(c
 + d*Tan[e + f*x]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
```

```
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^m}{(c+d\tan(e+fx))^2}\,dx = \frac{d^2(a+b\tan(e+fx))^{1+m}}{(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))} + \frac{\int \frac{(a+b\tan(e+fx))^m\left(-acd+b\left(c^2-d^2m\right)-d(bc\cdots\right.}{c+d\tan(e+\cdots}}{(bc-ad)\,(c\cdots}$$

$$= \frac{d^2(a+b\tan(e+fx))^{1+m}}{(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))} + \frac{\int (a+b\tan(e+fx))^m\left((bc-ad)\,(c\cdots\right.}{(bc-a\cdots}$$

$$= \frac{d^2(a+b\tan(e+fx))^{1+m}}{(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))} + \frac{\int (1+i\tan(e+fx))(a+b\tan(e+\cdots}{2(c-id)^2}$$

$$= -\frac{d^2\left(2acd-bc^2(2-m)+bd^2m\right)\,{}_2F_1\left(1,1+m;2+m;-\frac{d(a+b\tan(e+fx))}{bc-ad}\right)(a+b\cdots}{(bc-ad)^2\left(c^2+d^2\right)^2 f(1+m)}$$

$$= \frac{{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a-ib}\right)(a+b\tan(e+fx))^{1+m}}{2(ia+b)(c-id)^2f(1+m)} - \frac{{}_2F_1\left(1,1+m;2\cdots\right.}{2\cdots}$$

Mathematica [A]
time = 4.33, size = 266, normalized size = 0.88

$$\frac{(a+b\tan(e+fx))^{1+m}\left(-\frac{i\left(\frac{(c+id)^2(-bc+ad)\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a-ib}\right)}{a-ib}+\frac{(c-id)^2(bc-ad)\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a+ib}\right)}{a+ib}\right)}{(c^2+d^2)(1+m)}-\frac{2d^2\left(2acd+bc^2(-2+m)+bd^2m\right)\,{}_2F_1\left(1,1+m;2+m;\frac{d(a+b\tan(e+fx))}{-bc+ad}\right)}{(-bc+ad)(c^2+d^2)(1+m)}-\frac{2d^2}{c+d\tan(e+fx)}\right)}{2(-bc+ad)\left(c^2+d^2\right)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^m/(c + d*Tan[e + f*x])^2,x]
```

```
[Out] ((a + b*Tan[e + f*x])^(1 + m)*(((-I)*(((c + I*d)^2*(-(b*c) + a*d)*Hypergeom
etric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a - I*b)])/(a - I*b) + ((c
- I*d)^2*(b*c - a*d)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x]
)/(a + I*b)])/(a + I*b)))/((c^2 + d^2)*(1 + m)) - (2*d^2*(2*a*c*d + b*c^2*(
-2 + m) + b*d^2*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (d*(a + b*Tan[e + f*x
]))/(-(b*c) + a*d)])/((-(b*c) + a*d)*(c^2 + d^2)*(1 + m)) - (2*d^2)/(c + d*
Tan[e + f*x])))/(2*(-(b*c) + a*d)*(c^2 + d^2)*f)
```

Maple [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(fx+e))^m}{(c+d\tan(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^m/(c+d*tan(f*x+e))^2,x)
```

```
[Out] int((a+b*tan(f*x+e))^m/(c+d*tan(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m/(c+d*tan(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)^m/(d*tan(f*x + e) + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m/(c+d*tan(f*x+e))^2,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e) + a)^m/(d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) +
c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \tan{(e + fx)})^m}{(c + d \tan{(e + fx)})^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**m/(c+d*tan(f*x+e))**2,x)
[Out] Integral((a + b*tan(e + f*x))**m/(c + d*tan(e + f*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m/(c+d*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^m/(d*tan(f*x + e) + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \tan(e + f\,x))^m}{(c + d \tan(e + f\,x))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^m/(c + d*tan(e + f*x))^2,x)
[Out] int((a + b*tan(e + f*x))^m/(c + d*tan(e + f*x))^2, x)
```

3.1311 $\int \frac{(a+b\tan(e+fx))^m}{(c+d\tan(e+fx))^3}\,dx$

Optimal. Leaf size=455

$$\frac{{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a-ib}\right)(a+b\tan(e+fx))^{1+m}}{2(ia+b)(c-id)^3f(1+m)}+\frac{{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a+ib}\right)(a+b\tan(e$$

```
[Out] 1/2*hypergeom([1, 1+m],[2+m],(a+b*tan(f*x+e))/(a-I*b))*(a+b*tan(f*x+e))^(1+
m)/(I*a+b)/(c-I*d)^3/f/(1+m)+1/2*hypergeom([1, 1+m],[2+m],(a+b*tan(f*x+e))/
(a+I*b))*(a+b*tan(f*x+e))^(1+m)/(a+I*b)/(I*c-d)^3/f/(1+m)+1/2*d^2*(2*a^2*d^
2*(3*c^2-d^2)-4*a*b*c*d*(c^2*(3-m)-d^2*(1+m))-b^2*(d^4*(1-m)*m+2*c^2*d^2*(-
m^2+3*m+1)-c^4*(m^2-5*m+6)))*hypergeom([1, 1+m],[2+m],-d*(a+b*tan(f*x+e))/(
-a*d+b*c))*(a+b*tan(f*x+e))^(1+m)/(-a*d+b*c)^3/(c^2+d^2)^3/f/(1+m)+1/2*d^2*
(a+b*tan(f*x+e))^(1+m)/(-a*d+b*c)/(c^2+d^2)/f/(c+d*tan(f*x+e))^2-1/2*d^2*(4
*a*c*d-b*(d^2*(1-m)+c^2*(5-m)))*(a+b*tan(f*x+e))^(1+m)/(-a*d+b*c)^2/(c^2+d^
2)^2/f/(c+d*tan(f*x+e))
```

Rubi [A]

time = 1.06, antiderivative size = 455, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3650, 3730, 3734, 3620, 3618, 70, 3715}

$$\frac{d^2(2a^2d^2(3c^2-d^2)-4abcd(c^2(3-m)-d^2(m+1))-(b^2(-(c^4(m^2-5m+6))+2c^2d^2(-m^2+3m+1)+d^4(1-m)m)))(a+b\tan(e+fx))^{m+1}{}_2F_1\left(1,m+1;m+2;-\frac{d(a+b\tan(e+fx))}{bc-ad}\right)}{2f(m+1)(c^2+d^2)^3(bc-ad)^3}-\frac{d^2(4acd-bc^2(5-m)-bd^2(1-m))(a+b\tan(e+fx))^{m+1}}{2f(c^2+d^2)^2(bc-ad)^2(c+d\tan(e+fx))}+\frac{d^2(a+b\tan(e+fx))^{m+1}}{2f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))^2}+\frac{(a+b\tan(e+fx))^{m+1}{}_2F_1\left(1,m+1;m+2;\frac{a+b\tan(e+fx)}{a-ib}\right)}{2f(m+1)(b+ia)(c-id)^3}+\frac{(a+b\tan(e+fx))^{m+1}{}_2F_1\left(1,m+1;m+2;\frac{a+b\tan(e+fx)}{a+ib}\right)}{2f(m+1)(a+ib)(-d+ic)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^m/(c + d*Tan[e + f*x])^3,x]
```

```
[Out] (Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a - I*b)]*(a + b*
Tan[e + f*x])^(1 + m))/(2*(I*a + b)*(c - I*d)^3*f*(1 + m)) + (Hypergeometri
c2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a + I*b)]*(a + b*Tan[e + f*x])^
(1 + m))/(2*(a + I*b)*(I*c - d)^3*f*(1 + m)) + (d^2*(2*a^2*d^2*(3*c^2 - d^2
) - 4*a*b*c*d*(c^2*(3 - m) - d^2*(1 + m)) - b^2*(d^4*(1 - m)*m + 2*c^2*d^2*
(1 + 3*m - m^2) - c^4*(6 - 5*m + m^2)))*Hypergeometric2F1[1, 1 + m, 2 + m,
-((d*(a + b*Tan[e + f*x]))/(b*c - a*d))]*(a + b*Tan[e + f*x])^(1 + m))/(2*(
b*c - a*d)^3*(c^2 + d^2)^3*f*(1 + m)) + (d^2*(a + b*Tan[e + f*x])^(1 + m))/
(2*(b*c - a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^2) - (d^2*(4*a*c*d - b*d^
2*(1 - m) - b*c^2*(5 - m))*(a + b*Tan[e + f*x])^(1 + m))/(2*(b*c - a*d)^2*(
c^2 + d^2)^2*f*(c + d*Tan[e + f*x]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
```

```
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^m}{(c+d\tan(e+fx))^3}\,dx = \frac{d^2(a+b\tan(e+fx))^{1+m}}{2(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^2} + \frac{\int \frac{(a+b\tan(e+fx))^m\left(2c(bc-ad)+bd^2(1-m)\right.}{(c+d}}{2(bc-}$$

$$= \frac{d^2(a+b\tan(e+fx))^{1+m}}{2(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^2} - \frac{d^2(4acd-bd^2(1-m)-bc^2(5-}{2(bc-ad)^2\left(c^2+d^2\right)^2 f(}$$

$$= \frac{d^2(a+b\tan(e+fx))^{1+m}}{2(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^2} - \frac{d^2(4acd-bd^2(1-m)-bc^2(5-}{2(bc-ad)^2\left(c^2+d^2\right)^2 f(}$$

$$= \frac{d^2(a+b\tan(e+fx))^{1+m}}{2(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^2} - \frac{d^2(4acd-bd^2(1-m)-bc^2(5-}{2(bc-ad)^2\left(c^2+d^2\right)^2 f(}$$

$$= \frac{d^2(2a^2d^2(3c^2-d^2)-4abcd(c^2(3-m)-d^2(1+m))-b^2(d^4(1-m)m+2c^2d^2}{2(bc}$$

$$= -\frac{{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a-ib}\right)(a+b\tan(e+fx))^{1+m}}{2(a-ib)(ic+d)^3 f(1+m)} - \frac{{}_2F_1\left(1,1+m;\right.}{}$$

Mathematica [A]
time = 6.29, size = 670, normalized size = 1.47

$$-\frac{d^2(a+b\tan(e+fx))^{1+m}}{2(-bc+ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^2}-\frac{-\frac{\left(d^2\left(2c(bc-ad)+bd^2(1-m)\right)-c\left(-2d^2(bc-ad)-bcd^2(1-m)\right)\right)(a+b\tan(e+fx))^{1+m}}{(-bc+ad)(c^2+d^2)f(c+d\tan(e+fx))}-\frac{-\frac{\left(4c^2d^2(bc-ad)^2+bc^2d^2\left(4acd-bd^2(1-m)-bc^2(5-m)\right)m+d^2\left(-d^2(2ad-bc(3-m))(ad-bc(1+m))-\left(2bc^2-2acd+bd^2(1-m)\right)\left(acd-b\left(c^2-d^2m\right)\right)\right)\right){}_2F_1\left(1,1+m;2+m;\frac{d(a+b\tan(e+fx))}{-bc+ad}\right)(a+b\tan(e+fx))^{1+m}}{(-bc+ad)\left(c^2+d^2\right)f(1+m)}+\frac{\frac{i\left(2c(bc-ad)^2\left(c^2-3d^2\right)-2id(bc-ad)^2\left(3c^2-d^2\right)\right){}_2F_1\left(1,1+m;2+m;\frac{-ia-ib\tan(e+fx)}{-ia+b}\right)(a+b\tan(e+fx))^{1+m}}{2(a+ib)f(1+m)}-\frac{i\left(2c(bc-ad)^2\left(c^2-3d^2\right)+2id(bc-ad)^2\left(3c^2-d^2\right)\right){}_2F_1\left(1,1+m;2+m;-\frac{ia+ib\tan(e+fx)}{-ia-b}\right)(a+b\tan(e+fx))^{1+m}}{2(a-ib)f(1+m)}}{c^2+d^2}}{(-bc+ad)(c^2+d^2)}}{2(-bc+ad)\left(c^2+d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^m/(c + d*Tan[e + f*x])^3,x]
```

```
[Out] -1/2*(d^2*(a + b*Tan[e + f*x])^(1 + m))/((-(b*c) + a*d)*(c^2 + d^2)*f*(c +
d*Tan[e + f*x])^2) - (-(((d^2*(2*c*(b*c - a*d) + b*d^2*(1 - m)) - c*(-2*d^2
```

```
*(b*c - a*d) - b*c*d^2*(1 - m)))*(a + b*Tan[e + f*x])^(1 + m))/((-(b*c) + a
*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x]))) - (-(((4*c^2*d^2*(b*c - a*d)^2 + b
*c^2*d^2*(4*a*c*d - b*d^2*(1 - m) - b*c^2*(5 - m))*m + d^2*(-(d^2*(2*a*d -
b*c*(3 - m))*(a*d - b*c*(1 + m))) - (2*b*c^2 - 2*a*c*d + b*d^2*(1 - m))*(a*
c*d - b*(c^2 - d^2*m))))*Hypergeometric2F1[1, 1 + m, 2 + m, (d*(a + b*Tan[e
 + f*x]))/(-(b*c) + a*d)]*(a + b*Tan[e + f*x])^(1 + m))/((-(b*c) + a*d)*(c^
2 + d^2)*f*(1 + m))) + (((I/2)*(2*c*(b*c - a*d)^2*(c^2 - 3*d^2) - (2*I)*d*(
b*c - a*d)^2*(3*c^2 - d^2))*Hypergeometric2F1[1, 1 + m, 2 + m, ((-I)*a - I*
b*Tan[e + f*x])/((-I)*a + b)]*(a + b*Tan[e + f*x])^(1 + m))/((a + I*b)*f*(1
 + m)) - ((I/2)*(2*c*(b*c - a*d)^2*(c^2 - 3*d^2) + (2*I)*d*(b*c - a*d)^2*(3
*c^2 - d^2))*Hypergeometric2F1[1, 1 + m, 2 + m, -((I*a + I*b*Tan[e + f*x])/
((-I)*a - b))]*(a + b*Tan[e + f*x])^(1 + m))/((a - I*b)*f*(1 + m)))/(c^2 +
d^2))/((-(b*c) + a*d)*(c^2 + d^2)))/(2*(-(b*c) + a*d)*(c^2 + d^2))
```

Maple [F]
time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(fx+e))^m}{(c+d\tan(fx+e))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^m/(c+d*tan(f*x+e))^3,x)
[Out] int((a+b*tan(f*x+e))^m/(c+d*tan(f*x+e))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m/(c+d*tan(f*x+e))^3,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)^m/(d*tan(f*x + e) + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m/(c+d*tan(f*x+e))^3,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e) + a)^m/(d^3*tan(f*x + e)^3 + 3*c*d^2*tan(f*x + e)^
2 + 3*c^2*d*tan(f*x + e) + c^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(e+fx))^m}{(c+d\tan(e+fx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**m/(c+d*tan(f*x+e))**3,x)
[Out] Integral((a + b*tan(e + f*x))**m/(c + d*tan(e + f*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m/(c+d*tan(f*x+e))^3,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^m/(d*tan(f*x + e) + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\tan(e+f\,x))^m}{(c+d\tan(e+f\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^m/(c + d*tan(e + f*x))^3,x)
[Out] int((a + b*tan(e + f*x))^m/(c + d*tan(e + f*x))^3, x)
```

3.1312 $\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=283

$$\frac{(bc-ad)F_1\left(1+m;-\frac{3}{2},1;2+m;-\frac{d(a+b\tan(e+fx))}{bc-ad},\frac{a+b\tan(e+fx)}{a-ib}\right)(a+b\tan(e+fx))^{1+m}\sqrt{c+d\tan(e+fx)}}{2b(ia+b)f(1+m)\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}}$$

```
[Out] 1/2*(-a*d+b*c)*AppellF1(1+m,1,-3/2,2+m,(a+b*tan(f*x+e))/(a-I*b),-d*(a+b*tan
(f*x+e))/(-a*d+b*c))*(c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(1+m)/b/(I*a+b
)/f/(1+m)/(b*(c+d*tan(f*x+e))/(-a*d+b*c))^(1/2)-1/2*(-a*d+b*c)*AppellF1(1+m
,1,-3/2,2+m,(a+b*tan(f*x+e))/(a+I*b),-d*(a+b*tan(f*x+e))/(-a*d+b*c))*(c+d*t
an(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(1+m)/(I*a-b)/b/f/(1+m)/(b*(c+d*tan(f*x+e
))/(-a*d+b*c))^(1/2)
```

Rubi [A]

time = 0.26, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3656, 926, 142, 141}

$$\frac{(bc-ad)\sqrt{c+d\tan(e+fx)}\,(a+b\tan(e+fx))^{m+1}F_1\left(m+1;-\frac{3}{2},1;m+2;-\frac{d(a+b\tan(e+fx))}{bc-ad},\frac{a+b\tan(e+fx)}{a-ib}\right)}{2bf(m+1)(b+ia)\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}}-\frac{(bc-ad)\sqrt{c+d\tan(e+fx)}\,(a+b\tan(e+fx))^{m+1}F_1\left(m+1;-\frac{3}{2},1;m+2;-\frac{d(a+b\tan(e+fx))}{bc-ad},\frac{a+b\tan(e+fx)}{a+ib}\right)}{2bf(m+1)(-b+ia)\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((b*c - a*d)*AppellF1[1 + m, -3/2, 1, 2 + m, -((d*(a + b*Tan[e + f*x]))/(b*
c - a*d)), (a + b*Tan[e + f*x])/(a - I*b)]*(a + b*Tan[e + f*x])^(1 + m)*Sqr
t[c + d*Tan[e + f*x]])/(2*b*(I*a + b)*f*(1 + m)*Sqrt[(b*(c + d*Tan[e + f*x]
))/(b*c - a*d)]) - ((b*c - a*d)*AppellF1[1 + m, -3/2, 1, 2 + m, -((d*(a + b
*Tan[e + f*x]))/(b*c - a*d)), (a + b*Tan[e + f*x])/(a + I*b)]*(a + b*Tan[e
+ f*x])^(1 + m)*Sqrt[c + d*Tan[e + f*x]])/(2*(I*a - b)*b*f*(1 + m)*Sqrt[(b*
(c + d*Tan[e + f*x]))/(b*c - a*d)])
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
```

```
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^{3/2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{(a+bx)^m(c+dx)^{3/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{i(a+bx)^m(c+dx)^{3/2}}{2(i-x)} + \frac{i(a+bx)^m(c+dx)^{3/2}}{2(i+x)}\right)\,dx, x, \tan(\right.}{f}$$

$$= \frac{i\operatorname{Subst}\left(\int \frac{(a+bx)^m(c+dx)^{3/2}}{i-x}\,dx, x, \tan(e+fx)\right)}{2f} + \frac{i\operatorname{Subst}\left(\int\right.}{}$$

$$= \frac{\left(i(bc-ad)\sqrt{c+d\tan(e+fx)}\right)\operatorname{Subst}\left(\int \frac{(a+bx)^m\left(\frac{bc}{bc-ad}\right.}{i-x}\right.}{2bf\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}}$$

$$= \frac{(bc-ad)F_1\left(1+m; -\frac{3}{2}, 1; 2+m; -\frac{d(a+b\tan(e+fx))}{bc-ad}, \frac{a+b\tan}{a-}\right.}{2b(ia+b)f(1+m)\sqrt{\frac{b(c}{}}}$$

Mathematica [F]

time = 8.37, size = 0, normalized size = 0.00

$$\int (a + b\tan(e + fx))^m (c + d\tan(e + fx))^{3/2} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(3/2),x]
[Out] Integrate[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(3/2), x]
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \left(a + b\tan\left(fx + e\right)\right)^{m} \left(c + d\tan\left(fx + e\right)\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^(3/2),x)
[Out] int((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((d*tan(f*x + e) + c)^(3/2)*(b*tan(f*x + e) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral((d*tan(f*x + e) + c)^(3/2)*(b*tan(f*x + e) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan\left(e + fx\right)\right)^{m} \left(c + d\tan\left(e + fx\right)\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**m*(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**m*(c + d*tan(e + f*x))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\tan(e + f\,x))^m\,(c + d\tan(e + f\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^m*(c + d*tan(e + f*x))^(3/2),x)
```

```
[Out] int((a + b*tan(e + f*x))^m*(c + d*tan(e + f*x))^(3/2), x)
```

3.1313 $\int (a+b\tan(e+fx))^m \sqrt{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=261

$$\frac{F_1\left(1+m;-\frac{1}{2},1;2+m;-\frac{d(a+b\tan(e+fx))}{bc-ad},\frac{a+b\tan(e+fx)}{a-ib}\right)(a+b\tan(e+fx))^{1+m}\sqrt{c+d\tan(e+fx)}}{2(ia+b)f(1+m)\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}}-\frac{F_1\left(1\right.}{}$$

```
[Out] 1/2*AppellF1(1+m,1,-1/2,2+m,(a+b*tan(f*x+e))/(a-I*b),-d*(a+b*tan(f*x+e))/(-
a*d+b*c))*(c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(1+m)/(I*a+b)/f/(1+m)/(b*
(c+d*tan(f*x+e))/(-a*d+b*c))^(1/2)-1/2*AppellF1(1+m,1,-1/2,2+m,(a+b*tan(f*x
+e))/(a+I*b),-d*(a+b*tan(f*x+e))/(-a*d+b*c))*(c+d*tan(f*x+e))^(1/2)*(a+b*ta
n(f*x+e))^(1+m)/(I*a-b)/f/(1+m)/(b*(c+d*tan(f*x+e))/(-a*d+b*c))^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3656, 926, 142, 141}

$$\frac{\sqrt{c+d\tan(e+fx)}\,(a+b\tan(e+fx))^{m+1}F_1\left(m+1;-\frac{1}{2},1;m+2;-\frac{d(a+b\tan(e+fx))}{bc-ad},\frac{a+b\tan(e+fx)}{a-ib}\right)}{2f(m+1)(b+ia)\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}}-\frac{\sqrt{c+d\tan(e+fx)}\,(a+b\tan(e+fx))^{m+1}F_1\left(m+1;-\frac{1}{2},1;m+2;-\frac{d(a+b\tan(e+fx))}{bc-ad},\frac{a+b\tan(e+fx)}{a+ib}\right)}{2f(m+1)(-b+ia)\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^m*Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] (AppellF1[1 + m, -1/2, 1, 2 + m, -((d*(a + b*Tan[e + f*x]))/(b*c - a*d)), (
a + b*Tan[e + f*x])/(a - I*b)]*(a + b*Tan[e + f*x])^(1 + m)*Sqrt[c + d*Tan[
e + f*x]])/(2*(I*a + b)*f*(1 + m)*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c - a*d)
]) - (AppellF1[1 + m, -1/2, 1, 2 + m, -((d*(a + b*Tan[e + f*x]))/(b*c - a*d
)), (a + b*Tan[e + f*x])/(a + I*b)]*(a + b*Tan[e + f*x])^(1 + m)*Sqrt[c + d
*Tan[e + f*x]])/(2*(I*a - b)*f*(1 + m)*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c -
 a*d)])
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
```

```
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c
- a*d), 0] && !SimplerQ[c + d*x, a + b*x]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
& !IntegerQ[m] && !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int (a+b\tan(e+fx))^m\sqrt{c+d\tan(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx)^m\sqrt{c+dx}}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{i(a+bx)^m\sqrt{c+dx}}{2(i-x)}+\frac{i(a+bx)^m\sqrt{c+dx}}{2(i+x)}\right)dx,x,\text{t}\right.}{f}$$

$$= \frac{i\text{Subst}\left(\int \frac{(a+bx)^m\sqrt{c+dx}}{i-x}\,dx,x,\tan(e+fx)\right)}{2f}+\frac{i\text{Subst}\left(\right.}{}$$

$$= \frac{\left(i\sqrt{c+d\tan(e+fx)}\right)\text{Subst}\left(\int \frac{(a+bx)^m\sqrt{\frac{bc}{bc-ad}+\frac{b}{bc}}}{i-x}\right.}{2f\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}}$$

$$= \frac{F_1\left(1+m;-\frac{1}{2},1;2+m;-\frac{d(a+b\tan(e+fx))}{bc-ad},\frac{a+b\tan(e+fx)}{a-ib}\right)(a}{2(ia+b)f(1+m)\sqrt{\frac{b(c+d\text{t}}{bc}}}$$

Mathematica [F]
time = 0.81, size = 0, normalized size = 0.00

$$\int (a+b\tan(e+fx))^m\sqrt{c+d\tan(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Tan[e + f*x])^m*Sqrt[c + d*Tan[e + f*x]],x]
[Out] Integrate[(a + b*Tan[e + f*x])^m*Sqrt[c + d*Tan[e + f*x]], x]
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \sqrt{c+d\tan(fx+e)}\,(a+b\tan(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^m,x)
[Out] int((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^m,x, algorithm="maxima")
[Out] integrate(sqrt(d*tan(f*x + e) + c)*(b*tan(f*x + e) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^m,x, algorithm="fricas")
[Out] integral(sqrt(d*tan(f*x + e) + c)*(b*tan(f*x + e) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(e+fx))^m\sqrt{c+d\tan(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(a+b*tan(f*x+e))**m,x)
```

```
[Out] Integral((a + b*tan(e + f*x))**m*sqrt(c + d*tan(e + f*x)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^m,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\tan(e+f\,x))^m\,\sqrt{c+d\tan\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^m*(c + d*tan(e + f*x))^(1/2),x)
```

```
[Out] int((a + b*tan(e + f*x))^m*(c + d*tan(e + f*x))^(1/2), x)
```

3.1314 $\int \frac{(a+b\tan(e+fx))^m}{\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=261

$$\frac{F_1\left(1+m;\frac{1}{2},1;2+m;-\frac{d(a+b\tan(e+fx))}{bc-ad},\frac{a+b\tan(e+fx)}{a-ib}\right)(a+b\tan(e+fx))^{1+m}\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}}{2(ia+b)f(1+m)\sqrt{c+d\tan(e+fx)}}-\frac{F_1\left(\right.}{}$$

```
[Out] 1/2*AppellF1(1+m,1,1/2,2+m,(a+b*tan(f*x+e))/(a-I*b),-d*(a+b*tan(f*x+e))/(-a
*d+b*c))*(b*(c+d*tan(f*x+e))/(-a*d+b*c))^(1/2)*(a+b*tan(f*x+e))^(1+m)/(I*a+
b)/f/(1+m)/(c+d*tan(f*x+e))^(1/2)-1/2*AppellF1(1+m,1,1/2,2+m,(a+b*tan(f*x+e
))/(a+I*b),-d*(a+b*tan(f*x+e))/(-a*d+b*c))*(b*(c+d*tan(f*x+e))/(-a*d+b*c))^
(1/2)*(a+b*tan(f*x+e))^(1+m)/(I*a-b)/f/(1+m)/(c+d*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3656, 926, 142, 141}

$$\frac{(a+b\tan(e+fx))^{m+1}\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}\,F_1\left(m+1;\frac{1}{2},1;m+2;-\frac{d(a+b\tan(e+fx))}{bc-ad},\frac{a+b\tan(e+fx)}{a-ib}\right)}{2f(m+1)(b+ia)\sqrt{c+d\tan(e+fx)}}-\frac{(a+b\tan(e+fx))^{m+1}\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}\,F_1\left(m+1;\frac{1}{2},1;m+2;-\frac{d(a+b\tan(e+fx))}{bc-ad},\frac{a+b\tan(e+fx)}{a+ib}\right)}{2f(m+1)(-b+ia)\sqrt{c+d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^m/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] (AppellF1[1 + m, 1/2, 1, 2 + m, -((d*(a + b*Tan[e + f*x]))/(b*c - a*d)), (a
 + b*Tan[e + f*x])/(a - I*b)]*(a + b*Tan[e + f*x])^(1 + m)*Sqrt[(b*(c + d*T
an[e + f*x]))/(b*c - a*d)])/(2*(I*a + b)*f*(1 + m)*Sqrt[c + d*Tan[e + f*x]]
) - (AppellF1[1 + m, 1/2, 1, 2 + m, -((d*(a + b*Tan[e + f*x]))/(b*c - a*d))
, (a + b*Tan[e + f*x])/(a + I*b)]*(a + b*Tan[e + f*x])^(1 + m)*Sqrt[(b*(c +
 d*Tan[e + f*x]))/(b*c - a*d)])/(2*(I*a - b)*f*(1 + m)*Sqrt[c + d*Tan[e + f
*x]])
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
```

```
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c
- a*d), 0] && !SimplerQ[c + d*x, a + b*x]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
& !IntegerQ[m] && !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^m}{\sqrt{c+d\tan(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx)^m}{\sqrt{c+dx}\,(1+x^2)}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{i(a+bx)^m}{2(i-x)\sqrt{c+dx}}+\frac{i(a+bx)^m}{2(i+x)\sqrt{c+dx}}\right)dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{i\text{Subst}\left(\int \frac{(a+bx)^m}{(i-x)\sqrt{c+dx}}\,dx,x,\tan(e+fx)\right)}{2f} + \frac{i\text{Subst}\left(\int \frac{(a+bx)^m}{(i+x)\sqrt{c+dx}}\,dx,x,\ldots\right)}{2f}$$

$$= \frac{\left(i\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}\right)\text{Subst}\left(\int \frac{(a+bx)^m}{(i-x)\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}\,dx,x,\tan(e\ldots\right)}{2f\sqrt{c+d\tan(e+fx)}}$$

$$= \frac{F_1\left(1+m;\frac{1}{2},1;2+m;-\frac{d(a+b\tan(e+fx))}{bc-ad},\frac{a+b\tan(e+fx)}{a-ib}\right)(a+b\tan(e+fx))^{1+m}\sqrt{\ldots}}{2(ia+b)f(1+m)\sqrt{c+d\tan(e+fx)}}$$

Mathematica [F]

time = 5.45, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan(e + fx))^m}{\sqrt{c + d\tan(e + fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Tan[e + f*x])^m/Sqrt[c + d*Tan[e + f*x]],x]
[Out] Integrate[(a + b*Tan[e + f*x])^m/Sqrt[c + d*Tan[e + f*x]], x]
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan(fx + e))^m}{\sqrt{c + d\tan(fx + e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^m/(c+d*tan(f*x+e))^(1/2),x)
[Out] int((a+b*tan(f*x+e))^m/(c+d*tan(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m/(c+d*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)^m/sqrt(d*tan(f*x + e) + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m/(c+d*tan(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral((b*tan(f*x + e) + a)^m/sqrt(d*tan(f*x + e) + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan(e + fx))^m}{\sqrt{c + d\tan(e + fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**m/(c+d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**m/sqrt(c + d*tan(e + f*x)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m/(c+d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\tan(e+f\,x))^m}{\sqrt{c+d\tan\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^m/(c + d*tan(e + f*x))^(1/2),x)
```

```
[Out] int((a + b*tan(e + f*x))^m/(c + d*tan(e + f*x))^(1/2), x)
```

3.1315 $\int \frac{(a+b\tan(e+fx))^m}{(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=283

$$\frac{bF_1\left(1+m;\frac{3}{2},1;2+m;-\frac{d(a+b\tan(e+fx))}{bc-ad},\frac{a+b\tan(e+fx)}{a-ib}\right)(a+b\tan(e+fx))^{1+m}\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}}{2(ia+b)(bc-ad)f(1+m)\sqrt{c+d\tan(e+fx)}}-\frac{bF_1}{}$$

```
[Out] 1/2*b*AppellF1(1+m,1,3/2,2+m,(a+b*tan(f*x+e))/(a-I*b),-d*(a+b*tan(f*x+e))/(
-a*d+b*c))*(b*(c+d*tan(f*x+e))/(-a*d+b*c))^(1/2)*(a+b*tan(f*x+e))^(1+m)/(I*
a+b)/(-a*d+b*c)/f/(1+m)/(c+d*tan(f*x+e))^(1/2)-1/2*b*AppellF1(1+m,1,3/2,2+m
,(a+b*tan(f*x+e))/(a+I*b),-d*(a+b*tan(f*x+e))/(-a*d+b*c))*(b*(c+d*tan(f*x+e
))/(-a*d+b*c))^(1/2)*(a+b*tan(f*x+e))^(1+m)/(I*a-b)/(-a*d+b*c)/f/(1+m)/(c+d
*tan(f*x+e))^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3656, 926, 142, 141}

$$\frac{b(a+b\tan(e+fx))^{m+1}\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}\,F_1\left(m+1;\frac{3}{2},1;m+2;-\frac{d(a+b\tan(e+fx))}{bc-ad},\frac{a+b\tan(e+fx)}{a-ib}\right)}{2f(m+1)(b+ia)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{b(a+b\tan(e+fx))^{m+1}\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}\,F_1\left(m+1;\frac{3}{2},1;m+2;-\frac{d(a+b\tan(e+fx))}{bc-ad},\frac{a+b\tan(e+fx)}{a+ib}\right)}{2f(m+1)(-b+ia)(bc-ad)\sqrt{c+d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^m/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (b*AppellF1[1 + m, 3/2, 1, 2 + m, -((d*(a + b*Tan[e + f*x]))/(b*c - a*d)),
(a + b*Tan[e + f*x])/(a - I*b)]*(a + b*Tan[e + f*x])^(1 + m)*Sqrt[(b*(c + d
*Tan[e + f*x]))/(b*c - a*d)])/(2*(I*a + b)*(b*c - a*d)*f*(1 + m)*Sqrt[c + d
*Tan[e + f*x]]) - (b*AppellF1[1 + m, 3/2, 1, 2 + m, -((d*(a + b*Tan[e + f*x
]))/(b*c - a*d)), (a + b*Tan[e + f*x])/(a + I*b)]*(a + b*Tan[e + f*x])^(1 +
 m)*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c - a*d)])/(2*(I*a - b)*(b*c - a*d)*f*
(1 + m)*Sqrt[c + d*Tan[e + f*x]])
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
```

```
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c
- a*d), 0] && !SimplerQ[c + d*x, a + b*x]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
& !IntegerQ[m] && !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+b\tan(e+fx))^m}{(c+d\tan(e+fx))^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^m}{(c+dx)^{3/2}(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(\frac{i(a+bx)^m}{2(i-x)(c+dx)^{3/2}}+\frac{i(a+bx)^m}{2(i+x)(c+dx)^{3/2}}\right)dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{i\text{Subst}\left(\int \frac{(a+bx)^m}{(i-x)(c+dx)^{3/2}}\,dx, x, \tan(e+fx)\right)}{2f}+\frac{i\text{Subst}\left(\int \frac{(a+bx)^m}{(i+x)(c+dx)^{3/2}}\,dx, x, \tan\right.}{2f}\\
&= \frac{\left(ib\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}\right)\text{Subst}\left(\int \frac{(a+bx)^m}{(i-x)\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{3/2}}\,dx, x, \tan(e+fx\right.}{2(bc-ad)f\sqrt{c+d\tan(e+fx)}}\\
&= \frac{bF_1\left(1+m;\frac{3}{2},1;2+m;-\frac{d(a+b\tan(e+fx))}{bc-ad},\frac{a+b\tan(e+fx)}{a-ib}\right)(a+b\tan(e+fx))^{1+m}}{2(ia+b)(bc-ad)f(1+m)\sqrt{c+d\tan(e+fx)}}
\end{aligned}$$

Mathematica [F]
time = 9.84, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(e+fx))^m}{(c+d\tan(e+fx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Tan[e + f*x])^m/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] Integrate[(a + b*Tan[e + f*x])^m/(c + d*Tan[e + f*x])^(3/2), x]
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(fx+e))^m}{(c+d\tan(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^m/(c+d*tan(f*x+e))^(3/2),x)
```

```
[Out] int((a+b*tan(f*x+e))^m/(c+d*tan(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m/(c+d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^m/(d*tan(f*x + e) + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m/(c+d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(f*x + e) + c)*(b*tan(f*x + e) + a)^m/(d^2*tan(f*x + e)^
2 + 2*c*d*tan(f*x + e) + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(e+fx))^m}{(c+d\tan(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**m/(c+d*tan(f*x+e))**(3/2),x)
```

[Out] Integral((a + b*tan(e + f*x))**m/(c + d*tan(e + f*x))**(3/2), x)

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m/(c+d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\tan(e+f\,x))^m}{(c+d\tan\left(e+f\,x\right))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^m/(c + d*tan(e + f*x))^(3/2),x)
```

```
[Out] int((a + b*tan(e + f*x))^m/(c + d*tan(e + f*x))^(3/2), x)
```

3.1316 $\int \frac{(a+b\tan(e+fx))^m}{(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=287

$$\frac{b^2F_1\left(1+m;\frac{5}{2},1;2+m;-\frac{d(a+b\tan(e+fx))}{bc-ad},\frac{a+b\tan(e+fx)}{a-ib}\right)(a+b\tan(e+fx))^{1+m}\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}}{2(ia+b)(bc-ad)^2f(1+m)\sqrt{c+d\tan(e+fx)}}-\frac{b^2\ldots}{\ldots}$$

```
[Out] 1/2*b^2*AppellF1(1+m,1,5/2,2+m,(a+b*tan(f*x+e))/(a-I*b),-d*(a+b*tan(f*x+e))
/(-a*d+b*c))*(b*(c+d*tan(f*x+e))/(-a*d+b*c))^(1/2)*(a+b*tan(f*x+e))^(1+m)/(
I*a+b)/(-a*d+b*c)^2/f/(1+m)/(c+d*tan(f*x+e))^(1/2)-1/2*b^2*AppellF1(1+m,1,5
/2,2+m,(a+b*tan(f*x+e))/(a+I*b),-d*(a+b*tan(f*x+e))/(-a*d+b*c))*(b*(c+d*tan
(f*x+e))/(-a*d+b*c))^(1/2)*(a+b*tan(f*x+e))^(1+m)/(I*a-b)/(-a*d+b*c)^2/f/(1
+m)/(c+d*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3656, 926, 142, 141}

$$\frac{b^2(a+b\tan(e+fx))^{m+1}\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}F_1\left(m+1;\frac{5}{2},1;m+2;-\frac{d(a+b\tan(e+fx))}{bc-ad},\frac{a+b\tan(e+fx)}{a-ib}\right)}{2f(m+1)(b+ia)(bc-ad)^2\sqrt{c+d\tan(e+fx)}}-\frac{b^2(a+b\tan(e+fx))^{m+1}\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}F_1\left(m+1;\frac{5}{2},1;m+2;-\frac{d(a+b\tan(e+fx))}{bc-ad},\frac{a+b\tan(e+fx)}{a+ib}\right)}{2f(m+1)(-b+ia)(bc-ad)^2\sqrt{c+d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^m/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (b^2*AppellF1[1 + m, 5/2, 1, 2 + m, -((d*(a + b*Tan[e + f*x]))/(b*c - a*d))
, (a + b*Tan[e + f*x])/(a - I*b)]*(a + b*Tan[e + f*x])^(1 + m)*Sqrt[(b*(c +
 d*Tan[e + f*x]))/(b*c - a*d)])/(2*(I*a + b)*(b*c - a*d)^2*f*(1 + m)*Sqrt[c
 + d*Tan[e + f*x]]) - (b^2*AppellF1[1 + m, 5/2, 1, 2 + m, -((d*(a + b*Tan[e
 + f*x]))/(b*c - a*d)), (a + b*Tan[e + f*x])/(a + I*b)]*(a + b*Tan[e + f*x]
)^(1 + m)*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c - a*d)])/(2*(I*a - b)*(b*c - a
*d)^2*f*(1 + m)*Sqrt[c + d*Tan[e + f*x]])
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
```

```
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c
- a*d), 0] && !SimplerQ[c + d*x, a + b*x]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
& !IntegerQ[m] && !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^m}{(c+d\tan(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx)^m}{(c+dx)^{5/2}(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{i(a+bx)^m}{2(i-x)(c+dx)^{5/2}}+\frac{i(a+bx)^m}{2(i+x)(c+dx)^{5/2}}\right)dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{i\text{Subst}\left(\int \frac{(a+bx)^m}{(i-x)(c+dx)^{5/2}}\,dx, x, \tan(e+fx)\right)}{2f} + \frac{i\text{Subst}\left(\int \frac{(a+bx)^m}{(i+x)(c+dx)^{5/2}}\,dx, x, \tan\right.}{2f}$$

$$= \frac{\left(ib^2\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}\right)\text{Subst}\left(\int \frac{(a+bx)^m}{(i-x)\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{5/2}}\,dx, x, \tan(e+f\right.}{2(bc-ad)^2 f\sqrt{c+d\tan(e+fx)}}$$

$$= \frac{b^2F_1\left(1+m;\frac{5}{2},1;2+m;-\frac{d(a+b\tan(e+fx))}{bc-ad},\frac{a+b\tan(e+fx)}{a-ib}\right)(a+b\tan(e+fx))^{1+}}{2(ia+b)(bc-ad)^2f(1+m)\sqrt{c+d\tan(e+fx)}}$$

Mathematica [F]
time = 20.86, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(e+fx))^m}{(c+d\tan(e+fx))^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Tan[e + f*x])^m/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] Integrate[(a + b*Tan[e + f*x])^m/(c + d*Tan[e + f*x])^(5/2), x]
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(fx+e))^m}{(c+d\tan(fx+e))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^m/(c+d*tan(f*x+e))^(5/2),x)
```

```
[Out] int((a+b*tan(f*x+e))^m/(c+d*tan(f*x+e))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m/(c+d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^m/(d*tan(f*x + e) + c)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m/(c+d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(f*x + e) + c)*(b*tan(f*x + e) + a)^m/(d^3*tan(f*x + e)^
3 + 3*c*d^2*tan(f*x + e)^2 + 3*c^2*d*tan(f*x + e) + c^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**m/(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m/(c+d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\tan(e+f\,x))^m}{(c+d\tan\left(e+f\,x\right))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^m/(c + d*tan(e + f*x))^(5/2),x)
```

```
[Out] int((a + b*tan(e + f*x))^m/(c + d*tan(e + f*x))^(5/2), x)
```

3.1317 $\int (c(d\tan(e+fx))^p)^n (a+ia\tan(e+fx))^m dx$

Optimal. Leaf size=99

$$\frac{F_1(1+np;1-m,1;2+np;-i\tan(e+fx),i\tan(e+fx))(1+i\tan(e+fx))^{-m}\tan(e+fx)\,(c(d\tan(e+f}{f(1+np)}$$

```
[Out] AppellF1(n*p+1,1-m,1,n*p+2,-I*tan(f*x+e),I*tan(f*x+e))*tan(f*x+e)*(c*(d*tan
(f*x+e))^p)^n*(a+I*a*tan(f*x+e))^m/f/(n*p+1)/((1+I*tan(f*x+e))^m)
```

Rubi [A]
time = 0.12, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3659, 3645, 140, 138}

$$\frac{\tan(e+fx)(1+i\tan(e+fx))^{-m}(a+ia\tan(e+fx))^m F_1(np+1;1-m,1;np+2;-i\tan(e+fx),i\tan(e+fx))\,(c(d\tan(e+fx))^p)^n}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Tan[e + f*x])^p)^n*(a + I*a*Tan[e + f*x])^m,x]
```

```
[Out] (AppellF1[1 + n*p, 1 - m, 1, 2 + n*p, (-I)*Tan[e + f*x], I*Tan[e + f*x]]*Ta
n[e + f*x]*(c*(d*Tan[e + f*x])^p)^n*(a + I*a*Tan[e + f*x])^m)/(f*(1 + n*p)*
(1 + I*Tan[e + f*x])^m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
 + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rule 3659

```
Int[((c_.)*((d_.)*tan[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Tan[e + f*x
])^p)^FracPart[n]/(d*Tan[e + f*x])^(p*FracPart[n])), Int[(a + b*Tan[e + f*x
])^m*(d*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && !IntegerQ[n] && !IntegerQ[m]
```

Rubi steps

$$\int (c(d\tan(e+fx))^p)^n\,(a+ia\tan(e+fx))^m\,dx = \left((d\tan(e+fx))^{-np}\,(c(d\tan(e+fx))^p)^n\right)\int (d\tan(e+f$$

$$= \frac{(ia^2(d\tan(e+fx))^{-np}\,(c(d\tan(e+fx))^p)^n)\,\mathrm{Subst}\Big(\int \frac{(-}{}}{f}$$

$$= \frac{(ia(1+i\tan(e+fx))^{-m}(d\tan(e+fx))^{-np}\,(c(d\tan(e+}{}$$

$$= \frac{F_1(1+np;1-m,1;2+np;-i\tan(e+fx),i\tan(e+fx}{}$$

Mathematica [F]
time = 5.74, size = 0, normalized size = 0.00

$$\int (c(d\tan(e+fx))^p)^n\,(a+ia\tan(e+fx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c*(d*Tan[e + f*x])^p)^n*(a + I*a*Tan[e + f*x])^m,x]
[Out] Integrate[(c*(d*Tan[e + f*x])^p)^n*(a + I*a*Tan[e + f*x])^m, x]
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int (c(d\tan(fx+e))^p)^n\,(a+ia\tan(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*tan(f*x+e))^p)^n*(a+I*a*tan(f*x+e))^m,x)
[Out] int((c*(d*tan(f*x+e))^p)^n*(a+I*a*tan(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))^p)^n*(a+I*a*tan(f*x+e))^m,x, algorithm="maxima"
)
```

```
[Out] integrate(((d*tan(f*x + e))^p*c)^n*(I*a*tan(f*x + e) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))^p)^n*(a+I*a*tan(f*x+e))^m,x, algorithm="fricas"
)
```

```
[Out] integral((2*a*e^(2*I*f*x + 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1))^m*e^(n*p*log((
-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)) + n*log(c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c(d\tan(e+fx))^p)^n (ia(\tan(e+fx)-i))^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))**p)**n*(a+I*a*tan(f*x+e))**m,x)
```

```
[Out] Integral((c*(d*tan(e + f*x))**p)**n*(I*a*(tan(e + f*x) - I))**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))^p)^n*(a+I*a*tan(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate(((d*tan(f*x + e))^p*c)^n*(I*a*tan(f*x + e) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c\left(d\tan(e+f\,x)\right)^p\right)^n\,\left(a+a\tan(e+f\,x)\,1\mathrm{i}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*tan(e + f*x))^p)^n*(a + a*tan(e + f*x)*1i)^m,x)
[Out] int((c*(d*tan(e + f*x))^p)^n*(a + a*tan(e + f*x)*1i)^m, x)
```

3.1318 $\int (c(d\tan(e+fx))^p)^n (a+ia\tan(e+fx))^3 \, dx$

Optimal. Leaf size=132

$$-\frac{3a^3\tan(e+fx)\left(c(d\tan(e+fx))^p\right)^n}{f(1+np)}+\frac{4a^3\,{}_2F_1(1,1+np;2+np;i\tan(e+fx))\tan(e+fx)\left(c(d\tan(e+fx))^p\right)^n}{f(1+np)}$$

[Out] -3*a^3*tan(f*x+e)*(c*(d*tan(f*x+e))^p)^n/f/(n*p+1)+4*a^3*hypergeom([1, n*p+1],[n*p+2],I*tan(f*x+e))*tan(f*x+e)*(c*(d*tan(f*x+e))^p)^n/f/(n*p+1)-I*a^3*tan(f*x+e)^2*(c*(d*tan(f*x+e))^p)^n/f/(n*p+2)

Rubi [A]

time = 0.17, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1600, 1970, 90, 66, 45}

$$\frac{4a^3\tan(e+fx)\,{}_2F_1(1,np+1;np+2;i\tan(e+fx))\left(c(d\tan(e+fx))^p\right)^n}{f(np+1)}-\frac{ia^3\tan^2(e+fx)\left(c(d\tan(e+fx))^p\right)^n}{f(np+2)}-\frac{3a^3\tan(e+fx)\left(c(d\tan(e+fx))^p\right)^n}{f(np+1)}$$

Antiderivative was successfully verified.

[In] Int[(c*(d*Tan[e + f*x])^p)^n*(a + I*a*Tan[e + f*x])^3,x]

[Out] (-3*a^3*Tan[e + f*x]*(c*(d*Tan[e + f*x])^p)^n)/(f*(1 + n*p)) + (4*a^3*Hypergeometric2F1[1, 1 + n*p, 2 + n*p, I*Tan[e + f*x]]*Tan[e + f*x]*(c*(d*Tan[e + f*x])^p)^n)/(f*(1 + n*p)) - (I*a^3*Tan[e + f*x]^2*(c*(d*Tan[e + f*x])^p)^n)/(f*(2 + n*p))

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1970

```
Int[(u_.)*((c_.)*((d_)*((a_.) + (b_.)*(x_)))^(q_))^(p_), x_Symbol] :> Dist[
(c*(d*(a + b*x))^q)^p/(a + b*x)^(p*q), Int[u*(a + b*x)^(p*q), x], x] /; Fre
eQ[{a, b, c, d, q, p}, x] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

Rubi steps

$$\int (c(d\tan(e+fx))^p)^n (a+ia\tan(e+fx))^3\,dx = \frac{\text{Subst}\left(\int \frac{(c(dx)^p)^n(a+iax)^3}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(c(dx)^p)^n(a+iax)^2}{\frac{1}{a}-\frac{ix}{a}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left((d\tan(e+fx))^{-np}(c(d\tan(e+fx))^p)^n\right)\text{Subst}\left(\int \frac{(dx)^{np}(a \ldots}{\frac{1}{a}- \ldots}\right.}{f}$$

$$= \frac{\left((d\tan(e+fx))^{-np}(c(d\tan(e+fx))^p)^n\right)\text{Subst}\left(\int \left(-2a^3 \ldots\right.\right.}{\ldots}$$

$$= -\frac{2a^3\tan(e+fx)(c(d\tan(e+fx))^p)^n}{f(1+np)} - \frac{(a^2(d\tan(e+f \ldots}{\ldots}$$

$$= -\frac{2a^3\tan(e+fx)(c(d\tan(e+fx))^p)^n}{f(1+np)} + \frac{4a^3\,{}_2F_1(1,1+n \ldots}{\ldots}$$

$$= -\frac{3a^3\tan(e+fx)(c(d\tan(e+fx))^p)^n}{f(1+np)} + \frac{4a^3\,{}_2F_1(1,1+n \ldots}{\ldots}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 981 vs. $2(132) = 264$.
time = 8.80, size = 981, normalized size = 7.43

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*(d*Tan[e + f*x])^p)^n*(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] (Cos[e + f*x]^3*((Sec[e + f*x]^2*((-I)*Cos[3*e] - Sin[3*e]))/(2 + n*p) + ((
-3 - 2*n*p + Cos[2*e])*Sec[e]^2*((-1/2*I)*Cos[3*e] - Sin[3*e]/2))/((1 + n*p
)*(2 + n*p)) + ((-Cos[e - f*x] + Cos[e + f*x])*Sec[e]^2*Sec[e + f*x]*((-1/2
*I)*Cos[3*e] - Sin[3*e]/2))/(1 + n*p))*(c*(d*Tan[e + f*x])^p)^n*(a + I*a*Ta
n[e + f*x])^3)/(f*(Cos[f*x] + I*Sin[f*x])^3) + (Cos[e + f*x]^3*((Sec[e]^2*(
-1 + Cos[2*e] + (3*I)*Sin[2*e])*((I/2)*Cos[3*e] + Sin[3*e]/2))/(1 + n*p) +
(Sec[e]^2*Sec[e + f*x]*((I/2)*Cos[3*e] + Sin[3*e]/2)*(-Cos[e - f*x] + Cos[e
 + f*x] - (3*I)*Sin[e - f*x] + (3*I)*Sin[e + f*x]))/(1 + n*p))*(c*(d*Tan[e
+ f*x])^p)^n*(a + I*a*Tan[e + f*x])^3)/(f*(Cos[f*x] + I*Sin[f*x])^3) + (I*2
^(2 - n*p)*(((-I)*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x))))^(n
*p)*Cos[e + f*x]^3*(2^(n*p)*Hypergeometric2F1[1, n*p, 1 + n*p, -((-1 + E^((
2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f*x))))] - (1 + E^((2*I)*(e + f*x)))^(n
*p)*Hypergeometric2F1[n*p, n*p, 1 + n*p, (1 - E^((2*I)*(e + f*x)))/2])*(c*(
d*Tan[e + f*x])^p)^n*(a + I*a*Tan[e + f*x])^3)/((E^(I*e) + E^((3*I)*e))*f*n
*p*(Cos[f*x] + I*Sin[f*x])^3*Tan[e + f*x]^(n*p)) - ((4*I)*(-1 + E^((2*I)*(e
 + f*x)))^(n*p)*(((-I)*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x))
))^(n*p)*Cos[e + f*x]^3*(-(Hypergeometric2F1[1, n*p, 1 + n*p, (1 - E^((2*I)
*(e + f*x)))/(1 + E^((2*I)*(e + f*x)))]/((1 + E^((2*I)*(e + f*x)))^(n*p)*n*
p)) - ((1 + E^((2*I)*e))*(-1 + E^((2*I)*(e + f*x)))*(1 + E^((2*I)*(e + f*x)
))^(-1 - n*p)*Hypergeometric2F1[1, 1 + n*p, 2 + n*p, (1 - E^((2*I)*(e + f*x
)))/(1 + E^((2*I)*(e + f*x)))])/(1 + n*p) + Hypergeometric2F1[n*p, n*p, 1 +
 n*p, (1 - E^((2*I)*(e + f*x)))/2]/(2^(n*p)*n*p))*(c*(d*Tan[e + f*x])^p)^n*
(a + I*a*Tan[e + f*x])^3)/(E^((3*I)*e)*(1 + E^((2*I)*e))*((-1 + E^((2*I)*(e
 + f*x)))/(1 + E^((2*I)*(e + f*x))))^(n*p)*f*(Cos[f*x] + I*Sin[f*x])^3*Tan[
e + f*x]^(n*p))
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \left(c (d \tan (f x+e))^{p}\right)^{n} \left(a+i a \tan (f x+e)\right)^{3} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*tan(f*x+e))^p)^n*(a+I*a*tan(f*x+e))^3,x)
```

```
[Out] int((c*(d*tan(f*x+e))^p)^n*(a+I*a*tan(f*x+e))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))^p)^n*(a+I*a*tan(f*x+e))^3,x, algorithm="maxima"
)
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^3*((d*tan(f*x + e))^p*c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))^p)^n*(a+I*a*tan(f*x+e))^3,x, algorithm="fricas"
)
```

```
[Out] integral(8*a^3*e^(n*p*log((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*
I*e) + 1)) + 6*I*f*x + n*log(c) + 6*I*e)/(e^(6*I*f*x + 6*I*e) + 3*e^(4*I*f*
x + 4*I*e) + 3*e^(2*I*f*x + 2*I*e) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia^3\left(\int i(c(d\tan(e+fx))^p)^n\,dx+\int\left(-3(c(d\tan(e+fx))^p)^n\tan(e+fx)\right)dx+\int(c(d\tan(e+fx))^p)^n\tan^3(e+fx)\,dx+\int\left(-3i(c(d\tan(e+fx))^p)^n\tan^2(e+fx)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))**p)**n*(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] -I*a**3*(Integral(I*(c*(d*tan(e + f*x))**p)**n, x) + Integral(-3*(c*(d*tan(
e + f*x))**p)**n*tan(e + f*x), x) + Integral((c*(d*tan(e + f*x))**p)**n*tan
(e + f*x)**3, x) + Integral(-3*I*(c*(d*tan(e + f*x))**p)**n*tan(e + f*x)**2
, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))^p)^n*(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^3*((d*tan(f*x + e))^p*c)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,{\left(d\,\tan(e+f\,x)\right)}^p\right)}^n\,{\left(a+a\,\tan(e+f\,x)\,1\mathrm{i}\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*tan(e + f*x))^p)^n*(a + a*tan(e + f*x)*1i)^3,x)
```

```
[Out] int((c*(d*tan(e + f*x))^p)^n*(a + a*tan(e + f*x)*1i)^3, x)
```

3.1319 $\int (c(d\tan(e+fx))^p)^n (a+ia\tan(e+fx))^2 \, dx$

Optimal. Leaf size=93

$$-\frac{a^2\tan(e+fx)\,(c(d\tan(e+fx))^p)^n}{f(1+np)}+\frac{2a^2\,{}_2F_1(1,1+np;2+np;i\tan(e+fx))\tan(e+fx)\,(c(d\tan(e+f$$

```
[Out] -a^2*tan(f*x+e)*(c*(d*tan(f*x+e))^p)^n/f/(n*p+1)+2*a^2*hypergeom([1, n*p+1]
,[n*p+2],I*tan(f*x+e))*tan(f*x+e)*(c*(d*tan(f*x+e))^p)^n/f/(n*p+1)
```

Rubi [A]
time = 0.11, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1600, 1970, 81, 66}

$$-\frac{a^2\tan(e+fx)\,(c(d\tan(e+fx))^p)^n}{f(np+1)}+\frac{2a^2\tan(e+fx)\,{}_2F_1(1,np+1;np+2;i\tan(e+fx))\,(c(d\tan(e+fx))^p)^n}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Tan[e + f*x])^p)^n*(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] -((a^2*Tan[e + f*x]*(c*(d*Tan[e + f*x])^p)^n)/(f*(1 + n*p))) + (2*a^2*Hyper
geometric2F1[1, 1 + n*p, 2 + n*p, I*Tan[e + f*x]]*Tan[e + f*x]*(c*(d*Tan[e
+ f*x])^p)^n)/(f*(1 + n*p))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1970

```
Int[(u_.)*((c_.)*((d_)*((a_.) + (b_.)*(x_)))^(q_))^(p_), x_Symbol] :> Dist[
(c*(d*(a + b*x))^q)^p/(a + b*x)^(p*q), Int[u*(a + b*x)^(p*q), x], x] /; Fre
eQ[{a, b, c, d, q, p}, x] && !IntegerQ[q] && !IntegerQ[p]
```

Rubi steps

$$\int (c(d\tan(e+fx))^p)^n (a+ia\tan(e+fx))^2\,dx = \frac{\text{Subst}\left(\int \frac{(c(dx)^p)^n(a+iax)^2}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(c(dx)^p)^n(a+iax)}{\frac{1}{a}-\frac{ix}{a}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left((d\tan(e+fx))^{-np}(c(d\tan(e+fx))^p)^n\right)\text{Subst}\left(\int \frac{(dx)^{np}(a}{\frac{1}{a}-}\right.}{f}$$

$$= -\frac{a^2\tan(e+fx)(c(d\tan(e+fx))^p)^n}{f(1+np)} + \frac{(2a(d\tan(e+fx}{}$$

$$= -\frac{a^2\tan(e+fx)(c(d\tan(e+fx))^p)^n}{f(1+np)} + \frac{2a^2\,{}_2F_1(1,1+np}{}$$

Mathematica [F]
time = 2.93, size = 0, normalized size = 0.00

$$\int (c(d\tan(e+fx))^p)^n (a+ia\tan(e+fx))^2\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c*(d*Tan[e + f*x])^p)^n*(a + I*a*Tan[e + f*x])^2,x]
[Out] Integrate[(c*(d*Tan[e + f*x])^p)^n*(a + I*a*Tan[e + f*x])^2, x]
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int (c(d\tan(fx+e))^p)^n (a+ia\tan(fx+e))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*tan(f*x+e))^p)^n*(a+I*a*tan(f*x+e))^2,x)
[Out] int((c*(d*tan(f*x+e))^p)^n*(a+I*a*tan(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))^p)^n*(a+I*a*tan(f*x+e))^2,x, algorithm="maxima"
)
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^2*((d*tan(f*x + e))^p*c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))^p)^n*(a+I*a*tan(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] integral(4*a^2*e^(n*p*log((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*
I*e) + 1)) + 4*I*f*x + n*log(c) + 4*I*e)/(e^(4*I*f*x + 4*I*e) + 2*e^(2*I*f*
x + 2*I*e) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^{2}\left(\int\left(-\left(c\left(d\tan\left(e+fx\right)\right)^{p}\right)^{n}\right)dx+\int\left(c\left(d\tan\left(e+fx\right)\right)^{p}\right)^{n}\tan^{2}\left(e+fx\right)dx+\int\left(-2i\left(c\left(d\tan\left(e+fx\right)\right)^{p}\right)^{n}\tan\left(e+fx\right)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))**p)**n*(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -a**2*(Integral(-(c*(d*tan(e + f*x))**p)**n, x) + Integral((c*(d*tan(e + f*
x))**p)**n*tan(e + f*x)**2, x) + Integral(-2*I*(c*(d*tan(e + f*x))**p)**n*t
an(e + f*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))^p)^n*(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^2*((d*tan(f*x + e))^p*c)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,(d\tan(e+f\,x))^p)^n\,(a+a\tan(e+f\,x)\ 1\mathrm{i})^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*tan(e + f*x))^p)^n*(a + a*tan(e + f*x)*1i)^2,x)
[Out] int((c*(d*tan(e + f*x))^p)^n*(a + a*tan(e + f*x)*1i)^2, x)
```

3.1320 $\int (c(d\tan(e+fx))^p)^n (a+ia\tan(e+fx))\,dx$

Optimal. Leaf size=54

$$\frac{a\,{}_2F_1(1,1+np;2+np;i\tan(e+fx))\tan(e+fx)\left(c(d\tan(e+fx))^p\right)^n}{f(1+np)}$$

```
[Out] a*hypergeom([1, n*p+1],[n*p+2],I*tan(f*x+e))*tan(f*x+e)*(c*(d*tan(f*x+e))^p
)^n/f/(n*p+1)
```

Rubi [A]
time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {12, 1970, 66}

$$\frac{a\tan(e+fx)\,{}_2F_1(1,np+1;np+2;i\tan(e+fx))\left(c(d\tan(e+fx))^p\right)^n}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Tan[e + f*x])^p)^n*(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (a*Hypergeometric2F1[1, 1 + n*p, 2 + n*p, I*Tan[e + f*x]]*Tan[e + f*x]*(c*(
d*Tan[e + f*x])^p)^n)/(f*(1 + n*p))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 1970

```
Int[(u_.)*((c_.)*((d_)*((a_.) + (b_.)*(x_)))^(q_))^(p_), x_Symbol] :> Dist[
(c*(d*(a + b*x))^q)^p/(a + b*x)^(p*q), Int[u*(a + b*x)^(p*q), x], x] /; Fre
eQ[{a, b, c, d, q, p}, x] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

Rubi steps

$$\int (c(d\tan(e+fx))^p)^n\,(a+ia\tan(e+fx))\,dx = \frac{i\text{Subst}\left(\int \frac{a(c(dx)^p)^n}{i+x}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{(ia)\text{Subst}\left(\int \frac{(c(dx)^p)^n}{i+x}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\left(ia(d\tan(e+fx))^{-np}\,(c(d\tan(e+fx))^p)^n\right)\text{Subst}\left(\int \frac{(dx)^{np}}{i+x}\right.}{f}$$
$$= \frac{a\,{}_2F_1(1,1+np;2+np;i\tan(e+fx))\tan(e+fx)\,(c(d\tan}{f(1+np)}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 173 vs. 2(54) = 108.
time = 0.91, size = 173, normalized size = 3.20

$$\frac{2^{-1-np}ae^{-ie}\left(-\frac{i(-1+e^{2i(e+fx)})}{1+e^{2i(e+fx)}}\right)^{1+np}\left(1+e^{2i(e+fx)}\right)^{1+np}\cos(e+fx)\,{}_2F_1\left(1+np,1+np;2+np;\frac{1}{2}\left(1-e^{2i(e+fx)}\right)\right)(\cos(fx)-i\sin(fx))(1+i\tan(e+fx))\tan^{-np}(e+fx)\,(c(d\tan(e+fx))^p)^n}{f+fnp}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Tan[e + f*x])^p)^n*(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (2^(-1 - n*p)*a*(((-I)*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x))
))^(1 + n*p)*(1 + E^((2*I)*(e + f*x)))^(1 + n*p)*Cos[e + f*x]*Hypergeometri
c2F1[1 + n*p, 1 + n*p, 2 + n*p, (1 - E^((2*I)*(e + f*x)))/2]*(Cos[f*x] - I*
Sin[f*x])*(1 + I*Tan[e + f*x])*(c*(d*Tan[e + f*x])^p)^n)/(E^(I*e)*(f + f*n*
p)*Tan[e + f*x]^(n*p))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(c\left(d\tan\left(fx+e\right)\right)^{p}\right)^{n}\left(a+ia\tan\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*tan(f*x+e))^p)^n*(a+I*a*tan(f*x+e)),x)
```

```
[Out] int((c*(d*tan(f*x+e))^p)^n*(a+I*a*tan(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*tan(f*x+e))^p)^n*(a+I*a*tan(f*x+e)),x, algorithm="maxima")

[Out] integrate((I*a*tan(f*x + e) + a)*((d*tan(f*x + e))^p*c)^n, x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*tan(f*x+e))^p)^n*(a+I*a*tan(f*x+e)),x, algorithm="fricas")

[Out] integral(2*a*e^(n*p*log((-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)) + 2*I*f*x + n*log(c) + 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-i(c(d\tan\left(e+fx\right))^{p})^{n}\right)dx+\int(c(d\tan\left(e+fx\right))^{p})^{n}\tan\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*tan(f*x+e))**p)**n*(a+I*a*tan(f*x+e)),x)

[Out] I*a*(Integral(-I*(c*(d*tan(e + f*x))**p)**n, x) + Integral((c*(d*tan(e + f*x))**p)**n*tan(e + f*x), x))

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*tan(f*x+e))^p)^n*(a+I*a*tan(f*x+e)),x, algorithm="giac")

[Out] integrate((I*a*tan(f*x + e) + a)*((d*tan(f*x + e))^p*c)^n, x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int{\left(c\,{\left(d\,\mathrm{tan}\left(e+f\,x\right)\right)}^p\right)}^n\,\left(a+a\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(d*tan(e + f*x))^p)^n*(a + a*tan(e + f*x)*1i),x)

[Out] int((c*(d*tan(e + f*x))^p)^n*(a + a*tan(e + f*x)*1i), x)

3.1321 $\int \frac{(c(d\tan(e+fx))^p)^n}{a+ia\tan(e+fx)}\,dx$

Optimal. Leaf size=134

$$\frac{{}_2F_1\left(2,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(e+fx)\right)\tan(e+fx)\left(c(d\tan(e+fx))^p\right)^n}{af(1+np)}-\frac{i\,{}_2F_1\left(2,\frac{1}{2}(2+np);\frac{1}{2}(4+np\right.}{}$$

```
[Out] hypergeom([2, 1/2*n*p+1/2],[1/2*n*p+3/2],-tan(f*x+e)^2)*tan(f*x+e)*(c*(d*ta
n(f*x+e))^p)^n/a/f/(n*p+1)-I*hypergeom([2, 1/2*n*p+1],[1/2*n*p+2],-tan(f*x+
e)^2)*tan(f*x+e)^2*(c*(d*tan(f*x+e))^p)^n/a/f/(n*p+2)
```

Rubi [A]
time = 0.15, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1970, 862, 83, 74, 371}

$$\frac{\tan(e+fx)\,{}_2F_1\left(2,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(e+fx)\right)\left(c(d\tan(e+fx))^p\right)^n}{af(np+1)}-\frac{i\tan^2(e+fx)\,{}_2F_1\left(2,\frac{1}{2}(np+2);\frac{1}{2}(np+4);-\tan^2(e+fx)\right)\left(c(d\tan(e+fx))^p\right)^n}{af(np+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Tan[e + f*x])^p)^n/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (Hypergeometric2F1[2, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(c*(d*Tan[e + f*x])^p)^n)/(a*f*(1 + n*p)) - (I*Hypergeometric2F1[2, (2 +
 n*p)/2, (4 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*x]^2*(c*(d*Tan[e + f*x])^p
)^n)/(a*f*(2 + n*p))
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 83

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[a, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^p, x], x] + Dist[
b/f, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n - 1, 0] &&  !RationalQ[
p] &&  !IGtQ[m, 0] && NeQ[m + n + p + 2, 0]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 1970

```
Int[(u_.)*((c_.)*((d_)*((a_.) + (b_.)*(x_)))^(q_))^(p_), x_Symbol] :> Dist[
(c*(d*(a + b*x))^q)^p/(a + b*x)^(p*q), Int[u*(a + b*x)^(p*q), x], x] /; Fre
eQ[{a, b, c, d, q, p}, x] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{(c(d\tan(e+fx))^p)^n}{a+ia\tan(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(c(dx)^p)^n}{(a+iax)(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left((d\tan(e+fx))^{-np}\,(c(d\tan(e+fx))^p)^n\right)\text{Subst}\left(\int \frac{(dx)^{np}}{(a+iax)(1+x^2)}\,dx, x, \tan(e+$$

$$= \frac{\left((d\tan(e+fx))^{-np}\,(c(d\tan(e+fx))^p)^n\right)\text{Subst}\left(\int \frac{(dx)^{np}}{\left(\frac{1}{a}-\frac{ix}{a}\right)(a+iax)^2}\,dx, x, \tan(e$$

$$= \frac{\left((d\tan(e+fx))^{-np}\,(c(d\tan(e+fx))^p)^n\right)\text{Subst}\left(\int \frac{(dx)^{np}}{\left(\frac{1}{a}-\frac{ix}{a}\right)^2(a+iax)^2}\,dx, x, \tan(e$$

$$= \frac{\left((d\tan(e+fx))^{-np}\,(c(d\tan(e+fx))^p)^n\right)\text{Subst}\left(\int \frac{(dx)^{np}}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)$$

$$= \frac{{}_2F_1\left(2, \frac{1}{2}(1+np); \frac{1}{2}(3+np); -\tan^2(e+fx)\right)\tan(e+fx)\,(c(d\tan(e+fx))^p)$$

Mathematica [F]
time = 18.97, size = 0, normalized size = 0.00

$$\int \frac{(c(d\tan(e+fx))^p)^n}{a+ia\tan(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c*(d*Tan[e + f*x])^p)^n/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] Integrate[(c*(d*Tan[e + f*x])^p)^n/(a + I*a*Tan[e + f*x]), x]
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{(c(d\tan(fx+e))^p)^n}{a+ia\tan(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*tan(f*x+e))^p)^n/(a+I*a*tan(f*x+e)),x)
[Out] int((c*(d*tan(f*x+e))^p)^n/(a+I*a*tan(f*x+e)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))^p)^n/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))^p)^n/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
[Out] integral(1/2*(e^(2*I*f*x + 2*I*e) + 1)*e^(n*p*log((-I*d*e^(2*I*f*x + 2*I*e)
 + I*d)/(e^(2*I*f*x + 2*I*e) + 1)) - 2*I*f*x + n*log(c) - 2*I*e)/a, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{(c(d\tan(e+fx))^p)^n}{\tan(e+fx)-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))**p)**n/(a+I*a*tan(f*x+e)),x)
[Out] -I*Integral((c*(d*tan(e + f*x))**p)**n/(tan(e + f*x) - I), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))^p)^n/(a+I*a*tan(f*x+e)),x, algorithm="giac")
[Out] integrate(((d*tan(f*x + e))^p*c)^n/(I*a*tan(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,(d\tan(e+f\,x))^p)^n}{a+a\tan\left(e+f\,x\right)\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*tan(e + f*x))^p)^n/(a + a*tan(e + f*x)*1i),x)
[Out] int((c*(d*tan(e + f*x))^p)^n/(a + a*tan(e + f*x)*1i), x)
```

3.1322 $\int \frac{(c(d\tan(e+fx))^p)^n}{(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=227

$$\frac{(1-4np+2n^2p^2)\ {}_2F_1(1,1+np;2+np;-i\tan(e+fx))\tan(e+fx)\,(c(d\tan(e+fx))^p)^n}{8a^2f(1+np)}+\frac{{}_2F_1(1,1+np;2$$

```
[Out] 1/8*(2*n^2*p^2-4*n*p+1)*hypergeom([1, n*p+1],[n*p+2],-I*tan(f*x+e))*tan(f*x
+e)*(c*(d*tan(f*x+e))^p)^n/a^2/f/(n*p+1)+1/8*hypergeom([1, n*p+1],[n*p+2],I
*tan(f*x+e))*tan(f*x+e)*(c*(d*tan(f*x+e))^p)^n/a^2/f/(n*p+1)+1/4*tan(f*x+e)
*(c*(d*tan(f*x+e))^p)^n/a^2/f/(1+I*tan(f*x+e))^2+1/4*(-n*p+2)*tan(f*x+e)*(c
*(d*tan(f*x+e))^p)^n/a^2/f/(1+I*tan(f*x+e))
```

Rubi [A]
time = 0.23, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1970, 862, 105, 156, 162, 66}

$$\frac{(2n^2p^2-4np+1)\tan(e+fx)\,{}_2F_1(1,np+1;np+2;-i\tan(e+fx))\,(c(d\tan(e+fx))^p)^n}{8a^2f(np+1)}+\frac{\tan(e+fx)\,{}_2F_1(1,np+1;np+2;i\tan(e+fx))\,(c(d\tan(e+fx))^p)^n}{8a^2f(np+1)}+\frac{(2-np)\tan(e+fx)\,(c(d\tan(e+fx))^p)^n}{4a^2f(1+i\tan(e+fx))}+\frac{\tan(e+fx)\,(c(d\tan(e+fx))^p)^n}{4a^2f(1+i\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Tan[e + f*x])^p)^n/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] ((1 - 4*n*p + 2*n^2*p^2)*Hypergeometric2F1[1, 1 + n*p, 2 + n*p, (-I)*Tan[e
+ f*x]]*Tan[e + f*x]*(c*(d*Tan[e + f*x])^p)^n)/(8*a^2*f*(1 + n*p)) + (Hyper
geometric2F1[1, 1 + n*p, 2 + n*p, I*Tan[e + f*x]]*Tan[e + f*x]*(c*(d*Tan[e
+ f*x])^p)^n)/(8*a^2*f*(1 + n*p)) + (Tan[e + f*x]*(c*(d*Tan[e + f*x])^p)^n)
/(4*a^2*f*(1 + I*Tan[e + f*x])^2) + ((2 - n*p)*Tan[e + f*x]*(c*(d*Tan[e + f
*x])^p)^n)/(4*a^2*f*(1 + I*Tan[e + f*x]))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
```

```
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 1970

```
Int[(u_.)*((c_.)*((d_)*((a_.) + (b_.)*(x_)))^(q_))^(p_), x_Symbol] :> Dist[
(c*(d*(a + b*x))^q)^p/(a + b*x)^(p*q), Int[u*(a + b*x)^(p*q), x], x] /; Fre
eQ[{a, b, c, d, q, p}, x] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{(c(d\tan(e+fx))^p)^n}{(a+ia\tan(e+fx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(c(dx)^p)^n}{(a+iax)^2(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{((d\tan(e+fx))^{-np}\,(c(d\tan(e+fx))^p)^n)\,\text{Subst}\left(\int \frac{(dx)^{np}}{(a+iax)^2(1+x^2)}\,dx, x, \tan(e+\right.}{f}$$

$$= \frac{((d\tan(e+fx))^{-np}\,(c(d\tan(e+fx))^p)^n)\,\text{Subst}\left(\int \frac{(dx)^{np}}{\left(\frac{1}{a}-\frac{ix}{a}\right)(a+iax)^3}\,dx, x, \tan(e\right.}{f}$$

$$= \frac{\tan(e+fx)\,(c(d\tan(e+fx))^p)^n}{4a^2f(1+i\tan(e+fx))^2} - \frac{(i(d\tan(e+fx))^{-np}\,(c(d\tan(e+fx))^p)^n}{}$$

$$= \frac{\tan(e+fx)\,(c(d\tan(e+fx))^p)^n}{4a^2f(1+i\tan(e+fx))^2} + \frac{(2-np)\tan(e+fx)\,(c(d\tan(e+fx))^p)^n}{4a^2f(1+i\tan(e+fx))}$$

$$= \frac{\tan(e+fx)\,(c(d\tan(e+fx))^p)^n}{4a^2f(1+i\tan(e+fx))^2} + \frac{(2-np)\tan(e+fx)\,(c(d\tan(e+fx))^p)^n}{4a^2f(1+i\tan(e+fx))}$$

$$= \frac{(1-4np+2n^2p^2)\,{}_2F_1(1,1+np;2+np;-i\tan(e+fx))\tan(e+fx)\,(c(d\tan}{8a^2f(1+np)}$$

Mathematica [F]
time = 6.41, size = 0, normalized size = 0.00

$$\int \frac{(c(d\tan(e+fx))^p)^n}{(a+ia\tan(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c*(d*Tan[e + f*x])^p)^n/(a + I*a*Tan[e + f*x])^2,x]
[Out] Integrate[(c*(d*Tan[e + f*x])^p)^n/(a + I*a*Tan[e + f*x])^2, x]
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{(c(d\tan(fx+e))^p)^n}{(a+ia\tan(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*tan(f*x+e))^p)^n/(a+I*a*tan(f*x+e))^2,x)
[Out] int((c*(d*tan(f*x+e))^p)^n/(a+I*a*tan(f*x+e))^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))^p)^n/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))^p)^n/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] integral(1/4*(e^(4*I*f*x + 4*I*e) + 2*e^(2*I*f*x + 2*I*e) + 1)*e^(n*p*log((
-I*d*e^(2*I*f*x + 2*I*e) + I*d)/(e^(2*I*f*x + 2*I*e) + 1)) - 4*I*f*x + n*lo
g(c) - 4*I*e)/a^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{(c(d\tan(e+fx))^p)^n}{\tan^2(e+fx)-2i\tan(e+fx)-1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))**p)**n/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -Integral((c*(d*tan(e + f*x))**p)**n/(tan(e + f*x)**2 - 2*I*tan(e + f*x) -
1), x)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))^p)^n/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(((d*tan(f*x + e))^p*c)^n/(I*a*tan(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\left(d\tan(e+fx)\right)^p\right)^n}{\left(a+a\tan\left(e+fx\right)\,1\mathrm{i}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*tan(e + f*x))^p)^n/(a + a*tan(e + f*x)*1i)^2,x)
[Out] int((c*(d*tan(e + f*x))^p)^n/(a + a*tan(e + f*x)*1i)^2, x)
```

3.1323 $\int (c(d\tan(e+fx))^p)^n (a+b\tan(e+fx))^m\,dx$

Optimal. Leaf size=201

$$\frac{F_1\left(1+np;-m,1;2+np;-\frac{b\tan(e+fx)}{a},-i\tan(e+fx)\right)\tan(e+fx)\,(c(d\tan(e+fx))^p)^n\,(a+b\tan(e+fx}{2f(1+np)}$$

```
[Out] 1/2*AppellF1(n*p+1,1,-m,n*p+2,-I*tan(f*x+e),-b*tan(f*x+e)/a)*tan(f*x+e)*(c*
(d*tan(f*x+e))^p)^n*(a+b*tan(f*x+e))^m/f/(n*p+1)/((1+b*tan(f*x+e)/a)^m)+1/2
*AppellF1(n*p+1,1,-m,n*p+2,I*tan(f*x+e),-b*tan(f*x+e)/a)*tan(f*x+e)*(c*(d*t
an(f*x+e))^p)^n*(a+b*tan(f*x+e))^m/f/(n*p+1)/((1+b*tan(f*x+e)/a)^m)
```

Rubi [A]

time = 0.18, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3659, 3656, 926, 140, 138}

$$\frac{\tan(e+fx)(a+b\tan(e+fx))^m\left(\frac{b\tan(e+fx)}{a}+1\right)^{-m}(c(d\tan(e+fx))^p)^n F_1\left(np+1;-m,1;np+2;-\frac{b\tan(e+fx)}{a},-i\tan(e+fx)\right)}{2f(np+1)}+\frac{\tan(e+fx)(a+b\tan(e+fx))^m\left(\frac{b\tan(e+fx)}{a}+1\right)^{-m}(c(d\tan(e+fx))^p)^n F_1\left(np+1;-m,1;np+2;-\frac{b\tan(e+fx)}{a},i\tan(e+fx)\right)}{2f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Tan[e + f*x])^p)^n*(a + b*Tan[e + f*x])^m,x]
```

```
[Out] (AppellF1[1 + n*p, -m, 1, 2 + n*p, -((b*Tan[e + f*x])/a), (-I)*Tan[e + f*x]
]*Tan[e + f*x]*(c*(d*Tan[e + f*x])^p)^n*(a + b*Tan[e + f*x])^m)/(2*f*(1 + n
*p)*(1 + (b*Tan[e + f*x])/a)^m) + (AppellF1[1 + n*p, -m, 1, 2 + n*p, -((b*T
an[e + f*x])/a), I*Tan[e + f*x]]*Tan[e + f*x]*(c*(d*Tan[e + f*x])^p)^n*(a +
 b*Tan[e + f*x])^m)/(2*f*(1 + n*p)*(1 + (b*Tan[e + f*x])/a)^m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
```

```
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
& !IntegerQ[m] && !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3659

```
Int[((c_.)*((d_.)*tan[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Tan[e + f*x
])^p)^FracPart[n]/(d*Tan[e + f*x])^(p*FracPart[n])), Int[(a + b*Tan[e + f*x
])^m*(d*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && !IntegerQ[n] && !IntegerQ[m]
```

Rubi steps

$$\int (c(d\tan(e+fx))^p)^n (a+b\tan(e+fx))^m\,dx = \left((d\tan(e+fx))^{-np}\left(c(d\tan(e+fx))^p\right)^n\right)\int (d\tan(e+fx$$

$$= \frac{\left((d\tan(e+fx))^{-np}\left(c(d\tan(e+fx))^p\right)^n\right)\text{Subst}\left(\int \frac{(dx)^{np}(a}{1+x}\right.}{f}$$

$$= \frac{\left((d\tan(e+fx))^{-np}\left(c(d\tan(e+fx))^p\right)^n\right)\text{Subst}\left(\int \left(\frac{i(dx)^{np}}{2(}\right.\right.}{f}$$

$$= \frac{\left(i(d\tan(e+fx))^{-np}\left(c(d\tan(e+fx))^p\right)^n\right)\text{Subst}\left(\int \frac{(dx)^{np}(}{i-}\right.}{2f}$$

$$= \frac{\left(i(d\tan(e+fx))^{-np}\left(c(d\tan(e+fx))^p\right)^n(a+b\tan(e+\right.}{}$$

$$= \frac{F_1\left(1+np;-m,1;2+np;-\frac{b\tan(e+fx)}{a},-i\tan(e+fx)\right)\tan}{}$$

Mathematica [F]
time = 1.66, size = 0, normalized size = 0.00

$$\int (c(d\tan(e+fx))^p)^n (a+b\tan(e+fx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c*(d*Tan[e + f*x])^p)^n*(a + b*Tan[e + f*x])^m,x]
```

```
[Out] Integrate[(c*(d*Tan[e + f*x])^p)^n*(a + b*Tan[e + f*x])^m, x]
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int \left(c \left(d \tan \left(f x+e\right)\right)^{p}\right)^{n} \left(a +b \tan \left(f x+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*tan(f*x+e))^p)^n*(a+b*tan(f*x+e))^m,x)
```

```
[Out] int((c*(d*tan(f*x+e))^p)^n*(a+b*tan(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))^p)^n*(a+b*tan(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate(((d*tan(f*x + e))^p*c)^n*(b*tan(f*x + e) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))^p)^n*(a+b*tan(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral(((d*tan(f*x + e))^p*c)^n*(b*tan(f*x + e) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c \left(d \tan{\left(e + f x \right)}\right)^{p}\right)^{n} \left(a + b \tan{\left(e + f x \right)}\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))**p)**n*(a+b*tan(f*x+e))**m,x)
```

```
[Out] Integral((c*(d*tan(e + f*x))**p)**n*(a + b*tan(e + f*x))**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))^p)^n*(a+b*tan(f*x+e))^m,x, algorithm="giac")
[Out] integrate(((d*tan(f*x + e))^p*c)^n*(b*tan(f*x + e) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c\,{\left(d\,\tan\left(e+f\,x\right)\right)}^p\right)^n\,{\left(a+b\,\tan\left(e+f\,x\right)\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*tan(e + f*x))^p)^n*(a + b*tan(e + f*x))^m,x)
[Out] int((c*(d*tan(e + f*x))^p)^n*(a + b*tan(e + f*x))^m, x)
```

3.1324 $\int (c(d\tan(e+fx))^p)^n (a+b\tan(e+fx))^3 dx$

Optimal. Leaf size=219

$$\frac{3ab^2\tan(e+fx)\left(c(d\tan(e+fx))^p\right)^n}{f(1+np)}+\frac{a(a^2-3b^2)\ {}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(e+fx)\right)\tan(e+f$$

```
[Out] 3*a*b^2*tan(f*x+e)*(c*(d*tan(f*x+e))^p)^n/f/(n*p+1)+a*(a^2-3*b^2)*hypergeom
([1, 1/2*n*p+1/2],[1/2*n*p+3/2],-tan(f*x+e)^2)*tan(f*x+e)*(c*(d*tan(f*x+e))
^p)^n/f/(n*p+1)+b^3*tan(f*x+e)^2*(c*(d*tan(f*x+e))^p)^n/f/(n*p+2)+b*(3*a^2-
b^2)*hypergeom([1, 1/2*n*p+1],[1/2*n*p+2],-tan(f*x+e)^2)*tan(f*x+e)^2*(c*(d
*tan(f*x+e))^p)^n/f/(n*p+2)
```

Rubi [A]
time = 0.20, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1970, 1816, 822, 371}

$$\frac{b(3a^2-b^2)\tan^2(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+2);\frac{1}{2}(np+4);-\tan^2(e+fx)\right)\left(c(d\tan(e+fx))^p\right)^n}{f(np+2)}+\frac{a(a^2-3b^2)\tan(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(e+fx)\right)\left(c(d\tan(e+fx))^p\right)^n}{f(np+1)}+\frac{3ab^2\tan(e+fx)\left(c(d\tan(e+fx))^p\right)^n}{f(np+1)}+\frac{b^3\tan^2(e+fx)\left(c(d\tan(e+fx))^p\right)^n}{f(np+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Tan[e + f*x])^p)^n*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (3*a*b^2*Tan[e + f*x]*(c*(d*Tan[e + f*x])^p)^n)/(f*(1 + n*p)) + (a*(a^2 - 3
*b^2)*Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e
 + f*x]*(c*(d*Tan[e + f*x])^p)^n)/(f*(1 + n*p)) + (b^3*Tan[e + f*x]^2*(c*(d
*Tan[e + f*x])^p)^n)/(f*(2 + n*p)) + (b*(3*a^2 - b^2)*Hypergeometric2F1[1,
(2 + n*p)/2, (4 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*x]^2*(c*(d*Tan[e + f*x
])^p)^n)/(f*(2 + n*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1970

```
Int[(u_.)*((c_.)*((d_)*((a_.) + (b_.)*(x_)))^(q_))^(p_), x_Symbol] :> Dist[
(c*(d*(a + b*x))^q)^p/(a + b*x)^(p*q), Int[u*(a + b*x)^(p*q), x], x] /; Fre
eQ[{a, b, c, d, q, p}, x] && !IntegerQ[q] && !IntegerQ[p]
```

Rubi steps

$$\int (c(d\tan(e+fx))^p)^n (a+b\tan(e+fx))^3\,dx = \frac{\text{Subst}\left(\int \frac{(c(dx)^p)^n(a+bx)^3}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left((d\tan(e+fx))^{-np}\left(c(d\tan(e+fx))^p\right)^n\right)\text{Subst}\left(\int \frac{(dx)^{np}(a-}{1+x^2}\right.}{f}$$

$$= \frac{\left((d\tan(e+fx))^{-np}\left(c(d\tan(e+fx))^p\right)^n\right)\text{Subst}\left(\int \left(3ab^2(a\right.\right.}{}$$

$$= \frac{3ab^2\tan(e+fx)\left(c(d\tan(e+fx))^p\right)^n}{f(1+np)} + \frac{b^3\tan^2(e+fx)\,(c}{f(2}$$

$$= \frac{3ab^2\tan(e+fx)\left(c(d\tan(e+fx))^p\right)^n}{f(1+np)} + \frac{b^3\tan^2(e+fx)\,(c}{f(2}$$

$$= \frac{3ab^2\tan(e+fx)\left(c(d\tan(e+fx))^p\right)^n}{f(1+np)} + \frac{a(a^2-3b^2)\,{}_2F_1(1}{}$$

Mathematica [A]

time = 0.98, size = 163, normalized size = 0.74

$$\frac{\tan(e+fx)\left(c(d\tan(e+fx))^p\right)^n\left(a(a^2-3b^2)(2+np)\,{}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(e+fx)\right)+b\left((3a^2-b^2)(1+np)\,{}_2F_1\left(1,1+\frac{np}{2};2+\frac{np}{2};-\tan^2(e+fx)\right)\tan(e+fx)+b(3a(2+np)+b(1+np)\tan(e+fx))\right)\right)}{f(1+np)(2+np)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Tan[e + f*x])^p)^n*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (Tan[e + f*x]*(c*(d*Tan[e + f*x])^p)^n*(a*(a^2 - 3*b^2)*(2 + n*p)*Hypergeom
etric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2] + b*((3*a^2 - b^2)*(
1 + n*p)*Hypergeometric2F1[1, 1 + (n*p)/2, 2 + (n*p)/2, -Tan[e + f*x]^2]*Ta
n[e + f*x] + b*(3*a*(2 + n*p) + b*(1 + n*p)*Tan[e + f*x]))))/(f*(1 + n*p)*(
2 + n*p))
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int \left(c(d\tan \left(fx+e\right))^{p}\right)^{n} \left(a+b\tan \left(fx+e\right)\right)^{3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*tan(f*x+e))^p)^n*(a+b*tan(f*x+e))^3,x)
[Out] int((c*(d*tan(f*x+e))^p)^n*(a+b*tan(f*x+e))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))^p)^n*(a+b*tan(f*x+e))^3,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)^3*((d*tan(f*x + e))^p*c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))^p)^n*(a+b*tan(f*x+e))^3,x, algorithm="fricas")
[Out] integral((b^3*tan(f*x + e)^3 + 3*a*b^2*tan(f*x + e)^2 + 3*a^2*b*tan(f*x + e
) + a^3)*((d*tan(f*x + e))^p*c)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c(d\tan \left(e+fx\right))^{p}\right)^{n} \left(a+b\tan \left(e+fx\right)\right)^{3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))**p)**n*(a+b*tan(f*x+e))**3,x)
[Out] Integral((c*(d*tan(e + f*x))**p)**n*(a + b*tan(e + f*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))^p)^n*(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)^3*((d*tan(f*x + e))^p*c)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,(d\tan(e+f\,x))^p)^n\,(a+b\tan(e+f\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*tan(e + f*x))^p)^n*(a + b*tan(e + f*x))^3,x)
```

```
[Out] int((c*(d*tan(e + f*x))^p)^n*(a + b*tan(e + f*x))^3, x)
```

3.1325 $\int (c(d\tan(e+fx))^p)^n (a+b\tan(e+fx))^2 \, dx$

Optimal. Leaf size=171

$$\frac{b^2\tan(e+fx)\,(c(d\tan(e+fx))^p)^n}{f(1+np)}+\frac{(a^2-b^2)\ {}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(e+fx)\right)\tan(e+fx)\,(c$$

```
[Out] b^2*tan(f*x+e)*(c*(d*tan(f*x+e))^p)^n/f/(n*p+1)+(a^2-b^2)*hypergeom([1, 1/2
*n*p+1/2],[1/2*n*p+3/2],-tan(f*x+e)^2)*tan(f*x+e)*(c*(d*tan(f*x+e))^p)^n/f/
(n*p+1)+2*a*b*hypergeom([1, 1/2*n*p+1],[1/2*n*p+2],-tan(f*x+e)^2)*tan(f*x+e
)^2*(c*(d*tan(f*x+e))^p)^n/f/(n*p+2)
```

Rubi [A]
time = 0.17, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1970, 1816, 822, 371}

$$\frac{(a^2-b^2)\tan(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(e+fx)\right)(c(d\tan(e+fx))^p)^n}{f(np+1)}+\frac{2ab\tan^2(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+2);\frac{1}{2}(np+4);-\tan^2(e+fx)\right)(c(d\tan(e+fx))^p)^n}{f(np+2)}+\frac{b^2\tan(e+fx)\,(c(d\tan(e+fx))^p)^n}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Tan[e + f*x])^p)^n*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (b^2*Tan[e + f*x]*(c*(d*Tan[e + f*x])^p)^n)/(f*(1 + n*p)) + ((a^2 - b^2)*Hy
pergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*x]*
(c*(d*Tan[e + f*x])^p)^n)/(f*(1 + n*p)) + (2*a*b*Hypergeometric2F1[1, (2 +
n*p)/2, (4 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*x]^2*(c*(d*Tan[e + f*x])^p)
^n)/(f*(2 + n*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1970

```
Int[(u_.)*((c_.)*((d_)*((a_.) + (b_.)*(x_)))^(q_))^(p_), x_Symbol] :> Dist[
(c*(d*(a + b*x))^q)^p/(a + b*x)^(p*q), Int[u*(a + b*x)^(p*q), x], x] /; Fre
eQ[{a, b, c, d, q, p}, x] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

Rubi steps

$$\int (c(d\tan(e+fx))^p)^n (a+b\tan(e+fx))^2\,dx = \frac{\text{Subst}\left(\int \frac{(c(dx)^p)^n(a+bx)^2}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left((d\tan(e+fx))^{-np}\left(c(d\tan(e+fx))^p\right)^n\right)\text{Subst}\left(\int \frac{(dx)^{np}(a-}{1+x^2}\right.}{f}$$

$$= \frac{\left((d\tan(e+fx))^{-np}\left(c(d\tan(e+fx))^p\right)^n\right)\text{Subst}\left(\int \left(b^2(dx\right.\right.}{f}$$

$$= \frac{b^2\tan(e+fx)\left(c(d\tan(e+fx))^p\right)^n}{f(1+np)} + \frac{((d\tan(e+fx))^{-np}}{}$$

$$= \frac{b^2\tan(e+fx)\left(c(d\tan(e+fx))^p\right)^n}{f(1+np)} + \frac{((a^2-b^2)\left(d\tan(e+\right.}{}$$

$$= \frac{b^2\tan(e+fx)\left(c(d\tan(e+fx))^p\right)^n}{f(1+np)} + \frac{(a^2-b^2)\ {}_2F_1\left(1,\frac{1}{2}(1\right.}{}$$

Mathematica [A]
time = 0.59, size = 136, normalized size = 0.80

$$\frac{\tan(e+fx)\left(c(d\tan(e+fx))^p\right)^n\left((a^2-b^2)(2+np)\,{}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(e+fx)\right)+b\left(b(2+np)+2a(1+np)\,{}_2F_1\left(1,1+\frac{np}{2};2+\frac{np}{2};-\tan^2(e+fx)\right)\tan(e+fx)\right)\right)}{f(1+np)(2+np)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Tan[e + f*x])^p)^n*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (Tan[e + f*x]*(c*(d*Tan[e + f*x])^p)^n*((a^2 - b^2)*(2 + n*p)*Hypergeometri
c2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2] + b*(b*(2 + n*p) + 2*a*(
1 + n*p)*Hypergeometric2F1[1, 1 + (n*p)/2, 2 + (n*p)/2, -Tan[e + f*x]^2]*Ta
n[e + f*x])))/(f*(1 + n*p)*(2 + n*p))
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \left(c\left(d\tan\left(fx+e\right)\right)^{p}\right)^{n}\left(a+b\tan\left(fx+e\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*tan(f*x+e))^p)^n*(a+b*tan(f*x+e))^2,x)
```

```
[Out] int((c*(d*tan(f*x+e))^p)^n*(a+b*tan(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))^p)^n*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2*((d*tan(f*x + e))^p*c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))^p)^n*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)*((d*tan(f*x + e))^
p*c)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c(d\tan\left(e+fx\right))^{p}\right)^{n}\left(a+b\tan\left(e+fx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))**p)**n*(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral((c*(d*tan(e + f*x))**p)**n*(a + b*tan(e + f*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))^p)^n*(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2*((d*tan(f*x + e))^p*c)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,(d\tan(e+f\,x))^p)^n\,(a+b\tan(e+f\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*tan(e + f*x))^p)^n*(a + b*tan(e + f*x))^2,x)
[Out] int((c*(d*tan(e + f*x))^p)^n*(a + b*tan(e + f*x))^2, x)
```

3.1326 $\int (c(d\tan(e+fx))^p)^n (a+b\tan(e+fx))\,dx$

Optimal. Leaf size=127

$$\frac{a\,{}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(e+fx)\right)\tan(e+fx)\left(c(d\tan(e+fx))^p\right)^n}{f(1+np)}+\frac{b\,{}_2F_1\left(1,\frac{1}{2}(2+np);\frac{1}{2}(4+\right.}{}$$

```
[Out] a*hypergeom([1, 1/2*n*p+1/2],[1/2*n*p+3/2],-tan(f*x+e)^2)*tan(f*x+e)*(c*(d*
tan(f*x+e))^p)^n/f/(n*p+1)+b*hypergeom([1, 1/2*n*p+1],[1/2*n*p+2],-tan(f*x+
e)^2)*tan(f*x+e)^2*(c*(d*tan(f*x+e))^p)^n/f/(n*p+2)
```

Rubi [A]
time = 0.09, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1970, 822, 371}

$$\frac{a\tan(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(e+fx)\right)\left(c(d\tan(e+fx))^p\right)^n}{f(np+1)}+\frac{b\tan^2(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+2);\frac{1}{2}(np+4);-\tan^2(e+fx)\right)\left(c(d\tan(e+fx))^p\right)^n}{f(np+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Tan[e + f*x])^p)^n*(a + b*Tan[e + f*x]),x]
```

```
[Out] (a*Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e +
f*x]*(c*(d*Tan[e + f*x])^p)^n)/(f*(1 + n*p)) + (b*Hypergeometric2F1[1, (2 +
 n*p)/2, (4 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*x]^2*(c*(d*Tan[e + f*x])^p
)^n)/(f*(2 + n*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 1970

```
Int[(u_.)*((c_.)*((d_)*((a_.) + (b_.)*(x_)))^(q_))^(p_), x_Symbol] :> Dist[
(c*(d*(a + b*x))^q)^p/(a + b*x)^(p*q), Int[u*(a + b*x)^(p*q), x], x] /; Fre
eQ[{a, b, c, d, q, p}, x] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

Rubi steps

$$\int (c(d\tan(e+fx))^p)^n (a+b\tan(e+fx))\,dx = \frac{\text{Subst}\left(\int \frac{(c(dx)^p)^n(a+bx)}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left((d\tan(e+fx))^{-np}\,(c(d\tan(e+fx))^p)^n\right)\text{Subst}\left(\int \frac{(dx)^{np}(a+\ldots}{1+x^2}\ldots\right.}{f}$$

$$= \frac{\left(a(d\tan(e+fx))^{-np}\,(c(d\tan(e+fx))^p)^n\right)\text{Subst}\left(\int \frac{(dx)^{np}}{1+x^2}\,d\ldots\right.}{f}$$

$$= \frac{a\,{}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(e+fx)\right)\tan(e+fx)\ldots}{f(1+np)}$$

Mathematica [A]
time = 0.33, size = 117, normalized size = 0.92

$$\frac{\tan(e+fx)\,(c(d\tan(e+fx))^p)^n\left(a(2+np)\,{}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(e+fx)\right)+b(1+np)\,{}_2F_1\left(1,1+\frac{np}{2};2+\frac{np}{2};-\tan^2(e+fx)\right)\tan(e+fx)\right)}{f(1+np)(2+np)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Tan[e + f*x])^p)^n*(a + b*Tan[e + f*x]),x]
[Out] (Tan[e + f*x]*(c*(d*Tan[e + f*x])^p)^n*(a*(2 + n*p)*Hypergeometric2F1[1, (1
 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2] + b*(1 + n*p)*Hypergeometric2F1[1,
 1 + (n*p)/2, 2 + (n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*x]))/(f*(1 + n*p)*(2
+ n*p))
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int (c(d\tan(fx+e))^p)^n (a+b\tan(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*tan(f*x+e))^p)^n*(a+b*tan(f*x+e)),x)
[Out] int((c*(d*tan(f*x+e))^p)^n*(a+b*tan(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))^p)^n*(a+b*tan(f*x+e)),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)*((d*tan(f*x + e))^p*c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))^p)^n*(a+b*tan(f*x+e)),x, algorithm="fricas")
[Out] integral((b*tan(f*x + e) + a)*((d*tan(f*x + e))^p*c)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c(d\tan (e+fx))^p)^n (a+b\tan (e+fx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))**p)**n*(a+b*tan(f*x+e)),x)
[Out] Integral((c*(d*tan(e + f*x))**p)**n*(a + b*tan(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))^p)^n*(a+b*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)*((d*tan(f*x + e))^p*c)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,{\left(d\,\tan(e+f\,x)\right)}^p\right)}^n\,\left(a+b\,\tan(e+f\,x)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*tan(e + f*x))^p)^n*(a + b*tan(e + f*x)),x)
[Out] int((c*(d*tan(e + f*x))^p)^n*(a + b*tan(e + f*x)), x)
```

3.1327 $\int \frac{(c(d\tan(e+fx))^p)^n}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=216

$$\frac{a\,{}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(e+fx)\right)\tan(e+fx)\,(c(d\tan(e+fx))^p)^n}{(a^2+b^2)\,f(1+np)}+\frac{b^2\,{}_2F_1\left(1,1+np;2+np;-\right.}{a}$$

```
[Out] a*hypergeom([1, 1/2*n*p+1/2],[1/2*n*p+3/2],-tan(f*x+e)^2)*tan(f*x+e)*(c*(d*
tan(f*x+e))^p)^n/(a^2+b^2)/f/(n*p+1)+b^2*hypergeom([1, n*p+1],[n*p+2],-b*ta
n(f*x+e)/a)*tan(f*x+e)*(c*(d*tan(f*x+e))^p)^n/a/(a^2+b^2)/f/(n*p+1)-b*hyper
geom([1, 1/2*n*p+1],[1/2*n*p+2],-tan(f*x+e)^2)*tan(f*x+e)^2*(c*(d*tan(f*x+e
))^p)^n/(a^2+b^2)/f/(n*p+2)
```

Rubi [A]
time = 0.21, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1970, 975, 66, 822, 371}

$$-\frac{b\tan^2(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+2);\frac{1}{2}(np+4);-\tan^2(e+fx)\right)(c(d\tan(e+fx))^p)^n}{f(a^2+b^2)(np+2)}+\frac{a\tan(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(e+fx)\right)(c(d\tan(e+fx))^p)^n}{f(a^2+b^2)(np+1)}+\frac{b^2\tan(e+fx)(c(d\tan(e+fx))^p)^n\,{}_2F_1\left(1,np+1;np+2;-\frac{b\tan(e+fx)}{a}\right)}{af(a^2+b^2)(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Tan[e + f*x])^p)^n/(a + b*Tan[e + f*x]),x]
```

```
[Out] (a*Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e +
f*x]*(c*(d*Tan[e + f*x])^p)^n)/((a^2 + b^2)*f*(1 + n*p)) + (b^2*Hypergeomet
ric2F1[1, 1 + n*p, 2 + n*p, -((b*Tan[e + f*x])/a)]*Tan[e + f*x]*(c*(d*Tan[e
 + f*x])^p)^n)/(a*(a^2 + b^2)*f*(1 + n*p)) - (b*Hypergeometric2F1[1, (2 + n
*p)/2, (4 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*x]^2*(c*(d*Tan[e + f*x])^p)^
n)/((a^2 + b^2)*f*(2 + n*p))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
 + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) && !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 1970

```
Int[(u_.)*((c_.)*((d_)*((a_.) + (b_.)*(x_)))^(q_))^(p_), x_Symbol] :> Dist[
(c*(d*(a + b*x))^q)^p/(a + b*x)^(p*q), Int[u*(a + b*x)^(p*q), x], x] /; Fre
eQ[{a, b, c, d, q, p}, x] && !IntegerQ[q] && !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c(d\tan(e+fx))^p)^n}{a+b\tan(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{(c(dx)^p)^n}{(a+bx)(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\left((d\tan(e+fx))^{-np}\,(c(d\tan(e+fx))^p)^n\right)\text{Subst}\left(\int \frac{(dx)^{np}}{(a+bx)(1+x^2)}\,dx, x, \tan(e+\right.}{f} \\
&= \frac{\left((d\tan(e+fx))^{-np}\,(c(d\tan(e+fx))^p)^n\right)\text{Subst}\left(\int \left(\frac{b^2(dx)^{np}}{(a^2+b^2)(a+bx)}+\frac{(dx)^{np}(a-bx}{(a^2+b^2)(1+x}\right.\right.}{f} \\
&= \frac{\left((d\tan(e+fx))^{-np}\,(c(d\tan(e+fx))^p)^n\right)\text{Subst}\left(\int \frac{(dx)^{np}(a-bx)}{1+x^2}\,dx, x, \tan(e+\right.}{(a^2+b^2)\,f} \\
&= \frac{b^2\,{}_2F_1\left(1,1+np;2+np;-\frac{b\tan(e+fx)}{a}\right)\tan(e+fx)\,(c(d\tan(e+fx))^p)^n}{a\,(a^2+b^2)\,f(1+np)} + \frac{(a(}{} \\
&= \frac{a\,{}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(e+fx)\right)\tan(e+fx)\,(c(d\tan(e+fx))}{(a^2+b^2)\,f(1+np)}
\end{aligned}
$$

Mathematica [A]
time = 0.76, size = 166, normalized size = 0.77

$$
\frac{\tan(e+fx)\,(c(d\tan(e+fx))^p)^n\left(a^2(2+np)\,{}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(e+fx)\right)+b\left(b(2+np)\,{}_2F_1\left(1,1+np;2+np;-\frac{b\tan(e+fx)}{a}\right)-a(1+np)\,{}_2F_1\left(1,1+\frac{np}{2};2+\frac{np}{2};-\tan^2(e+fx)\right)\tan(e+fx)\right)\right)}{a\,(a^2+b^2)\,f(1+np)(2+np)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Tan[e + f*x])^p)^n/(a + b*Tan[e + f*x]),x]
```

```
[Out] (Tan[e + f*x]*(c*(d*Tan[e + f*x])^p)^n*(a^2*(2 + n*p)*Hypergeometric2F1[1,
(1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2] + b*(b*(2 + n*p)*Hypergeometric2
F1[1, 1 + n*p, 2 + n*p, -((b*Tan[e + f*x])/a)] - a*(1 + n*p)*Hypergeometric
2F1[1, 1 + (n*p)/2, 2 + (n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*x])))/(a*(a^2 +
 b^2)*f*(1 + n*p)*(2 + n*p))
```

Maple [**F**]
time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\left(c (d \tan \left(f x +e \right))^{p}\right)^{n}}{a +b \tan \left(f x +e \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*tan(f*x+e))^p)^n/(a+b*tan(f*x+e)),x)
```

```
[Out] int((c*(d*tan(f*x+e))^p)^n/(a+b*tan(f*x+e)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))^p)^n/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(((d*tan(f*x + e))^p*c)^n/(b*tan(f*x + e) + a), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))^p)^n/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(((d*tan(f*x + e))^p*c)^n/(b*tan(f*x + e) + a), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c \left(d \tan{\left(e + f x \right)}\right)^{p}\right)^{n}}{a + b \tan{\left(e + f x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))**p)**n/(a+b*tan(f*x+e)),x)
```

```
[Out] Integral((c*(d*tan(e + f*x))**p)**n/(a + b*tan(e + f*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))^p)^n/(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(((d*tan(f*x + e))^p*c)^n/(b*tan(f*x + e) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,{\left(d\,\mathrm{tan}\left(e+f\,x\right)\right)}^p\right)^n}{a+b\,\mathrm{tan}\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*tan(e + f*x))^p)^n/(a + b*tan(e + f*x)),x)
```

```
[Out] int((c*(d*tan(e + f*x))^p)^n/(a + b*tan(e + f*x)), x)
```

3.1328 $\int \frac{(c(d\tan(e+fx))^p)^n}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=293

$$\frac{(a^2-b^2)\,{}_2F_1\left(1,\tfrac{1}{2}(1+np);\tfrac{1}{2}(3+np);-\tan^2(e+fx)\right)\tan(e+fx)\,(c(d\tan(e+fx))^p)^n}{(a^2+b^2)^2 f(1+np)}+\frac{2b^2\,{}_2F_1\Big(1,1+np;$$

```
[Out] (a^2-b^2)*hypergeom([1, 1/2*n*p+1/2],[1/2*n*p+3/2],-tan(f*x+e)^2)*tan(f*x+e
)*(c*(d*tan(f*x+e))^p)^n/(a^2+b^2)^2/f/(n*p+1)+2*b^2*hypergeom([1, n*p+1],[
n*p+2],-b*tan(f*x+e)/a)*tan(f*x+e)*(c*(d*tan(f*x+e))^p)^n/(a^2+b^2)^2/f/(n*
p+1)+b^2*hypergeom([2, n*p+1],[n*p+2],-b*tan(f*x+e)/a)*tan(f*x+e)*(c*(d*tan
(f*x+e))^p)^n/a^2/(a^2+b^2)/f/(n*p+1)-2*a*b*hypergeom([1, 1/2*n*p+1],[1/2*n
*p+2],-tan(f*x+e)^2)*tan(f*x+e)^2*(c*(d*tan(f*x+e))^p)^n/(a^2+b^2)^2/f/(n*p
+2)
```

Rubi [A]
time = 0.28, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1970, 975, 66, 822, 371}

$$-\frac{2ab\tan^2(e+fx)\,{}_2F_1\left(1,\tfrac{1}{2}(np+2);\tfrac{1}{2}(np+4);-\tan^2(e+fx)\right)(c(d\tan(e+fx))^p)^n}{f(a^2+b^2)^2(np+2)}+\frac{(a^2-b^2)\tan(e+fx)\,{}_2F_1\left(1,\tfrac{1}{2}(np+1);\tfrac{1}{2}(np+3);-\tan^2(e+fx)\right)(c(d\tan(e+fx))^p)^n}{f(a^2+b^2)^2(np+1)}+\frac{2b^2\tan(e+fx)(c(d\tan(e+fx))^p)^n\,{}_2F_1\left(1,np+1;np+2;-\frac{b\tan(e+fx)}{a}\right)}{f(a^2+b^2)^2(np+1)}+\frac{b^2\tan(e+fx)(c(d\tan(e+fx))^p)^n\,{}_2F_1\left(2,np+1;np+2;-\frac{b\tan(e+fx)}{a}\right)}{a^2f(a^2+b^2)(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Tan[e + f*x])^p)^n/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] ((a^2 - b^2)*Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2
]*Tan[e + f*x]*(c*(d*Tan[e + f*x])^p)^n)/((a^2 + b^2)^2*f*(1 + n*p)) + (2*b
^2*Hypergeometric2F1[1, 1 + n*p, 2 + n*p, -((b*Tan[e + f*x])/a)]*Tan[e + f*
x]*(c*(d*Tan[e + f*x])^p)^n)/((a^2 + b^2)^2*f*(1 + n*p)) + (b^2*Hypergeomet
ric2F1[2, 1 + n*p, 2 + n*p, -((b*Tan[e + f*x])/a)]*Tan[e + f*x]*(c*(d*Tan[e
 + f*x])^p)^n)/(a^2*(a^2 + b^2)*f*(1 + n*p)) - (2*a*b*Hypergeometric2F1[1,
(2 + n*p)/2, (4 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*x]^2*(c*(d*Tan[e + f*x
])^p)^n)/((a^2 + b^2)^2*f*(2 + n*p))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 1970

```
Int[(u_.)*((c_.)*((d_)*((a_.) + (b_.)*(x_)))^(q_))^(p_), x_Symbol] :> Dist[
(c*(d*(a + b*x))^q)^p/(a + b*x)^(p*q), Int[u*(a + b*x)^(p*q), x], x] /; Fre
eQ[{a, b, c, d, q, p}, x] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{(c(d\tan(e+fx))^p)^n}{(a+b\tan(e+fx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(c(dx)^p)^n}{(a+bx)^2(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left((d\tan(e+fx))^{-np}\,(c(d\tan(e+fx))^p)^n\right)\text{Subst}\left(\int \frac{(dx)^{np}}{(a+bx)^2(1+x^2)}\,dx, x, \tan(e+\right.}{f}$$

$$= \frac{\left((d\tan(e+fx))^{-np}\,(c(d\tan(e+fx))^p)^n\right)\text{Subst}\left(\int \left(\frac{b^2(dx)^{np}}{(a^2+b^2)(a+bx)^2} + \frac{2ab^2(dx)^{n}}{(a^2+b^2)^2(a+}\right.\right.}{f}$$

$$= \frac{\left((d\tan(e+fx))^{-np}\,(c(d\tan(e+fx))^p)^n\right)\text{Subst}\left(\int \frac{(dx)^{np}(a^2-b^2-2abx)}{1+x^2}\,dx, x, \tan\right.}{(a^2+b^2)^2 f}$$

$$= \frac{2b^2\,{}_2F_1\left(1, 1+np; 2+np; -\frac{b\tan(e+fx)}{a}\right)\tan(e+fx)\,(c(d\tan(e+fx))^p)^n}{(a^2+b^2)^2 f(1+np)} + \frac{b^2}{}$$

$$= \frac{(a^2-b^2)\,{}_2F_1\left(1, \frac{1}{2}(1+np); \frac{1}{2}(3+np); -\tan^2(e+fx)\right)\tan(e+fx)\,(c(d\tan(e}{(a^2+b^2)^2 f(1+np)}$$

Mathematica [A]
time = 1.94, size = 231, normalized size = 0.79

$$\frac{\tan(e+fx)\,(c(d\tan(e+fx))^p)^n\left(-\frac{b^2(b^2np+a^2(-2+np))\,{}_2F_1\left(1,1+np;2+np;-\frac{b\tan(e+fx)}{a}\right)}{a(a^2+b^2)(1+np)}+\frac{b^2}{a+b\tan(e+fx)}+\frac{a\left((a^2-b^2)(2+np)\,{}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(e+fx)\right)-2ab(1+np)\,{}_2F_1\left(1,1+\frac{np}{2};2+\frac{np}{2};-\tan^2(e+fx)\right)\tan(e+fx)\right)}{(a^2+b^2)(1+np)(2+np)}\right)}{a\,(a^2+b^2)\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Tan[e + f*x])^p)^n/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (Tan[e + f*x]*(c*(d*Tan[e + f*x])^p)^n*(-((b^2*(b^2*n*p + a^2*(-2 + n*p))*H
ypergeometric2F1[1, 1 + n*p, 2 + n*p, -((b*Tan[e + f*x])/a)])/(a*(a^2 + b^2
)*(1 + n*p))) + b^2/(a + b*Tan[e + f*x]) + (a*((a^2 - b^2)*(2 + n*p)*Hyperg
eometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2] - 2*a*b*(1 + n*p)
*Hypergeometric2F1[1, 1 + (n*p)/2, 2 + (n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]))/((a^2 + b^2)*(1 + n*p)*(2 + n*p))))/(a*(a^2 + b^2)*f)
```

Maple [F]
time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{(c(d\tan(fx+e))^p)^n}{(a+b\tan(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*tan(f*x+e))^p)^n/(a+b*tan(f*x+e))^2,x)
```

```
[Out] int((c*(d*tan(f*x+e))^p)^n/(a+b*tan(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))^p)^n/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(((d*tan(f*x + e))^p*c)^n/(b*tan(f*x + e) + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))^p)^n/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(((d*tan(f*x + e))^p*c)^n/(b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e)
+ a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(d\tan{(e+fx)})^p)^n}{(a+b\tan{(e+fx)})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))**p)**n/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral((c*(d*tan(e + f*x))**p)**n/(a + b*tan(e + f*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*tan(f*x+e))^p)^n/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(((d*tan(f*x + e))^p*c)^n/(b*tan(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,(d\tan(e+f\,x))^p)^n}{(a+b\tan{(e+f\,x)})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*tan(e + f*x))^p)^n/(a + b*tan(e + f*x))^2,x)
```

```
[Out] int((c*(d*tan(e + f*x))^p)^n/(a + b*tan(e + f*x))^2, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#           issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```